Computer algebra independent integration tests

9-Blake-problems/Blake-Problems

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [2443]. This is test number [39].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
IntegrateAlgebraic	99.30 (2426)	0.70 (17)
Fricas	79.29 (1937)	20.71 (506)
Mathematica	70.57 (1724)	29.43 (719)
Rubi	64.96 (1587)	35.04 (856)
Maple	61.81 (1510)	38.19 (933)
Mupad	32.79 (801)	67.21 (1642)
Giac	29.39 (718)	70.61 (1725)
Maxima	23.33 (570)	76.67 (1873)
Sympy	22.39 (547)	% 77.61 (1896)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
IntegrateAlgebraic	98.36	0.12	0.82	0.70
Fricas	53.09	26.03	0.16	20.71
Rubi	48.55	2.87	13.55	35.04
Mathematica	30.99	1.06	38.40	29.43
Giac	24.27	4.95	0.16	70.61
Maxima	19.98	3.23	0.12	76.67
Maple	16.74	3.60	41.47	38.19
Sympy	5.94	3.40	13.06	77.61
Mupad	N/A	32.79	0.00	67.21

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.

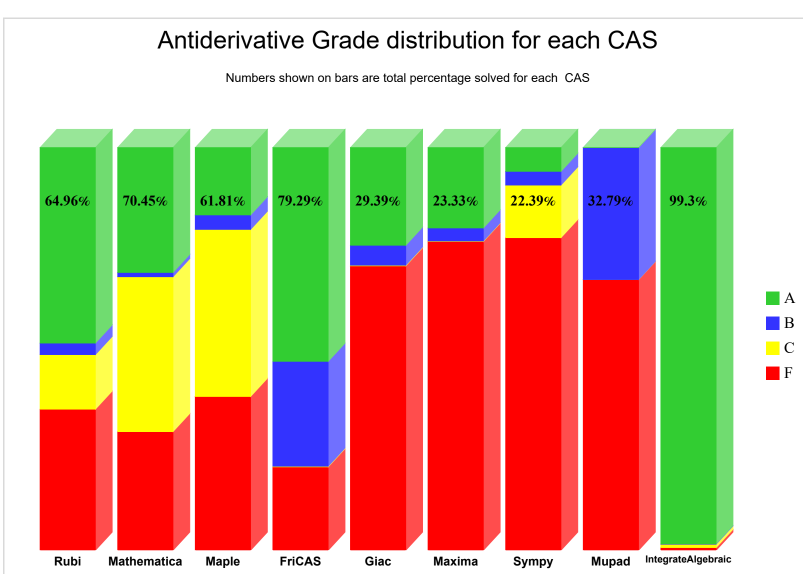

The figure below compares the CAS systems for each grade level.

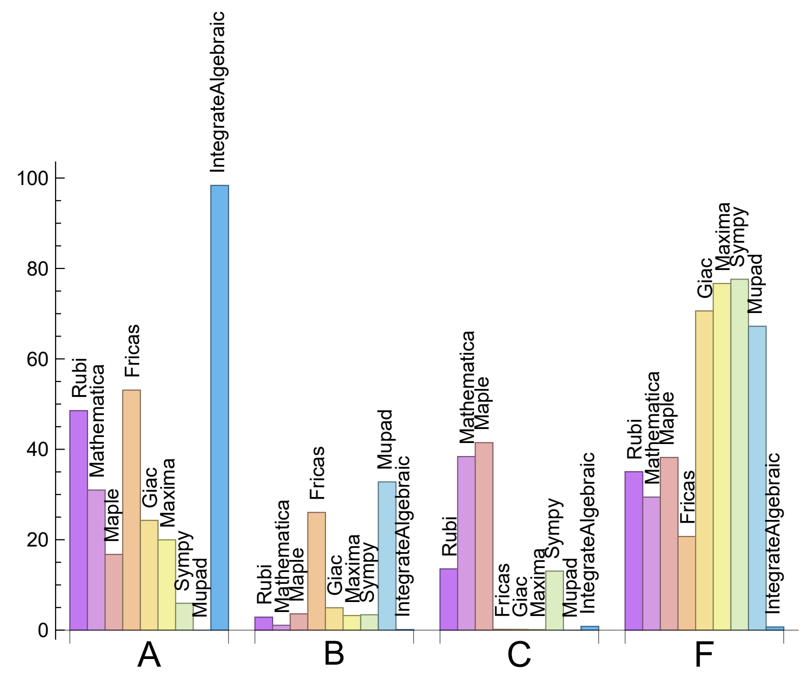

The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	856	99.18 %	0.82 %	0.00 %
Mathematica	719	99.30 %	0.70 %	0.00 %
Maple	933	91.75 %	2.04 %	6.22 %
Fricas	506	0.40 %	94.27 %	5.34 %
IntegrateAlgebraic	17	52.94 %	47.06 %	0.00 %
Giac	1725	91.77 %	4.93 %	3.30 %
Maxima	1873	98.83 %	0.00 %	1.17 %
Sympy	1896	72.31 %	27.69 %	0.00 %
Mupad	1642	97.50 %	2.50 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	1.10	206.87	2.34	101.00	1.09
Mathematica	1.18	1227.69	15.67	77.00	1.00
Maple	3.69	1149.26	11.65	174.00	2.26
Maxima	0.50	79.24	1.20	58.00	1.11
Fricas	6.40	313.58	2.31	117.00	1.29
Sympy	5.89	91.97	2.13	44.00	0.95
Giac	1.05	107.42	1.16	67.00	0.98
Mupad	1.34	277.16	2.73	46.00	0.91
IntegrateAlgebraic	1.90	128.43	0.99	96.00	1.00

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.

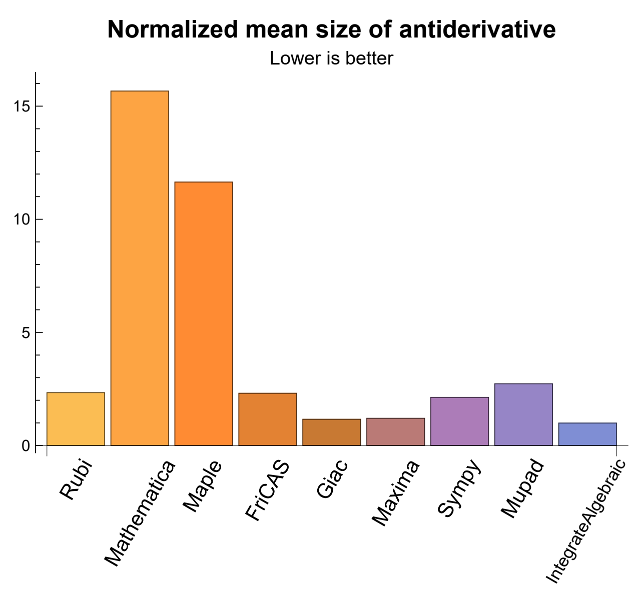

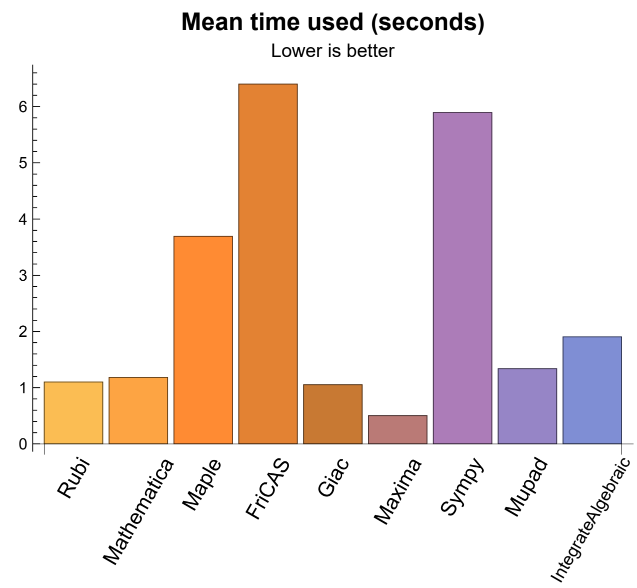
Mean time used (seconds)
Lower is better
0
1
2
3
4
5
6
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {81, 130, 193, 240, 278, 299, 300, 313, 323, 361, 417, 430, 451, 513, 515, 516, 517, 536, 563, 595, 650, 673, 676, 680, 681, 691, 713, 725, 728, 729, 730, 735, 748, 751, 754, 765, 772, 785, 796, 811, 812, 830, 834, 841, 847, 875, 883, 887, 901, 907, 914, 915, 917, 926, 928, 953, 957, 985, 986, 1010, 1015, 1020, 1064, 1066, 1076, 1080, 1083, 1118, 1139, 1142, 1146, 1168, 1169, 1170, 1191, 1192, 1198, 1219, 1239, 1246, 1250, 1256, 1257, 1261, 1269, 1271, 1272, 1275, 1276, 1280, 1286, 1305, 1307, 1316, 1334, 1341, 1351, 1368, 1374, 1375, 1377, 1420, 1428, 1441, 1442, 1457, 1458, 1478, 1479, 1480, 1481, 1482, 1497, 1501, 1502, 1503, 1504, 1505, 1523, 1542, 1554, 1572, 1581, 1583, 1585, 1587, 1588, 1589, 1595, 1603, 1607, 1612, 1627, 1628, 1629, 1642, 1643, 1668, 1680, 1681, 1684, 1695, 1707, 1708, 1709, 1737, 1738, 1748, 1770, 1777, 1779, 1781, 1787, 1804, 1805, 1806, 1807, 1808, 1809, 1810, 1811, 1813, 1814, 1815, 1816, 1830, 1841, 1847, 1856, 1861, 1880, 1889, 1893, 1899, 1907, 1908, 1913, 1915, 1917, 1918, 1919, 1920, 1930, 1931, 1932, 1933, 1934, 1935, 1937, 1941, 1945, 1958, 1959, 1960, 1981, 1998, 2001, 2016, 2021, 2036, 2038, 2041, 2042, 2050, 2056, 2057, 2078, 2088, 2091, 2092, 2117, 2137, 2140, 2152, 2182, 2183, 2186, 2191, 2192, 2197, 2209, 2215, 2219, 2226, 2229, 2231, 2236, 2249, 2259, 2280, 2286, 2289, 2300, 2301, 2302, 2313, 2315, 2322, 2357, 2359, 2365, 2366, 2367, 2375, 2376, 2377, 2389, 2390, 2391, 2392, 2393, 2413, 2431}

Mathematica {58, 67, 68, 113, 145, 162, 193, 199, 228, 265, 266, 276, 277, 299, 300, 313, 318, 323, 349, 352, 361, 364, 370, 373, 380, 381, 388, 392, 396, 397, 402, 404, 408, 409, 416, 420, 421, 424, 425, 436, 438, 452, 461, 480, 503, 508, 513, 514, 515, 516, 518, 519, 529, 538, 545, 547, 553, 568, 569, 570, 571, 572, 588, 589, 595, 596, 604, 609, 618, 625, 657, 659, 660, 665, 670, 672, 673, 677, 680, 699, 703, 708, 709, 714, 720, 723, 728, 729, 730, 731, 735, 736, 739, 740, 744, 747, 752, 753, 755, 767, 785, 786, 788, 789, 790, 793, 796, 805, 809, 811, 812, 818, 822, 824, 825, 826, 830, 832, 833, 844, 845, 847, 855, 864, 867, 893, 894, 907, 918, 919, 920, 921, 931, 934, 942, 952, 953, 955, 972, 1009, 1028, 1038, 1039, 1047, 1051, 1053, 1065, 1076, 1081, 1083, 1084, 1085, 1102, 1119, 1120, 1122, 1131, 1142, 1152, 1153, 1155, 1156, 1160, 1162, 1198, 1208, 1210, 1219, 1220, 1224, 1227, 1245, 1246, 1249, 1253, 1262, 1266, 1275, 1276, 1281, 1307, 1310, 1320, 1330, 1331, 1334, 1340, 1341, 1348, 1365, 1368, 1374, 1375, 1377, 1379, 1385, 1396, 1402, 1415, 1416, 1430, 1432, 1434, 1435, 1446, 1453, 1461, 1481, 1482, 1491, 1492, 1502, 1510, 1523, 1524, 1526, 1542, 1543, 1551, 1572, 1576, 1583, 1587, 1598, 1599, 1600, 1601, 1612, 1614, 1615, 1624, 1626, 1627, 1648, 1649, 1650, 1651, 1655, 1662, 1668, 1670, 1676, 1680, 1681, 1684, 1686, 1695, 1705, 1707, 1708, 1715, 1717, 1718, 1723, 1724, 1737, 1740, 1745, 1748, 1766, 1777, 1780, 1781, 1783, 1786, 1787, 1798, 1804, 1805, 1806, 1807, 1808, 1809, 1810, 1811, 1822, 1828, 1830, 1831, 1835, 1844, 1849, 1850, 1852, 1855, 1870, 1879, 1880, 1881, 1883, 1894, 1896, 1901, 1907, 1908, 1909, 1919, 1920, 1924, 1930, 1931, 1934, 1935, 1938, 1939, 1943, 1946, 1951, 1957, 1959, 1968, 1976, 1983, 1990, 1992, 1993, 1994, 2001, 2016, 2017, 2021, 2026, 2033, 2036, 2041, 2042, 2105, 2117, 2129, 2134, 2135, 2192, 2202, 2206, 2207, 2214, 2215, 2229, 2231, 2236, 2244, 2268, 2272, 2288, 2289, 2297, 2306, 2316, 2317, 2320, 2326, 2335, 2339, 2340, 2342, 2359, 2365, 2366, 2367, 2371, 2373, 2374, 2388, 2389, 2390, 2391, 2392, 2393, 2401, 2403, 2413, 2422, 2425, 2426}

IntegrateAlgebraic {2223, 2334, 2338, 2364, 2375, 2376, 2377, 2431}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.

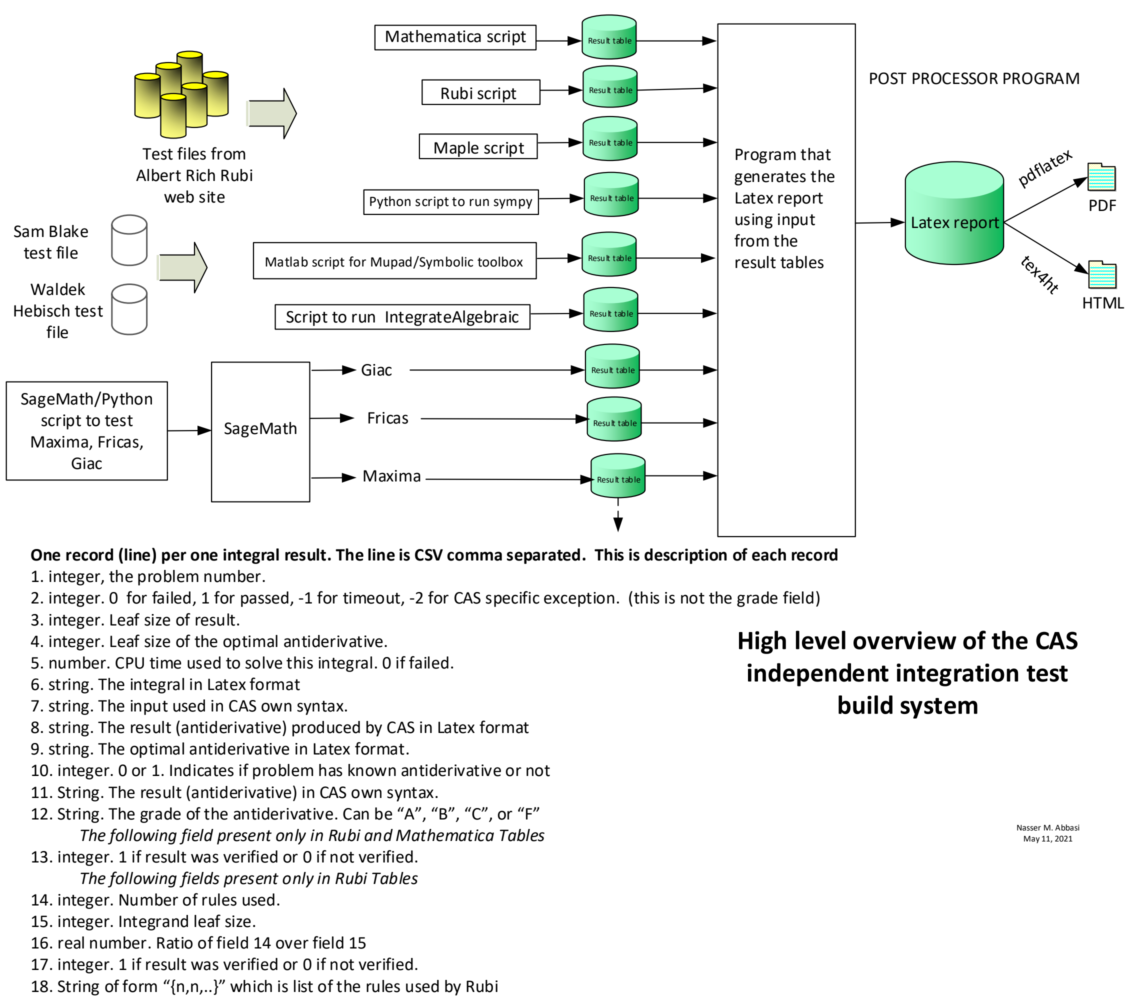

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 114, 115, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 146, 148, 149, 150, 152, 153, 155, 156, 157, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 184, 185, 186, 188, 189, 191, 192, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 279, 281, 282, 285, 286, 287, 291, 292, 294, 298, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 342, 344, 345, 346, 347, 348, 349, 350, 352, 353, 356, 357, 358, 360, 362, 363, 364, 365, 366, 367, 372, 373, 374, 375, 376, 377, 378, 380, 381, 382, 383, 384, 385, 386, 387, 389, 391, 392, 394, 395, 396, 397, 398, 399, 403, 405, 408, 409, 411, 412, 414, 415, 416, 418, 419, 420, 426, 428, 429, 432, 433, 434, 436, 438, 440, 441, 442, 443, 444, 445, 446, 448, 449, 450, 453, 457, 458, 459, 461, 463, 464, 465, 466, 467, 468, 470, 475, 476, 477, 478, 479, 481, 482, 483, 484, 490, 491, 492, 493, 494, 495, 496, 498, 499, 500, 505, 506, 507, 508, 510, 512, 523, 524, 526, 527, 528, 532, 533, 534, 537, 541, 542, 543, 549, 550, 551, 552, 554, 555, 559, 573, 574, 576, 577, 578, 579, 580, 583, 586, 587, 588, 592, 593, 601, 602, 604, 605, 607, 611, 612, 613, 619, 620, 621, 624, 630, 634, 636, 638, 644, 646, 647, 648, 656, 658, 659, 660, 661, 666, 668, 671, 677, 682, 684, 685, 686, 687, 689, 690, 692, 693, 694, 695, 696, 697, 700, 701, 702, 703, 704, 705, 706, 707, 715, 716, 718, 721, 722, 724, 731, 732, 733, 737, 738, 741, 745, 747, 756, 757, 758, 759, 768, 769, 771, 774, 775, 776, 780, 782, 783, 784, 787, 792, 794, 799, 801, 802, 803, 805, 806, 807, 808, 809, 810, 816, 817, 819, 822, 823, 826, 831, 832, 833, 835, 836, 839, 840, 842, 845, 846, 848, 852, 853, 854, 856, 857, 860, 861, 862, 866, 867, 868, 873, 879, 881, 891, 892, 895, 896, 898, 899, 900, 902, 904, 905, 908, 909, 910, 912, 916, 919, 922, 924, 927, 930, 932, 933, 935, 936, 938, 939, 941, 942, 944, 945, 946, 947, 948, 949, 955, 960, 966, 967, 969, 972, 973, 979, 980, 981, 982, 984, 988, 989, 990, 992, 993, 997, 999, 1000, 1001, 1002, 1003, 1004, 1007, 1008, 1011, 1012, 1017, 1021, 1022, 1023, 1026, 1031, 1032, 1033, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1048, 1051, 1052, 1055, 1056, 1059, 1060, 1061, 1063, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1077, 1081, 1082, 1086, 1087, 1089, 1090, 1091, 1092, 1095, 1096, 1102, 1103, 1104, 1105, 1108, 1109, 1110, 1115, 1120, 1121, 1122, 1124, 1125, 1126, 1127, 1128, 1130, 1132, 1133, 1134, 1135, 1136, 1137, 1139, 1140, 1141, 1143, 1145, 1148, 1150, 1154, 1155, 1156, 1157, 1158, 1160, 1163, 1164, 1167, 1171, 1183, 1184, 1185, 1188, 1190, 1195, 1196, 1197, 1199, 1200, 1201, 1202, 1203, 1204, 1206, 1207, 1208, 1211, 1212, 1213, 1214, 1217, 1218, 1221, 1222, 1225, 1226, 1230, 1232, 1233, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1247, 1248, 1249, 1251, 1253, 1254, 1263, 1264, 1265, 1266, 1268, 1274, 1277, 1278, 1279, 1281, 1282, 1283, 1284, 1285, 1289, 1290, 1291, 1292, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1302, 1310, 1311, 1315, 1317, 1318, 1319, 1320, 1323, 1325, 1326, 1327, 1328, 1330, 1335, 1336, 1337, 1339, 1340, 1342, 1346, 1350, 1354, 1355, 1356, 1357, 1360, 1361, 1362, 1364, 1365, 1366, 1367, 1376, 1379, 1380, 1381, 1382, 1383, 1387, 1388, 1389, 1390, 1393, 1397, 1398, 1399, 1402, 1404, 1407, 1408, 1409, 1410, 1411, 1413, 1415, 1416, 1418, 1421, 1422, 1423, 1424, 1425, 1427, 1429, 1432, 1438, 1439, 1440, 1444, 1445, 1446, 1449, 1450, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1469, 1470, 1473, 1474, 1475, 1476, 1482, 1483, 1486, 1490, 1494, 1496, 1498, 1499, 1508, 1509, 1510, 1513, 1515, 1516, 1517, 1522, 1525, 1528, 1530, 1532, 1536, 1537, 1538, 1543, 1544, 1546, 1549, 1551, 1555, 1556, 1557, 1559, 1560, 1562, 1563, 1564, 1565, 1569, 1570, 1571, 1573, 1575, 1576, 1580, 1582, 1586, 1588, 1589, 1596, 1600, 1602, 1610, 1611, 1614, 1615, 1616, 1617, 1619, 1622, 1623, 1624, 1625, 1626, 1630, 1631, 1632, 1634, 1635, 1636, 1637, 1640, 1641, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1657, 1658, 1659, 1662, 1663, 1664, 1665, 1669, 1670, 1671, 1676, 1681, 1689, 1690, 1692, 1694, 1697, 1698, 1700, 1706, 1711, 1712, 1713, 1714, 1715, 1717, 1718, 1722, 1723, 1724, 1725, 1726, 1728, 1729, 1734, 1742, 1745, 1749, 1750, 1752, 1754, 1755, 1756, 1758, 1759, 1764, 1766, 1771, 1775, 1780, 1783, 1789, 1793, 1800, 1812, 1820, 1823, 1827, 1828, 1831, 1832, 1834, 1835, 1839, 1840, 1843, 1845, 1849, 1851, 1852, 1855, 1859, 1866, 1867, 1870, 1872, 1875, 1876, 1879, 1882, 1883, 1887, 1891, 1892, 1894, 1895, 1896, 1898, 1905, 1909, 1912, 1914, 1922, 1926, 1936, 1943, 1946, 1953, 1954, 1958, 1959, 1960, 1961, 1962, 1963, 1964, 1965, 1968, 1970, 1971, 1972, 1976, 1980, 1983, 1984, 1986, 1989, 1990, 1992, 1993, 2001, 2005, 2008, 2013, 2017, 2026, 2031, 2033, 2035, 2048, 2054, 2056, 2057, 2059, 2060, 2064, 2067, 2073, 2080, 2081, 2083, 2084, 2085, 2086, 2095, 2105, 2108, 2115, 2124, 2127, 2129, 2132, 2136, 2146, 2147, 2148, 2149, 2150, 2155, 2167, 2168, 2169, 2170, 2172, 2173, 2176, 2177, 2186, 2187, 2192, 2199, 2205, 2211, 2213, 2217, 2219, 2225, 2234, 2235, 2237, 2240, 2244, 2252, 2256, 2269, 2270, 2272, 2273, 2274, 2275, 2278, 2279, 2288, 2289, 2290, 2295, 2297, 2301, 2303, 2304, 2305, 2307, 2308, 2309, 2320, 2323, 2325, 2326, 2335, 2337, 2339, 2340, 2345, 2346, 2348, 2350, 2351, 2353, 2358, 2359, 2360, 2361, 2363, 2368, 2370, 2372, 2374, 2380, 2381, 2383, 2384, 2387, 2388, 2389, 2394, 2398, 2400, 2401, 2403, 2404, 2405, 2406, 2408, 2410, 2413, 2415, 2417, 2420, 2422, 2423, 2424, 2425, 2429, 2430, 2431, 2432, 2438, 2439, 2440, 2441, 2442, 2443 }

B grade: { 77, 90, 97, 112, 129, 133, 144, 147, 151, 154, 177, 181, 182, 187, 343, 359, 400, 401, 406, 410, 435, 437, 460, 462, 485, 486, 487, 488, 489, 544, 562, 591, 603, 608, 631, 662, 664, 742, 764, 791, 837, 876, 877, 885, 940, 1258, 1412, 1443, 1471, 1511, 1512, 1524, 1587, 1685, 1686, 1801, 1951, 1956, 1969, 2038, 2044, 2045, 2091, 2094, 2104, 2164, 2197, 2310, 2334, 2364 }

C grade: { 81, 130, 159, 193, 240, 278, 280, 299, 300, 313, 323, 361, 407, 417, 430, 451, 455, 456, 471, 472, 473, 474, 502, 504, 513, 515, 516, 517, 520, 522, 530, 536, 546, 553, 556, 557, 563, 566, 567, 595, 610, 650, 663, 670, 673, 674, 675, 676, 678, 679, 680, 681, 683, 688, 691, 713, 719, 725, 728, 729, 730, 735, 743, 748, 749, 750, 751, 754, 761, 765, 766, 767, 772, 785, 790, 796, 811, 812, 815, 830, 834, 841, 847, 875, 882, 883, 887, 901, 907, 914, 915, 917, 923, 926, 928, 952, 953, 954, 957, 985, 986, 1010, 1013, 1014, 1015, 1019, 1020, 1028, 1054, 1064, 1066, 1076, 1080, 1083, 1114, 1118, 1142, 1146, 1168, 1169, 1170, 1191, 1192, 1194, 1198, 1219, 1220, 1228, 1229, 1245, 1246, 1250, 1256, 1257, 1261, 1269, 1271, 1272, 1275, 1276, 1280, 1286, 1288, 1305, 1306, 1307, 1316, 1331, 1334, 1341, 1343, 1348, 1351, 1352, 1368, 1374, 1375, 1377, 1378, 1396, 1417, 1420, 1428, 1434, 1441, 1442, 1457, 1458, 1478, 1479, 1480, 1481, 1493, 1497, 1501, 1502, 1503, 1504, 1505, 1518, 1519, 1523, 1531, 1541, 1542, 1552, 1554, 1572, 1578, 1581, 1583, 1585, 1595, 1598, 1601, 1603, 1604, 1607, 1612, 1627, 1628, 1629, 1639, 1642, 1643, 1668, 1672, 1680, 1684, 1695, 1707, 1708, 1709, 1731, 1732, 1737, 1738, 1748, 1760, 1770, 1777, 1779, 1781, 1787, 1790, 1798, 1804, 1805, 1806, 1807, 1808, 1809, 1810, 1811, 1813, 1814, 1815, 1816, 1830, 1836, 1837, 1841, 1844, 1847, 1856, 1861, 1863, 1880, 1889, 1890, 1893, 1899, 1907, 1908, 1913, 1915, 1917, 1918, 1919, 1920, 1928, 1929, 1930, 1931, 1932, 1933, 1934, 1935, 1937, 1941, 1945, 1955, 1981, 1994, 1998, 2016, 2021, 2036, 2041, 2042, 2050, 2078, 2088, 2092, 2100, 2101, 2117, 2121, 2137, 2140, 2152, 2156, 2158, 2182, 2183, 2191, 2206, 2209, 2215, 2223, 2226, 2229, 2231, 2236, 2249, 2259, 2260, 2267, 2268, 2271, 2280, 2284, 2286, 2300, 2302, 2313, 2315, 2322, 2357, 2365, 2366, 2367, 2375, 2376, 2377, 2390, 2391, 2392, 2393, 2399, 2428 }

F grade: { 58, 63, 64, 67, 99, 113, 116, 119, 145, 158, 161, 183, 190, 194, 195, 196, 197, 206, 219, 243, 244, 245, 246, 247, 248, 249, 276, 277, 283, 284, 288, 289, 290, 293, 295, 296, 297, 318, 339, 340, 341, 351, 354, 355, 368, 369, 370, 371, 379, 388, 390, 393, 402, 404, 413, 421, 422, 423, 424, 425, 427, 431, 439, 447, 452, 454, 469, 480, 497, 501, 503, 509, 511, 514, 518, 519, 521, 525, 529, 531, 535, 538, 539, 540, 545, 547, 548, 558, 560, 561, 564, 565, 568, 569, 570, 571, 572, 575, 581, 582, 584, 585, 589, 590, 594, 596, 597, 598, 599, 600, 606, 609, 614, 615, 616, 617, 618, 622, 623, 625, 626, 627, 628, 629, 632, 633, 635, 637, 639, 640, 641, 642, 643, 645, 649, 651, 652, 653, 654, 655, 657, 665, 667, 669, 672, 698, 699, 708, 709, 710, 711, 712, 714, 717, 720, 723, 726, 727, 734, 736, 739, 740, 744, 746, 752, 753, 755, 760, 762, 763, 770, 773, 777, 778, 779, 781, 786, 788, 789, 793, 795, 797, 798, 800, 804, 813, 814, 818, 820, 821, 824, 825, 827, 828, 829, 838, 843, 844, 849, 850, 851, 855, 858, 859, 863, 864, 865, 869, 870, 871, 872, 874, 878, 880, 884, 886, 888, 889, 890, 893, 894, 897, 903, 906, 911, 913, 918, 920, 921, 925, 929, 931, 934, 937, 943, 950, 951, 956, 958, 959, 961, 962, 963, 964, 965, 968, 970, 971, 974, 975, 976, 977, 978, 983, 987, 991, 994, 995, 996, 998, 1005, 1006, 1009, 1016, 1018, 1024, 1025, 1027, 1029, 1030, 1034, 1035, 1036, 1047, 1049, 1050, 1053, 1057, 1058, 1062, 1065, 1075, 1078, 1079, 1084, 1085, 1088, 1093, 1094, 1097, 1098, 1099, 1100, 1101, 1106, 1107, 1111, 1112, 1113, 1116, 1117, 1119, 1123, 1129, 1131, 1138, 1144, 1147, 1149, 1151, 1152, 1153, 1159, 1161, 1162, 1165, 1166, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1186, 1187, 1189, 1193, 1205, 1209, 1210, 1215, 1216, 1223, 1224, 1227, 1231, 1234, 1235, 1236, 1237, 1252, 1255, 1259, 1260, 1262, 1267, 1270, 1273, 1287, 1293, 1301, 1303, 1304, 1308, 1309, 1312, 1313, 1314, 1321, 1322, 1324, 1329, 1332, 1333, 1338, 1344, 1345, 1347, 1349, 1353, 1358, 1359, 1363, 1369, 1370, 1371, 1372, 1373, 1384, 1385, 1386, 1391, 1392, 1394, 1395, 1400, 1401, 1403, 1405, 1406, 1414, 1419, 1426, 1430, 1431, 1433, 1435, 1436, 1437, 1447, 1448, 1451, 1452, 1453, 1454, 1455, 1456, 1459, 1468, 1472, 1477, 1484, 1485, 1487, 1488, 1489, 1491, 1492, 1495, 1500, 1506, 1507, 1514, 1520, 1521, 1526, 1527, 1529, 1533, 1534, 1535, 1539, 1540, 1545, 1547, 1548, 1550, 1553, 1558, 1561, 1566, 1567, 1568, 1574, 1577, 1579, 1584, 1590, 1591, 1592, 1593, 1594, 1597, 1599, 1605, 1606, 1608, 1609, 1613, 1618, 1620, 1621, 1633, 1638, 1644, 1653, 1654, 1655, 1656, 1660, 1661, 1666, 1667, 1673, 1674, 1675, 1677, 1678, 1679, 1682, 1683, 1687, 1688, 1691, 1693, 1696, 1699, 1701, 1702, 1703, 1704, 1705, 1710, 1716, 1719, 1720, 1721, 1727, 1730, 1733, 1735, 1736, 1739, 1740, 1741, 1743, 1744, 1746, 1747, 1751, 1753, 1757, 1761, 1762, 1763, 1765, 1767, 1768, 1769, 1772, 1773, 1774, 1776, 1778, 1782, 1784, 1785, 1786, 1788, 1791, 1792, 1794, 1795, 1796, 1797, 1799, 1802, 1803, 1817, 1818, 1819, 1821, 1822, 1824, 1825, 1826, 1829, 1833, 1838, 1842, 1846, 1848, 1850, 1853, 1854, 1857, 1858, 1860, 1862, 1864, 1865, 1868, 1869, 1871, 1873, 1874, 1877, 1878, 1881, 1884, 1885, 1886, 1888, 1897, 1900, 1901, 1902, 1903, 1904, 1906, 1910, 1911, 1916, 1921, 1923, 1924, 1925, 1927, 1938, 1939, 1940, 1942, 1944, 1947, 1948, 1949, 1950, 1952, 1957, 1966, 1967, 1973, 1974, 1975, 1977, 1978, 1979, 1982, 1985, 1987, 1988, 1991, 1995, 1996, 1997, 1999, 2000, 2002, 2003, 2004, 2006, 2007, 2009, 2010, 2011, 2012, 2014, 2015, 2018, 2019, 2020, 2022, 2023, 2024, 2025, 2027, 2028, 2029, 2030, 2032, 2034, 2037, 2039, 2040, 2043, 2046, 2047, 2049, 2051, 2052, 2053, 2055, 2058, 2061, 2062, 2063, 2065, 2066, 2068, 2069, 2070, 2071, 2072, 2074, 2075, 2076, 2077, 2079, 2082, 2087, 2089, 2090, 2093, 2096, 2097, 2098, 2099, 2102, 2103, 2106, 2107, 2109, 2110, 2111, 2112, 2113, 2114, 2116, 2118, 2119, 2120, 2122, 2123, 2125, 2126, 2128, 2130, 2131, 2133, 2134, 2135, 2138, 2139, 2141, 2142, 2143, 2144, 2145, 2151, 2153, 2154, 2157, 2159, 2160, 2161, 2162, 2163, 2165, 2166, 2171, 2174, 2175, 2178, 2179, 2180, 2181, 2184, 2185, 2188, 2189, 2190, 2193, 2194, 2195, 2196, 2198, 2200, 2201, 2202, 2203, 2204, 2207, 2208, 2210, 2212, 2214, 2216, 2218, 2220, 2221, 2222, 2224, 2227, 2228, 2230, 2232, 2233, 2238, 2239, 2241, 2242, 2243, 2245, 2246, 2247, 2248, 2250, 2251, 2253, 2254, 2255, 2257, 2258, 2261, 2262, 2263, 2264, 2265, 2266, 2276, 2277, 2281, 2282, 2283, 2285, 2287, 2291, 2292, 2293, 2294, 2296, 2298, 2299, 2306, 2311, 2312, 2314, 2316, 2317, 2318, 2319, 2321, 2324, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2336, 2338, 2341, 2342, 2343, 2344, 2347, 2349, 2352, 2354, 2355, 2356, 2362, 2369, 2371, 2373, 2378, 2379, 2382, 2385, 2386, 2395, 2396, 2397, 2402, 2407, 2409, 2411, 2412, 2414, 2416, 2418, 2419, 2421, 2426, 2427, 2433, 2434, 2435, 2436, 2437 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 65, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 115, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 139, 140, 141, 142, 143, 144, 146, 147, 149, 150, 152, 153, 154, 155, 156, 157, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 189, 191, 194, 197, 198, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 220, 221, 222, 223, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 268, 269, 270, 271, 272, 273, 274, 275, 279, 287, 294, 298, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 315, 316, 319, 320, 321, 322, 327, 328, 331, 332, 333, 334, 337, 338, 342, 344, 345, 346, 347, 348, 350, 353, 356, 357, 358, 359, 362, 363, 365, 372, 375, 376, 377, 378, 382, 383, 384, 385, 386, 389, 391, 393, 394, 398, 399, 400, 401, 403, 405, 406, 410, 411, 412, 415, 426, 427, 428, 429, 431, 432, 433, 434, 435, 437, 440, 441, 442, 443, 444, 445, 446, 448, 449, 458, 459, 460, 463, 464, 465, 466, 467, 468, 470, 475, 479, 481, 491, 493, 497, 498, 500, 505, 506, 507, 510, 512, 520, 521, 522, 523, 524, 527, 528, 532, 533, 534, 537, 539, 540, 541, 542, 544, 550, 552, 554, 562, 565, 573, 576, 580, 586, 587, 591, 593, 594, 601, 602, 608, 611, 612, 613, 616, 621, 624, 630, 631, 632, 633, 634, 635, 636, 638, 644, 645, 648, 649, 650, 656, 658, 661, 664, 666, 668, 669, 671, 686, 689, 690, 692, 696, 700, 701, 702, 703, 704, 705, 706, 707, 713, 716, 718, 724, 727, 732, 737, 738, 747, 756, 757, 758, 759, 764, 768, 769, 771, 774, 780, 783, 784, 787, 794, 798, 799, 800, 805, 807, 808, 810, 816, 817, 819, 820, 830, 831, 832, 836, 840, 843, 846, 852, 853, 854, 857, 860, 861, 862, 865, 866, 868, 873, 874, 879, 881, 890, 891, 892, 895, 898, 899, 900, 924, 927, 932, 936, 937, 938, 940, 947, 948, 949, 954, 960, 966, 969, 976, 978, 979, 984, 988, 992, 997, 999, 1001, 1002, 1003, 1004, 1007, 1008, 1011, 1012, 1031, 1032, 1037, 1040, 1042, 1043, 1046, 1048, 1052, 1055, 1056, 1059, 1060, 1061, 1062, 1068, 1070, 1071, 1072, 1073, 1077, 1090, 1095, 1103, 1108, 1139, 1147, 1154, 1157, 1160, 1163, 1164, 1188, 1197, 1200, 1206, 1211, 1217, 1238, 1239, 1241, 1249, 1250, 1266, 1292, 1295, 1298, 1305, 1308, 1318, 1319, 1323, 1325, 1326, 1354, 1371, 1373, 1382, 1383, 1387, 1388, 1389, 1390, 1391, 1393, 1402, 1408, 1409, 1421, 1424, 1427, 1445, 1450, 1463, 1465, 1469, 1470, 1471, 1473, 1474, 1481, 1482, 1483, 1494, 1495, 1498, 1506, 1513, 1514, 1515, 1516, 1528, 1529, 1530, 1549, 1555, 1556, 1557, 1559, 1560, 1562, 1563, 1564, 1565, 1570, 1573, 1575, 1580, 1582, 1588, 1589, 1602, 1609, 1610, 1611, 1614, 1615, 1619, 1622, 1623, 1630, 1631, 1632, 1634, 1636, 1641, 1646, 1648, 1657, 1663, 1664, 1665, 1669, 1676, 1678, 1681, 1682, 1684, 1685, 1689, 1690, 1692, 1694, 1695, 1697, 1698, 1699, 1714, 1715, 1723, 1724, 1734, 1739, 1740, 1742, 1749, 1761, 1764, 1771, 1793, 1820, 1823, 1831, 1832, 1859, 1866, 1870, 1871, 1876, 1882, 1887, 1891, 1898, 1905, 1914, 1918, 1922, 1953, 1958, 1959, 1960, 1961, 1963, 1964, 1965, 1966, 1967, 1970, 1977, 1978, 1980, 1989, 1992, 1993, 2001, 2008, 2031, 2035, 2037, 2038, 2048, 2056, 2057, 2066, 2094, 2115, 2132, 2136, 2145, 2155, 2166, 2168, 2176, 2177, 2184, 2187, 2189, 2199, 2209, 2212, 2217, 2220, 2234, 2237, 2261, 2262, 2270, 2273, 2279, 2289, 2290, 2300, 2301, 2307, 2309, 2323, 2325, 2333, 2345, 2359, 2363, 2370, 2373, 2381, 2385, 2389, 2397, 2398, 2413, 2414, 2419, 2421, 2423, 2443 }

B grade: { 379, 499, 603, 782, 806, 835, 961, 962, 1026, 1260, 1410, 1497, 1569, 1587, 1824, 1825, 1854, 1971, 2167, 2240, 2339, 2340, 2401, 2422, 2425, 2426 }

C grade: { 58, 67, 68, 81, 84, 91, 112, 113, 114, 116, 117, 118, 135, 138, 145, 148, 151, 161, 162, 182, 192, 193, 195, 196, 199, 200, 218, 219, 228, 239, 240, 241, 242, 265, 266, 267, 276, 277, 278, 280, 281, 282, 285, 286, 291, 292, 299, 300, 312, 313, 314, 317, 318, 323, 324, 325, 326, 329, 330, 335, 336, 349, 352, 360, 361, 364, 366, 367, 370, 373, 374, 380, 381, 387, 388, 390, 392, 395, 396, 397, 402, 404, 408, 409, 414, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 430, 436, 438, 450, 451, 452, 453, 455, 456, 457, 461, 462, 471, 472, 473, 474, 476, 477, 478, 480, 482, 483, 484, 485, 486, 487, 488, 489, 490, 492, 494, 495, 496, 502, 503, 504, 508, 513, 514, 515, 516, 517, 518, 519, 529, 530, 538, 543, 545, 546, 547, 549, 551, 553, 555, 556, 557, 559, 566, 567, 568, 569, 570, 571, 572, 574, 577, 578, 579, 583, 588, 589, 592, 595, 596, 604, 605, 606, 607, 609, 610, 618, 619, 620, 625, 629, 637, 643, 646, 647, 657, 659, 660, 662, 663, 665, 670, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 684, 685, 687, 688, 691, 693, 694, 695, 697, 698, 699, 708, 709, 710, 714, 715, 719, 720, 721, 722, 723, 728, 729, 730, 731, 733, 735, 736, 739, 740, 741, 742, 743, 744, 745, 749, 750, 751, 752, 753, 754, 755, 761, 766, 767, 775, 776, 777, 785, 786, 788, 789, 790, 791, 792, 793, 796, 801, 802, 803, 804, 809, 811, 812, 815, 818, 821, 822, 823, 824, 825, 826, 833, 834, 837, 839, 842, 844, 845, 847, 848, 855, 856, 864, 867, 875, 876, 877, 882, 883, 885, 887, 893, 894, 896, 901, 902, 904, 907, 908, 909, 910, 912, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 925, 926, 928, 930, 931, 933, 934, 935, 939, 941, 942, 944, 945, 946, 950, 952, 953, 955, 956, 959, 964, 967, 972, 980, 981, 982, 985, 986, 989, 990, 993, 1000, 1006, 1009, 1013, 1014, 1015, 1017, 1019, 1021, 1022, 1023, 1028, 1030, 1033, 1038, 1039, 1041, 1044, 1045, 1047, 1051, 1053, 1054, 1063, 1064, 1065, 1067, 1069, 1074, 1076, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1089, 1091, 1092, 1096, 1102, 1104, 1105, 1109, 1110, 1114, 1115, 1119, 1120, 1121, 1122, 1124, 1125, 1126, 1128, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1140, 1141, 1142, 1143, 1145, 1148, 1150, 1152, 1153, 1155, 1156, 1158, 1162, 1165, 1166, 1167, 1171, 1183, 1184, 1185, 1190, 1194, 1195, 1196, 1198, 1199, 1201, 1202, 1203, 1204, 1207, 1208, 1210, 1212, 1213, 1214, 1218, 1219, 1220, 1221, 1222, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1232, 1233, 1240, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1251, 1253, 1254, 1255, 1258, 1261, 1262, 1263, 1264, 1265, 1268, 1274, 1275, 1276, 1277, 1278, 1279, 1281, 1282, 1283, 1284, 1285, 1286, 1288, 1289, 1290, 1291, 1294, 1296, 1297, 1299, 1300, 1302, 1306, 1307, 1310, 1311, 1315, 1317, 1320, 1327, 1328, 1330, 1331, 1334, 1335, 1336, 1337, 1339, 1340, 1341, 1342, 1343, 1346, 1348, 1350, 1351, 1352, 1355, 1356, 1357, 1360, 1361, 1362, 1364, 1365, 1366, 1367, 1368, 1370, 1372, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1385, 1396, 1397, 1398, 1399, 1404, 1407, 1411, 1412, 1413, 1415, 1416, 1417, 1418, 1422, 1423, 1425, 1428, 1429, 1430, 1432, 1434, 1435, 1438, 1439, 1440, 1441, 1443, 1444, 1446, 1447, 1449, 1453, 1460, 1461, 1462, 1464, 1466, 1467, 1475, 1476, 1486, 1490, 1491, 1492, 1493, 1496, 1499, 1502, 1504, 1508, 1509, 1510, 1511, 1512, 1517, 1518, 1519, 1522, 1523, 1524, 1525, 1526, 1531, 1532, 1536, 1537, 1538, 1541, 1542, 1543, 1544, 1546, 1551, 1552, 1554, 1571, 1572, 1576, 1579, 1583, 1590, 1596, 1598, 1599, 1600, 1601, 1604, 1612, 1616, 1617, 1624, 1625, 1626, 1627, 1635, 1637, 1640, 1643, 1645, 1647, 1649, 1650, 1651, 1652, 1655, 1656, 1658, 1659, 1662, 1668, 1670, 1671, 1680, 1683, 1686, 1688, 1700, 1705, 1706, 1707, 1708, 1711, 1713, 1717, 1718, 1720, 1722, 1725, 1726, 1728, 1729, 1731, 1732, 1733, 1737, 1745, 1746, 1748, 1752, 1754, 1755, 1756, 1758, 1759, 1760, 1766, 1770, 1775, 1777, 1780, 1781, 1783, 1785, 1786, 1787, 1788, 1789, 1790, 1798, 1800, 1801, 1804, 1805, 1806, 1807, 1808, 1809, 1810, 1811, 1818, 1819, 1822, 1828, 1830, 1834, 1835, 1836, 1837, 1839, 1840, 1843, 1844, 1845, 1847, 1848, 1849, 1850, 1852, 1855, 1861, 1863, 1872, 1874, 1879, 1880, 1881, 1883, 1894, 1895, 1896, 1899, 1901, 1907, 1908, 1909, 1912, 1919, 1920, 1924, 1926, 1928, 1929, 1930, 1931, 1934, 1935, 1936, 1937, 1938, 1939, 1943, 1946, 1951, 1955, 1956, 1957, 1962, 1968, 1969, 1972, 1973, 1976, 1983, 1984, 1986, 1987, 1988, 1990, 1994, 1996, 2005, 2013, 2016, 2017, 2021, 2026, 2033, 2036, 2041, 2042, 2044, 2045, 2054, 2059, 2060, 2064, 2067, 2069, 2073, 2080, 2081, 2083, 2085, 2091, 2095, 2100, 2101, 2104, 2105, 2108, 2113, 2116, 2117, 2121, 2124, 2127, 2129, 2134, 2135, 2137, 2146, 2147, 2148, 2150, 2153, 2156, 2158, 2164, 2169, 2170, 2172, 2173, 2181, 2185, 2186, 2192, 2194, 2197, 2202, 2205, 2206, 2207, 2210, 2213, 2214, 2215, 2216, 2219, 2223, 2225, 2229, 2231, 2235, 2236, 2244, 2250, 2252, 2256, 2259, 2267, 2268, 2269, 2271, 2272, 2274, 2275, 2278, 2284, 2288, 2295, 2296, 2297, 2298, 2303, 2304, 2305, 2306, 2310, 2316, 2317, 2320, 2326, 2332, 2335, 2337, 2342, 2346, 2348, 2350, 2351, 2353, 2358, 2360, 2361, 2365, 2366, 2367, 2371, 2374, 2383, 2387, 2388, 2390, 2391, 2392, 2393, 2394, 2399, 2400, 2403, 2404, 2405, 2406, 2408, 2410, 2415, 2417, 2420, 2424, 2428, 2429, 2430, 2432, 2438, 2439, 2440, 2441, 2442 }

F grade: { 63, 64, 99, 119, 190, 243, 244, 245, 246, 248, 249, 283, 284, 288, 289, 290, 293, 295, 296, 297, 339, 340, 341, 343, 351, 354, 355, 368, 369, 371, 407, 413, 439, 447, 454, 469, 501, 509, 511, 525, 526, 531, 535, 536, 548, 558, 560, 561, 563, 564, 575, 581, 582, 584, 585, 590, 597, 598, 599, 600, 614, 615, 617, 622, 623, 626, 627, 628, 639, 640, 641, 642, 651, 652, 653, 654, 655, 667, 683, 711, 712, 717, 725, 726, 734, 746, 748, 760, 762, 763, 765, 770, 772, 773, 778, 779, 781, 795, 797, 813, 814, 827, 828, 829, 838, 841, 849, 850, 851, 858, 859, 863, 869, 870, 871, 872, 878, 880, 884, 886, 888, 889, 897, 903, 905, 906, 911, 913, 929, 943, 951, 957, 958, 963, 965, 968, 970, 971, 973, 974, 975, 977, 983, 987, 991, 994, 995, 996, 998, 1005, 1010, 1016, 1018, 1020, 1024, 1025, 1027, 1029, 1034, 1035, 1036, 1049, 1050, 1057, 1058, 1066, 1075, 1078, 1079, 1088, 1093, 1094, 1097, 1098, 1099, 1100, 1101, 1106, 1107, 1111, 1112, 1113, 1116, 1117, 1118, 1123, 1127, 1129, 1138, 1144, 1146, 1149, 1151, 1159, 1161, 1168, 1169, 1170, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1186, 1187, 1189, 1191, 1192, 1193, 1205, 1209, 1215, 1216, 1223, 1231, 1234, 1235, 1236, 1237, 1252, 1256, 1257, 1259, 1267, 1269, 1270, 1271, 1272, 1273, 1280, 1287, 1293, 1301, 1303, 1304, 1309, 1312, 1313, 1314, 1316, 1321, 1322, 1324, 1329, 1332, 1333, 1338, 1344, 1345, 1347, 1349, 1353, 1358, 1359, 1363, 1369, 1384, 1386, 1392, 1394, 1395, 1400, 1401, 1403, 1405, 1406, 1414, 1419, 1420, 1426, 1431, 1433, 1436, 1437, 1442, 1448, 1451, 1452, 1454, 1455, 1456, 1457, 1458, 1459, 1468, 1472, 1477, 1478, 1479, 1480, 1484, 1485, 1487, 1488, 1489, 1500, 1501, 1503, 1505, 1507, 1520, 1521, 1527, 1533, 1534, 1535, 1539, 1540, 1545, 1547, 1548, 1550, 1553, 1558, 1561, 1566, 1567, 1568, 1574, 1577, 1578, 1581, 1584, 1585, 1586, 1591, 1592, 1593, 1594, 1595, 1597, 1603, 1605, 1606, 1607, 1608, 1613, 1618, 1620, 1621, 1628, 1629, 1633, 1638, 1639, 1642, 1644, 1653, 1654, 1660, 1661, 1666, 1667, 1672, 1673, 1674, 1675, 1677, 1679, 1687, 1691, 1693, 1696, 1701, 1702, 1703, 1704, 1709, 1710, 1712, 1716, 1719, 1721, 1727, 1730, 1735, 1736, 1738, 1741, 1743, 1744, 1747, 1750, 1751, 1753, 1757, 1762, 1763, 1765, 1767, 1768, 1769, 1772, 1773, 1774, 1776, 1778, 1779, 1782, 1784, 1791, 1792, 1794, 1795, 1796, 1797, 1799, 1802, 1803, 1812, 1813, 1814, 1815, 1816, 1817, 1821, 1826, 1827, 1829, 1833, 1838, 1841, 1842, 1846, 1851, 1853, 1856, 1857, 1858, 1860, 1862, 1864, 1865, 1867, 1868, 1869, 1873, 1875, 1877, 1878, 1884, 1885, 1886, 1888, 1889, 1890, 1892, 1893, 1900, 1902, 1903, 1904, 1906, 1910, 1911, 1913, 1915, 1916, 1917, 1921, 1923, 1925, 1927, 1932, 1933, 1940, 1941, 1942, 1944, 1945, 1947, 1948, 1949, 1950, 1952, 1954, 1974, 1975, 1979, 1981, 1982, 1985, 1991, 1995, 1997, 1998, 1999, 2000, 2002, 2003, 2004, 2006, 2007, 2009, 2010, 2011, 2012, 2014, 2015, 2018, 2019, 2020, 2022, 2023, 2024, 2025, 2027, 2028, 2029, 2030, 2032, 2034, 2039, 2040, 2043, 2046, 2047, 2049, 2050, 2051, 2052, 2053, 2055, 2058, 2061, 2062, 2063, 2065, 2068, 2070, 2071, 2072, 2074, 2075, 2076, 2077, 2078, 2079, 2082, 2084, 2086, 2087, 2088, 2089, 2090, 2092, 2093, 2096, 2097, 2098, 2099, 2102, 2103, 2106, 2107, 2109, 2110, 2111, 2112, 2114, 2119, 2120, 2122, 2123, 2125, 2126, 2128, 2130, 2131, 2133, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2149, 2151, 2152, 2154, 2157, 2159, 2160, 2161, 2162, 2163, 2165, 2171, 2174, 2175, 2178, 2180, 2182, 2183, 2188, 2190, 2191, 2193, 2195, 2196, 2198, 2200, 2201, 2203, 2204, 2208, 2211, 2218, 2221, 2222, 2224, 2226, 2227, 2228, 2230, 2232, 2233, 2238, 2239, 2241, 2242, 2243, 2245, 2246, 2247, 2248, 2249, 2251, 2253, 2254, 2255, 2257, 2258, 2260, 2263, 2264, 2265, 2266, 2276, 2277, 2280, 2281, 2282, 2283, 2285, 2286, 2287, 2291, 2292, 2293, 2294, 2299, 2302, 2308, 2311, 2312, 2313, 2314, 2315, 2318, 2319, 2321, 2322, 2324, 2327, 2328, 2329, 2330, 2331, 2334, 2336, 2338, 2341, 2343, 2344, 2347, 2349, 2352, 2354, 2355, 2356, 2357, 2362, 2364, 2368, 2369, 2372, 2375, 2376, 2377, 2378, 2379, 2380, 2382, 2384, 2386, 2395, 2396, 2402, 2407, 2409, 2411, 2412, 2416, 2418, 2427, 2431, 2433, 2434, 2435, 2436, 2437 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 25, 26, 31, 32, 33, 34, 35, 47, 48, 49, 50, 51, 52, 53, 54, 55, 59, 61, 65, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 97, 98, 114, 115, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 146, 148, 149, 150, 151, 153, 155, 156, 157, 159, 160, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 194, 197, 198, 201, 202, 204, 205, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 220, 221, 222, 223, 224, 225, 226, 227, 229, 230, 231, 232, 235, 236, 237, 238, 247, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 268, 269, 270, 271, 272, 273, 274, 275, 279, 281, 282, 285, 286, 287, 291, 292, 294, 298, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 330, 332, 334, 335, 336, 337, 342, 347, 348, 350, 353, 356, 357, 358, 359, 360, 362, 363, 365, 366, 367, 372, 375, 376, 378, 382, 383, 384, 385, 386, 387, 389, 391, 394, 398, 399, 400, 401, 410, 411, 412, 414, 415, 425, 428, 429, 431, 432, 433, 435, 437, 443, 444, 445, 446, 450, 453, 458, 460, 468, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 505, 521, 522, 524, 525, 527, 528, 532, 533, 539, 540, 543, 544, 549, 551, 554, 555, 559, 590, 594, 601, 608, 610, 613, 624, 631, 642, 644, 648, 649, 656, 658, 664, 674, 676, 685, 690, 707, 715, 716, 718, 756, 757, 761, 773, 780, 784, 815, 822, 846, 861, 862, 866, 879, 881, 914, 924, 936, 938, 940, 947, 948, 976, 997, 1001, 1002, 1003, 1011, 1059, 1060, 1061, 1095, 1102, 1119, 1139, 1154, 1206, 1306, 1325, 1388, 1389, 1409, 1482, 1483, 1494, 1498, 1528, 1570, 1573, 1610, 1685, 1859, 1882, 1887, 1914, 1926, 1980, 1989, 2026, 2033, 2035, 2038, 2067, 2100, 2168, 2173, 2217, 2325, 2373, 2398 }

B grade: { 18, 19, 20, 21, 22, 23, 24, 27, 28, 29, 30, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 56, 57, 60, 62, 63, 90, 93, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 119, 147, 154, 166, 233, 234, 333, 370, 520, 562, 705, 737, 738, 809, 867, 873, 900, 985, 1023, 1043, 1153, 1211, 1241, 1387, 1390, 1496, 1555, 1580, 1630, 1631, 1678, 1731, 1871, 1898, 1953, 2167, 2279, 2289, 2309, 2323, 2345, 2443 }

C grade: { 58, 64, 67, 68, 116, 117, 118, 137, 145, 152, 161, 162, 191, 192, 193, 195, 196, 199, 200, 203, 207, 218, 219, 228, 239, 240, 241, 242, 252, 265, 266, 267, 276, 277, 278, 280, 288, 293, 299, 300, 313, 318, 323, 331, 338, 343, 344, 345, 346, 349, 351, 352, 354, 355, 361, 364, 371, 373, 374, 377, 379, 380, 381, 388, 390, 392, 395, 396, 397, 402, 403, 404, 405, 406, 408, 409, 413, 416, 417, 418, 419, 420, 421, 422, 423, 424, 426, 430, 434, 436, 438, 440, 441, 442, 448, 449, 451, 452, 454, 455, 456, 457, 459, 461, 462, 463, 464, 465, 466, 467, 470, 471, 472, 473, 474, 480, 498, 499, 500, 501, 502, 503, 504, 506, 507, 508, 510, 513, 514, 515, 516, 517, 518, 519, 523, 529, 530, 534, 537, 538, 541, 542, 545, 546, 547, 550, 552, 553, 556, 557, 561, 566, 567, 568, 569, 570, 571, 572, 573, 577, 578, 579, 580, 582, 583, 584, 587, 588, 589, 591, 592, 593, 595, 596, 598, 600, 602, 604, 605, 606, 607, 609, 611, 612, 618, 619, 620, 621, 629, 630, 632, 633, 634, 635, 637, 638, 639, 641, 646, 647, 650, 657, 659, 660, 661, 662, 663, 665, 666, 668, 669, 670, 671, 673, 675, 677, 678, 679, 680, 681, 682, 684, 687, 688, 691, 692, 693, 694, 695, 696, 697, 698, 699, 703, 708, 709, 710, 713, 719, 720, 721, 722, 723, 726, 727, 728, 729, 730, 731, 732, 733, 735, 736, 739, 740, 741, 742, 743, 744, 745, 747, 749, 750, 751, 752, 753, 754, 755, 758, 759, 764, 765, 766, 767, 768, 770, 771, 775, 776, 777, 779, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 799, 801, 802, 803, 804, 805, 806, 807, 808, 811, 812, 816, 817, 818, 821, 824, 825, 826, 829, 830, 831, 832, 833, 834, 837, 838, 839, 842, 844, 845, 847, 848, 850, 851, 853, 855, 856, 858, 859, 864, 865, 868, 872, 875, 877, 880, 882, 883, 884, 885, 886, 887, 888, 889, 891, 892, 893, 894, 895, 896, 897, 899, 901, 906, 907, 911, 915, 916, 918, 919, 920, 921, 922, 923, 925, 926, 927, 928, 930, 931, 934, 935, 942, 943, 950, 952, 953, 954, 955, 956, 957, 958, 960, 961, 963, 964, 965, 969, 974, 977, 978, 983, 984, 986, 991, 993, 998, 1006, 1007, 1009, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1019, 1020, 1024, 1025, 1026, 1027, 1028, 1031, 1032, 1033, 1034, 1037, 1038, 1039, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1053, 1054, 1055, 1056, 1057, 1058, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1074, 1075, 1076, 1077, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1089, 1090, 1091, 1092, 1096, 1097, 1098, 1099, 1100, 1101, 1105, 1106, 1107, 1108, 1109, 1110, 1112, 1114, 1115, 1116, 1117, 1118, 1123, 1124, 1125, 1126, 1127, 1130, 1131, 1134, 1136, 1137, 1138, 1140, 1141, 1142, 1143, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1155, 1156, 1157, 1158, 1160, 1162, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1174, 1175, 1176, 1178, 1181, 1182, 1183, 1184, 1185, 1190, 1191, 1192, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1204, 1205, 1210, 1218, 1219, 1220, 1221, 1222, 1224, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1255, 1258, 1259, 1260, 1263, 1264, 1265, 1266, 1268, 1271, 1272, 1274, 1275, 1276, 1277, 1280, 1282, 1283, 1284, 1285, 1286, 1287, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1309, 1311, 1313, 1314, 1315, 1316, 1317, 1318, 1321, 1322, 1326, 1327, 1328, 1331, 1332, 1335, 1336, 1337, 1339, 1340, 1341, 1343, 1345, 1347, 1350, 1351, 1352, 1354, 1355, 1356, 1357, 1360, 1361, 1362, 1366, 1367, 1368, 1369, 1372, 1373, 1374, 1375, 1376, 1377, 1379, 1380, 1385, 1386, 1391, 1394, 1396, 1397, 1398, 1399, 1402, 1405, 1406, 1407, 1408, 1411, 1412, 1413, 1414, 1415, 1421, 1422, 1423, 1424, 1425, 1426, 1429, 1432, 1437, 1438, 1439, 1440, 1442, 1443, 1446, 1448, 1449, 1452, 1453, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1468, 1474, 1475, 1476, 1477, 1478, 1481, 1486, 1488, 1489, 1491, 1492, 1493, 1502, 1503, 1504, 1505, 1508, 1509, 1514, 1515, 1518, 1519, 1521, 1523, 1524, 1526, 1527, 1531, 1534, 1536, 1537, 1538, 1541, 1542, 1543, 1544, 1548, 1551, 1553, 1558, 1566, 1567, 1568, 1571, 1572, 1576, 1581, 1582, 1583, 1585, 1586, 1587, 1591, 1592, 1596, 1598, 1599, 1604, 1605, 1611, 1612, 1613, 1617, 1621, 1622, 1626, 1628, 1629, 1632, 1635, 1637, 1638, 1639, 1642, 1643, 1644, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1655, 1657, 1658, 1659, 1660, 1666, 1670, 1676, 1679, 1681, 1682, 1684, 1687, 1688, 1691, 1695, 1696, 1698, 1701, 1702, 1703, 1707, 1708, 1710, 1714, 1715, 1716, 1717, 1719, 1723, 1724, 1732, 1735, 1737, 1738, 1739, 1740, 1749, 1750, 1760, 1766, 1767, 1769, 1770, 1778, 1779, 1780, 1787, 1789, 1790, 1797, 1798, 1799, 1804, 1805, 1806, 1807, 1808, 1809, 1810, 1811, 1812, 1813, 1814, 1815, 1816, 1818, 1821, 1822, 1826, 1827, 1828, 1829, 1836, 1837, 1841, 1842, 1844, 1846, 1847, 1849, 1852, 1855, 1856, 1860, 1862, 1863, 1867, 1870, 1872, 1873, 1876, 1879, 1883, 1885, 1886, 1890, 1891, 1894, 1896, 1899, 1907, 1908, 1909, 1913, 1915, 1917, 1918, 1919, 1920, 1924, 1928, 1929, 1930, 1931, 1932, 1933, 1934, 1935, 1937, 1938, 1939, 1943, 1945, 1955, 1957, 1958, 1959, 1960, 1961, 1968, 1976, 1979, 1981, 1983, 1987, 1994, 1995, 2001, 2008, 2009, 2010, 2011, 2013, 2016, 2018, 2023, 2024, 2027, 2028, 2036, 2054, 2055, 2059, 2060, 2066, 2073, 2079, 2080, 2082, 2083, 2084, 2085, 2086, 2091, 2094, 2095, 2101, 2104, 2105, 2106, 2108, 2121, 2132, 2134, 2135, 2137, 2141, 2150, 2153, 2155, 2156, 2158, 2160, 2161, 2164, 2181, 2184, 2186, 2197, 2198, 2202, 2206, 2207, 2214, 2215, 2219, 2223, 2230, 2235, 2245, 2258, 2259, 2260, 2266, 2267, 2268, 2271, 2280, 2284, 2288, 2294, 2297, 2302, 2304, 2305, 2315, 2322, 2338, 2342, 2348, 2365, 2366, 2367, 2371, 2375, 2386, 2387, 2390, 2391, 2392, 2393, 2399, 2428 }

F grade: { 99, 158, 190, 206, 243, 244, 245, 246, 248, 249, 283, 284, 289, 290, 295, 296, 297, 339, 340, 341, 368, 369, 393, 407, 427, 439, 447, 469, 509, 511, 512, 526, 531, 535, 536, 548, 558, 560, 563, 564, 565, 574, 575, 576, 581, 585, 586, 597, 599, 603, 614, 615, 616, 617, 622, 623, 625, 626, 627, 628, 636, 640, 643, 645, 651, 652, 653, 654, 655, 667, 672, 683, 686, 689, 700, 701, 702, 704, 706, 711, 712, 714, 717, 724, 725, 734, 746, 748, 760, 762, 763, 769, 772, 774, 778, 781, 782, 783, 798, 800, 810, 813, 814, 819, 820, 823, 827, 828, 835, 836, 840, 841, 843, 849, 852, 854, 857, 860, 863, 869, 870, 871, 874, 876, 878, 890, 898, 902, 903, 904, 905, 908, 909, 910, 912, 913, 917, 929, 932, 933, 937, 939, 941, 944, 945, 946, 949, 951, 959, 962, 966, 967, 968, 970, 971, 972, 973, 975, 979, 980, 981, 982, 987, 988, 989, 990, 992, 994, 995, 996, 999, 1000, 1004, 1005, 1008, 1018, 1021, 1022, 1029, 1030, 1035, 1036, 1040, 1041, 1042, 1051, 1052, 1071, 1072, 1073, 1078, 1079, 1088, 1093, 1094, 1103, 1104, 1111, 1113, 1120, 1121, 1122, 1128, 1129, 1132, 1133, 1135, 1144, 1159, 1161, 1163, 1164, 1173, 1177, 1179, 1180, 1186, 1187, 1188, 1189, 1193, 1202, 1203, 1207, 1208, 1209, 1212, 1213, 1214, 1215, 1216, 1217, 1223, 1225, 1238, 1239, 1240, 1254, 1256, 1257, 1261, 1262, 1267, 1269, 1270, 1273, 1278, 1279, 1281, 1288, 1289, 1290, 1291, 1292, 1293, 1307, 1308, 1310, 1312, 1319, 1320, 1323, 1324, 1329, 1330, 1333, 1334, 1338, 1342, 1344, 1346, 1348, 1349, 1353, 1358, 1359, 1363, 1364, 1365, 1370, 1371, 1378, 1381, 1382, 1383, 1384, 1392, 1393, 1395, 1400, 1401, 1403, 1404, 1410, 1416, 1417, 1418, 1419, 1420, 1427, 1428, 1430, 1431, 1433, 1434, 1435, 1436, 1441, 1444, 1445, 1447, 1450, 1451, 1454, 1455, 1456, 1457, 1458, 1459, 1467, 1469, 1470, 1471, 1472, 1473, 1479, 1480, 1484, 1485, 1487, 1490, 1495, 1497, 1499, 1500, 1501, 1506, 1507, 1510, 1511, 1512, 1513, 1516, 1517, 1520, 1522, 1525, 1529, 1530, 1532, 1533, 1535, 1539, 1540, 1545, 1546, 1547, 1549, 1550, 1552, 1554, 1556, 1557, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1569, 1574, 1575, 1577, 1578, 1579, 1584, 1588, 1589, 1590, 1593, 1594, 1595, 1597, 1600, 1601, 1602, 1603, 1606, 1607, 1608, 1609, 1614, 1615, 1616, 1618, 1619, 1620, 1623, 1624, 1625, 1627, 1633, 1634, 1636, 1640, 1641, 1645, 1653, 1654, 1656, 1661, 1662, 1663, 1664, 1665, 1667, 1668, 1669, 1671, 1672, 1673, 1674, 1675, 1677, 1680, 1683, 1686, 1689, 1690, 1692, 1693, 1694, 1697, 1699, 1700, 1704, 1705, 1706, 1709, 1711, 1712, 1713, 1718, 1720, 1721, 1722, 1725, 1726, 1727, 1728, 1729, 1730, 1733, 1734, 1736, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1761, 1762, 1763, 1764, 1765, 1768, 1771, 1772, 1773, 1774, 1775, 1776, 1777, 1781, 1782, 1783, 1784, 1785, 1786, 1788, 1791, 1792, 1793, 1794, 1795, 1796, 1800, 1801, 1802, 1803, 1817, 1819, 1820, 1823, 1824, 1825, 1830, 1831, 1832, 1833, 1834, 1835, 1838, 1839, 1840, 1843, 1845, 1848, 1850, 1851, 1853, 1854, 1857, 1858, 1861, 1864, 1865, 1866, 1868, 1869, 1874, 1875, 1877, 1878, 1880, 1881, 1884, 1888, 1889, 1892, 1893, 1895, 1897, 1900, 1901, 1902, 1903, 1904, 1905, 1906, 1910, 1911, 1912, 1916, 1921, 1922, 1923, 1925, 1927, 1936, 1940, 1941, 1942, 1944, 1946, 1947, 1948, 1949, 1950, 1951, 1952, 1954, 1956, 1962, 1963, 1964, 1965, 1966, 1967, 1969, 1970, 1971, 1972, 1973, 1974, 1975, 1977, 1978, 1982, 1984, 1985, 1986, 1988, 1990, 1991, 1992, 1993, 1996, 1997, 1998, 1999, 2000, 2002, 2003, 2004, 2005, 2006, 2007, 2012, 2014, 2015, 2017, 2019, 2020, 2021, 2022, 2025, 2029, 2030, 2031, 2032, 2034, 2037, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2056, 2057, 2058, 2061, 2062, 2063, 2064, 2065, 2068, 2069, 2070, 2071, 2072, 2074, 2075, 2076, 2077, 2078, 2081, 2087, 2088, 2089, 2090, 2092, 2093, 2096, 2097, 2098, 2099, 2102, 2103, 2107, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2133, 2136, 2138, 2139, 2140, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2151, 2152, 2154, 2157, 2159, 2162, 2163, 2165, 2166, 2169, 2170, 2171, 2172, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2182, 2183, 2185, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2199, 2200, 2201, 2203, 2204, 2205, 2208, 2209, 2210, 2211, 2212, 2213, 2216, 2218, 2220, 2221, 2222, 2224, 2225, 2226, 2227, 2228, 2229, 2231, 2232, 2233, 2234, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2261, 2262, 2263, 2264, 2265, 2269, 2270, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2281, 2282, 2283, 2285, 2286, 2287, 2290, 2291, 2292, 2293, 2295, 2296, 2298, 2299, 2300, 2301, 2303, 2306, 2307, 2308, 2310, 2311, 2312, 2313, 2314, 2316, 2317, 2318, 2319, 2320, 2321, 2324, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2339, 2340, 2341, 2343, 2344, 2346, 2347, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2368, 2369, 2370, 2372, 2374, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2388, 2389, 2394, 2395, 2396, 2397, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442 }

2.1.4 Maxima

A grade: { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 49, 53, 54, 55, 57, 59, 60, 65, 66, 69, 70, 71, 72, 73, 75, 76, 78, 79, 80, 82, 83, 84, 85, 86, 92, 100, 101, 102, 103, 104, 106, 115, 120, 121, 122, 123, 124, 129, 131, 132, 134, 136, 140, 141, 155, 159, 163, 164, 165, 170, 171, 173, 174, 179, 183, 184, 188, 201, 202, 207, 208, 209, 214, 215, 216, 217, 220, 229, 230, 233, 234, 247, 250, 251, 252, 253, 254, 255, 256, 257, 259, 260, 264, 269, 273, 274, 279, 281, 282, 287, 292, 298, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 315, 316, 319, 320, 321, 322, 324, 325, 326, 329, 331, 332, 333, 334, 336, 337, 342, 346, 347, 348, 353, 356, 357, 358, 363, 372, 375, 376, 378, 382, 383, 384, 385, 386, 394, 398, 399, 403, 411, 412, 414, 415, 426, 429, 431, 432, 435, 441, 444, 449, 450, 453, 460, 463, 464, 465, 466, 467, 476, 479, 481, 482, 485, 486, 487, 488, 489, 490, 493, 494, 495, 496, 505, 520, 522, 524, 527, 528, 532, 537, 541, 549, 551, 562, 573, 577, 578, 579, 583, 592, 593, 602, 619, 620, 624, 636, 644, 646, 647, 648, 658, 682, 685, 689, 692, 700, 701, 702, 703, 704, 715, 724, 747, 756, 757, 758, 759, 764, 768, 769, 780, 805, 809, 810, 819, 840, 852, 853, 854, 857, 860, 868, 891, 892, 895, 898, 899, 902, 904, 908, 909, 910, 912, 916, 922, 924, 927, 930, 935, 938, 939, 941, 944, 945, 946, 947, 949, 966, 969, 979, 980, 981, 1007, 1011, 1012, 1021, 1022, 1033, 1037, 1040, 1042, 1044, 1045, 1046, 1048, 1055, 1056, 1059, 1061, 1067, 1068, 1069, 1070, 1072, 1074, 1077, 1082, 1086, 1087, 1089, 1091, 1092, 1095, 1096, 1105, 1108, 1109, 1110, 1124, 1125, 1126, 1128, 1130, 1132, 1134, 1135, 1136, 1137, 1139, 1140, 1141, 1143, 1145, 1150, 1155, 1156, 1157, 1171, 1183, 1184, 1195, 1196, 1197, 1201, 1204, 1217, 1221, 1222, 1226, 1233, 1241, 1242, 1243, 1244, 1248, 1263, 1264, 1265, 1268, 1277, 1282, 1283, 1285, 1292, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1311, 1315, 1317, 1325, 1335, 1336, 1355, 1356, 1357, 1360, 1361, 1366, 1367, 1380, 1390, 1407, 1409, 1413, 1421, 1422, 1423, 1424, 1425, 1438, 1449, 1450, 1463, 1464, 1465, 1466, 1473, 1482, 1486, 1494, 1498, 1516, 1517, 1530, 1544, 1559, 1560, 1562, 1563, 1564, 1565, 1619, 1625, 1634, 1636, 1641, 1647, 1671, 1689, 1690, 1692, 1694, 1697, 1722, 1725, 1726, 1728, 1729, 1734, 1752, 1754, 1755, 1756, 1758, 1759, 1800, 1834, 1839, 1840, 1843, 1845, 1882, 1912, 1953, 1989, 2073, 2081, 2168, 2205, 2217, 2252, 2278, 2303, 2337, 2443 }

B grade: { 3, 43, 48, 56, 61, 62, 74, 87, 88, 89, 90, 93, 94, 95, 96, 105, 107, 108, 109, 110, 114, 128, 133, 152, 153, 166, 172, 221, 222, 261, 291, 294, 350, 387, 400, 401, 440, 442, 443, 477, 478, 491, 492, 533, 542, 543, 550, 552, 554, 555, 559, 611, 612, 613, 668, 690, 693, 707, 721, 737, 738, 775, 776, 932, 940, 992, 1041, 1071, 1133, 1212, 1238, 1278, 1319, 1339, 1362, 1364, 1513, 1905, 2031 }

C grade: { 601, 1570, 1610 }

F grade: { 50, 51, 52, 58, 63, 64, 67, 68, 77, 81, 91, 97, 98, 99, 111, 112, 113, 116, 117, 118, 119, 125, 126, 127, 130, 135, 137, 138, 139, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 154, 156, 157, 158, 160, 161, 162, 167, 168, 169, 175, 176, 177, 178, 180, 181, 182, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 203, 204, 205, 206, 210, 211, 212, 213, 218, 219, 223, 224, 225, 226, 227, 228, 231, 232, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 248, 249, 258, 262, 263, 265, 266, 267, 268, 270, 271, 272, 275, 276, 277, 278, 280, 283, 284, 285, 286, 288, 289, 290, 293, 295, 296, 297, 299, 300, 312, 313, 314, 317, 318, 323, 327, 328, 330, 335, 338, 339, 340, 341, 343, 344, 345, 349, 351, 352, 354, 355, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 373, 374, 377, 379, 380, 381, 388, 389, 390, 391, 392, 393, 395, 396, 397, 402, 404, 405, 406, 407, 408, 409, 410, 413, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 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1914, 1922, 1928, 1929, 1936, 1937, 1940, 1948, 1953, 1954, 1956, 1963, 1964, 1965, 1966, 1967, 1971, 1974, 1976, 1977, 1978, 1980, 1982, 1983, 1989, 1992, 1996, 2008, 2009, 2010, 2011, 2013, 2026, 2027, 2028, 2031, 2035, 2038, 2044, 2045, 2054, 2059, 2060, 2061, 2062, 2067, 2073, 2080, 2087, 2106, 2112, 2116, 2124, 2125, 2126, 2127, 2145, 2146, 2147, 2148, 2149, 2150, 2168, 2169, 2170, 2173, 2176, 2177, 2185, 2194, 2206, 2213, 2216, 2217, 2220, 2225, 2226, 2232, 2234, 2235, 2236, 2240, 2242, 2243, 2246, 2255, 2256, 2259, 2261, 2268, 2269, 2270, 2274, 2275, 2285, 2286, 2288, 2289, 2292, 2295, 2296, 2297, 2298, 2301, 2302, 2304, 2305, 2309, 2310, 2314, 2322, 2323, 2333, 2339, 2340, 2348, 2350, 2351, 2353, 2356, 2358, 2359, 2361, 2362, 2369, 2373, 2378, 2384, 2385, 2387, 2389, 2394, 2395, 2397, 2398, 2400, 2401, 2403, 2404, 2405, 2406, 2408, 2410, 2412, 2413, 2414, 2415, 2416, 2418, 2419, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2433, 2434, 2435, 2436, 2438 }

B grade: { 48, 58, 61, 63, 64, 67, 68, 99, 113, 116, 118, 119, 145, 163, 190, 191, 200, 206, 218, 238, 242, 243, 244, 245, 246, 276, 277, 283, 284, 294, 296, 299, 300, 318, 338, 340, 370, 373, 377, 379, 390, 395, 397, 406, 413, 416, 417, 418, 419, 420, 426, 430, 454, 455, 456, 471, 472, 480, 500, 501, 516, 517, 518, 519, 526, 529, 538, 546, 556, 557, 561, 566, 567, 569, 570, 571, 572, 582, 584, 592, 595, 604, 609, 614, 615, 618, 626, 627, 628, 636, 637, 641, 644, 645, 652, 653, 654, 655, 657, 662, 666, 674, 675, 676, 678, 679, 682, 689, 692, 694, 695, 700, 701, 702, 704, 705, 706, 719, 723, 724, 726, 731, 741, 742, 744, 749, 750, 752, 755, 760, 762, 765, 766, 768, 769, 770, 779, 784, 788, 795, 797, 807, 810, 813, 814, 815, 823, 826, 827, 830, 831, 832, 834, 839, 841, 845, 849, 850, 851, 852, 853, 854, 855, 857, 859, 860, 872, 876, 882, 883, 887, 889, 893, 895, 896, 901, 902, 904, 905, 908, 910, 912, 914, 915, 917, 918, 920, 921, 926, 928, 930, 931, 932, 934, 935, 939, 941, 943, 944, 945, 946, 954, 958, 965, 966, 972, 973, 975, 980, 982, 983, 984, 985, 986, 989, 990, 1009, 1013, 1019, 1027, 1029, 1037, 1039, 1040, 1043, 1047, 1050, 1052, 1053, 1057, 1064, 1065, 1076, 1085, 1090, 1101, 1102, 1108, 1117, 1120, 1128, 1131, 1132, 1133, 1135, 1139, 1140, 1142, 1145, 1148, 1150, 1153, 1160, 1162, 1165, 1166, 1170, 1175, 1176, 1178, 1182, 1191, 1192, 1194, 1198, 1208, 1212, 1218, 1219, 1220, 1224, 1225, 1227, 1239, 1245, 1246, 1250, 1251, 1252, 1256, 1257, 1259, 1274, 1275, 1276, 1278, 1279, 1286, 1288, 1301, 1303, 1304, 1306, 1310, 1318, 1320, 1322, 1323, 1334, 1340, 1341, 1344, 1347, 1351, 1364, 1365, 1368, 1386, 1396, 1404, 1408, 1413, 1416, 1417, 1418, 1432, 1434, 1437, 1442, 1443, 1445, 1461, 1462, 1471, 1478, 1479, 1480, 1482, 1489, 1490, 1491, 1492, 1493, 1503, 1504, 1505, 1510, 1513, 1518, 1519, 1521, 1522, 1524, 1526, 1527, 1531, 1532, 1542, 1544, 1548, 1549, 1552, 1558, 1566, 1567, 1572, 1575, 1576, 1583, 1584, 1585, 1586, 1587, 1591, 1595, 1598, 1599, 1600, 1601, 1602, 1605, 1607, 1611, 1613, 1614, 1615, 1616, 1618, 1621, 1624, 1627, 1628, 1629, 1632, 1638, 1639, 1642, 1643, 1644, 1648, 1649, 1650, 1651, 1652, 1653, 1660, 1662, 1664, 1665, 1666, 1668, 1669, 1670, 1674, 1676, 1679, 1680, 1681, 1682, 1684, 1686, 1691, 1695, 1696, 1702, 1705, 1707, 1708, 1711, 1712, 1713, 1714, 1715, 1717, 1723, 1724, 1730, 1735, 1737, 1738, 1739, 1740, 1748, 1749, 1750, 1751, 1762, 1766, 1767, 1769, 1770, 1771, 1779, 1780, 1781, 1785, 1790, 1793, 1797, 1798, 1799, 1801, 1803, 1804, 1805, 1806, 1807, 1808, 1809, 1810, 1811, 1812, 1813, 1814, 1815, 1816, 1820, 1822, 1823, 1826, 1827, 1829, 1831, 1836, 1842, 1844, 1846, 1847, 1849, 1851, 1852, 1855, 1856, 1857, 1860, 1862, 1863, 1865, 1866, 1867, 1870, 1872, 1875, 1885, 1886, 1889, 1890, 1892, 1893, 1896, 1899, 1907, 1908, 1913, 1917, 1918, 1919, 1920, 1924, 1926, 1930, 1931, 1932, 1933, 1934, 1935, 1938, 1939, 1941, 1942, 1943, 1945, 1955, 1957, 1958, 1959, 1960, 1961, 1962, 1968, 1969, 1970, 1972, 1975, 1981, 1984, 1985, 1986, 1993, 1995, 2001, 2005, 2014, 2016, 2018, 2021, 2023, 2024, 2033, 2036, 2037, 2041, 2042, 2048, 2055, 2056, 2057, 2064, 2066, 2069, 2074, 2077, 2078, 2088, 2091, 2094, 2100, 2101, 2104, 2105, 2113, 2115, 2117, 2121, 2129, 2132, 2133, 2134, 2135, 2137, 2143, 2144, 2155, 2156, 2158, 2160, 2164, 2165, 2181, 2184, 2186, 2187, 2189, 2192, 2197, 2199, 2200, 2202, 2203, 2207, 2209, 2215, 2219, 2227, 2228, 2231, 2237, 2241, 2245, 2249, 2250, 2258, 2260, 2262, 2266, 2273, 2279, 2284, 2303, 2324, 2328, 2332, 2334, 2337, 2342, 2345, 2346, 2352, 2360, 2364, 2383, 2420, 2439, 2440, 2441, 2442 }

C grade: { 919, 1181, 2103, 2417 }

F grade: { 2, 278, 407, 439, 447, 469, 531, 535, 536, 563, 564, 575, 585, 598, 617, 622, 625, 640, 651, 667, 672, 683, 708, 711, 714, 717, 720, 725, 734, 736, 739, 740, 746, 748, 763, 772, 773, 786, 789, 821, 828, 838, 840, 869, 870, 871, 878, 898, 903, 909, 913, 925, 929, 933, 951, 967, 968, 970, 971, 974, 979, 981, 987, 992, 994, 995, 996, 998, 999, 1000, 1008, 1018, 1021, 1022, 1035, 1036, 1041, 1042, 1051, 1071, 1072, 1078, 1079, 1093, 1094, 1103, 1104, 1111, 1116, 1121, 1127, 1129, 1144, 1159, 1161, 1163, 1164, 1172, 1173, 1174, 1177, 1179, 1180, 1186, 1187, 1188, 1189, 1193, 1202, 1203, 1209, 1213, 1214, 1215, 1223, 1238, 1254, 1267, 1269, 1270, 1289, 1290, 1291, 1292, 1293, 1307, 1312, 1319, 1324, 1329, 1330, 1332, 1333, 1338, 1342, 1346, 1348, 1349, 1353, 1358, 1363, 1369, 1378, 1381, 1382, 1383, 1384, 1389, 1395, 1400, 1401, 1403, 1419, 1420, 1428, 1430, 1431, 1433, 1435, 1436, 1441, 1444, 1451, 1454, 1455, 1456, 1457, 1458, 1459, 1467, 1469, 1470, 1483, 1485, 1487, 1497, 1499, 1500, 1501, 1511, 1512, 1520, 1525, 1539, 1540, 1545, 1547, 1550, 1554, 1561, 1574, 1577, 1578, 1580, 1581, 1593, 1597, 1603, 1604, 1606, 1608, 1620, 1630, 1631, 1633, 1636, 1654, 1661, 1663, 1667, 1671, 1672, 1673, 1675, 1677, 1687, 1693, 1699, 1704, 1709, 1718, 1719, 1720, 1721, 1727, 1731, 1732, 1741, 1742, 1745, 1747, 1753, 1757, 1763, 1765, 1768, 1772, 1773, 1776, 1777, 1783, 1786, 1791, 1792, 1794, 1795, 1796, 1802, 1817, 1830, 1833, 1835, 1838, 1853, 1861, 1864, 1868, 1869, 1873, 1874, 1877, 1878, 1880, 1884, 1888, 1895, 1897, 1898, 1900, 1902, 1903, 1904, 1906, 1910, 1911, 1912, 1915, 1916, 1921, 1923, 1925, 1927, 1944, 1946, 1947, 1949, 1950, 1951, 1952, 1973, 1979, 1987, 1988, 1990, 1991, 1994, 1997, 1998, 1999, 2000, 2002, 2003, 2004, 2006, 2007, 2012, 2015, 2017, 2019, 2020, 2022, 2025, 2029, 2030, 2032, 2034, 2039, 2040, 2043, 2046, 2047, 2049, 2050, 2051, 2052, 2053, 2058, 2063, 2065, 2068, 2070, 2071, 2072, 2075, 2076, 2079, 2081, 2082, 2083, 2084, 2085, 2086, 2089, 2090, 2092, 2093, 2095, 2096, 2097, 2098, 2099, 2102, 2107, 2108, 2109, 2110, 2111, 2114, 2118, 2119, 2120, 2122, 2123, 2128, 2130, 2131, 2136, 2138, 2139, 2140, 2141, 2142, 2151, 2152, 2153, 2154, 2157, 2159, 2161, 2162, 2163, 2166, 2167, 2171, 2172, 2174, 2175, 2178, 2179, 2180, 2182, 2183, 2188, 2190, 2191, 2193, 2195, 2196, 2198, 2201, 2204, 2205, 2208, 2210, 2211, 2212, 2214, 2218, 2221, 2222, 2223, 2224, 2229, 2230, 2233, 2238, 2239, 2244, 2247, 2248, 2251, 2252, 2253, 2254, 2257, 2263, 2264, 2265, 2267, 2271, 2272, 2276, 2277, 2278, 2280, 2281, 2282, 2283, 2287, 2290, 2291, 2293, 2294, 2299, 2300, 2306, 2307, 2308, 2311, 2312, 2313, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2325, 2326, 2327, 2329, 2330, 2331, 2335, 2336, 2338, 2341, 2343, 2344, 2347, 2349, 2354, 2355, 2357, 2363, 2365, 2366, 2367, 2368, 2370, 2371, 2372, 2374, 2375, 2376, 2377, 2379, 2380, 2381, 2382, 2386, 2388, 2390, 2391, 2392, 2393, 2396, 2399, 2402, 2407, 2409, 2411, 2432, 2437, 2443 }

2.1.6 Sympy

A grade: { 1, 2, 4, 5, 10, 11, 15, 18, 19, 27, 32, 38, 44, 48, 55, 59, 61, 65, 79, 82, 100, 103, 104, 106, 120, 122, 152, 153, 170, 171, 209, 231, 233, 234, 250, 251, 253, 254, 255, 256, 257, 259, 287, 301, 303, 307, 320, 331, 332, 333, 342, 347, 348, 353, 372, 375, 376, 378, 382, 383, 384, 386, 393, 411, 427, 432, 440, 441, 442, 443, 444, 448, 453, 463, 464, 465, 466, 467, 470, 479, 481, 490, 492, 493, 498, 499, 505, 520, 523, 524, 527, 528, 534, 550, 552, 554, 573, 576, 593, 611, 612, 613, 624, 638, 648, 658, 668, 671, 673, 685, 707, 727, 756, 757, 771, 787, 796, 806, 816, 817, 835, 836, 864, 866, 947, 949, 960, 961, 1004, 1061, 1077, 1095, 1139, 1217, 1260, 1261, 1390, 1482, 1494, 1497, 1498, 1610, 1734, 1891, 2217 }

B grade: { 6, 7, 8, 9, 12, 13, 14, 16, 17, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 33, 34, 35, 36, 37, 39, 40, 41, 42, 45, 46, 53, 66, 70, 71, 72, 73, 75, 80, 86, 101, 102, 128, 158, 179, 206, 214, 215, 216, 217, 230, 274, 279, 298, 306, 310, 356, 357, 385, 398, 412, 415, 429, 450, 476, 477, 478, 532, 543, 549, 551, 559, 601, 634, 690, 718, 737, 738, 780, 799, 1062, 1373, 1570, 2345 }

C grade: { 31, 43, 47, 49, 54, 56, 57, 60, 62, 69, 74, 83, 84, 85, 87, 93, 94, 95, 96, 105, 107, 108, 109, 110, 114, 115, 172, 207, 229, 252, 269, 281, 282, 285, 286, 291, 292, 305, 308, 309, 315, 316, 319, 324, 325, 326, 329, 334, 336, 337, 346, 387, 394, 399, 403, 414, 426, 449, 482, 483, 484, 485, 486, 487, 488, 489, 494, 495, 496, 500, 537, 541, 542, 577, 578, 579, 583, 592, 602, 619, 620, 629, 635, 636, 643, 646, 647, 677, 682, 687, 689, 692, 693, 700, 701, 702, 703, 704, 715, 721, 724, 747, 758, 759, 768, 769, 775, 776, 805, 810, 819, 840, 852, 853, 854, 857, 860, 865, 868, 891, 892, 895, 898, 899, 902, 904, 908, 909, 910, 912, 916, 922, 927, 930, 932, 935, 939, 941, 944, 945, 946, 966, 969, 979, 980, 981, 992, 1007, 1012, 1021, 1022, 1023, 1033, 1037, 1040, 1041, 1042, 1044, 1045, 1046, 1048, 1055, 1056, 1067, 1068, 1069, 1070, 1071, 1072, 1074, 1082, 1086, 1087, 1089, 1090, 1091, 1092, 1096, 1105, 1108, 1109, 1110, 1124, 1125, 1126, 1128, 1130, 1132, 1133, 1134, 1135, 1136, 1137, 1140, 1141, 1143, 1145, 1150, 1155, 1156, 1157, 1171, 1183, 1184, 1195, 1196, 1197, 1204, 1212, 1226, 1238, 1242, 1243, 1244, 1248, 1263, 1264, 1265, 1277, 1278, 1282, 1283, 1292, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1311, 1315, 1317, 1319, 1335, 1336, 1355, 1356, 1357, 1360, 1361, 1364, 1366, 1367, 1372, 1380, 1391, 1407, 1413, 1421, 1422, 1423, 1424, 1425, 1438, 1446, 1447, 1449, 1450, 1463, 1464, 1465, 1466, 1473, 1486, 1496, 1513, 1516, 1517, 1530, 1544, 1559, 1560, 1562, 1563, 1564, 1565, 1619, 1625, 1634, 1636, 1641, 1647, 1671, 1688, 1689, 1690, 1692, 1694, 1697, 1722, 1725, 1726, 1728, 1729, 1752, 1754, 1755, 1756, 1758, 1759, 1800, 1818, 1819, 1834, 1839, 1840, 1843, 1845, 1905, 1912, 1987, 1988, 2031, 2081, 2153, 2205, 2252, 2278 }

F grade: { 3, 50, 51, 52, 58, 63, 64, 67, 68, 76, 77, 78, 81, 88, 89, 90, 91, 92, 97, 98, 99, 111, 112, 113, 116, 117, 118, 119, 121, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 208, 210, 211, 212, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 232, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 270, 271, 272, 273, 275, 276, 277, 278, 280, 283, 284, 288, 289, 290, 293, 294, 295, 296, 297, 299, 300, 302, 304, 311, 312, 313, 314, 317, 318, 321, 322, 323, 327, 328, 330, 335, 338, 339, 340, 341, 343, 344, 345, 349, 350, 351, 352, 354, 355, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 373, 374, 377, 379, 380, 381, 388, 389, 390, 391, 392, 395, 396, 397, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 413, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 428, 430, 431, 433, 434, 435, 436, 437, 438, 439, 445, 446, 447, 451, 452, 454, 455, 456, 457, 458, 459, 460, 461, 462, 468, 469, 471, 472, 473, 474, 475, 480, 491, 497, 501, 502, 503, 504, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 521, 522, 525, 526, 529, 530, 531, 533, 535, 536, 538, 539, 540, 544, 545, 546, 547, 548, 553, 555, 556, 557, 558, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 574, 575, 580, 581, 582, 584, 585, 586, 587, 588, 589, 590, 591, 594, 595, 596, 597, 598, 599, 600, 603, 604, 605, 606, 607, 608, 609, 610, 614, 615, 616, 617, 618, 621, 622, 623, 625, 626, 627, 628, 630, 631, 632, 633, 637, 639, 640, 641, 642, 644, 645, 649, 650, 651, 652, 653, 654, 655, 656, 657, 659, 660, 661, 662, 663, 664, 665, 666, 667, 669, 670, 672, 674, 675, 676, 678, 679, 680, 681, 683, 684, 686, 688, 691, 694, 695, 696, 697, 698, 699, 705, 706, 708, 709, 710, 711, 712, 713, 714, 716, 717, 719, 720, 722, 723, 725, 726, 728, 729, 730, 731, 732, 733, 734, 735, 736, 739, 740, 741, 742, 743, 744, 745, 746, 748, 749, 750, 751, 752, 753, 754, 755, 760, 761, 762, 763, 764, 765, 766, 767, 770, 772, 773, 774, 777, 778, 779, 781, 782, 783, 784, 785, 786, 788, 789, 790, 791, 792, 793, 794, 795, 797, 798, 800, 801, 802, 803, 804, 807, 808, 809, 811, 812, 813, 814, 815, 818, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 837, 838, 839, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 855, 856, 858, 859, 861, 862, 863, 867, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 893, 894, 896, 897, 900, 901, 903, 905, 906, 907, 911, 913, 914, 915, 917, 918, 919, 920, 921, 923, 924, 925, 926, 928, 929, 931, 933, 934, 936, 937, 938, 940, 942, 943, 948, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 962, 963, 964, 965, 967, 968, 970, 971, 972, 973, 974, 975, 976, 977, 978, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1005, 1006, 1008, 1009, 1010, 1011, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1034, 1035, 1036, 1038, 1039, 1043, 1047, 1049, 1050, 1051, 1052, 1053, 1054, 1057, 1058, 1059, 1060, 1063, 1064, 1065, 1066, 1073, 1075, 1076, 1078, 1079, 1080, 1081, 1083, 1084, 1085, 1088, 1093, 1094, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1106, 1107, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1127, 1129, 1131, 1138, 1142, 1144, 1146, 1147, 1148, 1149, 1151, 1152, 1153, 1154, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 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2148, 2149, 2150, 2151, 2152, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 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2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 50, 52, 54, 55, 59, 65, 66, 69, 75, 76, 77, 78, 81, 83, 84, 88, 89, 90, 92, 93, 96, 100, 102, 103, 106, 120, 122, 123, 124, 126, 127, 129, 130, 131, 132, 133, 134, 136, 137, 140, 141, 143, 144, 152, 153, 155, 168, 169, 170, 171, 173, 174, 176, 177, 178, 179, 184, 198, 204, 205, 207, 209, 210, 211, 212, 213, 220, 221, 222, 224, 225, 226, 227, 228, 231, 238, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 263, 264, 265, 266, 267, 268, 273, 287, 298, 301, 302, 303, 304, 305, 307, 311, 314, 317, 320, 321, 322, 327, 328, 331, 332, 333, 346, 347, 348, 350, 352, 353, 356, 357, 358, 359, 362, 363, 364, 365, 372, 375, 376, 378, 380, 381, 382, 383, 384, 385, 386, 389, 391, 392, 393, 403, 405, 406, 408, 409, 410, 411, 414, 415, 427, 428, 429, 432, 433, 434, 435, 436, 437, 438, 441, 442, 444, 445, 449, 458, 459, 460, 461, 462, 468, 476, 477, 479, 481, 490, 491, 493, 497, 498, 499, 505, 506, 507, 508, 520, 523, 524, 525, 527, 528, 532, 533, 534, 537, 541, 543, 544, 549, 551, 555, 559, 573, 577, 578, 579, 580, 583, 587, 590, 591, 592, 593, 594, 602, 607, 608, 619, 620, 621, 630, 631, 638, 644, 645, 646, 647, 648, 650, 658, 661, 662, 664, 671, 673, 682, 684, 690, 693, 694, 695, 696, 697, 705, 707, 713, 715, 718, 721, 722, 732, 737, 738, 739, 740, 741, 742, 757, 771, 780, 784, 787, 791, 792, 794, 796, 806, 809, 816, 817, 823, 837, 839, 842, 845, 861, 862, 867, 868, 876, 877, 879, 881, 885, 891, 892, 896, 899, 900, 901, 905, 916, 922, 930, 935, 936, 938, 947, 948, 949, 954, 955, 969, 973, 993, 997, 1001, 1002, 1003, 1007, 1011, 1012, 1033, 1037, 1038, 1039, 1043, 1044, 1045, 1046, 1048, 1055, 1056, 1059, 1060, 1061, 1074, 1077, 1082, 1086, 1087, 1089, 1091, 1092, 1096, 1105, 1108, 1109, 1110, 1115, 1124, 1125, 1130, 1134, 1136, 1137, 1139, 1140, 1145, 1147, 1148, 1150, 1154, 1158, 1185, 1190, 1195, 1196, 1197, 1199, 1201, 1204, 1206, 1217, 1221, 1222, 1226, 1230, 1232, 1233, 1241, 1247, 1250, 1251, 1258, 1268, 1282, 1284, 1285, 1294, 1302, 1327, 1328, 1334, 1337, 1339, 1340, 1350, 1354, 1362, 1368, 1376, 1379, 1388, 1389, 1390, 1397, 1398, 1399, 1408, 1409, 1411, 1412, 1415, 1416, 1429, 1439, 1440, 1443, 1450, 1460, 1471, 1474, 1475, 1476, 1482, 1494, 1498, 1508, 1510, 1514, 1516, 1517, 1524, 1530, 1543, 1551, 1555, 1559, 1560, 1562, 1563, 1564, 1565, 1571, 1573, 1580, 1582, 1612, 1622, 1630, 1631, 1634, 1635, 1637, 1640, 1641, 1645, 1649, 1668, 1670, 1686, 1689, 1690, 1692, 1694, 1697, 1698, 1700, 1706, 1722, 1725, 1726, 1728, 1729, 1734, 1736, 1746, 1752, 1754, 1755, 1756, 1758, 1759, 1765, 1770, 1776, 1782, 1789, 1794, 1796, 1834, 1839, 1840, 1843, 1845, 1859, 1872, 1882, 1887, 1891, 1898, 1946, 1951, 1956, 1969, 1980, 1990, 2005, 2013, 2034, 2044, 2045, 2054, 2059, 2060, 2073, 2076, 2080, 2091, 2093, 2104, 2115, 2117, 2127, 2146, 2147, 2148, 2150, 2167, 2168, 2169, 2170, 2186, 2192, 2197, 2216, 2217, 2219, 2221, 2233, 2244, 2257, 2262, 2272, 2279, 2289, 2290, 2307, 2310, 2325, 2326, 2335, 2345, 2353, 2358, 2398, 2400 }

B grade: { 48, 61, 74, 114, 115, 128, 172, 203, 338, 450, 453, 478, 492, 636, 689, 700, 701, 702, 704, 756, 775, 776, 810, 819, 852, 854, 857, 860, 866, 869, 873, 902, 903, 904, 910, 912, 933, 939, 941, 944, 945, 946, 967, 976, 982, 989, 990, 1000, 1008, 1095, 1103, 1104, 1121, 1128, 1132, 1135, 1163, 1164, 1188, 1202, 1203, 1213, 1214, 1225, 1254, 1279, 1289, 1290, 1291, 1342, 1346, 1365, 1381, 1382, 1383, 1387, 1401, 1404, 1417, 1418, 1434, 1444, 1467, 1469, 1470, 1490, 1507, 1511, 1512, 1522, 1525, 1532, 1552, 1587, 1601, 1616, 1627, 1663, 1680, 1713, 1718, 1742, 1783, 1801, 1835, 1880, 1936, 1953, 1962, 1972, 1984, 1986, 1989, 2014, 2017, 2064, 2231, 2256, 2303, 2337, 2346 }

C grade: { 601, 846, 1570, 1610 }

F grade: { 26, 31, 43, 47, 49, 51, 53, 56, 57, 58, 60, 62, 63, 64, 67, 68, 70, 71, 72, 73, 79, 80, 82, 85, 86, 87, 91, 94, 95, 97, 98, 99, 101, 104, 105, 107, 108, 109, 110, 111, 112, 113, 116, 117, 118, 119, 121, 125, 135, 138, 139, 142, 145, 146, 147, 148, 149, 150, 151, 154, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 175, 180, 181, 182, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 206, 208, 214, 215, 216, 217, 218, 219, 223, 229, 230, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 262, 269, 270, 271, 272, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 306, 308, 309, 310, 312, 313, 315, 316, 318, 319, 323, 324, 325, 326, 329, 330, 334, 335, 336, 337, 339, 340, 341, 342, 343, 344, 345, 349, 351, 354, 355, 360, 361, 366, 367, 368, 369, 370, 371, 373, 374, 377, 379, 387, 388, 390, 394, 395, 396, 397, 398, 399, 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717, 719, 720, 723, 724, 725, 726, 727, 728, 729, 730, 731, 733, 734, 735, 736, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 772, 773, 774, 777, 778, 779, 781, 782, 783, 785, 786, 788, 789, 790, 793, 795, 797, 798, 799, 800, 801, 802, 803, 804, 805, 807, 808, 811, 812, 813, 814, 815, 818, 820, 821, 822, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 838, 840, 841, 843, 844, 847, 848, 849, 850, 851, 853, 855, 856, 858, 859, 863, 864, 865, 870, 871, 872, 874, 875, 878, 880, 882, 883, 884, 886, 887, 888, 889, 890, 893, 894, 895, 897, 898, 906, 907, 908, 909, 911, 913, 914, 915, 917, 918, 919, 920, 921, 923, 924, 925, 926, 927, 928, 929, 931, 932, 934, 937, 940, 942, 943, 950, 951, 952, 953, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 968, 970, 971, 972, 974, 975, 977, 978, 979, 980, 981, 983, 984, 985, 986, 987, 988, 991, 992, 994, 995, 996, 998, 999, 1004, 1005, 1006, 1009, 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1256, 1257, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1280, 1281, 1283, 1286, 1287, 1288, 1292, 1293, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1329, 1330, 1331, 1332, 1333, 1335, 1336, 1338, 1341, 1343, 1344, 1345, 1347, 1348, 1349, 1351, 1352, 1353, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1363, 1364, 1366, 1367, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1377, 1378, 1380, 1384, 1385, 1386, 1391, 1392, 1393, 1394, 1395, 1396, 1400, 1402, 1403, 1405, 1406, 1407, 1410, 1413, 1414, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1430, 1431, 1432, 1433, 1435, 1436, 1437, 1438, 1441, 1442, 1445, 1446, 1447, 1448, 1449, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1461, 1462, 1463, 1464, 1465, 1466, 1468, 1472, 1473, 1477, 1478, 1479, 1480, 1481, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1491, 1492, 1493, 1495, 1496, 1497, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1509, 1513, 1515, 1518, 1519, 1520, 1521, 1523, 1526, 1527, 1528, 1529, 1531, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1553, 1554, 1556, 1557, 1558, 1561, 1566, 1567, 1568, 1569, 1572, 1574, 1575, 1576, 1577, 1578, 1579, 1581, 1583, 1584, 1585, 1586, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1602, 1603, 1604, 1605, 1606, 1607, 1608, 1609, 1611, 1613, 1614, 1615, 1617, 1618, 1619, 1620, 1621, 1623, 1624, 1625, 1626, 1628, 1629, 1632, 1633, 1636, 1638, 1639, 1642, 1643, 1644, 1646, 1647, 1648, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1664, 1665, 1666, 1667, 1669, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1681, 1682, 1683, 1684, 1685, 1687, 1688, 1691, 1693, 1695, 1696, 1699, 1701, 1702, 1703, 1704, 1705, 1707, 1708, 1709, 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1915, 1916, 1917, 1918, 1919, 1920, 1921, 1922, 1923, 1924, 1925, 1926, 1927, 1928, 1929, 1930, 1931, 1932, 1933, 1934, 1935, 1937, 1938, 1939, 1940, 1941, 1942, 1943, 1944, 1945, 1947, 1948, 1949, 1950, 1952, 1954, 1955, 1957, 1958, 1959, 1960, 1961, 1963, 1964, 1965, 1966, 1967, 1968, 1970, 1971, 1973, 1974, 1975, 1976, 1977, 1978, 1979, 1981, 1982, 1983, 1985, 1987, 1988, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2015, 2016, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2055, 2056, 2057, 2058, 2061, 2062, 2063, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2074, 2075, 2077, 2078, 2079, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2092, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2116, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2149, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2187, 2188, 2189, 2190, 2191, 2193, 2194, 2195, 2196, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2218, 2220, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2232, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2258, 2259, 2260, 2261, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2273, 2274, 2275, 2276, 2277, 2278, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2304, 2305, 2306, 2308, 2309, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2336, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2347, 2348, 2349, 2350, 2351, 2352, 2354, 2355, 2356, 2357, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2399, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 154, 155, 156, 157, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 191, 192, 193, 194, 197, 198, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 220, 221, 222, 223, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 281, 282, 283, 284, 285, 286, 287, 291, 292, 294, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 340, 342, 343, 346, 347, 348, 350, 353, 356, 357, 358, 359, 360, 361, 362, 363, 365, 366, 367, 368, 369, 372, 373, 374, 375, 376, 377, 378, 383, 384, 385, 386, 387, 389, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 405, 406, 410, 412, 413, 414, 415, 416, 417, 427, 428, 429, 430, 431, 432, 433, 435, 437, 445, 449, 450, 451, 453, 458, 459, 460, 463, 464, 465, 466, 467, 468, 471, 472, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 498, 499, 500, 502, 503, 504, 509, 513, 514, 515, 516, 517, 518, 519, 520, 522, 523, 524, 527, 528, 529, 532, 533, 534, 537, 538, 541, 543, 544, 545, 548, 549, 551, 555, 559, 566, 567, 568, 569, 570, 572, 577, 578, 579, 581, 582, 583, 592, 593, 597, 600, 601, 608, 610, 618, 623, 624, 626, 627, 628, 631, 636, 638, 645, 646, 647, 648, 650, 652, 653, 657, 658, 662, 664, 671, 673, 685, 688, 689, 690, 691, 692, 697, 700, 701, 702, 703, 704, 707, 708, 711, 713, 715, 718, 719, 728, 729, 730, 735, 737, 738, 741, 747, 749, 750, 751, 752, 753, 754, 755, 756, 757, 760, 762, 766, 768, 770, 771, 778, 785, 787, 788, 792, 796, 800, 806, 809, 810, 811, 816, 817, 819, 821, 842, 844, 852, 853, 854, 857, 860, 866, 868, 872, 873, 881, 882, 883, 886, 887, 891, 892, 893, 895, 899, 900, 902, 904, 908, 910, 912, 915, 916, 918, 920, 921, 922, 924, 925, 927, 928, 930, 931, 933, 935, 936, 939, 940, 941, 944, 945, 946, 947, 949, 950, 952, 964, 967, 969, 979, 980, 982, 993, 1000, 1006, 1007, 1011, 1012, 1014, 1015, 1019, 1033, 1037, 1044, 1045, 1046, 1048, 1053, 1055, 1056, 1059, 1061, 1064, 1072, 1074, 1076, 1077, 1080, 1081, 1082, 1083, 1086, 1087, 1089, 1090, 1091, 1092, 1095, 1096, 1105, 1108, 1109, 1110, 1115, 1121, 1124, 1125, 1126, 1128, 1130, 1131, 1132, 1134, 1135, 1136, 1137, 1140, 1142, 1145, 1150, 1152, 1155, 1156, 1158, 1162, 1163, 1164, 1171, 1189, 1190, 1195, 1196, 1197, 1198, 1201, 1202, 1203, 1204, 1213, 1214, 1217, 1218, 1219, 1220, 1221, 1222, 1225, 1226, 1232, 1233, 1242, 1244, 1245, 1246, 1248, 1254, 1268, 1274, 1275, 1276, 1282, 1284, 1285, 1289, 1290, 1292, 1294, 1302, 1311, 1339, 1342, 1346, 1350, 1354, 1355, 1357, 1362, 1381, 1382, 1383, 1390, 1396, 1397, 1398, 1413, 1444, 1449, 1450, 1467, 1469, 1470, 1475, 1490, 1504, 1507, 1514, 1516, 1517, 1523, 1528, 1530, 1542, 1544, 1559, 1560, 1562, 1563, 1564, 1565, 1572, 1582, 1583, 1598, 1619, 1622, 1634, 1636, 1641, 1647, 1671, 1689, 1690, 1692, 1694, 1697, 1713, 1722, 1725, 1726, 1728, 1729, 1734, 1752, 1754, 1755, 1756, 1758, 1759, 1785, 1790, 1800, 1834, 1836, 1839, 1840, 1843, 1845, 1847, 1851, 1891, 1895, 1912, 1953, 1962, 1963, 1964, 1965, 1980, 1992, 2013, 2073, 2081, 2094, 2156, 2164, 2168, 2176, 2177, 2185, 2194, 2205, 2217, 2303, 2337, 2346 }

C grade: { }

F grade: { 63, 64, 68, 113, 116, 117, 118, 119, 137, 145, 152, 153, 158, 161, 162, 189, 190, 195, 196, 199, 200, 206, 218, 219, 228, 238, 241, 242, 248, 249, 265, 266, 267, 278, 280, 288, 289, 290, 293, 295, 297, 323, 327, 339, 341, 344, 345, 349, 351, 352, 354, 355, 364, 370, 371, 379, 380, 381, 382, 388, 390, 392, 404, 407, 408, 409, 411, 418, 419, 420, 421, 422, 423, 424, 425, 426, 434, 436, 438, 439, 440, 441, 442, 443, 444, 446, 447, 448, 452, 454, 455, 456, 457, 461, 462, 469, 470, 473, 474, 497, 501, 505, 506, 507, 508, 510, 511, 512, 521, 525, 526, 530, 531, 535, 536, 539, 540, 542, 546, 547, 550, 552, 553, 554, 556, 557, 558, 560, 561, 562, 563, 564, 565, 571, 573, 574, 575, 576, 580, 584, 585, 586, 587, 588, 589, 590, 591, 594, 595, 596, 598, 599, 602, 603, 604, 605, 606, 607, 609, 611, 612, 613, 614, 615, 616, 617, 619, 620, 621, 622, 625, 629, 630, 632, 633, 634, 635, 637, 639, 640, 641, 642, 643, 644, 649, 651, 654, 655, 656, 659, 660, 661, 663, 665, 666, 667, 668, 669, 670, 672, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 686, 687, 693, 694, 695, 696, 698, 699, 705, 706, 709, 710, 712, 714, 716, 717, 720, 721, 722, 723, 724, 725, 726, 727, 731, 732, 733, 734, 736, 739, 740, 742, 743, 744, 745, 746, 748, 758, 759, 761, 763, 764, 765, 767, 769, 772, 773, 774, 775, 776, 777, 779, 780, 781, 782, 783, 784, 786, 789, 790, 791, 793, 794, 795, 797, 798, 799, 801, 802, 803, 804, 805, 807, 808, 812, 813, 814, 815, 818, 820, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 843, 845, 846, 847, 848, 849, 850, 851, 855, 856, 858, 859, 861, 862, 863, 864, 865, 867, 869, 870, 871, 874, 875, 876, 877, 878, 879, 880, 884, 885, 888, 889, 890, 894, 896, 897, 898, 901, 903, 905, 906, 907, 909, 911, 913, 914, 917, 919, 923, 926, 929, 932, 934, 937, 938, 942, 943, 948, 951, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 965, 966, 968, 970, 971, 972, 973, 974, 975, 976, 977, 978, 981, 983, 984, 985, 986, 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2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443 }

2.1.9 IntegrateAlgebraic

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 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2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2215, 2217, 2218, 2219, 2220, 2221, 2222, 2224, 2225, 2226, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2264, 2265, 2266, 2268, 2269, 2270, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2441, 2442 }

B grade: { 2303, 2337, 2443 }

C grade: { 815, 822, 1029, 1303, 1746, 1788, 1863, 2137, 2223, 2227, 2267, 2271, 2365, 2366, 2367, 2379, 2390, 2391, 2392, 2393 }

F grade: { 1455, 1472, 1712, 1855, 1943, 1968, 2105, 2214, 2216, 2263, 2347, 2389, 2413, 2429, 2438, 2439, 2440 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	16	9	9	8	9	9	11
N.S.	1	1.00	1.00	1.45	0.82	0.82	0.73	0.82	0.82	1.00
time (sec)	N/A	0.018	0.011	0.028	0.718	1.837	0.174	0.487	0.405	0.011

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	0	10	10	10	12
N.S.	1	1.00	1.00	0.92	0.83	0.00	0.83	0.83	0.83	1.00
time (sec)	N/A	0.058	0.048	0.006	0.607	0.000	0.136	0.405	0.096	0.015

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	27	10	0	10	10	12
N.S.	1	1.00	1.00	0.92	2.25	0.83	0.00	0.83	0.83	1.00
time (sec)	N/A	0.009	0.006	0.008	1.395	0.460	0.000	0.554	0.165	0.227

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	16	9	9	10	9	9	13
N.S.	1	1.00	1.00	1.23	0.69	0.69	0.77	0.69	0.69	1.00
time (sec)	N/A	0.002	0.002	0.003	0.448	0.392	0.124	0.634	0.111	0.012

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	16	9	9	10	9	9	13
N.S.	1	1.00	1.00	1.23	0.69	0.69	0.77	0.69	0.69	1.00
time (sec)	N/A	0.002	0.002	0.003	0.427	0.393	0.147	0.419	0.103	0.011

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	16	9	9	26	9	9	13
N.S.	1	1.00	1.00	1.23	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.002	0.002	0.004	0.486	0.381	0.168	0.454	0.078	0.010

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	16	9	9	26	9	9	13
N.S.	1	1.00	1.00	1.23	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.002	0.002	0.003	0.642	0.389	0.138	0.440	0.078	0.012

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	16	9	9	26	9	9	13
N.S.	1	1.00	1.00	1.23	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.002	0.002	0.003	0.622	0.378	0.216	0.282	0.079	0.012

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	16	9	9	26	9	9	13
N.S.	1	1.00	1.00	1.23	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.002	0.002	0.003	0.666	0.385	0.456	0.304	0.080	0.012

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	10	9	9	10	9	9	13
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.77	0.69	0.69	1.00
time (sec)	N/A	0.002	0.002	0.003	0.837	0.376	0.125	0.407	0.084	0.011

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	10	9	9	10	9	9	13
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.77	0.69	0.69	1.00
time (sec)	N/A	0.002	0.002	0.003	0.379	0.381	0.146	0.259	0.090	0.013

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	10	9	9	26	9	9	13
N.S.	1	1.00	1.00	0.77	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.002	0.002	0.002	0.453	0.392	0.167	0.328	0.077	0.012

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	10	9	9	26	9	9	13
N.S.	1	1.00	1.00	0.77	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.002	0.002	0.003	0.617	0.387	0.136	0.663	0.048	0.011

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	10	9	9	26	9	9	13
N.S.	1	1.00	1.00	0.77	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.002	0.002	0.001	0.347	0.386	0.458	0.361	0.049	0.012

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	19	9	9	10	9	9	13
N.S.	1	1.00	1.00	1.46	0.69	0.69	0.77	0.69	0.69	1.00
time (sec)	N/A	0.003	0.002	0.005	0.682	0.377	0.149	0.381	0.242	0.015

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	19	9	9	26	9	9	13
N.S.	1	1.00	1.00	1.46	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.002	0.003	0.006	0.431	0.380	0.211	0.359	0.127	0.012

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	19	9	9	26	9	9	13
N.S.	1	1.00	1.00	1.46	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.003	0.003	0.005	0.402	0.380	0.577	0.436	0.129	0.017

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	21	9	9	10	9	9	13
N.S.	1	1.00	1.00	1.62	0.69	0.69	0.77	0.69	0.69	1.00
time (sec)	N/A	0.002	0.002	0.006	0.710	0.387	0.126	0.748	0.069	0.001

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	21	9	9	10	9	9	13
N.S.	1	1.00	1.00	1.62	0.69	0.69	0.77	0.69	0.69	1.00
time (sec)	N/A	0.003	0.002	0.006	0.580	0.375	0.146	0.361	0.174	0.014

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	21	9	9	26	9	9	13
N.S.	1	1.00	1.00	1.62	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.002	0.003	0.003	0.648	0.376	0.206	0.355	0.122	0.015

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	21	9	9	22	9	9	13
N.S.	1	1.00	1.00	1.62	0.69	0.69	1.69	0.69	0.69	1.00
time (sec)	N/A	0.003	0.003	0.006	0.780	0.372	0.162	0.444	0.134	0.012

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	21	9	9	26	9	9	13
N.S.	1	1.00	1.00	1.62	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.002	0.003	0.004	0.701	0.390	0.139	0.438	0.022	0.014

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	21	9	9	22	9	9	13
N.S.	1	1.00	1.00	1.62	0.69	0.69	1.69	0.69	0.69	1.00
time (sec)	N/A	0.003	0.003	0.006	0.328	0.384	0.273	0.588	0.128	0.015

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	21	9	9	26	9	9	13
N.S.	1	1.00	1.00	1.62	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.003	0.003	0.004	0.460	0.374	0.568	0.502	0.121	0.016

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	19	16	9	9	27	9	9	13
N.S.	1	1.00	1.46	1.23	0.69	0.69	2.08	0.69	0.69	1.00
time (sec)	N/A	0.007	0.033	0.004	0.368	0.398	0.174	0.294	0.092	0.017

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	23	11	11	61	0	11	13
N.S.	1	1.00	1.00	1.77	0.85	0.85	4.69	0.00	0.85	1.00
time (sec)	N/A	0.004	0.002	0.004	0.472	0.391	0.582	0.000	0.209	0.167

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	21	9	9	10	9	9	13
N.S.	1	1.00	1.00	1.62	0.69	0.69	0.77	0.69	0.69	1.00
time (sec)	N/A	0.002	0.002	0.007	0.425	0.387	0.146	0.694	0.168	0.015

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	21	9	9	26	9	9	13
N.S.	1	1.00	1.00	1.62	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.003	0.003	0.007	0.450	0.374	0.211	0.424	0.126	0.014

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	21	9	9	26	9	9	13
N.S.	1	1.00	1.00	1.62	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.003	0.003	0.006	0.492	0.387	0.361	0.277	0.127	0.016

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	21	9	9	22	9	9	13
N.S.	1	1.00	1.00	1.62	0.69	0.69	1.69	0.69	0.69	1.00
time (sec)	N/A	0.003	0.003	0.004	0.323	0.389	0.727	0.294	0.131	0.017

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	11	11	61	0	11	13
N.S.	1	1.00	1.00	0.92	0.85	0.85	4.69	0.00	0.85	1.00
time (sec)	N/A	0.006	0.005	0.005	0.518	0.388	1.515	0.000	0.047	0.623

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	10	9	9	10	9	9	13
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.77	0.69	0.69	1.00
time (sec)	N/A	0.002	0.002	0.005	0.416	0.384	0.147	0.606	0.108	0.001

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	10	9	9	22	9	9	13
N.S.	1	1.00	1.00	0.77	0.69	0.69	1.69	0.69	0.69	1.00
time (sec)	N/A	0.003	0.002	0.004	0.324	0.377	0.260	0.278	0.134	0.014

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	10	9	9	26	9	9	13
N.S.	1	1.00	1.00	0.77	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.002	0.003	0.005	0.320	0.376	0.208	0.373	0.127	0.014

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	10	9	9	26	9	9	13
N.S.	1	1.00	1.00	0.77	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.003	0.003	0.003	0.335	0.389	0.361	0.407	0.131	0.017

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	25	9	9	26	9	9	13
N.S.	1	1.00	1.00	1.92	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.003	0.003	0.006	0.331	0.386	0.424	0.439	0.164	0.017

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	29	9	9	26	9	9	13
N.S.	1	1.00	1.00	2.23	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.003	0.003	0.005	0.339	0.388	0.423	0.425	0.145	0.019

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	30	9	9	8	9	9	13
N.S.	1	1.00	1.00	2.31	0.69	0.69	0.62	0.69	0.69	1.00
time (sec)	N/A	0.003	0.002	0.007	0.325	0.380	0.212	0.277	0.226	0.016

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	30	9	9	26	9	9	13
N.S.	1	1.00	1.00	2.31	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.003	0.003	0.008	0.349	0.383	0.425	0.377	0.140	0.015

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	30	9	9	22	9	9	13
N.S.	1	1.00	1.00	2.31	0.69	0.69	1.69	0.69	0.69	1.00
time (sec)	N/A	0.003	0.003	0.006	0.343	0.399	0.347	0.320	0.136	0.013

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	30	9	9	22	9	9	13
N.S.	1	1.00	1.00	2.31	0.69	0.69	1.69	0.69	0.69	1.00
time (sec)	N/A	0.002	0.003	0.007	0.318	0.385	0.268	0.511	0.188	0.014

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	30	9	9	26	9	9	13
N.S.	1	1.00	1.00	2.31	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.003	0.003	0.007	0.315	0.394	1.124	0.417	0.132	0.018

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	27	23	11	63	0	11	13
N.S.	1	1.00	1.00	2.08	1.77	0.85	4.85	0.00	0.85	1.00
time (sec)	N/A	0.006	0.006	0.007	0.486	0.384	2.070	0.000	0.095	2.038

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	25	9	9	8	9	9	13
N.S.	1	1.00	1.00	1.92	0.69	0.69	0.62	0.69	0.69	1.00
time (sec)	N/A	0.002	0.002	0.004	0.406	0.391	0.215	0.343	0.080	0.014

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	25	9	9	26	9	9	13
N.S.	1	1.00	1.00	1.92	0.69	0.69	2.00	0.69	0.69	1.00
time (sec)	N/A	0.002	0.003	0.007	0.318	0.411	0.423	0.315	0.130	0.014

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	25	9	9	22	9	9	13
N.S.	1	1.00	1.00	1.92	0.69	0.69	1.69	0.69	0.69	1.00
time (sec)	N/A	0.002	0.002	0.005	0.318	0.393	0.337	0.354	0.129	0.012

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	22	18	12	68	0	12	14
N.S.	1	1.00	1.00	1.57	1.29	0.86	4.86	0.00	0.86	1.00
time (sec)	N/A	0.006	0.005	0.004	0.591	0.435	1.693	0.000	0.119	2.444

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	11	25	25	12	26	164	14
N.S.	1	1.00	1.00	0.79	1.79	1.79	0.86	1.86	11.71	1.00
time (sec)	N/A	0.008	0.002	0.293	0.491	0.398	0.783	0.337	0.254	0.001

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	24	20	12	63	0	12	14
N.S.	1	1.00	1.00	1.71	1.43	0.86	4.50	0.00	0.86	1.00
time (sec)	N/A	0.006	0.005	0.006	0.732	0.393	1.602	0.000	0.073	2.423

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	11	0	10	0	12	10	14
N.S.	1	1.00	1.00	0.79	0.00	0.71	0.00	0.86	0.71	1.00
time (sec)	N/A	0.084	0.038	0.006	0.000	0.407	0.000	0.420	0.186	0.157

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	16	14	15	0	22	0	0	21	14
N.S.	1	1.14	1.00	1.07	0.00	1.57	0.00	0.00	1.50	1.00
time (sec)	N/A	0.063	0.009	0.003	0.000	0.391	0.000	0.000	0.163	6.973

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	0	18	0	11	19	14
N.S.	1	1.00	1.00	0.93	0.00	1.29	0.00	0.79	1.36	1.00
time (sec)	N/A	0.060	0.008	0.005	0.000	0.391	0.000	0.446	0.126	7.012

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	12	22	0	12	14
N.S.	1	1.00	1.00	0.93	0.86	0.86	1.57	0.00	0.86	1.00
time (sec)	N/A	0.003	0.003	0.003	0.428	0.407	0.555	0.000	0.153	0.171

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	12	63	12	12	14
N.S.	1	1.00	1.00	0.93	0.86	0.86	4.50	0.86	0.86	1.00
time (sec)	N/A	0.005	0.005	0.005	0.572	0.388	1.546	0.573	0.083	0.703

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	12	12	12	12	14
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.86	0.86	0.86	1.00
time (sec)	N/A	0.010	0.007	0.008	0.444	0.388	0.164	0.439	0.202	0.015

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	32	28	12	63	0	12	14
N.S.	1	1.00	1.00	2.29	2.00	0.86	4.50	0.00	0.86	1.00
time (sec)	N/A	0.007	0.006	0.006	0.528	0.398	1.609	0.000	0.097	0.224

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	28	24	12	68	0	12	14
N.S.	1	1.00	1.00	2.00	1.71	0.86	4.86	0.00	0.86	1.00
time (sec)	N/A	0.006	0.006	0.006	0.658	0.395	1.693	0.000	0.129	0.219

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	14	0	2691	197	0	25	0	0	163	14
N.S.	1	0.00	192.21	14.07	0.00	1.79	0.00	0.00	11.64	1.00
time (sec)	N/A	0.544	4.920	0.155	0.000	0.509	0.000	0.000	1.530	1.820

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	7	10	10	24	10	10	14
N.S.	1	1.00	1.00	0.50	0.71	0.71	1.71	0.71	0.71	1.00
time (sec)	N/A	0.007	0.002	0.008	0.597	0.411	0.816	0.459	0.179	0.020

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	28	24	12	61	0	12	14
N.S.	1	1.00	1.00	2.00	1.71	0.86	4.36	0.00	0.86	1.00
time (sec)	N/A	0.006	0.006	0.006	0.783	0.387	1.775	0.000	0.107	0.318

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	19	25	25	8	25	10	14
N.S.	1	1.00	1.00	1.36	1.79	1.79	0.57	1.79	0.71	1.00
time (sec)	N/A	0.007	0.002	0.006	0.507	0.391	0.784	0.312	0.116	0.021

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	33	33	12	66	0	12	14
N.S.	1	1.00	1.00	2.36	2.36	0.86	4.71	0.00	0.86	1.00
time (sec)	N/A	0.007	0.006	0.008	0.503	0.405	1.854	0.000	0.141	0.328

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	B	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	14	0	0	36	0	34	0	0	-1	14
N.S.	1	0.00	0.00	2.57	0.00	2.43	0.00	0.00	-0.07	1.00
time (sec)	N/A	0.558	0.141	0.401	0.000	0.495	0.000	0.000	0.000	3.705

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	14	0	0	62	0	25	0	0	-1	14
N.S.	1	0.00	0.00	4.43	0.00	1.79	0.00	0.00	-0.07	1.00
time (sec)	N/A	0.519	0.135	0.487	0.000	0.580	0.000	0.000	0.000	3.839

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	19	11	11	10	11	11	15
N.S.	1	1.00	1.00	1.27	0.73	0.73	0.67	0.73	0.73	1.00
time (sec)	N/A	0.008	0.024	0.004	0.480	0.391	0.141	0.286	0.294	0.020

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	19	11	11	27	11	11	15
N.S.	1	1.00	1.00	1.27	0.73	0.73	1.80	0.73	0.73	1.00
time (sec)	N/A	0.007	0.031	0.004	0.364	0.381	0.180	0.312	0.114	0.019

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	15	0	3866	1905	0	35	0	0	2490	15
N.S.	1	0.00	257.73	127.00	0.00	2.33	0.00	0.00	166.00	1.00
time (sec)	N/A	2.081	3.523	2.613	0.000	0.416	0.000	0.000	1.883	0.299

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	15	15	1547	265	0	34	0	0	-1	15
N.S.	1	1.00	103.13	17.67	0.00	2.27	0.00	0.00	-0.07	1.00
time (sec)	N/A	0.087	3.505	0.177	0.000	0.445	0.000	0.000	0.000	0.272

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	22	12	12	68	12	12	16
N.S.	1	1.00	1.00	1.38	0.75	0.75	4.25	0.75	0.75	1.00
time (sec)	N/A	0.010	0.010	0.006	0.682	0.381	14.056	0.356	0.159	0.132

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	22	12	12	129	0	25	16
N.S.	1	1.00	1.00	1.38	0.75	0.75	8.06	0.00	1.56	1.00
time (sec)	N/A	0.003	0.003	0.004	0.430	0.384	0.690	0.000	0.191	0.086

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	22	12	12	126	0	25	16
N.S.	1	1.00	1.00	1.38	0.75	0.75	7.88	0.00	1.56	1.00
time (sec)	N/A	0.003	0.003	0.003	0.403	0.414	0.769	0.000	0.191	0.084

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	24	12	12	53	0	24	16
N.S.	1	1.00	1.00	1.50	0.75	0.75	3.31	0.00	1.50	1.00
time (sec)	N/A	0.003	0.002	0.004	0.549	0.406	0.599	0.000	0.187	0.089

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	24	12	12	53	0	24	16
N.S.	1	1.00	1.00	1.50	0.75	0.75	3.31	0.00	1.50	1.00
time (sec)	N/A	0.003	0.003	0.004	0.517	0.392	0.674	0.000	0.185	0.091

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	24	30	22	105	27	12	16
N.S.	1	1.00	1.00	1.50	1.88	1.38	6.56	1.69	0.75	1.00
time (sec)	N/A	0.007	0.008	0.007	0.511	0.407	2.892	0.497	0.037	0.076

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	24	12	12	49	12	12	16
N.S.	1	1.00	1.00	1.50	0.75	0.75	3.06	0.75	0.75	1.00
time (sec)	N/A	0.010	0.009	0.004	0.565	0.397	46.795	0.306	0.150	0.040

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	18	17	12	0	9	12	16
N.S.	1	1.00	1.00	1.12	1.06	0.75	0.00	0.56	0.75	1.00
time (sec)	N/A	0.020	0.008	0.005	0.503	0.388	0.000	0.505	0.147	0.172

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	33	21	20	0	17	0	11	27	16
N.S.	1	2.06	1.31	1.25	0.00	1.06	0.00	0.69	1.69	1.00
time (sec)	N/A	0.259	0.015	0.005	0.000	0.399	0.000	0.415	0.349	0.399

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	21	18	17	17	0	9	27	16
N.S.	1	1.00	1.31	1.12	1.06	1.06	0.00	0.56	1.69	1.00
time (sec)	N/A	0.017	0.004	0.004	0.507	0.399	0.000	0.517	0.162	0.173

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	24	12	12	63	0	12	16
N.S.	1	1.00	1.00	1.50	0.75	0.75	3.94	0.00	0.75	1.00
time (sec)	N/A	0.003	0.002	0.004	0.364	0.380	0.621	0.000	0.217	0.148

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	24	12	12	126	0	25	16
N.S.	1	1.00	1.00	1.50	0.75	0.75	7.88	0.00	1.56	1.00
time (sec)	N/A	0.003	0.003	0.005	0.312	0.394	0.980	0.000	0.206	0.133

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	A	F	A	F	A	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	78	77	20	0	17	0	21	9	16
N.S.	1	4.88	4.81	1.25	0.00	1.06	0.00	1.31	0.56	1.00
time (sec)	N/A	0.090	0.060	0.005	0.000	0.403	0.000	0.411	0.026	0.189

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	12	22	0	12	16
N.S.	1	1.00	1.00	0.81	0.75	0.75	1.38	0.00	0.75	1.00
time (sec)	N/A	0.003	0.003	0.003	0.418	0.381	0.575	0.000	0.163	0.134

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	22	12	71	22	12	16
N.S.	1	1.00	1.00	0.81	1.38	0.75	4.44	1.38	0.75	1.00
time (sec)	N/A	0.009	0.009	0.005	0.463	0.408	178.661	0.271	0.099	0.047

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	40	13	12	12	65	23	12	16
N.S.	1	1.00	2.50	0.81	0.75	0.75	4.06	1.44	0.75	1.00
time (sec)	N/A	0.019	0.017	0.006	0.545	0.389	2.051	0.502	0.184	0.155

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	12	73	0	27	16
N.S.	1	1.00	1.00	0.81	0.75	0.75	4.56	0.00	1.69	1.00
time (sec)	N/A	0.006	0.006	0.006	0.625	0.409	2.298	0.000	0.191	0.102

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	12	53	0	24	16
N.S.	1	1.00	1.00	0.81	0.75	0.75	3.31	0.00	1.50	1.00
time (sec)	N/A	0.003	0.003	0.003	0.496	0.380	0.905	0.000	0.208	0.134

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	24	27	12	78	0	27	16
N.S.	1	1.00	1.00	1.50	1.69	0.75	4.88	0.00	1.69	1.00
time (sec)	N/A	0.006	0.007	0.007	0.602	0.399	2.403	0.000	0.194	0.099

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	50	12	0	12	12	16
N.S.	1	1.00	1.00	0.81	3.12	0.75	0.00	0.75	0.75	1.00
time (sec)	N/A	0.009	0.008	0.005	0.592	0.379	0.000	0.545	0.226	0.048

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	24	25	12	0	9	12	16
N.S.	1	1.00	1.00	1.50	1.56	0.75	0.00	0.56	0.75	1.00
time (sec)	N/A	0.019	0.007	0.005	0.674	0.415	0.000	0.393	0.175	0.387

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	B	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	33	21	29	58	17	0	9	27	16
N.S.	1	2.06	1.31	1.81	3.62	1.06	0.00	0.56	1.69	1.00
time (sec)	N/A	0.111	0.007	0.006	0.439	0.407	0.000	0.472	0.267	0.261

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	21	61	18	0	17	0	0	19	16
N.S.	1	1.31	3.81	1.12	0.00	1.06	0.00	0.00	1.19	1.00
time (sec)	N/A	0.061	0.043	0.005	0.000	0.390	0.000	0.000	0.133	0.197

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	17	14	0	9	14	16
N.S.	1	1.00	1.00	0.94	1.06	0.88	0.00	0.56	0.88	1.00
time (sec)	N/A	0.032	0.006	0.006	0.668	0.466	0.000	0.342	0.126	0.104

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	28	50	12	71	12	12	16
N.S.	1	1.00	1.00	1.75	3.12	0.75	4.44	0.75	0.75	1.00
time (sec)	N/A	0.010	0.009	0.008	0.663	0.401	4.158	0.431	0.267	0.087

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	32	33	12	75	0	27	16
N.S.	1	1.00	1.00	2.00	2.06	0.75	4.69	0.00	1.69	1.00
time (sec)	N/A	0.006	0.006	0.008	0.895	0.427	2.687	0.000	0.235	0.137

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	28	29	12	80	0	27	16
N.S.	1	1.00	1.00	1.75	1.81	0.75	5.00	0.00	1.69	1.00
time (sec)	N/A	0.006	0.009	0.007	0.654	0.413	2.769	0.000	0.221	0.140

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	32	50	12	70	12	12	16
N.S.	1	1.00	1.00	2.00	3.12	0.75	4.38	0.75	0.75	1.00
time (sec)	N/A	0.010	0.007	0.007	0.574	0.388	3.951	0.741	0.248	0.077

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	33	21	20	0	17	0	0	27	16
N.S.	1	2.06	1.31	1.25	0.00	1.06	0.00	0.00	1.69	1.00
time (sec)	N/A	0.232	0.011	0.006	0.000	0.412	0.000	0.000	0.293	0.267

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	0	14	0	0	14	16
N.S.	1	1.00	1.00	0.94	0.00	0.88	0.00	0.00	0.88	1.00
time (sec)	N/A	0.141	0.009	0.006	0.000	0.436	0.000	0.000	0.188	0.095

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	16	0	0	0	0	35	0	0	42	16
N.S.	1	0.00	0.00	0.00	0.00	2.19	0.00	0.00	2.62	1.00
time (sec)	N/A	0.835	0.262	0.204	0.000	0.509	0.000	0.000	2.261	4.307

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	33	12	16	29	20	12	16
N.S.	1	1.00	1.00	2.06	0.75	1.00	1.81	1.25	0.75	1.00
time (sec)	N/A	0.003	0.002	0.006	0.453	0.407	0.576	0.356	0.297	0.160

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	33	12	12	129	0	12	16
N.S.	1	1.00	1.00	2.06	0.75	0.75	8.06	0.00	0.75	1.00
time (sec)	N/A	0.003	0.002	0.018	0.453	0.424	0.871	0.000	0.235	0.624

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	33	12	16	61	20	12	16
N.S.	1	1.00	1.00	2.06	0.75	1.00	3.81	1.25	0.75	1.00
time (sec)	N/A	0.003	0.002	0.006	0.534	0.403	0.868	0.617	0.284	0.135

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	28	12	16	12	18	12	16
N.S.	1	1.00	1.00	1.75	0.75	1.00	0.75	1.12	0.75	1.00
time (sec)	N/A	0.003	0.002	0.006	0.375	0.387	0.547	0.370	0.218	0.148

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	28	12	12	22	0	12	16
N.S.	1	1.00	1.00	1.75	0.75	0.75	1.38	0.00	0.75	1.00
time (sec)	N/A	0.003	0.003	0.005	0.374	0.420	0.583	0.000	0.181	0.771

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	28	29	12	42	0	12	16
N.S.	1	1.00	1.00	1.75	1.81	0.75	2.62	0.00	0.75	1.00
time (sec)	N/A	0.006	0.009	0.007	0.549	0.411	2.586	0.000	0.043	0.172

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	28	12	16	27	18	12	16
N.S.	1	1.00	1.00	1.75	0.75	1.00	1.69	1.12	0.75	1.00
time (sec)	N/A	0.003	0.003	0.006	0.354	0.448	0.793	0.447	0.278	0.144

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	28	29	12	71	0	27	16
N.S.	1	1.00	1.00	1.75	1.81	0.75	4.44	0.00	1.69	1.00
time (sec)	N/A	0.006	0.006	0.006	0.542	0.419	3.052	0.000	0.225	0.173

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	33	38	12	71	0	12	16
N.S.	1	1.00	1.00	2.06	2.38	0.75	4.44	0.00	0.75	1.00
time (sec)	N/A	0.005	0.005	0.007	0.490	0.401	3.007	0.000	0.205	0.151

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	33	38	12	48	0	12	16
N.S.	1	1.00	1.00	2.06	2.38	0.75	3.00	0.00	0.75	1.00
time (sec)	N/A	0.006	0.007	0.009	0.553	0.398	2.618	0.000	0.151	0.167

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	33	38	12	76	0	27	16
N.S.	1	1.00	1.00	2.06	2.38	0.75	4.75	0.00	1.69	1.00
time (sec)	N/A	0.007	0.005	0.007	0.575	0.397	3.108	0.000	0.221	0.171

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	32	0	12	0	0	12	16
N.S.	1	1.00	1.00	2.00	0.00	0.75	0.00	0.00	0.75	1.00
time (sec)	N/A	0.015	0.011	0.006	0.000	0.410	0.000	0.000	0.252	0.219

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	B	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	47	83	29	0	22	0	0	19	16
N.S.	1	2.94	5.19	1.81	0.00	1.38	0.00	0.00	1.19	1.00
time (sec)	N/A	0.193	0.024	0.007	0.000	0.402	0.000	0.000	0.333	0.204

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	B	F	B	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	16	0	933	42	0	36	0	0	-1	16
N.S.	1	0.00	58.31	2.62	0.00	2.25	0.00	0.00	-0.06	1.00
time (sec)	N/A	0.754	5.621	0.299	0.000	0.410	0.000	0.000	0.000	17.258

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	76	24	27	12	56	25	12	16
N.S.	1	1.00	4.75	1.50	1.69	0.75	3.50	1.56	0.75	1.00
time (sec)	N/A	0.013	0.036	0.007	0.965	0.406	1.852	0.648	0.208	0.170

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	12	66	25	12	16
N.S.	1	1.00	1.00	0.81	0.75	0.75	4.12	1.56	0.75	1.00
time (sec)	N/A	0.006	0.006	0.007	0.581	0.392	3.198	0.686	0.228	2.645

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	0	249	781	0	70	0	0	-1	17
N.S.	1	0.00	14.65	45.94	0.00	4.12	0.00	0.00	-0.06	1.00
time (sec)	N/A	0.748	1.231	0.359	0.000	0.481	0.000	0.000	0.000	1.231

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	93	188	0	15	0	0	-1	17
N.S.	1	1.00	5.47	11.06	0.00	0.88	0.00	0.00	-0.06	1.00
time (sec)	N/A	0.037	0.125	0.245	0.000	0.423	0.000	0.000	0.000	0.206

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	120	250	0	34	0	0	-1	17
N.S.	1	1.00	7.06	14.71	0.00	2.00	0.00	0.00	-0.06	1.00
time (sec)	N/A	0.082	0.234	0.174	0.000	0.409	0.000	0.000	0.000	0.280

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	B	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	17	0	0	32	0	72	0	0	-1	17
N.S.	1	0.00	0.00	1.88	0.00	4.24	0.00	0.00	-0.06	1.00
time (sec)	N/A	0.394	0.144	0.356	0.000	0.445	0.000	0.000	0.000	9.086

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	25	18	21	19	14	75	19	14	18
N.S.	1	1.39	1.00	1.17	1.06	0.78	4.17	1.06	0.78	1.00
time (sec)	N/A	0.010	0.006	0.004	0.340	0.382	0.827	0.285	0.214	0.021

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	20	21	18	12	19	0	0	14	18
N.S.	1	1.11	1.17	1.00	0.67	1.06	0.00	0.00	0.78	1.00
time (sec)	N/A	0.008	0.036	0.004	0.538	0.394	0.000	0.000	0.142	0.071

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	27	18	15	19	14	26	19	14	18
N.S.	1	1.50	1.00	0.83	1.06	0.78	1.44	1.06	0.78	1.00
time (sec)	N/A	0.010	0.005	0.003	0.530	0.392	0.776	0.290	0.197	0.022

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	21	22	14	0	11	14	18
N.S.	1	1.00	1.00	1.17	1.22	0.78	0.00	0.61	0.78	1.00
time (sec)	N/A	0.020	0.009	0.004	0.567	0.387	0.000	0.426	0.167	0.179

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	21	22	19	0	11	31	18
N.S.	1	1.00	1.00	1.17	1.22	1.06	0.00	0.61	1.72	1.00
time (sec)	N/A	0.019	0.007	0.005	0.408	0.403	0.000	0.329	0.169	0.167

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	12	12	11	0	16	0	0	10	18
N.S.	1	0.67	0.67	0.61	0.00	0.89	0.00	0.00	0.56	1.00
time (sec)	N/A	0.070	0.022	0.006	0.000	0.391	0.000	0.000	0.051	0.001

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	0	14	0	9	14	18
N.S.	1	1.00	1.00	0.83	0.00	0.78	0.00	0.50	0.78	1.00
time (sec)	N/A	0.022	0.007	0.003	0.000	0.376	0.000	0.383	0.173	0.138

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	18	0	14	0	11	14	18
N.S.	1	1.00	1.00	1.00	0.00	0.78	0.00	0.61	0.78	1.00
time (sec)	N/A	0.016	0.011	0.003	0.000	0.396	0.000	0.412	0.229	0.171

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	107	14	78	34	14	18
N.S.	1	1.00	1.00	0.83	5.94	0.78	4.33	1.89	0.78	1.00
time (sec)	N/A	0.014	0.011	0.006	0.454	0.378	26.236	0.309	0.254	0.058

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	68	33	23	28	19	0	27	14	18
N.S.	1	3.78	1.83	1.28	1.56	1.06	0.00	1.50	0.78	1.00
time (sec)	N/A	0.407	0.142	0.006	0.593	0.390	0.000	0.445	0.139	3.650

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	A	F	A	F	A	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	96	18	26	0	19	0	25	1	18
N.S.	1	5.33	1.00	1.44	0.00	1.06	0.00	1.39	0.06	1.00
time (sec)	N/A	0.095	0.012	0.006	0.000	0.394	0.000	0.538	0.011	0.184

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	24	25	14	0	11	14	18
N.S.	1	1.00	1.00	1.33	1.39	0.78	0.00	0.61	0.78	1.00
time (sec)	N/A	0.021	0.009	0.003	0.699	0.393	0.000	0.577	0.189	0.379

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	24	25	14	0	11	14	18
N.S.	1	1.00	1.00	1.33	1.39	0.78	0.00	0.61	0.78	1.00
time (sec)	N/A	0.021	0.008	0.005	0.576	0.396	0.000	0.472	0.184	0.217

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	37	18	29	58	19	0	11	31	18
N.S.	1	2.06	1.00	1.61	3.22	1.06	0.00	0.61	1.72	1.00
time (sec)	N/A	0.122	0.009	0.005	0.752	0.420	0.000	0.504	0.243	0.260

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	24	25	19	0	11	31	18
N.S.	1	1.00	1.00	1.33	1.39	1.06	0.00	0.61	1.72	1.00
time (sec)	N/A	0.021	0.007	0.004	0.672	0.406	0.000	0.506	0.193	0.188

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	25	61	20	0	19	0	0	21	18
N.S.	1	1.39	3.39	1.11	0.00	1.06	0.00	0.00	1.17	1.00
time (sec)	N/A	0.069	0.047	0.006	0.000	0.395	0.000	0.000	0.153	0.201

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	24	25	19	0	11	19	18
N.S.	1	1.00	1.00	1.33	1.39	1.06	0.00	0.61	1.06	1.00
time (sec)	N/A	0.020	0.007	0.003	0.772	0.401	0.000	0.380	0.201	0.381

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	33	290	0	20	0	26	-1	18
N.S.	1	1.00	1.83	16.11	0.00	1.11	0.00	1.44	-0.06	1.00
time (sec)	N/A	0.016	0.006	0.006	0.000	0.456	0.000	0.353	0.000	0.308

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	12	60	11	0	16	0	0	16	18
N.S.	1	0.67	3.33	0.61	0.00	0.89	0.00	0.00	0.89	1.00
time (sec)	N/A	0.075	0.044	0.004	0.000	0.415	0.000	0.000	0.146	0.215

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	20	17	0	14	0	0	14	18
N.S.	1	1.00	1.11	0.94	0.00	0.78	0.00	0.00	0.78	1.00
time (sec)	N/A	0.065	0.011	0.005	0.000	0.420	0.000	0.000	0.153	0.079

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	20	17	14	0	9	14	18
N.S.	1	1.00	1.00	1.11	0.94	0.78	0.00	0.50	0.78	1.00
time (sec)	N/A	0.023	0.008	0.005	0.542	0.412	0.000	0.353	0.122	0.091

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	23	20	17	19	0	9	29	18
N.S.	1	1.00	1.28	1.11	0.94	1.06	0.00	0.50	1.61	1.00
time (sec)	N/A	0.021	0.004	0.004	0.508	0.415	0.000	0.298	0.173	0.091

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	23	20	0	19	0	0	19	18
N.S.	1	1.00	1.28	1.11	0.00	1.06	0.00	0.00	1.06	1.00
time (sec)	N/A	0.017	0.007	0.004	0.000	0.395	0.000	0.000	0.211	0.074

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	0	14	0	9	14	18
N.S.	1	1.00	1.00	0.83	0.00	0.78	0.00	0.50	0.78	1.00
time (sec)	N/A	0.023	0.007	0.003	0.000	0.403	0.000	0.577	0.261	0.204

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	37	18	20	0	22	0	9	43	18
N.S.	1	2.06	1.00	1.11	0.00	1.22	0.00	0.50	2.39	1.00
time (sec)	N/A	0.121	0.010	0.003	0.000	0.395	0.000	1.000	0.243	0.202

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	18	0	17638	4880	0	35	0	0	-1	18
N.S.	1	0.00	979.89	271.11	0.00	1.94	0.00	0.00	-0.06	1.00
time (sec)	N/A	0.459	6.205	5.317	0.000	0.442	0.000	0.000	0.000	0.176

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	26	0	14	0	0	14	18
N.S.	1	1.00	1.00	1.44	0.00	0.78	0.00	0.00	0.78	1.00
time (sec)	N/A	0.017	0.011	0.008	0.000	0.403	0.000	0.000	0.249	0.087

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	B	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	37	18	31	0	19	0	0	31	18
N.S.	1	2.06	1.00	1.72	0.00	1.06	0.00	0.00	1.72	1.00
time (sec)	N/A	0.259	0.011	0.007	0.000	0.408	0.000	0.000	0.281	0.273

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	12	60	11	0	16	0	0	16	18
N.S.	1	0.67	3.33	0.61	0.00	0.89	0.00	0.00	0.89	1.00
time (sec)	N/A	0.068	0.042	0.006	0.000	0.402	0.000	0.000	0.162	0.143

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	23	26	0	19	0	0	19	18
N.S.	1	1.00	1.28	1.44	0.00	1.06	0.00	0.00	1.06	1.00
time (sec)	N/A	0.017	0.008	0.007	0.000	0.406	0.000	0.000	0.211	0.104

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	20	17	0	14	0	0	14	18
N.S.	1	1.00	1.11	0.94	0.00	0.78	0.00	0.00	0.78	1.00
time (sec)	N/A	0.057	0.010	0.010	0.000	0.387	0.000	0.000	0.191	0.092

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	53	65	22	0	24	0	0	41	18
N.S.	1	2.94	3.61	1.22	0.00	1.33	0.00	0.00	2.28	1.00
time (sec)	N/A	0.181	0.042	0.007	0.000	0.416	0.000	0.000	0.268	0.109

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	25	33	16	19	17	-1	18
N.S.	1	1.00	1.00	1.39	1.83	0.89	1.06	0.94	-0.06	1.00
time (sec)	N/A	0.008	0.002	0.252	0.486	0.412	0.790	0.396	0.000	0.127

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	8	8	7	33	16	5	16	-1	18
N.S.	1	0.44	0.44	0.39	1.83	0.89	0.28	0.89	-0.06	1.00
time (sec)	N/A	0.004	0.002	0.182	0.507	0.408	0.754	0.448	0.000	0.118

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	B	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	37	18	35	0	19	0	0	31	18
N.S.	1	2.06	1.00	1.94	0.00	1.06	0.00	0.00	1.72	1.00
time (sec)	N/A	0.265	0.014	0.009	0.000	0.411	0.000	0.000	0.284	0.312

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	20	21	14	0	9	14	18
N.S.	1	1.00	1.00	1.11	1.17	0.78	0.00	0.50	0.78	1.00
time (sec)	N/A	0.023	0.008	0.003	0.694	0.400	0.000	0.610	0.152	0.347

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	20	0	14	0	0	14	18
N.S.	1	1.00	1.00	1.11	0.00	0.78	0.00	0.00	0.78	1.00
time (sec)	N/A	0.016	0.011	0.006	0.000	0.398	0.000	0.000	0.130	0.243

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	23	20	0	19	0	0	19	18
N.S.	1	1.00	1.28	1.11	0.00	1.06	0.00	0.00	1.06	1.00
time (sec)	N/A	0.016	0.007	0.006	0.000	0.410	0.000	0.000	0.207	0.254

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	B	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	0	31	0	0	25	31	0	-1	18
N.S.	1	0.00	1.72	0.00	0.00	1.39	1.72	0.00	-0.06	1.00
time (sec)	N/A	0.084	0.113	0.204	0.000	0.445	0.726	0.000	0.000	0.054

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	719	19	13	15	17	0	0	276	19
N.S.	1	37.84	1.00	0.68	0.79	0.89	0.00	0.00	14.53	1.00
time (sec)	N/A	1.919	0.124	0.006	0.762	0.416	0.000	0.000	0.189	0.511

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	24	0	17	0	0	17	19
N.S.	1	1.00	1.00	1.26	0.00	0.89	0.00	0.00	0.89	1.00
time (sec)	N/A	0.101	0.020	0.007	0.000	0.410	0.000	0.000	0.212	0.216

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	0	307	13532	0	17	0	0	-1	19
N.S.	1	0.00	16.16	712.21	0.00	0.89	0.00	0.00	-0.05	1.00
time (sec)	N/A	1.898	1.488	0.081	0.000	0.458	0.000	0.000	0.000	1.283

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	19	19	1511	273	0	30	0	0	-1	19
N.S.	1	1.00	79.53	14.37	0.00	1.58	0.00	0.00	-0.05	1.00
time (sec)	N/A	0.092	4.992	0.149	0.000	0.461	0.000	0.000	0.000	0.264

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	33	19	29	15	40	0	0	30	19
N.S.	1	1.74	1.00	1.53	0.79	2.11	0.00	0.00	1.58	1.00
time (sec)	N/A	0.023	0.012	0.003	0.322	0.385	0.000	0.000	0.199	5.786

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	15	15	0	0	15	19
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.00	0.00	0.79	1.00
time (sec)	N/A	0.022	0.059	0.008	0.582	0.410	0.000	0.000	0.142	0.573

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	15	15	0	0	15	19
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.00	0.00	0.79	1.00
time (sec)	N/A	0.019	0.044	0.007	0.534	0.391	0.000	0.000	0.220	0.472

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	40	43	15	0	0	15	19
N.S.	1	1.00	1.00	2.11	2.26	0.79	0.00	0.00	0.79	1.00
time (sec)	N/A	0.020	0.049	0.011	0.558	0.412	0.000	0.000	0.146	0.468

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	14	14	13	0	18	0	0	12	20
N.S.	1	0.70	0.70	0.65	0.00	0.90	0.00	0.00	0.60	1.00
time (sec)	N/A	0.075	0.026	0.006	0.000	0.405	0.000	0.000	0.067	0.125

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	14	14	11	0	16	0	9	18	20
N.S.	1	0.70	0.70	0.55	0.00	0.80	0.00	0.45	0.90	1.00
time (sec)	N/A	0.034	0.006	0.004	0.000	0.403	0.000	0.302	0.153	0.110

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	18	17	0	16	0	11	16	20
N.S.	1	1.00	0.90	0.85	0.00	0.80	0.00	0.55	0.80	1.00
time (sec)	N/A	0.023	0.006	0.002	0.000	0.376	0.000	0.280	0.198	0.145

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	17	16	16	26	16	16	20
N.S.	1	1.00	1.00	0.85	0.80	0.80	1.30	0.80	0.80	1.00
time (sec)	N/A	0.005	0.006	0.004	0.420	0.386	0.365	0.280	0.304	0.021

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	17	16	16	44	16	16	20
N.S.	1	1.00	1.00	0.85	0.80	0.80	2.20	0.80	0.80	1.00
time (sec)	N/A	0.004	0.007	0.006	0.555	0.392	0.179	0.193	0.240	0.019

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	17	79	16	309	39	16	20
N.S.	1	1.00	1.00	0.85	3.95	0.80	15.45	1.95	0.80	1.00
time (sec)	N/A	0.016	0.008	0.006	0.732	0.396	22.397	0.608	0.235	0.048

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	23	22	16	0	11	16	20
N.S.	1	1.00	1.00	1.15	1.10	0.80	0.00	0.55	0.80	1.00
time (sec)	N/A	0.025	0.009	0.005	0.606	0.403	0.000	0.359	0.365	0.121

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	23	22	21	0	11	33	20
N.S.	1	1.00	1.00	1.15	1.10	1.05	0.00	0.55	1.65	1.00
time (sec)	N/A	0.023	0.007	0.003	0.421	0.422	0.000	0.293	0.177	0.105

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	23	0	21	0	0	21	20
N.S.	1	1.00	1.00	1.15	0.00	1.05	0.00	0.00	1.05	1.00
time (sec)	N/A	0.018	0.010	0.006	0.000	0.399	0.000	0.000	0.205	0.083

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	18	17	0	16	0	11	16	20
N.S.	1	1.00	0.90	0.85	0.00	0.80	0.00	0.55	0.80	1.00
time (sec)	N/A	0.023	0.006	0.004	0.000	0.409	0.000	0.671	0.301	0.217

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	41	18	22	0	24	0	18	49	20
N.S.	1	2.05	0.90	1.10	0.00	1.20	0.00	0.90	2.45	1.00
time (sec)	N/A	0.126	0.011	0.003	0.000	0.397	0.000	0.228	0.248	0.212

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	18	20	0	19	0	11	33	20
N.S.	1	1.00	0.90	1.00	0.00	0.95	0.00	0.55	1.65	1.00
time (sec)	N/A	0.022	0.005	0.003	0.000	0.403	0.000	0.250	0.166	0.186

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	17	16	16	56	16	16	20
N.S.	1	1.00	1.00	0.85	0.80	0.80	2.80	0.80	0.80	1.00
time (sec)	N/A	0.016	0.008	0.003	0.552	0.391	0.195	0.264	0.168	0.267

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	26	0	21	0	0	21	20
N.S.	1	1.00	1.00	1.30	0.00	1.05	0.00	0.00	1.05	1.00
time (sec)	N/A	0.018	0.011	0.007	0.000	0.397	0.000	0.000	0.222	0.112

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	41	20	28	0	21	0	0	35	20
N.S.	1	2.05	1.00	1.40	0.00	1.05	0.00	0.00	1.75	1.00
time (sec)	N/A	0.148	0.015	0.004	0.000	0.409	0.000	0.000	0.283	0.170

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	59	65	28	0	26	0	0	47	20
N.S.	1	2.95	3.25	1.40	0.00	1.30	0.00	0.00	2.35	1.00
time (sec)	N/A	0.196	0.044	0.004	0.000	0.408	0.000	0.000	0.262	0.118

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	A	A	A	F	F	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	0	17	22	23	15	0	0	18	20
N.S.	1	0.00	0.85	1.10	1.15	0.75	0.00	0.00	0.90	1.00
time (sec)	N/A	0.594	0.108	0.009	0.815	0.410	0.000	0.000	0.167	10.700

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	28	30	16	0	11	16	20
N.S.	1	1.00	1.00	1.40	1.50	0.80	0.00	0.55	0.80	1.00
time (sec)	N/A	0.025	0.009	0.006	0.414	0.393	0.000	0.427	0.239	0.359

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	28	0	16	0	0	16	20
N.S.	1	1.00	1.00	1.40	0.00	0.80	0.00	0.00	0.80	1.00
time (sec)	N/A	0.020	0.013	0.006	0.000	0.417	0.000	0.000	0.214	0.250

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	28	0	21	0	0	21	20
N.S.	1	1.00	1.00	1.40	0.00	1.05	0.00	0.00	1.05	1.00
time (sec)	N/A	0.018	0.011	0.004	0.000	0.412	0.000	0.000	0.210	0.250

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	41	20	33	0	21	0	0	33	20
N.S.	1	2.05	1.00	1.65	0.00	1.05	0.00	0.00	1.65	1.00
time (sec)	N/A	0.161	0.012	0.006	0.000	0.399	0.000	0.000	0.255	0.328

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	14	14	13	29	18	0	0	12	20
N.S.	1	0.70	0.70	0.65	1.45	0.90	0.00	0.00	0.60	1.00
time (sec)	N/A	0.004	0.007	0.007	0.603	0.386	0.000	0.000	0.241	0.564

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	37	26	0	18	0	0	-1	20
N.S.	1	1.00	1.85	1.30	0.00	0.90	0.00	0.00	-0.05	1.00
time (sec)	N/A	0.005	0.007	0.010	0.000	0.411	0.000	0.000	0.000	0.057

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	20	0	0	0	0	55	0	0	-1	20
N.S.	1	0.00	0.00	0.00	0.00	2.75	0.00	0.00	-0.05	1.00
time (sec)	N/A	0.231	0.076	0.185	0.000	1.701	0.000	0.000	0.000	0.065

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	25	25	240	0	40	0	0	206	21
N.S.	1	1.19	1.19	11.43	0.00	1.90	0.00	0.00	9.81	1.00
time (sec)	N/A	0.056	0.016	0.230	0.000	0.436	0.000	0.000	0.302	0.881

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	18	298	1656	0	25	0	0	275	21
N.S.	1	0.86	14.19	78.86	0.00	1.19	0.00	0.00	13.10	1.00
time (sec)	N/A	0.064	0.988	0.051	0.000	0.417	0.000	0.000	0.172	1.009

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	21	402	234	250	0	15	0	0	252	21
N.S.	1	19.14	11.14	11.90	0.00	0.71	0.00	0.00	12.00	1.00
time (sec)	N/A	0.538	0.255	0.027	0.000	0.426	0.000	0.000	0.156	0.993

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	A	F	A	F	F	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	0	19	18	0	20	0	0	19	21
N.S.	1	0.00	0.90	0.86	0.00	0.95	0.00	0.00	0.90	1.00
time (sec)	N/A	1.189	0.115	0.006	0.000	0.391	0.000	0.000	0.193	1.206

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	0	320	13534	0	19	0	0	-1	21
N.S.	1	0.00	15.24	644.48	0.00	0.90	0.00	0.00	-0.05	1.00
time (sec)	N/A	1.764	1.281	0.196	0.000	0.452	0.000	0.000	0.000	1.290

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	0	250	780	0	25	0	0	-1	21
N.S.	1	0.00	11.90	37.14	0.00	1.19	0.00	0.00	-0.05	1.00
time (sec)	N/A	0.692	1.135	0.011	0.000	0.431	0.000	0.000	0.000	1.207

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	A	F	A	F	F	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	0	15	14	0	19	0	0	19	21
N.S.	1	0.00	0.71	0.67	0.00	0.90	0.00	0.00	0.90	1.00
time (sec)	N/A	2.283	0.422	0.007	0.000	0.399	0.000	0.000	0.171	0.674

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	14	14	13	0	20	0	9	19	21
N.S.	1	0.67	0.67	0.62	0.00	0.95	0.00	0.43	0.90	1.00
time (sec)	N/A	0.030	0.006	0.003	0.000	0.382	0.000	0.622	0.166	0.202

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	21	21	99	179	0	34	0	0	-1	21
N.S.	1	1.00	4.71	8.52	0.00	1.62	0.00	0.00	-0.05	1.00
time (sec)	N/A	0.107	0.266	0.068	0.000	0.441	0.000	0.000	0.000	0.369

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	107	170	0	47	0	0	-1	21
N.S.	1	1.00	5.10	8.10	0.00	2.24	0.00	0.00	-0.05	1.00
time (sec)	N/A	0.122	0.335	0.046	0.000	0.435	0.000	0.000	0.000	0.624

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	27	27	0	0	17	21
N.S.	1	1.00	1.00	0.86	1.29	1.29	0.00	0.00	0.81	1.00
time (sec)	N/A	0.021	0.053	0.007	0.737	0.410	0.000	0.000	0.323	0.312

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	0	0	17	21
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.00	0.00	0.81	1.00
time (sec)	N/A	0.020	0.041	0.008	0.639	0.417	0.000	0.000	0.361	0.693

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	19	27	417	0	15	0	67	2737	37
N.S.	1	0.90	1.29	19.86	0.00	0.71	0.00	3.19	130.33	1.76
time (sec)	N/A	0.441	0.053	0.214	0.000	0.416	0.000	0.319	0.100	0.053

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	16	16	13	0	18	0	11	20	22
N.S.	1	0.73	0.73	0.59	0.00	0.82	0.00	0.50	0.91	1.00
time (sec)	N/A	0.041	0.006	0.006	0.000	0.400	0.000	0.643	0.165	0.114

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	16	16	13	0	18	0	11	20	22
N.S.	1	0.73	0.73	0.59	0.00	0.82	0.00	0.50	0.91	1.00
time (sec)	N/A	0.042	0.007	0.005	0.000	0.403	0.000	0.389	0.152	0.220

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	B	B	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	0	22	0	0	38	41	0	-1	22
N.S.	1	0.00	1.00	0.00	0.00	1.73	1.86	0.00	-0.05	1.00
time (sec)	N/A	0.177	0.129	0.246	0.000	0.466	0.715	0.000	0.000	0.101

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	59	33	33	32	33	19	23
N.S.	1	1.00	1.00	2.57	1.43	1.43	1.39	1.43	0.83	1.00
time (sec)	N/A	0.014	0.003	0.206	0.454	0.406	0.744	0.367	0.250	0.027

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	36	21	21	23	17	0	0	24	39
N.S.	1	1.57	0.91	0.91	1.00	0.74	0.00	0.00	1.04	1.70
time (sec)	N/A	0.006	0.011	0.001	0.320	0.383	0.000	0.000	0.106	4.732

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	27	20	28	19	19	37	19	20	20
N.S.	1	1.17	0.87	1.22	0.83	0.83	1.61	0.83	0.87	0.87
time (sec)	N/A	0.011	0.005	0.004	0.372	0.381	0.361	0.617	0.194	0.017

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	18	18	21	0	19	0	26	19	18
N.S.	1	0.78	0.78	0.91	0.00	0.83	0.00	1.13	0.83	0.78
time (sec)	N/A	0.016	0.012	0.007	0.000	0.386	0.000	0.296	0.175	0.227

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	16	14	15	0	22	0	11	21	23
N.S.	1	0.70	0.61	0.65	0.00	0.96	0.00	0.48	0.91	1.00
time (sec)	N/A	0.031	0.006	0.004	0.000	0.372	0.000	0.275	0.159	0.154

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	37	23	23	0	19	0	19	29	23
N.S.	1	1.61	1.00	1.00	0.00	0.83	0.00	0.83	1.26	1.00
time (sec)	N/A	0.048	0.011	0.004	0.000	0.384	0.000	0.492	0.142	0.121

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	22	0	21	0	13	14	23
N.S.	1	1.00	1.00	0.96	0.00	0.91	0.00	0.57	0.61	1.00
time (sec)	N/A	0.083	0.040	0.005	0.000	0.407	0.000	0.311	0.166	0.172

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	33	23	31	24	19	124	0	25	23
N.S.	1	1.43	1.00	1.35	1.04	0.83	5.39	0.00	1.09	1.00
time (sec)	N/A	0.007	0.007	0.005	0.315	0.386	0.817	0.000	0.220	0.185

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	31	23	20	23	25	75	0	19	23
N.S.	1	1.35	1.00	0.87	1.00	1.09	3.26	0.00	0.83	1.00
time (sec)	N/A	0.005	0.005	0.003	0.335	0.401	0.769	0.000	0.202	0.186

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	33	23	20	24	19	48	0	25	23
N.S.	1	1.43	1.00	0.87	1.04	0.83	2.09	0.00	1.09	1.00
time (sec)	N/A	0.007	0.006	0.003	0.380	0.390	0.774	0.000	0.197	0.174

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	33	23	20	25	19	71	0	25	23
N.S.	1	1.43	1.00	0.87	1.09	0.83	3.09	0.00	1.09	1.00
time (sec)	N/A	0.011	0.007	0.005	0.326	0.400	1.796	0.000	0.075	0.207

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	36	112	0	42	0	0	-1	23
N.S.	1	1.00	1.57	4.87	0.00	1.83	0.00	0.00	-0.04	1.00
time (sec)	N/A	0.035	0.101	0.066	0.000	0.447	0.000	0.000	0.000	0.382

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	0	167	774	0	28	0	0	-1	23
N.S.	1	0.00	7.26	33.65	0.00	1.22	0.00	0.00	-0.04	1.00
time (sec)	N/A	0.715	0.644	0.066	0.000	0.426	0.000	0.000	0.000	1.233

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	33	23	31	32	19	0	19	19	23
N.S.	1	1.43	1.00	1.35	1.39	0.83	0.00	0.83	0.83	1.00
time (sec)	N/A	0.040	0.011	0.004	0.910	0.404	0.000	0.586	0.158	0.405

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	33	23	31	58	19	0	19	27	23
N.S.	1	1.43	1.00	1.35	2.52	0.83	0.00	0.83	1.17	1.00
time (sec)	N/A	0.075	0.014	0.007	0.634	0.399	0.000	0.418	0.179	0.273

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	33	23	31	58	19	0	19	27	23
N.S.	1	1.43	1.00	1.35	2.52	0.83	0.00	0.83	1.17	1.00
time (sec)	N/A	0.075	0.009	0.007	0.839	0.396	0.000	0.416	0.165	0.278

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	14	14	13	0	18	0	0	21	23
N.S.	1	0.61	0.61	0.57	0.00	0.78	0.00	0.00	0.91	1.00
time (sec)	N/A	0.082	0.025	0.005	0.000	0.394	0.000	0.000	0.153	0.105

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	14	14	13	0	18	0	9	21	23
N.S.	1	0.61	0.61	0.57	0.00	0.78	0.00	0.39	0.91	1.00
time (sec)	N/A	0.011	0.006	0.004	0.000	0.408	0.000	0.229	0.123	0.101

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	16	14	15	0	24	0	11	21	23
N.S.	1	0.70	0.61	0.65	0.00	1.04	0.00	0.48	0.91	1.00
time (sec)	N/A	0.032	0.006	0.003	0.000	0.392	0.000	0.324	0.173	0.206

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	37	23	20	0	26	0	19	29	23
N.S.	1	1.61	1.00	0.87	0.00	1.13	0.00	0.83	1.26	1.00
time (sec)	N/A	0.029	0.007	0.003	0.000	0.424	0.000	0.368	0.132	0.129

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	37	23	23	0	19	0	19	29	23
N.S.	1	1.61	1.00	1.00	0.00	0.83	0.00	0.83	1.26	1.00
time (sec)	N/A	0.050	0.008	0.003	0.000	0.430	0.000	0.323	0.128	0.205

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	23	11	305	528	0	21	0	23	-1	23
N.S.	1	0.48	13.26	22.96	0.00	0.91	0.00	1.00	-0.04	1.00
time (sec)	N/A	0.053	0.589	0.332	0.000	0.440	0.000	0.396	0.000	0.083

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	33	23	31	25	19	190	0	27	23
N.S.	1	1.43	1.00	1.35	1.09	0.83	8.26	0.00	1.17	1.00
time (sec)	N/A	0.010	0.007	0.006	0.614	0.400	2.002	0.000	0.166	0.179

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	31	23	20	22	25	97	0	19	23
N.S.	1	1.35	1.00	0.87	0.96	1.09	4.22	0.00	0.83	1.00
time (sec)	N/A	0.009	0.009	0.004	0.377	0.392	5.019	0.000	0.071	0.232

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	42	23	32	0	19	37	19	19	23
N.S.	1	1.83	1.00	1.39	0.00	0.83	1.61	0.83	0.83	1.00
time (sec)	N/A	0.364	0.053	0.009	0.000	0.387	0.429	0.272	0.189	0.064

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	14	14	13	0	20	0	0	21	23
N.S.	1	0.61	0.61	0.57	0.00	0.87	0.00	0.00	0.91	1.00
time (sec)	N/A	0.079	0.026	0.005	0.000	0.405	0.000	0.000	0.152	0.150

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	33	23	40	25	26	63	0	25	23
N.S.	1	1.43	1.00	1.74	1.09	1.13	2.74	0.00	1.09	1.00
time (sec)	N/A	0.007	0.005	0.008	0.325	0.389	0.925	0.000	0.287	0.183

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	33	23	40	25	26	49	0	25	23
N.S.	1	1.43	1.00	1.74	1.09	1.13	2.13	0.00	1.09	1.00
time (sec)	N/A	0.010	0.006	0.010	0.672	0.386	2.891	0.000	0.173	0.227

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	14	14	13	0	18	0	0	21	23
N.S.	1	0.61	0.61	0.57	0.00	0.78	0.00	0.00	0.91	1.00
time (sec)	N/A	0.079	0.027	0.004	0.000	0.388	0.000	0.000	0.161	0.268

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	14	14	13	0	18	0	0	21	23
N.S.	1	0.61	0.61	0.57	0.00	0.78	0.00	0.00	0.91	1.00
time (sec)	N/A	0.074	0.022	0.006	0.000	0.397	0.000	0.000	0.132	0.273

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	25	22	0	21	0	0	21	23
N.S.	1	1.00	1.09	0.96	0.00	0.91	0.00	0.00	0.91	1.00
time (sec)	N/A	0.032	0.213	0.010	0.000	0.425	0.000	0.000	0.249	5.091

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	40	37	43	0	58	0	34	-1	24
N.S.	1	1.67	1.54	1.79	0.00	2.42	0.00	1.42	-0.04	1.00
time (sec)	N/A	0.042	0.008	0.010	0.000	0.396	0.000	0.338	0.000	0.060

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	46	107	271	0	18	0	0	179	24
N.S.	1	1.92	4.46	11.29	0.00	0.75	0.00	0.00	7.46	1.00
time (sec)	N/A	0.400	0.300	0.079	0.000	0.434	0.000	0.000	0.298	0.072

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	320	136	781	0	29	0	0	223	24
N.S.	1	13.33	5.67	32.54	0.00	1.21	0.00	0.00	9.29	1.00
time (sec)	N/A	1.028	0.526	0.052	0.000	0.419	0.000	0.000	0.151	0.075

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	41	111	0	18	0	0	-1	24
N.S.	1	1.00	1.71	4.62	0.00	0.75	0.00	0.00	-0.04	1.00
time (sec)	N/A	0.034	0.093	0.022	0.000	0.450	0.000	0.000	0.000	0.351

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	37	111	0	42	0	0	-1	24
N.S.	1	1.00	1.54	4.62	0.00	1.75	0.00	0.00	-0.04	1.00
time (sec)	N/A	0.035	0.101	0.007	0.000	0.447	0.000	0.000	0.000	0.369

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	24	0	0	0	0	136	0	0	44	24
N.S.	1	0.00	0.00	0.00	0.00	5.67	0.00	0.00	1.83	1.00
time (sec)	N/A	0.707	0.175	0.233	0.000	0.477	0.000	0.000	0.753	5.888

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	24	0	0	0	0	138	0	0	44	24
N.S.	1	0.00	0.00	0.00	0.00	5.75	0.00	0.00	1.83	1.00
time (sec)	N/A	0.700	0.182	0.236	0.000	0.465	0.000	0.000	0.748	5.830

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	24	0	0	0	0	151	0	0	52	24
N.S.	1	0.00	0.00	0.00	0.00	6.29	0.00	0.00	2.17	1.00
time (sec)	N/A	0.678	0.239	0.262	0.000	0.476	0.000	0.000	0.856	2.417

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	24	0	0	0	0	147	0	0	46	24
N.S.	1	0.00	0.00	0.00	0.00	6.12	0.00	0.00	1.92	1.00
time (sec)	N/A	0.622	0.223	0.207	0.000	0.461	0.000	0.000	0.807	2.291

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	A	A	A	F	F	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	0	24	16	26	22	0	0	22	24
N.S.	1	0.00	1.00	0.67	1.08	0.92	0.00	0.00	0.92	1.00
time (sec)	N/A	0.973	0.352	0.007	0.541	0.390	0.000	0.000	0.083	9.074

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	24	0	0	0	0	119	0	0	-1	24
N.S.	1	0.00	0.00	0.00	0.00	4.96	0.00	0.00	-0.04	1.00
time (sec)	N/A	1.074	0.470	0.291	0.000	0.584	0.000	0.000	0.000	2.688

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	24	0	0	0	0	129	0	0	-1	24
N.S.	1	0.00	0.00	0.00	0.00	5.38	0.00	0.00	-0.04	1.00
time (sec)	N/A	0.961	0.455	0.315	0.000	0.581	0.000	0.000	0.000	2.660

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	27	20	23	19	21	41	19	21	20
N.S.	1	1.08	0.80	0.92	0.76	0.84	1.64	0.76	0.84	0.80
time (sec)	N/A	0.012	0.007	0.005	0.340	0.377	0.371	0.294	0.211	0.018

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	27	20	23	19	21	41	19	21	20
N.S.	1	1.08	0.80	0.92	0.76	0.84	1.64	0.76	0.84	0.80
time (sec)	N/A	0.013	0.007	0.004	0.343	0.383	0.764	0.525	0.195	0.020

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	43	35	35	32	35	21	25
N.S.	1	1.00	1.00	1.72	1.40	1.40	1.28	1.40	0.84	1.00
time (sec)	N/A	0.014	0.003	0.198	0.527	0.401	0.850	0.445	0.226	0.029

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	27	20	17	19	21	41	19	20	20
N.S.	1	1.08	0.80	0.68	0.76	0.84	1.64	0.76	0.80	0.80
time (sec)	N/A	0.016	0.006	0.005	0.324	0.420	0.799	0.435	0.192	0.019

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	27	20	26	19	21	37	19	21	20
N.S.	1	1.08	0.80	1.04	0.76	0.84	1.48	0.76	0.84	0.80
time (sec)	N/A	0.013	0.006	0.003	0.574	0.384	0.407	0.276	0.202	0.016

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	27	20	26	19	21	41	19	21	20
N.S.	1	1.08	0.80	1.04	0.76	0.84	1.64	0.76	0.84	0.80
time (sec)	N/A	0.012	0.007	0.006	0.458	0.390	1.250	0.613	0.200	0.019

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	40	25	33	28	21	41	28	20	25
N.S.	1	1.60	1.00	1.32	1.12	0.84	1.64	1.12	0.80	1.00
time (sec)	N/A	0.016	0.007	0.004	0.322	0.401	1.245	0.521	0.228	0.019

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	27	20	28	19	21	37	19	20	20
N.S.	1	1.08	0.80	1.12	0.76	0.84	1.48	0.76	0.80	0.80
time (sec)	N/A	0.011	0.006	0.005	0.422	0.395	0.631	0.406	0.186	0.018

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	41	23	25	0	21	0	23	33	25
N.S.	1	1.64	0.92	1.00	0.00	0.84	0.00	0.92	1.32	1.00
time (sec)	N/A	0.053	0.010	0.003	0.000	0.385	0.000	0.468	0.163	0.130

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	27	20	17	19	21	41	19	20	20
N.S.	1	1.08	0.80	0.68	0.76	0.84	1.64	0.76	0.80	0.80
time (sec)	N/A	0.012	0.006	0.002	0.442	0.385	0.996	0.454	0.216	0.034

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	37	25	31	32	21	0	23	21	25
N.S.	1	1.48	1.00	1.24	1.28	0.84	0.00	0.92	0.84	1.00
time (sec)	N/A	0.047	0.010	0.003	0.797	0.402	0.000	0.396	0.190	0.408

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	37	25	31	58	21	0	23	31	25
N.S.	1	1.48	1.00	1.24	2.32	0.84	0.00	0.92	1.24	1.00
time (sec)	N/A	0.079	0.012	0.006	0.827	0.404	0.000	0.550	0.208	0.303

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	16	16	15	0	22	0	0	23	25
N.S.	1	0.64	0.64	0.60	0.00	0.88	0.00	0.00	0.92	1.00
time (sec)	N/A	0.085	0.027	0.006	0.000	0.403	0.000	0.000	0.193	0.101

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	16	16	15	0	22	0	11	23	25
N.S.	1	0.64	0.64	0.60	0.00	0.88	0.00	0.44	0.92	1.00
time (sec)	N/A	0.012	0.006	0.004	0.000	0.399	0.000	0.331	0.157	0.125

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	37	25	27	24	21	0	19	31	25
N.S.	1	1.48	1.00	1.08	0.96	0.84	0.00	0.76	1.24	1.00
time (sec)	N/A	0.048	0.014	0.003	0.439	0.375	0.000	0.302	0.176	0.105

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	25	27	728	1358	0	23	0	24	-1	25
N.S.	1	1.08	29.12	54.32	0.00	0.92	0.00	0.96	-0.04	1.00
time (sec)	N/A	0.054	2.373	0.612	0.000	0.426	0.000	0.407	0.000	0.096

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	25	9	613	882	0	23	0	23	-1	25
N.S.	1	0.36	24.52	35.28	0.00	0.92	0.00	0.92	-0.04	1.00
time (sec)	N/A	0.041	2.358	0.323	0.000	0.414	0.000	0.682	0.000	0.152

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	15	803	1352	0	23	0	23	-1	25
N.S.	1	0.60	32.12	54.08	0.00	0.92	0.00	0.92	-0.04	1.00
time (sec)	N/A	0.056	3.044	0.428	0.000	0.424	0.000	0.296	0.000	0.280

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	41	23	25	0	21	0	23	33	25
N.S.	1	1.64	0.92	1.00	0.00	0.84	0.00	0.92	1.32	1.00
time (sec)	N/A	0.051	0.008	0.003	0.000	0.386	0.000	0.371	0.162	0.232

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	37	23	20	29	19	233	0	30	25
N.S.	1	1.48	0.92	0.80	1.16	0.76	9.32	0.00	1.20	1.00
time (sec)	N/A	0.011	0.008	0.006	0.420	0.393	2.111	0.000	0.201	0.194

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	16	16	15	0	24	0	0	23	25
N.S.	1	0.64	0.64	0.60	0.00	0.96	0.00	0.00	0.92	1.00
time (sec)	N/A	0.088	0.028	0.006	0.000	0.389	0.000	0.000	0.193	0.163

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	16	16	15	0	22	0	0	23	25
N.S.	1	0.64	0.64	0.60	0.00	0.88	0.00	0.00	0.92	1.00
time (sec)	N/A	0.085	0.029	0.008	0.000	0.397	0.000	0.000	0.208	0.274

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	16	16	15	0	22	0	0	23	25
N.S.	1	0.64	0.64	0.60	0.00	0.88	0.00	0.00	0.92	1.00
time (sec)	N/A	0.081	0.026	0.007	0.000	0.406	0.000	0.000	0.168	0.290

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	37	25	27	28	21	0	19	31	25
N.S.	1	1.48	1.00	1.08	1.12	0.84	0.00	0.76	1.24	1.00
time (sec)	N/A	0.047	0.012	0.004	0.585	0.388	0.000	0.493	0.207	0.447

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	33	23	29	25	22	289	0	38	26
N.S.	1	1.27	0.88	1.12	0.96	0.85	11.12	0.00	1.46	1.00
time (sec)	N/A	0.006	0.005	0.003	0.365	0.406	0.869	0.000	0.246	0.101

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	17	17	16	0	25	0	0	24	26
N.S.	1	0.65	0.65	0.62	0.00	0.96	0.00	0.00	0.92	1.00
time (sec)	N/A	0.151	0.047	0.007	0.000	0.413	0.000	0.000	0.236	0.198

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	26	0	2764	845	0	167	0	0	40	26
N.S.	1	0.00	106.31	32.50	0.00	6.42	0.00	0.00	1.54	1.00
time (sec)	N/A	1.177	6.282	0.554	0.000	0.465	0.000	0.000	1.109	0.610

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	26	0	2725	842	0	169	0	0	51	26
N.S.	1	0.00	104.81	32.38	0.00	6.50	0.00	0.00	1.96	1.00
time (sec)	N/A	0.911	6.317	0.167	0.000	0.468	0.000	0.000	3.383	0.618

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	F(-2)	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	131	74	136	0	0	0	0	-1	26
N.S.	1	5.04	2.85	5.23	0.00	0.00	0.00	0.00	-0.04	1.00
time (sec)	N/A	0.653	0.227	0.105	0.000	0.000	0.000	0.000	0.000	13.443

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	33	23	20	25	24	136	0	38	23
N.S.	1	1.27	0.88	0.77	0.96	0.92	5.23	0.00	1.46	0.88
time (sec)	N/A	0.012	0.009	0.006	0.359	0.415	3.371	0.000	0.315	0.168

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	291	92	146	0	40	0	0	-1	26
N.S.	1	11.19	3.54	5.62	0.00	1.54	0.00	0.00	-0.04	1.00
time (sec)	N/A	0.981	0.417	0.042	0.000	0.545	0.000	0.000	0.000	0.402

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	44	93	23	22	22	126	0	25	26
N.S.	1	1.69	3.58	0.88	0.85	0.85	4.85	0.00	0.96	1.00
time (sec)	N/A	0.116	0.058	0.009	0.767	0.396	3.106	0.000	0.241	0.469

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	47	115	32	38	22	178	0	39	26
N.S.	1	1.81	4.42	1.23	1.46	0.85	6.85	0.00	1.50	1.00
time (sec)	N/A	0.131	0.075	0.006	0.762	0.402	4.045	0.000	0.243	4.023

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	26	0	0	0	0	132	0	0	40	26
N.S.	1	0.00	0.00	0.00	0.00	5.08	0.00	0.00	1.54	1.00
time (sec)	N/A	1.364	0.575	0.359	0.000	0.475	0.000	0.000	0.458	0.608

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	26	0	0	0	0	146	0	0	42	26
N.S.	1	0.00	0.00	0.00	0.00	5.62	0.00	0.00	1.62	1.00
time (sec)	N/A	1.342	0.751	0.303	0.000	0.464	0.000	0.000	0.442	1.364

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	44	238	38	0	22	151	0	27	26
N.S.	1	1.69	9.15	1.46	0.00	0.85	5.81	0.00	1.04	1.00
time (sec)	N/A	0.103	0.298	0.007	0.000	0.404	4.565	0.000	0.283	2.570

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	44	107	42	0	22	143	0	27	26
N.S.	1	1.69	4.12	1.62	0.00	0.85	5.50	0.00	1.04	1.00
time (sec)	N/A	0.099	0.081	0.010	0.000	0.399	4.413	0.000	0.176	2.206

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	33	23	40	25	29	92	41	24	26
N.S.	1	1.27	0.88	1.54	0.96	1.12	3.54	1.58	0.92	1.00
time (sec)	N/A	0.007	0.008	0.007	0.596	0.395	1.367	0.420	0.385	0.148

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	26	0	0	51	0	22	0	0	-1	26
N.S.	1	0.00	0.00	1.96	0.00	0.85	0.00	0.00	-0.04	1.00
time (sec)	N/A	0.889	0.267	0.406	0.000	0.447	0.000	0.000	0.000	13.088

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	26	0	0	0	0	123	0	0	-1	26
N.S.	1	0.00	0.00	0.00	0.00	4.73	0.00	0.00	-0.04	1.00
time (sec)	N/A	1.195	0.502	0.348	0.000	0.572	0.000	0.000	0.000	2.679

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	26	0	0	0	0	137	0	0	-1	26
N.S.	1	0.00	0.00	0.00	0.00	5.27	0.00	0.00	-0.04	1.00
time (sec)	N/A	1.208	0.527	0.388	0.000	0.614	0.000	0.000	0.000	2.671

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	31	117	43	53	22	168	0	27	26
N.S.	1	1.19	4.50	1.65	2.04	0.85	6.46	0.00	1.04	1.00
time (sec)	N/A	0.104	0.065	0.008	0.709	0.431	4.498	0.000	0.305	4.613

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	31	105	38	44	22	155	0	27	26
N.S.	1	1.19	4.04	1.46	1.69	0.85	5.96	0.00	1.04	1.00
time (sec)	N/A	0.098	0.037	0.009	0.576	0.410	4.454	0.000	0.296	4.552

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	26	0	0	73	0	40	0	0	-1	26
N.S.	1	0.00	0.00	2.81	0.00	1.54	0.00	0.00	-0.04	1.00
time (sec)	N/A	0.771	0.241	0.458	0.000	0.777	0.000	0.000	0.000	3.710

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	23	62	62	0	0	147	26
N.S.	1	1.00	1.00	0.88	2.38	2.38	0.00	0.00	5.65	1.00
time (sec)	N/A	0.323	0.125	0.009	0.671	0.418	0.000	0.000	1.520	0.134

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	26	0	0	0	0	144	0	0	-1	26
N.S.	1	0.00	0.00	0.00	0.00	5.54	0.00	0.00	-0.04	1.00
time (sec)	N/A	1.713	0.313	0.464	0.000	1.053	0.000	0.000	0.000	17.406

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	26	0	0	0	0	139	0	0	46	26
N.S.	1	0.00	0.00	0.00	0.00	5.35	0.00	0.00	1.77	1.00
time (sec)	N/A	1.419	0.269	0.416	0.000	0.601	0.000	0.000	0.932	4.801

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	26	0	0	0	0	132	0	0	-1	26
N.S.	1	0.00	0.00	0.00	0.00	5.08	0.00	0.00	-0.04	1.00
time (sec)	N/A	1.536	0.340	0.467	0.000	1.095	0.000	0.000	0.000	9.872

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	31	22	19	23	23	42	32	34	22
N.S.	1	1.15	0.81	0.70	0.85	0.85	1.56	1.19	1.26	0.81
time (sec)	N/A	0.016	0.011	0.004	0.322	0.377	0.294	0.306	0.150	0.017

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	27	1340	812	1075	0	97	0	0	1872	27
N.S.	1	49.63	30.07	39.81	0.00	3.59	0.00	0.00	69.33	1.00
time (sec)	N/A	3.944	1.060	0.417	0.000	0.438	0.000	0.000	1.367	0.222

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	27	537	148	250	0	51	0	0	167	27
N.S.	1	19.89	5.48	9.26	0.00	1.89	0.00	0.00	6.19	1.00
time (sec)	N/A	1.024	0.362	0.063	0.000	0.432	0.000	0.000	0.284	0.095

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	38	27	24	30	23	46	30	24	27
N.S.	1	1.41	1.00	0.89	1.11	0.85	1.70	1.11	0.89	1.00
time (sec)	N/A	0.027	0.015	0.006	0.443	0.385	0.603	0.588	0.309	0.027

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	41	27	30	29	23	0	23	35	27
N.S.	1	1.52	1.00	1.11	1.07	0.85	0.00	0.85	1.30	1.00
time (sec)	N/A	0.054	0.013	0.003	0.650	0.388	0.000	0.421	0.244	0.127

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	29	37	63	22	23	19	27
N.S.	1	1.00	1.00	1.07	1.37	2.33	0.81	0.85	0.70	1.00
time (sec)	N/A	0.021	0.007	0.012	0.802	0.406	0.874	0.359	0.385	0.001

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	41	27	35	37	23	0	23	35	27
N.S.	1	1.52	1.00	1.30	1.37	0.85	0.00	0.85	1.30	1.00
time (sec)	N/A	0.056	0.010	0.005	0.475	0.387	0.000	0.325	0.244	0.438

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	21	20	20	85	20	174	28
N.S.	1	1.00	1.00	0.75	0.71	0.71	3.04	0.71	6.21	1.00
time (sec)	N/A	0.011	0.005	0.025	0.497	0.380	1.039	0.645	0.157	0.021

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	33	23	31	25	24	134	0	39	23
N.S.	1	1.18	0.82	1.11	0.89	0.86	4.79	0.00	1.39	0.82
time (sec)	N/A	0.009	0.008	0.005	0.615	0.389	2.138	0.000	0.160	0.095

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	21	34	34	48	35	174	28
N.S.	1	1.00	1.00	0.75	1.21	1.21	1.71	1.25	6.21	1.00
time (sec)	N/A	0.010	0.005	0.022	0.466	0.394	0.949	0.226	0.157	0.022

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	33	23	29	25	24	416	0	39	28
N.S.	1	1.18	0.82	1.04	0.89	0.86	14.86	0.00	1.39	1.00
time (sec)	N/A	0.009	0.007	0.006	0.375	0.398	2.227	0.000	0.250	0.101

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	33	23	29	25	24	420	0	24	28
N.S.	1	1.18	0.82	1.04	0.89	0.86	15.00	0.00	0.86	1.00
time (sec)	N/A	0.009	0.008	0.005	0.322	0.395	2.564	0.000	0.290	0.103

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	33	21	29	25	22	139	0	22	28
N.S.	1	1.18	0.75	1.04	0.89	0.79	4.96	0.00	0.79	1.00
time (sec)	N/A	0.010	0.007	0.005	0.646	0.396	2.378	0.000	0.291	0.106

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	49	28	30	31	24	0	28	24	28
N.S.	1	1.75	1.00	1.07	1.11	0.86	0.00	1.00	0.86	1.00
time (sec)	N/A	0.061	0.016	0.005	0.710	0.397	0.000	0.373	0.211	0.223

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	36	70	19	0	18	0	0	17	28
N.S.	1	1.29	2.50	0.68	0.00	0.64	0.00	0.00	0.61	1.00
time (sec)	N/A	0.134	0.027	0.006	0.000	0.408	0.000	0.000	0.248	0.451

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	28	533	1600	250	0	47	0	0	165	28
N.S.	1	19.04	57.14	8.93	0.00	1.68	0.00	0.00	5.89	1.00
time (sec)	N/A	0.894	7.306	0.028	0.000	0.435	0.000	0.000	0.253	0.116

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	49	124	30	0	24	0	23	39	28
N.S.	1	1.75	4.43	1.07	0.00	0.86	0.00	0.82	1.39	1.00
time (sec)	N/A	0.362	0.092	0.007	0.000	0.412	0.000	0.563	0.237	0.735

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	49	31	34	25	27	564	0	49	28
N.S.	1	1.75	1.11	1.21	0.89	0.96	20.14	0.00	1.75	1.00
time (sec)	N/A	0.013	0.008	0.006	0.355	0.396	3.045	0.000	0.356	0.105

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	33	23	29	25	24	418	0	39	23
N.S.	1	1.18	0.82	1.04	0.89	0.86	14.93	0.00	1.39	0.82
time (sec)	N/A	0.010	0.006	0.005	0.569	0.402	2.483	0.000	0.271	0.101

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	49	125	30	0	24	0	23	39	28
N.S.	1	1.75	4.46	1.07	0.00	0.86	0.00	0.82	1.39	1.00
time (sec)	N/A	0.364	0.076	0.007	0.000	0.400	0.000	0.572	0.242	0.732

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	28	0	2752	788	0	40	0	0	45	28
N.S.	1	0.00	98.29	28.14	0.00	1.43	0.00	0.00	1.61	1.00
time (sec)	N/A	1.035	6.249	0.522	0.000	0.443	0.000	0.000	2.059	0.560

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	33	23	31	25	24	420	0	39	23
N.S.	1	1.18	0.82	1.11	0.89	0.86	15.00	0.00	1.39	0.82
time (sec)	N/A	0.012	0.008	0.009	0.317	0.384	3.608	0.000	0.357	0.157

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	21	34	34	39	34	20	28
N.S.	1	1.00	1.00	0.75	1.21	1.21	1.39	1.21	0.71	1.00
time (sec)	N/A	0.011	0.004	0.010	0.425	0.389	0.963	0.686	0.281	0.025

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	37	25	31	30	24	0	30	24	25
N.S.	1	1.32	0.89	1.11	1.07	0.86	0.00	1.07	0.86	0.89
time (sec)	N/A	0.041	0.008	0.003	0.574	0.412	0.000	0.385	0.349	0.238

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	33	28	31	34	24	0	19	22	23
N.S.	1	1.18	1.00	1.11	1.21	0.86	0.00	0.68	0.79	0.82
time (sec)	N/A	0.038	0.006	0.005	0.777	0.392	0.000	0.260	0.339	0.230

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	28	520	1430	174	0	41	0	0	-1	28
N.S.	1	18.57	51.07	6.21	0.00	1.46	0.00	0.00	-0.04	1.00
time (sec)	N/A	0.324	4.240	0.013	0.000	0.469	0.000	0.000	0.000	0.951

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	47	105	36	42	24	167	0	39	28
N.S.	1	1.68	3.75	1.29	1.50	0.86	5.96	0.00	1.39	1.00
time (sec)	N/A	0.121	0.050	0.006	0.766	0.408	4.093	0.000	0.245	4.094

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	49	118	36	42	24	177	0	39	28
N.S.	1	1.75	4.21	1.29	1.50	0.86	6.32	0.00	1.39	1.00
time (sec)	N/A	0.113	0.121	0.008	0.608	0.389	4.797	0.000	0.140	0.688

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	49	145	34	40	24	189	0	39	28
N.S.	1	1.75	5.18	1.21	1.43	0.86	6.75	0.00	1.39	1.00
time (sec)	N/A	0.112	0.105	0.005	0.662	0.448	4.960	0.000	0.257	0.668

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	18	31	30	0	26	0	24	-1	28
N.S.	1	0.64	1.11	1.07	0.00	0.93	0.00	0.86	-0.04	1.00
time (sec)	N/A	0.022	0.008	0.006	0.000	0.407	0.000	0.368	0.000	0.140

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	32	20	17	0	16	0	19	16	28
N.S.	1	1.14	0.71	0.61	0.00	0.57	0.00	0.68	0.57	1.00
time (sec)	N/A	0.098	0.012	0.004	0.000	0.402	0.000	0.655	0.173	0.219

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	47	102	36	42	24	168	0	39	28
N.S.	1	1.68	3.64	1.29	1.50	0.86	6.00	0.00	1.39	1.00
time (sec)	N/A	0.124	0.039	0.008	0.615	0.463	4.264	0.000	0.237	4.086

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	49	123	30	0	24	0	0	39	28
N.S.	1	1.75	4.39	1.07	0.00	0.86	0.00	0.00	1.39	1.00
time (sec)	N/A	0.303	0.096	0.007	0.000	0.425	0.000	0.000	0.294	2.605

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	82	20	20	88	20	20	28
N.S.	1	1.00	1.00	2.93	0.71	0.71	3.14	0.71	0.71	1.00
time (sec)	N/A	0.010	0.004	0.231	0.506	0.401	1.033	0.356	0.294	0.026

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	24	29	34	34	39	34	20	28
N.S.	1	1.00	0.86	1.04	1.21	1.21	1.39	1.21	0.71	1.00
time (sec)	N/A	0.013	0.006	0.010	0.553	0.394	28.445	0.291	0.298	0.058

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	56	34	34	39	34	20	28
N.S.	1	1.00	1.00	2.00	1.21	1.21	1.39	1.21	0.71	1.00
time (sec)	N/A	0.011	0.004	0.204	0.369	0.392	0.968	0.443	0.270	0.024

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	33	23	40	25	24	416	0	24	23
N.S.	1	1.18	0.82	1.43	0.89	0.86	14.86	0.00	0.86	0.82
time (sec)	N/A	0.010	0.009	0.007	0.436	0.394	4.285	0.000	0.339	0.609

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	49	126	44	0	24	0	0	39	28
N.S.	1	1.75	4.50	1.57	0.00	0.86	0.00	0.00	1.39	1.00
time (sec)	N/A	0.320	0.103	0.007	0.000	0.411	0.000	0.000	0.324	2.643

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	33	108	40	46	24	143	0	39	28
N.S.	1	1.18	3.86	1.43	1.64	0.86	5.11	0.00	1.39	1.00
time (sec)	N/A	0.080	0.045	0.010	0.670	0.397	4.608	0.000	0.185	2.293

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	33	23	40	25	24	418	0	24	23
N.S.	1	1.18	0.82	1.43	0.89	0.86	14.93	0.00	0.86	0.82
time (sec)	N/A	0.010	0.009	0.007	0.564	0.410	4.708	0.000	0.363	1.427

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	298	0	53	0	58	236	28
N.S.	1	1.00	1.00	10.64	0.00	1.89	0.00	2.07	8.43	1.00
time (sec)	N/A	0.384	0.061	0.156	0.000	0.387	0.000	0.287	1.489	0.087

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	28	0	0	0	0	151	0	0	-1	28
N.S.	1	0.00	0.00	0.00	0.00	5.39	0.00	0.00	-0.04	1.00
time (sec)	N/A	1.670	0.315	0.716	0.000	1.033	0.000	0.000	0.000	18.726

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	28	0	0	0	0	146	0	0	48	28
N.S.	1	0.00	0.00	0.00	0.00	5.21	0.00	0.00	1.71	1.00
time (sec)	N/A	1.378	0.247	0.648	0.000	0.616	0.000	0.000	1.001	4.977

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	28	0	0	0	0	138	0	0	-1	28
N.S.	1	0.00	0.00	0.00	0.00	4.93	0.00	0.00	-0.04	1.00
time (sec)	N/A	1.548	0.265	0.731	0.000	1.073	0.000	0.000	0.000	10.700

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	49	33	45	37	31	65	0	24	28
N.S.	1	1.75	1.18	1.61	1.32	1.11	2.32	0.00	0.86	1.00
time (sec)	N/A	0.030	0.026	0.010	0.473	0.395	3.689	0.000	0.325	0.328

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	F	C	F	A	F	F	B	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	29	262	0	149	0	42	0	0	25	29
N.S.	1	9.03	0.00	5.14	0.00	1.45	0.00	0.00	0.86	1.00
time (sec)	N/A	1.121	0.245	1.556	0.000	0.414	0.000	0.000	0.394	0.060

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	27	27	143	0	42	0	0	-1	29
N.S.	1	0.93	0.93	4.93	0.00	1.45	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.023	0.006	1.470	0.000	0.398	0.000	0.000	0.000	0.073

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	27	27	145	0	42	0	0	-1	29
N.S.	1	0.93	0.93	5.00	0.00	1.45	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.024	0.006	1.490	0.000	0.392	0.000	0.000	0.000	0.058

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	59	35	35	32	36	21	29
N.S.	1	1.00	1.00	2.03	1.21	1.21	1.10	1.24	0.72	1.00
time (sec)	N/A	0.015	0.003	0.230	0.520	0.394	0.780	0.364	0.250	0.032

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	26	21	64	60	21	37	29
N.S.	1	1.00	1.00	0.90	0.72	2.21	2.07	0.72	1.28	1.00
time (sec)	N/A	0.019	0.007	0.021	0.776	0.392	0.965	0.533	1.081	0.032

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	42	29	26	34	25	48	34	25	29
N.S.	1	1.45	1.00	0.90	1.17	0.86	1.66	1.17	0.86	1.00
time (sec)	N/A	0.025	0.013	0.004	0.341	0.379	0.631	0.412	0.396	0.031

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	29	46	742	210	0	36	0	0	-1	29
N.S.	1	1.59	25.59	7.24	0.00	1.24	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.111	1.717	0.323	0.000	0.446	0.000	0.000	0.000	0.426

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	35	29	40	56	48	0	26	25	29
N.S.	1	1.21	1.00	1.38	1.93	1.66	0.00	0.90	0.86	1.00
time (sec)	N/A	0.050	0.045	0.010	0.515	0.389	0.000	0.423	0.126	0.417

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	29	0	0	150	0	48	0	0	-1	29
N.S.	1	0.00	0.00	5.17	0.00	1.66	0.00	0.00	-0.03	1.00
time (sec)	N/A	1.038	0.196	0.949	0.000	0.392	0.000	0.000	0.000	2.704

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	29	39	736	1382	0	27	0	33	-1	29
N.S.	1	1.34	25.38	47.66	0.00	0.93	0.00	1.14	-0.03	1.00
time (sec)	N/A	0.051	2.483	0.673	0.000	0.421	0.000	0.385	0.000	0.173

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	35	21	64	53	21	21	29
N.S.	1	1.00	1.00	1.21	0.72	2.21	1.83	0.72	0.72	1.00
time (sec)	N/A	0.020	0.008	0.012	0.701	0.409	0.934	0.287	0.382	0.039

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	29	0	0	74	0	36	0	0	-1	29
N.S.	1	0.00	0.00	2.55	0.00	1.24	0.00	0.00	-0.03	1.00
time (sec)	N/A	1.554	0.338	0.402	0.000	0.467	0.000	0.000	0.000	1.231

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	29	0	0	85	0	36	0	0	-1	29
N.S.	1	0.00	0.00	2.93	0.00	1.24	0.00	0.00	-0.03	1.00
time (sec)	N/A	1.151	0.261	0.406	0.000	0.431	0.000	0.000	0.000	1.240

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	40	25	31	28	26	53	28	26	25
N.S.	1	1.33	0.83	1.03	0.93	0.87	1.77	0.93	0.87	0.83
time (sec)	N/A	0.016	0.007	0.005	0.543	0.389	0.816	0.277	0.220	0.032

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	40	25	33	28	26	56	28	25	30
N.S.	1	1.33	0.83	1.10	0.93	0.87	1.87	0.93	0.83	1.00
time (sec)	N/A	0.015	0.008	0.006	0.395	0.391	1.407	0.449	0.211	0.024

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	55	30	33	34	26	0	41	26	30
N.S.	1	1.83	1.00	1.10	1.13	0.87	0.00	1.37	0.87	1.00
time (sec)	N/A	0.067	0.015	0.005	0.629	0.401	0.000	0.445	0.231	0.253

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	61	28	30	0	26	0	41	49	30
N.S.	1	2.03	0.93	1.00	0.00	0.87	0.00	1.37	1.63	1.00
time (sec)	N/A	0.077	0.011	0.003	0.000	0.394	0.000	0.727	0.193	0.220

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	55	119	33	0	26	0	0	45	30
N.S.	1	1.83	3.97	1.10	0.00	0.87	0.00	0.00	1.50	1.00
time (sec)	N/A	0.434	0.109	0.006	0.000	0.441	0.000	0.000	0.250	0.824

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	30	373	1680	387	0	25	0	0	206	30
N.S.	1	12.43	56.00	12.90	0.00	0.83	0.00	0.00	6.87	1.00
time (sec)	N/A	0.997	6.708	0.019	0.000	0.448	0.000	0.000	0.138	0.085

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	31	23	20	0	25	0	19	26	30
N.S.	1	1.03	0.77	0.67	0.00	0.83	0.00	0.63	0.87	1.00
time (sec)	N/A	0.146	0.013	0.007	0.000	0.390	0.000	0.447	0.231	0.284

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	41	27	30	27	26	0	30	26	30
N.S.	1	1.37	0.90	1.00	0.90	0.87	0.00	1.00	0.87	1.00
time (sec)	N/A	0.048	0.015	0.004	0.716	0.412	0.000	0.428	0.332	0.133

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	30	23	717	1352	0	26	0	34	-1	30
N.S.	1	0.77	23.90	45.07	0.00	0.87	0.00	1.13	-0.03	1.00
time (sec)	N/A	0.053	3.367	0.423	0.000	0.421	0.000	0.317	0.000	0.400

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	36	20	19	0	18	0	23	18	30
N.S.	1	1.20	0.67	0.63	0.00	0.60	0.00	0.77	0.60	1.00
time (sec)	N/A	0.156	0.012	0.004	0.000	0.390	0.000	0.768	0.227	0.234

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	55	117	38	0	26	0	0	45	30
N.S.	1	1.83	3.90	1.27	0.00	0.87	0.00	0.00	1.50	1.00
time (sec)	N/A	0.358	0.104	0.007	0.000	0.404	0.000	0.000	0.302	2.632

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	55	117	38	0	26	0	0	45	30
N.S.	1	1.83	3.90	1.27	0.00	0.87	0.00	0.00	1.50	1.00
time (sec)	N/A	0.350	0.073	0.007	0.000	0.398	0.000	0.000	0.254	2.610

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	30	0	0	0	0	127	0	0	37	30
N.S.	1	0.00	0.00	0.00	0.00	4.23	0.00	0.00	1.23	1.00
time (sec)	N/A	1.223	0.469	0.298	0.000	0.434	0.000	0.000	0.418	0.577

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	30	0	0	0	0	137	0	0	38	30
N.S.	1	0.00	0.00	0.00	0.00	4.57	0.00	0.00	1.27	1.00
time (sec)	N/A	1.121	0.444	0.276	0.000	0.457	0.000	0.000	0.422	1.154

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	B	F	B	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	30	0	947	54	0	53	0	0	-1	30
N.S.	1	0.00	31.57	1.80	0.00	1.77	0.00	0.00	-0.03	1.00
time (sec)	N/A	1.005	5.566	0.036	0.000	0.453	0.000	0.000	0.000	7.344

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	30	0	0	85	0	42	0	0	-1	30
N.S.	1	0.00	0.00	2.83	0.00	1.40	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.838	0.313	0.512	0.000	0.789	0.000	0.000	0.000	3.376

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	41	24	23	25	82	23	177	31
N.S.	1	1.00	1.32	0.77	0.74	0.81	2.65	0.74	5.71	1.00
time (sec)	N/A	0.013	0.016	0.197	0.485	0.392	1.447	0.234	0.171	0.001

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	31	27	496	423	0	65	0	0	275	31
N.S.	1	0.87	16.00	13.65	0.00	2.10	0.00	0.00	8.87	1.00
time (sec)	N/A	0.075	0.721	0.095	0.000	0.432	0.000	0.000	0.203	1.125

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	27	291	1641	0	26	0	0	273	31
N.S.	1	0.87	9.39	52.94	0.00	0.84	0.00	0.00	8.81	1.00
time (sec)	N/A	0.070	0.588	0.066	0.000	0.421	0.000	0.000	0.097	1.149

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	48	24	23	27	82	23	177	31
N.S.	1	1.00	1.55	0.77	0.74	0.87	2.65	0.74	5.71	1.00
time (sec)	N/A	0.010	0.012	0.025	0.732	0.400	1.288	0.233	0.171	0.040

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	24	37	44	26	38	176	31
N.S.	1	1.00	1.00	0.77	1.19	1.42	0.84	1.23	5.68	1.00
time (sec)	N/A	0.012	0.008	0.025	0.348	0.395	1.463	0.245	0.159	0.001

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	23	46	240	0	44	0	0	204	31
N.S.	1	0.74	1.48	7.74	0.00	1.42	0.00	0.00	6.58	1.00
time (sec)	N/A	0.058	0.007	0.040	0.000	0.431	0.000	0.000	0.227	0.899

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	44	24	37	44	27	38	177	31
N.S.	1	1.00	1.42	0.77	1.19	1.42	0.87	1.23	5.71	1.00
time (sec)	N/A	0.012	0.010	0.025	0.319	0.391	1.251	0.389	0.064	0.065

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	B	C	F	B	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	0	69	491	0	48	0	0	-1	31
N.S.	1	0.00	2.23	15.84	0.00	1.55	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.736	0.196	0.009	0.000	0.456	0.000	0.000	0.000	1.027

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	31	33	701	1110	0	27	0	26	-1	31
N.S.	1	1.06	22.61	35.81	0.00	0.87	0.00	0.84	-0.03	1.00
time (sec)	N/A	0.040	1.562	0.588	0.000	0.418	0.000	0.464	0.000	0.077

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	31	20	898	1252	0	27	0	25	-1	31
N.S.	1	0.65	28.97	40.39	0.00	0.87	0.00	0.81	-0.03	1.00
time (sec)	N/A	0.048	2.276	0.413	0.000	0.427	0.000	0.660	0.000	0.156

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	30	30	24	45	63	20	25	-1	31
N.S.	1	0.97	0.97	0.77	1.45	2.03	0.65	0.81	-0.03	1.00
time (sec)	N/A	0.018	0.008	0.009	0.552	0.406	0.863	0.508	0.000	0.134

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	41	20	23	25	73	23	23	31
N.S.	1	1.00	1.32	0.65	0.74	0.81	2.35	0.74	0.74	1.00
time (sec)	N/A	0.011	0.018	0.035	0.471	0.399	1.585	0.256	0.374	0.033

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	48	20	23	27	73	23	23	31
N.S.	1	1.00	1.55	0.65	0.74	0.87	2.35	0.74	0.74	1.00
time (sec)	N/A	0.011	0.010	0.018	0.492	0.403	1.375	0.283	0.335	0.029

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	30	32	37	45	56	37	23	31
N.S.	1	1.00	0.97	1.03	1.19	1.45	1.81	1.19	0.74	1.00
time (sec)	N/A	0.015	0.009	0.038	0.449	0.400	98.974	0.318	0.516	0.059

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	44	32	37	44	24	37	23	31
N.S.	1	1.00	1.42	1.03	1.19	1.42	0.77	1.19	0.74	1.00
time (sec)	N/A	0.012	0.012	0.022	0.578	0.386	1.351	0.231	0.327	0.025

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	168	48	58	27	139	0	49	31
N.S.	1	1.00	5.42	1.55	1.87	0.87	4.48	0.00	1.58	1.00
time (sec)	N/A	0.107	0.120	0.013	0.895	0.417	6.021	0.000	0.402	8.908

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	32	0	533	828	0	28	0	0	-1	32
N.S.	1	0.00	16.66	25.88	0.00	0.88	0.00	0.00	-0.03	1.00
time (sec)	N/A	2.411	2.154	0.265	0.000	0.464	0.000	0.000	0.000	1.478

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	35	25	22	0	29	0	23	28	32
N.S.	1	1.09	0.78	0.69	0.00	0.91	0.00	0.72	0.88	1.00
time (sec)	N/A	0.157	0.013	0.008	0.000	0.390	0.000	0.322	0.237	0.311

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	0	298	13798	0	68	0	0	-1	32
N.S.	1	0.00	9.31	431.19	0.00	2.12	0.00	0.00	-0.03	1.00
time (sec)	N/A	2.076	1.239	0.076	0.000	0.454	0.000	0.000	0.000	1.502

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	35	25	22	0	27	0	19	28	32
N.S.	1	1.09	0.78	0.69	0.00	0.84	0.00	0.59	0.88	1.00
time (sec)	N/A	0.162	0.011	0.006	0.000	0.399	0.000	0.363	0.222	0.129

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	32	37	292	487	0	30	0	33	-1	32
N.S.	1	1.16	9.12	15.22	0.00	0.94	0.00	1.03	-0.03	1.00
time (sec)	N/A	0.051	0.542	0.327	0.000	0.436	0.000	0.727	0.000	0.172

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	A	A	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	0	33	0	0	45	44	46	29	33
N.S.	1	0.00	1.00	0.00	0.00	1.36	1.33	1.39	0.88	1.00
time (sec)	N/A	0.026	0.273	0.451	0.000	0.403	12.218	0.422	0.545	0.035

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	49	33	34	37	29	563	0	49	33
N.S.	1	1.48	1.00	1.03	1.12	0.88	17.06	0.00	1.48	1.00
time (sec)	N/A	0.016	0.010	0.006	0.398	0.395	3.342	0.000	0.373	0.109

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	26	290	1625	0	63	0	0	272	33
N.S.	1	0.79	8.79	49.24	0.00	1.91	0.00	0.00	8.24	1.00
time (sec)	N/A	0.071	0.636	0.055	0.000	0.411	0.000	0.000	0.306	1.151

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	33	26	515	432	0	28	0	0	274	33
N.S.	1	0.79	15.61	13.09	0.00	0.85	0.00	0.00	8.30	1.00
time (sec)	N/A	0.078	0.867	0.099	0.000	0.432	0.000	0.000	0.096	1.137

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	33	26	394	435	0	66	0	0	274	33
N.S.	1	0.79	11.94	13.18	0.00	2.00	0.00	0.00	8.30	1.00
time (sec)	N/A	0.073	1.449	0.102	0.000	0.440	0.000	0.000	0.207	1.133

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	49	28	36	37	29	199	0	49	28
N.S.	1	1.48	0.85	1.09	1.12	0.88	6.03	0.00	1.48	0.85
time (sec)	N/A	0.014	0.008	0.006	0.339	0.391	2.696	0.000	0.327	0.110

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	49	33	34	37	29	563	0	49	28
N.S.	1	1.48	1.00	1.03	1.12	0.88	17.06	0.00	1.48	0.85
time (sec)	N/A	0.013	0.008	0.005	0.353	0.400	2.820	0.000	0.322	0.101

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	B	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	70	31	36	72	42	0	0	41	57
N.S.	1	2.12	0.94	1.09	2.18	1.27	0.00	0.00	1.24	1.73
time (sec)	N/A	0.194	0.019	0.003	0.338	0.382	0.000	0.000	0.260	5.100

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	B	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	72	31	36	74	37	0	0	35	57
N.S.	1	2.18	0.94	1.09	2.24	1.12	0.00	0.00	1.06	1.73
time (sec)	N/A	0.185	0.015	0.003	0.339	0.378	0.000	0.000	0.198	5.272

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	33	0	2741	841	0	56	0	0	43	33
N.S.	1	0.00	83.06	25.48	0.00	1.70	0.00	0.00	1.30	1.00
time (sec)	N/A	1.224	6.240	0.182	0.000	26.702	0.000	0.000	0.925	0.565

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	54	14	47	50	27	47	12	33
N.S.	1	1.00	1.64	0.42	1.42	1.52	0.82	1.42	0.36	1.00
time (sec)	N/A	0.005	0.005	0.232	0.590	0.425	0.717	0.350	0.203	0.168

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	33	0	287	830	0	28	0	0	-1	33
N.S.	1	0.00	8.70	25.15	0.00	0.85	0.00	0.00	-0.03	1.00
time (sec)	N/A	2.488	1.899	0.180	0.000	0.474	0.000	0.000	0.000	1.131

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	40	301	0	31	0	36	27	33
N.S.	1	1.00	1.21	9.12	0.00	0.94	0.00	1.09	0.82	1.00
time (sec)	N/A	0.020	0.022	0.032	0.000	0.450	0.000	0.503	0.297	0.378

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	C	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	89	61	33	0	95	0	41	31	33
N.S.	1	2.70	1.85	1.00	0.00	2.88	0.00	1.24	0.94	1.00
time (sec)	N/A	0.024	0.014	0.255	0.000	2.872	0.000	0.351	0.396	0.141

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	33	489	0	0	0	0	0	0	-1	33
N.S.	1	14.82	0.00	0.00	0.00	0.00	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.641	0.244	0.237	0.000	0.000	0.000	0.000	0.000	0.201

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	33	37	702	1358	0	31	0	34	-1	33
N.S.	1	1.12	21.27	41.15	0.00	0.94	0.00	1.03	-0.03	1.00
time (sec)	N/A	0.053	3.196	0.665	0.000	0.646	0.000	0.389	0.000	0.157

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	33	37	702	1382	0	29	0	34	-1	33
N.S.	1	1.12	21.27	41.88	0.00	0.88	0.00	1.03	-0.03	1.00
time (sec)	N/A	0.053	3.000	0.679	0.000	0.450	0.000	0.387	0.000	0.164

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	73	33	33	0	29	0	37	57	33
N.S.	1	2.21	1.00	1.00	0.00	0.88	0.00	1.12	1.73	1.00
time (sec)	N/A	0.103	0.015	0.003	0.000	0.398	0.000	0.306	0.196	0.242

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	32	32	26	49	67	53	27	-1	33
N.S.	1	0.97	0.97	0.79	1.48	2.03	1.61	0.82	-0.03	1.00
time (sec)	N/A	0.019	0.009	0.007	0.786	0.409	0.926	0.295	0.000	0.160

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	49	33	36	37	29	260	0	49	33
N.S.	1	1.48	1.00	1.09	1.12	0.88	7.88	0.00	1.48	1.00
time (sec)	N/A	0.031	0.016	0.007	0.452	0.386	3.945	0.000	0.412	0.131

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	33	0	0	101	0	71	0	0	75	33
N.S.	1	0.00	0.00	3.06	0.00	2.15	0.00	0.00	2.27	1.00
time (sec)	N/A	2.093	0.563	0.780	0.000	0.434	0.000	0.000	3.243	0.909

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	47	85	36	39	29	60	39	29	33
N.S.	1	1.42	2.58	1.09	1.18	0.88	1.82	1.18	0.88	1.00
time (sec)	N/A	0.067	0.040	0.008	0.528	0.420	2.729	0.428	0.342	0.236

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	19	19	16	15	28	56	15	15	19
N.S.	1	0.58	0.58	0.48	0.45	0.85	1.70	0.45	0.45	0.58
time (sec)	N/A	0.289	0.017	0.003	0.340	0.387	4.910	0.393	0.278	0.440

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	34	56	632	271	0	68	0	0	179	34
N.S.	1	1.65	18.59	7.97	0.00	2.00	0.00	0.00	5.26	1.00
time (sec)	N/A	0.409	2.283	0.197	0.000	0.430	0.000	0.000	0.322	0.108

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	315	132	425	0	68	0	0	227	34
N.S.	1	9.26	3.88	12.50	0.00	2.00	0.00	0.00	6.68	1.00
time (sec)	N/A	1.211	0.962	0.087	0.000	0.423	0.000	0.000	0.082	0.119

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	52	175	136	0	66	0	0	-1	34
N.S.	1	1.53	5.15	4.00	0.00	1.94	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.090	0.305	0.009	0.000	0.444	0.000	0.000	0.000	0.689

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	53	178	136	0	68	0	0	-1	34
N.S.	1	1.56	5.24	4.00	0.00	2.00	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.089	0.258	0.007	0.000	0.450	0.000	0.000	0.000	0.657

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	34	62	221	212	0	73	0	0	-1	34
N.S.	1	1.82	6.50	6.24	0.00	2.15	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.113	0.408	0.200	0.000	0.443	0.000	0.000	0.000	0.489

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	34	0	3217	4112	0	38	0	0	-1	34
N.S.	1	0.00	94.62	120.94	0.00	1.12	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.474	1.628	1.812	0.000	0.433	0.000	0.000	0.000	0.168

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	0	759	1625	0	40	0	0	-1	34
N.S.	1	0.00	22.32	47.79	0.00	1.18	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.058	0.559	1.317	0.000	0.429	0.000	0.000	0.000	0.160

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	0	830	2934	0	38	0	0	-1	34
N.S.	1	0.00	24.41	86.29	0.00	1.12	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.154	0.538	0.786	0.000	0.429	0.000	0.000	0.000	0.193

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	34	0	768	1633	0	40	0	0	-1	34
N.S.	1	0.00	22.59	48.03	0.00	1.18	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.067	0.933	1.252	0.000	0.429	0.000	0.000	0.000	0.157

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	A	F	A	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	34	0	892	60	0	58	0	0	-1	34
N.S.	1	0.00	26.24	1.76	0.00	1.71	0.00	0.00	-0.03	1.00
time (sec)	N/A	1.078	5.541	0.335	0.000	0.443	0.000	0.000	0.000	7.324

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	42	45	83	41	0	-1	35
N.S.	1	1.00	1.00	1.20	1.29	2.37	1.17	0.00	-0.03	1.00
time (sec)	N/A	0.020	0.004	0.230	0.534	0.813	0.865	0.000	0.000	0.215

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	A	A	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	0	35	0	0	51	56	60	31	35
N.S.	1	0.00	1.00	0.00	0.00	1.46	1.60	1.71	0.89	1.00
time (sec)	N/A	0.024	0.271	0.381	0.000	0.400	11.810	0.462	0.610	0.048

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	36	34	30	0	106	0	36	29	35
N.S.	1	1.03	0.97	0.86	0.00	3.03	0.00	1.03	0.83	1.00
time (sec)	N/A	0.012	0.032	0.004	0.000	0.401	0.000	0.638	0.280	0.126

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	53	30	36	37	31	68	37	31	35
N.S.	1	1.51	0.86	1.03	1.06	0.89	1.94	1.06	0.89	1.00
time (sec)	N/A	0.020	0.009	0.004	0.525	0.386	1.606	0.373	0.236	0.021

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	229	112	167	0	73	0	0	175	35
N.S.	1	6.54	3.20	4.77	0.00	2.09	0.00	0.00	5.00	1.00
time (sec)	N/A	0.916	1.036	0.047	0.000	0.415	0.000	0.000	0.219	0.313

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	A	A	A	F	F	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	0	35	34	31	33	0	0	33	35
N.S.	1	0.00	1.00	0.97	0.89	0.94	0.00	0.00	0.94	1.00
time (sec)	N/A	2.688	0.472	0.007	0.463	0.394	0.000	0.000	0.286	0.364

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	25	22	20	45	38	19	38	19	37
N.S.	1	0.71	0.63	0.57	1.29	1.09	0.54	1.09	0.54	1.06
time (sec)	N/A	0.010	0.009	0.019	0.533	0.392	1.525	0.309	0.393	0.079

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	47	28	25	0	30	0	28	31	35
N.S.	1	1.34	0.80	0.71	0.00	0.86	0.00	0.80	0.89	1.00
time (sec)	N/A	0.169	0.012	0.007	0.000	0.407	0.000	0.347	0.306	0.399

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	50	303	0	37	0	35	-1	35
N.S.	1	1.00	1.43	8.66	0.00	1.06	0.00	1.00	-0.03	1.00
time (sec)	N/A	0.035	0.017	0.230	0.000	0.447	0.000	0.263	0.000	0.373

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	73	35	37	36	31	0	37	57	35
N.S.	1	2.09	1.00	1.06	1.03	0.89	0.00	1.06	1.63	1.00
time (sec)	N/A	0.099	0.019	0.005	0.595	0.401	0.000	0.799	0.254	0.191

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	35	37	789	1182	0	33	0	35	-1	35
N.S.	1	1.06	22.54	33.77	0.00	0.94	0.00	1.00	-0.03	1.00
time (sec)	N/A	0.053	1.637	0.323	0.000	0.429	0.000	0.461	0.000	0.270

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	81	33	35	0	31	0	59	65	35
N.S.	1	2.31	0.94	1.00	0.00	0.89	0.00	1.69	1.86	1.00
time (sec)	N/A	0.104	0.013	0.003	0.000	0.393	0.000	0.270	0.219	0.258

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	35	39	505	782	0	33	0	35	-1	35
N.S.	1	1.11	14.43	22.34	0.00	0.94	0.00	1.00	-0.03	1.00
time (sec)	N/A	0.055	0.981	0.391	0.000	0.431	0.000	0.730	0.000	0.185

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	35	0	0	0	0	0	0	0	-1	35
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.739	0.293	0.236	0.000	0.000	0.000	0.000	0.000	0.699

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	32	38	58	29	61	0	-1	35
N.S.	1	1.00	0.91	1.09	1.66	0.83	1.74	0.00	-0.03	1.00
time (sec)	N/A	0.015	0.007	0.205	0.342	0.398	1.558	0.000	0.000	0.190

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	36	38	45	34	76	46	-1	35
N.S.	1	1.00	1.03	1.09	1.29	0.97	2.17	1.31	-0.03	1.00
time (sec)	N/A	0.014	0.016	0.015	0.400	0.400	1.057	0.288	0.000	0.156

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	42	38	58	29	60	30	-1	35
N.S.	1	1.00	1.20	1.09	1.66	0.83	1.71	0.86	-0.03	1.00
time (sec)	N/A	0.014	0.015	0.233	0.395	0.416	1.297	0.294	0.000	0.136

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	25	25	20	58	29	19	0	-1	35
N.S.	1	0.71	0.71	0.57	1.66	0.83	0.54	0.00	-0.03	1.00
time (sec)	N/A	0.010	0.005	0.207	0.335	0.390	1.523	0.000	0.000	0.175

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	25	25	20	45	34	34	38	-1	35
N.S.	1	0.71	0.71	0.57	1.29	0.97	0.97	1.09	-0.03	1.00
time (sec)	N/A	0.010	0.007	0.195	0.374	0.402	0.982	0.310	0.000	0.144

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	47	28	25	0	30	0	28	31	35
N.S.	1	1.34	0.80	0.71	0.00	0.86	0.00	0.80	0.89	1.00
time (sec)	N/A	0.165	0.053	0.008	0.000	0.400	0.000	0.276	0.301	0.479

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	40	27	0	35	0	0	-1	35
N.S.	1	1.00	1.14	0.77	0.00	1.00	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.037	0.030	0.195	0.000	0.537	0.000	0.000	0.000	0.303

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	35	0	0	0	0	0	0	0	-1	35
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03	1.00
time (sec)	N/A	1.144	0.281	0.286	0.000	0.000	0.000	0.000	0.000	5.154

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	62	0	28	42	0	-1	35
N.S.	1	1.00	1.00	1.77	0.00	0.80	1.20	0.00	-0.03	1.00
time (sec)	N/A	0.014	0.013	0.035	0.000	0.396	0.340	0.000	0.000	0.066

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	36	45	44	44	37	44	30	36
N.S.	1	1.00	1.00	1.25	1.22	1.22	1.03	1.22	0.83	1.00
time (sec)	N/A	0.016	0.007	0.244	0.552	0.401	0.813	0.336	0.250	0.043

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	47	28	36	46	31	138	63	189	36
N.S.	1	1.31	0.78	1.00	1.28	0.86	3.83	1.75	5.25	1.00
time (sec)	N/A	0.017	0.005	0.238	0.461	0.390	2.278	0.299	0.074	0.061

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F(-2)	A	F	F	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	289	214	1161	0	159	0	0	51	36
N.S.	1	8.03	5.94	32.25	0.00	4.42	0.00	0.00	1.42	1.00
time (sec)	N/A	1.989	1.642	0.087	0.000	0.475	0.000	0.000	2.251	0.352

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	36	0	640	740	0	43	0	0	-1	36
N.S.	1	0.00	17.78	20.56	0.00	1.19	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.136	0.827	0.434	0.000	0.432	0.000	0.000	0.000	0.174

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	47	28	30	46	31	124	63	35	36
N.S.	1	1.31	0.78	0.83	1.28	0.86	3.44	1.75	0.97	1.00
time (sec)	N/A	0.017	0.005	0.030	0.737	0.396	2.581	0.212	0.437	0.034

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	36	0	0	70	0	78	0	0	-1	36
N.S.	1	0.00	0.00	1.94	0.00	2.17	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.694	0.244	0.440	0.000	0.471	0.000	0.000	0.000	15.208

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	154	114	206	0	81	0	0	-1	37
N.S.	1	4.16	3.08	5.57	0.00	2.19	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.638	0.853	0.055	0.000	0.458	0.000	0.000	0.000	0.180

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	159	116	210	0	87	0	0	-1	37
N.S.	1	4.30	3.14	5.68	0.00	2.35	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.553	0.753	0.023	0.000	0.447	0.000	0.000	0.000	0.166

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	72	326	0	30	0	0	-1	37
N.S.	1	1.00	1.95	8.81	0.00	0.81	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.334	0.118	0.037	0.000	0.461	0.000	0.000	0.000	0.299

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	53	30	27	0	34	0	34	33	37
N.S.	1	1.43	0.81	0.73	0.00	0.92	0.00	0.92	0.89	1.00
time (sec)	N/A	0.165	0.014	0.007	0.000	0.401	0.000	0.703	0.326	0.419

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	44	301	0	35	0	42	29	37
N.S.	1	1.00	1.19	8.14	0.00	0.95	0.00	1.14	0.78	1.00
time (sec)	N/A	0.023	0.021	0.220	0.000	0.422	0.000	0.322	0.301	0.386

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	81	37	40	39	33	0	59	65	37
N.S.	1	2.19	1.00	1.08	1.05	0.89	0.00	1.59	1.76	1.00
time (sec)	N/A	0.105	0.018	0.003	0.361	0.382	0.000	0.316	0.293	0.204

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	37	33	716	1020	0	31	0	35	-1	37
N.S.	1	0.89	19.35	27.57	0.00	0.84	0.00	0.95	-0.03	1.00
time (sec)	N/A	0.041	1.093	0.263	0.000	0.432	0.000	0.341	0.000	0.200

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	83	33	27	0	60	0	40	-1	37
N.S.	1	2.24	0.89	0.73	0.00	1.62	0.00	1.08	-0.03	1.00
time (sec)	N/A	0.110	0.011	0.232	0.000	0.384	0.000	0.306	0.000	0.249

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	33	31	86	43	33	20	0	25	41
N.S.	1	0.89	0.84	2.32	1.16	0.89	0.54	0.00	0.68	1.11
time (sec)	N/A	0.029	0.025	0.214	0.535	0.411	4.039	0.000	0.749	0.163

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	33	29	86	43	33	19	0	25	41
N.S.	1	0.89	0.78	2.32	1.16	0.89	0.51	0.00	0.68	1.11
time (sec)	N/A	0.033	0.014	0.235	0.490	0.400	6.823	0.000	0.290	0.132

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	33	33	86	43	33	20	0	25	41
N.S.	1	0.89	0.89	2.32	1.16	0.89	0.54	0.00	0.68	1.11
time (sec)	N/A	0.033	0.017	0.237	0.767	0.411	3.988	0.000	0.493	0.161

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	33	31	86	43	33	20	0	25	41
N.S.	1	0.89	0.84	2.32	1.16	0.89	0.54	0.00	0.68	1.11
time (sec)	N/A	0.031	0.015	0.220	0.923	0.396	7.051	0.000	0.293	0.150

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	33	31	86	43	33	20	0	25	41
N.S.	1	0.89	0.84	2.32	1.16	0.89	0.54	0.00	0.68	1.11
time (sec)	N/A	0.029	0.021	0.232	0.587	0.411	6.995	0.000	0.549	0.156

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	53	30	27	0	34	0	34	33	37
N.S.	1	1.43	0.81	0.73	0.00	0.92	0.00	0.92	0.89	1.00
time (sec)	N/A	0.167	0.054	0.007	0.000	0.388	0.000	0.857	0.422	0.492

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	37	0	0	0	0	0	0	0	-1	37
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.923	0.236	0.219	0.000	0.000	0.000	0.000	0.000	0.905

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	35	57	0	26	42	0	-1	37
N.S.	1	1.00	0.95	1.54	0.00	0.70	1.14	0.00	-0.03	1.00
time (sec)	N/A	0.013	0.011	0.014	0.000	0.421	0.286	0.000	0.000	0.044

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F(-2)	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	273	195	3392	0	250	0	0	532	38
N.S.	1	7.18	5.13	89.26	0.00	6.58	0.00	0.00	14.00	1.00
time (sec)	N/A	6.452	4.564	0.076	0.000	0.594	0.000	0.000	0.337	0.176

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F(-2)	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	308	210	4007	0	299	0	0	531	38
N.S.	1	8.11	5.53	105.45	0.00	7.87	0.00	0.00	13.97	1.00
time (sec)	N/A	8.699	5.771	0.059	0.000	0.744	0.000	0.000	0.354	0.199

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	113	63	104	0	37	0	0	-1	38
N.S.	1	2.97	1.66	2.74	0.00	0.97	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.974	0.265	0.066	0.000	0.464	0.000	0.000	0.000	1.925

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	111	61	102	0	37	0	0	-1	38
N.S.	1	2.92	1.61	2.68	0.00	0.97	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.779	0.223	0.042	0.000	0.468	0.000	0.000	0.000	1.838

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	26	21	20	0	36	0	0	28	38
N.S.	1	0.68	0.55	0.53	0.00	0.95	0.00	0.00	0.74	1.00
time (sec)	N/A	0.772	0.214	0.010	0.000	0.414	0.000	0.000	0.293	0.191

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	47	28	36	48	34	138	35	189	38
N.S.	1	1.24	0.74	0.95	1.26	0.89	3.63	0.92	4.97	1.00
time (sec)	N/A	0.017	0.005	0.024	0.422	0.400	2.426	0.324	0.200	0.001

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	47	26	36	64	52	65	50	189	38
N.S.	1	1.24	0.68	0.95	1.68	1.37	1.71	1.32	4.97	1.00
time (sec)	N/A	0.018	0.005	0.026	0.321	0.403	2.487	0.308	0.069	0.001

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	47	26	36	60	49	65	78	189	38
N.S.	1	1.24	0.68	0.95	1.58	1.29	1.71	2.05	4.97	1.00
time (sec)	N/A	0.018	0.006	0.022	0.321	0.403	2.263	0.332	0.063	0.104

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	47	50	36	60	34	63	35	189	38
N.S.	1	1.24	1.32	0.95	1.58	0.89	1.66	0.92	4.97	1.00
time (sec)	N/A	0.020	0.027	0.009	0.427	0.393	45.161	0.311	0.062	0.057

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	38	0	1701	306	0	99	0	0	56	38
N.S.	1	0.00	44.76	8.05	0.00	2.61	0.00	0.00	1.47	1.00
time (sec)	N/A	0.691	6.162	0.117	0.000	0.506	0.000	0.000	0.821	0.555

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	28	25	30	34	66	29	29	38
N.S.	1	1.00	0.74	0.66	0.79	0.89	1.74	0.76	0.76	1.00
time (sec)	N/A	0.028	0.016	0.006	0.321	0.396	0.473	0.361	0.333	0.039

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	33	169	25	34	34	178	0	50	28
N.S.	1	0.87	4.45	0.66	0.89	0.89	4.68	0.00	1.32	0.74
time (sec)	N/A	0.083	0.193	0.005	0.457	0.419	4.812	0.000	0.452	0.346

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	33	158	25	0	34	180	0	58	28
N.S.	1	0.87	4.16	0.66	0.00	0.89	4.74	0.00	1.53	0.74
time (sec)	N/A	0.087	0.227	0.006	0.000	0.396	5.456	0.000	0.471	0.291

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	33	155	36	0	34	187	0	58	28
N.S.	1	0.87	4.08	0.95	0.00	0.89	4.92	0.00	1.53	0.74
time (sec)	N/A	0.091	0.120	0.008	0.000	0.406	5.952	0.000	0.543	0.311

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	88	117	40	46	34	199	0	61	28
N.S.	1	2.32	3.08	1.05	1.21	0.89	5.24	0.00	1.61	0.74
time (sec)	N/A	0.192	0.073	0.010	0.485	0.392	6.701	0.000	0.826	3.116

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	89	153	44	50	34	184	0	52	28
N.S.	1	2.34	4.03	1.16	1.32	0.89	4.84	0.00	1.37	0.74
time (sec)	N/A	0.170	0.274	0.005	0.515	0.405	5.603	0.000	0.376	0.975

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	89	139	40	46	34	194	0	52	28
N.S.	1	2.34	3.66	1.05	1.21	0.89	5.11	0.00	1.37	0.74
time (sec)	N/A	0.171	0.124	0.008	0.513	0.399	5.710	0.000	0.492	0.981

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	88	106	44	50	34	192	0	61	28
N.S.	1	2.32	2.79	1.16	1.32	0.89	5.05	0.00	1.61	0.74
time (sec)	N/A	0.195	0.038	0.007	0.519	0.406	6.568	0.000	0.699	3.160

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	87	154	44	50	34	185	0	52	28
N.S.	1	2.29	4.05	1.16	1.32	0.89	4.87	0.00	1.37	0.74
time (sec)	N/A	0.174	0.203	0.007	0.500	0.399	5.644	0.000	0.488	1.145

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	47	28	32	48	34	124	35	35	38
N.S.	1	1.24	0.74	0.84	1.26	0.89	3.26	0.92	0.92	1.00
time (sec)	N/A	0.017	0.005	0.030	0.442	0.398	2.822	0.252	0.445	0.052

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	47	37	44	76	52	0	49	47	38
N.S.	1	1.24	0.97	1.16	2.00	1.37	0.00	1.29	1.24	1.00
time (sec)	N/A	0.021	0.019	0.041	0.436	0.394	0.000	0.318	0.845	0.079

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	47	26	42	60	49	58	77	49	38
N.S.	1	1.24	0.68	1.11	1.58	1.29	1.53	2.03	1.29	1.00
time (sec)	N/A	0.017	0.005	0.028	0.355	0.393	2.641	0.421	0.453	0.085

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	47	50	32	60	34	63	35	35	38
N.S.	1	1.24	1.32	0.84	1.58	0.89	1.66	0.92	0.92	1.00
time (sec)	N/A	0.019	0.026	0.033	0.606	0.384	138.355	0.276	0.580	0.047

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	49	100	45	55	34	148	0	56	28
N.S.	1	1.29	2.63	1.18	1.45	0.89	3.89	0.00	1.47	0.74
time (sec)	N/A	0.127	0.048	0.010	0.819	0.403	4.076	0.000	0.461	0.844

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	33	118	45	55	34	187	0	59	28
N.S.	1	0.87	3.11	1.18	1.45	0.89	4.92	0.00	1.55	0.74
time (sec)	N/A	0.088	0.065	0.012	0.768	0.431	6.773	0.000	0.593	3.038

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	33	103	40	46	34	177	0	59	28
N.S.	1	0.87	2.71	1.05	1.21	0.89	4.66	0.00	1.55	0.74
time (sec)	N/A	0.085	0.033	0.009	0.773	0.409	6.681	0.000	0.718	2.614

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	A	F	A	F	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	0	11	39	0	36	0	38	-1	38
N.S.	1	0.00	0.29	1.03	0.00	0.95	0.00	1.00	-0.03	1.00
time (sec)	N/A	0.397	0.063	0.196	0.000	0.437	0.000	0.690	0.000	0.187

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	76	0	29	36	29	29	38
N.S.	1	1.00	1.00	2.00	0.00	0.76	0.95	0.76	0.76	1.00
time (sec)	N/A	0.050	0.041	1.008	0.000	0.414	19.836	0.361	0.509	0.046

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	77	86	0	29	36	29	29	38
N.S.	1	1.00	2.03	2.26	0.00	0.76	0.95	0.76	0.76	1.00
time (sec)	N/A	0.050	0.062	1.037	0.000	0.402	22.743	0.306	0.485	0.051

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	56	0	77	68	0	48	39
N.S.	1	1.00	1.00	1.44	0.00	1.97	1.74	0.00	1.23	1.00
time (sec)	N/A	0.046	0.016	0.194	0.000	1.090	3.051	0.000	0.545	15.591

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	39	0	0	152	0	93	0	0	-1	39
N.S.	1	0.00	0.00	3.90	0.00	2.38	0.00	0.00	-0.03	1.00
time (sec)	N/A	1.150	0.541	1.856	0.000	7.516	0.000	0.000	0.000	0.116

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	2062	117	410	0	45	0	0	49	39
N.S.	1	52.87	3.00	10.51	0.00	1.15	0.00	0.00	1.26	1.00
time (sec)	N/A	9.424	1.032	0.079	0.000	0.415	0.000	0.000	0.726	0.274

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	39	0	10734	4717	0	66	0	0	2611	39
N.S.	1	0.00	275.23	120.95	0.00	1.69	0.00	0.00	66.95	1.00
time (sec)	N/A	74.089	6.342	3.438	0.000	0.420	0.000	0.000	1.595	0.379

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	254	204	1106	0	172	0	0	51	39
N.S.	1	6.51	5.23	28.36	0.00	4.41	0.00	0.00	1.31	1.00
time (sec)	N/A	1.994	1.654	0.085	0.000	0.477	0.000	0.000	2.187	0.339

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	35	42	28	58	29	60	29	-1	39
N.S.	1	0.90	1.08	0.72	1.49	0.74	1.54	0.74	-0.03	1.00
time (sec)	N/A	0.012	0.014	0.008	0.317	0.398	1.278	0.299	0.000	0.163

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	53	303	0	41	0	41	-1	39
N.S.	1	1.00	1.36	7.77	0.00	1.05	0.00	1.05	-0.03	1.00
time (sec)	N/A	0.038	0.015	0.037	0.000	0.437	0.000	0.258	0.000	0.371

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	49	48	313	0	37	0	43	-1	39
N.S.	1	1.26	1.23	8.03	0.00	0.95	0.00	1.10	-0.03	1.00
time (sec)	N/A	0.062	0.032	0.037	0.000	0.433	0.000	0.341	0.000	0.413

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	39	36	2092	1422	0	35	0	46	-1	39
N.S.	1	0.92	53.64	36.46	0.00	0.90	0.00	1.18	-0.03	1.00
time (sec)	N/A	0.041	6.078	0.533	0.000	0.422	0.000	0.397	0.000	0.192

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	39	0	0	0	0	157	0	0	56	39
N.S.	1	0.00	0.00	0.00	0.00	4.03	0.00	0.00	1.44	1.00
time (sec)	N/A	0.792	0.327	0.258	0.000	2.931	0.000	0.000	0.895	1.103

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	44	45	0	39	0	0	-1	39
N.S.	1	1.00	1.13	1.15	0.00	1.00	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.044	0.028	0.214	0.000	0.441	0.000	0.000	0.000	0.285

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	39	0	0	0	0	146	0	0	-1	39
N.S.	1	0.00	0.00	0.00	0.00	3.74	0.00	0.00	-0.03	1.00
time (sec)	N/A	1.235	0.654	0.297	0.000	3.873	0.000	0.000	0.000	0.659

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	0	0	29	0	0	-1	39
N.S.	1	1.00	1.00	0.00	0.00	0.74	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.016	0.018	0.240	0.000	0.393	0.000	0.000	0.000	0.070

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	40	387	215	1393	0	33	0	0	223	40
N.S.	1	9.68	5.38	34.82	0.00	0.82	0.00	0.00	5.58	1.00
time (sec)	N/A	1.280	0.966	0.069	0.000	0.430	0.000	0.000	0.155	0.102

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F(-1)	C	C	F	A	F	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	40	0	1451	5693	0	61	0	0	57	40
N.S.	1	0.00	36.28	142.32	0.00	1.52	0.00	0.00	1.42	1.00
time (sec)	N/A	180.004	2.951	1.364	0.000	0.418	0.000	0.000	0.700	0.316

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	40	355	173	677	0	64	0	0	227	40
N.S.	1	8.88	4.32	16.92	0.00	1.60	0.00	0.00	5.68	1.00
time (sec)	N/A	1.052	0.706	0.077	0.000	0.413	0.000	0.000	0.151	0.114

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	40	5379	1015	3006	0	225	0	0	62	40
N.S.	1	134.48	25.38	75.15	0.00	5.62	0.00	0.00	1.55	1.00
time (sec)	N/A	137.417	1.052	0.678	0.000	0.438	0.000	0.000	0.706	0.132

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	5437	824	3006	0	221	0	0	62	40
N.S.	1	135.92	20.60	75.15	0.00	5.52	0.00	0.00	1.55	1.00
time (sec)	N/A	131.813	0.761	0.749	0.000	0.454	0.000	0.000	0.693	0.126

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	40	0	2258	291	0	312	0	0	457	40
N.S.	1	0.00	56.45	7.28	0.00	7.80	0.00	0.00	11.42	1.00
time (sec)	N/A	16.430	5.445	0.055	0.000	0.787	0.000	0.000	0.746	0.550

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	40	0	1418	296	0	285	0	0	69	40
N.S.	1	0.00	35.45	7.40	0.00	7.12	0.00	0.00	1.72	1.00
time (sec)	N/A	11.965	4.303	0.050	0.000	1.764	0.000	0.000	3.307	0.521

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	74	58	79	34	34	49	34	18	29
N.S.	1	1.85	1.45	1.98	0.85	0.85	1.22	0.85	0.45	0.72
time (sec)	N/A	0.661	0.025	0.019	0.575	0.411	0.116	0.269	0.088	0.230

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	A	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	0	54	49	0	40	0	0	-1	40
N.S.	1	0.00	1.35	1.22	0.00	1.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.179	0.016	0.004	0.000	0.403	0.000	0.000	0.000	0.149

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	451	30	29	34	36	0	0	33	40
N.S.	1	11.28	0.75	0.72	0.85	0.90	0.00	0.00	0.82	1.00
time (sec)	N/A	1.712	0.143	0.013	0.695	0.410	0.000	0.000	0.326	5.671

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	76	0	29	36	29	29	40
N.S.	1	1.00	1.00	1.90	0.00	0.72	0.90	0.72	0.72	1.00
time (sec)	N/A	0.049	0.039	0.947	0.000	0.418	19.095	0.268	0.395	0.037

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	54	40	45	52	46	68	53	198	40
N.S.	1	1.35	1.00	1.12	1.30	1.15	1.70	1.32	4.95	1.00
time (sec)	N/A	0.097	0.045	0.039	0.462	0.397	27.310	0.338	0.201	0.034

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	A	F	A	F	A	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	40	0	0	39	0	38	0	41	-1	40
N.S.	1	0.00	0.00	0.98	0.00	0.95	0.00	1.02	-0.02	1.00
time (sec)	N/A	0.415	0.125	0.201	0.000	0.457	0.000	0.353	0.000	0.169

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F	F	B	F	F	F(-1)	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	40	40	0	0	0	465	0	0	-1	40
N.S.	1	1.00	0.00	0.00	0.00	11.62	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.673	0.369	0.229	0.000	1.156	0.000	0.000	0.000	3.123

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	65	41	38	53	37	71	53	37	41
N.S.	1	1.59	1.00	0.93	1.29	0.90	1.73	1.29	0.90	1.00
time (sec)	N/A	0.044	0.021	0.006	0.464	0.412	1.247	0.299	0.617	0.032

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	42	30	27	34	37	68	33	33	41
N.S.	1	1.02	0.73	0.66	0.83	0.90	1.66	0.80	0.80	1.00
time (sec)	N/A	0.028	0.014	0.004	0.402	0.390	0.482	0.338	0.553	0.027

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	41	0	2865	790	0	123	0	0	56	41
N.S.	1	0.00	69.88	19.27	0.00	3.00	0.00	0.00	1.37	1.00
time (sec)	N/A	1.228	6.329	0.180	0.000	0.441	0.000	0.000	1.638	0.593

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	334	145	205	0	62	0	0	-1	41
N.S.	1	8.15	3.54	5.00	0.00	1.51	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.189	0.482	0.081	0.000	0.453	0.000	0.000	0.000	0.692

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	41	0	0	0	0	0	0	0	-1	41
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.225	0.445	0.362	0.000	0.000	0.000	0.000	0.000	0.384

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	57	40	45	55	55	83	56	198	41
N.S.	1	1.39	0.98	1.10	1.34	1.34	2.02	1.37	4.83	1.00
time (sec)	N/A	0.119	0.048	0.010	0.673	0.406	74.242	0.396	0.053	0.053

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	63	41	48	113	38	0	44	201	41
N.S.	1	1.54	1.00	1.17	2.76	0.93	0.00	1.07	4.90	1.00
time (sec)	N/A	0.068	0.039	0.035	0.682	0.425	0.000	0.641	0.220	0.067

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	73	88	0	29	37	29	29	41
N.S.	1	1.00	1.78	2.15	0.00	0.71	0.90	0.71	0.71	1.00
time (sec)	N/A	0.056	0.066	1.053	0.000	0.426	22.450	0.324	0.538	0.040

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	41	0	0	0	0	0	0	0	-1	41
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.715	0.330	0.256	0.000	0.000	0.000	0.000	0.000	2.726

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	41	187	0	0	0	0	0	0	-1	41
N.S.	1	4.56	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.437	0.249	0.245	0.000	0.000	0.000	0.000	0.000	19.864

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	45	44	44	37	45	30	42
N.S.	1	1.00	1.00	1.07	1.05	1.05	0.88	1.07	0.71	1.00
time (sec)	N/A	0.017	0.008	0.222	0.588	0.727	0.790	0.332	0.280	0.027

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F(-1)	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	42	0	5105	2264	0	459	0	0	456	42
N.S.	1	0.00	121.55	53.90	0.00	10.93	0.00	0.00	10.86	1.00
time (sec)	N/A	2.634	6.788	0.820	0.000	2.551	0.000	0.000	10.902	6.845

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	A	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	0	37	37	0	46	0	0	-1	44
N.S.	1	0.00	0.88	0.88	0.00	1.10	0.00	0.00	-0.02	1.05
time (sec)	N/A	0.680	0.148	0.040	0.000	0.638	0.000	0.000	0.000	0.180

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	A	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	0	37	35	0	45	0	0	-1	44
N.S.	1	0.00	0.88	0.83	0.00	1.07	0.00	0.00	-0.02	1.05
time (sec)	N/A	0.627	0.119	0.032	0.000	0.814	0.000	0.000	0.000	0.163

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	45	44	44	37	44	30	42
N.S.	1	1.00	1.00	1.07	1.05	1.05	0.88	1.05	0.71	1.00
time (sec)	N/A	0.018	0.008	0.221	0.560	0.738	0.789	1.931	0.309	0.026

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	51	37	46	70	43	92	0	-1	42
N.S.	1	1.21	0.88	1.10	1.67	1.02	2.19	0.00	-0.02	1.00
time (sec)	N/A	0.029	0.029	0.027	0.544	0.826	3.545	0.000	0.000	0.251

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	63	26	48	80	57	85	59	201	43
N.S.	1	1.47	0.60	1.12	1.86	1.33	1.98	1.37	4.67	1.00
time (sec)	N/A	0.025	0.005	0.026	0.399	0.708	3.928	0.302	0.045	0.001

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	109	43	43	0	39	0	55	85	43
N.S.	1	2.53	1.00	1.00	0.00	0.91	0.00	1.28	1.98	1.00
time (sec)	N/A	0.318	0.056	0.006	0.000	0.655	0.000	0.455	0.301	0.280

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	43	0	1638	302	0	55	0	0	70	43
N.S.	1	0.00	38.09	7.02	0.00	1.28	0.00	0.00	1.63	1.00
time (sec)	N/A	0.960	6.175	0.075	0.000	0.473	0.000	0.000	1.307	0.596

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	350	284	436	0	111	0	0	-1	43
N.S.	1	8.14	6.60	10.14	0.00	2.58	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.432	0.787	0.268	0.000	0.458	0.000	0.000	0.000	0.365

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	43	0	17667	9105	0	67	0	0	-1	43
N.S.	1	0.00	410.86	211.74	0.00	1.56	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.592	6.211	0.475	0.000	0.432	0.000	0.000	0.000	0.140

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	43	0	0	0	0	154	0	0	55	43
N.S.	1	0.00	0.00	0.00	0.00	3.58	0.00	0.00	1.28	1.00
time (sec)	N/A	1.404	0.668	0.299	0.000	2.396	0.000	0.000	0.566	0.367

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	63	28	37	66	39	165	44	47	43
N.S.	1	1.47	0.65	0.86	1.53	0.91	3.84	1.02	1.09	1.00
time (sec)	N/A	0.026	0.005	0.035	0.420	0.385	4.770	0.360	0.594	0.038

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	51	41	45	86	37	104	0	-1	43
N.S.	1	1.19	0.95	1.05	2.00	0.86	2.42	0.00	-0.02	1.00
time (sec)	N/A	0.025	0.013	0.230	0.329	0.397	2.738	0.000	0.000	0.179

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	63	28	37	66	39	160	44	47	43
N.S.	1	1.47	0.65	0.86	1.53	0.91	3.72	1.02	1.09	1.00
time (sec)	N/A	0.025	0.005	0.033	0.492	0.387	4.329	0.345	0.550	0.036

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	51	49	45	84	37	104	0	-1	43
N.S.	1	1.19	1.14	1.05	1.95	0.86	2.42	0.00	-0.02	1.00
time (sec)	N/A	0.025	0.024	0.222	0.345	0.392	2.265	0.000	0.000	0.115

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	43	216	125	220	0	53	0	0	-1	43
N.S.	1	5.02	2.91	5.12	0.00	1.23	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.684	0.347	0.030	0.000	0.454	0.000	0.000	0.000	0.697

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	41	31	27	86	37	46	0	-1	43
N.S.	1	0.95	0.72	0.63	2.00	0.86	1.07	0.00	-0.02	1.00
time (sec)	N/A	0.016	0.011	0.201	0.317	0.394	2.739	0.000	0.000	0.163

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	63	36	49	119	57	0	58	85	43
N.S.	1	1.47	0.84	1.14	2.77	1.33	0.00	1.35	1.98	1.00
time (sec)	N/A	0.025	0.010	0.048	0.489	0.402	0.000	0.322	1.012	0.062

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	354	114	572	0	68	0	0	-1	43
N.S.	1	8.23	2.65	13.30	0.00	1.58	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.548	0.266	0.068	0.000	0.447	0.000	0.000	0.000	0.681

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	354	114	571	0	68	0	0	-1	43
N.S.	1	8.23	2.65	13.28	0.00	1.58	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.319	0.185	0.010	0.000	0.470	0.000	0.000	0.000	0.708

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	43	0	0	0	0	154	0	0	-1	43
N.S.	1	0.00	0.00	0.00	0.00	3.58	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.253	0.614	0.394	0.000	3.894	0.000	0.000	0.000	0.590

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	60	61	45	85	57	105	59	198	43
N.S.	1	1.40	1.42	1.05	1.98	1.33	2.44	1.37	4.60	1.00
time (sec)	N/A	0.114	0.064	0.011	0.503	0.398	151.829	0.340	0.237	0.051

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F(-1)	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	43	0	0	0	0	172	0	0	-1	43
N.S.	1	0.00	0.00	0.00	0.00	4.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.754	0.641	0.336	0.000	0.565	0.000	0.000	0.000	2.698

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	43	0	0	346	0	167	0	0	-1	43
N.S.	1	0.00	0.00	8.05	0.00	3.88	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.142	0.237	112.802	0.000	12.491	0.000	0.000	0.000	15.626

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	B	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	102	43	78	14	14	0	0	-1	43
N.S.	1	2.37	1.00	1.81	0.33	0.33	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.062	0.020	0.006	0.349	0.375	0.000	0.000	0.000	19.097

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	43	189	0	0	0	0	0	0	-1	43
N.S.	1	4.40	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.484	0.272	0.312	0.000	0.000	0.000	0.000	0.000	20.012

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	43	0	0	0	0	0	0	0	-1	43
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.787	0.321	0.248	0.000	0.000	0.000	0.000	0.000	18.501

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	0	72	0	0	37	0	0	-1	43
N.S.	1	0.00	1.67	0.00	0.00	0.86	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.769	0.649	0.276	0.000	0.444	0.000	0.000	0.000	2.885

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F(-2)	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	377	346	2321	0	224	0	0	465	44
N.S.	1	8.57	7.86	52.75	0.00	5.09	0.00	0.00	10.57	1.00
time (sec)	N/A	6.246	4.805	0.060	0.000	0.638	0.000	0.000	0.373	0.349

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F(-2)	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	376	238	2508	0	231	0	0	437	44
N.S.	1	8.55	5.41	57.00	0.00	5.25	0.00	0.00	9.93	1.00
time (sec)	N/A	7.286	7.215	0.078	0.000	0.772	0.000	0.000	0.380	0.323

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F(-1)	C	C	F	A	F	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	44	0	1609	5687	0	68	0	0	61	44
N.S.	1	0.00	36.57	129.25	0.00	1.55	0.00	0.00	1.39	1.00
time (sec)	N/A	180.004	3.127	0.677	0.000	0.402	0.000	0.000	1.131	0.296

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	44	0	2730	336	0	407	0	0	589	44
N.S.	1	0.00	62.05	7.64	0.00	9.25	0.00	0.00	13.39	1.00
time (sec)	N/A	7.297	4.992	0.065	0.000	0.806	0.000	0.000	0.688	0.639

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	44	0	1571	344	0	441	0	0	368	44
N.S.	1	0.00	35.70	7.82	0.00	10.02	0.00	0.00	8.36	1.00
time (sec)	N/A	7.745	5.182	0.057	0.000	1.149	0.000	0.000	5.924	0.673

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	44	0	4253	1659	0	93	0	0	-1	44
N.S.	1	0.00	96.66	37.70	0.00	2.11	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.379	4.598	1.878	0.000	0.446	0.000	0.000	0.000	0.411

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F(-1)	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	44	0	8005	3173	0	471	0	0	1058	44
N.S.	1	0.00	181.93	72.11	0.00	10.70	0.00	0.00	24.05	1.00
time (sec)	N/A	4.075	7.088	0.716	0.000	3.861	0.000	0.000	80.208	18.163

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	51	41	50	57	46	37	59	-1	44
N.S.	1	1.16	0.93	1.14	1.30	1.05	0.84	1.34	-0.02	1.00
time (sec)	N/A	0.030	0.035	0.234	0.451	0.387	2.618	0.418	0.000	0.296

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	61	0	0	45	0	0	-1	44
N.S.	1	1.00	1.39	0.00	0.00	1.02	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.400	0.107	0.457	0.000	0.432	0.000	0.000	0.000	20.392

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	44	0	0	0	0	0	0	0	-1	44
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.540	0.171	0.206	0.000	0.000	0.000	0.000	0.000	0.133

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	31	31	0	0	60	15	0	-1	44
N.S.	1	0.70	0.70	0.00	0.00	1.36	0.34	0.00	-0.02	1.00
time (sec)	N/A	0.056	0.012	0.228	0.000	0.597	0.995	0.000	0.000	0.198

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	24	56	47	58	32	47	33	45
N.S.	1	1.00	0.53	1.24	1.04	1.29	0.71	1.04	0.73	1.00
time (sec)	N/A	0.018	0.005	0.211	0.449	0.404	0.916	0.345	0.564	0.062

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	26	72	47	57	32	48	33	45
N.S.	1	1.00	0.58	1.60	1.04	1.27	0.71	1.07	0.73	1.00
time (sec)	N/A	0.019	0.005	0.232	0.444	0.418	0.903	0.168	0.582	0.053

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	26	56	47	57	34	48	33	45
N.S.	1	1.00	0.58	1.24	1.04	1.27	0.76	1.07	0.73	1.00
time (sec)	N/A	0.018	0.005	0.305	0.521	0.402	0.901	0.223	0.543	0.042

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	55	62	315	0	43	0	49	-1	45
N.S.	1	1.22	1.38	7.00	0.00	0.96	0.00	1.09	-0.02	1.00
time (sec)	N/A	0.066	0.026	0.367	0.000	0.449	0.000	0.197	0.000	0.410

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	45	0	0	0	0	146	0	0	50	45
N.S.	1	0.00	0.00	0.00	0.00	3.24	0.00	0.00	1.11	1.00
time (sec)	N/A	1.244	0.623	0.293	0.000	1.651	0.000	0.000	0.713	0.371

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	45	0	0	121	0	115	0	0	77	45
N.S.	1	0.00	0.00	2.69	0.00	2.56	0.00	0.00	1.71	1.00
time (sec)	N/A	1.640	0.319	0.835	0.000	0.439	0.000	0.000	3.553	2.661

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	26	56	47	57	34	47	33	45
N.S.	1	1.00	0.58	1.24	1.04	1.27	0.76	1.04	0.73	1.00
time (sec)	N/A	0.019	0.005	0.292	0.448	0.405	1.046	0.209	0.601	0.039

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	45	0	0	805	0	256	0	0	-1	45
N.S.	1	0.00	0.00	17.89	0.00	5.69	0.00	0.00	-0.02	1.00
time (sec)	N/A	14.354	2.339	5.134	0.000	46.525	0.000	0.000	0.000	4.503

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	45	0	0	0	0	0	0	0	-1	45
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.981	1.956	0.349	0.000	0.000	0.000	0.000	0.000	2.040

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	18	70	0	0	57	0	0	-1	45
N.S.	1	0.40	1.56	0.00	0.00	1.27	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.181	0.059	0.316	0.000	0.729	0.000	0.000	0.000	1.426

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	65	54	325	0	44	0	50	-1	46
N.S.	1	1.41	1.17	7.07	0.00	0.96	0.00	1.09	-0.02	1.00
time (sec)	N/A	0.083	0.035	0.335	0.000	0.438	0.000	0.203	0.000	0.458

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	46	39	709	266	0	41	0	0	-1	46
N.S.	1	0.85	15.41	5.78	0.00	0.89	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.204	5.043	0.072	0.000	0.438	0.000	0.000	0.000	0.661

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	46	0	3173	100440	0	39	0	0	-1	46
N.S.	1	0.00	68.98	2183.48	0.00	0.85	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.481	3.217	0.921	0.000	0.431	0.000	0.000	0.000	0.198

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	A	F	A	F	A	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	46	0	0	45	0	44	0	46	-1	46
N.S.	1	0.00	0.00	0.98	0.00	0.96	0.00	1.00	-0.02	1.00
time (sec)	N/A	0.310	0.165	0.286	0.000	0.436	0.000	0.225	0.000	4.072

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	113	53	390	0	52	0	42	-1	46
N.S.	1	2.46	1.15	8.48	0.00	1.13	0.00	0.91	-0.02	1.00
time (sec)	N/A	0.611	0.022	0.551	0.000	0.433	0.000	0.167	0.000	0.408

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	38	46	59	85	34	59	29	47
N.S.	1	1.00	0.81	0.98	1.26	1.81	0.72	1.26	0.62	1.00
time (sec)	N/A	0.012	0.006	0.313	0.527	2.221	0.835	0.208	0.681	0.172

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	53	45	62	53	49	56	53	39	47
N.S.	1	1.13	0.96	1.32	1.13	1.04	1.19	1.13	0.83	1.00
time (sec)	N/A	0.024	0.019	0.314	0.486	0.397	37.101	0.292	0.733	0.073

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	A	F	A	F	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	0	57	48	0	66	0	61	-1	47
N.S.	1	0.00	1.21	1.02	0.00	1.40	0.00	1.30	-0.02	1.00
time (sec)	N/A	0.013	0.024	0.010	0.000	0.382	0.000	0.478	0.000	0.276

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	47	1670	1280	712	0	109	0	0	-1	47
N.S.	1	35.53	27.23	15.15	0.00	2.32	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.611	5.037	0.340	0.000	0.455	0.000	0.000	0.000	0.450

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	47	0	12187	6119	0	64	0	0	-1	47
N.S.	1	0.00	259.30	130.19	0.00	1.36	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.603	6.320	13.224	0.000	0.465	0.000	0.000	0.000	0.169

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	47	0	0	0	0	193	0	0	61	47
N.S.	1	0.00	0.00	0.00	0.00	4.11	0.00	0.00	1.30	1.00
time (sec)	N/A	2.233	1.487	0.411	0.000	0.460	0.000	0.000	0.785	2.330

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-2)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	47	0	0	120	0	0	0	0	-1	47
N.S.	1	0.00	0.00	2.55	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.927	0.537	0.663	0.000	0.000	0.000	0.000	0.000	2.746

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F(-1)	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	47	0	0	0	0	184	0	0	-1	47
N.S.	1	0.00	0.00	0.00	0.00	3.91	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.950	1.512	0.467	0.000	0.543	0.000	0.000	0.000	2.716

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	47	0	0	97	0	60	0	0	72	47
N.S.	1	0.00	0.00	2.06	0.00	1.28	0.00	0.00	1.53	1.00
time (sec)	N/A	1.804	0.570	0.984	0.000	0.431	0.000	0.000	1.341	0.524

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	B	C	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	35	28	24	23	21	184	23	16	28
N.S.	1	0.74	0.60	0.51	0.49	0.45	3.91	0.49	0.34	0.60
time (sec)	N/A	0.013	0.010	0.010	0.319	0.384	0.803	0.229	0.517	0.020

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	51	56	67	49	56	-1	47
N.S.	1	1.00	1.00	1.09	1.19	1.43	1.04	1.19	-0.02	1.00
time (sec)	N/A	0.114	0.040	0.076	0.450	0.818	1.140	0.431	0.000	0.105

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	B	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	215	138	0	0	28	0	0	-1	47
N.S.	1	4.57	2.94	0.00	0.00	0.60	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.199	0.236	0.066	0.000	0.386	0.000	0.000	0.000	0.110

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	48	48	48	121	0	91	0	0	-1	48
N.S.	1	1.00	1.00	2.52	0.00	1.90	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.015	0.035	1.958	0.000	0.396	0.000	0.000	0.000	2.172

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	72	68	331	0	52	0	0	-1	48
N.S.	1	1.50	1.42	6.90	0.00	1.08	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.363	0.253	0.059	0.000	0.425	0.000	0.000	0.000	0.431

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	0	196	3042	0	49	0	0	-1	48
N.S.	1	0.00	4.08	63.38	0.00	1.02	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.770	0.456	0.116	0.000	0.457	0.000	0.000	0.000	0.237

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	94	45	366	0	65	0	45	-1	48
N.S.	1	1.96	0.94	7.62	0.00	1.35	0.00	0.94	-0.02	1.00
time (sec)	N/A	0.156	0.015	0.594	0.000	0.396	0.000	0.388	0.000	0.279

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	109	38	40	0	44	0	46	99	48
N.S.	1	2.27	0.79	0.83	0.00	0.92	0.00	0.96	2.06	1.00
time (sec)	N/A	0.257	0.040	0.007	0.000	0.384	0.000	0.360	1.102	0.370

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	48	0	10080	505	0	150	0	0	-1	48
N.S.	1	0.00	210.00	10.52	0.00	3.12	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.585	13.037	0.125	0.000	0.493	0.000	0.000	0.000	0.366

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	208	203	38	0	43	0	0	53	48
N.S.	1	4.33	4.23	0.79	0.00	0.90	0.00	0.00	1.10	1.00
time (sec)	N/A	0.565	0.156	0.013	0.000	0.397	0.000	0.000	0.658	2.652

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	67	46	50	109	42	143	0	-1	48
N.S.	1	1.40	0.96	1.04	2.27	0.88	2.98	0.00	-0.02	1.00
time (sec)	N/A	0.030	0.016	0.312	0.315	0.382	4.681	0.000	0.000	0.199

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	67	56	50	109	42	136	0	-1	48
N.S.	1	1.40	1.17	1.04	2.27	0.88	2.83	0.00	-0.02	1.00
time (sec)	N/A	0.030	0.031	0.310	0.319	0.390	3.835	0.000	0.000	0.162

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	57	36	32	109	42	65	0	-1	48
N.S.	1	1.19	0.75	0.67	2.27	0.88	1.35	0.00	-0.02	1.00
time (sec)	N/A	0.023	0.014	0.290	0.330	0.377	4.721	0.000	0.000	0.204

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	48	0	0	0	0	321	0	0	-1	48
N.S.	1	0.00	0.00	0.00	0.00	6.69	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.512	0.479	0.711	0.000	1.128	0.000	0.000	0.000	2.740

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	48	0	0	0	0	264	0	0	-1	48
N.S.	1	0.00	0.00	0.00	0.00	5.50	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.599	0.486	0.603	0.000	1.153	0.000	0.000	0.000	2.749

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	0	48	0	0	58	0	0	-1	48
N.S.	1	0.00	1.00	0.00	0.00	1.21	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.145	0.055	0.329	0.000	0.397	0.000	0.000	0.000	0.144

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	49	0	0	0	0	0	0	0	-1	49
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.107	1.482	0.339	0.000	0.000	0.000	0.000	0.000	0.233

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	49	0	5117	278	0	88	0	0	235	49
N.S.	1	0.00	104.43	5.67	0.00	1.80	0.00	0.00	4.80	1.00
time (sec)	N/A	51.170	6.403	0.096	0.000	0.411	0.000	0.000	0.597	0.390

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	40	46	72	62	36	60	-1	49
N.S.	1	1.00	0.82	0.94	1.47	1.27	0.73	1.22	-0.02	1.00
time (sec)	N/A	0.013	0.006	0.303	0.406	0.407	0.869	0.192	0.000	0.179

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	22	30	72	62	31	59	-1	49
N.S.	1	1.00	0.45	0.61	1.47	1.27	0.63	1.20	-0.02	1.00
time (sec)	N/A	0.013	0.002	0.301	0.422	0.397	0.823	0.510	0.000	0.194

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	60	62	609	0	51	0	57	-1	49
N.S.	1	1.22	1.27	12.43	0.00	1.04	0.00	1.16	-0.02	1.00
time (sec)	N/A	0.084	0.031	0.022	0.000	0.433	0.000	0.355	0.000	0.478

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	49	0	0	0	0	0	0	0	-1	49
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.844	0.254	0.304	0.000	0.000	0.000	0.000	0.000	0.644

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	49	0	0	0	0	180	0	0	55	49
N.S.	1	0.00	0.00	0.00	0.00	3.67	0.00	0.00	1.12	1.00
time (sec)	N/A	1.998	0.642	0.332	0.000	0.458	0.000	0.000	0.772	2.205

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	47	65	42	35	51	49	0	35	51
N.S.	1	0.96	1.33	0.86	0.71	1.04	1.00	0.00	0.71	1.04
time (sec)	N/A	0.035	0.031	0.052	0.528	0.418	4.251	0.000	1.096	0.203

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	49	0	57707	0	0	0	0	0	-1	49
N.S.	1	0.00	1177.69	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.841	17.678	0.156	0.000	0.000	0.000	0.000	0.000	0.671

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	49	0	0	0	0	195	0	0	98	49
N.S.	1	0.00	0.00	0.00	0.00	3.98	0.00	0.00	2.00	1.00
time (sec)	N/A	0.540	0.170	0.186	0.000	1.000	0.000	0.000	8.020	4.141

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	49	0	0	0	0	242	0	0	311	49
N.S.	1	0.00	0.00	0.00	0.00	4.94	0.00	0.00	6.35	1.00
time (sec)	N/A	0.531	0.169	0.198	0.000	0.957	0.000	0.000	14.909	7.569

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	49	0	0	0	0	238	0	0	309	49
N.S.	1	0.00	0.00	0.00	0.00	4.86	0.00	0.00	6.31	1.00
time (sec)	N/A	0.547	0.178	0.186	0.000	0.888	0.000	0.000	14.589	7.538

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	C	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	0	106	20	0	51	32	0	-1	49
N.S.	1	0.00	2.16	0.41	0.00	1.04	0.65	0.00	-0.02	1.00
time (sec)	N/A	0.019	0.224	0.028	0.000	1.874	0.727	0.000	0.000	0.109

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	73	58	329	0	48	0	56	-1	50
N.S.	1	1.46	1.16	6.58	0.00	0.96	0.00	1.12	-0.02	1.00
time (sec)	N/A	0.094	0.037	0.187	0.000	0.658	0.000	0.199	0.000	0.435

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	121	38	42	0	46	0	91	113	50
N.S.	1	2.42	0.76	0.84	0.00	0.92	0.00	1.82	2.26	1.00
time (sec)	N/A	0.312	0.019	0.005	0.000	0.384	0.000	0.205	1.233	0.283

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	0	55	52	0	42	0	0	-1	54
N.S.	1	0.00	1.10	1.04	0.00	0.84	0.00	0.00	-0.02	1.08
time (sec)	N/A	0.568	0.155	0.528	0.000	0.414	0.000	0.000	0.000	0.169

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	0	55	54	0	42	0	0	-1	54
N.S.	1	0.00	1.10	1.08	0.00	0.84	0.00	0.00	-0.02	1.08
time (sec)	N/A	0.529	0.132	0.555	0.000	0.382	0.000	0.000	0.000	0.144

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	B	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	41	44	55	0	28	197	0	-1	50
N.S.	1	0.82	0.88	1.10	0.00	0.56	3.94	0.00	-0.02	1.00
time (sec)	N/A	0.008	0.055	0.020	0.000	1.200	0.968	0.000	0.000	0.001

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	F	A	C	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	0	78	22	0	43	37	0	-1	50
N.S.	1	0.00	1.56	0.44	0.00	0.86	0.74	0.00	-0.02	1.00
time (sec)	N/A	0.077	0.118	0.033	0.000	2.748	0.877	0.000	0.000	0.118

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	44	0	57	110	41	186	34	51
N.S.	1	1.00	0.86	0.00	1.12	2.16	0.80	3.65	0.67	1.00
time (sec)	N/A	0.041	0.012	0.165	0.427	0.411	0.925	0.501	0.701	0.082

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	0	131	188	0	91	0	0	-1	51
N.S.	1	0.00	2.57	3.69	0.00	1.78	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.322	0.242	0.030	0.000	0.439	0.000	0.000	0.000	0.130

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	47	83	0	38	46	38	38	51
N.S.	1	1.00	0.92	1.63	0.00	0.75	0.90	0.75	0.75	1.00
time (sec)	N/A	0.052	0.048	1.002	0.000	0.393	19.828	0.182	0.856	0.039

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	51	0	0	155	0	75	0	0	-1	51
N.S.	1	0.00	0.00	3.04	0.00	1.47	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.558	0.437	0.652	0.000	0.511	0.000	0.000	0.000	2.530

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	51	0	0	0	0	0	0	0	-1	51
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.997	0.251	0.163	0.000	0.000	0.000	0.000	0.000	0.897

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	51	0	0	1016	0	331	0	0	-1	51
N.S.	1	0.00	0.00	19.92	0.00	6.49	0.00	0.00	-0.02	1.00
time (sec)	N/A	9.301	2.079	10.754	0.000	97.323	0.000	0.000	0.000	0.202

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	A	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	51	0	0	50	0	45	0	0	-1	51
N.S.	1	0.00	0.00	0.98	0.00	0.88	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.063	0.504	0.326	0.000	1.235	0.000	0.000	0.000	0.566

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	C	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	0	45	0	0	43	34	0	-1	51
N.S.	1	0.00	0.88	0.00	0.00	0.84	0.67	0.00	-0.02	1.00
time (sec)	N/A	0.325	0.113	0.127	0.000	1.346	1.191	0.000	0.000	0.086

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	67	48	54	71	106	0	71	-1	52
N.S.	1	1.31	0.94	1.06	1.39	2.08	0.00	1.39	-0.02	1.02
time (sec)	N/A	0.305	0.079	0.009	0.429	0.391	0.000	0.182	0.000	0.088

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	B	F	A	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	0	45	0	0	94	0	65	39	52
N.S.	1	0.00	0.87	0.00	0.00	1.81	0.00	1.25	0.75	1.00
time (sec)	N/A	0.023	0.278	0.357	0.000	0.411	0.000	0.304	0.988	0.137

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	61	26	82	74	65	32	60	45	52
N.S.	1	1.17	0.50	1.58	1.42	1.25	0.62	1.15	0.87	1.00
time (sec)	N/A	0.025	0.005	0.138	0.424	0.396	1.164	0.215	0.793	0.075

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	61	26	66	72	63	34	58	45	52
N.S.	1	1.17	0.50	1.27	1.38	1.21	0.65	1.12	0.87	1.00
time (sec)	N/A	0.023	0.005	0.138	0.417	0.389	1.156	0.201	0.689	0.078

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	59	51	47	63	103	78	50	68	52
N.S.	1	1.13	0.98	0.90	1.21	1.98	1.50	0.96	1.31	1.00
time (sec)	N/A	0.043	0.029	0.051	0.415	0.402	20.740	0.139	0.700	0.045

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	A	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	0	47	47	0	55	0	0	-1	54
N.S.	1	0.00	0.90	0.90	0.00	1.06	0.00	0.00	-0.02	1.04
time (sec)	N/A	0.616	0.142	0.060	0.000	0.386	0.000	0.000	0.000	0.235

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	C	F	A	F	A	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	163	65	332	0	55	0	49	47	52
N.S.	1	3.13	1.25	6.38	0.00	1.06	0.00	0.94	0.90	1.00
time (sec)	N/A	0.295	0.069	0.194	0.000	0.434	0.000	0.397	0.993	0.499

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	52	0	0	0	0	0	0	0	-1	52
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.256	0.302	0.181	0.000	0.000	0.000	0.000	0.000	2.269

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	52	0	0	0	0	231	0	0	134	52
N.S.	1	0.00	0.00	0.00	0.00	4.44	0.00	0.00	2.58	1.00
time (sec)	N/A	1.458	0.504	0.424	0.000	0.526	0.000	0.000	4.014	2.033

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	52	0	0	0	0	202	0	0	103	52
N.S.	1	0.00	0.00	0.00	0.00	3.88	0.00	0.00	1.98	1.00
time (sec)	N/A	1.506	0.506	0.356	0.000	0.517	0.000	0.000	3.557	2.670

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	52	0	0	0	0	334	0	0	-1	52
N.S.	1	0.00	0.00	0.00	0.00	6.42	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.517	0.493	0.739	0.000	1.130	0.000	0.000	0.000	2.727

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	52	0	0	0	0	280	0	0	-1	52
N.S.	1	0.00	0.00	0.00	0.00	5.38	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.549	0.495	0.596	0.000	1.219	0.000	0.000	0.000	2.746

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	66	59	48	0	60	0	0	-1	56
N.S.	1	1.25	1.11	0.91	0.00	1.13	0.00	0.00	-0.02	1.06
time (sec)	N/A	0.019	0.024	0.042	0.000	0.403	0.000	0.000	0.000	4.929

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	53	0	1719	330	0	112	0	0	76	53
N.S.	1	0.00	32.43	6.23	0.00	2.11	0.00	0.00	1.43	1.00
time (sec)	N/A	0.993	6.151	0.240	0.000	0.472	0.000	0.000	1.135	0.732

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	59	52	47	63	103	75	50	68	53
N.S.	1	1.11	0.98	0.89	1.19	1.94	1.42	0.94	1.28	1.00
time (sec)	N/A	0.044	0.028	0.013	0.666	0.440	21.162	0.185	0.707	0.079

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	53	53	26	102	0	61	0	0	-1	53
N.S.	1	1.00	0.49	1.92	0.00	1.15	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.114	0.018	0.025	0.000	0.496	0.000	0.000	0.000	0.331

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	53	53	108	362	0	61	0	0	-1	53
N.S.	1	1.00	2.04	6.83	0.00	1.15	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.014	0.091	0.031	0.000	0.470	0.000	0.000	0.000	0.251

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	86	58	618	0	49	0	53	-1	53
N.S.	1	1.62	1.09	11.66	0.00	0.92	0.00	1.00	-0.02	1.00
time (sec)	N/A	0.112	0.064	0.013	0.000	0.446	0.000	0.279	0.000	0.490

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	107	40	17	0	95	0	47	29	53
N.S.	1	2.02	0.75	0.32	0.00	1.79	0.00	0.89	0.55	1.00
time (sec)	N/A	0.057	0.006	0.148	0.000	0.990	0.000	0.154	0.572	0.158

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	1165	257	334	0	65	0	0	-1	53
N.S.	1	21.98	4.85	6.30	0.00	1.23	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.880	1.372	0.098	0.000	0.473	0.000	0.000	0.000	0.657

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	145	53	53	0	49	0	73	113	53
N.S.	1	2.74	1.00	1.00	0.00	0.92	0.00	1.38	2.13	1.00
time (sec)	N/A	0.216	0.020	0.004	0.000	0.389	0.000	0.352	0.577	0.328

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	53	0	484	2700	0	47	0	0	-1	53
N.S.	1	0.00	9.13	50.94	0.00	0.89	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.137	1.395	1.138	0.000	0.436	0.000	0.000	0.000	5.069

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	44	158	0	144	0	0	-1	53
N.S.	1	1.00	0.83	2.98	0.00	2.72	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.014	0.010	1.286	0.000	7.581	0.000	0.000	0.000	0.231

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	53	0	0	0	0	0	0	0	-1	53
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.295	0.574	0.183	0.000	0.000	0.000	0.000	0.000	1.164

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	83	61	55	134	47	175	0	-1	53
N.S.	1	1.57	1.15	1.04	2.53	0.89	3.30	0.00	-0.02	1.00
time (sec)	N/A	0.040	0.034	0.155	0.312	0.396	6.226	0.000	0.000	0.177

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	0	44	98	0	57	0	0	-1	57
N.S.	1	0.00	0.83	1.85	0.00	1.08	0.00	0.00	-0.02	1.08
time (sec)	N/A	0.600	0.135	0.158	0.000	0.418	0.000	0.000	0.000	0.195

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	53	228	542	944	0	62	0	0	-1	53
N.S.	1	4.30	10.23	17.81	0.00	1.17	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.150	1.804	0.165	0.000	0.474	0.000	0.000	0.000	0.296

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	96	90	0	47	48	41	41	53
N.S.	1	1.00	1.81	1.70	0.00	0.89	0.91	0.77	0.77	1.00
time (sec)	N/A	0.054	0.066	1.028	0.000	0.409	26.202	0.142	1.017	0.065

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	53	0	58426	0	0	0	0	0	-1	53
N.S.	1	0.00	1102.38	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.990	18.226	0.183	0.000	0.000	0.000	0.000	0.000	0.699

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	A	A	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	53	1215	85	434	0	65	102	72	775	53
N.S.	1	22.92	1.60	8.19	0.00	1.23	1.92	1.36	14.62	1.00
time (sec)	N/A	3.536	0.200	0.257	0.000	0.422	28.447	0.136	0.045	0.074

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	A	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	314	106	74	0	199	0	0	-1	53
N.S.	1	5.92	2.00	1.40	0.00	3.75	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.651	0.438	0.033	0.000	0.564	0.000	0.000	0.000	0.277

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	314	106	161	0	199	0	0	-1	53
N.S.	1	5.92	2.00	3.04	0.00	3.75	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.539	0.330	0.064	0.000	0.538	0.000	0.000	0.000	0.275

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	A	F	B	F(-1)	F(-2)	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	647	154	74	0	226	0	0	-1	53
N.S.	1	12.21	2.91	1.40	0.00	4.26	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.431	0.284	0.036	0.000	0.574	0.000	0.000	0.000	0.335

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	C	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	53	53	983	22	0	78	44	0	-1	53
N.S.	1	1.00	18.55	0.42	0.00	1.47	0.83	0.00	-0.02	1.00
time (sec)	N/A	0.059	14.557	0.049	0.000	0.416	1.758	0.000	0.000	0.146

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	199	209	221	0	100	0	0	-1	54
N.S.	1	3.69	3.87	4.09	0.00	1.85	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.392	0.341	0.132	0.000	0.643	0.000	0.000	0.000	3.388

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	199	208	222	0	100	0	0	-1	54
N.S.	1	3.69	3.85	4.11	0.00	1.85	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.174	0.244	0.098	0.000	0.656	0.000	0.000	0.000	3.395

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	54	5514	1148	3008	0	234	0	0	-1	54
N.S.	1	102.11	21.26	55.70	0.00	4.33	0.00	0.00	-0.02	1.00
time (sec)	N/A	137.715	1.055	0.704	0.000	0.486	0.000	0.000	0.000	0.132

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	5395	933	3008	0	234	0	0	-1	54
N.S.	1	99.91	17.28	55.70	0.00	4.33	0.00	0.00	-0.02	1.00
time (sec)	N/A	133.556	0.801	0.716	0.000	0.473	0.000	0.000	0.000	0.119

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	B	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	62	34	40	83	92	65	70	-1	54
N.S.	1	1.15	0.63	0.74	1.54	1.70	1.20	1.30	-0.02	1.00
time (sec)	N/A	0.022	0.010	0.139	0.421	2.422	2.465	0.179	0.000	0.187

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	54	443	0	0	0	0	0	0	-1	54
N.S.	1	8.20	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.598	0.235	0.165	0.000	0.000	0.000	0.000	0.000	0.221

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	100	35	390	0	73	0	54	-1	54
N.S.	1	1.85	0.65	7.22	0.00	1.35	0.00	1.00	-0.02	1.00
time (sec)	N/A	0.075	0.010	0.467	0.000	0.395	0.000	0.198	0.000	0.234

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	63	37	47	58	56	56	0	46	56
N.S.	1	1.17	0.69	0.87	1.07	1.04	1.04	0.00	0.85	1.04
time (sec)	N/A	0.046	0.021	0.052	0.431	0.391	4.819	0.000	1.362	0.183

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	79	70	0	0	65	0	0	-1	54
N.S.	1	1.46	1.30	0.00	0.00	1.20	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.025	0.032	0.033	0.000	0.392	0.000	0.000	0.000	16.236

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	175	22	0	69	44	0	-1	54
N.S.	1	1.00	3.24	0.41	0.00	1.28	0.81	0.00	-0.02	1.00
time (sec)	N/A	0.050	0.918	0.040	0.000	0.396	1.338	0.000	0.000	0.109

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F(-2)	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	396	202	2705	0	269	0	0	84	55
N.S.	1	7.20	3.67	49.18	0.00	4.89	0.00	0.00	1.53	1.00
time (sec)	N/A	5.071	2.916	0.075	0.000	0.511	0.000	0.000	3.102	0.264

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	46	0	57	110	41	186	39	55
N.S.	1	1.00	0.84	0.00	1.04	2.00	0.75	3.38	0.71	1.00
time (sec)	N/A	0.038	0.012	0.256	0.413	0.424	0.925	0.134	0.714	0.065

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	63	54	73	107	115	102	61	69	55
N.S.	1	1.15	0.98	1.33	1.95	2.09	1.85	1.11	1.25	1.00
time (sec)	N/A	0.042	0.033	0.051	0.429	0.429	25.803	0.188	0.813	0.103

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F(-2)	A	F	F	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	508	202	2705	0	269	0	0	84	55
N.S.	1	9.24	3.67	49.18	0.00	4.89	0.00	0.00	1.53	1.00
time (sec)	N/A	2.755	0.764	0.035	0.000	0.505	0.000	0.000	1.826	0.207

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	73	79	134	56	0	31	55
N.S.	1	1.00	1.00	1.33	1.44	2.44	1.02	0.00	0.56	1.00
time (sec)	N/A	0.045	0.019	0.246	0.429	9.523	2.399	0.000	0.777	4.549

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	B	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	65	34	37	99	68	31	81	-1	55
N.S.	1	1.18	0.62	0.67	1.80	1.24	0.56	1.47	-0.02	1.00
time (sec)	N/A	0.019	0.009	0.270	0.420	0.396	1.042	0.167	0.000	0.192

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	109	42	442	0	93	0	53	-1	55
N.S.	1	1.98	0.76	8.04	0.00	1.69	0.00	0.96	-0.02	1.00
time (sec)	N/A	0.078	0.008	3.784	0.000	1.213	0.000	0.241	0.000	0.217

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	109	42	444	0	91	0	56	-1	55
N.S.	1	1.98	0.76	8.07	0.00	1.65	0.00	1.02	-0.02	1.00
time (sec)	N/A	0.076	0.007	3.740	0.000	1.245	0.000	0.510	0.000	0.221

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	66	68	609	0	55	0	65	-1	55
N.S.	1	1.20	1.24	11.07	0.00	1.00	0.00	1.18	-0.02	1.00
time (sec)	N/A	0.086	0.036	0.016	0.000	0.433	0.000	0.230	0.000	0.465

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	101	35	394	0	81	0	51	29	55
N.S.	1	1.84	0.64	7.16	0.00	1.47	0.00	0.93	0.53	1.00
time (sec)	N/A	0.068	0.010	0.411	0.000	0.377	0.000	0.193	0.717	0.218

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	0	1522	1610	0	50	0	0	-1	55
N.S.	1	0.00	27.67	29.27	0.00	0.91	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.059	0.881	1.046	0.000	0.436	0.000	0.000	0.000	4.712

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	55	0	59389	7840	0	77	0	0	-1	55
N.S.	1	0.00	1079.80	142.55	0.00	1.40	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.713	6.664	14.237	0.000	0.521	0.000	0.000	0.000	0.231

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	46	0	57	109	39	186	34	55
N.S.	1	1.00	0.84	0.00	1.04	1.98	0.71	3.38	0.62	1.00
time (sec)	N/A	0.038	0.012	0.276	0.430	0.408	0.914	0.177	0.697	0.047

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	48	0	57	81	37	186	36	55
N.S.	1	1.00	0.87	0.00	1.04	1.47	0.67	3.38	0.65	1.00
time (sec)	N/A	0.037	0.006	0.246	0.429	0.405	0.866	0.171	0.666	0.050

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	46	0	57	110	41	186	39	55
N.S.	1	1.00	0.84	0.00	1.04	2.00	0.75	3.38	0.71	1.00
time (sec)	N/A	0.037	0.010	0.293	0.429	0.404	0.930	0.199	0.709	0.050

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	55	51	77	136	77	47	151	0	54	63
N.S.	1	0.93	1.40	2.47	1.40	0.85	2.75	0.00	0.98	1.15
time (sec)	N/A	0.043	0.057	0.235	0.432	0.385	10.475	0.000	1.123	0.164

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	46	0	57	110	41	186	34	55
N.S.	1	1.00	0.84	0.00	1.04	2.00	0.75	3.38	0.62	1.00
time (sec)	N/A	0.037	0.012	0.276	0.424	0.395	0.920	0.142	0.721	0.051

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	79	67	164	0	98	0	74	-1	60
N.S.	1	1.41	1.20	2.93	0.00	1.75	0.00	1.32	-0.02	1.07
time (sec)	N/A	0.071	0.065	0.030	0.000	1.965	0.000	0.511	0.000	0.217

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	42	39	0	0	102	0	0	-1	56
N.S.	1	0.75	0.70	0.00	0.00	1.82	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.045	0.016	1.512	0.000	0.439	0.000	0.000	0.000	0.114

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	63	55	73	107	117	105	62	69	56
N.S.	1	1.12	0.98	1.30	1.91	2.09	1.88	1.11	1.23	1.00
time (sec)	N/A	0.046	0.036	0.018	0.436	0.422	26.738	0.189	0.777	0.078

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	F(-1)	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	56	0	2888	1747	0	0	0	0	102	56
N.S.	1	0.00	51.57	31.20	0.00	0.00	0.00	0.00	1.82	1.00
time (sec)	N/A	1.547	6.293	0.246	0.000	0.000	0.000	0.000	5.991	0.552

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	56	0	1333	2702	0	50	0	0	-1	56
N.S.	1	0.00	23.80	48.25	0.00	0.89	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.142	2.129	1.141	0.000	0.439	0.000	0.000	0.000	5.030

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	0	2609	2700	0	53	0	0	-1	56
N.S.	1	0.00	46.59	48.21	0.00	0.95	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.131	6.079	1.162	0.000	0.445	0.000	0.000	0.000	4.792

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	56	0	0	0	0	0	0	0	102	56
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.82	1.00
time (sec)	N/A	1.415	0.696	0.283	0.000	0.000	0.000	0.000	6.400	0.818

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F(-1)	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	56	0	0	0	0	223	0	0	-1	56
N.S.	1	0.00	0.00	0.00	0.00	3.98	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.922	0.714	0.301	0.000	0.562	0.000	0.000	0.000	2.727

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	C	F	A	F	A	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	178	71	336	0	59	0	57	51	56
N.S.	1	3.18	1.27	6.00	0.00	1.05	0.00	1.02	0.91	1.00
time (sec)	N/A	0.316	0.078	0.291	0.000	0.438	0.000	0.381	1.047	0.483

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	56	0	2948	0	0	0	0	0	-1	56
N.S.	1	0.00	52.64	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.214	6.795	0.266	0.000	0.000	0.000	0.000	0.000	11.660

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	39	248	33	52	52	223	65	76	56
N.S.	1	0.70	4.43	0.59	0.93	0.93	3.98	1.16	1.36	1.00
time (sec)	N/A	0.142	0.580	0.007	0.367	0.429	10.908	0.664	1.690	18.751

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	64	52	49	0	61	0	0	-1	56
N.S.	1	1.14	0.93	0.88	0.00	1.09	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.152	0.037	0.037	0.000	0.384	0.000	0.000	0.000	11.249

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	56	0	0	0	0	0	0	0	-1	56
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.221	0.338	0.258	0.000	0.000	0.000	0.000	0.000	12.798

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	52	49	60	0	44	745	37	48	56
N.S.	1	0.93	0.88	1.07	0.00	0.79	13.30	0.66	0.86	1.00
time (sec)	N/A	0.008	0.025	0.009	0.000	0.417	1.651	0.202	0.098	0.060

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	338	227	4588	0	345	0	0	122	57
N.S.	1	5.93	3.98	80.49	0.00	6.05	0.00	0.00	2.14	1.00
time (sec)	N/A	8.511	4.224	0.082	0.000	0.812	0.000	0.000	3.842	0.314

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	57	0	8060	568	0	0	0	0	-1	59
N.S.	1	0.00	141.40	9.96	0.00	0.00	0.00	0.00	-0.02	1.04
time (sec)	N/A	4.765	13.030	0.123	0.000	0.000	0.000	0.000	0.000	0.823

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	B	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	65	54	53	99	70	32	82	-1	57
N.S.	1	1.14	0.95	0.93	1.74	1.23	0.56	1.44	-0.02	1.00
time (sec)	N/A	0.020	0.019	0.249	0.429	0.412	1.085	0.301	0.000	0.188

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	113	47	373	0	72	0	60	-1	57
N.S.	1	1.98	0.82	6.54	0.00	1.26	0.00	1.05	-0.02	1.00
time (sec)	N/A	0.138	0.016	0.406	0.000	0.394	0.000	0.585	0.000	0.260

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	57	0	59573	1574	0	180	0	0	-1	57
N.S.	1	0.00	1045.14	27.61	0.00	3.16	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.745	6.680	1.023	0.000	0.518	0.000	0.000	0.000	0.430

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	50	0	68	134	37	0	-1	57
N.S.	1	1.00	0.88	0.00	1.19	2.35	0.65	0.00	-0.02	1.00
time (sec)	N/A	0.021	0.013	0.273	0.424	0.412	0.878	0.000	0.000	0.277

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	57	461	0	0	0	0	0	0	-1	57
N.S.	1	8.09	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.480	1.125	0.246	0.000	0.000	0.000	0.000	0.000	11.597

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	57	0	0	185	0	124	0	0	-1	57
N.S.	1	0.00	0.00	3.25	0.00	2.18	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.536	0.710	4.387	0.000	75.970	0.000	0.000	0.000	2.764

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	F	A	A	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	0	92	22	0	61	17	0	-1	57
N.S.	1	0.00	1.61	0.39	0.00	1.07	0.30	0.00	-0.02	1.00
time (sec)	N/A	0.020	0.183	0.031	0.000	0.760	0.696	0.000	0.000	0.119

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	58	182	95	210	0	76	0	0	100	58
N.S.	1	3.14	1.64	3.62	0.00	1.31	0.00	0.00	1.72	1.00
time (sec)	N/A	0.715	0.277	0.032	0.000	0.460	0.000	0.000	0.081	0.266

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	58	182	95	210	0	76	0	0	100	58
N.S.	1	3.14	1.64	3.62	0.00	1.31	0.00	0.00	1.72	1.00
time (sec)	N/A	0.685	0.194	0.030	0.000	0.479	0.000	0.000	0.560	0.260

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	58	183	95	211	0	76	0	0	100	58
N.S.	1	3.16	1.64	3.64	0.00	1.31	0.00	0.00	1.72	1.00
time (sec)	N/A	0.688	0.526	0.031	0.000	0.468	0.000	0.000	0.563	0.267

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	58	102	40	198	0	193	0	0	-1	58
N.S.	1	1.76	0.69	3.41	0.00	3.33	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.159	0.025	2.408	0.000	2.865	0.000	0.000	0.000	0.212

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	96	63	620	0	54	0	61	-1	58
N.S.	1	1.66	1.09	10.69	0.00	0.93	0.00	1.05	-0.02	1.00
time (sec)	N/A	0.124	0.060	0.013	0.000	0.443	0.000	0.384	0.000	0.444

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	46	170	219	0	63	0	0	-1	58
N.S.	1	0.79	2.93	3.78	0.00	1.09	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.104	0.206	0.047	0.000	0.509	0.000	0.000	0.000	0.525

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	58	0	0	0	0	0	0	0	-1	58
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.638	0.350	0.274	0.000	0.000	0.000	0.000	0.000	15.120

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	59	322	667	420	0	89	0	0	223	59
N.S.	1	5.46	11.31	7.12	0.00	1.51	0.00	0.00	3.78	1.00
time (sec)	N/A	1.005	2.394	0.020	0.000	0.461	0.000	0.000	0.170	0.119

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	59	0	6921	525	0	0	0	0	-1	59
N.S.	1	0.00	117.31	8.90	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	4.304	13.098	0.089	0.000	0.000	0.000	0.000	0.000	0.632

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	68	76	97	158	138	128	70	74	59
N.S.	1	1.15	1.29	1.64	2.68	2.34	2.17	1.19	1.25	1.00
time (sec)	N/A	0.052	0.035	0.056	0.421	0.410	65.159	0.471	0.956	0.114

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	68	75	97	158	136	128	69	74	59
N.S.	1	1.15	1.27	1.64	2.68	2.31	2.17	1.17	1.25	1.00
time (sec)	N/A	0.044	0.038	0.013	0.434	0.454	68.218	0.283	0.931	0.092

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	F(-1)	F(-1)	A	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	59	0	28348	628	0	0	0	62	-1	59
N.S.	1	0.00	480.47	10.64	0.00	0.00	0.00	1.05	-0.02	1.00
time (sec)	N/A	6.437	12.642	0.120	0.000	0.000	0.000	4.217	0.000	0.751

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	F(-1)	F(-1)	A	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	59	0	32877	674	0	0	0	69	-1	59
N.S.	1	0.00	557.24	11.42	0.00	0.00	0.00	1.17	-0.02	1.00
time (sec)	N/A	7.241	12.273	0.118	0.000	0.000	0.000	4.029	0.000	0.981

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	113	42	33	0	110	0	57	31	59
N.S.	1	1.92	0.71	0.56	0.00	1.86	0.00	0.97	0.53	1.00
time (sec)	N/A	0.064	0.008	0.253	0.000	2.177	0.000	0.261	0.768	0.119

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	125	52	417	0	102	0	57	-1	59
N.S.	1	2.12	0.88	7.07	0.00	1.73	0.00	0.97	-0.02	1.00
time (sec)	N/A	0.113	0.014	2.136	0.000	1.726	0.000	0.291	0.000	0.142

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	253	199	340	0	92	0	0	-1	59
N.S.	1	4.29	3.37	5.76	0.00	1.56	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.435	0.871	0.059	0.000	0.426	0.000	0.000	0.000	0.655

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	59	0	109133	12512	0	168	0	0	-1	59
N.S.	1	0.00	1849.71	212.07	0.00	2.85	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.856	6.678	9.232	0.000	0.540	0.000	0.000	0.000	0.230

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	50	95	152	0	132	0	0	-1	50
N.S.	1	0.85	1.61	2.58	0.00	2.24	0.00	0.00	-0.02	0.85
time (sec)	N/A	0.081	0.224	0.077	0.000	0.471	0.000	0.000	0.000	0.483

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	59	0	0	0	0	0	0	0	-1	59
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.504	0.637	0.267	0.000	0.000	0.000	0.000	0.000	1.189

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	59	53	80	136	77	47	150	0	54	63
N.S.	1	0.90	1.36	2.31	1.31	0.80	2.54	0.00	0.92	1.07
time (sec)	N/A	0.044	0.062	0.260	0.429	0.390	10.288	0.000	1.038	0.171

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	59	460	0	0	0	0	0	0	-1	59
N.S.	1	7.80	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.496	1.159	0.353	0.000	0.000	0.000	0.000	0.000	12.272

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F(-2)	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	440	289	10248	0	349	0	0	711	60
N.S.	1	7.33	4.82	170.80	0.00	5.82	0.00	0.00	11.85	1.00
time (sec)	N/A	15.466	6.908	0.108	0.000	21.827	0.000	0.000	0.822	2.799

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F(-2)	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	455	308	11540	0	379	0	0	690	60
N.S.	1	7.58	5.13	192.33	0.00	6.32	0.00	0.00	11.50	1.00
time (sec)	N/A	19.384	8.965	0.095	0.000	29.665	0.000	0.000	0.792	2.805

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	262	165	341	0	92	0	0	64	60
N.S.	1	4.37	2.75	5.68	0.00	1.53	0.00	0.00	1.07	1.00
time (sec)	N/A	2.349	2.491	0.069	0.000	0.444	0.000	0.000	2.761	0.142

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F(-1)	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	60	0	3908	486	0	638	0	0	946	60
N.S.	1	0.00	65.13	8.10	0.00	10.63	0.00	0.00	15.77	1.00
time (sec)	N/A	48.216	6.210	0.103	0.000	47.544	0.000	0.000	1.735	3.143

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F(-1)	C	C	F	A	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	60	0	1788	10183	0	244	0	0	79	60
N.S.	1	0.00	29.80	169.72	0.00	4.07	0.00	0.00	1.32	1.00
time (sec)	N/A	180.001	4.085	2.629	0.000	0.468	0.000	0.000	1.473	0.498

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F(-2)	A	F	F	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	290	204	1096	0	219	0	0	78	60
N.S.	1	4.83	3.40	18.27	0.00	3.65	0.00	0.00	1.30	1.00
time (sec)	N/A	2.118	2.307	0.107	0.000	0.451	0.000	0.000	3.281	0.340

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F(-2)	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	60	0	4752	489	0	651	0	0	569	60
N.S.	1	0.00	79.20	8.15	0.00	10.85	0.00	0.00	9.48	1.00
time (sec)	N/A	53.090	4.949	0.102	0.000	86.731	0.000	0.000	19.258	3.111

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	46	41	63	81	85	58	118	45	64
N.S.	1	0.77	0.68	1.05	1.35	1.42	0.97	1.97	0.75	1.07
time (sec)	N/A	0.043	0.072	0.028	0.423	0.417	4.702	0.705	0.966	0.168

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	44	41	51	78	73	75	81	42	58
N.S.	1	0.73	0.68	0.85	1.30	1.22	1.25	1.35	0.70	0.97
time (sec)	N/A	0.039	0.033	0.027	0.417	0.404	4.896	0.349	1.024	0.135

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	56	71	113	67	65	153	0	-1	68
N.S.	1	0.93	1.18	1.88	1.12	1.08	2.55	0.00	-0.02	1.13
time (sec)	N/A	0.047	0.066	0.342	0.422	0.403	5.301	0.000	0.000	0.173

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	56	63	141	64	62	168	0	-1	68
N.S.	1	0.93	1.05	2.35	1.07	1.03	2.80	0.00	-0.02	1.13
time (sec)	N/A	0.046	0.074	0.341	0.419	0.399	5.217	0.000	0.000	0.184

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	60	0	0	0	0	220	0	0	217	60
N.S.	1	0.00	0.00	0.00	0.00	3.67	0.00	0.00	3.62	1.00
time (sec)	N/A	1.436	0.492	0.503	0.000	0.527	0.000	0.000	4.757	1.967

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	335	149	85	0	75	0	0	-1	60
N.S.	1	5.58	2.48	1.42	0.00	1.25	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.892	0.303	0.535	0.000	0.468	0.000	0.000	0.000	22.255

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	60	0	0	0	0	190	0	0	98	60
N.S.	1	0.00	0.00	0.00	0.00	3.17	0.00	0.00	1.63	1.00
time (sec)	N/A	1.450	0.507	0.475	0.000	0.521	0.000	0.000	3.427	2.663

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	60	0	0	0	0	0	0	0	-1	60
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.378	0.293	0.372	0.000	0.000	0.000	0.000	0.000	15.676

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	C	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	141	59	130	27	27	0	0	-1	60
N.S.	1	2.35	0.98	2.17	0.45	0.45	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.952	0.045	0.049	0.405	0.410	0.000	0.000	0.000	19.168

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	61	87	0	513	0	332	0	0	-1	61
N.S.	1	1.43	0.00	8.41	0.00	5.44	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.102	0.275	11.457	0.000	5.843	0.000	0.000	0.000	0.262

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	386	146	151	0	141	0	0	234	61
N.S.	1	6.33	2.39	2.48	0.00	2.31	0.00	0.00	3.84	1.00
time (sec)	N/A	1.086	0.796	0.079	0.000	0.441	0.000	0.000	0.593	0.283

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	61	384	525	358	0	90	0	0	-1	61
N.S.	1	6.30	8.61	5.87	0.00	1.48	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.031	0.654	0.029	0.000	0.463	0.000	0.000	0.000	1.081

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	61	73	81	101	56	0	33	61
N.S.	1	1.00	1.00	1.20	1.33	1.66	0.92	0.00	0.54	1.00
time (sec)	N/A	0.043	0.017	0.339	0.418	4.497	2.389	0.000	0.772	4.109

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	54	0	74	142	41	0	-1	61
N.S.	1	1.00	0.89	0.00	1.21	2.33	0.67	0.00	-0.02	1.00
time (sec)	N/A	0.021	0.014	0.360	0.404	0.440	0.934	0.000	0.000	0.286

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	61	0	0	111	0	121	0	0	81	61
N.S.	1	0.00	0.00	1.82	0.00	1.98	0.00	0.00	1.33	1.00
time (sec)	N/A	7.952	1.935	0.972	0.000	0.452	0.000	0.000	3.443	0.166

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	67	100	104	0	45	56	47	47	61
N.S.	1	1.10	1.64	1.70	0.00	0.74	0.92	0.77	0.77	1.00
time (sec)	N/A	0.102	0.109	1.461	0.000	0.423	43.418	0.284	0.783	0.053

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	61	444	0	0	0	0	0	0	-1	61
N.S.	1	7.28	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.113	1.837	0.343	0.000	0.000	0.000	0.000	0.000	1.417

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	A	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	61	0	0	73	0	0	0	0	-1	61
N.S.	1	0.00	0.00	1.20	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.276	0.186	0.536	0.000	0.000	0.000	0.000	0.000	0.308

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F(-2)	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	57	0	0	57	0	0	-1	61
N.S.	1	1.00	0.93	0.00	0.00	0.93	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.030	0.032	180.000	0.000	0.394	0.000	0.000	0.000	0.095

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	B	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	81	67	58	123	75	36	105	-1	62
N.S.	1	1.31	1.08	0.94	1.98	1.21	0.58	1.69	-0.02	1.00
time (sec)	N/A	0.025	0.021	0.338	0.403	0.408	1.401	0.288	0.000	0.192

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	B	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	81	45	42	123	75	31	104	-1	62
N.S.	1	1.31	0.73	0.68	1.98	1.21	0.50	1.68	-0.02	1.00
time (sec)	N/A	0.024	0.015	0.308	0.409	0.410	1.361	0.269	0.000	0.187

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	0	2477	3365	0	148	0	0	-1	62
N.S.	1	0.00	39.95	54.27	0.00	2.39	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.676	1.884	0.178	0.000	1.923	0.000	0.000	0.000	0.299

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	62	0	0	0	0	231	0	0	73	62
N.S.	1	0.00	0.00	0.00	0.00	3.73	0.00	0.00	1.18	1.00
time (sec)	N/A	1.917	0.693	0.385	0.000	0.751	0.000	0.000	0.929	2.670

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	62	0	0	175	0	118	0	0	-1	62
N.S.	1	0.00	0.00	2.82	0.00	1.90	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.714	0.631	3.687	0.000	41.711	0.000	0.000	0.000	2.672

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	101	58	68	67	38	6166	67	-1	58
N.S.	1	1.63	0.94	1.10	1.08	0.61	99.45	1.08	-0.02	0.94
time (sec)	N/A	0.062	0.043	0.004	1.053	0.386	5.287	0.146	0.000	0.035

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	62	0	0	0	0	47	0	0	-1	62
N.S.	1	0.00	0.00	0.00	0.00	0.76	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.505	0.314	0.342	0.000	0.488	0.000	0.000	0.000	0.438

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	510	0	0	56	0	0	-1	62
N.S.	1	1.00	8.23	0.00	0.00	0.90	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.252	2.436	180.000	0.000	0.407	0.000	0.000	0.000	0.124

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F(-2)	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	59	0	0	44	0	0	-1	62
N.S.	1	1.00	0.95	0.00	0.00	0.71	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.027	0.026	180.000	0.000	0.391	0.000	0.000	0.000	0.087

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	61	79	67	0	102	0	76	-1	63
N.S.	1	0.97	1.25	1.06	0.00	1.62	0.00	1.21	-0.02	1.00
time (sec)	N/A	0.042	0.062	0.019	0.000	1.527	0.000	0.858	0.000	0.204

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	63	273	627	300	0	89	0	0	187	63
N.S.	1	4.33	9.95	4.76	0.00	1.41	0.00	0.00	2.97	1.00
time (sec)	N/A	0.810	2.278	0.023	0.000	0.444	0.000	0.000	0.761	0.113

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	63	0	8886	489	0	0	0	0	-1	63
N.S.	1	0.00	141.05	7.76	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	3.315	13.326	0.100	0.000	0.000	0.000	0.000	0.000	0.711

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	60	433	0	154	66	54	89	63
N.S.	1	1.00	0.95	6.87	0.00	2.44	1.05	0.86	1.41	1.00
time (sec)	N/A	0.093	0.083	0.121	0.000	0.429	37.514	0.134	1.321	0.065

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	63	0	21715	16170	0	293	0	0	125	63
N.S.	1	0.00	344.68	256.67	0.00	4.65	0.00	0.00	1.98	1.00
time (sec)	N/A	3.012	6.916	5.558	0.000	0.695	0.000	0.000	34.980	2.912

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	63	0	8822	502	0	0	0	0	-1	63
N.S.	1	0.00	140.03	7.97	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	4.092	13.324	0.104	0.000	0.000	0.000	0.000	0.000	0.535

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	63	380	525	374	0	88	0	0	-1	63
N.S.	1	6.03	8.33	5.94	0.00	1.40	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.040	0.725	0.025	0.000	0.629	0.000	0.000	0.000	1.051

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	127	56	451	0	99	0	68	-1	63
N.S.	1	2.02	0.89	7.16	0.00	1.57	0.00	1.08	-0.02	1.00
time (sec)	N/A	0.117	0.018	4.010	0.000	2.029	0.000	0.242	0.000	0.252

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	122	35	27	0	80	0	68	27	63
N.S.	1	1.94	0.56	0.43	0.00	1.27	0.00	1.08	0.43	1.00
time (sec)	N/A	0.091	0.009	0.328	0.000	0.457	0.000	0.147	0.838	0.245

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	63	0	61074	5959	0	75	0	0	-1	63
N.S.	1	0.00	969.43	94.59	0.00	1.19	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.693	6.628	4.169	0.000	0.618	0.000	0.000	0.000	1.340

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	49	47	65	0	82	0	72	-1	67
N.S.	1	0.78	0.75	1.03	0.00	1.30	0.00	1.14	-0.02	1.06
time (sec)	N/A	0.057	0.065	0.605	0.000	0.447	0.000	0.202	0.000	0.139

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	63	0	0	197	0	137	0	0	-1	63
N.S.	1	0.00	0.00	3.13	0.00	2.17	0.00	0.00	-0.02	1.00
time (sec)	N/A	2.324	0.686	7.921	0.000	79.492	0.000	0.000	0.000	2.736

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	A	A	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	63	1234	84	449	0	86	148	87	724	63
N.S.	1	19.59	1.33	7.13	0.00	1.37	2.35	1.38	11.49	1.00
time (sec)	N/A	3.489	0.233	0.039	0.000	0.450	135.611	0.189	0.051	0.103

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	63	0	0	485	0	3547	0	0	-1	63
N.S.	1	0.00	0.00	7.70	0.00	56.30	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.478	0.139	4.227	0.000	1.566	0.000	0.000	0.000	2.851

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	0	87	0	0	34	0	0	-1	63
N.S.	1	0.00	1.38	0.00	0.00	0.54	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.708	0.595	0.303	0.000	0.449	0.000	0.000	0.000	3.849

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	B	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	51	41	118	0	47	286	0	-1	63
N.S.	1	0.81	0.65	1.87	0.00	0.75	4.54	0.00	-0.02	1.00
time (sec)	N/A	0.017	0.102	0.038	0.000	0.490	1.179	0.000	0.000	0.124

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F(-1)	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	0	117	0	0	78	0	0	121	63
N.S.	1	0.00	1.86	0.00	0.00	1.24	0.00	0.00	1.92	1.00
time (sec)	N/A	0.453	0.114	0.313	0.000	0.455	0.000	0.000	1.488	0.126

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	91	110	231	0	213	0	0	-1	64
N.S.	1	1.42	1.72	3.61	0.00	3.33	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.752	0.468	0.096	0.000	0.493	0.000	0.000	0.000	0.272

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	91	110	233	0	213	0	0	-1	64
N.S.	1	1.42	1.72	3.64	0.00	3.33	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.677	0.404	0.045	0.000	0.509	0.000	0.000	0.000	0.273

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	91	110	231	0	213	0	0	-1	64
N.S.	1	1.42	1.72	3.61	0.00	3.33	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.710	0.145	0.015	0.000	0.507	0.000	0.000	0.000	0.285

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	0	1144	2769	0	58	0	0	-1	64
N.S.	1	0.00	17.88	43.27	0.00	0.91	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.137	1.119	1.300	0.000	0.496	0.000	0.000	0.000	5.057

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	64	56	70	115	67	64	151	0	-1	68
N.S.	1	0.88	1.09	1.80	1.05	1.00	2.36	0.00	-0.02	1.06
time (sec)	N/A	0.045	0.041	0.349	0.427	0.443	5.616	0.000	0.000	0.200

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	70	135	108	0	54	60	50	50	64
N.S.	1	1.09	2.11	1.69	0.00	0.84	0.94	0.78	0.78	1.00
time (sec)	N/A	0.098	0.091	1.253	0.000	0.444	50.595	0.416	0.983	0.080

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	82	225	0	248	0	0	-1	64
N.S.	1	1.00	1.28	3.52	0.00	3.88	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.047	0.220	2.000	0.000	5.372	0.000	0.000	0.000	0.332

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	47	47	77	0	51	0	0	-1	66
N.S.	1	0.73	0.73	1.20	0.00	0.80	0.00	0.00	-0.02	1.03
time (sec)	N/A	0.086	0.058	0.614	0.000	0.496	0.000	0.000	0.000	0.113

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	65	45	159	139	80	66	0	83	111	65
N.S.	1	0.69	2.45	2.14	1.23	1.02	0.00	1.28	1.71	1.00
time (sec)	N/A	0.021	1.309	0.021	0.416	0.473	0.000	0.350	0.832	0.220

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	65	0	71	132	46	185	49	65
N.S.	1	1.00	1.00	0.00	1.09	2.03	0.71	2.85	0.75	1.00
time (sec)	N/A	0.050	0.020	0.324	0.413	0.461	1.061	0.360	0.812	0.067

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	65	172	177	166	0	92	0	0	116	65
N.S.	1	2.65	2.72	2.55	0.00	1.42	0.00	0.00	1.78	1.00
time (sec)	N/A	0.785	0.550	0.036	0.000	0.475	0.000	0.000	0.717	0.281

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	65	520	131	178	0	41	0	0	-1	65
N.S.	1	8.00	2.02	2.74	0.00	0.63	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.410	0.196	0.019	0.000	0.484	0.000	0.000	0.000	3.749

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	65	0	0	0	0	135	0	0	-1	65
N.S.	1	0.00	0.00	0.00	0.00	2.08	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.082	0.294	0.359	0.000	0.448	0.000	0.000	0.000	0.433

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	65	0	0	0	0	130	0	0	-1	65
N.S.	1	0.00	0.00	0.00	0.00	2.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	1.089	0.408	0.325	0.000	0.470	0.000	0.000	0.000	0.799

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	A	F	B	F	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	155	107	102	0	130	0	0	-1	57
N.S.	1	2.38	1.65	1.57	0.00	2.00	0.00	0.00	-0.02	0.88
time (sec)	N/A	0.538	0.629	0.041	0.000	0.614	0.000	0.000	0.000	0.259

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	62	458	0	151	63	50	96	66
N.S.	1	1.00	0.94	6.94	0.00	2.29	0.95	0.76	1.45	1.00
time (sec)	N/A	0.062	0.026	0.142	0.000	0.458	6.492	0.258	3.503	0.061

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	62	418	0	158	56	49	96	66
N.S.	1	1.00	0.94	6.33	0.00	2.39	0.85	0.74	1.45	1.00
time (sec)	N/A	0.088	0.079	0.080	0.000	0.452	18.264	0.996	2.947	0.058

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	66	0	641	222	0	106	0	0	-1	66
N.S.	1	0.00	9.71	3.36	0.00	1.61	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.566	2.692	0.181	0.000	0.485	0.000	0.000	0.000	0.635

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	66	0	66	93	42	183	48	66
N.S.	1	1.00	1.00	0.00	1.00	1.41	0.64	2.77	0.73	1.00
time (sec)	N/A	0.048	0.015	0.326	0.409	0.456	0.930	0.870	0.823	0.068

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	0	89	0	0	39	0	0	-1	66
N.S.	1	0.00	1.35	0.00	0.00	0.59	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.030	0.110	0.046	0.000	0.697	0.000	0.000	0.000	3.263

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	F(-1)	F(-1)	F	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	0	504	3791	0	0	0	0	722	67
N.S.	1	0.00	7.52	56.58	0.00	0.00	0.00	0.00	10.78	1.00
time (sec)	N/A	7.404	8.497	0.060	0.000	0.000	0.000	0.000	0.107	0.176

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	67	45	108	88	0	51	0	0	-1	53
N.S.	1	0.67	1.61	1.31	0.00	0.76	0.00	0.00	-0.01	0.79
time (sec)	N/A	0.007	0.105	0.028	0.000	0.467	0.000	0.000	0.000	0.194

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F(-1)	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	133	56	0	0	118	0	69	-1	67
N.S.	1	1.99	0.84	0.00	0.00	1.76	0.00	1.03	-0.01	1.00
time (sec)	N/A	0.122	0.019	180.000	0.000	1.239	0.000	0.196	0.000	0.175

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	67	0	1105	1609	0	70	0	0	-1	67
N.S.	1	0.00	16.49	24.01	0.00	1.04	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.061	2.249	0.973	0.000	0.457	0.000	0.000	0.000	13.305

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	67	0	1189	1609	0	70	0	0	-1	67
N.S.	1	0.00	17.75	24.01	0.00	1.04	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.059	1.966	1.049	0.000	0.464	0.000	0.000	0.000	13.192

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	67	65	119	127	0	120	0	0	-1	67
N.S.	1	0.97	1.78	1.90	0.00	1.79	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.391	0.248	1.323	0.000	0.411	0.000	0.000	0.000	0.386

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	67	0	0	0	0	134	0	0	-1	67
N.S.	1	0.00	0.00	0.00	0.00	2.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.921	0.346	0.325	0.000	0.655	0.000	0.000	0.000	0.484

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	67	0	0	0	0	0	0	0	-1	67
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.501	0.537	0.334	0.000	0.000	0.000	0.000	0.000	3.080

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	67	0	0	133	0	66	0	0	-1	67
N.S.	1	0.00	0.00	1.99	0.00	0.99	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.386	0.306	0.717	0.000	0.635	0.000	0.000	0.000	2.649

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	67	58	91	218	0	242	0	0	-1	67
N.S.	1	0.87	1.36	3.25	0.00	3.61	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.077	0.177	1.654	0.000	5.242	0.000	0.000	0.000	0.321

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	85	236	0	242	0	0	-1	67
N.S.	1	1.00	1.27	3.52	0.00	3.61	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.043	0.248	1.681	0.000	5.337	0.000	0.000	0.000	0.277

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	67	67	91	217	0	243	0	0	-1	67
N.S.	1	1.00	1.36	3.24	0.00	3.63	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.064	0.196	1.522	0.000	5.833	0.000	0.000	0.000	0.320

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	67	67	119	172	0	90	0	0	-1	67
N.S.	1	1.00	1.78	2.57	0.00	1.34	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.347	0.285	0.035	0.000	0.524	0.000	0.000	0.000	0.364

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	1382	175	193	0	247	0	0	-1	67
N.S.	1	20.63	2.61	2.88	0.00	3.69	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.224	0.805	0.102	0.000	0.627	0.000	0.000	0.000	0.509

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	56	350	0	0	38	63	0	-1	67
N.S.	1	0.84	5.22	0.00	0.00	0.57	0.94	0.00	-0.01	1.00
time (sec)	N/A	0.804	1.321	0.301	0.000	0.424	0.582	0.000	0.000	0.079

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	56	51	0	0	38	63	0	-1	67
N.S.	1	0.84	0.76	0.00	0.00	0.57	0.94	0.00	-0.01	1.00
time (sec)	N/A	0.106	0.027	0.295	0.000	0.433	0.515	0.000	0.000	0.069

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	145	69	454	0	104	0	88	-1	68
N.S.	1	2.13	1.01	6.68	0.00	1.53	0.00	1.29	-0.01	1.00
time (sec)	N/A	0.183	0.027	3.913	0.000	2.306	0.000	0.237	0.000	0.270

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-2)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	68	0	0	700	0	0	0	0	-1	68
N.S.	1	0.00	0.00	10.29	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	6.839	4.939	6.624	0.000	0.000	0.000	0.000	0.000	0.256

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	114	42	541	0	128	0	65	-1	68
N.S.	1	1.68	0.62	7.96	0.00	1.88	0.00	0.96	-0.01	1.00
time (sec)	N/A	0.188	0.019	1.538	0.000	0.449	0.000	0.198	0.000	0.336

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	68	0	86	0	70	0	-1	68
N.S.	1	1.00	1.00	0.00	1.26	0.00	1.03	0.00	-0.01	1.00
time (sec)	N/A	0.054	0.037	0.347	0.412	0.000	1.979	0.000	0.000	0.334

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	68	147	0	0	0	262	0	0	-1	68
N.S.	1	2.16	0.00	0.00	0.00	3.85	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.756	0.730	5.238	0.000	12.943	0.000	0.000	0.000	0.386

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	88	146	218	0	61	0	62	107	68
N.S.	1	1.29	2.15	3.21	0.00	0.90	0.00	0.91	1.57	1.00
time (sec)	N/A	0.505	0.185	0.407	0.000	0.433	0.000	0.194	2.508	0.115

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	0	107	0	0	60	0	0	-1	68
N.S.	1	0.00	1.57	0.00	0.00	0.88	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.915	0.471	0.296	0.000	0.437	0.000	0.000	0.000	3.926

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	69	0	4112	817	0	338	0	0	97	69
N.S.	1	0.00	59.59	11.84	0.00	4.90	0.00	0.00	1.41	1.00
time (sec)	N/A	23.714	9.719	0.058	0.000	1.800	0.000	0.000	4.614	0.953

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	69	111	58	229	0	218	0	62	-1	69
N.S.	1	1.61	0.84	3.32	0.00	3.16	0.00	0.90	-0.01	1.00
time (sec)	N/A	0.138	0.013	6.099	0.000	1.935	0.000	0.342	0.000	0.318

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	C	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	79	77	101	0	94	0	119	-1	80
N.S.	1	1.14	1.12	1.46	0.00	1.36	0.00	1.72	-0.01	1.16
time (sec)	N/A	0.178	0.039	0.067	0.000	0.431	0.000	0.490	0.000	0.250

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	69	6713	477	293	0	87	0	0	-1	69
N.S.	1	97.29	6.91	4.25	0.00	1.26	0.00	0.00	-0.01	1.00
time (sec)	N/A	13.497	1.818	0.254	0.000	0.478	0.000	0.000	0.000	0.754

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	147	398	0	162	0	0	-1	69
N.S.	1	1.00	2.13	5.77	0.00	2.35	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.970	0.293	0.060	0.000	1.788	0.000	0.000	0.000	0.410

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	69	0	0	0	0	138	0	0	-1	69
N.S.	1	0.00	0.00	0.00	0.00	2.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.542	0.456	0.350	0.000	0.679	0.000	0.000	0.000	0.821

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	69	0	0	187	0	125	0	0	-1	69
N.S.	1	0.00	0.00	2.71	0.00	1.81	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.620	0.462	3.552	0.000	46.484	0.000	0.000	0.000	2.671

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	69	0	0	183	0	125	0	0	-1	69
N.S.	1	0.00	0.00	2.65	0.00	1.81	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.650	0.334	3.271	0.000	42.857	0.000	0.000	0.000	2.672

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	70	0	71	132	46	185	50	70
N.S.	1	1.00	1.00	0.00	1.01	1.89	0.66	2.64	0.71	1.00
time (sec)	N/A	0.079	0.021	0.335	0.411	0.424	1.138	0.752	0.839	0.067

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	66	90	88	141	66	0	41	70
N.S.	1	1.00	0.94	1.29	1.26	2.01	0.94	0.00	0.59	1.00
time (sec)	N/A	0.081	0.030	0.313	0.403	10.510	3.070	0.000	1.028	4.420

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	70	0	71	132	44	185	50	70
N.S.	1	1.00	1.00	0.00	1.01	1.89	0.63	2.64	0.71	1.00
time (sec)	N/A	0.059	0.019	0.309	0.415	0.416	1.089	0.272	0.813	0.069

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	70	0	3600	79345	0	291	0	0	-1	70
N.S.	1	0.00	51.43	1133.50	0.00	4.16	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.188	2.169	0.152	0.000	0.716	0.000	0.000	0.000	0.543

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	147	397	0	159	0	0	-1	70
N.S.	1	1.00	2.10	5.67	0.00	2.27	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.971	0.296	0.052	0.000	1.143	0.000	0.000	0.000	0.391

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	70	0	71	132	46	185	50	70
N.S.	1	1.00	1.00	0.00	1.01	1.89	0.66	2.64	0.71	1.00
time (sec)	N/A	0.088	0.023	0.344	0.444	0.416	1.060	0.273	0.878	0.057

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	70	0	0	115	0	69	0	0	-1	70
N.S.	1	0.00	0.00	1.64	0.00	0.99	0.00	0.00	-0.01	1.00
time (sec)	N/A	5.724	2.552	0.767	0.000	0.454	0.000	0.000	0.000	0.247

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	70	0	0	1242	0	154	0	0	-1	70
N.S.	1	0.00	0.00	17.74	0.00	2.20	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.441	0.993	2.069	0.000	160.788	0.000	0.000	0.000	2.596

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	70	0	71	132	46	185	50	70
N.S.	1	1.00	1.00	0.00	1.01	1.89	0.66	2.64	0.71	1.00
time (sec)	N/A	0.076	0.021	0.320	0.401	0.436	1.052	0.283	0.834	0.066

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	72	62	47	0	56	0	44	-1	60
N.S.	1	1.03	0.89	0.67	0.00	0.80	0.00	0.63	-0.01	0.86
time (sec)	N/A	0.117	0.042	0.007	0.000	0.833	0.000	0.362	0.000	0.126

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	62	64	41	0	56	0	44	-1	60
N.S.	1	0.89	0.91	0.59	0.00	0.80	0.00	0.63	-0.01	0.86
time (sec)	N/A	0.129	0.026	0.005	0.000	0.833	0.000	0.265	0.000	0.101

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	70	0	0	0	0	87	0	0	-1	70
N.S.	1	0.00	0.00	0.00	0.00	1.24	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.570	0.126	0.296	0.000	1.533	0.000	0.000	0.000	0.097

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	A	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	70	0	1665	51	0	90	15	0	-1	70
N.S.	1	0.00	23.79	0.73	0.00	1.29	0.21	0.00	-0.01	1.00
time (sec)	N/A	0.028	11.733	0.073	0.000	0.694	0.985	0.000	0.000	0.323

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	F	A	C	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	0	82	51	0	81	53	0	-1	70
N.S.	1	0.00	1.17	0.73	0.00	1.16	0.76	0.00	-0.01	1.00
time (sec)	N/A	0.143	0.224	0.058	0.000	0.820	1.111	0.000	0.000	0.318

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	82	78	67	0	61	12	132	96	71
N.S.	1	1.15	1.10	0.94	0.00	0.86	0.17	1.86	1.35	1.00
time (sec)	N/A	0.225	0.103	0.026	0.000	0.400	20.561	0.620	1.956	0.343

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	71	43	167	172	0	80	0	94	-1	71
N.S.	1	0.61	2.35	2.42	0.00	1.13	0.00	1.32	-0.01	1.00
time (sec)	N/A	0.015	3.246	0.072	0.000	0.408	0.000	0.321	0.000	0.104

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	51	51	63	54	56	29	55	74	71
N.S.	1	0.72	0.72	0.89	0.76	0.79	0.41	0.77	1.04	1.00
time (sec)	N/A	0.065	0.015	0.298	0.471	0.408	0.752	0.288	0.899	0.049

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	B	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	71	0	0	0	0	0	0	264	-1	71
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	3.72	-0.01	1.00
time (sec)	N/A	18.110	1.144	0.114	0.000	0.000	0.000	1.416	0.000	3.075

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	71	0	0	0	0	0	0	0	-1	71
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.750	0.467	0.342	0.000	0.000	0.000	0.000	0.000	3.106

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	71	0	0	0	0	0	0	0	-1	71
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.947	0.619	0.350	0.000	0.000	0.000	0.000	0.000	2.790

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	71	0	0	151	0	387	0	0	173	82
N.S.	1	0.00	0.00	2.13	0.00	5.45	0.00	0.00	2.44	1.15
time (sec)	N/A	0.601	0.133	1.982	0.000	0.490	0.000	0.000	12.785	4.616

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	82	99	370	0	100	0	177	204	71
N.S.	1	1.15	1.39	5.21	0.00	1.41	0.00	2.49	2.87	1.00
time (sec)	N/A	0.118	0.136	0.069	0.000	0.410	0.000	0.224	0.210	0.349

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	0	104	0	0	142	0	0	-1	71
N.S.	1	0.00	1.46	0.00	0.00	2.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.187	0.143	0.335	0.000	18.862	0.000	0.000	0.000	2.039

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	2595	350	753	0	73	0	0	-1	72
N.S.	1	36.04	4.86	10.46	0.00	1.01	0.00	0.00	-0.01	1.00
time (sec)	N/A	9.865	1.440	0.074	0.000	0.490	0.000	0.000	0.000	1.616

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	F(-1)	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	153	69	0	0	123	0	89	-1	72
N.S.	1	2.12	0.96	0.00	0.00	1.71	0.00	1.24	-0.01	1.00
time (sec)	N/A	0.195	0.027	180.000	0.000	1.405	0.000	0.296	0.000	0.182

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	168	38	390	0	87	0	87	-1	72
N.S.	1	2.33	0.53	5.42	0.00	1.21	0.00	1.21	-0.01	1.00
time (sec)	N/A	0.152	0.012	0.421	0.000	0.412	0.000	0.316	0.000	0.352

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	72	0	0	0	0	0	0	0	-1	72
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.446	0.525	0.272	0.000	0.000	0.000	0.000	0.000	1.666

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	67	72	64	0	58	0	90	-1	72
N.S.	1	0.93	1.00	0.89	0.00	0.81	0.00	1.25	-0.01	1.00
time (sec)	N/A	0.188	0.053	0.009	0.000	0.668	0.000	0.531	0.000	0.322

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	72	0	0	91	0	120	0	0	-1	72
N.S.	1	0.00	0.00	1.26	0.00	1.67	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.167	0.581	0.431	0.000	0.948	0.000	0.000	0.000	0.416

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	75	71	89	0	177	0	68	63	77
N.S.	1	1.03	0.97	1.22	0.00	2.42	0.00	0.93	0.86	1.05
time (sec)	N/A	0.024	0.077	0.003	0.000	0.661	0.000	0.383	0.117	0.212

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	103	89	116	0	126	0	0	102	85
N.S.	1	1.41	1.22	1.59	0.00	1.73	0.00	0.00	1.40	1.16
time (sec)	N/A	0.511	0.798	0.032	0.000	0.686	0.000	0.000	0.108	0.615

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	180	93	273	0	126	0	0	159	85
N.S.	1	2.47	1.27	3.74	0.00	1.73	0.00	0.00	2.18	1.16
time (sec)	N/A	0.819	0.665	0.030	0.000	0.697	0.000	0.000	0.072	0.632

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	73	0	0	227	0	74	0	0	-1	73
N.S.	1	0.00	0.00	3.11	0.00	1.01	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.909	0.177	1.171	0.000	0.532	0.000	0.000	0.000	1.758

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	162	35	407	0	90	0	106	-1	73
N.S.	1	2.22	0.48	5.58	0.00	1.23	0.00	1.45	-0.01	1.00
time (sec)	N/A	0.191	0.010	0.380	0.000	0.403	0.000	0.585	0.000	0.313

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	73	0	0	113	0	122	0	0	81	73
N.S.	1	0.00	0.00	1.55	0.00	1.67	0.00	0.00	1.11	1.00
time (sec)	N/A	5.679	1.153	0.785	0.000	0.449	0.000	0.000	3.473	0.160

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	2513	372	530	0	221	0	0	119	73
N.S.	1	34.42	5.10	7.26	0.00	3.03	0.00	0.00	1.63	1.00
time (sec)	N/A	15.301	1.418	0.108	0.000	0.541	0.000	0.000	4.364	0.412

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	73	0	0	121	0	111	0	0	-1	73
N.S.	1	0.00	0.00	1.66	0.00	1.52	0.00	0.00	-0.01	1.00
time (sec)	N/A	5.597	0.935	0.865	0.000	0.485	0.000	0.000	0.000	0.320

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	73	0	0	589	0	6596	0	0	-1	73
N.S.	1	0.00	0.00	8.07	0.00	90.36	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.965	0.233	3.661	0.000	8.824	0.000	0.000	0.000	0.471

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	0	58	0	0	37	0	0	-1	73
N.S.	1	0.00	0.79	0.00	0.00	0.51	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.174	0.208	0.047	0.000	0.413	0.000	0.000	0.000	3.420

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	54	73	46	54	56	32	54	70	74
N.S.	1	0.73	0.99	0.62	0.73	0.76	0.43	0.73	0.95	1.00
time (sec)	N/A	0.035	0.017	0.255	0.445	0.434	0.816	0.339	0.871	0.049

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	54	53	63	54	56	32	54	80	74
N.S.	1	0.73	0.72	0.85	0.73	0.76	0.43	0.73	1.08	1.00
time (sec)	N/A	0.038	0.007	0.239	0.470	0.425	0.807	0.434	0.873	0.047

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	74	0	1814	364	0	141	0	0	117	74
N.S.	1	0.00	24.51	4.92	0.00	1.91	0.00	0.00	1.58	1.00
time (sec)	N/A	1.349	6.220	0.400	0.000	0.533	0.000	0.000	1.879	1.260

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	74	0	1735	1431	0	103	0	0	-1	74
N.S.	1	0.00	23.45	19.34	0.00	1.39	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.114	6.175	0.536	0.000	0.482	0.000	0.000	0.000	1.175

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	68	90	90	123	66	0	43	74
N.S.	1	1.00	0.92	1.22	1.22	1.66	0.89	0.00	0.58	1.00
time (sec)	N/A	0.057	0.032	0.237	0.419	9.120	4.506	0.000	1.237	4.480

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	120	36	566	0	138	0	72	-1	74
N.S.	1	1.62	0.49	7.65	0.00	1.86	0.00	0.97	-0.01	1.00
time (sec)	N/A	0.155	0.014	1.377	0.000	0.459	0.000	0.430	0.000	0.347

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	74	0	0	287	0	110	0	0	-1	74
N.S.	1	0.00	0.00	3.88	0.00	1.49	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.594	0.143	2.629	0.000	1.749	0.000	0.000	0.000	1.145

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	74	0	94	0	126	0	-1	74
N.S.	1	1.00	1.00	0.00	1.27	0.00	1.70	0.00	-0.01	1.00
time (sec)	N/A	0.077	0.039	0.281	0.495	0.000	2.223	0.000	0.000	0.362

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	54	53	63	54	56	31	54	80	74
N.S.	1	0.73	0.72	0.85	0.73	0.76	0.42	0.73	1.08	1.00
time (sec)	N/A	0.051	0.014	0.244	0.535	0.775	0.785	0.495	0.865	0.042

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	129	80	169	0	107	0	44	270	74
N.S.	1	1.74	1.08	2.28	0.00	1.45	0.00	0.59	3.65	1.00
time (sec)	N/A	3.702	0.245	0.098	0.000	0.545	0.000	0.546	1.421	0.120

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	198	92	471	0	151	0	108	-1	74
N.S.	1	2.68	1.24	6.36	0.00	2.04	0.00	1.46	-0.01	1.00
time (sec)	N/A	2.882	0.233	1.070	0.000	4.112	0.000	0.440	0.000	0.280

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	37	0	74	185	42	209	55	75
N.S.	1	1.00	0.49	0.00	0.99	2.47	0.56	2.79	0.73	1.00
time (sec)	N/A	0.067	0.010	0.270	0.740	0.416	1.184	0.311	0.961	0.133

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	B	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	75	0	0	0	0	0	0	288	-1	75
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	3.84	-0.01	1.00
time (sec)	N/A	19.455	3.456	0.064	0.000	0.000	0.000	0.683	0.000	0.238

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	37	0	74	182	42	209	55	75
N.S.	1	1.00	0.49	0.00	0.99	2.43	0.56	2.79	0.73	1.00
time (sec)	N/A	0.069	0.011	0.276	0.442	0.436	1.234	0.613	0.966	0.077

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F(-1)	F	B	F	A	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	75	129	0	0	0	253	0	65	-1	75
N.S.	1	1.72	0.00	0.00	0.00	3.37	0.00	0.87	-0.01	1.00
time (sec)	N/A	0.235	0.555	180.000	0.000	1.989	0.000	0.337	0.000	0.247

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	75	0	0	226	0	102	0	0	-1	75
N.S.	1	0.00	0.00	3.01	0.00	1.36	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.984	0.172	2.684	0.000	1.482	0.000	0.000	0.000	1.473

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	75	475	2667	823	0	71	0	0	-1	75
N.S.	1	6.33	35.56	10.97	0.00	0.95	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.792	7.038	0.250	0.000	0.513	0.000	0.000	0.000	0.427

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	46	0	102	192	37	0	37	75
N.S.	1	1.00	0.61	0.00	1.36	2.56	0.49	0.00	0.49	1.00
time (sec)	N/A	0.023	0.010	0.295	0.440	0.440	1.177	0.000	0.785	0.303

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	51	0	85	0	42	0	-1	75
N.S.	1	1.00	0.68	0.00	1.13	0.00	0.56	0.00	-0.01	1.00
time (sec)	N/A	0.024	0.010	0.269	0.439	0.000	1.239	0.000	0.000	0.233

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	37	0	74	185	42	209	55	75
N.S.	1	1.00	0.49	0.00	0.99	2.47	0.56	2.79	0.73	1.00
time (sec)	N/A	0.053	0.012	0.273	0.436	0.493	1.444	0.307	1.029	0.117

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	75	0	0	347	0	100	0	0	-1	75
N.S.	1	0.00	0.00	4.63	0.00	1.33	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.245	0.401	5.943	0.000	2.348	0.000	0.000	0.000	2.690

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	37	0	74	182	42	209	55	75
N.S.	1	1.00	0.49	0.00	0.99	2.43	0.56	2.79	0.73	1.00
time (sec)	N/A	0.052	0.011	0.277	0.414	0.482	1.526	0.435	1.035	0.075

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	75	0	0	0	0	0	0	0	-1	75
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.091	0.299	0.286	0.000	0.000	0.000	0.000	0.000	4.652

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	A	F	B	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	1128	146	110	0	430	0	0	-1	75
N.S.	1	15.04	1.95	1.47	0.00	5.73	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.921	0.550	0.034	0.000	0.589	0.000	0.000	0.000	0.312

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F(-2)	B	F(-1)	F	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	299	345	1178	0	413	0	0	90	76
N.S.	1	3.93	4.54	15.50	0.00	5.43	0.00	0.00	1.18	1.00
time (sec)	N/A	3.224	6.042	0.070	0.000	0.754	0.000	0.000	3.765	0.195

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	52	28	83	56	58	34	57	80	76
N.S.	1	0.68	0.37	1.09	0.74	0.76	0.45	0.75	1.05	1.00
time (sec)	N/A	0.034	0.006	0.260	0.417	0.436	0.806	0.447	0.980	0.047

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	2421	360	0	0	113	0	0	-1	76
N.S.	1	31.86	4.74	0.00	0.00	1.49	0.00	0.00	-0.01	1.00
time (sec)	N/A	26.563	0.602	0.267	0.000	0.469	0.000	0.000	0.000	2.781

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	76	0	31019	289	0	444	0	0	714	76
N.S.	1	0.00	408.14	3.80	0.00	5.84	0.00	0.00	9.39	1.00
time (sec)	N/A	17.663	15.242	0.058	0.000	1.190	0.000	0.000	1.532	0.279

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	76	73	78	429	0	199	0	0	-1	76
N.S.	1	0.96	1.03	5.64	0.00	2.62	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.097	0.082	0.985	0.000	3.549	0.000	0.000	0.000	0.211

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	76	0	32987	304	0	434	0	0	175	76
N.S.	1	0.00	434.04	4.00	0.00	5.71	0.00	0.00	2.30	1.00
time (sec)	N/A	21.991	14.419	0.059	0.000	2.035	0.000	0.000	7.780	0.340

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	76	0	32986	303	0	433	0	0	175	76
N.S.	1	0.00	434.03	3.99	0.00	5.70	0.00	0.00	2.30	1.00
time (sec)	N/A	23.922	6.421	0.017	0.000	1.925	0.000	0.000	7.316	0.310

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	52	28	83	56	58	32	57	80	76
N.S.	1	0.68	0.37	1.09	0.74	0.76	0.42	0.75	1.05	1.00
time (sec)	N/A	0.037	0.005	0.267	0.427	0.420	0.802	0.257	1.096	0.050

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	1134	605	671	0	114	0	0	-1	76
N.S.	1	14.92	7.96	8.83	0.00	1.50	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.674	2.544	0.604	0.000	0.592	0.000	0.000	0.000	0.781

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	139	44	73	29	64	0	0	64	71
N.S.	1	1.83	0.58	0.96	0.38	0.84	0.00	0.00	0.84	0.93
time (sec)	N/A	0.063	0.038	0.004	0.433	0.399	0.000	0.000	0.775	0.069

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	F(-1)	F(-1)	F	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	0	226	2792	0	0	0	0	754	77
N.S.	1	0.00	2.94	36.26	0.00	0.00	0.00	0.00	9.79	1.00
time (sec)	N/A	9.680	7.599	0.046	0.000	0.000	0.000	0.000	0.870	0.759

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F(-1)	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	250	152	323	0	174	0	0	-1	77
N.S.	1	3.25	1.97	4.19	0.00	2.26	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.684	1.798	0.032	0.000	0.514	0.000	0.000	0.000	0.184

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	46	78	14	69	76	27	0	12	77
N.S.	1	0.60	1.01	0.18	0.90	0.99	0.35	0.00	0.16	1.00
time (sec)	N/A	0.005	0.040	0.258	0.432	0.422	0.743	0.000	0.772	0.149

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	251	88	273	0	126	0	0	159	89
N.S.	1	3.26	1.14	3.55	0.00	1.64	0.00	0.00	2.06	1.16
time (sec)	N/A	0.691	0.814	0.028	0.000	0.488	0.000	0.000	0.817	0.424

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	77	0	0	0	0	0	0	0	-1	77
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	13.647	1.379	0.102	0.000	0.000	0.000	0.000	0.000	3.200

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	129	28	79	102	155	31	102	45	82
N.S.	1	1.68	0.36	1.03	1.32	2.01	0.40	1.32	0.58	1.06
time (sec)	N/A	0.096	0.006	0.310	0.436	0.466	0.779	0.214	0.891	0.088

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	77	0	22729	427	0	406	0	0	1150	77
N.S.	1	0.00	295.18	5.55	0.00	5.27	0.00	0.00	14.94	1.00
time (sec)	N/A	19.426	13.752	0.059	0.000	0.927	0.000	0.000	1.783	0.282

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	77	0	185	192	78	0	-1	77
N.S.	1	1.00	1.00	0.00	2.40	2.49	1.01	0.00	-0.01	1.00
time (sec)	N/A	0.044	0.046	0.276	0.442	0.505	2.737	0.000	0.000	0.426

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	123	51	0	0	0	0	186	40	77
N.S.	1	1.60	0.66	0.00	0.00	0.00	0.00	2.42	0.52	1.00
time (sec)	N/A	0.073	0.013	0.290	0.000	0.000	0.000	0.295	0.835	0.276

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	77	0	24546	454	0	483	0	0	-1	77
N.S.	1	0.00	318.78	5.90	0.00	6.27	0.00	0.00	-0.01	1.00
time (sec)	N/A	31.543	13.847	0.060	0.000	1.259	0.000	0.000	0.000	0.294

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	129	28	79	102	155	34	102	45	82
N.S.	1	1.68	0.36	1.03	1.32	2.01	0.44	1.32	0.58	1.06
time (sec)	N/A	0.108	0.005	0.285	0.433	0.468	0.790	0.252	0.900	0.088

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	74	89	98	0	73	0	52	111	77
N.S.	1	0.96	1.16	1.27	0.00	0.95	0.00	0.68	1.44	1.00
time (sec)	N/A	0.188	0.057	0.021	0.000	0.418	0.000	0.446	0.403	0.345

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	0	74	0	0	202	0	0	-1	77
N.S.	1	0.00	0.96	0.00	0.00	2.62	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.327	0.130	0.284	0.000	0.508	0.000	0.000	0.000	0.211

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	64	59	125	53	92	0	84	-1	78
N.S.	1	0.82	0.76	1.60	0.68	1.18	0.00	1.08	-0.01	1.00
time (sec)	N/A	0.031	0.059	0.043	0.440	0.446	0.000	0.379	0.000	0.315

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	35	0	94	194	41	221	58	78
N.S.	1	1.00	0.45	0.00	1.21	2.49	0.53	2.83	0.74	1.00
time (sec)	N/A	0.062	0.009	0.290	0.421	0.455	1.443	0.361	0.986	0.128

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	B	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	248	156	81	540	87	0	0	86	138
N.S.	1	3.18	2.00	1.04	6.92	1.12	0.00	0.00	1.10	1.77
time (sec)	N/A	0.996	0.100	0.008	0.358	0.432	0.000	0.000	0.902	5.063

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	37	0	94	191	41	221	58	78
N.S.	1	1.00	0.47	0.00	1.21	2.45	0.53	2.83	0.74	1.00
time (sec)	N/A	0.057	0.009	0.319	0.426	0.439	1.198	0.966	1.009	0.078

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	78	76	1226	1290	0	72	0	0	-1	78
N.S.	1	0.97	15.72	16.54	0.00	0.92	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.034	2.682	0.902	0.000	0.500	0.000	0.000	0.000	5.043

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	78	0	0	1594	0	287	0	0	-1	78
N.S.	1	0.00	0.00	20.44	0.00	3.68	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.072	0.415	3.046	0.000	14.720	0.000	0.000	0.000	0.535

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	34	0	94	194	39	221	58	78
N.S.	1	1.00	0.44	0.00	1.21	2.49	0.50	2.83	0.74	1.00
time (sec)	N/A	0.061	0.009	0.316	0.431	0.593	1.398	0.461	1.173	0.070

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	37	0	94	194	41	221	58	78
N.S.	1	1.00	0.47	0.00	1.21	2.49	0.53	2.83	0.74	1.00
time (sec)	N/A	0.057	0.009	0.324	0.431	0.613	1.328	0.984	1.038	0.064

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	37	0	94	191	41	221	58	78
N.S.	1	1.00	0.47	0.00	1.21	2.45	0.53	2.83	0.74	1.00
time (sec)	N/A	0.063	0.009	0.314	0.435	0.456	1.407	0.298	1.068	0.067

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	56	53	103	114	61	65	86	47	81
N.S.	1	0.72	0.68	1.32	1.46	0.78	0.83	1.10	0.60	1.04
time (sec)	N/A	0.062	0.037	0.276	0.432	0.418	31.276	0.322	1.240	0.155

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	79	65	52	0	59	0	53	-1	65
N.S.	1	1.01	0.83	0.67	0.00	0.76	0.00	0.68	-0.01	0.83
time (sec)	N/A	0.047	0.031	0.007	0.000	0.884	0.000	0.286	0.000	0.113

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	79	0	92	264	76	69	65	79
N.S.	1	1.00	1.00	0.00	1.16	3.34	0.96	0.87	0.82	1.00
time (sec)	N/A	0.068	0.039	0.300	0.419	0.432	7.865	0.348	1.137	0.068

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F(-1)	F	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	0	280	4446	0	395	0	0	695	79
N.S.	1	0.00	3.54	56.28	0.00	5.00	0.00	0.00	8.80	1.00
time (sec)	N/A	13.143	6.957	0.043	0.000	0.771	0.000	0.000	0.872	0.303

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	79	0	0	0	0	0	0	0	-1	79
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	12.581	3.322	0.061	0.000	0.000	0.000	0.000	0.000	0.247

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	79	119	63	251	0	105	0	0	232	79
N.S.	1	1.51	0.80	3.18	0.00	1.33	0.00	0.00	2.94	1.00
time (sec)	N/A	0.684	0.254	0.049	0.000	0.492	0.000	0.000	0.759	0.279

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	79	938	390	915	0	115	0	0	-1	79
N.S.	1	11.87	4.94	11.58	0.00	1.46	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.135	0.913	0.074	0.000	0.584	0.000	0.000	0.000	0.587

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	C	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	45	68	241	0	258	0	63	-1	79
N.S.	1	0.57	0.86	3.05	0.00	3.27	0.00	0.80	-0.01	1.00
time (sec)	N/A	0.166	0.319	3.947	0.000	1.741	0.000	0.506	0.000	0.252

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	79	65	94	80	0	106	0	97	-1	79
N.S.	1	0.82	1.19	1.01	0.00	1.34	0.00	1.23	-0.01	1.00
time (sec)	N/A	0.093	0.096	0.536	0.000	0.432	0.000	0.307	0.000	0.362

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	0	354	133	0	96	0	0	-1	79
N.S.	1	0.00	4.48	1.68	0.00	1.22	0.00	0.00	-0.01	1.00
time (sec)	N/A	4.393	1.700	0.507	0.000	0.557	0.000	0.000	0.000	3.087

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	A	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	79	601	0	396	0	102	0	0	-1	79
N.S.	1	7.61	0.00	5.01	0.00	1.29	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.741	0.165	1.666	0.000	2.437	0.000	0.000	0.000	1.512

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	79	0	0	157	0	512	0	0	-1	91
N.S.	1	0.00	0.00	1.99	0.00	6.48	0.00	0.00	-0.01	1.15
time (sec)	N/A	0.511	0.134	1.678	0.000	0.726	0.000	0.000	0.000	4.736

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	0	111	0	0	58	0	0	-1	79
N.S.	1	0.00	1.41	0.00	0.00	0.73	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.099	0.181	0.250	0.000	1.166	0.000	0.000	0.000	0.141

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	77	66	84	0	43	92	0	-1	79
N.S.	1	0.97	0.84	1.06	0.00	0.54	1.16	0.00	-0.01	1.00
time (sec)	N/A	0.044	0.041	0.048	0.000	0.438	0.623	0.000	0.000	0.100

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	B	C	F	A	A	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	0	194	22	0	81	15	0	-1	79
N.S.	1	0.00	2.46	0.28	0.00	1.03	0.19	0.00	-0.01	1.00
time (sec)	N/A	0.100	3.297	0.033	0.000	0.807	0.727	0.000	0.000	0.174

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	B	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	0	200	0	0	81	0	0	-1	79
N.S.	1	0.00	2.53	0.00	0.00	1.03	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.100	0.958	0.284	0.000	0.909	0.000	0.000	0.000	0.184

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	80	0	0	255	0	100	0	0	-1	80
N.S.	1	0.00	0.00	3.19	0.00	1.25	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.413	0.116	1.503	0.000	1.298	0.000	0.000	0.000	0.135

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F(-1)	F	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	0	277	2958	0	312	0	0	628	80
N.S.	1	0.00	3.46	36.98	0.00	3.90	0.00	0.00	7.85	1.00
time (sec)	N/A	12.216	4.758	0.029	0.000	0.847	0.000	0.000	0.869	1.175

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	80	0	0	207	0	189	0	0	-1	80
N.S.	1	0.00	0.00	2.59	0.00	2.36	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.139	0.199	1.600	0.000	0.438	0.000	0.000	0.000	2.621

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	75	0	110	204	37	0	-1	80
N.S.	1	1.00	0.94	0.00	1.38	2.55	0.46	0.00	-0.01	1.00
time (sec)	N/A	0.036	0.022	0.283	0.419	0.474	1.080	0.000	0.000	0.401

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	142	53	0	0	0	0	185	44	80
N.S.	1	1.78	0.66	0.00	0.00	0.00	0.00	2.31	0.55	1.00
time (sec)	N/A	0.148	0.017	0.102	0.000	0.000	0.000	0.372	0.983	0.231

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	81	0	0	0	0	0	0	0	-1	81
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	5.906	8.402	0.487	0.000	0.000	0.000	0.000	0.000	0.368

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	61	80	48	61	63	34	62	75	81
N.S.	1	0.75	0.99	0.59	0.75	0.78	0.42	0.77	0.93	1.00
time (sec)	N/A	0.046	0.020	0.267	0.430	0.452	0.940	0.303	0.827	0.052

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	81	0	0	0	0	0	0	0	-1	81
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	16.810	2.934	0.065	0.000	0.000	0.000	0.000	0.000	0.284

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	81	0	0	0	0	0	0	0	-1	81
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	5.346	11.068	0.477	0.000	0.000	0.000	0.000	0.000	2.523

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	81	81	109	0	0	149	0	0	-1	81
N.S.	1	1.00	1.35	0.00	0.00	1.84	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.035	0.114	0.819	0.000	0.430	0.000	0.000	0.000	0.284

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F(-1)	F	B	F	A	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	81	137	0	0	0	273	0	72	-1	81
N.S.	1	1.69	0.00	0.00	0.00	3.37	0.00	0.89	-0.01	1.00
time (sec)	N/A	0.219	0.705	180.000	0.000	1.926	0.000	1.143	0.000	0.243

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-2)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	81	0	0	383	0	0	0	0	-1	81
N.S.	1	0.00	0.00	4.73	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.080	0.235	3.410	0.000	0.000	0.000	0.000	0.000	5.350

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	81	0	0	0	0	382	0	0	-1	81
N.S.	1	0.00	0.00	0.00	0.00	4.72	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.079	0.097	2.401	0.000	8.094	0.000	0.000	0.000	1.015

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	A	F	A	F	B	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	0	76	70	0	79	0	183	-1	81
N.S.	1	0.00	0.94	0.86	0.00	0.98	0.00	2.26	-0.01	1.00
time (sec)	N/A	0.175	0.028	0.013	0.000	0.405	0.000	0.460	0.000	0.295

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	81	0	0	320	0	112	0	0	-1	81
N.S.	1	0.00	0.00	3.95	0.00	1.38	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.637	0.155	3.109	0.000	2.299	0.000	0.000	0.000	1.170

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	0	85	1689	0	132	0	0	-1	81
N.S.	1	0.00	1.05	20.85	0.00	1.63	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.258	0.466	0.429	0.000	0.460	0.000	0.000	0.000	0.454

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	F(-1)	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	81	0	93	0	126	0	57	81
N.S.	1	1.00	1.00	0.00	1.15	0.00	1.56	0.00	0.70	1.00
time (sec)	N/A	0.036	0.043	0.282	0.429	0.000	2.595	0.000	1.151	0.302

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	48	0	112	202	41	0	39	81
N.S.	1	1.00	0.59	0.00	1.38	2.49	0.51	0.00	0.48	1.00
time (sec)	N/A	0.024	0.011	0.312	0.429	0.416	1.416	0.000	0.783	0.287

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	53	0	93	0	46	0	-1	81
N.S.	1	1.00	0.65	0.00	1.15	0.00	0.57	0.00	-0.01	1.00
time (sec)	N/A	0.025	0.012	0.020	0.437	0.000	1.269	0.000	0.000	0.234

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	135	47	0	0	148	0	183	40	81
N.S.	1	1.67	0.58	0.00	0.00	1.83	0.00	2.26	0.49	1.00
time (sec)	N/A	0.149	0.016	0.082	0.000	0.426	0.000	0.423	0.993	0.310

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	81	0	0	250	0	196	0	0	-1	81
N.S.	1	0.00	0.00	3.09	0.00	2.42	0.00	0.00	-0.01	1.00
time (sec)	N/A	18.764	1.946	5.367	0.000	52.043	0.000	0.000	0.000	0.194

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	102	220	0	245	0	0	-1	81
N.S.	1	1.00	1.26	2.72	0.00	3.02	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.076	0.277	1.696	0.000	6.363	0.000	0.000	0.000	0.407

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	B	F	B	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	1639	201	157	0	500	0	0	-1	81
N.S.	1	20.23	2.48	1.94	0.00	6.17	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.530	0.849	0.046	0.000	5.099	0.000	0.000	0.000	0.435

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	1639	201	287	0	500	0	0	-1	81
N.S.	1	20.23	2.48	3.54	0.00	6.17	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.737	0.564	0.109	0.000	5.367	0.000	0.000	0.000	0.548

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	81	0	0	0	0	0	0	0	-1	81
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.211	0.093	0.270	0.000	0.000	0.000	0.000	0.000	0.222

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F(-2)	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	47	47	0	0	135	0	0	-1	81
N.S.	1	0.58	0.58	0.00	0.00	1.67	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.116	0.024	180.000	0.000	1.886	0.000	0.000	0.000	0.394

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	136	46	0	0	206	0	211	-1	82
N.S.	1	1.66	0.56	0.00	0.00	2.51	0.00	2.57	-0.01	1.00
time (sec)	N/A	0.153	0.015	0.057	0.000	0.428	0.000	0.206	0.000	0.361

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	136	60	0	0	207	0	207	-1	82
N.S.	1	1.66	0.73	0.00	0.00	2.52	0.00	2.52	-0.01	1.00
time (sec)	N/A	0.358	0.029	0.062	0.000	0.435	0.000	0.381	0.000	0.387

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	82	0	0	337	0	112	0	0	-1	82
N.S.	1	0.00	0.00	4.11	0.00	1.37	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.056	0.203	4.426	0.000	1.708	0.000	0.000	0.000	2.671

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	92	82	0	127	0	70	0	-1	82
N.S.	1	1.12	1.00	0.00	1.55	0.00	0.85	0.00	-0.01	1.00
time (sec)	N/A	0.060	0.076	0.326	0.422	0.000	2.368	0.000	0.000	0.563

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	129	34	15	0	84	0	55	25	83
N.S.	1	1.55	0.41	0.18	0.00	1.01	0.00	0.66	0.30	1.00
time (sec)	N/A	0.017	0.007	0.313	0.000	0.612	0.000	3.027	0.955	0.169

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	83	0	0	0	0	0	0	0	-1	83
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	6.757	1.840	0.316	0.000	0.000	0.000	0.000	0.000	0.389

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	83	0	0	0	0	0	0	0	-1	83
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	7.556	0.775	0.304	0.000	0.000	0.000	0.000	0.000	0.627

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	83	0	0	0	0	0	0	0	-1	83
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	15.009	3.751	0.065	0.000	0.000	0.000	0.000	0.000	0.258

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	78	89	125	0	79	0	91	-1	83
N.S.	1	0.94	1.07	1.51	0.00	0.95	0.00	1.10	-0.01	1.00
time (sec)	N/A	0.111	0.021	0.033	0.000	0.403	0.000	0.181	0.000	0.184

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-2)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	83	0	0	1059	0	0	0	0	-1	83
N.S.	1	0.00	0.00	12.76	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.513	0.423	111.572	0.000	0.000	0.000	0.000	0.000	0.250

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	66	0	0	0	0	0	-1	83
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.048	0.029	0.300	0.000	0.000	0.000	0.000	0.000	0.351

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	153	54	0	0	0	0	192	45	83
N.S.	1	1.84	0.65	0.00	0.00	0.00	0.00	2.31	0.54	1.00
time (sec)	N/A	0.168	0.020	0.102	0.000	0.000	0.000	0.186	1.027	0.211

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	103	76	68	0	64	0	82	-1	74
N.S.	1	1.24	0.92	0.82	0.00	0.77	0.00	0.99	-0.01	0.89
time (sec)	N/A	0.232	0.077	0.009	0.000	1.632	0.000	0.354	0.000	0.180

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	103	76	68	0	64	0	62	-1	74
N.S.	1	1.24	0.92	0.82	0.00	0.77	0.00	0.75	-0.01	0.89
time (sec)	N/A	0.213	0.055	0.006	0.000	1.716	0.000	0.148	0.000	0.207

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	103	76	68	0	64	0	82	-1	74
N.S.	1	1.24	0.92	0.82	0.00	0.77	0.00	0.99	-0.01	0.89
time (sec)	N/A	0.111	0.044	0.005	0.000	1.573	0.000	0.158	0.000	0.131

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	90	85	0	0	47	107	0	-1	83
N.S.	1	1.08	1.02	0.00	0.00	0.57	1.29	0.00	-0.01	1.00
time (sec)	N/A	0.167	0.058	0.300	0.000	0.392	0.548	0.000	0.000	0.097

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	83	0	0	0	0	52	0	0	-1	83
N.S.	1	0.00	0.00	0.00	0.00	0.63	0.00	0.00	-0.01	1.00
time (sec)	N/A	5.254	0.748	0.288	0.000	0.420	0.000	0.000	0.000	4.045

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F(-1)	F	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	0	277	2956	0	313	0	0	628	84
N.S.	1	0.00	3.30	35.19	0.00	3.73	0.00	0.00	7.48	1.00
time (sec)	N/A	12.659	2.567	0.030	0.000	0.821	0.000	0.000	0.176	0.997

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	64	59	64	63	65	36	64	83	84
N.S.	1	0.76	0.70	0.76	0.75	0.77	0.43	0.76	0.99	1.00
time (sec)	N/A	0.047	0.025	0.303	0.489	0.405	0.837	0.210	0.815	0.055

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	146	123	0	0	0	0	209	-1	84
N.S.	1	1.74	1.46	0.00	0.00	0.00	0.00	2.49	-0.01	1.00
time (sec)	N/A	0.220	0.128	0.079	0.000	0.000	0.000	0.194	0.000	0.313

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	84	0	345	862	0	171	0	0	-1	84
N.S.	1	0.00	4.11	10.26	0.00	2.04	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.456	1.456	0.178	0.000	0.530	0.000	0.000	0.000	1.550

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	A	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	84	666	0	122	0	120	0	0	-1	84
N.S.	1	7.93	0.00	1.45	0.00	1.43	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.340	0.431	1.144	0.000	0.523	0.000	0.000	0.000	37.154

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	56	43	41	43	50	0	99	44	55
N.S.	1	0.67	0.51	0.49	0.51	0.60	0.00	1.18	0.52	0.65
time (sec)	N/A	0.039	0.030	0.007	0.321	0.526	0.000	0.225	0.857	0.031

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	65	84	62	63	65	41	63	79	85
N.S.	1	0.76	0.99	0.73	0.74	0.76	0.48	0.74	0.93	1.00
time (sec)	N/A	0.046	0.023	0.277	0.426	0.412	7.386	0.244	0.998	0.049

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	203	154	232	0	177	0	0	-1	85
N.S.	1	2.39	1.81	2.73	0.00	2.08	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.688	1.386	0.026	0.000	0.474	0.000	0.000	0.000	0.194

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	137	89	96	0	101	0	0	102	73
N.S.	1	1.61	1.05	1.13	0.00	1.19	0.00	0.00	1.20	0.86
time (sec)	N/A	0.562	0.783	0.031	0.000	0.487	0.000	0.000	0.108	0.587

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	257	88	222	0	87	0	0	159	73
N.S.	1	3.02	1.04	2.61	0.00	1.02	0.00	0.00	1.87	0.86
time (sec)	N/A	0.726	0.727	0.029	0.000	0.486	0.000	0.000	0.799	0.392

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	85	0	0	267	0	100	0	0	-1	85
N.S.	1	0.00	0.00	3.14	0.00	1.18	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.434	0.118	1.395	0.000	1.159	0.000	0.000	0.000	0.124

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	86	40	298	0	104	0	0	-1	85
N.S.	1	1.01	0.47	3.51	0.00	1.22	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.046	0.012	0.914	0.000	0.847	0.000	0.000	0.000	0.191

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	85	0	0	0	0	0	0	0	-1	85
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	15.030	1.083	0.087	0.000	0.000	0.000	0.000	0.000	3.175

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	654	96	119	0	385	0	0	205	85
N.S.	1	7.69	1.13	1.40	0.00	4.53	0.00	0.00	2.41	1.00
time (sec)	N/A	1.656	0.725	0.033	0.000	0.479	0.000	0.000	0.031	0.358

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	A	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	85	297	0	307	0	106	0	0	-1	85
N.S.	1	3.49	0.00	3.61	0.00	1.25	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.149	0.765	1.793	0.000	1.203	0.000	0.000	0.000	0.328

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	94	67	0	103	0	114	0	-1	85
N.S.	1	1.11	0.79	0.00	1.21	0.00	1.34	0.00	-0.01	1.00
time (sec)	N/A	0.042	0.063	0.293	0.415	0.000	3.256	0.000	0.000	0.262

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	96	69	0	113	0	209	0	-1	85
N.S.	1	1.13	0.81	0.00	1.33	0.00	2.46	0.00	-0.01	1.00
time (sec)	N/A	0.045	0.086	0.269	0.417	0.000	3.438	0.000	0.000	0.263

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	109	226	190	0	83	223	0	-1	85
N.S.	1	1.28	2.66	2.24	0.00	0.98	2.62	0.00	-0.01	1.00
time (sec)	N/A	0.241	0.280	0.085	0.000	0.707	5.449	0.000	0.000	1.176

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	85	0	0	555	0	108	0	0	-1	85
N.S.	1	0.00	0.00	6.53	0.00	1.27	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.998	0.200	3.050	0.000	1.216	0.000	0.000	0.000	0.912

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	85	0	0	360	0	104	0	0	-1	85
N.S.	1	0.00	0.00	4.24	0.00	1.22	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.192	0.384	7.780	0.000	2.073	0.000	0.000	0.000	2.690

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	79	201	97	0	80	0	0	-1	87
N.S.	1	0.93	2.36	1.14	0.00	0.94	0.00	0.00	-0.01	1.02
time (sec)	N/A	0.226	0.181	0.536	0.000	0.407	0.000	0.000	0.000	0.202

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	85	0	0	1334	0	161	0	0	-1	85
N.S.	1	0.00	0.00	15.69	0.00	1.89	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.964	0.667	4.349	0.000	47.986	0.000	0.000	0.000	2.667

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	85	420	204	730	0	120	0	0	-1	85
N.S.	1	4.94	2.40	8.59	0.00	1.41	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.066	0.915	0.041	0.000	0.496	0.000	0.000	0.000	0.424

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-1)	F	B	F(-1)	F	F	C
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	85	0	0	0	0	440	0	0	-1	105
N.S.	1	0.00	0.00	0.00	0.00	5.18	0.00	0.00	-0.01	1.24
time (sec)	N/A	2.676	0.618	180.000	0.000	1.161	0.000	0.000	0.000	18.210

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	0	125	0	0	65	0	0	-1	85
N.S.	1	0.00	1.47	0.00	0.00	0.76	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.279	0.212	0.286	0.000	1.057	0.000	0.000	0.000	0.403

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	62	61	285	0	68	0	0	-1	86
N.S.	1	0.72	0.71	3.31	0.00	0.79	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.054	0.021	1.901	0.000	0.408	0.000	0.000	0.000	0.085

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	60	59	285	0	68	0	0	-1	86
N.S.	1	0.70	0.69	3.31	0.00	0.79	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.047	0.019	1.895	0.000	0.411	0.000	0.000	0.000	0.084

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	62	30	84	65	67	37	66	83	86
N.S.	1	0.72	0.35	0.98	0.76	0.78	0.43	0.77	0.97	1.00
time (sec)	N/A	0.041	0.008	0.283	0.721	0.409	0.873	0.413	0.855	0.058

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	86	0	0	260	0	104	0	0	-1	86
N.S.	1	0.00	0.00	3.02	0.00	1.21	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.439	0.123	1.558	0.000	1.143	0.000	0.000	0.000	0.123

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	86	0	0	0	0	0	0	0	-1	86
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	5.401	5.188	0.097	0.000	0.000	0.000	0.000	0.000	12.912

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	86	0	0	0	0	0	0	0	-1	86
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	5.181	4.974	0.085	0.000	0.000	0.000	0.000	0.000	12.932

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	138	135	64	109	162	36	109	52	91
N.S.	1	1.60	1.57	0.74	1.27	1.88	0.42	1.27	0.60	1.06
time (sec)	N/A	0.108	0.030	0.295	0.486	0.430	0.832	0.160	0.850	0.113

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	86	125	52	636	0	219	0	80	-1	86
N.S.	1	1.45	0.60	7.40	0.00	2.55	0.00	0.93	-0.01	1.00
time (sec)	N/A	0.146	0.026	0.395	0.000	0.734	0.000	0.297	0.000	0.523

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	86	131	69	247	0	286	0	70	-1	86
N.S.	1	1.52	0.80	2.87	0.00	3.33	0.00	0.81	-0.01	1.00
time (sec)	N/A	0.117	0.227	4.022	0.000	1.653	0.000	0.235	0.000	0.256

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	81	0	120	214	41	0	-1	86
N.S.	1	1.00	0.94	0.00	1.40	2.49	0.48	0.00	-0.01	1.00
time (sec)	N/A	0.036	0.025	0.297	0.414	0.462	1.090	0.000	0.000	0.391

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	B	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	93	61	0	190	0	83	0	-1	86
N.S.	1	1.08	0.71	0.00	2.21	0.00	0.97	0.00	-0.01	1.00
time (sec)	N/A	0.045	0.059	0.279	0.417	0.000	3.164	0.000	0.000	0.330

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	97	84	0	107	0	105	0	-1	86
N.S.	1	1.13	0.98	0.00	1.24	0.00	1.22	0.00	-0.01	1.00
time (sec)	N/A	0.067	0.101	0.312	0.494	0.000	2.318	0.000	0.000	0.517

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	123	97	219	0	132	0	96	-1	87
N.S.	1	1.41	1.11	2.52	0.00	1.52	0.00	1.10	-0.01	1.00
time (sec)	N/A	0.149	0.088	0.021	0.000	1.973	0.000	0.858	0.000	0.275

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	67	26	59	66	78	32	66	80	87
N.S.	1	0.77	0.30	0.68	0.76	0.90	0.37	0.76	0.92	1.00
time (sec)	N/A	0.042	0.005	0.276	0.570	0.407	0.918	0.293	0.894	0.090

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	67	26	76	66	79	34	66	86	87
N.S.	1	0.77	0.30	0.87	0.76	0.91	0.39	0.76	0.99	1.00
time (sec)	N/A	0.038	0.006	0.310	0.406	0.407	0.900	0.348	0.947	0.082

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	67	62	64	63	65	37	63	89	87
N.S.	1	0.77	0.71	0.74	0.72	0.75	0.43	0.72	1.02	1.00
time (sec)	N/A	0.035	0.024	0.282	0.414	0.405	0.812	0.218	0.867	0.049

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	87	0	18077	6278	0	168	0	0	-1	87
N.S.	1	0.00	207.78	72.16	0.00	1.93	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.382	6.699	4.607	0.000	0.578	0.000	0.000	0.000	0.415

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	67	62	64	63	65	37	63	89	87
N.S.	1	0.77	0.71	0.74	0.72	0.75	0.43	0.72	1.02	1.00
time (sec)	N/A	0.038	0.025	0.266	0.436	0.414	0.832	0.256	0.823	0.043

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	87	0	0	754	0	132	0	0	-1	87
N.S.	1	0.00	0.00	8.67	0.00	1.52	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.752	0.292	3.032	0.000	3.823	0.000	0.000	0.000	0.830

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	87	0	0	202	0	189	0	0	-1	87
N.S.	1	0.00	0.00	2.32	0.00	2.17	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.060	0.211	1.622	0.000	0.425	0.000	0.000	0.000	2.616

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	87	87	77	0	0	0	0	0	-1	87
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.059	0.036	0.313	0.000	0.000	0.000	0.000	0.000	0.531

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	70	0	0	431	0	0	-1	87
N.S.	1	1.00	0.80	0.00	0.00	4.95	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.038	0.025	0.303	0.000	172.990	0.000	0.000	0.000	0.333

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	87	0	5341	330	0	340	0	0	135	87
N.S.	1	0.00	61.39	3.79	0.00	3.91	0.00	0.00	1.55	1.00
time (sec)	N/A	9.954	8.076	0.063	0.000	1.594	0.000	0.000	7.319	2.060

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	169	72	155	0	80	0	0	-1	87
N.S.	1	1.94	0.83	1.78	0.00	0.92	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.457	0.518	0.039	0.000	0.521	0.000	0.000	0.000	1.843

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	67	62	64	63	65	37	64	89	87
N.S.	1	0.77	0.71	0.74	0.72	0.75	0.43	0.74	1.02	1.00
time (sec)	N/A	0.039	0.028	0.263	0.451	0.411	0.831	0.174	1.003	0.048

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	67	62	64	63	65	36	63	89	87
N.S.	1	0.77	0.71	0.74	0.72	0.75	0.41	0.72	1.02	1.00
time (sec)	N/A	0.034	0.022	0.289	0.419	0.426	0.808	0.285	0.843	0.045

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	87	0	0	174	0	184	0	0	-1	87
N.S.	1	0.00	0.00	2.00	0.00	2.11	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.593	0.391	1.122	0.000	0.570	0.000	0.000	0.000	0.205

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	87	0	0	725	0	131	0	0	-1	87
N.S.	1	0.00	0.00	8.33	0.00	1.51	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.795	0.296	6.776	0.000	38.498	0.000	0.000	0.000	20.307

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	77	68	60	62	94	0	126	80	87
N.S.	1	0.89	0.78	0.69	0.71	1.08	0.00	1.45	0.92	1.00
time (sec)	N/A	0.078	0.043	0.536	0.448	0.418	0.000	0.178	1.973	0.148

Problem 1060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	93	93	127	0	93	0	87	-1	87
N.S.	1	1.07	1.07	1.46	0.00	1.07	0.00	1.00	-0.01	1.00
time (sec)	N/A	0.154	0.073	0.007	0.000	1.156	0.000	0.610	0.000	0.227

Problem 1061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	81	73	60	88	183	78	82	63	87
N.S.	1	0.93	0.84	0.69	1.01	2.10	0.90	0.94	0.72	1.00
time (sec)	N/A	0.062	0.080	0.010	0.414	0.433	3.539	0.161	0.963	0.112

Problem 1062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	F	A	B	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	0	85	153	0	70	581	0	-1	87
N.S.	1	0.00	0.98	1.76	0.00	0.80	6.68	0.00	-0.01	1.00
time (sec)	N/A	0.282	0.205	0.043	0.000	0.474	3.707	0.000	0.000	0.157

Problem 1063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	60	36	522	0	70	0	0	-1	88
N.S.	1	0.68	0.41	5.93	0.00	0.80	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.052	0.013	1.981	0.000	0.418	0.000	0.000	0.000	0.086

Problem 1064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	253	169	315	0	639	0	0	103	88
N.S.	1	2.88	1.92	3.58	0.00	7.26	0.00	0.00	1.17	1.00
time (sec)	N/A	6.830	1.876	0.066	0.000	1.538	0.000	0.000	4.312	0.287

Problem 1065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	88	0	31196	437	0	384	0	0	-1	88
N.S.	1	0.00	354.50	4.97	0.00	4.36	0.00	0.00	-0.01	1.00
time (sec)	N/A	16.931	15.225	0.060	0.000	1.203	0.000	0.000	0.000	0.928

Problem 1066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	A	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	88	305	0	591	0	109	0	0	-1	88
N.S.	1	3.47	0.00	6.72	0.00	1.24	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.157	1.145	5.477	0.000	1.202	0.000	0.000	0.000	0.453

Problem 1067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	87	35	33	69	82	32	0	-1	88
N.S.	1	0.99	0.40	0.38	0.78	0.93	0.36	0.00	-0.01	1.00
time (sec)	N/A	0.065	0.008	0.320	0.491	0.414	0.794	0.000	0.000	0.671

Problem 1068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	53	83	33	69	82	31	0	-1	88
N.S.	1	0.60	0.94	0.38	0.78	0.93	0.35	0.00	-0.01	1.00
time (sec)	N/A	0.019	0.033	0.310	0.405	0.411	0.808	0.000	0.000	0.551

Problem 1069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	87	35	17	69	82	29	0	-1	88
N.S.	1	0.99	0.40	0.19	0.78	0.93	0.33	0.00	-0.01	1.00
time (sec)	N/A	0.064	0.007	0.276	0.448	0.411	0.783	0.000	0.000	0.712

Problem 1070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	53	83	17	69	82	29	0	-1	88
N.S.	1	0.60	0.94	0.19	0.78	0.93	0.33	0.00	-0.01	1.00
time (sec)	N/A	0.019	0.031	0.284	0.439	0.415	0.741	0.000	0.000	0.565

Problem 1071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	101	88	0	140	0	126	0	-1	88
N.S.	1	1.15	1.00	0.00	1.59	0.00	1.43	0.00	-0.01	1.00
time (sec)	N/A	0.060	0.076	0.308	0.450	0.000	2.412	0.000	0.000	0.559

Problem 1072	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	F(-1)	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	99	88	0	116	0	320	0	71	88
N.S.	1	1.12	1.00	0.00	1.32	0.00	3.64	0.00	0.81	1.00
time (sec)	N/A	0.061	0.084	0.306	0.426	0.000	2.523	0.000	1.305	0.358

Problem 1073	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	46	148	0	0	161	0	0	-1	88
N.S.	1	0.52	1.68	0.00	0.00	1.83	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.644	1.084	0.254	0.000	12.919	0.000	0.000	0.000	0.004

Problem 1074	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	65	30	84	65	67	39	66	89	89
N.S.	1	0.73	0.34	0.94	0.73	0.75	0.44	0.74	1.00	1.00
time (sec)	N/A	0.042	0.007	0.301	0.440	0.412	0.872	0.350	0.829	0.057

Problem 1075	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	89	0	0	334	0	95	0	0	-1	89
N.S.	1	0.00	0.00	3.75	0.00	1.07	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.009	0.134	2.078	0.000	0.855	0.000	0.000	0.000	0.121

Problem 1076	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	89	433	279	1516	0	230	0	0	509	89
N.S.	1	4.87	3.13	17.03	0.00	2.58	0.00	0.00	5.72	1.00
time (sec)	N/A	0.938	0.797	0.318	0.000	0.483	0.000	0.000	0.203	1.543

Problem 1077	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	75	96	65	72	70	82	73	86	89
N.S.	1	0.84	1.08	0.73	0.81	0.79	0.92	0.82	0.97	1.00
time (sec)	N/A	0.048	0.024	0.320	0.439	0.436	26.654	0.290	0.952	0.058

Problem 1078	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	89	0	0	0	0	0	0	0	-1	89
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	13.164	3.347	0.068	0.000	0.000	0.000	0.000	0.000	0.275

Problem 1079	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	89	0	0	0	0	0	0	0	-1	89
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	13.054	1.017	0.081	0.000	0.000	0.000	0.000	0.000	3.136

Problem 1080	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	541	153	960	0	120	0	0	565	89
N.S.	1	6.08	1.72	10.79	0.00	1.35	0.00	0.00	6.35	1.00
time (sec)	N/A	2.386	0.624	0.026	0.000	0.573	0.000	0.000	0.070	0.138

Problem 1081	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	89	155	743	1788	0	120	0	0	565	89
N.S.	1	1.74	8.35	20.09	0.00	1.35	0.00	0.00	6.35	1.00
time (sec)	N/A	3.129	3.218	0.336	0.000	0.724	0.000	0.000	0.058	0.127

Problem 1082	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	65	30	84	65	67	39	66	89	89
N.S.	1	0.73	0.34	0.94	0.73	0.75	0.44	0.74	1.00	1.00
time (sec)	N/A	0.040	0.007	0.288	0.562	0.758	0.855	0.133	0.832	0.053

Problem 1083	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	89	592	774	1080	0	120	0	0	698	89
N.S.	1	6.65	8.70	12.13	0.00	1.35	0.00	0.00	7.84	1.00
time (sec)	N/A	2.228	3.074	0.026	0.000	0.693	0.000	0.000	0.033	0.148

Problem 1084	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	89	0	743	2852	0	121	0	0	-1	89
N.S.	1	0.00	8.35	32.04	0.00	1.36	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.623	1.593	0.728	0.000	0.513	0.000	0.000	0.000	0.445

Problem 1085	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	89	0	2703	314	0	195	0	0	-1	89
N.S.	1	0.00	30.37	3.53	0.00	2.19	0.00	0.00	-0.01	1.00
time (sec)	N/A	11.536	10.468	0.507	0.000	0.506	0.000	0.000	0.000	0.413

Problem 1086	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	65	30	84	65	67	39	66	89	89
N.S.	1	0.73	0.34	0.94	0.73	0.75	0.44	0.74	1.00	1.00
time (sec)	N/A	0.046	0.008	0.309	0.413	0.420	0.913	0.296	0.846	0.051

Problem 1087	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	65	30	84	65	67	37	66	89	89
N.S.	1	0.73	0.34	0.94	0.73	0.75	0.42	0.74	1.00	1.00
time (sec)	N/A	0.035	0.009	0.342	0.410	0.412	0.840	0.337	0.841	0.049

Problem 1088	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	89	0	0	0	0	249	0	0	-1	89
N.S.	1	0.00	0.00	0.00	0.00	2.80	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.038	0.037	180.000	0.000	3.764	0.000	0.000	0.000	0.170

Problem 1089	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	66	28	96	68	80	36	69	86	90
N.S.	1	0.73	0.31	1.07	0.76	0.89	0.40	0.77	0.96	1.00
time (sec)	N/A	0.038	0.006	0.339	0.414	0.416	0.932	0.448	0.885	0.106

Problem 1090	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	138	138	76	0	451	75	0	89	86
N.S.	1	1.53	1.53	0.84	0.00	5.01	0.83	0.00	0.99	0.96
time (sec)	N/A	0.125	0.049	0.365	0.000	1.343	2.898	0.000	1.333	18.598

Problem 1091	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	70	26	59	66	78	32	67	78	90
N.S.	1	0.78	0.29	0.66	0.73	0.87	0.36	0.74	0.87	1.00
time (sec)	N/A	0.041	0.006	0.291	0.439	0.410	0.885	0.155	0.896	0.082

Problem 1092	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	70	26	76	66	79	31	67	92	90
N.S.	1	0.78	0.29	0.84	0.73	0.88	0.34	0.74	1.02	1.00
time (sec)	N/A	0.038	0.006	0.280	0.493	0.417	0.928	0.164	0.895	0.094

Problem 1093	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	90	0	0	0	0	0	0	0	-1	90
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	5.846	1.506	0.099	0.000	0.000	0.000	0.000	0.000	12.966

Problem 1094	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	90	0	0	0	0	0	0	0	-1	90
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	5.815	1.173	0.086	0.000	0.000	0.000	0.000	0.000	12.965

Problem 1095	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	59	59	80	80	78	230	148	71	84
N.S.	1	0.66	0.66	0.89	0.89	0.87	2.56	1.64	0.79	0.93
time (sec)	N/A	0.064	0.050	0.026	0.448	0.583	5.125	0.252	1.113	0.170

Problem 1096	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	70	26	76	66	79	34	66	92	90
N.S.	1	0.78	0.29	0.84	0.73	0.88	0.38	0.73	1.02	1.00
time (sec)	N/A	0.049	0.006	0.288	0.414	0.499	1.013	0.328	0.887	0.060

Problem 1097	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	90	0	0	272	0	134	0	0	-1	90
N.S.	1	0.00	0.00	3.02	0.00	1.49	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.101	0.224	3.082	0.000	3.027	0.000	0.000	0.000	1.002

Problem 1098	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	90	0	0	806	0	128	0	0	-1	90
N.S.	1	0.00	0.00	8.96	0.00	1.42	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.778	0.305	3.630	0.000	4.347	0.000	0.000	0.000	0.852

Problem 1099	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	90	0	0	293	0	131	0	0	-1	90
N.S.	1	0.00	0.00	3.26	0.00	1.46	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.008	0.235	2.885	0.000	5.506	0.000	0.000	0.000	1.002

Problem 1100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	90	0	0	477	0	122	0	0	-1	90
N.S.	1	0.00	0.00	5.30	0.00	1.36	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.742	0.258	3.500	0.000	2.789	0.000	0.000	0.000	0.854

Problem 1101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	90	0	0	1633	0	304	0	0	-1	90
N.S.	1	0.00	0.00	18.14	0.00	3.38	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.707	0.467	3.138	0.000	13.541	0.000	0.000	0.000	2.424

Problem 1102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	90	77	120	87	0	146	0	0	-1	77
N.S.	1	0.86	1.33	0.97	0.00	1.62	0.00	0.00	-0.01	0.86
time (sec)	N/A	0.032	0.099	0.024	0.000	0.481	0.000	0.000	0.000	0.407

Problem 1103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	160	125	0	0	0	0	222	-1	90
N.S.	1	1.78	1.39	0.00	0.00	0.00	0.00	2.47	-0.01	1.00
time (sec)	N/A	0.236	0.114	0.079	0.000	0.000	0.000	0.349	0.000	0.333

Problem 1104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	141	66	0	0	0	0	195	-1	90
N.S.	1	1.57	0.73	0.00	0.00	0.00	0.00	2.17	-0.01	1.00
time (sec)	N/A	0.208	0.080	0.334	0.000	0.000	0.000	0.215	0.000	0.278

Problem 1105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	70	26	76	66	79	34	67	92	90
N.S.	1	0.78	0.29	0.84	0.73	0.88	0.38	0.74	1.02	1.00
time (sec)	N/A	0.045	0.006	0.302	0.418	0.415	1.025	0.158	0.943	0.086

Problem 1106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	90	0	0	289	0	135	0	0	-1	90
N.S.	1	0.00	0.00	3.21	0.00	1.50	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.763	0.283	3.825	0.000	5.374	0.000	0.000	0.000	1.498

Problem 1107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	90	0	0	288	0	135	0	0	-1	90
N.S.	1	0.00	0.00	3.20	0.00	1.50	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.787	0.274	3.965	0.000	5.374	0.000	0.000	0.000	1.384

Problem 1108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	142	135	64	111	164	39	111	54	95
N.S.	1	1.58	1.50	0.71	1.23	1.82	0.43	1.23	0.60	1.06
time (sec)	N/A	0.106	0.030	0.312	0.411	0.412	0.858	0.157	0.910	0.092

Problem 1109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	70	26	76	66	79	31	66	92	90
N.S.	1	0.78	0.29	0.84	0.73	0.88	0.34	0.73	1.02	1.00
time (sec)	N/A	0.043	0.006	0.283	0.418	0.412	1.007	0.306	0.941	0.085

Problem 1110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	70	26	76	66	79	32	66	92	90
N.S.	1	0.78	0.29	0.84	0.73	0.88	0.36	0.73	1.02	1.00
time (sec)	N/A	0.039	0.006	0.337	0.411	0.408	1.087	0.158	0.901	0.056

Problem 1111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	90	0	0	0	0	0	0	0	-1	90
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	6.480	2.911	0.075	0.000	0.000	0.000	0.000	0.000	12.926

Problem 1112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	90	0	0	292	0	131	0	0	-1	90
N.S.	1	0.00	0.00	3.24	0.00	1.46	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.723	0.265	5.128	0.000	16.091	0.000	0.000	0.000	17.786

Problem 1113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	90	0	0	0	0	53	0	0	-1	90
N.S.	1	0.00	0.00	0.00	0.00	0.59	0.00	0.00	-0.01	1.00
time (sec)	N/A	4.391	1.427	0.289	0.000	0.410	0.000	0.000	0.000	5.020

Problem 1114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	146	101	564	0	791	0	0	-1	91
N.S.	1	1.60	1.11	6.20	0.00	8.69	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.865	0.581	0.111	0.000	0.565	0.000	0.000	0.000	0.763

Problem 1115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	135	35	27	0	92	0	63	27	91
N.S.	1	1.48	0.38	0.30	0.00	1.01	0.00	0.69	0.30	1.00
time (sec)	N/A	0.018	0.009	0.305	0.000	0.396	0.000	0.179	0.975	0.147

Problem 1116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-2)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	91	0	0	232	0	0	0	0	-1	91
N.S.	1	0.00	0.00	2.55	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.054	0.147	4.390	0.000	0.000	0.000	0.000	0.000	4.950

Problem 1117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	91	0	0	226	0	228	0	0	-1	91
N.S.	1	0.00	0.00	2.48	0.00	2.51	0.00	0.00	-0.01	1.00
time (sec)	N/A	8.196	1.944	4.553	0.000	84.159	0.000	0.000	0.000	0.151

Problem 1118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	A	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	91	593	0	865	0	128	0	0	-1	91
N.S.	1	6.52	0.00	9.51	0.00	1.41	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.703	0.258	1.707	0.000	14.347	0.000	0.000	0.000	1.010

Problem 1119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	A	F	A	F(-1)	F(-2)	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	91	0	2093	121	0	95	0	0	-1	65
N.S.	1	0.00	23.00	1.33	0.00	1.04	0.00	0.00	-0.01	0.71
time (sec)	N/A	0.490	6.245	0.071	0.000	0.547	0.000	0.000	0.000	0.590

Problem 1120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	91	91	111	0	0	197	0	0	-1	91
N.S.	1	1.00	1.22	0.00	0.00	2.16	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.203	0.293	0.823	0.000	0.429	0.000	0.000	0.000	0.384

Problem 1121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	149	53	0	0	0	0	213	42	91
N.S.	1	1.64	0.58	0.00	0.00	0.00	0.00	2.34	0.46	1.00
time (sec)	N/A	0.135	0.016	0.438	0.000	0.000	0.000	0.294	0.927	0.450

Problem 1122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	91	83	1129	0	0	50	0	0	-1	91
N.S.	1	0.91	12.41	0.00	0.00	0.55	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.435	14.166	0.288	0.000	0.447	0.000	0.000	0.000	0.101

Problem 1123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	92	0	0	315	0	98	0	0	-1	92
N.S.	1	0.00	0.00	3.42	0.00	1.07	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.333	0.171	2.390	0.000	0.875	0.000	0.000	0.000	0.140

Problem 1124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	68	28	79	68	81	34	69	80	92
N.S.	1	0.74	0.30	0.86	0.74	0.88	0.37	0.75	0.87	1.00
time (sec)	N/A	0.041	0.005	0.310	0.624	0.414	0.949	0.351	0.892	0.109

Problem 1125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	68	28	96	68	80	32	69	92	92
N.S.	1	0.74	0.30	1.04	0.74	0.87	0.35	0.75	1.00	1.00
time (sec)	N/A	0.041	0.006	0.370	0.941	0.411	0.962	0.280	0.888	0.084

Problem 1126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	93	20	30	81	100	32	0	15	92
N.S.	1	1.01	0.22	0.33	0.88	1.09	0.35	0.00	0.16	1.00
time (sec)	N/A	0.046	0.002	0.313	1.024	0.632	0.823	0.000	0.897	0.131

Problem 1127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	C	F	F(-2)	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	92	53	0	543	0	0	0	0	-1	92
N.S.	1	0.58	0.00	5.90	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.059	0.145	3.694	0.000	0.000	0.000	0.000	0.000	0.839

Problem 1128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	104	39	0	132	216	41	244	82	92
N.S.	1	1.13	0.42	0.00	1.43	2.35	0.45	2.65	0.89	1.00
time (sec)	N/A	0.080	0.009	0.389	0.499	0.431	1.468	0.239	1.093	0.153

Problem 1129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	92	0	0	0	0	0	0	0	-1	92
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	28.764	2.074	0.128	0.000	0.000	0.000	0.000	0.000	15.680

Problem 1130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	68	28	96	68	80	34	69	92	92
N.S.	1	0.74	0.30	1.04	0.74	0.87	0.37	0.75	1.00	1.00
time (sec)	N/A	0.047	0.005	0.307	0.499	0.416	0.986	0.434	0.943	0.073

Problem 1131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	92	0	28005	411	0	339	0	0	705	92
N.S.	1	0.00	304.40	4.47	0.00	3.68	0.00	0.00	7.66	1.00
time (sec)	N/A	19.115	14.412	0.066	0.000	0.928	0.000	0.000	1.380	1.026

Problem 1132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	104	37	0	132	216	39	244	82	92
N.S.	1	1.13	0.40	0.00	1.43	2.35	0.42	2.65	0.89	1.00
time (sec)	N/A	0.080	0.008	0.390	0.554	0.428	1.596	0.179	1.195	0.084

Problem 1133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	B	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	101	62	0	148	218	39	0	-1	92
N.S.	1	1.10	0.67	0.00	1.61	2.37	0.42	0.00	-0.01	1.00
time (sec)	N/A	0.043	0.052	0.306	0.699	0.451	1.351	0.000	0.000	0.363

Problem 1134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	68	28	96	68	80	36	69	92	92
N.S.	1	0.74	0.30	1.04	0.74	0.87	0.39	0.75	1.00	1.00
time (sec)	N/A	0.042	0.005	0.311	0.472	0.486	1.091	0.299	0.941	0.069

Problem 1135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	104	39	0	132	216	41	244	82	92
N.S.	1	1.13	0.42	0.00	1.43	2.35	0.45	2.65	0.89	1.00
time (sec)	N/A	0.076	0.009	0.359	0.484	0.489	1.821	0.247	1.184	0.110

Problem 1136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	68	28	96	68	80	32	69	92	92
N.S.	1	0.74	0.30	1.04	0.74	0.87	0.35	0.75	1.00	1.00
time (sec)	N/A	0.041	0.004	0.428	0.629	0.429	1.047	0.145	0.913	0.059

Problem 1137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	68	28	96	68	80	34	69	92	92
N.S.	1	0.74	0.30	1.04	0.74	0.87	0.37	0.75	1.00	1.00
time (sec)	N/A	0.040	0.006	0.351	0.515	0.438	1.119	0.165	0.905	0.064

Problem 1138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	92	0	0	489	0	127	0	0	-1	92
N.S.	1	0.00	0.00	5.32	0.00	1.38	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.394	0.606	8.385	0.000	3.067	0.000	0.000	0.000	0.187

Problem 1139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	114	125	97	132	234	260	136	-1	92
N.S.	1	1.24	1.36	1.05	1.43	2.54	2.83	1.48	-0.01	1.00
time (sec)	N/A	0.288	0.283	0.012	1.057	0.428	12.346	0.557	0.000	0.152

Problem 1140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	145	26	76	114	181	34	114	57	98
N.S.	1	1.56	0.28	0.82	1.23	1.95	0.37	1.23	0.61	1.05
time (sec)	N/A	0.114	0.004	0.350	0.755	0.438	0.948	0.280	0.947	0.176

Problem 1141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	62	40	43	81	105	37	0	-1	93
N.S.	1	0.67	0.43	0.46	0.87	1.13	0.40	0.00	-0.01	1.00
time (sec)	N/A	0.008	0.006	0.352	0.568	0.749	0.908	0.000	0.000	0.151

Problem 1142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	93	386	274	1500	0	232	0	0	509	93
N.S.	1	4.15	2.95	16.13	0.00	2.49	0.00	0.00	5.47	1.00
time (sec)	N/A	0.941	0.794	0.332	0.000	0.487	0.000	0.000	0.198	1.512

Problem 1143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	62	22	27	81	105	34	0	-1	93
N.S.	1	0.67	0.24	0.29	0.87	1.13	0.37	0.00	-0.01	1.00
time (sec)	N/A	0.008	0.003	0.330	1.515	0.728	0.856	0.000	0.000	0.134

Problem 1144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	93	0	0	0	0	0	0	0	-1	93
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	10.181	4.034	0.075	0.000	0.000	0.000	0.000	0.000	2.878

Problem 1145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	145	28	76	114	180	34	114	57	98
N.S.	1	1.56	0.30	0.82	1.23	1.94	0.37	1.23	0.61	1.05
time (sec)	N/A	0.116	0.006	0.332	0.661	0.432	0.982	0.168	0.970	0.168

Problem 1146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	A	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	93	293	0	386	0	130	0	0	-1	93
N.S.	1	3.15	0.00	4.15	0.00	1.40	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.253	0.636	2.802	0.000	1.235	0.000	0.000	0.000	0.350

Problem 1147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	F	A	F	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	0	88	3475	0	92	0	100	-1	93
N.S.	1	0.00	0.95	37.37	0.00	0.99	0.00	1.08	-0.01	1.00
time (sec)	N/A	1.827	0.178	1.411	0.000	0.487	0.000	0.348	0.000	0.270

Problem 1148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	185	58	387	0	174	0	100	-1	93
N.S.	1	1.99	0.62	4.16	0.00	1.87	0.00	1.08	-0.01	1.00
time (sec)	N/A	0.150	0.014	1.696	0.000	0.436	0.000	0.232	0.000	0.322

Problem 1149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	93	0	0	385	0	124	0	0	-1	93
N.S.	1	0.00	0.00	4.14	0.00	1.33	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.010	0.199	4.368	0.000	1.366	0.000	0.000	0.000	1.048

Problem 1150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	145	28	76	114	180	36	114	57	98
N.S.	1	1.56	0.30	0.82	1.23	1.94	0.39	1.23	0.61	1.05
time (sec)	N/A	0.110	0.005	0.368	0.582	0.432	1.107	0.197	0.982	0.190

Problem 1151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	93	0	0	580	0	124	0	0	-1	93
N.S.	1	0.00	0.00	6.24	0.00	1.33	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.991	0.196	5.482	0.000	1.702	0.000	0.000	0.000	2.698

Problem 1152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	93	0	25746	2877	0	79	0	0	2803	81
N.S.	1	0.00	276.84	30.94	0.00	0.85	0.00	0.00	30.14	0.87
time (sec)	N/A	0.673	5.178	5.740	0.000	0.521	0.000	0.000	3.196	0.542

Problem 1153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	B	F	B	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	93	0	5470	216	0	313	0	0	-1	81
N.S.	1	0.00	58.82	2.32	0.00	3.37	0.00	0.00	-0.01	0.87
time (sec)	N/A	0.590	6.480	0.087	0.000	0.635	0.000	0.000	0.000	0.421

Problem 1154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	147	92	98	0	74	0	80	-1	90
N.S.	1	1.58	0.99	1.05	0.00	0.80	0.00	0.86	-0.01	0.97
time (sec)	N/A	0.415	0.109	0.008	0.000	1.150	0.000	0.163	0.000	0.255

Problem 1155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	94	63	99	30	94	86	32	0	26	94
N.S.	1	0.67	1.05	0.32	1.00	0.91	0.34	0.00	0.28	1.00
time (sec)	N/A	0.010	0.100	0.334	0.780	0.428	0.865	0.000	0.849	0.194

Problem 1156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	94	63	99	14	94	86	29	0	12	94
N.S.	1	0.67	1.05	0.15	1.00	0.91	0.31	0.00	0.13	1.00
time (sec)	N/A	0.010	0.061	0.318	0.757	0.437	0.815	0.000	0.768	0.189

Problem 1157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	63	91	29	94	86	71	0	-1	94
N.S.	1	0.67	0.97	0.31	1.00	0.91	0.76	0.00	-0.01	1.00
time (sec)	N/A	0.012	0.038	0.313	0.900	0.431	1.613	0.000	0.000	0.206

Problem 1158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	178	38	17	0	90	0	67	27	94
N.S.	1	1.89	0.40	0.18	0.00	0.96	0.00	0.71	0.29	1.00
time (sec)	N/A	0.131	0.007	0.304	0.000	0.623	0.000	0.359	0.795	0.176

Problem 1159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	94	0	0	0	0	0	0	0	-1	94
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	35.351	1.883	0.112	0.000	0.000	0.000	0.000	0.000	15.737

Problem 1160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	94	105	118	247	0	277	0	0	-1	94
N.S.	1	1.12	1.26	2.63	0.00	2.95	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.119	0.257	1.839	0.000	6.275	0.000	0.000	0.000	0.455

Problem 1161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	94	0	0	0	0	0	0	0	-1	94
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.718	0.213	0.356	0.000	0.000	0.000	0.000	0.000	0.746

Problem 1162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	B	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	94	0	4636	307	0	379	0	0	453	91
N.S.	1	0.00	49.32	3.27	0.00	4.03	0.00	0.00	4.82	0.97
time (sec)	N/A	10.493	7.414	0.057	0.000	1.103	0.000	0.000	1.443	1.896

Problem 1163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	169	142	0	0	0	0	209	75	94
N.S.	1	1.80	1.51	0.00	0.00	0.00	0.00	2.22	0.80	1.00
time (sec)	N/A	0.294	0.147	0.329	0.000	0.000	0.000	0.225	1.252	0.506

Problem 1164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	169	142	0	0	0	0	209	75	94
N.S.	1	1.80	1.51	0.00	0.00	0.00	0.00	2.22	0.80	1.00
time (sec)	N/A	0.285	0.083	0.030	0.000	0.000	0.000	0.230	1.053	0.472

Problem 1165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	0	6023	2813	0	392	0	0	-1	94
N.S.	1	0.00	64.07	29.93	0.00	4.17	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.573	6.486	0.101	0.000	0.932	0.000	0.000	0.000	0.737

Problem 1166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	0	6023	2813	0	382	0	0	-1	94
N.S.	1	0.00	64.07	29.93	0.00	4.06	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.505	6.399	0.089	0.000	0.975	0.000	0.000	0.000	0.748

Problem 1167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	87	70	112	0	83	0	0	-1	94
N.S.	1	0.93	0.74	1.19	0.00	0.88	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.101	0.393	0.034	0.000	0.935	0.000	0.000	0.000	0.984

Problem 1168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	A	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	94	313	0	413	0	136	0	0	-1	94
N.S.	1	3.33	0.00	4.39	0.00	1.45	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.914	0.874	2.099	0.000	17.210	0.000	0.000	0.000	0.943

Problem 1169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	A	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	94	247	0	282	0	135	0	0	-1	94
N.S.	1	2.63	0.00	3.00	0.00	1.44	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.792	1.091	1.900	0.000	9.024	0.000	0.000	0.000	0.914

Problem 1170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	94	347	0	624	0	227	0	0	-1	115
N.S.	1	3.69	0.00	6.64	0.00	2.41	0.00	0.00	-0.01	1.22
time (sec)	N/A	0.795	7.108	2.378	0.000	19.352	0.000	0.000	0.000	0.507

Problem 1171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	86	35	91	78	104	53	0	104	95
N.S.	1	0.91	0.37	0.96	0.82	1.09	0.56	0.00	1.09	1.00
time (sec)	N/A	0.081	0.022	0.403	0.470	0.701	1.815	0.000	1.337	7.951

Problem 1172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-2)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	95	0	0	1096	0	0	0	0	-1	95
N.S.	1	0.00	0.00	11.54	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.474	0.203	3.109	0.000	0.000	0.000	0.000	0.000	0.255

Problem 1173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	95	0	0	0	0	0	0	0	-1	95
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	11.595	1.305	0.093	0.000	0.000	0.000	0.000	0.000	3.381

Problem 1174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-2)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	95	0	0	232	0	0	0	0	-1	95
N.S.	1	0.00	0.00	2.44	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.071	0.236	3.735	0.000	0.000	0.000	0.000	0.000	5.321

Problem 1175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	95	0	0	593	0	459	0	0	-1	95
N.S.	1	0.00	0.00	6.24	0.00	4.83	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.413	0.457	2.310	0.000	24.613	0.000	0.000	0.000	4.359

Problem 1176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	95	0	0	285	0	280	0	0	-1	95
N.S.	1	0.00	0.00	3.00	0.00	2.95	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.452	0.645	11.081	0.000	102.442	0.000	0.000	0.000	2.765

Problem 1177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	95	0	0	0	0	0	0	0	-1	95
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	4.957	1.039	0.358	0.000	0.000	0.000	0.000	0.000	12.991

Problem 1178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	95	0	0	217	0	702	0	0	-1	95
N.S.	1	0.00	0.00	2.28	0.00	7.39	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.032	0.267	14.881	0.000	173.492	0.000	0.000	0.000	6.671

Problem 1179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	95	0	0	0	0	0	0	0	-1	95
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.941	0.758	0.388	0.000	0.000	0.000	0.000	0.000	15.543

Problem 1180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	95	0	0	0	0	0	0	0	-1	95
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.776	0.547	0.341	0.000	0.000	0.000	0.000	0.000	15.407

Problem 1181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	C	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	95	0	0	195	0	241	0	0	-1	95
N.S.	1	0.00	0.00	2.05	0.00	2.54	0.00	0.00	-0.01	1.00
time (sec)	N/A	4.338	0.687	1.167	0.000	1.590	0.000	0.000	0.000	0.519

Problem 1182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	95	0	0	578	0	438	0	0	-1	95
N.S.	1	0.00	0.00	6.08	0.00	4.61	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.558	0.159	3.730	0.000	0.865	0.000	0.000	0.000	2.194

Problem 1183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	97	40	46	94	88	36	0	-1	96
N.S.	1	1.01	0.42	0.48	0.98	0.92	0.38	0.00	-0.01	1.00
time (sec)	N/A	0.049	0.006	0.366	0.416	0.562	0.866	0.000	0.000	0.164

Problem 1184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	97	22	30	94	88	31	0	-1	96
N.S.	1	1.01	0.23	0.31	0.98	0.92	0.32	0.00	-0.01	1.00
time (sec)	N/A	0.047	0.002	0.339	0.446	0.580	0.834	0.000	0.000	0.163

Problem 1185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	180	40	729	0	95	0	64	-1	96
N.S.	1	1.88	0.42	7.59	0.00	0.99	0.00	0.67	-0.01	1.00
time (sec)	N/A	0.127	0.009	2.268	0.000	0.712	0.000	0.190	0.000	0.207

Problem 1186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	96	0	0	0	0	0	0	0	-1	96
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	16.646	1.699	0.178	0.000	0.000	0.000	0.000	0.000	4.044

Problem 1187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F(-1)	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	96	0	0	0	0	0	0	0	-1	96
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	180.007	2.129	0.118	0.000	0.000	0.000	0.000	0.000	15.705

Problem 1188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	153	136	0	0	0	0	207	-1	96
N.S.	1	1.59	1.42	0.00	0.00	0.00	0.00	2.16	-0.01	1.00
time (sec)	N/A	0.329	0.194	0.407	0.000	0.000	0.000	0.222	0.000	0.278

Problem 1189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	96	0	0	0	0	0	0	0	184	96
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.92	1.00
time (sec)	N/A	5.485	1.025	0.332	0.000	0.000	0.000	0.000	8.379	12.891

Problem 1190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	115	39	17	0	92	0	55	31	96
N.S.	1	1.20	0.41	0.18	0.00	0.96	0.00	0.57	0.32	1.00
time (sec)	N/A	0.094	0.010	0.359	0.000	0.943	0.000	0.156	0.944	0.318

Problem 1191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	96	319	0	238	0	678	0	0	-1	96
N.S.	1	3.32	0.00	2.48	0.00	7.06	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.074	1.432	5.061	0.000	60.581	0.000	0.000	0.000	0.701

Problem 1192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	96	319	0	240	0	678	0	0	-1	96
N.S.	1	3.32	0.00	2.50	0.00	7.06	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.880	0.428	4.612	0.000	58.126	0.000	0.000	0.000	0.001

Problem 1193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	96	0	0	0	0	0	0	0	-1	96
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.085	0.400	0.374	0.000	0.000	0.000	0.000	0.000	15.654

Problem 1194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	269	169	333	0	197	0	0	-1	97
N.S.	1	2.77	1.74	3.43	0.00	2.03	0.00	0.00	-0.01	1.00
time (sec)	N/A	5.290	2.015	0.026	0.000	0.732	0.000	0.000	0.000	0.234

Problem 1195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	84	28	89	91	86	34	79	107	97
N.S.	1	0.87	0.29	0.92	0.94	0.89	0.35	0.81	1.10	1.00
time (sec)	N/A	0.056	0.005	0.370	0.764	0.487	1.091	0.162	0.956	0.130

Problem 1196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	86	26	69	91	83	32	77	108	97
N.S.	1	0.89	0.27	0.71	0.94	0.86	0.33	0.79	1.11	1.00
time (sec)	N/A	0.055	0.006	0.323	0.590	0.477	1.053	0.157	0.958	0.114

Problem 1197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	83	98	71	73	85	65	74	152	97
N.S.	1	0.86	1.01	0.73	0.75	0.88	0.67	0.76	1.57	1.00
time (sec)	N/A	0.049	0.034	0.319	0.430	0.508	87.873	0.175	1.236	0.084

Problem 1198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	97	375	205	735	0	2458	0	0	210	97
N.S.	1	3.87	2.11	7.58	0.00	25.34	0.00	0.00	2.16	1.00
time (sec)	N/A	1.168	0.662	0.063	0.000	0.927	0.000	0.000	0.885	0.286

Problem 1199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	142	35	443	0	93	0	64	-1	97
N.S.	1	1.46	0.36	4.57	0.00	0.96	0.00	0.66	-0.01	1.00
time (sec)	N/A	0.065	0.008	0.516	0.000	0.404	0.000	0.329	0.000	0.251

Problem 1200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	107	106	425	0	124	0	0	-1	97
N.S.	1	1.10	1.09	4.38	0.00	1.28	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.135	0.114	1.207	0.000	0.840	0.000	0.000	0.000	0.196

Problem 1201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	86	35	69	103	84	0	76	121	97
N.S.	1	0.89	0.36	0.71	1.06	0.87	0.00	0.78	1.25	1.00
time (sec)	N/A	0.056	0.010	0.348	0.414	0.407	0.000	0.236	1.266	0.095

Problem 1202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	143	66	0	0	0	0	185	58	97
N.S.	1	1.47	0.68	0.00	0.00	0.00	0.00	1.91	0.60	1.00
time (sec)	N/A	0.177	0.030	0.385	0.000	0.000	0.000	0.226	1.145	0.329

Problem 1203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	143	53	0	0	0	0	185	58	97
N.S.	1	1.47	0.55	0.00	0.00	0.00	0.00	1.91	0.60	1.00
time (sec)	N/A	0.229	0.017	0.023	0.000	0.000	0.000	0.221	0.962	0.314

Problem 1204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	86	26	86	93	86	31	78	118	97
N.S.	1	0.89	0.27	0.89	0.96	0.89	0.32	0.80	1.22	1.00
time (sec)	N/A	0.049	0.005	0.421	0.416	0.403	1.492	0.181	0.986	0.123

Problem 1205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	97	0	0	362	0	121	0	0	-1	97
N.S.	1	0.00	0.00	3.73	0.00	1.25	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.957	0.403	2.276	0.000	1.921	0.000	0.000	0.000	2.853

Problem 1206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	91	93	125	0	82	0	65	-1	87
N.S.	1	0.94	0.96	1.29	0.00	0.85	0.00	0.67	-0.01	0.90
time (sec)	N/A	0.148	0.072	0.011	0.000	1.861	0.000	0.167	0.000	0.200

Problem 1207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	91	304	0	0	64	0	0	-1	97
N.S.	1	0.94	3.13	0.00	0.00	0.66	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.862	1.407	0.348	0.000	0.428	0.000	0.000	0.000	0.134

Problem 1208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	98	96	68	0	0	207	0	0	-1	98
N.S.	1	0.98	0.69	0.00	0.00	2.11	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.043	0.068	0.416	0.000	0.428	0.000	0.000	0.000	2.407

Problem 1209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	98	0	0	0	0	0	0	0	-1	98
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	49.307	1.715	0.118	0.000	0.000	0.000	0.000	0.000	15.751

Problem 1210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	98	0	880	637	0	150	0	0	-1	98
N.S.	1	0.00	8.98	6.50	0.00	1.53	0.00	0.00	-0.01	1.00
time (sec)	N/A	6.891	3.861	0.210	0.000	0.513	0.000	0.000	0.000	0.980

Problem 1211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	93	105	163	0	91	0	0	-1	98
N.S.	1	0.95	1.07	1.66	0.00	0.93	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.139	0.253	0.393	0.000	0.469	0.000	0.000	0.000	0.611

Problem 1212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	B	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	109	65	0	162	228	42	0	-1	98
N.S.	1	1.11	0.66	0.00	1.65	2.33	0.43	0.00	-0.01	1.00
time (sec)	N/A	0.039	0.075	0.405	0.417	0.420	1.322	0.000	0.000	0.368

Problem 1213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	170	71	0	0	0	0	221	89	98
N.S.	1	1.73	0.72	0.00	0.00	0.00	0.00	2.26	0.91	1.00
time (sec)	N/A	0.224	0.070	0.095	0.000	0.000	0.000	0.199	1.285	0.322

Problem 1214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	170	69	0	0	0	0	221	89	98
N.S.	1	1.73	0.70	0.00	0.00	0.00	0.00	2.26	0.91	1.00
time (sec)	N/A	0.223	0.071	0.084	0.000	0.000	0.000	0.183	1.102	0.335

Problem 1215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	98	0	0	0	0	0	0	0	-1	98
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	10.237	1.949	0.075	0.000	0.000	0.000	0.000	0.000	12.075

Problem 1216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	98	0	0	0	0	229	0	0	-1	98
N.S.	1	0.00	0.00	0.00	0.00	2.34	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.041	0.059	180.000	0.000	3.008	0.000	0.000	0.000	0.573

Problem 1217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	99	0	107	299	88	97	81	99
N.S.	1	1.00	1.00	0.00	1.08	3.02	0.89	0.98	0.82	1.00
time (sec)	N/A	0.078	0.067	0.276	0.442	0.454	9.770	0.295	1.123	0.102

Problem 1218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	97	220	702	0	233	0	0	221	99
N.S.	1	0.98	2.22	7.09	0.00	2.35	0.00	0.00	2.23	1.00
time (sec)	N/A	0.618	0.230	0.271	0.000	0.494	0.000	0.000	0.225	1.293

Problem 1219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	99	457	290	1517	0	246	0	0	505	99
N.S.	1	4.62	2.93	15.32	0.00	2.48	0.00	0.00	5.10	1.00
time (sec)	N/A	0.750	0.801	0.268	0.000	0.475	0.000	0.000	0.087	1.631

Problem 1220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	99	450	291	1517	0	218	0	0	509	99
N.S.	1	4.55	2.94	15.32	0.00	2.20	0.00	0.00	5.14	1.00
time (sec)	N/A	0.997	0.731	0.267	0.000	0.482	0.000	0.000	0.830	1.357

Problem 1221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	84	38	91	103	88	0	81	186	99
N.S.	1	0.85	0.38	0.92	1.04	0.89	0.00	0.82	1.88	1.00
time (sec)	N/A	0.061	0.009	0.266	0.478	0.392	0.000	0.179	1.039	0.143

Problem 1222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	84	38	91	103	88	0	81	187	99
N.S.	1	0.85	0.38	0.92	1.04	0.89	0.00	0.82	1.89	1.00
time (sec)	N/A	0.054	0.009	0.263	0.637	0.407	0.000	0.177	1.203	0.109

Problem 1223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	99	0	0	0	0	0	0	0	-1	99
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	13.124	4.245	0.099	0.000	0.000	0.000	0.000	0.000	5.910

Problem 1224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	99	0	11770	7672	0	5095	0	0	-1	97
N.S.	1	0.00	118.89	77.49	0.00	51.46	0.00	0.00	-0.01	0.98
time (sec)	N/A	4.179	6.718	0.441	0.000	6.434	0.000	0.000	0.000	0.544

Problem 1225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	168	47	0	0	234	0	243	38	99
N.S.	1	1.70	0.47	0.00	0.00	2.36	0.00	2.45	0.38	1.00
time (sec)	N/A	0.197	0.014	0.305	0.000	0.508	0.000	0.192	0.856	0.426

Problem 1226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	84	28	108	93	88	32	81	116	99
N.S.	1	0.85	0.28	1.09	0.94	0.89	0.32	0.82	1.17	1.00
time (sec)	N/A	0.061	0.005	0.273	0.417	0.443	1.601	0.160	0.988	0.194

Problem 1227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	99	0	64755	867	0	708	0	0	-1	99
N.S.	1	0.00	654.09	8.76	0.00	7.15	0.00	0.00	-0.01	1.00
time (sec)	N/A	8.162	60.485	0.076	0.000	0.937	0.000	0.000	0.000	0.910

Problem 1228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	697	2045	11680	0	344	0	0	-1	100
N.S.	1	6.97	20.45	116.80	0.00	3.44	0.00	0.00	-0.01	1.00
time (sec)	N/A	4.729	7.487	0.090	0.000	0.901	0.000	0.000	0.000	4.103

Problem 1229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	691	2156	11374	0	342	0	0	-1	100
N.S.	1	6.91	21.56	113.74	0.00	3.42	0.00	0.00	-0.01	1.00
time (sec)	N/A	5.124	8.334	0.089	0.000	0.914	0.000	0.000	0.000	3.718

Problem 1230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	194	50	735	0	96	0	77	-1	100
N.S.	1	1.94	0.50	7.35	0.00	0.96	0.00	0.77	-0.01	1.00
time (sec)	N/A	0.182	0.016	2.102	0.000	0.577	0.000	0.323	0.000	0.237

Problem 1231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	100	0	0	772	0	138	0	0	-1	100
N.S.	1	0.00	0.00	7.72	0.00	1.38	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.594	0.156	3.078	0.000	1.169	0.000	0.000	0.000	0.134

Problem 1232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	147	36	448	0	102	0	63	27	100
N.S.	1	1.47	0.36	4.48	0.00	1.02	0.00	0.63	0.27	1.00
time (sec)	N/A	0.063	0.010	0.506	0.000	0.387	0.000	0.316	1.029	0.170

Problem 1233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	102	35	74	146	88	0	85	232	100
N.S.	1	1.02	0.35	0.74	1.46	0.88	0.00	0.85	2.32	1.00
time (sec)	N/A	0.063	0.011	0.250	0.458	0.388	0.000	0.159	1.533	0.193

Problem 1234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	100	0	0	321	0	147	0	0	-1	100
N.S.	1	0.00	0.00	3.21	0.00	1.47	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.350	0.306	2.864	0.000	4.862	0.000	0.000	0.000	3.588

Problem 1235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	100	0	0	320	0	144	0	0	-1	100
N.S.	1	0.00	0.00	3.20	0.00	1.44	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.330	0.259	2.691	0.000	3.000	0.000	0.000	0.000	3.558

Problem 1236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	100	0	0	778	0	141	0	0	-1	100
N.S.	1	0.00	0.00	7.78	0.00	1.41	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.873	0.326	5.996	0.000	13.778	0.000	0.000	0.000	17.818

Problem 1237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	100	0	0	125	0	94	0	0	-1	100
N.S.	1	0.00	0.00	1.25	0.00	0.94	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.448	0.583	0.845	0.000	2.817	0.000	0.000	0.000	0.395

Problem 1238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	113	99	0	217	0	168	0	-1	100
N.S.	1	1.13	0.99	0.00	2.17	0.00	1.68	0.00	-0.01	1.00
time (sec)	N/A	0.082	0.074	0.274	0.421	0.000	3.172	0.000	0.000	1.094

Problem 1239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	100	0	0	154	0	0	-1	100
N.S.	1	1.00	1.00	0.00	0.00	1.54	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.153	0.147	0.240	0.000	0.441	0.000	0.000	0.000	0.206

Problem 1240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F(-2)	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	204	0	0	321	0	0	-1	100
N.S.	1	1.00	2.04	0.00	0.00	3.21	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.123	2.188	180.000	0.000	0.440	0.000	0.000	0.000	0.132

Problem 1241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	199	107	197	168	91	0	92	-1	101
N.S.	1	1.97	1.06	1.95	1.66	0.90	0.00	0.91	-0.01	1.00
time (sec)	N/A	0.374	0.213	0.026	0.418	0.414	0.000	0.180	0.000	0.557

Problem 1242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	109	37	42	93	112	87	0	40	101
N.S.	1	1.08	0.37	0.42	0.92	1.11	0.86	0.00	0.40	1.00
time (sec)	N/A	0.048	0.009	0.240	0.419	0.637	2.053	0.000	1.144	0.158

Problem 1243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	110	34	40	105	105	65	0	-1	101
N.S.	1	1.09	0.34	0.40	1.04	1.04	0.64	0.00	-0.01	1.00
time (sec)	N/A	0.053	0.010	0.258	0.458	0.675	2.161	0.000	0.000	0.191

Problem 1244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	109	50	58	93	112	165	0	55	101
N.S.	1	1.08	0.50	0.57	0.92	1.11	1.63	0.00	0.54	1.00
time (sec)	N/A	0.049	0.022	0.267	0.442	0.637	2.134	0.000	1.172	0.161

Problem 1245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	101	239	152	620	0	2408	0	0	178	101
N.S.	1	2.37	1.50	6.14	0.00	23.84	0.00	0.00	1.76	1.00
time (sec)	N/A	0.968	0.643	0.047	0.000	0.944	0.000	0.000	1.043	0.325

Problem 1246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	101	383	211	1353	0	2504	0	0	226	101
N.S.	1	3.79	2.09	13.40	0.00	24.79	0.00	0.00	2.24	1.00
time (sec)	N/A	1.202	0.627	0.043	0.000	0.941	0.000	0.000	0.843	0.324

Problem 1247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	143	35	30	0	103	0	64	-1	101
N.S.	1	1.42	0.35	0.30	0.00	1.02	0.00	0.63	-0.01	1.00
time (sec)	N/A	0.044	0.011	0.247	0.000	0.406	0.000	0.234	0.000	0.186

Problem 1248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	109	52	57	93	112	167	0	55	101
N.S.	1	1.08	0.51	0.56	0.92	1.11	1.65	0.00	0.54	1.00
time (sec)	N/A	0.061	0.026	0.267	0.444	0.650	2.347	0.000	1.120	0.157

Problem 1249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	101	111	111	393	0	124	0	0	-1	101
N.S.	1	1.10	1.10	3.89	0.00	1.23	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.130	0.122	1.154	0.000	0.940	0.000	0.000	0.000	0.195

Problem 1250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	C	F	B	F	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	109	112	268	0	311	0	81	-1	101
N.S.	1	1.08	1.11	2.65	0.00	3.08	0.00	0.80	-0.01	1.00
time (sec)	N/A	9.823	0.330	4.161	0.000	1.752	0.000	0.355	0.000	0.465

Problem 1251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	193	77	974	0	182	0	97	-1	101
N.S.	1	1.91	0.76	9.64	0.00	1.80	0.00	0.96	-0.01	1.00
time (sec)	N/A	1.248	0.063	2.231	0.000	0.435	0.000	0.256	0.000	0.391

Problem 1252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	101	0	0	593	0	709	0	0	-1	101
N.S.	1	0.00	0.00	5.87	0.00	7.02	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.499	0.453	2.181	0.000	25.890	0.000	0.000	0.000	4.285

Problem 1253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	101	73	127	291	0	137	0	0	-1	101
N.S.	1	0.72	1.26	2.88	0.00	1.36	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.266	0.198	1.381	0.000	0.418	0.000	0.000	0.000	0.399

Problem 1254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	183	55	0	0	0	0	229	91	101
N.S.	1	1.81	0.54	0.00	0.00	0.00	0.00	2.27	0.90	1.00
time (sec)	N/A	0.472	0.022	0.082	0.000	0.000	0.000	0.197	1.361	0.364

Problem 1255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	0	3897	3402	0	216	0	0	-1	105
N.S.	1	0.00	38.58	33.68	0.00	2.14	0.00	0.00	-0.01	1.04
time (sec)	N/A	0.914	6.212	0.108	0.000	2.282	0.000	0.000	0.000	0.340

Problem 1256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	101	127	0	0	0	526	0	0	-1	101
N.S.	1	1.26	0.00	0.00	0.00	5.21	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.640	0.419	2.908	0.000	24.996	0.000	0.000	0.000	0.500

Problem 1257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	101	127	0	0	0	526	0	0	-1	101
N.S.	1	1.26	0.00	0.00	0.00	5.21	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.506	0.111	0.001	0.000	24.046	0.000	0.000	0.000	0.001

Problem 1258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	242	47	445	0	129	0	171	-1	101
N.S.	1	2.40	0.47	4.41	0.00	1.28	0.00	1.69	-0.01	1.00
time (sec)	N/A	0.483	0.018	0.393	0.000	0.412	0.000	0.268	0.000	0.631

Problem 1259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	101	0	0	187	0	1041	0	0	-1	101
N.S.	1	0.00	0.00	1.85	0.00	10.31	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.569	0.149	2.546	0.000	0.868	0.000	0.000	0.000	5.289

Problem 1260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	B	C	F	A	A	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	0	250	62	0	92	17	0	-1	101
N.S.	1	0.00	2.48	0.61	0.00	0.91	0.17	0.00	-0.01	1.00
time (sec)	N/A	0.108	0.726	0.073	0.000	0.661	1.255	0.000	0.000	0.375

Problem 1261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	A	A	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	161	286	0	0	90	32	0	-1	114
N.S.	1	1.59	2.83	0.00	0.00	0.89	0.32	0.00	-0.01	1.13
time (sec)	N/A	0.499	2.561	0.246	0.000	0.780	4.522	0.000	0.000	0.312

Problem 1262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	101	0	1439	0	0	90	0	0	-1	101
N.S.	1	0.00	14.25	0.00	0.00	0.89	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.109	0.389	0.242	0.000	0.733	0.000	0.000	0.000	0.427

Problem 1263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	79	44	49	121	94	34	0	-1	102
N.S.	1	0.77	0.43	0.48	1.19	0.92	0.33	0.00	-0.01	1.00
time (sec)	N/A	0.020	0.021	0.267	0.420	0.401	1.035	0.000	0.000	0.215

Problem 1264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	113	34	37	121	94	31	0	-1	102
N.S.	1	1.11	0.33	0.36	1.19	0.92	0.30	0.00	-0.01	1.00
time (sec)	N/A	0.056	0.010	0.247	0.422	0.423	0.970	0.000	0.000	0.200

Problem 1265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	79	32	33	121	94	31	0	-1	102
N.S.	1	0.77	0.31	0.32	1.19	0.92	0.30	0.00	-0.01	1.00
time (sec)	N/A	0.022	0.010	0.252	0.413	0.404	1.004	0.000	0.000	0.222

Problem 1266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	102	111	111	268	0	122	0	0	-1	102
N.S.	1	1.09	1.09	2.63	0.00	1.20	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.136	0.192	1.257	0.000	0.989	0.000	0.000	0.000	0.199

Problem 1267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	102	0	0	0	0	0	0	0	-1	102
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.001	0.503	0.290	0.000	0.000	0.000	0.000	0.000	0.519

Problem 1268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	36	76	146	91	0	87	231	102
N.S.	1	1.00	0.35	0.75	1.43	0.89	0.00	0.85	2.26	1.00
time (sec)	N/A	0.073	0.011	0.254	0.447	0.446	0.000	0.267	1.279	0.168

Problem 1269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	102	2432	0	0	0	0	0	0	-1	102
N.S.	1	23.84	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	11.756	0.215	0.273	0.000	0.000	0.000	0.000	0.000	0.307

Problem 1270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	102	0	0	0	0	0	0	0	-1	102
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.865	0.276	0.276	0.000	0.000	0.000	0.000	0.000	0.752

Problem 1271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	A	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	102	288	0	496	0	147	0	0	-1	102
N.S.	1	2.82	0.00	4.86	0.00	1.44	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.289	1.123	1.749	0.000	9.141	0.000	0.000	0.000	2.344

Problem 1272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	A	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	102	389	0	490	0	148	0	0	-1	102
N.S.	1	3.81	0.00	4.80	0.00	1.45	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.288	0.913	1.925	0.000	12.773	0.000	0.000	0.000	2.335

Problem 1273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	102	0	0	0	0	60	0	0	-1	102
N.S.	1	0.00	0.00	0.00	0.00	0.59	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.345	0.306	0.056	0.000	0.776	0.000	0.000	0.000	7.127

Problem 1274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	93	218	694	0	237	0	0	220	103
N.S.	1	0.90	2.12	6.74	0.00	2.30	0.00	0.00	2.14	1.00
time (sec)	N/A	0.648	0.211	0.273	0.000	0.479	0.000	0.000	0.213	1.260

Problem 1275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	103	406	286	1501	0	250	0	0	505	103
N.S.	1	3.94	2.78	14.57	0.00	2.43	0.00	0.00	4.90	1.00
time (sec)	N/A	0.788	0.818	0.263	0.000	0.494	0.000	0.000	0.080	1.561

Problem 1276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	103	406	286	1501	0	250	0	0	509	103
N.S.	1	3.94	2.78	14.57	0.00	2.43	0.00	0.00	4.94	1.00
time (sec)	N/A	0.889	0.815	0.291	0.000	0.485	0.000	0.000	0.839	1.540

Problem 1277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	79	50	56	106	112	70	0	-1	103
N.S.	1	0.77	0.49	0.54	1.03	1.09	0.68	0.00	-0.01	1.00
time (sec)	N/A	0.024	0.024	0.273	0.559	0.848	2.263	0.000	0.000	0.205

Problem 1278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	B	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	125	94	0	189	239	39	0	-1	103
N.S.	1	1.21	0.91	0.00	1.83	2.32	0.38	0.00	-0.01	1.00
time (sec)	N/A	0.055	0.054	0.395	0.514	0.433	1.753	0.000	0.000	0.428

Problem 1279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	149	44	0	0	422	0	203	-1	103
N.S.	1	1.45	0.43	0.00	0.00	4.10	0.00	1.97	-0.01	1.00
time (sec)	N/A	0.210	0.019	0.368	0.000	107.333	0.000	0.419	0.000	0.403

Problem 1280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	A	F(-1)	F(-2)	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	103	117	0	613	0	112	0	0	-1	103
N.S.	1	1.14	0.00	5.95	0.00	1.09	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.770	0.163	44.095	0.000	3.725	0.000	0.000	0.000	0.523

Problem 1281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F(-2)	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	103	130	9150	0	0	335	0	0	-1	103
N.S.	1	1.26	88.83	0.00	0.00	3.25	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.212	23.021	180.000	0.000	0.446	0.000	0.000	0.000	0.183

Problem 1282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	100	28	96	111	93	34	90	124	104
N.S.	1	0.96	0.27	0.92	1.07	0.89	0.33	0.87	1.19	1.00
time (sec)	N/A	0.111	0.005	0.284	0.409	0.432	1.418	0.239	0.986	0.188

Problem 1283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	113	54	53	121	96	32	0	-1	104
N.S.	1	1.09	0.52	0.51	1.16	0.92	0.31	0.00	-0.01	1.00
time (sec)	N/A	0.067	0.017	0.273	0.461	0.408	1.003	0.000	0.000	0.179

Problem 1284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	186	40	33	0	99	0	77	29	104
N.S.	1	1.79	0.38	0.32	0.00	0.95	0.00	0.74	0.28	1.00
time (sec)	N/A	0.156	0.009	0.288	0.000	0.603	0.000	0.344	0.934	0.178

Problem 1285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	100	38	96	146	93	0	90	231	104
N.S.	1	0.96	0.37	0.92	1.40	0.89	0.00	0.87	2.22	1.00
time (sec)	N/A	0.077	0.018	0.288	0.406	0.412	0.000	0.176	1.376	0.173

Problem 1286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F(-1)	F	F(-1)	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	362	310	363	0	207	0	0	-1	104
N.S.	1	3.48	2.98	3.49	0.00	1.99	0.00	0.00	-0.01	1.00
time (sec)	N/A	4.134	6.619	0.069	0.000	0.509	0.000	0.000	0.000	0.360

Problem 1287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	104	0	0	292	0	146	0	0	-1	104
N.S.	1	0.00	0.00	2.81	0.00	1.40	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.413	0.265	2.914	0.000	3.253	0.000	0.000	0.000	3.598

Problem 1288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	46	77	0	0	474	0	0	-1	104
N.S.	1	0.44	0.74	0.00	0.00	4.56	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.068	0.038	0.293	0.000	60.440	0.000	0.000	0.000	0.354

Problem 1289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	154	54	0	0	0	0	192	60	104
N.S.	1	1.48	0.52	0.00	0.00	0.00	0.00	1.85	0.58	1.00
time (sec)	N/A	0.257	0.025	0.302	0.000	0.000	0.000	0.214	1.207	0.329

Problem 1290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	154	69	0	0	0	0	192	60	104
N.S.	1	1.48	0.66	0.00	0.00	0.00	0.00	1.85	0.58	1.00
time (sec)	N/A	0.193	0.034	0.027	0.000	0.000	0.000	0.301	0.961	0.347

Problem 1291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	154	54	0	0	0	0	192	-1	104
N.S.	1	1.48	0.52	0.00	0.00	0.00	0.00	1.85	-0.01	1.00
time (sec)	N/A	0.170	0.015	0.115	0.000	0.000	0.000	0.321	0.000	0.296

Problem 1292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	F(-1)	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	104	0	115	0	141	0	60	104
N.S.	1	1.00	1.00	0.00	1.11	0.00	1.36	0.00	0.58	1.00
time (sec)	N/A	0.053	0.080	0.294	0.413	0.000	2.178	0.000	1.174	0.381

Problem 1293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	104	0	0	0	0	0	0	0	-1	104
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.760	2.975	0.302	0.000	0.000	0.000	0.000	0.000	3.098

Problem 1294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	100	28	113	111	93	32	90	134	104
N.S.	1	0.96	0.27	1.09	1.07	0.89	0.31	0.87	1.29	1.00
time (sec)	N/A	0.069	0.005	0.289	0.427	0.647	2.411	0.348	1.062	0.201

Problem 1295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	69	97	46	94	94	34	0	-1	104
N.S.	1	0.66	0.93	0.44	0.90	0.90	0.33	0.00	-0.01	1.00
time (sec)	N/A	0.034	0.042	0.284	0.432	0.632	0.943	0.000	0.000	1.008

Problem 1296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	103	40	46	94	94	36	0	-1	104
N.S.	1	0.99	0.38	0.44	0.90	0.90	0.35	0.00	-0.01	1.00
time (sec)	N/A	0.081	0.007	0.277	0.415	0.491	0.914	0.000	0.000	0.636

Problem 1297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	69	40	46	94	94	34	0	-1	104
N.S.	1	0.66	0.38	0.44	0.90	0.90	0.33	0.00	-0.01	1.00
time (sec)	N/A	0.032	0.007	0.283	0.425	0.395	0.905	0.000	0.000	0.608

Problem 1298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	69	99	30	94	94	29	0	-1	104
N.S.	1	0.66	0.95	0.29	0.90	0.90	0.28	0.00	-0.01	1.00
time (sec)	N/A	0.032	0.040	0.262	0.416	0.400	0.925	0.000	0.000	0.969

Problem 1299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	103	22	30	94	94	31	0	-1	104
N.S.	1	0.99	0.21	0.29	0.90	0.90	0.30	0.00	-0.01	1.00
time (sec)	N/A	0.072	0.003	0.263	0.415	0.405	0.861	0.000	0.000	0.613

Problem 1300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	69	22	30	94	94	31	0	-1	104
N.S.	1	0.66	0.21	0.29	0.90	0.90	0.30	0.00	-0.01	1.00
time (sec)	N/A	0.030	0.003	0.258	0.423	0.407	0.876	0.000	0.000	0.601

Problem 1301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	104	0	0	220	0	682	0	0	-1	104
N.S.	1	0.00	0.00	2.12	0.00	6.56	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.429	0.623	9.508	0.000	129.216	0.000	0.000	0.000	2.832

Problem 1302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	119	44	33	0	99	0	63	33	104
N.S.	1	1.14	0.42	0.32	0.00	0.95	0.00	0.61	0.32	1.00
time (sec)	N/A	0.099	0.010	0.282	0.000	0.739	0.000	0.174	1.118	0.337

Problem 1303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	C
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	104	0	0	3488	0	2090	0	0	-1	106
N.S.	1	0.00	0.00	33.54	0.00	20.10	0.00	0.00	-0.01	1.02
time (sec)	N/A	0.517	0.178	40.639	0.000	5.143	0.000	0.000	0.000	0.429

Problem 1304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	104	0	0	1513	0	380	0	0	-1	104
N.S.	1	0.00	0.00	14.55	0.00	3.65	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.730	0.492	4.311	0.000	133.644	0.000	0.000	0.000	2.797

Problem 1305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	C	F	A	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	119	104	425	0	127	0	0	-1	104
N.S.	1	1.14	1.00	4.09	0.00	1.22	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.399	0.123	1.079	0.000	0.941	0.000	0.000	0.000	0.198

Problem 1306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	A	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	139	51	122	0	466	0	0	-1	71
N.S.	1	1.34	0.49	1.17	0.00	4.48	0.00	0.00	-0.01	0.68
time (sec)	N/A	0.469	0.233	0.039	0.000	0.544	0.000	0.000	0.000	0.335

Problem 1307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	104	122	93	0	0	0	0	0	-1	104
N.S.	1	1.17	0.89	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	4.057	0.121	0.313	0.000	0.000	0.000	0.000	0.000	0.521

Problem 1308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	0	72	0	0	48	0	0	-1	104
N.S.	1	0.00	0.69	0.00	0.00	0.46	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.166	0.130	0.026	0.000	0.423	0.000	0.000	0.000	0.392

Problem 1309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	105	0	0	641	0	139	0	0	-1	105
N.S.	1	0.00	0.00	6.10	0.00	1.32	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.519	0.261	1.914	0.000	1.088	0.000	0.000	0.000	0.113

Problem 1310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	105	101	163	0	0	338	0	0	-1	105
N.S.	1	0.96	1.55	0.00	0.00	3.22	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.029	0.153	0.292	0.000	113.083	0.000	0.000	0.000	0.175

Problem 1311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	118	41	101	117	114	82	0	146	105
N.S.	1	1.12	0.39	0.96	1.11	1.09	0.78	0.00	1.39	1.00
time (sec)	N/A	0.098	0.026	0.261	0.429	0.764	2.161	0.000	1.145	15.645

Problem 1312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	105	0	0	0	0	0	0	0	-1	105
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.126	0.384	0.390	0.000	0.000	0.000	0.000	0.000	0.542

Problem 1313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	105	0	0	282	0	103	0	0	-1	105
N.S.	1	0.00	0.00	2.69	0.00	0.98	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.551	0.128	3.615	0.000	2.802	0.000	0.000	0.000	2.562

Problem 1314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	105	0	0	326	0	144	0	0	-1	105
N.S.	1	0.00	0.00	3.10	0.00	1.37	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.226	0.288	3.529	0.000	6.140	0.000	0.000	0.000	3.500

Problem 1315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	94	50	44	105	117	175	0	-1	105
N.S.	1	0.90	0.48	0.42	1.00	1.11	1.67	0.00	-0.01	1.00
time (sec)	N/A	0.048	0.019	0.260	0.412	0.731	3.221	0.000	0.000	0.181

Problem 1316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	A	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	105	380	0	483	0	142	0	0	-1	105
N.S.	1	3.62	0.00	4.60	0.00	1.35	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.194	0.926	2.010	0.000	17.229	0.000	0.000	0.000	2.301

Problem 1317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	94	57	56	93	115	461	0	-1	105
N.S.	1	0.90	0.54	0.53	0.89	1.10	4.39	0.00	-0.01	1.00
time (sec)	N/A	0.048	0.032	0.285	0.417	0.722	3.576	0.000	0.000	0.162

Problem 1318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	94	456	0	416	0	0	-1	105
N.S.	1	1.00	0.90	4.34	0.00	3.96	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.201	0.064	2.945	0.000	9.752	0.000	0.000	0.000	0.411

Problem 1319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	108	102	0	192	0	107	0	-1	105
N.S.	1	1.03	0.97	0.00	1.83	0.00	1.02	0.00	-0.01	1.00
time (sec)	N/A	0.069	0.071	0.294	0.416	0.000	2.950	0.000	0.000	0.501

Problem 1320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	105	105	132	0	0	205	0	0	-1	105
N.S.	1	1.00	1.26	0.00	0.00	1.95	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.250	0.332	0.888	0.000	0.824	0.000	0.000	0.000	0.467

Problem 1321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	105	0	0	336	0	142	0	0	-1	105
N.S.	1	0.00	0.00	3.20	0.00	1.35	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.071	0.681	2.106	0.000	17.136	0.000	0.000	0.000	2.685

Problem 1322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	105	0	0	580	0	336	0	0	-1	105
N.S.	1	0.00	0.00	5.52	0.00	3.20	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.609	0.170	3.845	0.000	0.653	0.000	0.000	0.000	2.216

Problem 1323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	145	131	0	0	189	0	0	-1	105
N.S.	1	1.38	1.25	0.00	0.00	1.80	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.148	0.038	0.261	0.000	0.433	0.000	0.000	0.000	0.141

Problem 1324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	105	0	0	0	0	0	0	0	-1	105
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.200	0.091	180.000	0.000	0.000	0.000	0.000	0.000	0.548

Problem 1325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	124	106	86	107	150	0	0	-1	102
N.S.	1	1.18	1.01	0.82	1.02	1.43	0.00	0.00	-0.01	0.97
time (sec)	N/A	0.663	0.216	0.019	0.413	0.421	0.000	0.000	0.000	0.145

Problem 1326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	127	100	942	0	139	0	0	-1	153
N.S.	1	1.20	0.94	8.89	0.00	1.31	0.00	0.00	-0.01	1.44
time (sec)	N/A	0.055	0.048	2.694	0.000	0.433	0.000	0.000	0.000	0.219

Problem 1327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	188	42	787	0	104	0	74	-1	106
N.S.	1	1.77	0.40	7.42	0.00	0.98	0.00	0.70	-0.01	1.00
time (sec)	N/A	0.148	0.010	1.927	0.000	0.598	0.000	0.191	0.000	0.201

Problem 1328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	149	35	434	0	103	0	74	-1	106
N.S.	1	1.41	0.33	4.09	0.00	0.97	0.00	0.70	-0.01	1.00
time (sec)	N/A	0.072	0.010	0.495	0.000	0.419	0.000	0.144	0.000	0.215

Problem 1329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	106	0	0	0	0	0	0	0	-1	106
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.240	0.590	0.294	0.000	0.000	0.000	0.000	0.000	0.546

Problem 1330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	106	170	61	0	0	0	0	0	-1	106
N.S.	1	1.60	0.58	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.361	0.046	0.308	0.000	0.000	0.000	0.000	0.000	0.324

Problem 1331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	106	662	566	443	0	166	0	0	-1	106
N.S.	1	6.25	5.34	4.18	0.00	1.57	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.855	3.871	0.074	0.000	0.880	0.000	0.000	0.000	2.302

Problem 1332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	106	0	0	1584	0	0	0	0	-1	106
N.S.	1	0.00	0.00	14.94	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.882	1.260	3.451	0.000	0.000	0.000	0.000	0.000	2.761

Problem 1333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	106	0	0	0	0	0	0	0	-1	106
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	12.188	1.023	0.096	0.000	0.000	0.000	0.000	0.000	15.954

Problem 1334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	A	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	106	67	67	0	0	330	0	143	-1	106
N.S.	1	0.63	0.63	0.00	0.00	3.11	0.00	1.35	-0.01	1.00
time (sec)	N/A	0.653	0.137	4.210	0.000	24.810	0.000	0.220	0.000	0.337

Problem 1335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	95	65	54	145	99	34	0	-1	107
N.S.	1	0.89	0.61	0.50	1.36	0.93	0.32	0.00	-0.01	1.00
time (sec)	N/A	0.026	0.023	0.284	0.413	0.420	1.317	0.000	0.000	0.274

Problem 1336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	95	43	38	145	99	31	0	-1	107
N.S.	1	0.89	0.40	0.36	1.36	0.93	0.29	0.00	-0.01	1.00
time (sec)	N/A	0.030	0.016	0.266	0.409	0.418	1.243	0.000	0.000	0.243

Problem 1337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	210	61	508	0	103	0	88	-1	107
N.S.	1	1.96	0.57	4.75	0.00	0.96	0.00	0.82	-0.01	1.00
time (sec)	N/A	0.227	0.023	2.347	0.000	0.637	0.000	0.412	0.000	0.330

Problem 1338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	107	0	0	0	0	0	0	0	-1	107
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	19.182	4.752	0.117	0.000	0.000	0.000	0.000	0.000	3.648

Problem 1339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	B	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	118	35	81	182	96	0	96	268	107
N.S.	1	1.10	0.33	0.76	1.70	0.90	0.00	0.90	2.50	1.00
time (sec)	N/A	0.073	0.011	0.276	0.430	0.433	0.000	0.132	1.824	0.245

Problem 1340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	107	194	92	273	0	331	0	104	-1	107
N.S.	1	1.81	0.86	2.55	0.00	3.09	0.00	0.97	-0.01	1.00
time (sec)	N/A	0.159	0.555	3.863	0.000	2.301	0.000	0.317	0.000	0.420

Problem 1341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	107	5419	767	1206	0	184	0	0	-1	107
N.S.	1	50.64	7.17	11.27	0.00	1.72	0.00	0.00	-0.01	1.00
time (sec)	N/A	10.756	1.705	0.144	0.000	0.501	0.000	0.000	0.000	2.156

Problem 1342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	165	69	0	0	0	0	201	104	107
N.S.	1	1.54	0.64	0.00	0.00	0.00	0.00	1.88	0.97	1.00
time (sec)	N/A	0.350	0.077	0.306	0.000	0.000	0.000	0.278	1.810	0.415

Problem 1343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	662	562	457	0	165	0	0	-1	107
N.S.	1	6.19	5.25	4.27	0.00	1.54	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.664	3.400	0.074	0.000	0.570	0.000	0.000	0.000	2.449

Problem 1344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	107	0	0	0	0	228	0	0	-1	107
N.S.	1	0.00	0.00	0.00	0.00	2.13	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.155	0.941	0.332	0.000	0.513	0.000	0.000	0.000	1.038

Problem 1345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	107	0	0	783	0	154	0	0	-1	107
N.S.	1	0.00	0.00	7.32	0.00	1.44	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.098	0.533	5.925	0.000	2.039	0.000	0.000	0.000	2.325

Problem 1346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	167	72	0	0	0	0	191	77	107
N.S.	1	1.56	0.67	0.00	0.00	0.00	0.00	1.79	0.72	1.00
time (sec)	N/A	0.283	0.047	0.311	0.000	0.000	0.000	0.190	1.315	0.416

Problem 1347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	107	0	0	369	0	784	0	0	-1	107
N.S.	1	0.00	0.00	3.45	0.00	7.33	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.167	0.236	3.155	0.000	62.911	0.000	0.000	0.000	1.173

Problem 1348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	107	44	97	0	0	0	0	0	-1	107
N.S.	1	0.41	0.91	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.087	0.126	0.334	0.000	0.000	0.000	0.000	0.000	0.676

Problem 1349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	107	0	0	0	0	0	0	0	-1	107
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.939	0.259	0.287	0.000	0.000	0.000	0.000	0.000	0.240

Problem 1350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	164	36	15	0	100	0	77	25	108
N.S.	1	1.52	0.33	0.14	0.00	0.93	0.00	0.71	0.23	1.00
time (sec)	N/A	0.086	0.007	0.296	0.000	0.563	0.000	0.176	1.134	0.202

Problem 1351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F(-1)	F	F(-1)	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	364	314	368	0	217	0	0	-1	108
N.S.	1	3.37	2.91	3.41	0.00	2.01	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.560	7.663	0.076	0.000	0.797	0.000	0.000	0.000	0.411

Problem 1352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	192	93	174	0	107	0	0	-1	108
N.S.	1	1.78	0.86	1.61	0.00	0.99	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.621	0.512	0.037	0.000	1.241	0.000	0.000	0.000	3.271

Problem 1353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	108	0	0	0	0	0	0	0	-1	108
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.619	0.609	0.321	0.000	0.000	0.000	0.000	0.000	1.240

Problem 1354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	151	163	49	0	112	0	82	52	108
N.S.	1	1.40	1.51	0.45	0.00	1.04	0.00	0.76	0.48	1.00
time (sec)	N/A	0.187	0.107	0.291	0.000	0.638	0.000	0.391	1.189	0.360

Problem 1355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	97	52	59	106	117	160	0	54	108
N.S.	1	0.90	0.48	0.55	0.98	1.08	1.48	0.00	0.50	1.00
time (sec)	N/A	0.033	0.040	0.309	0.414	0.851	2.525	0.000	1.129	0.211

Problem 1356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	95	77	61	118	117	199	0	-1	108
N.S.	1	0.88	0.71	0.56	1.09	1.08	1.84	0.00	-0.01	1.00
time (sec)	N/A	0.036	0.032	0.297	0.417	0.858	3.296	0.000	0.000	0.223

Problem 1357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	97	35	45	106	117	85	0	38	108
N.S.	1	0.90	0.32	0.42	0.98	1.08	0.79	0.00	0.35	1.00
time (sec)	N/A	0.032	0.029	0.283	0.413	0.860	2.393	0.000	1.115	0.211

Problem 1358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	108	0	0	0	0	0	0	0	-1	108
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.620	0.676	0.317	0.000	0.000	0.000	0.000	0.000	3.056

Problem 1359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	108	0	0	0	0	70	0	0	-1	108
N.S.	1	0.00	0.00	0.00	0.00	0.65	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.590	1.771	0.294	0.000	0.410	0.000	0.000	0.000	4.929

Problem 1360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	129	67	58	145	101	36	0	-1	109
N.S.	1	1.18	0.61	0.53	1.33	0.93	0.33	0.00	-0.01	1.00
time (sec)	N/A	0.068	0.021	0.299	0.420	0.414	1.240	0.000	0.000	0.243

Problem 1361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	129	45	42	145	101	31	0	-1	109
N.S.	1	1.18	0.41	0.39	1.33	0.93	0.28	0.00	-0.01	1.00
time (sec)	N/A	0.065	0.016	0.317	0.430	0.404	1.195	0.000	0.000	0.242

Problem 1362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	B	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	116	38	101	184	98	0	99	265	109
N.S.	1	1.06	0.35	0.93	1.69	0.90	0.00	0.91	2.43	1.00
time (sec)	N/A	0.070	0.011	0.304	0.421	0.422	0.000	0.353	1.607	0.192

Problem 1363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	109	0	0	0	0	0	0	0	-1	109
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	10.444	3.826	0.368	0.000	0.000	0.000	0.000	0.000	7.280

Problem 1364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	B	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	135	97	0	207	249	42	0	-1	109
N.S.	1	1.24	0.89	0.00	1.90	2.28	0.39	0.00	-0.01	1.00
time (sec)	N/A	0.056	0.064	0.310	0.410	0.456	1.737	0.000	0.000	0.427

Problem 1365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	109	159	76	0	0	430	0	209	-1	109
N.S.	1	1.46	0.70	0.00	0.00	3.94	0.00	1.92	-0.01	1.00
time (sec)	N/A	0.207	0.043	0.322	0.000	106.372	0.000	0.238	0.000	0.399

Problem 1366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	119	50	56	106	113	71	0	-1	109
N.S.	1	1.09	0.46	0.51	0.97	1.04	0.65	0.00	-0.01	1.00
time (sec)	N/A	0.082	0.027	0.303	0.424	0.907	3.252	0.000	0.000	0.483

Problem 1367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	115	52	58	93	116	167	0	-1	109
N.S.	1	1.06	0.48	0.53	0.85	1.06	1.53	0.00	-0.01	1.00
time (sec)	N/A	0.078	0.025	0.302	0.425	0.626	3.721	0.000	0.000	0.729

Problem 1368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	A	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	109	121	97	508	0	444	0	172	-1	109
N.S.	1	1.11	0.89	4.66	0.00	4.07	0.00	1.58	-0.01	1.00
time (sec)	N/A	0.383	0.091	1.820	0.000	2.610	0.000	0.345	0.000	0.306

Problem 1369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	109	0	0	239	0	0	0	0	-1	109
N.S.	1	0.00	0.00	2.19	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.206	0.311	15.635	0.000	0.000	0.000	0.000	0.000	15.406

Problem 1370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	0	97	0	0	96	0	0	-1	109
N.S.	1	0.00	0.89	0.00	0.00	0.88	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.234	0.477	0.280	0.000	0.430	0.000	0.000	0.000	4.221

Problem 1371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	0	140	0	0	96	0	0	-1	109
N.S.	1	0.00	1.28	0.00	0.00	0.88	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.774	0.357	0.281	0.000	0.408	0.000	0.000	0.000	4.907

Problem 1372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	C	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	0	149	29	0	239	42	0	-1	77
N.S.	1	0.00	1.37	0.27	0.00	2.19	0.39	0.00	-0.01	0.71
time (sec)	N/A	0.046	0.392	0.036	0.000	46.120	0.814	0.000	0.000	0.163

Problem 1373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	F	A	B	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	0	144	189	0	93	1100	0	-1	109
N.S.	1	0.00	1.32	1.73	0.00	0.85	10.09	0.00	-0.01	1.00
time (sec)	N/A	0.485	0.288	0.326	0.000	0.509	7.332	0.000	0.000	0.231

Problem 1374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	110	220	81	667	0	117	0	0	-1	110
N.S.	1	2.00	0.74	6.06	0.00	1.06	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.224	0.298	2.151	0.000	0.825	0.000	0.000	0.000	0.303

Problem 1375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	110	220	81	668	0	117	0	0	-1	110
N.S.	1	2.00	0.74	6.07	0.00	1.06	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.207	0.265	2.579	0.000	0.813	0.000	0.000	0.000	0.316

Problem 1376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	152	35	41	0	113	0	74	-1	110
N.S.	1	1.38	0.32	0.37	0.00	1.03	0.00	0.67	-0.01	1.00
time (sec)	N/A	0.043	0.009	0.316	0.000	0.412	0.000	0.170	0.000	0.274

Problem 1377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	110	42	42	278	0	136	0	0	-1	110
N.S.	1	0.38	0.38	2.53	0.00	1.24	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.021	0.016	1.294	0.000	0.935	0.000	0.000	0.000	0.264

Problem 1378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	573	585	0	0	0	0	0	-1	110
N.S.	1	5.21	5.32	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.388	7.211	0.406	0.000	0.000	0.000	0.000	0.000	0.648

Problem 1379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	110	129	156	963	0	233	0	101	-1	110
N.S.	1	1.17	1.42	8.75	0.00	2.12	0.00	0.92	-0.01	1.00
time (sec)	N/A	0.263	0.195	0.393	0.000	1.720	0.000	0.329	0.000	0.659

Problem 1380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	119	34	37	121	100	31	0	-1	110
N.S.	1	1.08	0.31	0.34	1.10	0.91	0.28	0.00	-0.01	1.00
time (sec)	N/A	0.090	0.011	0.286	0.423	0.412	1.181	0.000	0.000	0.881

Problem 1381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	167	83	0	0	0	0	212	77	110
N.S.	1	1.52	0.75	0.00	0.00	0.00	0.00	1.93	0.70	1.00
time (sec)	N/A	0.273	0.049	0.037	0.000	0.000	0.000	0.282	1.196	0.550

Problem 1382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	167	138	0	0	0	0	209	71	110
N.S.	1	1.52	1.25	0.00	0.00	0.00	0.00	1.90	0.65	1.00
time (sec)	N/A	0.253	0.126	0.034	0.000	0.000	0.000	0.249	1.302	0.706

Problem 1383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	167	138	0	0	0	0	209	71	110
N.S.	1	1.52	1.25	0.00	0.00	0.00	0.00	1.90	0.65	1.00
time (sec)	N/A	0.259	0.081	0.032	0.000	0.000	0.000	0.285	1.238	0.732

Problem 1384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	110	0	0	0	0	0	0	0	-1	110
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.412	0.648	0.329	0.000	0.000	0.000	0.000	0.000	2.994

Problem 1385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	110	0	1525	356	0	146	0	0	-1	73
N.S.	1	0.00	13.86	3.24	0.00	1.33	0.00	0.00	-0.01	0.66
time (sec)	N/A	1.619	2.288	0.263	0.000	0.553	0.000	0.000	0.000	1.214

Problem 1386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	110	0	0	941	0	566	0	0	-1	110
N.S.	1	0.00	0.00	8.55	0.00	5.15	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.338	0.676	8.332	0.000	0.837	0.000	0.000	0.000	9.316

Problem 1387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	96	88	298	0	87	0	188	-1	84
N.S.	1	0.87	0.80	2.71	0.00	0.79	0.00	1.71	-0.01	0.76
time (sec)	N/A	0.369	0.133	0.025	0.000	2.287	0.000	0.425	0.000	0.269

Problem 1388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	90	90	170	0	74	0	86	-1	84
N.S.	1	0.82	0.82	1.55	0.00	0.67	0.00	0.78	-0.01	0.76
time (sec)	N/A	0.265	0.042	0.016	0.000	2.686	0.000	0.430	0.000	0.181

Problem 1389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	93	89	114	0	0	0	74	-1	95
N.S.	1	0.85	0.81	1.04	0.00	0.00	0.00	0.67	-0.01	0.86
time (sec)	N/A	0.195	0.092	0.011	0.000	0.000	0.000	0.757	0.000	0.167

Problem 1390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	104	106	201	125	392	144	100	279	124
N.S.	1	0.95	0.96	1.83	1.14	3.56	1.31	0.91	2.54	1.13
time (sec)	N/A	0.556	0.215	0.013	0.310	0.441	92.604	0.361	1.190	0.120

Problem 1391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	F	A	C	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	0	99	31	0	213	48	0	-1	78
N.S.	1	0.00	0.90	0.28	0.00	1.94	0.44	0.00	-0.01	0.71
time (sec)	N/A	0.156	0.264	0.036	0.000	39.347	0.906	0.000	0.000	0.188

Problem 1392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	110	0	0	0	0	71	0	0	-1	110
N.S.	1	0.00	0.00	0.00	0.00	0.65	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.582	1.418	0.299	0.000	0.416	0.000	0.000	0.000	5.260

Problem 1393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	84	84	0	0	66	0	0	-1	110
N.S.	1	0.76	0.76	0.00	0.00	0.60	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.063	0.043	0.023	0.000	0.413	0.000	0.000	0.000	0.118

Problem 1394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	111	0	0	552	0	107	0	0	-1	111
N.S.	1	0.00	0.00	4.97	0.00	0.96	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.010	0.176	2.323	0.000	1.051	0.000	0.000	0.000	0.116

Problem 1395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	111	0	0	0	0	0	0	0	-1	111
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	6.936	24.681	0.609	0.000	0.000	0.000	0.000	0.000	0.771

Problem 1396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	111	412	278	1501	0	229	0	0	509	111
N.S.	1	3.71	2.50	13.52	0.00	2.06	0.00	0.00	4.59	1.00
time (sec)	N/A	0.920	0.914	0.322	0.000	0.513	0.000	0.000	0.969	1.032

Problem 1397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	155	35	42	0	119	0	74	44	111
N.S.	1	1.40	0.32	0.38	0.00	1.07	0.00	0.67	0.40	1.00
time (sec)	N/A	0.114	0.015	0.320	0.000	0.434	0.000	0.212	1.171	0.230

Problem 1398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	155	41	42	0	119	0	74	44	111
N.S.	1	1.40	0.37	0.38	0.00	1.07	0.00	0.67	0.40	1.00
time (sec)	N/A	0.083	0.016	0.324	0.000	0.421	0.000	0.181	0.886	0.228

Problem 1399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	164	38	36	0	108	0	79	-1	111
N.S.	1	1.48	0.34	0.32	0.00	0.97	0.00	0.71	-0.01	1.00
time (sec)	N/A	0.078	0.011	0.294	0.000	0.426	0.000	0.198	0.000	0.263

Problem 1400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	111	0	0	0	0	0	0	0	-1	111
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	18.096	4.759	0.392	0.000	0.000	0.000	0.000	0.000	1.040

Problem 1401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	B	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	111	0	0	0	0	0	0	312	-1	111
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	2.81	-0.01	1.00
time (sec)	N/A	38.574	7.895	0.363	0.000	0.000	0.000	0.382	0.000	0.880

Problem 1402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	111	123	102	478	0	133	0	0	-1	111
N.S.	1	1.11	0.92	4.31	0.00	1.20	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.143	0.216	1.362	0.000	1.382	0.000	0.000	0.000	0.254

Problem 1403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	111	0	0	0	0	0	0	0	-1	111
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	17.464	4.410	0.083	0.000	0.000	0.000	0.000	0.000	0.456

Problem 1404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	198	87	0	0	693	0	257	-1	111
N.S.	1	1.78	0.78	0.00	0.00	6.24	0.00	2.32	-0.01	1.00
time (sec)	N/A	0.285	0.050	0.098	0.000	0.433	0.000	0.215	0.000	0.617

Problem 1405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	111	0	0	1119	0	136	0	0	-1	111
N.S.	1	0.00	0.00	10.08	0.00	1.23	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.120	0.549	1.987	0.000	5.318	0.000	0.000	0.000	2.182

Problem 1406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	111	0	0	442	0	141	0	0	-1	111
N.S.	1	0.00	0.00	3.98	0.00	1.27	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.184	0.529	2.715	0.000	7.364	0.000	0.000	0.000	2.653

Problem 1407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	119	50	58	93	116	167	0	-1	111
N.S.	1	1.07	0.45	0.52	0.84	1.05	1.50	0.00	-0.01	1.00
time (sec)	N/A	0.073	0.024	0.318	0.449	0.700	3.373	0.000	0.000	0.489

Problem 1408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	121	75	635	0	503	0	65	-1	111
N.S.	1	1.09	0.68	5.72	0.00	4.53	0.00	0.59	-0.01	1.00
time (sec)	N/A	0.057	0.028	2.141	0.000	4.876	0.000	0.351	0.000	0.427

Problem 1409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	138	102	160	162	84	0	160	-1	110
N.S.	1	1.24	0.92	1.44	1.46	0.76	0.00	1.44	-0.01	0.99
time (sec)	N/A	0.469	0.193	0.005	0.310	0.451	0.000	0.180	0.000	0.096

Problem 1410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	93	629	0	0	68	0	0	-1	111
N.S.	1	0.84	5.67	0.00	0.00	0.61	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.353	2.521	180.000	0.000	0.423	0.000	0.000	0.000	0.171

Problem 1411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	204	54	543	0	107	0	89	-1	112
N.S.	1	1.82	0.48	4.85	0.00	0.96	0.00	0.79	-0.01	1.00
time (sec)	N/A	0.187	0.019	2.184	0.000	0.624	0.000	2.008	0.000	0.227

Problem 1412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	226	68	745	0	108	0	97	-1	112
N.S.	1	2.02	0.61	6.65	0.00	0.96	0.00	0.87	-0.01	1.00
time (sec)	N/A	0.243	0.026	2.052	0.000	0.652	0.000	1.003	0.000	0.414

Problem 1413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	B	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	153	51	110	148	508	61	0	72	107
N.S.	1	1.37	0.46	0.98	1.32	4.54	0.54	0.00	0.64	0.96
time (sec)	N/A	0.143	0.028	0.332	0.419	18.677	2.530	0.000	1.138	4.402

Problem 1414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	112	0	0	689	0	137	0	0	-1	112
N.S.	1	0.00	0.00	6.15	0.00	1.22	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.183	0.547	2.583	0.000	2.104	0.000	0.000	0.000	0.598

Problem 1415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	112	149	136	946	0	247	0	100	-1	112
N.S.	1	1.33	1.21	8.45	0.00	2.21	0.00	0.89	-0.01	1.00
time (sec)	N/A	0.255	0.535	0.021	0.000	1.721	0.000	0.561	0.000	0.575

Problem 1416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	112	211	85	0	0	351	0	97	-1	112
N.S.	1	1.88	0.76	0.00	0.00	3.13	0.00	0.87	-0.01	1.00
time (sec)	N/A	0.625	0.444	1.377	0.000	17.149	0.000	0.437	0.000	0.333

Problem 1417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	55	48	0	0	504	0	208	-1	112
N.S.	1	0.49	0.43	0.00	0.00	4.50	0.00	1.86	-0.01	1.00
time (sec)	N/A	0.367	0.031	0.344	0.000	79.944	0.000	0.377	0.000	0.397

Problem 1418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	196	49	0	0	253	0	261	-1	112
N.S.	1	1.75	0.44	0.00	0.00	2.26	0.00	2.33	-0.01	1.00
time (sec)	N/A	0.276	0.016	0.066	0.000	0.436	0.000	0.376	0.000	0.456

Problem 1419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	112	0	0	0	0	0	0	0	-1	112
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.616	0.604	0.345	0.000	0.000	0.000	0.000	0.000	2.920

Problem 1420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	112	392	0	0	0	0	0	0	-1	112
N.S.	1	3.50	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.961	1.135	0.449	0.000	0.000	0.000	0.000	0.000	2.874

Problem 1421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	85	113	53	122	102	31	0	-1	112
N.S.	1	0.76	1.01	0.47	1.09	0.91	0.28	0.00	-0.01	1.00
time (sec)	N/A	0.040	0.047	0.312	0.412	0.666	1.330	0.000	0.000	2.759

Problem 1422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	119	54	53	121	102	32	0	-1	112
N.S.	1	1.06	0.48	0.47	1.08	0.91	0.29	0.00	-0.01	1.00
time (sec)	N/A	0.086	0.019	0.318	0.418	0.718	1.203	0.000	0.000	0.801

Problem 1423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	85	54	53	121	102	34	0	-1	112
N.S.	1	0.76	0.48	0.47	1.08	0.91	0.30	0.00	-0.01	1.00
time (sec)	N/A	0.039	0.022	0.323	0.417	0.648	1.215	0.000	0.000	1.014

Problem 1424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	85	113	37	122	102	29	0	-1	112
N.S.	1	0.76	1.01	0.33	1.09	0.91	0.26	0.00	-0.01	1.00
time (sec)	N/A	0.043	0.046	0.294	0.425	0.795	1.311	0.000	0.000	2.521

Problem 1425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	85	34	37	121	102	31	0	-1	112
N.S.	1	0.76	0.30	0.33	1.08	0.91	0.28	0.00	-0.01	1.00
time (sec)	N/A	0.037	0.014	0.299	0.409	0.420	1.167	0.000	0.000	1.008

Problem 1426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	112	0	0	1136	0	152	0	0	-1	112
N.S.	1	0.00	0.00	10.14	0.00	1.36	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.995	0.368	6.580	0.000	17.209	0.000	0.000	0.000	2.736

Problem 1427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F(-2)	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	95	93	0	0	72	0	0	-1	112
N.S.	1	0.85	0.83	0.00	0.00	0.64	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.135	0.056	180.000	0.000	0.414	0.000	0.000	0.000	0.156

Problem 1428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	3261	675	0	0	0	0	0	-1	113
N.S.	1	28.86	5.97	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	65.518	25.685	0.508	0.000	0.000	0.000	0.000	0.000	0.266

Problem 1429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	182	35	663	0	105	0	88	-1	113
N.S.	1	1.61	0.31	5.87	0.00	0.93	0.00	0.78	-0.01	1.00
time (sec)	N/A	0.129	0.012	0.447	0.000	0.420	0.000	0.182	0.000	0.264

Problem 1430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	113	0	2458	0	0	0	0	0	-1	113
N.S.	1	0.00	21.75	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	7.249	7.338	0.070	0.000	0.000	0.000	0.000	0.000	0.313

Problem 1431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	113	0	0	0	0	0	0	0	-1	113
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	26.454	5.963	0.151	0.000	0.000	0.000	0.000	0.000	0.891

Problem 1432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	113	159	60	417	0	326	0	0	-1	113
N.S.	1	1.41	0.53	3.69	0.00	2.88	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.235	0.045	0.052	0.000	0.720	0.000	0.000	0.000	0.372

Problem 1433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	113	0	0	0	0	0	0	0	-1	113
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	9.255	3.549	0.124	0.000	0.000	0.000	0.000	0.000	3.644

Problem 1434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	113	58	80	0	0	514	0	213	-1	113
N.S.	1	0.51	0.71	0.00	0.00	4.55	0.00	1.88	-0.01	1.00
time (sec)	N/A	0.384	0.047	0.341	0.000	79.499	0.000	0.209	0.000	0.403

Problem 1435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	113	0	333050	0	0	0	0	0	-1	113
N.S.	1	0.00	2947.35	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	28.011	22.591	0.066	0.000	0.000	0.000	0.000	0.000	0.462

Problem 1436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	113	0	0	0	0	0	0	0	-1	113
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	52.450	2.345	0.085	0.000	0.000	0.000	0.000	0.000	0.476

Problem 1437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	113	0	0	425	0	236	0	0	-1	113
N.S.	1	0.00	0.00	3.76	0.00	2.09	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.177	0.358	3.176	0.000	1.301	0.000	0.000	0.000	3.774

Problem 1438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	145	72	63	170	106	32	0	-1	114
N.S.	1	1.27	0.63	0.55	1.49	0.93	0.28	0.00	-0.01	1.00
time (sec)	N/A	0.079	0.025	0.500	0.428	0.441	1.621	0.000	0.000	0.329

Problem 1439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	151	33	40	0	137	0	74	-1	114
N.S.	1	1.32	0.29	0.35	0.00	1.20	0.00	0.65	-0.01	1.00
time (sec)	N/A	0.089	0.009	0.318	0.000	0.711	0.000	0.246	0.000	0.244

Problem 1440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	151	36	40	0	137	0	74	-1	114
N.S.	1	1.32	0.32	0.35	0.00	1.20	0.00	0.65	-0.01	1.00
time (sec)	N/A	0.087	0.013	0.313	0.000	0.751	0.000	0.155	0.000	0.250

Problem 1441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	2624	611	0	0	0	0	0	-1	114
N.S.	1	23.02	5.36	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	14.509	7.427	0.433	0.000	0.000	0.000	0.000	0.000	0.562

Problem 1442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	114	797	0	390	0	1098	0	0	-1	114
N.S.	1	6.99	0.00	3.42	0.00	9.63	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.524	0.129	5.635	0.000	6.966	0.000	0.000	0.000	1.553

Problem 1443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	230	124	878	0	183	0	123	-1	114
N.S.	1	2.02	1.09	7.70	0.00	1.61	0.00	1.08	-0.01	1.00
time (sec)	N/A	0.208	0.071	1.877	0.000	0.429	0.000	0.528	0.000	0.558

Problem 1444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	177	71	0	0	0	0	209	72	114
N.S.	1	1.55	0.62	0.00	0.00	0.00	0.00	1.83	0.63	1.00
time (sec)	N/A	0.401	0.088	0.323	0.000	0.000	0.000	0.303	1.879	0.473

Problem 1445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	126	114	0	0	461	0	0	-1	114
N.S.	1	1.11	1.00	0.00	0.00	4.04	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.675	0.124	0.332	0.000	166.002	0.000	0.000	0.000	0.924

Problem 1446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	C	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	114	141	9486	84	0	109	90	0	-1	114
N.S.	1	1.24	83.21	0.74	0.00	0.96	0.79	0.00	-0.01	1.00
time (sec)	N/A	0.360	24.910	0.061	0.000	0.428	8.120	0.000	0.000	0.226

Problem 1447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	C	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	0	59	0	0	219	44	0	-1	82
N.S.	1	0.00	0.52	0.00	0.00	1.92	0.39	0.00	-0.01	0.72
time (sec)	N/A	0.655	0.236	0.296	0.000	38.627	1.025	0.000	0.000	0.167

Problem 1448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	115	0	0	615	0	147	0	0	-1	115
N.S.	1	0.00	0.00	5.35	0.00	1.28	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.475	0.230	1.851	0.000	1.067	0.000	0.000	0.000	0.133

Problem 1449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	150	46	111	145	124	112	0	158	115
N.S.	1	1.30	0.40	0.97	1.26	1.08	0.97	0.00	1.37	1.00
time (sec)	N/A	0.099	0.032	0.298	0.429	0.843	2.848	0.000	1.290	17.303

Problem 1450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	83	70	0	86	236	37	87	100	115
N.S.	1	0.72	0.61	0.00	0.75	2.05	0.32	0.76	0.87	1.00
time (sec)	N/A	0.062	0.055	0.322	0.429	0.432	0.886	0.686	1.051	0.096

Problem 1451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	115	0	0	0	0	0	0	0	-1	115
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	38.087	7.085	0.141	0.000	0.000	0.000	0.000	0.000	0.918

Problem 1452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	115	0	0	433	0	150	0	0	-1	115
N.S.	1	0.00	0.00	3.77	0.00	1.30	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.922	0.358	1.983	0.000	4.719	0.000	0.000	0.000	0.574

Problem 1453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	116	0	71	594	0	127	0	0	-1	116
N.S.	1	0.00	0.61	5.12	0.00	1.09	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.369	0.173	2.029	0.000	0.843	0.000	0.000	0.000	0.369

Problem 1454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	116	0	0	0	0	0	0	0	-1	116
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	20.566	3.876	0.383	0.000	0.000	0.000	0.000	0.000	4.310

Problem 1455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	116	0	0	0	0	0	0	0	-1	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	21.076	3.977	0.333	0.000	0.000	0.000	0.000	0.000	180.022

Problem 1456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	116	0	0	0	0	0	0	0	-1	116
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.784	0.704	0.355	0.000	0.000	0.000	0.000	0.000	2.953

Problem 1457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	116	397	0	0	0	0	0	0	-1	116
N.S.	1	3.42	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.109	1.226	0.474	0.000	0.000	0.000	0.000	0.000	2.915

Problem 1458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	116	447	0	0	0	0	0	0	-1	116
N.S.	1	3.85	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.168	1.968	0.414	0.000	0.000	0.000	0.000	0.000	1.509

Problem 1459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	116	0	0	0	0	0	0	0	-1	116
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.766	0.374	0.389	0.000	0.000	0.000	0.000	0.000	2.928

Problem 1460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	222	67	796	0	112	0	109	-1	117
N.S.	1	1.90	0.57	6.80	0.00	0.96	0.00	0.93	-0.01	1.00
time (sec)	N/A	0.233	0.031	1.947	0.000	0.824	0.000	0.185	0.000	0.332

Problem 1461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	117	163	66	296	0	344	0	0	-1	117
N.S.	1	1.39	0.56	2.53	0.00	2.94	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.463	0.044	0.043	0.000	0.563	0.000	0.000	0.000	0.374

Problem 1462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	163	111	631	0	344	0	0	-1	117
N.S.	1	1.39	0.95	5.39	0.00	2.94	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.381	0.907	0.058	0.000	0.533	0.000	0.000	0.000	0.346

Problem 1463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	101	127	58	145	107	34	0	-1	117
N.S.	1	0.86	1.09	0.50	1.24	0.91	0.29	0.00	-0.01	1.00
time (sec)	N/A	0.050	0.055	0.382	0.406	0.416	2.384	0.000	0.000	7.476

Problem 1464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	135	67	58	145	107	36	0	-1	117
N.S.	1	1.15	0.57	0.50	1.24	0.91	0.31	0.00	-0.01	1.00
time (sec)	N/A	0.089	0.028	0.380	0.406	0.414	1.908	0.000	0.000	1.568

Problem 1465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	101	127	42	145	107	29	0	-1	117
N.S.	1	0.86	1.09	0.36	1.24	0.91	0.25	0.00	-0.01	1.00
time (sec)	N/A	0.049	0.053	0.342	0.412	0.411	2.296	0.000	0.000	7.517

Problem 1466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	135	45	42	145	107	31	0	-1	117
N.S.	1	1.15	0.38	0.36	1.24	0.91	0.26	0.00	-0.01	1.00
time (sec)	N/A	0.096	0.016	0.349	0.411	0.410	1.871	0.000	0.000	1.565

Problem 1467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	179	84	0	0	0	0	220	80	117
N.S.	1	1.53	0.72	0.00	0.00	0.00	0.00	1.88	0.68	1.00
time (sec)	N/A	0.308	0.066	0.330	0.000	0.000	0.000	0.311	1.413	0.570

Problem 1468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	117	0	0	582	0	157	0	0	-1	117
N.S.	1	0.00	0.00	4.97	0.00	1.34	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.893	0.246	3.381	0.000	2.939	0.000	0.000	0.000	0.425

Problem 1469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	179	147	0	0	0	0	217	73	117
N.S.	1	1.53	1.26	0.00	0.00	0.00	0.00	1.85	0.62	1.00
time (sec)	N/A	0.280	0.151	0.032	0.000	0.000	0.000	0.354	1.373	0.723

Problem 1470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	179	147	0	0	0	0	217	73	117
N.S.	1	1.53	1.26	0.00	0.00	0.00	0.00	1.85	0.62	1.00
time (sec)	N/A	0.274	0.096	0.031	0.000	0.000	0.000	0.290	1.178	0.738

Problem 1471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	F	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	248	117	0	0	330	0	123	-1	117
N.S.	1	2.12	1.00	0.00	0.00	2.82	0.00	1.05	-0.01	1.00
time (sec)	N/A	0.582	0.592	1.635	0.000	8.947	0.000	0.199	0.000	0.505

Problem 1472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	117	0	0	0	0	78	0	0	-1	0
N.S.	1	0.00	0.00	0.00	0.00	0.67	0.00	0.00	-0.01	0.00
time (sec)	N/A	3.924	1.078	0.378	0.000	0.425	0.000	0.000	0.000	180.015

Problem 1473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	73	98	0	170	101	73	0	-1	118
N.S.	1	0.62	0.83	0.00	1.44	0.86	0.62	0.00	-0.01	1.00
time (sec)	N/A	0.019	0.092	0.375	0.411	0.411	1.904	0.000	0.000	0.275

Problem 1474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	223	136	54	0	100	0	83	-1	118
N.S.	1	1.89	1.15	0.46	0.00	0.85	0.00	0.70	-0.01	1.00
time (sec)	N/A	0.160	0.201	0.305	0.000	83.613	0.000	0.243	0.000	0.341

Problem 1475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	174	35	27	0	110	0	89	27	118
N.S.	1	1.47	0.30	0.23	0.00	0.93	0.00	0.75	0.23	1.00
time (sec)	N/A	0.087	0.010	0.337	0.000	0.402	0.000	0.177	1.035	0.197

Problem 1476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	200	38	461	0	110	0	97	-1	118
N.S.	1	1.69	0.32	3.91	0.00	0.93	0.00	0.82	-0.01	1.00
time (sec)	N/A	0.190	0.014	0.497	0.000	0.404	0.000	0.188	0.000	0.349

Problem 1477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	118	0	0	445	0	153	0	0	-1	118
N.S.	1	0.00	0.00	3.77	0.00	1.30	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.631	0.461	2.072	0.000	2.810	0.000	0.000	0.000	1.997

Problem 1478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	118	123	0	951	0	294	0	0	-1	118
N.S.	1	1.04	0.00	8.06	0.00	2.49	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.595	0.378	20.514	0.000	4.141	0.000	0.000	0.000	0.394

Problem 1479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	118	147	0	0	0	545	0	0	-1	118
N.S.	1	1.25	0.00	0.00	0.00	4.62	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.586	1.074	5.812	0.000	7.317	0.000	0.000	0.000	0.479

Problem 1480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	118	147	0	0	0	545	0	0	-1	118
N.S.	1	1.25	0.00	0.00	0.00	4.62	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.550	0.399	0.002	0.000	7.396	0.000	0.000	0.000	0.001

Problem 1481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	C	F	A	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	118	224	125	407	0	155	0	0	-1	118
N.S.	1	1.90	1.06	3.45	0.00	1.31	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.481	0.220	1.368	0.000	1.075	0.000	0.000	0.000	0.268

Problem 1482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	118	131	143	110	145	243	279	149	-1	103
N.S.	1	1.11	1.21	0.93	1.23	2.06	2.36	1.26	-0.01	0.87
time (sec)	N/A	0.487	0.255	0.010	0.436	0.452	18.314	1.707	0.000	0.177

Problem 1483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	148	102	161	0	0	0	0	-1	126
N.S.	1	1.25	0.86	1.36	0.00	0.00	0.00	0.00	-0.01	1.07
time (sec)	N/A	0.248	0.134	0.014	0.000	0.000	0.000	0.000	0.000	0.263

Problem 1484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	118	0	0	0	0	319	0	0	-1	118
N.S.	1	0.00	0.00	0.00	0.00	2.70	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.218	0.143	180.000	0.000	4.447	0.000	0.000	0.000	0.296

Problem 1485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	119	0	0	0	0	0	0	0	-1	119
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.913	6.983	0.648	0.000	0.000	0.000	0.000	0.000	0.387

Problem 1486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	161	77	68	193	111	36	0	-1	119
N.S.	1	1.35	0.65	0.57	1.62	0.93	0.30	0.00	-0.01	1.00
time (sec)	N/A	0.102	0.029	0.324	0.423	0.434	2.778	0.000	0.000	0.353

Problem 1487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	119	0	0	0	0	0	0	0	-1	119
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	9.559	4.951	0.079	0.000	0.000	0.000	0.000	0.000	0.436

Problem 1488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	119	0	0	573	0	149	0	0	-1	119
N.S.	1	0.00	0.00	4.82	0.00	1.25	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.536	0.564	1.842	0.000	17.557	0.000	0.000	0.000	0.350

Problem 1489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	119	0	0	1372	0	780	0	0	-1	119
N.S.	1	0.00	0.00	11.53	0.00	6.55	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.747	0.304	2.806	0.000	101.293	0.000	0.000	0.000	1.027

Problem 1490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	196	47	0	0	265	0	261	38	119
N.S.	1	1.65	0.39	0.00	0.00	2.23	0.00	2.19	0.32	1.00
time (sec)	N/A	0.253	0.016	0.454	0.000	0.414	0.000	0.338	0.926	0.547

Problem 1491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	119	0	375	260	0	395	0	0	-1	119
N.S.	1	0.00	3.15	2.18	0.00	3.32	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.208	0.760	0.091	0.000	0.559	0.000	0.000	0.000	2.260

Problem 1492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	119	0	371	377	0	431	0	0	-1	119
N.S.	1	0.00	3.12	3.17	0.00	3.62	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.880	0.662	0.079	0.000	0.566	0.000	0.000	0.000	2.172

Problem 1493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	404	227	272	0	302	0	0	-1	119
N.S.	1	3.39	1.91	2.29	0.00	2.54	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.047	1.181	0.044	0.000	0.586	0.000	0.000	0.000	0.620

Problem 1494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	106	96	85	95	200	90	107	-1	108
N.S.	1	0.89	0.81	0.71	0.80	1.68	0.76	0.90	-0.01	0.91
time (sec)	N/A	0.119	0.201	0.009	1.801	0.449	5.577	0.152	0.000	0.150

Problem 1495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	0	238	0	0	100	0	0	-1	119
N.S.	1	0.00	2.00	0.00	0.00	0.84	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.333	2.119	0.313	0.000	0.985	0.000	0.000	0.000	1.491

Problem 1496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	163	354	261	0	117	323	0	-1	119
N.S.	1	1.37	2.97	2.19	0.00	0.98	2.71	0.00	-0.01	1.00
time (sec)	N/A	0.491	0.274	0.063	0.000	0.692	8.298	0.000	0.000	3.266

Problem 1497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	B	F	F	F(-1)	A	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	113	247	0	0	0	32	0	-1	145
N.S.	1	0.95	2.08	0.00	0.00	0.00	0.27	0.00	-0.01	1.22
time (sec)	N/A	0.477	1.757	0.379	0.000	0.000	4.133	0.000	0.000	0.398

Problem 1498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	127	128	63	102	130	175	102	-1	99
N.S.	1	1.07	1.08	0.53	0.86	1.09	1.47	0.86	-0.01	0.83
time (sec)	N/A	0.685	0.172	0.008	0.860	0.431	138.848	0.161	0.000	0.118

Problem 1499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	132	65	0	0	0	0	0	-1	120
N.S.	1	1.10	0.54	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.741	0.120	0.624	0.000	0.000	0.000	0.000	0.000	1.070

Problem 1500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	121	0	0	0	0	0	0	0	-1	121
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	63.201	6.897	0.095	0.000	0.000	0.000	0.000	0.000	0.445

Problem 1501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	121	150	0	0	0	0	0	0	-1	121
N.S.	1	1.24	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.295	0.620	0.456	0.000	0.000	0.000	0.000	0.000	0.385

Problem 1502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	121	917	871	1643	0	150	0	0	-1	121
N.S.	1	7.58	7.20	13.58	0.00	1.24	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.652	1.602	0.655	0.000	0.806	0.000	0.000	0.000	1.120

Problem 1503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	121	127	0	971	0	307	0	0	-1	121
N.S.	1	1.05	0.00	8.02	0.00	2.54	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.649	0.198	9.502	0.000	5.340	0.000	0.000	0.000	0.395

Problem 1504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	1906	234	303	0	344	0	0	164	121
N.S.	1	15.75	1.93	2.50	0.00	2.84	0.00	0.00	1.36	1.00
time (sec)	N/A	6.232	1.293	0.076	0.000	0.940	0.000	0.000	5.625	0.402

Problem 1505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	121	123	0	1135	0	320	0	0	-1	121
N.S.	1	1.02	0.00	9.38	0.00	2.64	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.623	0.422	16.713	0.000	4.409	0.000	0.000	0.000	0.379

Problem 1506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	0	175	0	0	114	0	0	-1	121
N.S.	1	0.00	1.45	0.00	0.00	0.94	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.009	0.883	1.032	0.000	0.787	0.000	0.000	0.000	4.108

Problem 1507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	B	B	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	122	0	0	0	0	154	0	202	57	122
N.S.	1	0.00	0.00	0.00	0.00	1.26	0.00	1.66	0.47	1.00
time (sec)	N/A	0.032	0.653	1.489	0.000	0.803	0.000	0.407	1.331	0.295

Problem 1508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	240	72	801	0	117	0	127	-1	122
N.S.	1	1.97	0.59	6.57	0.00	0.96	0.00	1.04	-0.01	1.00
time (sec)	N/A	0.263	0.033	1.649	0.000	2.117	0.000	0.195	0.000	0.470

Problem 1509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	167	48	558	0	120	0	0	-1	122
N.S.	1	1.37	0.39	4.57	0.00	0.98	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.138	0.019	0.391	0.000	0.664	0.000	0.000	0.000	0.289

Problem 1510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	122	221	109	0	0	395	0	112	-1	122
N.S.	1	1.81	0.89	0.00	0.00	3.24	0.00	0.92	-0.01	1.00
time (sec)	N/A	0.606	0.301	2.003	0.000	17.893	0.000	0.357	0.000	0.379

Problem 1511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	F	F	F(-1)	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	321	108	0	0	0	0	277	-1	122
N.S.	1	2.63	0.89	0.00	0.00	0.00	0.00	2.27	-0.01	1.00
time (sec)	N/A	0.495	0.155	0.083	0.000	0.000	0.000	0.381	0.000	0.544

Problem 1512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	F	F	F(-1)	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	321	108	0	0	0	0	277	-1	122
N.S.	1	2.63	0.89	0.00	0.00	0.00	0.00	2.27	-0.01	1.00
time (sec)	N/A	0.507	0.083	0.075	0.000	0.000	0.000	0.301	0.000	0.510

Problem 1513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	130	112	0	361	726	122	0	-1	122
N.S.	1	1.07	0.92	0.00	2.96	5.95	1.00	0.00	-0.01	1.00
time (sec)	N/A	0.078	0.095	1.141	0.734	0.628	3.358	0.000	0.000	0.768

Problem 1514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	F	A	F	A	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	0	78	1042	0	93	0	84	109	123
N.S.	1	0.00	0.63	8.47	0.00	0.76	0.00	0.68	0.89	1.00
time (sec)	N/A	0.014	0.078	9.904	0.000	0.673	0.000	0.296	1.188	0.234

Problem 1515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	83	72	805	0	93	0	0	-1	123
N.S.	1	0.67	0.59	6.54	0.00	0.76	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.071	0.031	9.082	0.000	0.589	0.000	0.000	0.000	0.224

Problem 1516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	196	165	0	162	134	42	162	44	121
N.S.	1	1.59	1.34	0.00	1.32	1.09	0.34	1.32	0.36	0.98
time (sec)	N/A	0.205	0.073	1.030	2.055	0.619	0.952	0.171	0.999	0.220

Problem 1517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	86	39	0	95	308	39	120	118	123
N.S.	1	0.70	0.32	0.00	0.77	2.50	0.32	0.98	0.96	1.00
time (sec)	N/A	0.068	0.014	1.147	1.449	0.684	0.893	2.056	1.045	0.111

Problem 1518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	728	208	391	0	421	0	0	-1	123
N.S.	1	5.92	1.69	3.18	0.00	3.42	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.478	1.224	0.091	0.000	0.796	0.000	0.000	0.000	0.526

Problem 1519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	747	212	387	0	423	0	0	-1	123
N.S.	1	6.07	1.72	3.15	0.00	3.44	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.391	1.058	0.086	0.000	0.708	0.000	0.000	0.000	0.495

Problem 1520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	123	0	0	0	0	0	0	0	-1	123
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.322	0.652	0.316	0.000	0.000	0.000	0.000	0.000	1.723

Problem 1521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	123	0	0	848	0	559	0	0	-1	123
N.S.	1	0.00	0.00	6.89	0.00	4.54	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.037	0.687	2.218	0.000	8.644	0.000	0.000	0.000	0.368

Problem 1522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	224	49	0	0	264	0	278	-1	123
N.S.	1	1.82	0.40	0.00	0.00	2.15	0.00	2.26	-0.01	1.00
time (sec)	N/A	0.355	0.016	0.059	0.000	0.449	0.000	0.267	0.000	0.522

Problem 1523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	123	785	939	736	0	196	0	0	1195	123
N.S.	1	6.38	7.63	5.98	0.00	1.59	0.00	0.00	9.72	1.00
time (sec)	N/A	2.507	5.022	0.303	0.000	0.445	0.000	0.000	0.877	0.220

Problem 1524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	123	338	192	187	0	479	0	125	-1	123
N.S.	1	2.75	1.56	1.52	0.00	3.89	0.00	1.02	-0.01	1.00
time (sec)	N/A	0.820	1.366	4.270	0.000	2.023	0.000	0.238	0.000	0.681

Problem 1525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	183	73	0	0	0	0	217	-1	123
N.S.	1	1.49	0.59	0.00	0.00	0.00	0.00	1.76	-0.01	1.00
time (sec)	N/A	0.368	0.121	0.352	0.000	0.000	0.000	0.389	0.000	0.474

Problem 1526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	123	0	109075	8123	0	935	0	0	-1	117
N.S.	1	0.00	886.79	66.04	0.00	7.60	0.00	0.00	-0.01	0.95
time (sec)	N/A	3.041	6.891	8.681	0.000	0.962	0.000	0.000	0.000	3.802

Problem 1527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	123	0	0	599	0	555	0	0	-1	123
N.S.	1	0.00	0.00	4.87	0.00	4.51	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.974	0.852	4.319	0.000	0.828	0.000	0.000	0.000	3.446

Problem 1528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	74	126	115	0	105	0	0	82	78
N.S.	1	0.60	1.02	0.93	0.00	0.85	0.00	0.00	0.67	0.63
time (sec)	N/A	0.062	0.131	0.019	0.000	0.421	0.000	0.000	2.189	0.099

Problem 1529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	0	98	0	0	79	0	0	-1	123
N.S.	1	0.00	0.80	0.00	0.00	0.64	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.406	4.686	0.033	0.000	0.445	0.000	0.000	0.000	2.993

Problem 1530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	196	54	0	162	127	42	162	45	122
N.S.	1	1.58	0.44	0.00	1.31	1.02	0.34	1.31	0.36	0.98
time (sec)	N/A	0.209	0.026	0.312	0.557	0.418	0.925	0.180	1.035	0.212

Problem 1531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	610	444	1772	0	213	0	0	-1	124
N.S.	1	4.92	3.58	14.29	0.00	1.72	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.221	4.790	0.308	0.000	0.821	0.000	0.000	0.000	4.204

Problem 1532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	193	81	0	0	771	0	322	-1	124
N.S.	1	1.56	0.65	0.00	0.00	6.22	0.00	2.60	-0.01	1.00
time (sec)	N/A	0.286	0.052	0.062	0.000	0.445	0.000	0.217	0.000	0.696

Problem 1533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	124	0	0	0	0	148	0	0	-1	128
N.S.	1	0.00	0.00	0.00	0.00	1.19	0.00	0.00	-0.01	1.03
time (sec)	N/A	2.056	0.430	0.311	0.000	4.767	0.000	0.000	0.000	1.068

Problem 1534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	124	0	0	536	0	174	0	0	-1	124
N.S.	1	0.00	0.00	4.32	0.00	1.40	0.00	0.00	-0.01	1.00
time (sec)	N/A	4.655	1.276	23.036	0.000	2.669	0.000	0.000	0.000	0.522

Problem 1535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	124	0	0	0	0	323	0	0	-1	124
N.S.	1	0.00	0.00	0.00	0.00	2.60	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.203	0.127	180.000	0.000	3.104	0.000	0.000	0.000	0.279

Problem 1536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	84	36	1199	0	113	0	0	-1	125
N.S.	1	0.67	0.29	9.59	0.00	0.90	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.064	0.014	7.995	0.000	0.398	0.000	0.000	0.000	0.216

Problem 1537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	84	36	803	0	116	0	0	-1	125
N.S.	1	0.67	0.29	6.42	0.00	0.93	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.067	0.012	7.951	0.000	0.395	0.000	0.000	0.000	0.227

Problem 1538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	82	36	802	0	116	0	0	-1	125
N.S.	1	0.66	0.29	6.42	0.00	0.93	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.061	0.012	7.326	0.000	0.404	0.000	0.000	0.000	0.218

Problem 1539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	125	0	0	0	0	0	0	0	-1	125
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	19.786	15.031	0.526	0.000	0.000	0.000	0.000	0.000	0.734

Problem 1540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	125	0	0	0	0	0	0	0	-1	125
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	8.440	5.482	0.106	0.000	0.000	0.000	0.000	0.000	3.725

Problem 1541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	610	444	1771	0	212	0	0	-1	125
N.S.	1	4.88	3.55	14.17	0.00	1.70	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.940	4.923	0.256	0.000	0.809	0.000	0.000	0.000	4.256

Problem 1542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	125	1650	2469	2289	0	2484	0	0	537	125
N.S.	1	13.20	19.75	18.31	0.00	19.87	0.00	0.00	4.30	1.00
time (sec)	N/A	5.082	13.086	0.083	0.000	0.932	0.000	0.000	0.073	0.233

Problem 1543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	125	144	170	970	0	266	0	128	-1	125
N.S.	1	1.15	1.36	7.76	0.00	2.13	0.00	1.02	-0.01	1.00
time (sec)	N/A	0.353	0.345	0.394	0.000	4.061	0.000	0.572	0.000	0.758

Problem 1544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	B	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	170	70	142	155	517	71	0	81	120
N.S.	1	1.36	0.56	1.14	1.24	4.14	0.57	0.00	0.65	0.96
time (sec)	N/A	0.140	0.036	0.320	0.431	17.868	3.184	0.000	1.302	4.726

Problem 1545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	125	0	0	0	0	0	0	0	-1	125
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.001	0.407	0.322	0.000	0.000	0.000	0.000	0.000	13.122

Problem 1546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	159	53	0	0	188	0	0	-1	122
N.S.	1	1.27	0.42	0.00	0.00	1.50	0.00	0.00	-0.01	0.98
time (sec)	N/A	0.045	0.015	0.033	0.000	0.391	0.000	0.000	0.000	11.528

Problem 1547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	125	0	0	0	0	0	0	0	-1	125
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.627	0.330	0.320	0.000	0.000	0.000	0.000	0.000	15.737

Problem 1548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	125	0	0	436	0	816	0	0	-1	125
N.S.	1	0.00	0.00	3.49	0.00	6.53	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.592	0.454	4.027	0.000	47.170	0.000	0.000	0.000	14.713

Problem 1549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	125	0	0	255	0	0	-1	125
N.S.	1	1.00	1.00	0.00	0.00	2.04	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.350	0.102	1.143	0.000	0.419	0.000	0.000	0.000	0.494

Problem 1550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	125	0	0	0	0	0	0	0	-1	125
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.173	0.367	0.296	0.000	0.000	0.000	0.000	0.000	0.285

Problem 1551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	126	164	186	953	0	288	0	136	-1	126
N.S.	1	1.30	1.48	7.56	0.00	2.29	0.00	1.08	-0.01	1.00
time (sec)	N/A	0.372	0.606	0.380	0.000	3.994	0.000	0.718	0.000	0.699

Problem 1552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	53	46	0	0	479	0	208	-1	126
N.S.	1	0.42	0.37	0.00	0.00	3.80	0.00	1.65	-0.01	1.00
time (sec)	N/A	0.379	0.025	0.365	0.000	89.713	0.000	0.400	0.000	0.452

Problem 1553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	126	0	0	1137	0	151	0	0	-1	126
N.S.	1	0.00	0.00	9.02	0.00	1.20	0.00	0.00	-0.01	1.00
time (sec)	N/A	8.914	2.666	19.718	0.000	1.081	0.000	0.000	0.000	0.224

Problem 1554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	283	488	0	0	0	0	0	-1	127
N.S.	1	2.23	3.84	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.730	1.554	0.475	0.000	0.000	0.000	0.000	0.000	3.559

Problem 1555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	83	85	298	0	81	0	188	-1	77
N.S.	1	0.65	0.67	2.35	0.00	0.64	0.00	1.48	-0.01	0.61
time (sec)	N/A	0.136	0.052	0.020	0.000	5.967	0.000	1.025	0.000	0.277

Problem 1556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	136	150	0	0	158	0	0	-1	127
N.S.	1	1.07	1.18	0.00	0.00	1.24	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.322	0.338	0.291	0.000	0.426	0.000	0.000	0.000	0.329

Problem 1557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	136	150	0	0	158	0	0	-1	127
N.S.	1	1.07	1.18	0.00	0.00	1.24	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.271	0.305	0.296	0.000	0.429	0.000	0.000	0.000	0.276

Problem 1558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	128	0	0	671	0	407	0	0	-1	128
N.S.	1	0.00	0.00	5.24	0.00	3.18	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.446	0.299	5.553	0.000	17.302	0.000	0.000	0.000	0.200

Problem 1559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	201	165	0	162	134	42	162	51	127
N.S.	1	1.57	1.29	0.00	1.27	1.05	0.33	1.27	0.40	0.99
time (sec)	N/A	0.211	0.077	0.325	0.432	0.407	1.068	0.322	1.022	0.223

Problem 1560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	201	57	0	162	127	42	162	51	127
N.S.	1	1.57	0.45	0.00	1.27	0.99	0.33	1.27	0.40	0.99
time (sec)	N/A	0.193	0.022	0.312	0.437	0.412	1.020	0.333	0.960	0.192

Problem 1561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	128	0	0	0	0	0	0	0	-1	128
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	5.966	2.220	0.325	0.000	0.000	0.000	0.000	0.000	1.310

Problem 1562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	201	165	0	162	133	41	162	44	127
N.S.	1	1.57	1.29	0.00	1.27	1.04	0.32	1.27	0.34	0.99
time (sec)	N/A	0.191	0.071	0.325	0.437	0.416	1.166	0.278	1.003	0.197

Problem 1563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	201	57	0	162	127	39	162	46	127
N.S.	1	1.57	0.45	0.00	1.27	0.99	0.30	1.27	0.36	0.99
time (sec)	N/A	0.179	0.022	0.315	0.429	0.408	1.022	0.370	0.966	0.166

Problem 1564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	201	165	0	162	134	42	162	51	127
N.S.	1	1.57	1.29	0.00	1.27	1.05	0.33	1.27	0.40	0.99
time (sec)	N/A	0.191	0.072	0.329	0.410	0.411	1.047	0.325	1.018	0.180

Problem 1565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	201	165	0	162	134	42	162	44	127
N.S.	1	1.57	1.29	0.00	1.27	1.05	0.33	1.27	0.34	0.99
time (sec)	N/A	0.195	0.071	0.325	0.408	0.404	1.048	0.384	1.017	0.174

Problem 1566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	128	0	0	871	0	4669	0	0	-1	128
N.S.	1	0.00	0.00	6.80	0.00	36.48	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.314	0.592	0.172	0.000	4.234	0.000	0.000	0.000	1.439

Problem 1567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	128	0	0	1037	0	322	0	0	-1	128
N.S.	1	0.00	0.00	8.10	0.00	2.52	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.433	1.228	7.693	0.000	10.655	0.000	0.000	0.000	0.553

Problem 1568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	128	0	0	165	0	105	0	0	-1	128
N.S.	1	0.00	0.00	1.29	0.00	0.82	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.322	1.163	1.507	0.000	0.515	0.000	0.000	0.000	3.848

Problem 1569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F(-2)	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	1053	0	0	98	0	0	-1	128
N.S.	1	1.00	8.23	0.00	0.00	0.77	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.114	5.094	180.000	0.000	0.396	0.000	0.000	0.000	0.208

Problem 1570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	B	C	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	70	49	47	46	43	445	79	-1	82
N.S.	1	0.55	0.38	0.37	0.36	0.34	3.48	0.62	-0.01	0.64
time (sec)	N/A	0.071	0.033	0.009	0.322	0.398	1.721	1.318	0.000	0.070

Problem 1571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	215	62	512	0	114	0	93	-1	129
N.S.	1	1.67	0.48	3.97	0.00	0.88	0.00	0.72	-0.01	1.00
time (sec)	N/A	0.207	0.024	1.943	0.000	69.455	0.000	0.482	0.000	0.341

Problem 1572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	129	625	329	288	0	2368	0	0	682	129
N.S.	1	4.84	2.55	2.23	0.00	18.36	0.00	0.00	5.29	1.00
time (sec)	N/A	1.984	1.428	0.121	0.000	0.826	0.000	0.000	0.050	0.448

Problem 1573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	209	101	120	0	139	0	97	-1	129
N.S.	1	1.62	0.78	0.93	0.00	1.08	0.00	0.75	-0.01	1.00
time (sec)	N/A	0.076	0.090	0.013	0.000	0.394	0.000	0.302	0.000	0.261

Problem 1574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	129	0	0	0	0	0	0	0	-1	129
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.335	0.615	0.156	0.000	0.000	0.000	0.000	0.000	0.479

Problem 1575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	129	0	0	255	0	0	-1	129
N.S.	1	1.00	1.00	0.00	0.00	1.98	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.348	0.107	1.254	0.000	0.439	0.000	0.000	0.000	0.496

Problem 1576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	130	176	75	643	0	1139	0	0	-1	130
N.S.	1	1.35	0.58	4.95	0.00	8.76	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.603	0.072	0.043	0.000	74.883	0.000	0.000	0.000	0.406

Problem 1577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	130	0	0	0	0	0	0	0	-1	130
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	28.980	7.861	0.171	0.000	0.000	0.000	0.000	0.000	3.796

Problem 1578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	130	571	0	0	0	0	0	0	-1	130
N.S.	1	4.39	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.841	0.298	0.321	0.000	0.000	0.000	0.000	0.000	0.458

Problem 1579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	0	137	0	0	118	0	0	-1	130
N.S.	1	0.00	1.05	0.00	0.00	0.91	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.627	0.431	0.293	0.000	0.416	0.000	0.000	0.000	4.640

Problem 1580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	182	170	238	0	0	0	157	-1	127
N.S.	1	1.40	1.31	1.83	0.00	0.00	0.00	1.21	-0.01	0.98
time (sec)	N/A	0.341	0.132	0.022	0.000	0.000	0.000	0.734	0.000	0.352

Problem 1581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	F(-2)	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	131	265	0	581	0	0	0	0	-1	131
N.S.	1	2.02	0.00	4.44	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.614	1.121	28.237	0.000	0.000	0.000	0.000	0.000	0.208

Problem 1582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	128	160	55	0	112	0	78	46	110
N.S.	1	0.98	1.22	0.42	0.00	0.85	0.00	0.60	0.35	0.84
time (sec)	N/A	0.119	0.071	0.325	0.000	1.216	0.000	0.242	1.250	0.274

Problem 1583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	131	611	1848	1009	0	2485	0	0	658	131
N.S.	1	4.66	14.11	7.70	0.00	18.97	0.00	0.00	5.02	1.00
time (sec)	N/A	1.910	8.745	0.033	0.000	0.909	0.000	0.000	0.853	0.325

Problem 1584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	131	0	0	0	0	327	0	0	-1	131
N.S.	1	0.00	0.00	0.00	0.00	2.50	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.591	0.387	2.418	0.000	31.701	0.000	0.000	0.000	0.721

Problem 1585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	131	195	0	1600	0	334	0	0	-1	131
N.S.	1	1.49	0.00	12.21	0.00	2.55	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.183	0.321	4.873	0.000	174.842	0.000	0.000	0.000	1.179

Problem 1586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	C	F	B	F(-1)	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	131	134	0	613	0	292	0	0	-1	131
N.S.	1	1.02	0.00	4.68	0.00	2.23	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.527	0.333	2.796	0.000	2.757	0.000	0.000	0.000	0.349

Problem 1587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	B	C	F	B	F(-1)	B	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	131	323	1192	667	0	2741	0	207	-1	121
N.S.	1	2.47	9.10	5.09	0.00	20.92	0.00	1.58	-0.01	0.92
time (sec)	N/A	0.860	3.317	0.457	0.000	12.098	0.000	0.352	0.000	1.250

Problem 1588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	134	148	0	0	164	0	0	-1	131
N.S.	1	1.02	1.13	0.00	0.00	1.25	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.367	0.365	0.297	0.000	0.431	0.000	0.000	0.000	0.244

Problem 1589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	134	148	0	0	164	0	0	-1	131
N.S.	1	1.02	1.13	0.00	0.00	1.25	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.314	0.357	0.303	0.000	0.435	0.000	0.000	0.000	0.225

Problem 1590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	0	160	0	0	123	0	0	-1	131
N.S.	1	0.00	1.22	0.00	0.00	0.94	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.888	0.388	0.340	0.000	0.434	0.000	0.000	0.000	4.548

Problem 1591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	132	0	0	662	0	395	0	0	-1	132
N.S.	1	0.00	0.00	5.02	0.00	2.99	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.381	0.500	5.439	0.000	10.733	0.000	0.000	0.000	0.193

Problem 1592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	132	0	0	455	0	123	0	0	-1	132
N.S.	1	0.00	0.00	3.45	0.00	0.93	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.521	0.102	1.189	0.000	0.681	0.000	0.000	0.000	0.280

Problem 1593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	132	0	0	0	0	0	0	0	-1	132
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.366	0.653	0.135	0.000	0.000	0.000	0.000	0.000	0.535

Problem 1594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	132	0	0	0	0	93	0	0	-1	132
N.S.	1	0.00	0.00	0.00	0.00	0.70	0.00	0.00	-0.01	1.00
time (sec)	N/A	4.780	1.624	0.308	0.000	0.413	0.000	0.000	0.000	5.449

Problem 1595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	132	161	0	0	0	289	0	0	-1	190
N.S.	1	1.22	0.00	0.00	0.00	2.19	0.00	0.00	-0.01	1.44
time (sec)	N/A	0.721	0.157	0.309	0.000	5.303	0.000	0.000	0.000	0.623

Problem 1596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	88	40	1244	0	124	0	0	-1	133
N.S.	1	0.66	0.30	9.35	0.00	0.93	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.086	0.012	8.079	0.000	0.400	0.000	0.000	0.000	0.241

Problem 1597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	133	0	0	0	0	0	0	0	-1	133
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	26.531	5.342	0.355	0.000	0.000	0.000	0.000	0.000	6.786

Problem 1598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	133	467	430	898	0	2485	0	0	533	133
N.S.	1	3.51	3.23	6.75	0.00	18.68	0.00	0.00	4.01	1.00
time (sec)	N/A	3.083	0.924	0.030	0.000	0.929	0.000	0.000	0.057	0.319

Problem 1599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	133	0	4461	712	0	306	0	0	-1	133
N.S.	1	0.00	33.54	5.35	0.00	2.30	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.004	139.374	0.105	0.000	0.635	0.000	0.000	0.000	1.142

Problem 1600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	133	133	63	0	0	302	0	0	-1	133
N.S.	1	1.00	0.47	0.00	0.00	2.27	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.113	0.059	0.339	0.000	0.438	0.000	0.000	0.000	0.652

Problem 1601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	133	56	78	0	0	488	0	215	-1	133
N.S.	1	0.42	0.59	0.00	0.00	3.67	0.00	1.62	-0.01	1.00
time (sec)	N/A	0.414	0.051	0.344	0.000	90.947	0.000	0.228	0.000	0.477

Problem 1602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	108	0	0	237	0	0	-1	133
N.S.	1	1.00	0.81	0.00	0.00	1.78	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.071	0.122	0.338	0.000	0.432	0.000	0.000	0.000	0.451

Problem 1603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	133	2670	0	0	0	0	0	0	-1	133
N.S.	1	20.08	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	15.784	0.282	0.328	0.000	0.000	0.000	0.000	0.000	0.380

Problem 1604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	F(-1)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	1209	3835	6694	0	0	0	0	-1	133
N.S.	1	9.09	28.83	50.33	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	7.105	8.741	0.099	0.000	0.000	0.000	0.000	0.000	1.626

Problem 1605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	133	0	0	1656	0	384	0	0	-1	133
N.S.	1	0.00	0.00	12.45	0.00	2.89	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.855	0.365	8.290	0.000	100.238	0.000	0.000	0.000	2.679

Problem 1606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	133	0	0	0	0	0	0	0	-1	133
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.149	0.538	0.335	0.000	0.000	0.000	0.000	0.000	13.372

Problem 1607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	B	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	133	241	0	0	0	349	0	0	-1	133
N.S.	1	1.81	0.00	0.00	0.00	2.62	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.887	1.402	4.817	0.000	146.000	0.000	0.000	0.000	1.178

Problem 1608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	133	0	0	0	0	0	0	0	-1	133
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.019	0.407	0.332	0.000	0.000	0.000	0.000	0.000	15.825

Problem 1609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	0	189	0	0	81	0	0	-1	133
N.S.	1	0.00	1.42	0.00	0.00	0.61	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.345	0.234	0.297	0.000	0.888	0.000	0.000	0.000	0.198

Problem 1610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	A	C	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	70	49	47	46	44	230	312	-1	85
N.S.	1	0.53	0.37	0.35	0.35	0.33	1.73	2.35	-0.01	0.64
time (sec)	N/A	0.070	0.033	0.006	0.310	0.405	2.273	1.755	0.000	0.069

Problem 1611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	137	124	612	0	274	0	0	-1	134
N.S.	1	1.02	0.93	4.57	0.00	2.04	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.163	0.180	2.253	0.000	2.271	0.000	0.000	0.000	0.330

Problem 1612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	A	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	134	223	118	931	0	209	0	184	-1	134
N.S.	1	1.66	0.88	6.95	0.00	1.56	0.00	1.37	-0.01	1.00
time (sec)	N/A	0.778	0.201	1.791	0.000	0.411	0.000	0.222	0.000	0.888

Problem 1613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	134	0	0	644	0	391	0	0	-1	134
N.S.	1	0.00	0.00	4.81	0.00	2.92	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.064	0.297	68.464	0.000	96.453	0.000	0.000	0.000	1.219

Problem 1614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	134	143	151	0	0	494	0	0	-1	134
N.S.	1	1.07	1.13	0.00	0.00	3.69	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.192	0.261	0.343	0.000	85.323	0.000	0.000	0.000	0.658

Problem 1615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	134	143	151	0	0	495	0	0	-1	134
N.S.	1	1.07	1.13	0.00	0.00	3.69	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.175	0.162	0.338	0.000	91.961	0.000	0.000	0.000	0.611

Problem 1616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	255	49	0	0	275	0	295	-1	134
N.S.	1	1.90	0.37	0.00	0.00	2.05	0.00	2.20	-0.01	1.00
time (sec)	N/A	0.316	0.024	0.065	0.000	0.435	0.000	0.737	0.000	0.549

Problem 1617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	50	95	152	0	132	0	0	-1	50
N.S.	1	0.37	0.71	1.13	0.00	0.99	0.00	0.00	-0.01	0.37
time (sec)	N/A	0.097	0.217	0.039	0.000	0.468	0.000	0.000	0.000	0.511

Problem 1618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	134	0	0	0	0	444	0	0	-1	134
N.S.	1	0.00	0.00	0.00	0.00	3.31	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.458	0.138	0.310	0.000	4.118	0.000	0.000	0.000	0.397

Problem 1619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	74	118	0	108	350	37	0	39	135
N.S.	1	0.55	0.87	0.00	0.80	2.59	0.27	0.00	0.29	1.00
time (sec)	N/A	0.010	0.073	0.344	0.431	0.424	0.857	0.000	0.967	0.250

Problem 1620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	135	0	0	0	0	0	0	0	-1	135
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	67.841	1.391	0.115	0.000	0.000	0.000	0.000	0.000	4.159

Problem 1621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	135	0	0	1652	0	809	0	0	-1	135
N.S.	1	0.00	0.00	12.24	0.00	5.99	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.361	0.649	26.984	0.000	93.568	0.000	0.000	0.000	1.965

Problem 1622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	156	149	51	0	118	0	88	54	112
N.S.	1	1.16	1.10	0.38	0.00	0.87	0.00	0.65	0.40	0.83
time (sec)	N/A	0.181	0.194	0.313	0.000	1.255	0.000	0.216	1.296	0.301

Problem 1623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F(-2)	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	130	111	0	0	83	0	0	-1	135
N.S.	1	0.96	0.82	0.00	0.00	0.61	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.088	0.126	180.000	0.000	0.411	0.000	0.000	0.000	0.176

Problem 1624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	136	115	67	0	0	207	0	0	-1	136
N.S.	1	0.85	0.49	0.00	0.00	1.52	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.038	0.059	0.349	0.000	0.429	0.000	0.000	0.000	2.415

Problem 1625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	130	44	0	108	181	41	0	-1	136
N.S.	1	0.96	0.32	0.00	0.79	1.33	0.30	0.00	-0.01	1.00
time (sec)	N/A	0.073	0.012	0.329	0.427	0.399	0.939	0.000	0.000	0.232

Problem 1626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	136	109	57	1333	0	128	0	0	-1	136
N.S.	1	0.80	0.42	9.80	0.00	0.94	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.205	0.027	19.822	0.000	0.406	0.000	0.000	0.000	0.267

Problem 1627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	B	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	136	132	123	0	0	511	0	233	-1	136
N.S.	1	0.97	0.90	0.00	0.00	3.76	0.00	1.71	-0.01	1.00
time (sec)	N/A	4.753	4.885	0.353	0.000	152.068	0.000	0.205	0.000	0.508

Problem 1628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	136	163	0	420	0	780	0	0	-1	136
N.S.	1	1.20	0.00	3.09	0.00	5.74	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.466	0.864	7.293	0.000	12.137	0.000	0.000	0.000	0.485

Problem 1629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	136	163	0	488	0	780	0	0	-1	136
N.S.	1	1.20	0.00	3.59	0.00	5.74	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.431	0.100	10.486	0.000	12.430	0.000	0.000	0.000	0.001

Problem 1630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	166	132	251	0	0	0	112	-1	135
N.S.	1	1.22	0.97	1.85	0.00	0.00	0.00	0.82	-0.01	0.99
time (sec)	N/A	0.543	0.208	0.007	0.000	0.000	0.000	0.752	0.000	0.316

Problem 1631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	166	132	251	0	0	0	181	-1	139
N.S.	1	1.22	0.97	1.85	0.00	0.00	0.00	1.33	-0.01	1.02
time (sec)	N/A	0.455	0.165	0.007	0.000	0.000	0.000	1.282	0.000	0.416

Problem 1632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	193	171	569	0	610	0	0	-1	137
N.S.	1	1.41	1.25	4.15	0.00	4.45	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.268	0.118	3.902	0.000	9.965	0.000	0.000	0.000	0.444

Problem 1633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	137	0	0	0	0	0	0	0	-1	137
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.197	0.094	0.313	0.000	0.000	0.000	0.000	0.000	0.291

Problem 1634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	211	211	0	175	122	48	175	64	138
N.S.	1	1.53	1.53	0.00	1.27	0.88	0.35	1.27	0.46	1.00
time (sec)	N/A	0.211	0.080	0.339	0.418	0.435	0.964	0.272	1.026	0.219

Problem 1635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	225	66	800	0	123	0	105	-1	138
N.S.	1	1.63	0.48	5.80	0.00	0.89	0.00	0.76	-0.01	1.00
time (sec)	N/A	0.216	0.028	1.603	0.000	77.586	0.000	0.865	0.000	0.357

Problem 1636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	F(-1)	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	126	169	0	213	0	76	0	160	138
N.S.	1	0.91	1.22	0.00	1.54	0.00	0.55	0.00	1.16	1.00
time (sec)	N/A	0.148	0.128	0.347	0.413	0.000	4.307	0.000	1.353	0.625

Problem 1637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	258	124	1030	0	207	0	149	-1	138
N.S.	1	1.87	0.90	7.46	0.00	1.50	0.00	1.08	-0.01	1.00
time (sec)	N/A	0.225	0.042	2.446	0.000	0.429	0.000	0.632	0.000	0.645

Problem 1638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	138	0	0	2147	0	278	0	0	-1	138
N.S.	1	0.00	0.00	15.56	0.00	2.01	0.00	0.00	-0.01	1.00
time (sec)	N/A	10.096	0.622	4.908	0.000	3.362	0.000	0.000	0.000	3.167

Problem 1639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	138	402	0	929	0	301	0	0	-1	138
N.S.	1	2.91	0.00	6.73	0.00	2.18	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.777	0.411	2.683	0.000	2.453	0.000	0.000	0.000	0.371

Problem 1640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	194	46	0	0	338	0	107	-1	139
N.S.	1	1.40	0.33	0.00	0.00	2.43	0.00	0.77	-0.01	1.00
time (sec)	N/A	0.364	0.026	0.365	0.000	0.420	0.000	0.224	0.000	0.408

Problem 1641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	213	79	0	178	159	48	175	63	139
N.S.	1	1.53	0.57	0.00	1.28	1.14	0.35	1.26	0.45	1.00
time (sec)	N/A	0.237	0.027	0.340	0.418	0.403	1.072	0.129	1.058	0.215

Problem 1642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	139	851	0	591	0	1172	0	0	-1	139
N.S.	1	6.12	0.00	4.25	0.00	8.43	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.851	0.156	8.953	0.000	12.146	0.000	0.000	0.000	12.173

Problem 1643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F(-1)	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	450	247	570	0	286	0	0	-1	139
N.S.	1	3.24	1.78	4.10	0.00	2.06	0.00	0.00	-0.01	1.00
time (sec)	N/A	4.734	4.055	0.045	0.000	0.522	0.000	0.000	0.000	0.309

Problem 1644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F(-2)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	139	0	0	286	0	658	0	0	-1	139
N.S.	1	0.00	0.00	2.06	0.00	4.73	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.328	2.235	3.661	0.000	26.730	0.000	0.000	0.000	2.429

Problem 1645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	248	42	0	0	153	0	103	-1	140
N.S.	1	1.77	0.30	0.00	0.00	1.09	0.00	0.74	-0.01	1.00
time (sec)	N/A	0.381	0.027	0.353	0.000	0.400	0.000	0.445	0.000	0.352

Problem 1646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	103	131	1820	0	102	0	0	-1	140
N.S.	1	0.74	0.94	13.00	0.00	0.73	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.091	0.057	8.349	0.000	0.395	0.000	0.000	0.000	0.232

Problem 1647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	94	103	113	124	196	60	0	100	140
N.S.	1	0.67	0.74	0.81	0.89	1.40	0.43	0.00	0.71	1.00
time (sec)	N/A	0.069	0.063	0.337	0.411	0.781	2.417	0.000	1.241	3.581

Problem 1648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	140	145	135	889	0	266	0	0	-1	140
N.S.	1	1.04	0.96	6.35	0.00	1.90	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.142	0.155	2.704	0.000	2.854	0.000	0.000	0.000	0.331

Problem 1649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	140	228	102	1587	0	288	0	97	-1	140
N.S.	1	1.63	0.73	11.34	0.00	2.06	0.00	0.69	-0.01	1.00
time (sec)	N/A	0.385	0.092	8.745	0.000	2.111	0.000	0.179	0.000	0.413

Problem 1650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	140	181	68	284	0	363	0	0	-1	150
N.S.	1	1.29	0.49	2.03	0.00	2.59	0.00	0.00	-0.01	1.07
time (sec)	N/A	0.562	0.046	0.046	0.000	0.527	0.000	0.000	0.000	0.449

Problem 1651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	140	176	62	400	0	344	0	0	-1	150
N.S.	1	1.26	0.44	2.86	0.00	2.46	0.00	0.00	-0.01	1.07
time (sec)	N/A	0.222	0.048	0.042	0.000	0.526	0.000	0.000	0.000	0.416

Problem 1652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	181	121	601	0	363	0	0	-1	150
N.S.	1	1.29	0.86	4.29	0.00	2.59	0.00	0.00	-0.01	1.07
time (sec)	N/A	0.440	1.026	0.054	0.000	0.529	0.000	0.000	0.000	0.419

Problem 1653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-1)	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	140	0	0	0	0	399	0	0	-1	140
N.S.	1	0.00	0.00	0.00	0.00	2.85	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.295	0.334	180.000	0.000	174.901	0.000	0.000	0.000	3.571

Problem 1654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	140	0	0	0	0	0	0	0	-1	144
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.03
time (sec)	N/A	0.790	0.601	0.336	0.000	0.000	0.000	0.000	0.000	2.791

Problem 1655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	140	0	687	172	0	165	0	0	-1	140
N.S.	1	0.00	4.91	1.23	0.00	1.18	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.139	3.156	0.982	0.000	0.495	0.000	0.000	0.000	0.503

Problem 1656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	0	139	0	0	128	0	0	-1	140
N.S.	1	0.00	0.99	0.00	0.00	0.91	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.726	0.498	0.309	0.000	0.403	0.000	0.000	0.000	4.920

Problem 1657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	103	129	2698	0	102	0	0	-1	141
N.S.	1	0.73	0.91	19.13	0.00	0.72	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.089	0.069	8.146	0.000	0.389	0.000	0.000	0.000	0.306

Problem 1658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	214	48	361	0	161	0	0	-1	141
N.S.	1	1.52	0.34	2.56	0.00	1.14	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.202	0.020	0.426	0.000	0.398	0.000	0.000	0.000	5.100

Problem 1659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	191	58	343	0	151	0	0	-1	152
N.S.	1	1.35	0.41	2.43	0.00	1.07	0.00	0.00	-0.01	1.08
time (sec)	N/A	0.196	0.019	0.415	0.000	0.406	0.000	0.000	0.000	5.277

Problem 1660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	141	0	0	695	0	380	0	0	-1	141
N.S.	1	0.00	0.00	4.93	0.00	2.70	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.761	0.322	4.488	0.000	10.652	0.000	0.000	0.000	0.505

Problem 1661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	141	0	0	0	0	0	0	0	-1	141
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	29.434	4.262	0.134	0.000	0.000	0.000	0.000	0.000	3.981

Problem 1662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	141	141	67	0	0	318	0	0	-1	141
N.S.	1	1.00	0.48	0.00	0.00	2.26	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.127	0.065	0.356	0.000	0.441	0.000	0.000	0.000	0.809

Problem 1663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	265	161	0	0	0	0	377	-1	141
N.S.	1	1.88	1.14	0.00	0.00	0.00	0.00	2.67	-0.01	1.00
time (sec)	N/A	0.346	0.182	0.361	0.000	0.000	0.000	0.233	0.000	0.425

Problem 1664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	128	0	0	241	0	0	-1	141
N.S.	1	1.00	0.91	0.00	0.00	1.71	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.111	0.119	0.353	0.000	0.435	0.000	0.000	0.000	0.590

Problem 1665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	116	0	0	253	0	0	-1	141
N.S.	1	1.00	0.82	0.00	0.00	1.79	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.076	0.086	0.351	0.000	0.433	0.000	0.000	0.000	0.445

Problem 1666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	141	0	0	321	0	852	0	0	-1	141
N.S.	1	0.00	0.00	2.28	0.00	6.04	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.735	1.029	9.945	0.000	10.153	0.000	0.000	0.000	2.879

Problem 1667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	141	0	0	0	0	0	0	0	-1	141
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.239	0.710	0.369	0.000	0.000	0.000	0.000	0.000	15.649

Problem 1668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	A	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	141	123	99	0	0	519	0	177	-1	141
N.S.	1	0.87	0.70	0.00	0.00	3.68	0.00	1.26	-0.01	1.00
time (sec)	N/A	0.370	0.142	2.136	0.000	32.413	0.000	0.196	0.000	0.497

Problem 1669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	180	180	0	0	216	0	0	-1	141
N.S.	1	1.28	1.28	0.00	0.00	1.53	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.406	0.119	0.319	0.000	0.415	0.000	0.000	0.000	0.180

Problem 1670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	142	242	103	1861	0	263	0	90	-1	142
N.S.	1	1.70	0.73	13.11	0.00	1.85	0.00	0.63	-0.01	1.00
time (sec)	N/A	0.420	4.232	9.390	0.000	1.626	0.000	0.227	0.000	0.384

Problem 1671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	F(-1)	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	151	74	0	133	0	114	0	66	142
N.S.	1	1.06	0.52	0.00	0.94	0.00	0.80	0.00	0.46	1.00
time (sec)	N/A	0.098	0.065	0.354	0.422	0.000	2.628	0.000	1.625	0.249

Problem 1672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	142	539	0	0	0	0	0	0	-1	142
N.S.	1	3.80	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.844	0.251	0.355	0.000	0.000	0.000	0.000	0.000	0.465

Problem 1673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	142	0	0	0	0	0	0	0	-1	142
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.456	0.862	0.365	0.000	0.000	0.000	0.000	0.000	13.175

Problem 1674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	142	0	0	0	0	344	0	0	-1	142
N.S.	1	0.00	0.00	0.00	0.00	2.42	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.934	0.359	2.206	0.000	51.277	0.000	0.000	0.000	1.492

Problem 1675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	142	0	0	0	0	0	0	0	-1	142
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.283	0.590	0.333	0.000	0.000	0.000	0.000	0.000	0.356

Problem 1676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	143	148	128	843	0	281	0	0	-1	143
N.S.	1	1.03	0.90	5.90	0.00	1.97	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.171	0.279	2.840	0.000	2.385	0.000	0.000	0.000	0.344

Problem 1677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-1)	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	143	0	0	0	0	0	0	0	-1	143
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.428	0.415	180.000	0.000	0.000	0.000	0.000	0.000	0.329

Problem 1678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	B	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	0	121	253	0	174	0	0	-1	143
N.S.	1	0.00	0.85	1.77	0.00	1.22	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.117	0.139	0.030	0.000	0.398	0.000	0.000	0.000	9.269

Problem 1679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	143	0	0	669	0	404	0	0	-1	143
N.S.	1	0.00	0.00	4.68	0.00	2.83	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.168	0.329	85.356	0.000	104.811	0.000	0.000	0.000	1.176

Problem 1680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	B	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	143	132	132	0	0	520	0	241	-1	143
N.S.	1	0.92	0.92	0.00	0.00	3.64	0.00	1.69	-0.01	1.00
time (sec)	N/A	0.571	0.358	0.361	0.000	152.898	0.000	0.215	0.000	0.533

Problem 1681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	143	277	139	628	0	276	0	0	-1	143
N.S.	1	1.94	0.97	4.39	0.00	1.93	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.418	0.268	2.615	0.000	2.547	0.000	0.000	0.000	0.370

Problem 1682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	F	B	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	0	131	931	0	271	0	0	-1	143
N.S.	1	0.00	0.92	6.51	0.00	1.90	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.710	0.238	2.450	0.000	2.532	0.000	0.000	0.000	0.374

Problem 1683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	0	126	0	0	64	0	0	-1	143
N.S.	1	0.00	0.88	0.00	0.00	0.45	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.208	0.380	0.340	0.000	1.160	0.000	0.000	0.000	0.431

Problem 1684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	144	141	140	564	0	267	0	0	-1	144
N.S.	1	0.98	0.97	3.92	0.00	1.85	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.611	0.194	2.496	0.000	2.798	0.000	0.000	0.000	0.362

Problem 1685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	303	126	113	0	168	0	0	-1	144
N.S.	1	2.10	0.88	0.78	0.00	1.17	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.232	0.068	0.023	0.000	0.412	0.000	0.000	0.000	1.360

Problem 1686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	F	F	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	144	351	75	0	0	355	0	162	-1	144
N.S.	1	2.44	0.52	0.00	0.00	2.47	0.00	1.12	-0.01	1.00
time (sec)	N/A	0.401	0.042	0.359	0.000	125.455	0.000	0.207	0.000	0.479

Problem 1687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	144	0	0	1664	0	0	0	0	-1	144
N.S.	1	0.00	0.00	11.56	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.596	0.416	60.156	0.000	0.000	0.000	0.000	0.000	18.133

Problem 1688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	C	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	0	143	31	0	282	46	0	-1	112
N.S.	1	0.00	0.99	0.22	0.00	1.96	0.32	0.00	-0.01	0.78
time (sec)	N/A	0.163	0.400	0.037	0.000	44.197	0.993	0.000	0.000	0.270

Problem 1689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	218	211	0	175	122	48	175	64	144
N.S.	1	1.50	1.46	0.00	1.21	0.84	0.33	1.21	0.44	0.99
time (sec)	N/A	0.232	0.084	0.357	0.429	0.400	0.969	0.475	1.021	0.250

Problem 1690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	218	84	0	178	159	48	175	64	144
N.S.	1	1.50	0.58	0.00	1.23	1.10	0.33	1.21	0.44	0.99
time (sec)	N/A	0.201	0.026	0.351	0.514	0.403	1.086	0.309	0.994	0.200

Problem 1691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	145	0	0	809	0	433	0	0	-1	145
N.S.	1	0.00	0.00	5.58	0.00	2.99	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.410	0.397	68.529	0.000	126.148	0.000	0.000	0.000	3.527

Problem 1692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	218	84	0	178	158	46	175	64	144
N.S.	1	1.50	0.58	0.00	1.23	1.09	0.32	1.21	0.44	0.99
time (sec)	N/A	0.211	0.028	0.352	0.411	0.410	1.062	0.296	1.023	0.192

Problem 1693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	145	0	0	0	0	0	0	0	-1	145
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	37.229	2.587	0.096	0.000	0.000	0.000	0.000	0.000	0.513

Problem 1694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	218	84	0	178	159	48	175	64	144
N.S.	1	1.50	0.58	0.00	1.23	1.10	0.33	1.21	0.44	0.99
time (sec)	N/A	0.209	0.029	0.395	0.420	0.409	1.079	0.144	1.230	0.188

Problem 1695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	145	143	138	826	0	277	0	0	-1	145
N.S.	1	0.99	0.95	5.70	0.00	1.91	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.346	0.181	2.605	0.000	2.957	0.000	0.000	0.000	0.347

Problem 1696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	145	0	0	1188	0	478	0	0	-1	145
N.S.	1	0.00	0.00	8.19	0.00	3.30	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.941	0.426	31.115	0.000	43.548	0.000	0.000	0.000	0.553

Problem 1697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	218	84	0	178	159	48	175	64	144
N.S.	1	1.50	0.58	0.00	1.23	1.10	0.33	1.21	0.44	0.99
time (sec)	N/A	0.205	0.028	0.384	0.423	0.417	1.082	0.253	1.021	0.185

Problem 1698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	271	220	98	0	128	0	133	-1	145
N.S.	1	1.87	1.52	0.68	0.00	0.88	0.00	0.92	-0.01	1.00
time (sec)	N/A	0.228	0.150	0.378	0.000	155.525	0.000	0.246	0.000	0.483

Problem 1699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	0	231	0	0	0	0	0	-1	206
N.S.	1	0.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.01	1.42
time (sec)	N/A	0.500	1.130	0.351	0.000	0.000	0.000	0.000	0.000	3.756

Problem 1700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	246	37	0	0	377	0	100	-1	146
N.S.	1	1.68	0.25	0.00	0.00	2.58	0.00	0.68	-0.01	1.00
time (sec)	N/A	0.341	0.022	0.391	0.000	0.442	0.000	0.225	0.000	0.377

Problem 1701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	146	0	0	698	0	204	0	0	-1	146
N.S.	1	0.00	0.00	4.78	0.00	1.40	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.682	1.246	3.805	0.000	3.234	0.000	0.000	0.000	0.227

Problem 1702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	146	0	0	668	0	410	0	0	-1	146
N.S.	1	0.00	0.00	4.58	0.00	2.81	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.440	0.342	103.809	0.000	108.406	0.000	0.000	0.000	3.588

Problem 1703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	146	0	0	1306	0	198	0	0	-1	146
N.S.	1	0.00	0.00	8.95	0.00	1.36	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.225	1.120	9.221	0.000	3.050	0.000	0.000	0.000	4.921

Problem 1704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	146	0	0	0	0	0	0	0	-1	146
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.376	0.513	0.380	0.000	0.000	0.000	0.000	0.000	1.815

Problem 1705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	B	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	146	0	808	0	0	516	0	0	-1	146
N.S.	1	0.00	5.53	0.00	0.00	3.53	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.076	2.718	0.366	0.000	22.967	0.000	0.000	0.000	2.103

Problem 1706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	192	41	0	0	185	0	99	-1	147
N.S.	1	1.31	0.28	0.00	0.00	1.26	0.00	0.67	-0.01	1.00
time (sec)	N/A	0.364	0.020	0.385	0.000	0.398	0.000	0.223	0.000	0.400

Problem 1707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F(-1)	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	147	284	133	710	0	1553	0	0	-1	147
N.S.	1	1.93	0.90	4.83	0.00	10.56	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.712	0.514	0.051	0.000	0.822	0.000	0.000	0.000	0.442

Problem 1708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F(-1)	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	147	284	133	710	0	1557	0	0	-1	147
N.S.	1	1.93	0.90	4.83	0.00	10.59	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.505	0.429	0.046	0.000	0.821	0.000	0.000	0.000	0.429

Problem 1709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	147	1003	0	0	0	0	0	0	-1	147
N.S.	1	6.82	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.837	0.672	0.403	0.000	0.000	0.000	0.000	0.000	2.207

Problem 1710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	147	0	0	319	0	103	0	0	-1	147
N.S.	1	0.00	0.00	2.17	0.00	0.70	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.235	0.208	10.546	0.000	3.213	0.000	0.000	0.000	10.228

Problem 1711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	200	102	0	0	226	0	0	-1	135
N.S.	1	1.36	0.69	0.00	0.00	1.54	0.00	0.00	-0.01	0.92
time (sec)	N/A	0.143	0.233	0.651	0.000	0.423	0.000	0.000	0.000	0.291

Problem 1712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F(-2)	F	B	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	147	147	0	0	0	268	0	0	-1	0
N.S.	1	1.00	0.00	0.00	0.00	1.82	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.234	1.521	180.000	0.000	0.432	0.000	0.000	0.000	16.295

Problem 1713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	239	237	0	0	334	0	296	111	148
N.S.	1	1.61	1.60	0.00	0.00	2.26	0.00	2.00	0.75	1.00
time (sec)	N/A	0.439	0.279	0.081	0.000	0.428	0.000	1.256	1.846	0.819

Problem 1714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	162	135	485	0	303	0	0	-1	149
N.S.	1	1.09	0.91	3.26	0.00	2.03	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.204	0.273	2.626	0.000	3.112	0.000	0.000	0.000	0.380

Problem 1715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	149	158	130	958	0	279	0	0	-1	149
N.S.	1	1.06	0.87	6.43	0.00	1.87	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.179	0.270	2.743	0.000	2.744	0.000	0.000	0.000	0.363

Problem 1716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	149	0	0	2146	0	279	0	0	-1	149
N.S.	1	0.00	0.00	14.40	0.00	1.87	0.00	0.00	-0.01	1.00
time (sec)	N/A	7.565	0.622	5.121	0.000	3.946	0.000	0.000	0.000	0.384

Problem 1717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	149	187	93	448	0	426	0	0	-1	149
N.S.	1	1.26	0.62	3.01	0.00	2.86	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.345	0.692	0.050	0.000	0.530	0.000	0.000	0.000	0.427

Problem 1718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	149	253	97	0	0	0	0	272	-1	149
N.S.	1	1.70	0.65	0.00	0.00	0.00	0.00	1.83	-0.01	1.00
time (sec)	N/A	1.158	5.221	0.408	0.000	0.000	0.000	0.300	0.000	0.904

Problem 1719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	149	0	0	2335	0	0	0	0	-1	149
N.S.	1	0.00	0.00	15.67	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.590	0.719	9.677	0.000	0.000	0.000	0.000	0.000	2.826

Problem 1720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	0	212	0	0	0	0	0	-1	81
N.S.	1	0.00	1.42	0.00	0.00	0.00	0.00	0.00	-0.01	0.54
time (sec)	N/A	0.206	0.430	0.366	0.000	0.000	0.000	0.000	0.000	0.222

Problem 1721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	149	0	0	0	0	0	0	0	-1	149
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.578	0.537	0.363	0.000	0.000	0.000	0.000	0.000	0.326

Problem 1722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	225	40	0	181	204	44	196	69	149
N.S.	1	1.50	0.27	0.00	1.21	1.36	0.29	1.31	0.46	0.99
time (sec)	N/A	0.229	0.014	0.384	0.408	0.422	1.176	0.172	1.169	0.359

Problem 1723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	150	159	139	631	0	290	0	0	-1	150
N.S.	1	1.06	0.93	4.21	0.00	1.93	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.182	0.173	2.838	0.000	2.595	0.000	0.000	0.000	0.377

Problem 1724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	150	157	129	906	0	278	0	0	-1	150
N.S.	1	1.05	0.86	6.04	0.00	1.85	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.158	0.181	2.671	0.000	2.925	0.000	0.000	0.000	0.343

Problem 1725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	225	40	0	182	198	44	195	69	149
N.S.	1	1.50	0.27	0.00	1.21	1.32	0.29	1.30	0.46	0.99
time (sec)	N/A	0.222	0.013	0.430	0.406	0.411	1.135	0.183	1.184	0.334

Problem 1726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	225	40	0	181	201	44	196	69	149
N.S.	1	1.50	0.27	0.00	1.21	1.34	0.29	1.31	0.46	0.99
time (sec)	N/A	0.212	0.013	0.388	0.406	0.425	1.228	0.319	1.108	0.232

Problem 1727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	150	0	0	0	0	0	0	0	-1	150
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	36.928	7.115	0.237	0.000	0.000	0.000	0.000	0.000	4.719

Problem 1728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	225	40	0	181	204	44	196	69	149
N.S.	1	1.50	0.27	0.00	1.21	1.36	0.29	1.31	0.46	0.99
time (sec)	N/A	0.208	0.014	0.390	0.407	0.407	1.351	0.167	1.200	0.219

Problem 1729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	225	40	0	181	201	44	196	69	149
N.S.	1	1.50	0.27	0.00	1.21	1.34	0.29	1.31	0.46	0.99
time (sec)	N/A	0.198	0.012	0.388	0.514	0.419	1.428	0.294	1.240	0.203

Problem 1730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-1)	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	150	0	0	0	0	390	0	0	-1	150
N.S.	1	0.00	0.00	0.00	0.00	2.60	0.00	0.00	-0.01	1.00
time (sec)	N/A	14.568	3.154	180.000	0.000	7.939	0.000	0.000	0.000	0.414

Problem 1731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	B	F	F(-1)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	400	192	289	0	0	0	0	-1	156
N.S.	1	2.67	1.28	1.93	0.00	0.00	0.00	0.00	-0.01	1.04
time (sec)	N/A	0.732	0.868	0.050	0.000	0.000	0.000	0.000	0.000	0.632

Problem 1732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	F(-1)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	400	192	281	0	0	0	0	-1	156
N.S.	1	2.67	1.28	1.87	0.00	0.00	0.00	0.00	-0.01	1.04
time (sec)	N/A	0.648	0.599	0.062	0.000	0.000	0.000	0.000	0.000	0.708

Problem 1733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	0	156	0	0	138	0	0	-1	150
N.S.	1	0.00	1.04	0.00	0.00	0.92	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.761	0.572	0.365	0.000	0.576	0.000	0.000	0.000	5.123

Problem 1734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	230	91	0	195	173	80	189	78	150
N.S.	1	1.52	0.60	0.00	1.29	1.15	0.53	1.25	0.52	0.99
time (sec)	N/A	0.243	0.057	0.402	0.410	0.756	10.019	0.256	1.235	0.240

Problem 1735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	151	0	0	661	0	420	0	0	-1	151
N.S.	1	0.00	0.00	4.38	0.00	2.78	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.879	0.399	4.928	0.000	15.178	0.000	0.000	0.000	0.475

Problem 1736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F(-1)	A	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	151	0	0	0	0	338	0	235	-1	151
N.S.	1	0.00	0.00	0.00	0.00	2.24	0.00	1.56	-0.01	1.00
time (sec)	N/A	2.333	0.470	0.396	0.000	0.413	0.000	0.273	0.000	0.522

Problem 1737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	151	487	10871	255	0	1197	0	0	-1	151
N.S.	1	3.23	71.99	1.69	0.00	7.93	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.844	14.605	0.067	0.000	0.550	0.000	0.000	0.000	0.347

Problem 1738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	151	97	0	2918	0	2372	0	0	-1	151
N.S.	1	0.64	0.00	19.32	0.00	15.71	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.663	0.851	53.884	0.000	23.566	0.000	0.000	0.000	0.549

Problem 1739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	F	B	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	0	138	939	0	297	0	0	-1	151
N.S.	1	0.00	0.91	6.22	0.00	1.97	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.858	0.503	2.378	0.000	2.828	0.000	0.000	0.000	0.461

Problem 1740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	F	B	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	151	0	146	572	0	294	0	0	-1	151
N.S.	1	0.00	0.97	3.79	0.00	1.95	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.879	0.592	2.450	0.000	2.920	0.000	0.000	0.000	0.455

Problem 1741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	151	0	0	0	0	0	0	0	-1	151
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	4.373	1.102	0.417	0.000	0.000	0.000	0.000	0.000	15.654

Problem 1742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	122	122	0	0	0	0	288	-1	151
N.S.	1	0.81	0.81	0.00	0.00	0.00	0.00	1.91	-0.01	1.00
time (sec)	N/A	0.391	0.220	0.033	0.000	0.000	0.000	0.198	0.000	0.369

Problem 1743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	151	0	0	0	0	236	0	0	-1	151
N.S.	1	0.00	0.00	0.00	0.00	1.56	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.245	0.292	180.000	0.000	2.442	0.000	0.000	0.000	0.828

Problem 1744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	A	F	F(-2)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	151	0	0	0	0	236	0	0	-1	151
N.S.	1	0.00	0.00	0.00	0.00	1.56	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.180	0.088	180.000	0.000	2.211	0.000	0.000	0.000	0.743

Problem 1745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	152	172	58	0	0	0	0	0	-1	152
N.S.	1	1.13	0.38	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.375	0.048	0.419	0.000	0.000	0.000	0.000	0.000	0.435

Problem 1746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F(-1)	A	F	C
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	0	646	0	0	156	0	1	-1	78
N.S.	1	0.00	4.25	0.00	0.00	1.03	0.00	0.01	-0.01	0.51
time (sec)	N/A	1.896	1.430	0.399	0.000	0.442	0.000	0.259	0.000	0.591

Problem 1747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	152	0	0	0	0	0	0	0	-1	152
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.229	0.735	0.400	0.000	0.000	0.000	0.000	0.000	1.161

Problem 1748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	152	22	110	0	0	677	0	0	-1	152
N.S.	1	0.14	0.72	0.00	0.00	4.45	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.009	0.123	16.951	0.000	16.980	0.000	0.000	0.000	0.521

Problem 1749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	282	175	1593	0	2047	0	0	-1	152
N.S.	1	1.86	1.15	10.48	0.00	13.47	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.265	0.255	17.879	0.000	13.020	0.000	0.000	0.000	0.498

Problem 1750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	C	F	B	F	F(-2)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	152	282	0	1163	0	1933	0	0	-1	152
N.S.	1	1.86	0.00	7.65	0.00	12.72	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.219	0.051	12.348	0.000	11.994	0.000	0.000	0.000	0.446

Problem 1751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	152	0	0	0	0	520	0	0	-1	152
N.S.	1	0.00	0.00	0.00	0.00	3.42	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.876	0.257	0.376	0.000	20.615	0.000	0.000	0.000	0.491

Problem 1752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	228	38	0	202	212	42	199	72	152
N.S.	1	1.49	0.25	0.00	1.32	1.39	0.27	1.30	0.47	0.99
time (sec)	N/A	0.221	0.013	0.428	0.507	0.409	1.150	0.341	1.255	0.364

Problem 1753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	153	0	0	0	0	0	0	0	-1	153
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	7.485	6.224	0.590	0.000	0.000	0.000	0.000	0.000	0.387

Problem 1754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	228	40	0	202	209	42	199	72	152
N.S.	1	1.49	0.26	0.00	1.32	1.37	0.27	1.30	0.47	0.99
time (sec)	N/A	0.231	0.012	0.414	0.403	0.448	1.210	0.177	1.348	0.186

Problem 1755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	228	40	0	199	214	42	198	72	152
N.S.	1	1.49	0.26	0.00	1.30	1.40	0.27	1.29	0.47	0.99
time (sec)	N/A	0.208	0.009	0.395	0.410	0.422	1.137	0.300	1.255	0.340

Problem 1756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	228	37	0	202	212	41	199	72	152
N.S.	1	1.49	0.24	0.00	1.32	1.39	0.27	1.30	0.47	0.99
time (sec)	N/A	0.223	0.014	0.444	0.640	0.421	1.243	0.176	1.301	0.180

Problem 1757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	153	0	0	0	0	0	0	0	-1	153
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	19.216	2.608	0.105	0.000	0.000	0.000	0.000	0.000	0.478

Problem 1758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	228	40	0	202	212	42	199	72	152
N.S.	1	1.49	0.26	0.00	1.32	1.39	0.27	1.30	0.47	0.99
time (sec)	N/A	0.221	0.011	0.453	0.582	0.456	1.342	0.323	1.306	0.180

Problem 1759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	228	40	0	202	209	42	199	72	152
N.S.	1	1.49	0.26	0.00	1.32	1.37	0.27	1.30	0.47	0.99
time (sec)	N/A	0.212	0.011	0.451	0.426	0.442	1.409	0.178	1.324	0.184

Problem 1760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	340	268	825	0	127	0	0	-1	71
N.S.	1	2.22	1.75	5.39	0.00	0.83	0.00	0.00	-0.01	0.46
time (sec)	N/A	3.931	0.706	0.052	0.000	0.552	0.000	0.000	0.000	1.397

Problem 1761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	0	150	0	0	153	0	0	-1	153
N.S.	1	0.00	0.98	0.00	0.00	1.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.374	0.735	0.398	0.000	3.606	0.000	0.000	0.000	2.423

Problem 1762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	153	0	0	0	0	499	0	0	-1	153
N.S.	1	0.00	0.00	0.00	0.00	3.26	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.787	0.190	0.376	0.000	10.953	0.000	0.000	0.000	0.273

Problem 1763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	153	0	0	0	0	0	0	0	-1	120
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.78
time (sec)	N/A	1.028	0.626	0.382	0.000	0.000	0.000	0.000	0.000	0.255

Problem 1764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	109	217	0	0	66	0	0	-1	153
N.S.	1	0.71	1.42	0.00	0.00	0.43	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.218	1.055	0.020	0.000	0.421	0.000	0.000	0.000	0.266

Problem 1765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	A	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	154	0	0	0	0	0	0	130	-1	154
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.84	-0.01	1.00
time (sec)	N/A	4.799	7.179	0.635	0.000	0.000	0.000	0.579	0.000	0.391

Problem 1766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	154	195	88	617	0	1180	0	0	-1	165
N.S.	1	1.27	0.57	4.01	0.00	7.66	0.00	0.00	-0.01	1.07
time (sec)	N/A	0.746	0.059	0.046	0.000	71.914	0.000	0.000	0.000	0.447

Problem 1767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	154	0	0	725	0	413	0	0	-1	154
N.S.	1	0.00	0.00	4.71	0.00	2.68	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.820	0.334	51.398	0.000	71.808	0.000	0.000	0.000	0.780

Problem 1768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	154	0	0	0	0	0	0	0	-1	154
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.188	0.565	0.411	0.000	0.000	0.000	0.000	0.000	1.371

Problem 1769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	154	0	0	7488	0	1749	0	0	-1	154
N.S.	1	0.00	0.00	48.62	0.00	11.36	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.215	1.022	33.326	0.000	3.065	0.000	0.000	0.000	0.546

Problem 1770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	157	94	987	0	724	0	221	-1	154
N.S.	1	1.02	0.61	6.41	0.00	4.70	0.00	1.44	-0.01	1.00
time (sec)	N/A	0.971	0.209	3.552	0.000	5.784	0.000	0.173	0.000	0.422

Problem 1771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	212	157	0	0	1214	0	0	-1	154
N.S.	1	1.38	1.02	0.00	0.00	7.88	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.845	0.305	0.417	0.000	0.505	0.000	0.000	0.000	20.248

Problem 1772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-1)	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	154	0	0	0	0	0	0	0	-1	154
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	10.597	1.074	180.000	0.000	0.000	0.000	0.000	0.000	3.022

Problem 1773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	154	0	0	0	0	0	0	0	-1	154
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.725	0.657	0.409	0.000	0.000	0.000	0.000	0.000	3.055

Problem 1774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	154	0	0	0	0	115	0	0	-1	154
N.S.	1	0.00	0.00	0.00	0.00	0.75	0.00	0.00	-0.01	1.00
time (sec)	N/A	4.768	1.774	0.413	0.000	0.401	0.000	0.000	0.000	5.571

Problem 1775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	208	482	0	0	84	0	0	-1	154
N.S.	1	1.35	3.13	0.00	0.00	0.55	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.723	2.075	0.378	0.000	0.396	0.000	0.000	0.000	0.236

Problem 1776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	A	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	155	0	0	0	0	0	0	129	-1	155
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.83	-0.01	1.00
time (sec)	N/A	4.430	4.453	0.544	0.000	0.000	0.000	0.592	0.000	0.292

Problem 1777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	155	232	129	0	0	0	0	0	-1	155
N.S.	1	1.50	0.83	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.703	0.334	0.426	0.000	0.000	0.000	0.000	0.000	0.485

Problem 1778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	156	0	0	1291	0	212	0	0	-1	156
N.S.	1	0.00	0.00	8.28	0.00	1.36	0.00	0.00	-0.01	1.00
time (sec)	N/A	9.805	0.659	1.625	0.000	2.763	0.000	0.000	0.000	0.949

Problem 1779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	156	320	0	741	0	953	0	0	-1	156
N.S.	1	2.05	0.00	4.75	0.00	6.11	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.745	1.444	13.233	0.000	5.599	0.000	0.000	0.000	0.532

Problem 1780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	157	176	71	941	0	291	0	0	-1	157
N.S.	1	1.12	0.45	5.99	0.00	1.85	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.207	0.097	2.622	0.000	2.054	0.000	0.000	0.000	0.352

Problem 1781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	157	46	48	0	0	2094	0	0	-1	157
N.S.	1	0.29	0.31	0.00	0.00	13.34	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.115	0.025	4.640	0.000	2.859	0.000	0.000	0.000	0.432

Problem 1782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F(-1)	A	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	157	0	0	0	0	158	0	1	-1	157
N.S.	1	0.00	0.00	0.00	0.00	1.01	0.00	0.01	-0.01	1.00
time (sec)	N/A	2.218	0.521	0.401	0.000	0.402	0.000	0.498	0.000	0.550

Problem 1783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	157	269	114	0	0	0	0	279	-1	157
N.S.	1	1.71	0.73	0.00	0.00	0.00	0.00	1.78	-0.01	1.00
time (sec)	N/A	1.192	5.182	0.422	0.000	0.000	0.000	0.321	0.000	0.914

Problem 1784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	A	F	F(-2)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	157	0	0	0	0	236	0	0	-1	157
N.S.	1	0.00	0.00	0.00	0.00	1.50	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.188	0.100	180.000	0.000	1.759	0.000	0.000	0.000	0.548

Problem 1785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F(-2)	F	B	F	F(-1)	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	0	74	0	0	291	0	0	99	157
N.S.	1	0.00	0.47	0.00	0.00	1.85	0.00	0.00	0.63	1.00
time (sec)	N/A	1.256	0.258	180.000	0.000	0.560	0.000	0.000	1.977	0.393

Problem 1786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	158	0	2491	0	0	0	0	0	-1	158
N.S.	1	0.00	15.77	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	28.415	12.482	0.434	0.000	0.000	0.000	0.000	0.000	1.084

Problem 1787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	158	467	251	862	0	75	0	0	-1	79
N.S.	1	2.96	1.59	5.46	0.00	0.47	0.00	0.00	-0.01	0.50
time (sec)	N/A	1.472	0.943	0.286	0.000	0.531	0.000	0.000	0.000	0.474

Problem 1788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F(-1)	F(-2)	F	C
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	0	625	0	0	330	0	0	-1	72
N.S.	1	0.00	3.96	0.00	0.00	2.09	0.00	0.00	-0.01	0.46
time (sec)	N/A	1.785	1.197	0.406	0.000	0.465	0.000	0.000	0.000	0.461

Problem 1789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	182	41	714	0	163	0	98	-1	159
N.S.	1	1.14	0.26	4.49	0.00	1.03	0.00	0.62	-0.01	1.00
time (sec)	N/A	0.079	0.014	2.906	0.000	0.417	0.000	0.205	0.000	0.417

Problem 1790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	880	396	375	0	401	0	0	165	159
N.S.	1	5.53	2.49	2.36	0.00	2.52	0.00	0.00	1.04	1.00
time (sec)	N/A	3.221	1.492	0.092	0.000	0.680	0.000	0.000	4.806	0.974

Problem 1791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	159	0	0	0	0	0	0	0	-1	159
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	5.499	0.671	0.077	0.000	0.000	0.000	0.000	0.000	0.740

Problem 1792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	159	0	0	0	0	0	0	0	-1	159
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	6.060	0.700	0.079	0.000	0.000	0.000	0.000	0.000	0.783

Problem 1793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	148	148	0	0	1028	0	0	-1	160
N.S.	1	0.92	0.92	0.00	0.00	6.42	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.030	0.257	0.121	0.000	127.129	0.000	0.000	0.000	0.889

Problem 1794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	A	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	160	0	0	0	0	0	0	122	-1	160
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.76	-0.01	1.00
time (sec)	N/A	3.879	7.325	0.563	0.000	0.000	0.000	0.526	0.000	0.354

Problem 1795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	161	0	0	0	0	0	0	0	-1	161
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.632	1.603	0.406	0.000	0.000	0.000	0.000	0.000	0.252

Problem 1796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	A	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	161	0	0	0	0	0	0	121	-1	161
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.75	-0.01	1.00
time (sec)	N/A	3.495	6.571	0.486	0.000	0.000	0.000	0.295	0.000	0.286

Problem 1797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	161	0	0	1001	0	438	0	0	-1	161
N.S.	1	0.00	0.00	6.22	0.00	2.72	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.593	0.377	4.815	0.000	18.751	0.000	0.000	0.000	0.439

Problem 1798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	161	257	136	681	0	1593	0	0	-1	159
N.S.	1	1.60	0.84	4.23	0.00	9.89	0.00	0.00	-0.01	0.99
time (sec)	N/A	1.696	0.596	0.055	0.000	0.856	0.000	0.000	0.000	0.522

Problem 1799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	161	0	0	1065	0	1858	0	0	-1	161
N.S.	1	0.00	0.00	6.61	0.00	11.54	0.00	0.00	-0.01	1.00
time (sec)	N/A	6.126	0.561	7.597	0.000	46.389	0.000	0.000	0.000	0.519

Problem 1800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	142	53	0	152	352	126	0	184	162
N.S.	1	0.88	0.33	0.00	0.94	2.17	0.78	0.00	1.14	1.00
time (sec)	N/A	0.180	0.055	0.416	0.453	83.141	2.700	0.000	1.784	9.104

Problem 1801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	F	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	392	167	0	0	810	0	350	-1	162
N.S.	1	2.42	1.03	0.00	0.00	5.00	0.00	2.16	-0.01	1.00
time (sec)	N/A	0.698	0.124	0.084	0.000	0.451	0.000	0.501	0.000	0.676

Problem 1802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	162	0	0	0	0	0	0	0	-1	162
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	23.783	1.274	0.062	0.000	0.000	0.000	0.000	0.000	6.395

Problem 1803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-1)	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	162	0	0	0	0	1060	0	0	-1	167
N.S.	1	0.00	0.00	0.00	0.00	6.54	0.00	0.00	-0.01	1.03
time (sec)	N/A	1.531	0.515	180.000	0.000	88.790	0.000	0.000	0.000	2.967

Problem 1804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	162	46	44	631	0	712	0	0	-1	162
N.S.	1	0.28	0.27	3.90	0.00	4.40	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.138	0.028	22.355	0.000	13.727	0.000	0.000	0.000	0.514

Problem 1805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	162	46	44	632	0	712	0	0	-1	162
N.S.	1	0.28	0.27	3.90	0.00	4.40	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.132	0.016	22.575	0.000	13.513	0.000	0.000	0.000	0.001

Problem 1806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	162	42	42	636	0	1002	0	0	-1	162
N.S.	1	0.26	0.26	3.93	0.00	6.19	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.094	0.019	21.050	0.000	49.883	0.000	0.000	0.000	0.512

Problem 1807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	162	42	42	633	0	1002	0	0	-1	162
N.S.	1	0.26	0.26	3.91	0.00	6.19	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.099	0.011	21.343	0.000	50.469	0.000	0.000	0.000	0.001

Problem 1808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	162	127	95	627	0	1002	0	0	-1	162
N.S.	1	0.78	0.59	3.87	0.00	6.19	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.772	0.284	23.247	0.000	99.264	0.000	0.000	0.000	0.780

Problem 1809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	162	127	95	633	0	1002	0	0	-1	162
N.S.	1	0.78	0.59	3.91	0.00	6.19	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.623	0.181	23.584	0.000	98.759	0.000	0.000	0.000	0.002

Problem 1810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	162	127	95	633	0	1002	0	0	-1	162
N.S.	1	0.78	0.59	3.91	0.00	6.19	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.642	0.197	23.740	0.000	97.404	0.000	0.000	0.000	0.779

Problem 1811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	162	127	95	636	0	1002	0	0	-1	162
N.S.	1	0.78	0.59	3.93	0.00	6.19	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.575	0.195	23.082	0.000	97.634	0.000	0.000	0.000	0.001

Problem 1812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	C	F	B	F(-1)	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	162	157	0	910	0	291	0	0	-1	162
N.S.	1	0.97	0.00	5.62	0.00	1.80	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.596	0.335	3.050	0.000	2.705	0.000	0.000	0.000	0.447

Problem 1813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	162	105	0	2866	0	1273	0	0	-1	162
N.S.	1	0.65	0.00	17.69	0.00	7.86	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.385	1.482	114.533	0.000	13.396	0.000	0.000	0.000	0.608

Problem 1814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	162	105	0	2859	0	1273	0	0	-1	162
N.S.	1	0.65	0.00	17.65	0.00	7.86	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.306	1.029	111.254	0.000	13.396	0.000	0.000	0.000	0.001

Problem 1815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	162	97	0	2799	0	2347	0	0	-1	162
N.S.	1	0.60	0.00	17.28	0.00	14.49	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.529	1.321	133.740	0.000	115.495	0.000	0.000	0.000	0.664

Problem 1816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	162	97	0	2801	0	2347	0	0	-1	162
N.S.	1	0.60	0.00	17.29	0.00	14.49	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.444	0.498	129.413	0.000	116.033	0.000	0.000	0.000	0.001

Problem 1817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	162	0	0	0	0	0	0	0	-1	162
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	27.473	0.891	0.059	0.000	0.000	0.000	0.000	0.000	8.210

Problem 1818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	A	C	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	0	81	31	0	280	51	0	-1	130
N.S.	1	0.00	0.50	0.19	0.00	1.73	0.31	0.00	-0.01	0.80
time (sec)	N/A	0.143	0.186	0.039	0.000	96.577	1.115	0.000	0.000	0.275

Problem 1819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	C	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	0	61	0	0	280	49	0	-1	130
N.S.	1	0.00	0.38	0.00	0.00	1.73	0.30	0.00	-0.01	0.80
time (sec)	N/A	0.640	0.202	0.391	0.000	138.229	1.436	0.000	0.000	0.294

Problem 1820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	224	326	0	0	289	0	0	-1	162
N.S.	1	1.37	2.00	0.00	0.00	1.77	0.00	0.00	-0.01	0.99
time (sec)	N/A	0.962	0.340	1.776	0.000	0.422	0.000	0.000	0.000	0.515

Problem 1821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	163	0	0	516	0	119	0	0	-1	163
N.S.	1	0.00	0.00	3.17	0.00	0.73	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.334	0.214	17.455	0.000	3.182	0.000	0.000	0.000	9.850

Problem 1822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	163	0	11852	488	0	485	0	0	-1	163
N.S.	1	0.00	72.71	2.99	0.00	2.98	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.445	7.788	0.319	0.000	1.046	0.000	0.000	0.000	1.091

Problem 1823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	198	0	0	251	0	0	-1	163
N.S.	1	1.00	1.21	0.00	0.00	1.54	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.337	0.345	0.398	0.000	0.615	0.000	0.000	0.000	0.289

Problem 1824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	B	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	0	11560	0	0	168	0	0	-1	163
N.S.	1	0.00	70.92	0.00	0.00	1.03	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.537	21.921	0.398	0.000	15.916	0.000	0.000	0.000	3.161

Problem 1825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	B	F	F	A	F	F(-2)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	0	714	0	0	168	0	0	-1	223
N.S.	1	0.00	4.38	0.00	0.00	1.03	0.00	0.00	-0.01	1.37
time (sec)	N/A	0.428	4.603	0.374	0.000	20.414	0.000	0.000	0.000	3.341

Problem 1826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	164	0	0	743	0	424	0	0	-1	164
N.S.	1	0.00	0.00	4.53	0.00	2.59	0.00	0.00	-0.01	1.00
time (sec)	N/A	4.036	0.391	4.765	0.000	10.309	0.000	0.000	0.000	0.569

Problem 1827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	C	F	B	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	164	166	0	606	0	253	0	0	-1	164
N.S.	1	1.01	0.00	3.70	0.00	1.54	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.241	0.309	3.103	0.000	2.643	0.000	0.000	0.000	0.434

Problem 1828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	165	84	118	1377	0	165	0	0	-1	165
N.S.	1	0.51	0.72	8.35	0.00	1.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.015	0.064	14.234	0.000	2.211	0.000	0.000	0.000	0.204

Problem 1829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	165	0	0	2200	0	406	0	0	-1	165
N.S.	1	0.00	0.00	13.33	0.00	2.46	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.576	0.325	4.313	0.000	13.487	0.000	0.000	0.000	0.515

Problem 1830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	165	325	736	0	0	0	0	0	-1	165
N.S.	1	1.97	4.46	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.338	18.731	0.562	0.000	0.000	0.000	0.000	0.000	0.947

Problem 1831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	165	255	202	0	0	429	0	0	-1	165
N.S.	1	1.55	1.22	0.00	0.00	2.60	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.293	0.294	0.424	0.000	67.712	0.000	0.000	0.000	0.525

Problem 1832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	166	0	0	243	0	0	-1	166
N.S.	1	1.00	1.00	0.00	0.00	1.46	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.983	0.264	0.115	0.000	0.476	0.000	0.000	0.000	1.284

Problem 1833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	166	0	0	0	0	0	0	0	-1	166
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	6.078	10.172	0.576	0.000	0.000	0.000	0.000	0.000	0.849

Problem 1834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	257	42	0	247	224	44	223	95	165
N.S.	1	1.55	0.25	0.00	1.49	1.35	0.27	1.34	0.57	0.99
time (sec)	N/A	0.255	0.012	0.430	0.455	0.489	1.362	0.353	1.390	0.511

Problem 1835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	166	290	156	0	0	0	0	394	-1	166
N.S.	1	1.75	0.94	0.00	0.00	0.00	0.00	2.37	-0.01	1.00
time (sec)	N/A	0.416	0.291	0.416	0.000	0.000	0.000	0.279	0.000	0.587

Problem 1836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	518	204	247	0	979	0	0	202	166
N.S.	1	3.12	1.23	1.49	0.00	5.90	0.00	0.00	1.22	1.00
time (sec)	N/A	1.705	1.480	0.066	0.000	0.667	0.000	0.000	7.933	0.571

Problem 1837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	405	195	441	0	101	0	0	-1	84
N.S.	1	2.44	1.17	2.66	0.00	0.61	0.00	0.00	-0.01	0.51
time (sec)	N/A	2.204	0.711	0.094	0.000	0.598	0.000	0.000	0.000	1.205

Problem 1838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	167	0	0	0	0	0	0	0	-1	167
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	5.304	9.892	0.523	0.000	0.000	0.000	0.000	0.000	0.468

Problem 1839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	260	42	0	250	234	42	224	98	166
N.S.	1	1.56	0.25	0.00	1.50	1.40	0.25	1.34	0.59	0.99
time (sec)	N/A	0.262	0.012	0.437	0.421	0.493	1.688	0.189	1.401	0.282

Problem 1840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	260	42	0	241	240	42	224	98	166
N.S.	1	1.56	0.25	0.00	1.44	1.44	0.25	1.34	0.59	0.99
time (sec)	N/A	0.273	0.009	0.415	0.435	0.479	1.443	0.329	1.319	0.286

Problem 1841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	A	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	167	42	0	2591	0	232	0	0	-1	167
N.S.	1	0.25	0.00	15.51	0.00	1.39	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.099	0.071	25.524	0.000	1.111	0.000	0.000	0.000	0.520

Problem 1842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	167	0	0	808	0	418	0	0	-1	167
N.S.	1	0.00	0.00	4.84	0.00	2.50	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.495	0.330	89.033	0.000	124.258	0.000	0.000	0.000	3.521

Problem 1843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	260	39	0	250	234	41	224	98	166
N.S.	1	1.56	0.23	0.00	1.50	1.40	0.25	1.34	0.59	0.99
time (sec)	N/A	0.249	0.011	0.425	0.419	0.483	1.654	0.271	1.395	0.261

Problem 1844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	167	503	105	567	0	1675	0	0	-1	167
N.S.	1	3.01	0.63	3.40	0.00	10.03	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.745	0.700	0.025	0.000	0.925	0.000	0.000	0.000	0.643

Problem 1845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	260	42	0	250	234	42	224	98	166
N.S.	1	1.56	0.25	0.00	1.50	1.40	0.25	1.34	0.59	0.99
time (sec)	N/A	0.249	0.011	0.431	0.491	0.480	1.878	0.256	1.465	0.208

Problem 1846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	167	0	0	758	0	2054	0	0	-1	167
N.S.	1	0.00	0.00	4.54	0.00	12.30	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.639	0.776	5.162	0.000	6.604	0.000	0.000	0.000	0.542

Problem 1847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	2697	345	447	0	456	0	0	201	167
N.S.	1	16.15	2.07	2.68	0.00	2.73	0.00	0.00	1.20	1.00
time (sec)	N/A	20.798	2.268	0.129	0.000	0.615	0.000	0.000	5.788	0.550

Problem 1848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	0	341	0	0	175	0	0	-1	223
N.S.	1	0.00	2.04	0.00	0.00	1.05	0.00	0.00	-0.01	1.34
time (sec)	N/A	1.777	5.406	0.409	0.000	32.325	0.000	0.000	0.000	3.307

Problem 1849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	167	242	13567	1164	0	354	0	0	-1	167
N.S.	1	1.45	81.24	6.97	0.00	2.12	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.456	6.506	0.316	0.000	1.191	0.000	0.000	0.000	1.194

Problem 1850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	167	0	1550	0	0	105	0	0	-1	167
N.S.	1	0.00	9.28	0.00	0.00	0.63	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.352	7.291	0.400	0.000	0.884	0.000	0.000	0.000	0.592

Problem 1851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F	F	B	F	F(-1)	B	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	167	242	0	0	0	354	0	0	498	167
N.S.	1	1.45	0.00	0.00	0.00	2.12	0.00	0.00	2.98	1.00
time (sec)	N/A	0.473	0.310	0.439	0.000	1.624	0.000	0.000	150.611	2.104

Problem 1852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	168	95	143	1553	0	315	0	0	-1	168
N.S.	1	0.57	0.85	9.24	0.00	1.88	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.018	0.220	7.914	0.000	5.524	0.000	0.000	0.000	0.235

Problem 1853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	168	0	0	0	0	0	0	0	-1	168
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	73.775	7.308	0.456	0.000	0.000	0.000	0.000	0.000	1.318

Problem 1854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	B	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	0	423	0	0	122	0	0	-1	168
N.S.	1	0.00	2.52	0.00	0.00	0.73	0.00	0.00	-0.01	1.00
time (sec)	N/A	13.173	3.792	0.428	0.000	0.486	0.000	0.000	0.000	0.319

Problem 1855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	169	81	126	446	0	345	0	0	-1	0
N.S.	1	0.48	0.75	2.64	0.00	2.04	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.014	0.095	2.945	0.000	1.670	0.000	0.000	0.000	3.979

Problem 1856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	169	44	0	1756	0	1053	0	0	-1	169
N.S.	1	0.26	0.00	10.39	0.00	6.23	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.182	0.113	15.263	0.000	5.671	0.000	0.000	0.000	0.450

Problem 1857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-1)	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	170	0	0	0	0	1274	0	0	-1	170
N.S.	1	0.00	0.00	0.00	0.00	7.49	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.125	0.569	180.000	0.000	83.420	0.000	0.000	0.000	0.387

Problem 1858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	170	0	0	0	0	164	0	0	-1	170
N.S.	1	0.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.150	0.788	0.391	0.000	1.630	0.000	0.000	0.000	0.445

Problem 1859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	102	113	72	0	100	0	66	-1	86
N.S.	1	0.60	0.66	0.42	0.00	0.58	0.00	0.39	-0.01	0.50
time (sec)	N/A	0.058	0.091	0.016	0.000	1.341	0.000	0.193	0.000	0.615

Problem 1860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	171	0	0	941	0	300	0	0	-1	171
N.S.	1	0.00	0.00	5.50	0.00	1.75	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.014	0.168	9.437	0.000	7.614	0.000	0.000	0.000	0.243

Problem 1861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	325	492	0	0	0	0	0	-1	171
N.S.	1	1.90	2.88	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.051	1.658	0.531	0.000	0.000	0.000	0.000	0.000	4.015

Problem 1862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	171	0	0	1158	0	316	0	0	-1	171
N.S.	1	0.00	0.00	6.77	0.00	1.85	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.283	0.190	8.921	0.000	10.215	0.000	0.000	0.000	1.138

Problem 1863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F(-1)	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	833	161	281	0	740	0	0	-1	153
N.S.	1	4.87	0.94	1.64	0.00	4.33	0.00	0.00	-0.01	0.89
time (sec)	N/A	1.231	0.670	0.085	0.000	1.186	0.000	0.000	0.000	0.812

Problem 1864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	171	0	0	0	0	0	0	0	-1	171
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.974	0.650	0.375	0.000	0.000	0.000	0.000	0.000	0.387

Problem 1865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	171	0	0	0	0	333	0	0	-1	242
N.S.	1	0.00	0.00	0.00	0.00	1.95	0.00	0.00	-0.01	1.42
time (sec)	N/A	0.308	0.114	0.369	0.000	1.675	0.000	0.000	0.000	0.835

Problem 1866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	164	164	0	0	1037	0	0	-1	172
N.S.	1	0.95	0.95	0.00	0.00	6.03	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.182	0.324	0.116	0.000	151.925	0.000	0.000	0.000	1.241

Problem 1867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	C	F	B	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	172	97	0	1162	0	301	0	0	-1	172
N.S.	1	0.56	0.00	6.76	0.00	1.75	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.047	0.081	8.109	0.000	3.452	0.000	0.000	0.000	0.992

Problem 1868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	173	0	0	0	0	0	0	0	-1	173
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	6.825	5.817	0.563	0.000	0.000	0.000	0.000	0.000	3.850

Problem 1869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	173	0	0	0	0	0	0	0	-1	173
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	4.003	4.926	0.560	0.000	0.000	0.000	0.000	0.000	0.487

Problem 1870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	173	168	156	824	0	289	0	0	-1	173
N.S.	1	0.97	0.90	4.76	0.00	1.67	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.190	0.218	2.454	0.000	5.691	0.000	0.000	0.000	0.405

Problem 1871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	B	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	0	146	287	0	213	0	0	-1	173
N.S.	1	0.00	0.84	1.66	0.00	1.23	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.145	0.153	0.031	0.000	0.786	0.000	0.000	0.000	10.624

Problem 1872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	185	280	365	0	1083	0	207	-1	173
N.S.	1	1.07	1.62	2.11	0.00	6.26	0.00	1.20	-0.01	1.00
time (sec)	N/A	0.283	0.255	0.469	0.000	1.184	0.000	1.097	0.000	0.796

Problem 1873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	173	0	0	1128	0	0	0	0	-1	173
N.S.	1	0.00	0.00	6.52	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	3.223	0.357	3.249	0.000	0.000	0.000	0.000	0.000	2.865

Problem 1874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	0	260	0	0	0	0	0	-1	140
N.S.	1	0.00	1.50	0.00	0.00	0.00	0.00	0.00	-0.01	0.81
time (sec)	N/A	0.239	0.643	0.384	0.000	0.000	0.000	0.000	0.000	0.271

Problem 1875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F	F	B	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	173	255	0	0	0	361	0	0	-1	173
N.S.	1	1.47	0.00	0.00	0.00	2.09	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.593	0.363	0.411	0.000	1.653	0.000	0.000	0.000	27.738

Problem 1876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	119	152	2661	0	146	0	0	-1	174
N.S.	1	0.68	0.87	15.29	0.00	0.84	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.160	0.122	8.249	0.000	0.840	0.000	0.000	0.000	0.435

Problem 1877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	174	0	0	0	0	0	0	0	-1	174
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	4.632	10.759	0.507	0.000	0.000	0.000	0.000	0.000	1.111

Problem 1878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	174	0	0	0	0	0	0	0	-1	174
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	9.836	7.423	0.630	0.000	0.000	0.000	0.000	0.000	0.592

Problem 1879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	174	143	146	801	0	214	0	0	-1	174
N.S.	1	0.82	0.84	4.60	0.00	1.23	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.103	0.236	8.053	0.000	6.679	0.000	0.000	0.000	0.267

Problem 1880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F	B	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	174	64	164	0	0	0	0	403	-1	174
N.S.	1	0.37	0.94	0.00	0.00	0.00	0.00	2.32	-0.01	1.00
time (sec)	N/A	0.257	0.171	0.430	0.000	0.000	0.000	0.383	0.000	0.681

Problem 1881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	174	0	5076	0	0	1497	0	0	-1	174
N.S.	1	0.00	29.17	0.00	0.00	8.60	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.536	4.891	0.408	0.000	3.194	0.000	0.000	0.000	1.307

Problem 1882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	127	114	93	92	71	0	190	-1	114
N.S.	1	0.73	0.66	0.53	0.53	0.41	0.00	1.09	-0.01	0.66
time (sec)	N/A	0.163	0.144	0.007	0.314	0.605	0.000	4.500	0.000	0.138

Problem 1883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	175	83	114	2320	0	156	0	0	-1	175
N.S.	1	0.47	0.65	13.26	0.00	0.89	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.018	0.053	14.178	0.000	1.767	0.000	0.000	0.000	0.245

Problem 1884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	175	0	0	0	0	0	0	0	-1	175
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	9.145	14.407	0.528	0.000	0.000	0.000	0.000	0.000	0.444

Problem 1885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	175	0	0	873	0	426	0	0	-1	175
N.S.	1	0.00	0.00	4.99	0.00	2.43	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.144	0.335	4.483	0.000	15.469	0.000	0.000	0.000	0.601

Problem 1886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	175	0	0	1054	0	427	0	0	-1	175
N.S.	1	0.00	0.00	6.02	0.00	2.44	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.073	0.333	4.681	0.000	17.271	0.000	0.000	0.000	0.577

Problem 1887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	175	208	161	120	0	202	0	62	-1	175
N.S.	1	1.19	0.92	0.69	0.00	1.15	0.00	0.35	-0.01	1.00
time (sec)	N/A	0.327	0.090	0.016	0.000	0.428	0.000	0.377	0.000	0.475

Problem 1888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	175	0	0	0	0	0	0	0	-1	175
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.912	1.850	0.145	0.000	0.000	0.000	0.000	0.000	1.004

Problem 1889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	175	192	0	0	0	374	0	0	-1	246
N.S.	1	1.10	0.00	0.00	0.00	2.14	0.00	0.00	-0.01	1.41
time (sec)	N/A	1.083	0.219	0.411	0.000	3.950	0.000	0.000	0.000	0.783

Problem 1890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	176	378	0	407	0	986	0	0	-1	176
N.S.	1	2.15	0.00	2.31	0.00	5.60	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.680	0.354	5.276	0.000	9.021	0.000	0.000	0.000	0.328

Problem 1891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	176	223	194	943	0	735	162	183	204	176
N.S.	1	1.27	1.10	5.36	0.00	4.18	0.92	1.04	1.16	1.00
time (sec)	N/A	0.777	0.226	0.768	0.000	0.707	114.318	0.299	9.525	0.263

Problem 1892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F	F	B	F(-1)	F(-2)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	176	306	0	0	0	353	0	0	-1	176
N.S.	1	1.74	0.00	0.00	0.00	2.01	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.487	0.156	4.200	0.000	0.433	0.000	0.000	0.000	0.645

Problem 1893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	176	180	0	0	0	368	0	0	-1	247
N.S.	1	1.02	0.00	0.00	0.00	2.09	0.00	0.00	-0.01	1.40
time (sec)	N/A	1.007	0.233	0.398	0.000	3.952	0.000	0.000	0.000	0.926

Problem 1894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	177	88	120	1417	0	175	0	0	-1	177
N.S.	1	0.50	0.68	8.01	0.00	0.99	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.020	0.062	14.198	0.000	2.247	0.000	0.000	0.000	0.205

Problem 1895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	267	69	0	0	0	0	0	65	177
N.S.	1	1.51	0.39	0.00	0.00	0.00	0.00	0.00	0.37	1.00
time (sec)	N/A	0.415	0.139	0.094	0.000	0.000	0.000	0.000	1.853	0.555

Problem 1896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	177	234	73	787	0	1141	0	0	-1	177
N.S.	1	1.32	0.41	4.45	0.00	6.45	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.627	0.563	0.100	0.000	0.639	0.000	0.000	0.000	0.488

Problem 1897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	0	56	0	0	0	0	0	-1	176
N.S.	1	0.00	0.32	0.00	0.00	0.00	0.00	0.00	-0.01	0.99
time (sec)	N/A	4.258	1.207	0.441	0.000	0.000	0.000	0.000	0.000	14.269

Problem 1898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	255	199	442	0	0	0	224	-1	209
N.S.	1	1.44	1.12	2.50	0.00	0.00	0.00	1.27	-0.01	1.18
time (sec)	N/A	0.895	0.369	0.010	0.000	0.000	0.000	2.087	0.000	0.529

Problem 1899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F(-1)	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	178	495	282	604	0	356	0	0	-1	178
N.S.	1	2.78	1.58	3.39	0.00	2.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	7.964	5.000	0.036	0.000	1.587	0.000	0.000	0.000	0.397

Problem 1900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	178	0	0	0	0	0	0	0	-1	178
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.614	0.782	0.155	0.000	0.000	0.000	0.000	0.000	0.413

Problem 1901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	179	0	4912	0	0	1508	0	0	-1	179
N.S.	1	0.00	27.44	0.00	0.00	8.42	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.523	3.439	0.398	0.000	3.223	0.000	0.000	0.000	1.293

Problem 1902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	179	0	0	0	0	0	0	0	-1	179
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	13.584	3.132	0.058	0.000	0.000	0.000	0.000	0.000	1.108

Problem 1903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	179	0	0	0	0	0	0	0	-1	179
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	14.968	5.931	0.056	0.000	0.000	0.000	0.000	0.000	2.973

Problem 1904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	180	0	0	0	0	0	0	0	-1	180
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	6.856	5.475	0.593	0.000	0.000	0.000	0.000	0.000	0.539

Problem 1905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	180	124	259	0	412	158	153	0	-1	180
N.S.	1	0.69	1.44	0.00	2.29	0.88	0.85	0.00	-0.01	1.00
time (sec)	N/A	0.091	0.147	0.411	0.431	0.795	3.679	0.000	0.000	0.733

Problem 1906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	180	0	0	0	0	0	0	0	-1	180
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.916	3.270	0.168	0.000	0.000	0.000	0.000	0.000	1.008

Problem 1907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	180	121	63	1588	0	1099	0	0	-1	180
N.S.	1	0.67	0.35	8.82	0.00	6.11	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.519	0.084	30.051	0.000	12.685	0.000	0.000	0.000	0.753

Problem 1908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	180	121	63	1591	0	1099	0	0	-1	180
N.S.	1	0.67	0.35	8.84	0.00	6.11	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.394	0.045	28.212	0.000	13.077	0.000	0.000	0.000	0.001

Problem 1909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	181	176	59	1612	0	277	0	0	-1	181
N.S.	1	0.97	0.33	8.91	0.00	1.53	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.027	0.025	8.616	0.000	1.813	0.000	0.000	0.000	0.231

Problem 1910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	181	0	0	0	0	0	0	0	-1	181
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	6.334	13.528	0.545	0.000	0.000	0.000	0.000	0.000	0.401

Problem 1911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	181	0	0	0	0	0	0	0	-1	181
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.261	0.766	0.162	0.000	0.000	0.000	0.000	0.000	0.371

Problem 1912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	F(-1)	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	184	198	65	0	219	0	320	0	250	184
N.S.	1	1.08	0.35	0.00	1.19	0.00	1.74	0.00	1.36	1.00
time (sec)	N/A	0.247	0.071	0.415	0.673	0.000	2.959	0.000	2.285	18.220

Problem 1913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	184	94	0	733	0	1102	0	0	-1	184
N.S.	1	0.51	0.00	3.98	0.00	5.99	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.653	0.579	13.331	0.000	15.411	0.000	0.000	0.000	0.661

Problem 1914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	184	162	146	115	0	186	0	0	-1	165
N.S.	1	0.88	0.79	0.62	0.00	1.01	0.00	0.00	-0.01	0.90
time (sec)	N/A	1.754	3.528	0.048	0.000	0.440	0.000	0.000	0.000	0.306

Problem 1915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	F(-2)	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	185	238	0	1371	0	0	0	0	-1	185
N.S.	1	1.29	0.00	7.41	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.572	0.558	16.309	0.000	0.000	0.000	0.000	0.000	0.271

Problem 1916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	185	0	0	0	0	0	0	0	-1	185
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	15.978	2.068	0.057	0.000	0.000	0.000	0.000	0.000	2.901

Problem 1917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	185	81	0	737	0	1100	0	0	-1	185
N.S.	1	0.44	0.00	3.98	0.00	5.95	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.580	0.525	13.523	0.000	13.500	0.000	0.000	0.000	0.628

Problem 1918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	C	F	B	F(-1)	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	264	162	900	0	319	0	0	-1	185
N.S.	1	1.43	0.88	4.86	0.00	1.72	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.500	0.303	2.985	0.000	3.567	0.000	0.000	0.000	0.601

Problem 1919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	185	81	45	651	0	1055	0	0	-1	185
N.S.	1	0.44	0.24	3.52	0.00	5.70	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.538	0.371	19.424	0.000	51.813	0.000	0.000	0.000	0.779

Problem 1920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	185	81	45	646	0	1055	0	0	-1	185
N.S.	1	0.44	0.24	3.49	0.00	5.70	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.409	0.293	18.625	0.000	52.564	0.000	0.000	0.000	0.001

Problem 1921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	185	0	0	0	0	0	0	0	-1	185
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	20.456	4.388	0.059	0.000	0.000	0.000	0.000	0.000	2.996

Problem 1922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	225	220	0	0	248	0	0	-1	185
N.S.	1	1.22	1.19	0.00	0.00	1.34	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.164	0.226	0.391	0.000	0.453	0.000	0.000	0.000	0.330

Problem 1923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	186	0	0	0	0	0	0	0	-1	186
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	4.781	15.457	0.615	0.000	0.000	0.000	0.000	0.000	4.224

Problem 1924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	186	0	26530	14911	0	723	0	0	-1	188
N.S.	1	0.00	142.63	80.17	0.00	3.89	0.00	0.00	-0.01	1.01
time (sec)	N/A	6.140	6.663	0.573	0.000	0.769	0.000	0.000	0.000	0.739

Problem 1925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	186	0	0	0	0	0	0	0	-1	186
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	18.268	1.175	0.069	0.000	0.000	0.000	0.000	0.000	8.946

Problem 1926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	186	299	63	137	0	399	0	0	-1	190
N.S.	1	1.61	0.34	0.74	0.00	2.15	0.00	0.00	-0.01	1.02
time (sec)	N/A	0.365	0.029	0.038	0.000	0.478	0.000	0.000	0.000	23.384

Problem 1927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	186	0	0	0	0	0	0	0	-1	186
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.176	0.222	180.000	0.000	0.000	0.000	0.000	0.000	0.558

Problem 1928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	201	208	211	0	249	0	0	-1	56
N.S.	1	1.07	1.11	1.13	0.00	1.33	0.00	0.00	-0.01	0.30
time (sec)	N/A	1.451	0.357	0.172	0.000	1.037	0.000	0.000	0.000	3.663

Problem 1929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	201	208	211	0	249	0	0	-1	56
N.S.	1	1.07	1.11	1.13	0.00	1.33	0.00	0.00	-0.01	0.30
time (sec)	N/A	1.218	0.265	0.164	0.000	1.065	0.000	0.000	0.000	3.696

Problem 1930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	187	46	48	651	0	765	0	0	-1	187
N.S.	1	0.25	0.26	3.48	0.00	4.09	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.151	1.128	18.697	0.000	6.415	0.000	0.000	0.000	0.689

Problem 1931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	187	46	48	648	0	765	0	0	-1	187
N.S.	1	0.25	0.26	3.47	0.00	4.09	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.150	0.890	19.631	0.000	6.422	0.000	0.000	0.000	0.002

Problem 1932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	187	163	0	1509	0	1348	0	0	-1	187
N.S.	1	0.87	0.00	8.07	0.00	7.21	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.509	3.858	74.329	0.000	18.303	0.000	0.000	0.000	0.966

Problem 1933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	187	163	0	1508	0	1348	0	0	-1	187
N.S.	1	0.87	0.00	8.06	0.00	7.21	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.441	0.074	71.040	0.000	17.943	0.000	0.000	0.000	0.001

Problem 1934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	187	99	47	649	0	1055	0	0	-1	187
N.S.	1	0.53	0.25	3.47	0.00	5.64	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.556	0.425	18.792	0.000	53.134	0.000	0.000	0.000	0.954

Problem 1935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	187	99	47	649	0	1055	0	0	-1	187
N.S.	1	0.53	0.25	3.47	0.00	5.64	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.450	0.276	18.835	0.000	53.592	0.000	0.000	0.000	0.001

Problem 1936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	139	49	0	0	262	0	317	-1	188
N.S.	1	0.74	0.26	0.00	0.00	1.39	0.00	1.69	-0.01	1.00
time (sec)	N/A	0.114	0.018	0.480	0.000	0.438	0.000	0.224	0.000	1.366

Problem 1937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F(-2)	A	F(-1)	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	455	227	1121	0	467	0	0	-1	189
N.S.	1	2.41	1.20	5.93	0.00	2.47	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.387	2.224	0.060	0.000	0.667	0.000	0.000	0.000	0.555

Problem 1938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	189	0	3575	4541	0	1659	0	0	-1	189
N.S.	1	0.00	18.92	24.03	0.00	8.78	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.770	2.116	0.027	0.000	0.981	0.000	0.000	0.000	1.320

Problem 1939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	189	0	16759	1362	0	1321	0	0	-1	189
N.S.	1	0.00	88.67	7.21	0.00	6.99	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.343	6.684	0.454	0.000	4.534	0.000	0.000	0.000	1.654

Problem 1940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	189	0	0	0	0	180	0	0	-1	202
N.S.	1	0.00	0.00	0.00	0.00	0.95	0.00	0.00	-0.01	1.07
time (sec)	N/A	2.755	0.756	0.451	0.000	1.028	0.000	0.000	0.000	0.765

Problem 1941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	189	405	0	0	0	384	0	0	-1	247
N.S.	1	2.14	0.00	0.00	0.00	2.03	0.00	0.00	-0.01	1.31
time (sec)	N/A	1.461	0.251	0.416	0.000	4.284	0.000	0.000	0.000	1.462

Problem 1942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	189	0	0	0	0	1321	0	0	-1	189
N.S.	1	0.00	0.00	0.00	0.00	6.99	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.372	0.412	8.875	0.000	86.733	0.000	0.000	0.000	13.420

Problem 1943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	190	70	124	512	0	1395	0	0	-1	0
N.S.	1	0.37	0.65	2.69	0.00	7.34	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.010	0.092	10.570	0.000	1.746	0.000	0.000	0.000	0.001

Problem 1944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	190	0	0	0	0	0	0	0	-1	190
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	8.090	3.588	0.102	0.000	0.000	0.000	0.000	0.000	8.115

Problem 1945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	190	211	0	1780	0	1133	0	0	-1	190
N.S.	1	1.11	0.00	9.37	0.00	5.96	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.608	0.553	16.102	0.000	6.081	0.000	0.000	0.000	0.691

Problem 1946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	191	264	68	0	0	0	0	187	-1	191
N.S.	1	1.38	0.36	0.00	0.00	0.00	0.00	0.98	-0.01	1.00
time (sec)	N/A	0.332	0.029	0.686	0.000	0.000	0.000	0.381	0.000	0.484

Problem 1947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-2)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	191	0	0	0	0	0	0	0	-1	191
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	6.541	0.964	2.452	0.000	0.000	0.000	0.000	0.000	3.029

Problem 1948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	191	0	0	0	0	188	0	0	-1	191
N.S.	1	0.00	0.00	0.00	0.00	0.98	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.317	0.616	0.184	0.000	55.097	0.000	0.000	0.000	0.491

Problem 1949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	192	0	0	0	0	0	0	0	-1	192
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.697	7.523	0.599	0.000	0.000	0.000	0.000	0.000	0.712

Problem 1950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	192	0	0	0	0	0	0	0	-1	192
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	7.831	3.535	0.084	0.000	0.000	0.000	0.000	0.000	8.090

Problem 1951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	F	F	F(-1)	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	192	617	244	0	0	0	0	237	-1	192
N.S.	1	3.21	1.27	0.00	0.00	0.00	0.00	1.23	-0.01	1.00
time (sec)	N/A	1.636	0.919	0.452	0.000	0.000	0.000	0.514	0.000	1.086

Problem 1952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	192	0	0	0	0	0	0	0	-1	192
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.332	0.654	0.181	0.000	0.000	0.000	0.000	0.000	0.577

Problem 1953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	266	201	588	355	419	0	551	305	193
N.S.	1	1.38	1.04	3.05	1.84	2.17	0.00	2.85	1.58	1.00
time (sec)	N/A	0.277	0.445	0.046	0.450	0.627	0.000	0.363	0.856	0.313

Problem 1954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F	F	A	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	193	245	0	0	0	240	0	0	-1	193
N.S.	1	1.27	0.00	0.00	0.00	1.24	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.282	0.397	0.629	0.000	1.241	0.000	0.000	0.000	0.602

Problem 1955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	411	202	337	0	1809	0	0	-1	193
N.S.	1	2.13	1.05	1.75	0.00	9.37	0.00	0.00	-0.01	1.00
time (sec)	N/A	2.796	0.498	0.049	0.000	4.111	0.000	0.000	0.000	3.794

Problem 1956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	F	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	416	192	0	0	185	0	197	-1	193
N.S.	1	2.16	0.99	0.00	0.00	0.96	0.00	1.02	-0.01	1.00
time (sec)	N/A	0.415	0.103	0.454	0.000	1.138	0.000	0.612	0.000	0.766

Problem 1957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	193	0	3036	3458	0	1661	0	0	-1	193
N.S.	1	0.00	15.73	17.92	0.00	8.61	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.749	1.487	0.025	0.000	1.353	0.000	0.000	0.000	1.286

Problem 1958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F(-2)	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	209	214	1722	0	953	0	0	-1	193
N.S.	1	1.08	1.11	8.92	0.00	4.94	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.200	0.401	11.049	0.000	28.675	0.000	0.000	0.000	0.650

Problem 1959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	F(-2)	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	193	189	172	1753	0	990	0	0	-1	193
N.S.	1	0.98	0.89	9.08	0.00	5.13	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.287	0.163	10.621	0.000	31.715	0.000	0.000	0.000	0.740

Problem 1960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F(-2)	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	209	214	1722	0	913	0	0	-1	193
N.S.	1	1.08	1.11	8.92	0.00	4.73	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.125	0.302	11.269	0.000	32.043	0.000	0.000	0.000	0.615

Problem 1961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	189	170	1692	0	985	0	0	-1	193
N.S.	1	0.98	0.88	8.77	0.00	5.10	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.311	0.117	10.344	0.000	31.334	0.000	0.000	0.000	0.721

Problem 1962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	341	424	0	0	377	0	358	207	193
N.S.	1	1.77	2.20	0.00	0.00	1.95	0.00	1.85	1.07	1.00
time (sec)	N/A	0.607	0.444	0.081	0.000	0.613	0.000	0.666	4.815	2.758

Problem 1963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	73	158	0	0	74	0	0	47	193
N.S.	1	0.38	0.82	0.00	0.00	0.38	0.00	0.00	0.24	1.00
time (sec)	N/A	0.404	0.147	0.409	0.000	0.509	0.000	0.000	2.291	0.330

Problem 1964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	81	158	0	0	74	0	0	47	193
N.S.	1	0.42	0.82	0.00	0.00	0.38	0.00	0.00	0.24	1.00
time (sec)	N/A	0.379	0.189	0.429	0.000	0.537	0.000	0.000	2.243	0.321

Problem 1965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	73	158	0	0	92	0	0	73	193
N.S.	1	0.38	0.82	0.00	0.00	0.48	0.00	0.00	0.38	1.00
time (sec)	N/A	0.471	0.213	0.410	0.000	0.627	0.000	0.000	2.667	0.330

Problem 1966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	0	174	0	0	180	0	0	-1	193
N.S.	1	0.00	0.90	0.00	0.00	0.93	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.379	0.254	0.442	0.000	1.180	0.000	0.000	0.000	0.718

Problem 1967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	0	127	0	0	532	0	0	-1	193
N.S.	1	0.00	0.66	0.00	0.00	2.76	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.297	0.110	0.403	0.000	1.059	0.000	0.000	0.000	0.653

Problem 1968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	194	81	126	643	0	1790	0	0	-1	0
N.S.	1	0.42	0.65	3.31	0.00	9.23	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.013	0.101	6.244	0.000	3.295	0.000	0.000	0.000	4.087

Problem 1969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	F	F	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	194	418	69	0	0	535	0	234	-1	194
N.S.	1	2.15	0.36	0.00	0.00	2.76	0.00	1.21	-0.01	1.00
time (sec)	N/A	0.653	0.152	0.585	0.000	0.606	0.000	5.284	0.000	0.588

Problem 1970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	194	256	221	0	0	1231	0	0	-1	194
N.S.	1	1.32	1.14	0.00	0.00	6.35	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.433	0.167	0.433	0.000	0.663	0.000	0.000	0.000	1.166

Problem 1971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F(-2)	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	194	196	952	0	0	112	0	0	-1	194
N.S.	1	1.01	4.91	0.00	0.00	0.58	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.324	7.390	180.000	0.000	0.473	0.000	0.000	0.000	0.304

Problem 1972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	341	323	0	0	385	0	359	-1	195
N.S.	1	1.75	1.66	0.00	0.00	1.97	0.00	1.84	-0.01	1.00
time (sec)	N/A	0.752	0.358	0.080	0.000	0.660	0.000	0.857	0.000	2.117

Problem 1973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	0	388	0	0	0	0	0	-1	162
N.S.	1	0.00	1.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.83
time (sec)	N/A	0.207	0.687	0.404	0.000	0.000	0.000	0.000	0.000	0.304

Problem 1974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	195	0	0	0	0	98	0	0	-1	195
N.S.	1	0.00	0.00	0.00	0.00	0.50	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.164	1.904	0.015	0.000	0.809	0.000	0.000	0.000	0.251

Problem 1975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	196	0	0	0	0	368	0	0	-1	196
N.S.	1	0.00	0.00	0.00	0.00	1.88	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.500	0.847	33.095	0.000	6.209	0.000	0.000	0.000	1.207

Problem 1976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	196	101	199	595	0	159	0	0	-1	114
N.S.	1	0.52	1.02	3.04	0.00	0.81	0.00	0.00	-0.01	0.58
time (sec)	N/A	0.643	0.582	0.058	0.000	0.711	0.000	0.000	0.000	0.892

Problem 1977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	196	0	384	0	0	295	0	0	-1	196
N.S.	1	0.00	1.96	0.00	0.00	1.51	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.711	11.664	0.410	0.000	35.110	0.000	0.000	0.000	3.259

Problem 1978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	196	0	179	0	0	98	0	0	-1	196
N.S.	1	0.00	0.91	0.00	0.00	0.50	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.033	0.268	0.011	0.000	0.547	0.000	0.000	0.000	0.197

Problem 1979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-2)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	197	0	0	1411	0	0	0	0	-1	197
N.S.	1	0.00	0.00	7.16	0.00	0.00	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.013	1.223	16.642	0.000	0.000	0.000	0.000	0.000	0.304

Problem 1980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	197	149	162	360	0	425	0	208	143	197
N.S.	1	0.76	0.82	1.83	0.00	2.16	0.00	1.06	0.73	1.00
time (sec)	N/A	0.067	0.638	0.005	0.000	0.827	0.000	0.487	1.764	0.691

Problem 1981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F(-1)	F(-2)	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	197	157	0	3500	0	971	0	0	-1	197
N.S.	1	0.80	0.00	17.77	0.00	4.93	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.549	7.215	11.687	0.000	18.203	0.000	0.000	0.000	0.970

Problem 1982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	197	0	0	0	0	108	0	0	-1	197
N.S.	1	0.00	0.00	0.00	0.00	0.55	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.135	2.970	0.020	0.000	0.644	0.000	0.000	0.000	0.249

Problem 1983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	198	97	111	2395	0	171	0	0	-1	198
N.S.	1	0.49	0.56	12.10	0.00	0.86	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.015	0.056	13.523	0.000	2.485	0.000	0.000	0.000	0.227

Problem 1984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	339	183	0	0	489	0	461	-1	198
N.S.	1	1.71	0.92	0.00	0.00	2.47	0.00	2.33	-0.01	1.00
time (sec)	N/A	0.464	0.169	0.473	0.000	0.661	0.000	2.865	0.000	0.862

Problem 1985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	198	0	0	0	0	397	0	0	-1	198
N.S.	1	0.00	0.00	0.00	0.00	2.01	0.00	0.00	-0.01	1.00
time (sec)	N/A	1.503	0.824	22.586	0.000	5.080	0.000	0.000	0.000	1.158

Problem 1986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	359	325	0	0	390	0	373	-1	198
N.S.	1	1.81	1.64	0.00	0.00	1.97	0.00	1.88	-0.01	1.00
time (sec)	N/A	0.801	0.390	0.082	0.000	0.618	0.000	0.671	0.000	1.484

Problem 1987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	F(-1)	C	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	0	95	31	0	0	51	0	-1	166
N.S.	1	0.00	0.48	0.16	0.00	0.00	0.26	0.00	-0.01	0.84
time (sec)	N/A	0.146	0.248	0.037	0.000	0.000	1.584	0.000	0.000	0.432

Problem 1988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	F(-1)	C	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	0	81	0	0	0	49	0	-1	166
N.S.	1	0.00	0.41	0.00	0.00	0.00	0.25	0.00	-0.01	0.84
time (sec)	N/A	0.683	0.250	0.423	0.000	0.000	1.986	0.000	0.000	0.397

Problem 1989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	129	114	93	92	94	0	444	-1	144
N.S.	1	0.65	0.58	0.47	0.46	0.47	0.00	2.24	-0.01	0.73
time (sec)	N/A	0.150	0.126	0.007	0.338	0.789	0.000	2.657	0.000	0.130

Problem 1990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	199	270	75	0	0	0	0	195	-1	199
N.S.	1	1.36	0.38	0.00	0.00	0.00	0.00	0.98	-0.01	1.00
time (sec)	N/A	0.338	0.034	0.849	0.000	0.000	0.000	0.517	0.000	0.509

Problem 1991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	199	0	0	0	0	0	0	0	-1	207
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.04
time (sec)	N/A	0.616	0.405	0.465	0.000	0.000	0.000	0.000	0.000	3.225

Problem 1992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	199	60	161	0	0	86	0	0	47	199
N.S.	1	0.30	0.81	0.00	0.00	0.43	0.00	0.00	0.24	1.00
time (sec)	N/A	0.450	0.273	0.475	0.000	0.896	0.000	0.000	2.238	0.404

Problem 1993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	199	199	233	0	0	287	0	0	-1	199
N.S.	1	1.00	1.17	0.00	0.00	1.44	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.560	0.456	0.434	0.000	1.192	0.000	0.000	0.000	0.321

Problem 1994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	F(-1)	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	200	1405	7717	13623	0	0	0	0	-1	200
N.S.	1	7.02	38.58	68.12	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	9.279	9.152	0.114	0.000	0.000	0.000	0.000	0.000	4.278

Problem 1995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	200	0	0	933	0	344	0	0	-1	200
N.S.	1	0.00	0.00	4.66	0.00	1.72	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.491	0.136	50.931	0.000	82.371	0.000	0.000	0.000	19.990

Problem 1996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	0	205	0	0	323	0	0	-1	200
N.S.	1	0.00	1.02	0.00	0.00	1.62	0.00	0.00	-0.00	1.00
time (sec)	N/A	3.073	1.808	0.427	0.000	0.460	0.000	0.000	0.000	5.690

Problem 1997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	201	0	0	0	0	0	0	0	-1	201
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.163	11.114	0.649	0.000	0.000	0.000	0.000	0.000	0.476

Problem 1998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	201	191	0	0	0	0	0	0	-1	201
N.S.	1	0.95	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.556	1.072	0.694	0.000	0.000	0.000	0.000	0.000	0.774

Problem 1999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	201	0	0	0	0	0	0	0	-1	201
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	3.674	5.197	0.640	0.000	0.000	0.000	0.000	0.000	0.668

Problem 2000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	201	0	0	0	0	0	0	0	-1	201
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	8.015	0.570	0.096	0.000	0.000	0.000	0.000	0.000	8.110

Problem 2001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F(-1)	F(-2)	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	201	213	181	1608	0	1049	0	0	-1	201
N.S.	1	1.06	0.90	8.00	0.00	5.22	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.401	0.284	11.296	0.000	43.776	0.000	0.000	0.000	1.133

Problem 2002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	203	0	0	0	0	0	0	0	-1	203
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	54.849	14.565	0.660	0.000	0.000	0.000	0.000	0.000	0.812

Problem 2003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	203	0	0	0	0	0	0	0	-1	203
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	8.045	0.556	0.091	0.000	0.000	0.000	0.000	0.000	8.120

Problem 2004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	204	0	0	0	0	0	0	0	-1	204
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	12.294	3.089	0.068	0.000	0.000	0.000	0.000	0.000	0.926

Problem 2005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	408	75	0	0	521	0	209	-1	205
N.S.	1	1.99	0.37	0.00	0.00	2.54	0.00	1.02	-0.00	1.00
time (sec)	N/A	0.832	0.147	0.566	0.000	0.461	0.000	0.221	0.000	0.690

Problem 2006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	205	0	0	0	0	0	0	0	-1	205
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	24.996	14.007	0.601	0.000	0.000	0.000	0.000	0.000	0.470

Problem 2007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	206	0	0	0	0	0	0	0	-1	206
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	11.502	4.491	0.078	0.000	0.000	0.000	0.000	0.000	4.196

Problem 2008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	206	170	100	2039	0	189	0	0	-1	206
N.S.	1	0.83	0.49	9.90	0.00	0.92	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.073	0.087	2.576	0.000	0.833	0.000	0.000	0.000	0.315

Problem 2009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	206	0	0	1807	0	223	0	0	-1	206
N.S.	1	0.00	0.00	8.77	0.00	1.08	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.425	0.199	30.576	0.000	33.828	0.000	0.000	0.000	6.835

Problem 2010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	206	0	0	2702	0	223	0	0	-1	206
N.S.	1	0.00	0.00	13.12	0.00	1.08	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.328	0.079	30.286	0.000	37.047	0.000	0.000	0.000	0.001

Problem 2011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	206	0	0	3644	0	268	0	0	-1	206
N.S.	1	0.00	0.00	17.69	0.00	1.30	0.00	0.00	-0.00	1.00
time (sec)	N/A	4.532	2.161	12.621	0.000	11.413	0.000	0.000	0.000	5.556

Problem 2012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	207	0	0	0	0	0	0	0	-1	207
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	2.655	5.661	0.655	0.000	0.000	0.000	0.000	0.000	0.730

Problem 2013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	207	359	321	1088	0	1051	0	308	248	207
N.S.	1	1.73	1.55	5.26	0.00	5.08	0.00	1.49	1.20	1.00
time (sec)	N/A	0.909	1.088	0.091	0.000	1.279	0.000	0.191	9.502	0.702

Problem 2014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	B	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	207	0	0	0	0	431	0	399	-1	207
N.S.	1	0.00	0.00	0.00	0.00	2.08	0.00	1.93	-0.00	1.00
time (sec)	N/A	3.377	1.012	0.473	0.000	0.736	0.000	0.961	0.000	0.892

Problem 2015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	208	0	0	0	0	0	0	0	-1	208
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	3.507	5.500	0.707	0.000	0.000	0.000	0.000	0.000	1.021

Problem 2016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	208	370	47	1684	0	365	0	0	-1	208
N.S.	1	1.78	0.23	8.10	0.00	1.75	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.602	0.029	19.806	0.000	1.926	0.000	0.000	0.000	0.433

Problem 2017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	208	343	188	0	0	0	0	437	-1	208
N.S.	1	1.65	0.90	0.00	0.00	0.00	0.00	2.10	-0.00	1.00
time (sec)	N/A	1.148	3.243	0.460	0.000	0.000	0.000	0.214	0.000	1.021

Problem 2018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	209	0	0	1374	0	313	0	0	-1	209
N.S.	1	0.00	0.00	6.57	0.00	1.50	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.011	0.224	9.638	0.000	9.618	0.000	0.000	0.000	0.300

Problem 2019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	209	0	0	0	0	0	0	0	-1	209
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	23.853	12.807	0.568	0.000	0.000	0.000	0.000	0.000	0.473

Problem 2020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-2)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	209	0	0	0	0	0	0	0	-1	209
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	6.213	2.601	1.440	0.000	0.000	0.000	0.000	0.000	0.509

Problem 2021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	209	99	164	0	0	382	0	0	-1	209
N.S.	1	0.47	0.78	0.00	0.00	1.83	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.386	0.409	0.976	0.000	29.761	0.000	0.000	0.000	0.453

Problem 2022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	209	0	0	0	0	0	0	0	-1	209
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.627	0.456	0.444	0.000	0.000	0.000	0.000	0.000	3.618

Problem 2023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	210	0	0	5143	0	532	0	0	-1	210
N.S.	1	0.00	0.00	24.49	0.00	2.53	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.513	1.899	15.180	0.000	25.526	0.000	0.000	0.000	1.139

Problem 2024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	210	0	0	2327	0	424	0	0	-1	210
N.S.	1	0.00	0.00	11.08	0.00	2.02	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.534	0.467	21.346	0.000	5.953	0.000	0.000	0.000	0.784

Problem 2025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	210	0	0	0	0	0	0	0	-1	210
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	16.811	3.611	0.066	0.000	0.000	0.000	0.000	0.000	0.935

Problem 2026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	210	97	152	97	0	259	0	0	-1	97
N.S.	1	0.46	0.72	0.46	0.00	1.23	0.00	0.00	-0.00	0.46
time (sec)	N/A	0.044	0.209	0.036	0.000	0.769	0.000	0.000	0.000	0.366

Problem 2027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	210	0	0	2712	0	251	0	0	-1	210
N.S.	1	0.00	0.00	12.91	0.00	1.20	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.408	0.177	31.387	0.000	24.134	0.000	0.000	0.000	6.953

Problem 2028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	210	0	0	2695	0	251	0	0	-1	210
N.S.	1	0.00	0.00	12.83	0.00	1.20	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.321	0.073	30.970	0.000	20.643	0.000	0.000	0.000	0.001

Problem 2029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	211	0	0	0	0	0	0	0	-1	179
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.85
time (sec)	N/A	2.480	0.744	0.438	0.000	0.000	0.000	0.000	0.000	0.481

Problem 2030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	212	0	0	0	0	0	0	0	-1	212
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	55.133	9.444	0.666	0.000	0.000	0.000	0.000	0.000	0.500

Problem 2031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	171	151	0	480	190	151	0	-1	212
N.S.	1	0.81	0.71	0.00	2.26	0.90	0.71	0.00	-0.00	1.00
time (sec)	N/A	0.120	5.149	0.488	0.597	0.773	3.993	0.000	0.000	1.337

Problem 2032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	212	0	0	0	0	0	0	0	-1	212
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	6.322	4.041	0.077	0.000	0.000	0.000	0.000	0.000	5.639

Problem 2033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	212	97	64	109	0	288	0	0	-1	97
N.S.	1	0.46	0.30	0.51	0.00	1.36	0.00	0.00	-0.00	0.46
time (sec)	N/A	0.302	0.048	0.033	0.000	1.158	0.000	0.000	0.000	0.514

Problem 2034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	A	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	212	0	0	0	0	0	0	290	-1	212
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	1.37	-0.00	1.00
time (sec)	N/A	18.567	4.549	0.082	0.000	0.000	0.000	0.476	0.000	1.039

Problem 2035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	213	94	67	71	0	75	0	0	-1	184
N.S.	1	0.44	0.31	0.33	0.00	0.35	0.00	0.00	-0.00	0.86
time (sec)	N/A	0.347	0.142	0.065	0.000	0.827	0.000	0.000	0.000	2.215

Problem 2036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	213	382	46	1110	0	326	0	0	-1	213
N.S.	1	1.79	0.22	5.21	0.00	1.53	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.676	0.058	20.082	0.000	3.448	0.000	0.000	0.000	0.489

Problem 2037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F(-2)	F	B	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	213	0	317	0	0	877	0	0	-1	213
N.S.	1	0.00	1.49	0.00	0.00	4.12	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.881	0.740	180.000	0.000	0.599	0.000	0.000	0.000	0.596

Problem 2038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	F	A	F(-1)	F(-2)	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	213	432	357	267	0	255	0	0	-1	210
N.S.	1	2.03	1.68	1.25	0.00	1.20	0.00	0.00	-0.00	0.99
time (sec)	N/A	3.403	1.040	0.061	0.000	0.479	0.000	0.000	0.000	1.054

Problem 2039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	214	0	0	0	0	0	0	0	-1	214
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	4.140	5.752	0.633	0.000	0.000	0.000	0.000	0.000	0.654

Problem 2040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	214	0	0	0	0	0	0	0	-1	214
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	6.407	3.872	0.076	0.000	0.000	0.000	0.000	0.000	5.639

Problem 2041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	214	103	154	0	0	361	0	0	-1	214
N.S.	1	0.48	0.72	0.00	0.00	1.69	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.386	0.228	1.079	0.000	27.886	0.000	0.000	0.000	0.484

Problem 2042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	214	123	181	0	0	360	0	0	-1	214
N.S.	1	0.57	0.85	0.00	0.00	1.68	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.392	0.269	0.976	0.000	26.723	0.000	0.000	0.000	0.496

Problem 2043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	214	0	0	0	0	0	0	0	-1	214
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.914	1.272	0.169	0.000	0.000	0.000	0.000	0.000	0.736

Problem 2044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	F	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	466	119	0	0	206	0	195	-1	215
N.S.	1	2.17	0.55	0.00	0.00	0.96	0.00	0.91	-0.00	1.00
time (sec)	N/A	0.343	0.117	0.449	0.000	0.485	0.000	0.311	0.000	0.622

Problem 2045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	F	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	467	117	0	0	206	0	195	-1	221
N.S.	1	2.17	0.54	0.00	0.00	0.96	0.00	0.91	-0.00	1.03
time (sec)	N/A	0.360	0.082	0.448	0.000	0.462	0.000	0.287	0.000	0.627

Problem 2046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	215	0	0	0	0	0	0	0	-1	215
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	14.669	4.028	0.140	0.000	0.000	0.000	0.000	0.000	0.558

Problem 2047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	215	0	0	0	0	0	0	0	-1	215
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	27.448	4.741	0.117	0.000	0.000	0.000	0.000	0.000	0.618

Problem 2048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F(-2)	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	215	333	0	0	673	0	0	-1	215
N.S.	1	1.00	1.55	0.00	0.00	3.13	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.499	0.641	180.000	0.000	0.553	0.000	0.000	0.000	0.696

Problem 2049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	215	0	0	0	0	0	0	0	-1	215
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.876	1.146	0.186	0.000	0.000	0.000	0.000	0.000	0.653

Problem 2050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	216	209	0	0	0	0	0	0	-1	216
N.S.	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.657	1.194	0.619	0.000	0.000	0.000	0.000	0.000	0.647

Problem 2051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	216	0	0	0	0	0	0	0	-1	216
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.170	11.461	0.674	0.000	0.000	0.000	0.000	0.000	0.565

Problem 2052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	216	0	0	0	0	0	0	0	-1	216
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.169	11.056	0.592	0.000	0.000	0.000	0.000	0.000	0.540

Problem 2053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	217	0	0	0	0	0	0	0	-1	217
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	3.564	8.402	0.628	0.000	0.000	0.000	0.000	0.000	1.781

Problem 2054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	217	291	60	2015	0	206	0	147	-1	217
N.S.	1	1.34	0.28	9.29	0.00	0.95	0.00	0.68	-0.00	1.00
time (sec)	N/A	0.105	0.024	5.411	0.000	0.803	0.000	0.247	0.000	0.555

Problem 2055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	217	0	0	2035	0	615	0	0	-1	217
N.S.	1	0.00	0.00	9.38	0.00	2.83	0.00	0.00	-0.00	1.00
time (sec)	N/A	2.287	0.606	54.072	0.000	52.339	0.000	0.000	0.000	0.797

Problem 2056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F(-1)	F(-2)	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	217	233	239	0	0	406	0	0	-1	217
N.S.	1	1.07	1.10	0.00	0.00	1.87	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.379	0.549	2.880	0.000	0.458	0.000	0.000	0.000	0.727

Problem 2057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F(-1)	F(-2)	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	217	233	239	0	0	406	0	0	-1	217
N.S.	1	1.07	1.10	0.00	0.00	1.87	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.302	0.492	2.810	0.000	0.442	0.000	0.000	0.000	0.711

Problem 2058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	218	0	0	0	0	0	0	0	-1	218
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	3.701	6.194	0.643	0.000	0.000	0.000	0.000	0.000	3.016

Problem 2059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	219	295	61	1508	0	241	0	149	-1	219
N.S.	1	1.35	0.28	6.89	0.00	1.10	0.00	0.68	-0.00	1.00
time (sec)	N/A	0.105	0.039	2.865	0.000	2.719	0.000	0.323	0.000	0.489

Problem 2060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	219	298	61	1215	0	210	0	148	-1	219
N.S.	1	1.36	0.28	5.55	0.00	0.96	0.00	0.68	-0.00	1.00
time (sec)	N/A	0.082	0.019	3.058	0.000	0.716	0.000	0.214	0.000	0.471

Problem 2061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	A	F	F(-2)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	219	0	0	0	0	319	0	0	-1	219
N.S.	1	0.00	0.00	0.00	0.00	1.46	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.668	0.182	180.000	0.000	5.579	0.000	0.000	0.000	0.401

Problem 2062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	220	0	0	0	0	367	0	0	-1	220
N.S.	1	0.00	0.00	0.00	0.00	1.67	0.00	0.00	-0.00	1.00
time (sec)	N/A	4.128	0.409	0.430	0.000	0.490	0.000	0.000	0.000	7.435

Problem 2063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	221	0	0	0	0	0	0	0	-1	221
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	7.553	1.821	0.075	0.000	0.000	0.000	0.000	0.000	5.672

Problem 2064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	221	179	69	0	0	1021	0	386	-1	221
N.S.	1	0.81	0.31	0.00	0.00	4.62	0.00	1.75	-0.00	1.00
time (sec)	N/A	0.336	0.049	0.534	0.000	1.542	0.000	0.213	0.000	3.140

Problem 2065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	221	0	0	0	0	0	0	0	-1	221
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	3.722	0.596	0.503	0.000	0.000	0.000	0.000	0.000	1.087

Problem 2066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	F	B	F(-1)	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	221	0	214	1246	0	367	0	0	-1	221
N.S.	1	0.00	0.97	5.64	0.00	1.66	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.782	0.519	5.616	0.000	4.977	0.000	0.000	0.000	0.542

Problem 2067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	222	196	40	116	0	220	0	0	-1	222
N.S.	1	0.88	0.18	0.52	0.00	0.99	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.107	0.035	0.044	0.000	0.539	0.000	0.000	0.000	0.661

Problem 2068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	222	0	0	0	0	0	0	0	-1	222
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	13.141	4.356	0.062	0.000	0.000	0.000	0.000	0.000	3.018

Problem 2069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	B	F(-1)	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	0	55	0	0	1383	0	0	-1	223
N.S.	1	0.00	0.25	0.00	0.00	6.20	0.00	0.00	-0.00	1.00
time (sec)	N/A	22.005	0.231	0.052	0.000	46.427	0.000	0.000	0.000	0.530

Problem 2070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	223	0	0	0	0	0	0	0	-1	223
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	3.595	4.671	0.628	0.000	0.000	0.000	0.000	0.000	1.001

Problem 2071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	223	0	0	0	0	0	0	0	-1	223
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	7.201	1.542	0.079	0.000	0.000	0.000	0.000	0.000	5.666

Problem 2072	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	223	0	0	0	0	0	0	0	-1	223
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	7.940	2.563	0.083	0.000	0.000	0.000	0.000	0.000	8.171

Problem 2073	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	373	398	1759	243	350	0	250	1310	216
N.S.	1	1.67	1.78	7.89	1.09	1.57	0.00	1.12	5.87	0.97
time (sec)	N/A	1.335	0.602	5.285	0.420	1.085	0.000	0.649	2.269	0.636

Problem 2074	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	223	0	0	0	0	385	0	0	-1	294
N.S.	1	0.00	0.00	0.00	0.00	1.73	0.00	0.00	-0.00	1.32
time (sec)	N/A	3.358	0.550	0.503	0.000	5.291	0.000	0.000	0.000	1.092

Problem 2075	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	224	0	0	0	0	0	0	0	-1	224
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	8.298	3.610	0.085	0.000	0.000	0.000	0.000	0.000	8.344

Problem 2076	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	A	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	224	0	0	0	0	0	0	262	-1	224
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	1.17	-0.00	1.00
time (sec)	N/A	20.165	1.955	0.092	0.000	0.000	0.000	0.761	0.000	1.529

Problem 2077	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	224	0	0	0	0	389	0	0	-1	295
N.S.	1	0.00	0.00	0.00	0.00	1.74	0.00	0.00	-0.00	1.32
time (sec)	N/A	1.886	0.418	0.470	0.000	5.010	0.000	0.000	0.000	1.311

Problem 2078	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	224	426	0	0	0	391	0	0	-1	295
N.S.	1	1.90	0.00	0.00	0.00	1.75	0.00	0.00	-0.00	1.32
time (sec)	N/A	2.612	0.470	0.467	0.000	4.044	0.000	0.000	0.000	1.272

Problem 2079	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-2)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	225	0	0	1645	0	0	0	0	-1	225
N.S.	1	0.00	0.00	7.31	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.738	1.627	10.947	0.000	0.000	0.000	0.000	0.000	0.586

Problem 2080	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	369	84	2348	0	204	0	154	-1	225
N.S.	1	1.64	0.37	10.44	0.00	0.91	0.00	0.68	-0.00	1.00
time (sec)	N/A	1.006	0.079	3.370	0.000	0.455	0.000	0.315	0.000	0.755

Problem 2081	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	F(-1)	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	131	158	0	227	0	80	0	166	225
N.S.	1	0.58	0.70	0.00	1.01	0.00	0.36	0.00	0.74	1.00
time (sec)	N/A	0.139	0.185	0.469	0.422	0.000	4.550	0.000	2.018	5.112

Problem 2082	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-2)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	225	0	0	2504	0	0	0	0	-1	225
N.S.	1	0.00	0.00	11.13	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	6.270	2.525	31.237	0.000	0.000	0.000	0.000	0.000	0.614

Problem 2083	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	242	201	544	0	0	0	0	-1	225
N.S.	1	1.08	0.89	2.42	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.201	0.183	5.595	0.000	0.000	0.000	0.000	0.000	0.900

Problem 2084	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	C	F	F(-1)	F(-1)	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	225	242	0	463	0	0	0	0	-1	225
N.S.	1	1.08	0.00	2.06	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.192	0.044	4.985	0.000	0.000	0.000	0.000	0.000	0.795

Problem 2085	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	242	201	529	0	0	0	0	-1	225
N.S.	1	1.08	0.89	2.35	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.179	0.173	5.484	0.000	0.000	0.000	0.000	0.000	0.831

Problem 2086	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	C	F	F(-1)	F(-1)	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	225	242	0	463	0	0	0	0	-1	225
N.S.	1	1.08	0.00	2.06	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.187	0.043	5.300	0.000	0.000	0.000	0.000	0.000	0.795

Problem 2087	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	225	0	0	0	0	367	0	0	-1	225
N.S.	1	0.00	0.00	0.00	0.00	1.63	0.00	0.00	-0.00	1.00
time (sec)	N/A	4.403	1.476	0.444	0.000	0.867	0.000	0.000	0.000	7.038

Problem 2088	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	225	81	0	0	0	369	0	0	-1	225
N.S.	1	0.36	0.00	0.00	0.00	1.64	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.176	0.360	0.443	0.000	5.510	0.000	0.000	0.000	1.625

Problem 2089	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	226	0	0	0	0	0	0	0	-1	226
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	13.258	4.242	0.068	0.000	0.000	0.000	0.000	0.000	1.513

Problem 2090	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	226	0	0	0	0	0	0	0	-1	226
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.799	12.421	0.071	0.000	0.000	0.000	0.000	0.000	3.986

Problem 2091	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	B	F	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	226	496	147	3822	0	415	0	225	-1	226
N.S.	1	2.19	0.65	16.91	0.00	1.84	0.00	1.00	-0.00	1.00
time (sec)	N/A	0.744	0.248	10.829	0.000	0.580	0.000	0.398	0.000	1.103

Problem 2092	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	226	423	0	0	0	0	0	0	-1	226
N.S.	1	1.87	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	2.912	1.833	0.079	0.000	0.000	0.000	0.000	0.000	5.630

Problem 2093	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	A	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	227	0	0	0	0	0	0	318	-1	227
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	1.40	-0.00	1.00
time (sec)	N/A	41.617	3.711	0.096	0.000	0.000	0.000	0.589	0.000	1.319

Problem 2094	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	C	F	B	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	227	609	267	703	0	9711	0	0	246	223
N.S.	1	2.68	1.18	3.10	0.00	42.78	0.00	0.00	1.08	0.98
time (sec)	N/A	3.156	0.443	0.164	0.000	20.949	0.000	0.000	13.682	0.512

Problem 2095	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	227	250	441	686	0	0	0	0	-1	227
N.S.	1	1.10	1.94	3.02	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.735	0.793	0.567	0.000	0.000	0.000	0.000	0.000	5.869

Problem 2096	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	228	0	0	0	0	0	0	0	-1	228
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	10.445	3.192	0.089	0.000	0.000	0.000	0.000	0.000	3.161

Problem 2097	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	228	0	0	0	0	0	0	0	-1	228
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	4.140	1.743	0.474	0.000	0.000	0.000	0.000	0.000	21.047

Problem 2098	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	229	0	0	0	0	0	0	0	-1	229
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	5.922	3.740	0.075	0.000	0.000	0.000	0.000	0.000	5.714

Problem 2099	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	229	0	0	0	0	0	0	0	-1	229
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	5.171	3.723	0.071	0.000	0.000	0.000	0.000	0.000	5.690

Problem 2100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	A	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	229	508	326	286	0	746	0	0	-1	228
N.S.	1	2.22	1.42	1.25	0.00	3.26	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.638	1.639	0.057	0.000	47.634	0.000	0.000	0.000	1.222

Problem 2101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	229	508	326	332	0	746	0	0	-1	228
N.S.	1	2.22	1.42	1.45	0.00	3.26	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.218	1.305	0.062	0.000	40.907	0.000	0.000	0.000	1.458

Problem 2102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	230	0	0	0	0	0	0	0	-1	230
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	4.180	8.572	0.652	0.000	0.000	0.000	0.000	0.000	1.062

Problem 2103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	C	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	230	0	0	0	0	471	0	0	-1	230
N.S.	1	0.00	0.00	0.00	0.00	2.05	0.00	0.00	-0.00	1.00
time (sec)	N/A	6.347	0.353	1.098	0.000	2.640	0.000	0.000	0.000	1.008

Problem 2104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	480	126	2431	0	423	0	244	-1	230
N.S.	1	2.09	0.55	10.57	0.00	1.84	0.00	1.06	-0.00	1.00
time (sec)	N/A	1.048	0.176	6.602	0.000	0.542	0.000	0.483	0.000	0.807

Problem 2105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	231	136	116	874	0	1896	0	0	-1	0
N.S.	1	0.59	0.50	3.78	0.00	8.21	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.023	0.155	18.425	0.000	1.492	0.000	0.000	0.000	4.036

Problem 2106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	231	0	0	1175	0	324	0	0	-1	231
N.S.	1	0.00	0.00	5.09	0.00	1.40	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.305	0.205	9.193	0.000	10.164	0.000	0.000	0.000	1.260

Problem 2107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	231	0	0	0	0	0	0	0	-1	231
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	9.488	0.639	0.091	0.000	0.000	0.000	0.000	0.000	8.184

Problem 2108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	231	268	487	681	0	0	0	0	-1	261
N.S.	1	1.16	2.11	2.95	0.00	0.00	0.00	0.00	-0.00	1.13
time (sec)	N/A	0.849	1.597	0.582	0.000	0.000	0.000	0.000	0.000	5.772

Problem 2109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	232	0	0	0	0	0	0	0	-1	232
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	4.151	9.041	0.673	0.000	0.000	0.000	0.000	0.000	3.184

Problem 2110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	232	0	0	0	0	0	0	0	-1	232
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	15.935	5.292	0.105	0.000	0.000	0.000	0.000	0.000	3.540

Problem 2111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	232	0	0	0	0	0	0	0	-1	232
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	9.001	0.646	0.086	0.000	0.000	0.000	0.000	0.000	8.141

Problem 2112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	232	0	0	0	0	438	0	0	-1	232
N.S.	1	0.00	0.00	0.00	0.00	1.89	0.00	0.00	-0.00	1.00
time (sec)	N/A	4.564	0.362	0.467	0.000	0.626	0.000	0.000	0.000	7.392

Problem 2113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	B	F(-1)	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	233	0	55	0	0	1496	0	0	-1	233
N.S.	1	0.00	0.24	0.00	0.00	6.42	0.00	0.00	-0.00	1.00
time (sec)	N/A	22.316	0.189	0.055	0.000	24.478	0.000	0.000	0.000	0.541

Problem 2114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	233	0	0	0	0	0	0	0	-1	233
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	3.689	4.908	0.633	0.000	0.000	0.000	0.000	0.000	2.938

Problem 2115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	233	393	268	0	0	1300	0	232	-1	233
N.S.	1	1.69	1.15	0.00	0.00	5.58	0.00	1.00	-0.00	1.00
time (sec)	N/A	1.283	0.286	10.010	0.000	106.330	0.000	0.476	0.000	0.768

Problem 2116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F	F(-2)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	233	0	733	0	0	359	0	0	-1	233
N.S.	1	0.00	3.15	0.00	0.00	1.54	0.00	0.00	-0.00	1.00
time (sec)	N/A	2.098	12.150	0.439	0.000	37.469	0.000	0.000	0.000	3.530

Problem 2117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	A	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	234	316	137	0	0	387	0	171	-1	234
N.S.	1	1.35	0.59	0.00	0.00	1.65	0.00	0.73	-0.00	1.00
time (sec)	N/A	0.803	0.118	1.566	0.000	7.758	0.000	0.498	0.000	0.576

Problem 2118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	0	79	0	0	0	0	0	-1	234
N.S.	1	0.00	0.34	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	3.804	2.200	0.464	0.000	0.000	0.000	0.000	0.000	15.330

Problem 2119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	235	0	0	0	0	0	0	0	-1	235
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	10.028	3.995	0.123	0.000	0.000	0.000	0.000	0.000	4.291

Problem 2120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	235	0	0	0	0	0	0	0	-1	235
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	38.438	8.232	0.615	0.000	0.000	0.000	0.000	0.000	0.553

Problem 2121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	389	204	364	0	1793	0	0	-1	236
N.S.	1	1.65	0.86	1.54	0.00	7.60	0.00	0.00	-0.00	1.00
time (sec)	N/A	2.376	0.463	0.050	0.000	1.829	0.000	0.000	0.000	3.667

Problem 2122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	236	0	0	0	0	0	0	0	-1	236
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	6.506	1.330	0.073	0.000	0.000	0.000	0.000	0.000	5.640

Problem 2123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	236	0	0	0	0	0	0	0	-1	236
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	6.183	1.354	0.071	0.000	0.000	0.000	0.000	0.000	5.636

Problem 2124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F(-2)	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	236	915	0	0	482	0	0	-1	236
N.S.	1	1.00	3.88	0.00	0.00	2.04	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.728	8.042	180.000	0.000	0.602	0.000	0.000	0.000	0.468

Problem 2125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	236	0	0	0	0	129	0	0	-1	236
N.S.	1	0.00	0.00	0.00	0.00	0.55	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.468	4.511	0.477	0.000	0.473	0.000	0.000	0.000	0.352

Problem 2126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	236	0	0	0	0	119	0	0	-1	236
N.S.	1	0.00	0.00	0.00	0.00	0.50	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.338	2.741	0.441	0.000	0.399	0.000	0.000	0.000	0.322

Problem 2127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	237	233	68	0	0	297	0	325	-1	237
N.S.	1	0.98	0.29	0.00	0.00	1.25	0.00	1.37	-0.00	1.00
time (sec)	N/A	0.245	0.022	0.513	0.000	0.453	0.000	3.005	0.000	1.057

Problem 2128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	237	0	0	0	0	0	0	0	-1	237
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	31.660	0.620	0.102	0.000	0.000	0.000	0.000	0.000	6.876

Problem 2129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	238	226	108	0	0	418	0	0	-1	238
N.S.	1	0.95	0.45	0.00	0.00	1.76	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.353	0.243	0.501	0.000	21.863	0.000	0.000	0.000	0.583

Problem 2130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	238	0	0	0	0	0	0	0	-1	238
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	13.757	4.157	0.079	0.000	0.000	0.000	0.000	0.000	4.108

Problem 2131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	238	0	0	0	0	0	0	0	-1	238
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	32.195	6.614	0.578	0.000	0.000	0.000	0.000	0.000	0.513

Problem 2132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	238	253	176	676	0	1784	0	0	-1	233
N.S.	1	1.06	0.74	2.84	0.00	7.50	0.00	0.00	-0.00	0.98
time (sec)	N/A	0.633	2.793	0.579	0.000	173.972	0.000	0.000	0.000	1.353

Problem 2133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-1)	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	238	0	0	0	0	635	0	0	-1	238
N.S.	1	0.00	0.00	0.00	0.00	2.67	0.00	0.00	-0.00	1.00
time (sec)	N/A	2.920	1.105	180.000	0.000	65.916	0.000	0.000	0.000	5.342

Problem 2134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	F(-1)	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	239	0	574	487	0	2091	0	0	-1	239
N.S.	1	0.00	2.40	2.04	0.00	8.75	0.00	0.00	-0.00	1.00
time (sec)	N/A	3.668	2.176	0.118	0.000	0.815	0.000	0.000	0.000	1.283

Problem 2135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	F(-1)	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	239	0	574	482	0	2075	0	0	-1	239
N.S.	1	0.00	2.40	2.02	0.00	8.68	0.00	0.00	-0.00	1.00
time (sec)	N/A	3.379	1.921	0.115	0.000	0.838	0.000	0.000	0.000	1.212

Problem 2136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	239	346	292	0	0	0	0	0	-1	239
N.S.	1	1.45	1.22	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.887	0.315	0.497	0.000	0.000	0.000	0.000	0.000	1.007

Problem 2137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F	C
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	239	853	195	685	0	1173	0	0	-1	278
N.S.	1	3.57	0.82	2.87	0.00	4.91	0.00	0.00	-0.00	1.16
time (sec)	N/A	4.445	2.751	0.107	0.000	1.045	0.000	0.000	0.000	2.280

Problem 2138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	241	0	0	0	0	0	0	0	-1	241
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	15.690	3.292	0.065	0.000	0.000	0.000	0.000	0.000	1.122

Problem 2139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	241	0	0	0	0	0	0	0	-1	241
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.824	11.804	0.068	0.000	0.000	0.000	0.000	0.000	5.454

Problem 2140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	241	451	0	0	0	0	0	0	-1	241
N.S.	1	1.87	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	2.872	1.891	0.074	0.000	0.000	0.000	0.000	0.000	5.526

Problem 2141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	241	0	0	682	0	0	0	0	-1	241
N.S.	1	0.00	0.00	2.83	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.398	0.347	26.684	0.000	0.000	0.000	0.000	0.000	16.056

Problem 2142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F(-2)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	242	0	0	0	0	0	0	0	-1	242
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	18.807	2.324	0.195	0.000	0.000	0.000	0.000	0.000	1.157

Problem 2143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	242	0	0	0	0	420	0	0	-1	326
N.S.	1	0.00	0.00	0.00	0.00	1.74	0.00	0.00	-0.00	1.35
time (sec)	N/A	0.535	0.253	0.446	0.000	7.802	0.000	0.000	0.000	0.956

Problem 2144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	242	0	0	0	0	399	0	0	-1	326
N.S.	1	0.00	0.00	0.00	0.00	1.65	0.00	0.00	-0.00	1.35
time (sec)	N/A	0.456	0.150	0.450	0.000	3.778	0.000	0.000	0.000	1.072

Problem 2145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	242	0	373	0	0	118	0	0	-1	242
N.S.	1	0.00	1.54	0.00	0.00	0.49	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.154	0.646	0.468	0.000	0.457	0.000	0.000	0.000	0.309

Problem 2146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	243	418	71	0	0	167	0	108	-1	243
N.S.	1	1.72	0.29	0.00	0.00	0.69	0.00	0.44	-0.00	1.00
time (sec)	N/A	0.841	0.177	0.639	0.000	0.445	0.000	2.640	0.000	0.549

Problem 2147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	243	418	73	0	0	372	0	108	-1	243
N.S.	1	1.72	0.30	0.00	0.00	1.53	0.00	0.44	-0.00	1.00
time (sec)	N/A	0.613	0.125	0.576	0.000	0.507	0.000	0.499	0.000	0.501

Problem 2148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	243	418	71	0	0	167	0	108	-1	243
N.S.	1	1.72	0.29	0.00	0.00	0.69	0.00	0.44	-0.00	1.00
time (sec)	N/A	0.783	0.117	0.577	0.000	0.419	0.000	0.493	0.000	0.551

Problem 2149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F	F	A	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	243	139	0	0	0	370	0	0	-1	243
N.S.	1	0.57	0.00	0.00	0.00	1.52	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.104	0.188	0.945	0.000	2.446	0.000	0.000	0.000	4.976

Problem 2150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	243	385	85	2018	0	222	0	174	-1	243
N.S.	1	1.58	0.35	8.30	0.00	0.91	0.00	0.72	-0.00	1.00
time (sec)	N/A	1.012	0.082	5.511	0.000	0.435	0.000	0.856	0.000	0.647

Problem 2151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	243	0	0	0	0	0	0	0	-1	243
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.851	13.136	0.066	0.000	0.000	0.000	0.000	0.000	3.855

Problem 2152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	243	251	0	0	0	0	0	0	-1	243
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	2.439	1.882	0.077	0.000	0.000	0.000	0.000	0.000	5.551

Problem 2153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	F(-1)	C	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	243	0	196	31	0	0	49	0	-1	211
N.S.	1	0.00	0.81	0.13	0.00	0.00	0.20	0.00	-0.00	0.87
time (sec)	N/A	0.158	0.606	0.039	0.000	0.000	1.283	0.000	0.000	0.392

Problem 2154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	244	0	0	0	0	0	0	0	-1	244
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	7.404	3.870	0.524	0.000	0.000	0.000	0.000	0.000	0.659

Problem 2155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	244	271	181	669	0	1800	0	0	-1	266
N.S.	1	1.11	0.74	2.74	0.00	7.38	0.00	0.00	-0.00	1.09
time (sec)	N/A	0.637	3.495	0.564	0.000	112.402	0.000	0.000	0.000	1.300

Problem 2156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	244	519	255	828	0	1145	0	0	236	244
N.S.	1	2.13	1.05	3.39	0.00	4.69	0.00	0.00	0.97	1.00
time (sec)	N/A	3.095	2.110	0.113	0.000	0.708	0.000	0.000	8.575	0.941

Problem 2157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	245	0	0	0	0	0	0	0	-1	245
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	3.517	9.845	0.661	0.000	0.000	0.000	0.000	0.000	3.218

Problem 2158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	245	956	296	681	0	828	0	0	-1	245
N.S.	1	3.90	1.21	2.78	0.00	3.38	0.00	0.00	-0.00	1.00
time (sec)	N/A	8.420	2.615	0.049	0.000	0.834	0.000	0.000	0.000	0.612

Problem 2159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F(-1)	F	F	F(-2)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	245	0	0	0	0	0	0	0	-1	245
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	9.536	180.089	2.588	0.000	0.000	0.000	0.000	0.000	0.716

Problem 2160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	245	0	0	461	0	1425	0	0	-1	225
N.S.	1	0.00	0.00	1.88	0.00	5.82	0.00	0.00	-0.00	0.92
time (sec)	N/A	1.417	0.290	6.781	0.000	0.860	0.000	0.000	0.000	4.076

Problem 2161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	245	0	0	462	0	0	0	0	-1	225
N.S.	1	0.00	0.00	1.89	0.00	0.00	0.00	0.00	-0.00	0.92
time (sec)	N/A	1.137	0.279	11.334	0.000	0.000	0.000	0.000	0.000	13.338

Problem 2162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	245	0	0	0	0	0	0	0	-1	329
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.34
time (sec)	N/A	0.834	0.182	0.482	0.000	0.000	0.000	0.000	0.000	1.382

Problem 2163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	247	0	0	0	0	0	0	0	-1	247
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	6.221	2.133	0.487	0.000	0.000	0.000	0.000	0.000	23.144

Problem 2164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	B	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	247	744	442	864	0	10278	0	0	274	247
N.S.	1	3.01	1.79	3.50	0.00	41.61	0.00	0.00	1.11	1.00
time (sec)	N/A	3.597	0.813	0.037	0.000	15.871	0.000	0.000	14.496	0.887

Problem 2165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	248	0	0	0	0	1328	0	0	-1	253
N.S.	1	0.00	0.00	0.00	0.00	5.35	0.00	0.00	-0.00	1.02
time (sec)	N/A	8.719	0.310	1.523	0.000	26.594	0.000	0.000	0.000	1.874

Problem 2166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	0	225	0	0	0	0	0	-1	248
N.S.	1	0.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.332	13.648	0.471	0.000	0.000	0.000	0.000	0.000	2.472

Problem 2167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	F(-2)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	442	503	1634	0	0	0	395	-1	232
N.S.	1	1.78	2.03	6.59	0.00	0.00	0.00	1.59	-0.00	0.94
time (sec)	N/A	0.708	0.793	0.048	0.000	0.000	0.000	1.068	0.000	0.578

Problem 2168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	249	99	86	74	107	64	0	77	76	91
N.S.	1	0.40	0.35	0.30	0.43	0.26	0.00	0.31	0.31	0.37
time (sec)	N/A	0.108	0.099	0.047	0.518	0.523	0.000	0.155	1.688	0.357

Problem 2169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	249	408	68	0	0	357	0	100	-1	247
N.S.	1	1.64	0.27	0.00	0.00	1.43	0.00	0.40	-0.00	0.99
time (sec)	N/A	0.900	0.175	0.604	0.000	0.572	0.000	0.254	0.000	0.630

Problem 2170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	249	408	73	0	0	194	0	100	-1	247
N.S.	1	1.64	0.29	0.00	0.00	0.78	0.00	0.40	-0.00	0.99
time (sec)	N/A	0.619	0.108	0.579	0.000	1.137	0.000	0.249	0.000	0.525

Problem 2171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	249	0	0	0	0	0	0	0	-1	249
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	19.265	3.251	0.073	0.000	0.000	0.000	0.000	0.000	1.447

Problem 2172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	249	244	84	0	0	0	0	0	-1	237
N.S.	1	0.98	0.34	0.00	0.00	0.00	0.00	0.00	-0.00	0.95
time (sec)	N/A	0.414	0.049	0.114	0.000	0.000	0.000	0.000	0.000	0.305

Problem 2173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	250	179	37	110	0	169	0	0	-1	250
N.S.	1	0.72	0.15	0.44	0.00	0.68	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.089	0.038	0.036	0.000	0.850	0.000	0.000	0.000	5.527

Problem 2174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	250	0	0	0	0	0	0	0	-1	250
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	3.575	1.536	0.476	0.000	0.000	0.000	0.000	0.000	17.994

Problem 2175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	250	0	0	0	0	0	0	0	-1	250
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	7.405	1.906	0.530	0.000	0.000	0.000	0.000	0.000	1.156

Problem 2176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	250	89	204	0	0	103	0	0	60	250
N.S.	1	0.36	0.82	0.00	0.00	0.41	0.00	0.00	0.24	1.00
time (sec)	N/A	0.807	0.438	0.474	0.000	0.813	0.000	0.000	2.560	0.635

Problem 2177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	250	97	238	0	0	103	0	0	60	250
N.S.	1	0.39	0.95	0.00	0.00	0.41	0.00	0.00	0.24	1.00
time (sec)	N/A	0.730	0.751	0.483	0.000	1.866	0.000	0.000	2.539	0.422

Problem 2178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	251	0	0	0	0	0	0	0	-1	251
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	13.936	4.421	0.204	0.000	0.000	0.000	0.000	0.000	1.350

Problem 2179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	0	87	0	0	0	0	0	-1	252
N.S.	1	0.00	0.35	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	10.806	3.618	0.096	0.000	0.000	0.000	0.000	0.000	3.230

Problem 2180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	252	0	0	0	0	0	0	0	-1	252
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.770	0.988	0.201	0.000	0.000	0.000	0.000	0.000	0.509

Problem 2181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	0	27	11302	0	1084	0	0	-1	411
N.S.	1	0.00	0.11	44.67	0.00	4.28	0.00	0.00	-0.00	1.62
time (sec)	N/A	0.083	0.011	11.893	0.000	1.806	0.000	0.000	0.000	1.211

Problem 2182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	254	127	0	0	0	0	0	0	-1	254
N.S.	1	0.50	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.871	0.332	6.848	0.000	7.466	0.000	0.000	0.000	0.728

Problem 2183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	254	127	0	0	0	0	0	0	-1	254
N.S.	1	0.50	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.757	0.194	6.909	0.000	7.122	0.000	0.000	0.000	0.678

Problem 2184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	F	B	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	254	0	236	1771	0	521	0	0	-1	254
N.S.	1	0.00	0.93	6.97	0.00	2.05	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.737	0.503	4.948	0.000	5.968	0.000	0.000	0.000	0.659

Problem 2185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F(-2)	F	A	F	F(-1)	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	254	0	74	0	0	255	0	0	99	254
N.S.	1	0.00	0.29	0.00	0.00	1.00	0.00	0.00	0.39	1.00
time (sec)	N/A	1.094	0.344	180.000	0.000	0.686	0.000	0.000	2.186	0.965

Problem 2186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	416	186	4380	0	438	0	238	-1	255
N.S.	1	1.63	0.73	17.18	0.00	1.72	0.00	0.93	-0.00	1.00
time (sec)	N/A	0.454	0.433	11.671	0.000	0.603	0.000	1.121	0.000	1.037

Problem 2187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	137	77	0	0	498	0	0	-1	255
N.S.	1	0.54	0.30	0.00	0.00	1.95	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.516	0.138	0.490	0.000	0.593	0.000	0.000	0.000	0.473

Problem 2188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	255	0	0	0	0	0	0	0	-1	255
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	2.212	1.362	0.212	0.000	0.000	0.000	0.000	0.000	0.452

Problem 2189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	B	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	0	298	0	0	590	0	0	-1	255
N.S.	1	0.00	1.17	0.00	0.00	2.31	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.355	0.519	0.480	0.000	0.847	0.000	0.000	0.000	1.258

Problem 2190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	256	0	0	0	0	0	0	0	-1	256
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.874	14.271	0.072	0.000	0.000	0.000	0.000	0.000	4.969

Problem 2191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	256	275	0	0	0	0	0	0	-1	256
N.S.	1	1.07	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	2.572	2.502	0.082	0.000	0.000	0.000	0.000	0.000	5.020

Problem 2192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	A	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	257	435	231	0	0	1255	0	248	-1	257
N.S.	1	1.69	0.90	0.00	0.00	4.88	0.00	0.96	-0.00	1.00
time (sec)	N/A	0.805	2.068	12.334	0.000	70.418	0.000	1.918	0.000	0.964

Problem 2193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	257	0	0	0	0	0	0	0	-1	257
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	11.278	8.800	0.060	0.000	0.000	0.000	0.000	0.000	3.034

Problem 2194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F(-2)	F	A	F	F(-1)	B	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	257	0	74	0	0	758	0	0	99	257
N.S.	1	0.00	0.29	0.00	0.00	2.95	0.00	0.00	0.39	1.00
time (sec)	N/A	1.243	0.296	180.000	0.000	0.909	0.000	0.000	2.600	0.566

Problem 2195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	258	0	0	0	0	0	0	0	-1	258
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	8.073	7.304	0.658	0.000	0.000	0.000	0.000	0.000	3.461

Problem 2196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	258	0	0	0	0	0	0	0	-1	258
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	9.106	1.264	0.536	0.000	0.000	0.000	0.000	0.000	5.432

Problem 2197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	B	F	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	261	672	168	4137	0	474	0	261	-1	261
N.S.	1	2.57	0.64	15.85	0.00	1.82	0.00	1.00	-0.00	1.00
time (sec)	N/A	1.149	0.258	10.513	0.000	0.908	0.000	1.098	0.000	1.592

Problem 2198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	261	0	0	684	0	0	0	0	-1	241
N.S.	1	0.00	0.00	2.62	0.00	0.00	0.00	0.00	-0.00	0.92
time (sec)	N/A	1.384	0.308	26.274	0.000	0.000	0.000	0.000	0.000	16.138

Problem 2199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F(-2)	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	261	310	273	0	0	1174	0	0	-1	261
N.S.	1	1.19	1.05	0.00	0.00	4.50	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.612	0.887	180.000	0.000	0.708	0.000	0.000	0.000	1.126

Problem 2200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	261	0	0	0	0	517	0	0	-1	341
N.S.	1	0.00	0.00	0.00	0.00	1.98	0.00	0.00	-0.00	1.31
time (sec)	N/A	0.721	0.302	0.479	0.000	8.833	0.000	0.000	0.000	2.400

Problem 2201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	262	0	0	0	0	0	0	0	-1	262
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	12.898	3.504	0.823	0.000	0.000	0.000	0.000	0.000	7.205

Problem 2202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	262	0	6061	82800	0	4887	0	0	-1	262
N.S.	1	0.00	23.13	316.03	0.00	18.65	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.355	3.136	0.033	0.000	7.108	0.000	0.000	0.000	2.333

Problem 2203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	262	0	0	0	0	517	0	0	-1	342
N.S.	1	0.00	0.00	0.00	0.00	1.97	0.00	0.00	-0.00	1.31
time (sec)	N/A	1.215	0.340	0.516	0.000	11.462	0.000	0.000	0.000	1.993

Problem 2204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	263	0	0	0	0	0	0	0	-1	263
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	11.198	3.601	0.074	0.000	0.000	0.000	0.000	0.000	0.669

Problem 2205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	F(-1)	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	263	163	159	0	209	0	80	0	165	263
N.S.	1	0.62	0.60	0.00	0.79	0.00	0.30	0.00	0.63	1.00
time (sec)	N/A	0.149	0.177	0.520	0.438	0.000	3.107	0.000	2.134	4.647

Problem 2206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	263	350	305	268	0	131	0	0	-1	227
N.S.	1	1.33	1.16	1.02	0.00	0.50	0.00	0.00	-0.00	0.86
time (sec)	N/A	1.077	0.646	0.064	0.000	5.939	0.000	0.000	0.000	1.549

Problem 2207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	265	0	5410	81718	0	4886	0	0	-1	265
N.S.	1	0.00	20.42	308.37	0.00	18.44	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.460	2.691	0.033	0.000	3.707	0.000	0.000	0.000	2.159

Problem 2208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	265	0	0	0	0	0	0	0	-1	265
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	14.056	1.929	0.102	0.000	0.000	0.000	0.000	0.000	3.407

Problem 2209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	F	F	B	F(-1)	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	265	173	259	0	0	552	0	0	-1	265
N.S.	1	0.65	0.98	0.00	0.00	2.08	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.851	0.408	1.464	0.000	11.219	0.000	0.000	0.000	0.765

Problem 2210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	F(-1)	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	265	0	199	0	0	0	0	0	-1	288
N.S.	1	0.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00	1.09
time (sec)	N/A	0.396	0.357	0.487	0.000	0.000	0.000	0.000	0.000	0.953

Problem 2211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F	F	F(-1)	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	266	134	0	0	0	0	0	0	-1	266
N.S.	1	0.50	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.073	0.104	0.512	0.000	0.000	0.000	0.000	0.000	1.415

Problem 2212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	266	0	331	0	0	0	0	0	-1	266
N.S.	1	0.00	1.24	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.496	1.438	0.476	0.000	0.000	0.000	0.000	0.000	3.850

Problem 2213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	267	239	62	0	0	715	0	0	-1	267
N.S.	1	0.90	0.23	0.00	0.00	2.68	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.634	0.064	0.524	0.000	0.904	0.000	0.000	0.000	5.865

Problem 2214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	F(-1)	F	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	267	0	52549	7274	0	0	0	0	-1	0
N.S.	1	0.00	196.81	27.24	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	13.392	7.396	2.391	0.000	0.000	0.000	0.000	0.000	180.001

Problem 2215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	268	583	13957	314	0	2407	0	0	-1	268
N.S.	1	2.18	52.08	1.17	0.00	8.98	0.00	0.00	-0.00	1.00
time (sec)	N/A	5.963	18.669	0.072	0.000	6.995	0.000	0.000	0.000	1.092

Problem 2216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F	A	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	268	0	738	0	0	167	0	1	-1	0
N.S.	1	0.00	2.75	0.00	0.00	0.62	0.00	0.00	-0.00	0.00
time (sec)	N/A	2.965	1.883	0.490	0.000	0.527	0.000	1.136	0.000	180.002

Problem 2217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	41	23	18	64	54	162	133	49	34
N.S.	1	0.15	0.09	0.07	0.24	0.20	0.60	0.49	0.18	0.13
time (sec)	N/A	0.007	0.013	0.003	0.309	0.645	5.165	0.191	2.128	0.060

Problem 2218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	269	0	0	0	0	0	0	0	-1	269
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	15.309	3.389	0.076	0.000	0.000	0.000	0.000	0.000	0.738

Problem 2219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	436	226	4526	0	462	0	260	-1	269
N.S.	1	1.62	0.84	16.83	0.00	1.72	0.00	0.97	-0.00	1.00
time (sec)	N/A	0.454	0.190	11.621	0.000	0.494	0.000	0.696	0.000	1.057

Problem 2220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	0	188	0	0	152	0	0	-1	269
N.S.	1	0.00	0.70	0.00	0.00	0.57	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.078	1.229	0.072	0.000	0.447	0.000	0.000	0.000	0.457

Problem 2221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	A	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	270	0	0	0	0	0	0	289	-1	270
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	1.07	-0.00	1.00
time (sec)	N/A	21.235	3.904	0.094	0.000	0.000	0.000	2.632	0.000	0.631

Problem 2222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	270	0	0	0	0	0	0	0	-1	270
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	10.261	1.548	0.069	0.000	0.000	0.000	0.000	0.000	6.987

Problem 2223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	F(-1)	F(-1)	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	270	1043	276	541	0	0	0	0	-1	598
N.S.	1	3.86	1.02	2.00	0.00	0.00	0.00	0.00	-0.00	2.21
time (sec)	N/A	9.287	1.799	0.104	0.000	0.000	0.000	0.000	0.000	26.424

Problem 2224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	272	0	0	0	0	0	0	0	-1	272
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	7.482	4.352	0.570	0.000	0.000	0.000	0.000	0.000	0.792

Problem 2225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	273	513	88	0	0	740	0	0	-1	273
N.S.	1	1.88	0.32	0.00	0.00	2.71	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.128	0.279	0.687	0.000	0.649	0.000	0.000	0.000	6.541

Problem 2226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	A	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	273	309	0	0	0	373	0	0	-1	389
N.S.	1	1.13	0.00	0.00	0.00	1.37	0.00	0.00	-0.00	1.42
time (sec)	N/A	1.687	0.243	0.506	0.000	14.708	0.000	0.000	0.000	1.824

Problem 2227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	C
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	274	0	0	0	0	3724	0	0	-1	269
N.S.	1	0.00	0.00	0.00	0.00	13.59	0.00	0.00	-0.00	0.98
time (sec)	N/A	2.735	0.887	98.645	0.000	16.936	0.000	0.000	0.000	1.017

Problem 2228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	274	0	0	0	0	436	0	0	-1	274
N.S.	1	0.00	0.00	0.00	0.00	1.59	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.105	0.176	0.466	0.000	2.257	0.000	0.000	0.000	1.509

Problem 2229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	275	70	65	0	0	0	0	0	-1	318
N.S.	1	0.25	0.24	0.00	0.00	0.00	0.00	0.00	-0.00	1.16
time (sec)	N/A	0.230	0.051	0.517	0.000	0.000	0.000	0.000	0.000	0.994

Problem 2230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	275	0	0	694	0	0	0	0	-1	257
N.S.	1	0.00	0.00	2.52	0.00	0.00	0.00	0.00	-0.00	0.93
time (sec)	N/A	1.231	0.248	12.634	0.000	0.000	0.000	0.000	0.000	2.561

Problem 2231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	B	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	276	707	232	0	0	1239	0	704	-1	276
N.S.	1	2.56	0.84	0.00	0.00	4.49	0.00	2.55	-0.00	1.00
time (sec)	N/A	1.760	0.750	0.510	0.000	0.640	0.000	0.331	0.000	1.537

Problem 2232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	276	0	0	0	0	351	0	0	-1	276
N.S.	1	0.00	0.00	0.00	0.00	1.27	0.00	0.00	-0.00	1.00
time (sec)	N/A	8.634	2.845	0.070	0.000	0.772	0.000	0.000	0.000	3.243

Problem 2233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	A	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	276	0	0	0	0	0	0	123	-1	274
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.45	-0.00	0.99
time (sec)	N/A	15.637	5.416	0.082	0.000	0.000	0.000	0.369	0.000	0.684

Problem 2234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	276	169	157	0	0	134	0	0	-1	276
N.S.	1	0.61	0.57	0.00	0.00	0.49	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.131	0.187	0.053	0.000	0.814	0.000	0.000	0.000	0.484

Problem 2235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	278	273	56	1046	0	236	0	0	-1	218
N.S.	1	0.98	0.20	3.76	0.00	0.85	0.00	0.00	-0.00	0.78
time (sec)	N/A	1.446	0.031	2.961	0.000	0.639	0.000	0.000	0.000	17.535

Problem 2236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	A	F	F(-2)	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	278	1306	156	0	0	959	0	0	-1	278
N.S.	1	4.70	0.56	0.00	0.00	3.45	0.00	0.00	-0.00	1.00
time (sec)	N/A	147.513	0.877	0.593	0.000	1.427	0.000	0.000	0.000	1.238

Problem 2237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F(-2)	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	278	278	374	0	0	781	0	0	-1	278
N.S.	1	1.00	1.35	0.00	0.00	2.81	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.087	1.275	180.000	0.000	0.996	0.000	0.000	0.000	0.737

Problem 2238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	279	0	0	0	0	0	0	0	-1	279
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	11.770	4.409	0.075	0.000	0.000	0.000	0.000	0.000	3.914

Problem 2239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	279	0	0	0	0	0	0	0	-1	279
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	11.025	5.499	0.066	0.000	0.000	0.000	0.000	0.000	3.083

Problem 2240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F(-2)	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	282	232	1949	0	0	167	0	0	-1	282
N.S.	1	0.82	6.91	0.00	0.00	0.59	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.226	7.646	180.000	0.000	1.470	0.000	0.000	0.000	0.359

Problem 2241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	285	0	0	0	0	932	0	0	-1	285
N.S.	1	0.00	0.00	0.00	0.00	3.27	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.495	0.658	0.503	0.000	94.953	0.000	0.000	0.000	3.211

Problem 2242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	286	0	0	0	0	359	0	0	-1	286
N.S.	1	0.00	0.00	0.00	0.00	1.26	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.340	9.266	180.000	0.000	2.965	0.000	0.000	0.000	0.720

Problem 2243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	A	F	F(-2)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	287	0	0	0	0	284	0	0	-1	287
N.S.	1	0.00	0.00	0.00	0.00	0.99	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.698	0.144	180.000	0.000	3.245	0.000	0.000	0.000	1.101

Problem 2244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	288	445	53	0	0	0	0	256	-1	291
N.S.	1	1.55	0.18	0.00	0.00	0.00	0.00	0.89	-0.00	1.01
time (sec)	N/A	0.579	0.041	0.605	0.000	0.000	0.000	0.412	0.000	0.804

Problem 2245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	288	0	0	7357	0	456	0	0	-1	288
N.S.	1	0.00	0.00	25.55	0.00	1.58	0.00	0.00	-0.00	1.00
time (sec)	N/A	2.464	0.723	55.671	0.000	7.011	0.000	0.000	0.000	0.900

Problem 2246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	288	0	0	0	0	566	0	0	-1	288
N.S.	1	0.00	0.00	0.00	0.00	1.97	0.00	0.00	-0.00	1.00
time (sec)	N/A	4.135	0.436	0.484	0.000	0.778	0.000	0.000	0.000	8.728

Problem 2247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	289	0	0	0	0	0	0	0	-1	289
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	6.618	2.580	0.551	0.000	0.000	0.000	0.000	0.000	1.239

Problem 2248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	289	0	0	0	0	0	0	0	-1	289
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	12.307	4.857	0.083	0.000	0.000	0.000	0.000	0.000	3.873

Problem 2249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	B	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	289	109	0	0	0	534	0	0	-1	289
N.S.	1	0.38	0.00	0.00	0.00	1.85	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.336	0.347	1.429	0.000	13.888	0.000	0.000	0.000	0.736

Problem 2250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	B	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	0	73	0	0	584	0	0	-1	289
N.S.	1	0.00	0.25	0.00	0.00	2.02	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.314	0.183	0.492	0.000	2.044	0.000	0.000	0.000	1.301

Problem 2251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	290	0	0	0	0	0	0	0	-1	290
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	6.181	7.296	0.626	0.000	0.000	0.000	0.000	0.000	3.501

Problem 2252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	290	190	184	0	250	0	83	0	-1	290
N.S.	1	0.66	0.63	0.00	0.86	0.00	0.29	0.00	-0.00	1.00
time (sec)	N/A	0.169	0.265	0.508	0.878	0.000	3.089	0.000	0.000	4.879

Problem 2253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	291	0	0	0	0	0	0	0	-1	291
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	7.247	5.768	0.657	0.000	0.000	0.000	0.000	0.000	0.618

Problem 2254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	291	0	0	0	0	0	0	0	-1	291
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	8.958	3.634	0.079	0.000	0.000	0.000	0.000	0.000	0.722

Problem 2255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	291	0	0	0	0	798	0	0	-1	291
N.S.	1	0.00	0.00	0.00	0.00	2.74	0.00	0.00	-0.00	1.00
time (sec)	N/A	7.491	2.879	0.068	0.000	1.223	0.000	0.000	0.000	3.168

Problem 2256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	291	309	105	0	0	441	0	505	-1	293
N.S.	1	1.06	0.36	0.00	0.00	1.52	0.00	1.74	-0.00	1.01
time (sec)	N/A	0.398	0.066	0.533	0.000	0.676	0.000	0.513	0.000	1.455

Problem 2257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	A	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	291	0	0	0	0	0	0	317	-1	291
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	1.09	-0.00	1.00
time (sec)	N/A	48.536	1.863	0.111	0.000	0.000	0.000	1.254	0.000	1.281

Problem 2258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F(-1)	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	291	0	0	1434	0	1837	0	0	-1	291
N.S.	1	0.00	0.00	4.93	0.00	6.31	0.00	0.00	-0.00	1.00
time (sec)	N/A	180.009	0.996	28.144	0.000	78.741	0.000	0.000	0.000	3.165

Problem 2259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F(-2)	A	F(-1)	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	293	385	244	1907	0	491	0	0	-1	293
N.S.	1	1.31	0.83	6.51	0.00	1.68	0.00	0.00	-0.00	1.00
time (sec)	N/A	3.755	2.813	0.061	0.000	1.689	0.000	0.000	0.000	2.732

Problem 2260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	B	F(-1)	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	293	298	0	1017	0	1982	0	0	-1	273
N.S.	1	1.02	0.00	3.47	0.00	6.76	0.00	0.00	-0.00	0.93
time (sec)	N/A	0.285	0.066	11.168	0.000	0.797	0.000	0.000	0.000	1.273

Problem 2261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	293	0	220	0	0	359	0	0	-1	293
N.S.	1	0.00	0.75	0.00	0.00	1.23	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.075	0.591	0.013	0.000	0.775	0.000	0.000	0.000	0.588

Problem 2262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	B	F(-1)	A	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	294	0	219	0	0	557	0	209	-1	426
N.S.	1	0.00	0.74	0.00	0.00	1.89	0.00	0.71	-0.00	1.45
time (sec)	N/A	1.786	0.702	0.512	0.000	0.756	0.000	0.288	0.000	1.314

Problem 2263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F(-2)	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	294	0	0	0	0	0	0	0	-1	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.437	0.280	0.525	0.000	0.000	0.000	0.000	0.000	4.313

Problem 2264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	296	0	0	0	0	0	0	0	-1	296
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	8.972	3.289	0.778	0.000	0.000	0.000	0.000	0.000	7.270

Problem 2265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	297	0	0	0	0	0	0	0	-1	295
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.99
time (sec)	N/A	24.683	2.659	0.096	0.000	0.000	0.000	0.000	0.000	1.166

Problem 2266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F(-1)	F	C	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	297	0	0	1422	0	1837	0	0	-1	297
N.S.	1	0.00	0.00	4.79	0.00	6.19	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.000	0.883	24.181	0.000	59.493	0.000	0.000	0.000	1.338

Problem 2267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	F(-1)	F(-1)	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	297	506	373	1190	0	0	0	0	-1	624
N.S.	1	1.70	1.26	4.01	0.00	0.00	0.00	0.00	-0.00	2.10
time (sec)	N/A	1.708	1.321	0.119	0.000	0.000	0.000	0.000	0.000	4.621

Problem 2268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	299	338	221	1031	0	215	0	0	-1	178
N.S.	1	1.13	0.74	3.45	0.00	0.72	0.00	0.00	-0.00	0.60
time (sec)	N/A	0.766	0.743	0.050	0.000	2.508	0.000	0.000	0.000	1.379

Problem 2269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	301	240	57	0	0	290	0	0	-1	301
N.S.	1	0.80	0.19	0.00	0.00	0.96	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.599	0.062	0.528	0.000	0.469	0.000	0.000	0.000	5.886

Problem 2270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	301	391	524	0	0	269	0	0	-1	301
N.S.	1	1.30	1.74	0.00	0.00	0.89	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.252	8.628	0.503	0.000	0.583	0.000	0.000	0.000	0.690

Problem 2271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	F(-1)	F(-1)	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	303	522	379	1130	0	0	0	0	-1	612
N.S.	1	1.72	1.25	3.73	0.00	0.00	0.00	0.00	-0.00	2.02
time (sec)	N/A	1.245	1.404	0.064	0.000	0.000	0.000	0.000	0.000	2.978

Problem 2272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	304	457	54	0	0	0	0	272	-1	307
N.S.	1	1.50	0.18	0.00	0.00	0.00	0.00	0.89	-0.00	1.01
time (sec)	N/A	0.530	0.043	0.579	0.000	0.000	0.000	0.247	0.000	0.775

Problem 2273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	304	343	379	0	0	463	0	0	-1	304
N.S.	1	1.13	1.25	0.00	0.00	1.52	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.765	1.080	0.489	0.000	0.525	0.000	0.000	0.000	0.691

Problem 2274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	305	513	91	0	0	331	0	0	-1	305
N.S.	1	1.68	0.30	0.00	0.00	1.09	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.227	0.406	0.714	0.000	3.396	0.000	0.000	0.000	6.536

Problem 2275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	305	513	91	0	0	331	0	0	-1	305
N.S.	1	1.68	0.30	0.00	0.00	1.09	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.858	0.270	0.639	0.000	0.655	0.000	0.000	0.000	6.454

Problem 2276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	306	0	0	0	0	0	0	0	-1	306
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	5.658	8.167	0.674	0.000	0.000	0.000	0.000	0.000	0.630

Problem 2277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	306	0	0	0	0	0	0	0	-1	306
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	8.920	3.543	0.079	0.000	0.000	0.000	0.000	0.000	0.715

Problem 2278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	306	219	216	0	301	0	82	0	-1	306
N.S.	1	0.72	0.71	0.00	0.98	0.00	0.27	0.00	-0.00	1.00
time (sec)	N/A	0.240	0.297	0.543	1.188	0.000	3.404	0.000	0.000	4.914

Problem 2279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	306	256	255	645	0	1106	0	210	-1	306
N.S.	1	0.84	0.83	2.11	0.00	3.61	0.00	0.69	-0.00	1.00
time (sec)	N/A	1.738	0.457	0.062	0.000	0.881	0.000	0.195	0.000	0.727

Problem 2280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	F(-2)	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	308	454	0	2482	0	0	0	0	-1	308
N.S.	1	1.47	0.00	8.06	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.016	0.241	18.744	0.000	0.000	0.000	0.000	0.000	0.385

Problem 2281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	310	0	0	0	0	0	0	0	-1	274
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.88
time (sec)	N/A	12.787	3.362	0.085	0.000	0.000	0.000	0.000	0.000	6.927

Problem 2282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	311	0	0	0	0	0	0	0	-1	311
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	9.851	9.124	0.702	0.000	0.000	0.000	0.000	0.000	3.171

Problem 2283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	311	0	0	0	0	0	0	0	-1	309
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.99
time (sec)	N/A	15.346	3.162	0.063	0.000	0.000	0.000	0.000	0.000	0.832

Problem 2284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	B	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	311	1295	294	475	0	2045	0	0	-1	311
N.S.	1	4.16	0.95	1.53	0.00	6.58	0.00	0.00	-0.00	1.00
time (sec)	N/A	8.751	3.059	0.083	0.000	1.779	0.000	0.000	0.000	0.910

Problem 2285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	311	0	0	0	0	433	0	0	-1	440
N.S.	1	0.00	0.00	0.00	0.00	1.39	0.00	0.00	-0.00	1.41
time (sec)	N/A	1.786	0.273	0.493	0.000	5.150	0.000	0.000	0.000	2.009

Problem 2286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	A	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	312	125	0	0	0	394	0	0	-1	312
N.S.	1	0.40	0.00	0.00	0.00	1.26	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.194	0.266	0.481	0.000	2.068	0.000	0.000	0.000	3.224

Problem 2287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	313	0	0	0	0	0	0	0	-1	313
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	13.530	3.566	0.069	0.000	0.000	0.000	0.000	0.000	0.743

Problem 2288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	313	135	201	981	0	162	0	0	-1	247
N.S.	1	0.43	0.64	3.13	0.00	0.52	0.00	0.00	-0.00	0.79
time (sec)	N/A	0.713	0.632	0.050	0.000	0.845	0.000	0.000	0.000	1.183

Problem 2289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	313	365	340	902	0	424	0	367	-1	231
N.S.	1	1.17	1.09	2.88	0.00	1.35	0.00	1.17	-0.00	0.74
time (sec)	N/A	0.565	0.808	0.145	0.000	0.457	0.000	6.008	0.000	8.917

Problem 2290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	315	407	308	0	0	0	0	284	-1	318
N.S.	1	1.29	0.98	0.00	0.00	0.00	0.00	0.90	-0.00	1.01
time (sec)	N/A	0.524	0.594	0.519	0.000	0.000	0.000	0.373	0.000	2.909

Problem 2291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	315	0	0	0	0	0	0	0	-1	315
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	19.096	3.378	0.085	0.000	0.000	0.000	0.000	0.000	2.015

Problem 2292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	316	0	0	0	0	159	0	0	-1	316
N.S.	1	0.00	0.00	0.00	0.00	0.50	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.429	5.190	0.479	0.000	0.409	0.000	0.000	0.000	0.650

Problem 2293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	317	0	0	0	0	0	0	0	-1	317
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	8.665	1.277	0.597	0.000	0.000	0.000	0.000	0.000	5.384

Problem 2294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	318	0	0	3484	0	0	0	0	-1	318
N.S.	1	0.00	0.00	10.96	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	15.704	1.928	107.009	0.000	0.000	0.000	0.000	0.000	5.099

Problem 2295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	319	291	56	0	0	549	0	0	-1	365
N.S.	1	0.91	0.18	0.00	0.00	1.72	0.00	0.00	-0.00	1.14
time (sec)	N/A	0.137	0.032	0.569	0.000	0.868	0.000	0.000	0.000	2.069

Problem 2296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	A	F(-1)	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	319	0	893	0	0	337	0	0	-1	319
N.S.	1	0.00	2.80	0.00	0.00	1.06	0.00	0.00	-0.00	1.00
time (sec)	N/A	7.076	1.758	0.056	0.000	1.206	0.000	0.000	0.000	3.149

Problem 2297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	319	120	205	921	0	166	0	0	-1	253
N.S.	1	0.38	0.64	2.89	0.00	0.52	0.00	0.00	-0.00	0.79
time (sec)	N/A	0.076	0.472	0.048	0.000	2.443	0.000	0.000	0.000	0.934

Problem 2298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	319	0	74	0	0	770	0	0	-1	319
N.S.	1	0.00	0.23	0.00	0.00	2.41	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.264	0.306	180.000	0.000	1.228	0.000	0.000	0.000	0.970

Problem 2299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	324	0	0	0	0	0	0	0	-1	324
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	52.337	3.337	0.096	0.000	0.000	0.000	0.000	0.000	4.388

Problem 2300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	F(-1)	F	F(-2)	F(-1)	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	325	176	265	0	0	0	0	0	-1	325
N.S.	1	0.54	0.82	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.564	0.752	180.000	0.000	0.000	0.000	0.000	0.000	2.176

Problem 2301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	326	341	383	0	0	467	0	0	-1	303
N.S.	1	1.05	1.17	0.00	0.00	1.43	0.00	0.00	-0.00	0.93
time (sec)	N/A	0.817	0.960	0.522	0.000	0.501	0.000	0.000	0.000	0.493

Problem 2302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	A	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	328	160	0	2437	0	470	0	0	-1	312
N.S.	1	0.49	0.00	7.43	0.00	1.43	0.00	0.00	-0.00	0.95
time (sec)	N/A	2.089	1.790	46.884	0.000	2.577	0.000	0.000	0.000	0.725

Problem 2303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	329	624	171	0	487	6305	0	1490	1566	711
N.S.	1	1.90	0.52	0.00	1.48	19.16	0.00	4.53	4.76	2.16
time (sec)	N/A	0.767	0.171	0.105	1.653	2.145	0.000	7.673	3.768	1.165

Problem 2304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	329	20	199	298	0	51	0	0	-1	52
N.S.	1	0.06	0.60	0.91	0.00	0.16	0.00	0.00	-0.00	0.16
time (sec)	N/A	0.149	0.955	0.115	0.000	0.548	0.000	0.000	0.000	1.375

Problem 2305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	330	488	188	6304	0	305	0	0	-1	308
N.S.	1	1.48	0.57	19.10	0.00	0.92	0.00	0.00	-0.00	0.93
time (sec)	N/A	0.551	0.115	49.128	0.000	0.442	0.000	0.000	0.000	8.000

Problem 2306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	F(-1)	F	F(-2)	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	330	0	10907	0	0	0	0	0	-1	333
N.S.	1	0.00	33.05	0.00	0.00	0.00	0.00	0.00	-0.00	1.01
time (sec)	N/A	0.467	20.844	0.486	0.000	0.000	0.000	0.000	0.000	3.717

Problem 2307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	331	417	310	0	0	0	0	300	-1	334
N.S.	1	1.26	0.94	0.00	0.00	0.00	0.00	0.91	-0.00	1.01
time (sec)	N/A	0.546	0.602	0.538	0.000	0.000	0.000	0.275	0.000	3.192

Problem 2308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F	F	F(-1)	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	332	139	0	0	0	0	0	0	-1	385
N.S.	1	0.42	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.16
time (sec)	N/A	0.076	0.085	0.537	0.000	0.000	0.000	0.000	0.000	2.408

Problem 2309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	334	309	275	1232	0	707	0	0	-1	334
N.S.	1	0.93	0.82	3.69	0.00	2.12	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.334	0.531	0.023	0.000	81.286	0.000	0.000	0.000	2.193

Problem 2310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	F	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	339	993	319	0	0	325	0	572	-1	339
N.S.	1	2.93	0.94	0.00	0.00	0.96	0.00	1.69	-0.00	1.00
time (sec)	N/A	1.066	0.282	0.517	0.000	0.424	0.000	0.486	0.000	4.716

Problem 2311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	340	0	0	0	0	0	0	0	-1	340
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	23.643	3.365	0.090	0.000	0.000	0.000	0.000	0.000	0.842

Problem 2312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	343	0	0	0	0	0	0	0	-1	276
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.80
time (sec)	N/A	10.656	4.589	0.086	0.000	0.000	0.000	0.000	0.000	5.337

Problem 2313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	345	150	0	0	0	0	0	0	-1	345
N.S.	1	0.43	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.296	0.827	0.564	0.000	0.000	0.000	0.000	0.000	6.154

Problem 2314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	347	0	0	0	0	811	0	0	-1	347
N.S.	1	0.00	0.00	0.00	0.00	2.34	0.00	0.00	-0.00	1.00
time (sec)	N/A	7.863	2.305	0.058	0.000	1.236	0.000	0.000	0.000	3.211

Problem 2315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	F(-1)	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	351	167	0	2164	0	0	0	0	-1	377
N.S.	1	0.48	0.00	6.17	0.00	0.00	0.00	0.00	-0.00	1.07
time (sec)	N/A	0.761	0.249	53.726	0.000	0.000	0.000	0.000	0.000	3.643

Problem 2316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	351	0	12131	0	0	0	0	0	-1	351
N.S.	1	0.00	34.56	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.313	24.907	0.530	0.000	0.000	0.000	0.000	0.000	3.597

Problem 2317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F	F	F(-1)	F	F(-1)	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	352	0	12183	0	0	0	0	0	-1	352
N.S.	1	0.00	34.61	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	2.748	7.139	0.513	0.000	0.000	0.000	0.000	0.000	4.873

Problem 2318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	352	0	0	0	0	0	0	0	-1	352
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.761	1.248	0.248	0.000	0.000	0.000	0.000	0.000	0.552

Problem 2319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	352	0	0	0	0	0	0	0	-1	352
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.823	1.285	0.268	0.000	0.000	0.000	0.000	0.000	0.609

Problem 2320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	354	229	250	0	0	0	0	0	-1	354
N.S.	1	0.65	0.71	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.082	0.469	0.535	0.000	0.000	0.000	0.000	0.000	0.597

Problem 2321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	355	0	0	0	0	0	0	0	-1	355
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	27.691	3.481	0.073	0.000	0.000	0.000	0.000	0.000	1.281

Problem 2322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	A	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	356	152	0	2987	0	418	0	0	-1	360
N.S.	1	0.43	0.00	8.39	0.00	1.17	0.00	0.00	-0.00	1.01
time (sec)	N/A	1.576	1.342	46.145	0.000	2.485	0.000	0.000	0.000	0.950

Problem 2323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	362	277	271	1306	0	854	0	0	-1	366
N.S.	1	0.77	0.75	3.61	0.00	2.36	0.00	0.00	-0.00	1.01
time (sec)	N/A	0.317	0.635	0.020	0.000	27.541	0.000	0.000	0.000	1.869

Problem 2324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	362	0	0	0	0	553	0	0	-1	483
N.S.	1	0.00	0.00	0.00	0.00	1.53	0.00	0.00	-0.00	1.33
time (sec)	N/A	1.705	0.530	0.508	0.000	5.676	0.000	0.000	0.000	7.324

Problem 2325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	363	148	143	266	0	0	0	127	-1	155
N.S.	1	0.41	0.39	0.73	0.00	0.00	0.00	0.35	-0.00	0.43
time (sec)	N/A	0.305	0.195	0.010	0.000	0.000	0.000	6.044	0.000	0.397

Problem 2326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	367	523	168	0	0	0	0	354	-1	367
N.S.	1	1.43	0.46	0.00	0.00	0.00	0.00	0.96	-0.00	1.00
time (sec)	N/A	0.864	0.255	0.523	0.000	0.000	0.000	0.893	0.000	1.222

Problem 2327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	369	0	0	0	0	0	0	0	-1	455
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.23
time (sec)	N/A	10.149	9.929	0.739	0.000	0.000	0.000	0.000	0.000	6.378

Problem 2328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	370	0	0	0	0	735	0	0	-1	370
N.S.	1	0.00	0.00	0.00	0.00	1.99	0.00	0.00	-0.00	1.00
time (sec)	N/A	4.935	1.392	1.832	0.000	18.008	0.000	0.000	0.000	1.343

Problem 2329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	371	0	0	0	0	0	0	0	-1	371
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	2.313	0.618	0.523	0.000	0.000	0.000	0.000	0.000	16.744

Problem 2330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-2)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	383	0	0	0	0	0	0	0	-1	383
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	35.027	1.669	0.102	0.000	0.000	0.000	0.000	0.000	1.870

Problem 2331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-2)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	383	0	0	0	0	0	0	0	-1	383
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	28.933	7.618	0.083	0.000	0.000	0.000	0.000	0.000	1.422

Problem 2332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	F(-2)	F	B	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	383	0	76	0	0	761	0	0	-1	355
N.S.	1	0.00	0.20	0.00	0.00	1.99	0.00	0.00	-0.00	0.93
time (sec)	N/A	1.219	0.360	180.000	0.000	0.647	0.000	0.000	0.000	22.293

Problem 2333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	383	0	444	0	0	210	0	0	-1	383
N.S.	1	0.00	1.16	0.00	0.00	0.55	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.197	2.143	0.015	0.000	0.448	0.000	0.000	0.000	0.952

Problem 2334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	F	F(-2)	F	B	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	384	869	0	0	0	5796	0	0	-1	446
N.S.	1	2.26	0.00	0.00	0.00	15.09	0.00	0.00	-0.00	1.16
time (sec)	N/A	1.741	0.654	180.000	0.000	0.639	0.000	0.000	0.000	4.300

Problem 2335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	385	537	183	0	0	0	0	372	-1	385
N.S.	1	1.39	0.48	0.00	0.00	0.00	0.00	0.97	-0.00	1.00
time (sec)	N/A	0.877	0.688	0.526	0.000	0.000	0.000	3.068	0.000	1.225

Problem 2336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	387	0	0	0	0	0	0	0	-1	387
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	13.232	6.928	0.086	0.000	0.000	0.000	0.000	0.000	5.948

Problem 2337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	388	397	155	0	535	8984	0	1632	532	815
N.S.	1	1.02	0.40	0.00	1.38	23.15	0.00	4.21	1.37	2.10
time (sec)	N/A	0.578	0.338	0.120	0.431	4.101	0.000	6.052	4.222	1.134

Problem 2338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F(-1)	F	C	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	390	0	0	2154	0	0	0	0	-1	416
N.S.	1	0.00	0.00	5.52	0.00	0.00	0.00	0.00	-0.00	1.07
time (sec)	N/A	180.004	1.149	48.506	0.000	0.000	0.000	0.000	0.000	4.700

Problem 2339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F(-2)	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	390	433	11755	0	0	329	0	0	-1	390
N.S.	1	1.11	30.14	0.00	0.00	0.84	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.974	23.412	180.000	0.000	0.841	0.000	0.000	0.000	0.920

Problem 2340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F(-2)	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	397	499	9604	0	0	341	0	0	-1	397
N.S.	1	1.26	24.19	0.00	0.00	0.86	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.723	24.235	180.000	0.000	0.455	0.000	0.000	0.000	0.936

Problem 2341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	399	0	0	0	0	0	0	0	-1	399
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	2.416	0.386	0.529	0.000	0.000	0.000	0.000	0.000	20.120

Problem 2342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	B	F	F(-1)	F(-1)	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	399	0	65821	1596	0	2875	0	0	-1	367
N.S.	1	0.00	164.96	4.00	0.00	7.21	0.00	0.00	-0.00	0.92
time (sec)	N/A	2.333	7.815	0.697	0.000	17.194	0.000	0.000	0.000	3.478

Problem 2343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	404	0	0	0	0	0	0	0	-1	322
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.80
time (sec)	N/A	11.403	6.199	0.083	0.000	0.000	0.000	0.000	0.000	6.512

Problem 2344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	405	0	0	0	0	0	0	0	-1	405
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	9.106	4.635	0.575	0.000	0.000	0.000	0.000	0.000	3.585

Problem 2345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	407	255	328	919	0	2014	1095	259	-1	388
N.S.	1	0.63	0.81	2.26	0.00	4.95	2.69	0.64	-0.00	0.95
time (sec)	N/A	4.652	4.446	0.052	0.000	0.504	123.161	1.046	0.000	0.967

Problem 2346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	411	425	151	0	0	4062	0	1194	132915	426
N.S.	1	1.03	0.37	0.00	0.00	9.88	0.00	2.91	323.39	1.04
time (sec)	N/A	1.182	0.185	0.542	0.000	3.609	0.000	8.007	44.071	1.285

Problem 2347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	413	0	0	0	0	0	0	0	-1	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	2.709	0.417	0.536	0.000	0.000	0.000	0.000	0.000	78.279

Problem 2348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	415	22	190	289	0	40	0	0	-1	31
N.S.	1	0.05	0.46	0.70	0.00	0.10	0.00	0.00	-0.00	0.07
time (sec)	N/A	0.154	1.079	0.073	0.000	1.389	0.000	0.000	0.000	1.226

Problem 2349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	415	0	0	0	0	0	0	0	-1	415
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	17.535	1.623	0.078	0.000	0.000	0.000	0.000	0.000	2.313

Problem 2350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	417	453	85	0	0	790	0	0	-1	466
N.S.	1	1.09	0.20	0.00	0.00	1.89	0.00	0.00	-0.00	1.12
time (sec)	N/A	0.221	0.056	0.543	0.000	0.493	0.000	0.000	0.000	3.132

Problem 2351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	423	376	58	0	0	301	0	0	-1	423
N.S.	1	0.89	0.14	0.00	0.00	0.71	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.681	0.064	0.534	0.000	0.406	0.000	0.000	0.000	3.189

Problem 2352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	B	F(-1)	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	423	0	0	0	0	960	0	0	-1	423
N.S.	1	0.00	0.00	0.00	0.00	2.27	0.00	0.00	-0.00	1.00
time (sec)	N/A	10.032	1.169	2.819	0.000	63.508	0.000	0.000	0.000	1.723

Problem 2353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	425	477	89	0	0	806	0	255	-1	475
N.S.	1	1.12	0.21	0.00	0.00	1.90	0.00	0.60	-0.00	1.12
time (sec)	N/A	0.232	0.062	0.541	0.000	0.445	0.000	143.517	0.000	3.148

Problem 2354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-1)	F	F(-2)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	429	0	0	0	0	0	0	0	-1	429
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	2.545	0.584	180.000	0.000	0.000	0.000	0.000	0.000	1.585

Problem 2355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-1)	F	F(-2)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	429	0	0	0	0	0	0	0	-1	429
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	2.178	0.377	180.000	0.000	0.000	0.000	0.000	0.000	1.418

Problem 2356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	431	0	0	0	0	564	0	0	-1	431
N.S.	1	0.00	0.00	0.00	0.00	1.31	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.092	10.902	180.000	0.000	0.512	0.000	0.000	0.000	1.206

Problem 2357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	432	160	0	0	0	0	0	0	-1	486
N.S.	1	0.37	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.12
time (sec)	N/A	0.384	0.237	0.526	0.000	0.000	0.000	0.000	0.000	14.884

Problem 2358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	433	461	87	0	0	823	0	255	-1	482
N.S.	1	1.06	0.20	0.00	0.00	1.90	0.00	0.59	-0.00	1.11
time (sec)	N/A	0.237	0.046	0.546	0.000	0.477	0.000	141.946	0.000	3.257

Problem 2359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	434	463	249	0	0	399	0	0	-1	241
N.S.	1	1.07	0.57	0.00	0.00	0.92	0.00	0.00	-0.00	0.56
time (sec)	N/A	0.796	0.839	0.550	0.000	0.466	0.000	0.000	0.000	1.242

Problem 2360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	438	513	95	0	0	2217	0	0	-1	438
N.S.	1	1.17	0.22	0.00	0.00	5.06	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.320	0.279	0.684	0.000	0.475	0.000	0.000	0.000	14.675

Problem 2361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	441	489	91	0	0	843	0	0	-1	491
N.S.	1	1.11	0.21	0.00	0.00	1.91	0.00	0.00	-0.00	1.11
time (sec)	N/A	0.237	0.058	0.548	0.000	0.436	0.000	0.000	0.000	3.369

Problem 2362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F(-1)	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	445	0	0	0	0	469	0	0	-1	445
N.S.	1	0.00	0.00	0.00	0.00	1.05	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.376	180.001	180.000	0.000	0.555	0.000	0.000	0.000	1.438

Problem 2363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	448	409	376	0	0	0	0	0	-1	510
N.S.	1	0.91	0.84	0.00	0.00	0.00	0.00	0.00	-0.00	1.14
time (sec)	N/A	0.710	0.715	0.523	0.000	0.000	0.000	0.000	0.000	41.421

Problem 2364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	F	F(-2)	F	B	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	448	1077	0	0	0	4991	0	0	-1	510
N.S.	1	2.40	0.00	0.00	0.00	11.14	0.00	0.00	-0.00	1.14
time (sec)	N/A	1.870	0.376	180.000	0.000	0.621	0.000	0.000	0.000	5.521

Problem 2365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	F(-1)	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	452	50	50	2929	0	0	0	0	-1	153
N.S.	1	0.11	0.11	6.48	0.00	0.00	0.00	0.00	-0.00	0.34
time (sec)	N/A	0.158	0.029	91.794	0.000	0.000	0.000	0.000	0.000	0.808

Problem 2366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	F(-1)	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	452	50	50	2926	0	0	0	0	-1	153
N.S.	1	0.11	0.11	6.47	0.00	0.00	0.00	0.00	-0.00	0.34
time (sec)	N/A	0.159	0.019	92.090	0.000	0.000	0.000	0.000	0.000	0.001

Problem 2367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	F(-1)	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	452	46	48	2857	0	0	0	0	-1	153
N.S.	1	0.10	0.11	6.32	0.00	0.00	0.00	0.00	-0.00	0.34
time (sec)	N/A	0.112	0.026	78.415	0.000	0.000	0.000	0.000	0.000	0.784

Problem 2368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	452	409	0	0	0	0	0	0	-1	510
N.S.	1	0.90	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.13
time (sec)	N/A	0.520	0.074	0.523	0.000	0.000	0.000	0.000	0.000	36.686

Problem 2369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	455	0	0	0	0	535	0	0	-1	455
N.S.	1	0.00	0.00	0.00	0.00	1.18	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.890	15.654	180.000	0.000	0.501	0.000	0.000	0.000	1.185

Problem 2370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	456	457	418	0	0	0	0	0	-1	510
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00	1.12
time (sec)	N/A	0.768	0.860	0.523	0.000	0.000	0.000	0.000	0.000	40.400

Problem 2371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	F(-1)	F(-1)	F(-1)	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	457	0	15065	47	0	0	0	0	-1	425
N.S.	1	0.00	32.96	0.10	0.00	0.00	0.00	0.00	-0.00	0.93
time (sec)	N/A	2.624	7.802	0.089	0.000	0.000	0.000	0.000	0.000	24.807

Problem 2372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	460	457	0	0	0	0	0	0	-1	510
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.11
time (sec)	N/A	0.586	0.109	0.528	0.000	0.000	0.000	0.000	0.000	37.482

Problem 2373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	A	F	A	F(-1)	F(-2)	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	466	0	238	475	0	1023	0	0	-1	511
N.S.	1	0.00	0.51	1.02	0.00	2.20	0.00	0.00	-0.00	1.10
time (sec)	N/A	0.065	0.647	0.066	0.000	0.651	0.000	0.000	0.000	1.964

Problem 2374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F(-1)	F	F(-2)	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	468	191	151	0	0	0	0	0	-1	626
N.S.	1	0.41	0.32	0.00	0.00	0.00	0.00	0.00	-0.00	1.34
time (sec)	N/A	0.074	0.265	180.000	0.000	0.000	0.000	0.000	0.000	9.354

Problem 2375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	C	F	F(-1)	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	469	101	0	4192	0	0	0	0	-1	394
N.S.	1	0.22	0.00	8.94	0.00	0.00	0.00	0.00	-0.00	0.84
time (sec)	N/A	0.653	0.829	80.866	0.000	0.000	0.000	0.000	0.000	24.182

Problem 2376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F(-1)	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	469	101	0	0	0	0	0	0	-1	394
N.S.	1	0.22	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.84
time (sec)	N/A	0.563	1.287	180.000	0.000	0.000	0.000	0.000	0.000	27.546

Problem 2377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	F	F(-1)	F	F(-1)	F(-1)	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	469	101	0	0	0	0	0	0	-1	394
N.S.	1	0.22	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.84
time (sec)	N/A	0.484	0.598	180.000	0.000	0.000	0.000	0.000	0.000	0.001

Problem 2378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	470	0	0	0	0	433	0	0	-1	470
N.S.	1	0.00	0.00	0.00	0.00	0.92	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.313	174.923	180.000	0.000	1.024	0.000	0.000	0.000	1.026

Problem 2379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	C
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	471	0	0	0	0	0	0	0	-1	1138
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	2.42
time (sec)	N/A	0.669	0.326	0.512	0.000	0.000	0.000	0.000	0.000	4.575

Problem 2380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	472	465	0	0	0	0	0	0	-1	534
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.13
time (sec)	N/A	0.602	0.093	0.528	0.000	0.000	0.000	0.000	0.000	42.143

Problem 2381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	476	465	426	0	0	0	0	0	-1	534
N.S.	1	0.98	0.89	0.00	0.00	0.00	0.00	0.00	-0.00	1.12
time (sec)	N/A	0.384	0.729	0.530	0.000	0.000	0.000	0.000	0.000	45.775

Problem 2382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	477	0	0	0	0	0	0	0	-1	477
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	3.005	2.092	0.536	0.000	0.000	0.000	0.000	0.000	8.252

Problem 2383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	481	513	89	0	0	2613	0	0	-1	481
N.S.	1	1.07	0.19	0.00	0.00	5.43	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.057	0.279	0.680	0.000	0.623	0.000	0.000	0.000	15.055

Problem 2384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F	F	A	F(-1)	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	482	451	0	0	0	567	0	0	-1	536
N.S.	1	0.94	0.00	0.00	0.00	1.18	0.00	0.00	-0.00	1.11
time (sec)	N/A	1.045	0.295	0.527	0.000	0.637	0.000	0.000	0.000	136.806

Problem 2385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	495	0	536	0	0	581	0	0	-1	495
N.S.	1	0.00	1.08	0.00	0.00	1.17	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.332	3.324	0.530	0.000	0.721	0.000	0.000	0.000	1.052

Problem 2386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	F(-2)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	496	0	0	3644	0	0	0	0	-1	496
N.S.	1	0.00	0.00	7.35	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.344	0.715	19.656	0.000	0.000	0.000	0.000	0.000	1.056

Problem 2387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	497	45	78	269	0	62	0	0	-1	497
N.S.	1	0.09	0.16	0.54	0.00	0.12	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.155	0.475	0.067	0.000	0.459	0.000	0.000	0.000	2.894

Problem 2388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F(-1)	F	F(-2)	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	499	213	151	0	0	0	0	0	-1	499
N.S.	1	0.43	0.30	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.089	0.175	180.000	0.000	0.000	0.000	0.000	0.000	7.992

Problem 2389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	501	758	467	0	0	344	0	0	-1	0
N.S.	1	1.51	0.93	0.00	0.00	0.69	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.497	1.405	0.203	0.000	0.447	0.000	0.000	0.000	3.703

Problem 2390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	F(-1)	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	501	131	69	7288	0	0	0	0	-1	162
N.S.	1	0.26	0.14	14.55	0.00	0.00	0.00	0.00	-0.00	0.32
time (sec)	N/A	0.548	0.106	130.128	0.000	0.000	0.000	0.000	0.000	1.016

Problem 2391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	F(-1)	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	501	131	69	7288	0	0	0	0	-1	162
N.S.	1	0.26	0.14	14.55	0.00	0.00	0.00	0.00	-0.00	0.32
time (sec)	N/A	0.454	0.063	131.383	0.000	0.000	0.000	0.000	0.000	0.001

Problem 2392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	F(-1)	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	501	111	69	7288	0	0	0	0	-1	162
N.S.	1	0.22	0.14	14.55	0.00	0.00	0.00	0.00	-0.00	0.32
time (sec)	N/A	0.442	0.070	118.480	0.000	0.000	0.000	0.000	0.000	0.954

Problem 2393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	F(-1)	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	501	111	69	7288	0	0	0	0	-1	162
N.S.	1	0.22	0.14	14.55	0.00	0.00	0.00	0.00	-0.00	0.32
time (sec)	N/A	0.398	0.034	118.868	0.000	0.000	0.000	0.000	0.000	0.001

Problem 2394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	506	510	158	0	0	408	0	0	-1	551
N.S.	1	1.01	0.31	0.00	0.00	0.81	0.00	0.00	-0.00	1.09
time (sec)	N/A	0.268	0.139	0.435	0.000	0.681	0.000	0.000	0.000	4.051

Problem 2395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	507	0	0	0	0	679	0	0	-1	507
N.S.	1	0.00	0.00	0.00	0.00	1.34	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.210	20.900	180.000	0.000	1.089	0.000	0.000	0.000	1.446

Problem 2396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F(-1)	F	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	514	0	0	0	0	0	0	0	-1	514
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	5.384	1.822	0.306	0.000	0.000	0.000	0.000	0.000	2.140

Problem 2397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	515	0	392	0	0	554	0	0	-1	515
N.S.	1	0.00	0.76	0.00	0.00	1.08	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.310	1.263	0.415	0.000	0.495	0.000	0.000	0.000	1.359

Problem 2398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	518	459	144	205	0	652	0	92	-1	518
N.S.	1	0.89	0.28	0.40	0.00	1.26	0.00	0.18	-0.00	1.00
time (sec)	N/A	0.935	0.396	0.044	0.000	0.421	0.000	0.335	0.000	0.468

Problem 2399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	F(-1)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	524	1029	293	528	0	0	0	0	-1	54
N.S.	1	1.96	0.56	1.01	0.00	0.00	0.00	0.00	-0.00	0.10
time (sec)	N/A	3.468	1.512	0.080	0.000	0.000	0.000	0.000	0.000	0.917

Problem 2400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	526	524	162	0	0	437	0	339	-1	571
N.S.	1	1.00	0.31	0.00	0.00	0.83	0.00	0.64	-0.00	1.09
time (sec)	N/A	0.269	0.154	0.418	0.000	0.446	0.000	126.674	0.000	4.196

Problem 2401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	526	468	14841	0	0	683	0	0	-1	501
N.S.	1	0.89	28.21	0.00	0.00	1.30	0.00	0.00	-0.00	0.95
time (sec)	N/A	0.657	23.492	180.000	0.000	0.440	0.000	0.000	0.000	1.779

Problem 2402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	530	0	0	0	0	0	0	0	-1	195
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.37
time (sec)	N/A	0.920	0.303	0.434	0.000	0.000	0.000	0.000	0.000	0.528

Problem 2403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	535	490	742	0	0	842	0	0	-1	510
N.S.	1	0.92	1.39	0.00	0.00	1.57	0.00	0.00	-0.00	0.95
time (sec)	N/A	2.106	5.176	180.000	0.000	0.460	0.000	0.000	0.000	2.000

Problem 2404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	540	539	226	0	0	770	0	0	-1	514
N.S.	1	1.00	0.42	0.00	0.00	1.43	0.00	0.00	-0.00	0.95
time (sec)	N/A	0.700	3.905	180.000	0.000	0.481	0.000	0.000	0.000	2.077

Problem 2405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	541	513	93	0	0	323	0	0	-1	541
N.S.	1	0.95	0.17	0.00	0.00	0.60	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.079	0.296	0.523	0.000	0.410	0.000	0.000	0.000	3.771

Problem 2406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	541	513	93	0	0	323	0	0	-1	541
N.S.	1	0.95	0.17	0.00	0.00	0.60	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.962	0.150	0.441	0.000	0.430	0.000	0.000	0.000	3.649

Problem 2407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	541	0	0	0	0	0	0	0	-1	251
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.46
time (sec)	N/A	0.968	0.368	0.394	0.000	0.000	0.000	0.000	0.000	0.766

Problem 2408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	543	377	53	0	0	662	0	0	-1	703
N.S.	1	0.69	0.10	0.00	0.00	1.22	0.00	0.00	-0.00	1.29
time (sec)	N/A	0.673	0.045	0.418	0.000	0.447	0.000	0.000	0.000	7.721

Problem 2409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	553	0	0	0	0	0	0	0	-1	553
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.103	0.057	0.394	0.000	0.000	0.000	0.000	0.000	5.654

Problem 2410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	569	513	93	0	0	755	0	0	-1	569
N.S.	1	0.90	0.16	0.00	0.00	1.33	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.051	0.288	0.508	0.000	0.813	0.000	0.000	0.000	3.826

Problem 2411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	569	0	0	0	0	0	0	0	-1	241
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.42
time (sec)	N/A	2.330	0.717	0.389	0.000	0.000	0.000	0.000	0.000	1.533

Problem 2412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	604	0	0	0	0	562	0	0	-1	600
N.S.	1	0.00	0.00	0.00	0.00	0.93	0.00	0.00	-0.00	0.99
time (sec)	N/A	0.336	0.349	0.392	0.000	5.756	0.000	0.000	0.000	8.374

Problem 2413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	617	1229	547	0	0	374	0	0	-1	0
N.S.	1	1.99	0.89	0.00	0.00	0.61	0.00	0.00	-0.00	0.00
time (sec)	N/A	2.030	1.978	0.121	0.000	0.456	0.000	0.000	0.000	3.699

Problem 2414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	650	0	655	0	0	317	0	0	-1	650
N.S.	1	0.00	1.01	0.00	0.00	0.49	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.282	1.830	0.055	0.000	0.433	0.000	0.000	0.000	2.210

Problem 2415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	669	473	72	0	0	3267	0	0	-1	669
N.S.	1	0.71	0.11	0.00	0.00	4.88	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.200	0.048	0.102	0.000	0.505	0.000	0.000	0.000	2.051

Problem 2416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	674	0	0	0	0	562	0	0	-1	674
N.S.	1	0.00	0.00	0.00	0.00	0.83	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.051	157.125	180.000	0.000	0.542	0.000	0.000	0.000	1.939

Problem 2417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F(-2)	F	C	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	678	760	317	0	0	1065	0	0	-1	1211
N.S.	1	1.12	0.47	0.00	0.00	1.57	0.00	0.00	-0.00	1.79
time (sec)	N/A	1.256	2.356	180.000	0.000	1.189	0.000	0.000	0.000	7.510

Problem 2418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F(-1)	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	697	0	0	0	0	1036	0	0	-1	697
N.S.	1	0.00	0.00	0.00	0.00	1.49	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.355	180.001	180.000	0.000	0.586	0.000	0.000	0.000	1.547

Problem 2419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	699	0	575	0	0	1039	0	0	-1	699
N.S.	1	0.00	0.82	0.00	0.00	1.49	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.053	1.757	0.040	0.000	0.577	0.000	0.000	0.000	1.188

Problem 2420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	708	471	171	0	0	4114	0	0	-1	708
N.S.	1	0.67	0.24	0.00	0.00	5.81	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.405	0.237	0.098	0.000	0.770	0.000	0.000	0.000	2.015

Problem 2421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	719	0	569	0	0	396	0	0	-1	719
N.S.	1	0.00	0.79	0.00	0.00	0.55	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.053	0.860	0.008	0.000	0.573	0.000	0.000	0.000	1.239

Problem 2422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	725	746	2000	0	0	50	0	0	-1	698
N.S.	1	1.03	2.76	0.00	0.00	0.07	0.00	0.00	-0.00	0.96
time (sec)	N/A	0.912	3.945	180.000	0.000	0.434	0.000	0.000	0.000	3.229

Problem 2423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F(-2)	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	747	279	388	0	0	511	0	0	-1	747
N.S.	1	0.37	0.52	0.00	0.00	0.68	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.891	0.549	180.000	0.000	0.433	0.000	0.000	0.000	1.891

Problem 2424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	752	609	95	0	0	3627	0	0	-1	752
N.S.	1	0.81	0.13	0.00	0.00	4.82	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.539	0.069	0.447	0.000	0.505	0.000	0.000	0.000	3.771

Problem 2425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	757	772	2041	0	0	80	0	0	-1	731
N.S.	1	1.02	2.70	0.00	0.00	0.11	0.00	0.00	-0.00	0.97
time (sec)	N/A	0.873	2.354	180.000	0.000	0.422	0.000	0.000	0.000	5.800

Problem 2426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	B	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	787	0	2441	0	0	1060	0	0	-1	787
N.S.	1	0.00	3.10	0.00	0.00	1.35	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.333	7.385	0.439	0.000	0.576	0.000	0.000	0.000	1.937

Problem 2427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F(-1)	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	803	0	0	0	0	1121	0	0	-1	803
N.S.	1	0.00	0.00	0.00	0.00	1.40	0.00	0.00	-0.00	1.00
time (sec)	N/A	1.132	180.001	180.000	0.000	0.563	0.000	0.000	0.000	2.561

Problem 2428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	827	908	416	899	0	246	0	0	-1	87
N.S.	1	1.10	0.50	1.09	0.00	0.30	0.00	0.00	-0.00	0.11
time (sec)	N/A	5.570	2.956	0.079	0.000	0.524	0.000	0.000	0.000	0.884

Problem 2429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	849	286	72	0	0	389	0	0	-1	0
N.S.	1	0.34	0.08	0.00	0.00	0.46	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.876	0.073	0.424	0.000	0.424	0.000	0.000	0.000	180.043

Problem 2430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	857	283	65	0	0	976	0	0	-1	242
N.S.	1	0.33	0.08	0.00	0.00	1.14	0.00	0.00	-0.00	0.28
time (sec)	N/A	1.444	0.052	0.427	0.000	0.456	0.000	0.000	0.000	11.625

Problem 2431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F(-2)	F	A	F(-1)	F(-1)	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	876	1407	0	0	0	160	0	0	-1	938
N.S.	1	1.61	0.00	0.00	0.00	0.18	0.00	0.00	-0.00	1.07
time (sec)	N/A	3.190	0.587	180.000	0.000	24.530	0.000	0.000	0.000	15.501

Problem 2432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	F(-1)	F(-1)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	887	1549	322	0	0	0	0	0	-1	898
N.S.	1	1.75	0.36	0.00	0.00	0.00	0.00	0.00	-0.00	1.01
time (sec)	N/A	5.280	0.484	0.601	0.000	0.000	0.000	0.000	0.000	3.513

Problem 2433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	963	0	0	0	0	617	0	0	-1	1246
N.S.	1	0.00	0.00	0.00	0.00	0.64	0.00	0.00	-0.00	1.29
time (sec)	N/A	1.128	0.961	180.000	0.000	0.575	0.000	0.000	0.000	5.958

Problem 2434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	1186	0	0	0	0	644	0	0	-1	1186
N.S.	1	0.00	0.00	0.00	0.00	0.54	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.297	0.740	0.398	0.000	0.590	0.000	0.000	0.000	3.309

Problem 2435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	1202	0	0	0	0	1193	0	0	-1	1202
N.S.	1	0.00	0.00	0.00	0.00	0.99	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.879	22.261	180.000	0.000	0.521	0.000	0.000	0.000	4.567

Problem 2436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F(-1)	F(-2)	F	A	F	F(-1)	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	1225	0	0	0	0	719	0	0	-1	1225
N.S.	1	0.00	0.00	0.00	0.00	0.59	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.946	180.002	180.000	0.000	0.596	0.000	0.000	0.000	4.271

Problem 2437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	1310	0	0	0	0	0	0	0	-1	262
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.20
time (sec)	N/A	2.067	0.624	0.421	0.000	0.000	0.000	0.000	0.000	1.032

Problem 2438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1387	280	66	0	0	473	0	0	-1	0
N.S.	1	0.20	0.05	0.00	0.00	0.34	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.471	0.078	0.517	0.000	0.778	0.000	0.000	0.000	180.694

Problem 2439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1655	561	109	0	0	11598	0	0	-1	0
N.S.	1	0.34	0.07	0.00	0.00	7.01	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.964	0.361	0.465	0.000	1.671	0.000	0.000	0.000	180.014

Problem 2440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1707	561	107	0	0	11788	0	0	-1	0
N.S.	1	0.33	0.06	0.00	0.00	6.91	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.872	0.359	0.460	0.000	1.864	0.000	0.000	0.000	180.014

Problem 2441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1835	561	111	0	0	9684	0	0	-1	571
N.S.	1	0.31	0.06	0.00	0.00	5.28	0.00	0.00	-0.00	0.31
time (sec)	N/A	2.285	0.337	0.445	0.000	1.090	0.000	0.000	0.000	18.966

Problem 2442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1886	561	109	0	0	9468	0	0	-1	587
N.S.	1	0.30	0.06	0.00	0.00	5.02	0.00	0.00	-0.00	0.31
time (sec)	N/A	2.160	0.833	0.460	0.000	1.014	0.000	0.000	0.000	22.359

Problem 2443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1916	1063	1379	384279	2606	0	0	0	-1	4104
N.S.	1	0.55	0.72	200.56	1.36	0.00	0.00	0.00	-0.00	2.14
time (sec)	N/A	4.678	5.328	0.480	0.967	0.000	0.000	0.000	0.000	3.440

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [901] had the largest ratio of [1.222]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	11	0.091
2	A	1	1	1.00	18	0.056
3	A	3	3	1.00	27	0.111
4	A	1	1	1.00	11	0.091
5	A	1	1	1.00	11	0.091
6	A	1	1	1.00	11	0.091
7	A	1	1	1.00	11	0.091
8	A	1	1	1.00	11	0.091
9	A	1	1	1.00	11	0.091
10	A	1	1	1.00	11	0.091
11	A	1	1	1.00	11	0.091
12	A	1	1	1.00	11	0.091
13	A	1	1	1.00	11	0.091
14	A	1	1	1.00	11	0.091
15	A	1	1	1.00	13	0.077
16	A	1	1	1.00	13	0.077
17	A	1	1	1.00	13	0.077
18	A	1	1	1.00	13	0.077
19	A	1	1	1.00	13	0.077
20	A	1	1	1.00	13	0.077
21	A	1	1	1.00	13	0.077
22	A	1	1	1.00	13	0.077
23	A	1	1	1.00	13	0.077
24	A	1	1	1.00	13	0.077
25	A	1	1	1.00	17	0.059
26	A	1	1	1.00	13	0.077
27	A	1	1	1.00	13	0.077
28	A	1	1	1.00	13	0.077
29	A	1	1	1.00	13	0.077
30	A	1	1	1.00	13	0.077
31	A	1	1	1.00	18	0.056
32	A	1	1	1.00	13	0.077
33	A	1	1	1.00	13	0.077
34	A	1	1	1.00	13	0.077
35	A	1	1	1.00	13	0.077
36	A	1	1	1.00	13	0.077
37	A	1	1	1.00	13	0.077
38	A	1	1	1.00	13	0.077
39	A	1	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
40	A	1	1	1.00	13	0.077
41	A	1	1	1.00	13	0.077
42	A	1	1	1.00	13	0.077
43	A	1	1	1.00	18	0.056
44	A	1	1	1.00	13	0.077
45	A	1	1	1.00	13	0.077
46	A	1	1	1.00	13	0.077
47	A	1	1	1.00	18	0.056
48	A	3	3	1.00	13	0.231
49	A	1	1	1.00	18	0.056
50	A	2	2	1.00	24	0.083
51	A	2	2	1.14	22	0.091
52	A	2	2	1.00	20	0.100
53	A	1	1	1.00	13	0.077
54	A	1	1	1.00	18	0.056
55	A	1	1	1.00	22	0.045
56	A	1	1	1.00	18	0.056
57	A	1	1	1.00	18	0.056
58	F	0	0	N/A	0	N/A
59	A	3	3	1.00	13	0.231
60	A	1	1	1.00	18	0.056
61	A	3	3	1.00	13	0.231
62	A	1	1	1.00	18	0.056
63	F	0	0	N/A	0	N/A
64	F	0	0	N/A	0	N/A
65	A	1	1	1.00	19	0.053
66	A	1	1	1.00	19	0.053
67	F	0	0	N/A	0	N/A
68	A	2	2	1.00	27	0.074
69	A	2	2	1.00	18	0.111
70	A	1	1	1.00	13	0.077
71	A	1	1	1.00	13	0.077
72	A	1	1	1.00	13	0.077
73	A	1	1	1.00	13	0.077
74	A	1	1	1.00	18	0.056
75	A	2	2	1.00	18	0.111
76	A	1	1	1.00	13	0.077
77	B	14	4	2.06	23	0.174
78	A	1	1	1.00	13	0.077
79	A	1	1	1.00	13	0.077
80	A	1	1	1.00	13	0.077
81	C	5	5	4.88	18	0.278
82	A	1	1	1.00	13	0.077
83	A	2	2	1.00	18	0.111
84	A	2	2	1.00	23	0.087
85	A	1	1	1.00	18	0.056
86	A	1	1	1.00	13	0.077
87	A	1	1	1.00	18	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
88	A	2	2	1.00	18	0.111
89	A	1	1	1.00	13	0.077
90	B	5	3	2.06	18	0.167
91	A	2	2	1.31	22	0.091
92	A	5	5	1.00	22	0.227
93	A	2	2	1.00	18	0.111
94	A	1	1	1.00	18	0.056
95	A	1	1	1.00	18	0.056
96	A	2	2	1.00	18	0.111
97	B	11	4	2.06	23	0.174
98	A	2	2	1.00	27	0.074
99	F	0	0	N/A	0	N/A
100	A	1	1	1.00	13	0.077
101	A	1	1	1.00	13	0.077
102	A	1	1	1.00	13	0.077
103	A	1	1	1.00	13	0.077
104	A	1	1	1.00	13	0.077
105	A	1	1	1.00	18	0.056
106	A	1	1	1.00	13	0.077
107	A	1	1	1.00	18	0.056
108	A	1	1	1.00	18	0.056
109	A	1	1	1.00	18	0.056
110	A	1	1	1.00	18	0.056
111	A	1	1	1.00	20	0.050
112	B	12	6	2.94	23	0.261
113	F	0	0	N/A	0	N/A
114	A	2	2	1.00	18	0.111
115	A	1	1	1.00	18	0.056
116	F	0	0	N/A	0	N/A
117	A	2	2	1.00	25	0.080
118	A	2	2	1.00	25	0.080
119	F	0	0	N/A	0	N/A
120	A	3	2	1.39	13	0.154
121	A	1	1	1.11	18	0.056
122	A	3	2	1.50	13	0.154
123	A	1	1	1.00	15	0.067
124	A	1	1	1.00	15	0.067
125	A	2	2	0.67	22	0.091
126	A	1	1	1.00	15	0.067
127	A	1	1	1.00	18	0.056
128	A	2	2	1.00	24	0.083
129	B	32	19	3.78	21	0.905
130	C	5	5	5.33	20	0.250
131	A	1	1	1.00	15	0.067
132	A	1	1	1.00	15	0.067
133	B	5	3	2.06	20	0.150
134	A	1	1	1.00	15	0.067
135	A	2	2	1.39	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	1	1	1.00	15	0.067
137	A	2	2	1.00	11	0.182
138	A	2	2	0.67	22	0.091
139	A	2	2	1.00	20	0.100
140	A	1	1	1.00	15	0.067
141	A	1	1	1.00	15	0.067
142	A	1	1	1.00	20	0.050
143	A	1	1	1.00	15	0.067
144	B	5	3	2.06	18	0.167
145	F	0	0	N/A	0	N/A
146	A	1	1	1.00	20	0.050
147	B	14	5	2.06	25	0.200
148	A	2	2	0.67	22	0.091
149	A	1	1	1.00	22	0.045
150	A	1	1	1.00	20	0.050
151	B	9	6	2.94	20	0.300
152	A	3	3	1.00	13	0.231
153	A	2	2	0.44	13	0.154
154	B	14	5	2.06	27	0.185
155	A	1	1	1.00	15	0.067
156	A	1	1	1.00	20	0.050
157	A	1	1	1.00	20	0.050
158	F	0	0	N/A	0	N/A
159	C	27	10	37.84	26	0.385
160	A	1	1	1.00	25	0.040
161	F	0	0	N/A	0	N/A
162	A	2	2	1.00	27	0.074
163	A	4	3	1.74	24	0.125
164	A	1	1	1.00	21	0.048
165	A	1	1	1.00	21	0.048
166	A	1	1	1.00	21	0.048
167	A	2	2	0.70	24	0.083
168	A	2	2	0.70	17	0.118
169	A	1	1	1.00	17	0.059
170	A	1	1	1.00	17	0.059
171	A	1	1	1.00	17	0.059
172	A	2	2	1.00	26	0.077
173	A	1	1	1.00	17	0.059
174	A	1	1	1.00	17	0.059
175	A	1	1	1.00	22	0.045
176	A	1	1	1.00	17	0.059
177	B	5	3	2.05	20	0.150
178	A	1	1	1.00	17	0.059
179	A	2	2	1.00	25	0.080
180	A	1	1	1.00	24	0.042
181	B	10	6	2.05	22	0.273
182	B	11	7	2.95	22	0.318
183	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
184	A	1	1	1.00	17	0.059
185	A	1	1	1.00	22	0.045
186	A	1	1	1.00	22	0.045
187	B	12	7	2.05	22	0.318
188	A	2	2	0.70	24	0.083
189	A	2	2	1.00	13	0.154
190	F	0	0	N/A	0	N/A
191	A	2	2	1.19	18	0.111
192	A	2	2	0.86	25	0.080
193	C	13	9	19.14	27	0.333
194	F	0	0	N/A	0	N/A
195	F	0	0	N/A	0	N/A
196	F	0	0	N/A	0	N/A
197	F	0	0	N/A	0	N/A
198	A	2	2	0.67	17	0.118
199	A	2	2	1.00	33	0.061
200	A	2	2	1.00	38	0.053
201	A	1	1	1.00	23	0.043
202	A	1	1	1.00	23	0.043
203	A	4	3	0.90	37	0.081
204	A	2	2	0.73	19	0.105
205	A	2	2	0.73	19	0.105
206	F	0	0	N/A	0	N/A
207	A	5	5	1.00	13	0.385
208	A	2	2	1.57	18	0.111
209	A	3	2	1.17	13	0.154
210	A	1	1	0.78	22	0.045
211	A	2	2	0.70	19	0.105
212	A	2	2	1.61	15	0.133
213	A	2	2	1.00	28	0.071
214	A	2	2	1.43	13	0.154
215	A	2	2	1.35	13	0.154
216	A	2	2	1.43	13	0.154
217	A	2	2	1.43	18	0.111
218	A	2	2	1.00	24	0.083
219	F	0	0	N/A	0	N/A
220	A	2	2	1.43	13	0.154
221	A	2	2	1.43	18	0.111
222	A	2	2	1.43	20	0.100
223	A	2	2	0.61	24	0.083
224	A	2	2	0.61	19	0.105
225	A	2	2	0.70	19	0.105
226	A	2	2	1.61	15	0.133
227	A	2	2	1.61	15	0.133
228	A	5	5	0.48	23	0.217
229	A	2	2	1.43	20	0.100
230	A	2	2	1.35	20	0.100
231	A	18	7	1.83	27	0.259

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
232	A	2	2	0.61	24	0.083
233	A	2	2	1.43	13	0.154
234	A	2	2	1.43	18	0.111
235	A	2	2	0.61	26	0.077
236	A	2	2	0.61	24	0.083
237	A	1	1	1.00	40	0.025
238	A	4	4	1.67	15	0.267
239	A	4	4	1.92	21	0.190
240	C	17	8	13.33	25	0.320
241	A	2	2	1.00	22	0.091
242	A	2	2	1.00	22	0.091
243	F	0	0	N/A	0	N/A
244	F	0	0	N/A	0	N/A
245	F	0	0	N/A	0	N/A
246	F	0	0	N/A	0	N/A
247	F	0	0	N/A	0	N/A
248	F	0	0	N/A	0	N/A
249	F	0	0	N/A	0	N/A
250	A	3	2	1.08	13	0.154
251	A	3	2	1.08	13	0.154
252	A	5	5	1.00	13	0.385
253	A	3	2	1.08	13	0.154
254	A	3	2	1.08	13	0.154
255	A	3	2	1.08	13	0.154
256	A	3	2	1.60	13	0.154
257	A	3	2	1.08	13	0.154
258	A	2	2	1.64	17	0.118
259	A	3	2	1.08	13	0.154
260	A	2	2	1.48	15	0.133
261	A	2	2	1.48	20	0.100
262	A	2	2	0.64	26	0.077
263	A	2	2	0.64	21	0.095
264	A	2	2	1.48	15	0.133
265	A	5	5	1.08	23	0.217
266	A	5	5	0.36	23	0.217
267	A	5	5	0.60	23	0.217
268	A	2	2	1.64	17	0.118
269	A	2	2	1.48	20	0.100
270	A	2	2	0.64	26	0.077
271	A	2	2	0.64	28	0.071
272	A	2	2	0.64	26	0.077
273	A	2	2	1.48	15	0.133
274	A	2	2	1.27	13	0.154
275	A	2	2	0.65	33	0.061
276	F	0	0	N/A	0	N/A
277	F	0	0	N/A	0	N/A
278	C	18	8	5.04	29	0.276
279	A	2	2	1.27	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
280	C	32	13	11.19	28	0.464
281	A	5	4	1.69	26	0.154
282	A	5	3	1.81	26	0.115
283	F	0	0	N/A	0	N/A
284	F	0	0	N/A	0	N/A
285	A	5	4	1.69	26	0.154
286	A	5	4	1.69	26	0.154
287	A	2	2	1.27	13	0.154
288	F	0	0	N/A	0	N/A
289	F	0	0	N/A	0	N/A
290	F	0	0	N/A	0	N/A
291	A	6	4	1.19	28	0.143
292	A	6	4	1.19	28	0.143
293	F	0	0	N/A	0	N/A
294	A	2	2	1.00	50	0.040
295	F	0	0	N/A	0	N/A
296	F	0	0	N/A	0	N/A
297	F	0	0	N/A	0	N/A
298	A	3	2	1.15	15	0.133
299	C	11	9	49.63	21	0.429
300	C	12	11	19.89	27	0.407
301	A	3	2	1.41	15	0.133
302	A	2	2	1.52	17	0.118
303	A	3	3	1.00	15	0.200
304	A	2	2	1.52	17	0.118
305	A	4	4	1.00	13	0.308
306	A	2	2	1.18	18	0.111
307	A	4	4	1.00	13	0.308
308	A	2	2	1.18	18	0.111
309	A	2	2	1.18	18	0.111
310	A	2	2	1.18	18	0.111
311	A	3	2	1.75	13	0.154
312	A	3	3	1.29	32	0.094
313	C	12	11	19.04	25	0.440
314	A	21	5	1.75	26	0.192
315	A	3	3	1.75	20	0.150
316	A	2	2	1.18	20	0.100
317	A	21	5	1.75	30	0.167
318	F	0	0	N/A	0	N/A
319	A	2	2	1.18	18	0.111
320	A	4	4	1.00	13	0.308
321	A	2	2	1.32	15	0.133
322	A	2	2	1.18	13	0.154
323	C	10	9	18.57	27	0.333
324	A	5	3	1.68	28	0.107
325	A	6	4	1.75	28	0.143
326	A	6	4	1.75	28	0.143
327	A	2	2	0.64	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
328	A	3	3	1.14	20	0.150
329	A	5	3	1.68	30	0.100
330	A	16	4	1.75	28	0.143
331	A	4	4	1.00	13	0.308
332	A	4	4	1.00	18	0.222
333	A	4	4	1.00	13	0.308
334	A	2	2	1.18	18	0.111
335	A	16	4	1.75	30	0.133
336	A	6	4	1.18	28	0.143
337	A	2	2	1.18	20	0.100
338	A	4	3	1.00	30	0.100
339	F	0	0	N/A	0	N/A
340	F	0	0	N/A	0	N/A
341	F	0	0	N/A	0	N/A
342	A	4	4	1.75	18	0.222
343	B	42	16	9.03	26	0.615
344	A	6	6	0.93	18	0.333
345	A	6	6	0.93	18	0.333
346	A	5	5	1.00	13	0.385
347	A	3	3	1.00	17	0.176
348	A	3	2	1.45	17	0.118
349	A	7	6	1.59	23	0.261
350	A	2	2	1.21	23	0.087
351	F	0	0	N/A	0	N/A
352	A	5	5	1.34	28	0.179
353	A	3	3	1.00	17	0.176
354	F	0	0	N/A	0	N/A
355	F	0	0	N/A	0	N/A
356	A	3	2	1.33	13	0.154
357	A	3	2	1.33	13	0.154
358	A	3	2	1.83	15	0.133
359	B	3	2	2.03	17	0.118
360	A	26	6	1.83	30	0.200
361	C	15	8	12.43	29	0.276
362	A	3	3	1.03	25	0.120
363	A	2	2	1.37	17	0.118
364	A	5	5	0.77	21	0.238
365	A	3	3	1.20	25	0.120
366	A	21	5	1.83	30	0.167
367	A	21	5	1.83	28	0.179
368	F	0	0	N/A	0	N/A
369	F	0	0	N/A	0	N/A
370	F	0	0	N/A	0	N/A
371	F	0	0	N/A	0	N/A
372	A	4	4	1.00	13	0.308
373	A	2	2	0.87	28	0.071
374	A	2	2	0.87	28	0.071
375	A	4	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
376	A	4	4	1.00	13	0.308
377	A	2	2	0.74	18	0.111
378	A	4	4	1.00	13	0.308
379	F	0	0	N/A	0	N/A
380	A	4	4	1.06	23	0.174
381	A	4	4	0.65	23	0.174
382	A	3	3	0.97	13	0.231
383	A	4	4	1.00	13	0.308
384	A	4	4	1.00	13	0.308
385	A	4	4	1.00	18	0.222
386	A	4	4	1.00	13	0.308
387	A	7	5	1.00	31	0.161
388	F	0	0	N/A	0	N/A
389	A	3	3	1.09	27	0.111
390	F	0	0	N/A	0	N/A
391	A	3	3	1.09	27	0.111
392	A	5	5	1.16	27	0.185
393	F	0	0	N/A	0	N/A
394	A	3	3	1.48	18	0.167
395	A	2	2	0.79	28	0.071
396	A	2	2	0.79	30	0.067
397	A	2	2	0.79	30	0.067
398	A	3	3	1.48	18	0.167
399	A	3	3	1.48	18	0.167
400	B	17	6	2.12	31	0.194
401	B	17	6	2.18	31	0.194
402	F	0	0	N/A	0	N/A
403	A	4	4	1.00	9	0.444
404	F	0	0	N/A	0	N/A
405	A	3	3	1.00	9	0.333
406	B	6	6	2.70	13	0.462
407	C	10	4	14.82	36	0.111
408	A	5	5	1.12	28	0.179
409	A	5	5	1.12	28	0.179
410	B	4	2	2.21	15	0.133
411	A	3	3	0.97	15	0.200
412	A	4	4	1.48	25	0.160
413	F	0	0	N/A	0	N/A
414	A	5	5	1.42	18	0.278
415	A	2	1	0.58	30	0.033
416	A	4	4	1.65	21	0.190
417	C	17	8	9.26	28	0.286
418	A	7	6	1.53	18	0.333
419	A	7	6	1.56	18	0.333
420	A	7	6	1.82	21	0.286
421	F	0	0	N/A	0	N/A
422	F	0	0	N/A	0	N/A
423	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
424	F	0	0	N/A	0	N/A
425	F	0	0	N/A	0	N/A
426	A	5	5	1.00	15	0.333
427	F	0	0	N/A	0	N/A
428	A	2	2	1.03	14	0.143
429	A	3	2	1.51	13	0.154
430	C	13	7	6.54	32	0.219
431	F	0	0	N/A	0	N/A
432	A	3	3	0.71	18	0.167
433	A	4	4	1.34	25	0.160
434	A	3	3	1.00	13	0.231
435	B	4	2	2.09	15	0.133
436	A	5	5	1.06	28	0.179
437	B	4	2	2.31	17	0.118
438	A	5	5	1.11	28	0.179
439	F	0	0	N/A	0	N/A
440	A	4	4	1.00	13	0.308
441	A	4	4	1.00	13	0.308
442	A	4	4	1.00	13	0.308
443	A	3	3	0.71	13	0.231
444	A	3	3	0.71	13	0.231
445	A	5	5	1.34	25	0.200
446	A	3	3	1.00	11	0.273
447	F	0	0	N/A	0	N/A
448	A	3	2	1.00	15	0.133
449	A	6	6	1.00	13	0.462
450	A	5	5	1.31	13	0.385
451	C	13	7	8.03	35	0.200
452	F	0	0	N/A	0	N/A
453	A	5	5	1.31	13	0.385
454	F	0	0	N/A	0	N/A
455	C	6	6	4.16	32	0.188
456	C	6	6	4.30	32	0.188
457	A	12	9	1.00	25	0.360
458	A	4	4	1.43	27	0.148
459	A	3	3	1.00	11	0.273
460	B	4	2	2.19	17	0.118
461	A	4	4	0.89	26	0.154
462	B	6	6	2.24	22	0.273
463	A	7	7	0.89	18	0.389
464	A	7	7	0.89	18	0.389
465	A	7	7	0.89	20	0.350
466	A	7	7	0.89	20	0.350
467	A	7	7	0.89	20	0.350
468	A	5	5	1.43	27	0.185
469	F	0	0	N/A	0	N/A
470	A	3	2	1.00	15	0.133
471	C	15	7	7.18	42	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
472	C	15	7	8.11	49	0.143
473	C	5	4	2.97	39	0.103
474	C	5	4	2.92	39	0.103
475	A	4	4	0.68	47	0.085
476	A	5	4	1.24	13	0.308
477	A	5	4	1.24	13	0.308
478	A	5	5	1.24	13	0.385
479	A	5	5	1.24	18	0.278
480	F	0	0	N/A	0	N/A
481	A	3	2	1.00	15	0.133
482	A	7	6	0.87	26	0.231
483	A	7	6	0.87	26	0.231
484	A	7	6	0.87	30	0.200
485	B	7	5	2.32	28	0.179
486	B	7	5	2.34	28	0.179
487	B	7	5	2.34	28	0.179
488	B	7	5	2.32	30	0.167
489	B	7	5	2.29	32	0.156
490	A	5	4	1.24	13	0.308
491	A	5	5	1.24	18	0.278
492	A	5	5	1.24	13	0.385
493	A	5	5	1.24	18	0.278
494	A	6	4	1.29	26	0.154
495	A	6	4	0.87	28	0.143
496	A	6	4	0.87	28	0.143
497	F	0	0	N/A	0	N/A
498	A	6	4	1.00	27	0.148
499	A	6	4	1.00	29	0.138
500	A	6	6	1.00	19	0.316
501	F	0	0	N/A	0	N/A
502	C	196	18	52.87	30	0.600
503	F	0	0	N/A	0	N/A
504	C	14	8	6.51	36	0.222
505	A	4	4	0.90	11	0.364
506	A	3	3	1.00	15	0.200
507	A	4	4	1.26	13	0.308
508	A	4	4	0.92	26	0.154
509	F	0	0	N/A	0	N/A
510	A	3	3	1.00	13	0.231
511	F	0	0	N/A	0	N/A
512	A	3	2	1.00	17	0.118
513	C	15	8	9.68	32	0.250
514	F	0	0	N/A	0	N/A
515	C	15	8	8.88	29	0.276
516	C	13	11	134.48	26	0.423
517	C	13	11	135.92	26	0.423
518	F	0	0	N/A	0	N/A
519	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
520	C	12	7	1.85	34	0.206
521	F	0	0	N/A	0	N/A
522	C	14	10	11.28	54	0.185
523	A	6	4	1.00	25	0.160
524	A	5	4	1.35	26	0.154
525	F	0	0	N/A	0	N/A
526	A	2	2	1.00	44	0.045
527	A	3	2	1.59	17	0.118
528	A	3	2	1.02	17	0.118
529	F	0	0	N/A	0	N/A
530	C	26	10	8.15	38	0.263
531	F	0	0	N/A	0	N/A
532	A	8	5	1.39	26	0.192
533	A	7	7	1.54	23	0.304
534	A	6	4	1.00	29	0.138
535	F	0	0	N/A	0	N/A
536	C	11	6	4.56	39	0.154
537	A	6	6	1.00	13	0.462
538	F	0	0	N/A	0	N/A
539	F	0	0	N/A	0	N/A
540	F	0	0	N/A	0	N/A
541	A	6	6	1.00	13	0.462
542	A	5	5	1.21	18	0.278
543	A	6	4	1.47	13	0.308
544	B	11	3	2.53	20	0.150
545	F	0	0	N/A	0	N/A
546	C	34	8	8.14	37	0.216
547	F	0	0	N/A	0	N/A
548	F	0	0	N/A	0	N/A
549	A	6	4	1.47	13	0.308
550	A	5	4	1.19	13	0.308
551	A	6	5	1.47	13	0.385
552	A	5	5	1.19	13	0.385
553	C	17	9	5.02	22	0.409
554	A	4	3	0.95	13	0.231
555	A	6	5	1.47	18	0.278
556	C	41	13	8.23	24	0.542
557	C	41	13	8.23	22	0.591
558	F	0	0	N/A	0	N/A
559	A	12	5	1.40	26	0.192
560	F	0	0	N/A	0	N/A
561	F	0	0	N/A	0	N/A
562	B	3	2	2.37	54	0.037
563	C	12	7	4.40	43	0.163
564	F	0	0	N/A	0	N/A
565	F	0	0	N/A	0	N/A
566	C	15	7	8.57	43	0.163

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
567	C	15	7	8.55	45	0.156
568	F	0	0	N/A	0	N/A
569	F	0	0	N/A	0	N/A
570	F	0	0	N/A	0	N/A
571	F	0	0	N/A	0	N/A
572	F	0	0	N/A	0	N/A
573	A	7	6	1.16	18	0.333
574	A	4	4	1.00	42	0.095
575	F	0	0	N/A	0	N/A
576	A	2	2	0.70	27	0.074
577	A	6	6	1.00	13	0.462
578	A	6	6	1.00	13	0.462
579	A	6	6	1.00	13	0.462
580	A	4	4	1.22	15	0.267
581	F	0	0	N/A	0	N/A
582	F	0	0	N/A	0	N/A
583	A	6	6	1.00	13	0.462
584	F	0	0	N/A	0	N/A
585	F	0	0	N/A	0	N/A
586	A	2	2	0.40	32	0.062
587	A	5	4	1.41	13	0.308
588	A	6	6	0.85	28	0.214
589	F	0	0	N/A	0	N/A
590	F	0	0	N/A	0	N/A
591	B	8	8	2.46	37	0.216
592	A	5	5	1.00	13	0.385
593	A	7	7	1.13	18	0.389
594	F	0	0	N/A	0	N/A
595	C	52	13	35.53	37	0.351
596	F	0	0	N/A	0	N/A
597	F	0	0	N/A	0	N/A
598	F	0	0	N/A	0	N/A
599	F	0	0	N/A	0	N/A
600	F	0	0	N/A	0	N/A
601	A	4	3	0.74	13	0.231
602	A	7	6	1.00	19	0.316
603	B	26	7	4.57	16	0.438
604	A	1	1	1.00	20	0.050
605	A	20	10	1.50	22	0.454
606	F	0	0	N/A	0	N/A
607	A	7	7	1.96	23	0.304
608	B	10	3	2.27	20	0.150
609	F	0	0	N/A	0	N/A
610	C	18	9	4.33	45	0.200
611	A	6	4	1.40	13	0.308
612	A	6	5	1.40	13	0.385
613	A	5	3	1.19	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
614	F	0	0	N/A	0	N/A
615	F	0	0	N/A	0	N/A
616	F	0	0	N/A	0	N/A
617	F	0	0	N/A	0	N/A
618	F	0	0	N/A	0	N/A
619	A	5	5	1.00	13	0.385
620	A	5	5	1.00	13	0.385
621	A	4	4	1.22	22	0.182
622	F	0	0	N/A	0	N/A
623	F	0	0	N/A	0	N/A
624	A	6	6	0.96	18	0.333
625	F	0	0	N/A	0	N/A
626	F	0	0	N/A	0	N/A
627	F	0	0	N/A	0	N/A
628	F	0	0	N/A	0	N/A
629	F	0	0	N/A	0	N/A
630	A	5	4	1.46	15	0.267
631	B	12	3	2.42	22	0.136
632	F	0	0	N/A	0	N/A
633	F	0	0	N/A	0	N/A
634	A	1	1	0.82	15	0.067
635	F	0	0	N/A	0	N/A
636	A	5	5	1.00	15	0.333
637	F	0	0	N/A	0	N/A
638	A	7	5	1.00	25	0.200
639	F	0	0	N/A	0	N/A
640	F	0	0	N/A	0	N/A
641	F	0	0	N/A	0	N/A
642	F	0	0	N/A	0	N/A
643	F	0	0	N/A	0	N/A
644	A	6	3	1.31	19	0.158
645	F	0	0	N/A	0	N/A
646	A	7	6	1.17	13	0.462
647	A	7	7	1.17	13	0.538
648	A	5	5	1.13	24	0.208
649	F	0	0	N/A	0	N/A
650	C	8	7	3.13	22	0.318
651	F	0	0	N/A	0	N/A
652	F	0	0	N/A	0	N/A
653	F	0	0	N/A	0	N/A
654	F	0	0	N/A	0	N/A
655	F	0	0	N/A	0	N/A
656	A	4	4	1.25	20	0.200
657	F	0	0	N/A	0	N/A
658	A	5	5	1.11	24	0.208
659	A	9	6	1.00	20	0.300
660	A	4	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
661	A	9	6	1.62	17	0.353
662	B	7	7	2.02	11	0.636
663	C	80	9	21.98	27	0.333
664	B	8	2	2.74	15	0.133
665	F	0	0	N/A	0	N/A
666	A	4	4	1.00	19	0.210
667	F	0	0	N/A	0	N/A
668	A	7	5	1.57	13	0.385
669	F	0	0	N/A	0	N/A
670	C	95	19	4.30	29	0.655
671	A	7	5	1.00	25	0.200
672	F	0	0	N/A	0	N/A
673	C	33	16	22.92	30	0.533
674	C	20	6	5.92	22	0.273
675	C	21	7	5.92	22	0.318
676	C	40	10	12.21	27	0.370
677	A	6	6	1.00	19	0.316
678	C	9	8	3.69	43	0.186
679	C	9	8	3.69	43	0.186
680	C	13	11	102.11	27	0.407
681	C	13	11	99.91	27	0.407
682	A	6	6	1.15	18	0.333
683	C	10	4	8.20	36	0.111
684	A	7	7	1.85	17	0.412
685	A	7	7	1.17	18	0.389
686	A	5	5	1.46	14	0.357
687	A	6	6	1.00	19	0.316
688	C	12	6	7.20	48	0.125
689	A	5	5	1.00	15	0.333
690	A	5	5	1.15	24	0.208
691	C	17	9	9.24	58	0.155
692	A	11	8	1.00	16	0.500
693	A	6	6	1.18	13	0.462
694	A	8	8	1.98	15	0.533
695	A	8	8	1.98	13	0.615
696	A	4	4	1.20	24	0.167
697	A	7	7	1.84	17	0.412
698	F	0	0	N/A	0	N/A
699	F	0	0	N/A	0	N/A
700	A	5	5	1.00	15	0.333
701	A	5	5	1.00	15	0.333
702	A	5	5	1.00	15	0.333
703	A	8	8	0.93	20	0.400
704	A	5	5	1.00	15	0.333
705	A	6	6	1.41	19	0.316
706	A	6	6	0.75	22	0.273
707	A	5	5	1.12	24	0.208
708	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
709	F	0	0	N/A	0	N/A
710	F	0	0	N/A	0	N/A
711	F	0	0	N/A	0	N/A
712	F	0	0	N/A	0	N/A
713	C	8	7	3.18	24	0.292
714	F	0	0	N/A	0	N/A
715	A	8	7	0.70	40	0.175
716	A	6	6	1.14	28	0.214
717	F	0	0	N/A	0	N/A
718	A	3	3	0.93	16	0.188
719	C	12	6	5.93	60	0.100
720	F	0	0	N/A	0	N/A
721	A	6	6	1.14	13	0.462
722	A	8	8	1.98	20	0.400
723	F	0	0	N/A	0	N/A
724	A	4	4	1.00	15	0.267
725	C	18	8	8.09	36	0.222
726	F	0	0	N/A	0	N/A
727	F	0	0	N/A	0	N/A
728	C	13	7	3.14	27	0.259
729	C	13	7	3.14	25	0.280
730	C	13	7	3.16	27	0.259
731	A	11	8	1.76	19	0.421
732	A	9	6	1.66	19	0.316
733	A	6	6	0.79	30	0.200
734	F	0	0	N/A	0	N/A
735	C	17	8	5.46	25	0.320
736	F	0	0	N/A	0	N/A
737	A	5	5	1.15	24	0.208
738	A	5	5	1.15	24	0.208
739	F	0	0	N/A	0	N/A
740	F	0	0	N/A	0	N/A
741	A	7	7	1.92	13	0.538
742	B	8	8	2.12	15	0.533
743	C	90	10	4.29	38	0.263
744	F	0	0	N/A	0	N/A
745	A	2	2	0.85	34	0.059
746	F	0	0	N/A	0	N/A
747	A	8	8	0.90	18	0.444
748	C	18	8	7.80	36	0.222
749	C	16	7	7.33	58	0.121
750	C	16	7	7.58	63	0.111
751	C	16	9	4.37	40	0.225
752	F	0	0	N/A	0	N/A
753	F	0	0	N/A	0	N/A
754	C	13	7	4.83	43	0.163
755	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
756	A	7	7	0.77	18	0.389
757	A	7	7	0.73	18	0.389
758	A	8	8	0.93	20	0.400
759	A	8	8	0.93	20	0.400
760	F	0	0	N/A	0	N/A
761	C	47	17	5.58	41	0.415
762	F	0	0	N/A	0	N/A
763	F	0	0	N/A	0	N/A
764	B	9	8	2.35	65	0.123
765	C	6	4	1.43	15	0.267
766	C	23	9	6.33	22	0.409
767	C	29	12	6.30	22	0.546
768	A	11	8	1.00	16	0.500
769	A	4	4	1.00	17	0.235
770	F	0	0	N/A	0	N/A
771	A	10	5	1.10	31	0.161
772	C	20	10	7.28	39	0.256
773	F	0	0	N/A	0	N/A
774	A	3	2	1.00	23	0.087
775	A	7	6	1.31	13	0.462
776	A	7	6	1.31	13	0.462
777	F	0	0	N/A	0	N/A
778	F	0	0	N/A	0	N/A
779	F	0	0	N/A	0	N/A
780	A	3	2	1.63	19	0.105
781	F	0	0	N/A	0	N/A
782	A	3	2	1.00	40	0.050
783	A	3	2	1.00	23	0.087
784	A	6	5	0.97	23	0.217
785	C	19	9	4.33	21	0.429
786	F	0	0	N/A	0	N/A
787	A	6	5	1.00	35	0.143
788	F	0	0	N/A	0	N/A
789	F	0	0	N/A	0	N/A
790	C	29	12	6.03	22	0.546
791	B	9	9	2.02	15	0.600
792	A	8	8	1.94	13	0.615
793	F	0	0	N/A	0	N/A
794	A	6	5	0.78	25	0.200
795	F	0	0	N/A	0	N/A
796	C	41	16	19.59	30	0.533
797	F	0	0	N/A	0	N/A
798	F	0	0	N/A	0	N/A
799	A	1	1	0.81	19	0.053
800	F	0	0	N/A	0	N/A
801	A	4	4	1.42	32	0.125
802	A	4	4	1.42	32	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
803	A	6	6	1.42	43	0.140
804	F	0	0	N/A	0	N/A
805	A	8	8	0.88	18	0.444
806	A	9	6	1.09	31	0.194
807	A	7	7	1.00	27	0.259
808	A	7	7	0.73	30	0.233
809	A	4	4	0.69	24	0.167
810	A	6	6	1.00	15	0.400
811	C	15	8	2.65	25	0.320
812	C	10	9	8.00	29	0.310
813	F	0	0	N/A	0	N/A
814	F	0	0	N/A	0	N/A
815	C	16	6	2.38	31	0.194
816	A	6	5	1.00	28	0.179
817	A	6	5	1.00	38	0.132
818	F	0	0	N/A	0	N/A
819	A	6	6	1.00	15	0.400
820	F	0	0	N/A	0	N/A
821	F	0	0	N/A	0	N/A
822	A	1	1	0.67	17	0.059
823	A	8	8	1.99	17	0.471
824	F	0	0	N/A	0	N/A
825	F	0	0	N/A	0	N/A
826	A	20	10	0.97	34	0.294
827	F	0	0	N/A	0	N/A
828	F	0	0	N/A	0	N/A
829	F	0	0	N/A	0	N/A
830	C	3	3	0.87	20	0.150
831	A	6	6	1.00	17	0.353
832	A	7	7	1.00	24	0.292
833	A	16	10	1.00	22	0.454
834	C	25	8	20.63	22	0.364
835	A	7	5	0.84	30	0.167
836	A	3	2	0.84	25	0.080
837	B	10	9	2.13	15	0.600
838	F	0	0	N/A	0	N/A
839	A	7	7	1.68	22	0.318
840	A	5	5	1.00	23	0.217
841	C	20	11	2.16	27	0.407
842	A	25	8	1.29	30	0.267
843	F	0	0	N/A	0	N/A
844	F	0	0	N/A	0	N/A
845	A	7	7	1.61	19	0.368
846	A	7	6	1.14	24	0.250
847	C	156	14	97.29	51	0.274
848	A	12	9	1.00	37	0.243
849	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
850	F	0	0	N/A	0	N/A
851	F	0	0	N/A	0	N/A
852	A	6	6	1.00	15	0.400
853	A	12	9	1.00	23	0.391
854	A	6	6	1.00	15	0.400
855	F	0	0	N/A	0	N/A
856	A	14	10	1.00	37	0.270
857	A	6	6	1.00	15	0.400
858	F	0	0	N/A	0	N/A
859	F	0	0	N/A	0	N/A
860	A	6	6	1.00	15	0.400
861	A	5	4	1.03	21	0.190
862	A	4	3	0.89	21	0.143
863	F	0	0	N/A	0	N/A
864	F	0	0	N/A	0	N/A
865	F	0	0	N/A	0	N/A
866	A	10	5	1.15	25	0.200
867	A	3	3	0.61	19	0.158
868	A	5	5	0.72	13	0.385
869	F	0	0	N/A	0	N/A
870	F	0	0	N/A	0	N/A
871	F	0	0	N/A	0	N/A
872	F	0	0	N/A	0	N/A
873	A	9	6	1.15	15	0.400
874	F	0	0	N/A	0	N/A
875	C	248	20	36.04	33	0.606
876	B	9	8	2.12	17	0.471
877	B	11	7	2.33	20	0.350
878	F	0	0	N/A	0	N/A
879	A	7	7	0.93	36	0.194
880	F	0	0	N/A	0	N/A
881	A	3	3	1.03	14	0.214
882	C	9	5	1.41	25	0.200
883	C	13	7	2.47	30	0.233
884	F	0	0	N/A	0	N/A
885	B	10	8	2.22	17	0.471
886	F	0	0	N/A	0	N/A
887	C	23	9	34.42	48	0.188
888	F	0	0	N/A	0	N/A
889	F	0	0	N/A	0	N/A
890	F	0	0	N/A	0	N/A
891	A	5	5	0.73	13	0.385
892	A	5	5	0.73	13	0.385
893	F	0	0	N/A	0	N/A
894	F	0	0	N/A	0	N/A
895	A	12	9	1.00	21	0.429
896	A	7	7	1.62	24	0.292

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
897	F	0	0	N/A	0	N/A
898	A	5	5	1.00	27	0.185
899	A	5	5	0.73	13	0.385
900	A	17	12	1.74	62	0.194
901	C	176	33	2.68	27	1.222
902	A	6	6	1.00	15	0.400
903	F	0	0	N/A	0	N/A
904	A	6	6	1.00	15	0.400
905	A	9	8	1.72	22	0.364
906	F	0	0	N/A	0	N/A
907	C	32	8	6.33	47	0.170
908	A	5	5	1.00	11	0.454
909	A	5	5	1.00	15	0.333
910	A	6	6	1.00	15	0.400
911	F	0	0	N/A	0	N/A
912	A	6	6	1.00	15	0.400
913	F	0	0	N/A	0	N/A
914	C	20	6	15.04	27	0.222
915	C	16	9	3.93	44	0.204
916	A	5	5	0.68	13	0.385
917	C	24	8	31.86	41	0.195
918	F	0	0	N/A	0	N/A
919	A	7	7	0.96	29	0.241
920	F	0	0	N/A	0	N/A
921	F	0	0	N/A	0	N/A
922	A	5	5	0.68	13	0.385
923	C	50	8	14.92	53	0.151
924	A	4	3	1.83	37	0.081
925	F	0	0	N/A	0	N/A
926	C	15	8	3.25	40	0.200
927	A	1	1	0.60	9	0.111
928	C	16	9	3.26	27	0.333
929	F	0	0	N/A	0	N/A
930	A	11	8	1.68	13	0.615
931	F	0	0	N/A	0	N/A
932	A	5	5	1.00	24	0.208
933	A	7	7	1.60	13	0.538
934	F	0	0	N/A	0	N/A
935	A	11	8	1.68	13	0.615
936	A	13	8	0.96	26	0.308
937	F	0	0	N/A	0	N/A
938	A	6	6	0.82	17	0.353
939	A	6	6	1.00	15	0.400
940	B	53	6	3.18	33	0.182
941	A	6	6	1.00	15	0.400
942	A	1	1	0.97	19	0.053
943	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
944	A	6	6	1.00	15	0.400
945	A	6	6	1.00	15	0.400
946	A	6	6	1.00	15	0.400
947	A	7	7	0.72	22	0.318
948	A	5	4	1.01	13	0.308
949	A	6	6	1.00	28	0.214
950	F	0	0	N/A	0	N/A
951	F	0	0	N/A	0	N/A
952	C	18	12	1.51	24	0.500
953	C	57	21	11.87	33	0.636
954	C	4	4	0.57	22	0.182
955	A	7	6	0.82	25	0.240
956	F	0	0	N/A	0	N/A
957	C	40	11	7.61	25	0.440
958	F	0	0	N/A	0	N/A
959	F	0	0	N/A	0	N/A
960	A	3	2	0.97	21	0.095
961	F	0	0	N/A	0	N/A
962	F	0	0	N/A	0	N/A
963	F	0	0	N/A	0	N/A
964	F	0	0	N/A	0	N/A
965	F	0	0	N/A	0	N/A
966	A	5	5	1.00	15	0.333
967	A	7	7	1.78	19	0.368
968	F	0	0	N/A	0	N/A
969	A	6	6	0.75	13	0.462
970	F	0	0	N/A	0	N/A
971	F	0	0	N/A	0	N/A
972	A	9	6	1.00	17	0.353
973	A	9	8	1.69	24	0.333
974	F	0	0	N/A	0	N/A
975	F	0	0	N/A	0	N/A
976	F	0	0	N/A	0	N/A
977	F	0	0	N/A	0	N/A
978	F	0	0	N/A	0	N/A
979	A	5	5	1.00	26	0.192
980	A	5	5	1.00	13	0.385
981	A	5	5	1.00	17	0.294
982	A	7	7	1.67	19	0.368
983	F	0	0	N/A	0	N/A
984	A	7	7	1.00	29	0.241
985	C	20	6	20.23	42	0.143
986	C	21	7	20.23	42	0.167
987	F	0	0	N/A	0	N/A
988	A	2	2	0.58	37	0.054
989	A	7	7	1.66	19	0.368
990	A	7	7	1.66	33	0.212

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
991	F	0	0	N/A	0	N/A
992	A	6	6	1.12	24	0.250
993	A	2	2	1.55	11	0.182
994	F	0	0	N/A	0	N/A
995	F	0	0	N/A	0	N/A
996	F	0	0	N/A	0	N/A
997	A	9	7	0.94	25	0.280
998	F	0	0	N/A	0	N/A
999	A	4	4	1.00	23	0.174
1000	A	7	7	1.84	20	0.350
1001	A	6	5	1.24	24	0.208
1002	A	6	5	1.24	22	0.227
1003	A	6	5	1.24	21	0.238
1004	A	8	5	1.08	27	0.185
1005	F	0	0	N/A	0	N/A
1006	F	0	0	N/A	0	N/A
1007	A	6	6	0.76	13	0.462
1008	A	7	7	1.74	35	0.200
1009	F	0	0	N/A	0	N/A
1010	C	45	22	7.93	38	0.579
1011	A	4	3	0.67	17	0.176
1012	A	6	6	0.76	20	0.300
1013	C	13	7	2.39	32	0.219
1014	C	9	5	1.61	25	0.200
1015	C	16	9	3.02	29	0.310
1016	F	0	0	N/A	0	N/A
1017	A	7	7	1.01	20	0.350
1018	F	0	0	N/A	0	N/A
1019	C	43	13	7.69	24	0.542
1020	C	18	8	3.49	25	0.320
1021	A	6	6	1.11	24	0.250
1022	A	6	6	1.13	27	0.222
1023	A	11	10	1.28	41	0.244
1024	F	0	0	N/A	0	N/A
1025	F	0	0	N/A	0	N/A
1026	A	14	9	0.93	31	0.290
1027	F	0	0	N/A	0	N/A
1028	C	171	18	4.94	22	0.818
1029	F	0	0	N/A	0	N/A
1030	F	0	0	N/A	0	N/A
1031	A	6	6	0.72	18	0.333
1032	A	6	6	0.70	18	0.333
1033	A	6	6	0.72	13	0.462
1034	F	0	0	N/A	0	N/A
1035	F	0	0	N/A	0	N/A
1036	F	0	0	N/A	0	N/A
1037	A	12	9	1.60	13	0.692

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1038	A	8	8	1.45	23	0.348
1039	A	8	8	1.52	21	0.381
1040	A	5	5	1.00	17	0.294
1041	A	6	6	1.08	24	0.250
1042	A	8	7	1.13	24	0.292
1043	A	9	9	1.41	24	0.375
1044	A	6	6	0.77	13	0.462
1045	A	6	6	0.77	13	0.462
1046	A	6	6	0.77	13	0.462
1047	F	0	0	N/A	0	N/A
1048	A	6	6	0.77	13	0.462
1049	F	0	0	N/A	0	N/A
1050	F	0	0	N/A	0	N/A
1051	A	4	4	1.00	26	0.154
1052	A	4	4	1.00	23	0.174
1053	F	0	0	N/A	0	N/A
1054	C	8	4	1.94	43	0.093
1055	A	6	6	0.77	13	0.462
1056	A	6	6	0.77	13	0.462
1057	F	0	0	N/A	0	N/A
1058	F	0	0	N/A	0	N/A
1059	A	7	7	0.89	23	0.304
1060	A	7	5	1.07	27	0.185
1061	A	6	6	0.93	31	0.194
1062	F	0	0	N/A	0	N/A
1063	A	6	6	0.68	18	0.333
1064	C	14	9	2.88	40	0.225
1065	F	0	0	N/A	0	N/A
1066	C	22	11	3.47	29	0.379
1067	A	8	8	0.99	13	0.615
1068	A	2	2	0.60	11	0.182
1069	A	8	8	0.99	13	0.615
1070	A	2	2	0.60	11	0.182
1071	A	6	6	1.15	26	0.231
1072	A	8	7	1.12	27	0.259
1073	A	2	2	0.52	59	0.034
1074	A	6	6	0.73	13	0.462
1075	F	0	0	N/A	0	N/A
1076	C	13	6	4.87	25	0.240
1077	A	7	7	0.84	18	0.389
1078	F	0	0	N/A	0	N/A
1079	F	0	0	N/A	0	N/A
1080	C	59	11	6.08	23	0.478
1081	A	41	18	1.74	20	0.900
1082	A	6	6	0.73	13	0.462
1083	C	31	8	6.65	30	0.267
1084	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1085	F	0	0	N/A	0	N/A
1086	A	6	6	0.73	13	0.462
1087	A	6	6	0.73	13	0.462
1088	F	0	0	N/A	0	N/A
1089	A	6	6	0.73	13	0.462
1090	A	12	9	1.53	19	0.474
1091	A	6	6	0.78	13	0.462
1092	A	6	6	0.78	13	0.462
1093	F	0	0	N/A	0	N/A
1094	F	0	0	N/A	0	N/A
1095	A	7	7	0.66	20	0.350
1096	A	6	6	0.78	13	0.462
1097	F	0	0	N/A	0	N/A
1098	F	0	0	N/A	0	N/A
1099	F	0	0	N/A	0	N/A
1100	F	0	0	N/A	0	N/A
1101	F	0	0	N/A	0	N/A
1102	A	4	4	0.86	21	0.190
1103	A	7	7	1.78	35	0.200
1104	A	7	7	1.57	35	0.200
1105	A	6	6	0.78	13	0.462
1106	F	0	0	N/A	0	N/A
1107	F	0	0	N/A	0	N/A
1108	A	12	9	1.58	13	0.692
1109	A	6	6	0.78	13	0.462
1110	A	6	6	0.78	13	0.462
1111	F	0	0	N/A	0	N/A
1112	F	0	0	N/A	0	N/A
1113	F	0	0	N/A	0	N/A
1114	C	8	7	1.60	35	0.200
1115	A	2	2	1.48	13	0.154
1116	F	0	0	N/A	0	N/A
1117	F	0	0	N/A	0	N/A
1118	C	38	9	6.52	28	0.321
1119	F	0	0	N/A	0	N/A
1120	A	12	8	1.00	24	0.333
1121	A	8	8	1.64	15	0.533
1122	A	16	12	0.91	33	0.364
1123	F	0	0	N/A	0	N/A
1124	A	6	6	0.74	13	0.462
1125	A	6	6	0.74	13	0.462
1126	A	8	8	1.01	13	0.615
1127	A	1	1	0.58	18	0.056
1128	A	7	6	1.13	15	0.400
1129	F	0	0	N/A	0	N/A
1130	A	6	6	0.74	13	0.462
1131	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1132	A	7	6	1.13	15	0.400
1133	A	6	6	1.10	15	0.400
1134	A	6	6	0.74	13	0.462
1135	A	7	6	1.13	15	0.400
1136	A	6	6	0.74	13	0.462
1137	A	6	6	0.74	13	0.462
1138	F	0	0	N/A	0	N/A
1139	A	6	4	1.24	27	0.148
1140	A	12	9	1.56	13	0.692
1141	A	2	2	0.67	13	0.154
1142	C	13	6	4.15	25	0.240
1143	A	2	2	0.67	13	0.154
1144	F	0	0	N/A	0	N/A
1145	A	12	9	1.56	13	0.692
1146	C	23	11	3.15	27	0.407
1147	F	0	0	N/A	0	N/A
1148	A	11	9	1.99	22	0.409
1149	F	0	0	N/A	0	N/A
1150	A	12	9	1.56	13	0.692
1151	F	0	0	N/A	0	N/A
1152	F	0	0	N/A	0	N/A
1153	F	0	0	N/A	0	N/A
1154	A	8	5	1.58	24	0.208
1155	A	2	2	0.67	9	0.222
1156	A	2	2	0.67	9	0.222
1157	A	2	2	0.67	15	0.133
1158	A	11	11	1.89	9	1.222
1159	F	0	0	N/A	0	N/A
1160	A	7	7	1.12	25	0.280
1161	F	0	0	N/A	0	N/A
1162	F	0	0	N/A	0	N/A
1163	A	10	9	1.80	28	0.321
1164	A	10	9	1.80	28	0.321
1165	F	0	0	N/A	0	N/A
1166	F	0	0	N/A	0	N/A
1167	A	11	7	0.93	37	0.189
1168	C	21	10	3.33	30	0.333
1169	C	16	7	2.63	30	0.233
1170	C	13	7	3.69	37	0.189
1171	A	9	8	0.91	16	0.500
1172	F	0	0	N/A	0	N/A
1173	F	0	0	N/A	0	N/A
1174	F	0	0	N/A	0	N/A
1175	F	0	0	N/A	0	N/A
1176	F	0	0	N/A	0	N/A
1177	F	0	0	N/A	0	N/A
1178	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1179	F	0	0	N/A	0	N/A
1180	F	0	0	N/A	0	N/A
1181	F	0	0	N/A	0	N/A
1182	F	0	0	N/A	0	N/A
1183	A	8	8	1.01	11	0.727
1184	A	8	8	1.01	11	0.727
1185	A	11	11	1.88	13	0.846
1186	F	0	0	N/A	0	N/A
1187	F	0	0	N/A	0	N/A
1188	A	7	7	1.59	31	0.226
1189	F	0	0	N/A	0	N/A
1190	A	5	5	1.20	13	0.385
1191	C	15	7	3.32	27	0.259
1192	C	15	7	3.32	27	0.259
1193	F	0	0	N/A	0	N/A
1194	C	12	6	2.77	43	0.140
1195	A	7	7	0.87	13	0.538
1196	A	7	7	0.89	13	0.538
1197	A	7	7	0.86	18	0.389
1198	C	15	8	3.87	29	0.276
1199	A	3	3	1.46	15	0.200
1200	A	10	10	1.10	27	0.370
1201	A	7	7	0.89	18	0.389
1202	A	8	8	1.47	26	0.308
1203	A	10	9	1.47	24	0.375
1204	A	7	6	0.89	13	0.462
1205	F	0	0	N/A	0	N/A
1206	A	7	6	0.94	25	0.240
1207	A	41	22	0.94	31	0.710
1208	A	1	1	0.98	28	0.036
1209	F	0	0	N/A	0	N/A
1210	F	0	0	N/A	0	N/A
1211	A	6	6	0.95	33	0.182
1212	A	6	6	1.11	17	0.353
1213	A	8	8	1.73	28	0.286
1214	A	8	8	1.73	28	0.286
1215	F	0	0	N/A	0	N/A
1216	F	0	0	N/A	0	N/A
1217	A	7	7	1.00	29	0.241
1218	A	10	6	0.98	23	0.261
1219	C	13	6	4.62	28	0.214
1220	C	14	7	4.55	27	0.259
1221	A	7	7	0.85	18	0.389
1222	A	7	7	0.85	20	0.350
1223	F	0	0	N/A	0	N/A
1224	F	0	0	N/A	0	N/A
1225	A	8	8	1.70	15	0.533

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1226	A	7	6	0.85	13	0.462
1227	F	0	0	N/A	0	N/A
1228	C	16	8	6.97	43	0.186
1229	C	16	8	6.91	43	0.186
1230	A	12	12	1.94	13	0.923
1231	F	0	0	N/A	0	N/A
1232	A	3	3	1.47	15	0.200
1233	A	8	8	1.02	18	0.444
1234	F	0	0	N/A	0	N/A
1235	F	0	0	N/A	0	N/A
1236	F	0	0	N/A	0	N/A
1237	F	0	0	N/A	0	N/A
1238	A	6	6	1.13	31	0.194
1239	A	8	7	1.00	21	0.333
1240	A	6	6	1.00	27	0.222
1241	A	12	7	1.97	23	0.304
1242	A	9	9	1.08	18	0.500
1243	A	9	9	1.09	18	0.500
1244	A	9	9	1.08	18	0.500
1245	C	17	9	2.37	25	0.360
1246	C	15	8	3.79	30	0.267
1247	A	3	3	1.42	13	0.231
1248	A	9	9	1.08	20	0.450
1249	A	10	10	1.10	29	0.345
1250	C	4	4	1.08	22	0.182
1251	A	44	20	1.91	27	0.741
1252	F	0	0	N/A	0	N/A
1253	A	9	6	0.72	32	0.188
1254	A	16	9	1.81	29	0.310
1255	F	0	0	N/A	0	N/A
1256	C	9	7	1.26	24	0.292
1257	C	9	7	1.26	24	0.292
1258	B	16	11	2.40	22	0.500
1259	F	0	0	N/A	0	N/A
1260	F	0	0	N/A	0	N/A
1261	C	11	7	1.59	32	0.219
1262	F	0	0	N/A	0	N/A
1263	A	3	3	0.77	13	0.231
1264	A	9	9	1.11	13	0.692
1265	A	3	3	0.77	13	0.231
1266	A	10	10	1.09	25	0.400
1267	F	0	0	N/A	0	N/A
1268	A	8	8	1.00	18	0.444
1269	C	18	6	23.84	37	0.162
1270	F	0	0	N/A	0	N/A
1271	C	18	8	2.82	37	0.216
1272	C	25	12	3.81	40	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1273	F	0	0	N/A	0	N/A
1274	A	10	6	0.90	23	0.261
1275	C	13	6	3.94	26	0.231
1276	C	13	6	3.94	30	0.200
1277	A	3	3	0.77	18	0.167
1278	A	7	6	1.21	15	0.400
1279	A	7	7	1.45	25	0.280
1280	C	11	6	1.14	30	0.200
1281	A	6	6	1.26	27	0.222
1282	A	8	7	0.96	13	0.538
1283	A	9	9	1.09	13	0.692
1284	A	11	11	1.79	11	1.000
1285	A	8	8	0.96	20	0.400
1286	C	19	8	3.48	40	0.200
1287	F	0	0	N/A	0	N/A
1288	C	2	2	0.44	26	0.077
1289	A	10	9	1.48	27	0.333
1290	A	8	8	1.48	25	0.320
1291	A	8	8	1.48	18	0.444
1292	A	5	5	1.00	27	0.185
1293	F	0	0	N/A	0	N/A
1294	A	8	6	0.96	13	0.462
1295	A	3	3	0.66	13	0.231
1296	A	9	9	0.99	13	0.692
1297	A	3	3	0.66	11	0.273
1298	A	3	3	0.66	13	0.231
1299	A	9	9	0.99	13	0.692
1300	A	3	3	0.66	11	0.273
1301	F	0	0	N/A	0	N/A
1302	A	5	5	1.14	15	0.333
1303	F	0	0	N/A	0	N/A
1304	F	0	0	N/A	0	N/A
1305	C	8	6	1.14	29	0.207
1306	C	16	6	1.34	29	0.207
1307	C	3	3	1.17	30	0.100
1308	F	0	0	N/A	0	N/A
1309	F	0	0	N/A	0	N/A
1310	A	1	1	0.96	25	0.040
1311	A	11	9	1.12	16	0.562
1312	F	0	0	N/A	0	N/A
1313	F	0	0	N/A	0	N/A
1314	F	0	0	N/A	0	N/A
1315	A	7	5	0.90	21	0.238
1316	C	25	12	3.62	38	0.316
1317	A	7	5	0.90	23	0.217
1318	A	10	5	1.00	25	0.200
1319	A	6	6	1.03	28	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1320	A	12	8	1.00	34	0.235
1321	F	0	0	N/A	0	N/A
1322	F	0	0	N/A	0	N/A
1323	A	11	8	1.38	23	0.348
1324	F	0	0	N/A	0	N/A
1325	A	6	3	1.18	28	0.107
1326	A	5	5	1.20	18	0.278
1327	A	11	11	1.77	15	0.733
1328	A	3	3	1.41	17	0.176
1329	F	0	0	N/A	0	N/A
1330	A	12	9	1.60	26	0.346
1331	C	29	13	6.25	42	0.310
1332	F	0	0	N/A	0	N/A
1333	F	0	0	N/A	0	N/A
1334	C	23	9	0.63	30	0.300
1335	A	4	3	0.89	13	0.231
1336	A	4	3	0.89	13	0.231
1337	A	13	12	1.96	13	0.923
1338	F	0	0	N/A	0	N/A
1339	A	9	8	1.10	18	0.444
1340	A	11	10	1.81	21	0.476
1341	C	136	18	50.64	54	0.333
1342	A	12	10	1.54	32	0.312
1343	C	29	13	6.19	42	0.310
1344	F	0	0	N/A	0	N/A
1345	F	0	0	N/A	0	N/A
1346	A	12	10	1.56	30	0.333
1347	F	0	0	N/A	0	N/A
1348	C	3	3	0.41	30	0.100
1349	F	0	0	N/A	0	N/A
1350	A	4	4	1.52	11	0.364
1351	C	20	9	3.37	40	0.225
1352	C	8	4	1.78	49	0.082
1353	F	0	0	N/A	0	N/A
1354	A	9	7	1.40	18	0.389
1355	A	4	4	0.90	18	0.222
1356	A	5	5	0.88	21	0.238
1357	A	4	4	0.90	20	0.200
1358	F	0	0	N/A	0	N/A
1359	F	0	0	N/A	0	N/A
1360	A	10	9	1.18	13	0.692
1361	A	10	9	1.18	13	0.692
1362	A	9	8	1.06	18	0.444
1363	F	0	0	N/A	0	N/A
1364	A	7	6	1.24	17	0.353
1365	A	7	7	1.46	24	0.292
1366	A	10	10	1.09	18	0.556

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1367	A	10	10	1.06	20	0.500
1368	C	12	10	1.11	34	0.294
1369	F	0	0	N/A	0	N/A
1370	F	0	0	N/A	0	N/A
1371	F	0	0	N/A	0	N/A
1372	F	0	0	N/A	0	N/A
1373	F	0	0	N/A	0	N/A
1374	C	14	13	2.00	19	0.684
1375	C	14	13	2.00	19	0.684
1376	A	3	3	1.38	15	0.200
1377	C	2	2	0.38	29	0.069
1378	C	13	6	5.21	33	0.182
1379	A	8	8	1.17	24	0.333
1380	A	10	10	1.08	13	0.769
1381	A	12	10	1.52	28	0.357
1382	A	11	10	1.52	26	0.385
1383	A	11	10	1.52	24	0.417
1384	F	0	0	N/A	0	N/A
1385	F	0	0	N/A	0	N/A
1386	F	0	0	N/A	0	N/A
1387	A	8	6	0.87	24	0.250
1388	A	9	6	0.82	24	0.250
1389	A	4	3	0.85	31	0.097
1390	A	8	6	0.95	43	0.140
1391	F	0	0	N/A	0	N/A
1392	F	0	0	N/A	0	N/A
1393	A	4	3	0.76	19	0.158
1394	F	0	0	N/A	0	N/A
1395	F	0	0	N/A	0	N/A
1396	C	13	6	3.71	28	0.214
1397	A	5	4	1.40	20	0.200
1398	A	3	3	1.40	20	0.150
1399	A	4	3	1.48	15	0.200
1400	F	0	0	N/A	0	N/A
1401	F	0	0	N/A	0	N/A
1402	A	10	10	1.11	27	0.370
1403	F	0	0	N/A	0	N/A
1404	A	8	8	1.78	25	0.320
1405	F	0	0	N/A	0	N/A
1406	F	0	0	N/A	0	N/A
1407	A	10	10	1.07	18	0.556
1408	A	6	6	1.09	29	0.207
1409	A	3	1	1.24	30	0.033
1410	A	3	2	0.84	42	0.048
1411	A	12	12	1.82	15	0.800
1412	B	14	12	2.02	13	0.923
1413	A	17	13	1.37	18	0.722

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1414	F	0	0	N/A	0	N/A
1415	A	9	9	1.33	26	0.346
1416	A	14	10	1.88	26	0.385
1417	C	4	4	0.49	37	0.108
1418	A	9	8	1.75	17	0.471
1419	F	0	0	N/A	0	N/A
1420	C	21	10	3.50	35	0.286
1421	A	4	3	0.76	13	0.231
1422	A	10	10	1.06	13	0.769
1423	A	4	4	0.76	13	0.308
1424	A	4	3	0.76	13	0.231
1425	A	4	4	0.76	13	0.308
1426	F	0	0	N/A	0	N/A
1427	A	3	2	0.85	39	0.051
1428	C	21	8	28.86	44	0.182
1429	A	5	4	1.61	13	0.308
1430	F	0	0	N/A	0	N/A
1431	F	0	0	N/A	0	N/A
1432	A	6	6	1.41	32	0.188
1433	F	0	0	N/A	0	N/A
1434	C	4	4	0.51	38	0.105
1435	F	0	0	N/A	0	N/A
1436	F	0	0	N/A	0	N/A
1437	F	0	0	N/A	0	N/A
1438	A	11	9	1.27	13	0.692
1439	A	3	3	1.32	20	0.150
1440	A	3	3	1.32	22	0.136
1441	C	21	8	23.02	36	0.222
1442	C	46	19	6.99	21	0.905
1443	B	27	11	2.02	20	0.550
1444	A	12	10	1.55	37	0.270
1445	A	8	7	1.11	38	0.184
1446	A	13	11	1.24	24	0.458
1447	F	0	0	N/A	0	N/A
1448	F	0	0	N/A	0	N/A
1449	A	13	9	1.30	16	0.562
1450	A	5	5	0.72	15	0.333
1451	F	0	0	N/A	0	N/A
1452	F	0	0	N/A	0	N/A
1453	F	0	0	N/A	0	N/A
1454	F	0	0	N/A	0	N/A
1455	F	0	0	N/A	0	N/A
1456	F	0	0	N/A	0	N/A
1457	C	21	10	3.42	36	0.278
1458	C	20	10	3.85	38	0.263
1459	F	0	0	N/A	0	N/A
1460	A	13	12	1.90	15	0.800

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1461	A	11	8	1.39	26	0.308
1462	A	12	9	1.39	29	0.310
1463	A	5	3	0.86	13	0.231
1464	A	11	10	1.15	13	0.769
1465	A	5	3	0.86	13	0.231
1466	A	11	10	1.15	13	0.769
1467	A	12	10	1.53	31	0.323
1468	F	0	0	N/A	0	N/A
1469	A	11	10	1.53	27	0.370
1470	A	11	10	1.53	25	0.400
1471	B	24	11	2.12	29	0.379
1472	F	0	0	N/A	0	N/A
1473	A	2	2	0.62	15	0.133
1474	A	11	6	1.89	17	0.353
1475	A	4	4	1.47	13	0.308
1476	A	6	4	1.69	15	0.267
1477	F	0	0	N/A	0	N/A
1478	C	10	8	1.04	22	0.364
1479	C	20	11	1.25	27	0.407
1480	C	20	11	1.25	27	0.407
1481	C	15	14	1.90	34	0.412
1482	A	7	4	1.11	34	0.118
1483	A	5	4	1.25	28	0.143
1484	F	0	0	N/A	0	N/A
1485	F	0	0	N/A	0	N/A
1486	A	12	9	1.35	13	0.692
1487	F	0	0	N/A	0	N/A
1488	F	0	0	N/A	0	N/A
1489	F	0	0	N/A	0	N/A
1490	A	9	8	1.65	15	0.533
1491	F	0	0	N/A	0	N/A
1492	F	0	0	N/A	0	N/A
1493	C	37	16	3.39	22	0.727
1494	A	7	6	0.89	40	0.150
1495	F	0	0	N/A	0	N/A
1496	A	16	13	1.37	45	0.289
1497	C	9	7	0.95	32	0.219
1498	A	7	3	1.07	26	0.115
1499	A	9	7	1.10	33	0.212
1500	F	0	0	N/A	0	N/A
1501	C	8	5	1.24	27	0.185
1502	C	53	21	7.58	48	0.438
1503	C	10	8	1.05	20	0.400
1504	C	23	9	15.75	42	0.214
1505	C	10	8	1.02	22	0.364
1506	F	0	0	N/A	0	N/A
1507	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1508	A	14	12	1.97	15	0.800
1509	A	3	3	1.37	15	0.200
1510	A	14	10	1.81	28	0.357
1511	B	17	10	2.63	28	0.357
1512	B	17	10	2.63	26	0.385
1513	A	6	6	1.07	29	0.207
1514	F	0	0	N/A	0	N/A
1515	A	6	6	0.67	18	0.333
1516	A	11	8	1.59	17	0.471
1517	A	5	5	0.70	17	0.294
1518	C	19	11	5.92	44	0.250
1519	C	19	11	6.07	44	0.250
1520	F	0	0	N/A	0	N/A
1521	F	0	0	N/A	0	N/A
1522	A	10	8	1.82	19	0.421
1523	C	43	13	6.38	27	0.482
1524	B	20	14	2.75	36	0.389
1525	A	12	11	1.49	33	0.333
1526	F	0	0	N/A	0	N/A
1527	F	0	0	N/A	0	N/A
1528	A	8	7	0.60	17	0.412
1529	F	0	0	N/A	0	N/A
1530	A	11	8	1.58	17	0.471
1531	C	21	9	4.92	47	0.192
1532	A	8	8	1.56	27	0.296
1533	F	0	0	N/A	0	N/A
1534	F	0	0	N/A	0	N/A
1535	F	0	0	N/A	0	N/A
1536	A	6	6	0.67	18	0.333
1537	A	6	6	0.67	18	0.333
1538	A	6	6	0.66	18	0.333
1539	F	0	0	N/A	0	N/A
1540	F	0	0	N/A	0	N/A
1541	C	21	9	4.88	47	0.192
1542	C	55	19	13.20	24	0.792
1543	A	11	10	1.15	28	0.357
1544	A	18	14	1.36	23	0.609
1545	F	0	0	N/A	0	N/A
1546	A	3	3	1.27	32	0.094
1547	F	0	0	N/A	0	N/A
1548	F	0	0	N/A	0	N/A
1549	A	16	6	1.00	32	0.188
1550	F	0	0	N/A	0	N/A
1551	A	12	11	1.30	30	0.367
1552	C	5	5	0.42	35	0.143
1553	F	0	0	N/A	0	N/A
1554	C	8	5	2.23	77	0.065

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1555	A	7	5	0.65	17	0.294
1556	A	12	9	1.07	24	0.375
1557	A	12	9	1.07	24	0.375
1558	F	0	0	N/A	0	N/A
1559	A	11	8	1.57	17	0.471
1560	A	11	8	1.57	17	0.471
1561	F	0	0	N/A	0	N/A
1562	A	11	8	1.57	17	0.471
1563	A	11	8	1.57	17	0.471
1564	A	11	8	1.57	17	0.471
1565	A	11	8	1.57	17	0.471
1566	F	0	0	N/A	0	N/A
1567	F	0	0	N/A	0	N/A
1568	F	0	0	N/A	0	N/A
1569	A	3	2	1.00	29	0.069
1570	A	5	3	0.55	19	0.158
1571	A	12	12	1.67	22	0.546
1572	C	27	8	4.84	29	0.276
1573	A	7	7	1.62	32	0.219
1574	F	0	0	N/A	0	N/A
1575	A	16	6	1.00	34	0.176
1576	A	13	10	1.35	35	0.286
1577	F	0	0	N/A	0	N/A
1578	C	10	4	4.39	38	0.105
1579	F	0	0	N/A	0	N/A
1580	A	18	7	1.40	25	0.280
1581	C	20	10	2.02	27	0.370
1582	A	5	5	0.98	24	0.208
1583	C	27	8	4.66	29	0.276
1584	F	0	0	N/A	0	N/A
1585	C	29	6	1.49	30	0.200
1586	A	13	11	1.02	37	0.297
1587	B	12	7	2.47	31	0.226
1588	A	13	10	1.02	24	0.417
1589	A	13	10	1.02	24	0.417
1590	F	0	0	N/A	0	N/A
1591	F	0	0	N/A	0	N/A
1592	F	0	0	N/A	0	N/A
1593	F	0	0	N/A	0	N/A
1594	F	0	0	N/A	0	N/A
1595	C	12	4	1.22	34	0.118
1596	A	6	6	0.66	22	0.273
1597	F	0	0	N/A	0	N/A
1598	C	55	11	3.51	27	0.407
1599	F	0	0	N/A	0	N/A
1600	A	8	5	1.00	26	0.192
1601	C	5	5	0.42	36	0.139

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1602	A	9	6	1.00	33	0.182
1603	C	18	6	20.08	39	0.154
1604	C	42	20	9.09	43	0.465
1605	F	0	0	N/A	0	N/A
1606	F	0	0	N/A	0	N/A
1607	C	16	7	1.81	32	0.219
1608	F	0	0	N/A	0	N/A
1609	F	0	0	N/A	0	N/A
1610	A	5	3	0.53	19	0.158
1611	A	10	10	1.02	27	0.370
1612	C	35	11	1.66	25	0.440
1613	F	0	0	N/A	0	N/A
1614	A	7	7	1.07	35	0.200
1615	A	7	7	1.07	35	0.200
1616	A	11	7	1.90	26	0.269
1617	A	2	2	0.37	34	0.059
1618	F	0	0	N/A	0	N/A
1619	A	1	1	0.55	13	0.077
1620	F	0	0	N/A	0	N/A
1621	F	0	0	N/A	0	N/A
1622	A	9	7	1.16	20	0.350
1623	A	3	2	0.96	35	0.057
1624	A	1	1	0.85	26	0.038
1625	A	7	7	0.96	15	0.467
1626	A	8	8	0.80	22	0.364
1627	C	5	5	0.97	35	0.143
1628	C	7	4	1.20	27	0.148
1629	C	7	4	1.20	27	0.148
1630	A	6	5	1.22	32	0.156
1631	A	6	5	1.22	36	0.139
1632	A	16	11	1.41	29	0.379
1633	F	0	0	N/A	0	N/A
1634	A	12	9	1.53	17	0.529
1635	A	12	12	1.63	22	0.546
1636	A	8	7	0.91	26	0.269
1637	A	27	12	1.87	22	0.546
1638	F	0	0	N/A	0	N/A
1639	C	10	6	2.91	28	0.214
1640	A	2	2	1.40	27	0.074
1641	A	12	9	1.53	17	0.529
1642	C	53	21	6.12	27	0.778
1643	C	27	8	3.24	40	0.200
1644	F	0	0	N/A	0	N/A
1645	A	3	3	1.77	26	0.115
1646	A	7	7	0.74	18	0.389
1647	A	8	7	0.67	16	0.438
1648	A	10	10	1.04	25	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1649	A	13	13	1.63	25	0.520
1650	A	13	9	1.29	25	0.360
1651	A	3	3	1.26	33	0.091
1652	A	14	10	1.29	30	0.333
1653	F	0	0	N/A	0	N/A
1654	F	0	0	N/A	0	N/A
1655	F	0	0	N/A	0	N/A
1656	F	0	0	N/A	0	N/A
1657	A	7	7	0.73	18	0.389
1658	A	5	5	1.52	21	0.238
1659	A	4	4	1.35	17	0.235
1660	F	0	0	N/A	0	N/A
1661	F	0	0	N/A	0	N/A
1662	A	8	5	1.00	26	0.192
1663	A	13	9	1.88	37	0.243
1664	A	9	6	1.00	33	0.182
1665	A	9	6	1.00	31	0.194
1666	F	0	0	N/A	0	N/A
1667	F	0	0	N/A	0	N/A
1668	C	9	8	0.87	34	0.235
1669	A	16	11	1.28	28	0.393
1670	A	13	13	1.70	27	0.482
1671	A	9	9	1.06	28	0.321
1672	C	10	4	3.80	41	0.098
1673	F	0	0	N/A	0	N/A
1674	F	0	0	N/A	0	N/A
1675	F	0	0	N/A	0	N/A
1676	A	10	10	1.03	27	0.370
1677	F	0	0	N/A	0	N/A
1678	F	0	0	N/A	0	N/A
1679	F	0	0	N/A	0	N/A
1680	C	5	5	0.92	36	0.139
1681	A	23	12	1.94	30	0.400
1682	F	0	0	N/A	0	N/A
1683	F	0	0	N/A	0	N/A
1684	C	9	7	0.98	32	0.219
1685	B	20	8	2.10	24	0.333
1686	B	13	10	2.44	24	0.417
1687	F	0	0	N/A	0	N/A
1688	F	0	0	N/A	0	N/A
1689	A	12	9	1.50	17	0.529
1690	A	12	9	1.50	17	0.529
1691	F	0	0	N/A	0	N/A
1692	A	12	9	1.50	17	0.529
1693	F	0	0	N/A	0	N/A
1694	A	12	9	1.50	17	0.529
1695	C	9	7	0.99	28	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1696	F	0	0	N/A	0	N/A
1697	A	12	9	1.50	17	0.529
1698	A	13	6	1.87	19	0.316
1699	F	0	0	N/A	0	N/A
1700	A	3	3	1.68	23	0.130
1701	F	0	0	N/A	0	N/A
1702	F	0	0	N/A	0	N/A
1703	F	0	0	N/A	0	N/A
1704	F	0	0	N/A	0	N/A
1705	F	0	0	N/A	0	N/A
1706	A	2	2	1.31	24	0.083
1707	C	13	7	1.93	36	0.194
1708	C	13	7	1.93	35	0.200
1709	C	38	9	6.82	39	0.231
1710	F	0	0	N/A	0	N/A
1711	A	12	9	1.36	21	0.429
1712	A	6	6	1.00	41	0.146
1713	A	12	11	1.61	29	0.379
1714	A	10	10	1.09	25	0.400
1715	A	10	10	1.06	27	0.370
1716	F	0	0	N/A	0	N/A
1717	A	9	9	1.26	32	0.281
1718	A	14	11	1.70	35	0.314
1719	F	0	0	N/A	0	N/A
1720	F	0	0	N/A	0	N/A
1721	F	0	0	N/A	0	N/A
1722	A	12	9	1.50	17	0.529
1723	A	10	10	1.06	29	0.345
1724	A	10	10	1.05	31	0.323
1725	A	12	9	1.50	17	0.529
1726	A	12	9	1.50	17	0.529
1727	F	0	0	N/A	0	N/A
1728	A	12	9	1.50	17	0.529
1729	A	12	9	1.50	17	0.529
1730	F	0	0	N/A	0	N/A
1731	C	18	7	2.67	42	0.167
1732	C	19	8	2.67	44	0.182
1733	F	0	0	N/A	0	N/A
1734	A	13	10	1.52	27	0.370
1735	F	0	0	N/A	0	N/A
1736	F	0	0	N/A	0	N/A
1737	C	28	9	3.23	40	0.225
1738	C	7	5	0.64	26	0.192
1739	F	0	0	N/A	0	N/A
1740	F	0	0	N/A	0	N/A
1741	F	0	0	N/A	0	N/A
1742	A	8	6	0.81	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1743	F	0	0	N/A	0	N/A
1744	F	0	0	N/A	0	N/A
1745	A	12	9	1.13	25	0.360
1746	F	0	0	N/A	0	N/A
1747	F	0	0	N/A	0	N/A
1748	C	1	1	0.14	17	0.059
1749	A	16	11	1.86	25	0.440
1750	A	25	13	1.86	22	0.591
1751	F	0	0	N/A	0	N/A
1752	A	12	9	1.49	17	0.529
1753	F	0	0	N/A	0	N/A
1754	A	12	9	1.49	17	0.529
1755	A	12	9	1.49	17	0.529
1756	A	12	9	1.49	17	0.529
1757	F	0	0	N/A	0	N/A
1758	A	12	9	1.49	17	0.529
1759	A	12	9	1.49	17	0.529
1760	C	41	13	2.22	42	0.310
1761	F	0	0	N/A	0	N/A
1762	F	0	0	N/A	0	N/A
1763	F	0	0	N/A	0	N/A
1764	A	13	9	0.71	21	0.429
1765	F	0	0	N/A	0	N/A
1766	A	15	11	1.27	36	0.306
1767	F	0	0	N/A	0	N/A
1768	F	0	0	N/A	0	N/A
1769	F	0	0	N/A	0	N/A
1770	C	10	8	1.02	41	0.195
1771	A	19	14	1.38	32	0.438
1772	F	0	0	N/A	0	N/A
1773	F	0	0	N/A	0	N/A
1774	F	0	0	N/A	0	N/A
1775	A	32	22	1.35	31	0.710
1776	F	0	0	N/A	0	N/A
1777	C	12	10	1.50	35	0.286
1778	F	0	0	N/A	0	N/A
1779	C	18	5	2.05	25	0.200
1780	A	11	10	1.12	31	0.323
1781	C	4	4	0.29	26	0.154
1782	F	0	0	N/A	0	N/A
1783	A	14	11	1.71	34	0.324
1784	F	0	0	N/A	0	N/A
1785	F	0	0	N/A	0	N/A
1786	F	0	0	N/A	0	N/A
1787	C	32	8	2.96	50	0.160
1788	F	0	0	N/A	0	N/A
1789	A	4	4	1.14	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1790	C	21	10	5.53	46	0.217
1791	F	0	0	N/A	0	N/A
1792	F	0	0	N/A	0	N/A
1793	A	12	5	0.92	27	0.185
1794	F	0	0	N/A	0	N/A
1795	F	0	0	N/A	0	N/A
1796	F	0	0	N/A	0	N/A
1797	F	0	0	N/A	0	N/A
1798	C	14	8	1.60	36	0.222
1799	F	0	0	N/A	0	N/A
1800	A	9	8	0.88	24	0.333
1801	B	19	10	2.42	29	0.345
1802	F	0	0	N/A	0	N/A
1803	F	0	0	N/A	0	N/A
1804	C	3	3	0.28	22	0.136
1805	C	3	3	0.28	22	0.136
1806	C	3	3	0.26	24	0.125
1807	C	3	3	0.26	24	0.125
1808	C	15	7	0.78	29	0.241
1809	C	15	7	0.78	29	0.241
1810	C	15	7	0.78	27	0.259
1811	C	15	7	0.78	27	0.259
1812	A	13	11	0.97	33	0.333
1813	C	7	4	0.65	24	0.167
1814	C	7	4	0.65	24	0.167
1815	C	7	5	0.60	24	0.208
1816	C	7	5	0.60	24	0.208
1817	F	0	0	N/A	0	N/A
1818	F	0	0	N/A	0	N/A
1819	F	0	0	N/A	0	N/A
1820	A	26	13	1.37	35	0.371
1821	F	0	0	N/A	0	N/A
1822	F	0	0	N/A	0	N/A
1823	A	18	10	1.00	23	0.435
1824	F	0	0	N/A	0	N/A
1825	F	0	0	N/A	0	N/A
1826	F	0	0	N/A	0	N/A
1827	A	11	10	1.01	30	0.333
1828	A	1	1	0.51	16	0.062
1829	F	0	0	N/A	0	N/A
1830	C	8	5	1.97	81	0.062
1831	A	13	10	1.55	34	0.294
1832	A	11	5	1.00	31	0.161
1833	F	0	0	N/A	0	N/A
1834	A	13	10	1.55	17	0.588
1835	A	14	10	1.75	35	0.286
1836	C	21	10	3.12	45	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1837	C	17	9	2.44	42	0.214
1838	F	0	0	N/A	0	N/A
1839	A	13	9	1.56	17	0.529
1840	A	13	9	1.56	17	0.529
1841	C	3	3	0.25	24	0.125
1842	F	0	0	N/A	0	N/A
1843	A	13	9	1.56	17	0.529
1844	C	18	12	3.01	45	0.267
1845	A	13	9	1.56	17	0.529
1846	F	0	0	N/A	0	N/A
1847	C	27	10	16.15	44	0.227
1848	F	0	0	N/A	0	N/A
1849	A	10	7	1.45	40	0.175
1850	F	0	0	N/A	0	N/A
1851	A	10	7	1.45	41	0.171
1852	A	1	1	0.57	22	0.045
1853	F	0	0	N/A	0	N/A
1854	F	0	0	N/A	0	N/A
1855	A	1	1	0.48	19	0.053
1856	C	3	3	0.26	27	0.111
1857	F	0	0	N/A	0	N/A
1858	F	0	0	N/A	0	N/A
1859	A	9	7	0.60	32	0.219
1860	F	0	0	N/A	0	N/A
1861	C	7	4	1.90	52	0.077
1862	F	0	0	N/A	0	N/A
1863	C	57	19	4.87	20	0.950
1864	F	0	0	N/A	0	N/A
1865	F	0	0	N/A	0	N/A
1866	A	12	5	0.95	31	0.161
1867	A	1	1	0.56	17	0.059
1868	F	0	0	N/A	0	N/A
1869	F	0	0	N/A	0	N/A
1870	A	10	10	0.97	25	0.400
1871	F	0	0	N/A	0	N/A
1872	A	14	10	1.07	23	0.435
1873	F	0	0	N/A	0	N/A
1874	F	0	0	N/A	0	N/A
1875	A	10	7	1.47	41	0.171
1876	A	7	7	0.68	22	0.318
1877	F	0	0	N/A	0	N/A
1878	F	0	0	N/A	0	N/A
1879	A	10	10	0.82	27	0.370
1880	C	4	4	0.37	35	0.114
1881	F	0	0	N/A	0	N/A
1882	A	5	3	0.73	21	0.143
1883	A	1	1	0.47	16	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1884	F	0	0	N/A	0	N/A
1885	F	0	0	N/A	0	N/A
1886	F	0	0	N/A	0	N/A
1887	A	9	9	1.19	38	0.237
1888	F	0	0	N/A	0	N/A
1889	C	16	5	1.10	39	0.128
1890	C	32	18	2.15	27	0.667
1891	A	14	7	1.27	52	0.135
1892	A	31	14	1.74	34	0.412
1893	C	16	5	1.02	39	0.128
1894	A	1	1	0.50	18	0.056
1895	A	14	13	1.51	24	0.542
1896	A	22	13	1.32	45	0.289
1897	F	0	0	N/A	0	N/A
1898	A	8	5	1.44	34	0.147
1899	C	28	8	2.78	46	0.174
1900	F	0	0	N/A	0	N/A
1901	F	0	0	N/A	0	N/A
1902	F	0	0	N/A	0	N/A
1903	F	0	0	N/A	0	N/A
1904	F	0	0	N/A	0	N/A
1905	A	3	3	0.69	20	0.150
1906	F	0	0	N/A	0	N/A
1907	C	11	8	0.67	27	0.296
1908	C	11	8	0.67	27	0.296
1909	A	2	2	0.97	16	0.125
1910	F	0	0	N/A	0	N/A
1911	F	0	0	N/A	0	N/A
1912	A	11	9	1.08	24	0.375
1913	C	10	8	0.51	29	0.276
1914	A	10	6	0.88	43	0.140
1915	C	18	8	1.29	25	0.320
1916	F	0	0	N/A	0	N/A
1917	C	7	6	0.44	26	0.231
1918	C	14	13	1.43	28	0.464
1919	C	7	6	0.44	24	0.250
1920	C	7	6	0.44	24	0.250
1921	F	0	0	N/A	0	N/A
1922	A	13	10	1.22	23	0.435
1923	F	0	0	N/A	0	N/A
1924	F	0	0	N/A	0	N/A
1925	F	0	0	N/A	0	N/A
1926	A	12	11	1.61	46	0.239
1927	F	0	0	N/A	0	N/A
1928	C	9	8	1.07	47	0.170
1929	C	9	8	1.07	47	0.170
1930	C	4	4	0.25	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1931	C	4	4	0.25	22	0.182
1932	C	7	4	0.87	29	0.138
1933	C	7	4	0.87	29	0.138
1934	C	10	8	0.53	29	0.276
1935	C	10	8	0.53	29	0.276
1936	A	5	5	0.74	23	0.217
1937	C	26	14	2.41	41	0.342
1938	F	0	0	N/A	0	N/A
1939	F	0	0	N/A	0	N/A
1940	F	0	0	N/A	0	N/A
1941	C	28	6	2.14	34	0.176
1942	F	0	0	N/A	0	N/A
1943	A	1	1	0.37	17	0.059
1944	F	0	0	N/A	0	N/A
1945	C	10	6	1.11	30	0.200
1946	A	10	10	1.38	19	0.526
1947	F	0	0	N/A	0	N/A
1948	F	0	0	N/A	0	N/A
1949	F	0	0	N/A	0	N/A
1950	F	0	0	N/A	0	N/A
1951	B	20	14	3.21	43	0.326
1952	F	0	0	N/A	0	N/A
1953	A	5	5	1.38	21	0.238
1954	A	30	13	1.27	39	0.333
1955	C	16	8	2.13	43	0.186
1956	B	12	4	2.16	26	0.154
1957	F	0	0	N/A	0	N/A
1958	A	9	5	1.08	22	0.227
1959	A	10	5	0.98	29	0.172
1960	A	9	5	1.08	20	0.250
1961	A	10	5	0.98	25	0.200
1962	A	16	11	1.77	28	0.393
1963	A	7	5	0.38	32	0.156
1964	A	5	4	0.42	32	0.125
1965	A	7	5	0.38	38	0.132
1966	F	0	0	N/A	0	N/A
1967	F	0	0	N/A	0	N/A
1968	A	1	1	0.42	19	0.053
1969	B	9	5	2.15	39	0.128
1970	A	15	7	1.32	37	0.189
1971	A	8	5	1.01	31	0.161
1972	A	16	11	1.75	28	0.393
1973	F	0	0	N/A	0	N/A
1974	F	0	0	N/A	0	N/A
1975	F	0	0	N/A	0	N/A
1976	A	16	10	0.52	42	0.238
1977	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1978	F	0	0	N/A	0	N/A
1979	F	0	0	N/A	0	N/A
1980	A	5	3	0.76	14	0.214
1981	C	9	5	0.80	35	0.143
1982	F	0	0	N/A	0	N/A
1983	A	1	1	0.49	18	0.056
1984	A	27	11	1.71	28	0.393
1985	F	0	0	N/A	0	N/A
1986	A	16	11	1.81	29	0.379
1987	F	0	0	N/A	0	N/A
1988	F	0	0	N/A	0	N/A
1989	A	5	3	0.65	21	0.143
1990	A	10	10	1.36	23	0.435
1991	F	0	0	N/A	0	N/A
1992	A	6	5	0.30	34	0.147
1993	A	23	12	1.00	28	0.429
1994	C	46	21	7.02	56	0.375
1995	F	0	0	N/A	0	N/A
1996	F	0	0	N/A	0	N/A
1997	F	0	0	N/A	0	N/A
1998	C	8	5	0.95	55	0.091
1999	F	0	0	N/A	0	N/A
2000	F	0	0	N/A	0	N/A
2001	A	10	5	1.06	27	0.185
2002	F	0	0	N/A	0	N/A
2003	F	0	0	N/A	0	N/A
2004	F	0	0	N/A	0	N/A
2005	A	9	5	1.99	36	0.139
2006	F	0	0	N/A	0	N/A
2007	F	0	0	N/A	0	N/A
2008	A	6	6	0.83	24	0.250
2009	F	0	0	N/A	0	N/A
2010	F	0	0	N/A	0	N/A
2011	F	0	0	N/A	0	N/A
2012	F	0	0	N/A	0	N/A
2013	A	23	9	1.73	52	0.173
2014	F	0	0	N/A	0	N/A
2015	F	0	0	N/A	0	N/A
2016	C	9	9	1.78	24	0.375
2017	A	17	13	1.65	35	0.371
2018	F	0	0	N/A	0	N/A
2019	F	0	0	N/A	0	N/A
2020	F	0	0	N/A	0	N/A
2021	C	6	5	0.47	29	0.172
2022	F	0	0	N/A	0	N/A
2023	F	0	0	N/A	0	N/A
2024	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
2025	F	0	0	N/A	0	N/A
2026	A	4	4	0.46	23	0.174
2027	F	0	0	N/A	0	N/A
2028	F	0	0	N/A	0	N/A
2029	F	0	0	N/A	0	N/A
2030	F	0	0	N/A	0	N/A
2031	A	4	4	0.81	17	0.235
2032	F	0	0	N/A	0	N/A
2033	A	9	6	0.46	26	0.231
2034	F	0	0	N/A	0	N/A
2035	A	6	4	0.44	31	0.129
2036	C	9	9	1.79	22	0.409
2037	F	0	0	N/A	0	N/A
2038	B	19	9	2.03	43	0.209
2039	F	0	0	N/A	0	N/A
2040	F	0	0	N/A	0	N/A
2041	C	6	5	0.48	30	0.167
2042	C	7	6	0.57	30	0.200
2043	F	0	0	N/A	0	N/A
2044	B	8	4	2.17	29	0.138
2045	B	8	4	2.17	27	0.148
2046	F	0	0	N/A	0	N/A
2047	F	0	0	N/A	0	N/A
2048	A	8	6	1.00	31	0.194
2049	F	0	0	N/A	0	N/A
2050	C	8	5	0.97	52	0.096
2051	F	0	0	N/A	0	N/A
2052	F	0	0	N/A	0	N/A
2053	F	0	0	N/A	0	N/A
2054	A	4	4	1.34	22	0.182
2055	F	0	0	N/A	0	N/A
2056	A	15	7	1.07	34	0.206
2057	A	15	7	1.07	32	0.219
2058	F	0	0	N/A	0	N/A
2059	A	4	4	1.35	20	0.200
2060	A	5	5	1.36	21	0.238
2061	F	0	0	N/A	0	N/A
2062	F	0	0	N/A	0	N/A
2063	F	0	0	N/A	0	N/A
2064	A	7	7	0.81	31	0.226
2065	F	0	0	N/A	0	N/A
2066	F	0	0	N/A	0	N/A
2067	A	7	7	0.88	21	0.333
2068	F	0	0	N/A	0	N/A
2069	F	0	0	N/A	0	N/A
2070	F	0	0	N/A	0	N/A
2071	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
2072	F	0	0	N/A	0	N/A
2073	A	18	9	1.67	51	0.176
2074	F	0	0	N/A	0	N/A
2075	F	0	0	N/A	0	N/A
2076	F	0	0	N/A	0	N/A
2077	F	0	0	N/A	0	N/A
2078	C	44	7	1.90	41	0.171
2079	F	0	0	N/A	0	N/A
2080	A	36	14	1.64	24	0.583
2081	A	8	7	0.58	28	0.250
2082	F	0	0	N/A	0	N/A
2083	A	16	11	1.08	27	0.407
2084	A	25	13	1.08	24	0.542
2085	A	16	11	1.08	27	0.407
2086	A	25	13	1.08	24	0.542
2087	F	0	0	N/A	0	N/A
2088	C	5	3	0.36	32	0.094
2089	F	0	0	N/A	0	N/A
2090	F	0	0	N/A	0	N/A
2091	B	25	10	2.19	25	0.400
2092	C	10	7	1.87	73	0.096
2093	F	0	0	N/A	0	N/A
2094	B	18	13	2.68	50	0.260
2095	A	15	10	1.10	28	0.357
2096	F	0	0	N/A	0	N/A
2097	F	0	0	N/A	0	N/A
2098	F	0	0	N/A	0	N/A
2099	F	0	0	N/A	0	N/A
2100	C	18	7	2.22	48	0.146
2101	C	19	8	2.22	50	0.160
2102	F	0	0	N/A	0	N/A
2103	F	0	0	N/A	0	N/A
2104	B	40	18	2.09	29	0.621
2105	A	1	1	0.59	17	0.059
2106	F	0	0	N/A	0	N/A
2107	F	0	0	N/A	0	N/A
2108	A	18	11	1.16	30	0.367
2109	F	0	0	N/A	0	N/A
2110	F	0	0	N/A	0	N/A
2111	F	0	0	N/A	0	N/A
2112	F	0	0	N/A	0	N/A
2113	F	0	0	N/A	0	N/A
2114	F	0	0	N/A	0	N/A
2115	A	18	8	1.69	31	0.258
2116	F	0	0	N/A	0	N/A
2117	C	18	17	1.35	27	0.630
2118	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
2119	F	0	0	N/A	0	N/A
2120	F	0	0	N/A	0	N/A
2121	C	16	8	1.65	43	0.186
2122	F	0	0	N/A	0	N/A
2123	F	0	0	N/A	0	N/A
2124	A	11	8	1.00	34	0.235
2125	F	0	0	N/A	0	N/A
2126	F	0	0	N/A	0	N/A
2127	A	11	11	0.98	26	0.423
2128	F	0	0	N/A	0	N/A
2129	A	10	10	0.95	35	0.286
2130	F	0	0	N/A	0	N/A
2131	F	0	0	N/A	0	N/A
2132	A	15	10	1.06	22	0.454
2133	F	0	0	N/A	0	N/A
2134	F	0	0	N/A	0	N/A
2135	F	0	0	N/A	0	N/A
2136	A	17	13	1.45	44	0.296
2137	C	153	22	3.57	25	0.880
2138	F	0	0	N/A	0	N/A
2139	F	0	0	N/A	0	N/A
2140	C	10	7	1.87	73	0.096
2141	F	0	0	N/A	0	N/A
2142	F	0	0	N/A	0	N/A
2143	F	0	0	N/A	0	N/A
2144	F	0	0	N/A	0	N/A
2145	F	0	0	N/A	0	N/A
2146	A	9	5	1.72	40	0.125
2147	A	9	5	1.72	35	0.143
2148	A	10	6	1.72	42	0.143
2149	A	3	3	0.57	18	0.167
2150	A	36	14	1.58	26	0.538
2151	F	0	0	N/A	0	N/A
2152	C	10	7	1.03	66	0.106
2153	F	0	0	N/A	0	N/A
2154	F	0	0	N/A	0	N/A
2155	A	18	11	1.11	24	0.458
2156	C	36	17	2.13	45	0.378
2157	F	0	0	N/A	0	N/A
2158	C	185	21	3.90	44	0.477
2159	F	0	0	N/A	0	N/A
2160	F	0	0	N/A	0	N/A
2161	F	0	0	N/A	0	N/A
2162	F	0	0	N/A	0	N/A
2163	F	0	0	N/A	0	N/A
2164	B	27	15	3.01	50	0.300
2165	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
2166	F	0	0	N/A	0	N/A
2167	A	25	14	1.78	35	0.400
2168	A	4	4	0.40	23	0.174
2169	A	10	6	1.64	41	0.146
2170	A	9	5	1.64	34	0.147
2171	F	0	0	N/A	0	N/A
2172	A	9	9	0.98	60	0.150
2173	A	6	6	0.72	20	0.300
2174	F	0	0	N/A	0	N/A
2175	F	0	0	N/A	0	N/A
2176	A	18	12	0.36	32	0.375
2177	A	16	11	0.39	32	0.344
2178	F	0	0	N/A	0	N/A
2179	F	0	0	N/A	0	N/A
2180	F	0	0	N/A	0	N/A
2181	F	0	0	N/A	0	N/A
2182	C	18	8	0.50	27	0.296
2183	C	18	8	0.50	25	0.320
2184	F	0	0	N/A	0	N/A
2185	F	0	0	N/A	0	N/A
2186	A	26	12	1.63	25	0.480
2187	A	11	8	0.54	80	0.100
2188	F	0	0	N/A	0	N/A
2189	F	0	0	N/A	0	N/A
2190	F	0	0	N/A	0	N/A
2191	C	10	7	1.07	66	0.106
2192	A	25	12	1.69	31	0.387
2193	F	0	0	N/A	0	N/A
2194	F	0	0	N/A	0	N/A
2195	F	0	0	N/A	0	N/A
2196	F	0	0	N/A	0	N/A
2197	B	34	12	2.57	32	0.375
2198	F	0	0	N/A	0	N/A
2199	A	7	6	1.19	31	0.194
2200	F	0	0	N/A	0	N/A
2201	F	0	0	N/A	0	N/A
2202	F	0	0	N/A	0	N/A
2203	F	0	0	N/A	0	N/A
2204	F	0	0	N/A	0	N/A
2205	A	8	7	0.62	24	0.292
2206	C	16	7	1.33	51	0.137
2207	F	0	0	N/A	0	N/A
2208	F	0	0	N/A	0	N/A
2209	C	9	6	0.65	37	0.162
2210	F	0	0	N/A	0	N/A
2211	A	1	1	0.50	25	0.040
2212	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
2213	A	2	2	0.90	36	0.056
2214	F	0	0	N/A	0	N/A
2215	C	28	10	2.18	46	0.217
2216	F	0	0	N/A	0	N/A
2217	A	3	3	0.15	20	0.150
2218	F	0	0	N/A	0	N/A
2219	A	26	13	1.62	31	0.419
2220	F	0	0	N/A	0	N/A
2221	F	0	0	N/A	0	N/A
2222	F	0	0	N/A	0	N/A
2223	C	207	18	3.86	44	0.409
2224	F	0	0	N/A	0	N/A
2225	A	10	6	1.88	61	0.098
2226	C	26	4	1.13	34	0.118
2227	F	0	0	N/A	0	N/A
2228	F	0	0	N/A	0	N/A
2229	C	4	4	0.25	33	0.121
2230	F	0	0	N/A	0	N/A
2231	C	41	11	2.56	36	0.306
2232	F	0	0	N/A	0	N/A
2233	F	0	0	N/A	0	N/A
2234	A	4	3	0.61	27	0.111
2235	A	22	11	0.98	31	0.355
2236	C	10	6	4.70	43	0.140
2237	A	13	9	1.00	38	0.237
2238	F	0	0	N/A	0	N/A
2239	F	0	0	N/A	0	N/A
2240	A	3	2	0.82	31	0.065
2241	F	0	0	N/A	0	N/A
2242	F	0	0	N/A	0	N/A
2243	F	0	0	N/A	0	N/A
2244	A	17	12	1.55	22	0.546
2245	F	0	0	N/A	0	N/A
2246	F	0	0	N/A	0	N/A
2247	F	0	0	N/A	0	N/A
2248	F	0	0	N/A	0	N/A
2249	C	7	4	0.38	25	0.160
2250	F	0	0	N/A	0	N/A
2251	F	0	0	N/A	0	N/A
2252	A	9	8	0.66	26	0.308
2253	F	0	0	N/A	0	N/A
2254	F	0	0	N/A	0	N/A
2255	F	0	0	N/A	0	N/A
2256	A	11	10	1.06	29	0.345
2257	F	0	0	N/A	0	N/A
2258	F	0	0	N/A	0	N/A
2259	C	17	10	1.31	53	0.189

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
2260	C	28	15	1.02	34	0.441
2261	F	0	0	N/A	0	N/A
2262	F	0	0	N/A	0	N/A
2263	F	0	0	N/A	0	N/A
2264	F	0	0	N/A	0	N/A
2265	F	0	0	N/A	0	N/A
2266	F	0	0	N/A	0	N/A
2267	C	44	20	1.70	44	0.454
2268	C	20	11	1.13	47	0.234
2269	A	2	2	0.80	36	0.056
2270	A	18	12	1.30	37	0.324
2271	C	40	19	1.72	36	0.528
2272	A	17	12	1.50	26	0.462
2273	A	34	18	1.13	28	0.643
2274	A	9	5	1.68	57	0.088
2275	A	10	6	1.68	60	0.100
2276	F	0	0	N/A	0	N/A
2277	F	0	0	N/A	0	N/A
2278	A	10	9	0.72	26	0.346
2279	A	6	4	0.84	31	0.129
2280	C	20	10	1.47	38	0.263
2281	F	0	0	N/A	0	N/A
2282	F	0	0	N/A	0	N/A
2283	F	0	0	N/A	0	N/A
2284	C	209	20	4.16	45	0.444
2285	F	0	0	N/A	0	N/A
2286	C	7	4	0.40	32	0.125
2287	F	0	0	N/A	0	N/A
2288	A	20	11	0.43	44	0.250
2289	A	18	12	1.17	27	0.444
2290	A	15	14	1.29	26	0.538
2291	F	0	0	N/A	0	N/A
2292	F	0	0	N/A	0	N/A
2293	F	0	0	N/A	0	N/A
2294	F	0	0	N/A	0	N/A
2295	A	2	2	0.91	30	0.067
2296	F	0	0	N/A	0	N/A
2297	A	4	4	0.38	36	0.111
2298	F	0	0	N/A	0	N/A
2299	F	0	0	N/A	0	N/A
2300	C	9	5	0.54	32	0.156
2301	A	36	19	1.05	28	0.679
2302	C	55	7	0.49	24	0.292
2303	A	18	6	1.90	23	0.261
2304	A	2	2	0.06	38	0.053
2305	A	9	7	1.48	26	0.269
2306	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
2307	A	15	14	1.26	32	0.438
2308	A	1	1	0.42	25	0.040
2309	A	8	6	0.93	22	0.273
2310	B	28	4	2.93	33	0.121
2311	F	0	0	N/A	0	N/A
2312	F	0	0	N/A	0	N/A
2313	C	8	5	0.43	25	0.200
2314	F	0	0	N/A	0	N/A
2315	C	9	6	0.48	31	0.194
2316	F	0	0	N/A	0	N/A
2317	F	0	0	N/A	0	N/A
2318	F	0	0	N/A	0	N/A
2319	F	0	0	N/A	0	N/A
2320	A	2	2	0.65	39	0.051
2321	F	0	0	N/A	0	N/A
2322	C	43	6	0.43	22	0.273
2323	A	8	6	0.77	22	0.273
2324	F	0	0	N/A	0	N/A
2325	A	7	5	0.41	35	0.143
2326	A	22	16	1.43	26	0.615
2327	F	0	0	N/A	0	N/A
2328	F	0	0	N/A	0	N/A
2329	F	0	0	N/A	0	N/A
2330	F	0	0	N/A	0	N/A
2331	F	0	0	N/A	0	N/A
2332	F	0	0	N/A	0	N/A
2333	F	0	0	N/A	0	N/A
2334	B	35	17	2.26	46	0.370
2335	A	22	16	1.39	32	0.500
2336	F	0	0	N/A	0	N/A
2337	A	7	7	1.02	26	0.269
2338	F	0	0	N/A	0	N/A
2339	A	20	14	1.11	49	0.286
2340	A	18	12	1.26	49	0.245
2341	F	0	0	N/A	0	N/A
2342	F	0	0	N/A	0	N/A
2343	F	0	0	N/A	0	N/A
2344	F	0	0	N/A	0	N/A
2345	A	7	4	0.63	40	0.100
2346	A	17	12	1.03	28	0.429
2347	F	0	0	N/A	0	N/A
2348	A	2	2	0.05	39	0.051
2349	F	0	0	N/A	0	N/A
2350	A	4	4	1.09	34	0.118
2351	A	3	3	0.89	31	0.097
2352	F	0	0	N/A	0	N/A
2353	A	4	4	1.12	35	0.114

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
2354	F	0	0	N/A	0	N/A
2355	F	0	0	N/A	0	N/A
2356	F	0	0	N/A	0	N/A
2357	C	4	2	0.37	33	0.061
2358	A	4	4	1.06	34	0.118
2359	A	34	15	1.07	23	0.652
2360	A	9	5	1.17	54	0.093
2361	A	4	4	1.11	35	0.114
2362	F	0	0	N/A	0	N/A
2363	A	10	5	0.91	32	0.156
2364	B	38	18	2.40	46	0.391
2365	C	4	4	0.11	24	0.167
2366	C	4	4	0.11	24	0.167
2367	C	4	4	0.10	26	0.154
2368	A	19	9	0.90	29	0.310
2369	F	0	0	N/A	0	N/A
2370	A	10	5	1.00	30	0.167
2371	F	0	0	N/A	0	N/A
2372	A	19	9	0.99	27	0.333
2373	F	0	0	N/A	0	N/A
2374	A	7	7	0.41	20	0.350
2375	C	9	6	0.22	28	0.214
2376	C	9	6	0.22	26	0.231
2377	C	9	6	0.22	26	0.231
2378	F	0	0	N/A	0	N/A
2379	F	0	0	N/A	0	N/A
2380	A	19	9	0.99	29	0.310
2381	A	10	5	0.98	32	0.156
2382	F	0	0	N/A	0	N/A
2383	A	9	5	1.07	52	0.096
2384	A	25	10	0.94	40	0.250
2385	F	0	0	N/A	0	N/A
2386	F	0	0	N/A	0	N/A
2387	A	2	2	0.09	43	0.047
2388	A	7	7	0.43	22	0.318
2389	A	16	6	1.51	29	0.207
2390	C	14	10	0.26	34	0.294
2391	C	14	10	0.26	34	0.294
2392	C	10	9	0.22	32	0.281
2393	C	10	9	0.22	32	0.281
2394	A	5	5	1.01	29	0.172
2395	F	0	0	N/A	0	N/A
2396	F	0	0	N/A	0	N/A
2397	F	0	0	N/A	0	N/A
2398	A	14	9	0.89	40	0.225
2399	C	22	9	1.96	41	0.220
2400	A	5	5	1.00	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2401	A	17	13	0.89	42	0.310
2402	F	0	0	N/A	0	N/A
2403	A	20	14	0.92	49	0.286
2404	A	18	14	1.00	42	0.333
2405	A	10	6	0.95	53	0.113
2406	A	9	5	0.95	52	0.096
2407	F	0	0	N/A	0	N/A
2408	A	3	3	0.69	31	0.097
2409	F	0	0	N/A	0	N/A
2410	A	9	5	0.90	52	0.096
2411	F	0	0	N/A	0	N/A
2412	F	0	0	N/A	0	N/A
2413	A	22	6	1.99	31	0.194
2414	F	0	0	N/A	0	N/A
2415	A	10	6	0.71	44	0.136
2416	F	0	0	N/A	0	N/A
2417	A	25	13	1.12	42	0.310
2418	F	0	0	N/A	0	N/A
2419	F	0	0	N/A	0	N/A
2420	A	10	10	0.67	60	0.167
2421	F	0	0	N/A	0	N/A
2422	A	31	14	1.03	42	0.333
2423	A	12	8	0.37	38	0.210
2424	A	12	7	0.81	48	0.146
2425	A	31	14	1.02	42	0.333
2426	F	0	0	N/A	0	N/A
2427	F	0	0	N/A	0	N/A
2428	C	50	10	1.10	38	0.263
2429	A	4	4	0.34	44	0.091
2430	A	4	4	0.33	51	0.078
2431	A	52	20	1.61	49	0.408
2432	A	45	10	1.75	38	0.263
2433	F	0	0	N/A	0	N/A
2434	F	0	0	N/A	0	N/A
2435	F	0	0	N/A	0	N/A
2436	F	0	0	N/A	0	N/A
2437	F	0	0	N/A	0	N/A
2438	A	4	4	0.20	51	0.078
2439	A	5	3	0.34	60	0.050
2440	A	5	3	0.33	60	0.050
2441	A	5	3	0.31	65	0.046
2442	A	5	3	0.30	65	0.046
2443	A	9	5	0.55	52	0.096

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{x}{\left(-1+x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=11

$$2\sqrt[4]{x^2-1}$$

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$2\sqrt[4]{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x^2)^(3/4),x]
[Out] 2*(-1 + x^2)^(1/4)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(-1+x^2\right)^{3/4}}\,dx = 2\sqrt[4]{-1+x^2}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$2\sqrt[4]{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x^2)^(3/4),x]
[Out] 2*(-1 + x^2)^(1/4)
```

IntegrateAlgebraic [A] time = 0.01, size = 11, normalized size = 1.00

$$2\sqrt[4]{x^2-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(-1 + x^2)^(3/4),x]
[Out] 2*(-1 + x^2)^(1/4)
```

fricas [A] time = 1.84, size = 9, normalized size = 0.82

$$2\left(x^2-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1)^(3/4),x, algorithm="fricas")
[Out] 2*(x^2 - 1)^(1/4)
```

giac [A] time = 0.49, size = 9, normalized size = 0.82

$$2\left(x^{2}-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] 2*(x^2 - 1)^(1/4)
```

maple [A] time = 0.03, size = 16, normalized size = 1.45

$$\frac{2\left(-1+x\right)\left(1+x\right)}{\left(x^{2}-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-1)^(3/4),x)
```

```
[Out] 2*(-1+x)*(1+x)/(x^2-1)^(3/4)
```

maxima [A] time = 0.72, size = 9, normalized size = 0.82

$$2\left(x^{2}-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] 2*(x^2 - 1)^(1/4)
```

mupad [B] time = 0.40, size = 9, normalized size = 0.82

$$2\left(x^{2}-1\right)^{1/4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 - 1)^(3/4),x)
```

```
[Out] 2*(x^2 - 1)^(1/4)
```

sympy [A] time = 0.17, size = 8, normalized size = 0.73

$$2\sqrt[4]{x^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-1)**(3/4),x)
```

```
[Out] 2*(x**2 - 1)**(1/4)
```

3.2 $\int \frac{1+3x^2}{\sqrt{-1+x+x^3}}\,dx$

Optimal. Leaf size=12

$$2\sqrt{x^3+x-1}$$

Rubi [A] time = 0.06, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1588}

$$2\sqrt{x^3+x-1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x^2)/Sqrt[-1 + x + x^3],x]
```

```
[Out] 2*Sqrt[-1 + x + x^3]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1+3x^2}{\sqrt{-1+x+x^3}}\,dx = 2\sqrt{-1+x+x^3}$$

Mathematica [A] time = 0.05, size = 12, normalized size = 1.00

$$2\sqrt{x^3+x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x^2)/Sqrt[-1 + x + x^3],x]
```

```
[Out] 2*Sqrt[-1 + x + x^3]
```

IntegrateAlgebraic [A] time = 0.01, size = 12, normalized size = 1.00

$$2\sqrt{x^3+x-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 3*x^2)/Sqrt[-1 + x + x^3],x]
```

```
[Out] 2*Sqrt[-1 + x + x^3]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)/(x^3+x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.41, size = 10, normalized size = 0.83

$$2\sqrt{x^3+x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)/(x^3+x-1)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x^3 + x - 1)
```

maple [A] time = 0.01, size = 11, normalized size = 0.92

$$2\sqrt{x^3+x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+1)/(x^3+x-1)^(1/2),x)
[Out] 2*(x^3+x-1)^(1/2)
```

maxima [A] time = 0.61, size = 10, normalized size = 0.83

$$2\sqrt{x^3+x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)/(x^3+x-1)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x^3 + x - 1)
```

mupad [B] time = 0.10, size = 10, normalized size = 0.83

$$2\sqrt{x^3+x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 1)/(x + x^3 - 1)^(1/2),x)
[Out] 2*(x + x^3 - 1)^(1/2)
```

sympy [A] time = 0.14, size = 10, normalized size = 0.83

$$2\sqrt{x^3+x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+1)/(x**3+x-1)**(1/2),x)
[Out] 2*sqrt(x**3 + x - 1)
```

3.3 $\int \frac{-1+x^8}{\sqrt{-1+x^4}\left(1-2x^4+x^8\right)}\,dx$

Optimal. Leaf size=12

$$-\frac{x}{\sqrt{x^4-1}}$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {28, 1404, 383}

$$-\frac{x}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^8)/(Sqrt[-1 + x^4]*(1 - 2*x^4 + x^8)),x]
[Out] -(x/Sqrt[-1 + x^4])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 1404

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^8}{\sqrt{-1+x^4}\left(1-2x^4+x^8\right)}\,dx &= \int \frac{-1+x^8}{\left(-1+x^4\right)^{5/2}}\,dx\\ &= \int \frac{1+x^4}{\left(-1+x^4\right)^{3/2}}\,dx\\ &= -\frac{x}{\sqrt{-1+x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\frac{x}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^8)/(Sqrt[-1 + x^4]*(1 - 2*x^4 + x^8)),x]
[Out] -(x/Sqrt[-1 + x^4])
```

IntegrateAlgebraic [A] time = 0.23, size = 12, normalized size = 1.00

$$-\frac{x}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^8)/(Sqrt[-1 + x^4]*(1 - 2*x^4 + x^8)),x]
[Out] -(x/Sqrt[-1 + x^4])
```

fricas [A] time = 0.46, size = 10, normalized size = 0.83

$$-\frac{x}{\sqrt{x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/(x^4-1)^(1/2)/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -x/sqrt(x^4 - 1)
```

giac [A] time = 0.55, size = 10, normalized size = 0.83

$$-\frac{x}{\sqrt{x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/(x^4-1)^(1/2)/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -x/sqrt(x^4 - 1)
```

maple [A] time = 0.01, size = 11, normalized size = 0.92

$$-\frac{x}{\sqrt{x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-1)/(x^4-1)^(1/2)/(x^8-2*x^4+1),x)
[Out] -x/(x^4-1)^(1/2)
```

maxima [B] time = 1.40, size = 27, normalized size = 2.25

$$-\frac{\sqrt{x^2+1}\,\sqrt{x+1}\,\sqrt{x-1}\,x}{x^4-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/(x^4-1)^(1/2)/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -sqrt(x^2 + 1)*sqrt(x + 1)*sqrt(x - 1)*x/(x^4 - 1)
```

mupad [B] time = 0.16, size = 10, normalized size = 0.83

$$-\frac{x}{\sqrt{x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 - 1)/((x^4 - 1)^(1/2)*(x^8 - 2*x^4 + 1)),x)
[Out] -x/(x^4 - 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\sqrt{(x-1)(x+1)\left(x^2+1\right)}\,(x-1)(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-1)/(x**4-1)**(1/2)/(x**8-2*x**4+1),x)
```

```
[Out] Integral((x**4 + 1)/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x - 1)*(x + 1)*(x**2
 + 1)), x)
```

3.4 $\int \frac{x}{\sqrt[3]{-1+x^2}}\,dx$

Optimal. Leaf size=13

$$\frac{3}{4}\left(x^2-1\right)^{2/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{3}{4}\left(x^2-1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x^2)^(1/3),x]
```

```
[Out] (3*(-1 + x^2)^(2/3))/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{-1+x^2}}\,dx=\frac{3}{4}\left(-1+x^2\right)^{2/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{4}\left(x^2-1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x^2)^(1/3),x]
```

```
[Out] (3*(-1 + x^2)^(2/3))/4
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{3}{4}\left(x^2-1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(-1 + x^2)^(1/3),x]
```

```
[Out] (3*(-1 + x^2)^(2/3))/4
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{3}{4}\left(x^2-1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*(x^2 - 1)^(2/3)
```

giac [A] time = 0.63, size = 9, normalized size = 0.69

$$\frac{3}{4}\left(x^2-1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1)^(1/3),x, algorithm="giac")
[Out] 3/4*(x^2 - 1)^(2/3)
```

maple [A] time = 0.00, size = 16, normalized size = 1.23

$$\frac{3(-1+x)(1+x)}{4\left(x^2-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-1)^(1/3),x)
[Out] 3/4*(-1+x)*(1+x)/(x^2-1)^(1/3)
```

maxima [A] time = 0.45, size = 9, normalized size = 0.69

$$\frac{3}{4}\left(x^2-1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1)^(1/3),x, algorithm="maxima")
[Out] 3/4*(x^2 - 1)^(2/3)
```

mupad [B] time = 0.11, size = 9, normalized size = 0.69

$$\frac{3\left(x^2-1\right)^{2/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 - 1)^(1/3),x)
[Out] (3*(x^2 - 1)^(2/3))/4
```

sympy [A] time = 0.12, size = 10, normalized size = 0.77

$$\frac{3\left(x^2-1\right)^{\frac{2}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-1)**(1/3),x)
[Out] 3*(x**2 - 1)**(2/3)/4
```

3.5 $\int \frac{x}{\sqrt[4]{-1+x^2}} \, dx$

Optimal. Leaf size=13

$$\frac{2}{3}\left(x^2-1\right)^{3/4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{2}{3}\left(x^2-1\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x^2)^(1/4),x]
[Out] (2*(-1 + x^2)^(3/4))/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt[4]{-1+x^2}} \, dx = \frac{2}{3}\left(-1+x^2\right)^{3/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{2}{3}\left(x^2-1\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x^2)^(1/4),x]
[Out] (2*(-1 + x^2)^(3/4))/3
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{2}{3}\left(x^2-1\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(-1 + x^2)^(1/4),x]
[Out] (2*(-1 + x^2)^(3/4))/3
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{2}{3}\left(x^2-1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 - 1)^(3/4)
```

giac [A] time = 0.42, size = 9, normalized size = 0.69

$$\frac{2}{3}\left(x^2-1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1)^(1/4),x, algorithm="giac")
[Out] 2/3*(x^2 - 1)^(3/4)
```

maple [A] time = 0.00, size = 16, normalized size = 1.23

$$\frac{2\left(-1+x\right)\left(1+x\right)}{3\left(x^2-1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-1)^(1/4),x)
[Out] 2/3*(-1+x)*(1+x)/(x^2-1)^(1/4)
```

maxima [A] time = 0.43, size = 9, normalized size = 0.69

$$\frac{2}{3}\left(x^2-1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1)^(1/4),x, algorithm="maxima")
[Out] 2/3*(x^2 - 1)^(3/4)
```

mupad [B] time = 0.10, size = 9, normalized size = 0.69

$$\frac{2\left(x^2-1\right)^{3/4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 - 1)^(1/4),x)
[Out] (2*(x^2 - 1)^(3/4))/3
```

sympy [A] time = 0.15, size = 10, normalized size = 0.77

$$\frac{2\left(x^2-1\right)^{\frac{3}{4}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-1)**(1/4),x)
[Out] 2*(x**2 - 1)**(3/4)/3
```

3.6 $\int x\sqrt[4]{-1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{2}{5}\left(x^2-1\right)^{5/4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{2}{5}\left(x^2-1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(-1 + x^2)^(1/4),x]
[Out] (2*(-1 + x^2)^(5/4))/5
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt[4]{-1+x^2}\,dx=\frac{2}{5}\left(-1+x^2\right)^{5/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{2}{5}\left(x^2-1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-1 + x^2)^(1/4),x]
[Out] (2*(-1 + x^2)^(5/4))/5
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{2}{5}\left(x^2-1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(-1 + x^2)^(1/4),x]
[Out] (2*(-1 + x^2)^(5/4))/5
```

fricas [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{2}{5}\left(x^2-1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/5*(x^2 - 1)^(5/4)
```

giac [**A**] time = 0.45, size = 9, normalized size = 0.69

$$\frac{2}{5}\left(x^2-1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(1/4),x, algorithm="giac")
[Out] 2/5*(x^2 - 1)^(5/4)
```

maple [**A**] time = 0.00, size = 16, normalized size = 1.23

$$\frac{2\left(-1+x\right)\left(1+x\right)\left(x^2-1\right)^{\frac{1}{4}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2-1)^(1/4),x)
[Out] 2/5*(-1+x)*(1+x)*(x^2-1)^(1/4)
```

maxima [**A**] time = 0.49, size = 9, normalized size = 0.69

$$\frac{2}{5}\left(x^2-1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(1/4),x, algorithm="maxima")
[Out] 2/5*(x^2 - 1)^(5/4)
```

mupad [**B**] time = 0.08, size = 9, normalized size = 0.69

$$\frac{2\left(x^2-1\right)^{5/4}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 - 1)^(1/4),x)
[Out] (2*(x^2 - 1)^(5/4))/5
```

sympy [**B**] time = 0.17, size = 26, normalized size = 2.00

$$\frac{2x^2\sqrt[4]{x^2-1}}{5}-\frac{2\sqrt[4]{x^2-1}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2-1)**(1/4),x)
[Out] 2*x**2*(x**2 - 1)**(1/4)/5 - 2*(x**2 - 1)**(1/4)/5
```

3.7 $\int x\sqrt[3]{-1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{3}{8}\left(x^2-1\right)^{4/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{3}{8}\left(x^2-1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(-1 + x^2)^(1/3),x]
```

```
[Out] (3*(-1 + x^2)^(4/3))/8
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt[3]{-1+x^2}\,dx=\frac{3}{8}\left(-1+x^2\right)^{4/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{8}\left(x^2-1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-1 + x^2)^(1/3),x]
```

```
[Out] (3*(-1 + x^2)^(4/3))/8
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{3}{8}\left(x^2-1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(-1 + x^2)^(1/3),x]
```

```
[Out] (3*(-1 + x^2)^(4/3))/8
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{3}{8}\left(x^2-1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/8*(x^2 - 1)^(4/3)
```

giac [A] time = 0.44, size = 9, normalized size = 0.69

$$\frac{3}{8}\left(x^2-1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(1/3),x, algorithm="giac")
[Out] 3/8*(x^2 - 1)^(4/3)
```

maple [A] time = 0.00, size = 16, normalized size = 1.23

$$\frac{3\left(-1+x\right)\left(1+x\right)\left(x^2-1\right)^{\frac{1}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2-1)^(1/3),x)
[Out] 3/8*(-1+x)*(1+x)*(x^2-1)^(1/3)
```

maxima [A] time = 0.64, size = 9, normalized size = 0.69

$$\frac{3}{8}\left(x^2-1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(1/3),x, algorithm="maxima")
[Out] 3/8*(x^2 - 1)^(4/3)
```

mupad [B] time = 0.08, size = 9, normalized size = 0.69

$$\frac{3\left(x^2-1\right)^{4/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 - 1)^(1/3),x)
[Out] (3*(x^2 - 1)^(4/3))/8
```

sympy [B] time = 0.14, size = 26, normalized size = 2.00

$$\frac{3x^2\sqrt[3]{x^2-1}}{8}-\frac{3\sqrt[3]{x^2-1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2-1)**(1/3),x)
[Out] 3*x**2*(x**2 - 1)**(1/3)/8 - 3*(x**2 - 1)**(1/3)/8
```

3.8 $\int x\left(-1+x^2\right)^{2/3}\,dx$

Optimal. Leaf size=13

$$\frac{3}{10}\left(x^2-1\right)^{5/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{3}{10}\left(x^2-1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(-1 + x^2)^(2/3),x]
[Out] (3*(-1 + x^2)^(5/3))/10
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(-1+x^2\right)^{2/3}\,dx=\frac{3}{10}\left(-1+x^2\right)^{5/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{10}\left(x^2-1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-1 + x^2)^(2/3),x]
[Out] (3*(-1 + x^2)^(5/3))/10
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{3}{10}\left(x^2-1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(-1 + x^2)^(2/3),x]
[Out] (3*(-1 + x^2)^(5/3))/10
```

fricas [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{3}{10}\left(x^2-1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/10*(x^2 - 1)^(5/3)
```

giac [A] time = 0.28, size = 9, normalized size = 0.69

$$\frac{3}{10}\left(x^2-1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(2/3),x, algorithm="giac")
[Out] 3/10*(x^2 - 1)^(5/3)
```

maple [A] time = 0.00, size = 16, normalized size = 1.23

$$\frac{3\left(-1+x\right)\left(1+x\right)\left(x^2-1\right)^{\frac{2}{3}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2-1)^(2/3),x)
[Out] 3/10*(-1+x)*(1+x)*(x^2-1)^(2/3)
```

maxima [A] time = 0.62, size = 9, normalized size = 0.69

$$\frac{3}{10}\left(x^2-1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(2/3),x, algorithm="maxima")
[Out] 3/10*(x^2 - 1)^(5/3)
```

mupad [B] time = 0.08, size = 9, normalized size = 0.69

$$\frac{3\left(x^2-1\right)^{5/3}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 - 1)^(2/3),x)
[Out] (3*(x^2 - 1)^(5/3))/10
```

sympy [B] time = 0.22, size = 26, normalized size = 2.00

$$\frac{3x^2\left(x^2-1\right)^{\frac{2}{3}}}{10}-\frac{3\left(x^2-1\right)^{\frac{2}{3}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2-1)**(2/3),x)
[Out] 3*x**2*(x**2 - 1)**(2/3)/10 - 3*(x**2 - 1)**(2/3)/10
```

3.9 $\int x\left(-1+x^2\right)^{3/4}\,dx$

Optimal. Leaf size=13

$$\frac{2}{7}\left(x^2-1\right)^{7/4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{2}{7}\left(x^2-1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(-1 + x^2)^(3/4),x]
```

```
[Out] (2*(-1 + x^2)^(7/4))/7
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(-1+x^2\right)^{3/4}\,dx=\frac{2}{7}\left(-1+x^2\right)^{7/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{2}{7}\left(x^2-1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-1 + x^2)^(3/4),x]
```

```
[Out] (2*(-1 + x^2)^(7/4))/7
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{2}{7}\left(x^2-1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(-1 + x^2)^(3/4),x]
```

```
[Out] (2*(-1 + x^2)^(7/4))/7
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{2}{7}\left(x^2-1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/7*(x^2 - 1)^(7/4)
```

giac [A] time = 0.30, size = 9, normalized size = 0.69

$$\frac{2}{7}\left(x^2-1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(3/4),x, algorithm="giac")
[Out] 2/7*(x^2 - 1)^(7/4)
```

maple [A] time = 0.00, size = 16, normalized size = 1.23

$$\frac{2\left(-1+x\right)\left(1+x\right)\left(x^2-1\right)^{\frac{3}{4}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2-1)^(3/4),x)
[Out] 2/7*(-1+x)*(1+x)*(x^2-1)^(3/4)
```

maxima [A] time = 0.67, size = 9, normalized size = 0.69

$$\frac{2}{7}\left(x^2-1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(3/4),x, algorithm="maxima")
[Out] 2/7*(x^2 - 1)^(7/4)
```

mupad [B] time = 0.08, size = 9, normalized size = 0.69

$$\frac{2\left(x^2-1\right)^{7/4}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 - 1)^(3/4),x)
[Out] (2*(x^2 - 1)^(7/4))/7
```

sympy [B] time = 0.46, size = 26, normalized size = 2.00

$$\frac{2x^2\left(x^2-1\right)^{\frac{3}{4}}}{7}-\frac{2\left(x^2-1\right)^{\frac{3}{4}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2-1)**(3/4),x)
[Out] 2*x**2*(x**2 - 1)**(3/4)/7 - 2*(x**2 - 1)**(3/4)/7
```

3.10 $\int \frac{x}{\sqrt[3]{1+x^2}} \, dx$

Optimal. Leaf size=13

$$\frac{3}{4}\left(x^2+1\right)^{2/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{3}{4}\left(x^2+1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^2)^(1/3),x]
[Out] (3*(1 + x^2)^(2/3))/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{1+x^2}} \, dx = \frac{3}{4}\left(1+x^2\right)^{2/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{4}\left(x^2+1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^2)^(1/3),x]
[Out] (3*(1 + x^2)^(2/3))/4
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{3}{4}\left(x^2+1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(1 + x^2)^(1/3),x]
[Out] (3*(1 + x^2)^(2/3))/4
```

fricas [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{3}{4}\left(x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*(x^2 + 1)^(2/3)
```

giac [A] time = 0.41, size = 9, normalized size = 0.69

$$\frac{3}{4}\left(x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/3),x, algorithm="giac")
[Out] 3/4*(x^2 + 1)^(2/3)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{3\left(x^2+1\right)^{\frac{2}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+1)^(1/3),x)
[Out] 3/4*(x^2+1)^(2/3)
```

maxima [A] time = 0.84, size = 9, normalized size = 0.69

$$\frac{3}{4}\left(x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/3),x, algorithm="maxima")
[Out] 3/4*(x^2 + 1)^(2/3)
```

mupad [B] time = 0.08, size = 9, normalized size = 0.69

$$\frac{3\left(x^2+1\right)^{2/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 + 1)^(1/3),x)
[Out] (3*(x^2 + 1)^(2/3))/4
```

sympy [A] time = 0.12, size = 10, normalized size = 0.77

$$\frac{3\left(x^2+1\right)^{\frac{2}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+1)**(1/3),x)
[Out] 3*(x**2 + 1)**(2/3)/4
```

3.11 $\int \frac{x}{\sqrt[4]{1+x^2}}\,dx$

Optimal. Leaf size=13

$$\frac{2}{3}\left(x^2+1\right)^{3/4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{2}{3}\left(x^2+1\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^2)^(1/4),x]
```

```
[Out] (2*(1 + x^2)^(3/4))/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt[4]{1+x^2}}\,dx=\frac{2}{3}\left(1+x^2\right)^{3/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{2}{3}\left(x^2+1\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^2)^(1/4),x]
```

```
[Out] (2*(1 + x^2)^(3/4))/3
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{2}{3}\left(x^2+1\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(1 + x^2)^(1/4),x]
```

```
[Out] (2*(1 + x^2)^(3/4))/3
```

fricas [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{2}{3}\left(x^2+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 + 1)^(3/4)
```

giac [A] time = 0.26, size = 9, normalized size = 0.69

$$\frac{2}{3}\left(x^2+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/4),x, algorithm="giac")
[Out] 2/3*(x^2 + 1)^(3/4)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{2\left(x^2+1\right)^{\frac{3}{4}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+1)^(1/4),x)
[Out] 2/3*(x^2+1)^(3/4)
```

maxima [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{2}{3}\left(x^2+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/4),x, algorithm="maxima")
[Out] 2/3*(x^2 + 1)^(3/4)
```

mupad [B] time = 0.09, size = 9, normalized size = 0.69

$$\frac{2\left(x^2+1\right)^{3/4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 + 1)^(1/4),x)
[Out] (2*(x^2 + 1)^(3/4))/3
```

sympy [A] time = 0.15, size = 10, normalized size = 0.77

$$\frac{2\left(x^2+1\right)^{\frac{3}{4}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+1)**(1/4),x)
[Out] 2*(x**2 + 1)**(3/4)/3
```

3.12 $\int x\sqrt[4]{1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{2}{5}\left(x^2+1\right)^{5/4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{2}{5}\left(x^2+1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + x^2)^(1/4),x]
[Out] (2*(1 + x^2)^(5/4))/5
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt[4]{1+x^2}\,dx=\frac{2}{5}\left(1+x^2\right)^{5/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{2}{5}\left(x^2+1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + x^2)^(1/4),x]
[Out] (2*(1 + x^2)^(5/4))/5
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{2}{5}\left(x^2+1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(1 + x^2)^(1/4),x]
[Out] (2*(1 + x^2)^(5/4))/5
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{2}{5}\left(x^2+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/5*(x^2 + 1)^(5/4)
```

giac [A] time = 0.33, size = 9, normalized size = 0.69

$$\frac{2}{5}\left(x^2+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/4),x, algorithm="giac")
[Out] 2/5*(x^2 + 1)^(5/4)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{2\left(x^2+1\right)^{\frac{5}{4}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^(1/4),x)
[Out] 2/5*(x^2+1)^(5/4)
```

maxima [A] time = 0.45, size = 9, normalized size = 0.69

$$\frac{2}{5}\left(x^2+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/4),x, algorithm="maxima")
[Out] 2/5*(x^2 + 1)^(5/4)
```

mupad [B] time = 0.08, size = 9, normalized size = 0.69

$$\frac{2\left(x^2+1\right)^{5/4}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 + 1)^(1/4),x)
[Out] (2*(x^2 + 1)^(5/4))/5
```

sympy [B] time = 0.17, size = 26, normalized size = 2.00

$$\frac{2x^2\sqrt[4]{x^2+1}}{5}+\frac{2\sqrt[4]{x^2+1}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**(1/4),x)
[Out] 2*x**2*(x**2 + 1)**(1/4)/5 + 2*(x**2 + 1)**(1/4)/5
```

3.13 $\int x\sqrt[3]{1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{3}{8}\left(x^2+1\right)^{4/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{3}{8}\left(x^2+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + x^2)^(1/3),x]
```

```
[Out] (3*(1 + x^2)^(4/3))/8
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt[3]{1+x^2}\,dx=\frac{3}{8}\left(1+x^2\right)^{4/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{8}\left(x^2+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + x^2)^(1/3),x]
```

```
[Out] (3*(1 + x^2)^(4/3))/8
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{3}{8}\left(x^2+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(1 + x^2)^(1/3),x]
```

```
[Out] (3*(1 + x^2)^(4/3))/8
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{3}{8}\left(x^2+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/8*(x^2 + 1)^(4/3)
```

giac [A] time = 0.66, size = 9, normalized size = 0.69

$$\frac{3}{8}\left(x^2+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] 3/8*(x^2 + 1)^(4/3)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{3\left(x^2+1\right)^{\frac{4}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^(1/3),x)
```

```
[Out] 3/8*(x^2+1)^(4/3)
```

maxima [A] time = 0.62, size = 9, normalized size = 0.69

$$\frac{3}{8}\left(x^2+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*(x^2 + 1)^(4/3)
```

mupad [B] time = 0.05, size = 9, normalized size = 0.69

$$\frac{3\left(x^2+1\right)^{4/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 + 1)^(1/3),x)
```

```
[Out] (3*(x^2 + 1)^(4/3))/8
```

sympy [B] time = 0.14, size = 26, normalized size = 2.00

$$\frac{3x^2\sqrt[3]{x^2+1}}{8}+\frac{3\sqrt[3]{x^2+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**(1/3),x)
```

```
[Out] 3*x**2*(x**2 + 1)**(1/3)/8 + 3*(x**2 + 1)**(1/3)/8
```

3.14 $\int x\left(1+x^2\right)^{3/4}\,dx$

Optimal. Leaf size=13

$$\frac{2}{7}\left(x^2+1\right)^{7/4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{2}{7}\left(x^2+1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + x^2)^(3/4),x]
[Out] (2*(1 + x^2)^(7/4))/7
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(1+x^2\right)^{3/4}\,dx=\frac{2}{7}\left(1+x^2\right)^{7/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{2}{7}\left(x^2+1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + x^2)^(3/4),x]
[Out] (2*(1 + x^2)^(7/4))/7
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{2}{7}\left(x^2+1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(1 + x^2)^(3/4),x]
[Out] (2*(1 + x^2)^(7/4))/7
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{2}{7}\left(x^2+1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/7*(x^2 + 1)^(7/4)
```

giac [A] time = 0.36, size = 9, normalized size = 0.69

$$\frac{2}{7}\left(x^2+1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(3/4),x, algorithm="giac")
[Out] 2/7*(x^2 + 1)^(7/4)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{2\left(x^2+1\right)^{\frac{7}{4}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^(3/4),x)
[Out] 2/7*(x^2+1)^(7/4)
```

maxima [A] time = 0.35, size = 9, normalized size = 0.69

$$\frac{2}{7}\left(x^2+1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(3/4),x, algorithm="maxima")
[Out] 2/7*(x^2 + 1)^(7/4)
```

mupad [B] time = 0.05, size = 9, normalized size = 0.69

$$\frac{2\left(x^2+1\right)^{7/4}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 + 1)^(3/4),x)
[Out] (2*(x^2 + 1)^(7/4))/7
```

sympy [B] time = 0.46, size = 26, normalized size = 2.00

$$\frac{2x^2\left(x^2+1\right)^{\frac{3}{4}}}{7}+\frac{2\left(x^2+1\right)^{\frac{3}{4}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**(3/4),x)
[Out] 2*x**2*(x**2 + 1)**(3/4)/7 + 2*(x**2 + 1)**(3/4)/7
```

3.15 $\int \frac{x^2}{\sqrt[4]{-1+x^3}} \, dx$

Optimal. Leaf size=13

$$\frac{4}{9}\left(x^3-1\right)^{3/4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{4}{9}\left(x^3-1\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-1 + x^3)^(1/4),x]
[Out] (4*(-1 + x^3)^(3/4))/9
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[4]{-1+x^3}} \, dx = \frac{4}{9}\left(-1+x^3\right)^{3/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{4}{9}\left(x^3-1\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-1 + x^3)^(1/4),x]
[Out] (4*(-1 + x^3)^(3/4))/9
```

IntegrateAlgebraic [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{4}{9}\left(x^3-1\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(-1 + x^3)^(1/4),x]
[Out] (4*(-1 + x^3)^(3/4))/9
```

fricas [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{4}{9}\left(x^3-1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3-1)^(1/4),x, algorithm="fricas")
[Out] 4/9*(x^3 - 1)^(3/4)
```

giac [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{4}{9}\left(x^3-1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3-1)^(1/4),x, algorithm="giac")
[Out] 4/9*(x^3 - 1)^(3/4)
```

maple [A] time = 0.00, size = 19, normalized size = 1.46

$$\frac{4\left(-1+x\right)\left(x^2+x+1\right)}{9\left(x^3-1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3-1)^(1/4),x)
[Out] 4/9*(-1+x)*(x^2+x+1)/(x^3-1)^(1/4)
```

maxima [A] time = 0.68, size = 9, normalized size = 0.69

$$\frac{4}{9}\left(x^3-1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3-1)^(1/4),x, algorithm="maxima")
[Out] 4/9*(x^3 - 1)^(3/4)
```

mupad [B] time = 0.24, size = 9, normalized size = 0.69

$$\frac{4\left(x^3-1\right)^{3/4}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3 - 1)^(1/4),x)
[Out] (4*(x^3 - 1)^(3/4))/9
```

sympy [A] time = 0.15, size = 10, normalized size = 0.77

$$\frac{4\left(x^3-1\right)^{\frac{3}{4}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**3-1)**(1/4),x)
[Out] 4*(x**3 - 1)**(3/4)/9
```

3.16 $\int x^2 \sqrt[4]{-1+x^3}\,dx$

Optimal. Leaf size=13

$$\frac{4}{15}\left(x^3-1\right)^{5/4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{4}{15}\left(x^3-1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(-1 + x^3)^(1/4),x]
```

```
[Out] (4*(-1 + x^3)^(5/4))/15
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \sqrt[4]{-1+x^3}\,dx = \frac{4}{15}\left(-1+x^3\right)^{5/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{4}{15}\left(x^3-1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(-1 + x^3)^(1/4),x]
```

```
[Out] (4*(-1 + x^3)^(5/4))/15
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{4}{15}\left(x^3-1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(-1 + x^3)^(1/4),x]
```

```
[Out] (4*(-1 + x^3)^(5/4))/15
```

fricas [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{4}{15}\left(x^3-1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/15*(x^3 - 1)^(5/4)
```

giac [A] time = 0.36, size = 9, normalized size = 0.69

$$\frac{4}{15}\left(x^3-1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-1)^(1/4),x, algorithm="giac")
[Out] 4/15*(x^3 - 1)^(5/4)
```

maple [A] time = 0.01, size = 19, normalized size = 1.46

$$\frac{4\left(-1+x\right)\left(x^2+x+1\right)\left(x^3-1\right)^{\frac{1}{4}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3-1)^(1/4),x)
[Out] 4/15*(-1+x)*(x^2+x+1)*(x^3-1)^(1/4)
```

maxima [A] time = 0.43, size = 9, normalized size = 0.69

$$\frac{4}{15}\left(x^3-1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-1)^(1/4),x, algorithm="maxima")
[Out] 4/15*(x^3 - 1)^(5/4)
```

mupad [B] time = 0.13, size = 9, normalized size = 0.69

$$\frac{4\left(x^3-1\right)^{5/4}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3 - 1)^(1/4),x)
[Out] (4*(x^3 - 1)^(5/4))/15
```

sympy [B] time = 0.21, size = 26, normalized size = 2.00

$$\frac{4x^3\sqrt[4]{x^3-1}}{15}-\frac{4\sqrt[4]{x^3-1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3-1)**(1/4),x)
[Out] 4*x**3*(x**3 - 1)**(1/4)/15 - 4*(x**3 - 1)**(1/4)/15
```

3.17 $\int x^2 \left(-1+x^3\right)^{3/4} \, dx$

Optimal. Leaf size=13

$$\frac{4}{21}\left(x^3-1\right)^{7/4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{4}{21}\left(x^3-1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(-1 + x^3)^(3/4),x]
```

```
[Out] (4*(-1 + x^3)^(7/4))/21
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \left(-1+x^3\right)^{3/4} \, dx = \frac{4}{21}\left(-1+x^3\right)^{7/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{4}{21}\left(x^3-1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(-1 + x^3)^(3/4),x]
```

```
[Out] (4*(-1 + x^3)^(7/4))/21
```

IntegrateAlgebraic [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{4}{21}\left(x^3-1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(-1 + x^3)^(3/4),x]
```

```
[Out] (4*(-1 + x^3)^(7/4))/21
```

fricas [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{4}{21}\left(x^3-1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/21*(x^3 - 1)^(7/4)
```

giac [A] time = 0.44, size = 9, normalized size = 0.69

$$\frac{4}{21} \left(x^3 - 1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-1)^(3/4),x, algorithm="giac")
[Out] 4/21*(x^3 - 1)^(7/4)
```

maple [A] time = 0.00, size = 19, normalized size = 1.46

$$\frac{4 (-1 + x) \left(x^2 + x + 1\right) \left(x^3 - 1\right)^{\frac{3}{4}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3-1)^(3/4),x)
[Out] 4/21*(-1+x)*(x^2+x+1)*(x^3-1)^(3/4)
```

maxima [A] time = 0.40, size = 9, normalized size = 0.69

$$\frac{4}{21} \left(x^3 - 1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-1)^(3/4),x, algorithm="maxima")
[Out] 4/21*(x^3 - 1)^(7/4)
```

mupad [B] time = 0.13, size = 9, normalized size = 0.69

$$\frac{4 \left(x^3 - 1\right)^{7/4}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3 - 1)^(3/4),x)
[Out] (4*(x^3 - 1)^(7/4))/21
```

sympy [B] time = 0.58, size = 26, normalized size = 2.00

$$\frac{4x^3 \left(x^3 - 1\right)^{\frac{3}{4}}}{21} - \frac{4 \left(x^3 - 1\right)^{\frac{3}{4}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3-1)**(3/4),x)
[Out] 4*x**3*(x**3 - 1)**(3/4)/21 - 4*(x**3 - 1)**(3/4)/21
```

3.18 $\int \frac{x^2}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=13

$$\frac{2\sqrt{x^3+1}}{3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{2\sqrt{x^3+1}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[1 + x^3])/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{1+x^3}}{3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{2\sqrt{x^3+1}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[1 + x^3])/3
```

IntegrateAlgebraic [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{2\sqrt{x^3+1}}{3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[1 + x^3])/3
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{2}{3}\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(x^3 + 1)
```

giac [A] time = 0.75, size = 9, normalized size = 0.69

$$\frac{2}{3}\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+1)^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt(x^3 + 1)
```

maple [B] time = 0.01, size = 21, normalized size = 1.62

$$\frac{2\,(1+x)\left(x^2-x+1\right)}{3\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3+1)^(1/2),x)
[Out] 2/3*(1+x)*(x^2-x+1)/(x^3+1)^(1/2)
```

maxima [A] time = 0.71, size = 9, normalized size = 0.69

$$\frac{2}{3}\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] 2/3*sqrt(x^3 + 1)
```

mupad [B] time = 0.07, size = 9, normalized size = 0.69

$$\frac{2\,\sqrt{x^3+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3 + 1)^(1/2),x)
[Out] (2*(x^3 + 1)^(1/2))/3
```

sympy [A] time = 0.13, size = 10, normalized size = 0.77

$$\frac{2\sqrt{x^3+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**3+1)**(1/2),x)
[Out] 2*sqrt(x**3 + 1)/3
```

3.19 $\int \frac{x^2}{\sqrt[4]{1+x^3}}\,dx$

Optimal. Leaf size=13

$$\frac{4}{9}\left(x^3+1\right)^{3/4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{4}{9}\left(x^3+1\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x^3)^(1/4),x]
[Out] (4*(1 + x^3)^(3/4))/9
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[4]{1+x^3}}\,dx = \frac{4}{9}\left(1+x^3\right)^{3/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{4}{9}\left(x^3+1\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x^3)^(1/4),x]
[Out] (4*(1 + x^3)^(3/4))/9
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{4}{9}\left(x^3+1\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(1 + x^3)^(1/4),x]
[Out] (4*(1 + x^3)^(3/4))/9
```

fricas [A] time = 0.37, size = 9, normalized size = 0.69

$$\frac{4}{9}\left(x^3+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+1)^(1/4),x, algorithm="fricas")
[Out] 4/9*(x^3 + 1)^(3/4)
```

giac [A] time = 0.36, size = 9, normalized size = 0.69

$$\frac{4}{9}\left(x^3+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+1)^(1/4),x, algorithm="giac")
[Out] 4/9*(x^3 + 1)^(3/4)
```

maple [B] time = 0.01, size = 21, normalized size = 1.62

$$\frac{4\,(1+x)\left(x^2-x+1\right)}{9\left(x^3+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3+1)^(1/4),x)
[Out] 4/9*(1+x)*(x^2-x+1)/(x^3+1)^(1/4)
```

maxima [A] time = 0.58, size = 9, normalized size = 0.69

$$\frac{4}{9}\left(x^3+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+1)^(1/4),x, algorithm="maxima")
[Out] 4/9*(x^3 + 1)^(3/4)
```

mupad [B] time = 0.17, size = 9, normalized size = 0.69

$$\frac{4\left(x^3+1\right)^{3/4}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3 + 1)^(1/4),x)
[Out] (4*(x^3 + 1)^(3/4))/9
```

sympy [A] time = 0.15, size = 10, normalized size = 0.77

$$\frac{4\left(x^3+1\right)^{\frac{3}{4}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**3+1)**(1/4),x)
[Out] 4*(x**3 + 1)**(3/4)/9
```

3.20 $\int x^2 \sqrt[4]{1+x^3}\,dx$

Optimal. Leaf size=13

$$\frac{4}{15}\left(x^3+1\right)^{5/4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{4}{15}\left(x^3+1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 + x^3)^(1/4),x]
[Out] (4*(1 + x^3)^(5/4))/15
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \sqrt[4]{1+x^3}\,dx = \frac{4}{15}\left(1+x^3\right)^{5/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{4}{15}\left(x^3+1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 + x^3)^(1/4),x]
[Out] (4*(1 + x^3)^(5/4))/15
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{4}{15}\left(x^3+1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(1 + x^3)^(1/4),x]
[Out] (4*(1 + x^3)^(5/4))/15
```

fricas [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{4}{15}\left(x^3+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/15*(x^3 + 1)^(5/4)
```

giac [A] time = 0.36, size = 9, normalized size = 0.69

$$\frac{4}{15}\left(x^3+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^(1/4),x, algorithm="giac")
[Out] 4/15*(x^3 + 1)^(5/4)
```

maple [B] time = 0.00, size = 21, normalized size = 1.62

$$\frac{4\,(1+x)\left(x^2-x+1\right)\left(x^3+1\right)^{\frac{1}{4}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3+1)^(1/4),x)
[Out] 4/15*(1+x)*(x^2-x+1)*(x^3+1)^(1/4)
```

maxima [A] time = 0.65, size = 9, normalized size = 0.69

$$\frac{4}{15}\left(x^3+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^(1/4),x, algorithm="maxima")
[Out] 4/15*(x^3 + 1)^(5/4)
```

mupad [B] time = 0.12, size = 9, normalized size = 0.69

$$\frac{4\left(x^3+1\right)^{5/4}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3 + 1)^(1/4),x)
[Out] (4*(x^3 + 1)^(5/4))/15
```

sympy [B] time = 0.21, size = 26, normalized size = 2.00

$$\frac{4x^3\sqrt[4]{x^3+1}}{15}+\frac{4\sqrt[4]{x^3+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3+1)**(1/4),x)
[Out] 4*x**3*(x**3 + 1)**(1/4)/15 + 4*(x**3 + 1)**(1/4)/15
```

3.21 $\int x^2 \sqrt[3]{1+x^3}\, dx$

Optimal. Leaf size=13

$$\frac{1}{4}\left(x^3+1\right)^{4/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{4}\left(x^3+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 + x^3)^(1/3),x]
```

```
[Out] (1 + x^3)^(4/3)/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \sqrt[3]{1+x^3}\, dx = \frac{1}{4}\left(1+x^3\right)^{4/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{4}\left(x^3+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 + x^3)^(1/3),x]
```

```
[Out] (1 + x^3)^(4/3)/4
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{4}\left(x^3+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(1 + x^3)^(1/3),x]
```

```
[Out] (1 + x^3)^(4/3)/4
```

fricas [A] time = 0.37, size = 9, normalized size = 0.69

$$\frac{1}{4}\left(x^3+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*(x^3 + 1)^(4/3)
```

giac [A] time = 0.44, size = 9, normalized size = 0.69

$$\frac{1}{4}\left(x^3+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^(1/3),x, algorithm="giac")
[Out] 1/4*(x^3 + 1)^(4/3)
```

maple [B] time = 0.01, size = 21, normalized size = 1.62

$$\frac{(1+x)\left(x^2-x+1\right)\left(x^3+1\right)^{\frac{1}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3+1)^(1/3),x)
[Out] 1/4*(1+x)*(x^2-x+1)*(x^3+1)^(1/3)
```

maxima [A] time = 0.78, size = 9, normalized size = 0.69

$$\frac{1}{4}\left(x^3+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^(1/3),x, algorithm="maxima")
[Out] 1/4*(x^3 + 1)^(4/3)
```

mupad [B] time = 0.13, size = 9, normalized size = 0.69

$$\frac{\left(x^3+1\right)^{4/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3 + 1)^(1/3),x)
[Out] (x^3 + 1)^(4/3)/4
```

sympy [B] time = 0.16, size = 22, normalized size = 1.69

$$\frac{x^3\sqrt[3]{x^3+1}}{4}+\frac{\sqrt[3]{x^3+1}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3+1)**(1/3),x)
[Out] x**3*(x**3 + 1)**(1/3)/4 + (x**3 + 1)**(1/3)/4
```

3.22 $\int x^2\sqrt{1+x^3}\,dx$

Optimal. Leaf size=13

$$\frac{2}{9}\left(x^3+1\right)^{3/2}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{2}{9}\left(x^3+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[1 + x^3],x]
[Out] (2*(1 + x^3)^(3/2))/9
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2\sqrt{1+x^3}\,dx=\frac{2}{9}\left(1+x^3\right)^{3/2}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{2}{9}\left(x^3+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[1 + x^3],x]
[Out] (2*(1 + x^3)^(3/2))/9
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{2}{9}\left(x^3+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[1 + x^3],x]
[Out] (2*(1 + x^3)^(3/2))/9
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{2}{9}\left(x^3+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(x^3 + 1)^(3/2)
```

giac [A] time = 0.44, size = 9, normalized size = 0.69

$$\frac{2}{9}\left(x^3+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^(1/2),x, algorithm="giac")
[Out] 2/9*(x^3 + 1)^(3/2)
```

maple [B] time = 0.00, size = 21, normalized size = 1.62

$$\frac{2\,(1+x)\left(x^2-x+1\right)\sqrt{x^3+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3+1)^(1/2),x)
[Out] 2/9*(1+x)*(x^2-x+1)*(x^3+1)^(1/2)
```

maxima [A] time = 0.70, size = 9, normalized size = 0.69

$$\frac{2}{9}\left(x^3+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^(1/2),x, algorithm="maxima")
[Out] 2/9*(x^3 + 1)^(3/2)
```

mupad [B] time = 0.02, size = 9, normalized size = 0.69

$$\frac{2\left(x^3+1\right)^{3/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3 + 1)^(1/2),x)
[Out] (2*(x^3 + 1)^(3/2))/9
```

sympy [B] time = 0.14, size = 26, normalized size = 2.00

$$\frac{2x^3\sqrt{x^3+1}}{9}+\frac{2\sqrt{x^3+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3+1)**(1/2),x)
[Out] 2*x**3*sqrt(x**3 + 1)/9 + 2*sqrt(x**3 + 1)/9
```

3.23 $\int x^2 \left(1+x^3\right)^{2/3} dx$

Optimal. Leaf size=13

$$\frac{1}{5}\left(x^3+1\right)^{5/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{5}\left(x^3+1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 + x^3)^(2/3),x]
[Out] (1 + x^3)^(5/3)/5
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \left(1+x^3\right)^{2/3} dx = \frac{1}{5}\left(1+x^3\right)^{5/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{5}\left(x^3+1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 + x^3)^(2/3),x]
[Out] (1 + x^3)^(5/3)/5
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{5}\left(x^3+1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(1 + x^3)^(2/3),x]
[Out] (1 + x^3)^(5/3)/5
```

fricas [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{1}{5}\left(x^3+1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/5*(x^3 + 1)^(5/3)
```

giac [A] time = 0.59, size = 9, normalized size = 0.69

$$\frac{1}{5}\left(x^3+1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^(2/3),x, algorithm="giac")
[Out] 1/5*(x^3 + 1)^(5/3)
```

maple [B] time = 0.01, size = 21, normalized size = 1.62

$$\frac{(1+x)\left(x^2-x+1\right)\left(x^3+1\right)^{\frac{2}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3+1)^(2/3),x)
[Out] 1/5*(1+x)*(x^2-x+1)*(x^3+1)^(2/3)
```

maxima [A] time = 0.33, size = 9, normalized size = 0.69

$$\frac{1}{5}\left(x^3+1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^(2/3),x, algorithm="maxima")
[Out] 1/5*(x^3 + 1)^(5/3)
```

mupad [B] time = 0.13, size = 9, normalized size = 0.69

$$\frac{\left(x^3+1\right)^{5/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3 + 1)^(2/3),x)
[Out] (x^3 + 1)^(5/3)/5
```

sympy [B] time = 0.27, size = 22, normalized size = 1.69

$$\frac{x^3\left(x^3+1\right)^{\frac{2}{3}}}{5}+\frac{\left(x^3+1\right)^{\frac{2}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3+1)**(2/3),x)
[Out] x**3*(x**3 + 1)**(2/3)/5 + (x**3 + 1)**(2/3)/5
```

3.24 $\int x^2 \left(1+x^3\right)^{3/4} dx$

Optimal. Leaf size=13

$$\frac{4}{21}\left(x^3+1\right)^{7/4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{4}{21}\left(x^3+1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 + x^3)^(3/4),x]
```

```
[Out] (4*(1 + x^3)^(7/4))/21
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \left(1+x^3\right)^{3/4} dx = \frac{4}{21}\left(1+x^3\right)^{7/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{4}{21}\left(x^3+1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 + x^3)^(3/4),x]
```

```
[Out] (4*(1 + x^3)^(7/4))/21
```

IntegrateAlgebraic [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{4}{21}\left(x^3+1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(1 + x^3)^(3/4),x]
```

```
[Out] (4*(1 + x^3)^(7/4))/21
```

fricas [A] time = 0.37, size = 9, normalized size = 0.69

$$\frac{4}{21}\left(x^3+1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/21*(x^3 + 1)^(7/4)
```

giac [A] time = 0.50, size = 9, normalized size = 0.69

$$\frac{4}{21}\left(x^3+1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^(3/4),x, algorithm="giac")
[Out] 4/21*(x^3 + 1)^(7/4)
```

maple [B] time = 0.00, size = 21, normalized size = 1.62

$$\frac{4\left(1+x\right)\left(x^2-x+1\right)\left(x^3+1\right)^{\frac{3}{4}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3+1)^(3/4),x)
[Out] 4/21*(1+x)*(x^2-x+1)*(x^3+1)^(3/4)
```

maxima [A] time = 0.46, size = 9, normalized size = 0.69

$$\frac{4}{21}\left(x^3+1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^(3/4),x, algorithm="maxima")
[Out] 4/21*(x^3 + 1)^(7/4)
```

mupad [B] time = 0.12, size = 9, normalized size = 0.69

$$\frac{4\left(x^3+1\right)^{7/4}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3 + 1)^(3/4),x)
[Out] (4*(x^3 + 1)^(7/4))/21
```

sympy [B] time = 0.57, size = 26, normalized size = 2.00

$$\frac{4x^3\left(x^3+1\right)^{\frac{3}{4}}}{21}+\frac{4\left(x^3+1\right)^{\frac{3}{4}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3+1)**(3/4),x)
[Out] 4*x**3*(x**3 + 1)**(3/4)/21 + 4*(x**3 + 1)**(3/4)/21
```

3.25 $\int (1+3x^2)\sqrt[3]{x+x^3}\,dx$

Optimal. Leaf size=13

$$\frac{3}{4}\left(x^3+x\right)^{4/3}$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$\frac{3}{4}\left(x^3+x\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x^2)*(x + x^3)^(1/3),x]
```

```
[Out] (3*(x + x^3)^(4/3))/4
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(1+3x^2\right)\sqrt[3]{x+x^3}\,dx=\frac{3}{4}\left(x+x^3\right)^{4/3}$$

Mathematica [A] time = 0.03, size = 19, normalized size = 1.46

$$\frac{3}{4}x\left(x^2+1\right)\sqrt[3]{x^3+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x^2)*(x + x^3)^(1/3),x]
```

```
[Out] (3*x*(1 + x^2)*(x + x^3)^(1/3))/4
```

IntegrateAlgebraic [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{3}{4}\left(x^3+x\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 3*x^2)*(x + x^3)^(1/3),x]
```

```
[Out] (3*(x + x^3)^(4/3))/4
```

fricas [A] time = 0.40, size = 9, normalized size = 0.69

$$\frac{3}{4}\left(x^3+x\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)*(x^3+x)^(1/3),x, algorithm="fricas")
[Out] 3/4*(x^3 + x)^(4/3)
```

giac [A] time = 0.29, size = 9, normalized size = 0.69

$$\frac{3}{4}\left(x^3+x\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)*(x^3+x)^(1/3),x, algorithm="giac")
[Out] 3/4*(x^3 + x)^(4/3)
```

maple [A] time = 0.00, size = 16, normalized size = 1.23

$$\frac{3\left(x^2+1\right)x\left(x^3+x\right)^{\frac{1}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+1)*(x^3+x)^(1/3),x)
[Out] 3/4*(x^2+1)*x*(x^3+x)^(1/3)
```

maxima [A] time = 0.37, size = 9, normalized size = 0.69

$$\frac{3}{4}\left(x^3+x\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)*(x^3+x)^(1/3),x, algorithm="maxima")
[Out] 3/4*(x^3 + x)^(4/3)
```

mupad [B] time = 0.09, size = 9, normalized size = 0.69

$$\frac{3\left(x^3+x\right)^{4/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 1)*(x + x^3)^(1/3),x)
[Out] (3*(x + x^3)^(4/3))/4
```

sympy [B] time = 0.17, size = 27, normalized size = 2.08

$$\frac{3x^3\sqrt[3]{x^3+x}}{4}+\frac{3x\sqrt[3]{x^3+x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+1)*(x**3+x)**(1/3),x)
[Out] 3*x**3*(x**3 + x)**(1/3)/4 + 3*x*(x**3 + x)**(1/3)/4
```

3.26 $\int \frac{1}{x^2(-1+x^4)^{3/4}}\,dx$

Optimal. Leaf size=13

$$\frac{\sqrt[4]{x^4-1}}{x}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$\frac{\sqrt[4]{x^4-1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-1 + x^4)^(3/4)),x]
```

```
[Out] (-1 + x^4)^(1/4)/x
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(-1+x^4\right)^{3/4}}\,dx=\frac{\sqrt[4]{-1+x^4}}{x}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{\sqrt[4]{x^4-1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-1 + x^4)^(3/4)),x]
```

```
[Out] (-1 + x^4)^(1/4)/x
```

IntegrateAlgebraic [A] time = 0.17, size = 13, normalized size = 1.00

$$\frac{\sqrt[4]{x^4-1}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(-1 + x^4)^(3/4)),x]
```

```
[Out] (-1 + x^4)^(1/4)/x
```

fricas [A] time = 0.39, size = 11, normalized size = 0.85

$$\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4-1)^(3/4),x, algorithm="fricas")
[Out] (x^4 - 1)^(1/4)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4-1\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4-1)^(3/4),x, algorithm="giac")
[Out] integrate(1/((x^4 - 1)^(3/4)*x^2), x)
```

maple [A] time = 0.00, size = 23, normalized size = 1.77

$$\frac{(-1+x)\,(1+x)\left(x^2+1\right)}{x\left(x^4-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^4-1)^(3/4),x)
[Out] 1/x*(-1+x)*(1+x)*(x^2+1)/(x^4-1)^(3/4)
```

maxima [A] time = 0.47, size = 11, normalized size = 0.85

$$\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4-1)^(3/4),x, algorithm="maxima")
[Out] (x^4 - 1)^(1/4)/x
```

mupad [B] time = 0.21, size = 11, normalized size = 0.85

$$\frac{\left(x^4-1\right)^{1/4}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4 - 1)^(3/4)),x)
[Out] (x^4 - 1)^(1/4)/x
```

sympy [B] time = 0.58, size = 61, normalized size = 4.69

$$\begin{cases} -\frac{\sqrt[4]{-1+\frac{1}{x^4}}\,e^{\frac{i\pi}{4}}\,\Gamma\left(-\frac{1}{4}\right)}{4\Gamma\left(\frac{3}{4}\right)} & \text{for}\ \frac{1}{|x^4|}>1 \\ -\frac{\sqrt[4]{1-\frac{1}{x^4}}\,\Gamma\left(-\frac{1}{4}\right)}{4\Gamma\left(\frac{3}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**4-1)**(3/4),x)
```

```
[Out] Piecewise((-(-1 + x**(-4))**(1/4)*exp(I*pi/4)*gamma(-1/4)/(4*gamma(3/4)), 1
/Abs(x**4) > 1), (-(1 - 1/x**4)**(1/4)*gamma(-1/4)/(4*gamma(3/4)), True))
```

3.27 $\int \frac{x^3}{\sqrt[3]{-1+x^4}} \, dx$

Optimal. Leaf size=13

$$\frac{3}{8}\left(x^4-1\right)^{2/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3}{8}\left(x^4-1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(-1 + x^4)^(1/3),x]
```

```
[Out] (3*(-1 + x^4)^(2/3))/8
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt[3]{-1+x^4}} \, dx = \frac{3}{8}\left(-1+x^4\right)^{2/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{8}\left(x^4-1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(-1 + x^4)^(1/3),x]
```

```
[Out] (3*(-1 + x^4)^(2/3))/8
```

IntegrateAlgebraic [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{3}{8}\left(x^4-1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(-1 + x^4)^(1/3),x]
```

```
[Out] (3*(-1 + x^4)^(2/3))/8
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{3}{8}\left(x^4-1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-1)^(1/3),x, algorithm="fricas")
[Out] 3/8*(x^4 - 1)^(2/3)
```

giac [A] time = 0.69, size = 9, normalized size = 0.69

$$\frac{3}{8}\left(x^4-1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-1)^(1/3),x, algorithm="giac")
[Out] 3/8*(x^4 - 1)^(2/3)
```

maple [B] time = 0.01, size = 21, normalized size = 1.62

$$\frac{3\left(-1+x\right)\left(1+x\right)\left(x^2+1\right)}{8\left(x^4-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4-1)^(1/3),x)
[Out] 3/8*(-1+x)*(1+x)*(x^2+1)/(x^4-1)^(1/3)
```

maxima [A] time = 0.42, size = 9, normalized size = 0.69

$$\frac{3}{8}\left(x^4-1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-1)^(1/3),x, algorithm="maxima")
[Out] 3/8*(x^4 - 1)^(2/3)
```

mupad [B] time = 0.17, size = 9, normalized size = 0.69

$$\frac{3\left(x^4-1\right)^{2/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4 - 1)^(1/3),x)
[Out] (3*(x^4 - 1)^(2/3))/8
```

sympy [A] time = 0.15, size = 10, normalized size = 0.77

$$\frac{3\left(x^4-1\right)^{\frac{2}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4-1)**(1/3),x)
[Out] 3*(x**4 - 1)**(2/3)/8
```

3.28 $\int x^3 \sqrt[3]{-1+x^4}\,dx$

Optimal. Leaf size=13

$$\frac{3}{16}\left(x^4-1\right)^{4/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3}{16}\left(x^4-1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(-1 + x^4)^(1/3),x]
```

```
[Out] (3*(-1 + x^4)^(4/3))/16
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^3 \sqrt[3]{-1+x^4}\,dx=\frac{3}{16}\left(-1+x^4\right)^{4/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{16}\left(x^4-1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(-1 + x^4)^(1/3),x]
```

```
[Out] (3*(-1 + x^4)^(4/3))/16
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{3}{16}\left(x^4-1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(-1 + x^4)^(1/3),x]
```

```
[Out] (3*(-1 + x^4)^(4/3))/16
```

fricas [A] time = 0.37, size = 9, normalized size = 0.69

$$\frac{3}{16}\left(x^4-1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/16*(x^4 - 1)^(4/3)
```

giac [A] time = 0.42, size = 9, normalized size = 0.69

$$\frac{3}{16}\left(x^4-1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4-1)^(1/3),x, algorithm="giac")
[Out] 3/16*(x^4 - 1)^(4/3)
```

maple [B] time = 0.01, size = 21, normalized size = 1.62

$$\frac{3\,(-1+x)\,(1+x)\left(x^2+1\right)\left(x^4-1\right)^{\frac{1}{3}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4-1)^(1/3),x)
[Out] 3/16*(-1+x)*(1+x)*(x^2+1)*(x^4-1)^(1/3)
```

maxima [A] time = 0.45, size = 9, normalized size = 0.69

$$\frac{3}{16}\left(x^4-1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4-1)^(1/3),x, algorithm="maxima")
[Out] 3/16*(x^4 - 1)^(4/3)
```

mupad [B] time = 0.13, size = 9, normalized size = 0.69

$$\frac{3\left(x^4-1\right)^{4/3}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4 - 1)^(1/3),x)
[Out] (3*(x^4 - 1)^(4/3))/16
```

sympy [B] time = 0.21, size = 26, normalized size = 2.00

$$\frac{3x^4\sqrt[3]{x^4-1}}{16}-\frac{3\sqrt[3]{x^4-1}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**4-1)**(1/3),x)
[Out] 3*x**4*(x**4 - 1)**(1/3)/16 - 3*(x**4 - 1)**(1/3)/16
```

3.29 $\int x^3 \left(-1+x^4\right)^{2/3} \, dx$

Optimal. Leaf size=13

$$\frac{3}{20} \left(x^4-1\right)^{5/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3}{20} \left(x^4-1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(-1 + x^4)^(2/3),x]
```

```
[Out] (3*(-1 + x^4)^(5/3))/20
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^3 \left(-1+x^4\right)^{2/3} \, dx = \frac{3}{20} \left(-1+x^4\right)^{5/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{20} \left(x^4-1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(-1 + x^4)^(2/3),x]
```

```
[Out] (3*(-1 + x^4)^(5/3))/20
```

IntegrateAlgebraic [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{3}{20} \left(x^4-1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(-1 + x^4)^(2/3),x]
```

```
[Out] (3*(-1 + x^4)^(5/3))/20
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{3}{20} \, {\left(x^{4} - 1\right)}^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4-1)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/20*(x^4 - 1)^(5/3)
```

giac [A] time = 0.28, size = 9, normalized size = 0.69

$$\frac{3}{20}\left(x^4-1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4-1)^(2/3),x, algorithm="giac")
[Out] 3/20*(x^4 - 1)^(5/3)
```

maple [B] time = 0.01, size = 21, normalized size = 1.62

$$\frac{3\left(-1+x\right)\left(1+x\right)\left(x^2+1\right)\left(x^4-1\right)^{\frac{2}{3}}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4-1)^(2/3),x)
[Out] 3/20*(-1+x)*(1+x)*(x^2+1)*(x^4-1)^(2/3)
```

maxima [A] time = 0.49, size = 9, normalized size = 0.69

$$\frac{3}{20}\left(x^4-1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4-1)^(2/3),x, algorithm="maxima")
[Out] 3/20*(x^4 - 1)^(5/3)
```

mupad [B] time = 0.13, size = 9, normalized size = 0.69

$$\frac{3\left(x^4-1\right)^{5/3}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4 - 1)^(2/3),x)
[Out] (3*(x^4 - 1)^(5/3))/20
```

sympy [B] time = 0.36, size = 26, normalized size = 2.00

$$\frac{3x^4\left(x^4-1\right)^{\frac{2}{3}}}{20}-\frac{3\left(x^4-1\right)^{\frac{2}{3}}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**4-1)**(2/3),x)
[Out] 3*x**4*(x**4 - 1)**(2/3)/20 - 3*(x**4 - 1)**(2/3)/20
```

3.30 $\int x^3 \left(-1+x^4\right)^{3/4}\,dx$

Optimal. Leaf size=13

$$\frac{1}{7}\left(x^4-1\right)^{7/4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{7}\left(x^4-1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(-1 + x^4)^(3/4),x]
```

```
[Out] (-1 + x^4)^(7/4)/7
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^3 \left(-1+x^4\right)^{3/4}\,dx=\frac{1}{7}\left(-1+x^4\right)^{7/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{7}\left(x^4-1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(-1 + x^4)^(3/4),x]
```

```
[Out] (-1 + x^4)^(7/4)/7
```

IntegrateAlgebraic [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{1}{7}\left(x^4-1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(-1 + x^4)^(3/4),x]
```

```
[Out] (-1 + x^4)^(7/4)/7
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{1}{7}\left(x^4-1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/7*(x^4 - 1)^(7/4)
```

giac [A] time = 0.29, size = 9, normalized size = 0.69

$$\frac{1}{7}\left(x^4-1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4-1)^(3/4),x, algorithm="giac")
[Out] 1/7*(x^4 - 1)^(7/4)
```

maple [B] time = 0.00, size = 21, normalized size = 1.62

$$\frac{(-1+x)(1+x)\left(x^2+1\right)\left(x^4-1\right)^{\frac{3}{4}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4-1)^(3/4),x)
[Out] 1/7*(-1+x)*(1+x)*(x^2+1)*(x^4-1)^(3/4)
```

maxima [A] time = 0.32, size = 9, normalized size = 0.69

$$\frac{1}{7}\left(x^4-1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4-1)^(3/4),x, algorithm="maxima")
[Out] 1/7*(x^4 - 1)^(7/4)
```

mupad [B] time = 0.13, size = 9, normalized size = 0.69

$$\frac{\left(x^4-1\right)^{7/4}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4 - 1)^(3/4),x)
[Out] (x^4 - 1)^(7/4)/7
```

sympy [B] time = 0.73, size = 22, normalized size = 1.69

$$\frac{x^4\left(x^4-1\right)^{\frac{3}{4}}}{7}-\frac{\left(x^4-1\right)^{\frac{3}{4}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**4-1)**(3/4),x)
[Out] x**4*(x**4 - 1)**(3/4)/7 - (x**4 - 1)**(3/4)/7
```

3.31 $\int \frac{-1+x^4}{x^2\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=13

$$\frac{\sqrt{x^4+1}}{x}$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$\frac{\sqrt{x^4+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)/(x^2*Sqrt[1 + x^4]),x]
```

```
[Out] Sqrt[1 + x^4]/x
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^4}}\,dx=\frac{\sqrt{1+x^4}}{x}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{\sqrt{x^4+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)/(x^2*Sqrt[1 + x^4]),x]
```

```
[Out] Sqrt[1 + x^4]/x
```

IntegrateAlgebraic [A] time = 0.62, size = 13, normalized size = 1.00

$$\frac{\sqrt{x^4+1}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/(x^2*Sqrt[1 + x^4]),x]
```

```
[Out] Sqrt[1 + x^4]/x
```

fricas [A] time = 0.39, size = 11, normalized size = 0.85

$$\frac{\sqrt{x^4+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^4 + 1)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 - 1}{\sqrt{x^4 + 1}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 1)/(sqrt(x^4 + 1)*x^2), x)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{\sqrt{x^4 + 1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/x^2/(x^4+1)^(1/2),x)
```

```
[Out] (x^4+1)^(1/2)/x
```

maxima [A] time = 0.52, size = 11, normalized size = 0.85

$$\frac{\sqrt{x^4 + 1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^4 + 1)/x
```

mupad [B] time = 0.05, size = 11, normalized size = 0.85

$$\frac{\sqrt{x^4 + 1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/(x^2*(x^4 + 1)^(1/2)),x)
```

```
[Out] (x^4 + 1)^(1/2)/x
```

sympy [C] time = 1.51, size = 61, normalized size = 4.69

$$\frac{x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\left.\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\right| x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4} \end{matrix}\right| x^4 e^{i\pi}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/x**2/(x**4+1)**(1/2),x)
```

```
[Out] x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(I*pi))/(4*gamma(7/
4)) - gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_polar(I*pi))/(4*x*gam
ma(3/4))
```

3.32 $\int \frac{x^3}{\sqrt[3]{1+x^4}} \, dx$

Optimal. Leaf size=13

$$\frac{3}{8}\left(x^4+1\right)^{2/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3}{8}\left(x^4+1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^4)^(1/3),x]
[Out] (3*(1 + x^4)^(2/3))/8
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt[3]{1+x^4}} \, dx = \frac{3}{8}\left(1+x^4\right)^{2/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{8}\left(x^4+1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^4)^(1/3),x]
[Out] (3*(1 + x^4)^(2/3))/8
```

IntegrateAlgebraic [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{8}\left(x^4+1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(1 + x^4)^(1/3),x]
[Out] (3*(1 + x^4)^(2/3))/8
```

fricas [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{3}{8}\left(x^4+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+1)^(1/3),x, algorithm="fricas")
[Out] 3/8*(x^4 + 1)^(2/3)
```

giac [A] time = 0.61, size = 9, normalized size = 0.69

$$\frac{3}{8}\left(x^{4}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+1)^(1/3),x, algorithm="giac")
[Out] 3/8*(x^4 + 1)^(2/3)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{3\left(x^{4}+1\right)^{\frac{2}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4+1)^(1/3),x)
[Out] 3/8*(x^4+1)^(2/3)
```

maxima [A] time = 0.42, size = 9, normalized size = 0.69

$$\frac{3}{8}\left(x^{4}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+1)^(1/3),x, algorithm="maxima")
[Out] 3/8*(x^4 + 1)^(2/3)
```

mupad [B] time = 0.11, size = 9, normalized size = 0.69

$$\frac{3\left(x^{4}+1\right)^{2/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4 + 1)^(1/3),x)
[Out] (3*(x^4 + 1)^(2/3))/8
```

sympy [A] time = 0.15, size = 10, normalized size = 0.77

$$\frac{3\left(x^{4}+1\right)^{\frac{2}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4+1)**(1/3),x)
[Out] 3*(x**4 + 1)**(2/3)/8
```

3.33 $\int x^3 \sqrt[4]{1+x^4}\,dx$

Optimal. Leaf size=13

$$\frac{1}{5}\left(x^4+1\right)^{5/4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{5}\left(x^4+1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x^4)^(1/4),x]
[Out] (1 + x^4)^(5/4)/5
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^3 \sqrt[4]{1+x^4}\,dx=\frac{1}{5}\left(1+x^4\right)^{5/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{5}\left(x^4+1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x^4)^(1/4),x]
[Out] (1 + x^4)^(5/4)/5
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{5}\left(x^4+1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(1 + x^4)^(1/4),x]
[Out] (1 + x^4)^(5/4)/5
```

fricas [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{1}{5}\left(x^4+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/5*(x^4 + 1)^(5/4)
```

giac [A] time = 0.28, size = 9, normalized size = 0.69

$$\frac{1}{5}\left(x^4+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+1)^(1/4),x, algorithm="giac")
[Out] 1/5*(x^4 + 1)^(5/4)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{\left(x^4+1\right)^{\frac{5}{4}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4+1)^(1/4),x)
[Out] 1/5*(x^4+1)^(5/4)
```

maxima [A] time = 0.32, size = 9, normalized size = 0.69

$$\frac{1}{5}\left(x^4+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+1)^(1/4),x, algorithm="maxima")
[Out] 1/5*(x^4 + 1)^(5/4)
```

mupad [B] time = 0.13, size = 9, normalized size = 0.69

$$\frac{\left(x^4+1\right)^{5/4}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4 + 1)^(1/4),x)
[Out] (x^4 + 1)^(5/4)/5
```

sympy [B] time = 0.26, size = 22, normalized size = 1.69

$$\frac{x^4\sqrt[4]{x^4+1}}{5}+\frac{\sqrt[4]{x^4+1}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**4+1)**(1/4),x)
[Out] x**4*(x**4 + 1)**(1/4)/5 + (x**4 + 1)**(1/4)/5
```

3.34 $\int x^3 \sqrt[3]{1+x^4}\,dx$

Optimal. Leaf size=13

$$\frac{3}{16}\left(x^4+1\right)^{4/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3}{16}\left(x^4+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x^4)^(1/3),x]
```

```
[Out] (3*(1 + x^4)^(4/3))/16
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^3 \sqrt[3]{1+x^4}\,dx = \frac{3}{16}\left(1+x^4\right)^{4/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{16}\left(x^4+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x^4)^(1/3),x]
```

```
[Out] (3*(1 + x^4)^(4/3))/16
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{3}{16}\left(x^4+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(1 + x^4)^(1/3),x]
```

```
[Out] (3*(1 + x^4)^(4/3))/16
```

fricas [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{3}{16}\left(x^4+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/16*(x^4 + 1)^(4/3)
```

giac [A] time = 0.37, size = 9, normalized size = 0.69

$$\frac{3}{16}\left(x^4+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+1)^(1/3),x, algorithm="giac")
[Out] 3/16*(x^4 + 1)^(4/3)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{3\left(x^4+1\right)^{\frac{4}{3}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4+1)^(1/3),x)
[Out] 3/16*(x^4+1)^(4/3)
```

maxima [A] time = 0.32, size = 9, normalized size = 0.69

$$\frac{3}{16}\left(x^4+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+1)^(1/3),x, algorithm="maxima")
[Out] 3/16*(x^4 + 1)^(4/3)
```

mupad [B] time = 0.13, size = 9, normalized size = 0.69

$$\frac{3\left(x^4+1\right)^{4/3}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4 + 1)^(1/3),x)
[Out] (3*(x^4 + 1)^(4/3))/16
```

sympy [B] time = 0.21, size = 26, normalized size = 2.00

$$\frac{3x^4\sqrt[3]{x^4+1}}{16}+\frac{3\sqrt[3]{x^4+1}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**4+1)**(1/3),x)
[Out] 3*x**4*(x**4 + 1)**(1/3)/16 + 3*(x**4 + 1)**(1/3)/16
```

3.35 $\int x^3 \left(1+x^4\right)^{2/3} dx$

Optimal. Leaf size=13

$$\frac{3}{20}\left(x^4+1\right)^{5/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3}{20}\left(x^4+1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x^4)^(2/3),x]
```

```
[Out] (3*(1 + x^4)^(5/3))/20
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^3 \left(1+x^4\right)^{2/3} dx = \frac{3}{20}\left(1+x^4\right)^{5/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{20}\left(x^4+1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x^4)^(2/3),x]
```

```
[Out] (3*(1 + x^4)^(5/3))/20
```

IntegrateAlgebraic [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{3}{20}\left(x^4+1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(1 + x^4)^(2/3),x]
```

```
[Out] (3*(1 + x^4)^(5/3))/20
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{3}{20}\left(x^4+1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/20*(x^4 + 1)^(5/3)
```

giac [A] time = 0.41, size = 9, normalized size = 0.69

$$\frac{3}{20}\left(x^4+1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+1)^(2/3),x, algorithm="giac")
[Out] 3/20*(x^4 + 1)^(5/3)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{3\left(x^4+1\right)^{\frac{5}{3}}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4+1)^(2/3),x)
[Out] 3/20*(x^4+1)^(5/3)
```

maxima [A] time = 0.34, size = 9, normalized size = 0.69

$$\frac{3}{20}\left(x^4+1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+1)^(2/3),x, algorithm="maxima")
[Out] 3/20*(x^4 + 1)^(5/3)
```

mupad [B] time = 0.13, size = 9, normalized size = 0.69

$$\frac{3\left(x^4+1\right)^{5/3}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4 + 1)^(2/3),x)
[Out] (3*(x^4 + 1)^(5/3))/20
```

sympy [B] time = 0.36, size = 26, normalized size = 2.00

$$\frac{3x^4\left(x^4+1\right)^{\frac{2}{3}}}{20}+\frac{3\left(x^4+1\right)^{\frac{2}{3}}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**4+1)**(2/3),x)
[Out] 3*x**4*(x**4 + 1)**(2/3)/20 + 3*(x**4 + 1)**(2/3)/20
```

3.36 $\int x^4 \left(-1 + x^5\right)^{2/3} dx$

Optimal. Leaf size=13

$$\frac{3}{25}\left(x^5-1\right)^{5/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3}{25}\left(x^5-1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(-1 + x^5)^(2/3),x]
[Out] (3*(-1 + x^5)^(5/3))/25
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^4 \left(-1+x^5\right)^{2/3} dx = \frac{3}{25}\left(-1+x^5\right)^{5/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{25}\left(x^5-1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(-1 + x^5)^(2/3),x]
[Out] (3*(-1 + x^5)^(5/3))/25
```

IntegrateAlgebraic [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{3}{25}\left(x^5-1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(-1 + x^5)^(2/3),x]
[Out] (3*(-1 + x^5)^(5/3))/25
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{3}{25}\left(x^5-1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5-1)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/25*(x^5 - 1)^(5/3)
```

giac [A] time = 0.44, size = 9, normalized size = 0.69

$$\frac{3}{25}\left(x^5-1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5-1)^(2/3),x, algorithm="giac")
[Out] 3/25*(x^5 - 1)^(5/3)
```

maple [B] time = 0.01, size = 25, normalized size = 1.92

$$\frac{3\left(-1+x\right)\left(x^4+x^3+x^2+x+1\right)\left(x^5-1\right)^{\frac{2}{3}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^5-1)^(2/3),x)
[Out] 3/25*(-1+x)*(x^4+x^3+x^2+x+1)*(x^5-1)^(2/3)
```

maxima [A] time = 0.33, size = 9, normalized size = 0.69

$$\frac{3}{25}\left(x^5-1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5-1)^(2/3),x, algorithm="maxima")
[Out] 3/25*(x^5 - 1)^(5/3)
```

mupad [B] time = 0.16, size = 9, normalized size = 0.69

$$\frac{3\left(x^5-1\right)^{5/3}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^5 - 1)^(2/3),x)
[Out] (3*(x^5 - 1)^(5/3))/25
```

sympy [B] time = 0.42, size = 26, normalized size = 2.00

$$\frac{3x^5\left(x^5-1\right)^{\frac{2}{3}}}{25}-\frac{3\left(x^5-1\right)^{\frac{2}{3}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**5-1)**(2/3),x)
[Out] 3*x**5*(x**5 - 1)**(2/3)/25 - 3*(x**5 - 1)**(2/3)/25
```

3.37 $\int x^4 \left(1+x^5\right)^{2/3}\,dx$

Optimal. Leaf size=13

$$\frac{3}{25}\left(x^5+1\right)^{5/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3}{25}\left(x^5+1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 + x^5)^(2/3),x]
[Out] (3*(1 + x^5)^(5/3))/25
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^4 \left(1+x^5\right)^{2/3}\,dx = \frac{3}{25}\left(1+x^5\right)^{5/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{25}\left(x^5+1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 + x^5)^(2/3),x]
[Out] (3*(1 + x^5)^(5/3))/25
```

IntegrateAlgebraic [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{3}{25}\left(x^5+1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(1 + x^5)^(2/3),x]
[Out] (3*(1 + x^5)^(5/3))/25
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{3}{25}\left(x^5+1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/25*(x^5 + 1)^(5/3)
```

giac [A] time = 0.43, size = 9, normalized size = 0.69

$$\frac{3}{25}\left(x^{5}+1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+1)^(2/3),x, algorithm="giac")
[Out] 3/25*(x^5 + 1)^(5/3)
```

maple [B] time = 0.00, size = 29, normalized size = 2.23

$$\frac{3\left(1+x\right)\left(x^{4}-x^{3}+x^{2}-x+1\right)\left(x^{5}+1\right)^{\frac{2}{3}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^5+1)^(2/3),x)
[Out] 3/25*(1+x)*(x^4-x^3+x^2-x+1)*(x^5+1)^(2/3)
```

maxima [A] time = 0.34, size = 9, normalized size = 0.69

$$\frac{3}{25}\left(x^{5}+1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+1)^(2/3),x, algorithm="maxima")
[Out] 3/25*(x^5 + 1)^(5/3)
```

mupad [B] time = 0.14, size = 9, normalized size = 0.69

$$\frac{3\left(x^{5}+1\right)^{5/3}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^5 + 1)^(2/3),x)
[Out] (3*(x^5 + 1)^(5/3))/25
```

sympy [B] time = 0.42, size = 26, normalized size = 2.00

$$\frac{3x^{5}\left(x^{5}+1\right)^{\frac{2}{3}}}{25}+\frac{3\left(x^{5}+1\right)^{\frac{2}{3}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**5+1)**(2/3),x)
[Out] 3*x**5*(x**5 + 1)**(2/3)/25 + 3*(x**5 + 1)**(2/3)/25
```

3.38 $\int \frac{x^5}{\sqrt[3]{-1+x^6}} \, dx$

Optimal. Leaf size=13

$$\frac{1}{4}\left(x^6-1\right)^{2/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{4}\left(x^6-1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(-1 + x^6)^(1/3),x]
```

```
[Out] (-1 + x^6)^(2/3)/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5}{\sqrt[3]{-1+x^6}} \, dx = \frac{1}{4}\left(-1+x^6\right)^{2/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{4}\left(x^6-1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(-1 + x^6)^(1/3),x]
```

```
[Out] (-1 + x^6)^(2/3)/4
```

IntegrateAlgebraic [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{1}{4}\left(x^6-1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(-1 + x^6)^(1/3),x]
```

```
[Out] (-1 + x^6)^(2/3)/4
```

fricas [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{1}{4}\left(x^6-1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6-1)^(1/3),x, algorithm="fricas")
[Out] 1/4*(x^6 - 1)^(2/3)
```

giac [A] time = 0.28, size = 9, normalized size = 0.69

$$\frac{1}{4}\left(x^6-1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6-1)^(1/3),x, algorithm="giac")
[Out] 1/4*(x^6 - 1)^(2/3)
```

maple [B] time = 0.01, size = 30, normalized size = 2.31

$$\frac{(-1+x)(1+x)\left(x^2+x+1\right)\left(x^2-x+1\right)}{4\left(x^6-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6-1)^(1/3),x)
[Out] 1/4*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)/(x^6-1)^(1/3)
```

maxima [A] time = 0.32, size = 9, normalized size = 0.69

$$\frac{1}{4}\left(x^6-1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6-1)^(1/3),x, algorithm="maxima")
[Out] 1/4*(x^6 - 1)^(2/3)
```

mupad [B] time = 0.23, size = 9, normalized size = 0.69

$$\frac{\left(x^6-1\right)^{2/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6 - 1)^(1/3),x)
[Out] (x^6 - 1)^(2/3)/4
```

sympy [A] time = 0.21, size = 8, normalized size = 0.62

$$\frac{\left(x^6-1\right)^{\frac{2}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6-1)**(1/3),x)
[Out] (x**6 - 1)**(2/3)/4
```

3.39 $\int x^5 \sqrt[4]{-1+x^6}\,dx$

Optimal. Leaf size=13

$$\frac{2}{15}\left(x^6-1\right)^{5/4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{2}{15}\left(x^6-1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(-1 + x^6)^(1/4),x]
```

```
[Out] (2*(-1 + x^6)^(5/4))/15
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^5 \sqrt[4]{-1+x^6}\,dx=\frac{2}{15}\left(-1+x^6\right)^{5/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{2}{15}\left(x^6-1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(-1 + x^6)^(1/4),x]
```

```
[Out] (2*(-1 + x^6)^(5/4))/15
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{2}{15}\left(x^6-1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(-1 + x^6)^(1/4),x]
```

```
[Out] (2*(-1 + x^6)^(5/4))/15
```

fricas [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{2}{15}\left(x^6-1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/15*(x^6 - 1)^(5/4)
```

giac [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{2}{15}\left(x^6-1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6-1)^(1/4),x, algorithm="giac")
[Out] 2/15*(x^6 - 1)^(5/4)
```

maple [B] time = 0.01, size = 30, normalized size = 2.31

$$\frac{2\left(-1+x\right)\left(1+x\right)\left(x^2+x+1\right)\left(x^2-x+1\right)\left(x^6-1\right)^{\frac{1}{4}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^6-1)^(1/4),x)
[Out] 2/15*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)*(x^6-1)^(1/4)
```

maxima [A] time = 0.35, size = 9, normalized size = 0.69

$$\frac{2}{15}\left(x^6-1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6-1)^(1/4),x, algorithm="maxima")
[Out] 2/15*(x^6 - 1)^(5/4)
```

mupad [B] time = 0.14, size = 9, normalized size = 0.69

$$\frac{2\left(x^6-1\right)^{5/4}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^6 - 1)^(1/4),x)
[Out] (2*(x^6 - 1)^(5/4))/15
```

sympy [B] time = 0.42, size = 26, normalized size = 2.00

$$\frac{2x^6\sqrt[4]{x^6-1}}{15}-\frac{2\sqrt[4]{x^6-1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**6-1)**(1/4),x)
[Out] 2*x**6*(x**6 - 1)**(1/4)/15 - 2*(x**6 - 1)**(1/4)/15
```

3.40 $\int x^5 \sqrt[3]{-1+x^6}\, dx$

Optimal. Leaf size=13

$$\frac{1}{8}\left(x^6-1\right)^{4/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{8}\left(x^6-1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(-1 + x^6)^(1/3),x]
```

```
[Out] (-1 + x^6)^(4/3)/8
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^5 \sqrt[3]{-1+x^6}\, dx = \frac{1}{8}\left(-1+x^6\right)^{4/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{8}\left(x^6-1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(-1 + x^6)^(1/3),x]
```

```
[Out] (-1 + x^6)^(4/3)/8
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{8}\left(x^6-1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(-1 + x^6)^(1/3),x]
```

```
[Out] (-1 + x^6)^(4/3)/8
```

fricas [A] time = 0.40, size = 9, normalized size = 0.69

$$\frac{1}{8}\left(x^6-1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/8*(x^6 - 1)^(4/3)
```

giac [A] time = 0.32, size = 9, normalized size = 0.69

$$\frac{1}{8}\left(x^6-1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6-1)^(1/3),x, algorithm="giac")
[Out] 1/8*(x^6 - 1)^(4/3)
```

maple [B] time = 0.01, size = 30, normalized size = 2.31

$$\frac{(-1+x)(1+x)\left(x^2+x+1\right)\left(x^2-x+1\right)\left(x^6-1\right)^{\frac{1}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^6-1)^(1/3),x)
[Out] 1/8*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)*(x^6-1)^(1/3)
```

maxima [A] time = 0.34, size = 9, normalized size = 0.69

$$\frac{1}{8}\left(x^6-1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6-1)^(1/3),x, algorithm="maxima")
[Out] 1/8*(x^6 - 1)^(4/3)
```

mupad [B] time = 0.14, size = 9, normalized size = 0.69

$$\frac{\left(x^6-1\right)^{4/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^6 - 1)^(1/3),x)
[Out] (x^6 - 1)^(4/3)/8
```

sympy [B] time = 0.35, size = 22, normalized size = 1.69

$$\frac{x^6\sqrt[3]{x^6-1}}{8}-\frac{\sqrt[3]{x^6-1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**6-1)**(1/3),x)
[Out] x**6*(x**6 - 1)**(1/3)/8 - (x**6 - 1)**(1/3)/8
```

3.41 $\int x^5\sqrt{-1+x^6}\,dx$

Optimal. Leaf size=13

$$\frac{1}{9}\left(x^6-1\right)^{3/2}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{9}\left(x^6-1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[-1 + x^6],x]
```

```
[Out] (-1 + x^6)^(3/2)/9
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^5\sqrt{-1+x^6}\,dx=\frac{1}{9}\left(-1+x^6\right)^{3/2}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{9}\left(x^6-1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[-1 + x^6],x]
```

```
[Out] (-1 + x^6)^(3/2)/9
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{9}\left(x^6-1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[-1 + x^6],x]
```

```
[Out] (-1 + x^6)^(3/2)/9
```

fricas [A] time = 0.38, size = 9, normalized size = 0.69

$$\frac{1}{9}\left(x^6-1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(x^6 - 1)^(3/2)
```

giac [A] time = 0.51, size = 9, normalized size = 0.69

$$\frac{1}{9}\left(x^6-1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6-1)^(1/2),x, algorithm="giac")
[Out] 1/9*(x^6 - 1)^(3/2)
```

maple [B] time = 0.01, size = 30, normalized size = 2.31

$$\frac{(-1+x)(1+x)\left(x^2+x+1\right)\left(x^2-x+1\right)\sqrt{x^6-1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^6-1)^(1/2),x)
[Out] 1/9*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)*(x^6-1)^(1/2)
```

maxima [A] time = 0.32, size = 9, normalized size = 0.69

$$\frac{1}{9}\left(x^6-1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6-1)^(1/2),x, algorithm="maxima")
[Out] 1/9*(x^6 - 1)^(3/2)
```

mupad [B] time = 0.19, size = 9, normalized size = 0.69

$$\frac{\left(x^6-1\right)^{3/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^6 - 1)^(1/2),x)
[Out] (x^6 - 1)^(3/2)/9
```

sympy [B] time = 0.27, size = 22, normalized size = 1.69

$$\frac{x^6\sqrt{x^6-1}}{9}-\frac{\sqrt{x^6-1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**6-1)**(1/2),x)
[Out] x**6*sqrt(x**6 - 1)/9 - sqrt(x**6 - 1)/9
```

3.42 $\int x^5 \left(-1+x^6\right)^{3/4} \, dx$

Optimal. Leaf size=13

$$\frac{2}{21} \left(x^6-1\right)^{7/4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{2}{21} \left(x^6-1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(-1 + x^6)^(3/4),x]
```

```
[Out] (2*(-1 + x^6)^(7/4))/21
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^5 \left(-1+x^6\right)^{3/4} \, dx = \frac{2}{21} \left(-1+x^6\right)^{7/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{2}{21} \left(x^6-1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(-1 + x^6)^(3/4),x]
```

```
[Out] (2*(-1 + x^6)^(7/4))/21
```

IntegrateAlgebraic [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{2}{21} \left(x^6-1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(-1 + x^6)^(3/4),x]
```

```
[Out] (2*(-1 + x^6)^(7/4))/21
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{2}{21} \left(x^6-1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/21*(x^6 - 1)^(7/4)
```

giac [A] time = 0.42, size = 9, normalized size = 0.69

$$\frac{2}{21}\left(x^6-1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6-1)^(3/4),x, algorithm="giac")
[Out] 2/21*(x^6 - 1)^(7/4)
```

maple [B] time = 0.01, size = 30, normalized size = 2.31

$$\frac{2\left(-1+x\right)\left(1+x\right)\left(x^2+x+1\right)\left(x^2-x+1\right)\left(x^6-1\right)^{\frac{3}{4}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^6-1)^(3/4),x)
[Out] 2/21*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)*(x^6-1)^(3/4)
```

maxima [A] time = 0.32, size = 9, normalized size = 0.69

$$\frac{2}{21}\left(x^6-1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6-1)^(3/4),x, algorithm="maxima")
[Out] 2/21*(x^6 - 1)^(7/4)
```

mupad [B] time = 0.13, size = 9, normalized size = 0.69

$$\frac{2\left(x^6-1\right)^{7/4}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^6 - 1)^(3/4),x)
[Out] (2*(x^6 - 1)^(7/4))/21
```

sympy [B] time = 1.12, size = 26, normalized size = 2.00

$$\frac{2x^6\left(x^6-1\right)^{\frac{3}{4}}}{21}-\frac{2\left(x^6-1\right)^{\frac{3}{4}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**6-1)**(3/4),x)
[Out] 2*x**6*(x**6 - 1)**(3/4)/21 - 2*(x**6 - 1)**(3/4)/21
```

3.43 $\int \frac{-2+x^6}{x^3\sqrt{1+x^6}}\,dx$

Optimal. Leaf size=13

$$\frac{\sqrt{x^6+1}}{x^2}$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$\frac{\sqrt{x^6+1}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^6)/(x^3*Sqrt[1 + x^6]),x]
[Out] Sqrt[1 + x^6]/x^2
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-2+x^6}{x^3\sqrt{1+x^6}}\,dx = \frac{\sqrt{1+x^6}}{x^2}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{\sqrt{x^6+1}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^6)/(x^3*Sqrt[1 + x^6]),x]
[Out] Sqrt[1 + x^6]/x^2
```

IntegrateAlgebraic [A] time = 2.04, size = 13, normalized size = 1.00

$$\frac{\sqrt{x^6+1}}{x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x^6)/(x^3*Sqrt[1 + x^6]),x]
[Out] Sqrt[1 + x^6]/x^2
```

fricas [A] time = 0.38, size = 11, normalized size = 0.85

$$\frac{\sqrt{x^6+1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)/x^3/(x^6+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^6 + 1)/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6-2}{\sqrt{x^6+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)/x^3/(x^6+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^6 - 2)/(sqrt(x^6 + 1)*x^3), x)
```

maple [B] time = 0.01, size = 27, normalized size = 2.08

$$\frac{\left(x^4-x^2+1\right)\left(x^2+1\right)}{x^2\sqrt{x^6+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-2)/x^3/(x^6+1)^(1/2),x)
```

```
[Out] (x^4-x^2+1)*(x^2+1)/x^2/(x^6+1)^(1/2)
```

maxima [B] time = 0.49, size = 23, normalized size = 1.77

$$\frac{\sqrt{x^4-x^2+1}\,\sqrt{x^2+1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)/x^3/(x^6+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^4 - x^2 + 1)*sqrt(x^2 + 1)/x^2
```

mupad [B] time = 0.10, size = 11, normalized size = 0.85

$$\frac{\sqrt{x^6+1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 2)/(x^3*(x^6 + 1)^(1/2)),x)
```

```
[Out] (x^6 + 1)^(1/2)/x^2
```

sympy [C] time = 2.07, size = 63, normalized size = 4.85

$$\frac{x^4\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|x^6e^{i\pi}\right)}{6\Gamma\left(\frac{5}{3}\right)}-\frac{\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\right|x^6e^{i\pi}\right)}{3x^2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-2)/x**3/(x**6+1)**(1/2),x)
```

```
[Out] x**4*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**6*exp_polar(I*pi))/(6*gamma(5/
3)) - gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), x**6*exp_polar(I*pi))/(3*x**2*
gamma(2/3))
```

3.44 $\int \frac{x^5}{\sqrt[3]{1+x^6}} \, dx$

Optimal. Leaf size=13

$$\frac{1}{4}\left(x^6+1\right)^{2/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{4}\left(x^6+1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + x^6)^(1/3),x]
```

```
[Out] (1 + x^6)^(2/3)/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5}{\sqrt[3]{1+x^6}} \, dx = \frac{1}{4}\left(1+x^6\right)^{2/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{4}\left(x^6+1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + x^6)^(1/3),x]
```

```
[Out] (1 + x^6)^(2/3)/4
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{4}\left(x^6+1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(1 + x^6)^(1/3),x]
```

```
[Out] (1 + x^6)^(2/3)/4
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{1}{4}\left(x^6+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*(x^6 + 1)^(2/3)
```

giac [A] time = 0.34, size = 9, normalized size = 0.69

$$\frac{1}{4}\left(x^6+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/4*(x^6 + 1)^(2/3)
```

maple [B] time = 0.00, size = 25, normalized size = 1.92

$$\frac{\left(x^2+1\right)\left(x^4-x^2+1\right)}{4\left(x^6+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6+1)^(1/3),x)
```

```
[Out] 1/4*(x^2+1)*(x^4-x^2+1)/(x^6+1)^(1/3)
```

maxima [A] time = 0.41, size = 9, normalized size = 0.69

$$\frac{1}{4}\left(x^6+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/4*(x^6 + 1)^(2/3)
```

mupad [B] time = 0.08, size = 9, normalized size = 0.69

$$\frac{\left(x^6+1\right)^{2/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6 + 1)^(1/3),x)
```

```
[Out] (x^6 + 1)^(2/3)/4
```

sympy [A] time = 0.22, size = 8, normalized size = 0.62

$$\frac{\left(x^6+1\right)^{\frac{2}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6+1)**(1/3),x)
```

```
[Out] (x**6 + 1)**(2/3)/4
```

3.45 $\int x^5 \sqrt[4]{1+x^6}\,dx$

Optimal. Leaf size=13

$$\frac{2}{15}\left(x^6+1\right)^{5/4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{2}{15}\left(x^6+1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(1 + x^6)^(1/4),x]
[Out] (2*(1 + x^6)^(5/4))/15
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^5 \sqrt[4]{1+x^6}\,dx=\frac{2}{15}\left(1+x^6\right)^{5/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{2}{15}\left(x^6+1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(1 + x^6)^(1/4),x]
[Out] (2*(1 + x^6)^(5/4))/15
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{2}{15}\left(x^6+1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(1 + x^6)^(1/4),x]
[Out] (2*(1 + x^6)^(5/4))/15
```

fricas [A] time = 0.41, size = 9, normalized size = 0.69

$$\frac{2}{15}\left(x^6+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6+1)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/15*(x^6 + 1)^(5/4)
```

giac [A] time = 0.31, size = 9, normalized size = 0.69

$$\frac{2}{15} \left(x^6 + 1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6+1)^(1/4),x, algorithm="giac")
[Out] 2/15*(x^6 + 1)^(5/4)
```

maple [B] time = 0.01, size = 25, normalized size = 1.92

$$\frac{2 \left(x^2 + 1\right) \left(x^4 - x^2 + 1\right) \left(x^6 + 1\right)^{\frac{1}{4}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^6+1)^(1/4),x)
[Out] 2/15*(x^2+1)*(x^4-x^2+1)*(x^6+1)^(1/4)
```

maxima [A] time = 0.32, size = 9, normalized size = 0.69

$$\frac{2}{15} \left(x^6 + 1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6+1)^(1/4),x, algorithm="maxima")
[Out] 2/15*(x^6 + 1)^(5/4)
```

mupad [B] time = 0.13, size = 9, normalized size = 0.69

$$\frac{2 \left(x^6 + 1\right)^{5/4}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^6 + 1)^(1/4),x)
[Out] (2*(x^6 + 1)^(5/4))/15
```

sympy [B] time = 0.42, size = 26, normalized size = 2.00

$$\frac{2x^6 \sqrt[4]{x^6 + 1}}{15} + \frac{2 \sqrt[4]{x^6 + 1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**6+1)**(1/4),x)
[Out] 2*x**6*(x**6 + 1)**(1/4)/15 + 2*(x**6 + 1)**(1/4)/15
```

3.46 $\int x^5 \sqrt[3]{1+x^6}\,dx$

Optimal. Leaf size=13

$$\frac{1}{8}\left(x^6+1\right)^{4/3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{1}{8}\left(x^6+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(1 + x^6)^(1/3),x]
```

```
[Out] (1 + x^6)^(4/3)/8
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^5 \sqrt[3]{1+x^6}\,dx=\frac{1}{8}\left(1+x^6\right)^{4/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{8}\left(x^6+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(1 + x^6)^(1/3),x]
```

```
[Out] (1 + x^6)^(4/3)/8
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{8}\left(x^6+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(1 + x^6)^(1/3),x]
```

```
[Out] (1 + x^6)^(4/3)/8
```

fricas [A] time = 0.39, size = 9, normalized size = 0.69

$$\frac{1}{8}\left(x^6+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/8*(x^6 + 1)^(4/3)
```

giac [A] time = 0.35, size = 9, normalized size = 0.69

$$\frac{1}{8}\left(x^6+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6+1)^(1/3),x, algorithm="giac")
[Out] 1/8*(x^6 + 1)^(4/3)
```

maple [B] time = 0.00, size = 25, normalized size = 1.92

$$\frac{\left(x^2+1\right)\left(x^4-x^2+1\right)\left(x^6+1\right)^{\frac{1}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^6+1)^(1/3),x)
[Out] 1/8*(x^2+1)*(x^4-x^2+1)*(x^6+1)^(1/3)
```

maxima [A] time = 0.32, size = 9, normalized size = 0.69

$$\frac{1}{8}\left(x^6+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^6+1)^(1/3),x, algorithm="maxima")
[Out] 1/8*(x^6 + 1)^(4/3)
```

mupad [B] time = 0.13, size = 9, normalized size = 0.69

$$\frac{\left(x^6+1\right)^{4/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^6 + 1)^(1/3),x)
[Out] (x^6 + 1)^(4/3)/8
```

sympy [B] time = 0.34, size = 22, normalized size = 1.69

$$\frac{x^6\sqrt[3]{x^6+1}}{8}+\frac{\sqrt[3]{x^6+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**6+1)**(1/3),x)
[Out] x**6*(x**6 + 1)**(1/3)/8 + (x**6 + 1)**(1/3)/8
```

3.47 $\int \frac{-4+x^3}{x^2\left(-1+x^3\right)^{3/4}}\,dx$

Optimal. Leaf size=14

$$-\frac{4\sqrt[4]{x^3-1}}{x}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$-\frac{4\sqrt[4]{x^3-1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + x^3)/(x^2*(-1 + x^3)^(3/4)),x]
```

```
[Out] (-4*(-1 + x^3)^(1/4))/x
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-4+x^3}{x^2\left(-1+x^3\right)^{3/4}}\,dx = -\frac{4\sqrt[4]{-1+x^3}}{x}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$-\frac{4\sqrt[4]{x^3-1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + x^3)/(x^2*(-1 + x^3)^(3/4)),x]
```

```
[Out] (-4*(-1 + x^3)^(1/4))/x
```

IntegrateAlgebraic [A] time = 2.44, size = 14, normalized size = 1.00

$$-\frac{4\sqrt[4]{x^3-1}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-4 + x^3)/(x^2*(-1 + x^3)^(3/4)),x]
```

```
[Out] (-4*(-1 + x^3)^(1/4))/x
```

fricas [A] time = 0.43, size = 12, normalized size = 0.86

$$-\frac{4\left(x^3-1\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)/x^2/(x^3-1)^(3/4),x, algorithm="fricas")
```

```
[Out] -4*(x^3 - 1)^(1/4)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3-4}{\left(x^3-1\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)/x^2/(x^3-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 4)/((x^3 - 1)^(3/4)*x^2), x)
```

maple [A] time = 0.00, size = 22, normalized size = 1.57

$$-\frac{4\left(-1+x\right)\left(x^2+x+1\right)}{x\left(x^3-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-4)/x^2/(x^3-1)^(3/4),x)
```

```
[Out] -4/x*(-1+x)*(x^2+x+1)/(x^3-1)^(3/4)
```

maxima [A] time = 0.59, size = 18, normalized size = 1.29

$$-\frac{4\left(x^2+x+1\right)^{\frac{1}{4}}\left(x-1\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)/x^2/(x^3-1)^(3/4),x, algorithm="maxima")
```

```
[Out] -4*(x^2 + x + 1)^(1/4)*(x - 1)^(1/4)/x
```

mupad [B] time = 0.12, size = 12, normalized size = 0.86

$$-\frac{4\left(x^3-1\right)^{1/4}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 4)/(x^2*(x^3 - 1)^(3/4)),x)
```

```
[Out] -(4*(x^3 - 1)^(1/4))/x
```

sympy [C] time = 1.69, size = 68, normalized size = 4.86

$$\frac{x^2 e^{-\frac{3i\pi}{4}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{4}\\ \frac{5}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{4e^{\frac{i\pi}{4}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{4}\\ \frac{2}{3}\end{matrix}\middle|x^3\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-4)/x**2/(x**3-1)**(3/4),x)
```

```
[Out] x**2*exp(-3*I*pi/4)*gamma(2/3)*hyper((2/3, 3/4), (5/3,), x**3)/(3*gamma(5/3
)) + 4*exp(I*pi/4)*gamma(-1/3)*hyper((-1/3, 3/4), (2/3,), x**3)/(3*x*gamma(
2/3))
```

3.48 $\int \frac{1}{x\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=14

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 63, 207}

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 + x^3]])/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{1+x^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)\\
&= \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)\\
&= -\frac{2}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 + x^3]])/3
```

IntegrateAlgebraic [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[1 + x^3]),x]
[Out] (-2*ArcTanh[Sqrt[1 + x^3]])/3
```

fricas [B] time = 0.40, size = 25, normalized size = 1.79

$$-\frac{1}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] -1/3*log(sqrt(x^3 + 1) + 1) + 1/3*log(sqrt(x^3 + 1) - 1)
```

giac [B] time = 0.34, size = 26, normalized size = 1.86

$$-\frac{1}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{1}{3}\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3+1)^(1/2),x, algorithm="giac")
[Out] -1/3*log(sqrt(x^3 + 1) + 1) + 1/3*log(abs(sqrt(x^3 + 1) - 1))
```

maple [A] time = 0.29, size = 11, normalized size = 0.79

$$-\frac{2\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^3+1)^(1/2),x)
[Out] -2/3*arctanh((x^3+1)^(1/2))
```

maxima [B] time = 0.49, size = 25, normalized size = 1.79

$$-\frac{1}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] -1/3*log(sqrt(x^3 + 1) + 1) + 1/3*log(sqrt(x^3 + 1) - 1)
```

mupad [B] time = 0.25, size = 164, normalized size = 11.71

$$-\frac{\left(3+\sqrt{3}\,1i\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^3 + 1)^(1/2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
)/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [**A**] time = 0.78, size = 12, normalized size = 0.86

$$-\frac{2\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**3+1)**(1/2),x)
```

```
[Out] -2*asinh(x**(-3/2))/3
```

3.49 $\int \frac{4+x^3}{x^2(1+x^3)^{3/4}}\,dx$

Optimal. Leaf size=14

$$-\frac{4\sqrt[4]{x^3+1}}{x}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$-\frac{4\sqrt[4]{x^3+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^3)/(x^2*(1 + x^3)^(3/4)),x]
```

```
[Out] (-4*(1 + x^3)^(1/4))/x
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{4+x^3}{x^2\left(1+x^3\right)^{3/4}}\,dx = -\frac{4\sqrt[4]{1+x^3}}{x}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$-\frac{4\sqrt[4]{x^3+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^3)/(x^2*(1 + x^3)^(3/4)),x]
```

```
[Out] (-4*(1 + x^3)^(1/4))/x
```

IntegrateAlgebraic [A] time = 2.42, size = 14, normalized size = 1.00

$$-\frac{4\sqrt[4]{x^3+1}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4 + x^3)/(x^2*(1 + x^3)^(3/4)),x]
```

```
[Out] (-4*(1 + x^3)^(1/4))/x
```

fricas [A] time = 0.39, size = 12, normalized size = 0.86

$$-\frac{4\left(x^3+1\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)/x^2/(x^3+1)^(3/4),x, algorithm="fricas")
```

```
[Out] -4*(x^3 + 1)^(1/4)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3+4}{\left(x^3+1\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)/x^2/(x^3+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 4)/((x^3 + 1)^(3/4)*x^2), x)
```

maple [A] time = 0.01, size = 24, normalized size = 1.71

$$-\frac{4\,(1+x)\left(x^2-x+1\right)}{x\left(x^3+1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+4)/x^2/(x^3+1)^(3/4),x)
```

```
[Out] -4/x*(1+x)*(x^2-x+1)/(x^3+1)^(3/4)
```

maxima [A] time = 0.73, size = 20, normalized size = 1.43

$$-\frac{4\left(x^2-x+1\right)^{\frac{1}{4}}(x+1)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)/x^2/(x^3+1)^(3/4),x, algorithm="maxima")
```

```
[Out] -4*(x^2 - x + 1)^(1/4)*(x + 1)^(1/4)/x
```

mupad [B] time = 0.07, size = 12, normalized size = 0.86

$$-\frac{4\left(x^3+1\right)^{1/4}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 4)/(x^2*(x^3 + 1)^(3/4)),x)
```

```
[Out] -(4*(x^3 + 1)^(1/4))/x
```

sympy [C] time = 1.60, size = 63, normalized size = 4.50

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{3}{4}\\\frac{5}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{4\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{3}{4}\\\frac{2}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+4)/x**2/(x**3+1)**(3/4),x)
```

```
[Out] x**2*gamma(2/3)*hyper((2/3, 3/4), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5/
3)) + 4*gamma(-1/3)*hyper((-1/3, 3/4), (2/3,), x**3*exp_polar(I*pi))/(3*x*g
amma(2/3))
```

3.50 $\int \frac{(2+3x^2)\sqrt[3]{x+x^3}}{1+x^2}\,dx$

Optimal. Leaf size=14

$$\frac{3}{2}x\sqrt[3]{x^3+x}$$

Rubi [A] time = 0.08, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2056, 449}

$$\frac{3}{2}x\sqrt[3]{x^3+x}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(x + x^3)^(1/3))/(1 + x^2),x]
```

```
[Out] (3*x*(x + x^3)^(1/3))/2
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+3x^2)\sqrt[3]{x+x^3}}{1+x^2}\,dx &= \frac{\sqrt[3]{x+x^3}\int \frac{\sqrt[3]{x}(2+3x^2)}{(1+x^2)^{2/3}}\,dx}{\sqrt[3]{x}\sqrt[3]{1+x^2}}\\ &= \frac{3}{2}x\sqrt[3]{x+x^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 14, normalized size = 1.00

$$\frac{3}{2}x\sqrt[3]{x^3+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(x + x^3)^(1/3))/(1 + x^2),x]
```

```
[Out] (3*x*(x + x^3)^(1/3))/2
```

IntegrateAlgebraic [A] time = 0.16, size = 14, normalized size = 1.00

$$\frac{3}{2}x\sqrt[3]{x^3+x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^2)*(x + x^3)^(1/3))/(1 + x^2),x]
[Out] (3*x*(x + x^3)^(1/3))/2
```

fricas [A] time = 0.41, size = 10, normalized size = 0.71

$$\frac{3}{2}\left(x^{3}+x\right)^{\frac{1}{3}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^3+x)^(1/3)/(x^2+1),x, algorithm="fricas")
[Out] 3/2*(x^3 + x)^(1/3)*x
```

giac [A] time = 0.42, size = 12, normalized size = 0.86

$$\frac{3}{2}x^{2}\left(\frac{1}{x^{2}}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^3+x)^(1/3)/(x^2+1),x, algorithm="giac")
[Out] 3/2*x^2*(1/x^2 + 1)^(1/3)
```

maple [A] time = 0.01, size = 11, normalized size = 0.79

$$\frac{3x\left(x^{3}+x\right)^{\frac{1}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^3+x)^(1/3)/(x^2+1),x)
[Out] 3/2*x*(x^3+x)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^{3}+x\right)^{\frac{1}{3}}\left(3\,x^{2}+2\right)}{x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^3+x)^(1/3)/(x^2+1),x, algorithm="maxima")
[Out] integrate((x^3 + x)^(1/3)*(3*x^2 + 2)/(x^2 + 1), x)
```

mupad [B] time = 0.19, size = 10, normalized size = 0.71

$$\frac{3\,x\,\left(x^3+x\right)^{1/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(x + x^3)^(1/3))/(x^2 + 1),x)
[Out] (3*x*(x + x^3)^(1/3))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[3]{x\left(x^{2}+1\right)}\left(3x^{2}+2\right)}{x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**3+x)**(1/3)/(x**2+1),x)
```

```
[Out] Integral((x*(x**2 + 1))**(1/3)*(3*x**2 + 2)/(x**2 + 1), x)
```

3.51 $\int \frac{-2+x}{(-1+x)\sqrt[4]{-x^2+x^3}}\,dx$

Optimal. Leaf size=14

$$\frac{4x}{\sqrt[4]{(x-1)x^2}}$$

Rubi [A] time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2056, 74}

$$\frac{4x}{\sqrt[4]{x^3-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x)/((-1 + x)*(-x^2 + x^3)^(1/4)),x]
```

```
[Out] (4*x)/(-x^2 + x^3)^(1/4)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-2+x}{(-1+x)\sqrt[4]{-x^2+x^3}}\,dx &= \frac{\left(\sqrt[4]{-1+x}\,\sqrt{x}\right)\int \frac{-2+x}{(-1+x)^{5/4}\sqrt{x}}\,dx}{\sqrt[4]{-x^2+x^3}}\\ &= \frac{4x}{\sqrt[4]{-x^2+x^3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{4x}{\sqrt[4]{(x-1)x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x)/((-1 + x)*(-x^2 + x^3)^(1/4)),x]
```

```
[Out] (4*x)/((-1 + x)*x^2)^(1/4)
```

IntegrateAlgebraic [A] time = 6.97, size = 14, normalized size = 1.00

$$\frac{4x}{\sqrt[4]{(x-1)x^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x)/((-1 + x)*(-x^2 + x^3)^(1/4)),x]
[Out] (4*x)/((-1 + x)*x^2)^(1/4)
```

fricas [A] time = 0.39, size = 22, normalized size = 1.57

$$\frac{4\left(x^3-x^2\right)^{\frac{3}{4}}}{x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(-1+x)/(x^3-x^2)^(1/4),x, algorithm="fricas")
[Out] 4*(x^3 - x^2)^(3/4)/(x^2 - x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-2}{\left(x^3-x^2\right)^{\frac{1}{4}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(-1+x)/(x^3-x^2)^(1/4),x, algorithm="giac")
[Out] integrate((x - 2)/((x^3 - x^2)^(1/4)*(x - 1)), x)
```

maple [A] time = 0.00, size = 15, normalized size = 1.07

$$\frac{4x}{\left(x^3-x^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+x)/(-1+x)/(x^3-x^2)^(1/4),x)
[Out] 4*x/(x^3-x^2)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-2}{\left(x^3-x^2\right)^{\frac{1}{4}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(-1+x)/(x^3-x^2)^(1/4),x, algorithm="maxima")
[Out] integrate((x - 2)/((x^3 - x^2)^(1/4)*(x - 1)), x)
```

mupad [B] time = 0.16, size = 21, normalized size = 1.50

$$\frac{4\left(x^3-x^2\right)^{3/4}}{x\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2)/((x^3 - x^2)^(1/4)*(x - 1)),x)
[Out] (4*(x^3 - x^2)^(3/4))/(x*(x - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-2}{\sqrt[4]{x^2 (x-1)} (x-1)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(-1+x)/(x**3-x**2)**(1/4),x)
```

```
[Out] Integral((x - 2)/((x**2*(x - 1))**(1/4)*(x - 1)), x)
```

3.52 $\int \frac{2+x}{(1+x)\sqrt[4]{x^2+x^3}}\,dx$

Optimal. Leaf size=14

$$\frac{4x}{\sqrt[4]{x^2(x+1)}}$$

Rubi [A] time = 0.06, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2056, 74}

$$\frac{4x}{\sqrt[4]{x^3+x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((1 + x)*(x^2 + x^3)^(1/4)),x]
```

```
[Out] (4*x)/(x^2 + x^3)^(1/4)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{(1+x)\sqrt[4]{x^2+x^3}}\,dx &= \frac{\left(\sqrt{x}\sqrt[4]{1+x}\right)\int \frac{2+x}{\sqrt{x}(1+x)^{5/4}}\,dx}{\sqrt[4]{x^2+x^3}}\\ &= \frac{4x}{\sqrt[4]{x^2+x^3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{4x}{\sqrt[4]{x^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((1 + x)*(x^2 + x^3)^(1/4)),x]
```

```
[Out] (4*x)/(x^2*(1 + x))^(1/4)
```

IntegrateAlgebraic [A] time = 7.01, size = 14, normalized size = 1.00

$$\frac{4x}{\sqrt[4]{x^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)/((1 + x)*(x^2 + x^3)^(1/4)),x]
```

```
[Out] (4*x)/(x^2*(1 + x))^(1/4)
```

fricas [A] time = 0.39, size = 18, normalized size = 1.29

$$\frac{4 \left(x^{3}+x^{2}\right)^{\frac{3}{4}}}{x^{2}+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(1+x)/(x^3+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] 4*(x^3 + x^2)^(3/4)/(x^2 + x)
```

giac [A] time = 0.45, size = 11, normalized size = 0.79

$$\frac{4}{\left(\frac{1}{x}+\frac{1}{x^{2}}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(1+x)/(x^3+x^2)^(1/4),x, algorithm="giac")
```

```
[Out] 4/(1/x + 1/x^2)^(1/4)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{4x}{\left(x^{3}+x^{2}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(1+x)/(x^3+x^2)^(1/4),x)
```

```
[Out] 4*x/(x^3+x^2)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\left(x^{3}+x^{2}\right)^{\frac{1}{4}} (x+1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(1+x)/(x^3+x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x + 2)/((x^3 + x^2)^(1/4)*(x + 1)), x)
```

mupad [B] time = 0.13, size = 19, normalized size = 1.36

$$\frac{4 \left(x^{3}+x^{2}\right)^{3/4}}{x\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((x^2 + x^3)^(1/4)*(x + 1)),x)
```

```
[Out] (4*(x^2 + x^3)^(3/4))/(x*(x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\sqrt[4]{x^2\,(x+1)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(1+x)/(x**3+x**2)**(1/4),x)
[Out] Integral((x + 2)/((x**2*(x + 1))**(1/4)*(x + 1)), x)
```

3.53 $\int \frac{1}{x^2\left(1+x^4\right)^{3/4}}\,dx$

Optimal. Leaf size=14

$$-\frac{\sqrt[4]{x^4+1}}{x}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{\sqrt[4]{x^4+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x^4)^(3/4)),x]
```

```
[Out] -((1 + x^4)^(1/4)/x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(1+x^4\right)^{3/4}}\,dx = -\frac{\sqrt[4]{1+x^4}}{x}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{\sqrt[4]{x^4+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + x^4)^(3/4)),x]
```

```
[Out] -((1 + x^4)^(1/4)/x)
```

IntegrateAlgebraic [A] time = 0.17, size = 14, normalized size = 1.00

$$-\frac{\sqrt[4]{x^4+1}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(1 + x^4)^(3/4)),x]
```

```
[Out] -((1 + x^4)^(1/4)/x)
```

fricas [A] time = 0.41, size = 12, normalized size = 0.86

$$-\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+1)^(3/4),x, algorithm="fricas")
```

```
[Out] -(x^4 + 1)^(1/4)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)^{\frac{3}{4}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 1)^(3/4)*x^2), x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^4+1)^(3/4),x)
```

```
[Out] -(x^4+1)^(1/4)/x
```

maxima [A] time = 0.43, size = 12, normalized size = 0.86

$$-\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+1)^(3/4),x, algorithm="maxima")
```

```
[Out] -(x^4 + 1)^(1/4)/x
```

mupad [B] time = 0.15, size = 12, normalized size = 0.86

$$-\frac{\left(x^4+1\right)^{1/4}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4 + 1)^(3/4)),x)
```

```
[Out] -(x^4 + 1)^(1/4)/x
```

sympy [B] time = 0.55, size = 22, normalized size = 1.57

$$\frac{\sqrt[4]{1+\frac{1}{x^4}}\,\Gamma\left(-\frac{1}{4}\right)}{4\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**4+1)**(3/4),x)
```

```
[Out] (1 + x**(-4))**(1/4)*gamma(-1/4)/(4*gamma(3/4))
```

3.54 $\int \frac{3+x^4}{x^4\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=14

$$-\frac{\sqrt{x^4+1}}{x^3}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$-\frac{\sqrt{x^4+1}}{x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x^4)/(x^4*Sqrt[1 + x^4]),x]
```

```
[Out] -(Sqrt[1 + x^4]/x^3)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{3+x^4}{x^4\sqrt{1+x^4}}\,dx = -\frac{\sqrt{1+x^4}}{x^3}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{\sqrt{x^4+1}}{x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x^4)/(x^4*Sqrt[1 + x^4]),x]
```

```
[Out] -(Sqrt[1 + x^4]/x^3)
```

IntegrateAlgebraic [A] time = 0.70, size = 14, normalized size = 1.00

$$-\frac{\sqrt{x^4+1}}{x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + x^4)/(x^4*Sqrt[1 + x^4]),x]
```

```
[Out] -(Sqrt[1 + x^4]/x^3)
```

fricas [A] time = 0.39, size = 12, normalized size = 0.86

$$-\frac{\sqrt{x^4+1}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)/x^4/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(x^4 + 1)/x^3
```

giac [A] time = 0.57, size = 12, normalized size = 0.86

$$-\frac{\sqrt{\frac{1}{x^4}+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)/x^4/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(1/x^4 + 1)/x
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{\sqrt{x^4+1}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3)/x^4/(x^4+1)^(1/2),x)
```

```
[Out] -(x^4+1)^(1/2)/x^3
```

maxima [A] time = 0.57, size = 12, normalized size = 0.86

$$-\frac{\sqrt{x^4+1}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)/x^4/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x^4 + 1)/x^3
```

mupad [B] time = 0.08, size = 12, normalized size = 0.86

$$-\frac{\sqrt{x^4+1}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 3)/(x^4*(x^4 + 1)^(1/2)),x)
```

```
[Out] -(x^4 + 1)^(1/2)/x^3
```

sympy [C] time = 1.55, size = 63, normalized size = 4.50

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{3\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3)/x**4/(x**4+1)**(1/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(I*pi))/(4*gamma(5/4))
 + 3*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), x**4*exp_polar(I*pi))/(4*x**3*g
amma(1/4))
```

3.55 $\int \frac{-1+4x^3}{\sqrt{-1-2x+2x^4}}\,dx$

Optimal. Leaf size=14

$$\sqrt{2x^4-2x-1}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1588}

$$\sqrt{2x^4-2x-1}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 4*x^3)/Sqrt[-1 - 2*x + 2*x^4],x]
```

```
[Out] Sqrt[-1 - 2*x + 2*x^4]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+4x^3}{\sqrt{-1-2x+2x^4}}\,dx = \sqrt{-1-2x+2x^4}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\sqrt{2x^4-2x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 4*x^3)/Sqrt[-1 - 2*x + 2*x^4],x]
```

```
[Out] Sqrt[-1 - 2*x + 2*x^4]
```

IntegrateAlgebraic [A] time = 0.01, size = 14, normalized size = 1.00

$$\sqrt{2x^4-2x-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 4*x^3)/Sqrt[-1 - 2*x + 2*x^4],x]
```

```
[Out] Sqrt[-1 - 2*x + 2*x^4]
```

fricas [A] time = 0.39, size = 12, normalized size = 0.86

$$\sqrt{2\,x^4-2\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-1)/(2*x^4-2*x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2*x^4 - 2*x - 1)
```

giac [A] time = 0.44, size = 12, normalized size = 0.86

$$\sqrt{2\,x^4 - 2\,x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-1)/(2*x^4-2*x-1)^(1/2),x, algorithm="giac")
[Out] sqrt(2*x^4 - 2*x - 1)
```

maple [A] time = 0.01, size = 13, normalized size = 0.93

$$\sqrt{2x^4 - 2x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-1)/(2*x^4-2*x-1)^(1/2),x)
[Out] (2*x^4-2*x-1)^(1/2)
```

maxima [A] time = 0.44, size = 12, normalized size = 0.86

$$\sqrt{2\,x^4 - 2\,x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-1)/(2*x^4-2*x-1)^(1/2),x, algorithm="maxima")
[Out] sqrt(2*x^4 - 2*x - 1)
```

mupad [B] time = 0.20, size = 12, normalized size = 0.86

$$\sqrt{2\,x^4 - 2\,x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3 - 1)/(2*x^4 - 2*x - 1)^(1/2),x)
[Out] (2*x^4 - 2*x - 1)^(1/2)
```

sympy [A] time = 0.16, size = 12, normalized size = 0.86

$$\sqrt{2x^4 - 2x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-1)/(2*x**4-2*x-1)**(1/2),x)
[Out] sqrt(2*x**4 - 2*x - 1)
```

3.56 $\int \frac{-4+x^5}{x^2(1+x^5)^{3/4}} \, dx$

Optimal. Leaf size=14

$$\frac{4\sqrt[4]{x^5+1}}{x}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$\frac{4\sqrt[4]{x^5+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + x^5)/(x^2*(1 + x^5)^(3/4)),x]
```

```
[Out] (4*(1 + x^5)^(1/4))/x
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-4+x^5}{x^2\left(1+x^5\right)^{3/4}} \, dx = \frac{4\sqrt[4]{1+x^5}}{x}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{4\sqrt[4]{x^5+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + x^5)/(x^2*(1 + x^5)^(3/4)),x]
```

```
[Out] (4*(1 + x^5)^(1/4))/x
```

IntegrateAlgebraic [A] time = 0.22, size = 14, normalized size = 1.00

$$\frac{4\sqrt[4]{x^5+1}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-4 + x^5)/(x^2*(1 + x^5)^(3/4)),x]
```

```
[Out] (4*(1 + x^5)^(1/4))/x
```

fricas [A] time = 0.40, size = 12, normalized size = 0.86

$$\frac{4\left(x^5+1\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-4)/x^2/(x^5+1)^(3/4),x, algorithm="fricas")
```

```
[Out] 4*(x^5 + 1)^(1/4)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 - 4}{\left(x^5 + 1\right)^{\frac{3}{4}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-4)/x^2/(x^5+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((x^5 - 4)/((x^5 + 1)^(3/4)*x^2), x)
```

maple [B] time = 0.01, size = 32, normalized size = 2.29

$$\frac{4\,(1+x)\left(x^4 - x^3 + x^2 - x + 1\right)}{x\left(x^5+1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-4)/x^2/(x^5+1)^(3/4),x)
```

```
[Out] 4/x*(1+x)*(x^4-x^3+x^2-x+1)/(x^5+1)^(3/4)
```

maxima [B] time = 0.53, size = 28, normalized size = 2.00

$$\frac{4\left(x^4 - x^3 + x^2 - x + 1\right)^{\frac{1}{4}}(x+1)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-4)/x^2/(x^5+1)^(3/4),x, algorithm="maxima")
```

```
[Out] 4*(x^4 - x^3 + x^2 - x + 1)^(1/4)*(x + 1)^(1/4)/x
```

mupad [B] time = 0.10, size = 12, normalized size = 0.86

$$\frac{4\left(x^5+1\right)^{1/4}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5 - 4)/(x^2*(x^5 + 1)^(3/4)),x)
```

```
[Out] (4*(x^5 + 1)^(1/4))/x
```

sympy [C] time = 1.61, size = 63, normalized size = 4.50

$$\frac{x^4\Gamma\left(\frac{4}{5}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{4}{5}\\ \frac{9}{5}\end{matrix}\middle| x^5 e^{i\pi}\right)}{5\Gamma\left(\frac{9}{5}\right)} - \frac{4\Gamma\left(-\frac{1}{5}\right){}_2F_1\left(\begin{matrix}-\frac{1}{5},\frac{3}{4}\\ \frac{4}{5}\end{matrix}\middle| x^5 e^{i\pi}\right)}{5x\Gamma\left(\frac{4}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-4)/x**2/(x**5+1)**(3/4),x)
```

```
[Out] x**4*gamma(4/5)*hyper((3/4, 4/5), (9/5,), x**5*exp_polar(I*pi))/(5*gamma(9/
5)) - 4*gamma(-1/5)*hyper((-1/5, 3/4), (4/5,), x**5*exp_polar(I*pi))/(5*x*g
amma(4/5))
```

3.57 $\int \frac{4+x^5}{x^2\left(-1+x^5\right)^{3/4}}\,dx$

Optimal. Leaf size=14

$$\frac{4\sqrt[4]{x^5-1}}{x}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$\frac{4\sqrt[4]{x^5-1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^5)/(x^2*(-1 + x^5)^(3/4)),x]
```

```
[Out] (4*(-1 + x^5)^(1/4))/x
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{4+x^5}{x^2\left(-1+x^5\right)^{3/4}}\,dx=\frac{4\sqrt[4]{-1+x^5}}{x}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{4\sqrt[4]{x^5-1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^5)/(x^2*(-1 + x^5)^(3/4)),x]
```

```
[Out] (4*(-1 + x^5)^(1/4))/x
```

IntegrateAlgebraic [A] time = 0.22, size = 14, normalized size = 1.00

$$\frac{4\sqrt[4]{x^5-1}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4 + x^5)/(x^2*(-1 + x^5)^(3/4)),x]
```

```
[Out] (4*(-1 + x^5)^(1/4))/x
```

fricas [A] time = 0.39, size = 12, normalized size = 0.86

$$\frac{4\left(x^5-1\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+4)/x^2/(x^5-1)^(3/4),x, algorithm="fricas")
[Out] 4*(x^5 - 1)^(1/4)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5+4}{\left(x^5-1\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+4)/x^2/(x^5-1)^(3/4),x, algorithm="giac")
[Out] integrate((x^5 + 4)/((x^5 - 1)^(3/4)*x^2), x)
```

maple [B] time = 0.01, size = 28, normalized size = 2.00

$$\frac{4\left(-1+x\right)\left(x^4+x^3+x^2+x+1\right)}{x\left(x^5-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+4)/x^2/(x^5-1)^(3/4),x)
[Out] 4/x*(-1+x)*(x^4+x^3+x^2+x+1)/(x^5-1)^(3/4)
```

maxima [A] time = 0.66, size = 24, normalized size = 1.71

$$\frac{4\left(x^4+x^3+x^2+x+1\right)^{\frac{1}{4}}(x-1)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+4)/x^2/(x^5-1)^(3/4),x, algorithm="maxima")
[Out] 4*(x^4 + x^3 + x^2 + x + 1)^(1/4)*(x - 1)^(1/4)/x
```

mupad [B] time = 0.13, size = 12, normalized size = 0.86

$$\frac{4\left(x^5-1\right)^{1/4}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5 + 4)/(x^2*(x^5 - 1)^(3/4)),x)
[Out] (4*(x^5 - 1)^(1/4))/x
```

sympy [C] time = 1.69, size = 68, normalized size = 4.86

$$\frac{x^4e^{-\frac{3i\pi}{4}}\Gamma\left(\frac{4}{5}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{4}{5}\\\frac{9}{5}\end{matrix}\middle|x^5\right)}{5\Gamma\left(\frac{9}{5}\right)}-\frac{4e^{\frac{i\pi}{4}}\Gamma\left(-\frac{1}{5}\right){}_2F_1\left(\begin{matrix}-\frac{1}{5},\frac{3}{4}\\\frac{4}{5}\end{matrix}\middle|x^5\right)}{5x\Gamma\left(\frac{4}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+4)/x**2/(x**5-1)**(3/4),x)
[Out] x**4*exp(-3*I*pi/4)*gamma(4/5)*hyper((3/4, 4/5), (9/5,), x**5)/(5*gamma(9/5
)) - 4*exp(I*pi/4)*gamma(-1/5)*hyper((-1/5, 3/4), (4/5,), x**5)/(5*x*gamma(
4/5))
```

3.58 $\int \frac{2+5x^3}{\sqrt{1+x^3}\left(1+x^2+x^5\right)} \, dx$

Optimal. Leaf size=14

$$2\tan^{-1}\left(x\sqrt{x^3+1}\right)$$

Rubi [F] time = 0.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2+5x^3}{\sqrt{1+x^3}\left(1+x^2+x^5\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(2 + 5*x^3)/(Sqrt[1 + x^3]*(1 + x^2 + x^5)),x]
```

```
[Out] 2*Defer[Int][1/(Sqrt[1 + x^3]*(1 + x^2 + x^5)), x] + 5*Defer[Int][x^3/(Sqrt
[1 + x^3]*(1 + x^2 + x^5)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+5x^3}{\sqrt{1+x^3}\left(1+x^2+x^5\right)} \, dx &= \int \left(\frac{2}{\sqrt{1+x^3}\left(1+x^2+x^5\right)}+\frac{5x^3}{\sqrt{1+x^3}\left(1+x^2+x^5\right)}\right) dx \\
&= 2\int \frac{1}{\sqrt{1+x^3}\left(1+x^2+x^5\right)} \, dx+5\int \frac{x^3}{\sqrt{1+x^3}\left(1+x^2+x^5\right)} \, dx
\end{aligned}$$

Mathematica [C] time = 4.92, size = 2691, normalized size = 192.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 5*x^3)/(Sqrt[1 + x^3]*(1 + x^2 + x^5)),x]
```

```
[Out] ((-4*I)*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt
[3])/((-1)^(1/3) - Root[1 + #1^2 + #1^5 & , 1, 0]), ArcSin[Sqrt[(1 + (-1)^(
2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(Sqrt[1 + x^3]*((-1)^(1/3) - Root[
1 + #1^2 + #1^5 & , 1, 0])*(Root[1 + #1^2 + #1^5 & , 1, 0] - Root[1 + #1^2
+ #1^5 & , 2, 0])*(Root[1 + #1^2 + #1^5 & , 1, 0] - Root[1 + #1^2 + #1^5 &
, 3, 0])*(Root[1 + #1^2 + #1^5 & , 1, 0] - Root[1 + #1^2 + #1^5 & , 4, 0])*
(Root[1 + #1^2 + #1^5 & , 1, 0] - Root[1 + #1^2 + #1^5 & , 5, 0])) - ((10*I
)*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/(
(-1)^(1/3) - Root[1 + #1^2 + #1^5 & , 1, 0]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x
)/(1 + (-1)^(1/3))]], (-1)^(1/3)]*Root[1 + #1^2 + #1^5 & , 1, 0]^3)/(Sqrt[1
 + x^3]*((-1)^(1/3) - Root[1 + #1^2 + #1^5 & , 1, 0])*(Root[1 + #1^2 + #1^5
 & , 1, 0] - Root[1 + #1^2 + #1^5 & , 2, 0])*(Root[1 + #1^2 + #1^5 & , 1, 0
] - Root[1 + #1^2 + #1^5 & , 3, 0])*(Root[1 + #1^2 + #1^5 & , 1, 0] - Root[
1 + #1^2 + #1^5 & , 4, 0])*(Root[1 + #1^2 + #1^5 & , 1, 0] - Root[1 + #1^2
+ #1^5 & , 5, 0])) - ((4*I)*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2
]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) - Root[1 + #1^2 + #1^5 & , 2, 0]), Arc
Sin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(Sqrt[1 + x^3]
*((-1)^(1/3) - Root[1 + #1^2 + #1^5 & , 2, 0])*(-Root[1 + #1^2 + #1^5 & , 1
, 0] + Root[1 + #1^2 + #1^5 & , 2, 0])*(Root[1 + #1^2 + #1^5 & , 2, 0] - Ro
ot[1 + #1^2 + #1^5 & , 3, 0])*(Root[1 + #1^2 + #1^5 & , 2, 0] - Root[1 + #1
```

```
^2 + #1^5 & , 4, 0])*(Root[1 + #1^2 + #1^5 & , 2, 0] - Root[1 + #1^2 + #1^5
 & , 5, 0])) - ((10*I)*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*Ell
ipticPi[(I*Sqrt[3])/((-1)^(1/3) - Root[1 + #1^2 + #1^5 & , 2, 0]), ArcSin[S
qrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]*Root[1 + #1^2 + #1^5
 & , 2, 0]^3)/(Sqrt[1 + x^3]*((-1)^(1/3) - Root[1 + #1^2 + #1^5 & , 2, 0])*
(-Root[1 + #1^2 + #1^5 & , 1, 0] + Root[1 + #1^2 + #1^5 & , 2, 0])*(Root[1
+ #1^2 + #1^5 & , 2, 0] - Root[1 + #1^2 + #1^5 & , 3, 0])*(Root[1 + #1^2 +
#1^5 & , 2, 0] - Root[1 + #1^2 + #1^5 & , 4, 0])*(Root[1 + #1^2 + #1^5 & ,
2, 0] - Root[1 + #1^2 + #1^5 & , 5, 0])) - ((4*I)*Sqrt[(1 + x)/(1 + (-1)^(1
/3))]*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) - Root[1 + #1^2
+ #1^5 & , 3, 0]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^
(1/3)])/(Sqrt[1 + x^3]*((-1)^(1/3) - Root[1 + #1^2 + #1^5 & , 3, 0])*(-Root
[1 + #1^2 + #1^5 & , 1, 0] + Root[1 + #1^2 + #1^5 & , 3, 0])*(-Root[1 + #1^
2 + #1^5 & , 2, 0] + Root[1 + #1^2 + #1^5 & , 3, 0])*(Root[1 + #1^2 + #1^5
& , 3, 0] - Root[1 + #1^2 + #1^5 & , 4, 0])*(Root[1 + #1^2 + #1^5 & , 3, 0]
 - Root[1 + #1^2 + #1^5 & , 5, 0])) - ((10*I)*Sqrt[(1 + x)/(1 + (-1)^(1/3))
]*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) - Root[1 + #1^2 + #1
^5 & , 3, 0]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3
)]*Root[1 + #1^2 + #1^5 & , 3, 0]^3)/(Sqrt[1 + x^3]*((-1)^(1/3) - Root[1 +
#1^2 + #1^5 & , 3, 0])*(-Root[1 + #1^2 + #1^5 & , 1, 0] + Root[1 + #1^2 + #
1^5 & , 3, 0])*(-Root[1 + #1^2 + #1^5 & , 2, 0] + Root[1 + #1^2 + #1^5 & ,
3, 0])*(Root[1 + #1^2 + #1^5 & , 3, 0] - Root[1 + #1^2 + #1^5 & , 4, 0])*(R
oot[1 + #1^2 + #1^5 & , 3, 0] - Root[1 + #1^2 + #1^5 & , 5, 0])) - ((4*I)*S
qrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/((-1
)^(1/3) - Root[1 + #1^2 + #1^5 & , 4, 0]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/(Sqrt[1 + x^3]*((-1)^(1/3) - Root[1 + #1^2
+ #1^5 & , 4, 0])*(-Root[1 + #1^2 + #1^5 & , 1, 0] + Root[1 + #1^2 + #1^5 &
 , 4, 0])*(-Root[1 + #1^2 + #1^5 & , 2, 0] + Root[1 + #1^2 + #1^5 & , 4, 0]
)*(-Root[1 + #1^2 + #1^5 & , 3, 0] + Root[1 + #1^2 + #1^5 & , 4, 0])*(Root[
1 + #1^2 + #1^5 & , 4, 0] - Root[1 + #1^2 + #1^5 & , 5, 0])) - ((10*I)*Sqrt
[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(
1/3) - Root[1 + #1^2 + #1^5 & , 4, 0]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
 (-1)^(1/3))]], (-1)^(1/3)]*Root[1 + #1^2 + #1^5 & , 4, 0]^3)/(Sqrt[1 + x^3
]*((-1)^(1/3) - Root[1 + #1^2 + #1^5 & , 4, 0])*(-Root[1 + #1^2 + #1^5 & ,
1, 0] + Root[1 + #1^2 + #1^5 & , 4, 0])*(-Root[1 + #1^2 + #1^5 & , 2, 0] +
Root[1 + #1^2 + #1^5 & , 4, 0])*(-Root[1 + #1^2 + #1^5 & , 3, 0] + Root[1 +
 #1^2 + #1^5 & , 4, 0])*(Root[1 + #1^2 + #1^5 & , 4, 0] - Root[1 + #1^2 + #
1^5 & , 5, 0])) - ((4*I)*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*E
llipticPi[(I*Sqrt[3])/((-1)^(1/3) - Root[1 + #1^2 + #1^5 & , 5, 0]), ArcSin
[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(Sqrt[1 + x^3]*((
-1)^(1/3) - Root[1 + #1^2 + #1^5 & , 5, 0])*(-Root[1 + #1^2 + #1^5 & , 1, 0
] + Root[1 + #1^2 + #1^5 & , 5, 0])*(-Root[1 + #1^2 + #1^5 & , 2, 0] + Root
[1 + #1^2 + #1^5 & , 5, 0])*(-Root[1 + #1^2 + #1^5 & , 3, 0] + Root[1 + #1^
2 + #1^5 & , 5, 0])*(-Root[1 + #1^2 + #1^5 & , 4, 0] + Root[1 + #1^2 + #1^5
 & , 5, 0])) - ((10*I)*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*Ell
ipticPi[(I*Sqrt[3])/((-1)^(1/3) - Root[1 + #1^2 + #1^5 & , 5, 0]), ArcSin[S
qrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]*Root[1 + #1^2 + #1^5
 & , 5, 0]^3)/(Sqrt[1 + x^3]*((-1)^(1/3) - Root[1 + #1^2 + #1^5 & , 5, 0])*
(-Root[1 + #1^2 + #1^5 & , 1, 0] + Root[1 + #1^2 + #1^5 & , 5, 0])*(-Root[1
 + #1^2 + #1^5 & , 2, 0] + Root[1 + #1^2 + #1^5 & , 5, 0])*(-Root[1 + #1^2
+ #1^5 & , 3, 0] + Root[1 + #1^2 + #1^5 & , 5, 0])*(-Root[1 + #1^2 + #1^5 &
 , 4, 0] + Root[1 + #1^2 + #1^5 & , 5, 0]))
```

IntegrateAlgebraic [A] time = 1.82, size = 14, normalized size = 1.00

$$2\tan^{-1}\left(x\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 5*x^3)/(Sqrt[1 + x^3]*(1 + x^2 + x^5)),x]
```

```
[Out] 2*ArcTan[x*Sqrt[1 + x^3]]
```

fricas [B] time = 0.51, size = 25, normalized size = 1.79

$$\arctan\left(\frac{\left(x^5+x^2-1\right)\sqrt{x^3+1}}{2\left(x^4+x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3+2)/(x^3+1)^(1/2)/(x^5+x^2+1),x, algorithm="fricas")
[Out] arctan(1/2*(x^5 + x^2 - 1)*sqrt(x^3 + 1)/(x^4 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,x^3+2}{\left(x^5+x^2+1\right)\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3+2)/(x^3+1)^(1/2)/(x^5+x^2+1),x, algorithm="giac")
[Out] integrate((5*x^3 + 2)/((x^5 + x^2 + 1)*sqrt(x^3 + 1)), x)
```

maple [C] time = 0.16, size = 197, normalized size = 14.07

$$-\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^5+\_Z^2+1\right)}\frac{\_\alpha\left(\_\alpha^3+1\right)\left(\_\alpha^4-\_\alpha^3+\_\alpha^2\right)\left(3-i\sqrt{3}\right)\sqrt{\frac{1+x}{3-i\sqrt{3}}}\,\sqrt{\frac{-1+2x-i\sqrt{3}}{-3-i\sqrt{3}}}\,\sqrt{\frac{-1+2x+i\sqrt{3}}{-3+i\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{3\_\alpha^4}{2}+\frac{3\_\alpha^3}{2}-\frac{3\_\alpha^2}{2}+\frac{i\_\alpha^4\sqrt{3}}{2}-\frac{i\_\alpha^3\sqrt{3}}{2}+\frac{i\_\alpha^2\sqrt{3}}{2},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^3+2)/(x^3+1)^(1/2)/(x^5+x^2+1),x)
[Out] -2^(1/2)*sum(_alpha*(_alpha^3+1)*(_alpha^4-_alpha^3+_alpha^2)*(3-I*3^(1/2))
*((1+x)/(3-I*3^(1/2)))^(1/2)*((-1+2*x-I*3^(1/2))/(-3-I*3^(1/2)))^(1/2)*((-1
+2*x+I*3^(1/2))/(-3+I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-
1/2*I*3^(1/2)))^(1/2),-3/2*_alpha^4+3/2*_alpha^3-3/2*_alpha^2+1/2*I*_alpha^
4*3^(1/2)-1/2*I*_alpha^3*3^(1/2)+1/2*I*_alpha^2*3^(1/2),((-3/2+1/2*I*3^(1/2
))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^5+_Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,x^3+2}{\left(x^5+x^2+1\right)\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3+2)/(x^3+1)^(1/2)/(x^5+x^2+1),x, algorithm="maxima")
[Out] integrate((5*x^3 + 2)/((x^5 + x^2 + 1)*sqrt(x^3 + 1)), x)
```

mupad [B] time = 1.53, size = 163, normalized size = 11.64

$$\sum_{k=1}^{5}\left(-\frac{\sqrt{6}\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\left(-3+\sqrt{3}\,1\mathrm{i}\right)\,\left(x+1\right)}\;\Pi\left(\frac{3+\sqrt{3}\,1\mathrm{i}}{2\left(\mathrm{root}\left(z^5+z^2+1,z,k\right)+1\right)};\mathrm{asin}\left(\frac{\sqrt{6}\,\sqrt{-\left(-3+\sqrt{3}\,1\mathrm{i}\right)\left(x+1\right)}}{6}\right)\middle|\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{3-3\,x+\sqrt{3}\,x\,1\mathrm{i}+\sqrt{3}\,1\mathrm{i}}\,\sqrt{3-3\,x-\sqrt{3}\,x\,1\mathrm{i}-\sqrt{3}\,1\mathrm{i}}}{18\,\sqrt{x^3+1}\,\left(\mathrm{root}\left(z^5+z^2+1,z,k\right)+1\right)\,\mathrm{root}\left(z^5+z^2+1,z,k\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^3 + 2)/((x^3 + 1)^(1/2)*(x^2 + x^5 + 1)),x)
```

```
[Out] symsum(-(6^(1/2)*((3^(1/2)*1i)/2 + 3/2)*(-(3^(1/2)*1i - 3)*(x + 1))^(1/2)*e
llipticPi((3^(1/2)*1i + 3)/(2*(root(z^5 + z^2 + 1, z, k) + 1)), asin((6^(1/
2)*(-(3^(1/2)*1i - 3)*(x + 1))^(1/2))/6), (3^(1/2)*1i)/2 + 1/2)*(3^(1/2)*x*
1i - 3*x + 3^(1/2)*1i + 3)^(1/2)*(3 - 3^(1/2)*x*1i - 3^(1/2)*1i - 3*x)^(1/2
))/(18*(x^3 + 1)^(1/2)*(root(z^5 + z^2 + 1, z, k) + 1)*root(z^5 + z^2 + 1,
z, k)), k, 1, 5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^3 + 2}{\sqrt{(x + 1)\left(x^2 - x + 1\right)}\left(x^5 + x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**3+2)/(x**3+1)**(1/2)/(x**5+x**2+1),x)
```

```
[Out] Integral((5*x**3 + 2)/(sqrt((x + 1)*(x**2 - x + 1))*(x**5 + x**2 + 1)), x)
```

3.59 $\int \frac{1}{x\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=14

$$\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 63, 203}

$$\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-1 + x^6]),x]

[Out] ArcTan[Sqrt[-1 + x^6]]/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{-1+x^6}}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^6}\right)\\
&= \frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-1 + x^6]),x]

[Out] ArcTan[Sqrt[-1 + x^6]]/3
```

IntegrateAlgebraic [A] time = 0.02, size = 14, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[-1 + x^6]),x]
[Out] ArcTan[Sqrt[-1 + x^6]]/3
```

fricas [A] time = 0.41, size = 10, normalized size = 0.71

$$\frac{1}{3}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-1)^(1/2),x, algorithm="fricas")
[Out] 1/3*arctan(sqrt(x^6 - 1))
```

giac [A] time = 0.46, size = 10, normalized size = 0.71

$$\frac{1}{3}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-1)^(1/2),x, algorithm="giac")
[Out] 1/3*arctan(sqrt(x^6 - 1))
```

maple [A] time = 0.01, size = 7, normalized size = 0.50

$$-\frac{\arcsin\left(\frac{1}{x^3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6-1)^(1/2),x)
[Out] -1/3*arcsin(1/x^3)
```

maxima [A] time = 0.60, size = 10, normalized size = 0.71

$$\frac{1}{3}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-1)^(1/2),x, algorithm="maxima")
[Out] 1/3*arctan(sqrt(x^6 - 1))
```

mupad [B] time = 0.18, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\sqrt{x^6-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^6 - 1)^(1/2)),x)
```

```
[Out] atan((x^6 - 1)^(1/2))/3
```

sympy [A] time = 0.82, size = 24, normalized size = 1.71

$$\begin{cases} \frac{i\operatorname{acosh}\left(\frac{1}{x^3}\right)}{3} & \text{for } \frac{1}{|x^6|} > 1 \\ -\frac{\operatorname{asin}\left(\frac{1}{x^3}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6-1)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(x**(-3))/3, 1/Abs(x**6) > 1), (-asin(x**(-3))/3, True))
```

3.60 $\int \frac{-2+x^6}{x^2\left(1+x^6\right)^{3/4}} \, dx$

Optimal. Leaf size=14

$$\frac{2\sqrt[4]{x^6+1}}{x}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$\frac{2\sqrt[4]{x^6+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^6)/(x^2*(1 + x^6)^(3/4)),x]
```

```
[Out] (2*(1 + x^6)^(1/4))/x
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-2+x^6}{x^2\left(1+x^6\right)^{3/4}} \, dx = \frac{2\sqrt[4]{1+x^6}}{x}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{2\sqrt[4]{x^6+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^6)/(x^2*(1 + x^6)^(3/4)),x]
```

```
[Out] (2*(1 + x^6)^(1/4))/x
```

IntegrateAlgebraic [A] time = 0.32, size = 14, normalized size = 1.00

$$\frac{2\sqrt[4]{x^6+1}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x^6)/(x^2*(1 + x^6)^(3/4)),x]
```

```
[Out] (2*(1 + x^6)^(1/4))/x
```

fricas [A] time = 0.39, size = 12, normalized size = 0.86

$$\frac{2\left(x^6+1\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)/x^2/(x^6+1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*(x^6 + 1)^(1/4)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 - 2}{\left(x^6 + 1\right)^{\frac{3}{4}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)/x^2/(x^6+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((x^6 - 2)/((x^6 + 1)^(3/4)*x^2), x)
```

maple [B] time = 0.01, size = 28, normalized size = 2.00

$$\frac{2\left(x^2 + 1\right)\left(x^4 - x^2 + 1\right)}{x\left(x^6 + 1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-2)/x^2/(x^6+1)^(3/4),x)
```

```
[Out] 2/x*(x^2+1)*(x^4-x^2+1)/(x^6+1)^(3/4)
```

maxima [A] time = 0.78, size = 24, normalized size = 1.71

$$\frac{2\left(x^4 - x^2 + 1\right)^{\frac{1}{4}}\left(x^2 + 1\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)/x^2/(x^6+1)^(3/4),x, algorithm="maxima")
```

```
[Out] 2*(x^4 - x^2 + 1)^(1/4)*(x^2 + 1)^(1/4)/x
```

mupad [B] time = 0.11, size = 12, normalized size = 0.86

$$\frac{2\left(x^6 + 1\right)^{1/4}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 2)/(x^2*(x^6 + 1)^(3/4)),x)
```

```
[Out] (2*(x^6 + 1)^(1/4))/x
```

sympy [C] time = 1.78, size = 61, normalized size = 4.36

$$\frac{x^5 \Gamma\left(\frac{5}{6}\right) {}_2F_1\left(\left.\begin{matrix} \frac{3}{4}, \frac{5}{6} \\ \frac{11}{6} \end{matrix}\right| x^6 e^{i\pi}\right)}{6\Gamma\left(\frac{11}{6}\right)} - \frac{\Gamma\left(-\frac{1}{6}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{6}, \frac{3}{4} \\ \frac{5}{6} \end{matrix}\right| x^6 e^{i\pi}\right)}{3x\Gamma\left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-2)/x**2/(x**6+1)**(3/4),x)
```

```
[Out] x**5*gamma(5/6)*hyper((3/4, 5/6), (11/6,), x**6*exp_polar(I*pi))/(6*gamma(1
1/6)) - gamma(-1/6)*hyper((-1/6, 3/4), (5/6,), x**6*exp_polar(I*pi))/(3*x*g
amma(5/6))
```

3.61 $\int \frac{1}{x\sqrt{1+x^6}}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{3}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 63, 207}

$$-\frac{1}{3}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 + x^6]),x]
[Out] -1/3*ArcTanh[Sqrt[1 + x^6]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{1+x^6}}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^6\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^6}\right)\\ &= -\frac{1}{3}\tanh^{-1}\left(\sqrt{1+x^6}\right) \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{3}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[1 + x^6]),x]
[Out] -1/3*ArcTanh[Sqrt[1 + x^6]]
```

IntegrateAlgebraic [**A**] time = 0.02, size = 14, normalized size = 1.00

$$-\frac{1}{3}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[1 + x^6]),x]
[Out] -1/3*ArcTanh[Sqrt[1 + x^6]]
```

fricas [**B**] time = 0.39, size = 25, normalized size = 1.79

$$-\frac{1}{6}\log\left(\sqrt{x^6+1}+1\right)+\frac{1}{6}\log\left(\sqrt{x^6+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+1)^(1/2),x, algorithm="fricas")
[Out] -1/6*log(sqrt(x^6 + 1) + 1) + 1/6*log(sqrt(x^6 + 1) - 1)
```

giac [**B**] time = 0.31, size = 25, normalized size = 1.79

$$-\frac{1}{6}\log\left(\sqrt{x^6+1}+1\right)+\frac{1}{6}\log\left(\sqrt{x^6+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+1)^(1/2),x, algorithm="giac")
[Out] -1/6*log(sqrt(x^6 + 1) + 1) + 1/6*log(sqrt(x^6 + 1) - 1)
```

maple [**A**] time = 0.01, size = 19, normalized size = 1.36

$$\frac{\ln\left(\frac{\sqrt{x^6+1}-1}{\sqrt{x^6}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6+1)^(1/2),x)
[Out] 1/3*ln(((x^6+1)^(1/2)-1)/(x^6)^(1/2))
```

maxima [**B**] time = 0.51, size = 25, normalized size = 1.79

$$-\frac{1}{6}\log\left(\sqrt{x^6+1}+1\right)+\frac{1}{6}\log\left(\sqrt{x^6+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+1)^(1/2),x, algorithm="maxima")
[Out] -1/6*log(sqrt(x^6 + 1) + 1) + 1/6*log(sqrt(x^6 + 1) - 1)
```

mupad [**B**] time = 0.12, size = 10, normalized size = 0.71

$$-\frac{\mathrm{atanh}\left(\sqrt{x^6+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^6 + 1)^(1/2)),x)
```

```
[Out] -atanh((x^6 + 1)^(1/2))/3
```

sympy [A] time = 0.78, size = 8, normalized size = 0.57

$$-\frac{\operatorname{asinh}\left(\frac{1}{x^3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6+1)**(1/2),x)
```

```
[Out] -asinh(x**(-3))/3
```

3.62 $\int \frac{2+x^6}{x^2\left(-1+x^6\right)^{3/4}}\,dx$

Optimal. Leaf size=14

$$\frac{2\sqrt[4]{x^6-1}}{x}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$\frac{2\sqrt[4]{x^6-1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^6)/(x^2*(-1 + x^6)^(3/4)),x]
```

```
[Out] (2*(-1 + x^6)^(1/4))/x
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{2+x^6}{x^2\left(-1+x^6\right)^{3/4}}\,dx = \frac{2\sqrt[4]{-1+x^6}}{x}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{2\sqrt[4]{x^6-1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^6)/(x^2*(-1 + x^6)^(3/4)),x]
```

```
[Out] (2*(-1 + x^6)^(1/4))/x
```

IntegrateAlgebraic [A] time = 0.33, size = 14, normalized size = 1.00

$$\frac{2\sqrt[4]{x^6-1}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x^6)/(x^2*(-1 + x^6)^(3/4)),x]
```

```
[Out] (2*(-1 + x^6)^(1/4))/x
```

fricas [A] time = 0.40, size = 12, normalized size = 0.86

$$\frac{2\left(x^6-1\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)/x^2/(x^6-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*(x^6 - 1)^(1/4)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+2}{\left(x^6-1\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)/x^2/(x^6-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 2)/((x^6 - 1)^(3/4)*x^2), x)
```

maple [B] time = 0.01, size = 33, normalized size = 2.36

$$\frac{2\left(-1+x\right)\left(1+x\right)\left(x^2+x+1\right)\left(x^2-x+1\right)}{x\left(x^6-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+2)/x^2/(x^6-1)^(3/4),x)
```

```
[Out] 2/x*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)/(x^6-1)^(3/4)
```

maxima [B] time = 0.50, size = 33, normalized size = 2.36

$$\frac{2\left(x^2+x+1\right)^{\frac{1}{4}}\left(x^2-x+1\right)^{\frac{1}{4}}\left(x+1\right)^{\frac{1}{4}}\left(x-1\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)/x^2/(x^6-1)^(3/4),x, algorithm="maxima")
```

```
[Out] 2*(x^2 + x + 1)^(1/4)*(x^2 - x + 1)^(1/4)*(x + 1)^(1/4)*(x - 1)^(1/4)/x
```

mupad [B] time = 0.14, size = 12, normalized size = 0.86

$$\frac{2\left(x^6-1\right)^{1/4}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 2)/(x^2*(x^6 - 1)^(3/4)),x)
```

```
[Out] (2*(x^6 - 1)^(1/4))/x
```

sympy [C] time = 1.85, size = 66, normalized size = 4.71

$$\frac{x^5e^{-\frac{3i\pi}{4}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{6}\\\frac{11}{6}\end{matrix}\right|x^6\right)}{6\Gamma\left(\frac{11}{6}\right)}-\frac{e^{\frac{i\pi}{4}}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{6},\frac{3}{4}\\\frac{5}{6}\end{matrix}\right|x^6\right)}{3x\Gamma\left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+2)/x**2/(x**6-1)**(3/4),x)
```

```
[Out] x**5*exp(-3*I*pi/4)*gamma(5/6)*hyper((3/4, 5/6), (11/6,), x**6)/(6*gamma(11
/6)) - exp(I*pi/4)*gamma(-1/6)*hyper((-1/6, 3/4), (5/6,), x**6)/(3*x*gamma(
5/6))
```

3.63 $\int \frac{-1+2x^6}{\sqrt{1+x^6}\left(1-x^2+x^6\right)}\,dx$

Optimal. Leaf size=14

$$-\tanh^{-1}\left(\frac{x}{\sqrt{x^6+1}}\right)$$

Rubi [F] time = 0.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+2x^6}{\sqrt{1+x^6}\left(1-x^2+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 2*x^6)/(Sqrt[1 + x^6]*(1 - x^2 + x^6)),x]
```

```
[Out] (x*(1 + x^2)*Sqrt[(1 - x^2 + x^4)/(1 + (1 + Sqrt[3])*x^2)^2]*EllipticF[ArcC
os[(1 + (1 - Sqrt[3])*x^2)/(1 + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(3^(
1/4)*Sqrt[(x^2*(1 + x^2))/(1 + (1 + Sqrt[3])*x^2)^2]*Sqrt[1 + x^6]) - 3*Def
er[Int][1/(Sqrt[1 + x^6]*(1 - x^2 + x^6)), x] + 2*Defer[Int][x^2/(Sqrt[1 +
x^6]*(1 - x^2 + x^6)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+2x^6}{\sqrt{1+x^6}\left(1-x^2+x^6\right)}\,dx &= \int \left(\frac{2}{\sqrt{1+x^6}}-\frac{3-2x^2}{\sqrt{1+x^6}\left(1-x^2+x^6\right)}\right)dx\\
&= 2\int \frac{1}{\sqrt{1+x^6}}\,dx-\int \frac{3-2x^2}{\sqrt{1+x^6}\left(1-x^2+x^6\right)}\,dx\\
&= \frac{x\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x^2}{1+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x^2\left(1+x^2\right)}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1+x^6}}-\int \left(\frac{3}{\sqrt{1+x^6}\left(1-\right.}\right.\\
&= \frac{x\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x^2}{1+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x^2\left(1+x^2\right)}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1+x^6}}+2\int \frac{x^2}{\sqrt{1+x^6}\left(1-\right.}
\end{aligned}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{-1+2x^6}{\sqrt{1+x^6}\left(1-x^2+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + 2*x^6)/(Sqrt[1 + x^6]*(1 - x^2 + x^6)),x]
```

```
[Out] Integrate[(-1 + 2*x^6)/(Sqrt[1 + x^6]*(1 - x^2 + x^6)), x]
```

IntegrateAlgebraic [A] time = 3.71, size = 14, normalized size = 1.00

$$-\tanh^{-1}\left(\frac{x}{\sqrt{x^6+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x^6)/(Sqrt[1 + x^6]*(1 - x^2 + x^6)),x]
[Out] -ArcTanh[x/Sqrt[1 + x^6]]
```

fricas [B] time = 0.49, size = 34, normalized size = 2.43

$$\frac{1}{2}\log\left(\frac{x^6+x^2-2\sqrt{x^6+1}\,x+1}{x^6-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6-1)/(x^6+1)^(1/2)/(x^6-x^2+1),x, algorithm="fricas")
[Out] 1/2*log((x^6 + x^2 - 2*sqrt(x^6 + 1)*x + 1)/(x^6 - x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^6-1}{\left(x^6-x^2+1\right)\sqrt{x^6+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6-1)/(x^6+1)^(1/2)/(x^6-x^2+1),x, algorithm="giac")
[Out] integrate((2*x^6 - 1)/((x^6 - x^2 + 1)*sqrt(x^6 + 1)), x)
```

maple [B] time = 0.40, size = 36, normalized size = 2.57

$$-\frac{\ln\left(-\frac{x^6+2\sqrt{x^6+1}\,x+x^2+1}{x^6-x^2+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6-1)/(x^6+1)^(1/2)/(x^6-x^2+1),x)
[Out] -1/2*ln(-(x^6+2*(x^6+1)^(1/2)*x+x^2+1)/(x^6-x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^6-1}{\left(x^6-x^2+1\right)\sqrt{x^6+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6-1)/(x^6+1)^(1/2)/(x^6-x^2+1),x, algorithm="maxima")
[Out] integrate((2*x^6 - 1)/((x^6 - x^2 + 1)*sqrt(x^6 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{2\,x^6-1}{\sqrt{x^6+1}\,\left(x^6-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6 - 1)/((x^6 + 1)^(1/2)*(x^6 - x^2 + 1)),x)
[Out] int((2*x^6 - 1)/((x^6 + 1)^(1/2)*(x^6 - x^2 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**6-1)/(x**6+1)**(1/2)/(x**6-x**2+1),x)
```

```
[Out] Timed out
```

3.64 $\int \frac{1+2x^6}{\sqrt{-1+x^6}\left(-1+x^2+x^6\right)}\,dx$

Optimal. Leaf size=14

$$-\tan^{-1}\left(\frac{x}{\sqrt{x^6-1}}\right)$$

Rubi [F] time = 0.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+2x^6}{\sqrt{-1+x^6}\left(-1+x^2+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + 2*x^6)/(Sqrt[-1 + x^6]*(-1 + x^2 + x^6)),x]
```

```
[Out] (x*(1 - x^2)*Sqrt[(1 + x^2 + x^4)/(1 - (1 + Sqrt[3])*x^2)^2]*EllipticF[ArcC
os[(1 - (1 - Sqrt[3])*x^2)/(1 - (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(3^(
1/4)*Sqrt[-((x^2*(1 - x^2))/(1 - (1 + Sqrt[3])*x^2)^2)]*Sqrt[-1 + x^6]) + 3
*Defer[Int][1/(Sqrt[-1 + x^6]*(-1 + x^2 + x^6)), x] - 2*Defer[Int][x^2/(Sqr
t[-1 + x^6]*(-1 + x^2 + x^6)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^6}{\sqrt{-1+x^6}\left(-1+x^2+x^6\right)}\,dx &= \int \left(\frac{2}{\sqrt{-1+x^6}}+\frac{3-2x^2}{\sqrt{-1+x^6}\left(-1+x^2+x^6\right)}\right)dx\\
&= 2\int \frac{1}{\sqrt{-1+x^6}}\,dx+\int \frac{3-2x^2}{\sqrt{-1+x^6}\left(-1+x^2+x^6\right)}\,dx\\
&= \frac{x\left(1-x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x^2}{1-\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{-\frac{x^2\left(1-x^2\right)}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{-1+x^6}}+\int \Bigg(\frac{}{\sqrt{-1+}}\\
&= \frac{x\left(1-x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x^2}{1-\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{-\frac{x^2\left(1-x^2\right)}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{-1+x^6}}-2\int \frac{}{\sqrt{-1+}}
\end{aligned}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1+2x^6}{\sqrt{-1+x^6}\left(-1+x^2+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + 2*x^6)/(Sqrt[-1 + x^6]*(-1 + x^2 + x^6)),x]
```

```
[Out] Integrate[(1 + 2*x^6)/(Sqrt[-1 + x^6]*(-1 + x^2 + x^6)), x]
```

IntegrateAlgebraic [A] time = 3.84, size = 14, normalized size = 1.00

$$-\tan^{-1}\left(\frac{x}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x^6)/(Sqrt[-1 + x^6]*(-1 + x^2 + x^6)),x]
[Out] -ArcTan[x/Sqrt[-1 + x^6]]
```

fricas [B] time = 0.58, size = 25, normalized size = 1.79

$$-\frac{1}{2}\arctan\left(\frac{2\sqrt{x^6-1}\,x}{x^6-x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+1)/(x^6-1)^(1/2)/(x^6+x^2-1),x, algorithm="fricas")
[Out] -1/2*arctan(2*sqrt(x^6 - 1)*x/(x^6 - x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^6+1}{\left(x^6+x^2-1\right)\sqrt{x^6-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+1)/(x^6-1)^(1/2)/(x^6+x^2-1),x, algorithm="giac")
[Out] integrate((2*x^6 + 1)/((x^6 + x^2 - 1)*sqrt(x^6 - 1)), x)
```

maple [C] time = 0.49, size = 62, normalized size = 4.43

$$-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x^6-\operatorname{RootOf}\left(\_Z^2+1\right)x^2+2\sqrt{x^6-1}\,x-\operatorname{RootOf}\left(\_Z^2+1\right)}{x^6+x^2-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6+1)/(x^6-1)^(1/2)/(x^6+x^2-1),x)
[Out] -1/2*RootOf(_Z^2+1)*ln(-(RootOf(_Z^2+1)*x^6-RootOf(_Z^2+1)*x^2+2*(x^6-1)^(1
/2)*x-RootOf(_Z^2+1))/(x^6+x^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^6+1}{\left(x^6+x^2-1\right)\sqrt{x^6-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+1)/(x^6-1)^(1/2)/(x^6+x^2-1),x, algorithm="maxima")
[Out] integrate((2*x^6 + 1)/((x^6 + x^2 - 1)*sqrt(x^6 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{2\,x^6+1}{\sqrt{x^6-1}\,\left(x^6+x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6 + 1)/((x^6 - 1)^(1/2)*(x^2 + x^6 - 1)),x)
[Out] int((2*x^6 + 1)/((x^6 - 1)^(1/2)*(x^2 + x^6 - 1)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**6+1)/(x**6-1)**(1/2)/(x**6+x**2-1),x)
[Out] Timed out
```

3.65 $\int \frac{-1+3x^2}{\sqrt[3]{-x+x^3}}\,dx$

Optimal. Leaf size=15

$$\frac{3}{2}\left(x^3-x\right)^{2/3}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$\frac{3}{2}\left(x^3-x\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x^2)/(-x + x^3)^(1/3),x]
```

```
[Out] (3*(-x + x^3)^(2/3))/2
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+3x^2}{\sqrt[3]{-x+x^3}}\,dx=\frac{3}{2}\left(-x+x^3\right)^{2/3}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{3}{2}\left(x\left(x^2-1\right)\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x^2)/(-x + x^3)^(1/3),x]
```

```
[Out] (3*(x*(-1 + x^2))^(2/3))/2
```

IntegrateAlgebraic [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{3}{2}\left(x^3-x\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 3*x^2)/(-x + x^3)^(1/3),x]
```

```
[Out] (3*(-x + x^3)^(2/3))/2
```

fricas [A] time = 0.39, size = 11, normalized size = 0.73

$$\frac{3}{2}\left(x^3-x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-1)/(x^3-x)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*(x^3 - x)^(2/3)
```

giac [A] time = 0.29, size = 11, normalized size = 0.73

$$\frac{3}{2}\left(x^{3}-x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-1)/(x^3-x)^(1/3),x, algorithm="giac")
```

```
[Out] 3/2*(x^3 - x)^(2/3)
```

maple [A] time = 0.00, size = 19, normalized size = 1.27

$$\frac{3x\left(-1+x\right)\left(1+x\right)}{2\left(x^{3}-x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-1)/(x^3-x)^(1/3),x)
```

```
[Out] 3/2*x*(-1+x)*(1+x)/(x^3-x)^(1/3)
```

maxima [A] time = 0.48, size = 11, normalized size = 0.73

$$\frac{3}{2}\left(x^{3}-x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-1)/(x^3-x)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/2*(x^3 - x)^(2/3)
```

mupad [B] time = 0.29, size = 11, normalized size = 0.73

$$\frac{3\left(x^{3}-x\right)^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 1)/(x^3 - x)^(1/3),x)
```

```
[Out] (3*(x^3 - x)^(2/3))/2
```

sympy [A] time = 0.14, size = 10, normalized size = 0.67

$$\frac{3\left(x^{3}-x\right)^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-1)/(x**3-x)**(1/3),x)
```

```
[Out] 3*(x**3 - x)**(2/3)/2
```

3.66 $\int \left(-1+3x^2\right)\sqrt[3]{-x+x^3}\,dx$

Optimal. Leaf size=15

$$\frac{3}{4}\left(x^3-x\right)^{4/3}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$\frac{3}{4}\left(x^3-x\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x^2)*(-x + x^3)^(1/3),x]
```

```
[Out] (3*(-x + x^3)^(4/3))/4
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(-1+3x^2\right)\sqrt[3]{-x+x^3}\,dx=\frac{3}{4}\left(-x+x^3\right)^{4/3}$$

Mathematica [A] time = 0.03, size = 15, normalized size = 1.00

$$\frac{3}{4}\left(x\left(x^2-1\right)\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x^2)*(-x + x^3)^(1/3),x]
```

```
[Out] (3*(x*(-1 + x^2))^(4/3))/4
```

IntegrateAlgebraic [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{3}{4}\left(x^3-x\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 3*x^2)*(-x + x^3)^(1/3),x]
```

```
[Out] (3*(-x + x^3)^(4/3))/4
```

fricas [A] time = 0.38, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^3-x\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-1)*(x^3-x)^(1/3),x, algorithm="fricas")
[Out] 3/4*(x^3 - x)^(4/3)
```

giac [A] time = 0.31, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^3 - x\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-1)*(x^3-x)^(1/3),x, algorithm="giac")
[Out] 3/4*(x^3 - x)^(4/3)
```

maple [A] time = 0.00, size = 19, normalized size = 1.27

$$\frac{3x\left(-1+x\right)\left(1+x\right)\left(x^3 - x\right)^{\frac{1}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-1)*(x^3-x)^(1/3),x)
[Out] 3/4*x*(-1+x)*(1+x)*(x^3-x)^(1/3)
```

maxima [A] time = 0.36, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^3 - x\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-1)*(x^3-x)^(1/3),x, algorithm="maxima")
[Out] 3/4*(x^3 - x)^(4/3)
```

mupad [B] time = 0.11, size = 11, normalized size = 0.73

$$\frac{3\left(x^3 - x\right)^{4/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - x)^(1/3)*(3*x^2 - 1),x)
[Out] (3*(x^3 - x)^(4/3))/4
```

sympy [B] time = 0.18, size = 27, normalized size = 1.80

$$\frac{3x^3\sqrt[3]{x^3 - x}}{4} - \frac{3x\sqrt[3]{x^3 - x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-1)*(x**3-x)**(1/3),x)
[Out] 3*x**3*(x**3 - x)**(1/3)/4 - 3*x*(x**3 - x)**(1/3)/4
```

3.67 $\int \frac{2+x-x^3}{\sqrt{1+x+x^3}\left(1+x-x^2+x^3\right)} \, dx$

Optimal. Leaf size=15

$$2 \tanh^{-1}\left(\frac{x}{\sqrt{x^3+x+1}}\right)$$

Rubi [F] time = 2.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2+x-x^3}{\sqrt{1+x+x^3}\left(1+x-x^2+x^3\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(2 + x - x^3)/(Sqrt[1 + x + x^3]*(1 + x - x^2 + x^3)),x]
```

```
[Out] (((-2*I)/3)*Sqrt[(2*(3/(-9 + Sqrt[93]))^(1/3) - (2*(-9 + Sqrt[93]))^(1/3) +
 6^(2/3)*x)/(6*(3/(-9 + Sqrt[93]))^(1/3) - 3*(2*(-9 + Sqrt[93]))^(1/3) - I*
6^(1/6)*Sqrt[12 + 6*3^(1/3)*(2/(-9 + Sqrt[93]))^(2/3) + 2^(1/3)*(3*(-9 + Sq
rt[93]))^(2/3)])]*Sqrt[6 + 6*3^(1/3)*(2/(-9 + Sqrt[93]))^(2/3) + 2^(1/3)*(3
*(-9 + Sqrt[93]))^(2/3) - 6*3^(1/3)*((6/(-9 + Sqrt[93]))^(1/3) - ((-9 + Sqr
t[93])/2)^(1/3))*x + 18*x^2]*EllipticF[ArcSin[Sqrt[I*(6^(1/3)*(2*(3/(-9 + S
qrt[93]))^(1/3) - (2*(-9 + Sqrt[93]))^(1/3) - I*6^(1/6)*Sqrt[12 + 6*3^(1/3)
*(2/(-9 + Sqrt[93]))^(2/3) + 2^(1/3)*(3*(-9 + Sqrt[93]))^(2/3)]) - 12*x)]/(
2^(3/4)*(3*(12 + 6*3^(1/3)*(2/(-9 + Sqrt[93]))^(2/3) + 2^(1/3)*(3*(-9 + Sqr
t[93]))^(2/3)))^(1/4))], (2*6^(1/6)*Sqrt[12 + 6*3^(1/3)*(2/(-9 + Sqrt[93]))
^(2/3) + 2^(1/3)*(3*(-9 + Sqrt[93]))^(2/3)])/(I*(6*(3/(-9 + Sqrt[93]))^(1/3
) - 3*(2*(-9 + Sqrt[93]))^(1/3)) + 6^(1/6)*Sqrt[12 + 6*3^(1/3)*(2/(-9 + Sqr
t[93]))^(2/3) + 2^(1/3)*(3*(-9 + Sqrt[93]))^(2/3)])])/Sqrt[1 + x + x^3] + 3
*Defer[Int][1/(Sqrt[1 + x + x^3]*(1 + x - x^2 + x^3)), x] + 2*Defer[Int][x/
(Sqrt[1 + x + x^3]*(1 + x - x^2 + x^3)), x] - Defer[Int][x^2/(Sqrt[1 + x +
x^3]*(1 + x - x^2 + x^3)), x]
```

Rubi steps

$$\int \frac{2+x-x^3}{\sqrt{1+x+x^3}\left(1+x-x^2+x^3\right)}dx = \int \left(-\frac{1}{\sqrt{1+x+x^3}}+\frac{3+2x-x^2}{\sqrt{1+x+x^3}\left(1+x-x^2+x^3\right)}\right)dx$$

$$= -\int \frac{1}{\sqrt{1+x+x^3}}dx + \int \frac{3+2x-x^2}{\sqrt{1+x+x^3}\left(1+x-x^2+x^3\right)}dx$$

$$= -\frac{\left(\sqrt{\frac{2\sqrt[3]{\frac{3}{-9+\sqrt{93}}}-\sqrt[3]{2\left(-9+\sqrt{93}\right)}}{6^{2/3}}+x}\sqrt{\frac{1}{18}\left(6+6\sqrt[3]{3}\left(\frac{2}{-9+\sqrt{93}}\right)^{2/3}+\sqrt[3]{2}\left(3\left(\cdots\right.\right.\right.}}{\cdots}$$

$$= 2\int \frac{x}{\sqrt{1+x+x^3}\left(1+x-x^2+x^3\right)}dx + 3\int \frac{1}{\sqrt{1+x+x^3}\left(1+x-x^2\cdots\right)}$$

$$= -\frac{2i\sqrt{\frac{2\sqrt[3]{\frac{3}{-9+\sqrt{93}}}-\sqrt[3]{2\left(-9+\sqrt{93}\right)}+6^{2/3}x}{6\sqrt[3]{\frac{3}{-9+\sqrt{93}}}-3\sqrt[3]{2\left(-9+\sqrt{93}\right)}-i\sqrt[6]{6}\sqrt{12+6\sqrt[3]{3}\left(\frac{2}{-9+\sqrt{93}}\right)^{2/3}+\sqrt[3]{2}\left(3\left(-9+\sqrt{93}\right)\right)^{2/3}}}}\cdots}{\cdots}$$

Mathematica [C] time = 3.52, size = 3866, normalized size = 257.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + x - x^3)/(Sqrt[1 + x + x^3]*(1 + x - x^2 + x^3)),x]
```

```
[Out] (2*Sqrt[(-x + Root[1 + #1 + #1^3 & , 1, 0])/(Root[1 + #1 + #1^3 & , 1, 0] -
 Root[1 + #1 + #1^3 & , 3, 0])]*(-((EllipticF[ArcSin[Sqrt[(-x + Root[1 + #1
 + #1^3 & , 3, 0])/(-Root[1 + #1 + #1^3 & , 2, 0] + Root[1 + #1 + #1^3 & ,
3, 0])]], (Root[1 + #1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3, 0])/(Ro
ot[1 + #1 + #1^3 & , 1, 0] - Root[1 + #1 + #1^3 & , 3, 0])]*(x - Root[1 + #
1 + #1^3 & , 3, 0])*Sqrt[(-x + Root[1 + #1 + #1^3 & , 2, 0])/(Root[1 + #1 +
 #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3, 0])]])/Sqrt[(x - Root[1 + #1 + #
1^3 & , 3, 0])/(Root[1 + #1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3, 0]
)]) + (2*EllipticPi[(-Root[1 + #1 + #1^3 & , 2, 0] + Root[1 + #1 + #1^3 & ,
 3, 0])/(Root[1 + #1 + #1^3 & , 3, 0] - Root[1 + #1 - #1^2 + #1^3 & , 1, 0]
), ArcSin[Sqrt[(-x + Root[1 + #1 + #1^3 & , 3, 0])/(-Root[1 + #1 + #1^3 & ,
 2, 0] + Root[1 + #1 + #1^3 & , 3, 0])]], (Root[1 + #1 + #1^3 & , 2, 0] - R
oot[1 + #1 + #1^3 & , 3, 0])/(Root[1 + #1 + #1^3 & , 1, 0] - Root[1 + #1 +
#1^3 & , 3, 0])]*Sqrt[-(((x - Root[1 + #1 + #1^3 & , 2, 0])*(x - Root[1 + #
1 + #1^3 & , 3, 0]))/(Root[1 + #1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & ,
 3, 0])^2)]*(Root[1 + #1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3, 0]))/
((Root[1 + #1 + #1^3 & , 3, 0] - Root[1 + #1 - #1^2 + #1^3 & , 1, 0])*(Root
[1 + #1 - #1^2 + #1^3 & , 1, 0] - Root[1 + #1 - #1^2 + #1^3 & , 2, 0])*(Roo
t[1 + #1 - #1^2 + #1^3 & , 1, 0] - Root[1 + #1 - #1^2 + #1^3 & , 3, 0])) +
```

```
(EllipticPi[(-Root[1 + #1 + #1^3 & , 2, 0] + Root[1 + #1 + #1^3 & , 3, 0])/
(Root[1 + #1 + #1^3 & , 3, 0] - Root[1 + #1 - #1^2 + #1^3 & , 1, 0]), ArcSi
n[Sqrt[(-x + Root[1 + #1 + #1^3 & , 3, 0])/(-Root[1 + #1 + #1^3 & , 2, 0] +
 Root[1 + #1 + #1^3 & , 3, 0])]], (Root[1 + #1 + #1^3 & , 2, 0] - Root[1 +
#1 + #1^3 & , 3, 0])/(Root[1 + #1 + #1^3 & , 1, 0] - Root[1 + #1 + #1^3 & ,
 3, 0])]*Sqrt[-(((x - Root[1 + #1 + #1^3 & , 2, 0])*(x - Root[1 + #1 + #1^3
 & , 3, 0]))/(Root[1 + #1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3, 0])^
2)]*(Root[1 + #1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3, 0])*Root[1 +
#1 - #1^2 + #1^3 & , 1, 0])/((Root[1 + #1 + #1^3 & , 3, 0] - Root[1 + #1 -
#1^2 + #1^3 & , 1, 0])*(Root[1 + #1 - #1^2 + #1^3 & , 1, 0] - Root[1 + #1 -
 #1^2 + #1^3 & , 2, 0])*(Root[1 + #1 - #1^2 + #1^3 & , 1, 0] - Root[1 + #1
- #1^2 + #1^3 & , 3, 0])) - (EllipticPi[(-Root[1 + #1 + #1^3 & , 2, 0] + Ro
ot[1 + #1 + #1^3 & , 3, 0])/(Root[1 + #1 + #1^3 & , 3, 0] - Root[1 + #1 - #
1^2 + #1^3 & , 1, 0]), ArcSin[Sqrt[(-x + Root[1 + #1 + #1^3 & , 3, 0])/(-Ro
ot[1 + #1 + #1^3 & , 2, 0] + Root[1 + #1 + #1^3 & , 3, 0])]], (Root[1 + #1
+ #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3, 0])/(Root[1 + #1 + #1^3 & , 1,
 0] - Root[1 + #1 + #1^3 & , 3, 0])]*Sqrt[-(((x - Root[1 + #1 + #1^3 & , 2,
 0])*(x - Root[1 + #1 + #1^3 & , 3, 0]))/(Root[1 + #1 + #1^3 & , 2, 0] - Ro
ot[1 + #1 + #1^3 & , 3, 0])^2)]*(Root[1 + #1 + #1^3 & , 2, 0] - Root[1 + #1
 + #1^3 & , 3, 0])*Root[1 + #1 - #1^2 + #1^3 & , 1, 0]^3)/((Root[1 + #1 + #
1^3 & , 3, 0] - Root[1 + #1 - #1^2 + #1^3 & , 1, 0])*(Root[1 + #1 - #1^2 +
#1^3 & , 1, 0] - Root[1 + #1 - #1^2 + #1^3 & , 2, 0])*(Root[1 + #1 - #1^2 +
 #1^3 & , 1, 0] - Root[1 + #1 - #1^2 + #1^3 & , 3, 0])) + (2*EllipticPi[(-R
oot[1 + #1 + #1^3 & , 2, 0] + Root[1 + #1 + #1^3 & , 3, 0])/(Root[1 + #1 +
#1^3 & , 3, 0] - Root[1 + #1 - #1^2 + #1^3 & , 2, 0]), ArcSin[Sqrt[(-x + Ro
ot[1 + #1 + #1^3 & , 3, 0])/(-Root[1 + #1 + #1^3 & , 2, 0] + Root[1 + #1 +
#1^3 & , 3, 0])]], (Root[1 + #1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3
, 0])/(Root[1 + #1 + #1^3 & , 1, 0] - Root[1 + #1 + #1^3 & , 3, 0])]*Sqrt[-
(((x - Root[1 + #1 + #1^3 & , 2, 0])*(x - Root[1 + #1 + #1^3 & , 3, 0]))/(R
oot[1 + #1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3, 0])^2)]*(Root[1 + #
1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3, 0]))/((Root[1 + #1 + #1^3 &
, 3, 0] - Root[1 + #1 - #1^2 + #1^3 & , 2, 0])*(-Root[1 + #1 - #1^2 + #1^3
& , 1, 0] + Root[1 + #1 - #1^2 + #1^3 & , 2, 0])*(Root[1 + #1 - #1^2 + #1^3
 & , 2, 0] - Root[1 + #1 - #1^2 + #1^3 & , 3, 0])) + (EllipticPi[(-Root[1 +
 #1 + #1^3 & , 2, 0] + Root[1 + #1 + #1^3 & , 3, 0])/(Root[1 + #1 + #1^3 &
, 3, 0] - Root[1 + #1 - #1^2 + #1^3 & , 2, 0]), ArcSin[Sqrt[(-x + Root[1 +
#1 + #1^3 & , 3, 0])/(-Root[1 + #1 + #1^3 & , 2, 0] + Root[1 + #1 + #1^3 &
, 3, 0])]], (Root[1 + #1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3, 0])/(
Root[1 + #1 + #1^3 & , 1, 0] - Root[1 + #1 + #1^3 & , 3, 0])]*Sqrt[-(((x -
Root[1 + #1 + #1^3 & , 2, 0])*(x - Root[1 + #1 + #1^3 & , 3, 0]))/(Root[1 +
 #1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3, 0])^2)]*(Root[1 + #1 + #1^
3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3, 0])*Root[1 + #1 - #1^2 + #1^3 & , 2
, 0])/((Root[1 + #1 + #1^3 & , 3, 0] - Root[1 + #1 - #1^2 + #1^3 & , 2, 0])
*(-Root[1 + #1 - #1^2 + #1^3 & , 1, 0] + Root[1 + #1 - #1^2 + #1^3 & , 2, 0
])*(Root[1 + #1 - #1^2 + #1^3 & , 2, 0] - Root[1 + #1 - #1^2 + #1^3 & , 3,
0])) - (EllipticPi[(-Root[1 + #1 + #1^3 & , 2, 0] + Root[1 + #1 + #1^3 & ,
3, 0])/(Root[1 + #1 + #1^3 & , 3, 0] - Root[1 + #1 - #1^2 + #1^3 & , 2, 0])
, ArcSin[Sqrt[(-x + Root[1 + #1 + #1^3 & , 3, 0])/(-Root[1 + #1 + #1^3 & ,
2, 0] + Root[1 + #1 + #1^3 & , 3, 0])]], (Root[1 + #1 + #1^3 & , 2, 0] - Ro
ot[1 + #1 + #1^3 & , 3, 0])/(Root[1 + #1 + #1^3 & , 1, 0] - Root[1 + #1 + #
1^3 & , 3, 0])]*Sqrt[-(((x - Root[1 + #1 + #1^3 & , 2, 0])*(x - Root[1 + #1
 + #1^3 & , 3, 0]))/(Root[1 + #1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & ,
3, 0])^2)]*(Root[1 + #1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3, 0])*Ro
ot[1 + #1 - #1^2 + #1^3 & , 2, 0]^3)/((Root[1 + #1 + #1^3 & , 3, 0] - Root[
1 + #1 - #1^2 + #1^3 & , 2, 0])*(-Root[1 + #1 - #1^2 + #1^3 & , 1, 0] + Roo
t[1 + #1 - #1^2 + #1^3 & , 2, 0])*(Root[1 + #1 - #1^2 + #1^3 & , 2, 0] - Ro
ot[1 + #1 - #1^2 + #1^3 & , 3, 0])) + (2*EllipticPi[(-Root[1 + #1 + #1^3 &
, 2, 0] + Root[1 + #1 + #1^3 & , 3, 0])/(Root[1 + #1 + #1^3 & , 3, 0] - Roo
t[1 + #1 - #1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[(-x + Root[1 + #1 + #1^3 & ,
```

```
 3, 0])/(-Root[1 + #1 + #1^3 & , 2, 0] + Root[1 + #1 + #1^3 & , 3, 0])]], (
Root[1 + #1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3, 0])/(Root[1 + #1 +
 #1^3 & , 1, 0] - Root[1 + #1 + #1^3 & , 3, 0])]*Sqrt[-(((x - Root[1 + #1 +
 #1^3 & , 2, 0])*(x - Root[1 + #1 + #1^3 & , 3, 0]))/(Root[1 + #1 + #1^3 &
, 2, 0] - Root[1 + #1 + #1^3 & , 3, 0])^2)]*(Root[1 + #1 + #1^3 & , 2, 0] -
 Root[1 + #1 + #1^3 & , 3, 0]))/((Root[1 + #1 + #1^3 & , 3, 0] - Root[1 + #
1 - #1^2 + #1^3 & , 3, 0])*(-Root[1 + #1 - #1^2 + #1^3 & , 1, 0] + Root[1 +
 #1 - #1^2 + #1^3 & , 3, 0])*(-Root[1 + #1 - #1^2 + #1^3 & , 2, 0] + Root[1
 + #1 - #1^2 + #1^3 & , 3, 0])) + (EllipticPi[(-Root[1 + #1 + #1^3 & , 2, 0
] + Root[1 + #1 + #1^3 & , 3, 0])/(Root[1 + #1 + #1^3 & , 3, 0] - Root[1 +
#1 - #1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[(-x + Root[1 + #1 + #1^3 & , 3, 0]
)/(-Root[1 + #1 + #1^3 & , 2, 0] + Root[1 + #1 + #1^3 & , 3, 0])]], (Root[1
 + #1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3, 0])/(Root[1 + #1 + #1^3
& , 1, 0] - Root[1 + #1 + #1^3 & , 3, 0])]*Sqrt[-(((x - Root[1 + #1 + #1^3
& , 2, 0])*(x - Root[1 + #1 + #1^3 & , 3, 0]))/(Root[1 + #1 + #1^3 & , 2, 0
] - Root[1 + #1 + #1^3 & , 3, 0])^2)]*(Root[1 + #1 + #1^3 & , 2, 0] - Root[
1 + #1 + #1^3 & , 3, 0])*Root[1 + #1 - #1^2 + #1^3 & , 3, 0])/((Root[1 + #1
 + #1^3 & , 3, 0] - Root[1 + #1 - #1^2 + #1^3 & , 3, 0])*(-Root[1 + #1 - #1
^2 + #1^3 & , 1, 0] + Root[1 + #1 - #1^2 + #1^3 & , 3, 0])*(-Root[1 + #1 -
#1^2 + #1^3 & , 2, 0] + Root[1 + #1 - #1^2 + #1^3 & , 3, 0])) - (EllipticPi
[(-Root[1 + #1 + #1^3 & , 2, 0] + Root[1 + #1 + #1^3 & , 3, 0])/(Root[1 + #
1 + #1^3 & , 3, 0] - Root[1 + #1 - #1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[(-x
+ Root[1 + #1 + #1^3 & , 3, 0])/(-Root[1 + #1 + #1^3 & , 2, 0] + Root[1 + #
1 + #1^3 & , 3, 0])]], (Root[1 + #1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 &
 , 3, 0])/(Root[1 + #1 + #1^3 & , 1, 0] - Root[1 + #1 + #1^3 & , 3, 0])]*Sq
rt[-(((x - Root[1 + #1 + #1^3 & , 2, 0])*(x - Root[1 + #1 + #1^3 & , 3, 0])
)/(Root[1 + #1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3, 0])^2)]*(Root[1
 + #1 + #1^3 & , 2, 0] - Root[1 + #1 + #1^3 & , 3, 0])*Root[1 + #1 - #1^2 +
 #1^3 & , 3, 0]^3)/((Root[1 + #1 + #1^3 & , 3, 0] - Root[1 + #1 - #1^2 + #1
^3 & , 3, 0])*(-Root[1 + #1 - #1^2 + #1^3 & , 1, 0] + Root[1 + #1 - #1^2 +
#1^3 & , 3, 0])*(-Root[1 + #1 - #1^2 + #1^3 & , 2, 0] + Root[1 + #1 - #1^2
+ #1^3 & , 3, 0]))))/Sqrt[1 + x + x^3]
```

IntegrateAlgebraic [A] time = 0.30, size = 15, normalized size = 1.00

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^3+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x - x^3)/(Sqrt[1 + x + x^3]*(1 + x - x^2 + x^3)),x]
[Out] 2*ArcTanh[x/Sqrt[1 + x + x^3]]
```

fricas [B] time = 0.42, size = 35, normalized size = 2.33

$$\log\left(\frac{x^3+x^2+2\sqrt{x^3+x+1}\,x+x+1}{x^3-x^2+x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+x+2)/(x^3+x+1)^(1/2)/(x^3-x^2+x+1),x, algorithm="fricas")
[Out] log((x^3 + x^2 + 2*sqrt(x^3 + x + 1)*x + x + 1)/(x^3 - x^2 + x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^3-x-2}{\left(x^3-x^2+x+1\right)\sqrt{x^3+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+x+2)/(x^3+x+1)^(1/2)/(x^3-x^2+x+1),x, algorithm="giac")
```

```
[Out] integrate(-(x^3 - x - 2)/((x^3 - x^2 + x + 1)*sqrt(x^3 + x + 1)), x)
```

maple [C] time = 2.61, size = 1905, normalized size = 127.00

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+x+2)/(x^3+x+1)^(1/2)/(x^3-x^2+x+1),x)
```

```
[Out] -2/3*I*3^(1/2)*(-1/6*(108+12*93^(1/2))^(1/3)-2/(108+12*93^(1/2))^(1/3))*(I*
(x-1/12*(108+12*93^(1/2))^(1/3)+1/(108+12*93^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1
/6*(108+12*93^(1/2))^(1/3)-2/(108+12*93^(1/2))^(1/3)))*3^(1/2)/(1/6*(108+12
*93^(1/2))^(1/3)+2/(108+12*93^(1/2))^(1/3)))^(1/2)*((x+1/6*(108+12*93^(1/2)
)^(1/3)-2/(108+12*93^(1/2))^(1/3))/(1/4*(108+12*93^(1/2))^(1/3)-3/(108+12*9
3^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/6*(108+12*93^(1/2))^(1/3)-2/(108+12*93^(1/
2))^(1/3))))^(1/2)*(-I*(x-1/12*(108+12*93^(1/2))^(1/3)+1/(108+12*93^(1/2))^
(1/3)-1/2*I*3^(1/2)*(-1/6*(108+12*93^(1/2))^(1/3)-2/(108+12*93^(1/2))^(1/3)
))*3^(1/2)/(1/6*(108+12*93^(1/2))^(1/3)+2/(108+12*93^(1/2))^(1/3)))^(1/2)/(
x^3+x+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/12*(108+12*93^(1/2))^(1/3)+1/(
108+12*93^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/6*(108+12*93^(1/2))^(1/3)-2/(108+1
2*93^(1/2))^(1/3)))*3^(1/2)/(1/6*(108+12*93^(1/2))^(1/3)+2/(108+12*93^(1/2)
)^(1/3)))^(1/2),(I*3^(1/2)*(1/6*(108+12*93^(1/2))^(1/3)+2/(108+12*93^(1/2))
^(1/3))/(1/4*(108+12*93^(1/2))^(1/3)-3/(108+12*93^(1/2))^(1/3)-1/2*I*3^(1/2
)*(-1/6*(108+12*93^(1/2))^(1/3)-2/(108+12*93^(1/2))^(1/3))))^(1/2))-1/216*I
*6^(1/2)*12^(1/3)*sum(_alpha*(-(9+93^(1/2))^(1/3)-12^(1/3)/(9+93^(1/2))^(1/
3))*(1/24*I*(12*x+12^(1/3)*(-(9+93^(1/2))^(1/3)+12^(1/3)/(9+93^(1/2))^(1/3)
+I*3^(1/2)*(-(9+93^(1/2))^(1/3)-12^(1/3)/(9+93^(1/2))^(1/3))))/((9+93^(1/2)
)^(1/3)+12^(1/3)/(9+93^(1/2))^(1/3)))^(1/2)*((6*x+12^(1/3)*((9+93^(1/2))^(1
/3)-12^(1/3)/(9+93^(1/2))^(1/3)))/(3*(9+93^(1/2))^(1/3)-3*12^(1/3)/(9+93^(1
/2))^(1/3)-I*3^(1/2)*(-(9+93^(1/2))^(1/3)-12^(1/3)/(9+93^(1/2))^(1/3))))^(1
/2)*(-1/24*I*(12*x+12^(1/3)*(-(9+93^(1/2))^(1/3)+12^(1/3)/(9+93^(1/2))^(1/3
)-I*3^(1/2)*(-(9+93^(1/2))^(1/3)-12^(1/3)/(9+93^(1/2))^(1/3))))/((9+93^(1/2
))^(1/3)+12^(1/3)/(9+93^(1/2))^(1/3)))^(1/2)/(x^3+x+1)^(1/2)*(-48*_alpha^2+
96*_alpha-240+12^(1/3)*(31^(1/2)*3^(1/2)*12^(1/3)*_alpha*(3^(1/2)*31^(1/2)+
9)^(2/3)+2*3^(1/2)*31^(1/2)*_alpha^2*(3^(1/2)*31^(1/2)+9)^(1/3)-31^(1/2)*3^
(1/2)*12^(1/3)*_alpha^2*(3^(1/2)*31^(1/2)+9)^(2/3)-4*3^(1/2)*31^(1/2)*_alph
a*(3^(1/2)*31^(1/2)+9)^(1/3)+3*I*12^(1/3)*31^(1/2)*(3^(1/2)*31^(1/2)+9)^(2/
3)-13*I*3^(1/2)*12^(1/3)*(3^(1/2)*31^(1/2)+9)^(2/3)-3*I*12^(1/3)*31^(1/2)*_
alpha*(3^(1/2)*31^(1/2)+9)^(2/3)+24*I*3^(1/2)*_alpha*(3^(1/2)*31^(1/2)+9)^(
1/3)-11*I*3^(1/2)*12^(1/3)*_alpha^2*(3^(1/2)*31^(1/2)+9)^(2/3)-12*I*31^(1/2
)*_alpha*(3^(1/2)*31^(1/2)+9)^(1/3)-6*I*3^(1/2)*_alpha^2*(3^(1/2)*31^(1/2)+
9)^(1/3)-24*I*3^(1/2)*(3^(1/2)*31^(1/2)+9)^(1/3)-31^(1/2)*3^(1/2)*12^(1/3)*
(3^(1/2)*31^(1/2)+9)^(2/3)+13*12^(1/3)*(3^(1/2)*31^(1/2)+9)^(2/3)+6*I*31^(1
/2)*_alpha^2*(3^(1/2)*31^(1/2)+9)^(1/3)+12*I*31^(1/2)*(3^(1/2)*31^(1/2)+9)^
(1/3)+3*I*12^(1/3)*31^(1/2)*_alpha^2*(3^(1/2)*31^(1/2)+9)^(2/3)+13*I*3^(1/2
)*12^(1/3)*_alpha*(3^(1/2)*31^(1/2)+9)^(2/3)-24*(3^(1/2)*31^(1/2)+9)^(1/3)+
24*_alpha*(3^(1/2)*31^(1/2)+9)^(1/3)+11*12^(1/3)*_alpha^2*(3^(1/2)*31^(1/2)
+9)^(2/3)-13*12^(1/3)*_alpha*(3^(1/2)*31^(1/2)+9)^(2/3)-6*_alpha^2*(3^(1/2)
*31^(1/2)+9)^(1/3)+4*3^(1/2)*31^(1/2)*(3^(1/2)*31^(1/2)+9)^(1/3)))*Elliptic
Pi(1/3*3^(1/2)*(I*(x-1/12*(108+12*93^(1/2))^(1/3)+1/(108+12*93^(1/2))^(1/3)
+1/2*I*3^(1/2)*(-1/6*(108+12*93^(1/2))^(1/3)-2/(108+12*93^(1/2))^(1/3)))*3^
(1/2)/(1/6*(108+12*93^(1/2))^(1/3)+2/(108+12*93^(1/2))^(1/3)))^(1/2),2+1/72
*_alpha*31^(1/2)*3^(1/2)*(108+12*3^(1/2)*31^(1/2))^(2/3)-1/24*_alpha^2*31^(
1/2)*3^(1/2)*(108+12*3^(1/2)*31^(1/2))^(1/3)-1/144*_alpha^2*(108+12*3^(1/2)
*31^(1/2))^(2/3)*31^(1/2)*3^(1/2)+1/24*_alpha*3^(1/2)*(108+12*3^(1/2)*31^(1
/2))^(1/3)*31^(1/2)+5/144*I*_alpha^2*3^(1/2)*(108+12*3^(1/2)*31^(1/2))^(2/3
)-1/24*I*_alpha^2*(108+12*3^(1/2)*31^(1/2))^(1/3)*31^(1/2)-1/18*I*_alpha*3^
(1/2)*(108+12*3^(1/2)*31^(1/2))^(2/3)+7/72*I*_alpha^2*3^(1/2)*(108+12*3^(1/
```

```
2)*31^(1/2))^(1/3)+1/24*I*_alpha*(108+12*3^(1/2)*31^(1/2))^(1/3)*31^(1/2)-5
/72*I*_alpha*3^(1/2)*(108+12*3^(1/2)*31^(1/2))^(1/3)+1/72*I*_alpha*(108+12*
3^(1/2)*31^(1/2))^(2/3)*31^(1/2)-1/144*I*_alpha^2*(108+12*3^(1/2)*31^(1/2))
^(2/3)*31^(1/2)-1/72*(108+12*3^(1/2)*31^(1/2))^(2/3)*31^(1/2)*3^(1/2)+1/12*
(108+12*3^(1/2)*31^(1/2))^(2/3)-1/24*I*(108+12*3^(1/2)*31^(1/2))^(1/3)*31^(
1/2)-7/72*I*(108+12*3^(1/2)*31^(1/2))^(1/3)*3^(1/2)+13/72*I*(108+12*3^(1/2)
*31^(1/2))^(2/3)*3^(1/2)-1/18*I*(108+12*3^(1/2)*31^(1/2))^(2/3)*31^(1/2)+1/
3*I*31^(1/2)+13/24*(108+12*3^(1/2)*31^(1/2))^(1/3)-1/3*I*_alpha*31^(1/2)+1/
6*I*_alpha^2*31^(1/2)-13/24*_alpha*(108+12*3^(1/2)*31^(1/2))^(1/3)+1/48*(10
8+12*3^(1/2)*31^(1/2))^(2/3)*_alpha^2-1/12*(108+12*3^(1/2)*31^(1/2))^(2/3)*
_alpha+11/24*(108+12*3^(1/2)*31^(1/2))^(1/3)*_alpha^2+1/2*_alpha^2-1/24*(10
8+12*3^(1/2)*31^(1/2))^(1/3)*31^(1/2)*3^(1/2)-2*_alpha,(I*3^(1/2)*(1/6*(108
+12*93^(1/2))^(1/3)+2/(108+12*93^(1/2))^(1/3))/(1/4*(108+12*93^(1/2))^(1/3)
-3/(108+12*93^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/6*(108+12*93^(1/2))^(1/3)-2/(1
08+12*93^(1/2))^(1/3))))^(1/2)),_alpha=RootOf(_Z^3-_Z^2+_Z+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3 - x - 2}{\left(x^3 - x^2 + x + 1\right)\sqrt{x^3 + x + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+x+2)/(x^3+x+1)^(1/2)/(x^3-x^2+x+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^3 - x - 2)/((x^3 - x^2 + x + 1)*sqrt(x^3 + x + 1)), x)
```

mupad [B] time = 1.88, size = 2490, normalized size = 166.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - x^3 + 2)/((x + x^3 + 1)^(1/2)*(x - x^2 + x^3 + 1)),x)
```

```
[Out] symsum(-(2*(-(x + 1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) - ((31^(1/2)
*108^(1/2))/108 - 1/2)^(1/3))/((3^(1/2)*(1/(3*((31^(1/2)*108^(1/2))/108 - 1
/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 - 1/(2*((31^(1/2
)*108^(1/2))/108 - 1/2)^(1/3)) + (3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))
/2))^(1/2)*((x + (3^(1/2)*(1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + (
(31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 - 1/(6*((31^(1/2)*108^(1/2))/1
08 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)/2)/((3^(1/2)*(1/(
3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2
)^(1/3))*1i)/2 - 1/(2*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + (3*((31^(1/
2)*108^(1/2))/108 - 1/2)^(1/3))/2))^(1/2)*((3^(1/2)*(1/(3*((31^(1/2)*108^(1
/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 - 1/
(2*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + (3*((31^(1/2)*108^(1/2))/108 -
 1/2)^(1/3))/2)*ellipticPi(((3^(1/2)*(1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)
^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 - 1/(2*((31^(1/2)*1
08^(1/2))/108 - 1/2)^(1/3)) + (3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))/2)
/(root(z^3 - z^2 + z + 1, z, k) + (3^(1/2)*(1/(3*((31^(1/2)*108^(1/2))/108
- 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 - 1/(6*((31^(
1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)/
2), asin(((x + (3^(1/2)*(1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((3
1^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 - 1/(6*((31^(1/2)*108^(1/2))/108
 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)/2)/((3^(1/2)*(1/(3*
((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^
(1/3))*1i)/2 - 1/(2*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + (3*((31^(1/2)
*108^(1/2))/108 - 1/2)^(1/3))/2))^(1/2)), -(3^(1/2)*((3^(1/2)*(1/(3*((31^(1
/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*
1i)/2 - 1/(2*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + (3*((31^(1/2)*108^(1
/2))/108 - 1/2)^(1/3))/2)*1i)/(3*(1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/
```

3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))))*(2*root(z^3 - z^2 + z + 1,
z, k) - root(z^3 - z^2 + z + 1, z, k)^2 + 3)*((3^(1/2)*(x - (3^(1/2)*(1/(3*
((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^
(1/3))*1i)/2 - 1/(6*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*10
8^(1/2))/108 - 1/2)^(1/3)/2)*1i)/(3*(1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^
(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))))^(1/2))/((x^3 - x*((1/(3*
((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) - ((31^(1/2)*108^(1/2))/108 - 1/2)^
(1/3))*((3^(1/2)*(1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)
*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 + 1/(6*((31^(1/2)*108^(1/2))/108 - 1/2)
^(1/3)) - ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)/2) - (1/(3*((31^(1/2)*108^
(1/2))/108 - 1/2)^(1/3)) - ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*((3^(1/2
)*(1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108
 - 1/2)^(1/3))*1i)/2 - 1/(6*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^
(1/2)*108^(1/2))/108 - 1/2)^(1/3)/2) + ((3^(1/2)*(1/(3*((31^(1/2)*108^(1/2)
)/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 - 1/(6*
((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^
(1/3)/2)*((3^(1/2)*(1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/
2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 + 1/(6*((31^(1/2)*108^(1/2))/108 - 1/
2)^(1/3)) - ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)/2)) - (1/(3*((31^(1/2)*1
08^(1/2))/108 - 1/2)^(1/3)) - ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*((3^(
1/2)*(1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/
108 - 1/2)^(1/3))*1i)/2 - 1/(6*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((
31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)/2)*((3^(1/2)*(1/(3*((31^(1/2)*108^(1/2
))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 + 1/(6
*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) - ((31^(1/2)*108^(1/2))/108 - 1/2)
^(1/3)/2))^(1/2)*(3*root(z^3 - z^2 + z + 1, z, k)^2 - 2*root(z^3 - z^2 + z
+ 1, z, k) + 1)*(root(z^3 - z^2 + z + 1, z, k) + (3^(1/2)*(1/(3*((31^(1/2)*
108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/
2 - 1/(6*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/10
8 - 1/2)^(1/3)/2)), k, 1, 3) - (2*(-(x + 1/(3*((31^(1/2)*108^(1/2))/108 - 1
/2)^(1/3)) - ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))/((3^(1/2)*(1/(3*((31^(
1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))
1i)/2 - 1/(2((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + (3*((31^(1/2)*108^(
1/2))/108 - 1/2)^(1/3))/2))^(1/2)*ellipticF(asin(((x + (3^(1/2)*(1/(3*((31^
(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)
)*1i)/2 - 1/(6*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/
2))/108 - 1/2)^(1/3)/2)/((3^(1/2)*(1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1
/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 - 1/(2*((31^(1/2)*108^
(1/2))/108 - 1/2)^(1/3)) + (3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))/2))^(
1/2)), -(3^(1/2)*((3^(1/2)*(1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) +
((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 - 1/(2*((31^(1/2)*108^(1/2))/
108 - 1/2)^(1/3)) + (3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))/2)*1i)/(3*(1
/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1
/2)^(1/3))))*((x + (3^(1/2)*(1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) +
 ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 - 1/(6*((31^(1/2)*108^(1/2))
/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)/2)/((3^(1/2)*(1
/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1
/2)^(1/3))*1i)/2 - 1/(2*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + (3*((31^(
1/2)*108^(1/2))/108 - 1/2)^(1/3))/2))^(1/2)*((3^(1/2)*(1/(3*((31^(1/2)*108^
(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 -
1/(2*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + (3*((31^(1/2)*108^(1/2))/108
 - 1/2)^(1/3))/2)*((3^(1/2)*(x - (3^(1/2)*(1/(3*((31^(1/2)*108^(1/2))/108 -
 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 - 1/(6*((31^(1
/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)/2
)*1i)/(3*(1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/
2))/108 - 1/2)^(1/3))))^(1/2))/(x^3 - x*((1/(3*((31^(1/2)*108^(1/2))/108 -
1/2)^(1/3)) - ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*((3^(1/2)*(1/(3*((31^
(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)
)*1i)/2 + 1/(6*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) - ((31^(1/2)*108^(1/

```
2))/108 - 1/2)^(1/3)/2) - (1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) - (
(31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*((3^(1/2)*(1/(3*((31^(1/2)*108^(1/2)
)/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 - 1/(6*
((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^
(1/3)/2) + ((3^(1/2)*(1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(
1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 - 1/(6*((31^(1/2)*108^(1/2))/108 -
1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)/2)*((3^(1/2)*(1/(3*((3
1^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/
3))*1i)/2 + 1/(6*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) - ((31^(1/2)*108^(
1/2))/108 - 1/2)^(1/3)/2)) - (1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))
- ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*((3^(1/2)*(1/(3*((31^(1/2)*108^(1
/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 - 1/
(6*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^(1/2)*108^(1/2))/108 - 1/
2)^(1/3)/2)*((3^(1/2)*(1/(3*((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)) + ((31^
(1/2)*108^(1/2))/108 - 1/2)^(1/3))*1i)/2 + 1/(6*((31^(1/2)*108^(1/2))/108 -
 1/2)^(1/3)) - ((31^(1/2)*108^(1/2))/108 - 1/2)^(1/3)/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{x}{x^3\sqrt{x^3+x+1}-x^2\sqrt{x^3+x+1}+x\sqrt{x^3+x+1}+\sqrt{x^3+x+1}}\right)dx-\int\frac{x^3}{x^3\sqrt{x^3+x+1}-x^2\sqrt{x^3+x+1}+x\sqrt{x^3+x+1}+\sqrt{x^3+x+1}}\,dx-\int\left(-\frac{2}{x^3\sqrt{x^3+x+1}-x^2\sqrt{x^3+x+1}+x\sqrt{x^3+x+1}+\sqrt{x^3+x+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+x+2)/(x**3+x+1)**(1/2)/(x**3-x**2+x+1),x)
```

```
[Out] -Integral(-x/(x**3*sqrt(x**3 + x + 1) - x**2*sqrt(x**3 + x + 1) + x*sqrt(x*
*3 + x + 1) + sqrt(x**3 + x + 1)), x) - Integral(x**3/(x**3*sqrt(x**3 + x +
 1) - x**2*sqrt(x**3 + x + 1) + x*sqrt(x**3 + x + 1) + sqrt(x**3 + x + 1)),
 x) - Integral(-2/(x**3*sqrt(x**3 + x + 1) - x**2*sqrt(x**3 + x + 1) + x*sq
rt(x**3 + x + 1) + sqrt(x**3 + x + 1)), x)
```

3.68 $\int \frac{1+x^4}{(1-x^4)\sqrt{-1+x^2+x^4}}\,dx$

Optimal. Leaf size=15

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^4+x^2-1}}\right)$$

Rubi [A] time = 0.09, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2112, 206}

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^4+x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/((1 - x^4)*Sqrt[-1 + x^2 + x^4]),x]
```

```
[Out] ArcTanh[x/Sqrt[-1 + x^2 + x^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2112

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^4}{(1-x^4)\sqrt{-1+x^2+x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2+x^4}}\right)\\ &= \tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2+x^4}}\right)\end{aligned}$$

Mathematica [C] time = 3.51, size = 1547, normalized size = 103.13

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^4)/((1 - x^4)*Sqrt[-1 + x^2 + x^4]),x]
```

```
[Out] (((2*I)*Sqrt[1 - x^2 - x^4]*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]*x], -
3/2 - Sqrt[5]/2])/Sqrt[-1 + Sqrt[5]] - ((2*I)*Sqrt[1 - x^2 - x^4]*EllipticP
i[(1 + Sqrt[5])/2, I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/2])/S
qrt[-1 + Sqrt[5]] - ((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1
 + Sqrt[5])] - 2*x)^2*Sqrt[(I*Sqrt[2*(1 + Sqrt[5])] + 2*x)/((1 + 2*I)*Sqrt[
2] - Sqrt[10] + 2*Sqrt[-1 + Sqrt[5]]*x - (2*I)*Sqrt[1 + Sqrt[5]]*x)]*Sqrt[(
I*Sqrt[2*(1 + Sqrt[5])] - 2*x)/(Sqrt[2]*((-1 + 2*I) + Sqrt[5]) - 2*(Sqrt[-1
 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*x)]*Sqrt[(Sqrt[2]*((-1 - 2*I) + Sqrt[5])
 + 2*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*x)/(Sqrt[2]*((-1 + 2*I) + S
```

```
qrt[5]) - 2*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*x)]*((2 + Sqrt[2*(-1
 + Sqrt[5])])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[
5]])*(Sqrt[2*(-1 + Sqrt[5])] + 2*x))/((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt
[5]])*(Sqrt[2*(-1 + Sqrt[5])] - 2*x))]], -3/5 - (4*I)/5] - 2*Sqrt[2*(-1 + S
qrt[5])]*EllipticPi[((-2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] + I*
Sqrt[1 + Sqrt[5]]))/((2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*S
qrt[1 + Sqrt[5]])), ArcSin[Sqrt[((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])
*(Sqrt[2*(-1 + Sqrt[5])] + 2*x))/((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]]
)*(Sqrt[2*(-1 + Sqrt[5])] - 2*x))]], -3/5 - (4*I)/5]))/((-2 + Sqrt[2*(-1 +
Sqrt[5])])*(2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sq
rt[5]])) + ((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5
])] - 2*x)^2*Sqrt[(I*Sqrt[2*(1 + Sqrt[5])] + 2*x)/((1 + 2*I)*Sqrt[2] - Sqrt
[10] + 2*Sqrt[-1 + Sqrt[5]]*x - (2*I)*Sqrt[1 + Sqrt[5]]*x)]*Sqrt[(I*Sqrt[2*
(1 + Sqrt[5])] - 2*x)/(Sqrt[2]*((-1 + 2*I) + Sqrt[5]) - 2*(Sqrt[-1 + Sqrt[5
]] + I*Sqrt[1 + Sqrt[5]])*x)]*Sqrt[(Sqrt[2]*((-1 - 2*I) + Sqrt[5]) + 2*(Sqr
t[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*x)/(Sqrt[2]*((-1 + 2*I) + Sqrt[5]) -
 2*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*x)]*((-2 + Sqrt[2*(-1 + Sqrt[
5])])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(Sq
rt[2*(-1 + Sqrt[5])] + 2*x))/((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(S
qrt[2*(-1 + Sqrt[5])] - 2*x))]], -3/5 - (4*I)/5] - 2*Sqrt[2*(-1 + Sqrt[5])]
*EllipticPi[((2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 +
Sqrt[5]]))/((-2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 +
Sqrt[5]])), ArcSin[Sqrt[((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(Sqrt[2
*(-1 + Sqrt[5])] + 2*x))/((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt[
2*(-1 + Sqrt[5])] - 2*x))]], -3/5 - (4*I)/5]))/((-2 + Sqrt[2*(-1 + Sqrt[5])
])*(2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]]))
)/(Sqrt[2]*Sqrt[-1 + x^2 + x^4])
```

IntegrateAlgebraic [A] time = 0.27, size = 15, normalized size = 1.00

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^4+x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)/((1 - x^4)*Sqrt[-1 + x^2 + x^4]),x]
```

```
[Out] ArcTanh[x/Sqrt[-1 + x^2 + x^4]]
```

fricas [B] time = 0.45, size = 34, normalized size = 2.27

$$\frac{1}{2}\log\left(\frac{x^4+2x^2+2\sqrt{x^4+x^2-1}\,x-1}{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(-x^4+1)/(x^4+x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log((x^4 + 2*x^2 + 2*sqrt(x^4 + x^2 - 1)*x - 1)/(x^4 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^4+1}{\sqrt{x^4+x^2-1}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(-x^4+1)/(x^4+x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^4 + 1)/(sqrt(x^4 + x^2 - 1)*(x^4 - 1)), x)
```

maple [C] time = 0.18, size = 265, normalized size = 17.67

$$-\frac{2\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)x^{2}}\,\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)x^{2}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2-2\sqrt{5}}}{2},\frac{i}{2}+\frac{i\sqrt{5}}{2}\right)}{\sqrt{2-2\sqrt{5}}\,\sqrt{x^{4}+x^{2}-1}}+\frac{\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)x^{2}}\,\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)x^{2}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}\,x,\frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}},\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}}{\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}\right)}{\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}\,\sqrt{x^{4}+x^{2}-1}}+\frac{\sqrt{1-\frac{x^{2}}{2}+\frac{\sqrt{5}\,x^{2}}{2}}\,\sqrt{1-\frac{x^{2}}{2}-\frac{\sqrt{5}\,x^{2}}{2}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}\,x,-\frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}},\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}}{\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}\right)}{\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}\,\sqrt{x^{4}+x^{2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(-x^4+1)/(x^4+x^2-1)^(1/2),x)
```

```
[Out] -2/(2-2*5^(1/2))^(1/2)*(1-(1/2-1/2*5^(1/2))*x^2)^(1/2)*(1-(1/2+1/2*5^(1/2))
*x^2)^(1/2)/(x^4+x^2-1)^(1/2)*EllipticF(1/2*x*(2-2*5^(1/2))^(1/2),1/2*I+1/2
*I*5^(1/2))+1/(1/2-1/2*5^(1/2))^(1/2)*(1-(1/2-1/2*5^(1/2))*x^2)^(1/2)*(1-(1
/2+1/2*5^(1/2))*x^2)^(1/2)/(x^4+x^2-1)^(1/2)*EllipticPi((1/2-1/2*5^(1/2))^(
1/2)*x,1/(1/2-1/2*5^(1/2)),(1/2+1/2*5^(1/2))^(1/2)/(1/2-1/2*5^(1/2))^(1/2))
+1/(1/2-1/2*5^(1/2))^(1/2)*(1-1/2*x^2+1/2*5^(1/2)*x^2)^(1/2)*(1-1/2*x^2-1/2
*5^(1/2)*x^2)^(1/2)/(x^4+x^2-1)^(1/2)*EllipticPi((1/2-1/2*5^(1/2))^(1/2)*x,
-1/(1/2-1/2*5^(1/2)),(1/2+1/2*5^(1/2))^(1/2)/(1/2-1/2*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{4}+1}{\sqrt{x^{4}+x^{2}-1}\left(x^{4}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(-x^4+1)/(x^4+x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 + 1)/(sqrt(x^4 + x^2 - 1)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int -\frac{x^{4}+1}{\left(x^{4}-1\right)\sqrt{x^{4}+x^{2}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 + 1)/((x^4 - 1)*(x^2 + x^4 - 1)^(1/2)),x)
```

```
[Out] int(-(x^4 + 1)/((x^4 - 1)*(x^2 + x^4 - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{4}}{x^{4}\sqrt{x^{4}+x^{2}-1}-\sqrt{x^{4}+x^{2}-1}}\,dx-\int \frac{1}{x^{4}\sqrt{x^{4}+x^{2}-1}-\sqrt{x^{4}+x^{2}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(-x**4+1)/(x**4+x**2-1)**(1/2),x)
```

```
[Out] -Integral(x**4/(x**4*sqrt(x**4 + x**2 - 1) - sqrt(x**4 + x**2 - 1)), x) - I
ntegral(1/(x**4*sqrt(x**4 + x**2 - 1) - sqrt(x**4 + x**2 - 1)), x)
```

3.69 $\int \frac{-4+x^3}{x^4 \sqrt[4]{-1+x^3}} \, dx$

Optimal. Leaf size=16

$$-\frac{4\left(x^3-1\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 74}

$$-\frac{4\left(x^3-1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + x^3)/(x^4*(-1 + x^3)^(1/4)),x]
```

```
[Out] (-4*(-1 + x^3)^(3/4))/(3*x^3)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{-4+x^3}{x^4\sqrt[4]{-1+x^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{-4+x}{\sqrt[4]{-1+x}\,x^2}\,dx,x,x^3\right)\\ &= -\frac{4\left(-1+x^3\right)^{3/4}}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{4\left(x^3-1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + x^3)/(x^4*(-1 + x^3)^(1/4)),x]
```

```
[Out] (-4*(-1 + x^3)^(3/4))/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.13, size = 16, normalized size = 1.00

$$-\frac{4\left(x^3-1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-4 + x^3)/(x^4*(-1 + x^3)^(1/4)),x]
[Out] (-4*(-1 + x^3)^(3/4))/(3*x^3)
```

fricas [A] time = 0.38, size = 12, normalized size = 0.75

$$-\frac{4\left(x^3-1\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)/x^4/(x^3-1)^(1/4),x, algorithm="fricas")
[Out] -4/3*(x^3 - 1)^(3/4)/x^3
```

giac [A] time = 0.36, size = 12, normalized size = 0.75

$$-\frac{4\left(x^3-1\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)/x^4/(x^3-1)^(1/4),x, algorithm="giac")
[Out] -4/3*(x^3 - 1)^(3/4)/x^3
```

maple [A] time = 0.01, size = 22, normalized size = 1.38

$$-\frac{4\,(-1+x)\left(x^2+x+1\right)}{3x^3\left(x^3-1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-4)/x^4/(x^3-1)^(1/4),x)
[Out] -4/3/x^3*(-1+x)*(x^2+x+1)/(x^3-1)^(1/4)
```

maxima [A] time = 0.68, size = 12, normalized size = 0.75

$$-\frac{4\left(x^3-1\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)/x^4/(x^3-1)^(1/4),x, algorithm="maxima")
[Out] -4/3*(x^3 - 1)^(3/4)/x^3
```

mupad [B] time = 0.16, size = 12, normalized size = 0.75

$$-\frac{4\left(x^3-1\right)^{3/4}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 4)/(x^4*(x^3 - 1)^(1/4)),x)
[Out] -(4*(x^3 - 1)^(3/4))/(3*x^3)
```

sympy [C] time = 14.06, size = 68, normalized size = 4.25

$$-\frac{\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^{2i\pi}}{x^3}\right)}{3x^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)}+\frac{4\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{e^{2i\pi}}{x^3}\right)}{3x^{\frac{15}{4}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-4)/x**4/(x**3-1)**(1/4),x)
```

```
[Out] -gamma(1/4)*hyper((1/4, 1/4), (5/4,), exp_polar(2*I*pi)/x**3)/(3*x**(3/4)*g
amma(5/4)) + 4*gamma(5/4)*hyper((1/4, 5/4), (9/4,), exp_polar(2*I*pi)/x**3)
/(3*x**(15/4)*gamma(9/4))
```

3.70 $\int \frac{\sqrt[3]{-1+x^3}}{x^5} \, dx$

Optimal. Leaf size=16

$$\frac{\left(x^3-1\right)^{4/3}}{4x^4}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$\frac{\left(x^3-1\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)^(1/3)/x^5,x]
[Out] (-1 + x^3)^(4/3)/(4*x^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{-1+x^3}}{x^5} \, dx = \frac{\left(-1+x^3\right)^{4/3}}{4x^4}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\left(x^3-1\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)^(1/3)/x^5,x]
[Out] (-1 + x^3)^(4/3)/(4*x^4)
```

IntegrateAlgebraic [A] time = 0.09, size = 16, normalized size = 1.00

$$\frac{\left(x^3-1\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)^(1/3)/x^5,x]
[Out] (-1 + x^3)^(4/3)/(4*x^4)
```

fricas [A] time = 0.38, size = 12, normalized size = 0.75

$$\frac{\left(x^3-1\right)^{\frac{4}{3}}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)/x^5,x, algorithm="fricas")
[Out] 1/4*(x^3 - 1)^(4/3)/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-1\right)^{\frac{1}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)/x^5,x, algorithm="giac")
[Out] integrate((x^3 - 1)^(1/3)/x^5, x)
```

maple [A] time = 0.00, size = 22, normalized size = 1.38

$$\frac{(-1+x)\left(x^2+x+1\right)\left(x^3-1\right)^{\frac{1}{3}}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/3)/x^5,x)
[Out] 1/4/x^4*(-1+x)*(x^2+x+1)*(x^3-1)^(1/3)
```

maxima [A] time = 0.43, size = 12, normalized size = 0.75

$$\frac{\left(x^3-1\right)^{\frac{4}{3}}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)/x^5,x, algorithm="maxima")
[Out] 1/4*(x^3 - 1)^(4/3)/x^4
```

mupad [B] time = 0.19, size = 25, normalized size = 1.56

$$-\frac{\left(x^3-1\right)^{1/3}-x^3\left(x^3-1\right)^{1/3}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)^(1/3)/x^5,x)
[Out] -((x^3 - 1)^(1/3) - x^3*(x^3 - 1)^(1/3))/(4*x^4)
```

sympy [B] time = 0.69, size = 129, normalized size = 8.06

$$\begin{cases} \frac{\sqrt[3]{-1+\frac{1}{x^3}}\,e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)} - \frac{\sqrt[3]{-1+\frac{1}{x^3}}\,e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{3x^3\Gamma\left(-\frac{1}{3}\right)} & \text{for}\ \frac{1}{\left|x^3\right|}>1 \\ -\frac{\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{4}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{4}{3}\right)}{3x^3\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/3)/x**5,x)
```

```
[Out] Piecewise(((-1 + x**(-3))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(3*gamma(-1/3))
 - (-1 + x**(-3))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(3*x**3*gamma(-1/3)), 1
/Abs(x**3) > 1), (-(1 - 1/x**3)**(1/3)*gamma(-4/3)/(3*gamma(-1/3)) + (1 - 1
/x**3)**(1/3)*gamma(-4/3)/(3*x**3*gamma(-1/3)), True))
```

3.71 $\int \frac{(-1+x^3)^{2/3}}{x^6}\,dx$

Optimal. Leaf size=16

$$\frac{(x^3-1)^{5/3}}{5x^5}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$\frac{(x^3-1)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)^(2/3)/x^6,x]
```

```
[Out] (-1 + x^3)^(5/3)/(5*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(-1+x^3)^{2/3}}{x^6}\,dx = \frac{(-1+x^3)^{5/3}}{5x^5}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{(x^3-1)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)^(2/3)/x^6,x]
```

```
[Out] (-1 + x^3)^(5/3)/(5*x^5)
```

IntegrateAlgebraic [A] time = 0.08, size = 16, normalized size = 1.00

$$\frac{(x^3-1)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)^(2/3)/x^6,x]
```

```
[Out] (-1 + x^3)^(5/3)/(5*x^5)
```

fricas [A] time = 0.41, size = 12, normalized size = 0.75

$$\frac{(x^3-1)^{\frac{5}{3}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)/x^6,x, algorithm="fricas")
[Out] 1/5*(x^3 - 1)^(5/3)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-1\right)^{\frac{2}{3}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)/x^6,x, algorithm="giac")
[Out] integrate((x^3 - 1)^(2/3)/x^6, x)
```

maple [A] time = 0.00, size = 22, normalized size = 1.38

$$\frac{(-1+x)\left(x^2+x+1\right)\left(x^3-1\right)^{\frac{2}{3}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)/x^6,x)
[Out] 1/5/x^5*(-1+x)*(x^2+x+1)*(x^3-1)^(2/3)
```

maxima [A] time = 0.40, size = 12, normalized size = 0.75

$$\frac{\left(x^3-1\right)^{\frac{5}{3}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)/x^6,x, algorithm="maxima")
[Out] 1/5*(x^3 - 1)^(5/3)/x^5
```

mupad [B] time = 0.19, size = 25, normalized size = 1.56

$$-\frac{\left(x^3-1\right)^{2/3}-x^3\left(x^3-1\right)^{2/3}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)^(2/3)/x^6,x)
[Out] -((x^3 - 1)^(2/3) - x^3*(x^3 - 1)^(2/3))/(5*x^5)
```

sympy [B] time = 0.77, size = 126, normalized size = 7.88

$$\begin{cases} \frac{\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}} e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{5}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)} - \frac{\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}} e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)} & \text{for}\ \frac{1}{|x^3|}>1 \\ -\frac{\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)} + \frac{\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)/x**6,x)
```

```
[Out] Piecewise(((-1 + x**(-3))**(2/3)*exp(-I*pi/3)*gamma(-5/3)/(3*gamma(-2/3)) -
 (-1 + x**(-3))**(2/3)*exp(-I*pi/3)*gamma(-5/3)/(3*x**3*gamma(-2/3)), 1/Abs
(x**3) > 1), (-(1 - 1/x**3)**(2/3)*gamma(-5/3)/(3*gamma(-2/3)) + (1 - 1/x**
3)**(2/3)*gamma(-5/3)/(3*x**3*gamma(-2/3)), True))
```

3.72 $\int \frac{\sqrt[3]{1+x^3}}{x^5} dx$

Optimal. Leaf size=16

$$-\frac{\left(x^3+1\right)^{4/3}}{4x^4}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{\left(x^3+1\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)^(1/3)/x^5,x]
```

```
[Out] -1/4*(1 + x^3)^(4/3)/x^4
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{1+x^3}}{x^5} dx = -\frac{\left(1+x^3\right)^{4/3}}{4x^4}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\left(x^3+1\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)^(1/3)/x^5,x]
```

```
[Out] -1/4*(1 + x^3)^(4/3)/x^4
```

IntegrateAlgebraic [A] time = 0.09, size = 16, normalized size = 1.00

$$-\frac{\left(x^3+1\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)^(1/3)/x^5,x]
```

```
[Out] -1/4*(1 + x^3)^(4/3)/x^4
```

fricas [A] time = 0.41, size = 12, normalized size = 0.75

$$-\frac{\left(x^3+1\right)^{\frac{4}{3}}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/3)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(x^3 + 1)^(4/3)/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+1\right)^{\frac{1}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/3)/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)^(1/3)/x^5, x)
```

maple [A] time = 0.00, size = 24, normalized size = 1.50

$$-\frac{(1+x)\left(x^2-x+1\right)\left(x^3+1\right)^{\frac{1}{3}}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(1/3)/x^5,x)
```

```
[Out] -1/4/x^4*(1+x)*(x^2-x+1)*(x^3+1)^(1/3)
```

maxima [A] time = 0.55, size = 12, normalized size = 0.75

$$-\frac{\left(x^3+1\right)^{\frac{4}{3}}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/3)/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*(x^3 + 1)^(4/3)/x^4
```

mupad [B] time = 0.19, size = 24, normalized size = 1.50

$$-\frac{\left(x^3+1\right)^{1/3}+x^3\left(x^3+1\right)^{1/3}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)^(1/3)/x^5,x)
```

```
[Out] -((x^3 + 1)^(1/3) + x^3*(x^3 + 1)^(1/3))/(4*x^4)
```

sympy [B] time = 0.60, size = 53, normalized size = 3.31

$$\frac{\sqrt[3]{1+\frac{1}{x^3}}\,\Gamma\left(-\frac{4}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)}+\frac{\sqrt[3]{1+\frac{1}{x^3}}\,\Gamma\left(-\frac{4}{3}\right)}{3x^3\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(1/3)/x**5,x)
```

```
[Out] (1 + x**(-3))**(1/3)*gamma(-4/3)/(3*gamma(-1/3)) + (1 + x**(-3))**(1/3)*gam
ma(-4/3)/(3*x**3*gamma(-1/3))
```

3.73 $\int \frac{(1+x^3)^{2/3}}{x^6}\,dx$

Optimal. Leaf size=16

$$-\frac{(x^3+1)^{5/3}}{5x^5}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{(x^3+1)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)^(2/3)/x^6,x]
```

```
[Out] -1/5*(1 + x^3)^(5/3)/x^5
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(1+x^3)^{2/3}}{x^6}\,dx = -\frac{(1+x^3)^{5/3}}{5x^5}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{(x^3+1)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)^(2/3)/x^6,x]
```

```
[Out] -1/5*(1 + x^3)^(5/3)/x^5
```

IntegrateAlgebraic [A] time = 0.09, size = 16, normalized size = 1.00

$$-\frac{(x^3+1)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)^(2/3)/x^6,x]
```

```
[Out] -1/5*(1 + x^3)^(5/3)/x^5
```

fricas [A] time = 0.39, size = 12, normalized size = 0.75

$$-\frac{(x^3+1)^{\frac{5}{3}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)/x^6,x, algorithm="fricas")
```

```
[Out] -1/5*(x^3 + 1)^(5/3)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+1\right)^{\frac{2}{3}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)/x^6,x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)^(2/3)/x^6, x)
```

maple [A] time = 0.00, size = 24, normalized size = 1.50

$$-\frac{(1+x)\left(x^2-x+1\right)\left(x^3+1\right)^{\frac{2}{3}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)/x^6,x)
```

```
[Out] -1/5/x^5*(1+x)*(x^2-x+1)*(x^3+1)^(2/3)
```

maxima [A] time = 0.52, size = 12, normalized size = 0.75

$$-\frac{\left(x^3+1\right)^{\frac{5}{3}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*(x^3 + 1)^(5/3)/x^5
```

mupad [B] time = 0.19, size = 24, normalized size = 1.50

$$-\frac{\left(x^3+1\right)^{2/3}+x^3\left(x^3+1\right)^{2/3}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)^(2/3)/x^6,x)
```

```
[Out] -((x^3 + 1)^(2/3) + x^3*(x^3 + 1)^(2/3))/(5*x^5)
```

sympy [B] time = 0.67, size = 53, normalized size = 3.31

$$\frac{\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}+\frac{\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)/x**6,x)
```

```
[Out] (1 + x**(-3))**(2/3)*gamma(-5/3)/(3*gamma(-2/3)) + (1 + x**(-3))**(2/3)*gam
ma(-5/3)/(3*x**3*gamma(-2/3))
```

3.74 $\int \frac{(-2+x^3)(1+x^3)^{3/2}}{x^6} \, dx$

Optimal. Leaf size=16

$$\frac{2(x^3+1)^{5/2}}{5x^5}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$\frac{2(x^3+1)^{5/2}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((-2 + x^3)*(1 + x^3)^(3/2))/x^6,x]
```

```
[Out] (2*(1 + x^3)^(5/2))/(5*x^5)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(-2+x^3)(1+x^3)^{3/2}}{x^6} \, dx = \frac{2(1+x^3)^{5/2}}{5x^5}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(x^3+1)^{5/2}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-2 + x^3)*(1 + x^3)^(3/2))/x^6,x]
```

```
[Out] (2*(1 + x^3)^(5/2))/(5*x^5)
```

IntegrateAlgebraic [A] time = 0.08, size = 16, normalized size = 1.00

$$\frac{2(x^3+1)^{5/2}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^3)*(1 + x^3)^(3/2))/x^6,x]
```

```
[Out] (2*(1 + x^3)^(5/2))/(5*x^5)
```

fricas [A] time = 0.41, size = 22, normalized size = 1.38

$$\frac{2(x^6+2x^3+1)\sqrt{x^3+1}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3+1)^(3/2)/x^6,x, algorithm="fricas")
[Out] 2/5*(x^6 + 2*x^3 + 1)*sqrt(x^3 + 1)/x^5
```

giac [B] time = 0.50, size = 27, normalized size = 1.69

$$\frac{2}{5}\sqrt{x^3+1}\,x+\frac{2}{5}\sqrt{\frac{1}{x}+\frac{1}{x^4}}\left(\frac{1}{x^3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3+1)^(3/2)/x^6,x, algorithm="giac")
[Out] 2/5*sqrt(x^3 + 1)*x + 2/5*sqrt(1/x + 1/x^4)*(1/x^3 + 2)
```

maple [A] time = 0.01, size = 24, normalized size = 1.50

$$\frac{2\,(1+x)\left(x^2-x+1\right)\left(x^3+1\right)^{\frac{3}{2}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2)*(x^3+1)^(3/2)/x^6,x)
[Out] 2/5/x^5*(1+x)*(x^2-x+1)*(x^3+1)^(3/2)
```

maxima [B] time = 0.51, size = 30, normalized size = 1.88

$$\frac{2\left(x^6+2\,x^3+1\right)\sqrt{x^2-x+1}\,\sqrt{x+1}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3+1)^(3/2)/x^6,x, algorithm="maxima")
[Out] 2/5*(x^6 + 2*x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)/x^5
```

mupad [B] time = 0.04, size = 12, normalized size = 0.75

$$\frac{2\left(x^3+1\right)^{5/2}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)^(3/2)*(x^3 - 2))/x^6,x)
[Out] (2*(x^3 + 1)^(5/2))/(5*x^5)
```

sympy [C] time = 2.89, size = 105, normalized size = 6.56

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}-\frac{2\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\ -\frac{2}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2)*(x**3+1)**(3/2)/x**6,x)
[Out] x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3)
) - gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), x**3*exp_polar(I*pi))/(3*x**2*g
amma(1/3)) - 2*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), x**3*exp_polar(I*pi
))/(3*x**5*gamma(-2/3))
```

3.75 $\int \frac{4+x^3}{x^4 \sqrt[4]{1+x^3}} \, dx$

Optimal. Leaf size=16

$$-\frac{4\left(x^3+1\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {446, 74}

$$-\frac{4\left(x^3+1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^3)/(x^4*(1 + x^3)^(1/4)),x]
```

```
[Out] (-4*(1 + x^3)^(3/4))/(3*x^3)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{4+x^3}{x^4 \sqrt[4]{1+x^3}} \, dx &= \frac{1}{3} \text{Subst}\left(\int \frac{4+x}{x^2 \sqrt[4]{1+x}} \, dx, x, x^3\right) \\ &= -\frac{4\left(1+x^3\right)^{3/4}}{3x^3} \end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{4\left(x^3+1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^3)/(x^4*(1 + x^3)^(1/4)),x]
```

```
[Out] (-4*(1 + x^3)^(3/4))/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 16, normalized size = 1.00

$$-\frac{4\left(x^3+1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4 + x^3)/(x^4*(1 + x^3)^(1/4)),x]
[Out] (-4*(1 + x^3)^(3/4))/(3*x^3)
```

fricas [A] time = 0.40, size = 12, normalized size = 0.75

$$-\frac{4\left(x^3+1\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)/x^4/(x^3+1)^(1/4),x, algorithm="fricas")
[Out] -4/3*(x^3 + 1)^(3/4)/x^3
```

giac [A] time = 0.31, size = 12, normalized size = 0.75

$$-\frac{4\left(x^3+1\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)/x^4/(x^3+1)^(1/4),x, algorithm="giac")
[Out] -4/3*(x^3 + 1)^(3/4)/x^3
```

maple [A] time = 0.00, size = 24, normalized size = 1.50

$$-\frac{4\,(1+x)\left(x^2-x+1\right)}{3x^3\left(x^3+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+4)/x^4/(x^3+1)^(1/4),x)
[Out] -4/3/x^3*(1+x)*(x^2-x+1)/(x^3+1)^(1/4)
```

maxima [A] time = 0.56, size = 12, normalized size = 0.75

$$-\frac{4\left(x^3+1\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)/x^4/(x^3+1)^(1/4),x, algorithm="maxima")
[Out] -4/3*(x^3 + 1)^(3/4)/x^3
```

mupad [B] time = 0.15, size = 12, normalized size = 0.75

$$-\frac{4\left(x^3+1\right)^{3/4}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 4)/(x^4*(x^3 + 1)^(1/4)),x)
[Out] -(4*(x^3 + 1)^(3/4))/(3*x^3)
```

sympy [B] time = 46.79, size = 49, normalized size = 3.06

$$\frac{1}{3\left(1+\frac{1}{\sqrt[4]{x^3+1}}\right)}-\frac{2}{3\left(1+\frac{1}{\sqrt{x^3+1}}\right)\sqrt[4]{x^3+1}}+\frac{1}{3\left(-1+\frac{1}{\sqrt[4]{x^3+1}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+4)/x**4/(x**3+1)**(1/4),x)
```

```
[Out] 1/(3*(1 + (x**3 + 1)**(-1/4))) - 2/(3*(1 + 1/sqrt(x**3 + 1))*(x**3 + 1)**(1
/4)) + 1/(3*(-1 + (x**3 + 1)**(-1/4)))
```

3.76 $\int \frac{1}{x^2 \sqrt[3]{x+x^3}} \, dx$

Optimal. Leaf size=16

$$-\frac{3 \left(x^3+x\right)^{2/3}}{4 x^2}$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2014}

$$-\frac{3 \left(x^3+x\right)^{2/3}}{4 x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(x + x^3)^(1/3)),x]
```

```
[Out] (-3*(x + x^3)^(2/3))/(4*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2 \sqrt[3]{x+x^3}} \, dx = -\frac{3 \left(x+x^3\right)^{2/3}}{4 x^2}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{3 \left(x^3+x\right)^{2/3}}{4 x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(x + x^3)^(1/3)),x]
```

```
[Out] (-3*(x + x^3)^(2/3))/(4*x^2)
```

IntegrateAlgebraic [A] time = 0.17, size = 16, normalized size = 1.00

$$-\frac{3 \left(x^3+x\right)^{2/3}}{4 x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(x + x^3)^(1/3)),x]
```

```
[Out] (-3*(x + x^3)^(2/3))/(4*x^2)
```

fricas [A] time = 0.39, size = 12, normalized size = 0.75

$$-\frac{3 \left(x^3+x\right)^{\frac{2}{3}}}{4 \, x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3+x)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/4*(x^3 + x)^(2/3)/x^2
```

giac [A] time = 0.51, size = 9, normalized size = 0.56

$$-\frac{3}{4}\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3+x)^(1/3),x, algorithm="giac")
```

```
[Out] -3/4*(1/x^2 + 1)^(2/3)
```

maple [A] time = 0.00, size = 18, normalized size = 1.12

$$-\frac{3\left(x^2+1\right)}{4x\left(x^3+x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^3+x)^(1/3),x)
```

```
[Out] -3/4*(x^2+1)/x/(x^3+x)^(1/3)
```

maxima [A] time = 0.50, size = 17, normalized size = 1.06

$$-\frac{3\left(x^3+x\right)}{4\left(x^2+1\right)^{\frac{1}{3}}x^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3+x)^(1/3),x, algorithm="maxima")
```

```
[Out] -3/4*(x^3 + x)/((x^2 + 1)^(1/3)*x^(7/3))
```

mupad [B] time = 0.15, size = 12, normalized size = 0.75

$$-\frac{3\left(x^3+x\right)^{2/3}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x + x^3)^(1/3)),x)
```

```
[Out] -(3*(x + x^3)^(2/3))/(4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt[3]{x\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**3+x)**(1/3),x)
```

```
[Out] Integral(1/(x**2*(x*(x**2 + 1))**(1/3)), x)
```

3.77 $\int \frac{(1+x^2)(3+x^2)}{x^6 \sqrt[4]{x+x^3}} \, dx$

Optimal. Leaf size=16

$$-\frac{4\left(x^3+x\right)^{7/4}}{7x^7}$$

Rubi [B] time = 0.26, antiderivative size = 33, normalized size of antiderivative = 2.06, number of steps used = 14, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2052, 2025, 2011, 364}

$$-\frac{4\left(x^3+x\right)^{3/4}}{7x^6}-\frac{4\left(x^3+x\right)^{3/4}}{7x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(3 + x^2))/(x^6*(x + x^3)^(1/4)),x]
```

```
[Out] (-4*(x + x^3)^(3/4))/(7*x^6) - (4*(x + x^3)^(3/4))/(7*x^4)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{(1+x^2)(3+x^2)}{x^6\sqrt[4]{x+x^3}}\,dx = \int \left(\frac{3}{x^6\sqrt[4]{x+x^3}} + \frac{4}{x^4\sqrt[4]{x+x^3}} + \frac{1}{x^2\sqrt[4]{x+x^3}}\right) dx$$
$$= 3\int \frac{1}{x^6\sqrt[4]{x+x^3}}\,dx + 4\int \frac{1}{x^4\sqrt[4]{x+x^3}}\,dx + \int \frac{1}{x^2\sqrt[4]{x+x^3}}\,dx$$
$$= -\frac{4(x+x^3)^{3/4}}{7x^6} - \frac{16(x+x^3)^{3/4}}{13x^4} - \frac{4(x+x^3)^{3/4}}{5x^2} + \frac{1}{5}\int \frac{1}{\sqrt[4]{x+x^3}}\,dx - \frac{15}{7}\int \frac{1}{x^4\sqrt[4]{x+x^3}}$$
$$= -\frac{4(x+x^3)^{3/4}}{7x^6} - \frac{4(x+x^3)^{3/4}}{7x^4} + \frac{12(x+x^3)^{3/4}}{13x^2} - \frac{28}{65}\int \frac{1}{\sqrt[4]{x+x^3}}\,dx + \frac{15}{13}\int \frac{1}{x^2\sqrt[4]{x+x^3}}$$
$$= -\frac{4(x+x^3)^{3/4}}{7x^6} - \frac{4(x+x^3)^{3/4}}{7x^4} + \frac{4x\sqrt[4]{1+x^2}\,{}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-x^2\right)}{15\sqrt[4]{x+x^3}} + \frac{3}{13}\int \frac{1}{\sqrt[4]{x+x^3}}\,dx -$$
$$= -\frac{4(x+x^3)^{3/4}}{7x^6} - \frac{4(x+x^3)^{3/4}}{7x^4} - \frac{4x\sqrt[4]{1+x^2}\,{}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-x^2\right)}{13\sqrt[4]{x+x^3}} + \frac{\left(3\sqrt[4]{x}\sqrt[4]{1+x^2}\right)\int \frac{}{\sqrt[4]{x}}}{13\sqrt[4]{x+x^3}}$$
$$= -\frac{4(x+x^3)^{3/4}}{7x^6} - \frac{4(x+x^3)^{3/4}}{7x^4}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.31

$$-\frac{4(x^2+1)(x^3+x)^{3/4}}{7x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(3 + x^2))/(x^6*(x + x^3)^(1/4)),x]
[Out] (-4*(1 + x^2)*(x + x^3)^(3/4))/(7*x^6)
```

IntegrateAlgebraic [A] time = 0.40, size = 16, normalized size = 1.00

$$-\frac{4(x^3+x)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^2)*(3 + x^2))/(x^6*(x + x^3)^(1/4)),x]
[Out] (-4*(x + x^3)^(7/4))/(7*x^7)
```

fricas [A] time = 0.40, size = 17, normalized size = 1.06

$$-\frac{4(x^3+x)^{\frac{3}{4}}(x^2+1)}{7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^2+3)/x^6/(x^3+x)^(1/4),x, algorithm="fricas")
[Out] -4/7*(x^3 + x)^(3/4)*(x^2 + 1)/x^6
```

giac [A] time = 0.42, size = 11, normalized size = 0.69

$$-\frac{4}{7}\left(\frac{1}{x}+\frac{1}{x^3}\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^2+3)/x^6/(x^3+x)^(1/4),x, algorithm="giac")
[Out] -4/7*(1/x + 1/x^3)^(7/4)
```

maple [A] time = 0.00, size = 20, normalized size = 1.25

$$-\frac{4\left(x^{2}+1\right)^{2}}{7x^{5}\left(x^{3}+x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^2+3)/x^6/(x^3+x)^(1/4),x)
[Out] -4/7*(x^2+1)^2/x^5/(x^3+x)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}+3\right)\left(x^{2}+1\right)}{\left(x^{3}+x\right)^{\frac{1}{4}}x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^2+3)/x^6/(x^3+x)^(1/4),x, algorithm="maxima")
[Out] integrate((x^2 + 3)*(x^2 + 1)/((x^3 + x)^(1/4)*x^6), x)
```

mupad [B] time = 0.35, size = 27, normalized size = 1.69

$$-\frac{4\left(x^{3}+x\right)^{3/4}+4\,x^{2}\left(x^{3}+x\right)^{3/4}}{7\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*(x^2 + 3))/(x^6*(x + x^3)^(1/4)),x)
[Out] -(4*(x + x^3)^(3/4) + 4*x^2*(x + x^3)^(3/4))/(7*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}+1\right)\left(x^{2}+3\right)}{x^{6}\sqrt[4]{x\left(x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**2+3)/x**6/(x**3+x)**(1/4),x)
[Out] Integral((x**2 + 1)*(x**2 + 3)/(x**6*(x*(x**2 + 1))**(1/4)), x)
```

3.78 $\int \frac{\sqrt[3]{x+x^3}}{x^4} \, dx$

Optimal. Leaf size=16

$$-\frac{3\left(x^3+x\right)^{4/3}}{8x^4}$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2014}

$$-\frac{3\left(x^3+x\right)^{4/3}}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^3)^(1/3)/x^4,x]
```

```
[Out] (-3*(x + x^3)^(4/3))/(8*x^4)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{x+x^3}}{x^4} \, dx = -\frac{3\left(x+x^3\right)^{4/3}}{8x^4}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.31

$$-\frac{3\left(x^2+1\right)\sqrt[3]{x^3+x}}{8x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^3)^(1/3)/x^4,x]
```

```
[Out] (-3*(1 + x^2)*(x + x^3)^(1/3))/(8*x^3)
```

IntegrateAlgebraic [A] time = 0.17, size = 16, normalized size = 1.00

$$-\frac{3\left(x^3+x\right)^{4/3}}{8x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + x^3)^(1/3)/x^4,x]
```

```
[Out] (-3*(x + x^3)^(4/3))/(8*x^4)
```

fricas [A] time = 0.40, size = 17, normalized size = 1.06

$$-\frac{3\left(x^3+x\right)^{\frac{1}{3}}\left(x^2+1\right)}{8\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)^(1/3)/x^4,x, algorithm="fricas")
[Out] -3/8*(x^3 + x)^(1/3)*(x^2 + 1)/x^3
```

giac [A] time = 0.52, size = 9, normalized size = 0.56

$$-\frac{3}{8}\left(\frac{1}{x^2}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)^(1/3)/x^4,x, algorithm="giac")
[Out] -3/8*(1/x^2 + 1)^(4/3)
```

maple [A] time = 0.00, size = 18, normalized size = 1.12

$$-\frac{3\left(x^2+1\right)\left(x^3+x\right)^{\frac{1}{3}}}{8x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x)^(1/3)/x^4,x)
[Out] -3/8*(x^2+1)/x^3*(x^3+x)^(1/3)
```

maxima [A] time = 0.51, size = 17, normalized size = 1.06

$$-\frac{3\left(x^3+x\right)\left(x^2+1\right)^{\frac{1}{3}}}{8\,x^{\frac{11}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)^(1/3)/x^4,x, algorithm="maxima")
[Out] -3/8*(x^3 + x)*(x^2 + 1)^(1/3)/x^(11/3)
```

mupad [B] time = 0.16, size = 27, normalized size = 1.69

$$-\frac{3\left(x^3+x\right)^{1/3}+3\,x^2\left(x^3+x\right)^{1/3}}{8\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3)^(1/3)/x^4,x)
[Out] -(3*(x + x^3)^(1/3) + 3*x^2*(x + x^3)^(1/3))/(8*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x\left(x^2+1\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x)**(1/3)/x**4,x)
[Out] Integral((x*(x**2 + 1))**(1/3)/x**4, x)
```

3.79 $\int \frac{1}{x^4 \sqrt[4]{-1+x^4}} \, dx$

Optimal. Leaf size=16

$$\frac{(x^4-1)^{3/4}}{3x^3}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$\frac{(x^4-1)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-1 + x^4)^(1/4)),x]
```

```
[Out] (-1 + x^4)^(3/4)/(3*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4 \sqrt[4]{-1+x^4}} \, dx = \frac{(-1+x^4)^{3/4}}{3x^3}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{(x^4-1)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-1 + x^4)^(1/4)),x]
```

```
[Out] (-1 + x^4)^(3/4)/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.15, size = 16, normalized size = 1.00

$$\frac{(x^4-1)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(-1 + x^4)^(1/4)),x]
```

```
[Out] (-1 + x^4)^(3/4)/(3*x^3)
```

fricas [A] time = 0.38, size = 12, normalized size = 0.75

$$\frac{(x^4-1)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4-1)^(1/4),x, algorithm="fricas")
[Out] 1/3*(x^4 - 1)^(3/4)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4-1\right)^{\frac{1}{4}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4-1)^(1/4),x, algorithm="giac")
[Out] integrate(1/((x^4 - 1)^(1/4)*x^4), x)
```

maple [A] time = 0.00, size = 24, normalized size = 1.50

$$\frac{(-1+x)(1+x)\left(x^2+1\right)}{3x^3\left(x^4-1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^4-1)^(1/4),x)
[Out] 1/3/x^3*(-1+x)*(1+x)*(x^2+1)/(x^4-1)^(1/4)
```

maxima [A] time = 0.36, size = 12, normalized size = 0.75

$$\frac{\left(x^4-1\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4-1)^(1/4),x, algorithm="maxima")
[Out] 1/3*(x^4 - 1)^(3/4)/x^3
```

mupad [B] time = 0.22, size = 12, normalized size = 0.75

$$\frac{\left(x^4-1\right)^{3/4}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^4 - 1)^(1/4)),x)
[Out] (x^4 - 1)^(3/4)/(3*x^3)
```

sympy [A] time = 0.62, size = 63, normalized size = 3.94

$$\begin{cases} -\frac{\left(-1+\frac{1}{x^4}\right)^{\frac{3}{4}} e^{\frac{3i\pi}{4}}\Gamma\left(-\frac{3}{4}\right)}{4\Gamma\left(\frac{1}{4}\right)} & \text{for}\ \frac{1}{|x^4|}>1 \\ -\frac{\left(1-\frac{1}{x^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{4\Gamma\left(\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**4-1)**(1/4),x)

[Out] Piecewise((-(-1 + x**(-4))**(3/4)*exp(3*I*pi/4)*gamma(-3/4)/(4*gamma(1/4)),
 1/Abs(x**4) > 1), (-(1 - 1/x**4)**(3/4)*gamma(-3/4)/(4*gamma(1/4)), True))
```

3.80 $\int \frac{(-1+x^4)^{3/4}}{x^8} \, dx$

Optimal. Leaf size=16

$$\frac{(x^4-1)^{7/4}}{7x^7}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$\frac{(x^4-1)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)^(3/4)/x^8,x]
```

```
[Out] (-1 + x^4)^(7/4)/(7*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(-1+x^4)^{3/4}}{x^8} \, dx = \frac{(-1+x^4)^{7/4}}{7x^7}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{(x^4-1)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)^(3/4)/x^8,x]
```

```
[Out] (-1 + x^4)^(7/4)/(7*x^7)
```

IntegrateAlgebraic [A] time = 0.13, size = 16, normalized size = 1.00

$$\frac{(x^4-1)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)^(3/4)/x^8,x]
```

```
[Out] (-1 + x^4)^(7/4)/(7*x^7)
```

fricas [A] time = 0.39, size = 12, normalized size = 0.75

$$\frac{(x^4-1)^{\frac{7}{4}}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(3/4)/x^8,x, algorithm="fricas")
[Out] 1/7*(x^4 - 1)^(7/4)/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)^{\frac{3}{4}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(3/4)/x^8,x, algorithm="giac")
[Out] integrate((x^4 - 1)^(3/4)/x^8, x)
```

maple [A] time = 0.00, size = 24, normalized size = 1.50

$$\frac{(-1+x)(1+x)\left(x^2+1\right)\left(x^4-1\right)^{\frac{3}{4}}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(3/4)/x^8,x)
[Out] 1/7/x^7*(-1+x)*(1+x)*(x^2+1)*(x^4-1)^(3/4)
```

maxima [A] time = 0.31, size = 12, normalized size = 0.75

$$\frac{\left(x^4-1\right)^{\frac{7}{4}}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(3/4)/x^8,x, algorithm="maxima")
[Out] 1/7*(x^4 - 1)^(7/4)/x^7
```

mupad [B] time = 0.21, size = 25, normalized size = 1.56

$$-\frac{\left(x^4-1\right)^{3/4}-x^4\left(x^4-1\right)^{3/4}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)^(3/4)/x^8,x)
[Out] -((x^4 - 1)^(3/4) - x^4*(x^4 - 1)^(3/4))/(7*x^7)
```

sympy [B] time = 0.98, size = 126, normalized size = 7.88

$$\begin{cases} \frac{\left(-1+\frac{1}{x^4}\right)^{\frac{3}{4}} e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{7}{4}\right)}{4\Gamma\left(-\frac{3}{4}\right)} - \frac{\left(-1+\frac{1}{x^4}\right)^{\frac{3}{4}} e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{7}{4}\right)}{4x^4\Gamma\left(-\frac{3}{4}\right)} & \text{for}\ \frac{1}{\left|x^4\right|}>1 \\ -\frac{\left(1-\frac{1}{x^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4\Gamma\left(-\frac{3}{4}\right)} + \frac{\left(1-\frac{1}{x^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4x^4\Gamma\left(-\frac{3}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(3/4)/x**8,x)
```

```
[Out] Piecewise(((-1 + x**(-4))**(3/4)*exp(-I*pi/4)*gamma(-7/4)/(4*gamma(-3/4)) -
 (-1 + x**(-4))**(3/4)*exp(-I*pi/4)*gamma(-7/4)/(4*x**4*gamma(-3/4)), 1/Abs
(x**4) > 1), (-(1 - 1/x**4)**(3/4)*gamma(-7/4)/(4*gamma(-3/4)) + (1 - 1/x**
4)**(3/4)*gamma(-7/4)/(4*x**4*gamma(-3/4)), True))
```

3.81 $\int \frac{-1+x^4}{x^2\sqrt{x+x^3}}\,dx$

Optimal. Leaf size=16

$$\frac{2\left(x^3+x\right)^{3/2}}{3x^3}$$

Rubi [C] time = 0.09, antiderivative size = 78, normalized size of antiderivative = 4.88, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2048, 2025, 2011, 329, 220}

$$\frac{2\sqrt{x^3+x}}{3}+\frac{2\sqrt{x^3+x}}{3x^2}+\frac{\sqrt{x}\,(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{3\sqrt{x^3+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^4)/(x^2*Sqrt[x + x^3]),x]
```

```
[Out] (2*Sqrt[x + x^3])/3 + (2*Sqrt[x + x^3])/(3*x^2) + (Sqrt[x]*(1 + x)*Sqrt[(1
+ x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/(3*Sqrt[x + x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2048

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Int[(c*x)^
m*ExpandToSum[Pq - Pqq*x^q - (a*Pqq*(m + q - n + 1)*x^(q - n))/(b*(m + q +
n*p + 1)), x]*(a*x^j + b*x^n)^p, x] + Simp[(Pqq*(c*x)^(m + q - n + 1)*(a*x^
j + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /; GtQ[q, n -
1] && NeQ[m + q + n*p + 1, 0] && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*
n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] &&  !IntegerQ[p] && IGtQ
```

```
[j, 0] && IGtQ[n, 0] && LtQ[j, n]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^4}{x^2\sqrt{x+x^3}}\,dx &= \frac{2\sqrt{x+x^3}}{3}-\int \frac{1}{x^2\sqrt{x+x^3}}\,dx\\ &= \frac{2\sqrt{x+x^3}}{3}+\frac{2\sqrt{x+x^3}}{3x^2}+\frac{1}{3}\int \frac{1}{\sqrt{x+x^3}}\,dx\\ &= \frac{2\sqrt{x+x^3}}{3}+\frac{2\sqrt{x+x^3}}{3x^2}+\frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+x^2}}\,dx}{3\sqrt{x+x^3}}\\ &= \frac{2\sqrt{x+x^3}}{3}+\frac{2\sqrt{x+x^3}}{3x^2}+\frac{\left(2\sqrt{x}\sqrt{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{x+x^3}}\\ &= \frac{2\sqrt{x+x^3}}{3}+\frac{2\sqrt{x+x^3}}{3x^2}+\frac{\sqrt{x}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{3\sqrt{x+x^3}}\end{aligned}$$

Mathematica [C] time = 0.06, size = 77, normalized size = 4.81

$$\frac{2\left(x^2\left(-\sqrt{x^2+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^2\right)+x^2+1\right)+\sqrt{x^2+1}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-x^2\right)\right)}{3x\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)/(x^2*Sqrt[x + x^3]),x]
[Out] (2*(Sqrt[1 + x^2]*Hypergeometric2F1[-3/4, 1/2, 1/4, -x^2] + x^2*(1 + x^2 -
Sqrt[1 + x^2]*Hypergeometric2F1[1/4, 1/2, 5/4, -x^2])))/(3*x*Sqrt[x + x^3])
```

IntegrateAlgebraic [A] time = 0.19, size = 16, normalized size = 1.00

$$\frac{2\left(x^3+x\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/(x^2*Sqrt[x + x^3]),x]
[Out] (2*(x + x^3)^(3/2))/(3*x^3)
```

fricas [A] time = 0.40, size = 17, normalized size = 1.06

$$\frac{2\sqrt{x^3+x}\left(x^2+1\right)}{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^3+x)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(x^3 + x)*(x^2 + 1)/x^2
```

giac [A] time = 0.41, size = 21, normalized size = 1.31

$$\frac{2}{3}\sqrt{x^3+x}+\frac{2}{3}\sqrt{\frac{1}{x}+\frac{1}{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^3+x)^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt(x^3 + x) + 2/3*sqrt(1/x + 1/x^3)
```

maple [A] time = 0.00, size = 20, normalized size = 1.25

$$\frac{2\left(x^{2}+1\right)^{2}}{3\sqrt{x^{3}+x}\, x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/x^2/(x^3+x)^(1/2),x)
[Out] 2/3*(x^2+1)^2/(x^3+x)^(1/2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}-1}{\sqrt{x^{3}+x}\, x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^3+x)^(1/2),x, algorithm="maxima")
[Out] integrate((x^4 - 1)/(sqrt(x^3 + x)*x^2), x)
```

mupad [B] time = 0.03, size = 9, normalized size = 0.56

$$\frac{4\sqrt{x^{3}+x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/(x^2*(x + x^3)^(1/2)),x)
[Out] (4*(x + x^3)^(1/2))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^{2}+1\right)}{x^{2}\sqrt{x\left(x^{2}+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/x**2/(x**3+x)**(1/2),x)
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)/(x**2*sqrt(x*(x**2 + 1))), x)
```

3.82 $\int \frac{1}{x^4 \sqrt[4]{1+x^4}} \, dx$

Optimal. Leaf size=16

$$-\frac{\left(x^4+1\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {264}

$$-\frac{\left(x^4+1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + x^4)^(1/4)),x]
```

```
[Out] -1/3*(1 + x^4)^(3/4)/x^3
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4 \sqrt[4]{1+x^4}} \, dx = -\frac{\left(1+x^4\right)^{3/4}}{3x^3}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\left(x^4+1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + x^4)^(1/4)),x]
```

```
[Out] -1/3*(1 + x^4)^(3/4)/x^3
```

IntegrateAlgebraic [A] time = 0.13, size = 16, normalized size = 1.00

$$-\frac{\left(x^4+1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(1 + x^4)^(1/4)),x]
```

```
[Out] -1/3*(1 + x^4)^(3/4)/x^3
```

fricas [A] time = 0.38, size = 12, normalized size = 0.75

$$-\frac{\left(x^4+1\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+1)^(1/4),x, algorithm="fricas")
[Out] -1/3*(x^4 + 1)^(3/4)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)^{\frac{1}{4}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+1)^(1/4),x, algorithm="giac")
[Out] integrate(1/((x^4 + 1)^(1/4)*x^4), x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{\left(x^4+1\right)^{\frac{3}{4}}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^4+1)^(1/4),x)
[Out] -1/3*(x^4+1)^(3/4)/x^3
```

maxima [A] time = 0.42, size = 12, normalized size = 0.75

$$-\frac{\left(x^4+1\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+1)^(1/4),x, algorithm="maxima")
[Out] -1/3*(x^4 + 1)^(3/4)/x^3
```

mupad [B] time = 0.16, size = 12, normalized size = 0.75

$$-\frac{\left(x^4+1\right)^{3/4}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^4 + 1)^(1/4)),x)
[Out] -(x^4 + 1)^(3/4)/(3*x^3)
```

sympy [A] time = 0.58, size = 22, normalized size = 1.38

$$\frac{\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}} \Gamma\left(-\frac{3}{4}\right)}{4\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**4+1)**(1/4),x)
[Out] (1 + x**(-4))**(3/4)*gamma(-3/4)/(4*gamma(1/4))
```

3.83 $\int \frac{(-3+x^4)\sqrt[3]{1+x^4}}{x^5}\,dx$

Optimal. Leaf size=16

$$\frac{3(x^4+1)^{4/3}}{4x^4}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{446, 74\}$

$$\frac{3(x^4+1)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + x^4)*(1 + x^4)^(1/3))/x^5,x]
```

```
[Out] (3*(1 + x^4)^(4/3))/(4*x^4)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(-3+x^4)\sqrt[3]{1+x^4}}{x^5}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{(-3+x)\sqrt[3]{1+x}}{x^2}\,dx, x, x^4\right)\\ &= \frac{3(1+x^4)^{4/3}}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(x^4+1)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + x^4)*(1 + x^4)^(1/3))/x^5,x]
```

```
[Out] (3*(1 + x^4)^(4/3))/(4*x^4)
```

IntegrateAlgebraic [A] time = 0.05, size = 16, normalized size = 1.00

$$\frac{3(x^4+1)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 + x^4)^(1/3))/x^5,x]
```

```
[Out] (3*(1 + x^4)^(4/3))/(4*x^4)
```

fricas [A] time = 0.41, size = 12, normalized size = 0.75

$$\frac{3\left(x^4+1\right)^{\frac{4}{3}}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(1/3)/x^5,x, algorithm="fricas")
```

```
[Out] 3/4*(x^4 + 1)^(4/3)/x^4
```

giac [A] time = 0.27, size = 22, normalized size = 1.38

$$\frac{3}{4}\left(x^4+1\right)^{\frac{1}{3}}+\frac{3\left(x^4+1\right)^{\frac{1}{3}}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(1/3)/x^5,x, algorithm="giac")
```

```
[Out] 3/4*(x^4 + 1)^(1/3) + 3/4*(x^4 + 1)^(1/3)/x^4
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{3\left(x^4+1\right)^{\frac{4}{3}}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4+1)^(1/3)/x^5,x)
```

```
[Out] 3/4*(x^4+1)^(4/3)/x^4
```

maxima [A] time = 0.46, size = 22, normalized size = 1.38

$$\frac{3}{4}\left(x^4+1\right)^{\frac{1}{3}}+\frac{3\left(x^4+1\right)^{\frac{1}{3}}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(1/3)/x^5,x, algorithm="maxima")
```

```
[Out] 3/4*(x^4 + 1)^(1/3) + 3/4*(x^4 + 1)^(1/3)/x^4
```

mupad [B] time = 0.10, size = 12, normalized size = 0.75

$$\frac{3\left(x^4+1\right)^{4/3}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/3)*(x^4 - 3))/x^5,x)
```

```
[Out] (3*(x^4 + 1)^(4/3))/(4*x^4)
```

sympy [C] time = 178.66, size = 71, normalized size = 4.44

$$-\frac{x^{\frac{4}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{4\Gamma\left(\frac{2}{3}\right)}+\frac{3\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{4x^{\frac{8}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4+1)**(1/3)/x**5,x)
```

```
[Out] -x**(4/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), exp_polar(I*pi)/x**4)/(4*
gamma(2/3)) + 3*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), exp_polar(I*pi)/x**4)
/(4*x**(8/3)*gamma(5/3))
```

3.84 $\int \frac{(-1+x^2)(1+x^2)\sqrt{1+x^4}}{x^4}\,dx$

Optimal. Leaf size=16

$$\frac{(x^4+1)^{3/2}}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {517, 449}

$$\frac{(x^4+1)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^2)*(1 + x^2)*Sqrt[1 + x^4])/x^4,x]
```

```
[Out] (1 + x^4)^(3/2)/(3*x^3)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+x^2)(1+x^2)\sqrt{1+x^4}}{x^4}\,dx &= \int \frac{(-1+x^4)\sqrt{1+x^4}}{x^4}\,dx\\ &= \frac{(1+x^4)^{3/2}}{3x^3}\end{aligned}$$

Mathematica [C] time = 0.02, size = 40, normalized size = 2.50

$$x\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-x^4\right)+\frac{{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-x^4\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)*(1 + x^2)*Sqrt[1 + x^4])/x^4,x]
```

```
[Out] Hypergeometric2F1[-3/4, -1/2, 1/4, -x^4]/(3*x^3) + x*Hypergeometric2F1[-1/2
, 1/4, 5/4, -x^4]
```

IntegrateAlgebraic [A] time = 0.15, size = 16, normalized size = 1.00

$$\frac{\left(x^4+1\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*(1 + x^2)*Sqrt[1 + x^4])/x^4,x]
[Out] (1 + x^4)^(3/2)/(3*x^3)
```

fricas [A] time = 0.39, size = 12, normalized size = 0.75

$$\frac{\left(x^4+1\right)^{\frac{3}{2}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^2+1)*(x^4+1)^(1/2)/x^4,x, algorithm="fricas")
[Out] 1/3*(x^4 + 1)^(3/2)/x^3
```

giac [A] time = 0.50, size = 23, normalized size = 1.44

$$\frac{1}{3}\sqrt{x^4+1}\,x+\frac{\sqrt{\frac{1}{x^4}+1}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^2+1)*(x^4+1)^(1/2)/x^4,x, algorithm="giac")
[Out] 1/3*sqrt(x^4 + 1)*x + 1/3*sqrt(1/x^4 + 1)/x
```

maple [A] time = 0.01, size = 13, normalized size = 0.81

$$\frac{\left(x^4+1\right)^{\frac{3}{2}}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(x^2+1)*(x^4+1)^(1/2)/x^4,x)
[Out] 1/3*(x^4+1)^(3/2)/x^3
```

maxima [A] time = 0.54, size = 12, normalized size = 0.75

$$\frac{\left(x^4+1\right)^{\frac{3}{2}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^2+1)*(x^4+1)^(1/2)/x^4,x, algorithm="maxima")
[Out] 1/3*(x^4 + 1)^(3/2)/x^3
```

mupad [B] time = 0.18, size = 12, normalized size = 0.75

$$\frac{\left(x^4+1\right)^{3/2}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*(x^2 + 1)*(x^4 + 1)^(1/2))/x^4,x)
```

```
[Out] (x^4 + 1)^(3/2)/(3*x^3)
```

sympy [C] time = 2.05, size = 65, normalized size = 4.06

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|x^{4}e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}-\frac{\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|x^{4}e^{i\pi}\right)}{4x^{3}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**2+1)*(x**4+1)**(1/2)/x**4,x)
```

```
[Out] x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(I*pi))/(4*gamma(5/4)
) - gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), x**4*exp_polar(I*pi))/(4*x**3*g
amma(1/4))
```

3.85 $\int \frac{(-3+x^4)(1+x^4)^{2/3}}{x^6}\,dx$

Optimal. Leaf size=16

$$\frac{3(x^4+1)^{5/3}}{5x^5}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$\frac{3(x^4+1)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + x^4)*(1 + x^4)^(2/3))/x^6,x]
```

```
[Out] (3*(1 + x^4)^(5/3))/(5*x^5)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(-3+x^4)(1+x^4)^{2/3}}{x^6}\,dx = \frac{3(1+x^4)^{5/3}}{5x^5}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(x^4+1)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + x^4)*(1 + x^4)^(2/3))/x^6,x]
```

```
[Out] (3*(1 + x^4)^(5/3))/(5*x^5)
```

IntegrateAlgebraic [A] time = 0.10, size = 16, normalized size = 1.00

$$\frac{3(x^4+1)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 + x^4)^(2/3))/x^6,x]
```

```
[Out] (3*(1 + x^4)^(5/3))/(5*x^5)
```

fricas [A] time = 0.41, size = 12, normalized size = 0.75

$$\frac{3(x^4+1)^{\frac{5}{3}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)/x^6,x, algorithm="fricas")
```

```
[Out] 3/5*(x^4 + 1)^(5/3)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4+1)^{\frac{2}{3}}(x^4-3)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)/x^6,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)^(2/3)*(x^4 - 3)/x^6, x)
```

maple [A] time = 0.01, size = 13, normalized size = 0.81

$$\frac{3(x^4+1)^{\frac{5}{3}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4+1)^(2/3)/x^6,x)
```

```
[Out] 3/5*(x^4+1)^(5/3)/x^5
```

maxima [A] time = 0.62, size = 12, normalized size = 0.75

$$\frac{3(x^4+1)^{\frac{5}{3}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)/x^6,x, algorithm="maxima")
```

```
[Out] 3/5*(x^4 + 1)^(5/3)/x^5
```

mupad [B] time = 0.19, size = 27, normalized size = 1.69

$$\frac{3(x^4+1)^{2/3}+3x^4(x^4+1)^{2/3}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(2/3)*(x^4 - 3))/x^6,x)
```

```
[Out] (3*(x^4 + 1)^(2/3) + 3*x^4*(x^4 + 1)^(2/3))/(5*x^5)
```

sympy [C] time = 2.30, size = 73, normalized size = 4.56

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{3\Gamma\left(-\frac{5}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{5}{4},-\frac{2}{3}\\ -\frac{1}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4+1)**(2/3)/x**6,x)
```

```
[Out] gamma(-1/4)*hyper((-2/3, -1/4), (3/4,), x**4*exp_polar(I*pi))/(4*x*gamma(3/
4)) - 3*gamma(-5/4)*hyper((-5/4, -2/3), (-1/4,), x**4*exp_polar(I*pi))/(4*x
**5*gamma(-1/4))
```

3.86 $\int \frac{\left(1+x^4\right)^{3/4}}{x^8} dx$

Optimal. Leaf size=16

$$-\frac{\left(x^4+1\right)^{7/4}}{7x^7}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{\left(x^4+1\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)^(3/4)/x^8,x]
```

```
[Out] -1/7*(1 + x^4)^(7/4)/x^7
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(1+x^4\right)^{3/4}}{x^8} dx = -\frac{\left(1+x^4\right)^{7/4}}{7x^7}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\left(x^4+1\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)^(3/4)/x^8,x]
```

```
[Out] -1/7*(1 + x^4)^(7/4)/x^7
```

IntegrateAlgebraic [A] time = 0.13, size = 16, normalized size = 1.00

$$-\frac{\left(x^4+1\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)^(3/4)/x^8,x]
```

```
[Out] -1/7*(1 + x^4)^(7/4)/x^7
```

fricas [A] time = 0.38, size = 12, normalized size = 0.75

$$-\frac{\left(x^4+1\right)^{\frac{7}{4}}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(3/4)/x^8,x, algorithm="fricas")
[Out] -1/7*(x^4 + 1)^(7/4)/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}+1\right)^{\frac{3}{4}}}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(3/4)/x^8,x, algorithm="giac")
[Out] integrate((x^4 + 1)^(3/4)/x^8, x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{\left(x^{4}+1\right)^{\frac{7}{4}}}{7x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(3/4)/x^8,x)
[Out] -1/7*(x^4+1)^(7/4)/x^7
```

maxima [A] time = 0.50, size = 12, normalized size = 0.75

$$-\frac{\left(x^{4}+1\right)^{\frac{7}{4}}}{7\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(3/4)/x^8,x, algorithm="maxima")
[Out] -1/7*(x^4 + 1)^(7/4)/x^7
```

mupad [B] time = 0.21, size = 24, normalized size = 1.50

$$-\frac{\left(x^{4}+1\right)^{3/4}+x^{4}\left(x^{4}+1\right)^{3/4}}{7\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)^(3/4)/x^8,x)
[Out] -((x^4 + 1)^(3/4) + x^4*(x^4 + 1)^(3/4))/(7*x^7)
```

sympy [B] time = 0.90, size = 53, normalized size = 3.31

$$\frac{\left(1+\frac{1}{x^{4}}\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4\Gamma\left(-\frac{3}{4}\right)}+\frac{\left(1+\frac{1}{x^{4}}\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4x^{4}\Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(3/4)/x**8,x)
[Out] (1 + x**(-4))**(3/4)*gamma(-7/4)/(4*gamma(-3/4)) + (1 + x**(-4))**(3/4)*gam
ma(-7/4)/(4*x**4*gamma(-3/4))
```

3.87 $\int \frac{(-1+x^4)^{2/3}(3+x^4)}{x^6} dx$

Optimal. Leaf size=16

$$\frac{3(x^4-1)^{5/3}}{5x^5}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$\frac{3(x^4-1)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4)^(2/3)*(3 + x^4))/x^6,x]
```

```
[Out] (3*(-1 + x^4)^(5/3))/(5*x^5)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(-1+x^4)^{2/3}(3+x^4)}{x^6} dx = \frac{3(-1+x^4)^{5/3}}{5x^5}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(x^4-1)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)^(2/3)*(3 + x^4))/x^6,x]
```

```
[Out] (3*(-1 + x^4)^(5/3))/(5*x^5)
```

IntegrateAlgebraic [A] time = 0.10, size = 16, normalized size = 1.00

$$\frac{3(x^4-1)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)^(2/3)*(3 + x^4))/x^6,x]
```

```
[Out] (3*(-1 + x^4)^(5/3))/(5*x^5)
```

fricas [A] time = 0.40, size = 12, normalized size = 0.75

$$\frac{3(x^4-1)^{\frac{5}{3}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)/x^6,x, algorithm="fricas")
[Out] 3/5*(x^4 - 1)^(5/3)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4+3)(x^4-1)^{\frac{2}{3}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)/x^6,x, algorithm="giac")
[Out] integrate((x^4 + 3)*(x^4 - 1)^(2/3)/x^6, x)
```

maple [A] time = 0.01, size = 24, normalized size = 1.50

$$\frac{3\,(-1+x)\,(1+x)\,(x^2+1)\,(x^4-1)^{\frac{2}{3}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(2/3)*(x^4+3)/x^6,x)
[Out] 3/5/x^5*(-1+x)*(1+x)*(x^2+1)*(x^4-1)^(2/3)
```

maxima [B] time = 0.60, size = 27, normalized size = 1.69

$$\frac{3\,(x^4-1)(x^2+1)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}(x-1)^{\frac{2}{3}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)/x^6,x, algorithm="maxima")
[Out] 3/5*(x^4 - 1)*(x^2 + 1)^(2/3)*(x + 1)^(2/3)*(x - 1)^(2/3)/x^5
```

mupad [B] time = 0.19, size = 27, normalized size = 1.69

$$-\frac{3\,(x^4-1)^{2/3}-3\,x^4\,(x^4-1)^{2/3}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)^(2/3)*(x^4 + 3))/x^6,x)
[Out] -(3*(x^4 - 1)^(2/3) - 3*x^4*(x^4 - 1)^(2/3))/(5*x^5)
```

sympy [C] time = 2.40, size = 78, normalized size = 4.88

$$-\frac{e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|x^4\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{3e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},-\frac{2}{3}\\ -\frac{1}{4}\end{matrix}\right|x^4\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(2/3)*(x**4+3)/x**6,x)
[Out] -exp(-I*pi/3)*gamma(-1/4)*hyper((-2/3, -1/4), (3/4,), x**4)/(4*x*gamma(3/4)
) - 3*exp(-I*pi/3)*gamma(-5/4)*hyper((-5/4, -2/3), (-1/4,), x**4)/(4*x**5*g
amma(-1/4))
```

3.88 $\int \frac{\sqrt[3]{1+x^4}\left(3+x^4\right)}{x^9}\,dx$

Optimal. Leaf size=16

$$-\frac{3\left(x^4+1\right)^{4/3}}{8x^8}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 74}

$$-\frac{3\left(x^4+1\right)^{4/3}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^4)^(1/3)*(3 + x^4))/x^9,x]
```

```
[Out] (-3*(1 + x^4)^(4/3))/(8*x^8)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{1+x^4}\left(3+x^4\right)}{x^9}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{\sqrt[3]{1+x}\,(3+x)}{x^3}\,dx, x, x^4\right)\\ &= -\frac{3\left(1+x^4\right)^{4/3}}{8x^8}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{3\left(x^4+1\right)^{4/3}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^4)^(1/3)*(3 + x^4))/x^9,x]
```

```
[Out] (-3*(1 + x^4)^(4/3))/(8*x^8)
```

IntegrateAlgebraic [A] time = 0.05, size = 16, normalized size = 1.00

$$-\frac{3\left(x^4+1\right)^{4/3}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^4)^(1/3)*(3 + x^4))/x^9,x]
```

```
[Out] (-3*(1 + x^4)^(4/3))/(8*x^8)
```

fricas [A] time = 0.38, size = 12, normalized size = 0.75

$$-\frac{3\left(x^4+1\right)^{\frac{4}{3}}}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/3)*(x^4+3)/x^9,x, algorithm="fricas")
```

```
[Out] -3/8*(x^4 + 1)^(4/3)/x^8
```

giac [A] time = 0.55, size = 12, normalized size = 0.75

$$-\frac{3\left(x^4+1\right)^{\frac{4}{3}}}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/3)*(x^4+3)/x^9,x, algorithm="giac")
```

```
[Out] -3/8*(x^4 + 1)^(4/3)/x^8
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{3\left(x^4+1\right)^{\frac{4}{3}}}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(1/3)*(x^4+3)/x^9,x)
```

```
[Out] -3/8*(x^4+1)^(4/3)/x^8
```

maxima [B] time = 0.59, size = 50, normalized size = 3.12

$$\frac{\left(x^4+1\right)^{\frac{4}{3}}+2\left(x^4+1\right)^{\frac{1}{3}}}{8\left(2\,x^4-\left(x^4+1\right)^2+1\right)}-\frac{\left(x^4+1\right)^{\frac{1}{3}}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/3)*(x^4+3)/x^9,x, algorithm="maxima")
```

```
[Out] 1/8*((x^4 + 1)^(4/3) + 2*(x^4 + 1)^(1/3))/(2*x^4 - (x^4 + 1)^2 + 1) - 1/4*(
x^4 + 1)^(1/3)/x^4
```

mupad [B] time = 0.23, size = 12, normalized size = 0.75

$$-\frac{3\left(x^4+1\right)^{4/3}}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/3)*(x^4 + 3))/x^9,x)
```

```
[Out] -(3*(x^4 + 1)^(4/3))/(8*x^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(1/3)*(x**4+3)/x**9,x)
```

```
[Out] Timed out
```

3.89 $\int \frac{1}{x^2\sqrt{x+x^4}}\,dx$

Optimal. Leaf size=16

$$-\frac{2\sqrt{x^4+x}}{3x^2}$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2014}

$$-\frac{2\sqrt{x^4+x}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[x + x^4]),x]
[Out] (-2*Sqrt[x + x^4])/(3*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{x+x^4}}\,dx = -\frac{2\sqrt{x+x^4}}{3x^2}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2\sqrt{x^4+x}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[x + x^4]),x]
[Out] (-2*Sqrt[x + x^4])/(3*x^2)
```

IntegrateAlgebraic [A] time = 0.39, size = 16, normalized size = 1.00

$$-\frac{2\sqrt{x^4+x}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[x + x^4]),x]
[Out] (-2*Sqrt[x + x^4])/(3*x^2)
```

fricas [A] time = 0.41, size = 12, normalized size = 0.75

$$-\frac{2\sqrt{x^4+x}}{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+x)^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(x^4 + x)/x^2
```

giac [A] time = 0.39, size = 9, normalized size = 0.56

$$-\frac{2}{3}\sqrt{\frac{1}{x^3}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+x)^(1/2),x, algorithm="giac")
[Out] -2/3*sqrt(1/x^3 + 1)
```

maple [A] time = 0.00, size = 24, normalized size = 1.50

$$-\frac{2\,(1+x)\left(x^2-x+1\right)}{3x\sqrt{x^4+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^4+x)^(1/2),x)
[Out] -2/3/x*(1+x)*(x^2-x+1)/(x^4+x)^(1/2)
```

maxima [B] time = 0.67, size = 25, normalized size = 1.56

$$-\frac{2\left(x^4+x\right)}{3\sqrt{x^2-x+1}\,\sqrt{x+1}\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+x)^(1/2),x, algorithm="maxima")
[Out] -2/3*(x^4 + x)/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^(5/2))
```

mupad [B] time = 0.18, size = 12, normalized size = 0.75

$$-\frac{2\sqrt{x^4+x}}{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x + x^4)^(1/2)),x)
[Out] -(2*(x + x^4)^(1/2))/(3*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x\,(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**4+x)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(x*(x + 1)*(x**2 - x + 1))), x)
```

3.90 $\int \frac{1+x^3}{x^6 \sqrt[4]{x+x^4}} \, dx$

Optimal. Leaf size=16

$$-\frac{4\left(x^4+x\right)^{7/4}}{21x^7}$$

Rubi [B] time = 0.11, antiderivative size = 33, normalized size of antiderivative = 2.06, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2052, 2016, 2014}

$$-\frac{4\left(x^4+x\right)^{3/4}}{21x^6}-\frac{4\left(x^4+x\right)^{3/4}}{21x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(x^6*(x + x^4)^(1/4)),x]
```

```
[Out] (-4*(x + x^4)^(3/4))/(21*x^6) - (4*(x + x^4)^(3/4))/(21*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{x^6\sqrt[4]{x+x^4}}\,dx &= \int \left(\frac{1}{x^6\sqrt[4]{x+x^4}}+\frac{1}{x^3\sqrt[4]{x+x^4}}\right)dx\\
&= \int \frac{1}{x^6\sqrt[4]{x+x^4}}\,dx+\int \frac{1}{x^3\sqrt[4]{x+x^4}}\,dx\\
&= -\frac{4\left(x+x^4\right)^{3/4}}{21x^6}-\frac{4\left(x+x^4\right)^{3/4}}{9x^3}-\frac{4}{7}\int \frac{1}{x^3\sqrt[4]{x+x^4}}\,dx\\
&= -\frac{4\left(x+x^4\right)^{3/4}}{21x^6}-\frac{4\left(x+x^4\right)^{3/4}}{21x^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.31

$$-\frac{4\left(x^3+1\right)\left(x^4+x\right)^{3/4}}{21x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(x^6*(x + x^4)^(1/4)),x]
[Out] (-4*(1 + x^3)*(x + x^4)^(3/4))/(21*x^6)
```

IntegrateAlgebraic [A] time = 0.26, size = 16, normalized size = 1.00

$$-\frac{4\left(x^4+x\right)^{7/4}}{21x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)/(x^6*(x + x^4)^(1/4)),x]
[Out] (-4*(x + x^4)^(7/4))/(21*x^7)
```

fricas [A] time = 0.41, size = 17, normalized size = 1.06

$$-\frac{4\left(x^4+x\right)^{\frac{3}{4}}\left(x^3+1\right)}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x^6/(x^4+x)^(1/4),x, algorithm="fricas")
[Out] -4/21*(x^4 + x)^(3/4)*(x^3 + 1)/x^6
```

giac [A] time = 0.47, size = 9, normalized size = 0.56

$$-\frac{4}{21}\left(\frac{1}{x^3}+1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x^6/(x^4+x)^(1/4),x, algorithm="giac")
[Out] -4/21*(1/x^3 + 1)^(7/4)
```

maple [B] time = 0.01, size = 29, normalized size = 1.81

$$-\frac{4\,(1+x)\left(x^2-x+1\right)\left(x^3+1\right)}{21x^5\left(x^4+x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/x^6/(x^4+x)^(1/4),x)
[Out] -4/21/x^5*(1+x)*(x^2-x+1)*(x^3+1)/(x^4+x)^(1/4)
```

maxima [B] time = 0.44, size = 58, normalized size = 3.62

$$-\frac{4\left(x^4+x\right)}{9\left(x^2-x+1\right)^{\frac{1}{4}}(x+1)^{\frac{1}{4}}x^{\frac{13}{4}}}+\frac{4\left(4\,x^7+x^4-3\,x\right)}{63\left(x^2-x+1\right)^{\frac{1}{4}}(x+1)^{\frac{1}{4}}x^{\frac{25}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x^6/(x^4+x)^(1/4),x, algorithm="maxima")
```

```
[Out] -4/9*(x^4 + x)/((x^2 - x + 1)^(1/4)*(x + 1)^(1/4)*x^(13/4)) + 4/63*(4*x^7 +
 x^4 - 3*x)/((x^2 - x + 1)^(1/4)*(x + 1)^(1/4)*x^(25/4))
```

mupad [B] time = 0.27, size = 27, normalized size = 1.69

$$-\frac{4\left(x^4+x\right)^{3/4}+4\,x^3\left(x^4+x\right)^{3/4}}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/(x^6*(x + x^4)^(1/4)),x)
```

```
[Out] -(4*(x + x^4)^(3/4) + 4*x^3*(x + x^4)^(3/4))/(21*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)\left(x^2-x+1\right)}{x^6\sqrt[4]{x\,(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/x**6/(x**4+x)**(1/4),x)
```

```
[Out] Integral((x + 1)*(x**2 - x + 1)/(x**6*(x*(x + 1)*(x**2 - x + 1))**(1/4)), x
)
```

3.91 $\int \frac{\left(-2+x^3\right) \sqrt[3]{x+x^4}}{\left(1+x^3\right)^2} \, dx$

Optimal. Leaf size=16

$$-\frac{3x^2}{2\left(x^4+x\right)^{2/3}}$$

Rubi [A] time = 0.06, antiderivative size = 21, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2056, 449}

$$-\frac{3x\sqrt[3]{x^4+x}}{2\left(x^3+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((-2 + x^3)*(x + x^4)^(1/3))/(1 + x^3)^2,x]
```

```
[Out] (-3*x*(x + x^4)^(1/3))/(2*(1 + x^3))
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-2+x^3\right) \sqrt[3]{x+x^4}}{\left(1+x^3\right)^2} \, dx &= \frac{\sqrt[3]{x+x^4} \int \frac{\sqrt[3]{x}\left(-2+x^3\right)}{\left(1+x^3\right)^{5/3}} \, dx}{\sqrt[3]{x}\sqrt[3]{1+x^3}} \\
&= -\frac{3x\sqrt[3]{x+x^4}}{2\left(1+x^3\right)}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 61, normalized size = 3.81

$$\frac{3\sqrt[3]{x^4+x}\left(2x^4\,{}_2F_1\left(\frac{13}{9},\frac{5}{3};\frac{22}{9};-x^3\right)-13x\,{}_2F_1\left(\frac{4}{9},\frac{5}{3};\frac{13}{9};-x^3\right)\right)}{26\sqrt[3]{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-2 + x^3)*(x + x^4)^(1/3))/(1 + x^3)^2,x]
```

```
[Out] (3*(x + x^4)^(1/3)*(-13*x*Hypergeometric2F1[4/9, 5/3, 13/9, -x^3] + 2*x^4*H
ypergeometric2F1[13/9, 5/3, 22/9, -x^3]))/(26*(1 + x^3)^(1/3))
```

IntegrateAlgebraic [A] time = 0.20, size = 16, normalized size = 1.00

$$-\frac{3x^2}{2\left(x^4+x\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^3)*(x + x^4)^(1/3))/(1 + x^3)^2,x]
[Out] (-3*x^2)/(2*(x + x^4)^(2/3))
```

fricas [A] time = 0.39, size = 17, normalized size = 1.06

$$-\frac{3\left(x^4+x\right)^{\frac{1}{3}}x}{2\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^4+x)^(1/3)/(x^3+1)^2,x, algorithm="fricas")
[Out] -3/2*(x^4 + x)^(1/3)*x/(x^3 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x\right)^{\frac{1}{3}}\left(x^3-2\right)}{\left(x^3+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^4+x)^(1/3)/(x^3+1)^2,x, algorithm="giac")
[Out] integrate((x^4 + x)^(1/3)*(x^3 - 2)/(x^3 + 1)^2, x)
```

maple [A] time = 0.00, size = 18, normalized size = 1.12

$$-\frac{3x\left(x^4+x\right)^{\frac{1}{3}}}{2\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2)*(x^4+x)^(1/3)/(x^3+1)^2,x)
[Out] -3/2*x*(x^4+x)^(1/3)/(x^3+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x\right)^{\frac{1}{3}}\left(x^3-2\right)}{\left(x^3+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^4+x)^(1/3)/(x^3+1)^2,x, algorithm="maxima")
[Out] integrate((x^4 + x)^(1/3)*(x^3 - 2)/(x^3 + 1)^2, x)
```

mupad [B] time = 0.13, size = 19, normalized size = 1.19

$$-\frac{3\,x\,\left(x^4+x\right)^{1/3}}{2\,\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 2)*(x + x^4)^(1/3))/(x^3 + 1)^2,x)
```

```
[Out] -(3*x*(x + x^4)^(1/3))/(2*(x^3 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x(x+1)\left(x^2-x+1\right)}\left(x^3-2\right)}{(x+1)^2\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2)*(x**4+x)**(1/3)/(x**3+1)**2,x)
```

```
[Out] Integral((x*(x + 1)*(x**2 - x + 1))**(1/3)*(x**3 - 2)/((x + 1)**2*(x**2 - x
 + 1)**2), x)
```

3.92 $\int \frac{\sqrt[4]{x^2+x^4}}{x^2\left(1+x^2\right)}\,dx$

Optimal. Leaf size=16

$$-\frac{2\sqrt[4]{x^4+x^2}}{x}$$

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1311, 2000, 1146, 271, 264}

$$-\frac{2\sqrt[4]{x^4+x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2 + x^4)^(1/4)/(x^2*(1 + x^2)),x]
```

```
[Out] (-2*(x^2 + x^4)^(1/4))/x
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1146

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1311

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] - Dist[(c*d^2 - b*d*e + a*e^2)/(d*e*f^2),
Int[((f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1))/(d + e*x^2), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m, 0]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{x^2+x^4}}{x^2(1+x^2)}\,dx = \int \frac{1}{(x^2+x^4)^{3/4}}\,dx$$
$$= -\frac{2\sqrt[4]{x^2+x^4}}{x}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2\sqrt[4]{x^4+x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2 + x^4)^(1/4)/(x^2*(1 + x^2)),x]
[Out] (-2*(x^2 + x^4)^(1/4))/x
```

IntegrateAlgebraic [A] time = 0.10, size = 16, normalized size = 1.00

$$-\frac{2\sqrt[4]{x^4+x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2 + x^4)^(1/4)/(x^2*(1 + x^2)),x]
[Out] (-2*(x^2 + x^4)^(1/4))/x
```

fricas [A] time = 0.47, size = 14, normalized size = 0.88

$$-\frac{2\left(x^4+x^2\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2)^(1/4)/x^2/(x^2+1),x, algorithm="fricas")
[Out] -2*(x^4 + x^2)^(1/4)/x
```

giac [A] time = 0.34, size = 9, normalized size = 0.56

$$-2\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2)^(1/4)/x^2/(x^2+1),x, algorithm="giac")
[Out] -2*(1/x^2 + 1)^(1/4)
```

maple [A] time = 0.01, size = 15, normalized size = 0.94

$$-\frac{2\left(x^4+x^2\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2)^(1/4)/x^2/(x^2+1),x)
```

```
[Out] -2*(x^4+x^2)^(1/4)/x
```

maxima [A] time = 0.67, size = 17, normalized size = 1.06

$$-\frac{2\left(x^3+x\right)}{\left(x^2+1\right)^{\frac{3}{4}}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2)^(1/4)/x^2/(x^2+1),x, algorithm="maxima")
[Out] -2*(x^3 + x)/((x^2 + 1)^(3/4)*x^(3/2))
```

mupad [B] time = 0.13, size = 14, normalized size = 0.88

$$-\frac{2\left(x^4+x^2\right)^{1/4}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4)^(1/4)/(x^2*(x^2 + 1)),x)
[Out] -(2*(x^2 + x^4)^(1/4))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[4]{x^2\left(x^2+1\right)}}{x^2\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2)**(1/4)/x**2/(x**2+1),x)
[Out] Integral((x**2*(x**2 + 1))**(1/4)/(x**2*(x**2 + 1)), x)
```

3.93 $\int \frac{(-6+x^5)(-1+x^5)^{2/3}}{x^{11}} dx$

Optimal. Leaf size=16

$$-\frac{3(x^5-1)^{5/3}}{5x^{10}}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 74}

$$-\frac{3(x^5-1)^{5/3}}{5x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[((-6 + x^5)*(-1 + x^5)^(2/3))/x^11,x]
```

```
[Out] (-3*(-1 + x^5)^(5/3))/(5*x^10)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(-6+x^5)(-1+x^5)^{2/3}}{x^{11}} dx &= \frac{1}{5}\text{Subst}\left(\int \frac{(-6+x)(-1+x)^{2/3}}{x^3} dx, x, x^5\right)\\ &= -\frac{3(-1+x^5)^{5/3}}{5x^{10}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{3(x^5-1)^{5/3}}{5x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-6 + x^5)*(-1 + x^5)^(2/3))/x^11,x]
```

```
[Out] (-3*(-1 + x^5)^(5/3))/(5*x^10)
```

IntegrateAlgebraic [A] time = 0.09, size = 16, normalized size = 1.00

$$-\frac{3(x^5-1)^{5/3}}{5x^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-6 + x^5)*(-1 + x^5)^(2/3))/x^11,x]
```

```
[Out] (-3*(-1 + x^5)^(5/3))/(5*x^10)
```

fricas [A] time = 0.40, size = 12, normalized size = 0.75

$$-\frac{3\left(x^5-1\right)^{\frac{5}{3}}}{5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-6)*(x^5-1)^(2/3)/x^11,x, algorithm="fricas")
```

```
[Out] -3/5*(x^5 - 1)^(5/3)/x^10
```

giac [A] time = 0.43, size = 12, normalized size = 0.75

$$-\frac{3\left(x^5-1\right)^{\frac{5}{3}}}{5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-6)*(x^5-1)^(2/3)/x^11,x, algorithm="giac")
```

```
[Out] -3/5*(x^5 - 1)^(5/3)/x^10
```

maple [B] time = 0.01, size = 28, normalized size = 1.75

$$-\frac{3\left(-1+x\right)\left(x^4+x^3+x^2+x+1\right)\left(x^5-1\right)^{\frac{2}{3}}}{5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-6)*(x^5-1)^(2/3)/x^11,x)
```

```
[Out] -3/5/x^10*(-1+x)*(x^4+x^3+x^2+x+1)*(x^5-1)^(2/3)
```

maxima [B] time = 0.66, size = 50, normalized size = 3.12

$$-\frac{2\left(x^5-1\right)^{\frac{5}{3}}-\left(x^5-1\right)^{\frac{2}{3}}}{5\left(2\,x^5+\left(x^5-1\right)^2-1\right)}-\frac{\left(x^5-1\right)^{\frac{2}{3}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-6)*(x^5-1)^(2/3)/x^11,x, algorithm="maxima")
```

```
[Out] -1/5*(2*(x^5 - 1)^(5/3) - (x^5 - 1)^(2/3))/(2*x^5 + (x^5 - 1)^2 - 1) - 1/5*
(x^5 - 1)^(2/3)/x^5
```

mupad [B] time = 0.27, size = 12, normalized size = 0.75

$$-\frac{3\left(x^5-1\right)^{5/3}}{5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 - 1)^(2/3)*(x^5 - 6))/x^11,x)
```

[Out] -(3*(x^5 - 1)^(5/3))/(5*x^10)

sympy [C] time = 4.16, size = 71, normalized size = 4.44

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^{5}}\right)}{5x^{\frac{5}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{6\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^{5}}\right)}{5x^{\frac{20}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**5-6)*(x**5-1)**(2/3)/x**11,x)

[Out] -gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_polar(2*I*pi)/x**5)/(5*x**(5/3)*gamma(4/3)) + 6*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), exp_polar(2*I*pi)/x**5)/(5*x**(20/3)*gamma(7/3))

3.94 $\int \frac{(-4+x^5)(1+x^5)^{3/4}}{x^8} \, dx$

Optimal. Leaf size=16

$$\frac{4\left(x^5+1\right)^{7/4}}{7x^7}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$\frac{4\left(x^5+1\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((-4 + x^5)*(1 + x^5)^(3/4))/x^8,x]
```

```
[Out] (4*(1 + x^5)^(7/4))/(7*x^7)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(-4+x^5\right)\left(1+x^5\right)^{3/4}}{x^8} \, dx = \frac{4\left(1+x^5\right)^{7/4}}{7x^7}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{4\left(x^5+1\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-4 + x^5)*(1 + x^5)^(3/4))/x^8,x]
```

```
[Out] (4*(1 + x^5)^(7/4))/(7*x^7)
```

IntegrateAlgebraic [A] time = 0.14, size = 16, normalized size = 1.00

$$\frac{4\left(x^5+1\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + x^5)*(1 + x^5)^(3/4))/x^8,x]
```

```
[Out] (4*(1 + x^5)^(7/4))/(7*x^7)
```

fricas [A] time = 0.43, size = 12, normalized size = 0.75

$$\frac{4\left(x^5+1\right)^{\frac{7}{4}}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-4)*(x^5+1)^(3/4)/x^8,x, algorithm="fricas")
```

```
[Out] 4/7*(x^5 + 1)^(7/4)/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5+1\right)^{\frac{3}{4}}\left(x^5-4\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-4)*(x^5+1)^(3/4)/x^8,x, algorithm="giac")
```

```
[Out] integrate((x^5 + 1)^(3/4)*(x^5 - 4)/x^8, x)
```

maple [B] time = 0.01, size = 32, normalized size = 2.00

$$\frac{4\,(1+x)\left(x^4-x^3+x^2-x+1\right)\left(x^5+1\right)^{\frac{3}{4}}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-4)*(x^5+1)^(3/4)/x^8,x)
```

```
[Out] 4/7/x^7*(1+x)*(x^4-x^3+x^2-x+1)*(x^5+1)^(3/4)
```

maxima [B] time = 0.89, size = 33, normalized size = 2.06

$$\frac{4\left(x^5+1\right)\left(x^4-x^3+x^2-x+1\right)^{\frac{3}{4}}(x+1)^{\frac{3}{4}}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-4)*(x^5+1)^(3/4)/x^8,x, algorithm="maxima")
```

```
[Out] 4/7*(x^5 + 1)*(x^4 - x^3 + x^2 - x + 1)^(3/4)*(x + 1)^(3/4)/x^7
```

mupad [B] time = 0.24, size = 27, normalized size = 1.69

$$\frac{4\left(x^5+1\right)^{3/4}+4\,x^5\left(x^5+1\right)^{3/4}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 + 1)^(3/4)*(x^5 - 4))/x^8,x)
```

```
[Out] (4*(x^5 + 1)^(3/4) + 4*x^5*(x^5 + 1)^(3/4))/(7*x^7)
```

sympy [C] time = 2.69, size = 75, normalized size = 4.69

$$\frac{\Gamma\left(-\frac{2}{5}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{2}{5}\\ \frac{3}{5}\end{matrix}\middle| x^5e^{i\pi}\right)}{5x^2\Gamma\left(\frac{3}{5}\right)}-\frac{4\Gamma\left(-\frac{7}{5}\right){}_2F_1\left(\begin{matrix}-\frac{7}{5},-\frac{3}{4}\\ -\frac{2}{5}\end{matrix}\middle| x^5e^{i\pi}\right)}{5x^7\Gamma\left(-\frac{2}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-4)*(x**5+1)**(3/4)/x**8,x)
```

```
[Out] gamma(-2/5)*hyper((-3/4, -2/5), (3/5,), x**5*exp_polar(I*pi))/(5*x**2*gamma
(3/5)) - 4*gamma(-7/5)*hyper((-7/5, -3/4), (-2/5,), x**5*exp_polar(I*pi))/(
5*x**7*gamma(-2/5))
```

3.95 $\int \frac{(-1+x^5)^{3/4}(4+x^5)}{x^8} \, dx$

Optimal. Leaf size=16

$$\frac{4\left(x^5-1\right)^{7/4}}{7x^7}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$\frac{4\left(x^5-1\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^5)^(3/4)*(4 + x^5))/x^8,x]
```

```
[Out] (4*(-1 + x^5)^(7/4))/(7*x^7)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(-1+x^5\right)^{3/4} \left(4+x^5\right)}{x^8} \, dx = \frac{4\left(-1+x^5\right)^{7/4}}{7x^7}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{4\left(x^5-1\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^5)^(3/4)*(4 + x^5))/x^8,x]
```

```
[Out] (4*(-1 + x^5)^(7/4))/(7*x^7)
```

IntegrateAlgebraic [A] time = 0.14, size = 16, normalized size = 1.00

$$\frac{4\left(x^5-1\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^5)^(3/4)*(4 + x^5))/x^8,x]
```

```
[Out] (4*(-1 + x^5)^(7/4))/(7*x^7)
```

fricas [A] time = 0.41, size = 12, normalized size = 0.75

$$\frac{4\,\left(x^5-1\right)^{\frac{7}{4}}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(3/4)*(x^5+4)/x^8,x, algorithm="fricas")
```

```
[Out] 4/7*(x^5 - 1)^(7/4)/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^5+4)(x^5-1)^{\frac{3}{4}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(3/4)*(x^5+4)/x^8,x, algorithm="giac")
```

```
[Out] integrate((x^5 + 4)*(x^5 - 1)^(3/4)/x^8, x)
```

maple [B] time = 0.01, size = 28, normalized size = 1.75

$$\frac{4(-1+x)\left(x^4+x^3+x^2+x+1\right)\left(x^5-1\right)^{\frac{3}{4}}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)^(3/4)*(x^5+4)/x^8,x)
```

```
[Out] 4/7/x^7*(-1+x)*(x^4+x^3+x^2+x+1)*(x^5-1)^(3/4)
```

maxima [B] time = 0.65, size = 29, normalized size = 1.81

$$\frac{4\left(x^5-1\right)\left(x^4+x^3+x^2+x+1\right)^{\frac{3}{4}}(x-1)^{\frac{3}{4}}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(3/4)*(x^5+4)/x^8,x, algorithm="maxima")
```

```
[Out] 4/7*(x^5 - 1)*(x^4 + x^3 + x^2 + x + 1)^(3/4)*(x - 1)^(3/4)/x^7
```

mupad [B] time = 0.22, size = 27, normalized size = 1.69

$$-\frac{4\left(x^5-1\right)^{3/4}-4\,x^5\left(x^5-1\right)^{3/4}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 - 1)^(3/4)*(x^5 + 4))/x^8,x)
```

```
[Out] -(4*(x^5 - 1)^(3/4) - 4*x^5*(x^5 - 1)^(3/4))/(7*x^7)
```

sympy [C] time = 2.77, size = 80, normalized size = 5.00

$$-\frac{e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{2}{5}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{2}{5}\\ \frac{3}{5}\end{matrix}\middle| x^5\right)}{5x^2\Gamma\left(\frac{3}{5}\right)}-\frac{4e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{7}{5}\right){}_2F_1\left(\begin{matrix}-\frac{7}{5},-\frac{3}{4}\\ -\frac{2}{5}\end{matrix}\middle| x^5\right)}{5x^7\Gamma\left(-\frac{2}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)**(3/4)*(x**5+4)/x**8,x)
```

```
[Out] -exp(-I*pi/4)*gamma(-2/5)*hyper((-3/4, -2/5), (3/5,), x**5)/(5*x**2*gamma(3
/5)) - 4*exp(-I*pi/4)*gamma(-7/5)*hyper((-7/5, -3/4), (-2/5,), x**5)/(5*x**
7*gamma(-2/5))
```

3.96 $\int \frac{\left(1+x^5\right)^{2/3}\left(6+x^5\right)}{x^{11}}\,dx$

Optimal. Leaf size=16

$$-\frac{3\left(x^5+1\right)^{5/3}}{5x^{10}}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 74}

$$-\frac{3\left(x^5+1\right)^{5/3}}{5x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^5)^(2/3)*(6 + x^5))/x^11,x]
```

```
[Out] (-3*(1 + x^5)^(5/3))/(5*x^10)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1+x^5\right)^{2/3}\left(6+x^5\right)}{x^{11}}\,dx &= \frac{1}{5}\text{Subst}\left(\int \frac{(1+x)^{2/3}(6+x)}{x^3}\,dx, x, x^5\right)\\ &= -\frac{3\left(1+x^5\right)^{5/3}}{5x^{10}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{3\left(x^5+1\right)^{5/3}}{5x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^5)^(2/3)*(6 + x^5))/x^11,x]
```

```
[Out] (-3*(1 + x^5)^(5/3))/(5*x^10)
```

IntegrateAlgebraic [A] time = 0.08, size = 16, normalized size = 1.00

$$-\frac{3\left(x^5+1\right)^{5/3}}{5x^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^5)^(2/3)*(6 + x^5))/x^11,x]
```

```
[Out] (-3*(1 + x^5)^(5/3))/(5*x^10)
```

fricas [A] time = 0.39, size = 12, normalized size = 0.75

$$-\frac{3\left(x^5+1\right)^{\frac{5}{3}}}{5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)*(x^5+6)/x^11,x, algorithm="fricas")
```

```
[Out] -3/5*(x^5 + 1)^(5/3)/x^10
```

giac [A] time = 0.74, size = 12, normalized size = 0.75

$$-\frac{3\left(x^5+1\right)^{\frac{5}{3}}}{5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)*(x^5+6)/x^11,x, algorithm="giac")
```

```
[Out] -3/5*(x^5 + 1)^(5/3)/x^10
```

maple [B] time = 0.01, size = 32, normalized size = 2.00

$$-\frac{3\,(1+x)\left(x^4-x^3+x^2-x+1\right)\left(x^5+1\right)^{\frac{2}{3}}}{5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)^(2/3)*(x^5+6)/x^11,x)
```

```
[Out] -3/5/x^10*(1+x)*(x^4-x^3+x^2-x+1)*(x^5+1)^(2/3)
```

maxima [B] time = 0.57, size = 50, normalized size = 3.12

$$\frac{2\left(x^5+1\right)^{\frac{5}{3}}+\left(x^5+1\right)^{\frac{2}{3}}}{5\left(2\,x^5-\left(x^5+1\right)^2+1\right)}-\frac{\left(x^5+1\right)^{\frac{2}{3}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)*(x^5+6)/x^11,x, algorithm="maxima")
```

```
[Out] 1/5*(2*(x^5 + 1)^(5/3) + (x^5 + 1)^(2/3))/(2*x^5 - (x^5 + 1)^2 + 1) - 1/5*(
x^5 + 1)^(2/3)/x^5
```

mupad [B] time = 0.25, size = 12, normalized size = 0.75

$$-\frac{3\left(x^5+1\right)^{5/3}}{5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 + 1)^(2/3)*(x^5 + 6))/x^11,x)
```

```
[Out] -(3*(x^5 + 1)^(5/3))/(5*x^10)
```

sympy [C] time = 3.95, size = 70, normalized size = 4.38

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^5}\right)}{5x^{\frac{5}{3}}\Gamma\left(\frac{4}{3}\right)}-\frac{6\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^5}\right)}{5x^{\frac{20}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+1)**(2/3)*(x**5+6)/x**11,x)
```

```
[Out] -gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_polar(I*pi)/x**5)/(5*x**(5/3)*ga
mma(4/3)) - 6*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), exp_polar(I*pi)/x**5)/(
5*x**(20/3)*gamma(7/3))
```

3.97 $\int \frac{(-3+x^4)(1+x^4)}{x^6 \sqrt[4]{x+x^5}} \, dx$

Optimal. Leaf size=16

$$\frac{4(x^5+x)^{7/4}}{7x^7}$$

Rubi [B] time = 0.23, antiderivative size = 33, normalized size of antiderivative = 2.06, number of steps used = 11, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2052, 2025, 2032, 364}

$$\frac{4(x^5+x)^{3/4}}{7x^6} + \frac{4(x^5+x)^{3/4}}{7x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + x^4)*(1 + x^4))/(x^6*(x + x^5)^(1/4)),x]
```

```
[Out] (4*(x + x^5)^(3/4))/(7*x^6) + (4*(x + x^5)^(3/4))/(7*x^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{(-3+x^4)(1+x^4)}{x^6\sqrt[4]{x+x^5}}\,dx = \int \left(-\frac{3}{x^6\sqrt[4]{x+x^5}} - \frac{2}{x^2\sqrt[4]{x+x^5}} + \frac{x^2}{\sqrt[4]{x+x^5}}\right) dx$$
$$= -\left(2\int \frac{1}{x^2\sqrt[4]{x+x^5}}\,dx\right) - 3\int \frac{1}{x^6\sqrt[4]{x+x^5}}\,dx + \int \frac{x^2}{\sqrt[4]{x+x^5}}\,dx$$
$$= \frac{4(x+x^5)^{3/4}}{7x^6} + \frac{8(x+x^5)^{3/4}}{5x^2} + \frac{9}{7}\int \frac{1}{x^2\sqrt[4]{x+x^5}}\,dx - \frac{14}{5}\int \frac{x^2}{\sqrt[4]{x+x^5}}\,dx + \frac{\left(\sqrt[4]{x}\sqrt[4]{1+\right.}{\sqrt[4]{}}$$
$$= \frac{4(x+x^5)^{3/4}}{7x^6} + \frac{4(x+x^5)^{3/4}}{7x^2} + \frac{4x^3\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{4},\frac{11}{16};\frac{27}{16};-x^4\right)}{11\sqrt[4]{x+x^5}} + \frac{9}{5}\int \frac{x^2}{\sqrt[4]{x+x^5}}\,dx -$$
$$= \frac{4(x+x^5)^{3/4}}{7x^6} + \frac{4(x+x^5)^{3/4}}{7x^2} - \frac{36x^3\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{4},\frac{11}{16};\frac{27}{16};-x^4\right)}{55\sqrt[4]{x+x^5}} + \frac{\left(9\sqrt[4]{x}\sqrt[4]{1+x^4}\right)\int}{5\sqrt[4]{x+x^5}}$$
$$= \frac{4(x+x^5)^{3/4}}{7x^6} + \frac{4(x+x^5)^{3/4}}{7x^2}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.31

$$\frac{4(x^4+1)(x^5+x)^{3/4}}{7x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + x^4)*(1 + x^4))/(x^6*(x + x^5)^(1/4)),x]
[Out] (4*(1 + x^4)*(x + x^5)^(3/4))/(7*x^6)
```

IntegrateAlgebraic [A] time = 0.27, size = 16, normalized size = 1.00

$$\frac{4(x^5+x)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 + x^4))/(x^6*(x + x^5)^(1/4)),x]
[Out] (4*(x + x^5)^(7/4))/(7*x^7)
```

fricas [A] time = 0.41, size = 17, normalized size = 1.06

$$\frac{4(x^5+x)^{\frac{3}{4}}(x^4+1)}{7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)/x^6/(x^5+x)^(1/4),x, algorithm="fricas")
[Out] 4/7*(x^5 + x)^(3/4)*(x^4 + 1)/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4+1)(x^4-3)}{(x^5+x)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)/x^6/(x^5+x)^(1/4),x, algorithm="giac")
[Out] integrate((x^4 + 1)*(x^4 - 3)/((x^5 + x)^(1/4)*x^6), x)
```

maple [A] time = 0.01, size = 20, normalized size = 1.25

$$\frac{4\left(x^4+1\right)^2}{7x^5\left(x^5+x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4+1)/x^6/(x^5+x)^(1/4),x)
[Out] 4/7/x^5*(x^4+1)^2/(x^5+x)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)\left(x^4-3\right)}{\left(x^5+x\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)/x^6/(x^5+x)^(1/4),x, algorithm="maxima")
[Out] integrate((x^4 + 1)*(x^4 - 3)/((x^5 + x)^(1/4)*x^6), x)
```

mupad [B] time = 0.29, size = 27, normalized size = 1.69

$$\frac{4\left(x^5+x\right)^{3/4}+4\,x^4\left(x^5+x\right)^{3/4}}{7\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)*(x^4 - 3))/(x^6*(x + x^5)^(1/4)),x)
[Out] (4*(x + x^5)^(3/4) + 4*x^4*(x + x^5)^(3/4))/(7*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-3\right)\left(x^4+1\right)}{x^6\sqrt[4]{x\left(x^4+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4+1)/x**6/(x**5+x)**(1/4),x)
[Out] Integral((x**4 - 3)*(x**4 + 1)/(x**6*(x*(x**4 + 1))**(1/4)), x)
```

3.98 $\int \frac{(-1+x^2)\sqrt[4]{x^3+x^5}}{x^2(1+x^2)}\,dx$

Optimal. Leaf size=16

$$\frac{4\sqrt[4]{x^5+x^3}}{x}$$

Rubi [A] time = 0.14, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2056, 449}

$$\frac{4\sqrt[4]{x^5+x^3}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^2)*(x^3 + x^5)^(1/4))/(x^2*(1 + x^2)),x]
```

```
[Out] (4*(x^3 + x^5)^(1/4))/x
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+x^2)\sqrt[4]{x^3+x^5}}{x^2(1+x^2)}\,dx &= \frac{\sqrt[4]{x^3+x^5}\int \frac{-1+x^2}{x^{5/4}(1+x^2)^{3/4}}\,dx}{x^{3/4}\sqrt[4]{1+x^2}}\\ &= \frac{4\sqrt[4]{x^3+x^5}}{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{4\sqrt[4]{x^5+x^3}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)*(x^3 + x^5)^(1/4))/(x^2*(1 + x^2)),x]
```

```
[Out] (4*(x^3 + x^5)^(1/4))/x
```

IntegrateAlgebraic [A] time = 0.10, size = 16, normalized size = 1.00

$$\frac{4\sqrt[4]{x^5+x^3}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*(x^3 + x^5)^(1/4))/(x^2*(1 + x^2)),x]
[Out] (4*(x^3 + x^5)^(1/4))/x
```

fricas [A] time = 0.44, size = 14, normalized size = 0.88

$$\frac{4\left(x^5+x^3\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^5+x^3)^(1/4)/x^2/(x^2+1),x, algorithm="fricas")
[Out] 4*(x^5 + x^3)^(1/4)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5+x^3\right)^{\frac{1}{4}}\left(x^2-1\right)}{\left(x^2+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^5+x^3)^(1/4)/x^2/(x^2+1),x, algorithm="giac")
[Out] integrate((x^5 + x^3)^(1/4)*(x^2 - 1)/((x^2 + 1)*x^2), x)
```

maple [A] time = 0.01, size = 15, normalized size = 0.94

$$\frac{4\left(x^5+x^3\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(x^5+x^3)^(1/4)/x^2/(x^2+1),x)
[Out] 4*(x^5+x^3)^(1/4)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5+x^3\right)^{\frac{1}{4}}\left(x^2-1\right)}{\left(x^2+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^5+x^3)^(1/4)/x^2/(x^2+1),x, algorithm="maxima")
[Out] integrate((x^5 + x^3)^(1/4)*(x^2 - 1)/((x^2 + 1)*x^2), x)
```

mupad [B] time = 0.19, size = 14, normalized size = 0.88

$$\frac{4\left(x^5+x^3\right)^{1/4}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + x^5)^(1/4)*(x^2 - 1))/(x^2*(x^2 + 1)),x)
[Out] (4*(x^3 + x^5)^(1/4))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\left(x^2+1\right)}\,(x-1)\,(x+1)}{x^2\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**5+x**3)**(1/4)/x**2/(x**2+1),x)
```

```
[Out] Integral((x**3*(x**2 + 1))**(1/4)*(x - 1)*(x + 1)/(x**2*(x**2 + 1)), x)
```

3.99 $\int \frac{-2b+3ax^5}{\sqrt{b+ax^5}\left(b+x^2+ax^5\right)}\,dx$

Optimal. Leaf size=16

$$-2\tan^{-1}\left(\frac{x}{\sqrt{ax^5+b}}\right)$$

Rubi [F] time = 0.84, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2b+3ax^5}{\sqrt{b+ax^5}\left(b+x^2+ax^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*b + 3*a*x^5)/(Sqrt[b + a*x^5]*(b + x^2 + a*x^5)),x]
```

```
[Out] (3*x*Sqrt[1 + (a*x^5)/b]*Hypergeometric2F1[1/5, 1/2, 6/5, -((a*x^5)/b)])/Sq
rt[b + a*x^5] - 5*b*Defer[Int][1/(Sqrt[b + a*x^5]*(b + x^2 + a*x^5)), x] -
3*Defer[Int][x^2/(Sqrt[b + a*x^5]*(b + x^2 + a*x^5)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2b+3ax^5}{\sqrt{b+ax^5}\left(b+x^2+ax^5\right)}\,dx &= \int \left(\frac{3}{\sqrt{b+ax^5}}-\frac{5b+3x^2}{\sqrt{b+ax^5}\left(b+x^2+ax^5\right)}\right)dx\\
&= 3\int \frac{1}{\sqrt{b+ax^5}}\,dx-\int \frac{5b+3x^2}{\sqrt{b+ax^5}\left(b+x^2+ax^5\right)}\,dx\\
&= \frac{\left(3\sqrt{1+\frac{ax^5}{b}}\right)\int \frac{1}{\sqrt{1+\frac{ax^5}{b}}}\,dx}{\sqrt{b+ax^5}}-\int \left(\frac{5b}{\sqrt{b+ax^5}\left(b+x^2+ax^5\right)}+\frac{3x^2}{\sqrt{b+ax^5}\left(b+\right.}\right.\\
&= \frac{3x\sqrt{1+\frac{ax^5}{b}}\;{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{6}{5};-\frac{ax^5}{b}\right)}{\sqrt{b+ax^5}}-3\int \frac{x^2}{\sqrt{b+ax^5}\left(b+x^2+ax^5\right)}\,dx-(5b)\int
\end{aligned}$$

Mathematica [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{-2b+3ax^5}{\sqrt{b+ax^5}\left(b+x^2+ax^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*b + 3*a*x^5)/(Sqrt[b + a*x^5]*(b + x^2 + a*x^5)),x]
```

```
[Out] Integrate[(-2*b + 3*a*x^5)/(Sqrt[b + a*x^5]*(b + x^2 + a*x^5)), x]
```

IntegrateAlgebraic [A] time = 4.31, size = 16, normalized size = 1.00

$$-2\tan^{-1}\left(\frac{x}{\sqrt{ax^5+b}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*b + 3*a*x^5)/(Sqrt[b + a*x^5]*(b + x^2 + a*x^5)),x]
```

```
[Out] -2*ArcTan[x/Sqrt[b + a*x^5]]
```

fricas [B] time = 0.51, size = 35, normalized size = 2.19

$$\arctan\left(\frac{\left(ax^5 - x^2 + b\right)\sqrt{ax^5 + b}}{2\left(ax^6 + bx\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*x^5-2*b)/(a*x^5+b)^(1/2)/(a*x^5+x^2+b),x, algorithm="fricas"
)
```

```
[Out] arctan(1/2*(a*x^5 - x^2 + b)*sqrt(a*x^5 + b)/(a*x^6 + b*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,ax^5 - 2\,b}{\left(ax^5 + x^2 + b\right)\sqrt{ax^5 + b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*x^5-2*b)/(a*x^5+b)^(1/2)/(a*x^5+x^2+b),x, algorithm="giac")
```

```
[Out] integrate((3*a*x^5 - 2*b)/((a*x^5 + x^2 + b)*sqrt(a*x^5 + b)), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{3a\,x^5 - 2b}{\sqrt{a\,x^5 + b}\,\left(a\,x^5 + x^2 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a*x^5-2*b)/(a*x^5+b)^(1/2)/(a*x^5+x^2+b),x)
```

```
[Out] int((3*a*x^5-2*b)/(a*x^5+b)^(1/2)/(a*x^5+x^2+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,ax^5 - 2\,b}{\left(ax^5 + x^2 + b\right)\sqrt{ax^5 + b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*x^5-2*b)/(a*x^5+b)^(1/2)/(a*x^5+x^2+b),x, algorithm="maxima"
)
```

```
[Out] integrate((3*a*x^5 - 2*b)/((a*x^5 + x^2 + b)*sqrt(a*x^5 + b)), x)
```

mupad [B] time = 2.26, size = 42, normalized size = 2.62

$$\ln\left(\frac{b + a\,x^5 - x^2 + x\,\sqrt{a\,x^5 + b}\,2\mathrm{i}}{a\,x^5 + x^2 + b}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*b - 3*a*x^5)/((b + a*x^5)^(1/2)*(b + a*x^5 + x^2)),x)
```

```
[Out] log((b + x*(b + a*x^5)^(1/2)*2i + a*x^5 - x^2)/(b + a*x^5 + x^2))*1i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3ax^5 - 2b}{\sqrt{ax^5 + b}\,\left(ax^5 + b + x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*x**5-2*b)/(a*x**5+b)**(1/2)/(a*x**5+x**2+b),x)
```

```
[Out] Integral((3*a*x**5 - 2*b)/(sqrt(a*x**5 + b)*(a*x**5 + b + x**2)), x)
```

3.100 $\int \frac{1}{x^4\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=16

$$\frac{\sqrt{x^6-1}}{3x^3}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$\frac{\sqrt{x^6-1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[-1 + x^6]),x]
[Out] Sqrt[-1 + x^6]/(3*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{-1+x^6}}\,dx = \frac{\sqrt{-1+x^6}}{3x^3}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\sqrt{x^6-1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[-1 + x^6]),x]
[Out] Sqrt[-1 + x^6]/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.16, size = 16, normalized size = 1.00

$$\frac{\sqrt{x^6-1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[-1 + x^6]),x]
[Out] Sqrt[-1 + x^6]/(3*x^3)
```

fricas [A] time = 0.41, size = 16, normalized size = 1.00

$$\frac{x^3+\sqrt{x^6-1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + sqrt(x^6 - 1))/x^3
```

giac [A] time = 0.36, size = 20, normalized size = 1.25

$$\frac{\sqrt{-\frac{1}{x^6}+1}}{3\,\mathrm{sgn}(x)}-\frac{1}{3}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(-1/x^6 + 1)/sgn(x) - 1/3*sgn(x)
```

maple [B] time = 0.01, size = 33, normalized size = 2.06

$$\frac{(-1+x)\,(1+x)\left(x^2+x+1\right)\left(x^2-x+1\right)}{3x^3\sqrt{x^6-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^6-1)^(1/2),x)
```

```
[Out] 1/3/x^3*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)/(x^6-1)^(1/2)
```

maxima [A] time = 0.45, size = 12, normalized size = 0.75

$$\frac{\sqrt{x^6-1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^6 - 1)/x^3
```

mupad [B] time = 0.30, size = 12, normalized size = 0.75

$$\frac{\sqrt{x^6-1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^6 - 1)^(1/2)),x)
```

```
[Out] (x^6 - 1)^(1/2)/(3*x^3)
```

sympy [A] time = 0.58, size = 29, normalized size = 1.81

$$\begin{cases} \frac{i\sqrt{-1+\frac{1}{x^6}}}{3} & \text{for}\ \frac{1}{|x^6|}>1 \\ \frac{\sqrt{1-\frac{1}{x^6}}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**6-1)**(1/2),x)
```

```
[Out] Piecewise((I*sqrt(-1 + x**(-6)))/3, 1/Abs(x**6) > 1), (sqrt(1 - 1/x**6)/3, T
rue))
```

3.101 $\int \frac{\sqrt[3]{-1+x^6}}{x^9} dx$

Optimal. Leaf size=16

$$\frac{(x^6-1)^{4/3}}{8x^8}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$\frac{(x^6-1)^{4/3}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)^(1/3)/x^9,x]
```

```
[Out] (-1 + x^6)^(4/3)/(8*x^8)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{-1+x^6}}{x^9} dx = \frac{(-1+x^6)^{4/3}}{8x^8}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{(x^6-1)^{4/3}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)^(1/3)/x^9,x]
```

```
[Out] (-1 + x^6)^(4/3)/(8*x^8)
```

IntegrateAlgebraic [A] time = 0.62, size = 16, normalized size = 1.00

$$\frac{(x^6-1)^{4/3}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^6)^(1/3)/x^9,x]
```

```
[Out] (-1 + x^6)^(4/3)/(8*x^8)
```

fricas [A] time = 0.42, size = 12, normalized size = 0.75

$$\frac{(x^6-1)^{\frac{4}{3}}}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)/x^9,x, algorithm="fricas")
```

```
[Out] 1/8*(x^6 - 1)^(4/3)/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)/x^9,x, algorithm="giac")
```

```
[Out] integrate((x^6 - 1)^(1/3)/x^9, x)
```

maple [B] time = 0.02, size = 33, normalized size = 2.06

$$\frac{(-1+x)\,(1+x)\left(x^2+x+1\right)\left(x^2-x+1\right)\left(x^6-1\right)^{\frac{1}{3}}}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/3)/x^9,x)
```

```
[Out] 1/8/x^8*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)*(x^6-1)^(1/3)
```

maxima [A] time = 0.45, size = 12, normalized size = 0.75

$$\frac{\left(x^6-1\right)^{\frac{4}{3}}}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)/x^9,x, algorithm="maxima")
```

```
[Out] 1/8*(x^6 - 1)^(4/3)/x^8
```

mupad [B] time = 0.23, size = 12, normalized size = 0.75

$$\frac{\left(x^6-1\right)^{4/3}}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)^(1/3)/x^9,x)
```

```
[Out] (x^6 - 1)^(4/3)/(8*x^8)
```

sympy [B] time = 0.87, size = 129, normalized size = 8.06

$$\begin{cases} \frac{\sqrt[3]{-1+\frac{1}{x^6}}\,e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{6\Gamma\left(-\frac{1}{3}\right)} - \frac{\sqrt[3]{-1+\frac{1}{x^6}}\,e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{6x^6\Gamma\left(-\frac{1}{3}\right)} & \text{for}\ \frac{1}{|x^6|}>1 \\ -\frac{\sqrt[3]{1-\frac{1}{x^6}}\,\Gamma\left(-\frac{4}{3}\right)}{6\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt[3]{1-\frac{1}{x^6}}\,\Gamma\left(-\frac{4}{3}\right)}{6x^6\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/3)/x**9,x)
```

```
[Out] Piecewise(((-1 + x**(-6))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(6*gamma(-1/3))
 - (-1 + x**(-6))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(6*x**6*gamma(-1/3)), 1
/Abs(x**6) > 1), (-(1 - 1/x**6)**(1/3)*gamma(-4/3)/(6*gamma(-1/3)) + (1 - 1
/x**6)**(1/3)*gamma(-4/3)/(6*x**6*gamma(-1/3)), True))
```

3.102 $\int \frac{\sqrt{-1+x^6}}{x^{10}} \, dx$

Optimal. Leaf size=16

$$\frac{\left(x^6-1\right)^{3/2}}{9x^9}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$\frac{\left(x^6-1\right)^{3/2}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^6]/x^10,x]
```

```
[Out] (-1 + x^6)^(3/2)/(9*x^9)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{-1+x^6}}{x^{10}} \, dx = \frac{\left(-1+x^6\right)^{3/2}}{9x^9}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\left(x^6-1\right)^{3/2}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^6]/x^10,x]
```

```
[Out] (-1 + x^6)^(3/2)/(9*x^9)
```

IntegrateAlgebraic [A] time = 0.14, size = 16, normalized size = 1.00

$$\frac{\left(x^6-1\right)^{3/2}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x^6]/x^10,x]
```

```
[Out] (-1 + x^6)^(3/2)/(9*x^9)
```

fricas [A] time = 0.40, size = 16, normalized size = 1.00

$$\frac{x^9 + \left(x^6-1\right)^{\frac{3}{2}}}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/9*(x^9 + (x^6 - 1)^(3/2))/x^9
```

giac [A] time = 0.62, size = 20, normalized size = 1.25

$$\frac{\left(-\frac{1}{x^6}+1\right)^{\frac{3}{2}}}{9\,\mathrm{sgn}(x)}-\frac{1}{9}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/9*(-1/x^6 + 1)^(3/2)/sgn(x) - 1/9*sgn(x)
```

maple [B] time = 0.01, size = 33, normalized size = 2.06

$$\frac{(-1+x)\,(1+x)\left(x^2+x+1\right)\left(x^2-x+1\right)\sqrt{x^6-1}}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/2)/x^10,x)
```

```
[Out] 1/9/x^9*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)*(x^6-1)^(1/2)
```

maxima [A] time = 0.53, size = 12, normalized size = 0.75

$$\frac{\left(x^6-1\right)^{\frac{3}{2}}}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] 1/9*(x^6 - 1)^(3/2)/x^9
```

mupad [B] time = 0.28, size = 12, normalized size = 0.75

$$\frac{\left(x^6-1\right)^{3/2}}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)^(1/2)/x^10,x)
```

```
[Out] (x^6 - 1)^(3/2)/(9*x^9)
```

sympy [B] time = 0.87, size = 61, normalized size = 3.81

$$\begin{cases} \frac{i\sqrt{-1+\frac{1}{x^6}}}{9}-\frac{i\sqrt{-1+\frac{1}{x^6}}}{9x^6} & \text{for}\ \frac{1}{|x^6|}>1 \\ \frac{\sqrt{1-\frac{1}{x^6}}}{9}-\frac{\sqrt{1-\frac{1}{x^6}}}{9x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/2)/x**10,x)
```

```
[Out] Piecewise((I*sqrt(-1 + x**(-6))/9 - I*sqrt(-1 + x**(-6))/(9*x**6), 1/Abs(x*
*6) > 1), (sqrt(1 - 1/x**6)/9 - sqrt(1 - 1/x**6)/(9*x**6), True))
```

3.103 $\int \frac{1}{x^4\sqrt{1+x^6}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sqrt{x^6+1}}{3x^3}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{\sqrt{x^6+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[1 + x^6]),x]
[Out] -1/3*Sqrt[1 + x^6]/x^3
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{1+x^6}}\,dx = -\frac{\sqrt{1+x^6}}{3x^3}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\sqrt{x^6+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[1 + x^6]),x]
[Out] -1/3*Sqrt[1 + x^6]/x^3
```

IntegrateAlgebraic [A] time = 0.15, size = 16, normalized size = 1.00

$$-\frac{\sqrt{x^6+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[1 + x^6]),x]
[Out] -1/3*Sqrt[1 + x^6]/x^3
```

fricas [A] time = 0.39, size = 16, normalized size = 1.00

$$-\frac{x^3+\sqrt{x^6+1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+1)^(1/2),x, algorithm="fricas")
[Out] -1/3*(x^3 + sqrt(x^6 + 1))/x^3
```

giac [A] time = 0.37, size = 18, normalized size = 1.12

$$-\frac{\sqrt{\frac{1}{x^6}+1}}{3\,\mathrm{sgn}(x)}+\frac{1}{3}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+1)^(1/2),x, algorithm="giac")
[Out] -1/3*sqrt(1/x^6 + 1)/sgn(x) + 1/3*sgn(x)
```

maple [B] time = 0.01, size = 28, normalized size = 1.75

$$-\frac{\left(x^2+1\right)\left(x^4-x^2+1\right)}{3x^3\sqrt{x^6+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^6+1)^(1/2),x)
[Out] -1/3/x^3*(x^2+1)*(x^4-x^2+1)/(x^6+1)^(1/2)
```

maxima [A] time = 0.38, size = 12, normalized size = 0.75

$$-\frac{\sqrt{x^6+1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+1)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(x^6 + 1)/x^3
```

mupad [B] time = 0.22, size = 12, normalized size = 0.75

$$-\frac{\sqrt{x^6+1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^6 + 1)^(1/2)),x)
[Out] -(x^6 + 1)^(1/2)/(3*x^3)
```

sympy [A] time = 0.55, size = 12, normalized size = 0.75

$$-\frac{\sqrt{1+\frac{1}{x^6}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**6+1)**(1/2),x)
[Out] -sqrt(1 + x**(-6))/3
```

3.104 $\int \frac{1}{x^5\sqrt[3]{1+x^6}}\,dx$

Optimal. Leaf size=16

$$-\frac{\left(x^6+1\right)^{2/3}}{4x^4}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{\left(x^6+1\right)^{2/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 + x^6)^(1/3)),x]
```

```
[Out] -1/4*(1 + x^6)^(2/3)/x^4
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt[3]{1+x^6}}\,dx = -\frac{\left(1+x^6\right)^{2/3}}{4x^4}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\left(x^6+1\right)^{2/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 + x^6)^(1/3)),x]
```

```
[Out] -1/4*(1 + x^6)^(2/3)/x^4
```

IntegrateAlgebraic [A] time = 0.77, size = 16, normalized size = 1.00

$$-\frac{\left(x^6+1\right)^{2/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(1 + x^6)^(1/3)),x]
```

```
[Out] -1/4*(1 + x^6)^(2/3)/x^4
```

fricas [A] time = 0.42, size = 12, normalized size = 0.75

$$-\frac{\left(x^6+1\right)^{\frac{2}{3}}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(x^6 + 1)^(2/3)/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+1\right)^{\frac{1}{3}} x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^6 + 1)^(1/3)*x^5), x)
```

maple [B] time = 0.00, size = 28, normalized size = 1.75

$$-\frac{\left(x^2+1\right)\left(x^4-x^2+1\right)}{4x^4\left(x^6+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^6+1)^(1/3),x)
```

```
[Out] -1/4/x^4*(x^2+1)*(x^4-x^2+1)/(x^6+1)^(1/3)
```

maxima [A] time = 0.37, size = 12, normalized size = 0.75

$$-\frac{\left(x^6+1\right)^{\frac{2}{3}}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/4*(x^6 + 1)^(2/3)/x^4
```

mupad [B] time = 0.18, size = 12, normalized size = 0.75

$$-\frac{\left(x^6+1\right)^{2/3}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^6 + 1)^(1/3)),x)
```

```
[Out] -(x^6 + 1)^(2/3)/(4*x^4)
```

sympy [A] time = 0.58, size = 22, normalized size = 1.38

$$\frac{\left(1+\frac{1}{x^6}\right)^{\frac{2}{3}}\Gamma\left(-\frac{2}{3}\right)}{6\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**6+1)**(1/3),x)
```

```
[Out] (1 + x**(-6))**(2/3)*gamma(-2/3)/(6*gamma(1/3))
```

3.105 $\int \frac{-2+x^6}{x^4 \sqrt[4]{1+x^6}} \, dx$

Optimal. Leaf size=16

$$\frac{2\left(x^6+1\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$\frac{2\left(x^6+1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^6)/(x^4*(1 + x^6)^(1/4)),x]
```

```
[Out] (2*(1 + x^6)^(3/4))/(3*x^3)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-2+x^6}{x^4 \sqrt[4]{1+x^6}} \, dx = \frac{2\left(1+x^6\right)^{3/4}}{3x^3}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2\left(x^6+1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^6)/(x^4*(1 + x^6)^(1/4)),x]
```

```
[Out] (2*(1 + x^6)^(3/4))/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.17, size = 16, normalized size = 1.00

$$\frac{2\left(x^6+1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x^6)/(x^4*(1 + x^6)^(1/4)),x]
```

```
[Out] (2*(1 + x^6)^(3/4))/(3*x^3)
```

fricas [A] time = 0.41, size = 12, normalized size = 0.75

$$\frac{2\left(x^6+1\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)/x^4/(x^6+1)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/3*(x^6 + 1)^(3/4)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 - 2}{\left(x^6 + 1\right)^{\frac{1}{4}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)/x^4/(x^6+1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^6 - 2)/((x^6 + 1)^(1/4)*x^4), x)
```

maple [B] time = 0.01, size = 28, normalized size = 1.75

$$\frac{2\left(x^2+1\right)\left(x^4-x^2+1\right)}{3x^3\left(x^6+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-2)/x^4/(x^6+1)^(1/4),x)
```

```
[Out] 2/3/x^3*(x^2+1)*(x^4-x^2+1)/(x^6+1)^(1/4)
```

maxima [B] time = 0.55, size = 29, normalized size = 1.81

$$\frac{2\left(x^6+1\right)}{3\left(x^4-x^2+1\right)^{\frac{1}{4}}\left(x^2+1\right)^{\frac{1}{4}} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)/x^4/(x^6+1)^(1/4),x, algorithm="maxima")
```

```
[Out] 2/3*(x^6 + 1)/((x^4 - x^2 + 1)^(1/4)*(x^2 + 1)^(1/4)*x^3)
```

mupad [B] time = 0.04, size = 12, normalized size = 0.75

$$\frac{2\left(x^6+1\right)^{3/4}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 2)/(x^4*(x^6 + 1)^(1/4)),x)
```

```
[Out] (2*(x^6 + 1)^(3/4))/(3*x^3)
```

sympy [C] time = 2.59, size = 42, normalized size = 2.62

$$\frac{x^3 {}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{1}{2}\\ \frac{3}{2}\end{matrix}\right| x^6 e^{i\pi}\right)}{3} + \frac{2 {}_2F_1\left(\left.\begin{matrix}-\frac{1}{2}, \frac{1}{4}\\ \frac{1}{2}\end{matrix}\right| x^6 e^{i\pi}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-2)/x**4/(x**6+1)**(1/4),x)
```

```
[Out] x**3*hyper((1/4, 1/2), (3/2,), x**6*exp_polar(I*pi))/3 + 2*hyper((-1/2, 1/4
), (1/2,), x**6*exp_polar(I*pi))/(3*x**3)
```

3.106 $\int \frac{\sqrt{1+x^6}}{x^{10}} \, dx$

Optimal. Leaf size=16

$$-\frac{\left(x^6+1\right)^{3/2}}{9x^9}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{\left(x^6+1\right)^{3/2}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^6]/x^10,x]
```

```
[Out] -1/9*(1 + x^6)^(3/2)/x^9
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{1+x^6}}{x^{10}} \, dx = -\frac{\left(1+x^6\right)^{3/2}}{9x^9}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\left(x^6+1\right)^{3/2}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^6]/x^10,x]
```

```
[Out] -1/9*(1 + x^6)^(3/2)/x^9
```

IntegrateAlgebraic [A] time = 0.14, size = 16, normalized size = 1.00

$$-\frac{\left(x^6+1\right)^{3/2}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^6]/x^10,x]
```

```
[Out] -1/9*(1 + x^6)^(3/2)/x^9
```

fricas [A] time = 0.45, size = 16, normalized size = 1.00

$$-\frac{x^9+\left(x^6+1\right)^{\frac{3}{2}}}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/2)/x^10,x, algorithm="fricas")
[Out] -1/9*(x^9 + (x^6 + 1)^(3/2))/x^9
```

giac [A] time = 0.45, size = 18, normalized size = 1.12

$$-\frac{\left(\frac{1}{x^6}+1\right)^{\frac{3}{2}}}{9\,\mathrm{sgn}(x)}+\frac{1}{9}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/2)/x^10,x, algorithm="giac")
[Out] -1/9*(1/x^6 + 1)^(3/2)/sgn(x) + 1/9*sgn(x)
```

maple [B] time = 0.01, size = 28, normalized size = 1.75

$$-\frac{\left(x^2+1\right)\left(x^4-x^2+1\right)\sqrt{x^6+1}}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)^(1/2)/x^10,x)
[Out] -1/9/x^9*(x^2+1)*(x^4-x^2+1)*(x^6+1)^(1/2)
```

maxima [A] time = 0.35, size = 12, normalized size = 0.75

$$-\frac{\left(x^6+1\right)^{\frac{3}{2}}}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/2)/x^10,x, algorithm="maxima")
[Out] -1/9*(x^6 + 1)^(3/2)/x^9
```

mupad [B] time = 0.28, size = 12, normalized size = 0.75

$$-\frac{\left(x^6+1\right)^{3/2}}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)^(1/2)/x^10,x)
[Out] -(x^6 + 1)^(3/2)/(9*x^9)
```

sympy [A] time = 0.79, size = 27, normalized size = 1.69

$$-\frac{\sqrt{1+\frac{1}{x^6}}}{9}-\frac{\sqrt{1+\frac{1}{x^6}}}{9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)**(1/2)/x**10,x)
[Out] -sqrt(1 + x**(-6))/9 - sqrt(1 + x**(-6))/(9*x**6)
```

3.107 $\int \frac{(-2+x^6)(1+x^6)^{3/4}}{x^8}\,dx$

Optimal. Leaf size=16

$$\frac{2\left(x^6+1\right)^{7/4}}{7x^7}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$\frac{2\left(x^6+1\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((-2 + x^6)*(1 + x^6)^(3/4))/x^8,x]
```

```
[Out] (2*(1 + x^6)^(7/4))/(7*x^7)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(-2+x^6\right)\left(1+x^6\right)^{3/4}}{x^8}\,dx=\frac{2\left(1+x^6\right)^{7/4}}{7x^7}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2\left(x^6+1\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-2 + x^6)*(1 + x^6)^(3/4))/x^8,x]
```

```
[Out] (2*(1 + x^6)^(7/4))/(7*x^7)
```

IntegrateAlgebraic [A] time = 0.17, size = 16, normalized size = 1.00

$$\frac{2\left(x^6+1\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^6)*(1 + x^6)^(3/4))/x^8,x]
```

```
[Out] (2*(1 + x^6)^(7/4))/(7*x^7)
```

fricas [A] time = 0.42, size = 12, normalized size = 0.75

$$\frac{2\left(x^6+1\right)^{\frac{7}{4}}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6+1)^(3/4)/x^8,x, algorithm="fricas")
```

```
[Out] 2/7*(x^6 + 1)^(7/4)/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+1\right)^{\frac{3}{4}}\left(x^6-2\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6+1)^(3/4)/x^8,x, algorithm="giac")
```

```
[Out] integrate((x^6 + 1)^(3/4)*(x^6 - 2)/x^8, x)
```

maple [B] time = 0.01, size = 28, normalized size = 1.75

$$\frac{2\left(x^2+1\right)\left(x^4-x^2+1\right)\left(x^6+1\right)^{\frac{3}{4}}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-2)*(x^6+1)^(3/4)/x^8,x)
```

```
[Out] 2/7/x^7*(x^2+1)*(x^4-x^2+1)*(x^6+1)^(3/4)
```

maxima [B] time = 0.54, size = 29, normalized size = 1.81

$$\frac{2\left(x^6+1\right)\left(x^4-x^2+1\right)^{\frac{3}{4}}\left(x^2+1\right)^{\frac{3}{4}}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6+1)^(3/4)/x^8,x, algorithm="maxima")
```

```
[Out] 2/7*(x^6 + 1)*(x^4 - x^2 + 1)^(3/4)*(x^2 + 1)^(3/4)/x^7
```

mupad [B] time = 0.23, size = 27, normalized size = 1.69

$$\frac{2\left(x^6+1\right)^{3/4}+2\,x^6\left(x^6+1\right)^{3/4}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 + 1)^(3/4)*(x^6 - 2))/x^8,x)
```

```
[Out] (2*(x^6 + 1)^(3/4) + 2*x^6*(x^6 + 1)^(3/4))/(7*x^7)
```

sympy [C] time = 3.05, size = 71, normalized size = 4.44

$$\frac{\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|x^6e^{i\pi}\right)}{6x\Gamma\left(\frac{5}{6}\right)}-\frac{\Gamma\left(-\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{7}{6},-\frac{3}{4}\\ -\frac{1}{6}\end{matrix}\middle|x^6e^{i\pi}\right)}{3x^7\Gamma\left(-\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-2)*(x**6+1)**(3/4)/x**8,x)
```

```
[Out] gamma(-1/6)*hyper((-3/4, -1/6), (5/6,), x**6*exp_polar(I*pi))/(6*x*gamma(5/
6)) - gamma(-7/6)*hyper((-7/6, -3/4), (-1/6,), x**6*exp_polar(I*pi))/(3*x**
7*gamma(-1/6))
```

3.108 $\int \frac{\sqrt[3]{-1+x^6}\left(1+x^6\right)}{x^5} \, dx$

Optimal. Leaf size=16

$$\frac{\left(x^6-1\right)^{4/3}}{4x^4}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$\frac{\left(x^6-1\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^6)^(1/3)*(1 + x^6))/x^5,x]
```

```
[Out] (-1 + x^6)^(4/3)/(4*x^4)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{-1+x^6}\left(1+x^6\right)}{x^5} \, dx = \frac{\left(-1+x^6\right)^{4/3}}{4x^4}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{\left(x^6-1\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^6)^(1/3)*(1 + x^6))/x^5,x]
```

```
[Out] (-1 + x^6)^(4/3)/(4*x^4)
```

IntegrateAlgebraic [A] time = 0.15, size = 16, normalized size = 1.00

$$\frac{\left(x^6-1\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^6)^(1/3)*(1 + x^6))/x^5,x]
```

```
[Out] (-1 + x^6)^(4/3)/(4*x^4)
```

fricas [A] time = 0.40, size = 12, normalized size = 0.75

$$\frac{\left(x^6-1\right)^{\frac{4}{3}}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(x^6+1)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(x^6 - 1)^(4/3)/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+1\right)\left(x^6-1\right)^{\frac{1}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(x^6+1)/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^6 + 1)*(x^6 - 1)^(1/3)/x^5, x)
```

maple [B] time = 0.01, size = 33, normalized size = 2.06

$$\frac{\left(x^6-1\right)^{\frac{1}{3}}(-1+x)(1+x)\left(x^2+x+1\right)\left(x^2-x+1\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/3)*(x^6+1)/x^5,x)
```

```
[Out] 1/4*(x^6-1)^(1/3)*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)/x^4
```

maxima [B] time = 0.49, size = 38, normalized size = 2.38

$$\frac{\left(x^6-1\right)\left(x^2+x+1\right)^{\frac{1}{3}}\left(x^2-x+1\right)^{\frac{1}{3}}(x+1)^{\frac{1}{3}}(x-1)^{\frac{1}{3}}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(x^6+1)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(x^6 - 1)*(x^2 + x + 1)^(1/3)*(x^2 - x + 1)^(1/3)*(x + 1)^(1/3)*(x - 1)
^(1/3)/x^4
```

mupad [B] time = 0.20, size = 12, normalized size = 0.75

$$\frac{\left(x^6-1\right)^{4/3}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/3)*(x^6 + 1))/x^5,x)
```

```
[Out] (x^6 - 1)^(4/3)/(4*x^4)
```

sympy [C] time = 3.01, size = 71, normalized size = 4.44

$$\frac{x^2 e^{\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|x^6\right)}{6\Gamma\left(\frac{4}{3}\right)}-\frac{e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{3}\\\frac{1}{3}\end{matrix}\right|x^6\right)}{6x^4\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/3)*(x**6+1)/x**5,x)
```

```
[Out] x**2*exp(I*pi/3)*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), x**6)/(6*gamma(4/3))
 - exp(-2*I*pi/3)*gamma(-2/3)*hyper((-2/3, -1/3), (1/3,), x**6)/(6*x**4*gam
ma(1/3))
```

3.109 $\int \frac{2+x^6}{x^4 \sqrt[4]{-1+x^6}} \, dx$

Optimal. Leaf size=16

$$\frac{2\left(x^6-1\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$\frac{2\left(x^6-1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^6)/(x^4*(-1 + x^6)^(1/4)),x]
```

```
[Out] (2*(-1 + x^6)^(3/4))/(3*x^3)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{2+x^6}{x^4 \sqrt[4]{-1+x^6}} \, dx = \frac{2\left(-1+x^6\right)^{3/4}}{3x^3}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2\left(x^6-1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^6)/(x^4*(-1 + x^6)^(1/4)),x]
```

```
[Out] (2*(-1 + x^6)^(3/4))/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.17, size = 16, normalized size = 1.00

$$\frac{2\left(x^6-1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x^6)/(x^4*(-1 + x^6)^(1/4)),x]
```

```
[Out] (2*(-1 + x^6)^(3/4))/(3*x^3)
```

fricas [A] time = 0.40, size = 12, normalized size = 0.75

$$\frac{2\left(x^6-1\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)/x^4/(x^6-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/3*(x^6 - 1)^(3/4)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+2}{\left(x^6-1\right)^{\frac{1}{4}} x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)/x^4/(x^6-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 2)/((x^6 - 1)^(1/4)*x^4), x)
```

maple [B] time = 0.01, size = 33, normalized size = 2.06

$$\frac{2\left(-1+x\right)\left(1+x\right)\left(x^2+x+1\right)\left(x^2-x+1\right)}{3x^3\left(x^6-1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+2)/x^4/(x^6-1)^(1/4),x)
```

```
[Out] 2/3/x^3*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)/(x^6-1)^(1/4)
```

maxima [B] time = 0.55, size = 38, normalized size = 2.38

$$\frac{2\left(x^6-1\right)}{3\left(x^2+x+1\right)^{\frac{1}{4}}\left(x^2-x+1\right)^{\frac{1}{4}}\left(x+1\right)^{\frac{1}{4}}\left(x-1\right)^{\frac{1}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)/x^4/(x^6-1)^(1/4),x, algorithm="maxima")
```

```
[Out] 2/3*(x^6 - 1)/((x^2 + x + 1)^(1/4)*(x^2 - x + 1)^(1/4)*(x + 1)^(1/4)*(x - 1
)^(1/4)*x^3)
```

mupad [B] time = 0.15, size = 12, normalized size = 0.75

$$\frac{2\left(x^6-1\right)^{3/4}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 2)/(x^4*(x^6 - 1)^(1/4)),x)
```

```
[Out] (2*(x^6 - 1)^(3/4))/(3*x^3)
```

sympy [C] time = 2.62, size = 48, normalized size = 3.00

$$\frac{x^3 e^{-\frac{i\pi}{4}} {}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|x^6\right)}{3}+\frac{2e^{\frac{3i\pi}{4}} {}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{1}{2}\end{matrix}\middle|x^6\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+2)/x**4/(x**6-1)**(1/4),x)
```

```
[Out] x**3*exp(-I*pi/4)*hyper((1/4, 1/2), (3/2,), x**6)/3 + 2*exp(3*I*pi/4)*hyper
((-1/2, 1/4), (1/2,), x**6)/(3*x**3)
```

3.110 $\int \frac{\left(-1+x^6\right)^{3/4}\left(2+x^6\right)}{x^8}\,dx$

Optimal. Leaf size=16

$$\frac{2\left(x^6-1\right)^{7/4}}{7x^7}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {449}

$$\frac{2\left(x^6-1\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^6)^(3/4)*(2 + x^6))/x^8,x]
```

```
[Out] (2*(-1 + x^6)^(7/4))/(7*x^7)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(-1+x^6\right)^{3/4}\left(2+x^6\right)}{x^8}\,dx=\frac{2\left(-1+x^6\right)^{7/4}}{7x^7}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2\left(x^6-1\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^6)^(3/4)*(2 + x^6))/x^8,x]
```

```
[Out] (2*(-1 + x^6)^(7/4))/(7*x^7)
```

IntegrateAlgebraic [A] time = 0.17, size = 16, normalized size = 1.00

$$\frac{2\left(x^6-1\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^6)^(3/4)*(2 + x^6))/x^8,x]
```

```
[Out] (2*(-1 + x^6)^(7/4))/(7*x^7)
```

fricas [A] time = 0.40, size = 12, normalized size = 0.75

$$\frac{2\left(x^6-1\right)^{\frac{7}{4}}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(3/4)*(x^6+2)/x^8,x, algorithm="fricas")
```

```
[Out] 2/7*(x^6 - 1)^(7/4)/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^6+2)(x^6-1)^{\frac{3}{4}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(3/4)*(x^6+2)/x^8,x, algorithm="giac")
```

```
[Out] integrate((x^6 + 2)*(x^6 - 1)^(3/4)/x^8, x)
```

maple [B] time = 0.01, size = 33, normalized size = 2.06

$$\frac{2(-1+x)(1+x)\left(x^2+x+1\right)\left(x^2-x+1\right)\left(x^6-1\right)^{\frac{3}{4}}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(3/4)*(x^6+2)/x^8,x)
```

```
[Out] 2/7/x^7*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)*(x^6-1)^(3/4)
```

maxima [B] time = 0.58, size = 38, normalized size = 2.38

$$\frac{2\left(x^6-1\right)\left(x^2+x+1\right)^{\frac{3}{4}}\left(x^2-x+1\right)^{\frac{3}{4}}(x+1)^{\frac{3}{4}}(x-1)^{\frac{3}{4}}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(3/4)*(x^6+2)/x^8,x, algorithm="maxima")
```

```
[Out] 2/7*(x^6 - 1)*(x^2 + x + 1)^(3/4)*(x^2 - x + 1)^(3/4)*(x + 1)^(3/4)*(x - 1)
^(3/4)/x^7
```

mupad [B] time = 0.22, size = 27, normalized size = 1.69

$$-\frac{2\left(x^6-1\right)^{3/4}-2\,x^6\left(x^6-1\right)^{3/4}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(3/4)*(x^6 + 2))/x^8,x)
```

```
[Out] -(2*(x^6 - 1)^(3/4) - 2*x^6*(x^6 - 1)^(3/4))/(7*x^7)
```

sympy [C] time = 3.11, size = 76, normalized size = 4.75

$$-\frac{e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{1}{6}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{3}{4},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\right|x^6\right)}{6x\Gamma\left(\frac{5}{6}\right)}-\frac{e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{7}{6}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{7}{6},-\frac{3}{4}\\ -\frac{1}{6}\end{matrix}\right|x^6\right)}{3x^7\Gamma\left(-\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(3/4)*(x**6+2)/x**8,x)
```

```
[Out] -exp(-I*pi/4)*gamma(-1/6)*hyper((-3/4, -1/6), (5/6,), x**6)/(6*x*gamma(5/6)
) - exp(-I*pi/4)*gamma(-7/6)*hyper((-7/6, -3/4), (-1/6,), x**6)/(3*x**7*gam
ma(-1/6))
```

3.111 $\int \frac{-3+2x^5}{x^3 \sqrt[4]{x+x^6}} \, dx$

Optimal. Leaf size=16

$$\frac{4\left(x^6+x\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1590}

$$\frac{4\left(x^6+x\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 2*x^5)/(x^3*(x + x^6)^(1/4)),x]
```

```
[Out] (4*(x + x^6)^(3/4))/(3*x^3)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{-3+2x^5}{x^3 \sqrt[4]{x+x^6}} \, dx = \frac{4\left(x+x^6\right)^{3/4}}{3x^3}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{4\left(x^6+x\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 2*x^5)/(x^3*(x + x^6)^(1/4)),x]
```

```
[Out] (4*(x + x^6)^(3/4))/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.22, size = 16, normalized size = 1.00

$$\frac{4\left(x^6+x\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 + 2*x^5)/(x^3*(x + x^6)^(1/4)),x]
```

```
[Out] (4*(x + x^6)^(3/4))/(3*x^3)
```

fricas [A] time = 0.41, size = 12, normalized size = 0.75

$$\frac{4\left(x^6+x\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^5-3)/x^3/(x^6+x)^(1/4),x, algorithm="fricas")
[Out] 4/3*(x^6 + x)^(3/4)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^5-3}{\left(x^6+x\right)^{\frac{1}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^5-3)/x^3/(x^6+x)^(1/4),x, algorithm="giac")
[Out] integrate((2*x^5 - 3)/((x^6 + x)^(1/4)*x^3), x)
```

maple [B] time = 0.01, size = 32, normalized size = 2.00

$$\frac{4\,(1+x)\left(x^4-x^3+x^2-x+1\right)}{3x^2\left(x^6+x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^5-3)/x^3/(x^6+x)^(1/4),x)
[Out] 4/3/x^2*(1+x)*(x^4-x^3+x^2-x+1)/(x^6+x)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^5-3}{\left(x^6+x\right)^{\frac{1}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^5-3)/x^3/(x^6+x)^(1/4),x, algorithm="maxima")
[Out] integrate((2*x^5 - 3)/((x^6 + x)^(1/4)*x^3), x)
```

mupad [B] time = 0.25, size = 12, normalized size = 0.75

$$\frac{4\left(x^6+x\right)^{3/4}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^5 - 3)/(x^3*(x + x^6)^(1/4)),x)
[Out] (4*(x + x^6)^(3/4))/(3*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^5-3}{x^3\sqrt[4]{x\,(x+1)\left(x^4-x^3+x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**5-3)/x**3/(x**6+x)**(1/4),x)
```

```
[Out] Integral((2*x**5 - 3)/(x**3*(x*(x + 1)*(x**4 - x**3 + x**2 - x + 1))**(1/4)
), x)
```

3.112 $\int \frac{\sqrt[3]{x+x^4}\left(-2-x^3+x^6\right)}{x^6}\,dx$

Optimal. Leaf size=16

$$\frac{3\left(x^4+x\right)^{7/3}}{7x^7}$$

Rubi [B] time = 0.19, antiderivative size = 47, normalized size of antiderivative = 2.94, number of steps used = 12, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2052, 2004, 2032, 364, 2020, 2025}

$$\frac{3}{7}\sqrt[3]{x^4+x}\,x+\frac{3\sqrt[3]{x^4+x}}{7x^5}+\frac{6\sqrt[3]{x^4+x}}{7x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((x + x^4)^(1/3)*(-2 - x^3 + x^6))/x^6,x]
```

```
[Out] (3*(x + x^4)^(1/3))/(7*x^5) + (6*(x + x^4)^(1/3))/(7*x^2) + (3*x*(x + x^4)^
(1/3))/7
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{x+x^4}\left(-2-x^3+x^6\right)}{x^6}\,dx &= \int \left(\sqrt[3]{x+x^4}-\frac{2\sqrt[3]{x+x^4}}{x^6}-\frac{\sqrt[3]{x+x^4}}{x^3}\right)dx \\
&= -\left(2\int \frac{\sqrt[3]{x+x^4}}{x^6}\,dx\right)+\int \sqrt[3]{x+x^4}\,dx-\int \frac{\sqrt[3]{x+x^4}}{x^3}\,dx \\
&= \frac{3\sqrt[3]{x+x^4}}{7x^5}+\frac{3\sqrt[3]{x+x^4}}{5x^2}+\frac{3}{7}x\sqrt[3]{x+x^4}-\frac{3}{7}\int \frac{1}{x^2\left(x+x^4\right)^{2/3}}\,dx+\frac{3}{7}\int \frac{x}{\left(x+x\right.} \\
&= \frac{3\sqrt[3]{x+x^4}}{7x^5}+\frac{6\sqrt[3]{x+x^4}}{7x^2}+\frac{3}{7}x\sqrt[3]{x+x^4}+\frac{6}{35}\int \frac{x}{\left(x+x^4\right)^{2/3}}\,dx+\frac{\left(3x^{2/3}\left(1+x\right.\right.}{7\left(\right.} \\
&= \frac{3\sqrt[3]{x+x^4}}{7x^5}+\frac{6\sqrt[3]{x+x^4}}{7x^2}+\frac{3}{7}x\sqrt[3]{x+x^4}-\frac{9x^2\left(1+x^3\right)^{2/3}{}_2F_1\left(\frac{4}{9},\frac{2}{3};\frac{13}{9};-x^3\right)}{70\left(x+x^4\right)^{2/3}}+ \\
&= \frac{3\sqrt[3]{x+x^4}}{7x^5}+\frac{6\sqrt[3]{x+x^4}}{7x^2}+\frac{3}{7}x\sqrt[3]{x+x^4}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 83, normalized size = 5.19

$$
\frac{3\sqrt[3]{x^4+x}\left(28x^3\,{}_2F_1\left(-\frac{5}{9},-\frac{1}{3};\frac{4}{9};-x^3\right)+20\,{}_2F_1\left(-\frac{14}{9},-\frac{1}{3};-\frac{5}{9};-x^3\right)+35x^6\,{}_2F_1\left(-\frac{1}{3},\frac{4}{9};\frac{13}{9};-x^3\right)\right)}{140x^5\sqrt[3]{x^3+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((x + x^4)^(1/3)*(-2 - x^3 + x^6))/x^6,x]
```

```
[Out] (3*(x + x^4)^(1/3)*(20*Hypergeometric2F1[-14/9, -1/3, -5/9, -x^3] + 28*x^3*
Hypergeometric2F1[-5/9, -1/3, 4/9, -x^3] + 35*x^6*Hypergeometric2F1[-1/3, 4
/9, 13/9, -x^3]))/(140*x^5*(1 + x^3)^(1/3))
```

IntegrateAlgebraic [A] time = 0.20, size = 16, normalized size = 1.00

$$
\frac{3\left(x^4+x\right)^{7/3}}{7x^7}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((x + x^4)^(1/3)*(-2 - x^3 + x^6))/x^6,x]
```

```
[Out] (3*(x + x^4)^(7/3))/(7*x^7)
```

fricas [A] time = 0.40, size = 22, normalized size = 1.38

$$
\frac{3\left(x^6+2x^3+1\right)\left(x^4+x\right)^{\frac{1}{3}}}{7x^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/3)*(x^6-x^3-2)/x^6,x, algorithm="fricas")
[Out] 3/7*(x^6 + 2*x^3 + 1)*(x^4 + x)^(1/3)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^6 - x^3 - 2)(x^4 + x)^{\frac{1}{3}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/3)*(x^6-x^3-2)/x^6,x, algorithm="giac")
[Out] integrate((x^6 - x^3 - 2)*(x^4 + x)^(1/3)/x^6, x)
```

maple [B] time = 0.01, size = 29, normalized size = 1.81

$$\frac{3\left(x^3+1\right)\left(x^4+x\right)^{\frac{1}{3}}(1+x)\left(x^2-x+1\right)}{7x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x)^(1/3)*(x^6-x^3-2)/x^6,x)
[Out] 3/7*(x^3+1)*(x^4+x)^(1/3)*(1+x)*(x^2-x+1)/x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^6 - x^3 - 2)(x^4 + x)^{\frac{1}{3}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/3)*(x^6-x^3-2)/x^6,x, algorithm="maxima")
[Out] integrate((x^6 - x^3 - 2)*(x^4 + x)^(1/3)/x^6, x)
```

mupad [B] time = 0.33, size = 19, normalized size = 1.19

$$\frac{3\left(x^3+1\right)^2\left(x^4+x\right)^{1/3}}{7\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + x^4)^(1/3)*(x^3 - x^6 + 2))/x^6,x)
[Out] (3*(x^3 + 1)^2*(x + x^4)^(1/3))/(7*x^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x(x+1)\left(x^2-x+1\right)}\,(x+1)\left(x^3-2\right)\left(x^2-x+1\right)}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x)**(1/3)*(x**6-x**3-2)/x**6,x)
[Out] Integral((x*(x + 1)*(x**2 - x + 1))**(1/3)*(x + 1)*(x**3 - 2)*(x**2 - x + 1
)/x**6, x)
```

3.113 $\int \frac{x(2+x^6)}{\sqrt{-1+x^6}(-1-x^4+x^6)}\,dx$

Optimal. Leaf size=16

$$-\tanh^{-1}\left(\frac{x^2}{\sqrt{x^6-1}}\right)$$

Rubi [F] time = 0.75, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(2+x^6)}{\sqrt{-1+x^6}\left(-1-x^4+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(2 + x^6))/(Sqrt[-1 + x^6]*(-1 - x^4 + x^6)),x]
```

```
[Out] -((Sqrt[2 - Sqrt[3]]*(1 - x^2)*Sqrt[(1 + x^2 + x^4)/(1 - Sqrt[3] - x^2)^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - x^2)/(1 - Sqrt[3] - x^2)], -7 + 4*Sqrt[3]])
/(3^(1/4)*Sqrt[-((1 - x^2)/(1 - Sqrt[3] - x^2)^2)]*Sqrt[-1 + x^6])) + (3*De
fer[Subst][Defer[Int][1/(Sqrt[-1 + x^3]*(-1 - x^2 + x^3)), x], x, x^2])/2 +
 Defer[Subst][Defer[Int][x^2/(Sqrt[-1 + x^3]*(-1 - x^2 + x^3)), x], x, x^2]
/2
```

Rubi steps

$$\begin{aligned}
\int \frac{x(2+x^6)}{\sqrt{-1+x^6}\left(-1-x^4+x^6\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+x^3}{\sqrt{-1+x^3}\left(-1-x^2+x^3\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{\sqrt{-1+x^3}}+\frac{3+x^2}{\sqrt{-1+x^3}\left(-1-x^2+x^3\right)}\right)dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{3+x^2}{\sqrt{-1+x^3}\left(-1-x^2+x^3\right)}\,d
\end{aligned}$$

$$= -\frac{\sqrt{2-\sqrt{3}}\left(1-x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1-\sqrt{3}-x^2\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x^2}{1-\sqrt{3}-x^2}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x^2}{\left(1-\sqrt{3}-x^2\right)^2}}\sqrt{-1+x^6}}+\frac{1}{2}\text{Sub}$$

$$= -\frac{\sqrt{2-\sqrt{3}}\left(1-x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1-\sqrt{3}-x^2\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x^2}{1-\sqrt{3}-x^2}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x^2}{\left(1-\sqrt{3}-x^2\right)^2}}\sqrt{-1+x^6}}+\frac{1}{2}\text{Sub}$$

Mathematica [C] time = 5.62, size = 933, normalized size = 58.31

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(2 + x^6))/(Sqrt[-1 + x^6]*(-1 - x^4 + x^6)),x]
```

```
[Out] (Sqrt[(1 - x^2)/(1 + (-1)^(1/3))]*Sqrt[1 + x^2 + x^4]*(-((Sqrt[3]*(I*Sqrt[3
] + (1 + (-1)^(1/3))*x^2)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x^2)/(1 + (
```

```
-1)^(1/3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x^2)) - ((3*I)*((EllipticPi[(I*
Sqrt[3])/((-1)^(1/3) + Root[-1 - #1^2 + #1^3 & , 1, 0]), ArcSin[Sqrt[(1 - (
-1)^(2/3)*x^2)/(1 + (-1)^(1/3))]], (-1)^(1/3)]*(3 + Root[-1 - #1^2 + #1^3 &
 , 1, 0]^2))/((-1)^(1/3) + Root[-1 - #1^2 + #1^3 & , 1, 0]) + (2*EllipticPi
[(I*Sqrt[3])/((-1)^(1/3) + Root[-1 - #1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[(1
 - (-1)^(2/3)*x^2)/(1 + (-1)^(1/3))]], (-1)^(1/3)]*(Root[-1 - #1^2 + #1^3 &
 , 1, 0] - Root[-1 - #1^2 + #1^3 & , 2, 0])*((-1)^(1/3) + Root[-1 - #1^2 +
#1^3 & , 2, 0]) + EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + Root[-1 - #1^2 + #1^
3 & , 3, 0]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x^2)/(1 + (-1)^(1/3))]], (-1)^(1/
3)]*(Root[-1 - #1^2 + #1^3 & , 1, 0] - Root[-1 - #1^2 + #1^3 & , 2, 0])*((-
1)^(1/3) + Root[-1 - #1^2 + #1^3 & , 2, 0])*Root[-1 - #1^2 + #1^3 & , 3, 0]
^3 - 2*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + Root[-1 - #1^2 + #1^3 & , 2, 0]
), ArcSin[Sqrt[(1 - (-1)^(2/3)*x^2)/(1 + (-1)^(1/3))]], (-1)^(1/3)]*(Root[-
1 - #1^2 + #1^3 & , 1, 0] - Root[-1 - #1^2 + #1^3 & , 3, 0])*((-1)^(1/3) +
Root[-1 - #1^2 + #1^3 & , 3, 0]) - EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + Roo
t[-1 - #1^2 + #1^3 & , 2, 0]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x^2)/(1 + (-1)^(
1/3))]], (-1)^(1/3)]*Root[-1 - #1^2 + #1^3 & , 2, 0]^3*(Root[-1 - #1^2 + #1
^3 & , 1, 0] - Root[-1 - #1^2 + #1^3 & , 3, 0])*((-1)^(1/3) + Root[-1 - #1^
2 + #1^3 & , 3, 0]))/(((-1)^(1/3) + Root[-1 - #1^2 + #1^3 & , 2, 0])*(Root[
-1 - #1^2 + #1^3 & , 2, 0] - Root[-1 - #1^2 + #1^3 & , 3, 0])*((-1)^(1/3) +
 Root[-1 - #1^2 + #1^3 & , 3, 0]))))/((Root[-1 - #1^2 + #1^3 & , 1, 0] - Ro
ot[-1 - #1^2 + #1^3 & , 2, 0])*(Root[-1 - #1^2 + #1^3 & , 1, 0] - Root[-1 -
 #1^2 + #1^3 & , 3, 0]))))/(3*Sqrt[-1 + x^6])
```

IntegrateAlgebraic [A] time = 17.26, size = 16, normalized size = 1.00

$$-\tanh^{-1}\left(\frac{x^2}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(2 + x^6))/(Sqrt[-1 + x^6]*(-1 - x^4 + x^6)),x]
[Out] -ArcTanh[x^2/Sqrt[-1 + x^6]]
```

fricas [B] time = 0.41, size = 36, normalized size = 2.25

$$\frac{1}{2}\log\left(\frac{x^6+x^4-2\sqrt{x^6-1}\,x^2-1}{x^6-x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^6+2)/(x^6-1)^(1/2)/(x^6-x^4-1),x, algorithm="fricas")
[Out] 1/2*log((x^6 + x^4 - 2*sqrt(x^6 - 1)*x^2 - 1)/(x^6 - x^4 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^6+2\right)x}{\left(x^6-x^4-1\right)\sqrt{x^6-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^6+2)/(x^6-1)^(1/2)/(x^6-x^4-1),x, algorithm="giac")
[Out] integrate((x^6 + 2)*x/((x^6 - x^4 - 1)*sqrt(x^6 - 1)), x)
```

maple [B] time = 0.30, size = 42, normalized size = 2.62

$$\frac{\ln\left(-\frac{-x^6-x^4+2x^2\sqrt{x^6-1}+1}{x^6-x^4-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^6+2)/(x^6-1)^(1/2)/(x^6-x^4-1),x)
```

```
[Out] 1/2*ln(-(-x^6-x^4+2*x^2*(x^6-1)^(1/2)+1)/(x^6-x^4-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^6+2)x}{(x^6-x^4-1)\sqrt{x^6-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^6+2)/(x^6-1)^(1/2)/(x^6-x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + 2)*x/((x^6 - x^4 - 1)*sqrt(x^6 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int -\frac{x\,(x^6+2)}{\sqrt{x^6-1}\,(-x^6+x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(x^6 + 2))/((x^6 - 1)^(1/2)*(x^4 - x^6 + 1)),x)
```

```
[Out] int(-(x*(x^6 + 2))/((x^6 - 1)^(1/2)*(x^4 - x^6 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**6+2)/(x**6-1)**(1/2)/(x**6-x**4-1),x)
```

```
[Out] Timed out
```

3.114 $\int \frac{-1+x^8}{x^4\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=16

$$\frac{\left(x^4-1\right)^{3/2}}{3x^3}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1479, 449}

$$\frac{\left(x^4-1\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^8)/(x^4*Sqrt[-1 + x^4]),x]
```

```
[Out] (-1 + x^4)^(3/2)/(3*x^3)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1479

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n
2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*(a/d + (c*x^n)/e)^
p, x] /; FreeQ[{a, c, d, e, f, q, m, n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2
 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^8}{x^4\sqrt{-1+x^4}}\,dx &= \int \frac{\sqrt{-1+x^4}\left(1+x^4\right)}{x^4}\,dx\\ &= \frac{\left(-1+x^4\right)^{3/2}}{3x^3}\end{aligned}$$

Mathematica [C] time = 0.04, size = 76, normalized size = 4.75

$$\frac{x^4\left(\sqrt{1-x^4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};x^4\right)+x^4-1\right)+\sqrt{1-x^4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};x^4\right)}{3x^3\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^8)/(x^4*Sqrt[-1 + x^4]),x]
```

```
[Out] (Sqrt[1 - x^4]*Hypergeometric2F1[-3/4, 1/2, 1/4, x^4] + x^4*(-1 + x^4 + Sqr
t[1 - x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, x^4]))/(3*x^3*Sqrt[-1 + x^4])
```

IntegrateAlgebraic [A] time = 0.17, size = 16, normalized size = 1.00

$$\frac{\left(x^4-1\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^8)/(x^4*Sqrt[-1 + x^4]),x]
[Out] (-1 + x^4)^(3/2)/(3*x^3)
```

fricas [A] time = 0.41, size = 12, normalized size = 0.75

$$\frac{\left(x^4-1\right)^{\frac{3}{2}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/x^4/(x^4-1)^(1/2),x, algorithm="fricas")
[Out] 1/3*(x^4 - 1)^(3/2)/x^3
```

giac [B] time = 0.65, size = 25, normalized size = 1.56

$$\frac{1}{3}\sqrt{x^4-1}\,x-\frac{\sqrt{-\frac{1}{x^4}+1}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/x^4/(x^4-1)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(x^4 - 1)*x - 1/3*sqrt(-1/x^4 + 1)/x
```

maple [A] time = 0.01, size = 24, normalized size = 1.50

$$\frac{\sqrt{x^4-1}\,\left(-1+x\right)\left(1+x\right)\left(x^2+1\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-1)/x^4/(x^4-1)^(1/2),x)
[Out] 1/3*(x^4-1)^(1/2)*(-1+x)*(1+x)*(x^2+1)/x^3
```

maxima [B] time = 0.96, size = 27, normalized size = 1.69

$$\frac{\left(x^4-1\right)\sqrt{x^2+1}\,\sqrt{x+1}\,\sqrt{x-1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/x^4/(x^4-1)^(1/2),x, algorithm="maxima")
[Out] 1/3*(x^4 - 1)*sqrt(x^2 + 1)*sqrt(x + 1)*sqrt(x - 1)/x^3
```

mupad [B] time = 0.21, size = 12, normalized size = 0.75

$$\frac{\left(x^4-1\right)^{3/2}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 - 1)/(x^4*(x^4 - 1)^(1/2)),x)
[Out] (x^4 - 1)^(3/2)/(3*x^3)
```

sympy [C] time = 1.85, size = 56, normalized size = 3.50

$$-\frac{ix^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle| x^4\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{i\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle| x^4\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-1)/x**4/(x**4-1)**(1/2),x)
```

```
[Out] -I*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4)/(4*gamma(9/4)) + I*gamma
(-3/4)*hyper((-3/4, 1/2), (1/4,), x**4)/(4*x**3*gamma(1/4))
```

3.115 $\int \frac{(-1+x^8)\sqrt{1+x^8}}{x^7}\,dx$

Optimal. Leaf size=16

$$\frac{(x^8+1)^{3/2}}{6x^6}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {449}

$$\frac{(x^8+1)^{3/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^8)*Sqrt[1 + x^8])/x^7,x]
[Out] (1 + x^8)^(3/2)/(6*x^6)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(-1+x^8)\sqrt{1+x^8}}{x^7}\,dx = \frac{(1+x^8)^{3/2}}{6x^6}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{(x^8+1)^{3/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^8)*Sqrt[1 + x^8])/x^7,x]
[Out] (1 + x^8)^(3/2)/(6*x^6)
```

IntegrateAlgebraic [A] time = 2.64, size = 16, normalized size = 1.00

$$\frac{(x^8+1)^{3/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^8)*Sqrt[1 + x^8])/x^7,x]
[Out] (1 + x^8)^(3/2)/(6*x^6)
```

fricas [A] time = 0.39, size = 12, normalized size = 0.75

$$\frac{(x^8+1)^{\frac{3}{2}}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)*(x^8+1)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(x^8 + 1)^(3/2)/x^6
```

giac [B] time = 0.69, size = 25, normalized size = 1.56

$$\frac{1}{6}\sqrt{x^8+1}\,x^2 + \frac{\sqrt{\frac{1}{x^8}+1}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)*(x^8+1)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(x^8 + 1)*x^2 + 1/6*sqrt(1/x^8 + 1)/x^2
```

maple [A] time = 0.01, size = 13, normalized size = 0.81

$$\frac{(x^8+1)^{\frac{3}{2}}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-1)*(x^8+1)^(1/2)/x^7,x)
```

```
[Out] 1/6*(x^8+1)^(3/2)/x^6
```

maxima [A] time = 0.58, size = 12, normalized size = 0.75

$$\frac{(x^8+1)^{\frac{3}{2}}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)*(x^8+1)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*(x^8 + 1)^(3/2)/x^6
```

mupad [B] time = 0.23, size = 12, normalized size = 0.75

$$\frac{(x^8+1)^{3/2}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^8 - 1)*(x^8 + 1)^(1/2))/x^7,x)
```

```
[Out] (x^8 + 1)^(3/2)/(6*x^6)
```

sympy [C] time = 3.20, size = 66, normalized size = 4.12

$$\frac{x^2\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle| x^8e^{i\pi}\right)}{8\Gamma\left(\frac{5}{4}\right)} - \frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle| x^8e^{i\pi}\right)}{8x^6\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-1)*(x**8+1)**(1/2)/x**7,x)
```

```
[Out] x**2*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**8*exp_polar(I*pi))/(8*gamma(5
/4)) - gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), x**8*exp_polar(I*pi))/(8*x**
6*gamma(1/4))
```

3.116 $\int \frac{2+x+2x^2}{(-1+2x)\sqrt{x+x^4}}\,dx$

Optimal. Leaf size=17

$$2\tanh^{-1}\left(\frac{(x-2)x}{\sqrt{x^4+x}}\right)$$

Rubi [F] time = 0.75, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2+x+2x^2}{(-1+2x)\sqrt{x+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2 + x + 2*x^2)/((-1 + 2*x)*Sqrt[x + x^4]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 + x^3]*ArcSinh[x^(3/2)])/(3*Sqrt[x + x^4]) + (x*(1 + x)*S
qrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(1 + (1 - Sqrt[
3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3^(1/4)*Sqrt[(x*(1 + x))/
(1 + (1 + Sqrt[3])*x)^2]*Sqrt[x + x^4]) - (3*Sqrt[x]*Sqrt[1 + x^3]*Defer[Su
bst][Defer[Int][1/((1 - Sqrt[2]*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/Sqrt[x
+ x^4] - (3*Sqrt[x]*Sqrt[1 + x^3]*Defer[Subst][Defer[Int][1/((1 + Sqrt[2]*x
)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/Sqrt[x + x^4]
```

Rubi steps

$$\int \frac{2+x+2x^2}{(-1+2x)\sqrt{x+x^4}}\,dx = \frac{\left(\sqrt{x}\sqrt{1+x^3}\right)\int \frac{2+x+2x^2}{\sqrt{x}(-1+2x)\sqrt{1+x^3}}\,dx}{\sqrt{x+x^4}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{2+x^2+2x^4}{\left(-1+2x^2\right)\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{x+x^4}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{1+x^3}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt{1+x^6}}+\frac{x^2}{\sqrt{1+x^6}}+\frac{3}{\left(-1+2x^2\right)\sqrt{1+x^6}}\right)dx, x, \sqrt{x}\right)}{\sqrt{x+x^4}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{x+x^4}} + \frac{\left(2\sqrt{x}\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^6}}\,dx, x\right.}{\sqrt{x+x^4}}$$

$$= \frac{x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}} + \frac{\left(2\sqrt{x}\sqrt{1+x^3}\right)\text{Subst}}{3\sqrt{x}}$$

$$= \frac{2\sqrt{x}\sqrt{1+x^3}\sinh^{-1}\left(x^{3/2}\right)}{3\sqrt{x+x^4}} + \frac{x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\right.\right.}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}}$$

Mathematica [C] time = 1.23, size = 249, normalized size = 14.65

$$\frac{2\left(\sqrt{6}\left(1+\sqrt[3]{-1}\right)\sqrt{\frac{x-\sqrt[3]{-1}}{\left(1+\sqrt[3]{-1}\right)x}}\sqrt{\frac{(x+1)\left(2x+i\sqrt{3}-1\right)}{x^2}}\,x^2F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}(x+1)}{\left(-1+(-1)^{2/3}\right)x}}\right)|1+(-1)^{2/3}\right)-\sqrt{6}\left(1+\sqrt[3]{-1}\right)\sqrt{\frac{x-\sqrt[3]{-1}}{\left(1+\sqrt[3]{-1}\right)x}}\sqrt{\frac{(x+1)\left(2x+i\sqrt{3}-1\right)}{x^2}}\,x^2\Pi\left(\frac{1}{3}\left(1+\sqrt[3]{-1}\right);\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}(x+1)}{\left(-1+(-1)^{2/3}\right)x}}\right)|1+(-1)^{2/3}\right)+\sqrt{x^3+1}\sqrt{x}\sinh^{-1}\left(x^{3/2}\right)\right)}{3\sqrt{x^4+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 2*x^2)/((-1 + 2*x)*Sqrt[x + x^4]),x]
```

```
[Out] (2*(Sqrt[x]*Sqrt[1 + x^3]*ArcSinh[x^(3/2)] + Sqrt[6]*(1 + (-1)^(1/3))*x^2*S
qrt[(-(-1)^(1/3) + x)/((1 + (-1)^(1/3))*x)]*Sqrt[((1 + x)*(-1 + I*Sqrt[3] +
 2*x))/x^2]*EllipticF[ArcSin[Sqrt[((-1)^(2/3)*(1 + x))/((-1 + (-1)^(2/3))*x
)]], 1 + (-1)^(2/3)] - Sqrt[6]*(1 + (-1)^(1/3))*x^2*Sqrt[(-(-1)^(1/3) + x)/
((1 + (-1)^(1/3))*x)]*Sqrt[((1 + x)*(-1 + I*Sqrt[3] + 2*x))/x^2]*EllipticPi
[(1 + (-1)^(1/3))/3, ArcSin[Sqrt[((-1)^(2/3)*(1 + x))/((-1 + (-1)^(2/3))*x)
]], 1 + (-1)^(2/3)]))/(3*Sqrt[x + x^4])
```

IntegrateAlgebraic [A] time = 1.23, size = 17, normalized size = 1.00

$$2\tanh^{-1}\left(\frac{(x-2)x}{\sqrt{x^4+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x + 2*x^2)/((-1 + 2*x)*Sqrt[x + x^4]),x]
```

```
[Out] 2*ArcTanh[((-2 + x)*x)/Sqrt[x + x^4]]
```

fricas [B] time = 0.48, size = 70, normalized size = 4.12

$$\frac{1}{3}\log\left(-2x^3-2\sqrt{x^4+x}\,x-1\right)+\frac{2}{3}\log\left(-\frac{10x^3-6x^2-6\sqrt{x^4+x}\,(x+1)+12x+1}{8x^3-12x^2+6x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+x+2)/(-1+2*x)/(x^4+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log(-2*x^3 - 2*sqrt(x^4 + x)*x - 1) + 2/3*log(-(10*x^3 - 6*x^2 - 6*sqrt
(x^4 + x)*(x + 1) + 12*x + 1)/(8*x^3 - 12*x^2 + 6*x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2x^2+x+2}{\sqrt{x^4+x}\,(2x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+x+2)/(-1+2*x)/(x^4+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 + x + 2)/(sqrt(x^4 + x)*(2*x - 1)), x)
```

maple [C] time = 0.36, size = 781, normalized size = 45.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+x+2)/(-1+2*x)/(x^4+x)^(1/2),x)
```

```
[Out] -2*(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(
1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))/(1+x))^(1/2)*(-(x-
1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(3/2+1/2*I*3^(1/2))/(x*
(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1/2)*(-EllipticF(((3/2+
1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1
/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+Ellipti
cPi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),(1/2+1/2*I*3^(1
/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))-2*(-1/2-1/2*I*3^(1/2))*((3/2+1/
```

```
2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1
/2))/(1/2-1/2*I*3^(1/2))/(1+x))^(1/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^
(1/2))/(1+x))^(1/2)/(3/2+1/2*I*3^(1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1
/2-1/2*I*3^(1/2)))^(1/2)*EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2
))/(1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2
))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-
1/2*I*3^(1/2))/(1+x))^(1/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+
x))^(1/2)/(3/2+1/2*I*3^(1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3
^(1/2)))^(1/2)*(EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))
^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/
2-1/2*I*3^(1/2)))^(1/2))+2*EllipticPi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(
1/2))/(1+x))^(1/2),3*(1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3
^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/
2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2+x+2}{\sqrt{x^4+x}\,(2\,x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+x+2)/(-1+2*x)/(x^4+x)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x^2 + x + 2)/(sqrt(x^4 + x)*(2*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{2\,x^2+x+2}{(2\,x-1)\,\sqrt{x^4+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2*x^2 + 2)/((2*x - 1)*(x + x^4)^(1/2)),x)
[Out] int((x + 2*x^2 + 2)/((2*x - 1)*(x + x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2+x+2}{\sqrt{x\,(x+1)\left(x^2-x+1\right)}\,(2x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+x+2)/(-1+2*x)/(x**4+x)**(1/2),x)
[Out] Integral((2*x**2 + x + 2)/(sqrt(x*(x + 1)*(x**2 - x + 1))*(2*x - 1)), x)
```

3.117 $\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=17

$$-\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1698, 203}

$$-\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] -ArcTan[x/Sqrt[1 + x^2 + x^4]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx &= -\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right)\\ &= -\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)\end{aligned}$$

Mathematica [C] time = 0.13, size = 93, normalized size = 5.47

$$\frac{(-1)^{2/3}\sqrt{\sqrt[3]{-1}\,x^2+1}\,\sqrt{1-(-1)^{2/3}x^2}\,\left(F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-2\Pi\left(\sqrt[3]{-1};i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticF[I*
ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - 2*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1
)^(5/6)*x], (-1)^(2/3)]))/Sqrt[1 + x^2 + x^4]
```

IntegrateAlgebraic [A] time = 0.21, size = 17, normalized size = 1.00

$$-\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] -ArcTan[x/Sqrt[1 + x^2 + x^4]]
```

fricas [A] time = 0.42, size = 15, normalized size = 0.88

$$-\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(x/sqrt(x^4 + x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^4+x^2+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

maple [C] time = 0.24, size = 188, normalized size = 11.06

$$\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{2\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x,-\frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*
3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2
),1/2*(-2+2*I*3^(1/2))^(1/2))-2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I
*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*E
llipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*
3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^4+x^2+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x^2-1}{\left(x^2+1\right)\,\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

```
[Out] int((x^2 - 1)/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{\sqrt{(x^2-x+1)(x^2+x+1)}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)),
x)
```

3.118 $\int \frac{-1+x^4}{\left(1+x^4\right) \sqrt{1+x^2+x^4}} \, dx$

Optimal. Leaf size=17

$$-\tanh^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Rubi [A] time = 0.08, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2112, 206}

$$-\tanh^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)/((1 + x^4)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] -ArcTanh[x/Sqrt[1 + x^2 + x^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2112

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^4}{\left(1+x^4\right) \sqrt{1+x^2+x^4}} \, dx &= -\text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right)\\
&= -\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)
\end{aligned}$$

Mathematica [C] time = 0.23, size = 120, normalized size = 7.06

$$\frac{(-1)^{2/3} \sqrt{\sqrt[3]{-1} x^2+1} \sqrt{1-(-1)^{2/3} x^2} \left(F\left(i \sinh^{-1}\left((-1)^{5/6} x\right)|(-1)^{2/3}\right)-\Pi\left(-(-1)^{5/6};i \sinh^{-1}\left((-1)^{5/6} x\right)|(-1)^{2/3}\right)-\Pi\left((-1)^{5/6};i \sinh^{-1}\left((-1)^{5/6} x\right)|(-1)^{2/3}\right)\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)/((1 + x^4)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticF[I*
ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - EllipticPi[-(-1)^(5/6), I*ArcSinh[(-1)
^(5/6)*x], (-1)^(2/3)] - EllipticPi[(-1)^(5/6), I*ArcSinh[(-1)^(5/6)*x], (-
1)^(2/3)]))/Sqrt[1 + x^2 + x^4]
```

IntegrateAlgebraic [A] time = 0.28, size = 17, normalized size = 1.00

$$-\tanh^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/((1 + x^4)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] -ArcTanh[x/Sqrt[1 + x^2 + x^4]]
```

fricas [B] time = 0.41, size = 34, normalized size = 2.00

$$\frac{1}{2}\log\left(\frac{x^4+2x^2-2\sqrt{x^4+x^2+1}\,x+1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log((x^4 + 2*x^2 - 2*sqrt(x^4 + x^2 + 1)*x + 1)/(x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4-1}{\sqrt{x^4+x^2+1}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 1)/(sqrt(x^4 + x^2 + 1)*(x^4 + 1)), x)
```

maple [C] time = 0.17, size = 250, normalized size = 14.71

$$\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4+1\right)}\_\alpha\left(-\frac{\mathrm{arctanh}\left(\frac{\left(2\_\alpha^2+1\right)\left(-3\_\alpha^2+5x^2+4\right)}{10\sqrt{\_\alpha^2}\,\sqrt{x^4+x^2+1}}\right)}{\sqrt{\_\alpha^2}}+\frac{\sqrt{2}\,\_\alpha^3\sqrt{x^2+2-ix^2\sqrt{3}}\,\sqrt{x^2+2+ix^2\sqrt{3}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x,\frac{\_\alpha^2}{2}+\frac{i\_\alpha^2\sqrt{3}}{2},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{i\sqrt{3}-1}\,\sqrt{x^4+x^2+1}}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/(x^4+1)/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*
3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2
),1/2*(-2+2*I*3^(1/2))^(1/2))+1/4*sum(_alpha*(-1/(_alpha^2)^(1/2)*arctanh(1
/10*(2*_alpha^2+1)*(-3*_alpha^2+5*x^2+4)/(_alpha^2)^(1/2)/(x^4+x^2+1)^(1/2)
)+2^(1/2)*_alpha^3/(I*3^(1/2)-1)^(1/2)*(x^2+2-I*3^(1/2)*x^2)^(1/2)*(x^2+2+I
*3^(1/2)*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)
*x,1/2*_alpha^2+1/2*I*_alpha^2*3^(1/2),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2
*I*3^(1/2))^(1/2))),_alpha=RootOf(_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4-1}{\sqrt{x^4+x^2+1}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 1)/(sqrt(x^4 + x^2 + 1)*(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int\frac{x^4-1}{\left(x^4+1\right)\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/((x^4 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

```
[Out] int((x^4 - 1)/((x^4 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2+1\right)}{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/(x**4+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x
**4 + 1)), x)
```

3.119 $\int \frac{-1+2x^2+2x^4}{\left(1+2x^2\right)\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=17

$$\tanh^{-1}\left(\frac{x\left(x^2-1\right)}{\sqrt{x^6-1}}\right)$$

Rubi [F] time = 0.39, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+2x^2+2x^4}{\left(1+2x^2\right)\sqrt{-1+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 2*x^2 + 2*x^4)/((1 + 2*x^2)*Sqrt[-1 + x^6]),x]
```

```
[Out] ArcTanh[x^3/Sqrt[-1 + x^6]]/3 + (x*(1 - x^2)*Sqrt[(1 + x^2 + x^4)/(1 - (1 +
 Sqrt[3])*x^2)^2]*EllipticF[ArcCos[(1 - (1 - Sqrt[3])*x^2)/(1 - (1 + Sqrt[3
])*x^2)], (2 + Sqrt[3])/4])/(4*3^(1/4)*Sqrt[-((x^2*(1 - x^2))/(1 - (1 + Sqr
t[3])*x^2)^2)]*Sqrt[-1 + x^6]) - ((3*I)/4)*Defer[Int][1/((I - Sqrt[2]*x)*Sq
rt[-1 + x^6]), x] - ((3*I)/4)*Defer[Int][1/((I + Sqrt[2]*x)*Sqrt[-1 + x^6])
, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+2x^2+2x^4}{\left(1+2x^2\right)\sqrt{-1+x^6}}\,dx &= \int \left(\frac{1}{2\sqrt{-1+x^6}}+\frac{x^2}{\sqrt{-1+x^6}}-\frac{3}{2\left(1+2x^2\right)\sqrt{-1+x^6}}\right)dx\\
&= \frac{1}{2}\int \frac{1}{\sqrt{-1+x^6}}\,dx-\frac{3}{2}\int \frac{1}{\left(1+2x^2\right)\sqrt{-1+x^6}}\,dx+\int \frac{x^2}{\sqrt{-1+x^6}}\,dx\\
&= \frac{x\left(1-x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x^2}{1-\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt{-\frac{x^2\left(1-x^2\right)}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{-1+x^6}}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{-1+\ldots}}\right.\\
&= \frac{x\left(1-x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x^2}{1-\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt{-\frac{x^2\left(1-x^2\right)}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{-1+x^6}}-\frac{3}{4}i\int \frac{1}{\left(i-\sqrt{2}x\right)\sqrt{-\ldots}}\\
&= \frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)+\frac{x\left(1-x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x^2}{1-\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\ldots\right.\right.}{4\sqrt[4]{3}\sqrt{-\frac{x^2\left(1-x^2\right)}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{-1+x^6}}
\end{aligned}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{-1+2x^2+2x^4}{\left(1+2x^2\right)\sqrt{-1+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + 2*x^2 + 2*x^4)/((1 + 2*x^2)*Sqrt[-1 + x^6]),x]
[Out] Integrate[(-1 + 2*x^2 + 2*x^4)/((1 + 2*x^2)*Sqrt[-1 + x^6]), x]
```

IntegrateAlgebraic [A] time = 9.09, size = 17, normalized size = 1.00

$$\tanh^{-1}\left(\frac{x\left(x^2-1\right)}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x^2 + 2*x^4)/((1 + 2*x^2)*Sqrt[-1 + x^6]),x]
[Out] ArcTanh[(x*(-1 + x^2))/Sqrt[-1 + x^6]]
```

fricas [B] time = 0.45, size = 72, normalized size = 4.24

$$\frac{1}{3}\log\left(x^3+\sqrt{x^6-1}\right)+\frac{1}{6}\log\left(-\frac{10\,x^6+6\,x^4+12\,x^2+6\,\sqrt{x^6-1}\left(x^3-x\right)-1}{8\,x^6+12\,x^4+6\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2-1)/(2*x^2+1)/(x^6-1)^(1/2),x, algorithm="fricas")
[Out] 1/3*log(x^3 + sqrt(x^6 - 1)) + 1/6*log(-(10*x^6 + 6*x^4 + 12*x^2 + 6*sqrt(x
^6 - 1)*(x^3 - x) - 1)/(8*x^6 + 12*x^4 + 6*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2\,x^4+2\,x^2-1}{\sqrt{x^6-1}\left(2\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2-1)/(2*x^2+1)/(x^6-1)^(1/2),x, algorithm="giac")
[Out] integrate((2*x^4 + 2*x^2 - 1)/(sqrt(x^6 - 1)*(2*x^2 + 1)), x)
```

maple [B] time = 0.36, size = 32, normalized size = 1.88

$$-\frac{\ln\left(-\frac{-2x^4+2\sqrt{x^6-1}\,x-1}{2x^2+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2-1)/(2*x^2+1)/(x^6-1)^(1/2),x)
[Out] -1/2*ln(-(-2*x^4+2*(x^6-1)^(1/2)*x-1)/(2*x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2\,x^4+2\,x^2-1}{\sqrt{x^6-1}\left(2\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2-1)/(2*x^2+1)/(x^6-1)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x^4 + 2*x^2 - 1)/(sqrt(x^6 - 1)*(2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{2\,x^4+2\,x^2-1}{\sqrt{x^6-1}\,\left(2\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 2*x^4 - 1)/((x^6 - 1)^(1/2)*(2*x^2 + 1)),x)
[Out] int((2*x^2 + 2*x^4 - 1)/((x^6 - 1)^(1/2)*(2*x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^4+2x^2-1}{\sqrt{(x-1)\,(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,\left(2x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2-1)/(2*x**2+1)/(x**6-1)**(1/2),x)
[Out] Integral((2*x**4 + 2*x**2 - 1)/(sqrt((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 +
 x + 1))*(2*x**2 + 1)), x)
```

3.120 $\int \frac{x^3}{\left(-1+x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=18

$$\frac{2}{5}\sqrt[4]{x^2-1}\left(x^2+4\right)$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.39, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{2}{5}\left(x^2-1\right)^{5/4}+2\sqrt[4]{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(-1 + x^2)^(3/4),x]
```

```
[Out] 2*(-1 + x^2)^(1/4) + (2*(-1 + x^2)^(5/4))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(-1+x^2\right)^{3/4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(-1+x)^{3/4}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1+x)^{3/4}}+\sqrt[4]{-1+x}\right)dx,x,x^2\right)\\ &= 2\sqrt[4]{-1+x^2}+\frac{2}{5}\left(-1+x^2\right)^{5/4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{2}{5}\sqrt[4]{x^2-1}\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(-1 + x^2)^(3/4),x]
```

```
[Out] (2*(-1 + x^2)^(1/4)*(4 + x^2))/5
```

IntegrateAlgebraic [A] time = 0.02, size = 18, normalized size = 1.00

$$\frac{2}{5}\sqrt[4]{x^2-1}\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(-1 + x^2)^(3/4),x]
```

```
[Out] (2*(-1 + x^2)^(1/4)*(4 + x^2))/5
```

fricas [A] time = 0.38, size = 14, normalized size = 0.78

$$\frac{2}{5}\left(x^{2}+4\right)\left(x^{2}-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/5*(x^2 + 4)*(x^2 - 1)^(1/4)
```

giac [A] time = 0.29, size = 19, normalized size = 1.06

$$\frac{2}{5}\left(x^{2}-1\right)^{\frac{5}{4}}+2\left(x^{2}-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] 2/5*(x^2 - 1)^(5/4) + 2*(x^2 - 1)^(1/4)
```

maple [A] time = 0.00, size = 21, normalized size = 1.17

$$\frac{2\left(-1+x\right)\left(1+x\right)\left(x^{2}+4\right)}{5\left(x^{2}-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2-1)^(3/4),x)
```

```
[Out] 2/5*(-1+x)*(1+x)*(x^2+4)/(x^2-1)^(3/4)
```

maxima [A] time = 0.34, size = 19, normalized size = 1.06

$$\frac{2}{5}\left(x^{2}-1\right)^{\frac{5}{4}}+2\left(x^{2}-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] 2/5*(x^2 - 1)^(5/4) + 2*(x^2 - 1)^(1/4)
```

mupad [B] time = 0.21, size = 14, normalized size = 0.78

$$\frac{2\left(x^{2}-1\right)^{1/4}\left(x^{2}+4\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2 - 1)^(3/4),x)
```

```
[Out] (2*(x^2 - 1)^(1/4)*(x^2 + 4))/5
```

sympy [A] time = 0.83, size = 75, normalized size = 4.17

$$\begin{cases} \frac{2x^2\sqrt[4]{x^2-1}}{5} + \frac{8\sqrt[4]{x^2-1}}{5} & \text{for } |x^2| > 1 \\ -\frac{2x^2\sqrt[4]{1-x^2}e^{-\frac{3i\pi}{4}}}{5} - \frac{8\sqrt[4]{1-x^2}e^{-\frac{3i\pi}{4}}}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**2-1)**(3/4),x)
```

```
[Out] Piecewise((2*x**2*(x**2 - 1)**(1/4)/5 + 8*(x**2 - 1)**(1/4)/5, Abs(x**2) >
1), (-2*x**2*(1 - x**2)**(1/4)*exp(-3*I*pi/4)/5 - 8*(1 - x**2)**(1/4)*exp(-
3*I*pi/4)/5, True))
```

3.121 $\int \frac{3+x}{(-1+x)^2\sqrt[3]{-1+x^2}}\,dx$

Optimal. Leaf size=18

$$-\frac{3\left(x^2-1\right)^{2/3}}{2(x-1)^2}$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.11, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {787}

$$-\frac{3\left(x^2-1\right)^{2/3}}{2(1-x)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x)/((-1 + x)^2*(-1 + x^2)^(1/3)),x]
```

```
[Out] (-3*(-1 + x^2)^(2/3))/(2*(1 - x)^2)
```

Rule 787

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m
*(d*g + e*f) + 2*e*f*(p + 1), 0]
```

Rubi steps

$$\int \frac{3+x}{(-1+x)^2\sqrt[3]{-1+x^2}}\,dx = -\frac{3\left(-1+x^2\right)^{2/3}}{2(1-x)^2}$$

Mathematica [A] time = 0.04, size = 21, normalized size = 1.17

$$-\frac{3(x+1)}{2(x-1)\sqrt[3]{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x)/((-1 + x)^2*(-1 + x^2)^(1/3)),x]
```

```
[Out] (-3*(1 + x))/(2*(-1 + x)*(-1 + x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.07, size = 18, normalized size = 1.00

$$-\frac{3\left(x^2-1\right)^{2/3}}{2(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + x)/((-1 + x)^2*(-1 + x^2)^(1/3)),x]
```

```
[Out] (-3*(-1 + x^2)^(2/3))/(2*(-1 + x)^2)
```

fricas [A] time = 0.39, size = 19, normalized size = 1.06

$$-\frac{3\left(x^2-1\right)^{\frac{2}{3}}}{2\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-1+x)^2/(x^2-1)^(1/3),x, algorithm="fricas")
[Out] -3/2*(x^2 - 1)^(2/3)/(x^2 - 2*x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+3}{\left(x^2-1\right)^{\frac{1}{3}}(x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-1+x)^2/(x^2-1)^(1/3),x, algorithm="giac")
[Out] integrate((x + 3)/((x^2 - 1)^(1/3)*(x - 1)^2), x)
```

maple [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{3\,(1+x)}{2\,(-1+x)\left(x^2-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+x)/(-1+x)^2/(x^2-1)^(1/3),x)
[Out] -3/2/(-1+x)*(1+x)/(x^2-1)^(1/3)
```

maxima [A] time = 0.54, size = 12, normalized size = 0.67

$$-\frac{3\,(x+1)^{\frac{2}{3}}}{2\,(x-1)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-1+x)^2/(x^2-1)^(1/3),x, algorithm="maxima")
[Out] -3/2*(x + 1)^(2/3)/(x - 1)^(4/3)
```

mupad [B] time = 0.14, size = 14, normalized size = 0.78

$$-\frac{3\left(x^2-1\right)^{2/3}}{2\,(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3)/((x^2 - 1)^(1/3)*(x - 1)^2),x)
[Out] -(3*(x^2 - 1)^(2/3))/(2*(x - 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+3}{\sqrt[3]{(x-1)\,(x+1)}\,(x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(-1+x)**2/(x**2-1)**(1/3),x)
[Out] Integral((x + 3)/(((x - 1)*(x + 1))**(1/3)*(x - 1)**2), x)
```

3.122 $\int \frac{x^3}{\left(1+x^2\right)^{2/3}} \, dx$

Optimal. Leaf size=18

$$\frac{3}{8}\left(x^2-3\right)\sqrt[3]{x^2+1}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.50, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{8}\left(x^2+1\right)^{4/3}-\frac{3}{2}\sqrt[3]{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^2)^(2/3),x]
[Out] (-3*(1 + x^2)^(1/3))/2 + (3*(1 + x^2)^(4/3))/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(1+x^2\right)^{2/3}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)^{2/3}} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{(1+x)^{2/3}}+\sqrt[3]{1+x}\right) dx, x, x^2\right)\\
&= -\frac{3}{2}\sqrt[3]{1+x^2}+\frac{3}{8}\left(1+x^2\right)^{4/3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{3}{8}\left(x^2-3\right)\sqrt[3]{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^2)^(2/3),x]
[Out] (3*(-3 + x^2)*(1 + x^2)^(1/3))/8
```

IntegrateAlgebraic [A] time = 0.02, size = 18, normalized size = 1.00

$$\frac{3}{8}\left(x^2-3\right)\sqrt[3]{x^2+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(1 + x^2)^(2/3),x]
[Out] (3*(-3 + x^2)*(1 + x^2)^(1/3))/8
```

fricas [A] time = 0.39, size = 14, normalized size = 0.78

$$\frac{3}{8}\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1)^(2/3),x, algorithm="fricas")
[Out] 3/8*(x^2 + 1)^(1/3)*(x^2 - 3)
```

giac [A] time = 0.29, size = 19, normalized size = 1.06

$$\frac{3}{8}\left(x^2+1\right)^{\frac{4}{3}}-\frac{3}{2}\left(x^2+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1)^(2/3),x, algorithm="giac")
[Out] 3/8*(x^2 + 1)^(4/3) - 3/2*(x^2 + 1)^(1/3)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{3\left(x^2-3\right)\left(x^2+1\right)^{\frac{1}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2+1)^(2/3),x)
[Out] 3/8*(x^2-3)*(x^2+1)^(1/3)
```

maxima [A] time = 0.53, size = 19, normalized size = 1.06

$$\frac{3}{8}\left(x^2+1\right)^{\frac{4}{3}}-\frac{3}{2}\left(x^2+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1)^(2/3),x, algorithm="maxima")
[Out] 3/8*(x^2 + 1)^(4/3) - 3/2*(x^2 + 1)^(1/3)
```

mupad [B] time = 0.20, size = 14, normalized size = 0.78

$$\frac{3\left(x^2+1\right)^{1/3}\left(x^2-3\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2 + 1)^(2/3),x)
[Out] (3*(x^2 + 1)^(1/3)*(x^2 - 3))/8
```

sympy [A] time = 0.78, size = 26, normalized size = 1.44

$$\frac{3x^2\sqrt[3]{x^2+1}}{8}-\frac{9\sqrt[3]{x^2+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**2+1)**(2/3),x)
```

```
[Out] 3*x**2*(x**2 + 1)**(1/3)/8 - 9*(x**2 + 1)**(1/3)/8
```

3.123 $\int \frac{1}{x^2 \sqrt[3]{-x+x^3}}\,dx$

Optimal. Leaf size=18

$$\frac{3\left(x^3-x\right)^{2/3}}{4x^2}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2014}

$$\frac{3\left(x^3-x\right)^{2/3}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-x + x^3)^(1/3)),x]
```

```
[Out] (3*(-x + x^3)^(2/3))/(4*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2 \sqrt[3]{-x+x^3}}\,dx = \frac{3\left(-x+x^3\right)^{2/3}}{4x^2}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{3\left(x\left(x^2-1\right)\right)^{2/3}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-x + x^3)^(1/3)),x]
```

```
[Out] (3*(x*(-1 + x^2))^(2/3))/(4*x^2)
```

IntegrateAlgebraic [A] time = 0.18, size = 18, normalized size = 1.00

$$\frac{3\left(x^3-x\right)^{2/3}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(-x + x^3)^(1/3)),x]
```

```
[Out] (3*(-x + x^3)^(2/3))/(4*x^2)
```

fricas [A] time = 0.39, size = 14, normalized size = 0.78

$$\frac{3\left(x^3-x\right)^{\frac{2}{3}}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3-x)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*(x^3 - x)^(2/3)/x^2
```

giac [A] time = 0.43, size = 11, normalized size = 0.61

$$\frac{3}{4}\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3-x)^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*(-1/x^2 + 1)^(2/3)
```

maple [A] time = 0.00, size = 21, normalized size = 1.17

$$\frac{3\,(-1+x)\,(1+x)}{4x\left(x^3-x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^3-x)^(1/3),x)
```

```
[Out] 3/4/x*(-1+x)*(1+x)/(x^3-x)^(1/3)
```

maxima [A] time = 0.57, size = 22, normalized size = 1.22

$$\frac{3\left(x^3-x\right)}{4\,(x+1)^{\frac{1}{3}}(x-1)^{\frac{1}{3}}x^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3-x)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/4*(x^3 - x)/((x + 1)^(1/3)*(x - 1)^(1/3)*x^(7/3))
```

mupad [B] time = 0.17, size = 14, normalized size = 0.78

$$\frac{3\left(x^3-x\right)^{2/3}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^3 - x)^(1/3)),x)
```

```
[Out] (3*(x^3 - x)^(2/3))/(4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt[3]{x\,(x-1)\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**3-x)**(1/3),x)
```

```
[Out] Integral(1/(x**2*(x*(x - 1)*(x + 1))**(1/3)), x)
```

3.124 $\int \frac{\sqrt[3]{-x+x^3}}{x^4} \, dx$

Optimal. Leaf size=18

$$\frac{3 \left(x^3-x\right)^{4/3}}{8 x^4}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2014}

$$\frac{3 \left(x^3-x\right)^{4/3}}{8 x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)^(1/3)/x^4,x]
[Out] (3*(-x + x^3)^(4/3))/(8*x^4)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{-x+x^3}}{x^4} \, dx = \frac{3 \left(-x+x^3\right)^{4/3}}{8 x^4}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{3 \left(x \left(x^2-1\right)\right)^{4/3}}{8 x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)^(1/3)/x^4,x]
[Out] (3*(x*(-1 + x^2))^(4/3))/(8*x^4)
```

IntegrateAlgebraic [A] time = 0.17, size = 18, normalized size = 1.00

$$\frac{3 \left(x^3-x\right)^{4/3}}{8 x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x + x^3)^(1/3)/x^4,x]
[Out] (3*(-x + x^3)^(4/3))/(8*x^4)
```

fricas [A] time = 0.40, size = 19, normalized size = 1.06

$$\frac{3 \left(x^3-x\right)^{\frac{1}{3}} \left(x^2-1\right)}{8 \, x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)^(1/3)/x^4,x, algorithm="fricas")
[Out] 3/8*(x^3 - x)^(1/3)*(x^2 - 1)/x^3
```

giac [A] time = 0.33, size = 11, normalized size = 0.61

$$\frac{3}{8}\left(-\frac{1}{x^2}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)^(1/3)/x^4,x, algorithm="giac")
[Out] 3/8*(-1/x^2 + 1)^(4/3)
```

maple [A] time = 0.00, size = 21, normalized size = 1.17

$$\frac{3\,(-1+x)\,(1+x)\left(x^3-x\right)^{\frac{1}{3}}}{8x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x)^(1/3)/x^4,x)
[Out] 3/8/x^3*(-1+x)*(1+x)*(x^3-x)^(1/3)
```

maxima [A] time = 0.41, size = 22, normalized size = 1.22

$$\frac{3\left(x^3-x\right)(x+1)^{\frac{1}{3}}(x-1)^{\frac{1}{3}}}{8\,x^{\frac{11}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)^(1/3)/x^4,x, algorithm="maxima")
[Out] 3/8*(x^3 - x)*(x + 1)^(1/3)*(x - 1)^(1/3)/x^(11/3)
```

mupad [B] time = 0.17, size = 31, normalized size = 1.72

$$-\frac{3\left(x^3-x\right)^{1/3}-3\,x^2\left(x^3-x\right)^{1/3}}{8\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - x)^(1/3)/x^4,x)
[Out] -(3*(x^3 - x)^(1/3) - 3*x^2*(x^3 - x)^(1/3))/(8*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x\,(x-1)\,(x+1)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x)**(1/3)/x**4,x)
[Out] Integral((x*(x - 1)*(x + 1))**(1/3)/x**4, x)
```

3.125 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx$

Optimal. Leaf size=18

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Rubi [A] time = 0.07, antiderivative size = 12, normalized size of antiderivative = 0.67, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2056, 449}

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]
```

```
[Out] (-2*x)/Sqrt[x + x^3]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx &= \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}(1+x^2)^{3/2}}\,dx}{\sqrt{x+x^3}}\\ &= -\frac{2x}{\sqrt{x+x^3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 0.67

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]
```

```
[Out] (-2*x)/Sqrt[x + x^3]
```

IntegrateAlgebraic [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]
[Out] (-2*Sqrt[x + x^3])/(1 + x^2)
```

fricas [A] time = 0.39, size = 16, normalized size = 0.89

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(x^3 + x)/(x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^3+x}\left(x^2+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)
```

maple [A] time = 0.01, size = 11, normalized size = 0.61

$$-\frac{2x}{\sqrt{x^3+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x)
[Out] -2*x/(x^3+x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^3+x}\left(x^2+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)
```

mupad [B] time = 0.05, size = 10, normalized size = 0.56

$$-\frac{2\,x}{\sqrt{x^3+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + 1)*(x + x^3)^(1/2)),x)
[Out] -(2*x)/(x + x^3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)}{\sqrt{x\left(x^2+1\right)}\,\left(x^2+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**3+x)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + 1))*(x**2 + 1)), x)
```

3.126 $\int \frac{1}{x\sqrt[3]{x^2+x^3}}\,dx$

Optimal. Leaf size=18

$$-\frac{3\left(x^3+x^2\right)^{2/3}}{2x^2}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2014}

$$-\frac{3\left(x^3+x^2\right)^{2/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(x^2 + x^3)^(1/3)),x]
```

```
[Out] (-3*(x^2 + x^3)^(2/3))/(2*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{x^2+x^3}}\,dx=-\frac{3\left(x^2+x^3\right)^{2/3}}{2x^2}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{3(x+1)}{2\sqrt[3]{x^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(x^2 + x^3)^(1/3)),x]
```

```
[Out] (-3*(1 + x))/(2*(x^2*(1 + x))^(1/3))
```

IntegrateAlgebraic [A] time = 0.14, size = 18, normalized size = 1.00

$$-\frac{3\left(x^3+x^2\right)^{2/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(x^2 + x^3)^(1/3)),x]
```

```
[Out] (-3*(x^2 + x^3)^(2/3))/(2*x^2)
```

fricas [A] time = 0.38, size = 14, normalized size = 0.78

$$-\frac{3\left(x^3+x^2\right)^{\frac{2}{3}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/2*(x^3 + x^2)^(2/3)/x^2
```

giac [A] time = 0.38, size = 9, normalized size = 0.50

$$-\frac{3}{2}\left(\frac{1}{x}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] -3/2*(1/x + 1)^(2/3)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$-\frac{3\,(1+x)}{2\left(x^{3}+x^{2}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^3+x^2)^(1/3),x)
```

```
[Out] -3/2*(1+x)/(x^3+x^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{3}+x^{2}\right)^{\frac{1}{3}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3+x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + x^2)^(1/3)*x), x)
```

mupad [B] time = 0.17, size = 14, normalized size = 0.78

$$-\frac{3\left(x^{3}+x^{2}\right)^{2/3}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 + x^3)^(1/3)),x)
```

```
[Out] -(3*(x^2 + x^3)^(2/3))/(2*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{x^{2}\,(x+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**3+x**2)**(1/3),x)
```

```
[Out] Integral(1/(x*(x**2*(x + 1))**(1/3)), x)
```

3.127 $\int \frac{2+x}{x^2 \sqrt[4]{x^2+x^3}} \, dx$

Optimal. Leaf size=18

$$-\frac{4\left(x^3+x^2\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1590}

$$-\frac{4\left(x^3+x^2\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/(x^2*(x^2 + x^3)^(1/4)),x]

[Out] (-4*(x^2 + x^3)^(3/4))/(3*x^3)

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{2+x}{x^2 \sqrt[4]{x^2+x^3}} \, dx = -\frac{4\left(x^2+x^3\right)^{3/4}}{3x^3}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{4\left(x^2(x+1)\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x)/(x^2*(x^2 + x^3)^(1/4)),x]

[Out] (-4*(x^2*(1 + x))^(3/4))/(3*x^3)

IntegrateAlgebraic [A] time = 0.17, size = 18, normalized size = 1.00

$$-\frac{4\left(x^3+x^2\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x)/(x^2*(x^2 + x^3)^(1/4)),x]

[Out] (-4*(x^2 + x^3)^(3/4))/(3*x^3)

fricas [A] time = 0.40, size = 14, normalized size = 0.78

$$-\frac{4\left(x^3+x^2\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/x^2/(x^3+x^2)^(1/4),x, algorithm="fricas")
[Out] -4/3*(x^3 + x^2)^(3/4)/x^3
```

giac [A] time = 0.41, size = 11, normalized size = 0.61

$$-\frac{4}{3}\left(\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/x^2/(x^3+x^2)^(1/4),x, algorithm="giac")
[Out] -4/3*(1/x + 1/x^2)^(3/4)
```

maple [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{4\,(1+x)}{3x\left(x^3+x^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/x^2/(x^3+x^2)^(1/4),x)
[Out] -4/3/x*(1+x)/(x^3+x^2)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\left(x^3+x^2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/x^2/(x^3+x^2)^(1/4),x, algorithm="maxima")
[Out] integrate((x + 2)/((x^3 + x^2)^(1/4)*x^2), x)
```

mupad [B] time = 0.23, size = 14, normalized size = 0.78

$$-\frac{4\left(x^3+x^2\right)^{3/4}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/(x^2*(x^2 + x^3)^(1/4)),x)
[Out] -(4*(x^2 + x^3)^(3/4))/(3*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{x^2\sqrt[4]{x^2\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/x**2/(x**3+x**2)**(1/4),x)
[Out] Integral((x + 2)/(x**2*(x**2*(x + 1))**(1/4)), x)
```

3.128 $\int \frac{\left(-2b+ax^3\right)\sqrt{b+ax^3}}{x^4}\,dx$

Optimal. Leaf size=18

$$\frac{2\left(ax^3+b\right)^{3/2}}{3x^3}$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 74}

$$\frac{2\left(ax^3+b\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-2*b + a*x^3)*Sqrt[b + a*x^3])/x^4,x]
```

```
[Out] (2*(b + a*x^3)^(3/2))/(3*x^3)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-2b+ax^3\right)\sqrt{b+ax^3}}{x^4}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(-2b+ax)\sqrt{b+ax}}{x^2}\,dx, x, x^3\right)\\ &= \frac{2\left(b+ax^3\right)^{3/2}}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{2\left(ax^3+b\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-2*b + a*x^3)*Sqrt[b + a*x^3])/x^4,x]
```

```
[Out] (2*(b + a*x^3)^(3/2))/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.06, size = 18, normalized size = 1.00

$$\frac{2\left(ax^3+b\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*b + a*x^3)*Sqrt[b + a*x^3])/x^4,x]
```

```
[Out] (2*(b + a*x^3)^(3/2))/(3*x^3)
```

fricas [A] time = 0.38, size = 14, normalized size = 0.78

$$\frac{2\left(ax^3+b\right)^{\frac{3}{2}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*b)*(a*x^3+b)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 2/3*(a*x^3 + b)^(3/2)/x^3
```

giac [B] time = 0.31, size = 34, normalized size = 1.89

$$\frac{2\left(\sqrt{ax^3+b}\,a^2+\frac{\sqrt{ax^3+b}\,ab}{x^3}\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*b)*(a*x^3+b)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(sqrt(a*x^3 + b)*a^2 + sqrt(a*x^3 + b)*a*b/x^3)/a
```

maple [A] time = 0.01, size = 15, normalized size = 0.83

$$\frac{2\left(a\,x^3+b\right)^{\frac{3}{2}}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-2*b)*(a*x^3+b)^(1/2)/x^4,x)
```

```
[Out] 2/3*(a*x^3+b)^(3/2)/x^3
```

maxima [B] time = 0.45, size = 107, normalized size = 5.94

$$\frac{1}{3}\left(\sqrt{b}\,\log\left(\frac{\sqrt{ax^3+b}-\sqrt{b}}{\sqrt{ax^3+b}+\sqrt{b}}\right)+2\,\sqrt{ax^3+b}\right)a-\frac{1}{3}\left(\frac{a\log\left(\frac{\sqrt{ax^3+b}-\sqrt{b}}{\sqrt{ax^3+b}+\sqrt{b}}\right)}{\sqrt{b}}-\frac{2\,\sqrt{ax^3+b}}{x^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*b)*(a*x^3+b)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(b)*log((sqrt(a*x^3 + b) - sqrt(b))/(sqrt(a*x^3 + b) + sqrt(b))) +
 2*sqrt(a*x^3 + b))*a - 1/3*(a*log((sqrt(a*x^3 + b) - sqrt(b))/(sqrt(a*x^3
+ b) + sqrt(b)))/sqrt(b) - 2*sqrt(a*x^3 + b)/x^3)*b
```

mupad [B] time = 0.25, size = 14, normalized size = 0.78

$$\frac{2\left(a\,x^3+b\right)^{3/2}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b + a*x^3)^(1/2)*(2*b - a*x^3))/x^4,x)
```

```
[Out] (2*(b + a*x^3)^(3/2))/(3*x^3)
```

sympy [**B**] time = 26.24, size = 78, normalized size = 4.33

$$\frac{2a^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{1+\frac{b}{ax^3}}}+\frac{2\sqrt{a}\,b\sqrt{1+\frac{b}{ax^3}}}{3x^{\frac{3}{2}}}+\frac{2\sqrt{a}\,b}{3x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-2*b)*(a*x**3+b)**(1/2)/x**4,x)
```

```
[Out] 2*a**(3/2)*x**(3/2)/(3*sqrt(1 + b/(a*x**3))) + 2*sqrt(a)*b*sqrt(1 + b/(a*x*
*3))/(3*x**(3/2)) + 2*sqrt(a)*b/(3*x**(3/2)*sqrt(1 + b/(a*x**3)))
```

3.129 $\int \frac{1+x+x^2}{(-1+x)^2 \sqrt{-1+x^4}} \, dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{x^4-1}}{2(x-1)^2}$$

Rubi [B] time = 0.41, antiderivative size = 68, normalized size of antiderivative = 3.78, number of steps used = 32, number of rules used = 19, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.905, Rules used = {6742, 222, 2153, 1152, 414, 527, 524, 427, 424, 253, 1248, 659, 651, 1256, 471, 21, 1725, 423, 426}

$$\frac{x\left(x^2+1\right)}{\left(1-x^2\right)\sqrt{x^4-1}}+\frac{\sqrt{x^4-1}}{2\left(1-x^2\right)}-\frac{\sqrt{x^4-1}}{\left(1-x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/((-1 + x)^2*Sqrt[-1 + x^4]),x]

[Out] (x*(1 + x^2))/((1 - x^2)*Sqrt[-1 + x^4]) - Sqrt[-1 + x^4]/(1 - x^2)^2 + Sqr
t[-1 + x^4]/(2*(1 - x^2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
```

```
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1256

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d
+ (c*x^2)/e)^FracPart[p]), Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e
)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && EqQ[c*d^2 + a*e^2, 0]
&&  !IntegerQ[p]
```

Rule 1725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[d,
 Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Dist[e, Int[x/((d^2 - e^
2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Rule 2153

```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^nn)^p, (c/(c^2 - d^2*x^(2*n)) - (d*x^n)/
(c^2 - d^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !
IntegerQ[p] && ILtQ[q, 0] && IGtQ[Log[2, nn/n], 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1+x+x^2}{(-1+x)^2\sqrt{-1+x^4}}\,dx = \int\left(\frac{1}{\sqrt{-1+x^4}} + \frac{3}{(-1+x)^2\sqrt{-1+x^4}} + \frac{3}{(-1+x)\sqrt{-1+x^4}}\right)dx$$

$$= 3\int \frac{1}{(-1+x)^2\sqrt{-1+x^4}}\,dx + 3\int \frac{1}{(-1+x)\sqrt{-1+x^4}}\,dx + \int \frac{1}{\sqrt{-1+x^4}}\,dx$$

$$= \frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)\middle|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^4}} - 3\int \frac{1}{\left(1-x^2\right)\sqrt{-1+x^4}}\,dx - 3\int \frac{}{\left(1-x^2\right.}$$

$$= \frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)\middle|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^4}} - \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{-1+x^2}}\,dx, x, x^2\right) +$$

$$= -\frac{3x\left(1+x^2\right)}{2\sqrt{-1+x^4}} + \frac{3\sqrt{-1+x^4}}{2\left(1-x^2\right)} + \frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)\middle|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^4}} + 3\,\text{Subst}\left(\int\right.$$

$$= -\frac{3x\left(1+x^2\right)}{2\sqrt{-1+x^4}} + \frac{x\left(1+x^2\right)}{\left(1-x^2\right)\sqrt{-1+x^4}} - \frac{\sqrt{-1+x^4}}{\left(1-x^2\right)^2} + \frac{3\sqrt{-1+x^4}}{2\left(1-x^2\right)} + \frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}}{\sqrt{2}\,\sqrt{}}$$

$$= \frac{x\left(1+x^2\right)}{\left(1-x^2\right)\sqrt{-1+x^4}} - \frac{\sqrt{-1+x^4}}{\left(1-x^2\right)^2} + \frac{\sqrt{-1+x^4}}{2\left(1-x^2\right)} + \frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right.\right.}{\sqrt{2}\,\sqrt{-1+x^4}}$$

$$= \frac{x\left(1+x^2\right)}{\left(1-x^2\right)\sqrt{-1+x^4}} - \frac{\sqrt{-1+x^4}}{\left(1-x^2\right)^2} + \frac{\sqrt{-1+x^4}}{2\left(1-x^2\right)} + \frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right.\right.}{\sqrt{2}\,\sqrt{-1+x^4}}$$

$$= \frac{x\left(1+x^2\right)}{\left(1-x^2\right)\sqrt{-1+x^4}} - \frac{\sqrt{-1+x^4}}{\left(1-x^2\right)^2} + \frac{\sqrt{-1+x^4}}{2\left(1-x^2\right)} + \frac{3\sqrt{1-x^2}\,\sqrt{1+x^2}\,E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{-1+x^4}}$$

$$= \frac{x\left(1+x^2\right)}{\left(1-x^2\right)\sqrt{-1+x^4}} - \frac{\sqrt{-1+x^4}}{\left(1-x^2\right)^2} + \frac{\sqrt{-1+x^4}}{2\left(1-x^2\right)}$$

Mathematica [A] time = 0.14, size = 33, normalized size = 1.83

$$\frac{-x^3-x^2-x-1}{2(x-1)\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/((-1 + x)^2*Sqrt[-1 + x^4]),x]
[Out] (-1 - x - x^2 - x^3)/(2*(-1 + x)*Sqrt[-1 + x^4])
```

IntegrateAlgebraic [A] time = 3.65, size = 18, normalized size = 1.00

$$-\frac{\sqrt{x^4-1}}{2(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x + x^2)/((-1 + x)^2*Sqrt[-1 + x^4]),x]
[Out] -1/2*Sqrt[-1 + x^4]/(-1 + x)^2
```

fricas [A] time = 0.39, size = 19, normalized size = 1.06

$$-\frac{\sqrt{x^4-1}}{2\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(-1+x)^2/(x^4-1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(x^4 - 1)/(x^2 - 2*x + 1)
```

giac [A] time = 0.45, size = 27, normalized size = 1.50

$$-\frac{1}{2}\sqrt{\frac{4}{x-1}+\frac{6}{(x-1)^2}+\frac{4}{(x-1)^3}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(-1+x)^2/(x^4-1)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(4/(x - 1) + 6/(x - 1)^2 + 4/(x - 1)^3 + 1)
```

maple [A] time = 0.01, size = 23, normalized size = 1.28

$$-\frac{(1+x)\left(x^2+1\right)}{2(-1+x)\sqrt{x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/(-1+x)^2/(x^4-1)^(1/2),x)
[Out] -1/2/(-1+x)*(1+x)*(x^2+1)/(x^4-1)^(1/2)
```

maxima [A] time = 0.59, size = 28, normalized size = 1.56

$$-\frac{x^3+x^2+x+1}{2\sqrt{x^2+1}\sqrt{x+1}(x-1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(-1+x)^2/(x^4-1)^(1/2),x, algorithm="maxima")
[Out] -1/2*(x^3 + x^2 + x + 1)/(sqrt(x^2 + 1)*sqrt(x + 1)*(x - 1)^(3/2))
```

mupad [B] time = 0.14, size = 14, normalized size = 0.78

$$-\frac{\sqrt{x^4-1}}{2(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/((x^4 - 1)^(1/2)*(x - 1)^2),x)
[Out] -(x^4 - 1)^(1/2)/(2*(x - 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+x+1}{\sqrt{(x-1)(x+1)\left(x^2+1\right)}\,(x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(-1+x)**2/(x**4-1)**(1/2),x)
[Out] Integral((x**2 + x + 1)/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x - 1)**2), x)
```

3.130 $\int \frac{-1+x^4}{x^2\sqrt{-x+x^3}}\,dx$

Optimal. Leaf size=18

$$\frac{2\left(x^3-x\right)^{3/2}}{3x^3}$$

Rubi [C] time = 0.10, antiderivative size = 96, normalized size of antiderivative = 5.33, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2048, 2025, 2011, 329, 222}

$$\frac{2\sqrt{x^3-x}}{3}-\frac{\sqrt{2}\sqrt{x-1}\sqrt{x}\sqrt{x+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{x-1}}\right)|\frac{1}{2}\right)}{3\sqrt{x^3-x}}-\frac{2\sqrt{x^3-x}}{3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^4)/(x^2*Sqrt[-x + x^3]),x]
```

```
[Out] (2*Sqrt[-x + x^3])/3 - (2*Sqrt[-x + x^3])/(3*x^2) - (Sqrt[2]*Sqrt[-1 + x]*S
qrt[x]*Sqrt[1 + x]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[x])/Sqrt[-1 + x]], 1/2])/
(3*Sqrt[-x + x^3])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2048

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Int[(c*x)^
m*ExpandToSum[Pq - Pqq*x^q - (a*Pqq*(m + q - n + 1)*x^(q - n))/(b*(m + q +
n*p + 1)), x]*(a*x^j + b*x^n)^p, x] + Simp[(Pqq*(c*x)^(m + q - n + 1)*(a*x^
j + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /; GtQ[q, n -
```

```
1] && NeQ[m + q + n*p + 1, 0] && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*
n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] &&  !IntegerQ[p] && IGtQ
[j, 0] && IGtQ[n, 0] && LtQ[j, n]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^4}{x^2\sqrt{-x+x^3}}\,dx &= \frac{2}{3}\sqrt{-x+x^3}-\int \frac{1}{x^2\sqrt{-x+x^3}}\,dx\\
&= \frac{2}{3}\sqrt{-x+x^3}-\frac{2\sqrt{-x+x^3}}{3x^2}-\frac{1}{3}\int \frac{1}{\sqrt{-x+x^3}}\,dx\\
&= \frac{2}{3}\sqrt{-x+x^3}-\frac{2\sqrt{-x+x^3}}{3x^2}-\frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-1+x^2}}\,dx}{3\sqrt{-x+x^3}}\\
&= \frac{2}{3}\sqrt{-x+x^3}-\frac{2\sqrt{-x+x^3}}{3x^2}-\frac{\left(2\sqrt{x}\sqrt{-1+x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{-x+x^3}}\\
&= \frac{2}{3}\sqrt{-x+x^3}-\frac{2\sqrt{-x+x^3}}{3x^2}-\frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{3\sqrt{-x+x^3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{2\left(x\left(x^2-1\right)\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)/(x^2*Sqrt[-x + x^3]),x]
[Out] (2*(x*(-1 + x^2))^(3/2))/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.18, size = 18, normalized size = 1.00

$$\frac{2\left(x^3-x\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/(x^2*Sqrt[-x + x^3]),x]
[Out] (2*(-x + x^3)^(3/2))/(3*x^3)
```

fricas [A] time = 0.39, size = 19, normalized size = 1.06

$$\frac{2\sqrt{x^3-x}\left(x^2-1\right)}{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^3-x)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(x^3 - x)*(x^2 - 1)/x^2
```

giac [A] time = 0.54, size = 25, normalized size = 1.39

$$\frac{2}{3}\sqrt{x^3-x}-\frac{2}{3}\sqrt{\frac{1}{x}-\frac{1}{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^3-x)^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt(x^3 - x) - 2/3*sqrt(1/x - 1/x^3)
```

maple [A] time = 0.01, size = 26, normalized size = 1.44

$$\frac{2\left(x^2-1\right)(-1+x)(1+x)}{3\sqrt{x^3-x}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/x^2/(x^3-x)^(1/2),x)
[Out] 2/3*(x^2-1)*(-1+x)*(1+x)/(x^3-x)^(1/2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\sqrt{x^3-x}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^3-x)^(1/2),x, algorithm="maxima")
[Out] integrate((x^4 - 1)/(sqrt(x^3 - x)*x^2), x)
```

mupad [B] time = 0.01, size = 1, normalized size = 0.06

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/(x^2*(x^3 - x)^(1/2)),x)
[Out] 0
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2+1\right)}{x^2\sqrt{x(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/x**2/(x**3-x)**(1/2),x)
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)/(x**2*sqrt(x*(x - 1)*(x + 1))), x)
```

3.131 $\int \frac{1}{x^2\sqrt{-x+x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{2\sqrt{x^4-x}}{3x^2}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2014}

$$\frac{2\sqrt{x^4-x}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-x + x^4]),x]
[Out] (2*Sqrt[-x + x^4])/(3*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-x+x^4}}\,dx = \frac{2\sqrt{-x+x^4}}{3x^2}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{2\sqrt{x\left(x^3-1\right)}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-x + x^4]),x]
[Out] (2*Sqrt[x*(-1 + x^3)])/(3*x^2)
```

IntegrateAlgebraic [A] time = 0.38, size = 18, normalized size = 1.00

$$\frac{2\sqrt{x^4-x}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[-x + x^4]),x]
[Out] (2*Sqrt[-x + x^4])/(3*x^2)
```

fricas [A] time = 0.39, size = 14, normalized size = 0.78

$$\frac{2\,\sqrt{x^4-x}}{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(x^4 - x)/x^2
```

giac [A] time = 0.58, size = 11, normalized size = 0.61

$$\frac{2}{3}\sqrt{-\frac{1}{x^3}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4-x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(-1/x^3 + 1)
```

maple [A] time = 0.00, size = 24, normalized size = 1.33

$$\frac{2(-1+x)\left(x^2+x+1\right)}{3x\sqrt{x^4-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^4-x)^(1/2),x)
```

```
[Out] 2/3/x*(-1+x)*(x^2+x+1)/(x^4-x)^(1/2)
```

maxima [A] time = 0.70, size = 25, normalized size = 1.39

$$\frac{2\left(x^4-x\right)}{3\sqrt{x^2+x+1}\sqrt{x-1}\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4-x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x^4 - x)/(sqrt(x^2 + x + 1)*sqrt(x - 1)*x^(5/2))
```

mupad [B] time = 0.19, size = 14, normalized size = 0.78

$$\frac{2\sqrt{x^4-x}}{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4 - x)^(1/2)),x)
```

```
[Out] (2*(x^4 - x)^(1/2))/(3*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x(x-1)\left(x^2+x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**4-x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x*(x - 1)*(x**2 + x + 1))), x)
```

3.132 $\int \frac{1}{x^3 \sqrt[4]{-x+x^4}} \, dx$

Optimal. Leaf size=18

$$\frac{4\left(x^4-x\right)^{3/4}}{9x^3}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2014}

$$\frac{4\left(x^4-x\right)^{3/4}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-x + x^4)^(1/4)),x]
```

```
[Out] (4*(-x + x^4)^(3/4))/(9*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3 \sqrt[4]{-x+x^4}} \, dx = \frac{4\left(-x+x^4\right)^{3/4}}{9x^3}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{4\left(x\left(x^3-1\right)\right)^{3/4}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-x + x^4)^(1/4)),x]
```

```
[Out] (4*(x*(-1 + x^3))^(3/4))/(9*x^3)
```

IntegrateAlgebraic [A] time = 0.22, size = 18, normalized size = 1.00

$$\frac{4\left(x^4-x\right)^{3/4}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(-x + x^4)^(1/4)),x]
```

```
[Out] (4*(-x + x^4)^(3/4))/(9*x^3)
```

fricas [A] time = 0.40, size = 14, normalized size = 0.78

$$\frac{4\left(x^4-x\right)^{\frac{3}{4}}}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^4-x)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/9*(x^4 - x)^(3/4)/x^3
```

giac [A] time = 0.47, size = 11, normalized size = 0.61

$$-\frac{4}{9}\left(-\frac{1}{x^3}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^4-x)^(1/4),x, algorithm="giac")
```

```
[Out] -4/9*(-1/x^3 + 1)^(3/4)
```

maple [A] time = 0.00, size = 24, normalized size = 1.33

$$\frac{4\left(-1+x\right)\left(x^2+x+1\right)}{9x^2\left(x^4-x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^4-x)^(1/4),x)
```

```
[Out] 4/9/x^2*(-1+x)*(x^2+x+1)/(x^4-x)^(1/4)
```

maxima [A] time = 0.58, size = 25, normalized size = 1.39

$$\frac{4\left(x^4-x\right)}{9\left(x^2+x+1\right)^{\frac{1}{4}}\left(x-1\right)^{\frac{1}{4}}x^{\frac{13}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^4-x)^(1/4),x, algorithm="maxima")
```

```
[Out] 4/9*(x^4 - x)/((x^2 + x + 1)^(1/4)*(x - 1)^(1/4)*x^(13/4))
```

mupad [B] time = 0.18, size = 14, normalized size = 0.78

$$\frac{4\left(x^4-x\right)^{3/4}}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^4 - x)^(1/4)),x)
```

```
[Out] (4*(x^4 - x)^(3/4))/(9*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt[4]{x\left(x-1\right)\left(x^2+x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**4-x)**(1/4),x)
```

```
[Out] Integral(1/(x**3*(x*(x - 1)*(x**2 + x + 1))**(1/4)), x)
```

3.133 $\int \frac{-1+x^3}{x^6 \sqrt[4]{-x+x^4}} \, dx$

Optimal. Leaf size=18

$$\frac{4\left(x^4-x\right)^{7/4}}{21x^7}$$

Rubi [B] time = 0.12, antiderivative size = 37, normalized size of antiderivative = 2.06, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2052, 2016, 2014}

$$\frac{4\left(x^4-x\right)^{3/4}}{21x^3}-\frac{4\left(x^4-x\right)^{3/4}}{21x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(x^6*(-x + x^4)^(1/4)),x]
```

```
[Out] (-4*(-x + x^4)^(3/4))/(21*x^6) + (4*(-x + x^4)^(3/4))/(21*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^3}{x^6 \sqrt[4]{-x+x^4}} \, dx &= \int \left(-\frac{1}{x^6 \sqrt[4]{-x+x^4}}+\frac{1}{x^3 \sqrt[4]{-x+x^4}}\right) dx \\
&= -\int \frac{1}{x^6 \sqrt[4]{-x+x^4}} \, dx+\int \frac{1}{x^3 \sqrt[4]{-x+x^4}} \, dx \\
&= -\frac{4\left(-x+x^4\right)^{3/4}}{21x^6}+\frac{4\left(-x+x^4\right)^{3/4}}{9x^3}-\frac{4}{7}\int \frac{1}{x^3 \sqrt[4]{-x+x^4}} \, dx \\
&= -\frac{4\left(-x+x^4\right)^{3/4}}{21x^6}+\frac{4\left(-x+x^4\right)^{3/4}}{21x^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{4\left(x\left(x^3-1\right)\right)^{7/4}}{21x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(x^6*(-x + x^4)^(1/4)),x]
[Out] (4*(x*(-1 + x^3))^(7/4))/(21*x^7)
```

IntegrateAlgebraic [A] time = 0.26, size = 18, normalized size = 1.00

$$\frac{4\left(x^4-x\right)^{7/4}}{21x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)/(x^6*(-x + x^4)^(1/4)),x]
[Out] (4*(-x + x^4)^(7/4))/(21*x^7)
```

fricas [A] time = 0.42, size = 19, normalized size = 1.06

$$\frac{4\left(x^4-x\right)^{\frac{3}{4}}\left(x^3-1\right)}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/x^6/(x^4-x)^(1/4),x, algorithm="fricas")
[Out] 4/21*(x^4 - x)^(3/4)*(x^3 - 1)/x^6
```

giac [A] time = 0.50, size = 11, normalized size = 0.61

$$-\frac{4}{21}\left(-\frac{1}{x^3}+1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/x^6/(x^4-x)^(1/4),x, algorithm="giac")
[Out] -4/21*(-1/x^3 + 1)^(7/4)
```

maple [A] time = 0.00, size = 29, normalized size = 1.61

$$\frac{4\left(-1+x\right)\left(x^2+x+1\right)\left(x^3-1\right)}{21x^5\left(x^4-x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/x^6/(x^4-x)^(1/4),x)
[Out] 4/21/x^5*(-1+x)*(x^2+x+1)*(x^3-1)/(x^4-x)^(1/4)
```

maxima [B] time = 0.75, size = 58, normalized size = 3.22

$$\frac{4\left(x^4-x\right)}{9\left(x^2+x+1\right)^{\frac{1}{4}}(x-1)^{\frac{1}{4}}x^{\frac{13}{4}}}-\frac{4\left(4\,x^7-x^4-3\,x\right)}{63\left(x^2+x+1\right)^{\frac{1}{4}}(x-1)^{\frac{1}{4}}x^{\frac{25}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/x^6/(x^4-x)^(1/4),x, algorithm="maxima")
```

```
[Out] 4/9*(x^4 - x)/((x^2 + x + 1)^(1/4)*(x - 1)^(1/4)*x^(13/4)) - 4/63*(4*x^7 -
x^4 - 3*x)/((x^2 + x + 1)^(1/4)*(x - 1)^(1/4)*x^(25/4))
```

mupad [B] time = 0.24, size = 31, normalized size = 1.72

$$-\frac{4\left(x^4-x\right)^{3/4}-4\,x^3\,\left(x^4-x\right)^{3/4}}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x^6*(x^4 - x)^(1/4)),x)
```

```
[Out] -(4*(x^4 - x)^(3/4) - 4*x^3*(x^4 - x)^(3/4))/(21*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\left(x^2+x+1\right)}{x^6\sqrt[4]{x\,(x-1)\left(x^2+x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/x**6/(x**4-x)**(1/4),x)
```

```
[Out] Integral((x - 1)*(x**2 + x + 1)/(x**6*(x*(x - 1)*(x**2 + x + 1))**(1/4)), x
)
```

3.134 $\int \frac{\sqrt[4]{-x+x^4}}{x^5}\,dx$

Optimal. Leaf size=18

$$\frac{4\left(x^4-x\right)^{5/4}}{15x^5}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2014}

$$\frac{4\left(x^4-x\right)^{5/4}}{15x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^4)^(1/4)/x^5,x]
[Out] (4*(-x + x^4)^(5/4))/(15*x^5)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[4]{-x+x^4}}{x^5}\,dx=\frac{4\left(-x+x^4\right)^{5/4}}{15x^5}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{4\left(x\left(x^3-1\right)\right)^{5/4}}{15x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^4)^(1/4)/x^5,x]
[Out] (4*(x*(-1 + x^3))^(5/4))/(15*x^5)
```

IntegrateAlgebraic [A] time = 0.19, size = 18, normalized size = 1.00

$$\frac{4\left(x^4-x\right)^{5/4}}{15x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x + x^4)^(1/4)/x^5,x]
[Out] (4*(-x + x^4)^(5/4))/(15*x^5)
```

fricas [A] time = 0.41, size = 19, normalized size = 1.06

$$\frac{4\left(x^4-x\right)^{\frac{1}{4}}\left(x^3-1\right)}{15\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/4)/x^5,x, algorithm="fricas")
```

```
[Out] 4/15*(x^4 - x)^(1/4)*(x^3 - 1)/x^4
```

giac [A] time = 0.51, size = 11, normalized size = 0.61

$$-\frac{4}{15}\left(-\frac{1}{x^3}+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/4)/x^5,x, algorithm="giac")
```

```
[Out] -4/15*(-1/x^3 + 1)^(5/4)
```

maple [A] time = 0.00, size = 24, normalized size = 1.33

$$\frac{4\left(-1+x\right)\left(x^2+x+1\right)\left(x^4-x\right)^{\frac{1}{4}}}{15x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x)^(1/4)/x^5,x)
```

```
[Out] 4/15/x^4*(-1+x)*(x^2+x+1)*(x^4-x)^(1/4)
```

maxima [A] time = 0.67, size = 25, normalized size = 1.39

$$\frac{4\left(x^4-x\right)\left(x^2+x+1\right)^{\frac{1}{4}}\left(x-1\right)^{\frac{1}{4}}}{15\,x^{\frac{19}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/4)/x^5,x, algorithm="maxima")
```

```
[Out] 4/15*(x^4 - x)*(x^2 + x + 1)^(1/4)*(x - 1)^(1/4)/x^(19/4)
```

mupad [B] time = 0.19, size = 31, normalized size = 1.72

$$-\frac{4\left(x^4-x\right)^{1/4}-4\,x^3\left(x^4-x\right)^{1/4}}{15\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x)^(1/4)/x^5,x)
```

```
[Out] -(4*(x^4 - x)^(1/4) - 4*x^3*(x^4 - x)^(1/4))/(15*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x\left(x-1\right)\left(x^2+x+1\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x)**(1/4)/x**5,x)
```

```
[Out] Integral((x*(x - 1)*(x**2 + x + 1))**(1/4)/x**5, x)
```

3.135 $\int \frac{(2+x^3)\sqrt[3]{-x+x^4}}{(-1+x^3)^2}\,dx$

Optimal. Leaf size=18

$$-\frac{3x^2}{2\left(x^4-x\right)^{2/3}}$$

Rubi [A] time = 0.07, antiderivative size = 25, normalized size of antiderivative = 1.39, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2056, 449}

$$\frac{3x\sqrt[3]{x^4-x}}{2\left(1-x^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x^3)*(-x + x^4)^(1/3))/(-1 + x^3)^2,x]
```

```
[Out] (3*x*(-x + x^4)^(1/3))/(2*(1 - x^3))
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(2+x^3\right)\sqrt[3]{-x+x^4}}{\left(-1+x^3\right)^2}\,dx &= \frac{\sqrt[3]{-x+x^4}\int \frac{\sqrt[3]{x}\left(2+x^3\right)}{\left(-1+x^3\right)^{5/3}}\,dx}{\sqrt[3]{x}\sqrt[3]{-1+x^3}}\\ &= \frac{3x\sqrt[3]{-x+x^4}}{2\left(1-x^3\right)}\end{aligned}$$

Mathematica [C] time = 0.05, size = 61, normalized size = 3.39

$$\frac{3\sqrt[3]{x\left(x^3-1\right)}\left(13x\,{}_2F_1\left(\frac{4}{9},\frac{5}{3};\frac{13}{9};x^3\right)+2x^4\,{}_2F_1\left(\frac{13}{9},\frac{5}{3};\frac{22}{9};x^3\right)\right)}{26\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x^3)*(-x + x^4)^(1/3))/(-1 + x^3)^2,x]
```

```
[Out] (3*(x*(-1 + x^3))^(1/3)*(13*x*Hypergeometric2F1[4/9, 5/3, 13/9, x^3] + 2*x^
4*Hypergeometric2F1[13/9, 5/3, 22/9, x^3]))/(26*(1 - x^3)^(1/3))
```

IntegrateAlgebraic [A] time = 0.20, size = 18, normalized size = 1.00

$$-\frac{3x^2}{2\left(x^4-x\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^3)*(-x + x^4)^(1/3))/(-1 + x^3)^2,x]
[Out] (-3*x^2)/(2*(-x + x^4)^(2/3))
```

fricas [A] time = 0.39, size = 19, normalized size = 1.06

$$-\frac{3\left(x^4-x\right)^{\frac{1}{3}}x}{2\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^4-x)^(1/3)/(x^3-1)^2,x, algorithm="fricas")
[Out] -3/2*(x^4 - x)^(1/3)*x/(x^3 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x\right)^{\frac{1}{3}}\left(x^3+2\right)}{\left(x^3-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^4-x)^(1/3)/(x^3-1)^2,x, algorithm="giac")
[Out] integrate((x^4 - x)^(1/3)*(x^3 + 2)/(x^3 - 1)^2, x)
```

maple [A] time = 0.01, size = 20, normalized size = 1.11

$$-\frac{3x\left(x^4-x\right)^{\frac{1}{3}}}{2\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2)*(x^4-x)^(1/3)/(x^3-1)^2,x)
[Out] -3/2*x*(x^4-x)^(1/3)/(x^3-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x\right)^{\frac{1}{3}}\left(x^3+2\right)}{\left(x^3-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^4-x)^(1/3)/(x^3-1)^2,x, algorithm="maxima")
[Out] integrate((x^4 - x)^(1/3)*(x^3 + 2)/(x^3 - 1)^2, x)
```

mupad [B] time = 0.15, size = 21, normalized size = 1.17

$$-\frac{3\,x\,\left(x^4-x\right)^{1/3}}{2\,\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - x)^(1/3)*(x^3 + 2))/(x^3 - 1)^2,x)
```

```
[Out] -(3*x*(x^4 - x)^(1/3))/(2*(x^3 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x(x-1)\left(x^2+x+1\right)}\left(x^3+2\right)}{(x-1)^2\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2)*(x**4-x)**(1/3)/(x**3-1)**2,x)
```

```
[Out] Integral((x*(x - 1)*(x**2 + x + 1))**(1/3)*(x**3 + 2)/((x - 1)**2*(x**2 + x
 + 1)**2), x)
```

3.136 $\int \frac{\sqrt{-x+x^4}}{x^6} \, dx$

Optimal. Leaf size=18

$$\frac{2\left(x^4-x\right)^{3/2}}{9x^6}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2014}

$$\frac{2\left(x^4-x\right)^{3/2}}{9x^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-x + x^4]/x^6,x]
```

```
[Out] (2*(-x + x^4)^(3/2))/(9*x^6)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{-x+x^4}}{x^6} \, dx = \frac{2\left(-x+x^4\right)^{3/2}}{9x^6}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{2\left(x\left(x^3-1\right)\right)^{3/2}}{9x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x + x^4]/x^6,x]
```

```
[Out] (2*(x*(-1 + x^3))^(3/2))/(9*x^6)
```

IntegrateAlgebraic [A] time = 0.38, size = 18, normalized size = 1.00

$$\frac{2\left(x^4-x\right)^{3/2}}{9x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-x + x^4]/x^6,x]
```

```
[Out] (2*(-x + x^4)^(3/2))/(9*x^6)
```

fricas [A] time = 0.40, size = 19, normalized size = 1.06

$$\frac{2\sqrt{x^4-x}\left(x^3-1\right)}{9\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)/x^6,x, algorithm="fricas")
[Out] 2/9*sqrt(x^4 - x)*(x^3 - 1)/x^5
```

giac [A] time = 0.38, size = 11, normalized size = 0.61

$$\frac{2}{9}\left(-\frac{1}{x^3}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)/x^6,x, algorithm="giac")
[Out] 2/9*(-1/x^3 + 1)^(3/2)
```

maple [A] time = 0.00, size = 24, normalized size = 1.33

$$\frac{2\,(-1+x)\left(x^2+x+1\right)\sqrt{x^4-x}}{9x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x)^(1/2)/x^6,x)
[Out] 2/9/x^5*(-1+x)*(x^2+x+1)*(x^4-x)^(1/2)
```

maxima [A] time = 0.77, size = 25, normalized size = 1.39

$$\frac{2\left(x^4-x\right)\sqrt{x^2+x+1}\,\sqrt{x-1}}{9\,x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)/x^6,x, algorithm="maxima")
[Out] 2/9*(x^4 - x)*sqrt(x^2 + x + 1)*sqrt(x - 1)/x^(11/2)
```

mupad [B] time = 0.20, size = 19, normalized size = 1.06

$$\frac{2\,\sqrt{x^4-x}\,\left(x^3-1\right)}{9\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x)^(1/2)/x^6,x)
[Out] (2*(x^4 - x)^(1/2)*(x^3 - 1))/(9*x^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x\,(x-1)\left(x^2+x+1\right)}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x)**(1/2)/x**6,x)
[Out] Integral(sqrt(x*(x - 1)*(x**2 + x + 1))/x**6, x)
```

3.137 $\int \frac{x}{\sqrt{x+x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2029, 206}

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[x + x^4],x]
```

```
[Out] (2*ArcTanh[x^2/Sqrt[x + x^4]])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{x+x^4}}\,dx &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^2}{\sqrt{x+x^4}}\right)\\ &= \frac{2}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x+x^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.83

$$\frac{2\sqrt{x}\sqrt{x^3+1}\sinh^{-1}\left(x^{3/2}\right)}{3\sqrt{x^4+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[x + x^4],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 + x^3]*ArcSinh[x^(3/2)])/(3*Sqrt[x + x^4])
```

IntegrateAlgebraic [A] time = 0.31, size = 18, normalized size = 1.00

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[x + x^4],x]
```

```
[Out] (2*ArcTanh[x^2/Sqrt[x + x^4]])/3
```

fricas [A] time = 0.46, size = 20, normalized size = 1.11

$$\frac{1}{3}\log\left(-2x^3 - 2\sqrt{x^4 + x}\,x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log(-2*x^3 - 2*sqrt(x^4 + x)*x - 1)
```

giac [A] time = 0.35, size = 26, normalized size = 1.44

$$\frac{1}{3}\log\left(\sqrt{\frac{1}{x^3} + 1} + 1\right) - \frac{1}{3}\log\left(\left|\sqrt{\frac{1}{x^3} + 1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*log(sqrt(1/x^3 + 1) + 1) - 1/3*log(abs(sqrt(1/x^3 + 1) - 1))
```

maple [C] time = 0.01, size = 290, normalized size = 16.11

$$-\frac{2\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\,(1+x)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(1+x)}}\sqrt{-\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\left(-\mathrm{EllipticF}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}, \sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)+\mathrm{EllipticPi}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}, \frac{\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}, \sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(1+x)\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+x)^(1/2),x)
```

```
[Out] -2*(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(
1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))/(1+x))^(1/2)*(-(x-
1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(3/2+1/2*I*3^(1/2))/(x*
(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1/2)*(-EllipticF(((3/2+
1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1
/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+Ellipti
cPi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),(1/2+1/2*I*3^(1
/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^4 + x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(x^4 + x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x}{\sqrt{x^4 + x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + x^4)^(1/2),x)
```

```
[Out] int(x/(x + x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x (x+1) (x^2-x+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(x + 1)*(x**2 - x + 1)), x)
```

3.138 $\int \frac{-2+x^3}{\left(1+x^3\right)\sqrt[3]{x+x^4}}\,dx$

Optimal. Leaf size=18

$$-\frac{3\left(x^4+x\right)^{2/3}}{x^3+1}$$

Rubi [A] time = 0.08, antiderivative size = 12, normalized size of antiderivative = 0.67, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2056, 449}

$$-\frac{3x}{\sqrt[3]{x^4+x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^3)/((1 + x^3)*(x + x^4)^(1/3)),x]
```

```
[Out] (-3*x)/(x + x^4)^(1/3)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-2+x^3}{\left(1+x^3\right)\sqrt[3]{x+x^4}}\,dx &= \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^3}\right)\int \frac{-2+x^3}{\sqrt[3]{x}\left(1+x^3\right)^{4/3}}\,dx}{\sqrt[3]{x+x^4}}\\ &= -\frac{3x}{\sqrt[3]{x+x^4}}\end{aligned}$$

Mathematica [C] time = 0.04, size = 60, normalized size = 3.33

$$\frac{3\sqrt[3]{x^3+1}\left(x^4\,{}_2F_1\left(\frac{11}{9},\frac{4}{3};\frac{20}{9};-x^3\right)-11x\,{}_2F_1\left(\frac{2}{9},\frac{4}{3};\frac{11}{9};-x^3\right)\right)}{11\sqrt[3]{x^4+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^3)/((1 + x^3)*(x + x^4)^(1/3)),x]
```

```
[Out] (3*(1 + x^3)^(1/3)*(-11*x*Hypergeometric2F1[2/9, 4/3, 11/9, -x^3] + x^4*Hyp
ergeometric2F1[11/9, 4/3, 20/9, -x^3]))/(11*(x + x^4)^(1/3))
```

IntegrateAlgebraic [A] time = 0.22, size = 18, normalized size = 1.00

$$-\frac{3\left(x^4+x\right)^{2/3}}{x^3+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x^3)/((1 + x^3)*(x + x^4)^(1/3)),x]
[Out] (-3*(x + x^4)^(2/3))/(1 + x^3)
```

fricas [A] time = 0.41, size = 16, normalized size = 0.89

$$-\frac{3\left(x^4+x\right)^{\frac{2}{3}}}{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)/(x^3+1)/(x^4+x)^(1/3),x, algorithm="fricas")
[Out] -3*(x^4 + x)^(2/3)/(x^3 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3-2}{\left(x^4+x\right)^{\frac{1}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)/(x^3+1)/(x^4+x)^(1/3),x, algorithm="giac")
[Out] integrate((x^3 - 2)/((x^4 + x)^(1/3)*(x^3 + 1)), x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.61

$$-\frac{3x}{\left(x^4+x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2)/(x^3+1)/(x^4+x)^(1/3),x)
[Out] -3*x/(x^4+x)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3-2}{\left(x^4+x\right)^{\frac{1}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)/(x^3+1)/(x^4+x)^(1/3),x, algorithm="maxima")
[Out] integrate((x^3 - 2)/((x^4 + x)^(1/3)*(x^3 + 1)), x)
```

mupad [B] time = 0.15, size = 16, normalized size = 0.89

$$-\frac{3\left(x^4+x\right)^{2/3}}{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 2)/((x^3 + 1)*(x + x^4)^(1/3)),x)
```

```
[Out] -(3*(x + x^4)^(2/3))/(x^3 + 1)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 - 2}{\sqrt[3]{x(x+1)(x^2 - x + 1)}\,(x+1)(x^2 - x + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2)/(x**3+1)/(x**4+x)**(1/3),x)
```

```
[Out] Integral((x**3 - 2)/((x*(x + 1)*(x**2 - x + 1))**(1/3)*(x + 1)*(x**2 - x + 
1)), x)
```

3.139 $\int \frac{-1+x^2}{x\sqrt[3]{x^2+x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{3\left(x^4+x^2\right)^{2/3}}{2x^2}$$

Rubi [A] time = 0.07, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2034, 763}

$$\frac{3\left(x^4+x^2\right)^{2/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(x*(x^2 + x^4)^(1/3)),x]
```

```
[Out] (3*(x^2 + x^4)^(2/3))/(2*x^2)
```

Rule 763

```
Int[((e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.
), x_Symbol] :> Simp[(g*(e*x)^m*(b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x
] /; FreeQ[{b, c, e, f, g, m, p}, x] && EqQ[b*g*(m + p + 1) - c*f*(m + 2*p
+ 2), 0] && NeQ[m + 2*p + 2, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{x\sqrt[3]{x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x}{x\sqrt[3]{x+x^2}}\,dx, x, x^2\right)\\ &= \frac{3\left(x^2+x^4\right)^{2/3}}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.11

$$\frac{3\left(x^2+1\right)}{2\sqrt[3]{x^4+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(x*(x^2 + x^4)^(1/3)),x]
```

```
[Out] (3*(1 + x^2))/(2*(x^2 + x^4)^(1/3))
```

IntegrateAlgebraic [A] time = 0.08, size = 18, normalized size = 1.00

$$\frac{3\left(x^4+x^2\right)^{2/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/(x*(x^2 + x^4)^(1/3)),x]
```

```
[Out] (3*(x^2 + x^4)^(2/3))/(2*x^2)
```

fricas [A] time = 0.42, size = 14, normalized size = 0.78

$$\frac{3\left(x^{4}+x^{2}\right)^{\frac{2}{3}}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^4+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*(x^4 + x^2)^(2/3)/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}-1}{\left(x^{4}+x^{2}\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^4+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/((x^4 + x^2)^(1/3)*x), x)
```

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$\frac{\frac{3x^{2}}{2}+\frac{3}{2}}{\left(x^{4}+x^{2}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/x/(x^4+x^2)^(1/3),x)
```

```
[Out] 3/2*(x^2+1)/(x^4+x^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}-1}{\left(x^{4}+x^{2}\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^4+x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)/((x^4 + x^2)^(1/3)*x), x)
```

mupad [B] time = 0.15, size = 14, normalized size = 0.78

$$\frac{3\left(x^{4}+x^{2}\right)^{2/3}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x*(x^2 + x^4)^(1/3)),x)
```

```
[Out] (3*(x^2 + x^4)^(2/3))/(2*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{x\sqrt[3]{x^2(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/x/(x**4+x**2)**(1/3),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(x*(x**2*(x**2 + 1))**(1/3)), x)
```

3.140 $\int \frac{1}{x^2 \sqrt[4]{x^2+x^4}} \, dx$

Optimal. Leaf size=18

$$-\frac{2\left(x^4+x^2\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2014}

$$-\frac{2\left(x^4+x^2\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(x^2 + x^4)^(1/4)),x]

[Out] (-2*(x^2 + x^4)^(3/4))/(3*x^3)

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2 \sqrt[4]{x^2+x^4}} \, dx = -\frac{2\left(x^2+x^4\right)^{3/4}}{3x^3}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{2\left(x^4+x^2\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(x^2 + x^4)^(1/4)),x]

[Out] (-2*(x^2 + x^4)^(3/4))/(3*x^3)

IntegrateAlgebraic [A] time = 0.09, size = 18, normalized size = 1.00

$$-\frac{2\left(x^4+x^2\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*(x^2 + x^4)^(1/4)),x]

[Out] (-2*(x^2 + x^4)^(3/4))/(3*x^3)

fricas [A] time = 0.41, size = 14, normalized size = 0.78

$$-\frac{2\left(x^4+x^2\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+x^2)^(1/4),x, algorithm="fricas")
[Out] -2/3*(x^4 + x^2)^(3/4)/x^3
```

giac [A] time = 0.35, size = 9, normalized size = 0.50

$$-\frac{2}{3}\left(\frac{1}{x^2}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+x^2)^(1/4),x, algorithm="giac")
[Out] -2/3*(1/x^2 + 1)^(3/4)
```

maple [A] time = 0.00, size = 20, normalized size = 1.11

$$-\frac{2\left(x^2+1\right)}{3x\left(x^4+x^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^4+x^2)^(1/4),x)
[Out] -2/3*(x^2+1)/x/(x^4+x^2)^(1/4)
```

maxima [A] time = 0.54, size = 17, normalized size = 0.94

$$-\frac{2\left(x^3+x\right)}{3\left(x^2+1\right)^{\frac{1}{4}}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+x^2)^(1/4),x, algorithm="maxima")
[Out] -2/3*(x^3 + x)/((x^2 + 1)^(1/4)*x^(5/2))
```

mupad [B] time = 0.12, size = 14, normalized size = 0.78

$$-\frac{2\left(x^4+x^2\right)^{3/4}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^2 + x^4)^(1/4)),x)
[Out] -(2*(x^2 + x^4)^(3/4))/(3*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt[4]{x^2\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**4+x**2)**(1/4),x)
[Out] Integral(1/(x**2*(x**2*(x**2 + 1))**(1/4)), x)
```

3.141 $\int \frac{\sqrt[4]{x^2+x^4}}{x^4} \, dx$

Optimal. Leaf size=18

$$-\frac{2\left(x^4+x^2\right)^{5/4}}{5x^5}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2014}

$$-\frac{2\left(x^4+x^2\right)^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2 + x^4)^(1/4)/x^4,x]
[Out] (-2*(x^2 + x^4)^(5/4))/(5*x^5)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[4]{x^2+x^4}}{x^4} \, dx = -\frac{2\left(x^2+x^4\right)^{5/4}}{5x^5}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.28

$$-\frac{2\left(x^2+1\right)\sqrt[4]{x^4+x^2}}{5x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2 + x^4)^(1/4)/x^4,x]
[Out] (-2*(1 + x^2)*(x^2 + x^4)^(1/4))/(5*x^3)
```

IntegrateAlgebraic [A] time = 0.09, size = 18, normalized size = 1.00

$$-\frac{2\left(x^4+x^2\right)^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2 + x^4)^(1/4)/x^4,x]
[Out] (-2*(x^2 + x^4)^(5/4))/(5*x^5)
```

fricas [A] time = 0.42, size = 19, normalized size = 1.06

$$-\frac{2\left(x^4+x^2\right)^{\frac{1}{4}}\left(x^2+1\right)}{5\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2)^(1/4)/x^4,x, algorithm="fricas")
[Out] -2/5*(x^4 + x^2)^(1/4)*(x^2 + 1)/x^3
```

giac [A] time = 0.30, size = 9, normalized size = 0.50

$$-\frac{2}{5}\left(\frac{1}{x^2}+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2)^(1/4)/x^4,x, algorithm="giac")
[Out] -2/5*(1/x^2 + 1)^(5/4)
```

maple [A] time = 0.00, size = 20, normalized size = 1.11

$$-\frac{2\left(x^2+1\right)\left(x^4+x^2\right)^{\frac{1}{4}}}{5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2)^(1/4)/x^4,x)
[Out] -2/5*(x^2+1)*(x^4+x^2)^(1/4)/x^3
```

maxima [A] time = 0.51, size = 17, normalized size = 0.94

$$-\frac{2\left(x^3+x\right)\left(x^2+1\right)^{\frac{1}{4}}}{5\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2)^(1/4)/x^4,x, algorithm="maxima")
[Out] -2/5*(x^3 + x)*(x^2 + 1)^(1/4)/x^(7/2)
```

mupad [B] time = 0.17, size = 29, normalized size = 1.61

$$-\frac{2\left(x^4+x^2\right)^{1/4}}{5\,x}-\frac{2\left(x^4+x^2\right)^{1/4}}{5\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4)^(1/4)/x^4,x)
[Out] - (2*(x^2 + x^4)^(1/4))/(5*x) - (2*(x^2 + x^4)^(1/4))/(5*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[4]{x^2\left(x^2+1\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2)**(1/4)/x**4,x)
[Out] Integral((x**2*(x**2 + 1))**(1/4)/x**4, x)
```

3.142 $\int \frac{(-1+x^2)\sqrt[3]{x^2+x^4}}{x^3} \, dx$

Optimal. Leaf size=18

$$\frac{3(x^4+x^2)^{4/3}}{4x^4}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1590}

$$\frac{3(x^4+x^2)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^2)*(x^2 + x^4)^(1/3))/x^3,x]
[Out] (3*(x^2 + x^4)^(4/3))/(4*x^4)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(-1+x^2)\sqrt[3]{x^2+x^4}}{x^3} \, dx = \frac{3(x^2+x^4)^{4/3}}{4x^4}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.28

$$\frac{3(x^2+1)\sqrt[3]{x^4+x^2}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)*(x^2 + x^4)^(1/3))/x^3,x]
[Out] (3*(1 + x^2)*(x^2 + x^4)^(1/3))/(4*x^2)
```

IntegrateAlgebraic [A] time = 0.07, size = 18, normalized size = 1.00

$$\frac{3(x^4+x^2)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*(x^2 + x^4)^(1/3))/x^3,x]
[Out] (3*(x^2 + x^4)^(4/3))/(4*x^4)
```

fricas [A] time = 0.40, size = 19, normalized size = 1.06

$$\frac{3 \left(x^4 + x^2\right)^{\frac{1}{3}} \left(x^2 + 1\right)}{4 \, x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+x^2)^(1/3)/x^3,x, algorithm="fricas")
[Out] 3/4*(x^4 + x^2)^(1/3)*(x^2 + 1)/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4 + x^2\right)^{\frac{1}{3}} \left(x^2 - 1\right)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+x^2)^(1/3)/x^3,x, algorithm="giac")
[Out] integrate((x^4 + x^2)^(1/3)*(x^2 - 1)/x^3, x)
```

maple [A] time = 0.00, size = 20, normalized size = 1.11

$$\frac{3 \left(x^2 + 1\right) \left(x^4 + x^2\right)^{\frac{1}{3}}}{4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(x^4+x^2)^(1/3)/x^3,x)
[Out] 3/4*(x^2+1)*(x^4+x^2)^(1/3)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4 + x^2\right)^{\frac{1}{3}} \left(x^2 - 1\right)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+x^2)^(1/3)/x^3,x, algorithm="maxima")
[Out] integrate((x^4 + x^2)^(1/3)*(x^2 - 1)/x^3, x)
```

mupad [B] time = 0.21, size = 19, normalized size = 1.06

$$\frac{3 \left(x^4 + x^2\right)^{1/3} \left(x^2 + 1\right)}{4 \, x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + x^4)^(1/3)*(x^2 - 1))/x^3,x)
[Out] (3*(x^2 + x^4)^(1/3)*(x^2 + 1))/(4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x^2 \left(x^2 + 1\right)} \left(x - 1\right) \left(x + 1\right)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**4+x**2)**(1/3)/x**3,x)
[Out] Integral((x**2*(x**2 + 1))**(1/3)*(x - 1)*(x + 1)/x**3, x)
```

3.143 $\int \frac{1}{x\sqrt[4]{x^3+x^4}}\,dx$

Optimal. Leaf size=18

$$-\frac{4\left(x^4+x^3\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2014}

$$-\frac{4\left(x^4+x^3\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(x^3 + x^4)^(1/4)),x]
[Out] (-4*(x^3 + x^4)^(3/4))/(3*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt[4]{x^3+x^4}}\,dx=-\frac{4\left(x^3+x^4\right)^{3/4}}{3x^3}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{4(x+1)}{3\sqrt[4]{x^3(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(x^3 + x^4)^(1/4)),x]
[Out] (-4*(1 + x))/(3*(x^3*(1 + x))^(1/4))
```

IntegrateAlgebraic [A] time = 0.20, size = 18, normalized size = 1.00

$$-\frac{4\left(x^4+x^3\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(x^3 + x^4)^(1/4)),x]
[Out] (-4*(x^3 + x^4)^(3/4))/(3*x^3)
```

fricas [A] time = 0.40, size = 14, normalized size = 0.78

$$-\frac{4\left(x^4+x^3\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+x^3)^(1/4),x, algorithm="fricas")
[Out] -4/3*(x^4 + x^3)^(3/4)/x^3
```

giac [A] time = 0.58, size = 9, normalized size = 0.50

$$-\frac{4}{3}\left(\frac{1}{x}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+x^3)^(1/4),x, algorithm="giac")
[Out] -4/3*(1/x + 1)^(3/4)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$-\frac{4\,(1+x)}{3\left(x^{4}+x^{3}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4+x^3)^(1/4),x)
[Out] -4/3*(1+x)/(x^4+x^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{4}+x^{3}\right)^{\frac{1}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+x^3)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((x^4 + x^3)^(1/4)*x), x)
```

mupad [B] time = 0.26, size = 14, normalized size = 0.78

$$-\frac{4\left(x^{4}+x^{3}\right)^{3/4}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^3 + x^4)^(1/4)),x)
[Out] -(4*(x^3 + x^4)^(3/4))/(3*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[4]{x^{3}\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4+x**3)**(1/4),x)
[Out] Integral(1/(x*(x**3*(x + 1))**(1/4)), x)
```

3.144 $\int \frac{(1+x)\sqrt[4]{x^3+x^4}}{x^4}\,dx$

Optimal. Leaf size=18

$$-\frac{4\left(x^4+x^3\right)^{9/4}}{9x^9}$$

Rubi [B] time = 0.12, antiderivative size = 37, normalized size of antiderivative = 2.06, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2052, 2016, 2014}

$$-\frac{4\left(x^4+x^3\right)^{5/4}}{9x^6}-\frac{4\left(x^4+x^3\right)^{5/4}}{9x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)*(x^3 + x^4)^(1/4))/x^4,x]
```

```
[Out] (-4*(x^3 + x^4)^(5/4))/(9*x^6) - (4*(x^3 + x^4)^(5/4))/(9*x^5)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)\sqrt[4]{x^3+x^4}}{x^4}\,dx &= \int \left(\frac{\sqrt[4]{x^3+x^4}}{x^4}+\frac{\sqrt[4]{x^3+x^4}}{x^3}\right)dx\\
&= \int \frac{\sqrt[4]{x^3+x^4}}{x^4}\,dx+\int \frac{\sqrt[4]{x^3+x^4}}{x^3}\,dx\\
&= -\frac{4\left(x^3+x^4\right)^{5/4}}{9x^6}-\frac{4\left(x^3+x^4\right)^{5/4}}{5x^5}-\frac{4}{9}\int \frac{\sqrt[4]{x^3+x^4}}{x^3}\,dx\\
&= -\frac{4\left(x^3+x^4\right)^{5/4}}{9x^6}-\frac{4\left(x^3+x^4\right)^{5/4}}{9x^5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{4\left(x^3(x+1)\right)^{9/4}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)*(x^3 + x^4)^(1/4))/x^4,x]
[Out] (-4*(x^3*(1 + x))^(9/4))/(9*x^9)
```

IntegrateAlgebraic [A] time = 0.20, size = 18, normalized size = 1.00

$$-\frac{4\left(x^4+x^3\right)^{9/4}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x)*(x^3 + x^4)^(1/4))/x^4,x]
[Out] (-4*(x^3 + x^4)^(9/4))/(9*x^9)
```

fricas [A] time = 0.39, size = 22, normalized size = 1.22

$$-\frac{4\left(x^4+x^3\right)^{\frac{1}{4}}\left(x^2+2\,x+1\right)}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(x^4+x^3)^(1/4)/x^4,x, algorithm="fricas")
[Out] -4/9*(x^4 + x^3)^(1/4)*(x^2 + 2*x + 1)/x^3
```

giac [A] time = 1.00, size = 9, normalized size = 0.50

$$-\frac{4}{9}\left(\frac{1}{x}+1\right)^{\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(x^4+x^3)^(1/4)/x^4,x, algorithm="giac")
[Out] -4/9*(1/x + 1)^(9/4)
```

maple [A] time = 0.00, size = 20, normalized size = 1.11

$$-\frac{4\,(1+x)^2\left(x^4+x^3\right)^{\frac{1}{4}}}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)*(x^4+x^3)^(1/4)/x^4,x)
[Out] -4/9/x^3*(1+x)^2*(x^4+x^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^4+x^3\right)^{\frac{1}{4}}(x+1)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(x^4+x^3)^(1/4)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + x^3)^(1/4)*(x + 1)/x^4, x)
```

mupad [B] time = 0.24, size = 43, normalized size = 2.39

$$-\frac{8\,x\,{\left(x^4+x^3\right)}^{1/4}+4\,{\left(x^4+x^3\right)}^{1/4}+4\,x^2\,{\left(x^4+x^3\right)}^{1/4}}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + x^4)^(1/4)*(x + 1))/x^4,x)
```

```
[Out] -(8*x*(x^3 + x^4)^(1/4) + 4*(x^3 + x^4)^(1/4) + 4*x^2*(x^3 + x^4)^(1/4))/(9
*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^{3}\left(x+1\right)}\,\left(x+1\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(x**4+x**3)**(1/4)/x**4,x)
```

```
[Out] Integral((x**3*(x + 1))**(1/4)*(x + 1)/x**4, x)
```

3.145 $\int \frac{-2-x+2x^4}{(1+x+x^4)\sqrt{1+x+x^2+x^4}}\,dx$

Optimal. Leaf size=18

$$-2\tanh^{-1}\left(\frac{x}{\sqrt{x^4+x^2+x+1}}\right)$$

Rubi [F] time = 0.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2-x+2x^4}{(1+x+x^4)\sqrt{1+x+x^2+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2 - x + 2*x^4)/((1 + x + x^4)*Sqrt[1 + x + x^2 + x^4]),x]
```

```
[Out] 2*Defer[Int][1/Sqrt[1 + x + x^2 + x^4], x] - 4*Defer[Int][1/((1 + x + x^4)*
Sqrt[1 + x + x^2 + x^4]), x] - 3*Defer[Int][x/((1 + x + x^4)*Sqrt[1 + x + x
^2 + x^4]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2-x+2x^4}{(1+x+x^4)\sqrt{1+x+x^2+x^4}}\,dx &= \int\left(\frac{2}{\sqrt{1+x+x^2+x^4}}-\frac{4+3x}{(1+x+x^4)\sqrt{1+x+x^2+x^4}}\right)dx\\
&= 2\int\frac{1}{\sqrt{1+x+x^2+x^4}}\,dx-\int\frac{4+3x}{(1+x+x^4)\sqrt{1+x+x^2+x^4}}\,dx\\
&= 2\int\frac{1}{\sqrt{1+x+x^2+x^4}}\,dx-\int\left(\frac{4}{(1+x+x^4)\sqrt{1+x+x^2+x^4}}+\frac{}{(1+}\right.\\
&= 2\int\frac{1}{\sqrt{1+x+x^2+x^4}}\,dx-3\int\frac{x}{(1+x+x^4)\sqrt{1+x+x^2+x^4}}\,dx-4
\end{aligned}$$

Mathematica [C] time = 6.21, size = 17638, normalized size = 979.89

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-2 - x + 2*x^4)/((1 + x + x^4)*Sqrt[1 + x + x^2 + x^4]),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.18, size = 18, normalized size = 1.00

$$-2\tanh^{-1}\left(\frac{x}{\sqrt{x^4+x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 - x + 2*x^4)/((1 + x + x^4)*Sqrt[1 + x + x^2 + x^4])
,x]
```

```
[Out] -2*ArcTanh[x/Sqrt[1 + x + x^2 + x^4]]
```

fricas [B] time = 0.44, size = 35, normalized size = 1.94

$$\log\left(\frac{x^4+2\,x^2-2\,\sqrt{x^4+x^2+x+1}\,x+x+1}{x^4+x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x-2)/(x^4+x+1)/(x^4+x^2+x+1)^(1/2),x, algorithm="fricas")
[Out] log((x^4 + 2*x^2 - 2*sqrt(x^4 + x^2 + x + 1)*x + x + 1)/(x^4 + x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^4-x-2}{\sqrt{x^4+x^2+x+1}\,\left(x^4+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x-2)/(x^4+x+1)/(x^4+x^2+x+1)^(1/2),x, algorithm="giac")
[Out] integrate((2*x^4 - x - 2)/(sqrt(x^4 + x^2 + x + 1)*(x^4 + x + 1)), x)
```

maple [C] time = 5.32, size = 4880, normalized size = 271.11

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-x-2)/(x^4+x+1)/(x^4+x^2+x+1)^(1/2),x)
[Out] 4*(RootOf(_Z^4+_Z^2+_Z+1,index=1)-RootOf(_Z^4+_Z^2+_Z+1,index=4))*((RootOf(
_Z^4+_Z^2+_Z+1,index=4)-RootOf(_Z^4+_Z^2+_Z+1,index=2))*(x-RootOf(_Z^4+_Z^2
+_Z+1,index=1))/(RootOf(_Z^4+_Z^2+_Z+1,index=4)-RootOf(_Z^4+_Z^2+_Z+1,index
=1))/(x-RootOf(_Z^4+_Z^2+_Z+1,index=2)))^(1/2)*(x-RootOf(_Z^4+_Z^2+_Z+1,ind
ex=2))^2*((RootOf(_Z^4+_Z^2+_Z+1,index=2)-RootOf(_Z^4+_Z^2+_Z+1,index=1))*(
x-RootOf(_Z^4+_Z^2+_Z+1,index=3))/(RootOf(_Z^4+_Z^2+_Z+1,index=3)-RootOf(_Z
^4+_Z^2+_Z+1,index=1))/(x-RootOf(_Z^4+_Z^2+_Z+1,index=2)))^(1/2)*((RootOf(_
Z^4+_Z^2+_Z+1,index=2)-RootOf(_Z^4+_Z^2+_Z+1,index=1))*(x-RootOf(_Z^4+_Z^2+
_Z+1,index=4))/(RootOf(_Z^4+_Z^2+_Z+1,index=4)-RootOf(_Z^4+_Z^2+_Z+1,index=
1))/(x-RootOf(_Z^4+_Z^2+_Z+1,index=2)))^(1/2)/(RootOf(_Z^4+_Z^2+_Z+1,index=
4)-RootOf(_Z^4+_Z^2+_Z+1,index=2))/(RootOf(_Z^4+_Z^2+_Z+1,index=2)-RootOf(_
Z^4+_Z^2+_Z+1,index=1))/((x-RootOf(_Z^4+_Z^2+_Z+1,index=1))*(x-RootOf(_Z^4+
_Z^2+_Z+1,index=2))*(x-RootOf(_Z^4+_Z^2+_Z+1,index=3))*(x-RootOf(_Z^4+_Z^2+
_Z+1,index=4)))^(1/2)*EllipticF(((RootOf(_Z^4+_Z^2+_Z+1,index=4)-RootOf(_Z^
4+_Z^2+_Z+1,index=2))*(x-RootOf(_Z^4+_Z^2+_Z+1,index=1))/(RootOf(_Z^4+_Z^2+
_Z+1,index=4)-RootOf(_Z^4+_Z^2+_Z+1,index=1))/(x-RootOf(_Z^4+_Z^2+_Z+1,inde
x=2)))^(1/2),((RootOf(_Z^4+_Z^2+_Z+1,index=2)-RootOf(_Z^4+_Z^2+_Z+1,index=3
))*(RootOf(_Z^4+_Z^2+_Z+1,index=1)-RootOf(_Z^4+_Z^2+_Z+1,index=4))/(RootOf(
_Z^4+_Z^2+_Z+1,index=1)-RootOf(_Z^4+_Z^2+_Z+1,index=3))/(-RootOf(_Z^4+_Z^2+
_Z+1,index=4)+RootOf(_Z^4+_Z^2+_Z+1,index=2)))^(1/2))+2*sum(_alpha*(RootOf(
_Z^4+_Z^2+_Z+1,index=1)-RootOf(_Z^4+_Z^2+_Z+1,index=4))*((RootOf(_Z^4+_Z^2+
_Z+1,index=4)-RootOf(_Z^4+_Z^2+_Z+1,index=2))*(x-RootOf(_Z^4+_Z^2+_Z+1,inde
x=1))/(RootOf(_Z^4+_Z^2+_Z+1,index=4)-RootOf(_Z^4+_Z^2+_Z+1,index=1))/(x-Ro
otOf(_Z^4+_Z^2+_Z+1,index=2)))^(1/2)*(x-RootOf(_Z^4+_Z^2+_Z+1,index=2))^2*(
(RootOf(_Z^4+_Z^2+_Z+1,index=2)-RootOf(_Z^4+_Z^2+_Z+1,index=1))*(x-RootOf(_
Z^4+_Z^2+_Z+1,index=3))/(RootOf(_Z^4+_Z^2+_Z+1,index=3)-RootOf(_Z^4+_Z^2+_Z
+1,index=1))/(x-RootOf(_Z^4+_Z^2+_Z+1,index=2)))^(1/2)*((RootOf(_Z^4+_Z^2+_
Z+1,index=2)-RootOf(_Z^4+_Z^2+_Z+1,index=1))*(x-RootOf(_Z^4+_Z^2+_Z+1,index
=4))/(RootOf(_Z^4+_Z^2+_Z+1,index=4)-RootOf(_Z^4+_Z^2+_Z+1,index=1))/(x-Roo
tOf(_Z^4+_Z^2+_Z+1,index=2)))^(1/2)/(RootOf(_Z^4+_Z^2+_Z+1,index=4)-RootOf(
_Z^4+_Z^2+_Z+1,index=2))/(RootOf(_Z^4+_Z^2+_Z+1,index=2)-RootOf(_Z^4+_Z^2+_
```

Z+1,index=1))/((x-RootOf(_Z^4+_Z^2+_Z+1,index=1))*(x-RootOf(_Z^4+_Z^2+_Z+1,
index=2))*(x-RootOf(_Z^4+_Z^2+_Z+1,index=3))*(x-RootOf(_Z^4+_Z^2+_Z+1,index
=4)))^(1/2)*(-RootOf(_Z^4+_Z^2+_Z+1,index=2)^3*_alpha^3+RootOf(_Z^4+_Z^2+_Z
+1,index=2)^3*_alpha^2+RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*_alpha^3-RootOf(_Z^
4+_Z^2+_Z+1,index=2)*_alpha^3-RootOf(_Z^4+_Z^2+_Z+1,index=2)^3+RootOf(_Z^4+
_Z^2+_Z+1,index=2)*_alpha^2+RootOf(_Z^4+_Z^2+_Z+1,index=2)^2+_alpha^2-2*Roo
tOf(_Z^4+_Z^2+_Z+1,index=2)-_alpha)*(EllipticF(((RootOf(_Z^4+_Z^2+_Z+1,inde
x=4)-RootOf(_Z^4+_Z^2+_Z+1,index=2))*(x-RootOf(_Z^4+_Z^2+_Z+1,index=1))/(Ro
otOf(_Z^4+_Z^2+_Z+1,index=4)-RootOf(_Z^4+_Z^2+_Z+1,index=1))/(x-RootOf(_Z^4
+_Z^2+_Z+1,index=2)))^(1/2),((RootOf(_Z^4+_Z^2+_Z+1,index=2)-RootOf(_Z^4+_Z
^2+_Z+1,index=3))*(RootOf(_Z^4+_Z^2+_Z+1,index=1)-RootOf(_Z^4+_Z^2+_Z+1,ind
ex=4))/(RootOf(_Z^4+_Z^2+_Z+1,index=1)-RootOf(_Z^4+_Z^2+_Z+1,index=3))/(-Ro
otOf(_Z^4+_Z^2+_Z+1,index=4)+RootOf(_Z^4+_Z^2+_Z+1,index=2)))^(1/2))+(RootO
f(_Z^4+_Z^2+_Z+1,index=2)-RootOf(_Z^4+_Z^2+_Z+1,index=1))*(-RootOf(_Z^4+_Z^
2+_Z+1,index=1)^3*_alpha^3+RootOf(_Z^4+_Z^2+_Z+1,index=1)^3*_alpha^2+RootOf
(_Z^4+_Z^2+_Z+1,index=1)^2*_alpha^3-RootOf(_Z^4+_Z^2+_Z+1,index=1)*_alpha^3
-RootOf(_Z^4+_Z^2+_Z+1,index=1)^3+RootOf(_Z^4+_Z^2+_Z+1,index=1)*_alpha^2+R
ootOf(_Z^4+_Z^2+_Z+1,index=1)^2+_alpha^2-2*RootOf(_Z^4+_Z^2+_Z+1,index=1)-_
alpha)*EllipticPi(((RootOf(_Z^4+_Z^2+_Z+1,index=4)-RootOf(_Z^4+_Z^2+_Z+1,in
dex=2))*(x-RootOf(_Z^4+_Z^2+_Z+1,index=1))/(RootOf(_Z^4+_Z^2+_Z+1,index=4)-
RootOf(_Z^4+_Z^2+_Z+1,index=1))/(x-RootOf(_Z^4+_Z^2+_Z+1,index=2)))^(1/2),-
191/257*RootOf(_Z^4+_Z^2+_Z+1,index=4)+175/257*RootOf(_Z^4+_Z^2+_Z+1,index=
2)+284/257*RootOf(_Z^4+_Z^2+_Z+1,index=1)+90/257*RootOf(_Z^4+_Z^2+_Z+1,inde
x=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)+125/257*RootOf(_Z^4+_Z^2+_Z+1,index=4)*
RootOf(_Z^4+_Z^2+_Z+1,index=1)-363/257*RootOf(_Z^4+_Z^2+_Z+1,index=1)*RootO
f(_Z^4+_Z^2+_Z+1,index=2)-312/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=4)*Roo
tOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2+104/257*_alp
ha*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_
Z^4+_Z^2+_Z+1,index=1)-96/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(
_Z^4+_Z^2+_Z+1,index=2)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3+122/257*_alpha*Roo
tOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)*RootOf(_Z^4+_Z^2
+_Z+1,index=1)^2+126/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+
_Z^2+_Z+1,index=1)*RootOf(_Z^4+_Z^2+_Z+1,index=2)-78/257*_alpha^3*RootOf(_Z
^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+
1,index=1)^3-26/257*_alpha^3*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^
2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2-286/257*_alpha^3*RootOf(
_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_
Z+1,index=1)+55/257*_alpha^3*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^
2+_Z+1,index=2)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3-181/257*_alpha^3*RootOf(_Z
^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)*RootOf(_Z^4+_Z^2+_Z+1,
index=1)^2-41/257*_alpha^3*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+
_Z+1,index=1)*RootOf(_Z^4+_Z^2+_Z+1,index=2)-26/257*_alpha^2*RootOf(_Z^4+_Z
^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,ind
ex=1)^3-208/257*_alpha^2*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z
+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2-338/257*_alpha^2*RootOf(_Z^4
+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,
index=1)-181/257*_alpha^2*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_
Z+1,index=2)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3-96/257*_alpha^2*RootOf(_Z^4+_
Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)*RootOf(_Z^4+_Z^2+_Z+1,inde
x=1)^2-59/257*_alpha^2*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1
,index=1)*RootOf(_Z^4+_Z^2+_Z+1,index=2)-208/257*_alpha*RootOf(_Z^4+_Z^2+_Z
+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)
^3-416/257*_alpha^3*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,in
dex=2)^2-4/257*_alpha^3*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+
1,index=2)-96/257*_alpha^3*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+
_Z+1,index=1)^3-239/257*_alpha^3*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4
+_Z^2+_Z+1,index=1)^2-90/257*_alpha^3*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf
(_Z^4+_Z^2+_Z+1,index=1)+136/257*_alpha^3*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*
RootOf(_Z^4+_Z^2+_Z+1,index=1)^3-154/257*_alpha^3*RootOf(_Z^4+_Z^2+_Z+1,ind

ex=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2+256/257*_alpha^3*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)-58/257*_alpha^3*RootOf(_Z^4+_Z^2+_Z+1,index=2)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3+359/257*_alpha^3*RootOf(_Z^4+_Z^2+_Z+1,index=2)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2-132/257*_alpha^3*RootOf(_Z^4+_Z^2+_Z+1,index=1)*RootOf(_Z^4+_Z^2+_Z+1,index=2)+78/257*_alpha^2*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2-55/257*_alpha^2*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)-239/257*_alpha^2*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3+6/257*_alpha^2*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2-191/257*_alpha^2*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=1)-154/257*_alpha^2*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3+120/257*_alpha^2*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2-222/257*_alpha^2*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)+359/257*_alpha^2*RootOf(_Z^4+_Z^2+_Z+1,index=2)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3-74/257*_alpha^2*RootOf(_Z^4+_Z^2+_Z+1,index=2)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2+209/257*_alpha^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)*RootOf(_Z^4+_Z^2+_Z+1,index=2)+26/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2+181/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)+6/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3+48/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2+335/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=1)+120/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3-68/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2+18/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)-74/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=2)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3-150/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=2)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2-301/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=1)*RootOf(_Z^4+_Z^2+_Z+1,index=2)-320/257*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2-222/257*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2-402/257*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3+130/257*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2-214/257*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)+73/257*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3+53/257*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2+202/257*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=1)*RootOf(_Z^4+_Z^2+_Z+1,index=2)+186/257*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3+99/257*_alpha^3-336/257*_alpha^2+150/257*_alpha-185/257*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2-102/257*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3+73/257*RootOf(_Z^4+_Z^2+_Z+1,index=4)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2+16/257*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3-82/257*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2+187/257*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)+20/257*RootOf(_Z^4+_Z^2+_Z+1,index=2)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3+98/257*RootOf(_Z^4+_Z^2+_Z+1,index=2)*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2-86/257*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2*_alpha^3+151/257*RootOf(_Z^4+_Z^2+_Z+1,index=2)*_alpha^3+58/257*RootOf(_Z^4+_Z^2+_Z+1,index=2)*_alpha^2+336/257*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3*_alpha^3-150/257*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3*_alpha^2-150/257*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2*_alpha^3+332/257*RootOf(_Z^4+_Z^2+_Z+1,index=1)*_alpha^3-237/257*RootOf(_Z^4+_Z^2+_Z+1,index=1)*_alpha^2-4/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=1)^3-87/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2-186/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=1)+239/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=4)+154/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2-359/257*_alpha*RootOf(_Z^4+_Z^2+_Z+1,index=2)-287/257*_alpha^3*RootOf(_Z^4+_Z^2+_Z+1,index=4)+96/257*_alpha^2*RootOf(_Z^4+_Z^2+_Z+1,index=4)-136/257*_alpha^2*RootOf(_Z^4+_Z^2+_Z+1,index=2)^2-4/257*_alpha^2*RootOf(_Z^4+_Z^2+_Z+1,index=1)^2+44/257,((RootOf(_Z^4+_Z^2+_Z+1,index=2)-RootOf(_Z^4+_Z^2+_Z+1,index=3))*(RootOf(_Z^4+_Z^2+_Z+

```
1,index=1)-RootOf(_Z^4+_Z^2+_Z+1,index=4))/(RootOf(_Z^4+_Z^2+_Z+1,index=1)-
RootOf(_Z^4+_Z^2+_Z+1,index=3))/(-RootOf(_Z^4+_Z^2+_Z+1,index=4)+RootOf(_Z^
4+_Z^2+_Z+1,index=2)))^(1/2))),_alpha=RootOf(_Z^4+_Z+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^4 - x - 2}{\sqrt{x^4 + x^2 + x + 1}\,\left(x^4 + x + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x-2)/(x^4+x+1)/(x^4+x^2+x+1)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x^4 - x - 2)/(sqrt(x^4 + x^2 + x + 1)*(x^4 + x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int -\frac{-2\,x^4 + x + 2}{\left(x^4 + x + 1\right)\,\sqrt{x^4 + x^2 + x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 2*x^4 + 2)/((x + x^4 + 1)*(x + x^2 + x^4 + 1)^(1/2)),x)
[Out] int(-(x - 2*x^4 + 2)/((x + x^4 + 1)*(x + x^2 + x^4 + 1)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-x-2)/(x**4+x+1)/(x**4+x**2+x+1)**(1/2),x)
[Out] Timed out
```

3.146 $\int \frac{3+x^4}{x^3 \sqrt[4]{-x+x^5}} \, dx$

Optimal. Leaf size=18

$$\frac{4 \left(x^5 - x\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1590}

$$\frac{4 \left(x^5 - x\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x^4)/(x^3*(-x + x^5)^(1/4)),x]
[Out] (4*(-x + x^5)^(3/4))/(3*x^3)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{3 + x^4}{x^3 \sqrt[4]{-x + x^5}} \, dx = \frac{4 \left(-x + x^5\right)^{3/4}}{3x^3}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{4 \left(x \left(x^4 - 1\right)\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x^4)/(x^3*(-x + x^5)^(1/4)),x]
[Out] (4*(x*(-1 + x^4))^(3/4))/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.09, size = 18, normalized size = 1.00

$$\frac{4 \left(x^5 - x\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + x^4)/(x^3*(-x + x^5)^(1/4)),x]
[Out] (4*(-x + x^5)^(3/4))/(3*x^3)
```

fricas [A] time = 0.40, size = 14, normalized size = 0.78

$$\frac{4\left(x^5-x\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)/x^3/(x^5-x)^(1/4),x, algorithm="fricas")
[Out] 4/3*(x^5 - x)^(3/4)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+3}{\left(x^5-x\right)^{\frac{1}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)/x^3/(x^5-x)^(1/4),x, algorithm="giac")
[Out] integrate((x^4 + 3)/((x^5 - x)^(1/4)*x^3), x)
```

maple [A] time = 0.01, size = 26, normalized size = 1.44

$$\frac{4\,(-1+x)\,(1+x)\left(x^2+1\right)}{3x^2\left(x^5-x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3)/x^3/(x^5-x)^(1/4),x)
[Out] 4/3/x^2*(-1+x)*(1+x)*(x^2+1)/(x^5-x)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+3}{\left(x^5-x\right)^{\frac{1}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)/x^3/(x^5-x)^(1/4),x, algorithm="maxima")
[Out] integrate((x^4 + 3)/((x^5 - x)^(1/4)*x^3), x)
```

mupad [B] time = 0.25, size = 14, normalized size = 0.78

$$\frac{4\left(x^5-x\right)^{3/4}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 3)/(x^3*(x^5 - x)^(1/4)),x)
[Out] (4*(x^5 - x)^(3/4))/(3*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+3}{x^3\sqrt[4]{x\,(x-1)\,(x+1)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3)/x**3/(x**5-x)**(1/4),x)
```

```
[Out] Integral((x**4 + 3)/(x**3*(x*(x - 1)*(x + 1)*(x**2 + 1))**(1/4)), x)
```

3.147 $\int \frac{(-1+x^4)(3+x^4)}{x^6 \sqrt[4]{-x+x^5}} \, dx$

Optimal. Leaf size=18

$$\frac{4 \left(x^5-x\right)^{7/4}}{7x^7}$$

Rubi [B] time = 0.26, antiderivative size = 37, normalized size of antiderivative = 2.06, number of steps used = 14, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2052, 2025, 2032, 365, 364}

$$\frac{4 \left(x^5-x\right)^{3/4}}{7x^2}-\frac{4 \left(x^5-x\right)^{3/4}}{7x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4)*(3 + x^4))/(x^6*(-x + x^5)^(1/4)),x]
```

```
[Out] (-4*(-x + x^5)^(3/4))/(7*x^6) + (4*(-x + x^5)^(3/4))/(7*x^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{(-1+x^4)(3+x^4)}{x^6\sqrt[4]{-x+x^5}}\,dx = \int \left(-\frac{3}{x^6\sqrt[4]{-x+x^5}} + \frac{2}{x^2\sqrt[4]{-x+x^5}} + \frac{x^2}{\sqrt[4]{-x+x^5}}\right) dx$$
$$= 2\int \frac{1}{x^2\sqrt[4]{-x+x^5}}\,dx - 3\int \frac{1}{x^6\sqrt[4]{-x+x^5}}\,dx + \int \frac{x^2}{\sqrt[4]{-x+x^5}}\,dx$$
$$= -\frac{4(-x+x^5)^{3/4}}{7x^6} + \frac{8(-x+x^5)^{3/4}}{5x^2} - \frac{9}{7}\int \frac{1}{x^2\sqrt[4]{-x+x^5}}\,dx - \frac{14}{5}\int \frac{x^2}{\sqrt[4]{-x+x^5}}\,dx + \frac{(\sqrt[4]{}}{}$$
$$= -\frac{4(-x+x^5)^{3/4}}{7x^6} + \frac{4(-x+x^5)^{3/4}}{7x^2} + \frac{9}{5}\int \frac{x^2}{\sqrt[4]{-x+x^5}}\,dx + \frac{\left(\sqrt[4]{x}\sqrt[4]{1-x^4}\right)\int \frac{x^{7/4}}{\sqrt[4]{1-x^4}}\,dx}{\sqrt[4]{-x+x^5}} -$$
$$= -\frac{4(-x+x^5)^{3/4}}{7x^6} + \frac{4(-x+x^5)^{3/4}}{7x^2} + \frac{4x^3\sqrt[4]{1-x^4}\,{}_2F_1\left(\frac{1}{4},\frac{11}{16};\frac{27}{16};x^4\right)}{11\sqrt[4]{-x+x^5}} - \frac{\left(14\sqrt[4]{x}\sqrt[4]{1-x^4}\right)}{5\sqrt[4]{-x+}}$$
$$= -\frac{4(-x+x^5)^{3/4}}{7x^6} + \frac{4(-x+x^5)^{3/4}}{7x^2} - \frac{36x^3\sqrt[4]{1-x^4}\,{}_2F_1\left(\frac{1}{4},\frac{11}{16};\frac{27}{16};x^4\right)}{55\sqrt[4]{-x+x^5}} + \frac{\left(9\sqrt[4]{x}\sqrt[4]{1-x^4}\right)}{5\sqrt[4]{-x-}}$$
$$= -\frac{4(-x+x^5)^{3/4}}{7x^6} + \frac{4(-x+x^5)^{3/4}}{7x^2}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{4\left(x\left(x^4-1\right)\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)*(3 + x^4))/(x^6*(-x + x^5)^(1/4)),x]
[Out] (4*(x*(-1 + x^4))^(7/4))/(7*x^7)
```

IntegrateAlgebraic [A] time = 0.27, size = 18, normalized size = 1.00

$$\frac{4\left(x^5-x\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(3 + x^4))/(x^6*(-x + x^5)^(1/4)),x]
[Out] (4*(-x + x^5)^(7/4))/(7*x^7)
```

fricas [A] time = 0.41, size = 19, normalized size = 1.06

$$\frac{4\left(x^5-x\right)^{\frac{3}{4}}\left(x^4-1\right)}{7\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+3)/x^6/(x^5-x)^(1/4),x, algorithm="fricas")
[Out] 4/7*(x^5 - x)^(3/4)*(x^4 - 1)/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\right)\left(x^4-1\right)}{\left(x^5-x\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+3)/x^6/(x^5-x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3)*(x^4 - 1)/((x^5 - x)^(1/4)*x^6), x)
```

maple [B] time = 0.01, size = 31, normalized size = 1.72

$$\frac{4\left(-1+x\right)\left(1+x\right)\left(x^{2}+1\right)\left(x^{4}-1\right)}{7x^{5}\left(x^{5}-x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^4+3)/x^6/(x^5-x)^(1/4),x)
```

```
[Out] 4/7/x^5*(-1+x)*(1+x)*(x^2+1)*(x^4-1)/(x^5-x)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}+3\right)\left(x^{4}-1\right)}{\left(x^{5}-x\right)^{\frac{1}{4}}x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+3)/x^6/(x^5-x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3)*(x^4 - 1)/((x^5 - x)^(1/4)*x^6), x)
```

mupad [B] time = 0.28, size = 31, normalized size = 1.72

$$-\frac{4\left(x^{5}-x\right)^{3/4}-4\,x^{4}\left(x^{5}-x\right)^{3/4}}{7\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(x^4 + 3))/(x^6*(x^5 - x)^(1/4)),x)
```

```
[Out] -(4*(x^5 - x)^(3/4) - 4*x^4*(x^5 - x)^(3/4))/(7*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-1\right)\left(x+1\right)\left(x^{2}+1\right)\left(x^{4}+3\right)}{x^{6}\sqrt[4]{x\left(x-1\right)\left(x+1\right)\left(x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**4+3)/x**6/(x**5-x)**(1/4),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)*(x**4 + 3)/(x**6*(x*(x - 1)*(x + 1)*(x*
*2 + 1))**(1/4)), x)
```

3.148 $\int \frac{-3+x^4}{\left(1+x^4\right)\sqrt[4]{x+x^5}}\,dx$

Optimal. Leaf size=18

$$-\frac{4\left(x^5+x\right)^{3/4}}{x^4+1}$$

Rubi [A] time = 0.07, antiderivative size = 12, normalized size of antiderivative = 0.67, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2056, 449}

$$-\frac{4x}{\sqrt[4]{x^5+x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x^4)/((1 + x^4)*(x + x^5)^(1/4)),x]
```

```
[Out] (-4*x)/(x + x^5)^(1/4)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3+x^4}{\left(1+x^4\right)\sqrt[4]{x+x^5}}\,dx &= \frac{\left(\sqrt[4]{x}\sqrt[4]{1+x^4}\right)\int \frac{-3+x^4}{\sqrt[4]{x}\left(1+x^4\right)^{5/4}}\,dx}{\sqrt[4]{x+x^5}}\\
&= -\frac{4x}{\sqrt[4]{x+x^5}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 60, normalized size = 3.33

$$\frac{4\sqrt[4]{x^4+1}\left(x^5\,{}_2F_1\left(\frac{19}{16},\frac{5}{4};\frac{35}{16};-x^4\right)-19x\,{}_2F_1\left(\frac{3}{16},\frac{5}{4};\frac{19}{16};-x^4\right)\right)}{19\sqrt[4]{x^5+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x^4)/((1 + x^4)*(x + x^5)^(1/4)),x]
```

```
[Out] (4*(1 + x^4)^(1/4)*(-19*x*Hypergeometric2F1[3/16, 5/4, 19/16, -x^4] + x^5*H
ypergeometric2F1[19/16, 5/4, 35/16, -x^4]))/(19*(x + x^5)^(1/4))
```

IntegrateAlgebraic [A] time = 0.14, size = 18, normalized size = 1.00

$$-\frac{4\left(x^{5}+x\right)^{3/4}}{x^{4}+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 + x^4)/((1 + x^4)*(x + x^5)^(1/4)),x]
[Out] (-4*(x + x^5)^(3/4))/(1 + x^4)
```

fricas [A] time = 0.40, size = 16, normalized size = 0.89

$$-\frac{4\left(x^{5}+x\right)^{\frac{3}{4}}}{x^{4}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)/(x^4+1)/(x^5+x)^(1/4),x, algorithm="fricas")
[Out] -4*(x^5 + x)^(3/4)/(x^4 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}-3}{\left(x^{5}+x\right)^{\frac{1}{4}}\left(x^{4}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)/(x^4+1)/(x^5+x)^(1/4),x, algorithm="giac")
[Out] integrate((x^4 - 3)/((x^5 + x)^(1/4)*(x^4 + 1)), x)
```

maple [A] time = 0.01, size = 11, normalized size = 0.61

$$-\frac{4x}{\left(x^{5}+x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)/(x^4+1)/(x^5+x)^(1/4),x)
[Out] -4*x/(x^5+x)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}-3}{\left(x^{5}+x\right)^{\frac{1}{4}}\left(x^{4}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)/(x^4+1)/(x^5+x)^(1/4),x, algorithm="maxima")
[Out] integrate((x^4 - 3)/((x^5 + x)^(1/4)*(x^4 + 1)), x)
```

mupad [B] time = 0.16, size = 16, normalized size = 0.89

$$-\frac{4\left(x^{5}+x\right)^{3/4}}{x^{4}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 3)/((x^4 + 1)*(x + x^5)^(1/4)),x)
```

```
[Out] -(4*(x + x^5)^(3/4))/(x^4 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-3}{\sqrt[4]{x\left(x^4+1\right)}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)/(x**4+1)/(x**5+x)**(1/4),x)
```

```
[Out] Integral((x**4 - 3)/((x*(x**4 + 1))**(1/4)*(x**4 + 1)), x)
```

3.149 $\int \frac{(-1+2x^3)\sqrt[3]{x^2+x^5}}{x^3}\,dx$

Optimal. Leaf size=18

$$\frac{3(x^5+x^2)^{4/3}}{4x^4}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1590}

$$\frac{3(x^5+x^2)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + 2*x^3)*(x^2 + x^5)^(1/3))/x^3,x]
```

```
[Out] (3*(x^2 + x^5)^(4/3))/(4*x^4)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(-1+2x^3)\sqrt[3]{x^2+x^5}}{x^3}\,dx = \frac{3(x^2+x^5)^{4/3}}{4x^4}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.28

$$\frac{3(x^3+1)\sqrt[3]{x^5+x^2}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + 2*x^3)*(x^2 + x^5)^(1/3))/x^3,x]
```

```
[Out] (3*(1 + x^3)*(x^2 + x^5)^(1/3))/(4*x^2)
```

IntegrateAlgebraic [A] time = 0.10, size = 18, normalized size = 1.00

$$\frac{3(x^5+x^2)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + 2*x^3)*(x^2 + x^5)^(1/3))/x^3,x]
```

```
[Out] (3*(x^2 + x^5)^(4/3))/(4*x^4)
```

fricas [A] time = 0.41, size = 19, normalized size = 1.06

$$\frac{3\left(x^{5}+x^{2}\right)^{\frac{1}{3}}\left(x^{3}+1\right)}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-1)*(x^5+x^2)^(1/3)/x^3,x, algorithm="fricas")
[Out] 3/4*(x^5 + x^2)^(1/3)*(x^3 + 1)/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{5}+x^{2}\right)^{\frac{1}{3}}\left(2\,x^{3}-1\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-1)*(x^5+x^2)^(1/3)/x^3,x, algorithm="giac")
[Out] integrate((x^5 + x^2)^(1/3)*(2*x^3 - 1)/x^3, x)
```

maple [A] time = 0.01, size = 26, normalized size = 1.44

$$\frac{3\left(x^{2}-x+1\right)(1+x)\left(x^{5}+x^{2}\right)^{\frac{1}{3}}}{4x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-1)*(x^5+x^2)^(1/3)/x^3,x)
[Out] 3/4*(x^2-x+1)/x^2*(1+x)*(x^5+x^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{5}+x^{2}\right)^{\frac{1}{3}}\left(2\,x^{3}-1\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-1)*(x^5+x^2)^(1/3)/x^3,x, algorithm="maxima")
[Out] integrate((x^5 + x^2)^(1/3)*(2*x^3 - 1)/x^3, x)
```

mupad [B] time = 0.21, size = 19, normalized size = 1.06

$$\frac{3\left(x^{5}+x^{2}\right)^{1/3}\left(x^{3}+1\right)}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + x^5)^(1/3)*(2*x^3 - 1))/x^3,x)
[Out] (3*(x^2 + x^5)^(1/3)*(x^3 + 1))/(4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x^{2}\left(x+1\right)\left(x^{2}-x+1\right)}\left(2x^{3}-1\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-1)*(x**5+x**2)**(1/3)/x**3,x)
[Out] Integral((x**2*(x + 1)*(x**2 - x + 1))**(1/3)*(2*x**3 - 1)/x**3, x)
```

3.150 $\int \frac{-1+x^2}{x\sqrt[4]{x^3+x^5}}\,dx$

Optimal. Leaf size=18

$$\frac{4\left(x^5+x^3\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.06, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2036}

$$\frac{4\left(x^5+x^3\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(x*(x^3 + x^5)^(1/4)),x]
```

```
[Out] (4*(x^3 + x^5)^(3/4))/(3*x^3)
```

Rule 2036

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] /; FreeQ[{a, b, c, d, e, j, m
, n, p}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && EqQ[
a*d*(m + j*p + 1) - b*c*(m + n + p*(j + n) + 1), 0] && (GtQ[e, 0] || Intege
rsQ[j]) && NeQ[m + j*p + 1, 0]
```

Rubi steps

$$\int \frac{-1+x^2}{x\sqrt[4]{x^3+x^5}}\,dx=\frac{4\left(x^3+x^5\right)^{3/4}}{3x^3}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.11

$$\frac{4\left(x^2+1\right)}{3\sqrt[4]{x^5+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(x*(x^3 + x^5)^(1/4)),x]
```

```
[Out] (4*(1 + x^2))/(3*(x^3 + x^5)^(1/4))
```

IntegrateAlgebraic [A] time = 0.09, size = 18, normalized size = 1.00

$$\frac{4\left(x^5+x^3\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/(x*(x^3 + x^5)^(1/4)),x]
```

```
[Out] (4*(x^3 + x^5)^(3/4))/(3*x^3)
```

fricas [A] time = 0.39, size = 14, normalized size = 0.78

$$\frac{4\left(x^{5}+x^{3}\right)^{\frac{3}{4}}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^5+x^3)^(1/4),x, algorithm="fricas")
[Out] 4/3*(x^5 + x^3)^(3/4)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}-1}{\left(x^{5}+x^{3}\right)^{\frac{1}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^5+x^3)^(1/4),x, algorithm="giac")
[Out] integrate((x^2 - 1)/((x^5 + x^3)^(1/4)*x), x)
```

maple [A] time = 0.01, size = 17, normalized size = 0.94

$$\frac{\frac{4x^{2}}{3}+\frac{4}{3}}{\left(x^{5}+x^{3}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/x/(x^5+x^3)^(1/4),x)
[Out] 4/3*(x^2+1)/(x^5+x^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}-1}{\left(x^{5}+x^{3}\right)^{\frac{1}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^5+x^3)^(1/4),x, algorithm="maxima")
[Out] integrate((x^2 - 1)/((x^5 + x^3)^(1/4)*x), x)
```

mupad [B] time = 0.19, size = 14, normalized size = 0.78

$$\frac{4\left(x^{5}+x^{3}\right)^{3/4}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x*(x^3 + x^5)^(1/4)),x)
[Out] (4*(x^3 + x^5)^(3/4))/(3*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)}{x\sqrt[4]{x^{3}\left(x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/x/(x**5+x**3)**(1/4),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(x*(x**3*(x**2 + 1))**(1/4)), x)
```

3.151 $\int \frac{(-1+x^4)\sqrt[4]{x^3+x^5}}{x^4}\,dx$

Optimal. Leaf size=18

$$\frac{4\left(x^5+x^3\right)^{9/4}}{9x^9}$$

Rubi [B] time = 0.18, antiderivative size = 53, normalized size of antiderivative = 2.94, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2052, 2004, 2032, 364, 2020, 2025}

$$\frac{4}{9}\sqrt[4]{x^5+x^3}\,x+\frac{8\sqrt[4]{x^5+x^3}}{9x}+\frac{4\sqrt[4]{x^5+x^3}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4)*(x^3 + x^5)^(1/4))/x^4,x]
```

```
[Out] (4*(x^3 + x^5)^(1/4))/(9*x^3) + (8*(x^3 + x^5)^(1/4))/(9*x) + (4*x*(x^3 + x
^5)^(1/4))/9
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^3+x^5}}{x^4}\,dx = \int \left(\sqrt[4]{x^3+x^5}-\frac{\sqrt[4]{x^3+x^5}}{x^4}\right)dx$$

$$= \int \sqrt[4]{x^3+x^5}\,dx - \int \frac{\sqrt[4]{x^3+x^5}}{x^4}\,dx$$

$$= \frac{4\sqrt[4]{x^3+x^5}}{9x^3} + \frac{4}{9}x\sqrt[4]{x^3+x^5} - \frac{2}{9}\int \frac{x}{\left(x^3+x^5\right)^{3/4}}\,dx + \frac{2}{9}\int \frac{x^3}{\left(x^3+x^5\right)^{3/4}}\,dx$$

$$= \frac{4\sqrt[4]{x^3+x^5}}{9x^3} + \frac{8\sqrt[4]{x^3+x^5}}{9x} + \frac{4}{9}x\sqrt[4]{x^3+x^5} - \frac{2}{9}\int \frac{x^3}{\left(x^3+x^5\right)^{3/4}}\,dx + \frac{\left(2x^{9/4}\left(1+x^2\right.\right.}{9\left(x^3\right.}$$

$$= \frac{4\sqrt[4]{x^3+x^5}}{9x^3} + \frac{8\sqrt[4]{x^3+x^5}}{9x} + \frac{4}{9}x\sqrt[4]{x^3+x^5} + \frac{8x^4\left(1+x^2\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{7}{8};\frac{15}{8};-x^2\right)}{63\left(x^3+x^5\right)^{3/4}} - $$

$$= \frac{4\sqrt[4]{x^3+x^5}}{9x^3} + \frac{8\sqrt[4]{x^3+x^5}}{9x} + \frac{4}{9}x\sqrt[4]{x^3+x^5}$$

Mathematica [C] time = 0.04, size = 65, normalized size = 3.61

$$\frac{4\sqrt[4]{x^5+x^3}\left(7\,{}_2F_1\left(-\frac{9}{8},-\frac{1}{4};-\frac{1}{8};-x^2\right)+9x^4\,{}_2F_1\left(-\frac{1}{4},\frac{7}{8};\frac{15}{8};-x^2\right)\right)}{63x^3\sqrt[4]{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)*(x^3 + x^5)^(1/4))/x^4,x]
```

```
[Out] (4*(x^3 + x^5)^(1/4)*(7*Hypergeometric2F1[-9/8, -1/4, -1/8, -x^2] + 9*x^4*H
ypergeometric2F1[-1/4, 7/8, 15/8, -x^2]))/(63*x^3*(1 + x^2)^(1/4))
```

IntegrateAlgebraic [A] time = 0.11, size = 18, normalized size = 1.00

$$\frac{4\left(x^5+x^3\right)^{9/4}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(x^3 + x^5)^(1/4))/x^4,x]
```

```
[Out] (4*(x^3 + x^5)^(9/4))/(9*x^9)
```

fricas [A] time = 0.42, size = 24, normalized size = 1.33

$$\frac{4\left(x^5+x^3\right)^{\frac{1}{4}}\left(x^4+2\,x^2+1\right)}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^5+x^3)^(1/4)/x^4,x, algorithm="fricas")
```

```
[Out] 4/9*(x^5 + x^3)^(1/4)*(x^4 + 2*x^2 + 1)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5 + x^3\right)^{\frac{1}{4}} \left(x^4 - 1\right)}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^5+x^3)^(1/4)/x^4,x, algorithm="giac")
```

```
[Out] integrate((x^5 + x^3)^(1/4)*(x^4 - 1)/x^4, x)
```

maple [A] time = 0.01, size = 22, normalized size = 1.22

$$\frac{4 \left(x^2 + 1\right)^2 \left(x^5 + x^3\right)^{\frac{1}{4}}}{9 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^5+x^3)^(1/4)/x^4,x)
```

```
[Out] 4/9*(x^2+1)^2*(x^5+x^3)^(1/4)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5 + x^3\right)^{\frac{1}{4}} \left(x^4 - 1\right)}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^5+x^3)^(1/4)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((x^5 + x^3)^(1/4)*(x^4 - 1)/x^4, x)
```

mupad [B] time = 0.27, size = 41, normalized size = 2.28

$$\frac{4\, x \left(x^5 + x^3\right)^{1/4}}{9} + \frac{8 \left(x^5 + x^3\right)^{1/4}}{9\, x} + \frac{4 \left(x^5 + x^3\right)^{1/4}}{9\, x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + x^5)^(1/4)*(x^4 - 1))/x^4,x)
```

```
[Out] (4*x*(x^3 + x^5)^(1/4))/9 + (8*(x^3 + x^5)^(1/4))/(9*x) + (4*(x^3 + x^5)^(1
/4))/(9*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3 \left(x^2 + 1\right)} \left(x - 1\right) \left(x + 1\right) \left(x^2 + 1\right)}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**5+x**3)**(1/4)/x**4,x)
```

```
[Out] Integral((x**3*(x**2 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)/x**4, x)
```

3.152 $\int \frac{x^2}{\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=18

$$\frac{1}{3}\log\left(\sqrt{x^6-1}+x^3\right)$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {275, 217, 206}

$$\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[-1 + x^6],x]
[Out] ArcTanh[x^3/Sqrt[-1 + x^6]]/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{-1+x^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^3}{\sqrt{-1+x^6}}\right)\\
&= \frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[-1 + x^6],x]
[Out] ArcTanh[x^3/Sqrt[-1 + x^6]]/3
```

IntegrateAlgebraic [A] time = 0.13, size = 18, normalized size = 1.00

$$\frac{1}{3}\log\left(\sqrt{x^6-1}+x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[-1 + x^6],x]
[Out] Log[x^3 + Sqrt[-1 + x^6]]/3
```

fricas [A] time = 0.41, size = 16, normalized size = 0.89

$$-\frac{1}{3}\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-1)^(1/2),x, algorithm="fricas")
[Out] -1/3*log(-x^3 + sqrt(x^6 - 1))
```

giac [A] time = 0.40, size = 17, normalized size = 0.94

$$-\frac{1}{3}\log\left(\left|-x^3+\sqrt{x^6-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-1)^(1/2),x, algorithm="giac")
[Out] -1/3*log(abs(-x^3 + sqrt(x^6 - 1)))
```

maple [C] time = 0.25, size = 25, normalized size = 1.39

$$\frac{\sqrt{-\mathrm{signum}\left(x^6-1\right)}\,\arcsin\left(x^3\right)}{3\sqrt{\mathrm{signum}\left(x^6-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6-1)^(1/2),x)
[Out] 1/3/signum(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*arcsin(x^3)
```

maxima [B] time = 0.49, size = 33, normalized size = 1.83

$$\frac{1}{6}\log\left(\frac{\sqrt{x^6-1}}{x^3}+1\right)-\frac{1}{6}\log\left(\frac{\sqrt{x^6-1}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-1)^(1/2),x, algorithm="maxima")
[Out] 1/6*log(sqrt(x^6 - 1)/x^3 + 1) - 1/6*log(sqrt(x^6 - 1)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x^2}{\sqrt{x^6-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6 - 1)^(1/2),x)
[Out] int(x^2/(x^6 - 1)^(1/2), x)
```

sympy [A] time = 0.79, size = 19, normalized size = 1.06

$$\begin{cases} \frac{\operatorname{acosh}\left(x^{3}\right)}{3} & \text{for}\ \left|x^{6}\right| > 1 \\ -\frac{i \operatorname{asin}\left(x^{3}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6-1)**(1/2),x)
[Out] Piecewise((acosh(x**3)/3, Abs(x**6) > 1), (-I*asin(x**3)/3, True))
```

3.153 $\int \frac{x^2}{\sqrt{1+x^6}} dx$

Optimal. Leaf size=18

$$\frac{1}{3}\log\left(\sqrt{x^6+1}+x^3\right)$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 0.44, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {275, 215}

$$\frac{1}{3}\sinh^{-1}\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 + x^6],x]
[Out] ArcSinh[x^3]/3
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{1+x^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\sinh^{-1}\left(x^3\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 0.44

$$\frac{1}{3}\sinh^{-1}\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 + x^6],x]
[Out] ArcSinh[x^3]/3
```

IntegrateAlgebraic [A] time = 0.12, size = 18, normalized size = 1.00

$$\frac{1}{3}\log\left(\sqrt{x^6+1}+x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[1 + x^6],x]
[Out] Log[x^3 + Sqrt[1 + x^6]]/3
```

fricas [A] time = 0.41, size = 16, normalized size = 0.89

$$-\frac{1}{3}\log\left(-x^3+\sqrt{x^6+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+1)^(1/2),x, algorithm="fricas")
[Out] -1/3*log(-x^3 + sqrt(x^6 + 1))
```

giac [A] time = 0.45, size = 16, normalized size = 0.89

$$-\frac{1}{3}\log\left(-x^3+\sqrt{x^6+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+1)^(1/2),x, algorithm="giac")
[Out] -1/3*log(-x^3 + sqrt(x^6 + 1))
```

maple [A] time = 0.18, size = 7, normalized size = 0.39

$$\frac{\operatorname{arcsinh}\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+1)^(1/2),x)
[Out] 1/3*arcsinh(x^3)
```

maxima [B] time = 0.51, size = 33, normalized size = 1.83

$$\frac{1}{6}\log\left(\frac{\sqrt{x^6+1}}{x^3}+1\right)-\frac{1}{6}\log\left(\frac{\sqrt{x^6+1}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+1)^(1/2),x, algorithm="maxima")
[Out] 1/6*log(sqrt(x^6 + 1)/x^3 + 1) - 1/6*log(sqrt(x^6 + 1)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x^2}{\sqrt{x^6+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6 + 1)^(1/2),x)
[Out] int(x^2/(x^6 + 1)^(1/2), x)
```

sympy [A] time = 0.75, size = 5, normalized size = 0.28

$$\frac{\operatorname{asinh}\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+1)**(1/2),x)
[Out] asinh(x**3)/3
```

3.154 $\int \frac{(-1+x^5)(3+2x^5)}{x^6\sqrt[4]{-x+x^6}}\,dx$

Optimal. Leaf size=18

$$\frac{4\left(x^6-x\right)^{7/4}}{7x^7}$$

Rubi [B] time = 0.27, antiderivative size = 37, normalized size of antiderivative = 2.06, number of steps used = 14, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2052, 2025, 2032, 365, 364}

$$\frac{4\left(x^6-x\right)^{3/4}}{7x}-\frac{4\left(x^6-x\right)^{3/4}}{7x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^5)*(3 + 2*x^5))/(x^6*(-x + x^6)^(1/4)),x]
```

```
[Out] (-4*(-x + x^6)^(3/4))/(7*x^6) + (4*(-x + x^6)^(3/4))/(7*x)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{(-1+x^5)(3+2x^5)}{x^6\sqrt[4]{-x+x^6}}\,dx = \int \left(-\frac{3}{x^6\sqrt[4]{-x+x^6}} + \frac{1}{x\sqrt[4]{-x+x^6}} + \frac{2x^4}{\sqrt[4]{-x+x^6}}\right) dx$$

$$= 2\int \frac{x^4}{\sqrt[4]{-x+x^6}}\,dx - 3\int \frac{1}{x^6\sqrt[4]{-x+x^6}}\,dx + \int \frac{1}{x\sqrt[4]{-x+x^6}}\,dx$$

$$= -\frac{4(-x+x^6)^{3/4}}{7x^6} + \frac{4(-x+x^6)^{3/4}}{x} - \frac{6}{7}\int \frac{1}{x\sqrt[4]{-x+x^6}}\,dx - 14\int \frac{x^4}{\sqrt[4]{-x+x^6}}\,dx + \cdots$$

$$= -\frac{4(-x+x^6)^{3/4}}{7x^6} + \frac{4(-x+x^6)^{3/4}}{7x} + 12\int \frac{x^4}{\sqrt[4]{-x+x^6}}\,dx + \frac{\left(2\sqrt[4]{x}\sqrt[4]{1-x^5}\right)\int \frac{x^{15/4}}{\sqrt[4]{1-x \cdots}}}{\sqrt[4]{-x+x^6}} \cdots$$

$$= -\frac{4(-x+x^6)^{3/4}}{7x^6} + \frac{4(-x+x^6)^{3/4}}{7x} + \frac{8x^5\sqrt[4]{1-x^5}\,{}_2F_1\left(\frac{1}{4},\frac{19}{20};\frac{39}{20};x^5\right)}{19\sqrt[4]{-x+x^6}} - \frac{\left(14\sqrt[4]{x}\sqrt[4]{1-\cdots}\right.}{\sqrt[4]{-\cdots}}$$

$$= -\frac{4(-x+x^6)^{3/4}}{7x^6} + \frac{4(-x+x^6)^{3/4}}{7x} - \frac{48x^5\sqrt[4]{1-x^5}\,{}_2F_1\left(\frac{1}{4},\frac{19}{20};\frac{39}{20};x^5\right)}{19\sqrt[4]{-x+x^6}} + \frac{\left(12\sqrt[4]{x}\sqrt[4]{1\cdots}\right.}{\sqrt[4]{\cdots}}$$

$$= -\frac{4(-x+x^6)^{3/4}}{7x^6} + \frac{4(-x+x^6)^{3/4}}{7x}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{4\left(x\left(x^5-1\right)\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^5)*(3 + 2*x^5))/(x^6*(-x + x^6)^(1/4)),x]
[Out] (4*(x*(-1 + x^5))^(7/4))/(7*x^7)
```

IntegrateAlgebraic [A] time = 0.31, size = 18, normalized size = 1.00

$$\frac{4\left(x^6-x\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^5)*(3 + 2*x^5))/(x^6*(-x + x^6)^(1/4)),x]
[Out] (4*(-x + x^6)^(7/4))/(7*x^7)
```

fricas [A] time = 0.41, size = 19, normalized size = 1.06

$$\frac{4\left(x^6-x\right)^{\frac{3}{4}}\left(x^5-1\right)}{7\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)*(2*x^5+3)/x^6/(x^6-x)^(1/4),x, algorithm="fricas")
[Out] 4/7*(x^6 - x)^(3/4)*(x^5 - 1)/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^5+3\right)\left(x^5-1\right)}{\left(x^6-x\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)*(2*x^5+3)/x^6/(x^6-x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((2*x^5 + 3)*(x^5 - 1)/((x^6 - x)^(1/4)*x^6), x)
```

maple [B] time = 0.01, size = 35, normalized size = 1.94

$$\frac{4\left(-1+x\right)\left(x^{4}+x^{3}+x^{2}+x+1\right)\left(x^{5}-1\right)}{7 x^{5} \left(x^{6}-x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)*(2*x^5+3)/x^6/(x^6-x)^(1/4),x)
```

```
[Out] 4/7/x^5*(-1+x)*(x^4+x^3+x^2+x+1)*(x^5-1)/(x^6-x)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\, x^{5}+3\right)\left(x^{5}-1\right)}{\left(x^{6}-x\right)^{\frac{1}{4}} x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)*(2*x^5+3)/x^6/(x^6-x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((2*x^5 + 3)*(x^5 - 1)/((x^6 - x)^(1/4)*x^6), x)
```

mupad [B] time = 0.28, size = 31, normalized size = 1.72

$$-\frac{4\left(x^{6}-x\right)^{3/4}-4\, x^{5}\left(x^{6}-x\right)^{3/4}}{7\, x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 - 1)*(2*x^5 + 3))/(x^6*(x^6 - x)^(1/4)),x)
```

```
[Out] -(4*(x^6 - x)^(3/4) - 4*x^5*(x^6 - x)^(3/4))/(7*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-1\right)\left(2 x^{5}+3\right)\left(x^{4}+x^{3}+x^{2}+x+1\right)}{x^{6} \sqrt[4]{x \left(x-1\right)\left(x^{4}+x^{3}+x^{2}+x+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)*(2*x**5+3)/x**6/(x**6-x)**(1/4),x)
```

```
[Out] Integral((x - 1)*(2*x**5 + 3)*(x**4 + x**3 + x**2 + x + 1)/(x**6*(x*(x - 1)
*(x**4 + x**3 + x**2 + x + 1))**(1/4)), x)
```

3.155 $\int \frac{1}{x^3 \sqrt[3]{x^2+x^6}} \, dx$

Optimal. Leaf size=18

$$-\frac{3\left(x^6+x^2\right)^{2/3}}{8x^4}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2014}

$$-\frac{3\left(x^6+x^2\right)^{2/3}}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(x^2 + x^6)^(1/3)),x]
```

```
[Out] (-3*(x^2 + x^6)^(2/3))/(8*x^4)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3 \sqrt[3]{x^2+x^6}} \, dx = -\frac{3\left(x^2+x^6\right)^{2/3}}{8x^4}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{3\left(x^6+x^2\right)^{2/3}}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(x^2 + x^6)^(1/3)),x]
```

```
[Out] (-3*(x^2 + x^6)^(2/3))/(8*x^4)
```

IntegrateAlgebraic [A] time = 0.35, size = 18, normalized size = 1.00

$$-\frac{3\left(x^6+x^2\right)^{2/3}}{8x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(x^2 + x^6)^(1/3)),x]
```

```
[Out] (-3*(x^2 + x^6)^(2/3))/(8*x^4)
```

fricas [A] time = 0.40, size = 14, normalized size = 0.78

$$-\frac{3\left(x^6+x^2\right)^{\frac{2}{3}}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/8*(x^6 + x^2)^(2/3)/x^4
```

giac [A] time = 0.61, size = 9, normalized size = 0.50

$$-\frac{3}{8}\left(\frac{1}{x^4}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] -3/8*(1/x^4 + 1)^(2/3)
```

maple [A] time = 0.00, size = 20, normalized size = 1.11

$$-\frac{3\left(x^4+1\right)}{8x^2\left(x^6+x^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^6+x^2)^(1/3),x)
```

```
[Out] -3/8/x^2*(x^4+1)/(x^6+x^2)^(1/3)
```

maxima [A] time = 0.69, size = 21, normalized size = 1.17

$$-\frac{3\left(x^6+x^2\right)}{8\left(x^4+1\right)^{\frac{1}{3}}\left(x^2\right)^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] -3/8*(x^6 + x^2)/((x^4 + 1)^(1/3)*(x^2)^(7/3))
```

mupad [B] time = 0.15, size = 14, normalized size = 0.78

$$-\frac{3\left(x^6+x^2\right)^{2/3}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^2 + x^6)^(1/3)),x)
```

```
[Out] -(3*(x^2 + x^6)^(2/3))/(8*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt[3]{x^2\left(x^4+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**6+x**2)**(1/3),x)
```

```
[Out] Integral(1/(x**3*(x**2*(x**4 + 1))**(1/3)), x)
```

3.156 $\int \frac{-1+x^4}{x^2 \sqrt[4]{x^2+x^6}} \, dx$

Optimal. Leaf size=18

$$\frac{2\left(x^6+x^2\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1590}

$$\frac{2\left(x^6+x^2\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)/(x^2*(x^2 + x^6)^(1/4)),x]
[Out] (2*(x^2 + x^6)^(3/4))/(3*x^3)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{-1+x^4}{x^2 \sqrt[4]{x^2+x^6}} \, dx = \frac{2\left(x^2+x^6\right)^{3/4}}{3x^3}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{2\left(x^6+x^2\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)/(x^2*(x^2 + x^6)^(1/4)),x]
[Out] (2*(x^2 + x^6)^(3/4))/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.24, size = 18, normalized size = 1.00

$$\frac{2\left(x^6+x^2\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/(x^2*(x^2 + x^6)^(1/4)),x]
[Out] (2*(x^2 + x^6)^(3/4))/(3*x^3)
```

fricas [A] time = 0.40, size = 14, normalized size = 0.78

$$\frac{2\left(x^{6}+x^{2}\right)^{\frac{3}{4}}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^6+x^2)^(1/4),x, algorithm="fricas")
[Out] 2/3*(x^6 + x^2)^(3/4)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}-1}{\left(x^{6}+x^{2}\right)^{\frac{1}{4}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^6+x^2)^(1/4),x, algorithm="giac")
[Out] integrate((x^4 - 1)/((x^6 + x^2)^(1/4)*x^2), x)
```

maple [A] time = 0.01, size = 20, normalized size = 1.11

$$\frac{\frac{2x^{4}}{3}+\frac{2}{3}}{\left(x^{6}+x^{2}\right)^{\frac{1}{4}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/x^2/(x^6+x^2)^(1/4),x)
[Out] 2/3*(x^4+1)/(x^6+x^2)^(1/4)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}-1}{\left(x^{6}+x^{2}\right)^{\frac{1}{4}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^6+x^2)^(1/4),x, algorithm="maxima")
[Out] integrate((x^4 - 1)/((x^6 + x^2)^(1/4)*x^2), x)
```

mupad [B] time = 0.13, size = 14, normalized size = 0.78

$$\frac{2\left(x^{6}+x^{2}\right)^{3/4}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/(x^2*(x^2 + x^6)^(1/4)),x)
[Out] (2*(x^2 + x^6)^(3/4))/(3*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)\left(x^{2}+1\right)}{x^{2}\sqrt[4]{x^{2}\left(x^{4}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/x**2/(x**6+x**2)**(1/4),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)/(x**2*(x**2*(x**4 + 1))**(1/4)), x)
```

3.157 $\int \frac{(-1+x^4)\sqrt[4]{x^2+x^6}}{x^4}\,dx$

Optimal. Leaf size=18

$$\frac{2\left(x^6+x^2\right)^{5/4}}{5x^5}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1590}

$$\frac{2\left(x^6+x^2\right)^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4)*(x^2 + x^6)^(1/4))/x^4,x]
[Out] (2*(x^2 + x^6)^(5/4))/(5*x^5)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{x^4}\,dx = \frac{2\left(x^2+x^6\right)^{5/4}}{5x^5}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.28

$$\frac{2\left(x^4+1\right)\sqrt[4]{x^6+x^2}}{5x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/x^4,x]
[Out] (2*(1 + x^4)*(x^2 + x^6)^(1/4))/(5*x^3)
```

IntegrateAlgebraic [A] time = 0.25, size = 18, normalized size = 1.00

$$\frac{2\left(x^6+x^2\right)^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(x^2 + x^6)^(1/4))/x^4,x]
[Out] (2*(x^2 + x^6)^(5/4))/(5*x^5)
```

fricas [A] time = 0.41, size = 19, normalized size = 1.06

$$\frac{2\left(x^{6}+x^{2}\right)^{\frac{1}{4}}\left(x^{4}+1\right)}{5\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/x^4,x, algorithm="fricas")
[Out] 2/5*(x^6 + x^2)^(1/4)*(x^4 + 1)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}+x^{2}\right)^{\frac{1}{4}}\left(x^{4}-1\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/x^4,x, algorithm="giac")
[Out] integrate((x^6 + x^2)^(1/4)*(x^4 - 1)/x^4, x)
```

maple [A] time = 0.01, size = 20, normalized size = 1.11

$$\frac{2\left(x^{4}+1\right)\left(x^{6}+x^{2}\right)^{\frac{1}{4}}}{5x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^6+x^2)^(1/4)/x^4,x)
[Out] 2/5*(x^4+1)*(x^6+x^2)^(1/4)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}+x^{2}\right)^{\frac{1}{4}}\left(x^{4}-1\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/x^4,x, algorithm="maxima")
[Out] integrate((x^6 + x^2)^(1/4)*(x^4 - 1)/x^4, x)
```

mupad [B] time = 0.21, size = 19, normalized size = 1.06

$$\frac{2\left(x^{6}+x^{2}\right)^{1/4}\left(x^{4}+1\right)}{5\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + x^6)^(1/4)*(x^4 - 1))/x^4,x)
[Out] (2*(x^2 + x^6)^(1/4)*(x^4 + 1))/(5*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^{2}\left(x^{4}+1\right)}\,(x-1)\,(x+1)\left(x^{2}+1\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**6+x**2)**(1/4)/x**4,x)
[Out] Integral((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)/x**4, x)
```

3.158 $\int \frac{\sqrt{1+\sqrt{1+x^2}}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=18

$$\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}$$

Rubi [F] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+\sqrt{1+x^2}}}{\sqrt{1+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + Sqrt[1 + x^2]]/Sqrt[1 + x^2],x]
[Out] Defer[Int][Sqrt[1 + Sqrt[1 + x^2]]/Sqrt[1 + x^2], x]
```

Rubi steps

$$\int \frac{\sqrt{1+\sqrt{1+x^2}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{1+\sqrt{1+x^2}}}{\sqrt{1+x^2}}\,dx$$

Mathematica [A] time = 0.11, size = 31, normalized size = 1.72

$$\frac{2\left(\sqrt{x^2+1}-1\right)\sqrt{\sqrt{x^2+1}+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + x^2]]/Sqrt[1 + x^2],x]
[Out] (2*(-1 + Sqrt[1 + x^2])*Sqrt[1 + Sqrt[1 + x^2]])/x
```

IntegrateAlgebraic [A] time = 0.05, size = 18, normalized size = 1.00

$$\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 + x^2]]/Sqrt[1 + x^2],x]
[Out] (2*x)/Sqrt[1 + Sqrt[1 + x^2]]
```

fricas [A] time = 0.44, size = 25, normalized size = 1.39

$$\frac{2\sqrt{\sqrt{x^2+1}+1}\left(\sqrt{x^2+1}-1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(sqrt(x^2 + 1) + 1)*(sqrt(x^2 + 1) - 1)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1)/sqrt(x^2 + 1), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\sqrt{x^2+1}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x^2+1)^(1/2))^(1/2)/(x^2+1)^(1/2),x)
[Out] int((1+(x^2+1)^(1/2))^(1/2)/(x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1)/sqrt(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)^(1/2) + 1)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int(((x^2 + 1)^(1/2) + 1)^(1/2)/(x^2 + 1)^(1/2), x)
```

sympy [B] time = 0.73, size = 31, normalized size = 1.72

$$\frac{\sqrt{2}\,x\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{\pi\sqrt{\sqrt{x^2+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x**2+1)**(1/2))**(1/2)/(x**2+1)**(1/2),x)
[Out] sqrt(2)*x*gamma(1/4)*gamma(3/4)/(pi*sqrt(sqrt(x**2 + 1) + 1))
```

3.159 $\int \frac{-2-2x+x^2}{(1+x+x^2)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=19

$$-\frac{2\sqrt{x^3-1}}{x^2+x+1}$$

Rubi [C] time = 1.92, antiderivative size = 719, normalized size of antiderivative = 37.84, number of steps used = 27, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6728, 219, 2136, 2142, 2113, 21, 414, 424, 444, 37}

$$\frac{2(1-x)}{\sqrt{x^3-1}}+\frac{2\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(\sqrt{3}+i\right)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}(1-x)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\left(3+(2+i)\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\sqrt[4]{3}\left(-\sqrt{3}+i\right)\sqrt{2-\sqrt{3}}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}(1-x)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\left(3+(2-i)\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\sqrt{2-\sqrt{3}}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}(1-x)F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{6\sqrt[4]{3}\left(\sqrt{3}+(-2+i)\right)\left(\sqrt{3}+i\right)\sqrt{26+15\sqrt{3}}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(1-x)E\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\left(3+(2+i)\sqrt{3}\right)^3\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{6\sqrt[4]{3}\left(1+i\sqrt{3}\right)\left(\sqrt{3}+(-2-i)\right)\sqrt{26+15\sqrt{3}}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(1-x)E\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\left(3i+(1+2i)\sqrt{3}\right)^3\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 - 2*x + x^2)/((1 + x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(1 - x))/Sqrt[-1 + x^3] + (6*3^(1/4)*(1 + I*Sqrt[3])*((-2 - I) + Sqrt[3]
)*Sqrt[26 + 15*Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ell
ipticE[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((3*I
+ (1 + 2*I)*Sqrt[3])^3*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3]) -
(6*3^(1/4)*((-2 + I) + Sqrt[3])*(I + Sqrt[3])*Sqrt[26 + 15*Sqrt[3]]*(1 - x)
*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)
/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((3 + (2 + I)*Sqrt[3])^3*Sqrt[(1 - x)
/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3]) - (2*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(
1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - S
qrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)
]*Sqrt[-1 + x^3]) - (2*3^(1/4)*(I - Sqrt[3])*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt
[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 -
 Sqrt[3] - x)], -7 + 4*Sqrt[3]])/((3 + (2 - I)*Sqrt[3])*Sqrt[-((1 - x)/(1 -
 Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (2*3^(1/4)*Sqrt[2 - Sqrt[3]]*(I + Sqrt[
3])*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + S
qrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/((3 + (2 + I)*Sqrt[3])*Sqr
t[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2136

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, -Dist[q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-2-2x+x^2}{\left(1+x+x^2\right)\sqrt{-1+x^3}}\,dx = \int\left(\frac{1}{\sqrt{-1+x^3}}-\frac{3(1+x)}{\left(1+x+x^2\right)\sqrt{-1+x^3}}\right)dx$$

$$= -\left(3\int \frac{1+x}{\left(1+x+x^2\right)\sqrt{-1+x^3}}\,dx\right)+\int\frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - 3\int\Bigg(\frac{\ }{\left(1-i\sqrt{3}\right.}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \left(3-i\sqrt{3}\right)\int$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(3i+\sqrt{3}\right)\int}{3i+(1+2}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{2\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{2\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{2\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{2\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{}$$

$$= \frac{1-x}{\sqrt{-1+x^3}} + \frac{6\left(i+\sqrt{3}\right)(1-x)}{\left(2-\sqrt{3}\right)\left(3+(2+i)\sqrt{3}\right)^2\sqrt{-1+x^3}} - \frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x}{(1-\sqrt{3}-}}}{\sqrt[4]{3}\sqrt{-\frac{}{(1-}}}$$

$$= \frac{1-x}{\sqrt{-1+x^3}} + \frac{6\left(i+\sqrt{3}\right)(1-x)}{\left(2-\sqrt{3}\right)\left(3+(2+i)\sqrt{3}\right)^2\sqrt{-1+x^3}} + \frac{i\sqrt[4]{3}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\,(1-x)\sqrt{\frac{1+}{(1+}}}{\sqrt{\frac{}{(1+}}}$$

Mathematica [A] time = 0.12, size = 19, normalized size = 1.00

$$-\frac{2\sqrt{x^3-1}}{x^2+x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 - 2*x + x^2)/((1 + x + x^2)*Sqrt[-1 + x^3]),x]
[Out] (-2*Sqrt[-1 + x^3])/(1 + x + x^2)
```

IntegrateAlgebraic [A] time = 0.51, size = 19, normalized size = 1.00

$$-\frac{2\sqrt{x^3-1}}{x^2+x+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 - 2*x + x^2)/((1 + x + x^2)*Sqrt[-1 + x^3]),x]
[Out] (-2*Sqrt[-1 + x^3])/(1 + x + x^2)
```

fricas [A] time = 0.42, size = 17, normalized size = 0.89

$$-\frac{2\sqrt{x^3-1}}{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-2)/(x^2+x+1)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(x^3 - 1)/(x^2 + x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-2x-2}{\sqrt{x^3-1}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-2)/(x^2+x+1)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + x + 1)), x)
```

maple [A] time = 0.01, size = 13, normalized size = 0.68

$$-\frac{2(-1+x)}{\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x-2)/(x^2+x+1)/(x^3-1)^(1/2),x)
[Out] -2*(-1+x)/(x^3-1)^(1/2)
```

maxima [A] time = 0.76, size = 15, normalized size = 0.79

$$-\frac{2\sqrt{x-1}}{\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-2)/(x^2+x+1)/(x^3-1)^(1/2),x, algorithm="maxima")
```

[Out] -2*sqrt(x - 1)/sqrt(x^2 + x + 1)

mupad [B] time = 0.19, size = 276, normalized size = 14.53

$$\frac{\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(6+9\,\sin\left(2\,\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\right)\,\sqrt{\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}+1}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}-6\,x+\sqrt{3}\,x\,2\mathrm{i}+\sqrt{3}\,2\mathrm{i}-\sqrt{3}\,x^2\,4\mathrm{i}-\sqrt{3}\,\sin\left(2\,\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\right)\,\sqrt{\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}+1}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,3\mathrm{i}\right)}{6\,\sqrt{1-\frac{x-1}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}+1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - x^2 + 2)/((x^3 - 1)^(1/2)*(x + x^2 + 1)),x)
```

```
[Out] ((-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*
1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3^(1/2)*x*2i - 6*x + 3^(1/2)*2i
 - 3^(1/2)*x^2*4i + 9*sin(2*asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)))*
((x - 1)/((3^(1/2)*1i)/2 + 3/2) + 1)^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2) - 3^(1/2)*sin(2*asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)))*((x
- 1)/((3^(1/2)*1i)/2 + 3/2) + 1)^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*3i + 6))/(6*(1 - (x - 1)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x - 1)/((3^(1/
2)*1i)/2 + 3/2) + 1)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) -
 x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-2x-2}{\sqrt{(x-1)\left(x^2+x+1\right)}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x-2)/(x**2+x+1)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x**2 - 2*x - 2)/(sqrt((x - 1)*(x**2 + x + 1))*(x**2 + x + 1)), x)
```

3.160 $\int \frac{b+ax^2}{x\sqrt{-bx+ax^3}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sqrt{ax^3-bx}}{x}$$

Rubi [A] time = 0.10, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2036}

$$\frac{2\sqrt{ax^3-bx}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^2)/(x*Sqrt[-(b*x) + a*x^3]),x]
```

```
[Out] (2*Sqrt[-(b*x) + a*x^3])/x
```

Rule 2036

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] /; FreeQ[{a, b, c, d, e, j, m
, n, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && EqQ[
a*d*(m + j*p + 1) - b*c*(m + n + p*(j + n) + 1), 0] && (GtQ[e, 0] || Intege
rsQ[j]) && NeQ[m + j*p + 1, 0]
```

Rubi steps

$$\int \frac{b+ax^2}{x\sqrt{-bx+ax^3}}\,dx = \frac{2\sqrt{-bx+ax^3}}{x}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$\frac{2\sqrt{ax^3-bx}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^2)/(x*Sqrt[-(b*x) + a*x^3]),x]
```

```
[Out] (2*Sqrt[-(b*x) + a*x^3])/x
```

IntegrateAlgebraic [A] time = 0.22, size = 19, normalized size = 1.00

$$\frac{2\sqrt{ax^3-bx}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^2)/(x*Sqrt[-(b*x) + a*x^3]),x]
```

```
[Out] (2*Sqrt[-(b*x) + a*x^3])/x
```

fricas [A] time = 0.41, size = 17, normalized size = 0.89

$$\frac{2\sqrt{ax^3-bx}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/x/(a*x^3-b*x)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(a*x^3 - b*x)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + b}{\sqrt{ax^3 - bx}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/x/(a*x^3-b*x)^(1/2),x, algorithm="giac")
[Out] integrate((a*x^2 + b)/(sqrt(a*x^3 - b*x)*x), x)
```

maple [A] time = 0.01, size = 24, normalized size = 1.26

$$\frac{2a\,x^2 - 2b}{\sqrt{a\,x^3 - bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)/x/(a*x^3-b*x)^(1/2),x)
[Out] 2*(a*x^2-b)/(a*x^3-b*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + b}{\sqrt{ax^3 - bx}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/x/(a*x^3-b*x)^(1/2),x, algorithm="maxima")
[Out] integrate((a*x^2 + b)/(sqrt(a*x^3 - b*x)*x), x)
```

mupad [B] time = 0.21, size = 17, normalized size = 0.89

$$\frac{2\,\sqrt{a\,x^3 - b\,x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^2)/(x*(a*x^3 - b*x)^(1/2)),x)
[Out] (2*(a*x^3 - b*x)^(1/2))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + b}{x\sqrt{x\left(ax^2 - b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b)/x/(a*x**3-b*x)**(1/2),x)
[Out] Integral((a*x**2 + b)/(x*sqrt(x*(a*x**2 - b))), x)
```

3.161 $\int \frac{-1-2x+2x^2}{(1+2x^2)\sqrt{x+x^4}}\,dx$

Optimal. Leaf size=19

$$\tan^{-1}\left(\frac{2\sqrt{x^4+x}}{2x-1}\right)$$

Rubi [F] time = 1.90, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1-2x+2x^2}{(1+2x^2)\sqrt{x+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 - 2*x + 2*x^2)/((1 + 2*x^2)*Sqrt[x + x^4]),x]
```

```
[Out] (x*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(1
+ (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3^(1/4)*Sqrt[
(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[x + x^4]) - ((1/2 + I/2)*(I + Sqr
t[2])*Sqrt[x]*Sqrt[1 + x^3]*Defer[Subst][Defer[Int][1/(((-1)^(1/4) - 2^(1/4
)*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/Sqrt[x + x^4] + ((1/2 + I/2)*(1 + I*S
qrt[2])*Sqrt[x]*Sqrt[1 + x^3]*Defer[Subst][Defer[Int][1/((-(-1)^(3/4) - 2^(
1/4)*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/Sqrt[x + x^4] - ((1/2 + I/2)*(I +
Sqrt[2])*Sqrt[x]*Sqrt[1 + x^3]*Defer[Subst][Defer[Int][1/(((-1)^(1/4) + 2^(
1/4)*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/Sqrt[x + x^4] + ((1/2 + I/2)*(1 +
I*Sqrt[2])*Sqrt[x]*Sqrt[1 + x^3]*Defer[Subst][Defer[Int][1/((-(-1)^(3/4) +
2^(1/4)*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/Sqrt[x + x^4]
```

Rubi steps

$$\int \frac{-1-2x+2x^2}{\left(1+2x^2\right)\sqrt{x+x^4}}\,dx = \frac{\left(\sqrt{x}\,\sqrt{1+x^3}\right)\int \frac{-1-2x+2x^2}{\sqrt{x}\left(1+2x^2\right)\sqrt{1+x^3}}\,dx}{\sqrt{x+x^4}}$$

$$= \frac{\left(\sqrt{x}\,\sqrt{1+x^3}\right)\int \left(\frac{1}{\sqrt{x}\,\sqrt{1+x^3}} - \frac{2(1+x)}{\sqrt{x}\left(1+2x^2\right)\sqrt{1+x^3}}\right)dx}{\sqrt{x+x^4}}$$

$$= \frac{\left(\sqrt{x}\,\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{x}\,\sqrt{1+x^3}}\,dx}{\sqrt{x+x^4}} - \frac{\left(2\sqrt{x}\,\sqrt{1+x^3}\right)\int \frac{1+x}{\sqrt{x}\left(1+2x^2\right)\sqrt{1+x^3}}\,dx}{\sqrt{x+x^4}}$$

$$= -\frac{\left(2\sqrt{x}\,\sqrt{1+x^3}\right)\int \left(\frac{i-\frac{1}{\sqrt{2}}}{2\sqrt{x}\left(i-\sqrt{2}\,x\right)\sqrt{1+x^3}} + \frac{i+\frac{1}{\sqrt{2}}}{2\sqrt{x}\left(i+\sqrt{2}\,x\right)\sqrt{1+x^3}}\right)dx}{\sqrt{x+x^4}} + \frac{\left(2\sqrt{x}\,\sqrt{1+x^3}\right)\text{Subs}}{\sqrt{x}}$$

$$= \frac{x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,\sqrt{x+x^4}} - \frac{\left(\left(2i-\sqrt{2}\right)\sqrt{x}\,\sqrt{1+x^3}\right)}{2\sqrt{x+}}$$

$$= \frac{x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,\sqrt{x+x^4}} - \frac{\left(\left(2i-\sqrt{2}\right)\sqrt{x}\,\sqrt{1+x^3}\right)}{}$$

$$= \frac{x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,\sqrt{x+x^4}} - \frac{\left(\left(2i-\sqrt{2}\right)\sqrt{x}\,\sqrt{1+x^3}\right)}{}$$

$$= \frac{x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,\sqrt{x+x^4}} + \frac{\left((-1)^{3/4}\left(2i-\sqrt{2}\right)\sqrt{x}\,\sqrt{\right.}{}$$

Mathematica [C] time = 1.49, size = 307, normalized size = 16.16

$$-\frac{2\sqrt{\frac{1}{x^2}-\frac{1}{x}+1}\sqrt{\frac{\frac{1}{x}+1}{1+\sqrt[3]{-1}}}\,x^2\left(\frac{i\sqrt{3}\left(\sqrt{3}x+(-1)^{5/6}+i\right)F\left(\sin^{-1}\left(\sqrt{\frac{x+(-1)^{2/3}}{\left(1+\sqrt[3]{-1}\right)x}}\right)|\sqrt[3]{-1}\right)}{x+(-1)^{2/3}} - \frac{3i\left(\sqrt{2}-i\right)\Pi\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{x+(-1)^{2/3}}{\left(1+\sqrt[3]{-1}\right)x}}\right)|\sqrt[3]{-1}\right)}{(-1)^{5/6}+\sqrt{2}} + \frac{3\left(5+i\sqrt{2}+i\sqrt{3}+\sqrt{6}\right)\Pi\left(\frac{2\sqrt{3}}{-i+2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{x+(-1)^{2/3}}{\left(1+\sqrt[3]{-1}\right)x}}\right)|\sqrt[3]{-1}\right)}{5i+2\sqrt{2}+\sqrt{3}+2i\sqrt{6}}\right)}{3\sqrt{x^4+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x + 2*x^2)/((1 + 2*x^2)*Sqrt[x + x^4]),x]
```

```
[Out] (-2*Sqrt[1 + x^(-2) - x^(-1)]*Sqrt[(1 + x^(-1))/(1 + (-1)^(1/3))]*x^2*((I*S
qrt[3]*(I + (-1)^(5/6) + Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[((-1)^(2/3) + x)/
((1 + (-1)^(1/3))*x)]], (-1)^(1/3)])/((-1)^(2/3) + x) - ((3*I)*(-I + Sqrt[2
])*EllipticPi[(2*Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[((-1)^(2/
3) + x)/((1 + (-1)^(1/3))*x)]], (-1)^(1/3)])/((-1)^(5/6) + Sqrt[2]) + (3*(5
 + I*Sqrt[2] + I*Sqrt[3] + Sqrt[6])*EllipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2]
+ Sqrt[3]), ArcSin[Sqrt[((-1)^(2/3) + x)/((1 + (-1)^(1/3))*x)]], (-1)^(1/3)
])/(5*I + 2*Sqrt[2] + Sqrt[3] + (2*I)*Sqrt[6])))/(3*Sqrt[x + x^4])
```

IntegrateAlgebraic [A] time = 1.28, size = 19, normalized size = 1.00

$$\tan^{-1}\left(\frac{2\sqrt{x^4+x}}{2x-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 2*x + 2*x^2)/((1 + 2*x^2)*Sqrt[x + x^4]),x]
[Out] ArcTan[(2*Sqrt[x + x^4])/(-1 + 2*x)]
```

fricas [A] time = 0.46, size = 17, normalized size = 0.89

$$-\arctan\left(\frac{2x-1}{2\sqrt{x^4+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-2*x-1)/(2*x^2+1)/(x^4+x)^(1/2),x, algorithm="fricas")
[Out] -arctan(1/2*(2*x - 1)/sqrt(x^4 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2-2x-1}{\sqrt{x^4+x}\,(2x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-2*x-1)/(2*x^2+1)/(x^4+x)^(1/2),x, algorithm="giac")
[Out] integrate((2*x^2 - 2*x - 1)/(sqrt(x^4 + x)*(2*x^2 + 1)), x)
```

maple [C] time = 0.08, size = 13532, normalized size = 712.21

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-2*x-1)/(2*x^2+1)/(x^4+x)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2-2x-1}{\sqrt{x^4+x}\,(2x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-2*x-1)/(2*x^2+1)/(x^4+x)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x^2 - 2*x - 1)/(sqrt(x^4 + x)*(2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int -\frac{-2x^2+2x+1}{(2x^2+1)\,\sqrt{x^4+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 2*x^2 + 1)/((2*x^2 + 1)*(x + x^4)^(1/2)),x)
```

[Out] int(-(2*x - 2*x^2 + 1)/((2*x^2 + 1)*(x + x^4)^(1/2)), x)

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2 - 2x - 1}{\sqrt{x(x+1)\left(x^2 - x + 1\right)}\left(2x^2 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-2*x-1)/(2*x**2+1)/(x**4+x)**(1/2),x)

[Out] Integral((2*x**2 - 2*x - 1)/(sqrt(x*(x + 1)*(x**2 - x + 1))*(2*x**2 + 1)), x)

3.162 $\int \frac{1+x^4}{\left(-1+x^4\right)\sqrt{-1-x^2+x^4}}\,dx$

Optimal. Leaf size=19

$$-\tan^{-1}\left(\frac{x}{\sqrt{x^4-x^2-1}}\right)$$

Rubi [A] time = 0.09, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2112, 204}

$$-\tan^{-1}\left(\frac{x}{\sqrt{x^4-x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/((-1 + x^4)*Sqrt[-1 - x^2 + x^4]),x]
```

```
[Out] -ArcTan[x/Sqrt[-1 - x^2 + x^4]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2112

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^4}{\left(-1+x^4\right)\sqrt{-1-x^2+x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \frac{x}{\sqrt{-1-x^2+x^4}}\right)\\ &= -\tan^{-1}\left(\frac{x}{\sqrt{-1-x^2+x^4}}\right)\end{aligned}$$

Mathematica [C] time = 4.99, size = 1511, normalized size = 79.53

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^4)/((-1 + x^4)*Sqrt[-1 - x^2 + x^4]),x]
```

```
[Out] ((-1/2*I)*((4*Sqrt[1 + x^2 - x^4]*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[5])
]*x], (-3 + Sqrt[5])/2])/Sqrt[1 + Sqrt[5]] - (4*Sqrt[1 + x^2 - x^4]*Ellipti
cPi[(-1 + Sqrt[5])/2, I*ArcSinh[Sqrt[2/(-1 + Sqrt[5])]*x], (-3 + Sqrt[5])/2
])/Sqrt[1 + Sqrt[5]] + ((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*
(-1 + Sqrt[5])] + (2*I)*x)^2*Sqrt[(I*(Sqrt[2*(1 + Sqrt[5])] + 2*x))/((-1 +
2*I)*Sqrt[2] + Sqrt[10] + (2*I)*Sqrt[-1 + Sqrt[5]]*x - 2*Sqrt[1 + Sqrt[5]]*
x)]*Sqrt[((-I)*(Sqrt[2*(1 + Sqrt[5])] - 2*x))/(Sqrt[2]*((-1 - 2*I) + Sqrt[5
]) + 2*(I*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)]*Sqrt[(Sqrt[2]*((-1 +
2*I) + Sqrt[5]) + 2*((-I)*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)/(Sqrt[
```

```
2]*((-1 - 2*I) + Sqrt[5]) + 2*(I*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)
]*((2 - I*Sqrt[2*(-1 + Sqrt[5])])*EllipticF[ArcSin[Sqrt[(Sqrt[2]*((-1 + 2*I
) + Sqrt[5]) + 2*((-I)*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)/(Sqrt[2]*
((-1 - 2*I) + Sqrt[5]) + 2*(I*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)]],
 -3/5 + (4*I)/5] + (2*I)*Sqrt[2*(-1 + Sqrt[5])]*EllipticPi[((-2*I + Sqrt[2*
(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]]))/((2*I + Sqrt[2
*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])), ArcSin[Sqrt[
(Sqrt[2]*((-1 + 2*I) + Sqrt[5]) + 2*((-I)*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqr
t[5]])*x)/(Sqrt[2]*((-1 - 2*I) + Sqrt[5]) + 2*(I*Sqrt[-1 + Sqrt[5]] + Sqrt[
1 + Sqrt[5]])*x)]], -3/5 + (4*I)/5]))/((1 + Sqrt[5])*(Sqrt[-1 + Sqrt[5]] +
I*Sqrt[1 + Sqrt[5]])) + ((I*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*(Sqrt[2
*(-1 + Sqrt[5])] + (2*I)*x)^2*Sqrt[(I*(Sqrt[2*(1 + Sqrt[5])] + 2*x))/((-1 +
 2*I)*Sqrt[2] + Sqrt[10] + (2*I)*Sqrt[-1 + Sqrt[5]]*x - 2*Sqrt[1 + Sqrt[5]]
*x)]*Sqrt[((-I)*(Sqrt[2*(1 + Sqrt[5])] - 2*x))/(Sqrt[2]*((-1 - 2*I) + Sqrt[
5]) + 2*(I*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)]*Sqrt[(Sqrt[2]*((-1 +
 2*I) + Sqrt[5]) + 2*((-I)*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)/(Sqrt
[2]*((-1 - 2*I) + Sqrt[5]) + 2*(I*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x
)]*((-2*I + Sqrt[2*(-1 + Sqrt[5])])*EllipticF[ArcSin[Sqrt[(Sqrt[2]*((-1 + 2
*I) + Sqrt[5]) + 2*((-I)*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)/(Sqrt[2
]*((-1 - 2*I) + Sqrt[5]) + 2*(I*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)]
], -3/5 + (4*I)/5] - 2*Sqrt[2*(-1 + Sqrt[5])]*EllipticPi[((2*I + Sqrt[2*(-1
 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]]))/((-2*I + Sqrt[2*(
-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])), ArcSin[Sqrt[(S
qrt[2]*((-1 + 2*I) + Sqrt[5]) + 2*((-I)*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[
5]])*x)/(Sqrt[2]*((-1 - 2*I) + Sqrt[5]) + 2*(I*Sqrt[-1 + Sqrt[5]] + Sqrt[1
+ Sqrt[5]])*x)]], -3/5 + (4*I)/5]))/((1 + Sqrt[5])*(Sqrt[-1 + Sqrt[5]] + I*
Sqrt[1 + Sqrt[5]]))))/(Sqrt[2]*Sqrt[-1 - x^2 + x^4])
```

IntegrateAlgebraic [A] time = 0.26, size = 19, normalized size = 1.00

$$-\tan^{-1}\left(\frac{x}{\sqrt{x^4-x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)/((-1 + x^4)*Sqrt[-1 - x^2 + x^4]),x]
[Out] -ArcTan[x/Sqrt[-1 - x^2 + x^4]]
```

fricas [A] time = 0.46, size = 30, normalized size = 1.58

$$-\frac{1}{2}\arctan\left(\frac{2\sqrt{x^4-x^2-1}\,x}{x^4-2\,x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(x^4-x^2-1)^(1/2),x, algorithm="fricas")
[Out] -1/2*arctan(2*sqrt(x^4 - x^2 - 1)*x/(x^4 - 2*x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\sqrt{x^4-x^2-1}\,\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(x^4-x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate((x^4 + 1)/(sqrt(x^4 - x^2 - 1)*(x^4 - 1)), x)
```

maple [C] time = 0.15, size = 273, normalized size = 14.37

$$\frac{2\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)x^{2}}\,\sqrt{1-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)x^{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2-2\sqrt{5}}}{2},\frac{i\sqrt{5}}{2}-\frac{i}{2}\right)}{\sqrt{-2-2\sqrt{5}}\,\sqrt{x^{4}-x^{2}-1}}-\frac{\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)x^{2}}\,\sqrt{1-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)x^{2}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}-\frac{\sqrt{5}}{2}}\,x,\frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}},\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}{\sqrt{-\frac{1}{2}-\frac{\sqrt{5}}{2}}}\right)}{\sqrt{-\frac{1}{2}-\frac{\sqrt{5}}{2}}\,\sqrt{x^{4}-x^{2}-1}}-\frac{\sqrt{1+\frac{x^{2}}{2}+\frac{\sqrt{5}\,x^{2}}{2}}\,\sqrt{1-\frac{\sqrt{5}\,x^{2}}{2}+\frac{x^{2}}{2}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}-\frac{\sqrt{5}}{2}}\,x,-\frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}},\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}{\sqrt{-\frac{1}{2}-\frac{\sqrt{5}}{2}}}\right)}{\sqrt{-\frac{1}{2}-\frac{\sqrt{5}}{2}}\,\sqrt{x^{4}-x^{2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^4-1)/(x^4-x^2-1)^(1/2),x)
```

```
[Out] 2/(-2-2*5^(1/2))^(1/2)*(1-(-1/2-1/2*5^(1/2))*x^2)^(1/2)*(1-(1/2*5^(1/2)-1/2
)*x^2)^(1/2)/(x^4-x^2-1)^(1/2)*EllipticF(1/2*x*(-2-2*5^(1/2))^(1/2),1/2*I*5
^(1/2)-1/2*I)-1/(-1/2-1/2*5^(1/2))^(1/2)*(1-(-1/2-1/2*5^(1/2))*x^2)^(1/2)*(
1-(1/2*5^(1/2)-1/2)*x^2)^(1/2)/(x^4-x^2-1)^(1/2)*EllipticPi((-1/2-1/2*5^(1/
2))^(1/2)*x,1/(-1/2-1/2*5^(1/2)),(1/2*5^(1/2)-1/2)^(1/2)/(-1/2-1/2*5^(1/2))
^(1/2))-1/(-1/2-1/2*5^(1/2))^(1/2)*(1+1/2*x^2+1/2*5^(1/2)*x^2)^(1/2)*(1-1/2
*5^(1/2)*x^2+1/2*x^2)^(1/2)/(x^4-x^2-1)^(1/2)*EllipticPi((-1/2-1/2*5^(1/2))
^(1/2)*x,-1/(-1/2-1/2*5^(1/2)),(1/2*5^(1/2)-1/2)^(1/2)/(-1/2-1/2*5^(1/2))^(
1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}+1}{\sqrt{x^{4}-x^{2}-1}\,\left(x^{4}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(x^4-x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(sqrt(x^4 - x^2 - 1)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^{4}+1}{\left(x^{4}-1\right)\,\sqrt{x^{4}-x^{2}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/((x^4 - 1)*(x^4 - x^2 - 1)^(1/2)),x)
```

```
[Out] int((x^4 + 1)/((x^4 - 1)*(x^4 - x^2 - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}+1}{\left(x-1\right)\left(x+1\right)\left(x^{2}+1\right)\sqrt{x^{4}-x^{2}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**4-1)/(x**4-x**2-1)**(1/2),x)
```

```
[Out] Integral((x**4 + 1)/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(x**4 - x**2 - 1)), x)
```

3.163 $\int \frac{x}{\sqrt[5]{1-4x+6x^2-4x^3+x^4}} \, dx$

Optimal. Leaf size=19

$$\frac{5(x-1)(x+5)}{6\sqrt[5]{(x-1)^4}}$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.74, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1680, 15, 43}

$$\frac{5(x-1)^2}{6\sqrt[5]{(x-1)^4}}+\frac{5(x-1)}{\sqrt[5]{(x-1)^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - 4*x + 6*x^2 - 4*x^3 + x^4)^(1/5),x]
```

```
[Out] (5*(-1 + x))/((-1 + x)^4)^(1/5) + (5*(-1 + x)^2)/(6*((-1 + x)^4)^(1/5))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt[5]{1-4x+6x^2-4x^3+x^4}} \, dx &= \text{Subst}\left(\int \frac{1+x}{\sqrt[5]{x^4}} \, dx, x, -1+x\right)\\
&= \frac{(-1+x)^{4/5}\,\text{Subst}\left(\int \frac{1+x}{x^{4/5}} \, dx, x, -1+x\right)}{\sqrt[5]{(-1+x)^4}}\\
&= \frac{(-1+x)^{4/5}\,\text{Subst}\left(\int \left(\frac{1}{x^{4/5}}+\sqrt[5]{x}\right) dx, x, -1+x\right)}{\sqrt[5]{(-1+x)^4}}\\
&= -\frac{5(1-x)}{\sqrt[5]{(-1+x)^4}}+\frac{5(1-x)^2}{6\sqrt[5]{(-1+x)^4}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{5(x-1)(x+5)}{6\sqrt[5]{(x-1)^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - 4*x + 6*x^2 - 4*x^3 + x^4)^(1/5),x]
[Out] (5*(-1 + x)*(5 + x))/(6*((-1 + x)^4)^(1/5))
```

IntegrateAlgebraic [A] time = 5.79, size = 19, normalized size = 1.00

$$\frac{5(x-1)(x+5)}{6\sqrt[5]{(x-1)^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(1 - 4*x + 6*x^2 - 4*x^3 + x^4)^(1/5),x]
[Out] (5*(-1 + x)*(5 + x))/(6*((-1 + x)^4)^(1/5))
```

fricas [B] time = 0.39, size = 40, normalized size = 2.11

$$\frac{5\left(x^4-4x^3+6x^2-4x+1\right)^{\frac{4}{5}}(x+5)}{6\left(x^3-3x^2+3x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-4*x^3+6*x^2-4*x+1)^(1/5),x, algorithm="fricas")
[Out] 5/6*(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)^(4/5)*(x + 5)/(x^3 - 3*x^2 + 3*x - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^4-4x^3+6x^2-4x+1\right)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-4*x^3+6*x^2-4*x+1)^(1/5),x, algorithm="giac")
[Out] integrate(x/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)^(1/5), x)
```

maple [A] time = 0.00, size = 29, normalized size = 1.53

$$\frac{5(-1+x)(5+x)}{6\left(x^4-4x^3+6x^2-4x+1\right)^{\frac{1}{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4-4*x^3+6*x^2-4*x+1)^(1/5),x)
[Out] 5/6*(-1+x)*(5+x)/(x^4-4*x^3+6*x^2-4*x+1)^(1/5)
```

maxima [A] time = 0.32, size = 15, normalized size = 0.79

$$\frac{5\left(x^2+4x-5\right)}{6(x-1)^{\frac{4}{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-4*x^3+6*x^2-4*x+1)^(1/5),x, algorithm="maxima")
```

```
[Out] 5/6*(x^2 + 4*x - 5)/(x - 1)^(4/5)
```

mupad [B] time = 0.20, size = 30, normalized size = 1.58

$$\frac{5\,\left(x+5\right)\,{\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}^{4/5}}{6\,{\left(x-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(6*x^2 - 4*x - 4*x^3 + x^4 + 1)^(1/5),x)
```

```
[Out] (5*(x + 5)*(6*x^2 - 4*x - 4*x^3 + x^4 + 1)^(4/5))/(6*(x - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[5]{\left(x-1\right)^{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4-4*x**3+6*x**2-4*x+1)**(1/5),x)
```

```
[Out] Integral(x/((x - 1)**4)**(1/5), x)
```

3.164 $\int \frac{-2+x^6}{x^4 \sqrt[4]{1+x^4+x^6}} \, dx$

Optimal. Leaf size=19

$$\frac{2\left(x^6+x^4+1\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1590}

$$\frac{2\left(x^6+x^4+1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^6)/(x^4*(1 + x^4 + x^6)^(1/4)),x]
```

```
[Out] (2*(1 + x^4 + x^6)^(3/4))/(3*x^3)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{-2+x^6}{x^4 \sqrt[4]{1+x^4+x^6}} \, dx = \frac{2\left(1+x^4+x^6\right)^{3/4}}{3x^3}$$

Mathematica [A] time = 0.06, size = 19, normalized size = 1.00

$$\frac{2\left(x^6+x^4+1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^6)/(x^4*(1 + x^4 + x^6)^(1/4)),x]
```

```
[Out] (2*(1 + x^4 + x^6)^(3/4))/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.57, size = 19, normalized size = 1.00

$$\frac{2\left(x^6+x^4+1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x^6)/(x^4*(1 + x^4 + x^6)^(1/4)),x]
```

```
[Out] (2*(1 + x^4 + x^6)^(3/4))/(3*x^3)
```

fricas [A] time = 0.41, size = 15, normalized size = 0.79

$$\frac{2\left(x^6+x^4+1\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)/x^4/(x^6+x^4+1)^(1/4),x, algorithm="fricas")
[Out] 2/3*(x^6 + x^4 + 1)^(3/4)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6-2}{\left(x^6+x^4+1\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)/x^4/(x^6+x^4+1)^(1/4),x, algorithm="giac")
[Out] integrate((x^6 - 2)/((x^6 + x^4 + 1)^(1/4)*x^4), x)
```

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$\frac{2\left(x^6+x^4+1\right)^{\frac{3}{4}}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-2)/x^4/(x^6+x^4+1)^(1/4),x)
[Out] 2/3*(x^6+x^4+1)^(3/4)/x^3
```

maxima [A] time = 0.58, size = 15, normalized size = 0.79

$$\frac{2\left(x^6+x^4+1\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)/x^4/(x^6+x^4+1)^(1/4),x, algorithm="maxima")
[Out] 2/3*(x^6 + x^4 + 1)^(3/4)/x^3
```

mupad [B] time = 0.14, size = 15, normalized size = 0.79

$$\frac{2\left(x^6+x^4+1\right)^{3/4}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 2)/(x^4*(x^4 + x^6 + 1)^(1/4)),x)
[Out] (2*(x^4 + x^6 + 1)^(3/4))/(3*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6-2}{x^4\sqrt[4]{x^6+x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-2)/x**4/(x**6+x**4+1)**(1/4),x)
[Out] Integral((x**6 - 2)/(x**4*(x**6 + x**4 + 1)**(1/4)), x)
```

3.165 $\int \frac{1+x^8}{x^4 \sqrt[4]{-1+x^4+x^8}} \, dx$

Optimal. Leaf size=19

$$\frac{\left(x^8+x^4-1\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1590}

$$\frac{\left(x^8+x^4-1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^8)/(x^4*(-1 + x^4 + x^8)^(1/4)),x]
```

```
[Out] (-1 + x^4 + x^8)^(3/4)/(3*x^3)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{1+x^8}{x^4 \sqrt[4]{-1+x^4+x^8}} \, dx = \frac{\left(-1+x^4+x^8\right)^{3/4}}{3x^3}$$

Mathematica [A] time = 0.04, size = 19, normalized size = 1.00

$$\frac{\left(x^8+x^4-1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^8)/(x^4*(-1 + x^4 + x^8)^(1/4)),x]
```

```
[Out] (-1 + x^4 + x^8)^(3/4)/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.47, size = 19, normalized size = 1.00

$$\frac{\left(x^8+x^4-1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^8)/(x^4*(-1 + x^4 + x^8)^(1/4)),x]
```

```
[Out] (-1 + x^4 + x^8)^(3/4)/(3*x^3)
```

fricas [A] time = 0.39, size = 15, normalized size = 0.79

$$\frac{\left(x^8+x^4-1\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)/x^4/(x^8+x^4-1)^(1/4),x, algorithm="fricas")
[Out] 1/3*(x^8 + x^4 - 1)^(3/4)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8+1}{\left(x^8+x^4-1\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)/x^4/(x^8+x^4-1)^(1/4),x, algorithm="giac")
[Out] integrate((x^8 + 1)/((x^8 + x^4 - 1)^(1/4)*x^4), x)
```

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$\frac{\left(x^8+x^4-1\right)^{\frac{3}{4}}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8+1)/x^4/(x^8+x^4-1)^(1/4),x)
[Out] 1/3*(x^8+x^4-1)^(3/4)/x^3
```

maxima [A] time = 0.53, size = 15, normalized size = 0.79

$$\frac{\left(x^8+x^4-1\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)/x^4/(x^8+x^4-1)^(1/4),x, algorithm="maxima")
[Out] 1/3*(x^8 + x^4 - 1)^(3/4)/x^3
```

mupad [B] time = 0.22, size = 15, normalized size = 0.79

$$\frac{\left(x^8+x^4-1\right)^{3/4}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 + 1)/(x^4*(x^4 + x^8 - 1)^(1/4)),x)
[Out] (x^4 + x^8 - 1)^(3/4)/(3*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8+1}{x^4\sqrt[4]{x^8+x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8+1)/x**4/(x**8+x**4-1)**(1/4),x)
[Out] Integral((x**8 + 1)/(x**4*(x**8 + x**4 - 1)**(1/4)), x)
```

3.166 $\int \frac{-1+x^8}{x^4 \sqrt[4]{1+x^4+x^8}} \, dx$

Optimal. Leaf size=19

$$\frac{\left(x^8+x^4+1\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1590}

$$\frac{\left(x^8+x^4+1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^8)/(x^4*(1 + x^4 + x^8)^(1/4)),x]
```

```
[Out] (1 + x^4 + x^8)^(3/4)/(3*x^3)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{-1+x^8}{x^4 \sqrt[4]{1+x^4+x^8}} \, dx = \frac{\left(1+x^4+x^8\right)^{3/4}}{3x^3}$$

Mathematica [A] time = 0.05, size = 19, normalized size = 1.00

$$\frac{\left(x^8+x^4+1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^8)/(x^4*(1 + x^4 + x^8)^(1/4)),x]
```

```
[Out] (1 + x^4 + x^8)^(3/4)/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.47, size = 19, normalized size = 1.00

$$\frac{\left(x^8+x^4+1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^8)/(x^4*(1 + x^4 + x^8)^(1/4)),x]
```

```
[Out] (1 + x^4 + x^8)^(3/4)/(3*x^3)
```

fricas [A] time = 0.41, size = 15, normalized size = 0.79

$$\frac{\left(x^8+x^4+1\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/x^4/(x^8+x^4+1)^(1/4),x, algorithm="fricas")
[Out] 1/3*(x^8 + x^4 + 1)^(3/4)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8-1}{\left(x^8+x^4+1\right)^{\frac{1}{4}} x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/x^4/(x^8+x^4+1)^(1/4),x, algorithm="giac")
[Out] integrate((x^8 - 1)/((x^8 + x^4 + 1)^(1/4)*x^4), x)
```

maple [B] time = 0.01, size = 40, normalized size = 2.11

$$\frac{\left(x^2+x+1\right)\left(x^2-x+1\right)\left(x^4-x^2+1\right)}{3\left(x^8+x^4+1\right)^{\frac{1}{4}} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-1)/x^4/(x^8+x^4+1)^(1/4),x)
[Out] 1/3*(x^2+x+1)*(x^2-x+1)*(x^4-x^2+1)/(x^8+x^4+1)^(1/4)/x^3
```

maxima [B] time = 0.56, size = 43, normalized size = 2.26

$$\frac{x^8+x^4+1}{3\left(x^4-x^2+1\right)^{\frac{1}{4}}\left(x^2+x+1\right)^{\frac{1}{4}}\left(x^2-x+1\right)^{\frac{1}{4}} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/x^4/(x^8+x^4+1)^(1/4),x, algorithm="maxima")
[Out] 1/3*(x^8 + x^4 + 1)/((x^4 - x^2 + 1)^(1/4)*(x^2 + x + 1)^(1/4)*(x^2 - x + 1
)^(1/4)*x^3)
```

mupad [B] time = 0.15, size = 15, normalized size = 0.79

$$\frac{\left(x^8+x^4+1\right)^{3/4}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 - 1)/(x^4*(x^4 + x^8 + 1)^(1/4)),x)
[Out] (x^4 + x^8 + 1)^(3/4)/(3*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2+1\right)\left(x^4+1\right)}{x^4\sqrt[4]{\left(x^2-x+1\right)\left(x^2+x+1\right)\left(x^4-x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-1)/x**4/(x**8+x**4+1)**(1/4),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)*(x**4 + 1)/(x**4*((x**2 - x + 1)*(x**2
+ x + 1)*(x**4 - x**2 + 1))**(1/4)), x)
```

3.167 $\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt{-x+x^3}}\,dx$

Optimal. Leaf size=20

$$-\frac{2\sqrt{x^3-x}}{x^2-1}$$

Rubi [A] time = 0.08, antiderivative size = 14, normalized size of antiderivative = 0.70, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2056, 449}

$$-\frac{2x}{\sqrt{x^3-x}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/((-1 + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] (-2*x)/Sqrt[-x + x^3]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt{-x+x^3}}\,dx &= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1+x^2}{\sqrt{x}\left(-1+x^2\right)^{3/2}}\,dx}{\sqrt{-x+x^3}}\\ &= -\frac{2x}{\sqrt{-x+x^3}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 14, normalized size = 0.70

$$-\frac{2x}{\sqrt{x\left(x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/((-1 + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] (-2*x)/Sqrt[x*(-1 + x^2)]
```

IntegrateAlgebraic [A] time = 0.12, size = 20, normalized size = 1.00

$$-\frac{2\sqrt{x^3-x}}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 + x^2)*Sqrt[-x + x^3]),x]
[Out] (-2*Sqrt[-x + x^3])/(-1 + x^2)
```

fricas [A] time = 0.40, size = 18, normalized size = 0.90

$$-\frac{2\sqrt{x^3-x}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^3-x)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(x^3 - x)/(x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x^3-x}\,(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^3-x)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 + 1)/(sqrt(x^3 - x)*(x^2 - 1)), x)
```

maple [A] time = 0.01, size = 13, normalized size = 0.65

$$-\frac{2x}{\sqrt{x^3-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-1)/(x^3-x)^(1/2),x)
[Out] -2*x/(x^3-x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x^3-x}\,(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^3-x)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/(sqrt(x^3 - x)*(x^2 - 1)), x)
```

mupad [B] time = 0.07, size = 12, normalized size = 0.60

$$-\frac{2\,x}{\sqrt{x^3-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((x^3 - x)^(1/2)*(x^2 - 1)),x)
[Out] -(2*x)/(x^3 - x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x\,(x-1)\,(x+1)}\,(x-1)\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-1)/(x**3-x)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)/(sqrt(x*(x - 1)*(x + 1))*(x - 1)*(x + 1)), x)
```

3.168 $\int \frac{1}{\left(1+x^2\right)\sqrt[3]{x+x^3}}\,dx$

Optimal. Leaf size=20

$$\frac{3\left(x^3+x\right)^{2/3}}{2\left(x^2+1\right)}$$

Rubi [A] time = 0.03, antiderivative size = 14, normalized size of antiderivative = 0.70, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2056, 264}

$$\frac{3x}{2\sqrt[3]{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*(x + x^3)^(1/3)),x]
```

```
[Out] (3*x)/(2*(x + x^3)^(1/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+x^2\right)\sqrt[3]{x+x^3}}\,dx &= \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\int \frac{1}{\sqrt[3]{x}\left(1+x^2\right)^{4/3}}\,dx}{\sqrt[3]{x+x^3}}\\ &= \frac{3x}{2\sqrt[3]{x+x^3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.70

$$\frac{3x}{2\sqrt[3]{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(x + x^3)^(1/3)),x]
```

```
[Out] (3*x)/(2*(x + x^3)^(1/3))
```

IntegrateAlgebraic [A] time = 0.11, size = 20, normalized size = 1.00

$$\frac{3\left(x^3+x\right)^{2/3}}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x^2)*(x + x^3)^(1/3)),x]
```

```
[Out] (3*(x + x^3)^(2/3))/(2*(1 + x^2))
```

fricas [A] time = 0.40, size = 16, normalized size = 0.80

$$\frac{3\left(x^3+x\right)^{\frac{2}{3}}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^3+x)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*(x^3 + x)^(2/3)/(x^2 + 1)
```

giac [A] time = 0.30, size = 9, normalized size = 0.45

$$\frac{3}{2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^3+x)^(1/3),x, algorithm="giac")
```

```
[Out] 3/2/(1/x^2 + 1)^(1/3)
```

maple [A] time = 0.00, size = 11, normalized size = 0.55

$$\frac{3x}{2\left(x^3+x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(x^3+x)^(1/3),x)
```

```
[Out] 3/2*x/(x^3+x)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3\left(x^3+x\right)}{4\left(x^{\frac{7}{3}}+x^{\frac{1}{3}}\right)\left(x^2+1\right)^{\frac{1}{3}}}+\int\frac{3\left(x^2+1\right)^{\frac{2}{3}}}{2\left(x^{\frac{13}{3}}+2\,x^{\frac{7}{3}}+x^{\frac{1}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^3+x)^(1/3),x, algorithm="maxima")
```

```
[Out] -3/4*(x^3 + x)/((x^(7/3) + x^(1/3))*(x^2 + 1)^(1/3)) + integrate(3/2*(x^2 +
 1)^(2/3)/(x^(13/3) + 2*x^(7/3) + x^(1/3)), x)
```

mupad [B] time = 0.15, size = 18, normalized size = 0.90

$$\frac{3\left(x^3+x\right)^{2/3}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(x + x^3)^(1/3)),x)
```

```
[Out] (3*(x + x^3)^(2/3))/(2*(x^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x\left(x^2+1\right)}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(x**3+x)**(1/3),x)
```

```
[Out] Integral(1/((x*(x**2 + 1))**(1/3)*(x**2 + 1)), x)
```

3.169 $\int \frac{1}{x\sqrt[3]{-x^2+x^3}}\,dx$

Optimal. Leaf size=20

$$\frac{3\left(x^3-x^2\right)^{2/3}}{2x^2}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2014}

$$\frac{3\left(x^3-x^2\right)^{2/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (3*(-x^2 + x^3)^(2/3))/(2*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-x^2+x^3}}\,dx=\frac{3\left(-x^2+x^3\right)^{2/3}}{2x^2}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.90

$$\frac{3(x-1)}{2\sqrt[3]{(x-1)x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (3*(-1 + x))/(2*((-1 + x)*x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.14, size = 20, normalized size = 1.00

$$\frac{3\left(x^3-x^2\right)^{2/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (3*(-x^2 + x^3)^(2/3))/(2*x^2)
```

fricas [A] time = 0.38, size = 16, normalized size = 0.80

$$\frac{3\left(x^3-x^2\right)^{\frac{2}{3}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3-x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*(x^3 - x^2)^(2/3)/x^2
```

giac [A] time = 0.28, size = 11, normalized size = 0.55

$$\frac{3}{2}\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3-x^2)^(1/3),x, algorithm="giac")
```

```
[Out] 3/2*(-1/x + 1)^(2/3)
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{-\frac{3}{2}+\frac{3x}{2}}{\left(x^{3}-x^{2}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^3-x^2)^(1/3),x)
```

```
[Out] 3/2*(-1+x)/(x^3-x^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{3}-x^{2}\right)^{\frac{1}{3}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3-x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 - x^2)^(1/3)*x), x)
```

mupad [B] time = 0.20, size = 16, normalized size = 0.80

$$\frac{3\left(x^{3}-x^{2}\right)^{2/3}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^3 - x^2)^(1/3)),x)
```

```
[Out] (3*(x^3 - x^2)^(2/3))/(2*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{x^{2}\left(x-1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**3-x**2)**(1/3),x)
```

```
[Out] Integral(1/(x*(x**2*(x - 1))**(1/3)), x)
```

3.170 $\int \frac{x^2}{\sqrt{-b+ax^3}}\,dx$

Optimal. Leaf size=20

$$\frac{2\sqrt{ax^3-b}}{3a}$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {261}

$$\frac{2\sqrt{ax^3-b}}{3a}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[-b + a*x^3],x]

[Out] (2*Sqrt[-b + a*x^3])/(3*a)

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{-b+ax^3}}\,dx = \frac{2\sqrt{-b+ax^3}}{3a}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{2\sqrt{ax^3-b}}{3a}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[-b + a*x^3],x]

[Out] (2*Sqrt[-b + a*x^3])/(3*a)

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 1.00

$$\frac{2\sqrt{ax^3-b}}{3a}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/Sqrt[-b + a*x^3],x]

[Out] (2*Sqrt[-b + a*x^3])/(3*a)

fricas [A] time = 0.39, size = 16, normalized size = 0.80

$$\frac{2\,\sqrt{ax^3-b}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^3-b)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(a*x^3 - b)/a
```

giac [A] time = 0.28, size = 16, normalized size = 0.80

$$\frac{2\sqrt{ax^3 - b}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^3-b)^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt(a*x^3 - b)/a
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{2\sqrt{a\,x^3 - b}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^3-b)^(1/2),x)
[Out] 2/3*(a*x^3-b)^(1/2)/a
```

maxima [A] time = 0.42, size = 16, normalized size = 0.80

$$\frac{2\sqrt{ax^3 - b}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^3-b)^(1/2),x, algorithm="maxima")
[Out] 2/3*sqrt(a*x^3 - b)/a
```

mupad [B] time = 0.30, size = 16, normalized size = 0.80

$$\frac{2\sqrt{a\,x^3 - b}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^3 - b)^(1/2),x)
[Out] (2*(a*x^3 - b)^(1/2))/(3*a)
```

sympy [A] time = 0.36, size = 26, normalized size = 1.30

$$\begin{cases} \frac{2\sqrt{ax^3-b}}{3a} & \text{for } a \neq 0 \\ \frac{x^3}{3\sqrt{-b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**3-b)**(1/2),x)
[Out] Piecewise((2*sqrt(a*x**3 - b)/(3*a), Ne(a, 0)), (x**3/(3*sqrt(-b)), True))
```

3.171 $\int x^2\sqrt{-b+ax^3}\,dx$

Optimal. Leaf size=20

$$\frac{2\left(ax^3-b\right)^{3/2}}{9a}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {261}

$$\frac{2\left(ax^3-b\right)^{3/2}}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[-b + a*x^3],x]
```

```
[Out] (2*(-b + a*x^3)^(3/2))/(9*a)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2\sqrt{-b+ax^3}\,dx=\frac{2\left(-b+ax^3\right)^{3/2}}{9a}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{2\left(ax^3-b\right)^{3/2}}{9a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[-b + a*x^3],x]
```

```
[Out] (2*(-b + a*x^3)^(3/2))/(9*a)
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 1.00

$$\frac{2\left(ax^3-b\right)^{3/2}}{9a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[-b + a*x^3],x]
```

```
[Out] (2*(-b + a*x^3)^(3/2))/(9*a)
```

fricas [A] time = 0.39, size = 16, normalized size = 0.80

$$\frac{2\left(ax^3-b\right)^{\frac{3}{2}}}{9\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^3-b)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(a*x^3 - b)^(3/2)/a
```

giac [A] time = 0.19, size = 16, normalized size = 0.80

$$\frac{2\left(ax^3-b\right)^{\frac{3}{2}}}{9\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^3-b)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*(a*x^3 - b)^(3/2)/a
```

maple [A] time = 0.01, size = 17, normalized size = 0.85

$$\frac{2\left(a\,x^3-b\right)^{\frac{3}{2}}}{9a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^3-b)^(1/2),x)
```

```
[Out] 2/9*(a*x^3-b)^(3/2)/a
```

maxima [A] time = 0.55, size = 16, normalized size = 0.80

$$\frac{2\left(ax^3-b\right)^{\frac{3}{2}}}{9\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^3-b)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(a*x^3 - b)^(3/2)/a
```

mupad [B] time = 0.24, size = 16, normalized size = 0.80

$$\frac{2\left(a\,x^3-b\right)^{3/2}}{9\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^3 - b)^(1/2),x)
```

```
[Out] (2*(a*x^3 - b)^(3/2))/(9*a)
```

sympy [A] time = 0.18, size = 44, normalized size = 2.20

$$\begin{cases} \frac{2x^3\sqrt{ax^3-b}}{9} - \frac{2b\sqrt{ax^3-b}}{9a} & \text{for}\ a \neq 0 \\ \frac{x^3\sqrt{-b}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x**3-b)**(1/2),x)
```

```
[Out] Piecewise((2*x**3*sqrt(a*x**3 - b)/9 - 2*b*sqrt(a*x**3 - b)/(9*a), Ne(a, 0)
), (x**3*sqrt(-b)/3, True))
```

3.172 $\int \frac{\sqrt{-b+ax^3}\left(2b+ax^3\right)}{x^4}\,dx$

Optimal. Leaf size=20

$$\frac{2\left(ax^3-b\right)^{3/2}}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 74}

$$\frac{2\left(ax^3-b\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-b + a*x^3]*(2*b + a*x^3))/x^4,x]
```

```
[Out] (2*(-b + a*x^3)^(3/2))/(3*x^3)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-b+ax^3}\left(2b+ax^3\right)}{x^4}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{-b+ax}\,(2b+ax)}{x^2}\,dx,x,x^3\right)\\ &= \frac{2\left(-b+ax^3\right)^{3/2}}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{2\left(ax^3-b\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-b + a*x^3]*(2*b + a*x^3))/x^4,x]
```

```
[Out] (2*(-b + a*x^3)^(3/2))/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.05, size = 20, normalized size = 1.00

$$\frac{2\left(ax^3-b\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-b + a*x^3]*(2*b + a*x^3))/x^4,x]
```

```
[Out] (2*(-b + a*x^3)^(3/2))/(3*x^3)
```

fricas [A] time = 0.40, size = 16, normalized size = 0.80

$$\frac{2\left(ax^{3}-b\right)^{\frac{3}{2}}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(1/2)*(a*x^3+2*b)/x^4,x, algorithm="fricas")
```

```
[Out] 2/3*(a*x^3 - b)^(3/2)/x^3
```

giac [B] time = 0.61, size = 39, normalized size = 1.95

$$\frac{2\left(\sqrt{ax^{3}-b}\,a^{2}-\frac{\sqrt{ax^{3}-b}\,ab}{x^{3}}\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(1/2)*(a*x^3+2*b)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(sqrt(a*x^3 - b)*a^2 - sqrt(a*x^3 - b)*a*b/x^3)/a
```

maple [A] time = 0.01, size = 17, normalized size = 0.85

$$\frac{2\left(a\,x^{3}-b\right)^{\frac{3}{2}}}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b)^(1/2)*(a*x^3+2*b)/x^4,x)
```

```
[Out] 2/3*(a*x^3-b)^(3/2)/x^3
```

maxima [B] time = 0.73, size = 79, normalized size = 3.95

$$-\frac{2}{3}\left(\sqrt{b}\,\arctan\left(\frac{\sqrt{ax^{3}-b}}{\sqrt{b}}\right)-\sqrt{ax^{3}-b}\right)a+\frac{2}{3}\left(\frac{a\arctan\left(\frac{\sqrt{ax^{3}-b}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{\sqrt{ax^{3}-b}}{x^{3}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(1/2)*(a*x^3+2*b)/x^4,x, algorithm="maxima")
```

```
[Out] -2/3*(sqrt(b)*arctan(sqrt(a*x^3 - b)/sqrt(b)) - sqrt(a*x^3 - b))*a + 2/3*(a
*arctan(sqrt(a*x^3 - b)/sqrt(b))/sqrt(b) - sqrt(a*x^3 - b)/x^3)*b
```

mupad [B] time = 0.24, size = 16, normalized size = 0.80

$$\frac{2\left(a\,x^{3}-b\right)^{3/2}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^3 - b)^(1/2)*(2*b + a*x^3))/x^4,x)
```

[Out] (2*(a*x^3 - b)^(3/2))/(3*x^3)

sympy [C] time = 22.40, size = 309, normalized size = 15.45

$$a\left(\begin{cases} -\frac{2i\sqrt{a}x^{\frac{3}{2}}}{3\sqrt{-1+\frac{b}{ax^3}}} - \frac{2i\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{b}}{\sqrt{a}x^{\frac{3}{2}}}\right)}{3} + \frac{2ib}{3\sqrt{a}x^{\frac{3}{2}}\sqrt{-1+\frac{b}{ax^3}}} & \text{for } \left|\frac{b}{ax^3}\right| > 1 \\ \frac{2\sqrt{a}x^{\frac{3}{2}}}{3\sqrt{1-\frac{b}{ax^3}}} + \frac{2\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{b}}{\sqrt{a}x^{\frac{3}{2}}}\right)}{3} - \frac{2b}{3\sqrt{a}x^{\frac{3}{2}}\sqrt{1-\frac{b}{ax^3}}} & \text{otherwise} \end{cases}\right) + 2b\left(\begin{cases} \frac{i\sqrt{a}}{3x^{\frac{3}{2}}\sqrt{-1+\frac{b}{ax^3}}} + \frac{ia\operatorname{acosh}\left(\frac{\sqrt{b}}{\sqrt{a}x^{\frac{3}{2}}}\right)}{3\sqrt{b}} - \frac{ib}{3\sqrt{a}x^{\frac{9}{2}}\sqrt{-1+\frac{b}{ax^3}}} & \text{for } \left|\frac{b}{ax^3}\right| > 1 \\ -\frac{\sqrt{a}\sqrt{1-\frac{b}{ax^3}}}{3x^{\frac{3}{2}}} - \frac{a\operatorname{asin}\left(\frac{\sqrt{b}}{\sqrt{a}x^{\frac{3}{2}}}\right)}{3\sqrt{b}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b)**(1/2)*(a*x**3+2*b)/x**4,x)
```

```
[Out] a*Piecewise((-2*I*sqrt(a)*x**(3/2)/(3*sqrt(-1 + b/(a*x**3))) - 2*I*sqrt(b)*
acosh(sqrt(b)/(sqrt(a)*x**(3/2)))/3 + 2*I*b/(3*sqrt(a)*x**(3/2)*sqrt(-1 + b
/(a*x**3))), Abs(b/(a*x**3)) > 1), (2*sqrt(a)*x**(3/2)/(3*sqrt(1 - b/(a*x**
3))) + 2*sqrt(b)*asin(sqrt(b)/(sqrt(a)*x**(3/2)))/3 - 2*b/(3*sqrt(a)*x**(3/
2)*sqrt(1 - b/(a*x**3))), True)) + 2*b*Piecewise((I*sqrt(a)/(3*x**(3/2)*sqr
t(-1 + b/(a*x**3))) + I*a*acosh(sqrt(b)/(sqrt(a)*x**(3/2)))/(3*sqrt(b)) - I
*b/(3*sqrt(a)*x**(9/2)*sqrt(-1 + b/(a*x**3))), Abs(b/(a*x**3)) > 1), (-sqrt
(a)*sqrt(1 - b/(a*x**3))/(3*x**(3/2)) - a*asin(sqrt(b)/(sqrt(a)*x**(3/2)))/
(3*sqrt(b)), True))
```

3.173 $\int \frac{1}{x^2 \sqrt[4]{-x^2+x^4}} \, dx$

Optimal. Leaf size=20

$$\frac{2\left(x^4-x^2\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2014}

$$\frac{2\left(x^4-x^2\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] (2*(-x^2 + x^4)^(3/4))/(3*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2 \sqrt[4]{-x^2+x^4}} \, dx = \frac{2\left(-x^2+x^4\right)^{3/4}}{3x^3}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{2\left(x^2\left(x^2-1\right)\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] (2*(x^2*(-1 + x^2))^(3/4))/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.12, size = 20, normalized size = 1.00

$$\frac{2\left(x^4-x^2\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] (2*(-x^2 + x^4)^(3/4))/(3*x^3)
```

fricas [A] time = 0.40, size = 16, normalized size = 0.80

$$\frac{2\left(x^4-x^2\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4-x^2)^(1/4),x, algorithm="fricas")
[Out] 2/3*(x^4 - x^2)^(3/4)/x^3
```

giac [A] time = 0.36, size = 11, normalized size = 0.55

$$-\frac{2}{3}\left(-\frac{1}{x^2}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4-x^2)^(1/4),x, algorithm="giac")
[Out] -2/3*(-1/x^2 + 1)^(3/4)
```

maple [A] time = 0.00, size = 23, normalized size = 1.15

$$\frac{2(-1+x)(1+x)}{3x\left(x^4-x^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^4-x^2)^(1/4),x)
[Out] 2/3/x*(-1+x)*(1+x)/(x^4-x^2)^(1/4)
```

maxima [A] time = 0.61, size = 22, normalized size = 1.10

$$\frac{2\left(x^3-x\right)}{3(x+1)^{\frac{1}{4}}(x-1)^{\frac{1}{4}}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4-x^2)^(1/4),x, algorithm="maxima")
[Out] 2/3*(x^3 - x)/((x + 1)^(1/4)*(x - 1)^(1/4)*x^(5/2))
```

mupad [B] time = 0.37, size = 16, normalized size = 0.80

$$\frac{2\left(x^4-x^2\right)^{3/4}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4 - x^2)^(1/4)),x)
[Out] (2*(x^4 - x^2)^(3/4))/(3*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt[4]{x^2(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**4-x**2)**(1/4),x)
[Out] Integral(1/(x**2*(x**2*(x - 1)*(x + 1))**(1/4)), x)
```

3.174 $\int \frac{\sqrt[4]{-x^2+x^4}}{x^4} \, dx$

Optimal. Leaf size=20

$$\frac{2(x^4-x^2)^{5/4}}{5x^5}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2014}

$$\frac{2(x^4-x^2)^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + x^4)^(1/4)/x^4,x]
```

```
[Out] (2*(-x^2 + x^4)^(5/4))/(5*x^5)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[4]{-x^2+x^4}}{x^4} \, dx = \frac{2(-x^2+x^4)^{5/4}}{5x^5}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{2(x^2(x^2-1))^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + x^4)^(1/4)/x^4,x]
```

```
[Out] (2*(x^2*(-1 + x^2))^(5/4))/(5*x^5)
```

IntegrateAlgebraic [A] time = 0.11, size = 20, normalized size = 1.00

$$\frac{2(x^4-x^2)^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^2 + x^4)^(1/4)/x^4,x]
```

```
[Out] (2*(-x^2 + x^4)^(5/4))/(5*x^5)
```

fricas [A] time = 0.42, size = 21, normalized size = 1.05

$$\frac{2(x^4-x^2)^{\frac{1}{4}}(x^2-1)}{5\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2)^(1/4)/x^4,x, algorithm="fricas")
[Out] 2/5*(x^4 - x^2)^(1/4)*(x^2 - 1)/x^3
```

giac [A] time = 0.29, size = 11, normalized size = 0.55

$$-\frac{2}{5}\left(-\frac{1}{x^2}+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2)^(1/4)/x^4,x, algorithm="giac")
[Out] -2/5*(-1/x^2 + 1)^(5/4)
```

maple [A] time = 0.00, size = 23, normalized size = 1.15

$$\frac{2\left(-1+x\right)\left(1+x\right)\left(x^4-x^2\right)^{\frac{1}{4}}}{5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^2)^(1/4)/x^4,x)
[Out] 2/5/x^3*(-1+x)*(1+x)*(x^4-x^2)^(1/4)
```

maxima [A] time = 0.42, size = 22, normalized size = 1.10

$$\frac{2\left(x^3-x\right)(x+1)^{\frac{1}{4}}(x-1)^{\frac{1}{4}}}{5\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2)^(1/4)/x^4,x, algorithm="maxima")
[Out] 2/5*(x^3 - x)*(x + 1)^(1/4)*(x - 1)^(1/4)/x^(7/2)
```

mupad [B] time = 0.18, size = 33, normalized size = 1.65

$$\frac{2\left(x^4-x^2\right)^{1/4}}{5\,x}-\frac{2\left(x^4-x^2\right)^{1/4}}{5\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x^2)^(1/4)/x^4,x)
[Out] (2*(x^4 - x^2)^(1/4))/(5*x) - (2*(x^4 - x^2)^(1/4))/(5*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[4]{x^2\,(x-1)\,(x+1)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**2)**(1/4)/x**4,x)
[Out] Integral((x**2*(x - 1)*(x + 1))**(1/4)/x**4, x)
```

3.175 $\int \frac{(1+x^2)\sqrt[3]{-x^2+x^4}}{x^3}\,dx$

Optimal. Leaf size=20

$$\frac{3\left(x^4-x^2\right)^{4/3}}{4x^4}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1590}

$$\frac{3\left(x^4-x^2\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(-x^2 + x^4)^(1/3))/x^3,x]
```

```
[Out] (3*(-x^2 + x^4)^(4/3))/(4*x^4)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\left(1+x^2\right)\sqrt[3]{-x^2+x^4}}{x^3}\,dx=\frac{3\left(-x^2+x^4\right)^{4/3}}{4x^4}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{3\left(x^2\left(x^2-1\right)\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(-x^2 + x^4)^(1/3))/x^3,x]
```

```
[Out] (3*(x^2*(-1 + x^2))^(4/3))/(4*x^4)
```

IntegrateAlgebraic [A] time = 0.08, size = 20, normalized size = 1.00

$$\frac{3\left(x^4-x^2\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^2)*(-x^2 + x^4)^(1/3))/x^3,x]
```

```
[Out] (3*(-x^2 + x^4)^(4/3))/(4*x^4)
```

fricas [A] time = 0.40, size = 21, normalized size = 1.05

$$\frac{3\left(x^4-x^2\right)^{\frac{1}{3}}\left(x^2-1\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4-x^2)^(1/3)/x^3,x, algorithm="fricas")
[Out] 3/4*(x^4 - x^2)^(1/3)*(x^2 - 1)/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^4-x^2\right)^{\frac{1}{3}}\left(x^2+1\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4-x^2)^(1/3)/x^3,x, algorithm="giac")
[Out] integrate((x^4 - x^2)^(1/3)*(x^2 + 1)/x^3, x)
```

maple [A] time = 0.01, size = 23, normalized size = 1.15

$$\frac{3\left(-1+x\right)\left(1+x\right)\left(x^4-x^2\right)^{\frac{1}{3}}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4-x^2)^(1/3)/x^3,x)
[Out] 3/4/x^2*(-1+x)*(1+x)*(x^4-x^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^4-x^2\right)^{\frac{1}{3}}\left(x^2+1\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4-x^2)^(1/3)/x^3,x, algorithm="maxima")
[Out] integrate((x^4 - x^2)^(1/3)*(x^2 + 1)/x^3, x)
```

mupad [B] time = 0.21, size = 21, normalized size = 1.05

$$\frac{3\,\left(x^2-1\right)\,{\left(x^4-x^2\right)}^{1/3}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*(x^4 - x^2)^(1/3))/x^3,x)
[Out] (3*(x^2 - 1)*(x^4 - x^2)^(1/3))/(4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[3]{x^2\left(x-1\right)\left(x+1\right)}\,\left(x^2+1\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4-x**2)**(1/3)/x**3,x)
[Out] Integral((x**2*(x - 1)*(x + 1))**(1/3)*(x**2 + 1)/x**3, x)
```

3.176 $\int \frac{1}{x\sqrt[4]{-x^3+x^4}}\,dx$

Optimal. Leaf size=20

$$\frac{4\left(x^4-x^3\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2014}

$$\frac{4\left(x^4-x^3\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (4*(-x^3 + x^4)^(3/4))/(3*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt[4]{-x^3+x^4}}\,dx = \frac{4\left(-x^3+x^4\right)^{3/4}}{3x^3}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.90

$$\frac{4(x-1)}{3\sqrt[4]{(x-1)x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (4*(-1 + x))/(3*((-1 + x)*x^3)^(1/4))
```

IntegrateAlgebraic [A] time = 0.22, size = 20, normalized size = 1.00

$$\frac{4\left(x^4-x^3\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (4*(-x^3 + x^4)^(3/4))/(3*x^3)
```

fricas [A] time = 0.41, size = 16, normalized size = 0.80

$$\frac{4\left(x^4-x^3\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-x^3)^(1/4),x, algorithm="fricas")
[Out] 4/3*(x^4 - x^3)^(3/4)/x^3
```

giac [A] time = 0.67, size = 11, normalized size = 0.55

$$-\frac{4}{3}\left(-\frac{1}{x}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-x^3)^(1/4),x, algorithm="giac")
[Out] -4/3*(-1/x + 1)^(3/4)
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{-\frac{4}{3}+\frac{4x}{3}}{\left(x^4-x^3\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4-x^3)^(1/4),x)
[Out] 4/3*(-1+x)/(x^4-x^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4-x^3\right)^{\frac{1}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-x^3)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((x^4 - x^3)^(1/4)*x), x)
```

mupad [B] time = 0.30, size = 16, normalized size = 0.80

$$\frac{4\left(x^4-x^3\right)^{3/4}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^4 - x^3)^(1/4)),x)
[Out] (4*(x^4 - x^3)^(3/4))/(3*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[4]{x^3\,(x-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4-x**3)**(1/4),x)
[Out] Integral(1/(x*(x**3*(x - 1))**(1/4)), x)
```

3.177 $\int \frac{(-1+x)\sqrt[4]{-x^3+x^4}}{x^4}\,dx$

Optimal. Leaf size=20

$$\frac{4\left(x^4-x^3\right)^{9/4}}{9x^9}$$

Rubi [B] time = 0.13, antiderivative size = 41, normalized size of antiderivative = 2.05, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2052, 2016, 2014}

$$\frac{4\left(x^4-x^3\right)^{5/4}}{9x^5}-\frac{4\left(x^4-x^3\right)^{5/4}}{9x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)*(-x^3 + x^4)^(1/4))/x^4,x]
```

```
[Out] (-4*(-x^3 + x^4)^(5/4))/(9*x^6) + (4*(-x^3 + x^4)^(5/4))/(9*x^5)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x)\sqrt[4]{-x^3+x^4}}{x^4}\,dx &= \int \left(-\frac{\sqrt[4]{-x^3+x^4}}{x^4}+\frac{\sqrt[4]{-x^3+x^4}}{x^3}\right)dx\\
&= -\int \frac{\sqrt[4]{-x^3+x^4}}{x^4}\,dx+\int \frac{\sqrt[4]{-x^3+x^4}}{x^3}\,dx\\
&= -\frac{4\left(-x^3+x^4\right)^{5/4}}{9x^6}+\frac{4\left(-x^3+x^4\right)^{5/4}}{5x^5}-\frac{4}{9}\int \frac{\sqrt[4]{-x^3+x^4}}{x^3}\,dx\\
&= -\frac{4\left(-x^3+x^4\right)^{5/4}}{9x^6}+\frac{4\left(-x^3+x^4\right)^{5/4}}{9x^5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.90

$$\frac{4\left((x-1)x^3\right)^{9/4}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)*(-x^3 + x^4)^(1/4))/x^4,x]
[Out] (4*((-1 + x)*x^3)^(9/4))/(9*x^9)
```

IntegrateAlgebraic [A] time = 0.21, size = 20, normalized size = 1.00

$$\frac{4\left(x^4-x^3\right)^{9/4}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)*(-x^3 + x^4)^(1/4))/x^4,x]
[Out] (4*(-x^3 + x^4)^(9/4))/(9*x^9)
```

fricas [A] time = 0.40, size = 24, normalized size = 1.20

$$\frac{4\left(x^4-x^3\right)^{\frac{1}{4}}\left(x^2-2x+1\right)}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(x^4-x^3)^(1/4)/x^4,x, algorithm="fricas")
[Out] 4/9*(x^4 - x^3)^(1/4)*(x^2 - 2*x + 1)/x^3
```

giac [A] time = 0.23, size = 18, normalized size = 0.90

$$-\frac{4}{9}\left(\frac{1}{x}-1\right)^2\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(x^4-x^3)^(1/4)/x^4,x, algorithm="giac")
[Out] -4/9*(1/x - 1)^2*(-1/x + 1)^(1/4)
```

maple [A] time = 0.00, size = 22, normalized size = 1.10

$$\frac{4\left(-1+x\right)^2\left(x^4-x^3\right)^{\frac{1}{4}}}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)*(x^4-x^3)^(1/4)/x^4,x)
[Out] 4/9/x^3*(-1+x)^2*(x^4-x^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^4-x^3\right)^{\frac{1}{4}}(x-1)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(x^4-x^3)^(1/4)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^3)^(1/4)*(x - 1)/x^4, x)
```

mupad [B] time = 0.25, size = 49, normalized size = 2.45

$$\frac{4\,x^2\,{\left(x^4-x^3\right)}^{1/4}-8\,x\,{\left(x^4-x^3\right)}^{1/4}+4\,{\left(x^4-x^3\right)}^{1/4}}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - x^3)^(1/4)*(x - 1))/x^4,x)
```

```
[Out] (4*x^2*(x^4 - x^3)^(1/4) - 8*x*(x^4 - x^3)^(1/4) + 4*(x^4 - x^3)^(1/4))/(9*
x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\,(x-1)}\,\,(x-1)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(x**4-x**3)**(1/4)/x**4,x)
```

```
[Out] Integral((x**3*(x - 1))**(1/4)*(x - 1)/x**4, x)
```

3.178 $\int \frac{\sqrt[4]{-x^3+x^4}}{x^3}\,dx$

Optimal. Leaf size=20

$$\frac{4\left(x^4-x^3\right)^{5/4}}{5x^5}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2014}

$$\frac{4\left(x^4-x^3\right)^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + x^4)^(1/4)/x^3,x]
```

```
[Out] (4*(-x^3 + x^4)^(5/4))/(5*x^5)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[4]{-x^3+x^4}}{x^3}\,dx=\frac{4\left(-x^3+x^4\right)^{5/4}}{5x^5}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.90

$$\frac{4\left((x-1)x^3\right)^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + x^4)^(1/4)/x^3,x]
```

```
[Out] (4*((-1 + x)*x^3)^(5/4))/(5*x^5)
```

IntegrateAlgebraic [A] time = 0.19, size = 20, normalized size = 1.00

$$\frac{4\left(x^4-x^3\right)^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^3 + x^4)^(1/4)/x^3,x]
```

```
[Out] (4*(-x^3 + x^4)^(5/4))/(5*x^5)
```

fricas [A] time = 0.40, size = 19, normalized size = 0.95

$$\frac{4\left(x^4-x^3\right)^{\frac{1}{4}}(x-1)}{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x^3,x, algorithm="fricas")
[Out] 4/5*(x^4 - x^3)^(1/4)*(x - 1)/x^2
```

giac [A] time = 0.25, size = 11, normalized size = 0.55

$$-\frac{4}{5}\left(-\frac{1}{x}+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x^3,x, algorithm="giac")
[Out] -4/5*(-1/x + 1)^(5/4)
```

maple [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{4\left(-1+x\right)\left(x^{4}-x^{3}\right)^{\frac{1}{4}}}{5x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3)^(1/4)/x^3,x)
[Out] 4/5/x^2*(-1+x)*(x^4-x^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}-x^{3}\right)^{\frac{1}{4}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x^3,x, algorithm="maxima")
[Out] integrate((x^4 - x^3)^(1/4)/x^3, x)
```

mupad [B] time = 0.17, size = 33, normalized size = 1.65

$$\frac{4\,x\,{\left(x^4-x^3\right)}^{1/4}-4\,{\left(x^4-x^3\right)}^{1/4}}{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x^3)^(1/4)/x^3,x)
[Out] (4*x*(x^4 - x^3)^(1/4) - 4*(x^4 - x^3)^(1/4))/(5*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^{3}\left(x-1\right)}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3)**(1/4)/x**3,x)
[Out] Integral((x**3*(x - 1))**(1/4)/x**3, x)
```

3.179 $\int x\left(1+2x^2\right)\sqrt{-1+2x^2+2x^4}\,dx$

Optimal. Leaf size=20

$$\frac{1}{6}\left(2x^4+2x^2-1\right)^{3/2}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1247, 629}

$$\frac{1}{6}\left(2x^4+2x^2-1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + 2*x^2)*Sqrt[-1 + 2*x^2 + 2*x^4],x]
```

```
[Out] (-1 + 2*x^2 + 2*x^4)^(3/2)/6
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(1+2x^2\right)\sqrt{-1+2x^2+2x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int (1+2x)\sqrt{-1+2x+2x^2}\,dx,x,x^2\right)\\ &= \frac{1}{6}\left(-1+2x^2+2x^4\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{1}{6}\left(2x^4+2x^2-1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + 2*x^2)*Sqrt[-1 + 2*x^2 + 2*x^4],x]
```

```
[Out] (-1 + 2*x^2 + 2*x^4)^(3/2)/6
```

IntegrateAlgebraic [A] time = 0.27, size = 20, normalized size = 1.00

$$\frac{1}{6}\left(2x^4+2x^2-1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(1 + 2*x^2)*Sqrt[-1 + 2*x^2 + 2*x^4],x]
```

```
[Out] (-1 + 2*x^2 + 2*x^4)^(3/2)/6
```

fricas [**A**] time = 0.39, size = 16, normalized size = 0.80

$$\frac{1}{6}\left(2x^4+2x^2-1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2+1)*(2*x^4+2*x^2-1)^(1/2),x, algorithm="fricas")
[Out] 1/6*(2*x^4 + 2*x^2 - 1)^(3/2)
```

giac [**A**] time = 0.26, size = 16, normalized size = 0.80

$$\frac{1}{6}\left(2x^4+2x^2-1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2+1)*(2*x^4+2*x^2-1)^(1/2),x, algorithm="giac")
[Out] 1/6*(2*x^4 + 2*x^2 - 1)^(3/2)
```

maple [**A**] time = 0.00, size = 17, normalized size = 0.85

$$\frac{\left(2x^4+2x^2-1\right)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^2+1)*(2*x^4+2*x^2-1)^(1/2),x)
[Out] 1/6*(2*x^4+2*x^2-1)^(3/2)
```

maxima [**A**] time = 0.55, size = 16, normalized size = 0.80

$$\frac{1}{6}\left(2x^4+2x^2-1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2+1)*(2*x^4+2*x^2-1)^(1/2),x, algorithm="maxima")
[Out] 1/6*(2*x^4 + 2*x^2 - 1)^(3/2)
```

mupad [**B**] time = 0.17, size = 16, normalized size = 0.80

$$\frac{\left(2x^4+2x^2-1\right)^{3/2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^2 + 1)*(2*x^2 + 2*x^4 - 1)^(1/2),x)
[Out] (2*x^2 + 2*x^4 - 1)^(3/2)/6
```

sympy [**B**] time = 0.19, size = 56, normalized size = 2.80

$$\frac{x^4\sqrt{2x^4+2x^2-1}}{3}+\frac{x^2\sqrt{2x^4+2x^2-1}}{3}-\frac{\sqrt{2x^4+2x^2-1}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**2+1)*(2*x**4+2*x**2-1)**(1/2),x)
[Out] x**4*sqrt(2*x**4 + 2*x**2 - 1)/3 + x**2*sqrt(2*x**4 + 2*x**2 - 1)/3 - sqrt(
2*x**4 + 2*x**2 - 1)/6
```

3.180 $\int \frac{(1+2x^3)\sqrt[3]{-x^2+x^5}}{x^3}\,dx$

Optimal. Leaf size=20

$$\frac{3\left(x^5-x^2\right)^{4/3}}{4x^4}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1590}

$$\frac{3\left(x^5-x^2\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x^3)*(-x^2 + x^5)^(1/3))/x^3,x]
[Out] (3*(-x^2 + x^5)^(4/3))/(4*x^4)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\left(1+2x^3\right)\sqrt[3]{-x^2+x^5}}{x^3}\,dx=\frac{3\left(-x^2+x^5\right)^{4/3}}{4x^4}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{3\left(x^2\left(x^3-1\right)\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x^3)*(-x^2 + x^5)^(1/3))/x^3,x]
[Out] (3*(x^2*(-1 + x^3))^(4/3))/(4*x^4)
```

IntegrateAlgebraic [A] time = 0.11, size = 20, normalized size = 1.00

$$\frac{3\left(x^5-x^2\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + 2*x^3)*(-x^2 + x^5)^(1/3))/x^3,x]
[Out] (3*(-x^2 + x^5)^(4/3))/(4*x^4)
```

fricas [A] time = 0.40, size = 21, normalized size = 1.05

$$\frac{3\left(x^5-x^2\right)^{\frac{1}{3}}\left(x^3-1\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)*(x^5-x^2)^(1/3)/x^3,x, algorithm="fricas")
[Out] 3/4*(x^5 - x^2)^(1/3)*(x^3 - 1)/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5-x^2\right)^{\frac{1}{3}}\left(2\,x^3+1\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)*(x^5-x^2)^(1/3)/x^3,x, algorithm="giac")
[Out] integrate((x^5 - x^2)^(1/3)*(2*x^3 + 1)/x^3, x)
```

maple [A] time = 0.01, size = 26, normalized size = 1.30

$$\frac{3\left(-1+x\right)\left(x^2+x+1\right)\left(x^5-x^2\right)^{\frac{1}{3}}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+1)*(x^5-x^2)^(1/3)/x^3,x)
[Out] 3/4/x^2*(-1+x)*(x^2+x+1)*(x^5-x^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5-x^2\right)^{\frac{1}{3}}\left(2\,x^3+1\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)*(x^5-x^2)^(1/3)/x^3,x, algorithm="maxima")
[Out] integrate((x^5 - x^2)^(1/3)*(2*x^3 + 1)/x^3, x)
```

mupad [B] time = 0.22, size = 21, normalized size = 1.05

$$\frac{3\,\left(x^3-1\right)\,\left(x^5-x^2\right)^{1/3}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 - x^2)^(1/3)*(2*x^3 + 1))/x^3,x)
[Out] (3*(x^3 - 1)*(x^5 - x^2)^(1/3))/(4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x^2\left(x-1\right)\left(x^2+x+1\right)}\left(2x^3+1\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+1)*(x**5-x**2)**(1/3)/x**3,x)
[Out] Integral((x**2*(x - 1)*(x**2 + x + 1))**(1/3)*(2*x**3 + 1)/x**3, x)
```

3.181 $\int \frac{-1+x^4}{x^2 \sqrt[4]{-x^3+x^5}} \, dx$

Optimal. Leaf size=20

$$\frac{4 \left(x^5-x^3\right)^{7/4}}{7 x^7}$$

Rubi [B] time = 0.15, antiderivative size = 41, normalized size of antiderivative = 2.05, number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2052, 2025, 2011, 365, 364, 2024}

$$\frac{4 \left(x^5-x^3\right)^{3/4}}{7 x^2}-\frac{4 \left(x^5-x^3\right)^{3/4}}{7 x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)/(x^2*(-x^3 + x^5)^(1/4)),x]
```

```
[Out] (-4*(-x^3 + x^5)^(3/4))/(7*x^4) + (4*(-x^3 + x^5)^(3/4))/(7*x^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^4}{x^2\sqrt[4]{-x^3+x^5}}\,dx &= \int \left(-\frac{1}{x^2\sqrt[4]{-x^3+x^5}}+\frac{x^2}{\sqrt[4]{-x^3+x^5}}\right)dx\\ &= -\int \frac{1}{x^2\sqrt[4]{-x^3+x^5}}\,dx+\int \frac{x^2}{\sqrt[4]{-x^3+x^5}}\,dx\\ &= -\frac{4\left(-x^3+x^5\right)^{3/4}}{7x^4}+\frac{4\left(-x^3+x^5\right)^{3/4}}{7x^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$\frac{4\left(x^3\left(x^2-1\right)\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)/(x^2*(-x^3 + x^5)^(1/4)),x]
[Out] (4*(x^3*(-1 + x^2))^(7/4))/(7*x^7)
```

IntegrateAlgebraic [A] time = 0.17, size = 20, normalized size = 1.00

$$\frac{4\left(x^5-x^3\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/(x^2*(-x^3 + x^5)^(1/4)),x]
[Out] (4*(-x^3 + x^5)^(7/4))/(7*x^7)
```

fricas [A] time = 0.41, size = 21, normalized size = 1.05

$$\frac{4\left(x^5-x^3\right)^{\frac{3}{4}}\left(x^2-1\right)}{7\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^5-x^3)^(1/4),x, algorithm="fricas")
[Out] 4/7*(x^5 - x^3)^(3/4)*(x^2 - 1)/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\left(x^5-x^3\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^5-x^3)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 1)/((x^5 - x^3)^(1/4)*x^2), x)
```

maple [A] time = 0.00, size = 28, normalized size = 1.40

$$\frac{4\left(x^{2}-1\right)\left(-1+x\right)\left(1+x\right)}{7\left(x^{5}-x^{3}\right)^{\frac{1}{4}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/x^2/(x^5-x^3)^(1/4),x)
```

```
[Out] 4/7*(x^2-1)*(-1+x)*(1+x)/(x^5-x^3)^(1/4)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}-1}{\left(x^{5}-x^{3}\right)^{\frac{1}{4}} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^5-x^3)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 1)/((x^5 - x^3)^(1/4)*x^2), x)
```

mupad [B] time = 0.28, size = 35, normalized size = 1.75

$$\frac{4\, x^{2} \left(x^{5}-x^{3}\right)^{3/4}-4\left(x^{5}-x^{3}\right)^{3/4}}{7\, x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/(x^2*(x^5 - x^3)^(1/4)),x)
```

```
[Out] (4*x^2*(x^5 - x^3)^(3/4) - 4*(x^5 - x^3)^(3/4))/(7*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-1\right)\left(x+1\right)\left(x^{2}+1\right)}{x^{2} \sqrt[4]{x^{3}\left(x-1\right)\left(x+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/x**2/(x**5-x**3)**(1/4),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)/(x**2*(x**3*(x - 1)*(x + 1))**(1/4)), x
)
```

3.182 $\int \frac{\left(-1+x^4\right)\sqrt[4]{-x^3+x^5}}{x^4}\,dx$

Optimal. Leaf size=20

$$\frac{4\left(x^5-x^3\right)^{9/4}}{9x^9}$$

Rubi [B] time = 0.20, antiderivative size = 59, normalized size of antiderivative = 2.95, number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2052, 2004, 2032, 365, 364, 2020, 2025}

$$\frac{4}{9}\sqrt[4]{x^5-x^3}\,x-\frac{8\sqrt[4]{x^5-x^3}}{9x}+\frac{4\sqrt[4]{x^5-x^3}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4)*(-x^3 + x^5)^(1/4))/x^4,x]
```

```
[Out] (4*(-x^3 + x^5)^(1/4))/(9*x^3) - (8*(-x^3 + x^5)^(1/4))/(9*x) + (4*x*(-x^3
+ x^5)^(1/4))/9
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^4)\sqrt[4]{-x^3+x^5}}{x^4}\,dx &= \int \left(\sqrt[4]{-x^3+x^5}-\frac{\sqrt[4]{-x^3+x^5}}{x^4}\right)dx \\
&= \int \sqrt[4]{-x^3+x^5}\,dx-\int \frac{\sqrt[4]{-x^3+x^5}}{x^4}\,dx \\
&= \frac{4\sqrt[4]{-x^3+x^5}}{9x^3}+\frac{4}{9}x\sqrt[4]{-x^3+x^5}-\frac{2}{9}\int \frac{x}{(-x^3+x^5)^{3/4}}\,dx-\frac{2}{9}\int \frac{x^3}{(-x^3+x^5)^{3/4}}\,dx \\
&= \frac{4\sqrt[4]{-x^3+x^5}}{9x^3}-\frac{8\sqrt[4]{-x^3+x^5}}{9x}+\frac{4}{9}x\sqrt[4]{-x^3+x^5}+\frac{2}{9}\int \frac{x^3}{(-x^3+x^5)^{3/4}}\,dx-\frac{\left(2x^{9/4}\left(-1\right.\right.}{} \\
&= \frac{4\sqrt[4]{-x^3+x^5}}{9x^3}-\frac{8\sqrt[4]{-x^3+x^5}}{9x}+\frac{4}{9}x\sqrt[4]{-x^3+x^5}-\frac{\left(2x^{9/4}(1-x^2)^{3/4}\right)\int \frac{x^{3/4}}{(1-x^2)^{3/4}}\,dx}{9(-x^3+x^5)^{3/4}}+ \\
&= \frac{4\sqrt[4]{-x^3+x^5}}{9x^3}-\frac{8\sqrt[4]{-x^3+x^5}}{9x}+\frac{4}{9}x\sqrt[4]{-x^3+x^5}-\frac{8x^4(1-x^2)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{7}{8};\frac{15}{8};x^2\right)}{63(-x^3+x^5)^{3/4}}+ \\
&= \frac{4\sqrt[4]{-x^3+x^5}}{9x^3}-\frac{8\sqrt[4]{-x^3+x^5}}{9x}+\frac{4}{9}x\sqrt[4]{-x^3+x^5}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 65, normalized size = 3.25

$$\frac{4\sqrt[4]{x^3(x^2-1)}\left(7\,{}_2F_1\left(-\frac{9}{8},-\frac{1}{4};-\frac{1}{8};x^2\right)+9x^4\,{}_2F_1\left(-\frac{1}{4},\frac{7}{8};\frac{15}{8};x^2\right)\right)}{63x^3\sqrt[4]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)*(-x^3 + x^5)^(1/4))/x^4,x]
```

```
[Out] (4*(x^3*(-1 + x^2))^(1/4)*(7*Hypergeometric2F1[-9/8, -1/4, -1/8, x^2] + 9*x
^4*Hypergeometric2F1[-1/4, 7/8, 15/8, x^2]))/(63*x^3*(1 - x^2)^(1/4))
```

IntegrateAlgebraic [A] time = 0.12, size = 20, normalized size = 1.00

$$\frac{4\left(x^5-x^3\right)^{9/4}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(-x^3 + x^5)^(1/4))/x^4,x]
[Out] (4*(-x^3 + x^5)^(9/4))/(9*x^9)
```

fricas [A] time = 0.41, size = 26, normalized size = 1.30

$$\frac{4\left(x^{5}-x^{3}\right)^{\frac{1}{4}}\left(x^{4}-2\,x^{2}+1\right)}{9\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^5-x^3)^(1/4)/x^4,x, algorithm="fricas")
[Out] 4/9*(x^5 - x^3)^(1/4)*(x^4 - 2*x^2 + 1)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{5}-x^{3}\right)^{\frac{1}{4}}\left(x^{4}-1\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^5-x^3)^(1/4)/x^4,x, algorithm="giac")
[Out] integrate((x^5 - x^3)^(1/4)*(x^4 - 1)/x^4, x)
```

maple [A] time = 0.00, size = 28, normalized size = 1.40

$$\frac{4\left(x^{5}-x^{3}\right)^{\frac{1}{4}}\left(x^{2}-1\right)(-1+x)(1+x)}{9x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^5-x^3)^(1/4)/x^4,x)
[Out] 4/9*(x^5-x^3)^(1/4)*(x^2-1)*(-1+x)*(1+x)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{5}-x^{3}\right)^{\frac{1}{4}}\left(x^{4}-1\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^5-x^3)^(1/4)/x^4,x, algorithm="maxima")
[Out] integrate((x^5 - x^3)^(1/4)*(x^4 - 1)/x^4, x)
```

mupad [B] time = 0.26, size = 47, normalized size = 2.35

$$\frac{4\left(x^{5}-x^{3}\right)^{1/4}}{9\,x^{3}}-\frac{8\left(x^{5}-x^{3}\right)^{1/4}}{9\,x}+\frac{4\,x\left(x^{5}-x^{3}\right)^{1/4}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(x^5 - x^3)^(1/4))/x^4,x)
[Out] (4*(x^5 - x^3)^(1/4))/(9*x^3) - (8*(x^5 - x^3)^(1/4))/(9*x) + (4*x*(x^5 - x
^3)^(1/4))/9
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3 (x-1)(x+1)}\,(x-1)(x+1)\left(x^2+1\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**5-x**3)**(1/4)/x**4,x)
```

```
[Out] Integral((x**3*(x - 1)*(x + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)/x**4, x)
```

3.183 $\int \frac{-1-2x^2+2x^4}{x^2(1+x^2)\sqrt{1+x^6}}\,dx$

Optimal. Leaf size=20

$$\frac{\sqrt{x^6+1}}{x\left(x^2+1\right)}$$

Rubi [F] time = 0.59, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1-2x^2+2x^4}{x^2\left(1+x^2\right)\sqrt{1+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 - 2*x^2 + 2*x^4)/(x^2*(1 + x^2)*Sqrt[1 + x^6]),x]
```

```
[Out] Sqrt[1 + x^6]/x - ((1 + Sqrt[3])*x*Sqrt[1 + x^6])/(1 + (1 + Sqrt[3])*x^2) +
 (3^(1/4)*x*(1 + x^2)*Sqrt[(1 - x^2 + x^4)/(1 + (1 + Sqrt[3])*x^2)^2]*Ellip
ticE[ArcCos[(1 + (1 - Sqrt[3])*x^2)/(1 + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])
/4])/(Sqrt[(x^2*(1 + x^2))/(1 + (1 + Sqrt[3])*x^2)^2]*Sqrt[1 + x^6]) + (x*(
1 + x^2)*Sqrt[(1 - x^2 + x^4)/(1 + (1 + Sqrt[3])*x^2)^2]*EllipticF[ArcCos[(
1 + (1 - Sqrt[3])*x^2)/(1 + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(3^(1/4)
*Sqrt[(x^2*(1 + x^2))/(1 + (1 + Sqrt[3])*x^2)^2]*Sqrt[1 + x^6]) + ((1 - Sqr
t[3])*x*(1 + x^2)*Sqrt[(1 - x^2 + x^4)/(1 + (1 + Sqrt[3])*x^2)^2]*EllipticF
[ArcCos[(1 + (1 - Sqrt[3])*x^2)/(1 + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])
/(2*3^(1/4)*Sqrt[(x^2*(1 + x^2))/(1 + (1 + Sqrt[3])*x^2)^2]*Sqrt[1 + x^6])
- ((3*I)/2)*Defer[Int][1/((I - x)*Sqrt[1 + x^6]), x] - ((3*I)/2)*Defer[Int]
[1/((I + x)*Sqrt[1 + x^6]), x]
```

Rubi steps

$$\int \frac{-1-2x^2+2x^4}{x^2\left(1+x^2\right)\sqrt{1+x^6}}\,dx = \int \left(\frac{2}{\sqrt{1+x^6}}-\frac{1}{x^2\sqrt{1+x^6}}-\frac{3}{\left(1+x^2\right)\sqrt{1+x^6}}\right)dx$$

$$= 2\int \frac{1}{\sqrt{1+x^6}}\,dx-3\int \frac{1}{\left(1+x^2\right)\sqrt{1+x^6}}\,dx-\int \frac{1}{x^2\sqrt{1+x^6}}\,dx$$

$$= \frac{\sqrt{1+x^6}}{x}+\frac{x\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x^2}{1+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x^2\left(1+x^2\right)}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1+x^6}}-2\int$$

$$= \frac{\sqrt{1+x^6}}{x}+\frac{x\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x^2}{1+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x^2\left(1+x^2\right)}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1+x^6}}-\frac{3}{2}i\int$$

$$= \frac{\sqrt{1+x^6}}{x}-\frac{\left(1+\sqrt{3}\right)x\sqrt{1+x^6}}{1+\left(1+\sqrt{3}\right)x^2}+\frac{\sqrt[4]{3}\,x\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\,E\left(\cos^{-1}\left(\frac{1+(1-\sqrt{}}{1+(1+\sqrt{}}\right.\right.}{\sqrt{\frac{x^2\left(1+x^2\right)}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1+x^6}}$$

Mathematica [A] time = 0.11, size = 17, normalized size = 0.85

$$\frac{\sqrt{x^6+1}}{x^3+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x^2 + 2*x^4)/(x^2*(1 + x^2)*Sqrt[1 + x^6]),x]
```

```
[Out] Sqrt[1 + x^6]/(x + x^3)
```

IntegrateAlgebraic [A] time = 10.70, size = 20, normalized size = 1.00

$$\frac{\sqrt{x^6+1}}{x\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 2*x^2 + 2*x^4)/(x^2*(1 + x^2)*Sqrt[1 + x^6]),x]
```

```
[Out] Sqrt[1 + x^6]/(x*(1 + x^2))
```

fricas [A] time = 0.41, size = 15, normalized size = 0.75

$$\frac{\sqrt{x^6+1}}{x^3+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x^2-1)/x^2/(x^2+1)/(x^6+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^6 + 1)/(x^3 + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^4-2\,x^2-1}{\sqrt{x^6+1}\left(x^2+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x^2-1)/x^2/(x^2+1)/(x^6+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*x^4 - 2*x^2 - 1)/(sqrt(x^6 + 1)*(x^2 + 1)*x^2), x)
```

maple [A] time = 0.01, size = 22, normalized size = 1.10

$$\frac{x^4-x^2+1}{x\sqrt{x^6+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-2*x^2-1)/x^2/(x^2+1)/(x^6+1)^(1/2),x)
```

```
[Out] 1/x/(x^6+1)^(1/2)*(x^4-x^2+1)
```

maxima [A] time = 0.82, size = 23, normalized size = 1.15

$$\frac{\sqrt{x^4-x^2+1}}{\sqrt{x^2+1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x^2-1)/x^2/(x^2+1)/(x^6+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^4 - x^2 + 1)/(sqrt(x^2 + 1)*x)
```

mupad [B] time = 0.17, size = 18, normalized size = 0.90

$$\frac{\sqrt{x^6+1}}{x\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 2*x^4 + 1)/(x^2*(x^2 + 1)*(x^6 + 1)^(1/2)),x)
```

```
[Out] (x^6 + 1)^(1/2)/(x*(x^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^4-2x^2-1}{x^2\sqrt{\left(x^2+1\right)\left(x^4-x^2+1\right)}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-2*x**2-1)/x**2/(x**2+1)/(x**6+1)**(1/2),x)
```

```
[Out] Integral((2*x**4 - 2*x**2 - 1)/(x**2*sqrt((x**2 + 1)*(x**4 - x**2 + 1))*(x*
*2 + 1)), x)
```

3.184 $\int \frac{1}{x^3 \sqrt[3]{-x^2+x^6}} dx$

Optimal. Leaf size=20

$$\frac{3 (x^6 - x^2)^{2/3}}{8x^4}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2014}

$$\frac{3 (x^6 - x^2)^{2/3}}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-x^2 + x^6)^(1/3)),x]
```

```
[Out] (3*(-x^2 + x^6)^(2/3))/(8*x^4)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3 \sqrt[3]{-x^2 + x^6}} dx = \frac{3 (-x^2 + x^6)^{2/3}}{8x^4}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{3 (x^2 (x^4 - 1))^{2/3}}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-x^2 + x^6)^(1/3)),x]
```

```
[Out] (3*(x^2*(-1 + x^4))^(2/3))/(8*x^4)
```

IntegrateAlgebraic [A] time = 0.36, size = 20, normalized size = 1.00

$$\frac{3 (x^6 - x^2)^{2/3}}{8x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(-x^2 + x^6)^(1/3)),x]
```

```
[Out] (3*(-x^2 + x^6)^(2/3))/(8*x^4)
```

fricas [A] time = 0.39, size = 16, normalized size = 0.80

$$\frac{3 (x^6 - x^2)^{\frac{2}{3}}}{8 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6-x^2)^(1/3),x, algorithm="fricas")
[Out] 3/8*(x^6 - x^2)^(2/3)/x^4
```

giac [A] time = 0.43, size = 11, normalized size = 0.55

$$\frac{3}{8}\left(-\frac{1}{x^4}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6-x^2)^(1/3),x, algorithm="giac")
[Out] 3/8*(-1/x^4 + 1)^(2/3)
```

maple [A] time = 0.01, size = 28, normalized size = 1.40

$$\frac{3\left(-1+x\right)\left(1+x\right)\left(x^2+1\right)}{8x^2\left(x^6-x^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^6-x^2)^(1/3),x)
[Out] 3/8/x^2*(-1+x)*(1+x)*(x^2+1)/(x^6-x^2)^(1/3)
```

maxima [A] time = 0.41, size = 30, normalized size = 1.50

$$\frac{3\left(x^6-x^2\right)}{8\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-1\right)^{\frac{1}{3}}\left(x^2\right)^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6-x^2)^(1/3),x, algorithm="maxima")
[Out] 3/8*(x^6 - x^2)/((x^2 + 1)^(1/3)*(x^2 - 1)^(1/3)*(x^2)^(7/3))
```

mupad [B] time = 0.24, size = 16, normalized size = 0.80

$$\frac{3\left(x^6-x^2\right)^{2/3}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^6 - x^2)^(1/3)),x)
[Out] (3*(x^6 - x^2)^(2/3))/(8*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt[3]{x^2\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**6-x**2)**(1/3),x)
[Out] Integral(1/(x**3*(x**2*(x - 1)*(x + 1)*(x**2 + 1))**(1/3)), x)
```

3.185 $\int \frac{1+x^4}{x^2 \sqrt[4]{-x^2+x^6}}\,dx$

Optimal. Leaf size=20

$$\frac{2\left(x^6-x^2\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1590}

$$\frac{2\left(x^6-x^2\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(x^2*(-x^2 + x^6)^(1/4)),x]
```

```
[Out] (2*(-x^2 + x^6)^(3/4))/(3*x^3)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{1+x^4}{x^2 \sqrt[4]{-x^2+x^6}}\,dx=\frac{2\left(-x^2+x^6\right)^{3/4}}{3x^3}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{2\left(x^2\left(x^4-1\right)\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(x^2*(-x^2 + x^6)^(1/4)),x]
```

```
[Out] (2*(x^2*(-1 + x^4))^(3/4))/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.25, size = 20, normalized size = 1.00

$$\frac{2\left(x^6-x^2\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)/(x^2*(-x^2 + x^6)^(1/4)),x]
```

```
[Out] (2*(-x^2 + x^6)^(3/4))/(3*x^3)
```

fricas [A] time = 0.42, size = 16, normalized size = 0.80

$$\frac{2\left(x^6-x^2\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x^2/(x^6-x^2)^(1/4),x, algorithm="fricas")
[Out] 2/3*(x^6 - x^2)^(3/4)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^6-x^2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x^2/(x^6-x^2)^(1/4),x, algorithm="giac")
[Out] integrate((x^4 + 1)/((x^6 - x^2)^(1/4)*x^2), x)
```

maple [A] time = 0.01, size = 28, normalized size = 1.40

$$\frac{2\,(-1+x)\,(1+x)\left(x^2+1\right)}{3x\left(x^6-x^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/x^2/(x^6-x^2)^(1/4),x)
[Out] 2/3/x*(-1+x)*(1+x)*(x^2+1)/(x^6-x^2)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^6-x^2\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x^2/(x^6-x^2)^(1/4),x, algorithm="maxima")
[Out] integrate((x^4 + 1)/((x^6 - x^2)^(1/4)*x^2), x)
```

mupad [B] time = 0.21, size = 16, normalized size = 0.80

$$\frac{2\left(x^6-x^2\right)^{3/4}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^2*(x^6 - x^2)^(1/4)),x)
[Out] (2*(x^6 - x^2)^(3/4))/(3*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{x^2\sqrt[4]{x^2\,(x-1)\,(x+1)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/x**2/(x**6-x**2)**(1/4),x)
```

```
[Out] Integral((x**4 + 1)/(x**2*(x**2*(x - 1)*(x + 1)*(x**2 + 1))**(1/4)), x)
```

3.186 $\int \frac{(1+x^4)\sqrt[4]{-x^2+x^6}}{x^4} \, dx$

Optimal. Leaf size=20

$$\frac{2(x^6-x^2)^{5/4}}{5x^5}$$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1590}

$$\frac{2(x^6-x^2)^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^4)*(-x^2 + x^6)^(1/4))/x^4,x]
```

```
[Out] (2*(-x^2 + x^6)^(5/4))/(5*x^5)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(1+x^4)\sqrt[4]{-x^2+x^6}}{x^4} \, dx = \frac{2(-x^2+x^6)^{5/4}}{5x^5}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{2(x^2(x^4-1))^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^4)*(-x^2 + x^6)^(1/4))/x^4,x]
```

```
[Out] (2*(x^2*(-1 + x^4))^(5/4))/(5*x^5)
```

IntegrateAlgebraic [A] time = 0.25, size = 20, normalized size = 1.00

$$\frac{2(x^6-x^2)^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^4)*(-x^2 + x^6)^(1/4))/x^4,x]
```

```
[Out] (2*(-x^2 + x^6)^(5/4))/(5*x^5)
```

fricas [A] time = 0.41, size = 21, normalized size = 1.05

$$\frac{2\left(x^{6}-x^{2}\right)^{\frac{1}{4}}\left(x^{4}-1\right)}{5\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^6-x^2)^(1/4)/x^4,x, algorithm="fricas")
[Out] 2/5*(x^6 - x^2)^(1/4)*(x^4 - 1)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}-x^{2}\right)^{\frac{1}{4}}\left(x^{4}+1\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^6-x^2)^(1/4)/x^4,x, algorithm="giac")
[Out] integrate((x^6 - x^2)^(1/4)*(x^4 + 1)/x^4, x)
```

maple [A] time = 0.00, size = 28, normalized size = 1.40

$$\frac{2\left(-1+x\right)\left(1+x\right)\left(x^{2}+1\right)\left(x^{6}-x^{2}\right)^{\frac{1}{4}}}{5x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)*(x^6-x^2)^(1/4)/x^4,x)
[Out] 2/5/x^3*(-1+x)*(1+x)*(x^2+1)*(x^6-x^2)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}-x^{2}\right)^{\frac{1}{4}}\left(x^{4}+1\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^6-x^2)^(1/4)/x^4,x, algorithm="maxima")
[Out] integrate((x^6 - x^2)^(1/4)*(x^4 + 1)/x^4, x)
```

mupad [B] time = 0.21, size = 21, normalized size = 1.05

$$\frac{2\,\left(x^4-1\right)\,{\left(x^6-x^2\right)}^{1/4}}{5\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)*(x^6 - x^2)^(1/4))/x^4,x)
[Out] (2*(x^4 - 1)*(x^6 - x^2)^(1/4))/(5*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^{2}\left(x-1\right)\left(x+1\right)\left(x^{2}+1\right)}\,\left(x^{4}+1\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)*(x**6-x**2)**(1/4)/x**4,x)
[Out] Integral((x**2*(x - 1)*(x + 1)*(x**2 + 1))**(1/4)*(x**4 + 1)/x**4, x)
```

3.187 $\int \frac{-1+x^8}{x^4 \sqrt[4]{-x^2+x^6}} \, dx$

Optimal. Leaf size=20

$$\frac{2\left(x^6-x^2\right)^{7/4}}{7x^7}$$

Rubi [B] time = 0.16, antiderivative size = 41, normalized size of antiderivative = 2.05, number of steps used = 12, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2052, 2025, 2011, 329, 246, 245, 2024}

$$\frac{2\left(x^6-x^2\right)^{3/4}}{7x} - \frac{2\left(x^6-x^2\right)^{3/4}}{7x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^8)/(x^4*(-x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*(-x^2 + x^6)^(3/4))/(7*x^5) + (2*(-x^2 + x^6)^(3/4))/(7*x)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^8}{x^4\sqrt[4]{-x^2+x^6}}\,dx &= \int \left(-\frac{1}{x^4\sqrt[4]{-x^2+x^6}}+\frac{x^4}{\sqrt[4]{-x^2+x^6}}\right)dx\\ &= -\int \frac{1}{x^4\sqrt[4]{-x^2+x^6}}\,dx+\int \frac{x^4}{\sqrt[4]{-x^2+x^6}}\,dx\\ &= -\frac{2\left(-x^2+x^6\right)^{3/4}}{7x^5}+\frac{2\left(-x^2+x^6\right)^{3/4}}{7x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{2\left(x^2\left(x^4-1\right)\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^8)/(x^4*(-x^2 + x^6)^(1/4)),x]
[Out] (2*(x^2*(-1 + x^4))^(7/4))/(7*x^7)
```

IntegrateAlgebraic [A] time = 0.33, size = 20, normalized size = 1.00

$$\frac{2\left(x^6-x^2\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^8)/(x^4*(-x^2 + x^6)^(1/4)),x]
[Out] (2*(-x^2 + x^6)^(7/4))/(7*x^7)
```

fricas [A] time = 0.40, size = 21, normalized size = 1.05

$$\frac{2\left(x^6-x^2\right)^{\frac{3}{4}}\left(x^4-1\right)}{7\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/x^4/(x^6-x^2)^(1/4),x, algorithm="fricas")
[Out] 2/7*(x^6 - x^2)^(3/4)*(x^4 - 1)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8-1}{\left(x^6-x^2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/x^4/(x^6-x^2)^(1/4),x, algorithm="giac")
[Out] integrate((x^8 - 1)/((x^6 - x^2)^(1/4)*x^4), x)
```

maple [A] time = 0.01, size = 33, normalized size = 1.65

$$\frac{2\left(x^4-1\right)(-1+x)(1+x)\left(x^2+1\right)}{7\left(x^6-x^2\right)^{\frac{1}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-1)/x^4/(x^6-x^2)^(1/4),x)
[Out] 2/7*(x^4-1)*(-1+x)*(1+x)*(x^2+1)/(x^6-x^2)^(1/4)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8-1}{\left(x^6-x^2\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/x^4/(x^6-x^2)^(1/4),x, algorithm="maxima")
[Out] integrate((x^8 - 1)/((x^6 - x^2)^(1/4)*x^4), x)
```

mupad [B] time = 0.26, size = 33, normalized size = 1.65

$$\frac{2\left(x^6-x^2\right)^{3/4}}{7\,x}-\frac{2\left(x^6-x^2\right)^{3/4}}{7\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 - 1)/(x^4*(x^6 - x^2)^(1/4)),x)
[Out] (2*(x^6 - x^2)^(3/4))/(7*x) - (2*(x^6 - x^2)^(3/4))/(7*x^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2+1\right)\left(x^4+1\right)}{x^4\sqrt[4]{x^2(x-1)(x+1)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-1)/x**4/(x**6-x**2)**(1/4),x)
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)*(x**4 + 1)/(x**4*(x**2*(x - 1)*(x + 1)*
(x**2 + 1))**(1/4)), x)
```

3.188 $\int \frac{1+x^8}{\sqrt[4]{1-x^8}\left(-1+x^8\right)}\,dx$

Optimal. Leaf size=20

$$\frac{x\left(1-x^8\right)^{3/4}}{x^8-1}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 0.70, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {21, 383}

$$-\frac{x}{\sqrt[4]{1-x^8}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^8)/((1 - x^8)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -(x/(1 - x^8)^(1/4))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^8}{\sqrt[4]{1-x^8}\left(-1+x^8\right)}\,dx &= -\int \frac{1+x^8}{\left(1-x^8\right)^{5/4}}\,dx\\ &= -\frac{x}{\sqrt[4]{1-x^8}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.70

$$-\frac{x}{\sqrt[4]{1-x^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^8)/((1 - x^8)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -(x/(1 - x^8)^(1/4))
```

IntegrateAlgebraic [A] time = 0.56, size = 20, normalized size = 1.00

$$\frac{x\left(1-x^8\right)^{3/4}}{x^8-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^8)/((1 - x^8)^(1/4)*(-1 + x^8)),x]
[Out] (x*(1 - x^8)^(3/4))/(-1 + x^8)
```

fricas [A] time = 0.39, size = 18, normalized size = 0.90

$$\frac{\left(-x^8+1\right)^{\frac{3}{4}}x}{x^8-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)/(-x^8+1)^(1/4)/(x^8-1),x, algorithm="fricas")
[Out] (-x^8 + 1)^(3/4)*x/(x^8 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8+1}{\left(x^8-1\right)\left(-x^8+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)/(-x^8+1)^(1/4)/(x^8-1),x, algorithm="giac")
[Out] integrate((x^8 + 1)/((x^8 - 1)*(-x^8 + 1)^(1/4)), x)
```

maple [A] time = 0.01, size = 13, normalized size = 0.65

$$-\frac{x}{\left(-x^8+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8+1)/(-x^8+1)^(1/4)/(x^8-1),x)
[Out] -x/(-x^8+1)^(1/4)
```

maxima [A] time = 0.60, size = 29, normalized size = 1.45

$$-\frac{x}{\left(x^4+1\right)^{\frac{1}{4}}\left(x^2+1\right)^{\frac{1}{4}}(x+1)^{\frac{1}{4}}(-x+1)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)/(-x^8+1)^(1/4)/(x^8-1),x, algorithm="maxima")
[Out] -x/((x^4 + 1)^(1/4)*(x^2 + 1)^(1/4)*(x + 1)^(1/4)*(-x + 1)^(1/4))
```

mupad [B] time = 0.24, size = 12, normalized size = 0.60

$$-\frac{x}{\left(1-x^8\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^8 + 1)/(1 - x^8)^(5/4),x)
[Out] -x/(1 - x^8)^(1/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8+1}{\sqrt[4]{-(x-1)(x+1)\left(x^2+1\right)\left(x^4+1\right)}\,(x-1)(x+1)\left(x^2+1\right)\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8+1)/(-x**8+1)**(1/4)/(x**8-1),x)
```

```
[Out] Integral((x**8 + 1)/((-(x - 1)*(x + 1)*(x**2 + 1)*(x**4 + 1))**(1/4)*(x - 1
)*(x + 1)*(x**2 + 1)*(x**4 + 1)), x)
```

3.189 $\int \frac{1}{\sqrt{-x^2+x^8}}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^8-x^2}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2008, 203}

$$-\frac{1}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^8-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-x^2 + x^8],x]
```

```
[Out] -1/3*ArcTan[x/Sqrt[-x^2 + x^8]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-x^2+x^8}}\,dx &= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{-x^2+x^8}}\right)\right)\\ &= -\frac{1}{3}\tan^{-1}\left(\frac{x}{\sqrt{-x^2+x^8}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.85

$$\frac{x\sqrt{x^6-1}\,\tan^{-1}\left(\sqrt{x^6-1}\right)}{3\sqrt{x^2\left(x^6-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-x^2 + x^8],x]
```

```
[Out] (x*Sqrt[-1 + x^6]*ArcTan[Sqrt[-1 + x^6]])/(3*Sqrt[x^2*(-1 + x^6)])
```

IntegrateAlgebraic [A] time = 0.06, size = 20, normalized size = 1.00

$$-\frac{1}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^8-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[-x^2 + x^8],x]
```

```
[Out] -1/3*ArcTan[x/Sqrt[-x^2 + x^8]]
```

fricas [A] time = 0.41, size = 18, normalized size = 0.90

$$\frac{1}{3}\arctan\left(\frac{\sqrt{x^8-x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*arctan(sqrt(x^8 - x^2)/x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: -atan
(i)/3*sign(x)+1/3*atan(sqrt(x^6-1))/sign(x)
```

maple [A] time = 0.01, size = 26, normalized size = 1.30

$$-\frac{x\sqrt{x^6-1}\,\arcsin\left(\frac{1}{x^3}\right)}{3\sqrt{x^8-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-x^2)^(1/2),x)
```

```
[Out] -1/3/(x^8-x^2)^(1/2)*x*(x^6-1)^(1/2)*arcsin(1/x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^8-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^8 - x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{x^8-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 - x^2)^(1/2),x)
```

```
[Out] int(1/(x^8 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^8-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**8 - x**2), x)
```

3.190 $\int \frac{\sqrt{1+\sqrt{1+x^2}}}{1+x^2} \, dx$

Optimal. Leaf size=20

$$2\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^2+1}+1}}\right)$$

Rubi [F] time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+\sqrt{1+x^2}}}{1+x^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + Sqrt[1 + x^2]]/(1 + x^2),x]
```

```
[Out] (I/2)*Defer[Int][Sqrt[1 + Sqrt[1 + x^2]]/(I - x), x] + (I/2)*Defer[Int][Sqr
t[1 + Sqrt[1 + x^2]]/(I + x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+\sqrt{1+x^2}}}{1+x^2} \, dx &= \int \left(\frac{i\sqrt{1+\sqrt{1+x^2}}}{2(i-x)}+\frac{i\sqrt{1+\sqrt{1+x^2}}}{2(i+x)}\right) dx \\
&= \frac{1}{2}i\int \frac{\sqrt{1+\sqrt{1+x^2}}}{i-x} \, dx+\frac{1}{2}i\int \frac{\sqrt{1+\sqrt{1+x^2}}}{i+x} \, dx
\end{aligned}$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\sqrt{1+x^2}}}{1+x^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + x^2]]/(1 + x^2),x]
```

```
[Out] Integrate[Sqrt[1 + Sqrt[1 + x^2]]/(1 + x^2), x]
```

IntegrateAlgebraic [A] time = 0.07, size = 20, normalized size = 1.00

$$2\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^2+1}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 + x^2]]/(1 + x^2),x]
```

```
[Out] 2*ArcTan[x/Sqrt[1 + Sqrt[1 + x^2]]]
```

fricas [B] time = 1.70, size = 55, normalized size = 2.75

$$-\frac{1}{2}\arctan\left(\frac{4\left(x^4-12\,x^2+\left(5\,x^2-3\right)\sqrt{x^2+1}+3\right)\sqrt{\sqrt{x^2+1}+1}}{x^5-46\,x^3+17\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(4*(x^4 - 12*x^2 + (5*x^2 - 3)*sqrt(x^2 + 1) + 3)*sqrt(sqrt(x^2
+ 1) + 1)/(x^5 - 46*x^3 + 17*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1)/(x^2 + 1), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\sqrt{x^2+1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x^2+1)^(1/2))^(1/2)/(x^2+1),x)
```

```
[Out] int((1+(x^2+1)^(1/2))^(1/2)/(x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1)/(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)^(1/2) + 1)^(1/2)/(x^2 + 1),x)
```

```
[Out] int(((x^2 + 1)^(1/2) + 1)^(1/2)/(x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x**2+1)**(1/2))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt(sqrt(x**2 + 1) + 1)/(x**2 + 1), x)
```

3.191 $\int \frac{-1+x}{(2+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=21

$$-\frac{2}{3}\tan^{-1}\left(\frac{3\sqrt{x^3-1}}{(x-1)^2}\right)$$

Rubi [A] time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2138, 203}

$$\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x}{(2+x)\sqrt{-1+x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \frac{(1-x)^2}{\sqrt{-1+x^3}}\right)\\ &= \frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.19

$$\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3
```

IntegrateAlgebraic [A] time = 0.88, size = 21, normalized size = 1.00

$$-\frac{2}{3}\tan^{-1}\left(\frac{3\sqrt{x^3-1}}{(x-1)^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/((2 + x)*Sqrt[-1 + x^3]),x]
[Out] (-2*ArcTan[(3*Sqrt[-1 + x^3])/(-1 + x)^2])/3
```

fricas [B] time = 0.44, size = 40, normalized size = 1.90

$$\frac{1}{3}\arctan\left(\frac{\left(x^3-12\,x^2-6\,x-10\right)\sqrt{x^3-1}}{6\left(x^4-x^3-x+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] 1/3*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x-1}{\sqrt{x^3-1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

maple [C] time = 0.23, size = 240, normalized size = 11.43

$$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\ \mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{i\sqrt{3}}{6}+\frac{1}{2},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(2+x)/(x^3-1)^(1/2),x)
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(
-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2
)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticP
i(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2)
)/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x-1}{\sqrt{x^3-1}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

mupad [B] time = 0.30, size = 206, normalized size = 9.81

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x^3 - 1)^(1/2)*(x + 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticF(asi
n((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/
2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*(-
(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i
)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(
1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(2+x)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x - 1)/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

3.192 $\int \frac{-2-2x+x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=21

$$-2\tanh^{-1}\left(\frac{\sqrt{x^3-1}}{x^2+x+1}\right)$$

Rubi [A] time = 0.06, antiderivative size = 18, normalized size of antiderivative = 0.86, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2146, 206}

$$2\tanh^{-1}\left(\frac{1-x}{\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 - 2*x + x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] 2*ArcTanh[(1 - x)/Sqrt[-1 + x^3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2146

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> -Dist[g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-2-2x+x^2}{\left(2+x^2\right)\sqrt{-1+x^3}}\,dx &= 2\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)\\ &= 2\tanh^{-1}\left(\frac{1-x}{\sqrt{-1+x^3}}\right)\end{aligned}$$

Mathematica [C] time = 0.99, size = 298, normalized size = 14.19

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(-\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}-\frac{3i\left(\sqrt{2}-i\right)\Pi\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{5/6}+\sqrt{2}}+\frac{3\left(5+i\sqrt{2}+i\sqrt{3}+\sqrt{6}\right)\Pi\left(\frac{2\sqrt{3}}{-i+2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{5i+2\sqrt{2}+\sqrt{3}+2i\sqrt{6}}\right)}{3\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 - 2*x + x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*(-((Sqrt[3]*(1 + (-1)^(
1/3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(
1/3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x)) - ((3*I)*(-I + Sqrt[2])*Elliptic
Pi[(2*Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(5/6) + Sqrt[2]) + (3*(5 + I*Sqrt[2]
+ I*Sqrt[3] + Sqrt[6])*EllipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), A
```

```
rcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(5*I + 2*Sqr
t[2] + Sqrt[3] + (2*I)*Sqrt[6])))/(3*Sqrt[-1 + x^3])
```

IntegrateAlgebraic [A] time = 1.01, size = 21, normalized size = 1.00

$$-2\tanh^{-1}\left(\frac{\sqrt{x^3-1}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 - 2*x + x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
[Out] -2*ArcTanh[Sqrt[-1 + x^3]/(1 + x + x^2)]
```

fricas [A] time = 0.42, size = 25, normalized size = 1.19

$$\log\left(\frac{x^2+2x-2\sqrt{x^3-1}}{x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] log((x^2 + 2*x - 2*sqrt(x^3 - 1))/(x^2 + 2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2-2x-2}{\sqrt{x^3-1}\,(x^2+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)
```

maple [C] time = 0.05, size = 1656, normalized size = 78.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x-2)/(x^2+2)/(x^3-1)^(1/2),x)
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-3*I*2^(1/2)*(1/(-3/2-1/2*I*3^(1/
2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I
*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x
+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(
1/2)/(1-I*2^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*
I*3^(1/2))/(1-I*2^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2
^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*
I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1-I*2^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*
I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1-I*2^(1/2)),((3/2+1/2*I*3^(1/2))/(3
/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I
*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(
1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1-I*2^(1/2))*
```

```
EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1-I*2^(
1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+I*(1/(-3/2-1/2*I*3^(
1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2
*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))
*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)
^(1/2)/(1-I*2^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/
2*I*3^(1/2))/(1-I*2^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
*3^(1/2)+3*I*2^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2
)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2)
)*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/
2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+I*2^(1/2))*EllipticPi(((-1
+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+I*2^(1/2)),((3/2+1/2
*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1
/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2
))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3
/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1
+I*2^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/
2))/(1+I*2^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+
3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(
1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1
/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^
(1/2))^(1/2)/(x^3-1)^(1/2)/(1+I*2^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(
1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+I*2^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/
2*I*3^(1/2)))^(1/2))+I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1
/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(
3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+I*2^(1/2))*EllipticPi(((
-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+I*2^(1/2)),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - 2\,x - 2}{\sqrt{x^3 - 1}\left(x^2 + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)
```

mupad [B] time = 0.17, size = 275, normalized size = 13.10

$$\frac{\left(3+\sqrt{3}\,1i\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{1+\sqrt{2}\,1i};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-1+\sqrt{2}\,1i};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - x^2 + 2)/((x^2 + 2)*(x^3 - 1)^(1/2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i + 1
), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/(
(3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(-((3^(1
/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) + x^3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - 2x - 2}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x-2)/(x**2+2)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x**2 - 2*x - 2)/(sqrt((x - 1)*(x**2 + x + 1))*(x**2 + 2)), x)
```

3.193 $\int \frac{-2-2x+x^2}{(2x+x^2)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=21

$$-2\tan^{-1}\left(\frac{\sqrt{x^3-1}}{x^2+x+1}\right)$$

Rubi [C] time = 0.54, antiderivative size = 402, normalized size of antiderivative = 19.14, number of steps used = 13, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 6742, 219, 266, 63, 203, 2136, 2139, 2138}

$$\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)-\frac{2}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2\left(7-4\sqrt{3}\right)}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{\sqrt{2\left(7-4\sqrt{3}\right)}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-2 - 2*x + x^2)/((2*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3 - (2*ArcTan[Sqrt[-1 + x^3]])/3 +
 (Sqrt[2*(7 - 4*Sqrt[3])]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*E
llipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(
1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (3^(1/4)*Sqrt[2
*(7 - 4*Sqrt[3])]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF
[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 -
 x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) - (2*Sqrt[2 - Sqrt[3]]*(1 - x)*Sq
rt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1
 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x
)^2)]*Sqrt[-1 + x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2136

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, -Dist[q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2139

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-2-2x+x^2}{\left(2x+x^2\right)\sqrt{-1+x^3}}\,dx = \int \frac{-2-2x+x^2}{x(2+x)\sqrt{-1+x^3}}\,dx$$
$$= \int \left(\frac{1}{\sqrt{-1+x^3}} - \frac{1}{x\sqrt{-1+x^3}} - \frac{3}{(2+x)\sqrt{-1+x^3}}\right)dx$$
$$= -\left(3\int \frac{1}{(2+x)\sqrt{-1+x^3}}\,dx\right) + \int \frac{1}{\sqrt{-1+x^3}}\,dx - \int \frac{1}{x\sqrt{-1+x^3}}\,dx$$
$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{1}{3}\,\text{Subst}\left(\int -\right.$$
$$= \frac{2\,3^{3/4}\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{2-\sqrt{3}}}{}$$
$$= -\frac{2}{3}\tan^{-1}\left(\sqrt{-1+x^3}\right) + \frac{2\sqrt[4]{3}\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-\right.}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$
$$= \frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{2}{3}\tan^{-1}\left(\sqrt{-1+x^3}\right) + \frac{2\sqrt[4]{3}\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)}}}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right.}}}$$

Mathematica [C] time = 0.25, size = 234, normalized size = 11.14

$$-\frac{2}{3}\tan^{-1}\left(\sqrt{x^3-1}\right) + \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{x^3-1}} + \frac{6i\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}-2\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-2 - 2*x + x^2)/((2*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-1)^(1/
3) + x)*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin
[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(Sqrt[(1 - (-1)^(
2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[-1 + x^3]) + ((6*I)*Sqrt[(1 - x)/(1 + (-1)^(
1/3))]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqr
t[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-2 + (-1)^(1/3))*Sq
rt[-1 + x^3])
```

IntegrateAlgebraic [A] time = 0.99, size = 21, normalized size = 1.00

$$-2\tan^{-1}\left(\frac{\sqrt{x^3-1}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 - 2*x + x^2)/((2*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] -2*ArcTan[Sqrt[-1 + x^3]/(1 + x + x^2)]
```

fricas [A]   time = 0.43, size = 15, normalized size = 0.71

$$\arctan\left(\frac{x^2+2}{2\sqrt{x^3-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-2)/(x^2+2*x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(1/2*(x^2 + 2)/sqrt(x^3 - 1))
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-2x-2}{\sqrt{x^3-1}\,(x^2+2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-2)/(x^2+2*x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2*x)), x)
```

maple [C]   time = 0.03, size = 250, normalized size = 11.90

$$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\ \mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{i\sqrt{3}}{6}+\frac{1}{2},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\arctan\left(\sqrt{x^3-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x-2)/(x^2+2*x)/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(
-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2
)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticP
i(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2)
)/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*arctan((x^3-1)^(1/2))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-2x-2}{\sqrt{x^3-1}\,(x^2+2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-2)/(x^2+2*x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2*x)), x)
```

mupad [B]   time = 0.16, size = 252, normalized size = 12.00

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - x^2 + 2)/((2*x + x^2)*(x^3 - 1)^(1/2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(
```

```
1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/
2)) - ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1
i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi((3^(1/2)*1i)/6 + 1/2, asin
((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2
)*1i)/2 - 3/2))))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - 2x - 2}{x\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x-2)/(x**2+2*x)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x**2 - 2*x - 2)/(x*sqrt((x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

3.194 $\int \frac{-1+2x+2x^2}{(-1+x)x\sqrt{-x+x^4}}\,dx$

Optimal. Leaf size=21

$$-\frac{2\sqrt{x^4-x}}{(x-1)x}$$

Rubi [F] time = 1.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+2x+2x^2}{(-1+x)x\sqrt{-x+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 2*x + 2*x^2)/((-1 + x)*x*Sqrt[-x + x^4]),x]
```

```
[Out] (-2*(1 - x^3))/Sqrt[-x + x^4] - (2*(1 + Sqrt[3])*x*(1 - x^3))/((1 - (1 + Sq
rt[3])*x)*Sqrt[-x + x^4]) + (2*3^(1/4)*(1 - x)*x*Sqrt[(1 + x + x^2)/(1 - (1
 + Sqrt[3])*x)^2]*EllipticE[ArcCos[(1 - (1 - Sqrt[3])*x)/(1 - (1 + Sqrt[3])
*x)], (2 + Sqrt[3])/4])/(Sqrt[-(((1 - x)*x)/(1 - (1 + Sqrt[3])*x)^2)]*Sqrt[
-x + x^4]) + (2*(1 - x)*x*Sqrt[(1 + x + x^2)/(1 - (1 + Sqrt[3])*x)^2]*Ellip
ticF[ArcCos[(1 - (1 - Sqrt[3])*x)/(1 - (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])
/(3^(1/4)*Sqrt[-(((1 - x)*x)/(1 - (1 + Sqrt[3])*x)^2)]*Sqrt[-x + x^4]) + ((
1 - Sqrt[3])*(1 - x)*x*Sqrt[(1 + x + x^2)/(1 - (1 + Sqrt[3])*x)^2]*Elliptic
F[ArcCos[(1 - (1 - Sqrt[3])*x)/(1 - (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3
^(1/4)*Sqrt[-(((1 - x)*x)/(1 - (1 + Sqrt[3])*x)^2)]*Sqrt[-x + x^4]) + (3*Sq
rt[x]*Sqrt[-1 + x^3]*Defer[Subst][Defer[Int][1/((-1 + x)*Sqrt[-1 + x^6]), x
], x, Sqrt[x]])/Sqrt[-x + x^4] - (3*Sqrt[x]*Sqrt[-1 + x^3]*Defer[Subst][Def
er[Int][1/((1 + x)*Sqrt[-1 + x^6]), x], x, Sqrt[x]])/Sqrt[-x + x^4]
```

Rubi steps

$$\int \frac{-1+2x+2x^2}{(-1+x)x\sqrt{-x+x^4}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1+x^3}\right)\int \frac{-1+2x+2x^2}{(-1+x)x^{3/2}\sqrt{-1+x^3}}\,dx}{\sqrt{-x+x^4}}$$
$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^3}\right)\text{Subst}\left(\int \frac{-1+2x^2+2x^4}{x^2\left(-1+x^2\right)\sqrt{-1+x^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{-x+x^4}}$$
$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^3}\right)\text{Subst}\left(\int \left(\frac{2}{\sqrt{-1+x^6}} + \frac{1}{x^2\sqrt{-1+x^6}} + \frac{3}{\left(-1+x^2\right)\sqrt{-1+x^6}}\right)dx, x, \sqrt{x}\right)}{\sqrt{-x+x^4}}$$
$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^3}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{-1+x^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{-x+x^4}} + \frac{\left(4\sqrt{x}\sqrt{-1+x^3}\right)\text{Subst}\left(\int \frac{}{\sqrt{-1}}\right.}{\sqrt{-x+x^4}}$$
$$= -\frac{2\left(1-x^3\right)}{\sqrt{-x+x^4}} + \frac{2(1-x)x\sqrt{\frac{1+x+x^2}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x}{1-\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{-\frac{(1-x)x}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{-x+x^4}} - \frac{(4\sqrt{}}{}$$
$$= -\frac{2\left(1-x^3\right)}{\sqrt{-x+x^4}} + \frac{2(1-x)x\sqrt{\frac{1+x+x^2}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x}{1-\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{-\frac{(1-x)x}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{-x+x^4}} + \frac{(2\sqrt{}}{}$$
$$= -\frac{2\left(1-x^3\right)}{\sqrt{-x+x^4}} - \frac{2\left(1+\sqrt{3}\right)x\left(1-x^3\right)}{\left(1-\left(1+\sqrt{3}\right)x\right)\sqrt{-x+x^4}} + \frac{2\sqrt[4]{3}(1-x)x\sqrt{\frac{1+x+x^2}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\,E\left(\cos^{-}\right.}{\sqrt{-\frac{(1-x)x}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}}$$

Mathematica [A] time = 0.12, size = 19, normalized size = 0.90

$$-\frac{2\left(x^2+x+1\right)}{\sqrt{x\left(x^3-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x + 2*x^2)/((-1 + x)*x*Sqrt[-x + x^4]),x]
[Out] (-2*(1 + x + x^2))/Sqrt[x*(-1 + x^3)]
```

IntegrateAlgebraic [A] time = 1.21, size = 21, normalized size = 1.00

$$-\frac{2\sqrt{x^4-x}}{(x-1)x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x + 2*x^2)/((-1 + x)*x*Sqrt[-x + x^4]),x]
[Out] (-2*Sqrt[-x + x^4])/((-1 + x)*x)
```

fricas [A] time = 0.39, size = 20, normalized size = 0.95

$$-\frac{2\sqrt{x^4-x}}{x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+2*x-1)/(-1+x)/x/(x^4-x)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(x^4 - x)/(x^2 - x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2+2\,x-1}{\sqrt{x^4-x}\,(x-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+2*x-1)/(-1+x)/x/(x^4-x)^(1/2),x, algorithm="giac")
[Out] integrate((2*x^2 + 2*x - 1)/(sqrt(x^4 - x)*(x - 1)*x), x)
```

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$-\frac{2\left(x^2+x+1\right)}{\sqrt{x^4-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+2*x-1)/(-1+x)/x/(x^4-x)^(1/2),x)
[Out] -2*(x^2+x+1)/(x^4-x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2+2\,x-1}{\sqrt{x^4-x}\,(x-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+2*x-1)/(-1+x)/x/(x^4-x)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x^2 + 2*x - 1)/(sqrt(x^4 - x)*(x - 1)*x), x)
```

mupad [B] time = 0.19, size = 19, normalized size = 0.90

$$-\frac{2\,\sqrt{x^4-x}}{x\,\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 2*x^2 - 1)/(x*(x^4 - x)^(1/2)*(x - 1)),x)
[Out] -(2*(x^4 - x)^(1/2))/(x*(x - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2+2x-1}{x\sqrt{x\,(x-1)\left(x^2+x+1\right)}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+2*x-1)/(-1+x)/x/(x**4-x)**(1/2),x)
[Out] Integral((2*x**2 + 2*x - 1)/(x*sqrt(x*(x - 1)*(x**2 + x + 1))*(x - 1)), x)
```

3.195 $\int \frac{-1+2x+2x^2}{(1+2x^2)\sqrt{-x+x^4}}\,dx$

Optimal. Leaf size=21

$$\tan^{-1}\left(\frac{2\sqrt{x^4-x}}{2x+1}\right)$$

Rubi [F] time = 1.76, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+2x+2x^2}{(1+2x^2)\sqrt{-x+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 2*x + 2*x^2)/((1 + 2*x^2)*Sqrt[-x + x^4]),x]
```

```
[Out] ((1 - x)*x*Sqrt[(1 + x + x^2)/(1 - (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(1
- (1 - Sqrt[3])*x)/(1 - (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3^(1/4)*Sqrt[
-(((1 - x)*x)/(1 - (1 + Sqrt[3])*x)^2)]*Sqrt[-x + x^4]) + ((1/2 + I/2)*(I -
 Sqrt[2])*Sqrt[x]*Sqrt[-1 + x^3]*Defer[Subst][Defer[Int][1/(((-1)^(1/4) - 2
^(1/4)*x)*Sqrt[-1 + x^6]), x], x, Sqrt[x]])/Sqrt[-x + x^4] - ((1/2 - I/2)*(
I + Sqrt[2])*Sqrt[x]*Sqrt[-1 + x^3]*Defer[Subst][Defer[Int][1/((-(-1)^(3/4)
 - 2^(1/4)*x)*Sqrt[-1 + x^6]), x], x, Sqrt[x]])/Sqrt[-x + x^4] + ((1/2 + I/
2)*(I - Sqrt[2])*Sqrt[x]*Sqrt[-1 + x^3]*Defer[Subst][Defer[Int][1/(((-1)^(1
/4) + 2^(1/4)*x)*Sqrt[-1 + x^6]), x], x, Sqrt[x]])/Sqrt[-x + x^4] - ((1/2 -
 I/2)*(I + Sqrt[2])*Sqrt[x]*Sqrt[-1 + x^3]*Defer[Subst][Defer[Int][1/((-(-1
)^(3/4) + 2^(1/4)*x)*Sqrt[-1 + x^6]), x], x, Sqrt[x]])/Sqrt[-x + x^4]
```

Rubi steps

$$\int \frac{-1+2x+2x^2}{\left(1+2x^2\right)\sqrt{-x+x^4}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1+x^3}\right)\int \frac{-1+2x+2x^2}{\sqrt{x}\left(1+2x^2\right)\sqrt{-1+x^3}}\,dx}{\sqrt{-x+x^4}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^3}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{-1+x^3}} - \frac{2(1-x)}{\sqrt{x}\left(1+2x^2\right)\sqrt{-1+x^3}}\right)dx}{\sqrt{-x+x^4}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^3}\right)\int \frac{1}{\sqrt{x}\sqrt{-1+x^3}}\,dx}{\sqrt{-x+x^4}} - \frac{\left(2\sqrt{x}\sqrt{-1+x^3}\right)\int \frac{1-x}{\sqrt{x}\left(1+2x^2\right)\sqrt{-1+x^3}}\,dx}{\sqrt{-x+x^4}}$$

$$= -\frac{\left(2\sqrt{x}\sqrt{-1+x^3}\right)\int \left(\frac{i+\frac{1}{\sqrt{2}}}{2\sqrt{x}\left(i-\sqrt{2}x\right)\sqrt{-1+x^3}} + \frac{i-\frac{1}{\sqrt{2}}}{2\sqrt{x}\left(i+\sqrt{2}x\right)\sqrt{-1+x^3}}\right)dx}{\sqrt{-x+x^4}} + \frac{\left(2\sqrt{x}\sqrt{-1+x^3}\right.}{}$$

$$= \frac{(1-x)x\sqrt{\frac{1+x+x^2}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x}{1-\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{-\frac{(1-x)x}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{-x+x^4}} - \frac{\left(\left(2i-\sqrt{2}\right)\sqrt{x}\sqrt{-1+x\right.}{2\sqrt{-}}$$

$$= \frac{(1-x)x\sqrt{\frac{1+x+x^2}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x}{1-\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{-\frac{(1-x)x}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{-x+x^4}} - \frac{\left(\left(2i-\sqrt{2}\right)\sqrt{x}\sqrt{-1+x\right.}{}$$

$$= \frac{(1-x)x\sqrt{\frac{1+x+x^2}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x}{1-\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{-\frac{(1-x)x}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{-x+x^4}} - \frac{\left(\left(2i-\sqrt{2}\right)\sqrt{x}\sqrt{-1+x\right.}{}$$

$$= \frac{(1-x)x\sqrt{\frac{1+x+x^2}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x}{1-\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{-\frac{(1-x)x}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{-x+x^4}} + \frac{\left(\sqrt[4]{-1}\left(2i-\sqrt{2}\right)\sqrt{x}\sqrt{-\right.}{}$$

Mathematica [C] time = 1.28, size = 320, normalized size = 15.24

$$-\frac{2\sqrt{\frac{1}{x^2}+\frac{1}{x}+1}\sqrt{\frac{x-1}{\left(1+\sqrt[3]{-1}\right)x}}\,x^2\left(\frac{\sqrt{3}\left(i\sqrt{3}x+\sqrt[3]{-1}+1\right)F\left(\sin^{-1}\left(\sqrt{\frac{x-(-1)^{2/3}}{\left(1+\sqrt[3]{-1}\right)x}}\right)|\sqrt[3]{-1}\right)}{(-1)^{2/3}-x} + \frac{6\left(\left(-2i+\sqrt[6]{-1}+2\sqrt{2}+(-1)^{2/3}\sqrt{2}\right)\Pi\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{x-(-1)^{2/3}}{\left(1+\sqrt[3]{-1}\right)x}}\right)|\sqrt[3]{-1}\right)+\frac{3}{2}\left(3i+\sqrt{2}-\sqrt{3}-i\sqrt{6}\right)\Pi\left(\frac{2\sqrt{3}}{-i+2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{x-(-1)^{2/3}}{\left(1+\sqrt[3]{-1}\right)x}}\right)|\sqrt[3]{-1}\right)\right)}{-3i\sqrt{2}+2i\sqrt{3}+\sqrt{6}}\right)}{3\sqrt{x\left(x^3-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x + 2*x^2)/((1 + 2*x^2)*Sqrt[-x + x^4]),x]
```

```
[Out] (-2*Sqrt[1 + x^(-2) + x^(-1)]*Sqrt[(-1 + x)/((1 + (-1)^(1/3))*x)]*x^2*((Sqr
t[3]*(1 + (-1)^(1/3) + I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[(-(-1)^(2/3) + x)
/((1 + (-1)^(1/3))*x)]], (-1)^(1/3)])/((-1)^(2/3) - x) + (6*((-2*I + (-1)^(
1/6) + 2*Sqrt[2] + (-1)^(2/3)*Sqrt[2])*EllipticPi[(2*Sqrt[3])/(-I - 2*Sqrt[
2] + Sqrt[3]), ArcSin[Sqrt[(-(-1)^(2/3) + x)/((1 + (-1)^(1/3))*x)]], (-1)^(
1/3)] + (3*(3*I + Sqrt[2] - Sqrt[3] - I*Sqrt[6])*EllipticPi[(2*Sqrt[3])/(-I
 + 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(-(-1)^(2/3) + x)/((1 + (-1)^(1/3))*x)
]], (-1)^(1/3)])/2))/((-3*I)*Sqrt[2] + (2*I)*Sqrt[3] + Sqrt[6])))/(3*Sqrt[x
*(-1 + x^3)])
```

IntegrateAlgebraic [A] time = 1.29, size = 21, normalized size = 1.00

$$\tan^{-1}\left(\frac{2\sqrt{x^4-x}}{2x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x + 2*x^2)/((1 + 2*x^2)*Sqrt[-x + x^4]),x]
[Out] ArcTan[(2*Sqrt[-x + x^4])/(1 + 2*x)]
```

fricas [A] time = 0.45, size = 19, normalized size = 0.90

$$-\arctan\left(\frac{2\,x+1}{2\,\sqrt{x^4-x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+2*x-1)/(2*x^2+1)/(x^4-x)^(1/2),x, algorithm="fricas")
[Out] -arctan(1/2*(2*x + 1)/sqrt(x^4 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2+2\,x-1}{\sqrt{x^4-x}\left(2\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+2*x-1)/(2*x^2+1)/(x^4-x)^(1/2),x, algorithm="giac")
[Out] integrate((2*x^2 + 2*x - 1)/(sqrt(x^4 - x)*(2*x^2 + 1)), x)
```

maple [C] time = 0.20, size = 13534, normalized size = 644.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+2*x-1)/(2*x^2+1)/(x^4-x)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2+2\,x-1}{\sqrt{x^4-x}\left(2\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+2*x-1)/(2*x^2+1)/(x^4-x)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x^2 + 2*x - 1)/(sqrt(x^4 - x)*(2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{2\,x^2+2\,x-1}{\sqrt{x^4-x}\,\left(2\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 2*x^2 - 1)/((x^4 - x)^(1/2)*(2*x^2 + 1)),x)
```

```
[Out] int((2*x + 2*x^2 - 1)/((x^4 - x)^(1/2)*(2*x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2 + 2x - 1}{\sqrt{x\,(x-1)\left(x^2 + x + 1\right)}\,\left(2x^2 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+2*x-1)/(2*x**2+1)/(x**4-x)**(1/2),x)
```

```
[Out] Integral((2*x**2 + 2*x - 1)/(sqrt(x*(x - 1)*(x**2 + x + 1))*(2*x**2 + 1)),
x)
```

3.196 $\int \frac{-1-2x+2x^2}{(-1+2x)\sqrt{x+x^4}}\,dx$

Optimal. Leaf size=21

$$\tanh^{-1}\left(\frac{2\sqrt{x^4+x}}{2x^2+1}\right)$$

Rubi [F] time = 0.69, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1-2x+2x^2}{(-1+2x)\sqrt{x+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 - 2*x + 2*x^2)/((-1 + 2*x)*Sqrt[x + x^4]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 + x^3]*ArcSinh[x^(3/2)])/(3*Sqrt[x + x^4]) - (x*(1 + x)*S
qrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(1 + (1 - Sqrt[
3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(2*3^(1/4)*Sqrt[(x*(1 + x)
)/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[x + x^4]) + (3*Sqrt[x]*Sqrt[1 + x^3]*Defer[
Subst][Defer[Int][1/((1 - Sqrt[2]*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/(2*Sq
rt[x + x^4]) + (3*Sqrt[x]*Sqrt[1 + x^3]*Defer[Subst][Defer[Int][1/((1 + Sqr
t[2]*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/(2*Sqrt[x + x^4])
```

Rubi steps

$$\begin{aligned}
\int \frac{-1-2x+2x^2}{(-1+2x)\sqrt{x+x^4}}\,dx &= \frac{\left(\sqrt{x}\sqrt{1+x^3}\right)\int \frac{-1-2x+2x^2}{\sqrt{x}(-1+2x)\sqrt{1+x^3}}\,dx}{\sqrt{x+x^4}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{-1-2x^2+2x^4}{\left(-1+2x^2\right)\sqrt{1+x^6}}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^4}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^3}\right)\text{Subst}\left(\int \left(-\frac{1}{2\sqrt{1+x^6}}+\frac{x^2}{\sqrt{1+x^6}}-\frac{3}{2\left(-1+2x^2\right)\sqrt{1+x^6}}\right)dx,x,\sqrt{x}\right)}{\sqrt{x+x^4}}\\
&= -\frac{\left(\sqrt{x}\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^6}}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^4}}+\frac{\left(2\sqrt{x}\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^6}}\,dx,\right.}{\sqrt{x+x^4}}\\
&= -\frac{x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}}+\frac{\left(2\sqrt{x}\sqrt{1+x^3}\right)\text{Sub}}{3\sqrt{}}\\
&= \frac{2\sqrt{x}\sqrt{1+x^3}\,\sinh^{-1}\left(x^{3/2}\right)}{3\sqrt{x+x^4}}-\frac{x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\right.\right.}{2\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}}
\end{aligned}$$

Mathematica [C] time = 1.13, size = 250, normalized size = 11.90

$$\frac{-\sqrt{6}\left(1+\sqrt[3]{-1}\right)\sqrt{\frac{x-\sqrt[3]{-1}}{\left(1+\sqrt[3]{-1}\right)x}}\sqrt{\frac{(x+1)\left(2x+i\sqrt{3}-1\right)}{x^2}}\,x^2F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}(x+1)}{\left(-1+(-1)^{2/3}\right)x}}\right)|1+(-1)^{2/3}\right)+\sqrt{6}\left(1+\sqrt[3]{-1}\right)\sqrt{\frac{x-\sqrt[3]{-1}}{\left(1+\sqrt[3]{-1}\right)x}}\sqrt{\frac{(x+1)\left(2x+i\sqrt{3}-1\right)}{x^2}}\,x^2\Pi\left(\frac{1}{3}\left(1+\sqrt[3]{-1}\right);\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}(x+1)}{\left(-1+(-1)^{2/3}\right)x}}\right)|1+(-1)^{2/3}\right)+2\sqrt{x^3+1}\sqrt{x}\sinh^{-1}\left(x^{3/2}\right)}{3\sqrt{x^4+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x + 2*x^2)/((-1 + 2*x)*Sqrt[x + x^4]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 + x^3]*ArcSinh[x^(3/2)] - Sqrt[6]*(1 + (-1)^(1/3))*x^2*Sq
rt[(-(-1)^(1/3) + x)/((1 + (-1)^(1/3))*x)]*Sqrt[((1 + x)*(-1 + I*Sqrt[3] +
2*x))/x^2]*EllipticF[ArcSin[Sqrt[((-1)^(2/3)*(1 + x))/((-1 + (-1)^(2/3))*x)
]], 1 + (-1)^(2/3)] + Sqrt[6]*(1 + (-1)^(1/3))*x^2*Sqrt[(-(-1)^(1/3) + x)/(
(1 + (-1)^(1/3))*x)]*Sqrt[((1 + x)*(-1 + I*Sqrt[3] + 2*x))/x^2]*EllipticPi[
(1 + (-1)^(1/3))/3, ArcSin[Sqrt[((-1)^(2/3)*(1 + x))/((-1 + (-1)^(2/3))*x)]
], 1 + (-1)^(2/3)])/(3*Sqrt[x + x^4])
```

IntegrateAlgebraic [A] time = 1.21, size = 21, normalized size = 1.00

$$\tanh^{-1}\left(\frac{2\sqrt{x^4+x}}{2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 2*x + 2*x^2)/((-1 + 2*x)*Sqrt[x + x^4]),x]
```

```
[Out] ArcTanh[(2*Sqrt[x + x^4])/(1 + 2*x^2)]
```

fricas [A] time = 0.43, size = 25, normalized size = 1.19

$$\log\left(\frac{2\,x^2+2\,\sqrt{x^4+x}\,+1}{2\,x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-2*x-1)/(-1+2*x)/(x^4+x)^(1/2),x, algorithm="fricas")
```

```
[Out] log((2*x^2 + 2*sqrt(x^4 + x) + 1)/(2*x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2-2\,x-1}{\sqrt{x^4+x}\,(2\,x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-2*x-1)/(-1+2*x)/(x^4+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 - 2*x - 1)/(sqrt(x^4 + x)*(2*x - 1)), x)
```

maple [C] time = 0.01, size = 780, normalized size = 37.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-2*x-1)/(-1+2*x)/(x^4+x)^(1/2),x)
```

```
[Out] -2*(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(
1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))/(1+x))^(1/2)*(-(x-
1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(3/2+1/2*I*3^(1/2))/(x*
(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1/2)*(-EllipticF(((3/2+
1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1
/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+Ellipti
cPi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),(1/2+1/2*I*3^(1
/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))+(-1/2-1/2*I*3^(1/2))*((3/2+1/2*
I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2
```

```
))/(1/2-1/2*I*3^(1/2))/(1+x))^(1/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1
/2))/(1+x))^(1/2)/(3/2+1/2*I*3^(1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2
-1/2*I*3^(1/2)))^(1/2)*EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))
/(1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2
))/(-3/2-1/2*I*3^(1/2)))^(1/2))-(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x
/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*
I*3^(1/2))/(1+x))^(1/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^
(1/2)/(3/2+1/2*I*3^(1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/
2)))^(1/2)*(EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/
2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/
2*I*3^(1/2)))^(1/2))+2*EllipticPi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2)
)/(1+x))^(1/2),3*(1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/
2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2-2\,x-1}{\sqrt{x^4+x}\,(2\,x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-2*x-1)/(-1+2*x)/(x^4+x)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x^2 - 2*x - 1)/(sqrt(x^4 + x)*(2*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int -\frac{-2\,x^2+2\,x+1}{(2\,x-1)\,\sqrt{x^4+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 2*x^2 + 1)/((2*x - 1)*(x + x^4)^(1/2)),x)
[Out] int(-(2*x - 2*x^2 + 1)/((2*x - 1)*(x + x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2-2x-1}{\sqrt{x\,(x+1)\left(x^2-x+1\right)}\,(2x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-2*x-1)/(-1+2*x)/(x**4+x)**(1/2),x)
[Out] Integral((2*x**2 - 2*x - 1)/(sqrt(x*(x + 1)*(x**2 - x + 1))*(2*x - 1)), x)
```

3.197 $\int \frac{-1-2x+2x^2}{(1-x+x^2)\sqrt{x+x^4}}\,dx$

Optimal. Leaf size=21

$$-\frac{2\sqrt{x^4+x}}{x^2-x+1}$$

Rubi [F] time = 2.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1-2x+2x^2}{(1-x+x^2)\sqrt{x+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 - 2*x + 2*x^2)/((1 - x + x^2)*Sqrt[x + x^4]),x]
```

```
[Out] (2*x*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(
1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3^(1/4)*Sqr
t[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[x + x^4]) + ((2*I)*Sqrt[3]*Sqrt
[x]*Sqrt[1 + x^3]*Defer[Subst][Defer[Int][1/((Sqrt[1 - I*Sqrt[3]] - Sqrt[2]
*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/(Sqrt[1 - I*Sqrt[3]]*Sqrt[x + x^4]) -
((2*I)*Sqrt[3]*Sqrt[x]*Sqrt[1 + x^3]*Defer[Subst][Defer[Int][1/((Sqrt[1 + I
*Sqrt[3]] - Sqrt[2]*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/(Sqrt[1 + I*Sqrt[3]
]*Sqrt[x + x^4]) + ((2*I)*Sqrt[3]*Sqrt[x]*Sqrt[1 + x^3]*Defer[Subst][Defer[
Int][1/((Sqrt[1 - I*Sqrt[3]] + Sqrt[2]*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/
(Sqrt[1 - I*Sqrt[3]]*Sqrt[x + x^4]) - ((2*I)*Sqrt[3]*Sqrt[x]*Sqrt[1 + x^3]*
Defer[Subst][Defer[Int][1/((Sqrt[1 + I*Sqrt[3]] + Sqrt[2]*x)*Sqrt[1 + x^6])
, x], x, Sqrt[x]])/(Sqrt[1 + I*Sqrt[3]]*Sqrt[x + x^4])
```

Rubi steps

$$\int \frac{-1-2x+2x^2}{\left(1-x+x^2\right)\sqrt{x+x^4}}\,dx = \frac{\left(\sqrt{x}\sqrt{1+x^3}\right)\int \frac{-1-2x+2x^2}{\sqrt{x}\left(1-x+x^2\right)\sqrt{1+x^3}}\,dx}{\sqrt{x+x^4}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x^3}\right)\int \left(\frac{2}{\sqrt{x}\sqrt{1+x^3}} - \frac{3}{\sqrt{x}\left(1-x+x^2\right)\sqrt{1+x^3}}\right)dx}{\sqrt{x+x^4}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{x}\sqrt{1+x^3}}\,dx}{\sqrt{x+x^4}} - \frac{\left(3\sqrt{x}\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{x}\left(1-x+x^2\right)\sqrt{1+x^3}}\,dx}{\sqrt{x+x^4}}$$

$$= -\frac{\left(3\sqrt{x}\sqrt{1+x^3}\right)\int \left(\frac{2i}{\sqrt{3}\left(1+i\sqrt{3}-2x\right)\sqrt{x}\sqrt{1+x^3}} + \frac{2i}{\sqrt{3}\sqrt{x}\left(-1+i\sqrt{3}+2x\right)\sqrt{1+x^3}}\right)dx}{\sqrt{x+x^4}} + \frac{\left(4\sqrt{\right.}}{}$$

$$= \frac{2x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}} - \frac{\left(2i\sqrt{3}\sqrt{x}\sqrt{1+x\right.}}{}$$

$$= \frac{2x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}} - \frac{\left(4i\sqrt{3}\sqrt{x}\sqrt{1+x\right.}}{}$$

$$= \frac{2x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}} - \frac{\left(4i\sqrt{3}\sqrt{x}\sqrt{1+x\right.}}{}$$

$$= \frac{2x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}} + \frac{\left(2i\sqrt{3}\sqrt{x}\sqrt{1+x\right.}}{}$$

Mathematica [A] time = 0.42, size = 15, normalized size = 0.71

$$-\frac{2x(x+1)}{\sqrt{x^4+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x + 2*x^2)/((1 - x + x^2)*Sqrt[x + x^4]),x]
[Out] (-2*x*(1 + x))/Sqrt[x + x^4]
```

IntegrateAlgebraic [A] time = 0.67, size = 21, normalized size = 1.00

$$-\frac{2\sqrt{x^4+x}}{x^2-x+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 2*x + 2*x^2)/((1 - x + x^2)*Sqrt[x + x^4]),x]
```

```
[Out] (-2*Sqrt[x + x^4])/(1 - x + x^2)
```

fricas [A] time = 0.40, size = 19, normalized size = 0.90

$$-\frac{2\sqrt{x^4+x}}{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-2*x-1)/(x^2-x+1)/(x^4+x)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(x^4 + x)/(x^2 - x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2-2x-1}{\sqrt{x^4+x}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-2*x-1)/(x^2-x+1)/(x^4+x)^(1/2),x, algorithm="giac")
[Out] integrate((2*x^2 - 2*x - 1)/(sqrt(x^4 + x)*(x^2 - x + 1)), x)
```

maple [A] time = 0.01, size = 14, normalized size = 0.67

$$-\frac{2x(1+x)}{\sqrt{x^4+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-2*x-1)/(x^2-x+1)/(x^4+x)^(1/2),x)
[Out] -2*x*(1+x)/(x^4+x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2-2x-1}{\sqrt{x^4+x}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-2*x-1)/(x^2-x+1)/(x^4+x)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x^2 - 2*x - 1)/(sqrt(x^4 + x)*(x^2 - x + 1)), x)
```

mupad [B] time = 0.17, size = 19, normalized size = 0.90

$$-\frac{2\sqrt{x^4+x}}{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 2*x^2 + 1)/((x + x^4)^(1/2)*(x^2 - x + 1)),x)
[Out] -(2*(x + x^4)^(1/2))/(x^2 - x + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2-2x-1}{\sqrt{x(x+1)\left(x^2-x+1\right)}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-2*x-1)/(x**2-x+1)/(x**4+x)**(1/2),x)
```

```
[Out] Integral((2*x**2 - 2*x - 1)/(sqrt(x*(x + 1)*(x**2 - x + 1))*(x**2 - x + 1))
, x)
```

3.198 $\int \frac{1}{(1+x)\sqrt[4]{x^3+x^4}}\,dx$

Optimal. Leaf size=21

$$\frac{4\left(x^4+x^3\right)^{3/4}}{x^2(x+1)}$$

Rubi [A] time = 0.03, antiderivative size = 14, normalized size of antiderivative = 0.67, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2056, 37}

$$\frac{4x}{\sqrt[4]{x^4+x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(x^3 + x^4)^(1/4)),x]
```

```
[Out] (4*x)/(x^3 + x^4)^(1/4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)\sqrt[4]{x^3+x^4}}\,dx &= \frac{\left(x^{3/4}\sqrt[4]{1+x}\right)\int \frac{1}{x^{3/4}(1+x)^{5/4}}\,dx}{\sqrt[4]{x^3+x^4}}\\ &= \frac{4x}{\sqrt[4]{x^3+x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.67

$$\frac{4x}{\sqrt[4]{x^3(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(x^3 + x^4)^(1/4)),x]
```

```
[Out] (4*x)/(x^3*(1 + x))^(1/4)
```

IntegrateAlgebraic [A] time = 0.20, size = 21, normalized size = 1.00

$$\frac{4\left(x^4+x^3\right)^{3/4}}{x^2(x+1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x)*(x^3 + x^4)^(1/4)),x]
[Out] (4*(x^3 + x^4)^(3/4))/(x^2*(1 + x))
```

fricas [A] time = 0.38, size = 20, normalized size = 0.95

$$\frac{4\left(x^4+x^3\right)^{\frac{3}{4}}}{x^3+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^4+x^3)^(1/4),x, algorithm="fricas")
[Out] 4*(x^4 + x^3)^(3/4)/(x^3 + x^2)
```

giac [A] time = 0.62, size = 9, normalized size = 0.43

$$\frac{4}{\left(\frac{1}{x}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^4+x^3)^(1/4),x, algorithm="giac")
[Out] 4/(1/x + 1)^(1/4)
```

maple [A] time = 0.00, size = 13, normalized size = 0.62

$$\frac{4x}{\left(x^4+x^3\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^4+x^3)^(1/4),x)
[Out] 4*x/(x^4+x^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+x^3\right)^{\frac{1}{4}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^4+x^3)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((x^4 + x^3)^(1/4)*(x + 1)), x)
```

mupad [B] time = 0.17, size = 19, normalized size = 0.90

$$\frac{4\left(x^4+x^3\right)^{3/4}}{x^2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 + x^4)^(1/4)*(x + 1)),x)
[Out] (4*(x^3 + x^4)^(3/4))/(x^2*(x + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^3\left(x+1\right)}\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**4+x**3)**(1/4),x)
```

```
[Out] Integral(1/((x**3*(x + 1))**(1/4)*(x + 1)), x)
```

3.199 $\int \frac{1+2x^4}{\left(-1+2x^4\right)\sqrt{-1-x^2+2x^4}}\,dx$

Optimal. Leaf size=21

$$-\tan^{-1}\left(\frac{x}{\sqrt{2x^4-x^2-1}}\right)$$

Rubi [A] time = 0.11, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2112, 204}

$$-\tan^{-1}\left(\frac{x}{\sqrt{2x^4-x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^4)/((-1 + 2*x^4)*Sqrt[-1 - x^2 + 2*x^4]),x]
```

```
[Out] -ArcTan[x/Sqrt[-1 - x^2 + 2*x^4]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 2112

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^4}{\left(-1+2x^4\right)\sqrt{-1-x^2+2x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \frac{x}{\sqrt{-1-x^2+2x^4}}\right)\\ &= -\tan^{-1}\left(\frac{x}{\sqrt{-1-x^2+2x^4}}\right)\end{aligned}$$

Mathematica [C] time = 0.27, size = 99, normalized size = 4.71

$$-\frac{i\sqrt{-2x^4+x^2+1}\left(F\left(i\sinh^{-1}\left(\sqrt{2}x\right)|-\frac{1}{2}\right)-\Pi\left(-\frac{1}{\sqrt{2}};i\sinh^{-1}\left(\sqrt{2}x\right)|-\frac{1}{2}\right)-\Pi\left(\frac{1}{\sqrt{2}};i\sinh^{-1}\left(\sqrt{2}x\right)|-\frac{1}{2}\right)\right)}{\sqrt{4x^4-2x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + 2*x^4)/((-1 + 2*x^4)*Sqrt[-1 - x^2 + 2*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2 - 2*x^4]*(EllipticF[I*ArcSinh[Sqrt[2]*x], -1/2] - Ellipt
icPi[-(1/Sqrt[2]), I*ArcSinh[Sqrt[2]*x], -1/2] - EllipticPi[1/Sqrt[2], I*Ar
cSinh[Sqrt[2]*x], -1/2]))/Sqrt[-2 - 2*x^2 + 4*x^4]
```

IntegrateAlgebraic [A] time = 0.37, size = 21, normalized size = 1.00

$$-\tan^{-1}\left(\frac{x}{\sqrt{2x^4-x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x^4)/((-1 + 2*x^4)*Sqrt[-1 - x^2 + 2*x^4]),x]
[Out] -ArcTan[x/Sqrt[-1 - x^2 + 2*x^4]]
```

fricas [A] time = 0.44, size = 34, normalized size = 1.62

$$-\frac{1}{2}\,\arctan\left(\frac{2\,\sqrt{2\,x^4-x^2-1}\,x}{2\,x^4-2\,x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+1)/(2*x^4-1)/(2*x^4-x^2-1)^(1/2),x, algorithm="fricas")
[Out] -1/2*arctan(2*sqrt(2*x^4 - x^2 - 1)*x/(2*x^4 - 2*x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^4+1}{\sqrt{2\,x^4-x^2-1}\,\left(2\,x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+1)/(2*x^4-1)/(2*x^4-x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate((2*x^4 + 1)/(sqrt(2*x^4 - x^2 - 1)*(2*x^4 - 1)), x)
```

maple [C] time = 0.07, size = 179, normalized size = 8.52

$$-\frac{i\sqrt{2}\,\sqrt{2x^2+1}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}\left(ix\sqrt{2},\frac{i\sqrt{2}}{2}\right)}{2\sqrt{2x^4-x^2-1}}+\frac{\left(\sum_{\_\alpha=\operatorname{RootOf}\left(2\_Z^4-1\right)}\_\alpha\left(-\frac{\operatorname{arctanh}\left(\frac{\left(4\_\alpha^2-1\right)\left(-9\_\alpha^2+7x^2-4\right)}{14\sqrt{-\_\alpha^2}\,\sqrt{2x^4-x^2-1}}\right)}{\sqrt{-\_\alpha^2}}+\frac{2i\sqrt{2}\,\_\alpha^3\sqrt{2x^2+1}\,\sqrt{-x^2+1}\,\operatorname{EllipticPi}\left(\sqrt{-2}\,x,-\_\alpha^2,\frac{\sqrt{-2}}{2}\right)}{\sqrt{2x^4-x^2-1}}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+1)/(2*x^4-1)/(2*x^4-x^2-1)^(1/2),x)
[Out] -1/2*I*2^(1/2)*(2*x^2+1)^(1/2)*(-x^2+1)^(1/2)/(2*x^4-x^2-1)^(1/2)*EllipticF
(I*x*2^(1/2),1/2*I*2^(1/2))+1/4*sum(_alpha*(-1/(-_alpha^2)^(1/2)*arctanh(1/
14*(4*_alpha^2-1)*(-9*_alpha^2+7*x^2-4)/(-_alpha^2)^(1/2)/(2*x^4-x^2-1)^(1/
2))+2*I*2^(1/2)*_alpha^3*(2*x^2+1)^(1/2)*(-x^2+1)^(1/2)/(2*x^4-x^2-1)^(1/2)
*EllipticPi((-2)^(1/2)*x,-_alpha^2,1/2*(-2)^(1/2))),_alpha=RootOf(2*_Z^4-1)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^4+1}{\sqrt{2\,x^4-x^2-1}\,\left(2\,x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+1)/(2*x^4-1)/(2*x^4-x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x^4 + 1)/(sqrt(2*x^4 - x^2 - 1)*(2*x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{2\,x^4+1}{\left(2\,x^4-1\right)\,\sqrt{2\,x^4-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4 + 1)/((2*x^4 - 1)*(2*x^4 - x^2 - 1)^(1/2)),x)
[Out] int((2*x^4 + 1)/((2*x^4 - 1)*(2*x^4 - x^2 - 1)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^4 + 1}{\sqrt{(x-1)(x+1)(2x^2+1)}\,(2x^4-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+1)/(2*x**4-1)/(2*x**4-x**2-1)**(1/2),x)
[Out] Integral((2*x**4 + 1)/(sqrt((x - 1)*(x + 1)*(2*x**2 + 1))*(2*x**4 - 1)), x)
```

3.200 $\int \frac{-1+2x^4}{\left(1+2x^2+2x^4\right)\sqrt{1+3x^2+2x^4}}\,dx$

Optimal. Leaf size=21

$$-\tanh^{-1}\left(\frac{x}{\sqrt{2x^4+3x^2+1}}\right)$$

Rubi [A] time = 0.12, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2112, 206}

$$-\tanh^{-1}\left(\frac{x}{\sqrt{2x^4+3x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x^4)/((1 + 2*x^2 + 2*x^4)*Sqrt[1 + 3*x^2 + 2*x^4]),x]
```

```
[Out] -ArcTanh[x/Sqrt[1 + 3*x^2 + 2*x^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2112

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+2x^4}{\left(1+2x^2+2x^4\right)\sqrt{1+3x^2+2x^4}}\,dx &= -\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{1+3x^2+2x^4}}\right)\\ &= -\tanh^{-1}\left(\frac{x}{\sqrt{1+3x^2+2x^4}}\right)\end{aligned}$$

Mathematica [C] time = 0.33, size = 107, normalized size = 5.10

$$-\frac{i\sqrt{x^2+1}\sqrt{2x^2+1}\left(F\left(i\sinh^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)-\Pi\left(\frac{1}{2}-\frac{i}{2};i\sinh^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)-\Pi\left(\frac{1}{2}+\frac{i}{2};i\sinh^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)\right)}{\sqrt{4x^4+6x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x^4)/((1 + 2*x^2 + 2*x^4)*Sqrt[1 + 3*x^2 + 2*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[1 + 2*x^2]*(EllipticF[I*ArcSinh[Sqrt[2]*x], 1/2] -
 EllipticPi[1/2 - I/2, I*ArcSinh[Sqrt[2]*x], 1/2] - EllipticPi[1/2 + I/2, I
*ArcSinh[Sqrt[2]*x], 1/2]))/Sqrt[2 + 6*x^2 + 4*x^4]
```

IntegrateAlgebraic [A] time = 0.62, size = 21, normalized size = 1.00

$$-\tanh^{-1}\left(\frac{x}{\sqrt{2x^4+3x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x^4)/((1 + 2*x^2 + 2*x^4)*Sqrt[1 + 3*x^2 + 2*x^4
]),x]
```

```
[Out] -ArcTanh[x/Sqrt[1 + 3*x^2 + 2*x^4]]
```

fricas [B] time = 0.43, size = 47, normalized size = 2.24

$$\frac{1}{2}\,\log\left(\frac{2\,x^4+4\,x^2-2\,\sqrt{2\,x^4+3\,x^2+1}\,x+1}{2\,x^4+2\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)/(2*x^4+2*x^2+1)/(2*x^4+3*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*log((2*x^4 + 4*x^2 - 2*sqrt(2*x^4 + 3*x^2 + 1)*x + 1)/(2*x^4 + 2*x^2 +
1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^4-1}{\sqrt{2\,x^4+3\,x^2+1}\,\left(2\,x^4+2\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)/(2*x^4+2*x^2+1)/(2*x^4+3*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((2*x^4 - 1)/(sqrt(2*x^4 + 3*x^2 + 1)*(2*x^4 + 2*x^2 + 1)), x)
```

maple [C] time = 0.05, size = 170, normalized size = 8.10

$$-\frac{i\sqrt{x^2+1}\,\sqrt{2x^2+1}\,\operatorname{EllipticF}\left(ix,\sqrt{2}\right)}{\sqrt{2x^4+3x^2+1}}+\frac{\left(\sum_{\_\alpha=\operatorname{RootOf}\left(2\_Z^4+2\_Z^2+1\right)}\_\alpha\left(-\frac{\operatorname{arctanh}\left(\frac{\left(4\_\alpha^2+3\right)\left(\_\alpha^2+5x^2+4\right)}{10\sqrt{\_\alpha^2}\,\sqrt{2x^4+3x^2+1}}\right)}{\sqrt{\_\alpha^2}}+\frac{4i\left(-\_\alpha^3-\_\alpha\right)\sqrt{x^2+1}\,\sqrt{2x^2+1}\,\operatorname{EllipticPi}\left(ix,2\_\alpha^2+2,i\sqrt{-2}\right)}{\sqrt{2x^4+3x^2+1}}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-1)/(2*x^4+2*x^2+1)/(2*x^4+3*x^2+1)^(1/2),x)
```

```
[Out] -I*(x^2+1)^(1/2)*(2*x^2+1)^(1/2)/(2*x^4+3*x^2+1)^(1/2)*EllipticF(I*x,2^(1/2
))+1/4*sum(_alpha*(-1/(_alpha^2)^(1/2)*arctanh(1/10*(4*_alpha^2+3)*(_alpha^
2+5*x^2+4)/(_alpha^2)^(1/2)/(2*x^4+3*x^2+1)^(1/2))+4*I*(-_alpha^3-_alpha)*(
x^2+1)^(1/2)*(2*x^2+1)^(1/2)/(2*x^4+3*x^2+1)^(1/2)*EllipticPi(I*x,2*_alpha^
2+2,I*(-2)^(1/2))),_alpha=RootOf(2*_Z^4+2*_Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^4-1}{\sqrt{2\,x^4+3\,x^2+1}\,\left(2\,x^4+2\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)/(2*x^4+2*x^2+1)/(2*x^4+3*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((2*x^4 - 1)/(sqrt(2*x^4 + 3*x^2 + 1)*(2*x^4 + 2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{2\,x^4-1}{\left(2\,x^4+2\,x^2+1\right)\,\sqrt{2\,x^4+3\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4 - 1)/((2*x^2 + 2*x^4 + 1)*(3*x^2 + 2*x^4 + 1)^(1/2)),x)
```

```
[Out] int((2*x^4 - 1)/((2*x^2 + 2*x^4 + 1)*(3*x^2 + 2*x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^4-1}{\sqrt{\left(x^2+1\right)\left(2x^2+1\right)}\,\left(2x^4+2x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-1)/(2*x**4+2*x**2+1)/(2*x**4+3*x**2+1)**(1/2),x)
```

```
[Out] Integral((2*x**4 - 1)/(sqrt((x**2 + 1)*(2*x**2 + 1))*(2*x**4 + 2*x**2 + 1))
, x)
```

3.201 $\int \frac{\sqrt[4]{-1+x^4-x^5}\left(-4+x^5\right)}{x^6}\,dx$

Optimal. Leaf size=21

$$-\frac{4\left(-x^5+x^4-1\right)^{5/4}}{5x^5}$$

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1590}

$$-\frac{4\left(-x^5+x^4-1\right)^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4 - x^5)^(1/4)*(-4 + x^5))/x^6,x]
```

```
[Out] (-4*(-1 + x^4 - x^5)^(5/4))/(5*x^5)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{-1+x^4-x^5}\left(-4+x^5\right)}{x^6}\,dx=-\frac{4\left(-1+x^4-x^5\right)^{5/4}}{5x^5}$$

Mathematica [A] time = 0.05, size = 21, normalized size = 1.00

$$-\frac{4\left(-x^5+x^4-1\right)^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4 - x^5)^(1/4)*(-4 + x^5))/x^6,x]
```

```
[Out] (-4*(-1 + x^4 - x^5)^(5/4))/(5*x^5)
```

IntegrateAlgebraic [A] time = 0.31, size = 21, normalized size = 1.00

$$-\frac{4\left(-x^5+x^4-1\right)^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4 - x^5)^(1/4)*(-4 + x^5))/x^6,x]
```

```
[Out] (-4*(-1 + x^4 - x^5)^(5/4))/(5*x^5)
```

fricas [A] time = 0.41, size = 27, normalized size = 1.29

$$\frac{4\left(x^5 - x^4 + 1\right)\left(-x^5 + x^4 - 1\right)^{\frac{1}{4}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^5+x^4-1)^(1/4)*(x^5-4)/x^6,x, algorithm="fricas")
[Out] 4/5*(x^5 - x^4 + 1)*(-x^5 + x^4 - 1)^(1/4)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5 - 4\right)\left(-x^5 + x^4 - 1\right)^{\frac{1}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^5+x^4-1)^(1/4)*(x^5-4)/x^6,x, algorithm="giac")
[Out] integrate((x^5 - 4)*(-x^5 + x^4 - 1)^(1/4)/x^6, x)
```

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$-\frac{4\left(-x^5 + x^4 - 1\right)^{\frac{5}{4}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^5+x^4-1)^(1/4)*(x^5-4)/x^6,x)
[Out] -4/5*(-x^5+x^4-1)^(5/4)/x^5
```

maxima [A] time = 0.74, size = 27, normalized size = 1.29

$$\frac{4\left(x^5 - x^4 + 1\right)\left(-x^5 + x^4 - 1\right)^{\frac{1}{4}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^5+x^4-1)^(1/4)*(x^5-4)/x^6,x, algorithm="maxima")
[Out] 4/5*(x^5 - x^4 + 1)*(-x^5 + x^4 - 1)^(1/4)/x^5
```

mupad [B] time = 0.32, size = 17, normalized size = 0.81

$$-\frac{4\left(-x^5 + x^4 - 1\right)^{5/4}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 - 4)*(x^4 - x^5 - 1)^(1/4))/x^6,x)
[Out] -(4*(x^4 - x^5 - 1)^(5/4))/(5*x^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5 - 4\right)\sqrt[4]{-x^5 + x^4 - 1}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**5+x**4-1)**(1/4)*(x**5-4)/x**6,x)
[Out] Integral((x**5 - 4)*(-x**5 + x**4 - 1)**(1/4)/x**6, x)
```

3.202 $\int \frac{(1+x^8)\sqrt{-1-2x^4+x^8}}{x^7}\,dx$

Optimal. Leaf size=21

$$\frac{(x^8-2x^4-1)^{3/2}}{6x^6}$$

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1590}

$$\frac{(x^8-2x^4-1)^{3/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^8)*Sqrt[-1 - 2*x^4 + x^8])/x^7,x]
```

```
[Out] (-1 - 2*x^4 + x^8)^(3/2)/(6*x^6)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(1+x^8)\sqrt{-1-2x^4+x^8}}{x^7}\,dx=\frac{(-1-2x^4+x^8)^{3/2}}{6x^6}$$

Mathematica [A] time = 0.04, size = 21, normalized size = 1.00

$$\frac{(x^8-2x^4-1)^{3/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^8)*Sqrt[-1 - 2*x^4 + x^8])/x^7,x]
```

```
[Out] (-1 - 2*x^4 + x^8)^(3/2)/(6*x^6)
```

IntegrateAlgebraic [A] time = 0.69, size = 21, normalized size = 1.00

$$\frac{(x^8-2x^4-1)^{3/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^8)*Sqrt[-1 - 2*x^4 + x^8])/x^7,x]
```

```
[Out] (-1 - 2*x^4 + x^8)^(3/2)/(6*x^6)
```

fricas [A] time = 0.42, size = 17, normalized size = 0.81

$$\frac{\left(x^{8}-2\,x^{4}-1\right)^{\frac{3}{2}}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)*(x^8-2*x^4-1)^(1/2)/x^7,x, algorithm="fricas")
[Out] 1/6*(x^8 - 2*x^4 - 1)^(3/2)/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{8}-2\,x^{4}-1}\,\left(x^{8}+1\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)*(x^8-2*x^4-1)^(1/2)/x^7,x, algorithm="giac")
[Out] integrate(sqrt(x^8 - 2*x^4 - 1)*(x^8 + 1)/x^7, x)
```

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$\frac{\left(x^{8}-2x^{4}-1\right)^{\frac{3}{2}}}{6x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8+1)*(x^8-2*x^4-1)^(1/2)/x^7,x)
[Out] 1/6*(x^8-2*x^4-1)^(3/2)/x^6
```

maxima [A] time = 0.64, size = 17, normalized size = 0.81

$$\frac{\left(x^{8}-2\,x^{4}-1\right)^{\frac{3}{2}}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)*(x^8-2*x^4-1)^(1/2)/x^7,x, algorithm="maxima")
[Out] 1/6*(x^8 - 2*x^4 - 1)^(3/2)/x^6
```

mupad [B] time = 0.36, size = 17, normalized size = 0.81

$$\frac{{\left(x^8-2\,x^4-1\right)}^{3/2}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^8 + 1)*(x^8 - 2*x^4 - 1)^(1/2))/x^7,x)
[Out] (x^8 - 2*x^4 - 1)^(3/2)/(6*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{8}+1\right)\sqrt{x^{8}-2x^{4}-1}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8+1)*(x**8-2*x**4-1)**(1/2)/x**7,x)
[Out] Integral((x**8 + 1)*sqrt(x**8 - 2*x**4 - 1)/x**7, x)
```

3.203 $\int \frac{x^2(-2+x^3)\sqrt{1+x^3}}{4+12x^3+13x^6+4x^9}\,dx$

Optimal. Leaf size=21

$$-\frac{1}{3}\tan^{-1}\left(\frac{x^3}{2\left(x^3+1\right)^{3/2}}\right)$$

Rubi [A] time = 0.44, antiderivative size = 19, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {6715, 2094, 203}

$$\frac{1}{3}\tan^{-1}\left(\frac{2\left(x^3+1\right)^{3/2}}{x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(-2 + x^3)*Sqrt[1 + x^3])/(4 + 12*x^3 + 13*x^6 + 4*x^9),x]
```

```
[Out] ArcTan[(2*(1 + x^3)^(3/2))/x^3]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(-2+x^3\right)\sqrt{1+x^3}}{4+12x^3+13x^6+4x^9}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(-2+x)\sqrt{1+x}}{4+12x+13x^2+4x^3}\,dx, x, x^3\right)\\
&= \frac{2}{3}\text{Subst}\left(\int \frac{x^2\left(-3+x^2\right)}{1-2x^2+x^4+4x^6}\,dx, x, \sqrt{1+x^3}\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{1+36x^2}\,dx, x, \frac{\left(1+x^3\right)^{3/2}}{3x^3}\right)\\
&= \frac{1}{3}\tan^{-1}\left(\frac{2\left(1+x^3\right)^{3/2}}{x^3}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 27, normalized size = 1.29

$$\frac{1}{3}\tan^{-1}\left(\frac{6\left(x^3+1\right)^{3/2}}{3\left(x^3+1\right)-3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(-2 + x^3)*Sqrt[1 + x^3])/(4 + 12*x^3 + 13*x^6 + 4*x^9),x]
[Out] ArcTan[(6*(1 + x^3)^(3/2))/(-3 + 3*(1 + x^3))]/3
```

IntegrateAlgebraic [A] time = 0.05, size = 37, normalized size = 1.76

$$\frac{1}{3}\tan^{-1}\left(\sqrt{x^3+1}\right)-\frac{1}{3}\tan^{-1}\left(\frac{2x^3+1}{\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(-2 + x^3)*Sqrt[1 + x^3])/(4 + 12*x^3 + 13*x^6 + 4*
x^9),x]
[Out] ArcTan[Sqrt[1 + x^3]]/3 - ArcTan[(1 + 2*x^3)/Sqrt[1 + x^3]]/3
```

fricas [A] time = 0.42, size = 15, normalized size = 0.71

$$\frac{1}{3}\arctan\left(\frac{2\left(x^3+1\right)^{\frac{3}{2}}}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)*(x^3+1)^(1/2)/(4*x^9+13*x^6+12*x^3+4),x, algorithm="f
ricas")
[Out] 1/3*arctan(2*(x^3 + 1)^(3/2)/x^3)
```

giac [B] time = 0.32, size = 67, normalized size = 3.19

$$-\frac{1}{3}\arctan\left(2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}+4\sqrt{x^3+1}\right)\right)-\frac{1}{3}\arctan\left(-2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}-4\sqrt{x^3+1}\right)\right)+\frac{1}{3}\arctan\left(\sqrt{x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)*(x^3+1)^(1/2)/(4*x^9+13*x^6+12*x^3+4),x, algorithm="g
iac")
[Out] -1/3*arctan(2*sqrt(2)*(1/4)^(3/4)*(sqrt(14)*(1/4)^(1/4) + 4*sqrt(x^3 + 1)))
 - 1/3*arctan(-2*sqrt(2)*(1/4)^(3/4)*(sqrt(14)*(1/4)^(1/4) - 4*sqrt(x^3 + 1
))) + 1/3*arctan(sqrt(x^3 + 1))
```

maple [C] time = 0.21, size = 417, normalized size = 19.86

$$\frac{\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}(\_Z^3+2)}\frac{(\_\alpha^2-\_\alpha+1)(3-i\sqrt{3})\sqrt{\frac{1+x}{3-i\sqrt{3}}}\sqrt{\frac{-1+2x-i\sqrt{3}}{-3-i\sqrt{3}}}\sqrt{\frac{-1+2x+i\sqrt{3}}{-3+i\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{3\_\alpha^2}{2}+\frac{3\_\alpha}{2}-\frac{3}{2}+\frac{i\_\alpha^2\sqrt{3}}{2}-\frac{i\sqrt{3}\_\alpha}{2}+\frac{i\sqrt{3}}{2},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}\right)}{6}+\frac{\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}(4\_Z^6+5\_Z^3+2)}\frac{(2\_\alpha^3+1)(4\_\alpha^5-4\_\alpha^4+4\_\alpha^3+\_\alpha^2-\_\alpha+1)(3-i\sqrt{3})\sqrt{\frac{1+x}{3-i\sqrt{3}}}\sqrt{\frac{-1+2x-i\sqrt{3}}{-3-i\sqrt{3}}}\sqrt{\frac{-1+2x+i\sqrt{3}}{-3+i\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-6\_\alpha^5+6\_\alpha^4-6\_\alpha^3-\frac{3\_\alpha^2}{2}+\frac{3\_\alpha}{2}-\frac{3}{2}+2i\sqrt{3}\_\alpha^5-2i\_\alpha^4\sqrt{3}+2i\_\alpha^3\sqrt{3}+\frac{i\_\alpha^2\sqrt{3}}{2}-\frac{i\sqrt{3}\_\alpha}{2}+\frac{i\sqrt{3}}{2},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3-2)*(x^3+1)^(1/2)/(4*x^9+13*x^6+12*x^3+4),x)
[Out] 1/6*2^(1/2)*sum((_alpha^2-_alpha+1)*(3-I*3^(1/2))*((1+x)/(3-I*3^(1/2)))^(1/
2)*((-1+2*x-I*3^(1/2))/(-3-I*3^(1/2)))^(1/2)*((-1+2*x+I*3^(1/2))/(-3+I*3^(1
```

```
/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-3/2
*_alpha^2+3/2*_alpha-3/2+1/2*I*_alpha^2*3^(1/2)-1/2*I*3^(1/2)*_alpha+1/2*I*
3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z
^3+2))+1/6*2^(1/2)*sum((2*_alpha^3+1)*(4*_alpha^5-4*_alpha^4+4*_alpha^3+_al
pha^2-_alpha+1)*(3-I*3^(1/2))*((1+x)/(3-I*3^(1/2)))^(1/2)*((-1+2*x-I*3^(1/2
))/(-3-I*3^(1/2)))^(1/2)*((-1+2*x+I*3^(1/2))/(-3+I*3^(1/2)))^(1/2)/(x^3+1)^
(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-6*_alpha^5+6*_alpha^4-6
*_alpha^3-3/2*_alpha^2+3/2*_alpha-3/2+2*I*3^(1/2)*_alpha^5-2*I*3^(1/2)*_alp
ha^4+2*I*3^(1/2)*_alpha^3+1/2*I*_alpha^2*3^(1/2)-1/2*I*3^(1/2)*_alpha+1/2*I
*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(4
*_Z^6+5*_Z^3+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^3+1}\left(x^3-2\right)x^2}{4\,x^9+13\,x^6+12\,x^3+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)*(x^3+1)^(1/2)/(4*x^9+13*x^6+12*x^3+4),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(x^3 + 1)*(x^3 - 2)*x^2/(4*x^9 + 13*x^6 + 12*x^3 + 4), x)
```

mupad [B] time = 0.10, size = 2737, normalized size = 130.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^3 + 1)^(1/2)*(x^3 - 2))/(12*x^3 + 13*x^6 + 4*x^9 + 4),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/((- (7^(1/
2)*1i)/8 - 5/8)^(1/3) + 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -
((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(2*(- (7^(1/2)*1i)/8 - 5/8)^
(2/3) + (- (7^(1/2)*1i)/8 - 5/8)^(5/3) - (- (7^(1/2)*1i)/8 - 5/8)^(8/3)))/(
((- (7^(1/2)*1i)/8 - 5/8)^(1/3) + 1)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/
2)*(36*(- (7^(1/2)*1i)/8 - 5/8)^(2/3) + 78*(- (7^(1/2)*1i)/8 - 5/8)^(5/3) +
 36*(- (7^(1/2)*1i)/8 - 5/8)^(8/3))) - (((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/
2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipti
cPi(((3^(1/2)*1i)/2 + 3/2)/((-1)^(1/3)*2^(1/3) + 1), asin(((x + 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3
*((-1)^(1/3)*2^(1/3) + 1)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) + (2*((
3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1
/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(((7^(1/2)*1i)/
8 - 5/8)^(1/3) + 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/
2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(2*((7^(1/2)*1i)/8 - 5/8)^(2/3) + (
(7^(1/2)*1i)/8 - 5/8)^(5/3) - ((7^(1/2)*1i)/8 - 5/8)^(8/3)))/((((7^(1/2)*1i
)/8 - 5/8)^(1/3) + 1)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/
2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)*(36*((7^(1/2
)*1i)/8 - 5/8)^(2/3) + 78*((7^(1/2)*1i)/8 - 5/8)^(5/3) + 36*((7^(1/2)*1i)/8
 - 5/8)^(8/3))) + (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2
)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2
 + 3/2)/(((3^(1/2)*1i)/2 - 1/2)*(- (7^(1/2)*1i)/8 - 5/8)^(1/3) + 1), asin((
(x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1
```

```
i)/2 - 3/2))*(2*((3^(1/2)*1i)/2 - 1/2)^2*(- (7^(1/2)*1i)/8 - 5/8)^(2/3) + (
(3^(1/2)*1i)/2 - 1/2)^5*(- (7^(1/2)*1i)/8 - 5/8)^(5/3) - ((3^(1/2)*1i)/2 -
1/2)^8*(- (7^(1/2)*1i)/8 - 5/8)^(8/3)))/((((3^(1/2)*1i)/2 - 1/2)*(- (7^(1/2
)*1i)/8 - 5/8)^(1/3) + 1)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)*(36*((3^
(1/2)*1i)/2 - 1/2)^2*(- (7^(1/2)*1i)/8 - 5/8)^(2/3) + 78*((3^(1/2)*1i)/2 -
1/2)^5*(- (7^(1/2)*1i)/8 - 5/8)^(5/3) + 36*((3^(1/2)*1i)/2 - 1/2)^8*(- (7^(
1/2)*1i)/8 - 5/8)^(8/3))) + (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2
- 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(
1/2)*1i)/2 + 3/2)/(((3^(1/2)*1i)/2 - 1/2)*((7^(1/2)*1i)/8 - 5/8)^(1/3) + 1)
, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3
^(1/2)*1i)/2 - 3/2))*(2*((3^(1/2)*1i)/2 - 1/2)^2*((7^(1/2)*1i)/8 - 5/8)^(2/
3) + ((3^(1/2)*1i)/2 - 1/2)^5*((7^(1/2)*1i)/8 - 5/8)^(5/3) - ((3^(1/2)*1i)/
2 - 1/2)^8*((7^(1/2)*1i)/8 - 5/8)^(8/3)))/((((3^(1/2)*1i)/2 - 1/2)*((7^(1/2
)*1i)/8 - 5/8)^(1/3) + 1)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)*(36*((3^
(1/2)*1i)/2 - 1/2)^2*((7^(1/2)*1i)/8 - 5/8)^(2/3) + 78*((3^(1/2)*1i)/2 - 1/
2)^5*((7^(1/2)*1i)/8 - 5/8)^(5/3) + 36*((3^(1/2)*1i)/2 - 1/2)^8*((7^(1/2)*1
i)/8 - 5/8)^(8/3))) + (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)
/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^
(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*
1i)/2 + 3/2)/(((3^(1/2)*1i)/2 + 1/2)*(- (7^(1/2)*1i)/8 - 5/8)^(1/3) - 1), a
sin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1
/2)*1i)/2 - 3/2))*(((3^(1/2)*1i)/2 + 1/2)^5*(- (7^(1/2)*1i)/8 - 5/8)^(5/3)
- 2*((3^(1/2)*1i)/2 + 1/2)^2*(- (7^(1/2)*1i)/8 - 5/8)^(2/3) + ((3^(1/2)*1i)
/2 + 1/2)^8*(- (7^(1/2)*1i)/8 - 5/8)^(8/3)))/((((3^(1/2)*1i)/2 + 1/2)*(- (7
^(1/2)*1i)/8 - 5/8)^(1/3) - 1)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)*(36
*((3^(1/2)*1i)/2 + 1/2)^2*(- (7^(1/2)*1i)/8 - 5/8)^(2/3) - 78*((3^(1/2)*1i)
/2 + 1/2)^5*(- (7^(1/2)*1i)/8 - 5/8)^(5/3) + 36*((3^(1/2)*1i)/2 + 1/2)^8*(-
 (7^(1/2)*1i)/8 - 5/8)^(8/3))) + (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1
i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(
-((3^(1/2)*1i)/2 + 3/2)/(((3^(1/2)*1i)/2 + 1/2)*((7^(1/2)*1i)/8 - 5/8)^(1/3
) - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/
2)/((3^(1/2)*1i)/2 - 3/2))*(((3^(1/2)*1i)/2 + 1/2)^5*((7^(1/2)*1i)/8 - 5/8)
^(5/3) - 2*((3^(1/2)*1i)/2 + 1/2)^2*((7^(1/2)*1i)/8 - 5/8)^(2/3) + ((3^(1/2
)*1i)/2 + 1/2)^8*((7^(1/2)*1i)/8 - 5/8)^(8/3)))/((((3^(1/2)*1i)/2 + 1/2)*((
7^(1/2)*1i)/8 - 5/8)^(1/3) - 1)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)*(3
6*((3^(1/2)*1i)/2 + 1/2)^2*((7^(1/2)*1i)/8 - 5/8)^(2/3) - 78*((3^(1/2)*1i)/
2 + 1/2)^5*((7^(1/2)*1i)/8 - 5/8)^(5/3) + 36*((3^(1/2)*1i)/2 + 1/2)^8*((7^(
1/2)*1i)/8 - 5/8)^(8/3))) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2
- 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(2*(-1)^(2/3)*2^
(2/3)*((3^(1/2)*1i)/2 - 1/2)^5 - 2*(-1)^(2/3)*2^(2/3)*((3^(1/2)*1i)/2 - 1/2
)^2 + 4*(-1)^(2/3)*2^(2/3)*((3^(1/2)*1i)/2 - 1/2)^8)*ellipticPi(((3^(1/2)*1
i)/2 + 3/2)/((-1)^(1/3)*2^(1/3)*((3^(1/2)*1i)/2 - 1/2) + 1), asin(((x + 1)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3
/2)))/(((-1)^(1/3)*2^(1/3)*((3^(1/2)*1i)/2 - 1/2) + 1)*(x^3 - x*(((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/2 + 1/2))^(1/2)*(36*(-1)^(2/3)*2^(2/3)*((3^(1/2)*1i)/2 - 1/2)^2 - 156*
(-1)^(2/3)*2^(2/3)*((3^(1/2)*1i)/2 - 1/2)^5 + 144*(-1)^(2/3)*2^(2/3)*((3^(1
/2)*1i)/2 - 1/2)^8)) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2
)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3
^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(2*(-1)^(2/3)*2^(2/3)
*((3^(1/2)*1i)/2 + 1/2)^2 + 2*(-1)^(2/3)*2^(2/3)*((3^(1/2)*1i)/2 + 1/2)^5 -
 4*(-1)^(2/3)*2^(2/3)*((3^(1/2)*1i)/2 + 1/2)^8)*ellipticPi(-((3^(1/2)*1i)/2
```

```
 + 3/2)/((-1)^(1/3)*2^(1/3)*((3^(1/2)*1i)/2 + 1/2) - 1), asin(((x + 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
)/(((-1)^(1/3)*2^(1/3)*((3^(1/2)*1i)/2 + 1/2) - 1)*(x^3 - x*(((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/2 + 1/2))^(1/2)*(36*(-1)^(2/3)*2^(2/3)*((3^(1/2)*1i)/2 + 1/2)^2 + 156*(-1)
^(2/3)*2^(2/3)*((3^(1/2)*1i)/2 + 1/2)^5 + 144*(-1)^(2/3)*2^(2/3)*((3^(1/2)*
1i)/2 + 1/2)^8))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3-2)*(x**3+1)**(1/2)/(4*x**9+13*x**6+12*x**3+4),x)
[Out] Timed out
```

3.204 $\int \frac{1}{(-1+x^2)\sqrt[3]{-x+x^3}}\,dx$

Optimal. Leaf size=22

$$-\frac{3\left(x^3-x\right)^{2/3}}{2\left(x^2-1\right)}$$

Rubi [A] time = 0.04, antiderivative size = 16, normalized size of antiderivative = 0.73, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2056, 264}

$$-\frac{3x}{2\sqrt[3]{x^3-x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x^2)*(-x + x^3)^(1/3)),x]
```

```
[Out] (-3*x)/(2*(-x + x^3)^(1/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-1+x^2\right)\sqrt[3]{-x+x^3}}\,dx &= \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\int \frac{1}{\sqrt[3]{x}\left(-1+x^2\right)^{4/3}}\,dx}{\sqrt[3]{-x+x^3}}\\ &= -\frac{3x}{2\sqrt[3]{-x+x^3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.73

$$-\frac{3x}{2\sqrt[3]{x\left(x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x^2)*(-x + x^3)^(1/3)),x]
```

```
[Out] (-3*x)/(2*(x*(-1 + x^2))^(1/3))
```

IntegrateAlgebraic [A] time = 0.11, size = 22, normalized size = 1.00

$$-\frac{3\left(x^3-x\right)^{2/3}}{2\left(x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x^2)*(-x + x^3)^(1/3)),x]
[Out] (-3*(-x + x^3)^(2/3))/(2*(-1 + x^2))
```

fricas [A] time = 0.40, size = 18, normalized size = 0.82

$$-\frac{3\left(x^3-x\right)^{\frac{2}{3}}}{2\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x^3-x)^(1/3),x, algorithm="fricas")
[Out] -3/2*(x^3 - x)^(2/3)/(x^2 - 1)
```

giac [A] time = 0.64, size = 11, normalized size = 0.50

$$-\frac{3}{2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x^3-x)^(1/3),x, algorithm="giac")
[Out] -3/2/(-1/x^2 + 1)^(1/3)
```

maple [A] time = 0.01, size = 13, normalized size = 0.59

$$-\frac{3x}{2\left(x^3-x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)/(x^3-x)^(1/3),x)
[Out] -3/2*x/(x^3-x)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3-x\right)^{\frac{1}{3}}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x^3-x)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^3 - x)^(1/3)*(x^2 - 1)), x)
```

mupad [B] time = 0.16, size = 20, normalized size = 0.91

$$-\frac{3\left(x^3-x\right)^{2/3}}{2\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 - x)^(1/3)*(x^2 - 1)),x)
```

```
[Out] -(3*(x^3 - x)^(2/3))/(2*(x^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x(x-1)(x+1)}\,(x-1)(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)/(x**3-x)**(1/3),x)
```

```
[Out] Integral(1/((x*(x - 1)*(x + 1))**(1/3)*(x - 1)*(x + 1)), x)
```

3.205 $\int \frac{1}{(-1+x^3)\sqrt[4]{-x+x^4}}\,dx$

Optimal. Leaf size=22

$$-\frac{4\left(x^4-x\right)^{3/4}}{3\left(x^3-1\right)}$$

Rubi [A] time = 0.04, antiderivative size = 16, normalized size of antiderivative = 0.73, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2056, 264}

$$-\frac{4x}{3\sqrt[4]{x^4-x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x^3)*(-x + x^4)^(1/4)),x]
```

```
[Out] (-4*x)/(3*(-x + x^4)^(1/4))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-1+x^3\right)\sqrt[4]{-x+x^4}}\,dx &= \frac{\left(\sqrt[4]{x}\,\sqrt[4]{-1+x^3}\right)\int \frac{1}{\sqrt[4]{x}\left(-1+x^3\right)^{5/4}}\,dx}{\sqrt[4]{-x+x^4}}\\ &= -\frac{4x}{3\sqrt[4]{-x+x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.73

$$-\frac{4x}{3\sqrt[4]{x\left(x^3-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x^3)*(-x + x^4)^(1/4)),x]
```

```
[Out] (-4*x)/(3*(x*(-1 + x^3))^(1/4))
```

IntegrateAlgebraic [A] time = 0.22, size = 22, normalized size = 1.00

$$-\frac{4\left(x^4-x\right)^{3/4}}{3\left(x^3-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x^3)*(-x + x^4)^(1/4)),x]
```

```
[Out] (-4*(-x + x^4)^(3/4))/(3*(-1 + x^3))
```

fricas [A] time = 0.40, size = 18, normalized size = 0.82

$$-\frac{4\left(x^4-x\right)^{\frac{3}{4}}}{3\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)/(x^4-x)^(1/4),x, algorithm="fricas")
```

```
[Out] -4/3*(x^4 - x)^(3/4)/(x^3 - 1)
```

giac [A] time = 0.39, size = 11, normalized size = 0.50

$$\frac{4}{3\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)/(x^4-x)^(1/4),x, algorithm="giac")
```

```
[Out] 4/3/(-1/x^3 + 1)^(1/4)
```

maple [A] time = 0.00, size = 13, normalized size = 0.59

$$-\frac{4x}{3\left(x^4-x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-1)/(x^4-x)^(1/4),x)
```

```
[Out] -4/3*x/(x^4-x)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4-x\right)^{\frac{1}{4}}\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)/(x^4-x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 - x)^(1/4)*(x^3 - 1)), x)
```

mupad [B] time = 0.15, size = 20, normalized size = 0.91

$$-\frac{4\left(x^4-x\right)^{3/4}}{3\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^4 - x)^(1/4)*(x^3 - 1)),x)
```

```
[Out] -(4*(x^4 - x)^(3/4))/(3*(x^3 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x(x-1)\left(x^2+x+1\right)}\,(x-1)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-1)/(x**4-x)**(1/4),x)
[Out] Integral(1/((x*(x - 1)*(x**2 + x + 1))**(1/4)*(x - 1)*(x**2 + x + 1)), x)
```

3.206 $\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\sqrt{b^2+ax^2}} \, dx$

Optimal. Leaf size=22

$$\frac{2x}{\sqrt{\sqrt{ax^2+b^2}+b}}$$

Rubi [F] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\sqrt{b^2+ax^2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b + Sqrt[b^2 + a*x^2]]/Sqrt[b^2 + a*x^2],x]
[Out] Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/Sqrt[b^2 + a*x^2], x]
```

Rubi steps

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\sqrt{b^2+ax^2}} \, dx = \int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\sqrt{b^2+ax^2}} \, dx$$

Mathematica [A] time = 0.13, size = 22, normalized size = 1.00

$$\frac{2x}{\sqrt{\sqrt{ax^2+b^2}+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/Sqrt[b^2 + a*x^2],x]
[Out] (2*x)/Sqrt[b + Sqrt[b^2 + a*x^2]]
```

IntegrateAlgebraic [A] time = 0.10, size = 22, normalized size = 1.00

$$\frac{2x}{\sqrt{\sqrt{ax^2+b^2}+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + Sqrt[b^2 + a*x^2]]/Sqrt[b^2 + a*x^2],x]
[Out] (2*x)/Sqrt[b + Sqrt[b^2 + a*x^2]]
```

fricas [B] time = 0.47, size = 38, normalized size = 1.73

$$-\frac{2\sqrt{b+\sqrt{ax^2+b^2}}\left(b-\sqrt{ax^2+b^2}\right)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] -2*sqrt(b + sqrt(a*x^2 + b^2))*(b - sqrt(a*x^2 + b^2))/(a*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\sqrt{ax^2+b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/sqrt(a*x^2 + b^2), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{\sqrt{a\,x^2+b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^(1/2),x)
```

```
[Out] int((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\sqrt{ax^2+b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/sqrt(a*x^2 + b^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{b+\sqrt{b^2+a\,x^2}}}{\sqrt{b^2+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(a*x^2 + b^2)^(1/2),x)
```

```
[Out] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(a*x^2 + b^2)^(1/2), x)
```

sympy [B] time = 0.72, size = 41, normalized size = 1.86

$$\frac{\sqrt{2}\,x\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{\pi\sqrt{b}\sqrt{\sqrt{\frac{ax^2}{b^2}+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x**2+b**2)**(1/2))**(1/2)/(a*x**2+b**2)**(1/2),x)
```

```
[Out] sqrt(2)*x*gamma(1/4)*gamma(3/4)/(pi*sqrt(b)*sqrt(sqrt(a*x**2/b**2 + 1) + 1)
)
```

3.207 $\int \frac{1}{x\sqrt[4]{1+x^2}}\,dx$

Optimal. Leaf size=23

$$\tan^{-1}\left(\sqrt[4]{x^2+1}\right)-\tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {266, 63, 298, 203, 206}

$$\tan^{-1}\left(\sqrt[4]{x^2+1}\right)-\tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^2)^(1/4)),x]
```

```
[Out] ArcTan[(1 + x^2)^(1/4)] - ArcTanh[(1 + x^2)^(1/4)]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt[4]{1+x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1+x}}\,dx, x, x^2\right)\\ &= 2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt[4]{1+x^2}\right)\\ &= -\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1+x^2}\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{1+x^2}\right)\\ &= \tan^{-1}\left(\sqrt[4]{1+x^2}\right) - \tanh^{-1}\left(\sqrt[4]{1+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\tan^{-1}\left(\sqrt[4]{x^2+1}\right) - \tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^2)^(1/4)),x]
[Out] ArcTan[(1 + x^2)^(1/4)] - ArcTanh[(1 + x^2)^(1/4)]
```

IntegrateAlgebraic [A] time = 0.03, size = 23, normalized size = 1.00

$$\tan^{-1}\left(\sqrt[4]{x^2+1}\right) - \tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(1 + x^2)^(1/4)),x]
[Out] ArcTan[(1 + x^2)^(1/4)] - ArcTanh[(1 + x^2)^(1/4)]
```

fricas [A] time = 0.41, size = 33, normalized size = 1.43

$$\arctan\left(\left(x^2+1\right)^{\frac{1}{4}}\right) - \frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}+1\right) + \frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^(1/4),x, algorithm="fricas")
[Out] arctan((x^2 + 1)^(1/4)) - 1/2*log((x^2 + 1)^(1/4) + 1) + 1/2*log((x^2 + 1)^
(1/4) - 1)
```

giac [A] time = 0.37, size = 33, normalized size = 1.43

$$\arctan\left(\left(x^2+1\right)^{\frac{1}{4}}\right) - \frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}+1\right) + \frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^(1/4),x, algorithm="giac")
[Out] arctan((x^2 + 1)^(1/4)) - 1/2*log((x^2 + 1)^(1/4) + 1) + 1/2*log((x^2 + 1)^
(1/4) - 1)
```

maple [C] time = 0.21, size = 59, normalized size = 2.57

$$\frac{\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)\left(-\frac{\pi\sqrt{2}\,x^2\,\text{hypergeom}\left(\left[1,1,\frac{5}{4}\right],[2,2],-x^2\right)}{4\Gamma\left(\frac{3}{4}\right)} + \frac{\left(-3\ln(2)-\frac{\pi}{2}+2\ln(x)\right)\pi\sqrt{2}}{\Gamma\left(\frac{3}{4}\right)}\right)}{4\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+1)^(1/4),x)
```

```
[Out] 1/4/Pi*2^(1/2)*GAMMA(3/4)*(-1/4*Pi*2^(1/2)/GAMMA(3/4)*x^2*hypergeom([1,1,5/
4],[2,2],-x^2)+(-3*ln(2)-1/2*Pi+2*ln(x))*Pi*2^(1/2)/GAMMA(3/4))
```

maxima [A] time = 0.45, size = 33, normalized size = 1.43

$$\arctan\left(\left(x^2+1\right)^{\frac{1}{4}}\right)-\frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^(1/4),x, algorithm="maxima")
```

```
[Out] arctan((x^2 + 1)^(1/4)) - 1/2*log((x^2 + 1)^(1/4) + 1) + 1/2*log((x^2 + 1)^
(1/4) - 1)
```

mupad [B] time = 0.25, size = 19, normalized size = 0.83

$$\mathrm{atan}\left(\left(x^2+1\right)^{1/4}\right)-\mathrm{atanh}\left(\left(x^2+1\right)^{1/4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 + 1)^(1/4)),x)
```

```
[Out] atan((x^2 + 1)^(1/4)) - atanh((x^2 + 1)^(1/4))
```

sympy [C] time = 0.74, size = 32, normalized size = 1.39

$$-\frac{\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^2}\right)}{2\sqrt{x}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+1)**(1/4),x)
```

```
[Out] -gamma(1/4)*hyper((1/4, 1/4), (5/4,), exp_polar(I*pi)/x**2)/(2*sqrt(x)*gamm
a(5/4))
```

3.208 $\int \frac{-5+2x}{\sqrt[4]{4-4x+x^2}}\,dx$

Optimal. Leaf size=23

$$\frac{2\left((x-2)^2\right)^{3/4}(2x-7)}{3(x-2)}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.57, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {640, 609}

$$\frac{2(2-x)}{\sqrt[4]{x^2-4x+4}}+\frac{4}{3}\left(x^2-4x+4\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 2*x)/(4 - 4*x + x^2)^(1/4),x]
```

```
[Out] (2*(2 - x))/(4 - 4*x + x^2)^(1/4) + (4*(4 - 4*x + x^2)^(3/4))/3
```

Rule 609

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)
*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{-5+2x}{\sqrt[4]{4-4x+x^2}}\,dx &= \frac{4}{3}\left(4-4x+x^2\right)^{3/4}-\int \frac{1}{\sqrt[4]{4-4x+x^2}}\,dx\\ &= \frac{2(2-x)}{\sqrt[4]{4-4x+x^2}}+\frac{4}{3}\left(4-4x+x^2\right)^{3/4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.91

$$\frac{2(x-2)(2x-7)}{3\sqrt[4]{(x-2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 2*x)/(4 - 4*x + x^2)^(1/4),x]
```

```
[Out] (2*(-2 + x)*(-7 + 2*x))/(3*((-2 + x)^2)^(1/4))
```

IntegrateAlgebraic [A] time = 4.73, size = 39, normalized size = 1.70

$$\frac{2\left(2(x-2)^{3/2}-3\sqrt{x-2}\right)\left((x-2)^2\right)^{3/4}}{3(x-2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-5 + 2*x)/(4 - 4*x + x^2)^(1/4),x]
```

```
[Out] (2*(-3*Sqrt[-2 + x] + 2*(-2 + x)^(3/2))*((-2 + x)^2)^(3/4))/(3*(-2 + x)^(3/
2))
```

fricas [A] time = 0.38, size = 17, normalized size = 0.74

$$\frac{2}{3}\left(x^2-4x+4\right)^{\frac{1}{4}}(2x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(x^2-4*x+4)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 - 4*x + 4)^(1/4)*(2*x - 7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-5}{\left(x^2-4x+4\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(x^2-4*x+4)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((2*x - 5)/(x^2 - 4*x + 4)^(1/4), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.91

$$\frac{2(-2+x)(-7+2x)}{3\left(x^2-4x+4\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+2*x)/(x^2-4*x+4)^(1/4),x)
```

```
[Out] 2/3*(-2+x)*(-7+2*x)/(x^2-4*x+4)^(1/4)
```

maxima [A] time = 0.32, size = 23, normalized size = 1.00

$$\frac{4\left(x^2+2x-8\right)}{3\sqrt{x-2}}-10\sqrt{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(x^2-4*x+4)^(1/4),x, algorithm="maxima")
```

```
[Out] 4/3*(x^2 + 2*x - 8)/sqrt(x - 2) - 10*sqrt(x - 2)
```

mupad [B] time = 0.11, size = 24, normalized size = 1.04

$$\frac{2\,(2x-7)\,\left(x^2-4x+4\right)^{3/4}}{3\,(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 5)/(x^2 - 4*x + 4)^(1/4),x)
```

```
[Out] (2*(2*x - 7)*(x^2 - 4*x + 4)^(3/4))/(3*(x - 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-5}{\sqrt[4]{(x-2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(x**2-4*x+4)**(1/4),x)
```

```
[Out] Integral((2*x - 5)/((x - 2)**2)**(1/4), x)
```

3.209 $\int x^5 \sqrt[3]{1+x^3}\, dx$

Optimal. Leaf size=23

$$\frac{1}{28}\sqrt[3]{x^3+1}\left(4x^6+x^3-3\right)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{1}{7}\left(x^3+1\right)^{7/3}-\frac{1}{4}\left(x^3+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(1 + x^3)^(1/3),x]
```

```
[Out] -1/4*(1 + x^3)^(4/3) + (1 + x^3)^(7/3)/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5 \sqrt[3]{1+x^3}\, dx &= \frac{1}{3}\text{Subst}\left(\int x\sqrt[3]{1+x}\, dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(-\sqrt[3]{1+x}+(1+x)^{4/3}\right) dx, x, x^3\right)\\ &= -\frac{1}{4}\left(1+x^3\right)^{4/3}+\frac{1}{7}\left(1+x^3\right)^{7/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{1}{28}\left(x^3+1\right)^{4/3}\left(4x^3-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(1 + x^3)^(1/3),x]
```

```
[Out] ((1 + x^3)^(4/3)*(-3 + 4*x^3))/28
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 0.87

$$\frac{1}{28}\left(x^3+1\right)^{4/3}\left(4x^3-3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(1 + x^3)^(1/3),x]
[Out] ((1 + x^3)^(4/3)*(-3 + 4*x^3))/28
```

fricas [A] time = 0.38, size = 19, normalized size = 0.83

$$\frac{1}{28}\left(4\,x^6 + x^3 - 3\right)\left(x^3 + 1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^3+1)^(1/3),x, algorithm="fricas")
[Out] 1/28*(4*x^6 + x^3 - 3)*(x^3 + 1)^(1/3)
```

giac [A] time = 0.62, size = 19, normalized size = 0.83

$$\frac{1}{7}\left(x^3 + 1\right)^{\frac{7}{3}} - \frac{1}{4}\left(x^3 + 1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^3+1)^(1/3),x, algorithm="giac")
[Out] 1/7*(x^3 + 1)^(7/3) - 1/4*(x^3 + 1)^(4/3)
```

maple [A] time = 0.00, size = 28, normalized size = 1.22

$$\frac{(1 + x)\left(x^2 - x + 1\right)\left(4x^3 - 3\right)\left(x^3 + 1\right)^{\frac{1}{3}}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^3+1)^(1/3),x)
[Out] 1/28*(1+x)*(x^2-x+1)*(4*x^3-3)*(x^3+1)^(1/3)
```

maxima [A] time = 0.37, size = 19, normalized size = 0.83

$$\frac{1}{7}\left(x^3 + 1\right)^{\frac{7}{3}} - \frac{1}{4}\left(x^3 + 1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^3+1)^(1/3),x, algorithm="maxima")
[Out] 1/7*(x^3 + 1)^(7/3) - 1/4*(x^3 + 1)^(4/3)
```

mupad [B] time = 0.19, size = 20, normalized size = 0.87

$$\left(x^3 + 1\right)^{1/3}\left(\frac{x^6}{7} + \frac{x^3}{28} - \frac{3}{28}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^3 + 1)^(1/3),x)
[Out] (x^3 + 1)^(1/3)*(x^3/28 + x^6/7 - 3/28)
```

sympy [A] time = 0.36, size = 37, normalized size = 1.61

$$\frac{x^6\sqrt[3]{x^3 + 1}}{7} + \frac{x^3\sqrt[3]{x^3 + 1}}{28} - \frac{3\sqrt[3]{x^3 + 1}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**3+1)**(1/3),x)
```

```
[Out] x**6*(x**3 + 1)**(1/3)/7 + x**3*(x**3 + 1)**(1/3)/28 - 3*(x**3 + 1)**(1/3)/
28
```

3.210 $\int \frac{(1+3x^2)\sqrt[3]{-x+x^3}}{x^2} \, dx$

Optimal. Leaf size=23

$$\frac{3(x^2-1)\sqrt[3]{x^3-x}}{2x}$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 0.78, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1590}

$$\frac{3(x^3-x)^{4/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 3*x^2)*(-x + x^3)^(1/3))/x^2,x]
```

```
[Out] (3*(-x + x^3)^(4/3))/(2*x^2)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(1+3x^2)\sqrt[3]{-x+x^3}}{x^2} \, dx = \frac{3(-x+x^3)^{4/3}}{2x^2}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.78

$$\frac{3(x(x^2-1))^{4/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 3*x^2)*(-x + x^3)^(1/3))/x^2,x]
```

```
[Out] (3*(x*(-1 + x^2))^(4/3))/(2*x^2)
```

IntegrateAlgebraic [A] time = 0.23, size = 18, normalized size = 0.78

$$\frac{3(x^3-x)^{4/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + 3*x^2)*(-x + x^3)^(1/3))/x^2,x]
```

```
[Out] (3*(-x + x^3)^(4/3))/(2*x^2)
```

fricas [A] time = 0.39, size = 19, normalized size = 0.83

$$\frac{3\left(x^3-x\right)^{\frac{1}{3}}\left(x^2-1\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)*(x^3-x)^(1/3)/x^2,x, algorithm="fricas")
[Out] 3/2*(x^3 - x)^(1/3)*(x^2 - 1)/x
```

giac [A] time = 0.30, size = 26, normalized size = 1.13

$$\frac{3}{2}\,x^2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}-\frac{3}{2}\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)*(x^3-x)^(1/3)/x^2,x, algorithm="giac")
[Out] 3/2*x^2*(-1/x^2 + 1)^(1/3) - 3/2*(-1/x^2 + 1)^(1/3)
```

maple [A] time = 0.01, size = 21, normalized size = 0.91

$$\frac{3\,(-1+x)\,(1+x)\left(x^3-x\right)^{\frac{1}{3}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+1)*(x^3-x)^(1/3)/x^2,x)
[Out] 3/2/x*(-1+x)*(1+x)*(x^3-x)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^3-x\right)^{\frac{1}{3}}\left(3\,x^2+1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)*(x^3-x)^(1/3)/x^2,x, algorithm="maxima")
[Out] integrate((x^3 - x)^(1/3)*(3*x^2 + 1)/x^2, x)
```

mupad [B] time = 0.18, size = 19, normalized size = 0.83

$$\frac{3\left(x^3-x\right)^{1/3}\,\left(x^2-1\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - x)^(1/3)*(3*x^2 + 1))/x^2,x)
[Out] (3*(x^3 - x)^(1/3)*(x^2 - 1))/(2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[3]{x\,(x-1)\,(x+1)}\,\left(3x^2+1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+1)*(x**3-x)**(1/3)/x**2,x)
[Out] Integral((x*(x - 1)*(x + 1))**(1/3)*(3*x**2 + 1)/x**2, x)
```

3.211 $\int \frac{1}{(-1+x)\sqrt[3]{-x^2+x^3}} \, dx$

Optimal. Leaf size=23

$$-\frac{3\left(x^3-x^2\right)^{2/3}}{(x-1)x}$$

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 0.70, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2056, 37}

$$-\frac{3x}{\sqrt[3]{x^3-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (-3*x)/(-x^2 + x^3)^(1/3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-1+x)\sqrt[3]{-x^2+x^3}} \, dx &= \frac{\left(\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{1}{(-1+x)^{4/3}x^{2/3}} \, dx}{\sqrt[3]{-x^2+x^3}}\\ &= -\frac{3x}{\sqrt[3]{-x^2+x^3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.61

$$-\frac{3x}{\sqrt[3]{(x-1)x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (-3*x)/((-1 + x)*x^2)^(1/3)
```

IntegrateAlgebraic [A] time = 0.15, size = 23, normalized size = 1.00

$$-\frac{3\left(x^3-x^2\right)^{2/3}}{(x-1)x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x)*(-x^2 + x^3)^(1/3)),x]
[Out] (-3*(-x^2 + x^3)^(2/3))/((-1 + x)*x)
```

fricas [A] time = 0.37, size = 22, normalized size = 0.96

$$-\frac{3\left(x^{3}-x^{2}\right)^{\frac{2}{3}}}{x^{2}-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^3-x^2)^(1/3),x, algorithm="fricas")
[Out] -3*(x^3 - x^2)^(2/3)/(x^2 - x)
```

giac [A] time = 0.27, size = 11, normalized size = 0.48

$$-\frac{3}{\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^3-x^2)^(1/3),x, algorithm="giac")
[Out] -3/(-1/x + 1)^(1/3)
```

maple [A] time = 0.00, size = 15, normalized size = 0.65

$$-\frac{3x}{\left(x^{3}-x^{2}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)/(x^3-x^2)^(1/3),x)
[Out] -3*x/(x^3-x^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{3}-x^{2}\right)^{\frac{1}{3}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^3-x^2)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^3 - x^2)^(1/3)*(x - 1)), x)
```

mupad [B] time = 0.16, size = 21, normalized size = 0.91

$$-\frac{3\left(x^{3}-x^{2}\right)^{2/3}}{x\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 - x^2)^(1/3)*(x - 1)),x)
[Out] -(3*(x^3 - x^2)^(2/3))/(x*(x - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x^2\,(x-1)}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x**3-x**2)**(1/3),x)
```

```
[Out] Integral(1/((x**2*(x - 1))**(1/3)*(x - 1)), x)
```

3.212 $\int \frac{1}{x^2 \sqrt[3]{x^2+x^3}}\,dx$

Optimal. Leaf size=23

$$\frac{3(3x-2)\left(x^3+x^2\right)^{2/3}}{10x^3}$$

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.61, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2016, 2014}

$$\frac{9\left(x^3+x^2\right)^{2/3}}{10x^2}-\frac{3\left(x^3+x^2\right)^{2/3}}{5x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(x^2 + x^3)^(1/3)),x]
```

```
[Out] (-3*(x^2 + x^3)^(2/3))/(5*x^3) + (9*(x^2 + x^3)^(2/3))/(10*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2 \sqrt[3]{x^2+x^3}}\,dx &= -\frac{3\left(x^2+x^3\right)^{2/3}}{5x^3}-\frac{3}{5}\int \frac{1}{x\sqrt[3]{x^2+x^3}}\,dx\\ &= -\frac{3\left(x^2+x^3\right)^{2/3}}{5x^3}+\frac{9\left(x^2+x^3\right)^{2/3}}{10x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{3\left(x^2(x+1)\right)^{2/3}(3x-2)}{10x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(x^2 + x^3)^(1/3)),x]
```

```
[Out] (3*(x^2*(1 + x))^(2/3)*(-2 + 3*x))/(10*x^3)
```

IntegrateAlgebraic [A] time = 0.12, size = 23, normalized size = 1.00

$$\frac{3(3x-2)\left(x^3+x^2\right)^{2/3}}{10x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(x^2 + x^3)^(1/3)),x]
[Out] (3*(-2 + 3*x)*(x^2 + x^3)^(2/3))/(10*x^3)
```

fricas [A] time = 0.38, size = 19, normalized size = 0.83

$$\frac{3\left(x^3+x^2\right)^{\frac{2}{3}}(3x-2)}{10x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3+x^2)^(1/3),x, algorithm="fricas")
[Out] 3/10*(x^3 + x^2)^(2/3)*(3*x - 2)/x^3
```

giac [A] time = 0.49, size = 19, normalized size = 0.83

$$-\frac{3}{5}\left(\frac{1}{x}+1\right)^{\frac{5}{3}}+\frac{3}{2}\left(\frac{1}{x}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3+x^2)^(1/3),x, algorithm="giac")
[Out] -3/5*(1/x + 1)^(5/3) + 3/2*(1/x + 1)^(2/3)
```

maple [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{3(1+x)(-2+3x)}{10x\left(x^3+x^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^3+x^2)^(1/3),x)
[Out] 3/10*(1+x)*(-2+3*x)/x/(x^3+x^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+x^2\right)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3+x^2)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^3 + x^2)^(1/3)*x^2), x)
```

mupad [B] time = 0.14, size = 29, normalized size = 1.26

$$\frac{9x\left(x^3+x^2\right)^{2/3}-6\left(x^3+x^2\right)^{2/3}}{10x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^2 + x^3)^(1/3)),x)
```

```
[Out] (9*x*(x^2 + x^3)^(2/3) - 6*(x^2 + x^3)^(2/3))/(10*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt[3]{x^2 (x+1)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**3+x**2)**(1/3),x)
```

```
[Out] Integral(1/(x**2*(x**2*(x + 1))**(1/3)), x)
```

3.213 $\int \frac{(3+2x^2)\sqrt{x+2x^3}}{(1+2x^2)^2}\,dx$

Optimal. Leaf size=23

$$\frac{2x\sqrt{2x^3+x}}{2x^2+1}$$

Rubi [A] time = 0.08, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2056, 449}

$$\frac{2x\sqrt{2x^3+x}}{2x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x^2)*Sqrt[x + 2*x^3])/(1 + 2*x^2)^2,x]
```

```
[Out] (2*x*Sqrt[x + 2*x^3])/(1 + 2*x^2)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(3+2x^2\right)\sqrt{x+2x^3}}{\left(1+2x^2\right)^2}\,dx &= \frac{\sqrt{x+2x^3}\int \frac{\sqrt{x}\left(3+2x^2\right)}{\left(1+2x^2\right)^{3/2}}\,dx}{\sqrt{x}\sqrt{1+2x^2}}\\ &= \frac{2x\sqrt{x+2x^3}}{1+2x^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 23, normalized size = 1.00

$$\frac{2x\sqrt{2x^3+x}}{2x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x^2)*Sqrt[x + 2*x^3])/(1 + 2*x^2)^2,x]
```

```
[Out] (2*x*Sqrt[x + 2*x^3])/(1 + 2*x^2)
```

IntegrateAlgebraic [A] time = 0.17, size = 23, normalized size = 1.00

$$\frac{2x\sqrt{2x^3+x}}{2x^2+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3 + 2*x^2)*Sqrt[x + 2*x^3])/(1 + 2*x^2)^2,x]
[Out] (2*x*Sqrt[x + 2*x^3])/(1 + 2*x^2)
```

fricas [A] time = 0.41, size = 21, normalized size = 0.91

$$\frac{2\sqrt{2x^3+x}\,x}{2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)*(2*x^3+x)^(1/2)/(2*x^2+1)^2,x, algorithm="fricas")
[Out] 2*sqrt(2*x^3 + x)*x/(2*x^2 + 1)
```

giac [A] time = 0.31, size = 13, normalized size = 0.57

$$\frac{2}{\sqrt{\frac{2}{x}+\frac{1}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)*(2*x^3+x)^(1/2)/(2*x^2+1)^2,x, algorithm="giac")
[Out] 2/sqrt(2/x + 1/x^3)
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{2x\sqrt{2x^3+x}}{2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+3)*(2*x^3+x)^(1/2)/(2*x^2+1)^2,x)
[Out] 2*x*(2*x^3+x)^(1/2)/(2*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^3+x}\left(2x^2+3\right)}{\left(2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)*(2*x^3+x)^(1/2)/(2*x^2+1)^2,x, algorithm="maxima")
[Out] integrate(sqrt(2*x^3 + x)*(2*x^2 + 3)/(2*x^2 + 1)^2, x)
```

mupad [B] time = 0.17, size = 14, normalized size = 0.61

$$\frac{2x^2}{\sqrt{2x^3+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 2*x^3)^(1/2)*(2*x^2 + 3))/(2*x^2 + 1)^2,x)
[Out] (2*x^2)/(x + 2*x^3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(2x^2+1\right)}\left(2x^2+3\right)}{\left(2x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+3)*(2*x**3+x)**(1/2)/(2*x**2+1)**2,x)
```

```
[Out] Integral(sqrt(x*(2*x**2 + 1))*(2*x**2 + 3)/(2*x**2 + 1)**2, x)
```

3.214 $\int \frac{1}{x^6\left(-1+x^4\right)^{3/4}}\,dx$

Optimal. Leaf size=23

$$\frac{\sqrt[4]{x^4-1}\left(4x^4+1\right)}{5x^5}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.43, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {271, 264}

$$\frac{4\sqrt[4]{x^4-1}}{5x}+\frac{\sqrt[4]{x^4-1}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(-1 + x^4)^(3/4)),x]
```

```
[Out] (-1 + x^4)^(1/4)/(5*x^5) + (4*(-1 + x^4)^(1/4))/(5*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(-1+x^4\right)^{3/4}}\,dx &= \frac{\sqrt[4]{-1+x^4}}{5x^5}+\frac{4}{5}\int \frac{1}{x^2\left(-1+x^4\right)^{3/4}}\,dx\\ &= \frac{\sqrt[4]{-1+x^4}}{5x^5}+\frac{4\sqrt[4]{-1+x^4}}{5x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\sqrt[4]{x^4-1}\left(4x^4+1\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(-1 + x^4)^(3/4)),x]
```

```
[Out] ((-1 + x^4)^(1/4)*(1 + 4*x^4))/(5*x^5)
```

IntegrateAlgebraic [A] time = 0.19, size = 23, normalized size = 1.00

$$\frac{\sqrt[4]{x^4-1}\left(4x^4+1\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^6*(-1 + x^4)^(3/4)),x]
[Out] ((-1 + x^4)^(1/4)*(1 + 4*x^4))/(5*x^5)
```

fricas [A] time = 0.39, size = 19, normalized size = 0.83

$$\frac{\left(4 x^4+1\right)\left(x^4-1\right)^{\frac{1}{4}}}{5 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^4-1)^(3/4),x, algorithm="fricas")
[Out] 1/5*(4*x^4 + 1)*(x^4 - 1)^(1/4)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4-1\right)^{\frac{3}{4}} x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^4-1)^(3/4),x, algorithm="giac")
[Out] integrate(1/((x^4 - 1)^(3/4)*x^6), x)
```

maple [A] time = 0.00, size = 31, normalized size = 1.35

$$\frac{(-1+x)(1+x)\left(x^2+1\right)\left(4x^4+1\right)}{5x^5\left(x^4-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^4-1)^(3/4),x)
[Out] 1/5*(-1+x)*(1+x)*(x^2+1)*(4*x^4+1)/x^5/(x^4-1)^(3/4)
```

maxima [A] time = 0.32, size = 24, normalized size = 1.04

$$\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}-\frac{\left(x^4-1\right)^{\frac{5}{4}}}{5 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^4-1)^(3/4),x, algorithm="maxima")
[Out] (x^4 - 1)^(1/4)/x - 1/5*(x^4 - 1)^(5/4)/x^5
```

mupad [B] time = 0.22, size = 25, normalized size = 1.09

$$\frac{\left(x^4-1\right)^{1/4}+4 x^4\left(x^4-1\right)^{1/4}}{5 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^4 - 1)^(3/4)),x)
[Out] ((x^4 - 1)^(1/4) + 4*x^4*(x^4 - 1)^(1/4))/(5*x^5)
```

sympy [**B**] time = 0.82, size = 124, normalized size = 5.39

$$\begin{cases} -\frac{\sqrt[4]{-1+\frac{1}{x^4}}\,e^{-\frac{3i\pi}{4}}\Gamma\left(-\frac{5}{4}\right)}{4\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt[4]{-1+\frac{1}{x^4}}\,e^{-\frac{3i\pi}{4}}\Gamma\left(-\frac{5}{4}\right)}{16x^4\Gamma\left(\frac{3}{4}\right)} & \text{for } \frac{1}{|x^4|} > 1 \\ \frac{\sqrt[4]{1-\frac{1}{x^4}}\,\Gamma\left(-\frac{5}{4}\right)}{4\Gamma\left(\frac{3}{4}\right)} + \frac{\sqrt[4]{1-\frac{1}{x^4}}\,\Gamma\left(-\frac{5}{4}\right)}{16x^4\Gamma\left(\frac{3}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**4-1)**(3/4),x)
```

```
[Out] Piecewise((-(-1 + x**(-4))**(1/4)*exp(-3*I*pi/4)*gamma(-5/4)/(4*gamma(3/4))
 - (-1 + x**(-4))**(1/4)*exp(-3*I*pi/4)*gamma(-5/4)/(16*x**4*gamma(3/4)), 1
/Abs(x**4) > 1), ((1 - 1/x**4)**(1/4)*gamma(-5/4)/(4*gamma(3/4)) + (1 - 1/x
**4)**(1/4)*gamma(-5/4)/(16*x**4*gamma(3/4)), True))
```

3.215 $\int \frac{1}{x^4(1+x^4)^{5/4}}\,dx$

Optimal. Leaf size=23

$$\frac{-4x^4-1}{3x^3\sqrt[4]{x^4+1}}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.35, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {271, 191}

$$-\frac{4x}{3\sqrt[4]{x^4+1}}-\frac{1}{3\sqrt[4]{x^4+1}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + x^4)^(5/4)),x]
```

```
[Out] -1/3*1/(x^3*(1 + x^4)^(1/4)) - (4*x)/(3*(1 + x^4)^(1/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(1+x^4\right)^{5/4}}\,dx &= -\frac{1}{3x^3\sqrt[4]{1+x^4}}-\frac{4}{3}\int \frac{1}{\left(1+x^4\right)^{5/4}}\,dx\\ &= -\frac{1}{3x^3\sqrt[4]{1+x^4}}-\frac{4x}{3\sqrt[4]{1+x^4}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{4x^4+1}{3x^3\sqrt[4]{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + x^4)^(5/4)),x]
```

```
[Out] -1/3*(1 + 4*x^4)/(x^3*(1 + x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.19, size = 23, normalized size = 1.00

$$\frac{-4x^4-1}{3x^3\sqrt[4]{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(1 + x^4)^(5/4)),x]
[Out] (-1 - 4*x^4)/(3*x^3*(1 + x^4)^(1/4))
```

fricas [**A**] time = 0.40, size = 25, normalized size = 1.09

$$-\frac{\left(4\,x^4+1\right)\left(x^4+1\right)^{\frac{3}{4}}}{3\left(x^7+x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+1)^(5/4),x, algorithm="fricas")
[Out] -1/3*(4*x^4 + 1)*(x^4 + 1)^(3/4)/(x^7 + x^3)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)^{\frac{5}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+1)^(5/4),x, algorithm="giac")
[Out] integrate(1/((x^4 + 1)^(5/4)*x^4), x)
```

maple [**A**] time = 0.00, size = 20, normalized size = 0.87

$$-\frac{4x^4+1}{3x^3\left(x^4+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^4+1)^(5/4),x)
[Out] -1/3*(4*x^4+1)/x^3/(x^4+1)^(1/4)
```

maxima [**A**] time = 0.34, size = 23, normalized size = 1.00

$$-\frac{x}{\left(x^4+1\right)^{\frac{1}{4}}}-\frac{\left(x^4+1\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+1)^(5/4),x, algorithm="maxima")
[Out] -x/(x^4 + 1)^(1/4) - 1/3*(x^4 + 1)^(3/4)/x^3
```

mupad [**B**] time = 0.20, size = 19, normalized size = 0.83

$$-\frac{4\,x^4+1}{3\,x^3\,\left(x^4+1\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^4 + 1)^(5/4)),x)
[Out] -(4*x^4 + 1)/(3*x^3*(x^4 + 1)^(1/4))
```

sympy [B] time = 0.77, size = 75, normalized size = 3.26

$$\frac{4x^4\left(x^4+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{16x^7\Gamma\left(\frac{5}{4}\right)+16x^3\Gamma\left(\frac{5}{4}\right)}+\frac{\left(x^4+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{16x^7\Gamma\left(\frac{5}{4}\right)+16x^3\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**4+1)**(5/4),x)
```

```
[Out] 4*x**4*(x**4 + 1)**(3/4)*gamma(-3/4)/(16*x**7*gamma(5/4) + 16*x**3*gamma(5/
4)) + (x**4 + 1)**(3/4)*gamma(-3/4)/(16*x**7*gamma(5/4) + 16*x**3*gamma(5/4
))
```

3.216 $\int \frac{1}{x^6(1+x^4)^{3/4}}\,dx$

Optimal. Leaf size=23

$$\frac{\sqrt[4]{x^4+1}\,(4x^4-1)}{5x^5}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.43, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {271, 264}

$$\frac{4\sqrt[4]{x^4+1}}{5x}-\frac{\sqrt[4]{x^4+1}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 + x^4)^(3/4)),x]
```

```
[Out] -1/5*(1 + x^4)^(1/4)/x^5 + (4*(1 + x^4)^(1/4))/(5*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(1+x^4\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{1+x^4}}{5x^5}-\frac{4}{5}\int \frac{1}{x^2\left(1+x^4\right)^{3/4}}\,dx\\ &= -\frac{\sqrt[4]{1+x^4}}{5x^5}+\frac{4\sqrt[4]{1+x^4}}{5x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\sqrt[4]{x^4+1}\,(4x^4-1)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 + x^4)^(3/4)),x]
```

```
[Out] ((1 + x^4)^(1/4)*(-1 + 4*x^4))/(5*x^5)
```

IntegrateAlgebraic [A] time = 0.17, size = 23, normalized size = 1.00

$$\frac{\sqrt[4]{x^4+1}\,(4x^4-1)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^6*(1 + x^4)^(3/4)),x]
[Out] ((1 + x^4)^(1/4)*(-1 + 4*x^4))/(5*x^5)
```

fricas [A] time = 0.39, size = 19, normalized size = 0.83

$$\frac{(4x^4-1)(x^4+1)^{\frac{1}{4}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^4+1)^(3/4),x, algorithm="fricas")
[Out] 1/5*(4*x^4 - 1)*(x^4 + 1)^(1/4)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^4+1)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^4+1)^(3/4),x, algorithm="giac")
[Out] integrate(1/((x^4 + 1)^(3/4)*x^6), x)
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$\frac{(x^4+1)^{\frac{1}{4}}(4x^4-1)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^4+1)^(3/4),x)
[Out] 1/5*(x^4+1)^(1/4)*(4*x^4-1)/x^5
```

maxima [A] time = 0.38, size = 24, normalized size = 1.04

$$\frac{(x^4+1)^{\frac{1}{4}}}{x} - \frac{(x^4+1)^{\frac{5}{4}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^4+1)^(3/4),x, algorithm="maxima")
[Out] (x^4 + 1)^(1/4)/x - 1/5*(x^4 + 1)^(5/4)/x^5
```

mupad [B] time = 0.20, size = 25, normalized size = 1.09

$$-\frac{(x^4+1)^{1/4} - 4x^4(x^4+1)^{1/4}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^4 + 1)^(3/4)),x)
[Out] -((x^4 + 1)^(1/4) - 4*x^4*(x^4 + 1)^(1/4))/(5*x^5)
```

sympy [B] time = 0.77, size = 48, normalized size = 2.09

$$\frac{\sqrt[4]{x^4+1}\,\Gamma\left(-\frac{5}{4}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{\sqrt[4]{x^4+1}\,\Gamma\left(-\frac{5}{4}\right)}{16x^5\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**4+1)**(3/4),x)
```

```
[Out] (x**4 + 1)**(1/4)*gamma(-5/4)/(4*x*gamma(3/4)) - (x**4 + 1)**(1/4)*gamma(-5
/4)/(16*x**5*gamma(3/4))
```

3.217 $\int \frac{-1+x^4}{x^6\left(1+x^4\right)^{3/4}}\,dx$

Optimal. Leaf size=23

$$\frac{\left(1-9x^4\right)\sqrt[4]{x^4+1}}{5x^5}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.43, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {453, 264}

$$\frac{\sqrt[4]{x^4+1}}{5x^5}-\frac{9\sqrt[4]{x^4+1}}{5x}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)/(x^6*(1 + x^4)^(3/4)),x]
```

```
[Out] (1 + x^4)^(1/4)/(5*x^5) - (9*(1 + x^4)^(1/4))/(5*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^4}{x^6\left(1+x^4\right)^{3/4}}\,dx &= \frac{\sqrt[4]{1+x^4}}{5x^5}+\frac{9}{5}\int \frac{1}{x^2\left(1+x^4\right)^{3/4}}\,dx\\ &= \frac{\sqrt[4]{1+x^4}}{5x^5}-\frac{9\sqrt[4]{1+x^4}}{5x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\left(1-9x^4\right)\sqrt[4]{x^4+1}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)/(x^6*(1 + x^4)^(3/4)),x]
```

```
[Out] ((1 - 9*x^4)*(1 + x^4)^(1/4))/(5*x^5)
```

IntegrateAlgebraic [A] time = 0.21, size = 23, normalized size = 1.00

$$\frac{\left(1-9x^4\right)\sqrt[4]{x^4+1}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/(x^6*(1 + x^4)^(3/4)),x]
[Out] ((1 - 9*x^4)*(1 + x^4)^(1/4))/(5*x^5)
```

fricas [A] time = 0.40, size = 19, normalized size = 0.83

$$-\frac{\left(9\,x^4-1\right)\left(x^4+1\right)^{\frac{1}{4}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^6/(x^4+1)^(3/4),x, algorithm="fricas")
[Out] -1/5*(9*x^4 - 1)*(x^4 + 1)^(1/4)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\left(x^4+1\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^6/(x^4+1)^(3/4),x, algorithm="giac")
[Out] integrate((x^4 - 1)/((x^4 + 1)^(3/4)*x^6), x)
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$-\frac{\left(9x^4-1\right)\left(x^4+1\right)^{\frac{1}{4}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/x^6/(x^4+1)^(3/4),x)
[Out] -1/5*(9*x^4-1)*(x^4+1)^(1/4)/x^5
```

maxima [A] time = 0.33, size = 25, normalized size = 1.09

$$-\frac{2\left(x^4+1\right)^{\frac{1}{4}}}{x}+\frac{\left(x^4+1\right)^{\frac{5}{4}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^6/(x^4+1)^(3/4),x, algorithm="maxima")
[Out] -2*(x^4 + 1)^(1/4)/x + 1/5*(x^4 + 1)^(5/4)/x^5
```

mupad [B] time = 0.08, size = 25, normalized size = 1.09

$$\frac{\left(x^4+1\right)^{1/4}-9\,x^4\left(x^4+1\right)^{1/4}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/(x^6*(x^4 + 1)^(3/4)),x)
[Out] ((x^4 + 1)^(1/4) - 9*x^4*(x^4 + 1)^(1/4))/(5*x^5)
```

sympy [B] time = 1.80, size = 71, normalized size = 3.09

$$\frac{\sqrt[4]{1+\frac{1}{x^4}}\,\Gamma\left(-\frac{1}{4}\right)}{4\Gamma\left(\frac{3}{4}\right)}-\frac{\sqrt[4]{x^4+1}\,\Gamma\left(-\frac{5}{4}\right)}{4x\Gamma\left(\frac{3}{4}\right)}+\frac{\sqrt[4]{x^4+1}\,\Gamma\left(-\frac{5}{4}\right)}{16x^5\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/x**6/(x**4+1)**(3/4),x)
```

```
[Out] (1 + x**(-4))**(1/4)*gamma(-1/4)/(4*gamma(3/4)) - (x**4 + 1)**(1/4)*gamma(-
5/4)/(4*x*gamma(3/4)) + (x**4 + 1)**(1/4)*gamma(-5/4)/(16*x**5*gamma(3/4))
```

3.218 $\int \frac{1+x^2}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=23

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1699, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.10, size = 36, normalized size = 1.57

$$\sqrt[4]{-1}\left(F\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)-2\Pi\left(i;\sin^{-1}\left((-1)^{3/4}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] (-1)^(1/4)*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - 2*EllipticPi[I, ArcSin
[(-1)^(3/4)*x], -1])
```

IntegrateAlgebraic [A] time = 0.38, size = 23, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

fricas [B] time = 0.45, size = 42, normalized size = 1.83

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\sqrt{x^4+1}\,x+2\,x^2+1}{x^4-2\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^4 + 2*sqrt(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2
+ 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2+1}{\sqrt{x^4+1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

maple [C] time = 0.07, size = 112, normalized size = 4.87

$$-\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{2\,(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticPi}\left((-1)^{\frac{1}{4}}x,-i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x)
```

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-2*(-1)^(3/4)*(1-I*x^2)^(1/2)*(
1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/
4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2+1}{\sqrt{x^4+1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{x^2+1}{\left(x^2-1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)),x)
```

```
[Out] int(-(x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx-\int \frac{1}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(-x**2+1)/(x**4+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x) - Integral(1/(x**
2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x)
```

3.219 $\int \frac{-1+2x+2x^2}{(1+2x)\sqrt{-x+x^4}}\,dx$

Optimal. Leaf size=23

$$\tanh^{-1}\left(\frac{2\sqrt{x^4-x}}{2x^2+1}\right)$$

Rubi [F] time = 0.72, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+2x+2x^2}{(1+2x)\sqrt{-x+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 2*x + 2*x^2)/((1 + 2*x)*Sqrt[-x + x^4]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[-1 + x^3]*ArcTanh[x^(3/2)/Sqrt[-1 + x^3]])/(3*Sqrt[-x + x^4
]) + ((1 - x)*x*Sqrt[(1 + x + x^2)/(1 - (1 + Sqrt[3])*x)^2]*EllipticF[ArcCo
s[(1 - (1 - Sqrt[3])*x)/(1 - (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(2*3^(1/4
)*Sqrt[-(((1 - x)*x)/(1 - (1 + Sqrt[3])*x)^2)]*Sqrt[-x + x^4]) - (((3*I)/2)
*Sqrt[x]*Sqrt[-1 + x^3]*Defer[Subst][Defer[Int][1/((I - Sqrt[2]*x)*Sqrt[-1
+ x^6]), x], x, Sqrt[x]])/Sqrt[-x + x^4] - (((3*I)/2)*Sqrt[x]*Sqrt[-1 + x^3
]*Defer[Subst][Defer[Int][1/((I + Sqrt[2]*x)*Sqrt[-1 + x^6]), x], x, Sqrt[x
]])/Sqrt[-x + x^4]
```

Rubi steps

$$\int \frac{-1+2x+2x^2}{(1+2x)\sqrt{-x+x^4}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1+x^3}\right)\int \frac{-1+2x+2x^2}{\sqrt{x}(1+2x)\sqrt{-1+x^3}}\,dx}{\sqrt{-x+x^4}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^3}\right)\text{Subst}\left(\int \frac{-1+2x^2+2x^4}{(1+2x^2)\sqrt{-1+x^6}}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^4}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^3}\right)\text{Subst}\left(\int \left(\frac{1}{2\sqrt{-1+x^6}}+\frac{x^2}{\sqrt{-1+x^6}}-\frac{3}{2(1+2x^2)\sqrt{-1+x^6}}\right)dx,x,\sqrt{x}\right)}{\sqrt{-x+x^4}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^6}}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^4}} + \frac{\left(2\sqrt{x}\sqrt{-1+x^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x^6}}\,dx,\right.}{\sqrt{-x+x^4}}$$

$$= \frac{(1-x)x\sqrt{\frac{1+x+x^2}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x}{1-\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\sqrt{-\frac{(1-x)x}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{-x+x^4}} + \frac{\left(2\sqrt{x}\sqrt{-1+x^3}\right)\text{Subst}\left(\right.}{3\sqrt{-x+}}$$

$$= \frac{(1-x)x\sqrt{\frac{1+x+x^2}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x}{1-\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\sqrt{-\frac{(1-x)x}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{-x+x^4}} - \frac{\left(3i\sqrt{x}\sqrt{-1+x^3}\right)\text{Subst}\left(\right.}{2\sqrt{-}}$$

$$= \frac{2\sqrt{x}\sqrt{-1+x^3}\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{-1+x^3}}\right)}{3\sqrt{-x+x^4}} + \frac{(1-x)x\sqrt{\frac{1+x+x^2}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x}{1-\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2-\right.\right.}{2\sqrt[4]{3}\sqrt{-\frac{(1-x)x}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{-x+x^4}}$$

Mathematica [C] time = 0.64, size = 167, normalized size = 7.26

$$\frac{-2\left((-1)^{2/3}-1\right)\sqrt{x}\sqrt{1-x^3}\sin^{-1}\left(x^{3/2}\right)-6(-1)^{2/3}\sqrt{-\frac{\left(1+\sqrt[3]{-1}\right)x\left(\sqrt[3]{-1}x+1\right)}{(x-1)^2}}\sqrt{\frac{(-1)^{2/3}x-1}{x-1}}(x-1)^2\Pi\left(\frac{1}{2}\left(3-i\sqrt{3}\right);\sin^{-1}\left(\sqrt{\frac{\left(1+\sqrt[3]{-1}\right)x}{x-1}}\right)|\frac{\sqrt[3]{-1}}{-1+\sqrt[3]{-1}}\right)}{3\left(1-(-1)^{2/3}\right)\sqrt{x\left(x^3-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x + 2*x^2)/((1 + 2*x)*Sqrt[-x + x^4]),x]
```

```
[Out] (-2*(-1 + (-1)^(2/3))*Sqrt[x]*Sqrt[1 - x^3]*ArcSin[x^(3/2)] - 6*(-1)^(2/3)*
(-1 + x)^2*Sqrt[-(((1 + (-1)^(1/3))*x*(1 + (-1)^(1/3)*x))/(-1 + x)^2)]*Sqrt
[(-1 + (-1)^(2/3)*x)/(-1 + x)]*EllipticPi[(3 - I*Sqrt[3])/2, ArcSin[Sqrt[((
1 + (-1)^(1/3))*x)/(-1 + x)]], (-1)^(1/3)/(-1 + (-1)^(1/3))])/(3*(1 - (-1)^
(2/3))*Sqrt[x*(-1 + x^3)])
```

IntegrateAlgebraic [A] time = 1.23, size = 23, normalized size = 1.00

$$\tanh^{-1}\left(\frac{2\sqrt{x^4-x}}{2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x + 2*x^2)/((1 + 2*x)*Sqrt[-x + x^4]),x]
```

```
[Out] ArcTanh[(2*Sqrt[-x + x^4])/(1 + 2*x^2)]
```

fricas [A] time = 0.43, size = 28, normalized size = 1.22

$$\log\left(-\frac{2\,x^2+2\,\sqrt{x^4-x}\,+1}{2\,x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+2*x-1)/(1+2*x)/(x^4-x)^(1/2),x, algorithm="fricas")
[Out] log(-(2*x^2 + 2*sqrt(x^4 - x) + 1)/(2*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2\,x^2+2\,x-1}{\sqrt{x^4-x}\,(2\,x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+2*x-1)/(1+2*x)/(x^4-x)^(1/2),x, algorithm="giac")
[Out] integrate((2*x^2 + 2*x - 1)/(sqrt(x^4 - x)*(2*x + 1)), x)
```

maple [C] time = 0.07, size = 774, normalized size = 33.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+2*x-1)/(1+2*x)/(x^4-x)^(1/2),x)
[Out] 2*(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^
(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2))/(-1+x))^(1/2)*((
x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2)/(-3/2+1/2*I*3^(1/2)
)/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/2)))^(1/2)*(EllipticF((
(-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3^(1/
2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-Ell
ipticPi(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),(-1/2+1/
2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/
(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)))+(1/2-1/2*I*3^(1/2))*((-3/2
+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*I
*3^(1/2))/(-1/2-1/2*I*3^(1/2))/(-1+x))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+1
/2*I*3^(1/2))/(-1+x))^(1/2)/(-3/2+1/2*I*3^(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^(
1/2))*(x+1/2-1/2*I*3^(1/2)))^(1/2)*EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+
1/2*I*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I
*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/
2))/(-1/2-1/2*I*3^(1/2))/(-1+x))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3
^(1/2))/(-1+x))^(1/2)/(-3/2+1/2*I*3^(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*
(x+1/2-1/2*I*3^(1/2)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I
*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*EllipticPi(((-3/2+1/2*I*3^(1/2))*x/
(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),3*(-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
2)),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I
*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2\,x^2+2\,x-1}{\sqrt{x^4-x}\,(2\,x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+2*x-1)/(1+2*x)/(x^4-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 2*x - 1)/(sqrt(x^4 - x)*(2*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{2\,x^2+2\,x-1}{\sqrt{x^4-x}\,\,(2\,x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 2*x^2 - 1)/((x^4 - x)^(1/2)*(2*x + 1)),x)
[Out] int((2*x + 2*x^2 - 1)/((x^4 - x)^(1/2)*(2*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2+2x-1}{\sqrt{x\,(x-1)\left(x^2+x+1\right)}\,\,(2x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+2*x-1)/(1+2*x)/(x**4-x)**(1/2),x)
[Out] Integral((2*x**2 + 2*x - 1)/(sqrt(x*(x - 1)*(x**2 + x + 1))*(2*x + 1)), x)
```

3.220 $\int \frac{1}{x^5\sqrt{x+x^4}}\,dx$

Optimal. Leaf size=23

$$\frac{2\left(2x^3-1\right)\sqrt{x^4+x}}{9x^5}$$

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.43, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2016, 2014}

$$\frac{4\sqrt{x^4+x}}{9x^2}-\frac{2\sqrt{x^4+x}}{9x^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[x + x^4]),x]
```

```
[Out] (-2*Sqrt[x + x^4])/(9*x^5) + (4*Sqrt[x + x^4])/(9*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{x+x^4}}\,dx &= -\frac{2\sqrt{x+x^4}}{9x^5}-\frac{2}{3}\int \frac{1}{x^2\sqrt{x+x^4}}\,dx\\ &= -\frac{2\sqrt{x+x^4}}{9x^5}+\frac{4\sqrt{x+x^4}}{9x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2\left(2x^3-1\right)\sqrt{x^4+x}}{9x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[x + x^4]),x]
```

```
[Out] (2*(-1 + 2*x^3)*Sqrt[x + x^4])/(9*x^5)
```

IntegrateAlgebraic [A] time = 0.41, size = 23, normalized size = 1.00

$$\frac{2\left(2x^3-1\right)\sqrt{x^4+x}}{9x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*Sqrt[x + x^4]),x]
[Out] (2*(-1 + 2*x^3)*Sqrt[x + x^4])/(9*x^5)
```

fricas [A] time = 0.40, size = 19, normalized size = 0.83

$$\frac{2\sqrt{x^4+x}\left(2x^3-1\right)}{9x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4+x)^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(x^4 + x)*(2*x^3 - 1)/x^5
```

giac [A] time = 0.59, size = 19, normalized size = 0.83

$$-\frac{2}{9}\left(\frac{1}{x^3}+1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{\frac{1}{x^3}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4+x)^(1/2),x, algorithm="giac")
[Out] -2/9*(1/x^3 + 1)^(3/2) + 2/3*sqrt(1/x^3 + 1)
```

maple [A] time = 0.00, size = 31, normalized size = 1.35

$$\frac{2(1+x)\left(x^2-x+1\right)\left(2x^3-1\right)}{9x^4\sqrt{x^4+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^4+x)^(1/2),x)
[Out] 2/9*(1+x)*(x^2-x+1)*(2*x^3-1)/x^4/(x^4+x)^(1/2)
```

maxima [A] time = 0.91, size = 32, normalized size = 1.39

$$\frac{2\left(2x^7+x^4-x\right)}{9\sqrt{x^2-x+1}\sqrt{x+1}\,x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4+x)^(1/2),x, algorithm="maxima")
[Out] 2/9*(2*x^7 + x^4 - x)/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^(11/2))
```

mupad [B] time = 0.16, size = 19, normalized size = 0.83

$$\frac{2\left(2x^3-1\right)\sqrt{x^4+x}}{9x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x + x^4)^(1/2)),x)
[Out] (2*(2*x^3 - 1)*(x + x^4)^(1/2))/(9*x^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{x\,(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**4+x)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(x*(x + 1)*(x**2 - x + 1))), x)
```

3.221 $\int \frac{-1+x^3}{x^6 \sqrt[4]{x+x^4}} \, dx$

Optimal. Leaf size=23

$$-\frac{4\left(11x^3-3\right)\left(x^4+x\right)^{3/4}}{63x^6}$$

Rubi [A] time = 0.07, antiderivative size = 33, normalized size of antiderivative = 1.43, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2038, 2014}

$$\frac{4\left(x^4+x\right)^{3/4}}{21x^6}-\frac{44\left(x^4+x\right)^{3/4}}{63x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(x^6*(x + x^4)^(1/4)),x]
```

```
[Out] (4*(x + x^4)^(3/4))/(21*x^6) - (44*(x + x^4)^(3/4))/(63*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^3}{x^6 \sqrt[4]{x+x^4}} \, dx &= \frac{4\left(x+x^4\right)^{3/4}}{21x^6}+\frac{11}{7}\int \frac{1}{x^3 \sqrt[4]{x+x^4}} \, dx\\ &= \frac{4\left(x+x^4\right)^{3/4}}{21x^6}-\frac{44\left(x+x^4\right)^{3/4}}{63x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{4\left(11x^3-3\right)\left(x^4+x\right)^{3/4}}{63x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(x^6*(x + x^4)^(1/4)),x]
```

```
[Out] (-4*(-3 + 11*x^3)*(x + x^4)^(3/4))/(63*x^6)
```

IntegrateAlgebraic [A] time = 0.27, size = 23, normalized size = 1.00

$$-\frac{4\left(11x^3-3\right)\left(x^4+x\right)^{3/4}}{63x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)/(x^6*(x + x^4)^(1/4)),x]
[Out] (-4*(-3 + 11*x^3)*(x + x^4)^(3/4))/(63*x^6)
```

fricas [A] time = 0.40, size = 19, normalized size = 0.83

$$-\frac{4\left(x^4+x\right)^{\frac{3}{4}}\left(11\,x^3-3\right)}{63\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/x^6/(x^4+x)^(1/4),x, algorithm="fricas")
[Out] -4/63*(x^4 + x)^(3/4)*(11*x^3 - 3)/x^6
```

giac [A] time = 0.42, size = 19, normalized size = 0.83

$$\frac{4}{21}\left(\frac{1}{x^3}+1\right)^{\frac{7}{4}}-\frac{8}{9}\left(\frac{1}{x^3}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/x^6/(x^4+x)^(1/4),x, algorithm="giac")
[Out] 4/21*(1/x^3 + 1)^(7/4) - 8/9*(1/x^3 + 1)^(3/4)
```

maple [A] time = 0.01, size = 31, normalized size = 1.35

$$-\frac{4\left(11x^3-3\right)(1+x)\left(x^2-x+1\right)}{63\left(x^4+x\right)^{\frac{1}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/x^6/(x^4+x)^(1/4),x)
[Out] -4/63*(11*x^3-3)*(1+x)*(x^2-x+1)/(x^4+x)^(1/4)/x^5
```

maxima [B] time = 0.63, size = 58, normalized size = 2.52

$$-\frac{4\left(x^4+x\right)}{9\left(x^2-x+1\right)^{\frac{1}{4}}(x+1)^{\frac{1}{4}}x^{\frac{13}{4}}}-\frac{4\left(4\,x^7+x^4-3\,x\right)}{63\left(x^2-x+1\right)^{\frac{1}{4}}(x+1)^{\frac{1}{4}}x^{\frac{25}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/x^6/(x^4+x)^(1/4),x, algorithm="maxima")
[Out] -4/9*(x^4 + x)/((x^2 - x + 1)^(1/4)*(x + 1)^(1/4)*x^(13/4)) - 4/63*(4*x^7 +
 x^4 - 3*x)/((x^2 - x + 1)^(1/4)*(x + 1)^(1/4)*x^(25/4))
```

mupad [B] time = 0.18, size = 27, normalized size = 1.17

$$\frac{12\left(x^4+x\right)^{3/4}-44\,x^3\left(x^4+x\right)^{3/4}}{63\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x^6*(x + x^4)^(1/4)),x)
```

```
[Out] (12*(x + x^4)^(3/4) - 44*x^3*(x + x^4)^(3/4))/(63*x^6)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\left(x^2+x+1\right)}{x^6\sqrt[4]{x(x+1)\left(x^2-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/x**6/(x**4+x)**(1/4),x)
```

```
[Out] Integral((x - 1)*(x**2 + x + 1)/(x**6*(x*(x + 1)*(x**2 - x + 1))**(1/4)), x
)
```

3.222 $\int \frac{1+2x^3}{x^6 \sqrt[4]{x+x^4}} \, dx$

Optimal. Leaf size=23

$$-\frac{4\left(10x^3+3\right)\left(x^4+x\right)^{3/4}}{63x^6}$$

Rubi [A] time = 0.07, antiderivative size = 33, normalized size of antiderivative = 1.43, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2038, 2014}

$$-\frac{4\left(x^4+x\right)^{3/4}}{21x^6}-\frac{40\left(x^4+x\right)^{3/4}}{63x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^3)/(x^6*(x + x^4)^(1/4)),x]
```

```
[Out] (-4*(x + x^4)^(3/4))/(21*x^6) - (40*(x + x^4)^(3/4))/(63*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^3}{x^6\sqrt[4]{x+x^4}}\,dx &= -\frac{4\left(x+x^4\right)^{3/4}}{21x^6}+\frac{10}{7}\int \frac{1}{x^3\sqrt[4]{x+x^4}}\,dx\\ &= -\frac{4\left(x+x^4\right)^{3/4}}{21x^6}-\frac{40\left(x+x^4\right)^{3/4}}{63x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{4\left(10x^3+3\right)\left(x^4+x\right)^{3/4}}{63x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^3)/(x^6*(x + x^4)^(1/4)),x]
```

```
[Out] (-4*(3 + 10*x^3)*(x + x^4)^(3/4))/(63*x^6)
```

IntegrateAlgebraic [A] time = 0.28, size = 23, normalized size = 1.00

$$-\frac{4\left(10x^3+3\right)\left(x^4+x\right)^{3/4}}{63x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x^3)/(x^6*(x + x^4)^(1/4)),x]
[Out] (-4*(3 + 10*x^3)*(x + x^4)^(3/4))/(63*x^6)
```

fricas [A] time = 0.40, size = 19, normalized size = 0.83

$$-\frac{4\left(x^4+x\right)^{\frac{3}{4}}\left(10\,x^3+3\right)}{63\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)/x^6/(x^4+x)^(1/4),x, algorithm="fricas")
[Out] -4/63*(x^4 + x)^(3/4)*(10*x^3 + 3)/x^6
```

giac [A] time = 0.42, size = 19, normalized size = 0.83

$$-\frac{4}{21}\left(\frac{1}{x^3}+1\right)^{\frac{7}{4}}-\frac{4}{9}\left(\frac{1}{x^3}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)/x^6/(x^4+x)^(1/4),x, algorithm="giac")
[Out] -4/21*(1/x^3 + 1)^(7/4) - 4/9*(1/x^3 + 1)^(3/4)
```

maple [A] time = 0.01, size = 31, normalized size = 1.35

$$-\frac{4\,(1+x)\left(x^2-x+1\right)\left(10x^3+3\right)}{63x^5\left(x^4+x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+1)/x^6/(x^4+x)^(1/4),x)
[Out] -4/63*(1+x)*(x^2-x+1)*(10*x^3+3)/x^5/(x^4+x)^(1/4)
```

maxima [B] time = 0.84, size = 58, normalized size = 2.52

$$-\frac{8\left(x^4+x\right)}{9\left(x^2-x+1\right)^{\frac{1}{4}}(x+1)^{\frac{1}{4}}x^{\frac{13}{4}}}+\frac{4\left(4\,x^7+x^4-3\,x\right)}{63\left(x^2-x+1\right)^{\frac{1}{4}}(x+1)^{\frac{1}{4}}x^{\frac{25}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)/x^6/(x^4+x)^(1/4),x, algorithm="maxima")
[Out] -8/9*(x^4 + x)/((x^2 - x + 1)^(1/4)*(x + 1)^(1/4)*x^(13/4)) + 4/63*(4*x^7 +
 x^4 - 3*x)/((x^2 - x + 1)^(1/4)*(x + 1)^(1/4)*x^(25/4))
```

mupad [B] time = 0.17, size = 27, normalized size = 1.17

$$-\frac{12\left(x^4+x\right)^{3/4}+40\,x^3\left(x^4+x\right)^{3/4}}{63\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3 + 1)/(x^6*(x + x^4)^(1/4)),x)
```

```
[Out] -(12*(x + x^4)^(3/4) + 40*x^3*(x + x^4)^(3/4))/(63*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3+1}{x^6\sqrt[4]{x(x+1)(x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+1)/x**6/(x**4+x)**(1/4),x)
```

```
[Out] Integral((2*x**3 + 1)/(x**6*(x*(x + 1)*(x**2 - x + 1))**(1/4)), x)
```

3.223 $\int \frac{-1+x^2}{(1+x^2)\sqrt[3]{x^2+x^4}}\,dx$

Optimal. Leaf size=23

$$-\frac{3\left(x^4+x^2\right)^{2/3}}{x\left(x^2+1\right)}$$

Rubi [A] time = 0.08, antiderivative size = 14, normalized size of antiderivative = 0.61, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2056, 449}

$$-\frac{3x}{\sqrt[3]{x^4+x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/((1 + x^2)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] (-3*x)/(x^2 + x^4)^(1/3)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt[3]{x^2+x^4}}\,dx &= \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{-1+x^2}{x^{2/3}\left(1+x^2\right)^{4/3}}\,dx}{\sqrt[3]{x^2+x^4}}\\ &= -\frac{3x}{\sqrt[3]{x^2+x^4}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 0.61

$$-\frac{3x}{\sqrt[3]{x^4+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] (-3*x)/(x^2 + x^4)^(1/3)
```

IntegrateAlgebraic [A] time = 0.10, size = 23, normalized size = 1.00

$$-\frac{3\left(x^4+x^2\right)^{2/3}}{x\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*(x^2 + x^4)^(1/3)),x]
[Out] (-3*(x^2 + x^4)^(2/3))/(x*(1 + x^2))
```

fricas [A] time = 0.39, size = 18, normalized size = 0.78

$$-\frac{3\left(x^4+x^2\right)^{\frac{2}{3}}}{x^3+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^4+x^2)^(1/3),x, algorithm="fricas")
[Out] -3*(x^4 + x^2)^(2/3)/(x^3 + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^4+x^2\right)^{\frac{1}{3}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^4+x^2)^(1/3),x, algorithm="giac")
[Out] integrate((x^2 - 1)/((x^4 + x^2)^(1/3)*(x^2 + 1)), x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.57

$$-\frac{3x}{\left(x^4+x^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^4+x^2)^(1/3),x)
[Out] -3*x/(x^4+x^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^4+x^2\right)^{\frac{1}{3}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^4+x^2)^(1/3),x, algorithm="maxima")
[Out] integrate((x^2 - 1)/((x^4 + x^2)^(1/3)*(x^2 + 1)), x)
```

mupad [B] time = 0.15, size = 21, normalized size = 0.91

$$-\frac{3\left(x^4+x^2\right)^{2/3}}{x\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + x^4)^(1/3)*(x^2 + 1)),x)
[Out] -(3*(x^2 + x^4)^(2/3))/(x*(x^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{\sqrt[3]{x^2\left(x^2+1\right)}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**4+x**2)**(1/3),x)
```

```
[Out] Integral((x - 1)*(x + 1)/((x**2*(x**2 + 1))**(1/3)*(x**2 + 1)), x)
```

3.224 $\int \frac{1}{\left(1+x^2\right)\sqrt[4]{x^2+x^4}}\,dx$

Optimal. Leaf size=23

$$\frac{2\left(x^4+x^2\right)^{3/4}}{x\left(x^2+1\right)}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 0.61, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1146, 264}

$$\frac{2x}{\sqrt[4]{x^4+x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*(x^2 + x^4)^(1/4)),x]
```

```
[Out] (2*x)/(x^2 + x^4)^(1/4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1146

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+x^2\right)\sqrt[4]{x^2+x^4}}\,dx &= \frac{\left(\sqrt{x}\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt{x}\left(1+x^2\right)^{5/4}}\,dx}{\sqrt[4]{x^2+x^4}}\\ &= \frac{2x}{\sqrt[4]{x^2+x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.61

$$\frac{2x}{\sqrt[4]{x^4+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(x^2 + x^4)^(1/4)),x]
```

```
[Out] (2*x)/(x^2 + x^4)^(1/4)
```

IntegrateAlgebraic [A] time = 0.10, size = 23, normalized size = 1.00

$$\frac{2\left(x^4+x^2\right)^{3/4}}{x\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x^2)*(x^2 + x^4)^(1/4)),x]
[Out] (2*(x^2 + x^4)^(3/4))/(x*(1 + x^2))
```

fricas [A] time = 0.41, size = 18, normalized size = 0.78

$$\frac{2\left(x^4+x^2\right)^{\frac{3}{4}}}{x^3+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2)^(1/4),x, algorithm="fricas")
[Out] 2*(x^4 + x^2)^(3/4)/(x^3 + x)
```

giac [A] time = 0.23, size = 9, normalized size = 0.39

$$\frac{2}{\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2)^(1/4),x, algorithm="giac")
[Out] 2/(1/x^2 + 1)^(1/4)
```

maple [A] time = 0.00, size = 13, normalized size = 0.57

$$\frac{2x}{\left(x^4+x^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(x^4+x^2)^(1/4),x)
[Out] 2*x/(x^4+x^2)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(x^3+x\right)}{3\left(x^{\frac{5}{2}}+\sqrt{x}\right)\left(x^2+1\right)^{\frac{1}{4}}}+\int\frac{4\left(x^2+1\right)^{\frac{3}{4}}}{3\left(x^{\frac{9}{2}}+2\,x^{\frac{5}{2}}+\sqrt{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2)^(1/4),x, algorithm="maxima")
[Out] -2/3*(x^3 + x)/((x^(5/2) + sqrt(x))*(x^2 + 1)^(1/4)) + integrate(4/3*(x^2 +
 1)^(3/4)/(x^(9/2) + 2*x^(5/2) + sqrt(x)), x)
```

mupad [B] time = 0.12, size = 21, normalized size = 0.91

$$\frac{2\left(x^4+x^2\right)^{3/4}}{x\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + x^4)^(1/4)*(x^2 + 1)),x)
```

```
[Out] (2*(x^2 + x^4)^(3/4))/(x*(x^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^2\left(x^2+1\right)}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(x**4+x**2)**(1/4),x)
```

```
[Out] Integral(1/((x**2*(x**2 + 1))**(1/4)*(x**2 + 1)), x)
```

3.225 $\int \frac{1}{(-1+x)\sqrt[4]{-x^3+x^4}}\,dx$

Optimal. Leaf size=23

$$-\frac{4\left(x^4-x^3\right)^{3/4}}{(x-1)x^2}$$

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 0.70, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2056, 37}

$$-\frac{4x}{\sqrt[4]{x^4-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (-4*x)/(-x^3 + x^4)^(1/4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-1+x)\sqrt[4]{-x^3+x^4}}\,dx &= \frac{\left(\sqrt[4]{-1+x}\,x^{3/4}\right)\int \frac{1}{(-1+x)^{5/4}x^{3/4}}\,dx}{\sqrt[4]{-x^3+x^4}}\\ &= -\frac{4x}{\sqrt[4]{-x^3+x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.61

$$-\frac{4x}{\sqrt[4]{(x-1)x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (-4*x)/((-1 + x)*x^3)^(1/4)
```

IntegrateAlgebraic [A] time = 0.21, size = 23, normalized size = 1.00

$$-\frac{4\left(x^4-x^3\right)^{3/4}}{(x-1)x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x)*(-x^3 + x^4)^(1/4)),x]
[Out] (-4*(-x^3 + x^4)^(3/4))/((-1 + x)*x^2)
```

fricas [A] time = 0.39, size = 24, normalized size = 1.04

$$-\frac{4\left(x^4-x^3\right)^{\frac{3}{4}}}{x^3-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^4-x^3)^(1/4),x, algorithm="fricas")
[Out] -4*(x^4 - x^3)^(3/4)/(x^3 - x^2)
```

giac [A] time = 0.32, size = 11, normalized size = 0.48

$$\frac{4}{\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^4-x^3)^(1/4),x, algorithm="giac")
[Out] 4/(-1/x + 1)^(1/4)
```

maple [A] time = 0.00, size = 15, normalized size = 0.65

$$-\frac{4x}{\left(x^4-x^3\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)/(x^4-x^3)^(1/4),x)
[Out] -4*x/(x^4-x^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4-x^3\right)^{\frac{1}{4}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^4-x^3)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((x^4 - x^3)^(1/4)*(x - 1)), x)
```

mupad [B] time = 0.17, size = 21, normalized size = 0.91

$$-\frac{4\left(x^4-x^3\right)^{3/4}}{x^2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^4 - x^3)^(1/4)*(x - 1)),x)
[Out] -(4*(x^4 - x^3)^(3/4))/(x^2*(x - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^3\,(x-1)}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x**4-x**3)**(1/4),x)
```

```
[Out] Integral(1/((x**3*(x - 1))**(1/4)*(x - 1)), x)
```

3.226 $\int \frac{1}{x\sqrt{x^3+x^4}}\,dx$

Optimal. Leaf size=23

$$\frac{2(2x-1)\sqrt{x^4+x^3}}{3x^3}$$

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.61, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2016, 2000}

$$\frac{4\sqrt{x^4+x^3}}{3x^2}-\frac{2\sqrt{x^4+x^3}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[x^3 + x^4]),x]
```

```
[Out] (-2*Sqrt[x^3 + x^4])/(3*x^3) + (4*Sqrt[x^3 + x^4])/(3*x^2)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{x^3+x^4}}\,dx &= -\frac{2\sqrt{x^3+x^4}}{3x^3}-\frac{2}{3}\int \frac{1}{\sqrt{x^3+x^4}}\,dx\\ &= -\frac{2\sqrt{x^3+x^4}}{3x^3}+\frac{4\sqrt{x^3+x^4}}{3x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2\left(2x^2+x-1\right)}{3\sqrt{x^3(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[x^3 + x^4]),x]
```

```
[Out] (2*(-1 + x + 2*x^2))/(3*Sqrt[x^3*(1 + x)])
```

IntegrateAlgebraic [A] time = 0.13, size = 23, normalized size = 1.00

$$\frac{2(2x-1)\sqrt{x^4+x^3}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[x^3 + x^4]),x]
[Out] (2*(-1 + 2*x)*Sqrt[x^3 + x^4])/(3*x^3)
```

fricas [A] time = 0.42, size = 26, normalized size = 1.13

$$\frac{2\left(2x^3 + \sqrt{x^4 + x^3}(2x - 1)\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+x^3)^(1/2),x, algorithm="fricas")
[Out] 2/3*(2*x^3 + sqrt(x^4 + x^3)*(2*x - 1))/x^3
```

giac [A] time = 0.37, size = 19, normalized size = 0.83

$$-\frac{2}{3}\left(\frac{1}{x} + 1\right)^{\frac{3}{2}} + 2\sqrt{\frac{1}{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+x^3)^(1/2),x, algorithm="giac")
[Out] -2/3*(1/x + 1)^(3/2) + 2*sqrt(1/x + 1)
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$\frac{2(1 + x)(-1 + 2x)}{3\sqrt{x^4 + x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4+x^3)^(1/2),x)
[Out] 2/3*(1+x)*(-1+2*x)/(x^4+x^3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4 + x^3}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^4 + x^3)*x), x)
```

mupad [B] time = 0.13, size = 29, normalized size = 1.26

$$\frac{4x\sqrt{x^4 + x^3} - 2\sqrt{x^4 + x^3}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^3 + x^4)^(1/2)),x)
[Out] (4*x*(x^3 + x^4)^(1/2) - 2*(x^3 + x^4)^(1/2))/(3*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^3(x + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4+x**3)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**3*(x + 1))), x)
```

3.227 $\int \frac{1}{x^2 \sqrt[4]{x^3+x^4}} \, dx$

Optimal. Leaf size=23

$$\frac{4(4x-3)\left(x^4+x^3\right)^{3/4}}{21x^4}$$

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.61, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2016, 2014}

$$\frac{16\left(x^4+x^3\right)^{3/4}}{21x^3} - \frac{4\left(x^4+x^3\right)^{3/4}}{7x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(x^3 + x^4)^(1/4)),x]
```

```
[Out] (-4*(x^3 + x^4)^(3/4))/(7*x^4) + (16*(x^3 + x^4)^(3/4))/(21*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2 \sqrt[4]{x^3+x^4}} \, dx &= -\frac{4\left(x^3+x^4\right)^{3/4}}{7x^4} - \frac{4}{7}\int \frac{1}{x\sqrt[4]{x^3+x^4}} \, dx \\ &= -\frac{4\left(x^3+x^4\right)^{3/4}}{7x^4} + \frac{16\left(x^3+x^4\right)^{3/4}}{21x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{4\left(x^3(x+1)\right)^{3/4}(4x-3)}{21x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(x^3 + x^4)^(1/4)),x]
```

```
[Out] (4*(x^3*(1 + x))^(3/4)*(-3 + 4*x))/(21*x^4)
```

IntegrateAlgebraic [A] time = 0.20, size = 23, normalized size = 1.00

$$\frac{4(4x-3)\left(x^4+x^3\right)^{3/4}}{21x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(x^3 + x^4)^(1/4)),x]
[Out] (4*(-3 + 4*x)*(x^3 + x^4)^(3/4))/(21*x^4)
```

fricas [A] time = 0.43, size = 19, normalized size = 0.83

$$\frac{4\left(x^4+x^3\right)^{\frac{3}{4}}(4\,x-3)}{21\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+x^3)^(1/4),x, algorithm="fricas")
[Out] 4/21*(x^4 + x^3)^(3/4)*(4*x - 3)/x^4
```

giac [A] time = 0.32, size = 19, normalized size = 0.83

$$-\frac{4}{7}\left(\frac{1}{x}+1\right)^{\frac{7}{4}}+\frac{4}{3}\left(\frac{1}{x}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+x^3)^(1/4),x, algorithm="giac")
[Out] -4/7*(1/x + 1)^(7/4) + 4/3*(1/x + 1)^(3/4)
```

maple [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{4\,(1+x)\,(-3+4x)}{21x\left(x^4+x^3\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^4+x^3)^(1/4),x)
[Out] 4/21*(1+x)*(-3+4*x)/x/(x^4+x^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(x^4+x^3\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+x^3)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((x^4 + x^3)^(1/4)*x^2), x)
```

mupad [B] time = 0.13, size = 29, normalized size = 1.26

$$\frac{16\,x\left(x^4+x^3\right)^{3/4}-12\left(x^4+x^3\right)^{3/4}}{21\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^3 + x^4)^(1/4)),x)
[Out] (16*x*(x^3 + x^4)^(3/4) - 12*(x^3 + x^4)^(3/4))/(21*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt[4]{x^3 (x+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**4+x**3)**(1/4),x)
```

```
[Out] Integral(1/(x**2*(x**3*(x + 1))**(1/4)), x)
```

3.228 $\int \frac{1+2x}{\sqrt{3+x^2+2x^3+x^4}}\,dx$

Optimal. Leaf size=23

$$\log\left(x^2+\sqrt{x^4+2x^3+x^2+3}+x\right)$$

Rubi [A] time = 0.05, antiderivative size = 11, normalized size of antiderivative = 0.48, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1680, 12, 1107, 619, 215}

$$\sinh^{-1}\left(\frac{x(x+1)}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/Sqrt[3 + x^2 + 2*x^3 + x^4],x]
```

```
[Out] ArcSinh[(x*(1 + x))/Sqrt[3]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+2x}{\sqrt{3+x^2+2x^3+x^4}}\,dx &= \text{Subst}\left(\int \frac{8x}{\sqrt{49-8x^2+16x^4}}\,dx, x, \frac{1}{2}+x\right) \\
&= 8\,\text{Subst}\left(\int \frac{x}{\sqrt{49-8x^2+16x^4}}\,dx, x, \frac{1}{2}+x\right) \\
&= 4\,\text{Subst}\left(\int \frac{1}{\sqrt{49-8x+16x^2}}\,dx, x, \left(\frac{1}{2}+x\right)^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3072}}}\,dx, x, 32x(1+x)\right)}{32\sqrt{3}} \\
&= \sinh^{-1}\left(\frac{x(1+x)}{\sqrt{3}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.59, size = 305, normalized size = 13.26

$$
-\frac{(-1)^{2/3}\sqrt{2}\sqrt[4]{3}\sqrt{\frac{2ix+\sqrt{3}+3i}{x+(-1)^{2/3}}}\left(x+(-1)^{2/3}\right)^2\sqrt{\frac{2\sqrt{3}x+3\sqrt{3}+3i}{\left(\sqrt{3}-2i\right)\left(x+(-1)^{2/3}\right)}}\sqrt{\frac{\left(4-2i\sqrt{3}\right)x-i\sqrt{3}-5}{x+(-1)^{2/3}}}\left(\left(\sqrt{3}+2i\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{2\left(2i+\sqrt{3}\right)x+\sqrt{3}-5i}{-2ix+\sqrt{3}+i}}}{\sqrt{2}}\right)|\frac{4}{7}\right)-2\sqrt{3}\Pi\left(\frac{2i}{2i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-\frac{2\left(2i+\sqrt{3}\right)x+\sqrt{3}-5i}{-2ix+\sqrt{3}+i}}}{\sqrt{2}}\right)|\frac{4}{7}\right)\right)}{\left(\sqrt{3}+9i\right)\sqrt{x^4+2x^3+x^2+3}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + 2*x)/Sqrt[3 + x^2 + 2*x^3 + x^4],x]
```

```
[Out] -(((-1)^(2/3)*Sqrt[2]*3^(1/4)*Sqrt[(3*I + Sqrt[3] + (2*I)*x)/((-1)^(2/3) +
x)]*((-1)^(2/3) + x)^2*Sqrt[(3*I + 3*Sqrt[3] + 2*Sqrt[3]*x)/((-2*I + Sqrt[3
])*((-1)^(2/3) + x))]*Sqrt[(-5 - I*Sqrt[3] + (4 - (2*I)*Sqrt[3])*x)/((-1)^(
2/3) + x)]*((2*I + Sqrt[3])*EllipticF[ArcSin[Sqrt[-((-5*I + Sqrt[3] + 2*(2*
I + Sqrt[3])*x)/(I + Sqrt[3] - (2*I)*x))]/Sqrt[2]], 4/7] - 2*Sqrt[3]*Ellipt
icPi[(2*I)/(2*I + Sqrt[3]), ArcSin[Sqrt[-((-5*I + Sqrt[3] + 2*(2*I + Sqrt[3
])*x)/(I + Sqrt[3] - (2*I)*x))]/Sqrt[2]], 4/7]))/((9*I + Sqrt[3])*Sqrt[3 +
x^2 + 2*x^3 + x^4]))
```

IntegrateAlgebraic [A] time = 0.08, size = 23, normalized size = 1.00

$$
\log\left(x^2+\sqrt{x^4+2x^3+x^2+3}+x\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x)/Sqrt[3 + x^2 + 2*x^3 + x^4],x]
```

```
[Out] Log[x + x^2 + Sqrt[3 + x^2 + 2*x^3 + x^4]]
```

fricas [A] time = 0.44, size = 21, normalized size = 0.91

$$
\log\left(x^2+x+\sqrt{x^4+2\,x^3+x^2+3}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^4+2*x^3+x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] log(x^2 + x + sqrt(x^4 + 2*x^3 + x^2 + 3))
```

giac [A] time = 0.40, size = 23, normalized size = 1.00

$$
-\log\left(-x^2-x+\sqrt{\left(x^2+x\right)^2+3}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^4+2*x^3+x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-x^2 - x + sqrt((x^2 + x)^2 + 3))
```

maple [C] time = 0.33, size = 528, normalized size = 22.96

$$\frac{i\left(2-i\sqrt{3}\right)\sqrt{-\frac{2\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{\left(-2+i\sqrt{3}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^{2}\sqrt{2}\sqrt{-\frac{i\sqrt{3}\left(x+\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)}{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{i\sqrt{3}\left(x+\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-2+i\sqrt{3}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}\sqrt{3}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{\left(-2+i\sqrt{3}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}},\frac{\sqrt{\left(2+i\sqrt{3}\right)\left(2-i\sqrt{3}\right)}}{2}\right)}{6\sqrt{\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(x+\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(x+\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}+\frac{i\left(2-i\sqrt{3}\right)\sqrt{-\frac{2\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{\left(-2+i\sqrt{3}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^{2}\sqrt{2}\sqrt{-\frac{i\sqrt{3}\left(x+\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)}{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{i\sqrt{3}\left(x+\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-2+i\sqrt{3}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}\sqrt{3}\left(\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticF}\left(\sqrt{-\frac{2\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{\left(-2+i\sqrt{3}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}},\frac{\sqrt{\left(2+i\sqrt{3}\right)\left(2-i\sqrt{3}\right)}}{2}\right)-i\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{2\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{\left(-2+i\sqrt{3}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}},1-\frac{i\sqrt{3}}{2},\frac{\sqrt{\left(2+i\sqrt{3}\right)\left(2-i\sqrt{3}\right)}}{2}\right)\right)}{3\sqrt{\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(x+\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(x+\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(x^4+2*x^3+x^2+3)^(1/2),x)
```

```
[Out] 1/6*I*(2-I*3^(1/2))*(-2*(x-1/2+1/2*I*3^(1/2))/(-2+I*3^(1/2))/(x-1/2-1/2*I*3
^(1/2)))^(1/2)*(x-1/2-1/2*I*3^(1/2))^2*2^(1/2)*(-I*3^(1/2)*(x+3/2+1/2*I*3^(
1/2))/(x-1/2-1/2*I*3^(1/2)))^(1/2)*(I*3^(1/2)*(x+3/2-1/2*I*3^(1/2))/(-2+I*3
^(1/2))/(x-1/2-1/2*I*3^(1/2)))^(1/2)*3^(1/2)/((x-1/2+1/2*I*3^(1/2))*(x-1/2-
1/2*I*3^(1/2))*(x+3/2+1/2*I*3^(1/2))*(x+3/2-1/2*I*3^(1/2)))^(1/2)*EllipticF
((-2*(x-1/2+1/2*I*3^(1/2))/(-2+I*3^(1/2))/(x-1/2-1/2*I*3^(1/2)))^(1/2),1/2*
((2+I*3^(1/2))*(2-I*3^(1/2)))^(1/2))+1/3*I*(2-I*3^(1/2))*(-2*(x-1/2+1/2*I*3
^(1/2))/(-2+I*3^(1/2))/(x-1/2-1/2*I*3^(1/2)))^(1/2)*(x-1/2-1/2*I*3^(1/2))^2
*2^(1/2)*(-I*3^(1/2)*(x+3/2+1/2*I*3^(1/2))/(x-1/2-1/2*I*3^(1/2)))^(1/2)*(I*
3^(1/2)*(x+3/2-1/2*I*3^(1/2))/(-2+I*3^(1/2))/(x-1/2-1/2*I*3^(1/2)))^(1/2)*3
^(1/2)/((x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2))*(x+3/2+1/2*I*3^(1/2))*(
x+3/2-1/2*I*3^(1/2)))^(1/2)*((1/2+1/2*I*3^(1/2))*EllipticF((-2*(x-1/2+1/2*I
*3^(1/2))/(-2+I*3^(1/2))/(x-1/2-1/2*I*3^(1/2)))^(1/2),1/2*((2+I*3^(1/2))*(2
-I*3^(1/2)))^(1/2))-I*3^(1/2)*EllipticPi((-2*(x-1/2+1/2*I*3^(1/2))/(-2+I*3^
(1/2))/(x-1/2-1/2*I*3^(1/2)))^(1/2),1-1/2*I*3^(1/2),1/2*((2+I*3^(1/2))*(2-I
*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x+1}{\sqrt{x^4+2\,x^3+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^4+2*x^3+x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)/sqrt(x^4 + 2*x^3 + x^2 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{2\,x+1}{\sqrt{x^4+2\,x^3+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/(x^2 + 2*x^3 + x^4 + 3)^(1/2),x)
```

```
[Out] int((2*x + 1)/(x^2 + 2*x^3 + x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\sqrt{\left(x^2-x+1\right)\left(x^2+3x+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**4+2*x**3+x**2+3)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/sqrt((x**2 - x + 1)*(x**2 + 3*x + 3)), x)
```

3.229 $\int \frac{-1+2x^4}{x^8 \sqrt[4]{-1+x^4}} \, dx$

Optimal. Leaf size=23

$$\frac{(x^4-1)^{3/4}(10x^4-3)}{21x^7}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.43, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {453, 264}

$$\frac{10(x^4-1)^{3/4}}{21x^3} - \frac{(x^4-1)^{3/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x^4)/(x^8*(-1 + x^4)^(1/4)),x]
```

```
[Out] -1/7*(-1 + x^4)^(3/4)/x^7 + (10*(-1 + x^4)^(3/4))/(21*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+2x^4}{x^8\sqrt[4]{-1+x^4}}\,dx &= -\frac{(-1+x^4)^{3/4}}{7x^7}+\frac{10}{7}\int \frac{1}{x^4\sqrt[4]{-1+x^4}}\,dx\\ &= -\frac{(-1+x^4)^{3/4}}{7x^7}+\frac{10(-1+x^4)^{3/4}}{21x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{(x^4-1)^{3/4}(10x^4-3)}{21x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x^4)/(x^8*(-1 + x^4)^(1/4)),x]
```

```
[Out] ((-1 + x^4)^(3/4)*(-3 + 10*x^4))/(21*x^7)
```

IntegrateAlgebraic [A] time = 0.18, size = 23, normalized size = 1.00

$$\frac{(x^4-1)^{3/4}(10x^4-3)}{21x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x^4)/(x^8*(-1 + x^4)^(1/4)),x]
[Out] ((-1 + x^4)^(3/4)*(-3 + 10*x^4))/(21*x^7)
```

fricas [A] time = 0.40, size = 19, normalized size = 0.83

$$\frac{\left(10 x^4 - 3\right)\left(x^4 - 1\right)^{\frac{3}{4}}}{21 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)/x^8/(x^4-1)^(1/4),x, algorithm="fricas")
[Out] 1/21*(10*x^4 - 3)*(x^4 - 1)^(3/4)/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 x^4 - 1}{\left(x^4 - 1\right)^{\frac{1}{4}} x^8} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)/x^8/(x^4-1)^(1/4),x, algorithm="giac")
[Out] integrate((2*x^4 - 1)/((x^4 - 1)^(1/4)*x^8), x)
```

maple [A] time = 0.01, size = 31, normalized size = 1.35

$$\frac{(-1 + x)(1 + x)\left(x^2 + 1\right)\left(10x^4 - 3\right)}{21x^7\left(x^4 - 1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-1)/x^8/(x^4-1)^(1/4),x)
[Out] 1/21*(-1+x)*(1+x)*(x^2+1)*(10*x^4-3)/x^7/(x^4-1)^(1/4)
```

maxima [A] time = 0.61, size = 25, normalized size = 1.09

$$\frac{\left(x^4 - 1\right)^{\frac{3}{4}}}{3 x^3} + \frac{\left(x^4 - 1\right)^{\frac{7}{4}}}{7 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)/x^8/(x^4-1)^(1/4),x, algorithm="maxima")
[Out] 1/3*(x^4 - 1)^(3/4)/x^3 + 1/7*(x^4 - 1)^(7/4)/x^7
```

mupad [B] time = 0.17, size = 27, normalized size = 1.17

$$-\frac{3\left(x^4 - 1\right)^{3/4} - 10 x^4 \left(x^4 - 1\right)^{3/4}}{21 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4 - 1)/(x^8*(x^4 - 1)^(1/4)),x)
[Out] -(3*(x^4 - 1)^(3/4) - 10*x^4*(x^4 - 1)^(3/4))/(21*x^7)
```

sympy [C] time = 2.00, size = 190, normalized size = 8.26

$$2\left(\begin{cases} -\frac{\left(-1+\frac{1}{x^4}\right)^{\frac{3}{4}} e^{\frac{3i\pi}{4}}\Gamma\left(-\frac{3}{4}\right)}{4\Gamma\left(\frac{1}{4}\right)} & \text{for } \frac{1}{|x^4|} > 1 \\ -\frac{\left(1-\frac{1}{x^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{4\Gamma\left(\frac{1}{4}\right)} & \text{otherwise} \end{cases}\right) - \begin{cases} -\frac{\left(-1+\frac{1}{x^4}\right)^{\frac{3}{4}} e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{7}{4}\right)}{4\Gamma\left(\frac{1}{4}\right)} - \frac{3\left(-1+\frac{1}{x^4}\right)^{\frac{3}{4}} e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{7}{4}\right)}{16x^4\Gamma\left(\frac{1}{4}\right)} & \text{for } \frac{1}{|x^4|} > 1 \\ \frac{\left(1-\frac{1}{x^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4\Gamma\left(\frac{1}{4}\right)} + \frac{3\left(1-\frac{1}{x^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{16x^4\Gamma\left(\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-1)/x**8/(x**4-1)**(1/4),x)
```

```
[Out] 2*Piecewise((-(-1 + x**(-4))**(3/4)*exp(3*I*pi/4)*gamma(-3/4)/(4*gamma(1/4)
), 1/Abs(x**4) > 1), (-(1 - 1/x**4)**(3/4)*gamma(-3/4)/(4*gamma(1/4)), True
)) - Piecewise((-(-1 + x**(-4))**(3/4)*exp(-I*pi/4)*gamma(-7/4)/(4*gamma(1/
4)) - 3*(-1 + x**(-4))**(3/4)*exp(-I*pi/4)*gamma(-7/4)/(16*x**4*gamma(1/4))
, 1/Abs(x**4) > 1), ((1 - 1/x**4)**(3/4)*gamma(-7/4)/(4*gamma(1/4)) + 3*(1
- 1/x**4)**(3/4)*gamma(-7/4)/(16*x**4*gamma(1/4)), True))
```

3.230 $\int \frac{1+2x^4}{x^4(1+x^4)^{5/4}}\,dx$

Optimal. Leaf size=23

$$\frac{2x^4-1}{3x^3\sqrt[4]{x^4+1}}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.35, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {453, 191}

$$\frac{2x}{3\sqrt[4]{x^4+1}}-\frac{1}{3x^3\sqrt[4]{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^4)/(x^4*(1 + x^4)^(5/4)),x]
```

```
[Out] -1/3*1/(x^3*(1 + x^4)^(1/4)) + (2*x)/(3*(1 + x^4)^(1/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^4}{x^4\left(1+x^4\right)^{5/4}}\,dx &= -\frac{1}{3x^3\sqrt[4]{1+x^4}}+\frac{2}{3}\int \frac{1}{\left(1+x^4\right)^{5/4}}\,dx\\ &= -\frac{1}{3x^3\sqrt[4]{1+x^4}}+\frac{2x}{3\sqrt[4]{1+x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2x^4-1}{3x^3\sqrt[4]{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^4)/(x^4*(1 + x^4)^(5/4)),x]
```

```
[Out] (-1 + 2*x^4)/(3*x^3*(1 + x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.23, size = 23, normalized size = 1.00

$$\frac{2x^4-1}{3x^3\sqrt[4]{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x^4)/(x^4*(1 + x^4)^(5/4)),x]
[Out] (-1 + 2*x^4)/(3*x^3*(1 + x^4)^(1/4))
```

fricas [A] time = 0.39, size = 25, normalized size = 1.09

$$\frac{(2x^4 - 1)(x^4 + 1)^{\frac{3}{4}}}{3(x^7 + x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+1)/x^4/(x^4+1)^(5/4),x, algorithm="fricas")
[Out] 1/3*(2*x^4 - 1)*(x^4 + 1)^(3/4)/(x^7 + x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^4 + 1}{(x^4 + 1)^{\frac{5}{4}} x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+1)/x^4/(x^4+1)^(5/4),x, algorithm="giac")
[Out] integrate((2*x^4 + 1)/((x^4 + 1)^(5/4)*x^4), x)
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$\frac{2x^4 - 1}{3x^3 (x^4 + 1)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+1)/x^4/(x^4+1)^(5/4),x)
[Out] 1/3*(2*x^4-1)/x^3/(x^4+1)^(1/4)
```

maxima [A] time = 0.38, size = 22, normalized size = 0.96

$$\frac{x}{(x^4 + 1)^{\frac{1}{4}}} - \frac{(x^4 + 1)^{\frac{3}{4}}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+1)/x^4/(x^4+1)^(5/4),x, algorithm="maxima")
[Out] x/(x^4 + 1)^(1/4) - 1/3*(x^4 + 1)^(3/4)/x^3
```

mupad [B] time = 0.07, size = 19, normalized size = 0.83

$$\frac{2x^4 - 1}{3x^3 (x^4 + 1)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4 + 1)/(x^4*(x^4 + 1)^(5/4)),x)
```

```
[Out] (2*x^4 - 1)/(3*x^3*(x^4 + 1)^(1/4))
```

sympy [B] time = 5.02, size = 97, normalized size = 4.22

$$\frac{4x^4\left(x^4+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{16x^7\Gamma\left(\frac{5}{4}\right)+16x^3\Gamma\left(\frac{5}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right)}{2\sqrt[4]{x^4+1}\,\Gamma\left(\frac{5}{4}\right)}+\frac{\left(x^4+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{16x^7\Gamma\left(\frac{5}{4}\right)+16x^3\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+1)/x**4/(x**4+1)**(5/4),x)
```

```
[Out] 4*x**4*(x**4 + 1)**(3/4)*gamma(-3/4)/(16*x**7*gamma(5/4) + 16*x**3*gamma(5/
4)) + x*gamma(1/4)/(2*(x**4 + 1)**(1/4)*gamma(5/4)) + (x**4 + 1)**(3/4)*gam
ma(-3/4)/(16*x**7*gamma(5/4) + 16*x**3*gamma(5/4))
```

3.231 $\int \frac{(1+4x^3)(1+2x+2x^4)}{\sqrt{x+x^4}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}\sqrt{x^4+x}\,(2x^4+2x+3)$$

Rubi [A] time = 0.36, antiderivative size = 42, normalized size of antiderivative = 1.83, number of steps used = 18, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2053, 2011, 329, 225, 2029, 206, 2024}

$$\frac{4}{3}\sqrt{x^4+x}\,x^4+\frac{4}{3}\sqrt{x^4+x}\,x+2\sqrt{x^4+x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x^3)*(1 + 2*x + 2*x^4))/Sqrt[x + x^4],x]
```

```
[Out] 2*Sqrt[x + x^4] + (4*x*Sqrt[x + x^4])/3 + (4*x^4*Sqrt[x + x^4])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2053

```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Expan
dIntegrand[Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && (Po
lyQ[Pq, x] || PolyQ[Pq, x^n]) && !IntegerQ[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{(1+4x^3)(1+2x+2x^4)}{\sqrt{x+x^4}}\,dx = \int \left(\frac{1}{\sqrt{x+x^4}}+\frac{2x}{\sqrt{x+x^4}}+\frac{4x^3}{\sqrt{x+x^4}}+\frac{10x^4}{\sqrt{x+x^4}}+\frac{8x^7}{\sqrt{x+x^4}}\right)dx$$

$$= 2\int \frac{x}{\sqrt{x+x^4}}\,dx+4\int \frac{x^3}{\sqrt{x+x^4}}\,dx+8\int \frac{x^7}{\sqrt{x+x^4}}\,dx+10\int \frac{x^4}{\sqrt{x+x^4}}\,dx$$

$$= 2\sqrt{x+x^4}+\frac{10}{3}x\sqrt{x+x^4}+\frac{4}{3}x^4\sqrt{x+x^4}+\frac{4}{3}\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x^2}{\sqrt{x+x}}\right.$$

$$= 2\sqrt{x+x^4}+\frac{4}{3}x\sqrt{x+x^4}+\frac{4}{3}x^4\sqrt{x+x^4}+\frac{4}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x+x^4}}\right)+3\int \frac{x}{\sqrt{x+}}$$

$$= 2\sqrt{x+x^4}+\frac{4}{3}x\sqrt{x+x^4}+\frac{4}{3}x^4\sqrt{x+x^4}-2\tanh^{-1}\left(\frac{x^2}{\sqrt{x+x^4}}\right)+\frac{x(1+x)\sqrt{}}{}$$

$$= 2\sqrt{x+x^4}+\frac{4}{3}x\sqrt{x+x^4}+\frac{4}{3}x^4\sqrt{x+x^4}$$

Mathematica [A] time = 0.05, size = 23, normalized size = 1.00

$$\frac{2}{3}\sqrt{x^4+x}\,\left(2x^4+2x+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x^3)*(1 + 2*x + 2*x^4))/Sqrt[x + x^4],x]
[Out] (2*Sqrt[x + x^4]*(3 + 2*x + 2*x^4))/3
```

IntegrateAlgebraic [A] time = 0.06, size = 23, normalized size = 1.00

$$\frac{2}{3}\sqrt{x^4+x}\,\left(2x^4+2x+3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + 4*x^3)*(1 + 2*x + 2*x^4))/Sqrt[x + x^4],x]
[Out] (2*Sqrt[x + x^4]*(3 + 2*x + 2*x^4))/3
```

fricas [A] time = 0.39, size = 19, normalized size = 0.83

$$\frac{2}{3}\left(2\,x^4+2\,x+3\right)\sqrt{x^4+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+1)*(2*x^4+2*x+1)/(x^4+x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(2*x^4 + 2*x + 3)*sqrt(x^4 + x)
```

giac [A]   time = 0.27, size = 19, normalized size = 0.83

$$\frac{4}{3}\left(x^{4}+x\right)^{\frac{3}{2}}+2\sqrt{x^{4}+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+1)*(2*x^4+2*x+1)/(x^4+x)^(1/2),x, algorithm="giac")
[Out] 4/3*(x^4 + x)^(3/2) + 2*sqrt(x^4 + x)
```

maple [A]   time = 0.01, size = 32, normalized size = 1.39

$$\frac{2x\left(1+x\right)\left(x^{2}-x+1\right)\left(2x^{4}+2x+3\right)}{3\sqrt{x^{4}+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3+1)*(2*x^4+2*x+1)/(x^4+x)^(1/2),x)
[Out] 2/3*x*(1+x)*(x^2-x+1)*(2*x^4+2*x+3)/(x^4+x)^(1/2)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{4}+2\,x+1\right)\left(4\,x^{3}+1\right)}{\sqrt{x^{4}+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+1)*(2*x^4+2*x+1)/(x^4+x)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x^4 + 2*x + 1)*(4*x^3 + 1)/sqrt(x^4 + x), x)
```

mupad [B]   time = 0.19, size = 19, normalized size = 0.83

$$\frac{2\,\sqrt{x^{4}+x}\,\left(2\,x^{4}+2\,x+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((4*x^3 + 1)*(2*x + 2*x^4 + 1))/(x + x^4)^(1/2),x)
[Out] (2*(x + x^4)^(1/2)*(2*x + 2*x^4 + 3))/3
```

sympy [A]   time = 0.43, size = 37, normalized size = 1.61

$$\frac{4x^{4}\sqrt{x^{4}+x}}{3}+\frac{4x\sqrt{x^{4}+x}}{3}+2\sqrt{x^{4}+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3+1)*(2*x**4+2*x+1)/(x**4+x)**(1/2),x)
[Out] 4*x**4*sqrt(x**4 + x)/3 + 4*x*sqrt(x**4 + x)/3 + 2*sqrt(x**4 + x)
```

3.232 $\int \frac{-1+x^2}{(1+x^2)\sqrt[4]{x^3+x^5}}\,dx$

Optimal. Leaf size=23

$$-\frac{4\left(x^5+x^3\right)^{3/4}}{x^2\left(x^2+1\right)}$$

Rubi [A] time = 0.08, antiderivative size = 14, normalized size of antiderivative = 0.61, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2056, 449}

$$-\frac{4x}{\sqrt[4]{x^5+x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/((1 + x^2)*(x^3 + x^5)^(1/4)),x]
```

```
[Out] (-4*x)/(x^3 + x^5)^(1/4)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt[4]{x^3+x^5}}\,dx &= \frac{\left(x^{3/4}\sqrt[4]{1+x^2}\right)\int \frac{-1+x^2}{x^{3/4}\left(1+x^2\right)^{5/4}}\,dx}{\sqrt[4]{x^3+x^5}}\\ &= -\frac{4x}{\sqrt[4]{x^3+x^5}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 14, normalized size = 0.61

$$-\frac{4x}{\sqrt[4]{x^5+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*(x^3 + x^5)^(1/4)),x]
```

```
[Out] (-4*x)/(x^3 + x^5)^(1/4)
```

IntegrateAlgebraic [A] time = 0.15, size = 23, normalized size = 1.00

$$-\frac{4\left(x^5+x^3\right)^{3/4}}{x^2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*(x^3 + x^5)^(1/4)),x]
[Out] (-4*(x^3 + x^5)^(3/4))/(x^2*(1 + x^2))
```

fricas [A] time = 0.40, size = 20, normalized size = 0.87

$$-\frac{4\left(x^5+x^3\right)^{\frac{3}{4}}}{x^4+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^5+x^3)^(1/4),x, algorithm="fricas")
[Out] -4*(x^5 + x^3)^(3/4)/(x^4 + x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^5+x^3\right)^{\frac{1}{4}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^5+x^3)^(1/4),x, algorithm="giac")
[Out] integrate((x^2 - 1)/((x^5 + x^3)^(1/4)*(x^2 + 1)), x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.57

$$-\frac{4x}{\left(x^5+x^3\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^5+x^3)^(1/4),x)
[Out] -4*x/(x^5+x^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^5+x^3\right)^{\frac{1}{4}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^5+x^3)^(1/4),x, algorithm="maxima")
[Out] integrate((x^2 - 1)/((x^5 + x^3)^(1/4)*(x^2 + 1)), x)
```

mupad [B] time = 0.15, size = 21, normalized size = 0.91

$$-\frac{4\left(x^5+x^3\right)^{3/4}}{x^2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^3 + x^5)^(1/4)*(x^2 + 1)),x)
[Out] -(4*(x^3 + x^5)^(3/4))/(x^2*(x^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{\sqrt[4]{x^3\left(x^2+1\right)}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**5+x**3)**(1/4),x)
```

```
[Out] Integral((x - 1)*(x + 1)/((x**3*(x**2 + 1))**(1/4)*(x**2 + 1)), x)
```

3.233 $\int \frac{1}{x^{10}\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=23

$$\frac{\sqrt{x^6-1}\,(2x^6+1)}{9x^9}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.43, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {271, 264}

$$\frac{\sqrt{x^6-1}}{9x^9}+\frac{2\sqrt{x^6-1}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*Sqrt[-1 + x^6]),x]
```

```
[Out] Sqrt[-1 + x^6]/(9*x^9) + (2*Sqrt[-1 + x^6])/(9*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}\sqrt{-1+x^6}}\,dx &= \frac{\sqrt{-1+x^6}}{9x^9}+\frac{2}{3}\int \frac{1}{x^4\sqrt{-1+x^6}}\,dx\\ &= \frac{\sqrt{-1+x^6}}{9x^9}+\frac{2\sqrt{-1+x^6}}{9x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{\sqrt{x^6-1}\,(2x^6+1)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*Sqrt[-1 + x^6]),x]
```

```
[Out] (Sqrt[-1 + x^6]*(1 + 2*x^6))/(9*x^9)
```

IntegrateAlgebraic [A] time = 0.18, size = 23, normalized size = 1.00

$$\frac{\sqrt{x^6-1}\,(2x^6+1)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^10*Sqrt[-1 + x^6]),x]
[Out] (Sqrt[-1 + x^6]*(1 + 2*x^6))/(9*x^9)
```

fricas [A] time = 0.39, size = 26, normalized size = 1.13

$$\frac{2\,x^{9}+\left(2\,x^{6}+1\right)\sqrt{x^{6}-1}}{9\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^6-1)^(1/2),x, algorithm="fricas")
[Out] 1/9*(2*x^9 + (2*x^6 + 1)*sqrt(x^6 - 1))/x^9
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^6-1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [abs(x)]sym2poly/r2sym(const gen
 & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.01, size = 40, normalized size = 1.74

$$\frac{(-1+x)\,(1+x)\left(x^{2}+x+1\right)\left(x^{2}-x+1\right)\left(2x^{6}+1\right)}{9x^{9}\sqrt{x^{6}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(x^6-1)^(1/2),x)
[Out] 1/9*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)*(2*x^6+1)/x^9/(x^6-1)^(1/2)
```

maxima [A] time = 0.33, size = 25, normalized size = 1.09

$$\frac{\sqrt{x^{6}-1}}{3\,x^{3}}-\frac{\left(x^{6}-1\right)^{\frac{3}{2}}}{9\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^6-1)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(x^6 - 1)/x^3 - 1/9*(x^6 - 1)^(3/2)/x^9
```

mupad [B] time = 0.29, size = 25, normalized size = 1.09

$$\frac{\sqrt{x^{6}-1}+2\,x^{6}\,\sqrt{x^{6}-1}}{9\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(x^6 - 1)^(1/2)),x)
[Out] ((x^6 - 1)^(1/2) + 2*x^6*(x^6 - 1)^(1/2))/(9*x^9)
```

sympy [A] time = 0.93, size = 63, normalized size = 2.74

$$\begin{cases} \frac{2\sqrt{x^6-1}}{9x^3} + \frac{\sqrt{x^6-1}}{9x^9} & \text{for } \left|x^6\right| > 1 \\ \frac{2i\sqrt{1-x^6}}{9x^3} + \frac{i\sqrt{1-x^6}}{9x^9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(x**6-1)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(x**6 - 1)/(9*x**3) + sqrt(x**6 - 1)/(9*x**9), Abs(x**6) >
 1), (2*I*sqrt(1 - x**6)/(9*x**3) + I*sqrt(1 - x**6)/(9*x**9), True))
```

3.234 $\int \frac{1+x^6}{x^{10}\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=23

$$\frac{\sqrt{x^6-1}\,(5x^6+1)}{9x^9}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.43, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {453, 264}

$$\frac{\sqrt{x^6-1}}{9x^9}+\frac{5\sqrt{x^6-1}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)/(x^10*Sqrt[-1 + x^6]),x]
```

```
[Out] Sqrt[-1 + x^6]/(9*x^9) + (5*Sqrt[-1 + x^6])/(9*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^6}{x^{10}\sqrt{-1+x^6}}\,dx &= \frac{\sqrt{-1+x^6}}{9x^9}+\frac{5}{3}\int \frac{1}{x^4\sqrt{-1+x^6}}\,dx\\ &= \frac{\sqrt{-1+x^6}}{9x^9}+\frac{5\sqrt{-1+x^6}}{9x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\sqrt{x^6-1}\,(5x^6+1)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)/(x^10*Sqrt[-1 + x^6]),x]
```

```
[Out] (Sqrt[-1 + x^6]*(1 + 5*x^6))/(9*x^9)
```

IntegrateAlgebraic [A] time = 0.23, size = 23, normalized size = 1.00

$$\frac{\sqrt{x^6-1}\,(5x^6+1)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^6)/(x^10*Sqrt[-1 + x^6]),x]
```

```
[Out] (Sqrt[-1 + x^6]*(1 + 5*x^6))/(9*x^9)
```

fricas [A] time = 0.39, size = 26, normalized size = 1.13

$$\frac{5\,x^{9}+\left(5\,x^{6}+1\right)\sqrt{x^{6}-1}}{9\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x^10/(x^6-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(5*x^9 + (5*x^6 + 1)*sqrt(x^6 - 1))/x^9
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x^10/(x^6-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [abs(x)]sym2poly/r2sym(const gen
 & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.01, size = 40, normalized size = 1.74

$$\frac{\left(5x^{6}+1\right)\left(-1+x\right)\left(1+x\right)\left(x^{2}+x+1\right)\left(x^{2}-x+1\right)}{9\sqrt{x^{6}-1}\;x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/x^10/(x^6-1)^(1/2),x)
```

```
[Out] 1/9*(5*x^6+1)*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)/(x^6-1)^(1/2)/x^9
```

maxima [A] time = 0.67, size = 25, normalized size = 1.09

$$\frac{2\,\sqrt{x^{6}-1}}{3\,x^{3}}-\frac{\left(x^{6}-1\right)^{\frac{3}{2}}}{9\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x^10/(x^6-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(x^6 - 1)/x^3 - 1/9*(x^6 - 1)^(3/2)/x^9
```

mupad [B] time = 0.17, size = 25, normalized size = 1.09

$$\frac{\sqrt{x^{6}-1}\;+5\,x^{6}\,\sqrt{x^{6}-1}}{9\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)/(x^10*(x^6 - 1)^(1/2)),x)
```

```
[Out] ((x^6 - 1)^(1/2) + 5*x^6*(x^6 - 1)^(1/2))/(9*x^9)
```

sympy [A] time = 2.89, size = 49, normalized size = 2.13

$$\frac{\begin{cases} \frac{\sqrt{x^6-1}}{x^3} & \text{for } x > -1 \wedge x < 1 \end{cases}}{3} + \frac{\begin{cases} \frac{\sqrt{x^6-1}}{x^3} - \frac{\left(x^6-1\right)^{\frac{3}{2}}}{3x^9} & \text{for } x > -1 \wedge x < 1 \end{cases}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/x**10/(x**6-1)**(1/2),x)
```

```
[Out] Piecewise((sqrt(x**6 - 1)/x**3, (x > -1) & (x < 1)))/3 + Piecewise((sqrt(x*
*6 - 1)/x**3 - (x**6 - 1)**(3/2)/(3*x**9), (x > -1) & (x < 1)))/3
```

3.235 $\int \frac{-1+3x^4}{(1+x^4)\sqrt[3]{x^2+x^6}}\,dx$

Optimal. Leaf size=23

$$-\frac{3\left(x^6+x^2\right)^{2/3}}{x\left(x^4+1\right)}$$

Rubi [A] time = 0.08, antiderivative size = 14, normalized size of antiderivative = 0.61, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2056, 449}

$$-\frac{3x}{\sqrt[3]{x^6+x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x^4)/((1 + x^4)*(x^2 + x^6)^(1/3)),x]
```

```
[Out] (-3*x)/(x^2 + x^6)^(1/3)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+3x^4}{\left(1+x^4\right)\sqrt[3]{x^2+x^6}}\,dx &= \frac{\left(x^{2/3}\sqrt[3]{1+x^4}\right)\int \frac{-1+3x^4}{x^{2/3}\left(1+x^4\right)^{4/3}}\,dx}{\sqrt[3]{x^2+x^6}}\\ &= -\frac{3x}{\sqrt[3]{x^2+x^6}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 14, normalized size = 0.61

$$-\frac{3x}{\sqrt[3]{x^6+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x^4)/((1 + x^4)*(x^2 + x^6)^(1/3)),x]
```

```
[Out] (-3*x)/(x^2 + x^6)^(1/3)
```

IntegrateAlgebraic [A] time = 0.27, size = 23, normalized size = 1.00

$$-\frac{3\left(x^6+x^2\right)^{2/3}}{x\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 3*x^4)/((1 + x^4)*(x^2 + x^6)^(1/3)),x]
[Out] (-3*(x^2 + x^6)^(2/3))/(x*(1 + x^4))
```

fricas [A] time = 0.39, size = 18, normalized size = 0.78

$$-\frac{3\left(x^6+x^2\right)^{\frac{2}{3}}}{x^5+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)/(x^4+1)/(x^6+x^2)^(1/3),x, algorithm="fricas")
[Out] -3*(x^6 + x^2)^(2/3)/(x^5 + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^4-1}{\left(x^6+x^2\right)^{\frac{1}{3}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)/(x^4+1)/(x^6+x^2)^(1/3),x, algorithm="giac")
[Out] integrate((3*x^4 - 1)/((x^6 + x^2)^(1/3)*(x^4 + 1)), x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.57

$$-\frac{3x}{\left(x^6+x^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4-1)/(x^4+1)/(x^6+x^2)^(1/3),x)
[Out] -3*x/(x^6+x^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^4-1}{\left(x^6+x^2\right)^{\frac{1}{3}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)/(x^4+1)/(x^6+x^2)^(1/3),x, algorithm="maxima")
[Out] integrate((3*x^4 - 1)/((x^6 + x^2)^(1/3)*(x^4 + 1)), x)
```

mupad [B] time = 0.16, size = 21, normalized size = 0.91

$$-\frac{3\left(x^6+x^2\right)^{2/3}}{x\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4 - 1)/((x^2 + x^6)^(1/3)*(x^4 + 1)),x)
[Out] -(3*(x^2 + x^6)^(2/3))/(x*(x^4 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^4-1}{\sqrt[3]{x^2\left(x^4+1\right)}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4-1)/(x**4+1)/(x**6+x**2)**(1/3),x)
```

```
[Out] Integral((3*x**4 - 1)/((x**2*(x**4 + 1))**(1/3)*(x**4 + 1)), x)
```

3.236 $\int \frac{-1+x^4}{\left(1+x^4\right)\sqrt[4]{x^2+x^6}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\left(x^6+x^2\right)^{3/4}}{x\left(x^4+1\right)}$$

Rubi [A] time = 0.07, antiderivative size = 14, normalized size of antiderivative = 0.61, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2056, 449}

$$-\frac{2x}{\sqrt[4]{x^6+x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)/((1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*x)/(x^2 + x^6)^(1/4)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^4}{\left(1+x^4\right)\sqrt[4]{x^2+x^6}}\,dx &= \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{-1+x^4}{\sqrt{x}\left(1+x^4\right)^{5/4}}\,dx}{\sqrt[4]{x^2+x^6}}\\ &= -\frac{2x}{\sqrt[4]{x^2+x^6}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 0.61

$$-\frac{2x}{\sqrt[4]{x^6+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)/((1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*x)/(x^2 + x^6)^(1/4)
```

IntegrateAlgebraic [A] time = 0.27, size = 23, normalized size = 1.00

$$-\frac{2\left(x^6+x^2\right)^{3/4}}{x\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/((1 + x^4)*(x^2 + x^6)^(1/4)),x]
[Out] (-2*(x^2 + x^6)^(3/4))/(x*(1 + x^4))
```

fricas [A] time = 0.40, size = 18, normalized size = 0.78

$$-\frac{2\left(x^6+x^2\right)^{\frac{3}{4}}}{x^5+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)/(x^6+x^2)^(1/4),x, algorithm="fricas")
[Out] -2*(x^6 + x^2)^(3/4)/(x^5 + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)/(x^6+x^2)^(1/4),x, algorithm="giac")
[Out] integrate((x^4 - 1)/((x^6 + x^2)^(1/4)*(x^4 + 1)), x)
```

maple [A] time = 0.01, size = 13, normalized size = 0.57

$$-\frac{2x}{\left(x^6+x^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/(x^4+1)/(x^6+x^2)^(1/4),x)
[Out] -2*x/(x^6+x^2)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)/(x^6+x^2)^(1/4),x, algorithm="maxima")
[Out] integrate((x^4 - 1)/((x^6 + x^2)^(1/4)*(x^4 + 1)), x)
```

mupad [B] time = 0.13, size = 21, normalized size = 0.91

$$-\frac{2\left(x^6+x^2\right)^{3/4}}{x\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/((x^2 + x^6)^(1/4)*(x^4 + 1)),x)
[Out] -(2*(x^2 + x^6)^(3/4))/(x*(x^4 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2+1\right)}{\sqrt[4]{x^2\left(x^4+1\right)}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/(x**4+1)/(x**6+x**2)**(1/4),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)/((x**2*(x**4 + 1))**(1/4)*(x**4 + 1)),
x)
```

3.237 $\int \frac{x^2(3+2x^2)(1+x^2+2x^6)}{(1+x^2)^2\sqrt{1+x^2+x^6}}\,dx$

Optimal. Leaf size=23

$$\frac{x^3\sqrt{x^6+x^2+1}}{x^2+1}$$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {1590}

$$\frac{x^3\sqrt{x^6+x^2+1}}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(3 + 2*x^2)*(1 + x^2 + 2*x^6))/((1 + x^2)^2*Sqrt[1 + x^2 + x^6]),x
]
```

```
[Out] (x^3*Sqrt[1 + x^2 + x^6])/(1 + x^2)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x^2\left(3+2x^2\right)\left(1+x^2+2x^6\right)}{\left(1+x^2\right)^2\sqrt{1+x^2+x^6}}\,dx=\frac{x^3\sqrt{1+x^2+x^6}}{1+x^2}$$

Mathematica [A] time = 0.21, size = 25, normalized size = 1.09

$$\sqrt{x^6+x^2+1}\left(x-\frac{x}{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(3 + 2*x^2)*(1 + x^2 + 2*x^6))/((1 + x^2)^2*Sqrt[1 + x^2 + x
^6]),x]
```

```
[Out] Sqrt[1 + x^2 + x^6]*(x - x/(1 + x^2))
```

IntegrateAlgebraic [A] time = 5.09, size = 23, normalized size = 1.00

$$\frac{x^3\sqrt{x^6+x^2+1}}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(3 + 2*x^2)*(1 + x^2 + 2*x^6))/((1 + x^2)^2*Sqrt[1
+ x^2 + x^6]),x]
```

```
[Out] (x^3*Sqrt[1 + x^2 + x^6])/(1 + x^2)
```

fricas [A] time = 0.42, size = 21, normalized size = 0.91

$$\frac{\sqrt{x^6+x^2+1}\,x^3}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^2+3)*(2*x^6+x^2+1)/(x^2+1)^2/(x^6+x^2+1)^(1/2),x, algori
thm="fricas")
```

```
[Out] sqrt(x^6 + x^2 + 1)*x^3/(x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^6+x^2+1\right)\left(2\,x^2+3\right)x^2}{\sqrt{x^6+x^2+1}\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^2+3)*(2*x^6+x^2+1)/(x^2+1)^2/(x^6+x^2+1)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((2*x^6 + x^2 + 1)*(2*x^2 + 3)*x^2/(sqrt(x^6 + x^2 + 1)*(x^2 + 1)^
2), x)
```

maple [A] time = 0.01, size = 22, normalized size = 0.96

$$\frac{x^3\sqrt{x^6+x^2+1}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^2+3)*(2*x^6+x^2+1)/(x^2+1)^2/(x^6+x^2+1)^(1/2),x)
```

```
[Out] x^3*(x^6+x^2+1)^(1/2)/(x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^6+x^2+1\right)\left(2\,x^2+3\right)x^2}{\sqrt{x^6+x^2+1}\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^2+3)*(2*x^6+x^2+1)/(x^2+1)^2/(x^6+x^2+1)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((2*x^6 + x^2 + 1)*(2*x^2 + 3)*x^2/(sqrt(x^6 + x^2 + 1)*(x^2 + 1)^
2), x)
```

mupad [B] time = 0.25, size = 21, normalized size = 0.91

$$\frac{x^3\,\sqrt{x^6+x^2+1}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(2*x^2 + 3)*(x^2 + 2*x^6 + 1))/((x^2 + 1)^2*(x^2 + x^6 + 1)^(1/2))
,x)
```

```
[Out] (x^3*(x^2 + x^6 + 1)^(1/2))/(x^2 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(2x^2+3\right) \left(2x^6+x^2+1\right)}{\left(x^2+1\right)^2 \sqrt{x^6+x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**2+3)*(2*x**6+x**2+1)/(x**2+1)**2/(x**6+x**2+1)**(1/2),
x)
```

```
[Out] Integral(x**2*(2*x**2 + 3)*(2*x**6 + x**2 + 1)/((x**2 + 1)**2*sqrt(x**6 + x
**2 + 1)), x)
```

3.238 $\int \frac{1}{\sqrt{3-5x+x^2+x^3}} \, dx$

Optimal. Leaf size=24

$$-\tanh^{-1}\left(\frac{2x-2}{\sqrt{x^3+x^2-5x+3}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.67, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2067, 2064, 63, 206}

$$\frac{(1-x)\sqrt{x+3}\tanh^{-1}\left(\frac{\sqrt{x+3}}{2}\right)}{\sqrt{x^3+x^2-5x+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 5*x + x^2 + x^3],x]
```

```
[Out] ((1 - x)*Sqrt[3 + x]*ArcTanh[Sqrt[3 + x]/2])/Sqrt[3 - 5*x + x^2 + x^3]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2064

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-5x+x^2+x^3}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{\frac{128}{27}-\frac{16x}{3}+x^3}}\,dx, x, \frac{1}{3}+x\right)$$
$$= \frac{\left(128(1-x)\sqrt{3+x}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)\sqrt{\frac{128}{9}+\frac{16x}{3}}}\,dx, x, \frac{1}{3}+x\right)}{3\sqrt{3}\sqrt{3-5x+x^2+x^3}}$$
$$= \frac{\left(16(1-x)\sqrt{3+x}\right)\text{Subst}\left(\int \frac{1}{\frac{128}{3}-2x^2}\,dx, x, \frac{4\sqrt{3+x}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{3-5x+x^2+x^3}}$$
$$= \frac{(1-x)\sqrt{3+x}\tanh^{-1}\left(\frac{\sqrt{3+x}}{2}\right)}{\sqrt{3-5x+x^2+x^3}}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.54

$$-\frac{(x-1)\sqrt{x+3}\tanh^{-1}\left(\frac{\sqrt{x+3}}{2}\right)}{\sqrt{(x-1)^2(x+3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 5*x + x^2 + x^3],x]
[Out] -(((-1 + x)*Sqrt[3 + x]*ArcTanh[Sqrt[3 + x]/2])/Sqrt[(-1 + x)^2*(3 + x)])
```

IntegrateAlgebraic [A] time = 0.06, size = 24, normalized size = 1.00

$$-\tanh^{-1}\left(\frac{2x-2}{\sqrt{x^3+x^2-5x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[3 - 5*x + x^2 + x^3],x]
[Out] -ArcTanh[(-2 + 2*x)/Sqrt[3 - 5*x + x^2 + x^3]]
```

fricas [B] time = 0.40, size = 58, normalized size = 2.42

$$-\frac{1}{2}\log\left(\frac{2x+\sqrt{x^3+x^2-5x+3}-2}{x-1}\right)+\frac{1}{2}\log\left(-\frac{2x-\sqrt{x^3+x^2-5x+3}-2}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2-5*x+3)^(1/2),x, algorithm="fricas")
[Out] -1/2*log((2*x + sqrt(x^3 + x^2 - 5*x + 3) - 2)/(x - 1)) + 1/2*log(-(2*x - s
qrt(x^3 + x^2 - 5*x + 3) - 2)/(x - 1))
```

giac [A] time = 0.34, size = 34, normalized size = 1.42

$$-\frac{\log\left(\sqrt{x+3}+2\right)}{2\,\text{sgn}(x-1)}+\frac{\log\left(\left|\sqrt{x+3}-2\right|\right)}{2\,\text{sgn}(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3+x^2-5*x+3)^(1/2),x, algorithm="giac")

[Out] -1/2*log(sqrt(x + 3) + 2)/sgn(x - 1) + 1/2*log(abs(sqrt(x + 3) - 2))/sgn(x - 1)

maple [A] time = 0.01, size = 43, normalized size = 1.79

$$\frac{(-1+x)\sqrt{3+x}\,\left(\ln\left(\sqrt{3+x}-2\right)-\ln\left(2+\sqrt{3+x}\right)\right)}{2\sqrt{x^3+x^2-5x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3+x^2-5*x+3)^(1/2),x)

[Out] 1/2*(-1+x)*(3+x)^(1/2)*(ln((3+x)^(1/2)-2)-ln(2+(3+x)^(1/2)))/(x^3+x^2-5*x+3)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3+x^2-5\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3+x^2-5*x+3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(x^3 + x^2 - 5*x + 3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{x^3+x^2-5\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2 - 5*x + x^3 + 3)^(1/2),x)

[Out] int(1/(x^2 - 5*x + x^3 + 3)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3+x^2-5x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**3+x**2-5*x+3)**(1/2),x)

[Out] Integral(1/sqrt(x**3 + x**2 - 5*x + 3), x)

3.239 $\int \frac{-1+x}{(1+x)\sqrt{x+x^2+x^3}}\,dx$

Optimal. Leaf size=24

$$-2\tan^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)$$

Rubi [A] time = 0.40, antiderivative size = 46, normalized size of antiderivative = 1.92, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2056, 6733, 1698, 203}

$$-\frac{2\sqrt{x}\sqrt{x^2+x+1}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x^2+x+1}}\right)}{\sqrt{x^3+x^2+x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/((1 + x)*Sqrt[x + x^2 + x^3]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[1 + x + x^2]*ArcTan[Sqrt[x]/Sqrt[1 + x + x^2]])/Sqrt[x + x
^2 + x^3]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{-1+x}{(1+x)\sqrt{x+x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{-1+x}{\sqrt{x}(1+x)\sqrt{1+x+x^2}}\,dx}{\sqrt{x+x^2+x^3}}$$
$$= \frac{\left(2\sqrt{x}\sqrt{1+x+x^2}\right)\text{Subst}\left(\int \frac{-1+x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{x+x^2+x^3}}$$
$$= -\frac{\left(2\sqrt{x}\sqrt{1+x+x^2}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{1+x+x^2}}\right)}{\sqrt{x+x^2+x^3}}$$
$$= -\frac{2\sqrt{x}\sqrt{1+x+x^2}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x+x^2}}\right)}{\sqrt{x+x^2+x^3}}$$

Mathematica [C] time = 0.30, size = 107, normalized size = 4.46

$$\frac{2(-1)^{2/3}\sqrt{\frac{\sqrt[3]{-1}}{x}+1}\sqrt{1-\frac{(-1)^{2/3}}{x}}\,x^{3/2}\left(F\left(i\sinh^{-1}\left(\frac{(-1)^{5/6}}{\sqrt{x}}\right)|(-1)^{2/3}\right)-2\Pi\left(\sqrt[3]{-1};i\sinh^{-1}\left(\frac{(-1)^{5/6}}{\sqrt{x}}\right)|(-1)^{2/3}\right)\right)}{\sqrt{x\left(x^2+x+1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/((1 + x)*Sqrt[x + x^2 + x^3]),x]
[Out] (2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)/x]*Sqrt[1 - (-1)^(2/3)/x]*x^(3/2)*(Ellipt
icF[I*ArcSinh[(-1)^(5/6)/Sqrt[x]], (-1)^(2/3)] - 2*EllipticPi[(-1)^(1/3), I
*ArcSinh[(-1)^(5/6)/Sqrt[x]], (-1)^(2/3)]))/Sqrt[x*(1 + x + x^2)]
```

IntegrateAlgebraic [A] time = 0.07, size = 24, normalized size = 1.00

$$-2\tan^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/((1 + x)*Sqrt[x + x^2 + x^3]),x]
[Out] -2*ArcTan[Sqrt[x + x^2 + x^3]/(1 + x + x^2)]
```

fricas [A] time = 0.43, size = 18, normalized size = 0.75

$$\arctan\left(\frac{x^2+1}{2\sqrt{x^3+x^2+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+x^2+x)^(1/2),x, algorithm="fricas")
[Out] arctan(1/2*(x^2 + 1)/sqrt(x^3 + x^2 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x^3+x^2+x}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/(sqrt(x^3 + x^2 + x)*(x + 1)), x)
```

maple [C] time = 0.08, size = 271, normalized size = 11.29

$$\frac{2\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}},\frac{\sqrt{3}\,\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{3\sqrt{x^{3}+x^{2}+x}}-\frac{4\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\ \mathrm{EllipticPi}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}},\frac{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{1}{2}-\frac{i\sqrt{3}}{2}},\frac{\sqrt{3}\,\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{3\sqrt{x^{3}+x^{2}+x}\,\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(1+x)/(x^3+x^2+x)^(1/2),x)
```

```
[Out] 2/3*(1/2+1/2*I*3^(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3
^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/
2)/(x^3+x^2+x)^(1/2)*EllipticF(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^
(1/2),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))-4/3*(1/2+1/2*I*3^
(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3^(1/2)*(I*(x+1/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1
/2)/(1/2-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2
)))^(1/2),(-1/2-1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x^{3}+x^{2}+x}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+x^2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/(sqrt(x^3 + x^2 + x)*(x + 1)), x)
```

mupad [B] time = 0.30, size = 179, normalized size = 7.46

$$\frac{\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(\sqrt{3}+1\mathrm{i}\right)\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-2\,\Pi\left(\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)1\mathrm{i}}{\sqrt{x^{3}+x^{2}-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x + 1)*(x + x^2 + x^3)^(1/2)),x)
```

```
[Out] ((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i
)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/2)
*(3^(1/2) + 1i)*(ellipticF(asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/
2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)) - 2*ellipticPi(1/2 - (3^(1/2)*1i)/2
, asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)
*1i)/2 + 1/2)))*1i)/(x^2 + x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x\left(x^{2}+x+1\right)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x**3+x**2+x)**(1/2),x)
```

```
[Out] Integral((x - 1)/(sqrt(x*(x**2 + x + 1))*(x + 1)), x)
```

3.240 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^2+x^3}}\,dx$

Optimal. Leaf size=24

$$-2\tanh^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)$$

Rubi [C] time = 1.03, antiderivative size = 320, normalized size of antiderivative = 13.33, number of steps used = 17, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2056, 6725, 716, 1103, 934, 169, 538, 537}

$$\frac{\sqrt{x}(x+1)\sqrt{\frac{x^2+x+1}{(x+1)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x^3+x^2+x}}-\frac{4\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\Pi\left(\frac{1}{2}\left(-i-\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}-\frac{4\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\Pi\left(\frac{1}{2}\left(i+\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^2 + x^3]),x]
```

```
[Out] (Sqrt[x]*(1 + x)*Sqrt[(1 + x + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]],
 1/4])/Sqrt[x + x^2 + x^3] - (4*Sqrt[x]*Sqrt[1 + (2*x)/(1 - I*Sqrt[3])]*Sqr
t[1 + (2*x)/(1 + I*Sqrt[3])]*EllipticPi[(-I - Sqrt[3])/2, ArcSin[((1 - I*Sq
rt[3])*Sqrt[x])/2], (I + Sqrt[3])/(I - Sqrt[3])])/((1 - I*Sqrt[3])*Sqrt[x +
 x^2 + x^3]) - (4*Sqrt[x]*Sqrt[1 + (2*x)/(1 - I*Sqrt[3])]*Sqrt[1 + (2*x)/(1
 + I*Sqrt[3])]*EllipticPi[(I + Sqrt[3])/2, ArcSin[((1 - I*Sqrt[3])*Sqrt[x])
/2], (I + Sqrt[3])/(I - Sqrt[3])])/((1 - I*Sqrt[3])*Sqrt[x + x^2 + x^3])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 716

```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2,
Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
```

```
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt{x+x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}\left(1+x^2\right)\sqrt{1+x+x^2}}\,dx}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{1+x+x^2}} - \frac{2}{\sqrt{x}\left(1+x^2\right)\sqrt{1+x+x^2}}\right)dx}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+x+x^2}}\,dx}{\sqrt{x+x^2+x^3}} - \frac{\left(2\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1}{\sqrt{x}\left(1+x^2\right)\sqrt{1+x+x^2}}\,d}{\sqrt{x+x^2+x^3}}$$

$$= -\frac{\left(2\sqrt{x}\sqrt{1+x+x^2}\right)\int \left(\frac{i}{2(i-x)\sqrt{x}\sqrt{1+x+x^2}} + \frac{i}{2\sqrt{x}(i+x)\sqrt{1+x+x^2}}\right)dx}{\sqrt{x+x^2+x^3}} + \frac{\left(2\sqrt{x}\sqrt{1+\right.}{}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} - \frac{\left(i\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1}{(i-x)\sqrt{x}\sqrt{1+x+x^2}}}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} - \frac{\left(i\sqrt{x}\sqrt{1-i\sqrt{3}+2x}\sqrt{1+i\sqrt{3}+2\right.}{\sqrt{x+x}}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} + \frac{\left(2i\sqrt{x}\sqrt{1-i\sqrt{3}+2x}\sqrt{1+i\sqrt{3}+\right.}{}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} + \frac{\left(2i\sqrt{x}\sqrt{1+i\sqrt{3}+2x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\right.}{}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} + \frac{\left(2i\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\right)\mathrm{Su}}{\sqrt{x}}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} - \frac{4\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\,\Pi\left(\frac{1}{2}\right.}{\left(1-i\sqrt{3}\right.}$$

Mathematica [C] time = 0.53, size = 136, normalized size = 5.67

$$-\frac{2(-1)^{2/3}\sqrt{\frac{\sqrt[3]{-1}}{x}+1}\sqrt{1-\frac{(-1)^{2/3}}{x}}\,x^{3/2}\left(-F\left(i\sinh^{-1}\left(\frac{(-1)^{5/6}}{\sqrt{x}}\right)|(-1)^{2/3}\right)+\Pi\left(-(-1)^{5/6};i\sinh^{-1}\left(\frac{(-1)^{5/6}}{\sqrt{x}}\right)|(-1)^{2/3}\right)+\Pi\left((-1)^{5/6};i\sinh^{-1}\left(\frac{(-1)^{5/6}}{\sqrt{x}}\right)|(-1)^{2/3}\right)\right)}{\sqrt{x\left(x^2+x+1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^2 + x^3]),x]
```

```
[Out] (-2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)/x]*Sqrt[1 - (-1)^(2/3)/x]*x^(3/2)*(-Elli
pticF[I*ArcSinh[(-1)^(5/6)/Sqrt[x]], (-1)^(2/3)] + EllipticPi[-(-1)^(5/6),
I*ArcSinh[(-1)^(5/6)/Sqrt[x]], (-1)^(2/3)] + EllipticPi[(-1)^(5/6), I*ArcSi
nh[(-1)^(5/6)/Sqrt[x]], (-1)^(2/3)]))/Sqrt[x*(1 + x + x^2)]
```

IntegrateAlgebraic [A] time = 0.07, size = 24, normalized size = 1.00

$$-2\tanh^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^2 + x^3]),x]
```

```
[Out] -2*ArcTanh[Sqrt[x + x^2 + x^3]/(1 + x + x^2)]
```

fricas [A] time = 0.42, size = 29, normalized size = 1.21

$$\log\left(\frac{x^2+2\,x-2\,\sqrt{x^3+x^2+x}+1}{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x^2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] log((x^2 + 2*x - 2*sqrt(x^3 + x^2 + x) + 1)/(x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^3+x^2+x}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^3 + x^2 + x)*(x^2 + 1)), x)
```

maple [C] time = 0.05, size = 781, normalized size = 32.54

$$\frac{2\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}, \frac{\sqrt{3}\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{3\sqrt{x^3+x^2+x}}+\frac{i\sqrt{\frac{x}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}+\frac{1}{1+i\sqrt{3}}+\frac{i\sqrt{3}}{1+i\sqrt{3}}}\,\sqrt{3}\,\sqrt{i\sqrt{3}\,x+\frac{i\sqrt{3}}{2}+\frac{3}{2}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}, \frac{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{1}{2}-i-\frac{i\sqrt{3}}{2}}, \frac{\sqrt{3}\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{3\sqrt{x^3+x^2+x}\left(-\frac{1}{2}-i-\frac{i\sqrt{3}}{2}\right)}-\frac{\sqrt{\frac{x}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}+\frac{1}{1+i\sqrt{3}}+\frac{i\sqrt{3}}{1+i\sqrt{3}}}\,\sqrt{i\sqrt{3}\,x+\frac{i\sqrt{3}}{2}+\frac{3}{2}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}, \frac{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{1}{2}-i-\frac{i\sqrt{3}}{2}}, \frac{\sqrt{3}\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{\sqrt{x^3+x^2+x}\left(-\frac{1}{2}-i-\frac{i\sqrt{3}}{2}\right)}-\frac{i\sqrt{\frac{x}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}+\frac{1}{1+i\sqrt{3}}+\frac{i\sqrt{3}}{1+i\sqrt{3}}}\,\sqrt{3}\,\sqrt{i\sqrt{3}\,x+\frac{i\sqrt{3}}{2}+\frac{3}{2}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}, \frac{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{1}{2}+i-\frac{i\sqrt{3}}{2}}, \frac{\sqrt{3}\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{3\sqrt{x^3+x^2+x}\left(-\frac{1}{2}+i-\frac{i\sqrt{3}}{2}\right)}+\frac{\sqrt{\frac{x}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}+\frac{1}{1+i\sqrt{3}}+\frac{i\sqrt{3}}{1+i\sqrt{3}}}\,\sqrt{i\sqrt{3}\,x+\frac{i\sqrt{3}}{2}+\frac{3}{2}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}, \frac{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{1}{2}+i-\frac{i\sqrt{3}}{2}}, \frac{\sqrt{3}\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{\sqrt{x^3+x^2+x}\left(-\frac{1}{2}+i-\frac{i\sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^3+x^2+x)^(1/2),x)
```

```
[Out] 2/3*(1/2+1/2*I*3^(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3
^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/
2)/(x^3+x^2+x)^(1/2)*EllipticF(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^
(1/2),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))+1/3*I*(1/(1/2+1/2
*I*3^(1/2))*x+1/2/(1/2+1/2*I*3^(1/2))+1/2*I/(1/2+1/2*I*3^(1/2))*3^(1/2))^(1
/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^
(1/2)/(x^3+x^2+x)^(1/2)/(-1/2-I-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(
1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-1/2-I-1/2*I*3^(1/2)
),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))-(1/(1/2+1/2*I*3^(1/2)
)*x+1/2/(1/2+1/2*I*3^(1/2))+1/2*I/(1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)*(I*3^(
1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^
(1/2)/(-1/2-I-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3
^(1/2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-1/2-I-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(
-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))-1/3*I*(1/(1/2+1/2*I*3^(1/2))*x+1/2/(1/2
+1/2*I*3^(1/2))+1/2*I/(1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)*3^(1/2)*(I*3^(1/2)
*x+1/2*I*3^(1/2)+3/2)^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2
)/(-1/2+I-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/
2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-1/2+I-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2
-1/2*I*3^(1/2))*3^(1/2))^(1/2))+(1/(1/2+1/2*I*3^(1/2))*x+1/2/(1/2+1/2*I*3^(
1/2))+1/2*I/(1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3
/2)^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2)/(-1/2+I-1/2*I*3^
(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1
/2*I*3^(1/2))/(-1/2+I-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^3+x^2+x}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x^2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^3 + x^2 + x)*(x^2 + 1)), x)
```

mupad [B]   time = 0.15, size = 223, normalized size = 9.29

$$-\frac{\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(\sqrt{3}+1\mathrm{i}\right)\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i};\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i};\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)1\mathrm{i}}{\sqrt{x^3+x^2-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + 1)*(x + x^2 + x^3)^(1/2)),x)
```

```
[Out] -((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/2
)*(3^(1/2) + 1i)*(ellipticPi(- 3^(1/2)/2 - 1i/2, asin((x/((3^(1/2)*1i)/2 -
1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)) - ellipticF(a
sin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i
)/2 + 1/2)) + ellipticPi(3^(1/2)/2 + 1i/2, asin((x/((3^(1/2)*1i)/2 - 1/2))^
(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))*1i)/(x^2 + x^3 - x
*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{\sqrt{x\left(x^2+x+1\right)}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**3+x**2+x)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + x + 1))*(x**2 + 1)), x)
```

3.241 $\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=24

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1699, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] -(ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt{1+x^4}}\,dx &= -\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.09, size = 41, normalized size = 1.71

$$-\sqrt[4]{-1}\left(F\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)-2\Pi\left(-i;i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] -((-1)^(1/4)*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - 2*EllipticPi[-I, I*A
rcSinh[(-1)^(1/4)*x], -1]))
```

IntegrateAlgebraic [A] time = 0.35, size = 24, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
[Out] -(ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2])
```

fricas [A] time = 0.45, size = 18, normalized size = 0.75

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^4+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

maple [C] time = 0.02, size = 111, normalized size = 4.62

$$\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\;\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{2\,(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\;\mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x,i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^4+1)^(1/2),x)
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+2*(-1)^(3/4)*(1-I*x^2)^(1/2)*(1
+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^4+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2-1}{\left(x^2+1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + 1)*(x^4 + 1)^(1/2)),x)
```

```
[Out] int((x^2 - 1)/((x^2 + 1)*(x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{(x^2+1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**4+1)**(1/2),x)
[Out] Integral((x - 1)*(x + 1)/((x**2 + 1)*sqrt(x**4 + 1)), x)
```

3.242 $\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=24

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1699, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/((-1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] -(ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt{1+x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.10, size = 37, normalized size = 1.54

$$-\sqrt[4]{-1}\left(F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right)-2\Pi\left(i;\sin^{-1}\left((-1)^{3/4}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/((-1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] -((-1)^(1/4)*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - 2*EllipticPi[I, ArcS
in[(-1)^(3/4)*x], -1]))
```

IntegrateAlgebraic [A] time = 0.37, size = 24, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 + x^2)*Sqrt[1 + x^4]),x]
[Out] -(ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2])
```

fricas [B] time = 0.45, size = 42, normalized size = 1.75

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^4-2\sqrt{2}\sqrt{x^4+1}\,x+2\,x^2+1}{x^4-2\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((x^4 - 2*sqrt(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2
+ 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+1}{\sqrt{x^4+1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

maple [C] time = 0.01, size = 111, normalized size = 4.62

$$\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\ \mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{2\,(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\ \mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x,-i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-1)/(x^4+1)^(1/2),x)
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+2*(-1)^(3/4)*(1-I*x^2)^(1/2)*(1
+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/4
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+1}{\sqrt{x^4+1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{x^2+1}{\left(x^2-1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)),x)
[Out] int((x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{(x-1)(x+1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-1)/(x**4+1)**(1/2),x)
[Out] Integral((x**2 + 1)/((x - 1)*(x + 1)*sqrt(x**4 + 1)), x)
```

3.243 $\int \frac{2-3x^5}{\sqrt{1+x^5}\left(1-ax^2+x^5\right)} dx$

Optimal. Leaf size=24

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^5+1}}\right)}{\sqrt{a}}$$

Rubi [F] time = 0.71, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2-3x^5}{\sqrt{1+x^5}\left(1-ax^2+x^5\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(2 - 3*x^5)/(Sqrt[1 + x^5]*(1 - a*x^2 + x^5)),x]
```

```
[Out] -3*x*Hypergeometric2F1[1/5, 1/2, 6/5, -x^5] - 5*Defer[Int][1/((-1 + a*x^2 -
 x^5)*Sqrt[1 + x^5]), x] + 3*a*Defer[Int][x^2/((-1 + a*x^2 - x^5)*Sqrt[1 +
x^5]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{2-3x^5}{\sqrt{1+x^5}\left(1-ax^2+x^5\right)} dx &= \int \left(-\frac{3}{\sqrt{1+x^5}}+\frac{5-3ax^2}{\sqrt{1+x^5}\left(1-ax^2+x^5\right)}\right) dx \\
&= -\left(3\int \frac{1}{\sqrt{1+x^5}} dx\right)+\int \frac{5-3ax^2}{\sqrt{1+x^5}\left(1-ax^2+x^5\right)} dx \\
&= -3x\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{6}{5};-x^5\right)+\int \left(-\frac{5}{\left(-1+ax^2-x^5\right)\sqrt{1+x^5}}+\frac{3ax^2}{\left(-1+ax^2-x^5\right)\sqrt{1+}\right. \\
&= -3x\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{6}{5};-x^5\right)-5\int \frac{1}{\left(-1+ax^2-x^5\right)\sqrt{1+x^5}} dx+(3a)\int \frac{x}{\left(-1+ax^2-\right.}
\end{aligned}$$

Mathematica [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{2-3x^5}{\sqrt{1+x^5}\left(1-ax^2+x^5\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2 - 3*x^5)/(Sqrt[1 + x^5]*(1 - a*x^2 + x^5)),x]
```

```
[Out] Integrate[(2 - 3*x^5)/(Sqrt[1 + x^5]*(1 - a*x^2 + x^5)), x]
```

IntegrateAlgebraic [A] time = 5.89, size = 24, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^5+1}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - 3*x^5)/(Sqrt[1 + x^5]*(1 - a*x^2 + x^5)),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*x)/Sqrt[1 + x^5]])/Sqrt[a]
```

fricas [B] time = 0.48, size = 136, normalized size = 5.67

$$\left[\frac{\log\left(\frac{x^{10}+6\,ax^{7}+a^{2}x^{4}+2\,x^{5}+6\,ax^{2}+4\,\left(x^{6}+ax^{3}+x\right)\sqrt{x^{5}+1}\,\sqrt{a}+1}{x^{10}-2\,ax^{7}+a^{2}x^{4}+2\,x^{5}-2\,ax^{2}+1}\right)}{2\,\sqrt{a}}, -\frac{\sqrt{-a}\,\arctan\left(\frac{\left(x^{5}+ax^{2}+1\right)\sqrt{x^{5}+1}\,\sqrt{-a}}{2\,\left(ax^{6}+ax\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^5+2)/(x^5+1)^(1/2)/(x^5-a*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/2*log((x^10 + 6*a*x^7 + a^2*x^4 + 2*x^5 + 6*a*x^2 + 4*(x^6 + a*x^3 + x)*
sqrt(x^5 + 1)*sqrt(a) + 1)/(x^10 - 2*a*x^7 + a^2*x^4 + 2*x^5 - 2*a*x^2 + 1)
)/sqrt(a), -sqrt(-a)*arctan(1/2*(x^5 + a*x^2 + 1)*sqrt(x^5 + 1)*sqrt(-a)/(a
*x^6 + a*x))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{3\,x^{5}-2}{\left(x^{5}-ax^{2}+1\right)\sqrt{x^{5}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^5+2)/(x^5+1)^(1/2)/(x^5-a*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(3*x^5 - 2)/((x^5 - a*x^2 + 1)*sqrt(x^5 + 1)), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{-3x^{5}+2}{\sqrt{x^{5}+1}\,\left(x^{5}-a\,x^{2}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^5+2)/(x^5+1)^(1/2)/(x^5-a*x^2+1),x)
```

```
[Out] int((-3*x^5+2)/(x^5+1)^(1/2)/(x^5-a*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3\,x^{5}-2}{\left(x^{5}-ax^{2}+1\right)\sqrt{x^{5}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^5+2)/(x^5+1)^(1/2)/(x^5-a*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((3*x^5 - 2)/((x^5 - a*x^2 + 1)*sqrt(x^5 + 1)), x)
```

mupad [B] time = 0.75, size = 44, normalized size = 1.83

$$\frac{\ln\left(\frac{a\,x^{2}+x^{5}+2\,\sqrt{a}\,x\,\sqrt{x^{5}+1}+1}{4\,x^{5}-4\,a\,x^{2}+4}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^5 - 2)/((x^5 + 1)^(1/2)*(x^5 - a*x^2 + 1)),x)
```

```
[Out] log((a*x^2 + x^5 + 2*a^(1/2)*x*(x^5 + 1)^(1/2) + 1)/(4*x^5 - 4*a*x^2 + 4))/
a^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3x^5}{-ax^2\sqrt{x^5+1}+x^5\sqrt{x^5+1}+\sqrt{x^5+1}}\,dx-\int \left(-\frac{2}{-ax^2\sqrt{x^5+1}+x^5\sqrt{x^5+1}+\sqrt{x^5+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**5+2)/(x**5+1)**(1/2)/(x**5-a*x**2+1),x)
```

```
[Out] -Integral(3*x**5/(-a*x**2*sqrt(x**5 + 1) + x**5*sqrt(x**5 + 1) + sqrt(x**5
+ 1)), x) - Integral(-2/(-a*x**2*sqrt(x**5 + 1) + x**5*sqrt(x**5 + 1) + sqr
t(x**5 + 1)), x)
```

3.244 $\int \frac{2+3x^5}{\sqrt{-1+x^5}\left(-1-ax^2+x^5\right)}\,dx$

Optimal. Leaf size=24

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^5-1}}\right)}{\sqrt{a}}$$

Rubi [F] time = 0.70, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2+3x^5}{\sqrt{-1+x^5}\left(-1-ax^2+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2 + 3*x^5)/(Sqrt[-1 + x^5]*(-1 - a*x^2 + x^5)),x]
```

```
[Out] (3*x*Sqrt[1 - x^5]*Hypergeometric2F1[1/5, 1/2, 6/5, x^5])/Sqrt[-1 + x^5] -
5*Defer[Int][1/((1 + a*x^2 - x^5)*Sqrt[-1 + x^5]), x] - 3*a*Defer[Int][x^2/
((1 + a*x^2 - x^5)*Sqrt[-1 + x^5]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^5}{\sqrt{-1+x^5}\left(-1-ax^2+x^5\right)}\,dx &= \int \left(\frac{3}{\sqrt{-1+x^5}}+\frac{5+3ax^2}{\sqrt{-1+x^5}\left(-1-ax^2+x^5\right)}\right)dx\\
&= 3\int \frac{1}{\sqrt{-1+x^5}}\,dx+\int \frac{5+3ax^2}{\sqrt{-1+x^5}\left(-1-ax^2+x^5\right)}\,dx\\
&= \frac{\left(3\sqrt{1-x^5}\right)\int \frac{1}{\sqrt{1-x^5}}\,dx}{\sqrt{-1+x^5}}+\int \left(-\frac{5}{\left(1+ax^2-x^5\right)\sqrt{-1+x^5}}-\frac{3ax^2}{\left(1+ax^2-x^5\right)}\right.\\
&= \frac{3x\sqrt{1-x^5}\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{6}{5};x^5\right)}{\sqrt{-1+x^5}}-5\int \frac{1}{\left(1+ax^2-x^5\right)\sqrt{-1+x^5}}\,dx-(3a)\int \frac{}{(1}
\end{aligned}$$

Mathematica [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{2+3x^5}{\sqrt{-1+x^5}\left(-1-ax^2+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2 + 3*x^5)/(Sqrt[-1 + x^5]*(-1 - a*x^2 + x^5)),x]
```

```
[Out] Integrate[(2 + 3*x^5)/(Sqrt[-1 + x^5]*(-1 - a*x^2 + x^5)), x]
```

IntegrateAlgebraic [A] time = 5.83, size = 24, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^5-1}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x^5)/(Sqrt[-1 + x^5]*(-1 - a*x^2 + x^5)),x]
[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[-1 + x^5]])/Sqrt[a]
```

fricas [B] time = 0.47, size = 138, normalized size = 5.75

$$\left[\frac{\log\left(\frac{x^{10}+6\,ax^{7}+a^{2}x^{4}-2\,x^{5}-6\,ax^{2}-4\,\left(x^{6}+ax^{3}-x\right)\sqrt{x^{5}-1}\,\sqrt{a}+1}{x^{10}-2\,ax^{7}+a^{2}x^{4}-2\,x^{5}+2\,ax^{2}+1}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\left(x^{5}+ax^{2}-1\right)\sqrt{x^{5}-1}\,\sqrt{-a}}{2\,\left(ax^{6}-ax\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+2)/(x^5-1)^(1/2)/(x^5-a*x^2-1),x, algorithm="fricas")
[Out] [1/2*log((x^10 + 6*a*x^7 + a^2*x^4 - 2*x^5 - 6*a*x^2 - 4*(x^6 + a*x^3 - x)*
sqrt(x^5 - 1)*sqrt(a) + 1)/(x^10 - 2*a*x^7 + a^2*x^4 - 2*x^5 + 2*a*x^2 + 1)
)/sqrt(a), sqrt(-a)*arctan(1/2*(x^5 + a*x^2 - 1)*sqrt(x^5 - 1)*sqrt(-a)/(a*
x^6 - a*x))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^{5}+2}{\left(x^{5}-ax^{2}-1\right)\sqrt{x^{5}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+2)/(x^5-1)^(1/2)/(x^5-a*x^2-1),x, algorithm="giac")
[Out] integrate((3*x^5 + 2)/((x^5 - a*x^2 - 1)*sqrt(x^5 - 1)), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{3x^{5}+2}{\sqrt{x^{5}-1}\,\left(x^{5}-a\,x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^5+2)/(x^5-1)^(1/2)/(x^5-a*x^2-1),x)
[Out] int((3*x^5+2)/(x^5-1)^(1/2)/(x^5-a*x^2-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^{5}+2}{\left(x^{5}-ax^{2}-1\right)\sqrt{x^{5}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+2)/(x^5-1)^(1/2)/(x^5-a*x^2-1),x, algorithm="maxima")
[Out] integrate((3*x^5 + 2)/((x^5 - a*x^2 - 1)*sqrt(x^5 - 1)), x)
```

mupad [B] time = 0.75, size = 44, normalized size = 1.83

$$\frac{\ln\left(\frac{a\,x^{2}+x^{5}-2\,\sqrt{a}\,x\,\sqrt{x^{5}-1}-1}{-4\,x^{5}+4\,a\,x^{2}+4}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^5 + 2)/((x^5 - 1)^(1/2)*(a*x^2 - x^5 + 1)),x)
```

```
[Out] log((a*x^2 + x^5 - 2*a^(1/2)*x*(x^5 - 1)^(1/2) - 1)/(4*a*x^2 - 4*x^5 + 4))/
a^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^5+2}{\sqrt{(x-1)\left(x^4+x^3+x^2+x+1\right)}\left(-ax^2+x^5-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**5+2)/(x**5-1)**(1/2)/(x**5-a*x**2-1),x)
```

```
[Out] Integral((3*x**5 + 2)/(sqrt((x - 1)*(x**4 + x**3 + x**2 + x + 1))*(-a*x**2
+ x**5 - 1)), x)
```

3.245 $\int \frac{2+3x^5}{\sqrt{-1+x^5}\,(-a-x^2+ax^5)}\,dx$

Optimal. Leaf size=24

$$-\frac{2\tanh^{-1}\left(\frac{x}{\sqrt{a}\sqrt{x^5-1}}\right)}{\sqrt{a}}$$

Rubi [F] time = 0.68, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2+3x^5}{\sqrt{-1+x^5}\,(-a-x^2+ax^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2 + 3*x^5)/(Sqrt[-1 + x^5]*(-a - x^2 + a*x^5)),x]
```

```
[Out] (3*x*Sqrt[1 - x^5]*Hypergeometric2F1[1/5, 1/2, 6/5, x^5])/(a*Sqrt[-1 + x^5]
) + 5*Defer[Int][1/(Sqrt[-1 + x^5]*(-a - x^2 + a*x^5)), x] + (3*Defer[Int][
x^2/(Sqrt[-1 + x^5]*(-a - x^2 + a*x^5)), x])/a
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^5}{\sqrt{-1+x^5}\,(-a-x^2+ax^5)}\,dx &= \int \left(\frac{3}{a\sqrt{-1+x^5}}+\frac{5a+3x^2}{a\sqrt{-1+x^5}\,(-a-x^2+ax^5)}\right)dx\\
&= \frac{\int \frac{5a+3x^2}{\sqrt{-1+x^5}\,(-a-x^2+ax^5)}\,dx}{a}+\frac{3\int \frac{1}{\sqrt{-1+x^5}}\,dx}{a}\\
&= \frac{\int \left(\frac{5a}{\sqrt{-1+x^5}\,(-a-x^2+ax^5)}+\frac{3x^2}{\sqrt{-1+x^5}\,(-a-x^2+ax^5)}\right)dx}{a}+\frac{\left(3\sqrt{1-x^5}\right)\int \frac{1}{\sqrt{1-x^5}}\,dx}{a\sqrt{-1+x^5}}\\
&= \frac{3x\sqrt{1-x^5}\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{6}{5};x^5\right)}{a\sqrt{-1+x^5}}+5\int \frac{1}{\sqrt{-1+x^5}\,(-a-x^2+ax^5)}\,dx+\frac{3\int \frac{}{\sqrt{-1+x^5}}}{}
\end{aligned}$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{2+3x^5}{\sqrt{-1+x^5}\,(-a-x^2+ax^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2 + 3*x^5)/(Sqrt[-1 + x^5]*(-a - x^2 + a*x^5)),x]
```

```
[Out] Integrate[(2 + 3*x^5)/(Sqrt[-1 + x^5]*(-a - x^2 + a*x^5)), x]
```

IntegrateAlgebraic [A] time = 2.42, size = 24, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{x}{\sqrt{a}\sqrt{x^5-1}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x^5)/(Sqrt[-1 + x^5]*(-a - x^2 + a*x^5)),x]
[Out] (-2*ArcTanh[x/(Sqrt[a]*Sqrt[-1 + x^5])])/Sqrt[a]
```

fricas [B] time = 0.48, size = 151, normalized size = 6.29

$$\left[\frac{\log\left(\frac{a^2x^{10}+6\,ax^7-2\,a^2x^5+x^4-6\,ax^2-4\left(ax^6+x^3-ax\right)\sqrt{x^5-1}\,\sqrt{a}+a^2}{a^2x^{10}-2\,ax^7-2\,a^2x^5+x^4+2\,ax^2+a^2}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\left(ax^5+x^2-a\right)\sqrt{x^5-1}\,\sqrt{-a}}{2\left(ax^6-ax\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+2)/(x^5-1)^(1/2)/(a*x^5-x^2-a),x, algorithm="fricas")
[Out] [1/2*log((a^2*x^10 + 6*a*x^7 - 2*a^2*x^5 + x^4 - 6*a*x^2 - 4*(a*x^6 + x^3 -
 a*x)*sqrt(x^5 - 1)*sqrt(a) + a^2)/(a^2*x^10 - 2*a*x^7 - 2*a^2*x^5 + x^4 +
2*a*x^2 + a^2))/sqrt(a), sqrt(-a)*arctan(1/2*(a*x^5 + x^2 - a)*sqrt(x^5 - 1
)*sqrt(-a)/(a*x^6 - a*x))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^5+2}{\left(ax^5-x^2-a\right)\sqrt{x^5-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+2)/(x^5-1)^(1/2)/(a*x^5-x^2-a),x, algorithm="giac")
[Out] integrate((3*x^5 + 2)/((a*x^5 - x^2 - a)*sqrt(x^5 - 1)), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{3x^5+2}{\sqrt{x^5-1}\,\left(a\,x^5-x^2-a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^5+2)/(x^5-1)^(1/2)/(a*x^5-x^2-a),x)
[Out] int((3*x^5+2)/(x^5-1)^(1/2)/(a*x^5-x^2-a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^5+2}{\left(ax^5-x^2-a\right)\sqrt{x^5-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+2)/(x^5-1)^(1/2)/(a*x^5-x^2-a),x, algorithm="maxima")
[Out] integrate((3*x^5 + 2)/((a*x^5 - x^2 - a)*sqrt(x^5 - 1)), x)
```

mupad [B] time = 0.86, size = 52, normalized size = 2.17

$$\frac{\ln\left(\frac{a^4\left(x^5-1\right)+a^3\,x^2-2\,a^{7/2}\,x\,\sqrt{x^5-1}}{4\,x^2-4\,a\left(x^5-1\right)}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^5 + 2)/((x^5 - 1)^(1/2)*(a - a*x^5 + x^2)),x)
```

```
[Out] log((a^4*(x^5 - 1) + a^3*x^2 - 2*a^(7/2)*x*(x^5 - 1)^(1/2))/(4*x^2 - 4*a*(x
^5 - 1)))/a^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^5 + 2}{\sqrt{(x-1)\left(x^4 + x^3 + x^2 + x + 1\right)}\left(ax^5 - a - x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**5+2)/(x**5-1)**(1/2)/(a*x**5-x**2-a),x)
```

```
[Out] Integral((3*x**5 + 2)/(sqrt((x - 1)*(x**4 + x**3 + x**2 + x + 1))*(a*x**5 -
 a - x**2)), x)
```

3.246 $\int \frac{-2+3x^5}{\sqrt{1+x^5}\left(a-x^2+ax^5\right)} dx$

Optimal. Leaf size=24

$$-\frac{2\tanh^{-1}\left(\frac{x}{\sqrt{a}\sqrt{x^5+1}}\right)}{\sqrt{a}}$$

Rubi [F] time = 0.62, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2+3x^5}{\sqrt{1+x^5}\left(a-x^2+ax^5\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(-2 + 3*x^5)/(Sqrt[1 + x^5]*(a - x^2 + a*x^5)),x]
```

```
[Out] (3*x*Hypergeometric2F1[1/5, 1/2, 6/5, -x^5])/a - 5*Defer[Int][1/(Sqrt[1 + x
^5]*(a - x^2 + a*x^5)), x] + (3*Defer[Int][x^2/(Sqrt[1 + x^5]*(a - x^2 + a*
x^5)), x])/a
```

Rubi steps

$$\begin{aligned}
\int \frac{-2+3x^5}{\sqrt{1+x^5}\left(a-x^2+ax^5\right)} dx &= \int \left(\frac{3}{a\sqrt{1+x^5}}-\frac{5a-3x^2}{a\sqrt{1+x^5}\left(a-x^2+ax^5\right)}\right) dx \\
&= -\frac{\int \frac{5a-3x^2}{\sqrt{1+x^5}\left(a-x^2+ax^5\right)} dx}{a}+\frac{3\int \frac{1}{\sqrt{1+x^5}} dx}{a} \\
&= \frac{3x\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{6}{5};-x^5\right)}{a}-\frac{\int \left(\frac{5a}{\sqrt{1+x^5}\left(a-x^2+ax^5\right)}-\frac{3x^2}{\sqrt{1+x^5}\left(a-x^2+ax^5\right)}\right) dx}{a} \\
&= \frac{3x\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{6}{5};-x^5\right)}{a}-5\int \frac{1}{\sqrt{1+x^5}\left(a-x^2+ax^5\right)} dx+\frac{3\int \frac{x^2}{\sqrt{1+x^5}\left(a-x^2+ax^5\right)} d}{a}
\end{aligned}$$

Mathematica [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{-2+3x^5}{\sqrt{1+x^5}\left(a-x^2+ax^5\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2 + 3*x^5)/(Sqrt[1 + x^5]*(a - x^2 + a*x^5)),x]
```

```
[Out] Integrate[(-2 + 3*x^5)/(Sqrt[1 + x^5]*(a - x^2 + a*x^5)), x]
```

IntegrateAlgebraic [A] time = 2.29, size = 24, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{x}{\sqrt{a}\sqrt{x^5+1}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + 3*x^5)/(Sqrt[1 + x^5]*(a - x^2 + a*x^5)),x]
[Out] (-2*ArcTanh[x/(Sqrt[a]*Sqrt[1 + x^5])])/Sqrt[a]
```

fricas [B] time = 0.46, size = 147, normalized size = 6.12

$$\left[\frac{\log\left(\frac{a^2x^{10}+6\,ax^7+2\,a^2x^5+x^4+6\,ax^2-4\left(ax^6+x^3+ax\right)\sqrt{x^5+1}\,\sqrt{a}+a^2}{a^2x^{10}-2\,ax^7+2\,a^2x^5+x^4-2\,ax^2+a^2}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\,\arctan\left(\frac{\left(ax^5+x^2+a\right)\sqrt{x^5+1}\,\sqrt{-a}}{2\left(ax^6+ax\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-2)/(x^5+1)^(1/2)/(a*x^5-x^2+a),x, algorithm="fricas")
[Out] [1/2*log((a^2*x^10 + 6*a*x^7 + 2*a^2*x^5 + x^4 + 6*a*x^2 - 4*(a*x^6 + x^3 +
 a*x)*sqrt(x^5 + 1)*sqrt(a) + a^2)/(a^2*x^10 - 2*a*x^7 + 2*a^2*x^5 + x^4 -
2*a*x^2 + a^2))/sqrt(a), sqrt(-a)*arctan(1/2*(a*x^5 + x^2 + a)*sqrt(x^5 + 1
)*sqrt(-a)/(a*x^6 + a*x))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^5-2}{\left(ax^5-x^2+a\right)\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-2)/(x^5+1)^(1/2)/(a*x^5-x^2+a),x, algorithm="giac")
[Out] integrate((3*x^5 - 2)/((a*x^5 - x^2 + a)*sqrt(x^5 + 1)), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{3x^5-2}{\sqrt{x^5+1}\,\left(a\,x^5-x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^5-2)/(x^5+1)^(1/2)/(a*x^5-x^2+a),x)
[Out] int((3*x^5-2)/(x^5+1)^(1/2)/(a*x^5-x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^5-2}{\left(ax^5-x^2+a\right)\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-2)/(x^5+1)^(1/2)/(a*x^5-x^2+a),x, algorithm="maxima")
[Out] integrate((3*x^5 - 2)/((a*x^5 - x^2 + a)*sqrt(x^5 + 1)), x)
```

mupad [B] time = 0.81, size = 46, normalized size = 1.92

$$\frac{\ln\left(\frac{a+a\,x^5+x^2-2\,\sqrt{a}\,x\,\sqrt{x^5+1}}{4\,a\,x^5-4\,x^2+4\,a}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^5 - 2)/((x^5 + 1)^(1/2)*(a + a*x^5 - x^2)),x)
```

```
[Out] log((a + a*x^5 + x^2 - 2*a^(1/2)*x*(x^5 + 1)^(1/2))/(4*a + 4*a*x^5 - 4*x^2)
)/a^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^5 - 2}{\sqrt{(x+1)\left(x^4 - x^3 + x^2 - x + 1\right)}\left(ax^5 + a - x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**5-2)/(x**5+1)**(1/2)/(a*x**5-x**2+a),x)
```

```
[Out] Integral((3*x**5 - 2)/(sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1))*(a*x**5 +
 a - x**2)), x)
```

3.247 $\int \frac{-1-2x^2+2x^4}{\left(1-x^2+x^4\right)\sqrt{1+x^6}}\,dx$

Optimal. Leaf size=24

$$-\frac{x\sqrt{x^6+1}}{x^4-x^2+1}$$

Rubi [F] time = 0.97, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1-2x^2+2x^4}{\left(1-x^2+x^4\right)\sqrt{1+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 - 2*x^2 + 2*x^4)/((1 - x^2 + x^4)*Sqrt[1 + x^6]),x]
```

```
[Out] (x*(1 + x^2)*Sqrt[(1 - x^2 + x^4)/(1 + (1 + Sqrt[3])*x^2)^2]*EllipticF[ArcC
os[(1 + (1 - Sqrt[3])*x^2)/(1 + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(3^(
1/4)*Sqrt[(x^2*(1 + x^2))/(1 + (1 + Sqrt[3])*x^2)^2]*Sqrt[1 + x^6]) + (I*De
fer[Int][1/((Sqrt[1 - I*Sqrt[3]] - Sqrt[2]*x)*Sqrt[1 + x^6]), x])/Sqrt[(1 -
 I*Sqrt[3])/3] - (I*Defer[Int][1/((Sqrt[1 + I*Sqrt[3]] - Sqrt[2]*x)*Sqrt[1
+ x^6]), x])/Sqrt[(1 + I*Sqrt[3])/3] + (I*Defer[Int][1/((Sqrt[1 - I*Sqrt[3]
] + Sqrt[2]*x)*Sqrt[1 + x^6]), x])/Sqrt[(1 - I*Sqrt[3])/3] - (I*Defer[Int][
1/((Sqrt[1 + I*Sqrt[3]] + Sqrt[2]*x)*Sqrt[1 + x^6]), x])/Sqrt[(1 + I*Sqrt[3
])/3]
```

Rubi steps

$$\int \frac{-1-2x^2+2x^4}{\left(1-x^2+x^4\right)\sqrt{1+x^6}}\,dx = \int \left(\frac{2}{\sqrt{1+x^6}}-\frac{3}{\left(1-x^2+x^4\right)\sqrt{1+x^6}}\right)dx$$

$$= 2\int \frac{1}{\sqrt{1+x^6}}\,dx-3\int \frac{1}{\left(1-x^2+x^4\right)\sqrt{1+x^6}}\,dx$$

$$= \frac{x\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x^2}{1+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x^2\left(1+x^2\right)}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1+x^6}}-3\int \Bigg(\frac{}{\sqrt{3}\left(1+i\sqrt{\right.}\right.}$$

$$= \frac{x\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x^2}{1+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x^2\left(1+x^2\right)}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1+x^6}}-\left(2i\sqrt{3}\right)\int \frac{}{\left(1+i\right.}$$

$$= \frac{x\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x^2}{1+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x^2\left(1+x^2\right)}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1+x^6}}-\left(2i\sqrt{3}\right)\int \Bigg(\frac{}{2\left(-1\right.}$$

$$= \frac{x\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x^2}{1+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x^2\left(1+x^2\right)}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1+x^6}}+\frac{i\int \frac{1}{\left(\sqrt{1-i\sqrt{3}}-\sqrt{2}x\right.}}{\sqrt{\frac{1}{3}}\left(1-i\sqrt{\right.}}$$

Mathematica [A] time = 0.35, size = 24, normalized size = 1.00

$$-\frac{x\sqrt{x^6+1}}{x^4-x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x^2 + 2*x^4)/((1 - x^2 + x^4)*Sqrt[1 + x^6]),x]
[Out] -((x*Sqrt[1 + x^6])/(1 - x^2 + x^4))
```

IntegrateAlgebraic [A] time = 9.07, size = 24, normalized size = 1.00

$$-\frac{x\sqrt{x^6+1}}{x^4-x^2+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 2*x^2 + 2*x^4)/((1 - x^2 + x^4)*Sqrt[1 + x^6]),x]
[Out] -((x*Sqrt[1 + x^6])/(1 - x^2 + x^4))
```

fricas [A] time = 0.39, size = 22, normalized size = 0.92

$$-\frac{\sqrt{x^6+1}\,x}{x^4-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x^2-1)/(x^4-x^2+1)/(x^6+1)^(1/2),x, algorithm="fricas")
[Out] -sqrt(x^6 + 1)*x/(x^4 - x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^4-2\,x^2-1}{\sqrt{x^6+1}\left(x^4-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x^2-1)/(x^4-x^2+1)/(x^6+1)^(1/2),x, algorithm="giac")
[Out] integrate((2*x^4 - 2*x^2 - 1)/(sqrt(x^6 + 1)*(x^4 - x^2 + 1)), x)
```

maple [A] time = 0.01, size = 16, normalized size = 0.67

$$-\frac{\left(x^2+1\right)x}{\sqrt{x^6+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-2*x^2-1)/(x^4-x^2+1)/(x^6+1)^(1/2),x)
[Out] -(x^2+1)*x/(x^6+1)^(1/2)
```

maxima [A] time = 0.54, size = 26, normalized size = 1.08

$$-\frac{x^3+x}{\sqrt{x^4-x^2+1}\,\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x^2-1)/(x^4-x^2+1)/(x^6+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(x^3 + x)/(sqrt(x^4 - x^2 + 1)*sqrt(x^2 + 1))
```

mupad [B] time = 0.08, size = 22, normalized size = 0.92

$$-\frac{x\sqrt{x^6+1}}{x^4-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 2*x^4 + 1)/((x^6 + 1)^(1/2)*(x^4 - x^2 + 1)),x)
```

```
[Out] -(x*(x^6 + 1)^(1/2))/(x^4 - x^2 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^4-2x^2-1}{\sqrt{\left(x^2+1\right)\left(x^4-x^2+1\right)}\left(x^4-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-2*x**2-1)/(x**4-x**2+1)/(x**6+1)**(1/2),x)
```

```
[Out] Integral((2*x**4 - 2*x**2 - 1)/(sqrt((x**2 + 1)*(x**4 - x**2 + 1))*(x**4 -
x**2 + 1)), x)
```

3.248 $\int \frac{-1+4x^5}{(1-ax+x^5)\sqrt{x+x^6}}\,dx$

Optimal. Leaf size=24

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^6+x}}\right)}{\sqrt{a}}$$

Rubi [F] time = 1.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+4x^5}{(1-ax+x^5)\sqrt{x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 4*x^5)/((1 - a*x + x^5)*Sqrt[x + x^6]),x]
```

```
[Out] (8*x*Sqrt[1 + x^5]*Hypergeometric2F1[1/10, 1/2, 11/10, -x^5])/Sqrt[x + x^6]
 + (10*Sqrt[x]*Sqrt[1 + x^5]*Defer[Subst][Defer[Int][1/((-1 + a*x^2 - x^10)
*Sqrt[1 + x^10]), x], x, Sqrt[x]])/Sqrt[x + x^6] - (8*a*Sqrt[x]*Sqrt[1 + x^
5]*Defer[Subst][Defer[Int][x^2/((-1 + a*x^2 - x^10)*Sqrt[1 + x^10]), x], x,
 Sqrt[x]])/Sqrt[x + x^6]
```

Rubi steps

$$\int \frac{-1+4x^5}{(1-ax+x^5)\sqrt{x+x^6}}\,dx = \frac{\left(\sqrt{x}\sqrt{1+x^5}\right)\int \frac{-1+4x^5}{\sqrt{x}\sqrt{1+x^5}\,(1-ax+x^5)}\,dx}{\sqrt{x+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{-1+4x^{10}}{\sqrt{1+x^{10}}\,(1-ax^2+x^{10})}\,dx, x, \sqrt{x}\right)}{\sqrt{x+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \left(\frac{4}{\sqrt{1+x^{10}}} - \frac{5-4ax^2}{\sqrt{1+x^{10}}\,(1-ax^2+x^{10})}\right) dx, x, \sqrt{x}\right)}{\sqrt{x+x^6}}$$

$$= -\frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{5-4ax^2}{\sqrt{1+x^{10}}\,(1-ax^2+x^{10})}\,dx, x, \sqrt{x}\right)}{\sqrt{x+x^6}} + \frac{\left(8\sqrt{x}\sqrt{1+x^5}\right)\text{Sub}}{\sqrt{x}}$$

$$= \frac{8x\sqrt{1+x^5}\,{}_2F_1\left(\frac{1}{10},\frac{1}{2};\frac{11}{10};-x^5\right)}{\sqrt{x+x^6}} - \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \left(-\frac{5}{(-1+ax^2-x^{10})\sqrt{1+x^1}}\right.\right.}{\sqrt{x+x^6}}$$

$$= \frac{8x\sqrt{1+x^5}\,{}_2F_1\left(\frac{1}{10},\frac{1}{2};\frac{11}{10};-x^5\right)}{\sqrt{x+x^6}} + \frac{\left(10\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{1}{(-1+ax^2-x^{10})\sqrt{1+x^{10}}}\right.}{\sqrt{x+x^6}}$$

Mathematica [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{-1+4x^5}{(1-ax+x^5)\sqrt{x+x^6}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(-1 + 4*x^5)/((1 - a*x + x^5)*Sqrt[x + x^6]),x]

[Out] Integrate[(-1 + 4*x^5)/((1 - a*x + x^5)*Sqrt[x + x^6]), x]

IntegrateAlgebraic [A] time = 2.69, size = 24, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^6+x}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-1 + 4*x^5)/((1 - a*x + x^5)*Sqrt[x + x^6]),x]

[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[x + x^6]])/Sqrt[a]

fricas [A] time = 0.58, size = 119, normalized size = 4.96

$$\left[\frac{\log\left(-\frac{x^{10}+6\,ax^6+2\,x^5+a^2x^2-4\,\sqrt{x^6+x}\left(x^5+ax+1\right)\sqrt{a}+6\,ax+1}{x^{10}-2\,ax^6+2\,x^5+a^2x^2-2\,ax+1}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{x^6+x}\,\sqrt{-a}}{x^5+ax+1}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^5-1)/(x^5-a*x+1)/(x^6+x)^(1/2),x, algorithm="fricas")

[Out] [1/2*log(-(x^10 + 6*a*x^6 + 2*x^5 + a^2*x^2 - 4*sqrt(x^6 + x)*(x^5 + a*x + 1)*sqrt(a) + 6*a*x + 1)/(x^10 - 2*a*x^6 + 2*x^5 + a^2*x^2 - 2*a*x + 1))/sqrt(a), sqrt(-a)*arctan(2*sqrt(x^6 + x)*sqrt(-a)/(x^5 + a*x + 1))/a]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x^5-1}{\sqrt{x^6+x}\left(x^5-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^5-1)/(x^5-a*x+1)/(x^6+x)^(1/2),x, algorithm="giac")

[Out] integrate((4*x^5 - 1)/(sqrt(x^6 + x)*(x^5 - a*x + 1)), x)

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{4x^5-1}{\left(x^5-ax+1\right)\sqrt{x^6+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^5-1)/(x^5-a*x+1)/(x^6+x)^(1/2),x)

[Out] int((4*x^5-1)/(x^5-a*x+1)/(x^6+x)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x^5-1}{\sqrt{x^6+x}\left(x^5-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^5-1)/(x^5-a*x+1)/(x^6+x)^(1/2),x, algorithm="maxima")

[Out] integrate((4*x^5 - 1)/(sqrt(x^6 + x)*(x^5 - a*x + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{4\,x^5-1}{\sqrt{x^6+x}\,\left(x^5-a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5 - 1)/((x + x^6)^(1/2)*(x^5 - a*x + 1)),x)
```

```
[Out] int((4*x^5 - 1)/((x + x^6)^(1/2)*(x^5 - a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^5-1}{\sqrt{x\,(x+1)\left(x^4-x^3+x^2-x+1\right)}\,\left(-ax+x^5+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**5-1)/(x**5-a*x+1)/(x**6+x)**(1/2),x)
```

```
[Out] Integral((4*x**5 - 1)/(sqrt(x*(x + 1)*(x**4 - x**3 + x**2 - x + 1))*(-a*x +
 x**5 + 1)), x)
```

3.249 $\int \frac{-1+4x^5}{(a-x+ax^5)\sqrt{x+x^6}}\,dx$

Optimal. Leaf size=24

$$-\frac{2\tanh^{-1}\left(\frac{x}{\sqrt{a}\sqrt{x^6+x}}\right)}{\sqrt{a}}$$

Rubi [F] time = 0.96, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+4x^5}{(a-x+ax^5)\sqrt{x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 4*x^5)/((a - x + a*x^5)*Sqrt[x + x^6]),x]
```

```
[Out] (8*x*Sqrt[1 + x^5]*Hypergeometric2F1[1/10, 1/2, 11/10, -x^5])/(a*Sqrt[x + x
^6]) - (10*Sqrt[x]*Sqrt[1 + x^5]*Defer[Subst][Defer[Int][1/(Sqrt[1 + x^10]*
(a - x^2 + a*x^10)), x], x, Sqrt[x]])/Sqrt[x + x^6] + (8*Sqrt[x]*Sqrt[1 + x
^5]*Defer[Subst][Defer[Int][x^2/(Sqrt[1 + x^10]*(a - x^2 + a*x^10)), x], x,
 Sqrt[x]])/(a*Sqrt[x + x^6])
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+4x^5}{(a-x+ax^5)\sqrt{x+x^6}}\,dx &= \frac{\left(\sqrt{x}\sqrt{1+x^5}\right)\int \frac{-1+4x^5}{\sqrt{x}\sqrt{1+x^5}(a-x+ax^5)}\,dx}{\sqrt{x+x^6}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{-1+4x^{10}}{\sqrt{1+x^{10}}(a-x^2+ax^{10})}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^6}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \left(\frac{4}{a\sqrt{1+x^{10}}}-\frac{5a-4x^2}{a\sqrt{1+x^{10}}(a-x^2+ax^{10})}\right)dx,x,\sqrt{x}\right)}{\sqrt{x+x^6}}\\
&= -\frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{5a-4x^2}{\sqrt{1+x^{10}}(a-x^2+ax^{10})}\,dx,x,\sqrt{x}\right)}{a\sqrt{x+x^6}}+\frac{\left(8\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}(\ldots}{a\sqrt{x+\ldots}}\\
&= \frac{8x\sqrt{1+x^5}\,{}_2F_1\left(\frac{1}{10},\frac{1}{2};\frac{11}{10};-x^5\right)}{a\sqrt{x+x^6}}-\frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int\left(\frac{5a}{\sqrt{1+x^{10}}(a-x^2+ax^{10})}-\ldots\right.\right.}{a\sqrt{x+x^6}}\\
&= \frac{8x\sqrt{1+x^5}\,{}_2F_1\left(\frac{1}{10},\frac{1}{2};\frac{11}{10};-x^5\right)}{a\sqrt{x+x^6}}-\frac{\left(10\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^{10}}(a-x^2+ax^{10})}\,dx,\ldots\right.}{\sqrt{x+x^6}}
\end{aligned}$$

Mathematica [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{-1+4x^5}{(a-x+ax^5)\sqrt{x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + 4*x^5)/((a - x + a*x^5)*Sqrt[x + x^6]),x]
[Out] Integrate[(-1 + 4*x^5)/((a - x + a*x^5)*Sqrt[x + x^6]), x]
```

IntegrateAlgebraic [A] time = 2.66, size = 24, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{x}{\sqrt{a}\sqrt{x^6+x}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 4*x^5)/((a - x + a*x^5)*Sqrt[x + x^6]),x]
[Out] (-2*ArcTanh[x/(Sqrt[a]*Sqrt[x + x^6])])/Sqrt[a]
```

fricas [A] time = 0.58, size = 129, normalized size = 5.38

$$\left[\frac{\log\left(-\frac{a^2x^{10}+2\,a^2x^5+6\,ax^6-4\left(ax^5+a+x\right)\sqrt{x^6+x}\,\sqrt{a}+a^2+6\,ax+x^2}{a^2x^{10}+2\,a^2x^5-2\,ax^6+a^2-2\,ax+x^2}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{x^6+x}\,\sqrt{-a}}{ax^5+a+x}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(a*x^5+a-x)/(x^6+x)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-(a^2*x^10 + 2*a^2*x^5 + 6*a*x^6 - 4*(a*x^5 + a + x)*sqrt(x^6 + x)
*sqrt(a) + a^2 + 6*a*x + x^2)/(a^2*x^10 + 2*a^2*x^5 - 2*a*x^6 + a^2 - 2*a*x
 + x^2))/sqrt(a), sqrt(-a)*arctan(2*sqrt(x^6 + x)*sqrt(-a)/(a*x^5 + a + x))
/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x^5-1}{\left(ax^5+a-x\right)\sqrt{x^6+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(a*x^5+a-x)/(x^6+x)^(1/2),x, algorithm="giac")
[Out] integrate((4*x^5 - 1)/((a*x^5 + a - x)*sqrt(x^6 + x)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{4x^5-1}{\left(a\,x^5+a-x\right)\sqrt{x^6+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5-1)/(a*x^5+a-x)/(x^6+x)^(1/2),x)
[Out] int((4*x^5-1)/(a*x^5+a-x)/(x^6+x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x^5-1}{\left(ax^5+a-x\right)\sqrt{x^6+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(a*x^5+a-x)/(x^6+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((4*x^5 - 1)/((a*x^5 + a - x)*sqrt(x^6 + x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{4\,x^5-1}{\sqrt{x^6+x}\,\left(a\,x^5-x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5 - 1)/((x + x^6)^(1/2)*(a - x + a*x^5)),x)
```

```
[Out] int((4*x^5 - 1)/((x + x^6)^(1/2)*(a - x + a*x^5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^5-1}{\sqrt{x\left(x+1\right)\left(x^4-x^3+x^2-x+1\right)}\,\left(ax^5+a-x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**5-1)/(a*x**5+a-x)/(x**6+x)**(1/2),x)
```

```
[Out] Integral((4*x**5 - 1)/(sqrt(x*(x + 1)*(x**4 - x**3 + x**2 - x + 1))*(a*x**5
 + a - x)), x)
```

3.250 $\int x^3\left(-1+x^2\right)^{2/3}\,dx$

Optimal. Leaf size=25

$$\frac{3}{80}\left(x^2-1\right)^{2/3}\left(5x^4-2x^2-3\right)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{16}\left(x^2-1\right)^{8/3}+\frac{3}{10}\left(x^2-1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(-1 + x^2)^(2/3),x]
[Out] (3*(-1 + x^2)^(5/3))/10 + (3*(-1 + x^2)^(8/3))/16
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(-1+x^2\right)^{2/3}\,dx &= \frac{1}{2}\text{Subst}\left(\int(-1+x)^{2/3}x\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left((-1+x)^{2/3}+(-1+x)^{5/3}\right)dx,x,x^2\right)\\ &= \frac{3}{10}\left(-1+x^2\right)^{5/3}+\frac{3}{16}\left(-1+x^2\right)^{8/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{3}{80}\left(x^2-1\right)^{5/3}\left(5x^2+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(-1 + x^2)^(2/3),x]
[Out] (3*(-1 + x^2)^(5/3)*(3 + 5*x^2))/80
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 0.80

$$\frac{3}{80}\left(x^2-1\right)^{5/3}\left(5x^2+3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(-1 + x^2)^(2/3),x]
[Out] (3*(-1 + x^2)^(5/3)*(3 + 5*x^2))/80
```

fricas [A] time = 0.38, size = 21, normalized size = 0.84

$$\frac{3}{80}\left(5\,x^4-2\,x^2-3\right)\left(x^2-1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2-1)^(2/3),x, algorithm="fricas")
[Out] 3/80*(5*x^4 - 2*x^2 - 3)*(x^2 - 1)^(2/3)
```

giac [A] time = 0.29, size = 19, normalized size = 0.76

$$\frac{3}{16}\left(x^2-1\right)^{\frac{8}{3}}+\frac{3}{10}\left(x^2-1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2-1)^(2/3),x, algorithm="giac")
[Out] 3/16*(x^2 - 1)^(8/3) + 3/10*(x^2 - 1)^(5/3)
```

maple [A] time = 0.00, size = 23, normalized size = 0.92

$$\frac{3\left(-1+x\right)\left(1+x\right)\left(5x^2+3\right)\left(x^2-1\right)^{\frac{2}{3}}}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2-1)^(2/3),x)
[Out] 3/80*(-1+x)*(1+x)*(5*x^2+3)*(x^2-1)^(2/3)
```

maxima [A] time = 0.34, size = 19, normalized size = 0.76

$$\frac{3}{16}\left(x^2-1\right)^{\frac{8}{3}}+\frac{3}{10}\left(x^2-1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2-1)^(2/3),x, algorithm="maxima")
[Out] 3/16*(x^2 - 1)^(8/3) + 3/10*(x^2 - 1)^(5/3)
```

mupad [B] time = 0.21, size = 21, normalized size = 0.84

$$-\left(x^2-1\right)^{2/3}\left(-\frac{3\,x^4}{16}+\frac{3\,x^2}{40}+\frac{9}{80}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2 - 1)^(2/3),x)
[Out] -(x^2 - 1)^(2/3)*((3*x^2)/40 - (3*x^4)/16 + 9/80)
```

sympy [A] time = 0.37, size = 41, normalized size = 1.64

$$\frac{3x^4\left(x^2-1\right)^{\frac{2}{3}}}{16}-\frac{3x^2\left(x^2-1\right)^{\frac{2}{3}}}{40}-\frac{9\left(x^2-1\right)^{\frac{2}{3}}}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2-1)**(2/3),x)
```

```
[Out] 3*x**4*(x**2 - 1)**(2/3)/16 - 3*x**2*(x**2 - 1)**(2/3)/40 - 9*(x**2 - 1)**(
2/3)/80
```

3.251 $\int x^3 \left(-1+x^2\right)^{3/4} dx$

Optimal. Leaf size=25

$$\frac{2}{77}\left(x^2-1\right)^{3/4}\left(7x^4-3x^2-4\right)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{2}{11}\left(x^2-1\right)^{11/4}+\frac{2}{7}\left(x^2-1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(-1 + x^2)^(3/4),x]
```

```
[Out] (2*(-1 + x^2)^(7/4))/7 + (2*(-1 + x^2)^(11/4))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(-1+x^2\right)^{3/4} dx &= \frac{1}{2}\operatorname{Subst}\left(\int(-1+x)^{3/4}x\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left((-1+x)^{3/4}+(-1+x)^{7/4}\right)dx,x,x^2\right)\\ &= \frac{2}{7}\left(-1+x^2\right)^{7/4}+\frac{2}{11}\left(-1+x^2\right)^{11/4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{2}{77}\left(x^2-1\right)^{7/4}\left(7x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(-1 + x^2)^(3/4),x]
```

```
[Out] (2*(-1 + x^2)^(7/4)*(4 + 7*x^2))/77
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 0.80

$$\frac{2}{77}\left(x^2-1\right)^{7/4}\left(7x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(-1 + x^2)^(3/4),x]
[Out] (2*(-1 + x^2)^(7/4)*(4 + 7*x^2))/77
```

fricas [A] time = 0.38, size = 21, normalized size = 0.84

$$\frac{2}{77}\left(7\,x^4-3\,x^2-4\right)\left(x^2-1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2-1)^(3/4),x, algorithm="fricas")
[Out] 2/77*(7*x^4 - 3*x^2 - 4)*(x^2 - 1)^(3/4)
```

giac [A] time = 0.53, size = 19, normalized size = 0.76

$$\frac{2}{11}\left(x^2-1\right)^{\frac{11}{4}}+\frac{2}{7}\left(x^2-1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2-1)^(3/4),x, algorithm="giac")
[Out] 2/11*(x^2 - 1)^(11/4) + 2/7*(x^2 - 1)^(7/4)
```

maple [A] time = 0.00, size = 23, normalized size = 0.92

$$\frac{2\left(-1+x\right)\left(1+x\right)\left(7x^2+4\right)\left(x^2-1\right)^{\frac{3}{4}}}{77}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2-1)^(3/4),x)
[Out] 2/77*(-1+x)*(1+x)*(7*x^2+4)*(x^2-1)^(3/4)
```

maxima [A] time = 0.34, size = 19, normalized size = 0.76

$$\frac{2}{11}\left(x^2-1\right)^{\frac{11}{4}}+\frac{2}{7}\left(x^2-1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2-1)^(3/4),x, algorithm="maxima")
[Out] 2/11*(x^2 - 1)^(11/4) + 2/7*(x^2 - 1)^(7/4)
```

mupad [B] time = 0.19, size = 21, normalized size = 0.84

$$-\left(x^2-1\right)^{3/4}\left(-\frac{2\,x^4}{11}+\frac{6\,x^2}{77}+\frac{8}{77}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2 - 1)^(3/4),x)
[Out] -(x^2 - 1)^(3/4)*((6*x^2)/77 - (2*x^4)/11 + 8/77)
```

sympy [A] time = 0.76, size = 41, normalized size = 1.64

$$\frac{2x^4\left(x^2-1\right)^{\frac{3}{4}}}{11}-\frac{6x^2\left(x^2-1\right)^{\frac{3}{4}}}{77}-\frac{8\left(x^2-1\right)^{\frac{3}{4}}}{77}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2-1)**(3/4),x)
```

```
[Out] 2*x**4*(x**2 - 1)**(3/4)/11 - 6*x**2*(x**2 - 1)**(3/4)/77 - 8*(x**2 - 1)**(
3/4)/77
```

3.252 $\int \frac{1}{x\left(1+x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=25

$$-\tan^{-1}\left(\sqrt[4]{x^2+1}\right)-\tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {266, 63, 212, 206, 203}

$$-\tan^{-1}\left(\sqrt[4]{x^2+1}\right)-\tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^2)^(3/4)),x]
```

```
[Out] -ArcTan[(1 + x^2)^(1/4)] - ArcTanh[(1 + x^2)^(1/4)]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1+x^2\right)^{3/4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{3/4}}\,dx, x, x^2\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt[4]{1+x^2}\right)\\ &= -\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1+x^2}\right) - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{1+x^2}\right)\\ &= -\tan^{-1}\left(\sqrt[4]{1+x^2}\right) - \tanh^{-1}\left(\sqrt[4]{1+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$-\tan^{-1}\left(\sqrt[4]{x^2+1}\right) - \tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^2)^(3/4)),x]
[Out] -ArcTan[(1 + x^2)^(1/4)] - ArcTanh[(1 + x^2)^(1/4)]
```

IntegrateAlgebraic [A] time = 0.03, size = 25, normalized size = 1.00

$$-\tan^{-1}\left(\sqrt[4]{x^2+1}\right) - \tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(1 + x^2)^(3/4)),x]
[Out] -ArcTan[(1 + x^2)^(1/4)] - ArcTanh[(1 + x^2)^(1/4)]
```

fricas [A] time = 0.40, size = 35, normalized size = 1.40

$$-\arctan\left(\left(x^2+1\right)^{\frac{1}{4}}\right) - \frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}+1\right) + \frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^(3/4),x, algorithm="fricas")
[Out] -arctan((x^2 + 1)^(1/4)) - 1/2*log((x^2 + 1)^(1/4) + 1) + 1/2*log((x^2 + 1)
^(1/4) - 1)
```

giac [A] time = 0.44, size = 35, normalized size = 1.40

$$-\arctan\left(\left(x^2+1\right)^{\frac{1}{4}}\right) - \frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}+1\right) + \frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^(3/4),x, algorithm="giac")
[Out] -arctan((x^2 + 1)^(1/4)) - 1/2*log((x^2 + 1)^(1/4) + 1) + 1/2*log((x^2 + 1)
^(1/4) - 1)
```

maple [C] time = 0.20, size = 43, normalized size = 1.72

$$\frac{-\frac{3\Gamma\left(\frac{3}{4}\right)x^2\,\text{hypergeom}\left(\left[1,1,\frac{7}{4}\right],[2,2],-x^2\right)}{4} + \left(-3\ln(2)+\frac{\pi}{2}+2\ln(x)\right)\Gamma\left(\frac{3}{4}\right)}{2\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+1)^(3/4),x)
```

```
[Out] 1/2/GAMMA(3/4)*(-3/4*GAMMA(3/4)*x^2*hypergeom([1,1,7/4],[2,2],-x^2)+(-3*ln(
2)+1/2*Pi+2*ln(x))*GAMMA(3/4))
```

maxima [A] time = 0.53, size = 35, normalized size = 1.40

$$-\arctan\left(\left(x^2+1\right)^{\frac{1}{4}}\right)-\frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^(3/4),x, algorithm="maxima")
```

```
[Out] -arctan((x^2 + 1)^(1/4)) - 1/2*log((x^2 + 1)^(1/4) + 1) + 1/2*log((x^2 + 1)
^(1/4) - 1)
```

mupad [B] time = 0.23, size = 21, normalized size = 0.84

$$-\mathrm{atan}\left(\left(x^2+1\right)^{1/4}\right)-\mathrm{atanh}\left(\left(x^2+1\right)^{1/4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 + 1)^(3/4)),x)
```

```
[Out] - atan((x^2 + 1)^(1/4)) - atanh((x^2 + 1)^(1/4))
```

sympy [C] time = 0.85, size = 32, normalized size = 1.28

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^2}\right)}{2x^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+1)**(3/4),x)
```

```
[Out] -gamma(3/4)*hyper((3/4, 3/4), (7/4,), exp_polar(I*pi)/x**2)/(2*x**(3/2)*gam
ma(7/4))
```

3.253 $\int x^3 \left(1+x^2\right)^{3/4} dx$

Optimal. Leaf size=25

$$\frac{2}{77}\left(x^2+1\right)^{3/4}\left(7x^4+3x^2-4\right)$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{2}{11}\left(x^2+1\right)^{11/4}-\frac{2}{7}\left(x^2+1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x^2)^(3/4),x]
[Out] (-2*(1 + x^2)^(7/4))/7 + (2*(1 + x^2)^(11/4))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(1+x^2\right)^{3/4} dx &= \frac{1}{2}\text{Subst}\left(\int x(1+x)^{3/4}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-(1+x)^{3/4}+(1+x)^{7/4}\right) dx, x, x^2\right)\\ &= -\frac{2}{7}\left(1+x^2\right)^{7/4}+\frac{2}{11}\left(1+x^2\right)^{11/4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{2}{77}\left(x^2+1\right)^{7/4}\left(7x^2-4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x^2)^(3/4),x]
[Out] (2*(1 + x^2)^(7/4)*(-4 + 7*x^2))/77
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 0.80

$$\frac{2}{77}\left(x^2+1\right)^{7/4}\left(7x^2-4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(1 + x^2)^(3/4),x]
[Out] (2*(1 + x^2)^(7/4)*(-4 + 7*x^2))/77
```

fricas [A] time = 0.42, size = 21, normalized size = 0.84

$$\frac{2}{77}\left(7\,x^4+3\,x^2-4\right)\left(x^2+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(3/4),x, algorithm="fricas")
[Out] 2/77*(7*x^4 + 3*x^2 - 4)*(x^2 + 1)^(3/4)
```

giac [A] time = 0.43, size = 19, normalized size = 0.76

$$\frac{2}{11}\left(x^2+1\right)^{\frac{11}{4}}-\frac{2}{7}\left(x^2+1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(3/4),x, algorithm="giac")
[Out] 2/11*(x^2 + 1)^(11/4) - 2/7*(x^2 + 1)^(7/4)
```

maple [A] time = 0.00, size = 17, normalized size = 0.68

$$\frac{2\left(x^2+1\right)^{\frac{7}{4}}\left(7x^2-4\right)}{77}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+1)^(3/4),x)
[Out] 2/77*(x^2+1)^(7/4)*(7*x^2-4)
```

maxima [A] time = 0.32, size = 19, normalized size = 0.76

$$\frac{2}{11}\left(x^2+1\right)^{\frac{11}{4}}-\frac{2}{7}\left(x^2+1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(3/4),x, algorithm="maxima")
[Out] 2/11*(x^2 + 1)^(11/4) - 2/7*(x^2 + 1)^(7/4)
```

mupad [B] time = 0.19, size = 20, normalized size = 0.80

$$\left(x^2+1\right)^{3/4}\,\left(\frac{2\,x^4}{11}+\frac{6\,x^2}{77}-\frac{8}{77}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2 + 1)^(3/4),x)
[Out] (x^2 + 1)^(3/4)*((6*x^2)/77 + (2*x^4)/11 - 8/77)
```

sympy [A] time = 0.80, size = 41, normalized size = 1.64

$$\frac{2x^4\left(x^2+1\right)^{\frac{3}{4}}}{11}+\frac{6x^2\left(x^2+1\right)^{\frac{3}{4}}}{77}-\frac{8\left(x^2+1\right)^{\frac{3}{4}}}{77}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+1)**(3/4),x)
```

```
[Out] 2*x**4*(x**2 + 1)**(3/4)/11 + 6*x**2*(x**2 + 1)**(3/4)/77 - 8*(x**2 + 1)**(
3/4)/77
```

3.254 $\int x^5 \sqrt[3]{-1+x^3}\,dx$

Optimal. Leaf size=25

$$\frac{1}{28}\sqrt[3]{x^3-1}\left(4x^6-x^3-3\right)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{7}\left(x^3-1\right)^{7/3}+\frac{1}{4}\left(x^3-1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(-1 + x^3)^(1/3),x]
```

```
[Out] (-1 + x^3)^(4/3)/4 + (-1 + x^3)^(7/3)/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \sqrt[3]{-1+x^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \sqrt[3]{-1+x}\,x\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\sqrt[3]{-1+x}+(-1+x)^{4/3}\right)dx,x,x^3\right)\\
&= \frac{1}{4}\left(-1+x^3\right)^{4/3}+\frac{1}{7}\left(-1+x^3\right)^{7/3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{1}{28}\left(x^3-1\right)^{4/3}\left(4x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(-1 + x^3)^(1/3),x]
```

```
[Out] ((-1 + x^3)^(4/3)*(3 + 4*x^3))/28
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 0.80

$$\frac{1}{28}\left(x^3-1\right)^{4/3}\left(4x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(-1 + x^3)^(1/3),x]
[Out] ((-1 + x^3)^(4/3)*(3 + 4*x^3))/28
```

fricas [A] time = 0.38, size = 21, normalized size = 0.84

$$\frac{1}{28}\left(4x^6 - x^3 - 3\right)\left(x^3 - 1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^3-1)^(1/3),x, algorithm="fricas")
[Out] 1/28*(4*x^6 - x^3 - 3)*(x^3 - 1)^(1/3)
```

giac [A] time = 0.28, size = 19, normalized size = 0.76

$$\frac{1}{7}\left(x^3 - 1\right)^{\frac{7}{3}} + \frac{1}{4}\left(x^3 - 1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^3-1)^(1/3),x, algorithm="giac")
[Out] 1/7*(x^3 - 1)^(7/3) + 1/4*(x^3 - 1)^(4/3)
```

maple [A] time = 0.00, size = 26, normalized size = 1.04

$$\frac{(-1+x)\left(x^2+x+1\right)\left(4x^3+3\right)\left(x^3-1\right)^{\frac{1}{3}}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^3-1)^(1/3),x)
[Out] 1/28*(-1+x)*(x^2+x+1)*(4*x^3+3)*(x^3-1)^(1/3)
```

maxima [A] time = 0.57, size = 19, normalized size = 0.76

$$\frac{1}{7}\left(x^3 - 1\right)^{\frac{7}{3}} + \frac{1}{4}\left(x^3 - 1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^3-1)^(1/3),x, algorithm="maxima")
[Out] 1/7*(x^3 - 1)^(7/3) + 1/4*(x^3 - 1)^(4/3)
```

mupad [B] time = 0.20, size = 21, normalized size = 0.84

$$-\left(x^3-1\right)^{1/3}\left(-\frac{x^6}{7}+\frac{x^3}{28}+\frac{3}{28}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^3 - 1)^(1/3),x)
[Out] -(x^3 - 1)^(1/3)*(x^3/28 - x^6/7 + 3/28)
```

sympy [A] time = 0.41, size = 37, normalized size = 1.48

$$\frac{x^6\sqrt[3]{x^3-1}}{7} - \frac{x^3\sqrt[3]{x^3-1}}{28} - \frac{3\sqrt[3]{x^3-1}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**3-1)**(1/3),x)
```

```
[Out] x**6*(x**3 - 1)**(1/3)/7 - x**3*(x**3 - 1)**(1/3)/28 - 3*(x**3 - 1)**(1/3)/
28
```

3.255 $\int x^5 \left(-1+x^3\right)^{3/4} dx$

Optimal. Leaf size=25

$$\frac{4}{231} \left(x^3-1\right)^{3/4} \left(7 x^6-3 x^3-4\right)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{4}{33} \left(x^3-1\right)^{11/4}+\frac{4}{21} \left(x^3-1\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(-1 + x^3)^(3/4),x]
```

```
[Out] (4*(-1 + x^3)^(7/4))/21 + (4*(-1 + x^3)^(11/4))/33
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(-1+x^3\right)^{3/4} dx &= \frac{1}{3} \text{Subst}\left(\int (-1+x)^{3/4} x \, dx, x, x^3\right)\\
&= \frac{1}{3} \text{Subst}\left(\int \left((-1+x)^{3/4}+(-1+x)^{7/4}\right) dx, x, x^3\right)\\
&= \frac{4}{21} \left(-1+x^3\right)^{7/4}+\frac{4}{33} \left(-1+x^3\right)^{11/4}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{4}{231} \left(x^3-1\right)^{7/4} \left(7 x^3+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(-1 + x^3)^(3/4),x]
```

```
[Out] (4*(-1 + x^3)^(7/4)*(4 + 7*x^3))/231
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 0.80

$$\frac{4}{231} \left(x^3-1\right)^{7/4} \left(7 x^3+4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(-1 + x^3)^(3/4),x]
[Out] (4*(-1 + x^3)^(7/4)*(4 + 7*x^3))/231
```

fricas [A] time = 0.39, size = 21, normalized size = 0.84

$$\frac{4}{231}\left(7x^6-3x^3-4\right)\left(x^3-1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^3-1)^(3/4),x, algorithm="fricas")
[Out] 4/231*(7*x^6 - 3*x^3 - 4)*(x^3 - 1)^(3/4)
```

giac [A] time = 0.61, size = 19, normalized size = 0.76

$$\frac{4}{33}\left(x^3-1\right)^{\frac{11}{4}}+\frac{4}{21}\left(x^3-1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^3-1)^(3/4),x, algorithm="giac")
[Out] 4/33*(x^3 - 1)^(11/4) + 4/21*(x^3 - 1)^(7/4)
```

maple [A] time = 0.01, size = 26, normalized size = 1.04

$$\frac{4\left(-1+x\right)\left(x^2+x+1\right)\left(7x^3+4\right)\left(x^3-1\right)^{\frac{3}{4}}}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^3-1)^(3/4),x)
[Out] 4/231*(-1+x)*(x^2+x+1)*(7*x^3+4)*(x^3-1)^(3/4)
```

maxima [A] time = 0.46, size = 19, normalized size = 0.76

$$\frac{4}{33}\left(x^3-1\right)^{\frac{11}{4}}+\frac{4}{21}\left(x^3-1\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^3-1)^(3/4),x, algorithm="maxima")
[Out] 4/33*(x^3 - 1)^(11/4) + 4/21*(x^3 - 1)^(7/4)
```

mupad [B] time = 0.20, size = 21, normalized size = 0.84

$$-\left(x^3-1\right)^{3/4}\left(-\frac{4x^6}{33}+\frac{4x^3}{77}+\frac{16}{231}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^3 - 1)^(3/4),x)
[Out] -(x^3 - 1)^(3/4)*((4*x^3)/77 - (4*x^6)/33 + 16/231)
```

sympy [A] time = 1.25, size = 41, normalized size = 1.64

$$\frac{4x^6\left(x^3-1\right)^{\frac{3}{4}}}{33}-\frac{4x^3\left(x^3-1\right)^{\frac{3}{4}}}{77}-\frac{16\left(x^3-1\right)^{\frac{3}{4}}}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**3-1)**(3/4),x)
```

```
[Out] 4*x**6*(x**3 - 1)**(3/4)/33 - 4*x**3*(x**3 - 1)**(3/4)/77 - 16*(x**3 - 1)**
(3/4)/231
```

3.256 $\int \frac{x^8}{\sqrt[4]{1+x^3}}\,dx$

Optimal. Leaf size=25

$$\frac{4}{693}\left(x^3+1\right)^{3/4}\left(21x^6-24x^3+32\right)$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.60, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{4}{33}\left(x^3+1\right)^{11/4}-\frac{8}{21}\left(x^3+1\right)^{7/4}+\frac{4}{9}\left(x^3+1\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 + x^3)^(1/4),x]
[Out] (4*(1 + x^3)^(3/4))/9 - (8*(1 + x^3)^(7/4))/21 + (4*(1 + x^3)^(11/4))/33
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\sqrt[4]{1+x^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{1+x}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{\sqrt[4]{1+x}}-2(1+x)^{3/4}+(1+x)^{7/4}\right)dx,x,x^3\right)\\ &= \frac{4}{9}\left(1+x^3\right)^{3/4}-\frac{8}{21}\left(1+x^3\right)^{7/4}+\frac{4}{33}\left(1+x^3\right)^{11/4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{4}{693}\left(x^3+1\right)^{3/4}\left(21x^6-24x^3+32\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 + x^3)^(1/4),x]
[Out] (4*(1 + x^3)^(3/4)*(32 - 24*x^3 + 21*x^6))/693
```

IntegrateAlgebraic [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{4}{693}\left(x^3+1\right)^{3/4}\left(21x^6-24x^3+32\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/(1 + x^3)^(1/4),x]
[Out] (4*(1 + x^3)^(3/4)*(32 - 24*x^3 + 21*x^6))/693
```

fricas [A] time = 0.40, size = 21, normalized size = 0.84

$$\frac{4}{693}\left(21\,x^{6}-24\,x^{3}+32\right)\left(x^{3}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^3+1)^(1/4),x, algorithm="fricas")
[Out] 4/693*(21*x^6 - 24*x^3 + 32)*(x^3 + 1)^(3/4)
```

giac [A] time = 0.52, size = 28, normalized size = 1.12

$$\frac{4}{33}\left(x^{3}+1\right)^{\frac{11}{4}}-\frac{8}{21}\left(x^{3}+1\right)^{\frac{7}{4}}+\frac{4}{9}\left(x^{3}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^3+1)^(1/4),x, algorithm="giac")
[Out] 4/33*(x^3 + 1)^(11/4) - 8/21*(x^3 + 1)^(7/4) + 4/9*(x^3 + 1)^(3/4)
```

maple [A] time = 0.00, size = 33, normalized size = 1.32

$$\frac{4\left(1+x\right)\left(x^{2}-x+1\right)\left(21x^{6}-24x^{3}+32\right)}{693\left(x^{3}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^3+1)^(1/4),x)
[Out] 4/693*(1+x)*(x^2-x+1)*(21*x^6-24*x^3+32)/(x^3+1)^(1/4)
```

maxima [A] time = 0.32, size = 28, normalized size = 1.12

$$\frac{4}{33}\left(x^{3}+1\right)^{\frac{11}{4}}-\frac{8}{21}\left(x^{3}+1\right)^{\frac{7}{4}}+\frac{4}{9}\left(x^{3}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^3+1)^(1/4),x, algorithm="maxima")
[Out] 4/33*(x^3 + 1)^(11/4) - 8/21*(x^3 + 1)^(7/4) + 4/9*(x^3 + 1)^(3/4)
```

mupad [B] time = 0.23, size = 20, normalized size = 0.80

$$\left(x^{3}+1\right)^{3/4}\left(\frac{4\,x^{6}}{33}-\frac{32\,x^{3}}{231}+\frac{128}{693}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^3 + 1)^(1/4),x)
[Out] (x^3 + 1)^(3/4)*((4*x^6)/33 - (32*x^3)/231 + 128/693)
```

sympy [A] time = 1.25, size = 41, normalized size = 1.64

$$\frac{4x^{6}\left(x^{3}+1\right)^{\frac{3}{4}}}{33}-\frac{32x^{3}\left(x^{3}+1\right)^{\frac{3}{4}}}{231}+\frac{128\left(x^{3}+1\right)^{\frac{3}{4}}}{693}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**3+1)**(1/4),x)
```

```
[Out] 4*x**6*(x**3 + 1)**(3/4)/33 - 32*x**3*(x**3 + 1)**(3/4)/231 + 128*(x**3 + 1
)**(3/4)/693
```

3.257 $\int x^5 \left(1+x^3\right)^{2/3} dx$

Optimal. Leaf size=25

$$\frac{1}{40}\left(x^3+1\right)^{2/3}\left(5x^6+2x^3-3\right)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{8}\left(x^3+1\right)^{8/3}-\frac{1}{5}\left(x^3+1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(1 + x^3)^(2/3),x]
```

```
[Out] -1/5*(1 + x^3)^(5/3) + (1 + x^3)^(8/3)/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5 \left(1+x^3\right)^{2/3} dx &= \frac{1}{3}\operatorname{Subst}\left(\int x(1+x)^{2/3}\, dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(-(1+x)^{2/3}+(1+x)^{5/3}\right) dx, x, x^3\right)\\ &= -\frac{1}{5}\left(1+x^3\right)^{5/3}+\frac{1}{8}\left(1+x^3\right)^{8/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{1}{40}\left(x^3+1\right)^{5/3}\left(5x^3-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(1 + x^3)^(2/3),x]
```

```
[Out] ((1 + x^3)^(5/3)*(-3 + 5*x^3))/40
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 0.80

$$\frac{1}{40}\left(x^3+1\right)^{5/3}\left(5x^3-3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(1 + x^3)^(2/3),x]
[Out] ((1 + x^3)^(5/3)*(-3 + 5*x^3))/40
```

fricas [A] time = 0.39, size = 21, normalized size = 0.84

$$\frac{1}{40}\left(5x^6+2x^3-3\right)\left(x^3+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^3+1)^(2/3),x, algorithm="fricas")
[Out] 1/40*(5*x^6 + 2*x^3 - 3)*(x^3 + 1)^(2/3)
```

giac [A] time = 0.41, size = 19, normalized size = 0.76

$$\frac{1}{8}\left(x^3+1\right)^{\frac{8}{3}}-\frac{1}{5}\left(x^3+1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^3+1)^(2/3),x, algorithm="giac")
[Out] 1/8*(x^3 + 1)^(8/3) - 1/5*(x^3 + 1)^(5/3)
```

maple [A] time = 0.00, size = 28, normalized size = 1.12

$$\frac{(1+x)\left(x^2-x+1\right)\left(5x^3-3\right)\left(x^3+1\right)^{\frac{2}{3}}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^3+1)^(2/3),x)
[Out] 1/40*(1+x)*(x^2-x+1)*(5*x^3-3)*(x^3+1)^(2/3)
```

maxima [A] time = 0.42, size = 19, normalized size = 0.76

$$\frac{1}{8}\left(x^3+1\right)^{\frac{8}{3}}-\frac{1}{5}\left(x^3+1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^3+1)^(2/3),x, algorithm="maxima")
[Out] 1/8*(x^3 + 1)^(8/3) - 1/5*(x^3 + 1)^(5/3)
```

mupad [B] time = 0.19, size = 20, normalized size = 0.80

$$\left(x^3+1\right)^{2/3}\left(\frac{x^6}{8}+\frac{x^3}{20}-\frac{3}{40}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^3 + 1)^(2/3),x)
[Out] (x^3 + 1)^(2/3)*(x^3/20 + x^6/8 - 3/40)
```

sympy [A] time = 0.63, size = 37, normalized size = 1.48

$$\frac{x^6\left(x^3+1\right)^{\frac{2}{3}}}{8}+\frac{x^3\left(x^3+1\right)^{\frac{2}{3}}}{20}-\frac{3\left(x^3+1\right)^{\frac{2}{3}}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**3+1)**(2/3),x)
```

```
[Out] x**6*(x**3 + 1)**(2/3)/8 + x**3*(x**3 + 1)**(2/3)/20 - 3*(x**3 + 1)**(2/3)/
40
```

3.258 $\int \frac{1}{x^2 \sqrt[3]{-x^2+x^3}} \, dx$

Optimal. Leaf size=25

$$\frac{3(3x+2)\left(x^3-x^2\right)^{2/3}}{10x^3}$$

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.64, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2016, 2014}

$$\frac{9\left(x^3-x^2\right)^{2/3}}{10x^2}+\frac{3\left(x^3-x^2\right)^{2/3}}{5x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (3*(-x^2 + x^3)^(2/3))/(5*x^3) + (9*(-x^2 + x^3)^(2/3))/(10*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2 \sqrt[3]{-x^2+x^3}} \, dx &= \frac{3\left(-x^2+x^3\right)^{2/3}}{5x^3}+\frac{3}{5}\int \frac{1}{x\sqrt[3]{-x^2+x^3}} \, dx\\ &= \frac{3\left(-x^2+x^3\right)^{2/3}}{5x^3}+\frac{9\left(-x^2+x^3\right)^{2/3}}{10x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.92

$$\frac{3\left((x-1)x^2\right)^{2/3}(3x+2)}{10x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (3*((-1 + x)*x^2)^(2/3)*(2 + 3*x))/(10*x^3)
```

IntegrateAlgebraic [A] time = 0.13, size = 25, normalized size = 1.00

$$\frac{3(3x+2)\left(x^3-x^2\right)^{2/3}}{10x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(-x^2 + x^3)^(1/3)),x]
[Out] (3*(2 + 3*x)*(-x^2 + x^3)^(2/3))/(10*x^3)
```

fricas [A] time = 0.39, size = 21, normalized size = 0.84

$$\frac{3\left(x^3-x^2\right)^{\frac{2}{3}}(3x+2)}{10x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3-x^2)^(1/3),x, algorithm="fricas")
[Out] 3/10*(x^3 - x^2)^(2/3)*(3*x + 2)/x^3
```

giac [A] time = 0.47, size = 23, normalized size = 0.92

$$-\frac{3}{5}\left(-\frac{1}{x}+1\right)^{\frac{5}{3}}+\frac{3}{2}\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3-x^2)^(1/3),x, algorithm="giac")
[Out] -3/5*(-1/x + 1)^(5/3) + 3/2*(-1/x + 1)^(2/3)
```

maple [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{3(-1+x)(2+3x)}{10x\left(x^3-x^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^3-x^2)^(1/3),x)
[Out] 3/10*(-1+x)*(2+3*x)/x/(x^3-x^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3-x^2\right)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3-x^2)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^3 - x^2)^(1/3)*x^2), x)
```

mupad [B] time = 0.16, size = 33, normalized size = 1.32

$$\frac{9x\left(x^3-x^2\right)^{2/3}+6\left(x^3-x^2\right)^{2/3}}{10x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^3 - x^2)^(1/3)),x)
```

```
[Out] (9*x*(x^3 - x^2)^(2/3) + 6*(x^3 - x^2)^(2/3))/(10*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt[3]{x^2 (x-1)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**3-x**2)**(1/3),x)
```

```
[Out] Integral(1/(x**2*(x**2*(x - 1))**(1/3)), x)
```

3.259 $\int x^7 \left(1+x^4\right)^{2/3} dx$

Optimal. Leaf size=25

$$\frac{3}{160}\left(x^4+1\right)^{2/3}\left(5x^8+2x^4-3\right)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{32}\left(x^4+1\right)^{8/3}-\frac{3}{20}\left(x^4+1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(1 + x^4)^(2/3),x]
[Out] (-3*(1 + x^4)^(5/3))/20 + (3*(1 + x^4)^(8/3))/32
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\left(1+x^4\right)^{2/3} dx &= \frac{1}{4}\text{Subst}\left(\int x(1+x)^{2/3}\,dx,x,x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(-(1+x)^{2/3}+(1+x)^{5/3}\right)dx,x,x^4\right)\\ &= -\frac{3}{20}\left(1+x^4\right)^{5/3}+\frac{3}{32}\left(1+x^4\right)^{8/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{3}{160}\left(x^4+1\right)^{5/3}\left(5x^4-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(1 + x^4)^(2/3),x]
[Out] (3*(1 + x^4)^(5/3)*(-3 + 5*x^4))/160
```

IntegrateAlgebraic [A] time = 0.03, size = 20, normalized size = 0.80

$$\frac{3}{160}\left(x^4+1\right)^{5/3}\left(5x^4-3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(1 + x^4)^(2/3),x]
[Out] (3*(1 + x^4)^(5/3)*(-3 + 5*x^4))/160
```

fricas [A] time = 0.38, size = 21, normalized size = 0.84

$$\frac{3}{160}\left(5\,x^{8}+2\,x^{4}-3\right)\left(x^{4}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^4+1)^(2/3),x, algorithm="fricas")
[Out] 3/160*(5*x^8 + 2*x^4 - 3)*(x^4 + 1)^(2/3)
```

giac [A] time = 0.45, size = 19, normalized size = 0.76

$$\frac{3}{32}\left(x^{4}+1\right)^{\frac{8}{3}}-\frac{3}{20}\left(x^{4}+1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^4+1)^(2/3),x, algorithm="giac")
[Out] 3/32*(x^4 + 1)^(8/3) - 3/20*(x^4 + 1)^(5/3)
```

maple [A] time = 0.00, size = 17, normalized size = 0.68

$$\frac{3\left(x^{4}+1\right)^{\frac{5}{3}}\left(5x^{4}-3\right)}{160}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(x^4+1)^(2/3),x)
[Out] 3/160*(x^4+1)^(5/3)*(5*x^4-3)
```

maxima [A] time = 0.44, size = 19, normalized size = 0.76

$$\frac{3}{32}\left(x^{4}+1\right)^{\frac{8}{3}}-\frac{3}{20}\left(x^{4}+1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^4+1)^(2/3),x, algorithm="maxima")
[Out] 3/32*(x^4 + 1)^(8/3) - 3/20*(x^4 + 1)^(5/3)
```

mupad [B] time = 0.22, size = 20, normalized size = 0.80

$$\left(x^{4}+1\right)^{2/3}\left(\frac{3\,x^{8}}{32}+\frac{3\,x^{4}}{80}-\frac{9}{160}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(x^4 + 1)^(2/3),x)
[Out] (x^4 + 1)^(2/3)*((3*x^4)/80 + (3*x^8)/32 - 9/160)
```

sympy [A] time = 1.00, size = 41, normalized size = 1.64

$$\frac{3x^{8}\left(x^{4}+1\right)^{\frac{2}{3}}}{32}+\frac{3x^{4}\left(x^{4}+1\right)^{\frac{2}{3}}}{80}-\frac{9\left(x^{4}+1\right)^{\frac{2}{3}}}{160}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(x**4+1)**(2/3),x)
```

```
[Out] 3*x**8*(x**4 + 1)**(2/3)/32 + 3*x**4*(x**4 + 1)**(2/3)/80 - 9*(x**4 + 1)**(
2/3)/160
```

3.260 $\int \frac{1}{x^5\sqrt{-x+x^4}}\,dx$

Optimal. Leaf size=25

$$\frac{2\left(2x^3+1\right)\sqrt{x^4-x}}{9x^5}$$

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.48, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2016, 2014}

$$\frac{2\sqrt{x^4-x}}{9x^5}+\frac{4\sqrt{x^4-x}}{9x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[-x + x^4]),x]
```

```
[Out] (2*Sqrt[-x + x^4])/(9*x^5) + (4*Sqrt[-x + x^4])/(9*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{-x+x^4}}\,dx &= \frac{2\sqrt{-x+x^4}}{9x^5}+\frac{2}{3}\int \frac{1}{x^2\sqrt{-x+x^4}}\,dx\\ &= \frac{2\sqrt{-x+x^4}}{9x^5}+\frac{4\sqrt{-x+x^4}}{9x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2\sqrt{x\left(x^3-1\right)}\left(2x^3+1\right)}{9x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[-x + x^4]),x]
```

```
[Out] (2*Sqrt[x*(-1 + x^3)]*(1 + 2*x^3))/(9*x^5)
```

IntegrateAlgebraic [A] time = 0.41, size = 25, normalized size = 1.00

$$\frac{2\left(2x^3+1\right)\sqrt{x^4-x}}{9x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*Sqrt[-x + x^4]),x]
```

```
[Out] (2*(1 + 2*x^3)*Sqrt[-x + x^4])/(9*x^5)
```

fricas [A] time = 0.40, size = 21, normalized size = 0.84

$$\frac{2\sqrt{x^4 - x}\left(2x^3 + 1\right)}{9x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*sqrt(x^4 - x)*(2*x^3 + 1)/x^5
```

giac [A] time = 0.40, size = 23, normalized size = 0.92

$$-\frac{2}{9}\left(-\frac{1}{x^3} + 1\right)^{\frac{3}{2}} + \frac{2}{3}\sqrt{-\frac{1}{x^3} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4-x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/9*(-1/x^3 + 1)^(3/2) + 2/3*sqrt(-1/x^3 + 1)
```

maple [A] time = 0.00, size = 31, normalized size = 1.24

$$\frac{2(-1 + x)\left(x^2 + x + 1\right)\left(2x^3 + 1\right)}{9x^4\sqrt{x^4 - x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^4-x)^(1/2),x)
```

```
[Out] 2/9*(-1+x)*(x^2+x+1)*(2*x^3+1)/x^4/(x^4-x)^(1/2)
```

maxima [A] time = 0.80, size = 32, normalized size = 1.28

$$\frac{2\left(2x^7 - x^4 - x\right)}{9\sqrt{x^2 + x + 1}\sqrt{x - 1}\,x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4-x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(2*x^7 - x^4 - x)/(sqrt(x^2 + x + 1)*sqrt(x - 1)*x^(11/2))
```

mupad [B] time = 0.19, size = 21, normalized size = 0.84

$$\frac{2\sqrt{x^4 - x}\,\left(2x^3 + 1\right)}{9x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^4 - x)^(1/2)),x)
```

```
[Out] (2*(x^4 - x)^(1/2)*(2*x^3 + 1))/(9*x^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{x\left(x-1\right)\left(x^2+x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**4-x)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(x*(x - 1)*(x**2 + x + 1))), x)
```

3.261 $\int \frac{1+x^3}{x^6 \sqrt[4]{-x+x^4}} \, dx$

Optimal. Leaf size=25

$$\frac{4\left(11x^3+3\right)\left(x^4-x\right)^{3/4}}{63x^6}$$

Rubi [A] time = 0.08, antiderivative size = 37, normalized size of antiderivative = 1.48, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2038, 2014}

$$\frac{4\left(x^4-x\right)^{3/4}}{21x^6}+\frac{44\left(x^4-x\right)^{3/4}}{63x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(x^6*(-x + x^4)^(1/4)),x]
```

```
[Out] (4*(-x + x^4)^(3/4))/(21*x^6) + (44*(-x + x^4)^(3/4))/(63*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^3}{x^6 \sqrt[4]{-x+x^4}} \, dx &= \frac{4\left(-x+x^4\right)^{3/4}}{21x^6}+\frac{11}{7}\int \frac{1}{x^3 \sqrt[4]{-x+x^4}} \, dx\\ &= \frac{4\left(-x+x^4\right)^{3/4}}{21x^6}+\frac{44\left(-x+x^4\right)^{3/4}}{63x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{4\left(x\left(x^3-1\right)\right)^{3/4}\left(11x^3+3\right)}{63x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(x^6*(-x + x^4)^(1/4)),x]
```

```
[Out] (4*(x*(-1 + x^3))^(3/4)*(3 + 11*x^3))/(63*x^6)
```

IntegrateAlgebraic [A] time = 0.30, size = 25, normalized size = 1.00

$$\frac{4\left(11x^3+3\right)\left(x^4-x\right)^{3/4}}{63x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)/(x^6*(-x + x^4)^(1/4)),x]
[Out] (4*(3 + 11*x^3)*(-x + x^4)^(3/4))/(63*x^6)
```

fricas [A] time = 0.40, size = 21, normalized size = 0.84

$$\frac{4\left(x^4-x\right)^{\frac{3}{4}}\left(11\,x^3+3\right)}{63\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x^6/(x^4-x)^(1/4),x, algorithm="fricas")
[Out] 4/63*(x^4 - x)^(3/4)*(11*x^3 + 3)/x^6
```

giac [A] time = 0.55, size = 23, normalized size = 0.92

$$\frac{4}{21}\left(-\frac{1}{x^3}+1\right)^{\frac{7}{4}}-\frac{8}{9}\left(-\frac{1}{x^3}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x^6/(x^4-x)^(1/4),x, algorithm="giac")
[Out] 4/21*(-1/x^3 + 1)^(7/4) - 8/9*(-1/x^3 + 1)^(3/4)
```

maple [A] time = 0.01, size = 31, normalized size = 1.24

$$\frac{4\left(11x^3+3\right)\left(-1+x\right)\left(x^2+x+1\right)}{63\left(x^4-x\right)^{\frac{1}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/x^6/(x^4-x)^(1/4),x)
[Out] 4/63*(11*x^3+3)*(-1+x)*(x^2+x+1)/(x^4-x)^(1/4)/x^5
```

maxima [B] time = 0.83, size = 58, normalized size = 2.32

$$\frac{4\left(x^4-x\right)}{9\left(x^2+x+1\right)^{\frac{1}{4}}(x-1)^{\frac{1}{4}}x^{\frac{13}{4}}}+\frac{4\left(4\,x^7-x^4-3\,x\right)}{63\left(x^2+x+1\right)^{\frac{1}{4}}(x-1)^{\frac{1}{4}}x^{\frac{25}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x^6/(x^4-x)^(1/4),x, algorithm="maxima")
[Out] 4/9*(x^4 - x)/((x^2 + x + 1)^(1/4)*(x - 1)^(1/4)*x^(13/4)) + 4/63*(4*x^7 -
x^4 - 3*x)/((x^2 + x + 1)^(1/4)*(x - 1)^(1/4)*x^(25/4))
```

mupad [B] time = 0.21, size = 31, normalized size = 1.24

$$\frac{12\left(x^4-x\right)^{3/4}+44\,x^3\left(x^4-x\right)^{3/4}}{63\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/(x^6*(x^4 - x)^(1/4)),x)
```

```
[Out] (12*(x^4 - x)^(3/4) + 44*x^3*(x^4 - x)^(3/4))/(63*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)\left(x^2-x+1\right)}{x^6\sqrt[4]{x(x-1)\left(x^2+x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/x**6/(x**4-x)**(1/4),x)
```

```
[Out] Integral((x + 1)*(x**2 - x + 1)/(x**6*(x*(x - 1)*(x**2 + x + 1))**(1/4)), x
)
```

3.262 $\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt[3]{-x^2+x^4}}\,dx$

Optimal. Leaf size=25

$$-\frac{3\left(x^4-x^2\right)^{2/3}}{x\left(x^2-1\right)}$$

Rubi [A] time = 0.08, antiderivative size = 16, normalized size of antiderivative = 0.64, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2056, 449}

$$-\frac{3x}{\sqrt[3]{x^4-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/((-1 + x^2)*(-x^2 + x^4)^(1/3)),x]
```

```
[Out] (-3*x)/(-x^2 + x^4)^(1/3)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt[3]{-x^2+x^4}}\,dx &= \frac{\left(x^{2/3}\sqrt[3]{-1+x^2}\right)\int \frac{1+x^2}{x^{2/3}\left(-1+x^2\right)^{4/3}}\,dx}{\sqrt[3]{-x^2+x^4}}\\ &= -\frac{3x}{\sqrt[3]{-x^2+x^4}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 0.64

$$-\frac{3x}{\sqrt[3]{x^2\left(x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/((-1 + x^2)*(-x^2 + x^4)^(1/3)),x]
```

```
[Out] (-3*x)/(x^2*(-1 + x^2))^(1/3)
```

IntegrateAlgebraic [A] time = 0.10, size = 25, normalized size = 1.00

$$-\frac{3\left(x^4-x^2\right)^{2/3}}{x\left(x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 + x^2)*(-x^2 + x^4)^(1/3)),x]
[Out] (-3*(-x^2 + x^4)^(2/3))/(x*(-1 + x^2))
```

fricas [A] time = 0.40, size = 22, normalized size = 0.88

$$-\frac{3\left(x^4-x^2\right)^{\frac{2}{3}}}{x^3-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^4-x^2)^(1/3),x, algorithm="fricas")
[Out] -3*(x^4 - x^2)^(2/3)/(x^3 - x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^4-x^2\right)^{\frac{1}{3}}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^4-x^2)^(1/3),x, algorithm="giac")
[Out] integrate((x^2 + 1)/((x^4 - x^2)^(1/3)*(x^2 - 1)), x)
```

maple [A] time = 0.01, size = 15, normalized size = 0.60

$$-\frac{3x}{\left(x^4-x^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-1)/(x^4-x^2)^(1/3),x)
[Out] -3*x/(x^4-x^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^4-x^2\right)^{\frac{1}{3}}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^4-x^2)^(1/3),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/((x^4 - x^2)^(1/3)*(x^2 - 1)), x)
```

mupad [B] time = 0.19, size = 23, normalized size = 0.92

$$-\frac{3\left(x^4-x^2\right)^{2/3}}{x\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((x^2 - 1)*(x^4 - x^2)^(1/3)),x)
[Out] -(3*(x^4 - x^2)^(2/3))/(x*(x^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt[3]{x^2\,(x-1)\,(x+1)}\;(x-1)\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-1)/(x**4-x**2)**(1/3),x)
```

```
[Out] Integral((x**2 + 1)/((x**2*(x - 1)*(x + 1))**(1/3)*(x - 1)*(x + 1)), x)
```

3.263 $\int \frac{1}{\left(-1+x^2\right)\sqrt[4]{-x^2+x^4}}\,dx$

Optimal. Leaf size=25

$$-\frac{2\left(x^4-x^2\right)^{3/4}}{x\left(x^2-1\right)}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 0.64, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1146, 264}

$$-\frac{2x}{\sqrt[4]{x^4-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x^2)*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] (-2*x)/(-x^2 + x^4)^(1/4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1146

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-1+x^2\right)\sqrt[4]{-x^2+x^4}}\,dx &= \frac{\left(\sqrt{x}\sqrt[4]{-1+x^2}\right)\int \frac{1}{\sqrt{x}\left(-1+x^2\right)^{5/4}}\,dx}{\sqrt[4]{-x^2+x^4}}\\ &= -\frac{2x}{\sqrt[4]{-x^2+x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.64

$$-\frac{2x}{\sqrt[4]{x^2\left(x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x^2)*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] (-2*x)/(x^2*(-1 + x^2))^(1/4)
```

IntegrateAlgebraic [A] time = 0.12, size = 25, normalized size = 1.00

$$-\frac{2\left(x^4-x^2\right)^{3/4}}{x\left(x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x^2)*(-x^2 + x^4)^(1/4)),x]
[Out] (-2*(-x^2 + x^4)^(3/4))/(x*(-1 + x^2))
```

fricas [A] time = 0.40, size = 22, normalized size = 0.88

$$-\frac{2\left(x^4-x^2\right)^{\frac{3}{4}}}{x^3-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x^4-x^2)^(1/4),x, algorithm="fricas")
[Out] -2*(x^4 - x^2)^(3/4)/(x^3 - x)
```

giac [A] time = 0.33, size = 11, normalized size = 0.44

$$\frac{2}{\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x^4-x^2)^(1/4),x, algorithm="giac")
[Out] 2/(-1/x^2 + 1)^(1/4)
```

maple [A] time = 0.00, size = 15, normalized size = 0.60

$$-\frac{2x}{\left(x^4-x^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)/(x^4-x^2)^(1/4),x)
[Out] -2*x/(x^4-x^2)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4-x^2\right)^{\frac{1}{4}}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x^4-x^2)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((x^4 - x^2)^(1/4)*(x^2 - 1)), x)
```

mupad [B] time = 0.16, size = 23, normalized size = 0.92

$$-\frac{2\left(x^4-x^2\right)^{3/4}}{x\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)*(x^4 - x^2)^(1/4)),x)
[Out] -(2*(x^4 - x^2)^(3/4))/(x*(x^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^2(x-1)(x+1)}\,(x-1)(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)/(x**4-x**2)**(1/4),x)
```

```
[Out] Integral(1/((x**2*(x - 1)*(x + 1))**(1/4)*(x - 1)*(x + 1)), x)
```

3.264 $\int \frac{1}{x^4 \sqrt[4]{x^2+x^4}} \, dx$

Optimal. Leaf size=25

$$\frac{2 \left(4x^2-3\right) \left(x^4+x^2\right)^{3/4}}{21x^5}$$

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.48, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2016, 2014}

$$\frac{8 \left(x^4+x^2\right)^{3/4}}{21x^3}-\frac{2 \left(x^4+x^2\right)^{3/4}}{7x^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(x^2 + x^4)^(1/4)),x]
```

```
[Out] (-2*(x^2 + x^4)^(3/4))/(7*x^5) + (8*(x^2 + x^4)^(3/4))/(21*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^4 \sqrt[4]{x^2+x^4}} \, dx &= -\frac{2 \left(x^2+x^4\right)^{3/4}}{7x^5}-\frac{4}{7} \int \frac{1}{x^2 \sqrt[4]{x^2+x^4}} \, dx \\ &= -\frac{2 \left(x^2+x^4\right)^{3/4}}{7x^5}+\frac{8 \left(x^2+x^4\right)^{3/4}}{21x^3} \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2 \left(4x^2-3\right) \left(x^4+x^2\right)^{3/4}}{21x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(x^2 + x^4)^(1/4)),x]
```

```
[Out] (2*(-3 + 4*x^2)*(x^2 + x^4)^(3/4))/(21*x^5)
```

IntegrateAlgebraic [A] time = 0.10, size = 25, normalized size = 1.00

$$\frac{2 \left(4x^2-3\right) \left(x^4+x^2\right)^{3/4}}{21x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(x^2 + x^4)^(1/4)),x]
```

```
[Out] (2*(-3 + 4*x^2)*(x^2 + x^4)^(3/4))/(21*x^5)
```

fricas [A] time = 0.38, size = 21, normalized size = 0.84

$$\frac{2\left(x^4+x^2\right)^{\frac{3}{4}}\left(4\,x^2-3\right)}{21\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/21*(x^4 + x^2)^(3/4)*(4*x^2 - 3)/x^5
```

giac [A] time = 0.30, size = 19, normalized size = 0.76

$$-\frac{2}{7}\left(\frac{1}{x^2}+1\right)^{\frac{7}{4}}+\frac{2}{3}\left(\frac{1}{x^2}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+x^2)^(1/4),x, algorithm="giac")
```

```
[Out] -2/7*(1/x^2 + 1)^(7/4) + 2/3*(1/x^2 + 1)^(3/4)
```

maple [A] time = 0.00, size = 27, normalized size = 1.08

$$\frac{2\left(x^2+1\right)\left(4x^2-3\right)}{21x^3\left(x^4+x^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^4+x^2)^(1/4),x)
```

```
[Out] 2/21*(x^2+1)*(4*x^2-3)/x^3/(x^4+x^2)^(1/4)
```

maxima [A] time = 0.44, size = 24, normalized size = 0.96

$$\frac{2\left(4\,x^5+x^3-3\,x\right)}{21\left(x^2+1\right)^{\frac{1}{4}}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] 2/21*(4*x^5 + x^3 - 3*x)/((x^2 + 1)^(1/4)*x^(9/2))
```

mupad [B] time = 0.18, size = 31, normalized size = 1.24

$$-\frac{6\left(x^4+x^2\right)^{3/4}-8\,x^2\left(x^4+x^2\right)^{3/4}}{21\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^2 + x^4)^(1/4)),x)
```

```
[Out] -(6*(x^2 + x^4)^(3/4) - 8*x^2*(x^2 + x^4)^(3/4))/(21*x^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt[4]{x^2 (x^2 + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**4+x**2)**(1/4),x)
```

```
[Out] Integral(1/(x**4*(x**2*(x**2 + 1))**(1/4)), x)
```

3.265 $\int \frac{-1+2x}{\sqrt{-3+x^2-2x^3+x^4}} \, dx$

Optimal. Leaf size=25

$$\log \left(x^2+\sqrt{x^4-2x^3+x^2-3}-x\right)$$

Rubi [A] time = 0.05, antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1680, 12, 1107, 621, 206}

$$-\tanh^{-1}\left(\frac{(1-x)x}{\sqrt{x^4-2x^3+x^2-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x)/Sqrt[-3 + x^2 - 2*x^3 + x^4],x]
```

```
[Out] -ArcTanh[((1 - x)*x)/Sqrt[-3 + x^2 - 2*x^3 + x^4]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{-1+2x}{\sqrt{-3+x^2-2x^3+x^4}}\,dx = \text{Subst}\left(\int \frac{8x}{\sqrt{-47-8x^2+16x^4}}\,dx, x, -\frac{1}{2}+x\right)$$
$$= 8\,\text{Subst}\left(\int \frac{x}{\sqrt{-47-8x^2+16x^4}}\,dx, x, -\frac{1}{2}+x\right)$$
$$= 4\,\text{Subst}\left(\int \frac{1}{\sqrt{-47-8x+16x^2}}\,dx, x, \left(-\frac{1}{2}+x\right)^2\right)$$
$$= 8\,\text{Subst}\left(\int \frac{1}{64-x^2}\,dx, x, \frac{8(-1+x)x}{\sqrt{-3+x^2-2x^3+x^4}}\right)$$
$$= -\tanh^{-1}\left(\frac{(1-x)x}{\sqrt{-3+x^2-2x^3+x^4}}\right)$$

Mathematica [C] time = 2.37, size = 728, normalized size = 29.12

$$\frac{\sqrt{4\sqrt{3}-1}\left(-2x+i\sqrt{4\sqrt{3}-1}+1\right)\left(-2x+\sqrt{1+4\sqrt{3}}+1\right)\sqrt{\frac{\left(\sqrt{4\sqrt{3}-1}+i\sqrt{1+4\sqrt{3}}\right)\left(-2ix+\sqrt{4\sqrt{3}-1}+i\right)}{\left(\sqrt{4\sqrt{3}-1}-i\sqrt{1+4\sqrt{3}}\right)\left(2ix+\sqrt{4\sqrt{3}-1}-i\right)}}\sqrt{\frac{2x+\sqrt{1+4\sqrt{3}}-1}{\left(\sqrt{1+4\sqrt{3}}-i\sqrt{4\sqrt{3}-1}\right)\left(2ix+\sqrt{4\sqrt{3}-1}-i\right)}}\left(F\left(\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{-1+4\sqrt{3}}+i\sqrt{1+4\sqrt{3}}\right)\left(-2ix+\sqrt{-1+4\sqrt{3}}+i\right)}{\left(\sqrt{-1+4\sqrt{3}}-i\sqrt{1+4\sqrt{3}}\right)\left(2ix+\sqrt{-1+4\sqrt{3}}-i\right)}}\right)\middle|\frac{i-\sqrt{47}}{i+\sqrt{47}}\right)-2\Pi\left(-\frac{\sqrt{-1+4\sqrt{3}}-i\sqrt{1+4\sqrt{3}}}{\sqrt{-1+4\sqrt{3}}+i\sqrt{1+4\sqrt{3}}};\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{-1+4\sqrt{3}}+i\sqrt{1+4\sqrt{3}}\right)\left(-2ix+\sqrt{-1+4\sqrt{3}}+i\right)}{\left(\sqrt{-1+4\sqrt{3}}-i\sqrt{1+4\sqrt{3}}\right)\left(2ix+\sqrt{-1+4\sqrt{3}}-i\right)}}\right)\middle|\frac{i-\sqrt{47}}{i+\sqrt{47}}\right)\right)}{\left(\sqrt{4\sqrt{3}-1}+i\sqrt{1+4\sqrt{3}}\right)\sqrt{\frac{-2x+\sqrt{1+4\sqrt{3}}+1}{\left(\sqrt{1+4\sqrt{3}}+i\sqrt{4\sqrt{3}-1}\right)\left(2ix+\sqrt{4\sqrt{3}-1}-i\right)}}\sqrt{x^4-2x^3+x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 2*x)/Sqrt[-3 + x^2 - 2*x^3 + x^4],x]
```

```
[Out] (Sqrt[-1 + 4*Sqrt[3]]*(1 + I*Sqrt[-1 + 4*Sqrt[3]] - 2*x)*(1 + Sqrt[1 + 4*Sq
rt[3]] - 2*x)*Sqrt[((Sqrt[-1 + 4*Sqrt[3]] + I*Sqrt[1 + 4*Sqrt[3]])*(I + Sqr
t[-1 + 4*Sqrt[3]] - (2*I)*x))/((Sqrt[-1 + 4*Sqrt[3]] - I*Sqrt[1 + 4*Sqrt[3]
])*(-I + Sqrt[-1 + 4*Sqrt[3]] + (2*I)*x))]*Sqrt[(-1 + Sqrt[1 + 4*Sqrt[3]] +
 2*x)/(((-I)*Sqrt[-1 + 4*Sqrt[3]] + Sqrt[1 + 4*Sqrt[3]])*(-I + Sqrt[-1 + 4*
Sqrt[3]] + (2*I)*x))]*(EllipticF[ArcSin[Sqrt[((Sqrt[-1 + 4*Sqrt[3]] + I*Sqr
t[1 + 4*Sqrt[3]])*(I + Sqrt[-1 + 4*Sqrt[3]] - (2*I)*x))/((Sqrt[-1 + 4*Sqrt[
3]] - I*Sqrt[1 + 4*Sqrt[3]])*(-I + Sqrt[-1 + 4*Sqrt[3]] + (2*I)*x))]], (I -
 Sqrt[47])/(I + Sqrt[47])] - 2*EllipticPi[-((Sqrt[-1 + 4*Sqrt[3]] - I*Sqrt[
1 + 4*Sqrt[3]])/(Sqrt[-1 + 4*Sqrt[3]] + I*Sqrt[1 + 4*Sqrt[3]])), ArcSin[Sqr
t[((Sqrt[-1 + 4*Sqrt[3]] + I*Sqrt[1 + 4*Sqrt[3]])*(I + Sqrt[-1 + 4*Sqrt[3]]
 - (2*I)*x))/((Sqrt[-1 + 4*Sqrt[3]] - I*Sqrt[1 + 4*Sqrt[3]])*(-I + Sqrt[-1
+ 4*Sqrt[3]] + (2*I)*x))]], (I - Sqrt[47])/(I + Sqrt[47])]))/((Sqrt[-1 + 4*
Sqrt[3]] + I*Sqrt[1 + 4*Sqrt[3]])*Sqrt[(1 + Sqrt[1 + 4*Sqrt[3]] - 2*x)/((I*
Sqrt[-1 + 4*Sqrt[3]] + Sqrt[1 + 4*Sqrt[3]])*(-I + Sqrt[-1 + 4*Sqrt[3]] + (2
*I)*x))]*Sqrt[-3 + x^2 - 2*x^3 + x^4])
```

IntegrateAlgebraic [A] time = 0.10, size = 25, normalized size = 1.00

$$\log\left(x^2+\sqrt{x^4-2x^3+x^2-3}-x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x)/Sqrt[-3 + x^2 - 2*x^3 + x^4],x]
```

```
[Out] Log[-x + x^2 + Sqrt[-3 + x^2 - 2*x^3 + x^4]]
```

fricas [A] time = 0.43, size = 23, normalized size = 0.92

$$\log\left(x^2-x+\sqrt{x^4-2\,x^3+x^2-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] log(x^2 - x + sqrt(x^4 - 2*x^3 + x^2 - 3))
```

giac [A] time = 0.41, size = 24, normalized size = 0.96

$$-\log\left(\left|-x^2 + x + \sqrt{\left(x^2 - x\right)^2 - 3}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-x^2 + x + sqrt((x^2 - x)^2 - 3)))
```

maple [C] time = 0.61, size = 1358, normalized size = 54.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x)/(x^4-2*x^3+x^2-3)^(1/2),x)
```

```
[Out] -2*(-1/2*(1+4*3^(1/2))^(1/2)-1/2*I*(-1+4*3^(1/2))^(1/2))*((1/2*I*(-1+4*3^(1
/2))^(1/2)-1/2*(1+4*3^(1/2))^(1/2))*(x-1/2+1/2*(1+4*3^(1/2))^(1/2))/(1/2*I*
(-1+4*3^(1/2))^(1/2)+1/2*(1+4*3^(1/2))^(1/2))/(x-1/2-1/2*(1+4*3^(1/2))^(1/2
)))^(1/2)*(x-1/2-1/2*(1+4*3^(1/2))^(1/2))^2*((1+4*3^(1/2))^(1/2)*(x-1/2+1/2
*I*(-1+4*3^(1/2))^(1/2))/(-1/2*I*(-1+4*3^(1/2))^(1/2)+1/2*(1+4*3^(1/2))^(1/
2))/(x-1/2-1/2*(1+4*3^(1/2))^(1/2)))^(1/2)*((1+4*3^(1/2))^(1/2)*(x-1/2-1/2*
I*(-1+4*3^(1/2))^(1/2))/(1/2*I*(-1+4*3^(1/2))^(1/2)+1/2*(1+4*3^(1/2))^(1/2)
)/(x-1/2-1/2*(1+4*3^(1/2))^(1/2)))^(1/2)/(1/2*I*(-1+4*3^(1/2))^(1/2)-1/2*(1
+4*3^(1/2))^(1/2))/(1+4*3^(1/2))^(1/2)/((x-1/2+1/2*(1+4*3^(1/2))^(1/2))*(x-
1/2-1/2*(1+4*3^(1/2))^(1/2))*(x-1/2+1/2*I*(-1+4*3^(1/2))^(1/2))*(x-1/2-1/2*
I*(-1+4*3^(1/2))^(1/2)))^(1/2)*EllipticF(((1/2*I*(-1+4*3^(1/2))^(1/2)-1/2*(
1+4*3^(1/2))^(1/2))*(x-1/2+1/2*(1+4*3^(1/2))^(1/2))/(1/2*I*(-1+4*3^(1/2))^(
1/2)+1/2*(1+4*3^(1/2))^(1/2))/(x-1/2-1/2*(1+4*3^(1/2))^(1/2)))^(1/2),((1/2*
I*(-1+4*3^(1/2))^(1/2)+1/2*(1+4*3^(1/2))^(1/2))*(-1/2*(1+4*3^(1/2))^(1/2)-1
/2*I*(-1+4*3^(1/2))^(1/2))/(1/2*I*(-1+4*3^(1/2))^(1/2)-1/2*(1+4*3^(1/2))^(1
/2))/(-1/2*I*(-1+4*3^(1/2))^(1/2)+1/2*(1+4*3^(1/2))^(1/2)))^(1/2))+4*(-1/2*
(1+4*3^(1/2))^(1/2)-1/2*I*(-1+4*3^(1/2))^(1/2))*((1/2*I*(-1+4*3^(1/2))^(1/2
)-1/2*(1+4*3^(1/2))^(1/2))*(x-1/2+1/2*(1+4*3^(1/2))^(1/2))/(1/2*I*(-1+4*3^(
1/2))^(1/2)+1/2*(1+4*3^(1/2))^(1/2))/(x-1/2-1/2*(1+4*3^(1/2))^(1/2)))^(1/2)
*(x-1/2-1/2*(1+4*3^(1/2))^(1/2))^2*((1+4*3^(1/2))^(1/2)*(x-1/2+1/2*I*(-1+4*
3^(1/2))^(1/2))/(-1/2*I*(-1+4*3^(1/2))^(1/2)+1/2*(1+4*3^(1/2))^(1/2))/(x-1/
2-1/2*(1+4*3^(1/2))^(1/2)))^(1/2)*((1+4*3^(1/2))^(1/2)*(x-1/2-1/2*I*(-1+4*3
^(1/2))^(1/2))/(1/2*I*(-1+4*3^(1/2))^(1/2)+1/2*(1+4*3^(1/2))^(1/2))/(x-1/2-
1/2*(1+4*3^(1/2))^(1/2)))^(1/2)/(1/2*I*(-1+4*3^(1/2))^(1/2)-1/2*(1+4*3^(1/2
))^(1/2))/(1+4*3^(1/2))^(1/2)/((x-1/2+1/2*(1+4*3^(1/2))^(1/2))*(x-1/2-1/2*(
1+4*3^(1/2))^(1/2))*(x-1/2+1/2*I*(-1+4*3^(1/2))^(1/2))*(x-1/2-1/2*I*(-1+4*3
^(1/2))^(1/2)))^(1/2)*((1/2+1/2*(1+4*3^(1/2))^(1/2))*EllipticF(((1/2*I*(-1+
4*3^(1/2))^(1/2)-1/2*(1+4*3^(1/2))^(1/2))*(x-1/2+1/2*(1+4*3^(1/2))^(1/2))/(
1/2*I*(-1+4*3^(1/2))^(1/2)+1/2*(1+4*3^(1/2))^(1/2))/(x-1/2-1/2*(1+4*3^(1/2)
)^(1/2)))^(1/2),((1/2*I*(-1+4*3^(1/2))^(1/2)+1/2*(1+4*3^(1/2))^(1/2))*(-1/2
*(1+4*3^(1/2))^(1/2)-1/2*I*(-1+4*3^(1/2))^(1/2))/(1/2*I*(-1+4*3^(1/2))^(1/2
)-1/2*(1+4*3^(1/2))^(1/2))/(-1/2*I*(-1+4*3^(1/2))^(1/2)+1/2*(1+4*3^(1/2))^(
1/2)))^(1/2))-(1+4*3^(1/2))^(1/2)*EllipticPi(((1/2*I*(-1+4*3^(1/2))^(1/2)-1
/2*(1+4*3^(1/2))^(1/2))*(x-1/2+1/2*(1+4*3^(1/2))^(1/2))/(1/2*I*(-1+4*3^(1/2
))^(1/2)+1/2*(1+4*3^(1/2))^(1/2))/(x-1/2-1/2*(1+4*3^(1/2))^(1/2)))^(1/2),(1
/2*I*(-1+4*3^(1/2))^(1/2)+1/2*(1+4*3^(1/2))^(1/2))/(1/2*I*(-1+4*3^(1/2))^(1
/2)-1/2*(1+4*3^(1/2))^(1/2)),((1/2*I*(-1+4*3^(1/2))^(1/2)+1/2*(1+4*3^(1/2))
^(1/2))*(-1/2*(1+4*3^(1/2))^(1/2)-1/2*I*(-1+4*3^(1/2))^(1/2))/(1/2*I*(-1+4*
3^(1/2))^(1/2)-1/2*(1+4*3^(1/2))^(1/2))/(-1/2*I*(-1+4*3^(1/2))^(1/2)+1/2*(1
+4*3^(1/2))^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x-1}{\sqrt{x^4-2\,x^3+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+x^2-3)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x - 1)/sqrt(x^4 - 2*x^3 + x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{2\,x-1}{\sqrt{x^4-2\,x^3+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 1)/(x^2 - 2*x^3 + x^4 - 3)^(1/2),x)
[Out] int((2*x - 1)/(x^2 - 2*x^3 + x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-1}{\sqrt{x^4-2x^3+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x**4-2*x**3+x**2-3)**(1/2),x)
[Out] Integral((2*x - 1)/sqrt(x**4 - 2*x**3 + x**2 - 3), x)
```

3.266 $\int \frac{-1+2x}{\sqrt{4+x^2-2x^3+x^4}}\,dx$

Optimal. Leaf size=25

$$\log\left(x^2+\sqrt{x^4-2x^3+x^2+4}-x\right)$$

Rubi [A] time = 0.04, antiderivative size = 9, normalized size of antiderivative = 0.36, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1680, 12, 1107, 619, 215}

$$\sinh^{-1}\left(\frac{1}{2}(x-1)x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x)/Sqrt[4 + x^2 - 2*x^3 + x^4],x]
```

```
[Out] ArcSinh[((-1 + x)*x)/2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+2x}{\sqrt{4+x^2-2x^3+x^4}}\,dx &= \text{Subst}\left(\int \frac{8x}{\sqrt{65-8x^2+16x^4}}\,dx,x,-\frac{1}{2}+x\right)\\
&= 8\,\text{Subst}\left(\int \frac{x}{\sqrt{65-8x^2+16x^4}}\,dx,x,-\frac{1}{2}+x\right)\\
&= 4\,\text{Subst}\left(\int \frac{1}{\sqrt{65-8x+16x^2}}\,dx,x,\left(-\frac{1}{2}+x\right)^2\right)\\
&= \frac{1}{64}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4096}}}\,dx,x,32(-1+x)x\right)\\
&= -\sinh^{-1}\left(\frac{1}{2}(1-x)x\right)
\end{aligned}
$$

Mathematica [C]   time = 2.36, size = 613, normalized size = 24.52

$$
\frac{\left(-2x+\sqrt{1-8i}+1\right)\sqrt{\frac{\sqrt{1-8i}\left(-2x+\sqrt{1+8i}+1\right)}{\left(\sqrt{1-8i}+\sqrt{1+8i}\right)\left(-2x+\sqrt{1-8i}+1\right)}}\left(2x+\sqrt{1-8i}-1\right)\sqrt{\frac{\sqrt{1-8i}\left(2x+\sqrt{1+8i}-1\right)}{\left(\sqrt{1+8i}-\sqrt{1-8i}\right)\left(-2x+\sqrt{1-8i}+1\right)}}\left(\left(1+\sqrt{1-8i}\right)F\left(\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{1-8i}-\sqrt{1+8i}\right)\left(2x+\sqrt{1-8i}-1\right)}{\left(\sqrt{1-8i}+\sqrt{1+8i}\right)\left(-2x+\sqrt{1-8i}+1\right)}}\right)\middle|\frac{\left(\sqrt{1-8i}+\sqrt{1+8i}\right)^2}{\left(\sqrt{1-8i}-\sqrt{1+8i}\right)^2}\right)-F\left(\sin^{-1}\left(\sqrt{-\frac{2\sqrt{1+8i}x-2\sqrt{1-8i}x+\sqrt{65}-\sqrt{1+8i}+\sqrt{1-8i}-(1-8i)}{\left(\sqrt{1-8i}+\sqrt{1+8i}\right)\left(-2x+\sqrt{1-8i}+1\right)}}\right)\middle|\frac{\left(\sqrt{1-8i}+\sqrt{1+8i}\right)^2}{\left(\sqrt{1-8i}-\sqrt{1+8i}\right)^2}\right)-2\sqrt{1-8i}\,\Pi\left(-\frac{\sqrt{1-8i}+\sqrt{1+8i}}{\sqrt{1-8i}-\sqrt{1+8i}};\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{1-8i}-\sqrt{1+8i}\right)\left(2x+\sqrt{1-8i}-1\right)}{\left(\sqrt{1-8i}+\sqrt{1+8i}\right)\left(-2x+\sqrt{1-8i}+1\right)}}\right)\middle|\frac{\left(\sqrt{1-8i}+\sqrt{1+8i}\right)^2}{\left(\sqrt{1-8i}-\sqrt{1+8i}\right)^2}\right)\right)}{\sqrt{1-8i}\sqrt{\frac{\left(\sqrt{1-8i}-\sqrt{1+8i}\right)\left(2x+\sqrt{1-8i}-1\right)}{\left(\sqrt{1-8i}+\sqrt{1+8i}\right)\left(-2x+\sqrt{1-8i}+1\right)}}\sqrt{x^4-2x^3+x^2+4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 2*x)/Sqrt[4 + x^2 - 2*x^3 + x^4],x]
```

```
[Out] ((1 + Sqrt[1 - 8*I] - 2*x)*Sqrt[(Sqrt[1 - 8*I]*(1 + Sqrt[1 + 8*I] - 2*x))/(
(Sqrt[1 - 8*I] + Sqrt[1 + 8*I])*(1 + Sqrt[1 - 8*I] - 2*x))]*(-1 + Sqrt[1 -
8*I] + 2*x)*Sqrt[(Sqrt[1 - 8*I]*(-1 + Sqrt[1 + 8*I] + 2*x))/((-Sqrt[1 - 8*I
] + Sqrt[1 + 8*I])*(1 + Sqrt[1 - 8*I] - 2*x))]*((1 + Sqrt[1 - 8*I])*Ellipti
cF[ArcSin[Sqrt[((Sqrt[1 - 8*I] - Sqrt[1 + 8*I])*(-1 + Sqrt[1 - 8*I] + 2*x))
/((Sqrt[1 - 8*I] + Sqrt[1 + 8*I])*(1 + Sqrt[1 - 8*I] - 2*x))]], (Sqrt[1 - 8
*I] + Sqrt[1 + 8*I])^2/(Sqrt[1 - 8*I] - Sqrt[1 + 8*I])^2] - EllipticF[ArcSi
n[Sqrt[-(((-1 + 8*I) + Sqrt[1 - 8*I] - Sqrt[1 + 8*I] + Sqrt[65] - 2*Sqrt[1
- 8*I]*x + 2*Sqrt[1 + 8*I]*x)/((Sqrt[1 - 8*I] + Sqrt[1 + 8*I])*(1 + Sqrt[1
- 8*I] - 2*x)))]], (Sqrt[1 - 8*I] + Sqrt[1 + 8*I])^2/(Sqrt[1 - 8*I] - Sqrt[
1 + 8*I])^2] - 2*Sqrt[1 - 8*I]*EllipticPi[-((Sqrt[1 - 8*I] + Sqrt[1 + 8*I])
/(Sqrt[1 - 8*I] - Sqrt[1 + 8*I])), ArcSin[Sqrt[((Sqrt[1 - 8*I] - Sqrt[1 + 8
*I])*(-1 + Sqrt[1 - 8*I] + 2*x))/((Sqrt[1 - 8*I] + Sqrt[1 + 8*I])*(1 + Sqrt
[1 - 8*I] - 2*x))]], (Sqrt[1 - 8*I] + Sqrt[1 + 8*I])^2/(Sqrt[1 - 8*I] - Sqr
t[1 + 8*I])^2]))/(Sqrt[1 - 8*I]*Sqrt[((Sqrt[1 - 8*I] - Sqrt[1 + 8*I])*(-1 +
 Sqrt[1 - 8*I] + 2*x))/((Sqrt[1 - 8*I] + Sqrt[1 + 8*I])*(1 + Sqrt[1 - 8*I]
- 2*x))]*Sqrt[4 + x^2 - 2*x^3 + x^4])
```

IntegrateAlgebraic [A]   time = 0.15, size = 25, normalized size = 1.00

$$
\log\left(x^2+\sqrt{x^4-2x^3+x^2+4}-x\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x)/Sqrt[4 + x^2 - 2*x^3 + x^4],x]
```

```
[Out] Log[-x + x^2 + Sqrt[4 + x^2 - 2*x^3 + x^4]]
```

fricas [A]   time = 0.41, size = 23, normalized size = 0.92

$$
\log\left(x^2-x+\sqrt{x^4-2\,x^3+x^2+4}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] log(x^2 - x + sqrt(x^4 - 2*x^3 + x^2 + 4))
```

giac [A] time = 0.68, size = 23, normalized size = 0.92

$$-\log\left(-x^2 + x + \sqrt{\left(x^2 - x\right)^2 + 4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-x^2 + x + sqrt((x^2 - x)^2 + 4))
```

maple [C] time = 0.32, size = 882, normalized size = 35.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x)/(x^4-2*x^3+x^2+4)^(1/2),x)
```

```
[Out] -2*(-1/2*(1+8*I)^(1/2)-1/2*(1-8*I)^(1/2))*((1/2*(1-8*I)^(1/2)-1/2*(1+8*I)^(
1/2))*(x-1/2+1/2*(1+8*I)^(1/2))/(1/2*(1-8*I)^(1/2)+1/2*(1+8*I)^(1/2))/(x-1/
2-1/2*(1+8*I)^(1/2)))^(1/2)*(x-1/2-1/2*(1+8*I)^(1/2))^2*((1+8*I)^(1/2)*(x-1
/2+1/2*(1-8*I)^(1/2))/(-1/2*(1-8*I)^(1/2)+1/2*(1+8*I)^(1/2))/(x-1/2-1/2*(1+
8*I)^(1/2)))^(1/2)*((1+8*I)^(1/2)*(x-1/2-1/2*(1-8*I)^(1/2))/(1/2*(1-8*I)^(1
/2)+1/2*(1+8*I)^(1/2))/(x-1/2-1/2*(1+8*I)^(1/2)))^(1/2)/(1/2*(1-8*I)^(1/2)-
1/2*(1+8*I)^(1/2))/(1+8*I)^(1/2)/((x-1/2+1/2*(1+8*I)^(1/2))*(x-1/2-1/2*(1+8
*I)^(1/2))*(x-1/2+1/2*(1-8*I)^(1/2))*(x-1/2-1/2*(1-8*I)^(1/2)))^(1/2)*Ellip
ticF(((1/2*(1-8*I)^(1/2)-1/2*(1+8*I)^(1/2))*(x-1/2+1/2*(1+8*I)^(1/2))/(1/2*
(1-8*I)^(1/2)+1/2*(1+8*I)^(1/2))/(x-1/2-1/2*(1+8*I)^(1/2)))^(1/2),(-(1/2*(1
-8*I)^(1/2)+1/2*(1+8*I)^(1/2))^2/(1/2*(1-8*I)^(1/2)-1/2*(1+8*I)^(1/2))/(-1/
2*(1-8*I)^(1/2)+1/2*(1+8*I)^(1/2)))^(1/2))+4*(-1/2*(1+8*I)^(1/2)-1/2*(1-8*I
)^(1/2))*((1/2*(1-8*I)^(1/2)-1/2*(1+8*I)^(1/2))*(x-1/2+1/2*(1+8*I)^(1/2))/(
1/2*(1-8*I)^(1/2)+1/2*(1+8*I)^(1/2))/(x-1/2-1/2*(1+8*I)^(1/2)))^(1/2)*(x-1/
2-1/2*(1+8*I)^(1/2))^2*((1+8*I)^(1/2)*(x-1/2+1/2*(1-8*I)^(1/2))/(-1/2*(1-8*
I)^(1/2)+1/2*(1+8*I)^(1/2))/(x-1/2-1/2*(1+8*I)^(1/2)))^(1/2)*((1+8*I)^(1/2)
*(x-1/2-1/2*(1-8*I)^(1/2))/(1/2*(1-8*I)^(1/2)+1/2*(1+8*I)^(1/2))/(x-1/2-1/2
*(1+8*I)^(1/2)))^(1/2)/(1/2*(1-8*I)^(1/2)-1/2*(1+8*I)^(1/2))/(1+8*I)^(1/2)/
((x-1/2+1/2*(1+8*I)^(1/2))*(x-1/2-1/2*(1+8*I)^(1/2))*(x-1/2+1/2*(1-8*I)^(1/
2))*(x-1/2-1/2*(1-8*I)^(1/2)))^(1/2)*((1/2+1/2*(1+8*I)^(1/2))*EllipticF(((1
/2*(1-8*I)^(1/2)-1/2*(1+8*I)^(1/2))*(x-1/2+1/2*(1+8*I)^(1/2))/(1/2*(1-8*I)^
(1/2)+1/2*(1+8*I)^(1/2))/(x-1/2-1/2*(1+8*I)^(1/2)))^(1/2),(-(1/2*(1-8*I)^(1
/2)+1/2*(1+8*I)^(1/2))^2/(1/2*(1-8*I)^(1/2)-1/2*(1+8*I)^(1/2))/(-1/2*(1-8*I
)^(1/2)+1/2*(1+8*I)^(1/2)))^(1/2))-(1+8*I)^(1/2)*EllipticPi(((1/2*(1-8*I)^(
1/2)-1/2*(1+8*I)^(1/2))*(x-1/2+1/2*(1+8*I)^(1/2))/(1/2*(1-8*I)^(1/2)+1/2*(1
+8*I)^(1/2))/(x-1/2-1/2*(1+8*I)^(1/2)))^(1/2),(1/2*(1-8*I)^(1/2)+1/2*(1+8*I
)^(1/2))/(1/2*(1-8*I)^(1/2)-1/2*(1+8*I)^(1/2)),(-(1/2*(1-8*I)^(1/2)+1/2*(1+
8*I)^(1/2))^2/(1/2*(1-8*I)^(1/2)-1/2*(1+8*I)^(1/2))/(-1/2*(1-8*I)^(1/2)+1/2
*(1+8*I)^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x - 1}{\sqrt{x^4 - 2x^3 + x^2 + 4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x - 1)/sqrt(x^4 - 2*x^3 + x^2 + 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{2\,x-1}{\sqrt{x^4-2\,x^3+x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 1)/(x^2 - 2*x^3 + x^4 + 4)^(1/2),x)
```

```
[Out] int((2*x - 1)/(x^2 - 2*x^3 + x^4 + 4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-1}{\sqrt{x^4-2x^3+x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x**4-2*x**3+x**2+4)**(1/2),x)
```

```
[Out] Integral((2*x - 1)/sqrt(x**4 - 2*x**3 + x**2 + 4), x)
```

3.267 $\int \frac{-1+2x}{\sqrt{13+x^2-2x^3+x^4}}\,dx$

Optimal. Leaf size=25

$$\log\left(x^2+\sqrt{x^4-2x^3+x^2+13}-x\right)$$

Rubi [A] time = 0.06, antiderivative size = 15, normalized size of antiderivative = 0.60, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1680, 12, 1107, 619, 215}

$$-\sinh^{-1}\left(\frac{(1-x)x}{\sqrt{13}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x)/Sqrt[13 + x^2 - 2*x^3 + x^4],x]
```

```
[Out] -ArcSinh[((1 - x)*x)/Sqrt[13]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+2x}{\sqrt{13+x^2-2x^3+x^4}}\,dx &= \text{Subst}\left(\int \frac{8x}{\sqrt{209-8x^2+16x^4}}\,dx, x, -\frac{1}{2}+x\right)\\
&= 8\,\text{Subst}\left(\int \frac{x}{\sqrt{209-8x^2+16x^4}}\,dx, x, -\frac{1}{2}+x\right)\\
&= 4\,\text{Subst}\left(\int \frac{1}{\sqrt{209-8x+16x^2}}\,dx, x, \left(-\frac{1}{2}+x\right)^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{13312}}}\,dx, x, 32(-1+x)x\right)}{32\sqrt{13}}\\
&= -\sinh^{-1}\left(\frac{(1-x)x}{\sqrt{13}}\right)
\end{aligned}
$$

Mathematica [C] time = 3.04, size = 803, normalized size = 32.12

$$
\frac{\left(-2x+\sqrt{1-4i\sqrt{13}}+1\right)\sqrt{\frac{\sqrt{1-4i\sqrt{13}}\left(-2x+\sqrt{1+4i\sqrt{13}}+1\right)}{\left(\sqrt{1-4i\sqrt{13}}+\sqrt{1+4i\sqrt{13}}\right)\left(-2x+\sqrt{1-4i\sqrt{13}}+1\right)}}\left(2x+\sqrt{1-4i\sqrt{13}}-1\right)\sqrt{-\frac{\sqrt{1-4i\sqrt{13}}\left(2x+\sqrt{1+4i\sqrt{13}}-1\right)}{\left(\sqrt{1-4i\sqrt{13}}-\sqrt{1+4i\sqrt{13}}\right)\left(-2x+\sqrt{1-4i\sqrt{13}}+1\right)}}\left(F\left(\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{1-4i\sqrt{13}}-\sqrt{1+4i\sqrt{13}}\right)\left(2x+\sqrt{1-4i\sqrt{13}}-1\right)}{\left(\sqrt{1-4i\sqrt{13}}+\sqrt{1+4i\sqrt{13}}\right)\left(-2x+\sqrt{1-4i\sqrt{13}}+1\right)}}\right)\middle|\frac{\left(\sqrt{1-4i\sqrt{13}}+\sqrt{1+4i\sqrt{13}}\right)^2}{\left(\sqrt{1-4i\sqrt{13}}-\sqrt{1+4i\sqrt{13}}\right)^2}\right)-2\Pi\left(-\frac{\sqrt{1-4i\sqrt{13}}+\sqrt{1+4i\sqrt{13}}}{\sqrt{1-4i\sqrt{13}}-\sqrt{1+4i\sqrt{13}}};\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{1-4i\sqrt{13}}-\sqrt{1+4i\sqrt{13}}\right)\left(2x+\sqrt{1-4i\sqrt{13}}-1\right)}{\left(\sqrt{1-4i\sqrt{13}}+\sqrt{1+4i\sqrt{13}}\right)\left(-2x+\sqrt{1-4i\sqrt{13}}+1\right)}}\right)\middle|\frac{\left(\sqrt{1-4i\sqrt{13}}+\sqrt{1+4i\sqrt{13}}\right)^2}{\left(\sqrt{1-4i\sqrt{13}}-\sqrt{1+4i\sqrt{13}}\right)^2}\right)\right)}{\sqrt{\frac{\left(\sqrt{1-4i\sqrt{13}}-\sqrt{1+4i\sqrt{13}}\right)\left(2x+\sqrt{1-4i\sqrt{13}}-1\right)}{\left(\sqrt{1-4i\sqrt{13}}+\sqrt{1+4i\sqrt{13}}\right)\left(-2x+\sqrt{1-4i\sqrt{13}}+1\right)}}\sqrt{x^4-2x^3+x^2+13}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x)/Sqrt[13 + x^2 - 2*x^3 + x^4],x]
```

```
[Out] ((1 + Sqrt[1 - (4*I)*Sqrt[13]] - 2*x)*Sqrt[(Sqrt[1 - (4*I)*Sqrt[13]]*(1 + S
qrt[1 + (4*I)*Sqrt[13]] - 2*x))/((Sqrt[1 - (4*I)*Sqrt[13]] + Sqrt[1 + (4*I)
*Sqrt[13]])*(1 + Sqrt[1 - (4*I)*Sqrt[13]] - 2*x))]*(-1 + Sqrt[1 - (4*I)*Sqr
t[13]] + 2*x)*Sqrt[-((Sqrt[1 - (4*I)*Sqrt[13]]*(-1 + Sqrt[1 + (4*I)*Sqrt[13
]] + 2*x))/((Sqrt[1 - (4*I)*Sqrt[13]] - Sqrt[1 + (4*I)*Sqrt[13]])*(1 + Sqrt
[1 - (4*I)*Sqrt[13]] - 2*x)))]*(EllipticF[ArcSin[Sqrt[((Sqrt[1 - (4*I)*Sqrt
[13]] - Sqrt[1 + (4*I)*Sqrt[13]])*(-1 + Sqrt[1 - (4*I)*Sqrt[13]] + 2*x))/((
Sqrt[1 - (4*I)*Sqrt[13]] + Sqrt[1 + (4*I)*Sqrt[13]])*(1 + Sqrt[1 - (4*I)*Sq
rt[13]] - 2*x))]], (Sqrt[1 - (4*I)*Sqrt[13]] + Sqrt[1 + (4*I)*Sqrt[13]])^2/
(Sqrt[1 - (4*I)*Sqrt[13]] - Sqrt[1 + (4*I)*Sqrt[13]])^2] - 2*EllipticPi[-((
Sqrt[1 - (4*I)*Sqrt[13]] + Sqrt[1 + (4*I)*Sqrt[13]])/(Sqrt[1 - (4*I)*Sqrt[1
3]] - Sqrt[1 + (4*I)*Sqrt[13]])), ArcSin[Sqrt[((Sqrt[1 - (4*I)*Sqrt[13]] -
Sqrt[1 + (4*I)*Sqrt[13]])*(-1 + Sqrt[1 - (4*I)*Sqrt[13]] + 2*x))/((Sqrt[1 -
 (4*I)*Sqrt[13]] + Sqrt[1 + (4*I)*Sqrt[13]])*(1 + Sqrt[1 - (4*I)*Sqrt[13]]
- 2*x))]], (Sqrt[1 - (4*I)*Sqrt[13]] + Sqrt[1 + (4*I)*Sqrt[13]])^2/(Sqrt[1
- (4*I)*Sqrt[13]] - Sqrt[1 + (4*I)*Sqrt[13]])^2]))/(Sqrt[((Sqrt[1 - (4*I)*S
qrt[13]] - Sqrt[1 + (4*I)*Sqrt[13]])*(-1 + Sqrt[1 - (4*I)*Sqrt[13]] + 2*x))
/((Sqrt[1 - (4*I)*Sqrt[13]] + Sqrt[1 + (4*I)*Sqrt[13]])*(1 + Sqrt[1 - (4*I)
*Sqrt[13]] - 2*x))]*Sqrt[13 + x^2 - 2*x^3 + x^4])
```

IntegrateAlgebraic [A] time = 0.28, size = 25, normalized size = 1.00

$$
\log\left(x^2+\sqrt{x^4-2x^3+x^2+13}-x\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x)/Sqrt[13 + x^2 - 2*x^3 + x^4],x]
```

```
[Out] Log[-x + x^2 + Sqrt[13 + x^2 - 2*x^3 + x^4]]
```

fricas [A] time = 0.42, size = 23, normalized size = 0.92

$$
\log\left(x^2-x+\sqrt{x^4-2\,x^3+x^2+13}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+x^2+13)^(1/2),x, algorithm="fricas")
```

```
[Out] log(x^2 - x + sqrt(x^4 - 2*x^3 + x^2 + 13))
```

giac [A] time = 0.30, size = 23, normalized size = 0.92

$$-\log\left(-x^2+x+\sqrt{\left(x^2-x\right)^2+13}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+x^2+13)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-x^2 + x + sqrt((x^2 - x)^2 + 13))
```

maple [C] time = 0.43, size = 1352, normalized size = 54.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x)/(x^4-2*x^3+x^2+13)^(1/2),x)
```

```
[Out] -2*(-1/2*(1+4*I*13^(1/2))^(1/2)-1/2*(1-4*I*13^(1/2))^(1/2))*((1/2*(1-4*I*13
^(1/2))^(1/2)-1/2*(1+4*I*13^(1/2))^(1/2))*(x-1/2+1/2*(1+4*I*13^(1/2))^(1/2)
)/(1/2*(1-4*I*13^(1/2))^(1/2)+1/2*(1+4*I*13^(1/2))^(1/2))/(x-1/2-1/2*(1+4*I
*13^(1/2))^(1/2)))^(1/2)*(x-1/2-1/2*(1+4*I*13^(1/2))^(1/2))^2*((1+4*I*13^(1
/2))^(1/2)*(x-1/2+1/2*(1-4*I*13^(1/2))^(1/2))/(-1/2*(1-4*I*13^(1/2))^(1/2)+
1/2*(1+4*I*13^(1/2))^(1/2))/(x-1/2-1/2*(1+4*I*13^(1/2))^(1/2)))^(1/2)*((1+4
*I*13^(1/2))^(1/2)*(x-1/2-1/2*(1-4*I*13^(1/2))^(1/2))/(1/2*(1-4*I*13^(1/2))
^(1/2)+1/2*(1+4*I*13^(1/2))^(1/2))/(x-1/2-1/2*(1+4*I*13^(1/2))^(1/2)))^(1/2
)/(1/2*(1-4*I*13^(1/2))^(1/2)-1/2*(1+4*I*13^(1/2))^(1/2))/(1+4*I*13^(1/2))^
(1/2)/((x-1/2+1/2*(1+4*I*13^(1/2))^(1/2))*(x-1/2-1/2*(1+4*I*13^(1/2))^(1/2)
)*(x-1/2+1/2*(1-4*I*13^(1/2))^(1/2))*(x-1/2-1/2*(1-4*I*13^(1/2))^(1/2)))^(1
/2)*EllipticF(((1/2*(1-4*I*13^(1/2))^(1/2)-1/2*(1+4*I*13^(1/2))^(1/2))*(x-1
/2+1/2*(1+4*I*13^(1/2))^(1/2))/(1/2*(1-4*I*13^(1/2))^(1/2)+1/2*(1+4*I*13^(1
/2))^(1/2))/(x-1/2-1/2*(1+4*I*13^(1/2))^(1/2)))^(1/2),(-(1/2*(1-4*I*13^(1/2
))^(1/2)+1/2*(1+4*I*13^(1/2))^(1/2))^2/(1/2*(1-4*I*13^(1/2))^(1/2)-1/2*(1+4
*I*13^(1/2))^(1/2))/(-1/2*(1-4*I*13^(1/2))^(1/2)+1/2*(1+4*I*13^(1/2))^(1/2)
))^(1/2))+4*(-1/2*(1+4*I*13^(1/2))^(1/2)-1/2*(1-4*I*13^(1/2))^(1/2))*((1/2*
(1-4*I*13^(1/2))^(1/2)-1/2*(1+4*I*13^(1/2))^(1/2))*(x-1/2+1/2*(1+4*I*13^(1/
2))^(1/2))/(1/2*(1-4*I*13^(1/2))^(1/2)+1/2*(1+4*I*13^(1/2))^(1/2))/(x-1/2-1
/2*(1+4*I*13^(1/2))^(1/2)))^(1/2)*(x-1/2-1/2*(1+4*I*13^(1/2))^(1/2))^2*((1+
4*I*13^(1/2))^(1/2)*(x-1/2+1/2*(1-4*I*13^(1/2))^(1/2))/(-1/2*(1-4*I*13^(1/2
))^(1/2)+1/2*(1+4*I*13^(1/2))^(1/2))/(x-1/2-1/2*(1+4*I*13^(1/2))^(1/2)))^(1
/2)*((1+4*I*13^(1/2))^(1/2)*(x-1/2-1/2*(1-4*I*13^(1/2))^(1/2))/(1/2*(1-4*I*
13^(1/2))^(1/2)+1/2*(1+4*I*13^(1/2))^(1/2))/(x-1/2-1/2*(1+4*I*13^(1/2))^(1/
2)))^(1/2)/(1/2*(1-4*I*13^(1/2))^(1/2)-1/2*(1+4*I*13^(1/2))^(1/2))/(1+4*I*1
3^(1/2))^(1/2)/((x-1/2+1/2*(1+4*I*13^(1/2))^(1/2))*(x-1/2-1/2*(1+4*I*13^(1/
2))^(1/2))*(x-1/2+1/2*(1-4*I*13^(1/2))^(1/2))*(x-1/2-1/2*(1-4*I*13^(1/2))^(
1/2)))^(1/2)*((1/2+1/2*(1+4*I*13^(1/2))^(1/2))*EllipticF(((1/2*(1-4*I*13^(1
/2))^(1/2)-1/2*(1+4*I*13^(1/2))^(1/2))*(x-1/2+1/2*(1+4*I*13^(1/2))^(1/2))/(
1/2*(1-4*I*13^(1/2))^(1/2)+1/2*(1+4*I*13^(1/2))^(1/2))/(x-1/2-1/2*(1+4*I*13
^(1/2))^(1/2)))^(1/2),(-(1/2*(1-4*I*13^(1/2))^(1/2)+1/2*(1+4*I*13^(1/2))^(1
/2))^2/(1/2*(1-4*I*13^(1/2))^(1/2)-1/2*(1+4*I*13^(1/2))^(1/2))/(-1/2*(1-4*I
*13^(1/2))^(1/2)+1/2*(1+4*I*13^(1/2))^(1/2)))^(1/2))-(1+4*I*13^(1/2))^(1/2)
*EllipticPi(((1/2*(1-4*I*13^(1/2))^(1/2)-1/2*(1+4*I*13^(1/2))^(1/2))*(x-1/2
+1/2*(1+4*I*13^(1/2))^(1/2))/(1/2*(1-4*I*13^(1/2))^(1/2)+1/2*(1+4*I*13^(1/2
))^(1/2))/(x-1/2-1/2*(1+4*I*13^(1/2))^(1/2)))^(1/2),(1/2*(1-4*I*13^(1/2))^(
1/2)+1/2*(1+4*I*13^(1/2))^(1/2))/(1/2*(1-4*I*13^(1/2))^(1/2)-1/2*(1+4*I*13^
```

```
(1/2))^(1/2)),(-(1/2*(1-4*I*13^(1/2))^(1/2)+1/2*(1+4*I*13^(1/2))^(1/2))^2/(
1/2*(1-4*I*13^(1/2))^(1/2)-1/2*(1+4*I*13^(1/2))^(1/2))/(-1/2*(1-4*I*13^(1/2
))^(1/2)+1/2*(1+4*I*13^(1/2))^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-1}{\sqrt{x^4-2x^3+x^2+13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+x^2+13)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x - 1)/sqrt(x^4 - 2*x^3 + x^2 + 13), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{2x-1}{\sqrt{x^4-2x^3+x^2+13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 1)/(x^2 - 2*x^3 + x^4 + 13)^(1/2),x)
[Out] int((2*x - 1)/(x^2 - 2*x^3 + x^4 + 13)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-1}{\sqrt{x^4-2x^3+x^2+13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x**4-2*x**3+x**2+13)**(1/2),x)
[Out] Integral((2*x - 1)/sqrt(x**4 - 2*x**3 + x**2 + 13), x)
```

3.268 $\int \frac{1}{x^2 \sqrt[4]{-x^3+x^4}} \, dx$

Optimal. Leaf size=25

$$\frac{4(4x+3)\left(x^4-x^3\right)^{3/4}}{21x^4}$$

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.64, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2016, 2014}

$$\frac{16\left(x^4-x^3\right)^{3/4}}{21x^3}+\frac{4\left(x^4-x^3\right)^{3/4}}{7x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (4*(-x^3 + x^4)^(3/4))/(7*x^4) + (16*(-x^3 + x^4)^(3/4))/(21*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2 \sqrt[4]{-x^3+x^4}} \, dx &= \frac{4\left(-x^3+x^4\right)^{3/4}}{7x^4}+\frac{4}{7}\int \frac{1}{x \sqrt[4]{-x^3+x^4}} \, dx\\ &= \frac{4\left(-x^3+x^4\right)^{3/4}}{7x^4}+\frac{16\left(-x^3+x^4\right)^{3/4}}{21x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.92

$$\frac{4\left((x-1)x^3\right)^{3/4}(4x+3)}{21x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (4*((-1 + x)*x^3)^(3/4)*(3 + 4*x))/(21*x^4)
```

IntegrateAlgebraic [A] time = 0.23, size = 25, normalized size = 1.00

$$\frac{4(4x+3)\left(x^4-x^3\right)^{3/4}}{21x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(-x^3 + x^4)^(1/4)),x]
[Out] (4*(3 + 4*x)*(-x^3 + x^4)^(3/4))/(21*x^4)
```

fricas [A] time = 0.39, size = 21, normalized size = 0.84

$$\frac{4\left(x^{4}-x^{3}\right)^{\frac{3}{4}}(4x+3)}{21x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4-x^3)^(1/4),x, algorithm="fricas")
[Out] 4/21*(x^4 - x^3)^(3/4)*(4*x + 3)/x^4
```

giac [A] time = 0.37, size = 23, normalized size = 0.92

$$\frac{4}{7}\left(-\frac{1}{x}+1\right)^{\frac{7}{4}}-\frac{4}{3}\left(-\frac{1}{x}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4-x^3)^(1/4),x, algorithm="giac")
[Out] 4/7*(-1/x + 1)^(7/4) - 4/3*(-1/x + 1)^(3/4)
```

maple [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{4(-1+x)(3+4x)}{21x\left(x^{4}-x^{3}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^4-x^3)^(1/4),x)
[Out] 4/21*(-1+x)*(3+4*x)/x/(x^4-x^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{4}-x^{3}\right)^{\frac{1}{4}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4-x^3)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((x^4 - x^3)^(1/4)*x^2), x)
```

mupad [B] time = 0.16, size = 33, normalized size = 1.32

$$\frac{16\,x\left(x^{4}-x^{3}\right)^{3/4}+12\left(x^{4}-x^{3}\right)^{3/4}}{21\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4 - x^3)^(1/4)),x)
[Out] (16*x*(x^4 - x^3)^(3/4) + 12*(x^4 - x^3)^(3/4))/(21*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\sqrt[4]{x^{3}\left(x-1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**4-x**3)**(1/4),x)
```

```
[Out] Integral(1/(x**2*(x**3*(x - 1))**(1/4)), x)
```

3.269 $\int \frac{-1+x^4}{x^8 \sqrt[4]{-1+2x^4}} \, dx$

Optimal. Leaf size=25

$$\frac{(-x^4-3)\left(2x^4-1\right)^{3/4}}{21x^7}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.48, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {453, 264}

$$-\frac{\left(2x^4-1\right)^{3/4}}{7x^7}-\frac{\left(2x^4-1\right)^{3/4}}{21x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)/(x^8*(-1 + 2*x^4)^(1/4)),x]
```

```
[Out] -1/7*(-1 + 2*x^4)^(3/4)/x^7 - (-1 + 2*x^4)^(3/4)/(21*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^4}{x^8 \sqrt[4]{-1+2x^4}} \, dx &= -\frac{\left(-1+2x^4\right)^{3/4}}{7x^7}-\frac{1}{7}\int \frac{1}{x^4 \sqrt[4]{-1+2x^4}} \, dx\\ &= -\frac{\left(-1+2x^4\right)^{3/4}}{7x^7}-\frac{\left(-1+2x^4\right)^{3/4}}{21x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.92

$$-\frac{\left(x^4+3\right)\left(2x^4-1\right)^{3/4}}{21x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)/(x^8*(-1 + 2*x^4)^(1/4)),x]
```

```
[Out] -1/21*((3 + x^4)*(-1 + 2*x^4)^(3/4))/x^7
```

IntegrateAlgebraic [A] time = 0.19, size = 25, normalized size = 1.00

$$\frac{(-x^4-3)\left(2x^4-1\right)^{3/4}}{21x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/(x^8*(-1 + 2*x^4)^(1/4)),x]
[Out] ((-3 - x^4)*(-1 + 2*x^4)^(3/4))/(21*x^7)
```

fricas [A] time = 0.39, size = 19, normalized size = 0.76

$$-\frac{\left(2 x^{4}-1\right)^{\frac{3}{4}}\left(x^{4}+3\right)}{21 x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^8/(2*x^4-1)^(1/4),x, algorithm="fricas")
[Out] -1/21*(2*x^4 - 1)^(3/4)*(x^4 + 3)/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}-1}{\left(2 x^{4}-1\right)^{\frac{1}{4}} x^{8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^8/(2*x^4-1)^(1/4),x, algorithm="giac")
[Out] integrate((x^4 - 1)/((2*x^4 - 1)^(1/4)*x^8), x)
```

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$-\frac{\left(x^{4}+3\right)\left(2x^{4}-1\right)^{\frac{3}{4}}}{21x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/x^8/(2*x^4-1)^(1/4),x)
[Out] -1/21*(x^4+3)*(2*x^4-1)^(3/4)/x^7
```

maxima [A] time = 0.42, size = 29, normalized size = 1.16

$$-\frac{\left(2 x^{4}-1\right)^{\frac{3}{4}}}{3 x^{3}}+\frac{\left(2 x^{4}-1\right)^{\frac{7}{4}}}{7 x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^8/(2*x^4-1)^(1/4),x, algorithm="maxima")
[Out] -1/3*(2*x^4 - 1)^(3/4)/x^3 + 1/7*(2*x^4 - 1)^(7/4)/x^7
```

mupad [B] time = 0.20, size = 30, normalized size = 1.20

$$-\frac{x^{4}\left(2 x^{4}-1\right)^{3/4}+3\left(2 x^{4}-1\right)^{3/4}}{21 x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/(x^8*(2*x^4 - 1)^(1/4)),x)
[Out] -(x^4*(2*x^4 - 1)^(3/4) + 3*(2*x^4 - 1)^(3/4))/(21*x^7)
```

sympy [C] time = 2.11, size = 233, normalized size = 9.32

$$\begin{cases} -\frac{2^{\frac{3}{4}}\left(-1+\frac{1}{2x^4}\right)^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\Gamma\left(-\frac{3}{4}\right)}{4\Gamma\left(\frac{1}{4}\right)} & \text{for } \frac{1}{2|x^4|}>1 \\ -\frac{2^{\frac{3}{4}}\left(1-\frac{1}{2x^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{4\Gamma\left(\frac{1}{4}\right)} & \text{otherwise} \end{cases} - \begin{cases} -\frac{2^{\frac{3}{4}}\left(-1+\frac{1}{2x^4}\right)^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{7}{4}\right)}{2\Gamma\left(\frac{1}{4}\right)} - \frac{3\cdot 2^{\frac{3}{4}}\left(-1+\frac{1}{2x^4}\right)^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{7}{4}\right)}{16x^4\Gamma\left(\frac{1}{4}\right)} & \text{for } \frac{1}{2|x^4|}>1 \\ \frac{2^{\frac{3}{4}}\left(1-\frac{1}{2x^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{2\Gamma\left(\frac{1}{4}\right)} + \frac{3\cdot 2^{\frac{3}{4}}\left(1-\frac{1}{2x^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{16x^4\Gamma\left(\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/x**8/(2*x**4-1)**(1/4),x)
```

```
[Out] Piecewise((-2**(3/4)*(-1 + 1/(2*x**4))**(3/4)*exp(3*I*pi/4)*gamma(-3/4)/(4*
gamma(1/4)), 1/(2*Abs(x**4)) > 1), (-2**(3/4)*(1 - 1/(2*x**4))**(3/4)*gamma
(-3/4)/(4*gamma(1/4)), True)) - Piecewise((-2**(3/4)*(-1 + 1/(2*x**4))**(3/
4)*exp(-I*pi/4)*gamma(-7/4)/(2*gamma(1/4)) - 3*2**(3/4)*(-1 + 1/(2*x**4))**
(3/4)*exp(-I*pi/4)*gamma(-7/4)/(16*x**4*gamma(1/4)), 1/(2*Abs(x**4)) > 1),
(2**(3/4)*(1 - 1/(2*x**4))**(3/4)*gamma(-7/4)/(2*gamma(1/4)) + 3*2**(3/4)*(
1 - 1/(2*x**4))**(3/4)*gamma(-7/4)/(16*x**4*gamma(1/4)), True))
```

3.270 $\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt[4]{-x^3+x^5}}\,dx$

Optimal. Leaf size=25

$$-\frac{4\left(x^5-x^3\right)^{3/4}}{x^2\left(x^2-1\right)}$$

Rubi [A] time = 0.09, antiderivative size = 16, normalized size of antiderivative = 0.64, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2056, 449}

$$-\frac{4x}{\sqrt[4]{x^5-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/((-1 + x^2)*(-x^3 + x^5)^(1/4)),x]
```

```
[Out] (-4*x)/(-x^3 + x^5)^(1/4)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt[4]{-x^3+x^5}}\,dx &= \frac{\left(x^{3/4}\sqrt[4]{-1+x^2}\right)\int \frac{1+x^2}{x^{3/4}\left(-1+x^2\right)^{5/4}}\,dx}{\sqrt[4]{-x^3+x^5}}\\ &= -\frac{4x}{\sqrt[4]{-x^3+x^5}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 0.64

$$-\frac{4x}{\sqrt[4]{x^3\left(x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/((-1 + x^2)*(-x^3 + x^5)^(1/4)),x]
```

```
[Out] (-4*x)/(x^3*(-1 + x^2))^(1/4)
```

IntegrateAlgebraic [A] time = 0.16, size = 25, normalized size = 1.00

$$-\frac{4\left(x^5-x^3\right)^{3/4}}{x^2\left(x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 + x^2)*(-x^3 + x^5)^(1/4)),x]
[Out] (-4*(-x^3 + x^5)^(3/4))/(x^2*(-1 + x^2))
```

fricas [A] time = 0.39, size = 24, normalized size = 0.96

$$-\frac{4\left(x^5-x^3\right)^{\frac{3}{4}}}{x^4-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^5-x^3)^(1/4),x, algorithm="fricas")
[Out] -4*(x^5 - x^3)^(3/4)/(x^4 - x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^5-x^3\right)^{\frac{1}{4}}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^5-x^3)^(1/4),x, algorithm="giac")
[Out] integrate((x^2 + 1)/((x^5 - x^3)^(1/4)*(x^2 - 1)), x)
```

maple [A] time = 0.01, size = 15, normalized size = 0.60

$$-\frac{4x}{\left(x^5-x^3\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-1)/(x^5-x^3)^(1/4),x)
[Out] -4*x/(x^5-x^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^5-x^3\right)^{\frac{1}{4}}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^5-x^3)^(1/4),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/((x^5 - x^3)^(1/4)*(x^2 - 1)), x)
```

mupad [B] time = 0.19, size = 23, normalized size = 0.92

$$-\frac{4\left(x^5-x^3\right)^{3/4}}{x^2\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((x^2 - 1)*(x^5 - x^3)^(1/4)),x)
[Out] -(4*(x^5 - x^3)^(3/4))/(x^2*(x^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt[4]{x^3\,(x-1)\,(x+1)}\,\,(x-1)\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-1)/(x**5-x**3)**(1/4),x)
```

```
[Out] Integral((x**2 + 1)/((x**3*(x - 1)*(x + 1))**(1/4)*(x - 1)*(x + 1)), x)
```

3.271 $\int \frac{1+3x^4}{(-1+x^4)\sqrt[3]{-x^2+x^6}}\,dx$

Optimal. Leaf size=25

$$-\frac{3\left(x^6-x^2\right)^{2/3}}{x\left(x^4-1\right)}$$

Rubi [A] time = 0.09, antiderivative size = 16, normalized size of antiderivative = 0.64, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2056, 449}

$$-\frac{3x}{\sqrt[3]{x^6-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x^4)/((-1 + x^4)*(-x^2 + x^6)^(1/3)),x]
```

```
[Out] (-3*x)/(-x^2 + x^6)^(1/3)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+3x^4}{\left(-1+x^4\right)\sqrt[3]{-x^2+x^6}}\,dx &= \frac{\left(x^{2/3}\sqrt[3]{-1+x^4}\right)\int \frac{1+3x^4}{x^{2/3}\left(-1+x^4\right)^{4/3}}\,dx}{\sqrt[3]{-x^2+x^6}}\\ &= -\frac{3x}{\sqrt[3]{-x^2+x^6}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 0.64

$$-\frac{3x}{\sqrt[3]{x^2\left(x^4-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x^4)/((-1 + x^4)*(-x^2 + x^6)^(1/3)),x]
```

```
[Out] (-3*x)/(x^2*(-1 + x^4))^(1/3)
```

IntegrateAlgebraic [A] time = 0.27, size = 25, normalized size = 1.00

$$-\frac{3\left(x^6-x^2\right)^{2/3}}{x\left(x^4-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 3*x^4)/((-1 + x^4)*(-x^2 + x^6)^(1/3)),x]
[Out] (-3*(-x^2 + x^6)^(2/3))/(x*(-1 + x^4))
```

fricas [A] time = 0.40, size = 22, normalized size = 0.88

$$-\frac{3\left(x^{6}-x^{2}\right)^{\frac{2}{3}}}{x^{5}-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+1)/(x^4-1)/(x^6-x^2)^(1/3),x, algorithm="fricas")
[Out] -3*(x^6 - x^2)^(2/3)/(x^5 - x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^{4}+1}{\left(x^{6}-x^{2}\right)^{\frac{1}{3}}\left(x^{4}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+1)/(x^4-1)/(x^6-x^2)^(1/3),x, algorithm="giac")
[Out] integrate((3*x^4 + 1)/((x^6 - x^2)^(1/3)*(x^4 - 1)), x)
```

maple [A] time = 0.01, size = 15, normalized size = 0.60

$$-\frac{3x}{\left(x^{6}-x^{2}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4+1)/(x^4-1)/(x^6-x^2)^(1/3),x)
[Out] -3*x/(x^6-x^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^{4}+1}{\left(x^{6}-x^{2}\right)^{\frac{1}{3}}\left(x^{4}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+1)/(x^4-1)/(x^6-x^2)^(1/3),x, algorithm="maxima")
[Out] integrate((3*x^4 + 1)/((x^6 - x^2)^(1/3)*(x^4 - 1)), x)
```

mupad [B] time = 0.21, size = 23, normalized size = 0.92

$$-\frac{3\left(x^{6}-x^{2}\right)^{2/3}}{x\left(x^{4}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4 + 1)/((x^4 - 1)*(x^6 - x^2)^(1/3)),x)
[Out] -(3*(x^6 - x^2)^(2/3))/(x*(x^4 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^4+1}{\sqrt[3]{x^2\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4+1)/(x**4-1)/(x**6-x**2)**(1/3),x)
```

```
[Out] Integral((3*x**4 + 1)/((x**2*(x - 1)*(x + 1)*(x**2 + 1))**(1/3)*(x - 1)*(x
+ 1)*(x**2 + 1)), x)
```

3.272 $\int \frac{1+x^4}{\left(-1+x^4\right)\sqrt[4]{-x^2+x^6}}\,dx$

Optimal. Leaf size=25

$$-\frac{2\left(x^6-x^2\right)^{3/4}}{x\left(x^4-1\right)}$$

Rubi [A] time = 0.08, antiderivative size = 16, normalized size of antiderivative = 0.64, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2056, 449}

$$-\frac{2x}{\sqrt[4]{x^6-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/((-1 + x^4)*(-x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*x)/(-x^2 + x^6)^(1/4)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^4}{\left(-1+x^4\right)\sqrt[4]{-x^2+x^6}}\,dx &= \frac{\left(\sqrt{x}\sqrt[4]{-1+x^4}\right)\int \frac{1+x^4}{\sqrt{x}\left(-1+x^4\right)^{5/4}}\,dx}{\sqrt[4]{-x^2+x^6}}\\ &= -\frac{2x}{\sqrt[4]{-x^2+x^6}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 0.64

$$-\frac{2x}{\sqrt[4]{x^2\left(x^4-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/((-1 + x^4)*(-x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*x)/(x^2*(-1 + x^4))^(1/4)
```

IntegrateAlgebraic [A] time = 0.29, size = 25, normalized size = 1.00

$$-\frac{2\left(x^6-x^2\right)^{3/4}}{x\left(x^4-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)/((-1 + x^4)*(-x^2 + x^6)^(1/4)),x]
[Out] (-2*(-x^2 + x^6)^(3/4))/(x*(-1 + x^4))
```

fricas [A] time = 0.41, size = 22, normalized size = 0.88

$$-\frac{2\left(x^{6}-x^{2}\right)^{\frac{3}{4}}}{x^{5}-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(x^6-x^2)^(1/4),x, algorithm="fricas")
[Out] -2*(x^6 - x^2)^(3/4)/(x^5 - x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}+1}{\left(x^{6}-x^{2}\right)^{\frac{1}{4}}\left(x^{4}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(x^6-x^2)^(1/4),x, algorithm="giac")
[Out] integrate((x^4 + 1)/((x^6 - x^2)^(1/4)*(x^4 - 1)), x)
```

maple [A] time = 0.01, size = 15, normalized size = 0.60

$$-\frac{2x}{\left(x^{6}-x^{2}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^4-1)/(x^6-x^2)^(1/4),x)
[Out] -2*x/(x^6-x^2)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}+1}{\left(x^{6}-x^{2}\right)^{\frac{1}{4}}\left(x^{4}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(x^6-x^2)^(1/4),x, algorithm="maxima")
[Out] integrate((x^4 + 1)/((x^6 - x^2)^(1/4)*(x^4 - 1)), x)
```

mupad [B] time = 0.17, size = 23, normalized size = 0.92

$$-\frac{2\left(x^{6}-x^{2}\right)^{3/4}}{x\left(x^{4}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/((x^4 - 1)*(x^6 - x^2)^(1/4)),x)
[Out] -(2*(x^6 - x^2)^(3/4))/(x*(x^4 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\sqrt[4]{x^2\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}\,\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**4-1)/(x**6-x**2)**(1/4),x)
```

```
[Out] Integral((x**4 + 1)/((x**2*(x - 1)*(x + 1)*(x**2 + 1))**(1/4)*(x - 1)*(x +
1)*(x**2 + 1)), x)
```

3.273 $\int \frac{1}{x^7 \sqrt[3]{x^2+x^6}} \, dx$

Optimal. Leaf size=25

$$\frac{3 \left(3x^4-2\right) \left(x^6+x^2\right)^{2/3}}{40x^8}$$

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.48, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2016, 2014}

$$\frac{9 \left(x^6+x^2\right)^{2/3}}{40x^4}-\frac{3 \left(x^6+x^2\right)^{2/3}}{20x^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(x^2 + x^6)^(1/3)),x]
[Out] (-3*(x^2 + x^6)^(2/3))/(20*x^8) + (9*(x^2 + x^6)^(2/3))/(40*x^4)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^7 \sqrt[3]{x^2+x^6}} \, dx &= -\frac{3 \left(x^2+x^6\right)^{2/3}}{20x^8}-\frac{3}{5} \int \frac{1}{x^3 \sqrt[3]{x^2+x^6}} \, dx\\ &= -\frac{3 \left(x^2+x^6\right)^{2/3}}{20x^8}+\frac{9 \left(x^2+x^6\right)^{2/3}}{40x^4} \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{3 \left(3x^4-2\right) \left(x^6+x^2\right)^{2/3}}{40x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(x^2 + x^6)^(1/3)),x]
[Out] (3*(-2 + 3*x^4)*(x^2 + x^6)^(2/3))/(40*x^8)
```

IntegrateAlgebraic [A] time = 0.45, size = 25, normalized size = 1.00

$$\frac{3(3x^4-2)(x^6+x^2)^{2/3}}{40x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(x^2 + x^6)^(1/3)),x]
[Out] (3*(-2 + 3*x^4)*(x^2 + x^6)^(2/3))/(40*x^8)
```

fricas [A] time = 0.39, size = 21, normalized size = 0.84

$$\frac{3(x^6+x^2)^{\frac{2}{3}}(3x^4-2)}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+x^2)^(1/3),x, algorithm="fricas")
[Out] 3/40*(x^6 + x^2)^(2/3)*(3*x^4 - 2)/x^8
```

giac [A] time = 0.49, size = 19, normalized size = 0.76

$$-\frac{3}{20}\left(\frac{1}{x^4}+1\right)^{\frac{5}{3}}+\frac{3}{8}\left(\frac{1}{x^4}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+x^2)^(1/3),x, algorithm="giac")
[Out] -3/20*(1/x^4 + 1)^(5/3) + 3/8*(1/x^4 + 1)^(2/3)
```

maple [A] time = 0.00, size = 27, normalized size = 1.08

$$\frac{3(x^4+1)(3x^4-2)}{40x^6(x^6+x^2)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^6+x^2)^(1/3),x)
[Out] 3/40*(x^4+1)*(3*x^4-2)/x^6/(x^6+x^2)^(1/3)
```

maxima [A] time = 0.59, size = 28, normalized size = 1.12

$$\frac{3(3x^{10}+x^6-2x^2)}{40(x^4+1)^{\frac{1}{3}}(x^2)^{\frac{13}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+x^2)^(1/3),x, algorithm="maxima")
[Out] 3/40*(3*x^10 + x^6 - 2*x^2)/((x^4 + 1)^(1/3)*(x^2)^(13/3))
```

mupad [B] time = 0.21, size = 31, normalized size = 1.24

$$-\frac{6(x^6+x^2)^{2/3}-9x^4(x^6+x^2)^{2/3}}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^2 + x^6)^(1/3)),x)
```

```
[Out] -(6*(x^2 + x^6)^(2/3) - 9*x^4*(x^2 + x^6)^(2/3))/(40*x^8)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 \sqrt[3]{x^2 \left(x^4+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**6+x**2)**(1/3),x)
```

```
[Out] Integral(1/(x**7*(x**2*(x**4 + 1))**(1/3)), x)
```

3.274 $\int \frac{\sqrt[3]{-1+x^3}}{x^8} \, dx$

Optimal. Leaf size=26

$$\frac{\sqrt[3]{x^3-1}\left(3x^6+x^3-4\right)}{28x^7}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{271, 264\}$

$$\frac{\left(x^3-1\right)^{4/3}}{7x^7}+\frac{3\left(x^3-1\right)^{4/3}}{28x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)^(1/3)/x^8,x]
```

```
[Out] (-1 + x^3)^(4/3)/(7*x^7) + (3*(-1 + x^3)^(4/3))/(28*x^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{-1+x^3}}{x^8} \, dx &= \frac{\left(-1+x^3\right)^{4/3}}{7x^7}+\frac{3}{7}\int \frac{\sqrt[3]{-1+x^3}}{x^5} \, dx\\ &= \frac{\left(-1+x^3\right)^{4/3}}{7x^7}+\frac{3\left(-1+x^3\right)^{4/3}}{28x^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.88

$$\frac{\left(x^3-1\right)^{4/3}\left(3x^3+4\right)}{28x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)^(1/3)/x^8,x]
```

```
[Out] ((-1 + x^3)^(4/3)*(4 + 3*x^3))/(28*x^7)
```

IntegrateAlgebraic [A] time = 0.10, size = 26, normalized size = 1.00

$$\frac{\sqrt[3]{x^3-1}\left(3x^6+x^3-4\right)}{28x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)^(1/3)/x^8,x]
[Out] ((-1 + x^3)^(1/3)*(-4 + x^3 + 3*x^6))/(28*x^7)
```

fricas [A] time = 0.41, size = 22, normalized size = 0.85

$$\frac{\left(3\,x^{6}+x^{3}-4\right)\left(x^{3}-1\right)^{\frac{1}{3}}}{28\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)/x^8,x, algorithm="fricas")
[Out] 1/28*(3*x^6 + x^3 - 4)*(x^3 - 1)^(1/3)/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}-1\right)^{\frac{1}{3}}}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)/x^8,x, algorithm="giac")
[Out] integrate((x^3 - 1)^(1/3)/x^8, x)
```

maple [A] time = 0.00, size = 29, normalized size = 1.12

$$\frac{(-1+x)\left(x^{2}+x+1\right)\left(3x^{3}+4\right)\left(x^{3}-1\right)^{\frac{1}{3}}}{28x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/3)/x^8,x)
[Out] 1/28*(-1+x)*(x^2+x+1)*(3*x^3+4)*(x^3-1)^(1/3)/x^7
```

maxima [A] time = 0.36, size = 25, normalized size = 0.96

$$\frac{\left(x^{3}-1\right)^{\frac{4}{3}}}{4\,x^{4}}-\frac{\left(x^{3}-1\right)^{\frac{7}{3}}}{7\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)/x^8,x, algorithm="maxima")
[Out] 1/4*(x^3 - 1)^(4/3)/x^4 - 1/7*(x^3 - 1)^(7/3)/x^7
```

mupad [B] time = 0.25, size = 38, normalized size = 1.46

$$\frac{x^{3}\left(x^{3}-1\right)^{1/3}-4\left(x^{3}-1\right)^{1/3}+3\,x^{6}\left(x^{3}-1\right)^{1/3}}{28\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)^(1/3)/x^8,x)
[Out] (x^3*(x^3 - 1)^(1/3) - 4*(x^3 - 1)^(1/3) + 3*x^6*(x^3 - 1)^(1/3))/(28*x^7)
```

sympy [B] time = 0.87, size = 289, normalized size = 11.12

$$\begin{cases} \frac{\sqrt[3]{-1+\frac{1}{x^3}}e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt[3]{-1+\frac{1}{x^3}}e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{9x^3\Gamma\left(-\frac{1}{3}\right)} - \frac{4\sqrt[3]{-1+\frac{1}{x^3}}e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)} & \text{for } \frac{1}{|x^3|} > 1 \\ \frac{3x^6\sqrt[3]{1-\frac{1}{x^3}}\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)-9x^3\Gamma\left(-\frac{1}{3}\right)} - \frac{2x^3\sqrt[3]{1-\frac{1}{x^3}}\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)-9x^3\Gamma\left(-\frac{1}{3}\right)} - \frac{5\sqrt[3]{1-\frac{1}{x^3}}\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)-9x^3\Gamma\left(-\frac{1}{3}\right)} + \frac{4\sqrt[3]{1-\frac{1}{x^3}}\Gamma\left(-\frac{7}{3}\right)}{x^3\left(9x^6\Gamma\left(-\frac{1}{3}\right)-9x^3\Gamma\left(-\frac{1}{3}\right)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/3)/x**8,x)
```

```
[Out] Piecewise(((-1 + x**(-3))**(1/3)*exp(I*pi/3)*gamma(-7/3)/(3*gamma(-1/3)) +
(-1 + x**(-3))**(1/3)*exp(I*pi/3)*gamma(-7/3)/(9*x**3*gamma(-1/3)) - 4*(-1
+ x**(-3))**(1/3)*exp(I*pi/3)*gamma(-7/3)/(9*x**6*gamma(-1/3)), 1/Abs(x**3)
 > 1), (3*x**6*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**6*gamma(-1/3) - 9*x**3
*gamma(-1/3)) - 2*x**3*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**6*gamma(-1/3)
- 9*x**3*gamma(-1/3)) - 5*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**6*gamma(-1/
3) - 9*x**3*gamma(-1/3)) + 4*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(x**3*(9*x**6*
gamma(-1/3) - 9*x**3*gamma(-1/3))), True))
```

3.275 $\int \frac{b+ax^2}{\left(-b+ax^2\right)\sqrt{-bx+ax^3}}\,dx$

Optimal. Leaf size=26

$$\frac{2\sqrt{ax^3-bx}}{b-ax^2}$$

Rubi [A] time = 0.15, antiderivative size = 17, normalized size of antiderivative = 0.65, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2056, 449}

$$-\frac{2x}{\sqrt{ax^3-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^2)/((-b + a*x^2)*Sqrt[-(b*x) + a*x^3]),x]
```

```
[Out] (-2*x)/Sqrt[-(b*x) + a*x^3]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{b+ax^2}{\left(-b+ax^2\right)\sqrt{-bx+ax^3}}\,dx &= \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\int \frac{b+ax^2}{\sqrt{x}\left(-b+ax^2\right)^{3/2}}\,dx}{\sqrt{-bx+ax^3}}\\ &= -\frac{2x}{\sqrt{-bx+ax^3}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 17, normalized size = 0.65

$$-\frac{2x}{\sqrt{ax^3-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^2)/((-b + a*x^2)*Sqrt[-(b*x) + a*x^3]),x]
```

```
[Out] (-2*x)/Sqrt[-(b*x) + a*x^3]
```

IntegrateAlgebraic [A] time = 0.20, size = 26, normalized size = 1.00

$$\frac{2\sqrt{ax^3-bx}}{b-ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^2)/((-b + a*x^2)*Sqrt[-(b*x) + a*x^3]),x]
[Out] (2*Sqrt[-(b*x) + a*x^3])/(b - a*x^2)
```

fricas [A] time = 0.41, size = 25, normalized size = 0.96

$$-\frac{2\sqrt{ax^3-bx}}{ax^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a*x^2-b)/(a*x^3-b*x)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(a*x^3 - b*x)/(a*x^2 - b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2+b}{\sqrt{ax^3-bx}\left(ax^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a*x^2-b)/(a*x^3-b*x)^(1/2),x, algorithm="giac")
[Out] integrate((a*x^2 + b)/(sqrt(a*x^3 - b*x)*(a*x^2 - b)), x)
```

maple [A] time = 0.01, size = 16, normalized size = 0.62

$$-\frac{2x}{\sqrt{a\,x^3-bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)/(a*x^2-b)/(a*x^3-b*x)^(1/2),x)
[Out] -2*x/(a*x^3-b*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2+b}{\sqrt{ax^3-bx}\left(ax^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a*x^2-b)/(a*x^3-b*x)^(1/2),x, algorithm="maxima")
[Out] integrate((a*x^2 + b)/(sqrt(a*x^3 - b*x)*(a*x^2 - b)), x)
```

mupad [B] time = 0.24, size = 24, normalized size = 0.92

$$\frac{2\sqrt{a\,x^3-b\,x}}{b-a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x^2)/((a*x^3 - b*x)^(1/2)*(b - a*x^2)),x)
[Out] (2*(a*x^3 - b*x)^(1/2))/(b - a*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2+b}{\sqrt{x\left(ax^2-b\right)}\left(ax^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b)/(a*x**2-b)/(a*x**3-b*x)**(1/2),x)
```

```
[Out] Integral((a*x**2 + b)/(sqrt(x*(a*x**2 - b))*(a*x**2 - b)), x)
```

3.276 $\int \frac{-2b+ax^3}{\sqrt{b+ax^3}\left(b-cx^2+ax^3\right)}\,dx$

Optimal. Leaf size=26

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{ax^3+b}}\right)}{\sqrt{c}}$$

Rubi [F] time = 1.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2b+ax^3}{\sqrt{b+ax^3}\left(b-cx^2+ax^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*b + a*x^3)/(Sqrt[b + a*x^3]*(b - c*x^2 + a*x^3)),x]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*(b^(1/3) + a^(1/3)*x)*Sqrt[(b^(2/3) - a^(1/3)*b^(1/3)*
x + a^(2/3)*x^2)/((1 + Sqrt[3])*b^(1/3) + a^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*b^(1/3) + a^(1/3)*x)/((1 + Sqrt[3])*b^(1/3) + a^(1/3)*x)], -7
- 4*Sqrt[3]])/(3^(1/4)*a^(1/3)*Sqrt[(b^(1/3)*(b^(1/3) + a^(1/3)*x))/((1 + S
qrt[3])*b^(1/3) + a^(1/3)*x)^2]*Sqrt[b + a*x^3]) - c*Defer[Int][x^2/((-b +
c*x^2 - a*x^3)*Sqrt[b + a*x^3]), x] - 3*b*Defer[Int][1/(Sqrt[b + a*x^3]*(b
- c*x^2 + a*x^3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2b+ax^3}{\sqrt{b+ax^3}\left(b-cx^2+ax^3\right)}\,dx &= \int \left(\frac{1}{\sqrt{b+ax^3}}-\frac{3b-cx^2}{\sqrt{b+ax^3}\left(b-cx^2+ax^3\right)}\right)dx\\
&= \int \frac{1}{\sqrt{b+ax^3}}\,dx-\int \frac{3b-cx^2}{\sqrt{b+ax^3}\left(b-cx^2+ax^3\right)}\,dx\\
&= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)\sqrt{\frac{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+a^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x}\right)\middle|-7-\right.}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x\right)^2}}\sqrt{b+ax^3}}\\
&= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)\sqrt{\frac{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+a^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x}\right)\middle|-7-\right.}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x\right)^2}}\sqrt{b+ax^3}}
\end{aligned}$$

Mathematica [C] time = 6.28, size = 2764, normalized size = 106.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-2*b + a*x^3)/(Sqrt[b + a*x^3]*(b - c*x^2 + a*x^3)),x]
```

```
[Out] (2*Sqrt[(b^(1/3)/a^(1/3) + x)/(b^(1/3)/a^(1/3) + ((-1)^(1/3)*b^(1/3))/a^(1/
3))]*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x)*Sqrt[(((-1)^(2/3)*b^(1/3))/a^(1/
```

```
3) + x)/(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))]*Elli
pticF[ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) - a^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3
))*b^(1/3))]], (-1)^(1/3)])/(Sqrt[(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x)/(-(
((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))]*Sqrt[b + a*x
^3]) - (4*(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))*b*S
qrt[(b^(1/3)/a^(1/3) + x)/(b^(1/3)/a^(1/3) + ((-1)^(1/3)*b^(1/3))/a^(1/3))]
*Sqrt[((-(((-1)^(2/3)*b^(1/3))/a^(1/3)) - x)*(-(((-1)^(1/3)*b^(1/3))/a^(1/3
)) + x))/(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*E
llipticPi[((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) - a
^(1/3)*Root[b - c*#1^2 + a*#1^3 & , 1]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) -
a^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)])/(a*Sqrt[b +
a*x^3]*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + Root[b - c*#1^2 + a*#1^3 & , 1])*
(Root[b - c*#1^2 + a*#1^3 & , 1] - Root[b - c*#1^2 + a*#1^3 & , 2])*(Root[b
 - c*#1^2 + a*#1^3 & , 1] - Root[b - c*#1^2 + a*#1^3 & , 3])) + (2*(((-1)^(
1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))*Sqrt[(b^(1/3)/a^(1/3)
 + x)/(b^(1/3)/a^(1/3) + ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[((-(((-1)^(2/3
)*b^(1/3))/a^(1/3)) - x)*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x))/(((-1)^(1/3
)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*EllipticPi[((-1)^(1/3
)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) - a^(1/3)*Root[b - c*#1
^2 + a*#1^3 & , 1]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) - a^(1/3)*x)/(((-1)^(1
/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)]*Root[b - c*#1^2 + a*#1^3 & , 1]^3
)/(Sqrt[b + a*x^3]*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + Root[b - c*#1^2 + a*#
1^3 & , 1])*(Root[b - c*#1^2 + a*#1^3 & , 1] - Root[b - c*#1^2 + a*#1^3 & ,
 2])*(Root[b - c*#1^2 + a*#1^3 & , 1] - Root[b - c*#1^2 + a*#1^3 & , 3])) -
 (4*(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))*b*Sqrt[(b
^(1/3)/a^(1/3) + x)/(b^(1/3)/a^(1/3) + ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[
((-(((-1)^(2/3)*b^(1/3))/a^(1/3)) - x)*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x
))/(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*Ellipti
cPi[((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) - a^(1/3)
*Root[b - c*#1^2 + a*#1^3 & , 2]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) - a^(1/3
)*x)/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)])/(a*Sqrt[b + a*x^3]
*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + Root[b - c*#1^2 + a*#1^3 & , 2])*(-Root
[b - c*#1^2 + a*#1^3 & , 1] + Root[b - c*#1^2 + a*#1^3 & , 2])*(Root[b - c*
#1^2 + a*#1^3 & , 2] - Root[b - c*#1^2 + a*#1^3 & , 3])) + (2*(((-1)^(1/3)*
b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))*Sqrt[(b^(1/3)/a^(1/3) + x)
/(b^(1/3)/a^(1/3) + ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[((-(((-1)^(2/3)*b^(
1/3))/a^(1/3)) - x)*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x))/(((-1)^(1/3)*b^(
1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*EllipticPi[((-1)^(1/3)*b^(
1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) - a^(1/3)*Root[b - c*#1^2 +
a*#1^3 & , 2]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) - a^(1/3)*x)/(((-1)^(1/3) +
 (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)]*Root[b - c*#1^2 + a*#1^3 & , 2]^3)/(Sq
rt[b + a*x^3]*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + Root[b - c*#1^2 + a*#1^3 &
 , 2])*(-Root[b - c*#1^2 + a*#1^3 & , 1] + Root[b - c*#1^2 + a*#1^3 & , 2])
*(Root[b - c*#1^2 + a*#1^3 & , 2] - Root[b - c*#1^2 + a*#1^3 & , 3])) - (4*
(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))*b*Sqrt[(b^(1/
3)/a^(1/3) + x)/(b^(1/3)/a^(1/3) + ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[((-(
((-1)^(2/3)*b^(1/3))/a^(1/3)) - x)*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x))/(
((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*EllipticPi[
((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) - a^(1/3)*Roo
t[b - c*#1^2 + a*#1^3 & , 3]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) - a^(1/3)*x)
/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)])/(a*Sqrt[b + a*x^3]*(-(
((-1)^(1/3)*b^(1/3))/a^(1/3)) + Root[b - c*#1^2 + a*#1^3 & , 3])*(-Root[b -
 c*#1^2 + a*#1^3 & , 1] + Root[b - c*#1^2 + a*#1^3 & , 3])*(-Root[b - c*#1^
2 + a*#1^3 & , 2] + Root[b - c*#1^2 + a*#1^3 & , 3])) + (2*(((-1)^(1/3)*b^(
1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))*Sqrt[(b^(1/3)/a^(1/3) + x)/(b
^(1/3)/a^(1/3) + ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[((-(((-1)^(2/3)*b^(1/3
))/a^(1/3)) - x)*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x))/(((-1)^(1/3)*b^(1/3
))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*EllipticPi[((-1)^(1/3)*b^(1/3
) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) - a^(1/3)*Root[b - c*#1^2 + a*#
```

```
1^3 & , 3]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) - a^(1/3)*x)/(((-1)^(1/3) + (-
1)^(2/3))*b^(1/3))]], (-1)^(1/3)]*Root[b - c*#1^2 + a*#1^3 & , 3]^3)/(Sqrt[
b + a*x^3]*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + Root[b - c*#1^2 + a*#1^3 & ,
3])*(-Root[b - c*#1^2 + a*#1^3 & , 1] + Root[b - c*#1^2 + a*#1^3 & , 3])*(-
Root[b - c*#1^2 + a*#1^3 & , 2] + Root[b - c*#1^2 + a*#1^3 & , 3]))
```

IntegrateAlgebraic [A] time = 0.61, size = 26, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{ax^3+b}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*b + a*x^3)/(Sqrt[b + a*x^3]*(b - c*x^2 + a*x^3)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[c]*x)/Sqrt[b + a*x^3]])/Sqrt[c]
```

fricas [B] time = 0.46, size = 167, normalized size = 6.42

$$\left[\frac{\log\left(\frac{a^2x^6+6\,acx^5+c^2x^4+2\,abx^3+6\,bcx^2+b^2-4\left(ax^4+cx^3+bx\right)\sqrt{ax^3+b}\,\sqrt{c}}{a^2x^6-2\,acx^5+c^2x^4+2\,abx^3-2\,bcx^2+b^2}\right)}{2\,\sqrt{c}}, \frac{\sqrt{-c}\,\arctan\left(\frac{\left(ax^3+cx^2+b\right)\sqrt{ax^3+b}\,\sqrt{-c}}{2\left(acx^4+bcx\right)}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*b)/(a*x^3+b)^(1/2)/(a*x^3-c*x^2+b),x, algorithm="fricas"
)
```

```
[Out] [1/2*log((a^2*x^6 + 6*a*c*x^5 + c^2*x^4 + 2*a*b*x^3 + 6*b*c*x^2 + b^2 - 4*(
a*x^4 + c*x^3 + b*x)*sqrt(a*x^3 + b)*sqrt(c))/(a^2*x^6 - 2*a*c*x^5 + c^2*x^
4 + 2*a*b*x^3 - 2*b*c*x^2 + b^2))/sqrt(c), sqrt(-c)*arctan(1/2*(a*x^3 + c*x
^2 + b)*sqrt(a*x^3 + b)*sqrt(-c)/(a*c*x^4 + b*c*x))/c]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3-2\,b}{\left(ax^3-cx^2+b\right)\sqrt{ax^3+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*b)/(a*x^3+b)^(1/2)/(a*x^3-c*x^2+b),x, algorithm="giac")
```

```
[Out] integrate((a*x^3 - 2*b)/((a*x^3 - c*x^2 + b)*sqrt(a*x^3 + b)), x)
```

maple [C] time = 0.55, size = 845, normalized size = 32.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-2*b)/(a*x^3+b)^(1/2)/(a*x^3-c*x^2+b),x)
```

```
[Out] -2/3*I*3^(1/2)/a*(-a^2*b)^(1/3)*(I*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*
(-a^2*b)^(1/3))*3^(1/2)*a/(-a^2*b)^(1/3))^(1/2)*((x-1/a*(-a^2*b)^(1/3))/(-3
/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2)*(-I*(x+1/2/a*(-a
^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*3^(1/2)*a/(-a^2*b)^(1/3))^(1/2)
/(a*x^3+b)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(
1/2)/a*(-a^2*b)^(1/3))*3^(1/2)*a/(-a^2*b)^(1/3))^(1/2),(I*3^(1/2)/a*(-a^2*b
)^(1/3)/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))-I/a^
2/b/c*2^(1/2)*sum((-_alpha^2*c+3*b)/_alpha/(3*_alpha*a-2*c)*(-a^2*b)^(1/3)*
(1/2*I*a*(2*x+1/a*(-I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3)))/(-a^2*b)^(1/3
```

```
))^(1/2)*(a*(x-1/a*(-a^2*b)^(1/3))/(-3*(-a^2*b)^(1/3)+I*3^(1/2)*(-a^2*b)^(1
/3)))^(1/2)*(-1/2*I*a*(2*x+1/a*(I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3)))/(
-a^2*b)^(1/3))^(1/2)/(a*x^3+b)^(1/2)*(-I*(-a^2*b)^(1/3)*3^(1/2)*_alpha^2*a^
2+I*(-a^2*b)^(2/3)*3^(1/2)*_alpha*a+I*(-a^2*b)^(1/3)*3^(1/2)*_alpha*a*c-I*(
-a^2*b)^(2/3)*3^(1/2)*c+(-a^2*b)^(1/3)*_alpha^2*a^2+_alpha*(-a^2*b)^(2/3)*a
-(-a^2*b)^(1/3)*_alpha*a*c-(-a^2*b)^(2/3)*c+2*a^2*b)*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*3^(1/2)*a/(-a^2
*b)^(1/3))^(1/2),1/2/a*(-I*(-a^2*b)^(2/3)*_alpha^2*3^(1/2)*a+I*(-a^2*b)^(2/
3)*_alpha*3^(1/2)*c+I*_alpha*3^(1/2)*a^2*b-3*(-a^2*b)^(2/3)*_alpha^2*a-2*I*
(-a^2*b)^(1/3)*3^(1/2)*a*b-I*3^(1/2)*a*b*c+3*(-a^2*b)^(2/3)*_alpha*c-3*_alp
ha*a^2*b+3*a*b*c)/b/c,(I*3^(1/2)/a*(-a^2*b)^(1/3)/(-3/2/a*(-a^2*b)^(1/3)+1/
2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*a-_Z^2*c+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3-2\,b}{\left(ax^3-cx^2+b\right)\sqrt{ax^3+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*b)/(a*x^3+b)^(1/2)/(a*x^3-c*x^2+b),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^3 - 2*b)/((a*x^3 - c*x^2 + b)*sqrt(a*x^3 + b)), x)
```

mupad [B] time = 1.11, size = 40, normalized size = 1.54

$$\frac{\ln\left(\frac{\sqrt{c}\,x-\sqrt{a\,x^3+b}}{\sqrt{c}\,x+\sqrt{a\,x^3+b}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*b - a*x^3)/((b + a*x^3)^(1/2)*(b + a*x^3 - c*x^2)),x)
```

```
[Out] log((c^(1/2)*x - (b + a*x^3)^(1/2))/(c^(1/2)*x + (b + a*x^3)^(1/2)))/c^(1/2
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3-2b}{\sqrt{ax^3+b}\,\left(ax^3+b-cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-2*b)/(a*x**3+b)**(1/2)/(a*x**3-c*x**2+b),x)
```

```
[Out] Integral((a*x**3 - 2*b)/(sqrt(a*x**3 + b)*(a*x**3 + b - c*x**2)), x)
```

3.277 $\int \frac{-2b+ax^3}{\sqrt{b+ax^3}\left(b+cx^2+ax^3\right)}\,dx$

Optimal. Leaf size=26

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{ax^3+b}}\right)}{\sqrt{c}}$$

Rubi [F] time = 0.91, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2b+ax^3}{\sqrt{b+ax^3}\left(b+cx^2+ax^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*b + a*x^3)/(Sqrt[b + a*x^3]*(b + c*x^2 + a*x^3)),x]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*(b^(1/3) + a^(1/3)*x)*Sqrt[(b^(2/3) - a^(1/3)*b^(1/3)*
x + a^(2/3)*x^2)/((1 + Sqrt[3])*b^(1/3) + a^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*b^(1/3) + a^(1/3)*x)/((1 + Sqrt[3])*b^(1/3) + a^(1/3)*x)], -7
- 4*Sqrt[3]])/(3^(1/4)*a^(1/3)*Sqrt[(b^(1/3)*(b^(1/3) + a^(1/3)*x))/((1 + S
qrt[3])*b^(1/3) + a^(1/3)*x)^2]*Sqrt[b + a*x^3]) - 3*b*Defer[Int][1/(Sqrt[b
 + a*x^3]*(b + c*x^2 + a*x^3)), x] - c*Defer[Int][x^2/(Sqrt[b + a*x^3]*(b +
 c*x^2 + a*x^3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2b+ax^3}{\sqrt{b+ax^3}\left(b+cx^2+ax^3\right)}\,dx &= \int \left(\frac{1}{\sqrt{b+ax^3}} - \frac{3b+cx^2}{\sqrt{b+ax^3}\left(b+cx^2+ax^3\right)}\right)dx \\
&= \int \frac{1}{\sqrt{b+ax^3}}\,dx - \int \frac{3b+cx^2}{\sqrt{b+ax^3}\left(b+cx^2+ax^3\right)}\,dx \\
&= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)\sqrt{\frac{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+a^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x}\right)\middle|-7-\right.}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x\right)^2}}\sqrt{b+ax^3}} \\
&= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)\sqrt{\frac{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+a^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x}\right)\middle|-7-\right.}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x\right)^2}}\sqrt{b+ax^3}}
\end{aligned}$$

Mathematica [C] time = 6.32, size = 2725, normalized size = 104.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-2*b + a*x^3)/(Sqrt[b + a*x^3]*(b + c*x^2 + a*x^3)),x]
```

```
[Out] (2*Sqrt[(b^(1/3)/a^(1/3) + x)/(b^(1/3)/a^(1/3) + ((-1)^(1/3)*b^(1/3))/a^(1/
3))]*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x)*Sqrt[(((-1)^(2/3)*b^(1/3))/a^(1/
```

```
3) + x)/(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))]*Elli
pticF[ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) - a^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3
))*b^(1/3))]], (-1)^(1/3)])/(Sqrt[(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x)/(-(
((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))]*Sqrt[b + a*x
^3]) - (4*(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))*b*S
qrt[(b^(1/3)/a^(1/3) + x)/(b^(1/3)/a^(1/3) + ((-1)^(1/3)*b^(1/3))/a^(1/3))]
*Sqrt[((-(((-1)^(2/3)*b^(1/3))/a^(1/3)) - x)*(-(((-1)^(1/3)*b^(1/3))/a^(1/3
)) + x))/(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*E
llipticPi[((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) - a
^(1/3)*Root[b + c*#1^2 + a*#1^3 & , 1]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) -
a^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)])/(a*Sqrt[b +
a*x^3]*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + Root[b + c*#1^2 + a*#1^3 & , 1])*
(Root[b + c*#1^2 + a*#1^3 & , 1] - Root[b + c*#1^2 + a*#1^3 & , 2])*(Root[b
 + c*#1^2 + a*#1^3 & , 1] - Root[b + c*#1^2 + a*#1^3 & , 3])) + (2*(((-1)^(
1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))*Sqrt[(b^(1/3)/a^(1/3)
 + x)/(b^(1/3)/a^(1/3) + ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[((-(((-1)^(2/3
)*b^(1/3))/a^(1/3)) - x)*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x))/(((-1)^(1/3
)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*EllipticPi[((-1)^(1/3
)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) - a^(1/3)*Root[b + c*#1
^2 + a*#1^3 & , 1]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) - a^(1/3)*x)/(((-1)^(1
/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)]*Root[b + c*#1^2 + a*#1^3 & , 1]^3
)/(Sqrt[b + a*x^3]*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + Root[b + c*#1^2 + a*#
1^3 & , 1])*(Root[b + c*#1^2 + a*#1^3 & , 1] - Root[b + c*#1^2 + a*#1^3 & ,
 2])*(Root[b + c*#1^2 + a*#1^3 & , 1] - Root[b + c*#1^2 + a*#1^3 & , 3])) -
 (4*(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))*b*Sqrt[(b
^(1/3)/a^(1/3) + x)/(b^(1/3)/a^(1/3) + ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[
((-(((-1)^(2/3)*b^(1/3))/a^(1/3)) - x)*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x
))/(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*Ellipti
cPi[((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) - a^(1/3)
*Root[b + c*#1^2 + a*#1^3 & , 2]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) - a^(1/3
)*x)/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)])/(a*Sqrt[b + a*x^3]
*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + Root[b + c*#1^2 + a*#1^3 & , 2])*(-Root
[b + c*#1^2 + a*#1^3 & , 1] + Root[b + c*#1^2 + a*#1^3 & , 2])*(Root[b + c*
#1^2 + a*#1^3 & , 2] - Root[b + c*#1^2 + a*#1^3 & , 3])) + (2*(((-1)^(1/3)*
b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))*Sqrt[(b^(1/3)/a^(1/3) + x)
/(b^(1/3)/a^(1/3) + ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[((-(((-1)^(2/3)*b^(
1/3))/a^(1/3)) - x)*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x))/(((-1)^(1/3)*b^(
1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*EllipticPi[((-1)^(1/3)*b^(
1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) - a^(1/3)*Root[b + c*#1^2 +
a*#1^3 & , 2]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) - a^(1/3)*x)/(((-1)^(1/3) +
 (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)]*Root[b + c*#1^2 + a*#1^3 & , 2]^3)/(Sq
rt[b + a*x^3]*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + Root[b + c*#1^2 + a*#1^3 &
 , 2])*(-Root[b + c*#1^2 + a*#1^3 & , 1] + Root[b + c*#1^2 + a*#1^3 & , 2])
*(Root[b + c*#1^2 + a*#1^3 & , 2] - Root[b + c*#1^2 + a*#1^3 & , 3])) - (4*
(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))*b*Sqrt[(b^(1/
3)/a^(1/3) + x)/(b^(1/3)/a^(1/3) + ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[((-(
((-1)^(2/3)*b^(1/3))/a^(1/3)) - x)*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x))/(
((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*EllipticPi[
((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) - a^(1/3)*Roo
t[b + c*#1^2 + a*#1^3 & , 3]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) - a^(1/3)*x)
/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)])/(a*Sqrt[b + a*x^3]*(-(
((-1)^(1/3)*b^(1/3))/a^(1/3)) + Root[b + c*#1^2 + a*#1^3 & , 3])*(-Root[b +
 c*#1^2 + a*#1^3 & , 1] + Root[b + c*#1^2 + a*#1^3 & , 3])*(-Root[b + c*#1^
2 + a*#1^3 & , 2] + Root[b + c*#1^2 + a*#1^3 & , 3])) + (2*(((-1)^(1/3)*b^(
1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))*Sqrt[(b^(1/3)/a^(1/3) + x)/(b
^(1/3)/a^(1/3) + ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[((-(((-1)^(2/3)*b^(1/3
))/a^(1/3)) - x)*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x))/(((-1)^(1/3)*b^(1/3
))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*EllipticPi[((-1)^(1/3)*b^(1/3
) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) - a^(1/3)*Root[b + c*#1^2 + a*#
```

```
1^3 & , 3]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) - a^(1/3)*x)/(((-1)^(1/3) + (-
1)^(2/3))*b^(1/3))]], (-1)^(1/3)]*Root[b + c*#1^2 + a*#1^3 & , 3]^3)/(Sqrt[
b + a*x^3]*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + Root[b + c*#1^2 + a*#1^3 & ,
3])*(-Root[b + c*#1^2 + a*#1^3 & , 1] + Root[b + c*#1^2 + a*#1^3 & , 3])*(-
Root[b + c*#1^2 + a*#1^3 & , 2] + Root[b + c*#1^2 + a*#1^3 & , 3]))
```

IntegrateAlgebraic [A] time = 0.62, size = 26, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{ax^3+b}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*b + a*x^3)/(Sqrt[b + a*x^3]*(b + c*x^2 + a*x^3)),x]
```

```
[Out] (-2*ArcTan[(Sqrt[c]*x)/Sqrt[b + a*x^3]])/Sqrt[c]
```

fricas [B] time = 0.47, size = 169, normalized size = 6.50

$$\left[-\frac{\sqrt{-c}\log\left(\frac{a^2x^6-6\,acx^5+c^2x^4+2\,abx^3-6\,bcx^2+b^2-4\left(ax^4-cx^3+bx\right)\sqrt{ax^3+b}\,\sqrt{-c}}{a^2x^6+2\,acx^5+c^2x^4+2\,abx^3+2\,bcx^2+b^2}\right)}{2\,c}, \frac{\arctan\left(\frac{\left(ax^3-cx^2+b\right)\sqrt{ax^3+b}\,\sqrt{c}}{2\left(acx^4+bcx\right)}\right)}{\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*b)/(a*x^3+b)^(1/2)/(a*x^3+c*x^2+b),x, algorithm="fricas"
)
```

```
[Out] [-1/2*sqrt(-c)*log((a^2*x^6 - 6*a*c*x^5 + c^2*x^4 + 2*a*b*x^3 - 6*b*c*x^2 +
 b^2 - 4*(a*x^4 - c*x^3 + b*x)*sqrt(a*x^3 + b)*sqrt(-c))/(a^2*x^6 + 2*a*c*x
^5 + c^2*x^4 + 2*a*b*x^3 + 2*b*c*x^2 + b^2))/c, arctan(1/2*(a*x^3 - c*x^2 +
 b)*sqrt(a*x^3 + b)*sqrt(c)/(a*c*x^4 + b*c*x))/sqrt(c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 - 2\,b}{\left(ax^3 + cx^2 + b\right)\sqrt{ax^3 + b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*b)/(a*x^3+b)^(1/2)/(a*x^3+c*x^2+b),x, algorithm="giac")
```

```
[Out] integrate((a*x^3 - 2*b)/((a*x^3 + c*x^2 + b)*sqrt(a*x^3 + b)), x)
```

maple [C] time = 0.17, size = 842, normalized size = 32.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-2*b)/(a*x^3+b)^(1/2)/(a*x^3+c*x^2+b),x)
```

```
[Out] -2/3*I*3^(1/2)/a*(-a^2*b)^(1/3)*(I*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*
(-a^2*b)^(1/3))*3^(1/2)*a/(-a^2*b)^(1/3))^(1/2)*((x-1/a*(-a^2*b)^(1/3))/(-3
/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2)*(-I*(x+1/2/a*(-a
^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*3^(1/2)*a/(-a^2*b)^(1/3))^(1/2)
/(a*x^3+b)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(
1/2)/a*(-a^2*b)^(1/3))*3^(1/2)*a/(-a^2*b)^(1/3))^(1/2),(I*3^(1/2)/a*(-a^2*b
)^(1/3)/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))-I/a^
2/b/c*2^(1/2)*sum((-_alpha^2*c-3*b)/_alpha/(3*_alpha*a+2*c)*(-a^2*b)^(1/3)*
(1/2*I*a*(2*x+1/a*(-I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3)))/(-a^2*b)^(1/3
```

```
))^(1/2)*(a*(x-1/a*(-a^2*b)^(1/3))/(-3*(-a^2*b)^(1/3)+I*3^(1/2)*(-a^2*b)^(1
/3)))^(1/2)*(-1/2*I*a*(2*x+1/a*(I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3)))/(
-a^2*b)^(1/3))^(1/2)/(a*x^3+b)^(1/2)*(-I*(-a^2*b)^(1/3)*3^(1/2)*_alpha^2*a^
2+I*(-a^2*b)^(2/3)*3^(1/2)*_alpha*a-I*(-a^2*b)^(1/3)*3^(1/2)*_alpha*a*c+I*(
-a^2*b)^(2/3)*3^(1/2)*c+(-a^2*b)^(1/3)*_alpha^2*a^2+_alpha*(-a^2*b)^(2/3)*a
+(-a^2*b)^(1/3)*_alpha*a*c+(-a^2*b)^(2/3)*c+2*a^2*b)*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*3^(1/2)*a/(-a^2
*b)^(1/3))^(1/2),-1/2/a*(-I*(-a^2*b)^(2/3)*_alpha^2*3^(1/2)*a-I*(-a^2*b)^(2
/3)*3^(1/2)*_alpha*c+I*_alpha*3^(1/2)*a^2*b-3*(-a^2*b)^(2/3)*_alpha^2*a-2*I
*(-a^2*b)^(1/3)*3^(1/2)*a*b+I*3^(1/2)*a*b*c-3*(-a^2*b)^(2/3)*_alpha*c-3*_al
pha*a^2*b-3*a*b*c)/b/c,(I*3^(1/2)/a*(-a^2*b)^(1/3)/(-3/2/a*(-a^2*b)^(1/3)+1
/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*a+_Z^2*c+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 - 2b}{\left(ax^3 + cx^2 + b\right)\sqrt{ax^3 + b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*b)/(a*x^3+b)^(1/2)/(a*x^3+c*x^2+b),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^3 - 2*b)/((a*x^3 + c*x^2 + b)*sqrt(a*x^3 + b)), x)
```

mupad [B] time = 3.38, size = 51, normalized size = 1.96

$$\frac{\ln\left(\frac{b+a\,x^3-c\,x^2+\sqrt{c}\,x\,\sqrt{a\,x^3+b}\,2\mathrm{i}}{a\,x^3+c\,x^2+b}\right)\,1\mathrm{i}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*b - a*x^3)/((b + a*x^3)^(1/2)*(b + a*x^3 + c*x^2)),x)
```

```
[Out] (log((b + a*x^3 - c*x^2 + c^(1/2)*x*(b + a*x^3)^(1/2)*2i)/(b + a*x^3 + c*x^
2))*1i)/c^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 - 2b}{\sqrt{ax^3 + b}\,\left(ax^3 + b + cx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-2*b)/(a*x**3+b)**(1/2)/(a*x**3+c*x**2+b),x)
```

```
[Out] Integral((a*x**3 - 2*b)/(sqrt(a*x**3 + b)*(a*x**3 + b + c*x**2)), x)
```

3.278 $\int \frac{1+x^2}{\left(-1+x^2\right)\left(2+x^2\right)\sqrt{-3+x^4}}\,dx$

Optimal. Leaf size=26

$$\frac{\tan^{-1}\left(\frac{x\sqrt{x^4-3}}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Rubi [C] time = 0.65, antiderivative size = 131, normalized size of antiderivative = 5.04, number of steps used = 18, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {6725, 1215, 223, 1457, 540, 253, 538, 537}

$$\frac{\sqrt{\sqrt{3}-x^2}\sqrt{\sqrt{3}x^2+3}\,\Pi\left(-\frac{\sqrt{3}}{2};\sin^{-1}\left(\frac{x}{\sqrt[4]{3}}\right)\middle|-1\right)}{6\sqrt{3}\sqrt{x^4-3}}-\frac{2\sqrt{\sqrt{3}-x^2}\sqrt{\sqrt{3}x^2+3}\,\Pi\left(\sqrt{3};\sin^{-1}\left(\frac{x}{\sqrt[4]{3}}\right)\middle|-1\right)}{3\sqrt{3}\sqrt{x^4-3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^2)/((-1 + x^2)*(2 + x^2)*Sqrt[-3 + x^4]),x]
```

```
[Out] (Sqrt[Sqrt[3] - x^2]*Sqrt[3 + Sqrt[3]*x^2]*EllipticPi[-1/2*Sqrt[3], ArcSin[
x/3^(1/4)], -1])/(6*Sqrt[3]*Sqrt[-3 + x^4]) - (2*Sqrt[Sqrt[3] - x^2]*Sqrt[3
 + Sqrt[3]*x^2]*EllipticPi[Sqrt[3], ArcSin[x/3^(1/4)], -1])/(3*Sqrt[3]*Sqrt
[-3 + x^4])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[(a - q*x^2)/(a + q*x^2)]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqr
t[(a + q*x^2)/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[a + b*x^4]*Sqrt[a/(a + q*x^2)]),
 x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 540

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x
] + Dist[(b*c - a*d)/b, Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

Rule 1215

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[c/(c*d + e*q), Int[1/Sqrt[a + c*x^4], x], x] + Di
st[e/(c*d + e*q), Int[(q - c*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
 FreeQ[{a, c, d, e}, x] && GtQ[-(a*c), 0] &&  !LtQ[c, 0]
```

Rule 1457

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + c*x^(2*n))^FracPart[p]/((d +
e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p]), Int[(d + e*x^n)^(p + q)*
(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, n, p,
 q, r}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^2}{\left(-1+x^2\right)\left(2+x^2\right)\sqrt{-3+x^4}}\,dx = \int \left(\frac{2}{3\left(-1+x^2\right)\sqrt{-3+x^4}}+\frac{1}{3\left(2+x^2\right)\sqrt{-3+x^4}}\right)dx$$

$$= \frac{1}{3}\int \frac{1}{\left(2+x^2\right)\sqrt{-3+x^4}}\,dx+\frac{2}{3}\int \frac{1}{\left(-1+x^2\right)\sqrt{-3+x^4}}\,dx$$

$$= \frac{2\int \frac{1}{\sqrt{-3+x^4}}\,dx}{3\left(-1+\sqrt{3}\right)}+\frac{2\int \frac{\sqrt{3}-x^2}{\left(-1+x^2\right)\sqrt{-3+x^4}}\,dx}{3\left(-1+\sqrt{3}\right)}+\frac{\int \frac{1}{\sqrt{-3+x^4}}\,dx}{3\left(2+\sqrt{3}\right)}+\frac{\int \frac{\sqrt{3}-x^2}{\left(2+x^2\right)\sqrt{-3+x^4}}}{3\left(2+\sqrt{3}\right)}\cdots$$

$$= -\frac{\sqrt{2}\sqrt{\frac{\sqrt{3}+x^2}{\sqrt{3}-x^2}}\sqrt{-3+\sqrt{3}x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}x}{\sqrt{-3+\sqrt{3}x^2}}\right)|\frac{1}{2}\right)}{3\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt{\frac{1}{3-\sqrt{3}x^2}}\sqrt{-3+x^4}}+\frac{\sqrt{\frac{\sqrt{3}+x^2}{\sqrt{3}-x^2}}\sqrt{-3+\cdots}}{3\sqrt{2}\,3^{3/4}\left(2\cdots\right.}$$

$$= -\frac{\sqrt{2}\sqrt{\frac{\sqrt{3}+x^2}{\sqrt{3}-x^2}}\sqrt{-3+\sqrt{3}x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}x}{\sqrt{-3+\sqrt{3}x^2}}\right)|\frac{1}{2}\right)}{3\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt{\frac{1}{3-\sqrt{3}x^2}}\sqrt{-3+x^4}}+\frac{\sqrt{\frac{\sqrt{3}+x^2}{\sqrt{3}-x^2}}\sqrt{-3+\cdots}}{3\sqrt{2}\,3^{3/4}\left(2\cdots\right.}$$

$$= -\frac{\sqrt{2}\sqrt{\frac{\sqrt{3}+x^2}{\sqrt{3}-x^2}}\sqrt{-3+\sqrt{3}x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}x}{\sqrt{-3+\sqrt{3}x^2}}\right)|\frac{1}{2}\right)}{3\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt{\frac{1}{3-\sqrt{3}x^2}}\sqrt{-3+x^4}}+\frac{\sqrt{\frac{\sqrt{3}+x^2}{\sqrt{3}-x^2}}\sqrt{-3+\cdots}}{3\sqrt{2}\,3^{3/4}\left(2\cdots\right.}$$

$$= \frac{\sqrt{\sqrt{3}-x^2}\sqrt{3+\sqrt{3}x^2}\,\Pi\left(-\frac{\sqrt{3}}{2};\sin^{-1}\left(\frac{x}{\sqrt[4]{3}}\right)\middle|-1\right)}{6\sqrt{3}\sqrt{-3+x^4}}-\frac{2\sqrt{\sqrt{3}-x^2}\sqrt{3+\cdots}}{3\cdots}$$

Mathematica [C] time = 0.23, size = 74, normalized size = 2.85

$$\frac{\sqrt{3-x^4}\left(\Pi\left(-\frac{\sqrt{3}}{2};\sin^{-1}\left(\frac{x}{\sqrt[4]{3}}\right)\middle|-1\right)+4i\Pi\left(-\sqrt{3};i\sinh^{-1}\left(\frac{x}{\sqrt[4]{3}}\right)\middle|-1\right)\right)}{6\sqrt[4]{3}\sqrt{x^4-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/((-1 + x^2)*(2 + x^2)*Sqrt[-3 + x^4]),x]
[Out] (Sqrt[3 - x^4]*((4*I)*EllipticPi[-Sqrt[3], I*ArcSinh[x/3^(1/4)], -1] + Elli
pticPi[-1/2*Sqrt[3], ArcSin[x/3^(1/4)], -1]))/(6*3^(1/4)*Sqrt[-3 + x^4])
```

IntegrateAlgebraic [A] time = 13.44, size = 26, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x\sqrt{x^4-3}}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 + x^2)*(2 + x^2)*Sqrt[-3 + x^4]),x]
[Out] ArcTan[(x*Sqrt[-3 + x^4])/Sqrt[2]]/(3*Sqrt[2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^2+2)/(x^4-3)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x^4-3}\left(x^2+2\right)\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^2+2)/(x^4-3)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 + 1)/(sqrt(x^4 - 3)*(x^2 + 2)*(x^2 - 1)), x)
```

maple [C] time = 0.10, size = 136, normalized size = 5.23

$$-\frac{2\sqrt{1+\frac{\sqrt{3}\,x^2}{3}}\,\sqrt{1-\frac{\sqrt{3}\,x^2}{3}}\;\mathrm{EllipticPi}\left(\sqrt{-\frac{\sqrt{3}}{3}}\,x,-\sqrt{3},\frac{3^{\frac{3}{4}}}{3\sqrt{-\frac{\sqrt{3}}{3}}}\right)}{3\sqrt{-\frac{\sqrt{3}}{3}}\,\sqrt{x^4-3}}+\frac{\sqrt{1+\frac{\sqrt{3}\,x^2}{3}}\,\sqrt{1-\frac{\sqrt{3}\,x^2}{3}}\;\mathrm{EllipticPi}\left(\sqrt{-\frac{\sqrt{3}}{3}}\,x,\frac{\sqrt{3}}{2},\frac{3^{\frac{3}{4}}}{3\sqrt{-\frac{\sqrt{3}}{3}}}\right)}{6\sqrt{-\frac{\sqrt{3}}{3}}\,\sqrt{x^4-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-1)/(x^2+2)/(x^4-3)^(1/2),x)
[Out] -2/3/(-1/3*3^(1/2))^(1/2)*(1+1/3*3^(1/2)*x^2)^(1/2)*(1-1/3*3^(1/2)*x^2)^(1/
2)/(x^4-3)^(1/2)*EllipticPi((-1/3*3^(1/2))^(1/2)*x,-3^(1/2),1/3*3^(3/4)/(-1
/3*3^(1/2))^(1/2))+1/6/(-1/3*3^(1/2))^(1/2)*(1+1/3*3^(1/2)*x^2)^(1/2)*(1-1/
```

```
3*3^(1/2)*x^2)^(1/2)/(x^4-3)^(1/2)*EllipticPi((-1/3*3^(1/2))^(1/2)*x,1/2*3^
(1/2),1/3*3^(3/4)/(-1/3*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x^4-3}\left(x^2+2\right)\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^2+2)/(x^4-3)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/(sqrt(x^4 - 3)*(x^2 + 2)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2+1}{\left(x^2-1\right)\left(x^2+2\right)\sqrt{x^4-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((x^2 - 1)*(x^2 + 2)*(x^4 - 3)^(1/2)),x)
[Out] int((x^2 + 1)/((x^2 - 1)*(x^2 + 2)*(x^4 - 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{(x-1)(x+1)\left(x^2+2\right)\sqrt{x^4-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-1)/(x**2+2)/(x**4-3)**(1/2),x)
[Out] Integral((x**2 + 1)/((x - 1)*(x + 1)*(x**2 + 2)*sqrt(x**4 - 3)), x)
```

3.279 $\int \frac{(-4+x^4)(1+x^4)^{3/4}}{x^{12}} dx$

Optimal. Leaf size=26

$$\frac{(x^4+1)^{3/4}(-27x^8+x^4+28)}{77x^{11}}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {453, 264}

$$\frac{4(x^4+1)^{7/4}}{11x^{11}} - \frac{27(x^4+1)^{7/4}}{77x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((-4 + x^4)*(1 + x^4)^(3/4))/x^12,x]
```

```
[Out] (4*(1 + x^4)^(7/4))/(11*x^11) - (27*(1 + x^4)^(7/4))/(77*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(-4+x^4)(1+x^4)^{3/4}}{x^{12}} dx &= \frac{4(1+x^4)^{7/4}}{11x^{11}} + \frac{27}{11}\int \frac{(1+x^4)^{3/4}}{x^8} dx \\ &= \frac{4(1+x^4)^{7/4}}{11x^{11}} - \frac{27(1+x^4)^{7/4}}{77x^7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.88

$$\frac{(28-27x^4)(x^4+1)^{7/4}}{77x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-4 + x^4)*(1 + x^4)^(3/4))/x^12,x]
```

```
[Out] ((28 - 27*x^4)*(1 + x^4)^(7/4))/(77*x^11)
```

IntegrateAlgebraic [A] time = 0.17, size = 23, normalized size = 0.88

$$\frac{(28-27x^4)(x^4+1)^{7/4}}{77x^{11}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + x^4)*(1 + x^4)^(3/4))/x^12,x]
[Out] ((28 - 27*x^4)*(1 + x^4)^(7/4))/(77*x^11)
```

fricas [A] time = 0.42, size = 24, normalized size = 0.92

$$-\frac{\left(27\,x^{8}-x^{4}-28\right)\left(x^{4}+1\right)^{\frac{3}{4}}}{77\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4)*(x^4+1)^(3/4)/x^12,x, algorithm="fricas")
[Out] -1/77*(27*x^8 - x^4 - 28)*(x^4 + 1)^(3/4)/x^11
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}+1\right)^{\frac{3}{4}}\left(x^{4}-4\right)}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4)*(x^4+1)^(3/4)/x^12,x, algorithm="giac")
[Out] integrate((x^4 + 1)^(3/4)*(x^4 - 4)/x^12, x)
```

maple [A] time = 0.01, size = 20, normalized size = 0.77

$$-\frac{\left(27x^{4}-28\right)\left(x^{4}+1\right)^{\frac{7}{4}}}{77x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-4)*(x^4+1)^(3/4)/x^12,x)
[Out] -1/77*(27*x^4-28)*(x^4+1)^(7/4)/x^11
```

maxima [A] time = 0.36, size = 25, normalized size = 0.96

$$-\frac{5\left(x^{4}+1\right)^{\frac{7}{4}}}{7\,x^{7}}+\frac{4\left(x^{4}+1\right)^{\frac{11}{4}}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4)*(x^4+1)^(3/4)/x^12,x, algorithm="maxima")
[Out] -5/7*(x^4 + 1)^(7/4)/x^7 + 4/11*(x^4 + 1)^(11/4)/x^11
```

mupad [B] time = 0.31, size = 38, normalized size = 1.46

$$\frac{28\left(x^{4}+1\right)^{3/4}+x^{4}\left(x^{4}+1\right)^{3/4}-27\,x^{8}\left(x^{4}+1\right)^{3/4}}{77\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(3/4)*(x^4 - 4))/x^12,x)
[Out] (28*(x^4 + 1)^(3/4) + x^4*(x^4 + 1)^(3/4) - 27*x^8*(x^4 + 1)^(3/4))/(77*x^1
1)
```

sympy [B] time = 3.37, size = 136, normalized size = 5.23

$$\frac{\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4\Gamma\left(-\frac{3}{4}\right)}-\frac{\left(x^4+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{x^3\Gamma\left(-\frac{3}{4}\right)}+\frac{\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4x^4\Gamma\left(-\frac{3}{4}\right)}+\frac{3\left(x^4+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}+\frac{7\left(x^4+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{4x^{11}\Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-4)*(x**4+1)**(3/4)/x**12,x)
```

```
[Out] (1 + x**(-4))**(3/4)*gamma(-7/4)/(4*gamma(-3/4)) - (x**4 + 1)**(3/4)*gamma(
-11/4)/(x**3*gamma(-3/4)) + (1 + x**(-4))**(3/4)*gamma(-7/4)/(4*x**4*gamma(
-3/4)) + 3*(x**4 + 1)**(3/4)*gamma(-11/4)/(4*x**7*gamma(-3/4)) + 7*(x**4 +
1)**(3/4)*gamma(-11/4)/(4*x**11*gamma(-3/4))
```

3.280 $\int \frac{\sqrt{-1+x^4}\left(1+x^4\right)}{x^2\left(-1+x^2+x^4\right)}\,dx$

Optimal. Leaf size=26

$$\frac{\sqrt{x^4-1}}{x}+\tan^{-1}\left(\frac{x}{\sqrt{x^4-1}}\right)$$

Rubi [C] time = 0.98, antiderivative size = 291, normalized size of antiderivative = 11.19, number of steps used = 32, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {6728, 277, 306, 222, 1185, 1209, 1188, 1215, 1457, 540, 253, 538, 537}

$$\frac{\sqrt{x^4-1}}{x}+\frac{\left(1+\sqrt{5}\right)\sqrt{x^2-1}\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}+\frac{\left(1-\sqrt{5}\right)\sqrt{x^2-1}\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}-\frac{\sqrt{2}\sqrt{x^2-1}\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^4-1}}+\frac{\sqrt{1-x^2}\sqrt{x^2+1}\,\Pi\left(\frac{1}{2}\left(1-\sqrt{5}\right);\sin^{-1}(x)|-1\right)}{\sqrt{x^4-1}}+\frac{\sqrt{1-x^2}\sqrt{x^2+1}\,\Pi\left(\frac{1}{2}\left(1+\sqrt{5}\right);\sin^{-1}(x)|-1\right)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^4]*(1 + x^4))/(x^2*(-1 + x^2 + x^4)),x]
```

```
[Out] Sqrt[-1 + x^4]/x - (Sqrt[2]*Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(
Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/Sqrt[-1 + x^4] + ((1 - Sqrt[5])*Sqrt[-1 +
 x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(2*
Sqrt[2]*Sqrt[-1 + x^4]) + ((1 + Sqrt[5])*Sqrt[-1 + x^2]*Sqrt[1 + x^2]*Ellip
ticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(2*Sqrt[2]*Sqrt[-1 + x^4]) +
 (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticPi[(1 - Sqrt[5])/2, ArcSin[x], -1])/S
qrt[-1 + x^4] + (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticPi[(1 + Sqrt[5])/2, Ar
cSin[x], -1])/Sqrt[-1 + x^4]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 306

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[
a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 540

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x
] + Dist[(b*c - a*d)/b, Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Simp[(e*x*(q + c*x^2))/(c*Sqrt[a + c*x^4]), x] - Simp[(Sqr
t[2]*e*q*Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticE[ArcSin[x/Sqrt[(a +
q*x^2)/(2*q)]], 1/2])/(Sqrt[-a]*c*Sqrt[a + c*x^4]), x] /; EqQ[c*d + e*q, 0]
 && IntegerQ[q]] /; FreeQ[{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[(c*d + e*q)/c, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e
/c, Int[(q - c*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[c*d + e*q, 0]] /; FreeQ[
{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1215

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[c/(c*d + e*q), Int[1/Sqrt[a + c*x^4], x], x] + Di
st[e/(c*d + e*q), Int[(q - c*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
 FreeQ[{a, c, d, e}, x] && GtQ[-(a*c), 0] &&  !LtQ[c, 0]
```

Rule 1457

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + c*x^(2*n))^FracPart[p]/((d +
e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p]), Int[(d + e*x^n)^(p + q)*
(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, n, p,
 q, r}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{-1+x^4}\left(1+x^4\right)}{x^2\left(-1+x^2+x^4\right)}\,dx = \int \left(-\frac{\sqrt{-1+x^4}}{x^2}+\frac{\left(1+2x^2\right)\sqrt{-1+x^4}}{-1+x^2+x^4}\right)dx$$

$$= -\int \frac{\sqrt{-1+x^4}}{x^2}\,dx + \int \frac{\left(1+2x^2\right)\sqrt{-1+x^4}}{-1+x^2+x^4}\,dx$$

$$= \frac{\sqrt{-1+x^4}}{x} - 2\int \frac{x^2}{\sqrt{-1+x^4}}\,dx + \int \left(\frac{2\sqrt{-1+x^4}}{1-\sqrt{5}+2x^2}+\frac{2\sqrt{-1+x^4}}{1+\sqrt{5}+2x^2}\right)dx$$

$$= \frac{\sqrt{-1+x^4}}{x} - 2\int \frac{1}{\sqrt{-1+x^4}}\,dx + 2\int \frac{1-x^2}{\sqrt{-1+x^4}}\,dx + 2\int \frac{\sqrt{-1+x^4}}{1-\sqrt{5}+2x^2}\,dx + 2\int$$

$$= -\frac{2x\left(1+x^2\right)}{\sqrt{-1+x^4}} + \frac{\sqrt{-1+x^4}}{x} + \frac{2\sqrt{2}\sqrt{-1+x^2}\sqrt{1+x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{\sqrt{-1+x^4}} - \sqrt{2}$$

$$= -\frac{2x\left(1+x^2\right)}{\sqrt{-1+x^4}} + \frac{\sqrt{-1+x^4}}{x} + \frac{2\sqrt{2}\sqrt{-1+x^2}\sqrt{1+x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{\sqrt{-1+x^4}} - \sqrt{2}$$

$$= -\frac{2x\left(1+x^2\right)}{\sqrt{-1+x^4}} + \frac{\sqrt{-1+x^4}}{x} + \frac{2\sqrt{2}\sqrt{-1+x^2}\sqrt{1+x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{\sqrt{-1+x^4}} - 2\Bigg(-$$

$$= -\frac{2x\left(1+x^2\right)}{\sqrt{-1+x^4}} + \frac{\sqrt{-1+x^4}}{x} + \frac{2\sqrt{2}\sqrt{-1+x^2}\sqrt{1+x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{\sqrt{-1+x^4}} - 2\Bigg(-$$

$$= -\frac{2x\left(1+x^2\right)}{\sqrt{-1+x^4}} + \frac{\sqrt{-1+x^4}}{x} + \frac{2\sqrt{2}\sqrt{-1+x^2}\sqrt{1+x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{\sqrt{-1+x^4}} - 2\Bigg(-$$

$$= -\frac{2x\left(1+x^2\right)}{\sqrt{-1+x^4}} + \frac{\sqrt{-1+x^4}}{x} + \frac{2\sqrt{2}\sqrt{-1+x^2}\sqrt{1+x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{\sqrt{-1+x^4}} - 2\Bigg(-$$

Mathematica [C] time = 0.42, size = 92, normalized size = 3.54

$$\frac{x^4-\sqrt{1-x^4}\,xF\left(\sin^{-1}(x)|-1\right)+\sqrt{1-x^4}\,x\Pi\left(-\frac{2}{1+\sqrt{5}};\sin^{-1}(x)|-1\right)+\sqrt{1-x^4}\,x\Pi\left(\frac{1}{2}\left(1+\sqrt{5}\right);\sin^{-1}(x)|-1\right)-1}{x\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^4]*(1 + x^4))/(x^2*(-1 + x^2 + x^4)),x]
```

```
[Out] (-1 + x^4 - x*Sqrt[1 - x^4]*EllipticF[ArcSin[x], -1] + x*Sqrt[1 - x^4]*Elli
pticPi[-2/(1 + Sqrt[5]), ArcSin[x], -1] + x*Sqrt[1 - x^4]*EllipticPi[(1 + S
qrt[5])/2, ArcSin[x], -1])/(x*Sqrt[-1 + x^4])
```

IntegrateAlgebraic [A] time = 0.40, size = 26, normalized size = 1.00

$$\frac{\sqrt{x^4-1}}{x}+\tan^{-1}\left(\frac{x}{\sqrt{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^4]*(1 + x^4))/(x^2*(-1 + x^2 + x^4)),x]
```

```
[Out] Sqrt[-1 + x^4]/x + ArcTan[x/Sqrt[-1 + x^4]]
```

fricas [A] time = 0.54, size = 40, normalized size = 1.54

$$\frac{x \arctan\left(\frac{2\sqrt{x^4-1}\,x}{x^4-x^2-1}\right)+2\sqrt{x^4-1}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/2)*(x^4+1)/x^2/(x^4+x^2-1),x, algorithm="fricas")
```

```
[Out] 1/2*(x*arctan(2*sqrt(x^4 - 1)*x/(x^4 - x^2 - 1)) + 2*sqrt(x^4 - 1))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)\sqrt{x^4-1}}{\left(x^4+x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/2)*(x^4+1)/x^2/(x^4+x^2-1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)*sqrt(x^4 - 1)/((x^4 + x^2 - 1)*x^2), x)
```

maple [C] time = 0.04, size = 146, normalized size = 5.62

$$\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\mathrm{EllipticF}(ix,i)}{\sqrt{x^4-1}}-\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}(\_Z^4+\_Z^2-1)}\_\alpha\left(-\frac{\mathrm{arctanh}\left(\frac{\_\alpha^2(-\_\alpha^2+x^2-1)}{\sqrt{-\_\alpha^2}\,\sqrt{x^4-1}}\right)}{\sqrt{-\_\alpha^2}}+\frac{2i(\_\alpha^3+\_\alpha)\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\mathrm{EllipticPi}(ix,-\_\alpha^2-1,i)}{\sqrt{x^4-1}}\right)\right)}{4}+\frac{\sqrt{x^4-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(1/2)*(x^4+1)/x^2/(x^4+x^2-1),x)
```

```
[Out] I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*EllipticF(I*x,I)-1/4*sum(_alph
a*(-1/(-_alpha^2)^(1/2)*arctanh(_alpha^2*(-_alpha^2+x^2-1)/(-_alpha^2)^(1/2
)/(x^4-1)^(1/2))+2*I*(_alpha^3+_alpha)*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)
^(1/2)*EllipticPi(I*x,-_alpha^2-1,I)),_alpha=RootOf(_Z^4+_Z^2-1))+(x^4-1)^(
1/2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)\sqrt{x^4-1}}{\left(x^4+x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/2)*(x^4+1)/x^2/(x^4+x^2-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)*sqrt(x^4 - 1)/((x^4 + x^2 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{x^4-1}\,\left(x^4+1\right)}{x^2\,\left(x^4+x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)^(1/2)*(x^4 + 1))/(x^2*(x^2 + x^4 - 1)),x)
```

```
[Out] int(((x^4 - 1)^(1/2)*(x^4 + 1))/(x^2*(x^2 + x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)\left(x^2+1\right)}\left(x^4+1\right)}{x^2\left(x^4+x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(1/2)*(x**4+1)/x**2/(x**4+x**2-1),x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x**4 + 1)/(x**2*(x**4 + x**2 - 1
)), x)
```

3.281 $\int \frac{(-1+x^4)(1+x^2+x^4)}{x^4\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=26

$$\frac{\sqrt{x^4+1}\,\left(x^4+3x^2+1\right)}{3x^3}$$

Rubi [A] time = 0.12, antiderivative size = 44, normalized size of antiderivative = 1.69, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1835, 1586, 1584, 383}

$$\frac{1}{3}\sqrt{x^4+1}\,x+\frac{\sqrt{x^4+1}}{x}+\frac{\sqrt{x^4+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4)*(1 + x^2 + x^4))/(x^4*Sqrt[1 + x^4]),x]
```

```
[Out] Sqrt[1 + x^4]/(3*x^3) + Sqrt[1 + x^4]/x + (x*Sqrt[1 + x^4])/3
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^4)(1+x^2+x^4)}{x^4\sqrt{1+x^4}}\,dx &= \frac{\sqrt{1+x^4}}{3x^3} - \frac{1}{6}\int \frac{6x-2x^3-6x^5-6x^7}{x^3\sqrt{1+x^4}}\,dx \\
&= \frac{\sqrt{1+x^4}}{3x^3} - \frac{1}{6}\int \frac{6-2x^2-6x^4-6x^6}{x^2\sqrt{1+x^4}}\,dx \\
&= \frac{\sqrt{1+x^4}}{3x^3} + \frac{\sqrt{1+x^4}}{x} + \frac{1}{12}\int \frac{4x+12x^5}{x\sqrt{1+x^4}}\,dx \\
&= \frac{\sqrt{1+x^4}}{3x^3} + \frac{\sqrt{1+x^4}}{x} + \frac{1}{12}\int \frac{4+12x^4}{\sqrt{1+x^4}}\,dx \\
&= \frac{\sqrt{1+x^4}}{3x^3} + \frac{\sqrt{1+x^4}}{x} + \frac{1}{3}x\sqrt{1+x^4}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 93, normalized size = 3.58

$$\frac{{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-x^4\right)+x^4\left(-{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^4\right)+x^2\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-x^4\right)+\sqrt{x^4+1}\right)+3x^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-x^4\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)*(1 + x^2 + x^4))/(x^4*Sqrt[1 + x^4]),x]
```

```
[Out] (Hypergeometric2F1[-3/4, 1/2, 1/4, -x^4] + 3*x^2*Hypergeometric2F1[-1/4, 1/
2, 3/4, -x^4] + x^4*(Sqrt[1 + x^4] - Hypergeometric2F1[1/4, 1/2, 5/4, -x^4]
 + x^2*Hypergeometric2F1[1/2, 3/4, 7/4, -x^4]))/(3*x^3)
```

IntegrateAlgebraic [A] time = 0.47, size = 26, normalized size = 1.00

$$\frac{\sqrt{x^4+1}\left(x^4+3x^2+1\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(1 + x^2 + x^4))/(x^4*Sqrt[1 + x^4]),x]
```

```
[Out] (Sqrt[1 + x^4]*(1 + 3*x^2 + x^4))/(3*x^3)
```

fricas [A] time = 0.40, size = 22, normalized size = 0.85

$$\frac{\left(x^4+3\,x^2+1\right)\sqrt{x^4+1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+x^2+1)/x^4/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^4 + 3*x^2 + 1)*sqrt(x^4 + 1)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^2+1\right)\left(x^4-1\right)}{\sqrt{x^4+1}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+x^2+1)/x^4/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + x^2 + 1)*(x^4 - 1)/(sqrt(x^4 + 1)*x^4), x)
```

maple [A] time = 0.01, size = 23, normalized size = 0.88

$$\frac{\sqrt{x^4+1}\,\left(x^4+3x^2+1\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^4+x^2+1)/x^4/(x^4+1)^(1/2),x)
[Out] 1/3*(x^4+1)^(1/2)*(x^4+3*x^2+1)/x^3
```

maxima [A] time = 0.77, size = 22, normalized size = 0.85

$$\frac{\left(x^4+3\,x^2+1\right)\sqrt{x^4+1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+x^2+1)/x^4/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] 1/3*(x^4 + 3*x^2 + 1)*sqrt(x^4 + 1)/x^3
```

mupad [B] time = 0.24, size = 25, normalized size = 0.96

$$\frac{\left(x^4+1\right)^{3/2}+3\,x^2\,\sqrt{x^4+1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(x^2 + x^4 + 1))/(x^4*(x^4 + 1)^(1/2)),x)
[Out] ((x^4 + 1)^(3/2) + 3*x^2*(x^4 + 1)^(1/2))/(3*x^3)
```

sympy [C] time = 3.11, size = 126, normalized size = 4.85

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**4+x**2+1)/x**4/(x**4+1)**(1/2),x)
[Out] x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4*exp_polar(I*pi))/(4*gamma(9/
4)) + x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(I*pi))/(4*ga
mma(7/4)) - gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_polar(I*pi))/(4
*x*gamma(3/4)) - gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), x**4*exp_polar(I*pi
))/(4*x**3*gamma(1/4))
```

3.282 $\int \frac{(-4+x^3)(-1+x^3+x^4)}{x^6(-1+x^3)^{3/4}} \, dx$

Optimal. Leaf size=26

$$-\frac{4\sqrt[4]{x^3-1}\left(5x^4+x^3-1\right)}{5x^5}$$

Rubi [A] time = 0.13, antiderivative size = 47, normalized size of antiderivative = 1.81, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1835, 1586, 449}

$$-\frac{4\sqrt[4]{x^3-1}}{x}+\frac{4\sqrt[4]{x^3-1}}{5x^5}-\frac{4\sqrt[4]{x^3-1}}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((-4 + x^3)*(-1 + x^3 + x^4))/(x^6*(-1 + x^3)^(3/4)),x]
```

```
[Out] (4*(-1 + x^3)^(1/4))/(5*x^5) - (4*(-1 + x^3)^(1/4))/(5*x^2) - (4*(-1 + x^3)
^(1/4))/x
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{(-4+x^3)(-1+x^3+x^4)}{x^6(-1+x^3)^{3/4}}\,dx &= \frac{4\sqrt[4]{-1+x^3}}{5x^5}+\frac{1}{10}\int \frac{-16x^2-40x^3+10x^5+10x^6}{x^5(-1+x^3)^{3/4}}\,dx\\ &= \frac{4\sqrt[4]{-1+x^3}}{5x^5}+\frac{1}{10}\int \frac{-16x-40x^2+10x^4+10x^5}{x^4(-1+x^3)^{3/4}}\,dx\\ &= \frac{4\sqrt[4]{-1+x^3}}{5x^5}+\frac{1}{10}\int \frac{-16-40x+10x^3+10x^4}{x^3(-1+x^3)^{3/4}}\,dx\\ &= \frac{4\sqrt[4]{-1+x^3}}{5x^5}-\frac{4\sqrt[4]{-1+x^3}}{5x^2}+\frac{1}{40}\int \frac{-160+40x^3}{x^2(-1+x^3)^{3/4}}\,dx\\ &= \frac{4\sqrt[4]{-1+x^3}}{5x^5}-\frac{4\sqrt[4]{-1+x^3}}{5x^2}-\frac{4\sqrt[4]{-1+x^3}}{x}\end{aligned}$$

Mathematica [C] time = 0.07, size = 115, normalized size = 4.42

$$\frac{(1-x^3)^{3/4}\left(5x^3\left(2x^3\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};x^3\right)+8x\,{}_2F_1\left(-\frac{1}{3},\frac{3}{4};\frac{2}{3};x^3\right)+5\,{}_2F_1\left(-\frac{2}{3},\frac{3}{4};\frac{1}{3};x^3\right)+x^4\,{}_2F_1\left(\frac{2}{3},\frac{3}{4};\frac{5}{3};x^3\right)\right)-8\,{}_2F_1\left(-\frac{5}{3},\frac{3}{4};-\frac{2}{3};x^3\right)\right)}{10x^5(x^3-1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-4 + x^3)*(-1 + x^3 + x^4))/(x^6*(-1 + x^3)^(3/4)),x]
```

```
[Out] ((1 - x^3)^(3/4)*(-8*Hypergeometric2F1[-5/3, 3/4, -2/3, x^3] + 5*x^3*(5*Hyp
ergeometric2F1[-2/3, 3/4, 1/3, x^3] + 8*x*Hypergeometric2F1[-1/3, 3/4, 2/3,
 x^3] + 2*x^3*Hypergeometric2F1[1/3, 3/4, 4/3, x^3] + x^4*Hypergeometric2F1
[2/3, 3/4, 5/3, x^3])))/(10*x^5*(-1 + x^3)^(3/4))
```

IntegrateAlgebraic [A] time = 4.02, size = 26, normalized size = 1.00

$$-\frac{4\sqrt[4]{x^3-1}\,(5x^4+x^3-1)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + x^3)*(-1 + x^3 + x^4))/(x^6*(-1 + x^3)^(3/4)),x]
```

```
[Out] (-4*(-1 + x^3)^(1/4)*(-1 + x^3 + 5*x^4))/(5*x^5)
```

fricas [A] time = 0.40, size = 22, normalized size = 0.85

$$-\frac{4\,(5\,x^4+x^3-1)(x^3-1)^{\frac{1}{4}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^4+x^3-1)/x^6/(x^3-1)^(3/4),x, algorithm="fricas")
```

```
[Out] -4/5*(5*x^4 + x^3 - 1)*(x^3 - 1)^(1/4)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4+x^3-1)(x^3-4)}{(x^3-1)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^4+x^3-1)/x^6/(x^3-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((x^4 + x^3 - 1)*(x^3 - 4)/((x^3 - 1)^(3/4)*x^6), x)
```

maple [A] time = 0.01, size = 32, normalized size = 1.23

$$-\frac{4\left(-1+x\right)\left(x^{2}+x+1\right)\left(5x^{4}+x^{3}-1\right)}{5x^{5}\left(x^{3}-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-4)*(x^4+x^3-1)/x^6/(x^3-1)^(3/4),x)
```

```
[Out] -4/5*(-1+x)*(x^2+x+1)*(5*x^4+x^3-1)/x^5/(x^3-1)^(3/4)
```

maxima [A] time = 0.76, size = 38, normalized size = 1.46

$$-\frac{4\left(5\,x^{7}+x^{6}-5\,x^{4}-2\,x^{3}+1\right)}{5\left(x^{2}+x+1\right)^{\frac{3}{4}}\left(x-1\right)^{\frac{3}{4}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^4+x^3-1)/x^6/(x^3-1)^(3/4),x, algorithm="maxima")
```

```
[Out] -4/5*(5*x^7 + x^6 - 5*x^4 - 2*x^3 + 1)/((x^2 + x + 1)^(3/4)*(x - 1)^(3/4)*x
^5)
```

mupad [B] time = 0.24, size = 39, normalized size = 1.50

$$-\frac{4\,x^{3}\left(x^{3}-1\right)^{1/4}-4\left(x^{3}-1\right)^{1/4}+20\,x^{4}\left(x^{3}-1\right)^{1/4}}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 4)*(x^3 + x^4 - 1))/(x^6*(x^3 - 1)^(3/4)),x)
```

```
[Out] -(4*x^3*(x^3 - 1)^(1/4) - 4*(x^3 - 1)^(1/4) + 20*x^4*(x^3 - 1)^(1/4))/(5*x^
5)
```

sympy [C] time = 4.05, size = 178, normalized size = 6.85

$$\frac{x^{2}e^{-\frac{3i\pi}{4}}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{2}{3},\frac{3}{4}\\\frac{5}{3}\end{matrix}\middle|x^{3}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{xe^{-\frac{3i\pi}{4}}\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{3}{4}\\\frac{4}{3}\end{matrix}\middle|x^{3}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{4e^{\frac{i\pi}{4}}\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{3}{4}\\\frac{2}{3}\end{matrix}\middle|x^{3}\right)}{3x\Gamma\left(\frac{2}{3}\right)}+\frac{5e^{\frac{i\pi}{4}}\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{3}{4}\\\frac{1}{3}\end{matrix}\middle|x^{3}\right)}{3x^{2}\Gamma\left(\frac{1}{3}\right)}-\frac{4e^{\frac{i\pi}{4}}\Gamma\left(-\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{3},\frac{3}{4}\\-\frac{2}{3}\end{matrix}\middle|x^{3}\right)}{3x^{5}\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-4)*(x**4+x**3-1)/x**6/(x**3-1)**(3/4),x)
```

```
[Out] x**2*exp(-3*I*pi/4)*gamma(2/3)*hyper((2/3, 3/4), (5/3,), x**3)/(3*gamma(5/3
)) + x*exp(-3*I*pi/4)*gamma(1/3)*hyper((1/3, 3/4), (4/3,), x**3)/(3*gamma(4
/3)) + 4*exp(I*pi/4)*gamma(-1/3)*hyper((-1/3, 3/4), (2/3,), x**3)/(3*x*gamm
a(2/3)) + 5*exp(I*pi/4)*gamma(-2/3)*hyper((-2/3, 3/4), (1/3,), x**3)/(3*x**
2*gamma(1/3)) - 4*exp(I*pi/4)*gamma(-5/3)*hyper((-5/3, 3/4), (-2/3,), x**3)
/(3*x**5*gamma(-2/3))
```

3.283 $\int \frac{1+3x^4}{\left(-1-ax+x^4\right)\sqrt{-x+x^5}}\,dx$

Optimal. Leaf size=26

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^5-x}}\right)}{\sqrt{a}}$$

Rubi [F] time = 1.36, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+3x^4}{\left(-1-ax+x^4\right)\sqrt{-x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + 3*x^4)/((-1 - a*x + x^4)*Sqrt[-x + x^5]),x]
```

```
[Out] ((3 + 3*I)*x^2*Sqrt[-(((-1)^(3/4)*(1 + (-1)^(1/4)*x)^2)/x)]*Sqrt[(I*(1 - x^
4))/x^2]*EllipticF[ArcSin[Sqrt[((-1)^(3/4)*(Sqrt[2] - 2*(-1)^(1/4)*x + I*Sq
rt[2]*x^2))/x]/2], -2*(1 - Sqrt[2])])/(Sqrt[2*(2 + Sqrt[2])]*(1 + (-1)^(1/4
)*x)*Sqrt[-x + x^5]) - ((3 + 3*I)*x^2*Sqrt[((-1)^(3/4)*(1 - (-1)^(1/4)*x)^2
)/x]*Sqrt[(I*(1 - x^4))/x^2]*EllipticF[ArcSin[Sqrt[-(((-1)^(3/4)*(Sqrt[2] +
 2*(-1)^(1/4)*x + I*Sqrt[2]*x^2))/x)]/2], -2*(1 - Sqrt[2])])/(Sqrt[2*(2 + S
qrt[2])]*(1 - (-1)^(1/4)*x)*Sqrt[-x + x^5]) - (8*Sqrt[x]*Sqrt[-1 + x^4]*Def
er[Subst][Defer[Int][1/((1 + a*x^2 - x^8)*Sqrt[-1 + x^8]), x], x, Sqrt[x]])
/Sqrt[-x + x^5] - (6*a*Sqrt[x]*Sqrt[-1 + x^4]*Defer[Subst][Defer[Int][x^2/(
(1 + a*x^2 - x^8)*Sqrt[-1 + x^8]), x], x, Sqrt[x]])/Sqrt[-x + x^5]
```

Rubi steps

$$\int \frac{1+3x^4}{\left(-1-ax+x^4\right)\sqrt{-x+x^5}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1+x^4}\right)\int \frac{1+3x^4}{\sqrt{x}\sqrt{-1+x^4}\left(-1-ax+x^4\right)}\,dx}{\sqrt{-x+x^5}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \frac{1+3x^8}{\sqrt{-1+x^8}\left(-1-ax^2+x^8\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{-x+x^5}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \left(\frac{3}{\sqrt{-1+x^8}}+\frac{4+3ax^2}{\sqrt{-1+x^8}\left(-1-ax^2+x^8\right)}\right)dx, x, \sqrt{x}\right)}{\sqrt{-x+x^5}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \frac{4+3ax^2}{\sqrt{-1+x^8}\left(-1-ax^2+x^8\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{-x+x^5}} + \frac{\left(6\sqrt{x}\sqrt{-1+x^4}\right)\text{Su}}{\sqrt{}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \left(-\frac{4}{\left(1+ax^2-x^8\right)\sqrt{-1+x^8}}-\frac{3ax^2}{\left(1+ax^2-x^8\right)\sqrt{-1+x^8}}\right)dx, x, \sqrt{x}\right)}{\sqrt{-x+x^5}}$$

$$= \frac{(3+3i)x^2\sqrt{-\frac{(-1)^{3/4}\left(1+\sqrt[4]{-1}\,x\right)^2}{x}}\sqrt{\frac{i\left(1-x^4\right)}{x^2}}\,F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{\frac{(-1)^{3/4}\left(\sqrt{2}-2\sqrt[4]{-1}\,x+i\sqrt{2}\,x^2\right)}{x}}\right)\right|}{\sqrt{2\left(2+\sqrt{2}\right)}\left(1+\sqrt[4]{-1}\,x\right)\sqrt{-x+x^5}}$$

Mathematica [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1+3x^4}{\left(-1-ax+x^4\right)\sqrt{-x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + 3*x^4)/((-1 - a*x + x^4)*Sqrt[-x + x^5]),x]
[Out] Integrate[(1 + 3*x^4)/((-1 - a*x + x^4)*Sqrt[-x + x^5]), x]
```

IntegrateAlgebraic [A] time = 0.61, size = 26, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^5-x}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 3*x^4)/((-1 - a*x + x^4)*Sqrt[-x + x^5]),x]
[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[-x + x^5]])/Sqrt[a]
```

fricas [B] time = 0.48, size = 132, normalized size = 5.08

$$\left[\frac{\log\left(\frac{x^8+6\,ax^5+a^2x^2-2\,x^4-4\,\sqrt{x^5-x}\left(x^4+ax-1\right)\sqrt{a}-6\,ax+1}{x^8-2\,ax^5+a^2x^2-2\,x^4+2\,ax+1}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{x^5-x}\left(x^4+ax-1\right)\sqrt{-a}}{2\left(ax^5-ax\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+1)/(x^4-a*x-1)/(x^5-x)^(1/2),x, algorithm="fricas")
[Out] [1/2*log((x^8 + 6*a*x^5 + a^2*x^2 - 2*x^4 - 4*sqrt(x^5 - x)*(x^4 + a*x - 1)
*sqrt(a) - 6*a*x + 1)/(x^8 - 2*a*x^5 + a^2*x^2 - 2*x^4 + 2*a*x + 1))/sqrt(a
), sqrt(-a)*arctan(1/2*sqrt(x^5 - x)*(x^4 + a*x - 1)*sqrt(-a)/(a*x^5 - a*x)
)/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^4+1}{\sqrt{x^5-x}\left(x^4-ax-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+1)/(x^4-a*x-1)/(x^5-x)^(1/2),x, algorithm="giac")
[Out] integrate((3*x^4 + 1)/(sqrt(x^5 - x)*(x^4 - a*x - 1)), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{3x^4+1}{\left(x^4-ax-1\right)\sqrt{x^5-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4+1)/(x^4-a*x-1)/(x^5-x)^(1/2),x)
[Out] int((3*x^4+1)/(x^4-a*x-1)/(x^5-x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^4+1}{\sqrt{x^5-x}\left(x^4-ax-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+1)/(x^4-a*x-1)/(x^5-x)^(1/2),x, algorithm="maxima")
[Out] integrate((3*x^4 + 1)/(sqrt(x^5 - x)*(x^4 - a*x - 1)), x)
```

mupad [B] time = 0.46, size = 40, normalized size = 1.54

$$\frac{\ln\left(\frac{a\,x-2\,\sqrt{a}\,\sqrt{x^5-x}+x^4-1}{-x^4+a\,x+1}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^4 + 1)/((x^5 - x)^(1/2)*(a*x - x^4 + 1)),x)
[Out] log((a*x - 2*a^(1/2)*(x^5 - x)^(1/2) + x^4 - 1)/(a*x - x^4 + 1))/a^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4+1)/(x**4-a*x-1)/(x**5-x)**(1/2),x)
[Out] Timed out
```

3.284 $\int \frac{1+3x^4}{\left(-a-x+ax^4\right)\sqrt{-x+x^5}}\,dx$

Optimal. Leaf size=26

$$-\frac{2\tanh^{-1}\left(\frac{x}{\sqrt{a}\sqrt{x^5-x}}\right)}{\sqrt{a}}$$

Rubi [F] time = 1.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+3x^4}{\left(-a-x+ax^4\right)\sqrt{-x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + 3*x^4)/((-a - x + a*x^4)*Sqrt[-x + x^5]),x]
```

```
[Out] ((3 + 3*I)*x^2*Sqrt[-(((-1)^(3/4)*(1 + (-1)^(1/4)*x)^2)/x)]*Sqrt[(I*(1 - x^
4))/x^2]*EllipticF[ArcSin[Sqrt[((-1)^(3/4)*(Sqrt[2] - 2*(-1)^(1/4)*x + I*Sq
rt[2]*x^2))/x]/2], -2*(1 - Sqrt[2])])/(Sqrt[2*(2 + Sqrt[2])]*a*(1 + (-1)^(1
/4)*x)*Sqrt[-x + x^5]) - ((3 + 3*I)*x^2*Sqrt[((-1)^(3/4)*(1 - (-1)^(1/4)*x)
^2)/x]*Sqrt[(I*(1 - x^4))/x^2]*EllipticF[ArcSin[Sqrt[-(((-1)^(3/4)*(Sqrt[2]
 + 2*(-1)^(1/4)*x + I*Sqrt[2]*x^2))/x)]/2], -2*(1 - Sqrt[2])])/(Sqrt[2*(2 +
 Sqrt[2])]*a*(1 - (-1)^(1/4)*x)*Sqrt[-x + x^5]) + (8*Sqrt[x]*Sqrt[-1 + x^4]
*Defer[Subst][Defer[Int][1/(Sqrt[-1 + x^8]*(-a - x^2 + a*x^8)), x], x, Sqrt
[x]])/Sqrt[-x + x^5] + (6*Sqrt[x]*Sqrt[-1 + x^4]*Defer[Subst][Defer[Int][x^
2/(Sqrt[-1 + x^8]*(-a - x^2 + a*x^8)), x], x, Sqrt[x]])/(a*Sqrt[-x + x^5])
```

Rubi steps

$$\int \frac{1+3x^4}{\left(-a-x+ax^4\right)\sqrt{-x+x^5}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1+x^4}\right)\int \frac{1+3x^4}{\sqrt{x}\sqrt{-1+x^4}\left(-a-x+ax^4\right)}\,dx}{\sqrt{-x+x^5}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \frac{1+3x^8}{\sqrt{-1+x^8}\left(-a-x^2+ax^8\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{-x+x^5}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \left(\frac{3}{a\sqrt{-1+x^8}}+\frac{4a+3x^2}{a\sqrt{-1+x^8}\left(-a-x^2+ax^8\right)}\right)dx, x, \sqrt{x}\right)}{\sqrt{-x+x^5}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \frac{4a+3x^2}{\sqrt{-1+x^8}\left(-a-x^2+ax^8\right)}\,dx, x, \sqrt{x}\right)}{a\sqrt{-x+x^5}} + \frac{\left(6\sqrt{x}\sqrt{-1+x^4}\right)}{}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \left(\frac{4a}{\sqrt{-1+x^8}\left(-a-x^2+ax^8\right)}+\frac{3x^2}{\sqrt{-1+x^8}\left(-a-x^2+ax^8\right)}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-x+x^5}}$$

$$= \frac{(3+3i)x^2\sqrt{-\frac{(-1)^{3/4}\left(1+\sqrt[4]{-1}\,x\right)^2}{x}}\sqrt{\frac{i\left(1-x^4\right)}{x^2}}\,F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{\frac{(-1)^{3/4}\left(\sqrt{2}-2\sqrt[4]{-1}\,x+i\sqrt{2}\,x^2\right)}{x}}\right.\right.}{\sqrt{2\left(2+\sqrt{2}\right)}\,a\left(1+\sqrt[4]{-1}\,x\right)\sqrt{-x+x^5}}$$

Mathematica [F] time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{1+3x^4}{\left(-a-x+ax^4\right)\sqrt{-x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + 3*x^4)/((-a - x + a*x^4)*Sqrt[-x + x^5]),x]
[Out] Integrate[(1 + 3*x^4)/((-a - x + a*x^4)*Sqrt[-x + x^5]), x]
```

IntegrateAlgebraic [A] time = 1.36, size = 26, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{x}{\sqrt{a}\sqrt{x^5-x}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 3*x^4)/((-a - x + a*x^4)*Sqrt[-x + x^5]),x]
[Out] (-2*ArcTanh[x/(Sqrt[a]*Sqrt[-x + x^5])])/Sqrt[a]
```

fricas [B] time = 0.46, size = 146, normalized size = 5.62

$$\left[\frac{\log\left(\frac{a^2x^8-2\,a^2x^4+6\,ax^5-4\left(ax^4-a+x\right)\sqrt{x^5-x}\,\sqrt{a}+a^2-6\,ax+x^2}{a^2x^8-2\,a^2x^4-2\,ax^5+a^2+2\,ax+x^2}\right)}{2\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\left(ax^4-a+x\right)\sqrt{x^5-x}\,\sqrt{-a}}{2\left(ax^5-ax\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+1)/(a*x^4-a-x)/(x^5-x)^(1/2),x, algorithm="fricas")
[Out] [1/2*log((a^2*x^8 - 2*a^2*x^4 + 6*a*x^5 - 4*(a*x^4 - a + x)*sqrt(x^5 - x)*s
qrt(a) + a^2 - 6*a*x + x^2)/(a^2*x^8 - 2*a^2*x^4 - 2*a*x^5 + a^2 + 2*a*x +
x^2))/sqrt(a), sqrt(-a)*arctan(1/2*(a*x^4 - a + x)*sqrt(x^5 - x)*sqrt(-a)/(
a*x^5 - a*x))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^4+1}{\left(ax^4-a-x\right)\sqrt{x^5-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+1)/(a*x^4-a-x)/(x^5-x)^(1/2),x, algorithm="giac")
[Out] integrate((3*x^4 + 1)/((a*x^4 - a - x)*sqrt(x^5 - x)), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{3x^4+1}{\left(a\,x^4-a-x\right)\sqrt{x^5-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4+1)/(a*x^4-a-x)/(x^5-x)^(1/2),x)
[Out] int((3*x^4+1)/(a*x^4-a-x)/(x^5-x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3 x^4 + 1}{\left(ax^4 - a - x\right)\sqrt{x^5 - x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+1)/(a*x^4-a-x)/(x^5-x)^(1/2),x, algorithm="maxima")
[Out] integrate((3*x^4 + 1)/((a*x^4 - a - x)*sqrt(x^5 - x)), x)
```

mupad [B] time = 0.44, size = 42, normalized size = 1.62

$$\frac{\ln\left(\frac{a-x+2\,\sqrt{a}\,\sqrt{x^5-x}-a\,x^4}{-a\,x^4+x+a}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^4 + 1)/((x^5 - x)^(1/2)*(a + x - a*x^4)),x)
[Out] log((a - x + 2*a^(1/2)*(x^5 - x)^(1/2) - a*x^4)/(a + x - a*x^4))/a^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4+1)/(a*x**4-a-x)/(x**5-x)**(1/2),x)
[Out] Timed out
```

3.285 $\int \frac{(4+x^5)(-1+x^4+x^5)}{x^6(-1+x^5)^{3/4}}\,dx$

Optimal. Leaf size=26

$$\frac{4\sqrt[4]{x^5-1}\left(x^5+5x^4-1\right)}{5x^5}$$

Rubi [A] time = 0.10, antiderivative size = 44, normalized size of antiderivative = 1.69, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1835, 1585, 12, 261}

$$\frac{4\sqrt[4]{x^5-1}}{x}-\frac{4\sqrt[4]{x^5-1}}{5x^5}+\frac{4}{5}\sqrt[4]{x^5-1}$$

Antiderivative was successfully verified.

```
[In] Int[((4 + x^5)*(-1 + x^4 + x^5))/(x^6*(-1 + x^5)^(3/4)),x]
```

```
[Out] (4*(-1 + x^5)^(1/4))/5 - (4*(-1 + x^5)^(1/4))/(5*x^5) + (4*(-1 + x^5)^(1/4)
)/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(4+x^5)(-1+x^4+x^5)}{x^6(-1+x^5)^{3/4}}\,dx &= -\frac{4\sqrt[4]{-1+x^5}}{5x^5}+\frac{1}{10}\int \frac{40x^3+10x^8+10x^9}{x^5(-1+x^5)^{3/4}}\,dx \\
&= -\frac{4\sqrt[4]{-1+x^5}}{5x^5}+\frac{1}{10}\int \frac{40+10x^5+10x^6}{x^2(-1+x^5)^{3/4}}\,dx \\
&= -\frac{4\sqrt[4]{-1+x^5}}{5x^5}+\frac{4\sqrt[4]{-1+x^5}}{x}+\frac{1}{20}\int \frac{20x^4}{(-1+x^5)^{3/4}}\,dx \\
&= -\frac{4\sqrt[4]{-1+x^5}}{5x^5}+\frac{4\sqrt[4]{-1+x^5}}{x}+\int \frac{x^4}{(-1+x^5)^{3/4}}\,dx \\
&= \frac{4}{5}\sqrt[4]{-1+x^5}-\frac{4\sqrt[4]{-1+x^5}}{5x^5}+\frac{4\sqrt[4]{-1+x^5}}{x}
\end{aligned}
$$

Mathematica [C] time = 0.30, size = 238, normalized size = 9.15

$$
\frac{1}{20}\left(-64\sqrt[4]{x^5-1}\,{}_2F_1\left(\frac{1}{4},2;\frac{5}{4};1-x^5\right)-\frac{80(1-x^5)^{3/4}\,{}_2F_1\left(-\frac{1}{5},\frac{3}{4};\frac{4}{5};x^5\right)}{(x^5-1)^{3/4}x}+\frac{5(1-x^5)^{3/4}x^4\,{}_2F_1\left(\frac{3}{4},\frac{4}{5};\frac{9}{5};x^5\right)}{(x^5-1)^{3/4}}+16\sqrt[4]{x^5-1}-6\sqrt{2}\log\left(\sqrt{x^5-1}-\sqrt{2}\sqrt[4]{x^5-1}+1\right)+6\sqrt{2}\log\left(\sqrt{x^5-1}+\sqrt{2}\sqrt[4]{x^5-1}+1\right)-12\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{x^5-1}\right)+12\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt[4]{x^5-1}+1\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((4 + x^5)*(-1 + x^4 + x^5))/(x^6*(-1 + x^5)^(3/4)),x]
```

```
[Out] (16*(-1 + x^5)^(1/4) - 12*Sqrt[2]*ArcTan[1 - Sqrt[2]*(-1 + x^5)^(1/4)] + 12
*Sqrt[2]*ArcTan[1 + Sqrt[2]*(-1 + x^5)^(1/4)] - (80*(1 - x^5)^(3/4)*Hyperge
ometric2F1[-1/5, 3/4, 4/5, x^5])/(x*(-1 + x^5)^(3/4)) - 64*(-1 + x^5)^(1/4)
*Hypergeometric2F1[1/4, 2, 5/4, 1 - x^5] + (5*x^4*(1 - x^5)^(3/4)*Hypergeom
etric2F1[3/4, 4/5, 9/5, x^5])/(-1 + x^5)^(3/4) - 6*Sqrt[2]*Log[1 - Sqrt[2]*
(-1 + x^5)^(1/4) + Sqrt[-1 + x^5]] + 6*Sqrt[2]*Log[1 + Sqrt[2]*(-1 + x^5)^(
1/4) + Sqrt[-1 + x^5]])/20
```

IntegrateAlgebraic [A] time = 2.57, size = 26, normalized size = 1.00

$$
\frac{4\sqrt[4]{x^5-1}\,\left(x^5+5x^4-1\right)}{5x^5}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((4 + x^5)*(-1 + x^4 + x^5))/(x^6*(-1 + x^5)^(3/4)),x]
```

```
[Out] (4*(-1 + x^5)^(1/4)*(-1 + 5*x^4 + x^5))/(5*x^5)
```

fricas [A] time = 0.40, size = 22, normalized size = 0.85

$$
\frac{4\left(x^5+5\,x^4-1\right)\left(x^5-1\right)^{\frac{1}{4}}}{5\,x^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+4)*(x^5+x^4-1)/x^6/(x^5-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/5*(x^5 + 5*x^4 - 1)*(x^5 - 1)^(1/4)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(x^5+x^4-1\right)\left(x^5+4\right)}{\left(x^5-1\right)^{\frac{3}{4}}x^6}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+4)*(x^5+x^4-1)/x^6/(x^5-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((x^5 + x^4 - 1)*(x^5 + 4)/((x^5 - 1)^(3/4)*x^6), x)
```

maple [A] time = 0.01, size = 38, normalized size = 1.46

$$\frac{4\left(x^{4}+x^{3}+x^{2}+x+1\right)\left(-1+x\right)\left(x^{5}+5x^{4}-1\right)}{5x^{5}\left(x^{5}-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+4)*(x^5+x^4-1)/x^6/(x^5-1)^(3/4),x)
```

```
[Out] 4/5*(x^4+x^3+x^2+x+1)*(-1+x)*(x^5+5*x^4-1)/x^5/(x^5-1)^(3/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3}{5}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(x^{5}-1\right)^{\frac{1}{4}}\right)\right)-\frac{3}{5}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(x^{5}-1\right)^{\frac{1}{4}}\right)\right)-\frac{3}{10}\sqrt{2}\log\left(\sqrt{2}\left(x^{5}-1\right)^{\frac{1}{4}}+\sqrt{x^{5}-1}+1\right)+\frac{3}{10}\sqrt{2}\log\left(-\sqrt{2}\left(x^{5}-1\right)^{\frac{1}{4}}+\sqrt{x^{5}-1}+1\right)-\frac{4\left(x^{5}-1\right)^{\frac{1}{4}}}{5x^{5}}+\int\frac{\left(x^{6}+x^{5}+3x+4\right)\left(x^{4}+x^{3}+x^{2}+x+1\right)^{\frac{1}{4}}\left(x-1\right)^{\frac{1}{4}}}{x^{7}-x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+4)*(x^5+x^4-1)/x^6/(x^5-1)^(3/4),x, algorithm="maxima")
```

```
[Out] -3/5*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^5 - 1)^(1/4))) - 3/5*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^5 - 1)^(1/4))) - 3/10*sqrt(2)*log(sqr
t(2)*(x^5 - 1)^(1/4) + sqrt(x^5 - 1) + 1) + 3/10*sqrt(2)*log(-sqrt(2)*(x^5
- 1)^(1/4) + sqrt(x^5 - 1) + 1) - 4/5*(x^5 - 1)^(1/4)/x^5 + integrate((x^6
+ x^5 + 3*x + 4)*(x^4 + x^3 + x^2 + x + 1)^(1/4)*(x - 1)^(1/4)/(x^7 - x^2),
 x)
```

mupad [B] time = 0.28, size = 27, normalized size = 1.04

$$\frac{4\left(x^{5}-1\right)^{5/4}+20\,x^{4}\left(x^{5}-1\right)^{1/4}}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 + 4)*(x^4 + x^5 - 1))/(x^6*(x^5 - 1)^(3/4)),x)
```

```
[Out] (4*(x^5 - 1)^(5/4) + 20*x^4*(x^5 - 1)^(1/4))/(5*x^5)
```

sympy [C] time = 4.56, size = 151, normalized size = 5.81

$$\frac{x^{4}e^{-\frac{3i\pi}{4}}\Gamma\left(\frac{4}{5}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{4}{5}\\\frac{9}{5}\end{matrix}\middle|x^{5}\right)}{5\Gamma\left(\frac{9}{5}\right)}+\frac{4\sqrt[4]{x^{5}-1}}{5}-\frac{4e^{\frac{i\pi}{4}}\Gamma\left(-\frac{1}{5}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{5},\frac{3}{4}\\\frac{4}{5}\end{matrix}\middle|x^{5}\right)}{5x\Gamma\left(\frac{4}{5}\right)}-\frac{3\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{e^{2i\pi}}{x^{5}}\right)}{5x^{\frac{15}{4}}\Gamma\left(\frac{7}{4}\right)}+\frac{4\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{e^{2i\pi}}{x^{5}}\right)}{5x^{\frac{35}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+4)*(x**5+x**4-1)/x**6/(x**5-1)**(3/4),x)
```

```
[Out] x**4*exp(-3*I*pi/4)*gamma(4/5)*hyper((3/4, 4/5), (9/5,), x**5)/(5*gamma(9/5
)) + 4*(x**5 - 1)**(1/4)/5 - 4*exp(I*pi/4)*gamma(-1/5)*hyper((-1/5, 3/4), (
4/5,), x**5)/(5*x*gamma(4/5)) - 3*gamma(3/4)*hyper((3/4, 3/4), (7/4,), exp_
polar(2*I*pi)/x**5)/(5*x**(15/4)*gamma(7/4)) + 4*gamma(7/4)*hyper((3/4, 7/4
), (11/4,), exp_polar(2*I*pi)/x**5)/(5*x**(35/4)*gamma(11/4))
```

3.286 $\int \frac{(-4+x^5)(1+x^4+x^5)}{x^6(1+x^5)^{3/4}}\,dx$

Optimal. Leaf size=26

$$\frac{4\sqrt[4]{x^5+1}\,(x^5+5x^4+1)}{5x^5}$$

Rubi [A] time = 0.10, antiderivative size = 44, normalized size of antiderivative = 1.69, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1835, 1585, 12, 261}

$$\frac{4\sqrt[4]{x^5+1}}{x}+\frac{4\sqrt[4]{x^5+1}}{5x^5}+\frac{4}{5}\sqrt[4]{x^5+1}$$

Antiderivative was successfully verified.

```
[In] Int[((-4 + x^5)*(1 + x^4 + x^5))/(x^6*(1 + x^5)^(3/4)),x]
```

```
[Out] (4*(1 + x^5)^(1/4))/5 + (4*(1 + x^5)^(1/4))/(5*x^5) + (4*(1 + x^5)^(1/4))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\int \frac{(-4+x^5)(1+x^4+x^5)}{x^6(1+x^5)^{3/4}}\,dx = \frac{4\sqrt[4]{1+x^5}}{5x^5} - \frac{1}{10}\int \frac{40x^3-10x^8-10x^9}{x^5(1+x^5)^{3/4}}\,dx$$
$$= \frac{4\sqrt[4]{1+x^5}}{5x^5} - \frac{1}{10}\int \frac{40-10x^5-10x^6}{x^2(1+x^5)^{3/4}}\,dx$$
$$= \frac{4\sqrt[4]{1+x^5}}{5x^5} + \frac{4\sqrt[4]{1+x^5}}{x} + \frac{1}{20}\int \frac{20x^4}{(1+x^5)^{3/4}}\,dx$$
$$= \frac{4\sqrt[4]{1+x^5}}{5x^5} + \frac{4\sqrt[4]{1+x^5}}{x} + \int \frac{x^4}{(1+x^5)^{3/4}}\,dx$$
$$= \frac{4}{5}\sqrt[4]{1+x^5} + \frac{4\sqrt[4]{1+x^5}}{5x^5} + \frac{4\sqrt[4]{1+x^5}}{x}$$

Mathematica [C] time = 0.08, size = 107, normalized size = 4.12

$$-\frac{16}{5}\sqrt[4]{x^5+1}\,{}_2F_1\left(\frac{1}{4},2;\frac{5}{4};x^5+1\right)+\frac{4\,{}_2F_1\left(-\frac{1}{5},\frac{3}{4};\frac{4}{5};-x^5\right)}{x}+\frac{1}{4}x^4\,{}_2F_1\left(\frac{3}{4},\frac{4}{5};\frac{9}{5};-x^5\right)+\frac{4}{5}\sqrt[4]{x^5+1}+\frac{6}{5}\left(\tan^{-1}\left(\sqrt[4]{x^5+1}\right)+\tanh^{-1}\left(\sqrt[4]{x^5+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-4 + x^5)*(1 + x^4 + x^5))/(x^6*(1 + x^5)^(3/4)),x]
```

```
[Out] (4*(1 + x^5)^(1/4))/5 + (6*(ArcTan[(1 + x^5)^(1/4)] + ArcTanh[(1 + x^5)^(1/
4)]))/5 + (4*Hypergeometric2F1[-1/5, 3/4, 4/5, -x^5])/x - (16*(1 + x^5)^(1/
4)*Hypergeometric2F1[1/4, 2, 5/4, 1 + x^5])/5 + (x^4*Hypergeometric2F1[3/4,
 4/5, 9/5, -x^5])/4
```

IntegrateAlgebraic [A] time = 2.21, size = 26, normalized size = 1.00

$$\frac{4\sqrt[4]{x^5+1}\left(x^5+5x^4+1\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + x^5)*(1 + x^4 + x^5))/(x^6*(1 + x^5)^(3/4)),x]
```

```
[Out] (4*(1 + x^5)^(1/4)*(1 + 5*x^4 + x^5))/(5*x^5)
```

fricas [A] time = 0.40, size = 22, normalized size = 0.85

$$\frac{4\left(x^5+5x^4+1\right)\left(x^5+1\right)^{\frac{1}{4}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-4)*(x^5+x^4+1)/x^6/(x^5+1)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/5*(x^5 + 5*x^4 + 1)*(x^5 + 1)^(1/4)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5+x^4+1\right)\left(x^5-4\right)}{\left(x^5+1\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-4)*(x^5+x^4+1)/x^6/(x^5+1)^(3/4),x, algorithm="giac")
[Out] integrate((x^5 + x^4 + 1)*(x^5 - 4)/((x^5 + 1)^(3/4)*x^6), x)
```

maple [A] time = 0.01, size = 42, normalized size = 1.62

$$\frac{4\left(x^{5}+5x^{4}+1\right)(1+x)\left(x^{4}-x^{3}+x^{2}-x+1\right)}{5\left(x^{5}+1\right)^{\frac{3}{4}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-4)*(x^5+x^4+1)/x^6/(x^5+1)^(3/4),x)
[Out] 4/5*(x^5+5*x^4+1)*(1+x)*(x^4-x^3+x^2-x+1)/(x^5+1)^(3/4)/x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4\left(x^{5}+1\right)^{\frac{1}{4}}}{5x^{5}}-\frac{6}{5}\arctan\left(\left(x^{5}+1\right)^{\frac{1}{4}}\right)+\int\frac{\left(x^{6}+x^{5}-3x-4\right)\left(x^{4}-x^{3}+x^{2}-x+1\right)^{\frac{1}{4}}(x+1)^{\frac{1}{4}}}{x^{7}+x^{2}}\,dx-\frac{3}{5}\log\left(\left(x^{5}+1\right)^{\frac{1}{4}}+1\right)+\frac{3}{5}\log\left(\left(x^{5}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-4)*(x^5+x^4+1)/x^6/(x^5+1)^(3/4),x, algorithm="maxima")
[Out] 4/5*(x^5 + 1)^(1/4)/x^5 - 6/5*arctan((x^5 + 1)^(1/4)) + integrate((x^6 + x^
5 - 3*x - 4)*(x^4 - x^3 + x^2 - x + 1)^(1/4)*(x + 1)^(1/4)/(x^7 + x^2), x)
- 3/5*log((x^5 + 1)^(1/4) + 1) + 3/5*log((x^5 + 1)^(1/4) - 1)
```

mupad [B] time = 0.18, size = 27, normalized size = 1.04

$$\frac{4\left(x^{5}+1\right)^{5/4}+20\,x^{4}\left(x^{5}+1\right)^{1/4}}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 - 4)*(x^4 + x^5 + 1))/(x^6*(x^5 + 1)^(3/4)),x)
[Out] (4*(x^5 + 1)^(5/4) + 20*x^4*(x^5 + 1)^(1/4))/(5*x^5)
```

sympy [C] time = 4.41, size = 143, normalized size = 5.50

$$\frac{x^{4}\Gamma\left(\frac{4}{5}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{4}{5}\\\frac{9}{5}\end{matrix}\middle|x^{5}e^{i\pi}\right)}{5\Gamma\left(\frac{9}{5}\right)}+\frac{4\sqrt[4]{x^{5}+1}}{5}-\frac{4\Gamma\left(-\frac{1}{5}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{5},\frac{3}{4}\\\frac{4}{5}\end{matrix}\middle|x^{5}e^{i\pi}\right)}{5x\Gamma\left(\frac{4}{5}\right)}+\frac{3\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^{5}}\right)}{5x^{\frac{15}{4}}\Gamma\left(\frac{7}{4}\right)}+\frac{4\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^{5}}\right)}{5x^{\frac{35}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-4)*(x**5+x**4+1)/x**6/(x**5+1)**(3/4),x)
[Out] x**4*gamma(4/5)*hyper((3/4, 4/5), (9/5,), x**5*exp_polar(I*pi))/(5*gamma(9/
5)) + 4*(x**5 + 1)**(1/4)/5 - 4*gamma(-1/5)*hyper((-1/5, 3/4), (4/5,), x**5
*exp_polar(I*pi))/(5*x*gamma(4/5)) + 3*gamma(3/4)*hyper((3/4, 3/4), (7/4,),
 exp_polar(I*pi)/x**5)/(5*x**(15/4)*gamma(7/4)) + 4*gamma(7/4)*hyper((3/4,
7/4), (11/4,), exp_polar(I*pi)/x**5)/(5*x**(35/4)*gamma(11/4))
```

3.287 $\int \frac{\sqrt{-1+x^6}}{x^{16}}\,dx$

Optimal. Leaf size=26

$$\frac{\sqrt{x^6-1}\,(2x^{12}+x^6-3)}{45x^{15}}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {271, 264}

$$\frac{(x^6-1)^{3/2}}{15x^{15}}+\frac{2(x^6-1)^{3/2}}{45x^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^6]/x^16,x]
```

```
[Out] (-1 + x^6)^(3/2)/(15*x^15) + (2*(-1 + x^6)^(3/2))/(45*x^9)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+x^6}}{x^{16}}\,dx &= \frac{(-1+x^6)^{3/2}}{15x^{15}}+\frac{2}{5}\int \frac{\sqrt{-1+x^6}}{x^{10}}\,dx\\ &= \frac{(-1+x^6)^{3/2}}{15x^{15}}+\frac{2(-1+x^6)^{3/2}}{45x^9}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.88

$$\frac{(x^6-1)^{3/2}(2x^6+3)}{45x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^6]/x^16,x]
```

```
[Out] ((-1 + x^6)^(3/2)*(3 + 2*x^6))/(45*x^15)
```

IntegrateAlgebraic [A] time = 0.15, size = 26, normalized size = 1.00

$$\frac{\sqrt{x^6-1}\,(2x^{12}+x^6-3)}{45x^{15}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x^6]/x^16,x]
```

```
[Out] (Sqrt[-1 + x^6]*(-3 + x^6 + 2*x^12))/(45*x^15)
```

fricas [A] time = 0.39, size = 29, normalized size = 1.12

$$\frac{2\,x^{15}+\left(2\,x^{12}+x^{6}-3\right)\sqrt{x^{6}-1}}{45\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^16,x, algorithm="fricas")
```

```
[Out] 1/45*(2*x^15 + (2*x^12 + x^6 - 3)*sqrt(x^6 - 1))/x^15
```

giac [A] time = 0.42, size = 41, normalized size = 1.58

$$-\frac{3\left(\frac{1}{x^{6}}-1\right)^{2}\sqrt{-\frac{1}{x^{6}}+1}-5\left(-\frac{1}{x^{6}}+1\right)^{\frac{3}{2}}}{45\,\mathrm{sgn}(x)}-\frac{2}{45}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^16,x, algorithm="giac")
```

```
[Out] -1/45*(3*(1/x^6 - 1)^2*sqrt(-1/x^6 + 1) - 5*(-1/x^6 + 1)^(3/2))/sgn(x) - 2/
45*sgn(x)
```

maple [A] time = 0.01, size = 40, normalized size = 1.54

$$\frac{(-1+x)(1+x)\left(x^{2}+x+1\right)\left(x^{2}-x+1\right)\left(2x^{6}+3\right)\sqrt{x^{6}-1}}{45x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/2)/x^16,x)
```

```
[Out] 1/45*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)*(2*x^6+3)*(x^6-1)^(1/2)/x^15
```

maxima [A] time = 0.60, size = 25, normalized size = 0.96

$$\frac{\left(x^{6}-1\right)^{\frac{3}{2}}}{9\,x^{9}}-\frac{\left(x^{6}-1\right)^{\frac{5}{2}}}{15\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^16,x, algorithm="maxima")
```

```
[Out] 1/9*(x^6 - 1)^(3/2)/x^9 - 1/15*(x^6 - 1)^(5/2)/x^15
```

mupad [B] time = 0.39, size = 24, normalized size = 0.92

$$\frac{5\left(x^{6}-1\right)^{3/2}+2\left(x^{6}-1\right)^{5/2}}{45\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)^(1/2)/x^16,x)
```

```
[Out] (5*(x^6 - 1)^(3/2) + 2*(x^6 - 1)^(5/2))/(45*x^15)
```

sympy [A] time = 1.37, size = 92, normalized size = 3.54

$$\begin{cases} \frac{2\sqrt{x^6-1}}{45x^3} + \frac{\sqrt{x^6-1}}{45x^9} - \frac{\sqrt{x^6-1}}{15x^{15}} & \text{for } |x^6| > 1 \\ \frac{2i\sqrt{1-x^6}}{45x^3} + \frac{i\sqrt{1-x^6}}{45x^9} - \frac{i\sqrt{1-x^6}}{15x^{15}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/2)/x**16,x)
```

```
[Out] Piecewise((2*sqrt(x**6 - 1)/(45*x**3) + sqrt(x**6 - 1)/(45*x**9) - sqrt(x**
6 - 1)/(15*x**15), Abs(x**6) > 1), (2*I*sqrt(1 - x**6)/(45*x**3) + I*sqrt(1
 - x**6)/(45*x**9) - I*sqrt(1 - x**6)/(15*x**15), True))
```

3.288 $\int \frac{-1-2x^2+2x^4}{\left(1+2x^4\right)\sqrt{1+x^6}}\,dx$

Optimal. Leaf size=26

$$-\tan^{-1}\left(\frac{x\sqrt{x^6+1}}{x^4-x^2+1}\right)$$

Rubi [F] time = 0.89, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1-2x^2+2x^4}{\left(1+2x^4\right)\sqrt{1+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 - 2*x^2 + 2*x^4)/((1 + 2*x^4)*Sqrt[1 + x^6]),x]
```

```
[Out] (x*(1 + x^2)*Sqrt[(1 - x^2 + x^4)/(1 + (1 + Sqrt[3])*x^2)^2]*EllipticF[ArcC
os[(1 + (1 - Sqrt[3])*x^2)/(1 + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(2*3
^(1/4)*Sqrt[(x^2*(1 + x^2))/(1 + (1 + Sqrt[3])*x^2)^2]*Sqrt[1 + x^6]) - (1/
4 + I/4)*(I + Sqrt[2])*Defer[Int][1/(((-1)^(1/4) - 2^(1/4)*x)*Sqrt[1 + x^6]
), x] + (1/4 + I/4)*(1 + I*Sqrt[2])*Defer[Int][1/((-(-1)^(3/4) - 2^(1/4)*x)
*Sqrt[1 + x^6]), x] - (1/4 + I/4)*(I + Sqrt[2])*Defer[Int][1/(((-1)^(1/4) +
 2^(1/4)*x)*Sqrt[1 + x^6]), x] + (1/4 + I/4)*(1 + I*Sqrt[2])*Defer[Int][1/(
(-(-1)^(3/4) + 2^(1/4)*x)*Sqrt[1 + x^6]), x]
```

Rubi steps

$$\int \frac{-1-2x^2+2x^4}{\left(1+2x^4\right)\sqrt{1+x^6}}\,dx = \int \left(\frac{1}{\sqrt{1+x^6}} - \frac{2\left(1+x^2\right)}{\left(1+2x^4\right)\sqrt{1+x^6}}\right)dx$$

$$= -\left(2\int \frac{1+x^2}{\left(1+2x^4\right)\sqrt{1+x^6}}\,dx\right) + \int \frac{1}{\sqrt{1+x^6}}\,dx$$

$$= \frac{x\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x^2}{1+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\sqrt{\frac{x^2\left(1+x^2\right)}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1+x^6}} - 2\int \left(\frac{i\left(i+\right.}{2\sqrt{2}\left(i-\sqrt{\right.}}\right.$$

$$= \frac{x\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x^2}{1+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\sqrt{\frac{x^2\left(1+x^2\right)}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1+x^6}} - \frac{1}{2}\left(2i-\sqrt{2}\right)\int \frac{}{(i}$$

$$= \frac{x\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x^2}{1+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\sqrt{\frac{x^2\left(1+x^2\right)}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1+x^6}} - \frac{1}{2}\left(2i-\sqrt{2}\right)\int \left(-\right.$$

$$= \frac{x\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x^2}{1+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\sqrt{\frac{x^2\left(1+x^2\right)}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1+x^6}} + \left(\left(\frac{1}{4}+\frac{i}{4}\right)\left(1+i\sqrt{\right.}\right.$$

Mathematica [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{-1-2x^2+2x^4}{\left(1+2x^4\right)\sqrt{1+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 - 2*x^2 + 2*x^4)/((1 + 2*x^4)*Sqrt[1 + x^6]),x]
[Out] Integrate[(-1 - 2*x^2 + 2*x^4)/((1 + 2*x^4)*Sqrt[1 + x^6]), x]
```

IntegrateAlgebraic [A] time = 13.09, size = 26, normalized size = 1.00

$$-\tan^{-1}\left(\frac{x\sqrt{x^6+1}}{x^4-x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 2*x^2 + 2*x^4)/((1 + 2*x^4)*Sqrt[1 + x^6]),x]
[Out] -ArcTan[(x*Sqrt[1 + x^6])/(1 - x^2 + x^4)]
```

fricas [A] time = 0.45, size = 22, normalized size = 0.85

$$\frac{1}{2}\arctan\left(\frac{2\sqrt{x^6+1}\,x}{2\,x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x^2-1)/(2*x^4+1)/(x^6+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*arctan(2*sqrt(x^6 + 1)*x/(2*x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^4-2\,x^2-1}{\sqrt{x^6+1}\left(2\,x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x^2-1)/(2*x^4+1)/(x^6+1)^(1/2),x, algorithm="giac")
[Out] integrate((2*x^4 - 2*x^2 - 1)/(sqrt(x^6 + 1)*(2*x^4 + 1)), x)
```

maple [C] time = 0.41, size = 51, normalized size = 1.96

$$-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{2\operatorname{RootOf}\left(\_Z^2+1\right)x^2-2\sqrt{x^6+1}\,x-\operatorname{RootOf}\left(\_Z^2+1\right)}{2x^4+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-2*x^2-1)/(2*x^4+1)/(x^6+1)^(1/2),x)
[Out] -1/2*RootOf(_Z^2+1)*ln(-(2*RootOf(_Z^2+1)*x^2-2*(x^6+1)^(1/2)*x-RootOf(_Z^2
+1))/(2*x^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^4-2\,x^2-1}{\sqrt{x^6+1}\left(2\,x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x^2-1)/(2*x^4+1)/(x^6+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^4 - 2*x^2 - 1)/(sqrt(x^6 + 1)*(2*x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{-2\,x^4+2\,x^2+1}{\sqrt{x^6+1}\,\left(2\,x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 2*x^4 + 1)/((x^6 + 1)^(1/2)*(2*x^4 + 1)),x)
```

```
[Out] int(-(2*x^2 - 2*x^4 + 1)/((x^6 + 1)^(1/2)*(2*x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^4-2x^2-1}{\sqrt{\left(x^2+1\right)\left(x^4-x^2+1\right)}\left(2x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-2*x**2-1)/(2*x**4+1)/(x**6+1)**(1/2),x)
```

```
[Out] Integral((2*x**4 - 2*x**2 - 1)/(sqrt((x**2 + 1)*(x**4 - x**2 + 1))*(2*x**4
+ 1)), x)
```

3.289 $\int \frac{1+4x^5}{(-1-ax+x^5)\sqrt{-x+x^6}}\,dx$

Optimal. Leaf size=26

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^6-x}}\right)}{\sqrt{a}}$$

Rubi [F] time = 1.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+4x^5}{(-1-ax+x^5)\sqrt{-x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + 4*x^5)/((-1 - a*x + x^5)*Sqrt[-x + x^6]),x]
```

```
[Out] (8*x*Sqrt[1 - x^5]*Hypergeometric2F1[1/10, 1/2, 11/10, x^5])/Sqrt[-x + x^6]
 - (10*Sqrt[x]*Sqrt[-1 + x^5]*Defer[Subst][Defer[Int][1/((1 + a*x^2 - x^10)
*Sqrt[-1 + x^10]), x], x, Sqrt[x]])/Sqrt[-x + x^6] - (8*a*Sqrt[x]*Sqrt[-1 +
 x^5]*Defer[Subst][Defer[Int][x^2/((1 + a*x^2 - x^10)*Sqrt[-1 + x^10]), x],
 x, Sqrt[x]])/Sqrt[-x + x^6]
```

Rubi steps

$$\int \frac{1+4x^5}{(-1-ax+x^5)\sqrt{-x+x^6}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1+x^5}\right)\int \frac{1+4x^5}{\sqrt{x}\sqrt{-1+x^5}(-1-ax+x^5)}\,dx}{\sqrt{-x+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{1+4x^{10}}{\sqrt{-1+x^{10}}(-1-ax^2+x^{10})}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \left(\frac{4}{\sqrt{-1+x^{10}}}+\frac{5+4ax^2}{\sqrt{-1+x^{10}}(-1-ax^2+x^{10})}\right)dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{5+4ax^2}{\sqrt{-1+x^{10}}(-1-ax^2+x^{10})}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}} + \frac{\left(8\sqrt{x}\sqrt{-1+x^5}\right)\text{S}}{}$$

$$= \frac{\left(8\sqrt{x}\sqrt{1-x^5}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^{10}}}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}} + \frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int\left(-\frac{}{(1}\right.\right.}{}$$

$$= \frac{8x\sqrt{1-x^5}\,{}_2F_1\left(\frac{1}{10},\frac{1}{2};\frac{11}{10};x^5\right)}{\sqrt{-x+x^6}} - \frac{\left(10\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{1}{(1+ax^2-x^{10})\sqrt{-1+x^{10}}}\right.}{\sqrt{-x+x^6}}$$

Mathematica [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{1+4x^5}{(-1-ax+x^5)\sqrt{-x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + 4*x^5)/((-1 - a*x + x^5)*Sqrt[-x + x^6]),x]
[Out] Integrate[(1 + 4*x^5)/((-1 - a*x + x^5)*Sqrt[-x + x^6]), x]
```

IntegrateAlgebraic [A] time = 2.68, size = 26, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^6-x}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 4*x^5)/((-1 - a*x + x^5)*Sqrt[-x + x^6]),x]
[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[-x + x^6]])/Sqrt[a]
```

fricas [A] time = 0.57, size = 123, normalized size = 4.73

$$\left[\frac{\log\left(-\frac{x^{10}+6\,ax^6-2\,x^5+a^2x^2-4\,\sqrt{x^6-x}\left(x^5+ax-1\right)\sqrt{a}-6\,ax+1}{x^{10}-2\,ax^6-2\,x^5+a^2x^2+2\,ax+1}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{2\,\sqrt{x^6-x}\,\sqrt{-a}}{x^5+ax-1}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+1)/(x^5-a*x-1)/(x^6-x)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-(x^10 + 6*a*x^6 - 2*x^5 + a^2*x^2 - 4*sqrt(x^6 - x)*(x^5 + a*x -
1)*sqrt(a) - 6*a*x + 1)/(x^10 - 2*a*x^6 - 2*x^5 + a^2*x^2 + 2*a*x + 1))/sqr
t(a), sqrt(-a)*arctan(2*sqrt(x^6 - x)*sqrt(-a)/(x^5 + a*x - 1))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x^5+1}{\sqrt{x^6-x}\left(x^5-ax-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+1)/(x^5-a*x-1)/(x^6-x)^(1/2),x, algorithm="giac")
[Out] integrate((4*x^5 + 1)/(sqrt(x^6 - x)*(x^5 - a*x - 1)), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{4x^5+1}{\left(x^5-ax-1\right)\sqrt{x^6-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5+1)/(x^5-a*x-1)/(x^6-x)^(1/2),x)
[Out] int((4*x^5+1)/(x^5-a*x-1)/(x^6-x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x^5+1}{\sqrt{x^6-x}\left(x^5-ax-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+1)/(x^5-a*x-1)/(x^6-x)^(1/2),x, algorithm="maxima")
[Out] integrate((4*x^5 + 1)/(sqrt(x^6 - x)*(x^5 - a*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{4\,x^5+1}{\sqrt{x^6-x}\,\left(-x^5+a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x^5 + 1)/((x^6 - x)^(1/2)*(a*x - x^5 + 1)),x)
```

```
[Out] int(-(4*x^5 + 1)/((x^6 - x)^(1/2)*(a*x - x^5 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^5+1}{\sqrt{x\,(x-1)\left(x^4+x^3+x^2+x+1\right)}\,\left(-ax+x^5-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**5+1)/(x**5-a*x-1)/(x**6-x)**(1/2),x)
```

```
[Out] Integral((4*x**5 + 1)/(sqrt(x*(x - 1)*(x**4 + x**3 + x**2 + x + 1))*(-a*x +
 x**5 - 1)), x)
```

3.290 $\int \frac{1+4x^5}{\left(-a-x+ax^5\right)\sqrt{-x+x^6}}\,dx$

Optimal. Leaf size=26

$$-\frac{2\tanh^{-1}\left(\frac{x}{\sqrt{a}\sqrt{x^6-x}}\right)}{\sqrt{a}}$$

Rubi [F] time = 1.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+4x^5}{\left(-a-x+ax^5\right)\sqrt{-x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + 4*x^5)/((-a - x + a*x^5)*Sqrt[-x + x^6]),x]
```

```
[Out] (8*x*Sqrt[1 - x^5]*Hypergeometric2F1[1/10, 1/2, 11/10, x^5])/(a*Sqrt[-x + x
^6]) + (10*Sqrt[x]*Sqrt[-1 + x^5]*Defer[Subst][Defer[Int][1/(Sqrt[-1 + x^10
]*(-a - x^2 + a*x^10)), x], x, Sqrt[x]])/Sqrt[-x + x^6] + (8*Sqrt[x]*Sqrt[-
1 + x^5]*Defer[Subst][Defer[Int][x^2/(Sqrt[-1 + x^10]*(-a - x^2 + a*x^10)),
 x], x, Sqrt[x]])/(a*Sqrt[-x + x^6])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+4x^5}{\left(-a-x+ax^5\right)\sqrt{-x+x^6}}\,dx &= \frac{\left(\sqrt{x}\sqrt{-1+x^5}\right)\int \frac{1+4x^5}{\sqrt{x}\sqrt{-1+x^5}\left(-a-x+ax^5\right)}\,dx}{\sqrt{-x+x^6}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{1+4x^{10}}{\sqrt{-1+x^{10}}\left(-a-x^2+ax^{10}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \left(\frac{4}{a\sqrt{-1+x^{10}}}+\frac{5a+4x^2}{a\sqrt{-1+x^{10}}\left(-a-x^2+ax^{10}\right)}\right)dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{5a+4x^2}{\sqrt{-1+x^{10}}\left(-a-x^2+ax^{10}\right)}\,dx,x,\sqrt{x}\right)}{a\sqrt{-x+x^6}}+\frac{\left(8\sqrt{x}\sqrt{-1+x^5}\right.}{}\\
&= \frac{\left(8\sqrt{x}\sqrt{1-x^5}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^{10}}}\,dx,x,\sqrt{x}\right)}{a\sqrt{-x+x^6}}+\frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int\left(\right.\right.}{}\\
&= \frac{8x\sqrt{1-x^5}\,{}_2F_1\left(\frac{1}{10},\frac{1}{2};\frac{11}{10};x^5\right)}{a\sqrt{-x+x^6}}+\frac{\left(10\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^{10}}\left(-a-x^2+\right.}\right.}{\sqrt{-x+x^6}}
\end{aligned}$$

Mathematica [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{1+4x^5}{\left(-a-x+ax^5\right)\sqrt{-x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + 4*x^5)/((-a - x + a*x^5)*Sqrt[-x + x^6]),x]
[Out] Integrate[(1 + 4*x^5)/((-a - x + a*x^5)*Sqrt[-x + x^6]), x]
```

IntegrateAlgebraic [A] time = 2.67, size = 26, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{x}{\sqrt{a}\,\sqrt{x^6-x}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 4*x^5)/((-a - x + a*x^5)*Sqrt[-x + x^6]),x]
[Out] (-2*ArcTanh[x/(Sqrt[a]*Sqrt[-x + x^6])])/Sqrt[a]
```

fricas [A] time = 0.61, size = 137, normalized size = 5.27

$$\left[\frac{\log\left(-\frac{a^2x^{10}-2\,a^2x^5+6\,ax^6-4\,\left(ax^5-a+x\right)\sqrt{x^6-x}\,\sqrt{a}+a^2-6\,ax+x^2}{a^2x^{10}-2\,a^2x^5-2\,ax^6+a^2+2\,ax+x^2}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{x^6-x}\,\sqrt{-a}}{ax^5-a+x}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+1)/(a*x^5-a-x)/(x^6-x)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-(a^2*x^10 - 2*a^2*x^5 + 6*a*x^6 - 4*(a*x^5 - a + x)*sqrt(x^6 - x)
*sqrt(a) + a^2 - 6*a*x + x^2)/(a^2*x^10 - 2*a^2*x^5 - 2*a*x^6 + a^2 + 2*a*x
 + x^2))/sqrt(a), sqrt(-a)*arctan(2*sqrt(x^6 - x)*sqrt(-a)/(a*x^5 - a + x))
/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x^5+1}{\left(ax^5-a-x\right)\sqrt{x^6-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+1)/(a*x^5-a-x)/(x^6-x)^(1/2),x, algorithm="giac")
[Out] integrate((4*x^5 + 1)/((a*x^5 - a - x)*sqrt(x^6 - x)), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{4x^5+1}{\left(a\,x^5-a-x\right)\sqrt{x^6-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5+1)/(a*x^5-a-x)/(x^6-x)^(1/2),x)
[Out] int((4*x^5+1)/(a*x^5-a-x)/(x^6-x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x^5+1}{\left(ax^5-a-x\right)\sqrt{x^6-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+1)/(a*x^5-a-x)/(x^6-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((4*x^5 + 1)/((a*x^5 - a - x)*sqrt(x^6 - x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{4\,x^5+1}{\sqrt{x^6-x}\,\left(-a\,x^5+x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x^5 + 1)/((x^6 - x)^(1/2)*(a + x - a*x^5)),x)
```

```
[Out] int(-(4*x^5 + 1)/((x^6 - x)^(1/2)*(a + x - a*x^5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^5+1}{\sqrt{x\,(x-1)\left(x^4+x^3+x^2+x+1\right)}\,\left(ax^5-a-x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**5+1)/(a*x**5-a-x)/(x**6-x)**(1/2),x)
```

```
[Out] Integral((4*x**5 + 1)/(sqrt(x*(x - 1)*(x**4 + x**3 + x**2 + x + 1))*(a*x**5
 - a - x)), x)
```

3.291 $\int \frac{(2+x^6)(-1-x^4+x^6)}{x^6(-1+x^6)^{3/4}} dx$

Optimal. Leaf size=26

$$\frac{2\sqrt[4]{x^6-1}\,(x^6-5x^4-1)}{5x^5}$$

Rubi [A] time = 0.10, antiderivative size = 31, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1833, 1584, 449, 1478}

$$\frac{2(x^6-1)^{5/4}}{5x^5}-\frac{2\sqrt[4]{x^6-1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x^6)*(-1 - x^4 + x^6))/(x^6*(-1 + x^6)^(3/4)),x]
```

```
[Out] (-2*(-1 + x^6)^(1/4))/x + (2*(-1 + x^6)^(5/4))/(5*x^5)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1478

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n
_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p
]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+x^6)(-1-x^4+x^6)}{x^6(-1+x^6)^{3/4}}\,dx &= \int \left(\frac{-2x^3-x^9}{x^5(-1+x^6)^{3/4}}+\frac{-2+x^6+x^{12}}{x^6(-1+x^6)^{3/4}}\right)dx\\ &= \int \frac{-2x^3-x^9}{x^5(-1+x^6)^{3/4}}\,dx+\int \frac{-2+x^6+x^{12}}{x^6(-1+x^6)^{3/4}}\,dx\\ &= \int \frac{-2-x^6}{x^2(-1+x^6)^{3/4}}\,dx+\int \frac{\sqrt[4]{-1+x^6}\,(2+x^6)}{x^6}\,dx\\ &= -\frac{2\sqrt[4]{-1+x^6}}{x}+\frac{2(-1+x^6)^{5/4}}{5x^5}\end{aligned}$$

Mathematica [C] time = 0.07, size = 117, normalized size = 4.50

$$\frac{(1-x^6)^{3/4}\left(14\,{}_2F_1\left(-\frac{5}{6},\frac{3}{4};\frac{1}{6};x^6\right)+x^4\left(-7x^6\,{}_2F_1\left(\frac{3}{4},\frac{5}{6};\frac{11}{6};x^6\right)+70\,{}_2F_1\left(-\frac{1}{6},\frac{3}{4};\frac{5}{6};x^6\right)+5x^8\,{}_2F_1\left(\frac{3}{4},\frac{7}{6};\frac{13}{6};x^6\right)+35x^2\,{}_2F_1\left(\frac{1}{6},\frac{3}{4};\frac{7}{6};x^6\right)\right)\right)}{35x^5(x^6-1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x^6)*(-1 - x^4 + x^6))/(x^6*(-1 + x^6)^(3/4)),x]
```

```
[Out] ((1 - x^6)^(3/4)*(14*Hypergeometric2F1[-5/6, 3/4, 1/6, x^6] + x^4*(70*Hyper
geometric2F1[-1/6, 3/4, 5/6, x^6] + 35*x^2*Hypergeometric2F1[1/6, 3/4, 7/6,
 x^6] - 7*x^6*Hypergeometric2F1[3/4, 5/6, 11/6, x^6] + 5*x^8*Hypergeometric
2F1[3/4, 7/6, 13/6, x^6])))/(35*x^5*(-1 + x^6)^(3/4))
```

IntegrateAlgebraic [A] time = 4.61, size = 26, normalized size = 1.00

$$\frac{2\sqrt[4]{x^6-1}\,(x^6-5x^4-1)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^6)*(-1 - x^4 + x^6))/(x^6*(-1 + x^6)^(3/4)),x]
```

```
[Out] (2*(-1 + x^6)^(1/4)*(-1 - 5*x^4 + x^6))/(5*x^5)
```

fricas [A] time = 0.43, size = 22, normalized size = 0.85

$$\frac{2\,(x^6-5\,x^4-1)(x^6-1)^{\frac{1}{4}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)*(x^6-x^4-1)/x^6/(x^6-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/5*(x^6 - 5*x^4 - 1)*(x^6 - 1)^(1/4)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^6-x^4-1)(x^6+2)}{(x^6-1)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)*(x^6-x^4-1)/x^6/(x^6-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((x^6 - x^4 - 1)*(x^6 + 2)/((x^6 - 1)^(3/4)*x^6), x)
```

maple [A] time = 0.01, size = 43, normalized size = 1.65

$$\frac{2(-1+x)(1+x)\left(x^{2}+x+1\right)\left(x^{2}-x+1\right)\left(x^{6}-5x^{4}-1\right)}{5x^{5}\left(x^{6}-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+2)*(x^6-x^4-1)/x^6/(x^6-1)^(3/4),x)
```

```
[Out] 2/5*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)*(x^6-5*x^4-1)/x^5/(x^6-1)^(3/4)
```

maxima [B] time = 0.71, size = 53, normalized size = 2.04

$$\frac{2\left(x^{12}-5\,x^{10}-2\,x^{6}+5\,x^{4}+1\right)}{5\left(x^{2}+x+1\right)^{\frac{3}{4}}\left(x^{2}-x+1\right)^{\frac{3}{4}}(x+1)^{\frac{3}{4}}(x-1)^{\frac{3}{4}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)*(x^6-x^4-1)/x^6/(x^6-1)^(3/4),x, algorithm="maxima")
```

```
[Out] 2/5*(x^12 - 5*x^10 - 2*x^6 + 5*x^4 + 1)/((x^2 + x + 1)^(3/4)*(x^2 - x + 1)^
(3/4)*(x + 1)^(3/4)*(x - 1)^(3/4)*x^5)
```

mupad [B] time = 0.30, size = 27, normalized size = 1.04

$$\frac{2\left(x^{6}-1\right)^{5/4}-10\,x^{4}\left(x^{6}-1\right)^{1/4}}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^6 + 2)*(x^4 - x^6 + 1))/(x^6*(x^6 - 1)^(3/4)),x)
```

```
[Out] (2*(x^6 - 1)^(5/4) - 10*x^4*(x^6 - 1)^(1/4))/(5*x^5)
```

sympy [C] time = 4.50, size = 168, normalized size = 6.46

$$\frac{x^{7}e^{-\frac{3i\pi}{4}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{7}{6}\\\frac{13}{6}\end{matrix}\middle|x^{6}\right)}{6\Gamma\left(\frac{13}{6}\right)}-\frac{x^{5}e^{-\frac{3i\pi}{4}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{5}{6}\\\frac{11}{6}\end{matrix}\middle|x^{6}\right)}{6\Gamma\left(\frac{11}{6}\right)}+\frac{xe^{-\frac{3i\pi}{4}}\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{6},\frac{3}{4}\\\frac{7}{6}\end{matrix}\middle|x^{6}\right)}{6\Gamma\left(\frac{7}{6}\right)}+\frac{e^{\frac{i\pi}{4}}\Gamma\left(-\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{6},\frac{3}{4}\\\frac{5}{6}\end{matrix}\middle|x^{6}\right)}{3x\Gamma\left(\frac{5}{6}\right)}+\frac{e^{\frac{i\pi}{4}}\Gamma\left(-\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{6},\frac{3}{4}\\\frac{1}{6}\end{matrix}\middle|x^{6}\right)}{3x^{5}\Gamma\left(\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+2)*(x**6-x**4-1)/x**6/(x**6-1)**(3/4),x)
```

```
[Out] x**7*exp(-3*I*pi/4)*gamma(7/6)*hyper((3/4, 7/6), (13/6,), x**6)/(6*gamma(13
/6)) - x**5*exp(-3*I*pi/4)*gamma(5/6)*hyper((3/4, 5/6), (11/6,), x**6)/(6*g
amma(11/6)) + x*exp(-3*I*pi/4)*gamma(1/6)*hyper((1/6, 3/4), (7/6,), x**6)/(
6*gamma(7/6)) + exp(I*pi/4)*gamma(-1/6)*hyper((-1/6, 3/4), (5/6,), x**6)/(3
*x*gamma(5/6)) + exp(I*pi/4)*gamma(-5/6)*hyper((-5/6, 3/4), (1/6,), x**6)/(
3*x**5*gamma(1/6))
```

3.292 $\int \frac{(-2+x^6)(1-x^4+x^6)}{x^6(1+x^6)^{3/4}}\,dx$

Optimal. Leaf size=26

$$\frac{2\sqrt[4]{x^6+1}\,\left(x^6-5x^4+1\right)}{5x^5}$$

Rubi [A] time = 0.10, antiderivative size = 31, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1833, 1584, 449, 1478}

$$\frac{2\left(x^6+1\right)^{5/4}}{5x^5}-\frac{2\sqrt[4]{x^6+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((-2 + x^6)*(1 - x^4 + x^6))/(x^6*(1 + x^6)^(3/4)),x]
```

```
[Out] (-2*(1 + x^6)^(1/4))/x + (2*(1 + x^6)^(5/4))/(5*x^5)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1478

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n
_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p
]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(-2+x^6)(1-x^4+x^6)}{x^6(1+x^6)^{3/4}}\,dx = \int \left(\frac{2x^3-x^9}{x^5(1+x^6)^{3/4}}+\frac{-2-x^6+x^{12}}{x^6(1+x^6)^{3/4}}\right)dx$$
$$= \int \frac{2x^3-x^9}{x^5(1+x^6)^{3/4}}\,dx + \int \frac{-2-x^6+x^{12}}{x^6(1+x^6)^{3/4}}\,dx$$
$$= \int \frac{2-x^6}{x^2(1+x^6)^{3/4}}\,dx + \int \frac{(-2+x^6)\sqrt[4]{1+x^6}}{x^6}\,dx$$
$$= -\frac{2\sqrt[4]{1+x^6}}{x} + \frac{2(1+x^6)^{5/4}}{5x^5}$$

Mathematica [C] time = 0.04, size = 105, normalized size = 4.04

$$-x\,{}_2F_1\left(\frac{1}{6},\frac{3}{4};\frac{7}{6};-x^6\right)-\frac{2\,{}_2F_1\left(-\frac{1}{6},\frac{3}{4};\frac{5}{6};-x^6\right)}{x}+\frac{1}{7}x^7\,{}_2F_1\left(\frac{3}{4},\frac{7}{6};\frac{13}{6};-x^6\right)-\frac{1}{5}x^5\,{}_2F_1\left(\frac{3}{4},\frac{5}{6};\frac{11}{6};-x^6\right)+\frac{2\,{}_2F_1\left(-\frac{5}{6},\frac{3}{4};\frac{1}{6};-x^6\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-2 + x^6)*(1 - x^4 + x^6))/(x^6*(1 + x^6)^(3/4)),x]
```

```
[Out] (2*Hypergeometric2F1[-5/6, 3/4, 1/6, -x^6])/(5*x^5) - (2*Hypergeometric2F1[
-1/6, 3/4, 5/6, -x^6])/x - x*Hypergeometric2F1[1/6, 3/4, 7/6, -x^6] - (x^5*
Hypergeometric2F1[3/4, 5/6, 11/6, -x^6])/5 + (x^7*Hypergeometric2F1[3/4, 7/
6, 13/6, -x^6])/7
```

IntegrateAlgebraic [A] time = 4.55, size = 26, normalized size = 1.00

$$\frac{2\sqrt[4]{x^6+1}\,(x^6-5x^4+1)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^6)*(1 - x^4 + x^6))/(x^6*(1 + x^6)^(3/4)),x]
```

```
[Out] (2*(1 + x^6)^(1/4)*(1 - 5*x^4 + x^6))/(5*x^5)
```

fricas [A] time = 0.41, size = 22, normalized size = 0.85

$$\frac{2\,(x^6-5\,x^4+1)(x^6+1)^{\frac{1}{4}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-x^4+1)/x^6/(x^6+1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/5*(x^6 - 5*x^4 + 1)*(x^6 + 1)^(1/4)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^6-x^4+1)(x^6-2)}{(x^6+1)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-x^4+1)/x^6/(x^6+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((x^6 - x^4 + 1)*(x^6 - 2)/((x^6 + 1)^(3/4)*x^6), x)
```

maple [A] time = 0.01, size = 38, normalized size = 1.46

$$\frac{2\left(x^{4}-x^{2}+1\right)\left(x^{2}+1\right)\left(x^{6}-5x^{4}+1\right)}{5x^{5}\left(x^{6}+1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-2)*(x^6-x^4+1)/x^6/(x^6+1)^(3/4),x)
```

```
[Out] 2/5*(x^4-x^2+1)*(x^2+1)*(x^6-5*x^4+1)/x^5/(x^6+1)^(3/4)
```

maxima [A] time = 0.58, size = 44, normalized size = 1.69

$$\frac{2\left(x^{12}-5\,x^{10}+2\,x^{6}-5\,x^{4}+1\right)}{5\left(x^{4}-x^{2}+1\right)^{\frac{3}{4}}\left(x^{2}+1\right)^{\frac{3}{4}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-x^4+1)/x^6/(x^6+1)^(3/4),x, algorithm="maxima")
```

```
[Out] 2/5*(x^12 - 5*x^10 + 2*x^6 - 5*x^4 + 1)/((x^4 - x^2 + 1)^(3/4)*(x^2 + 1)^(3
/4)*x^5)
```

mupad [B] time = 0.30, size = 27, normalized size = 1.04

$$\frac{2\left(x^{6}+1\right)^{5/4}-10\,x^{4}\left(x^{6}+1\right)^{1/4}}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 2)*(x^6 - x^4 + 1))/(x^6*(x^6 + 1)^(3/4)),x)
```

```
[Out] (2*(x^6 + 1)^(5/4) - 10*x^4*(x^6 + 1)^(1/4))/(5*x^5)
```

sympy [C] time = 4.45, size = 155, normalized size = 5.96

$$\frac{x^{7}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{7}{6}\\\frac{13}{6}\end{matrix}\middle|x^{6}e^{i\pi}\right)}{6\Gamma\left(\frac{13}{6}\right)}-\frac{x^{5}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{5}{6}\\\frac{11}{6}\end{matrix}\middle|x^{6}e^{i\pi}\right)}{6\Gamma\left(\frac{11}{6}\right)}-\frac{x\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{6},\frac{3}{4}\\\frac{7}{6}\end{matrix}\middle|x^{6}e^{i\pi}\right)}{6\Gamma\left(\frac{7}{6}\right)}+\frac{\Gamma\left(-\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{6},\frac{3}{4}\\\frac{5}{6}\end{matrix}\middle|x^{6}e^{i\pi}\right)}{3x\Gamma\left(\frac{5}{6}\right)}-\frac{\Gamma\left(-\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{6},\frac{3}{4}\\\frac{1}{6}\end{matrix}\middle|x^{6}e^{i\pi}\right)}{3x^{5}\Gamma\left(\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-2)*(x**6-x**4+1)/x**6/(x**6+1)**(3/4),x)
```

```
[Out] x**7*gamma(7/6)*hyper((3/4, 7/6), (13/6,), x**6*exp_polar(I*pi))/(6*gamma(1
3/6)) - x**5*gamma(5/6)*hyper((3/4, 5/6), (11/6,), x**6*exp_polar(I*pi))/(6
*gamma(11/6)) - x*gamma(1/6)*hyper((1/6, 3/4), (7/6,), x**6*exp_polar(I*pi)
)/(6*gamma(7/6)) + gamma(-1/6)*hyper((-1/6, 3/4), (5/6,), x**6*exp_polar(I*
pi))/(3*x*gamma(5/6)) - gamma(-5/6)*hyper((-5/6, 3/4), (1/6,), x**6*exp_pol
ar(I*pi))/(3*x**5*gamma(1/6))
```

3.293 $\int \frac{\sqrt{-1+x^6}\left(1+2x^6\right)}{x^2\left(-1+x^2+x^6\right)}\,dx$

Optimal. Leaf size=26

$$\frac{\sqrt{x^6-1}}{x}+\tan^{-1}\left(\frac{x}{\sqrt{x^6-1}}\right)$$

Rubi [F] time = 0.77, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-1+x^6}\left(1+2x^6\right)}{x^2\left(-1+x^2+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-1 + x^6]*(1 + 2*x^6))/(x^2*(-1 + x^2 + x^6)),x]
```

```
[Out] Sqrt[-1 + x^6]/x + (3*(1 + Sqrt[3])*x*Sqrt[-1 + x^6])/(2*(1 - (1 + Sqrt[3])
*x^2)) + (3*3^(1/4)*x*(1 - x^2)*Sqrt[(1 + x^2 + x^4)/(1 - (1 + Sqrt[3])*x^2
)^2]*EllipticE[ArcCos[(1 - (1 - Sqrt[3])*x^2)/(1 - (1 + Sqrt[3])*x^2)], (2
+ Sqrt[3])/4])/(2*Sqrt[-((x^2*(1 - x^2))/(1 - (1 + Sqrt[3])*x^2)^2)]*Sqrt[-
1 + x^6]) + (3^(3/4)*(1 - Sqrt[3])*x*(1 - x^2)*Sqrt[(1 + x^2 + x^4)/(1 - (1
 + Sqrt[3])*x^2)^2]*EllipticF[ArcCos[(1 - (1 - Sqrt[3])*x^2)/(1 - (1 + Sqrt
[3])*x^2)], (2 + Sqrt[3])/4])/(4*Sqrt[-((x^2*(1 - x^2))/(1 - (1 + Sqrt[3])*
x^2)^2)]*Sqrt[-1 + x^6]) + Defer[Int][Sqrt[-1 + x^6]/(-1 + x^2 + x^6), x] +
 3*Defer[Int][(x^4*Sqrt[-1 + x^6])/(-1 + x^2 + x^6), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x^6}\left(1+2x^6\right)}{x^2\left(-1+x^2+x^6\right)}\,dx &= \int \left(-\frac{\sqrt{-1+x^6}}{x^2}+\frac{\left(1+3x^4\right)\sqrt{-1+x^6}}{-1+x^2+x^6}\right)dx\\
&= -\int \frac{\sqrt{-1+x^6}}{x^2}\,dx+\int \frac{\left(1+3x^4\right)\sqrt{-1+x^6}}{-1+x^2+x^6}\,dx\\
&= \frac{\sqrt{-1+x^6}}{x}-3\int \frac{x^4}{\sqrt{-1+x^6}}\,dx+\int \left(\frac{\sqrt{-1+x^6}}{-1+x^2+x^6}+\frac{3x^4\sqrt{-1+x^6}}{-1+x^2+x^6}\right)dx\\
&= \frac{\sqrt{-1+x^6}}{x}+\frac{3}{2}\int \frac{-1+\sqrt{3}-2x^4}{\sqrt{-1+x^6}}\,dx+3\int \frac{x^4\sqrt{-1+x^6}}{-1+x^2+x^6}\,dx+\frac{1}{2}\left(3\left(1-\sqrt{3}\right)\right)\int \frac{}{\sqrt{}}\\
&= \frac{\sqrt{-1+x^6}}{x}+\frac{3\left(1+\sqrt{3}\right)x\sqrt{-1+x^6}}{2\left(1-\left(1+\sqrt{3}\right)x^2\right)}+\frac{3\sqrt[4]{3}\,x\left(1-x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\,E\left(\cos^{-1}\left(\frac{1-(}{1-(}\right.\right.}{2\sqrt{-\frac{x^2\left(1-x^2\right)}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{-1+}}
\end{aligned}$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-1+x^6}\left(1+2x^6\right)}{x^2\left(-1+x^2+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[-1 + x^6]*(1 + 2*x^6))/(x^2*(-1 + x^2 + x^6)),x]
```

```
[Out] Integrate[(Sqrt[-1 + x^6]*(1 + 2*x^6))/(x^2*(-1 + x^2 + x^6)), x]
```

IntegrateAlgebraic [A] time = 3.71, size = 26, normalized size = 1.00

$$\frac{\sqrt{x^6-1}}{x}+\tan^{-1}\left(\frac{x}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^6]*(1 + 2*x^6))/(x^2*(-1 + x^2 + x^6)),x]
[Out] Sqrt[-1 + x^6]/x + ArcTan[x/Sqrt[-1 + x^6]]
```

fricas [A] time = 0.78, size = 40, normalized size = 1.54

$$\frac{x\arctan\left(\frac{2\sqrt{x^6-1}\,x}{x^6-x^2-1}\right)+2\sqrt{x^6-1}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)*(2*x^6+1)/x^2/(x^6+x^2-1),x, algorithm="fricas")
[Out] 1/2*(x*arctan(2*sqrt(x^6 - 1)*x/(x^6 - x^2 - 1)) + 2*sqrt(x^6 - 1))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2\,x^6+1\right)\sqrt{x^6-1}}{\left(x^6+x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)*(2*x^6+1)/x^2/(x^6+x^2-1),x, algorithm="giac")
[Out] integrate((2*x^6 + 1)*sqrt(x^6 - 1)/((x^6 + x^2 - 1)*x^2), x)
```

maple [C] time = 0.46, size = 73, normalized size = 2.81

$$\frac{\sqrt{x^6-1}}{x}-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+1\right)x^6-\mathrm{RootOf}\left(\_Z^2+1\right)x^2-2\sqrt{x^6-1}\,x-\mathrm{RootOf}\left(\_Z^2+1\right)}{x^6+x^2-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/2)*(2*x^6+1)/x^2/(x^6+x^2-1),x)
[Out] (x^6-1)^(1/2)/x-1/2*RootOf(_Z^2+1)*ln((RootOf(_Z^2+1)*x^6-RootOf(_Z^2+1)*x^
2-2*(x^6-1)^(1/2)*x-RootOf(_Z^2+1))/(x^6+x^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2\,x^6+1\right)\sqrt{x^6-1}}{\left(x^6+x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)*(2*x^6+1)/x^2/(x^6+x^2-1),x, algorithm="maxima")
[Out] integrate((2*x^6 + 1)*sqrt(x^6 - 1)/((x^6 + x^2 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{x^6-1}\,\left(2\,x^6+1\right)}{x^2\,\left(x^6+x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/2)*(2*x^6 + 1))/(x^2*(x^2 + x^6 - 1)),x)
[Out] int(((x^6 - 1)^(1/2)*(2*x^6 + 1))/(x^2*(x^2 + x^6 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/2)*(2*x**6+1)/x**2/(x**6+x**2-1),x)
[Out] Timed out
```

3.294 $\int \frac{\left(-1+x^3-x^5-2x^7\right)^{2/3}\left(1-x^3+x^5+2x^7\right)\left(-3+2x^5+8x^7\right)}{x^9}\,dx$

Optimal. Leaf size=26

$$\frac{3\left(-2x^7-x^5+x^3-1\right)^{8/3}}{8x^8}$$

Rubi [A] time = 0.32, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {6688, 1590}

$$\frac{3\left(-2x^7-x^5+x^3-1\right)^{8/3}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3 - x^5 - 2*x^7)^(2/3)*(1 - x^3 + x^5 + 2*x^7)*(-3 + 2*x^5 + 8
*x^7))/x^9,x]
```

```
[Out] (3*(-1 + x^3 - x^5 - 2*x^7)^(8/3))/(8*x^8)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{\left(-1+x^3-x^5-2x^7\right)^{2/3}\left(1-x^3+x^5+2x^7\right)\left(-3+2x^5+8x^7\right)}{x^9}\,dx = \int \frac{\left(3-2x^5-8x^7\right)\left(-1+x^3-x^5-2x^7\right)}{x^9}$$

$$= \frac{3\left(-1+x^3-x^5-2x^7\right)^{8/3}}{8x^8}$$

Mathematica [A] time = 0.12, size = 26, normalized size = 1.00

$$\frac{3\left(-2x^7-x^5+x^3-1\right)^{8/3}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3 - x^5 - 2*x^7)^(2/3)*(1 - x^3 + x^5 + 2*x^7)*(-3 + 2*x
^5 + 8*x^7))/x^9,x]
```

```
[Out] (3*(-1 + x^3 - x^5 - 2*x^7)^(8/3))/(8*x^8)
```

IntegrateAlgebraic [A] time = 0.13, size = 26, normalized size = 1.00

$$\frac{3\left(-2x^7-x^5+x^3-1\right)^{8/3}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3 - x^5 - 2*x^7)^(2/3)*(1 - x^3 + x^5 + 2*x^7)*
(-3 + 2*x^5 + 8*x^7))/x^9,x]
```

```
[Out] (3*(-1 + x^3 - x^5 - 2*x^7)^(8/3))/(8*x^8)
```

fricas [B] time = 0.42, size = 62, normalized size = 2.38

$$\frac{3\left(4x^{14}+4x^{12}-3x^{10}-2x^8+4x^7+x^6+2x^5-2x^3+1\right)\left(-2x^7-x^5+x^3-1\right)^{\frac{2}{3}}}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^7-x^5+x^3-1)^(2/3)*(2*x^7+x^5-x^3+1)*(8*x^7+2*x^5-3)/x^9,x,
 algorithm="fricas")
```

```
[Out] 3/8*(4*x^14 + 4*x^12 - 3*x^10 - 2*x^8 + 4*x^7 + x^6 + 2*x^5 - 2*x^3 + 1)*(-
2*x^7 - x^5 + x^3 - 1)^(2/3)/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(8x^7+2x^5-3\right)\left(2x^7+x^5-x^3+1\right)\left(-2x^7-x^5+x^3-1\right)^{\frac{2}{3}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^7-x^5+x^3-1)^(2/3)*(2*x^7+x^5-x^3+1)*(8*x^7+2*x^5-3)/x^9,x,
 algorithm="giac")
```

```
[Out] integrate((8*x^7 + 2*x^5 - 3)*(2*x^7 + x^5 - x^3 + 1)*(-2*x^7 - x^5 + x^3 -
 1)^(2/3)/x^9, x)
```

maple [A] time = 0.01, size = 23, normalized size = 0.88

$$\frac{3\left(-2x^7-x^5+x^3-1\right)^{\frac{8}{3}}}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^7-x^5+x^3-1)^(2/3)*(2*x^7+x^5-x^3+1)*(8*x^7+2*x^5-3)/x^9,x)
```

```
[Out] 3/8*(-2*x^7-x^5+x^3-1)^(8/3)/x^8
```

maxima [B] time = 0.67, size = 62, normalized size = 2.38

$$\frac{3\left(4x^{14}+4x^{12}-3x^{10}-2x^8+4x^7+x^6+2x^5-2x^3+1\right)\left(-2x^7-x^5+x^3-1\right)^{\frac{2}{3}}}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^7-x^5+x^3-1)^(2/3)*(2*x^7+x^5-x^3+1)*(8*x^7+2*x^5-3)/x^9,x,
 algorithm="maxima")
```

```
[Out] 3/8*(4*x^14 + 4*x^12 - 3*x^10 - 2*x^8 + 4*x^7 + x^6 + 2*x^5 - 2*x^3 + 1)*(-
2*x^7 - x^5 + x^3 - 1)^(2/3)/x^8
```

mupad [B] time = 1.52, size = 147, normalized size = 5.65

$$\frac{3\left(-2x^7-x^5+x^3-1\right)^{2/3}}{2x}+\frac{3\left(-2x^7-x^5+x^3-1\right)^{2/3}}{8x^2}+\frac{3\left(-2x^7-x^5+x^3-1\right)^{2/3}}{4x^3}-\frac{3\left(-2x^7-x^5+x^3-1\right)^{2/3}}{4x^5}+\frac{3\left(-2x^7-x^5+x^3-1\right)^{2/3}}{8x^8}-\left(-\frac{3x^6}{2}-\frac{3x^4}{2}+\frac{9x^2}{8}+\frac{3}{4}\right)\left(-2x^7-x^5+x^3-1\right)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*x^5 + 8*x^7 - 3)*(x^3 - x^5 - 2*x^7 - 1)^(5/3))/x^9,x)
```

```
[Out] (3*(x^3 - x^5 - 2*x^7 - 1)^(2/3))/(2*x) + (3*(x^3 - x^5 - 2*x^7 - 1)^(2/3))
/(8*x^2) + (3*(x^3 - x^5 - 2*x^7 - 1)^(2/3))/(4*x^3) - (3*(x^3 - x^5 - 2*x^
7 - 1)^(2/3))/(4*x^5) + (3*(x^3 - x^5 - 2*x^7 - 1)^(2/3))/(8*x^8) - ((9*x^2
)/8 - (3*x^4)/2 - (3*x^6)/2 + 3/4)*(x^3 - x^5 - 2*x^7 - 1)^(2/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(8x^7+2x^5-3\right)\left(-2x^7-x^5+x^3-1\right)^{\frac{2}{3}}\left(2x^7+x^5-x^3+1\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**7-x**5+x**3-1)**(2/3)*(2*x**7+x**5-x**3+1)*(8*x**7+2*x**5-
3)/x**9,x)
```

```
[Out] Integral((8*x**7 + 2*x**5 - 3)*(-2*x**7 - x**5 + x**3 - 1)**(2/3)*(2*x**7 +
 x**5 - x**3 + 1)/x**9, x)
```

3.295 $\int \frac{x^4(9+4x^5)}{\sqrt{x+x^6}(-a-ax^5+x^9)}\,dx$

Optimal. Leaf size=26

$$-\frac{2\tanh^{-1}\left(\frac{x^5}{\sqrt{a}\sqrt{x^6+x}}\right)}{\sqrt{a}}$$

Rubi [F] time = 1.71, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4(9+4x^5)}{\sqrt{x+x^6}(-a-ax^5+x^9)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(9 + 4*x^5))/(Sqrt[x + x^6]*(-a - a*x^5 + x^9)),x]
```

```
[Out] (8*x*Sqrt[1 + x^5]*Hypergeometric2F1[1/10, 1/2, 11/10, -x^5])/Sqrt[x + x^6]
 - (8*a*Sqrt[x]*Sqrt[1 + x^5]*Defer[Subst][Defer[Int][1/(Sqrt[1 + x^10]*(a
+ a*x^10 - x^18)), x], x, Sqrt[x]])/Sqrt[x + x^6] - (8*a*Sqrt[x]*Sqrt[1 + x
^5]*Defer[Subst][Defer[Int][x^10/(Sqrt[1 + x^10]*(a + a*x^10 - x^18)), x],
x, Sqrt[x]])/Sqrt[x + x^6] + (18*Sqrt[x]*Sqrt[1 + x^5]*Defer[Subst][Defer[I
nt][x^8/(Sqrt[1 + x^10]*(-a - a*x^10 + x^18)), x], x, Sqrt[x]])/Sqrt[x + x^
6]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(9+4x^5)}{\sqrt{x+x^6}(-a-ax^5+x^9)}\,dx &= \frac{\left(\sqrt{x}\sqrt{1+x^5}\right)\int \frac{x^{7/2}(9+4x^5)}{\sqrt{1+x^5}(-a-ax^5+x^9)}\,dx}{\sqrt{x+x^6}} \\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{x^8(9+4x^{10})}{\sqrt{1+x^{10}}(-a-ax^{10}+x^{18})}\,dx, x, \sqrt{x}\right)}{\sqrt{x+x^6}} \\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \left(\frac{4}{\sqrt{1+x^{10}}}+\frac{4a+9x^8+4ax^{10}}{\sqrt{1+x^{10}}(-a-ax^{10}+x^{18})}\right)dx, x, \sqrt{x}\right)}{\sqrt{x+x^6}} \\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{4a+9x^8+4ax^{10}}{\sqrt{1+x^{10}}(-a-ax^{10}+x^{18})}\,dx, x, \sqrt{x}\right)}{\sqrt{x+x^6}} + \frac{\left(8\sqrt{x}\sqrt{1+x^5}\right)\text{Sub}}{\sqrt{x}} \\
&= \frac{8x\sqrt{1+x^5}\,{}_2F_1\left(\frac{1}{10},\frac{1}{2};\frac{11}{10};-x^5\right)}{\sqrt{x+x^6}} + \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \left(-\frac{4a}{\sqrt{1+x^{10}}(a+ax^{10}-x^{18})}\right.\right.}{} \\
&= \frac{8x\sqrt{1+x^5}\,{}_2F_1\left(\frac{1}{10},\frac{1}{2};\frac{11}{10};-x^5\right)}{\sqrt{x+x^6}} + \frac{\left(18\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{x^8}{\sqrt{1+x^{10}}(-a-ax^{10}+x^{18})}\right.}{\sqrt{x+x^6}}
\end{aligned}$$

Mathematica [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^4(9+4x^5)}{\sqrt{x+x^6}(-a-ax^5+x^9)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(9 + 4*x^5))/(Sqrt[x + x^6]*(-a - a*x^5 + x^9)),x]
[Out] Integrate[(x^4*(9 + 4*x^5))/(Sqrt[x + x^6]*(-a - a*x^5 + x^9)), x]
```

IntegrateAlgebraic [A] time = 17.41, size = 26, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{x^5}{\sqrt{a}\sqrt{x^6+x}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(9 + 4*x^5))/(Sqrt[x + x^6]*(-a - a*x^5 + x^9)),x]
[Out] (-2*ArcTanh[x^5/(Sqrt[a]*Sqrt[x + x^6])])/Sqrt[a]
```

fricas [A] time = 1.05, size = 144, normalized size = 5.54

$$\left[\frac{\log\left(-\frac{x^{18}+6\,ax^{14}+a^2x^{10}+6\,ax^9+2\,a^2x^5-4\left(x^{13}+ax^9+ax^4\right)\sqrt{x^6+x}\,\sqrt{a}+a^2}{x^{18}-2\,ax^{14}+a^2x^{10}-2\,ax^9+2\,a^2x^5+a^2}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{x^6+x}\,\sqrt{-a}\,x^4}{x^9+ax^5+a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^5+9)/(x^6+x)^(1/2)/(x^9-a*x^5-a),x, algorithm="fricas")
[Out] [1/2*log(-(x^18 + 6*a*x^14 + a^2*x^10 + 6*a*x^9 + 2*a^2*x^5 - 4*(x^13 + a*x
^9 + a*x^4)*sqrt(x^6 + x)*sqrt(a) + a^2)/(x^18 - 2*a*x^14 + a^2*x^10 - 2*a*
x^9 + 2*a^2*x^5 + a^2))/sqrt(a), sqrt(-a)*arctan(2*sqrt(x^6 + x)*sqrt(-a)*x
^4/(x^9 + a*x^5 + a))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^5+9\right)x^4}{\left(x^9-ax^5-a\right)\sqrt{x^6+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^5+9)/(x^6+x)^(1/2)/(x^9-a*x^5-a),x, algorithm="giac")
[Out] integrate((4*x^5 + 9)*x^4/((x^9 - a*x^5 - a)*sqrt(x^6 + x)), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(4x^5+9\right)}{\sqrt{x^6+x}\,\left(x^9-a\,x^5-a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^5+9)/(x^6+x)^(1/2)/(x^9-a*x^5-a),x)
[Out] int(x^4*(4*x^5+9)/(x^6+x)^(1/2)/(x^9-a*x^5-a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^5+9\right)x^4}{\left(x^9-ax^5-a\right)\sqrt{x^6+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^5+9)/(x^6+x)^(1/2)/(x^9-a*x^5-a),x, algorithm="maxima")
```

```
[Out] integrate((4*x^5 + 9)*x^4/((x^9 - a*x^5 - a)*sqrt(x^6 + x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{x^4\,\left(4\,x^5+9\right)}{\sqrt{x^6+x}\,\left(-x^9+a\,x^5+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(4*x^5 + 9))/((x + x^6)^(1/2)*(a + a*x^5 - x^9)),x)
```

```
[Out] int(-(x^4*(4*x^5 + 9))/((x + x^6)^(1/2)*(a + a*x^5 - x^9)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(4x^5+9\right)}{\sqrt{x\left(x+1\right)\left(x^4-x^3+x^2-x+1\right)}\,\left(-ax^5-a+x^9\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(4*x**5+9)/(x**6+x)**(1/2)/(x**9-a*x**5-a),x)
```

```
[Out] Integral(x**4*(4*x**5 + 9)/(sqrt(x*(x + 1)*(x**4 - x**3 + x**2 - x + 1))*(-
a*x**5 - a + x**9)), x)
```

3.296 $\int \frac{x^4(9+5x^4)}{\sqrt{x+x^5}\left(-1-x^4+ax^9\right)} dx$

Optimal. Leaf size=26

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^5+x}}{\sqrt{a}\,x^5}\right)}{\sqrt{a}}$$

Rubi [F] time = 1.42, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4\left(9+5x^4\right)}{\sqrt{x+x^5}\left(-1-x^4+ax^9\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(9 + 5*x^4))/(Sqrt[x + x^5]*(-1 - x^4 + a*x^9)),x]
```

```
[Out] (18*Sqrt[x]*Sqrt[1 + x^4]*Defer[Subst][Defer[Int][x^8/(Sqrt[1 + x^8]*(-1 -
x^8 + a*x^18)), x], x, Sqrt[x]])/Sqrt[x + x^5] + (10*Sqrt[x]*Sqrt[1 + x^4]*
Defer[Subst][Defer[Int][x^16/(Sqrt[1 + x^8]*(-1 - x^8 + a*x^18)), x], x, Sq
rt[x]])/Sqrt[x + x^5]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(9+5x^4\right)}{\sqrt{x+x^5}\left(-1-x^4+ax^9\right)} dx &= \frac{\left(\sqrt{x}\sqrt{1+x^4}\right)\int \frac{x^{7/2}\left(9+5x^4\right)}{\sqrt{1+x^4}\left(-1-x^4+ax^9\right)} dx}{\sqrt{x+x^5}} \\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \frac{x^8\left(9+5x^8\right)}{\sqrt{1+x^8}\left(-1-x^8+ax^{18}\right)} dx, x, \sqrt{x}\right)}{\sqrt{x+x^5}} \\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \left(\frac{9x^8}{\sqrt{1+x^8}\left(-1-x^8+ax^{18}\right)}+\frac{5x^{16}}{\sqrt{1+x^8}\left(-1-x^8+ax^{18}\right)}\right) dx, x, \sqrt{x}\right)}{\sqrt{x+x^5}} \\
&= \frac{\left(10\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \frac{x^{16}}{\sqrt{1+x^8}\left(-1-x^8+ax^{18}\right)} dx, x, \sqrt{x}\right)}{\sqrt{x+x^5}} + \frac{\left(18\sqrt{x}\sqrt{1+x^4}\right)\text{S}
\end{aligned}$$

Mathematica [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(9+5x^4\right)}{\sqrt{x+x^5}\left(-1-x^4+ax^9\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(9 + 5*x^4))/(Sqrt[x + x^5]*(-1 - x^4 + a*x^9)),x]
```

```
[Out] Integrate[(x^4*(9 + 5*x^4))/(Sqrt[x + x^5]*(-1 - x^4 + a*x^9)), x]
```

IntegrateAlgebraic [A] time = 4.80, size = 26, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^5+x}}{\sqrt{a}\,x^5}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(9 + 5*x^4))/(Sqrt[x + x^5]*(-1 - x^4 + a*x^9)),x]
```

```
[Out] (-2*ArcTanh[Sqrt[x + x^5]/(Sqrt[a]*x^5)])/Sqrt[a]
```

fricas [B] time = 0.60, size = 139, normalized size = 5.35

$$\left[\frac{\log\left(\frac{a^2x^{18}+6\,ax^{13}+6\,ax^9+x^8+2\,x^4-4\left(ax^{13}+x^8+x^4\right)\sqrt{x^5+x}\,\sqrt{a}+1}{a^2x^{18}-2\,ax^{13}-2\,ax^9+x^8+2\,x^4+1}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\left(ax^9+x^4+1\right)\sqrt{x^5+x}\,\sqrt{-a}}{2\left(ax^9+ax^5\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^4+9)/(x^5+x)^(1/2)/(a*x^9-x^4-1),x, algorithm="fricas")
```

```
[Out] [1/2*log((a^2*x^18 + 6*a*x^13 + 6*a*x^9 + x^8 + 2*x^4 - 4*(a*x^13 + x^8 + x
^4)*sqrt(x^5 + x)*sqrt(a) + 1)/(a^2*x^18 - 2*a*x^13 - 2*a*x^9 + x^8 + 2*x^4
 + 1))/sqrt(a), sqrt(-a)*arctan(1/2*(a*x^9 + x^4 + 1)*sqrt(x^5 + x)*sqrt(-a
)/(a*x^9 + a*x^5))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^4+9\right)x^4}{\left(ax^9-x^4-1\right)\sqrt{x^5+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^4+9)/(x^5+x)^(1/2)/(a*x^9-x^4-1),x, algorithm="giac")
```

```
[Out] integrate((5*x^4 + 9)*x^4/((a*x^9 - x^4 - 1)*sqrt(x^5 + x)), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(5x^4+9\right)}{\sqrt{x^5+x}\,\left(a\,x^9-x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(5*x^4+9)/(x^5+x)^(1/2)/(a*x^9-x^4-1),x)
```

```
[Out] int(x^4*(5*x^4+9)/(x^5+x)^(1/2)/(a*x^9-x^4-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^4+9\right)x^4}{\left(ax^9-x^4-1\right)\sqrt{x^5+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^4+9)/(x^5+x)^(1/2)/(a*x^9-x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((5*x^4 + 9)*x^4/((a*x^9 - x^4 - 1)*sqrt(x^5 + x)), x)
```

mupad [B] time = 0.93, size = 46, normalized size = 1.77

$$\frac{\ln\left(\frac{a\,x^9+x^4-2\,\sqrt{a}\,x^4\,\sqrt{x^5+x}+1}{-4\,a\,x^9+4\,x^4+4}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(5*x^4 + 9))/((x + x^5)^(1/2)*(x^4 - a*x^9 + 1)),x)
```

```
[Out] log((a*x^9 + x^4 - 2*a^(1/2)*x^4*(x + x^5)^(1/2) + 1)/(4*x^4 - 4*a*x^9 + 4)
)/a^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(5x^4 + 9\right)}{\sqrt{x \left(x^4 + 1\right)} \left(ax^9 - x^4 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(5*x**4+9)/(x**5+x)**(1/2)/(a*x**9-x**4-1),x)
```

```
[Out] Integral(x**4*(5*x**4 + 9)/(sqrt(x*(x**4 + 1))*(a*x**9 - x**4 - 1)), x)
```

3.297 $\int \frac{x^4(9+4x^5)}{\sqrt{x+x^6}\left(-1-x^5+ax^9\right)} dx$

Optimal. Leaf size=26

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^6+x}}{\sqrt{a}\,x^5}\right)}{\sqrt{a}}$$

Rubi [F] time = 1.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4\left(9+4x^5\right)}{\sqrt{x+x^6}\left(-1-x^5+ax^9\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(9 + 4*x^5))/(Sqrt[x + x^6]*(-1 - x^5 + a*x^9)),x]
```

```
[Out] (8*x*Sqrt[1 + x^5]*Hypergeometric2F1[1/10, 1/2, 11/10, -x^5])/(a*Sqrt[x + x
^6]) + (8*Sqrt[x]*Sqrt[1 + x^5]*Defer[Subst][Defer[Int][1/(Sqrt[1 + x^10]*(
-1 - x^10 + a*x^18)), x], x, Sqrt[x]])/(a*Sqrt[x + x^6]) + (18*Sqrt[x]*Sqrt
[1 + x^5]*Defer[Subst][Defer[Int][x^8/(Sqrt[1 + x^10]*(-1 - x^10 + a*x^18))
, x], x, Sqrt[x]])/Sqrt[x + x^6] + (8*Sqrt[x]*Sqrt[1 + x^5]*Defer[Subst][De
fer[Int][x^10/(Sqrt[1 + x^10]*(-1 - x^10 + a*x^18)), x], x, Sqrt[x]])/(a*Sq
rt[x + x^6])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(9+4x^5\right)}{\sqrt{x+x^6}\left(-1-x^5+ax^9\right)} dx &= \frac{\left(\sqrt{x}\sqrt{1+x^5}\right)\int \frac{x^{7/2}\left(9+4x^5\right)}{\sqrt{1+x^5}\left(-1-x^5+ax^9\right)} dx}{\sqrt{x+x^6}} \\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{x^8\left(9+4x^{10}\right)}{\sqrt{1+x^{10}}\left(-1-x^{10}+ax^{18}\right)} dx, x, \sqrt{x}\right)}{\sqrt{x+x^6}} \\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \left(\frac{4}{a\sqrt{1+x^{10}}}+\frac{4+9ax^8+4x^{10}}{a\sqrt{1+x^{10}}\left(-1-x^{10}+ax^{18}\right)}\right) dx, x, \sqrt{x}\right)}{\sqrt{x+x^6}} \\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{4+9ax^8+4x^{10}}{\sqrt{1+x^{10}}\left(-1-x^{10}+ax^{18}\right)} dx, x, \sqrt{x}\right)}{a\sqrt{x+x^6}} + \frac{\left(8\sqrt{x}\sqrt{1+x^5}\right)\text{Subs}}{a\sqrt{}} \\
&= \frac{8x\sqrt{1+x^5}\,{}_2F_1\left(\frac{1}{10},\frac{1}{2};\frac{11}{10};-x^5\right)}{a\sqrt{x+x^6}} + \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \left(\frac{4}{\sqrt{1+x^{10}}\left(-1-x^{10}+ax^{18}\right)}\right.\right.}{} \\
&= \frac{8x\sqrt{1+x^5}\,{}_2F_1\left(\frac{1}{10},\frac{1}{2};\frac{11}{10};-x^5\right)}{a\sqrt{x+x^6}} + \frac{\left(18\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{x^8}{\sqrt{1+x^{10}}\left(-1-x^{10}+ax^{18}\right)}\right.}{\sqrt{x+x^6}}
\end{aligned}$$

Mathematica [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(9+4x^5\right)}{\sqrt{x+x^6}\left(-1-x^5+ax^9\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(9 + 4*x^5))/(Sqrt[x + x^6]*(-1 - x^5 + a*x^9)),x]
[Out] Integrate[(x^4*(9 + 4*x^5))/(Sqrt[x + x^6]*(-1 - x^5 + a*x^9)), x]
```

IntegrateAlgebraic [A] time = 9.87, size = 26, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^6+x}}{\sqrt{a}\,x^5}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(9 + 4*x^5))/(Sqrt[x + x^6]*(-1 - x^5 + a*x^9)),x]
[Out] (-2*ArcTanh[Sqrt[x + x^6]/(Sqrt[a]*x^5)])/Sqrt[a]
```

fricas [A] time = 1.10, size = 132, normalized size = 5.08

$$\left[\frac{\log\left(-\frac{a^2x^{18}+6\,ax^{14}+6\,ax^9+x^{10}+2\,x^5-4\left(ax^{13}+x^9+x^4\right)\sqrt{x^6+x}\,\sqrt{a}+1}{a^2x^{18}-2\,ax^{14}-2\,ax^9+x^{10}+2\,x^5+1}\right)}{2\sqrt{a}}, \frac{\sqrt{-a}\,\arctan\left(\frac{2\sqrt{x^6+x}\,\sqrt{-a}\,x^4}{ax^9+x^5+1}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^5+9)/(x^6+x)^(1/2)/(a*x^9-x^5-1),x, algorithm="fricas")
[Out] [1/2*log(-(a^2*x^18 + 6*a*x^14 + 6*a*x^9 + x^10 + 2*x^5 - 4*(a*x^13 + x^9 +
 x^4)*sqrt(x^6 + x)*sqrt(a) + 1)/(a^2*x^18 - 2*a*x^14 - 2*a*x^9 + x^10 + 2*
x^5 + 1))/sqrt(a), sqrt(-a)*arctan(2*sqrt(x^6 + x)*sqrt(-a)*x^4/(a*x^9 + x^
5 + 1))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^5+9\right)x^4}{\left(ax^9-x^5-1\right)\sqrt{x^6+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^5+9)/(x^6+x)^(1/2)/(a*x^9-x^5-1),x, algorithm="giac")
[Out] integrate((4*x^5 + 9)*x^4/((a*x^9 - x^5 - 1)*sqrt(x^6 + x)), x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(4x^5+9\right)}{\sqrt{x^6+x}\,\left(a\,x^9-x^5-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^5+9)/(x^6+x)^(1/2)/(a*x^9-x^5-1),x)
[Out] int(x^4*(4*x^5+9)/(x^6+x)^(1/2)/(a*x^9-x^5-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^5+9\right)x^4}{\left(ax^9-x^5-1\right)\sqrt{x^6+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^5+9)/(x^6+x)^(1/2)/(a*x^9-x^5-1),x, algorithm="maxima")
```

```
[Out] integrate((4*x^5 + 9)*x^4/((a*x^9 - x^5 - 1)*sqrt(x^6 + x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{x^4 \left(4 x^5 + 9\right)}{\sqrt{x^6 + x} \left(-a x^9 + x^5 + 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(4*x^5 + 9))/((x + x^6)^(1/2)*(x^5 - a*x^9 + 1)),x)
```

```
[Out] int(-(x^4*(4*x^5 + 9))/((x + x^6)^(1/2)*(x^5 - a*x^9 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(4x^5 + 9\right)}{\sqrt{x (x + 1) \left(x^4 - x^3 + x^2 - x + 1\right)} \left(ax^9 - x^5 - 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(4*x**5+9)/(x**6+x)**(1/2)/(a*x**9-x**5-1),x)
```

```
[Out] Integral(x**4*(4*x**5 + 9)/(sqrt(x*(x + 1)*(x**4 - x**3 + x**2 - x + 1))*(a
*x**9 - x**5 - 1)), x)
```

3.298 $\int x^5\sqrt{1-2x^3}\,dx$

Optimal. Leaf size=27

$$\frac{1}{45}\sqrt{1-2x^3}\left(6x^6-x^3-1\right)$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{30}\left(1-2x^3\right)^{5/2}-\frac{1}{18}\left(1-2x^3\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[1 - 2*x^3],x]
```

```
[Out] -1/18*(1 - 2*x^3)^(3/2) + (1 - 2*x^3)^(5/2)/30
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{1-2x^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int\sqrt{1-2x}\,x\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(\frac{1}{2}\sqrt{1-2x}-\frac{1}{2}(1-2x)^{3/2}\right)dx,x,x^3\right)\\ &= -\frac{1}{18}\left(1-2x^3\right)^{3/2}+\frac{1}{30}\left(1-2x^3\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.81

$$-\frac{1}{45}\left(1-2x^3\right)^{3/2}\left(3x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[1 - 2*x^3],x]
```

```
[Out] -1/45*((1 - 2*x^3)^(3/2)*(1 + 3*x^3))
```

IntegrateAlgebraic [A] time = 0.02, size = 22, normalized size = 0.81

$$\frac{1}{45}\left(-3x^3-1\right)\left(1-2x^3\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[1 - 2*x^3],x]
[Out] ((-1 - 3*x^3)*(1 - 2*x^3)^(3/2))/45
```

fricas [A] time = 0.38, size = 23, normalized size = 0.85

$$\frac{1}{45} \left(6 x^6 - x^3 - 1\right) \sqrt{-2 x^3 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-2*x^3+1)^(1/2),x, algorithm="fricas")
[Out] 1/45*(6*x^6 - x^3 - 1)*sqrt(-2*x^3 + 1)
```

giac [A] time = 0.31, size = 32, normalized size = 1.19

$$\frac{1}{30} \left(2 x^3 - 1\right)^2 \sqrt{-2 x^3 + 1} - \frac{1}{18} \left(-2 x^3 + 1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-2*x^3+1)^(1/2),x, algorithm="giac")
[Out] 1/30*(2*x^3 - 1)^2*sqrt(-2*x^3 + 1) - 1/18*(-2*x^3 + 1)^(3/2)
```

maple [A] time = 0.00, size = 19, normalized size = 0.70

$$-\frac{\left(3x^3 + 1\right) \left(-2x^3 + 1\right)^{\frac{3}{2}}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-2*x^3+1)^(1/2),x)
[Out] -1/45*(3*x^3+1)*(-2*x^3+1)^(3/2)
```

maxima [A] time = 0.32, size = 23, normalized size = 0.85

$$\frac{1}{30} \left(-2 x^3 + 1\right)^{\frac{5}{2}} - \frac{1}{18} \left(-2 x^3 + 1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-2*x^3+1)^(1/2),x, algorithm="maxima")
[Out] 1/30*(-2*x^3 + 1)^(5/2) - 1/18*(-2*x^3 + 1)^(3/2)
```

mupad [B] time = 0.15, size = 34, normalized size = 1.26

$$-\frac{\frac{5 \left(2 x^3-1\right)^2}{2} + \frac{3 \left(2 x^3-1\right)^3}{2}}{45 \sqrt{1 - 2 x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(1 - 2*x^3)^(1/2),x)
[Out] -((5*(2*x^3 - 1)^2)/2 + (3*(2*x^3 - 1)^3)/2)/(45*(1 - 2*x^3)^(1/2))
```

sympy [B] time = 0.29, size = 42, normalized size = 1.56

$$\frac{2x^6 \sqrt{1 - 2x^3}}{15} - \frac{x^3 \sqrt{1 - 2x^3}}{45} - \frac{\sqrt{1 - 2x^3}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-2*x**3+1)**(1/2),x)
```

```
[Out] 2*x**6*sqrt(1 - 2*x**3)/15 - x**3*sqrt(1 - 2*x**3)/45 - sqrt(1 - 2*x**3)/45
```

3.299 $\int \frac{2+x}{(-1+x)\sqrt{-1+3x+x^3}}\,dx$

Optimal. Leaf size=27

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{x^3+3x-1}}\right)}{\sqrt{3}}$$

Rubi [C] time = 3.94, antiderivative size = 1340, normalized size of antiderivative = 49.63, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6742, 2066, 718, 419, 2080, 934, 169, 538, 537}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(2 + x)/((-1 + x)*Sqrt[-1 + 3*x + x^3]),x]
```

```
[Out] ((2*I)*2^(5/6)*Sqrt[((2/(1 + Sqrt[5]))^(1/3) - ((1 + Sqrt[5])/2)^(1/3) + x)
/(6/(1 + Sqrt[5])^(1/3) - 3*(2*(1 + Sqrt[5]))^(1/3) - I*2^(1/6)*Sqrt[3*(4 +
 2*(2/(1 + Sqrt[5]))^(2/3) + 2^(1/3)*(1 + Sqrt[5])^(2/3))])]*Sqrt[1 + (2/(1
 + Sqrt[5]))^(2/3) + ((1 + Sqrt[5])/2)^(2/3) - ((2/(1 + Sqrt[5]))^(1/3) - (
(1 + Sqrt[5])/2)^(1/3))*x + x^2]*EllipticF[ArcSin[Sqrt[(I*((2/(1 + Sqrt[5])
)^(1/3) - ((1 + Sqrt[5])/2)^(1/3) - I*Sqrt[6 + 3*(2/(1 + Sqrt[5]))^(2/3) +
3*((1 + Sqrt[5])/2)^(2/3)] - 2*x))/Sqrt[6*(4 + 2*(2/(1 + Sqrt[5]))^(2/3) +
2^(1/3)*(1 + Sqrt[5])^(2/3))]]], (2*2^(1/6)*Sqrt[3*(4 + 2*(2/(1 + Sqrt[5]))
^(2/3) + 2^(1/3)*(1 + Sqrt[5])^(2/3))])/((6*I)/(1 + Sqrt[5])^(1/3) - (3*I)*
(2*(1 + Sqrt[5]))^(1/3) + 2^(1/6)*Sqrt[3*(4 + 2*(2/(1 + Sqrt[5]))^(2/3) + 2
^(1/3)*(1 + Sqrt[5])^(2/3))])])/Sqrt[-1 + 3*x + x^3] - (3*(2/(1 + Sqrt[5]))
^(1/6)*Sqrt[6 - 3*2^(1/3)*(1 + Sqrt[5])^(2/3) + I*2^(1/6)*(1 + Sqrt[5])^(1/
3)*Sqrt[3*(4 + 2*(2/(1 + Sqrt[5]))^(2/3) + 2^(1/3)*(1 + Sqrt[5])^(2/3))]]*S
qrt[(2/(1 + Sqrt[5]))^(1/3) - ((1 + Sqrt[5])/2)^(1/3) + x]*Sqrt[1 - (2*((2/
(1 + Sqrt[5]))^(1/3) - ((1 + Sqrt[5])/2)^(1/3) + x))/(3*(2/(1 + Sqrt[5]))^(
1/3) - 3*((1 + Sqrt[5])/2)^(1/3) - I*Sqrt[6 + 3*(2/(1 + Sqrt[5]))^(2/3) + 3
*((1 + Sqrt[5])/2)^(2/3)])]*Sqrt[1 - (2*((2/(1 + Sqrt[5]))^(1/3) - ((1 + Sq
rt[5])/2)^(1/3) + x))/(3*(2/(1 + Sqrt[5]))^(1/3) - 3*((1 + Sqrt[5])/2)^(1/3
) + I*Sqrt[6 + 3*(2/(1 + Sqrt[5]))^(2/3) + 3*((1 + Sqrt[5])/2)^(2/3)])]*Ell
ipticPi[(3*(2/(1 + Sqrt[5]))^(1/3) - 3*((1 + Sqrt[5])/2)^(1/3) + I*Sqrt[6 +
 3*(2/(1 + Sqrt[5]))^(2/3) + 3*((1 + Sqrt[5])/2)^(2/3)])/(2*(1 + (2/(1 + Sq
rt[5]))^(1/3) - ((1 + Sqrt[5])/2)^(1/3))), ArcSin[(2^(5/6)*(1 + Sqrt[5])^(1
/6)*Sqrt[(2/(1 + Sqrt[5]))^(1/3) - ((1 + Sqrt[5])/2)^(1/3) + x])/Sqrt[6 - 3
*2^(1/3)*(1 + Sqrt[5])^(2/3) + I*2^(1/6)*(1 + Sqrt[5])^(1/3)*Sqrt[3*(4 + 2*
(2/(1 + Sqrt[5]))^(2/3) + 2^(1/3)*(1 + Sqrt[5])^(2/3))]]], (3*(2/(1 + Sqrt[
5]))^(1/3) - 3*((1 + Sqrt[5])/2)^(1/3) + I*Sqrt[6 + 3*(2/(1 + Sqrt[5]))^(2/
3) + 3*((1 + Sqrt[5])/2)^(2/3)])/(3*(2/(1 + Sqrt[5]))^(1/3) - 3*((1 + Sqrt[
5])/2)^(1/3) - I*Sqrt[6 + 3*(2/(1 + Sqrt[5]))^(2/3) + 3*((1 + Sqrt[5])/2)^(
2/3)])])/((1 + (2/(1 + Sqrt[5]))^(1/3) - ((1 + Sqrt[5])/2)^(1/3))*Sqrt[-1 +
 3*x + x^3])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 2066

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> With[{r = Rt[-9*
a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}, Dist[(a + b*x + d*x^3)^p
/(Simp[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d*x, x]^p*Simp[(b*d)/3 + (12^(1/
3)*b^2*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*((2^(1/3)*b*d)/(3^(1/3)*r) - r/1
8^(1/3))*x + d^2*x^2, x]^p), Int[Simp[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d
*x, x]^p*Simp[(b*d)/3 + (12^(1/3)*b^2*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*(
(2^(1/3)*b*d)/(3^(1/3)*r) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ
[{a, b, d, p}, x] && NeQ[4*b^3 + 27*a^2*d, 0] && !IntegerQ[p]
```

Rule 2080

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Dist[(a + b*x + d*x^3)^p/(Simp[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d*x, x
]^p*Simp[(b*d)/3 + (12^(1/3)*b^2*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*((2^(1
/3)*b*d)/(3^(1/3)*r) - r/18^(1/3))*x + d^2*x^2, x]^p), Int[(e + f*x)^m*Simp
[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d*x, x]^p*Simp[(b*d)/3 + (12^(1/3)*b^2
*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*((2^(1/3)*b*d)/(3^(1/3)*r) - r/18^(1/3
))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*b
```

```
^3 + 27*a^2*d, 0] && !IntegerQ[p]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{2+x}{(-1+x)\sqrt{-1+3x+x^3}}\,dx = \int\left(\frac{1}{\sqrt{-1+3x+x^3}}+\frac{3}{(-1+x)\sqrt{-1+3x+x^3}}\right)dx$$

$$= 3\int \frac{1}{(-1+x)\sqrt{-1+3x+x^3}}\,dx+\int\frac{1}{\sqrt{-1+3x+x^3}}\,dx$$

$$= \frac{\left(\sqrt{\sqrt[3]{\frac{2}{1+\sqrt{5}}}-\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}+x}\,\sqrt{1+\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}+\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{2/3}-\left(\sqrt[3]{\frac{2}{1+\cdots}}\right.\cdots}\right.\cdots}{\cdots}$$

$$= \frac{\left(3\sqrt{\sqrt[3]{\frac{2}{1+\sqrt{5}}}-\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}+x}\,\sqrt{-\sqrt[3]{\frac{2}{1+\sqrt{5}}}+\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}-i\sqrt{\frac{3}{2}\left(4+2\cdots\right.}\cdots}\right.\cdots}{\cdots}$$

$$= \frac{2i2^{5/6}\sqrt{\frac{\sqrt[3]{\frac{2}{1+\sqrt{5}}}-\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}+x}{\frac{6}{\sqrt[3]{1+\sqrt{5}}}-3\sqrt[3]{2\left(1+\sqrt{5}\right)}-i\sqrt[6]{2}\sqrt{3\left(4+2\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}+\sqrt[3]{2}\left(1+\sqrt{5}\right)^{2/3}\right)}}}\,\sqrt{1+\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\cdots}\cdots}{\cdots}$$

$$= \frac{2i2^{5/6}\sqrt{\frac{\sqrt[3]{\frac{2}{1+\sqrt{5}}}-\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}+x}{\frac{6}{\sqrt[3]{1+\sqrt{5}}}-3\sqrt[3]{2\left(1+\sqrt{5}\right)}-i\sqrt[6]{2}\sqrt{3\left(4+2\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}+\sqrt[3]{2}\left(1+\sqrt{5}\right)^{2/3}\right)}}}\,\sqrt{1+\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\cdots}\cdots}{\cdots}$$

$$= \frac{2i2^{5/6}\sqrt{\frac{\sqrt[3]{\frac{2}{1+\sqrt{5}}}-\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}+x}{\frac{6}{\sqrt[3]{1+\sqrt{5}}}-3\sqrt[3]{2\left(1+\sqrt{5}\right)}-i\sqrt[6]{2}\sqrt{3\left(4+2\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}+\sqrt[3]{2}\left(1+\sqrt{5}\right)^{2/3}\right)}}}\,\sqrt{1+\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\cdots}\cdots}{\cdots}$$

$$= \frac{2i2^{5/6}\sqrt{\frac{\sqrt[3]{\frac{2}{1+\sqrt{5}}}-\sqrt[3]{\frac{1}{2}\left(1+\sqrt{5}\right)}+x}{\frac{6}{\sqrt[3]{1+\sqrt{5}}}-3\sqrt[3]{2\left(1+\sqrt{5}\right)}-i\sqrt[6]{2}\sqrt{3\left(4+2\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}+\sqrt[3]{2}\left(1+\sqrt{5}\right)^{2/3}\right)}}}\,\sqrt{1+\left(\frac{2}{1+\sqrt{5}}\right)^{2/3}\cdots}\cdots}{\cdots}$$

Mathematica [C] time = 1.06, size = 812, normalized size = 30.07

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + x)/((-1 + x)*Sqrt[-1 + 3*x + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x + Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 1, 0])/(Root[3 + 6*#1 +
3*#1^2 + #1^3 & , 1, 0] - Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 3, 0])]*((3*Ell
ipticPi[1 - Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 2, 0]/Root[3 + 6*#1 + 3*#1^2
+ #1^3 & , 3, 0], ArcSin[Sqrt[(1 - x + Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 3,
 0])/(-Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 2, 0] + Root[3 + 6*#1 + 3*#1^2 + #
1^3 & , 3, 0])]], (Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 2, 0] - Root[3 + 6*#1
+ 3*#1^2 + #1^3 & , 3, 0])/(Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 1, 0] - Root[
3 + 6*#1 + 3*#1^2 + #1^3 & , 3, 0])]*Sqrt[-(((-1 + x - Root[3 + 6*#1 + 3*#1
^2 + #1^3 & , 2, 0])*(-1 + x - Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 3, 0]))/(R
oot[3 + 6*#1 + 3*#1^2 + #1^3 & , 2, 0] - Root[3 + 6*#1 + 3*#1^2 + #1^3 & ,
3, 0])^2)]*(Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 2, 0] - Root[3 + 6*#1 + 3*#1^
2 + #1^3 & , 3, 0]))/Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 3, 0] + (EllipticF[A
rcSin[Sqrt[(1 - x + Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 3, 0])/(-Root[3 + 6*#
1 + 3*#1^2 + #1^3 & , 2, 0] + Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 3, 0])]], (
Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 2, 0] - Root[3 + 6*#1 + 3*#1^2 + #1^3 & ,
 3, 0])/(Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 1, 0] - Root[3 + 6*#1 + 3*#1^2 +
 #1^3 & , 3, 0])]*(-1 + x - Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 3, 0])*Sqrt[(
1 - x + Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 2, 0])/(Root[3 + 6*#1 + 3*#1^2 +
#1^3 & , 2, 0] - Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 3, 0])])/Sqrt[(1 - x + R
oot[3 + 6*#1 + 3*#1^2 + #1^3 & , 3, 0])/(-Root[3 + 6*#1 + 3*#1^2 + #1^3 & ,
 2, 0] + Root[3 + 6*#1 + 3*#1^2 + #1^3 & , 3, 0])]))/Sqrt[-1 + 3*x + x^3]
```

IntegrateAlgebraic [A] time = 0.22, size = 27, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{x^3+3x-1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)/((-1 + x)*Sqrt[-1 + 3*x + x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3]*x)/Sqrt[-1 + 3*x + x^3]])/Sqrt[3]
```

fricas [B] time = 0.44, size = 97, normalized size = 3.59

$$\frac{1}{6}\sqrt{3}\log\left(\frac{x^6+18x^5+15x^4+52x^3-4\sqrt{3}\left(x^4+3x^3+3x^2-x\right)\sqrt{x^3+3x-1}-9x^2-6x+1}{x^6-6x^5+15x^4-20x^3+15x^2-6x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-1+x)/(x^3+3*x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((x^6 + 18*x^5 + 15*x^4 + 52*x^3 - 4*sqrt(3)*(x^4 + 3*x^3 +
3*x^2 - x)*sqrt(x^3 + 3*x - 1) - 9*x^2 - 6*x + 1)/(x^6 - 6*x^5 + 15*x^4 - 2
0*x^3 + 15*x^2 - 6*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+2}{\sqrt{x^3+3x-1}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-1+x)/(x^3+3*x-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 2)/(sqrt(x^3 + 3*x - 1)*(x - 1)), x)
```

maple [C] time = 0.42, size = 1075, normalized size = 39.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-1+x)/(x^3+3*x-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(1/2*(4+4*5^(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3))*(-I*(x+1/4*(4
+4*5^(1/2))^(1/3)-1/(4+4*5^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/2*(4+4*5^(1/2))^(1
/3)+2/(4+4*5^(1/2))^(1/3)))*3^(1/2)/(1/2*(4+4*5^(1/2))^(1/3)+2/(4+4*5^(1/2)
)^(1/3)))^(1/2)*((x-1/2*(4+4*5^(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3))/(-3/4*(4
+4*5^(1/2))^(1/3)+3/(4+4*5^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(4+4*5^(1/2))^(1
/3)+2/(4+4*5^(1/2))^(1/3))))^(1/2)*(I*(x+1/4*(4+4*5^(1/2))^(1/3)-1/(4+4*5^(
1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(4+4*5^(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3)))*
3^(1/2)/(1/2*(4+4*5^(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3)))^(1/2)/(x^3+3*x-1)^
(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/4*(4+4*5^(1/2))^(1/3)-1/(4+4*5^(1/2))^
(1/3)+1/2*I*3^(1/2)*(1/2*(4+4*5^(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3)))*3^(1/2
)/(1/2*(4+4*5^(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3)))^(1/2),(-I*3^(1/2)*(1/2*(
4+4*5^(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3))/(-3/4*(4+4*5^(1/2))^(1/3)+3/(4+4*
5^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(4+4*5^(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3)
)))^(1/2))+2*I*3^(1/2)*(1/2*(4+4*5^(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3))*(-I*
(x+1/4*(4+4*5^(1/2))^(1/3)-1/(4+4*5^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/2*(4+4*5^
(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3)))*3^(1/2)/(1/2*(4+4*5^(1/2))^(1/3)+2/(4+
4*5^(1/2))^(1/3)))^(1/2)*((x-1/2*(4+4*5^(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3))
/(-3/4*(4+4*5^(1/2))^(1/3)+3/(4+4*5^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(4+4*5^
(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3))))^(1/2)*(I*(x+1/4*(4+4*5^(1/2))^(1/3)-1
/(4+4*5^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(4+4*5^(1/2))^(1/3)+2/(4+4*5^(1/2))
^(1/3)))*3^(1/2)/(1/2*(4+4*5^(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3)))^(1/2)/(x^
3+3*x-1)^(1/2)/(-1/4*(4+4*5^(1/2))^(1/3)+1/(4+4*5^(1/2))^(1/3)-1/2*I*3^(1/2
)*(1/2*(4+4*5^(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3))-1)*EllipticPi(1/3*3^(1/2)
*(-I*(x+1/4*(4+4*5^(1/2))^(1/3)-1/(4+4*5^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/2*(4
+4*5^(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3)))*3^(1/2)/(1/2*(4+4*5^(1/2))^(1/3)+
2/(4+4*5^(1/2))^(1/3)))^(1/2),-I*3^(1/2)*(1/2*(4+4*5^(1/2))^(1/3)+2/(4+4*5^
(1/2))^(1/3))/(-1/4*(4+4*5^(1/2))^(1/3)+1/(4+4*5^(1/2))^(1/3)-1/2*I*3^(1/2)
*(1/2*(4+4*5^(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3))-1),(-I*3^(1/2)*(1/2*(4+4*5
^(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3))/(-3/4*(4+4*5^(1/2))^(1/3)+3/(4+4*5^(1/
2))^(1/3)-1/2*I*3^(1/2)*(1/2*(4+4*5^(1/2))^(1/3)+2/(4+4*5^(1/2))^(1/3))))^(
1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\sqrt{x^3+3x-1}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-1+x)/(x^3+3*x-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 2)/(sqrt(x^3 + 3*x - 1)*(x - 1)), x)
```

mupad [B] time = 1.37, size = 1872, normalized size = 69.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((x - 1)*(3*x + x^3 - 1)^(1/2)),x)
```

```
[Out] (2*(-(x + 1/(5^(1/2)/2 + 1/2)^(1/3) - (5^(1/2)/2 + 1/2)^(1/3))/((3^(1/2)*(1
/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 - 3/(2*(5^(1/2)/2
```

```
 + 1/2)^(1/3)) + (3*(5^(1/2)/2 + 1/2)^(1/3))/2))^(1/2)*ellipticF(asin(((x +
 (3^(1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 - 1/(
2*(5^(1/2)/2 + 1/2)^(1/3)) + (5^(1/2)/2 + 1/2)^(1/3)/2)/((3^(1/2)*(1/(5^(1/
2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 - 3/(2*(5^(1/2)/2 + 1/2)
^(1/3)) + (3*(5^(1/2)/2 + 1/2)^(1/3))/2))^(1/2)), -(3^(1/2)*((3^(1/2)*(1/(5
^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 - 3/(2*(5^(1/2)/2 +
1/2)^(1/3)) + (3*(5^(1/2)/2 + 1/2)^(1/3))/2)*1i)/(3*(1/(5^(1/2)/2 + 1/2)^(1
/3) + (5^(1/2)/2 + 1/2)^(1/3))))*((x + (3^(1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3)
+ (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 - 1/(2*(5^(1/2)/2 + 1/2)^(1/3)) + (5^(1/2)
/2 + 1/2)^(1/3)/2)/((3^(1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)
^(1/3))*1i)/2 - 3/(2*(5^(1/2)/2 + 1/2)^(1/3)) + (3*(5^(1/2)/2 + 1/2)^(1/3))
/2))^(1/2)*((3^(1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*
1i)/2 - 3/(2*(5^(1/2)/2 + 1/2)^(1/3)) + (3*(5^(1/2)/2 + 1/2)^(1/3))/2)*((3^
(1/2)*(x - (3^(1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1
i)/2 - 1/(2*(5^(1/2)/2 + 1/2)^(1/3)) + (5^(1/2)/2 + 1/2)^(1/3)/2)*1i)/(3*(1
/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))))^(1/2))/(x^3 - x*((1/(
5^(1/2)/2 + 1/2)^(1/3) - (5^(1/2)/2 + 1/2)^(1/3))*((3^(1/2)*(1/(5^(1/2)/2 +
 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 + 1/(2*(5^(1/2)/2 + 1/2)^(1/3)
) - (5^(1/2)/2 + 1/2)^(1/3)/2) - (1/(5^(1/2)/2 + 1/2)^(1/3) - (5^(1/2)/2 +
1/2)^(1/3))*((3^(1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))
*1i)/2 - 1/(2*(5^(1/2)/2 + 1/2)^(1/3)) + (5^(1/2)/2 + 1/2)^(1/3)/2) + ((3^(
1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 - 1/(2*(5^
(1/2)/2 + 1/2)^(1/3)) + (5^(1/2)/2 + 1/2)^(1/3)/2)*((3^(1/2)*(1/(5^(1/2)/2
+ 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 + 1/(2*(5^(1/2)/2 + 1/2)^(1/3
)) - (5^(1/2)/2 + 1/2)^(1/3)/2)) - (1/(5^(1/2)/2 + 1/2)^(1/3) - (5^(1/2)/2
+ 1/2)^(1/3))*((3^(1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3
))*1i)/2 - 1/(2*(5^(1/2)/2 + 1/2)^(1/3)) + (5^(1/2)/2 + 1/2)^(1/3)/2)*((3^(
1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 + 1/(2*(5^
(1/2)/2 + 1/2)^(1/3)) - (5^(1/2)/2 + 1/2)^(1/3)/2))^(1/2) - (6*(-(x + 1/(5^
(1/2)/2 + 1/2)^(1/3) - (5^(1/2)/2 + 1/2)^(1/3))/((3^(1/2)*(1/(5^(1/2)/2 + 1
/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 - 3/(2*(5^(1/2)/2 + 1/2)^(1/3))
+ (3*(5^(1/2)/2 + 1/2)^(1/3))/2))^(1/2)*((x + (3^(1/2)*(1/(5^(1/2)/2 + 1/2)
^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 - 1/(2*(5^(1/2)/2 + 1/2)^(1/3)) + (
5^(1/2)/2 + 1/2)^(1/3)/2)/((3^(1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2
 + 1/2)^(1/3))*1i)/2 - 3/(2*(5^(1/2)/2 + 1/2)^(1/3)) + (3*(5^(1/2)/2 + 1/2)
^(1/3))/2))^(1/2)*((3^(1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^
(1/3))*1i)/2 - 3/(2*(5^(1/2)/2 + 1/2)^(1/3)) + (3*(5^(1/2)/2 + 1/2)^(1/3))/
2)*ellipticPi(((3^(1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3
))*1i)/2 - 3/(2*(5^(1/2)/2 + 1/2)^(1/3)) + (3*(5^(1/2)/2 + 1/2)^(1/3))/2)/(
(3^(1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 - 1/(2
*(5^(1/2)/2 + 1/2)^(1/3)) + (5^(1/2)/2 + 1/2)^(1/3)/2 + 1), asin(((x + (3^(
1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 - 1/(2*(5^
(1/2)/2 + 1/2)^(1/3)) + (5^(1/2)/2 + 1/2)^(1/3)/2)/((3^(1/2)*(1/(5^(1/2)/2
+ 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 - 3/(2*(5^(1/2)/2 + 1/2)^(1/3
)) + (3*(5^(1/2)/2 + 1/2)^(1/3))/2))^(1/2)), -(3^(1/2)*((3^(1/2)*(1/(5^(1/2
)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 - 3/(2*(5^(1/2)/2 + 1/2)^
(1/3)) + (3*(5^(1/2)/2 + 1/2)^(1/3))/2)*1i)/(3*(1/(5^(1/2)/2 + 1/2)^(1/3) +
 (5^(1/2)/2 + 1/2)^(1/3))))*((3^(1/2)*(x - (3^(1/2)*(1/(5^(1/2)/2 + 1/2)^(1
/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 - 1/(2*(5^(1/2)/2 + 1/2)^(1/3)) + (5^(
1/2)/2 + 1/2)^(1/3)/2)*1i)/(3*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2
)^(1/3))))^(1/2))/((x^3 - x*((1/(5^(1/2)/2 + 1/2)^(1/3) - (5^(1/2)/2 + 1/2)
^(1/3))*((3^(1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)
/2 + 1/(2*(5^(1/2)/2 + 1/2)^(1/3)) - (5^(1/2)/2 + 1/2)^(1/3)/2) - (1/(5^(1/
2)/2 + 1/2)^(1/3) - (5^(1/2)/2 + 1/2)^(1/3))*((3^(1/2)*(1/(5^(1/2)/2 + 1/2)
^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 - 1/(2*(5^(1/2)/2 + 1/2)^(1/3)) + (
5^(1/2)/2 + 1/2)^(1/3)/2) + ((3^(1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)
/2 + 1/2)^(1/3))*1i)/2 - 1/(2*(5^(1/2)/2 + 1/2)^(1/3)) + (5^(1/2)/2 + 1/2)^
(1/3)/2)*((3^(1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i
)/2 + 1/(2*(5^(1/2)/2 + 1/2)^(1/3)) - (5^(1/2)/2 + 1/2)^(1/3)/2)) - (1/(5^(
```

```
1/2)/2 + 1/2)^(1/3) - (5^(1/2)/2 + 1/2)^(1/3))*((3^(1/2)*(1/(5^(1/2)/2 + 1/
2)^(1/3) + (5^(1/2)/2 + 1/2)^(1/3))*1i)/2 - 1/(2*(5^(1/2)/2 + 1/2)^(1/3)) +
 (5^(1/2)/2 + 1/2)^(1/3)/2)*((3^(1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)
/2 + 1/2)^(1/3))*1i)/2 + 1/(2*(5^(1/2)/2 + 1/2)^(1/3)) - (5^(1/2)/2 + 1/2)^
(1/3)/2))^(1/2)*((3^(1/2)*(1/(5^(1/2)/2 + 1/2)^(1/3) + (5^(1/2)/2 + 1/2)^(1
/3))*1i)/2 - 1/(2*(5^(1/2)/2 + 1/2)^(1/3)) + (5^(1/2)/2 + 1/2)^(1/3)/2 + 1)
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{(x-1)\sqrt{x^3+3x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-1+x)/(x**3+3*x-1)**(1/2),x)
```

```
[Out] Integral((x + 2)/((x - 1)*sqrt(x**3 + 3*x - 1)), x)
```

3.300 $\int \frac{-1+2x}{(1+x)\sqrt{-x-x^2+x^3}}\,dx$

Optimal. Leaf size=27

$$2\tan^{-1}\left(\frac{\sqrt{x^3-x^2-x}}{2x+1}\right)$$

Rubi [C] time = 1.02, antiderivative size = 537, normalized size of antiderivative = 19.89, number of steps used = 12, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2056, 6733, 1710, 1098, 1214, 1456, 540, 421, 419, 538, 537}

$$\frac{3\sqrt{2}\left(1+\sqrt{5}\right)\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\left(3+\sqrt{5}\right)\sqrt{x^3-x^2-x}}-\frac{6\sqrt{x}\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}\sqrt{\frac{\left(1+\sqrt{5}\right)x+2}{\left(1-\sqrt{5}\right)x+2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\left(3+\sqrt{5}\right)\sqrt{\frac{1}{\left(1-\sqrt{5}\right)x+2}}\sqrt{x^3-x^2-x}}+\frac{2\sqrt{x}\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}\sqrt{\frac{\left(1+\sqrt{5}\right)x+2}{\left(1-\sqrt{5}\right)x+2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{\left(1-\sqrt{5}\right)x+2}}\sqrt{x^3-x^2-x}}-\frac{6\sqrt{2}\left(2+\sqrt{5}\right)\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(\frac{1}{2}\left(-1-\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\left(3+\sqrt{5}\right)\sqrt{x^3-x^2-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + 2*x)/((1 + x)*Sqrt[-x - x^2 + x^3]),x]
```

```
[Out] (3*Sqrt[2]*(1 + Sqrt[5])*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1
 + Sqrt[5])]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]], (-3 - Sqrt[5]
)/2])/((3 + Sqrt[5])*Sqrt[-x - x^2 + x^3]) + (2*Sqrt[x]*Sqrt[-2 - (1 - Sqrt
[5])*x]*Sqrt[(2 + (1 + Sqrt[5])*x)/(2 + (1 - Sqrt[5])*x)]*EllipticF[ArcSin[
(Sqrt[2]*5^(1/4)*Sqrt[x])/Sqrt[-2 - (1 - Sqrt[5])*x]], (5 - Sqrt[5])/10])/(
5^(1/4)*Sqrt[(2 + (1 - Sqrt[5])*x)^(-1)]*Sqrt[-x - x^2 + x^3]) - (6*Sqrt[x]
*Sqrt[-2 - (1 - Sqrt[5])*x]*Sqrt[(2 + (1 + Sqrt[5])*x)/(2 + (1 - Sqrt[5])*x
)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*Sqrt[x])/Sqrt[-2 - (1 - Sqrt[5])*x]],
(5 - Sqrt[5])/10])/(5^(1/4)*(3 + Sqrt[5])*Sqrt[(2 + (1 - Sqrt[5])*x)^(-1)]*
Sqrt[-x - x^2 + x^3]) - (6*Sqrt[2]*(2 + Sqrt[5])*Sqrt[x]*Sqrt[-1 + Sqrt[5]
+ 2*x]*Sqrt[1 - (2*x)/(1 + Sqrt[5])]*EllipticPi[(-1 - Sqrt[5])/2, ArcSin[Sq
rt[2/(1 + Sqrt[5])]*Sqrt[x]], (-3 - Sqrt[5])/2])/((3 + Sqrt[5])*Sqrt[-x - x
^2 + x^3])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 540

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x
] + Dist[(b*c - a*d)/b, Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1710

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + b*x^2 + c*x^4], x],
 x] + Dist[(e*A - d*B)/e, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x],
x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{-1+2x}{(1+x)\sqrt{-x-x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{-1+2x}{\sqrt{x}(1+x)\sqrt{-1-x+x^2}}\,dx}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1-x+x^2}\right)\text{Subst}\left(\int \frac{-1+2x^2}{(1+x^2)\sqrt{-1-x^2+x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(4\sqrt{x}\sqrt{-1-x+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2+x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{-x-x^2+x^3}} - \frac{\left(6\sqrt{x}\sqrt{-1-x+x^2}\right)\text{Subs}}{\sqrt{-x}}$$

$$= \frac{2\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} + \frac{(6\sqrt{}}{}$$

$$= \frac{2\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - \frac{6\sqrt{x}}{}$$

$$= \frac{2\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - \frac{6\sqrt{x}}{}$$

$$= \frac{2\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - \frac{6\sqrt{x}}{}$$

$$= \frac{3\sqrt{2}\left(1+\sqrt{5}\right)\sqrt{x}\sqrt{-1+\sqrt{5}+2x}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{}\right.\right.}{\left(3+\sqrt{5}\right)\sqrt{-x-x^2+x^3}}$$

Mathematica [C] time = 0.36, size = 148, normalized size = 5.48

$$-\frac{2i\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{-\frac{1}{x^2}-\frac{1}{x}+1}\,x^{3/2}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{2}{1+\sqrt{5}}}}{\sqrt{x}}\right)|-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-3\Pi\left(\frac{1}{2}\left(1+\sqrt{5}\right);i\sinh^{-1}\left(\frac{\sqrt{\frac{2}{1+\sqrt{5}}}}{\sqrt{x}}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)\right)}{\sqrt{x\left(x^2-x-1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 2*x)/((1 + x)*Sqrt[-x - x^2 + x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 - x^(-2) - x^(-1)]*x^(3/2)*(EllipticF
[I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], -3/2 - Sqrt[5]/2] - 3*EllipticPi
[(1 + Sqrt[5])/2, I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], (-3 - Sqrt[5])/
2]))/Sqrt[x*(-1 - x + x^2)]
```

IntegrateAlgebraic [A] time = 0.09, size = 27, normalized size = 1.00

$$2\tan^{-1}\left(\frac{\sqrt{x^3-x^2-x}}{2x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x)/((1 + x)*Sqrt[-x - x^2 + x^3]),x]
```

```
[Out] 2*ArcTan[Sqrt[-x - x^2 + x^3]/(1 + 2*x)]
```

fricas [B] time = 0.43, size = 51, normalized size = 1.89

$$\arctan\left(\frac{\sqrt{x^3-x^2-x}\left(x^3-5\,x^2-5\,x-1\right)}{2\left(2\,x^4-x^3-3\,x^2-x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(1+x)/(x^3-x^2-x)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(1/2*sqrt(x^3 - x^2 - x)*(x^3 - 5*x^2 - 5*x - 1)/(2*x^4 - x^3 - 3*x^2
 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x-1}{\sqrt{x^3-x^2-x}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(1+x)/(x^3-x^2-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*x - 1)/(sqrt(x^3 - x^2 - x)*(x + 1)), x)
```

maple [C] time = 0.06, size = 250, normalized size = 9.26

$$\frac{4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\sqrt{-5\left(x-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\sqrt{5}}\,\sqrt{-\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}},\frac{\sqrt{5}\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)}{5\sqrt{x^3-x^2-x}}-\frac{6\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\sqrt{-5\left(x-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\sqrt{5}}\,\sqrt{-\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}},\frac{\frac{1}{2}-\frac{\sqrt{5}}{2}}{\frac{3}{2}-\frac{\sqrt{5}}{2}},\frac{\sqrt{5}\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)}{5\sqrt{x^3-x^2-x}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x)/(1+x)/(x^3-x^2-x)^(1/2),x)
```

```
[Out] 4/5*(1/2*5^(1/2)-1/2)*((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x-
1/2-1/2*5^(1/2))*5^(1/2))^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1
/2)*EllipticF(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),1/5*5^(1/2)*((1
/2*5^(1/2)-1/2)*5^(1/2))^(1/2))-6/5*(1/2*5^(1/2)-1/2)*((x-1/2+1/2*5^(1/2))/
(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x-1/2-1/2*5^(1/2))*5^(1/2))^(1/2)*(-x/(1/2*5^
(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(3/2-1/2*5^(1/2))*EllipticPi(((x-1/2+1/
2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(3/2-1/2*5^(1/2)),1/5
*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x-1}{\sqrt{x^3-x^2-x}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(1+x)/(x^3-x^2-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x - 1)/(sqrt(x^3 - x^2 - x)*(x + 1)), x)
```

mupad [B] time = 0.28, size = 167, normalized size = 6.19

$$\frac{\left(2\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)-3\,\Pi\left(-\frac{\sqrt{5}}{2}-\frac{1}{2};\operatorname{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)\right)\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\left(\sqrt{5}+1\right)\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}}{\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 1)/((x + 1)*(x^3 - x^2 - x)^(1/2)),x)
```

```
[Out] ((2*ellipticF(asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/2 + 1/2)/(5^(1/2
)/2 - 1/2)) - 3*ellipticPi(- 5^(1/2)/2 - 1/2, asin((x/(5^(1/2)/2 + 1/2))^(1
/2)), -(5^(1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2)))*(x/(5^(1/2)/2 + 1/2))^(1/2)*((
x + 5^(1/2)/2 - 1/2)/(5^(1/2)/2 - 1/2))^(1/2)*(5^(1/2) + 1)*((5^(1/2)/2 - x
 + 1/2)/(5^(1/2)/2 + 1/2))^(1/2))/(x^3 - x^2 - x*(5^(1/2)/2 - 1/2)*(5^(1/2)
/2 + 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-1}{\sqrt{x\left(x^2-x-1\right)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(1+x)/(x**3-x**2-x)**(1/2),x)
```

```
[Out] Integral((2*x - 1)/(sqrt(x*(x**2 - x - 1))*(x + 1)), x)
```

3.301 $\int \frac{x^5}{\sqrt{b+ax^3}}\,dx$

Optimal. Leaf size=27

$$\frac{2\left(ax^3-2b\right)\sqrt{ax^3+b}}{9a^2}$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.41, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{2\left(ax^3+b\right)^{3/2}}{9a^2}-\frac{2b\sqrt{ax^3+b}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[b + a*x^3],x]
```

```
[Out] (-2*b*Sqrt[b + a*x^3])/(3*a^2) + (2*(b + a*x^3)^(3/2))/(9*a^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{b+ax^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{\sqrt{b+ax}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{b}{a\sqrt{b+ax}}+\frac{\sqrt{b+ax}}{a}\right)dx, x, x^3\right)\\ &= -\frac{2b\sqrt{b+ax^3}}{3a^2}+\frac{2\left(b+ax^3\right)^{3/2}}{9a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{2\left(ax^3-2b\right)\sqrt{ax^3+b}}{9a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[b + a*x^3],x]
```

```
[Out] (2*(-2*b + a*x^3)*Sqrt[b + a*x^3])/(9*a^2)
```

IntegrateAlgebraic [A] time = 0.03, size = 27, normalized size = 1.00

$$\frac{2\left(ax^3-2b\right)\sqrt{ax^3+b}}{9a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/Sqrt[b + a*x^3],x]
```

```
[Out] (2*(-2*b + a*x^3)*Sqrt[b + a*x^3])/(9*a^2)
```

fricas [A] time = 0.39, size = 23, normalized size = 0.85

$$\frac{2\sqrt{ax^3+b}\left(ax^3-2b\right)}{9a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^3+b)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*sqrt(a*x^3 + b)*(a*x^3 - 2*b)/a^2
```

giac [A] time = 0.59, size = 30, normalized size = 1.11

$$\frac{2\left(ax^3+b\right)^{\frac{3}{2}}}{9a^2}-\frac{2\sqrt{ax^3+b}\,b}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^3+b)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*(a*x^3 + b)^(3/2)/a^2 - 2/3*sqrt(a*x^3 + b)*b/a^2
```

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$\frac{2\left(a\,x^3-2b\right)\sqrt{a\,x^3+b}}{9a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^3+b)^(1/2),x)
```

```
[Out] 2/9*(a*x^3-2*b)*(a*x^3+b)^(1/2)/a^2
```

maxima [A] time = 0.44, size = 30, normalized size = 1.11

$$\frac{2\left(ax^3+b\right)^{\frac{3}{2}}}{9a^2}-\frac{2\sqrt{ax^3+b}\,b}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^3+b)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(a*x^3 + b)^(3/2)/a^2 - 2/3*sqrt(a*x^3 + b)*b/a^2
```

mupad [B] time = 0.31, size = 24, normalized size = 0.89

$$-\frac{2\sqrt{a\,x^3+b}\,\left(2\,b-a\,x^3\right)}{9\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b + a*x^3)^(1/2),x)
```

```
[Out] -(2*(b + a*x^3)^(1/2)*(2*b - a*x^3))/(9*a^2)
```

sympy [**A**] time = 0.60, size = 46, normalized size = 1.70

$$\begin{cases} \frac{2x^3\sqrt{ax^3+b}}{9a} - \frac{4b\sqrt{ax^3+b}}{9a^2} & \text{for } a \neq 0 \\ \frac{x^6}{6\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*x**3+b)**(1/2),x)
```

```
[Out] Piecewise((2*x**3*sqrt(a*x**3 + b)/(9*a) - 4*b*sqrt(a*x**3 + b)/(9*a**2), N
e(a, 0)), (x**6/(6*sqrt(b)), True))
```

3.302 $\int \frac{1}{x^4 \sqrt[4]{-x^2+x^4}} \, dx$

Optimal. Leaf size=27

$$\frac{2 \left(4 x^2+3\right) \left(x^4-x^2\right)^{3/4}}{21 x^5}$$

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.52, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2016, 2014}

$$\frac{2 \left(x^4-x^2\right)^{3/4}}{7 x^5}+\frac{8 \left(x^4-x^2\right)^{3/4}}{21 x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] (2*(-x^2 + x^4)^(3/4))/(7*x^5) + (8*(-x^2 + x^4)^(3/4))/(21*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^4 \sqrt[4]{-x^2+x^4}} \, dx &= \frac{2 \left(-x^2+x^4\right)^{3/4}}{7 x^5}+\frac{4}{7} \int \frac{1}{x^2 \sqrt[4]{-x^2+x^4}} \, dx\\ &= \frac{2 \left(-x^2+x^4\right)^{3/4}}{7 x^5}+\frac{8 \left(-x^2+x^4\right)^{3/4}}{21 x^3}\\ \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{2 \left(x^2 \left(x^2-1\right)\right)^{3/4} \left(4 x^2+3\right)}{21 x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] (2*(x^2*(-1 + x^2))^(3/4)*(3 + 4*x^2))/(21*x^5)
```

IntegrateAlgebraic [A] time = 0.13, size = 27, normalized size = 1.00

$$\frac{2 \left(4 x^2+3\right) \left(x^4-x^2\right)^{3/4}}{21 x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(-x^2 + x^4)^(1/4)),x]
[Out] (2*(3 + 4*x^2)*(-x^2 + x^4)^(3/4))/(21*x^5)
```

fricas [A] time = 0.39, size = 23, normalized size = 0.85

$$\frac{2\left(x^4-x^2\right)^{\frac{3}{4}}\left(4\,x^2+3\right)}{21\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4-x^2)^(1/4),x, algorithm="fricas")
[Out] 2/21*(x^4 - x^2)^(3/4)*(4*x^2 + 3)/x^5
```

giac [A] time = 0.42, size = 23, normalized size = 0.85

$$\frac{2}{7}\left(-\frac{1}{x^2}+1\right)^{\frac{7}{4}}-\frac{2}{3}\left(-\frac{1}{x^2}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4-x^2)^(1/4),x, algorithm="giac")
[Out] 2/7*(-1/x^2 + 1)^(7/4) - 2/3*(-1/x^2 + 1)^(3/4)
```

maple [A] time = 0.00, size = 30, normalized size = 1.11

$$\frac{2\left(-1+x\right)\left(1+x\right)\left(4x^2+3\right)}{21x^3\left(x^4-x^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^4-x^2)^(1/4),x)
[Out] 2/21*(-1+x)*(1+x)*(4*x^2+3)/x^3/(x^4-x^2)^(1/4)
```

maxima [A] time = 0.65, size = 29, normalized size = 1.07

$$\frac{2\left(4\,x^5-x^3-3\,x\right)}{21\left(x+1\right)^{\frac{1}{4}}\left(x-1\right)^{\frac{1}{4}}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4-x^2)^(1/4),x, algorithm="maxima")
[Out] 2/21*(4*x^5 - x^3 - 3*x)/((x + 1)^(1/4)*(x - 1)^(1/4)*x^(9/2))
```

mupad [B] time = 0.24, size = 35, normalized size = 1.30

$$\frac{8\,x^2\left(x^4-x^2\right)^{3/4}+6\left(x^4-x^2\right)^{3/4}}{21\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^4 - x^2)^(1/4)),x)
[Out] (8*x^2*(x^4 - x^2)^(3/4) + 6*(x^4 - x^2)^(3/4))/(21*x^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt[4]{x^2 (x-1) (x+1)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**4-x**2)**(1/4),x)
```

```
[Out] Integral(1/(x**4*(x**2*(x - 1)*(x + 1))**(1/4)), x)
```

3.303 $\int \frac{1}{x\sqrt{b+ax^4}}\,dx$

Optimal. Leaf size=27

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{ax^4+b}}{\sqrt{b}}\right)}{2\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{ax^4+b}}{\sqrt{b}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b + a*x^4]),x]
```

```
[Out] -1/2*ArcTanh[Sqrt[b + a*x^4]/Sqrt[b]]/Sqrt[b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{b+ax^4}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{b+ax}}\,dx, x, x^4\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^2}{a}}\,dx, x, \sqrt{b+ax^4}\right)}{2a}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{b+ax^4}}{\sqrt{b}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{ax^4+b}}{\sqrt{b}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b + a*x^4]),x]
[Out] -1/2*ArcTanh[Sqrt[b + a*x^4]/Sqrt[b]]/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.00, size = 27, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{ax^4+b}}{\sqrt{b}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[b + a*x^4]),x]
[Out] -1/2*ArcTanh[Sqrt[b + a*x^4]/Sqrt[b]]/Sqrt[b]
```

fricas [A] time = 0.41, size = 63, normalized size = 2.33

$$\left[\frac{\log\left(\frac{ax^4-2\sqrt{ax^4+b}\sqrt{b}+2b}{x^4}\right)}{4\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{ax^4+b}\sqrt{-b}}{b}\right)}{2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+b)^(1/2),x, algorithm="fricas")
[Out] [1/4*log((a*x^4 - 2*sqrt(a*x^4 + b)*sqrt(b) + 2*b)/x^4)/sqrt(b), 1/2*sqrt(-
b)*arctan(sqrt(a*x^4 + b)*sqrt(-b)/b)/b]
```

giac [A] time = 0.36, size = 23, normalized size = 0.85

$$\frac{\arctan\left(\frac{\sqrt{ax^4+b}}{\sqrt{-b}}\right)}{2\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+b)^(1/2),x, algorithm="giac")
[Out] 1/2*arctan(sqrt(a*x^4 + b)/sqrt(-b))/sqrt(-b)
```

maple [A] time = 0.01, size = 29, normalized size = 1.07

$$-\frac{\ln\left(\frac{2b+2\sqrt{b}\sqrt{a x^4+b}}{x^2}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^4+b)^(1/2),x)
[Out] -1/2/b^(1/2)*ln((2*b+2*b^(1/2)*(a*x^4+b)^(1/2))/x^2)
```

maxima [A] time = 0.80, size = 37, normalized size = 1.37

$$\frac{\log\left(\frac{\sqrt{ax^4+b}-\sqrt{b}}{\sqrt{ax^4+b}+\sqrt{b}}\right)}{4\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+b)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*log((sqrt(a*x^4 + b) - sqrt(b))/(sqrt(a*x^4 + b) + sqrt(b)))/sqrt(b)
```

mupad [B] time = 0.39, size = 19, normalized size = 0.70

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a\,x^4+b}}{\sqrt{b}}\right)}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b + a*x^4)^(1/2)),x)
```

```
[Out] -atanh((b + a*x^4)^(1/2)/b^(1/2))/(2*b^(1/2))
```

sympy [A] time = 0.87, size = 22, normalized size = 0.81

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x^2}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**4+b)**(1/2),x)
```

```
[Out] -asinh(sqrt(b)/(sqrt(a)*x**2))/(2*sqrt(b))
```

3.304 $\int \frac{1}{x^7 \sqrt[3]{-x^2+x^6}} \, dx$

Optimal. Leaf size=27

$$\frac{3 \left(3 x^4+2\right) \left(x^6-x^2\right)^{2/3}}{40 x^8}$$

Rubi [A] time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.52, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2016, 2014}

$$\frac{3 \left(x^6-x^2\right)^{2/3}}{20 x^8}+\frac{9 \left(x^6-x^2\right)^{2/3}}{40 x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(-x^2 + x^6)^(1/3)),x]
```

```
[Out] (3*(-x^2 + x^6)^(2/3))/(20*x^8) + (9*(-x^2 + x^6)^(2/3))/(40*x^4)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7 \sqrt[3]{-x^2+x^6}} \, dx &= \frac{3 \left(-x^2+x^6\right)^{2/3}}{20 x^8}+\frac{3}{5} \int \frac{1}{x^3 \sqrt[3]{-x^2+x^6}} \, dx\\ &= \frac{3 \left(-x^2+x^6\right)^{2/3}}{20 x^8}+\frac{9 \left(-x^2+x^6\right)^{2/3}}{40 x^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{3 \left(x^2 \left(x^4-1\right)\right)^{2/3} \left(3 x^4+2\right)}{40 x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(-x^2 + x^6)^(1/3)),x]
```

```
[Out] (3*(x^2*(-1 + x^4))^(2/3)*(2 + 3*x^4))/(40*x^8)
```

IntegrateAlgebraic [A] time = 0.44, size = 27, normalized size = 1.00

$$\frac{3\left(3x^4+2\right)\left(x^6-x^2\right)^{2/3}}{40x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(-x^2 + x^6)^(1/3)),x]
[Out] (3*(2 + 3*x^4)*(-x^2 + x^6)^(2/3))/(40*x^8)
```

fricas [A] time = 0.39, size = 23, normalized size = 0.85

$$\frac{3\left(x^6-x^2\right)^{\frac{2}{3}}\left(3\,x^4+2\right)}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6-x^2)^(1/3),x, algorithm="fricas")
[Out] 3/40*(x^6 - x^2)^(2/3)*(3*x^4 + 2)/x^8
```

giac [A] time = 0.33, size = 23, normalized size = 0.85

$$-\frac{3}{20}\left(-\frac{1}{x^4}+1\right)^{\frac{5}{3}}+\frac{3}{8}\left(-\frac{1}{x^4}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6-x^2)^(1/3),x, algorithm="giac")
[Out] -3/20*(-1/x^4 + 1)^(5/3) + 3/8*(-1/x^4 + 1)^(2/3)
```

maple [A] time = 0.00, size = 35, normalized size = 1.30

$$\frac{3\left(-1+x\right)\left(1+x\right)\left(x^2+1\right)\left(3x^4+2\right)}{40x^6\left(x^6-x^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^6-x^2)^(1/3),x)
[Out] 3/40*(-1+x)*(1+x)*(x^2+1)*(3*x^4+2)/x^6/(x^6-x^2)^(1/3)
```

maxima [A] time = 0.48, size = 37, normalized size = 1.37

$$\frac{3\left(3\,x^{10}-x^6-2\,x^2\right)}{40\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-1\right)^{\frac{1}{3}}\left(x^2\right)^{\frac{13}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6-x^2)^(1/3),x, algorithm="maxima")
[Out] 3/40*(3*x^10 - x^6 - 2*x^2)/((x^2 + 1)^(1/3)*(x^2 - 1)^(1/3)*(x^2)^(13/3))
```

mupad [B] time = 0.24, size = 35, normalized size = 1.30

$$\frac{9\,x^4\left(x^6-x^2\right)^{2/3}+6\left(x^6-x^2\right)^{2/3}}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^6 - x^2)^(1/3)),x)
```

```
[Out] (9*x^4*(x^6 - x^2)^(2/3) + 6*(x^6 - x^2)^(2/3))/(40*x^8)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 \sqrt[3]{x^2 (x-1)(x+1)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**6-x**2)**(1/3),x)
```

```
[Out] Integral(1/(x**7*(x**2*(x - 1)*(x + 1)*(x**2 + 1))**(1/3)), x)
```

3.305 $\int \frac{\sqrt{-1+x^3}}{x} \, dx$

Optimal. Leaf size=28

$$\frac{2\sqrt{x^3-1}}{3}-\frac{2}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 50, 63, 203}

$$\frac{2\sqrt{x^3-1}}{3}-\frac{2}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^3]/x,x]
```

```
[Out] (2*Sqrt[-1 + x^3])/3 - (2*ArcTan[Sqrt[-1 + x^3]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{-1+x^3}}{x}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, x^3\right)$$
$$= \frac{2}{3}\sqrt{-1+x^3} - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^3\right)$$
$$= \frac{2}{3}\sqrt{-1+x^3} - \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^3}\right)$$
$$= \frac{2}{3}\sqrt{-1+x^3} - \frac{2}{3}\tan^{-1}\left(\sqrt{-1+x^3}\right)$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$\frac{2\sqrt{x^3-1}}{3} - \frac{2}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^3]/x,x]
[Out] (2*Sqrt[-1 + x^3])/3 - (2*ArcTan[Sqrt[-1 + x^3]])/3
```

IntegrateAlgebraic [A] time = 0.02, size = 28, normalized size = 1.00

$$\frac{2\sqrt{x^3-1}}{3} - \frac{2}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x^3]/x,x]
[Out] (2*Sqrt[-1 + x^3])/3 - (2*ArcTan[Sqrt[-1 + x^3]])/3
```

fricas [A] time = 0.38, size = 20, normalized size = 0.71

$$\frac{2}{3}\sqrt{x^3-1} - \frac{2}{3}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)/x,x, algorithm="fricas")
[Out] 2/3*sqrt(x^3 - 1) - 2/3*arctan(sqrt(x^3 - 1))
```

giac [A] time = 0.65, size = 20, normalized size = 0.71

$$\frac{2}{3}\sqrt{x^3-1} - \frac{2}{3}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)/x,x, algorithm="giac")
[Out] 2/3*sqrt(x^3 - 1) - 2/3*arctan(sqrt(x^3 - 1))
```

maple [A] time = 0.02, size = 21, normalized size = 0.75

$$\frac{2\sqrt{x^3-1}}{3} - \frac{2\arctan\left(\sqrt{x^3-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/2)/x,x)
```

```
[Out] 2/3*(x^3-1)^(1/2)-2/3*arctan((x^3-1)^(1/2))
```

maxima [A] time = 0.50, size = 20, normalized size = 0.71

$$\frac{2}{3}\sqrt{x^3-1}-\frac{2}{3}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(x^3 - 1) - 2/3*arctan(sqrt(x^3 - 1))
```

mupad [B] time = 0.16, size = 174, normalized size = 6.21

$$\frac{2\sqrt{x^3-1}}{3}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)^(1/2)/x,x)
```

```
[Out] (2*(x^3 - 1)^(1/2))/3 + (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2
)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i
)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2
)
```

sympy [C] time = 1.04, size = 85, normalized size = 3.04

$$\begin{cases}\frac{2\sqrt{x^3-1}}{3}-\frac{2i\log\left(x^{\frac{3}{2}}\right)}{3}+\frac{i\log\left(x^3\right)}{3}+\frac{2\,\mathrm{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \left|x^3\right|>1\\ \frac{2i\sqrt{1-x^3}}{3}+\frac{i\log\left(x^3\right)}{3}-\frac{2i\log\left(\sqrt{1-x^3}+1\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/2)/x,x)
```

```
[Out] Piecewise((2*sqrt(x**3 - 1)/3 - 2*I*log(x**(3/2))/3 + I*log(x**3)/3 + 2*asi
n(x**(-3/2))/3, Abs(x**3) > 1), (2*I*sqrt(1 - x**3)/3 + I*log(x**3)/3 - 2*I
*log(sqrt(1 - x**3) + 1)/3, True))
```

3.306 $\int \frac{(-1+x^3)\sqrt[3]{1+x^3}}{x^8}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt[3]{x^3+1}\,(-5x^6-3x^3+2)}{14x^7}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {453, 264}

$$\frac{(x^3+1)^{4/3}}{7x^7}-\frac{5(x^3+1)^{4/3}}{14x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)*(1 + x^3)^(1/3))/x^8,x]
```

```
[Out] (1 + x^3)^(4/3)/(7*x^7) - (5*(1 + x^3)^(4/3))/(14*x^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+x^3)\sqrt[3]{1+x^3}}{x^8}\,dx &= \frac{(1+x^3)^{4/3}}{7x^7}+\frac{10}{7}\int \frac{\sqrt[3]{1+x^3}}{x^5}\,dx\\ &= \frac{(1+x^3)^{4/3}}{7x^7}-\frac{5(1+x^3)^{4/3}}{14x^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.82

$$\frac{(2-5x^3)(x^3+1)^{4/3}}{14x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)*(1 + x^3)^(1/3))/x^8,x]
```

```
[Out] ((2 - 5*x^3)*(1 + x^3)^(4/3))/(14*x^7)
```

IntegrateAlgebraic [A] time = 0.10, size = 23, normalized size = 0.82

$$\frac{(2-5x^3)(x^3+1)^{4/3}}{14x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)*(1 + x^3)^(1/3))/x^8,x]
```

```
[Out] ((2 - 5*x^3)*(1 + x^3)^(4/3))/(14*x^7)
```

fricas [A] time = 0.39, size = 24, normalized size = 0.86

$$-\frac{\left(5 x^{6}+3 x^{3}-2\right)\left(x^{3}+1\right)^{\frac{1}{3}}}{14 x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x^8,x, algorithm="fricas")
```

```
[Out] -1/14*(5*x^6 + 3*x^3 - 2)*(x^3 + 1)^(1/3)/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}+1\right)^{\frac{1}{3}}\left(x^{3}-1\right)}{x^{8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x^8,x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)^(1/3)*(x^3 - 1)/x^8, x)
```

maple [A] time = 0.00, size = 31, normalized size = 1.11

$$-\frac{\left(x^{2}-x+1\right)\left(1+x\right)\left(5 x^{3}-2\right)\left(x^{3}+1\right)^{\frac{1}{3}}}{14 x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)*(x^3+1)^(1/3)/x^8,x)
```

```
[Out] -1/14*(x^2-x+1)*(1+x)*(5*x^3-2)*(x^3+1)^(1/3)/x^7
```

maxima [A] time = 0.61, size = 25, normalized size = 0.89

$$-\frac{\left(x^{3}+1\right)^{\frac{4}{3}}}{2 x^{4}}+\frac{\left(x^{3}+1\right)^{\frac{7}{3}}}{7 x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x^8,x, algorithm="maxima")
```

```
[Out] -1/2*(x^3 + 1)^(4/3)/x^4 + 1/7*(x^3 + 1)^(7/3)/x^7
```

mupad [B] time = 0.16, size = 39, normalized size = 1.39

$$-\frac{3 x^{3}\left(x^{3}+1\right)^{1/3}-2\left(x^{3}+1\right)^{1/3}+5 x^{6}\left(x^{3}+1\right)^{1/3}}{14 x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)*(x^3 + 1)^(1/3))/x^8,x)
```

```
[Out] -(3*x^3*(x^3 + 1)^(1/3) - 2*(x^3 + 1)^(1/3) + 5*x^6*(x^3 + 1)^(1/3))/(14*x^
7)
```

sympy [**B**] time = 2.14, size = 134, normalized size = 4.79

$$\frac{\sqrt[3]{1+\frac{1}{x^3}}\,\Gamma\left(-\frac{4}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)}-\frac{\sqrt[3]{x^3+1}\,\Gamma\left(-\frac{7}{3}\right)}{3x\Gamma\left(-\frac{1}{3}\right)}+\frac{\sqrt[3]{1+\frac{1}{x^3}}\,\Gamma\left(-\frac{4}{3}\right)}{3x^3\Gamma\left(-\frac{1}{3}\right)}+\frac{\sqrt[3]{x^3+1}\,\Gamma\left(-\frac{7}{3}\right)}{9x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{4\sqrt[3]{x^3+1}\,\Gamma\left(-\frac{7}{3}\right)}{9x^7\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)*(x**3+1)**(1/3)/x**8,x)
```

```
[Out] (1 + x**(-3))**(1/3)*gamma(-4/3)/(3*gamma(-1/3)) - (x**3 + 1)**(1/3)*gamma(
-7/3)/(3*x*gamma(-1/3)) + (1 + x**(-3))**(1/3)*gamma(-4/3)/(3*x**3*gamma(-1
/3)) + (x**3 + 1)**(1/3)*gamma(-7/3)/(9*x**4*gamma(-1/3)) + 4*(x**3 + 1)**(
1/3)*gamma(-7/3)/(9*x**7*gamma(-1/3))
```

3.307 $\int \frac{\sqrt{1+x^3}}{x} \, dx$

Optimal. Leaf size=28

$$\frac{2\sqrt{x^3+1}}{3} - \frac{2}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 50, 63, 207}

$$\frac{2\sqrt{x^3+1}}{3} - \frac{2}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^3]/x,x]
```

```
[Out] (2*Sqrt[1 + x^3])/3 - (2*ArcTanh[Sqrt[1 + x^3]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^3}}{x}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, x^3\right)\\ &= \frac{2\sqrt{1+x^3}}{3}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)\\ &= \frac{2\sqrt{1+x^3}}{3}+\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)\\ &= \frac{2\sqrt{1+x^3}}{3}-\frac{2}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$\frac{2\sqrt{x^3+1}}{3}-\frac{2}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^3]/x,x]
[Out] (2*Sqrt[1 + x^3])/3 - (2*ArcTanh[Sqrt[1 + x^3]])/3
```

IntegrateAlgebraic [A] time = 0.02, size = 28, normalized size = 1.00

$$\frac{2\sqrt{x^3+1}}{3}-\frac{2}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^3]/x,x]
[Out] (2*Sqrt[1 + x^3])/3 - (2*ArcTanh[Sqrt[1 + x^3]])/3
```

fricas [A] time = 0.39, size = 34, normalized size = 1.21

$$\frac{2}{3}\sqrt{x^3+1}-\frac{1}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)/x,x, algorithm="fricas")
[Out] 2/3*sqrt(x^3 + 1) - 1/3*log(sqrt(x^3 + 1) + 1) + 1/3*log(sqrt(x^3 + 1) - 1)
```

giac [A] time = 0.23, size = 35, normalized size = 1.25

$$\frac{2}{3}\sqrt{x^3+1}-\frac{1}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{1}{3}\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)/x,x, algorithm="giac")
[Out] 2/3*sqrt(x^3 + 1) - 1/3*log(sqrt(x^3 + 1) + 1) + 1/3*log(abs(sqrt(x^3 + 1)
- 1))
```

maple [A] time = 0.02, size = 21, normalized size = 0.75

$$\frac{2\sqrt{x^3+1}}{3}-\frac{2\,\text{arctanh}\left(\sqrt{x^3+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(1/2)/x,x)
```

```
[Out] 2/3*(x^3+1)^(1/2)-2/3*arctanh((x^3+1)^(1/2))
```

maxima [A] time = 0.47, size = 34, normalized size = 1.21

$$\frac{2}{3}\sqrt{x^3+1}-\frac{1}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(x^3 + 1) - 1/3*log(sqrt(x^3 + 1) + 1) + 1/3*log(sqrt(x^3 + 1) - 1)
```

mupad [B] time = 0.16, size = 174, normalized size = 6.21

$$\frac{2\sqrt{x^3+1}}{3}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)^(1/2)/x,x)
```

```
[Out] (2*(x^3 + 1)^(1/2))/3 - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/
2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((
3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*
1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [A] time = 0.95, size = 48, normalized size = 1.71

$$\frac{2x^{\frac{3}{2}}}{3\sqrt{1+\frac{1}{x^3}}}-\frac{2\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{2}{3x^{\frac{3}{2}}\sqrt{1+\frac{1}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(1/2)/x,x)
```

```
[Out] 2*x**(3/2)/(3*sqrt(1 + x**(-3))) - 2*asinh(x**(-3/2))/3 + 2/(3*x**(3/2)*sqr
t(1 + x**(-3)))
```

3.308 $\int \frac{\sqrt[3]{-1+x^3}\left(1+x^3\right)}{x^8}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt[3]{x^3-1}\left(5x^6-3x^3-2\right)}{14x^7}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {453, 264}

$$\frac{\left(x^3-1\right)^{4/3}}{7x^7}+\frac{5\left(x^3-1\right)^{4/3}}{14x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(1/3)*(1 + x^3))/x^8,x]
```

```
[Out] (-1 + x^3)^(4/3)/(7*x^7) + (5*(-1 + x^3)^(4/3))/(14*x^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{-1+x^3}\left(1+x^3\right)}{x^8}\,dx &= \frac{\left(-1+x^3\right)^{4/3}}{7x^7}+\frac{10}{7}\int \frac{\sqrt[3]{-1+x^3}}{x^5}\,dx\\ &= \frac{\left(-1+x^3\right)^{4/3}}{7x^7}+\frac{5\left(-1+x^3\right)^{4/3}}{14x^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.82

$$\frac{\left(x^3-1\right)^{4/3}\left(5x^3+2\right)}{14x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)^(1/3)*(1 + x^3))/x^8,x]
```

```
[Out] ((-1 + x^3)^(4/3)*(2 + 5*x^3))/(14*x^7)
```

IntegrateAlgebraic [A] time = 0.10, size = 28, normalized size = 1.00

$$\frac{\sqrt[3]{x^3-1}\left(5x^6-3x^3-2\right)}{14x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(1/3)*(1 + x^3))/x^8,x]
[Out] ((-1 + x^3)^(1/3)*(-2 - 3*x^3 + 5*x^6))/(14*x^7)
```

fricas [A] time = 0.40, size = 24, normalized size = 0.86

$$\frac{\left(5\,x^{6}-3\,x^{3}-2\right)\left(x^{3}-1\right)^{\frac{1}{3}}}{14\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(x^3+1)/x^8,x, algorithm="fricas")
[Out] 1/14*(5*x^6 - 3*x^3 - 2)*(x^3 - 1)^(1/3)/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}+1\right)\left(x^{3}-1\right)^{\frac{1}{3}}}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(x^3+1)/x^8,x, algorithm="giac")
[Out] integrate((x^3 + 1)*(x^3 - 1)^(1/3)/x^8, x)
```

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{\left(x^{3}-1\right)^{\frac{1}{3}}\left(5x^{3}+2\right)\left(-1+x\right)\left(x^{2}+x+1\right)}{14x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/3)*(x^3+1)/x^8,x)
[Out] 1/14*(x^3-1)^(1/3)*(5*x^3+2)*(-1+x)*(x^2+x+1)/x^7
```

maxima [A] time = 0.38, size = 25, normalized size = 0.89

$$\frac{\left(x^{3}-1\right)^{\frac{4}{3}}}{2\,x^{4}}-\frac{\left(x^{3}-1\right)^{\frac{7}{3}}}{7\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(x^3+1)/x^8,x, algorithm="maxima")
[Out] 1/2*(x^3 - 1)^(4/3)/x^4 - 1/7*(x^3 - 1)^(7/3)/x^7
```

mupad [B] time = 0.25, size = 39, normalized size = 1.39

$$-\frac{2\left(x^{3}-1\right)^{1/3}+3\,x^{3}\left(x^{3}-1\right)^{1/3}-5\,x^{6}\left(x^{3}-1\right)^{1/3}}{14\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(1/3)*(x^3 + 1))/x^8,x)
[Out] -(2*(x^3 - 1)^(1/3) + 3*x^3*(x^3 - 1)^(1/3) - 5*x^6*(x^3 - 1)^(1/3))/(14*x^
7)
```

sympy [C] time = 2.23, size = 416, normalized size = 14.86

$$\begin{cases} \frac{\sqrt[3]{-1+\frac{1}{x^3}}\,e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)} - \frac{\sqrt[3]{-1+\frac{1}{x^3}}\,e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{3x^3\Gamma\left(-\frac{1}{3}\right)} & \text{for } \frac{1}{|x^3|} > 1 \\ -\frac{\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{4}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{4}{3}\right)}{3x^3\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise} \end{cases} + \begin{cases} \frac{\sqrt[3]{-1+\frac{1}{x^3}}\,e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt[3]{-1+\frac{1}{x^3}}\,e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{9x^3\Gamma\left(-\frac{1}{3}\right)} - \frac{4\sqrt[3]{-1+\frac{1}{x^3}}\,e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)} & \text{for } \frac{1}{|x^3|} > 1 \\ \frac{3x^6\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)-9x^3\Gamma\left(-\frac{1}{3}\right)} - \frac{2x^3\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)-9x^3\Gamma\left(-\frac{1}{3}\right)} + \frac{4\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{7}{3}\right)}{9x^9\Gamma\left(-\frac{1}{3}\right)-9x^6\Gamma\left(-\frac{1}{3}\right)} - \frac{5\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)-9x^3\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/3)*(x**3+1)/x**8,x)
```

```
[Out] Piecewise(((-1 + x**(-3))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(3*gamma(-1/3))
 - (-1 + x**(-3))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(3*x**3*gamma(-1/3)), 1
/Abs(x**3) > 1), (-(1 - 1/x**3)**(1/3)*gamma(-4/3)/(3*gamma(-1/3)) + (1 - 1
/x**3)**(1/3)*gamma(-4/3)/(3*x**3*gamma(-1/3)), True)) + Piecewise(((-1 + x
**(-3))**(1/3)*exp(I*pi/3)*gamma(-7/3)/(3*gamma(-1/3)) + (-1 + x**(-3))**(1
/3)*exp(I*pi/3)*gamma(-7/3)/(9*x**3*gamma(-1/3)) - 4*(-1 + x**(-3))**(1/3)*
exp(I*pi/3)*gamma(-7/3)/(9*x**6*gamma(-1/3)), 1/Abs(x**3) > 1), (3*x**6*(1
- 1/x**3)**(1/3)*gamma(-7/3)/(9*x**6*gamma(-1/3) - 9*x**3*gamma(-1/3)) - 2*
x**3*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**6*gamma(-1/3) - 9*x**3*gamma(-1/
3)) + 4*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**9*gamma(-1/3) - 9*x**6*gamma(
-1/3)) - 5*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**6*gamma(-1/3) - 9*x**3*gam
ma(-1/3)), True))
```

3.309 $\int \frac{(-1+x^3)^{2/3}(2+x^3)}{x^9} \, dx$

Optimal. Leaf size=28

$$\frac{(x^3-1)^{2/3}(7x^6-2x^3-5)}{20x^8}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {453, 264}

$$\frac{(x^3-1)^{5/3}}{4x^8}+\frac{7(x^3-1)^{5/3}}{20x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(2/3)*(2 + x^3))/x^9,x]
```

```
[Out] (-1 + x^3)^(5/3)/(4*x^8) + (7*(-1 + x^3)^(5/3))/(20*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+x^3)^{2/3}(2+x^3)}{x^9} \, dx &= \frac{(-1+x^3)^{5/3}}{4x^8}+\frac{7}{4}\int \frac{(-1+x^3)^{2/3}}{x^6} \, dx\\ &= \frac{(-1+x^3)^{5/3}}{4x^8}+\frac{7(-1+x^3)^{5/3}}{20x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.82

$$\frac{(x^3-1)^{5/3}(7x^3+5)}{20x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)^(2/3)*(2 + x^3))/x^9,x]
```

```
[Out] ((-1 + x^3)^(5/3)*(5 + 7*x^3))/(20*x^8)
```

IntegrateAlgebraic [A] time = 0.10, size = 28, normalized size = 1.00

$$\frac{(x^3-1)^{2/3}(7x^6-2x^3-5)}{20x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(2/3)*(2 + x^3))/x^9,x]
[Out] ((-1 + x^3)^(2/3)*(-5 - 2*x^3 + 7*x^6))/(20*x^8)
```

fricas [A] time = 0.39, size = 24, normalized size = 0.86

$$\frac{\left(7\,x^{6}-2\,x^{3}-5\right)\left(x^{3}-1\right)^{\frac{2}{3}}}{20\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^3+2)/x^9,x, algorithm="fricas")
[Out] 1/20*(7*x^6 - 2*x^3 - 5)*(x^3 - 1)^(2/3)/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}+2\right)\left(x^{3}-1\right)^{\frac{2}{3}}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^3+2)/x^9,x, algorithm="giac")
[Out] integrate((x^3 + 2)*(x^3 - 1)^(2/3)/x^9, x)
```

maple [A] time = 0.00, size = 29, normalized size = 1.04

$$\frac{(-1+x)\left(x^{2}+x+1\right)\left(7x^{3}+5\right)\left(x^{3}-1\right)^{\frac{2}{3}}}{20x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)*(x^3+2)/x^9,x)
[Out] 1/20*(-1+x)*(x^2+x+1)*(7*x^3+5)*(x^3-1)^(2/3)/x^8
```

maxima [A] time = 0.32, size = 25, normalized size = 0.89

$$\frac{3\left(x^{3}-1\right)^{\frac{5}{3}}}{5\,x^{5}}-\frac{\left(x^{3}-1\right)^{\frac{8}{3}}}{4\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^3+2)/x^9,x, algorithm="maxima")
[Out] 3/5*(x^3 - 1)^(5/3)/x^5 - 1/4*(x^3 - 1)^(8/3)/x^8
```

mupad [B] time = 0.29, size = 24, normalized size = 0.86

$$-\frac{\left(x^{3}-1\right)^{2/3}\left(-7\,x^{6}+2\,x^{3}+5\right)}{20\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(x^3 + 2))/x^9,x)
[Out] -((x^3 - 1)^(2/3)*(2*x^3 - 7*x^6 + 5))/(20*x^8)
```

sympy [C] time = 2.56, size = 420, normalized size = 15.00

$$\begin{cases} \frac{\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}}e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{5}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)} - \frac{\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}}e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)} & \text{for } \frac{1}{|x^3|} > 1 \\ -\frac{\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)} + \frac{\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)} & \text{otherwise} \end{cases} + 2\left(\begin{cases} \frac{\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{8}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)} + \frac{2\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^3\Gamma\left(-\frac{2}{3}\right)} - \frac{5\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^6\Gamma\left(-\frac{2}{3}\right)} & \text{for } \frac{1}{|x^3|} > 1 \\ \frac{3x^6\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^6\Gamma\left(-\frac{2}{3}\right)-9x^3\Gamma\left(-\frac{2}{3}\right)} - \frac{x^3\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^6\Gamma\left(-\frac{2}{3}\right)-9x^3\Gamma\left(-\frac{2}{3}\right)} + \frac{5\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^9\Gamma\left(-\frac{2}{3}\right)-9x^6\Gamma\left(-\frac{2}{3}\right)} - \frac{7\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^6\Gamma\left(-\frac{2}{3}\right)-9x^3\Gamma\left(-\frac{2}{3}\right)} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)*(x**3+2)/x**9,x)
```

```
[Out] Piecewise(((-1 + x**(-3))**(2/3)*exp(-I*pi/3)*gamma(-5/3)/(3*gamma(-2/3)) -
 (-1 + x**(-3))**(2/3)*exp(-I*pi/3)*gamma(-5/3)/(3*x**3*gamma(-2/3)), 1/Abs
(x**3) > 1), (-(1 - 1/x**3)**(2/3)*gamma(-5/3)/(3*gamma(-2/3)) + (1 - 1/x**
3)**(2/3)*gamma(-5/3)/(3*x**3*gamma(-2/3)), True)) + 2*Piecewise(((-1 + x**
(-3))**(2/3)*exp(2*I*pi/3)*gamma(-8/3)/(3*gamma(-2/3)) + 2*(-1 + x**(-3))**
(2/3)*exp(2*I*pi/3)*gamma(-8/3)/(9*x**3*gamma(-2/3)) - 5*(-1 + x**(-3))**(2
/3)*exp(2*I*pi/3)*gamma(-8/3)/(9*x**6*gamma(-2/3)), 1/Abs(x**3) > 1), (3*x*
*6*(1 - 1/x**3)**(2/3)*gamma(-8/3)/(9*x**6*gamma(-2/3) - 9*x**3*gamma(-2/3)
) - x**3*(1 - 1/x**3)**(2/3)*gamma(-8/3)/(9*x**6*gamma(-2/3) - 9*x**3*gamma
(-2/3)) + 5*(1 - 1/x**3)**(2/3)*gamma(-8/3)/(9*x**9*gamma(-2/3) - 9*x**6*ga
mma(-2/3)) - 7*(1 - 1/x**3)**(2/3)*gamma(-8/3)/(9*x**6*gamma(-2/3) - 9*x**3
*gamma(-2/3)), True))
```

3.310 $\int \frac{(1+x^3)^{2/3}(2+x^3)}{x^9} \, dx$

Optimal. Leaf size=28

$$\frac{(x^3+1)^{2/3}(-x^6-6x^3-5)}{20x^8}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {453, 264}

$$-\frac{(x^3+1)^{5/3}}{4x^8} - \frac{(x^3+1)^{5/3}}{20x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^3)^(2/3)*(2 + x^3))/x^9,x]
```

```
[Out] -1/4*(1 + x^3)^(5/3)/x^8 - (1 + x^3)^(5/3)/(20*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^3)^{2/3}(2+x^3)}{x^9} \, dx &= -\frac{(1+x^3)^{5/3}}{4x^8} + \frac{1}{4}\int \frac{(1+x^3)^{2/3}}{x^6} \, dx \\
&= -\frac{(1+x^3)^{5/3}}{4x^8} - \frac{(1+x^3)^{5/3}}{20x^5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.75

$$-\frac{(x^3+1)^{5/3}(x^3+5)}{20x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)^(2/3)*(2 + x^3))/x^9,x]
```

```
[Out] -1/20*((1 + x^3)^(5/3)*(5 + x^3))/x^8
```

IntegrateAlgebraic [A] time = 0.11, size = 28, normalized size = 1.00

$$\frac{(x^3+1)^{2/3}(-x^6-6x^3-5)}{20x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)^(2/3)*(2 + x^3))/x^9,x]
[Out] ((1 + x^3)^(2/3)*(-5 - 6*x^3 - x^6))/(20*x^8)
```

fricas [A] time = 0.40, size = 22, normalized size = 0.79

$$-\frac{\left(x^6+6x^3+5\right)\left(x^3+1\right)^{\frac{2}{3}}}{20x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^3+2)/x^9,x, algorithm="fricas")
[Out] -1/20*(x^6 + 6*x^3 + 5)*(x^3 + 1)^(2/3)/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^3+2\right)\left(x^3+1\right)^{\frac{2}{3}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^3+2)/x^9,x, algorithm="giac")
[Out] integrate((x^3 + 2)*(x^3 + 1)^(2/3)/x^9, x)
```

maple [A] time = 0.00, size = 29, normalized size = 1.04

$$-\frac{(1+x)\left(x^2-x+1\right)\left(x^3+5\right)\left(x^3+1\right)^{\frac{2}{3}}}{20x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)*(x^3+2)/x^9,x)
[Out] -1/20*(1+x)*(x^2-x+1)*(x^3+5)*(x^3+1)^(2/3)/x^8
```

maxima [A] time = 0.65, size = 25, normalized size = 0.89

$$\frac{\left(x^3+1\right)^{\frac{5}{3}}}{5x^5}-\frac{\left(x^3+1\right)^{\frac{8}{3}}}{4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^3+2)/x^9,x, algorithm="maxima")
[Out] 1/5*(x^3 + 1)^(5/3)/x^5 - 1/4*(x^3 + 1)^(8/3)/x^8
```

mupad [B] time = 0.29, size = 22, normalized size = 0.79

$$-\frac{\left(x^3+1\right)^{2/3}\left(x^6+6x^3+5\right)}{20x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)^(2/3)*(x^3 + 2))/x^9,x)
[Out] -((x^3 + 1)^(2/3)*(6*x^3 + x^6 + 5))/(20*x^8)
```

sympy [B] time = 2.38, size = 139, normalized size = 4.96

$$\frac{\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}+\frac{2\left(x^3+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{3x^2\Gamma\left(-\frac{2}{3}\right)}+\frac{\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)}-\frac{4\left(x^3+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^5\Gamma\left(-\frac{2}{3}\right)}-\frac{10\left(x^3+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^8\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)*(x**3+2)/x**9,x)
```

```
[Out] (1 + x**(-3))**(2/3)*gamma(-5/3)/(3*gamma(-2/3)) + 2*(x**3 + 1)**(2/3)*gamm
a(-8/3)/(3*x**2*gamma(-2/3)) + (1 + x**(-3))**(2/3)*gamma(-5/3)/(3*x**3*gam
ma(-2/3)) - 4*(x**3 + 1)**(2/3)*gamma(-8/3)/(9*x**5*gamma(-2/3)) - 10*(x**3
 + 1)**(2/3)*gamma(-8/3)/(9*x**8*gamma(-2/3))
```

3.311 $\int \frac{1}{x^6 \sqrt[3]{x+x^3}} \, dx$

Optimal. Leaf size=28

$$-\frac{3\left(x^3+x\right)^{2/3}\left(9x^4-6x^2+5\right)}{80x^6}$$

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.75, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2016, 2014}

$$-\frac{3\left(x^3+x\right)^{2/3}}{16x^6}+\frac{9\left(x^3+x\right)^{2/3}}{40x^4}-\frac{27\left(x^3+x\right)^{2/3}}{80x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(x + x^3)^(1/3)),x]
```

```
[Out] (-3*(x + x^3)^(2/3))/(16*x^6) + (9*(x + x^3)^(2/3))/(40*x^4) - (27*(x + x^3
)^(2/3))/(80*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6 \sqrt[3]{x+x^3}} \, dx &= -\frac{3\left(x+x^3\right)^{2/3}}{16x^6}-\frac{3}{4}\int \frac{1}{x^4 \sqrt[3]{x+x^3}} \, dx \\
&= -\frac{3\left(x+x^3\right)^{2/3}}{16x^6}+\frac{9\left(x+x^3\right)^{2/3}}{40x^4}+\frac{9}{20}\int \frac{1}{x^2 \sqrt[3]{x+x^3}} \, dx \\
&= -\frac{3\left(x+x^3\right)^{2/3}}{16x^6}+\frac{9\left(x+x^3\right)^{2/3}}{40x^4}-\frac{27\left(x+x^3\right)^{2/3}}{80x^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 1.00

$$-\frac{3\left(x^3+x\right)^{2/3}\left(9x^4-6x^2+5\right)}{80x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(x + x^3)^(1/3)),x]
```

[Out] (-3*(x + x^3)^(2/3)*(5 - 6*x^2 + 9*x^4))/(80*x^6)

IntegrateAlgebraic [A] time = 0.22, size = 28, normalized size = 1.00

$$-\frac{3\left(x^3+x\right)^{2/3}\left(9x^4-6x^2+5\right)}{80x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^6*(x + x^3)^(1/3)),x]

[Out] (-3*(x + x^3)^(2/3)*(5 - 6*x^2 + 9*x^4))/(80*x^6)

fricas [A] time = 0.40, size = 24, normalized size = 0.86

$$-\frac{3\left(9\,x^4-6\,x^2+5\right)\left(x^3+x\right)^{\frac{2}{3}}}{80\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^3+x)^(1/3),x, algorithm="fricas")

[Out] -3/80*(9*x^4 - 6*x^2 + 5)*(x^3 + x)^(2/3)/x^6

giac [A] time = 0.37, size = 28, normalized size = 1.00

$$-\frac{3}{16}\left(\frac{1}{x^2}+1\right)^{\frac{8}{3}}+\frac{3}{5}\left(\frac{1}{x^2}+1\right)^{\frac{5}{3}}-\frac{3}{4}\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^3+x)^(1/3),x, algorithm="giac")

[Out] -3/16*(1/x^2 + 1)^(8/3) + 3/5*(1/x^2 + 1)^(5/3) - 3/4*(1/x^2 + 1)^(2/3)

maple [A] time = 0.00, size = 30, normalized size = 1.07

$$-\frac{3\left(x^2+1\right)\left(9x^4-6x^2+5\right)}{80x^5\left(x^3+x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(x^3+x)^(1/3),x)

[Out] -3/80*(x^2+1)*(9*x^4-6*x^2+5)/x^5/(x^3+x)^(1/3)

maxima [A] time = 0.71, size = 31, normalized size = 1.11

$$-\frac{3\left(9\,x^7+3\,x^5-x^3+5\,x\right)}{80\left(x^2+1\right)^{\frac{1}{3}}x^{\frac{19}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^6/(x^3+x)^(1/3),x, algorithm="maxima")

[Out] -3/80*(9*x^7 + 3*x^5 - x^3 + 5*x)/((x^2 + 1)^(1/3)*x^(19/3))

mupad [B] time = 0.21, size = 24, normalized size = 0.86

$$-\frac{3\left(x^3+x\right)^{2/3}\left(9\,x^4-6\,x^2+5\right)}{80\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x + x^3)^(1/3)),x)
```

```
[Out] -(3*(x + x^3)^(2/3)*(9*x^4 - 6*x^2 + 5))/(80*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \sqrt[3]{x\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**3+x)**(1/3),x)
```

```
[Out] Integral(1/(x**6*(x*(x**2 + 1))**(1/3)), x)
```

3.312 $\int \frac{-2-x+2x^2}{(-1+x)x\sqrt[4]{-x^2+x^3}}\,dx$

Optimal. Leaf size=28

$$\frac{4(2x-1)\left(x^3-x^2\right)^{3/4}}{(x-1)x^2}$$

Rubi [A] time = 0.13, antiderivative size = 36, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2056, 949, 74}

$$\frac{4}{\sqrt[4]{x^3-x^2}}-\frac{8(1-x)}{\sqrt[4]{x^3-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 - x + 2*x^2)/((-1 + x)*x*(-x^2 + x^3)^(1/4)),x]
```

```
[Out] 4/(-x^2 + x^3)^(1/4) - (8*(1 - x))/(-x^2 + x^3)^(1/4)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2-x+2x^2}{(-1+x)x\sqrt[4]{-x^2+x^3}}\,dx &= \frac{\left(\sqrt[4]{-1+x}\sqrt{x}\right)\int \frac{-2-x+2x^2}{(-1+x)^{5/4}x^{3/2}}\,dx}{\sqrt[4]{-x^2+x^3}}\\
&= \frac{4}{\sqrt[4]{-x^2+x^3}}-\frac{\left(4\sqrt[4]{-1+x}\sqrt{x}\right)\int \frac{-1-\frac{x}{2}}{\sqrt[4]{-1+x}\,x^{3/2}}\,dx}{\sqrt[4]{-x^2+x^3}}\\
&= \frac{4}{\sqrt[4]{-x^2+x^3}}-\frac{8(1-x)}{\sqrt[4]{-x^2+x^3}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 2.50

$$-\frac{4\sqrt{x}\left(2\,_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};1-x\right)-\,_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};1-x\right)-2\,_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};1-x\right)\right)}{\sqrt[4]{(x-1)x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 - x + 2*x^2)/((-1 + x)*x*(-x^2 + x^3)^(1/4)),x]
[Out] (-4*Sqrt[x]*(2*Hypergeometric2F1[-1/2, -1/4, 3/4, 1 - x] - Hypergeometric2F
1[-1/4, 1/2, 3/4, 1 - x] - 2*Hypergeometric2F1[-1/4, 3/2, 3/4, 1 - x]))/((-
1 + x)*x^2)^(1/4)
```

IntegrateAlgebraic [A] time = 0.45, size = 28, normalized size = 1.00

$$\frac{4(2x-1)\left(x^3-x^2\right)^{3/4}}{(x-1)x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 - x + 2*x^2)/((-1 + x)*x*(-x^2 + x^3)^(1/4)),x]
[Out] (4*(-1 + 2*x)*(-x^2 + x^3)^(3/4))/((-1 + x)*x^2)
```

fricas [A] time = 0.41, size = 18, normalized size = 0.64

$$\frac{4\,(2\,x-1)}{\left(x^3-x^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x-2)/(-1+x)/x/(x^3-x^2)^(1/4),x, algorithm="fricas")
[Out] 4*(2*x - 1)/(x^3 - x^2)^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2-x-2}{\left(x^3-x^2\right)^{\frac{1}{4}}(x-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x-2)/(-1+x)/x/(x^3-x^2)^(1/4),x, algorithm="giac")
[Out] integrate((2*x^2 - x - 2)/((x^3 - x^2)^(1/4)*(x - 1)*x), x)
```

maple [A] time = 0.01, size = 19, normalized size = 0.68

$$\frac{-4+8x}{\left(x^3-x^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x-2)/(-1+x)/x/(x^3-x^2)^(1/4),x)
[Out] 4*(-1+2*x)/(x^3-x^2)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2-x-2}{\left(x^3-x^2\right)^{\frac{1}{4}}(x-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x-2)/(-1+x)/x/(x^3-x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x - 2)/((x^3 - x^2)^(1/4)*(x - 1)*x), x)
```

mupad [B] time = 0.25, size = 17, normalized size = 0.61

$$\frac{8\,x-4}{{\left(x^3-x^2\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 2*x^2 + 2)/(x*(x^3 - x^2)^(1/4)*(x - 1)),x)
```

```
[Out] (8*x - 4)/(x^3 - x^2)^(1/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2-x-2}{x\sqrt[4]{x^2\,(x-1)}\,\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x-2)/(-1+x)/x/(x**3-x**2)**(1/4),x)
```

```
[Out] Integral((2*x**2 - x - 2)/(x*(x**2*(x - 1))**(1/4)*(x - 1)), x)
```

3.313 $\int \frac{2+x}{(-1+x)\sqrt{-x-x^2+x^3}}\,dx$

Optimal. Leaf size=28

$$-2\tan^{-1}\left(\frac{\sqrt{x^3-x^2-x}}{(x-2)x}\right)$$

Rubi [C] time = 0.89, antiderivative size = 533, normalized size of antiderivative = 19.04, number of steps used = 12, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2056, 6733, 1710, 1098, 1214, 1456, 540, 421, 419, 538, 537}

$$\frac{3\sqrt{2}\left(1+\sqrt{5}\right)\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\left(1-\sqrt{5}\right)\sqrt{x^3-x^2-x}}-\frac{6\sqrt{x}\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}\sqrt{\frac{\left(1+\sqrt{5}\right)x+2}{\left(1-\sqrt{5}\right)x+2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\left(1-\sqrt{5}\right)\sqrt{\frac{1}{\left(1-\sqrt{5}\right)x+2}}\sqrt{x^3-x^2-x}}+\frac{\sqrt{x}\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}\sqrt{\frac{\left(1+\sqrt{5}\right)x+2}{\left(1-\sqrt{5}\right)x+2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{\left(1-\sqrt{5}\right)x+2}}\sqrt{x^3-x^2-x}}+\frac{6\sqrt{2}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(\frac{1}{2}\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\left(1-\sqrt{5}\right)\sqrt{x^3-x^2-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(2 + x)/((-1 + x)*Sqrt[-x - x^2 + x^3]),x]
```

```
[Out] (3*Sqrt[2]*(1 + Sqrt[5])*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1
 + Sqrt[5])]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]], (-3 - Sqrt[5]
)/2])/((1 - Sqrt[5])*Sqrt[-x - x^2 + x^3]) + (Sqrt[x]*Sqrt[-2 - (1 - Sqrt[5
])*x]*Sqrt[(2 + (1 + Sqrt[5])*x)/(2 + (1 - Sqrt[5])*x)]*EllipticF[ArcSin[(S
qrt[2]*5^(1/4)*Sqrt[x])/Sqrt[-2 - (1 - Sqrt[5])*x]], (5 - Sqrt[5])/10])/(5^
(1/4)*Sqrt[(2 + (1 - Sqrt[5])*x)^(-1)]*Sqrt[-x - x^2 + x^3]) - (6*Sqrt[x]*S
qrt[-2 - (1 - Sqrt[5])*x]*Sqrt[(2 + (1 + Sqrt[5])*x)/(2 + (1 - Sqrt[5])*x)]
*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*Sqrt[x])/Sqrt[-2 - (1 - Sqrt[5])*x]], (5
 - Sqrt[5])/10])/(5^(1/4)*(1 - Sqrt[5])*Sqrt[(2 + (1 - Sqrt[5])*x)^(-1)]*Sq
rt[-x - x^2 + x^3]) + (6*Sqrt[2]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 -
(2*x)/(1 + Sqrt[5])]*EllipticPi[(1 + Sqrt[5])/2, ArcSin[Sqrt[2/(1 + Sqrt[5]
)]*Sqrt[x]], (-3 - Sqrt[5])/2])/((1 - Sqrt[5])*Sqrt[-x - x^2 + x^3])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 540

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x
] + Dist[(b*c - a*d)/b, Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1710

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + b*x^2 + c*x^4], x],
 x] + Dist[(e*A - d*B)/e, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x],
x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{2+x}{(-1+x)\sqrt{-x-x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{2+x}{(-1+x)\sqrt{x}\sqrt{-1-x+x^2}}\,dx}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1-x+x^2}\right)\mathrm{Subst}\left(\int \frac{2+x^2}{(-1+x^2)\sqrt{-1-x^2+x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1-x+x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1-x^2+x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{-x-x^2+x^3}} + \frac{\left(6\sqrt{x}\sqrt{-1-x+x^2}\right)\mathrm{S}}{}$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - \left(\right.$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - 6$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - 6$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - 6$$

$$= \frac{3\sqrt{2}\left(1+\sqrt{5}\right)\sqrt{x}\sqrt{-1+\sqrt{5}+2x}\sqrt{1-\frac{2x}{1+\sqrt{5}}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3\right.\right.}{\left(1-\sqrt{5}\right)\sqrt{-x-x^2+x^3}}$$

Mathematica [C] time = 7.31, size = 1600, normalized size = 57.14

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + x)/((-1 + x)*Sqrt[-x - x^2 + x^3]),x]
```

```
[Out] (-2*Sqrt[2]*(-1 + x^(-1))*x^(3/2)*(2 + x)*(((-2*I)*Sqrt[1 - x^(-2) - x^(-1)
]*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], -3/2 - Sqrt[5]/2])/Sq
rt[-1 + Sqrt[5]] + (3*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-
1 + Sqrt[5])] - 2/Sqrt[x])^2*Sqrt[(I*Sqrt[2*(1 + Sqrt[5])] + 2/Sqrt[x])/((1
 + 2*I)*Sqrt[2] - Sqrt[10] + (2*Sqrt[-1 + Sqrt[5]])/Sqrt[x] - ((2*I)*Sqrt[1
 + Sqrt[5]])/Sqrt[x])]*Sqrt[(I*Sqrt[2*(1 + Sqrt[5])] - 2/Sqrt[x])/(Sqrt[2]*
((-1 + 2*I) + Sqrt[5]) - (2*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]]))/Sqr
t[x])]*Sqrt[(Sqrt[2]*((-1 - 2*I) + Sqrt[5]) + (2*(Sqrt[-1 + Sqrt[5]] - I*Sq
rt[1 + Sqrt[5]]))/Sqrt[x])/(Sqrt[2]*((-1 + 2*I) + Sqrt[5]) - (2*(Sqrt[-1 +
Sqrt[5]] + I*Sqrt[1 + Sqrt[5]]))/Sqrt[x])]*((2 + Sqrt[2*(-1 + Sqrt[5])])*El
```

```
lipticF[ArcSin[Sqrt[((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1
 + Sqrt[5])] + 2/Sqrt[x]))/((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqr
t[2*(-1 + Sqrt[5])] - 2/Sqrt[x]))]], -3/5 - (4*I)/5] - 2*Sqrt[2*(-1 + Sqrt[
5])]*EllipticPi[((-2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] + I*Sqrt
[1 + Sqrt[5]]))/((2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[
1 + Sqrt[5]])), ArcSin[Sqrt[((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(Sq
rt[2*(-1 + Sqrt[5])] + 2/Sqrt[x]))/((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5
]])*(Sqrt[2*(-1 + Sqrt[5])] - 2/Sqrt[x]))]], -3/5 - (4*I)/5]))/(2*(-2 + Sqr
t[2*(-1 + Sqrt[5])])*(2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*S
qrt[1 + Sqrt[5]])) - (3*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*
(-1 + Sqrt[5])] - 2/Sqrt[x])^2*Sqrt[(I*Sqrt[2*(1 + Sqrt[5])] + 2/Sqrt[x])/(
(1 + 2*I)*Sqrt[2] - Sqrt[10] + (2*Sqrt[-1 + Sqrt[5]])/Sqrt[x] - ((2*I)*Sqrt
[1 + Sqrt[5]])/Sqrt[x])]*Sqrt[(I*Sqrt[2*(1 + Sqrt[5])] - 2/Sqrt[x])/(Sqrt[2
]*((-1 + 2*I) + Sqrt[5]) - (2*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]]))/S
qrt[x])]*Sqrt[(Sqrt[2]*((-1 - 2*I) + Sqrt[5]) + (2*(Sqrt[-1 + Sqrt[5]] - I*
Sqrt[1 + Sqrt[5]]))/Sqrt[x])/(Sqrt[2]*((-1 + 2*I) + Sqrt[5]) - (2*(Sqrt[-1
+ Sqrt[5]] + I*Sqrt[1 + Sqrt[5]]))/Sqrt[x])]*((-2 + Sqrt[2*(-1 + Sqrt[5])])
*EllipticF[ArcSin[Sqrt[((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*
(-1 + Sqrt[5])] + 2/Sqrt[x]))/((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(
Sqrt[2*(-1 + Sqrt[5])] - 2/Sqrt[x]))]], -3/5 - (4*I)/5] - 2*Sqrt[2*(-1 + Sq
rt[5])]*EllipticPi[((2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] + I*Sq
rt[1 + Sqrt[5]]))/((-2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*Sq
rt[1 + Sqrt[5]])), ArcSin[Sqrt[((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*
(Sqrt[2*(-1 + Sqrt[5])] + 2/Sqrt[x]))/((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqr
t[5]])*(Sqrt[2*(-1 + Sqrt[5])] - 2/Sqrt[x]))]], -3/5 - (4*I)/5]))/(2*(-2 +
Sqrt[2*(-1 + Sqrt[5])])*(2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] -
I*Sqrt[1 + Sqrt[5]]))))/((1 - 2/x + x)*Sqrt[x*(-1 - x + x^2)])
```

IntegrateAlgebraic [A] time = 0.12, size = 28, normalized size = 1.00

$$-2\tan^{-1}\left(\frac{\sqrt{x^3-x^2-x}}{(x-2)x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)/((-1 + x)*Sqrt[-x - x^2 + x^3]),x]
```

```
[Out] -2*ArcTan[Sqrt[-x - x^2 + x^3]/((-2 + x)*x)]
```

fricas [A] time = 0.43, size = 47, normalized size = 1.68

$$\arctan\left(\frac{\sqrt{x^3-x^2-x}\left(x^3-5x^2+5x+1\right)}{2\left(x^4-3x^3+x^2+2x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-1+x)/(x^3-x^2-x)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(1/2*sqrt(x^3 - x^2 - x)*(x^3 - 5*x^2 + 5*x + 1)/(x^4 - 3*x^3 + x^2 +
 2*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+2}{\sqrt{x^3-x^2-x}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-1+x)/(x^3-x^2-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 2)/(sqrt(x^3 - x^2 - x)*(x - 1)), x)
```

maple [C] time = 0.03, size = 250, normalized size = 8.93

$$\frac{2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\sqrt{-5\left(x-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\sqrt{5}}\,\sqrt{-\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}},\frac{\sqrt{5}\,\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)}{5\sqrt{x^3-x^2-x}}+\frac{6\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\sqrt{-5\left(x-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\sqrt{5}}\,\sqrt{-\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}},\frac{\frac{1}{2}-\frac{\sqrt{5}}{2}}{-\frac{1}{2}-\frac{\sqrt{5}}{2}},\frac{\sqrt{5}\,\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)}{5\sqrt{x^3-x^2-x}\,\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-1+x)/(x^3-x^2-x)^(1/2),x)
```

```
[Out] 2/5*(1/2*5^(1/2)-1/2)*((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x-
1/2-1/2*5^(1/2))*5^(1/2))^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1
/2)*EllipticF(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),1/5*5^(1/2)*((1
/2*5^(1/2)-1/2)*5^(1/2))^(1/2))+6/5*(1/2*5^(1/2)-1/2)*((x-1/2+1/2*5^(1/2))/
(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x-1/2-1/2*5^(1/2))*5^(1/2))^(1/2)*(-x/(1/2*5^
(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(-1/2-1/2*5^(1/2))*EllipticPi(((x-1/2+1
/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(-1/2-1/2*5^(1/2)),1
/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\sqrt{x^3-x^2-x}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-1+x)/(x^3-x^2-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 2)/(sqrt(x^3 - x^2 - x)*(x - 1)), x)
```

mupad [B] time = 0.25, size = 165, normalized size = 5.89

$$\frac{\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\left(\sqrt{5}+1\right)\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)-3\,\Pi\left(\frac{\sqrt{5}}{2}+\frac{1}{2};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)\right)\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}}{\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((x - 1)*(x^3 - x^2 - x)^(1/2)),x)
```

```
[Out] ((x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/2 - 1/2)/(5^(1/2)/2 - 1/2))^(1/2
)*(5^(1/2) + 1)*(ellipticF(asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/2 +
 1/2)/(5^(1/2)/2 - 1/2)) - 3*ellipticPi(5^(1/2)/2 + 1/2, asin((x/(5^(1/2)/2
 + 1/2))^(1/2)), -(5^(1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2)))*((5^(1/2)/2 - x + 1
/2)/(5^(1/2)/2 + 1/2))^(1/2))/(x^3 - x^2 - x*(5^(1/2)/2 - 1/2)*(5^(1/2)/2 +
 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\sqrt{x\left(x^2-x-1\right)}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-1+x)/(x**3-x**2-x)**(1/2),x)
```

```
[Out] Integral((x + 2)/(sqrt(x*(x**2 - x - 1))*(x - 1)), x)
```

3.314 $\int \frac{(3+x^2)(1+x^2+x^3)}{x^6 \sqrt[4]{x+x^3}} \, dx$

Optimal. Leaf size=28

$$-\frac{4\left(x^3+x\right)^{3/4}\left(7x^3+3x^2+3\right)}{21x^6}$$

Rubi [A] time = 0.36, antiderivative size = 49, normalized size of antiderivative = 1.75, number of steps used = 21, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2052, 2025, 2011, 364, 2032}

$$-\frac{4\left(x^3+x\right)^{3/4}}{3x^3}-\frac{4\left(x^3+x\right)^{3/4}}{7x^6}-\frac{4\left(x^3+x\right)^{3/4}}{7x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + x^2)*(1 + x^2 + x^3))/(x^6*(x + x^3)^(1/4)),x]
```

```
[Out] (-4*(x + x^3)^(3/4))/(7*x^6) - (4*(x + x^3)^(3/4))/(7*x^4) - (4*(x + x^3)^(
3/4))/(3*x^3)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(3+x^2)(1+x^2+x^3)}{x^6\sqrt[4]{x+x^3}}\,dx &= \int \left(\frac{3}{x^6\sqrt[4]{x+x^3}}+\frac{4}{x^4\sqrt[4]{x+x^3}}+\frac{3}{x^3\sqrt[4]{x+x^3}}+\frac{1}{x^2\sqrt[4]{x+x^3}}+\frac{1}{x\sqrt[4]{x+x^3}}\right)dx\\
&= 3\int \frac{1}{x^6\sqrt[4]{x+x^3}}\,dx+3\int \frac{1}{x^3\sqrt[4]{x+x^3}}\,dx+4\int \frac{1}{x^4\sqrt[4]{x+x^3}}\,dx+\int \frac{1}{x^2\sqrt[4]{x+x^3}}\\
&= -\frac{4(x+x^3)^{3/4}}{7x^6}-\frac{16(x+x^3)^{3/4}}{13x^4}-\frac{4(x+x^3)^{3/4}}{3x^3}-\frac{4(x+x^3)^{3/4}}{5x^2}-\frac{4(x+x^3)^{3/4}}{x}\\
&= -\frac{4(x+x^3)^{3/4}}{7x^6}-\frac{4(x+x^3)^{3/4}}{7x^4}-\frac{4(x+x^3)^{3/4}}{3x^3}+\frac{12(x+x^3)^{3/4}}{13x^2}-\frac{28}{65}\int \frac{1}{\sqrt[4]{x+x}}\\
&= -\frac{4(x+x^3)^{3/4}}{7x^6}-\frac{4(x+x^3)^{3/4}}{7x^4}-\frac{4(x+x^3)^{3/4}}{3x^3}+\frac{4x\sqrt[4]{1+x^2}\,{}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-x^2\right)}{15\sqrt[4]{x+x^3}}\\
&= -\frac{4(x+x^3)^{3/4}}{7x^6}-\frac{4(x+x^3)^{3/4}}{7x^4}-\frac{4(x+x^3)^{3/4}}{3x^3}-\frac{4x\sqrt[4]{1+x^2}\,{}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-x^2\right)}{13\sqrt[4]{x+x^3}}\\
&= -\frac{4(x+x^3)^{3/4}}{7x^6}-\frac{4(x+x^3)^{3/4}}{7x^4}-\frac{4(x+x^3)^{3/4}}{3x^3}
\end{aligned}
$$

Mathematica [C] time = 0.09, size = 124, normalized size = 4.43

$$
-\frac{4\sqrt[4]{x^2+1}\left(7x^2\left(60\,{}_2F_1\left(-\frac{13}{8},\frac{1}{4};-\frac{5}{8};-x^2\right)+13x\left(5\,{}_2F_1\left(-\frac{9}{8},\frac{1}{4};-\frac{1}{8};-x^2\right)+3x\left({}_2F_1\left(-\frac{5}{8},\frac{1}{4};\frac{3}{8};-x^2\right)+5x\,{}_2F_1\left(-\frac{1}{8},\frac{1}{4};\frac{7}{8};-x^2\right)\right)\right)\right)+195\,{}_2F_1\left(-\frac{21}{8},\frac{1}{4};-\frac{13}{8};-x^2\right)\right)}{1365x^5\sqrt[4]{x^3+x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + x^2)*(1 + x^2 + x^3))/(x^6*(x + x^3)^(1/4)),x]
```

```
[Out] (-4*(1 + x^2)^(1/4)*(195*Hypergeometric2F1[-21/8, 1/4, -13/8, -x^2] + 7*x^2
*(60*Hypergeometric2F1[-13/8, 1/4, -5/8, -x^2] + 13*x*(5*Hypergeometric2F1[
-9/8, 1/4, -1/8, -x^2] + 3*x*(Hypergeometric2F1[-5/8, 1/4, 3/8, -x^2] + 5*x
*Hypergeometric2F1[-1/8, 1/4, 7/8, -x^2])))))/(1365*x^5*(x + x^3)^(1/4))
```

IntegrateAlgebraic [A] time = 0.73, size = 28, normalized size = 1.00

$$
-\frac{4(x^3+x)^{3/4}(7x^3+3x^2+3)}{21x^6}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3 + x^2)*(1 + x^2 + x^3))/(x^6*(x + x^3)^(1/4)),x]
```

```
[Out] (-4*(x + x^3)^(3/4)*(3 + 3*x^2 + 7*x^3))/(21*x^6)
```

fricas [A] time = 0.41, size = 24, normalized size = 0.86

$$
-\frac{4\,(7\,x^3+3\,x^2+3)(x^3+x)^{\frac{3}{4}}}{21\,x^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)*(x^3+x^2+1)/x^6/(x^3+x)^(1/4),x, algorithm="fricas")
```

```
[Out] -4/21*(7*x^3 + 3*x^2 + 3)*(x^3 + x)^(3/4)/x^6
```

giac [A] time = 0.56, size = 23, normalized size = 0.82

$$-\frac{4}{7}\left(\frac{1}{x}+\frac{1}{x^3}\right)^{\frac{7}{4}}-\frac{4}{3}\left(\frac{1}{x}+\frac{1}{x^3}\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)*(x^3+x^2+1)/x^6/(x^3+x)^(1/4),x, algorithm="giac")
```

```
[Out] -4/7*(1/x + 1/x^3)^(7/4) - 4/3*(1/x + 1/x^3)^(3/4)
```

maple [A] time = 0.01, size = 30, normalized size = 1.07

$$-\frac{4\left(x^2+1\right)\left(7x^3+3x^2+3\right)}{21x^5\left(x^3+x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3)*(x^3+x^2+1)/x^6/(x^3+x)^(1/4),x)
```

```
[Out] -4/21*(x^2+1)*(7*x^3+3*x^2+3)/x^5/(x^3+x)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+x^2+1\right)\left(x^2+3\right)}{\left(x^3+x\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)*(x^3+x^2+1)/x^6/(x^3+x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + x^2 + 1)*(x^2 + 3)/((x^3 + x)^(1/4)*x^6), x)
```

mupad [B] time = 0.24, size = 39, normalized size = 1.39

$$-\frac{12\left(x^3+x\right)^{3/4}+12\,x^2\left(x^3+x\right)^{3/4}+28\,x^3\left(x^3+x\right)^{3/4}}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 3)*(x^2 + x^3 + 1))/(x^6*(x + x^3)^(1/4)),x)
```

```
[Out] -(12*(x + x^3)^(3/4) + 12*x^2*(x + x^3)^(3/4) + 28*x^3*(x + x^3)^(3/4))/(21
*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+3\right)\left(x^3+x^2+1\right)}{x^6\sqrt[4]{x\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3)*(x**3+x**2+1)/x**6/(x**3+x)**(1/4),x)
```

```
[Out] Integral((x**2 + 3)*(x**3 + x**2 + 1)/(x**6*(x*(x**2 + 1))**(1/4)), x)
```

3.315 $\int \frac{\sqrt[3]{-1+x^3}\left(-1+2x^3\right)}{x^{11}}\,dx$

Optimal. Leaf size=28

$$\frac{\left(x^3-1\right)^{4/3}\left(3x^6+4x^3-2\right)}{20x^{10}}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.75, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {453, 271, 264}

$$-\frac{\left(x^3-1\right)^{4/3}}{10x^{10}}+\frac{\left(x^3-1\right)^{4/3}}{5x^7}+\frac{3\left(x^3-1\right)^{4/3}}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(1/3)*(-1 + 2*x^3))/x^11,x]
```

```
[Out] -1/10*(-1 + x^3)^(4/3)/x^10 + (-1 + x^3)^(4/3)/(5*x^7) + (3*(-1 + x^3)^(4/3
))/(20*x^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{-1+x^3}\left(-1+2x^3\right)}{x^{11}}\,dx &= -\frac{\left(-1+x^3\right)^{4/3}}{10x^{10}}+\frac{7}{5}\int \frac{\sqrt[3]{-1+x^3}}{x^8}\,dx\\ &= -\frac{\left(-1+x^3\right)^{4/3}}{10x^{10}}+\frac{\left(-1+x^3\right)^{4/3}}{5x^7}+\frac{3}{5}\int \frac{\sqrt[3]{-1+x^3}}{x^5}\,dx\\ &= -\frac{\left(-1+x^3\right)^{4/3}}{10x^{10}}+\frac{\left(-1+x^3\right)^{4/3}}{5x^7}+\frac{3\left(-1+x^3\right)^{4/3}}{20x^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.11

$$\frac{\sqrt[3]{x^3-1}\left(3x^9+x^6-6x^3+2\right)}{20x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)^(1/3)*(-1 + 2*x^3))/x^11,x]
```

```
[Out] ((-1 + x^3)^(1/3)*(2 - 6*x^3 + x^6 + 3*x^9))/(20*x^10)
```

IntegrateAlgebraic [A] time = 0.11, size = 28, normalized size = 1.00

$$\frac{\left(x^3-1\right)^{4/3}\left(3x^6+4x^3-2\right)}{20x^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(1/3)*(-1 + 2*x^3))/x^11,x]
```

```
[Out] ((-1 + x^3)^(4/3)*(-2 + 4*x^3 + 3*x^6))/(20*x^10)
```

fricas [A] time = 0.40, size = 27, normalized size = 0.96

$$\frac{\left(3\,x^9+x^6-6\,x^3+2\right)\left(x^3-1\right)^{\frac{1}{3}}}{20\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(2*x^3-1)/x^11,x, algorithm="fricas")
```

```
[Out] 1/20*(3*x^9 + x^6 - 6*x^3 + 2)*(x^3 - 1)^(1/3)/x^10
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^3-1\right)\left(x^3-1\right)^{\frac{1}{3}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(2*x^3-1)/x^11,x, algorithm="giac")
```

```
[Out] integrate((2*x^3 - 1)*(x^3 - 1)^(1/3)/x^11, x)
```

maple [A] time = 0.01, size = 34, normalized size = 1.21

$$\frac{(-1+x)\left(x^2+x+1\right)\left(3x^6+4x^3-2\right)\left(x^3-1\right)^{\frac{1}{3}}}{20x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/3)*(2*x^3-1)/x^11,x)
```

```
[Out] 1/20*(-1+x)*(x^2+x+1)*(3*x^6+4*x^3-2)*(x^3-1)^(1/3)/x^10
```

maxima [A] time = 0.35, size = 25, normalized size = 0.89

$$\frac{\left(x^3-1\right)^{\frac{4}{3}}}{4\,x^4}-\frac{\left(x^3-1\right)^{\frac{10}{3}}}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(2*x^3-1)/x^11,x, algorithm="maxima")
```

```
[Out] 1/4*(x^3 - 1)^(4/3)/x^4 - 1/10*(x^3 - 1)^(10/3)/x^10
```

mupad [B] time = 0.36, size = 49, normalized size = 1.75

$$\frac{3\left(x^3-1\right)^{1/3}}{20\,x}+\frac{\left(x^3-1\right)^{1/3}}{20\,x^4}-\frac{3\left(x^3-1\right)^{1/3}}{10\,x^7}+\frac{\left(x^3-1\right)^{1/3}}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(1/3)*(2*x^3 - 1))/x^11,x)
```

```
[Out] (3*(x^3 - 1)^(1/3))/(20*x) + (x^3 - 1)^(1/3)/(20*x^4) - (3*(x^3 - 1)^(1/3))
/(10*x^7) + (x^3 - 1)^(1/3)/(10*x^10)
```

sympy [C] time = 3.05, size = 564, normalized size = 20.14

$$2\left(\begin{cases}\frac{\sqrt[3]{-1+\frac{1}{x^3}}\,e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)}+\frac{\sqrt[3]{-1+\frac{1}{x^3}}\,e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{9x^3\Gamma\left(-\frac{1}{3}\right)}-\frac{4\sqrt[3]{-1+\frac{1}{x^3}}\,e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)} & \text{for}\ \frac{1}{|x^3|}>1\\ \frac{3x^6\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)-9x^3\Gamma\left(-\frac{1}{3}\right)}-\frac{2x^3\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)-9x^3\Gamma\left(-\frac{1}{3}\right)}+\frac{4\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{7}{3}\right)}{9x^9\Gamma\left(-\frac{1}{3}\right)-9x^6\Gamma\left(-\frac{1}{3}\right)}-\frac{5\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)-9x^3\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise}\end{cases}\right)-\begin{cases}\frac{2\sqrt[3]{-1+\frac{1}{x^3}}\,e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{10}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)}+\frac{2\sqrt[3]{-1+\frac{1}{x^3}}\,e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{10}{3}\right)}{9x^3\Gamma\left(-\frac{1}{3}\right)}+\frac{4\sqrt[3]{-1+\frac{1}{x^3}}\,e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{10}{3}\right)}{27x^6\Gamma\left(-\frac{1}{3}\right)}-\frac{28\sqrt[3]{-1+\frac{1}{x^3}}\,e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{10}{3}\right)}{27x^9\Gamma\left(-\frac{1}{3}\right)} & \text{for}\ \frac{1}{|x^3|}>1\\ -\frac{2\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{10}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)}-\frac{2\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{10}{3}\right)}{9x^3\Gamma\left(-\frac{1}{3}\right)}-\frac{4\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{10}{3}\right)}{27x^6\Gamma\left(-\frac{1}{3}\right)}+\frac{28\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{10}{3}\right)}{27x^9\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/3)*(2*x**3-1)/x**11,x)
```

```
[Out] 2*Piecewise(((-1 + x**(-3))**(1/3)*exp(I*pi/3)*gamma(-7/3)/(3*gamma(-1/3))
+ (-1 + x**(-3))**(1/3)*exp(I*pi/3)*gamma(-7/3)/(9*x**3*gamma(-1/3)) - 4*(-
1 + x**(-3))**(1/3)*exp(I*pi/3)*gamma(-7/3)/(9*x**6*gamma(-1/3)), 1/Abs(x**
3) > 1), (3*x**6*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**6*gamma(-1/3) - 9*x*
*3*gamma(-1/3)) - 2*x**3*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**6*gamma(-1/3
) - 9*x**3*gamma(-1/3)) + 4*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**9*gamma(-
1/3) - 9*x**6*gamma(-1/3)) - 5*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**6*gamm
a(-1/3) - 9*x**3*gamma(-1/3)), True)) - Piecewise((2*(-1 + x**(-3))**(1/3)*
exp(-2*I*pi/3)*gamma(-10/3)/(3*gamma(-1/3)) + 2*(-1 + x**(-3))**(1/3)*exp(-
2*I*pi/3)*gamma(-10/3)/(9*x**3*gamma(-1/3)) + 4*(-1 + x**(-3))**(1/3)*exp(-
2*I*pi/3)*gamma(-10/3)/(27*x**6*gamma(-1/3)) - 28*(-1 + x**(-3))**(1/3)*exp
(-2*I*pi/3)*gamma(-10/3)/(27*x**9*gamma(-1/3)), 1/Abs(x**3) > 1), (-2*(1 -
1/x**3)**(1/3)*gamma(-10/3)/(3*gamma(-1/3)) - 2*(1 - 1/x**3)**(1/3)*gamma(-
10/3)/(9*x**3*gamma(-1/3)) - 4*(1 - 1/x**3)**(1/3)*gamma(-10/3)/(27*x**6*ga
mma(-1/3)) + 28*(1 - 1/x**3)**(1/3)*gamma(-10/3)/(27*x**9*gamma(-1/3)), Tru
e))
```

3.316 $\int \frac{\sqrt[3]{-1+x^3}\left(-1+2x^3\right)}{x^8}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt[3]{x^3-1}\left(11x^6-15x^3+4\right)}{28x^7}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {453, 264}

$$\frac{11\left(x^3-1\right)^{4/3}}{28x^4}-\frac{\left(x^3-1\right)^{4/3}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(1/3)*(-1 + 2*x^3))/x^8,x]
```

```
[Out] -1/7*(-1 + x^3)^(4/3)/x^7 + (11*(-1 + x^3)^(4/3))/(28*x^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{-1+x^3}\left(-1+2x^3\right)}{x^8}\,dx &= -\frac{\left(-1+x^3\right)^{4/3}}{7x^7}+\frac{11}{7}\int \frac{\sqrt[3]{-1+x^3}}{x^5}\,dx\\ &= -\frac{\left(-1+x^3\right)^{4/3}}{7x^7}+\frac{11\left(-1+x^3\right)^{4/3}}{28x^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.82

$$\frac{\left(x^3-1\right)^{4/3}\left(11x^3-4\right)}{28x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)^(1/3)*(-1 + 2*x^3))/x^8,x]
```

```
[Out] ((-1 + x^3)^(4/3)*(-4 + 11*x^3))/(28*x^7)
```

IntegrateAlgebraic [A] time = 0.10, size = 23, normalized size = 0.82

$$\frac{\left(x^3-1\right)^{4/3}\left(11x^3-4\right)}{28x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(1/3)*(-1 + 2*x^3))/x^8,x]
[Out] ((-1 + x^3)^(4/3)*(-4 + 11*x^3))/(28*x^7)
```

fricas [A] time = 0.40, size = 24, normalized size = 0.86

$$\frac{\left(11\,x^{6}-15\,x^{3}+4\right)\left(x^{3}-1\right)^{\frac{1}{3}}}{28\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(2*x^3-1)/x^8,x, algorithm="fricas")
[Out] 1/28*(11*x^6 - 15*x^3 + 4)*(x^3 - 1)^(1/3)/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{3}-1\right)\left(x^{3}-1\right)^{\frac{1}{3}}}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(2*x^3-1)/x^8,x, algorithm="giac")
[Out] integrate((2*x^3 - 1)*(x^3 - 1)^(1/3)/x^8, x)
```

maple [A] time = 0.00, size = 29, normalized size = 1.04

$$\frac{(-1+x)\left(x^{2}+x+1\right)\left(11x^{3}-4\right)\left(x^{3}-1\right)^{\frac{1}{3}}}{28x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/3)*(2*x^3-1)/x^8,x)
[Out] 1/28*(-1+x)*(x^2+x+1)*(11*x^3-4)*(x^3-1)^(1/3)/x^7
```

maxima [A] time = 0.57, size = 25, normalized size = 0.89

$$\frac{\left(x^{3}-1\right)^{\frac{4}{3}}}{4\,x^{4}}+\frac{\left(x^{3}-1\right)^{\frac{7}{3}}}{7\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(2*x^3-1)/x^8,x, algorithm="maxima")
[Out] 1/4*(x^3 - 1)^(4/3)/x^4 + 1/7*(x^3 - 1)^(7/3)/x^7
```

mupad [B] time = 0.27, size = 39, normalized size = 1.39

$$\frac{4\left(x^{3}-1\right)^{1/3}-15\,x^{3}\left(x^{3}-1\right)^{1/3}+11\,x^{6}\left(x^{3}-1\right)^{1/3}}{28\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(1/3)*(2*x^3 - 1))/x^8,x)
[Out] (4*(x^3 - 1)^(1/3) - 15*x^3*(x^3 - 1)^(1/3) + 11*x^6*(x^3 - 1)^(1/3))/(28*x
^7)
```

sympy [C] time = 2.48, size = 418, normalized size = 14.93

$$2\left(\begin{cases} \frac{\sqrt[3]{-1+\frac{1}{x^{3}}}e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)} - \frac{\sqrt[3]{-1+\frac{1}{x^{3}}}e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{3x^{3}\Gamma\left(-\frac{1}{3}\right)} & \text{for}\ \frac{1}{|x^{3}|} > 1 \\ -\frac{\sqrt[3]{1-\frac{1}{x^{3}}}\Gamma\left(-\frac{4}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt[3]{1-\frac{1}{x^{3}}}\Gamma\left(-\frac{4}{3}\right)}{3x^{3}\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise} \end{cases}\right) - \begin{cases} \frac{\sqrt[3]{-1+\frac{1}{x^{3}}}e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt[3]{-1+\frac{1}{x^{3}}}e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{9x^{3}\Gamma\left(-\frac{1}{3}\right)} - \frac{4\sqrt[3]{-1+\frac{1}{x^{3}}}e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{9x^{6}\Gamma\left(-\frac{1}{3}\right)} & \text{for}\ \frac{1}{|x^{3}|} > 1 \\ \frac{3x^{6}\sqrt[3]{1-\frac{1}{x^{3}}}\Gamma\left(-\frac{7}{3}\right)}{9x^{6}\Gamma\left(-\frac{1}{3}\right)-9x^{3}\Gamma\left(-\frac{1}{3}\right)} - \frac{2x^{3}\sqrt[3]{1-\frac{1}{x^{3}}}\Gamma\left(-\frac{7}{3}\right)}{9x^{6}\Gamma\left(-\frac{1}{3}\right)-9x^{3}\Gamma\left(-\frac{1}{3}\right)} + \frac{4\sqrt[3]{1-\frac{1}{x^{3}}}\Gamma\left(-\frac{7}{3}\right)}{9x^{9}\Gamma\left(-\frac{1}{3}\right)-9x^{6}\Gamma\left(-\frac{1}{3}\right)} - \frac{5\sqrt[3]{1-\frac{1}{x^{3}}}\Gamma\left(-\frac{7}{3}\right)}{9x^{6}\Gamma\left(-\frac{1}{3}\right)-9x^{3}\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/3)*(2*x**3-1)/x**8,x)
```

```
[Out] 2*Piecewise(((-1 + x**(-3))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(3*gamma(-1/3
)) - (-1 + x**(-3))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(3*x**3*gamma(-1/3)),
 1/Abs(x**3) > 1), (-(1 - 1/x**3)**(1/3)*gamma(-4/3)/(3*gamma(-1/3)) + (1 -
 1/x**3)**(1/3)*gamma(-4/3)/(3*x**3*gamma(-1/3)), True)) - Piecewise(((-1 +
 x**(-3))**(1/3)*exp(I*pi/3)*gamma(-7/3)/(3*gamma(-1/3)) + (-1 + x**(-3))**
(1/3)*exp(I*pi/3)*gamma(-7/3)/(9*x**3*gamma(-1/3)) - 4*(-1 + x**(-3))**(1/3
)*exp(I*pi/3)*gamma(-7/3)/(9*x**6*gamma(-1/3)), 1/Abs(x**3) > 1), (3*x**6*(
1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**6*gamma(-1/3) - 9*x**3*gamma(-1/3)) -
2*x**3*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**6*gamma(-1/3) - 9*x**3*gamma(-
1/3)) + 4*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**9*gamma(-1/3) - 9*x**6*gamm
a(-1/3)) - 5*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**6*gamma(-1/3) - 9*x**3*g
amma(-1/3)), True))
```

3.317 $\int \frac{(3+x^2)(-1-x^2+2x^3)}{x^6\sqrt[4]{x+x^3}}\,dx$

Optimal. Leaf size=28

$$-\frac{4\left(x^3+x\right)^{3/4}\left(14x^3-3x^2-3\right)}{21x^6}$$

Rubi [A] time = 0.36, antiderivative size = 49, normalized size of antiderivative = 1.75, number of steps used = 21, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2052, 2025, 2011, 364, 2032}

$$-\frac{8\left(x^3+x\right)^{3/4}}{3x^3}+\frac{4\left(x^3+x\right)^{3/4}}{7x^6}+\frac{4\left(x^3+x\right)^{3/4}}{7x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + x^2)*(-1 - x^2 + 2*x^3))/(x^6*(x + x^3)^(1/4)),x]
```

```
[Out] (4*(x + x^3)^(3/4))/(7*x^6) + (4*(x + x^3)^(3/4))/(7*x^4) - (8*(x + x^3)^(3
/4))/(3*x^3)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{(3+x^2)(-1-x^2+2x^3)}{x^6\sqrt[4]{x+x^3}}\,dx &= \int \left(-\frac{3}{x^6\sqrt[4]{x+x^3}}-\frac{4}{x^4\sqrt[4]{x+x^3}}+\frac{6}{x^3\sqrt[4]{x+x^3}}-\frac{1}{x^2\sqrt[4]{x+x^3}}+\frac{2}{x\sqrt[4]{x+x^3}}\right)dx\\
&= 2\int \frac{1}{x\sqrt[4]{x+x^3}}\,dx-3\int \frac{1}{x^6\sqrt[4]{x+x^3}}\,dx-4\int \frac{1}{x^4\sqrt[4]{x+x^3}}\,dx+6\int \frac{1}{x^3\sqrt[4]{x+x^3}}\\
&= \frac{4(x+x^3)^{3/4}}{7x^6}+\frac{16(x+x^3)^{3/4}}{13x^4}-\frac{8(x+x^3)^{3/4}}{3x^3}+\frac{4(x+x^3)^{3/4}}{5x^2}-\frac{8(x+x^3)^{3/4}}{x}-\\
&= \frac{4(x+x^3)^{3/4}}{7x^6}+\frac{4(x+x^3)^{3/4}}{7x^4}-\frac{8(x+x^3)^{3/4}}{3x^3}-\frac{12(x+x^3)^{3/4}}{13x^2}+\frac{28}{65}\int \frac{1}{\sqrt[4]{x+x^3}}\\
&= \frac{4(x+x^3)^{3/4}}{7x^6}+\frac{4(x+x^3)^{3/4}}{7x^4}-\frac{8(x+x^3)^{3/4}}{3x^3}-\frac{4x\sqrt[4]{1+x^2}\,{}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-x^2\right)}{15\sqrt[4]{x+x^3}}+\\
&= \frac{4(x+x^3)^{3/4}}{7x^6}+\frac{4(x+x^3)^{3/4}}{7x^4}-\frac{8(x+x^3)^{3/4}}{3x^3}+\frac{4x\sqrt[4]{1+x^2}\,{}_2F_1\left(\frac{1}{4},\frac{3}{8};\frac{11}{8};-x^2\right)}{13\sqrt[4]{x+x^3}}-\\
&= \frac{4(x+x^3)^{3/4}}{7x^6}+\frac{4(x+x^3)^{3/4}}{7x^4}-\frac{8(x+x^3)^{3/4}}{3x^3}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 125, normalized size = 4.46

$$-\frac{4\sqrt[4]{x^2+1}\left(7x^2\left(13x\left(30x^2\,{}_2F_1\left(-\frac{1}{8},\frac{1}{4};\frac{7}{8};-x^2\right)-3x\,{}_2F_1\left(-\frac{5}{8},\frac{1}{4};\frac{3}{8};-x^2\right)+10\,{}_2F_1\left(-\frac{9}{8},\frac{1}{4};-\frac{1}{8};-x^2\right)\right)-60\,{}_2F_1\left(-\frac{13}{8},\frac{1}{4};-\frac{5}{8};-x^2\right)\right)-195\,{}_2F_1\left(-\frac{21}{8},\frac{1}{4};-\frac{13}{8};-x^2\right)\right)}{1365x^5\sqrt[4]{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + x^2)*(-1 - x^2 + 2*x^3))/(x^6*(x + x^3)^(1/4)),x]
```

```
[Out] (-4*(1 + x^2)^(1/4)*(-195*Hypergeometric2F1[-21/8, 1/4, -13/8, -x^2] + 7*x^
2*(-60*Hypergeometric2F1[-13/8, 1/4, -5/8, -x^2] + 13*x*(10*Hypergeometric2
F1[-9/8, 1/4, -1/8, -x^2] - 3*x*Hypergeometric2F1[-5/8, 1/4, 3/8, -x^2] + 3
0*x^2*Hypergeometric2F1[-1/8, 1/4, 7/8, -x^2]))))/(1365*x^5*(x + x^3)^(1/4)
)
```

IntegrateAlgebraic [A] time = 0.73, size = 28, normalized size = 1.00

$$-\frac{4\left(x^3+x\right)^{3/4}\left(14x^3-3x^2-3\right)}{21x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3 + x^2)*(-1 - x^2 + 2*x^3))/(x^6*(x + x^3)^(1/4)),x]
```

```
[Out] (-4*(x + x^3)^(3/4)*(-3 - 3*x^2 + 14*x^3))/(21*x^6)
```

fricas [A] time = 0.40, size = 24, normalized size = 0.86

$$-\frac{4\left(14\,x^3-3\,x^2-3\right)\left(x^3+x\right)^{\frac{3}{4}}}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)*(2*x^3-x^2-1)/x^6/(x^3+x)^(1/4),x, algorithm="fricas")
```

```
[Out] -4/21*(14*x^3 - 3*x^2 - 3)*(x^3 + x)^(3/4)/x^6
```

giac [A] time = 0.57, size = 23, normalized size = 0.82

$$\frac{4}{7}\left(\frac{1}{x}+\frac{1}{x^3}\right)^{\frac{7}{4}}-\frac{8}{3}\left(\frac{1}{x}+\frac{1}{x^3}\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)*(2*x^3-x^2-1)/x^6/(x^3+x)^(1/4),x, algorithm="giac")
[Out] 4/7*(1/x + 1/x^3)^(7/4) - 8/3*(1/x + 1/x^3)^(3/4)
```

maple [A] time = 0.01, size = 30, normalized size = 1.07

$$-\frac{4\left(14x^3-3x^2-3\right)\left(x^2+1\right)}{21x^5\left(x^3+x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3)*(2*x^3-x^2-1)/x^6/(x^3+x)^(1/4),x)
[Out] -4/21*(14*x^3-3*x^2-3)*(x^2+1)/x^5/(x^3+x)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2\,x^3-x^2-1\right)\left(x^2+3\right)}{\left(x^3+x\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)*(2*x^3-x^2-1)/x^6/(x^3+x)^(1/4),x, algorithm="maxima")
[Out] integrate((2*x^3 - x^2 - 1)*(x^2 + 3)/((x^3 + x)^(1/4)*x^6), x)
```

mupad [B] time = 0.24, size = 39, normalized size = 1.39

$$\frac{12\left(x^3+x\right)^{3/4}+12\,x^2\left(x^3+x\right)^{3/4}-56\,x^3\left(x^3+x\right)^{3/4}}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^2 + 3)*(x^2 - 2*x^3 + 1))/(x^6*(x + x^3)^(1/4)),x)
[Out] (12*(x + x^3)^(3/4) + 12*x^2*(x + x^3)^(3/4) - 56*x^3*(x + x^3)^(3/4))/(21*
x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x-1)\left(x^2+3\right)\left(2x^2+x+1\right)}{x^6\sqrt[4]{x\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3)*(2*x**3-x**2-1)/x**6/(x**3+x)**(1/4),x)
[Out] Integral((x - 1)*(x**2 + 3)*(2*x**2 + x + 1)/(x**6*(x*(x**2 + 1))**(1/4)),
x)
```

3.318 $\int \frac{2b+ax^3}{\sqrt{-b+ax^3}\left(-b+x^2+ax^3\right)}\,dx$

Optimal. Leaf size=28

$$2\tan^{-1}\left(\frac{x\sqrt{ax^3-b}}{b-ax^3}\right)$$

Rubi [F] time = 1.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2b+ax^3}{\sqrt{-b+ax^3}\left(-b+x^2+ax^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2*b + a*x^3)/(Sqrt[-b + a*x^3]*(-b + x^2 + a*x^3)),x]
```

```
[Out] (-2*Sqrt[2 - Sqrt[3]]*(b^(1/3) - a^(1/3)*x)*Sqrt[(b^(2/3) + a^(1/3)*b^(1/3)
*x + a^(2/3)*x^2)/((1 - Sqrt[3])*b^(1/3) - a^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 + Sqrt[3])*b^(1/3) - a^(1/3)*x)/((1 - Sqrt[3])*b^(1/3) - a^(1/3)*x)], -7
 + 4*Sqrt[3]])/(3^(1/4)*a^(1/3)*Sqrt[-((b^(1/3)*(b^(1/3) - a^(1/3)*x))/((1
- Sqrt[3])*b^(1/3) - a^(1/3)*x)^2)]*Sqrt[-b + a*x^3]) - 3*b*Defer[Int][1/((
b - x^2 - a*x^3)*Sqrt[-b + a*x^3]), x] - Defer[Int][x^2/(Sqrt[-b + a*x^3]*(
-b + x^2 + a*x^3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{2b+ax^3}{\sqrt{-b+ax^3}\left(-b+x^2+ax^3\right)}\,dx &= \int \left(\frac{1}{\sqrt{-b+ax^3}}+\frac{3b-x^2}{\sqrt{-b+ax^3}\left(-b+x^2+ax^3\right)}\right)dx\\
&= \int \frac{1}{\sqrt{-b+ax^3}}\,dx+\int \frac{3b-x^2}{\sqrt{-b+ax^3}\left(-b+x^2+ax^3\right)}\,dx\\
&= -\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)\sqrt{\frac{b^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\sqrt[3]{a}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}-\sqrt[3]{a}x}{\left(1-\sqrt{3}\right)\sqrt[3]{b}-\sqrt[3]{a}x}\right)|-7\right.}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{-\frac{\sqrt[3]{b}\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\sqrt[3]{a}x\right)^2}}\sqrt{-b+ax^3}}\\
&= -\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)\sqrt{\frac{b^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\sqrt[3]{a}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}-\sqrt[3]{a}x}{\left(1-\sqrt{3}\right)\sqrt[3]{b}-\sqrt[3]{a}x}\right)|-7\right.}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{-\frac{\sqrt[3]{b}\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\sqrt[3]{a}x\right)^2}}\sqrt{-b+ax^3}}
\end{aligned}$$

Mathematica [C] time = 6.25, size = 2752, normalized size = 98.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*b + a*x^3)/(Sqrt[-b + a*x^3]*(-b + x^2 + a*x^3)),x]
```

```
[Out] (2*Sqrt[(-(b^(1/3)/a^(1/3)) + x)/(-(b^(1/3)/a^(1/3)) - ((-1)^(1/3)*b^(1/3))
/a^(1/3))]*(((-1)^(1/3)*b^(1/3))/a^(1/3) + x)*Sqrt[(-(((-1)^(2/3)*b^(1/3))/
```

```
a^(1/3)) + x)/(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/
3))]*EllipticF[ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) + a^(1/3)*x)/(((-1)^(1/3) +
(-1)^(2/3))*b^(1/3))]], (-1)^(1/3)])/(Sqrt[(((-1)^(1/3)*b^(1/3))/a^(1/3) +
x)/(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))]*Sqrt[-b +
 a*x^3]) + (4*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/
3))*b*Sqrt[(-(b^(1/3)/a^(1/3)) + x)/(-(b^(1/3)/a^(1/3)) - ((-1)^(1/3)*b^(1/
3))/a^(1/3))]*Sqrt[((((-1)^(2/3)*b^(1/3))/a^(1/3) - x)*(((-1)^(1/3)*b^(1/3)
)/a^(1/3) + x))/(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(
1/3))^2]*EllipticPi[((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b
^(1/3) + a^(1/3)*Root[-b + #1^2 + a*#1^3 & , 1]), ArcSin[Sqrt[((-1)^(1/3)*b
^(1/3) + a^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)])/(a*
Sqrt[-b + a*x^3]*(((-1)^(1/3)*b^(1/3))/a^(1/3) + Root[-b + #1^2 + a*#1^3 &
, 1])*(Root[-b + #1^2 + a*#1^3 & , 1] - Root[-b + #1^2 + a*#1^3 & , 2])*(Ro
ot[-b + #1^2 + a*#1^3 & , 1] - Root[-b + #1^2 + a*#1^3 & , 3])) + (2*(-(((-
1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))*Sqrt[(-(b^(1/3)/
a^(1/3)) + x)/(-(b^(1/3)/a^(1/3)) - ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[(((
(-1)^(2/3)*b^(1/3))/a^(1/3) - x)*(((-1)^(1/3)*b^(1/3))/a^(1/3) + x))/(-(((-
1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*EllipticPi[((
-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) + a^(1/3)*Root[
-b + #1^2 + a*#1^3 & , 1]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) + a^(1/3)*x)/((
(-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)]*Root[-b + #1^2 + a*#1^3 &
, 1]^3)/(Sqrt[-b + a*x^3]*(((-1)^(1/3)*b^(1/3))/a^(1/3) + Root[-b + #1^2 +
a*#1^3 & , 1])*(Root[-b + #1^2 + a*#1^3 & , 1] - Root[-b + #1^2 + a*#1^3 &
, 2])*(Root[-b + #1^2 + a*#1^3 & , 1] - Root[-b + #1^2 + a*#1^3 & , 3])) +
(4*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))*b*Sqrt[
(-(b^(1/3)/a^(1/3)) + x)/(-(b^(1/3)/a^(1/3)) - ((-1)^(1/3)*b^(1/3))/a^(1/3)
)]*Sqrt[((((-1)^(2/3)*b^(1/3))/a^(1/3) - x)*(((-1)^(1/3)*b^(1/3))/a^(1/3) +
 x))/(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*El
lipticPi[((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) + a^
(1/3)*Root[-b + #1^2 + a*#1^3 & , 2]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) + a^
(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)])/(a*Sqrt[-b + a
*x^3]*(((-1)^(1/3)*b^(1/3))/a^(1/3) + Root[-b + #1^2 + a*#1^3 & , 2])*(-Roo
t[-b + #1^2 + a*#1^3 & , 1] + Root[-b + #1^2 + a*#1^3 & , 2])*(Root[-b + #1
^2 + a*#1^3 & , 2] - Root[-b + #1^2 + a*#1^3 & , 3])) + (2*(-(((-1)^(1/3)*b
^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))*Sqrt[(-(b^(1/3)/a^(1/3)) +
 x)/(-(b^(1/3)/a^(1/3)) - ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[((((-1)^(2/3)
*b^(1/3))/a^(1/3) - x)*(((-1)^(1/3)*b^(1/3))/a^(1/3) + x))/(-(((-1)^(1/3)*b
^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*EllipticPi[((-1)^(1/3)*
b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) + a^(1/3)*Root[-b + #1^2
+ a*#1^3 & , 2]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) + a^(1/3)*x)/(((-1)^(1/3)
 + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)]*Root[-b + #1^2 + a*#1^3 & , 2]^3)/(S
qrt[-b + a*x^3]*(((-1)^(1/3)*b^(1/3))/a^(1/3) + Root[-b + #1^2 + a*#1^3 & ,
 2])*(-Root[-b + #1^2 + a*#1^3 & , 1] + Root[-b + #1^2 + a*#1^3 & , 2])*(Ro
ot[-b + #1^2 + a*#1^3 & , 2] - Root[-b + #1^2 + a*#1^3 & , 3])) + (4*(-(((-
1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))*b*Sqrt[(-(b^(1/3
)/a^(1/3)) + x)/(-(b^(1/3)/a^(1/3)) - ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[(
(((-1)^(2/3)*b^(1/3))/a^(1/3) - x)*(((-1)^(1/3)*b^(1/3))/a^(1/3) + x))/(-((
(-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*EllipticPi[
((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) + a^(1/3)*Roo
t[-b + #1^2 + a*#1^3 & , 3]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) + a^(1/3)*x)/
(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)])/(a*Sqrt[-b + a*x^3]*(((
-1)^(1/3)*b^(1/3))/a^(1/3) + Root[-b + #1^2 + a*#1^3 & , 3])*(-Root[-b + #1
^2 + a*#1^3 & , 1] + Root[-b + #1^2 + a*#1^3 & , 3])*(-Root[-b + #1^2 + a*#
1^3 & , 2] + Root[-b + #1^2 + a*#1^3 & , 3])) + (2*(-(((-1)^(1/3)*b^(1/3))/
a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))*Sqrt[(-(b^(1/3)/a^(1/3)) + x)/(-(b
^(1/3)/a^(1/3)) - ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[((((-1)^(2/3)*b^(1/3)
)/a^(1/3) - x)*(((-1)^(1/3)*b^(1/3))/a^(1/3) + x))/(-(((-1)^(1/3)*b^(1/3))/
a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*EllipticPi[((-1)^(1/3)*b^(1/3)
+ (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) + a^(1/3)*Root[-b + #1^2 + a*#1^3
```

```
 & , 3]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) + a^(1/3)*x)/(((-1)^(1/3) + (-1)^
(2/3))*b^(1/3))]], (-1)^(1/3)]*Root[-b + #1^2 + a*#1^3 & , 3]^3)/(Sqrt[-b +
 a*x^3]*(((-1)^(1/3)*b^(1/3))/a^(1/3) + Root[-b + #1^2 + a*#1^3 & , 3])*(-R
oot[-b + #1^2 + a*#1^3 & , 1] + Root[-b + #1^2 + a*#1^3 & , 3])*(-Root[-b +
 #1^2 + a*#1^3 & , 2] + Root[-b + #1^2 + a*#1^3 & , 3]))
```

IntegrateAlgebraic [A] time = 0.56, size = 28, normalized size = 1.00

$$2\tan^{-1}\left(\frac{x\sqrt{ax^3-b}}{b-ax^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*b + a*x^3)/(Sqrt[-b + a*x^3]*(-b + x^2 + a*x^3)),x]
[Out] 2*ArcTan[(x*Sqrt[-b + a*x^3])/(b - a*x^3)]
```

fricas [B] time = 0.44, size = 40, normalized size = 1.43

$$\arctan\left(\frac{\left(ax^3-x^2-b\right)\sqrt{ax^3-b}}{2\left(ax^4-bx\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b)/(a*x^3-b)^(1/2)/(a*x^3+x^2-b),x, algorithm="fricas")
[Out] arctan(1/2*(a*x^3 - x^2 - b)*sqrt(a*x^3 - b)/(a*x^4 - b*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3+2b}{\left(ax^3+x^2-b\right)\sqrt{ax^3-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b)/(a*x^3-b)^(1/2)/(a*x^3+x^2-b),x, algorithm="giac")
[Out] integrate((a*x^3 + 2*b)/((a*x^3 + x^2 - b)*sqrt(a*x^3 - b)), x)
```

maple [C] time = 0.52, size = 788, normalized size = 28.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+2*b)/(a*x^3-b)^(1/2)/(a*x^3+x^2-b),x)
[Out] 2/3*I*3^(1/2)/a*(a^2*b)^(1/3)*(-I*(x+1/2/a*(a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(a
^2*b)^(1/3))*3^(1/2)*a/(a^2*b)^(1/3))^(1/2)*((x-1/a*(a^2*b)^(1/3))/(-3/2/a*
(a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(a^2*b)^(1/3)))^(1/2)*(I*(x+1/2/a*(a^2*b)^(1/
3)-1/2*I*3^(1/2)/a*(a^2*b)^(1/3))*3^(1/2)*a/(a^2*b)^(1/3))^(1/2)/(a*x^3-b)^
(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/a*(a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(a^2
*b)^(1/3))*3^(1/2)*a/(a^2*b)^(1/3))^(1/2),(-I*3^(1/2)/a*(a^2*b)^(1/3)/(-3/2
/a*(a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(a^2*b)^(1/3)))^(1/2))+I/a^2/b*2^(1/2)*sum
((-_alpha^2+3*b)/_alpha/(3*_alpha*a+2)*(a^2*b)^(1/3)*(-1/2*I*a*(2*x+1/a*(I*
3^(1/2)*(a^2*b)^(1/3)+(a^2*b)^(1/3)))/(a^2*b)^(1/3))^(1/2)*(a*(x-1/a*(a^2*b
)^(1/3))/(-3*(a^2*b)^(1/3)-I*3^(1/2)*(a^2*b)^(1/3)))^(1/2)*(1/2*I*a*(2*x+1/
a*(-I*3^(1/2)*(a^2*b)^(1/3)+(a^2*b)^(1/3)))/(a^2*b)^(1/3))^(1/2)/(a*x^3-b)^
(1/2)*(-I*(a^2*b)^(1/3)*3^(1/2)*_alpha^2*a^2+I*(a^2*b)^(2/3)*3^(1/2)*_alpha
*a-(a^2*b)^(1/3)*_alpha^2*a^2-I*(a^2*b)^(1/3)*3^(1/2)*_alpha*a-_alpha*(a^2*
b)^(2/3)*a+I*(a^2*b)^(2/3)*3^(1/2)-(a^2*b)^(1/3)*_alpha*a+2*a^2*b-(a^2*b)^(
2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/a*(a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(a^
```

```
2*b)^(1/3))*3^(1/2)*a/(a^2*b)^(1/3))^(1/2),1/2/a*(I*(a^2*b)^(2/3)*_alpha^2*
3^(1/2)*a+I*_alpha*3^(1/2)*a^2*b-3*(a^2*b)^(2/3)*_alpha^2*a+I*(a^2*b)^(2/3)
*3^(1/2)*_alpha-2*I*(a^2*b)^(1/3)*3^(1/2)*a*b+3*_alpha*a^2*b+I*3^(1/2)*a*b-
3*(a^2*b)^(2/3)*_alpha+3*a*b)/b,(-I*3^(1/2)/a*(a^2*b)^(1/3)/(-3/2/a*(a^2*b)
^(1/3)-1/2*I*3^(1/2)/a*(a^2*b)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*a+_Z^2-b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 + 2\,b}{\left(ax^3 + x^2 - b\right)\sqrt{ax^3 - b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b)/(a*x^3-b)^(1/2)/(a*x^3+x^2-b),x, algorithm="maxima")
[Out] integrate((a*x^3 + 2*b)/((a*x^3 + x^2 - b)*sqrt(a*x^3 - b)), x)
```

mupad [B] time = 2.06, size = 45, normalized size = 1.61

$$\ln\left(\frac{b - a\,x^3 + x^2 - x\,\sqrt{a\,x^3 - b}\,2\mathrm{i}}{a\,x^3 + x^2 - b}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b + a*x^3)/((a*x^3 - b)^(1/2)*(a*x^3 - b + x^2)),x)
[Out] log((b - a*x^3 + x^2 - x*(a*x^3 - b)^(1/2)*2i)/(a*x^3 - b + x^2))*1i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+2*b)/(a*x**3-b)**(1/2)/(a*x**3+x**2-b),x)
[Out] Timed out
```

3.319 $\int \frac{(-4+x^4)(-1+x^4)^{3/4}}{x^{12}} \, dx$

Optimal. Leaf size=28

$$\frac{(x^4-1)^{3/4}(-5x^8-23x^4+28)}{77x^{11}}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {453, 264}

$$-\frac{4(x^4-1)^{7/4}}{11x^{11}}-\frac{5(x^4-1)^{7/4}}{77x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((-4 + x^4)*(-1 + x^4)^(3/4))/x^12,x]
```

```
[Out] (-4*(-1 + x^4)^(7/4))/(11*x^11) - (5*(-1 + x^4)^(7/4))/(77*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(-4+x^4)(-1+x^4)^{3/4}}{x^{12}} \, dx &= -\frac{4(-1+x^4)^{7/4}}{11x^{11}}-\frac{5}{11}\int \frac{(-1+x^4)^{3/4}}{x^8} \, dx\\ &= -\frac{4(-1+x^4)^{7/4}}{11x^{11}}-\frac{5(-1+x^4)^{7/4}}{77x^7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.82

$$-\frac{(x^4-1)^{7/4}(5x^4+28)}{77x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-4 + x^4)*(-1 + x^4)^(3/4))/x^12,x]
```

```
[Out] -1/77*((-1 + x^4)^(7/4)*(28 + 5*x^4))/x^11
```

IntegrateAlgebraic [A] time = 0.16, size = 23, normalized size = 0.82

$$\frac{(-5x^4-28)(x^4-1)^{7/4}}{77x^{11}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + x^4)*(-1 + x^4)^(3/4))/x^12,x]
[Out] ((-28 - 5*x^4)*(-1 + x^4)^(7/4))/(77*x^11)
```

fricas [A] time = 0.38, size = 24, normalized size = 0.86

$$-\frac{\left(5\,x^{8}+23\,x^{4}-28\right)\left(x^{4}-1\right)^{\frac{3}{4}}}{77\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4)*(x^4-1)^(3/4)/x^12,x, algorithm="fricas")
[Out] -1/77*(5*x^8 + 23*x^4 - 28)*(x^4 - 1)^(3/4)/x^11
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}-1\right)^{\frac{3}{4}}\left(x^{4}-4\right)}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4)*(x^4-1)^(3/4)/x^12,x, algorithm="giac")
[Out] integrate((x^4 - 1)^(3/4)*(x^4 - 4)/x^12, x)
```

maple [A] time = 0.01, size = 31, normalized size = 1.11

$$-\frac{\left(x^{2}+1\right)(1+x)(-1+x)\left(5x^{4}+28\right)\left(x^{4}-1\right)^{\frac{3}{4}}}{77x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-4)*(x^4-1)^(3/4)/x^12,x)
[Out] -1/77*(x^2+1)*(1+x)*(-1+x)*(5*x^4+28)*(x^4-1)^(3/4)/x^11
```

maxima [A] time = 0.32, size = 25, normalized size = 0.89

$$-\frac{3\left(x^{4}-1\right)^{\frac{7}{4}}}{7\,x^{7}}+\frac{4\left(x^{4}-1\right)^{\frac{11}{4}}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4)*(x^4-1)^(3/4)/x^12,x, algorithm="maxima")
[Out] -3/7*(x^4 - 1)^(7/4)/x^7 + 4/11*(x^4 - 1)^(11/4)/x^11
```

mupad [B] time = 0.36, size = 39, normalized size = 1.39

$$-\frac{23\,x^{4}\left(x^{4}-1\right)^{3/4}-28\left(x^{4}-1\right)^{3/4}+5\,x^{8}\left(x^{4}-1\right)^{3/4}}{77\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)^(3/4)*(x^4 - 4))/x^12,x)
[Out] -(23*x^4*(x^4 - 1)^(3/4) - 28*(x^4 - 1)^(3/4) + 5*x^8*(x^4 - 1)^(3/4))/(77*
x^11)
```

sympy [C] time = 3.61, size = 420, normalized size = 15.00

$$\begin{cases} \frac{\left(-1+\frac{1}{x^4}\right)^{\frac{3}{4}} e^{-\frac{i\pi}{4}} \Gamma\left(-\frac{7}{4}\right)}{4\Gamma\left(-\frac{3}{4}\right)} - \frac{\left(-1+\frac{1}{x^4}\right)^{\frac{3}{4}} e^{-\frac{i\pi}{4}} \Gamma\left(-\frac{7}{4}\right)}{4x^4\Gamma\left(-\frac{3}{4}\right)} & \text{for } \frac{1}{|x^4|} > 1 \\ -\frac{\left(1-\frac{1}{x^4}\right)^{\frac{3}{4}} \Gamma\left(-\frac{7}{4}\right)}{4\Gamma\left(-\frac{3}{4}\right)} + \frac{\left(1-\frac{1}{x^4}\right)^{\frac{3}{4}} \Gamma\left(-\frac{7}{4}\right)}{4x^4\Gamma\left(-\frac{3}{4}\right)} & \text{otherwise} \end{cases} - 4 \left(\begin{cases} \frac{\left(-1+\frac{1}{x^4}\right)^{\frac{3}{4}} e^{\frac{3i\pi}{4}} \Gamma\left(-\frac{11}{4}\right)}{4\Gamma\left(-\frac{3}{4}\right)} + \frac{3\left(-1+\frac{1}{x^4}\right)^{\frac{3}{4}} e^{\frac{3i\pi}{4}} \Gamma\left(-\frac{11}{4}\right)}{16x^4\Gamma\left(-\frac{3}{4}\right)} - \frac{7\left(-1+\frac{1}{x^4}\right)^{\frac{3}{4}} e^{\frac{3i\pi}{4}} \Gamma\left(-\frac{11}{4}\right)}{16x^8\Gamma\left(-\frac{3}{4}\right)} & \text{for } \frac{1}{|x^4|} > 1 \\ \frac{4x^8\left(1-\frac{1}{x^4}\right)^{\frac{3}{4}} \Gamma\left(-\frac{11}{4}\right)}{16x^8\Gamma\left(-\frac{3}{4}\right)-16x^4\Gamma\left(-\frac{3}{4}\right)} - \frac{x^4\left(1-\frac{1}{x^4}\right)^{\frac{3}{4}} \Gamma\left(-\frac{11}{4}\right)}{16x^8\Gamma\left(-\frac{3}{4}\right)-16x^4\Gamma\left(-\frac{3}{4}\right)} + \frac{7\left(1-\frac{1}{x^4}\right)^{\frac{3}{4}} \Gamma\left(-\frac{11}{4}\right)}{16x^{12}\Gamma\left(-\frac{3}{4}\right)-16x^8\Gamma\left(-\frac{3}{4}\right)} - \frac{10\left(1-\frac{1}{x^4}\right)^{\frac{3}{4}} \Gamma\left(-\frac{11}{4}\right)}{16x^8\Gamma\left(-\frac{3}{4}\right)-16x^4\Gamma\left(-\frac{3}{4}\right)} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-4)*(x**4-1)**(3/4)/x**12,x)
```

```
[Out] Piecewise(((-1 + x**(-4))**(3/4)*exp(-I*pi/4)*gamma(-7/4)/(4*gamma(-3/4)) -
 (-1 + x**(-4))**(3/4)*exp(-I*pi/4)*gamma(-7/4)/(4*x**4*gamma(-3/4)), 1/Abs
(x**4) > 1), (-(1 - 1/x**4)**(3/4)*gamma(-7/4)/(4*gamma(-3/4)) + (1 - 1/x**
4)**(3/4)*gamma(-7/4)/(4*x**4*gamma(-3/4)), True)) - 4*Piecewise(((-1 + x**
(-4))**(3/4)*exp(3*I*pi/4)*gamma(-11/4)/(4*gamma(-3/4)) + 3*(-1 + x**(-4))*
*(3/4)*exp(3*I*pi/4)*gamma(-11/4)/(16*x**4*gamma(-3/4)) - 7*(-1 + x**(-4))*
*(3/4)*exp(3*I*pi/4)*gamma(-11/4)/(16*x**8*gamma(-3/4)), 1/Abs(x**4) > 1),
(4*x**8*(1 - 1/x**4)**(3/4)*gamma(-11/4)/(16*x**8*gamma(-3/4) - 16*x**4*gam
ma(-3/4)) - x**4*(1 - 1/x**4)**(3/4)*gamma(-11/4)/(16*x**8*gamma(-3/4) - 16
*x**4*gamma(-3/4)) + 7*(1 - 1/x**4)**(3/4)*gamma(-11/4)/(16*x**12*gamma(-3/
4) - 16*x**8*gamma(-3/4)) - 10*(1 - 1/x**4)**(3/4)*gamma(-11/4)/(16*x**8*ga
mma(-3/4) - 16*x**4*gamma(-3/4)), True))
```

3.320 $\int \frac{\sqrt{1+x^4}}{x} \, dx$

Optimal. Leaf size=28

$$\frac{\sqrt{x^4+1}}{2} - \frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 50, 63, 207}

$$\frac{\sqrt{x^4+1}}{2} - \frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^4]/x,x]
```

```
[Out] Sqrt[1 + x^4]/2 - ArcTanh[Sqrt[1 + x^4]]/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{1+x^4}}{x}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, x^4\right)$$
$$= \frac{\sqrt{1+x^4}}{2} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^4\right)$$
$$= \frac{\sqrt{1+x^4}}{2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^4}\right)$$
$$= \frac{\sqrt{1+x^4}}{2} - \frac{1}{2}\tanh^{-1}\left(\sqrt{1+x^4}\right)$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$\frac{\sqrt{x^4+1}}{2} - \frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^4]/x,x]
[Out] Sqrt[1 + x^4]/2 - ArcTanh[Sqrt[1 + x^4]]/2
```

IntegrateAlgebraic [A] time = 0.03, size = 28, normalized size = 1.00

$$\frac{\sqrt{x^4+1}}{2} - \frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^4]/x,x]
[Out] Sqrt[1 + x^4]/2 - ArcTanh[Sqrt[1 + x^4]]/2
```

fricas [A] time = 0.39, size = 34, normalized size = 1.21

$$\frac{1}{2}\sqrt{x^4+1} - \frac{1}{4}\log\left(\sqrt{x^4+1}+1\right) + \frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/x,x, algorithm="fricas")
[Out] 1/2*sqrt(x^4 + 1) - 1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

giac [A] time = 0.69, size = 34, normalized size = 1.21

$$\frac{1}{2}\sqrt{x^4+1} - \frac{1}{4}\log\left(\sqrt{x^4+1}+1\right) + \frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/x,x, algorithm="giac")
[Out] 1/2*sqrt(x^4 + 1) - 1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

maple [A] time = 0.01, size = 21, normalized size = 0.75

$$\frac{\sqrt{x^4+1}}{2} - \frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(1/2)/x,x)
```

```
[Out] 1/2*(x^4+1)^(1/2)-1/2*arctanh(1/(x^4+1)^(1/2))
```

maxima [A] time = 0.42, size = 34, normalized size = 1.21

$$\frac{1}{2}\sqrt{x^4+1}-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^4 + 1) - 1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

mupad [B] time = 0.28, size = 20, normalized size = 0.71

$$\frac{\sqrt{x^4+1}}{2}-\frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)^(1/2)/x,x)
```

```
[Out] (x^4 + 1)^(1/2)/2 - atanh((x^4 + 1)^(1/2))/2
```

sympy [A] time = 0.96, size = 39, normalized size = 1.39

$$\frac{x^2}{2\sqrt{1+\frac{1}{x^4}}}-\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2}+\frac{1}{2x^2\sqrt{1+\frac{1}{x^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(1/2)/x,x)
```

```
[Out] x**2/(2*sqrt(1 + x**(-4))) - asinh(x**(-2))/2 + 1/(2*x**2*sqrt(1 + x**(-4))
)
```

3.321 $\int \frac{\sqrt[4]{-x+x^4}}{x^8} \, dx$

Optimal. Leaf size=28

$$\frac{4 \sqrt[4]{x^4-x} \left(4 x^6+x^3-5\right)}{135 x^7}$$

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2016, 2014}

$$\frac{4 \left(x^4-x\right)^{5/4}}{27 x^8}+\frac{16 \left(x^4-x\right)^{5/4}}{135 x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^4)^(1/4)/x^8,x]
```

```
[Out] (4*(-x + x^4)^(5/4))/(27*x^8) + (16*(-x + x^4)^(5/4))/(135*x^5)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt[4]{-x+x^4}}{x^8} \, dx &= \frac{4 \left(-x+x^4\right)^{5/4}}{27 x^8}+\frac{4}{9} \int \frac{\sqrt[4]{-x+x^4}}{x^5} \, dx \\ &= \frac{4 \left(-x+x^4\right)^{5/4}}{27 x^8}+\frac{16 \left(-x+x^4\right)^{5/4}}{135 x^5} \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.89

$$\frac{4 \left(x \left(x^3-1\right)\right)^{5/4} \left(4 x^3+5\right)}{135 x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^4)^(1/4)/x^8,x]
```

```
[Out] (4*(x*(-1 + x^3))^(5/4)*(5 + 4*x^3))/(135*x^8)
```

IntegrateAlgebraic [A] time = 0.24, size = 25, normalized size = 0.89

$$\frac{4 \left(4 x^3+5\right) \left(x^4-x\right)^{5/4}}{135 x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x + x^4)^(1/4)/x^8,x]
[Out] (4*(5 + 4*x^3)*(-x + x^4)^(5/4))/(135*x^8)
```

fricas [A] time = 0.41, size = 24, normalized size = 0.86

$$\frac{4\left(4\,x^{6}+x^{3}-5\right)\left(x^{4}-x\right)^{\frac{1}{4}}}{135\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/4)/x^8,x, algorithm="fricas")
[Out] 4/135*(4*x^6 + x^3 - 5)*(x^4 - x)^(1/4)/x^7
```

giac [A] time = 0.39, size = 30, normalized size = 1.07

$$\frac{4}{27}\left(\frac{1}{x^{3}}-1\right)^{2}\left(-\frac{1}{x^{3}}+1\right)^{\frac{1}{4}}-\frac{4}{15}\left(-\frac{1}{x^{3}}+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/4)/x^8,x, algorithm="giac")
[Out] 4/27*(1/x^3 - 1)^2*(-1/x^3 + 1)^(1/4) - 4/15*(-1/x^3 + 1)^(5/4)
```

maple [A] time = 0.00, size = 31, normalized size = 1.11

$$\frac{4\left(-1+x\right)\left(x^{2}+x+1\right)\left(4x^{3}+5\right)\left(x^{4}-x\right)^{\frac{1}{4}}}{135x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x)^(1/4)/x^8,x)
[Out] 4/135*(-1+x)*(x^2+x+1)*(4*x^3+5)*(x^4-x)^(1/4)/x^7
```

maxima [A] time = 0.57, size = 30, normalized size = 1.07

$$\frac{4\left(4\,x^{7}+x^{4}-5\,x\right)\left(x^{2}+x+1\right)^{\frac{1}{4}}\left(x-1\right)^{\frac{1}{4}}}{135\,x^{\frac{31}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/4)/x^8,x, algorithm="maxima")
[Out] 4/135*(4*x^7 + x^4 - 5*x)*(x^2 + x + 1)^(1/4)*(x - 1)^(1/4)/x^(31/4)
```

mupad [B] time = 0.35, size = 24, normalized size = 0.86

$$\frac{4\left(x^{4}-x\right)^{1/4}\left(4\,x^{6}+x^{3}-5\right)}{135\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x)^(1/4)/x^8,x)
[Out] (4*(x^4 - x)^(1/4)*(x^3 + 4*x^6 - 5))/(135*x^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x\,(x-1)\left(x^2+x+1\right)}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x)**(1/4)/x**8,x)
```

```
[Out] Integral((x*(x - 1)*(x**2 + x + 1))**(1/4)/x**8, x)
```

3.322 $\int \frac{\sqrt[4]{x+x^4}}{x^8}\,dx$

Optimal. Leaf size=28

$$\frac{4\sqrt[4]{x^4+x}\left(4x^6-x^3-5\right)}{135x^7}$$

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2016, 2014}

$$\frac{16\left(x^4+x\right)^{5/4}}{135x^5}-\frac{4\left(x^4+x\right)^{5/4}}{27x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^4)^(1/4)/x^8,x]
```

```
[Out] (-4*(x + x^4)^(5/4))/(27*x^8) + (16*(x + x^4)^(5/4))/(135*x^5)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{x+x^4}}{x^8}\,dx &= -\frac{4\left(x+x^4\right)^{5/4}}{27x^8}-\frac{4}{9}\int \frac{\sqrt[4]{x+x^4}}{x^5}\,dx\\ &= -\frac{4\left(x+x^4\right)^{5/4}}{27x^8}+\frac{16\left(x+x^4\right)^{5/4}}{135x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{4\left(5-4x^3\right)\left(x^3+1\right)\sqrt[4]{x^4+x}}{135x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^4)^(1/4)/x^8,x]
```

```
[Out] (-4*(5 - 4*x^3)*(1 + x^3)*(x + x^4)^(1/4))/(135*x^7)
```

IntegrateAlgebraic [A] time = 0.23, size = 23, normalized size = 0.82

$$\frac{4\left(4x^3-5\right)\left(x^4+x\right)^{5/4}}{135x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + x^4)^(1/4)/x^8,x]
```

```
[Out] (4*(-5 + 4*x^3)*(x + x^4)^(5/4))/(135*x^8)
```

fricas [A] time = 0.39, size = 24, normalized size = 0.86

$$\frac{4\left(4 x^{6}-x^{3}-5\right)\left(x^{4}+x\right)^{\frac{1}{4}}}{135 x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/4)/x^8,x, algorithm="fricas")
```

```
[Out] 4/135*(4*x^6 - x^3 - 5)*(x^4 + x)^(1/4)/x^7
```

giac [A] time = 0.26, size = 19, normalized size = 0.68

$$-\frac{4}{27}\left(\frac{1}{x^{3}}+1\right)^{\frac{9}{4}}+\frac{4}{15}\left(\frac{1}{x^{3}}+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/4)/x^8,x, algorithm="giac")
```

```
[Out] -4/27*(1/x^3 + 1)^(9/4) + 4/15*(1/x^3 + 1)^(5/4)
```

maple [A] time = 0.00, size = 31, normalized size = 1.11

$$\frac{4\left(1+x\right)\left(x^{2}-x+1\right)\left(4x^{3}-5\right)\left(x^{4}+x\right)^{\frac{1}{4}}}{135x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x)^(1/4)/x^8,x)
```

```
[Out] 4/135*(1+x)*(x^2-x+1)*(4*x^3-5)*(x^4+x)^(1/4)/x^7
```

maxima [A] time = 0.78, size = 34, normalized size = 1.21

$$\frac{4\left(4 x^{7}-x^{4}-5 x\right)\left(x^{2}-x+1\right)^{\frac{1}{4}}\left(x+1\right)^{\frac{1}{4}}}{135 x^{\frac{31}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/4)/x^8,x, algorithm="maxima")
```

```
[Out] 4/135*(4*x^7 - x^4 - 5*x)*(x^2 - x + 1)^(1/4)*(x + 1)^(1/4)/x^(31/4)
```

mupad [B] time = 0.34, size = 22, normalized size = 0.79

$$-\frac{4\left(x^{4}+x\right)^{1/4}\left(-4 x^{6}+x^{3}+5\right)}{135 x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^4)^(1/4)/x^8,x)
```

```
[Out] -(4*(x + x^4)^(1/4)*(x^3 - 4*x^6 + 5))/(135*x^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x\left(x+1\right)\left(x^{2}-x+1\right)}}{x^{8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x)**(1/4)/x**8,x)
```

```
[Out] Integral((x*(x + 1)*(x**2 - x + 1))**(1/4)/x**8, x)
```

3.323 $\int \frac{2+x^2}{\left(-1+x^2\right)\sqrt{-1-x^2+x^4}}\,dx$

Optimal. Leaf size=28

$$-\tan^{-1}\left(\frac{\sqrt{x^4-x^2-1}}{x\left(x^2-2\right)}\right)$$

Rubi [C] time = 0.32, antiderivative size = 520, normalized size of antiderivative = 18.57, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1710, 1098, 1214, 1456, 540, 421, 419, 538, 537}

$$\frac{3\left(1+\sqrt{5}\right)\sqrt{2x^2+\sqrt{5}-1}\sqrt{1-\frac{2x^2}{1+\sqrt{5}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\sqrt{2}\left(1-\sqrt{5}\right)\sqrt{x^4-x^2-1}}-\frac{3\sqrt{-\left(\left(1-\sqrt{5}\right)x^2\right)-2}\sqrt{\frac{\left(1+\sqrt{5}\right)x^2+2}{\left(1-\sqrt{5}\right)x^2+2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}x}{\sqrt{-\left(\left(1-\sqrt{5}\right)x^2\right)-2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\left(1-\sqrt{5}\right)\sqrt{\frac{1}{\left(1-\sqrt{5}\right)x^2+2}}\sqrt{x^4-x^2-1}}+\frac{\sqrt{-\left(\left(1-\sqrt{5}\right)x^2\right)-2}\sqrt{\frac{\left(1+\sqrt{5}\right)x^2+2}{\left(1-\sqrt{5}\right)x^2+2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}x}{\sqrt{-\left(\left(1-\sqrt{5}\right)x^2\right)-2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{\frac{1}{\left(1-\sqrt{5}\right)x^2+2}}\sqrt{x^4-x^2-1}}+\frac{3\sqrt{2}\sqrt{2x^2+\sqrt{5}-1}\sqrt{1-\frac{2x^2}{1+\sqrt{5}}}\,\Pi\left(\frac{1}{2}\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\left(1-\sqrt{5}\right)\sqrt{x^4-x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(2 + x^2)/((-1 + x^2)*Sqrt[-1 - x^2 + x^4]),x]
```

```
[Out] (3*(1 + Sqrt[5])*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[5])]
*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/2])/(Sqrt[2]*(1
- Sqrt[5])*Sqrt[-1 - x^2 + x^4]) + (Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 +
(1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/
4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(2*5^(1/4)*Sqrt[(2
+ (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) - (3*Sqrt[-2 - (1 - Sqrt[5
])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[Arc
Sin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(
5^(1/4)*(1 - Sqrt[5])*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^
4]) + (3*Sqrt[2]*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[5])]
*EllipticPi[(1 + Sqrt[5])/2, ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5]
)/2])/((1 - Sqrt[5])*Sqrt[-1 - x^2 + x^4])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 540

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x
] + Dist[(b*c - a*d)/b, Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1710

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + b*x^2 + c*x^4], x],
 x] + Dist[(e*A - d*B)/e, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x],
x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{2+x^2}{\left(-1+x^2\right)\sqrt{-1-x^2+x^4}}\,dx = 3\int \frac{1}{\left(-1+x^2\right)\sqrt{-1-x^2+x^4}}\,dx + \int \frac{1}{\sqrt{-1-x^2+x^4}}\,dx$$

$$= \frac{\sqrt{-2-\left(1-\sqrt{5}\right)x^2}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x^2}{2+\left(1-\sqrt{5}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\,x}{\sqrt{-2-\left(1-\sqrt{5}\right)x^2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x^2}}\sqrt{-1-x^2+x^4}} - 3\int$$

$$= \frac{\sqrt{-2-\left(1-\sqrt{5}\right)x^2}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x^2}{2+\left(1-\sqrt{5}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\,x}{\sqrt{-2-\left(1-\sqrt{5}\right)x^2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x^2}}\sqrt{-1-x^2+x^4}} - 3\sqrt{}$$

$$= \frac{\sqrt{-2-\left(1-\sqrt{5}\right)x^2}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x^2}{2+\left(1-\sqrt{5}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\,x}{\sqrt{-2-\left(1-\sqrt{5}\right)x^2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x^2}}\sqrt{-1-x^2+x^4}} - 3\sqrt{}$$

$$= \frac{\sqrt{-2-\left(1-\sqrt{5}\right)x^2}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x^2}{2+\left(1-\sqrt{5}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\,x}{\sqrt{-2-\left(1-\sqrt{5}\right)x^2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x^2}}\sqrt{-1-x^2+x^4}} - 3\sqrt{}$$

$$= \frac{3\left(1+\sqrt{5}\right)\sqrt{-1+\sqrt{5}+2x^2}\sqrt{1-\frac{2x^2}{1+\sqrt{5}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\sqrt{2}\left(1-\sqrt{5}\right)\sqrt{-1-x^2+x^4}} +$$

Mathematica [C] time = 4.24, size = 1430, normalized size = 51.07

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + x^2)/((-1 + x^2)*Sqrt[-1 - x^2 + x^4]),x]
```

```
[Out] ((-1/2*I)*(4*Sqrt[1 + Sqrt[5]]*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*S
qrt[1 + x^2 - x^4]*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[5])]*x], (-3 + Sqr
t[5])/2] + 3*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[
5])] + (2*I)*x)^2*Sqrt[(I*(Sqrt[2*(1 + Sqrt[5])] + 2*x))/((-1 + 2*I)*Sqrt[2
] + Sqrt[10] + (2*I)*Sqrt[-1 + Sqrt[5]]*x - 2*Sqrt[1 + Sqrt[5]]*x)]*Sqrt[((
-I)*(Sqrt[2*(1 + Sqrt[5])] - 2*x))/(Sqrt[2]*((-1 - 2*I) + Sqrt[5]) + 2*(I*S
qrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)]*Sqrt[(Sqrt[2]*((-1 + 2*I) + Sqrt
[5]) + 2*((-I)*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)/(Sqrt[2]*((-1 - 2
*I) + Sqrt[5]) + 2*(I*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)]*((2 - I*S
qrt[2*(-1 + Sqrt[5])])*EllipticF[ArcSin[Sqrt[(Sqrt[2]*((-1 + 2*I) + Sqrt[5]
) + 2*((-I)*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)/(Sqrt[2]*((-1 - 2*I)
 + Sqrt[5]) + 2*(I*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)]], -3/5 + (4*
I)/5] + (2*I)*Sqrt[2*(-1 + Sqrt[5])]*EllipticPi[((-2*I + Sqrt[2*(-1 + Sqrt[
5])])*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]]))/((2*I + Sqrt[2*(-1 + Sqrt
[5])])*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])), ArcSin[Sqrt[(Sqrt[2]*((
-1 + 2*I) + Sqrt[5]) + 2*((-I)*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)/(
Sqrt[2]*((-1 - 2*I) + Sqrt[5]) + 2*(I*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]
])*x)]], -3/5 + (4*I)/5]) + 3*(I*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*(S
qrt[2*(-1 + Sqrt[5])] + (2*I)*x)^2*Sqrt[(I*(Sqrt[2*(1 + Sqrt[5])] + 2*x))/(
```

```
(-1 + 2*I)*Sqrt[2] + Sqrt[10] + (2*I)*Sqrt[-1 + Sqrt[5]]*x - 2*Sqrt[1 + Sqr
t[5]]*x)]*Sqrt[((-I)*(Sqrt[2*(1 + Sqrt[5])] - 2*x))/(Sqrt[2]*((-1 - 2*I) +
Sqrt[5]) + 2*(I*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)]*Sqrt[(Sqrt[2]*(
(-1 + 2*I) + Sqrt[5]) + 2*((-I)*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)/
(Sqrt[2]*((-1 - 2*I) + Sqrt[5]) + 2*(I*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5
]])*x)]*((-2*I + Sqrt[2*(-1 + Sqrt[5])])*EllipticF[ArcSin[Sqrt[(Sqrt[2]*((-
1 + 2*I) + Sqrt[5]) + 2*((-I)*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*x)/(S
qrt[2]*((-1 - 2*I) + Sqrt[5]) + 2*(I*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]]
)*x)]], -3/5 + (4*I)/5] - 2*Sqrt[2*(-1 + Sqrt[5])]*EllipticPi[((2*I + Sqrt[
2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]]))/((-2*I + Sqr
t[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])), ArcSin[Sq
rt[(Sqrt[2]*((-1 + 2*I) + Sqrt[5]) + 2*((-I)*Sqrt[-1 + Sqrt[5]] + Sqrt[1 +
Sqrt[5]])*x)/(Sqrt[2]*((-1 - 2*I) + Sqrt[5]) + 2*(I*Sqrt[-1 + Sqrt[5]] + Sq
rt[1 + Sqrt[5]])*x)]], -3/5 + (4*I)/5])))/(Sqrt[2]*(1 + Sqrt[5])*(Sqrt[-1 +
 Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*Sqrt[-1 - x^2 + x^4])
```

IntegrateAlgebraic [A] time = 0.95, size = 28, normalized size = 1.00

$$-\tan^{-1}\left(\frac{\sqrt{x^4-x^2-1}}{x\left(x^2-2\right)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x^2)/((-1 + x^2)*Sqrt[-1 - x^2 + x^4]),x]
```

```
[Out] -ArcTan[Sqrt[-1 - x^2 + x^4]/(x*(-2 + x^2))]
```

fricas [A] time = 0.47, size = 41, normalized size = 1.46

$$-\frac{1}{2}\arctan\left(\frac{2\sqrt{x^4-x^2-1}\left(x^3-2x\right)}{x^6-5x^4+5x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(x^2-1)/(x^4-x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(2*sqrt(x^4 - x^2 - 1)*(x^3 - 2*x)/(x^6 - 5*x^4 + 5*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2}{\sqrt{x^4-x^2-1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(x^2-1)/(x^4-x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 2)/(sqrt(x^4 - x^2 - 1)*(x^2 - 1)), x)
```

maple [C] time = 0.01, size = 174, normalized size = 6.21

$$\frac{2\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2-2\sqrt{5}}}{2},\frac{i\sqrt{5}}{2}-\frac{i}{2}\right)}{\sqrt{-2-2\sqrt{5}}\,\sqrt{x^4-x^2-1}}-\frac{3\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)x^2}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}-\frac{\sqrt{5}}{2}}\,x,\frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}},\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}{\sqrt{-\frac{1}{2}-\frac{\sqrt{5}}{2}}}\right)}{\sqrt{-\frac{1}{2}-\frac{\sqrt{5}}{2}}\,\sqrt{x^4-x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)/(x^2-1)/(x^4-x^2-1)^(1/2),x)
```

```
[Out] 2/(-2-2*5^(1/2))^(1/2)*(1-(-1/2-1/2*5^(1/2))*x^2)^(1/2)*(1-(1/2*5^(1/2)-1/2
)*x^2)^(1/2)/(x^4-x^2-1)^(1/2)*EllipticF(1/2*x*(-2-2*5^(1/2))^(1/2),1/2*I*5
```

```
^(1/2)-1/2*I)-3/(-1/2-1/2*5^(1/2))^(1/2)*(1-(-1/2-1/2*5^(1/2))*x^2)^(1/2)*(
1-(1/2*5^(1/2)-1/2)*x^2)^(1/2)/(x^4-x^2-1)^(1/2)*EllipticPi((-1/2-1/2*5^(1/
2))^(1/2)*x,1/(-1/2-1/2*5^(1/2)),(1/2*5^(1/2)-1/2)^(1/2)/(-1/2-1/2*5^(1/2))
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2}{\sqrt{x^4-x^2-1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(x^2-1)/(x^4-x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 + 2)/(sqrt(x^4 - x^2 - 1)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2+2}{\left(x^2-1\right)\sqrt{x^4-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 2)/((x^2 - 1)*(x^4 - x^2 - 1)^(1/2)),x)
[Out] int((x^2 + 2)/((x^2 - 1)*(x^4 - x^2 - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2}{(x-1)(x+1)\sqrt{x^4-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)/(x**2-1)/(x**4-x**2-1)**(1/2),x)
[Out] Integral((x**2 + 2)/((x - 1)*(x + 1)*sqrt(x**4 - x**2 - 1)), x)
```

3.324 $\int \frac{(4+x^3)(-1-x^3+x^4)}{x^6(1+x^3)^{3/4}}\,dx$

Optimal. Leaf size=28

$$-\frac{4\sqrt[4]{x^3+1}\,(5x^4-x^3-1)}{5x^5}$$

Rubi [A] time = 0.12, antiderivative size = 47, normalized size of antiderivative = 1.68, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1835, 1586, 449}

$$-\frac{4\sqrt[4]{x^3+1}}{x}+\frac{4\sqrt[4]{x^3+1}}{5x^5}+\frac{4\sqrt[4]{x^3+1}}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((4 + x^3)*(-1 - x^3 + x^4))/(x^6*(1 + x^3)^(3/4)),x]
```

```
[Out] (4*(1 + x^3)^(1/4))/(5*x^5) + (4*(1 + x^3)^(1/4))/(5*x^2) - (4*(1 + x^3)^(1
/4))/x
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\int \frac{(4+x^3)(-1-x^3+x^4)}{x^6(1+x^3)^{3/4}}\,dx = \frac{4\sqrt[4]{1+x^3}}{5x^5} - \frac{1}{10}\int \frac{16x^2-40x^3+10x^5-10x^6}{x^5(1+x^3)^{3/4}}\,dx$$
$$= \frac{4\sqrt[4]{1+x^3}}{5x^5} - \frac{1}{10}\int \frac{16x-40x^2+10x^4-10x^5}{x^4(1+x^3)^{3/4}}\,dx$$
$$= \frac{4\sqrt[4]{1+x^3}}{5x^5} - \frac{1}{10}\int \frac{16-40x+10x^3-10x^4}{x^3(1+x^3)^{3/4}}\,dx$$
$$= \frac{4\sqrt[4]{1+x^3}}{5x^5} + \frac{4\sqrt[4]{1+x^3}}{5x^2} + \frac{1}{40}\int \frac{160+40x^3}{x^2(1+x^3)^{3/4}}\,dx$$
$$= \frac{4\sqrt[4]{1+x^3}}{5x^5} + \frac{4\sqrt[4]{1+x^3}}{5x^2} - \frac{4\sqrt[4]{1+x^3}}{x}$$

Mathematica [C] time = 0.05, size = 105, normalized size = 3.75

$$-x\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-x^3\right) - \frac{4\,{}_2F_1\left(-\frac{1}{3},\frac{3}{4};\frac{2}{3};-x^3\right)}{x} + \frac{4\,{}_2F_1\left(-\frac{5}{3},\frac{3}{4};-\frac{2}{3};-x^3\right)}{5x^5} + \frac{1}{2}x^2\,{}_2F_1\left(\frac{2}{3},\frac{3}{4};\frac{5}{3};-x^3\right) + \frac{5\,{}_2F_1\left(-\frac{2}{3},\frac{3}{4};\frac{1}{3};-x^3\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((4 + x^3)*(-1 - x^3 + x^4))/(x^6*(1 + x^3)^(3/4)),x]
```

```
[Out] (4*Hypergeometric2F1[-5/3, 3/4, -2/3, -x^3])/(5*x^5) + (5*Hypergeometric2F1
[-2/3, 3/4, 1/3, -x^3])/(2*x^2) - (4*Hypergeometric2F1[-1/3, 3/4, 2/3, -x^3
])/x - x*Hypergeometric2F1[1/3, 3/4, 4/3, -x^3] + (x^2*Hypergeometric2F1[2/
3, 3/4, 5/3, -x^3])/2
```

IntegrateAlgebraic [A] time = 4.09, size = 28, normalized size = 1.00

$$-\frac{4\sqrt[4]{x^3+1}\,(5x^4-x^3-1)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((4 + x^3)*(-1 - x^3 + x^4))/(x^6*(1 + x^3)^(3/4)),x]
```

```
[Out] (-4*(1 + x^3)^(1/4)*(-1 - x^3 + 5*x^4))/(5*x^5)
```

fricas [A] time = 0.41, size = 24, normalized size = 0.86

$$-\frac{4\,(5\,x^4-x^3-1)(x^3+1)^{\frac{1}{4}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)*(x^4-x^3-1)/x^6/(x^3+1)^(3/4),x, algorithm="fricas")
```

```
[Out] -4/5*(5*x^4 - x^3 - 1)*(x^3 + 1)^(1/4)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4-x^3-1)(x^3+4)}{(x^3+1)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)*(x^4-x^3-1)/x^6/(x^3+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((x^4 - x^3 - 1)*(x^3 + 4)/((x^3 + 1)^(3/4)*x^6), x)
```

maple [A] time = 0.01, size = 36, normalized size = 1.29

$$-\frac{4\left(1+x\right)\left(x^{2}-x+1\right)\left(5x^{4}-x^{3}-1\right)}{5x^{5}\left(x^{3}+1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+4)*(x^4-x^3-1)/x^6/(x^3+1)^(3/4),x)
```

```
[Out] -4/5*(1+x)*(x^2-x+1)*(5*x^4-x^3-1)/x^5/(x^3+1)^(3/4)
```

maxima [A] time = 0.77, size = 42, normalized size = 1.50

$$-\frac{4\left(5\,x^{7}-x^{6}+5\,x^{4}-2\,x^{3}-1\right)}{5\left(x^{2}-x+1\right)^{\frac{3}{4}}\left(x+1\right)^{\frac{3}{4}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)*(x^4-x^3-1)/x^6/(x^3+1)^(3/4),x, algorithm="maxima")
```

```
[Out] -4/5*(5*x^7 - x^6 + 5*x^4 - 2*x^3 - 1)/((x^2 - x + 1)^(3/4)*(x + 1)^(3/4)*x
^5)
```

mupad [B] time = 0.24, size = 39, normalized size = 1.39

$$\frac{4\left(x^{3}+1\right)^{1/4}+4\,x^{3}\left(x^{3}+1\right)^{1/4}-20\,x^{4}\left(x^{3}+1\right)^{1/4}}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3 + 4)*(x^3 - x^4 + 1))/(x^6*(x^3 + 1)^(3/4)),x)
```

```
[Out] (4*(x^3 + 1)^(1/4) + 4*x^3*(x^3 + 1)^(1/4) - 20*x^4*(x^3 + 1)^(1/4))/(5*x^5
)
```

sympy [C] time = 4.09, size = 167, normalized size = 5.96

$$\frac{x^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{2}{3},\frac{3}{4}\\\frac{5}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{3}{4}\\\frac{4}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{4\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{3}{4}\\\frac{2}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3x\Gamma\left(\frac{2}{3}\right)}-\frac{5\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{3}{4}\\\frac{1}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3x^{2}\Gamma\left(\frac{1}{3}\right)}-\frac{4\Gamma\left(-\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{3},\frac{3}{4}\\-\frac{2}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3x^{5}\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+4)*(x**4-x**3-1)/x**6/(x**3+1)**(3/4),x)
```

```
[Out] x**2*gamma(2/3)*hyper((2/3, 3/4), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5/
3)) - x*gamma(1/3)*hyper((1/3, 3/4), (4/3,), x**3*exp_polar(I*pi))/(3*gamma
(4/3)) + 4*gamma(-1/3)*hyper((-1/3, 3/4), (2/3,), x**3*exp_polar(I*pi))/(3*
x*gamma(2/3)) - 5*gamma(-2/3)*hyper((-2/3, 3/4), (1/3,), x**3*exp_polar(I*p
i))/(3*x**2*gamma(1/3)) - 4*gamma(-5/3)*hyper((-5/3, 3/4), (-2/3,), x**3*ex
p_polar(I*pi))/(3*x**5*gamma(-2/3))
```

3.325 $\int \frac{(4+x^3)(-1-x^3+x^4)}{x^8\sqrt[4]{1+x^3}}\,dx$

Optimal. Leaf size=28

$$-\frac{4\left(x^3+1\right)^{3/4}\left(7x^4-3x^3-3\right)}{21x^7}$$

Rubi [A] time = 0.11, antiderivative size = 49, normalized size of antiderivative = 1.75, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1835, 1586, 446, 74}

$$-\frac{4\left(x^3+1\right)^{3/4}}{3x^3}+\frac{4\left(x^3+1\right)^{3/4}}{7x^7}+\frac{4\left(x^3+1\right)^{3/4}}{7x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((4 + x^3)*(-1 - x^3 + x^4))/(x^8*(1 + x^3)^(1/4)),x]
```

```
[Out] (4*(1 + x^3)^(3/4))/(7*x^7) + (4*(1 + x^3)^(3/4))/(7*x^4) - (4*(1 + x^3)^(3
/4))/(3*x^3)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(4+x^3)(-1-x^3+x^4)}{x^8\sqrt[4]{1+x^3}}\,dx &= \frac{4(1+x^3)^{3/4}}{7x^7}-\frac{1}{14}\int \frac{32x^2-56x^3+14x^5-14x^6}{x^7\sqrt[4]{1+x^3}}\,dx \\
&= \frac{4(1+x^3)^{3/4}}{7x^7}-\frac{1}{14}\int \frac{32x-56x^2+14x^4-14x^5}{x^6\sqrt[4]{1+x^3}}\,dx \\
&= \frac{4(1+x^3)^{3/4}}{7x^7}-\frac{1}{14}\int \frac{32-56x+14x^3-14x^4}{x^5\sqrt[4]{1+x^3}}\,dx \\
&= \frac{4(1+x^3)^{3/4}}{7x^7}+\frac{4(1+x^3)^{3/4}}{7x^4}+\frac{1}{112}\int \frac{448+112x^3}{x^4\sqrt[4]{1+x^3}}\,dx \\
&= \frac{4(1+x^3)^{3/4}}{7x^7}+\frac{4(1+x^3)^{3/4}}{7x^4}+\frac{1}{336}\,\text{Subst}\left(\int \frac{448+112x}{x^2\sqrt[4]{1+x}}\,dx,x,x^3\right) \\
&= \frac{4(1+x^3)^{3/4}}{7x^7}+\frac{4(1+x^3)^{3/4}}{7x^4}-\frac{4(1+x^3)^{3/4}}{3x^3}
\end{aligned}
$$

Mathematica [C] time = 0.12, size = 118, normalized size = 4.21

$$
\frac{{}_2F_1\left(-\frac{1}{3},\frac{1}{4};\frac{2}{3};-x^3\right)}{x}+\frac{16}{9}\left(x^3+1\right)^{3/4}{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};x^3+1\right)+\frac{4\,{}_2F_1\left(-\frac{7}{3},\frac{1}{4};-\frac{4}{3};-x^3\right)}{7x^7}+\frac{5\,{}_2F_1\left(-\frac{4}{3},\frac{1}{4};-\frac{1}{3};-x^3\right)}{4x^4}+\frac{2}{3}\tan^{-1}\left(\sqrt[4]{x^3+1}\right)-\frac{2}{3}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((4 + x^3)*(-1 - x^3 + x^4))/(x^8*(1 + x^3)^(1/4)),x]
```

```
[Out] (2*ArcTan[(1 + x^3)^(1/4)])/3 - (2*ArcTanh[(1 + x^3)^(1/4)])/3 + (4*Hyperge
ometric2F1[-7/3, 1/4, -4/3, -x^3])/(7*x^7) + (5*Hypergeometric2F1[-4/3, 1/4
, -1/3, -x^3])/(4*x^4) + Hypergeometric2F1[-1/3, 1/4, 2/3, -x^3]/x + (16*(1
 + x^3)^(3/4)*Hypergeometric2F1[3/4, 2, 7/4, 1 + x^3])/9
```

IntegrateAlgebraic [A] time = 0.69, size = 28, normalized size = 1.00

$$
-\frac{4\left(x^3+1\right)^{3/4}\left(7x^4-3x^3-3\right)}{21x^7}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((4 + x^3)*(-1 - x^3 + x^4))/(x^8*(1 + x^3)^(1/4)),x]
```

```
[Out] (-4*(1 + x^3)^(3/4)*(-3 - 3*x^3 + 7*x^4))/(21*x^7)
```

fricas [A] time = 0.39, size = 24, normalized size = 0.86

$$
-\frac{4\left(7\,x^4-3\,x^3-3\right)\left(x^3+1\right)^{\frac{3}{4}}}{21\,x^7}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)*(x^4-x^3-1)/x^8/(x^3+1)^(1/4),x, algorithm="fricas")
```

```
[Out] -4/21*(7*x^4 - 3*x^3 - 3)*(x^3 + 1)^(3/4)/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(x^4-x^3-1\right)\left(x^3+4\right)}{\left(x^3+1\right)^{\frac{1}{4}}x^8}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)*(x^4-x^3-1)/x^8/(x^3+1)^(1/4),x, algorithm="giac")
[Out] integrate((x^4 - x^3 - 1)*(x^3 + 4)/((x^3 + 1)^(1/4)*x^8), x)
```

maple [A] time = 0.01, size = 36, normalized size = 1.29

$$-\frac{4\left(1+x\right)\left(x^{2}-x+1\right)\left(7x^{4}-3x^{3}-3\right)}{21x^{7}\left(x^{3}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+4)*(x^4-x^3-1)/x^8/(x^3+1)^(1/4),x)
[Out] -4/21*(1+x)*(x^2-x+1)*(7*x^4-3*x^3-3)/x^7/(x^3+1)^(1/4)
```

maxima [A] time = 0.61, size = 42, normalized size = 1.50

$$-\frac{4\left(7\,x^{7}-3\,x^{6}+7\,x^{4}-6\,x^{3}-3\right)}{21\left(x^{2}-x+1\right)^{\frac{1}{4}}\left(x+1\right)^{\frac{1}{4}}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)*(x^4-x^3-1)/x^8/(x^3+1)^(1/4),x, algorithm="maxima")
[Out] -4/21*(7*x^7 - 3*x^6 + 7*x^4 - 6*x^3 - 3)/((x^2 - x + 1)^(1/4)*(x + 1)^(1/4
)*x^7)
```

mupad [B] time = 0.14, size = 39, normalized size = 1.39

$$\frac{12\left(x^{3}+1\right)^{3/4}+12\,x^{3}\left(x^{3}+1\right)^{3/4}-28\,x^{4}\left(x^{3}+1\right)^{3/4}}{21\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3 + 4)*(x^3 - x^4 + 1))/(x^8*(x^3 + 1)^(1/4)),x)
[Out] (12*(x^3 + 1)^(3/4) + 12*x^3*(x^3 + 1)^(3/4) - 28*x^4*(x^3 + 1)^(3/4))/(21*
x^7)
```

sympy [C] time = 4.80, size = 177, normalized size = 6.32

$$-\frac{\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{1}{4}\\ \frac{2}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3x\Gamma\left(\frac{2}{3}\right)}-\frac{5\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},\frac{1}{4}\\ -\frac{1}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3x^{4}\Gamma\left(-\frac{1}{3}\right)}-\frac{4\Gamma\left(-\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{3},\frac{1}{4}\\ -\frac{4}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)}-\frac{\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^{3}}\right)}{3x^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)}-\frac{4\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^{3}}\right)}{3x^{\frac{15}{4}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+4)*(x**4-x**3-1)/x**8/(x**3+1)**(1/4),x)
[Out] -gamma(-1/3)*hyper((-1/3, 1/4), (2/3,), x**3*exp_polar(I*pi))/(3*x*gamma(2/
3)) - 5*gamma(-4/3)*hyper((-4/3, 1/4), (-1/3,), x**3*exp_polar(I*pi))/(3*x*
*4*gamma(-1/3)) - 4*gamma(-7/3)*hyper((-7/3, 1/4), (-4/3,), x**3*exp_polar(
I*pi))/(3*x**7*gamma(-4/3)) - gamma(1/4)*hyper((1/4, 1/4), (5/4,), exp_pola
r(I*pi)/x**3)/(3*x**(3/4)*gamma(5/4)) - 4*gamma(5/4)*hyper((1/4, 5/4), (9/4
,), exp_polar(I*pi)/x**3)/(3*x**(15/4)*gamma(9/4))
```

3.326 $\int \frac{(-4+x^3)(1-x^3+x^4)}{x^8\sqrt[4]{-1+x^3}}\,dx$

Optimal. Leaf size=28

$$-\frac{4\left(x^3-1\right)^{3/4}\left(7x^4-3x^3+3\right)}{21x^7}$$

Rubi [A] time = 0.11, antiderivative size = 49, normalized size of antiderivative = 1.75, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1835, 1586, 446, 74}

$$-\frac{4\left(x^3-1\right)^{3/4}}{3x^3}-\frac{4\left(x^3-1\right)^{3/4}}{7x^7}+\frac{4\left(x^3-1\right)^{3/4}}{7x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((-4 + x^3)*(1 - x^3 + x^4))/(x^8*(-1 + x^3)^(1/4)),x]
```

```
[Out] (-4*(-1 + x^3)^(3/4))/(7*x^7) + (4*(-1 + x^3)^(3/4))/(7*x^4) - (4*(-1 + x^3
)^(3/4))/(3*x^3)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-4+x^3)(1-x^3+x^4)}{x^8\sqrt[4]{-1+x^3}}\,dx &= -\frac{4(-1+x^3)^{3/4}}{7x^7}+\frac{1}{14}\int \frac{32x^2-56x^3-14x^5+14x^6}{x^7\sqrt[4]{-1+x^3}}\,dx \\
&= -\frac{4(-1+x^3)^{3/4}}{7x^7}+\frac{1}{14}\int \frac{32x-56x^2-14x^4+14x^5}{x^6\sqrt[4]{-1+x^3}}\,dx \\
&= -\frac{4(-1+x^3)^{3/4}}{7x^7}+\frac{1}{14}\int \frac{32-56x-14x^3+14x^4}{x^5\sqrt[4]{-1+x^3}}\,dx \\
&= -\frac{4(-1+x^3)^{3/4}}{7x^7}+\frac{4(-1+x^3)^{3/4}}{7x^4}+\frac{1}{112}\int \frac{-448+112x^3}{x^4\sqrt[4]{-1+x^3}}\,dx \\
&= -\frac{4(-1+x^3)^{3/4}}{7x^7}+\frac{4(-1+x^3)^{3/4}}{7x^4}+\frac{1}{336}\operatorname{Subst}\left(\int \frac{-448+112x}{\sqrt[4]{-1+x}\,x^2}\,dx,x,x^3\right) \\
&= -\frac{4(-1+x^3)^{3/4}}{7x^7}+\frac{4(-1+x^3)^{3/4}}{7x^4}-\frac{4(-1+x^3)^{3/4}}{3x^3}
\end{aligned}
$$

Mathematica [C] time = 0.11, size = 145, normalized size = 5.18

$$
\frac{-315\sqrt[4]{1-x^3}\,x^3\,{}_2F_1\left(-\frac{4}{3},\frac{1}{4};-\frac{1}{3};x^3\right)+144\sqrt[4]{1-x^3}\,{}_2F_1\left(-\frac{7}{3},\frac{1}{4};-\frac{4}{3};x^3\right)+28x^6\left(9\sqrt[4]{1-x^3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{4};\frac{2}{3};x^3\right)+4x\left(x^3-1\right)\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};1-x^3\right)-4\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};1-x^3\right)\right)\right)}{252x^7\sqrt[4]{x^3-1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((-4 + x^3)*(1 - x^3 + x^4))/(x^8*(-1 + x^3)^(1/4)),x]
```

```
[Out] (144*(1 - x^3)^(1/4)*Hypergeometric2F1[-7/3, 1/4, -4/3, x^3] - 315*x^3*(1 -
 x^3)^(1/4)*Hypergeometric2F1[-4/3, 1/4, -1/3, x^3] + 28*x^6*(9*(1 - x^3)^(
1/4)*Hypergeometric2F1[-1/3, 1/4, 2/3, x^3] + 4*x*(-1 + x^3)*(Hypergeometri
c2F1[3/4, 1, 7/4, 1 - x^3] - 4*Hypergeometric2F1[3/4, 2, 7/4, 1 - x^3])))/(
252*x^7*(-1 + x^3)^(1/4))
```

IntegrateAlgebraic [A] time = 0.67, size = 28, normalized size = 1.00

$$
-\frac{4\left(x^3-1\right)^{3/4}\left(7x^4-3x^3+3\right)}{21x^7}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + x^3)*(1 - x^3 + x^4))/(x^8*(-1 + x^3)^(1/4)),x]
```

```
[Out] (-4*(-1 + x^3)^(3/4)*(3 - 3*x^3 + 7*x^4))/(21*x^7)
```

fricas [A] time = 0.45, size = 24, normalized size = 0.86

$$
-\frac{4\left(7\,x^4-3\,x^3+3\right)\left(x^3-1\right)^{\frac{3}{4}}}{21\,x^7}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^4-x^3+1)/x^8/(x^3-1)^(1/4),x, algorithm="fricas")
```

```
[Out] -4/21*(7*x^4 - 3*x^3 + 3)*(x^3 - 1)^(3/4)/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(x^4-x^3+1\right)\left(x^3-4\right)}{\left(x^3-1\right)^{\frac{1}{4}}x^8}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^4-x^3+1)/x^8/(x^3-1)^(1/4),x, algorithm="giac")
[Out] integrate((x^4 - x^3 + 1)*(x^3 - 4)/((x^3 - 1)^(1/4)*x^8), x)
```

maple [A] time = 0.00, size = 34, normalized size = 1.21

$$-\frac{4 (-1+x) \left(x^{2}+x+1\right) \left(7 x^{4}-3 x^{3}+3\right)}{21 x^{7} \left(x^{3}-1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-4)*(x^4-x^3+1)/x^8/(x^3-1)^(1/4),x)
[Out] -4/21*(-1+x)*(x^2+x+1)*(7*x^4-3*x^3+3)/x^7/(x^3-1)^(1/4)
```

maxima [A] time = 0.66, size = 40, normalized size = 1.43

$$-\frac{4 \left(7 x^{7}-3 x^{6}-7 x^{4}+6 x^{3}-3\right)}{21 \left(x^{2}+x+1\right)^{\frac{1}{4}} (x-1)^{\frac{1}{4}} x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^4-x^3+1)/x^8/(x^3-1)^(1/4),x, algorithm="maxima")
[Out] -4/21*(7*x^7 - 3*x^6 - 7*x^4 + 6*x^3 - 3)/((x^2 + x + 1)^(1/4)*(x - 1)^(1/4
)*x^7)
```

mupad [B] time = 0.26, size = 39, normalized size = 1.39

$$-\frac{12 \left(x^{3}-1\right)^{3/4}-12 x^{3} \left(x^{3}-1\right)^{3/4}+28 x^{4} \left(x^{3}-1\right)^{3/4}}{21 x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 4)*(x^4 - x^3 + 1))/(x^8*(x^3 - 1)^(1/4)),x)
[Out] -(12*(x^3 - 1)^(3/4) - 12*x^3*(x^3 - 1)^(3/4) + 28*x^4*(x^3 - 1)^(3/4))/(21
*x^7)
```

sympy [C] time = 4.96, size = 189, normalized size = 6.75

$$\frac{e^{\frac{3i\pi}{4}} \Gamma\left(-\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{1}{4} \\ \frac{2}{3} \end{matrix}\middle| {x^{3}} \right)}{3x\Gamma\left(\frac{2}{3}\right)} - \frac{5e^{\frac{3i\pi}{4}} \Gamma\left(-\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, \frac{1}{4} \\ -\frac{1}{3} \end{matrix}\middle| {x^{3}} \right)}{3x^{4}\Gamma\left(-\frac{1}{3}\right)} + \frac{4e^{\frac{3i\pi}{4}} \Gamma\left(-\frac{7}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{7}{3}, \frac{1}{4} \\ -\frac{4}{3} \end{matrix}\middle| {x^{3}} \right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)} - \frac{\Gamma\left(\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| {\frac{e^{2i\pi}}{x^{3}}} \right)}{3x^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)} + \frac{4\Gamma\left(\frac{5}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| {\frac{e^{2i\pi}}{x^{3}}} \right)}{3x^{\frac{15}{4}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-4)*(x**4-x**3+1)/x**8/(x**3-1)**(1/4),x)
[Out] exp(3*I*pi/4)*gamma(-1/3)*hyper((-1/3, 1/4), (2/3,), x**3)/(3*x*gamma(2/3))
 - 5*exp(3*I*pi/4)*gamma(-4/3)*hyper((-4/3, 1/4), (-1/3,), x**3)/(3*x**4*ga
mma(-1/3)) + 4*exp(3*I*pi/4)*gamma(-7/3)*hyper((-7/3, 1/4), (-4/3,), x**3)/
(3*x**7*gamma(-4/3)) - gamma(1/4)*hyper((1/4, 1/4), (5/4,), exp_polar(2*I*p
i)/x**3)/(3*x**(3/4)*gamma(5/4)) + 4*gamma(5/4)*hyper((1/4, 5/4), (9/4,), e
xp_polar(2*I*pi)/x**3)/(3*x**(15/4)*gamma(9/4))
```

3.327 $\int \frac{x}{\sqrt{x^3+x^4}}\,dx$

Optimal. Leaf size=28

$$\log(x)-\log\left(-2x^2+2\sqrt{x^4+x^3}-x\right)$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 0.64, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2029, 206}

$$2\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[x^3 + x^4],x]
```

```
[Out] 2*ArcTanh[x^2/Sqrt[x^3 + x^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{x^3+x^4}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^2}{\sqrt{x^3+x^4}}\right)\\ &= 2\tanh^{-1}\left(\frac{x^2}{\sqrt{x^3+x^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.11

$$\frac{2x^{3/2}\sqrt{x+1}\,\sinh^{-1}\left(\sqrt{x}\right)}{\sqrt{x^3(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[x^3 + x^4],x]
```

```
[Out] (2*x^(3/2)*Sqrt[1 + x]*ArcSinh[Sqrt[x]])/Sqrt[x^3*(1 + x)]
```

IntegrateAlgebraic [A] time = 0.14, size = 28, normalized size = 1.00

$$\log(x)-\log\left(-2x^2+2\sqrt{x^4+x^3}-x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[x^3 + x^4],x]
[Out] Log[x] - Log[-x - 2*x^2 + 2*Sqrt[x^3 + x^4]]
```

fricas [A] time = 0.41, size = 26, normalized size = 0.93

$$-\log\left(-\frac{2x^2 + x - 2\sqrt{x^4 + x^3}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^3)^(1/2),x, algorithm="fricas")
[Out] -log(-(2*x^2 + x - 2*sqrt(x^4 + x^3))/x)
```

giac [A] time = 0.37, size = 24, normalized size = 0.86

$$\log\left(\sqrt{\frac{1}{x} + 1} + 1\right) - \log\left(\left|\sqrt{\frac{1}{x} + 1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^3)^(1/2),x, algorithm="giac")
[Out] log(sqrt(1/x + 1) + 1) - log(abs(sqrt(1/x + 1) - 1))
```

maple [A] time = 0.01, size = 30, normalized size = 1.07

$$\frac{x\sqrt{x(1+x)}\ \ln\left(x + \frac{1}{2} + \sqrt{x^2 + x}\right)}{\sqrt{x^4 + x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+x^3)^(1/2),x)
[Out] 1/(x^4+x^3)^(1/2)*x*(x*(1+x))^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^4 + x^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(x^4 + x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{\sqrt{x^4 + x^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3 + x^4)^(1/2),x)
[Out] int(x/(x^3 + x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3 (x+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+x**3)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**3*(x + 1)), x)
```

3.328 $\int \frac{1}{x(1+x)\sqrt[4]{x^3+x^4}}\,dx$

Optimal. Leaf size=28

$$-\frac{4(4x+1)\left(x^4+x^3\right)^{3/4}}{3x^3(x+1)}$$

Rubi [A] time = 0.10, antiderivative size = 32, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2056, 45, 37}

$$\frac{4}{\sqrt[4]{x^4+x^3}}-\frac{16(x+1)}{3\sqrt[4]{x^4+x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x)*(x^3 + x^4)^(1/4)),x]
```

```
[Out] 4/(x^3 + x^4)^(1/4) - (16*(1 + x))/(3*(x^3 + x^4)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1+x)\sqrt[4]{x^3+x^4}}\,dx &= \frac{\left(x^{3/4}\sqrt[4]{1+x}\right)\int \frac{1}{x^{7/4}(1+x)^{5/4}}\,dx}{\sqrt[4]{x^3+x^4}}\\ &= \frac{4}{\sqrt[4]{x^3+x^4}}+\frac{\left(4x^{3/4}\sqrt[4]{1+x}\right)\int \frac{1}{x^{7/4}\sqrt[4]{1+x}}\,dx}{\sqrt[4]{x^3+x^4}}\\ &= \frac{4}{\sqrt[4]{x^3+x^4}}-\frac{16(1+x)}{3\sqrt[4]{x^3+x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.71

$$-\frac{4(4x+1)}{3\sqrt[4]{x^3(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x)*(x^3 + x^4)^(1/4)),x]
[Out] (-4*(1 + 4*x))/(3*(x^3*(1 + x))^(1/4))
```

IntegrateAlgebraic [A] time = 0.22, size = 28, normalized size = 1.00

$$-\frac{4(4x+1)\left(x^4+x^3\right)^{3/4}}{3x^3(x+1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(1 + x)*(x^3 + x^4)^(1/4)),x]
[Out] (-4*(1 + 4*x)*(x^3 + x^4)^(3/4))/(3*x^3*(1 + x))
```

fricas [A] time = 0.40, size = 16, normalized size = 0.57

$$-\frac{4\,(4\,x+1)}{3\,\left(x^4+x^3\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(x^4+x^3)^(1/4),x, algorithm="fricas")
[Out] -4/3*(4*x + 1)/(x^4 + x^3)^(1/4)
```

giac [A] time = 0.66, size = 19, normalized size = 0.68

$$-\frac{4}{3}\left(\frac{1}{x}+1\right)^{\frac{3}{4}}-\frac{4}{\left(\frac{1}{x}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(x^4+x^3)^(1/4),x, algorithm="giac")
[Out] -4/3*(1/x + 1)^(3/4) - 4/(1/x + 1)^(1/4)
```

maple [A] time = 0.00, size = 17, normalized size = 0.61

$$-\frac{4\,(1+4x)}{3\,\left(x^4+x^3\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+x)/(x^4+x^3)^(1/4),x)
[Out] -4/3*(1+4*x)/(x^4+x^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+x^3\right)^{\frac{1}{4}}(x+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(x^4+x^3)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + x^3)^(1/4)*(x + 1)*x), x)
```

mupad [B] time = 0.17, size = 16, normalized size = 0.57

$$-\frac{16\,x+4}{3\,{\left(x^4+x^3\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^3 + x^4)^(1/4)*(x + 1)),x)
[Out] -(16*x + 4)/(3*(x^3 + x^4)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[4]{x^3\,(x+1)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(x**4+x**3)**(1/4),x)
[Out] Integral(1/(x*(x**3*(x + 1))**(1/4)*(x + 1)), x)
```

3.329 $\int \frac{(4+x^3)(-1-x^3+2x^4)}{x^6(1+x^3)^{3/4}}\,dx$

Optimal. Leaf size=28

$$-\frac{4\sqrt[4]{x^3+1}\,(10x^4-x^3-1)}{5x^5}$$

Rubi [A] time = 0.12, antiderivative size = 47, normalized size of antiderivative = 1.68, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1835, 1586, 449}

$$-\frac{8\sqrt[4]{x^3+1}}{x}+\frac{4\sqrt[4]{x^3+1}}{5x^5}+\frac{4\sqrt[4]{x^3+1}}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((4 + x^3)*(-1 - x^3 + 2*x^4))/(x^6*(1 + x^3)^(3/4)),x]
```

```
[Out] (4*(1 + x^3)^(1/4))/(5*x^5) + (4*(1 + x^3)^(1/4))/(5*x^2) - (8*(1 + x^3)^(1
/4))/x
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\int \frac{(4+x^3)(-1-x^3+2x^4)}{x^6(1+x^3)^{3/4}}\,dx = \frac{4\sqrt[4]{1+x^3}}{5x^5} - \frac{1}{10}\int \frac{16x^2-80x^3+10x^5-20x^6}{x^5(1+x^3)^{3/4}}\,dx$$
$$= \frac{4\sqrt[4]{1+x^3}}{5x^5} - \frac{1}{10}\int \frac{16x-80x^2+10x^4-20x^5}{x^4(1+x^3)^{3/4}}\,dx$$
$$= \frac{4\sqrt[4]{1+x^3}}{5x^5} - \frac{1}{10}\int \frac{16-80x+10x^3-20x^4}{x^3(1+x^3)^{3/4}}\,dx$$
$$= \frac{4\sqrt[4]{1+x^3}}{5x^5} + \frac{4\sqrt[4]{1+x^3}}{5x^2} + \frac{1}{40}\int \frac{320+80x^3}{x^2(1+x^3)^{3/4}}\,dx$$
$$= \frac{4\sqrt[4]{1+x^3}}{5x^5} + \frac{4\sqrt[4]{1+x^3}}{5x^2} - \frac{8\sqrt[4]{1+x^3}}{x}$$

Mathematica [C] time = 0.04, size = 102, normalized size = 3.64

$$-x\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-x^3\right) - \frac{8\,{}_2F_1\left(-\frac{1}{3},\frac{3}{4};\frac{2}{3};-x^3\right)}{x} + \frac{4\,{}_2F_1\left(-\frac{5}{3},\frac{3}{4};-\frac{2}{3};-x^3\right)}{5x^5} + x^2\,{}_2F_1\left(\frac{2}{3},\frac{3}{4};\frac{5}{3};-x^3\right) + \frac{5\,{}_2F_1\left(-\frac{2}{3},\frac{3}{4};\frac{1}{3};-x^3\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((4 + x^3)*(-1 - x^3 + 2*x^4))/(x^6*(1 + x^3)^(3/4)),x]
```

```
[Out] (4*Hypergeometric2F1[-5/3, 3/4, -2/3, -x^3])/(5*x^5) + (5*Hypergeometric2F1
[-2/3, 3/4, 1/3, -x^3])/(2*x^2) - (8*Hypergeometric2F1[-1/3, 3/4, 2/3, -x^3
])/x - x*Hypergeometric2F1[1/3, 3/4, 4/3, -x^3] + x^2*Hypergeometric2F1[2/3
, 3/4, 5/3, -x^3]
```

IntegrateAlgebraic [A] time = 4.09, size = 28, normalized size = 1.00

$$-\frac{4\sqrt[4]{x^3+1}\,(10x^4-x^3-1)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((4 + x^3)*(-1 - x^3 + 2*x^4))/(x^6*(1 + x^3)^(3/4)),x]
```

```
[Out] (-4*(1 + x^3)^(1/4)*(-1 - x^3 + 10*x^4))/(5*x^5)
```

fricas [A] time = 0.46, size = 24, normalized size = 0.86

$$-\frac{4\,(10\,x^4-x^3-1)(x^3+1)^{\frac{1}{4}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)*(2*x^4-x^3-1)/x^6/(x^3+1)^(3/4),x, algorithm="fricas")
```

```
[Out] -4/5*(10*x^4 - x^3 - 1)*(x^3 + 1)^(1/4)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2\,x^4-x^3-1)(x^3+4)}{(x^3+1)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)*(2*x^4-x^3-1)/x^6/(x^3+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((2*x^4 - x^3 - 1)*(x^3 + 4)/((x^3 + 1)^(3/4)*x^6), x)
```

maple [A] time = 0.01, size = 36, normalized size = 1.29

$$-\frac{4\left(10x^4-x^3-1\right)(1+x)\left(x^2-x+1\right)}{5\left(x^3+1\right)^{\frac{3}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+4)*(2*x^4-x^3-1)/x^6/(x^3+1)^(3/4),x)
```

```
[Out] -4/5*(10*x^4-x^3-1)*(1+x)*(x^2-x+1)/(x^3+1)^(3/4)/x^5
```

maxima [A] time = 0.62, size = 42, normalized size = 1.50

$$-\frac{4\left(10\,x^7-x^6+10\,x^4-2\,x^3-1\right)}{5\left(x^2-x+1\right)^{\frac{3}{4}}(x+1)^{\frac{3}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)*(2*x^4-x^3-1)/x^6/(x^3+1)^(3/4),x, algorithm="maxima")
```

```
[Out] -4/5*(10*x^7 - x^6 + 10*x^4 - 2*x^3 - 1)/((x^2 - x + 1)^(3/4)*(x + 1)^(3/4)
*x^5)
```

mupad [B] time = 0.24, size = 39, normalized size = 1.39

$$\frac{4\left(x^3+1\right)^{1/4}+4\,x^3\left(x^3+1\right)^{1/4}-40\,x^4\left(x^3+1\right)^{1/4}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3 + 4)*(x^3 - 2*x^4 + 1))/(x^6*(x^3 + 1)^(3/4)),x)
```

```
[Out] (4*(x^3 + 1)^(1/4) + 4*x^3*(x^3 + 1)^(1/4) - 40*x^4*(x^3 + 1)^(1/4))/(5*x^5
)
```

sympy [C] time = 4.26, size = 168, normalized size = 6.00

$$\frac{2x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{4}\\ \frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{4}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{8\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{4}\\ \frac{2}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3x\Gamma\left(\frac{2}{3}\right)}-\frac{5\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{4}\\ \frac{1}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}-\frac{4\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},\frac{3}{4}\\ -\frac{2}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+4)*(2*x**4-x**3-1)/x**6/(x**3+1)**(3/4),x)
```

```
[Out] 2*x**2*gamma(2/3)*hyper((2/3, 3/4), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(
5/3)) - x*gamma(1/3)*hyper((1/3, 3/4), (4/3,), x**3*exp_polar(I*pi))/(3*gam
ma(4/3)) + 8*gamma(-1/3)*hyper((-1/3, 3/4), (2/3,), x**3*exp_polar(I*pi))/(
3*x*gamma(2/3)) - 5*gamma(-2/3)*hyper((-2/3, 3/4), (1/3,), x**3*exp_polar(I
*pi))/(3*x**2*gamma(1/3)) - 4*gamma(-5/3)*hyper((-5/3, 3/4), (-2/3,), x**3*
exp_polar(I*pi))/(3*x**5*gamma(-2/3))
```

3.330 $\int \frac{(-3+x^4)(1-x^3+x^4)}{x^6\sqrt[4]{x+x^5}}\,dx$

Optimal. Leaf size=28

$$\frac{4\left(3x^4-7x^3+3\right)\left(x^5+x\right)^{3/4}}{21x^6}$$

Rubi [A] time = 0.30, antiderivative size = 49, normalized size of antiderivative = 1.75, number of steps used = 16, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2052, 2025, 2032, 364}

$$\frac{4\left(x^5+x\right)^{3/4}}{7x^6}-\frac{4\left(x^5+x\right)^{3/4}}{3x^3}+\frac{4\left(x^5+x\right)^{3/4}}{7x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + x^4)*(1 - x^3 + x^4))/(x^6*(x + x^5)^(1/4)),x]
```

```
[Out] (4*(x + x^5)^(3/4))/(7*x^6) - (4*(x + x^5)^(3/4))/(3*x^3) + (4*(x + x^5)^(3
/4))/(7*x^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{(-3+x^4)(1-x^3+x^4)}{x^6\sqrt[4]{x+x^5}}\,dx = \int \left(-\frac{3}{x^6\sqrt[4]{x+x^5}}+\frac{3}{x^3\sqrt[4]{x+x^5}}-\frac{2}{x^2\sqrt[4]{x+x^5}}-\frac{x}{\sqrt[4]{x+x^5}}+\frac{x^2}{\sqrt[4]{x+x^5}}\right)dx$$

$$= -\left(2\int \frac{1}{x^2\sqrt[4]{x+x^5}}\,dx\right)-3\int \frac{1}{x^6\sqrt[4]{x+x^5}}\,dx+3\int \frac{1}{x^3\sqrt[4]{x+x^5}}\,dx-\int \frac{x}{\sqrt[4]{x+x^5}}$$

$$= \frac{4(x+x^5)^{3/4}}{7x^6}-\frac{4(x+x^5)^{3/4}}{3x^3}+\frac{8(x+x^5)^{3/4}}{5x^2}+\frac{9}{7}\int \frac{1}{x^2\sqrt[4]{x+x^5}}\,dx-\frac{14}{5}\int \frac{x^2}{\sqrt[4]{x+}}$$

$$= \frac{4(x+x^5)^{3/4}}{7x^6}-\frac{4(x+x^5)^{3/4}}{3x^3}+\frac{4(x+x^5)^{3/4}}{7x^2}-\frac{4x^2\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{4},\frac{7}{16};\frac{23}{16};-x^4\right)}{7\sqrt[4]{x+x^5}}+$$

$$= \frac{4(x+x^5)^{3/4}}{7x^6}-\frac{4(x+x^5)^{3/4}}{3x^3}+\frac{4(x+x^5)^{3/4}}{7x^2}-\frac{36x^3\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{4},\frac{11}{16};\frac{27}{16};-x^4\right)}{55\sqrt[4]{x+x^5}}$$

$$= \frac{4(x+x^5)^{3/4}}{7x^6}-\frac{4(x+x^5)^{3/4}}{3x^3}+\frac{4(x+x^5)^{3/4}}{7x^2}$$

Mathematica [C] time = 0.10, size = 123, normalized size = 4.39

$$\frac{4\sqrt[4]{x^4+1}\left(165\,{}_2F_1\left(-\frac{21}{16},\frac{1}{4};-\frac{5}{16};-x^4\right)+x^3\left(-165x^4\,{}_2F_1\left(\frac{1}{4},\frac{7}{16};\frac{23}{16};-x^4\right)+462x\,{}_2F_1\left(-\frac{5}{16},\frac{1}{4};\frac{11}{16};-x^4\right)-385\,{}_2F_1\left(-\frac{9}{16},\frac{1}{4};\frac{7}{16};-x^4\right)+105x^5\,{}_2F_1\left(\frac{1}{4},\frac{11}{16};\frac{27}{16};-x^4\right)\right)\right)}{1155x^5\sqrt[4]{x^5+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + x^4)*(1 - x^3 + x^4))/(x^6*(x + x^5)^(1/4)),x]
```

```
[Out] (4*(1 + x^4)^(1/4)*(165*Hypergeometric2F1[-21/16, 1/4, -5/16, -x^4] + x^3*(
-385*Hypergeometric2F1[-9/16, 1/4, 7/16, -x^4] + 462*x*Hypergeometric2F1[-5
/16, 1/4, 11/16, -x^4] - 165*x^4*Hypergeometric2F1[1/4, 7/16, 23/16, -x^4]
+ 105*x^5*Hypergeometric2F1[1/4, 11/16, 27/16, -x^4])))/(1155*x^5*(x + x^5)
^(1/4))
```

IntegrateAlgebraic [A] time = 2.61, size = 28, normalized size = 1.00

$$\frac{4(3x^4-7x^3+3)(x^5+x)^{3/4}}{21x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 - x^3 + x^4))/(x^6*(x + x^5)^(1/4)),x]
```

```
[Out] (4*(3 - 7*x^3 + 3*x^4)*(x + x^5)^(3/4))/(21*x^6)
```

fricas [A] time = 0.42, size = 24, normalized size = 0.86

$$\frac{4\,(x^5+x)^{\frac{3}{4}}(3\,x^4-7\,x^3+3)}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4-x^3+1)/x^6/(x^5+x)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/21*(x^5 + x)^(3/4)*(3*x^4 - 7*x^3 + 3)/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4-x^3+1)(x^4-3)}{(x^5+x)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4-x^3+1)/x^6/(x^5+x)^(1/4),x, algorithm="giac")
[Out] integrate((x^4 - x^3 + 1)*(x^4 - 3)/((x^5 + x)^(1/4)*x^6), x)
```

maple [A] time = 0.01, size = 30, normalized size = 1.07

$$\frac{4\left(x^{4}+1\right)\left(3x^{4}-7x^{3}+3\right)}{21x^{5}\left(x^{5}+x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4-x^3+1)/x^6/(x^5+x)^(1/4),x)
[Out] 4/21*(x^4+1)*(3*x^4-7*x^3+3)/x^5/(x^5+x)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}-x^{3}+1\right)\left(x^{4}-3\right)}{\left(x^{5}+x\right)^{\frac{1}{4}}x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4-x^3+1)/x^6/(x^5+x)^(1/4),x, algorithm="maxima")
[Out] integrate((x^4 - x^3 + 1)*(x^4 - 3)/((x^5 + x)^(1/4)*x^6), x)
```

mupad [B] time = 0.29, size = 39, normalized size = 1.39

$$\frac{12\left(x^{5}+x\right)^{3/4}-28\,x^{3}\left(x^{5}+x\right)^{3/4}+12\,x^{4}\left(x^{5}+x\right)^{3/4}}{21\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 3)*(x^4 - x^3 + 1))/(x^6*(x + x^5)^(1/4)),x)
[Out] (12*(x + x^5)^(3/4) - 28*x^3*(x + x^5)^(3/4) + 12*x^4*(x + x^5)^(3/4))/(21*
x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}-3\right)\left(x^{4}-x^{3}+1\right)}{x^{6}\sqrt[4]{x\left(x^{4}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4-x**3+1)/x**6/(x**5+x)**(1/4),x)
[Out] Integral((x**4 - 3)*(x**4 - x**3 + 1)/(x**6*(x*(x**4 + 1))**(1/4)), x)
```

3.331 $\int \frac{\sqrt{-1+x^6}}{x} dx$

Optimal. Leaf size=28

$$\frac{\sqrt{x^6-1}}{3}-\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 50, 63, 203}

$$\frac{\sqrt{x^6-1}}{3}-\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^6]/x,x]
```

```
[Out] Sqrt[-1 + x^6]/3 - ArcTan[Sqrt[-1 + x^6]]/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{-1+x^6}}{x}\,dx = \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, x^6\right)$$
$$= \frac{1}{3}\sqrt{-1+x^6} - \frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right)$$
$$= \frac{1}{3}\sqrt{-1+x^6} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^6}\right)$$
$$= \frac{1}{3}\sqrt{-1+x^6} - \frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right)$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$\frac{\sqrt{x^6-1}}{3} - \frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^6]/x,x]
[Out] Sqrt[-1 + x^6]/3 - ArcTan[Sqrt[-1 + x^6]]/3
```

IntegrateAlgebraic [A] time = 0.03, size = 28, normalized size = 1.00

$$\frac{\sqrt{x^6-1}}{3} - \frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x^6]/x,x]
[Out] Sqrt[-1 + x^6]/3 - ArcTan[Sqrt[-1 + x^6]]/3
```

fricas [A] time = 0.40, size = 20, normalized size = 0.71

$$\frac{1}{3}\sqrt{x^6-1} - \frac{1}{3}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x,x, algorithm="fricas")
[Out] 1/3*sqrt(x^6 - 1) - 1/3*arctan(sqrt(x^6 - 1))
```

giac [A] time = 0.36, size = 20, normalized size = 0.71

$$\frac{1}{3}\sqrt{x^6-1} - \frac{1}{3}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x,x, algorithm="giac")
[Out] 1/3*sqrt(x^6 - 1) - 1/3*arctan(sqrt(x^6 - 1))
```

maple [C] time = 0.23, size = 82, normalized size = 2.93

$$-\frac{\sqrt{\mathrm{signum}\left(x^6-1\right)}\,\left(4\sqrt{\pi} - 4\sqrt{\pi}\,\sqrt{-x^6+1} + 4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^6+1}}{2}\right) - 2\left(2-2\ln(2)+6\ln(x)+i\pi\right)\sqrt{\pi}\right)}{12\sqrt{\pi}\,\sqrt{-\mathrm{signum}\left(x^6-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/2)/x,x)
```

```
[Out] -1/12/Pi^(1/2)*signum(x^6-1)^(1/2)/(-signum(x^6-1))^(1/2)*(4*Pi^(1/2)-4*Pi^
(1/2)*(-x^6+1)^(1/2)+4*Pi^(1/2)*ln(1/2+1/2*(-x^6+1)^(1/2))-2*(2-2*ln(2)+6*l
n(x)+I*Pi)*Pi^(1/2))
```

maxima [A] time = 0.51, size = 20, normalized size = 0.71

$$\frac{1}{3}\sqrt{x^6-1}-\frac{1}{3}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^6 - 1) - 1/3*arctan(sqrt(x^6 - 1))
```

mupad [B] time = 0.29, size = 20, normalized size = 0.71

$$\frac{\sqrt{x^6-1}}{3}-\frac{\operatorname{atan}\left(\sqrt{x^6-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)^(1/2)/x,x)
```

```
[Out] (x^6 - 1)^(1/2)/3 - atan((x^6 - 1)^(1/2))/3
```

sympy [A] time = 1.03, size = 88, normalized size = 3.14

$$\begin{cases} -\frac{ix^3}{3\sqrt{-1+\frac{1}{x^6}}}-\frac{i\operatorname{acosh}\left(\frac{1}{x^3}\right)}{3}+\frac{i}{3x^3\sqrt{-1+\frac{1}{x^6}}} & \text{for}\ \frac{1}{|x^6|}>1 \\ \frac{x^3}{3\sqrt{1-\frac{1}{x^6}}}+\frac{\operatorname{asin}\left(\frac{1}{x^3}\right)}{3}-\frac{1}{3x^3\sqrt{1-\frac{1}{x^6}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/2)/x,x)
```

```
[Out] Piecewise((-I*x**3/(3*sqrt(-1 + x**(-6))) - I*acosh(x**(-3))/3 + I/(3*x**3*
sqrt(-1 + x**(-6))), 1/Abs(x**6) > 1), (x**3/(3*sqrt(1 - 1/x**6)) + asin(x*
*(-3))/3 - 1/(3*x**3*sqrt(1 - 1/x**6)), True))
```

3.332 $\int \frac{-1+x^6}{x\sqrt{1+x^6}}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{x^6+1}}{3}+\frac{1}{3}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 80, 63, 207}

$$\frac{\sqrt{x^6+1}}{3}+\frac{1}{3}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)/(x*Sqrt[1 + x^6]),x]
```

```
[Out] Sqrt[1 + x^6]/3 + ArcTanh[Sqrt[1 + x^6]]/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^6}{x\sqrt{1+x^6}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{-1+x}{x\sqrt{1+x}}\,dx,x,x^6\right)\\ &= \frac{\sqrt{1+x^6}}{3}-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^6\right)\\ &= \frac{\sqrt{1+x^6}}{3}-\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^6}\right)\\ &= \frac{\sqrt{1+x^6}}{3}+\frac{1}{3}\tanh^{-1}\left(\sqrt{1+x^6}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.86

$$\frac{1}{3}\left(\sqrt{x^6+1}+\tanh^{-1}\left(\sqrt{x^6+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)/(x*Sqrt[1 + x^6]),x]
[Out] (Sqrt[1 + x^6] + ArcTanh[Sqrt[1 + x^6]])/3
```

IntegrateAlgebraic [A] time = 0.06, size = 28, normalized size = 1.00

$$\frac{\sqrt{x^6+1}}{3}+\frac{1}{3}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^6)/(x*Sqrt[1 + x^6]),x]
[Out] Sqrt[1 + x^6]/3 + ArcTanh[Sqrt[1 + x^6]]/3
```

fricas [A] time = 0.39, size = 34, normalized size = 1.21

$$\frac{1}{3}\sqrt{x^6+1}+\frac{1}{6}\log\left(\sqrt{x^6+1}+1\right)-\frac{1}{6}\log\left(\sqrt{x^6+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/x/(x^6+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(x^6 + 1) + 1/6*log(sqrt(x^6 + 1) + 1) - 1/6*log(sqrt(x^6 + 1) - 1)
```

giac [A] time = 0.29, size = 34, normalized size = 1.21

$$\frac{1}{3}\sqrt{x^6+1}+\frac{1}{6}\log\left(\sqrt{x^6+1}+1\right)-\frac{1}{6}\log\left(\sqrt{x^6+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/x/(x^6+1)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(x^6 + 1) + 1/6*log(sqrt(x^6 + 1) + 1) - 1/6*log(sqrt(x^6 + 1) - 1)
```

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{\sqrt{x^6+1}}{3}-\frac{\ln\left(\frac{\sqrt{x^6+1}-1}{\sqrt{x^6}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)/x/(x^6+1)^(1/2),x)
```

```
[Out] 1/3*(x^6+1)^(1/2)-1/3*ln(((x^6+1)^(1/2)-1)/(x^6)^(1/2))
```

maxima [A] time = 0.55, size = 34, normalized size = 1.21

$$\frac{1}{3}\sqrt{x^6+1}+\frac{1}{6}\log\left(\sqrt{x^6+1}+1\right)-\frac{1}{6}\log\left(\sqrt{x^6+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/x/(x^6+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^6 + 1) + 1/6*log(sqrt(x^6 + 1) + 1) - 1/6*log(sqrt(x^6 + 1) - 1)
```

mupad [B] time = 0.30, size = 20, normalized size = 0.71

$$\frac{\operatorname{atanh}\left(\sqrt{x^6+1}\right)}{3}+\frac{\sqrt{x^6+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)/(x*(x^6 + 1)^(1/2)),x)
```

```
[Out] atanh((x^6 + 1)^(1/2))/3 + (x^6 + 1)^(1/2)/3
```

sympy [A] time = 28.44, size = 39, normalized size = 1.39

$$\frac{\sqrt{x^6+1}}{3}-\frac{\log\left(-1+\frac{1}{\sqrt{x^6+1}}\right)}{6}+\frac{\log\left(1+\frac{1}{\sqrt{x^6+1}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)/x/(x**6+1)**(1/2),x)
```

```
[Out] sqrt(x**6 + 1)/3 - log(-1 + 1/sqrt(x**6 + 1))/6 + log(1 + 1/sqrt(x**6 + 1))
/6
```

3.333 $\int \frac{\sqrt{1+x^6}}{x} \, dx$

Optimal. Leaf size=28

$$\frac{\sqrt{x^6+1}}{3}-\frac{1}{3}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 50, 63, 207}

$$\frac{\sqrt{x^6+1}}{3}-\frac{1}{3}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^6]/x,x]
```

```
[Out] Sqrt[1 + x^6]/3 - ArcTanh[Sqrt[1 + x^6]]/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+x^6}}{x}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, x^6\right)\\
&= \frac{\sqrt{1+x^6}}{3} + \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^6\right)\\
&= \frac{\sqrt{1+x^6}}{3} + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^6}\right)\\
&= \frac{\sqrt{1+x^6}}{3} - \frac{1}{3}\tanh^{-1}\left(\sqrt{1+x^6}\right)
\end{aligned}
$$

Mathematica [**A**] time = 0.00, size = 28, normalized size = 1.00

$$\frac{\sqrt{x^6+1}}{3} - \frac{1}{3}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^6]/x,x]
[Out] Sqrt[1 + x^6]/3 - ArcTanh[Sqrt[1 + x^6]]/3
```

IntegrateAlgebraic [**A**] time = 0.02, size = 28, normalized size = 1.00

$$\frac{\sqrt{x^6+1}}{3} - \frac{1}{3}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^6]/x,x]
[Out] Sqrt[1 + x^6]/3 - ArcTanh[Sqrt[1 + x^6]]/3
```

fricas [**A**] time = 0.39, size = 34, normalized size = 1.21

$$\frac{1}{3}\sqrt{x^6+1} - \frac{1}{6}\log\left(\sqrt{x^6+1}+1\right) + \frac{1}{6}\log\left(\sqrt{x^6+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/2)/x,x, algorithm="fricas")
[Out] 1/3*sqrt(x^6 + 1) - 1/6*log(sqrt(x^6 + 1) + 1) + 1/6*log(sqrt(x^6 + 1) - 1)
```

giac [**A**] time = 0.44, size = 34, normalized size = 1.21

$$\frac{1}{3}\sqrt{x^6+1} - \frac{1}{6}\log\left(\sqrt{x^6+1}+1\right) + \frac{1}{6}\log\left(\sqrt{x^6+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/2)/x,x, algorithm="giac")
[Out] 1/3*sqrt(x^6 + 1) - 1/6*log(sqrt(x^6 + 1) + 1) + 1/6*log(sqrt(x^6 + 1) - 1)
```

maple [**B**] time = 0.20, size = 56, normalized size = 2.00

$$-\frac{4\sqrt{\pi} - 4\sqrt{\pi}\,\sqrt{x^6+1} + 4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^6+1}}{2}\right) - 2\left(2-2\ln(2)+6\ln(x)\right)\sqrt{\pi}}{12\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)^(1/2)/x,x)
```

```
[Out] -1/12/Pi^(1/2)*(4*Pi^(1/2)-4*Pi^(1/2)*(x^6+1)^(1/2)+4*Pi^(1/2)*ln(1/2+1/2*(
x^6+1)^(1/2))-2*(2-2*ln(2)+6*ln(x))*Pi^(1/2))
```

maxima [A] time = 0.37, size = 34, normalized size = 1.21

$$\frac{1}{3}\sqrt{x^6+1}-\frac{1}{6}\log\left(\sqrt{x^6+1}+1\right)+\frac{1}{6}\log\left(\sqrt{x^6+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^6 + 1) - 1/6*log(sqrt(x^6 + 1) + 1) + 1/6*log(sqrt(x^6 + 1) - 1)
```

mupad [B] time = 0.27, size = 20, normalized size = 0.71

$$\frac{\sqrt{x^6+1}}{3}-\frac{\operatorname{atanh}\left(\sqrt{x^6+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)^(1/2)/x,x)
```

```
[Out] (x^6 + 1)^(1/2)/3 - atanh((x^6 + 1)^(1/2))/3
```

sympy [A] time = 0.97, size = 39, normalized size = 1.39

$$\frac{x^3}{3\sqrt{1+\frac{1}{x^6}}}-\frac{\operatorname{asinh}\left(\frac{1}{x^3}\right)}{3}+\frac{1}{3x^3\sqrt{1+\frac{1}{x^6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)**(1/2)/x,x)
```

```
[Out] x**3/(3*sqrt(1 + x**(-6))) - asinh(x**(-3))/3 + 1/(3*x**3*sqrt(1 + x**(-6))
)
```

3.334 $\int \frac{\sqrt[3]{-1+x^6}\left(1+x^6\right)}{x^{15}} dx$

Optimal. Leaf size=28

$$\frac{\sqrt[3]{x^6-1}\left(5x^{12}-3x^6-2\right)}{28x^{14}}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {453, 264}

$$\frac{\left(x^6-1\right)^{4/3}}{14x^{14}}+\frac{5\left(x^6-1\right)^{4/3}}{28x^8}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^6)^(1/3)*(1 + x^6))/x^15,x]
```

```
[Out] (-1 + x^6)^(4/3)/(14*x^14) + (5*(-1 + x^6)^(4/3))/(28*x^8)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{-1+x^6}\left(1+x^6\right)}{x^{15}} dx &= \frac{\left(-1+x^6\right)^{4/3}}{14x^{14}}+\frac{10}{7}\int \frac{\sqrt[3]{-1+x^6}}{x^9} dx\\ &= \frac{\left(-1+x^6\right)^{4/3}}{14x^{14}}+\frac{5\left(-1+x^6\right)^{4/3}}{28x^8}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.82

$$\frac{\left(x^6-1\right)^{4/3}\left(5x^6+2\right)}{28x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^6)^(1/3)*(1 + x^6))/x^15,x]
```

```
[Out] ((-1 + x^6)^(4/3)*(2 + 5*x^6))/(28*x^14)
```

IntegrateAlgebraic [A] time = 0.61, size = 23, normalized size = 0.82

$$\frac{\left(x^6-1\right)^{4/3}\left(5x^6+2\right)}{28x^{14}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^6)^(1/3)*(1 + x^6))/x^15,x]
```

```
[Out] ((-1 + x^6)^(4/3)*(2 + 5*x^6))/(28*x^14)
```

fricas [A] time = 0.39, size = 24, normalized size = 0.86

$$\frac{\left(5\,x^{12}-3\,x^{6}-2\right)\left(x^{6}-1\right)^{\frac{1}{3}}}{28\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(x^6+1)/x^15,x, algorithm="fricas")
```

```
[Out] 1/28*(5*x^12 - 3*x^6 - 2)*(x^6 - 1)^(1/3)/x^14
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}+1\right)\left(x^{6}-1\right)^{\frac{1}{3}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(x^6+1)/x^15,x, algorithm="giac")
```

```
[Out] integrate((x^6 + 1)*(x^6 - 1)^(1/3)/x^15, x)
```

maple [A] time = 0.01, size = 40, normalized size = 1.43

$$\frac{\left(x^{6}-1\right)^{\frac{1}{3}}\left(5x^{6}+2\right)\left(-1+x\right)\left(1+x\right)\left(x^{2}+x+1\right)\left(x^{2}-x+1\right)}{28x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/3)*(x^6+1)/x^15,x)
```

```
[Out] 1/28*(x^6-1)^(1/3)*(5*x^6+2)*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)/x^14
```

maxima [A] time = 0.44, size = 25, normalized size = 0.89

$$\frac{\left(x^{6}-1\right)^{\frac{4}{3}}}{4\,x^{8}}-\frac{\left(x^{6}-1\right)^{\frac{7}{3}}}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(x^6+1)/x^15,x, algorithm="maxima")
```

```
[Out] 1/4*(x^6 - 1)^(4/3)/x^8 - 1/14*(x^6 - 1)^(7/3)/x^14
```

mupad [B] time = 0.34, size = 24, normalized size = 0.86

$$\frac{7\left(x^{6}-1\right)^{4/3}+5\left(x^{6}-1\right)^{7/3}}{28\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/3)*(x^6 + 1))/x^15,x)
```

```
[Out] (7*(x^6 - 1)^(4/3) + 5*(x^6 - 1)^(7/3))/(28*x^14)
```

sympy [C] time = 4.28, size = 416, normalized size = 14.86

$$\begin{cases} \frac{\sqrt[3]{-1+\frac{1}{x^6}}e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{6\Gamma\left(-\frac{1}{3}\right)} - \frac{\sqrt[3]{-1+\frac{1}{x^6}}e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{6x^6\Gamma\left(-\frac{1}{3}\right)} & \text{for } \frac{1}{|x^6|} > 1 \\ -\frac{\sqrt[3]{1-\frac{1}{x^6}}\Gamma\left(-\frac{4}{3}\right)}{6\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt[3]{1-\frac{1}{x^6}}\Gamma\left(-\frac{4}{3}\right)}{6x^6\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise} \end{cases} + \begin{cases} \frac{\sqrt[3]{-1+\frac{1}{x^6}}e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{6\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt[3]{-1+\frac{1}{x^6}}e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{18x^6\Gamma\left(-\frac{1}{3}\right)} - \frac{2\sqrt[3]{-1+\frac{1}{x^6}}e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{9x^{12}\Gamma\left(-\frac{1}{3}\right)} & \text{for } \frac{1}{|x^6|} > 1 \\ \frac{3x^{12}\sqrt[3]{1-\frac{1}{x^6}}\Gamma\left(-\frac{7}{3}\right)}{18x^{12}\Gamma\left(-\frac{1}{3}\right)-18x^6\Gamma\left(-\frac{1}{3}\right)} - \frac{2x^6\sqrt[3]{1-\frac{1}{x^6}}\Gamma\left(-\frac{7}{3}\right)}{18x^{12}\Gamma\left(-\frac{1}{3}\right)-18x^6\Gamma\left(-\frac{1}{3}\right)} + \frac{4\sqrt[3]{1-\frac{1}{x^6}}\Gamma\left(-\frac{7}{3}\right)}{18x^{18}\Gamma\left(-\frac{1}{3}\right)-18x^{12}\Gamma\left(-\frac{1}{3}\right)} - \frac{5\sqrt[3]{1-\frac{1}{x^6}}\Gamma\left(-\frac{7}{3}\right)}{18x^{12}\Gamma\left(-\frac{1}{3}\right)-18x^6\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/3)*(x**6+1)/x**15,x)
```

```
[Out] Piecewise(((-1 + x**(-6))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(6*gamma(-1/3))
 - (-1 + x**(-6))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(6*x**6*gamma(-1/3)), 1
/Abs(x**6) > 1), (-(1 - 1/x**6)**(1/3)*gamma(-4/3)/(6*gamma(-1/3)) + (1 - 1
/x**6)**(1/3)*gamma(-4/3)/(6*x**6*gamma(-1/3)), True)) + Piecewise(((-1 + x
**(-6))**(1/3)*exp(I*pi/3)*gamma(-7/3)/(6*gamma(-1/3)) + (-1 + x**(-6))**(1
/3)*exp(I*pi/3)*gamma(-7/3)/(18*x**6*gamma(-1/3)) - 2*(-1 + x**(-6))**(1/3)
*exp(I*pi/3)*gamma(-7/3)/(9*x**12*gamma(-1/3)), 1/Abs(x**6) > 1), (3*x**12*
(1 - 1/x**6)**(1/3)*gamma(-7/3)/(18*x**12*gamma(-1/3) - 18*x**6*gamma(-1/3)
) - 2*x**6*(1 - 1/x**6)**(1/3)*gamma(-7/3)/(18*x**12*gamma(-1/3) - 18*x**6*
gamma(-1/3)) + 4*(1 - 1/x**6)**(1/3)*gamma(-7/3)/(18*x**18*gamma(-1/3) - 18
*x**12*gamma(-1/3)) - 5*(1 - 1/x**6)**(1/3)*gamma(-7/3)/(18*x**12*gamma(-1/
3) - 18*x**6*gamma(-1/3)), True))
```

3.335 $\int \frac{(1-x^3+x^5)(-3+2x^5)}{x^6\sqrt[4]{x+x^6}}\,dx$

Optimal. Leaf size=28

$$\frac{4\left(3x^5-7x^3+3\right)\left(x^6+x\right)^{3/4}}{21x^6}$$

Rubi [A] time = 0.32, antiderivative size = 49, normalized size of antiderivative = 1.75, number of steps used = 16, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2052, 2025, 2032, 364}

$$\frac{4\left(x^6+x\right)^{3/4}}{7x}+\frac{4\left(x^6+x\right)^{3/4}}{7x^6}-\frac{4\left(x^6+x\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x^3 + x^5)*(-3 + 2*x^5))/(x^6*(x + x^6)^(1/4)),x]
```

```
[Out] (4*(x + x^6)^(3/4))/(7*x^6) - (4*(x + x^6)^(3/4))/(3*x^3) + (4*(x + x^6)^(3
/4))/(7*x)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\left(1-x^3+x^5\right)\left(-3+2x^5\right)}{x^6\sqrt[4]{x+x^6}}\,dx = \int \left(-\frac{3}{x^6\sqrt[4]{x+x^6}}+\frac{3}{x^3\sqrt[4]{x+x^6}}-\frac{1}{x\sqrt[4]{x+x^6}}-\frac{2x^2}{\sqrt[4]{x+x^6}}+\frac{2x^4}{\sqrt[4]{x+x^6}}\right)dx$$

$$= -\left(2\int \frac{x^2}{\sqrt[4]{x+x^6}}\,dx\right)+2\int \frac{x^4}{\sqrt[4]{x+x^6}}\,dx-3\int \frac{1}{x^6\sqrt[4]{x+x^6}}\,dx+3\int \frac{1}{x^3\sqrt[4]{x-}}$$

$$= \frac{4\left(x+x^6\right)^{3/4}}{7x^6}-\frac{4\left(x+x^6\right)^{3/4}}{3x^3}+\frac{4\left(x+x^6\right)^{3/4}}{x}+\frac{6}{7}\int \frac{1}{x\sqrt[4]{x+x^6}}\,dx+2\int \frac{}{\sqrt[4]{x}}$$

$$= \frac{4\left(x+x^6\right)^{3/4}}{7x^6}-\frac{4\left(x+x^6\right)^{3/4}}{3x^3}+\frac{4\left(x+x^6\right)^{3/4}}{7x}-\frac{8x^3\sqrt[4]{1+x^5}\,{}_2F_1\left(\frac{1}{4},\frac{11}{20};\frac{31}{20};-x\right.}{11\sqrt[4]{x+x^6}}$$

$$= \frac{4\left(x+x^6\right)^{3/4}}{7x^6}-\frac{4\left(x+x^6\right)^{3/4}}{3x^3}+\frac{4\left(x+x^6\right)^{3/4}}{7x}-\frac{48x^5\sqrt[4]{1+x^5}\,{}_2F_1\left(\frac{1}{4},\frac{19}{20};\frac{39}{20};-\right.}{19\sqrt[4]{x+x^6}}$$

$$= \frac{4\left(x+x^6\right)^{3/4}}{7x^6}-\frac{4\left(x+x^6\right)^{3/4}}{3x^3}+\frac{4\left(x+x^6\right)^{3/4}}{7x}$$

Mathematica [C] time = 0.10, size = 126, normalized size = 4.50

$$\frac{4\sqrt[4]{x^5+1}\left(627\,{}_2F_1\left(-\frac{21}{20},\frac{1}{4};-\frac{1}{20};-x^5\right)+7x^3\left(-114x^5\,{}_2F_1\left(\frac{1}{4},\frac{11}{20};\frac{31}{20};-x^5\right)-209\,{}_2F_1\left(-\frac{9}{20},\frac{1}{4};\frac{11}{20};-x^5\right)+66x^7\,{}_2F_1\left(\frac{1}{4},\frac{19}{20};\frac{39}{20};-x^5\right)+627x^2\,{}_2F_1\left(-\frac{1}{20},\frac{1}{4};\frac{19}{20};-x^5\right)\right)\right)}{4389x^5\sqrt[4]{x^6+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x^3 + x^5)*(-3 + 2*x^5))/(x^6*(x + x^6)^(1/4)),x]
```

```
[Out] (4*(1 + x^5)^(1/4)*(627*Hypergeometric2F1[-21/20, 1/4, -1/20, -x^5] + 7*x^3
*(-209*Hypergeometric2F1[-9/20, 1/4, 11/20, -x^5] + 627*x^2*Hypergeometric2
F1[-1/20, 1/4, 19/20, -x^5] - 114*x^5*Hypergeometric2F1[1/4, 11/20, 31/20,
-x^5] + 66*x^7*Hypergeometric2F1[1/4, 19/20, 39/20, -x^5])))/(4389*x^5*(x +
 x^6)^(1/4))
```

IntegrateAlgebraic [A] time = 2.64, size = 28, normalized size = 1.00

$$\frac{4\left(3x^5-7x^3+3\right)\left(x^6+x\right)^{3/4}}{21x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 - x^3 + x^5)*(-3 + 2*x^5))/(x^6*(x + x^6)^(1/4)),x]
```

```
[Out] (4*(3 - 7*x^3 + 3*x^5)*(x + x^6)^(3/4))/(21*x^6)
```

fricas [A] time = 0.41, size = 24, normalized size = 0.86

$$\frac{4\left(x^6+x\right)^{\frac{3}{4}}\left(3\,x^5-7\,x^3+3\right)}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^3+1)*(2*x^5-3)/x^6/(x^6+x)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/21*(x^6 + x)^(3/4)*(3*x^5 - 7*x^3 + 3)/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^5-3\right)\left(x^5-x^3+1\right)}{\left(x^6+x\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^3+1)*(2*x^5-3)/x^6/(x^6+x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((2*x^5 - 3)*(x^5 - x^3 + 1)/((x^6 + x)^(1/4)*x^6), x)
```

maple [A] time = 0.01, size = 44, normalized size = 1.57

$$\frac{4\left(x^{4}-x^{3}+x^{2}-x+1\right)(1+x)\left(3x^{5}-7x^{3}+3\right)}{21x^{5}\left(x^{6}+x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-x^3+1)*(2*x^5-3)/x^6/(x^6+x)^(1/4),x)
```

```
[Out] 4/21*(x^4-x^3+x^2-x+1)*(1+x)*(3*x^5-7*x^3+3)/x^5/(x^6+x)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{5}-3\right)\left(x^{5}-x^{3}+1\right)}{\left(x^{6}+x\right)^{\frac{1}{4}}x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^3+1)*(2*x^5-3)/x^6/(x^6+x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((2*x^5 - 3)*(x^5 - x^3 + 1)/((x^6 + x)^(1/4)*x^6), x)
```

mupad [B] time = 0.32, size = 39, normalized size = 1.39

$$\frac{12\left(x^{6}+x\right)^{3/4}-28\,x^{3}\left(x^{6}+x\right)^{3/4}+12\,x^{5}\left(x^{6}+x\right)^{3/4}}{21\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^5 - 3)*(x^5 - x^3 + 1))/(x^6*(x + x^6)^(1/4)),x)
```

```
[Out] (12*(x + x^6)^(3/4) - 28*x^3*(x + x^6)^(3/4) + 12*x^5*(x + x^6)^(3/4))/(21*
x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{5}-3\right)\left(x^{5}-x^{3}+1\right)}{x^{6}\sqrt[4]{x\left(x+1\right)\left(x^{4}-x^{3}+x^{2}-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-x**3+1)*(2*x**5-3)/x**6/(x**6+x)**(1/4),x)
```

```
[Out] Integral((2*x**5 - 3)*(x**5 - x**3 + 1)/(x**6*(x*(x + 1)*(x**4 - x**3 + x**
2 - x + 1))**(1/4)), x)
```

3.336 $\int \frac{(-2+x^6)(1-x^4+x^6)}{x^8 \sqrt[4]{1+x^6}} dx$

Optimal. Leaf size=28

$$\frac{2\left(x^6+1\right)^{3/4}\left(3x^6-7x^4+3\right)}{21x^7}$$

Rubi [A] time = 0.08, antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1833, 1584, 449, 1478}

$$\frac{2\left(x^6+1\right)^{7/4}}{7x^7}-\frac{2\left(x^6+1\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-2 + x^6)*(1 - x^4 + x^6))/(x^8*(1 + x^6)^(1/4)),x]
```

```
[Out] (-2*(1 + x^6)^(3/4))/(3*x^3) + (2*(1 + x^6)^(7/4))/(7*x^7)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1478

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n
_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p
]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{\left(-2+x^6\right)\left(1-x^4+x^6\right)}{x^8\sqrt[4]{1+x^6}}\,dx=\int \left(\frac{2x^3-x^9}{x^7\sqrt[4]{1+x^6}}+\frac{-2-x^6+x^{12}}{x^8\sqrt[4]{1+x^6}}\right)dx$$
$$=\int \frac{2x^3-x^9}{x^7\sqrt[4]{1+x^6}}\,dx+\int \frac{-2-x^6+x^{12}}{x^8\sqrt[4]{1+x^6}}\,dx$$
$$=\int \frac{2-x^6}{x^4\sqrt[4]{1+x^6}}\,dx+\int \frac{\left(-2+x^6\right)\left(1+x^6\right)^{3/4}}{x^8}\,dx$$
$$=-\frac{2\left(1+x^6\right)^{3/4}}{3x^3}+\frac{2\left(1+x^6\right)^{7/4}}{7x^7}$$

Mathematica [C] time = 0.05, size = 108, normalized size = 3.86

$$\frac{{}_2F_1\left(-\frac{1}{6},\frac{1}{4};\frac{5}{6};-x^6\right)}{x}+\frac{2\,{}_2F_1\left(-\frac{7}{6},\frac{1}{4};-\frac{1}{6};-x^6\right)}{7x^7}+\frac{1}{5}x^5\,{}_2F_1\left(\frac{1}{4},\frac{5}{6};\frac{11}{6};-x^6\right)-\frac{1}{3}x^3\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-x^6\right)-\frac{2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};-x^6\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-2 + x^6)*(1 - x^4 + x^6))/(x^8*(1 + x^6)^(1/4)),x]
```

```
[Out] (2*Hypergeometric2F1[-7/6, 1/4, -1/6, -x^6])/(7*x^7) - (2*Hypergeometric2F1
[-1/2, 1/4, 1/2, -x^6])/(3*x^3) + Hypergeometric2F1[-1/6, 1/4, 5/6, -x^6]/x
 - (x^3*Hypergeometric2F1[1/4, 1/2, 3/2, -x^6])/3 + (x^5*Hypergeometric2F1[
1/4, 5/6, 11/6, -x^6])/5
```

IntegrateAlgebraic [A] time = 2.29, size = 28, normalized size = 1.00

$$\frac{2\left(x^6+1\right)^{3/4}\left(3x^6-7x^4+3\right)}{21x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^6)*(1 - x^4 + x^6))/(x^8*(1 + x^6)^(1/4)),x]
```

```
[Out] (2*(1 + x^6)^(3/4)*(3 - 7*x^4 + 3*x^6))/(21*x^7)
```

fricas [A] time = 0.40, size = 24, normalized size = 0.86

$$\frac{2\left(3\,x^6-7\,x^4+3\right)\left(x^6+1\right)^{\frac{3}{4}}}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-x^4+1)/x^8/(x^6+1)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/21*(3*x^6 - 7*x^4 + 3)*(x^6 + 1)^(3/4)/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6-x^4+1\right)\left(x^6-2\right)}{\left(x^6+1\right)^{\frac{1}{4}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-x^4+1)/x^8/(x^6+1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^6 - x^4 + 1)*(x^6 - 2)/((x^6 + 1)^(1/4)*x^8), x)
```

maple [A] time = 0.01, size = 40, normalized size = 1.43

$$\frac{2\left(x^{2}+1\right)\left(x^{4}-x^{2}+1\right)\left(3x^{6}-7x^{4}+3\right)}{21x^{7}\left(x^{6}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-2)*(x^6-x^4+1)/x^8/(x^6+1)^(1/4),x)
```

```
[Out] 2/21*(x^2+1)*(x^4-x^2+1)*(3*x^6-7*x^4+3)/x^7/(x^6+1)^(1/4)
```

maxima [A] time = 0.67, size = 46, normalized size = 1.64

$$\frac{2\left(3\,x^{12}-7\,x^{10}+6\,x^{6}-7\,x^{4}+3\right)}{21\left(x^{4}-x^{2}+1\right)^{\frac{1}{4}}\left(x^{2}+1\right)^{\frac{1}{4}}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-x^4+1)/x^8/(x^6+1)^(1/4),x, algorithm="maxima")
```

```
[Out] 2/21*(3*x^12 - 7*x^10 + 6*x^6 - 7*x^4 + 3)/((x^4 - x^2 + 1)^(1/4)*(x^2 + 1)
^(1/4)*x^7)
```

mupad [B] time = 0.18, size = 39, normalized size = 1.39

$$\frac{6\left(x^{6}+1\right)^{3/4}-14\,x^{4}\left(x^{6}+1\right)^{3/4}+6\,x^{6}\left(x^{6}+1\right)^{3/4}}{21\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 2)*(x^6 - x^4 + 1))/(x^8*(x^6 + 1)^(1/4)),x)
```

```
[Out] (6*(x^6 + 1)^(3/4) - 14*x^4*(x^6 + 1)^(3/4) + 6*x^6*(x^6 + 1)^(3/4))/(21*x^
7)
```

sympy [C] time = 4.61, size = 143, normalized size = 5.11

$$\frac{x^{5}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|x^{6}e^{i\pi}\right)}{6\Gamma\left(\frac{11}{6}\right)}-\frac{x^{3}{}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|x^{6}e^{i\pi}\right)}{3}-\frac{\Gamma\left(-\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{6},\frac{1}{4}\\ \frac{5}{6}\end{matrix}\middle|x^{6}e^{i\pi}\right)}{6x\Gamma\left(\frac{5}{6}\right)}-\frac{2{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|x^{6}e^{i\pi}\right)}{3x^{3}}-\frac{\Gamma\left(-\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{6},\frac{1}{4}\\ -\frac{1}{6}\end{matrix}\middle|x^{6}e^{i\pi}\right)}{3x^{7}\Gamma\left(-\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-2)*(x**6-x**4+1)/x**8/(x**6+1)**(1/4),x)
```

```
[Out] x**5*gamma(5/6)*hyper((1/4, 5/6), (11/6,), x**6*exp_polar(I*pi))/(6*gamma(1
1/6)) - x**3*hyper((1/4, 1/2), (3/2,), x**6*exp_polar(I*pi))/3 - gamma(-1/6
)*hyper((-1/6, 1/4), (5/6,), x**6*exp_polar(I*pi))/(6*x*gamma(5/6)) - 2*hyp
er((-1/2, 1/4), (1/2,), x**6*exp_polar(I*pi))/(3*x**3) - gamma(-7/6)*hyper(
(-7/6, 1/4), (-1/6,), x**6*exp_polar(I*pi))/(3*x**7*gamma(-1/6))
```

3.337 $\int \frac{\sqrt[3]{-1+x^6}\left(-1+2x^6\right)}{x^{15}}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt[3]{x^6-1}\left(11x^{12}-15x^6+4\right)}{56x^{14}}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {453, 264}

$$\frac{11\left(x^6-1\right)^{4/3}}{56x^8}-\frac{\left(x^6-1\right)^{4/3}}{14x^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^6)^(1/3)*(-1 + 2*x^6))/x^15,x]
```

```
[Out] -1/14*(-1 + x^6)^(4/3)/x^14 + (11*(-1 + x^6)^(4/3))/(56*x^8)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{-1+x^6}\left(-1+2x^6\right)}{x^{15}}\,dx &= -\frac{\left(-1+x^6\right)^{4/3}}{14x^{14}}+\frac{11}{7}\int \frac{\sqrt[3]{-1+x^6}}{x^9}\,dx\\ &= -\frac{\left(-1+x^6\right)^{4/3}}{14x^{14}}+\frac{11\left(-1+x^6\right)^{4/3}}{56x^8}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.82

$$\frac{\left(x^6-1\right)^{4/3}\left(11x^6-4\right)}{56x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^6)^(1/3)*(-1 + 2*x^6))/x^15,x]
```

```
[Out] ((-1 + x^6)^(4/3)*(-4 + 11*x^6))/(56*x^14)
```

IntegrateAlgebraic [A] time = 1.43, size = 23, normalized size = 0.82

$$\frac{\left(x^6-1\right)^{4/3}\left(11x^6-4\right)}{56x^{14}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^6)^(1/3)*(-1 + 2*x^6))/x^15,x]
[Out] ((-1 + x^6)^(4/3)*(-4 + 11*x^6))/(56*x^14)
```

fricas [A] time = 0.41, size = 24, normalized size = 0.86

$$\frac{\left(11\,x^{12}-15\,x^{6}+4\right)\left(x^{6}-1\right)^{\frac{1}{3}}}{56\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(2*x^6-1)/x^15,x, algorithm="fricas")
[Out] 1/56*(11*x^12 - 15*x^6 + 4)*(x^6 - 1)^(1/3)/x^14
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{6}-1\right)\left(x^{6}-1\right)^{\frac{1}{3}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(2*x^6-1)/x^15,x, algorithm="giac")
[Out] integrate((2*x^6 - 1)*(x^6 - 1)^(1/3)/x^15, x)
```

maple [A] time = 0.01, size = 40, normalized size = 1.43

$$\frac{(-1+x)\,(1+x)\left(x^{2}+x+1\right)\left(x^{2}-x+1\right)\left(11x^{6}-4\right)\left(x^{6}-1\right)^{\frac{1}{3}}}{56x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/3)*(2*x^6-1)/x^15,x)
[Out] 1/56*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)*(11*x^6-4)*(x^6-1)^(1/3)/x^14
```

maxima [A] time = 0.56, size = 25, normalized size = 0.89

$$\frac{\left(x^{6}-1\right)^{\frac{4}{3}}}{8\,x^{8}}+\frac{\left(x^{6}-1\right)^{\frac{7}{3}}}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(2*x^6-1)/x^15,x, algorithm="maxima")
[Out] 1/8*(x^6 - 1)^(4/3)/x^8 + 1/14*(x^6 - 1)^(7/3)/x^14
```

mupad [B] time = 0.36, size = 24, normalized size = 0.86

$$\frac{7\left(x^{6}-1\right)^{4/3}+11\left(x^{6}-1\right)^{7/3}}{56\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/3)*(2*x^6 - 1))/x^15,x)
[Out] (7*(x^6 - 1)^(4/3) + 11*(x^6 - 1)^(7/3))/(56*x^14)
```

sympy [C] time = 4.71, size = 418, normalized size = 14.93

$$2\left(\begin{cases} \frac{\sqrt[3]{-1+\frac{1}{x^6}}e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{6\Gamma\left(-\frac{1}{3}\right)} - \frac{\sqrt[3]{-1+\frac{1}{x^6}}e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{6x^6\Gamma\left(-\frac{1}{3}\right)} & \text{for } \frac{1}{|x^6|} > 1 \\ -\frac{\sqrt[3]{1-\frac{1}{x^6}}\Gamma\left(-\frac{4}{3}\right)}{6\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt[3]{1-\frac{1}{x^6}}\Gamma\left(-\frac{4}{3}\right)}{6x^6\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise} \end{cases}\right) - \begin{cases} \frac{\sqrt[3]{-1+\frac{1}{x^6}}e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{6\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt[3]{-1+\frac{1}{x^6}}e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{18x^6\Gamma\left(-\frac{1}{3}\right)} - \frac{2\sqrt[3]{-1+\frac{1}{x^6}}e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{9x^{12}\Gamma\left(-\frac{1}{3}\right)} & \text{for } \frac{1}{|x^6|} > 1 \\ \frac{3x^{12}\sqrt[3]{1-\frac{1}{x^6}}\Gamma\left(-\frac{7}{3}\right)}{18x^{12}\Gamma\left(-\frac{1}{3}\right)-18x^6\Gamma\left(-\frac{1}{3}\right)} - \frac{2x^6\sqrt[3]{1-\frac{1}{x^6}}\Gamma\left(-\frac{7}{3}\right)}{18x^{12}\Gamma\left(-\frac{1}{3}\right)-18x^6\Gamma\left(-\frac{1}{3}\right)} + \frac{4\sqrt[3]{1-\frac{1}{x^6}}\Gamma\left(-\frac{7}{3}\right)}{18x^{18}\Gamma\left(-\frac{1}{3}\right)-18x^{12}\Gamma\left(-\frac{1}{3}\right)} - \frac{5\sqrt[3]{1-\frac{1}{x^6}}\Gamma\left(-\frac{7}{3}\right)}{18x^{12}\Gamma\left(-\frac{1}{3}\right)-18x^6\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/3)*(2*x**6-1)/x**15,x)
```

```
[Out] 2*Piecewise(((-1 + x**(-6))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(6*gamma(-1/3
)) - (-1 + x**(-6))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(6*x**6*gamma(-1/3)),
 1/Abs(x**6) > 1), (-(1 - 1/x**6)**(1/3)*gamma(-4/3)/(6*gamma(-1/3)) + (1 -
 1/x**6)**(1/3)*gamma(-4/3)/(6*x**6*gamma(-1/3)), True)) - Piecewise(((-1 +
 x**(-6))**(1/3)*exp(I*pi/3)*gamma(-7/3)/(6*gamma(-1/3)) + (-1 + x**(-6))**
(1/3)*exp(I*pi/3)*gamma(-7/3)/(18*x**6*gamma(-1/3)) - 2*(-1 + x**(-6))**(1/
3)*exp(I*pi/3)*gamma(-7/3)/(9*x**12*gamma(-1/3)), 1/Abs(x**6) > 1), (3*x**1
2*(1 - 1/x**6)**(1/3)*gamma(-7/3)/(18*x**12*gamma(-1/3) - 18*x**6*gamma(-1/
3)) - 2*x**6*(1 - 1/x**6)**(1/3)*gamma(-7/3)/(18*x**12*gamma(-1/3) - 18*x**
6*gamma(-1/3)) + 4*(1 - 1/x**6)**(1/3)*gamma(-7/3)/(18*x**18*gamma(-1/3) -
18*x**12*gamma(-1/3)) - 5*(1 - 1/x**6)**(1/3)*gamma(-7/3)/(18*x**12*gamma(-
1/3) - 18*x**6*gamma(-1/3)), True))
```

3.338 $\int \frac{x^2(-2+x^3)\sqrt{1+x^3}}{1+3x^3+x^9}\,dx$

Optimal. Leaf size=28

$$-\frac{2\tanh^{-1}\left(\frac{(x^3+1)^{3/2}}{\sqrt{3}x^3}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.38, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6715, 2094, 207}

$$-\frac{2\tanh^{-1}\left(\frac{(x^3+1)^{3/2}}{\sqrt{3}x^3}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(-2 + x^3)*Sqrt[1 + x^3])/(1 + 3*x^3 + x^9),x]
```

```
[Out] (-2*ArcTanh[(1 + x^3)^(3/2)/(Sqrt[3]*x^3)])/(3*Sqrt[3])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(-2+x^3\right)\sqrt{1+x^3}}{1+3x^3+x^9}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(-2+x)\sqrt{1+x}}{1+3x+x^3}\,dx, x, x^3\right)\\
&= \frac{2}{3}\text{Subst}\left(\int \frac{x^2\left(-3+x^2\right)}{-3+6x^2-3x^4+x^6}\,dx, x, \sqrt{1+x^3}\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{-3+9x^2}\,dx, x, \frac{\left(1+x^3\right)^{3/2}}{3x^3}\right)\\
&= -\frac{2\tanh^{-1}\left(\frac{(1+x^3)^{3/2}}{\sqrt{3}x^3}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 28, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\left(x^3+1\right)^{3/2}}{\sqrt{3}\,x^3}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(-2 + x^3)*Sqrt[1 + x^3])/(1 + 3*x^3 + x^9),x]
```

```
[Out] (-2*ArcTanh[(1 + x^3)^(3/2)/(Sqrt[3]*x^3)])/(3*Sqrt[3])
```

IntegrateAlgebraic [A] time = 0.09, size = 28, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\,x^3}{\left(x^3+1\right)^{3/2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(-2 + x^3)*Sqrt[1 + x^3])/(1 + 3*x^3 + x^9),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3]*x^3)/(1 + x^3)^(3/2)])/(3*Sqrt[3])
```

fricas [B] time = 0.39, size = 53, normalized size = 1.89

$$\frac{1}{9}\sqrt{3}\log\left(\frac{x^9+6\,x^6+3\,x^3-2\sqrt{3}\left(x^6+x^3\right)\sqrt{x^3+1}+1}{x^9+3\,x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)*(x^3+1)^(1/2)/(x^9+3*x^3+1),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*log((x^9 + 6*x^6 + 3*x^3 - 2*sqrt(3)*(x^6 + x^3)*sqrt(x^3 + 1)
+ 1)/(x^9 + 3*x^3 + 1))
```

giac [B] time = 0.29, size = 58, normalized size = 2.07

$$-\frac{1}{9}\sqrt{3}\log\left(\left|\sqrt{3}\left(x^3+1\right)+\left(x^3+1\right)^{\frac{3}{2}}-\sqrt{3}\right|\right)+\frac{1}{9}\sqrt{3}\log\left(\left|-\sqrt{3}\left(x^3+1\right)+\left(x^3+1\right)^{\frac{3}{2}}+\sqrt{3}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)*(x^3+1)^(1/2)/(x^9+3*x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*log(abs(sqrt(3)*(x^3 + 1) + (x^3 + 1)^(3/2) - sqrt(3))) + 1/9*
sqrt(3)*log(abs(-sqrt(3)*(x^3 + 1) + (x^3 + 1)^(3/2) + sqrt(3)))
```

maple [C] time = 0.16, size = 298, normalized size = 10.64

$$\frac{\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^9+3\_Z^3+1\right)}\frac{\left(-\_\alpha^6+\_\alpha^3-1\right)\left(-\_\alpha^8+\_\alpha^7-\_\alpha^6+\_\alpha^5-\_\alpha^4+\_\alpha^3-4\_\alpha^2+4\_\alpha-4\right)\left(3-i\sqrt{3}\right)\sqrt{\frac{1+x}{3-i\sqrt{3}}}\sqrt{\frac{-1+2x-i\sqrt{3}}{-3-i\sqrt{3}}}\sqrt{\frac{-1+2x+i\sqrt{3}}{-3+i\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\_\alpha^8}{2}-\frac{\_\alpha^7}{2}+\frac{\_\alpha^6}{2}-\frac{\_\alpha^5}{2}+\frac{\_\alpha^4}{2}-\frac{\_\alpha^3}{2}+2\_\alpha^2-2\_\alpha+2+\frac{i\sqrt{3}\,\_\alpha^7}{6}-\frac{i\sqrt{3}\,\_\alpha^6}{6}-\frac{2i\_\alpha^2\sqrt{3}}{3}-\frac{i\sqrt{3}\,\_\alpha^8}{6}+\frac{2i\sqrt{3}\,\_\alpha}{3}+\frac{i\sqrt{3}\,\_\alpha^5}{6}-\frac{i\_\alpha^4\sqrt{3}}{6}+\frac{i\_\alpha^3\sqrt{3}}{6}-\frac{2i\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3-2)*(x^3+1)^(1/2)/(x^9+3*x^3+1),x)
```

```
[Out] 1/27*2^(1/2)*sum((-_alpha^6+_alpha^3-1)*(-_alpha^8+_alpha^7-_alpha^6+_alpha
^5-_alpha^4+_alpha^3-4*_alpha^2+4*_alpha-4)*(3-I*3^(1/2))*((1+x)/(3-I*3^(1/
2)))^(1/2)*((-1+2*x-I*3^(1/2))/(-3-I*3^(1/2)))^(1/2)*((-1+2*x+I*3^(1/2))/(-
3+I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1
```

```
/2),1/2*_alpha^8-1/2*_alpha^7+1/2*_alpha^6-1/2*_alpha^5+1/2*_alpha^4-1/2*_a
lpha^3+2*_alpha^2-2*_alpha+2+1/6*I*3^(1/2)*_alpha^7-1/6*I*3^(1/2)*_alpha^6-
2/3*I*3^(1/2)*_alpha^2-1/6*I*3^(1/2)*_alpha^8+2/3*I*3^(1/2)*_alpha+1/6*I*3^
(1/2)*_alpha^5-1/6*I*3^(1/2)*_alpha^4+1/6*I*3^(1/2)*_alpha^3-2/3*I*3^(1/2),
((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^9+3*_Z^
3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^3+1}\left(x^3-2\right)x^2}{x^9+3\,x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)*(x^3+1)^(1/2)/(x^9+3*x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^3 + 1)*(x^3 - 2)*x^2/(x^9 + 3*x^3 + 1), x)
```

mupad [B] time = 1.49, size = 236, normalized size = 8.43

$$\sum_{k=1}^{9} \frac{\sqrt{6}\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{-\left(-3+\sqrt{3}\,1\mathrm{i}\right)\,\left(x+1\right)}\,\Pi\left(\frac{3+\sqrt{3}\,1\mathrm{i}}{2\left(\mathrm{root}\left(z^9+3\,z^3+1,z,k\right)+1\right)};\mathrm{asin}\left(\frac{\sqrt{6}\,\sqrt{-\left(-3+\sqrt{3}\,1\mathrm{i}\right)\left(x+1\right)}}{6}\right)\middle|\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\mathrm{root}\left(z^9+3\,z^3+1,z,k\right)^6+\mathrm{root}\left(z^9+3\,z^3+1,z,k\right)^3+2\right)\,\mathrm{root}\left(z^9+3\,z^3+1,z,k\right)^2\,\sqrt{3-3\,x+\sqrt{3}\,x\,1\mathrm{i}+\sqrt{3}\,1\mathrm{i}}\,\sqrt{3-3\,x-\sqrt{3}\,x\,1\mathrm{i}-\sqrt{3}\,1\mathrm{i}}}{162\,\left(\mathrm{root}\left(z^9+3\,z^3+1,z,k\right)+1\right)\,\sqrt{x^3+1}\,\left(\mathrm{root}\left(z^9+3\,z^3+1,z,k\right)^8+\mathrm{root}\left(z^9+3\,z^3+1,z,k\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^3 + 1)^(1/2)*(x^3 - 2))/(3*x^3 + x^9 + 1),x)
```

```
[Out] symsum((6^(1/2)*((3^(1/2)*1i)/2 + 3/2)*(-(3^(1/2)*1i - 3)*(x + 1))^(1/2)*el
lipticPi((3^(1/2)*1i + 3)/(2*(root(z^9 + 3*z^3 + 1, z, k) + 1)), asin((6^(1
/2)*(-(3^(1/2)*1i - 3)*(x + 1))^(1/2))/6), (3^(1/2)*1i)/2 + 1/2)*(root(z^9
+ 3*z^3 + 1, z, k)^3 - root(z^9 + 3*z^3 + 1, z, k)^6 + 2)*root(z^9 + 3*z^3
+ 1, z, k)^2*(3^(1/2)*x*1i - 3*x + 3^(1/2)*1i + 3)^(1/2)*(3 - 3^(1/2)*x*1i
- 3^(1/2)*1i - 3*x)^(1/2))/(162*(root(z^9 + 3*z^3 + 1, z, k) + 1)*(x^3 + 1)
^(1/2)*(root(z^9 + 3*z^3 + 1, z, k)^2 + root(z^9 + 3*z^3 + 1, z, k)^8)), k,
 1, 9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3-2)*(x**3+1)**(1/2)/(x**9+3*x**3+1),x)
```

```
[Out] Timed out
```

3.339 $\int \frac{x^4(-9+4x^5)}{\sqrt{-x+x^6}\,(a-ax^5+x^9)}\,dx$

Optimal. Leaf size=28

$$-\frac{2\tanh^{-1}\left(\frac{x^5}{\sqrt{a}\,\sqrt{x^6-x}}\right)}{\sqrt{a}}$$

Rubi [F] time = 1.67, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4\left(-9+4x^5\right)}{\sqrt{-x+x^6}\,\left(a-ax^5+x^9\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(-9 + 4*x^5))/(Sqrt[-x + x^6]*(a - a*x^5 + x^9)),x]
```

```
[Out] (8*x*Sqrt[1 - x^5]*Hypergeometric2F1[1/10, 1/2, 11/10, x^5])/Sqrt[-x + x^6]
 + (8*a*Sqrt[x]*Sqrt[-1 + x^5]*Defer[Subst][Defer[Int][1/(Sqrt[-1 + x^10]*(
-a + a*x^10 - x^18)), x], x, Sqrt[x]])/Sqrt[-x + x^6] - (8*a*Sqrt[x]*Sqrt[-
1 + x^5]*Defer[Subst][Defer[Int][x^10/(Sqrt[-1 + x^10]*(-a + a*x^10 - x^18)
), x], x, Sqrt[x]])/Sqrt[-x + x^6] - (18*Sqrt[x]*Sqrt[-1 + x^5]*Defer[Subst
][Defer[Int][x^8/(Sqrt[-1 + x^10]*(a - a*x^10 + x^18)), x], x, Sqrt[x]])/Sq
rt[-x + x^6]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(-9+4x^5\right)}{\sqrt{-x+x^6}\,\left(a-ax^5+x^9\right)}\,dx &= \frac{\left(\sqrt{x}\,\sqrt{-1+x^5}\right)\int \frac{x^{7/2}\left(-9+4x^5\right)}{\sqrt{-1+x^5}\left(a-ax^5+x^9\right)}\,dx}{\sqrt{-x+x^6}}\\
&= \frac{\left(2\sqrt{x}\,\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{x^8\left(-9+4x^{10}\right)}{\sqrt{-1+x^{10}}\left(a-ax^{10}+x^{18}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}}\\
&= \frac{\left(2\sqrt{x}\,\sqrt{-1+x^5}\right)\text{Subst}\left(\int \left(\frac{4}{\sqrt{-1+x^{10}}}-\frac{4a+9x^8-4ax^{10}}{\sqrt{-1+x^{10}}\left(a-ax^{10}+x^{18}\right)}\right)dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}}\\
&= -\frac{\left(2\sqrt{x}\,\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{4a+9x^8-4ax^{10}}{\sqrt{-1+x^{10}}\left(a-ax^{10}+x^{18}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}}+\frac{\left(8\sqrt{x}\,\sqrt{-1+x^5}\right)}{}\\
&= \frac{\left(8\sqrt{x}\,\sqrt{1-x^5}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^{10}}}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}}-\frac{\left(2\sqrt{x}\,\sqrt{-1+x^5}\right)\text{Subst}\left(\int\left(-\right.\right.}{}\\
&= \frac{8x\sqrt{1-x^5}\;{}_2F_1\left(\frac{1}{10},\frac{1}{2};\frac{11}{10};x^5\right)}{\sqrt{-x+x^6}}-\frac{\left(18\sqrt{x}\,\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{x^8}{\sqrt{-1+x^{10}}\left(a-ax^{10}+x^{18}\right)}\right.}{\sqrt{-x+x^6}}
\end{aligned}$$

Mathematica [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(-9+4x^5\right)}{\sqrt{-x+x^6}\,\left(a-ax^5+x^9\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(-9 + 4*x^5))/(Sqrt[-x + x^6]*(a - a*x^5 + x^9)),x]
[Out] Integrate[(x^4*(-9 + 4*x^5))/(Sqrt[-x + x^6]*(a - a*x^5 + x^9)), x]
```

IntegrateAlgebraic [A] time = 18.73, size = 28, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{x^5}{\sqrt{a}\sqrt{x^6-x}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(-9 + 4*x^5))/(Sqrt[-x + x^6]*(a - a*x^5 + x^9)),x]
[Out] (-2*ArcTanh[x^5/(Sqrt[a]*Sqrt[-x + x^6])])/Sqrt[a]
```

fricas [A] time = 1.03, size = 151, normalized size = 5.39

$$\left[\frac{\log\left(-\frac{x^{18}+6\,ax^{14}+a^2x^{10}-6\,ax^9-2\,a^2x^5-4\left(x^{13}+ax^9-ax^4\right)\sqrt{x^6-x}\,\sqrt{a}+a^2}{x^{18}-2\,ax^{14}+a^2x^{10}+2\,ax^9-2\,a^2x^5+a^2}\right)}{2\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{x^6-x}\,\sqrt{-a}\,x^4}{x^9+ax^5-a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^5-9)/(x^6-x)^(1/2)/(x^9-a*x^5+a),x, algorithm="fricas")
[Out] [1/2*log(-(x^18 + 6*a*x^14 + a^2*x^10 - 6*a*x^9 - 2*a^2*x^5 - 4*(x^13 + a*x
^9 - a*x^4)*sqrt(x^6 - x)*sqrt(a) + a^2)/(x^18 - 2*a*x^14 + a^2*x^10 + 2*a*
x^9 - 2*a^2*x^5 + a^2))/sqrt(a), sqrt(-a)*arctan(2*sqrt(x^6 - x)*sqrt(-a)*x
^4/(x^9 + a*x^5 - a))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^5-9\right)x^4}{\left(x^9-ax^5+a\right)\sqrt{x^6-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^5-9)/(x^6-x)^(1/2)/(x^9-a*x^5+a),x, algorithm="giac")
[Out] integrate((4*x^5 - 9)*x^4/((x^9 - a*x^5 + a)*sqrt(x^6 - x)), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(4x^5-9\right)}{\sqrt{x^6-x}\,\left(x^9-a\,x^5+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^5-9)/(x^6-x)^(1/2)/(x^9-a*x^5+a),x)
[Out] int(x^4*(4*x^5-9)/(x^6-x)^(1/2)/(x^9-a*x^5+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^5-9\right)x^4}{\left(x^9-ax^5+a\right)\sqrt{x^6-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^5-9)/(x^6-x)^(1/2)/(x^9-a*x^5+a),x, algorithm="maxima")
[Out] integrate((4*x^5 - 9)*x^4/((x^9 - a*x^5 + a)*sqrt(x^6 - x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4\,\left(4\,x^5-9\right)}{\sqrt{x^6-x}\,\left(x^9-a\,x^5+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(4*x^5 - 9))/((x^6 - x)^(1/2)*(a - a*x^5 + x^9)),x)
[Out] int((x^4*(4*x^5 - 9))/((x^6 - x)^(1/2)*(a - a*x^5 + x^9)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(4x^5-9\right)}{\sqrt{x\left(x-1\right)\left(x^4+x^3+x^2+x+1\right)}\left(-ax^5+a+x^9\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(4*x**5-9)/(x**6-x)**(1/2)/(x**9-a*x**5+a),x)
[Out] Integral(x**4*(4*x**5 - 9)/(sqrt(x*(x - 1)*(x**4 + x**3 + x**2 + x + 1))*(-
a*x**5 + a + x**9)), x)
```

3.340 $\int \frac{x^4(-9+5x^4)}{\sqrt{-x+x^5}\,(1-x^4+ax^9)}\,dx$

Optimal. Leaf size=28

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^5-x}}{\sqrt{a}\,x^5}\right)}{\sqrt{a}}$$

Rubi [F] time = 1.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4\left(-9+5x^4\right)}{\sqrt{-x+x^5}\left(1-x^4+ax^9\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(-9 + 5*x^4))/(Sqrt[-x + x^5]*(1 - x^4 + a*x^9)),x]
```

```
[Out] (-18*Sqrt[x]*Sqrt[-1 + x^4]*Defer[Subst][Defer[Int][x^8/(Sqrt[-1 + x^8]*(1
- x^8 + a*x^18)), x], x, Sqrt[x]])/Sqrt[-x + x^5] + (10*Sqrt[x]*Sqrt[-1 + x
^4]*Defer[Subst][Defer[Int][x^16/(Sqrt[-1 + x^8]*(1 - x^8 + a*x^18)), x], x
, Sqrt[x]])/Sqrt[-x + x^5]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(-9+5x^4\right)}{\sqrt{-x+x^5}\left(1-x^4+ax^9\right)}\,dx &= \frac{\left(\sqrt{x}\sqrt{-1+x^4}\right)\int \frac{x^{7/2}\left(-9+5x^4\right)}{\sqrt{-1+x^4}\left(1-x^4+ax^9\right)}\,dx}{\sqrt{-x+x^5}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \frac{x^8\left(-9+5x^8\right)}{\sqrt{-1+x^8}\left(1-x^8+ax^{18}\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{-x+x^5}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \left(-\frac{9x^8}{\sqrt{-1+x^8}\left(1-x^8+ax^{18}\right)}+\frac{5x^{16}}{\sqrt{-1+x^8}\left(1-x^8+ax^{18}\right)}\right)dx, x, \sqrt{\right.}{\sqrt{-x+x^5}}\\
&= \frac{\left(10\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \frac{x^{16}}{\sqrt{-1+x^8}\left(1-x^8+ax^{18}\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{-x+x^5}} - \frac{\left(18\sqrt{x}\sqrt{-1+x^4}\right.}{}
\end{aligned}$$

Mathematica [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(-9+5x^4\right)}{\sqrt{-x+x^5}\left(1-x^4+ax^9\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(-9 + 5*x^4))/(Sqrt[-x + x^5]*(1 - x^4 + a*x^9)),x]
```

```
[Out] Integrate[(x^4*(-9 + 5*x^4))/(Sqrt[-x + x^5]*(1 - x^4 + a*x^9)), x]
```

IntegrateAlgebraic [A] time = 4.98, size = 28, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^5-x}}{\sqrt{a}\,x^5}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(-9 + 5*x^4))/(Sqrt[-x + x^5]*(1 - x^4 + a*x^9)),x]
[Out] (-2*ArcTanh[Sqrt[-x + x^5]/(Sqrt[a]*x^5)])/Sqrt[a]
```

fricas [B] time = 0.62, size = 146, normalized size = 5.21

$$\left[\frac{\log\left(\frac{a^2x^{18}+6\,ax^{13}-6\,ax^9+x^8-2\,x^4-4\left(ax^{13}+x^8-x^4\right)\sqrt{x^5-x}\,\sqrt{a}+1}{a^2x^{18}-2\,ax^{13}+2\,ax^9+x^8-2\,x^4+1}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\left(ax^9+x^4-1\right)\sqrt{x^5-x}\,\sqrt{-a}}{2\left(ax^9-ax^5\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^4-9)/(x^5-x)^(1/2)/(a*x^9-x^4+1),x, algorithm="fricas")
[Out] [1/2*log((a^2*x^18 + 6*a*x^13 - 6*a*x^9 + x^8 - 2*x^4 - 4*(a*x^13 + x^8 - x
^4)*sqrt(x^5 - x)*sqrt(a) + 1)/(a^2*x^18 - 2*a*x^13 + 2*a*x^9 + x^8 - 2*x^4
 + 1))/sqrt(a), sqrt(-a)*arctan(1/2*(a*x^9 + x^4 - 1)*sqrt(x^5 - x)*sqrt(-a
)/(a*x^9 - a*x^5))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^4-9\right)x^4}{\left(ax^9-x^4+1\right)\sqrt{x^5-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^4-9)/(x^5-x)^(1/2)/(a*x^9-x^4+1),x, algorithm="giac")
[Out] integrate((5*x^4 - 9)*x^4/((a*x^9 - x^4 + 1)*sqrt(x^5 - x)), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(5x^4-9\right)}{\sqrt{x^5-x}\,\left(a\,x^9-x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(5*x^4-9)/(x^5-x)^(1/2)/(a*x^9-x^4+1),x)
[Out] int(x^4*(5*x^4-9)/(x^5-x)^(1/2)/(a*x^9-x^4+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^4-9\right)x^4}{\left(ax^9-x^4+1\right)\sqrt{x^5-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^4-9)/(x^5-x)^(1/2)/(a*x^9-x^4+1),x, algorithm="maxima")
[Out] integrate((5*x^4 - 9)*x^4/((a*x^9 - x^4 + 1)*sqrt(x^5 - x)), x)
```

mupad [B] time = 1.00, size = 48, normalized size = 1.71

$$\frac{\ln\left(\frac{a\,x^9+x^4-2\,\sqrt{a}\,x^4\,\sqrt{x\left(x^4-1\right)}-1}{4\,a\,x^9-4\,x^4+4}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(5*x^4 - 9))/((x^5 - x)^(1/2)*(a*x^9 - x^4 + 1)),x)
```

```
[Out] log((a*x^9 + x^4 - 2*a^(1/2)*x^4*(x*(x^4 - 1))^(1/2) - 1)/(4*a*x^9 - 4*x^4
+ 4))/a^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(5x^4 - 9\right)}{\sqrt{x\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}\left(ax^9 - x^4 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(5*x**4-9)/(x**5-x)**(1/2)/(a*x**9-x**4+1),x)
```

```
[Out] Integral(x**4*(5*x**4 - 9)/(sqrt(x*(x - 1)*(x + 1)*(x**2 + 1))*(a*x**9 - x*
*4 + 1)), x)
```

3.341 $\int \frac{x^4(-9+4x^5)}{\sqrt{-x+x^6}\,(1-x^5+ax^9)}\,dx$

Optimal. Leaf size=28

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^6-x}}{\sqrt{a}\,x^5}\right)}{\sqrt{a}}$$

Rubi [F] time = 1.55, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4\left(-9+4x^5\right)}{\sqrt{-x+x^6}\,\left(1-x^5+ax^9\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(-9 + 4*x^5))/(Sqrt[-x + x^6]*(1 - x^5 + a*x^9)),x]
```

```
[Out] (8*x*Sqrt[1 - x^5]*Hypergeometric2F1[1/10, 1/2, 11/10, x^5])/(a*Sqrt[-x + x
^6]) - (8*Sqrt[x]*Sqrt[-1 + x^5]*Defer[Subst][Defer[Int][1/(Sqrt[-1 + x^10]
*(1 - x^10 + a*x^18)), x], x, Sqrt[x]])/(a*Sqrt[-x + x^6]) - (18*Sqrt[x]*Sq
rt[-1 + x^5]*Defer[Subst][Defer[Int][x^8/(Sqrt[-1 + x^10]*(1 - x^10 + a*x^1
8)), x], x, Sqrt[x]])/Sqrt[-x + x^6] + (8*Sqrt[x]*Sqrt[-1 + x^5]*Defer[Subs
t][Defer[Int][x^10/(Sqrt[-1 + x^10]*(1 - x^10 + a*x^18)), x], x, Sqrt[x]])/
(a*Sqrt[-x + x^6])
```

Rubi steps

$$\int \frac{x^4\left(-9+4x^5\right)}{\sqrt{-x+x^6}\,\left(1-x^5+ax^9\right)}\,dx = \frac{\left(\sqrt{x}\,\sqrt{-1+x^5}\right)\int \frac{x^{7/2}\left(-9+4x^5\right)}{\sqrt{-1+x^5}\,\left(1-x^5+ax^9\right)}\,dx}{\sqrt{-x+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{x^8\left(-9+4x^{10}\right)}{\sqrt{-1+x^{10}}\,\left(1-x^{10}+ax^{18}\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{-x+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt{-1+x^5}\right)\text{Subst}\left(\int \left(\frac{4}{a\sqrt{-1+x^{10}}}-\frac{4+9ax^8-4x^{10}}{a\sqrt{-1+x^{10}}\,\left(1-x^{10}+ax^{18}\right)}\right)dx, x, \sqrt{x}\right)}{\sqrt{-x+x^6}}$$

$$= -\frac{\left(2\sqrt{x}\,\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{4+9ax^8-4x^{10}}{\sqrt{-1+x^{10}}\,\left(1-x^{10}+ax^{18}\right)}\,dx, x, \sqrt{x}\right)}{a\sqrt{-x+x^6}} + \frac{\left(8\sqrt{x}\,\sqrt{-1+x^5}\right)}{}$$

$$= \frac{\left(8\sqrt{x}\,\sqrt{1-x^5}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^{10}}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-x+x^6}} - \frac{\left(2\sqrt{x}\,\sqrt{-1+x^5}\right)\text{Subst}\left(\int \left(\frac{}{\sqrt{-1-}}\right.\right.}{}$$

$$= \frac{8x\sqrt{1-x^5}\,{}_2F_1\left(\frac{1}{10},\frac{1}{2};\frac{11}{10};x^5\right)}{a\sqrt{-x+x^6}} - \frac{\left(18\sqrt{x}\,\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{x^8}{\sqrt{-1+x^{10}}\,\left(1-x^{10}+ax^{18}\right)}\right.}{\sqrt{-x+x^6}}$$

Mathematica [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(-9+4x^5\right)}{\sqrt{-x+x^6}\,\left(1-x^5+ax^9\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(-9 + 4*x^5))/(Sqrt[-x + x^6]*(1 - x^5 + a*x^9)),x]
[Out] Integrate[(x^4*(-9 + 4*x^5))/(Sqrt[-x + x^6]*(1 - x^5 + a*x^9)), x]
```

IntegrateAlgebraic [A] time = 10.70, size = 28, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^6-x}}{\sqrt{a}\,x^5}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(-9 + 4*x^5))/(Sqrt[-x + x^6]*(1 - x^5 + a*x^9)),x]
[Out] (-2*ArcTanh[Sqrt[-x + x^6]/(Sqrt[a]*x^5)])/Sqrt[a]
```

fricas [A] time = 1.07, size = 138, normalized size = 4.93

$$\left[\frac{\log\left(-\frac{a^2x^{18}+6\,ax^{14}-6\,ax^9+x^{10}-2\,x^5-4\left(ax^{13}+x^9-x^4\right)\sqrt{x^6-x}\,\sqrt{a}+1}{a^2x^{18}-2\,ax^{14}+2\,ax^9+x^{10}-2\,x^5+1}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{x^6-x}\,\sqrt{-a}\,x^4}{ax^9+x^5-1}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^5-9)/(x^6-x)^(1/2)/(a*x^9-x^5+1),x, algorithm="fricas")
[Out] [1/2*log(-(a^2*x^18 + 6*a*x^14 - 6*a*x^9 + x^10 - 2*x^5 - 4*(a*x^13 + x^9 -
 x^4)*sqrt(x^6 - x)*sqrt(a) + 1)/(a^2*x^18 - 2*a*x^14 + 2*a*x^9 + x^10 - 2*
x^5 + 1))/sqrt(a), sqrt(-a)*arctan(2*sqrt(x^6 - x)*sqrt(-a)*x^4/(a*x^9 + x^
5 - 1))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^5-9\right)x^4}{\left(ax^9-x^5+1\right)\sqrt{x^6-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^5-9)/(x^6-x)^(1/2)/(a*x^9-x^5+1),x, algorithm="giac")
[Out] integrate((4*x^5 - 9)*x^4/((a*x^9 - x^5 + 1)*sqrt(x^6 - x)), x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(4x^5-9\right)}{\sqrt{x^6-x}\,\left(a\,x^9-x^5+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(4*x^5-9)/(x^6-x)^(1/2)/(a*x^9-x^5+1),x)
[Out] int(x^4*(4*x^5-9)/(x^6-x)^(1/2)/(a*x^9-x^5+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^5-9\right)x^4}{\left(ax^9-x^5+1\right)\sqrt{x^6-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(4*x^5-9)/(x^6-x)^(1/2)/(a*x^9-x^5+1),x, algorithm="maxima")
[Out] integrate((4*x^5 - 9)*x^4/((a*x^9 - x^5 + 1)*sqrt(x^6 - x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4\,\left(4\,x^5-9\right)}{\sqrt{x^6-x}\,\left(a\,x^9-x^5+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(4*x^5 - 9))/((x^6 - x)^(1/2)*(a*x^9 - x^5 + 1)),x)
[Out] int((x^4*(4*x^5 - 9))/((x^6 - x)^(1/2)*(a*x^9 - x^5 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(4x^5-9\right)}{\sqrt{x\left(x-1\right)\left(x^4+x^3+x^2+x+1\right)}\left(ax^9-x^5+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(4*x**5-9)/(x**6-x)**(1/2)/(a*x**9-x**5+1),x)
[Out] Integral(x**4*(4*x**5 - 9)/(sqrt(x*(x - 1)*(x**4 + x**3 + x**2 + x + 1))*(a
*x**9 - x**5 + 1)), x)
```

3.342 $\int \frac{1+x^{12}}{x^{16}\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{x^6-1}\left(23x^{12}+4x^6+3\right)}{45x^{15}}$$

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.75, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1487, 453, 271, 264}

$$\frac{\sqrt{x^6-1}}{15x^{15}}+\frac{4\sqrt{x^6-1}}{45x^9}+\frac{23\sqrt{x^6-1}}{45x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^12)/(x^16*Sqrt[-1 + x^6]),x]
```

```
[Out] Sqrt[-1 + x^6]/(15*x^15) + (4*Sqrt[-1 + x^6])/(45*x^9) + (23*Sqrt[-1 + x^6]
)/(45*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1487

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 2*n*p - n + 1)*(d + e*x^n)^(
q + 1))/(e*f^(2*n*p - n + 1)*(m + 2*n*p + n*q + 1)), x] + Dist[1/(e*(m + 2*
n*p + n*q + 1)), Int[(f*x)^m*(d + e*x^n)^q*ExpandToSum[e*(m + 2*n*p + n*q +
 1)*((a + c*x^(2*n))^p - c^p*x^(2*n*p)) - d*c^p*(m + 2*n*p - n + 1)*x^(2*n*
p - n), x], x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IG
tQ[n, 0] && IGtQ[p, 0] && GtQ[2*n*p, n - 1] &&  !IntegerQ[q] && NeQ[m + 2*n
*p + n*q + 1, 0]
```

Rubi steps

$$\int \frac{1+x^{12}}{x^{16}\sqrt{-1+x^6}}\,dx = -\frac{\sqrt{-1+x^6}}{6x^9} - \frac{1}{6}\int \frac{-6-9x^6}{x^{16}\sqrt{-1+x^6}}\,dx$$
$$= \frac{\sqrt{-1+x^6}}{15x^{15}} - \frac{\sqrt{-1+x^6}}{6x^9} + \frac{23}{10}\int \frac{1}{x^{10}\sqrt{-1+x^6}}\,dx$$
$$= \frac{\sqrt{-1+x^6}}{15x^{15}} + \frac{4\sqrt{-1+x^6}}{45x^9} + \frac{23}{15}\int \frac{1}{x^4\sqrt{-1+x^6}}\,dx$$
$$= \frac{\sqrt{-1+x^6}}{15x^{15}} + \frac{4\sqrt{-1+x^6}}{45x^9} + \frac{23\sqrt{-1+x^6}}{45x^3}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 1.18

$$\frac{23x^{18}-19x^{12}-x^6-3}{45x^{15}\sqrt{x^6-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^12)/(x^16*Sqrt[-1 + x^6]),x]
[Out] (-3 - x^6 - 19*x^12 + 23*x^18)/(45*x^15*Sqrt[-1 + x^6])
```

IntegrateAlgebraic [A] time = 0.33, size = 28, normalized size = 1.00

$$\frac{\sqrt{x^6-1}\left(23x^{12}+4x^6+3\right)}{45x^{15}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^12)/(x^16*Sqrt[-1 + x^6]),x]
[Out] (Sqrt[-1 + x^6]*(3 + 4*x^6 + 23*x^12))/(45*x^15)
```

fricas [A] time = 0.40, size = 31, normalized size = 1.11

$$\frac{23\,x^{15}+\left(23\,x^{12}+4\,x^6+3\right)\sqrt{x^6-1}}{45\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^12+1)/x^16/(x^6-1)^(1/2),x, algorithm="fricas")
[Out] 1/45*(23*x^15 + (23*x^12 + 4*x^6 + 3)*sqrt(x^6 - 1))/x^15
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^12+1)/x^16/(x^6-1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [abs(x)]sym2poly/r2sym(const gen
 & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 45, normalized size = 1.61

$$\frac{\left(23x^{12}+4x^{6}+3\right)(-1+x)(1+x)\left(x^{2}+x+1\right)\left(x^{2}-x+1\right)}{45\sqrt{x^{6}-1}\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^12+1)/x^16/(x^6-1)^(1/2),x)
```

```
[Out] 1/45*(23*x^12+4*x^6+3)*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)/(x^6-1)^(1/2)/x^15
```

maxima [A] time = 0.47, size = 37, normalized size = 1.32

$$\frac{2\sqrt{x^{6}-1}}{3\,x^{3}}-\frac{2\left(x^{6}-1\right)^{\frac{3}{2}}}{9\,x^{9}}+\frac{\left(x^{6}-1\right)^{\frac{5}{2}}}{15\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^12+1)/x^16/(x^6-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(x^6 - 1)/x^3 - 2/9*(x^6 - 1)^(3/2)/x^9 + 1/15*(x^6 - 1)^(5/2)/x^15
```

mupad [B] time = 0.32, size = 24, normalized size = 0.86

$$\frac{\sqrt{x^{6}-1}\,\left(23\,x^{12}+4\,x^{6}+3\right)}{45\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^12 + 1)/(x^16*(x^6 - 1)^(1/2)),x)
```

```
[Out] ((x^6 - 1)^(1/2)*(4*x^6 + 23*x^12 + 3))/(45*x^15)
```

sympy [A] time = 3.69, size = 65, normalized size = 2.32

$$\frac{\begin{cases}\frac{\sqrt{x^{6}-1}}{x^{3}} & \text{for}\: x>-1\wedge x<1\end{cases}}{3}+\frac{\begin{cases}\frac{\sqrt{x^{6}-1}}{x^{3}}-\frac{2\left(x^{6}-1\right)^{\frac{3}{2}}}{3x^{9}}+\frac{\left(x^{6}-1\right)^{\frac{5}{2}}}{5x^{15}} & \text{for}\: x>-1\wedge x<1\end{cases}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**12+1)/x**16/(x**6-1)**(1/2),x)
```

```
[Out] Piecewise((sqrt(x**6 - 1)/x**3, (x > -1) & (x < 1)))/3 + Piecewise((sqrt(x*
*6 - 1)/x**3 - 2*(x**6 - 1)**(3/2)/(3*x**9) + (x**6 - 1)**(5/2)/(5*x**15),
(x > -1) & (x < 1)))/3
```

3.343 $\int \frac{-1+x}{(-3+x)(1+x)\sqrt[4]{-2-2x+x^2}}\,dx$

Optimal. Leaf size=29

$$\tan^{-1}\left(\sqrt[4]{x^2-2x-2}\right)-\tanh^{-1}\left(\sqrt[4]{x^2-2x-2}\right)$$

Rubi [B] time = 1.12, antiderivative size = 262, normalized size of antiderivative = 9.03, number of steps used = 42, number of rules used = 16, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {6742, 749, 748, 746, 399, 490, 1213, 537, 444, 63, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{-x^2+2x+2}\log\left(\sqrt{-x^2+2x+2}+\sqrt{2}\sqrt[4]{-x^2+2x+2}+1\right)}{2\sqrt{2}\sqrt[4]{x^2-2x-2}}+\frac{\sqrt[4]{-x^2+2x+2}\log\left(3\sqrt{-x^2+2x+2}-3\sqrt{2}\sqrt[4]{-x^2+2x+2}+3\right)}{2\sqrt{2}\sqrt[4]{x^2-2x-2}}-\frac{\sqrt[4]{-x^2+2x+2}\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3-(1-x)^2}\right)}{\sqrt{2}\sqrt[4]{x^2-2x-2}}+\frac{\sqrt[4]{-x^2+2x+2}\tan^{-1}\left(\sqrt{2}\sqrt[4]{3-(1-x)^2}+1\right)}{\sqrt{2}\sqrt[4]{x^2-2x-2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/((-3 + x)*(1 + x)*(-2 - 2*x + x^2)^(1/4)),x]
```

```
[Out] -(((2 + 2*x - x^2)^(1/4)*ArcTan[1 - Sqrt[2]*(3 - (1 - x)^2)^(1/4)])/(Sqrt[2
]*(-2 - 2*x + x^2)^(1/4))) + ((2 + 2*x - x^2)^(1/4)*ArcTan[1 + Sqrt[2]*(3 -
 (1 - x)^2)^(1/4)])/(Sqrt[2]*(-2 - 2*x + x^2)^(1/4)) - ((2 + 2*x - x^2)^(1/
4)*Log[1 + Sqrt[2]*(2 + 2*x - x^2)^(1/4) + Sqrt[2 + 2*x - x^2]])/(2*Sqrt[2]
*(-2 - 2*x + x^2)^(1/4)) + ((2 + 2*x - x^2)^(1/4)*Log[3 - 3*Sqrt[2]*(2 + 2*
x - x^2)^(1/4) + 3*Sqrt[2 + 2*x - x^2]])/(2*Sqrt[2]*(-2 - 2*x + x^2)^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 748

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[1/((-4*c)/(b^2 - 4*a*c))^p, Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c),
 x]^p/Simp[2*c*d - b*e + e*x, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && GtQ[4*a - b^2/c, 0] && IntegerQ[4*p]
```

Rule 749

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[(a + b*x + c*x^2)^p/(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^p, I
nt[(-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) - (c^2*x^2)/(b^2 - 4*a*c
))^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&  !GtQ[4*a - b^2/c
, 0] && IntegerQ[4*p]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-1+x}{(-3+x)(1+x)\sqrt[4]{-2-2x+x^2}}\,dx = \int \left(\frac{1}{2(-3+x)\sqrt[4]{-2-2x+x^2}} + \frac{1}{2(1+x)\sqrt[4]{-2-2x+x^2}}\right)dx$$

$$= \frac{1}{2}\int \frac{1}{(-3+x)\sqrt[4]{-2-2x+x^2}}\,dx + \frac{1}{2}\int \frac{1}{(1+x)\sqrt[4]{-2-2x+x^2}}\,dx$$

$$= \frac{\sqrt[4]{2+2x-x^2}\int \frac{1}{(-3+x)\sqrt[4]{\frac{1}{6}+\frac{x}{6}-\frac{x^2}{12}}}\,dx}{2\sqrt{2}\,\sqrt[4]{3}\,\sqrt[4]{-2-2x+x^2}} + \frac{\sqrt[4]{2+2x-x^2}\int \frac{1}{(1+x)\sqrt[4]{\frac{1}{6}+\frac{x}{6}-\frac{x^2}{12}}}\,d}{2\sqrt{2}\,\sqrt[4]{3}\,\sqrt[4]{-2-2x+x^2}}$$

$$= \frac{\sqrt[4]{2+2x-x^2}\,\text{Subst}\left(\int \frac{1}{\left(-\frac{1}{3}+x\right)\sqrt[4]{1-12x^2}}\,dx, x, \frac{1}{6}-\frac{x}{6}\right)}{2\sqrt[4]{3}\,\sqrt[4]{-2-2x+x^2}} + \frac{\sqrt[4]{2+2x-x^2}\,\text{S}}{2}$$

$$= -2\frac{\sqrt[4]{2+2x-x^2}\,\text{Subst}\left(\int \frac{x}{\sqrt[4]{1-12x^2}\left(\frac{1}{9}-x^2\right)}\,dx, x, \frac{1}{6}-\frac{x}{6}\right)}{2\sqrt[4]{3}\,\sqrt[4]{-2-2x+x^2}}$$

$$= -2\frac{\sqrt[4]{2+2x-x^2}\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-12x}\left(\frac{1}{9}-x\right)}\,dx, x, \left(\frac{1}{6}-\frac{x}{6}\right)^2\right)}{4\sqrt[4]{3}\,\sqrt[4]{-2-2x+x^2}}$$

$$= 2\frac{\sqrt[4]{2+2x-x^2}\,\text{Subst}\left(\int \frac{x^2}{\frac{1}{36}+\frac{x^4}{12}}\,dx, x, \frac{\sqrt[4]{3-(-1+x)^2}}{\sqrt[4]{3}}\right)}{12\sqrt[4]{3}\,\sqrt[4]{-2-2x+x^2}}$$

$$= 2\left(-\frac{\sqrt[4]{2+2x-x^2}\,\text{Subst}\left(\int \frac{1-\sqrt{3}x^2}{\frac{1}{36}+\frac{x^4}{12}}\,dx, x, \frac{\sqrt[4]{3-(-1+x)^2}}{\sqrt[4]{3}}\right)}{24\,3^{3/4}\sqrt[4]{-2-2x+x^2}} + \frac{\sqrt[4]{2+2x-x^2}}{}\right.$$

$$= 2\left(\frac{\sqrt[4]{2+2x-x^2}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}}{\sqrt[4]{3}}+2x}{-\frac{1}{\sqrt{3}}-\frac{\sqrt{2}x}{\sqrt[4]{3}}-x^2}\,dx, x, \frac{\sqrt[4]{3-(-1+x)^2}}{\sqrt[4]{3}}\right)}{4\sqrt{2}\,\sqrt[4]{-2-2x+x^2}} + \frac{\sqrt[4]{2+2x-}}{}\right.$$

$$= 2\left(\frac{\sqrt[4]{2+2x-x^2}\,\log\left(\sqrt{3}-\sqrt{6}\,\sqrt[4]{3-(1-x)^2}+\sqrt{3}\,\sqrt{3-(1-x)^2}\right)}{4\sqrt{2}\,\sqrt[4]{-2-2x+x^2}} - \right.$$

$$= 2\left(-\frac{\sqrt[4]{2+2x-x^2}\,\tan^{-1}\left(1-\sqrt{2}\,\sqrt[4]{3-(1-x)^2}\right)}{2\sqrt{2}\,\sqrt[4]{-2-2x+x^2}} + \frac{\sqrt[4]{2+2x-x^2}\,\tan^{-}}{2\sqrt{2}\,\sqrt[4]{}}\right.$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{-1+x}{(-3+x)(1+x)\sqrt[4]{-2-2x+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x)/((-3 + x)*(1 + x)*(-2 - 2*x + x^2)^(1/4)),x]
[Out] Integrate[(-1 + x)/((-3 + x)*(1 + x)*(-2 - 2*x + x^2)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.06, size = 29, normalized size = 1.00

$$\tan^{-1}\left(\sqrt[4]{x^2-2x-2}\right)-\tanh^{-1}\left(\sqrt[4]{x^2-2x-2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/((-3 + x)*(1 + x)*(-2 - 2*x + x^2)^(1/4)),x]
[Out] ArcTan[(-2 - 2*x + x^2)^(1/4)] - ArcTanh[(-2 - 2*x + x^2)^(1/4)]
```

fricas [A] time = 0.41, size = 42, normalized size = 1.45

$$\arctan\left(\left(x^2-2x-2\right)^{\frac{1}{4}}\right)-\frac{1}{2}\log\left(\left(x^2-2x-2\right)^{\frac{1}{4}}+1\right)+\frac{1}{2}\log\left(\left(x^2-2x-2\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-3+x)/(1+x)/(x^2-2*x-2)^(1/4),x, algorithm="fricas")
[Out] arctan((x^2 - 2*x - 2)^(1/4)) - 1/2*log((x^2 - 2*x - 2)^(1/4) + 1) + 1/2*lo
g((x^2 - 2*x - 2)^(1/4) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x-1}{\left(x^2-2x-2\right)^{\frac{1}{4}}(x+1)(x-3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-3+x)/(1+x)/(x^2-2*x-2)^(1/4),x, algorithm="giac")
[Out] integrate((x - 1)/((x^2 - 2*x - 2)^(1/4)*(x + 1)*(x - 3)), x)
```

maple [C] time = 1.56, size = 149, normalized size = 5.14

$$-\frac{\ln\left(\frac{2\left(x^2-2x-2\right)^{\frac{3}{4}}+2\sqrt{x^2-2x-2}+x^2+2\left(x^2-2x-2\right)^{\frac{1}{4}}-2x-1}{(-3+x)(1+x)}\right)}{2}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\left(x^2-2x-2\right)^{\frac{3}{4}}+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{x^2-2x-2}-\mathrm{RootOf}\left(\_Z^2+1\right)x^2+2\,\mathrm{RootOf}\left(\_Z^2+1\right)x-2\left(x^2-2x-2\right)^{\frac{1}{4}}+\mathrm{RootOf}\left(\_Z^2+1\right)}{(-3+x)(1+x)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(-3+x)/(1+x)/(x^2-2*x-2)^(1/4),x)
[Out] -1/2*ln((2*(x^2-2*x-2)^(3/4)+2*(x^2-2*x-2)^(1/2)+x^2+2*(x^2-2*x-2)^(1/4)-2*
x-1)/(-3+x)/(1+x))+1/2*RootOf(_Z^2+1)*ln((2*(x^2-2*x-2)^(3/4)+2*RootOf(_Z^2
+1)*(x^2-2*x-2)^(1/2)-RootOf(_Z^2+1)*x^2+2*RootOf(_Z^2+1)*x-2*(x^2-2*x-2)^(
1/4)+RootOf(_Z^2+1))/(-3+x)/(1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x-1}{\left(x^2-2x-2\right)^{\frac{1}{4}}(x+1)(x-3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-3+x)/(1+x)/(x^2-2*x-2)^(1/4),x, algorithm="maxima")
[Out] integrate((x - 1)/((x^2 - 2*x - 2)^(1/4)*(x + 1)*(x - 3)), x)
```

mupad [B] time = 0.39, size = 25, normalized size = 0.86

$$\mathrm{atan}\left(\left(x^2-2x-2\right)^{1/4}\right)-\mathrm{atanh}\left(\left(x^2-2x-2\right)^{1/4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x + 1)*(x - 3)*(x^2 - 2*x - 2)^(1/4)),x)
[Out] atan((x^2 - 2*x - 2)^(1/4)) - atanh((x^2 - 2*x - 2)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{(x-3)(x+1)\sqrt[4]{x^2-2x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-3+x)/(1+x)/(x**2-2*x-2)**(1/4),x)
[Out] Integral((x - 1)/((x - 3)*(x + 1)*(x**2 - 2*x - 2)**(1/4)), x)
```

3.344 $\int \frac{1}{(-1+x)\sqrt[4]{2-2x+x^2}} \, dx$

Optimal. Leaf size=29

$$\tan^{-1}\left(\sqrt[4]{x^2-2x+2}\right)-\tanh^{-1}\left(\sqrt[4]{x^2-2x+2}\right)$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {694, 266, 63, 298, 203, 206}

$$\tan^{-1}\left(\sqrt[4]{(x-1)^2+1}\right)-\tanh^{-1}\left(\sqrt[4]{(x-1)^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)*(2 - 2*x + x^2)^(1/4)),x]
```

```
[Out] ArcTan[(1 + (-1 + x)^2)^(1/4)] - ArcTanh[(1 + (-1 + x)^2)^(1/4)]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-1+x)\sqrt[4]{2-2x+x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{x\sqrt[4]{1+x^2}}\,dx, x, -1+x\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1+x}}\,dx, x, (-1+x)^2\right)\\ &= 2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt[4]{1+(-1+x)^2}\right)\\ &= -\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1+(-1+x)^2}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{1+(-1\right.\\ &= \tan^{-1}\left(\sqrt[4]{1+(-1+x)^2}\right)-\tanh^{-1}\left(\sqrt[4]{1+(-1+x)^2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.93

$$\tan^{-1}\left(\sqrt[4]{(x-1)^2+1}\right)-\tanh^{-1}\left(\sqrt[4]{(x-1)^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)*(2 - 2*x + x^2)^(1/4)),x]
[Out] ArcTan[(1 + (-1 + x)^2)^(1/4)] - ArcTanh[(1 + (-1 + x)^2)^(1/4)]
```

IntegrateAlgebraic [A] time = 0.07, size = 29, normalized size = 1.00

$$\tan^{-1}\left(\sqrt[4]{x^2-2x+2}\right)-\tanh^{-1}\left(\sqrt[4]{x^2-2x+2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x)*(2 - 2*x + x^2)^(1/4)),x]
[Out] ArcTan[(2 - 2*x + x^2)^(1/4)] - ArcTanh[(2 - 2*x + x^2)^(1/4)]
```

fricas [A] time = 0.40, size = 42, normalized size = 1.45

$$\arctan\left(\left(x^2-2\,x+2\right)^{\frac{1}{4}}\right)-\frac{1}{2}\,\log\left(\left(x^2-2\,x+2\right)^{\frac{1}{4}}+1\right)+\frac{1}{2}\,\log\left(\left(x^2-2\,x+2\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^2-2*x+2)^(1/4),x, algorithm="fricas")
[Out] arctan((x^2 - 2*x + 2)^(1/4)) - 1/2*log((x^2 - 2*x + 2)^(1/4) + 1) + 1/2*lo
g((x^2 - 2*x + 2)^(1/4) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-2\,x+2\right)^{\frac{1}{4}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^2-2*x+2)^(1/4),x, algorithm="giac")
[Out] integrate(1/((x^2 - 2*x + 2)^(1/4)*(x - 1)), x)
```

maple [C] time = 1.47, size = 143, normalized size = 4.93

$$\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{x^2-2x+2}-\mathrm{RootOf}\left(\_Z^2+1\right)x^2+2\left(x^2-2x+2\right)^{\frac{3}{4}}+2\,\mathrm{RootOf}\left(\_Z^2+1\right)x-3\,\mathrm{RootOf}\left(\_Z^2+1\right)-2\left(x^2-2x+2\right)^{\frac{1}{4}}}{(-1+x)^2}\right)}{2}-\frac{\ln\left(-\frac{2\left(x^2-2x+2\right)^{\frac{3}{4}}+2\sqrt{x^2-2x+2}+x^2+2\left(x^2-2x+2\right)^{\frac{1}{4}}-2x+3}{(-1+x)^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)/(x^2-2*x+2)^(1/4),x)
```

```
[Out] 1/2*RootOf(_Z^2+1)*ln(-(2*RootOf(_Z^2+1)*(x^2-2*x+2)^(1/2)-RootOf(_Z^2+1)*x
^2+2*(x^2-2*x+2)^(3/4)+2*RootOf(_Z^2+1)*x-3*RootOf(_Z^2+1)-2*(x^2-2*x+2)^(1
/4))/(-1+x)^2)-1/2*ln(-(2*(x^2-2*x+2)^(3/4)+2*(x^2-2*x+2)^(1/2)+x^2+2*(x^2-
2*x+2)^(1/4)-2*x+3)/(-1+x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-2\,x+2\right)^{\frac{1}{4}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^2-2*x+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - 2*x + 2)^(1/4)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(x-1)\,\left(x^2-2\,x+2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)*(x^2 - 2*x + 2)^(1/4)),x)
```

```
[Out] int(1/((x - 1)*(x^2 - 2*x + 2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x-1)\sqrt[4]{x^2-2x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x**2-2*x+2)**(1/4),x)
```

```
[Out] Integral(1/((x - 1)*(x**2 - 2*x + 2)**(1/4)), x)
```

3.345 $\int \frac{1}{(1+x)\sqrt[4]{2+2x+x^2}}\,dx$

Optimal. Leaf size=29

$$\tan^{-1}\left(\sqrt[4]{x^2+2x+2}\right)-\tanh^{-1}\left(\sqrt[4]{x^2+2x+2}\right)$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {694, 266, 63, 298, 203, 206}

$$\tan^{-1}\left(\sqrt[4]{(x+1)^2+1}\right)-\tanh^{-1}\left(\sqrt[4]{(x+1)^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + 2*x + x^2)^(1/4)),x]
```

```
[Out] ArcTan[(1 + (1 + x)^2)^(1/4)] - ArcTanh[(1 + (1 + x)^2)^(1/4)]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+x)\sqrt[4]{2+2x+x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{x\sqrt[4]{1+x^2}}\,dx, x, 1+x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1+x}}\,dx, x, (1+x)^2\right)\\
&= 2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt[4]{1+(1+x)^2}\right)\\
&= -\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1+(1+x)^2}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{1+(1+x)^2}\right)\\
&= \tan^{-1}\left(\sqrt[4]{1+(1+x)^2}\right)-\tanh^{-1}\left(\sqrt[4]{1+(1+x)^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.93

$$\tan^{-1}\left(\sqrt[4]{(x+1)^2+1}\right)-\tanh^{-1}\left(\sqrt[4]{(x+1)^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(2 + 2*x + x^2)^(1/4)),x]
[Out] ArcTan[(1 + (1 + x)^2)^(1/4)] - ArcTanh[(1 + (1 + x)^2)^(1/4)]
```

IntegrateAlgebraic [A] time = 0.06, size = 29, normalized size = 1.00

$$\tan^{-1}\left(\sqrt[4]{x^2+2x+2}\right)-\tanh^{-1}\left(\sqrt[4]{x^2+2x+2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x)*(2 + 2*x + x^2)^(1/4)),x]
[Out] ArcTan[(2 + 2*x + x^2)^(1/4)] - ArcTanh[(2 + 2*x + x^2)^(1/4)]
```

fricas [A] time = 0.39, size = 42, normalized size = 1.45

$$\arctan\left(\left(x^2+2\,x+2\right)^{\frac{1}{4}}\right)-\frac{1}{2}\,\log\left(\left(x^2+2\,x+2\right)^{\frac{1}{4}}+1\right)+\frac{1}{2}\,\log\left(\left(x^2+2\,x+2\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x+2)^(1/4),x, algorithm="fricas")
[Out] arctan((x^2 + 2*x + 2)^(1/4)) - 1/2*log((x^2 + 2*x + 2)^(1/4) + 1) + 1/2*lo
g((x^2 + 2*x + 2)^(1/4) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+2\,x+2\right)^{\frac{1}{4}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x+2)^(1/4),x, algorithm="giac")
[Out] integrate(1/((x^2 + 2*x + 2)^(1/4)*(x + 1)), x)
```

maple [C] time = 1.49, size = 145, normalized size = 5.00

$$\frac{\ln\left(-\frac{2(x^2+2x+2)^{\frac{3}{4}}-2\sqrt{x^2+2x+2}-x^2+2(x^2+2x+2)^{\frac{1}{4}}-2x-3}{(1+x)^2}\right)}{2}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{2\operatorname{RootOf}(\_Z^2+1)\sqrt{x^2+2x+2}-\operatorname{RootOf}(\_Z^2+1)x^2+2(x^2+2x+2)^{\frac{3}{4}}-2\operatorname{RootOf}(\_Z^2+1)x-3\operatorname{RootOf}(\_Z^2+1)-2(x^2+2x+2)^{\frac{1}{4}}}{(1+x)^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^2+2*x+2)^(1/4),x)
```

```
[Out] 1/2*ln(-(2*(x^2+2*x+2)^(3/4)-2*(x^2+2*x+2)^(1/2)-x^2+2*(x^2+2*x+2)^(1/4)-2*
x-3)/(1+x)^2)+1/2*RootOf(_Z^2+1)*ln(-(2*RootOf(_Z^2+1)*(x^2+2*x+2)^(1/2)-Ro
otOf(_Z^2+1)*x^2+2*(x^2+2*x+2)^(3/4)-2*RootOf(_Z^2+1)*x-3*RootOf(_Z^2+1)-2*
(x^2+2*x+2)^(1/4))/(1+x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+2\,x+2\right)^{\frac{1}{4}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 2*x + 2)^(1/4)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(x+1)\,\left(x^2+2\,x+2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)*(2*x + x^2 + 2)^(1/4)),x)
```

```
[Out] int(1/((x + 1)*(2*x + x^2 + 2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)\sqrt[4]{x^2+2x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+2*x+2)**(1/4),x)
```

```
[Out] Integral(1/((x + 1)*(x**2 + 2*x + 2)**(1/4)), x)
```

3.346 $\int \frac{1}{x\sqrt[4]{1+x^3}}\,dx$

Optimal. Leaf size=29

$$\frac{2}{3}\tan^{-1}\left(\sqrt[4]{x^3+1}\right)-\frac{2}{3}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {266, 63, 298, 203, 206}

$$\frac{2}{3}\tan^{-1}\left(\sqrt[4]{x^3+1}\right)-\frac{2}{3}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^3)^(1/4)),x]
```

```
[Out] (2*ArcTan[(1 + x^3)^(1/4)])/3 - (2*ArcTanh[(1 + x^3)^(1/4)])/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt[4]{1+x^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{1+x}}\,dx, x, x^3\right)\\ &= \frac{4}{3}\operatorname{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt[4]{1+x^3}\right)\\ &= -\left(\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1+x^3}\right)\right)+\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{1+x^3}\right)\\ &= \frac{2}{3}\tan^{-1}\left(\sqrt[4]{1+x^3}\right)-\frac{2}{3}\tanh^{-1}\left(\sqrt[4]{1+x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 29, normalized size = 1.00

$$\frac{2}{3}\tan^{-1}\left(\sqrt[4]{x^3+1}\right)-\frac{2}{3}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^3)^(1/4)),x]
[Out] (2*ArcTan[(1 + x^3)^(1/4)])/3 - (2*ArcTanh[(1 + x^3)^(1/4)])/3
```

IntegrateAlgebraic [A] time = 0.03, size = 29, normalized size = 1.00

$$\frac{2}{3}\tan^{-1}\left(\sqrt[4]{x^3+1}\right)-\frac{2}{3}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(1 + x^3)^(1/4)),x]
[Out] (2*ArcTan[(1 + x^3)^(1/4)])/3 - (2*ArcTanh[(1 + x^3)^(1/4)])/3
```

fricas [A] time = 0.39, size = 35, normalized size = 1.21

$$\frac{2}{3}\arctan\left(\left(x^3+1\right)^{\frac{1}{4}}\right)-\frac{1}{3}\log\left(\left(x^3+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{3}\log\left(\left(x^3+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3+1)^(1/4),x, algorithm="fricas")
[Out] 2/3*arctan((x^3 + 1)^(1/4)) - 1/3*log((x^3 + 1)^(1/4) + 1) + 1/3*log((x^3 +
 1)^(1/4) - 1)
```

giac [A] time = 0.36, size = 36, normalized size = 1.24

$$\frac{2}{3}\arctan\left(\left(x^3+1\right)^{\frac{1}{4}}\right)-\frac{1}{3}\log\left(\left(x^3+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{3}\log\left(\left|\left(x^3+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3+1)^(1/4),x, algorithm="giac")
[Out] 2/3*arctan((x^3 + 1)^(1/4)) - 1/3*log((x^3 + 1)^(1/4) + 1) + 1/3*log(abs((x
^3 + 1)^(1/4) - 1))
```

maple [C] time = 0.23, size = 59, normalized size = 2.03

$$\frac{\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)\left(-\frac{\pi\sqrt{2}\,x^3\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{4}\right],[2,2],-x^3\right)}{4\Gamma\left(\frac{3}{4}\right)}+\frac{\left(-3\ln(2)-\frac{\pi}{2}+3\ln(x)\right)\pi\sqrt{2}}{\Gamma\left(\frac{3}{4}\right)}\right)}{6\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^3+1)^(1/4),x)
```

```
[Out] 1/6/Pi*2^(1/2)*GAMMA(3/4)*(-1/4*Pi*2^(1/2)/GAMMA(3/4)*x^3*hypergeom([1,1,5/
4],[2,2],-x^3)+(-3*ln(2)-1/2*Pi+3*ln(x))*Pi*2^(1/2)/GAMMA(3/4))
```

maxima [A] time = 0.52, size = 35, normalized size = 1.21

$$\frac{2}{3}\arctan\left(\left(x^3+1\right)^{\frac{1}{4}}\right)-\frac{1}{3}\log\left(\left(x^3+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{3}\log\left(\left(x^3+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3+1)^(1/4),x, algorithm="maxima")
```

```
[Out] 2/3*arctan((x^3 + 1)^(1/4)) - 1/3*log((x^3 + 1)^(1/4) + 1) + 1/3*log((x^3 +
 1)^(1/4) - 1)
```

mupad [B] time = 0.25, size = 21, normalized size = 0.72

$$\frac{2\,\mathrm{atan}\left(\left(x^3+1\right)^{1/4}\right)}{3}-\frac{2\,\mathrm{atanh}\left(\left(x^3+1\right)^{1/4}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^3 + 1)^(1/4)),x)
```

```
[Out] (2*atan((x^3 + 1)^(1/4)))/3 - (2*atanh((x^3 + 1)^(1/4)))/3
```

sympy [C] time = 0.78, size = 32, normalized size = 1.10

$$-\frac{\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^3}\right)}{3x^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**3+1)**(1/4),x)
```

```
[Out] -gamma(1/4)*hyper((1/4, 1/4), (5/4,), exp_polar(I*pi)/x**3)/(3*x**(3/4)*gam
ma(5/4))
```

3.347 $\int \frac{1}{x\sqrt{-b+ax^3}}\,dx$

Optimal. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {266, 63, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-b + a*x^3]),x]
```

```
[Out] (2*ArcTan[Sqrt[-b + a*x^3]/Sqrt[b]])/(3*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-b+ax^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{-b+ax}}\,dx, x, x^3\right)\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^2}{a}}\,dx, x, \sqrt{-b+ax^3}\right)}{3a}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{-b+ax^3}}{\sqrt{b}}\right)}{3\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-b + a*x^3]),x]
[Out] (2*ArcTan[Sqrt[-b + a*x^3]/Sqrt[b]])/(3*Sqrt[b])
```

IntegrateAlgebraic [A] time = 0.03, size = 29, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[-b + a*x^3]),x]
[Out] (2*ArcTan[Sqrt[-b + a*x^3]/Sqrt[b]])/(3*Sqrt[b])
```

fricas [A] time = 0.39, size = 64, normalized size = 2.21

$$\left[-\frac{\sqrt{-b}\log\left(\frac{ax^3-2\sqrt{ax^3-b}\sqrt{-b}-2b}{x^3}\right)}{3b}, \frac{2\arctan\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3-b)^(1/2),x, algorithm="fricas")
[Out] [-1/3*sqrt(-b)*log((a*x^3 - 2*sqrt(a*x^3 - b)*sqrt(-b) - 2*b)/x^3)/b, 2/3*a
rctan(sqrt(a*x^3 - b)/sqrt(b))/sqrt(b)]
```

giac [A] time = 0.53, size = 21, normalized size = 0.72

$$\frac{2\arctan\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3-b)^(1/2),x, algorithm="giac")
[Out] 2/3*arctan(sqrt(a*x^3 - b)/sqrt(b))/sqrt(b)
```

maple [A] time = 0.02, size = 26, normalized size = 0.90

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^3-b}}{\sqrt{-b}}\right)}{3\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^3-b)^(1/2),x)
[Out] -2/3*arctanh((a*x^3-b)^(1/2)/(-b)^(1/2))/(-b)^(1/2)
```

maxima [A] time = 0.78, size = 21, normalized size = 0.72

$$\frac{2\arctan\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3-b)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*arctan(sqrt(a*x^3 - b)/sqrt(b))/sqrt(b)
```

mupad [B] time = 1.08, size = 37, normalized size = 1.28

$$\frac{\ln\left(\frac{a\,x^3-2\,b+\sqrt{b}\,\sqrt{a\,x^3-b}\,2\mathrm{i}}{x^3}\right)\,1\mathrm{i}}{3\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^3 - b)^(1/2)),x)
```

```
[Out] (log((b^(1/2)*(a*x^3 - b)^(1/2)*2i - 2*b + a*x^3)/x^3)*1i)/(3*b^(1/2))
```

sympy [A] time = 0.96, size = 60, normalized size = 2.07

$$\begin{cases} \frac{2i\operatorname{acosh}\left(\frac{\sqrt{b}}{\sqrt{a}x^{\frac{3}{2}}}\right)}{3\sqrt{b}} & \text{for}\: \left|\frac{b}{ax^{3}}\right| > 1 \\ -\frac{2\operatorname{asin}\left(\frac{\sqrt{b}}{\sqrt{a}x^{\frac{3}{2}}}\right)}{3\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**3-b)**(1/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(b)/(sqrt(a)*x**(3/2)))/(3*sqrt(b)), Abs(b/(a*x**3
)) > 1), (-2*asin(sqrt(b)/(sqrt(a)*x**(3/2)))/(3*sqrt(b)), True))
```

3.348 $\int \frac{x^5}{\sqrt{-b+ax^3}}\,dx$

Optimal. Leaf size=29

$$\frac{2\sqrt{ax^3-b}\left(ax^3+2b\right)}{9a^2}$$

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.45, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {266, 43}

$$\frac{2\left(ax^3-b\right)^{3/2}}{9a^2}+\frac{2b\sqrt{ax^3-b}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[-b + a*x^3],x]
[Out] (2*b*Sqrt[-b + a*x^3])/(3*a^2) + (2*(-b + a*x^3)^(3/2))/(9*a^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{-b+ax^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{\sqrt{-b+ax}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{b}{a\sqrt{-b+ax}}+\frac{\sqrt{-b+ax}}{a}\right)dx,x,x^3\right)\\ &= \frac{2b\sqrt{-b+ax^3}}{3a^2}+\frac{2\left(-b+ax^3\right)^{3/2}}{9a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{2\sqrt{ax^3-b}\left(ax^3+2b\right)}{9a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[-b + a*x^3],x]
[Out] (2*Sqrt[-b + a*x^3]*(2*b + a*x^3))/(9*a^2)
```

IntegrateAlgebraic [A] time = 0.03, size = 29, normalized size = 1.00

$$\frac{2\sqrt{ax^3-b}\left(ax^3+2b\right)}{9a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/Sqrt[-b + a*x^3],x]
[Out] (2*Sqrt[-b + a*x^3]*(2*b + a*x^3))/(9*a^2)
```

fricas [A] time = 0.38, size = 25, normalized size = 0.86

$$\frac{2\left(ax^3+2\,b\right)\sqrt{ax^3-b}}{9\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^3-b)^(1/2),x, algorithm="fricas")
[Out] 2/9*(a*x^3 + 2*b)*sqrt(a*x^3 - b)/a^2
```

giac [A] time = 0.41, size = 34, normalized size = 1.17

$$\frac{2\left(ax^3-b\right)^{\frac{3}{2}}}{9\,a^2}+\frac{2\,\sqrt{ax^3-b}\,b}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^3-b)^(1/2),x, algorithm="giac")
[Out] 2/9*(a*x^3 - b)^(3/2)/a^2 + 2/3*sqrt(a*x^3 - b)*b/a^2
```

maple [A] time = 0.00, size = 26, normalized size = 0.90

$$\frac{2\sqrt{a\,x^3-b}\,\left(a\,x^3+2b\right)}{9a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^3-b)^(1/2),x)
[Out] 2/9*(a*x^3-b)^(1/2)*(a*x^3+2*b)/a^2
```

maxima [A] time = 0.34, size = 34, normalized size = 1.17

$$\frac{2\left(ax^3-b\right)^{\frac{3}{2}}}{9\,a^2}+\frac{2\,\sqrt{ax^3-b}\,b}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^3-b)^(1/2),x, algorithm="maxima")
[Out] 2/9*(a*x^3 - b)^(3/2)/a^2 + 2/3*sqrt(a*x^3 - b)*b/a^2
```

mupad [B] time = 0.40, size = 25, normalized size = 0.86

$$\frac{2\,\sqrt{a\,x^3-b}\,\left(a\,x^3+2\,b\right)}{9\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^3 - b)^(1/2),x)
[Out] (2*(a*x^3 - b)^(1/2)*(2*b + a*x^3))/(9*a^2)
```

sympy [A] time = 0.63, size = 48, normalized size = 1.66

$$\begin{cases} \frac{2x^3\sqrt{ax^3-b}}{9a} + \frac{4b\sqrt{ax^3-b}}{9a^2} & \text{for } a \neq 0 \\ \frac{x^6}{6\sqrt{-b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*x**3-b)**(1/2),x)
```

```
[Out] Piecewise((2*x**3*sqrt(a*x**3 - b)/(9*a) + 4*b*sqrt(a*x**3 - b)/(9*a**2), N
e(a, 0)), (x**6/(6*sqrt(-b)), True))
```

3.349 $\int \frac{1+x}{(-1+x)\sqrt{1-x^2+x^4}}\,dx$

Optimal. Leaf size=29

$$-2\tanh^{-1}\left(\frac{x}{x^2+\sqrt{x^4-x^2+1}-2x+1}\right)$$

Rubi [A] time = 0.11, antiderivative size = 46, normalized size of antiderivative = 1.59, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1741, 12, 1247, 724, 206, 1698}

$$-\tanh^{-1}\left(\frac{x}{\sqrt{x^4-x^2+1}}\right)-\tanh^{-1}\left(\frac{x^2+1}{2\sqrt{x^4-x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-1 + x)*Sqrt[1 - x^2 + x^4]),x]

[Out] -ArcTanh[x/Sqrt[1 - x^2 + x^4]] - ArcTanh[(1 + x^2)/(2*Sqrt[1 - x^2 + x^4])
]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1741

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x
_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x
, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e
^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4
)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e},
```

```
x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4
, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x}{(-1+x)\sqrt{1-x^2+x^4}}\,dx &= \int -\frac{2x}{\left(1-x^2\right)\sqrt{1-x^2+x^4}}\,dx + \int \frac{-1-x^2}{\left(1-x^2\right)\sqrt{1-x^2+x^4}}\,dx \\
&= -\left(2\int \frac{x}{\left(1-x^2\right)\sqrt{1-x^2+x^4}}\,dx\right) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{1-x^2+x^4}}\right) \\
&= -\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2+x^4}}\right) - \text{Subst}\left(\int \frac{1}{(1-x)\sqrt{1-x+x^2}}\,dx, x, x^2\right) \\
&= -\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2+x^4}}\right) + 2\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-1-x^2}{\sqrt{1-x^2+x^4}}\right) \\
&= -\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2+x^4}}\right) - \tanh^{-1}\left(\frac{1+x^2}{2\sqrt{1-x^2+x^4}}\right)
\end{aligned}
$$

Mathematica [C] time = 1.72, size = 742, normalized size = 25.59

$$
\frac{2\sqrt[3]{-1}\left(\sqrt{1-\sqrt[3]{-1}}+\sqrt{1+(-1)^{2/3}}\right)\left(\sqrt{1-\sqrt[3]{-1}}-x\right)^2\sqrt{\frac{\sqrt{1-\sqrt[3]{-1}}\left(\sqrt{1+(-1)^{2/3}}-x\right)}{\left(\sqrt{1-\sqrt[3]{-1}}+\sqrt{1+(-1)^{2/3}}\right)\left(\sqrt{1-\sqrt[3]{-1}}-x\right)}}\sqrt{-\frac{\sqrt{1-\sqrt[3]{-1}}\left(x+\sqrt{1+(-1)^{2/3}}\right)}{\left(\sqrt{1-\sqrt[3]{-1}}-\sqrt{1+(-1)^{2/3}}\right)\left(\sqrt{1-\sqrt[3]{-1}}-x\right)}}\sqrt{\frac{\left(\sqrt{1-\sqrt[3]{-1}}-\sqrt{1+(-1)^{2/3}}\right)x-\sqrt[3]{-1}}{\left(\sqrt{1-\sqrt[3]{-1}}+\sqrt{1+(-1)^{2/3}}\right)\left(\sqrt{1-\sqrt[3]{-1}}-x\right)}}\left(\left(-2+\sqrt[3]{-1}-2\sqrt{1-\sqrt[3]{-1}}\right)F\left(\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{1-\sqrt[3]{-1}}-\sqrt{1+(-1)^{2/3}}\right)\left(x+\sqrt{1-\sqrt[3]{-1}}\right)}{\left(\sqrt{1-\sqrt[3]{-1}}+\sqrt{1+(-1)^{2/3}}\right)\left(\sqrt{1-\sqrt[3]{-1}}-x\right)}}\right)\middle|\frac{\left(\sqrt{1-\sqrt[3]{-1}}+\sqrt{1+(-1)^{2/3}}\right)^2}{\left(\sqrt{1-\sqrt[3]{-1}}-\sqrt{1+(-1)^{2/3}}\right)^2}\right)+4\sqrt{1-\sqrt[3]{-1}}\,\Pi\left(\frac{\left(-1+\sqrt{1-\sqrt[3]{-1}}\right)\left(\sqrt{1-\sqrt[3]{-1}}+\sqrt{1+(-1)^{2/3}}\right)}{\left(1+\sqrt{1-\sqrt[3]{-1}}\right)\left(\sqrt{1-\sqrt[3]{-1}}-\sqrt{1+(-1)^{2/3}}\right)};\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{1-\sqrt[3]{-1}}-\sqrt{1+(-1)^{2/3}}\right)\left(x+\sqrt{1-\sqrt[3]{-1}}\right)}{\left(\sqrt{1-\sqrt[3]{-1}}+\sqrt{1+(-1)^{2/3}}\right)\left(\sqrt{1-\sqrt[3]{-1}}-x\right)}}\right)\middle|\frac{\left(\sqrt{1-\sqrt[3]{-1}}+\sqrt{1+(-1)^{2/3}}\right)^2}{\left(\sqrt{1-\sqrt[3]{-1}}-\sqrt{1+(-1)^{2/3}}\right)^2}\right)\right)}{\sqrt{x^4-x^2+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x)/((-1 + x)*Sqrt[1 - x^2 + x^4]),x]
```

```
[Out] (2*(-1)^(1/3)*(Sqrt[1 - (-1)^(1/3)] + Sqrt[1 + (-1)^(2/3)])*(Sqrt[1 - (-1)^
(1/3)] - x)^2*Sqrt[(Sqrt[1 - (-1)^(1/3)]*(Sqrt[1 + (-1)^(2/3)] - x))/((Sqrt
[1 - (-1)^(1/3)] + Sqrt[1 + (-1)^(2/3)])*(Sqrt[1 - (-1)^(1/3)] - x))]*Sqrt[
-((Sqrt[1 - (-1)^(1/3)]*(Sqrt[1 + (-1)^(2/3)] + x))/((Sqrt[1 - (-1)^(1/3)]
- Sqrt[1 + (-1)^(2/3)])*(Sqrt[1 - (-1)^(1/3)] - x)))]*Sqrt[(-(-1)^(1/3) + (
Sqrt[1 - (-1)^(1/3)] - Sqrt[1 + (-1)^(2/3)])*x)/((Sqrt[1 - (-1)^(1/3)] + Sq
rt[1 + (-1)^(2/3)])*(Sqrt[1 - (-1)^(1/3)] - x))]*((-2 + (-1)^(1/3) - 2*Sqrt
[1 - (-1)^(1/3)])*EllipticF[ArcSin[Sqrt[((Sqrt[1 - (-1)^(1/3)] - Sqrt[1 + (
-1)^(2/3)])*(Sqrt[1 - (-1)^(1/3)] + x))/((Sqrt[1 - (-1)^(1/3)] + Sqrt[1 + (
-1)^(2/3)])*(Sqrt[1 - (-1)^(1/3)] - x))]], (Sqrt[1 - (-1)^(1/3)] + Sqrt[1 +
 (-1)^(2/3)])^2/(Sqrt[1 - (-1)^(1/3)] - Sqrt[1 + (-1)^(2/3)])^2] + 4*Sqrt[1
 - (-1)^(1/3)]*EllipticPi[((-1 + Sqrt[1 - (-1)^(1/3)])*(Sqrt[1 - (-1)^(1/3)
] + Sqrt[1 + (-1)^(2/3)]))/((1 + Sqrt[1 - (-1)^(1/3)])*(Sqrt[1 - (-1)^(1/3)
] - Sqrt[1 + (-1)^(2/3)])), ArcSin[Sqrt[((Sqrt[1 - (-1)^(1/3)] - Sqrt[1 + (
-1)^(2/3)])*(Sqrt[1 - (-1)^(1/3)] + x))/((Sqrt[1 - (-1)^(1/3)] + Sqrt[1 + (
-1)^(2/3)])*(Sqrt[1 - (-1)^(1/3)] - x))]], (Sqrt[1 - (-1)^(1/3)] + Sqrt[1 +
 (-1)^(2/3)])^2/(Sqrt[1 - (-1)^(1/3)] - Sqrt[1 + (-1)^(2/3)])^2]))/Sqrt[1 -
 x^2 + x^4]
```

IntegrateAlgebraic [A] time = 0.43, size = 29, normalized size = 1.00

$$
-2\tanh^{-1}\left(\frac{x}{x^2+\sqrt{x^4-x^2+1}-2x+1}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((-1 + x)*Sqrt[1 - x^2 + x^4]),x]
```

```
[Out] -2*ArcTanh[x/(1 - 2*x + x^2 + Sqrt[1 - x^2 + x^4])]
```

fricas [A] time = 0.45, size = 36, normalized size = 1.24

$$\log\left(\frac{2\,x^2-3\,x-\sqrt{x^4-x^2+1}+2}{x^2-2\,x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^4-x^2+1)^(1/2),x, algorithm="fricas")
[Out] log((2*x^2 - 3*x - sqrt(x^4 - x^2 + 1) + 2)/(x^2 - 2*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^4-x^2+1}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^4-x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate((x + 1)/(sqrt(x^4 - x^2 + 1)*(x - 1)), x)
```

maple [C] time = 0.32, size = 210, normalized size = 7.24

$$\frac{2\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2+2i\sqrt{3}}}{2},\frac{\sqrt{-2-2i\sqrt{3}}}{2}\right)}{\sqrt{2+2i\sqrt{3}}\,\sqrt{x^4-x^2+1}}-\operatorname{arctanh}\left(\frac{x^2+1}{2\sqrt{x^4-x^2+1}}\right)-\frac{2\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\operatorname{EllipticPi}\left(\sqrt{\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x,\frac{1}{\frac{1}{2}+\frac{i\sqrt{3}}{2}},\frac{\sqrt{\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-1+x)/(x^4-x^2+1)^(1/2),x)
[Out] 2/(2+2*I*3^(1/2))^(1/2)*(1-(1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(1/2-1/2*I*3^(
1/2))*x^2)^(1/2)/(x^4-x^2+1)^(1/2)*EllipticF(1/2*x*(2+2*I*3^(1/2))^(1/2),1/
2*(-2-2*I*3^(1/2))^(1/2))-arctanh(1/2*(x^2+1)/(x^4-x^2+1)^(1/2))-2/(1/2+1/2
*I*3^(1/2))^(1/2)*(1-(1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(1/2-1/2*I*3^(1/2))*
x^2)^(1/2)/(x^4-x^2+1)^(1/2)*EllipticPi((1/2+1/2*I*3^(1/2))^(1/2)*x,1/(1/2+
1/2*I*3^(1/2)),(1/2-1/2*I*3^(1/2))^(1/2)/(1/2+1/2*I*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^4-x^2+1}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^4-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + 1)/(sqrt(x^4 - x^2 + 1)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x+1}{(x-1)\,\sqrt{x^4-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x - 1)*(x^4 - x^2 + 1)^(1/2)),x)
[Out] int((x + 1)/((x - 1)*(x^4 - x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{(x-1)\sqrt{x^4-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x**4-x**2+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/((x - 1)*sqrt(x**4 - x**2 + 1)), x)
```

3.350 $\int \frac{(-1+x^4)(1+x^4)}{(1+x^2+x^4)^{5/2}} dx$

Optimal. Leaf size=29

$$-\frac{x(3x^4+2x^2+3)}{3(x^4+x^2+1)^{3/2}}$$

Rubi [A] time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1678, 1588}

$$\frac{x^3}{3(x^4+x^2+1)^{3/2}}-\frac{x}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4)*(1 + x^4))/(1 + x^2 + x^4)^(5/2),x]

[Out] x^3/(3*(1 + x^2 + x^4)^(3/2)) - x/Sqrt[1 + x^2 + x^4]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+x^4)(1+x^4)}{(1+x^2+x^4)^{5/2}} dx &= \frac{x^3}{3(1+x^2+x^4)^{3/2}}+\frac{1}{9}\int \frac{-9+9x^4}{(1+x^2+x^4)^{3/2}} dx\\ &= \frac{x^3}{3(1+x^2+x^4)^{3/2}}-\frac{x}{\sqrt{1+x^2+x^4}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 29, normalized size = 1.00

$$-\frac{x(3x^4+2x^2+3)}{3(x^4+x^2+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)*(1 + x^4))/(1 + x^2 + x^4)^(5/2),x]
```

```
[Out] -1/3*(x*(3 + 2*x^2 + 3*x^4))/(1 + x^2 + x^4)^(3/2)
```

IntegrateAlgebraic [A] time = 0.42, size = 29, normalized size = 1.00

$$-\frac{x\left(3x^4+2x^2+3\right)}{3\left(x^4+x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(1 + x^4))/(1 + x^2 + x^4)^(5/2),x]
```

```
[Out] -1/3*(x*(3 + 2*x^2 + 3*x^4))/(1 + x^2 + x^4)^(3/2)
```

fricas [A] time = 0.39, size = 48, normalized size = 1.66

$$-\frac{\left(3\,x^5+2\,x^3+3\,x\right)\sqrt{x^4+x^2+1}}{3\left(x^8+2\,x^6+3\,x^4+2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)/(x^4+x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*x^5 + 2*x^3 + 3*x)*sqrt(x^4 + x^2 + 1)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2
 + 1)
```

giac [A] time = 0.42, size = 26, normalized size = 0.90

$$-\frac{\left(\left(3\,x^2+2\right)x^2+3\right)x}{3\left(x^4+x^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)/(x^4+x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((3*x^2 + 2)*x^2 + 3)*x/(x^4 + x^2 + 1)^(3/2)
```

maple [A] time = 0.01, size = 40, normalized size = 1.38

$$-\frac{\left(3x^4+2x^2+3\right)x\left(x^2+x+1\right)\left(x^2-x+1\right)}{3\left(x^4+x^2+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^4+1)/(x^4+x^2+1)^(5/2),x)
```

```
[Out] -1/3*(3*x^4+2*x^2+3)*x*(x^2+x+1)*(x^2-x+1)/(x^4+x^2+1)^(5/2)
```

maxima [B] time = 0.51, size = 56, normalized size = 1.93

$$-\frac{\left(3\,x^5+2\,x^3+3\,x\right)\sqrt{x^2+x+1}\,\sqrt{x^2-x+1}}{3\left(x^8+2\,x^6+3\,x^4+2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)/(x^4+x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*x^5 + 2*x^3 + 3*x)*sqrt(x^2 + x + 1)*sqrt(x^2 - x + 1)/(x^8 + 2*x^6
 + 3*x^4 + 2*x^2 + 1)
```

mupad [B] time = 0.13, size = 25, normalized size = 0.86

$$-\frac{x\left(3x^4+2x^2+3\right)}{3\left(x^4+x^2+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(x^4 + 1))/(x^2 + x^4 + 1)^(5/2),x)
```

```
[Out] -(x*(2*x^2 + 3*x^4 + 3))/(3*(x^2 + x^4 + 1)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x-1)(x+1)\left(x^2+1\right)\left(x^4+1\right)}{\left(\left(x^2-x+1\right)\left(x^2+x+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**4+1)/(x**4+x**2+1)**(5/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)*(x**4 + 1)/((x**2 - x + 1)*(x**2 + x +
1))**(5/2), x)
```

3.351 $\int \frac{x^2(-4+x^3)}{(-1+x^3)^{3/4}(1-x^3+x^4)}\,dx$

Optimal. Leaf size=29

$$2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^3-1}}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^3-1}}\right)$$

Rubi [F] time = 1.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(-4+x^3\right)}{\left(-1+x^3\right)^{3/4}\left(1-x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(-4 + x^3))/((-1 + x^3)^(3/4)*(1 - x^3 + x^4)),x]
```

```
[Out] (x*(1 - x^3)^(3/4)*Hypergeometric2F1[1/3, 3/4, 4/3, x^3])/(-1 + x^3)^(3/4)
+ (x^2*(1 - x^3)^(3/4)*Hypergeometric2F1[2/3, 3/4, 5/3, x^3])/(2*(-1 + x^3)
^(3/4)) - Defer[Int][1/((-1 + x^3)^(3/4)*(1 - x^3 + x^4)), x] - Defer[Int][
x/((-1 + x^3)^(3/4)*(1 - x^3 + x^4)), x] - 4*Defer[Int][x^2/((-1 + x^3)^(3/
4)*(1 - x^3 + x^4)), x] + Defer[Int][x^3/((-1 + x^3)^(3/4)*(1 - x^3 + x^4))
, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(-4+x^3\right)}{\left(-1+x^3\right)^{3/4}\left(1-x^3+x^4\right)}\,dx &= \int \left(\frac{1}{\left(-1+x^3\right)^{3/4}}+\frac{x}{\left(-1+x^3\right)^{3/4}}-\frac{1+x+4x^2-x^3}{\left(-1+x^3\right)^{3/4}\left(1-x^3+x^4\right)}\right)dx\\
&= \int \frac{1}{\left(-1+x^3\right)^{3/4}}\,dx+\int \frac{x}{\left(-1+x^3\right)^{3/4}}\,dx-\int \frac{1+x+4x^2-x^3}{\left(-1+x^3\right)^{3/4}\left(1-x^3+x^4\right)}\,dx\\
&= \frac{\left(1-x^3\right)^{3/4}\int \frac{1}{\left(1-x^3\right)^{3/4}}\,dx}{\left(-1+x^3\right)^{3/4}}+\frac{\left(1-x^3\right)^{3/4}\int \frac{x}{\left(1-x^3\right)^{3/4}}\,dx}{\left(-1+x^3\right)^{3/4}}-\int \left(\frac{1}{\left(-1+x^3\right)^{3/4}\left(1-x^3\right.}\right.\\
&= \frac{x\left(1-x^3\right)^{3/4}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};x^3\right)}{\left(-1+x^3\right)^{3/4}}+\frac{x^2\left(1-x^3\right)^{3/4}\,{}_2F_1\left(\frac{2}{3},\frac{3}{4};\frac{5}{3};x^3\right)}{2\left(-1+x^3\right)^{3/4}}-4\int \frac{}{\left(-1+x^3\right.}
\end{aligned}$$

Mathematica [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-4+x^3\right)}{\left(-1+x^3\right)^{3/4}\left(1-x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(-4 + x^3))/((-1 + x^3)^(3/4)*(1 - x^3 + x^4)),x]
```

```
[Out] Integrate[(x^2*(-4 + x^3))/((-1 + x^3)^(3/4)*(1 - x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 2.70, size = 29, normalized size = 1.00

$$2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^3-1}}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(-4 + x^3))/((-1 + x^3)^(3/4)*(1 - x^3 + x^4)),x]
```

```
[Out] 2*ArcTan[x/(-1 + x^3)^(1/4)] - 2*ArcTanh[x/(-1 + x^3)^(1/4)]
```

fricas [A] time = 0.39, size = 48, normalized size = 1.66

$$-2 \arctan\left(\frac{(x^3-1)^{\frac{1}{4}}}{x}\right) - \log\left(\frac{x+(x^3-1)^{\frac{1}{4}}}{x}\right) + \log\left(-\frac{x-(x^3-1)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-4)/(x^3-1)^(3/4)/(x^4-x^3+1),x, algorithm="fricas")
```

```
[Out] -2*arctan((x^3 - 1)^(1/4)/x) - log((x + (x^3 - 1)^(1/4))/x) + log(-(x - (x^
3 - 1)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^3-4)x^2}{(x^4-x^3+1)(x^3-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-4)/(x^3-1)^(3/4)/(x^4-x^3+1),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 4)*x^2/((x^4 - x^3 + 1)*(x^3 - 1)^(3/4)), x)
```

maple [C] time = 0.95, size = 150, normalized size = 5.17

$$-\ln\left(\frac{2(x^3-1)^{\frac{3}{4}}x+2x^2\sqrt{x^3-1}+2(x^3-1)^{\frac{1}{4}}x^3+x^4+x^3-1}{x^4-x^3+1}\right)+\mathrm{RootOf}(\_Z^2+1)\ln\left(\frac{-2\,\mathrm{RootOf}(\_Z^2+1)\sqrt{x^3-1}\,x^2+\mathrm{RootOf}(\_Z^2+1)x^4+2(x^3-1)^{\frac{3}{4}}x-2(x^3-1)^{\frac{1}{4}}x^3+\mathrm{RootOf}(\_Z^2+1)x^3-\mathrm{RootOf}(\_Z^2+1)}{x^4-x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3-4)/(x^3-1)^(3/4)/(x^4-x^3+1),x)
```

```
[Out] -ln((2*(x^3-1)^(3/4)*x+2*x^2*(x^3-1)^(1/2)+2*(x^3-1)^(1/4)*x^3+x^4+x^3-1)/(
x^4-x^3+1))+RootOf(_Z^2+1)*ln((-2*RootOf(_Z^2+1)*(x^3-1)^(1/2)*x^2+RootOf(_
Z^2+1)*x^4+2*(x^3-1)^(3/4)*x-2*(x^3-1)^(1/4)*x^3+RootOf(_Z^2+1)*x^3-RootOf(
_Z^2+1))/(x^4-x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^3-4)x^2}{(x^4-x^3+1)(x^3-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-4)/(x^3-1)^(3/4)/(x^4-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 4)*x^2/((x^4 - x^3 + 1)*(x^3 - 1)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2\,(x^3-4)}{(x^3-1)^{3/4}\,(x^4-x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^3 - 4))/((x^3 - 1)^(3/4)*(x^4 - x^3 + 1)),x)
```

```
[Out] int((x^2*(x^3 - 4))/((x^3 - 1)^(3/4)*(x^4 - x^3 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3-4)/(x**3-1)**(3/4)/(x**4-x**3+1),x)
```

```
[Out] Timed out
```

3.352 $\int \frac{1+2x}{\sqrt{-4-4x-3x^2+2x^3+x^4}}\,dx$

Optimal. Leaf size=29

$$\log\left(x^2+\sqrt{x^4+2x^3-3x^2-4x-4}+x-2\right)$$

Rubi [A] time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.34, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1680, 12, 1107, 621, 206}

$$-\tanh^{-1}\left(\frac{9-4\left(x+\frac{1}{2}\right)^2}{\sqrt{16\left(x+\frac{1}{2}\right)^4-72\left(x+\frac{1}{2}\right)^2-47}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/Sqrt[-4 - 4*x - 3*x^2 + 2*x^3 + x^4],x]
```

```
[Out] -ArcTanh[(9 - 4*(1/2 + x)^2)/Sqrt[-47 - 72*(1/2 + x)^2 + 16*(1/2 + x)^4]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+2x}{\sqrt{-4-4x-3x^2+2x^3+x^4}}\,dx &= \text{Subst}\left(\int \frac{8x}{\sqrt{-47-72x^2+16x^4}}\,dx, x, \frac{1}{2}+x\right)\\
&= 8\,\text{Subst}\left(\int \frac{x}{\sqrt{-47-72x^2+16x^4}}\,dx, x, \frac{1}{2}+x\right)\\
&= 4\,\text{Subst}\left(\int \frac{1}{\sqrt{-47-72x+16x^2}}\,dx, x, \left(\frac{1}{2}+x\right)^2\right)\\
&= 8\,\text{Subst}\left(\int \frac{1}{64-x^2}\,dx, x, \frac{32\left(-2+x+x^2\right)}{\sqrt{-47-72\left(\frac{1}{2}+x\right)^2+(1+2x)^4}}\right)\\
&= -\tanh^{-1}\left(\frac{4\left(2-x-x^2\right)}{\sqrt{-47-18(1+2x)^2+(1+2x)^4}}\right)
\end{aligned}
$$

Mathematica [C] time = 2.48, size = 736, normalized size = 25.38

$$
\frac{i\sqrt{8\sqrt{2}-9}\left(-2x+\sqrt{9+8\sqrt{2}}-1\right)\sqrt{\frac{\left(\sqrt{8\sqrt{2}-9}+i\sqrt{9+8\sqrt{2}}\right)\left(-2ix+\sqrt{8\sqrt{2}-9}-i\right)}{\left(\sqrt{8\sqrt{2}-9}-i\sqrt{9+8\sqrt{2}}\right)\left(2ix+\sqrt{8\sqrt{2}-9}+i\right)}}\left(2ix+\sqrt{8\sqrt{2}-9}+i\right)\sqrt{\frac{2x+\sqrt{9+8\sqrt{2}}+1}{\left(\sqrt{9+8\sqrt{2}}-i\sqrt{8\sqrt{2}-9}\right)\left(2ix+\sqrt{8\sqrt{2}-9}+i\right)}}\left(F\left(\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{-9+8\sqrt{2}}+i\sqrt{9+8\sqrt{2}}\right)\left(-2ix+\sqrt{-9+8\sqrt{2}}-i\right)}{\left(\sqrt{-9+8\sqrt{2}}-i\sqrt{9+8\sqrt{2}}\right)\left(2ix+\sqrt{-9+8\sqrt{2}}+i\right)}}\right)\middle|\frac{9i-\sqrt{47}}{9i+\sqrt{47}}\right)-2\Pi\left(-\frac{\sqrt{-9+8\sqrt{2}}-i\sqrt{9+8\sqrt{2}}}{\sqrt{-9+8\sqrt{2}}+i\sqrt{9+8\sqrt{2}}};\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{-9+8\sqrt{2}}+i\sqrt{9+8\sqrt{2}}\right)\left(-2ix+\sqrt{-9+8\sqrt{2}}-i\right)}{\left(\sqrt{-9+8\sqrt{2}}-i\sqrt{9+8\sqrt{2}}\right)\left(2ix+\sqrt{-9+8\sqrt{2}}+i\right)}}\right)\middle|\frac{9i-\sqrt{47}}{9i+\sqrt{47}}\right)\right)}{\left(\sqrt{8\sqrt{2}-9}+i\sqrt{9+8\sqrt{2}}\right)\sqrt{\frac{-2x+\sqrt{9+8\sqrt{2}}-1}{\left(\sqrt{9+8\sqrt{2}}+i\sqrt{8\sqrt{2}-9}\right)\left(2ix+\sqrt{8\sqrt{2}-9}+i\right)}}\sqrt{x^4+2x^3-3x^2-4x-4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + 2*x)/Sqrt[-4 - 4*x - 3*x^2 + 2*x^3 + x^4],x]
```

```
[Out] (I*Sqrt[-9 + 8*Sqrt[2]]*(-1 + Sqrt[9 + 8*Sqrt[2]] - 2*x)*Sqrt[((Sqrt[-9 + 8
*Sqrt[2]] + I*Sqrt[9 + 8*Sqrt[2]])*(-I + Sqrt[-9 + 8*Sqrt[2]] - (2*I)*x))/(
(Sqrt[-9 + 8*Sqrt[2]] - I*Sqrt[9 + 8*Sqrt[2]])*(I + Sqrt[-9 + 8*Sqrt[2]] +
(2*I)*x))]*(I + Sqrt[-9 + 8*Sqrt[2]] + (2*I)*x)*Sqrt[(1 + Sqrt[9 + 8*Sqrt[2
]] + 2*x)/(((-I)*Sqrt[-9 + 8*Sqrt[2]] + Sqrt[9 + 8*Sqrt[2]])*(I + Sqrt[-9 +
 8*Sqrt[2]] + (2*I)*x))]*(EllipticF[ArcSin[Sqrt[((Sqrt[-9 + 8*Sqrt[2]] + I*
Sqrt[9 + 8*Sqrt[2]])*(-I + Sqrt[-9 + 8*Sqrt[2]] - (2*I)*x))/((Sqrt[-9 + 8*S
qrt[2]] - I*Sqrt[9 + 8*Sqrt[2]])*(I + Sqrt[-9 + 8*Sqrt[2]] + (2*I)*x))]], (
9*I - Sqrt[47])/(9*I + Sqrt[47])] - 2*EllipticPi[-((Sqrt[-9 + 8*Sqrt[2]] -
I*Sqrt[9 + 8*Sqrt[2]])/(Sqrt[-9 + 8*Sqrt[2]] + I*Sqrt[9 + 8*Sqrt[2]])), Arc
Sin[Sqrt[((Sqrt[-9 + 8*Sqrt[2]] + I*Sqrt[9 + 8*Sqrt[2]])*(-I + Sqrt[-9 + 8*
Sqrt[2]] - (2*I)*x))/((Sqrt[-9 + 8*Sqrt[2]] - I*Sqrt[9 + 8*Sqrt[2]])*(I + S
qrt[-9 + 8*Sqrt[2]] + (2*I)*x))]], (9*I - Sqrt[47])/(9*I + Sqrt[47])]))/((S
qrt[-9 + 8*Sqrt[2]] + I*Sqrt[9 + 8*Sqrt[2]])*Sqrt[(-1 + Sqrt[9 + 8*Sqrt[2]]
 - 2*x)/((I*Sqrt[-9 + 8*Sqrt[2]] + Sqrt[9 + 8*Sqrt[2]])*(I + Sqrt[-9 + 8*Sq
rt[2]] + (2*I)*x))]*Sqrt[-4 - 4*x - 3*x^2 + 2*x^3 + x^4])
```

IntegrateAlgebraic [A] time = 0.17, size = 29, normalized size = 1.00

$$
\log\left(x^2+\sqrt{x^4+2x^3-3x^2-4x-4}+x-2\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x)/Sqrt[-4 - 4*x - 3*x^2 + 2*x^3 + x^4],x]
```

```
[Out] Log[-2 + x + x^2 + Sqrt[-4 - 4*x - 3*x^2 + 2*x^3 + x^4]]
```

fricas [A] time = 0.42, size = 27, normalized size = 0.93

$$
\log\left(x^2+x+\sqrt{x^4+2\,x^3-3\,x^2-4\,x-4}-2\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^4+2*x^3-3*x^2-4*x-4)^(1/2),x, algorithm="fricas")
```

```
[Out] log(x^2 + x + sqrt(x^4 + 2*x^3 - 3*x^2 - 4*x - 4) - 2)
```

giac [A] time = 0.38, size = 33, normalized size = 1.14

$$-\log\left(\left|-x^2 - x + \sqrt{\left(x^2 + x\right)^2 - 4\,x^2 - 4\,x - 4} + 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^4+2*x^3-3*x^2-4*x-4)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-x^2 - x + sqrt((x^2 + x)^2 - 4*x^2 - 4*x - 4) + 2))
```

maple [C] time = 0.67, size = 1382, normalized size = 47.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(x^4+2*x^3-3*x^2-4*x-4)^(1/2),x)
```

```
[Out] -2*I*(-1/2*I*(-9+8*2^(1/2))^(1/2)-1/2*(9+8*2^(1/2))^(1/2))*((1/2*(9+8*2^(1/
2))^(1/2)-1/2*I*(-9+8*2^(1/2))^(1/2))*(x+1/2+1/2*I*(-9+8*2^(1/2))^(1/2))/(1
/2*(9+8*2^(1/2))^(1/2)+1/2*I*(-9+8*2^(1/2))^(1/2))/(x+1/2-1/2*I*(-9+8*2^(1/
2))^(1/2)))^(1/2)*(x+1/2-1/2*I*(-9+8*2^(1/2))^(1/2))^2*(I*(-9+8*2^(1/2))^(1
/2)*(x+1/2+1/2*(9+8*2^(1/2))^(1/2))/(-1/2*(9+8*2^(1/2))^(1/2)+1/2*I*(-9+8*2
^(1/2))^(1/2))/(x+1/2-1/2*I*(-9+8*2^(1/2))^(1/2)))^(1/2)*(I*(-9+8*2^(1/2))^
(1/2)*(x+1/2-1/2*(9+8*2^(1/2))^(1/2))/(1/2*(9+8*2^(1/2))^(1/2)+1/2*I*(-9+8*
2^(1/2))^(1/2))/(x+1/2-1/2*I*(-9+8*2^(1/2))^(1/2)))^(1/2)/(1/2*(9+8*2^(1/2)
)^(1/2)-1/2*I*(-9+8*2^(1/2))^(1/2))/(-9+8*2^(1/2))^(1/2)/((x+1/2+1/2*I*(-9+
8*2^(1/2))^(1/2))*(x+1/2-1/2*I*(-9+8*2^(1/2))^(1/2))*(x+1/2+1/2*(9+8*2^(1/2
))^(1/2))*(x+1/2-1/2*(9+8*2^(1/2))^(1/2)))^(1/2)*EllipticF(((1/2*(9+8*2^(1/
2))^(1/2)-1/2*I*(-9+8*2^(1/2))^(1/2))*(x+1/2+1/2*I*(-9+8*2^(1/2))^(1/2))/(1
/2*(9+8*2^(1/2))^(1/2)+1/2*I*(-9+8*2^(1/2))^(1/2))/(x+1/2-1/2*I*(-9+8*2^(1/
2))^(1/2)))^(1/2),((1/2*(9+8*2^(1/2))^(1/2)+1/2*I*(-9+8*2^(1/2))^(1/2))*(-1
/2*I*(-9+8*2^(1/2))^(1/2)-1/2*(9+8*2^(1/2))^(1/2))/(1/2*(9+8*2^(1/2))^(1/2)
-1/2*I*(-9+8*2^(1/2))^(1/2))/(-1/2*(9+8*2^(1/2))^(1/2)+1/2*I*(-9+8*2^(1/2))
^(1/2)))^(1/2))-4*I*(-1/2*I*(-9+8*2^(1/2))^(1/2)-1/2*(9+8*2^(1/2))^(1/2))*(
(1/2*(9+8*2^(1/2))^(1/2)-1/2*I*(-9+8*2^(1/2))^(1/2))*(x+1/2+1/2*I*(-9+8*2^(
1/2))^(1/2))/(1/2*(9+8*2^(1/2))^(1/2)+1/2*I*(-9+8*2^(1/2))^(1/2))/(x+1/2-1/
2*I*(-9+8*2^(1/2))^(1/2)))^(1/2)*(x+1/2-1/2*I*(-9+8*2^(1/2))^(1/2))^2*(I*(-
9+8*2^(1/2))^(1/2)*(x+1/2+1/2*(9+8*2^(1/2))^(1/2))/(-1/2*(9+8*2^(1/2))^(1/2
)+1/2*I*(-9+8*2^(1/2))^(1/2))/(x+1/2-1/2*I*(-9+8*2^(1/2))^(1/2)))^(1/2)*(I*
(-9+8*2^(1/2))^(1/2)*(x+1/2-1/2*(9+8*2^(1/2))^(1/2))/(1/2*(9+8*2^(1/2))^(1/
2)+1/2*I*(-9+8*2^(1/2))^(1/2))/(x+1/2-1/2*I*(-9+8*2^(1/2))^(1/2)))^(1/2)/(1
/2*(9+8*2^(1/2))^(1/2)-1/2*I*(-9+8*2^(1/2))^(1/2))/(-9+8*2^(1/2))^(1/2)/((x
+1/2+1/2*I*(-9+8*2^(1/2))^(1/2))*(x+1/2-1/2*I*(-9+8*2^(1/2))^(1/2))*(x+1/2+
1/2*(9+8*2^(1/2))^(1/2))*(x+1/2-1/2*(9+8*2^(1/2))^(1/2)))^(1/2)*((-1/2+1/2*
I*(-9+8*2^(1/2))^(1/2))*EllipticF(((1/2*(9+8*2^(1/2))^(1/2)-1/2*I*(-9+8*2^(
1/2))^(1/2))*(x+1/2+1/2*I*(-9+8*2^(1/2))^(1/2))/(1/2*(9+8*2^(1/2))^(1/2)+1/
2*I*(-9+8*2^(1/2))^(1/2))/(x+1/2-1/2*I*(-9+8*2^(1/2))^(1/2)))^(1/2),((1/2*(
9+8*2^(1/2))^(1/2)+1/2*I*(-9+8*2^(1/2))^(1/2))*(-1/2*I*(-9+8*2^(1/2))^(1/2)
-1/2*(9+8*2^(1/2))^(1/2))/(1/2*(9+8*2^(1/2))^(1/2)-1/2*I*(-9+8*2^(1/2))^(1/
2))/(-1/2*(9+8*2^(1/2))^(1/2)+1/2*I*(-9+8*2^(1/2))^(1/2)))^(1/2))-I*(-9+8*2
^(1/2))^(1/2)*EllipticPi(((1/2*(9+8*2^(1/2))^(1/2)-1/2*I*(-9+8*2^(1/2))^(1/
2))*(x+1/2+1/2*I*(-9+8*2^(1/2))^(1/2))/(1/2*(9+8*2^(1/2))^(1/2)+1/2*I*(-9+8
*2^(1/2))^(1/2))/(x+1/2-1/2*I*(-9+8*2^(1/2))^(1/2)))^(1/2),(1/2*(9+8*2^(1/2
))^(1/2)+1/2*I*(-9+8*2^(1/2))^(1/2))/(1/2*(9+8*2^(1/2))^(1/2)-1/2*I*(-9+8*2
^(1/2))^(1/2)),((1/2*(9+8*2^(1/2))^(1/2)+1/2*I*(-9+8*2^(1/2))^(1/2))*(-1/2*
I*(-9+8*2^(1/2))^(1/2)-1/2*(9+8*2^(1/2))^(1/2))/(1/2*(9+8*2^(1/2))^(1/2)-1/
```

```
2*I*(-9+8*2^(1/2))^(1/2))/(-1/2*(9+8*2^(1/2))^(1/2)+1/2*I*(-9+8*2^(1/2))^(1
/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x+1}{\sqrt{x^4+2\,x^3-3\,x^2-4\,x-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^4+2*x^3-3*x^2-4*x-4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)/sqrt(x^4 + 2*x^3 - 3*x^2 - 4*x - 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{2\,x+1}{\sqrt{x^4+2\,x^3-3\,x^2-4\,x-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/(2*x^3 - 3*x^2 - 4*x + x^4 - 4)^(1/2),x)
```

```
[Out] int((2*x + 1)/(2*x^3 - 3*x^2 - 4*x + x^4 - 4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\sqrt{x^4+2x^3-3x^2-4x-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**4+2*x**3-3*x**2-4*x-4)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/sqrt(x**4 + 2*x**3 - 3*x**2 - 4*x - 4), x)
```

3.353 $\int \frac{1}{x\sqrt{-b+ax^4}}\,dx$

Optimal. Leaf size=29

$$\frac{\tan^{-1}\left(\frac{\sqrt{ax^4-b}}{\sqrt{b}}\right)}{2\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {266, 63, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{ax^4-b}}{\sqrt{b}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-b + a*x^4]),x]
```

```
[Out] ArcTan[Sqrt[-b + a*x^4]/Sqrt[b]]/(2*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-b+ax^4}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{-b+ax}}\,dx, x, x^4\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^2}{a}}\,dx, x, \sqrt{-b+ax^4}\right)}{2a}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{-b+ax^4}}{\sqrt{b}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{ax^4-b}}{\sqrt{b}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-b + a*x^4]),x]
```

```
[Out] ArcTan[Sqrt[-b + a*x^4]/Sqrt[b]]/(2*Sqrt[b])
```

IntegrateAlgebraic [A] time = 0.04, size = 29, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{ax^4-b}}{\sqrt{b}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[-b + a*x^4]),x]
```

```
[Out] ArcTan[Sqrt[-b + a*x^4]/Sqrt[b]]/(2*Sqrt[b])
```

fricas [A] time = 0.41, size = 64, normalized size = 2.21

$$\left[-\frac{\sqrt{-b}\log\left(\frac{ax^4-2\sqrt{ax^4-b}\sqrt{-b}-2b}{x^4}\right)}{4b}, \frac{\arctan\left(\frac{\sqrt{ax^4-b}}{\sqrt{b}}\right)}{2\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4-b)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-b)*log((a*x^4 - 2*sqrt(a*x^4 - b)*sqrt(-b) - 2*b)/x^4)/b, 1/2*a
rctan(sqrt(a*x^4 - b)/sqrt(b))/sqrt(b)]
```

giac [A] time = 0.29, size = 21, normalized size = 0.72

$$\frac{\arctan\left(\frac{\sqrt{ax^4-b}}{\sqrt{b}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4-b)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(sqrt(a*x^4 - b)/sqrt(b))/sqrt(b)
```

maple [A] time = 0.01, size = 35, normalized size = 1.21

$$-\frac{\ln\left(\frac{-2b+2\sqrt{-b}\,\sqrt{a\,x^4-b}}{x^2}\right)}{2\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^4-b)^(1/2),x)
```

```
[Out] -1/2/(-b)^(1/2)*ln((-2*b+2*(-b)^(1/2)*(a*x^4-b)^(1/2))/x^2)
```

maxima [A] time = 0.70, size = 21, normalized size = 0.72

$$\frac{\arctan\left(\frac{\sqrt{ax^4-b}}{\sqrt{b}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4-b)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(sqrt(a*x^4 - b)/sqrt(b))/sqrt(b)
```

mupad [B] time = 0.38, size = 21, normalized size = 0.72

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a\,x^4-b}}{\sqrt{b}}\right)}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^4 - b)^(1/2)),x)
```

```
[Out] atan((a*x^4 - b)^(1/2)/b^(1/2))/(2*b^(1/2))
```

sympy [A] time = 0.93, size = 53, normalized size = 1.83

$$\begin{cases} \frac{i \operatorname{acosh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x^{2}}\right)}{2\sqrt{b}} & \text{for}\ \left|\frac{b}{ax^{4}}\right| > 1 \\ -\frac{\operatorname{asin}\left(\frac{\sqrt{b}}{\sqrt{a}\,x^{2}}\right)}{2\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**4-b)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(sqrt(b)/(sqrt(a)*x**2))/(2*sqrt(b)), Abs(b/(a*x**4)) > 1
), (-asin(sqrt(b)/(sqrt(a)*x**2))/(2*sqrt(b)), True))
```

3.354 $\int \frac{-1+2x^6}{\left(1+x^6\right)\sqrt{1-2x^2+x^6}}\,dx$

Optimal. Leaf size=29

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^6-2x^2+1}}\right)}{\sqrt{2}}$$

Rubi [F] time = 1.55, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+2x^6}{\left(1+x^6\right)\sqrt{1-2x^2+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 2*x^6)/((1 + x^6)*Sqrt[1 - 2*x^2 + x^6]),x]
```

```
[Out] 2*Defer[Int][1/Sqrt[1 - 2*x^2 + x^6], x] - (I/2)*Defer[Int][1/((I - x)*Sqrt
[1 - 2*x^2 + x^6]), x] - (I/2)*Defer[Int][1/((I + x)*Sqrt[1 - 2*x^2 + x^6])
, x] - (Sqrt[1 - I*Sqrt[3]]*Defer[Int][1/((Sqrt[1 - I*Sqrt[3]] - Sqrt[2]*x)
*Sqrt[1 - 2*x^2 + x^6]), x])/2 - (Sqrt[1 + I*Sqrt[3]]*Defer[Int][1/((Sqrt[1
 + I*Sqrt[3]] - Sqrt[2]*x)*Sqrt[1 - 2*x^2 + x^6]), x])/2 - (Sqrt[1 - I*Sqrt
[3]]*Defer[Int][1/((Sqrt[1 - I*Sqrt[3]] + Sqrt[2]*x)*Sqrt[1 - 2*x^2 + x^6])
, x])/2 - (Sqrt[1 + I*Sqrt[3]]*Defer[Int][1/((Sqrt[1 + I*Sqrt[3]] + Sqrt[2]
*x)*Sqrt[1 - 2*x^2 + x^6]), x])/2
```

Rubi steps

$$\int \frac{-1+2x^6}{\left(1+x^6\right)\sqrt{1-2x^2+x^6}}\,dx = \int \left(\frac{2}{\sqrt{1-2x^2+x^6}}-\frac{3}{\left(1+x^6\right)\sqrt{1-2x^2+x^6}}\right)dx$$

$$=2\int \frac{1}{\sqrt{1-2x^2+x^6}}\,dx-3\int \frac{1}{\left(1+x^6\right)\sqrt{1-2x^2+x^6}}\,dx$$

$$=2\int \frac{1}{\sqrt{1-2x^2+x^6}}\,dx-3\int \left(\frac{1}{3\left(1+x^2\right)\sqrt{1-2x^2+x^6}}+\frac{2-x^2}{3\left(1-x^2+x^4\right)\sqrt{1-2x^2+x^6}}\right)dx$$

$$=2\int \frac{1}{\sqrt{1-2x^2+x^6}}\,dx-\int \frac{1}{\left(1+x^2\right)\sqrt{1-2x^2+x^6}}\,dx-\int \frac{2-x^2}{\left(1-x^2+x^4\right)\sqrt{1-2x^2+x^6}}\,dx$$

$$=2\int \frac{1}{\sqrt{1-2x^2+x^6}}\,dx-\int \left(\frac{i}{2(i-x)\sqrt{1-2x^2+x^6}}+\frac{i}{2(i+x)\sqrt{1-2x^2+x^6}}\right)dx$$

$$=-\left(\frac{1}{2}i\int \frac{1}{(i-x)\sqrt{1-2x^2+x^6}}\,dx\right)-\frac{1}{2}i\int \frac{1}{(i+x)\sqrt{1-2x^2+x^6}}\,dx+2\int \frac{}{\sqrt{1-}}$$

$$=-\left(\frac{1}{2}i\int \frac{1}{(i-x)\sqrt{1-2x^2+x^6}}\,dx\right)-\frac{1}{2}i\int \frac{1}{(i+x)\sqrt{1-2x^2+x^6}}\,dx+2\int \frac{}{\sqrt{1-}}$$

$$=-\left(\frac{1}{2}i\int \frac{1}{(i-x)\sqrt{1-2x^2+x^6}}\,dx\right)-\frac{1}{2}i\int \frac{1}{(i+x)\sqrt{1-2x^2+x^6}}\,dx+2\int \frac{}{\sqrt{1-}}$$

Mathematica [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{-1+2x^6}{\left(1+x^6\right)\sqrt{1-2x^2+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + 2*x^6)/((1 + x^6)*Sqrt[1 - 2*x^2 + x^6]),x]
[Out] Integrate[(-1 + 2*x^6)/((1 + x^6)*Sqrt[1 - 2*x^2 + x^6]), x]
```

IntegrateAlgebraic [A] time = 1.23, size = 29, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^6-2x^2+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x^6)/((1 + x^6)*Sqrt[1 - 2*x^2 + x^6]),x]
[Out] -(ArcTan[(Sqrt[2]*x)/Sqrt[1 - 2*x^2 + x^6]]/Sqrt[2])
```

fricas [A] time = 0.47, size = 36, normalized size = 1.24

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{2\sqrt{2}\sqrt{x^6-2x^2+1}\,x}{x^6-4x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6-1)/(x^6+1)/(x^6-2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/4*sqrt(2)*arctan(2*sqrt(2)*sqrt(x^6 - 2*x^2 + 1)*x/(x^6 - 4*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^6-1}{\sqrt{x^6-2x^2+1}\left(x^6+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6-1)/(x^6+1)/(x^6-2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate((2*x^6 - 1)/(sqrt(x^6 - 2*x^2 + 1)*(x^6 + 1)), x)
```

maple [C] time = 0.40, size = 74, normalized size = 2.55

$$-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)x^6-4\,\mathrm{RootOf}\left(\_Z^2+2\right)x^2+4\sqrt{x^6-2x^2+1}\;x+\mathrm{RootOf}\left(\_Z^2+2\right)}{\left(x^2+1\right)\left(x^4-x^2+1\right)}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6-1)/(x^6+1)/(x^6-2*x^2+1)^(1/2),x)
[Out] -1/4*RootOf(_Z^2+2)*ln(-(RootOf(_Z^2+2)*x^6-4*RootOf(_Z^2+2)*x^2+4*(x^6-2*x
^2+1)^(1/2)*x+RootOf(_Z^2+2))/(x^2+1)/(x^4-x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^6-1}{\sqrt{x^6-2x^2+1}\left(x^6+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6-1)/(x^6+1)/(x^6-2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x^6 - 1)/(sqrt(x^6 - 2*x^2 + 1)*(x^6 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{2\,x^6-1}{\left(x^6+1\right)\,\sqrt{x^6-2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6 - 1)/((x^6 + 1)*(x^6 - 2*x^2 + 1)^(1/2)),x)
[Out] int((2*x^6 - 1)/((x^6 + 1)*(x^6 - 2*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^6-1}{\sqrt{(x-1)(x+1)\left(x^4+x^2-1\right)}\left(x^2+1\right)\left(x^4-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**6-1)/(x**6+1)/(x**6-2*x**2+1)**(1/2),x)
[Out] Integral((2*x**6 - 1)/(sqrt((x - 1)*(x + 1)*(x**4 + x**2 - 1))*(x**2 + 1)*(
x**4 - x**2 + 1)), x)
```

3.355 $\int \frac{1+2x^6}{(-1+x^6)\sqrt{-1-2x^2+x^6}}\,dx$

Optimal. Leaf size=29

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^6-2x^2-1}}\right)}{\sqrt{2}}$$

Rubi [F] time = 1.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+2x^6}{(-1+x^6)\sqrt{-1-2x^2+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + 2*x^6)/((-1 + x^6)*Sqrt[-1 - 2*x^2 + x^6]),x]
```

```
[Out] 2*Defer[Int][1/Sqrt[-1 - 2*x^2 + x^6], x] + Defer[Int][1/((-1 + x)*Sqrt[-1
- 2*x^2 + x^6]), x]/2 - Defer[Int][1/((1 + x)*Sqrt[-1 - 2*x^2 + x^6]), x]/2
 + ((1 + I*Sqrt[3])*Defer[Int][1/((-1 - I*Sqrt[3] + 2*x)*Sqrt[-1 - 2*x^2 +
x^6]), x])/2 - ((1 - I*Sqrt[3])*Defer[Int][1/((1 - I*Sqrt[3] + 2*x)*Sqrt[-1
 - 2*x^2 + x^6]), x])/2 + ((1 - I*Sqrt[3])*Defer[Int][1/((-1 + I*Sqrt[3] +
2*x)*Sqrt[-1 - 2*x^2 + x^6]), x])/2 - ((1 + I*Sqrt[3])*Defer[Int][1/((1 + I
*Sqrt[3] + 2*x)*Sqrt[-1 - 2*x^2 + x^6]), x])/2
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^6}{(-1+x^6)\sqrt{-1-2x^2+x^6}}\,dx &= \int \left(\frac{2}{\sqrt{-1-2x^2+x^6}}+\frac{3}{(-1+x^6)\sqrt{-1-2x^2+x^6}}\right)dx\\
&= 2\int \frac{1}{\sqrt{-1-2x^2+x^6}}\,dx+3\int \frac{1}{(-1+x^6)\sqrt{-1-2x^2+x^6}}\,dx\\
&= 2\int \frac{1}{\sqrt{-1-2x^2+x^6}}\,dx+3\int \left(\frac{1}{3(-1+x^2)\sqrt{-1-2x^2+x^6}}+\frac{}{6(1-x+}\right.\\
&= \frac{1}{2}\int \frac{-2+x}{(1-x+x^2)\sqrt{-1-2x^2+x^6}}\,dx+\frac{1}{2}\int \frac{-2-x}{(1+x+x^2)\sqrt{-1-2x^2+x^6}}\\
&= \frac{1}{2}\int \left(\frac{1+i\sqrt{3}}{(-1-i\sqrt{3}+2x)\sqrt{-1-2x^2+x^6}}+\frac{1-i\sqrt{3}}{(-1+i\sqrt{3}+2x)\sqrt{-1-2x^2+x}}\right.\\
&= \frac{1}{2}\int \frac{1}{(-1+x)\sqrt{-1-2x^2+x^6}}\,dx-\frac{1}{2}\int \frac{1}{(1+x)\sqrt{-1-2x^2+x^6}}\,dx+2\int
\end{aligned}$$

Mathematica [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1+2x^6}{(-1+x^6)\sqrt{-1-2x^2+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + 2*x^6)/((-1 + x^6)*Sqrt[-1 - 2*x^2 + x^6]),x]
```

```
[Out] Integrate[(1 + 2*x^6)/((-1 + x^6)*Sqrt[-1 - 2*x^2 + x^6]), x]
```

IntegrateAlgebraic [A] time = 1.24, size = 29, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^6-2x^2-1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x^6)/((-1 + x^6)*Sqrt[-1 - 2*x^2 + x^6]),x]
[Out] -(ArcTan[(Sqrt[2]*x)/Sqrt[-1 - 2*x^2 + x^6]]/Sqrt[2])
```

fricas [A] time = 0.43, size = 36, normalized size = 1.24

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{2\sqrt{2}\sqrt{x^6-2x^2-1}\,x}{x^6-4x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+1)/(x^6-1)/(x^6-2*x^2-1)^(1/2),x, algorithm="fricas")
[Out] -1/4*sqrt(2)*arctan(2*sqrt(2)*sqrt(x^6 - 2*x^2 - 1)*x/(x^6 - 4*x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^6+1}{\sqrt{x^6-2x^2-1}\,(x^6-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+1)/(x^6-1)/(x^6-2*x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate((2*x^6 + 1)/(sqrt(x^6 - 2*x^2 - 1)*(x^6 - 1)), x)
```

maple [C] time = 0.41, size = 85, normalized size = 2.93

$$-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)x^6-4\operatorname{RootOf}\left(\_Z^2+2\right)x^2+4\sqrt{x^6-2x^2-1}\,x-\operatorname{RootOf}\left(\_Z^2+2\right)}{(-1+x)(1+x)\left(x^2+x+1\right)\left(x^2-x+1\right)}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6+1)/(x^6-1)/(x^6-2*x^2-1)^(1/2),x)
[Out] -1/4*RootOf(_Z^2+2)*ln(-(RootOf(_Z^2+2)*x^6-4*RootOf(_Z^2+2)*x^2+4*(x^6-2*x
^2-1)^(1/2)*x-RootOf(_Z^2+2))/(-1+x)/(1+x)/(x^2+x+1)/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^6+1}{\sqrt{x^6-2x^2-1}\,(x^6-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+1)/(x^6-1)/(x^6-2*x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x^6 + 1)/(sqrt(x^6 - 2*x^2 - 1)*(x^6 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{2x^6+1}{(x^6-1)\sqrt{x^6-2x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6 + 1)/((x^6 - 1)*(x^6 - 2*x^2 - 1)^(1/2)),x)
```

```
[Out] int((2*x^6 + 1)/((x^6 - 1)*(x^6 - 2*x^2 - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^6+1}{\sqrt{(x^2+1)(x^4-x^2-1)}\,(x-1)(x+1)(x^2-x+1)(x^2+x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**6+1)/(x**6-1)/(x**6-2*x**2-1)**(1/2),x)
```

```
[Out] Integral((2*x**6 + 1)/(sqrt((x**2 + 1)*(x**4 - x**2 - 1))*(x - 1)*(x + 1)*(
x**2 - x + 1)*(x**2 + x + 1)), x)
```

3.356 $\int x^8 \sqrt[3]{-1+x^3}\,dx$

Optimal. Leaf size=30

$$\frac{1}{140}\sqrt[3]{x^3-1}\left(14x^9-2x^6-3x^3-9\right)$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.33, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{1}{10}\left(x^3-1\right)^{10/3}+\frac{2}{7}\left(x^3-1\right)^{7/3}+\frac{1}{4}\left(x^3-1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(-1 + x^3)^(1/3),x]
```

```
[Out] (-1 + x^3)^(4/3)/4 + (2*(-1 + x^3)^(7/3))/7 + (-1 + x^3)^(10/3)/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^8 \sqrt[3]{-1+x^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \sqrt[3]{-1+x}\,x^2\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\sqrt[3]{-1+x}+2(-1+x)^{4/3}+(-1+x)^{7/3}\right)dx,x,x^3\right)\\ &= \frac{1}{4}\left(-1+x^3\right)^{4/3}+\frac{2}{7}\left(-1+x^3\right)^{7/3}+\frac{1}{10}\left(-1+x^3\right)^{10/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.83

$$\frac{1}{140}\left(x^3-1\right)^{4/3}\left(14x^6+12x^3+9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(-1 + x^3)^(1/3),x]
```

```
[Out] ((-1 + x^3)^(4/3)*(9 + 12*x^3 + 14*x^6))/140
```

IntegrateAlgebraic [A] time = 0.03, size = 25, normalized size = 0.83

$$\frac{1}{140}\left(x^3-1\right)^{4/3}\left(14x^6+12x^3+9\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8*(-1 + x^3)^(1/3),x]
[Out] ((-1 + x^3)^(4/3)*(9 + 12*x^3 + 14*x^6))/140
```

fricas [A] time = 0.39, size = 26, normalized size = 0.87

$$\frac{1}{140} \left(14 x^9 - 2 x^6 - 3 x^3 - 9\right) \left(x^3 - 1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(x^3-1)^(1/3),x, algorithm="fricas")
[Out] 1/140*(14*x^9 - 2*x^6 - 3*x^3 - 9)*(x^3 - 1)^(1/3)
```

giac [A] time = 0.28, size = 28, normalized size = 0.93

$$\frac{1}{10} \left(x^3 - 1\right)^{\frac{10}{3}} + \frac{2}{7} \left(x^3 - 1\right)^{\frac{7}{3}} + \frac{1}{4} \left(x^3 - 1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(x^3-1)^(1/3),x, algorithm="giac")
[Out] 1/10*(x^3 - 1)^(10/3) + 2/7*(x^3 - 1)^(7/3) + 1/4*(x^3 - 1)^(4/3)
```

maple [A] time = 0.00, size = 31, normalized size = 1.03

$$\frac{(-1 + x) \left(x^2 + x + 1\right) \left(14x^6 + 12x^3 + 9\right) \left(x^3 - 1\right)^{\frac{1}{3}}}{140}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(x^3-1)^(1/3),x)
[Out] 1/140*(-1+x)*(x^2+x+1)*(14*x^6+12*x^3+9)*(x^3-1)^(1/3)
```

maxima [A] time = 0.54, size = 28, normalized size = 0.93

$$\frac{1}{10} \left(x^3 - 1\right)^{\frac{10}{3}} + \frac{2}{7} \left(x^3 - 1\right)^{\frac{7}{3}} + \frac{1}{4} \left(x^3 - 1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(x^3-1)^(1/3),x, algorithm="maxima")
[Out] 1/10*(x^3 - 1)^(10/3) + 2/7*(x^3 - 1)^(7/3) + 1/4*(x^3 - 1)^(4/3)
```

mupad [B] time = 0.22, size = 26, normalized size = 0.87

$$-\left(x^3 - 1\right)^{1/3} \left(-\frac{x^9}{10} + \frac{x^6}{70} + \frac{3 x^3}{140} + \frac{9}{140}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(x^3 - 1)^(1/3),x)
[Out] -(x^3 - 1)^(1/3)*((3*x^3)/140 + x^6/70 - x^9/10 + 9/140)
```

sympy [B] time = 0.82, size = 53, normalized size = 1.77

$$\frac{x^9 \sqrt[3]{x^3 - 1}}{10} - \frac{x^6 \sqrt[3]{x^3 - 1}}{70} - \frac{3x^3 \sqrt[3]{x^3 - 1}}{140} - \frac{9 \sqrt[3]{x^3 - 1}}{140}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(x**3-1)**(1/3),x)
```

```
[Out] x**9*(x**3 - 1)**(1/3)/10 - x**6*(x**3 - 1)**(1/3)/70 - 3*x**3*(x**3 - 1)**
(1/3)/140 - 9*(x**3 - 1)**(1/3)/140
```

3.357 $\int x^8 \sqrt[4]{1+x^3}\, dx$

Optimal. Leaf size=30

$$\frac{4\sqrt[4]{x^3+1}\left(45x^9+5x^6-8x^3+32\right)}{1755}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.33, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{4}{39}\left(x^3+1\right)^{13/4}-\frac{8}{27}\left(x^3+1\right)^{9/4}+\frac{4}{15}\left(x^3+1\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(1 + x^3)^(1/4),x]
```

```
[Out] (4*(1 + x^3)^(5/4))/15 - (8*(1 + x^3)^(9/4))/27 + (4*(1 + x^3)^(13/4))/39
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^8 \sqrt[4]{1+x^3}\, dx &= \frac{1}{3}\text{Subst}\left(\int x^2\sqrt[4]{1+x}\, dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\sqrt[4]{1+x}-2(1+x)^{5/4}+(1+x)^{9/4}\right) dx, x, x^3\right)\\ &= \frac{4}{15}\left(1+x^3\right)^{5/4}-\frac{8}{27}\left(1+x^3\right)^{9/4}+\frac{4}{39}\left(1+x^3\right)^{13/4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.83

$$\frac{4\left(x^3+1\right)^{5/4}\left(45x^6-40x^3+32\right)}{1755}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(1 + x^3)^(1/4),x]
```

```
[Out] (4*(1 + x^3)^(5/4)*(32 - 40*x^3 + 45*x^6))/1755
```

IntegrateAlgebraic [A] time = 0.02, size = 30, normalized size = 1.00

$$\frac{4\sqrt[4]{x^3+1}\left(45x^9+5x^6-8x^3+32\right)}{1755}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8*(1 + x^3)^(1/4),x]
[Out] (4*(1 + x^3)^(1/4)*(32 - 8*x^3 + 5*x^6 + 45*x^9))/1755
```

fricas [A] time = 0.39, size = 26, normalized size = 0.87

$$\frac{4}{1755}\left(45\,x^{9}+5\,x^{6}-8\,x^{3}+32\right)\left(x^{3}+1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(x^3+1)^(1/4),x, algorithm="fricas")
[Out] 4/1755*(45*x^9 + 5*x^6 - 8*x^3 + 32)*(x^3 + 1)^(1/4)
```

giac [A] time = 0.45, size = 28, normalized size = 0.93

$$\frac{4}{39}\left(x^{3}+1\right)^{\frac{13}{4}}-\frac{8}{27}\left(x^{3}+1\right)^{\frac{9}{4}}+\frac{4}{15}\left(x^{3}+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(x^3+1)^(1/4),x, algorithm="giac")
[Out] 4/39*(x^3 + 1)^(13/4) - 8/27*(x^3 + 1)^(9/4) + 4/15*(x^3 + 1)^(5/4)
```

maple [A] time = 0.01, size = 33, normalized size = 1.10

$$\frac{4\left(1+x\right)\left(x^{2}-x+1\right)\left(45x^{6}-40x^{3}+32\right)\left(x^{3}+1\right)^{\frac{1}{4}}}{1755}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(x^3+1)^(1/4),x)
[Out] 4/1755*(1+x)*(x^2-x+1)*(45*x^6-40*x^3+32)*(x^3+1)^(1/4)
```

maxima [A] time = 0.40, size = 28, normalized size = 0.93

$$\frac{4}{39}\left(x^{3}+1\right)^{\frac{13}{4}}-\frac{8}{27}\left(x^{3}+1\right)^{\frac{9}{4}}+\frac{4}{15}\left(x^{3}+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(x^3+1)^(1/4),x, algorithm="maxima")
[Out] 4/39*(x^3 + 1)^(13/4) - 8/27*(x^3 + 1)^(9/4) + 4/15*(x^3 + 1)^(5/4)
```

mupad [B] time = 0.21, size = 25, normalized size = 0.83

$$\left(x^{3}+1\right)^{1/4}\left(\frac{4\,x^{9}}{39}+\frac{4\,x^{6}}{351}-\frac{32\,x^{3}}{1755}+\frac{128}{1755}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(x^3 + 1)^(1/4),x)
[Out] (x^3 + 1)^(1/4)*((4*x^6)/351 - (32*x^3)/1755 + (4*x^9)/39 + 128/1755)
```

sympy [B] time = 1.41, size = 56, normalized size = 1.87

$$\frac{4x^{9}\sqrt[4]{x^{3}+1}}{39}+\frac{4x^{6}\sqrt[4]{x^{3}+1}}{351}-\frac{32x^{3}\sqrt[4]{x^{3}+1}}{1755}+\frac{128\sqrt[4]{x^{3}+1}}{1755}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(x**3+1)**(1/4),x)
```

```
[Out] 4*x**9*(x**3 + 1)**(1/4)/39 + 4*x**6*(x**3 + 1)**(1/4)/351 - 32*x**3*(x**3
+ 1)**(1/4)/1755 + 128*(x**3 + 1)**(1/4)/1755
```

3.358 $\int \frac{1}{x^6 \sqrt[3]{-x+x^3}}\, dx$

Optimal. Leaf size=30

$$\frac{3\left(x^3-x\right)^{2/3}\left(9x^4+6x^2+5\right)}{80x^6}$$

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.83, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2016, 2014}

$$\frac{3\left(x^3-x\right)^{2/3}}{16x^6}+\frac{9\left(x^3-x\right)^{2/3}}{40x^4}+\frac{27\left(x^3-x\right)^{2/3}}{80x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(-x + x^3)^(1/3)),x]
```

```
[Out] (3*(-x + x^3)^(2/3))/(16*x^6) + (9*(-x + x^3)^(2/3))/(40*x^4) + (27*(-x + x
^3)^(2/3))/(80*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6 \sqrt[3]{-x+x^3}}\, dx &= \frac{3\left(-x+x^3\right)^{2/3}}{16x^6}+\frac{3}{4}\int \frac{1}{x^4 \sqrt[3]{-x+x^3}}\, dx\\
&= \frac{3\left(-x+x^3\right)^{2/3}}{16x^6}+\frac{9\left(-x+x^3\right)^{2/3}}{40x^4}+\frac{9}{20}\int \frac{1}{x^2 \sqrt[3]{-x+x^3}}\, dx\\
&= \frac{3\left(-x+x^3\right)^{2/3}}{16x^6}+\frac{9\left(-x+x^3\right)^{2/3}}{40x^4}+\frac{27\left(-x+x^3\right)^{2/3}}{80x^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 1.00

$$\frac{3\left(x\left(x^2-1\right)\right)^{2/3}\left(9x^4+6x^2+5\right)}{80x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(-x + x^3)^(1/3)),x]
```

```
[Out] (3*(x*(-1 + x^2))^(2/3)*(5 + 6*x^2 + 9*x^4))/(80*x^6)
```

IntegrateAlgebraic [A] time = 0.25, size = 30, normalized size = 1.00

$$\frac{3\left(x^3-x\right)^{2/3}\left(9x^4+6x^2+5\right)}{80x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^6*(-x + x^3)^(1/3)),x]
[Out] (3*(-x + x^3)^(2/3)*(5 + 6*x^2 + 9*x^4))/(80*x^6)
```

fricas [A] time = 0.40, size = 26, normalized size = 0.87

$$\frac{3\left(9\,x^4+6\,x^2+5\right)\left(x^3-x\right)^{\frac{2}{3}}}{80\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^3-x)^(1/3),x, algorithm="fricas")
[Out] 3/80*(9*x^4 + 6*x^2 + 5)*(x^3 - x)^(2/3)/x^6
```

giac [A] time = 0.44, size = 41, normalized size = 1.37

$$\frac{3}{16}\left(\frac{1}{x^2}-1\right)^2\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}-\frac{3}{5}\left(-\frac{1}{x^2}+1\right)^{\frac{5}{3}}+\frac{3}{4}\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^3-x)^(1/3),x, algorithm="giac")
[Out] 3/16*(1/x^2 - 1)^2*(-1/x^2 + 1)^(2/3) - 3/5*(-1/x^2 + 1)^(5/3) + 3/4*(-1/x^
2 + 1)^(2/3)
```

maple [A] time = 0.00, size = 33, normalized size = 1.10

$$\frac{3\left(-1+x\right)\left(1+x\right)\left(9x^4+6x^2+5\right)}{80x^5\left(x^3-x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^3-x)^(1/3),x)
[Out] 3/80*(-1+x)*(1+x)*(9*x^4+6*x^2+5)/x^5/(x^3-x)^(1/3)
```

maxima [A] time = 0.63, size = 34, normalized size = 1.13

$$\frac{3\left(9\,x^7-3\,x^5-x^3-5\,x\right)}{80\left(x+1\right)^{\frac{1}{3}}\left(x-1\right)^{\frac{1}{3}}x^{\frac{19}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^3-x)^(1/3),x, algorithm="maxima")
[Out] 3/80*(9*x^7 - 3*x^5 - x^3 - 5*x)/((x + 1)^(1/3)*(x - 1)^(1/3)*x^(19/3))
```

mupad [B] time = 0.23, size = 26, normalized size = 0.87

$$\frac{3\left(x^3-x\right)^{2/3}\left(9\,x^4+6\,x^2+5\right)}{80\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^3 - x)^(1/3)),x)
```

```
[Out] (3*(x^3 - x)^(2/3)*(6*x^2 + 9*x^4 + 5))/(80*x^6)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \sqrt[3]{x(x-1)(x+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**3-x)**(1/3),x)
```

```
[Out] Integral(1/(x**6*(x*(x - 1)*(x + 1))**(1/3)), x)
```

3.359 $\int \frac{1}{x^3 \sqrt[3]{-x^2+x^3}} \, dx$

Optimal. Leaf size=30

$$\frac{3\left(9x^2+6x+5\right)\left(x^3-x^2\right)^{2/3}}{40x^4}$$

Rubi [B] time = 0.08, antiderivative size = 61, normalized size of antiderivative = 2.03, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2016, 2014}

$$\frac{27\left(x^3-x^2\right)^{2/3}}{40x^2}+\frac{9\left(x^3-x^2\right)^{2/3}}{20x^3}+\frac{3\left(x^3-x^2\right)^{2/3}}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (3*(-x^2 + x^3)^(2/3))/(8*x^4) + (9*(-x^2 + x^3)^(2/3))/(20*x^3) + (27*(-x^
2 + x^3)^(2/3))/(40*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \sqrt[3]{-x^2+x^3}} \, dx &= \frac{3\left(-x^2+x^3\right)^{2/3}}{8x^4}+\frac{3}{4}\int \frac{1}{x^2 \sqrt[3]{-x^2+x^3}} \, dx\\
&= \frac{3\left(-x^2+x^3\right)^{2/3}}{8x^4}+\frac{9\left(-x^2+x^3\right)^{2/3}}{20x^3}+\frac{9}{20}\int \frac{1}{x \sqrt[3]{-x^2+x^3}} \, dx\\
&= \frac{3\left(-x^2+x^3\right)^{2/3}}{8x^4}+\frac{9\left(-x^2+x^3\right)^{2/3}}{20x^3}+\frac{27\left(-x^2+x^3\right)^{2/3}}{40x^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.93

$$\frac{3\left((x-1)x^2\right)^{2/3}\left(9x^2+6x+5\right)}{40x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-x^2 + x^3)^(1/3)),x]
```

[Out] (3*((-1 + x)*x^2)^(2/3)*(5 + 6*x + 9*x^2))/(40*x^4)

IntegrateAlgebraic [A] time = 0.22, size = 30, normalized size = 1.00

$$\frac{3\left(9x^2+6x+5\right)\left(x^3-x^2\right)^{2/3}}{40x^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*(-x^2 + x^3)^(1/3)),x]

[Out] (3*(5 + 6*x + 9*x^2)*(-x^2 + x^3)^(2/3))/(40*x^4)

fricas [A] time = 0.39, size = 26, normalized size = 0.87

$$\frac{3\left(x^3-x^2\right)^{\frac{2}{3}}\left(9x^2+6x+5\right)}{40x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^3-x^2)^(1/3),x, algorithm="fricas")

[Out] 3/40*(x^3 - x^2)^(2/3)*(9*x^2 + 6*x + 5)/x^4

giac [A] time = 0.73, size = 41, normalized size = 1.37

$$\frac{3}{8}\left(\frac{1}{x}-1\right)^2\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}-\frac{6}{5}\left(-\frac{1}{x}+1\right)^{\frac{5}{3}}+\frac{3}{2}\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^3-x^2)^(1/3),x, algorithm="giac")

[Out] 3/8*(1/x - 1)^2*(-1/x + 1)^(2/3) - 6/5*(-1/x + 1)^(5/3) + 3/2*(-1/x + 1)^(2/3)

maple [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{3(-1+x)\left(9x^2+6x+5\right)}{40x^2\left(x^3-x^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(x^3-x^2)^(1/3),x)

[Out] 3/40*(-1+x)*(9*x^2+6*x+5)/x^2/(x^3-x^2)^(1/3)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3-x^2\right)^{\frac{1}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(x^3-x^2)^(1/3),x, algorithm="maxima")

[Out] integrate(1/((x^3 - x^2)^(1/3)*x^3), x)

mupad [B] time = 0.19, size = 49, normalized size = 1.63

$$\frac{27x^2\left(x^3-x^2\right)^{2/3}+18x\left(x^3-x^2\right)^{2/3}+15\left(x^3-x^2\right)^{2/3}}{40x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^3 - x^2)^(1/3)),x)
```

```
[Out] (27*x^2*(x^3 - x^2)^(2/3) + 18*x*(x^3 - x^2)^(2/3) + 15*(x^3 - x^2)^(2/3))/
(40*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt[3]{x^2 (x-1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**3-x**2)**(1/3),x)
```

```
[Out] Integral(1/(x**3*(x**2*(x - 1))**(1/3)), x)
```

3.360 $\int \frac{(-3+x^2)(1-x^2+x^3)}{x^6\sqrt[4]{-x+x^3}}\,dx$

Optimal. Leaf size=30

$$-\frac{4\left(x^3-x\right)^{3/4}\left(7x^3-3x^2+3\right)}{21x^6}$$

Rubi [A] time = 0.43, antiderivative size = 55, normalized size of antiderivative = 1.83, number of steps used = 26, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2052, 2025, 2011, 365, 364, 2032}

$$-\frac{4\left(x^3-x\right)^{3/4}}{3x^3}-\frac{4\left(x^3-x\right)^{3/4}}{7x^6}+\frac{4\left(x^3-x\right)^{3/4}}{7x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + x^2)*(1 - x^2 + x^3))/(x^6*(-x + x^3)^(1/4)),x]
```

```
[Out] (-4*(-x + x^3)^(3/4))/(7*x^6) + (4*(-x + x^3)^(3/4))/(7*x^4) - (4*(-x + x^3
)^(3/4))/(3*x^3)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\left(-3+x^2\right)\left(1-x^2+x^3\right)}{x^6\sqrt[4]{-x+x^3}}\,dx = \int \left(-\frac{3}{x^6\sqrt[4]{-x+x^3}}+\frac{4}{x^4\sqrt[4]{-x+x^3}}-\frac{3}{x^3\sqrt[4]{-x+x^3}}-\frac{1}{x^2\sqrt[4]{-x+x^3}}+\frac{1}{x\sqrt[4]{-x+\cdots}}\right.$$

$$= -\left(3\int \frac{1}{x^6\sqrt[4]{-x+x^3}}\,dx\right)-3\int \frac{1}{x^3\sqrt[4]{-x+x^3}}\,dx+4\int \frac{1}{x^4\sqrt[4]{-x+x^3}}\,dx-\int \cdots$$

$$= -\frac{4\left(-x+x^3\right)^{3/4}}{7x^6}+\frac{16\left(-x+x^3\right)^{3/4}}{13x^4}-\frac{4\left(-x+x^3\right)^{3/4}}{3x^3}-\frac{4\left(-x+x^3\right)^{3/4}}{5x^2}+\frac{4\left(-x\cdots\right.}{\cdots}$$

$$= -\frac{4\left(-x+x^3\right)^{3/4}}{7x^6}+\frac{4\left(-x+x^3\right)^{3/4}}{7x^4}-\frac{4\left(-x+x^3\right)^{3/4}}{3x^3}+\frac{12\left(-x+x^3\right)^{3/4}}{13x^2}-\frac{28}{65}\int\cdots$$

$$= -\frac{4\left(-x+x^3\right)^{3/4}}{7x^6}+\frac{4\left(-x+x^3\right)^{3/4}}{7x^4}-\frac{4\left(-x+x^3\right)^{3/4}}{3x^3}+\frac{3}{13}\int \frac{1}{\sqrt[4]{-x+x^3}}\,dx+\cdots$$

$$= -\frac{4\left(-x+x^3\right)^{3/4}}{7x^6}+\frac{4\left(-x+x^3\right)^{3/4}}{7x^4}-\frac{4\left(-x+x^3\right)^{3/4}}{3x^3}+\frac{4x\sqrt[4]{1-x^2}\,{}_2F_1\left(\frac14,\frac38;\frac{11}{8};\cdots\right)}{15\sqrt[4]{-x+x^3}}$$

$$= -\frac{4\left(-x+x^3\right)^{3/4}}{7x^6}+\frac{4\left(-x+x^3\right)^{3/4}}{7x^4}-\frac{4\left(-x+x^3\right)^{3/4}}{3x^3}-\frac{4x\sqrt[4]{1-x^2}\,{}_2F_1\left(\frac14,\frac38;\frac{11}{8};\cdots\right)}{13\sqrt[4]{-x+x^3}}$$

$$= -\frac{4\left(-x+x^3\right)^{3/4}}{7x^6}+\frac{4\left(-x+x^3\right)^{3/4}}{7x^4}-\frac{4\left(-x+x^3\right)^{3/4}}{3x^3}$$

Mathematica [C] time = 0.11, size = 119, normalized size = 3.97

$$-\frac{4\sqrt[4]{1-x^2}\left(7x^2\left(60\,{}_2F_1\left(-\frac{13}{8},\frac14;-\frac58;x^2\right)+13x\left(15x^2\,{}_2F_1\left(-\frac18,\frac14;\frac78;x^2\right)-3x\,{}_2F_1\left(-\frac58,\frac14;\frac38;x^2\right)-5\,{}_2F_1\left(-\frac98,\frac14;-\frac18;x^2\right)\right)\right)-195\,{}_2F_1\left(-\frac{21}{8},\frac14;-\frac{13}{8};x^2\right)\right)}{1365x^5\sqrt[4]{x\left(x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + x^2)*(1 - x^2 + x^3))/(x^6*(-x + x^3)^(1/4)),x]
```

```
[Out] (-4*(1 - x^2)^(1/4)*(-195*Hypergeometric2F1[-21/8, 1/4, -13/8, x^2] + 7*x^2
*(60*Hypergeometric2F1[-13/8, 1/4, -5/8, x^2] + 13*x*(-5*Hypergeometric2F1[
-9/8, 1/4, -1/8, x^2] - 3*x*Hypergeometric2F1[-5/8, 1/4, 3/8, x^2] + 15*x^2
*Hypergeometric2F1[-1/8, 1/4, 7/8, x^2]))))/(1365*x^5*(x*(-1 + x^2))^(1/4))
```

IntegrateAlgebraic [A] time = 0.82, size = 30, normalized size = 1.00

$$-\frac{4\left(x^3-x\right)^{3/4}\left(7x^3-3x^2+3\right)}{21x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^2)*(1 - x^2 + x^3))/(x^6*(-x + x^3)^(1/4)),x]
```

```
[Out] (-4*(-x + x^3)^(3/4)*(3 - 3*x^2 + 7*x^3))/(21*x^6)
```

fricas [A] time = 0.44, size = 26, normalized size = 0.87

$$-\frac{4\left(7\,x^3-3\,x^2+3\right)\left(x^3-x\right)^{\frac{3}{4}}}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)*(x^3-x^2+1)/x^6/(x^3-x)^(1/4),x, algorithm="fricas")
[Out] -4/21*(7*x^3 - 3*x^2 + 3)*(x^3 - x)^(3/4)/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-x^2+1\right)\left(x^2-3\right)}{\left(x^3-x\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)*(x^3-x^2+1)/x^6/(x^3-x)^(1/4),x, algorithm="giac")
[Out] integrate((x^3 - x^2 + 1)*(x^2 - 3)/((x^3 - x)^(1/4)*x^6), x)
```

maple [A] time = 0.01, size = 33, normalized size = 1.10

$$-\frac{4\,(-1+x)\,(1+x)\left(7x^3-3x^2+3\right)}{21x^5\left(x^3-x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3)*(x^3-x^2+1)/x^6/(x^3-x)^(1/4),x)
[Out] -4/21*(-1+x)*(1+x)*(7*x^3-3*x^2+3)/x^5/(x^3-x)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-x^2+1\right)\left(x^2-3\right)}{\left(x^3-x\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)*(x^3-x^2+1)/x^6/(x^3-x)^(1/4),x, algorithm="maxima")
[Out] integrate((x^3 - x^2 + 1)*(x^2 - 3)/((x^3 - x)^(1/4)*x^6), x)
```

mupad [B] time = 0.25, size = 45, normalized size = 1.50

$$-\frac{12\left(x^3-x\right)^{3/4}-12\,x^2\left(x^3-x\right)^{3/4}+28\,x^3\left(x^3-x\right)^{3/4}}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 3)*(x^3 - x^2 + 1))/(x^6*(x^3 - x)^(1/4)),x)
[Out] -(12*(x^3 - x)^(3/4) - 12*x^2*(x^3 - x)^(3/4) + 28*x^3*(x^3 - x)^(3/4))/(21
*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-3\right)\left(x^3-x^2+1\right)}{x^6\sqrt[4]{x\,(x-1)\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3)*(x**3-x**2+1)/x**6/(x**3-x)**(1/4),x)
[Out] Integral((x**2 - 3)*(x**3 - x**2 + 1)/(x**6*(x*(x - 1)*(x + 1))**(1/4)), x)
```

3.361 $\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt{-x-x^2+x^3}}\,dx$

Optimal. Leaf size=30

$$-2\tan^{-1}\left(\frac{\sqrt{x^3-x^2-x}}{x^2-x-1}\right)$$

Rubi [C] time = 1.00, antiderivative size = 373, normalized size of antiderivative = 12.43, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2056, 6725, 716, 1098, 934, 168, 538, 537}

$$\frac{\sqrt{x}\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}\sqrt{\frac{\left(1+\sqrt{5}\right)x+2}{\left(1-\sqrt{5}\right)x+2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{\left(1-\sqrt{5}\right)x+2}}\sqrt{x^3-x^2-x}}-\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(\frac{1}{2}\left(-1-\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\sqrt{x^3-x^2-x}}-\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(\frac{1}{2}\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\sqrt{x^3-x^2-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^2)/((-1 + x^2)*Sqrt[-x - x^2 + x^3]),x]
```

```
[Out] (Sqrt[x]*Sqrt[-2 - (1 - Sqrt[5])*x]*Sqrt[(2 + (1 + Sqrt[5])*x)/(2 + (1 - Sq
rt[5])*x)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*Sqrt[x])/Sqrt[-2 - (1 - Sqrt[5
])*x]], (5 - Sqrt[5])/10])/(5^(1/4)*Sqrt[(2 + (1 - Sqrt[5])*x)^(-1)]*Sqrt[-
x - x^2 + x^3]) - (Sqrt[3 + Sqrt[5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[
1 - (2*x)/(1 + Sqrt[5])]*EllipticPi[(-1 - Sqrt[5])/2, ArcSin[Sqrt[2/(1 + Sq
rt[5])]*Sqrt[x]], (-3 - Sqrt[5])/2])/Sqrt[-x - x^2 + x^3] - (Sqrt[3 + Sqrt[
5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1 + Sqrt[5])]*Elliptic
Pi[(1 + Sqrt[5])/2, ArcSin[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]], (-3 - Sqrt[5])/2
])/Sqrt[-x - x^2 + x^3]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 716

```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2,
Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
```

```
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt{-x-x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{1+x^2}{\sqrt{x}\left(-1+x^2\right)\sqrt{-1-x+x^2}}\,dx}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{-1-x+x^2}}+\frac{2}{\sqrt{x}\left(-1+x^2\right)\sqrt{-1-x+x^2}}\right)dx}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-1-x+x^2}}\,dx}{\sqrt{-x-x^2+x^3}} + \frac{\left(2\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{1}{\sqrt{x}\left(-1+x^2\right)\sqrt{-1-x}}}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1-x+x^2}\right)\int \left(-\frac{1}{2(1-x)\sqrt{x}\sqrt{-1-x+x^2}}-\frac{1}{2\sqrt{x}(1+x)\sqrt{-1-x+x^2}}\right)dx}{\sqrt{-x-x^2+x^3}} + \frac{\left(2\sqrt{x}\sqrt{\right.}{}}{}$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - \frac{\left(\sqrt{\right.}{}}{}$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - \frac{\left(\sqrt{\right.}{}}{}$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} + \frac{\left(2\right.}{}$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} + \frac{\left(2\right.}{}$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - \frac{\sqrt{}}{}$$

Mathematica [C] time = 6.71, size = 1680, normalized size = 56.00

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2)/((-1 + x^2)*Sqrt[-x - x^2 + x^3]),x]
```

```
[Out] -((Sqrt[2]*(-1 + x^(-2))*x^(3/2)*(1 + x^2)*(((-2*I)*Sqrt[1 - x^(-2) - x^(-1
)]*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], -3/2 - Sqrt[5]/2])/S
qrt[-1 + Sqrt[5]] + ((2*I)*Sqrt[1 - x^(-2) - x^(-1)]*EllipticPi[(1 + Sqrt[5
])/2, I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], (-3 - Sqrt[5])/2])/Sqrt[-1
+ Sqrt[5]] + ((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt
[5])] - 2/Sqrt[x])^2*Sqrt[(I*Sqrt[2*(1 + Sqrt[5])] + 2/Sqrt[x])/((1 + 2*I)*
Sqrt[2] - Sqrt[10] + (2*Sqrt[-1 + Sqrt[5]])/Sqrt[x] - ((2*I)*Sqrt[1 + Sqrt[
```

```
5]])/Sqrt[x])]*Sqrt[(I*Sqrt[2*(1 + Sqrt[5])] - 2/Sqrt[x])/(Sqrt[2]*((-1 + 2
*I) + Sqrt[5]) - (2*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]]))/Sqrt[x])]*S
qrt[(Sqrt[2]*((-1 - 2*I) + Sqrt[5]) + (2*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + S
qrt[5]]))/Sqrt[x])/(Sqrt[2]*((-1 + 2*I) + Sqrt[5]) - (2*(Sqrt[-1 + Sqrt[5]]
 + I*Sqrt[1 + Sqrt[5]]))/Sqrt[x])]*((2 + Sqrt[2*(-1 + Sqrt[5])])*EllipticF[
ArcSin[Sqrt[((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[
5])] + 2/Sqrt[x]))/((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1
+ Sqrt[5])] - 2/Sqrt[x]))]], -3/5 - (4*I)/5] - 2*Sqrt[2*(-1 + Sqrt[5])]*Ell
ipticPi[((-2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqr
t[5]]))/((2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt
[5]])), ArcSin[Sqrt[((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1
 + Sqrt[5])] + 2/Sqrt[x]))/((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqr
t[2*(-1 + Sqrt[5])] - 2/Sqrt[x]))]], -3/5 - (4*I)/5]))/((-2 + Sqrt[2*(-1 +
Sqrt[5])])*(2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sq
rt[5]])) - ((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5
])] - 2/Sqrt[x])^2*Sqrt[(I*Sqrt[2*(1 + Sqrt[5])] + 2/Sqrt[x])/((1 + 2*I)*Sq
rt[2] - Sqrt[10] + (2*Sqrt[-1 + Sqrt[5]])/Sqrt[x] - ((2*I)*Sqrt[1 + Sqrt[5]
])/Sqrt[x])]*Sqrt[(I*Sqrt[2*(1 + Sqrt[5])] - 2/Sqrt[x])/(Sqrt[2]*((-1 + 2*I
) + Sqrt[5]) - (2*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]]))/Sqrt[x])]*Sqr
t[(Sqrt[2]*((-1 - 2*I) + Sqrt[5]) + (2*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqr
t[5]]))/Sqrt[x])/(Sqrt[2]*((-1 + 2*I) + Sqrt[5]) - (2*(Sqrt[-1 + Sqrt[5]] +
 I*Sqrt[1 + Sqrt[5]]))/Sqrt[x])]*((-2 + Sqrt[2*(-1 + Sqrt[5])])*EllipticF[A
rcSin[Sqrt[((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5
])] + 2/Sqrt[x]))/((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 +
 Sqrt[5])] - 2/Sqrt[x]))]], -3/5 - (4*I)/5] - 2*Sqrt[2*(-1 + Sqrt[5])]*Elli
pticPi[((2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[
5]]))/((-2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[
5]])), ArcSin[Sqrt[((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1
+ Sqrt[5])] + 2/Sqrt[x]))/((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt
[2*(-1 + Sqrt[5])] - 2/Sqrt[x]))]], -3/5 - (4*I)/5]))/((-2 + Sqrt[2*(-1 + S
qrt[5])])*(2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqr
t[5]]))))/((-x^(-2) + x^2)*Sqrt[x*(-1 - x + x^2)]))
```

IntegrateAlgebraic [A] time = 0.08, size = 30, normalized size = 1.00

$$-2\tan^{-1}\left(\frac{\sqrt{x^3-x^2-x}}{x^2-x-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 + x^2)*Sqrt[-x - x^2 + x^3]),x]
[Out] -2*ArcTan[Sqrt[-x - x^2 + x^3]/(-1 - x + x^2)]
```

fricas [A] time = 0.45, size = 25, normalized size = 0.83

$$\arctan\left(\frac{x^2-2x-1}{2\sqrt{x^3-x^2-x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^3-x^2-x)^(1/2),x, algorithm="fricas")
[Out] arctan(1/2*(x^2 - 2*x - 1)/sqrt(x^3 - x^2 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x^3-x^2-x}\,(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^3-x^2-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/(sqrt(x^3 - x^2 - x)*(x^2 - 1)), x)
```

maple [C] time = 0.02, size = 387, normalized size = 12.90

$$\frac{2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\sqrt{-5\left(x-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\sqrt{5}}\,\sqrt{-\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}},\frac{\sqrt{5}\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)}{5\sqrt{x^3-x^2-x}}+\frac{2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\sqrt{-5\left(x-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\sqrt{5}}\,\sqrt{-\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}},\frac{\frac{1}{2}-\frac{\sqrt{5}}{2}}{-\frac{1}{2}-\frac{\sqrt{5}}{2}},\frac{\sqrt{5}\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)}{5\sqrt{x^3-x^2-x}\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)}-\frac{2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\sqrt{-5\left(x-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\sqrt{5}}\,\sqrt{-\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}},\frac{\frac{1}{2}-\frac{\sqrt{5}}{2}}{\frac{3}{2}-\frac{\sqrt{5}}{2}},\frac{\sqrt{5}\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)}{5\sqrt{x^3-x^2-x}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-1)/(x^3-x^2-x)^(1/2),x)
```

```
[Out] 2/5*(1/2*5^(1/2)-1/2)*((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x-
1/2-1/2*5^(1/2))*5^(1/2))^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1
/2)*EllipticF(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),1/5*5^(1/2)*((1
/2*5^(1/2)-1/2)*5^(1/2))^(1/2))+2/5*(1/2*5^(1/2)-1/2)*((x-1/2+1/2*5^(1/2))/
(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x-1/2-1/2*5^(1/2))*5^(1/2))^(1/2)*(-x/(1/2*5^
(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(-1/2-1/2*5^(1/2))*EllipticPi(((x-1/2+1
/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(-1/2-1/2*5^(1/2)),1
/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))-2/5*(1/2*5^(1/2)-1/2)*((x-1/2
+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x-1/2-1/2*5^(1/2))*5^(1/2))^(1/
2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(3/2-1/2*5^(1/2))*Ellipti
cPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(3/2-1/
2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x^3-x^2-x}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^3-x^2-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(sqrt(x^3 - x^2 - x)*(x^2 - 1)), x)
```

mupad [B] time = 0.14, size = 206, normalized size = 6.87

$$-\frac{\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\left(\sqrt{5}+1\right)\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\left(\Pi\left(-\frac{\sqrt{5}}{2}-\frac{1}{2};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)+\Pi\left(\frac{\sqrt{5}}{2}+\frac{1}{2};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)\right)}{\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((x^2 - 1)*(x^3 - x^2 - x)^(1/2)),x)
```

```
[Out] -((x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/2 - 1/2)/(5^(1/2)/2 - 1/2))^(1/
2)*(5^(1/2) + 1)*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 + 1/2))^(1/2)*(ellipticP
i(- 5^(1/2)/2 - 1/2, asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/2 + 1/2)/
(5^(1/2)/2 - 1/2)) - ellipticF(asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2)
/2 + 1/2)/(5^(1/2)/2 - 1/2)) + ellipticPi(5^(1/2)/2 + 1/2, asin((x/(5^(1/2)
/2 + 1/2))^(1/2)), -(5^(1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2))))/(x^3 - x^2 - x*(
5^(1/2)/2 - 1/2)*(5^(1/2)/2 + 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x\left(x^2-x-1\right)}\,(x-1)\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-1)/(x**3-x**2-x)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)/(sqrt(x*(x**2 - x - 1))*(x - 1)*(x + 1)), x)
```

3.362 $\int \frac{-1+x^3}{x^3(1+x^3)\sqrt[4]{x+x^4}}\,dx$

Optimal. Leaf size=30

$$\frac{4\left(7x^3+1\right)\left(x^4+x\right)^{3/4}}{9x^3\left(x^3+1\right)}$$

Rubi [A] time = 0.15, antiderivative size = 31, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2056, 453, 264}

$$\frac{28x}{9\sqrt[4]{x^4+x}} + \frac{4}{9\sqrt[4]{x^4+x}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(x^3*(1 + x^3)*(x + x^4)^(1/4)),x]
```

```
[Out] 4/(9*x^2*(x + x^4)^(1/4)) + (28*x)/(9*(x + x^4)^(1/4))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^3}{x^3\left(1+x^3\right)\sqrt[4]{x+x^4}}\,dx &= \frac{\left(\sqrt[4]{x}\sqrt[4]{1+x^3}\right)\int \frac{-1+x^3}{x^{13/4}\left(1+x^3\right)^{5/4}}\,dx}{\sqrt[4]{x+x^4}}\\
&= \frac{4}{9x^2\sqrt[4]{x+x^4}} + \frac{\left(7\sqrt[4]{x}\sqrt[4]{1+x^3}\right)\int \frac{1}{\sqrt[4]{x}\left(1+x^3\right)^{5/4}}\,dx}{3\sqrt[4]{x+x^4}}\\
&= \frac{4}{9x^2\sqrt[4]{x+x^4}} + \frac{28x}{9\sqrt[4]{x+x^4}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.77

$$\frac{28x^3+4}{9x^2\sqrt[4]{x^4+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(x^3*(1 + x^3)*(x + x^4)^(1/4)),x]
[Out] (4 + 28*x^3)/(9*x^2*(x + x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.28, size = 30, normalized size = 1.00

$$\frac{4\left(7x^3+1\right)\left(x^4+x\right)^{3/4}}{9x^3\left(x^3+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)/(x^3*(1 + x^3)*(x + x^4)^(1/4)),x]
[Out] (4*(1 + 7*x^3)*(x + x^4)^(3/4))/(9*x^3*(1 + x^3))
```

fricas [A] time = 0.39, size = 25, normalized size = 0.83

$$\frac{4\left(x^4+x\right)^{\frac{3}{4}}\left(7x^3+1\right)}{9\left(x^6+x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/x^3/(x^3+1)/(x^4+x)^(1/4),x, algorithm="fricas")
[Out] 4/9*(x^4 + x)^(3/4)*(7*x^3 + 1)/(x^6 + x^3)
```

giac [A] time = 0.45, size = 19, normalized size = 0.63

$$\frac{4}{9}\left(\frac{1}{x^3}+1\right)^{\frac{3}{4}}+\frac{8}{3\left(\frac{1}{x^3}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/x^3/(x^3+1)/(x^4+x)^(1/4),x, algorithm="giac")
[Out] 4/9*(1/x^3 + 1)^(3/4) + 8/3/(1/x^3 + 1)^(1/4)
```

maple [A] time = 0.01, size = 20, normalized size = 0.67

$$\frac{\frac{28x^3}{9}+\frac{4}{9}}{\left(x^4+x\right)^{\frac{1}{4}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/x^3/(x^3+1)/(x^4+x)^(1/4),x)
[Out] 4/9*(7*x^3+1)/(x^4+x)^(1/4)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3-1}{\left(x^4+x\right)^{\frac{1}{4}}\left(x^3+1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/x^3/(x^3+1)/(x^4+x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 1)/((x^4 + x)^(1/4)*(x^3 + 1)*x^3), x)
```

mupad [B] time = 0.23, size = 26, normalized size = 0.87

$$\frac{4\,\left(7\,x^3+1\right)\,{\left(x^4+x\right)}^{3/4}}{9\,x^3\,\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x^3*(x^3 + 1)*(x + x^4)^(1/4)),x)
```

```
[Out] (4*(7*x^3 + 1)*(x + x^4)^(3/4))/(9*x^3*(x^3 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\left(x^2+x+1\right)}{x^3\sqrt[4]{x(x+1)\left(x^2-x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/x**3/(x**3+1)/(x**4+x)**(1/4),x)
```

```
[Out] Integral((x - 1)*(x**2 + x + 1)/(x**3*(x*(x + 1)*(x**2 - x + 1))**(1/4)*(x
+ 1)*(x**2 - x + 1)), x)
```

3.363 $\int \frac{\sqrt[4]{-x^2+x^4}}{x^6}\,dx$

Optimal. Leaf size=30

$$\frac{2\sqrt[4]{x^4-x^2}\left(4x^4+x^2-5\right)}{45x^5}$$

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.37, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2016, 2014}

$$\frac{2\left(x^4-x^2\right)^{5/4}}{9x^7}+\frac{8\left(x^4-x^2\right)^{5/4}}{45x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + x^4)^(1/4)/x^6,x]
[Out] (2*(-x^2 + x^4)^(5/4))/(9*x^7) + (8*(-x^2 + x^4)^(5/4))/(45*x^5)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{-x^2+x^4}}{x^6}\,dx &= \frac{2\left(-x^2+x^4\right)^{5/4}}{9x^7}+\frac{4}{9}\int \frac{\sqrt[4]{-x^2+x^4}}{x^4}\,dx\\ &= \frac{2\left(-x^2+x^4\right)^{5/4}}{9x^7}+\frac{8\left(-x^2+x^4\right)^{5/4}}{45x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.90

$$\frac{2\left(x^2\left(x^2-1\right)\right)^{5/4}\left(4x^2+5\right)}{45x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + x^4)^(1/4)/x^6,x]
[Out] (2*(x^2*(-1 + x^2))^(5/4)*(5 + 4*x^2))/(45*x^7)
```

IntegrateAlgebraic [A] time = 0.13, size = 30, normalized size = 1.00

$$\frac{2\sqrt[4]{x^4-x^2}\left(4x^4+x^2-5\right)}{45x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^2 + x^4)^(1/4)/x^6,x]
[Out] (2*(-x^2 + x^4)^(1/4)*(-5 + x^2 + 4*x^4))/(45*x^5)
```

fricas [A] time = 0.41, size = 26, normalized size = 0.87

$$\frac{2\left(4\,x^4+x^2-5\right)\left(x^4-x^2\right)^{\frac{1}{4}}}{45\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2)^(1/4)/x^6,x, algorithm="fricas")
[Out] 2/45*(4*x^4 + x^2 - 5)*(x^4 - x^2)^(1/4)/x^5
```

giac [A] time = 0.43, size = 30, normalized size = 1.00

$$\frac{2}{9}\left(\frac{1}{x^2}-1\right)^2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}-\frac{2}{5}\left(-\frac{1}{x^2}+1\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2)^(1/4)/x^6,x, algorithm="giac")
[Out] 2/9*(1/x^2 - 1)^2*(-1/x^2 + 1)^(1/4) - 2/5*(-1/x^2 + 1)^(5/4)
```

maple [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{2\left(-1+x\right)\left(1+x\right)\left(4x^2+5\right)\left(x^4-x^2\right)^{\frac{1}{4}}}{45x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^2)^(1/4)/x^6,x)
[Out] 2/45*(-1+x)*(1+x)*(4*x^2+5)*(x^4-x^2)^(1/4)/x^5
```

maxima [A] time = 0.72, size = 27, normalized size = 0.90

$$\frac{2\left(4\,x^5+x^3-5\,x\right)\left(x+1\right)^{\frac{1}{4}}\left(x-1\right)^{\frac{1}{4}}}{45\,x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2)^(1/4)/x^6,x, algorithm="maxima")
[Out] 2/45*(4*x^5 + x^3 - 5*x)*(x + 1)^(1/4)*(x - 1)^(1/4)/x^(11/2)
```

mupad [B] time = 0.33, size = 26, normalized size = 0.87

$$\frac{2\left(x^4-x^2\right)^{1/4}\left(4\,x^4+x^2-5\right)}{45\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x^2)^(1/4)/x^6,x)
[Out] (2*(x^4 - x^2)^(1/4)*(x^2 + 4*x^4 - 5))/(45*x^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2 (x-1)(x+1)}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**2)**(1/4)/x**6,x)
```

```
[Out] Integral((x**2*(x - 1)*(x + 1))**(1/4)/x**6, x)
```

3.364 $\int \frac{-1+2x}{\sqrt{1+x-2x^3+x^4}}\,dx$

Optimal. Leaf size=30

$$-\log\left(-2x^2+2\sqrt{x^4-2x^3+x+1}+2x+1\right)$$

Rubi [A] time = 0.05, antiderivative size = 23, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1680, 12, 1107, 619, 215}

$$-\sinh^{-1}\left(\frac{3-4\left(x-\frac{1}{2}\right)^2}{2\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x)/Sqrt[1 + x - 2*x^3 + x^4],x]
```

```
[Out] -ArcSinh[(3 - 4*(-1/2 + x)^2)/(2*Sqrt[3])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+2x}{\sqrt{1+x-2x^3+x^4}}\,dx &= \text{Subst}\left(\int \frac{8x}{\sqrt{21-24x^2+16x^4}}\,dx, x, -\frac{1}{2}+x\right) \\
&= 8\,\text{Subst}\left(\int \frac{x}{\sqrt{21-24x^2+16x^4}}\,dx, x, -\frac{1}{2}+x\right) \\
&= 4\,\text{Subst}\left(\int \frac{1}{\sqrt{21-24x+16x^2}}\,dx, x, \left(-\frac{1}{2}+x\right)^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{768}}}\,dx, x, 8\left(-3+4\left(-\frac{1}{2}+x\right)^2\right)\right)}{16\sqrt{3}} \\
&= -\sinh^{-1}\left(\frac{3-(-1+2x)^2}{2\sqrt{3}}\right)
\end{aligned}
$$

Mathematica [C] time = 3.37, size = 717, normalized size = 23.90

$$
\frac{\left(-2x+\sqrt{1+4\sqrt[3]{-1}}+1\right)\sqrt{\frac{\sqrt{1+4\sqrt[3]{-1}}\left(-2x+\sqrt{1-4(-1)^{2/3}}+1\right)}{\left(\sqrt{1+4\sqrt[3]{-1}}+\sqrt{1-4(-1)^{2/3}}\right)\left(-2x+\sqrt{1+4\sqrt[3]{-1}}+1\right)}}\left(2x+\sqrt{1+4\sqrt[3]{-1}}-1\right)\sqrt{-\frac{\sqrt{1+4\sqrt[3]{-1}}\left(2x+\sqrt{1-4(-1)^{2/3}}-1\right)}{\left(\sqrt{1+4\sqrt[3]{-1}}-\sqrt{1-4(-1)^{2/3}}\right)\left(-2x+\sqrt{1+4\sqrt[3]{-1}}+1\right)}}\left(F\left(\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{1+4\sqrt[3]{-1}}-\sqrt{1-4(-1)^{2/3}}\right)\left(2x+\sqrt{1+4\sqrt[3]{-1}}-1\right)}{\left(\sqrt{1+4\sqrt[3]{-1}}+\sqrt{1-4(-1)^{2/3}}\right)\left(-2x+\sqrt{1+4\sqrt[3]{-1}}+1\right)}}\right)\middle|\frac{\left(\sqrt{1+4\sqrt[3]{-1}}+\sqrt{1-4(-1)^{2/3}}\right)^2}{\left(\sqrt{1+4\sqrt[3]{-1}}-\sqrt{1-4(-1)^{2/3}}\right)^2}\right)-2\Pi\left(-\frac{\sqrt{1+4\sqrt[3]{-1}}+\sqrt{1-4(-1)^{2/3}}}{\sqrt{1+4\sqrt[3]{-1}}-\sqrt{1-4(-1)^{2/3}}};\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{1+4\sqrt[3]{-1}}-\sqrt{1-4(-1)^{2/3}}\right)\left(2x+\sqrt{1+4\sqrt[3]{-1}}-1\right)}{\left(\sqrt{1+4\sqrt[3]{-1}}+\sqrt{1-4(-1)^{2/3}}\right)\left(-2x+\sqrt{1+4\sqrt[3]{-1}}+1\right)}}\right)\middle|\frac{\left(\sqrt{1+4\sqrt[3]{-1}}+\sqrt{1-4(-1)^{2/3}}\right)^2}{\left(\sqrt{1+4\sqrt[3]{-1}}-\sqrt{1-4(-1)^{2/3}}\right)^2}\right)\right)}{\sqrt{\frac{\left(\sqrt{1+4\sqrt[3]{-1}}-\sqrt{1-4(-1)^{2/3}}\right)\left(2x+\sqrt{1+4\sqrt[3]{-1}}-1\right)}{\left(\sqrt{1+4\sqrt[3]{-1}}+\sqrt{1-4(-1)^{2/3}}\right)\left(-2x+\sqrt{1+4\sqrt[3]{-1}}+1\right)}}\sqrt{x^4-2x^3+x+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 2*x)/Sqrt[1 + x - 2*x^3 + x^4],x]
```

```
[Out] ((1 + Sqrt[1 + 4*(-1)^(1/3)] - 2*x)*Sqrt[(Sqrt[1 + 4*(-1)^(1/3)]*(1 + Sqrt[
1 - 4*(-1)^(2/3)] - 2*x))/((Sqrt[1 + 4*(-1)^(1/3)] + Sqrt[1 - 4*(-1)^(2/3)]
)*(1 + Sqrt[1 + 4*(-1)^(1/3)] - 2*x))]*(-1 + Sqrt[1 + 4*(-1)^(1/3)] + 2*x)*
Sqrt[-((Sqrt[1 + 4*(-1)^(1/3)]*(-1 + Sqrt[1 - 4*(-1)^(2/3)] + 2*x))/((Sqrt[
1 + 4*(-1)^(1/3)] - Sqrt[1 - 4*(-1)^(2/3)])*(1 + Sqrt[1 + 4*(-1)^(1/3)] - 2
*x)))]*(EllipticF[ArcSin[Sqrt[((Sqrt[1 + 4*(-1)^(1/3)] - Sqrt[1 - 4*(-1)^(2
/3)])*(-1 + Sqrt[1 + 4*(-1)^(1/3)] + 2*x))/((Sqrt[1 + 4*(-1)^(1/3)] + Sqrt[
1 - 4*(-1)^(2/3)])*(1 + Sqrt[1 + 4*(-1)^(1/3)] - 2*x))]], (Sqrt[1 + 4*(-1)^
(1/3)] + Sqrt[1 - 4*(-1)^(2/3)])^2/(Sqrt[1 + 4*(-1)^(1/3)] - Sqrt[1 - 4*(-1
)^(2/3)])^2] - 2*EllipticPi[-((Sqrt[1 + 4*(-1)^(1/3)] + Sqrt[1 - 4*(-1)^(2/
3)])/(Sqrt[1 + 4*(-1)^(1/3)] - Sqrt[1 - 4*(-1)^(2/3)])), ArcSin[Sqrt[((Sqrt
[1 + 4*(-1)^(1/3)] - Sqrt[1 - 4*(-1)^(2/3)])*(-1 + Sqrt[1 + 4*(-1)^(1/3)] +
 2*x))/((Sqrt[1 + 4*(-1)^(1/3)] + Sqrt[1 - 4*(-1)^(2/3)])*(1 + Sqrt[1 + 4*(
-1)^(1/3)] - 2*x))]], (Sqrt[1 + 4*(-1)^(1/3)] + Sqrt[1 - 4*(-1)^(2/3)])^2/(
Sqrt[1 + 4*(-1)^(1/3)] - Sqrt[1 - 4*(-1)^(2/3)])^2]))/(Sqrt[((Sqrt[1 + 4*(-
1)^(1/3)] - Sqrt[1 - 4*(-1)^(2/3)])*(-1 + Sqrt[1 + 4*(-1)^(1/3)] + 2*x))/((
Sqrt[1 + 4*(-1)^(1/3)] + Sqrt[1 - 4*(-1)^(2/3)])*(1 + Sqrt[1 + 4*(-1)^(1/3)
] - 2*x))]*Sqrt[1 + x - 2*x^3 + x^4])
```

IntegrateAlgebraic [A] time = 0.40, size = 30, normalized size = 1.00

$$
-\log\left(-2x^2+2\sqrt{x^4-2x^3+x+1}+2x+1\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x)/Sqrt[1 + x - 2*x^3 + x^4],x]
```

```
[Out] -Log[1 + 2*x - 2*x^2 + 2*Sqrt[1 + x - 2*x^3 + x^4]]
```

fricas [A] time = 0.42, size = 26, normalized size = 0.87

$$
\log\left(2\,x^2-2\,x+2\,\sqrt{x^4-2\,x^3+x+1}-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] log(2*x^2 - 2*x + 2*sqrt(x^4 - 2*x^3 + x + 1) - 1)
```

giac [A] time = 0.32, size = 34, normalized size = 1.13

$$-\log\left(-2\,x^2+2\,x+2\,\sqrt{\left(x^2-x\right)^2-x^2+x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-2*x^2 + 2*x + 2*sqrt((x^2 - x)^2 - x^2 + x + 1) + 1)
```

maple [C] time = 0.42, size = 1352, normalized size = 45.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x)/(x^4-2*x^3+x+1)^(1/2),x)
```

```
[Out] -2*(-1/2*(3-2*I*3^(1/2))^(1/2)-1/2*(3+2*I*3^(1/2))^(1/2))*((1/2*(3+2*I*3^(1
/2))^(1/2)-1/2*(3-2*I*3^(1/2))^(1/2))*(x-1/2+1/2*(3-2*I*3^(1/2))^(1/2))/(1/
2*(3+2*I*3^(1/2))^(1/2)+1/2*(3-2*I*3^(1/2))^(1/2))/(x-1/2-1/2*(3-2*I*3^(1/2
))^(1/2)))^(1/2)*(x-1/2-1/2*(3-2*I*3^(1/2))^(1/2))^2*((3-2*I*3^(1/2))^(1/2)
*(x-1/2+1/2*(3+2*I*3^(1/2))^(1/2))/(-1/2*(3+2*I*3^(1/2))^(1/2)+1/2*(3-2*I*3
^(1/2))^(1/2))/(x-1/2-1/2*(3-2*I*3^(1/2))^(1/2)))^(1/2)*((3-2*I*3^(1/2))^(1
/2)*(x-1/2-1/2*(3+2*I*3^(1/2))^(1/2))/(1/2*(3+2*I*3^(1/2))^(1/2)+1/2*(3-2*I
*3^(1/2))^(1/2))/(x-1/2-1/2*(3-2*I*3^(1/2))^(1/2)))^(1/2)/(1/2*(3+2*I*3^(1/
2))^(1/2)-1/2*(3-2*I*3^(1/2))^(1/2))/(3-2*I*3^(1/2))^(1/2)/((x-1/2+1/2*(3-2
*I*3^(1/2))^(1/2))*(x-1/2-1/2*(3-2*I*3^(1/2))^(1/2))*(x-1/2+1/2*(3+2*I*3^(1
/2))^(1/2))*(x-1/2-1/2*(3+2*I*3^(1/2))^(1/2)))^(1/2)*EllipticF(((1/2*(3+2*I
*3^(1/2))^(1/2)-1/2*(3-2*I*3^(1/2))^(1/2))*(x-1/2+1/2*(3-2*I*3^(1/2))^(1/2)
)/(1/2*(3+2*I*3^(1/2))^(1/2)+1/2*(3-2*I*3^(1/2))^(1/2))/(x-1/2-1/2*(3-2*I*3
^(1/2))^(1/2)))^(1/2),(-(1/2*(3+2*I*3^(1/2))^(1/2)+1/2*(3-2*I*3^(1/2))^(1/2
))^2/(1/2*(3+2*I*3^(1/2))^(1/2)-1/2*(3-2*I*3^(1/2))^(1/2))/(-1/2*(3+2*I*3^(
1/2))^(1/2)+1/2*(3-2*I*3^(1/2))^(1/2)))^(1/2))+4*(-1/2*(3-2*I*3^(1/2))^(1/2
)-1/2*(3+2*I*3^(1/2))^(1/2))*((1/2*(3+2*I*3^(1/2))^(1/2)-1/2*(3-2*I*3^(1/2)
)^(1/2))*(x-1/2+1/2*(3-2*I*3^(1/2))^(1/2))/(1/2*(3+2*I*3^(1/2))^(1/2)+1/2*(
3-2*I*3^(1/2))^(1/2))/(x-1/2-1/2*(3-2*I*3^(1/2))^(1/2)))^(1/2)*(x-1/2-1/2*(
3-2*I*3^(1/2))^(1/2))^2*((3-2*I*3^(1/2))^(1/2)*(x-1/2+1/2*(3+2*I*3^(1/2))^(
1/2))/(-1/2*(3+2*I*3^(1/2))^(1/2)+1/2*(3-2*I*3^(1/2))^(1/2))/(x-1/2-1/2*(3-
2*I*3^(1/2))^(1/2)))^(1/2)*((3-2*I*3^(1/2))^(1/2)*(x-1/2-1/2*(3+2*I*3^(1/2)
)^(1/2))/(1/2*(3+2*I*3^(1/2))^(1/2)+1/2*(3-2*I*3^(1/2))^(1/2))/(x-1/2-1/2*(
3-2*I*3^(1/2))^(1/2)))^(1/2)/(1/2*(3+2*I*3^(1/2))^(1/2)-1/2*(3-2*I*3^(1/2))
^(1/2))/(3-2*I*3^(1/2))^(1/2)/((x-1/2+1/2*(3-2*I*3^(1/2))^(1/2))*(x-1/2-1/2
*(3-2*I*3^(1/2))^(1/2))*(x-1/2+1/2*(3+2*I*3^(1/2))^(1/2))*(x-1/2-1/2*(3+2*I
*3^(1/2))^(1/2)))^(1/2)*((1/2+1/2*(3-2*I*3^(1/2))^(1/2))*EllipticF(((1/2*(3
+2*I*3^(1/2))^(1/2)-1/2*(3-2*I*3^(1/2))^(1/2))*(x-1/2+1/2*(3-2*I*3^(1/2))^(
1/2))/(1/2*(3+2*I*3^(1/2))^(1/2)+1/2*(3-2*I*3^(1/2))^(1/2))/(x-1/2-1/2*(3-2
*I*3^(1/2))^(1/2)))^(1/2),(-(1/2*(3+2*I*3^(1/2))^(1/2)+1/2*(3-2*I*3^(1/2))^
(1/2))^2/(1/2*(3+2*I*3^(1/2))^(1/2)-1/2*(3-2*I*3^(1/2))^(1/2))/(-1/2*(3+2*I
*3^(1/2))^(1/2)+1/2*(3-2*I*3^(1/2))^(1/2)))^(1/2))-(3-2*I*3^(1/2))^(1/2)*El
lipticPi(((1/2*(3+2*I*3^(1/2))^(1/2)-1/2*(3-2*I*3^(1/2))^(1/2))*(x-1/2+1/2*
(3-2*I*3^(1/2))^(1/2))/(1/2*(3+2*I*3^(1/2))^(1/2)+1/2*(3-2*I*3^(1/2))^(1/2)
)/(x-1/2-1/2*(3-2*I*3^(1/2))^(1/2)))^(1/2),(1/2*(3+2*I*3^(1/2))^(1/2)+1/2*(
3-2*I*3^(1/2))^(1/2))/(1/2*(3+2*I*3^(1/2))^(1/2)-1/2*(3-2*I*3^(1/2))^(1/2))
,(-(1/2*(3+2*I*3^(1/2))^(1/2)+1/2*(3-2*I*3^(1/2))^(1/2))^2/(1/2*(3+2*I*3^(1
```

```
/2))^(1/2)-1/2*(3-2*I*3^(1/2))^(1/2))/(-1/2*(3+2*I*3^(1/2))^(1/2)+1/2*(3-2*
I*3^(1/2))^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x-1}{\sqrt{x^4-2\,x^3+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x - 1)/sqrt(x^4 - 2*x^3 + x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{2\,x-1}{\sqrt{x^4-2\,x^3+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 1)/(x - 2*x^3 + x^4 + 1)^(1/2),x)
```

```
[Out] int((2*x - 1)/(x - 2*x^3 + x^4 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-1}{\sqrt{x^4-2x^3+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x**4-2*x**3+x+1)**(1/2),x)
```

```
[Out] Integral((2*x - 1)/sqrt(x**4 - 2*x**3 + x + 1), x)
```

3.365 $\int \frac{1+x}{(-1+x)x\sqrt[4]{-x^3+x^4}}\,dx$

Optimal. Leaf size=30

$$-\frac{4(7x-1)\left(x^4-x^3\right)^{3/4}}{3(x-1)x^3}$$

Rubi [A] time = 0.16, antiderivative size = 36, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2056, 78, 37}

$$\frac{4}{3\sqrt[4]{x^4-x^3}}-\frac{28x}{3\sqrt[4]{x^4-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-1 + x)*x*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] 4/(3*(-x^3 + x^4)^(1/4)) - (28*x)/(3*(-x^3 + x^4)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x}{(-1+x)x\sqrt[4]{-x^3+x^4}}\,dx &= \frac{\left(\sqrt[4]{-1+x}\,x^{3/4}\right)\int \frac{1+x}{(-1+x)^{5/4}x^{7/4}}\,dx}{\sqrt[4]{-x^3+x^4}}\\
&= \frac{4}{3\sqrt[4]{-x^3+x^4}}+\frac{\left(7\sqrt[4]{-1+x}\,x^{3/4}\right)\int \frac{1}{(-1+x)^{5/4}x^{3/4}}\,dx}{3\sqrt[4]{-x^3+x^4}}\\
&= \frac{4}{3\sqrt[4]{-x^3+x^4}}-\frac{28x}{3\sqrt[4]{-x^3+x^4}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.67

$$-\frac{4(7x-1)}{3\sqrt[4]{(x-1)x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-1 + x)*x*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (-4*(-1 + 7*x))/(3*((-1 + x)*x^3)^(1/4))
```

IntegrateAlgebraic [A] time = 0.23, size = 30, normalized size = 1.00

$$-\frac{4(7x-1)\left(x^4-x^3\right)^{3/4}}{3(x-1)x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((-1 + x)*x*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (-4*(-1 + 7*x)*(-x^3 + x^4)^(3/4))/(3*(-1 + x)*x^3)
```

fricas [A] time = 0.39, size = 18, normalized size = 0.60

$$-\frac{4\,(7\,x-1)}{3\,\left(x^4-x^3\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x/(x^4-x^3)^(1/4),x, algorithm="fricas")
```

```
[Out] -4/3*(7*x - 1)/(x^4 - x^3)^(1/4)
```

giac [A] time = 0.77, size = 23, normalized size = 0.77

$$\frac{4}{3}\left(-\frac{1}{x}+1\right)^{\frac{3}{4}}+\frac{8}{\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x/(x^4-x^3)^(1/4),x, algorithm="giac")
```

```
[Out] 4/3*(-1/x + 1)^(3/4) + 8/(-1/x + 1)^(1/4)
```

maple [A] time = 0.00, size = 19, normalized size = 0.63

$$-\frac{4\,(-1+7x)}{3\,\left(x^4-x^3\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-1+x)/x/(x^4-x^3)^(1/4),x)
```

```
[Out] -4/3*(-1+7*x)/(x^4-x^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(x^4-x^3\right)^{\frac{1}{4}}(x-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x/(x^4-x^3)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/((x^4 - x^3)^(1/4)*(x - 1)*x), x)
```

mupad [B] time = 0.23, size = 18, normalized size = 0.60

$$-\frac{28\,x-4}{3\,{\left(x^4-x^3\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x*(x^4 - x^3)^(1/4)*(x - 1)),x)
[Out] -(28*x - 4)/(3*(x^4 - x^3)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{x\sqrt[4]{x^3\,\left(x-1\right)}\,\left(x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x/(x**4-x**3)**(1/4),x)
[Out] Integral((x + 1)/(x*(x**3*(x - 1))**(1/4)*(x - 1)), x)
```

3.366 $\int \frac{(3+x^4)(-1-x^3+x^4)}{x^6\sqrt[4]{-x+x^5}}\,dx$

Optimal. Leaf size=30

$$\frac{4\left(3x^4-7x^3-3\right)\left(x^5-x\right)^{3/4}}{21x^6}$$

Rubi [A] time = 0.36, antiderivative size = 55, normalized size of antiderivative = 1.83, number of steps used = 21, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2052, 2025, 2032, 365, 364}

$$-\frac{4\left(x^5-x\right)^{3/4}}{7x^6}-\frac{4\left(x^5-x\right)^{3/4}}{3x^3}+\frac{4\left(x^5-x\right)^{3/4}}{7x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + x^4)*(-1 - x^3 + x^4))/(x^6*(-x + x^5)^(1/4)),x]
```

```
[Out] (-4*(-x + x^5)^(3/4))/(7*x^6) - (4*(-x + x^5)^(3/4))/(3*x^3) + (4*(-x + x^5
)^(3/4))/(7*x^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(3+x^4)(-1-x^3+x^4)}{x^6\sqrt[4]{-x+x^5}}\,dx &= \int \left(-\frac{3}{x^6\sqrt[4]{-x+x^5}}-\frac{3}{x^3\sqrt[4]{-x+x^5}}+\frac{2}{x^2\sqrt[4]{-x+x^5}}-\frac{x}{\sqrt[4]{-x+x^5}}+\frac{x^2}{\sqrt[4]{-x+x^5}}\right) \\
&= 2\int \frac{1}{x^2\sqrt[4]{-x+x^5}}\,dx-3\int \frac{1}{x^6\sqrt[4]{-x+x^5}}\,dx-3\int \frac{1}{x^3\sqrt[4]{-x+x^5}}\,dx-\int \frac{x}{\sqrt[4]{-x}} \\
&= -\frac{4\left(-x+x^5\right)^{3/4}}{7x^6}-\frac{4\left(-x+x^5\right)^{3/4}}{3x^3}+\frac{8\left(-x+x^5\right)^{3/4}}{5x^2}-\frac{9}{7}\int \frac{1}{x^2\sqrt[4]{-x+x^5}}\,dx-\frac{1}{} \\
&= -\frac{4\left(-x+x^5\right)^{3/4}}{7x^6}-\frac{4\left(-x+x^5\right)^{3/4}}{3x^3}+\frac{4\left(-x+x^5\right)^{3/4}}{7x^2}+\frac{9}{5}\int \frac{x^2}{\sqrt[4]{-x+x^5}}\,dx-\frac{\left(\sqrt[4]{}\right.}{} \\
&= -\frac{4\left(-x+x^5\right)^{3/4}}{7x^6}-\frac{4\left(-x+x^5\right)^{3/4}}{3x^3}+\frac{4\left(-x+x^5\right)^{3/4}}{7x^2}-\frac{4x^2\sqrt[4]{1-x^4}\,{}_2F_1\left(\frac{1}{4},\frac{7}{16};\frac{2}{1}\right.}{7\sqrt[4]{-x+x^5}} \\
&= -\frac{4\left(-x+x^5\right)^{3/4}}{7x^6}-\frac{4\left(-x+x^5\right)^{3/4}}{3x^3}+\frac{4\left(-x+x^5\right)^{3/4}}{7x^2}-\frac{36x^3\sqrt[4]{1-x^4}\,{}_2F_1\left(\frac{1}{4},\frac{11}{16};\right.}{55\sqrt[4]{-x+x^5}} \\
&= -\frac{4\left(-x+x^5\right)^{3/4}}{7x^6}-\frac{4\left(-x+x^5\right)^{3/4}}{3x^3}+\frac{4\left(-x+x^5\right)^{3/4}}{7x^2}
\end{aligned}
$$

Mathematica [C] time = 0.10, size = 117, normalized size = 3.90

$$
\frac{4\sqrt[4]{1-x^4}\left(165\,{}_2F_1\left(-\frac{21}{16},\frac{1}{4};-\frac{5}{16};x^4\right)+x^3\left(-165x^4\,{}_2F_1\left(\frac{1}{4},\frac{7}{16};\frac{23}{16};x^4\right)-462x\,{}_2F_1\left(-\frac{5}{16},\frac{1}{4};\frac{11}{16};x^4\right)+385\,{}_2F_1\left(-\frac{9}{16},\frac{1}{4};\frac{7}{16};x^4\right)+105x^5\,{}_2F_1\left(\frac{1}{4},\frac{11}{16};\frac{27}{16};x^4\right)\right)\right)}{1155x^5\sqrt[4]{x\left(x^4-1\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + x^4)*(-1 - x^3 + x^4))/(x^6*(-x + x^5)^(1/4)),x]
```

```
[Out] (4*(1 - x^4)^(1/4)*(165*Hypergeometric2F1[-21/16, 1/4, -5/16, x^4] + x^3*(3
85*Hypergeometric2F1[-9/16, 1/4, 7/16, x^4] - 462*x*Hypergeometric2F1[-5/16
, 1/4, 11/16, x^4] - 165*x^4*Hypergeometric2F1[1/4, 7/16, 23/16, x^4] + 105
*x^5*Hypergeometric2F1[1/4, 11/16, 27/16, x^4])))/(1155*x^5*(x*(-1 + x^4))^
(1/4))
```

IntegrateAlgebraic [A] time = 2.63, size = 30, normalized size = 1.00

$$
\frac{4\left(3x^4-7x^3-3\right)\left(x^5-x\right)^{3/4}}{21x^6}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3 + x^4)*(-1 - x^3 + x^4))/(x^6*(-x + x^5)^(1/4)),x]
```

```
[Out] (4*(-3 - 7*x^3 + 3*x^4)*(-x + x^5)^(3/4))/(21*x^6)
```

fricas [A] time = 0.40, size = 26, normalized size = 0.87

$$
\frac{4\left(x^5-x\right)^{\frac{3}{4}}\left(3\,x^4-7\,x^3-3\right)}{21\,x^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)*(x^4-x^3-1)/x^6/(x^5-x)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/21*(x^5 - x)^(3/4)*(3*x^4 - 7*x^3 - 3)/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4 - x^3 - 1)(x^4 + 3)}{(x^5 - x)^{\frac{1}{4}} x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)*(x^4-x^3-1)/x^6/(x^5-x)^(1/4),x, algorithm="giac")
[Out] integrate((x^4 - x^3 - 1)*(x^4 + 3)/((x^5 - x)^(1/4)*x^6), x)
```

maple [A] time = 0.01, size = 38, normalized size = 1.27

$$\frac{4(-1+x)(1+x)(x^2+1)(3x^4 - 7x^3 - 3)}{21x^5 (x^5 - x)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3)*(x^4-x^3-1)/x^6/(x^5-x)^(1/4),x)
[Out] 4/21*(-1+x)*(1+x)*(x^2+1)*(3*x^4-7*x^3-3)/x^5/(x^5-x)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4 - x^3 - 1)(x^4 + 3)}{(x^5 - x)^{\frac{1}{4}} x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)*(x^4-x^3-1)/x^6/(x^5-x)^(1/4),x, algorithm="maxima")
[Out] integrate((x^4 - x^3 - 1)*(x^4 + 3)/((x^5 - x)^(1/4)*x^6), x)
```

mupad [B] time = 0.30, size = 45, normalized size = 1.50

$$-\frac{12 (x^5 - x)^{3/4} + 28 x^3 (x^5 - x)^{3/4} - 12 x^4 (x^5 - x)^{3/4}}{21 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^4 + 3)*(x^3 - x^4 + 1))/(x^6*(x^5 - x)^(1/4)),x)
[Out] -(12*(x^5 - x)^(3/4) + 28*x^3*(x^5 - x)^(3/4) - 12*x^4*(x^5 - x)^(3/4))/(21
*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4 + 3)(x^4 - x^3 - 1)}{x^6 \sqrt[4]{x(x-1)(x+1)(x^2+1)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3)*(x**4-x**3-1)/x**6/(x**5-x)**(1/4),x)
[Out] Integral((x**4 + 3)*(x**4 - x**3 - 1)/(x**6*(x*(x - 1)*(x + 1)*(x**2 + 1))*
*(1/4)), x)
```

3.367 $\int \frac{(3+x^4)(-1+x^3+x^4)}{x^6\sqrt[4]{-x+x^5}}\,dx$

Optimal. Leaf size=30

$$\frac{4\left(3x^4+7x^3-3\right)\left(x^5-x\right)^{3/4}}{21x^6}$$

Rubi [A] time = 0.35, antiderivative size = 55, normalized size of antiderivative = 1.83, number of steps used = 21, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2052, 2025, 2032, 365, 364}

$$-\frac{4\left(x^5-x\right)^{3/4}}{7x^6}+\frac{4\left(x^5-x\right)^{3/4}}{3x^3}+\frac{4\left(x^5-x\right)^{3/4}}{7x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + x^4)*(-1 + x^3 + x^4))/(x^6*(-x + x^5)^(1/4)),x]
```

```
[Out] (-4*(-x + x^5)^(3/4))/(7*x^6) + (4*(-x + x^5)^(3/4))/(3*x^3) + (4*(-x + x^5
)^(3/4))/(7*x^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{(3+x^4)(-1+x^3+x^4)}{x^6\sqrt[4]{-x+x^5}}\,dx = \int \left(-\frac{3}{x^6\sqrt[4]{-x+x^5}}+\frac{3}{x^3\sqrt[4]{-x+x^5}}+\frac{2}{x^2\sqrt[4]{-x+x^5}}+\frac{x}{\sqrt[4]{-x+x^5}}+\frac{x^2}{\sqrt[4]{-x+x^5}}\right) d$$

$$= 2\int \frac{1}{x^2\sqrt[4]{-x+x^5}}\,dx - 3\int \frac{1}{x^6\sqrt[4]{-x+x^5}}\,dx + 3\int \frac{1}{x^3\sqrt[4]{-x+x^5}}\,dx + \int \frac{x}{\sqrt[4]{-x+}}$$

$$= -\frac{4(-x+x^5)^{3/4}}{7x^6}+\frac{4(-x+x^5)^{3/4}}{3x^3}+\frac{8(-x+x^5)^{3/4}}{5x^2}-\frac{9}{7}\int \frac{1}{x^2\sqrt[4]{-x+x^5}}\,dx-\frac{14}{5}$$

$$= -\frac{4(-x+x^5)^{3/4}}{7x^6}+\frac{4(-x+x^5)^{3/4}}{3x^3}+\frac{4(-x+x^5)^{3/4}}{7x^2}+\frac{9}{5}\int \frac{x^2}{\sqrt[4]{-x+x^5}}\,dx+\frac{(\sqrt[4]{x}}{}$$

$$= -\frac{4(-x+x^5)^{3/4}}{7x^6}+\frac{4(-x+x^5)^{3/4}}{3x^3}+\frac{4(-x+x^5)^{3/4}}{7x^2}+\frac{4x^2\sqrt[4]{1-x^4}\,{}_2F_1\left(\frac{1}{4},\frac{7}{16};\frac{23}{16};\right.}{7\sqrt[4]{-x+x^5}}$$

$$= -\frac{4(-x+x^5)^{3/4}}{7x^6}+\frac{4(-x+x^5)^{3/4}}{3x^3}+\frac{4(-x+x^5)^{3/4}}{7x^2}-\frac{36x^3\sqrt[4]{1-x^4}\,{}_2F_1\left(\frac{1}{4},\frac{11}{16};\frac{27}{16};\right.}{55\sqrt[4]{-x+x^5}}$$

$$= -\frac{4(-x+x^5)^{3/4}}{7x^6}+\frac{4(-x+x^5)^{3/4}}{3x^3}+\frac{4(-x+x^5)^{3/4}}{7x^2}$$

Mathematica [C] time = 0.07, size = 117, normalized size = 3.90

$$\frac{4\sqrt[4]{1-x^4}\left(165\,{}_2F_1\left(-\frac{21}{16},\frac{1}{4};-\frac{5}{16};x^4\right)+x^3\left(165x^4\,{}_2F_1\left(\frac{1}{4},\frac{7}{16};\frac{23}{16};x^4\right)-462x\,{}_2F_1\left(-\frac{5}{16},\frac{1}{4};\frac{11}{16};x^4\right)-385\,{}_2F_1\left(-\frac{9}{16},\frac{1}{4};\frac{7}{16};x^4\right)+105x^5\,{}_2F_1\left(\frac{1}{4},\frac{11}{16};\frac{27}{16};x^4\right)\right)\right)}{1155x^5\sqrt[4]{x(x^4-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + x^4)*(-1 + x^3 + x^4))/(x^6*(-x + x^5)^(1/4)),x]
```

```
[Out] (4*(1 - x^4)^(1/4)*(165*Hypergeometric2F1[-21/16, 1/4, -5/16, x^4] + x^3*(-
385*Hypergeometric2F1[-9/16, 1/4, 7/16, x^4] - 462*x*Hypergeometric2F1[-5/1
6, 1/4, 11/16, x^4] + 165*x^4*Hypergeometric2F1[1/4, 7/16, 23/16, x^4] + 10
5*x^5*Hypergeometric2F1[1/4, 11/16, 27/16, x^4])))/(1155*x^5*(x*(-1 + x^4))
^(1/4))
```

IntegrateAlgebraic [A] time = 2.61, size = 30, normalized size = 1.00

$$\frac{4(3x^4+7x^3-3)(x^5-x)^{3/4}}{21x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3 + x^4)*(-1 + x^3 + x^4))/(x^6*(-x + x^5)^(1/4)),x]
```

```
[Out] (4*(-3 + 7*x^3 + 3*x^4)*(-x + x^5)^(3/4))/(21*x^6)
```

fricas [A] time = 0.40, size = 26, normalized size = 0.87

$$\frac{4\,(x^5-x)^{\frac{3}{4}}(3\,x^4+7\,x^3-3)}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)*(x^4+x^3-1)/x^6/(x^5-x)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/21*(x^5 - x)^(3/4)*(3*x^4 + 7*x^3 - 3)/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4 + x^3 - 1\right)\left(x^4 + 3\right)}{\left(x^5 - x\right)^{\frac{1}{4}} x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)*(x^4+x^3-1)/x^6/(x^5-x)^(1/4),x, algorithm="giac")
[Out] integrate((x^4 + x^3 - 1)*(x^4 + 3)/((x^5 - x)^(1/4)*x^6), x)
```

maple [A] time = 0.01, size = 38, normalized size = 1.27

$$\frac{4\left(-1+x\right)\left(1+x\right)\left(x^2+1\right)\left(3x^4+7x^3-3\right)}{21x^5\left(x^5-x\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3)*(x^4+x^3-1)/x^6/(x^5-x)^(1/4),x)
[Out] 4/21*(-1+x)*(1+x)*(x^2+1)*(3*x^4+7*x^3-3)/x^5/(x^5-x)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4 + x^3 - 1\right)\left(x^4 + 3\right)}{\left(x^5 - x\right)^{\frac{1}{4}} x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)*(x^4+x^3-1)/x^6/(x^5-x)^(1/4),x, algorithm="maxima")
[Out] integrate((x^4 + x^3 - 1)*(x^4 + 3)/((x^5 - x)^(1/4)*x^6), x)
```

mupad [B] time = 0.25, size = 45, normalized size = 1.50

$$\frac{28\,x^3\,{\left(x^5-x\right)}^{3/4}-12\,{\left(x^5-x\right)}^{3/4}+12\,x^4\,{\left(x^5-x\right)}^{3/4}}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 3)*(x^3 + x^4 - 1))/(x^6*(x^5 - x)^(1/4)),x)
[Out] (28*x^3*(x^5 - x)^(3/4) - 12*(x^5 - x)^(3/4) + 12*x^4*(x^5 - x)^(3/4))/(21*
x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4} + 3\right) \left(x^{4} + x^{3} - 1\right)}{x^{6} \sqrt[4]{x \left(x - 1\right) \left(x + 1\right) \left(x^{2} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3)*(x**4+x**3-1)/x**6/(x**5-x)**(1/4),x)
[Out] Integral((x**4 + 3)*(x**4 + x**3 - 1)/(x**6*(x*(x - 1)*(x + 1)*(x**2 + 1))*
*(1/4)), x)
```

3.368 $\int \frac{-1+3x^4}{(1-ax+x^4)\sqrt{x+x^5}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x^5+x}}{x^4+1}\right)}{\sqrt{a}}$$

Rubi [F] time = 1.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+3x^4}{(1-ax+x^4)\sqrt{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 3*x^4)/((1 - a*x + x^4)*Sqrt[x + x^5]),x]
```

```
[Out] (3*x^2*Sqrt[(1 + x)^2/x]*Sqrt[-((1 + x^4)/x^2)]*EllipticF[ArcSin[Sqrt[-((Sq
rt[2] - 2*x + Sqrt[2]*x^2)/x)]/2], -2*(1 - Sqrt[2])])/(Sqrt[2 + Sqrt[2]]*(1
 + x)*Sqrt[x + x^5]) - (3*Sqrt[-((1 - x)^2/x)]*x^2*Sqrt[-((1 + x^4)/x^2)]*E
llipticF[ArcSin[Sqrt[(Sqrt[2] + 2*x + Sqrt[2]*x^2)/x]/2], -2*(1 - Sqrt[2])]
)/(Sqrt[2 + Sqrt[2]]*(1 - x)*Sqrt[x + x^5]) + (8*Sqrt[x]*Sqrt[1 + x^4]*Defe
r[Subst][Defer[Int][1/((-1 + a*x^2 - x^8)*Sqrt[1 + x^8]), x], x, Sqrt[x]])/
Sqrt[x + x^5] - (6*a*Sqrt[x]*Sqrt[1 + x^4]*Defer[Subst][Defer[Int][x^2/((-1
 + a*x^2 - x^8)*Sqrt[1 + x^8]), x], x, Sqrt[x]])/Sqrt[x + x^5]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+3x^4}{(1-ax+x^4)\sqrt{x+x^5}}\,dx &= \frac{\left(\sqrt{x}\sqrt{1+x^4}\right)\int \frac{-1+3x^4}{\sqrt{x}\sqrt{1+x^4}(1-ax+x^4)}\,dx}{\sqrt{x+x^5}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \frac{-1+3x^8}{\sqrt{1+x^8}(1-ax^2+x^8)}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^5}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \left(\frac{3}{\sqrt{1+x^8}}-\frac{4-3ax^2}{\sqrt{1+x^8}(1-ax^2+x^8)}\right)dx,x,\sqrt{x}\right)}{\sqrt{x+x^5}}\\
&= -\frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \frac{4-3ax^2}{\sqrt{1+x^8}(1-ax^2+x^8)}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^5}}+\frac{\left(6\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int\right.}{\sqrt{x+x^5}}\\
&= -\frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \left(-\frac{4}{(-1+ax^2-x^8)\sqrt{1+x^8}}+\frac{3ax^2}{(-1+ax^2-x^8)\sqrt{1+x^8}}\right)dx,x,\sqrt{x}\right)}{\sqrt{x+x^5}}+\\
&= \frac{3x^2\sqrt{\frac{(1+x)^2}{x}}\sqrt{-\frac{1+x^4}{x^2}}\,F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}-2x+\sqrt{2}x^2}{x}}\right)\middle|-2\left(1-\sqrt{2}\right)\right)}{\sqrt{2+\sqrt{2}}\,(1+x)\sqrt{x+x^5}}-\frac{3\sqrt{-\frac{(1-x)^2}{x}}\,x^2}{}
\end{aligned}
$$

Mathematica [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{-1+3x^4}{(1-ax+x^4)\sqrt{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + 3*x^4)/((1 - a*x + x^4)*Sqrt[x + x^5]),x]
[Out] Integrate[(-1 + 3*x^4)/((1 - a*x + x^4)*Sqrt[x + x^5]), x]
```

IntegrateAlgebraic [A] time = 0.58, size = 30, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x^5+x}}{x^4+1}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 3*x^4)/((1 - a*x + x^4)*Sqrt[x + x^5]),x]
[Out] (-2*ArcTanh[(Sqrt[a]*Sqrt[x + x^5])/(1 + x^4)])/Sqrt[a]
```

fricas [A] time = 0.43, size = 127, normalized size = 4.23

$$\left[\frac{\log\left(\frac{x^8+6\,ax^5+a^2x^2+2\,x^4-4\,\sqrt{x^5+x}\left(x^4+ax+1\right)\sqrt{a}+6\,ax+1}{x^8-2\,ax^5+a^2x^2+2\,x^4-2\,ax+1}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{x^5+x}\left(x^4+ax+1\right)\sqrt{-a}}{2\left(ax^5+ax\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)/(x^4-a*x+1)/(x^5+x)^(1/2),x, algorithm="fricas")
[Out] [1/2*log((x^8 + 6*a*x^5 + a^2*x^2 + 2*x^4 - 4*sqrt(x^5 + x)*(x^4 + a*x + 1)
*sqrt(a) + 6*a*x + 1)/(x^8 - 2*a*x^5 + a^2*x^2 + 2*x^4 - 2*a*x + 1))/sqrt(a
), sqrt(-a)*arctan(1/2*sqrt(x^5 + x)*(x^4 + a*x + 1)*sqrt(-a)/(a*x^5 + a*x)
)/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^4-1}{\sqrt{x^5+x}\left(x^4-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)/(x^4-a*x+1)/(x^5+x)^(1/2),x, algorithm="giac")
[Out] integrate((3*x^4 - 1)/(sqrt(x^5 + x)*(x^4 - a*x + 1)), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{3x^4-1}{\left(x^4-ax+1\right)\sqrt{x^5+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4-1)/(x^4-a*x+1)/(x^5+x)^(1/2),x)
[Out] int((3*x^4-1)/(x^4-a*x+1)/(x^5+x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^4-1}{\sqrt{x^5+x}\left(x^4-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)/(x^4-a*x+1)/(x^5+x)^(1/2),x, algorithm="maxima")
[Out] integrate((3*x^4 - 1)/(sqrt(x^5 + x)*(x^4 - a*x + 1)), x)
```

mupad [B] time = 0.42, size = 37, normalized size = 1.23

$$\frac{\ln\left(\frac{a\,x-2\,\sqrt{a}\,\sqrt{x^5+x}+x^4+1}{x^4-a\,x+1}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4 - 1)/((x + x^5)^(1/2)*(x^4 - a*x + 1)),x)
[Out] log((a*x - 2*a^(1/2)*(x + x^5)^(1/2) + x^4 + 1)/(x^4 - a*x + 1))/a^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4-1)/(x**4-a*x+1)/(x**5+x)**(1/2),x)
[Out] Timed out
```

3.369 $\int \frac{-1+3x^4}{(a-x+ax^4)\sqrt{x+x^5}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^5+x}}{\sqrt{a}\,(x^4+1)}\right)}{\sqrt{a}}$$

Rubi [F] time = 1.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+3x^4}{(a-x+ax^4)\sqrt{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 3*x^4)/((a - x + a*x^4)*Sqrt[x + x^5]),x]
```

```
[Out] (3*x^2*Sqrt[(1 + x)^2/x]*Sqrt[-((1 + x^4)/x^2)]*EllipticF[ArcSin[Sqrt[-((Sq
rt[2] - 2*x + Sqrt[2]*x^2)/x)]/2], -2*(1 - Sqrt[2])])/(Sqrt[2 + Sqrt[2]]*a*
(1 + x)*Sqrt[x + x^5]) - (3*Sqrt[-((1 - x)^2/x)]*x^2*Sqrt[-((1 + x^4)/x^2)]
*EllipticF[ArcSin[Sqrt[(Sqrt[2] + 2*x + Sqrt[2]*x^2)/x]/2], -2*(1 - Sqrt[2]
)])/(Sqrt[2 + Sqrt[2]]*a*(1 - x)*Sqrt[x + x^5]) - (8*Sqrt[x]*Sqrt[1 + x^4]*
Defer[Subst][Defer[Int][1/(Sqrt[1 + x^8]*(a - x^2 + a*x^8)), x], x, Sqrt[x]
])/Sqrt[x + x^5] + (6*Sqrt[x]*Sqrt[1 + x^4]*Defer[Subst][Defer[Int][x^2/(Sq
rt[1 + x^8]*(a - x^2 + a*x^8)), x], x, Sqrt[x]])/(a*Sqrt[x + x^5])
```

Rubi steps

$$\int \frac{-1+3x^4}{(a-x+ax^4)\sqrt{x+x^5}}\,dx = \frac{\left(\sqrt{x}\sqrt{1+x^4}\right)\int \frac{-1+3x^4}{\sqrt{x}\sqrt{1+x^4}\,(a-x+ax^4)}\,dx}{\sqrt{x+x^5}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \frac{-1+3x^8}{\sqrt{1+x^8}\,(a-x^2+ax^8)}\,dx, x, \sqrt{x}\right)}{\sqrt{x+x^5}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \left(\frac{3}{a\sqrt{1+x^8}} - \frac{4a-3x^2}{a\sqrt{1+x^8}\,(a-x^2+ax^8)}\right)dx, x, \sqrt{x}\right)}{\sqrt{x+x^5}}$$

$$= -\frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \frac{4a-3x^2}{\sqrt{1+x^8}\,(a-x^2+ax^8)}\,dx, x, \sqrt{x}\right)}{a\sqrt{x+x^5}} + \frac{\left(6\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}}{a\sqrt{x}}$$

$$= -\frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \left(\frac{4a}{\sqrt{1+x^8}\,(a-x^2+ax^8)} - \frac{3x^2}{\sqrt{1+x^8}\,(a-x^2+ax^8)}\right)dx, x, \sqrt{x}\right)}{a\sqrt{x+x^5}} + \ldots$$

$$= \frac{3x^2\sqrt{\frac{(1+x)^2}{x}}\sqrt{-\frac{1+x^4}{x^2}}\,F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}-2x+\sqrt{2}\,x^2}{x}}\right)\middle|-2\left(1-\sqrt{2}\right)\right)}{\sqrt{2+\sqrt{2}}\,a(1+x)\sqrt{x+x^5}} - \frac{3\sqrt{-\frac{(1-x)^2}{x}}}{}$$

Mathematica [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{-1+3x^4}{(a-x+ax^4)\sqrt{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + 3*x^4)/((a - x + a*x^4)*Sqrt[x + x^5]),x]
[Out] Integrate[(-1 + 3*x^4)/((a - x + a*x^4)*Sqrt[x + x^5]), x]
```

IntegrateAlgebraic [A] time = 1.15, size = 30, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^5+x}}{\sqrt{a}\left(x^4+1\right)}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 3*x^4)/((a - x + a*x^4)*Sqrt[x + x^5]),x]
[Out] (-2*ArcTanh[Sqrt[x + x^5]/(Sqrt[a]*(1 + x^4))])/Sqrt[a]
```

fricas [A] time = 0.46, size = 137, normalized size = 4.57

$$\left[\frac{\log\left(\frac{a^2x^8+2\,a^2x^4+6\,ax^5-4\left(ax^4+a+x\right)\sqrt{x^5+x}\,\sqrt{a}+a^2+6\,ax+x^2}{a^2x^8+2\,a^2x^4-2\,ax^5+a^2-2\,ax+x^2}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\,\arctan\left(\frac{\left(ax^4+a+x\right)\sqrt{x^5+x}\,\sqrt{-a}}{2\left(ax^5+ax\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)/(a*x^4+a-x)/(x^5+x)^(1/2),x, algorithm="fricas")
[Out] [1/2*log((a^2*x^8 + 2*a^2*x^4 + 6*a*x^5 - 4*(a*x^4 + a + x)*sqrt(x^5 + x)*s
qrt(a) + a^2 + 6*a*x + x^2)/(a^2*x^8 + 2*a^2*x^4 - 2*a*x^5 + a^2 - 2*a*x +
x^2))/sqrt(a), sqrt(-a)*arctan(1/2*(a*x^4 + a + x)*sqrt(x^5 + x)*sqrt(-a)/(
a*x^5 + a*x))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^4-1}{\left(ax^4+a-x\right)\sqrt{x^5+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)/(a*x^4+a-x)/(x^5+x)^(1/2),x, algorithm="giac")
[Out] integrate((3*x^4 - 1)/((a*x^4 + a - x)*sqrt(x^5 + x)), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{3x^4-1}{\left(a\,x^4+a-x\right)\sqrt{x^5+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4-1)/(a*x^4+a-x)/(x^5+x)^(1/2),x)
[Out] int((3*x^4-1)/(a*x^4+a-x)/(x^5+x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^4-1}{\left(ax^4+a-x\right)\sqrt{x^5+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)/(a*x^4+a-x)/(x^5+x)^(1/2),x, algorithm="maxima")
[Out] integrate((3*x^4 - 1)/((a*x^4 + a - x)*sqrt(x^5 + x)), x)
```

mupad [B] time = 0.42, size = 38, normalized size = 1.27

$$\frac{\ln\left(\frac{a+x-2\,\sqrt{a}\,\sqrt{x^5+x}+a\,x^4}{a\,x^4-x+a}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4 - 1)/((x + x^5)^(1/2)*(a - x + a*x^4)),x)
[Out] log((a + x - 2*a^(1/2)*(x + x^5)^(1/2) + a*x^4)/(a - x + a*x^4))/a^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4-1)/(a*x**4+a-x)/(x**5+x)**(1/2),x)
[Out] Timed out
```

3.370 $\int \frac{\sqrt{-1+x^6}\left(2+x^6\right)}{x^3\left(-1-x^4+x^6\right)}\,dx$

Optimal. Leaf size=30

$$\frac{\sqrt{x^6-1}}{x^2}-\tanh^{-1}\left(\frac{x^2}{\sqrt{x^6-1}}\right)$$

Rubi [F] time = 1.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-1+x^6}\left(2+x^6\right)}{x^3\left(-1-x^4+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-1 + x^6]*(2 + x^6))/(x^3*(-1 - x^4 + x^6)),x]
```

```
[Out] Sqrt[-1 + x^6]/x^2 + (3*Sqrt[-1 + x^6])/(1 - Sqrt[3] - x^2) - (3*3^(1/4)*Sq
rt[2 + Sqrt[3]]*(1 - x^2)*Sqrt[(1 + x^2 + x^4)/(1 - Sqrt[3] - x^2)^2]*Ellip
ticE[ArcSin[(1 + Sqrt[3] - x^2)/(1 - Sqrt[3] - x^2)], -7 + 4*Sqrt[3]])/(2*S
qrt[-((1 - x^2)/(1 - Sqrt[3] - x^2)^2)]*Sqrt[-1 + x^6]) + (Sqrt[2]*3^(3/4)*
(1 - x^2)*Sqrt[(1 + x^2 + x^4)/(1 - Sqrt[3] - x^2)^2]*EllipticF[ArcSin[(1 +
 Sqrt[3] - x^2)/(1 - Sqrt[3] - x^2)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x^2)/(1
 - Sqrt[3] - x^2)^2)]*Sqrt[-1 + x^6]) - Defer[Subst][Defer[Int][Sqrt[-1 + x
^3]/(-1 - x^2 + x^3), x], x, x^2] + (3*Defer[Subst][Defer[Int][(x*Sqrt[-1 +
 x^3])/(-1 - x^2 + x^3), x], x, x^2])/2
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x^6}\left(2+x^6\right)}{x^3\left(-1-x^4+x^6\right)}\,dx &= \int \left(-\frac{2\sqrt{-1+x^6}}{x^3}+\frac{x\left(2-3x^2\right)\sqrt{-1+x^6}}{1+x^4-x^6}\right)dx\\
&= -\left(2\int \frac{\sqrt{-1+x^6}}{x^3}\,dx\right)+\int \frac{x\left(2-3x^2\right)\sqrt{-1+x^6}}{1+x^4-x^6}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(2-3x)\sqrt{-1+x^3}}{1+x^2-x^3}\,dx,x,x^2\right)-\text{Subst}\left(\int \frac{\sqrt{-1+x^3}}{x^2}\,dx,x,x^2\right)\\
&= \frac{\sqrt{-1+x^6}}{x^2}+\frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{2\sqrt{-1+x^3}}{-1-x^2+x^3}+\frac{3x\sqrt{-1+x^3}}{-1-x^2+x^3}\right)dx,x,x^2\right)-\frac{3}{2}\,\text{Subst}\left(\int -\right.\\
&= \frac{\sqrt{-1+x^6}}{x^2}+\frac{3}{2}\,\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx,x,x^2\right)+\frac{3}{2}\,\text{Subst}\left(\int \frac{x\sqrt{-1+x^3}}{-1-x^2+x^3}\,dx,x,x^2\right.\\
&= \frac{\sqrt{-1+x^6}}{x^2}+\frac{3\sqrt{-1+x^6}}{1-\sqrt{3}-x^2}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1-\sqrt{3}-x^2\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x^2}{1-\sqrt{3}-x^2}\right)\right.}{2\sqrt{-\frac{1-x^2}{\left(1-\sqrt{3}-x^2\right)^2}}\sqrt{-1+x^6}}
\end{aligned}$$

Mathematica [C] time = 5.57, size = 947, normalized size = 31.57

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[-1 + x^6]*(2 + x^6))/(x^3*(-1 - x^4 + x^6)),x]
```

```
[Out] Sqrt[-1 + x^6]/x^2 + (Sqrt[(1 - x^2)/(1 + (-1)^(1/3))]*Sqrt[1 + x^2 + x^4]*
(-((Sqrt[3]*(I*Sqrt[3] + (1 + (-1)^(1/3))*x^2)*EllipticF[ArcSin[Sqrt[(1 - (
-1)^(2/3)*x^2)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x^2)) - ((
3*I)*((EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + Root[-1 - #1^2 + #1^3 & , 1, 0]
), ArcSin[Sqrt[(1 - (-1)^(2/3)*x^2)/(1 + (-1)^(1/3))]], (-1)^(1/3)]*(3 + Ro
ot[-1 - #1^2 + #1^3 & , 1, 0]^2))/((-1)^(1/3) + Root[-1 - #1^2 + #1^3 & , 1
, 0]) + (2*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + Root[-1 - #1^2 + #1^3 & , 3
, 0]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x^2)/(1 + (-1)^(1/3))]], (-1)^(1/3)]*(Ro
ot[-1 - #1^2 + #1^3 & , 1, 0] - Root[-1 - #1^2 + #1^3 & , 2, 0])*((-1)^(1/3
) + Root[-1 - #1^2 + #1^3 & , 2, 0]) + EllipticPi[(I*Sqrt[3])/((-1)^(1/3) +
 Root[-1 - #1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x^2)/(1 + (-
1)^(1/3))]], (-1)^(1/3)]*(Root[-1 - #1^2 + #1^3 & , 1, 0] - Root[-1 - #1^2
+ #1^3 & , 2, 0])*((-1)^(1/3) + Root[-1 - #1^2 + #1^3 & , 2, 0])*Root[-1 -
#1^2 + #1^3 & , 3, 0]^3 - 2*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + Root[-1 -
#1^2 + #1^3 & , 2, 0]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x^2)/(1 + (-1)^(1/3))]]
, (-1)^(1/3)]*(Root[-1 - #1^2 + #1^3 & , 1, 0] - Root[-1 - #1^2 + #1^3 & ,
3, 0])*((-1)^(1/3) + Root[-1 - #1^2 + #1^3 & , 3, 0]) - EllipticPi[(I*Sqrt[
3])/((-1)^(1/3) + Root[-1 - #1^2 + #1^3 & , 2, 0]), ArcSin[Sqrt[(1 - (-1)^(
2/3)*x^2)/(1 + (-1)^(1/3))]], (-1)^(1/3)]*Root[-1 - #1^2 + #1^3 & , 2, 0]^3
*(Root[-1 - #1^2 + #1^3 & , 1, 0] - Root[-1 - #1^2 + #1^3 & , 3, 0])*((-1)^
(1/3) + Root[-1 - #1^2 + #1^3 & , 3, 0]))/(((-1)^(1/3) + Root[-1 - #1^2 + #
1^3 & , 2, 0])*(Root[-1 - #1^2 + #1^3 & , 2, 0] - Root[-1 - #1^2 + #1^3 & ,
 3, 0])*((-1)^(1/3) + Root[-1 - #1^2 + #1^3 & , 3, 0]))))/((Root[-1 - #1^2
+ #1^3 & , 1, 0] - Root[-1 - #1^2 + #1^3 & , 2, 0])*(Root[-1 - #1^2 + #1^3
& , 1, 0] - Root[-1 - #1^2 + #1^3 & , 3, 0]))))/(3*Sqrt[-1 + x^6])
```

IntegrateAlgebraic [A] time = 7.34, size = 30, normalized size = 1.00

$$\frac{\sqrt{x^6-1}}{x^2}-\tanh^{-1}\left(\frac{x^2}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^6]*(2 + x^6))/(x^3*(-1 - x^4 + x^6)),x]
```

```
[Out] Sqrt[-1 + x^6]/x^2 - ArcTanh[x^2/Sqrt[-1 + x^6]]
```

fricas [B] time = 0.45, size = 53, normalized size = 1.77

$$\frac{x^2\log\left(\frac{x^6+x^4-2\sqrt{x^6-1}\,x^2-1}{x^6-x^4-1}\right)+2\sqrt{x^6-1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)*(x^6+2)/x^3/(x^6-x^4-1),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2*log((x^6 + x^4 - 2*sqrt(x^6 - 1)*x^2 - 1)/(x^6 - x^4 - 1)) + 2*sqr
t(x^6 - 1))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^6+2\right)\sqrt{x^6-1}}{\left(x^6-x^4-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)*(x^6+2)/x^3/(x^6-x^4-1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 2)*sqrt(x^6 - 1)/((x^6 - x^4 - 1)*x^3), x)
```

maple [B] time = 0.04, size = 54, normalized size = 1.80

$$\frac{\sqrt{x^6-1}}{x^2}+\frac{\ln\left(-\frac{-x^6-x^4+2x^2\sqrt{x^6-1}+1}{x^6-x^4-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/2)*(x^6+2)/x^3/(x^6-x^4-1),x)
[Out] (x^6-1)^(1/2)/x^2+1/2*ln(-(-x^6-x^4+2*x^2*(x^6-1)^(1/2)+1)/(x^6-x^4-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^6+2)\sqrt{x^6-1}}{(x^6-x^4-1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)*(x^6+2)/x^3/(x^6-x^4-1),x, algorithm="maxima")
[Out] integrate((x^6 + 2)*sqrt(x^6 - 1)/((x^6 - x^4 - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{\sqrt{x^6-1}\,(x^6+2)}{x^3\,(-x^6+x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^6 - 1)^(1/2)*(x^6 + 2))/(x^3*(x^4 - x^6 + 1)),x)
[Out] int(-((x^6 - 1)^(1/2)*(x^6 + 2))/(x^3*(x^4 - x^6 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/2)*(x**6+2)/x**3/(x**6-x**4-1),x)
[Out] Timed out
```

3.371 $\int \frac{\sqrt{1-x^6}\left(1+2x^6\right)}{x^2\left(-1-x^2+x^6\right)}\,dx$

Optimal. Leaf size=30

$$\frac{\sqrt{1-x^6}}{x}+\tan^{-1}\left(\frac{x}{\sqrt{1-x^6}}\right)$$

Rubi [F] time = 0.84, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1-x^6}\left(1+2x^6\right)}{x^2\left(-1-x^2+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[1 - x^6]*(1 + 2*x^6))/(x^2*(-1 - x^2 + x^6)),x]
```

```
[Out] Sqrt[1 - x^6]/x + (3*(1 + Sqrt[3])*x*Sqrt[1 - x^6])/(2*(1 - (1 + Sqrt[3])*x
^2)) - (3*3^(1/4)*x*(1 - x^2)*Sqrt[(1 + x^2 + x^4)/(1 - (1 + Sqrt[3])*x^2)^
2]*EllipticE[ArcCos[(1 - (1 - Sqrt[3])*x^2)/(1 - (1 + Sqrt[3])*x^2)], (2 +
Sqrt[3])/4])/(2*Sqrt[-((x^2*(1 - x^2))/(1 - (1 + Sqrt[3])*x^2)^2)]*Sqrt[1 -
 x^6]) - (3^(3/4)*(1 - Sqrt[3])*x*(1 - x^2)*Sqrt[(1 + x^2 + x^4)/(1 - (1 +
Sqrt[3])*x^2)^2]*EllipticF[ArcCos[(1 - (1 - Sqrt[3])*x^2)/(1 - (1 + Sqrt[3]
)*x^2)], (2 + Sqrt[3])/4])/(4*Sqrt[-((x^2*(1 - x^2))/(1 - (1 + Sqrt[3])*x^2
)^2)]*Sqrt[1 - x^6]) + Defer[Int][Sqrt[1 - x^6]/(1 + x^2 - x^6), x] + 3*Def
er[Int][(x^4*Sqrt[1 - x^6])/(-1 - x^2 + x^6), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-x^6}\left(1+2x^6\right)}{x^2\left(-1-x^2+x^6\right)}\,dx &= \int \left(-\frac{\sqrt{1-x^6}}{x^2}+\frac{\left(-1+3x^4\right)\sqrt{1-x^6}}{-1-x^2+x^6}\right)dx\\
&= -\int \frac{\sqrt{1-x^6}}{x^2}\,dx+\int \frac{\left(-1+3x^4\right)\sqrt{1-x^6}}{-1-x^2+x^6}\,dx\\
&= \frac{\sqrt{1-x^6}}{x}+3\int \frac{x^4}{\sqrt{1-x^6}}\,dx+\int \left(\frac{\sqrt{1-x^6}}{1+x^2-x^6}+\frac{3x^4\sqrt{1-x^6}}{-1-x^2+x^6}\right)dx\\
&= \frac{\sqrt{1-x^6}}{x}-\frac{3}{2}\int \frac{-1+\sqrt{3}-2x^4}{\sqrt{1-x^6}}\,dx+3\int \frac{x^4\sqrt{1-x^6}}{-1-x^2+x^6}\,dx-\frac{1}{2}\left(3\left(1-\sqrt{3}\right)\right)\int \frac{}{\sqrt{1}}\\
&= \frac{\sqrt{1-x^6}}{x}+\frac{3\left(1+\sqrt{3}\right)x\sqrt{1-x^6}}{2\left(1-\left(1+\sqrt{3}\right)x^2\right)}-\frac{3\sqrt[4]{3}\,x\left(1-x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\,E\left(\cos^{-1}\left(\frac{1-(1-}{1-(1+}\right.\right.}{2\sqrt{-\frac{x^2\left(1-x^2\right)}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1-x^6}}
\end{aligned}$$

Mathematica [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x^6}\left(1+2x^6\right)}{x^2\left(-1-x^2+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[1 - x^6]*(1 + 2*x^6))/(x^2*(-1 - x^2 + x^6)),x]
```

```
[Out] Integrate[(Sqrt[1 - x^6]*(1 + 2*x^6))/(x^2*(-1 - x^2 + x^6)), x]
```

IntegrateAlgebraic [A] time = 3.38, size = 30, normalized size = 1.00

$$\frac{\sqrt{1-x^6}}{x}+\tan^{-1}\left(\frac{x}{\sqrt{1-x^6}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 - x^6]*(1 + 2*x^6))/(x^2*(-1 - x^2 + x^6)),x]
[Out] Sqrt[1 - x^6]/x + ArcTan[x/Sqrt[1 - x^6]]
```

fricas [A] time = 0.79, size = 42, normalized size = 1.40

$$-\frac{x\arctan\left(\frac{2\sqrt{-x^6+1}\,x}{x^6+x^2-1}\right)-2\sqrt{-x^6+1}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(1/2)*(2*x^6+1)/x^2/(x^6-x^2-1),x, algorithm="fricas")
[Out] -1/2*(x*arctan(2*sqrt(-x^6 + 1)*x/(x^6 + x^2 - 1)) - 2*sqrt(-x^6 + 1))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2\,x^6+1\right)\sqrt{-x^6+1}}{\left(x^6-x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(1/2)*(2*x^6+1)/x^2/(x^6-x^2-1),x, algorithm="giac")
[Out] integrate((2*x^6 + 1)*sqrt(-x^6 + 1)/((x^6 - x^2 - 1)*x^2), x)
```

maple [C] time = 0.51, size = 85, normalized size = 2.83

$$-\frac{x^6-1}{x\sqrt{-x^6+1}}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^2+1\right)x^6-\mathrm{RootOf}\left(\_Z^2+1\right)x^2+2\sqrt{-x^6+1}\,x+\mathrm{RootOf}\left(\_Z^2+1\right)}{x^6-x^2-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^6+1)^(1/2)*(2*x^6+1)/x^2/(x^6-x^2-1),x)
[Out] -(x^6-1)/x/(-x^6+1)^(1/2)+1/2*RootOf(_Z^2+1)*ln(-(-RootOf(_Z^2+1)*x^6-RootO
f(_Z^2+1)*x^2+2*(-x^6+1)^(1/2)*x+RootOf(_Z^2+1))/(x^6-x^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2\,x^6+1\right)\sqrt{-x^6+1}}{\left(x^6-x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(1/2)*(2*x^6+1)/x^2/(x^6-x^2-1),x, algorithm="maxima")
[Out] integrate((2*x^6 + 1)*sqrt(-x^6 + 1)/((x^6 - x^2 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{\sqrt{1-x^6}\,\left(2\,x^6+1\right)}{x^2\,\left(-x^6+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1 - x^6)^(1/2)*(2*x^6 + 1))/(x^2*(x^2 - x^6 + 1)),x)
[Out] int(-((1 - x^6)^(1/2)*(2*x^6 + 1))/(x^2*(x^2 - x^6 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**6+1)**(1/2)*(2*x**6+1)/x**2/(x**6-x**2-1),x)
[Out] Timed out
```

3.372 $\int \frac{1}{x^4\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{x^3-1}}{3x^3}+\frac{1}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 51, 63, 203}

$$\frac{\sqrt{x^3-1}}{3x^3}+\frac{1}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[-1 + x^3]),x]
```

```
[Out] Sqrt[-1 + x^3]/(3*x^3) + ArcTan[Sqrt[-1 + x^3]]/3
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt{-1+x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx,x,x^3\right)\\
&= \frac{\sqrt{-1+x^3}}{3x^3}+\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx,x,x^3\right)\\
&= \frac{\sqrt{-1+x^3}}{3x^3}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x^3}\right)\\
&= \frac{\sqrt{-1+x^3}}{3x^3}+\frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^3}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.32

$$
\frac{1}{3}\sqrt{x^3-1}\left(\frac{1}{x^3}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{\sqrt{1-x^3}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[-1 + x^3]),x]
[Out] (Sqrt[-1 + x^3]*(x^(-3) + ArcTanh[Sqrt[1 - x^3]]/Sqrt[1 - x^3]))/3
```

IntegrateAlgebraic [A] time = 0.00, size = 31, normalized size = 1.00

$$
\frac{\sqrt{x^3-1}}{3x^3}+\frac{1}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[-1 + x^3]),x]
[Out] Sqrt[-1 + x^3]/(3*x^3) + ArcTan[Sqrt[-1 + x^3]]/3
```

fricas [A] time = 0.39, size = 25, normalized size = 0.81

$$
\frac{x^3\arctan\left(\sqrt{x^3-1}\right)+\sqrt{x^3-1}}{3\,x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] 1/3*(x^3*arctan(sqrt(x^3 - 1)) + sqrt(x^3 - 1))/x^3
```

giac [A] time = 0.23, size = 23, normalized size = 0.74

$$
\frac{\sqrt{x^3-1}}{3\,x^3}+\frac{1}{3}\arctan\left(\sqrt{x^3-1}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^3-1)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(x^3 - 1)/x^3 + 1/3*arctan(sqrt(x^3 - 1))
```

maple [A] time = 0.20, size = 24, normalized size = 0.77

$$
\frac{\sqrt{x^3-1}}{3x^3}+\frac{\arctan\left(\sqrt{x^3-1}\right)}{3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^3-1)^(1/2),x)
```

```
[Out] 1/3*(x^3-1)^(1/2)/x^3+1/3*arctan((x^3-1)^(1/2))
```

maxima [A] time = 0.48, size = 23, normalized size = 0.74

$$\frac{\sqrt{x^3-1}}{3\,x^3}+\frac{1}{3}\,\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^3 - 1)/x^3 + 1/3*arctan(sqrt(x^3 - 1))
```

mupad [B] time = 0.17, size = 177, normalized size = 5.71

$$\frac{\sqrt{x^3-1}}{3\,x^3}-\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^3 - 1)^(1/2)),x)
```

```
[Out] (x^3 - 1)^(1/2)/(3*x^3) - (((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2
)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i
)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2
)
```

sympy [A] time = 1.45, size = 82, normalized size = 2.65

$$\begin{cases} \frac{i\,\mathrm{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{i\sqrt{-1+\frac{1}{x^3}}}{3x^{\frac{3}{2}}} & \text{for}\ \frac{1}{|x^3|}>1 \\ -\frac{\mathrm{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{1}{3x^{\frac{3}{2}}\sqrt{1-\frac{1}{x^3}}}-\frac{1}{3x^{\frac{9}{2}}\sqrt{1-\frac{1}{x^3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**3-1)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(x**(-3/2))/3 + I*sqrt(-1 + x**(-3))/(3*x**(3/2)), 1/Abs(
x**3) > 1), (-asin(x**(-3/2))/3 + 1/(3*x**(3/2)*sqrt(1 - 1/x**3)) - 1/(3*x*
*(9/2)*sqrt(1 - 1/x**3)), True))
```

3.373 $\int \frac{-2-2x+x^2}{\left(3-x+x^2\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=31

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x^3-1}}{x^2+x+1}\right)$$

Rubi [A] time = 0.07, antiderivative size = 27, normalized size of antiderivative = 0.87, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2145, 206}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}(1-x)}{\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 - 2*x + x^2)/((3 - x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] Sqrt[2]*ArcTanh[(Sqrt[2]*(1 - x))/Sqrt[-1 + x^3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-2-2x+x^2}{\left(3-x+x^2\right)\sqrt{-1+x^3}}\,dx &= 4\,\text{Subst}\left(\int \frac{1}{2-4x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)\\ &= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}(1-x)}{\sqrt{-1+x^3}}\right)\end{aligned}$$

Mathematica [C] time = 0.72, size = 496, normalized size = 16.00

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{(-1)^{5/6}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{-2i+\sqrt{3}-\sqrt{11}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{-3+2i\sqrt{3}+\sqrt{33}}-\frac{i\sqrt{11}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{-2i+\sqrt{3}-\sqrt{11}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{2i-\sqrt{3}+\sqrt{11}}-\frac{(-1)^{5/6}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{-2i+\sqrt{3}+\sqrt{11}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{3-2i\sqrt{3}+\sqrt{33}}-\frac{i\sqrt{11}\sqrt{x^2+x+1}\,\Pi\left(\frac{2\sqrt{3}}{-2i+\sqrt{3}+\sqrt{11}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{-2i+\sqrt{3}+\sqrt{11}}+\frac{\left(x+\sqrt[3]{-1}\right)\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}\right)}{\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-2 - 2*x + x^2)/((3 - x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] - (I*
Sqrt[11]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(-2*I + Sqrt[3] - Sqrt[11
]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(2*I -
Sqrt[3] + Sqrt[11]) + ((-1)^(5/6)*(1 + (-1)^(1/3))*Sqrt[1 + x + x^2]*Ellipt
```

```
icPi[(2*Sqrt[3])/(-2*I + Sqrt[3] - Sqrt[11]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x
)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-3 + (2*I)*Sqrt[3] + Sqrt[33]) - (I*Sqr
t[11]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(-2*I + Sqrt[3] + Sqrt[11]),
 ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2*I + Sq
rt[3] + Sqrt[11]) - ((-1)^(5/6)*(1 + (-1)^(1/3))*Sqrt[1 + x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(-2*I + Sqrt[3] + Sqrt[11]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)])/(3 - (2*I)*Sqrt[3] + Sqrt[33])))/Sqrt[-1 +
 x^3]
```

IntegrateAlgebraic [A] time = 1.12, size = 31, normalized size = 1.00

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x^3-1}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 - 2*x + x^2)/((3 - x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[-1 + x^3])/(1 + x + x^2)])
```

fricas [B] time = 0.43, size = 65, normalized size = 2.10

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^4+14x^3-4\sqrt{2}\sqrt{x^3-1}\left(x^2+3x-1\right)+7x^2-6x-7}{x^4-2x^3+7x^2-6x+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-2)/(x^2-x+3)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^4 + 14*x^3 - 4*sqrt(2)*sqrt(x^3 - 1)*(x^2 + 3*x - 1) + 7
*x^2 - 6*x - 7)/(x^4 - 2*x^3 + 7*x^2 - 6*x + 9))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2-2x-2}{\sqrt{x^3-1}\left(x^2-x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-2)/(x^2-x+3)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 - x + 3)), x)
```

maple [C] time = 0.10, size = 423, normalized size = 13.65

$$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\left(\frac{1}{2}-\frac{i\sqrt{11}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\frac{1}{2}+\frac{i\sqrt{11}}{2}\right)\operatorname{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{1}{4}+\frac{i\sqrt{11}}{4}+\frac{i\left(\frac{1}{2}+\frac{i\sqrt{11}}{2}\right)\sqrt{3}}{6},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3-1}}-\frac{2\left(\frac{1}{2}+\frac{i\sqrt{11}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\frac{1}{2}-\frac{i\sqrt{11}}{2}\right)\operatorname{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{1}{4}-\frac{i\sqrt{11}}{4}+\frac{i\left(\frac{1}{2}-\frac{i\sqrt{11}}{2}\right)\sqrt{3}}{6},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x-2)/(x^2-x+3)/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(1/2-1/2*I*11^(1/2))*(-3/2-1
/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3
/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/
(x^3-1)^(1/2)*(1/2+1/2*I*11^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))
^(1/2),1/4+1/4*I*11^(1/2)+1/6*I*(1/2+1/2*I*11^(1/2))*3^(1/2),((3/2+1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(1/2+1/2*I*11^(1/2))*(-3/2-1/2*I*3^(
1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I
```

```
*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^
(1/2)*(1/2-1/2*I*11^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1
/4-1/4*I*11^(1/2)+1/6*I*(1/2-1/2*I*11^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(
3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - 2\,x - 2}{\sqrt{x^3 - 1}\left(x^2 - x + 3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-2)/(x^2-x+3)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 - x + 3)), x)
```

mupad [B] time = 0.20, size = 275, normalized size = 8.87

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{1}{2}+\frac{\sqrt{11}\,1\mathrm{i}}{2}};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{1}{2}+\frac{\sqrt{11}\,1\mathrm{i}}{2}};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - x^2 + 2)/((x^3 - 1)^(1/2)*(x^2 - x + 3)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/((11^(1/2)*1i)/
2 + 1/2), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(
-((3^(1/2)*1i)/2 + 3/2)/((11^(1/2)*1i)/2 - 1/2), asin((-(x - 1)/((3^(1/2)*1
i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - 2x - 2}{\sqrt{(x-1)\left(x^2 + x + 1\right)}\left(x^2 - x + 3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x-2)/(x**2-x+3)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x**2 - 2*x - 2)/(sqrt((x - 1)*(x**2 + x + 1))*(x**2 - x + 3)), x)
```

3.374 $\int \frac{-2-2x+x^2}{(-1+3x+x^2)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=31

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x^3-1}}{x^2+x+1}\right)$$

Rubi [A] time = 0.07, antiderivative size = 27, normalized size of antiderivative = 0.87, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2145, 203}

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}(1-x)}{\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 - 2*x + x^2)/((-1 + 3*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] Sqrt[2]*ArcTan[(Sqrt[2]*(1 - x))/Sqrt[-1 + x^3]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-2-2x+x^2}{(-1+3x+x^2)\sqrt{-1+x^3}}\,dx &= 4\,\text{Subst}\left(\int \frac{1}{2+4x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)\\ &= \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}(1-x)}{\sqrt{-1+x^3}}\right)\end{aligned}$$

Mathematica [C] time = 0.59, size = 291, normalized size = 9.39

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(-\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(x+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1}+\frac{6i\left(\left(-1+5\sqrt[3]{-1}+\sqrt{13}+\sqrt[3]{-1}\sqrt{13}\right)\Pi\left(\frac{2\sqrt{3}}{2i+\sqrt{3}+i\sqrt{13}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)-\left(1-5\sqrt[3]{-1}+\sqrt{13}+\sqrt[3]{-1}\sqrt{13}\right)\Pi\left(\frac{2i\sqrt{3}}{-3+2\sqrt[3]{-1}+\sqrt{13}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)\right)}{-12-4i\sqrt{3}}\right)}{3\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 - 2*x + x^2)/((-1 + 3*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*(-((Sqrt[3]*(1 + (-1)^(
1/3))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(
1/3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x)) + ((6*I)*((-1 + 5*(-1)^(1/3) + S
qrt[13] + (-1)^(1/3)*Sqrt[13])*EllipticPi[(2*Sqrt[3])/(2*I + Sqrt[3] + I*Sq
rt[13]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)] - (
```

```
1 - 5*(-1)^(1/3) + Sqrt[13] + (-1)^(1/3)*Sqrt[13])*EllipticPi[((2*I)*Sqrt[3
])/(-3 + 2*(-1)^(1/3) + Sqrt[13]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)
^(1/3))]], (-1)^(1/3)]))/(-12 - (4*I)*Sqrt[3])))/(3*Sqrt[-1 + x^3])
```

IntegrateAlgebraic [A] time = 1.15, size = 31, normalized size = 1.00

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x^3-1}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 - 2*x + x^2)/((-1 + 3*x + x^2)*Sqrt[-1 + x^3]),x]
[Out] -(Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[-1 + x^3])/(1 + x + x^2)])
```

fricas [A] time = 0.42, size = 26, normalized size = 0.84

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2-x+3\right)}{4\sqrt{x^3-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-2)/(x^2+3*x-1)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*arctan(1/4*sqrt(2)*(x^2 - x + 3)/sqrt(x^3 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2-2x-2}{\sqrt{x^3-1}\left(x^2+3x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-2)/(x^2+3*x-1)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 3*x - 1)), x)
```

maple [C] time = 0.07, size = 1641, normalized size = 52.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x-2)/(x^2+3*x-1)/(x^3-1)^(1/2),x)
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-3/2*13^(1/2)*(1/(-3/2-1/2*I*3^(1
/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*
I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*
x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^
(1/2)/(5/2-1/2*13^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/
2+1/2*I*3^(1/2))/(5/2-1/2*13^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)
))^(1/2))-1/2*I*13^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^
(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I
/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(5/2-1/2*13^(1/2))*Ellipt
icPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(5/2-1/2*13^(1
/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+15/2*(1/(-3/2
-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/
2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*
```

```
I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(x^3-1)^(1/2)/(5/2-1/2*13^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2))
)^(1/2),(3/2+1/2*I*3^(1/2))/(5/2-1/2*13^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/
2*I*3^(1/2)))^(1/2))+5/2*I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2))
)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2
*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(5/2-1/2*13^(1/2))*Elli
pticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(5/2-1/2*13^
(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+3/2*13^(1/2
)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(
1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1
/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^
(1/2))^(1/2)/(x^3-1)^(1/2)/(5/2+1/2*13^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*
I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(5/2+1/2*13^(1/2)),((3/2+1/2*I*3^(1/2
))/(3/2-1/2*I*3^(1/2)))^(1/2))+1/2*I*13^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(
-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))
-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2
+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(5/2
+1/2*13^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^
(1/2))/(5/2+1/2*13^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*
3^(1/2)+15/2*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/
2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*
3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(5/2+1/2*13^(1/2))*EllipticPi(((-1+x)
/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(5/2+1/2*13^(1/2)),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+5/2*I*(1/(-3/2-1/2*I*3^(1/2))*x-1
/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2
))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3
/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(5
/2+1/2*13^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*
3^(1/2))/(5/2+1/2*13^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)
)*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - 2x - 2}{\sqrt{x^3 - 1}\left(x^2 + 3x - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-2)/(x^2+3*x-1)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 3*x - 1)), x)
```

mupad [B] time = 0.10, size = 273, normalized size = 8.81

$$\frac{\left(3+\sqrt{3}\,1i\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{\sqrt{13}}{2}+\frac{5}{2}};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{\sqrt{13}}{2}-\frac{5}{2}};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - x^2 + 2)/((x^3 - 1)^(1/2)*(3*x + x^2 - 1)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(13^(1/2)/2 + 5
/2), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)
/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(-((3^
(1/2)*1i)/2 + 3/2)/(13^(1/2)/2 - 5/2), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
```

```
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
2 + 1/2) + 1) + x^3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - 2x - 2}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x^2+3x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x-2)/(x**2+3*x-1)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x**2 - 2*x - 2)/(sqrt((x - 1)*(x**2 + x + 1))*(x**2 + 3*x - 1)),
x)
```

3.375 $\int \frac{\sqrt{-1+x^3}}{x^4}\,dx$

Optimal. Leaf size=31

$$\frac{1}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)-\frac{\sqrt{x^3-1}}{3x^3}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 47, 63, 203}

$$\frac{1}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)-\frac{\sqrt{x^3-1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^3]/x^4,x]
```

```
[Out] -1/3*Sqrt[-1 + x^3]/x^3 + ArcTan[Sqrt[-1 + x^3]]/3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{-1+x^3}}{x^4}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x^2}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{-1+x^3}}{3x^3} + \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{-1+x^3}}{3x^3} + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^3}\right)$$
$$= -\frac{\sqrt{-1+x^3}}{3x^3} + \frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^3}\right)$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.55

$$-\frac{x^3 + \sqrt{1-x^3}\,x^3\tanh^{-1}\left(\sqrt{1-x^3}\right) - 1}{3x^3\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^3]/x^4,x]
[Out] -1/3*(-1 + x^3 + x^3*Sqrt[1 - x^3]*ArcTanh[Sqrt[1 - x^3]])/(x^3*Sqrt[-1 + x
^3])
```

IntegrateAlgebraic [A] time = 0.04, size = 31, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\sqrt{x^3-1}\right) - \frac{\sqrt{x^3-1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x^3]/x^4,x]
[Out] -1/3*Sqrt[-1 + x^3]/x^3 + ArcTan[Sqrt[-1 + x^3]]/3
```

fricas [A] time = 0.40, size = 27, normalized size = 0.87

$$\frac{x^3\arctan\left(\sqrt{x^3-1}\right) - \sqrt{x^3-1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)/x^4,x, algorithm="fricas")
[Out] 1/3*(x^3*arctan(sqrt(x^3 - 1)) - sqrt(x^3 - 1))/x^3
```

giac [A] time = 0.23, size = 23, normalized size = 0.74

$$-\frac{\sqrt{x^3-1}}{3\,x^3} + \frac{1}{3}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)/x^4,x, algorithm="giac")
[Out] -1/3*sqrt(x^3 - 1)/x^3 + 1/3*arctan(sqrt(x^3 - 1))
```

maple [A] time = 0.02, size = 24, normalized size = 0.77

$$-\frac{\sqrt{x^3-1}}{3x^3}+\frac{\arctan\left(\sqrt{x^3-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/2)/x^4,x)
[Out] -1/3*(x^3-1)^(1/2)/x^3+1/3*arctan((x^3-1)^(1/2))
```

maxima [A] time = 0.73, size = 23, normalized size = 0.74

$$-\frac{\sqrt{x^3-1}}{3\,x^3}+\frac{1}{3}\,\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)/x^4,x, algorithm="maxima")
[Out] -1/3*sqrt(x^3 - 1)/x^3 + 1/3*arctan(sqrt(x^3 - 1))
```

mupad [B] time = 0.17, size = 177, normalized size = 5.71

$$-\frac{\sqrt{x^3-1}}{3\,x^3}-\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)^(1/2)/x^4,x)
[Out] - (x^3 - 1)^(1/2)/(3*x^3) - (((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 +
 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1
/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*
1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1
/2)
```

sympy [A] time = 1.29, size = 82, normalized size = 2.65

$$\begin{cases}\frac{i\,\mathrm{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{i}{3x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x^3}}}-\frac{i}{3x^{\frac{9}{2}}\sqrt{-1+\frac{1}{x^3}}} & \text{for}\ \frac{1}{|x^3|}>1\\ -\frac{\mathrm{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}-\frac{\sqrt{1-\frac{1}{x^3}}}{3x^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/2)/x**4,x)
[Out] Piecewise((I*acosh(x**(-3/2))/3 + I/(3*x**(3/2)*sqrt(-1 + x**(-3))) - I/(3*
x**(9/2)*sqrt(-1 + x**(-3))), 1/Abs(x**3) > 1), (-asin(x**(-3/2))/3 - sqrt(
1 - 1/x**3)/(3*x**(3/2)), True))
```

3.376 $\int \frac{1}{x^4\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{\sqrt{x^3+1}}{3x^3}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 51, 63, 207}

$$\frac{1}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{\sqrt{x^3+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[1 + x^3]),x]
```

```
[Out] -1/3*Sqrt[1 + x^3]/x^3 + ArcTanh[Sqrt[1 + x^3]]/3
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt{1+x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx,x,x^3\right)\\
&= -\frac{\sqrt{1+x^3}}{3x^3}-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^3\right)\\
&= -\frac{\sqrt{1+x^3}}{3x^3}-\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^3}\right)\\
&= -\frac{\sqrt{1+x^3}}{3x^3}+\frac{1}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{1}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{\sqrt{x^3+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[1 + x^3]),x]
[Out] -1/3*Sqrt[1 + x^3]/x^3 + ArcTanh[Sqrt[1 + x^3]]/3
```

IntegrateAlgebraic [A] time = 0.00, size = 31, normalized size = 1.00

$$\frac{1}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{\sqrt{x^3+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[1 + x^3]),x]
[Out] -1/3*Sqrt[1 + x^3]/x^3 + ArcTanh[Sqrt[1 + x^3]]/3
```

fricas [A] time = 0.39, size = 44, normalized size = 1.42

$$\frac{x^3\log\left(\sqrt{x^3+1}+1\right)-x^3\log\left(\sqrt{x^3+1}-1\right)-2\sqrt{x^3+1}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] 1/6*(x^3*log(sqrt(x^3 + 1) + 1) - x^3*log(sqrt(x^3 + 1) - 1) - 2*sqrt(x^3 +
 1))/x^3
```

giac [A] time = 0.24, size = 38, normalized size = 1.23

$$-\frac{\sqrt{x^3+1}}{3\,x^3}+\frac{1}{6}\log\left(\sqrt{x^3+1}+1\right)-\frac{1}{6}\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^3+1)^(1/2),x, algorithm="giac")
[Out] -1/3*sqrt(x^3 + 1)/x^3 + 1/6*log(sqrt(x^3 + 1) + 1) - 1/6*log(abs(sqrt(x^3
+ 1) - 1))
```

maple [A] time = 0.02, size = 24, normalized size = 0.77

$$-\frac{\sqrt{x^3+1}}{3x^3}+\frac{\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^3+1)^(1/2),x)
```

```
[Out] -1/3*(x^3+1)^(1/2)/x^3+1/3*arctanh((x^3+1)^(1/2))
```

maxima [A] time = 0.35, size = 37, normalized size = 1.19

$$-\frac{\sqrt{x^3+1}}{3\,x^3}+\frac{1}{6}\,\log\left(\sqrt{x^3+1}\,+1\right)-\frac{1}{6}\,\log\left(\sqrt{x^3+1}\,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(x^3 + 1)/x^3 + 1/6*log(sqrt(x^3 + 1) + 1) - 1/6*log(sqrt(x^3 + 1)
 - 1)
```

mupad [B] time = 0.16, size = 176, normalized size = 5.68

$$-\frac{\sqrt{x^3+1}}{3\,x^3}+\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^3 + 1)^(1/2)),x)
```

```
[Out] (((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))
^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (x^3 + 1)^(1/2)/(3*x^3)
```

sympy [A] time = 1.46, size = 26, normalized size = 0.84

$$\frac{\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}-\frac{\sqrt{1+\frac{1}{x^3}}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**3+1)**(1/2),x)
```

```
[Out] asinh(x**(-3/2))/3 - sqrt(1 + x**(-3))/(3*x**(3/2))
```

3.377 $\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=31

$$-\frac{2}{3}\tanh^{-1}\left(\frac{\frac{x^2}{3}+\frac{2x}{3}+\frac{1}{3}}{\sqrt{x^3+1}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 23, normalized size of antiderivative = 0.74, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2138, 206}

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{(1+x)^2}{\sqrt{1+x^3}}\right)\right)\\ &= -\frac{2}{3}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.48

$$\frac{1}{3}\log\left(3-\frac{(x+1)^2}{\sqrt{x^3+1}}\right)-\frac{1}{3}\log\left(\frac{(x+1)^2}{\sqrt{x^3+1}}+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] Log[3 - (1 + x)^2/Sqrt[1 + x^3]]/3 - Log[3 + (1 + x)^2/Sqrt[1 + x^3]]/3
```

IntegrateAlgebraic [A] time = 0.90, size = 31, normalized size = 1.00

$$-\frac{2}{3}\tanh^{-1}\left(\frac{\frac{x^2}{3}+\frac{2x}{3}+\frac{1}{3}}{\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(1/3 + (2*x)/3 + x^2/3)/Sqrt[1 + x^3]])/3
```

fricas [B] time = 0.43, size = 44, normalized size = 1.42

$$\frac{1}{3}\,\log\left(\frac{x^{3}+12\,x^{2}-6\,\sqrt{x^{3}+1}\,(x+1)-6\,x+10}{x^{3}-6\,x^{2}+12\,x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log((x^3 + 12*x^2 - 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
12*x - 8))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^{3}+1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

maple [C] time = 0.04, size = 240, normalized size = 7.74

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{1}{2}-\frac{i\sqrt{3}}{6},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-2+x)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2*I*3^(1/2))/(
-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^{3}+1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

mupad [B] time = 0.23, size = 204, normalized size = 6.58

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{\sqrt{x^{3}+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^3 + 1)^(1/2)*(x - 2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
+ 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin(((x
+ 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/
2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x - 2)), x)
```

3.378 $\int \frac{\sqrt{1+x^3}}{x^4} dx$

Optimal. Leaf size=31

$$-\frac{\sqrt{x^3+1}}{3x^3}-\frac{1}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 47, 63, 207}

$$-\frac{\sqrt{x^3+1}}{3x^3}-\frac{1}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^3]/x^4,x]
```

```
[Out] -1/3*Sqrt[1 + x^3]/x^3 - ArcTanh[Sqrt[1 + x^3]]/3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{1+x^3}}{x^4}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{1+x^3}}{3x^3} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{1+x^3}}{3x^3} + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)$$
$$= -\frac{\sqrt{1+x^3}}{3x^3} - \frac{1}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right)$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.42

$$-\frac{1}{3x^3\sqrt{x^3+1}} - \frac{1}{3\sqrt{x^3+1}} - \frac{1}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^3]/x^4,x]
```

```
[Out] -1/3*1/Sqrt[1 + x^3] - 1/(3*x^3*Sqrt[1 + x^3]) - ArcTanh[Sqrt[1 + x^3]]/3
```

IntegrateAlgebraic [A] time = 0.07, size = 31, normalized size = 1.00

$$-\frac{\sqrt{x^3+1}}{3x^3} - \frac{1}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^3]/x^4,x]
```

```
[Out] -1/3*Sqrt[1 + x^3]/x^3 - ArcTanh[Sqrt[1 + x^3]]/3
```

fricas [A] time = 0.39, size = 44, normalized size = 1.42

$$-\frac{x^3\log\left(\sqrt{x^3+1}+1\right) - x^3\log\left(\sqrt{x^3+1}-1\right) + 2\sqrt{x^3+1}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(x^3*log(sqrt(x^3 + 1) + 1) - x^3*log(sqrt(x^3 + 1) - 1) + 2*sqrt(x^3
+ 1))/x^3
```

giac [A] time = 0.39, size = 38, normalized size = 1.23

$$-\frac{\sqrt{x^3+1}}{3\,x^3} - \frac{1}{6}\log\left(\sqrt{x^3+1}+1\right) + \frac{1}{6}\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(x^3 + 1)/x^3 - 1/6*log(sqrt(x^3 + 1) + 1) + 1/6*log(abs(sqrt(x^3
+ 1) - 1))
```

maple [A] time = 0.02, size = 24, normalized size = 0.77

$$-\frac{\sqrt{x^3+1}}{3x^3}-\frac{\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(1/2)/x^4,x)
```

```
[Out] -1/3*(x^3+1)^(1/2)/x^3-1/3*arctanh((x^3+1)^(1/2))
```

maxima [A] time = 0.32, size = 37, normalized size = 1.19

$$-\frac{\sqrt{x^3+1}}{3\,x^3}-\frac{1}{6}\,\log\left(\sqrt{x^3+1}\,+1\right)+\frac{1}{6}\,\log\left(\sqrt{x^3+1}\,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(x^3 + 1)/x^3 - 1/6*log(sqrt(x^3 + 1) + 1) + 1/6*log(sqrt(x^3 + 1)
 - 1)
```

mupad [B] time = 0.06, size = 177, normalized size = 5.71

$$-\frac{\sqrt{x^3+1}}{3\,x^3}-\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)^(1/2)/x^4,x)
```

```
[Out] - (x^3 + 1)^(1/2)/(3*x^3) - (((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 -
1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(
((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2
)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)
/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [A] time = 1.25, size = 27, normalized size = 0.87

$$-\frac{\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}-\frac{\sqrt{1+\frac{1}{x^3}}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(1/2)/x**4,x)
```

```
[Out] -asinh(x**(-3/2))/3 - sqrt(1 + x**(-3))/(3*x**(3/2))
```

3.379 $\int \frac{1+x}{(-1+2x)\sqrt{x+x^4}}\,dx$

Optimal. Leaf size=31

$$-\frac{2}{3}\tanh^{-1}\left(\frac{\frac{x^2}{3}+\frac{2x}{3}+\frac{1}{3}}{\sqrt{x^4+x}}\right)$$

Rubi [F] time = 0.74, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x}{(-1+2x)\sqrt{x+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x)/((-1 + 2*x)*Sqrt[x + x^4]),x]
```

```
[Out] (x*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(1
+ (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(2*3^(1/4)*Sqr
t[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[x + x^4]) - (3*Sqrt[x]*Sqrt[1 +
 x^3]*Defer[Subst][Defer[Int][1/((1 - Sqrt[2]*x)*Sqrt[1 + x^6]), x], x, Sqr
t[x]])/(2*Sqrt[x + x^4]) - (3*Sqrt[x]*Sqrt[1 + x^3]*Defer[Subst][Defer[Int]
[1/((1 + Sqrt[2]*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/(2*Sqrt[x + x^4])
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x}{(-1+2x)\sqrt{x+x^4}}\,dx &= \frac{\left(\sqrt{x}\sqrt{1+x^3}\right)\int \frac{1+x}{\sqrt{x}(-1+2x)\sqrt{1+x^3}}\,dx}{\sqrt{x+x^4}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{1+x^2}{\left(-1+2x^2\right)\sqrt{1+x^6}}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^4}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^3}\right)\text{Subst}\left(\int \left(\frac{1}{2\sqrt{1+x^6}}+\frac{3}{2\left(-1+2x^2\right)\sqrt{1+x^6}}\right)dx,x,\sqrt{x}\right)}{\sqrt{x+x^4}}\\
&= \frac{\left(\sqrt{x}\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^6}}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^4}}+\frac{\left(3\sqrt{x}\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{1}{\left(-1+2x^2\right)\sqrt{1+x^6}}\,d\right.}{\sqrt{x+x^4}}\\
&= \frac{x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}}+\frac{\left(3\sqrt{x}\sqrt{1+x^3}\right)\text{Subst}\left(\int\right.}{}\\
&= \frac{x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}}-\frac{\left(3\sqrt{x}\sqrt{1+x^3}\right)\text{Subst}\left(\int\right.}{2\sqrt{x}}
\end{aligned}$$

Mathematica [B] time = 0.20, size = 69, normalized size = 2.23

$$-\frac{\sqrt{\frac{1}{x^3}+1}\,x^2\left(\log\left(\frac{\left(\frac{1}{x}+1\right)^2}{\sqrt{\frac{1}{x^3}+1}}+3\right)-\log\left(3-\frac{\left(\frac{1}{x}+1\right)^2}{\sqrt{\frac{1}{x^3}+1}}\right)\right)}{3\sqrt{x^4+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-1 + 2*x)*Sqrt[x + x^4]),x]
```

```
[Out] -1/3*(Sqrt[1 + x^(-3)]*x^2*(-Log[3 - (1 + x^(-1))^2/Sqrt[1 + x^(-3)]] + Log
[3 + (1 + x^(-1))^2/Sqrt[1 + x^(-3)]]))/Sqrt[x + x^4]
```

IntegrateAlgebraic [A] time = 1.03, size = 31, normalized size = 1.00

$$-\frac{2}{3}\tanh^{-1}\left(\frac{\frac{x^2}{3}+\frac{2x}{3}+\frac{1}{3}}{\sqrt{x^4+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((-1 + 2*x)*Sqrt[x + x^4]),x]
```

```
[Out] (-2*ArcTanh[(1/3 + (2*x)/3 + x^2/3)/Sqrt[x + x^4]])/3
```

fricas [B] time = 0.46, size = 48, normalized size = 1.55

$$\frac{1}{3}\log\left(\frac{10\,x^3-6\,x^2-6\,\sqrt{x^4+x}\,(x+1)+12\,x+1}{8\,x^3-12\,x^2+6\,x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+2*x)/(x^4+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log((10*x^3 - 6*x^2 - 6*sqrt(x^4 + x)*(x + 1) + 12*x + 1)/(8*x^3 - 12*x
^2 + 6*x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+1}{\sqrt{x^4+x}\,(2\,x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+2*x)/(x^4+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^4 + x)*(2*x - 1)), x)
```

maple [C] time = 0.01, size = 491, normalized size = 15.84

$$-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\,(1+x)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(1+x)}}\sqrt{-\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)}{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(1+x)\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}+\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\,(1+x)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(1+x)}}\sqrt{-\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)+2\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\frac{\frac{3}{2}+\frac{3i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(1+x)\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-1+2*x)/(x^4+x)^(1/2),x)
```

```
[Out] -(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/
2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))/(1+x))^(1/2)*(-(x-1/
2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(3/2+1/2*I*3^(1/2))/(x*(1
+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1/2)*EllipticF(((3/2+1/2*
```

```
I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1
/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2-1/2*I
*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)*(1+x)^2*(
-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))/(1+x))^(1/2)*(-(x-1/2-1/2*I*3^(1
/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(3/2+1/2*I*3^(1/2))/(x*(1+x)*(x-1/2+1
/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1/2)*(EllipticF(((3/2+1/2*I*3^(1/2))*
x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2
))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*EllipticPi(((3/2+1/2
*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),3*(1/2+1/2*I*3^(1/2))/(3/2+1
/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2
))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^4+x}\,(2\,x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+2*x)/(x^4+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^4 + x)*(2*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x+1}{(2\,x-1)\,\sqrt{x^4+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((2*x - 1)*(x + x^4)^(1/2)),x)
```

```
[Out] int((x + 1)/((2*x - 1)*(x + x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x\,(x+1)\left(x^2-x+1\right)}\,(2x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+2*x)/(x**4+x)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt(x*(x + 1)*(x**2 - x + 1))*(2*x - 1)), x)
```

3.380 $\int \frac{-1+x}{\sqrt{-5+4x^2-4x^3+x^4}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}\log\left(x^2+\sqrt{x^4-4x^3+4x^2-5}-2x\right)$$

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1680, 1107, 621, 206}

$$-\frac{1}{2}\tanh^{-1}\left(\frac{1-(x-1)^2}{\sqrt{(x-1)^4-2(x-1)^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/Sqrt[-5 + 4*x^2 - 4*x^3 + x^4],x]
```

```
[Out] -1/2*ArcTanh[(1 - (-1 + x)^2)/Sqrt[-4 - 2*(-1 + x)^2 + (-1 + x)^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x}{\sqrt{-5+4x^2-4x^3+x^4}}\,dx &= \text{Subst}\left(\int \frac{x}{\sqrt{-4-2x^2+x^4}}\,dx, x, -1+x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-4-2x+x^2}}\,dx, x, (-1+x)^2\right)\\ &= \text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{2(-2+x)x}{\sqrt{-4-2(-1+x)^2+(-1+x)^4}}\right)\\ &= \frac{1}{2}\tanh^{-1}\left(\frac{(-2+x)x}{\sqrt{-4-2(-1+x)^2+(-1+x)^4}}\right)\end{aligned}$$

Mathematica [C] time = 1.56, size = 701, normalized size = 22.61

$$\frac{2\sqrt{\sqrt{5}-1}\left(-x+i\sqrt{\sqrt{5}-1}+1\right)\left(-x+\sqrt{1+\sqrt{5}}+1\right)\sqrt{\frac{\left(\sqrt{1+\sqrt{5}}-i\sqrt{\sqrt{5}-1}\right)\left(x+i\sqrt{\sqrt{5}-1}-1\right)}{\left(\sqrt{1+\sqrt{5}}+i\sqrt{\sqrt{5}-1}\right)\left(x-i\sqrt{\sqrt{5}-1}-1\right)}}\sqrt{\frac{i\left(x+\sqrt{1+\sqrt{5}}-1\right)}{\left(\sqrt{\sqrt{5}-1}+i\sqrt{1+\sqrt{5}}\right)\left(ix+\sqrt{\sqrt{5}-1}-i\right)}}\left(F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt{1+\sqrt{5}}x-i\sqrt{-1+\sqrt{5}}x-\sqrt{1+\sqrt{5}}+i\sqrt{-1+\sqrt{5}}+\sqrt{5}-(1-2i)}{\sqrt{1+\sqrt{5}}x+i\sqrt{-1+\sqrt{5}}x-\sqrt{1+\sqrt{5}}-i\sqrt{-1+\sqrt{5}}+\sqrt{5}-(1+2i)}}\right)\middle|-\frac{3}{5}+\frac{4i}{5}\right)-2\Pi\left(-\frac{\sqrt{-1+\sqrt{5}}-i\sqrt{1+\sqrt{5}}}{\sqrt{-1+\sqrt{5}}+i\sqrt{1+\sqrt{5}}};\sin^{-1}\left(\sqrt{\frac{\sqrt{1+\sqrt{5}}x-i\sqrt{-1+\sqrt{5}}x-\sqrt{1+\sqrt{5}}+i\sqrt{-1+\sqrt{5}}+\sqrt{5}-(1-2i)}{\sqrt{1+\sqrt{5}}x+i\sqrt{-1+\sqrt{5}}x-\sqrt{1+\sqrt{5}}-i\sqrt{-1+\sqrt{5}}+\sqrt{5}-(1+2i)}}\right)\middle|-\frac{3}{5}+\frac{4i}{5}\right)\right)}{\left(\sqrt{\sqrt{5}-1}+i\sqrt{1+\sqrt{5}}\right)\sqrt{-\frac{i\left(-x+\sqrt{1+\sqrt{5}}+1\right)}{\left(\sqrt{\sqrt{5}-1}-i\sqrt{1+\sqrt{5}}\right)\left(ix+\sqrt{\sqrt{5}-1}-i\right)}}\sqrt{x^4-4x^3+4x^2-5}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + x)/Sqrt[-5 + 4*x^2 - 4*x^3 + x^4],x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[5]]*(1 + I*Sqrt[-1 + Sqrt[5]] - x)*(1 + Sqrt[1 + Sqrt[5]]
 - x)*Sqrt[(((-I)*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*(-1 + I*Sqrt[-1 +
 Sqrt[5]] + x))/((I*Sqrt[-1 + Sqrt[5]] + Sqrt[1 + Sqrt[5]])*(-1 - I*Sqrt[-1
 + Sqrt[5]] + x))]*Sqrt[(I*(-1 + Sqrt[1 + Sqrt[5]] + x))/((Sqrt[-1 + Sqrt[5
]] + I*Sqrt[1 + Sqrt[5]])*(-I + Sqrt[-1 + Sqrt[5]] + I*x))]*(EllipticF[ArcS
in[Sqrt[((-1 + 2*I) + Sqrt[5] + I*Sqrt[-1 + Sqrt[5]] - Sqrt[1 + Sqrt[5]] -
I*Sqrt[-1 + Sqrt[5]]*x + Sqrt[1 + Sqrt[5]]*x)/((-1 - 2*I) + Sqrt[5] - I*Sqr
t[-1 + Sqrt[5]] - Sqrt[1 + Sqrt[5]] + I*Sqrt[-1 + Sqrt[5]]*x + Sqrt[1 + Sqr
t[5]]*x)]], -3/5 + (4*I)/5] - 2*EllipticPi[-((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1
 + Sqrt[5]])/(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])), ArcSin[Sqrt[((-1
+ 2*I) + Sqrt[5] + I*Sqrt[-1 + Sqrt[5]] - Sqrt[1 + Sqrt[5]] - I*Sqrt[-1 + S
qrt[5]]*x + Sqrt[1 + Sqrt[5]]*x)/((-1 - 2*I) + Sqrt[5] - I*Sqrt[-1 + Sqrt[5
]] - Sqrt[1 + Sqrt[5]] + I*Sqrt[-1 + Sqrt[5]]*x + Sqrt[1 + Sqrt[5]]*x)]], -
3/5 + (4*I)/5]))/((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*Sqrt[((-I)*(1
+ Sqrt[1 + Sqrt[5]] - x))/((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(-I +
 Sqrt[-1 + Sqrt[5]] + I*x))]*Sqrt[-5 + 4*x^2 - 4*x^3 + x^4])
```

IntegrateAlgebraic [A] time = 0.08, size = 31, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+\sqrt{x^4-4x^3+4x^2-5}-2x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/Sqrt[-5 + 4*x^2 - 4*x^3 + x^4],x]
```

```
[Out] Log[-2*x + x^2 + Sqrt[-5 + 4*x^2 - 4*x^3 + x^4]]/2
```

fricas [A] time = 0.42, size = 27, normalized size = 0.87

$$\frac{1}{2}\log\left(x^2-2x+\sqrt{x^4-4x^3+4x^2-5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^4-4*x^3+4*x^2-5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - 2*x + sqrt(x^4 - 4*x^3 + 4*x^2 - 5))
```

giac [A] time = 0.46, size = 26, normalized size = 0.84

$$-\frac{1}{2}\log\left(\left|-x^2+2x+\sqrt{\left(x^2-2x\right)^2-5}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^4-4*x^3+4*x^2-5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-x^2 + 2*x + sqrt((x^2 - 2*x)^2 - 5)))
```

maple [C] time = 0.59, size = 1110, normalized size = 35.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(x^4-4*x^3+4*x^2-5)^(1/2),x)
```

```
[Out] 2*(-(5^(1/2)+1)^(1/2)-I*(5^(1/2)-1)^(1/2))*((I*(5^(1/2)-1)^(1/2)-(5^(1/2)+1
)^(1/2))*(x-1+(5^(1/2)+1)^(1/2))/(I*(5^(1/2)-1)^(1/2)+(5^(1/2)+1)^(1/2))/(x
-1-(5^(1/2)+1)^(1/2)))^(1/2)*(x-1-(5^(1/2)+1)^(1/2))^2*((5^(1/2)+1)^(1/2)*(
x-1+I*(5^(1/2)-1)^(1/2))/(-I*(5^(1/2)-1)^(1/2)+(5^(1/2)+1)^(1/2))/(x-1-(5^(
1/2)+1)^(1/2)))^(1/2)*((5^(1/2)+1)^(1/2)*(x-1-I*(5^(1/2)-1)^(1/2))/(I*(5^(1
/2)-1)^(1/2)+(5^(1/2)+1)^(1/2))/(x-1-(5^(1/2)+1)^(1/2)))^(1/2)/(I*(5^(1/2)-
1)^(1/2)-(5^(1/2)+1)^(1/2))/(5^(1/2)+1)^(1/2)/((x-1+(5^(1/2)+1)^(1/2))*(x-1
-(5^(1/2)+1)^(1/2))*(x-1+I*(5^(1/2)-1)^(1/2))*(x-1-I*(5^(1/2)-1)^(1/2)))^(1
/2)*((1+(5^(1/2)+1)^(1/2))*EllipticF(((I*(5^(1/2)-1)^(1/2)-(5^(1/2)+1)^(1/2
))*(x-1+(5^(1/2)+1)^(1/2))/(I*(5^(1/2)-1)^(1/2)+(5^(1/2)+1)^(1/2))/(x-1-(5^
(1/2)+1)^(1/2)))^(1/2),((I*(5^(1/2)-1)^(1/2)+(5^(1/2)+1)^(1/2))*(-(5^(1/2)+
1)^(1/2)-I*(5^(1/2)-1)^(1/2))/(I*(5^(1/2)-1)^(1/2)-(5^(1/2)+1)^(1/2))/(-I*(
5^(1/2)-1)^(1/2)+(5^(1/2)+1)^(1/2)))^(1/2))-2*(5^(1/2)+1)^(1/2)*EllipticPi(
((I*(5^(1/2)-1)^(1/2)-(5^(1/2)+1)^(1/2))*(x-1+(5^(1/2)+1)^(1/2))/(I*(5^(1/2
)-1)^(1/2)+(5^(1/2)+1)^(1/2))/(x-1-(5^(1/2)+1)^(1/2)))^(1/2),(I*(5^(1/2)-1)
^(1/2)+(5^(1/2)+1)^(1/2))/(I*(5^(1/2)-1)^(1/2)-(5^(1/2)+1)^(1/2)),((I*(5^(1
/2)-1)^(1/2)+(5^(1/2)+1)^(1/2))*(-(5^(1/2)+1)^(1/2)-I*(5^(1/2)-1)^(1/2))/(I
*(5^(1/2)-1)^(1/2)-(5^(1/2)+1)^(1/2))/(-I*(5^(1/2)-1)^(1/2)+(5^(1/2)+1)^(1/
2)))^(1/2)))-2*(-(5^(1/2)+1)^(1/2)-I*(5^(1/2)-1)^(1/2))*((I*(5^(1/2)-1)^(1/
2)-(5^(1/2)+1)^(1/2))*(x-1+(5^(1/2)+1)^(1/2))/(I*(5^(1/2)-1)^(1/2)+(5^(1/2)
+1)^(1/2))/(x-1-(5^(1/2)+1)^(1/2)))^(1/2)*(x-1-(5^(1/2)+1)^(1/2))^2*((5^(1/
2)+1)^(1/2)*(x-1+I*(5^(1/2)-1)^(1/2))/(-I*(5^(1/2)-1)^(1/2)+(5^(1/2)+1)^(1/
2))/(x-1-(5^(1/2)+1)^(1/2)))^(1/2)*((5^(1/2)+1)^(1/2)*(x-1-I*(5^(1/2)-1)^(1
/2))/(I*(5^(1/2)-1)^(1/2)+(5^(1/2)+1)^(1/2))/(x-1-(5^(1/2)+1)^(1/2)))^(1/2)
/(I*(5^(1/2)-1)^(1/2)-(5^(1/2)+1)^(1/2))/(5^(1/2)+1)^(1/2)/((x-1+(5^(1/2)+1
)^(1/2))*(x-1-(5^(1/2)+1)^(1/2))*(x-1+I*(5^(1/2)-1)^(1/2))*(x-1-I*(5^(1/2)-
1)^(1/2)))^(1/2)*EllipticF(((I*(5^(1/2)-1)^(1/2)-(5^(1/2)+1)^(1/2))*(x-1+(5
^(1/2)+1)^(1/2))/(I*(5^(1/2)-1)^(1/2)+(5^(1/2)+1)^(1/2))/(x-1-(5^(1/2)+1)^(
1/2)))^(1/2),((I*(5^(1/2)-1)^(1/2)+(5^(1/2)+1)^(1/2))*(-(5^(1/2)+1)^(1/2)-I
*(5^(1/2)-1)^(1/2))/(I*(5^(1/2)-1)^(1/2)-(5^(1/2)+1)^(1/2))/(-I*(5^(1/2)-1)
^(1/2)+(5^(1/2)+1)^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x^4-4\,x^3+4\,x^2-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^4-4*x^3+4*x^2-5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/sqrt(x^4 - 4*x^3 + 4*x^2 - 5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x-1}{\sqrt{x^4-4\,x^3+4\,x^2-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(4*x^2 - 4*x^3 + x^4 - 5)^(1/2),x)
```

```
[Out] int((x - 1)/(4*x^2 - 4*x^3 + x^4 - 5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x^4-4x^3+4x^2-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**4-4*x**3+4*x**2-5)**(1/2),x)
[Out] Integral((x - 1)/sqrt(x**4 - 4*x**3 + 4*x**2 - 5), x)
```

3.381 $\int \frac{2+x}{\sqrt{13+16x^2+8x^3+x^4}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}\log\left(x^2+\sqrt{x^4+8x^3+16x^2+13}+4x\right)$$

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 0.65, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1680, 1107, 619, 215}

$$-\frac{1}{2}\sinh^{-1}\left(\frac{4-(x+2)^2}{\sqrt{13}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/Sqrt[13 + 16*x^2 + 8*x^3 + x^4],x]
```

```
[Out] -1/2*ArcSinh[(4 - (2 + x)^2)/Sqrt[13]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x}{\sqrt{13+16x^2+8x^3+x^4}}\,dx &= \text{Subst}\left(\int \frac{x}{\sqrt{29-8x^2+x^4}}\,dx, x, 2+x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{29-8x+x^2}}\,dx, x, (2+x)^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{52}}}\,dx, x, 2x(4+x)\right)}{4\sqrt{13}}\\
&= \frac{1}{2}\sinh^{-1}\left(\frac{x(4+x)}{\sqrt{13}}\right)
\end{aligned}$$

Mathematica [C] time = 2.28, size = 898, normalized size = 28.97

$$\frac{2\left(\sqrt{4-i\sqrt{13}}+\sqrt{4+i\sqrt{13}}\right)\sqrt{\frac{\sqrt{4-i\sqrt{13}}\left(-x+\sqrt{4+i\sqrt{13}}-2\right)}{\left(\sqrt{4-i\sqrt{13}}+\sqrt{4+i\sqrt{13}}\right)\left(-x+\sqrt{4-i\sqrt{13}}-2\right)}}\left(x-\sqrt{4-i\sqrt{13}}+2\right)^2\sqrt{\frac{\left(-\sqrt{4-i\sqrt{13}}+\sqrt{4+i\sqrt{13}}\right)\left(x+\sqrt{4-i\sqrt{13}}+2\right)}{\left(\sqrt{4-i\sqrt{13}}+\sqrt{4+i\sqrt{13}}\right)\left(x-\sqrt{4-i\sqrt{13}}+2\right)}}\sqrt{-\frac{\sqrt{4-i\sqrt{13}}\left(x+\sqrt{4+i\sqrt{13}}+2\right)}{\left(\sqrt{4-i\sqrt{13}}-\sqrt{4+i\sqrt{13}}\right)\left(-x+\sqrt{4-i\sqrt{13}}-2\right)}}\left(F\left(\sin^{-1}\left(\sqrt{\frac{i\left(\sqrt{4-i\sqrt{13}}-\sqrt{4+i\sqrt{13}}\right)x-2i\sqrt{4+i\sqrt{13}}+2i\sqrt{4-i\sqrt{13}}-i\sqrt{29}+\sqrt{13}+4i}{-i\left(\sqrt{4-i\sqrt{13}}+\sqrt{4+i\sqrt{13}}\right)x-2i\sqrt{4+i\sqrt{13}}-2i\sqrt{4-i\sqrt{13}}+i\sqrt{29}+\sqrt{13}+4i}}\right)\middle|\frac{1}{13}\left(-45-8\sqrt{29}\right)\right)-2\Pi\left(-\frac{\sqrt{4-i\sqrt{13}}+\sqrt{4+i\sqrt{13}}}{\sqrt{4-i\sqrt{13}}-\sqrt{4+i\sqrt{13}}};\sin^{-1}\left(\sqrt{\frac{i\left(\sqrt{4-i\sqrt{13}}-\sqrt{4+i\sqrt{13}}\right)x-2i\sqrt{4+i\sqrt{13}}+2i\sqrt{4-i\sqrt{13}}-i\sqrt{29}+\sqrt{13}+4i}{-i\left(\sqrt{4-i\sqrt{13}}+\sqrt{4+i\sqrt{13}}\right)x-2i\sqrt{4+i\sqrt{13}}-2i\sqrt{4-i\sqrt{13}}+i\sqrt{29}+\sqrt{13}+4i}}\right)\middle|\frac{1}{13}\left(-45-8\sqrt{29}\right)\right)\right)}{\left(\sqrt{4-i\sqrt{13}}-\sqrt{4+i\sqrt{13}}\right)\sqrt{x^4+8x^3+16x^2+13}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + x)/Sqrt[13 + 16*x^2 + 8*x^3 + x^4],x]
```

```
[Out] (2*(Sqrt[4 - I*Sqrt[13]] + Sqrt[4 + I*Sqrt[13]])*Sqrt[(Sqrt[4 - I*Sqrt[13]]
*(-2 + Sqrt[4 + I*Sqrt[13]] - x))/((Sqrt[4 - I*Sqrt[13]] + Sqrt[4 + I*Sqrt[
13]])*(-2 + Sqrt[4 - I*Sqrt[13]] - x))]*(2 - Sqrt[4 - I*Sqrt[13]] + x)^2*Sq
rt[((-Sqrt[4 - I*Sqrt[13]] + Sqrt[4 + I*Sqrt[13]])*(2 + Sqrt[4 - I*Sqrt[13]
] + x))/((Sqrt[4 - I*Sqrt[13]] + Sqrt[4 + I*Sqrt[13]])*(2 - Sqrt[4 - I*Sqrt
[13]] + x))]*Sqrt[-((Sqrt[4 - I*Sqrt[13]]*(2 + Sqrt[4 + I*Sqrt[13]] + x))/(
(Sqrt[4 - I*Sqrt[13]] - Sqrt[4 + I*Sqrt[13]])*(-2 + Sqrt[4 - I*Sqrt[13]] -
x)))]*(EllipticF[ArcSin[Sqrt[(4*I + Sqrt[13] - I*Sqrt[29] + (2*I)*Sqrt[4 -
I*Sqrt[13]] - (2*I)*Sqrt[4 + I*Sqrt[13]] + I*(Sqrt[4 - I*Sqrt[13]] - Sqrt[4
 + I*Sqrt[13]])*x)/(4*I + Sqrt[13] + I*Sqrt[29] - (2*I)*Sqrt[4 - I*Sqrt[13]
] - (2*I)*Sqrt[4 + I*Sqrt[13]] - I*(Sqrt[4 - I*Sqrt[13]] + Sqrt[4 + I*Sqrt[
13]])*x)]], (-45 - 8*Sqrt[29])/13] - 2*EllipticPi[-((Sqrt[4 - I*Sqrt[13]] +
 Sqrt[4 + I*Sqrt[13]])/(Sqrt[4 - I*Sqrt[13]] - Sqrt[4 + I*Sqrt[13]])), ArcS
in[Sqrt[(4*I + Sqrt[13] - I*Sqrt[29] + (2*I)*Sqrt[4 - I*Sqrt[13]] - (2*I)*S
qrt[4 + I*Sqrt[13]] + I*(Sqrt[4 - I*Sqrt[13]] - Sqrt[4 + I*Sqrt[13]])*x)/(4
*I + Sqrt[13] + I*Sqrt[29] - (2*I)*Sqrt[4 - I*Sqrt[13]] - (2*I)*Sqrt[4 + I*
Sqrt[13]] - I*(Sqrt[4 - I*Sqrt[13]] + Sqrt[4 + I*Sqrt[13]])*x)]], (-45 - 8*
Sqrt[29])/13]))/((Sqrt[4 - I*Sqrt[13]] - Sqrt[4 + I*Sqrt[13]])*Sqrt[13 + 16
*x^2 + 8*x^3 + x^4])
```

IntegrateAlgebraic [A] time = 0.16, size = 31, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+\sqrt{x^4+8x^3+16x^2+13}+4x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)/Sqrt[13 + 16*x^2 + 8*x^3 + x^4],x]
```

```
[Out] Log[4*x + x^2 + Sqrt[13 + 16*x^2 + 8*x^3 + x^4]]/2
```

fricas [A] time = 0.43, size = 27, normalized size = 0.87

$$\frac{1}{2}\log\left(x^2+4x+\sqrt{x^4+8x^3+16x^2+13}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^4+8*x^3+16*x^2+13)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 + 4*x + sqrt(x^4 + 8*x^3 + 16*x^2 + 13))
```

giac [A] time = 0.66, size = 25, normalized size = 0.81

$$-\frac{1}{2}\log\left(-x^2-4x+\sqrt{\left(x^2+4x\right)^2+13}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^4+8*x^3+16*x^2+13)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(-x^2 - 4*x + sqrt((x^2 + 4*x)^2 + 13))
```

maple [C] time = 0.41, size = 1252, normalized size = 40.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(x^4+8*x^3+16*x^2+13)^(1/2),x)
```

```
[Out] 4*(-(4+I*13^(1/2))^(1/2)-(4-I*13^(1/2))^(1/2))*(((4-I*13^(1/2))^(1/2)-(4+I*
13^(1/2))^(1/2))*(x+2+(4+I*13^(1/2))^(1/2))/((4-I*13^(1/2))^(1/2)+(4+I*13^(
1/2))^(1/2))/(x+2-(4+I*13^(1/2))^(1/2)))^(1/2)*(x+2-(4+I*13^(1/2))^(1/2))^2
*((4+I*13^(1/2))^(1/2)*(x+2+(4-I*13^(1/2))^(1/2))/(-(4-I*13^(1/2))^(1/2)+(4
+I*13^(1/2))^(1/2))/(x+2-(4+I*13^(1/2))^(1/2)))^(1/2)*((4+I*13^(1/2))^(1/2)
*(x+2-(4-I*13^(1/2))^(1/2))/((4-I*13^(1/2))^(1/2)+(4+I*13^(1/2))^(1/2))/(x+
2-(4+I*13^(1/2))^(1/2)))^(1/2)/((4-I*13^(1/2))^(1/2)-(4+I*13^(1/2))^(1/2))/
(4+I*13^(1/2))^(1/2)/((x+2+(4+I*13^(1/2))^(1/2))*(x+2-(4+I*13^(1/2))^(1/2))
*(x+2+(4-I*13^(1/2))^(1/2))*(x+2-(4-I*13^(1/2))^(1/2)))^(1/2)*EllipticF((((
4-I*13^(1/2))^(1/2)-(4+I*13^(1/2))^(1/2))*(x+2+(4+I*13^(1/2))^(1/2))/((4-I*
13^(1/2))^(1/2)+(4+I*13^(1/2))^(1/2))/(x+2-(4+I*13^(1/2))^(1/2)))^(1/2),(-(
(4-I*13^(1/2))^(1/2)+(4+I*13^(1/2))^(1/2))^2/((4-I*13^(1/2))^(1/2)-(4+I*13^
(1/2))^(1/2))/(-(4-I*13^(1/2))^(1/2)+(4+I*13^(1/2))^(1/2)))^(1/2))+2*(-(4+I
*13^(1/2))^(1/2)-(4-I*13^(1/2))^(1/2))*(((4-I*13^(1/2))^(1/2)-(4+I*13^(1/2)
)^(1/2))*(x+2+(4+I*13^(1/2))^(1/2))/((4-I*13^(1/2))^(1/2)+(4+I*13^(1/2))^(1
/2))/(x+2-(4+I*13^(1/2))^(1/2)))^(1/2)*(x+2-(4+I*13^(1/2))^(1/2))^2*((4+I*1
3^(1/2))^(1/2)*(x+2+(4-I*13^(1/2))^(1/2))/(-(4-I*13^(1/2))^(1/2)+(4+I*13^(1
/2))^(1/2))/(x+2-(4+I*13^(1/2))^(1/2)))^(1/2)*((4+I*13^(1/2))^(1/2)*(x+2-(4
-I*13^(1/2))^(1/2))/((4-I*13^(1/2))^(1/2)+(4+I*13^(1/2))^(1/2))/(x+2-(4+I*1
3^(1/2))^(1/2)))^(1/2)/((4-I*13^(1/2))^(1/2)-(4+I*13^(1/2))^(1/2))/(4+I*13^
(1/2))^(1/2)/((x+2+(4+I*13^(1/2))^(1/2))*(x+2-(4+I*13^(1/2))^(1/2))*(x+2+(4
-I*13^(1/2))^(1/2))*(x+2-(4-I*13^(1/2))^(1/2)))^(1/2)*((-2+(4+I*13^(1/2))^(
1/2))*EllipticF((((4-I*13^(1/2))^(1/2)-(4+I*13^(1/2))^(1/2))*(x+2+(4+I*13^(
1/2))^(1/2))/((4-I*13^(1/2))^(1/2)+(4+I*13^(1/2))^(1/2))/(x+2-(4+I*13^(1/2)
)^(1/2)))^(1/2),(-((4-I*13^(1/2))^(1/2)+(4+I*13^(1/2))^(1/2))^2/((4-I*13^(1
/2))^(1/2)-(4+I*13^(1/2))^(1/2))/(-(4-I*13^(1/2))^(1/2)+(4+I*13^(1/2))^(1/2
)))^(1/2))-2*(4+I*13^(1/2))^(1/2)*EllipticPi((((4-I*13^(1/2))^(1/2)-(4+I*13
^(1/2))^(1/2))*(x+2+(4+I*13^(1/2))^(1/2))/((4-I*13^(1/2))^(1/2)+(4+I*13^(1/
2))^(1/2))/(x+2-(4+I*13^(1/2))^(1/2)))^(1/2),((4-I*13^(1/2))^(1/2)+(4+I*13^
(1/2))^(1/2))/((4-I*13^(1/2))^(1/2)-(4+I*13^(1/2))^(1/2)),(-((4-I*13^(1/2))
^(1/2)+(4+I*13^(1/2))^(1/2))^2/((4-I*13^(1/2))^(1/2)-(4+I*13^(1/2))^(1/2))/
(-(4-I*13^(1/2))^(1/2)+(4+I*13^(1/2))^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\sqrt{x^4+8\,x^3+16\,x^2+13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^4+8*x^3+16*x^2+13)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 2)/sqrt(x^4 + 8*x^3 + 16*x^2 + 13), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x+2}{\sqrt{x^4+8\,x^3+16\,x^2+13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/(16*x^2 + 8*x^3 + x^4 + 13)^(1/2),x)
```

```
[Out] int((x + 2)/(16*x^2 + 8*x^3 + x^4 + 13)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\sqrt{x^4+8x^3+16x^2+13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**4+8*x**3+16*x**2+13)**(1/2),x)
```

```
[Out] Integral((x + 2)/sqrt(x**4 + 8*x**3 + 16*x**2 + 13), x)
```

3.382 $\int \frac{x}{\sqrt{b+ax^4}}\,dx$

Optimal. Leaf size=31

$$\frac{\log\left(\sqrt{ax^4+b}+\sqrt{a}\,x^2\right)}{2\sqrt{a}}$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {275, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,x^2}{\sqrt{ax^4+b}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b + a*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[a]*x^2)/Sqrt[b + a*x^4]]/(2*Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{b+ax^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^2}{\sqrt{b+ax^4}}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\,x^2}{\sqrt{b+ax^4}}\right)}{2\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.97

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,x^2}{\sqrt{ax^4+b}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b + a*x^4],x]
```

[Out] ArcTanh[(Sqrt[a]*x^2)/Sqrt[b + a*x^4]]/(2*Sqrt[a])

IntegrateAlgebraic [A] time = 0.13, size = 31, normalized size = 1.00

$$\frac{\log\left(\sqrt{ax^4+b}+\sqrt{a}\,x^2\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/Sqrt[b + a*x^4],x]

[Out] Log[Sqrt[a]*x^2 + Sqrt[b + a*x^4]]/(2*Sqrt[a])

fricas [A] time = 0.41, size = 63, normalized size = 2.03

$$\left[\frac{\log\left(-2\,ax^4-2\,\sqrt{ax^4+b}\,\sqrt{a}\,x^2-b\right)}{4\,\sqrt{a}},-\frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,x^2}{\sqrt{ax^4+b}}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a*x^4+b)^(1/2),x, algorithm="fricas")

[Out] [1/4*log(-2*a*x^4 - 2*sqrt(a*x^4 + b)*sqrt(a)*x^2 - b)/sqrt(a), -1/2*sqrt(-a)*arctan(sqrt(-a)*x^2/sqrt(a*x^4 + b))/a]

giac [A] time = 0.51, size = 25, normalized size = 0.81

$$-\frac{\log\left(\left|-\sqrt{a}\,x^2+\sqrt{ax^4+b}\right|\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a*x^4+b)^(1/2),x, algorithm="giac")

[Out] -1/2*log(abs(-sqrt(a)*x^2 + sqrt(a*x^4 + b)))/sqrt(a)

maple [A] time = 0.01, size = 24, normalized size = 0.77

$$\frac{\ln\left(\sqrt{a}\;x^2+\sqrt{a\,x^4+b}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a*x^4+b)^(1/2),x)

[Out] 1/2*ln(a^(1/2)*x^2+(a*x^4+b)^(1/2))/a^(1/2)

maxima [A] time = 0.55, size = 45, normalized size = 1.45

$$-\frac{\log\left(-\frac{\sqrt{a}-\frac{\sqrt{ax^4+b}}{x^2}}{\sqrt{a}+\frac{\sqrt{ax^4+b}}{x^2}}\right)}{4\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a*x^4+b)^(1/2),x, algorithm="maxima")

[Out] -1/4*log(-(sqrt(a) - sqrt(a*x^4 + b)/x^2)/(sqrt(a) + sqrt(a*x^4 + b)/x^2))/sqrt(a)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{a\,x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b + a*x^4)^(1/2),x)
```

```
[Out] int(x/(b + a*x^4)^(1/2), x)
```

sympy [A] time = 0.86, size = 20, normalized size = 0.65

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{a}\,x^2}{\sqrt{b}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x**4+b)**(1/2),x)
```

```
[Out] asinh(sqrt(a)*x**2/sqrt(b))/(2*sqrt(a))
```

3.383 $\int \frac{1}{x^7\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{x^6-1}}{6x^6}+\frac{1}{6}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 51, 63, 203}

$$\frac{\sqrt{x^6-1}}{6x^6}+\frac{1}{6}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[-1 + x^6]),x]
```

```
[Out] Sqrt[-1 + x^6]/(6*x^6) + ArcTan[Sqrt[-1 + x^6]]/6
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\sqrt{-1+x^6}}\,dx &= \frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx, x, x^6\right)\\
&= \frac{\sqrt{-1+x^6}}{6x^6} + \frac{1}{12}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right)\\
&= \frac{\sqrt{-1+x^6}}{6x^6} + \frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^6}\right)\\
&= \frac{\sqrt{-1+x^6}}{6x^6} + \frac{1}{6}\tan^{-1}\left(\sqrt{-1+x^6}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.32

$$\frac{1}{6}\sqrt{x^6-1}\left(\frac{1}{x^6}+\frac{\tanh^{-1}\left(\sqrt{1-x^6}\right)}{\sqrt{1-x^6}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[-1 + x^6]),x]
[Out] (Sqrt[-1 + x^6]*(x^(-6) + ArcTanh[Sqrt[1 - x^6]]/Sqrt[1 - x^6]))/6
```

IntegrateAlgebraic [A] time = 0.03, size = 31, normalized size = 1.00

$$\frac{\sqrt{x^6-1}}{6x^6}+\frac{1}{6}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*Sqrt[-1 + x^6]),x]
[Out] Sqrt[-1 + x^6]/(6*x^6) + ArcTan[Sqrt[-1 + x^6]]/6
```

fricas [A] time = 0.40, size = 25, normalized size = 0.81

$$\frac{x^6\arctan\left(\sqrt{x^6-1}\right)+\sqrt{x^6-1}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6-1)^(1/2),x, algorithm="fricas")
[Out] 1/6*(x^6*arctan(sqrt(x^6 - 1)) + sqrt(x^6 - 1))/x^6
```

giac [A] time = 0.26, size = 23, normalized size = 0.74

$$\frac{\sqrt{x^6-1}}{6\,x^6}+\frac{1}{6}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6-1)^(1/2),x, algorithm="giac")
[Out] 1/6*sqrt(x^6 - 1)/x^6 + 1/6*arctan(sqrt(x^6 - 1))
```

maple [A] time = 0.04, size = 20, normalized size = 0.65

$$\frac{\sqrt{x^6-1}}{6x^6}-\frac{\arcsin\left(\frac{1}{x^3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^6-1)^(1/2),x)
```

```
[Out] 1/6*(x^6-1)^(1/2)/x^6-1/6*arcsin(1/x^3)
```

maxima [A] time = 0.47, size = 23, normalized size = 0.74

$$\frac{\sqrt{x^6-1}}{6\,x^6}+\frac{1}{6}\,\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(x^6 - 1)/x^6 + 1/6*arctan(sqrt(x^6 - 1))
```

mupad [B] time = 0.37, size = 23, normalized size = 0.74

$$\frac{\mathrm{atan}\left(\sqrt{x^6-1}\right)}{6}+\frac{\sqrt{x^6-1}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^6 - 1)^(1/2)),x)
```

```
[Out] atan((x^6 - 1)^(1/2))/6 + (x^6 - 1)^(1/2)/(6*x^6)
```

sympy [A] time = 1.58, size = 73, normalized size = 2.35

$$\begin{cases} \frac{i\,\mathrm{acosh}\left(\frac{1}{x^3}\right)}{6}+\frac{i\sqrt{-1+\frac{1}{x^6}}}{6x^3} & \text{for}\ \frac{1}{|x^6|}>1 \\ -\frac{\mathrm{asin}\left(\frac{1}{x^3}\right)}{6}+\frac{1}{6x^3\sqrt{1-\frac{1}{x^6}}}-\frac{1}{6x^9\sqrt{1-\frac{1}{x^6}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**6-1)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(x**(-3))/6 + I*sqrt(-1 + x**(-6))/(6*x**3), 1/Abs(x**6)
> 1), (-asin(x**(-3))/6 + 1/(6*x**3*sqrt(1 - 1/x**6)) - 1/(6*x**9*sqrt(1 -
1/x**6)), True))
```

3.384 $\int \frac{\sqrt{-1+x^6}}{x^7}\,dx$

Optimal. Leaf size=31

$$\frac{1}{6}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{\sqrt{x^6-1}}{6x^6}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 47, 63, 203}

$$\frac{1}{6}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{\sqrt{x^6-1}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^6]/x^7,x]
```

```
[Out] -1/6*Sqrt[-1 + x^6]/x^6 + ArcTan[Sqrt[-1 + x^6]]/6
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{-1+x^6}}{x^7}\,dx = \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x^2}\,dx, x, x^6\right)$$
$$= -\frac{\sqrt{-1+x^6}}{6x^6} + \frac{1}{12}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right)$$
$$= -\frac{\sqrt{-1+x^6}}{6x^6} + \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^6}\right)$$
$$= -\frac{\sqrt{-1+x^6}}{6x^6} + \frac{1}{6}\tan^{-1}\left(\sqrt{-1+x^6}\right)$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.55

$$-\frac{x^6 + \sqrt{1-x^6}\,x^6 \tanh^{-1}\left(\sqrt{1-x^6}\right) - 1}{6x^6\sqrt{x^6-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^6]/x^7,x]
[Out] -1/6*(-1 + x^6 + x^6*Sqrt[1 - x^6]*ArcTanh[Sqrt[1 - x^6]])/(x^6*Sqrt[-1 + x
^6])
```

IntegrateAlgebraic [A] time = 0.03, size = 31, normalized size = 1.00

$$\frac{1}{6}\tan^{-1}\left(\sqrt{x^6-1}\right) - \frac{\sqrt{x^6-1}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x^6]/x^7,x]
[Out] -1/6*Sqrt[-1 + x^6]/x^6 + ArcTan[Sqrt[-1 + x^6]]/6
```

fricas [A] time = 0.40, size = 27, normalized size = 0.87

$$\frac{x^6 \arctan\left(\sqrt{x^6-1}\right) - \sqrt{x^6-1}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^7,x, algorithm="fricas")
[Out] 1/6*(x^6*arctan(sqrt(x^6 - 1)) - sqrt(x^6 - 1))/x^6
```

giac [A] time = 0.28, size = 23, normalized size = 0.74

$$-\frac{\sqrt{x^6-1}}{6\,x^6} + \frac{1}{6}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^7,x, algorithm="giac")
[Out] -1/6*sqrt(x^6 - 1)/x^6 + 1/6*arctan(sqrt(x^6 - 1))
```

maple [A] time = 0.02, size = 20, normalized size = 0.65

$$-\frac{\sqrt{x^6-1}}{6x^6}-\frac{\arcsin\left(\frac{1}{x^3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/2)/x^7,x)
```

```
[Out] -1/6*(x^6-1)^(1/2)/x^6-1/6*arcsin(1/x^3)
```

maxima [A] time = 0.49, size = 23, normalized size = 0.74

$$-\frac{\sqrt{x^6-1}}{6\,x^6}+\frac{1}{6}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(x^6 - 1)/x^6 + 1/6*arctan(sqrt(x^6 - 1))
```

mupad [B] time = 0.34, size = 23, normalized size = 0.74

$$\frac{\operatorname{atan}\left(\sqrt{x^6-1}\right)}{6}-\frac{\sqrt{x^6-1}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)^(1/2)/x^7,x)
```

```
[Out] atan((x^6 - 1)^(1/2))/6 - (x^6 - 1)^(1/2)/(6*x^6)
```

sympy [A] time = 1.38, size = 73, normalized size = 2.35

$$\begin{cases} \frac{i\operatorname{acosh}\left(\frac{1}{x^3}\right)}{6}+\frac{i}{6x^3\sqrt{-1+\frac{1}{x^6}}}-\frac{i}{6x^9\sqrt{-1+\frac{1}{x^6}}} & \text{for}\ \frac{1}{|x^6|}>1 \\ -\frac{\operatorname{asin}\left(\frac{1}{x^3}\right)}{6}-\frac{\sqrt{1-\frac{1}{x^6}}}{6x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/2)/x**7,x)
```

```
[Out] Piecewise((I*acosh(x**(-3))/6 + I/(6*x**3*sqrt(-1 + x**(-6))) - I/(6*x**9*s
qrt(-1 + x**(-6))), 1/Abs(x**6) > 1), (-asin(x**(-3))/6 - sqrt(1 - 1/x**6)/
(6*x**3), True))
```

3.385 $\int \frac{-1+x^6}{x^7 \sqrt{1+x^6}} \, dx$

Optimal. Leaf size=31

$$\frac{\sqrt{x^6+1}}{6x^6} - \frac{1}{2}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 78, 63, 207}

$$\frac{\sqrt{x^6+1}}{6x^6} - \frac{1}{2}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)/(x^7*Sqrt[1 + x^6]),x]
```

```
[Out] Sqrt[1 + x^6]/(6*x^6) - ArcTanh[Sqrt[1 + x^6]]/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x^6}{x^7\sqrt{1+x^6}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{-1+x}{x^2\sqrt{1+x}}\,dx,x,x^6\right)\\
&= \frac{\sqrt{1+x^6}}{6x^6}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^6\right)\\
&= \frac{\sqrt{1+x^6}}{6x^6}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^6}\right)\\
&= \frac{\sqrt{1+x^6}}{6x^6}-\frac{1}{2}\tanh^{-1}\left(\sqrt{1+x^6}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.97

$$\frac{1}{6}\left(\frac{\sqrt{x^6+1}}{x^6}-3\tanh^{-1}\left(\sqrt{x^6+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)/(x^7*Sqrt[1 + x^6]),x]
```

```
[Out] (Sqrt[1 + x^6]/x^6 - 3*ArcTanh[Sqrt[1 + x^6]])/6
```

IntegrateAlgebraic [A] time = 0.06, size = 31, normalized size = 1.00

$$\frac{\sqrt{x^6+1}}{6x^6}-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^6)/(x^7*Sqrt[1 + x^6]),x]
```

```
[Out] Sqrt[1 + x^6]/(6*x^6) - ArcTanh[Sqrt[1 + x^6]]/2
```

fricas [A] time = 0.40, size = 45, normalized size = 1.45

$$-\frac{3\,x^6\log\left(\sqrt{x^6+1}+1\right)-3\,x^6\log\left(\sqrt{x^6+1}-1\right)-2\,\sqrt{x^6+1}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/x^7/(x^6+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*x^6*log(sqrt(x^6 + 1) + 1) - 3*x^6*log(sqrt(x^6 + 1) - 1) - 2*sqrt
(x^6 + 1))/x^6
```

giac [A] time = 0.32, size = 37, normalized size = 1.19

$$\frac{\sqrt{x^6+1}}{6\,x^6}-\frac{1}{4}\,\log\left(\sqrt{x^6+1}+1\right)+\frac{1}{4}\,\log\left(\sqrt{x^6+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/x^7/(x^6+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(x^6 + 1)/x^6 - 1/4*log(sqrt(x^6 + 1) + 1) + 1/4*log(sqrt(x^6 + 1)
- 1)
```

maple [A] time = 0.04, size = 32, normalized size = 1.03

$$\frac{\sqrt{x^6+1}}{6x^6}+\frac{\ln\left(\frac{\sqrt{x^6+1}-1}{\sqrt{x^6}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)/x^7/(x^6+1)^(1/2),x)
```

```
[Out] 1/6*(x^6+1)^(1/2)/x^6+1/2*ln(((x^6+1)^(1/2)-1)/(x^6)^(1/2))
```

maxima [A] time = 0.45, size = 37, normalized size = 1.19

$$\frac{\sqrt{x^6+1}}{6\,x^6}-\frac{1}{4}\,\log\left(\sqrt{x^6+1}\,+1\right)+\frac{1}{4}\,\log\left(\sqrt{x^6+1}\,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/x^7/(x^6+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(x^6 + 1)/x^6 - 1/4*log(sqrt(x^6 + 1) + 1) + 1/4*log(sqrt(x^6 + 1)
- 1)
```

mupad [B] time = 0.52, size = 23, normalized size = 0.74

$$\frac{\sqrt{x^6+1}}{6\,x^6}-\frac{\mathrm{atanh}\left(\sqrt{x^6+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)/(x^7*(x^6 + 1)^(1/2)),x)
```

```
[Out] (x^6 + 1)^(1/2)/(6*x^6) - atanh((x^6 + 1)^(1/2))/2
```

sympy [B] time = 98.97, size = 56, normalized size = 1.81

$$\frac{\log\left(-1+\frac{1}{\sqrt{x^6+1}}\right)}{4}-\frac{\log\left(1+\frac{1}{\sqrt{x^6+1}}\right)}{4}-\frac{1}{12\left(1+\frac{1}{\sqrt{x^6+1}}\right)}-\frac{1}{12\left(-1+\frac{1}{\sqrt{x^6+1}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)/x**7/(x**6+1)**(1/2),x)
```

```
[Out] log(-1 + 1/sqrt(x**6 + 1))/4 - log(1 + 1/sqrt(x**6 + 1))/4 - 1/(12*(1 + 1/s
qrt(x**6 + 1))) - 1/(12*(-1 + 1/sqrt(x**6 + 1)))
```

3.386 $\int \frac{\sqrt{1+x^6}}{x^7} \, dx$

Optimal. Leaf size=31

$$-\frac{\sqrt{x^6+1}}{6x^6}-\frac{1}{6}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 47, 63, 207}

$$-\frac{\sqrt{x^6+1}}{6x^6}-\frac{1}{6}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^6]/x^7,x]
```

```
[Out] -1/6*Sqrt[1 + x^6]/x^6 - ArcTanh[Sqrt[1 + x^6]]/6
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{1+x^6}}{x^7} dx = \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2} dx, x, x^6\right)$$
$$= -\frac{\sqrt{1+x^6}}{6x^6} + \frac{1}{12}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}} dx, x, x^6\right)$$
$$= -\frac{\sqrt{1+x^6}}{6x^6} + \frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1+x^6}\right)$$
$$= -\frac{\sqrt{1+x^6}}{6x^6} - \frac{1}{6}\tanh^{-1}\left(\sqrt{1+x^6}\right)$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.42

$$-\frac{1}{6x^6\sqrt{x^6+1}} - \frac{1}{6\sqrt{x^6+1}} - \frac{1}{6}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^6]/x^7,x]
[Out] -1/6*1/Sqrt[1 + x^6] - 1/(6*x^6*Sqrt[1 + x^6]) - ArcTanh[Sqrt[1 + x^6]]/6
```

IntegrateAlgebraic [A] time = 0.02, size = 31, normalized size = 1.00

$$-\frac{\sqrt{x^6+1}}{6x^6} - \frac{1}{6}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^6]/x^7,x]
[Out] -1/6*Sqrt[1 + x^6]/x^6 - ArcTanh[Sqrt[1 + x^6]]/6
```

fricas [A] time = 0.39, size = 44, normalized size = 1.42

$$-\frac{x^6\log\left(\sqrt{x^6+1}+1\right) - x^6\log\left(\sqrt{x^6+1}-1\right) + 2\sqrt{x^6+1}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/2)/x^7,x, algorithm="fricas")
[Out] -1/12*(x^6*log(sqrt(x^6 + 1) + 1) - x^6*log(sqrt(x^6 + 1) - 1) + 2*sqrt(x^6
 + 1))/x^6
```

giac [A] time = 0.23, size = 37, normalized size = 1.19

$$-\frac{\sqrt{x^6+1}}{6\,x^6} - \frac{1}{12}\log\left(\sqrt{x^6+1}+1\right) + \frac{1}{12}\log\left(\sqrt{x^6+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/2)/x^7,x, algorithm="giac")
[Out] -1/6*sqrt(x^6 + 1)/x^6 - 1/12*log(sqrt(x^6 + 1) + 1) + 1/12*log(sqrt(x^6 +
1) - 1)
```

maple [A] time = 0.02, size = 32, normalized size = 1.03

$$-\frac{\sqrt{x^6+1}}{6x^6}+\frac{\ln\left(\frac{\sqrt{x^6+1}-1}{\sqrt{x^6}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)^(1/2)/x^7,x)
```

```
[Out] -1/6*(x^6+1)^(1/2)/x^6+1/6*ln(((x^6+1)^(1/2)-1)/(x^6)^(1/2))
```

maxima [A] time = 0.58, size = 37, normalized size = 1.19

$$-\frac{\sqrt{x^6+1}}{6\,x^6}-\frac{1}{12}\,\log\left(\sqrt{x^6+1}+1\right)+\frac{1}{12}\,\log\left(\sqrt{x^6+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(x^6 + 1)/x^6 - 1/12*log(sqrt(x^6 + 1) + 1) + 1/12*log(sqrt(x^6 +
1) - 1)
```

mupad [B] time = 0.33, size = 23, normalized size = 0.74

$$-\frac{\mathrm{atanh}\left(\sqrt{x^6+1}\right)}{6}-\frac{\sqrt{x^6+1}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)^(1/2)/x^7,x)
```

```
[Out] - atanh((x^6 + 1)^(1/2))/6 - (x^6 + 1)^(1/2)/(6*x^6)
```

sympy [A] time = 1.35, size = 24, normalized size = 0.77

$$-\frac{\mathrm{asinh}\left(\frac{1}{x^3}\right)}{6}-\frac{\sqrt{1+\frac{1}{x^6}}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)**(1/2)/x**7,x)
```

```
[Out] -asinh(x**(-3))/6 - sqrt(1 + x**(-6))/(6*x**3)
```

3.387 $\int \frac{(2+x^6)(1-2x^6+x^8+x^{12})}{x^{10}(-1+x^6)^{3/4}}\,dx$

Optimal. Leaf size=31

$$\frac{2\sqrt[4]{x^6-1}\,(x^{12}+9x^8-2x^6+1)}{9x^9}$$

Rubi [A] time = 0.11, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1833, 1584, 449, 1586, 1478}

$$\frac{2\sqrt[4]{x^6-1}}{x}+\frac{2(x^6-1)^{9/4}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x^6)*(1 - 2*x^6 + x^8 + x^12))/(x^10*(-1 + x^6)^(3/4)),x]
```

```
[Out] (2*(-1 + x^6)^(1/4))/x + (2*(-1 + x^6)^(9/4))/(9*x^9)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1478

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n
_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p
]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(2+x^6\right)\left(1-2x^6+x^8+x^{12}\right)}{x^{10}\left(-1+x^6\right)^{3/4}}\,dx &= \int \left(\frac{2x^6+x^{12}}{x^8\left(-1+x^6\right)^{3/4}}+\frac{2-3x^6+x^{18}}{x^{10}\left(-1+x^6\right)^{3/4}}\right)dx \\
&= \int \frac{2x^6+x^{12}}{x^8\left(-1+x^6\right)^{3/4}}\,dx+\int \frac{2-3x^6+x^{18}}{x^{10}\left(-1+x^6\right)^{3/4}}\,dx \\
&= \int \frac{2+x^6}{x^2\left(-1+x^6\right)^{3/4}}\,dx+\int \frac{\sqrt[4]{-1+x^6}\left(-2+x^6+x^{12}\right)}{x^{10}}\,dx \\
&= \frac{2\sqrt[4]{-1+x^6}}{x}+\int \frac{\left(-1+x^6\right)^{5/4}\left(2+x^6\right)}{x^{10}}\,dx \\
&= \frac{2\sqrt[4]{-1+x^6}}{x}+\frac{2\left(-1+x^6\right)^{9/4}}{9x^9}
\end{aligned}
$$

Mathematica [C] time = 0.12, size = 168, normalized size = 5.42

$$
\frac{45\left(1-x^6\right)^{3/4}x^6\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};x^6\right)-10\left(1-x^6\right)^{3/4}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};x^6\right)-90\left(1-x^6\right)^{3/4}x^8\,{}_2F_1\left(-\frac{1}{6},\frac{3}{4};\frac{5}{6};x^6\right)+x^{12}\left(10\left(1-x^6\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};x^6\right)+9\left(1-x^6\right)^{3/4}x^2\,{}_2F_1\left(\frac{3}{4},\frac{5}{6};\frac{11}{6};x^6\right)+10\left(x^6-1\right)\right)}{45x^9\left(x^6-1\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x^6)*(1 - 2*x^6 + x^8 + x^12))/(x^10*(-1 + x^6)^(3/4)),x]
```

```
[Out] (-10*(1 - x^6)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, x^6] + 45*x^6*(1 -
x^6)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, x^6] - 90*x^8*(1 - x^6)^(3/4)*
Hypergeometric2F1[-1/6, 3/4, 5/6, x^6] + x^12*(10*(-1 + x^6) + 10*(1 - x^6)
^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, x^6] + 9*x^2*(1 - x^6)^(3/4)*Hyperg
eometric2F1[3/4, 5/6, 11/6, x^6]))/(45*x^9*(-1 + x^6)^(3/4))
```

IntegrateAlgebraic [A] time = 8.91, size = 31, normalized size = 1.00

$$
\frac{2\sqrt[4]{x^6-1}\left(x^{12}+9x^8-2x^6+1\right)}{9x^9}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^6)*(1 - 2*x^6 + x^8 + x^12))/(x^10*(-1 + x^6)^(3
/4)),x]
```

```
[Out] (2*(-1 + x^6)^(1/4)*(1 - 2*x^6 + 9*x^8 + x^12))/(9*x^9)
```

fricas [A] time = 0.42, size = 27, normalized size = 0.87

$$
\frac{2\left(x^{12}+9\,x^8-2\,x^6+1\right)\left(x^6-1\right)^{\frac{1}{4}}}{9\,x^9}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)*(x^12+x^8-2*x^6+1)/x^10/(x^6-1)^(3/4),x, algorithm="frica
s")
```

```
[Out] 2/9*(x^12 + 9*x^8 - 2*x^6 + 1)*(x^6 - 1)^(1/4)/x^9
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(x^{12}+x^8-2\,x^6+1\right)\left(x^6+2\right)}{\left(x^6-1\right)^{\frac{3}{4}}x^{10}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)*(x^12+x^8-2*x^6+1)/x^10/(x^6-1)^(3/4),x, algorithm="giac"
)
```

```
[Out] integrate((x^12 + x^8 - 2*x^6 + 1)*(x^6 + 2)/((x^6 - 1)^(3/4)*x^10), x)
```

maple [A] time = 0.01, size = 48, normalized size = 1.55

$$\frac{2\left(-1+x\right)\left(1+x\right)\left(x^{2}+x+1\right)\left(x^{2}-x+1\right)\left(x^{12}+9x^{8}-2x^{6}+1\right)}{9x^{9}\left(x^{6}-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+2)*(x^12+x^8-2*x^6+1)/x^10/(x^6-1)^(3/4),x)
```

```
[Out] 2/9*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)*(x^12+9*x^8-2*x^6+1)/x^9/(x^6-1)^(3/4)
```

maxima [B] time = 0.90, size = 58, normalized size = 1.87

$$\frac{2\left(x^{18}+9\,x^{14}-3\,x^{12}-9\,x^{8}+3\,x^{6}-1\right)}{9\left(x^{2}+x+1\right)^{\frac{3}{4}}\left(x^{2}-x+1\right)^{\frac{3}{4}}\left(x+1\right)^{\frac{3}{4}}\left(x-1\right)^{\frac{3}{4}}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)*(x^12+x^8-2*x^6+1)/x^10/(x^6-1)^(3/4),x, algorithm="maxim
a")
```

```
[Out] 2/9*(x^18 + 9*x^14 - 3*x^12 - 9*x^8 + 3*x^6 - 1)/((x^2 + x + 1)^(3/4)*(x^2
- x + 1)^(3/4)*(x + 1)^(3/4)*(x - 1)^(3/4)*x^9)
```

mupad [B] time = 0.40, size = 49, normalized size = 1.58

$$\frac{2\left(x^{6}-1\right)^{1/4}}{x}-\frac{4\left(x^{6}-1\right)^{1/4}}{9\,x^{3}}+\frac{2\,x^{3}\left(x^{6}-1\right)^{1/4}}{9}+\frac{2\left(x^{6}-1\right)^{1/4}}{9\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 + 2)*(x^8 - 2*x^6 + x^12 + 1))/(x^10*(x^6 - 1)^(3/4)),x)
```

```
[Out] (2*(x^6 - 1)^(1/4))/x - (4*(x^6 - 1)^(1/4))/(9*x^3) + (2*x^3*(x^6 - 1)^(1/4
))/9 + (2*(x^6 - 1)^(1/4))/(9*x^9)
```

sympy [C] time = 6.02, size = 139, normalized size = 4.48

$$\frac{x^{9}e^{-\frac{3i\pi}{4}}{}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\right|x^{6}\right)}{9}+\frac{x^{5}e^{-\frac{3i\pi}{4}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{6}\\\frac{11}{6}\end{matrix}\right|x^{6}\right)}{6\Gamma\left(\frac{11}{6}\right)}-\frac{e^{\frac{i\pi}{4}}\Gamma\left(-\frac{1}{6}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{6},\frac{3}{4}\\\frac{5}{6}\end{matrix}\right|x^{6}\right)}{3x\Gamma\left(\frac{5}{6}\right)}-\frac{e^{\frac{i\pi}{4}}{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{1}{2}\end{matrix}\right|x^{6}\right)}{x^{3}}+\frac{2e^{\frac{i\pi}{4}}{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{3}{2},\frac{3}{4}\\-\frac{1}{2}\end{matrix}\right|x^{6}\right)}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+2)*(x**12+x**8-2*x**6+1)/x**10/(x**6-1)**(3/4),x)
```

```
[Out] x**9*exp(-3*I*pi/4)*hyper((3/4, 3/2), (5/2,), x**6)/9 + x**5*exp(-3*I*pi/4)
*gamma(5/6)*hyper((3/4, 5/6), (11/6,), x**6)/(6*gamma(11/6)) - exp(I*pi/4)*
gamma(-1/6)*hyper((-1/6, 3/4), (5/6,), x**6)/(3*x*gamma(5/6)) - exp(I*pi/4)
*hyper((-1/2, 3/4), (1/2,), x**6)/x**3 + 2*exp(I*pi/4)*hyper((-3/2, 3/4), (
-1/2,), x**6)/(9*x**9)
```

3.388 $\int \frac{-1+2x+2x^2}{\left(1-x+3x^2\right)\sqrt{-x+x^4}}\,dx$

Optimal. Leaf size=32

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x^4-x}}{x^2+x+1}\right)$$

Rubi [F] time = 2.41, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+2x+2x^2}{\left(1-x+3x^2\right)\sqrt{-x+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 2*x + 2*x^2)/((1 - x + 3*x^2)*Sqrt[-x + x^4]),x]
```

```
[Out] (2*(1 - x)*x*Sqrt[(1 + x + x^2)/(1 - (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(
1 - (1 - Sqrt[3])*x)/(1 - (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3*3^(1/4)*S
qrt[-(((1 - x)*x)/(1 - (1 + Sqrt[3])*x)^2)]*Sqrt[-x + x^4]) - (2*(4 - I*Sqr
t[11])*Sqrt[x]*Sqrt[-1 + x^3]*Defer[Subst][Defer[Int][1/((Sqrt[1 - I*Sqrt[1
1]] - Sqrt[6]*x)*Sqrt[-1 + x^6]), x], x, Sqrt[x]])/(3*Sqrt[1 - I*Sqrt[11]]*
Sqrt[-x + x^4]) - (2*(4 + I*Sqrt[11])*Sqrt[x]*Sqrt[-1 + x^3]*Defer[Subst][D
efer[Int][1/((Sqrt[1 + I*Sqrt[11]] - Sqrt[6]*x)*Sqrt[-1 + x^6]), x], x, Sqr
t[x]])/(3*Sqrt[1 + I*Sqrt[11]]*Sqrt[-x + x^4]) - (2*(4 - I*Sqrt[11])*Sqrt[x
]*Sqrt[-1 + x^3]*Defer[Subst][Defer[Int][1/((Sqrt[1 - I*Sqrt[11]] + Sqrt[6]
*x)*Sqrt[-1 + x^6]), x], x, Sqrt[x]])/(3*Sqrt[1 - I*Sqrt[11]]*Sqrt[-x + x^4
]) - (2*(4 + I*Sqrt[11])*Sqrt[x]*Sqrt[-1 + x^3]*Defer[Subst][Defer[Int][1/(
(Sqrt[1 + I*Sqrt[11]] + Sqrt[6]*x)*Sqrt[-1 + x^6]), x], x, Sqrt[x]])/(3*Sqr
t[1 + I*Sqrt[11]]*Sqrt[-x + x^4])
```

Rubi steps

$$\int \frac{-1+2x+2x^2}{(1-x+3x^2)\sqrt{-x+x^4}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1+x^3}\right)\int \frac{-1+2x+2x^2}{\sqrt{x}(1-x+3x^2)\sqrt{-1+x^3}}\,dx}{\sqrt{-x+x^4}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^3}\right)\int \left(\frac{2}{3\sqrt{x}\sqrt{-1+x^3}} - \frac{5-8x}{3\sqrt{x}(1-x+3x^2)\sqrt{-1+x^3}}\right)dx}{\sqrt{-x+x^4}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-1+x^3}\right)\int \frac{5-8x}{\sqrt{x}(1-x+3x^2)\sqrt{-1+x^3}}\,dx}{3\sqrt{-x+x^4}} + \frac{\left(2\sqrt{x}\sqrt{-1+x^3}\right)\int \frac{1}{\sqrt{x}\sqrt{-1+x^3}}\,dx}{3\sqrt{-x+x^4}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-1+x^3}\right)\int \left(\frac{-8-2i\sqrt{11}}{\sqrt{x}(-1-i\sqrt{11}+6x)\sqrt{-1+x^3}} + \frac{-8+2i\sqrt{11}}{\sqrt{x}(-1+i\sqrt{11}+6x)\sqrt{-1+x^3}}\right)dx}{3\sqrt{-x+x^4}} + \frac{(4\sqrt{\cdots}}{\cdots}$$

$$= \frac{2(1-x)x\sqrt{\frac{1+x+x^2}{(1-(1+\sqrt{3})x)^2}}\,F\left(\cos^{-1}\left(\frac{1-(1-\sqrt{3})x}{1-(1+\sqrt{3})x}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{3\sqrt[4]{3}\sqrt{-\frac{(1-x)x}{(1-(1+\sqrt{3})x)^2}}\sqrt{-x+x^4}} + \frac{\left(2\left(4-i\sqrt{11}\right)\sqrt{x}\cdots\right.}{\cdots}$$

$$= \frac{2(1-x)x\sqrt{\frac{1+x+x^2}{(1-(1+\sqrt{3})x)^2}}\,F\left(\cos^{-1}\left(\frac{1-(1-\sqrt{3})x}{1-(1+\sqrt{3})x}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{3\sqrt[4]{3}\sqrt{-\frac{(1-x)x}{(1-(1+\sqrt{3})x)^2}}\sqrt{-x+x^4}} + \frac{\left(4\left(4-i\sqrt{11}\right)\sqrt{x}\cdots\right.}{\cdots}$$

$$= \frac{2(1-x)x\sqrt{\frac{1+x+x^2}{(1-(1+\sqrt{3})x)^2}}\,F\left(\cos^{-1}\left(\frac{1-(1-\sqrt{3})x}{1-(1+\sqrt{3})x}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{3\sqrt[4]{3}\sqrt{-\frac{(1-x)x}{(1-(1+\sqrt{3})x)^2}}\sqrt{-x+x^4}} + \frac{\left(4\left(4-i\sqrt{11}\right)\sqrt{x}\cdots\right.}{\cdots}$$

$$= \frac{2(1-x)x\sqrt{\frac{1+x+x^2}{(1-(1+\sqrt{3})x)^2}}\,F\left(\cos^{-1}\left(\frac{1-(1-\sqrt{3})x}{1-(1+\sqrt{3})x}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{3\sqrt[4]{3}\sqrt{-\frac{(1-x)x}{(1-(1+\sqrt{3})x)^2}}\sqrt{-x+x^4}} - \frac{\left(2\left(4-i\sqrt{11}\right)\sqrt{x}\cdots\right.}{\cdots}$$

Mathematica [C] time = 2.15, size = 533, normalized size = 16.66

$$-\frac{2\sqrt{\frac{1-\frac{1}{x}}{1+\sqrt[3]{-1}}}\,x^2\left(\frac{i\sqrt{11}\sqrt{\frac{1}{x^2}+\frac{1}{x}+1}\,\Pi\left(\frac{2\sqrt{3}}{-2i+\sqrt{3}-\sqrt{11}};\sin^{-1}\left(\sqrt{\frac{1-\frac{(-1)^{2/3}}{x}}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{2i-\sqrt{3}+\sqrt{11}}+\frac{\sqrt{\frac{1}{x^2}+\frac{1}{x}+1}\,\Pi\left(\frac{2\sqrt{3}}{-2i+\sqrt{3}-\sqrt{11}};\sin^{-1}\left(\sqrt{-\frac{2i+\frac{i+\sqrt{3}}{x}}{-3i+\sqrt{3}}}\right)|\frac{1}{2}(1+i\sqrt{3})\right)}{2i-\sqrt{3}+\sqrt{11}}+\frac{(-1)^{5/6}(1+\sqrt[3]{-1})\sqrt{\frac{1}{x^2}+\frac{1}{x}+1}\,\Pi\left(\frac{2\sqrt{3}}{-2i+\sqrt{3}+\sqrt{11}};\sin^{-1}\left(\sqrt{\frac{1-\frac{(-1)^{2/3}}{x}}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{3-2i\sqrt{3}+\sqrt{33}}+\frac{i\sqrt{11}\sqrt{\frac{1}{x^2}+\frac{1}{x}+1}\,\Pi\left(\frac{2\sqrt{3}}{-2i+\sqrt{3}+\sqrt{11}};\sin^{-1}\left(\sqrt{\frac{1-\frac{(-1)^{2/3}}{x}}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{-2i+\sqrt{3}+\sqrt{11}}-\frac{\left(\frac{1}{x}+\sqrt[3]{-1}\right)\sqrt{\frac{\frac{(-1)^{2/3}}{x}+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-\frac{(-1)^{2/3}}{x}}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-\frac{(-1)^{2/3}}{x}}{1+\sqrt[3]{-1}}}}\right)}{\sqrt{x(x^3-1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 2*x + 2*x^2)/((1 - x + 3*x^2)*Sqrt[-x + x^4]),x]
```

```
[Out] (-2*Sqrt[(1 - x^(-1))/(1 + (-1)^(1/3))]*x^2*(-((((-1)^(1/3) + x^(-1))*Sqrt[
((-1)^(1/3) + (-1)^(2/3)/x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-
1)^(2/3)/x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)/x)/(1 + (
-1)^(1/3))]) + (I*Sqrt[11]*Sqrt[1 + x^(-2) + x^(-1)]*EllipticPi[(2*Sqrt[3])
/(-2*I + Sqrt[3] - Sqrt[11]), ArcSin[Sqrt[(1 - (-1)^(2/3)/x)/(1 + (-1)^(1/3
))]], (-1)^(1/3)])/(2*I - Sqrt[3] + Sqrt[11]) + (Sqrt[1 + x^(-2) + x^(-1)]*
EllipticPi[(2*Sqrt[3])/(-2*I + Sqrt[3] - Sqrt[11]), ArcSin[Sqrt[-((2*I + (I
 + Sqrt[3])/x)/(-3*I + Sqrt[3]))]], (1 + I*Sqrt[3])/2])/(2*I - Sqrt[3] + Sq
rt[11]) + (I*Sqrt[11]*Sqrt[1 + x^(-2) + x^(-1)]*EllipticPi[(2*Sqrt[3])/(-2*
I + Sqrt[3] + Sqrt[11]), ArcSin[Sqrt[(1 - (-1)^(2/3)/x)/(1 + (-1)^(1/3))]],
```

```
(-1)^(1/3)])/(-2*I + Sqrt[3] + Sqrt[11]) + ((-1)^(5/6)*(1 + (-1)^(1/3))*Sq
rt[1 + x^(-2) + x^(-1)]*EllipticPi[(2*Sqrt[3])/(-2*I + Sqrt[3] + Sqrt[11]),
 ArcSin[Sqrt[(1 - (-1)^(2/3)/x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(3 - (2*I)
*Sqrt[3] + Sqrt[33])))/Sqrt[x*(-1 + x^3)]
```

IntegrateAlgebraic [A] time = 1.48, size = 32, normalized size = 1.00

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x^4-x}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x + 2*x^2)/((1 - x + 3*x^2)*Sqrt[-x + x^4]),x]
[Out] Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[-x + x^4])/(1 + x + x^2)]
```

fricas [A] time = 0.46, size = 28, normalized size = 0.88

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2-3x-1\right)}{4\sqrt{x^4-x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+2*x-1)/(3*x^2-x+1)/(x^4-x)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*arctan(1/4*sqrt(2)*(x^2 - 3*x - 1)/sqrt(x^4 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2+2x-1}{\sqrt{x^4-x}\left(3x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+2*x-1)/(3*x^2-x+1)/(x^4-x)^(1/2),x, algorithm="giac")
[Out] integrate((2*x^2 + 2*x - 1)/(sqrt(x^4 - x)*(3*x^2 - x + 1)), x)
```

maple [C] time = 0.26, size = 828, normalized size = 25.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+2*x-1)/(3*x^2-x+1)/(x^4-x)^(1/2),x)
[Out] 4/3*(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x)
)^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2))/(-1+x))^(1/2)*
((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2)/(-3/2+1/2*I*3^(1/
2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/2)))^(1/2)*EllipticF(
((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3^(1
/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2/
3*(4/3+1/3*I*11^(1/2))*(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/
2*I*3^(1/2))/(-1+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1
/2))/(-1+x))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2
)/(-3/2+1/2*I*3^(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/2)
))^(1/2)*(5/6+1/6*I*11^(1/2))*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*
I*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*
I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-(1/2-1/2*I*11^(1/2))*EllipticPi(((-3
/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),-1/2*I*3^(1/2)*(1/6+
1/6*I*11^(1/2))+1/4+1/4*I*11^(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))
/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)))+2/3*(4/3-1/3*I*11^(1/2))*
```

```
(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1
/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2))/(-1+x))^(1/2)*((x+
1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2)/(-3/2+1/2*I*3^(1/2))/
(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/2)))^(1/2)*(5/6-1/6*I*11^
(1/2))*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2
),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3
^(1/2)))^(1/2))-(1/2+1/2*I*11^(1/2))*EllipticPi(((-3/2+1/2*I*3^(1/2))*x/(-1
/2+1/2*I*3^(1/2))/(-1+x))^(1/2),-1/2*I*3^(1/2)*(1/6-1/6*I*11^(1/2))+1/4-1/4
*I*11^(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3
/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2+2\,x-1}{\sqrt{x^4-x}\left(3\,x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+2*x-1)/(3*x^2-x+1)/(x^4-x)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x^2 + 2*x - 1)/(sqrt(x^4 - x)*(3*x^2 - x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{2\,x^2+2\,x-1}{\sqrt{x^4-x}\,\left(3\,x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 2*x^2 - 1)/((x^4 - x)^(1/2)*(3*x^2 - x + 1)),x)
[Out] int((2*x + 2*x^2 - 1)/((x^4 - x)^(1/2)*(3*x^2 - x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2+2x-1}{\sqrt{x\,(x-1)\left(x^2+x+1\right)}\,\left(3x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+2*x-1)/(3*x**2-x+1)/(x**4-x)**(1/2),x)
[Out] Integral((2*x**2 + 2*x - 1)/(sqrt(x*(x - 1)*(x**2 + x + 1))*(3*x**2 - x + 1
)), x)
```

3.389 $\int \frac{1+x^3}{x^3\left(-1+x^3\right)\sqrt[4]{-x+x^4}}\,dx$

Optimal. Leaf size=32

$$-\frac{4\left(7x^3-1\right)\left(x^4-x\right)^{3/4}}{9x^3\left(x^3-1\right)}$$

Rubi [A] time = 0.16, antiderivative size = 35, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2056, 453, 264}

$$\frac{4}{9x^2\sqrt[4]{x^4-x}}-\frac{28x}{9\sqrt[4]{x^4-x}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(x^3*(-1 + x^3)*(-x + x^4)^(1/4)),x]
```

```
[Out] 4/(9*x^2*(-x + x^4)^(1/4)) - (28*x)/(9*(-x + x^4)^(1/4))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{x^3\left(-1+x^3\right)\sqrt[4]{-x+x^4}}\,dx &= \frac{\left(\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\int \frac{1+x^3}{x^{13/4}\left(-1+x^3\right)^{5/4}}\,dx}{\sqrt[4]{-x+x^4}}\\
&= \frac{4}{9x^2\sqrt[4]{-x+x^4}}+\frac{\left(7\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\int \frac{1}{\sqrt[4]{x}\left(-1+x^3\right)^{5/4}}\,dx}{3\sqrt[4]{-x+x^4}}\\
&= \frac{4}{9x^2\sqrt[4]{-x+x^4}}-\frac{28x}{9\sqrt[4]{-x+x^4}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.78

$$-\frac{4\left(7x^3-1\right)}{9x^2\sqrt[4]{x\left(x^3-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(x^3*(-1 + x^3)*(-x + x^4)^(1/4)),x]
[Out] (-4*(-1 + 7*x^3))/(9*x^2*(x*(-1 + x^3))^(1/4))
```

IntegrateAlgebraic [A] time = 0.31, size = 32, normalized size = 1.00

$$-\frac{4\left(7x^3-1\right)\left(x^4-x\right)^{3/4}}{9x^3\left(x^3-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)/(x^3*(-1 + x^3)*(-x + x^4)^(1/4)),x]
[Out] (-4*(-1 + 7*x^3)*(-x + x^4)^(3/4))/(9*x^3*(-1 + x^3))
```

fricas [A] time = 0.39, size = 29, normalized size = 0.91

$$-\frac{4\left(x^4-x\right)^{\frac{3}{4}}\left(7x^3-1\right)}{9\left(x^6-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x^3/(x^3-1)/(x^4-x)^(1/4),x, algorithm="fricas")
[Out] -4/9*(x^4 - x)^(3/4)*(7*x^3 - 1)/(x^6 - x^3)
```

giac [A] time = 0.32, size = 23, normalized size = 0.72

$$\frac{4}{9}\left(-\frac{1}{x^3}+1\right)^{\frac{3}{4}}+\frac{8}{3\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x^3/(x^3-1)/(x^4-x)^(1/4),x, algorithm="giac")
[Out] 4/9*(-1/x^3 + 1)^(3/4) + 8/3/(-1/x^3 + 1)^(1/4)
```

maple [A] time = 0.01, size = 22, normalized size = 0.69

$$-\frac{4\left(7x^3-1\right)}{9\left(x^4-x\right)^{\frac{1}{4}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/x^3/(x^3-1)/(x^4-x)^(1/4),x)
[Out] -4/9*(7*x^3-1)/(x^4-x)^(1/4)/x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3+1}{\left(x^4-x\right)^{\frac{1}{4}}\left(x^3-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x^3/(x^3-1)/(x^4-x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 1)/((x^4 - x)^(1/4)*(x^3 - 1)*x^3), x)
```

mupad [B] time = 0.24, size = 28, normalized size = 0.88

$$-\frac{4\left(x^4-x\right)^{3/4}\left(7x^3-1\right)}{9x^3\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/(x^3*(x^4 - x)^(1/4)*(x^3 - 1)),x)
```

```
[Out] -(4*(x^4 - x)^(3/4)*(7*x^3 - 1))/(9*x^3*(x^3 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)\left(x^2-x+1\right)}{x^3\sqrt[4]{x(x-1)\left(x^2+x+1\right)}\,(x-1)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/x**3/(x**3-1)/(x**4-x)**(1/4),x)
```

```
[Out] Integral((x + 1)*(x**2 - x + 1)/(x**3*(x*(x - 1)*(x**2 + x + 1))**(1/4)*(x
- 1)*(x**2 + x + 1)), x)
```

3.390 $\int \frac{-1-2x+2x^2}{\left(-1+3x+x^2\right)\sqrt{x+x^4}}\,dx$

Optimal. Leaf size=32

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x^4+x}}{x^2-x+1}\right)$$

Rubi [F] time = 2.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1-2x+2x^2}{\left(-1+3x+x^2\right)\sqrt{x+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 - 2*x + 2*x^2)/((-1 + 3*x + x^2)*Sqrt[x + x^4]),x]
```

```
[Out] (2*x*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(
1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3^(1/4)*Sqr
t[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[x + x^4]) + (Sqrt[-17 + 5*Sqrt[
13]]*Sqrt[x]*Sqrt[1 + x^3]*Defer[Subst][Defer[Int][1/((Sqrt[-3 + Sqrt[13]]
- Sqrt[2]*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/Sqrt[x + x^4] - (I*Sqrt[17 +
5*Sqrt[13]]*Sqrt[x]*Sqrt[1 + x^3]*Defer[Subst][Defer[Int][1/((I*Sqrt[3 + Sq
rt[13]] - Sqrt[2]*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/Sqrt[x + x^4] + (Sqrt
[-17 + 5*Sqrt[13]]*Sqrt[x]*Sqrt[1 + x^3]*Defer[Subst][Defer[Int][1/((Sqrt[-
3 + Sqrt[13]] + Sqrt[2]*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/Sqrt[x + x^4] -
 (I*Sqrt[17 + 5*Sqrt[13]]*Sqrt[x]*Sqrt[1 + x^3]*Defer[Subst][Defer[Int][1/(
(I*Sqrt[3 + Sqrt[13]] + Sqrt[2]*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/Sqrt[x
+ x^4]
```

Rubi steps

$$\int \frac{-1-2x+2x^2}{\left(-1+3x+x^2\right)\sqrt{x+x^4}}\,dx = \frac{\left(\sqrt{x}\sqrt{1+x^3}\right)\int \frac{-1-2x+2x^2}{\sqrt{x}\left(-1+3x+x^2\right)\sqrt{1+x^3}}\,dx}{\sqrt{x+x^4}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x^3}\right)\int \left(\frac{2}{\sqrt{x}\sqrt{1+x^3}}+\frac{1-8x}{\sqrt{x}\left(-1+3x+x^2\right)\sqrt{1+x^3}}\right)dx}{\sqrt{x+x^4}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x^3}\right)\int \frac{1-8x}{\sqrt{x}\left(-1+3x+x^2\right)\sqrt{1+x^3}}\,dx}{\sqrt{x+x^4}} + \frac{\left(2\sqrt{x}\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{x}\sqrt{1+x^3}}\,dx}{\sqrt{x+x^4}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x^3}\right)\int \left(\frac{-8+2\sqrt{13}}{\sqrt{x}\left(3-\sqrt{13}+2x\right)\sqrt{1+x^3}}+\frac{-8-2\sqrt{13}}{\sqrt{x}\left(3+\sqrt{13}+2x\right)\sqrt{1+x^3}}\right)dx}{\sqrt{x+x^4}} + \frac{\left(4\sqrt{x}\sqrt{1\ \cdots\right.}{\cdots}$$

$$= \frac{2x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}} - \frac{\left(2\left(4-\sqrt{13}\right)\cdots\right.}{\cdots}$$

$$= \frac{2x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}} - \frac{\left(4\left(4-\sqrt{13}\right)\cdots\right.}{\cdots}$$

$$= \frac{2x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}} - \frac{\left(4\left(4-\sqrt{13}\right)\cdots\right.}{\cdots}$$

$$= \frac{2x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}} + \frac{\left(2\left(4-\sqrt{13}\right)\cdots\right.}{\cdots}$$

Mathematica [C] time = 1.24, size = 298, normalized size = 9.31

$$-\frac{2\sqrt{\frac{1}{x^2}-\frac{1}{x}+1}\sqrt{\frac{\frac{1}{x}+1}{1+\sqrt[3]{-1}}}\,x^2\left(\frac{\sqrt{3}\left(-i\sqrt{3}x+\sqrt[3]{-1}+1\right)F\left(\sin^{-1}\left(\sqrt{\frac{x+(-1)^{2/3}}{\left(1+\sqrt[3]{-1}\right)x}}\right)|\sqrt[3]{-1}\right)}{x+(-1)^{2/3}}-\frac{6i\left(\left(-1+5\sqrt[3]{-1}+\sqrt{13}+\sqrt[3]{-1}\sqrt{13}\right)\Pi\left(\frac{2\sqrt{3}}{2i+\sqrt{3}+i\sqrt{13}};\sin^{-1}\left(\sqrt{\frac{x+(-1)^{2/3}}{\left(1+\sqrt[3]{-1}\right)x}}\right)|\sqrt[3]{-1}\right)-\left(1-5\sqrt[3]{-1}+\sqrt{13}+\sqrt[3]{-1}\sqrt{13}\right)\Pi\left(\frac{2i\sqrt{3}}{-3+2\sqrt[3]{-1}+\sqrt{13}};\sin^{-1}\left(\sqrt{\frac{x+(-1)^{2/3}}{\left(1+\sqrt[3]{-1}\right)x}}\right)|\sqrt[3]{-1}\right)\right)}{-12-4i\sqrt{3}}\right)}{3\sqrt{x^4+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x + 2*x^2)/((-1 + 3*x + x^2)*Sqrt[x + x^4]),x]
```

```
[Out] (-2*Sqrt[1 + x^(-2) - x^(-1)]*Sqrt[(1 + x^(-1))/(1 + (-1)^(1/3))]*x^2*((Sqr
t[3]*(1 + (-1)^(1/3) - I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[((-1)^(2/3) + x)/
((1 + (-1)^(1/3))*x)]], (-1)^(1/3)])/((-1)^(2/3) + x) - ((6*I)*((-1 + 5*(-1
)^(1/3) + Sqrt[13] + (-1)^(1/3)*Sqrt[13])*EllipticPi[(2*Sqrt[3])/(2*I + Sqr
t[3] + I*Sqrt[13]), ArcSin[Sqrt[((-1)^(2/3) + x)/((1 + (-1)^(1/3))*x)]], (-
1)^(1/3)] - (1 - 5*(-1)^(1/3) + Sqrt[13] + (-1)^(1/3)*Sqrt[13])*EllipticPi[
((2*I)*Sqrt[3])/(-3 + 2*(-1)^(1/3) + Sqrt[13]), ArcSin[Sqrt[((-1)^(2/3) + x
)/((1 + (-1)^(1/3))*x)]], (-1)^(1/3)]))/(-12 - (4*I)*Sqrt[3])))/(3*Sqrt[x +
 x^4])
```

IntegrateAlgebraic [A] time = 1.50, size = 32, normalized size = 1.00

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x^4+x}}{x^2-x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 2*x + 2*x^2)/((-1 + 3*x + x^2)*Sqrt[x + x^4]),x]
[Out] Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x + x^4])/(1 - x + x^2)]
```

fricas [B] time = 0.45, size = 68, normalized size = 2.12

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{17x^4+6x^3+4\sqrt{2}\sqrt{x^4+x}\left(3x^2+x+1\right)+7x^2+10x+1}{x^4+6x^3+7x^2-6x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-2*x-1)/(x^2+3*x-1)/(x^4+x)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log(-(17*x^4 + 6*x^3 + 4*sqrt(2)*sqrt(x^4 + x)*(3*x^2 + x + 1)
+ 7*x^2 + 10*x + 1)/(x^4 + 6*x^3 + 7*x^2 - 6*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2x^2-2x-1}{\sqrt{x^4+x}\left(x^2+3x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-2*x-1)/(x^2+3*x-1)/(x^4+x)^(1/2),x, algorithm="giac")
[Out] integrate((2*x^2 - 2*x - 1)/(sqrt(x^4 + x)*(x^2 + 3*x - 1)), x)
```

maple [C] time = 0.08, size = 13798, normalized size = 431.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-2*x-1)/(x^2+3*x-1)/(x^4+x)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2x^2-2x-1}{\sqrt{x^4+x}\left(x^2+3x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-2*x-1)/(x^2+3*x-1)/(x^4+x)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x^2 - 2*x - 1)/(sqrt(x^4 + x)*(x^2 + 3*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int-\frac{-2x^2+2x+1}{\sqrt{x^4+x}\,\left(x^2+3x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 2*x^2 + 1)/((x + x^4)^(1/2)*(3*x + x^2 - 1)),x)
```

```
[Out] int(-(2*x - 2*x^2 + 1)/((x + x^4)^(1/2)*(3*x + x^2 - 1)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2 - 2x - 1}{\sqrt{x(x+1)\left(x^2 - x + 1\right)}\left(x^2 + 3x - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-2*x-1)/(x**2+3*x-1)/(x**4+x)**(1/2),x)
```

```
[Out] Integral((2*x**2 - 2*x - 1)/(sqrt(x*(x + 1)*(x**2 - x + 1))*(x**2 + 3*x - 1
)), x)
```

3.391 $\int \frac{-1+x^2}{x^2(1+x^2)\sqrt[4]{x^2+x^4}}\,dx$

Optimal. Leaf size=32

$$\frac{2(7x^2+1)(x^4+x^2)^{3/4}}{3x^3(x^2+1)}$$

Rubi [A] time = 0.16, antiderivative size = 35, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2056, 453, 264}

$$\frac{14x}{3\sqrt[4]{x^4+x^2}}+\frac{2}{3\sqrt[4]{x^4+x^2}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(x^2*(1 + x^2)*(x^2 + x^4)^(1/4)),x]
```

```
[Out] 2/(3*x*(x^2 + x^4)^(1/4)) + (14*x)/(3*(x^2 + x^4)^(1/4))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{x^2(1+x^2)\sqrt[4]{x^2+x^4}}\,dx &= \frac{\left(\sqrt{x}\sqrt[4]{1+x^2}\right)\int \frac{-1+x^2}{x^{5/2}(1+x^2)^{5/4}}\,dx}{\sqrt[4]{x^2+x^4}}\\ &= \frac{2}{3x\sqrt[4]{x^2+x^4}}+\frac{\left(7\sqrt{x}\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt{x}(1+x^2)^{5/4}}\,dx}{3\sqrt[4]{x^2+x^4}}\\ &= \frac{2}{3x\sqrt[4]{x^2+x^4}}+\frac{14x}{3\sqrt[4]{x^2+x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.78

$$\frac{14x^2+2}{3x\sqrt[4]{x^4+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(x^2*(1 + x^2)*(x^2 + x^4)^(1/4)),x]
[Out] (2 + 14*x^2)/(3*x*(x^2 + x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.13, size = 32, normalized size = 1.00

$$\frac{2\left(7x^2+1\right)\left(x^4+x^2\right)^{3/4}}{3x^3\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/(x^2*(1 + x^2)*(x^2 + x^4)^(1/4)),x]
[Out] (2*(1 + 7*x^2)*(x^2 + x^4)^(3/4))/(3*x^3*(1 + x^2))
```

fricas [A] time = 0.40, size = 27, normalized size = 0.84

$$\frac{2\left(x^4+x^2\right)^{\frac{3}{4}}\left(7x^2+1\right)}{3\left(x^5+x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^2/(x^2+1)/(x^4+x^2)^(1/4),x, algorithm="fricas")
[Out] 2/3*(x^4 + x^2)^(3/4)*(7*x^2 + 1)/(x^5 + x^3)
```

giac [A] time = 0.36, size = 19, normalized size = 0.59

$$\frac{2}{3}\left(\frac{1}{x^2}+1\right)^{\frac{3}{4}}+\frac{4}{\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^2/(x^2+1)/(x^4+x^2)^(1/4),x, algorithm="giac")
[Out] 2/3*(1/x^2 + 1)^(3/4) + 4/(1/x^2 + 1)^(1/4)
```

maple [A] time = 0.01, size = 22, normalized size = 0.69

$$\frac{\frac{14x^2}{3}+\frac{2}{3}}{x\left(x^4+x^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/x^2/(x^2+1)/(x^4+x^2)^(1/4),x)
[Out] 2/3*(7*x^2+1)/(x^4+x^2)^(1/4)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(8x^5+7\left(x^3+x\right)x^2+9x^3+x\right)}{21\left(x^{\frac{9}{2}}+x^{\frac{5}{2}}\right)\left(x^2+1\right)^{\frac{1}{4}}}+\int\frac{8\left(4x^4+x^2-3\right)}{21\left(x^{\frac{13}{2}}+2x^{\frac{9}{2}}+x^{\frac{5}{2}}\right)\left(x^2+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^2/(x^2+1)/(x^4+x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] -2/21*(8*x^5 + 7*(x^3 + x)*x^2 + 9*x^3 + x)/((x^(9/2) + x^(5/2))*(x^2 + 1)^
(1/4)) + integrate(8/21*(4*x^4 + x^2 - 3)/((x^(13/2) + 2*x^(9/2) + x^(5/2))
*(x^2 + 1)^(1/4)), x)
```

mupad [B] time = 0.22, size = 28, normalized size = 0.88

$$\frac{2\left(x^4+x^2\right)^{3/4}\left(7x^2+1\right)}{3x^3\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^2*(x^2 + x^4)^(1/4)*(x^2 + 1)),x)
```

```
[Out] (2*(x^2 + x^4)^(3/4)*(7*x^2 + 1))/(3*x^3*(x^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{x^2\sqrt[4]{x^2\left(x^2+1\right)}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/x**2/(x**2+1)/(x**4+x**2)**(1/4),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(x**2*(x**2*(x**2 + 1))**(1/4)*(x**2 + 1)), x)
```

3.392 $\int \frac{-1+2x}{\sqrt{-8x+9x^2-2x^3+x^4}} \, dx$

Optimal. Leaf size=32

$$-\log\left(-x^2+\sqrt{x^4-2x^3+9x^2-8x}+x-4\right)$$

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1680, 12, 1107, 621, 206}

$$\tanh^{-1}\left(\frac{4\left(x-\frac{1}{2}\right)^2+15}{\sqrt{16\left(x-\frac{1}{2}\right)^4+120\left(x-\frac{1}{2}\right)^2-31}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x)/Sqrt[-8*x + 9*x^2 - 2*x^3 + x^4],x]
```

```
[Out] ArcTanh[(15 + 4*(-1/2 + x)^2)/Sqrt[-31 + 120*(-1/2 + x)^2 + 16*(-1/2 + x)^4
]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{-1+2x}{\sqrt{-8x+9x^2-2x^3+x^4}}\,dx = \text{Subst}\left(\int \frac{8x}{\sqrt{-31+120x^2+16x^4}}\,dx, x, -\frac{1}{2}+x\right)$$
$$= 8\,\text{Subst}\left(\int \frac{x}{\sqrt{-31+120x^2+16x^4}}\,dx, x, -\frac{1}{2}+x\right)$$
$$= 4\,\text{Subst}\left(\int \frac{1}{\sqrt{-31+120x+16x^2}}\,dx, x, \left(-\frac{1}{2}+x\right)^2\right)$$
$$= 8\,\text{Subst}\left(\int \frac{1}{64-x^2}\,dx, x, \frac{2\left(15+4\left(-\frac{1}{2}+x\right)^2\right)}{\sqrt{x\left(-8+9x-2x^2+x^3\right)}}\right)$$
$$= \tanh^{-1}\left(\frac{15+(-1+2x)^2}{4\sqrt{-x\left(8-9x+2x^2-x^3\right)}}\right)$$

Mathematica [C] time = 0.54, size = 292, normalized size = 9.12

$$\frac{\sqrt{-\frac{i(x-1)}{\left(\sqrt{31}-15i\right)x}}\,x\left(64\sqrt{62}\sqrt{-\frac{16i}{x}+\sqrt{31}+i}\,x\sqrt{\frac{x^2-x+8}{x^2}}\,\Pi\left(\frac{2\sqrt{31}}{i+\sqrt{31}};\sin^{-1}\left(\frac{\sqrt{\sqrt{31}+i-\frac{16i}{x}}}{\sqrt{2}\sqrt[4]{31}}\right)|\frac{2\sqrt{31}}{-15i+\sqrt{31}}\right)+\sqrt{\frac{16i}{x}+\sqrt{31}-i}\left(\left(-31+15i\sqrt{31}\right)x+8i\sqrt{31}+248\right)F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{31}+i-\frac{16i}{x}}}{\sqrt{2}\sqrt[4]{31}}\right)|\frac{2\sqrt{31}}{-15i+\sqrt{31}}\right)\right)}{\sqrt{31}\left(\sqrt{31}+i\right)\sqrt{-\frac{16i}{x}+\sqrt{31}+i}\sqrt{x\left(x^3-2x^2+9x-8\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 2*x)/Sqrt[-8*x + 9*x^2 - 2*x^3 + x^4],x]
```

```
[Out] (Sqrt[((-I)*(-1 + x))/((-15*I + Sqrt[31])*x)]*x*(Sqrt[-I + Sqrt[31] + (16*I
)/x]*(248 + (8*I)*Sqrt[31] + (-31 + (15*I)*Sqrt[31])*x)*EllipticF[ArcSin[Sq
rt[I + Sqrt[31] - (16*I)/x]/(Sqrt[2]*31^(1/4))], (2*Sqrt[31])/(-15*I + Sqrt
[31])] + 64*Sqrt[62]*Sqrt[I + Sqrt[31] - (16*I)/x]*x*Sqrt[(8 - x + x^2)/x^2
]*EllipticPi[(2*Sqrt[31])/(I + Sqrt[31]), ArcSin[Sqrt[I + Sqrt[31] - (16*I)
/x]/(Sqrt[2]*31^(1/4))], (2*Sqrt[31])/(-15*I + Sqrt[31])]))/(Sqrt[31]*(I +
Sqrt[31])*Sqrt[I + Sqrt[31] - (16*I)/x]*Sqrt[x*(-8 + 9*x - 2*x^2 + x^3)])
```

IntegrateAlgebraic [A] time = 0.17, size = 32, normalized size = 1.00

$$-\log\left(-x^2+\sqrt{x^4-2x^3+9x^2-8x}+x-4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x)/Sqrt[-8*x + 9*x^2 - 2*x^3 + x^4],x]
```

```
[Out] -Log[-4 + x - x^2 + Sqrt[-8*x + 9*x^2 - 2*x^3 + x^4]]
```

fricas [A] time = 0.44, size = 30, normalized size = 0.94

$$\log\left(-x^2+x-\sqrt{x^4-2\,x^3+9\,x^2-8\,x}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+9*x^2-8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] log(-x^2 + x - sqrt(x^4 - 2*x^3 + 9*x^2 - 8*x) - 4)
```

giac [A] time = 0.73, size = 33, normalized size = 1.03

$$-\log\left(x^2-x-\sqrt{\left(x^2-x\right)^2+8\,x^2-8\,x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+9*x^2-8*x)^(1/2),x, algorithm="giac")
```

```
[Out] -log(x^2 - x - sqrt((x^2 - x)^2 + 8*x^2 - 8*x) + 4)
```

maple [C] time = 0.33, size = 487, normalized size = 15.22

$$-\frac{2\left(-\frac{1}{2}-\frac{i\sqrt{31}}{2}\right)\sqrt{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)(-1+x)}}\,(-1+x)^{2}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{31}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{31}}{2}\right)(-1+x)}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{31}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)(-1+x)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)(-1+x)}},\sqrt{\frac{\left(\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{31}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{31}}{2}\right)}}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)\sqrt{x\,(-1+x)\left(x-\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{31}}{2}\right)}}+\frac{4\left(-\frac{1}{2}-\frac{i\sqrt{31}}{2}\right)\sqrt{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)(-1+x)}}\,(-1+x)^{2}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{31}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{31}}{2}\right)(-1+x)}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{31}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)(-1+x)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)(-1+x)}},\sqrt{\frac{\left(\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{31}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{31}}{2}\right)}}\right)-\mathrm{EllipticPi}\left(\sqrt{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)(-1+x)}},\frac{\frac{1}{2}+\frac{i\sqrt{31}}{2}}{-\frac{1}{2}+\frac{i\sqrt{31}}{2}},\sqrt{\frac{\left(\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{31}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{31}}{2}\right)}}\right)\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)\sqrt{x\,(-1+x)\left(x-\frac{1}{2}+\frac{i\sqrt{31}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{31}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x)/(x^4-2*x^3+9*x^2-8*x)^(1/2),x)
```

```
[Out] -2*(-1/2-1/2*I*31^(1/2))*((-1/2+1/2*I*31^(1/2))*x/(1/2+1/2*I*31^(1/2))/(-1+
x))^(1/2)*(-1+x)^2*((x-1/2+1/2*I*31^(1/2))/(1/2-1/2*I*31^(1/2))/(-1+x))^(1/
2)*((x-1/2-1/2*I*31^(1/2))/(1/2+1/2*I*31^(1/2))/(-1+x))^(1/2)/(-1/2+1/2*I*3
1^(1/2))/(x*(-1+x)*(x-1/2+1/2*I*31^(1/2))*(x-1/2-1/2*I*31^(1/2)))^(1/2)*Ell
ipticF(((-1/2+1/2*I*31^(1/2))*x/(1/2+1/2*I*31^(1/2))/(-1+x))^(1/2),((1/2+1/
2*I*31^(1/2))*(-1/2-1/2*I*31^(1/2))/(-1/2+1/2*I*31^(1/2))/(1/2-1/2*I*31^(1/
2)))^(1/2))+4*(-1/2-1/2*I*31^(1/2))*((-1/2+1/2*I*31^(1/2))*x/(1/2+1/2*I*31^
(1/2))/(-1+x))^(1/2)*(-1+x)^2*((x-1/2+1/2*I*31^(1/2))/(1/2-1/2*I*31^(1/2))/
(-1+x))^(1/2)*((x-1/2-1/2*I*31^(1/2))/(1/2+1/2*I*31^(1/2))/(-1+x))^(1/2)/(-
1/2+1/2*I*31^(1/2))/(x*(-1+x)*(x-1/2+1/2*I*31^(1/2))*(x-1/2-1/2*I*31^(1/2))
)^(1/2)*(EllipticF(((-1/2+1/2*I*31^(1/2))*x/(1/2+1/2*I*31^(1/2))/(-1+x))^(1
/2),((1/2+1/2*I*31^(1/2))*(-1/2-1/2*I*31^(1/2))/(-1/2+1/2*I*31^(1/2))/(1/2-
1/2*I*31^(1/2)))^(1/2))-EllipticPi(((-1/2+1/2*I*31^(1/2))*x/(1/2+1/2*I*31^(
1/2))/(-1+x))^(1/2),(1/2+1/2*I*31^(1/2))/(-1/2+1/2*I*31^(1/2)),((1/2+1/2*I*
31^(1/2))*(-1/2-1/2*I*31^(1/2))/(-1/2+1/2*I*31^(1/2))/(1/2-1/2*I*31^(1/2)))
^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x-1}{\sqrt{x^4-2\,x^3+9\,x^2-8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+9*x^2-8*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x - 1)/sqrt(x^4 - 2*x^3 + 9*x^2 - 8*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{2\,x-1}{\sqrt{x^4-2\,x^3+9\,x^2-8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 1)/(9*x^2 - 8*x - 2*x^3 + x^4)^(1/2),x)
```

```
[Out] int((2*x - 1)/(9*x^2 - 8*x - 2*x^3 + x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-1}{\sqrt{x\,(x-1)\left(x^2-x+8\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x**4-2*x**3+9*x**2-8*x)**(1/2),x)
```

```
[Out] Integral((2*x - 1)/sqrt(x*(x - 1)*(x**2 - x + 8)), x)
```

3.393 $\int \frac{-a+2x}{\left(-1+b-ax+x^2\right)\sqrt[4]{b-ax+x^2}}\,dx$

Optimal. Leaf size=33

$$2\tan^{-1}\left(\sqrt[4]{-ax+b+x^2}\right)-2\tanh^{-1}\left(\sqrt[4]{-ax+b+x^2}\right)$$

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-a+2x}{\left(-1+b-ax+x^2\right)\sqrt[4]{b-ax+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-a + 2*x)/((-1 + b - a*x + x^2)*(b - a*x + x^2)^(1/4)),x]
[Out] Defer[Int][(-a + 2*x)/((-1 + b - a*x + x^2)*(b - a*x + x^2)^(1/4)), x]
```

Rubi steps

$$\int \frac{-a+2x}{\left(-1+b-ax+x^2\right)\sqrt[4]{b-ax+x^2}}\,dx=\int \frac{-a+2x}{\left(-1+b-ax+x^2\right)\sqrt[4]{b-ax+x^2}}\,dx$$

Mathematica [A] time = 0.27, size = 33, normalized size = 1.00

$$2\left(\tan^{-1}\left(\sqrt[4]{-ax+b+x^2}\right)-\tanh^{-1}\left(\sqrt[4]{-ax+b+x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + 2*x)/((-1 + b - a*x + x^2)*(b - a*x + x^2)^(1/4)),x]
[Out] 2*(ArcTan[(b - a*x + x^2)^(1/4)] - ArcTanh[(b - a*x + x^2)^(1/4)])
```

IntegrateAlgebraic [A] time = 0.03, size = 33, normalized size = 1.00

$$2\tan^{-1}\left(\sqrt[4]{-ax+b+x^2}\right)-2\tanh^{-1}\left(\sqrt[4]{-ax+b+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + 2*x)/((-1 + b - a*x + x^2)*(b - a*x + x^2)^(1/4)),
x]
[Out] 2*ArcTan[(b - a*x + x^2)^(1/4)] - 2*ArcTanh[(b - a*x + x^2)^(1/4)]
```

fricas [A] time = 0.40, size = 45, normalized size = 1.36

$$2\arctan\left(\left(-ax+x^2+b\right)^{\frac{1}{4}}\right)-\log\left(\left(-ax+x^2+b\right)^{\frac{1}{4}}+1\right)+\log\left(\left(-ax+x^2+b\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+2*x)/(-a*x+x^2+b-1)/(-a*x+x^2+b)^(1/4),x, algorithm="fricas")
[Out] 2*arctan((-a*x + x^2 + b)^(1/4)) - log((-a*x + x^2 + b)^(1/4) + 1) + log((-
a*x + x^2 + b)^(1/4) - 1)
```

giac [A] time = 0.42, size = 46, normalized size = 1.39

$$2 \arctan\left(\left(-ax + x^2 + b\right)^{\frac{1}{4}}\right) - \log\left(\left(-ax + x^2 + b\right)^{\frac{1}{4}} + 1\right) + \log\left(\left|\left(-ax + x^2 + b\right)^{\frac{1}{4}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+2*x)/(-a*x+x^2+b-1)/(-a*x+x^2+b)^(1/4),x, algorithm="giac")
[Out] 2*arctan((-a*x + x^2 + b)^(1/4)) - log((-a*x + x^2 + b)^(1/4) + 1) + log(ab
s((-a*x + x^2 + b)^(1/4) - 1))
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{-a + 2x}{\left(-ax + x^2 + b - 1\right)\left(-ax + x^2 + b\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+2*x)/(-a*x+x^2+b-1)/(-a*x+x^2+b)^(1/4),x)
[Out] int((-a+2*x)/(-a*x+x^2+b-1)/(-a*x+x^2+b)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a - 2\,x}{\left(ax - x^2 - b + 1\right)\left(-ax + x^2 + b\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+2*x)/(-a*x+x^2+b-1)/(-a*x+x^2+b)^(1/4),x, algorithm="maxima")
[Out] integrate((a - 2*x)/((a*x - x^2 - b + 1)*(-a*x + x^2 + b)^(1/4)), x)
```

mupad [B] time = 0.55, size = 29, normalized size = 0.88

$$2\,\mathrm{atan}\left(\left(x^2 - a\,x + b\right)^{1/4}\right) - 2\,\mathrm{atanh}\left(\left(x^2 - a\,x + b\right)^{1/4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a - 2*x)/((b - a*x + x^2)^(1/4)*(b - a*x + x^2 - 1)),x)
[Out] 2*atan((b - a*x + x^2)^(1/4)) - 2*atanh((b - a*x + x^2)^(1/4))
```

sympy [A] time = 12.22, size = 44, normalized size = 1.33

$$\log\left(\sqrt[4]{-ax + b + x^2} - 1\right) - \log\left(\sqrt[4]{-ax + b + x^2} + 1\right) + 2\,\mathrm{atan}\left(\sqrt[4]{-ax + b + x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+2*x)/(-a*x+x**2+b-1)/(-a*x+x**2+b)**(1/4),x)
[Out] log((-a*x + b + x**2)**(1/4) - 1) - log((-a*x + b + x**2)**(1/4) + 1) + 2*a
tan((-a*x + b + x**2)**(1/4))
```

3.394 $\int \frac{(-4+x^3)(-1+x^3)^{2/3}}{x^{12}}\,dx$

Optimal. Leaf size=33

$$\frac{(x^3-1)^{2/3}(-39x^9-26x^6-95x^3+160)}{440x^{11}}$$

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.48, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {453, 271, 264}

$$-\frac{4(x^3-1)^{5/3}}{11x^{11}}-\frac{13(x^3-1)^{5/3}}{88x^8}-\frac{39(x^3-1)^{5/3}}{440x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((-4 + x^3)*(-1 + x^3)^(2/3))/x^12,x]
```

```
[Out] (-4*(-1 + x^3)^(5/3))/(11*x^11) - (13*(-1 + x^3)^(5/3))/(88*x^8) - (39*(-1
+ x^3)^(5/3))/(440*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(-4+x^3)(-1+x^3)^{2/3}}{x^{12}}\,dx &= -\frac{4(-1+x^3)^{5/3}}{11x^{11}}-\frac{13}{11}\int \frac{(-1+x^3)^{2/3}}{x^9}\,dx\\ &= -\frac{4(-1+x^3)^{5/3}}{11x^{11}}-\frac{13(-1+x^3)^{5/3}}{88x^8}-\frac{39}{88}\int \frac{(-1+x^3)^{2/3}}{x^6}\,dx\\ &= -\frac{4(-1+x^3)^{5/3}}{11x^{11}}-\frac{13(-1+x^3)^{5/3}}{88x^8}-\frac{39(-1+x^3)^{5/3}}{440x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$-\frac{(x^3-1)^{2/3}(39x^9+26x^6+95x^3-160)}{440x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-4 + x^3)*(-1 + x^3)^(2/3))/x^12,x]
[Out] -1/440*((-1 + x^3)^(2/3)*(-160 + 95*x^3 + 26*x^6 + 39*x^9))/x^11
```

IntegrateAlgebraic [A]   time = 0.11, size = 33, normalized size = 1.00

$$\frac{(x^3-1)^{2/3}(-39x^9-26x^6-95x^3+160)}{440x^{11}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + x^3)*(-1 + x^3)^(2/3))/x^12,x]
[Out] ((-1 + x^3)^(2/3)*(160 - 95*x^3 - 26*x^6 - 39*x^9))/(440*x^11)
```

fricas [A]   time = 0.39, size = 29, normalized size = 0.88

$$-\frac{(39\,x^9+26\,x^6+95\,x^3-160)(x^3-1)^{\frac{2}{3}}}{440\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^3-1)^(2/3)/x^12,x, algorithm="fricas")
[Out] -1/440*(39*x^9 + 26*x^6 + 95*x^3 - 160)*(x^3 - 1)^(2/3)/x^11
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x^3-1)^{\frac{2}{3}}(x^3-4)}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^3-1)^(2/3)/x^12,x, algorithm="giac")
[Out] integrate((x^3 - 1)^(2/3)*(x^3 - 4)/x^12, x)
```

maple [A]   time = 0.01, size = 34, normalized size = 1.03

$$-\frac{(-1+x)(x^2+x+1)(39x^6+65x^3+160)(x^3-1)^{\frac{2}{3}}}{440x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-4)*(x^3-1)^(2/3)/x^12,x)
[Out] -1/440*(-1+x)*(x^2+x+1)*(39*x^6+65*x^3+160)*(x^3-1)^(2/3)/x^11
```

maxima [A]   time = 0.40, size = 37, normalized size = 1.12

$$-\frac{3\,(x^3-1)^{\frac{5}{3}}}{5\,x^5}+\frac{7\,(x^3-1)^{\frac{8}{3}}}{8\,x^8}-\frac{4\,(x^3-1)^{\frac{11}{3}}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^3-1)^(2/3)/x^12,x, algorithm="maxima")
[Out] -3/5*(x^3 - 1)^(5/3)/x^5 + 7/8*(x^3 - 1)^(8/3)/x^8 - 4/11*(x^3 - 1)^(11/3)/
x^11
```

mupad [B] time = 0.37, size = 49, normalized size = 1.48

$$\frac{4\left(x^3-1\right)^{2/3}}{11\,x^{11}}-\frac{13\left(x^3-1\right)^{2/3}}{220\,x^5}-\frac{19\left(x^3-1\right)^{2/3}}{88\,x^8}-\frac{39\left(x^3-1\right)^{2/3}}{440\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(x^3 - 4))/x^12,x)
```

```
[Out] (4*(x^3 - 1)^(2/3))/(11*x^11) - (13*(x^3 - 1)^(2/3))/(220*x^5) - (19*(x^3 -
 1)^(2/3))/(88*x^8) - (39*(x^3 - 1)^(2/3))/(440*x^2)
```

sympy [C] time = 3.34, size = 563, normalized size = 17.06

$$\begin{cases} \frac{\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}} e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{8}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}+\frac{2\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}} e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^3\Gamma\left(-\frac{2}{3}\right)}-\frac{5\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}} e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^6\Gamma\left(-\frac{2}{3}\right)} & \text{for}\ \frac{1}{|x^3|}>1 \\ \frac{3x^6\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^6\Gamma\left(-\frac{2}{3}\right)-9x^3\Gamma\left(-\frac{2}{3}\right)}-\frac{x^3\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^6\Gamma\left(-\frac{2}{3}\right)-9x^3\Gamma\left(-\frac{2}{3}\right)}+\frac{5\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^9\Gamma\left(-\frac{2}{3}\right)-9x^6\Gamma\left(-\frac{2}{3}\right)}-\frac{7\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^6\Gamma\left(-\frac{2}{3}\right)-9x^3\Gamma\left(-\frac{2}{3}\right)} & \text{otherwise} \end{cases} - 4\left(\begin{cases} \frac{2\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}} e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{11}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}+\frac{4\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}} e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{11}{3}\right)}{9x^3\Gamma\left(-\frac{2}{3}\right)}+\frac{10\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}} e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{11}{3}\right)}{27x^6\Gamma\left(-\frac{2}{3}\right)}-\frac{40\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}} e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{11}{3}\right)}{27x^9\Gamma\left(-\frac{2}{3}\right)} & \text{for}\ \frac{1}{|x^3|}>1 \\ -\frac{2\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{11}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}-\frac{4\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{11}{3}\right)}{9x^3\Gamma\left(-\frac{2}{3}\right)}-\frac{10\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{11}{3}\right)}{27x^6\Gamma\left(-\frac{2}{3}\right)}+\frac{40\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{11}{3}\right)}{27x^9\Gamma\left(-\frac{2}{3}\right)} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-4)*(x**3-1)**(2/3)/x**12,x)
```

```
[Out] Piecewise(((-1 + x**(-3))**(2/3)*exp(2*I*pi/3)*gamma(-8/3)/(3*gamma(-2/3))
+ 2*(-1 + x**(-3))**(2/3)*exp(2*I*pi/3)*gamma(-8/3)/(9*x**3*gamma(-2/3)) -
5*(-1 + x**(-3))**(2/3)*exp(2*I*pi/3)*gamma(-8/3)/(9*x**6*gamma(-2/3)), 1/A
bs(x**3) > 1), (3*x**6*(1 - 1/x**3)**(2/3)*gamma(-8/3)/(9*x**6*gamma(-2/3)
- 9*x**3*gamma(-2/3)) - x**3*(1 - 1/x**3)**(2/3)*gamma(-8/3)/(9*x**6*gamma(
-2/3) - 9*x**3*gamma(-2/3)) + 5*(1 - 1/x**3)**(2/3)*gamma(-8/3)/(9*x**9*gam
ma(-2/3) - 9*x**6*gamma(-2/3)) - 7*(1 - 1/x**3)**(2/3)*gamma(-8/3)/(9*x**6*
gamma(-2/3) - 9*x**3*gamma(-2/3)), True)) - 4*Piecewise((2*(-1 + x**(-3))**
(2/3)*exp(-I*pi/3)*gamma(-11/3)/(3*gamma(-2/3)) + 4*(-1 + x**(-3))**(2/3)*e
xp(-I*pi/3)*gamma(-11/3)/(9*x**3*gamma(-2/3)) + 10*(-1 + x**(-3))**(2/3)*ex
p(-I*pi/3)*gamma(-11/3)/(27*x**6*gamma(-2/3)) - 40*(-1 + x**(-3))**(2/3)*ex
p(-I*pi/3)*gamma(-11/3)/(27*x**9*gamma(-2/3)), 1/Abs(x**3) > 1), (-2*(1 - 1
/x**3)**(2/3)*gamma(-11/3)/(3*gamma(-2/3)) - 4*(1 - 1/x**3)**(2/3)*gamma(-1
1/3)/(9*x**3*gamma(-2/3)) - 10*(1 - 1/x**3)**(2/3)*gamma(-11/3)/(27*x**6*ga
mma(-2/3)) + 40*(1 - 1/x**3)**(2/3)*gamma(-11/3)/(27*x**9*gamma(-2/3)), Tru
e))
```

3.395 $\int \frac{-2+2x+x^2}{\left(-1-3x+x^2\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=33

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x^3+1}}{x^2-x+1}\right)$$

Rubi [A] time = 0.07, antiderivative size = 26, normalized size of antiderivative = 0.79, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2145, 206}

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 2*x + x^2)/((-1 - 3*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[(Sqrt[2]*(1 + x))/Sqrt[1 + x^3]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-2+2x+x^2}{\left(-1-3x+x^2\right)\sqrt{1+x^3}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{1}{2-4x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right)\\ &= -\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}(1+x)}{\sqrt{1+x^3}}\right)\end{aligned}$$

Mathematica [C] time = 0.64, size = 290, normalized size = 8.79

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1}\left(-\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}-\frac{6i\left(\left(-1+5\sqrt[3]{-1}+\sqrt{13}+\sqrt[3]{-1}\sqrt{13}\right)\Pi\left(\frac{2\sqrt{3}}{2i+\sqrt{3}+i\sqrt{13}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)-\left(1-5\sqrt[3]{-1}+\sqrt{13}+\sqrt[3]{-1}\sqrt{13}\right)\Pi\left(\frac{2i\sqrt{3}}{-3+2\sqrt[3]{-1}+\sqrt{13}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)\right)}{-12-4i\sqrt{3}}\right)}{3\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 2*x + x^2)/((-1 - 3*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*(-((Sqrt[3]*(1 + (-1)^(
1/3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(
1/3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x)) - ((6*I)*((-1 + 5*(-1)^(1/3) + Sq
rt[13] + (-1)^(1/3)*Sqrt[13])*EllipticPi[(2*Sqrt[3])/(2*I + Sqrt[3] + I*Sqr
t[13]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)] - (1
```

```
- 5*(-1)^(1/3) + Sqrt[13] + (-1)^(1/3)*Sqrt[13])*EllipticPi[((2*I)*Sqrt[3]
)/(-3 + 2*(-1)^(1/3) + Sqrt[13]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^
(1/3))]], (-1)^(1/3)]))/(-12 - (4*I)*Sqrt[3])))/(3*Sqrt[1 + x^3])
```

IntegrateAlgebraic [A] time = 1.15, size = 33, normalized size = 1.00

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x^3+1}}{x^2-x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + 2*x + x^2)/((-1 - 3*x + x^2)*Sqrt[1 + x^3]),x]
[Out] -(Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[1 + x^3])/(1 - x + x^2)])
```

fricas [B] time = 0.41, size = 63, normalized size = 1.91

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^4+10x^3-4\sqrt{2}\sqrt{x^3+1}\left(x^2+x+3\right)+7x^2+6x+17}{x^4-6x^3+7x^2+6x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-2)/(x^2-3*x-1)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((x^4 + 10*x^3 - 4*sqrt(2)*sqrt(x^3 + 1)*(x^2 + x + 3) + 7*x
^2 + 6*x + 17)/(x^4 - 6*x^3 + 7*x^2 + 6*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+2x-2}{\sqrt{x^3+1}\left(x^2-3x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-2)/(x^2-3*x-1)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 - 3*x - 1)), x)
```

maple [C] time = 0.06, size = 1625, normalized size = 49.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x-2)/(x^2-3*x-1)/(x^3+1)^(1/2),x)
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+3/2*13^(1/2)*(1/(3/2-1/2*I*3^(1/2
))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I
*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))
*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+
1)^(1/2)/(-5/2-1/2*13^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(
-3/2+1/2*I*3^(1/2))/(-5/2-1/2*13^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3
^(1/2)))^(1/2))-1/2*I*13^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2
)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/
2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/
2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-5/2-1/2*13^(1
/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-5/
2-1/2*13^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+
15/2*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3
```

```
^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1
/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-5/2-1/2*13^(1/2))*EllipticPi(((1+x)/(3/
2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-5/2-1/2*13^(1/2)),((-3/2+1/2
*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-5/2*I*(1/(3/2-1/2*I*3^(1/2))*x+1/(
3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2)
)-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(
-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)
/(-5/2-1/2*13^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2
*I*3^(1/2))/(-5/2-1/2*13^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2))*3^(1/2)-3/2*13^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2))
)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*
I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2)
)+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-5/2+1/2*13^(1/2
))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-5/2+
1/2*13^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+1/2*I*13^(
1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3
^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1
/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-5/2+1/2*13^(1/2))*EllipticPi(((1+x)/(3/
2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-5/2+1/2*13^(1/2)),((-3/2+1/2
*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+15/2*(1/(3/2-1/2*I*3^(1/2)
)*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*
3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*
x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1
)^(1/2)/(-5/2+1/2*13^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-
3/2+1/2*I*3^(1/2))/(-5/2+1/2*13^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2))-5/2*I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*
(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/
(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-5/2+1/2*13^(1/2))*Ellip
ticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-5/2+1/2*13^(
1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2x-2}{\sqrt{x^3+1}\left(x^2-3x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-2)/(x^2-3*x-1)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 - 3*x - 1)), x)
```

mupad [B] time = 0.31, size = 272, normalized size = 8.24

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{\sqrt{13}}{2}+\frac{5}{2}};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{\sqrt{13}}{2}-\frac{5}{2}};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + x^2 - 2)/((x^3 + 1)^(1/2)*(3*x - x^2 + 1)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(13^(1/2)/2 + 5/
2), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/(
(3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(-((3^(1/
```

```
2)*1i)/2 + 3/2)/(13^(1/2)/2 - 5/2), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(x^3 - x*(((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2x-2}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x^2-3x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x-2)/(x**2-3*x-1)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x**2 + 2*x - 2)/(sqrt((x + 1)*(x**2 - x + 1))*(x**2 - 3*x - 1)),
x)
```

3.396 $\int \frac{-2+2x+x^2}{\left(3-x+2x^2\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=33

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{x^3+1}}{\sqrt{2}\left(x^2-x+1\right)}\right)$$

Rubi [A] time = 0.08, antiderivative size = 26, normalized size of antiderivative = 0.79, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2145, 203}

$$-\sqrt{2}\tan^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 2*x + x^2)/((3 - x + 2*x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(1 + x)/(Sqrt[2]*Sqrt[1 + x^3])])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-2+2x+x^2}{\left(3-x+2x^2\right)\sqrt{1+x^3}}\,dx &= -\left(4\operatorname{Subst}\left(\int \frac{1}{4+2x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right)\\ &= -\sqrt{2}\tan^{-1}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x^3}}\right)\end{aligned}$$

Mathematica [C] time = 0.87, size = 515, normalized size = 15.61

$$\frac{\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(\frac{2i\sqrt{23}\sqrt{x^2-x+1}\,\Pi\left(-\frac{4\sqrt{3}}{i-2\sqrt{3}+\sqrt{23}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{i-2\sqrt{3}+\sqrt{23}}+\frac{10\sqrt{x^2-x+1}\,\Pi\left(-\frac{4\sqrt{3}}{i-2\sqrt{3}+\sqrt{23}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)x-2i}{-3i+\sqrt{3}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)}{i-2\sqrt{3}+\sqrt{23}}+\frac{10(-1)^{5/6}\left(1+\sqrt[3]{-1}\right)\sqrt{x^2-x+1}\,\Pi\left(\frac{4\sqrt{3}}{-i+2\sqrt{3}+\sqrt{23}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{6-i\sqrt{3}+\sqrt{69}}+\frac{2i\sqrt{23}\sqrt{x^2-x+1}\,\Pi\left(\frac{4\sqrt{3}}{-i+2\sqrt{3}+\sqrt{23}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{-i+2\sqrt{3}+\sqrt{23}}-\frac{2\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}\right)}{2\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-2 + 2*x + x^2)/((3 - x + 2*x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((-2*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (-
1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
 (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((
2*I)*Sqrt[23]*Sqrt[1 - x + x^2]*EllipticPi[(-4*Sqrt[3])/(I - 2*Sqrt[3] + Sq
rt[23]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(I
 - 2*Sqrt[3] + Sqrt[23]) + (10*Sqrt[1 - x + x^2]*EllipticPi[(-4*Sqrt[3])/(I
```

```
 - 2*Sqrt[3] + Sqrt[23]), ArcSin[Sqrt[(-2*I + (I + Sqrt[3])*x)/(-3*I + Sqrt
[3])]], (1 + I*Sqrt[3])/2])/(I - 2*Sqrt[3] + Sqrt[23]) + ((2*I)*Sqrt[23]*Sq
rt[1 - x + x^2]*EllipticPi[(4*Sqrt[3])/(-I + 2*Sqrt[3] + Sqrt[23]), ArcSin[
Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-I + 2*Sqrt[3] +
Sqrt[23]) + (10*(-1)^(5/6)*(1 + (-1)^(1/3))*Sqrt[1 - x + x^2]*EllipticPi[(4
*Sqrt[3])/(-I + 2*Sqrt[3] + Sqrt[23]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/(6 - I*Sqrt[3] + Sqrt[69])))/(2*Sqrt[1 + x^3])
```

IntegrateAlgebraic [A] time = 1.14, size = 33, normalized size = 1.00

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{x^3+1}}{\sqrt{2}\left(x^2-x+1\right)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + 2*x + x^2)/((3 - x + 2*x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] -(Sqrt[2]*ArcTan[Sqrt[1 + x^3]/(Sqrt[2]*(1 - x + x^2))])
```

fricas [A] time = 0.43, size = 28, normalized size = 0.85

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2x^2-3x+1\right)}{4\sqrt{x^3+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-2)/(2*x^2-x+3)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/4*sqrt(2)*(2*x^2 - 3*x + 1)/sqrt(x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2x-2}{\sqrt{x^3+1}\left(2x^2-x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-2)/(2*x^2-x+3)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(2*x^2 - x + 3)), x)
```

maple [C] time = 0.10, size = 432, normalized size = 13.09

$$\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{\left(\frac{5}{4}+\frac{i\sqrt{23}}{4}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{5}{12}+\frac{i\sqrt{23}}{12}\right)\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{5}{8}-\frac{i\sqrt{23}}{8}-\frac{i\sqrt{3}}{4}+\frac{i\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)\sqrt{3}}{6},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{\left(\frac{5}{4}-\frac{i\sqrt{23}}{4}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{5}{12}-\frac{i\sqrt{23}}{12}\right)\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{5}{8}+\frac{i\sqrt{23}}{8}-\frac{i\sqrt{3}}{4}+\frac{i\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)\sqrt{3}}{6},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x-2)/(2*x^2-x+3)/(x^3+1)^(1/2),x)
```

```
[Out] (3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2)
)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^
(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*
I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(5/4+1/4*I*23^(1/2))*(3/2-1/2*I*3^(
1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*
3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^
(1/2)*(-5/12+1/12*I*23^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),
5/8-1/8*I*23^(1/2)-1/4*I*3^(1/2)+1/6*I*(1/4+1/4*I*23^(1/2))*3^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(5/4-1/4*I*23^(1/2))*(3/2-1/2*I
*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/
2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3
+1)^(1/2)*(-5/12-1/12*I*23^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1
```

```
/2),5/8+1/8*I*23^(1/2)-1/4*I*3^(1/2)+1/6*I*(1/4-1/4*I*23^(1/2))*3^(1/2),((-
3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 2\,x - 2}{\sqrt{x^3 + 1}\left(2\,x^2 - x + 3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-2)/(2*x^2-x+3)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(2*x^2 - x + 3)), x)
```

mupad [B] time = 0.10, size = 274, normalized size = 8.30

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{5}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{5}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{2\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 - 2)/((x^3 + 1)^(1/2)*(2*x^2 - x + 3)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/((23^(1/2)*1i)/4
 + 5/4), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3
/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(-((
3^(1/2)*1i)/2 + 3/2)/((23^(1/2)*1i)/4 - 5/4), asin(((x + 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(2*(x^3
- x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 2x - 2}{\sqrt{(x+1)\left(x^2 - x + 1\right)}\left(2x^2 - x + 3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x-2)/(2*x**2-x+3)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x**2 + 2*x - 2)/(sqrt((x + 1)*(x**2 - x + 1))*(2*x**2 - x + 3)),
x)
```

3.397 $\int \frac{-2+2x+x^2}{(2-4x+3x^2)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=33

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^3+1}}{\sqrt{3}\,(x^2-x+1)}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.07, antiderivative size = 26, normalized size of antiderivative = 0.79, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2145, 206}

$$-\frac{2\tanh^{-1}\left(\frac{x+1}{\sqrt{3}\,\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 2*x + x^2)/((2 - 4*x + 3*x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(1 + x)/(Sqrt[3]*Sqrt[1 + x^3])])/Sqrt[3]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-2+2x+x^2}{\left(2-4x+3x^2\right)\sqrt{1+x^3}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{1}{6-2x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{1+x}{\sqrt{3}\,\sqrt{1+x^3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 1.45, size = 394, normalized size = 11.94

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\,\sqrt{x^2-x+1}\left(-\frac{\sqrt{3}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{2/3}x+1}+\frac{3i\sqrt{2}\,\Pi\left(\frac{6\sqrt{3}}{i-2\sqrt{2}+3\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{-2i+3(-1)^{5/6}+\sqrt{2}}+\frac{15\Pi\left(\frac{6\sqrt{3}}{i-2\sqrt{2}+3\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)x-2i}{-3i+\sqrt{3}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)}{-2i+3(-1)^{5/6}+\sqrt{2}}+\frac{6i\left(\sqrt{2}+5i\right)\Pi\left(\frac{6\sqrt{3}}{i+2\sqrt{2}+3\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)x-2i}{-3i+\sqrt{3}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)}{i+2\sqrt{2}+3\sqrt{3}}\right)}{9\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-2 + 2*x + x^2)/((2 - 4*x + 3*x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*(-((Sqrt[3]*(1 + (-1)^(
1/3))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(
1/3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x)) + ((3*I)*Sqrt[2]*EllipticPi[(6*Sq
```

```
rt[3])/(I - 2*Sqrt[2] + 3*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1
)^(1/3))]], (-1)^(1/3)])/(-2*I + 3*(-1)^(5/6) + Sqrt[2]) + (15*EllipticPi[(
6*Sqrt[3])/(I - 2*Sqrt[2] + 3*Sqrt[3]), ArcSin[Sqrt[(-2*I + (I + Sqrt[3])*x
)/(-3*I + Sqrt[3])]], (1 + I*Sqrt[3])/2])/(-2*I + 3*(-1)^(5/6) + Sqrt[2]) +
 ((6*I)*(5*I + Sqrt[2])*EllipticPi[(6*Sqrt[3])/(I + 2*Sqrt[2] + 3*Sqrt[3]),
 ArcSin[Sqrt[(-2*I + (I + Sqrt[3])*x)/(-3*I + Sqrt[3])]], (1 + I*Sqrt[3])/2
])/(I + 2*Sqrt[2] + 3*Sqrt[3])))/(9*Sqrt[1 + x^3])
```

IntegrateAlgebraic [A] time = 1.13, size = 33, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^3+1}}{\sqrt{3}\left(x^2-x+1\right)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + 2*x + x^2)/((2 - 4*x + 3*x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 + x^3]/(Sqrt[3]*(1 - x + x^2))])/Sqrt[3]
```

fricas [B] time = 0.44, size = 66, normalized size = 2.00

$$\frac{1}{6}\sqrt{3}\log\left(\frac{9x^4-4\sqrt{3}\sqrt{x^3+1}\left(3x^2-2x+4\right)+28x^2-16x+28}{9x^4-24x^3+28x^2-16x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-2)/(3*x^2-4*x+2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((9*x^4 - 4*sqrt(3)*sqrt(x^3 + 1)*(3*x^2 - 2*x + 4) + 28*x^2
 - 16*x + 28)/(9*x^4 - 24*x^3 + 28*x^2 - 16*x + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2x-2}{\sqrt{x^3+1}\left(3x^2-4x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-2)/(3*x^2-4*x+2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(3*x^2 - 4*x + 2)), x)
```

maple [C] time = 0.10, size = 435, normalized size = 13.18

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}+\frac{4\left(\frac{5}{6}+\frac{i\sqrt{2}}{6}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{5}{9}+\frac{i\sqrt{2}}{9}\right)\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{5}{6}-\frac{i\sqrt{2}}{6}-\frac{7i\sqrt{3}}{18}+\frac{i\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)\sqrt{3}}{6},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}+\frac{4\left(\frac{5}{6}-\frac{i\sqrt{2}}{6}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{5}{9}-\frac{i\sqrt{2}}{9}\right)\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{5}{6}+\frac{i\sqrt{2}}{6}-\frac{7i\sqrt{3}}{18}+\frac{i\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)\sqrt{3}}{6},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x-2)/(3*x^2-4*x+2)/(x^3+1)^(1/2),x)
```

```
[Out] 2/3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+4/3*(5/6+1/6*I*2^(1/2))*(3/2-1/
2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(
x^3+1)^(1/2)*(-5/9+1/9*I*2^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1
/2),5/6-1/6*I*2^(1/2)-7/18*I*3^(1/2)+1/6*I*(2/3+1/3*I*2^(1/2))*3^(1/2),((-3
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+4/3*(5/6-1/6*I*2^(1/2))*(3/2
-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-
```

```
3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2
)/(x^3+1)^(1/2)*(-5/9-1/9*I*2^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))
^(1/2),5/6+1/6*I*2^(1/2)-7/18*I*3^(1/2)+1/6*I*(2/3-1/3*I*2^(1/2))*3^(1/2),(
(-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 2\,x - 2}{\sqrt{x^3 + 1}\left(3\,x^2 - 4\,x + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-2)/(3*x^2-4*x+2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(3*x^2 - 4*x + 2)), x)
```

mupad [B] time = 0.21, size = 274, normalized size = 8.30

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{5}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{5}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 - 2)/((x^3 + 1)^(1/2)*(3*x^2 - 4*x + 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/((2^(1/2)*1i)/3
+ 5/3), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/
2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(-((3
^(1/2)*1i)/2 + 3/2)/((2^(1/2)*1i)/3 - 5/3), asin(((x + 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(x^3 -
x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 2x - 2}{\sqrt{(x+1)\left(x^2 - x + 1\right)}\left(3x^2 - 4x + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x-2)/(3*x**2-4*x+2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x**2 + 2*x - 2)/(sqrt((x + 1)*(x**2 - x + 1))*(3*x**2 - 4*x + 2))
, x)
```

3.398 $\int \frac{(-1+x^3)\sqrt[3]{1+x^3}}{x^{11}}\,dx$

Optimal. Leaf size=33

$$\frac{\sqrt[3]{x^3+1}\,(12x^9-4x^6-9x^3+7)}{70x^{10}}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.48, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {453, 271, 264}

$$\frac{(x^3+1)^{4/3}}{10x^{10}}-\frac{8(x^3+1)^{4/3}}{35x^7}+\frac{6(x^3+1)^{4/3}}{35x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)*(1 + x^3)^(1/3))/x^11,x]
```

```
[Out] (1 + x^3)^(4/3)/(10*x^10) - (8*(1 + x^3)^(4/3))/(35*x^7) + (6*(1 + x^3)^(4/
3))/(35*x^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^3)\sqrt[3]{1+x^3}}{x^{11}}\,dx &= \frac{(1+x^3)^{4/3}}{10x^{10}}+\frac{8}{5}\int \frac{\sqrt[3]{1+x^3}}{x^8}\,dx\\
&= \frac{(1+x^3)^{4/3}}{10x^{10}}-\frac{8(1+x^3)^{4/3}}{35x^7}-\frac{24}{35}\int \frac{\sqrt[3]{1+x^3}}{x^5}\,dx\\
&= \frac{(1+x^3)^{4/3}}{10x^{10}}-\frac{8(1+x^3)^{4/3}}{35x^7}+\frac{6(1+x^3)^{4/3}}{35x^4}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.85

$$\frac{(x^3+1)^{4/3}(12x^6-16x^3+7)}{70x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)*(1 + x^3)^(1/3))/x^11,x]
```

```
[Out] ((1 + x^3)^(4/3)*(7 - 16*x^3 + 12*x^6))/(70*x^10)
```

IntegrateAlgebraic [A] time = 0.11, size = 28, normalized size = 0.85

$$\frac{(x^3+1)^{4/3}(12x^6-16x^3+7)}{70x^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)*(1 + x^3)^(1/3))/x^11,x]
```

```
[Out] ((1 + x^3)^(4/3)*(7 - 16*x^3 + 12*x^6))/(70*x^10)
```

fricas [A] time = 0.39, size = 29, normalized size = 0.88

$$\frac{\left(12\,x^9-4\,x^6-9\,x^3+7\right)\left(x^3+1\right)^{\frac{1}{3}}}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x^11,x, algorithm="fricas")
```

```
[Out] 1/70*(12*x^9 - 4*x^6 - 9*x^3 + 7)*(x^3 + 1)^(1/3)/x^10
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+1\right)^{\frac{1}{3}}\left(x^3-1\right)}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x^11,x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)^(1/3)*(x^3 - 1)/x^11, x)
```

maple [A] time = 0.01, size = 36, normalized size = 1.09

$$\frac{\left(x^2-x+1\right)(1+x)\left(12x^6-16x^3+7\right)\left(x^3+1\right)^{\frac{1}{3}}}{70x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)*(x^3+1)^(1/3)/x^11,x)
```

```
[Out] 1/70*(x^2-x+1)*(1+x)*(12*x^6-16*x^3+7)*(x^3+1)^(1/3)/x^10
```

maxima [A] time = 0.34, size = 37, normalized size = 1.12

$$\frac{\left(x^3+1\right)^{\frac{4}{3}}}{2\,x^4}-\frac{3\left(x^3+1\right)^{\frac{7}{3}}}{7\,x^7}+\frac{\left(x^3+1\right)^{\frac{10}{3}}}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*(x^3 + 1)^(4/3)/x^4 - 3/7*(x^3 + 1)^(7/3)/x^7 + 1/10*(x^3 + 1)^(10/3)/x
^10
```

mupad [B] time = 0.33, size = 49, normalized size = 1.48

$$\frac{6\left(x^3+1\right)^{1/3}}{35\,x}-\frac{2\left(x^3+1\right)^{1/3}}{35\,x^4}-\frac{9\left(x^3+1\right)^{1/3}}{70\,x^7}+\frac{\left(x^3+1\right)^{1/3}}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)*(x^3 + 1)^(1/3))/x^11,x)
```

```
[Out] (6*(x^3 + 1)^(1/3))/(35*x) - (2*(x^3 + 1)^(1/3))/(35*x^4) - (9*(x^3 + 1)^(1
/3))/(70*x^7) + (x^3 + 1)^(1/3)/(10*x^10)
```

sympy [B] time = 2.70, size = 199, normalized size = 6.03

$$-\frac{2\sqrt[3]{1+\frac{1}{x^3}}\,\Gamma\left(-\frac{10}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)}+\frac{\sqrt[3]{x^3+1}\,\Gamma\left(-\frac{7}{3}\right)}{3x\Gamma\left(-\frac{1}{3}\right)}+\frac{2\sqrt[3]{1+\frac{1}{x^3}}\,\Gamma\left(-\frac{10}{3}\right)}{9x^3\Gamma\left(-\frac{1}{3}\right)}-\frac{\sqrt[3]{x^3+1}\,\Gamma\left(-\frac{7}{3}\right)}{9x^4\Gamma\left(-\frac{1}{3}\right)}-\frac{4\sqrt[3]{1+\frac{1}{x^3}}\,\Gamma\left(-\frac{10}{3}\right)}{27x^6\Gamma\left(-\frac{1}{3}\right)}-\frac{4\sqrt[3]{x^3+1}\,\Gamma\left(-\frac{7}{3}\right)}{9x^7\Gamma\left(-\frac{1}{3}\right)}-\frac{28\sqrt[3]{1+\frac{1}{x^3}}\,\Gamma\left(-\frac{10}{3}\right)}{27x^9\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)*(x**3+1)**(1/3)/x**11,x)
```

```
[Out] -2*(1 + x**(-3))**(1/3)*gamma(-10/3)/(3*gamma(-1/3)) + (x**3 + 1)**(1/3)*ga
mma(-7/3)/(3*x*gamma(-1/3)) + 2*(1 + x**(-3))**(1/3)*gamma(-10/3)/(9*x**3*g
amma(-1/3)) - (x**3 + 1)**(1/3)*gamma(-7/3)/(9*x**4*gamma(-1/3)) - 4*(1 + x
**(-3))**(1/3)*gamma(-10/3)/(27*x**6*gamma(-1/3)) - 4*(x**3 + 1)**(1/3)*gam
ma(-7/3)/(9*x**7*gamma(-1/3)) - 28*(1 + x**(-3))**(1/3)*gamma(-10/3)/(27*x*
*9*gamma(-1/3))
```

3.399 $\int \frac{\sqrt[3]{-1+x^3}\left(1+x^3\right)}{x^{11}}\,dx$

Optimal. Leaf size=33

$$\frac{\sqrt[3]{x^3-1}\left(12x^9+4x^6-9x^3-7\right)}{70x^{10}}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.48, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {453, 271, 264}

$$\frac{\left(x^3-1\right)^{4/3}}{10x^{10}}+\frac{8\left(x^3-1\right)^{4/3}}{35x^7}+\frac{6\left(x^3-1\right)^{4/3}}{35x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(1/3)*(1 + x^3))/x^11,x]
```

```
[Out] (-1 + x^3)^(4/3)/(10*x^10) + (8*(-1 + x^3)^(4/3))/(35*x^7) + (6*(-1 + x^3)^
(4/3))/(35*x^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{-1+x^3}\left(1+x^3\right)}{x^{11}}\,dx &= \frac{\left(-1+x^3\right)^{4/3}}{10x^{10}}+\frac{8}{5}\int \frac{\sqrt[3]{-1+x^3}}{x^8}\,dx\\
&= \frac{\left(-1+x^3\right)^{4/3}}{10x^{10}}+\frac{8\left(-1+x^3\right)^{4/3}}{35x^7}+\frac{24}{35}\int \frac{\sqrt[3]{-1+x^3}}{x^5}\,dx\\
&= \frac{\left(-1+x^3\right)^{4/3}}{10x^{10}}+\frac{8\left(-1+x^3\right)^{4/3}}{35x^7}+\frac{6\left(-1+x^3\right)^{4/3}}{35x^4}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{\sqrt[3]{x^3-1}\left(12x^9+4x^6-9x^3-7\right)}{70x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)^(1/3)*(1 + x^3))/x^11,x]
[Out] ((-1 + x^3)^(1/3)*(-7 - 9*x^3 + 4*x^6 + 12*x^9))/(70*x^10)
```

IntegrateAlgebraic [A] time = 0.10, size = 28, normalized size = 0.85

$$\frac{(x^3-1)^{4/3}(12x^6+16x^3+7)}{70x^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(1/3)*(1 + x^3))/x^11,x]
[Out] ((-1 + x^3)^(4/3)*(7 + 16*x^3 + 12*x^6))/(70*x^10)
```

fricas [A] time = 0.40, size = 29, normalized size = 0.88

$$\frac{(12\,x^9+4\,x^6-9\,x^3-7)(x^3-1)^{\frac{1}{3}}}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(x^3+1)/x^11,x, algorithm="fricas")
[Out] 1/70*(12*x^9 + 4*x^6 - 9*x^3 - 7)*(x^3 - 1)^(1/3)/x^10
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x^3+1)(x^3-1)^{\frac{1}{3}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(x^3+1)/x^11,x, algorithm="giac")
[Out] integrate((x^3 + 1)*(x^3 - 1)^(1/3)/x^11, x)
```

maple [A] time = 0.00, size = 34, normalized size = 1.03

$$\frac{(x^3-1)^{\frac{1}{3}}(12x^6+16x^3+7)(-1+x)(x^2+x+1)}{70x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/3)*(x^3+1)/x^11,x)
[Out] 1/70*(x^3-1)^(1/3)*(12*x^6+16*x^3+7)*(-1+x)*(x^2+x+1)/x^10
```

maxima [A] time = 0.35, size = 37, normalized size = 1.12

$$\frac{(x^3-1)^{\frac{4}{3}}}{2\,x^4}-\frac{3\,(x^3-1)^{\frac{7}{3}}}{7\,x^7}+\frac{(x^3-1)^{\frac{10}{3}}}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(x^3+1)/x^11,x, algorithm="maxima")
[Out] 1/2*(x^3 - 1)^(4/3)/x^4 - 3/7*(x^3 - 1)^(7/3)/x^7 + 1/10*(x^3 - 1)^(10/3)/x
^10
```

mupad [B] time = 0.32, size = 49, normalized size = 1.48

$$\frac{6\left(x^3-1\right)^{1/3}}{35\,x}+\frac{2\left(x^3-1\right)^{1/3}}{35\,x^4}-\frac{9\left(x^3-1\right)^{1/3}}{70\,x^7}-\frac{\left(x^3-1\right)^{1/3}}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(1/3)*(x^3 + 1))/x^11,x)
```

```
[Out] (6*(x^3 - 1)^(1/3))/(35*x) + (2*(x^3 - 1)^(1/3))/(35*x^4) - (9*(x^3 - 1)^(1
/3))/(70*x^7) - (x^3 - 1)^(1/3)/(10*x^10)
```

sympy [C] time = 2.82, size = 563, normalized size = 17.06

$$\begin{cases} \frac{\sqrt[3]{-1+\frac{1}{x^3}}\, e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)}+\frac{\sqrt[3]{-1+\frac{1}{x^3}}\, e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{9x^3\Gamma\left(-\frac{1}{3}\right)}-\frac{4\sqrt[3]{-1+\frac{1}{x^3}}\, e^{\frac{i\pi}{3}}\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)} & \text{for}\ \frac{1}{|x^3|}>1 \\ \frac{3x^6\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)-9x^3\Gamma\left(-\frac{1}{3}\right)}-\frac{2x^3\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)-9x^3\Gamma\left(-\frac{1}{3}\right)}+\frac{4\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{7}{3}\right)}{9x^9\Gamma\left(-\frac{1}{3}\right)-9x^6\Gamma\left(-\frac{1}{3}\right)}-\frac{5\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)-9x^3\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise} \end{cases} + \begin{cases} \frac{2\sqrt[3]{-1+\frac{1}{x^3}}\, e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{10}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)}+\frac{2\sqrt[3]{-1+\frac{1}{x^3}}\, e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{10}{3}\right)}{9x^3\Gamma\left(-\frac{1}{3}\right)}+\frac{4\sqrt[3]{-1+\frac{1}{x^3}}\, e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{10}{3}\right)}{27x^6\Gamma\left(-\frac{1}{3}\right)}-\frac{28\sqrt[3]{-1+\frac{1}{x^3}}\, e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{10}{3}\right)}{27x^9\Gamma\left(-\frac{1}{3}\right)} & \text{for}\ \frac{1}{|x^3|}>1 \\ -\frac{2\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{10}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)}-\frac{2\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{10}{3}\right)}{9x^3\Gamma\left(-\frac{1}{3}\right)}-\frac{4\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{10}{3}\right)}{27x^6\Gamma\left(-\frac{1}{3}\right)}+\frac{28\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{10}{3}\right)}{27x^9\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/3)*(x**3+1)/x**11,x)
```

```
[Out] Piecewise(((-1 + x**(-3))**(1/3)*exp(I*pi/3)*gamma(-7/3)/(3*gamma(-1/3)) +
(-1 + x**(-3))**(1/3)*exp(I*pi/3)*gamma(-7/3)/(9*x**3*gamma(-1/3)) - 4*(-1
+ x**(-3))**(1/3)*exp(I*pi/3)*gamma(-7/3)/(9*x**6*gamma(-1/3)), 1/Abs(x**3)
 > 1), (3*x**6*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**6*gamma(-1/3) - 9*x**3
*gamma(-1/3)) - 2*x**3*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**6*gamma(-1/3)
- 9*x**3*gamma(-1/3)) + 4*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**9*gamma(-1/
3) - 9*x**6*gamma(-1/3)) - 5*(1 - 1/x**3)**(1/3)*gamma(-7/3)/(9*x**6*gamma(
-1/3) - 9*x**3*gamma(-1/3)), True)) + Piecewise((2*(-1 + x**(-3))**(1/3)*ex
p(-2*I*pi/3)*gamma(-10/3)/(3*gamma(-1/3)) + 2*(-1 + x**(-3))**(1/3)*exp(-2*
I*pi/3)*gamma(-10/3)/(9*x**3*gamma(-1/3)) + 4*(-1 + x**(-3))**(1/3)*exp(-2*
I*pi/3)*gamma(-10/3)/(27*x**6*gamma(-1/3)) - 28*(-1 + x**(-3))**(1/3)*exp(-
2*I*pi/3)*gamma(-10/3)/(27*x**9*gamma(-1/3)), 1/Abs(x**3) > 1), (-2*(1 - 1/
x**3)**(1/3)*gamma(-10/3)/(3*gamma(-1/3)) - 2*(1 - 1/x**3)**(1/3)*gamma(-10
/3)/(9*x**3*gamma(-1/3)) - 4*(1 - 1/x**3)**(1/3)*gamma(-10/3)/(27*x**6*gamm
a(-1/3)) + 28*(1 - 1/x**3)**(1/3)*gamma(-10/3)/(27*x**9*gamma(-1/3)), True)
)
```

3.400 $\int \frac{-1-2x+x^2+3x^3}{\sqrt[4]{-1+3x-3x^2+x^3}}\,dx$

Optimal. Leaf size=33

$$\frac{4\left((x-1)^3\right)^{3/4}\left(135x^3+245x^2+158x+47\right)}{585(x-1)^2}$$

Rubi [B] time = 0.19, antiderivative size = 70, normalized size of antiderivative = 2.12, number of steps used = 17, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {6742, 2067, 15, 30, 2081, 43}

$$\frac{12(1-x)^4}{13\sqrt[4]{(x-1)^3}}+\frac{36(1-x)^2}{5\sqrt[4]{(x-1)^3}}-\frac{4(1-x)}{\sqrt[4]{(x-1)^3}}+\frac{40}{9}\left((x-1)^3\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 2*x + x^2 + 3*x^3)/(-1 + 3*x - 3*x^2 + x^3)^(1/4),x]
```

```
[Out] (-4*(1 - x))/((-1 + x)^3)^(1/4) + (36*(1 - x)^2)/(5*((-1 + x)^3)^(1/4)) + (
12*(1 - x)^4)/(13*((-1 + x)^3)^(1/4)) + (40*((-1 + x)^3)^(3/4))/9
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 2081

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-1-2x+x^2+3x^3}{\sqrt[4]{-1+3x-3x^2+x^3}}\,dx = \int \left(-\frac{1}{\sqrt[4]{-1+3x-3x^2+x^3}} - \frac{2x}{\sqrt[4]{-1+3x-3x^2+x^3}} + \frac{x^2}{\sqrt[4]{-1+3x-3x^2+x^3}} + \frac{}{\sqrt[4]{}}\right.$$
$$= -\left(2\int \frac{x}{\sqrt[4]{-1+3x-3x^2+x^3}}\,dx\right) + 3\int \frac{x^3}{\sqrt[4]{-1+3x-3x^2+x^3}}\,dx - \int \frac{}{\sqrt[4]{-1+3x}}$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1+x}{\sqrt[4]{x^3}}\,dx, x, -1+x\right)\right) + 3\,\text{Subst}\left(\int \frac{(1+x)^3}{\sqrt[4]{x^3}}\,dx, x, -1+x\right) - \text{Subst}$$
$$= -\frac{(-1+x)^{3/4}\,\text{Subst}\left(\int \frac{1}{x^{3/4}}\,dx, x, -1+x\right)}{\sqrt[4]{(-1+x)^3}} + \frac{(-1+x)^{3/4}\,\text{Subst}\left(\int \frac{(1+x)^2}{x^{3/4}}\,dx, x, -1+x\right)}{\sqrt[4]{(-1+x)^3}}$$
$$= \frac{4(1-x)}{\sqrt[4]{(-1+x)^3}} + \frac{(-1+x)^{3/4}\,\text{Subst}\left(\int \left(\frac{1}{x^{3/4}} + 2\sqrt[4]{x} + x^{5/4}\right)dx, x, -1+x\right)}{\sqrt[4]{(-1+x)^3}} - \frac{(2(-1+}{}$$
$$= -\frac{4(1-x)}{\sqrt[4]{(-1+x)^3}} + \frac{36(1-x)^2}{5\sqrt[4]{(-1+x)^3}} + \frac{12(1-x)^4}{13\sqrt[4]{(-1+x)^3}} + \frac{40}{9}\left((-1+x)^3\right)^{3/4}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.94

$$\frac{4(x-1)\left(135x^3+245x^2+158x+47\right)}{585\sqrt[4]{(x-1)^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x + x^2 + 3*x^3)/(-1 + 3*x - 3*x^2 + x^3)^(1/4),x]
```

```
[Out] (4*(-1 + x)*(47 + 158*x + 245*x^2 + 135*x^3))/(585*((-1 + x)^3)^(1/4))
```

IntegrateAlgebraic [A] time = 5.10, size = 57, normalized size = 1.73

$$\frac{4\left(135(x-1)^{13/4}+650(x-1)^{9/4}+1053(x-1)^{5/4}+585\sqrt[4]{x-1}\right)\left((x-1)^3\right)^{3/4}}{585(x-1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 2*x + x^2 + 3*x^3)/(-1 + 3*x - 3*x^2 + x^3)^(1/4),
x]
```

```
[Out] (4*(585*(-1 + x)^(1/4) + 1053*(-1 + x)^(5/4) + 650*(-1 + x)^(9/4) + 135*(-1
 + x)^(13/4))*((-1 + x)^3)^(3/4))/(585*(-1 + x)^(9/4))
```

fricas [A] time = 0.38, size = 42, normalized size = 1.27

$$\frac{4\left(135\,x^3+245\,x^2+158\,x+47\right)\left(x^3-3\,x^2+3\,x-1\right)^{\frac{3}{4}}}{585\left(x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3+x^2-2*x-1)/(x^3-3*x^2+3*x-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/585*(135*x^3 + 245*x^2 + 158*x + 47)*(x^3 - 3*x^2 + 3*x - 1)^(3/4)/(x^2 -
 2*x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^3+x^2-2\,x-1}{\left(x^3-3\,x^2+3\,x-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3+x^2-2*x-1)/(x^3-3*x^2+3*x-1)^(1/4),x, algorithm="giac")
[Out] integrate((3*x^3 + x^2 - 2*x - 1)/(x^3 - 3*x^2 + 3*x - 1)^(1/4), x)
```

maple [A] time = 0.00, size = 36, normalized size = 1.09

$$\frac{4\,(-1+x)\left(135x^3+245x^2+158x+47\right)}{585\left(x^3-3x^2+3x-1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3+x^2-2*x-1)/(x^3-3*x^2+3*x-1)^(1/4),x)
[Out] 4/585*(-1+x)*(135*x^3+245*x^2+158*x+47)/(x^3-3*x^2+3*x-1)^(1/4)
```

maxima [B] time = 0.34, size = 72, normalized size = 2.18

$$\frac{4\left(15\,x^4+5\,x^3+12\,x^2+96\,x-128\right)}{65\,(x-1)^{\frac{3}{4}}}+\frac{4\left(5\,x^3+3\,x^2+24\,x-32\right)}{45\,(x-1)^{\frac{3}{4}}}-\frac{8\left(x^2+3\,x-4\right)}{5\,(x-1)^{\frac{3}{4}}}-4\,(x-1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3+x^2-2*x-1)/(x^3-3*x^2+3*x-1)^(1/4),x, algorithm="maxima")
[Out] 4/65*(15*x^4 + 5*x^3 + 12*x^2 + 96*x - 128)/(x - 1)^(3/4) + 4/45*(5*x^3 + 3
*x^2 + 24*x - 32)/(x - 1)^(3/4) - 8/5*(x^2 + 3*x - 4)/(x - 1)^(3/4) - 4*(x
- 1)^(1/4)
```

mupad [B] time = 0.26, size = 41, normalized size = 1.24

$$\frac{\left(x^3-3\,x^2+3\,x-1\right)^{3/4}\,\left(\frac{12\,x^3}{13}+\frac{196\,x^2}{117}+\frac{632\,x}{585}+\frac{188}{585}\right)}{x^2-2\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - x^2 - 3*x^3 + 1)/(3*x - 3*x^2 + x^3 - 1)^(1/4),x)
[Out] ((3*x - 3*x^2 + x^3 - 1)^(3/4)*((632*x)/585 + (196*x^2)/117 + (12*x^3)/13 +
 188/585))/(x^2 - 2*x + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^3+x^2-2x-1}{\sqrt[4]{(x-1)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3+x**2-2*x-1)/(x**3-3*x**2+3*x-1)**(1/4),x)
[Out] Integral((3*x**3 + x**2 - 2*x - 1)/((x - 1)**3)**(1/4), x)
```

3.401 $\int \sqrt[4]{-1+3x-3x^2+x^3}\,\left(-1-2x+x^2+3x^3\right)\,dx$

Optimal. Leaf size=33

$$\frac{4\sqrt[4]{(x-1)^3}\,\left(693x^4+154x^3-1029x^2-549x+731\right)}{4389}$$

Rubi [B] time = 0.19, antiderivative size = 72, normalized size of antiderivative = 2.18, number of steps used = 17, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {6742, 2067, 15, 30, 2081, 43}

$$\frac{12}{19}\sqrt[4]{(x-1)^3}\,(1-x)^4+\frac{36}{11}\sqrt[4]{(x-1)^3}\,(1-x)^2-\frac{4}{7}\sqrt[4]{(x-1)^3}\,(1-x)+\frac{8}{3}\left((x-1)^3\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x - 3*x^2 + x^3)^(1/4)*(-1 - 2*x + x^2 + 3*x^3),x]
```

```
[Out] (-4*(1 - x)*((-1 + x)^3)^(1/4))/7 + (36*(1 - x)^2*((-1 + x)^3)^(1/4))/11 +
(12*(1 - x)^4*((-1 + x)^3)^(1/4))/19 + (8*((-1 + x)^3)^(5/4))/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 2081

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \sqrt[4]{-1+3x-3x^2+x^3}\left(-1-2x+x^2+3x^3\right)dx = \int \left(-\sqrt[4]{-1+3x-3x^2+x^3} - 2x\sqrt[4]{-1+3x-3x^2+x^3} + x\right.$$
$$= -\left(2\int x\sqrt[4]{-1+3x-3x^2+x^3}\,dx\right) + 3\int x^3\sqrt[4]{-1+3x-3$$
$$= -\left(2\,\text{Subst}\left(\int \sqrt[4]{x^3}\,(1+x)\,dx, x, -1+x\right)\right) + 3\,\text{Subst}\left(\int \sqrt[4]{}\right.$$
$$= -\frac{\sqrt[4]{(-1+x)^3}\,\text{Subst}\left(\int x^{3/4}\,dx, x, -1+x\right)}{(-1+x)^{3/4}} + \frac{\sqrt[4]{(-1+x)^3}\,\text{S}}{}$$
$$= \frac{4}{7}(1-x)\sqrt[4]{(-1+x)^3} + \frac{\sqrt[4]{(-1+x)^3}\,\text{Subst}\left(\int \left(x^{3/4}+2x^{7/4}\right.\right.}{(-1+x)^{3/4}}$$
$$= -\frac{4}{7}(1-x)\sqrt[4]{(-1+x)^3} + \frac{36}{11}(1-x)^2\sqrt[4]{(-1+x)^3} + \frac{12}{19}(1-x$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.94

$$\frac{4(x-1)\sqrt[4]{(x-1)^3}\left(693x^3+847x^2-182x-731\right)}{4389}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x - 3*x^2 + x^3)^(1/4)*(-1 - 2*x + x^2 + 3*x^3),x]
[Out] (4*(-1 + x)*((-1 + x)^3)^(1/4)*(-731 - 182*x + 847*x^2 + 693*x^3))/4389
```

IntegrateAlgebraic [A] time = 5.27, size = 57, normalized size = 1.73

$$\frac{4\left(693(x-1)^{19/4}+2926(x-1)^{15/4}+3591(x-1)^{11/4}+627(x-1)^{7/4}\right)\sqrt[4]{(x-1)^3}}{4389(x-1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 3*x - 3*x^2 + x^3)^(1/4)*(-1 - 2*x + x^2 + 3*x^3),
x]
[Out] (4*(627*(-1 + x)^(7/4) + 3591*(-1 + x)^(11/4) + 2926*(-1 + x)^(15/4) + 693*
(-1 + x)^(19/4))*((-1 + x)^3)^(1/4))/(4389*(-1 + x)^(3/4))
```

fricas [A] time = 0.38, size = 37, normalized size = 1.12

$$\frac{4}{4389}\left(693\,x^4+154\,x^3-1029\,x^2-549\,x+731\right)\left(x^3-3\,x^2+3\,x-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)^(1/4)*(3*x^3+x^2-2*x-1),x, algorithm="fricas")
[Out] 4/4389*(693*x^4 + 154*x^3 - 1029*x^2 - 549*x + 731)*(x^3 - 3*x^2 + 3*x - 1)
^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3\,x^3+x^2-2\,x-1\right)\left(x^3-3\,x^2+3\,x-1\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)^(1/4)*(3*x^3+x^2-2*x-1),x, algorithm="giac")
```

```
[Out] integrate((3*x^3 + x^2 - 2*x - 1)*(x^3 - 3*x^2 + 3*x - 1)^(1/4), x)
```

maple [A] time = 0.00, size = 36, normalized size = 1.09

$$\frac{4\left(-1+x\right)\left(693x^{3}+847x^{2}-182x-731\right)\left(x^{3}-3x^{2}+3x-1\right)^{\frac{1}{4}}}{4389}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3*x^2+3*x-1)^(1/4)*(3*x^3+x^2-2*x-1),x)
```

```
[Out] 4/4389*(-1+x)*(693*x^3+847*x^2-182*x-731)*(x^3-3*x^2+3*x-1)^(1/4)
```

maxima [B] time = 0.34, size = 74, normalized size = 2.24

$$\frac{12}{7315}\left(385\,x^{4}-77\,x^{3}-84\,x^{2}-96\,x-128\right)(x-1)^{\frac{3}{4}}+\frac{4}{1155}\left(77\,x^{3}-21\,x^{2}-24\,x-32\right)(x-1)^{\frac{3}{4}}-\frac{8}{77}\left(7\,x^{2}-3\,x-4\right)(x-1)^{\frac{3}{4}}-\frac{4}{7}\,(x-1)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)^(1/4)*(3*x^3+x^2-2*x-1),x, algorithm="maxima")
```

```
[Out] 12/7315*(385*x^4 - 77*x^3 - 84*x^2 - 96*x - 128)*(x - 1)^(3/4) + 4/1155*(77
*x^3 - 21*x^2 - 24*x - 32)*(x - 1)^(3/4) - 8/77*(7*x^2 - 3*x - 4)*(x - 1)^(
3/4) - 4/7*(x - 1)^(7/4)
```

mupad [B] time = 0.20, size = 35, normalized size = 1.06

$$-\frac{4\,\left(x-1\right)\,{\left(x^3-3\,x^2+3\,x-1\right)}^{1/4}\,\left(-693\,x^3-847\,x^2+182\,x+731\right)}{4389}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - 3*x^2 + x^3 - 1)^(1/4)*(2*x - x^2 - 3*x^3 + 1),x)
```

```
[Out] -(4*(x - 1)*(3*x - 3*x^2 + x^3 - 1)^(1/4)*(182*x - 847*x^2 - 693*x^3 + 731)
)/4389
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3x^{3}+x^{2}-2x-1\right)\sqrt[4]{\left(x-1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3*x**2+3*x-1)**(1/4)*(3*x**3+x**2-2*x-1),x)
```

```
[Out] Integral((3*x**3 + x**2 - 2*x - 1)*((x - 1)**3)**(1/4), x)
```

3.402 $\int \frac{(-2b+ax^3)\sqrt{b+ax^3}}{x^2(b-x^2+ax^3)}\,dx$

Optimal. Leaf size=33

$$\frac{2\sqrt{ax^3+b}}{x}-2\tanh^{-1}\left(\frac{x}{\sqrt{ax^3+b}}\right)$$

Rubi [F] time = 1.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2b+ax^3)\sqrt{b+ax^3}}{x^2(b-x^2+ax^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*b + a*x^3)*Sqrt[b + a*x^3])/(x^2*(b - x^2 + a*x^3)),x]
```

```
[Out] (2*Sqrt[b + a*x^3])/x - (6*a^(1/3)*Sqrt[b + a*x^3])/((1 + Sqrt[3])*b^(1/3)
+ a^(1/3)*x) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(b^(1/3) + a^(1
/3)*x)*Sqrt[(b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2)/((1 + Sqrt[3])*b^(1
/3) + a^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*b^(1/3) + a^(1/3)*x)/((
1 + Sqrt[3])*b^(1/3) + a^(1/3)*x)], -7 - 4*Sqrt[3]])/(Sqrt[(b^(1/3)*(b^(1/3
) + a^(1/3)*x))/((1 + Sqrt[3])*b^(1/3) + a^(1/3)*x)^2]*Sqrt[b + a*x^3]) - (
2*Sqrt[2]*3^(3/4)*a^(1/3)*b^(1/3)*(b^(1/3) + a^(1/3)*x)*Sqrt[(b^(2/3) - a^(
1/3)*b^(1/3)*x + a^(2/3)*x^2)/((1 + Sqrt[3])*b^(1/3) + a^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*b^(1/3) + a^(1/3)*x)/((1 + Sqrt[3])*b^(1/3) + a^(
1/3)*x)], -7 - 4*Sqrt[3]])/(Sqrt[(b^(1/3)*(b^(1/3) + a^(1/3)*x))/((1 + Sqrt
[3])*b^(1/3) + a^(1/3)*x)^2]*Sqrt[b + a*x^3]) - 2*Defer[Int][Sqrt[b + a*x^3
]/(b - x^2 + a*x^3), x] + 3*a*Defer[Int][(x*Sqrt[b + a*x^3])/(b - x^2 + a*x
^3), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-2b+ax^3)\sqrt{b+ax^3}}{x^2(b-x^2+ax^3)}\,dx &= \int \left(-\frac{2\sqrt{b+ax^3}}{x^2}+\frac{(-2+3ax)\sqrt{b+ax^3}}{b-x^2+ax^3}\right)dx\\
&= -\left(2\int \frac{\sqrt{b+ax^3}}{x^2}\,dx\right)+\int \frac{(-2+3ax)\sqrt{b+ax^3}}{b-x^2+ax^3}\,dx\\
&= \frac{2\sqrt{b+ax^3}}{x}-(3a)\int \frac{x}{\sqrt{b+ax^3}}\,dx+\int \left(-\frac{2\sqrt{b+ax^3}}{b-x^2+ax^3}+\frac{3ax\sqrt{b+ax^3}}{b-x^2+ax^3}\right)dx\\
&= \frac{2\sqrt{b+ax^3}}{x}-2\int \frac{\sqrt{b+ax^3}}{b-x^2+ax^3}\,dx-\left(3a^{2/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x}{\sqrt{b+ax^3}}\,dx+(3a)\\
&= \frac{2\sqrt{b+ax^3}}{x}-\frac{6\sqrt[3]{a}\sqrt{b+ax^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}+\sqrt[3]{a}\,x}+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)\sqrt{\frac{b}{(}}}{\sqrt{\frac{\sqrt[3]{b}}{((1+}}}
\end{aligned}$$

Mathematica [C] time = 6.24, size = 2741, normalized size = 83.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-2*b + a*x^3)*Sqrt[b + a*x^3])/(x^2*(b - x^2 + a*x^3)),x]
```

```
[Out] (2*Sqrt[b + a*x^3])/x + (2*Sqrt[(b^(1/3)/a^(1/3) + x)/(b^(1/3)/a^(1/3) + ((
-1)^(1/3)*b^(1/3))/a^(1/3))]*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x)*Sqrt[(((
-1)^(2/3)*b^(1/3))/a^(1/3) + x)/(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)
*b^(1/3))/a^(1/3))]*EllipticF[ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) - a^(1/3)*x)/
(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)])/(Sqrt[(-(((-1)^(1/3)*b^
(1/3))/a^(1/3)) + x)/(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3)
)/a^(1/3))]*Sqrt[b + a*x^3]) - (4*(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/
3)*b^(1/3))/a^(1/3))*b*Sqrt[(b^(1/3)/a^(1/3) + x)/(b^(1/3)/a^(1/3) + ((-1)^
(1/3)*b^(1/3))/a^(1/3))]*Sqrt[((-(((-1)^(2/3)*b^(1/3))/a^(1/3)) - x)*(-(((-
1)^(1/3)*b^(1/3))/a^(1/3)) + x))/(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3
)*b^(1/3))/a^(1/3))^2]*EllipticPi[((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))
/((-1)^(1/3)*b^(1/3) - a^(1/3)*Root[b - #1^2 + a*#1^3 & , 1]), ArcSin[Sqrt[
((-1)^(1/3)*b^(1/3) - a^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1
)^(1/3)])/(a*Sqrt[b + a*x^3]*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + Root[b - #1
^2 + a*#1^3 & , 1])*(Root[b - #1^2 + a*#1^3 & , 1] - Root[b - #1^2 + a*#1^3
 & , 2])*(Root[b - #1^2 + a*#1^3 & , 1] - Root[b - #1^2 + a*#1^3 & , 3])) +
 (2*(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))*Sqrt[(b^(
1/3)/a^(1/3) + x)/(b^(1/3)/a^(1/3) + ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[((
-(((-1)^(2/3)*b^(1/3))/a^(1/3)) - x)*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x))
/(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*EllipticP
i[((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) - a^(1/3)*R
oot[b - #1^2 + a*#1^3 & , 1]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) - a^(1/3)*x)
/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)]*Root[b - #1^2 + a*#1^3
& , 1]^3)/(Sqrt[b + a*x^3]*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + Root[b - #1^2
 + a*#1^3 & , 1])*(Root[b - #1^2 + a*#1^3 & , 1] - Root[b - #1^2 + a*#1^3 &
 , 2])*(Root[b - #1^2 + a*#1^3 & , 1] - Root[b - #1^2 + a*#1^3 & , 3])) - (
4*(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))*b*Sqrt[(b^(
1/3)/a^(1/3) + x)/(b^(1/3)/a^(1/3) + ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[((
-(((-1)^(2/3)*b^(1/3))/a^(1/3)) - x)*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x))
/(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*EllipticP
i[((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) - a^(1/3)*R
oot[b - #1^2 + a*#1^3 & , 2]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) - a^(1/3)*x)
/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)])/(a*Sqrt[b + a*x^3]*(-(
((-1)^(1/3)*b^(1/3))/a^(1/3)) + Root[b - #1^2 + a*#1^3 & , 2])*(-Root[b - #
1^2 + a*#1^3 & , 1] + Root[b - #1^2 + a*#1^3 & , 2])*(Root[b - #1^2 + a*#1^
3 & , 2] - Root[b - #1^2 + a*#1^3 & , 3])) + (2*(((-1)^(1/3)*b^(1/3))/a^(1/
3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))*Sqrt[(b^(1/3)/a^(1/3) + x)/(b^(1/3)/a^(1
/3) + ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[((-(((-1)^(2/3)*b^(1/3))/a^(1/3))
 - x)*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x))/(((-1)^(1/3)*b^(1/3))/a^(1/3)
+ ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*EllipticPi[((-1)^(1/3)*b^(1/3) + (-1)^(2
/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) - a^(1/3)*Root[b - #1^2 + a*#1^3 & , 2]),
ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) - a^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3))*b^(
1/3))]], (-1)^(1/3)]*Root[b - #1^2 + a*#1^3 & , 2]^3)/(Sqrt[b + a*x^3]*(-((
(-1)^(1/3)*b^(1/3))/a^(1/3)) + Root[b - #1^2 + a*#1^3 & , 2])*(-Root[b - #1
^2 + a*#1^3 & , 1] + Root[b - #1^2 + a*#1^3 & , 2])*(Root[b - #1^2 + a*#1^3
 & , 2] - Root[b - #1^2 + a*#1^3 & , 3])) - (4*(((-1)^(1/3)*b^(1/3))/a^(1/3
) + ((-1)^(2/3)*b^(1/3))/a^(1/3))*b*Sqrt[(b^(1/3)/a^(1/3) + x)/(b^(1/3)/a^(
1/3) + ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[((-(((-1)^(2/3)*b^(1/3))/a^(1/3)
) - x)*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) + x))/(((-1)^(1/3)*b^(1/3))/a^(1/3)
 + ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*EllipticPi[((-1)^(1/3)*b^(1/3) + (-1)^(
2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) - a^(1/3)*Root[b - #1^2 + a*#1^3 & , 3]),
 ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) - a^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3))*b^
(1/3))]], (-1)^(1/3)])/(a*Sqrt[b + a*x^3]*(-(((-1)^(1/3)*b^(1/3))/a^(1/3))
+ Root[b - #1^2 + a*#1^3 & , 3])*(-Root[b - #1^2 + a*#1^3 & , 1] + Root[b -
 #1^2 + a*#1^3 & , 3])*(-Root[b - #1^2 + a*#1^3 & , 2] + Root[b - #1^2 + a*
#1^3 & , 3])) + (2*(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(
```

```
1/3))*Sqrt[(b^(1/3)/a^(1/3) + x)/(b^(1/3)/a^(1/3) + ((-1)^(1/3)*b^(1/3))/a^
(1/3))]*Sqrt[((-(((-1)^(2/3)*b^(1/3))/a^(1/3)) - x)*(-(((-1)^(1/3)*b^(1/3))
/a^(1/3)) + x))/(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3
))^2]*EllipticPi[((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1
/3) - a^(1/3)*Root[b - #1^2 + a*#1^3 & , 3]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/
3) - a^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)]*Root[b -
 #1^2 + a*#1^3 & , 3]^3)/(Sqrt[b + a*x^3]*(-(((-1)^(1/3)*b^(1/3))/a^(1/3))
+ Root[b - #1^2 + a*#1^3 & , 3])*(-Root[b - #1^2 + a*#1^3 & , 1] + Root[b -
 #1^2 + a*#1^3 & , 3])*(-Root[b - #1^2 + a*#1^3 & , 2] + Root[b - #1^2 + a*
#1^3 & , 3]))
```

IntegrateAlgebraic [A] time = 0.56, size = 33, normalized size = 1.00

$$\frac{2\sqrt{ax^3+b}}{x}-2\tanh^{-1}\left(\frac{x}{\sqrt{ax^3+b}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*b + a*x^3)*Sqrt[b + a*x^3])/(x^2*(b - x^2 + a*x^3))
,x]
```

```
[Out] (2*Sqrt[b + a*x^3])/x - 2*ArcTanh[x/Sqrt[b + a*x^3]]
```

fricas [A] time = 26.70, size = 56, normalized size = 1.70

$$\frac{x\log\left(\frac{ax^3+x^2-2\sqrt{ax^3+b}\,x+b}{ax^3-x^2+b}\right)+2\sqrt{ax^3+b}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*b)*(a*x^3+b)^(1/2)/x^2/(a*x^3-x^2+b),x, algorithm="frica
s")
```

```
[Out] (x*log((a*x^3 + x^2 - 2*sqrt(a*x^3 + b)*x + b)/(a*x^3 - x^2 + b)) + 2*sqrt(
a*x^3 + b))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ax^3+b}\left(ax^3-2\,b\right)}{\left(ax^3-x^2+b\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*b)*(a*x^3+b)^(1/2)/x^2/(a*x^3-x^2+b),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*x^3 + b)*(a*x^3 - 2*b)/((a*x^3 - x^2 + b)*x^2), x)
```

maple [C] time = 0.18, size = 841, normalized size = 25.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-2*b)*(a*x^3+b)^(1/2)/x^2/(a*x^3-x^2+b),x)
```

```
[Out] -2/3*I*3^(1/2)/a*(-a^2*b)^(1/3)*(I*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*
(-a^2*b)^(1/3))*3^(1/2)*a/(-a^2*b)^(1/3))^(1/2)*((x-1/a*(-a^2*b)^(1/3))/(-3
/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2)*(-I*(x+1/2/a*(-a
^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*3^(1/2)*a/(-a^2*b)^(1/3))^(1/2)
/(a*x^3+b)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(
1/2)/a*(-a^2*b)^(1/3))*3^(1/2)*a/(-a^2*b)^(1/3))^(1/2),(I*3^(1/2)/a*(-a^2*b
```

```
)^(1/3)/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))-I/a^
2/b*2^(1/2)*sum((-_alpha^2+3*b)/_alpha/(3*_alpha*a-2)*(-a^2*b)^(1/3)*(1/2*I
*a*(2*x+1/a*(-I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3)))/(-a^2*b)^(1/3))^(1/
2)*(a*(x-1/a*(-a^2*b)^(1/3))/(-3*(-a^2*b)^(1/3)+I*3^(1/2)*(-a^2*b)^(1/3)))^
(1/2)*(-1/2*I*a*(2*x+1/a*(I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3)))/(-a^2*b
)^(1/3))^(1/2)/(a*x^3+b)^(1/2)*(-I*(-a^2*b)^(1/3)*3^(1/2)*_alpha^2*a^2+I*(-
a^2*b)^(2/3)*3^(1/2)*_alpha*a+I*(-a^2*b)^(1/3)*3^(1/2)*_alpha*a+(-a^2*b)^(1
/3)*_alpha^2*a^2-I*(-a^2*b)^(2/3)*3^(1/2)+_alpha*(-a^2*b)^(2/3)*a-(-a^2*b)^
(1/3)*_alpha*a+2*a^2*b-(-a^2*b)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/a*(
-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*3^(1/2)*a/(-a^2*b)^(1/3))^(1/
2),1/2/a*(-I*(-a^2*b)^(2/3)*_alpha^2*3^(1/2)*a+I*_alpha*3^(1/2)*a^2*b+I*(-a
^2*b)^(2/3)*3^(1/2)*_alpha-3*(-a^2*b)^(2/3)*_alpha^2*a-2*I*(-a^2*b)^(1/3)*3
^(1/2)*a*b-I*3^(1/2)*a*b-3*_alpha*a^2*b+3*(-a^2*b)^(2/3)*_alpha+3*a*b)/b,(I
*3^(1/2)/a*(-a^2*b)^(1/3)/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(
1/3)))^(1/2)),_alpha=RootOf(_Z^3*a-_Z^2+b))+2*(a*x^3+b)^(1/2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3+b}\,\left(ax^3-2\,b\right)}{\left(ax^3-x^2+b\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*b)*(a*x^3+b)^(1/2)/x^2/(a*x^3-x^2+b),x, algorithm="maxim
a")
[Out] integrate(sqrt(a*x^3 + b)*(a*x^3 - 2*b)/((a*x^3 - x^2 + b)*x^2), x)
```

mupad [B] time = 0.92, size = 43, normalized size = 1.30

$$\ln\left(\frac{x-\sqrt{a\,x^3+b}}{x+\sqrt{a\,x^3+b}}\right)+\frac{2\,\sqrt{a\,x^3+b}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b + a*x^3)^(1/2)*(2*b - a*x^3))/(x^2*(b + a*x^3 - x^2)),x)
[Out] log((x - (b + a*x^3)^(1/2))/(x + (b + a*x^3)^(1/2))) + (2*(b + a*x^3)^(1/2)
)/x
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-2*b)*(a*x**3+b)**(1/2)/x**2/(a*x**3-x**2+b),x)
[Out] Timed out
```

3.403 $\int \frac{1}{\sqrt[4]{1+x^4}}\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {240, 212, 206, 203}

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)^(-1/4),x]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)]/2 + ArcTanh[x/(1 + x^4)^(1/4)]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{1+x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)\\ &= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 54, normalized size = 1.64

$$-\frac{1}{4}\log\left(1-\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{4}\log\left(\frac{x}{\sqrt[4]{x^4+1}}+1\right)+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)^(-1/4),x]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)]/2 - Log[1 - x/(1 + x^4)^(1/4)]/4 + Log[1 + x/(1 +
 x^4)^(1/4)]/4
```

IntegrateAlgebraic [A] time = 0.17, size = 33, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)^(-1/4),x]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)]/2 + ArcTanh[x/(1 + x^4)^(1/4)]/2
```

fricas [A] time = 0.42, size = 50, normalized size = 1.52

$$-\frac{1}{2}\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{4}\log\left(\frac{x+\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{4}\log\left(-\frac{x-\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/2*arctan((x^4 + 1)^(1/4)/x) + 1/4*log((x + (x^4 + 1)^(1/4))/x) - 1/4*log
(-(x - (x^4 + 1)^(1/4))/x)
```

giac [A] time = 0.35, size = 47, normalized size = 1.42

$$-\frac{1}{2}\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{4}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}+1\right)-\frac{1}{4}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4),x, algorithm="giac")
```

```
[Out] -1/2*arctan((x^4 + 1)^(1/4)/x) + 1/4*log((x^4 + 1)^(1/4)/x + 1) - 1/4*log((
x^4 + 1)^(1/4)/x - 1)
```

maple [C] time = 0.23, size = 14, normalized size = 0.42

$$x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{4}\right],\left[\frac{5}{4}\right],-x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1)^(1/4),x)
```

```
[Out] x*hypergeom([1/4,1/4],[5/4],-x^4)
```

maxima [A] time = 0.59, size = 47, normalized size = 1.42

$$-\frac{1}{2}\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{4}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}+1\right)-\frac{1}{4}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/2*arctan((x^4 + 1)^(1/4)/x) + 1/4*log((x^4 + 1)^(1/4)/x + 1) - 1/4*log((
x^4 + 1)^(1/4)/x - 1)
```

mupad [B] time = 0.20, size = 12, normalized size = 0.36

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 + 1)^(1/4),x)
```

```
[Out] x*hypergeom([1/4, 1/4], 5/4, -x^4)
```

sympy [C] time = 0.72, size = 27, normalized size = 0.82

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)**(1/4),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/4), (5/4,), x**4*exp_polar(I*pi))/(4*gamma(5/4))
```

3.404 $\int \frac{-1-2x+2x^2}{(1+2x+4x^2)\sqrt{x+x^4}}\,dx$

Optimal. Leaf size=33

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{x^4+x}}{x^2-x+1}\right)}{\sqrt{3}}$$

Rubi [F] time = 2.49, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1-2x+2x^2}{(1+2x+4x^2)\sqrt{x+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 - 2*x + 2*x^2)/((1 + 2*x + 4*x^2)*Sqrt[x + x^4]),x]
```

```
[Out] (x*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(1
+ (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(2*3^(1/4)*Sqr
t[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[x + x^4]) + (Sqrt[3]*(I + Sqrt[
3])*Sqrt[x]*Sqrt[1 + x^3]*Defer[Subst][Defer[Int][1/((Sqrt[-1 - I*Sqrt[3]]
- 2*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/(2*Sqrt[-1 - I*Sqrt[3]]*Sqrt[x + x^
4]) - (Sqrt[3]*(I - Sqrt[3])*Sqrt[x]*Sqrt[1 + x^3]*Defer[Subst][Defer[Int][
1/((Sqrt[-1 + I*Sqrt[3]] - 2*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/(2*Sqrt[I*
(I + Sqrt[3])]*Sqrt[x + x^4]) + (Sqrt[3]*(I + Sqrt[3])*Sqrt[x]*Sqrt[1 + x^3
]*Defer[Subst][Defer[Int][1/((Sqrt[-1 - I*Sqrt[3]] + 2*x)*Sqrt[1 + x^6]), x
], x, Sqrt[x]])/(2*Sqrt[-1 - I*Sqrt[3]]*Sqrt[x + x^4]) - (Sqrt[3]*(I - Sqrt
[3])*Sqrt[x]*Sqrt[1 + x^3]*Defer[Subst][Defer[Int][1/((Sqrt[-1 + I*Sqrt[3]]
 + 2*x)*Sqrt[1 + x^6]), x], x, Sqrt[x]])/(2*Sqrt[I*(I + Sqrt[3])]*Sqrt[x +
x^4])
```

Rubi steps

$$\int \frac{-1-2x+2x^2}{\left(1+2x+4x^2\right)\sqrt{x+x^4}}\,dx = \frac{\left(\sqrt{x}\sqrt{1+x^3}\right)\int \frac{-1-2x+2x^2}{\sqrt{x}\left(1+2x+4x^2\right)\sqrt{1+x^3}}\,dx}{\sqrt{x+x^4}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x^3}\right)\int \left(\frac{1}{2\sqrt{x}\sqrt{1+x^3}} - \frac{3(1+2x)}{2\sqrt{x}\left(1+2x+4x^2\right)\sqrt{1+x^3}}\right)dx}{\sqrt{x+x^4}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{x}\sqrt{1+x^3}}\,dx}{2\sqrt{x+x^4}} - \frac{\left(3\sqrt{x}\sqrt{1+x^3}\right)\int \frac{1+2x}{\sqrt{x}\left(1+2x+4x^2\right)\sqrt{1+x^3}}\,dx}{2\sqrt{x+x^4}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{x+x^4}} - \frac{\left(3\sqrt{x}\sqrt{1+x^3}\right)\int \left(\frac{2-\frac{2i}{\sqrt{3}}}{\sqrt{x}\left(2-2i\sqrt{3}+8x\right. \cdots}\right. \cdots}{2\sqrt{x \cdots}}$$

$$= \frac{x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}} - \frac{\left(\left(3-i\sqrt{3}\right)\sqrt{x} \cdots\right.}{\cdots}$$

$$= \frac{x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}} - \frac{\left(2\left(3-i\sqrt{3}\right)\sqrt{x} \cdots\right.}{\cdots}$$

$$= \frac{x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}} - \frac{\left(2\left(3-i\sqrt{3}\right)\sqrt{x} \cdots\right.}{\cdots}$$

$$= \frac{x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x+x^4}} + \frac{\left(\left(3-i\sqrt{3}\right)\sqrt{x} \cdots\right.}{\cdots}$$

Mathematica [C] time = 1.90, size = 287, normalized size = 8.70

$$-\frac{2\sqrt{\frac{\frac{1}{x}+1}{1+\sqrt[3]{-1}}}\,x^2\left(\frac{2\sqrt{\frac{1}{x^2}-\frac{1}{x}+1}\,\Pi\left(\frac{2i}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{x+(-1)^{2/3}}{\left(1+\sqrt[3]{-1}\right)x}}\right)|\sqrt[3]{-1}\right)}{\sqrt{3}+3i} - \frac{2i\sqrt{3}\sqrt{\frac{1}{x^2}-\frac{1}{x}+1}\,\Pi\left(-\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{x+(-1)^{2/3}}{\left(1+\sqrt[3]{-1}\right)x}}\right)|\sqrt[3]{-1}\right)}{\sqrt{3}+3i} + \frac{\left(\sqrt[3]{-1}-\frac{1}{x}\right)\sqrt{\frac{\sqrt[3]{-1}-\frac{(-1)^{2/3}}{x}}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{x+(-1)^{2/3}}{\left(1+\sqrt[3]{-1}\right)x}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{x+(-1)^{2/3}}{\left(1+\sqrt[3]{-1}\right)x}}}\right)}{\sqrt{x^4+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 - 2*x + 2*x^2)/((1 + 2*x + 4*x^2)*Sqrt[x + x^4]),x]
```

```
[Out] (-2*Sqrt[(1 + x^(-1))/(1 + (-1)^(1/3))]*x^2*((((-1)^(1/3) - x^(-1))*Sqrt[((
-1)^(1/3) - (-1)^(2/3)/x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[((-1)^(2/
3) + x)/((1 + (-1)^(1/3))*x)]], (-1)^(1/3)])/Sqrt[((-1)^(2/3) + x)/((1 + (-
1)^(1/3))*x)] + (2*Sqrt[1 + x^(-2) - x^(-1)]*EllipticPi[(2*I)/(3*I + Sqrt[3
]), ArcSin[Sqrt[((-1)^(2/3) + x)/((1 + (-1)^(1/3))*x)]], (-1)^(1/3)])/(3*I
+ Sqrt[3]) - ((2*I)*Sqrt[3]*Sqrt[1 + x^(-2) - x^(-1)]*EllipticPi[(-2*Sqrt[3
])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-1)^(2/3) + x)/((1 + (-1)^(1/3))*x)]], (-
1)^(1/3)])/(3*I + Sqrt[3])))/Sqrt[x + x^4]
```

IntegrateAlgebraic [A] time = 1.13, size = 33, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{x^4+x}}{x^2-x+1}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 2*x + 2*x^2)/((1 + 2*x + 4*x^2)*Sqrt[x + x^4]),x]
[Out] (-2*ArcTan[(Sqrt[3]*Sqrt[x + x^4])/(1 - x + x^2)])/Sqrt[3]
```

fricas [A] time = 0.47, size = 28, normalized size = 0.85

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^2+4x-1\right)}{6\sqrt{x^4+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-2*x-1)/(4*x^2+2*x+1)/(x^4+x)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*(2*x^2 + 4*x - 1)/sqrt(x^4 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2x^2-2x-1}{\sqrt{x^4+x}\left(4x^2+2x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-2*x-1)/(4*x^2+2*x+1)/(x^4+x)^(1/2),x, algorithm="giac")
[Out] integrate((2*x^2 - 2*x - 1)/(sqrt(x^4 + x)*(4*x^2 + 2*x + 1)), x)
```

maple [C] time = 0.18, size = 830, normalized size = 25.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-2*x-1)/(4*x^2+2*x+1)/(x^4+x)^(1/2),x)
[Out] -(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/
2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))/(1+x))^(1/2)*(-(x-1/
2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(3/2+1/2*I*3^(1/2))/(x*(1
+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1/2)*EllipticF(((3/2+1/2*
I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1
/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+3*(1/4-1/12
*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))
/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))/(1+x))^(1
/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(3/2+1/2*I*3^(
1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1/2)*(-1+1/3*I
*3^(1/2))*(EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2
),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2
*I*3^(1/2)))^(1/2))+(-1-I*3^(1/2))*EllipticPi(((3/2+1/2*I*3^(1/2))*x/(1/2+1
/2*I*3^(1/2))/(1+x))^(1/2),-2/3*I*3^(1/2)*(-1/4+1/4*I*3^(1/2))-2/3*I*3^(1/2
),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2
*I*3^(1/2)))^(1/2)))+3*(1/4+1/12*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))*((3/2+1/2*
I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2
))/(1/2-1/2*I*3^(1/2))/(1+x))^(1/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1
/2))/(1+x))^(1/2)/(3/2+1/2*I*3^(1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2
-1/2*I*3^(1/2)))^(1/2)*(-1-1/3*I*3^(1/2))*(EllipticF(((3/2+1/2*I*3^(1/2))*x
```

```
/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2)
)/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(I*3^(1/2)-1)*EllipticP
i(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),-2/3*I*3^(1/2)*(-
1/4-1/4*I*3^(1/2))+1/3*I*3^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))
/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2-2\,x-1}{\sqrt{x^4+x}\left(4\,x^2+2\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-2*x-1)/(4*x^2+2*x+1)/(x^4+x)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x^2 - 2*x - 1)/(sqrt(x^4 + x)*(4*x^2 + 2*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{-2\,x^2+2\,x+1}{\sqrt{x^4+x}\,\left(4\,x^2+2\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 2*x^2 + 1)/((x + x^4)^(1/2)*(2*x + 4*x^2 + 1)),x)
[Out] int(-(2*x - 2*x^2 + 1)/((x + x^4)^(1/2)*(2*x + 4*x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2-2x-1}{\sqrt{x\,(x+1)\left(x^2-x+1\right)}\,\left(4x^2+2x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-2*x-1)/(4*x**2+2*x+1)/(x**4+x)**(1/2),x)
[Out] Integral((2*x**2 - 2*x - 1)/(sqrt(x*(x + 1)*(x**2 - x + 1))*(4*x**2 + 2*x +
 1)), x)
```

3.405 $\int \sqrt{x+x^4}\,dx$

Optimal. Leaf size=33

$$\frac{1}{3}\sqrt{x^4+x}\,x+\frac{1}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2004, 2029, 206}

$$\frac{1}{3}\sqrt{x^4+x}\,x+\frac{1}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + x^4],x]
```

```
[Out] (x*Sqrt[x + x^4])/3 + ArcTanh[x^2/Sqrt[x + x^4]]/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x+x^4}\,dx&=\frac{1}{3}x\sqrt{x+x^4}+\frac{1}{2}\int \frac{x}{\sqrt{x+x^4}}\,dx\\&=\frac{1}{3}x\sqrt{x+x^4}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x^2}{\sqrt{x+x^4}}\right)\\&=\frac{1}{3}x\sqrt{x+x^4}+\frac{1}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x+x^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.21

$$\frac{\sqrt{x^4+x}\left(x^{3/2}+\frac{\sinh^{-1}\left(x^{3/2}\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + x^4],x]
```

```
[Out] (Sqrt[x + x^4]*(x^(3/2) + ArcSinh[x^(3/2)]/Sqrt[1 + x^3]))/(3*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.38, size = 33, normalized size = 1.00

$$\frac{1}{3}\sqrt{x^4+x}\,x+\frac{1}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x + x^4],x]
```

```
[Out] (x*Sqrt[x + x^4])/3 + ArcTanh[x^2/Sqrt[x + x^4]]/3
```

fricas [A] time = 0.45, size = 31, normalized size = 0.94

$$\frac{1}{3}\sqrt{x^4+x}\,x+\frac{1}{6}\log\left(-2\,x^3-2\,\sqrt{x^4+x}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(x^4 + x)*x + 1/6*log(-2*x^3 - 2*sqrt(x^4 + x)*x - 1)
```

giac [A] time = 0.50, size = 36, normalized size = 1.09

$$\frac{1}{3}\sqrt{x^4+x}\,x+\frac{1}{6}\log\left(\sqrt{\frac{1}{x^3}+1}+1\right)-\frac{1}{6}\log\left(\left|\sqrt{\frac{1}{x^3}+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(x^4 + x)*x + 1/6*log(sqrt(1/x^3 + 1) + 1) - 1/6*log(abs(sqrt(1/x^3
 + 1) - 1))
```

maple [C] time = 0.03, size = 301, normalized size = 9.12

$$\frac{x\sqrt{x^4+x}}{3}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\,(1+x)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(1+x)}}\sqrt{-\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\left(-\mathrm{EllipticF}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)+\mathrm{EllipticPi}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\frac{\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x\,(1+x)\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x)^(1/2),x)
```

```
[Out] 1/3*x*(x^4+x)^(1/2)-(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*
3^(1/2))/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))/(
1+x))^(1/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(3/2+1
/2*I*3^(1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1/2)*(
-EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-3/2+1
/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)
))^(1/2))+EllipticPi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2
),(1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I
*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^4+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + x), x)
```

mupad [B] time = 0.30, size = 27, normalized size = 0.82

$$\frac{2\,x\,\sqrt{x^4+x}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2};\ \frac{3}{2};\ -x^3\right)}{3\,\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^4)^(1/2),x)
[Out] (2*x*(x + x^4)^(1/2)*hypergeom([-1/2, 1/2], 3/2, -x^3))/(3*(x^3 + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^4+x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x)**(1/2),x)
[Out] Integral(sqrt(x**4 + x), x)
```

3.406 $\int \frac{1}{\sqrt[4]{-x^2+x^4}}\,dx$

Optimal. Leaf size=33

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)$$

Rubi [B] time = 0.02, antiderivative size = 89, normalized size of antiderivative = 2.70, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2011, 329, 240, 212, 206, 203}

$$\frac{\sqrt{x}\,\sqrt[4]{x^2-1}\,\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{\sqrt[4]{x^4-x^2}}+\frac{\sqrt{x}\,\sqrt[4]{x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{\sqrt[4]{x^4-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + x^4)^(-1/4),x]
```

```
[Out] (Sqrt[x]*(-1 + x^2)^(1/4)*ArcTan[Sqrt[x]/(-1 + x^2)^(1/4)])/(-x^2 + x^4)^(1
/4) + (Sqrt[x]*(-1 + x^2)^(1/4)*ArcTanh[Sqrt[x]/(-1 + x^2)^(1/4)])/(-x^2 +
x^4)^(1/4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
```

```
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[4]{-x^2+x^4}}\,dx &= \frac{\left(\sqrt{x}\sqrt[4]{-1+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt[4]{-1+x^2}}\,dx}{\sqrt[4]{-x^2+x^4}}\\
&= \frac{\left(2\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-1+x^4}}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{-x^2+x^4}}\\
&= \frac{\left(2\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}}\\
&= \frac{\left(\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}}+\frac{\left(\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}}\\
&= \frac{\sqrt{x}\sqrt[4]{-1+x^2}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}}+\frac{\sqrt{x}\sqrt[4]{-1+x^2}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 1.85

$$\frac{\sqrt{x}\sqrt[4]{x^2-1}\left(\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)+\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)\right)}{\sqrt[4]{x^2\left(x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + x^4)^(-1/4),x]
```

```
[Out] (Sqrt[x]*(-1 + x^2)^(1/4)*(ArcTan[Sqrt[x]/(-1 + x^2)^(1/4)] + ArcTanh[Sqrt[
x]/(-1 + x^2)^(1/4)]))/(x^2*(-1 + x^2))^(1/4)
```

IntegrateAlgebraic [A] time = 0.14, size = 33, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^2 + x^4)^(-1/4),x]
```

```
[Out] ArcTan[x/(-x^2 + x^4)^(1/4)] + ArcTanh[x/(-x^2 + x^4)^(1/4)]
```

fricas [B] time = 2.87, size = 95, normalized size = 2.88

$$-\frac{1}{2}\arctan\left(\frac{2\left(\left(x^4-x^2\right)^{\frac{1}{4}}x^2+\left(x^4-x^2\right)^{\frac{3}{4}}\right)}{x}\right)+\frac{1}{2}\log\left(\frac{2\,x^3+2\left(x^4-x^2\right)^{\frac{1}{4}}x^2+2\,\sqrt{x^4-x^2}\,x-x+2\left(x^4-x^2\right)^{\frac{3}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(2*((x^4 - x^2)^(1/4)*x^2 + (x^4 - x^2)^(3/4))/x) + 1/2*log((2*x
^3 + 2*(x^4 - x^2)^(1/4)*x^2 + 2*sqrt(x^4 - x^2)*x - x + 2*(x^4 - x^2)^(3/4
))/x)
```

giac [A] time = 0.35, size = 41, normalized size = 1.24

$$\arctan\left(\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)-\frac{1}{2}\log\left(\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{2}\log\left(-\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2)^(1/4),x, algorithm="giac")
[Out] arctan((-1/x^2 + 1)^(1/4)) - 1/2*log((-1/x^2 + 1)^(1/4) + 1) + 1/2*log(-(-1
/x^2 + 1)^(1/4) + 1)
```

maple [C] time = 0.26, size = 33, normalized size = 1.00

$$\frac{2\left(-\mathrm{signum}\left(x^2-1\right)\right)^{\frac{1}{4}}\sqrt{x}\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{4}\right],\left[\frac{5}{4}\right],x^2\right)}{\mathrm{signum}\left(x^2-1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-x^2)^(1/4),x)
[Out] 2/signum(x^2-1)^(1/4)*(-signum(x^2-1))^(1/4)*x^(1/2)*hypergeom([1/4,1/4],[5
/4],x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4-x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2)^(1/4),x, algorithm="maxima")
[Out] integrate((x^4 - x^2)^(-1/4), x)
```

mupad [B] time = 0.40, size = 31, normalized size = 0.94

$$\frac{2\,x\,\left(1-x^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ x^2\right)}{\left(x^4-x^2\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - x^2)^(1/4),x)
[Out] (2*x*(1 - x^2)^(1/4)*hypergeom([1/4, 1/4], 5/4, x^2))/(x^4 - x^2)^(1/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^4-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-x**2)**(1/4),x)
[Out] Integral((x**4 - x**2)**(-1/4), x)
```

3.407 $\int \frac{2b+ax^2}{\sqrt[4]{b+ax^2}\left(-b-ax^2+x^4\right)}\,dx$

Optimal. Leaf size=33

$$\tan^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{x}\right)-\tanh^{-1}\left(\frac{x}{\sqrt[4]{ax^2+b}}\right)$$

Rubi [C] time = 0.64, antiderivative size = 489, normalized size of antiderivative = 14.82, number of steps used = 10, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1692, 399, 490, 1218}

$$\frac{\sqrt[4]{b}\left(a-\sqrt{a^2+4b}\right)\sqrt{-\frac{ax^2}{b}}\,\Pi\left(-\frac{\sqrt{2}\sqrt{b}}{\sqrt{a^2-\sqrt{a^2+4b}\,a+2b}};\sin^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt{2}x\sqrt{-a\sqrt{a^2+4b}+a^2+2b}}-\frac{\sqrt[4]{b}\left(a-\sqrt{a^2+4b}\right)\sqrt{-\frac{ax^2}{b}}\,\Pi\left(\frac{\sqrt{2}\sqrt{b}}{\sqrt{a^2-\sqrt{a^2+4b}\,a+2b}};\sin^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt{2}x\sqrt{-a\sqrt{a^2+4b}+a^2+2b}}+\frac{\sqrt[4]{b}\left(\sqrt{a^2+4b}+a\right)\sqrt{-\frac{ax^2}{b}}\,\Pi\left(-\frac{\sqrt{2}\sqrt{b}}{\sqrt{a^2+\sqrt{a^2+4b}\,a+2b}};\sin^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt{2}x\sqrt{a\sqrt{a^2+4b}+a^2+2b}}-\frac{\sqrt[4]{b}\left(\sqrt{a^2+4b}+a\right)\sqrt{-\frac{ax^2}{b}}\,\Pi\left(\frac{\sqrt{2}\sqrt{b}}{\sqrt{a^2+\sqrt{a^2+4b}\,a+2b}};\sin^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt{2}x\sqrt{a\sqrt{a^2+4b}+a^2+2b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*b + a*x^2)/((b + a*x^2)^(1/4)*(-b - a*x^2 + x^4)),x]
```

```
[Out] (b^(1/4)*(a - Sqrt[a^2 + 4*b])*Sqrt[-((a*x^2)/b)]*EllipticPi[-((Sqrt[2]*Sqr
t[b])/Sqrt[a^2 + 2*b - a*Sqrt[a^2 + 4*b]]), ArcSin[(b + a*x^2)^(1/4)/b^(1/4
)], -1])/(Sqrt[2]*Sqrt[a^2 + 2*b - a*Sqrt[a^2 + 4*b]]*x) - (b^(1/4)*(a - Sq
rt[a^2 + 4*b])*Sqrt[-((a*x^2)/b)]*EllipticPi[(Sqrt[2]*Sqrt[b])/Sqrt[a^2 + 2
*b - a*Sqrt[a^2 + 4*b]], ArcSin[(b + a*x^2)^(1/4)/b^(1/4)], -1])/(Sqrt[2]*S
qrt[a^2 + 2*b - a*Sqrt[a^2 + 4*b]]*x) + (b^(1/4)*(a + Sqrt[a^2 + 4*b])*Sqrt
[-((a*x^2)/b)]*EllipticPi[-((Sqrt[2]*Sqrt[b])/Sqrt[a^2 + 2*b + a*Sqrt[a^2 +
 4*b]]), ArcSin[(b + a*x^2)^(1/4)/b^(1/4)], -1])/(Sqrt[2]*Sqrt[a^2 + 2*b +
a*Sqrt[a^2 + 4*b]]*x) - (b^(1/4)*(a + Sqrt[a^2 + 4*b])*Sqrt[-((a*x^2)/b)]*E
llipticPi[(Sqrt[2]*Sqrt[b])/Sqrt[a^2 + 2*b + a*Sqrt[a^2 + 4*b]], ArcSin[(b
+ a*x^2)^(1/4)/b^(1/4)], -1])/(Sqrt[2]*Sqrt[a^2 + 2*b + a*Sqrt[a^2 + 4*b]]*
x)
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{2b+ax^2}{\sqrt[4]{b+ax^2}\left(-b-ax^2+x^4\right)}dx = \int \left(\frac{a+\sqrt{a^2+4b}}{\left(-a-\sqrt{a^2+4b}+2x^2\right)\sqrt[4]{b+ax^2}} + \frac{a-\sqrt{a^2+4b}}{\left(-a+\sqrt{a^2+4b}+2x^2\right)\sqrt[4]{b+ax^2}}\right.$$

$$= \left(a-\sqrt{a^2+4b}\right)\int \frac{1}{\left(-a+\sqrt{a^2+4b}+2x^2\right)\sqrt[4]{b+ax^2}}dx + \left(a+\sqrt{a^2+4b}\right)$$

$$= \frac{\left(2\left(a-\sqrt{a^2+4b}\right)\sqrt{-\frac{ax^2}{b}}\right)\text{Subst}\left(\int \frac{x^2}{\left(-2b+a\left(-a+\sqrt{a^2+4b}\right)+2x^4\right)\sqrt{1-\frac{x^4}{b}}}dx, x, \sqrt[4]{b}\right.}{x}$$

$$= -\frac{\left(\left(a-\sqrt{a^2+4b}\right)\sqrt{-\frac{ax^2}{b}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a^2+2b-a\sqrt{a^2+4b}}-\sqrt{2}x^2\right)\sqrt{1-\frac{x^4}{b}}}dx, x, \sqrt[4]{\right.}{\sqrt{2}x}$$

$$= \frac{\sqrt[4]{b}\left(a-\sqrt{a^2+4b}\right)\sqrt{-\frac{ax^2}{b}}\,\Pi\left(-\frac{\sqrt{2}\sqrt{b}}{\sqrt{a^2+2b-a\sqrt{a^2+4b}}};\sin^{-1}\left(\frac{\sqrt[4]{b+ax^2}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt{2}\sqrt{a^2+2b-a\sqrt{a^2+4b}}\,x} - \cdots$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{2b+ax^2}{\sqrt[4]{b+ax^2}\left(-b-ax^2+x^4\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2*b + a*x^2)/((b + a*x^2)^(1/4)*(-b - a*x^2 + x^4)),x]
[Out] Integrate[(2*b + a*x^2)/((b + a*x^2)^(1/4)*(-b - a*x^2 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.20, size = 33, normalized size = 1.00

$$\tan^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{x}\right) - \tanh^{-1}\left(\frac{x}{\sqrt[4]{ax^2+b}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*b + a*x^2)/((b + a*x^2)^(1/4)*(-b - a*x^2 + x^4)),x]
[Out] ArcTan[(b + a*x^2)^(1/4)/x] - ArcTanh[x/(b + a*x^2)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b)/(a*x^2+b)^(1/4)/(x^4-a*x^2-b),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2+2b}{\left(x^4-ax^2-b\right)\left(ax^2+b\right)^{\frac{1}{4}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b)/(a*x^2+b)^(1/4)/(x^4-a*x^2-b),x, algorithm="giac")
[Out] integrate((a*x^2 + 2*b)/((x^4 - a*x^2 - b)*(a*x^2 + b)^(1/4)), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{a\,x^{2}+2b}{\left(a\,x^{2}+b\right)^{\frac{1}{4}} \left(x^{4}-a\,x^{2}-b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+2*b)/(a*x^2+b)^(1/4)/(x^4-a*x^2-b),x)
[Out] int((a*x^2+2*b)/(a*x^2+b)^(1/4)/(x^4-a*x^2-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{2}+2\,b}{\left(x^{4}-ax^{2}-b\right)\left(ax^{2}+b\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b)/(a*x^2+b)^(1/4)/(x^4-a*x^2-b),x, algorithm="maxima")
[Out] integrate((a*x^2 + 2*b)/((x^4 - a*x^2 - b)*(a*x^2 + b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{a\,x^2+2\,b}{{\left(a\,x^2+b\right)}^{1/4}\,\left(-x^4+a\,x^2+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*b + a*x^2)/((b + a*x^2)^(1/4)*(b + a*x^2 - x^4)),x)
[Out] int(-(2*b + a*x^2)/((b + a*x^2)^(1/4)*(b + a*x^2 - x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{2} + 2b}{\sqrt[4]{ax^{2} + b} \left(- ax^{2} - b + x^{4}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+2*b)/(a*x**2+b)**(1/4)/(x**4-a*x**2-b),x)
[Out] Integral((a*x**2 + 2*b)/((a*x**2 + b)**(1/4)*(-a*x**2 - b + x**4)), x)
```

3.408 $\int \frac{-1+2x}{\sqrt{-2-2x+3x^2-2x^3+x^4}}\,dx$

Optimal. Leaf size=33

$$-\log\left(-x^2+\sqrt{x^4-2x^3+3x^2-2x-2}+x-1\right)$$

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1680, 12, 1107, 621, 206}

$$\tanh^{-1}\left(\frac{4\left(x-\frac{1}{2}\right)^2+3}{\sqrt{16\left(x-\frac{1}{2}\right)^4+24\left(x-\frac{1}{2}\right)^2-39}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x)/Sqrt[-2 - 2*x + 3*x^2 - 2*x^3 + x^4],x]
```

```
[Out] ArcTanh[(3 + 4*(-1/2 + x)^2)/Sqrt[-39 + 24*(-1/2 + x)^2 + 16*(-1/2 + x)^4]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+2x}{\sqrt{-2-2x+3x^2-2x^3+x^4}}\,dx &= \text{Subst}\left(\int \frac{8x}{\sqrt{-39+24x^2+16x^4}}\,dx, x, -\frac{1}{2}+x\right) \\
&= 8\,\text{Subst}\left(\int \frac{x}{\sqrt{-39+24x^2+16x^4}}\,dx, x, -\frac{1}{2}+x\right) \\
&= 4\,\text{Subst}\left(\int \frac{1}{\sqrt{-39+24x+16x^2}}\,dx, x, \left(-\frac{1}{2}+x\right)^2\right) \\
&= 8\,\text{Subst}\left(\int \frac{1}{64-x^2}\,dx, x, \frac{8\left(3+4\left(-\frac{1}{2}+x\right)^2\right)}{\sqrt{-39+(1-2x)^4+24\left(-\frac{1}{2}+x\right)^2}}\right) \\
&= \tanh^{-1}\left(\frac{3+(-1+2x)^2}{\sqrt{-39+6(1-2x)^2+(1-2x)^4}}\right)
\end{aligned}$$

Mathematica [C] time = 3.20, size = 702, normalized size = 21.27

$$\frac{\sqrt{4\sqrt{3}-3}\left(-2x+\sqrt{4\sqrt{3}-3}+1\right)^2\left(\frac{-2x-i\sqrt{3+4\sqrt{3}}+1}{\left(\sqrt{4\sqrt{3}-3}-i\sqrt{3+4\sqrt{3}}\right)\left(-2x+\sqrt{4\sqrt{3}-3}+1\right)}\right)^{3/2}\left(-2x+i\sqrt{3+4\sqrt{3}}+1\right)\sqrt{\frac{\left(\sqrt{4\sqrt{3}-3}-i\sqrt{3+4\sqrt{3}}\right)\left(2x+\sqrt{4\sqrt{3}-3}-1\right)}{\left(\sqrt{4\sqrt{3}-3}+i\sqrt{3+4\sqrt{3}}\right)\left(-2x+\sqrt{4\sqrt{3}-3}+1\right)}}\left(F\left(\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{-3+4\sqrt{3}}-i\sqrt{3+4\sqrt{3}}\right)\left(2x+\sqrt{-3+4\sqrt{3}}-1\right)}{\left(\sqrt{-3+4\sqrt{3}}+i\sqrt{3+4\sqrt{3}}\right)\left(-2x+\sqrt{-3+4\sqrt{3}}+1\right)}}\right)\middle|\frac{3i+\sqrt{39}}{3i-\sqrt{39}}\right)-2\Pi\left(-\frac{\sqrt{-3+4\sqrt{3}}+i\sqrt{3+4\sqrt{3}}}{\sqrt{-3+4\sqrt{3}}-i\sqrt{3+4\sqrt{3}}};\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{-3+4\sqrt{3}}-i\sqrt{3+4\sqrt{3}}\right)\left(2x+\sqrt{-3+4\sqrt{3}}-1\right)}{\left(\sqrt{-3+4\sqrt{3}}+i\sqrt{3+4\sqrt{3}}\right)\left(-2x+\sqrt{-3+4\sqrt{3}}+1\right)}}\right)\middle|\frac{3i+\sqrt{39}}{3i-\sqrt{39}}\right)\right)}{\left(-2x-i\sqrt{3+4\sqrt{3}}+1\right)\sqrt{\frac{-2x+i\sqrt{3+4\sqrt{3}}+1}{\left(\sqrt{4\sqrt{3}-3}+i\sqrt{3+4\sqrt{3}}\right)\left(-2x+\sqrt{4\sqrt{3}-3}+1\right)}}\sqrt{x^4-2x^3+3x^2-2x-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 2*x)/Sqrt[-2 - 2*x + 3*x^2 - 2*x^3 + x^4],x]
```

```
[Out] (Sqrt[-3 + 4*Sqrt[3]]*(1 + Sqrt[-3 + 4*Sqrt[3]] - 2*x)^2*((1 - I*Sqrt[3 + 4
*Sqrt[3]] - 2*x)/((Sqrt[-3 + 4*Sqrt[3]] - I*Sqrt[3 + 4*Sqrt[3]])*(1 + Sqrt[
-3 + 4*Sqrt[3]] - 2*x)))^(3/2)*(1 + I*Sqrt[3 + 4*Sqrt[3]] - 2*x)*Sqrt[((Sqr
t[-3 + 4*Sqrt[3]] - I*Sqrt[3 + 4*Sqrt[3]])*(-1 + Sqrt[-3 + 4*Sqrt[3]] + 2*x
))/((Sqrt[-3 + 4*Sqrt[3]] + I*Sqrt[3 + 4*Sqrt[3]])*(1 + Sqrt[-3 + 4*Sqrt[3]
] - 2*x))]*(EllipticF[ArcSin[Sqrt[((Sqrt[-3 + 4*Sqrt[3]] - I*Sqrt[3 + 4*Sqr
t[3]])*(-1 + Sqrt[-3 + 4*Sqrt[3]] + 2*x))/((Sqrt[-3 + 4*Sqrt[3]] + I*Sqrt[3
 + 4*Sqrt[3]])*(1 + Sqrt[-3 + 4*Sqrt[3]] - 2*x))]], (3*I + Sqrt[39])/(3*I -
 Sqrt[39])] - 2*EllipticPi[-((Sqrt[-3 + 4*Sqrt[3]] + I*Sqrt[3 + 4*Sqrt[3]])
/(Sqrt[-3 + 4*Sqrt[3]] - I*Sqrt[3 + 4*Sqrt[3]])), ArcSin[Sqrt[((Sqrt[-3 + 4
*Sqrt[3]] - I*Sqrt[3 + 4*Sqrt[3]])*(-1 + Sqrt[-3 + 4*Sqrt[3]] + 2*x))/((Sqr
t[-3 + 4*Sqrt[3]] + I*Sqrt[3 + 4*Sqrt[3]])*(1 + Sqrt[-3 + 4*Sqrt[3]] - 2*x)
)]], (3*I + Sqrt[39])/(3*I - Sqrt[39])]))/((1 - I*Sqrt[3 + 4*Sqrt[3]] - 2*x
)*Sqrt[(1 + I*Sqrt[3 + 4*Sqrt[3]] - 2*x)/((Sqrt[-3 + 4*Sqrt[3]] + I*Sqrt[3
+ 4*Sqrt[3]])*(1 + Sqrt[-3 + 4*Sqrt[3]] - 2*x))]*Sqrt[-2 - 2*x + 3*x^2 - 2*
x^3 + x^4])
```

IntegrateAlgebraic [A] time = 0.16, size = 33, normalized size = 1.00

$$-\log\left(-x^2+\sqrt{x^4-2x^3+3x^2-2x-2}+x-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x)/Sqrt[-2 - 2*x + 3*x^2 - 2*x^3 + x^4],x]
```

```
[Out] -Log[-1 + x - x^2 + Sqrt[-2 - 2*x + 3*x^2 - 2*x^3 + x^4]]
```

fricas [A] time = 0.65, size = 31, normalized size = 0.94

$$\log\left(-x^2+x-\sqrt{x^4-2\,x^3+3\,x^2-2\,x-2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+3*x^2-2*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] log(-x^2 + x - sqrt(x^4 - 2*x^3 + 3*x^2 - 2*x - 2) - 1)
```

giac [A] time = 0.39, size = 34, normalized size = 1.03

$$-\log\left(x^2 - x - \sqrt{\left(x^2 - x\right)^2 + 2\,x^2 - 2\,x - 2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+3*x^2-2*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(x^2 - x - sqrt((x^2 - x)^2 + 2*x^2 - 2*x - 2) + 1)
```

maple [C] time = 0.66, size = 1358, normalized size = 41.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x)/(x^4-2*x^3+3*x^2-2*x-2)^(1/2),x)
```

```
[Out] -2*(-1/2*(-3+4*3^(1/2))^(1/2)-1/2*I*(3+4*3^(1/2))^(1/2))*((1/2*I*(3+4*3^(1/
2))^(1/2)-1/2*(-3+4*3^(1/2))^(1/2))*(x-1/2+1/2*(-3+4*3^(1/2))^(1/2))/(1/2*I
*(3+4*3^(1/2))^(1/2)+1/2*(-3+4*3^(1/2))^(1/2))/(x-1/2-1/2*(-3+4*3^(1/2))^(1
/2)))^(1/2)*(x-1/2-1/2*(-3+4*3^(1/2))^(1/2))^2*((-3+4*3^(1/2))^(1/2)*(x-1/2
+1/2*I*(3+4*3^(1/2))^(1/2))/(-1/2*I*(3+4*3^(1/2))^(1/2)+1/2*(-3+4*3^(1/2))^
(1/2))/(x-1/2-1/2*(-3+4*3^(1/2))^(1/2)))^(1/2)*((-3+4*3^(1/2))^(1/2)*(x-1/2
-1/2*I*(3+4*3^(1/2))^(1/2))/(1/2*I*(3+4*3^(1/2))^(1/2)+1/2*(-3+4*3^(1/2))^(
1/2))/(x-1/2-1/2*(-3+4*3^(1/2))^(1/2)))^(1/2)/(1/2*I*(3+4*3^(1/2))^(1/2)-1/
2*(-3+4*3^(1/2))^(1/2))/(-3+4*3^(1/2))^(1/2)/((x-1/2+1/2*(-3+4*3^(1/2))^(1/
2))*(x-1/2-1/2*(-3+4*3^(1/2))^(1/2))*(x-1/2+1/2*I*(3+4*3^(1/2))^(1/2))*(x-1
/2-1/2*I*(3+4*3^(1/2))^(1/2)))^(1/2)*EllipticF(((1/2*I*(3+4*3^(1/2))^(1/2)-
1/2*(-3+4*3^(1/2))^(1/2))*(x-1/2+1/2*(-3+4*3^(1/2))^(1/2))/(1/2*I*(3+4*3^(1
/2))^(1/2)+1/2*(-3+4*3^(1/2))^(1/2))/(x-1/2-1/2*(-3+4*3^(1/2))^(1/2)))^(1/2
),((1/2*I*(3+4*3^(1/2))^(1/2)+1/2*(-3+4*3^(1/2))^(1/2))*(-1/2*(-3+4*3^(1/2)
)^(1/2)-1/2*I*(3+4*3^(1/2))^(1/2))/(1/2*I*(3+4*3^(1/2))^(1/2)-1/2*(-3+4*3^(
1/2))^(1/2))/(-1/2*I*(3+4*3^(1/2))^(1/2)+1/2*(-3+4*3^(1/2))^(1/2)))^(1/2))+
4*(-1/2*(-3+4*3^(1/2))^(1/2)-1/2*I*(3+4*3^(1/2))^(1/2))*((1/2*I*(3+4*3^(1/2
))^(1/2)-1/2*(-3+4*3^(1/2))^(1/2))*(x-1/2+1/2*(-3+4*3^(1/2))^(1/2))/(1/2*I*
(3+4*3^(1/2))^(1/2)+1/2*(-3+4*3^(1/2))^(1/2))/(x-1/2-1/2*(-3+4*3^(1/2))^(1/
2)))^(1/2)*(x-1/2-1/2*(-3+4*3^(1/2))^(1/2))^2*((-3+4*3^(1/2))^(1/2)*(x-1/2+
1/2*I*(3+4*3^(1/2))^(1/2))/(-1/2*I*(3+4*3^(1/2))^(1/2)+1/2*(-3+4*3^(1/2))^(
1/2))/(x-1/2-1/2*(-3+4*3^(1/2))^(1/2)))^(1/2)*((-3+4*3^(1/2))^(1/2)*(x-1/2-
1/2*I*(3+4*3^(1/2))^(1/2))/(1/2*I*(3+4*3^(1/2))^(1/2)+1/2*(-3+4*3^(1/2))^(1
/2))/(x-1/2-1/2*(-3+4*3^(1/2))^(1/2)))^(1/2)/(1/2*I*(3+4*3^(1/2))^(1/2)-1/2
*(-3+4*3^(1/2))^(1/2))/(-3+4*3^(1/2))^(1/2)/((x-1/2+1/2*(-3+4*3^(1/2))^(1/2
))*(x-1/2-1/2*(-3+4*3^(1/2))^(1/2))*(x-1/2+1/2*I*(3+4*3^(1/2))^(1/2))*(x-1/
2-1/2*I*(3+4*3^(1/2))^(1/2)))^(1/2)*((1/2+1/2*(-3+4*3^(1/2))^(1/2))*Ellipti
cF(((1/2*I*(3+4*3^(1/2))^(1/2)-1/2*(-3+4*3^(1/2))^(1/2))*(x-1/2+1/2*(-3+4*3
^(1/2))^(1/2))/(1/2*I*(3+4*3^(1/2))^(1/2)+1/2*(-3+4*3^(1/2))^(1/2))/(x-1/2-
1/2*(-3+4*3^(1/2))^(1/2)))^(1/2),((1/2*I*(3+4*3^(1/2))^(1/2)+1/2*(-3+4*3^(1
/2))^(1/2))*(-1/2*(-3+4*3^(1/2))^(1/2)-1/2*I*(3+4*3^(1/2))^(1/2))/(1/2*I*(3
+4*3^(1/2))^(1/2)-1/2*(-3+4*3^(1/2))^(1/2))/(-1/2*I*(3+4*3^(1/2))^(1/2)+1/2
*(-3+4*3^(1/2))^(1/2)))^(1/2))-(-3+4*3^(1/2))^(1/2)*EllipticPi(((1/2*I*(3+4
*3^(1/2))^(1/2)-1/2*(-3+4*3^(1/2))^(1/2))*(x-1/2+1/2*(-3+4*3^(1/2))^(1/2))/
(1/2*I*(3+4*3^(1/2))^(1/2)+1/2*(-3+4*3^(1/2))^(1/2))/(x-1/2-1/2*(-3+4*3^(1/
2))^(1/2)))^(1/2),(1/2*I*(3+4*3^(1/2))^(1/2)+1/2*(-3+4*3^(1/2))^(1/2))/(1/2
*I*(3+4*3^(1/2))^(1/2)-1/2*(-3+4*3^(1/2))^(1/2)),((1/2*I*(3+4*3^(1/2))^(1/2
)+1/2*(-3+4*3^(1/2))^(1/2))*(-1/2*(-3+4*3^(1/2))^(1/2)-1/2*I*(3+4*3^(1/2))^
(1/2))/(1/2*I*(3+4*3^(1/2))^(1/2)-1/2*(-3+4*3^(1/2))^(1/2))/(-1/2*I*(3+4*3^
(1/2))^(1/2)+1/2*(-3+4*3^(1/2))^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x-1}{\sqrt{x^4-2\,x^3+3\,x^2-2\,x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+3*x^2-2*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x - 1)/sqrt(x^4 - 2*x^3 + 3*x^2 - 2*x - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{2\,x-1}{\sqrt{x^4-2\,x^3+3\,x^2-2\,x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 1)/(3*x^2 - 2*x - 2*x^3 + x^4 - 2)^(1/2),x)
```

```
[Out] int((2*x - 1)/(3*x^2 - 2*x - 2*x^3 + x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-1}{\sqrt{x^4-2x^3+3x^2-2x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x**4-2*x**3+3*x**2-2*x-2)**(1/2),x)
```

```
[Out] Integral((2*x - 1)/sqrt(x**4 - 2*x**3 + 3*x**2 - 2*x - 2), x)
```

3.409 $\int \frac{-1+2x}{\sqrt{-4-4x+5x^2-2x^3+x^4}}\,dx$

Optimal. Leaf size=33

$$-\log\left(-x^2+\sqrt{x^4-2x^3+5x^2-4x-4}+x-2\right)$$

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1680, 12, 1107, 621, 206}

$$\tanh^{-1}\left(\frac{4\left(x-\frac{1}{2}\right)^2+7}{\sqrt{16\left(x-\frac{1}{2}\right)^4+56\left(x-\frac{1}{2}\right)^2-79}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x)/Sqrt[-4 - 4*x + 5*x^2 - 2*x^3 + x^4],x]
```

```
[Out] ArcTanh[(7 + 4*(-1/2 + x)^2)/Sqrt[-79 + 56*(-1/2 + x)^2 + 16*(-1/2 + x)^4]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+2x}{\sqrt{-4-4x+5x^2-2x^3+x^4}}\,dx &= \text{Subst}\left(\int \frac{8x}{\sqrt{-79+56x^2+16x^4}}\,dx,x,-\frac{1}{2}+x\right)\\
&= 8\,\text{Subst}\left(\int \frac{x}{\sqrt{-79+56x^2+16x^4}}\,dx,x,-\frac{1}{2}+x\right)\\
&= 4\,\text{Subst}\left(\int \frac{1}{\sqrt{-79+56x+16x^2}}\,dx,x,\left(-\frac{1}{2}+x\right)^2\right)\\
&= 8\,\text{Subst}\left(\int \frac{1}{64-x^2}\,dx,x,\frac{8\left(7+4\left(-\frac{1}{2}+x\right)^2\right)}{\sqrt{-79+(1-2x)^4+56\left(-\frac{1}{2}+x\right)^2}}\right)\\
&= \tanh^{-1}\left(\frac{7+(-1+2x)^2}{\sqrt{-79+14(1-2x)^2+(1-2x)^4}}\right)
\end{aligned}$$

Mathematica [C] time = 3.00, size = 702, normalized size = 21.27

$$\frac{\sqrt{8\sqrt{2}-7}\left(-2x+\sqrt{8\sqrt{2}-7}+1\right)^2\left(\frac{-2x-i\sqrt{7+8\sqrt{2}}+1}{\left(\sqrt{8\sqrt{2}-7}-i\sqrt{7+8\sqrt{2}}\right)\left(-2x+\sqrt{8\sqrt{2}-7}+1\right)}\right)^{3/2}\left(-2x+i\sqrt{7+8\sqrt{2}}+1\right)\sqrt{\frac{\left(\sqrt{8\sqrt{2}-7}-i\sqrt{7+8\sqrt{2}}\right)\left(2x+\sqrt{8\sqrt{2}-7}-1\right)}{\left(\sqrt{8\sqrt{2}-7}+i\sqrt{7+8\sqrt{2}}\right)\left(-2x+\sqrt{8\sqrt{2}-7}+1\right)}}\left(F\left(\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{-7+8\sqrt{2}}-i\sqrt{7+8\sqrt{2}}\right)\left(2x+\sqrt{-7+8\sqrt{2}}-1\right)}{\left(\sqrt{-7+8\sqrt{2}}+i\sqrt{7+8\sqrt{2}}\right)\left(-2x+\sqrt{-7+8\sqrt{2}}+1\right)}}\right)|\frac{7i+\sqrt{79}}{7i-\sqrt{79}}\right)-2\Pi\left(-\frac{\sqrt{-7+8\sqrt{2}}+i\sqrt{7+8\sqrt{2}}}{\sqrt{-7+8\sqrt{2}}-i\sqrt{7+8\sqrt{2}}};\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{-7+8\sqrt{2}}-i\sqrt{7+8\sqrt{2}}\right)\left(2x+\sqrt{-7+8\sqrt{2}}-1\right)}{\left(\sqrt{-7+8\sqrt{2}}+i\sqrt{7+8\sqrt{2}}\right)\left(-2x+\sqrt{-7+8\sqrt{2}}+1\right)}}\right)|\frac{7i+\sqrt{79}}{7i-\sqrt{79}}\right)\right)}{\left(-2x-i\sqrt{7+8\sqrt{2}}+1\right)\sqrt{\frac{-2x+i\sqrt{7+8\sqrt{2}}+1}{\left(\sqrt{8\sqrt{2}-7}+i\sqrt{7+8\sqrt{2}}\right)\left(-2x+\sqrt{8\sqrt{2}-7}+1\right)}}\sqrt{x^4-2x^3+5x^2-4x-4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 2*x)/Sqrt[-4 - 4*x + 5*x^2 - 2*x^3 + x^4],x]
```

```
[Out] (Sqrt[-7 + 8*Sqrt[2]]*(1 + Sqrt[-7 + 8*Sqrt[2]] - 2*x)^2*((1 - I*Sqrt[7 + 8
*Sqrt[2]] - 2*x)/((Sqrt[-7 + 8*Sqrt[2]] - I*Sqrt[7 + 8*Sqrt[2]])*(1 + Sqrt[
-7 + 8*Sqrt[2]] - 2*x)))^(3/2)*(1 + I*Sqrt[7 + 8*Sqrt[2]] - 2*x)*Sqrt[((Sqr
t[-7 + 8*Sqrt[2]] - I*Sqrt[7 + 8*Sqrt[2]])*(-1 + Sqrt[-7 + 8*Sqrt[2]] + 2*x
))/((Sqrt[-7 + 8*Sqrt[2]] + I*Sqrt[7 + 8*Sqrt[2]])*(1 + Sqrt[-7 + 8*Sqrt[2]
] - 2*x))]*(EllipticF[ArcSin[Sqrt[((Sqrt[-7 + 8*Sqrt[2]] - I*Sqrt[7 + 8*Sqr
t[2]])*(-1 + Sqrt[-7 + 8*Sqrt[2]] + 2*x))/((Sqrt[-7 + 8*Sqrt[2]] + I*Sqrt[7
 + 8*Sqrt[2]])*(1 + Sqrt[-7 + 8*Sqrt[2]] - 2*x))]], (7*I + Sqrt[79])/(7*I -
 Sqrt[79])] - 2*EllipticPi[-((Sqrt[-7 + 8*Sqrt[2]] + I*Sqrt[7 + 8*Sqrt[2]])
/(Sqrt[-7 + 8*Sqrt[2]] - I*Sqrt[7 + 8*Sqrt[2]])), ArcSin[Sqrt[((Sqrt[-7 + 8
*Sqrt[2]] - I*Sqrt[7 + 8*Sqrt[2]])*(-1 + Sqrt[-7 + 8*Sqrt[2]] + 2*x))/((Sqr
t[-7 + 8*Sqrt[2]] + I*Sqrt[7 + 8*Sqrt[2]])*(1 + Sqrt[-7 + 8*Sqrt[2]] - 2*x)
)]], (7*I + Sqrt[79])/(7*I - Sqrt[79])]))/((1 - I*Sqrt[7 + 8*Sqrt[2]] - 2*x
)*Sqrt[(1 + I*Sqrt[7 + 8*Sqrt[2]] - 2*x)/((Sqrt[-7 + 8*Sqrt[2]] + I*Sqrt[7
+ 8*Sqrt[2]])*(1 + Sqrt[-7 + 8*Sqrt[2]] - 2*x))]*Sqrt[-4 - 4*x + 5*x^2 - 2*
x^3 + x^4])
```

IntegrateAlgebraic [A] time = 0.16, size = 33, normalized size = 1.00

$$-\log\left(-x^2+\sqrt{x^4-2x^3+5x^2-4x-4}+x-2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x)/Sqrt[-4 - 4*x + 5*x^2 - 2*x^3 + x^4],x]
```

```
[Out] -Log[-2 + x - x^2 + Sqrt[-4 - 4*x + 5*x^2 - 2*x^3 + x^4]]
```

fricas [A] time = 0.45, size = 29, normalized size = 0.88

$$\log\left(x^2-x+\sqrt{x^4-2\,x^3+5\,x^2-4\,x-4}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+5*x^2-4*x-4)^(1/2),x, algorithm="fricas")
```

```
[Out] log(x^2 - x + sqrt(x^4 - 2*x^3 + 5*x^2 - 4*x - 4) + 2)
```

giac [A] time = 0.39, size = 34, normalized size = 1.03

$$-\log\left(x^2 - x - \sqrt{\left(x^2 - x\right)^2 + 4\,x^2 - 4\,x - 4} + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+5*x^2-4*x-4)^(1/2),x, algorithm="giac")
```

```
[Out] -log(x^2 - x - sqrt((x^2 - x)^2 + 4*x^2 - 4*x - 4) + 2)
```

maple [C] time = 0.68, size = 1382, normalized size = 41.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x)/(x^4-2*x^3+5*x^2-4*x-4)^(1/2),x)
```

```
[Out] 2*I*(-1/2*I*(7+8*2^(1/2))^(1/2)-1/2*(-7+8*2^(1/2))^(1/2))*((1/2*(-7+8*2^(1/
2))^(1/2)-1/2*I*(7+8*2^(1/2))^(1/2))*(x-1/2+1/2*I*(7+8*2^(1/2))^(1/2))/(1/2
*(-7+8*2^(1/2))^(1/2)+1/2*I*(7+8*2^(1/2))^(1/2))/(x-1/2-1/2*I*(7+8*2^(1/2))
^(1/2)))^(1/2)*(x-1/2-1/2*I*(7+8*2^(1/2))^(1/2))^2*(I*(7+8*2^(1/2))^(1/2)*(
x-1/2+1/2*(-7+8*2^(1/2))^(1/2))/(-1/2*(-7+8*2^(1/2))^(1/2)+1/2*I*(7+8*2^(1/
2))^(1/2))/(x-1/2-1/2*I*(7+8*2^(1/2))^(1/2)))^(1/2)*(I*(7+8*2^(1/2))^(1/2)*
(x-1/2-1/2*(-7+8*2^(1/2))^(1/2))/(1/2*(-7+8*2^(1/2))^(1/2)+1/2*I*(7+8*2^(1/
2))^(1/2))/(x-1/2-1/2*I*(7+8*2^(1/2))^(1/2)))^(1/2)/(1/2*(-7+8*2^(1/2))^(1/
2)-1/2*I*(7+8*2^(1/2))^(1/2))/(7+8*2^(1/2))^(1/2)/((x-1/2+1/2*I*(7+8*2^(1/2
))^(1/2))*(x-1/2-1/2*I*(7+8*2^(1/2))^(1/2))*(x-1/2+1/2*(-7+8*2^(1/2))^(1/2)
)*(x-1/2-1/2*(-7+8*2^(1/2))^(1/2)))^(1/2)*EllipticF(((1/2*(-7+8*2^(1/2))^(1
/2)-1/2*I*(7+8*2^(1/2))^(1/2))*(x-1/2+1/2*I*(7+8*2^(1/2))^(1/2))/(1/2*(-7+8
*2^(1/2))^(1/2)+1/2*I*(7+8*2^(1/2))^(1/2))/(x-1/2-1/2*I*(7+8*2^(1/2))^(1/2)
))^(1/2),((1/2*(-7+8*2^(1/2))^(1/2)+1/2*I*(7+8*2^(1/2))^(1/2))*(-1/2*I*(7+8
*2^(1/2))^(1/2)-1/2*(-7+8*2^(1/2))^(1/2))/(1/2*(-7+8*2^(1/2))^(1/2)-1/2*I*(
7+8*2^(1/2))^(1/2))/(-1/2*(-7+8*2^(1/2))^(1/2)+1/2*I*(7+8*2^(1/2))^(1/2)))^
(1/2))-4*I*(-1/2*I*(7+8*2^(1/2))^(1/2)-1/2*(-7+8*2^(1/2))^(1/2))*((1/2*(-7+
8*2^(1/2))^(1/2)-1/2*I*(7+8*2^(1/2))^(1/2))*(x-1/2+1/2*I*(7+8*2^(1/2))^(1/2
))/(1/2*(-7+8*2^(1/2))^(1/2)+1/2*I*(7+8*2^(1/2))^(1/2))/(x-1/2-1/2*I*(7+8*2
^(1/2))^(1/2)))^(1/2)*(x-1/2-1/2*I*(7+8*2^(1/2))^(1/2))^2*(I*(7+8*2^(1/2))^
(1/2)*(x-1/2+1/2*(-7+8*2^(1/2))^(1/2))/(-1/2*(-7+8*2^(1/2))^(1/2)+1/2*I*(7+
8*2^(1/2))^(1/2))/(x-1/2-1/2*I*(7+8*2^(1/2))^(1/2)))^(1/2)*(I*(7+8*2^(1/2))
^(1/2)*(x-1/2-1/2*(-7+8*2^(1/2))^(1/2))/(1/2*(-7+8*2^(1/2))^(1/2)+1/2*I*(7+
8*2^(1/2))^(1/2))/(x-1/2-1/2*I*(7+8*2^(1/2))^(1/2)))^(1/2)/(1/2*(-7+8*2^(1/
2))^(1/2)-1/2*I*(7+8*2^(1/2))^(1/2))/(7+8*2^(1/2))^(1/2)/((x-1/2+1/2*I*(7+8
*2^(1/2))^(1/2))*(x-1/2-1/2*I*(7+8*2^(1/2))^(1/2))*(x-1/2+1/2*(-7+8*2^(1/2)
)^(1/2))*(x-1/2-1/2*(-7+8*2^(1/2))^(1/2)))^(1/2)*((1/2+1/2*I*(7+8*2^(1/2))^
(1/2))*EllipticF(((1/2*(-7+8*2^(1/2))^(1/2)-1/2*I*(7+8*2^(1/2))^(1/2))*(x-1
/2+1/2*I*(7+8*2^(1/2))^(1/2))/(1/2*(-7+8*2^(1/2))^(1/2)+1/2*I*(7+8*2^(1/2))
^(1/2))/(x-1/2-1/2*I*(7+8*2^(1/2))^(1/2)))^(1/2),((1/2*(-7+8*2^(1/2))^(1/2)
+1/2*I*(7+8*2^(1/2))^(1/2))*(-1/2*I*(7+8*2^(1/2))^(1/2)-1/2*(-7+8*2^(1/2))^
(1/2))/(1/2*(-7+8*2^(1/2))^(1/2)-1/2*I*(7+8*2^(1/2))^(1/2))/(-1/2*(-7+8*2^(
1/2))^(1/2)+1/2*I*(7+8*2^(1/2))^(1/2)))^(1/2))-I*(7+8*2^(1/2))^(1/2)*Ellipt
icPi(((1/2*(-7+8*2^(1/2))^(1/2)-1/2*I*(7+8*2^(1/2))^(1/2))*(x-1/2+1/2*I*(7+
8*2^(1/2))^(1/2))/(1/2*(-7+8*2^(1/2))^(1/2)+1/2*I*(7+8*2^(1/2))^(1/2))/(x-1
/2-1/2*I*(7+8*2^(1/2))^(1/2)))^(1/2),(1/2*(-7+8*2^(1/2))^(1/2)+1/2*I*(7+8*2
^(1/2))^(1/2))/(1/2*(-7+8*2^(1/2))^(1/2)-1/2*I*(7+8*2^(1/2))^(1/2)),((1/2*(
-7+8*2^(1/2))^(1/2)+1/2*I*(7+8*2^(1/2))^(1/2))*(-1/2*I*(7+8*2^(1/2))^(1/2)-
1/2*(-7+8*2^(1/2))^(1/2))/(1/2*(-7+8*2^(1/2))^(1/2)-1/2*I*(7+8*2^(1/2))^(1/
2))/(-1/2*(-7+8*2^(1/2))^(1/2)+1/2*I*(7+8*2^(1/2))^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x-1}{\sqrt{x^4-2\,x^3+5\,x^2-4\,x-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-2*x^3+5*x^2-4*x-4)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x - 1)/sqrt(x^4 - 2*x^3 + 5*x^2 - 4*x - 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{2\,x-1}{\sqrt{x^4-2\,x^3+5\,x^2-4\,x-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 1)/(5*x^2 - 4*x - 2*x^3 + x^4 - 4)^(1/2),x)
[Out] int((2*x - 1)/(5*x^2 - 4*x - 2*x^3 + x^4 - 4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-1}{\sqrt{x^4-2x^3+5x^2-4x-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x**4-2*x**3+5*x**2-4*x-4)**(1/2),x)
[Out] Integral((2*x - 1)/sqrt(x**4 - 2*x**3 + 5*x**2 - 4*x - 4), x)
```

3.410 $\int \frac{1}{x^4 \sqrt[4]{x^3+x^4}} \, dx$

Optimal. Leaf size=33

$$\frac{4\left(128x^3-96x^2+84x-77\right)\left(x^4+x^3\right)^{3/4}}{1155x^6}$$

Rubi [B] time = 0.10, antiderivative size = 73, normalized size of antiderivative = 2.21, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2016, 2014}

$$\frac{512\left(x^4+x^3\right)^{3/4}}{1155x^3}-\frac{128\left(x^4+x^3\right)^{3/4}}{385x^4}-\frac{4\left(x^4+x^3\right)^{3/4}}{15x^6}+\frac{16\left(x^4+x^3\right)^{3/4}}{55x^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(x^3 + x^4)^(1/4)),x]
```

```
[Out] (-4*(x^3 + x^4)^(3/4))/(15*x^6) + (16*(x^3 + x^4)^(3/4))/(55*x^5) - (128*(x
^3 + x^4)^(3/4))/(385*x^4) + (512*(x^3 + x^4)^(3/4))/(1155*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 \sqrt[4]{x^3+x^4}} \, dx &= -\frac{4\left(x^3+x^4\right)^{3/4}}{15x^6}-\frac{4}{5}\int \frac{1}{x^3 \sqrt[4]{x^3+x^4}} \, dx \\
&= -\frac{4\left(x^3+x^4\right)^{3/4}}{15x^6}+\frac{16\left(x^3+x^4\right)^{3/4}}{55x^5}+\frac{32}{55}\int \frac{1}{x^2 \sqrt[4]{x^3+x^4}} \, dx \\
&= -\frac{4\left(x^3+x^4\right)^{3/4}}{15x^6}+\frac{16\left(x^3+x^4\right)^{3/4}}{55x^5}-\frac{128\left(x^3+x^4\right)^{3/4}}{385x^4}-\frac{128}{385}\int \frac{1}{x \sqrt[4]{x^3+x^4}} \, dx \\
&= -\frac{4\left(x^3+x^4\right)^{3/4}}{15x^6}+\frac{16\left(x^3+x^4\right)^{3/4}}{55x^5}-\frac{128\left(x^3+x^4\right)^{3/4}}{385x^4}+\frac{512\left(x^3+x^4\right)^{3/4}}{1155x^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{4\left(x^3(x+1)\right)^{3/4}\left(128x^3-96x^2+84x-77\right)}{1155x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(x^3 + x^4)^(1/4)),x]
```

```
[Out] (4*(x^3*(1 + x))^(3/4)*(-77 + 84*x - 96*x^2 + 128*x^3))/(1155*x^6)
```

IntegrateAlgebraic [A] time = 0.24, size = 33, normalized size = 1.00

$$\frac{4\left(128x^3-96x^2+84x-77\right)\left(x^4+x^3\right)^{3/4}}{1155x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(x^3 + x^4)^(1/4)),x]
```

```
[Out] (4*(-77 + 84*x - 96*x^2 + 128*x^3)*(x^3 + x^4)^(3/4))/(1155*x^6)
```

fricas [A] time = 0.40, size = 29, normalized size = 0.88

$$\frac{4\left(x^4+x^3\right)^{\frac{3}{4}}\left(128\,x^3-96\,x^2+84\,x-77\right)}{1155\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+x^3)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/1155*(x^4 + x^3)^(3/4)*(128*x^3 - 96*x^2 + 84*x - 77)/x^6
```

giac [A] time = 0.31, size = 37, normalized size = 1.12

$$-\frac{4}{15}\left(\frac{1}{x}+1\right)^{\frac{15}{4}}+\frac{12}{11}\left(\frac{1}{x}+1\right)^{\frac{11}{4}}-\frac{12}{7}\left(\frac{1}{x}+1\right)^{\frac{7}{4}}+\frac{4}{3}\left(\frac{1}{x}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+x^3)^(1/4),x, algorithm="giac")
```

```
[Out] -4/15*(1/x + 1)^(15/4) + 12/11*(1/x + 1)^(11/4) - 12/7*(1/x + 1)^(7/4) + 4/
3*(1/x + 1)^(3/4)
```

maple [A] time = 0.00, size = 33, normalized size = 1.00

$$\frac{4\left(1+x\right)\left(128x^3-96x^2+84x-77\right)}{1155x^3\left(x^4+x^3\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^4+x^3)^(1/4),x)
```

```
[Out] 4/1155*(1+x)*(128*x^3-96*x^2+84*x-77)/x^3/(x^4+x^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+x^3\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+x^3)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + x^3)^(1/4)*x^4), x)
```

mupad [B] time = 0.20, size = 57, normalized size = 1.73

$$\frac{512\left(x^4+x^3\right)^{3/4}}{1155\,x^3}-\frac{128\left(x^4+x^3\right)^{3/4}}{385\,x^4}+\frac{16\left(x^4+x^3\right)^{3/4}}{55\,x^5}-\frac{4\left(x^4+x^3\right)^{3/4}}{15\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^3 + x^4)^(1/4)),x)
```

```
[Out] (512*(x^3 + x^4)^(3/4))/(1155*x^3) - (128*(x^3 + x^4)^(3/4))/(385*x^4) + (1
6*(x^3 + x^4)^(3/4))/(55*x^5) - (4*(x^3 + x^4)^(3/4))/(15*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt[4]{x^3\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**4+x**3)**(1/4),x)
```

```
[Out] Integral(1/(x**4*(x**3*(x + 1))**(1/4)), x)
```

3.411 $\int \frac{x}{\sqrt{-b+ax^4}}\,dx$

Optimal. Leaf size=33

$$\frac{\log\left(\sqrt{ax^4-b}+\sqrt{a}\,x^2\right)}{2\sqrt{a}}$$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {275, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,x^2}{\sqrt{ax^4-b}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-b + a*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[a]*x^2)/Sqrt[-b + a*x^4]]/(2*Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{-b+ax^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-b+ax^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^2}{\sqrt{-b+ax^4}}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\,x^2}{\sqrt{-b+ax^4}}\right)}{2\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.97

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,x^2}{\sqrt{ax^4-b}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-b + a*x^4],x]
```

[Out] ArcTanh[(Sqrt[a]*x^2)/Sqrt[-b + a*x^4]]/(2*Sqrt[a])

IntegrateAlgebraic [A] time = 0.16, size = 33, normalized size = 1.00

$$\frac{\log\left(\sqrt{ax^4-b}+\sqrt{a}\,x^2\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/Sqrt[-b + a*x^4],x]

[Out] Log[Sqrt[a]*x^2 + Sqrt[-b + a*x^4]]/(2*Sqrt[a])

fricas [A] time = 0.41, size = 67, normalized size = 2.03

$$\left[\frac{\log\left(2\,ax^4+2\,\sqrt{ax^4-b}\,\sqrt{a}\,x^2-b\right)}{4\,\sqrt{a}},-\frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,x^2}{\sqrt{ax^4-b}}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a*x^4-b)^(1/2),x, algorithm="fricas")

[Out] [1/4*log(2*a*x^4 + 2*sqrt(a*x^4 - b)*sqrt(a)*x^2 - b)/sqrt(a), -1/2*sqrt(-a)*arctan(sqrt(-a)*x^2/sqrt(a*x^4 - b))/a]

giac [A] time = 0.29, size = 27, normalized size = 0.82

$$-\frac{\log\left(\left|-\sqrt{a}\,x^2+\sqrt{ax^4-b}\right|\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a*x^4-b)^(1/2),x, algorithm="giac")

[Out] -1/2*log(abs(-sqrt(a)*x^2 + sqrt(a*x^4 - b)))/sqrt(a)

maple [A] time = 0.01, size = 26, normalized size = 0.79

$$\frac{\ln\left(\sqrt{a}\;x^2+\sqrt{a\,x^4-b}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a*x^4-b)^(1/2),x)

[Out] 1/2*ln(a^(1/2)*x^2+(a*x^4-b)^(1/2))/a^(1/2)

maxima [A] time = 0.79, size = 49, normalized size = 1.48

$$-\frac{\log\left(-\frac{\sqrt{a}-\frac{\sqrt{ax^4-b}}{x^2}}{\sqrt{a}+\frac{\sqrt{ax^4-b}}{x^2}}\right)}{4\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a*x^4-b)^(1/2),x, algorithm="maxima")

[Out] -1/4*log(-(sqrt(a) - sqrt(a*x^4 - b)/x^2)/(sqrt(a) + sqrt(a*x^4 - b)/x^2))/sqrt(a)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{a\,x^4-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^4 - b)^(1/2),x)
```

```
[Out] int(x/(a*x^4 - b)^(1/2), x)
```

sympy [A] time = 0.93, size = 53, normalized size = 1.61

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{a}\,x^2}{\sqrt{b}}\right)}{2\sqrt{a}} & \text{for}\ \left|\frac{ax^4}{b}\right| > 1 \\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{a}\,x^2}{\sqrt{b}}\right)}{2\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x**4-b)**(1/2),x)
```

```
[Out] Piecewise((acosh(sqrt(a)*x**2/sqrt(b))/(2*sqrt(a)), Abs(a*x**4/b) > 1), (-I
*asin(sqrt(a)*x**2/sqrt(b))/(2*sqrt(a)), True))
```

3.412 $\int \frac{(1+x^3)^{2/3}(1-x^3+2x^6)}{x^{12}}\,dx$

Optimal. Leaf size=33

$$\frac{(x^3+1)^{2/3}(-227x^9-142x^6+45x^3-40)}{440x^{11}}$$

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.48, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1486, 453, 271, 264}

$$-\frac{(x^3+1)^{5/3}}{11x^{11}}+\frac{17(x^3+1)^{5/3}}{88x^8}-\frac{227(x^3+1)^{5/3}}{440x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^3)^(2/3)*(1 - x^3 + 2*x^6))/x^12,x]
```

```
[Out] -1/11*(1 + x^3)^(5/3)/x^11 + (17*(1 + x^3)^(5/3))/(88*x^8) - (227*(1 + x^3)
^(5/3))/(440*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1486

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 2*n*p - n
+ 1)*(d + e*x^n)^(q + 1))/(e*f^(2*n*p - n + 1)*(m + 2*n*p + n*q + 1)), x] +
 Dist[1/(e*(m + 2*n*p + n*q + 1)), Int[(f*x)^m*(d + e*x^n)^q*ExpandToSum[e*
(m + 2*n*p + n*q + 1)*((a + b*x^n + c*x^(2*n))^p - c^p*x^(2*n*p)) - d*c^p*(
m + 2*n*p - n + 1)*x^(2*n*p - n), x], x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGtQ[p, 0]
&& GtQ[2*n*p, n - 1] &&  !IntegerQ[q] && NeQ[m + 2*n*p + n*q + 1, 0]
```

Rubi steps

$$\int \frac{(1+x^3)^{2/3}(1-x^3+2x^6)}{x^{12}}\,dx = -\frac{2(1+x^3)^{5/3}}{3x^8} - \frac{1}{3}\int \frac{(1+x^3)^{2/3}(-3+19x^3)}{x^{12}}\,dx$$
$$= -\frac{(1+x^3)^{5/3}}{11x^{11}} - \frac{2(1+x^3)^{5/3}}{3x^8} - \frac{227}{33}\int \frac{(1+x^3)^{2/3}}{x^9}\,dx$$
$$= -\frac{(1+x^3)^{5/3}}{11x^{11}} + \frac{17(1+x^3)^{5/3}}{88x^8} + \frac{227}{88}\int \frac{(1+x^3)^{2/3}}{x^6}\,dx$$
$$= -\frac{(1+x^3)^{5/3}}{11x^{11}} + \frac{17(1+x^3)^{5/3}}{88x^8} - \frac{227(1+x^3)^{5/3}}{440x^5}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$-\frac{(x^3+1)^{2/3}(227x^9+142x^6-45x^3+40)}{440x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)^(2/3)*(1 - x^3 + 2*x^6))/x^12,x]
[Out] -1/440*((1 + x^3)^(2/3)*(40 - 45*x^3 + 142*x^6 + 227*x^9))/x^11
```

IntegrateAlgebraic [A] time = 0.13, size = 33, normalized size = 1.00

$$\frac{(x^3+1)^{2/3}(-227x^9-142x^6+45x^3-40)}{440x^{11}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)^(2/3)*(1 - x^3 + 2*x^6))/x^12,x]
[Out] ((1 + x^3)^(2/3)*(-40 + 45*x^3 - 142*x^6 - 227*x^9))/(440*x^11)
```

fricas [A] time = 0.39, size = 29, normalized size = 0.88

$$-\frac{(227\,x^9+142\,x^6-45\,x^3+40)(x^3+1)^{\frac{2}{3}}}{440\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6-x^3+1)/x^12,x, algorithm="fricas")
[Out] -1/440*(227*x^9 + 142*x^6 - 45*x^3 + 40)*(x^3 + 1)^(2/3)/x^11
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2\,x^6-x^3+1)(x^3+1)^{\frac{2}{3}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6-x^3+1)/x^12,x, algorithm="giac")
[Out] integrate((2*x^6 - x^3 + 1)*(x^3 + 1)^(2/3)/x^12, x)
```

maple [A] time = 0.01, size = 36, normalized size = 1.09

$$-\frac{(1+x)(x^2-x+1)(227x^6-85x^3+40)(x^3+1)^{\frac{2}{3}}}{440x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)*(2*x^6-x^3+1)/x^12,x)
```

```
[Out] -1/440*(1+x)*(x^2-x+1)*(227*x^6-85*x^3+40)*(x^3+1)^(2/3)/x^11
```

maxima [A] time = 0.45, size = 37, normalized size = 1.12

$$-\frac{4\left(x^3+1\right)^{\frac{5}{3}}}{5\,x^5}+\frac{3\left(x^3+1\right)^{\frac{8}{3}}}{8\,x^8}-\frac{\left(x^3+1\right)^{\frac{11}{3}}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6-x^3+1)/x^12,x, algorithm="maxima")
```

```
[Out] -4/5*(x^3 + 1)^(5/3)/x^5 + 3/8*(x^3 + 1)^(8/3)/x^8 - 1/11*(x^3 + 1)^(11/3)/
x^11
```

mupad [B] time = 0.41, size = 49, normalized size = 1.48

$$\frac{9\left(x^3+1\right)^{2/3}}{88\,x^8}-\frac{71\left(x^3+1\right)^{2/3}}{220\,x^5}-\frac{227\left(x^3+1\right)^{2/3}}{440\,x^2}-\frac{\left(x^3+1\right)^{2/3}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)^(2/3)*(2*x^6 - x^3 + 1))/x^12,x)
```

```
[Out] (9*(x^3 + 1)^(2/3))/(88*x^8) - (71*(x^3 + 1)^(2/3))/(220*x^5) - (227*(x^3 +
 1)^(2/3))/(440*x^2) - (x^3 + 1)^(2/3)/(11*x^11)
```

sympy [B] time = 3.94, size = 260, normalized size = 7.88

$$\frac{2\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}+\frac{2\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{11}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}-\frac{\left(x^3+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{3x^2\Gamma\left(-\frac{2}{3}\right)}+\frac{2\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)}-\frac{4\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{11}{3}\right)}{9x^3\Gamma\left(-\frac{2}{3}\right)}+\frac{2\left(x^3+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{10\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{11}{3}\right)}{27x^6\Gamma\left(-\frac{2}{3}\right)}+\frac{5\left(x^3+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^8\Gamma\left(-\frac{2}{3}\right)}+\frac{40\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{11}{3}\right)}{27x^9\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)*(2*x**6-x**3+1)/x**12,x)
```

```
[Out] 2*(1 + x**(-3))**(2/3)*gamma(-5/3)/(3*gamma(-2/3)) + 2*(1 + x**(-3))**(2/3)
*gamma(-11/3)/(3*gamma(-2/3)) - (x**3 + 1)**(2/3)*gamma(-8/3)/(3*x**2*gamma
(-2/3)) + 2*(1 + x**(-3))**(2/3)*gamma(-5/3)/(3*x**3*gamma(-2/3)) - 4*(1 +
x**(-3))**(2/3)*gamma(-11/3)/(9*x**3*gamma(-2/3)) + 2*(x**3 + 1)**(2/3)*gam
ma(-8/3)/(9*x**5*gamma(-2/3)) + 10*(1 + x**(-3))**(2/3)*gamma(-11/3)/(27*x*
*6*gamma(-2/3)) + 5*(x**3 + 1)**(2/3)*gamma(-8/3)/(9*x**8*gamma(-2/3)) + 40
*(1 + x**(-3))**(2/3)*gamma(-11/3)/(27*x**9*gamma(-2/3))
```

3.413 $\int \frac{(3+x^4)\sqrt{x-x^5}}{1-2x^4-x^6+x^8}\,dx$

Optimal. Leaf size=33

$$\tan^{-1}\left(\frac{x^2}{\sqrt{x-x^5}}\right)+\tanh^{-1}\left(\frac{x^2}{\sqrt{x-x^5}}\right)$$

Rubi [F] time = 2.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(3+x^4)\sqrt{x-x^5}}{1-2x^4-x^6+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((3 + x^4)*Sqrt[x - x^5])/(1 - 2*x^4 - x^6 + x^8),x]
```

```
[Out] (-3*Sqrt[x - x^5]*Defer[Subst][Defer[Int][(x^2*Sqrt[1 - x^8])/(-1 - x^6 + x
^8), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 - x^4]) + (4*Sqrt[x - x^5]*Defer[Subs
t][Defer[Int][(x^4*Sqrt[1 - x^8])/(-1 - x^6 + x^8), x], x, Sqrt[x]])/(Sqrt[
x]*Sqrt[1 - x^4]) - (3*Sqrt[x - x^5]*Defer[Subst][Defer[Int][(x^2*Sqrt[1 -
x^8])/(-1 + x^6 + x^8), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 - x^4]) - (4*Sqrt[
x - x^5]*Defer[Subst][Defer[Int][(x^4*Sqrt[1 - x^8])/(-1 + x^6 + x^8), x],
x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 - x^4])
```

Rubi steps

$$\int \frac{(3+x^4)\sqrt{x-x^5}}{1-2x^4-x^6+x^8}\,dx = \frac{\sqrt{x-x^5}\int \frac{\sqrt{x}\sqrt{1-x^4}(3+x^4)}{1-2x^4-x^6+x^8}\,dx}{\sqrt{x}\sqrt{1-x^4}}$$

$$= \frac{\left(2\sqrt{x-x^5}\right)\text{Subst}\left(\int \frac{x^2\sqrt{1-x^8}(3+x^8)}{1-2x^8-x^{12}+x^{16}}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1-x^4}}$$

$$= \frac{\left(2\sqrt{x-x^5}\right)\text{Subst}\left(\int \left(\frac{x^2(3-4x^2)\sqrt{1-x^8}}{2(1+x^6-x^8)}-\frac{x^2(3+4x^2)\sqrt{1-x^8}}{2(-1+x^6+x^8)}\right)dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1-x^4}}$$

$$= \frac{\sqrt{x-x^5}\,\text{Subst}\left(\int \frac{x^2(3-4x^2)\sqrt{1-x^8}}{1+x^6-x^8}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1-x^4}}-\frac{\sqrt{x-x^5}\,\text{Subst}\left(\int \frac{x^2(3+4x^2)\sqrt{1-x^8}}{-1+x^6+x^8}\,dx,x,\ldots\right.}{\sqrt{x}\sqrt{1-x^4}}$$

$$= \frac{\sqrt{x-x^5}\,\text{Subst}\left(\int \left(-\frac{3x^2\sqrt{1-x^8}}{-1-x^6+x^8}+\frac{4x^4\sqrt{1-x^8}}{-1-x^6+x^8}\right)dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1-x^4}}-\frac{\sqrt{x-x^5}\,\text{Subst}\left(\int \left(\frac{3x^2\sqrt{1-x^8}}{-1+x^6+x^8}\ldots\right.\right.}{\sqrt{x}\sqrt{1-\ldots}}$$

$$= -\frac{\left(3\sqrt{x-x^5}\right)\text{Subst}\left(\int \frac{x^2\sqrt{1-x^8}}{-1-x^6+x^8}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1-x^4}}-\frac{\left(3\sqrt{x-x^5}\right)\text{Subst}\left(\int \frac{x^2\sqrt{1-x^8}}{-1+x^6+x^8}\,dx,x,\sqrt{x}\ldots\right.}{\sqrt{x}\sqrt{1-x^4}}$$

Mathematica [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{(3+x^4)\sqrt{x-x^5}}{1-2x^4-x^6+x^8}\,dx$$

Verification is not applicable to the result.

[In] Integrate[((3 + x^4)*Sqrt[x - x^5])/(1 - 2*x^4 - x^6 + x^8),x]

[Out] Integrate[((3 + x^4)*Sqrt[x - x^5])/(1 - 2*x^4 - x^6 + x^8), x]

IntegrateAlgebraic [A] time = 0.91, size = 33, normalized size = 1.00

$$\tan^{-1}\left(\frac{x^2}{\sqrt{x-x^5}}\right)+\tanh^{-1}\left(\frac{x^2}{\sqrt{x-x^5}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((3 + x^4)*Sqrt[x - x^5])/(1 - 2*x^4 - x^6 + x^8),x]

[Out] ArcTan[x^2/Sqrt[x - x^5]] + ArcTanh[x^2/Sqrt[x - x^5]]

fricas [B] time = 0.43, size = 71, normalized size = 2.15

$$-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^5+x}\left(x^4+x^3-1\right)}{2\left(x^6-x^2\right)}\right)+\frac{1}{2}\log\left(-\frac{x^4-x^3-2\sqrt{-x^5+x}\,x-1}{x^4+x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3)*(-x^5+x)^(1/2)/(x^8-x^6-2*x^4+1),x, algorithm="fricas")

[Out] -1/2*arctan(1/2*sqrt(-x^5 + x)*(x^4 + x^3 - 1)/(x^6 - x^2)) + 1/2*log(-(x^4 - x^3 - 2*sqrt(-x^5 + x)*x - 1)/(x^4 + x^3 - 1))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^5+x}\left(x^4+3\right)}{x^8-x^6-2x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3)*(-x^5+x)^(1/2)/(x^8-x^6-2*x^4+1),x, algorithm="giac")

[Out] integrate(sqrt(-x^5 + x)*(x^4 + 3)/(x^8 - x^6 - 2*x^4 + 1), x)

maple [C] time = 0.78, size = 101, normalized size = 3.06

$$\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2+1\right)x^4-\mathrm{RootOf}\left(\_Z^2+1\right)x^3+2\sqrt{-x^5+x}\,x+\mathrm{RootOf}\left(\_Z^2+1\right)}{x^4-x^3-1}\right)}{2}+\frac{\ln\left(\frac{-x^4+x^3+2\sqrt{-x^5+x}\,x+1}{x^4+x^3-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+3)*(-x^5+x)^(1/2)/(x^8-x^6-2*x^4+1),x)

[Out] 1/2*RootOf(_Z^2+1)*ln((-RootOf(_Z^2+1)*x^4-RootOf(_Z^2+1)*x^3+2*(-x^5+x)^(1/2)*x+RootOf(_Z^2+1))/(x^4-x^3-1))+1/2*ln((-x^4+x^3+2*(-x^5+x)^(1/2)*x+1)/(x^4+x^3-1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^5+x}\left(x^4+3\right)}{x^8-x^6-2x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3)*(-x^5+x)^(1/2)/(x^8-x^6-2*x^4+1),x, algorithm="maxima")

[Out] integrate(sqrt(-x^5 + x)*(x^4 + 3)/(x^8 - x^6 - 2*x^4 + 1), x)

mupad [B] time = 3.24, size = 75, normalized size = 2.27

$$\frac{\ln\left(\frac{2\,x\,\sqrt{x-x^5}+x^3-x^4+1}{x^4+x^3-1}\right)}{2}+\frac{\ln\left(\frac{x^3+x^4-1+x\,\sqrt{x-x^5}\,2\mathrm{i}}{-x^4+x^3+1}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x - x^5)^(1/2)*(x^4 + 3))/(2*x^4 + x^6 - x^8 - 1),x)
```

```
[Out] log((2*x*(x - x^5)^(1/2) + x^3 - x^4 + 1)/(x^3 + x^4 - 1))/2 + (log((x*(x -
 x^5)^(1/2)*2i + x^3 + x^4 - 1)/(x^3 - x^4 + 1))*1i)/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3)*(-x**5+x)**(1/2)/(x**8-x**6-2*x**4+1),x)
```

```
[Out] Timed out
```

3.414 $\int \frac{-1+x^{16}}{x^8\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=33

$$\frac{\sqrt{x^4-1}\,\left(3x^{12}+5x^8-5x^4-3\right)}{21x^7}$$

Rubi [A] time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.42, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1586, 1835, 1585, 1486, 449}

$$\frac{1}{7}x\left(x^4-1\right)^{3/2}+\frac{\left(x^4-1\right)^{3/2}}{7x^7}+\frac{8\left(x^4-1\right)^{3/2}}{21x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^16)/(x^8*Sqrt[-1 + x^4]),x]
```

```
[Out] (-1 + x^4)^(3/2)/(7*x^7) + (8*(-1 + x^4)^(3/2))/(21*x^3) + (x*(-1 + x^4)^(3
/2))/7
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1486

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 2*n*p - n
+ 1)*(d + e*x^n)^(q + 1))/(e*f^(2*n*p - n + 1)*(m + 2*n*p + n*q + 1)), x] +
 Dist[1/(e*(m + 2*n*p + n*q + 1)), Int[(f*x)^m*(d + e*x^n)^q*ExpandToSum[e*
(m + 2*n*p + n*q + 1)*((a + b*x^n + c*x^(2*n))^p - c^p*x^(2*n*p)) - d*c^p*(
m + 2*n*p - n + 1)*x^(2*n*p - n), x], x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGtQ[p, 0]
&& GtQ[2*n*p, n - 1] &&  !IntegerQ[q] && NeQ[m + 2*n*p + n*q + 1, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x^{16}}{x^8\sqrt{-1+x^4}}\,dx &= \int \frac{\sqrt{-1+x^4}\left(1+x^4+x^8+x^{12}\right)}{x^8}\,dx\\
&= \frac{\left(-1+x^4\right)^{3/2}}{7x^7}+\frac{1}{14}\int \frac{\sqrt{-1+x^4}\left(16x^3+14x^7+14x^{11}\right)}{x^7}\,dx\\
&= \frac{\left(-1+x^4\right)^{3/2}}{7x^7}+\frac{1}{14}\int \frac{\sqrt{-1+x^4}\left(16+14x^4+14x^8\right)}{x^4}\,dx\\
&= \frac{\left(-1+x^4\right)^{3/2}}{7x^7}+\frac{1}{7}x\left(-1+x^4\right)^{3/2}+\frac{1}{98}\int \frac{\sqrt{-1+x^4}\left(112+112x^4\right)}{x^4}\,dx\\
&= \frac{\left(-1+x^4\right)^{3/2}}{7x^7}+\frac{8\left(-1+x^4\right)^{3/2}}{21x^3}+\frac{1}{7}x\left(-1+x^4\right)^{3/2}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 85, normalized size = 2.58

$$
\frac{3\sqrt{1-x^4}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};-\frac{3}{4};x^4\right)+x^8\left(5\sqrt{1-x^4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};x^4\right)+3x^8+2x^4-5\right)}{21x^7\sqrt{x^4-1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^16)/(x^8*Sqrt[-1 + x^4]),x]
```

```
[Out] (3*Sqrt[1 - x^4]*Hypergeometric2F1[-7/4, 1/2, -3/4, x^4] + x^8*(-5 + 2*x^4
+ 3*x^8 + 5*Sqrt[1 - x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, x^4]))/(21*x^7*S
qrt[-1 + x^4])
```

IntegrateAlgebraic [A] time = 0.24, size = 33, normalized size = 1.00

$$
\frac{\sqrt{x^4-1}\left(3x^{12}+5x^8-5x^4-3\right)}{21x^7}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^16)/(x^8*Sqrt[-1 + x^4]),x]
```

```
[Out] (Sqrt[-1 + x^4]*(-3 - 5*x^4 + 5*x^8 + 3*x^12))/(21*x^7)
```

fricas [A] time = 0.42, size = 29, normalized size = 0.88

$$
\frac{\left(3\,x^{12}+5\,x^8-5\,x^4-3\right)\sqrt{x^4-1}}{21\,x^7}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^16-1)/x^8/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/21*(3*x^12 + 5*x^8 - 5*x^4 - 3)*sqrt(x^4 - 1)/x^7
```

giac [A] time = 0.43, size = 39, normalized size = 1.18

$$
\frac{1}{21}\left(3\,x^4+5\right)\sqrt{x^4-1}\,x-\frac{\left(\frac{3}{x^4}+5\right)\sqrt{-\frac{1}{x^4}+1}}{21\,x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^16-1)/x^8/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/21*(3*x^4 + 5)*sqrt(x^4 - 1)*x - 1/21*(3/x^4 + 5)*sqrt(-1/x^4 + 1)/x
```

maple [A] time = 0.01, size = 36, normalized size = 1.09

$$\frac{\left(x^{2}+1\right)\left(1+x\right)\left(-1+x\right)\left(3x^{8}+8x^{4}+3\right)\sqrt{x^{4}-1}}{21x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^16-1)/x^8/(x^4-1)^(1/2),x)
```

```
[Out] 1/21*(x^2+1)*(1+x)*(-1+x)*(3*x^8+8*x^4+3)*(x^4-1)^(1/2)/x^7
```

maxima [A] time = 0.53, size = 39, normalized size = 1.18

$$\frac{\left(3\,x^{12}+5\,x^{8}-5\,x^{4}-3\right)\sqrt{x^{2}+1}\,\sqrt{x+1}\,\sqrt{x-1}}{21\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^16-1)/x^8/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/21*(3*x^12 + 5*x^8 - 5*x^4 - 3)*sqrt(x^2 + 1)*sqrt(x + 1)*sqrt(x - 1)/x^7
```

mupad [B] time = 0.34, size = 29, normalized size = 0.88

$$-\frac{\sqrt{x^{4}-1}\,\left(-3\,x^{12}-5\,x^{8}+5\,x^{4}+3\right)}{21\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^16 - 1)/(x^8*(x^4 - 1)^(1/2)),x)
```

```
[Out] -((x^4 - 1)^(1/2)*(5*x^4 - 5*x^8 - 3*x^12 + 3))/(21*x^7)
```

sympy [C] time = 2.73, size = 60, normalized size = 1.82

$$-\frac{ix^{9}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|x^{4}\right)}{4\Gamma\left(\frac{13}{4}\right)}+\frac{i\Gamma\left(-\frac{7}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{7}{4},\frac{1}{2}\\-\frac{3}{4}\end{matrix}\right|x^{4}\right)}{4x^{7}\Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**16-1)/x**8/(x**4-1)**(1/2),x)
```

```
[Out] -I*x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), x**4)/(4*gamma(13/4)) + I*gam
ma(-7/4)*hyper((-7/4, 1/2), (-3/4,), x**4)/(4*x**7*gamma(-3/4))
```

3.415 $\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=33

$$\frac{2\sqrt{x-1}}{1-x}+\frac{2\sqrt{x+1}}{-x-1}$$

Rubi [A] time = 0.29, antiderivative size = 19, normalized size of antiderivative = 0.58, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {6688}

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx &= \int \left(\frac{1}{(-1+x)^{3/2}}+\frac{1}{(1+x)^{3/2}}\right)dx\\ &= -\frac{2}{\sqrt{-1+x}}-\frac{2}{\sqrt{1+x}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 0.58

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),x
]
```

```
[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

IntegrateAlgebraic [A] time = 0.44, size = 19, normalized size = 0.58

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)
^(3/2)),x]
```

```
[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

fricas [A] time = 0.39, size = 28, normalized size = 0.85

$$-\frac{2\left((x+1)\sqrt{x-1}+\sqrt{x+1}\,(x-1)\right)}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"fricas")
```

```
[Out] -2*((x + 1)*sqrt(x - 1) + sqrt(x + 1)*(x - 1))/(x^2 - 1)
```

giac [A] time = 0.39, size = 15, normalized size = 0.45

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"giac")
```

```
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

maple [A] time = 0.00, size = 16, normalized size = 0.48

$$-\frac{2}{\sqrt{1+x}}-\frac{2}{\sqrt{-1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x)
```

```
[Out] -2/(1+x)^(1/2)-2/(-1+x)^(1/2)
```

maxima [A] time = 0.34, size = 15, normalized size = 0.45

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

mupad [B] time = 0.28, size = 15, normalized size = 0.45

$$-\frac{2}{\sqrt{x-1}}-\frac{2}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)^(3/2) + (x + 1)^(3/2))/((x - 1)^(3/2)*(x + 1)^(3/2)),x)
```

```
[Out] - 2/(x - 1)^(1/2) - 2/(x + 1)^(1/2)
```

sympy [B] time = 4.91, size = 56, normalized size = 1.70

$$-\frac{2x\sqrt{x-1}}{x^2-1}-\frac{2x\sqrt{x+1}}{x^2-1}-\frac{2\sqrt{x-1}}{x^2-1}+\frac{2\sqrt{x+1}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)**(3/2)+(1+x)**(3/2))/(-1+x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] -2*x*sqrt(x - 1)/(x**2 - 1) - 2*x*sqrt(x + 1)/(x**2 - 1) - 2*sqrt(x - 1)/(x
**2 - 1) + 2*sqrt(x + 1)/(x**2 - 1)
```

3.416 $\int \frac{1+x}{(-1+x)\sqrt{x+x^2+x^3}}\,dx$

Optimal. Leaf size=34

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x^3+x^2+x}}{x^2+x+1}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.41, antiderivative size = 56, normalized size of antiderivative = 1.65, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2056, 6733, 1698, 207}

$$-\frac{2\sqrt{x}\sqrt{x^2+x+1}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x}}{\sqrt{x^2+x+1}}\right)}{\sqrt{3}\sqrt{x^3+x^2+x}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-1 + x)*Sqrt[x + x^2 + x^3]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[1 + x + x^2]*ArcTanh[(Sqrt[3]*Sqrt[x])/Sqrt[1 + x + x^2]])
/(Sqrt[3]*Sqrt[x + x^2 + x^3])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{1+x}{(-1+x)\sqrt{x+x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1+x}{(-1+x)\sqrt{x}\sqrt{1+x+x^2}}\,dx}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{1+x+x^2}\right)\text{Subst}\left(\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt{1+x^2+x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{1+x+x^2}\right)\text{Subst}\left(\int \frac{1}{-1+3x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{1+x+x^2}}\right)}{\sqrt{x+x^2+x^3}}$$

$$= -\frac{2\sqrt{x}\sqrt{1+x+x^2}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x}}{\sqrt{1+x+x^2}}\right)}{\sqrt{3}\sqrt{x+x^2+x^3}}$$

Mathematica [C] time = 2.28, size = 632, normalized size = 18.59

$$-\frac{2(-1)^{2/3}\left(\frac{1}{x}-1\right)\sqrt{x}(x+1)\left(\sqrt[3]{-1}\left(\sqrt[3]{-1}-1\right)^2\left(1+\sqrt[3]{-1}\right)\sqrt{1-\frac{(-1)^{2/3}}{x}}\sqrt{\frac{x+\sqrt[3]{-1}}{x}}F\left(i\sinh^{-1}\left(\frac{(-1)^{5/6}}{\sqrt{x}}\right)\middle|(-1)^{2/3}\right)+\frac{2i\sqrt{3}\sqrt{\frac{\sqrt{x}-\sqrt[3]{-1}+1}{\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt{x}-1\right)}}\sqrt{\frac{(-1)^{2/3}\sqrt{x}-1}{\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt{x}-1\right)}}\sqrt{-\frac{(-1)^{2/3}\sqrt{x}+1}{\sqrt{x}+\sqrt[3]{-1}-1}}\left(\sqrt[3]{-1}\sqrt{x}-1\right)^2\left(\left(1+\sqrt[3]{-1}\right)F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt{x}-\sqrt[3]{-1}+1}{\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt{x}-1\right)}}\right)\middle|-3\right)-2\sqrt[3]{-1}\Pi\left(-1;\sin^{-1}\left(\sqrt{\frac{\sqrt{x}-\sqrt[3]{-1}+1}{\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt{x}-1\right)}}\right)\middle|-3\right)\right)}{x}-\frac{2i\sqrt{3}\sqrt{\frac{\sqrt{x}-\sqrt[3]{-1}+1}{\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt{x}-1\right)}}\sqrt{\frac{(-1)^{2/3}\sqrt{x}-1}{\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt{x}-1\right)}}\sqrt{-\frac{(-1)^{2/3}\sqrt{x}+1}{\sqrt{x}+\sqrt[3]{-1}-1}}\left(\sqrt[3]{-1}\sqrt{x}-1\right)^2\left(\left(\sqrt[3]{-1}-1\right)F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt{x}-\sqrt[3]{-1}+1}{\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt{x}-1\right)}}\right)\middle|-3\right)-2\sqrt[3]{-1}\Pi\left(3;\sin^{-1}\left(\sqrt{\frac{\sqrt{x}-\sqrt[3]{-1}+1}{\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt{x}-1\right)}}\right)\middle|-3\right)\right)}{x}\right)}{\left((-1)^{2/3}-1\right)\left(\frac{1}{x^2}-1\right)\sqrt{x\left(x^2+x+1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x)/((-1 + x)*Sqrt[x + x^2 + x^3]),x]
```

```
[Out] (-2*(-1)^(2/3)*(-1 + x^(-1))*Sqrt[x]*(1 + x)*((-1)^(1/3)*(-1 + (-1)^(1/3))^
2*(1 + (-1)^(1/3))*Sqrt[1 - (-1)^(2/3)/x]*Sqrt[((-1)^(1/3) + x)/x]*Elliptic
F[I*ArcSinh[(-1)^(5/6)/Sqrt[x]], (-1)^(2/3)] + ((2*I)*Sqrt[3]*Sqrt[(1 - (-1
)^(1/3) + Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x]))]*(-1 + (-1)
^(1/3)*Sqrt[x])^2*Sqrt[(-1 + (-1)^(2/3)*Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-
1)^(1/3)*Sqrt[x]))]*Sqrt[-((1 + (-1)^(2/3)*Sqrt[x])/(-1 + (-1)^(1/3) + Sqrt
[x]))]*((1 + (-1)^(1/3))*EllipticF[ArcSin[Sqrt[(1 - (-1)^(1/3) + Sqrt[x])/(
(1 + (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x]))]], -3] - 2*(-1)^(1/3)*EllipticP
i[-1, ArcSin[Sqrt[(1 - (-1)^(1/3) + Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-1)^(
1/3)*Sqrt[x]))]], -3]))/x - ((2*I)*Sqrt[3]*Sqrt[(1 - (-1)^(1/3) + Sqrt[x])/
((1 + (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x]))]*(-1 + (-1)^(1/3)*Sqrt[x])^2*S
qrt[(-1 + (-1)^(2/3)*Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x]))]
*Sqrt[-((1 + (-1)^(2/3)*Sqrt[x])/(-1 + (-1)^(1/3) + Sqrt[x]))]*((-1 + (-1)^
(1/3))*EllipticF[ArcSin[Sqrt[(1 - (-1)^(1/3) + Sqrt[x])/((1 + (-1)^(1/3))*(
-1 + (-1)^(1/3)*Sqrt[x]))]], -3] - 2*(-1)^(1/3)*EllipticPi[3, ArcSin[Sqrt[(
1 - (-1)^(1/3) + Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x]))]], -
3]))/x))/((-1 + (-1)^(2/3))*(-1 + x^(-2))*Sqrt[x*(1 + x + x^2)])
```

IntegrateAlgebraic [A] time = 0.11, size = 34, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x^3+x^2+x}}{x^2+x+1}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((-1 + x)*Sqrt[x + x^2 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3]*Sqrt[x + x^2 + x^3])/(1 + x + x^2)])/Sqrt[3]
```

fricas [B] time = 0.43, size = 68, normalized size = 2.00

$$\frac{1}{6}\sqrt{3}\log\left(\frac{x^4+20\,x^3-4\,\sqrt{3}\sqrt{x^3+x^2+x}\left(x^2+4\,x+1\right)+30\,x^2+20\,x+1}{x^4-4\,x^3+6\,x^2-4\,x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^3+x^2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((x^4 + 20*x^3 - 4*sqrt(3)*sqrt(x^3 + x^2 + x)*(x^2 + 4*x +
1) + 30*x^2 + 20*x + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^3+x^2+x}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^3+x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + x^2 + x)*(x - 1)), x)
```

maple [C] time = 0.20, size = 271, normalized size = 7.97

$$\frac{2\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}},\frac{\sqrt{3}\,\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{3\sqrt{x^3+x^2+x}}+\frac{4\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}},\frac{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}},\frac{\sqrt{3}\,\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{3\sqrt{x^3+x^2+x}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-1+x)/(x^3+x^2+x)^(1/2),x)
```

```
[Out] 2/3*(1/2+1/2*I*3^(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3
^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/
2)/(x^3+x^2+x)^(1/2)*EllipticF(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^
(1/2),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))+4/3*(1/2+1/2*I*3^
(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3^(1/2)*(I*(x+1/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1
/2)/(-3/2-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/
2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^3+x^2+x}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^3+x^2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + x^2 + x)*(x - 1)), x)
```

mupad [B] time = 0.32, size = 179, normalized size = 5.26

$$\frac{\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\left(\sqrt{3}+1i\right)\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-2\,\Pi\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\right)1i}{\sqrt{x^3+x^2-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x - 1)*(x + x^2 + x^3)^(1/2)),x)
```

```
[Out] ((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i
)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/2)
*(3^(1/2) + 1i)*(ellipticF(asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/
2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)) - 2*ellipticPi((3^(1/2)*1i)/2 - 1/2
```

```
, asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)
*1i)/2 + 1/2)))*1i)/(x^2 + x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x\left(x^2+x+1\right)}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x**3+x**2+x)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt(x*(x**2 + x + 1))*(x - 1)), x)
```

3.417 $\int \frac{-1+x^2}{\left(1-x+x^2\right)\sqrt{x+x^2+x^3}}\,dx$

Optimal. Leaf size=34

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x^3+x^2+x}}{x^2+x+1}\right)$$

Rubi [C] time = 1.21, antiderivative size = 315, normalized size of antiderivative = 9.26, number of steps used = 17, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2056, 6728, 716, 1103, 934, 169, 538, 537}

$$\frac{\sqrt{x}(x+1)\sqrt{\frac{x^2+x+1}{(x+1)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x^3+x^2+x}}-\frac{4\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\Pi\left(-1;\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}-\frac{\left(1+i\sqrt{3}\right)\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\Pi\left(-\frac{i+\sqrt{3}}{i-\sqrt{3}};\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{\sqrt{x^3+x^2+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^2)/((1 - x + x^2)*Sqrt[x + x^2 + x^3]),x]
```

```
[Out] (Sqrt[x]*(1 + x)*Sqrt[(1 + x + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]],
 1/4])/Sqrt[x + x^2 + x^3] - (4*Sqrt[x]*Sqrt[1 + (2*x)/(1 - I*Sqrt[3])]*Sqr
t[1 + (2*x)/(1 + I*Sqrt[3])]*EllipticPi[-1, ArcSin[((1 - I*Sqrt[3])*Sqrt[x]
)/2], (I + Sqrt[3])/(I - Sqrt[3])])/((1 - I*Sqrt[3])*Sqrt[x + x^2 + x^3]) -
 ((1 + I*Sqrt[3])*Sqrt[x]*Sqrt[1 + (2*x)/(1 - I*Sqrt[3])]*Sqrt[1 + (2*x)/(1
 + I*Sqrt[3])]*EllipticPi[-((I + Sqrt[3])/(I - Sqrt[3])), ArcSin[((1 - I*Sq
rt[3])*Sqrt[x])/2], (I + Sqrt[3])/(I - Sqrt[3])])/Sqrt[x + x^2 + x^3]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 716

```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2,
Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
```

```
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-1+x^2}{\left(1-x+x^2\right)\sqrt{x+x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}\left(1-x+x^2\right)\sqrt{1+x+x^2}}\,dx}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{1+x+x^2}} - \frac{2-x}{\sqrt{x}\left(1-x+x^2\right)\sqrt{1+x+x^2}}\right)dx}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+x+x^2}}\,dx}{\sqrt{x+x^2+x^3}} - \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{2-x}{\sqrt{x}\left(1-x+x^2\right)\sqrt{1+x+x^2}}}{\sqrt{x+x^2+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \left(\frac{-1-i\sqrt{3}}{\sqrt{x}\left(-1-i\sqrt{3}+2x\right)\sqrt{1+x+x^2}} + \frac{-1+i\sqrt{3}}{\sqrt{x}\left(-1+i\sqrt{3}+2x\right)\sqrt{1+x+x^2}}\right)dx}{\sqrt{x+x^2+x^3}} +$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} - \frac{\left(\left(-1-i\sqrt{3}\right)\sqrt{x}\sqrt{1+x+x^2}\right)\int}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} - \frac{\left(\left(-1-i\sqrt{3}\right)\sqrt{x}\sqrt{1-i\sqrt{3}+2x}\right.}{}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} + \frac{\left(2\left(-1-i\sqrt{3}\right)\sqrt{x}\sqrt{1-i\sqrt{3}+2x}\right.}{}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} + \frac{\left(2\left(-1-i\sqrt{3}\right)\sqrt{x}\sqrt{1+i\sqrt{3}+2x}\right.}{}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} + \frac{\left(2\left(-1-i\sqrt{3}\right)\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1\right.}{}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} - \frac{4\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\,\Pi\left(-\right.}{\left(1-i\sqrt{3}\right)\sqrt{x}}$$

Mathematica [C] time = 0.96, size = 132, normalized size = 3.88

$$-\frac{2(-1)^{2/3}\sqrt{\frac{\sqrt[3]{-1}}{x}+1}\sqrt{1-\frac{(-1)^{2/3}}{x}}\,x^{3/2}\left(-F\left(i\sinh^{-1}\left(\frac{(-1)^{5/6}}{\sqrt{x}}\right)|(-1)^{2/3}\right)+\Pi\left(-1;i\sinh^{-1}\left(\frac{(-1)^{5/6}}{\sqrt{x}}\right)|(-1)^{2/3}\right)+\Pi\left(-(-1)^{2/3};i\sinh^{-1}\left(\frac{(-1)^{5/6}}{\sqrt{x}}\right)|(-1)^{2/3}\right)\right)}{\sqrt{x\left(x^2+x+1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((1 - x + x^2)*Sqrt[x + x^2 + x^3]),x]
```

```
[Out] (-2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)/x]*Sqrt[1 - (-1)^(2/3)/x]*x^(3/2)*(-Elli
pticF[I*ArcSinh[(-1)^(5/6)/Sqrt[x]], (-1)^(2/3)] + EllipticPi[-1, I*ArcSinh
[(-1)^(5/6)/Sqrt[x]], (-1)^(2/3)] + EllipticPi[-(-1)^(2/3), I*ArcSinh[(-1)^
(5/6)/Sqrt[x]], (-1)^(2/3)]))/Sqrt[x*(1 + x + x^2)]
```

IntegrateAlgebraic [A] time = 0.12, size = 34, normalized size = 1.00

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{x^3+x^2+x}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 - x + x^2)*Sqrt[x + x^2 + x^3]),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x + x^2 + x^3])/(1 + x + x^2)])
```

fricas [B] time = 0.42, size = 68, normalized size = 2.00

$$\frac{1}{4}\,\sqrt{2}\,\log\left(\frac{x^4+14\,x^3-4\,\sqrt{2}\,\sqrt{x^3+x^2+x}\left(x^2+3\,x+1\right)+19\,x^2+14\,x+1}{x^4-2\,x^3+3\,x^2-2\,x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2-x+1)/(x^3+x^2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^4 + 14*x^3 - 4*sqrt(2)*sqrt(x^3 + x^2 + x)*(x^2 + 3*x +
1) + 19*x^2 + 14*x + 1)/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^3+x^2+x}\,\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2-x+1)/(x^3+x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^3 + x^2 + x)*(x^2 - x + 1)), x)
```

maple [C] time = 0.09, size = 425, normalized size = 12.50

$$\frac{2\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}, \frac{\sqrt{3}\,\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{3\sqrt{x^3+x^2+x}}+\frac{2\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\left(-\frac{i\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{4}-\frac{5}{8}+\frac{3i\sqrt{3}}{8}\right)\mathrm{EllipticPi}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}, \frac{1}{2}, \frac{\sqrt{3}\,\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{3\sqrt{x^3+x^2+x}}+\frac{2\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\left(-\frac{i\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{4}-\frac{5}{8}+\frac{i\sqrt{3}}{8}\right)\mathrm{EllipticPi}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}, \frac{1}{2}+\frac{i\sqrt{3}}{2}, \frac{\sqrt{3}\,\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{3\sqrt{x^3+x^2+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2-x+1)/(x^3+x^2+x)^(1/2),x)
```

```
[Out] 2/3*(1/2+1/2*I*3^(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3
^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/
2)/(x^3+x^2+x)^(1/2)*EllipticF(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^
(1/2),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))+2/3*(1/2+1/2*I*3^
(1/2))^2*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3^(1/2)*(I*(x+1/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^
(1/2)*(-1/4*I*(1/2+1/2*I*3^(1/2))*3^(1/2)-5/8+3/8*I*3^(1/2))*EllipticPi(((x
+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),1/2,1/3*3^(1/2)*(I*(-1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2))+2/3*(1/2-1/2*I*3^(1/2))*(1/2+1/2*I*3^(1/2))*((x
+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2)*(-1/4*
I*(1/2-1/2*I*3^(1/2))*3^(1/2)-5/8+1/8*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3
^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),1/2+1/2*I*3^(1/2),1/3*3^(1/2)*(I*(-1/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^3+x^2+x}\,\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2-x+1)/(x^3+x^2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^3 + x^2 + x)*(x^2 - x + 1)), x)
```

mupad [B] time = 0.08, size = 227, normalized size = 6.68

$$-\frac{\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(\sqrt{3}+1\mathrm{i}\right)\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}};\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(-1;\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)1\mathrm{i}}{\sqrt{x^3+x^2-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 - x + 1)*(x + x^2 + x^3)^(1/2)),x)
```

```
[Out] -((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/2
)*(3^(1/2) + 1i)*(ellipticPi(((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2),
 asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*
1i)/2 + 1/2)) - ellipticF(asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/2
)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)) + ellipticPi(-1, asin((x/((3^(1/2)*1
i)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))*1i)/(
x^2 + x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{\sqrt{x\left(x^2+x+1\right)}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2-x+1)/(x**3+x**2+x)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + x + 1))*(x**2 - x + 1)), x)
```

3.418 $\int \frac{-1+x}{(1+x)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=34

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}+x^2+2x+1}\right)$$

Rubi [A] time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.53, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1742, 12, 1248, 725, 206, 1699}

$$\frac{\tanh^{-1}\left(\frac{x^2+1}{\sqrt{2}\sqrt{x^4+1}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] -(ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]) + ArcTanh[(1 + x^2)/(Sqrt[2]*
Sqrt[1 + x^4])]/Sqrt[2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 1742

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
```

```
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x}{(1+x)\sqrt{1+x^4}}\,dx &= \int \frac{2x}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx + \int \frac{-1-x^2}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx\\
&= 2\int \frac{x}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx - \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}} + \text{Subst}\left(\int \frac{1}{(1-x)\sqrt{1+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}} - \text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \frac{-1-x^2}{\sqrt{1+x^4}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{-1-x^2}{\sqrt{2}\sqrt{1+x^4}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.30, size = 175, normalized size = 5.15

$$\frac{(-1)^{3/4}\sqrt{2}\sqrt{-\frac{i\left(\sqrt{2}+(1-i)x\right)}{\sqrt{2}-(1-i)x}}\left(x^2+i\right)\left(\left(\sqrt{2}-1\right)F\left(\sin^{-1}\left(\sqrt{-\frac{i\left((1-i)x+\sqrt{2}\right)}{\sqrt{2}-(1-i)x}}\right)\middle|-1\right)-2\sqrt{2}\,\Pi\left(1+\sqrt{2};\sin^{-1}\left(\sqrt{-\frac{i\left((1-i)x+\sqrt{2}\right)}{\sqrt{2}-(1-i)x}}\right)\middle|-1\right)\right)}{\sqrt{\frac{x^2+i}{\left(\sqrt[4]{-1}-x\right)^2}}\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] ((-1)^(3/4)*Sqrt[2]*Sqrt[((-I)*(Sqrt[2] + (1 - I)*x))/(Sqrt[2] - (1 - I)*x)
]*(I + x^2)*((-1 + Sqrt[2])*EllipticF[ArcSin[Sqrt[((-I)*(Sqrt[2] + (1 - I)*
x))/(Sqrt[2] - (1 - I)*x)]], -1] - 2*Sqrt[2]*EllipticPi[1 + Sqrt[2], ArcSin
[Sqrt[((-I)*(Sqrt[2] + (1 - I)*x))/(Sqrt[2] - (1 - I)*x)]], -1]))/(Sqrt[(I
+ x^2)/((-1)^(1/4) - x)^2]*Sqrt[1 + x^4])
```

IntegrateAlgebraic [A] time = 0.69, size = 34, normalized size = 1.00

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}+x^2+2x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[(Sqrt[2]*x)/(1 + 2*x + x^2 + Sqrt[1 + x^4])])
```

fricas [B] time = 0.44, size = 66, normalized size = 1.94

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{3x^4+4x^3+2\sqrt{2}\sqrt{x^4+1}\left(x^2+x+1\right)+6x^2+4x+3}{x^4+4x^3+6x^2+4x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(3*x^4 + 4*x^3 + 2*sqrt(2)*sqrt(x^4 + 1)*(x^2 + x + 1) + 6
*x^2 + 4*x + 3)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x^4+1}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/(sqrt(x^4 + 1)*(x + 1)), x)
```

maple [C] time = 0.01, size = 136, normalized size = 4.00

$$\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right), i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x^2+2)\sqrt{2}}{4\sqrt{x^4+1}}\right)}{2}+\frac{2(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticPi}\left((-1)^{\frac{1}{4}}x, -i, -\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(1+x)/(x^4+1)^(1/2),x)
```

```
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+1/2*2^(1/2)*arctanh(1/4*(2*x^2+
2)*2^(1/2)/(x^4+1)^(1/2))+2*(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4
+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x^4+1}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/(sqrt(x^4 + 1)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x-1}{\sqrt{x^4+1}\,\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x^4 + 1)^(1/2)*(x + 1)),x)
```

```
[Out] int((x - 1)/((x^4 + 1)^(1/2)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{(x+1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x**4+1)**(1/2),x)
```

```
[Out] Integral((x - 1)/((x + 1)*sqrt(x**4 + 1)), x)
```

3.419 $\int \frac{1+x}{(-1+x)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=34

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}+x^2-2x+1}\right)$$

Rubi [A] time = 0.09, antiderivative size = 53, normalized size of antiderivative = 1.56, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1742, 12, 1248, 725, 206, 1699}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x^2+1}{\sqrt{2}\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] -(ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]) - ArcTanh[(1 + x^2)/(Sqrt[2]*
Sqrt[1 + x^4])]/Sqrt[2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 1742

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
```

```
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x}{(-1+x)\sqrt{1+x^4}}\,dx &= \int -\frac{2x}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx + \int \frac{-1-x^2}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx \\
&= -\left(2\int \frac{x}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx\right) - \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right) \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}} - \text{Subst}\left(\int \frac{1}{(1-x)\sqrt{1+x^2}}\,dx, x, x^2\right) \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}} + \text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \frac{-1-x^2}{\sqrt{1+x^4}}\right) \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{-1-x^2}{\sqrt{2}\sqrt{1+x^4}}\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.26, size = 178, normalized size = 5.24

$$
-\frac{(-1)^{3/4}\sqrt{2}\sqrt{-\frac{i\left(\sqrt{2}+(1-i)x\right)}{\sqrt{2}-(1-i)x}}\left(x^2+i\right)\left(\left(1+\sqrt{2}\right)F\left(\sin^{-1}\left(\sqrt{-\frac{i\left((1-i)x+\sqrt{2}\right)}{\sqrt{2}-(1-i)x}}\right)\middle|-1\right)-2\sqrt{2}\,\Pi\left(1-\sqrt{2};\sin^{-1}\left(\sqrt{-\frac{i\left((1-i)x+\sqrt{2}\right)}{\sqrt{2}-(1-i)x}}\right)\middle|-1\right)\right)}{\sqrt{\frac{x^2+i}{\left(\sqrt[4]{-1}-x\right)^2}}\sqrt{x^4+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] -(((-1)^(3/4)*Sqrt[2]*Sqrt[((-I)*(Sqrt[2] + (1 - I)*x))/(Sqrt[2] - (1 - I)*
x)]*(I + x^2)*((1 + Sqrt[2])*EllipticF[ArcSin[Sqrt[((-I)*(Sqrt[2] + (1 - I)
*x))/(Sqrt[2] - (1 - I)*x)]], -1] - 2*Sqrt[2]*EllipticPi[1 - Sqrt[2], ArcSi
n[Sqrt[((-I)*(Sqrt[2] + (1 - I)*x))/(Sqrt[2] - (1 - I)*x)]], -1]))/(Sqrt[(I
 + x^2)/((-1)^(1/4) - x)^2]*Sqrt[1 + x^4]))
```

IntegrateAlgebraic [A] time = 0.66, size = 34, normalized size = 1.00

$$
-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}+x^2-2x+1}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((-1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[(Sqrt[2]*x)/(1 - 2*x + x^2 + Sqrt[1 + x^4])])
```

fricas [B] time = 0.45, size = 68, normalized size = 2.00

$$
\frac{1}{4}\sqrt{2}\log\left(-\frac{3x^4-4x^3-2\sqrt{2}\sqrt{x^4+1}\left(x^2-x+1\right)+6x^2-4x+3}{x^4-4x^3+6x^2-4x+1}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(3*x^4 - 4*x^3 - 2*sqrt(2)*sqrt(x^4 + 1)*(x^2 - x + 1) + 6
*x^2 - 4*x + 3)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^4+1}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^4 + 1)*(x - 1)), x)
```

maple [C] time = 0.01, size = 136, normalized size = 4.00

$$\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x^2+2)\sqrt{2}}{4\sqrt{x^4+1}}\right)}{2}+\frac{2\,(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticPi}\left((-1)^{\frac{1}{4}}x,-i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-1+x)/(x^4+1)^(1/2),x)
```

```
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-1/2*2^(1/2)*arctanh(1/4*(2*x^2+
2)*2^(1/2)/(x^4+1)^(1/2))+2*(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4
+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^4+1}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^4 + 1)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x+1}{\sqrt{x^4+1}\,\left(x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^4 + 1)^(1/2)*(x - 1)),x)
```

```
[Out] int((x + 1)/((x^4 + 1)^(1/2)*(x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{(x-1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x**4+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/((x - 1)*sqrt(x**4 + 1)), x)
```

3.420 $\int \frac{1+x}{(-1+x)\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=34

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x^4+x^2+1}}{x^2+x+1}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.82, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1741, 12, 1247, 724, 206, 1698}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{x^4+x^2+1}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(x^2+1)}{2\sqrt{x^4+x^2+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-1 + x)*Sqrt[1 + x^2 + x^4]),x]

[Out] -(ArcTanh[(Sqrt[3]*x)/Sqrt[1 + x^2 + x^4]]/Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 +
 x^2))/(2*Sqrt[1 + x^2 + x^4])]/Sqrt[3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1741

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x
_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x
, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e
```

```
^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4
)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e},
x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4
, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x}{(-1+x)\sqrt{1+x^2+x^4}}\,dx &= \int -\frac{2x}{\left(1-x^2\right)\sqrt{1+x^2+x^4}}\,dx + \int \frac{-1-x^2}{\left(1-x^2\right)\sqrt{1+x^2+x^4}}\,dx \\
&= -\left(2\int \frac{x}{\left(1-x^2\right)\sqrt{1+x^2+x^4}}\,dx\right) - \text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right) \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}\,x}{\sqrt{1+x^2+x^4}}\right)}{\sqrt{3}} - \text{Subst}\left(\int \frac{1}{(1-x)\sqrt{1+x+x^2}}\,dx, x, x^2\right) \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}\,x}{\sqrt{1+x^2+x^4}}\right)}{\sqrt{3}} + 2\,\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, -\frac{3\left(1+x^2\right)}{\sqrt{1+x^2+x^4}}\right) \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}\,x}{\sqrt{1+x^2+x^4}}\right)}{\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1+x^2\right)}{2\sqrt{1+x^2+x^4}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.41, size = 221, normalized size = 6.50

$$\frac{\sqrt[6]{-1}\,\sqrt{6}\left(1-i\sqrt{3}\right)\left(-x+(-1)^{2/3}+1\right)^2\sqrt{\frac{2\sqrt{3}x+\sqrt{3}+3i}{2ix+\sqrt{3}-i}}\sqrt{\frac{i\left(2\sqrt{3}x^2+\sqrt{3}+3i\right)}{\left(\left(\sqrt{3}+i\right)x-2i\right)^2}}\left(F\left(\sin^{-1}\left(\sqrt{\frac{-2ix+\sqrt{3}+i}{4i-2\left(i+\sqrt{3}\right)x}}\right)\middle|4\right)-2\Pi\left(-2;\sin^{-1}\left(\sqrt{\frac{-2ix+\sqrt{3}+i}{4i-2\left(i+\sqrt{3}\right)x}}\right)\middle|4\right)\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt{x^4+x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x)/((-1 + x)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((-1)^(1/6)*Sqrt[6]*(1 - I*Sqrt[3])*(1 + (-1)^(2/3) - x)^2*Sqrt[(3*I + Sqrt
[3] + 2*Sqrt[3]*x)/(-I + Sqrt[3] + (2*I)*x)]*Sqrt[(I*(3*I + Sqrt[3] + 2*Sqr
t[3]*x^2))/(-2*I + (I + Sqrt[3])*x)^2]*(EllipticF[ArcSin[Sqrt[(I + Sqrt[3]
- (2*I)*x)/(4*I - 2*(I + Sqrt[3])*x)]], 4] - 2*EllipticPi[-2, ArcSin[Sqrt[(
I + Sqrt[3] - (2*I)*x)/(4*I - 2*(I + Sqrt[3])*x)]], 4]))/((1 + (-1)^(1/3))*
Sqrt[1 + x^2 + x^4])
```

IntegrateAlgebraic [A] time = 0.49, size = 34, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x^4+x^2+1}}{x^2+x+1}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((-1 + x)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3]*Sqrt[1 + x^2 + x^4])/(1 + x + x^2)])/Sqrt[3]
```

fricas [B] time = 0.44, size = 73, normalized size = 2.15

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{7x^4-4x^3-2\sqrt{3}\sqrt{x^4+x^2+1}\left(2x^2-x+2\right)+12x^2-4x+7}{x^4-4x^3+6x^2-4x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(7*x^4 - 4*x^3 - 2*sqrt(3)*sqrt(x^4 + x^2 + 1)*(2*x^2 - x
+ 2) + 12*x^2 - 4*x + 7)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^4+x^2+1}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^4 + x^2 + 1)*(x - 1)), x)
```

maple [C] time = 0.20, size = 212, normalized size = 6.24

$$\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(3x^2+3)\sqrt{3}}{6\sqrt{x^4+x^2+1}}\right)}{3}-\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x,\frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-1+x)/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*
3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2
),1/2*(-2+2*I*3^(1/2))^(1/2))-1/3*3^(1/2)*arctanh(1/6*(3*x^2+3)*3^(1/2)/(x^
4+x^2+1)^(1/2))-2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(
1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+
1/2*I*3^(1/2))^(1/2)*x,1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-
1/2+1/2*I*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^4+x^2+1}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^4 + x^2 + 1)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x+1}{(x-1)\,\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x - 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

```
[Out] int((x + 1)/((x - 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x - 1)), x)
```

3.421 $\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt{1-x-x^2+x^3+x^4}}\,dx$

Optimal. Leaf size=34

$$2\tanh^{-1}\left(\frac{x}{x^2-\sqrt{x^4+x^3-x^2-x+1}-1}\right)$$

Rubi [F] time = 0.47, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt{1-x-x^2+x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^2)/((-1 + x^2)*Sqrt[1 - x - x^2 + x^3 + x^4]),x]
```

```
[Out] Defer[Int][1/Sqrt[1 - x - x^2 + x^3 + x^4], x] + Defer[Int][1/((-1 + x)*Sqr
t[1 - x - x^2 + x^3 + x^4]), x] - Defer[Int][1/((1 + x)*Sqrt[1 - x - x^2 +
x^3 + x^4]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt{1-x-x^2+x^3+x^4}}\,dx &= \int \left(\frac{1}{\sqrt{1-x-x^2+x^3+x^4}}+\frac{2}{\left(-1+x^2\right)\sqrt{1-x-x^2+x^3+x^4}}\right)dx\\
&= 2\int \frac{1}{\left(-1+x^2\right)\sqrt{1-x-x^2+x^3+x^4}}\,dx+\int \frac{1}{\sqrt{1-x-x^2+x^3+x^4}}\\
&= 2\int \left(\frac{1}{2(-1+x)\sqrt{1-x-x^2+x^3+x^4}}-\frac{1}{2(1+x)\sqrt{1-x-x^2+x^3+}}\right.\\
&= \int \frac{1}{\sqrt{1-x-x^2+x^3+x^4}}\,dx+\int \frac{1}{(-1+x)\sqrt{1-x-x^2+x^3+x^4}}\,dx
\end{aligned}$$

Mathematica [C] time = 1.63, size = 3217, normalized size = 94.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2)/((-1 + x^2)*Sqrt[1 - x - x^2 + x^3 + x^4]),x]
```

```
[Out] (2*(x - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0])^2*Sqrt[((Root[1 - #1 -
#1^2 + #1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0])*
(x - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 3, 0]))/((x - Root[1 - #1 - #1^2
+ #1^3 + #1^4 & , 2, 0])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0] - Root
[1 - #1 - #1^2 + #1^3 + #1^4 & , 3, 0]))]*(Root[1 - #1 - #1^2 + #1^3 + #1^4
 & , 1, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 4, 0])*Sqrt[((x - Root[1
- #1 - #1^2 + #1^3 + #1^4 & , 1, 0])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 & ,
1, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0])*(x - Root[1 - #1 - #1^2
 + #1^3 + #1^4 & , 4, 0])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0] - Roo
t[1 - #1 - #1^2 + #1^3 + #1^4 & , 4, 0]))/((x - Root[1 - #1 - #1^2 + #1^3 +
 #1^4 & , 2, 0])^2*(Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0] - Root[1 - #
1 - #1^2 + #1^3 + #1^4 & , 4, 0])^2)]*(-((EllipticF[ArcSin[Sqrt[((x - Root[
1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 &
```

```
, 2, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 4, 0]))/((x - Root[1 - #1 -
#1^2 + #1^3 + #1^4 & , 2, 0])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0] -
 Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 4, 0]))]], -(((Root[1 - #1 - #1^2 + #
1^3 + #1^4 & , 2, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 3, 0])*(Root[1
- #1 - #1^2 + #1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^4 & ,
4, 0]))/((-Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0] + Root[1 - #1 - #1^2
+ #1^3 + #1^4 & , 3, 0])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0] - Root
[1 - #1 - #1^2 + #1^3 + #1^4 & , 4, 0]))))]*(-1 + Root[1 - #1 - #1^2 + #1^3
+ #1^4 & , 1, 0]) + EllipticPi[((-1 + Root[1 - #1 - #1^2 + #1^3 + #1^4 & ,
2, 0])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2 + #
1^3 + #1^4 & , 4, 0]))/((-1 + Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0])*(
Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0] - Root[1 - #1 - #1^2 + #1^3 + #1
^4 & , 4, 0])), ArcSin[Sqrt[((x - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0
])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0] - Root[1 - #1 - #1^2 + #1^3
+ #1^4 & , 4, 0]))/((x - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0])*(Root[
1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^4 &
, 4, 0]))]], -(((Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0] - Root[1 - #1 -
 #1^2 + #1^3 + #1^4 & , 3, 0])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0]
- Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 4, 0]))/((-Root[1 - #1 - #1^2 + #1^3
 + #1^4 & , 1, 0] + Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 3, 0])*(Root[1 - #
1 - #1^2 + #1^3 + #1^4 & , 2, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 4,
0]))))]*(-Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0] + Root[1 - #1 - #1^2 +
#1^3 + #1^4 & , 2, 0]))/((-1 + Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0])*
(-1 + Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0])*(Root[1 - #1 - #1^2 + #1^
3 + #1^4 & , 2, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 4, 0]))) + (Ellip
ticF[ArcSin[Sqrt[((x - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0])*(Root[1
- #1 - #1^2 + #1^3 + #1^4 & , 2, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^4 & ,
4, 0]))/((x - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0])*(Root[1 - #1 - #1
^2 + #1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 4, 0]))]]
, -(((Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0] - Root[1 - #1 - #1^2 + #1^
3 + #1^4 & , 3, 0])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0] - Root[1 -
#1 - #1^2 + #1^3 + #1^4 & , 4, 0]))/((-Root[1 - #1 - #1^2 + #1^3 + #1^4 & ,
 1, 0] + Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 3, 0])*(Root[1 - #1 - #1^2 +
#1^3 + #1^4 & , 2, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 4, 0]))))]*(1 +
 Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0]) + EllipticPi[((1 + Root[1 - #1
 - #1^2 + #1^3 + #1^4 & , 2, 0])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0
] - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 4, 0]))/((1 + Root[1 - #1 - #1^2 +
 #1^3 + #1^4 & , 1, 0])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0] - Root[
1 - #1 - #1^2 + #1^3 + #1^4 & , 4, 0])), ArcSin[Sqrt[((x - Root[1 - #1 - #1
^2 + #1^3 + #1^4 & , 1, 0])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0] - R
oot[1 - #1 - #1^2 + #1^3 + #1^4 & , 4, 0]))/((x - Root[1 - #1 - #1^2 + #1^3
 + #1^4 & , 2, 0])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0] - Root[1 - #
1 - #1^2 + #1^3 + #1^4 & , 4, 0]))]], -(((Root[1 - #1 - #1^2 + #1^3 + #1^4
& , 2, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 3, 0])*(Root[1 - #1 - #1^2
 + #1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 4, 0]))/((-
Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0] + Root[1 - #1 - #1^2 + #1^3 + #1
^4 & , 3, 0])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0] - Root[1 - #1 - #
1^2 + #1^3 + #1^4 & , 4, 0]))))]*(-Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0
] + Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0]))/((1 + Root[1 - #1 - #1^2 +
 #1^3 + #1^4 & , 1, 0])*(1 + Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0])*(R
oot[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^
4 & , 4, 0])) + EllipticF[ArcSin[Sqrt[((x - Root[1 - #1 - #1^2 + #1^3 + #1^
4 & , 1, 0])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0] - Root[1 - #1 - #1
^2 + #1^3 + #1^4 & , 4, 0]))/((x - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2,
0])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2 + #1^3
 + #1^4 & , 4, 0]))]], ((Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0] - Root[
1 - #1 - #1^2 + #1^3 + #1^4 & , 3, 0])*(Root[1 - #1 - #1^2 + #1^3 + #1^4 &
, 1, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 4, 0]))/((Root[1 - #1 - #1^2
 + #1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 3, 0])*(Roo
```

```
t[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0] - Root[1 - #1 - #1^2 + #1^3 + #1^4
& , 4, 0]))]/(-Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0] + Root[1 - #1 - #
1^2 + #1^3 + #1^4 & , 4, 0])))/(Sqrt[1 - x - x^2 + x^3 + x^4]*(-Root[1 - #1
 - #1^2 + #1^3 + #1^4 & , 1, 0] + Root[1 - #1 - #1^2 + #1^3 + #1^4 & , 2, 0
]))
```

IntegrateAlgebraic [A] time = 0.17, size = 34, normalized size = 1.00

$$2\tanh^{-1}\left(\frac{x}{x^2-\sqrt{x^4+x^3-x^2-x+1}-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 + x^2)*Sqrt[1 - x - x^2 + x^3 + x^4]),x]
[Out] 2*ArcTanh[x/(-1 + x^2 - Sqrt[1 - x - x^2 + x^3 + x^4])]
```

fricas [A] time = 0.43, size = 38, normalized size = 1.12

$$\log\left(-\frac{x^2+2x-2\sqrt{x^4+x^3-x^2-x+1}-1}{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^4+x^3-x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] log(-(x^2 + 2*x - 2*sqrt(x^4 + x^3 - x^2 - x + 1) - 1)/(x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x^4+x^3-x^2-x+1}\,(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^4+x^3-x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 + 1)/(sqrt(x^4 + x^3 - x^2 - x + 1)*(x^2 - 1)), x)
```

maple [C] time = 1.81, size = 4112, normalized size = 120.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-1)/(x^4+x^3-x^2-x+1)^(1/2),x)
```

```
[Out] 2*(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4))
*((RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2))
*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=
4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index
=2)))^(1/2)*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2))^2*((RootOf(_Z^4+_Z^3-_Z
^2-_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))*(x-RootOf(_Z^4+_Z^3-_
Z^2-_Z+1,index=3))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3)-RootOf(_Z^4+_Z^3-_Z
^2-_Z+1,index=1))/(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)))^(1/2)*((RootOf(_
Z^4+_Z^3-_Z^2-_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))*(x-RootOf(
_Z^4+_Z^3-_Z^2-_Z+1,index=4))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_
Z^4+_Z^3-_Z^2-_Z+1,index=1))/(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)))^(1/2)
/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2))/
(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(
(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index
=2))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,
```

index=4)))^(1/2)*EllipticF(((RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)))^(1/2),((RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3))*(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)))^(1/2))+2*(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4))*((RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)))^(1/2)*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2))^2*((RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)))^(1/2)*((RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)))^(1/2)/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/((x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)))^(1/2)/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)-1)*(EllipticF(((RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)))^(1/2),((RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3))*(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)))^(1/2))+(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1)-1)*EllipticPi(((RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)))^(1/2),(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)-1)*(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1)-1)/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)),((RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3))*(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)))^(1/2)))-2*(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4))*((RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)))^(1/2)*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2))^2*((RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)))^(1/2)*((RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)))^(1/2)/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/((x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)))^(1/2)/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)+1)*(EllipticF(((RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=

```
4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index
=1))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=
1))/(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)))^(1/2),((RootOf(_Z^4+_Z^3-_Z^2-
_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3))*(RootOf(_Z^4+_Z^3-_Z^2-_
Z+1,index=1)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4))/(RootOf(_Z^4+_Z^3-_Z^2-_Z
+1,index=1)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+
1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)))^(1/2))+(RootOf(_Z^4+_Z^3-_
Z^2-_Z+1,index=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(RootOf(_Z^4+_Z^3-_Z
^2-_Z+1,index=1)+1)*EllipticPi(((RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf
(_Z^4+_Z^3-_Z^2-_Z+1,index=2))*(x-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(Roo
tOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1))/(x-Ro
otOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)))^(1/2),(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index
=2)+1)*(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,inde
x=1))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=1)+1)/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,i
ndex=4)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=2)),((RootOf(_Z^4+_Z^3-_Z^2-_Z+1,i
ndex=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3))*(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,in
dex=1)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,ind
ex=1)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=3))/(RootOf(_Z^4+_Z^3-_Z^2-_Z+1,inde
x=2)-RootOf(_Z^4+_Z^3-_Z^2-_Z+1,index=4)))^(1/2)))
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x^4+x^3-x^2-x+1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^4+x^3-x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/(sqrt(x^4 + x^3 - x^2 - x + 1)*(x^2 - 1)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2+1}{\left(x^2-1\right)\sqrt{x^4+x^3-x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((x^2 - 1)*(x^3 - x^2 - x + x^4 + 1)^(1/2)),x)
[Out] int((x^2 + 1)/((x^2 - 1)*(x^3 - x^2 - x + x^4 + 1)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{(x-1)(x+1)\sqrt{x^4+x^3-x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-1)/(x**4+x**3-x**2-x+1)**(1/2),x)
[Out] Integral((x**2 + 1)/((x - 1)*(x + 1)*sqrt(x**4 + x**3 - x**2 - x + 1)), x)
```

3.422 $\int \frac{x}{\sqrt{11-11x-3x^2+2x^3+x^4}}\,dx$

Optimal. Leaf size=34

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^2+x-2}{\sqrt{x^4+2x^3-3x^2-11x+11}}\right)$$

Rubi [F] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\sqrt{11-11x-3x^2+2x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/Sqrt[11 - 11*x - 3*x^2 + 2*x^3 + x^4],x]
```

```
[Out] Defer[Int][x/Sqrt[11 - 11*x - 3*x^2 + 2*x^3 + x^4], x]
```

Rubi steps

$$\int \frac{x}{\sqrt{11-11x-3x^2+2x^3+x^4}}\,dx=\int \frac{x}{\sqrt{11-11x-3x^2+2x^3+x^4}}\,dx$$

Mathematica [C] time = 0.56, size = 759, normalized size = 22.32

$$\frac{2(x-1)\left(x-\text{Root}\left[\#1^3+3\#1^2-11\&,1\right]\right)\sqrt{\frac{x-\text{Root}\left[\#1^3+3\#1^2-11\&,2\right]}{\left(-1+\text{Root}\left[\#1^3+3\#1^2-11\&,2\right]\right)\left(x-\text{Root}\left[\#1^3+3\#1^2-11\&,1\right]\right)}}\sqrt{\frac{x-\text{Root}\left[\#1^3+3\#1^2-11\&,3\right]}{\left(-1+\text{Root}\left[\#1^3+3\#1^2-11\&,3\right]\right)\left(x-\text{Root}\left[\#1^3+3\#1^2-11\&,1\right]\right)}}\left(\left(\text{Root}\left[\#1^3+3\#1^2-11\&,1\right]-1\right)\Pi\left(\frac{-1+\text{Root}\left[\#1^3+3\#1^2-11\&,3\right]}{-\text{Root}\left[\#1^3+3\#1^2-11\&,1\right]+\text{Root}\left[\#1^3+3\#1^2-11\&,3\right]};\sin^{-1}\left(\sqrt{-\frac{(x-1)\left(\text{Root}\left[\#1^3+3\#1^2-11\&,1\right]-\text{Root}\left[\#1^3+3\#1^2-11\&,3\right]\right)}{\left(x-\text{Root}\left[\#1^3+3\#1^2-11\&,1\right]\right)\left(-1+\text{Root}\left[\#1^3+3\#1^2-11\&,3\right]\right)}}\right)|\frac{\left(\text{Root}\left[\#1^3+3\#1^2-11\&,1\right]-\text{Root}\left[\#1^3+3\#1^2-11\&,2\right]\right)\left(-1+\text{Root}\left[\#1^3+3\#1^2-11\&,3\right]\right)}{\left(-1+\text{Root}\left[\#1^3+3\#1^2-11\&,2\right]\right)\left(\text{Root}\left[\#1^3+3\#1^2-11\&,1\right]-\text{Root}\left[\#1^3+3\#1^2-11\&,3\right]\right)}\right)-\text{Root}\left[\#1^3+3\#1^2-11\&,1\right]F\left(\sin^{-1}\left(\sqrt{-\frac{(x-1)\left(\text{Root}\left[\#1^3+3\#1^2-11\&,1\right]-\text{Root}\left[\#1^3+3\#1^2-11\&,3\right]\right)}{\left(x-\text{Root}\left[\#1^3+3\#1^2-11\&,1\right]\right)\left(-1+\text{Root}\left[\#1^3+3\#1^2-11\&,3\right]\right)}}\right)|\frac{\left(\text{Root}\left[\#1^3+3\#1^2-11\&,1\right]-\text{Root}\left[\#1^3+3\#1^2-11\&,2\right]\right)\left(-1+\text{Root}\left[\#1^3+3\#1^2-11\&,3\right]\right)}{\left(-1+\text{Root}\left[\#1^3+3\#1^2-11\&,2\right]\right)\left(\text{Root}\left[\#1^3+3\#1^2-11\&,1\right]-\text{Root}\left[\#1^3+3\#1^2-11\&,3\right]\right)}\right)\right)}{\sqrt{x^4+2x^3-3x^2-11x+11}\sqrt{-\frac{(x-1)\left(\text{Root}\left[\#1^3+3\#1^2-11\&,1\right]-\text{Root}\left[\#1^3+3\#1^2-11\&,3\right]\right)}{\left(-1+\text{Root}\left[\#1^3+3\#1^2-11\&,3\right]\right)\left(x-\text{Root}\left[\#1^3+3\#1^2-11\&,1\right]\right)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[11 - 11*x - 3*x^2 + 2*x^3 + x^4],x]
```

```
[Out] (2*(-1 + x)*(x - Root[-11 + 3*#1^2 + #1^3 & , 1, 0])*(EllipticPi[(-1 + Root
[-11 + 3*#1^2 + #1^3 & , 3, 0])/(-Root[-11 + 3*#1^2 + #1^3 & , 1, 0] + Root
[-11 + 3*#1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[-(((-1 + x)*(Root[-11 + 3*#1^2
 + #1^3 & , 1, 0] - Root[-11 + 3*#1^2 + #1^3 & , 3, 0]))/((x - Root[-11 + 3
*#1^2 + #1^3 & , 1, 0])*(-1 + Root[-11 + 3*#1^2 + #1^3 & , 3, 0])))]], ((Ro
ot[-11 + 3*#1^2 + #1^3 & , 1, 0] - Root[-11 + 3*#1^2 + #1^3 & , 2, 0])*(-1
+ Root[-11 + 3*#1^2 + #1^3 & , 3, 0]))/((-1 + Root[-11 + 3*#1^2 + #1^3 & ,
2, 0])*(Root[-11 + 3*#1^2 + #1^3 & , 1, 0] - Root[-11 + 3*#1^2 + #1^3 & , 3
, 0]))]*(-1 + Root[-11 + 3*#1^2 + #1^3 & , 1, 0]) - EllipticF[ArcSin[Sqrt[-
(((-1 + x)*(Root[-11 + 3*#1^2 + #1^3 & , 1, 0] - Root[-11 + 3*#1^2 + #1^3 &
 , 3, 0]))/((x - Root[-11 + 3*#1^2 + #1^3 & , 1, 0])*(-1 + Root[-11 + 3*#1^
2 + #1^3 & , 3, 0])))]], ((Root[-11 + 3*#1^2 + #1^3 & , 1, 0] - Root[-11 +
3*#1^2 + #1^3 & , 2, 0])*(-1 + Root[-11 + 3*#1^2 + #1^3 & , 3, 0]))/((-1 +
Root[-11 + 3*#1^2 + #1^3 & , 2, 0])*(Root[-11 + 3*#1^2 + #1^3 & , 1, 0] - R
oot[-11 + 3*#1^2 + #1^3 & , 3, 0]))]*Root[-11 + 3*#1^2 + #1^3 & , 1, 0])*Sq
rt[(x - Root[-11 + 3*#1^2 + #1^3 & , 2, 0])/((x - Root[-11 + 3*#1^2 + #1^3
& , 1, 0])*(-1 + Root[-11 + 3*#1^2 + #1^3 & , 2, 0]))]*Sqrt[(x - Root[-11 +
 3*#1^2 + #1^3 & , 3, 0])/((x - Root[-11 + 3*#1^2 + #1^3 & , 1, 0])*(-1 + R
oot[-11 + 3*#1^2 + #1^3 & , 3, 0]))])/(Sqrt[11 - 11*x - 3*x^2 + 2*x^3 + x^4
]*Sqrt[-(((-1 + x)*(Root[-11 + 3*#1^2 + #1^3 & , 1, 0] - Root[-11 + 3*#1^2
+ #1^3 & , 3, 0]))/((x - Root[-11 + 3*#1^2 + #1^3 & , 1, 0])*(-1 + Root[-11
 + 3*#1^2 + #1^3 & , 3, 0]))))])
```

IntegrateAlgebraic [A] time = 0.16, size = 34, normalized size = 1.00

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^2+x-2}{\sqrt{x^4+2x^3-3x^2-11x+11}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[11 - 11*x - 3*x^2 + 2*x^3 + x^4],x]
```

```
[Out] (2*ArcTanh[(-2 + x + x^2)/Sqrt[11 - 11*x - 3*x^2 + 2*x^3 + x^4]])/3
```

fricas [A] time = 0.43, size = 40, normalized size = 1.18

$$\frac{1}{3}\log\left(2x^3+6x^2+2\sqrt{x^4+2x^3-3x^2-11x+11}(x+2)-15\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^3-3*x^2-11*x+11)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log(2*x^3 + 6*x^2 + 2*sqrt(x^4 + 2*x^3 - 3*x^2 - 11*x + 11)*(x + 2) - 1
5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^4+2x^3-3x^2-11x+11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^3-3*x^2-11*x+11)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(x^4 + 2*x^3 - 3*x^2 - 11*x + 11), x)
```

maple [C] time = 1.32, size = 1625, normalized size = 47.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+2*x^3-3*x^2-11*x+11)^(1/2),x)
```

```
[Out] 2*(2+1/4*(36+4*77^(1/2))^(1/3)+1/(36+4*77^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/2*(
36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3)))*((-3/4*(36+4*77^(1/2))^(1/3)
-3/(36+4*77^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(36+4*77^(1/2))^(1/3)-2/(36+4*7
7^(1/2))^(1/3)))*(-1+x)/(-1/4*(36+4*77^(1/2))^(1/3)-1/(36+4*77^(1/2))^(1/3)
-2-1/2*I*3^(1/2)*(1/2*(36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3)))/(x-1/
2*(36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3)+1))^(1/2)*(x-1/2*(36+4*77^(
1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3)+1)^2*((1/2*(36+4*77^(1/2))^(1/3)+2/(36+
4*77^(1/2))^(1/3)-2)*(x+1/4*(36+4*77^(1/2))^(1/3)+1/(36+4*77^(1/2))^(1/3)+1
-1/2*I*3^(1/2)*(1/2*(36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3)))/(-1/4*(
36+4*77^(1/2))^(1/3)-1/(36+4*77^(1/2))^(1/3)-2+1/2*I*3^(1/2)*(1/2*(36+4*77^
(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3)))/(x-1/2*(36+4*77^(1/2))^(1/3)-2/(36+4
*77^(1/2))^(1/3)+1))^(1/2)*((1/2*(36+4*77^(1/2))^(1/3)+2/(36+4*77^(1/2))^(1
/3)-2)*(x+1/4*(36+4*77^(1/2))^(1/3)+1/(36+4*77^(1/2))^(1/3)+1+1/2*I*3^(1/2)
*(1/2*(36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3)))/(-1/4*(36+4*77^(1/2))
^(1/3)-1/(36+4*77^(1/2))^(1/3)-2-1/2*I*3^(1/2)*(1/2*(36+4*77^(1/2))^(1/3)-2
/(36+4*77^(1/2))^(1/3)))/(x-1/2*(36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/
3)+1))^(1/2)/(-3/4*(36+4*77^(1/2))^(1/3)-3/(36+4*77^(1/2))^(1/3)-1/2*I*3^(1
/2)*(1/2*(36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3)))/(1/2*(36+4*77^(1/2
))^(1/3)+2/(36+4*77^(1/2))^(1/3)-2)/((-1+x)*(x-1/2*(36+4*77^(1/2))^(1/3)-2/
(36+4*77^(1/2))^(1/3)+1)*(x+1/4*(36+4*77^(1/2))^(1/3)+1/(36+4*77^(1/2))^(1/
3)+1-1/2*I*3^(1/2)*(1/2*(36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3)))*(x+
1/4*(36+4*77^(1/2))^(1/3)+1/(36+4*77^(1/2))^(1/3)+1+1/2*I*3^(1/2)*(1/2*(36+
4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3))))^(1/2)*((1/2*(36+4*77^(1/2))^(1
/3)+2/(36+4*77^(1/2))^(1/3)-1)*EllipticF(((-3/4*(36+4*77^(1/2))^(1/3)-3/(36
+4*77^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2
))^(1/3)))*(-1+x)/(-1/4*(36+4*77^(1/2))^(1/3)-1/(36+4*77^(1/2))^(1/3)-2-1/2
*I*3^(1/2)*(1/2*(36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3)))/(x-1/2*(36+
```

```
4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3)+1))^(1/2),((3/4*(36+4*77^(1/2))^(
1/3)+3/(36+4*77^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(36+4*77^(1/2))^(1/3)-2/(36
+4*77^(1/2))^(1/3)))*(2+1/4*(36+4*77^(1/2))^(1/3)+1/(36+4*77^(1/2))^(1/3)+1
/2*I*3^(1/2)*(1/2*(36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3)))/(2+1/4*(3
6+4*77^(1/2))^(1/3)+1/(36+4*77^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(36+4*77^(1/
2))^(1/3)-2/(36+4*77^(1/2))^(1/3)))/(3/4*(36+4*77^(1/2))^(1/3)+3/(36+4*77^(
1/2))^(1/3)+1/2*I*3^(1/2)*(1/2*(36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3
))))^(1/2))+(2-1/2*(36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3))*EllipticP
i(((-3/4*(36+4*77^(1/2))^(1/3)-3/(36+4*77^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(
36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3)))*(-1+x)/(-1/4*(36+4*77^(1/2))
^(1/3)-1/(36+4*77^(1/2))^(1/3)-2-1/2*I*3^(1/2)*(1/2*(36+4*77^(1/2))^(1/3)-2
/(36+4*77^(1/2))^(1/3)))/(x-1/2*(36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/
3)+1))^(1/2),(-1/4*(36+4*77^(1/2))^(1/3)-1/(36+4*77^(1/2))^(1/3)-2-1/2*I*3^
(1/2)*(1/2*(36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3)))/(-3/4*(36+4*77^(
1/2))^(1/3)-3/(36+4*77^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(36+4*77^(1/2))^(1/3
)-2/(36+4*77^(1/2))^(1/3))),((3/4*(36+4*77^(1/2))^(1/3)+3/(36+4*77^(1/2))^(
1/3)-1/2*I*3^(1/2)*(1/2*(36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3)))*(2+
1/4*(36+4*77^(1/2))^(1/3)+1/(36+4*77^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/2*(36+4*
77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3)))/(2+1/4*(36+4*77^(1/2))^(1/3)+1/(3
6+4*77^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/
2))^(1/3)))/(3/4*(36+4*77^(1/2))^(1/3)+3/(36+4*77^(1/2))^(1/3)+1/2*I*3^(1/2
)*(1/2*(36+4*77^(1/2))^(1/3)-2/(36+4*77^(1/2))^(1/3))))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^4 + 2\,x^3 - 3\,x^2 - 11\,x + 11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^3-3*x^2-11*x+11)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(x^4 + 2*x^3 - 3*x^2 - 11*x + 11), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{x^4 + 2\,x^3 - 3\,x^2 - 11\,x + 11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x^3 - 3*x^2 - 11*x + x^4 + 11)^(1/2),x)
[Out] int(x/(2*x^3 - 3*x^2 - 11*x + x^4 + 11)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x-1)\left(x^3 + 3x^2 - 11\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+2*x**3-3*x**2-11*x+11)**(1/2),x)
[Out] Integral(x/sqrt((x - 1)*(x**3 + 3*x**2 - 11)), x)
```

3.423 $\int \frac{1+x}{\sqrt{2-5x-3x^2+2x^3+x^4}} \, dx$

Optimal. Leaf size=34

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^2+x-2}{\sqrt{x^4+2x^3-3x^2-5x+2}}\right)$$

Rubi [F] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x}{\sqrt{2-5x-3x^2+2x^3+x^4}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x)/Sqrt[2 - 5*x - 3*x^2 + 2*x^3 + x^4],x]
```

```
[Out] Defer[Int][1/Sqrt[2 - 5*x - 3*x^2 + 2*x^3 + x^4], x] + Defer[Int][x/Sqrt[2
- 5*x - 3*x^2 + 2*x^3 + x^4], x]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x}{\sqrt{2-5x-3x^2+2x^3+x^4}} \, dx &= \int \left(\frac{1}{\sqrt{2-5x-3x^2+2x^3+x^4}}+\frac{x}{\sqrt{2-5x-3x^2+2x^3+x^4}}\right) dx \\ &= \int \frac{1}{\sqrt{2-5x-3x^2+2x^3+x^4}} \, dx+\int \frac{x}{\sqrt{2-5x-3x^2+2x^3+x^4}} \, dx \end{aligned}$$

Mathematica [C] time = 0.54, size = 830, normalized size = 24.41

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/Sqrt[2 - 5*x - 3*x^2 + 2*x^3 + x^4],x]
```

```
[Out] (-2*Sqrt[(2 + x)/(x - Root[1 - 3*#1 + #1^3 & , 1, 0])]*(x - Root[1 - 3*#1 +
 #1^3 & , 1, 0])^2*(EllipticF[ArcSin[Sqrt[((2 + x)*(-Root[1 - 3*#1 + #1^3 &
 , 1, 0] + Root[1 - 3*#1 + #1^3 & , 3, 0]))/((x - Root[1 - 3*#1 + #1^3 & ,
1, 0])*(2 + Root[1 - 3*#1 + #1^3 & , 3, 0]))]], ((Root[1 - 3*#1 + #1^3 & ,
1, 0] - Root[1 - 3*#1 + #1^3 & , 2, 0])*(2 + Root[1 - 3*#1 + #1^3 & , 3, 0]
))/((2 + Root[1 - 3*#1 + #1^3 & , 2, 0])*(Root[1 - 3*#1 + #1^3 & , 1, 0] -
Root[1 - 3*#1 + #1^3 & , 3, 0]))] + EllipticF[ArcSin[Sqrt[-(((2 + x)*(Root[
1 - 3*#1 + #1^3 & , 1, 0] - Root[1 - 3*#1 + #1^3 & , 3, 0]))/((x - Root[1 -
 3*#1 + #1^3 & , 1, 0])*(2 + Root[1 - 3*#1 + #1^3 & , 3, 0])))]], ((Root[1
- 3*#1 + #1^3 & , 1, 0] - Root[1 - 3*#1 + #1^3 & , 2, 0])*(2 + Root[1 - 3*#
1 + #1^3 & , 3, 0]))/((2 + Root[1 - 3*#1 + #1^3 & , 2, 0])*(Root[1 - 3*#1 +
 #1^3 & , 1, 0] - Root[1 - 3*#1 + #1^3 & , 3, 0]))]*Root[1 - 3*#1 + #1^3 &
, 1, 0] - EllipticPi[(2 + Root[1 - 3*#1 + #1^3 & , 3, 0])/(-Root[1 - 3*#1 +
 #1^3 & , 1, 0] + Root[1 - 3*#1 + #1^3 & , 3, 0]), ArcSin[Sqrt[-(((2 + x)*(
Root[1 - 3*#1 + #1^3 & , 1, 0] - Root[1 - 3*#1 + #1^3 & , 3, 0]))/((x - Roo
t[1 - 3*#1 + #1^3 & , 1, 0])*(2 + Root[1 - 3*#1 + #1^3 & , 3, 0])))]], ((Ro
ot[1 - 3*#1 + #1^3 & , 1, 0] - Root[1 - 3*#1 + #1^3 & , 2, 0])*(2 + Root[1
- 3*#1 + #1^3 & , 3, 0]))/((2 + Root[1 - 3*#1 + #1^3 & , 2, 0])*(Root[1 - 3
*#1 + #1^3 & , 1, 0] - Root[1 - 3*#1 + #1^3 & , 3, 0]))]*(2 + Root[1 - 3*#1
 + #1^3 & , 1, 0]))*Sqrt[(x - Root[1 - 3*#1 + #1^3 & , 2, 0])/(x - Root[1 -
 3*#1 + #1^3 & , 1, 0])]*Sqrt[(x - Root[1 - 3*#1 + #1^3 & , 3, 0])/(x - Roo
t[1 - 3*#1 + #1^3 & , 1, 0])])/(Sqrt[2 - 5*x - 3*x^2 + 2*x^3 + x^4]*Sqrt[(2
```

```
 + Root[1 - 3*#1 + #1^3 & , 2, 0])*(-Root[1 - 3*#1 + #1^3 & , 1, 0] + Root[
1 - 3*#1 + #1^3 & , 3, 0])])
```

IntegrateAlgebraic [A] time = 0.19, size = 34, normalized size = 1.00

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^2+x-2}{\sqrt{x^4+2x^3-3x^2-5x+2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/Sqrt[2 - 5*x - 3*x^2 + 2*x^3 + x^4],x]
[Out] (2*ArcTanh[(-2 + x + x^2)/Sqrt[2 - 5*x - 3*x^2 + 2*x^3 + x^4]])/3
```

fricas [A] time = 0.43, size = 38, normalized size = 1.12

$$\frac{1}{3}\log\left(2x^3+2\sqrt{x^4+2x^3-3x^2-5x+2}\,(x-1)-6x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^4+2*x^3-3*x^2-5*x+2)^(1/2),x, algorithm="fricas")
[Out] 1/3*log(2*x^3 + 2*sqrt(x^4 + 2*x^3 - 3*x^2 - 5*x + 2)*(x - 1) - 6*x + 3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+1}{\sqrt{x^4+2x^3-3x^2-5x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^4+2*x^3-3*x^2-5*x+2)^(1/2),x, algorithm="giac")
[Out] integrate((x + 1)/sqrt(x^4 + 2*x^3 - 3*x^2 - 5*x + 2), x)
```

maple [C] time = 0.79, size = 2934, normalized size = 86.29

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^4+2*x^3-3*x^2-5*x+2)^(1/2),x)
[Out] 2*(-2+1/4*(-4+4*I*3^(1/2))^(1/3)+1/(-4+4*I*3^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/
2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3)))*(-(3/4*(-4+4*I*3^(1/2))
^(1/3)+3/(-4+4*I*3^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2
/(-4+4*I*3^(1/2))^(1/3)))*(2+x)/(-1/4*(-4+4*I*3^(1/2))^(1/3)-1/(-4+4*I*3^(1
/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/
3))+2)/(x-1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3)))^(1/2)*(x-1/
2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3))^2*((1/2*(-4+4*I*3^(1/2))
^(1/3)+2/(-4+4*I*3^(1/2))^(1/3)+2)*(x+1/4*(-4+4*I*3^(1/2))^(1/3)+1/(-4+4*I*
3^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))
^(1/3)))/(-1/4*(-4+4*I*3^(1/2))^(1/3)-1/(-4+4*I*3^(1/2))^(1/3)+1/2*I*3^(1/2
)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3))+2)/(x-1/2*(-4+4*I*3
^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3)))^(1/2)*((1/2*(-4+4*I*3^(1/2))^(1/3)
+2/(-4+4*I*3^(1/2))^(1/3)+2)*(x+1/4*(-4+4*I*3^(1/2))^(1/3)+1/(-4+4*I*3^(1/2
))^(1/3)+1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3)
))/(-1/4*(-4+4*I*3^(1/2))^(1/3)-1/(-4+4*I*3^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2
*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3))+2)/(x-1/2*(-4+4*I*3^(1/2)
)^(1/3)-2/(-4+4*I*3^(1/2))^(1/3)))^(1/2)/(-3/4*(-4+4*I*3^(1/2))^(1/3)-3/(-4
+4*I*3^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(
1/2))^(1/3)))/(1/2*(-4+4*I*3^(1/2))^(1/3)+2/(-4+4*I*3^(1/2))^(1/3)+2)/((2+x
```

```
)*(x-1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3))*(x+1/4*(-4+4*I*3^
(1/2))^(1/3)+1/(-4+4*I*3^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(
1/3)-2/(-4+4*I*3^(1/2))^(1/3)))*(x+1/4*(-4+4*I*3^(1/2))^(1/3)+1/(-4+4*I*3^(
1/2))^(1/3)+1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1
/3))))^(1/2)*((1/2*(-4+4*I*3^(1/2))^(1/3)+2/(-4+4*I*3^(1/2))^(1/3))*Ellipti
cF((-(3/4*(-4+4*I*3^(1/2))^(1/3)+3/(-4+4*I*3^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/
2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3)))*(2+x)/(-1/4*(-4+4*I*3^(
1/2))^(1/3)-1/(-4+4*I*3^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1
/3)-2/(-4+4*I*3^(1/2))^(1/3))+2)/(x-1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^
(1/2))^(1/3)))^(1/2),((3/4*(-4+4*I*3^(1/2))^(1/3)+3/(-4+4*I*3^(1/2))^(1/3)-
1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3)))*(-1/4*
(-4+4*I*3^(1/2))^(1/3)-1/(-4+4*I*3^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(-4+4*I*
3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3))+2)/(-1/4*(-4+4*I*3^(1/2))^(1/3)-1/
(-4+4*I*3^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*
3^(1/2))^(1/3))+2)/(3/4*(-4+4*I*3^(1/2))^(1/3)+3/(-4+4*I*3^(1/2))^(1/3)+1/2
*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3))))^(1/2))+(
-2-1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3))*EllipticPi((-(3/4*(
-4+4*I*3^(1/2))^(1/3)+3/(-4+4*I*3^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/2*(-4+4*I*3
^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3)))*(2+x)/(-1/4*(-4+4*I*3^(1/2))^(1/3)
-1/(-4+4*I*3^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4
*I*3^(1/2))^(1/3))+2)/(x-1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3
)))^(1/2),(-1/4*(-4+4*I*3^(1/2))^(1/3)-1/(-4+4*I*3^(1/2))^(1/3)-1/2*I*3^(1/
2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3))+2)/(-3/4*(-4+4*I*3
^(1/2))^(1/3)-3/(-4+4*I*3^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^
(1/3)-2/(-4+4*I*3^(1/2))^(1/3))),((3/4*(-4+4*I*3^(1/2))^(1/3)+3/(-4+4*I*3^(
1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1
/3)))*(-1/4*(-4+4*I*3^(1/2))^(1/3)-1/(-4+4*I*3^(1/2))^(1/3)-1/2*I*3^(1/2)*(
1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3))+2)/(-1/4*(-4+4*I*3^(1/
2))^(1/3)-1/(-4+4*I*3^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3
)-2/(-4+4*I*3^(1/2))^(1/3))+2)/(3/4*(-4+4*I*3^(1/2))^(1/3)+3/(-4+4*I*3^(1/2
))^(1/3)+1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3)
)))^(1/2)))+2*(-2+1/4*(-4+4*I*3^(1/2))^(1/3)+1/(-4+4*I*3^(1/2))^(1/3)+1/2*I
*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3)))*(-(3/4*(-4+
4*I*3^(1/2))^(1/3)+3/(-4+4*I*3^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1
/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3)))*(2+x)/(-1/4*(-4+4*I*3^(1/2))^(1/3)-1/
(-4+4*I*3^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*
3^(1/2))^(1/3))+2)/(x-1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3)))
^(1/2)*(x-1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3))^2*((1/2*(-4+
4*I*3^(1/2))^(1/3)+2/(-4+4*I*3^(1/2))^(1/3)+2)*(x+1/4*(-4+4*I*3^(1/2))^(1/3
)+1/(-4+4*I*3^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+
4*I*3^(1/2))^(1/3)))/(-1/4*(-4+4*I*3^(1/2))^(1/3)-1/(-4+4*I*3^(1/2))^(1/3)+
1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3))+2)/(x-1
/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3)))^(1/2)*((1/2*(-4+4*I*3^
(1/2))^(1/3)+2/(-4+4*I*3^(1/2))^(1/3)+2)*(x+1/4*(-4+4*I*3^(1/2))^(1/3)+1/(-
4+4*I*3^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^
(1/2))^(1/3)))/(-1/4*(-4+4*I*3^(1/2))^(1/3)-1/(-4+4*I*3^(1/2))^(1/3)-1/2*I*
3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3))+2)/(x-1/2*(-4
+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3)))^(1/2)/(-3/4*(-4+4*I*3^(1/2))
^(1/3)-3/(-4+4*I*3^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2
/(-4+4*I*3^(1/2))^(1/3)))/(1/2*(-4+4*I*3^(1/2))^(1/3)+2/(-4+4*I*3^(1/2))^(1
/3)+2)/((2+x)*(x-1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3))*(x+1/
4*(-4+4*I*3^(1/2))^(1/3)+1/(-4+4*I*3^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(-4+4*
I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3)))*(x+1/4*(-4+4*I*3^(1/2))^(1/3)+1
/(-4+4*I*3^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I
*3^(1/2))^(1/3))))^(1/2)*EllipticF((-(3/4*(-4+4*I*3^(1/2))^(1/3)+3/(-4+4*I*
3^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))
^(1/3)))*(2+x)/(-1/4*(-4+4*I*3^(1/2))^(1/3)-1/(-4+4*I*3^(1/2))^(1/3)-1/2*I*
3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3))+2)/(x-1/2*(-4
+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3)))^(1/2),((3/4*(-4+4*I*3^(1/2))
```

```
^(1/3)+3/(-4+4*I*3^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2
/(-4+4*I*3^(1/2))^(1/3)))*(-1/4*(-4+4*I*3^(1/2))^(1/3)-1/(-4+4*I*3^(1/2))^(
1/3)-1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3))+2)
/(-1/4*(-4+4*I*3^(1/2))^(1/3)-1/(-4+4*I*3^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/2*(
-4+4*I*3^(1/2))^(1/3)-2/(-4+4*I*3^(1/2))^(1/3))+2)/(3/4*(-4+4*I*3^(1/2))^(1
/3)+3/(-4+4*I*3^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/2*(-4+4*I*3^(1/2))^(1/3)-2/(-
4+4*I*3^(1/2))^(1/3))))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^4+2\,x^3-3\,x^2-5\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^4+2*x^3-3*x^2-5*x+2)^(1/2),x, algorithm="maxima")
[Out] integrate((x + 1)/sqrt(x^4 + 2*x^3 - 3*x^2 - 5*x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x+1}{\sqrt{x^4+2\,x^3-3\,x^2-5\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(2*x^3 - 3*x^2 - 5*x + x^4 + 2)^(1/2),x)
[Out] int((x + 1)/(2*x^3 - 3*x^2 - 5*x + x^4 + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{(x+2)\left(x^3-3x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**4+2*x**3-3*x**2-5*x+2)**(1/2),x)
[Out] Integral((x + 1)/sqrt((x + 2)*(x**3 - 3*x + 1)), x)
```

3.424 $\int \frac{x}{\sqrt{-3+3x-3x^2+2x^3+x^4}}\,dx$

Optimal. Leaf size=34

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^2+x-2}{\sqrt{x^4+2x^3-3x^2+3x-3}}\right)$$

Rubi [F] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\sqrt{-3+3x-3x^2+2x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/Sqrt[-3 + 3*x - 3*x^2 + 2*x^3 + x^4],x]
```

```
[Out] Defer[Int][x/Sqrt[-3 + 3*x - 3*x^2 + 2*x^3 + x^4], x]
```

Rubi steps

$$\int \frac{x}{\sqrt{-3+3x-3x^2+2x^3+x^4}}\,dx = \int \frac{x}{\sqrt{-3+3x-3x^2+2x^3+x^4}}\,dx$$

Mathematica [C] time = 0.93, size = 768, normalized size = 22.59

$$\frac{2(x-1)\left(x-\text{Root}\left[\#1^3+3\#1^2+3\&,3\right]\right)\sqrt{\frac{\left(\text{Root}\left[\#1^3+3\#1^2+3\&,1\right]-1\right)\left(x-\text{Root}\left[\#1^3+3\#1^2+3\&,2\right]\right)}{\left(-1+\text{Root}\left[\#1^3+3\#1^2+3\&,2\right]\right)\left(x-\text{Root}\left[\#1^3+3\#1^2+3\&,1\right]\right)}}\left(\left(\text{Root}\left[\#1^3+3\#1^2+3\&,1\right]-1\right)\Pi\left(\frac{-1+\text{Root}\left[\#1^3+3\#1^2+3\&,3\right]}{-\text{Root}\left[\#1^3+3\#1^2+3\&,1\right]+\text{Root}\left[\#1^3+3\#1^2+3\&,3\right]};\sin^{-1}\left(\sqrt{-\frac{(x-1)\left(\text{Root}\left[\#1^3+3\#1^2+3\&,1\right]-\text{Root}\left[\#1^3+3\#1^2+3\&,3\right]\right)}{\left(x-\text{Root}\left[\#1^3+3\#1^2+3\&,1\right]\right)\left(-1+\text{Root}\left[\#1^3+3\#1^2+3\&,3\right]\right)}}\right)\middle|\frac{\left(\text{Root}\left[\#1^3+3\#1^2+3\&,1\right]-\text{Root}\left[\#1^3+3\#1^2+3\&,2\right]\right)\left(-1+\text{Root}\left[\#1^3+3\#1^2+3\&,3\right]\right)}{\left(-1+\text{Root}\left[\#1^3+3\#1^2+3\&,2\right]\right)\left(\text{Root}\left[\#1^3+3\#1^2+3\&,1\right]-\text{Root}\left[\#1^3+3\#1^2+3\&,3\right]\right)}\right)-\text{Root}\left[\#1^3+3\#1^2+3\&,1\right]F\left(\sin^{-1}\left(\sqrt{-\frac{(x-1)\left(\text{Root}\left[\#1^3+3\#1^2+3\&,1\right]-\text{Root}\left[\#1^3+3\#1^2+3\&,3\right]\right)}{\left(x-\text{Root}\left[\#1^3+3\#1^2+3\&,1\right]\right)\left(-1+\text{Root}\left[\#1^3+3\#1^2+3\&,3\right]\right)}}\right)\middle|\frac{\left(\text{Root}\left[\#1^3+3\#1^2+3\&,1\right]-\text{Root}\left[\#1^3+3\#1^2+3\&,2\right]\right)\left(-1+\text{Root}\left[\#1^3+3\#1^2+3\&,3\right]\right)}{\left(-1+\text{Root}\left[\#1^3+3\#1^2+3\&,2\right]\right)\left(\text{Root}\left[\#1^3+3\#1^2+3\&,1\right]-\text{Root}\left[\#1^3+3\#1^2+3\&,3\right]\right)}\right)\right)}{\sqrt{x^4+2x^3-3x^2+3x-3}\left(-1+\text{Root}\left[\#1^3+3\#1^2+3\&,3\right]\right)\sqrt{-\frac{(x-1)\left(\text{Root}\left[\#1^3+3\#1^2+3\&,1\right]-1\right)\left(\text{Root}\left[\#1^3+3\#1^2+3\&,1\right]-\text{Root}\left[\#1^3+3\#1^2+3\&,3\right]\right)\left(x-\text{Root}\left[\#1^3+3\#1^2+3\&,3\right]\right)}{\left(-1+\text{Root}\left[\#1^3+3\#1^2+3\&,3\right]\right)^2\left(x-\text{Root}\left[\#1^3+3\#1^2+3\&,1\right]\right)^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[-3 + 3*x - 3*x^2 + 2*x^3 + x^4],x]
```

```
[Out] (2*(-1 + x)*(EllipticPi[(-1 + Root[3 + 3*#1^2 + #1^3 & , 3, 0])/(-Root[3 +
3*#1^2 + #1^3 & , 1, 0] + Root[3 + 3*#1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[-(
((-1 + x)*(Root[3 + 3*#1^2 + #1^3 & , 1, 0] - Root[3 + 3*#1^2 + #1^3 & , 3,
 0]))/((x - Root[3 + 3*#1^2 + #1^3 & , 1, 0])*(-1 + Root[3 + 3*#1^2 + #1^3
& , 3, 0])))]], ((Root[3 + 3*#1^2 + #1^3 & , 1, 0] - Root[3 + 3*#1^2 + #1^3
 & , 2, 0])*(-1 + Root[3 + 3*#1^2 + #1^3 & , 3, 0]))/((-1 + Root[3 + 3*#1^2
 + #1^3 & , 2, 0])*(Root[3 + 3*#1^2 + #1^3 & , 1, 0] - Root[3 + 3*#1^2 + #1
^3 & , 3, 0]))]*(-1 + Root[3 + 3*#1^2 + #1^3 & , 1, 0]) - EllipticF[ArcSin[
Sqrt[-(((-1 + x)*(Root[3 + 3*#1^2 + #1^3 & , 1, 0] - Root[3 + 3*#1^2 + #1^3
 & , 3, 0]))/((x - Root[3 + 3*#1^2 + #1^3 & , 1, 0])*(-1 + Root[3 + 3*#1^2
+ #1^3 & , 3, 0])))]], ((Root[3 + 3*#1^2 + #1^3 & , 1, 0] - Root[3 + 3*#1^2
 + #1^3 & , 2, 0])*(-1 + Root[3 + 3*#1^2 + #1^3 & , 3, 0]))/((-1 + Root[3 +
 3*#1^2 + #1^3 & , 2, 0])*(Root[3 + 3*#1^2 + #1^3 & , 1, 0] - Root[3 + 3*#1
^2 + #1^3 & , 3, 0]))]*Root[3 + 3*#1^2 + #1^3 & , 1, 0])*Sqrt[((-1 + Root[3
 + 3*#1^2 + #1^3 & , 1, 0])*(x - Root[3 + 3*#1^2 + #1^3 & , 2, 0]))/((x - R
oot[3 + 3*#1^2 + #1^3 & , 1, 0])*(-1 + Root[3 + 3*#1^2 + #1^3 & , 2, 0]))]*
(x - Root[3 + 3*#1^2 + #1^3 & , 3, 0]))/(Sqrt[-3 + 3*x - 3*x^2 + 2*x^3 + x^
4]*Sqrt[-(((-1 + x)*(-1 + Root[3 + 3*#1^2 + #1^3 & , 1, 0])*(x - Root[3 + 3
*#1^2 + #1^3 & , 3, 0])*(Root[3 + 3*#1^2 + #1^3 & , 1, 0] - Root[3 + 3*#1^2
 + #1^3 & , 3, 0]))/((x - Root[3 + 3*#1^2 + #1^3 & , 1, 0])^2*(-1 + Root[3
+ 3*#1^2 + #1^3 & , 3, 0])^2))]*(-1 + Root[3 + 3*#1^2 + #1^3 & , 3, 0]))
```

IntegrateAlgebraic [A] time = 0.16, size = 34, normalized size = 1.00

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^2+x-2}{\sqrt{x^4+2x^3-3x^2+3x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[-3 + 3*x - 3*x^2 + 2*x^3 + x^4],x]
[Out] (2*ArcTanh[(-2 + x + x^2)/Sqrt[-3 + 3*x - 3*x^2 + 2*x^3 + x^4]])/3
```

fricas [A] time = 0.43, size = 40, normalized size = 1.18

$$\frac{1}{3}\log\left(2x^3+6x^2+2\sqrt{x^4+2x^3-3x^2+3x-3}(x+2)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^3-3*x^2+3*x-3)^(1/2),x, algorithm="fricas")
[Out] 1/3*log(2*x^3 + 6*x^2 + 2*sqrt(x^4 + 2*x^3 - 3*x^2 + 3*x - 3)*(x + 2) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^4+2x^3-3x^2+3x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^3-3*x^2+3*x-3)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(x^4 + 2*x^3 - 3*x^2 + 3*x - 3), x)
```

maple [C] time = 1.25, size = 1633, normalized size = 48.03

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+2*x^3-3*x^2+3*x-3)^(1/2),x)
[Out] 2*(2-1/4*(20+4*21^(1/2))^(1/3)-1/(20+4*21^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/2*
(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3)))*((3/4*(20+4*21^(1/2))^(1/3)
+3/(20+4*21^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*
21^(1/2))^(1/3)))*(-1+x)/(1/4*(20+4*21^(1/2))^(1/3)+1/(20+4*21^(1/2))^(1/3)
-2-1/2*I*3^(1/2)*(-1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3)))/(x+1
/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3)+1))^(1/2)*(x+1/2*(20+4*21^
(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3)+1)^2*(-(x-1/4*(20+4*21^(1/2))^(1/3)-1/
(20+4*21^(1/2))^(1/3)+1-1/2*I*3^(1/2)*(-1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*2
1^(1/2))^(1/3)))/(1/4*(20+4*21^(1/2))^(1/3)+1/(20+4*21^(1/2))^(1/3)-2+1/2*I
*3^(1/2)*(-1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3)))/(x+1/2*(20+4
*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3)+1)*(1/2*(20+4*21^(1/2))^(1/3)+2/(2
0+4*21^(1/2))^(1/3)+2))^(1/2)*(-(x-1/4*(20+4*21^(1/2))^(1/3)-1/(20+4*21^(1/
2))^(1/3)+1+1/2*I*3^(1/2)*(-1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/
3)))/(1/4*(20+4*21^(1/2))^(1/3)+1/(20+4*21^(1/2))^(1/3)-2-1/2*I*3^(1/2)*(-1
/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3)))/(x+1/2*(20+4*21^(1/2))^(
1/3)+2/(20+4*21^(1/2))^(1/3)+1)*(1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2)
)^(1/3)+2))^(1/2)/(3/4*(20+4*21^(1/2))^(1/3)+3/(20+4*21^(1/2))^(1/3)-1/2*I*
3^(1/2)*(-1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3)))/(-1/2*(20+4*2
1^(1/2))^(1/3)-2/(20+4*21^(1/2))^(1/3)-2)/((-1+x)*(x+1/2*(20+4*21^(1/2))^(1
/3)+2/(20+4*21^(1/2))^(1/3)+1)*(x-1/4*(20+4*21^(1/2))^(1/3)-1/(20+4*21^(1/2
))^(1/3)+1-1/2*I*3^(1/2)*(-1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3
)))*(x-1/4*(20+4*21^(1/2))^(1/3)-1/(20+4*21^(1/2))^(1/3)+1+1/2*I*3^(1/2)*(-
1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3))))^(1/2)*((-1/2*(20+4*21^
(1/2))^(1/3)-2/(20+4*21^(1/2))^(1/3)-1)*EllipticF(((3/4*(20+4*21^(1/2))^(1/
3)+3/(20+4*21^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/2*(20+4*21^(1/2))^(1/3)+2/(20+
4*21^(1/2))^(1/3)))*(-1+x)/(1/4*(20+4*21^(1/2))^(1/3)+1/(20+4*21^(1/2))^(1/
3)-2-1/2*I*3^(1/2)*(-1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3)))/(x
```

```
+1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3)+1))^(1/2),((-3/4*(20+4*2
1^(1/2))^(1/3)-3/(20+4*21^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/2*(20+4*21^(1/2))^
(1/3)+2/(20+4*21^(1/2))^(1/3)))*(2-1/4*(20+4*21^(1/2))^(1/3)-1/(20+4*21^(1/
2))^(1/3)+1/2*I*3^(1/2)*(-1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3)
))/(2-1/4*(20+4*21^(1/2))^(1/3)-1/(20+4*21^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/2
*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3)))/(-3/4*(20+4*21^(1/2))^(1/3
)-3/(20+4*21^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/2*(20+4*21^(1/2))^(1/3)+2/(20+4
*21^(1/2))^(1/3))))^(1/2))+(1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/
3)+2)*EllipticPi(((3/4*(20+4*21^(1/2))^(1/3)+3/(20+4*21^(1/2))^(1/3)-1/2*I*
3^(1/2)*(-1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3)))*(-1+x)/(1/4*(
20+4*21^(1/2))^(1/3)+1/(20+4*21^(1/2))^(1/3)-2-1/2*I*3^(1/2)*(-1/2*(20+4*21
^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3)))/(x+1/2*(20+4*21^(1/2))^(1/3)+2/(20+
4*21^(1/2))^(1/3)+1))^(1/2),(1/4*(20+4*21^(1/2))^(1/3)+1/(20+4*21^(1/2))^(1
/3)-2-1/2*I*3^(1/2)*(-1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3)))/(
3/4*(20+4*21^(1/2))^(1/3)+3/(20+4*21^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/2*(20+4
*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3))),((-3/4*(20+4*21^(1/2))^(1/3)-3/(
20+4*21^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(
1/2))^(1/3)))*(2-1/4*(20+4*21^(1/2))^(1/3)-1/(20+4*21^(1/2))^(1/3)+1/2*I*3^
(1/2)*(-1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3)))/(2-1/4*(20+4*21
^(1/2))^(1/3)-1/(20+4*21^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/2*(20+4*21^(1/2))^(
1/3)+2/(20+4*21^(1/2))^(1/3)))/(-3/4*(20+4*21^(1/2))^(1/3)-3/(20+4*21^(1/2)
)^(1/3)+1/2*I*3^(1/2)*(-1/2*(20+4*21^(1/2))^(1/3)+2/(20+4*21^(1/2))^(1/3)))
)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^4 + 2\,x^3 - 3\,x^2 + 3\,x - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^3-3*x^2+3*x-3)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(x^4 + 2*x^3 - 3*x^2 + 3*x - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{x^4 + 2\,x^3 - 3\,x^2 + 3\,x - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x - 3*x^2 + 2*x^3 + x^4 - 3)^(1/2),x)
[Out] int(x/(3*x - 3*x^2 + 2*x^3 + x^4 - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x-1)\left(x^3 + 3x^2 + 3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+2*x**3-3*x**2+3*x-3)**(1/2),x)
[Out] Integral(x/sqrt((x - 1)*(x**3 + 3*x**2 + 3)), x)
```

3.425 $\int \frac{\sqrt{1-x^6}\left(2+x^6\right)}{x^3\left(-1+x^4+x^6\right)}\,dx$

Optimal. Leaf size=34

$$\frac{\sqrt{1-x^6}}{x^2}-\tanh^{-1}\left(\frac{x^2}{\sqrt{1-x^6}}\right)$$

Rubi [F] time = 1.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1-x^6}\left(2+x^6\right)}{x^3\left(-1+x^4+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[1 - x^6]*(2 + x^6))/(x^3*(-1 + x^4 + x^6)),x]
```

```
[Out] Sqrt[1 - x^6]/x^2 + (3*Sqrt[1 - x^6])/(1 + Sqrt[3] - x^2) - (3*3^(1/4)*Sqrt
[2 - Sqrt[3]]*(1 - x^2)*Sqrt[(1 + x^2 + x^4)/(1 + Sqrt[3] - x^2)^2]*Ellipti
cE[ArcSin[(1 - Sqrt[3] - x^2)/(1 + Sqrt[3] - x^2)], -7 - 4*Sqrt[3]])/(2*Sqr
t[(1 - x^2)/(1 + Sqrt[3] - x^2)^2]*Sqrt[1 - x^6]) + (Sqrt[2]*3^(3/4)*(1 - x
^2)*Sqrt[(1 + x^2 + x^4)/(1 + Sqrt[3] - x^2)^2]*EllipticF[ArcSin[(1 - Sqrt[
3] - x^2)/(1 + Sqrt[3] - x^2)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x^2)/(1 + Sqrt[
3] - x^2)^2]*Sqrt[1 - x^6]) + Defer[Subst][Defer[Int][Sqrt[1 - x^3]/(-1 + x
^2 + x^3), x], x, x^2] + (3*Defer[Subst][Defer[Int][(x*Sqrt[1 - x^3])/(-1 +
 x^2 + x^3), x], x, x^2])/2
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-x^6}\left(2+x^6\right)}{x^3\left(-1+x^4+x^6\right)}\,dx &= \int \left(-\frac{2\sqrt{1-x^6}}{x^3}+\frac{x\left(2+3x^2\right)\sqrt{1-x^6}}{-1+x^4+x^6}\right)dx\\
&= -\left(2\int \frac{\sqrt{1-x^6}}{x^3}\,dx\right)+\int \frac{x\left(2+3x^2\right)\sqrt{1-x^6}}{-1+x^4+x^6}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(2+3x)\sqrt{1-x^3}}{-1+x^2+x^3}\,dx,x,x^2\right)-\text{Subst}\left(\int \frac{\sqrt{1-x^3}}{x^2}\,dx,x,x^2\right)\\
&= \frac{\sqrt{1-x^6}}{x^2}+\frac{1}{2}\,\text{Subst}\left(\int \left(\frac{2\sqrt{1-x^3}}{-1+x^2+x^3}+\frac{3x\sqrt{1-x^3}}{-1+x^2+x^3}\right)dx,x,x^2\right)+\frac{3}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{1-\cdots}}\right.\\
&= \frac{\sqrt{1-x^6}}{x^2}-\frac{3}{2}\,\text{Subst}\left(\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx,x,x^2\right)+\frac{3}{2}\,\text{Subst}\left(\int \frac{x\sqrt{1-x^3}}{-1+x^2+x^3}\,dx,x,x^2\right)-\cdots\\
&= \frac{\sqrt{1-x^6}}{x^2}+\frac{3\sqrt{1-x^6}}{1+\sqrt{3}-x^2}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1-x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+\sqrt{3}-x^2\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x^2}{1+\sqrt{3}-x^2}\right)|-\cdots\right.}{2\sqrt{\frac{1-x^2}{\left(1+\sqrt{3}-x^2\right)^2}}\sqrt{1-x^6}}
\end{aligned}$$

Mathematica [C] time = 5.54, size = 892, normalized size = 26.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - x^6]*(2 + x^6))/(x^3*(-1 + x^4 + x^6)),x]
```

```
[Out] Sqrt[1 - x^6]/x^2 - (Sqrt[(1 - x^2)/(1 + (-1)^(1/3))]*Sqrt[1 + x^2 + x^4]*(
(Sqrt[3]*(I*Sqrt[3] + (1 + (-1)^(1/3))*x^2)*EllipticF[ArcSin[Sqrt[(1 - (-1)
^(2/3)*x^2)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x^2) + ((3*I)
*((EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + Root[-1 + #1^2 + #1^3 & , 1, 0]), A
rcSin[Sqrt[(1 - (-1)^(2/3)*x^2)/(1 + (-1)^(1/3))]], (-1)^(1/3)]*(2 + Root[-
1 + #1^2 + #1^3 & , 1, 0]^3))/((-1)^(1/3) + Root[-1 + #1^2 + #1^3 & , 1, 0]
) + (2*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + Root[-1 + #1^2 + #1^3 & , 3, 0]
), ArcSin[Sqrt[(1 - (-1)^(2/3)*x^2)/(1 + (-1)^(1/3))]], (-1)^(1/3)]*(Root[-
1 + #1^2 + #1^3 & , 1, 0] - Root[-1 + #1^2 + #1^3 & , 2, 0])*((-1)^(1/3) +
Root[-1 + #1^2 + #1^3 & , 2, 0]) + EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + Roo
t[-1 + #1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x^2)/(1 + (-1)^(
1/3))]], (-1)^(1/3)]*(Root[-1 + #1^2 + #1^3 & , 1, 0] - Root[-1 + #1^2 + #1
^3 & , 2, 0])*((-1)^(1/3) + Root[-1 + #1^2 + #1^3 & , 2, 0])*Root[-1 + #1^2
 + #1^3 & , 3, 0]^3 - 2*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + Root[-1 + #1^2
 + #1^3 & , 2, 0]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x^2)/(1 + (-1)^(1/3))]], (-
1)^(1/3)]*(Root[-1 + #1^2 + #1^3 & , 1, 0] - Root[-1 + #1^2 + #1^3 & , 3, 0
])*((-1)^(1/3) + Root[-1 + #1^2 + #1^3 & , 3, 0]) - EllipticPi[(I*Sqrt[3])/
((-1)^(1/3) + Root[-1 + #1^2 + #1^3 & , 2, 0]), ArcSin[Sqrt[(1 - (-1)^(2/3)
*x^2)/(1 + (-1)^(1/3))]], (-1)^(1/3)]*Root[-1 + #1^2 + #1^3 & , 2, 0]^3*(Ro
ot[-1 + #1^2 + #1^3 & , 1, 0] - Root[-1 + #1^2 + #1^3 & , 3, 0])*((-1)^(1/3
) + Root[-1 + #1^2 + #1^3 & , 3, 0]))/(((-1)^(1/3) + Root[-1 + #1^2 + #1^3
& , 2, 0])*(Root[-1 + #1^2 + #1^3 & , 2, 0] - Root[-1 + #1^2 + #1^3 & , 3,
0])*((-1)^(1/3) + Root[-1 + #1^2 + #1^3 & , 3, 0]))))/((Root[-1 + #1^2 + #1
^3 & , 1, 0] - Root[-1 + #1^2 + #1^3 & , 2, 0])*(Root[-1 + #1^2 + #1^3 & ,
1, 0] - Root[-1 + #1^2 + #1^3 & , 3, 0]))))/(3*Sqrt[1 - x^6])
```

IntegrateAlgebraic [A] time = 7.32, size = 34, normalized size = 1.00

$$\frac{\sqrt{1-x^6}}{x^2}-\tanh^{-1}\left(\frac{x^2}{\sqrt{1-x^6}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 - x^6]*(2 + x^6))/(x^3*(-1 + x^4 + x^6)),x]
```

```
[Out] Sqrt[1 - x^6]/x^2 - ArcTanh[x^2/Sqrt[1 - x^6]]
```

fricas [A] time = 0.44, size = 58, normalized size = 1.71

$$\frac{x^2\log\left(-\frac{x^6-x^4+2\sqrt{-x^6+1}\,x^2-1}{x^6+x^4-1}\right)+2\sqrt{-x^6+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(1/2)*(x^6+2)/x^3/(x^6+x^4-1),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2*log(-(x^6 - x^4 + 2*sqrt(-x^6 + 1)*x^2 - 1)/(x^6 + x^4 - 1)) + 2*s
qrt(-x^6 + 1))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^6+2\right)\sqrt{-x^6+1}}{\left(x^6+x^4-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(1/2)*(x^6+2)/x^3/(x^6+x^4-1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 2)*sqrt(-x^6 + 1)/((x^6 + x^4 - 1)*x^3), x)
```

maple [A] time = 0.34, size = 60, normalized size = 1.76

$$-\frac{x^{6}-1}{x^{2}\sqrt{-x^{6}+1}}+\frac{\ln\left(-\frac{x^{6}-x^{4}+2\sqrt{-x^{6}+1}\,x^{2}-1}{x^{6}+x^{4}-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^6+1)^(1/2)*(x^6+2)/x^3/(x^6+x^4-1),x)
```

```
[Out] -(x^6-1)/x^2/(-x^6+1)^(1/2)+1/2*ln(-(x^6-x^4+2*(-x^6+1)^(1/2)*x^2-1)/(x^6+x
^4-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}+2\right)\sqrt{-x^{6}+1}}{\left(x^{6}+x^{4}-1\right)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(1/2)*(x^6+2)/x^3/(x^6+x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + 2)*sqrt(-x^6 + 1)/((x^6 + x^4 - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-x^{6}}\,\left(x^{6}+2\right)}{x^{3}\,\left(x^{6}+x^{4}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - x^6)^(1/2)*(x^6 + 2))/(x^3*(x^4 + x^6 - 1)),x)
```

```
[Out] int(((1 - x^6)^(1/2)*(x^6 + 2))/(x^3*(x^4 + x^6 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**6+1)**(1/2)*(x**6+2)/x**3/(x**6+x**4-1),x)
```

```
[Out] Timed out
```

3.426 $\int \frac{\sqrt{x}}{\left(-2+x^2\right)^{3/4}} dx$

Optimal. Leaf size=35

$$\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-2}}\right)-\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-2}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {329, 331, 298, 203, 206}

$$\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-2}}\right)-\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-2 + x^2)^(3/4),x]
```

```
[Out] -ArcTan[Sqrt[x]/(-2 + x^2)^(1/4)] + ArcTanh[Sqrt[x]/(-2 + x^2)^(1/4)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(-2+x^2\right)^{3/4}}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{\left(-2+x^4\right)^{3/4}}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{-2+x^2}}\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{-2+x^2}}\right) - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{-2+x^2}}\right)$$
$$= -\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-2+x^2}}\right) + \tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-2+x^2}}\right)$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-2}}\right) - \tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-2 + x^2)^(3/4),x]
```

```
[Out] -ArcTan[Sqrt[x]/(-2 + x^2)^(1/4)] + ArcTanh[Sqrt[x]/(-2 + x^2)^(1/4)]
```

IntegrateAlgebraic [A] time = 0.21, size = 35, normalized size = 1.00

$$\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-2}}\right) - \tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(-2 + x^2)^(3/4),x]
```

```
[Out] -ArcTan[Sqrt[x]/(-2 + x^2)^(1/4)] + ArcTanh[Sqrt[x]/(-2 + x^2)^(1/4)]
```

fricas [B] time = 0.81, size = 83, normalized size = 2.37

$$-\frac{1}{2}\arctan\left(\frac{\left(x^2-2\right)^{\frac{3}{4}}x^{\frac{3}{2}} - \left(x^2-2\right)^{\frac{5}{4}}\sqrt{x}}{2\left(x^3-2x\right)}\right) + \frac{1}{2}\log\left(-x^2 - \left(x^2-2\right)^{\frac{1}{4}}x^{\frac{3}{2}} - \sqrt{x^2-2}\,x - \left(x^2-2\right)^{\frac{3}{4}}\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2-2)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*((x^2 - 2)^(3/4)*x^(3/2) - (x^2 - 2)^(5/4)*sqrt(x))/(x^3 -
2*x)) + 1/2*log(-x^2 - (x^2 - 2)^(1/4)*x^(3/2) - sqrt(x^2 - 2)*x - (x^2 - 2
)^(3/4)*sqrt(x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(x^2-2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2-2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/(x^2 - 2)^(3/4), x)
```

maple [C] time = 0.23, size = 42, normalized size = 1.20

$$\frac{2^{\frac{1}{4}} \left(-\mathrm{signum}\left(-1+\frac{x^2}{2}\right)\right)^{\frac{3}{4}} x^{\frac{3}{2}} \,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],\frac{x^2}{2}\right)}{3\mathrm{signum}\left(-1+\frac{x^2}{2}\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2-2)^(3/4),x)
```

```
[Out] 1/3*2^(1/4)/signum(-1+1/2*x^2)^(3/4)*(-signum(-1+1/2*x^2))^(3/4)*x^(3/2)*hy
pergeom([3/4,3/4],[7/4],1/2*x^2)
```

maxima [A] time = 0.53, size = 45, normalized size = 1.29

$$\arctan\left(\frac{\left(x^2-2\right)^{\frac{1}{4}}}{\sqrt{x}}\right)+\frac{1}{2}\log\left(\frac{\left(x^2-2\right)^{\frac{1}{4}}}{\sqrt{x}}+1\right)-\frac{1}{2}\log\left(\frac{\left(x^2-2\right)^{\frac{1}{4}}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2-2)^(3/4),x, algorithm="maxima")
```

```
[Out] arctan((x^2 - 2)^(1/4)/sqrt(x)) + 1/2*log((x^2 - 2)^(1/4)/sqrt(x) + 1) - 1/
2*log((x^2 - 2)^(1/4)/sqrt(x) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{x}}{\left(x^2-2\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2 - 2)^(3/4),x)
```

```
[Out] int(x^(1/2)/(x^2 - 2)^(3/4), x)
```

sympy [C] time = 0.87, size = 41, normalized size = 1.17

$$\frac{\sqrt[4]{2}\,x^{\frac{3}{2}} e^{-\frac{3i\pi}{4}}\,\Gamma\left(\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\,\frac{x^2}{2}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2-2)**(3/4),x)
```

```
[Out] 2**(1/4)*x**(3/2)*exp(-3*I*pi/4)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), x**2/
2)/(4*gamma(7/4))
```

3.427 $\int \frac{a+x}{\left(-1+2b+2ax+x^2\right) \sqrt[4]{2b+2ax+x^2}} \, dx$

Optimal. Leaf size=35

$$\tan^{-1}\left(\sqrt[4]{2ax+2b+x^2}\right)-\tanh^{-1}\left(\sqrt[4]{2ax+2b+x^2}\right)$$

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+x}{\left(-1+2b+2ax+x^2\right) \sqrt[4]{2b+2ax+x^2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(a + x)/((-1 + 2*b + 2*a*x + x^2)*(2*b + 2*a*x + x^2)^(1/4)),x]
```

```
[Out] Defer[Int][(a + x)/((-1 + 2*b + 2*a*x + x^2)*(2*b + 2*a*x + x^2)^(1/4)), x]
```

Rubi steps

$$\int \frac{a+x}{\left(-1+2b+2ax+x^2\right) \sqrt[4]{2b+2ax+x^2}} \, dx=\int \frac{a+x}{\left(-1+2b+2ax+x^2\right) \sqrt[4]{2b+2ax+x^2}} \, dx$$

Mathematica [A] time = 0.27, size = 35, normalized size = 1.00

$$\tan^{-1}\left(\sqrt[4]{2ax+2b+x^2}\right)-\tanh^{-1}\left(\sqrt[4]{2ax+2b+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x)/((-1 + 2*b + 2*a*x + x^2)*(2*b + 2*a*x + x^2)^(1/4)),x]
```

```
[Out] ArcTan[(2*b + 2*a*x + x^2)^(1/4)] - ArcTanh[(2*b + 2*a*x + x^2)^(1/4)]
```

IntegrateAlgebraic [A] time = 0.05, size = 35, normalized size = 1.00

$$\tan^{-1}\left(\sqrt[4]{2ax+2b+x^2}\right)-\tanh^{-1}\left(\sqrt[4]{2ax+2b+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + x)/((-1 + 2*b + 2*a*x + x^2)*(2*b + 2*a*x + x^2)^(1
/4)),x]
```

```
[Out] ArcTan[(2*b + 2*a*x + x^2)^(1/4)] - ArcTanh[(2*b + 2*a*x + x^2)^(1/4)]
```

fricas [A] time = 0.40, size = 51, normalized size = 1.46

$$\arctan\left(\left(2\,ax+x^2+2\,b\right)^{\frac{1}{4}}\right)-\frac{1}{2}\,\log\left(\left(2\,ax+x^2+2\,b\right)^{\frac{1}{4}}+1\right)+\frac{1}{2}\,\log\left(\left(2\,ax+x^2+2\,b\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(2*a*x+x^2+2*b-1)/(2*a*x+x^2+2*b)^(1/4),x, algorithm="frica
s")
```

```
[Out] arctan((2*a*x + x^2 + 2*b)^(1/4)) - 1/2*log((2*a*x + x^2 + 2*b)^(1/4) + 1)
+ 1/2*log((2*a*x + x^2 + 2*b)^(1/4) - 1)
```

giac [A] time = 0.46, size = 60, normalized size = 1.71

$$\arctan\left(2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(ax+\frac{1}{2}x^2+b\right)^{\frac{1}{4}}\right)-\frac{1}{2}\log\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}+\left(ax+\frac{1}{2}x^2+b\right)^{\frac{1}{4}}\right)+\frac{1}{2}\log\left(\left|-\left(\frac{1}{2}\right)^{\frac{1}{4}}+\left(ax+\frac{1}{2}x^2+b\right)^{\frac{1}{4}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(2*a*x+x^2+2*b-1)/(2*a*x+x^2+2*b)^(1/4),x, algorithm="giac"
)
```

```
[Out] arctan(2*(1/2)^(3/4)*(a*x + 1/2*x^2 + b)^(1/4)) - 1/2*log((1/2)^(1/4) + (a*
x + 1/2*x^2 + b)^(1/4)) + 1/2*log(abs(-(1/2)^(1/4) + (a*x + 1/2*x^2 + b)^(1
/4)))
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{a+x}{\left(2ax+x^2+2b-1\right)\left(2ax+x^2+2b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+x)/(2*a*x+x^2+2*b-1)/(2*a*x+x^2+2*b)^(1/4),x)
```

```
[Out] int((a+x)/(2*a*x+x^2+2*b-1)/(2*a*x+x^2+2*b)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+x}{\left(2\,ax+x^2+2\,b\right)^{\frac{1}{4}}\left(2\,ax+x^2+2\,b-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(2*a*x+x^2+2*b-1)/(2*a*x+x^2+2*b)^(1/4),x, algorithm="maxim
a")
```

```
[Out] integrate((a + x)/((2*a*x + x^2 + 2*b)^(1/4)*(2*a*x + x^2 + 2*b - 1)), x)
```

mupad [B] time = 0.61, size = 31, normalized size = 0.89

$$\mathrm{atan}\left(\left(x^2+2\,a\,x+2\,b\right)^{1/4}\right)-\mathrm{atanh}\left(\left(x^2+2\,a\,x+2\,b\right)^{1/4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x)/((2*b + 2*a*x + x^2)^(1/4)*(2*b + 2*a*x + x^2 - 1)),x)
```

```
[Out] atan((2*b + 2*a*x + x^2)^(1/4)) - atanh((2*b + 2*a*x + x^2)^(1/4))
```

sympy [A] time = 11.81, size = 56, normalized size = 1.60

$$\frac{\log\left(\sqrt[4]{2ax+2b+x^2}-1\right)}{2}-\frac{\log\left(\sqrt[4]{2ax+2b+x^2}+1\right)}{2}+\mathrm{atan}\left(\sqrt[4]{2ax+2b+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(2*a*x+x**2+2*b-1)/(2*a*x+x**2+2*b)**(1/4),x)
```

```
[Out] log((2*a*x + 2*b + x**2)**(1/4) - 1)/2 - log((2*a*x + 2*b + x**2)**(1/4) +
1)/2 + atan((2*a*x + 2*b + x**2)**(1/4))
```

3.428 $\int \frac{1}{\sqrt{c+bx+ax^2}}\,dx$

Optimal. Leaf size=35

$$-\frac{\log\left(-2\sqrt{a}\sqrt{ax^2+bx+c}+2ax+b\right)}{\sqrt{a}}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {621, 206}

$$\frac{\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{ax^2+bx+c}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c + b*x + a*x^2],x]
```

```
[Out] ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c + b*x + a*x^2])]/Sqrt[a]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+bx+ax^2}}\,dx = 2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{b+2ax}{\sqrt{c+bx+ax^2}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{b+2ax}{2\sqrt{a}\sqrt{c+bx+ax^2}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 0.97

$$\frac{\log\left(2\sqrt{a}\sqrt{ax^2+bx+c}+2ax+b\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c + b*x + a*x^2],x]
```

```
[Out] Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[c + b*x + a*x^2]]/Sqrt[a]
```

IntegrateAlgebraic [A] time = 0.13, size = 35, normalized size = 1.00

$$-\frac{\log\left(-2\sqrt{a}\sqrt{ax^2+bx+c}+2ax+b\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[c + b*x + a*x^2],x]
```

```
[Out] -(Log[b + 2*a*x - 2*Sqrt[a]*Sqrt[c + b*x + a*x^2]]/Sqrt[a])
```

fricas [A] time = 0.40, size = 106, normalized size = 3.03

$$\left[\frac{\log\left(-8\,a^2x^2 - 8\,abx - 4\,\sqrt{ax^2+bx+c}\,(2\,ax+b)\sqrt{a} - b^2 - 4\,ac\right)}{2\,\sqrt{a}}, -\frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{ax^2+bx+c}\,(2\,ax+b)\sqrt{-a}}{2\,(a^2x^2+abx+ac)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-8*a^2*x^2 - 8*a*b*x - 4*sqrt(a*x^2 + b*x + c)*(2*a*x + b)*sqrt(a)
 - b^2 - 4*a*c)/sqrt(a), -sqrt(-a)*arctan(1/2*sqrt(a*x^2 + b*x + c)*(2*a*x
+ b)*sqrt(-a)/(a^2*x^2 + a*b*x + a*c))/a]
```

giac [A] time = 0.64, size = 36, normalized size = 1.03

$$-\frac{\log\left(\left|-2\left(\sqrt{a}x - \sqrt{ax^2+bx+c}\right)\sqrt{a} - b\right|\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x + c))*sqrt(a) - b))/sqrt(a)
```

maple [A] time = 0.00, size = 30, normalized size = 0.86

$$\frac{\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}} + \sqrt{a\,x^2+bx+c}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+b*x+c)^(1/2),x)
```

```
[Out] ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x+c)^(1/2))/a^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 0.28, size = 29, normalized size = 0.83

$$\frac{\ln\left(\frac{\frac{b}{2}+a\,x}{\sqrt{a}} + \sqrt{a\,x^2+b\,x+c}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + b*x + a*x^2)^(1/2),x)
```

```
[Out] log((b/2 + a*x)/a^(1/2) + (c + b*x + a*x^2)^(1/2))/a^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + bx + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+b*x+c)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x + c), x)
```

3.429 $\int x^{11} \sqrt[3]{-1+x^3}\, dx$

Optimal. Leaf size=35

$$\frac{\sqrt[3]{x^3-1}\left(140x^{12}-14x^9-18x^6-27x^3-81\right)}{1820}$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.51, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{13}\left(x^3-1\right)^{13/3}+\frac{3}{10}\left(x^3-1\right)^{10/3}+\frac{3}{7}\left(x^3-1\right)^{7/3}+\frac{1}{4}\left(x^3-1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(-1 + x^3)^(1/3),x]
```

```
[Out] (-1 + x^3)^(4/3)/4 + (3*(-1 + x^3)^(7/3))/7 + (3*(-1 + x^3)^(10/3))/10 + (-
1 + x^3)^(13/3)/13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^{11} \sqrt[3]{-1+x^3}\, dx &= \frac{1}{3}\text{Subst}\left(\int \sqrt[3]{-1+x}\, x^3\, dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\sqrt[3]{-1+x}+3(-1+x)^{4/3}+3(-1+x)^{7/3}+(-1+x)^{10/3}\right) dx, x, x^3\right)\\
&= \frac{1}{4}\left(-1+x^3\right)^{4/3}+\frac{3}{7}\left(-1+x^3\right)^{7/3}+\frac{3}{10}\left(-1+x^3\right)^{10/3}+\frac{1}{13}\left(-1+x^3\right)^{13/3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.86

$$\frac{\left(x^3-1\right)^{4/3}\left(140x^9+126x^6+108x^3+81\right)}{1820}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(-1 + x^3)^(1/3),x]
```

```
[Out] ((-1 + x^3)^(4/3)*(81 + 108*x^3 + 126*x^6 + 140*x^9))/1820
```

IntegrateAlgebraic [A] time = 0.02, size = 35, normalized size = 1.00

$$\frac{\sqrt[3]{x^3-1}\left(140x^{12}-14x^9-18x^6-27x^3-81\right)}{1820}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11*(-1 + x^3)^(1/3),x]
```

```
[Out] ((-1 + x^3)^(1/3)*(-81 - 27*x^3 - 18*x^6 - 14*x^9 + 140*x^12))/1820
```

fricas [A] time = 0.39, size = 31, normalized size = 0.89

$$\frac{1}{1820}\left(140\,x^{12}-14\,x^{9}-18\,x^{6}-27\,x^{3}-81\right)\left(x^{3}-1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(x^3-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/1820*(140*x^12 - 14*x^9 - 18*x^6 - 27*x^3 - 81)*(x^3 - 1)^(1/3)
```

giac [A] time = 0.37, size = 37, normalized size = 1.06

$$\frac{1}{13}\left(x^{3}-1\right)^{\frac{13}{3}}+\frac{3}{10}\left(x^{3}-1\right)^{\frac{10}{3}}+\frac{3}{7}\left(x^{3}-1\right)^{\frac{7}{3}}+\frac{1}{4}\left(x^{3}-1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(x^3-1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/13*(x^3 - 1)^(13/3) + 3/10*(x^3 - 1)^(10/3) + 3/7*(x^3 - 1)^(7/3) + 1/4*(
x^3 - 1)^(4/3)
```

maple [A] time = 0.00, size = 36, normalized size = 1.03

$$\frac{(-1+x)\left(x^{2}+x+1\right)\left(140x^{9}+126x^{6}+108x^{3}+81\right)\left(x^{3}-1\right)^{\frac{1}{3}}}{1820}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(x^3-1)^(1/3),x)
```

```
[Out] 1/1820*(-1+x)*(x^2+x+1)*(140*x^9+126*x^6+108*x^3+81)*(x^3-1)^(1/3)
```

maxima [A] time = 0.53, size = 37, normalized size = 1.06

$$\frac{1}{13}\left(x^{3}-1\right)^{\frac{13}{3}}+\frac{3}{10}\left(x^{3}-1\right)^{\frac{10}{3}}+\frac{3}{7}\left(x^{3}-1\right)^{\frac{7}{3}}+\frac{1}{4}\left(x^{3}-1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(x^3-1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/13*(x^3 - 1)^(13/3) + 3/10*(x^3 - 1)^(10/3) + 3/7*(x^3 - 1)^(7/3) + 1/4*(
x^3 - 1)^(4/3)
```

mupad [B] time = 0.24, size = 31, normalized size = 0.89

$$-\left(x^{3}-1\right)^{1/3}\left(-\frac{x^{12}}{13}+\frac{x^{9}}{130}+\frac{9\,x^{6}}{910}+\frac{27\,x^{3}}{1820}+\frac{81}{1820}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(x^3 - 1)^(1/3),x)
```

```
[Out] -(x^3 - 1)^(1/3)*((27*x^3)/1820 + (9*x^6)/910 + x^9/130 - x^12/13 + 81/1820
)
```

sympy [B] time = 1.61, size = 68, normalized size = 1.94

$$\frac{x^{12}\sqrt[3]{x^3-1}}{13}-\frac{x^9\sqrt[3]{x^3-1}}{130}-\frac{9x^6\sqrt[3]{x^3-1}}{910}-\frac{27x^3\sqrt[3]{x^3-1}}{1820}-\frac{81\sqrt[3]{x^3-1}}{1820}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(x**3-1)**(1/3),x)
```

```
[Out] x**12*(x**3 - 1)**(1/3)/13 - x**9*(x**3 - 1)**(1/3)/130 - 9*x**6*(x**3 - 1)
**(1/3)/910 - 27*x**3*(x**3 - 1)**(1/3)/1820 - 81*(x**3 - 1)**(1/3)/1820
```

3.430 $\int \frac{-1-2x+x^2}{(1+2x+3x^2)\sqrt{-x+x^3}}\,dx$

Optimal. Leaf size=35

$$-\frac{2\tanh^{-1}\left(\frac{\frac{x}{\sqrt{3}}-\frac{1}{\sqrt{3}}}{\sqrt{x^3-x}}\right)}{\sqrt{3}}$$

Rubi [C] time = 0.92, antiderivative size = 229, normalized size of antiderivative = 6.54, number of steps used = 13, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2056, 6728, 329, 222, 933, 168, 537}

$$\frac{\sqrt{2}\sqrt{x-1}\sqrt{x}\sqrt{x+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{x-1}}\right)|\frac{1}{2}\right)}{3\sqrt{x^3-x}}+\frac{2\left(1+2i\sqrt{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{3}{4-i\sqrt{2}};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{3\left(\sqrt{2}+4i\right)\sqrt{x^3-x}}-\frac{2\left(1-2i\sqrt{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{3}{4+i\sqrt{2}};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{3\left(-\sqrt{2}+4i\right)\sqrt{x^3-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 - 2*x + x^2)/((1 + 2*x + 3*x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] (Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]*Sqrt[1 + x]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[x]
)/Sqrt[-1 + x]], 1/2])/(3*Sqrt[-x + x^3]) + (2*(1 + (2*I)*Sqrt[2])*Sqrt[x]*
Sqrt[1 - x^2]*EllipticPi[3/(4 - I*Sqrt[2]), ArcSin[Sqrt[1 - x]], 1/2])/(3*(
4*I + Sqrt[2])*Sqrt[-x + x^3]) - (2*(1 - (2*I)*Sqrt[2])*Sqrt[x]*Sqrt[1 - x^
2]*EllipticPi[3/(4 + I*Sqrt[2]), ArcSin[Sqrt[1 - x]], 1/2])/(3*(4*I - Sqrt[
2])*Sqrt[-x + x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
```

```
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-1-2x+x^2}{\left(1+2x+3x^2\right)\sqrt{-x+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{-1-2x+x^2}{\sqrt{x}\sqrt{-1+x^2}\left(1+2x+3x^2\right)}\,dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \left(\frac{1}{3\sqrt{x}\sqrt{-1+x^2}}-\frac{4(1+2x)}{3\sqrt{x}\sqrt{-1+x^2}\left(1+2x+3x^2\right)}\right)dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-1+x^2}}\,dx}{3\sqrt{-x+x^3}} - \frac{\left(4\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1+2x}{\sqrt{x}\sqrt{-1+x^2}\left(1+2x+3x^2\right)}\,dx}{3\sqrt{-x+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{-x+x^3}} - \frac{\left(4\sqrt{x}\sqrt{-1+x^2}\right)\int \Big(\frac{}{\sqrt{x}\left(2-\right.}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{3\sqrt{-x+x^3}} - \frac{\left(2\left(4-i\sqrt{2}\right)\sqrt{x}\sqrt{-1+x^2}\right)}{3\sqrt{-x}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{3\sqrt{-x+x^3}} - \frac{\left(2\left(4-i\sqrt{2}\right)\sqrt{x}\sqrt{1-x^2}\right)\int}{3\sqrt{-x}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{3\sqrt{-x+x^3}} + \frac{\left(4\left(4-i\sqrt{2}\right)\sqrt{x}\sqrt{1-x^2}\right)\text{S}}{}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{3\sqrt{-x+x^3}} + \frac{\sqrt{2}\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{3}{4-i\sqrt{2}};\text{si}\right.}{3\sqrt{-x+x^3}}$$

Mathematica [C] time = 1.04, size = 112, normalized size = 3.20

$$-\frac{2\sqrt{1-\frac{1}{x^2}}\,x^{3/2}\left(-3F\left(\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)\middle|-1\right)+\left(2+i\sqrt{2}\right)\Pi\left(\frac{i}{-i+\sqrt{2}};\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)\middle|-1\right)+\left(2-i\sqrt{2}\right)\Pi\left(-\frac{i}{i+\sqrt{2}};\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)\middle|-1\right)\right)}{3\sqrt{x\left(x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x + x^2)/((1 + 2*x + 3*x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] (-2*Sqrt[1 - x^(-2)]*x^(3/2)*(-3*EllipticF[ArcSin[1/Sqrt[x]], -1] + (2 + I*
Sqrt[2])*EllipticPi[I/(-I + Sqrt[2]), ArcSin[1/Sqrt[x]], -1] + (2 - I*Sqrt[
2])*EllipticPi[(-I)/(I + Sqrt[2]), ArcSin[1/Sqrt[x]], -1]))/(3*Sqrt[x*(-1 +
 x^2)])
```

IntegrateAlgebraic [A] time = 0.31, size = 35, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\frac{x}{\sqrt{3}}-\frac{1}{\sqrt{3}}}{\sqrt{x^3-x}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 2*x + x^2)/((1 + 2*x + 3*x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] (-2*ArcTanh[(-(1/Sqrt[3]) + x/Sqrt[3])/Sqrt[-x + x^3]])/Sqrt[3]
```

fricas [B] time = 0.42, size = 73, normalized size = 2.09

$$\frac{1}{6}\sqrt{3}\log\left(\frac{9x^4+36x^3-4\sqrt{3}\sqrt{x^3-x}\left(3x^2+4x-1\right)+10x^2-20x+1}{9x^4+12x^3+10x^2+4x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(3*x^2+2*x+1)/(x^3-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((9*x^4 + 36*x^3 - 4*sqrt(3)*sqrt(x^3 - x)*(3*x^2 + 4*x - 1)
 + 10*x^2 - 20*x + 1)/(9*x^4 + 12*x^3 + 10*x^2 + 4*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2-2x-1}{\sqrt{x^3-x}\left(3x^2+2x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(3*x^2+2*x+1)/(x^3-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 2*x - 1)/(sqrt(x^3 - x)*(3*x^2 + 2*x + 1)), x)
```

maple [C] time = 0.05, size = 167, normalized size = 4.77

$$\frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticF}\left(\sqrt{1+x},\frac{\sqrt{2}}{2}\right)}{3\sqrt{x^3-x}}-\frac{4\left(\frac{1}{3}-\frac{i\sqrt{2}}{12}\right)\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\left(-1+\frac{i\sqrt{2}}{2}\right)\mathrm{EllipticPi}\left(\sqrt{1+x},-\frac{i\sqrt{2}}{2}+1,\frac{\sqrt{2}}{2}\right)}{3\sqrt{x^3-x}}-\frac{4\left(\frac{1}{3}+\frac{i\sqrt{2}}{12}\right)\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\left(-1-\frac{i\sqrt{2}}{2}\right)\mathrm{EllipticPi}\left(\sqrt{1+x},\frac{i\sqrt{2}}{2}+1,\frac{\sqrt{2}}{2}\right)}{3\sqrt{x^3-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x-1)/(3*x^2+2*x+1)/(x^3-x)^(1/2),x)
```

```
[Out] 1/3*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticF((1+x)^(1/2
),1/2*2^(1/2))-4/3*(1/3-1/12*I*2^(1/2))*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2
)/(x^3-x)^(1/2)*(-1+1/2*I*2^(1/2))*EllipticPi((1+x)^(1/2),-1/2*I*2^(1/2)+1,
1/2*2^(1/2))-4/3*(1/3+1/12*I*2^(1/2))*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/
(x^3-x)^(1/2)*(-1-1/2*I*2^(1/2))*EllipticPi((1+x)^(1/2),1/2*I*2^(1/2)+1,1/2
*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2-2x-1}{\sqrt{x^3-x}\left(3x^2+2x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(3*x^2+2*x+1)/(x^3-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 2*x - 1)/(sqrt(x^3 - x)*(3*x^2 + 2*x + 1)), x)
```

mupad [B] time = 0.22, size = 175, normalized size = 5.00

$$-\frac{2\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{3\sqrt{x^3-x}}-\frac{\sqrt{2}\,\sqrt{-x}\,\left(-\frac{4}{9}+\frac{\sqrt{2}\,8\mathrm{i}}{9}\right)\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(\frac{1}{\frac{1}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}};\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)1\mathrm{i}}{2\sqrt{x^3-x}\,\left(\frac{1}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}\right)}+\frac{\sqrt{2}\,\sqrt{-x}\,\left(\frac{4}{9}+\frac{\sqrt{2}\,8\mathrm{i}}{9}\right)\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(-\frac{1}{-\frac{1}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}};\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)1\mathrm{i}}{2\sqrt{x^3-x}\,\left(-\frac{1}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - x^2 + 1)/((x^3 - x)^(1/2)*(2*x + 3*x^2 + 1)),x)
```

```
[Out] (2^(1/2)*(-x)^(1/2)*((2^(1/2)*8i)/9 + 4/9)*(1 - x)^(1/2)*(x + 1)^(1/2)*elli
pticPi(-1/((2^(1/2)*1i)/3 - 1/3), asin((-x)^(1/2)), -1)*1i)/(2*(x^3 - x)^(1
/2)*((2^(1/2)*1i)/3 - 1/3)) - (2^(1/2)*(-x)^(1/2)*((2^(1/2)*8i)/9 - 4/9)*(1
 - x)^(1/2)*(x + 1)^(1/2)*ellipticPi(1/((2^(1/2)*1i)/3 + 1/3), asin((-x)^(1
/2)), -1)*1i)/(2*(x^3 - x)^(1/2)*((2^(1/2)*1i)/3 + 1/3)) - (2*(-x)^(1/2)*(1
 - x)^(1/2)*(x + 1)^(1/2)*ellipticF(asin((-x)^(1/2)), -1))/(3*(x^3 - x)^(1/
2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-2x-1}{\sqrt{x\,(x-1)\,(x+1)}\,\left(3x^2+2x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x-1)/(3*x**2+2*x+1)/(x**3-x)**(1/2),x)
```

```
[Out] Integral((x**2 - 2*x - 1)/(sqrt(x*(x - 1)*(x + 1))*(3*x**2 + 2*x + 1)), x)
```

3.431 $\int \frac{(-1+x^2)\sqrt{-1-4x-5x^2-4x^3-x^4}}{(1+x+x^2)(1+3x+x^2)^2}\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{-x^4-4x^3-5x^2-4x-1}}{x^2+3x+1}$$

Rubi [F] time = 2.69, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1+x^2\right)\sqrt{-1-4x-5x^2-4x^3-x^4}}{\left(1+x+x^2\right)\left(1+3x+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^2)*Sqrt[-1 - 4*x - 5*x^2 - 4*x^3 - x^4])/((1 + x + x^2)*(1 + 3
*x + x^2)^2),x]
```

```
[Out] (-6*Defer[Int][Sqrt[-1 - 4*x - 5*x^2 - 4*x^3 - x^4]/(-3 + Sqrt[5] - 2*x)^2,
 x])/5 + (2*(3 - Sqrt[5])*Defer[Int][Sqrt[-1 - 4*x - 5*x^2 - 4*x^3 - x^4]/(
-3 + Sqrt[5] - 2*x)^2, x])/5 - ((1 + I*Sqrt[3])*Defer[Int][Sqrt[-1 - 4*x -
5*x^2 - 4*x^3 - x^4]/(1 - I*Sqrt[3] + 2*x), x])/4 - ((1 - I*Sqrt[3])*Defer[
Int][Sqrt[-1 - 4*x - 5*x^2 - 4*x^3 - x^4]/(1 + I*Sqrt[3] + 2*x), x])/4 + ((
5 - Sqrt[5])*Defer[Int][Sqrt[-1 - 4*x - 5*x^2 - 4*x^3 - x^4]/(3 - Sqrt[5] +
 2*x), x])/20 - (6*Defer[Int][Sqrt[-1 - 4*x - 5*x^2 - 4*x^3 - x^4]/(3 + Sqr
t[5] + 2*x)^2, x])/5 + (2*(3 + Sqrt[5])*Defer[Int][Sqrt[-1 - 4*x - 5*x^2 -
4*x^3 - x^4]/(3 + Sqrt[5] + 2*x)^2, x])/5 + ((5 + Sqrt[5])*Defer[Int][Sqrt[
-1 - 4*x - 5*x^2 - 4*x^3 - x^4]/(3 + Sqrt[5] + 2*x), x])/20
```

Rubi steps

$$\int \frac{\left(-1+x^2\right)\sqrt{-1-4x-5x^2-4x^3-x^4}}{\left(1+x+x^2\right)\left(1+3x+x^2\right)^2}\,dx = \int \left(\frac{(1-x)\sqrt{-1-4x-5x^2-4x^3-x^4}}{4\left(1+x+x^2\right)} + \frac{(-3-2x)\sqrt{-1-4x-5x^2}}{2\left(1+3x+x^2\right)}\right.$$

$$= \frac{1}{4}\int \frac{(1-x)\sqrt{-1-4x-5x^2-4x^3-x^4}}{1+x+x^2}\,dx + \frac{1}{4}\int \frac{(1+x)\sqrt{-1-4x}}{1+3}$$

$$= \frac{1}{4}\int \left(\frac{\left(-1-i\sqrt{3}\right)\sqrt{-1-4x-5x^2-4x^3-x^4}}{1-i\sqrt{3}+2x} + \frac{\left(-1+i\sqrt{3}\right)\sqrt{-1}}{1+i}\right.$$

$$= -\left(\frac{3}{2}\int \frac{\sqrt{-1-4x-5x^2-4x^3-x^4}}{\left(1+3x+x^2\right)^2}\,dx\right) + \frac{1}{4}\left(-1-i\sqrt{3}\right)\int \frac{\sqrt{-1-}}{}$$

$$= -\left(\frac{3}{2}\int \left(\frac{4\sqrt{-1-4x-5x^2-4x^3-x^4}}{5\left(-3+\sqrt{5}-2x\right)^2} + \frac{4\sqrt{-1-4x-5x^2-4x^3-}}{5\sqrt{5}\left(-3+\sqrt{5}-2x\right)}\right.\right.$$

$$= -\left(\frac{6}{5}\int \frac{\sqrt{-1-4x-5x^2-4x^3-x^4}}{\left(-3+\sqrt{5}-2x\right)^2}\,dx\right) - \frac{6}{5}\int \frac{\sqrt{-1-4x-5x^2-}}{\left(3+\sqrt{5}+2\right.}$$

Mathematica [A] time = 0.47, size = 35, normalized size = 1.00

$$\frac{\sqrt{-x^4-4x^3-5x^2-4x-1}}{x^2+3x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)*Sqrt[-1 - 4*x - 5*x^2 - 4*x^3 - x^4])/((1 + x + x^2)*
(1 + 3*x + x^2)^2),x]
```

```
[Out] Sqrt[-1 - 4*x - 5*x^2 - 4*x^3 - x^4]/(1 + 3*x + x^2)
```

IntegrateAlgebraic [A] time = 0.36, size = 35, normalized size = 1.00

$$\frac{\sqrt{-x^4-4x^3-5x^2-4x-1}}{x^2+3x+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*Sqrt[-1 - 4*x - 5*x^2 - 4*x^3 - x^4])/((1 +
x + x^2)*(1 + 3*x + x^2)^2),x]
```

```
[Out] Sqrt[-1 - 4*x - 5*x^2 - 4*x^3 - x^4]/(1 + 3*x + x^2)
```

fricas [A] time = 0.39, size = 33, normalized size = 0.94

$$\frac{\sqrt{-x^4-4\,x^3-5\,x^2-4\,x-1}}{x^2+3\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(-x^4-4*x^3-5*x^2-4*x-1)^(1/2)/(x^2+x+1)/(x^2+3*x+1)^2,x,
 algorithm="fricas")
```

```
[Out] sqrt(-x^4 - 4*x^3 - 5*x^2 - 4*x - 1)/(x^2 + 3*x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4-4\,x^3-5\,x^2-4\,x-1}\left(x^2-1\right)}{\left(x^2+3\,x+1\right)^2\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(-x^4-4*x^3-5*x^2-4*x-1)^(1/2)/(x^2+x+1)/(x^2+3*x+1)^2,x,
 algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 - 4*x^3 - 5*x^2 - 4*x - 1)*(x^2 - 1)/((x^2 + 3*x + 1)^2
*(x^2 + x + 1)), x)
```

maple [A] time = 0.01, size = 34, normalized size = 0.97

$$\frac{\sqrt{-x^4-4x^3-5x^2-4x-1}}{x^2+3x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(-x^4-4*x^3-5*x^2-4*x-1)^(1/2)/(x^2+x+1)/(x^2+3*x+1)^2,x)
```

```
[Out] (-x^4-4*x^3-5*x^2-4*x-1)^(1/2)/(x^2+3*x+1)
```

maxima [A] time = 0.46, size = 31, normalized size = 0.89

$$\frac{\sqrt{x^2+x+1}\,\sqrt{-x^2-3\,x-1}}{x^2+3\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(-x^4-4*x^3-5*x^2-4*x-1)^(1/2)/(x^2+x+1)/(x^2+3*x+1)^2,x,
 algorithm="maxima")
```

```
[Out] sqrt(x^2 + x + 1)*sqrt(-x^2 - 3*x - 1)/(x^2 + 3*x + 1)
```

mupad [B] time = 0.29, size = 33, normalized size = 0.94

$$\frac{\sqrt{-x^4-4\,x^3-5\,x^2-4\,x-1}}{x^2+3\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*(- 4*x - 5*x^2 - 4*x^3 - x^4 - 1)^(1/2))/((3*x + x^2 + 1)^2*
(x + x^2 + 1)),x)
```

```
[Out] (- 4*x - 5*x^2 - 4*x^3 - x^4 - 1)^(1/2)/(3*x + x^2 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(x^2+x+1\right)\left(x^2+3x+1\right)}\,(x-1)\,(x+1)}{\left(x^2+x+1\right)\left(x^2+3x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(-x**4-4*x**3-5*x**2-4*x-1)**(1/2)/(x**2+x+1)/(x**2+3*x+
1)**2,x)
```

```
[Out] Integral(sqrt(-(x**2 + x + 1)*(x**2 + 3*x + 1))*(x - 1)*(x + 1)/((x**2 + x
+ 1)*(x**2 + 3*x + 1)**2), x)
```

3.432 $\int \frac{-1+x^4}{x^3\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{x^4+1}}{2x^2}+\frac{1}{2}\log\left(\sqrt{x^4+1}+x^2\right)$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 0.71, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {451, 275, 215}

$$\frac{1}{2}\sinh^{-1}\left(x^2\right)+\frac{\sqrt{x^4+1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)/(x^3*Sqrt[1 + x^4]),x]
```

```
[Out] Sqrt[1 + x^4]/(2*x^2) + ArcSinh[x^2]/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^4}{x^3\sqrt{1+x^4}}\,dx &= \frac{\sqrt{1+x^4}}{2x^2}+\int \frac{x}{\sqrt{1+x^4}}\,dx\\ &= \frac{\sqrt{1+x^4}}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{1+x^4}}{2x^2}+\frac{1}{2}\sinh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.63

$$\frac{1}{2}\left(\sinh^{-1}\left(x^2\right)+\frac{\sqrt{x^4+1}}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)/(x^3*Sqrt[1 + x^4]),x]
[Out] (Sqrt[1 + x^4]/x^2 + ArcSinh[x^2])/2
```

IntegrateAlgebraic [A] time = 0.08, size = 37, normalized size = 1.06

$$\frac{\sqrt{x^4+1}}{2x^2}-\frac{1}{2}\log\left(\sqrt{x^4+1}-x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/(x^3*Sqrt[1 + x^4]),x]
[Out] Sqrt[1 + x^4]/(2*x^2) - Log[-x^2 + Sqrt[1 + x^4]]/2
```

fricas [A] time = 0.39, size = 38, normalized size = 1.09

$$-\frac{x^2\log\left(-x^2+\sqrt{x^4+1}\right)-x^2-\sqrt{x^4+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^3/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*(x^2*log(-x^2 + sqrt(x^4 + 1)) - x^2 - sqrt(x^4 + 1))/x^2
```

giac [A] time = 0.31, size = 38, normalized size = 1.09

$$-\frac{1}{\left(x^2-\sqrt{x^4+1}\right)^2-1}-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^3/(x^4+1)^(1/2),x, algorithm="giac")
[Out] -1/((x^2 - sqrt(x^4 + 1))^2 - 1) - 1/2*log(-x^2 + sqrt(x^4 + 1))
```

maple [A] time = 0.02, size = 20, normalized size = 0.57

$$\frac{\operatorname{arcsinh}\left(x^2\right)}{2}+\frac{\sqrt{x^4+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/x^3/(x^4+1)^(1/2),x)
[Out] 1/2*arcsinh(x^2)+1/2*(x^4+1)^(1/2)/x^2
```

maxima [A] time = 0.53, size = 45, normalized size = 1.29

$$\frac{\sqrt{x^4+1}}{2\,x^2}+\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right)-\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^3/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(x^4 + 1)/x^2 + 1/4*log(sqrt(x^4 + 1)/x^2 + 1) - 1/4*log(sqrt(x^4 +
 1)/x^2 - 1)
```

mupad [B] time = 0.39, size = 19, normalized size = 0.54

$$\frac{\operatorname{asinh}\left(x^2\right)}{2}+\frac{\sqrt{x^4+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/(x^3*(x^4 + 1)^(1/2)),x)
[Out] asinh(x^2)/2 + (x^4 + 1)^(1/2)/(2*x^2)
```

sympy [A] time = 1.53, size = 19, normalized size = 0.54

$$\frac{\operatorname{asinh}\left(x^2\right)}{2}+\frac{\sqrt{x^4+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/x**3/(x**4+1)**(1/2),x)
[Out] asinh(x**2)/2 + sqrt(x**4 + 1)/(2*x**2)
```

3.433 $\int \frac{-1+x^3}{x^6(1+x^3)\sqrt[4]{x+x^4}}\,dx$

Optimal. Leaf size=35

$$-\frac{4\left(x^4+x\right)^{3/4}\left(20x^6+5x^3-1\right)}{21x^6\left(x^3+1\right)}$$

Rubi [A] time = 0.17, antiderivative size = 47, normalized size of antiderivative = 1.34, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2056, 453, 271, 264}

$$-\frac{80x}{21\sqrt[4]{x^4+x}}+\frac{4}{21\sqrt[4]{x^4+x}\,x^5}-\frac{20}{21\sqrt[4]{x^4+x}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(x^6*(1 + x^3)*(x + x^4)^(1/4)),x]
```

```
[Out] 4/(21*x^5*(x + x^4)^(1/4)) - 20/(21*x^2*(x + x^4)^(1/4)) - (80*x)/(21*(x + 
x^4)^(1/4))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{-1+x^3}{x^6\left(1+x^3\right)\sqrt[4]{x+x^4}}\,dx = \frac{\left(\sqrt[4]{x}\,\sqrt[4]{1+x^3}\right)\int \frac{-1+x^3}{x^{25/4}\left(1+x^3\right)^{5/4}}\,dx}{\sqrt[4]{x+x^4}}$$
$$= \frac{4}{21x^5\sqrt[4]{x+x^4}} + \frac{\left(15\sqrt[4]{x}\,\sqrt[4]{1+x^3}\right)\int \frac{1}{x^{13/4}\left(1+x^3\right)^{5/4}}\,dx}{7\sqrt[4]{x+x^4}}$$
$$= \frac{4}{21x^5\sqrt[4]{x+x^4}} - \frac{20}{21x^2\sqrt[4]{x+x^4}} - \frac{\left(20\sqrt[4]{x}\,\sqrt[4]{1+x^3}\right)\int \frac{1}{\sqrt[4]{x}\left(1+x^3\right)^{5/4}}\,dx}{7\sqrt[4]{x+x^4}}$$
$$= \frac{4}{21x^5\sqrt[4]{x+x^4}} - \frac{20}{21x^2\sqrt[4]{x+x^4}} - \frac{80x}{21\sqrt[4]{x+x^4}}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.80

$$\frac{-80x^6-20x^3+4}{21x^5\sqrt[4]{x^4+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(x^6*(1 + x^3)*(x + x^4)^(1/4)),x]
[Out] (4 - 20*x^3 - 80*x^6)/(21*x^5*(x + x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.40, size = 35, normalized size = 1.00

$$-\frac{4\left(x^4+x\right)^{3/4}\left(20x^6+5x^3-1\right)}{21x^6\left(x^3+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)/(x^6*(1 + x^3)*(x + x^4)^(1/4)),x]
[Out] (-4*(x + x^4)^(3/4)*(-1 + 5*x^3 + 20*x^6))/(21*x^6*(1 + x^3))
```

fricas [A] time = 0.41, size = 30, normalized size = 0.86

$$-\frac{4\left(20\,x^6+5\,x^3-1\right)\left(x^4+x\right)^{\frac{3}{4}}}{21\left(x^9+x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/x^6/(x^3+1)/(x^4+x)^(1/4),x, algorithm="fricas")
[Out] -4/21*(20*x^6 + 5*x^3 - 1)*(x^4 + x)^(3/4)/(x^9 + x^6)
```

giac [A] time = 0.35, size = 28, normalized size = 0.80

$$\frac{4}{21}\left(\frac{1}{x^3}+1\right)^{\frac{7}{4}} - \frac{4}{3}\left(\frac{1}{x^3}+1\right)^{\frac{3}{4}} - \frac{8}{3\left(\frac{1}{x^3}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/x^6/(x^3+1)/(x^4+x)^(1/4),x, algorithm="giac")
```

```
[Out] 4/21*(1/x^3 + 1)^(7/4) - 4/3*(1/x^3 + 1)^(3/4) - 8/3/(1/x^3 + 1)^(1/4)
```

maple [A] time = 0.01, size = 25, normalized size = 0.71

$$-\frac{4\left(20x^6+5x^3-1\right)}{21\left(x^4+x\right)^{\frac{1}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/x^6/(x^3+1)/(x^4+x)^(1/4),x)
[Out] -4/21*(20*x^6+5*x^3-1)/(x^4+x)^(1/4)/x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3-1}{\left(x^4+x\right)^{\frac{1}{4}}\left(x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/x^6/(x^3+1)/(x^4+x)^(1/4),x, algorithm="maxima")
[Out] integrate((x^3 - 1)/((x^4 + x)^(1/4)*(x^3 + 1)*x^6), x)
```

mupad [B] time = 0.31, size = 31, normalized size = 0.89

$$-\frac{4\left(x^4+x\right)^{3/4}\left(20\,x^6+5\,x^3-1\right)}{21\,x^6\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x^6*(x^3 + 1)*(x + x^4)^(1/4)),x)
[Out] -(4*(x + x^4)^(3/4)*(5*x^3 + 20*x^6 - 1))/(21*x^6*(x^3 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\left(x^2+x+1\right)}{x^6\sqrt[4]{x(x+1)\left(x^2-x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/x**6/(x**3+1)/(x**4+x)**(1/4),x)
[Out] Integral((x - 1)*(x**2 + x + 1)/(x**6*(x*(x + 1)*(x**2 - x + 1))**(1/4)*(x
+ 1)*(x**2 - x + 1)), x)
```

3.434 $\int \frac{\sqrt{x+x^4}}{x^3} dx$

Optimal. Leaf size=35

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)-\frac{2\sqrt{x^4+x}}{3x^2}$$

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2020, 2029, 206}

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)-\frac{2\sqrt{x^4+x}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + x^4]/x^3,x]
```

```
[Out] (-2*Sqrt[x + x^4])/(3*x^2) + (2*ArcTanh[x^2/Sqrt[x + x^4]])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x+x^4}}{x^3} dx &= -\frac{2\sqrt{x+x^4}}{3x^2}+\int \frac{x}{\sqrt{x+x^4}} dx\\ &= -\frac{2\sqrt{x+x^4}}{3x^2}+\frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{x^2}{\sqrt{x+x^4}}\right)\\ &= -\frac{2\sqrt{x+x^4}}{3x^2}+\frac{2}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x+x^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.43

$$\frac{2\sqrt{x^4+x}\sinh^{-1}\left(x^{3/2}\right)}{3\sqrt{x}\sqrt{x^3+1}}-\frac{2\sqrt{x^4+x}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + x^4]/x^3,x]
```

```
[Out] (-2*Sqrt[x + x^4])/(3*x^2) + (2*Sqrt[x + x^4]*ArcSinh[x^(3/2)])/(3*Sqrt[x]*
Sqrt[1 + x^3])
```

IntegrateAlgebraic [A] time = 0.37, size = 35, normalized size = 1.00

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)-\frac{2\sqrt{x^4+x}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x + x^4]/x^3,x]
```

```
[Out] (-2*Sqrt[x + x^4])/(3*x^2) + (2*ArcTanh[x^2/Sqrt[x + x^4]])/3
```

fricas [A] time = 0.45, size = 37, normalized size = 1.06

$$\frac{x^2\log\left(-2\,x^3-2\,\sqrt{x^4+x}\,x-1\right)-2\,\sqrt{x^4+x}}{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/3*(x^2*log(-2*x^3 - 2*sqrt(x^4 + x)*x - 1) - 2*sqrt(x^4 + x))/x^2
```

giac [A] time = 0.26, size = 35, normalized size = 1.00

$$-\frac{2}{3}\sqrt{\frac{1}{x^3}+1}+\frac{1}{3}\log\left(\sqrt{\frac{1}{x^3}+1}+1\right)-\frac{1}{3}\log\left(\left|\sqrt{\frac{1}{x^3}+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -2/3*sqrt(1/x^3 + 1) + 1/3*log(sqrt(1/x^3 + 1) + 1) - 1/3*log(abs(sqrt(1/x^
3 + 1) - 1))
```

maple [C] time = 0.23, size = 303, normalized size = 8.66

$$-\frac{2\sqrt{x^4+x}}{3x^2}-\frac{2\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\,(1+x)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(1+x)}}\sqrt{-\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\left(-\mathrm{EllipticF}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)+\mathrm{EllipticPi}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\frac{\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(1+x)\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x)^(1/2)/x^3,x)
```

```
[Out] -2/3*(x^4+x)^(1/2)/x^2-2*(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1
/2*I*3^(1/2))/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/
2))/(1+x))^(1/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(
3/2+1/2*I*3^(1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1
/2)*(-EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-
3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2))+EllipticPi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))
^(1/2),(1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-
1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^4+x}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{x^4 + x}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^4)^(1/2)/x^3,x)
[Out] int((x + x^4)^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\,(x+1)\left(x^2 - x + 1\right)}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x)**(1/2)/x**3,x)
[Out] Integral(sqrt(x*(x + 1)*(x**2 - x + 1))/x**3, x)
```

3.435 $\int \frac{1}{x^8 \sqrt[4]{x^2+x^4}} \, dx$

Optimal. Leaf size=35

$$\frac{2 \left(x^4+x^2\right)^{3/4} \left(128 x^6-96 x^4+84 x^2-77\right)}{1155 x^9}$$

Rubi [B] time = 0.10, antiderivative size = 73, normalized size of antiderivative = 2.09, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2016, 2014}

$$-\frac{2 \left(x^4+x^2\right)^{3/4}}{15 x^9}+\frac{8 \left(x^4+x^2\right)^{3/4}}{55 x^7}-\frac{64 \left(x^4+x^2\right)^{3/4}}{385 x^5}+\frac{256 \left(x^4+x^2\right)^{3/4}}{1155 x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(x^2 + x^4)^(1/4)),x]
```

```
[Out] (-2*(x^2 + x^4)^(3/4))/(15*x^9) + (8*(x^2 + x^4)^(3/4))/(55*x^7) - (64*(x^2
 + x^4)^(3/4))/(385*x^5) + (256*(x^2 + x^4)^(3/4))/(1155*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8 \sqrt[4]{x^2+x^4}} \, dx &= -\frac{2 \left(x^2+x^4\right)^{3/4}}{15 x^9}-\frac{4}{5} \int \frac{1}{x^6 \sqrt[4]{x^2+x^4}} \, dx\\
&= -\frac{2 \left(x^2+x^4\right)^{3/4}}{15 x^9}+\frac{8 \left(x^2+x^4\right)^{3/4}}{55 x^7}+\frac{32}{55} \int \frac{1}{x^4 \sqrt[4]{x^2+x^4}} \, dx\\
&= -\frac{2 \left(x^2+x^4\right)^{3/4}}{15 x^9}+\frac{8 \left(x^2+x^4\right)^{3/4}}{55 x^7}-\frac{64 \left(x^2+x^4\right)^{3/4}}{385 x^5}-\frac{128}{385} \int \frac{1}{x^2 \sqrt[4]{x^2+x^4}} \, dx\\
&= -\frac{2 \left(x^2+x^4\right)^{3/4}}{15 x^9}+\frac{8 \left(x^2+x^4\right)^{3/4}}{55 x^7}-\frac{64 \left(x^2+x^4\right)^{3/4}}{385 x^5}+\frac{256 \left(x^2+x^4\right)^{3/4}}{1155 x^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.00

$$\frac{2 \left(x^4+x^2\right)^{3/4} \left(128 x^6-96 x^4+84 x^2-77\right)}{1155 x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(x^2 + x^4)^(1/4)),x]
[Out] (2*(x^2 + x^4)^(3/4)*(-77 + 84*x^2 - 96*x^4 + 128*x^6))/(1155*x^9)
```

IntegrateAlgebraic [A] time = 0.19, size = 35, normalized size = 1.00

$$\frac{2\left(x^4+x^2\right)^{3/4}\left(128x^6-96x^4+84x^2-77\right)}{1155x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^8*(x^2 + x^4)^(1/4)),x]
[Out] (2*(x^2 + x^4)^(3/4)*(-77 + 84*x^2 - 96*x^4 + 128*x^6))/(1155*x^9)
```

fricas [A] time = 0.40, size = 31, normalized size = 0.89

$$\frac{2\left(128\,x^6-96\,x^4+84\,x^2-77\right)\left(x^4+x^2\right)^{\frac{3}{4}}}{1155\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^4+x^2)^(1/4),x, algorithm="fricas")
[Out] 2/1155*(128*x^6 - 96*x^4 + 84*x^2 - 77)*(x^4 + x^2)^(3/4)/x^9
```

giac [A] time = 0.80, size = 37, normalized size = 1.06

$$-\frac{2}{15}\left(\frac{1}{x^2}+1\right)^{\frac{15}{4}}+\frac{6}{11}\left(\frac{1}{x^2}+1\right)^{\frac{11}{4}}-\frac{6}{7}\left(\frac{1}{x^2}+1\right)^{\frac{7}{4}}+\frac{2}{3}\left(\frac{1}{x^2}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^4+x^2)^(1/4),x, algorithm="giac")
[Out] -2/15*(1/x^2 + 1)^(15/4) + 6/11*(1/x^2 + 1)^(11/4) - 6/7*(1/x^2 + 1)^(7/4)
+ 2/3*(1/x^2 + 1)^(3/4)
```

maple [A] time = 0.00, size = 37, normalized size = 1.06

$$\frac{2\left(x^2+1\right)\left(128x^6-96x^4+84x^2-77\right)}{1155x^7\left(x^4+x^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^4+x^2)^(1/4),x)
[Out] 2/1155*(x^2+1)*(128*x^6-96*x^4+84*x^2-77)/x^7/(x^4+x^2)^(1/4)
```

maxima [A] time = 0.60, size = 36, normalized size = 1.03

$$\frac{2\left(128\,x^9+32\,x^7-12\,x^5+7\,x^3-77\,x\right)}{1155\left(x^2+1\right)^{\frac{1}{4}}x^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^4+x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] 2/1155*(128*x^9 + 32*x^7 - 12*x^5 + 7*x^3 - 77*x)/((x^2 + 1)^(1/4)*x^(17/2)
)
```

mupad [B] time = 0.25, size = 57, normalized size = 1.63

$$\frac{256\left(x^4+x^2\right)^{3/4}}{1155\,x^3}-\frac{64\left(x^4+x^2\right)^{3/4}}{385\,x^5}+\frac{8\left(x^4+x^2\right)^{3/4}}{55\,x^7}-\frac{2\left(x^4+x^2\right)^{3/4}}{15\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^2 + x^4)^(1/4)),x)
```

```
[Out] (256*(x^2 + x^4)^(3/4))/(1155*x^3) - (64*(x^2 + x^4)^(3/4))/(385*x^5) + (8*
(x^2 + x^4)^(3/4))/(55*x^7) - (2*(x^2 + x^4)^(3/4))/(15*x^9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8\sqrt[4]{x^2\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**4+x**2)**(1/4),x)
```

```
[Out] Integral(1/(x**8*(x**2*(x**2 + 1))**(1/4)), x)
```

3.436 $\int \frac{1-2x}{\sqrt{5+5x-4x^2-2x^3+x^4}} \, dx$

Optimal. Leaf size=35

$$\log\left(-2x^2+2\sqrt{x^4-2x^3-4x^2+5x+5}+2x+5\right)$$

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1680, 12, 1107, 621, 206}

$$\tanh^{-1}\left(\frac{11-4\left(x-\frac{1}{2}\right)^2}{\sqrt{16\left(x-\frac{1}{2}\right)^4-88\left(x-\frac{1}{2}\right)^2+101}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x)/Sqrt[5 + 5*x - 4*x^2 - 2*x^3 + x^4],x]
```

```
[Out] ArcTanh[(11 - 4*(-1/2 + x)^2)/Sqrt[101 - 88*(-1/2 + x)^2 + 16*(-1/2 + x)^4]
]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1-2x}{\sqrt{5+5x-4x^2-2x^3+x^4}}\,dx = \text{Subst}\left(\int -\frac{8x}{\sqrt{101-88x^2+16x^4}}\,dx, x, -\frac{1}{2}+x\right)$$
$$= -\left(8\,\text{Subst}\left(\int \frac{x}{\sqrt{101-88x^2+16x^4}}\,dx, x, -\frac{1}{2}+x\right)\right)$$
$$= -\left(4\,\text{Subst}\left(\int \frac{1}{\sqrt{101-88x+16x^2}}\,dx, x, \left(-\frac{1}{2}+x\right)^2\right)\right)$$
$$= -\left(8\,\text{Subst}\left(\int \frac{1}{64-x^2}\,dx, x, \frac{8\left(-11+4\left(-\frac{1}{2}+x\right)^2\right)}{\sqrt{101+(1-2x)^4-88\left(-\frac{1}{2}+x\right)^2}}\right)\right)$$
$$= \tanh^{-1}\left(\frac{11-(-1+2x)^2}{\sqrt{101-22(1-2x)^2+(1-2x)^4}}\right)$$

Mathematica [C] time = 1.64, size = 789, normalized size = 22.54

$$\frac{\left(-2x+\sqrt{11-2\sqrt{5}}+1\right)^2\sqrt{\frac{-2x+\sqrt{11+2\sqrt{5}}+1}{-2x+\sqrt{11-2\sqrt{5}}+1}}\sqrt{-\frac{2x+\sqrt{11+2\sqrt{5}}-1}{\left(\sqrt{11-2\sqrt{5}}-\sqrt{11+2\sqrt{5}}\right)\left(-2x+\sqrt{11-2\sqrt{5}}+1\right)}}\left(\left(-\sqrt{11+2\sqrt{5}}\sqrt{-\frac{2x+\sqrt{11-2\sqrt{5}}-1}{-2x+\sqrt{11-2\sqrt{5}}+1}}+\sqrt{\frac{\left(2\sqrt{5}-11\right)\left(2x+\sqrt{11-2\sqrt{5}}-1\right)}{-2x+\sqrt{11-2\sqrt{5}}+1}}+\sqrt{2}\sqrt{\frac{\left(\sqrt{101}-11\right)\left(2x+\sqrt{11-2\sqrt{5}}-1\right)}{-2x+\sqrt{11-2\sqrt{5}}+1}}+\sqrt{2}\sqrt{-\frac{\left(2\sqrt{5}-11\right)\left(\sqrt{101}-11\right)\left(2x+\sqrt{11-2\sqrt{5}}-1\right)}{-2x+\sqrt{11-2\sqrt{5}}+1}}\right)F\left(\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{11-2\sqrt{5}}-\sqrt{11+2\sqrt{5}}\right)\left(2x+\sqrt{11-2\sqrt{5}}-1\right)}{\left(\sqrt{11-2\sqrt{5}}+\sqrt{11+2\sqrt{5}}\right)\left(-2x+\sqrt{11-2\sqrt{5}}+1\right)}}\right)\middle|\frac{1}{20}\left(11+\sqrt{101}\right)^2\right)-2\sqrt{2}\sqrt{-\frac{\left(2\sqrt{5}-11\right)\left(\sqrt{101}-11\right)\left(2x+\sqrt{11-2\sqrt{5}}-1\right)}{-2x+\sqrt{11-2\sqrt{5}}+1}}\Pi\left(-\frac{\sqrt{11-2\sqrt{5}}+\sqrt{11+2\sqrt{5}}}{\sqrt{11-2\sqrt{5}}-\sqrt{11+2\sqrt{5}}};\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{11-2\sqrt{5}}-\sqrt{11+2\sqrt{5}}\right)\left(2x+\sqrt{11-2\sqrt{5}}-1\right)}{\left(\sqrt{11-2\sqrt{5}}+\sqrt{11+2\sqrt{5}}\right)\left(-2x+\sqrt{11-2\sqrt{5}}+1\right)}}\right)\middle|\frac{1}{20}\left(11+\sqrt{101}\right)^2\right)\right)}{\left(\sqrt{11+2\sqrt{5}}-\sqrt{11-2\sqrt{5}}\right)^{3/2}\sqrt{x^4-2x^3-4x^2+5x+5}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - 2*x)/Sqrt[5 + 5*x - 4*x^2 - 2*x^3 + x^4],x]
```

```
[Out] ((1 + Sqrt[11 - 2*Sqrt[5]] - 2*x)^2*Sqrt[(1 + Sqrt[11 + 2*Sqrt[5]] - 2*x)/(
1 + Sqrt[11 - 2*Sqrt[5]] - 2*x)]*Sqrt[-((-1 + Sqrt[11 + 2*Sqrt[5]] + 2*x)/(
(Sqrt[11 - 2*Sqrt[5]] - Sqrt[11 + 2*Sqrt[5]])*(1 + Sqrt[11 - 2*Sqrt[5]] - 2
*x)))]*((-(Sqrt[11 + 2*Sqrt[5]]*Sqrt[-((-1 + Sqrt[11 - 2*Sqrt[5]] + 2*x)/(1
 + Sqrt[11 - 2*Sqrt[5]] - 2*x))]) + Sqrt[((-11 + 2*Sqrt[5])*(-1 + Sqrt[11 -
 2*Sqrt[5]] + 2*x))/(1 + Sqrt[11 - 2*Sqrt[5]] - 2*x)] + Sqrt[2]*Sqrt[((-11
+ Sqrt[101])*(-1 + Sqrt[11 - 2*Sqrt[5]] + 2*x))/(1 + Sqrt[11 - 2*Sqrt[5]] -
 2*x)] + Sqrt[2]*Sqrt[-(((-11 + 2*Sqrt[5])*(-11 + Sqrt[101])*(-1 + Sqrt[11
- 2*Sqrt[5]] + 2*x))/(1 + Sqrt[11 - 2*Sqrt[5]] - 2*x))])*EllipticF[ArcSin[S
qrt[((Sqrt[11 - 2*Sqrt[5]] - Sqrt[11 + 2*Sqrt[5]])*(-1 + Sqrt[11 - 2*Sqrt[5
]] + 2*x))/((Sqrt[11 - 2*Sqrt[5]] + Sqrt[11 + 2*Sqrt[5]])*(1 + Sqrt[11 - 2*
Sqrt[5]] - 2*x))]], (11 + Sqrt[101])^2/20] - 2*Sqrt[2]*Sqrt[-(((-11 + 2*Sqr
t[5])*(-11 + Sqrt[101])*(-1 + Sqrt[11 - 2*Sqrt[5]] + 2*x))/(1 + Sqrt[11 - 2
*Sqrt[5]] - 2*x))]*EllipticPi[-((Sqrt[11 - 2*Sqrt[5]] + Sqrt[11 + 2*Sqrt[5]
])/(Sqrt[11 - 2*Sqrt[5]] - Sqrt[11 + 2*Sqrt[5]])), ArcSin[Sqrt[((Sqrt[11 -
2*Sqrt[5]] - Sqrt[11 + 2*Sqrt[5]])*(-1 + Sqrt[11 - 2*Sqrt[5]] + 2*x))/((Sqr
t[11 - 2*Sqrt[5]] + Sqrt[11 + 2*Sqrt[5]])*(1 + Sqrt[11 - 2*Sqrt[5]] - 2*x))
]], (11 + Sqrt[101])^2/20]))/((-Sqrt[11 - 2*Sqrt[5]] + Sqrt[11 + 2*Sqrt[5]]
)^(3/2)*Sqrt[5 + 5*x - 4*x^2 - 2*x^3 + x^4])
```

IntegrateAlgebraic [A] time = 0.27, size = 35, normalized size = 1.00

$$\log\left(-2x^2+2\sqrt{x^4-2x^3-4x^2+5x+5}+2x+5\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - 2*x)/Sqrt[5 + 5*x - 4*x^2 - 2*x^3 + x^4],x]
```

```
[Out] Log[5 + 2*x - 2*x^2 + 2*Sqrt[5 + 5*x - 4*x^2 - 2*x^3 + x^4]]
```

fricas [A] time = 0.43, size = 33, normalized size = 0.94

$$\log\left(-2\,x^2+2\,x+2\,\sqrt{x^4-2\,x^3-4\,x^2+5\,x+5}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)/(x^4-2*x^3-4*x^2+5*x+5)^(1/2),x, algorithm="fricas")
[Out] log(-2*x^2 + 2*x + 2*sqrt(x^4 - 2*x^3 - 4*x^2 + 5*x + 5) + 5)
```

giac [A] time = 0.46, size = 35, normalized size = 1.00

$$\log\left(\left|-2\,x^2+2\,x+2\,\sqrt{\left(x^2-x\right)^2-5\,x^2+5\,x+5}+5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)/(x^4-2*x^3-4*x^2+5*x+5)^(1/2),x, algorithm="giac")
[Out] log(abs(-2*x^2 + 2*x + 2*sqrt((x^2 - x)^2 - 5*x^2 + 5*x + 5) + 5))
```

maple [C] time = 0.32, size = 1182, normalized size = 33.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2*x)/(x^4-2*x^3-4*x^2+5*x+5)^(1/2),x)
[Out] 2*(-1/2*(11-2*5^(1/2))^(1/2)-1/2*(11+2*5^(1/2))^(1/2))*((1/2*(11+2*5^(1/2))
^(1/2)-1/2*(11-2*5^(1/2))^(1/2))*(x-1/2+1/2*(11-2*5^(1/2))^(1/2))/(1/2*(11+
2*5^(1/2))^(1/2)+1/2*(11-2*5^(1/2))^(1/2))/(x-1/2-1/2*(11-2*5^(1/2))^(1/2))
)^(1/2)*(x-1/2-1/2*(11-2*5^(1/2))^(1/2))^2*(-(x-1/2+1/2*(11+2*5^(1/2))^(1/2
))/(1/2*(11+2*5^(1/2))^(1/2)-1/2*(11-2*5^(1/2))^(1/2))/(x-1/2-1/2*(11-2*5^(
1/2))^(1/2))*(11-2*5^(1/2))^(1/2))^(1/2)*((11-2*5^(1/2))^(1/2)*(x-1/2-1/2*(
11+2*5^(1/2))^(1/2))/(1/2*(11+2*5^(1/2))^(1/2)+1/2*(11-2*5^(1/2))^(1/2))/(x
-1/2-1/2*(11-2*5^(1/2))^(1/2)))^(1/2)/(1/2*(11+2*5^(1/2))^(1/2)-1/2*(11-2*5
^(1/2))^(1/2))/(11-2*5^(1/2))^(1/2)/((x-1/2+1/2*(11-2*5^(1/2))^(1/2))*(x-1/
2-1/2*(11-2*5^(1/2))^(1/2))*(x-1/2+1/2*(11+2*5^(1/2))^(1/2))*(x-1/2-1/2*(11
+2*5^(1/2))^(1/2)))^(1/2)*EllipticF(((1/2*(11+2*5^(1/2))^(1/2)-1/2*(11-2*5^
(1/2))^(1/2))*(x-1/2+1/2*(11-2*5^(1/2))^(1/2))/(1/2*(11+2*5^(1/2))^(1/2)+1/
2*(11-2*5^(1/2))^(1/2))/(x-1/2-1/2*(11-2*5^(1/2))^(1/2)))^(1/2),((1/2*(11+2
*5^(1/2))^(1/2)+1/2*(11-2*5^(1/2))^(1/2))^2/(1/2*(11+2*5^(1/2))^(1/2)-1/2*(
11-2*5^(1/2))^(1/2))^2)^(1/2))-4*(-1/2*(11-2*5^(1/2))^(1/2)-1/2*(11+2*5^(1/
2))^(1/2))*((1/2*(11+2*5^(1/2))^(1/2)-1/2*(11-2*5^(1/2))^(1/2))*(x-1/2+1/2*
(11-2*5^(1/2))^(1/2))/(1/2*(11+2*5^(1/2))^(1/2)+1/2*(11-2*5^(1/2))^(1/2))/(
x-1/2-1/2*(11-2*5^(1/2))^(1/2)))^(1/2)*(x-1/2-1/2*(11-2*5^(1/2))^(1/2))^2*(
-(x-1/2+1/2*(11+2*5^(1/2))^(1/2))/(1/2*(11+2*5^(1/2))^(1/2)-1/2*(11-2*5^(1/
2))^(1/2))/(x-1/2-1/2*(11-2*5^(1/2))^(1/2))*(11-2*5^(1/2))^(1/2))^(1/2)*((1
1-2*5^(1/2))^(1/2)*(x-1/2-1/2*(11+2*5^(1/2))^(1/2))/(1/2*(11+2*5^(1/2))^(1/
2)+1/2*(11-2*5^(1/2))^(1/2))/(x-1/2-1/2*(11-2*5^(1/2))^(1/2)))^(1/2)/(1/2*(
11+2*5^(1/2))^(1/2)-1/2*(11-2*5^(1/2))^(1/2))/(11-2*5^(1/2))^(1/2)/((x-1/2+
1/2*(11-2*5^(1/2))^(1/2))*(x-1/2-1/2*(11-2*5^(1/2))^(1/2))*(x-1/2+1/2*(11+2
*5^(1/2))^(1/2))*(x-1/2-1/2*(11+2*5^(1/2))^(1/2)))^(1/2)*((1/2+1/2*(11-2*5^
(1/2))^(1/2))*EllipticF(((1/2*(11+2*5^(1/2))^(1/2)-1/2*(11-2*5^(1/2))^(1/2)
)*(x-1/2+1/2*(11-2*5^(1/2))^(1/2))/(1/2*(11+2*5^(1/2))^(1/2)+1/2*(11-2*5^(1
/2))^(1/2))/(x-1/2-1/2*(11-2*5^(1/2))^(1/2)))^(1/2),((1/2*(11+2*5^(1/2))^(1
/2)+1/2*(11-2*5^(1/2))^(1/2))^2/(1/2*(11+2*5^(1/2))^(1/2)-1/2*(11-2*5^(1/2)
)^(1/2))^2)^(1/2))-(11-2*5^(1/2))^(1/2)*EllipticPi(((1/2*(11+2*5^(1/2))^(1/
2)-1/2*(11-2*5^(1/2))^(1/2))*(x-1/2+1/2*(11-2*5^(1/2))^(1/2))/(1/2*(11+2*5^
(1/2))^(1/2)+1/2*(11-2*5^(1/2))^(1/2))/(x-1/2-1/2*(11-2*5^(1/2))^(1/2)))^(1
/2),(1/2*(11+2*5^(1/2))^(1/2)+1/2*(11-2*5^(1/2))^(1/2))/(1/2*(11+2*5^(1/2))
^(1/2)-1/2*(11-2*5^(1/2))^(1/2)),((1/2*(11+2*5^(1/2))^(1/2)+1/2*(11-2*5^(1/
2))^(1/2))^2/(1/2*(11+2*5^(1/2))^(1/2)-1/2*(11-2*5^(1/2))^(1/2))^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,x-1}{\sqrt{x^4-2\,x^3-4\,x^2+5\,x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)/(x^4-2*x^3-4*x^2+5*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x - 1)/sqrt(x^4 - 2*x^3 - 4*x^2 + 5*x + 5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{2\,x-1}{\sqrt{x^4-2\,x^3-4\,x^2+5\,x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 1)/(5*x - 4*x^2 - 2*x^3 + x^4 + 5)^(1/2),x)
```

```
[Out] int(-(2*x - 1)/(5*x - 4*x^2 - 2*x^3 + x^4 + 5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x}{\sqrt{x^4-2x^3-4x^2+5x+5}}\,dx-\int \left(-\frac{1}{\sqrt{x^4-2x^3-4x^2+5x+5}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)/(x**4-2*x**3-4*x**2+5*x+5)**(1/2),x)
```

```
[Out] -Integral(2*x/sqrt(x**4 - 2*x**3 - 4*x**2 + 5*x + 5), x) - Integral(-1/sqrt
(x**4 - 2*x**3 - 4*x**2 + 5*x + 5), x)
```

3.437 $\int \frac{1}{x^4 \sqrt[4]{-x^3+x^4}}\,dx$

Optimal. Leaf size=35

$$\frac{4\left(128x^3+96x^2+84x+77\right)\left(x^4-x^3\right)^{3/4}}{1155x^6}$$

Rubi [B] time = 0.10, antiderivative size = 81, normalized size of antiderivative = 2.31, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2016, 2014}

$$\frac{512\left(x^4-x^3\right)^{3/4}}{1155x^3}+\frac{128\left(x^4-x^3\right)^{3/4}}{385x^4}+\frac{4\left(x^4-x^3\right)^{3/4}}{15x^6}+\frac{16\left(x^4-x^3\right)^{3/4}}{55x^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (4*(-x^3 + x^4)^(3/4))/(15*x^6) + (16*(-x^3 + x^4)^(3/4))/(55*x^5) + (128*(
-x^3 + x^4)^(3/4))/(385*x^4) + (512*(-x^3 + x^4)^(3/4))/(1155*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 \sqrt[4]{-x^3+x^4}}\,dx &= \frac{4\left(-x^3+x^4\right)^{3/4}}{15x^6}+\frac{4}{5}\int \frac{1}{x^3 \sqrt[4]{-x^3+x^4}}\,dx\\
&= \frac{4\left(-x^3+x^4\right)^{3/4}}{15x^6}+\frac{16\left(-x^3+x^4\right)^{3/4}}{55x^5}+\frac{32}{55}\int \frac{1}{x^2 \sqrt[4]{-x^3+x^4}}\,dx\\
&= \frac{4\left(-x^3+x^4\right)^{3/4}}{15x^6}+\frac{16\left(-x^3+x^4\right)^{3/4}}{55x^5}+\frac{128\left(-x^3+x^4\right)^{3/4}}{385x^4}+\frac{128}{385}\int \frac{1}{x \sqrt[4]{-x^3+x^4}}\,dx\\
&= \frac{4\left(-x^3+x^4\right)^{3/4}}{15x^6}+\frac{16\left(-x^3+x^4\right)^{3/4}}{55x^5}+\frac{128\left(-x^3+x^4\right)^{3/4}}{385x^4}+\frac{512\left(-x^3+x^4\right)^{3/4}}{1155x^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.94

$$\frac{4\left((x-1)x^3\right)^{3/4}\left(128x^3+96x^2+84x+77\right)}{1155x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (4*((-1 + x)*x^3)^(3/4)*(77 + 84*x + 96*x^2 + 128*x^3))/(1155*x^6)
```

IntegrateAlgebraic [A] time = 0.26, size = 35, normalized size = 1.00

$$\frac{4\left(128x^3+96x^2+84x+77\right)\left(x^4-x^3\right)^{3/4}}{1155x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (4*(77 + 84*x + 96*x^2 + 128*x^3)*(-x^3 + x^4)^(3/4))/(1155*x^6)
```

fricas [A] time = 0.39, size = 31, normalized size = 0.89

$$\frac{4\left(x^4-x^3\right)^{\frac{3}{4}}\left(128\,x^3+96\,x^2+84\,x+77\right)}{1155\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4-x^3)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/1155*(x^4 - x^3)^(3/4)*(128*x^3 + 96*x^2 + 84*x + 77)/x^6
```

giac [A] time = 0.27, size = 59, normalized size = 1.69

$$-\frac{4}{15}\left(\frac{1}{x}-1\right)^3\left(-\frac{1}{x}+1\right)^{\frac{3}{4}}-\frac{12}{11}\left(\frac{1}{x}-1\right)^2\left(-\frac{1}{x}+1\right)^{\frac{3}{4}}+\frac{12}{7}\left(-\frac{1}{x}+1\right)^{\frac{7}{4}}-\frac{4}{3}\left(-\frac{1}{x}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4-x^3)^(1/4),x, algorithm="giac")
```

```
[Out] -4/15*(1/x - 1)^3*(-1/x + 1)^(3/4) - 12/11*(1/x - 1)^2*(-1/x + 1)^(3/4) + 1
2/7*(-1/x + 1)^(7/4) - 4/3*(-1/x + 1)^(3/4)
```

maple [A] time = 0.00, size = 35, normalized size = 1.00

$$\frac{4\left(-1+x\right)\left(128x^3+96x^2+84x+77\right)}{1155x^3\left(x^4-x^3\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^4-x^3)^(1/4),x)
```

```
[Out] 4/1155*(-1+x)*(128*x^3+96*x^2+84*x+77)/x^3/(x^4-x^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4-x^3\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4-x^3)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 - x^3)^(1/4)*x^4), x)
```

mupad [B] time = 0.22, size = 65, normalized size = 1.86

$$\frac{512\,{\left(x^4-x^3\right)}^{3/4}}{1155\,x^3}+\frac{128\,{\left(x^4-x^3\right)}^{3/4}}{385\,x^4}+\frac{16\,{\left(x^4-x^3\right)}^{3/4}}{55\,x^5}+\frac{4\,{\left(x^4-x^3\right)}^{3/4}}{15\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^4 - x^3)^(1/4)),x)
```

```
[Out] (512*(x^4 - x^3)^(3/4))/(1155*x^3) + (128*(x^4 - x^3)^(3/4))/(385*x^4) + (1
6*(x^4 - x^3)^(3/4))/(55*x^5) + (4*(x^4 - x^3)^(3/4))/(15*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\,\sqrt[4]{x^3\,(x-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**4-x**3)**(1/4),x)
```

```
[Out] Integral(1/(x**4*(x**3*(x - 1))**(1/4)), x)
```

3.438 $\int \frac{1+2x}{\sqrt{-4-3x-2x^2+2x^3+x^4}}\,dx$

Optimal. Leaf size=35

$$\log\left(2x^2+2\sqrt{x^4+2x^3-2x^2-3x-4}+2x-3\right)$$

Rubi [A] time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1680, 12, 1107, 621, 206}

$$-\tanh^{-1}\left(\frac{7-4\left(x+\frac{1}{2}\right)^2}{\sqrt{16\left(x+\frac{1}{2}\right)^4-56\left(x+\frac{1}{2}\right)^2-51}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/Sqrt[-4 - 3*x - 2*x^2 + 2*x^3 + x^4],x]
```

```
[Out] -ArcTanh[(7 - 4*(1/2 + x)^2)/Sqrt[-51 - 56*(1/2 + x)^2 + 16*(1/2 + x)^4]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+2x}{\sqrt{-4-3x-2x^2+2x^3+x^4}}\,dx &= \text{Subst}\left(\int \frac{8x}{\sqrt{-51-56x^2+16x^4}}\,dx, x, \frac{1}{2}+x\right)\\
&= 8\,\text{Subst}\left(\int \frac{x}{\sqrt{-51-56x^2+16x^4}}\,dx, x, \frac{1}{2}+x\right)\\
&= 4\,\text{Subst}\left(\int \frac{1}{\sqrt{-51-56x+16x^2}}\,dx, x, \left(\frac{1}{2}+x\right)^2\right)\\
&= 8\,\text{Subst}\left(\int \frac{1}{64-x^2}\,dx, x, \frac{8\left(-7+4\left(\frac{1}{2}+x\right)^2\right)}{\sqrt{-51-56\left(\frac{1}{2}+x\right)^2+(1+2x)^4}}\right)\\
&= -\tanh^{-1}\left(\frac{7-(1+2x)^2}{\sqrt{-51-14(1+2x)^2+(1+2x)^4}}\right)
\end{aligned}
$$

Mathematica [**C**] time = 0.98, size = 505, normalized size = 14.43

$$
\frac{6\sqrt[6]{-1}\left(-2x+\sqrt{17}-1\right)\left((-1)^{2/3}-x\right)\sqrt{\frac{i\left(2x+\sqrt{17}+1\right)}{\left(\sqrt{17}-i\sqrt{3}\right)\left((-1)^{2/3}-x\right)}}\sqrt{\frac{-\sqrt{17}x+2(-1)^{2/3}x+x-\sqrt[3]{-1}\sqrt{17}+\sqrt[3]{-1}-2}{\left(\sqrt{17}+i\sqrt{3}\right)\left((-1)^{2/3}-x\right)}}\left(F\left(\sin^{-1}\left(\sqrt{\frac{2i\sqrt{17}x+2\sqrt{3}x-\sqrt{51}+i\sqrt{17}+\sqrt{3}+3i}{2i\sqrt{17}x-2\sqrt{3}x+\sqrt{51}+i\sqrt{17}-\sqrt{3}+3i}}\right)|\frac{7i-\sqrt{51}}{7i+\sqrt{51}}\right)-2\Pi\left(-\frac{\sqrt{3}-i\sqrt{17}}{\sqrt{3}+i\sqrt{17}};\sin^{-1}\left(\sqrt{\frac{2i\sqrt{17}x+2\sqrt{3}x-\sqrt{51}+i\sqrt{17}+\sqrt{3}+3i}{2i\sqrt{17}x-2\sqrt{3}x+\sqrt{51}+i\sqrt{17}-\sqrt{3}+3i}}\right)|\frac{7i-\sqrt{51}}{7i+\sqrt{51}}\right)\right)}{\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}+i\sqrt{17}\right)\sqrt{\frac{-2x+\sqrt{17}-1}{\left(\sqrt{3}-i\sqrt{17}\right)\left((-1)^{2/3}-x\right)}}\sqrt{x^4+2x^3-2x^2-3x-4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + 2*x)/Sqrt[-4 - 3*x - 2*x^2 + 2*x^3 + x^4],x]
```

```
[Out] (6*(-1)^(1/6)*(-1 + Sqrt[17] - 2*x)*((-1)^(2/3) - x)*Sqrt[(I*(1 + Sqrt[17]
+ 2*x))/(((-I)*Sqrt[3] + Sqrt[17])*((-1)^(2/3) - x))]*Sqrt[(-2 + (-1)^(1/3)
 - (-1)^(1/3)*Sqrt[17] + x + 2*(-1)^(2/3)*x - Sqrt[17]*x)/((I*Sqrt[3] + Sqr
t[17])*((-1)^(2/3) - x))]*(EllipticF[ArcSin[Sqrt[(3*I + Sqrt[3] + I*Sqrt[17
] - Sqrt[51] + 2*Sqrt[3]*x + (2*I)*Sqrt[17]*x)/(3*I - Sqrt[3] + I*Sqrt[17]
+ Sqrt[51] - 2*Sqrt[3]*x + (2*I)*Sqrt[17]*x)]], (7*I - Sqrt[51])/(7*I + Sqr
t[51])] - 2*EllipticPi[-((Sqrt[3] - I*Sqrt[17])/(Sqrt[3] + I*Sqrt[17])), Ar
cSin[Sqrt[(3*I + Sqrt[3] + I*Sqrt[17] - Sqrt[51] + 2*Sqrt[3]*x + (2*I)*Sqrt
[17]*x)/(3*I - Sqrt[3] + I*Sqrt[17] + Sqrt[51] - 2*Sqrt[3]*x + (2*I)*Sqrt[1
7]*x)]], (7*I - Sqrt[51])/(7*I + Sqrt[51])]))/((1 + (-1)^(1/3))*(Sqrt[3] +
I*Sqrt[17])*Sqrt[(-1 + Sqrt[17] - 2*x)/((Sqrt[3] - I*Sqrt[17])*((-1)^(2/3)
- x))]*Sqrt[-4 - 3*x - 2*x^2 + 2*x^3 + x^4])
```

IntegrateAlgebraic [**A**] time = 0.19, size = 35, normalized size = 1.00

$$
\log\left(2x^2+2\sqrt{x^4+2x^3-2x^2-3x-4}+2x-3\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x)/Sqrt[-4 - 3*x - 2*x^2 + 2*x^3 + x^4],x]
```

```
[Out] Log[-3 + 2*x + 2*x^2 + 2*Sqrt[-4 - 3*x - 2*x^2 + 2*x^3 + x^4]]
```

fricas [**A**] time = 0.43, size = 33, normalized size = 0.94

$$
\log\left(2\,x^2+2\,x+2\,\sqrt{x^4+2\,x^3-2\,x^2-3\,x-4}-3\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^4+2*x^3-2*x^2-3*x-4)^(1/2),x, algorithm="fricas")
```

```
[Out] log(2*x^2 + 2*x + 2*sqrt(x^4 + 2*x^3 - 2*x^2 - 3*x - 4) - 3)
```

giac [A] time = 0.73, size = 35, normalized size = 1.00

$$-\log\left(\left|-2\,x^2-2\,x+2\,\sqrt{\left(x^2+x\right)^2-3\,x^2-3\,x-4}+3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^4+2*x^3-2*x^2-3*x-4)^(1/2),x, algorithm="giac")
[Out] -log(abs(-2*x^2 - 2*x + 2*sqrt((x^2 + x)^2 - 3*x^2 - 3*x - 4) + 3))
```

maple [C] time = 0.39, size = 782, normalized size = 22.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(x^4+2*x^3-2*x^2-3*x-4)^(1/2),x)
[Out] -2/3*I*(-1/2*I*3^(1/2)-1/2*17^(1/2))*((1/2*17^(1/2)-1/2*I*3^(1/2))*(x+1/2+1
/2*I*3^(1/2))/(1/2*17^(1/2)+1/2*I*3^(1/2))/(x+1/2-1/2*I*3^(1/2)))^(1/2)*(x+
1/2-1/2*I*3^(1/2))^2*(I*3^(1/2)*(x+1/2+1/2*17^(1/2))/(-1/2*17^(1/2)+1/2*I*3
^(1/2))/(x+1/2-1/2*I*3^(1/2)))^(1/2)*(I*3^(1/2)*(x+1/2-1/2*17^(1/2))/(1/2*1
7^(1/2)+1/2*I*3^(1/2))/(x+1/2-1/2*I*3^(1/2)))^(1/2)/(1/2*17^(1/2)-1/2*I*3^(
1/2))*3^(1/2)/((x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/2))*(x+1/2+1/2*17^(1
/2))*(x+1/2-1/2*17^(1/2)))^(1/2)*EllipticF(((1/2*17^(1/2)-1/2*I*3^(1/2))*(x
+1/2+1/2*I*3^(1/2))/(1/2*17^(1/2)+1/2*I*3^(1/2))/(x+1/2-1/2*I*3^(1/2)))^(1/
2),((1/2*17^(1/2)+1/2*I*3^(1/2))*(-1/2*I*3^(1/2)-1/2*17^(1/2))/(1/2*17^(1/2
)-1/2*I*3^(1/2))/(-1/2*17^(1/2)+1/2*I*3^(1/2)))^(1/2))-4/3*I*(-1/2*I*3^(1/2
)-1/2*17^(1/2))*((1/2*17^(1/2)-1/2*I*3^(1/2))*(x+1/2+1/2*I*3^(1/2))/(1/2*17
^(1/2)+1/2*I*3^(1/2))/(x+1/2-1/2*I*3^(1/2)))^(1/2)*(x+1/2-1/2*I*3^(1/2))^2*
(I*3^(1/2)*(x+1/2+1/2*17^(1/2))/(-1/2*17^(1/2)+1/2*I*3^(1/2))/(x+1/2-1/2*I*
3^(1/2)))^(1/2)*(I*3^(1/2)*(x+1/2-1/2*17^(1/2))/(1/2*17^(1/2)+1/2*I*3^(1/2)
)/(x+1/2-1/2*I*3^(1/2)))^(1/2)/(1/2*17^(1/2)-1/2*I*3^(1/2))*3^(1/2)/((x+1/2
+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/2))*(x+1/2+1/2*17^(1/2))*(x+1/2-1/2*17^(1
/2)))^(1/2)*((-1/2+1/2*I*3^(1/2))*EllipticF(((1/2*17^(1/2)-1/2*I*3^(1/2))*(
x+1/2+1/2*I*3^(1/2))/(1/2*17^(1/2)+1/2*I*3^(1/2))/(x+1/2-1/2*I*3^(1/2)))^(1
/2),((1/2*17^(1/2)+1/2*I*3^(1/2))*(-1/2*I*3^(1/2)-1/2*17^(1/2))/(1/2*17^(1/
2)-1/2*I*3^(1/2))/(-1/2*17^(1/2)+1/2*I*3^(1/2)))^(1/2))-I*3^(1/2)*EllipticP
i(((1/2*17^(1/2)-1/2*I*3^(1/2))*(x+1/2+1/2*I*3^(1/2))/(1/2*17^(1/2)+1/2*I*3
^(1/2))/(x+1/2-1/2*I*3^(1/2)))^(1/2),(1/2*17^(1/2)+1/2*I*3^(1/2))/(1/2*17^(
1/2)-1/2*I*3^(1/2)),((1/2*17^(1/2)+1/2*I*3^(1/2))*(-1/2*I*3^(1/2)-1/2*17^(1
/2))/(1/2*17^(1/2)-1/2*I*3^(1/2))/(-1/2*17^(1/2)+1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x+1}{\sqrt{x^4+2\,x^3-2\,x^2-3\,x-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^4+2*x^3-2*x^2-3*x-4)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x + 1)/sqrt(x^4 + 2*x^3 - 2*x^2 - 3*x - 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{2\,x+1}{\sqrt{x^4+2\,x^3-2\,x^2-3\,x-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/(2*x^3 - 2*x^2 - 3*x + x^4 - 4)^(1/2),x)
```

```
[Out] int((2*x + 1)/(2*x^3 - 2*x^2 - 3*x + x^4 - 4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\sqrt{(x^2+x-4)(x^2+x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**4+2*x**3-2*x**2-3*x-4)**(1/2),x)
[Out] Integral((2*x + 1)/sqrt((x**2 + x - 4)*(x**2 + x + 1)), x)
```

3.439 $\int \frac{4b+ax^3}{\sqrt[4]{b+ax^3}\,(-b-ax^3+x^4)}\,dx$

Optimal. Leaf size=35

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{x}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt[4]{ax^3+b}}\right)$$

Rubi [F] time = 0.74, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{4b+ax^3}{\sqrt[4]{b+ax^3}\,(-b-ax^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(4*b + a*x^3)/((b + a*x^3)^(1/4)*(-b - a*x^3 + x^4)),x]
```

```
[Out] -4*b*Defer[Int][1/((b + a*x^3)^(1/4)*(b + a*x^3 - x^4)), x] - a*Defer[Int][
x^3/((b + a*x^3)^(1/4)*(b + a*x^3 - x^4)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{4b+ax^3}{\sqrt[4]{b+ax^3}\,(-b-ax^3+x^4)}\,dx &= \int \left(-\frac{4b}{\sqrt[4]{b+ax^3}\,(b+ax^3-x^4)}-\frac{ax^3}{\sqrt[4]{b+ax^3}\,(b+ax^3-x^4)}\right)dx\\ &= -\left(a\int \frac{x^3}{\sqrt[4]{b+ax^3}\,(b+ax^3-x^4)}\,dx\right)-(4b)\int \frac{1}{\sqrt[4]{b+ax^3}\,(b+ax^3-x^4)}\,dx\end{aligned}$$

Mathematica [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{4b+ax^3}{\sqrt[4]{b+ax^3}\,(-b-ax^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(4*b + a*x^3)/((b + a*x^3)^(1/4)*(-b - a*x^3 + x^4)),x]
```

```
[Out] Integrate[(4*b + a*x^3)/((b + a*x^3)^(1/4)*(-b - a*x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.70, size = 35, normalized size = 1.00

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{x}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt[4]{ax^3+b}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4*b + a*x^3)/((b + a*x^3)^(1/4)*(-b - a*x^3 + x^4)),x]
```

```
[Out] 2*ArcTan[(b + a*x^3)^(1/4)/x] - 2*ArcTanh[x/(b + a*x^3)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+4*b)/(a*x^3+b)^(1/4)/(-a*x^3+x^4-b),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ax^3 + 4\,b}{\left(ax^3 - x^4 + b\right)\left(ax^3 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+4*b)/(a*x^3+b)^(1/4)/(-a*x^3+x^4-b),x, algorithm="giac")
[Out] integrate(-(a*x^3 + 4*b)/((a*x^3 - x^4 + b)*(a*x^3 + b)^(1/4)), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{a\,x^3 + 4b}{\left(a\,x^3 + b\right)^{\frac{1}{4}}\left(-a\,x^3 + x^4 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+4*b)/(a*x^3+b)^(1/4)/(-a*x^3+x^4-b),x)
[Out] int((a*x^3+4*b)/(a*x^3+b)^(1/4)/(-a*x^3+x^4-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax^3 + 4\,b}{\left(ax^3 - x^4 + b\right)\left(ax^3 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+4*b)/(a*x^3+b)^(1/4)/(-a*x^3+x^4-b),x, algorithm="maxima")
[Out] -integrate((a*x^3 + 4*b)/((a*x^3 - x^4 + b)*(a*x^3 + b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{a\,x^3 + 4\,b}{\left(a\,x^3 + b\right)^{1/4}\left(-x^4 + a\,x^3 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*b + a*x^3)/((b + a*x^3)^(1/4)*(b + a*x^3 - x^4)),x)
[Out] int(-(4*b + a*x^3)/((b + a*x^3)^(1/4)*(b + a*x^3 - x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+4*b)/(a*x**3+b)**(1/4)/(-a*x**3+x**4-b),x)
[Out] Timed out
```

3.440 $\int \frac{x^8}{\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=35

$$\frac{1}{6}\sqrt{x^6-1}\,x^3+\frac{1}{6}\log\left(\sqrt{x^6-1}+x^3\right)$$

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {275, 321, 217, 206}

$$\frac{1}{6}\sqrt{x^6-1}\,x^3+\frac{1}{6}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[-1 + x^6],x]
```

```
[Out] (x^3*Sqrt[-1 + x^6])/6 + ArcTanh[x^3/Sqrt[-1 + x^6]]/6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\sqrt{-1+x^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x^2}}\,dx,x,x^3\right)\\
&= \frac{1}{6}x^3\sqrt{-1+x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx,x,x^3\right)\\
&= \frac{1}{6}x^3\sqrt{-1+x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x^3}{\sqrt{-1+x^6}}\right)\\
&= \frac{1}{6}x^3\sqrt{-1+x^6}+\frac{1}{6}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.91

$$\frac{1}{6}\left(\sqrt{x^6-1}\,x^3+\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[-1 + x^6],x]
[Out] (x^3*Sqrt[-1 + x^6] + ArcTanh[x^3/Sqrt[-1 + x^6]])/6
```

IntegrateAlgebraic [A] time = 0.19, size = 35, normalized size = 1.00

$$\frac{1}{6}\sqrt{x^6-1}\,x^3+\frac{1}{6}\log\left(\sqrt{x^6-1}+x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/Sqrt[-1 + x^6],x]
[Out] (x^3*Sqrt[-1 + x^6])/6 + Log[x^3 + Sqrt[-1 + x^6]]/6
```

fricas [A] time = 0.40, size = 29, normalized size = 0.83

$$\frac{1}{6}\sqrt{x^6-1}\,x^3-\frac{1}{6}\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6-1)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(x^6 - 1)*x^3 - 1/6*log(-x^3 + sqrt(x^6 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^8}{\sqrt{x^6-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6-1)^(1/2),x, algorithm="giac")
[Out] integrate(x^8/sqrt(x^6 - 1), x)
```

maple [C] time = 0.20, size = 38, normalized size = 1.09

$$\frac{x^3\sqrt{x^6-1}}{6}+\frac{\sqrt{-\mathrm{signum}\left(x^6-1\right)}\;\arcsin\left(x^3\right)}{6\sqrt{\mathrm{signum}\left(x^6-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6-1)^(1/2),x)
[Out] 1/6*x^3*(x^6-1)^(1/2)+1/6/signum(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*arcsin
(x^3)
```

maxima [B] time = 0.34, size = 58, normalized size = 1.66

$$-\frac{\sqrt{x^6-1}}{6\,x^3\left(\frac{x^6-1}{x^6}-1\right)}+\frac{1}{12}\log\left(\frac{\sqrt{x^6-1}}{x^3}+1\right)-\frac{1}{12}\log\left(\frac{\sqrt{x^6-1}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(x^6 - 1)/(x^3*((x^6 - 1)/x^6 - 1)) + 1/12*log(sqrt(x^6 - 1)/x^3 +
 1) - 1/12*log(sqrt(x^6 - 1)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^8}{\sqrt{x^6-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6 - 1)^(1/2),x)
```

```
[Out] int(x^8/(x^6 - 1)^(1/2), x)
```

sympy [A] time = 1.56, size = 61, normalized size = 1.74

$$\begin{cases} \frac{x^3\sqrt{x^6-1}}{6} + \frac{\operatorname{acosh}\left(x^3\right)}{6} & \text{for } \left|x^6\right| > 1 \\ -\frac{ix^9}{6\sqrt{1-x^6}} + \frac{ix^3}{6\sqrt{1-x^6}} - \frac{i\operatorname{asin}\left(x^3\right)}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**6-1)**(1/2),x)
```

```
[Out] Piecewise((x**3*sqrt(x**6 - 1)/6 + acosh(x**3)/6, Abs(x**6) > 1), (-I*x**9/
(6*sqrt(1 - x**6)) + I*x**3/(6*sqrt(1 - x**6)) - I*asin(x**3)/6, True))
```

3.441 $\int \frac{\sqrt{-1+x^6}}{x^4} \, dx$

Optimal. Leaf size=35

$$\frac{1}{3}\log\left(\sqrt{x^6-1}+x^3\right)-\frac{\sqrt{x^6-1}}{3x^3}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {275, 277, 217, 206}

$$\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)-\frac{\sqrt{x^6-1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^6]/x^4,x]
```

```
[Out] -1/3*Sqrt[-1 + x^6]/x^3 + ArcTanh[x^3/Sqrt[-1 + x^6]]/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x^6}}{x^4} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{-1+x^2}}{x^2} \, dx, x, x^3\right)\\
&= -\frac{\sqrt{-1+x^6}}{3x^3}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}} \, dx, x, x^3\right)\\
&= -\frac{\sqrt{-1+x^6}}{3x^3}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, \frac{x^3}{\sqrt{-1+x^6}}\right)\\
&= -\frac{\sqrt{-1+x^6}}{3x^3}+\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 1.03

$$\frac{1}{3}\sqrt{x^6-1}\left(-\frac{1}{x^3}-\frac{\sin^{-1}\left(x^3\right)}{\sqrt{1-x^6}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^6]/x^4,x]
[Out] (Sqrt[-1 + x^6]*(-x^(-3) - ArcSin[x^3]/Sqrt[1 - x^6]))/3
```

IntegrateAlgebraic [A] time = 0.16, size = 35, normalized size = 1.00

$$\frac{1}{3}\log\left(\sqrt{x^6-1}+x^3\right)-\frac{\sqrt{x^6-1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x^6]/x^4,x]
[Out] -1/3*Sqrt[-1 + x^6]/x^3 + Log[x^3 + Sqrt[-1 + x^6]]/3
```

fricas [A] time = 0.40, size = 34, normalized size = 0.97

$$-\frac{x^3\log\left(-x^3+\sqrt{x^6-1}\right)+x^3+\sqrt{x^6-1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^4,x, algorithm="fricas")
[Out] -1/3*(x^3*log(-x^3 + sqrt(x^6 - 1)) + x^3 + sqrt(x^6 - 1))/x^3
```

giac [A] time = 0.29, size = 46, normalized size = 1.31

$$-\frac{2\sqrt{-\frac{1}{x^6}+1}-\log\left(\sqrt{-\frac{1}{x^6}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{x^6}+1}+1\right)}{6\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^4,x, algorithm="giac")
[Out] -1/6*(2*sqrt(-1/x^6 + 1) - log(sqrt(-1/x^6 + 1) + 1) + log(-sqrt(-1/x^6 + 1
) + 1))/sgn(x)
```

maple [C] time = 0.02, size = 38, normalized size = 1.09

$$-\frac{\sqrt{x^6-1}}{3x^3}+\frac{\sqrt{-\mathrm{signum}\left(x^6-1\right)}\ \arcsin\left(x^3\right)}{3\sqrt{\mathrm{signum}\left(x^6-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/2)/x^4,x)
[Out] -1/3*(x^6-1)^(1/2)/x^3+1/3/signum(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*arcsi
n(x^3)
```

maxima [A] time = 0.40, size = 45, normalized size = 1.29

$$-\frac{\sqrt{x^6-1}}{3\,x^3}+\frac{1}{6}\log\left(\frac{\sqrt{x^6-1}}{x^3}+1\right)-\frac{1}{6}\log\left(\frac{\sqrt{x^6-1}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(x^6 - 1)/x^3 + 1/6*log(sqrt(x^6 - 1)/x^3 + 1) - 1/6*log(sqrt(x^6
- 1)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{x^6-1}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)^(1/2)/x^4,x)
```

```
[Out] int((x^6 - 1)^(1/2)/x^4, x)
```

sympy [A] time = 1.06, size = 76, normalized size = 2.17

$$\begin{cases} -\frac{x^3}{3\sqrt{x^6-1}} + \frac{\operatorname{acosh}\left(x^3\right)}{3} + \frac{1}{3x^3\sqrt{x^6-1}} & \text{for } \left|x^6\right| > 1 \\ \frac{ix^3}{3\sqrt{1-x^6}} - \frac{i\operatorname{asin}\left(x^3\right)}{3} - \frac{i}{3x^3\sqrt{1-x^6}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/2)/x**4,x)
```

```
[Out] Piecewise((-x**3/(3*sqrt(x**6 - 1)) + acosh(x**3)/3 + 1/(3*x**3*sqrt(x**6 -
 1)), Abs(x**6) > 1), (I*x**3/(3*sqrt(1 - x**6)) - I*asin(x**3)/3 - I/(3*x*
*3*sqrt(1 - x**6)), True))
```

3.442 $\int x^2\sqrt{-1+x^6}\,dx$

Optimal. Leaf size=35

$$\frac{1}{6}x^3\sqrt{x^6-1}-\frac{1}{6}\log\left(\sqrt{x^6-1}+x^3\right)$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {275, 195, 217, 206}

$$\frac{1}{6}x^3\sqrt{x^6-1}-\frac{1}{6}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[-1 + x^6],x]
```

```
[Out] (x^3*Sqrt[-1 + x^6])/6 - ArcTanh[x^3/Sqrt[-1 + x^6]]/6
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{-1+x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int\sqrt{-1+x^2}\,dx,x,x^3\right)\\
&= \frac{1}{6}x^3\sqrt{-1+x^6}-\frac{1}{6}\text{Subst}\left(\int\frac{1}{\sqrt{-1+x^2}}\,dx,x,x^3\right)\\
&= \frac{1}{6}x^3\sqrt{-1+x^6}-\frac{1}{6}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x^3}{\sqrt{-1+x^6}}\right)\\
&= \frac{1}{6}x^3\sqrt{-1+x^6}-\frac{1}{6}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.20

$$\frac{\left(x^6-1\right)\left(\sin^{-1}\left(x^3\right)+\sqrt{1-x^6}\,x^3\right)}{6\sqrt{-\left(x^6-1\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[-1 + x^6],x]
[Out] ((-1 + x^6)*(x^3*Sqrt[1 - x^6] + ArcSin[x^3]))/(6*Sqrt[-(-1 + x^6)^2])
```

IntegrateAlgebraic [A] time = 0.14, size = 35, normalized size = 1.00

$$\frac{1}{6}x^3\sqrt{x^6-1}-\frac{1}{6}\log\left(\sqrt{x^6-1}+x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[-1 + x^6],x]
[Out] (x^3*Sqrt[-1 + x^6])/6 - Log[x^3 + Sqrt[-1 + x^6]]/6
```

fricas [A] time = 0.42, size = 29, normalized size = 0.83

$$\frac{1}{6}\sqrt{x^6-1}\,x^3+\frac{1}{6}\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^6-1)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(x^6 - 1)*x^3 + 1/6*log(-x^3 + sqrt(x^6 - 1))
```

giac [A] time = 0.29, size = 30, normalized size = 0.86

$$\frac{1}{6}\sqrt{x^6-1}\,x^3+\frac{1}{6}\log\left(\left|-x^3+\sqrt{x^6-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^6-1)^(1/2),x, algorithm="giac")
[Out] 1/6*sqrt(x^6 - 1)*x^3 + 1/6*log(abs(-x^3 + sqrt(x^6 - 1)))
```

maple [C] time = 0.23, size = 38, normalized size = 1.09

$$\frac{x^3\sqrt{x^6-1}}{6}-\frac{\sqrt{-\mathrm{signum}\left(x^6-1\right)}\,\arcsin\left(x^3\right)}{6\sqrt{\mathrm{signum}\left(x^6-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^6-1)^(1/2),x)
[Out] 1/6*x^3*(x^6-1)^(1/2)-1/6/signum(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*arcsin
(x^3)
```

maxima [B] time = 0.40, size = 58, normalized size = 1.66

$$-\frac{\sqrt{x^6-1}}{6\,x^3\left(\frac{x^6-1}{x^6}-1\right)}-\frac{1}{12}\log\left(\frac{\sqrt{x^6-1}}{x^3}+1\right)+\frac{1}{12}\log\left(\frac{\sqrt{x^6-1}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^6-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(x^6 - 1)/(x^3*((x^6 - 1)/x^6 - 1)) - 1/12*log(sqrt(x^6 - 1)/x^3 +
 1) + 1/12*log(sqrt(x^6 - 1)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^2 \sqrt{x^6-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^6 - 1)^(1/2),x)
```

```
[Out] int(x^2*(x^6 - 1)^(1/2), x)
```

sympy [A] time = 1.30, size = 60, normalized size = 1.71

$$\begin{cases} \frac{x^9}{6\sqrt{x^6-1}} - \frac{x^3}{6\sqrt{x^6-1}} - \frac{\operatorname{acosh}\left(x^3\right)}{6} & \text{for}\ \left|x^6\right| > 1 \\ \frac{ix^3\sqrt{1-x^6}}{6} + \frac{i\operatorname{asin}\left(x^3\right)}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**6-1)**(1/2),x)
```

```
[Out] Piecewise((x**9/(6*sqrt(x**6 - 1)) - x**3/(6*sqrt(x**6 - 1)) - acosh(x**3)/
6, Abs(x**6) > 1), (I*x**3*sqrt(1 - x**6)/6 + I*asin(x**3)/6, True))
```

3.443 $\int \frac{x^8}{\sqrt{1+x^6}}\,dx$

Optimal. Leaf size=35

$$\frac{1}{6}x^3\sqrt{x^6+1}-\frac{1}{6}\log\left(\sqrt{x^6+1}+x^3\right)$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 0.71, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {275, 321, 215}

$$\frac{1}{6}x^3\sqrt{x^6+1}-\frac{1}{6}\sinh^{-1}\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[1 + x^6],x]
```

```
[Out] (x^3*Sqrt[1 + x^6])/6 - ArcSinh[x^3]/6
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\sqrt{1+x^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx,x,x^3\right)\\ &= \frac{1}{6}x^3\sqrt{1+x^6}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,x^3\right)\\ &= \frac{1}{6}x^3\sqrt{1+x^6}-\frac{1}{6}\sinh^{-1}\left(x^3\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.71

$$\frac{1}{6}x^3\sqrt{x^6+1}-\frac{1}{6}\sinh^{-1}\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[1 + x^6],x]
```

```
[Out] (x^3*Sqrt[1 + x^6])/6 - ArcSinh[x^3]/6
```

IntegrateAlgebraic [A] time = 0.18, size = 35, normalized size = 1.00

$$\frac{1}{6}x^3\sqrt{x^6+1}-\frac{1}{6}\log\left(\sqrt{x^6+1}+x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/Sqrt[1 + x^6],x]
[Out] (x^3*Sqrt[1 + x^6])/6 - Log[x^3 + Sqrt[1 + x^6]]/6
```

fricas [A] time = 0.39, size = 29, normalized size = 0.83

$$\frac{1}{6}\sqrt{x^6+1}\,x^3+\frac{1}{6}\log\left(-x^3+\sqrt{x^6+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+1)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(x^6 + 1)*x^3 + 1/6*log(-x^3 + sqrt(x^6 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{x^6+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^8/sqrt(x^6 + 1), x)
```

maple [A] time = 0.21, size = 20, normalized size = 0.57

$$\frac{x^3\sqrt{x^6+1}}{6}-\frac{\operatorname{arcsinh}\left(x^3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6+1)^(1/2),x)
[Out] 1/6*x^3*(x^6+1)^(1/2)-1/6*arcsinh(x^3)
```

maxima [B] time = 0.34, size = 58, normalized size = 1.66

$$\frac{\sqrt{x^6+1}}{6\,x^3\left(\frac{x^6+1}{x^6}-1\right)}-\frac{1}{12}\log\left(\frac{\sqrt{x^6+1}}{x^3}+1\right)+\frac{1}{12}\log\left(\frac{\sqrt{x^6+1}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+1)^(1/2),x, algorithm="maxima")
[Out] 1/6*sqrt(x^6 + 1)/(x^3*((x^6 + 1)/x^6 - 1)) - 1/12*log(sqrt(x^6 + 1)/x^3 +
1) + 1/12*log(sqrt(x^6 + 1)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^8}{\sqrt{x^6+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6 + 1)^(1/2),x)
```

```
[Out] int(x^8/(x^6 + 1)^(1/2), x)
```

sympy [A] time = 1.52, size = 19, normalized size = 0.54

$$\frac{x^3\sqrt{x^6+1}}{6}-\frac{\operatorname{asinh}\left(x^3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**6+1)**(1/2),x)
```

```
[Out] x**3*sqrt(x**6 + 1)/6 - asinh(x**3)/6
```

3.444 $\int \frac{\sqrt{1+x^6}}{x^4} \, dx$

Optimal. Leaf size=35

$$\frac{1}{3}\log\left(\sqrt{x^6+1}+x^3\right)-\frac{\sqrt{x^6+1}}{3x^3}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 0.71, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {275, 277, 215}

$$\frac{1}{3}\sinh^{-1}\left(x^3\right)-\frac{\sqrt{x^6+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^6]/x^4,x]
```

```
[Out] -1/3*Sqrt[1 + x^6]/x^3 + ArcSinh[x^3]/3
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x^6}}{x^4} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2} \, dx,x,x^3\right)\\
&= -\frac{\sqrt{1+x^6}}{3x^3}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} \, dx,x,x^3\right)\\
&= -\frac{\sqrt{1+x^6}}{3x^3}+\frac{1}{3}\sinh^{-1}\left(x^3\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.71

$$\frac{1}{3}\sinh^{-1}\left(x^3\right)-\frac{\sqrt{x^6+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^6]/x^4,x]
```

[Out] -1/3*Sqrt[1 + x^6]/x^3 + ArcSinh[x^3]/3

IntegrateAlgebraic [A] time = 0.14, size = 35, normalized size = 1.00

$$\frac{1}{3}\log\left(\sqrt{x^6+1}+x^3\right)-\frac{\sqrt{x^6+1}}{3x^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 + x^6]/x^4,x]

[Out] -1/3*Sqrt[1 + x^6]/x^3 + Log[x^3 + Sqrt[1 + x^6]]/3

fricas [A] time = 0.40, size = 34, normalized size = 0.97

$$-\frac{x^3\log\left(-x^3+\sqrt{x^6+1}\right)+x^3+\sqrt{x^6+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+1)^(1/2)/x^4,x, algorithm="fricas")

[Out] -1/3*(x^3*log(-x^3 + sqrt(x^6 + 1)) + x^3 + sqrt(x^6 + 1))/x^3

giac [A] time = 0.31, size = 38, normalized size = 1.09

$$-\frac{2\sqrt{\frac{1}{x^6}+1}-\log\left(\sqrt{\frac{1}{x^6}+1}+1\right)+\log\left(\sqrt{\frac{1}{x^6}+1}-1\right)}{6\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+1)^(1/2)/x^4,x, algorithm="giac")

[Out] -1/6*(2*sqrt(1/x^6 + 1) - log(sqrt(1/x^6 + 1) + 1) + log(sqrt(1/x^6 + 1) - 1))/sgn(x)

maple [A] time = 0.20, size = 20, normalized size = 0.57

$$-\frac{\sqrt{x^6+1}}{3x^3}+\frac{\operatorname{arcsinh}\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6+1)^(1/2)/x^4,x)

[Out] -1/3*(x^6+1)^(1/2)/x^3+1/3*arcsinh(x^3)

maxima [A] time = 0.37, size = 45, normalized size = 1.29

$$-\frac{\sqrt{x^6+1}}{3x^3}+\frac{1}{6}\log\left(\frac{\sqrt{x^6+1}}{x^3}+1\right)-\frac{1}{6}\log\left(\frac{\sqrt{x^6+1}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+1)^(1/2)/x^4,x, algorithm="maxima")

[Out] -1/3*sqrt(x^6 + 1)/x^3 + 1/6*log(sqrt(x^6 + 1)/x^3 + 1) - 1/6*log(sqrt(x^6 + 1)/x^3 - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\sqrt{x^6+1}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)^(1/2)/x^4,x)
```

```
[Out] int((x^6 + 1)^(1/2)/x^4, x)
```

sympy [A] time = 0.98, size = 34, normalized size = 0.97

$$-\frac{x^3}{3\sqrt{x^6+1}}+\frac{\operatorname{asinh}\left(x^3\right)}{3}-\frac{1}{3x^3\sqrt{x^6+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)**(1/2)/x**4,x)
```

```
[Out] -x**3/(3*sqrt(x**6 + 1)) + asinh(x**3)/3 - 1/(3*x**3*sqrt(x**6 + 1))
```

3.445 $\int \frac{1+x^6}{x^6(1+x^3)\sqrt[4]{x+x^4}}\,dx$

Optimal. Leaf size=35

$$\frac{4\left(x^4+x\right)^{3/4}\left(53x^6+8x^3-3\right)}{63x^6\left(x^3+1\right)}$$

Rubi [A] time = 0.17, antiderivative size = 47, normalized size of antiderivative = 1.34, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2056, 1487, 453, 271, 264}

$$\frac{212x}{63\sqrt[4]{x^4+x}}-\frac{4}{21\sqrt[4]{x^4+x}\,x^5}+\frac{32}{63\sqrt[4]{x^4+x}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)/(x^6*(1 + x^3)*(x + x^4)^(1/4)),x]
```

```
[Out] -4/(21*x^5*(x + x^4)^(1/4)) + 32/(63*x^2*(x + x^4)^(1/4)) + (212*x)/(63*(x
+ x^4)^(1/4))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1487

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 2*n*p - n + 1)*(d + e*x^n)^(
q + 1))/(e*f^(2*n*p - n + 1)*(m + 2*n*p + n*q + 1)), x] + Dist[1/(e*(m + 2*
n*p + n*q + 1)), Int[(f*x)^m*(d + e*x^n)^q*ExpandToSum[e*(m + 2*n*p + n*q +
 1)*((a + c*x^(2*n))^p - c^p*x^(2*n*p)) - d*c^p*(m + 2*n*p - n + 1)*x^(2*n*
p - n), x], x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IG
tQ[n, 0] && IGtQ[p, 0] && GtQ[2*n*p, n - 1] &&  !IntegerQ[q] && NeQ[m + 2*n
*p + n*q + 1, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
```

```
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^6}{x^6\left(1+x^3\right)\sqrt[4]{x+x^4}}\,dx &= \frac{\left(\sqrt[4]{x}\sqrt[4]{1+x^3}\right)\int \frac{1+x^6}{x^{25/4}\left(1+x^3\right)^{5/4}}\,dx}{\sqrt[4]{x+x^4}}\\
&= -\frac{1}{3x^2\sqrt[4]{x+x^4}} - \frac{\left(\sqrt[4]{x}\sqrt[4]{1+x^3}\right)\int \frac{-3+\frac{9x^3}{4}}{x^{25/4}\left(1+x^3\right)^{5/4}}\,dx}{3\sqrt[4]{x+x^4}}\\
&= -\frac{4}{21x^5\sqrt[4]{x+x^4}} - \frac{1}{3x^2\sqrt[4]{x+x^4}} - \frac{\left(53\sqrt[4]{x}\sqrt[4]{1+x^3}\right)\int \frac{1}{x^{13/4}\left(1+x^3\right)^{5/4}}\,dx}{28\sqrt[4]{x+x^4}}\\
&= -\frac{4}{21x^5\sqrt[4]{x+x^4}} + \frac{32}{63x^2\sqrt[4]{x+x^4}} + \frac{\left(53\sqrt[4]{x}\sqrt[4]{1+x^3}\right)\int \frac{1}{\sqrt[4]{x}\left(1+x^3\right)^{5/4}}\,dx}{21\sqrt[4]{x+x^4}}\\
&= -\frac{4}{21x^5\sqrt[4]{x+x^4}} + \frac{32}{63x^2\sqrt[4]{x+x^4}} + \frac{212x}{63\sqrt[4]{x+x^4}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 28, normalized size = 0.80

$$\frac{4\left(53x^6+8x^3-3\right)}{63x^5\sqrt[4]{x^4+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)/(x^6*(1 + x^3)*(x + x^4)^(1/4)),x]
[Out] (4*(-3 + 8*x^3 + 53*x^6))/(63*x^5*(x + x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.48, size = 35, normalized size = 1.00

$$\frac{4\left(x^4+x\right)^{3/4}\left(53x^6+8x^3-3\right)}{63x^6\left(x^3+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^6)/(x^6*(1 + x^3)*(x + x^4)^(1/4)),x]
[Out] (4*(x + x^4)^(3/4)*(-3 + 8*x^3 + 53*x^6))/(63*x^6*(1 + x^3))
```

fricas [A] time = 0.40, size = 30, normalized size = 0.86

$$\frac{4\left(53\,x^6+8\,x^3-3\right)\left(x^4+x\right)^{\frac{3}{4}}}{63\left(x^9+x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x^6/(x^3+1)/(x^4+x)^(1/4),x, algorithm="fricas")
[Out] 4/63*(53*x^6 + 8*x^3 - 3)*(x^4 + x)^(3/4)/(x^9 + x^6)
```

giac [A] time = 0.28, size = 28, normalized size = 0.80

$$-\frac{4}{21}\left(\frac{1}{x^3}+1\right)^{\frac{7}{4}}+\frac{8}{9}\left(\frac{1}{x^3}+1\right)^{\frac{3}{4}}+\frac{8}{3\left(\frac{1}{x^3}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x^6/(x^3+1)/(x^4+x)^(1/4),x, algorithm="giac")
[Out] -4/21*(1/x^3 + 1)^(7/4) + 8/9*(1/x^3 + 1)^(3/4) + 8/3/(1/x^3 + 1)^(1/4)
```

maple [A] time = 0.01, size = 25, normalized size = 0.71

$$\frac{\frac{212}{63}x^6+\frac{32}{63}x^3-\frac{4}{21}}{\left(x^4+x\right)^{\frac{1}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/x^6/(x^3+1)/(x^4+x)^(1/4),x)
[Out] 4/63*(53*x^6+8*x^3-3)/(x^4+x)^(1/4)/x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^6+1}{\left(x^4+x\right)^{\frac{1}{4}}\left(x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x^6/(x^3+1)/(x^4+x)^(1/4),x, algorithm="maxima")
[Out] integrate((x^6 + 1)/((x^4 + x)^(1/4)*(x^3 + 1)*x^6), x)
```

mupad [B] time = 0.30, size = 31, normalized size = 0.89

$$\frac{4\left(x^4+x\right)^{3/4}\left(53\,x^6+8\,x^3-3\right)}{63\,x^6\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)/(x^6*(x^3 + 1)*(x + x^4)^(1/4)),x)
[Out] (4*(x + x^4)^(3/4)*(8*x^3 + 53*x^6 - 3))/(63*x^6*(x^3 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2+1\right)\left(x^4-x^2+1\right)}{x^6\sqrt[4]{x\left(x+1\right)\left(x^2-x+1\right)}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/x**6/(x**3+1)/(x**4+x)**(1/4),x)
[Out] Integral((x**2 + 1)*(x**4 - x**2 + 1)/(x**6*(x*(x + 1)*(x**2 - x + 1))**(1/
4)*(x + 1)*(x**2 - x + 1)), x)
```

3.446 $\int x\sqrt{x+x^6}\,dx$

Optimal. Leaf size=35

$$\frac{1}{5}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6+x}}\right)+\frac{1}{5}\sqrt{x^6+x}\,x^2$$

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2021, 2029, 206}

$$\frac{1}{5}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6+x}}\right)+\frac{1}{5}\sqrt{x^6+x}\,x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[x + x^6],x]
```

```
[Out] (x^2*Sqrt[x + x^6])/5 + ArcTanh[x^3/Sqrt[x + x^6]]/5
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{x+x^6}\,dx &= \frac{1}{5}x^2\sqrt{x+x^6}+\frac{1}{2}\int\frac{x^2}{\sqrt{x+x^6}}\,dx\\
&= \frac{1}{5}x^2\sqrt{x+x^6}+\frac{1}{5}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x^3}{\sqrt{x+x^6}}\right)\\
&= \frac{1}{5}x^2\sqrt{x+x^6}+\frac{1}{5}\tanh^{-1}\left(\frac{x^3}{\sqrt{x+x^6}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 1.14

$$\frac{\sqrt{x^6+x}\left(x^{5/2}+\frac{\sinh^{-1}\left(x^{5/2}\right)}{\sqrt{x^5+1}}\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[x + x^6],x]
[Out] (Sqrt[x + x^6]*(x^(5/2) + ArcSinh[x^(5/2)]/Sqrt[1 + x^5]))/(5*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.30, size = 35, normalized size = 1.00

$$\frac{1}{5}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6+x}}\right)+\frac{1}{5}\sqrt{x^6+x}\,x^2$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[x + x^6],x]
[Out] (x^2*Sqrt[x + x^6])/5 + ArcTanh[x^3/Sqrt[x + x^6]]/5
```

fricas [A] time = 0.54, size = 35, normalized size = 1.00

$$\frac{1}{5}\sqrt{x^6+x}\,x^2+\frac{1}{10}\log\left(2\,x^5+2\,\sqrt{x^6+x}\,x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^6+x)^(1/2),x, algorithm="fricas")
[Out] 1/5*sqrt(x^6 + x)*x^2 + 1/10*log(2*x^5 + 2*sqrt(x^6 + x)*x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^6+x}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^6+x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^6 + x)*x, x)
```

maple [A] time = 0.20, size = 27, normalized size = 0.77

$$\frac{x^3\left(x^5+1\right)}{5\sqrt{x\left(x^5+1\right)}}+\frac{\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^6+x)^(1/2),x)
[Out] 1/5*x^3*(x^5+1)/(x*(x^5+1))^(1/2)+1/5*arcsinh(x^(5/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^6+x}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^6+x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^6 + x)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x\,\sqrt{x^6+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x + x^6)^(1/2),x)
```

```
[Out] int(x*(x + x^6)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x\left(x+1\right)\left(x^{4}-x^{3}+x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**6+x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x*(x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

3.447 $\int \frac{-2bc+acx^6}{\sqrt[4]{b+ax^6}\left(b-c^4x^4+ax^6\right)}\,dx$

Optimal. Leaf size=35

$$-\tan^{-1}\left(\frac{cx}{\sqrt[4]{ax^6+b}}\right)-\tanh^{-1}\left(\frac{cx}{\sqrt[4]{ax^6+b}}\right)$$

Rubi [F] time = 1.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2bc+acx^6}{\sqrt[4]{b+ax^6}\left(b-c^4x^4+ax^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*b*c + a*c*x^6)/((b + a*x^6)^(1/4)*(b - c^4*x^4 + a*x^6)),x]
```

```
[Out] (c*x*(1 + (a*x^6)/b)^(1/4)*Hypergeometric2F1[1/6, 1/4, 7/6, -((a*x^6)/b)])/
(b + a*x^6)^(1/4) - c^5*Defer[Int][x^4/((-b + c^4*x^4 - a*x^6)*(b + a*x^6)^
(1/4)), x] - 3*b*c*Defer[Int][1/((b + a*x^6)^(1/4)*(b - c^4*x^4 + a*x^6)),
x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2bc+acx^6}{\sqrt[4]{b+ax^6}\left(b-c^4x^4+ax^6\right)}\,dx &= \int \left(\frac{c}{\sqrt[4]{b+ax^6}}-\frac{3bc-c^5x^4}{\sqrt[4]{b+ax^6}\left(b-c^4x^4+ax^6\right)}\right)dx\\
&= c\int \frac{1}{\sqrt[4]{b+ax^6}}\,dx-\int \frac{3bc-c^5x^4}{\sqrt[4]{b+ax^6}\left(b-c^4x^4+ax^6\right)}\,dx\\
&= \frac{\left(c\sqrt[4]{1+\frac{ax^6}{b}}\right)\int \frac{1}{\sqrt[4]{1+\frac{ax^6}{b}}}\,dx}{\sqrt[4]{b+ax^6}}-\int \left(\frac{c^5x^4}{\left(-b+c^4x^4-ax^6\right)\sqrt[4]{b+ax^6}}+\frac{}{\sqrt[4]{b+ax^6}}\right.\\
&= \frac{cx\sqrt[4]{1+\frac{ax^6}{b}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{4};\frac{7}{6};-\frac{ax^6}{b}\right)}{\sqrt[4]{b+ax^6}}-(3bc)\int \frac{1}{\sqrt[4]{b+ax^6}\left(b-c^4x^4+ax^6\right)}\,dx-
\end{aligned}$$

Mathematica [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{-2bc+acx^6}{\sqrt[4]{b+ax^6}\left(b-c^4x^4+ax^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*b*c + a*c*x^6)/((b + a*x^6)^(1/4)*(b - c^4*x^4 + a*x^6)),x]
```

```
[Out] Integrate[(-2*b*c + a*c*x^6)/((b + a*x^6)^(1/4)*(b - c^4*x^4 + a*x^6)), x]
```

IntegrateAlgebraic [A] time = 5.15, size = 35, normalized size = 1.00

$$-\tan^{-1}\left(\frac{cx}{\sqrt[4]{ax^6+b}}\right)-\tanh^{-1}\left(\frac{cx}{\sqrt[4]{ax^6+b}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*b*c + a*c*x^6)/((b + a*x^6)^(1/4)*(b - c^4*x^4 + a*x
^6)),x]
```

```
[Out] -ArcTan[(c*x)/(b + a*x^6)^(1/4)] - ArcTanh[(c*x)/(b + a*x^6)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*x^6-2*b*c)/(a*x^6+b)^(1/4)/(-c^4*x^4+a*x^6+b),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{acx^6 - 2\,bc}{\left(c^4x^4 - ax^6 - b\right)\left(ax^6 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*x^6-2*b*c)/(a*x^6+b)^(1/4)/(-c^4*x^4+a*x^6+b),x, algorithm="
giac")
```

```
[Out] integrate(-(a*c*x^6 - 2*b*c)/((c^4*x^4 - a*x^6 - b)*(a*x^6 + b)^(1/4)), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{ac\,x^6 - 2bc}{\left(a\,x^6 + b\right)^{\frac{1}{4}}\left(-c^4x^4 + a\,x^6 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c*x^6-2*b*c)/(a*x^6+b)^(1/4)/(-c^4*x^4+a*x^6+b),x)
```

```
[Out] int((a*c*x^6-2*b*c)/(a*x^6+b)^(1/4)/(-c^4*x^4+a*x^6+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{acx^6 - 2\,bc}{\left(c^4x^4 - ax^6 - b\right)\left(ax^6 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*x^6-2*b*c)/(a*x^6+b)^(1/4)/(-c^4*x^4+a*x^6+b),x, algorithm="
maxima")
```

```
[Out] -integrate((a*c*x^6 - 2*b*c)/((c^4*x^4 - a*x^6 - b)*(a*x^6 + b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{2\,b\,c - a\,c\,x^6}{\left(a\,x^6 + b\right)^{1/4}\,\left(-c^4\,x^4 + a\,x^6 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*b*c - a*c*x^6)/((b + a*x^6)^(1/4)*(b + a*x^6 - c^4*x^4)),x)
```

```
[Out] int(-(2*b*c - a*c*x^6)/((b + a*x^6)^(1/4)*(b + a*x^6 - c^4*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*x**6-2*b*c)/(a*x**6+b)**(1/4)/(-c**4*x**4+a*x**6+b),x)
[Out] Timed out
```

3.448 $\int \frac{1}{\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=35

$$\sqrt{\sqrt{x^2+1}+x}-\frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2117, 14}

$$\sqrt{\sqrt{x^2+1}+x}-\frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x + Sqrt[1 + x^2]],x]
```

```
[Out] -1/3*1/(x + Sqrt[1 + x^2])^(3/2) + Sqrt[x + Sqrt[1 + x^2]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x+\sqrt{1+x^2}}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{x^{5/2}}\,dx,x,x+\sqrt{1+x^2}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^{5/2}}+\frac{1}{\sqrt{x}}\right)\,dx,x,x+\sqrt{1+x^2}\right)\\
&= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}}+\sqrt{x+\sqrt{1+x^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\sqrt{\sqrt{x^2+1}+x}-\frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x + Sqrt[1 + x^2]],x]
```

```
[Out] -1/3*1/(x + Sqrt[1 + x^2])^(3/2) + Sqrt[x + Sqrt[1 + x^2]]
```

IntegrateAlgebraic [A] time = 0.07, size = 35, normalized size = 1.00

$$\sqrt{\sqrt{x^2+1}+x}-\frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[x + Sqrt[1 + x^2]],x]
[Out] -1/3*1/(x + Sqrt[1 + x^2])^(3/2) + Sqrt[x + Sqrt[1 + x^2]]
```

fricas [A] time = 0.40, size = 28, normalized size = 0.80

$$-\frac{2}{3}\left(x^2-\sqrt{x^2+1}\,x-1\right)\sqrt{x+\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
[Out] -2/3*(x^2 - sqrt(x^2 + 1)*x - 1)*sqrt(x + sqrt(x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(x + sqrt(x^2 + 1)), x)
```

maple [C] time = 0.04, size = 62, normalized size = 1.77

$$-\frac{-\frac{32\sqrt{\pi}\,\sqrt{2}\,\cosh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{1}{x}\right)}{2}\right)}{3x^{\frac{3}{2}}}-\frac{8\sqrt{\pi}\,\sqrt{2}\,x^{\frac{3}{2}}\left(-\frac{4}{3x^4}-\frac{2}{3x^2}+\frac{2}{3}\right)\sinh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{1}{x}\right)}{2}\right)}{\sqrt{1+\frac{1}{x^2}}}}{8\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(x^2+1)^(1/2))^(1/2),x)
[Out] -1/8/Pi^(1/2)*(-32/3*Pi^(1/2)*2^(1/2)/x^(3/2)*cosh(3/2*arcsinh(1/x))-8*Pi^(
1/2)*2^(1/2)*x^(3/2)*(-4/3/x^4-2/3/x^2+2/3)*sinh(3/2*arcsinh(1/x))/(1+1/x^2
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(x + sqrt(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int(1/(x + (x^2 + 1)^(1/2))^(1/2), x)
```

sympy [A] time = 0.34, size = 42, normalized size = 1.20

$$\frac{4x}{3\sqrt{x+\sqrt{x^2+1}}}+\frac{2\sqrt{x^2+1}}{3\sqrt{x+\sqrt{x^2+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] 4*x/(3*sqrt(x + sqrt(x**2 + 1))) + 2*sqrt(x**2 + 1)/(3*sqrt(x + sqrt(x**2 +
 1)))
```

3.449 $\int \frac{\sqrt[4]{1+x^2}}{x} \, dx$

Optimal. Leaf size=36

$$2\sqrt[4]{x^2+1} - \tan^{-1}\left(\sqrt[4]{x^2+1}\right) - \tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 50, 63, 212, 206, 203}

$$2\sqrt[4]{x^2+1} - \tan^{-1}\left(\sqrt[4]{x^2+1}\right) - \tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^(1/4)/x,x]
```

```
[Out] 2*(1 + x^2)^(1/4) - ArcTan[(1 + x^2)^(1/4)] - ArcTanh[(1 + x^2)^(1/4)]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{1+x^2}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt[4]{1+x}}{x}\,dx, x, x^2\right)\\ &= 2\sqrt[4]{1+x^2} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{3/4}}\,dx, x, x^2\right)\\ &= 2\sqrt[4]{1+x^2} + 2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt[4]{1+x^2}\right)\\ &= 2\sqrt[4]{1+x^2} - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1+x^2}\right) - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{1+x^2}\right)\\ &= 2\sqrt[4]{1+x^2} - \tan^{-1}\left(\sqrt[4]{1+x^2}\right) - \tanh^{-1}\left(\sqrt[4]{1+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$2\sqrt[4]{x^2+1} - \tan^{-1}\left(\sqrt[4]{x^2+1}\right) - \tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^(1/4)/x,x]
[Out] 2*(1 + x^2)^(1/4) - ArcTan[(1 + x^2)^(1/4)] - ArcTanh[(1 + x^2)^(1/4)]
```

IntegrateAlgebraic [A] time = 0.04, size = 36, normalized size = 1.00

$$2\sqrt[4]{x^2+1} - \tan^{-1}\left(\sqrt[4]{x^2+1}\right) - \tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)^(1/4)/x,x]
[Out] 2*(1 + x^2)^(1/4) - ArcTan[(1 + x^2)^(1/4)] - ArcTanh[(1 + x^2)^(1/4)]
```

fricas [A] time = 0.40, size = 44, normalized size = 1.22

$$2\left(x^2+1\right)^{\frac{1}{4}} - \arctan\left(\left(x^2+1\right)^{\frac{1}{4}}\right) - \frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}+1\right) + \frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/4)/x,x, algorithm="fricas")
[Out] 2*(x^2 + 1)^(1/4) - arctan((x^2 + 1)^(1/4)) - 1/2*log((x^2 + 1)^(1/4) + 1)
+ 1/2*log((x^2 + 1)^(1/4) - 1)
```

giac [A] time = 0.34, size = 44, normalized size = 1.22

$$2\left(x^2+1\right)^{\frac{1}{4}} - \arctan\left(\left(x^2+1\right)^{\frac{1}{4}}\right) - \frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}+1\right) + \frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/4)/x,x, algorithm="giac")
```

```
[Out] 2*(x^2 + 1)^(1/4) - arctan((x^2 + 1)^(1/4)) - 1/2*log((x^2 + 1)^(1/4) + 1)
+ 1/2*log((x^2 + 1)^(1/4) - 1)
```

maple [C] time = 0.24, size = 45, normalized size = 1.25

$$-\frac{-\Gamma\left(\frac{3}{4}\right)x^2\,\mathrm{hypergeom}\left(\left[\frac{3}{4},1,1\right],[2,2],-x^2\right)-4\left(4-3\ln(2)+\frac{\pi}{2}+2\ln(x)\right)\Gamma\left(\frac{3}{4}\right)}{8\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/4)/x,x)
```

```
[Out] -1/8/GAMMA(3/4)*(-GAMMA(3/4)*x^2*hypergeom([3/4,1,1],[2,2],-x^2)-4*(4-3*ln(
2)+1/2*Pi+2*ln(x))*GAMMA(3/4))
```

maxima [A] time = 0.55, size = 44, normalized size = 1.22

$$2\left(x^2+1\right)^{\frac{1}{4}}-\arctan\left(\left(x^2+1\right)^{\frac{1}{4}}\right)-\frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/4)/x,x, algorithm="maxima")
```

```
[Out] 2*(x^2 + 1)^(1/4) - arctan((x^2 + 1)^(1/4)) - 1/2*log((x^2 + 1)^(1/4) + 1)
+ 1/2*log((x^2 + 1)^(1/4) - 1)
```

mupad [B] time = 0.25, size = 30, normalized size = 0.83

$$2\left(x^2+1\right)^{1/4}-\mathrm{atanh}\left(\left(x^2+1\right)^{1/4}\right)-\mathrm{atan}\left(\left(x^2+1\right)^{1/4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/4)/x,x)
```

```
[Out] 2*(x^2 + 1)^(1/4) - atanh((x^2 + 1)^(1/4)) - atan((x^2 + 1)^(1/4))
```

sympy [C] time = 0.81, size = 37, normalized size = 1.03

$$-\frac{\sqrt{x}\,\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\right|\frac{e^{i\pi}}{x^2}\right)}{2\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/4)/x,x)
```

```
[Out] -sqrt(x)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), exp_polar(I*pi)/x**2)/(2*g
amma(3/4))
```

3.450 $\int \frac{\sqrt{-1+x^3}}{x^7} \, dx$

Optimal. Leaf size=36

$$\frac{1}{12} \tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{\sqrt{x^3-1}\,\left(x^3-2\right)}{12x^6}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {266, 47, 51, 63, 203}

$$\frac{\sqrt{x^3-1}}{12x^3}+\frac{1}{12} \tan^{-1}\left(\sqrt{x^3-1}\right)-\frac{\sqrt{x^3-1}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^3]/x^7,x]
```

```
[Out] -1/6*Sqrt[-1 + x^3]/x^6 + Sqrt[-1 + x^3]/(12*x^3) + ArcTan[Sqrt[-1 + x^3]]/
12
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x^3}}{x^7}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x^3}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{-1+x^3}}{6x^6} + \frac{1}{12}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{-1+x^3}}{6x^6} + \frac{\sqrt{-1+x^3}}{12x^3} + \frac{1}{24}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{-1+x^3}}{6x^6} + \frac{\sqrt{-1+x^3}}{12x^3} + \frac{1}{12}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^3}\right)\\
&= -\frac{\sqrt{-1+x^3}}{6x^6} + \frac{\sqrt{-1+x^3}}{12x^3} + \frac{1}{12}\tan^{-1}\left(\sqrt{-1+x^3}\right)
\end{aligned}$$

Mathematica [C] time = 0.00, size = 28, normalized size = 0.78

$$\frac{2}{9}\left(x^3-1\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^3]/x^7,x]
```

```
[Out] (2*(-1 + x^3)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 - x^3])/9
```

IntegrateAlgebraic [A] time = 0.06, size = 36, normalized size = 1.00

$$\frac{1}{12}\tan^{-1}\left(\sqrt{x^3-1}\right) + \frac{\sqrt{x^3-1}\,\left(x^3-2\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x^3]/x^7,x]
```

```
[Out] ((-2 + x^3)*Sqrt[-1 + x^3])/(12*x^6) + ArcTan[Sqrt[-1 + x^3]]/12
```

fricas [A] time = 0.39, size = 31, normalized size = 0.86

$$\frac{x^6\arctan\left(\sqrt{x^3-1}\right) + \sqrt{x^3-1}\,\left(x^3-2\right)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(x^6*arctan(sqrt(x^3 - 1)) + sqrt(x^3 - 1)*(x^3 - 2))/x^6
```

giac [B] time = 0.30, size = 63, normalized size = 1.75

$$\frac{1}{48}\,\pi + \frac{\sqrt{x^3-1} - \frac{1}{\sqrt{x^3-1}}}{12\left(\left(\sqrt{x^3-1} - \frac{1}{\sqrt{x^3-1}}\right)^2 + 4\right)} + \frac{1}{24}\arctan\left(\frac{x^3-2}{2\sqrt{x^3-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*pi + 1/12*(sqrt(x^3 - 1) - 1/sqrt(x^3 - 1))/((sqrt(x^3 - 1) - 1/sqrt(x
^3 - 1))^2 + 4) + 1/24*arctan(1/2*(x^3 - 2)/sqrt(x^3 - 1))
```

maple [A] time = 0.24, size = 36, normalized size = 1.00

$$-\frac{\sqrt{x^3-1}}{6x^6}+\frac{\sqrt{x^3-1}}{12x^3}+\frac{\arctan\left(\sqrt{x^3-1}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/2)/x^7,x)
[Out] -1/6*(x^3-1)^(1/2)/x^6+1/12*(x^3-1)^(1/2)/x^3+1/12*arctan((x^3-1)^(1/2))
```

maxima [A] time = 0.46, size = 46, normalized size = 1.28

$$\frac{\left(x^3-1\right)^{\frac{3}{2}}-\sqrt{x^3-1}}{12\left(2\,x^3+\left(x^3-1\right)^2-1\right)}+\frac{1}{12}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)/x^7,x, algorithm="maxima")
[Out] 1/12*((x^3 - 1)^(3/2) - sqrt(x^3 - 1))/(2*x^3 + (x^3 - 1)^2 - 1) + 1/12*arc
tan(sqrt(x^3 - 1))
```

mupad [B] time = 0.07, size = 189, normalized size = 5.25

$$\frac{\sqrt{x^3-1}}{12\,x^3}-\frac{\sqrt{x^3-1}}{6\,x^6}-\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)^(1/2)/x^7,x)
[Out] (x^3 - 1)^(1/2)/(12*x^3) - (x^3 - 1)^(1/2)/(6*x^6) - (((3^(1/2)*1i)/2 + 3/2
)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)
*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(4*(((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [B] time = 2.28, size = 138, normalized size = 3.83

$$\begin{cases} \frac{i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{12}-\frac{i}{12x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x^3}}}+\frac{i}{4x^{\frac{9}{2}}\sqrt{-1+\frac{1}{x^3}}}-\frac{i}{6x^{\frac{15}{2}}\sqrt{-1+\frac{1}{x^3}}} & \text{for}\ \frac{1}{|x^3|}>1 \\ -\frac{\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{12}+\frac{1}{12x^{\frac{3}{2}}\sqrt{1-\frac{1}{x^3}}}-\frac{1}{4x^{\frac{9}{2}}\sqrt{1-\frac{1}{x^3}}}+\frac{1}{6x^{\frac{15}{2}}\sqrt{1-\frac{1}{x^3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/2)/x**7,x)
```

```
[Out] Piecewise((I*acosh(x**(-3/2))/12 - I/(12*x**(3/2)*sqrt(-1 + x**(-3))) + I/(
4*x**(9/2)*sqrt(-1 + x**(-3))) - I/(6*x**(15/2)*sqrt(-1 + x**(-3))), 1/Abs(
x**3) > 1), (-asin(x**(-3/2))/12 + 1/(12*x**(3/2)*sqrt(1 - 1/x**3)) - 1/(4*
x**(9/2)*sqrt(1 - 1/x**3)) + 1/(6*x**(15/2)*sqrt(1 - 1/x**3)), True))
```

3.451 $\int \frac{-b+ax^2}{(b+cx+ax^2)\sqrt{bx+ax^3}}\,dx$

Optimal. Leaf size=36

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{ax^3+bx}}{ax^2+b}\right)}{\sqrt{c}}$$

Rubi [C] time = 1.99, antiderivative size = 289, normalized size of antiderivative = 8.03, number of steps used = 13, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2056, 6728, 329, 220, 933, 168, 537}

$$-\frac{2\sqrt[4]{b}\,\sqrt{x}\,\sqrt{\frac{ax^2}{b}+1}\,\Pi\left(\frac{2\sqrt{-a}\,\sqrt{b}}{c-\sqrt{c^2-4ab}};\sin^{-1}\left(\frac{\sqrt[4]{-a}\,\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{-a}\,\sqrt{ax^3+bx}}-\frac{2\sqrt[4]{b}\,\sqrt{x}\,\sqrt{\frac{ax^2}{b}+1}\,\Pi\left(\frac{2\sqrt{-a}\,\sqrt{b}}{c+\sqrt{c^2-4ab}};\sin^{-1}\left(\frac{\sqrt[4]{-a}\,\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{-a}\,\sqrt{ax^3+bx}}+\frac{\sqrt{x}\left(\sqrt{a}\,x+\sqrt{b}\right)\sqrt{\frac{ax^2+b}{\left(\sqrt{a}\,x+\sqrt{b}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{ax^3+bx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-b + a*x^2)/((b + c*x + a*x^2)*Sqrt[b*x + a*x^3]),x]
```

```
[Out] (Sqrt[x]*(Sqrt[b] + Sqrt[a]*x)*Sqrt[(b + a*x^2)/(Sqrt[b] + Sqrt[a]*x)^2]*El
lipticF[2*ArcTan[(a^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(a^(1/4)*b^(1/4)*Sqrt[b*
x + a*x^3]) - (2*b^(1/4)*Sqrt[x]*Sqrt[1 + (a*x^2)/b]*EllipticPi[(2*Sqrt[-a]
*Sqrt[b])/(c - Sqrt[-4*a*b + c^2]), ArcSin[((-a)^(1/4)*Sqrt[x])/b^(1/4)], -
1])/((-a)^(1/4)*Sqrt[b*x + a*x^3]) - (2*b^(1/4)*Sqrt[x]*Sqrt[1 + (a*x^2)/b]
*EllipticPi[(2*Sqrt[-a]*Sqrt[b])/(c + Sqrt[-4*a*b + c^2]), ArcSin[((-a)^(1/
4)*Sqrt[x])/b^(1/4)], -1])/((-a)^(1/4)*Sqrt[b*x + a*x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
```

```
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-b+ax^2}{\left(b+cx+ax^2\right)\sqrt{bx+ax^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \frac{-b+ax^2}{\sqrt{x}\sqrt{b+ax^2}\left(b+cx+ax^2\right)}\,dx}{\sqrt{bx+ax^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{b+ax^2}}-\frac{2b+cx}{\sqrt{x}\sqrt{b+ax^2}\left(b+cx+ax^2\right)}\right)dx}{\sqrt{bx+ax^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx}{\sqrt{bx+ax^3}} - \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \frac{2b+cx}{\sqrt{x}\sqrt{b+ax^2}\left(b+cx+ax^2\right)}\,dx}{\sqrt{bx+ax^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \left(\frac{c-\sqrt{-4ab+c^2}}{\sqrt{x}\left(c-\sqrt{-4ab+c^2}+2ax\right)\sqrt{b+ax^2}}+\frac{c+\sqrt{-4ab+c^2}}{\sqrt{x}\left(c+\sqrt{-4ab+c^2}+2ax\right)\sqrt{b+ax^2}}\right.}{\sqrt{bx+ax^3}}$$

$$= \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}\,x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} - \frac{\left(\left(c-\sqrt{-4ab+c^2}\right)\right.}{}$$

$$= \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}\,x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} - \frac{\left(\left(c-\sqrt{-4ab+c^2}\right)\right.}{}$$

$$= \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}\,x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} + \frac{\left(2\left(c-\sqrt{-4ab+c^2}\right.\right.}{}$$

$$= \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}\,x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} - \frac{2\sqrt[4]{b}\sqrt{x}\sqrt{1+\frac{ax^2}{b}}\,\mathrm{I}}{}$$

Mathematica [C] time = 1.64, size = 214, normalized size = 5.94

$$-\frac{2ix^{3/2}\sqrt{\frac{b}{ax^2}+1}\left(-\Pi\left(\frac{2i\sqrt{a}\sqrt{b}}{\sqrt{c^2-4ab}-c};i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(-\frac{2i\sqrt{a}\sqrt{b}}{c+\sqrt{c^2-4ab}};i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{x}}\right)\middle|-1\right)+F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{x\left(ax^2+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^2)/((b + c*x + a*x^2)*Sqrt[b*x + a*x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[1 + b/(a*x^2)]*x^(3/2)*(EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/S
qrt[a]]/Sqrt[x]], -1] - EllipticPi[((2*I)*Sqrt[a]*Sqrt[b])/(-c + Sqrt[-4*a*
b + c^2]), I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]/Sqrt[x]], -1] - EllipticPi[(
(-2*I)*Sqrt[a]*Sqrt[b])/(c + Sqrt[-4*a*b + c^2]), I*ArcSinh[Sqrt[(I*Sqrt[b]
)/Sqrt[a]]/Sqrt[x]], -1]))/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[x*(b + a*x^2)])
```

IntegrateAlgebraic [A] time = 0.35, size = 36, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax^3+bx}}{ax^2+b}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^2)/((b + c*x + a*x^2)*Sqrt[b*x + a*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[c]*Sqrt[b*x + a*x^3])/(b + a*x^2)])/Sqrt[c]
```

fricas [A] time = 0.47, size = 159, normalized size = 4.42

$$\left[-\frac{\sqrt{-c}\log\left(\frac{a^2x^4-6\,acx^3-6\,bcx+\left(2\,ab+c^2\right)x^2+b^2-4\,\sqrt{ax^3+bx}\left(ax^2-cx+b\right)\sqrt{-c}}{a^2x^4+2\,acx^3+2\,bcx+\left(2\,ab+c^2\right)x^2+b^2}\right)}{2\,c},\ \frac{\arctan\left(\frac{\sqrt{ax^3+bx}\left(ax^2-cx+b\right)\sqrt{c}}{2\left(acx^3+bcx\right)}\right)}{\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(a*x^2+c*x+b)/(a*x^3+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-c)*log((a^2*x^4 - 6*a*c*x^3 - 6*b*c*x + (2*a*b + c^2)*x^2 + b^2
 - 4*sqrt(a*x^3 + b*x)*(a*x^2 - c*x + b)*sqrt(-c))/(a^2*x^4 + 2*a*c*x^3 + 2
*b*c*x + (2*a*b + c^2)*x^2 + b^2))/c, arctan(1/2*sqrt(a*x^3 + b*x)*(a*x^2 -
 c*x + b)*sqrt(c)/(a*c*x^3 + b*c*x))/sqrt(c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2-b}{\sqrt{ax^3+bx}\left(ax^2+cx+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(a*x^2+c*x+b)/(a*x^3+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^2 - b)/(sqrt(a*x^3 + b*x)*(a*x^2 + c*x + b)), x)
```

maple [C] time = 0.09, size = 1161, normalized size = 32.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)/(a*x^2+c*x+b)/(a*x^3+b*x)^(1/2),x)
```

```
[Out] 1/a*(-a*b)^(1/2)*((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x-1/a*(-a
*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2
)*EllipticF(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/2/(-
4*a*b+c^2)^(1/2)/a^2*(-a*b)^(1/2)*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)
^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(-a*b)^(1
/2)+1/2*c/a-1/2/a*(-4*a*b+c^2)^(1/2))*EllipticPi(((x+1/a*(-a*b)^(1/2))*a/(-
a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2)/(-1/a*(-a*b)^(1/2)-1/2/a*(-c+(-4*a*b+c^
2)^(1/2))),1/2*2^(1/2))*c^2-1/2/a^2*(-a*b)^(1/2)*(x*a/(-a*b)^(1/2)+1)^(1/2)
*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)/
(-1/a*(-a*b)^(1/2)+1/2*c/a-1/2/a*(-4*a*b+c^2)^(1/2))*EllipticPi(((x+1/a*(-a
*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2)/(-1/a*(-a*b)^(1/2)-1/2/a
*(-c+(-4*a*b+c^2)^(1/2))),1/2*2^(1/2))*c-2/(-4*a*b+c^2)^(1/2)/a*(-a*b)^(1/2
)*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/
2))^(1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(-a*b)^(1/2)+1/2*c/a-1/2/a*(-4*a*b+c^2)^(
1/2))*EllipticPi(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1
/2)/(-1/a*(-a*b)^(1/2)-1/2/a*(-c+(-4*a*b+c^2)^(1/2))),1/2*2^(1/2))*b-1/2/(-
4*a*b+c^2)^(1/2)/a^2*(-a*b)^(1/2)*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)
^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(-a*b)^(1
/2)+1/2*c/a+1/2/a*(-4*a*b+c^2)^(1/2))*EllipticPi(((x+1/a*(-a*b)^(1/2))*a/(-
a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2)/(-1/a*(-a*b)^(1/2)+1/2*(c+(-4*a*b+c^2)^
(1/2))/a),1/2*2^(1/2))*c^2-1/2/a^2*(-a*b)^(1/2)*(x*a/(-a*b)^(1/2)+1)^(1/2)*
(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)/(
-1/a*(-a*b)^(1/2)+1/2*c/a+1/2/a*(-4*a*b+c^2)^(1/2))*EllipticPi(((x+1/a*(-a*
b)^(1/2))*a/(-a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2)/(-1/a*(-a*b)^(1/2)+1/2*(c
+(-4*a*b+c^2)^(1/2))/a),1/2*2^(1/2))*c+2/(-4*a*b+c^2)^(1/2)/a*(-a*b)^(1/2)*
(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2)
)^(1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(-a*b)^(1/2)+1/2*c/a+1/2/a*(-4*a*b+c^2)^(1/
2))*EllipticPi(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2
)/(-1/a*(-a*b)^(1/2)+1/2*(c+(-4*a*b+c^2)^(1/2))/a),1/2*2^(1/2))*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(a*x^2+c*x+b)/(a*x^3+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-4*a*b>0)', see `assume?` for mo
re details)Is c^2-4*a*b positive, negative or zero?
```

mupad [B] time = 2.25, size = 51, normalized size = 1.42

$$\frac{\ln\left(\frac{\frac{b}{2}-\frac{c\,x}{2}+\frac{a\,x^2}{2}+\sqrt{c}\,\sqrt{a\,x^3+b\,x}\,1\mathrm{i}}{a\,x^2+c\,x+b}\right)\,1\mathrm{i}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^2)/((b*x + a*x^3)^(1/2)*(b + c*x + a*x^2)),x)
```

```
[Out] (log((b/2 - (c*x)/2 + (a*x^2)/2 + c^(1/2)*(b*x + a*x^3)^(1/2)*1i)/(b + c*x
+ a*x^2))*1i)/c^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - b}{\sqrt{x\left(ax^2 + b\right)}\left(ax^2 + b + cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)/(a*x**2+c*x+b)/(a*x**3+b*x)**(1/2),x)
```

```
[Out] Integral((a*x**2 - b)/(sqrt(x*(a*x**2 + b))*(a*x**2 + b + c*x)), x)
```

3.452 $\int \frac{-1+x}{\sqrt{-2-x+6x^2-4x^3+x^4}}\,dx$

Optimal. Leaf size=36

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^2-2x+1}{\sqrt{x^4-4x^3+6x^2-x-2}}\right)$$

Rubi [F] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+x}{\sqrt{-2-x+6x^2-4x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x)/Sqrt[-2 - x + 6*x^2 - 4*x^3 + x^4],x]
```

```
[Out] -Defer[Int][1/Sqrt[-2 - x + 6*x^2 - 4*x^3 + x^4], x] + Defer[Int][x/Sqrt[-2
 - x + 6*x^2 - 4*x^3 + x^4], x]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x}{\sqrt{-2-x+6x^2-4x^3+x^4}}\,dx &= \int \left(-\frac{1}{\sqrt{-2-x+6x^2-4x^3+x^4}}+\frac{x}{\sqrt{-2-x+6x^2-4x^3+x^4}}\right)dx\\ &= -\int \frac{1}{\sqrt{-2-x+6x^2-4x^3+x^4}}\,dx+\int \frac{x}{\sqrt{-2-x+6x^2-4x^3+x^4}}\,dx\end{aligned}$$

Mathematica [C] time = 0.83, size = 640, normalized size = 17.78

$$\frac{2\sqrt[3]{3}(x-1)\sqrt{\frac{-x+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,2\right]}{\left(x+\sqrt[3]{3}-1\right)\left(-1+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,2\right]\right)}}\left(x-\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,3\right]\right)\left(F\left(\sin^{-1}\left(\sqrt{\frac{(x-1)\left(-1+\sqrt[3]{3}+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,3\right]\right)}{\left(x+\sqrt[3]{3}-1\right)\left(-1+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,3\right]\right)}}\right)|\frac{\left(-1+\sqrt[3]{3}+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,2\right]\right)\left(-1+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,3\right]\right)}{\left(-1+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,2\right]\right)\left(-1+\sqrt[3]{3}+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,3\right]\right)}\right)-\Pi\left(\frac{-1+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,3\right]}{-1+\sqrt[3]{3}+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,3\right]};\sin^{-1}\left(\sqrt{\frac{(x-1)\left(-1+\sqrt[3]{3}+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,3\right]\right)}{\left(x+\sqrt[3]{3}-1\right)\left(-1+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,3\right]\right)}}\right)|\frac{\left(-1+\sqrt[3]{3}+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,2\right]\right)\left(-1+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,3\right]\right)}{\left(-1+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,2\right]\right)\left(-1+\sqrt[3]{3}+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,3\right]\right)}\right)\right)}{\sqrt{x^4-4x^3+6x^2-x-2}\left(-1+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,3\right]\right)\sqrt{-\frac{(x-1)\left(\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,3\right]-1+\sqrt[3]{3}\right)\left(x-\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,3\right]\right)}{\left(x+\sqrt[3]{3}-1\right)^2\left(-1+\text{Root}\left[\#1^3-3\#1^2+3\#1+2\&,3\right]\right)^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + x)/Sqrt[-2 - x + 6*x^2 - 4*x^3 + x^4],x]
```

```
[Out] (2*3^(1/3)*(-1 + x)*(EllipticF[ArcSin[Sqrt[((-1 + x)*(-1 + 3^(1/3) + Root[2
 + 3*#1 - 3*#1^2 + #1^3 & , 3, 0]))/((-1 + 3^(1/3) + x)*(-1 + Root[2 + 3*#1
 - 3*#1^2 + #1^3 & , 3, 0]))]], ((-1 + 3^(1/3) + Root[2 + 3*#1 - 3*#1^2 + #
1^3 & , 2, 0])*(-1 + Root[2 + 3*#1 - 3*#1^2 + #1^3 & , 3, 0]))/((-1 + Root[
2 + 3*#1 - 3*#1^2 + #1^3 & , 2, 0])*(-1 + 3^(1/3) + Root[2 + 3*#1 - 3*#1^2
+ #1^3 & , 3, 0]))] - EllipticPi[(-1 + Root[2 + 3*#1 - 3*#1^2 + #1^3 & , 3,
 0])/(-1 + 3^(1/3) + Root[2 + 3*#1 - 3*#1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[
((-1 + x)*(-1 + 3^(1/3) + Root[2 + 3*#1 - 3*#1^2 + #1^3 & , 3, 0]))/((-1 +
3^(1/3) + x)*(-1 + Root[2 + 3*#1 - 3*#1^2 + #1^3 & , 3, 0]))]], ((-1 + 3^(1
/3) + Root[2 + 3*#1 - 3*#1^2 + #1^3 & , 2, 0])*(-1 + Root[2 + 3*#1 - 3*#1^2
 + #1^3 & , 3, 0]))/((-1 + Root[2 + 3*#1 - 3*#1^2 + #1^3 & , 2, 0])*(-1 + 3
^(1/3) + Root[2 + 3*#1 - 3*#1^2 + #1^3 & , 3, 0]))])*Sqrt[(-x + Root[2 + 3*
#1 - 3*#1^2 + #1^3 & , 2, 0])/((-1 + 3^(1/3) + x)*(-1 + Root[2 + 3*#1 - 3*#
1^2 + #1^3 & , 2, 0]))]*(x - Root[2 + 3*#1 - 3*#1^2 + #1^3 & , 3, 0]))/(Sqr
t[-2 - x + 6*x^2 - 4*x^3 + x^4]*(-1 + Root[2 + 3*#1 - 3*#1^2 + #1^3 & , 3,
0])*Sqrt[-(((-1 + x)*(x - Root[2 + 3*#1 - 3*#1^2 + #1^3 & , 3, 0])*(-1 + 3^
(1/3) + Root[2 + 3*#1 - 3*#1^2 + #1^3 & , 3, 0]))/((-1 + 3^(1/3) + x)^2*(-1
 + Root[2 + 3*#1 - 3*#1^2 + #1^3 & , 3, 0])^2))])
```

IntegrateAlgebraic [A] time = 0.17, size = 36, normalized size = 1.00

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^2-2x+1}{\sqrt{x^4-4x^3+6x^2-x-2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/Sqrt[-2 - x + 6*x^2 - 4*x^3 + x^4],x]
[Out] (2*ArcTanh[(1 - 2*x + x^2)/Sqrt[-2 - x + 6*x^2 - 4*x^3 + x^4]])/3
```

fricas [A] time = 0.43, size = 43, normalized size = 1.19

$$\frac{1}{3}\log\left(2x^3-6x^2+2\sqrt{x^4-4x^3+6x^2-x-2}(x-1)+6x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^4-4*x^3+6*x^2-x-2)^(1/2),x, algorithm="fricas")
[Out] 1/3*log(2*x^3 - 6*x^2 + 2*sqrt(x^4 - 4*x^3 + 6*x^2 - x - 2)*(x - 1) + 6*x +
 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x-1}{\sqrt{x^4-4x^3+6x^2-x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^4-4*x^3+6*x^2-x-2)^(1/2),x, algorithm="giac")
[Out] integrate((x - 1)/sqrt(x^4 - 4*x^3 + 6*x^2 - x - 2), x)
```

maple [C] time = 0.43, size = 740, normalized size = 20.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(x^4-4*x^3+6*x^2-x-2)^(1/2),x)
[Out] -2/3*(-1/2*3^(1/3)-1/2*I*3^(5/6))*((3/2*3^(1/3)+1/2*I*3^(5/6))*(-1+x)/(1/2*
3^(1/3)+1/2*I*3^(5/6))/(x+3^(1/3)-1))^(1/2)*(x+3^(1/3)-1)^2*(-3^(1/3)*(x-1/
2*3^(1/3)+1/2*I*3^(5/6)-1)/(1/2*3^(1/3)-1/2*I*3^(5/6))/(x+3^(1/3)-1))^(1/2)
*(-3^(1/3)*(x-1/2*3^(1/3)-1/2*I*3^(5/6)-1)/(1/2*3^(1/3)+1/2*I*3^(5/6))/(x+3
^(1/3)-1))^(1/2)/(3/2*3^(1/3)+1/2*I*3^(5/6))*3^(2/3)/((-1+x)*(x+3^(1/3)-1)*
(x-1/2*3^(1/3)+1/2*I*3^(5/6)-1)*(x-1/2*3^(1/3)-1/2*I*3^(5/6)-1))^(1/2)*((-3
^(1/3)+1)*EllipticF(((3/2*3^(1/3)+1/2*I*3^(5/6))*(-1+x)/(1/2*3^(1/3)+1/2*I*
3^(5/6))/(x+3^(1/3)-1))^(1/2),((-3/2*3^(1/3)+1/2*I*3^(5/6))*(-1/2*3^(1/3)-1
/2*I*3^(5/6))/(-1/2*3^(1/3)+1/2*I*3^(5/6))/(-3/2*3^(1/3)-1/2*I*3^(5/6)))^(1
/2))+3^(1/3)*EllipticPi(((3/2*3^(1/3)+1/2*I*3^(5/6))*(-1+x)/(1/2*3^(1/3)+1/
2*I*3^(5/6))/(x+3^(1/3)-1))^(1/2),(1/2*3^(1/3)+1/2*I*3^(5/6))/(3/2*3^(1/3)+
1/2*I*3^(5/6)),((-3/2*3^(1/3)+1/2*I*3^(5/6))*(-1/2*3^(1/3)-1/2*I*3^(5/6))/(
-1/2*3^(1/3)+1/2*I*3^(5/6))/(-3/2*3^(1/3)-1/2*I*3^(5/6)))^(1/2)))+2/3*(-1/2
*3^(1/3)-1/2*I*3^(5/6))*((3/2*3^(1/3)+1/2*I*3^(5/6))*(-1+x)/(1/2*3^(1/3)+1/
2*I*3^(5/6))/(x+3^(1/3)-1))^(1/2)*(x+3^(1/3)-1)^2*(-3^(1/3)*(x-1/2*3^(1/3)+
1/2*I*3^(5/6)-1)/(1/2*3^(1/3)-1/2*I*3^(5/6))/(x+3^(1/3)-1))^(1/2)*(-3^(1/3)
*(x-1/2*3^(1/3)-1/2*I*3^(5/6)-1)/(1/2*3^(1/3)+1/2*I*3^(5/6))/(x+3^(1/3)-1))
^(1/2)/(3/2*3^(1/3)+1/2*I*3^(5/6))*3^(2/3)/((-1+x)*(x+3^(1/3)-1)*(x-1/2*3^(
1/3)+1/2*I*3^(5/6)-1)*(x-1/2*3^(1/3)-1/2*I*3^(5/6)-1))^(1/2)*EllipticF(((3/
2*3^(1/3)+1/2*I*3^(5/6))*(-1+x)/(1/2*3^(1/3)+1/2*I*3^(5/6))/(x+3^(1/3)-1))^
(1/2),((-3/2*3^(1/3)+1/2*I*3^(5/6))*(-1/2*3^(1/3)-1/2*I*3^(5/6))/(-1/2*3^(1
/3)+1/2*I*3^(5/6))/(-3/2*3^(1/3)-1/2*I*3^(5/6)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x^4-4\,x^3+6\,x^2-x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^4-4*x^3+6*x^2-x-2)^(1/2),x, algorithm="maxima")
[Out] integrate((x - 1)/sqrt(x^4 - 4*x^3 + 6*x^2 - x - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x-1}{\sqrt{x^4-4\,x^3+6\,x^2-x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(6*x^2 - x - 4*x^3 + x^4 - 2)^(1/2),x)
[Out] int((x - 1)/(6*x^2 - x - 4*x^3 + x^4 - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{(x-1)\left(x^3-3x^2+3x+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**4-4*x**3+6*x**2-x-2)**(1/2),x)
[Out] Integral((x - 1)/sqrt((x - 1)*(x**3 - 3*x**2 + 3*x + 2)), x)
```

3.453 $\int \frac{\sqrt{-1+x^6}}{x^{13}} \, dx$

Optimal. Leaf size=36

$$\frac{1}{24}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}\,(x^6-2)}{24x^{12}}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {266, 47, 51, 63, 203}

$$\frac{\sqrt{x^6-1}}{24x^6}+\frac{1}{24}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{\sqrt{x^6-1}}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^6]/x^13,x]
```

```
[Out] -1/12*Sqrt[-1 + x^6]/x^12 + Sqrt[-1 + x^6]/(24*x^6) + ArcTan[Sqrt[-1 + x^6]
]/24
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+x^6}}{x^{13}}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{\sqrt{-1+x}}{x^3}\,dx, x, x^6\right)\\ &= -\frac{\sqrt{-1+x^6}}{12x^{12}} + \frac{1}{24}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx, x, x^6\right)\\ &= -\frac{\sqrt{-1+x^6}}{12x^{12}} + \frac{\sqrt{-1+x^6}}{24x^6} + \frac{1}{48}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right)\\ &= -\frac{\sqrt{-1+x^6}}{12x^{12}} + \frac{\sqrt{-1+x^6}}{24x^6} + \frac{1}{24}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^6}\right)\\ &= -\frac{\sqrt{-1+x^6}}{12x^{12}} + \frac{\sqrt{-1+x^6}}{24x^6} + \frac{1}{24}\tan^{-1}\left(\sqrt{-1+x^6}\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 28, normalized size = 0.78

$$\frac{1}{9}\left(x^6-1\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^6]/x^13,x]
[Out] ((-1 + x^6)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 - x^6])/9
```

IntegrateAlgebraic [A] time = 0.03, size = 36, normalized size = 1.00

$$\frac{1}{24}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}\left(x^6-2\right)}{24x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x^6]/x^13,x]
[Out] ((-2 + x^6)*Sqrt[-1 + x^6])/(24*x^12) + ArcTan[Sqrt[-1 + x^6]]/24
```

fricas [A] time = 0.40, size = 31, normalized size = 0.86

$$\frac{x^{12}\arctan\left(\sqrt{x^6-1}\right)+\sqrt{x^6-1}\left(x^6-2\right)}{24\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^13,x, algorithm="fricas")
[Out] 1/24*(x^12*arctan(sqrt(x^6 - 1)) + sqrt(x^6 - 1)*(x^6 - 2))/x^12
```

giac [B] time = 0.21, size = 63, normalized size = 1.75

$$\frac{1}{96}\pi+\frac{\sqrt{x^6-1}-\frac{1}{\sqrt{x^6-1}}}{24\left(\left(\sqrt{x^6-1}-\frac{1}{\sqrt{x^6-1}}\right)^2+4\right)}+\frac{1}{48}\arctan\left(\frac{x^6-2}{2\sqrt{x^6-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^13,x, algorithm="giac")
```

```
[Out] 1/96*pi + 1/24*(sqrt(x^6 - 1) - 1/sqrt(x^6 - 1))/((sqrt(x^6 - 1) - 1/sqrt(x
^6 - 1))^2 + 4) + 1/48*arctan(1/2*(x^6 - 2)/sqrt(x^6 - 1))
```

maple [A] time = 0.03, size = 30, normalized size = 0.83

$$\frac{x^{12}-3x^{6}+2}{24x^{12}\sqrt{x^{6}-1}}-\frac{\arcsin\left(\frac{1}{x^{3}}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/2)/x^13,x)
[Out] 1/24*(x^12-3*x^6+2)/x^12/(x^6-1)^(1/2)-1/24*arcsin(1/x^3)
```

maxima [A] time = 0.74, size = 46, normalized size = 1.28

$$\frac{\left(x^{6}-1\right)^{\frac{3}{2}}-\sqrt{x^{6}-1}}{24\left(2x^{6}+\left(x^{6}-1\right)^{2}-1\right)}+\frac{1}{24}\arctan\left(\sqrt{x^{6}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^13,x, algorithm="maxima")
[Out] 1/24*((x^6 - 1)^(3/2) - sqrt(x^6 - 1))/(2*x^6 + (x^6 - 1)^2 - 1) + 1/24*arc
tan(sqrt(x^6 - 1))
```

mupad [B] time = 0.44, size = 35, normalized size = 0.97

$$\frac{\operatorname{atan}\left(\sqrt{x^{6}-1}\right)}{24}-\frac{\frac{\sqrt{x^{6}-1}}{24}-\frac{\left(x^{6}-1\right)^{3/2}}{24}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)^(1/2)/x^13,x)
[Out] atan((x^6 - 1)^(1/2))/24 - ((x^6 - 1)^(1/2)/24 - (x^6 - 1)^(3/2)/24)/x^12
```

sympy [A] time = 2.58, size = 124, normalized size = 3.44

$$\begin{cases} \frac{i\operatorname{acosh}\left(\frac{1}{x^{3}}\right)}{24}-\frac{i}{24x^{3}\sqrt{-1+\frac{1}{x^{6}}}}+\frac{i}{8x^{9}\sqrt{-1+\frac{1}{x^{6}}}}-\frac{i}{12x^{15}\sqrt{-1+\frac{1}{x^{6}}}} & \text{for}\ \frac{1}{|x^{6}|}>1 \\ -\frac{\operatorname{asin}\left(\frac{1}{x^{3}}\right)}{24}+\frac{1}{24x^{3}\sqrt{1-\frac{1}{x^{6}}}}-\frac{1}{8x^{9}\sqrt{1-\frac{1}{x^{6}}}}+\frac{1}{12x^{15}\sqrt{1-\frac{1}{x^{6}}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/2)/x**13,x)
[Out] Piecewise((I*acosh(x**(-3))/24 - I/(24*x**3*sqrt(-1 + x**(-6))) + I/(8*x**9
*sqrt(-1 + x**(-6))) - I/(12*x**15*sqrt(-1 + x**(-6))), 1/Abs(x**6) > 1), (
-asin(x**(-3))/24 + 1/(24*x**3*sqrt(1 - 1/x**6)) - 1/(8*x**9*sqrt(1 - 1/x**
6)) + 1/(12*x**15*sqrt(1 - 1/x**6)), True))
```

3.454 $\int \frac{-1-2x^2+2x^4}{\left(2-3x^2+x^4\right)\sqrt{1+x^6}}\,dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{x\sqrt{x^6+1}}{\sqrt{2}\left(x^4-x^2+1\right)}\right)}{\sqrt{2}}$$

Rubi [F] time = 0.69, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1-2x^2+2x^4}{\left(2-3x^2+x^4\right)\sqrt{1+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 - 2*x^2 + 2*x^4)/((2 - 3*x^2 + x^4)*Sqrt[1 + x^6]),x]
```

```
[Out] (x*(1 + x^2)*Sqrt[(1 - x^2 + x^4)/(1 + (1 + Sqrt[3])*x^2)^2]*EllipticF[ArcC
os[(1 + (1 - Sqrt[3])*x^2)/(1 + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(3^(
1/4)*Sqrt[(x^2*(1 + x^2))/(1 + (1 + Sqrt[3])*x^2)^2]*Sqrt[1 + x^6]) - (3*De
fer[Int][1/((Sqrt[2] - x)*Sqrt[1 + x^6]), x])/(2*Sqrt[2]) + Defer[Int][1/((
-1 + x)*Sqrt[1 + x^6]), x]/2 - Defer[Int][1/((1 + x)*Sqrt[1 + x^6]), x]/2 -
 (3*Defer[Int][1/((Sqrt[2] + x)*Sqrt[1 + x^6]), x])/(2*Sqrt[2])
```

Rubi steps

$$\int \frac{-1-2x^2+2x^4}{\left(2-3x^2+x^4\right)\sqrt{1+x^6}}\,dx=\int \left(\frac{2}{\sqrt{1+x^6}}-\frac{5-4x^2}{\left(2-3x^2+x^4\right)\sqrt{1+x^6}}\right)dx$$

$$=2\int \frac{1}{\sqrt{1+x^6}}\,dx-\int \frac{5-4x^2}{\left(2-3x^2+x^4\right)\sqrt{1+x^6}}\,dx$$

$$=\frac{x\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x^2}{1+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x^2\left(1+x^2\right)}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1+x^6}}-\int \Bigg(-\frac{}{(-4+2}$$

$$=\frac{x\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x^2}{1+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x^2\left(1+x^2\right)}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1+x^6}}+2\int \frac{}{(-2+2}$$

$$=\frac{x\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x^2}{1+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x^2\left(1+x^2\right)}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1+x^6}}+2\int \Bigg(\frac{}{4(-1+}$$

$$=\frac{x\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x^2}{1+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x^2\left(1+x^2\right)}{\left(1+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1+x^6}}+\frac{1}{2}\int \frac{}{(-1+x}$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{-1-2x^2+2x^4}{\left(2-3x^2+x^4\right)\sqrt{1+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 - 2*x^2 + 2*x^4)/((2 - 3*x^2 + x^4)*Sqrt[1 + x^6]),x]
[Out] Integrate[(-1 - 2*x^2 + 2*x^4)/((2 - 3*x^2 + x^4)*Sqrt[1 + x^6]), x]
```

IntegrateAlgebraic [A] time = 15.21, size = 36, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{x\sqrt{x^6+1}}{\sqrt{2}\left(x^4-x^2+1\right)}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 2*x^2 + 2*x^4)/((2 - 3*x^2 + x^4)*Sqrt[1 + x^6]),x
]
[Out] -(ArcTanh[(x*Sqrt[1 + x^6])/(Sqrt[2]*(1 - x^2 + x^4))]/Sqrt[2])
```

fricas [B] time = 0.47, size = 78, normalized size = 2.17

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{17\,x^8-6\,x^6+13\,x^4-4\,\sqrt{2}\,\sqrt{x^6+1}\left(3\,x^5-x^3+2\,x\right)+4\,x^2+4}{x^8-6\,x^6+13\,x^4-12\,x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x^2-1)/(x^4-3*x^2+2)/(x^6+1)^(1/2),x, algorithm="fricas"
)
[Out] 1/8*sqrt(2)*log(-(17*x^8 - 6*x^6 + 13*x^4 - 4*sqrt(2)*sqrt(x^6 + 1)*(3*x^5
- x^3 + 2*x) + 4*x^2 + 4)/(x^8 - 6*x^6 + 13*x^4 - 12*x^2 + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^4-2\,x^2-1}{\sqrt{x^6+1}\left(x^4-3\,x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x^2-1)/(x^4-3*x^2+2)/(x^6+1)^(1/2),x, algorithm="giac")
[Out] integrate((2*x^4 - 2*x^2 - 1)/(sqrt(x^6 + 1)*(x^4 - 3*x^2 + 2)), x)
```

maple [C] time = 0.44, size = 70, normalized size = 1.94

$$-\frac{\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{3\,\mathrm{RootOf}\left(\_Z^2-2\right)x^4-\mathrm{RootOf}\left(\_Z^2-2\right)x^2+4\sqrt{x^6+1}\,x+2\,\mathrm{RootOf}\left(\_Z^2-2\right)}{(1+x)\left(x^2-2\right)(-1+x)}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-2*x^2-1)/(x^4-3*x^2+2)/(x^6+1)^(1/2),x)
[Out] -1/4*RootOf(_Z^2-2)*ln(-(3*RootOf(_Z^2-2)*x^4-RootOf(_Z^2-2)*x^2+4*(x^6+1)^
(1/2)*x+2*RootOf(_Z^2-2))/(1+x)/(x^2-2)/(-1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^4 - 2\,x^2 - 1}{\sqrt{x^6 + 1}\left(x^4 - 3\,x^2 + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x^2-1)/(x^4-3*x^2+2)/(x^6+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((2*x^4 - 2*x^2 - 1)/(sqrt(x^6 + 1)*(x^4 - 3*x^2 + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{-2\,x^4 + 2\,x^2 + 1}{\sqrt{x^6 + 1}\,\left(x^4 - 3\,x^2 + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 2*x^4 + 1)/((x^6 + 1)^(1/2)*(x^4 - 3*x^2 + 2)),x)
```

```
[Out] int(-(2*x^2 - 2*x^4 + 1)/((x^6 + 1)^(1/2)*(x^4 - 3*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^4 - 2x^2 - 1}{\sqrt{\left(x^2 + 1\right)\left(x^4 - x^2 + 1\right)}\,(x - 1)\,(x + 1)\left(x^2 - 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-2*x**2-1)/(x**4-3*x**2+2)/(x**6+1)**(1/2),x)
```

```
[Out] Integral((2*x**4 - 2*x**2 - 1)/(sqrt((x**2 + 1)*(x**4 - x**2 + 1))*(x - 1)*
(x + 1)*(x**2 - 2)), x)
```

3.455 $\int \frac{-1+kx}{(1+kx)\sqrt{(1-x)x(1-k^2x)}}\,dx$

Optimal. Leaf size=37

$$-\frac{2\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^3+(-k^2-1)x^2+x}}\right)}{k+1}$$

Rubi [C] time = 0.64, antiderivative size = 154, normalized size of antiderivative = 4.16, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6718, 1607, 168, 538, 537, 115}

$$\frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{4\sqrt{1-x}\sqrt{x}\sqrt{\frac{k^2(1-x)}{1-k^2}+1}\,\Pi\left(\frac{k}{k+1};\sin^{-1}\left(\sqrt{1-x}\right)|-\frac{k^2}{1-k^2}\right)}{(k+1)\sqrt{(1-x)x\left(1-k^2x\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + k*x)/((1 + k*x)*Sqrt[(1 - x)*x*(1 - k^2*x)]),x]
```

```
[Out] (2*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticF[ArcSin[Sqrt[x]], k^2])/Sqr
t[(1 - x)*x*(1 - k^2*x)] + (4*Sqrt[1 + (k^2*(1 - x))/(1 - k^2)]*Sqrt[1 - x]
*Sqrt[x]*EllipticPi[k/(1 + k), ArcSin[Sqrt[1 - x]], -(k^2/(1 - k^2))])/((1
+ k)*Sqrt[(1 - x)*x*(1 - k^2*x)])
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
```

```
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rubi steps

$$\int \frac{-1+kx}{(1+kx)\sqrt{(1-x)x\left(1-k^2x\right)}}\,dx = \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{-1+kx}{\sqrt{1-x}\sqrt{x}(1+kx)\sqrt{1-k^2x}}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int}{\sqrt{(1-x)x\left(1\right.}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(4\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\text{Subs}}{\sqrt{(}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(4\sqrt{1+\frac{k^2(-1+x)}{-1+k^2}}\sqrt{1-x}\sqrt{x}\right)}{\sqrt{}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{4\sqrt{1+\frac{k^2(1-x)}{1-k^2}}\sqrt{1-x}\sqrt{x}\,\Pi\left(\right.}{(1+k)\sqrt{(1-x}}$$

Mathematica [C] time = 0.85, size = 114, normalized size = 3.08

$$\frac{2i\sqrt{\frac{1}{x-1}+1}\,(x-1)^{3/2}\sqrt{\frac{1-\frac{1}{k^2}}{x-1}+1}\left((k-1)F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+2\Pi\left(1+\frac{1}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)}{(k+1)\sqrt{(x-1)x\left(k^2x-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + k*x)/((1 + k*x)*Sqrt[(1 - x)*x*(1 - k^2*x)]),x]
```

```
[Out] ((2*I)*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (1 - k^(-2))/(-1 + x)]*(-1 + x)^(3/
2)*((-1 + k)*EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] + 2*EllipticP
i[1 + k^(-1), I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)]))/((1 + k)*Sqrt[(-1 +
x)*x*(-1 + k^2*x)])
```

IntegrateAlgebraic [A] time = 0.18, size = 37, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}\right)}{k+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + k*x)/((1 + k*x)*Sqrt[(1 - x)*x*(1 - k^2*x)]),x]
```

```
[Out] (-2*ArcTan[((1 + k)*x)/Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3]])/(1 + k)
```

fricas [B] time = 0.46, size = 81, normalized size = 2.19

$$\frac{\arctan\left(\frac{\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\left(k^2x^2-2\left(k^2+k+1\right)x+1\right)}{2\left(\left(k^3+k^2\right)x^3-\left(k^3+k^2+k+1\right)x^2+(k+1)x\right)}\right)}{k+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x-1)/(k*x+1)/((1-x)*x*(-k^2*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2 - 2*(k^2 + k + 1)*x +
 1)/((k^3 + k^2)*x^3 - (k^3 + k^2 + k + 1)*x^2 + (k + 1)*x))/(k + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx-1}{\sqrt{\left(k^2x-1\right)(x-1)x}\,(kx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x-1)/(k*x+1)/((1-x)*x*(-k^2*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((k*x - 1)/(sqrt((k^2*x - 1)*(x - 1)*x)*(k*x + 1)), x)
```

maple [C] time = 0.06, size = 206, normalized size = 5.57

$$-\frac{2\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\mathrm{EllipticF}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^2\sqrt{k^2x^3-k^2x^2-x^2+x}}+\frac{4\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\mathrm{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}+\frac{1}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^3\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}+\frac{1}{k}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x-1)/(k*x+1)/((1-x)*x*(-k^2*x+1))^(1/2),x)
```

```
[Out] -2/k^2*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x
^3-k^2*x^2-x^2+x)^(1/2)*EllipticF((-(x-1/k^2)*k^2)^(1/2),(1/k^2/(1/k^2-1))^
(1/2))+4/k^3*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/
(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/k)*EllipticPi((-(x-1/k^2)*k^2)^(1/2)
,1/k^2/(1/k^2+1/k),(1/k^2/(1/k^2-1))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx-1}{\sqrt{\left(k^2x-1\right)(x-1)x}\,(kx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x-1)/(k*x+1)/((1-x)*x*(-k^2*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((k*x - 1)/(sqrt((k^2*x - 1)*(x - 1)*x)*(k*x + 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x - 1)/((k*x + 1)*(x*(k^2*x - 1)*(x - 1))^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx-1}{\sqrt{x\,(x-1)\left(k^2x-1\right)}\,(kx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x-1)/(k*x+1)/((1-x)*x*(-k**2*x+1))**(1/2),x)
```

```
[Out] Integral((k*x - 1)/(sqrt(x*(x - 1)*(k**2*x - 1))*(k*x + 1)), x)
```

3.456 $\int \frac{1+kx}{(-1+kx)\sqrt{(1-x)x(1-k^2x)}}\,dx$

Optimal. Leaf size=37

$$-\frac{2\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^3+(-k^2-1)x^2+x}}\right)}{k-1}$$

Rubi [C] time = 0.55, antiderivative size = 159, normalized size of antiderivative = 4.30, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {6718, 1607, 168, 538, 537, 115}

$$\frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{4\sqrt{1-x}\sqrt{x}\sqrt{\frac{k^2(1-x)}{1-k^2}+1}\,\Pi\left(-\frac{k}{1-k};\sin^{-1}\left(\sqrt{1-x}\right)|-\frac{k^2}{1-k^2}\right)}{(1-k)\sqrt{(1-x)x\left(1-k^2x\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + k*x)/((-1 + k*x)*Sqrt[(1 - x)*x*(1 - k^2*x)]),x]
```

```
[Out] (2*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticF[ArcSin[Sqrt[x]], k^2])/Sqr
t[(1 - x)*x*(1 - k^2*x)] + (4*Sqrt[1 + (k^2*(1 - x))/(1 - k^2)]*Sqrt[1 - x]
*Sqrt[x]*EllipticPi[-(k/(1 - k)), ArcSin[Sqrt[1 - x]], -(k^2/(1 - k^2))])/(
(1 - k)*Sqrt[(1 - x)*x*(1 - k^2*x)])
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
```

```
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rubi steps

$$\int \frac{1+kx}{(-1+kx)\sqrt{(1-x)x\left(1-k^2x\right)}}\,dx = \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{1+kx}{\sqrt{1-x}\sqrt{x}(-1+kx)\sqrt{1-k^2x}}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int}{\sqrt{(1-x)x}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(4\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\mathrm{Sub}}{}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(4\sqrt{1+\frac{k^2(-1+x)}{-1+k^2}}\sqrt{1-x}\sqrt{x}\right)}{}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{4\sqrt{1+\frac{k^2(1-x)}{1-k^2}}\sqrt{1-x}\sqrt{x}\,\Pi}{(1-k)\sqrt{(1-}}$$

Mathematica [C] time = 0.75, size = 116, normalized size = 3.14

$$\frac{2i\sqrt{\frac{1}{x-1}+1}\,(x-1)^{3/2}\sqrt{\frac{1-\frac{1}{k^2}}{x-1}+1}\left((k+1)F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)-2\Pi\left(\frac{k-1}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)}{(k-1)\sqrt{(x-1)x\left(k^2x-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + k*x)/((-1 + k*x)*Sqrt[(1 - x)*x*(1 - k^2*x)]),x]
```

```
[Out] ((2*I)*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (1 - k^(-2))/(-1 + x)]*(-1 + x)^(3/
2)*((1 + k)*EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] - 2*EllipticPi
[(-1 + k)/k, I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)]))/((-1 + k)*Sqrt[(-1 +
x)*x*(-1 + k^2*x)])
```

IntegrateAlgebraic [A] time = 0.17, size = 37, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}\right)}{k-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + k*x)/((-1 + k*x)*Sqrt[(1 - x)*x*(1 - k^2*x)]),x]
```

```
[Out] (-2*ArcTan[((-1 + k)*x)/Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3]])/(-1 + k)
```

fricas [B] time = 0.45, size = 87, normalized size = 2.35

$$\frac{\arctan\left(\frac{\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\left(k^2x^2-2\left(k^2-k+1\right)x+1\right)}{2\left(\left(k^3-k^2\right)x^3-\left(k^3-k^2+k-1\right)x^2+(k-1)x\right)}\right)}{k-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x+1)/(k*x-1)/((1-x)*x*(-k^2*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2 - 2*(k^2 - k + 1)*x +
 1)/((k^3 - k^2)*x^3 - (k^3 - k^2 + k - 1)*x^2 + (k - 1)*x))/(k - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx+1}{\sqrt{\left(k^2x-1\right)(x-1)x}\,(kx-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x+1)/(k*x-1)/((1-x)*x*(-k^2*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((k*x + 1)/(sqrt((k^2*x - 1)*(x - 1)*x)*(k*x - 1)), x)
```

maple [C] time = 0.02, size = 210, normalized size = 5.68

$$-\frac{2\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\operatorname{EllipticF}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^2\sqrt{k^2x^3-k^2x^2-x^2+x}}-\frac{4\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\operatorname{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}-\frac{1}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^3\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}-\frac{1}{k}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x+1)/(k*x-1)/((1-x)*x*(-k^2*x+1))^(1/2),x)
```

```
[Out] -2/k^2*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x
^3-k^2*x^2-x^2+x)^(1/2)*EllipticF((-(x-1/k^2)*k^2)^(1/2),(1/k^2/(1/k^2-1))^
(1/2))-4/k^3*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/
(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2-1/k)*EllipticPi((-(x-1/k^2)*k^2)^(1/2)
,1/k^2/(1/k^2-1/k),(1/k^2/(1/k^2-1))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx+1}{\sqrt{\left(k^2x-1\right)(x-1)x}\,(kx-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x+1)/(k*x-1)/((1-x)*x*(-k^2*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((k*x + 1)/(sqrt((k^2*x - 1)*(x - 1)*x)*(k*x - 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x + 1)/((k*x - 1)*(x*(k^2*x - 1)*(x - 1))^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx+1}{\sqrt{x(x-1)\left(k^2x-1\right)}\,(kx-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x+1)/(k*x-1)/((1-x)*x*(-k**2*x+1))**(1/2),x)
```

```
[Out] Integral((k*x + 1)/(sqrt(x*(x - 1)*(k**2*x - 1))*(k*x - 1)), x)
```

3.457 $\int \frac{(-1+x^2)\sqrt{1+x^4}}{x^2(1+x^2)}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{x^4+1}}{x}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)$$

Rubi [A] time = 0.33, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {6725, 277, 305, 220, 1196, 1209, 1211, 1699, 203}

$$\frac{\sqrt{x^4+1}}{x}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^2)*Sqrt[1 + x^4])/(x^2*(1 + x^2)),x]
```

```
[Out] Sqrt[1 + x^4]/x + Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-1+x^2\right)\sqrt{1+x^4}}{x^2\left(1+x^2\right)}\,dx &= \int \left(-\frac{\sqrt{1+x^4}}{x^2}+\frac{2\sqrt{1+x^4}}{1+x^2}\right)dx\\
&= 2\int \frac{\sqrt{1+x^4}}{1+x^2}\,dx-\int \frac{\sqrt{1+x^4}}{x^2}\,dx\\
&= \frac{\sqrt{1+x^4}}{x}-2\int \frac{x^2}{\sqrt{1+x^4}}\,dx-2\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx+4\int \frac{1}{\left(1+x^2\right)\sqrt{1+x^4}}\,dx\\
&= \frac{\sqrt{1+x^4}}{x}+\frac{2x\sqrt{1+x^4}}{1+x^2}-\frac{2\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{1+x^4}}+2\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx\\
&= \frac{\sqrt{1+x^4}}{x}+2\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\frac{x}{\sqrt{1+x^4}}\right)\\
&= \frac{\sqrt{1+x^4}}{x}+\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.12, size = 72, normalized size = 1.95

$$
\frac{1}{\sqrt{x^4+1}\,x}+\frac{x^3}{\sqrt{x^4+1}}+2\sqrt[4]{-1}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right)-4\sqrt[4]{-1}\,\Pi\left(-i;i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)*Sqrt[1 + x^4])/(x^2*(1 + x^2)),x]
```

```
[Out] 1/(x*Sqrt[1 + x^4]) + x^3/Sqrt[1 + x^4] + 2*(-1)^(1/4)*EllipticF[I*ArcSinh[
(-1)^(1/4)*x], -1] - 4*(-1)^(1/4)*EllipticPi[-I, I*ArcSinh[(-1)^(1/4)*x], -
1]
```

IntegrateAlgebraic [A] time = 0.30, size = 37, normalized size = 1.00

$$\frac{\sqrt{x^4+1}}{x}+\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*Sqrt[1 + x^4])/(x^2*(1 + x^2)),x]
[Out] Sqrt[1 + x^4]/x + Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]
```

fricas [A] time = 0.46, size = 30, normalized size = 0.81

$$\frac{\sqrt{2}\,x\arctan\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)+\sqrt{x^4+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+1)^(1/2)/x^2/(x^2+1),x, algorithm="fricas")
[Out] (sqrt(2)*x*arctan(sqrt(2)*x/sqrt(x^4 + 1)) + sqrt(x^4 + 1))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^4+1}\,\left(x^2-1\right)}{\left(x^2+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+1)^(1/2)/x^2/(x^2+1),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 1)*(x^2 - 1)/((x^2 + 1)*x^2), x)
```

maple [C] time = 0.04, size = 326, normalized size = 8.81

$$-\frac{2\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{2i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{2i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{4(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticPi}\left((-1)^{\frac{1}{4}}x,i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^4+1}}+\frac{\sqrt{x^4+1}}{x}-\frac{2i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\operatorname{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(x^4+1)^(1/2)/x^2/(x^2+1),x)
[Out] -2/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+2*I/(1/2*2^(1/2)+1/2*I*2^(1/2)
)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticF(x*(1/2*2^(1/2)+1/
2*I*2^(1/2)),I)-2*I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(
1/2)/(x^4+1)^(1/2)*EllipticE(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-4*(-1)^(3/4)*
(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I
)^(1/2)/(-1)^(1/4))+(x^4+1)^(1/2)/x-2*I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^
2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*(EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1
/2)),I)-EllipticE(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^4+1}\,\left(x^2-1\right)}{\left(x^2+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+1)^(1/2)/x^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 1)*(x^2 - 1)/((x^2 + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(x^2-1\right)\sqrt{x^4+1}}{x^2\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*(x^4 + 1)^(1/2))/(x^2*(x^2 + 1)),x)
[Out] int(((x^2 - 1)*(x^4 + 1)^(1/2))/(x^2*(x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\sqrt{x^4+1}}{x^2\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**4+1)**(1/2)/x**2/(x**2+1),x)
[Out] Integral((x - 1)*(x + 1)*sqrt(x**4 + 1)/(x**2*(x**2 + 1)), x)
```

3.458 $\int \frac{1+x^3}{x^6\left(-1+x^3\right)\sqrt[4]{-x+x^4}}\,dx$

Optimal. Leaf size=37

$$-\frac{4\left(x^4-x\right)^{3/4}\left(20x^6-5x^3-1\right)}{21x^6\left(x^3-1\right)}$$

Rubi [A] time = 0.16, antiderivative size = 53, normalized size of antiderivative = 1.43, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2056, 453, 271, 264}

$$-\frac{80x}{21\sqrt[4]{x^4-x}}+\frac{4}{21\sqrt[4]{x^4-x}\,x^5}+\frac{20}{21\sqrt[4]{x^4-x}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(x^6*(-1 + x^3)*(-x + x^4)^(1/4)),x]
```

```
[Out] 4/(21*x^5*(-x + x^4)^(1/4)) + 20/(21*x^2*(-x + x^4)^(1/4)) - (80*x)/(21*(-x
 + x^4)^(1/4))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1+x^3}{x^6\left(-1+x^3\right)\sqrt[4]{-x+x^4}}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\int \frac{1+x^3}{x^{25/4}\left(-1+x^3\right)^{5/4}}\,dx}{\sqrt[4]{-x+x^4}}$$

$$= \frac{4}{21x^5\sqrt[4]{-x+x^4}} + \frac{\left(15\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\int \frac{1}{x^{13/4}\left(-1+x^3\right)^{5/4}}\,dx}{7\sqrt[4]{-x+x^4}}$$

$$= \frac{4}{21x^5\sqrt[4]{-x+x^4}} + \frac{20}{21x^2\sqrt[4]{-x+x^4}} + \frac{\left(20\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\int \frac{1}{\sqrt[4]{x}\left(-1+x^3\right)^{5/4}}\,dx}{7\sqrt[4]{-x+x^4}}$$

$$= \frac{4}{21x^5\sqrt[4]{-x+x^4}} + \frac{20}{21x^2\sqrt[4]{-x+x^4}} - \frac{80x}{21\sqrt[4]{-x+x^4}}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.81

$$\frac{-80x^6+20x^3+4}{21x^5\sqrt[4]{x\left(x^3-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(x^6*(-1 + x^3)*(-x + x^4)^(1/4)),x]
[Out] (4 + 20*x^3 - 80*x^6)/(21*x^5*(x*(-1 + x^3))^(1/4))
```

IntegrateAlgebraic [A] time = 0.42, size = 37, normalized size = 1.00

$$-\frac{4\left(x^4-x\right)^{3/4}\left(20x^6-5x^3-1\right)}{21x^6\left(x^3-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)/(x^6*(-1 + x^3)*(-x + x^4)^(1/4)),x]
[Out] (-4*(-x + x^4)^(3/4)*(-1 - 5*x^3 + 20*x^6))/(21*x^6*(-1 + x^3))
```

fricas [A] time = 0.40, size = 34, normalized size = 0.92

$$-\frac{4\left(20\,x^6-5\,x^3-1\right)\left(x^4-x\right)^{\frac{3}{4}}}{21\left(x^9-x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x^6/(x^3-1)/(x^4-x)^(1/4),x, algorithm="fricas")
[Out] -4/21*(20*x^6 - 5*x^3 - 1)*(x^4 - x)^(3/4)/(x^9 - x^6)
```

giac [A] time = 0.70, size = 34, normalized size = 0.92

$$-\frac{4}{21}\left(-\frac{1}{x^3}+1\right)^{\frac{7}{4}} + \frac{4}{3}\left(-\frac{1}{x^3}+1\right)^{\frac{3}{4}} + \frac{8}{3\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x^6/(x^3-1)/(x^4-x)^(1/4),x, algorithm="giac")
[Out] -4/21*(-1/x^3 + 1)^(7/4) + 4/3*(-1/x^3 + 1)^(3/4) + 8/3/(-1/x^3 + 1)^(1/4)
```

maple [A] time = 0.01, size = 27, normalized size = 0.73

$$-\frac{4\left(20x^{6}-5x^{3}-1\right)}{21\left(x^{4}-x\right)^{\frac{1}{4}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/x^6/(x^3-1)/(x^4-x)^(1/4),x)
[Out] -4/21*(20*x^6-5*x^3-1)/(x^4-x)^(1/4)/x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}+1}{\left(x^{4}-x\right)^{\frac{1}{4}}\left(x^{3}-1\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x^6/(x^3-1)/(x^4-x)^(1/4),x, algorithm="maxima")
[Out] integrate((x^3 + 1)/((x^4 - x)^(1/4)*(x^3 - 1)*x^6), x)
```

mupad [B] time = 0.33, size = 33, normalized size = 0.89

$$\frac{4\left(x^{4}-x\right)^{3/4}\left(-20\,x^{6}+5\,x^{3}+1\right)}{21\,x^{6}\,\left(x^{3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/(x^6*(x^4 - x)^(1/4)*(x^3 - 1)),x)
[Out] (4*(x^4 - x)^(3/4)*(5*x^3 - 20*x^6 + 1))/(21*x^6*(x^3 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)\left(x^{2}-x+1\right)}{x^{6}\sqrt[4]{x(x-1)\left(x^{2}+x+1\right)}\,(x-1)\left(x^{2}+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/x**6/(x**3-1)/(x**4-x)**(1/4),x)
[Out] Integral((x + 1)*(x**2 - x + 1)/(x**6*(x*(x - 1)*(x**2 + x + 1))**(1/4)*(x
- 1)*(x**2 + x + 1)), x)
```

3.459 $\int \sqrt{-x+x^4}\,dx$

Optimal. Leaf size=37

$$\frac{1}{3}x\sqrt{x^4-x}-\frac{1}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2004, 2029, 206}

$$\frac{1}{3}x\sqrt{x^4-x}-\frac{1}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-x + x^4],x]
```

```
[Out] (x*Sqrt[-x + x^4])/3 - ArcTanh[x^2/Sqrt[-x + x^4]]/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{-x+x^4}\,dx &= \frac{1}{3}x\sqrt{-x+x^4}-\frac{1}{2}\int \frac{x}{\sqrt{-x+x^4}}\,dx\\
&= \frac{1}{3}x\sqrt{-x+x^4}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x^2}{\sqrt{-x+x^4}}\right)\\
&= \frac{1}{3}x\sqrt{-x+x^4}-\frac{1}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{-x+x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.19

$$\frac{\sqrt{x\left(x^3-1\right)}\left(x^{3/2}+\frac{\sin^{-1}\left(x^{3/2}\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x + x^4],x]
```

[Out] (Sqrt[x*(-1 + x^3)]*(x^(3/2) + ArcSin[x^(3/2)]/Sqrt[1 - x^3]))/(3*Sqrt[x])

IntegrateAlgebraic [A] time = 0.39, size = 37, normalized size = 1.00

$$\frac{1}{3}x\sqrt{x^4-x}-\frac{1}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-x + x^4],x]

[Out] (x*Sqrt[-x + x^4])/3 - ArcTanh[x^2/Sqrt[-x + x^4]]/3

fricas [A] time = 0.42, size = 35, normalized size = 0.95

$$\frac{1}{3}\sqrt{x^4-x}\,x+\frac{1}{6}\log\left(2x^3-2\sqrt{x^4-x}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x)^(1/2),x, algorithm="fricas")

[Out] 1/3*sqrt(x^4 - x)*x + 1/6*log(2*x^3 - 2*sqrt(x^4 - x)*x - 1)

giac [A] time = 0.32, size = 42, normalized size = 1.14

$$\frac{1}{3}\sqrt{x^4-x}\,x-\frac{1}{6}\log\left(\sqrt{-\frac{1}{x^3}+1}+1\right)+\frac{1}{6}\log\left(\left|\sqrt{-\frac{1}{x^3}+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x)^(1/2),x, algorithm="giac")

[Out] 1/3*sqrt(x^4 - x)*x - 1/6*log(sqrt(-1/x^3 + 1) + 1) + 1/6*log(abs(sqrt(-1/x^3 + 1) - 1))

maple [C] time = 0.22, size = 301, normalized size = 8.14

$$\frac{x\sqrt{x^4-x}}{3}-\frac{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}}\,(-1+x)^2\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(-1+x)}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)-\mathrm{EllipticPi}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\frac{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(-1+x)\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-x)^(1/2),x)

[Out] 1/3*x*(x^4-x)^(1/2)-(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2))/(-1+x))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2)/(-3/2+1/2*I*3^(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/2)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-EllipticPi(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),(-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^4-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x)^(1/2),x, algorithm="maxima")

```
[Out] integrate(sqrt(x^4 - x), x)
```

mupad [B] time = 0.30, size = 29, normalized size = 0.78

$$\frac{2\,x\,\sqrt{x^4-x}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{2};\ \frac{3}{2};\ x^3\right)}{3\,\sqrt{1-x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x)^(1/2),x)
[Out] (2*x*(x^4 - x)^(1/2)*hypergeom([-1/2, 1/2], 3/2, x^3))/(3*(1 - x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x)**(1/2),x)
[Out] Integral(sqrt(x**4 - x), x)
```

3.460 $\int \frac{1}{x^8 \sqrt[4]{-x^2+x^4}} \, dx$

Optimal. Leaf size=37

$$\frac{2 \left(x^4-x^2\right)^{3/4} \left(128 x^6+96 x^4+84 x^2+77\right)}{1155 x^9}$$

Rubi [B] time = 0.11, antiderivative size = 81, normalized size of antiderivative = 2.19, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2016, 2014}

$$\frac{2 \left(x^4-x^2\right)^{3/4}}{15 x^9}+\frac{8 \left(x^4-x^2\right)^{3/4}}{55 x^7}+\frac{64 \left(x^4-x^2\right)^{3/4}}{385 x^5}+\frac{256 \left(x^4-x^2\right)^{3/4}}{1155 x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] (2*(-x^2 + x^4)^(3/4))/(15*x^9) + (8*(-x^2 + x^4)^(3/4))/(55*x^7) + (64*(-x
^2 + x^4)^(3/4))/(385*x^5) + (256*(-x^2 + x^4)^(3/4))/(1155*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8 \sqrt[4]{-x^2+x^4}} \, dx &= \frac{2 \left(-x^2+x^4\right)^{3/4}}{15 x^9}+\frac{4}{5} \int \frac{1}{x^6 \sqrt[4]{-x^2+x^4}} \, dx \\
&= \frac{2 \left(-x^2+x^4\right)^{3/4}}{15 x^9}+\frac{8 \left(-x^2+x^4\right)^{3/4}}{55 x^7}+\frac{32}{55} \int \frac{1}{x^4 \sqrt[4]{-x^2+x^4}} \, dx \\
&= \frac{2 \left(-x^2+x^4\right)^{3/4}}{15 x^9}+\frac{8 \left(-x^2+x^4\right)^{3/4}}{55 x^7}+\frac{64 \left(-x^2+x^4\right)^{3/4}}{385 x^5}+\frac{128}{385} \int \frac{1}{x^2 \sqrt[4]{-x^2+x^4}} \, dx \\
&= \frac{2 \left(-x^2+x^4\right)^{3/4}}{15 x^9}+\frac{8 \left(-x^2+x^4\right)^{3/4}}{55 x^7}+\frac{64 \left(-x^2+x^4\right)^{3/4}}{385 x^5}+\frac{256 \left(-x^2+x^4\right)^{3/4}}{1155 x^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.00

$$\frac{2 \left(x^2 \left(x^2-1\right)\right)^{3/4} \left(128 x^6+96 x^4+84 x^2+77\right)}{1155 x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(-x^2 + x^4)^(1/4)),x]
[Out] (2*(x^2*(-1 + x^2))^(3/4)*(77 + 84*x^2 + 96*x^4 + 128*x^6))/(1155*x^9)
```

IntegrateAlgebraic [A] time = 0.20, size = 37, normalized size = 1.00

$$\frac{2\left(x^4-x^2\right)^{3/4}\left(128x^6+96x^4+84x^2+77\right)}{1155x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^8*(-x^2 + x^4)^(1/4)),x]
[Out] (2*(-x^2 + x^4)^(3/4)*(77 + 84*x^2 + 96*x^4 + 128*x^6))/(1155*x^9)
```

fricas [A] time = 0.38, size = 33, normalized size = 0.89

$$\frac{2\left(128\,x^6+96\,x^4+84\,x^2+77\right)\left(x^4-x^2\right)^{\frac{3}{4}}}{1155\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^4-x^2)^(1/4),x, algorithm="fricas")
[Out] 2/1155*(128*x^6 + 96*x^4 + 84*x^2 + 77)*(x^4 - x^2)^(3/4)/x^9
```

giac [A] time = 0.32, size = 59, normalized size = 1.59

$$-\frac{2}{15}\left(\frac{1}{x^2}-1\right)^3\left(-\frac{1}{x^2}+1\right)^{\frac{3}{4}}-\frac{6}{11}\left(\frac{1}{x^2}-1\right)^2\left(-\frac{1}{x^2}+1\right)^{\frac{3}{4}}+\frac{6}{7}\left(-\frac{1}{x^2}+1\right)^{\frac{7}{4}}-\frac{2}{3}\left(-\frac{1}{x^2}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^4-x^2)^(1/4),x, algorithm="giac")
[Out] -2/15*(1/x^2 - 1)^3*(-1/x^2 + 1)^(3/4) - 6/11*(1/x^2 - 1)^2*(-1/x^2 + 1)^(3
/4) + 6/7*(-1/x^2 + 1)^(7/4) - 2/3*(-1/x^2 + 1)^(3/4)
```

maple [A] time = 0.00, size = 40, normalized size = 1.08

$$\frac{2\left(-1+x\right)\left(1+x\right)\left(128x^6+96x^4+84x^2+77\right)}{1155x^7\left(x^4-x^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^4-x^2)^(1/4),x)
[Out] 2/1155*(-1+x)*(1+x)*(128*x^6+96*x^4+84*x^2+77)/x^7/(x^4-x^2)^(1/4)
```

maxima [A] time = 0.36, size = 39, normalized size = 1.05

$$\frac{2\left(128\,x^9-32\,x^7-12\,x^5-7\,x^3-77\,x\right)}{1155\left(x+1\right)^{\frac{1}{4}}\left(x-1\right)^{\frac{1}{4}}x^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^4-x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] 2/1155*(128*x^9 - 32*x^7 - 12*x^5 - 7*x^3 - 77*x)/((x + 1)^(1/4)*(x - 1)^(1
/4)*x^(17/2))
```

mupad [B] time = 0.29, size = 65, normalized size = 1.76

$$\frac{256\left(x^4-x^2\right)^{3/4}}{1155\,x^3}+\frac{64\left(x^4-x^2\right)^{3/4}}{385\,x^5}+\frac{8\left(x^4-x^2\right)^{3/4}}{55\,x^7}+\frac{2\left(x^4-x^2\right)^{3/4}}{15\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^4 - x^2)^(1/4)),x)
```

```
[Out] (256*(x^4 - x^2)^(3/4))/(1155*x^3) + (64*(x^4 - x^2)^(3/4))/(385*x^5) + (8*
(x^4 - x^2)^(3/4))/(55*x^7) + (2*(x^4 - x^2)^(3/4))/(15*x^9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8\sqrt[4]{x^2\,(x-1)\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**4-x**2)**(1/4),x)
```

```
[Out] Integral(1/(x**8*(x**2*(x - 1)*(x + 1))**(1/4)), x)
```

3.461 $\int \frac{-1+x}{\sqrt{-1+4x+2x^2-4x^3+x^4}}\,dx$

Optimal. Leaf size=37

$$-\frac{1}{2}\log\left(-x^2+\sqrt{x^4-4x^3+2x^2+4x-1}+2x+1\right)$$

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1680, 1107, 621, 206}

$$-\frac{1}{2}\tanh^{-1}\left(\frac{2-(x-1)^2}{\sqrt{(x-1)^4-4(x-1)^2+2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/Sqrt[-1 + 4*x + 2*x^2 - 4*x^3 + x^4],x]
```

```
[Out] -1/2*ArcTanh[(2 - (-1 + x)^2)/Sqrt[2 - 4*(-1 + x)^2 + (-1 + x)^4]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x}{\sqrt{-1+4x+2x^2-4x^3+x^4}}\,dx &= \text{Subst}\left(\int \frac{x}{\sqrt{2-4x^2+x^4}}\,dx, x, -1+x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{2-4x+x^2}}\,dx, x, (-1+x)^2\right)\\
&= \text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{2\left(-2+(-1+x)^2\right)}{\sqrt{2-4(-1+x)^2+(-1+x)^4}}\right)\\
&= \frac{1}{2}\tanh^{-1}\left(\frac{-2+(-1+x)^2}{\sqrt{2-4(-1+x)^2+(-1+x)^4}}\right)
\end{aligned}$$

Mathematica [C] time = 1.09, size = 716, normalized size = 19.35

$$-\frac{2\left(-x+\sqrt{2-\sqrt{2}}+1\right)^2\sqrt{\frac{-x+\sqrt{2+\sqrt{2}}+1}{-x+\sqrt{2-\sqrt{2}}+1}}\sqrt{-\frac{x+\sqrt{2+\sqrt{2}}-1}{\left(\sqrt{2-\sqrt{2}}-\sqrt{2+\sqrt{2}}\right)\left(-x+\sqrt{2-\sqrt{2}}+1\right)}}\left(\left(-\sqrt{2+\sqrt{2}}\sqrt{-\frac{x+\sqrt{2-\sqrt{2}}-1}{-x+\sqrt{2-\sqrt{2}}+1}}+\sqrt{2}\sqrt{\frac{\left(\sqrt{2}-2\right)\left(x+\sqrt{2-\sqrt{2}}-1\right)}{-x+\sqrt{2-\sqrt{2}}+1}}+\sqrt{\frac{\left(\sqrt{2}-2\right)\left(x+\sqrt{2-\sqrt{2}}-1\right)}{-x+\sqrt{2-\sqrt{2}}+1}}+2\sqrt{\frac{\left(2\sqrt{2}-3\right)\left(x+\sqrt{2-\sqrt{2}}-1\right)}{-x+\sqrt{2-\sqrt{2}}+1}}\right)F\left(\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{2-\sqrt{2}}-\sqrt{2+\sqrt{2}}\right)\left(x+\sqrt{2-\sqrt{2}}-1\right)}{\left(\sqrt{2-\sqrt{2}}+\sqrt{2+\sqrt{2}}\right)\left(-x+\sqrt{2-\sqrt{2}}+1\right)}}\right)|3+2\sqrt{2}\right)-4\sqrt{\frac{\left(2\sqrt{2}-3\right)\left(x+\sqrt{2-\sqrt{2}}-1\right)}{-x+\sqrt{2-\sqrt{2}}+1}}\Pi\left(\frac{\sqrt{2-\sqrt{2}}+\sqrt{2+\sqrt{2}}}{-\sqrt{2-\sqrt{2}}+\sqrt{2+\sqrt{2}}};\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{2-\sqrt{2}}-\sqrt{2+\sqrt{2}}\right)\left(x+\sqrt{2-\sqrt{2}}-1\right)}{\left(\sqrt{2-\sqrt{2}}+\sqrt{2+\sqrt{2}}\right)\left(-x+\sqrt{2-\sqrt{2}}+1\right)}}\right)|3+2\sqrt{2}\right)\right)}{\left(\sqrt{2+\sqrt{2}}-\sqrt{2-\sqrt{2}}\right)^{3/2}\sqrt{x^4-4x^3+2x^2+4x-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + x)/Sqrt[-1 + 4*x + 2*x^2 - 4*x^3 + x^4],x]
```

```
[Out] (-2*(1 + Sqrt[2 - Sqrt[2]] - x)^2*Sqrt[(1 + Sqrt[2 + Sqrt[2]] - x)/(1 + Sqr
t[2 - Sqrt[2]] - x)]*Sqrt[-((-1 + Sqrt[2 + Sqrt[2]] + x)/((Sqrt[2 - Sqrt[2]
] - Sqrt[2 + Sqrt[2]])*(1 + Sqrt[2 - Sqrt[2]] - x)))]*((-(Sqrt[2 + Sqrt[2]]
*Sqrt[-((-1 + Sqrt[2 - Sqrt[2]] + x)/(1 + Sqrt[2 - Sqrt[2]] - x))]) + Sqrt[
((-2 + Sqrt[2])*(-1 + Sqrt[2 - Sqrt[2]] + x))/(1 + Sqrt[2 - Sqrt[2]] - x)]
+ Sqrt[2]*Sqrt[((-2 + Sqrt[2])*(-1 + Sqrt[2 - Sqrt[2]] + x))/(1 + Sqrt[2 -
Sqrt[2]] - x)] + 2*Sqrt[((-3 + 2*Sqrt[2])*(-1 + Sqrt[2 - Sqrt[2]] + x))/(1
+ Sqrt[2 - Sqrt[2]] - x)])*EllipticF[ArcSin[Sqrt[((Sqrt[2 - Sqrt[2]] - Sqrt
[2 + Sqrt[2]])*(-1 + Sqrt[2 - Sqrt[2]] + x))/((Sqrt[2 - Sqrt[2]] + Sqrt[2 +
 Sqrt[2]])*(1 + Sqrt[2 - Sqrt[2]] - x))]], 3 + 2*Sqrt[2]] - 4*Sqrt[((-3 + 2
*Sqrt[2])*(-1 + Sqrt[2 - Sqrt[2]] + x))/(1 + Sqrt[2 - Sqrt[2]] - x)]*Ellipt
icPi[(Sqrt[2 - Sqrt[2]] + Sqrt[2 + Sqrt[2]])/(-Sqrt[2 - Sqrt[2]] + Sqrt[2 +
 Sqrt[2]]), ArcSin[Sqrt[((Sqrt[2 - Sqrt[2]] - Sqrt[2 + Sqrt[2]])*(-1 + Sqrt
[2 - Sqrt[2]] + x))/((Sqrt[2 - Sqrt[2]] + Sqrt[2 + Sqrt[2]])*(1 + Sqrt[2 -
Sqrt[2]] - x))]], 3 + 2*Sqrt[2]]))/((-Sqrt[2 - Sqrt[2]] + Sqrt[2 + Sqrt[2]]
)^(3/2)*Sqrt[-1 + 4*x + 2*x^2 - 4*x^3 + x^4])
```

IntegrateAlgebraic [A] time = 0.20, size = 37, normalized size = 1.00

$$-\frac{1}{2}\log\left(-x^2+\sqrt{x^4-4x^3+2x^2+4x-1}+2x+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/Sqrt[-1 + 4*x + 2*x^2 - 4*x^3 + x^4],x]
```

```
[Out] -1/2*Log[1 + 2*x - x^2 + Sqrt[-1 + 4*x + 2*x^2 - 4*x^3 + x^4]]
```

fricas [A] time = 0.43, size = 31, normalized size = 0.84

$$\frac{1}{2}\log\left(x^2-2\,x+\sqrt{x^4-4\,x^3+2\,x^2+4\,x-1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^4-4*x^3+2*x^2+4*x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - 2*x + sqrt(x^4 - 4*x^3 + 2*x^2 + 4*x - 1) - 1)
```

giac [A] time = 0.34, size = 35, normalized size = 0.95

$$-\frac{1}{2}\log\left(\left|-x^2+2\,x+\sqrt{\left(x^2-2\,x\right)^2-2\,x^2+4\,x-1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^4-4*x^3+2*x^2+4*x-1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-x^2 + 2*x + sqrt((x^2 - 2*x)^2 - 2*x^2 + 4*x - 1) + 1))
```

maple [C] time = 0.26, size = 1020, normalized size = 27.57

$$\frac{2\left(-\sqrt{2-\sqrt{2}}-\sqrt{2+\sqrt{2}}\right)\sqrt{\frac{\left(\sqrt{2+\sqrt{2}}-\sqrt{2-\sqrt{2}}\right)\left(x-1+\sqrt{2-\sqrt{2}}\right)}{\left(\sqrt{2+\sqrt{2}}+\sqrt{2-\sqrt{2}}\right)\left(x-1-\sqrt{2-\sqrt{2}}\right)}}\left(x-1-\sqrt{2-\sqrt{2}}\right)^2\sqrt{-\frac{\left(x-1+\sqrt{2+\sqrt{2}}\right)\sqrt{2-\sqrt{2}}}{\left(\sqrt{2+\sqrt{2}}-\sqrt{2-\sqrt{2}}\right)\left(x-1-\sqrt{2-\sqrt{2}}\right)}}\sqrt{\frac{\sqrt{2-\sqrt{2}}\left(x-1-\sqrt{2+\sqrt{2}}\right)}{\left(\sqrt{2+\sqrt{2}}+\sqrt{2-\sqrt{2}}\right)\left(x-1-\sqrt{2-\sqrt{2}}\right)}}\left(\left(1+\sqrt{2-\sqrt{2}}\right)\mathrm{EllipticF}\left(\sqrt{\frac{\left(\sqrt{2+\sqrt{2}}-\sqrt{2-\sqrt{2}}\right)\left(x-1+\sqrt{2-\sqrt{2}}\right)}{\left(\sqrt{2+\sqrt{2}}+\sqrt{2-\sqrt{2}}\right)\left(x-1-\sqrt{2-\sqrt{2}}\right)}},\sqrt{\frac{\left(\sqrt{2+\sqrt{2}}+\sqrt{2-\sqrt{2}}\right)^2}{\left(\sqrt{2+\sqrt{2}}-\sqrt{2-\sqrt{2}}\right)^2}}\right)-2\sqrt{2-\sqrt{2}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(\sqrt{2+\sqrt{2}}-\sqrt{2-\sqrt{2}}\right)\left(x-1+\sqrt{2-\sqrt{2}}\right)}{\left(\sqrt{2+\sqrt{2}}+\sqrt{2-\sqrt{2}}\right)\left(x-1-\sqrt{2-\sqrt{2}}\right)}},\frac{\sqrt{2+\sqrt{2}}+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}-\sqrt{2-\sqrt{2}}},\sqrt{\frac{\left(\sqrt{2+\sqrt{2}}+\sqrt{2-\sqrt{2}}\right)^2}{\left(\sqrt{2+\sqrt{2}}-\sqrt{2-\sqrt{2}}\right)^2}}\right)\right)}{\left(\sqrt{2+\sqrt{2}}-\sqrt{2-\sqrt{2}}\right)\sqrt{2-\sqrt{2}}\sqrt{\left(x-1+\sqrt{2-\sqrt{2}}\right)\left(x-1-\sqrt{2-\sqrt{2}}\right)\left(x-1+\sqrt{2+\sqrt{2}}\right)\left(x-1-\sqrt{2+\sqrt{2}}\right)}}-\frac{2\left(-\sqrt{2-\sqrt{2}}-\sqrt{2+\sqrt{2}}\right)\sqrt{\frac{\left(\sqrt{2+\sqrt{2}}-\sqrt{2-\sqrt{2}}\right)\left(x-1+\sqrt{2-\sqrt{2}}\right)}{\left(\sqrt{2+\sqrt{2}}+\sqrt{2-\sqrt{2}}\right)\left(x-1-\sqrt{2-\sqrt{2}}\right)}}\left(x-1-\sqrt{2-\sqrt{2}}\right)^2\sqrt{-\frac{\left(x-1+\sqrt{2+\sqrt{2}}\right)\sqrt{2-\sqrt{2}}}{\left(\sqrt{2+\sqrt{2}}-\sqrt{2-\sqrt{2}}\right)\left(x-1-\sqrt{2-\sqrt{2}}\right)}}\sqrt{\frac{\sqrt{2-\sqrt{2}}\left(x-1-\sqrt{2+\sqrt{2}}\right)}{\left(\sqrt{2+\sqrt{2}}+\sqrt{2-\sqrt{2}}\right)\left(x-1-\sqrt{2-\sqrt{2}}\right)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(\sqrt{2+\sqrt{2}}-\sqrt{2-\sqrt{2}}\right)\left(x-1+\sqrt{2-\sqrt{2}}\right)}{\left(\sqrt{2+\sqrt{2}}+\sqrt{2-\sqrt{2}}\right)\left(x-1-\sqrt{2-\sqrt{2}}\right)}},\sqrt{\frac{\left(\sqrt{2+\sqrt{2}}+\sqrt{2-\sqrt{2}}\right)^2}{\left(\sqrt{2+\sqrt{2}}-\sqrt{2-\sqrt{2}}\right)^2}}\right)}{\left(\sqrt{2+\sqrt{2}}-\sqrt{2-\sqrt{2}}\right)\sqrt{2-\sqrt{2}}\sqrt{\left(x-1+\sqrt{2-\sqrt{2}}\right)\left(x-1-\sqrt{2-\sqrt{2}}\right)\left(x-1+\sqrt{2+\sqrt{2}}\right)\left(x-1-\sqrt{2+\sqrt{2}}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(x^4-4*x^3+2*x^2+4*x-1)^(1/2),x)
```

```
[Out] 2*(-(2-2^(1/2))^(1/2)-(2+2^(1/2))^(1/2))*(((2+2^(1/2))^(1/2)-(2-2^(1/2))^(1
/2))*(x-1+(2-2^(1/2))^(1/2))/((2+2^(1/2))^(1/2)+(2-2^(1/2))^(1/2))/(x-1-(2-
2^(1/2))^(1/2)))^(1/2)*(x-1-(2-2^(1/2))^(1/2))^2*(-(x-1+(2+2^(1/2))^(1/2))/
((2+2^(1/2))^(1/2)-(2-2^(1/2))^(1/2))/(x-1-(2-2^(1/2))^(1/2))*(2-2^(1/2))^(
1/2))^(1/2)*((2-2^(1/2))^(1/2)*(x-1-(2+2^(1/2))^(1/2))/((2+2^(1/2))^(1/2)+(
2-2^(1/2))^(1/2))/(x-1-(2-2^(1/2))^(1/2)))^(1/2)/((2+2^(1/2))^(1/2)-(2-2^(1
/2))^(1/2))/(2-2^(1/2))^(1/2)/((x-1+(2-2^(1/2))^(1/2))*(x-1-(2-2^(1/2))^(1/
2))*(x-1+(2+2^(1/2))^(1/2))*(x-1-(2+2^(1/2))^(1/2)))^(1/2)*((1+(2-2^(1/2))^
(1/2))*EllipticF((((2+2^(1/2))^(1/2)-(2-2^(1/2))^(1/2))*(x-1+(2-2^(1/2))^(1
/2))/((2+2^(1/2))^(1/2)+(2-2^(1/2))^(1/2))/(x-1-(2-2^(1/2))^(1/2)))^(1/2),(
((2+2^(1/2))^(1/2)+(2-2^(1/2))^(1/2))^2/((2+2^(1/2))^(1/2)-(2-2^(1/2))^(1/2
))^2)^(1/2))-2*(2-2^(1/2))^(1/2)*EllipticPi((((2+2^(1/2))^(1/2)-(2-2^(1/2))
^(1/2))*(x-1+(2-2^(1/2))^(1/2))/((2+2^(1/2))^(1/2)+(2-2^(1/2))^(1/2))/(x-1-
(2-2^(1/2))^(1/2)))^(1/2),((2+2^(1/2))^(1/2)+(2-2^(1/2))^(1/2))/((2+2^(1/2)
)^(1/2)-(2-2^(1/2))^(1/2)),(((2+2^(1/2))^(1/2)+(2-2^(1/2))^(1/2))^2/((2+2^(
1/2))^(1/2)-(2-2^(1/2))^(1/2))^2)^(1/2)))-2*(-(2-2^(1/2))^(1/2)-(2+2^(1/2))
^(1/2))*(((2+2^(1/2))^(1/2)-(2-2^(1/2))^(1/2))*(x-1+(2-2^(1/2))^(1/2))/((2+
2^(1/2))^(1/2)+(2-2^(1/2))^(1/2))/(x-1-(2-2^(1/2))^(1/2)))^(1/2)*(x-1-(2-2^
(1/2))^(1/2))^2*(-(x-1+(2+2^(1/2))^(1/2))/((2+2^(1/2))^(1/2)-(2-2^(1/2))^(1
/2))/(x-1-(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2))^(1/2)*((2-2^(1/2))^(1/2)*(x
-1-(2+2^(1/2))^(1/2))/((2+2^(1/2))^(1/2)+(2-2^(1/2))^(1/2))/(x-1-(2-2^(1/2)
)^(1/2)))^(1/2)/((2+2^(1/2))^(1/2)-(2-2^(1/2))^(1/2))/(2-2^(1/2))^(1/2)/((x
-1+(2-2^(1/2))^(1/2))*(x-1-(2-2^(1/2))^(1/2))*(x-1+(2+2^(1/2))^(1/2))*(x-1-
(2+2^(1/2))^(1/2)))^(1/2)*EllipticF((((2+2^(1/2))^(1/2)-(2-2^(1/2))^(1/2))*
(x-1+(2-2^(1/2))^(1/2))/((2+2^(1/2))^(1/2)+(2-2^(1/2))^(1/2))/(x-1-(2-2^(1/
2))^(1/2)))^(1/2),(((2+2^(1/2))^(1/2)+(2-2^(1/2))^(1/2))^2/((2+2^(1/2))^(1/
2)-(2-2^(1/2))^(1/2))^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x^4-4\,x^3+2\,x^2+4\,x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^4-4*x^3+2*x^2+4*x-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/sqrt(x^4 - 4*x^3 + 2*x^2 + 4*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x-1}{\sqrt{x^4-4\,x^3+2\,x^2+4\,x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(4*x + 2*x^2 - 4*x^3 + x^4 - 1)^(1/2),x)
```

```
[Out] int((x - 1)/(4*x + 2*x^2 - 4*x^3 + x^4 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x^4-4x^3+2x^2+4x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**4-4*x**3+2*x**2+4*x-1)**(1/2),x)
```

```
[Out] Integral((x - 1)/sqrt(x**4 - 4*x**3 + 2*x**2 + 4*x - 1), x)
```

3.462 $\int \frac{\sqrt[4]{-x^3+x^4}}{(-1+x)x} \, dx$

Optimal. Leaf size=37

$$2\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)-2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)$$

Rubi [B] time = 0.11, antiderivative size = 83, normalized size of antiderivative = 2.24, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2056, 63, 240, 212, 206, 203}

$$\frac{2\sqrt[4]{x^4-x^3}\tan^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{\sqrt[4]{x-1}\,x^{3/4}}+\frac{2\sqrt[4]{x^4-x^3}\tanh^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{\sqrt[4]{x-1}\,x^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + x^4)^(1/4)/((-1 + x)*x),x]
```

```
[Out] (2*(-x^3 + x^4)^(1/4)*ArcTan[(-1 + x)^(1/4)/x^(1/4)])/((-1 + x)^(1/4)*x^(3/
4)) + (2*(-x^3 + x^4)^(1/4)*ArcTanh[(-1 + x)^(1/4)/x^(1/4)])/((-1 + x)^(1/4
)*x^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{-x^3+x^4}}{(-1+x)x}\,dx &= \frac{\sqrt[4]{-x^3+x^4}\int \frac{1}{(-1+x)^{3/4}\sqrt[4]{x}}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}}\\
&= \frac{\left(4\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^4}}\,dx, x, \sqrt[4]{-1+x}\right)}{\sqrt[4]{-1+x}\,x^{3/4}}\\
&= \frac{\left(4\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{\sqrt[4]{-1+x}\,x^{3/4}}\\
&= \frac{\left(2\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{\sqrt[4]{-1+x}\,x^{3/4}} + \frac{\left(2\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{\sqrt[4]{-1+x}\,x^{3/4}}\\
&= \frac{2\sqrt[4]{-x^3+x^4}\,\tan^{-1}\left(\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{\sqrt[4]{-1+x}\,x^{3/4}} + \frac{2\sqrt[4]{-x^3+x^4}\,\tanh^{-1}\left(\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{\sqrt[4]{-1+x}\,x^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.89

$$\frac{4\sqrt[4]{(x-1)x^3}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};1-x\right)}{x^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + x^4)^(1/4)/((-1 + x)*x),x]
[Out] (4*((-1 + x)*x^3)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, 1 - x])/x^(3/4)
```

IntegrateAlgebraic [A] time = 0.25, size = 37, normalized size = 1.00

$$2\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right) - 2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^3 + x^4)^(1/4)/((-1 + x)*x),x]
[Out] -2*ArcTan[x/(-x^3 + x^4)^(1/4)] + 2*ArcTanh[x/(-x^3 + x^4)^(1/4)]
```

fricas [A] time = 0.38, size = 60, normalized size = 1.62

$$2\arctan\left(\frac{\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right) + \log\left(\frac{x+\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right) - \log\left(-\frac{x-\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/(-1+x)/x,x, algorithm="fricas")
```

[Out] 2*arctan((x^4 - x^3)^(1/4)/x) + log((x + (x^4 - x^3)^(1/4))/x) - log(-(x - (x^4 - x^3)^(1/4))/x)

giac [A] time = 0.31, size = 40, normalized size = 1.08

$$-2\arctan\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)-\log\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right)+\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3)^(1/4)/(-1+x)/x,x, algorithm="giac")

[Out] -2*arctan((-1/x + 1)^(1/4)) - log((-1/x + 1)^(1/4) + 1) + log(abs((-1/x + 1)^(1/4) - 1))

maple [C] time = 0.23, size = 27, normalized size = 0.73

$$-\frac{4\mathrm{signum}\left(-1+x\right)^{\frac{1}{4}}x^{\frac{3}{4}}\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],x\right)}{3\left(-\mathrm{signum}\left(-1+x\right)\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-x^3)^(1/4)/(-1+x)/x,x)

[Out] -4/3*signum(-1+x)^(1/4)/(-signum(-1+x))^(1/4)*x^(3/4)*hypergeom([3/4,3/4],[7/4],x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^4-x^3\right)^{\frac{1}{4}}}{(x-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3)^(1/4)/(-1+x)/x,x, algorithm="maxima")

[Out] integrate((x^4 - x^3)^(1/4)/((x - 1)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\left(x^4-x^3\right)^{1/4}}{x\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4 - x^3)^(1/4)/(x*(x - 1)),x)

[Out] int((x^4 - x^3)^(1/4)/(x*(x - 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[4]{x^3\,(x-1)}}{x\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4-x**3)**(1/4)/(-1+x)/x,x)

[Out] Integral((x**3*(x - 1))**(1/4)/(x*(x - 1)), x)

3.463 $\int \frac{-2+x^3}{x\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=37

$$\frac{1}{3}\log\left(\sqrt{x^6-1}+x^3\right)-\frac{4}{3}\tan^{-1}\left(\sqrt{x^6-1}+x^3\right)$$

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1475, 844, 217, 206, 266, 63, 203}

$$\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)-\frac{2}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^3)/(x*Sqrt[-1 + x^6]),x]
```

```
[Out] (-2*ArcTan[Sqrt[-1 + x^6]])/3 + ArcTanh[x^3/Sqrt[-1 + x^6]]/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2+x^3}{x\sqrt{-1+x^6}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{-2+x}{x\sqrt{-1+x^2}}\,dx,x,x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx,x,x^3\right)-\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+x^2}}\,dx,x,x^3\right)\\
&= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x\,x}}\,dx,x,x^6\right)\right)+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x^3}{\sqrt{-1+x^6}}\right)\\
&= \frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)-\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x^6}\right)\\
&= -\frac{2}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right)+\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.84

$$\frac{1}{3}\left(\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)-2\tan^{-1}\left(\sqrt{x^6-1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^3)/(x*Sqrt[-1 + x^6]),x]
[Out] (-2*ArcTan[Sqrt[-1 + x^6]] + ArcTanh[x^3/Sqrt[-1 + x^6]])/3
```

IntegrateAlgebraic [A] time = 0.16, size = 41, normalized size = 1.11

$$\frac{4}{3}\tan^{-1}\left(x^3-\sqrt{x^6-1}\right)-\frac{1}{3}\log\left(\sqrt{x^6-1}-x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x^3)/(x*Sqrt[-1 + x^6]),x]
[Out] (4*ArcTan[x^3 - Sqrt[-1 + x^6]])/3 - Log[-x^3 + Sqrt[-1 + x^6]]/3
```

fricas [A] time = 0.41, size = 33, normalized size = 0.89

$$-\frac{4}{3}\,\arctan\left(-x^3+\sqrt{x^6-1}\right)-\frac{1}{3}\,\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)/x/(x^6-1)^(1/2),x, algorithm="fricas")
[Out] -4/3*arctan(-x^3 + sqrt(x^6 - 1)) - 1/3*log(-x^3 + sqrt(x^6 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3-2}{\sqrt{x^6-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)/x/(x^6-1)^(1/2),x, algorithm="giac")
[Out] integrate((x^3 - 2)/(sqrt(x^6 - 1)*x), x)
```

maple [C] time = 0.21, size = 86, normalized size = 2.32

$$\frac{\sqrt{-\mathrm{signum}\left(x^6-1\right)}\,\arcsin\left(x^3\right)}{3\sqrt{\mathrm{signum}\left(x^6-1\right)}}-\frac{\sqrt{-\mathrm{signum}\left(x^6-1\right)}\,\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^6+1}}{2}\right)+\left(-2\ln(2)+6\ln(x)+i\pi\right)\sqrt{\pi}\right)}{3\sqrt{\pi}\,\sqrt{\mathrm{signum}\left(x^6-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2)/x/(x^6-1)^(1/2),x)
[Out] 1/3/signum(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*arcsin(x^3)-1/3/Pi^(1/2)/sig
num(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(-x^6+1)^(1
/2))+(-2*ln(2)+6*ln(x)+I*Pi)*Pi^(1/2))
```

maxima [A] time = 0.53, size = 43, normalized size = 1.16

$$-\frac{2}{3}\arctan\left(\sqrt{x^6-1}\right)+\frac{1}{6}\log\left(\frac{\sqrt{x^6-1}}{x^3}+1\right)-\frac{1}{6}\log\left(\frac{\sqrt{x^6-1}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)/x/(x^6-1)^(1/2),x, algorithm="maxima")
[Out] -2/3*arctan(sqrt(x^6 - 1)) + 1/6*log(sqrt(x^6 - 1)/x^3 + 1) - 1/6*log(sqrt(
x^6 - 1)/x^3 - 1)
```

mupad [B] time = 0.75, size = 25, normalized size = 0.68

$$\frac{\ln\left(\sqrt{x^6-1}+x^3\right)}{3}-\frac{2\,\mathrm{atan}\left(\sqrt{x^6-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 2)/(x*(x^6 - 1)^(1/2)),x)
[Out] log((x^6 - 1)^(1/2) + x^3)/3 - (2*atan((x^6 - 1)^(1/2)))/3
```

sympy [A] time = 4.04, size = 20, normalized size = 0.54

$$-\frac{2\left(\begin{cases}\mathrm{acos}\left(\frac{1}{x^3}\right) & \text{for}\ x>-1\wedge x<1\end{cases}\right)}{3}+\frac{\mathrm{acosh}\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2)/x/(x**6-1)**(1/2),x)
[Out] -2*Piecewise((acos(x**(-3)), (x > -1) & (x < 1)))/3 + acosh(x**3)/3
```

3.464 $\int \frac{1+x^3}{x\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=37

$$\frac{1}{3}\log\left(\sqrt{x^6-1}+x^3\right)+\frac{2}{3}\tan^{-1}\left(\sqrt{x^6-1}+x^3\right)$$

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1475, 844, 217, 206, 266, 63, 203}

$$\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(x*Sqrt[-1 + x^6]),x]
```

```
[Out] ArcTan[Sqrt[-1 + x^6]]/3 + ArcTanh[x^3/Sqrt[-1 + x^6]]/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{x\sqrt{-1+x^6}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1+x}{x\sqrt{-1+x^2}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, x^3\right)+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+x^2}}\,dx, x, x^3\right)\\
&= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right)+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^3}{\sqrt{-1+x^6}}\right)\\
&= \frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^6}\right)\\
&= \frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right)+\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.78

$$\frac{1}{3}\left(\tan^{-1}\left(\sqrt{x^6-1}\right)+\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(x*Sqrt[-1 + x^6]),x]
[Out] (ArcTan[Sqrt[-1 + x^6]] + ArcTanh[x^3/Sqrt[-1 + x^6]])/3
```

IntegrateAlgebraic [A] time = 0.13, size = 41, normalized size = 1.11

$$-\frac{1}{3}\log\left(\sqrt{x^6-1}-x^3\right)-\frac{2}{3}\tan^{-1}\left(x^3-\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)/(x*Sqrt[-1 + x^6]),x]
[Out] (-2*ArcTan[x^3 - Sqrt[-1 + x^6]])/3 - Log[-x^3 + Sqrt[-1 + x^6]]/3
```

fricas [A] time = 0.40, size = 33, normalized size = 0.89

$$\frac{2}{3}\arctan\left(-x^3+\sqrt{x^6-1}\right)-\frac{1}{3}\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x/(x^6-1)^(1/2),x, algorithm="fricas")
[Out] 2/3*arctan(-x^3 + sqrt(x^6 - 1)) - 1/3*log(-x^3 + sqrt(x^6 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3+1}{\sqrt{x^6-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/x/(x^6-1)^(1/2),x, algorithm="giac")

[Out] integrate((x^3 + 1)/(sqrt(x^6 - 1)*x), x)

maple [C] time = 0.24, size = 86, normalized size = 2.32

$$\frac{\sqrt{-\mathrm{signum}\left(x^6-1\right)}\ \arcsin\left(x^3\right)}{3\sqrt{\mathrm{signum}\left(x^6-1\right)}}+\frac{\sqrt{-\mathrm{signum}\left(x^6-1\right)}\ \left(-2\sqrt{\pi}\ \ln\left(\frac{1}{2}+\frac{\sqrt{-x^6+1}}{2}\right)+\left(-2\ln(2)+6\ln(x)+i\pi\right)\sqrt{\pi}\right)}{6\sqrt{\pi}\ \sqrt{\mathrm{signum}\left(x^6-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+1)/x/(x^6-1)^(1/2),x)

[Out] 1/3/signum(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*arcsin(x^3)+1/6/Pi^(1/2)/signum(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(-x^6+1)^(1/2))+(-2*ln(2)+6*ln(x)+I*Pi)*Pi^(1/2))

maxima [A] time = 0.49, size = 43, normalized size = 1.16

$$\frac{1}{3}\ \arctan\left(\sqrt{x^6-1}\right)+\frac{1}{6}\ \log\left(\frac{\sqrt{x^6-1}}{x^3}+1\right)-\frac{1}{6}\ \log\left(\frac{\sqrt{x^6-1}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/x/(x^6-1)^(1/2),x, algorithm="maxima")

[Out] 1/3*arctan(sqrt(x^6 - 1)) + 1/6*log(sqrt(x^6 - 1)/x^3 + 1) - 1/6*log(sqrt(x^6 - 1)/x^3 - 1)

mupad [B] time = 0.29, size = 25, normalized size = 0.68

$$\frac{\ln\left(\sqrt{x^6-1}\ +x^3\right)}{3}+\frac{\mathrm{atan}\left(\sqrt{x^6-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 + 1)/(x*(x^6 - 1)^(1/2)),x)

[Out] log((x^6 - 1)^(1/2) + x^3)/3 + atan((x^6 - 1)^(1/2))/3

sympy [A] time = 6.82, size = 19, normalized size = 0.51

$$\frac{\begin{cases}\mathrm{acos}\left(\frac{1}{x^3}\right) & \text{for}\ x>-1\wedge x<1\end{cases}}{3}+\frac{\mathrm{acosh}\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+1)/x/(x**6-1)**(1/2),x)

[Out] Piecewise((acos(x**(-3)), (x > -1) & (x < 1)))/3 + acosh(x**3)/3

3.465 $\int \frac{-1+2x^3}{x\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{3}\log\left(\sqrt{x^6-1}+x^3\right)-\frac{2}{3}\tan^{-1}\left(\sqrt{x^6-1}+x^3\right)$$

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1475, 844, 217, 206, 266, 63, 203}

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)-\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x^3)/(x*Sqrt[-1 + x^6]),x]
```

```
[Out] -1/3*ArcTan[Sqrt[-1 + x^6]] + (2*ArcTanh[x^3/Sqrt[-1 + x^6]])/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+2x^3}{x\sqrt{-1+x^6}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{-1+2x}{x\sqrt{-1+x^2}}\,dx,x,x^3\right)\\ &= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1+x^2}}\,dx,x,x^3\right)\right)+\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx,x,x^3\right)\\ &= -\left(\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx,x,x^6\right)\right)+\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x^3}{\sqrt{-1+x^6}}\right)\\ &= \frac{2}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x^6}\right)\\ &= -\frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right)+\frac{2}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.89

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)-\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x^3)/(x*Sqrt[-1 + x^6]),x]
[Out] -1/3*ArcTan[Sqrt[-1 + x^6]] + (2*ArcTanh[x^3/Sqrt[-1 + x^6]])/3
```

IntegrateAlgebraic [A] time = 0.16, size = 41, normalized size = 1.11

$$\frac{2}{3}\tan^{-1}\left(x^3-\sqrt{x^6-1}\right)-\frac{2}{3}\log\left(\sqrt{x^6-1}-x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x^3)/(x*Sqrt[-1 + x^6]),x]
[Out] (2*ArcTan[x^3 - Sqrt[-1 + x^6]])/3 - (2*Log[-x^3 + Sqrt[-1 + x^6]])/3
```

fricas [A] time = 0.41, size = 33, normalized size = 0.89

$$-\frac{2}{3}\arctan\left(-x^3+\sqrt{x^6-1}\right)-\frac{2}{3}\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-1)/x/(x^6-1)^(1/2),x, algorithm="fricas")
[Out] -2/3*arctan(-x^3 + sqrt(x^6 - 1)) - 2/3*log(-x^3 + sqrt(x^6 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3-1}{\sqrt{x^6-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-1)/x/(x^6-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 - 1)/(sqrt(x^6 - 1)*x), x)
```

maple [C] time = 0.24, size = 86, normalized size = 2.32

$$\frac{2\sqrt{-\mathrm{signum}\left(x^{6}-1\right)}\, \arcsin\left(x^{3}\right)}{3\sqrt{\mathrm{signum}\left(x^{6}-1\right)}}-\frac{\sqrt{-\mathrm{signum}\left(x^{6}-1\right)}\, \left(-2\sqrt{\pi}\, \ln\left(\frac{1}{2}+\frac{\sqrt{-x^{6}+1}}{2}\right)+\left(-2\ln(2)+6\ln(x)+i\pi\right)\sqrt{\pi}\right)}{6\sqrt{\pi}\, \sqrt{\mathrm{signum}\left(x^{6}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-1)/x/(x^6-1)^(1/2),x)
```

```
[Out] 2/3/signum(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*arcsin(x^3)-1/6/Pi^(1/2)/sig
num(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(-x^6+1)^(1
/2))+(-2*ln(2)+6*ln(x)+I*Pi)*Pi^(1/2))
```

maxima [A] time = 0.77, size = 43, normalized size = 1.16

$$-\frac{1}{3}\arctan\left(\sqrt{x^{6}-1}\right)+\frac{1}{3}\log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}+1\right)-\frac{1}{3}\log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-1)/x/(x^6-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*arctan(sqrt(x^6 - 1)) + 1/3*log(sqrt(x^6 - 1)/x^3 + 1) - 1/3*log(sqrt(
x^6 - 1)/x^3 - 1)
```

mupad [B] time = 0.49, size = 25, normalized size = 0.68

$$\frac{2\ln\left(\sqrt{x^{6}-1}+x^{3}\right)}{3}-\frac{\mathrm{atan}\left(\sqrt{x^{6}-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3 - 1)/(x*(x^6 - 1)^(1/2)),x)
```

```
[Out] (2*log((x^6 - 1)^(1/2) + x^3))/3 - atan((x^6 - 1)^(1/2))/3
```

sympy [A] time = 3.99, size = 20, normalized size = 0.54

$$-\frac{\begin{cases}\mathrm{acos}\left(\frac{1}{x^{3}}\right) & \text{for}\: x>-1\wedge x<1\end{cases}}{3}+\frac{2\,\mathrm{acosh}\left(x^{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-1)/x/(x**6-1)**(1/2),x)
```

```
[Out] -Piecewise((acos(x**(-3)), (x > -1) & (x < 1)))/3 + 2*acosh(x**3)/3
```

3.466 $\int \frac{1+2x^3}{x\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{3}\log\left(\sqrt{x^6-1}+x^3\right)+\frac{2}{3}\tan^{-1}\left(\sqrt{x^6-1}+x^3\right)$$

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1475, 844, 217, 206, 266, 63, 203}

$$\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{2}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^3)/(x*Sqrt[-1 + x^6]),x]
```

```
[Out] ArcTan[Sqrt[-1 + x^6]]/3 + (2*ArcTanh[x^3/Sqrt[-1 + x^6]])/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^3}{x\sqrt{-1+x^6}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1+2x}{x\sqrt{-1+x^2}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+x^2}}\,dx,x,x^3\right)+\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx,x,x^3\right)\\ &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx,x,x^6\right)+\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x^3}{\sqrt{-1+x^6}}\right)\\ &= \frac{2}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x^6}\right)\\ &= \frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right)+\frac{2}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.84

$$\frac{1}{3}\left(\tan^{-1}\left(\sqrt{x^6-1}\right)+2\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^3)/(x*Sqrt[-1 + x^6]),x]
[Out] (ArcTan[Sqrt[-1 + x^6]] + 2*ArcTanh[x^3/Sqrt[-1 + x^6]])/3
```

IntegrateAlgebraic [A] time = 0.15, size = 41, normalized size = 1.11

$$-\frac{2}{3}\log\left(\sqrt{x^6-1}-x^3\right)-\frac{2}{3}\tan^{-1}\left(x^3-\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x^3)/(x*Sqrt[-1 + x^6]),x]
[Out] (-2*ArcTan[x^3 - Sqrt[-1 + x^6]])/3 - (2*Log[-x^3 + Sqrt[-1 + x^6]])/3
```

fricas [A] time = 0.40, size = 33, normalized size = 0.89

$$\frac{2}{3}\arctan\left(-x^3+\sqrt{x^6-1}\right)-\frac{2}{3}\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)/x/(x^6-1)^(1/2),x, algorithm="fricas")
[Out] 2/3*arctan(-x^3 + sqrt(x^6 - 1)) - 2/3*log(-x^3 + sqrt(x^6 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^3+1}{\sqrt{x^6-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+1)/x/(x^6-1)^(1/2),x, algorithm="giac")

[Out] integrate((2*x^3 + 1)/(sqrt(x^6 - 1)*x), x)

maple [C] time = 0.22, size = 86, normalized size = 2.32

$$\frac{2\sqrt{-\mathrm{signum}\left(x^{6}-1\right)}\,\arcsin\left(x^{3}\right)}{3\sqrt{\mathrm{signum}\left(x^{6}-1\right)}}+\frac{\sqrt{-\mathrm{signum}\left(x^{6}-1\right)}\,\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{6}+1}}{2}\right)+\left(-2\ln(2)+6\ln(x)+i\pi\right)\sqrt{\pi}\right)}{6\sqrt{\pi}\,\sqrt{\mathrm{signum}\left(x^{6}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+1)/x/(x^6-1)^(1/2),x)

[Out] 2/3/signum(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*arcsin(x^3)+1/6/Pi^(1/2)/signum(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(-x^6+1)^(1/2))+(-2*ln(2)+6*ln(x)+I*Pi)*Pi^(1/2))

maxima [A] time = 0.92, size = 43, normalized size = 1.16

$$\frac{1}{3}\arctan\left(\sqrt{x^{6}-1}\right)+\frac{1}{3}\log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}+1\right)-\frac{1}{3}\log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+1)/x/(x^6-1)^(1/2),x, algorithm="maxima")

[Out] 1/3*arctan(sqrt(x^6 - 1)) + 1/3*log(sqrt(x^6 - 1)/x^3 + 1) - 1/3*log(sqrt(x^6 - 1)/x^3 - 1)

mupad [B] time = 0.29, size = 25, normalized size = 0.68

$$\frac{2\ln\left(\sqrt{x^{6}-1}+x^{3}\right)}{3}+\frac{\mathrm{atan}\left(\sqrt{x^{6}-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3 + 1)/(x*(x^6 - 1)^(1/2)),x)

[Out] (2*log((x^6 - 1)^(1/2) + x^3))/3 + atan((x^6 - 1)^(1/2))/3

sympy [A] time = 7.05, size = 20, normalized size = 0.54

$$\frac{\begin{cases}\mathrm{acos}\left(\frac{1}{x^{3}}\right) & \text{for}\ x>-1\wedge x<1\end{cases}}{3}+\frac{2\,\mathrm{acosh}\left(x^{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+1)/x/(x**6-1)**(1/2),x)

[Out] Piecewise((acos(x**(-3)), (x > -1) & (x < 1)))/3 + 2*acosh(x**3)/3

3.467 $\int \frac{1+4x^3}{x\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=37

$$\frac{4}{3}\log\left(\sqrt{x^6-1}+x^3\right)+\frac{2}{3}\tan^{-1}\left(\sqrt{x^6-1}+x^3\right)$$

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1475, 844, 217, 206, 266, 63, 203}

$$\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{4}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x^3)/(x*Sqrt[-1 + x^6]),x]
```

```
[Out] ArcTan[Sqrt[-1 + x^6]]/3 + (4*ArcTanh[x^3/Sqrt[-1 + x^6]])/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1+4x^3}{x\sqrt{-1+x^6}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1+4x}{x\sqrt{-1+x^2}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+x^2}}\,dx, x, x^3\right)+\frac{4}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, x^3\right)\\ &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right)+\frac{4}{3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^3}{\sqrt{-1+x^6}}\right)\\ &= \frac{4}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^6}\right)\\ &= \frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right)+\frac{4}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.84

$$\frac{1}{3}\left(\tan^{-1}\left(\sqrt{x^6-1}\right)+4\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x^3)/(x*Sqrt[-1 + x^6]),x]
```

```
[Out] (ArcTan[Sqrt[-1 + x^6]] + 4*ArcTanh[x^3/Sqrt[-1 + x^6]])/3
```

IntegrateAlgebraic [A] time = 0.16, size = 41, normalized size = 1.11

$$-\frac{4}{3}\log\left(\sqrt{x^6-1}-x^3\right)-\frac{2}{3}\tan^{-1}\left(x^3-\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 4*x^3)/(x*Sqrt[-1 + x^6]),x]
```

```
[Out] (-2*ArcTan[x^3 - Sqrt[-1 + x^6]])/3 - (4*Log[-x^3 + Sqrt[-1 + x^6]])/3
```

fricas [A] time = 0.41, size = 33, normalized size = 0.89

$$\frac{2}{3}\arctan\left(-x^3+\sqrt{x^6-1}\right)-\frac{4}{3}\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+1)/x/(x^6-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*arctan(-x^3 + sqrt(x^6 - 1)) - 4/3*log(-x^3 + sqrt(x^6 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^3+1}{\sqrt{x^6-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+1)/x/(x^6-1)^(1/2),x, algorithm="giac")
[Out] integrate((4*x^3 + 1)/(sqrt(x^6 - 1)*x), x)
```

maple [C] time = 0.23, size = 86, normalized size = 2.32

$$\frac{4\sqrt{-\mathrm{signum}\left(x^{6}-1\right)}\,\arcsin\left(x^{3}\right)}{3\sqrt{\mathrm{signum}\left(x^{6}-1\right)}}+\frac{\sqrt{-\mathrm{signum}\left(x^{6}-1\right)}\,\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{6}+1}}{2}\right)+\left(-2\ln(2)+6\ln(x)+i\pi\right)\sqrt{\pi}\right)}{6\sqrt{\pi}\,\sqrt{\mathrm{signum}\left(x^{6}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3+1)/x/(x^6-1)^(1/2),x)
[Out] 4/3/signum(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*arcsin(x^3)+1/6/Pi^(1/2)/sig
num(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(-x^6+1)^(1
/2))+(-2*ln(2)+6*ln(x)+I*Pi)*Pi^(1/2))
```

maxima [A] time = 0.59, size = 43, normalized size = 1.16

$$\frac{1}{3}\arctan\left(\sqrt{x^{6}-1}\right)+\frac{2}{3}\log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}+1\right)-\frac{2}{3}\log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+1)/x/(x^6-1)^(1/2),x, algorithm="maxima")
[Out] 1/3*arctan(sqrt(x^6 - 1)) + 2/3*log(sqrt(x^6 - 1)/x^3 + 1) - 2/3*log(sqrt(x
^6 - 1)/x^3 - 1)
```

mupad [B] time = 0.55, size = 25, normalized size = 0.68

$$\frac{4\ln\left(\sqrt{x^{6}-1}+x^{3}\right)}{3}+\frac{\mathrm{atan}\left(\sqrt{x^{6}-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3 + 1)/(x*(x^6 - 1)^(1/2)),x)
[Out] (4*log((x^6 - 1)^(1/2) + x^3))/3 + atan((x^6 - 1)^(1/2))/3
```

sympy [A] time = 7.00, size = 20, normalized size = 0.54

$$\frac{\begin{cases}\mathrm{acos}\left(\frac{1}{x^{3}}\right) & \text{for}\ x>-1\wedge x<1\end{cases}}{3}+\frac{4\,\mathrm{acosh}\left(x^{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3+1)/x/(x**6-1)**(1/2),x)
[Out] Piecewise((acos(x**(-3)), (x > -1) & (x < 1)))/3 + 4*acosh(x**3)/3
```

3.468 $\int \frac{1+x^6}{x^6\left(-1+x^3\right)\sqrt[4]{-x+x^4}}\,dx$

Optimal. Leaf size=37

$$-\frac{4\left(x^4-x\right)^{3/4}\left(53x^6-8x^3-3\right)}{63x^6\left(x^3-1\right)}$$

Rubi [A] time = 0.17, antiderivative size = 53, normalized size of antiderivative = 1.43, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2056, 1487, 453, 271, 264}

$$-\frac{212x}{63\sqrt[4]{x^4-x}}+\frac{4}{21\sqrt[4]{x^4-x}\,x^5}+\frac{32}{63\sqrt[4]{x^4-x}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)/(x^6*(-1 + x^3)*(-x + x^4)^(1/4)),x]
```

```
[Out] 4/(21*x^5*(-x + x^4)^(1/4)) + 32/(63*x^2*(-x + x^4)^(1/4)) - (212*x)/(63*(-
x + x^4)^(1/4))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1487

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 2*n*p - n + 1)*(d + e*x^n)^(
q + 1))/(e*f^(2*n*p - n + 1)*(m + 2*n*p + n*q + 1)), x] + Dist[1/(e*(m + 2*
n*p + n*q + 1)), Int[(f*x)^m*(d + e*x^n)^q*ExpandToSum[e*(m + 2*n*p + n*q +
 1)*((a + c*x^(2*n))^p - c^p*x^(2*n*p)) - d*c^p*(m + 2*n*p - n + 1)*x^(2*n*
p - n), x], x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IG
tQ[n, 0] && IGtQ[p, 0] && GtQ[2*n*p, n - 1] &&  !IntegerQ[q] && NeQ[m + 2*n
*p + n*q + 1, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
```

```
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^6}{x^6\left(-1+x^3\right)\sqrt[4]{-x+x^4}}\,dx &= \frac{\left(\sqrt[4]{x}\,\sqrt[4]{-1+x^3}\right)\int \frac{1+x^6}{x^{25/4}\left(-1+x^3\right)^{5/4}}\,dx}{\sqrt[4]{-x+x^4}}\\
&= -\frac{1}{3x^2\sqrt[4]{-x+x^4}}-\frac{\left(\sqrt[4]{x}\,\sqrt[4]{-1+x^3}\right)\int \frac{-3-\frac{9x^3}{4}}{x^{25/4}\left(-1+x^3\right)^{5/4}}\,dx}{3\sqrt[4]{-x+x^4}}\\
&= \frac{4}{21x^5\sqrt[4]{-x+x^4}}-\frac{1}{3x^2\sqrt[4]{-x+x^4}}+\frac{\left(53\sqrt[4]{x}\,\sqrt[4]{-1+x^3}\right)\int \frac{1}{x^{13/4}\left(-1+x^3\right)^{5/4}}\,dx}{28\sqrt[4]{-x+x^4}}\\
&= \frac{4}{21x^5\sqrt[4]{-x+x^4}}+\frac{32}{63x^2\sqrt[4]{-x+x^4}}+\frac{\left(53\sqrt[4]{x}\,\sqrt[4]{-1+x^3}\right)\int \frac{1}{\sqrt[4]{x}\left(-1+x^3\right)^{5/4}}\,dx}{21\sqrt[4]{-x+x^4}}\\
&= \frac{4}{21x^5\sqrt[4]{-x+x^4}}+\frac{32}{63x^2\sqrt[4]{-x+x^4}}-\frac{212x}{63\sqrt[4]{-x+x^4}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 30, normalized size = 0.81

$$\frac{4\left(-53x^6+8x^3+3\right)}{63x^5\sqrt[4]{x\left(x^3-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)/(x^6*(-1 + x^3)*(-x + x^4)^(1/4)),x]
[Out] (4*(3 + 8*x^3 - 53*x^6))/(63*x^5*(x*(-1 + x^3))^(1/4))
```

IntegrateAlgebraic [A] time = 0.49, size = 37, normalized size = 1.00

$$-\frac{4\left(x^4-x\right)^{3/4}\left(53x^6-8x^3-3\right)}{63x^6\left(x^3-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^6)/(x^6*(-1 + x^3)*(-x + x^4)^(1/4)),x]
[Out] (-4*(-x + x^4)^(3/4)*(-3 - 8*x^3 + 53*x^6))/(63*x^6*(-1 + x^3))
```

fricas [A] time = 0.39, size = 34, normalized size = 0.92

$$-\frac{4\left(53\,x^6-8\,x^3-3\right)\left(x^4-x\right)^{\frac{3}{4}}}{63\left(x^9-x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x^6/(x^3-1)/(x^4-x)^(1/4),x, algorithm="fricas")
[Out] -4/63*(53*x^6 - 8*x^3 - 3)*(x^4 - x)^(3/4)/(x^9 - x^6)
```

giac [A] time = 0.86, size = 34, normalized size = 0.92

$$-\frac{4}{21}\left(-\frac{1}{x^3}+1\right)^{\frac{7}{4}}+\frac{8}{9}\left(-\frac{1}{x^3}+1\right)^{\frac{3}{4}}+\frac{8}{3\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x^6/(x^3-1)/(x^4-x)^(1/4),x, algorithm="giac")
[Out] -4/21*(-1/x^3 + 1)^(7/4) + 8/9*(-1/x^3 + 1)^(3/4) + 8/3/(-1/x^3 + 1)^(1/4)
```

maple [A] time = 0.01, size = 27, normalized size = 0.73

$$-\frac{4\left(53x^6-8x^3-3\right)}{63\left(x^4-x\right)^{\frac{1}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/x^6/(x^3-1)/(x^4-x)^(1/4),x)
[Out] -4/63*(53*x^6-8*x^3-3)/(x^4-x)^(1/4)/x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+1}{\left(x^4-x\right)^{\frac{1}{4}}\left(x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x^6/(x^3-1)/(x^4-x)^(1/4),x, algorithm="maxima")
[Out] integrate((x^6 + 1)/((x^4 - x)^(1/4)*(x^3 - 1)*x^6), x)
```

mupad [B] time = 0.42, size = 33, normalized size = 0.89

$$\frac{4\left(x^4-x\right)^{3/4}\left(-53\,x^6+8\,x^3+3\right)}{63\,x^6\,\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)/(x^6*(x^4 - x)^(1/4)*(x^3 - 1)),x)
[Out] (4*(x^4 - x)^(3/4)*(8*x^3 - 53*x^6 + 3))/(63*x^6*(x^3 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)\left(x^4-x^2+1\right)}{x^6\sqrt[4]{x\left(x-1\right)\left(x^2+x+1\right)}\,\left(x-1\right)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/x**6/(x**3-1)/(x**4-x)**(1/4),x)
[Out] Integral((x**2 + 1)*(x**4 - x**2 + 1)/(x**6*(x*(x - 1)*(x**2 + x + 1))**(1/
4)*(x - 1)*(x**2 + x + 1)), x)
```

3.469 $\int \frac{x(8b+5ax^3)}{\sqrt[4]{b+ax^3}\,(-b-ax^3+x^8)}\,dx$

Optimal. Leaf size=37

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{x^2}\right)-2\tanh^{-1}\left(\frac{x^2}{\sqrt[4]{ax^3+b}}\right)$$

Rubi [F] time = 0.92, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\left(8b+5ax^3\right)}{\sqrt[4]{b+ax^3}\left(-b-ax^3+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(8*b + 5*a*x^3))/((b + a*x^3)^(1/4)*(-b - a*x^3 + x^8)),x]
```

```
[Out] -8*b*Defer[Int][x/((b + a*x^3)^(1/4)*(b + a*x^3 - x^8)), x] - 5*a*Defer[Int
][x^4/((b + a*x^3)^(1/4)*(b + a*x^3 - x^8)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(8b+5ax^3\right)}{\sqrt[4]{b+ax^3}\left(-b-ax^3+x^8\right)}\,dx&=\int\left(-\frac{8bx}{\sqrt[4]{b+ax^3}\left(b+ax^3-x^8\right)}-\frac{5ax^4}{\sqrt[4]{b+ax^3}\left(b+ax^3-x^8\right)}\right)dx\\&=-\left((5a)\int\frac{x^4}{\sqrt[4]{b+ax^3}\left(b+ax^3-x^8\right)}\,dx\right)-(8b)\int\frac{x}{\sqrt[4]{b+ax^3}\left(b+ax^3-x^8\right)}\end{aligned}$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{x\left(8b+5ax^3\right)}{\sqrt[4]{b+ax^3}\left(-b-ax^3+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(8*b + 5*a*x^3))/((b + a*x^3)^(1/4)*(-b - a*x^3 + x^8)),x]
```

```
[Out] Integrate[(x*(8*b + 5*a*x^3))/((b + a*x^3)^(1/4)*(-b - a*x^3 + x^8)), x]
```

IntegrateAlgebraic [A] time = 0.91, size = 37, normalized size = 1.00

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{x^2}\right)-2\tanh^{-1}\left(\frac{x^2}{\sqrt[4]{ax^3+b}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(8*b + 5*a*x^3))/((b + a*x^3)^(1/4)*(-b - a*x^3 + x^8
)),x]
```

```
[Out] 2*ArcTan[(b + a*x^3)^(1/4)/x^2] - 2*ArcTanh[x^2/(b + a*x^3)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*a*x^3+8*b)/(a*x^3+b)^(1/4)/(x^8-a*x^3-b),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,ax^{3}+8\,b\right)x}{\left(x^{8}-ax^{3}-b\right)\left(ax^{3}+b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*a*x^3+8*b)/(a*x^3+b)^(1/4)/(x^8-a*x^3-b),x, algorithm="giac"
)
```

```
[Out] integrate((5*a*x^3 + 8*b)*x/((x^8 - a*x^3 - b)*(a*x^3 + b)^(1/4)), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x\left(5a\,x^{3}+8b\right)}{\left(a\,x^{3}+b\right)^{\frac{1}{4}}\left(x^{8}-a\,x^{3}-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(5*a*x^3+8*b)/(a*x^3+b)^(1/4)/(x^8-a*x^3-b),x)
```

```
[Out] int(x*(5*a*x^3+8*b)/(a*x^3+b)^(1/4)/(x^8-a*x^3-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,ax^{3}+8\,b\right)x}{\left(x^{8}-ax^{3}-b\right)\left(ax^{3}+b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*a*x^3+8*b)/(a*x^3+b)^(1/4)/(x^8-a*x^3-b),x, algorithm="maxim
a")
```

```
[Out] integrate((5*a*x^3 + 8*b)*x/((x^8 - a*x^3 - b)*(a*x^3 + b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{x\,\left(5\,a\,x^3+8\,b\right)}{{\left(a\,x^3+b\right)}^{1/4}\,\left(-x^8+a\,x^3+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(8*b + 5*a*x^3))/((b + a*x^3)^(1/4)*(b + a*x^3 - x^8)),x)
```

```
[Out] int(-(x*(8*b + 5*a*x^3))/((b + a*x^3)^(1/4)*(b + a*x^3 - x^8)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(5 a x^{3} + 8 b\right)}{\sqrt[4]{a x^{3} + b} \left(- a x^{3} - b + x^{8}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*a*x**3+8*b)/(a*x**3+b)**(1/4)/(x**8-a*x**3-b),x)
```

```
[Out] Integral(x*(5*a*x**3 + 8*b)/((a*x**3 + b)**(1/4)*(-a*x**3 - b + x**8)), x)
```

3.470 $\int \sqrt{x+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=37

$$\frac{1}{3}\left(\sqrt{x^2+1}+x\right)^{3/2}-\frac{1}{\sqrt{\sqrt{x^2+1}+x}}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2117, 14}

$$\frac{1}{3}\left(\sqrt{x^2+1}+x\right)^{3/2}-\frac{1}{\sqrt{\sqrt{x^2+1}+x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[1 + x^2]],x]
```

```
[Out] -(1/Sqrt[x + Sqrt[1 + x^2]]) + (x + Sqrt[1 + x^2])^(3/2)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x+\sqrt{1+x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{x^{3/2}}\,dx,x,x+\sqrt{1+x^2}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^{3/2}}+\sqrt{x}\right)dx,x,x+\sqrt{1+x^2}\right)\\
&= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}}+\frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.95

$$\frac{2\left(x^2+\sqrt{x^2+1}\,x-1\right)}{3\sqrt{\sqrt{x^2+1}+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[1 + x^2]],x]
```

```
[Out] (2*(-1 + x^2 + x*Sqrt[1 + x^2]))/(3*Sqrt[x + Sqrt[1 + x^2]])
```

IntegrateAlgebraic [A] time = 0.04, size = 37, normalized size = 1.00

$$\frac{1}{3}\left(\sqrt{x^2+1}+x\right)^{3/2}-\frac{1}{\sqrt{\sqrt{x^2+1}+x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x + Sqrt[1 + x^2]],x]
[Out] -(1/Sqrt[x + Sqrt[1 + x^2]]) + (x + Sqrt[1 + x^2])^(3/2)/3
```

fricas [A] time = 0.42, size = 26, normalized size = 0.70

$$\frac{2}{3}\left(2x-\sqrt{x^2+1}\right)\sqrt{x+\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2/3*(2*x - sqrt(x^2 + 1))*sqrt(x + sqrt(x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x+\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x + sqrt(x^2 + 1)), x)
```

maple [C] time = 0.01, size = 57, normalized size = 1.54

$$\frac{\frac{16\sqrt{\pi}\,\sqrt{2}\,x^{\frac{3}{2}}\left(1-\frac{1}{x^2}\right)\cosh\left(\frac{\operatorname{arcsinh}\left(\frac{1}{x}\right)}{2}\right)}{3}+\frac{16\sqrt{\pi}\,\sqrt{2}\,\sqrt{x}\,\sqrt{1+\frac{1}{x^2}}\,\sinh\left(\frac{\operatorname{arcsinh}\left(\frac{1}{x}\right)}{2}\right)}{3}}{8\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+1)^(1/2))^(1/2),x)
[Out] 1/8/Pi^(1/2)*(16/3*Pi^(1/2)*2^(1/2)*x^(3/2)*(1-1/x^2)*cosh(1/2*arcsinh(1/x)
)+16/3*Pi^(1/2)*2^(1/2)*x^(1/2)*(1+1/x^2)^(1/2)*sinh(1/2*arcsinh(1/x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x+\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x + sqrt(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\sqrt{x+\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int((x + (x^2 + 1)^(1/2))^(1/2), x)
```

sympy [A] time = 0.29, size = 42, normalized size = 1.14

$$\frac{4x\sqrt{x+\sqrt{x^2+1}}}{3}-\frac{2\sqrt{x+\sqrt{x^2+1}}\,\sqrt{x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] 4*x*sqrt(x + sqrt(x**2 + 1))/3 - 2*sqrt(x + sqrt(x**2 + 1))*sqrt(x**2 + 1)/
3
```

3.471 $\int \frac{ab-x^2}{\sqrt{x(-a+x)(-b+x)}\left(ab-(a+b+d)x+x^2\right)}\,dx$

Optimal. Leaf size=38

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{x^2(-a-b)+abx+x^3}}\right)}{\sqrt{d}}$$

Rubi [C] time = 6.45, antiderivative size = 273, normalized size of antiderivative = 7.18, number of steps used = 15, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6718, 6728, 117, 116, 169, 538, 537}

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\,\Pi\left(\frac{2a}{a+b+d-\sqrt{a^2-2(b-d)a+(b+d)^2}};\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{\sqrt{x(a-x)(b-x)}}+\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\,\Pi\left(\frac{2a}{a+b+d+\sqrt{a^2-2(b-d)a+(b+d)^2}};\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{\sqrt{x(a-x)(b-x)}}-\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{\sqrt{x(a-x)(b-x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b - x^2)/(Sqrt[x*(-a + x)*(-b + x)]*(a*b - (a + b + d)*x + x^2)),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[x]*Sqrt[1 - x/a]*Sqrt[1 - x/b]*EllipticF[ArcSin[Sqrt[x]/Sq
rt[a]], a/b])/Sqrt[(a - x)*(b - x)*x] + (2*Sqrt[a]*Sqrt[x]*Sqrt[1 - x/a]*Sq
rt[1 - x/b]*EllipticPi[(2*a)/(a + b + d - Sqrt[a^2 - 2*a*(b - d) + (b + d)^
2]), ArcSin[Sqrt[x]/Sqrt[a]], a/b])/Sqrt[(a - x)*(b - x)*x] + (2*Sqrt[a]*Sq
rt[x]*Sqrt[1 - x/a]*Sqrt[1 - x/b]*EllipticPi[(2*a)/(a + b + d + Sqrt[a^2 -
2*a*(b - d) + (b + d)^2]), ArcSin[Sqrt[x]/Sqrt[a]], a/b])/Sqrt[(a - x)*(b -
 x)*x]
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{ab-x^2}{\sqrt{x(-a+x)(-b+x)}\left(ab-(a+b+d)x+x^2\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{ab-x^2}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\left(ab-(a+b+d)x+x^2\right)}}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \left(-\frac{1}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}}+\frac{}{\sqrt{x}\sqrt{-a+x}}\right.}{\sqrt{x(-a+x)(-b+x)}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{1}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}}\,dx}{\sqrt{x(-a+x)(-b+x)}} + \frac{\left(\sqrt{x}\sqrt{\right.}{}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \left(\frac{-a-b-d-\sqrt{a^2-2ab+b^2+}}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\left(-a-b-d-\sqrt{a^2-2a}\right.}\right.}{}$$

$$= -\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{\sqrt{(a-x)(b-x)x}} + \frac{\left(\left(-a-b\right.\right.}{}$$

$$= -\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{\sqrt{(a-x)(b-x)x}} - \frac{\left(2\left(-a-b\right.\right.}{}$$

$$= -\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{\sqrt{(a-x)(b-x)x}} - \frac{\left(2\left(-a-b\right.\right.}{}$$

$$= -\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{\sqrt{(a-x)(b-x)x}} - \frac{\left(2\left(-a-b\right.\right.}{}$$

$$= -\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{\sqrt{(a-x)(b-x)x}} + \frac{2\sqrt{a}\sqrt{x}\sqrt{}}{}$$

Mathematica [C] time = 4.56, size = 195, normalized size = 5.13

$$\frac{2ix^{3/2}\sqrt{1-\frac{a}{x}}\sqrt{1-\frac{b}{x}}\left(-\Pi\left(\frac{2b}{a+b+d-\sqrt{a^2-2(b-d)a+(b+d)^2}};i\sinh^{-1}\left(\frac{\sqrt{-a}}{\sqrt{x}}\right)|\frac{b}{a}\right)-\Pi\left(\frac{2b}{a+b+d+\sqrt{a^2-2(b-d)a+(b+d)^2}};i\sinh^{-1}\left(\frac{\sqrt{-a}}{\sqrt{x}}\right)|\frac{b}{a}\right)+F\left(i\sinh^{-1}\left(\frac{\sqrt{-a}}{\sqrt{x}}\right)|\frac{b}{a}\right)\right)}{\sqrt{-a}\sqrt{x(x-a)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b - x^2)/(Sqrt[x*(-a + x)*(-b + x)]*(a*b - (a + b + d)*x + x^2
)),x]
```

```
[Out] ((2*I)*Sqrt[1 - a/x]*Sqrt[1 - b/x]*x^(3/2)*(EllipticF[I*ArcSinh[Sqrt[-a]/Sq
rt[x]], b/a] - EllipticPi[(2*b)/(a + b + d - Sqrt[a^2 - 2*a*(b - d) + (b +
d)^2]), I*ArcSinh[Sqrt[-a]/Sqrt[x]], b/a] - EllipticPi[(2*b)/(a + b + d + S
qrt[a^2 - 2*a*(b - d) + (b + d)^2]), I*ArcSinh[Sqrt[-a]/Sqrt[x]], b/a]))/(S
qrt[-a]*Sqrt[x*(-a + x)*(-b + x)])
```

IntegrateAlgebraic [A] time = 0.18, size = 38, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{x^2(-a-b)+abx+x^3}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b - x^2)/(Sqrt[x*(-a + x)*(-b + x)]*(a*b - (a + b + d
)*x + x^2)),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*x)/Sqrt[a*b*x + (-a - b)*x^2 + x^3]])/Sqrt[d]
```

fricas [B] time = 0.59, size = 250, normalized size = 6.58

$$\left[\frac{\log\left(\frac{a^2b^2-2\,(a+b-3\,d)x^3+x^4+\left(a^2+4\,ab+b^2-6\,(a+b)d+d^2\right)x^2+4\,\sqrt{abx-(a+b)x^2+x^3}\left(ab-(a+b-d)x+x^2\right)\sqrt{d}-2\left(a^2b+ab^2-3\,abd\right)x}{a^2b^2-2\,(a+b+d)x^3+x^4+\left(a^2+4\,ab+b^2+2\,(a+b)d+d^2\right)x^2-2\left(a^2b+ab^2+abd\right)x}\right)}{2\sqrt{d}},-\frac{\sqrt{-d}\,\arctan\left(\frac{\sqrt{abx-(a+b)x^2+x^3}\left(ab-(a+b-d)x+x^2\right)\sqrt{-d}}{2\left(abdx-(a+b)dx^2+dx^3\right)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a*b-(a+b+d)*x+x^2),x, algorith
m="fricas")
```

```
[Out] [1/2*log((a^2*b^2 - 2*(a + b - 3*d)*x^3 + x^4 + (a^2 + 4*a*b + b^2 - 6*(a +
 b)*d + d^2)*x^2 + 4*sqrt(a*b*x - (a + b)*x^2 + x^3)*(a*b - (a + b - d)*x +
 x^2)*sqrt(d) - 2*(a^2*b + a*b^2 - 3*a*b*d)*x)/(a^2*b^2 - 2*(a + b + d)*x^3
 + x^4 + (a^2 + 4*a*b + b^2 + 2*(a + b)*d + d^2)*x^2 - 2*(a^2*b + a*b^2 + a
*b*d)*x))/sqrt(d), -sqrt(-d)*arctan(1/2*sqrt(a*b*x - (a + b)*x^2 + x^3)*(a*
b - (a + b - d)*x + x^2)*sqrt(-d)/(a*b*d*x - (a + b)*d*x^2 + d*x^3))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab-x^2}{\sqrt{(a-x)(b-x)x}\left(ab-(a+b+d)x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a*b-(a+b+d)*x+x^2),x, algorith
m="giac")
```

```
[Out] integrate((a*b - x^2)/(sqrt((a - x)*(b - x)*x)*(a*b - (a + b + d)*x + x^2))
, x)
```

maple [C] time = 0.08, size = 3392, normalized size = 89.26

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b-x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a*b-(a+b+d)*x+x^2),x)
```

```
[Out] 2*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2
+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))+1/(a^2-2*a*b+2*a*d
+b^2+2*b*d+d^2)^(1/2)*a^3*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a
*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2
*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*
d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))-2/(a^2-2*a*b+
2*a*d+b^2+2*b*d+d^2)^(1/2)*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)
*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*
a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b
-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*b+2/(a^2
-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x
)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d-1/2*
(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*
a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*d
+a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^
2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(
1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*
d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))+1/(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^
(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a
*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2
)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2
*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*b^2+2/(a^2-2*a*b+2*a*d+b^2+2*b*
d+d^2)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(
a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*
b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d-1/2*(a^2-
2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*b*d+a*(1-1/a*x)^(1/2)*(-
1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2
*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+
x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))
,(a/(a-b))^(1/2))*b+1/(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)*a*(1-1/a*x)^(1/
2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)
/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((
-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(
1/2)),(a/(a-b))^(1/2))*d^2+a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(
1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*
b+2*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1
/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*d-1/(a^2-2
*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)*a^3*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^
(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d+1/2*(a
^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-
1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))+2/(
a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b
)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d+1
/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1
/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2)
)*b-2/(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b
+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-
1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/
2),a/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b)
)^(1/2))*d+a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(
a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*
b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d+1/2*(a^2-
2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))-1/(a^2-2*a*b+2*a*d+b^2+2
*b*d+d^2)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2
)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2
+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d+1/2*(a
```

```
^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*b^2-2/(a^2-2*a*b+2*a*
d+b^2+2*b*d+d^2)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*
x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*
a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d
+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*b*d+a*(1-1/a*x
)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^
(1/2)/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*Ellipti
cPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d
^2)^(1/2)),(a/(a-b))^(1/2))*b-1/(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)*a*(1-
1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+
x^3)^(1/2)/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*El
lipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*
b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*d^2+a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*
x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d+1/2
*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2
*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*
d
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a*b-(a+b+d)*x+x^2),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((d+b+a)^2-4*a*b>0)', see `assume?`
for more details)Is (d+b+a)^2-4*a*b    positive, negative or zero?
```

mupad [**B**] time = 0.34, size = 532, normalized size = 14.00

$$\frac{2\,b\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}}{\sqrt{x^3+\left(-a-b\right)\,x^2+a\,b\,x}}-\frac{2\,b\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\left(2\,a\,b-\left(a+b+d\right)\,\left(\frac{a}{2}+\frac{b}{2}+\frac{d}{2}-\frac{\sqrt{a^2-2\,a\,b+2\,a\,d+b^2+2\,b\,d+d^2}}{2}\right)\right)\,\Pi\left(-\frac{b}{\frac{a}{2}-\frac{b}{2}+\frac{d}{2}-\frac{\sqrt{a^2-2\,a\,b+2\,a\,d+b^2+2\,b\,d+d^2}}{2}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)}{\sqrt{x^3+\left(-a-b\right)\,x^2+a\,b\,x}\,\left(\frac{a}{2}-\frac{b}{2}+\frac{d}{2}-\frac{\sqrt{a^2-2\,a\,b+2\,a\,d+b^2+2\,b\,d+d^2}}{2}\right)\,\sqrt{a^2-2\,a\,b+2\,a\,d+b^2+2\,b\,d+d^2}}+\frac{2\,b\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\left(2\,a\,b-\left(a+b+d\right)\,\left(\frac{a}{2}+\frac{b}{2}+\frac{d}{2}+\frac{\sqrt{a^2-2\,a\,b+2\,a\,d+b^2+2\,b\,d+d^2}}{2}\right)\right)\,\Pi\left(-\frac{b}{\frac{a}{2}-\frac{b}{2}+\frac{d}{2}+\frac{\sqrt{a^2-2\,a\,b+2\,a\,d+b^2+2\,b\,d+d^2}}{2}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)}{\sqrt{x^3+\left(-a-b\right)\,x^2+a\,b\,x}\,\left(\frac{a}{2}-\frac{b}{2}+\frac{d}{2}+\frac{\sqrt{a^2-2\,a\,b+2\,a\,d+b^2+2\,b\,d+d^2}}{2}\right)\,\sqrt{a^2-2\,a\,b+2\,a\,d+b^2+2\,b\,d+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b - x^2)/((x*(a - x)*(b - x))^(1/2)*(a*b - x*(a + b + d) + x^2)),x)
```

```
[Out] (2*b*ellipticF(asin(((b - x)/b)^(1/2)), -b/(a - b))*(x/b)^(1/2)*((b - x)/b)
^(1/2)*((a - x)/(a - b))^(1/2))/(x^3 - x^2*(a + b) + a*b*x)^(1/2) - (2*b*(x
/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*(2*a*b - (a + b + d)*(a
/2 + b/2 + d/2 - (2*a*d - 2*a*b + 2*b*d + a^2 + b^2 + d^2)^(1/2)/2))*ellipt
icPi(-b/(a/2 - b/2 + d/2 - (2*a*d - 2*a*b + 2*b*d + a^2 + b^2 + d^2)^(1/2)/
2), asin(((b - x)/b)^(1/2)), -b/(a - b)))/((x^3 - x^2*(a + b) + a*b*x)^(1/2
)*(a/2 - b/2 + d/2 - (2*a*d - 2*a*b + 2*b*d + a^2 + b^2 + d^2)^(1/2)/2)*(2*
a*d - 2*a*b + 2*b*d + a^2 + b^2 + d^2)^(1/2)) + (2*b*(x/b)^(1/2)*((b - x)/b
)^(1/2)*((a - x)/(a - b))^(1/2)*(2*a*b - (a + b + d)*(a/2 + b/2 + d/2 + (2*
a*d - 2*a*b + 2*b*d + a^2 + b^2 + d^2)^(1/2)/2))*ellipticPi(-b/(a/2 - b/2 +
 d/2 + (2*a*d - 2*a*b + 2*b*d + a^2 + b^2 + d^2)^(1/2)/2), asin(((b - x)/b)
^(1/2)), -b/(a - b)))/((x^3 - x^2*(a + b) + a*b*x)^(1/2)*(a/2 - b/2 + d/2 +
 (2*a*d - 2*a*b + 2*b*d + a^2 + b^2 + d^2)^(1/2)/2)*(2*a*d - 2*a*b + 2*b*d
+ a^2 + b^2 + d^2)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-x**2)/(x*(-a+x)*(-b+x))**(1/2)/(a*b-(a+b+d)*x+x**2),x)
```

```
[Out] Timed out
```

3.472 $$\int \frac{ab-x^2}{\sqrt{x(-a+x)(-b+x)}\left(abd-(1+ad+bd)x+dx^2\right)}\,dx$$

Optimal. Leaf size=38

$$\frac{2\tanh^{-1}\left(\frac{x}{\sqrt{d}\sqrt{x^2(-a-b)+abx+x^3}}\right)}{\sqrt{d}}$$

Rubi [C] time = 8.70, antiderivative size = 308, normalized size of antiderivative = 8.11, number of steps used = 15, number of rules used = 7, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6718, 6728, 117, 116, 169, 538, 537}

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\,\Pi\left(\frac{2ad}{ad+bd-\sqrt{a^2d^2+2a(1-bd)d+(bd+1)^2}+1};\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{d\sqrt{x(a-x)(b-x)}}+\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\,\Pi\left(\frac{2ad}{ad+bd+\sqrt{a^2d^2+2a(1-bd)d+(bd+1)^2}+1};\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{d\sqrt{x(a-x)(b-x)}}-\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{d\sqrt{x(a-x)(b-x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b - x^2)/(Sqrt[x*(-a + x)*(-b + x)]*(a*b*d - (1 + a*d + b*d)*x + d*x
^2)),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[x]*Sqrt[1 - x/a]*Sqrt[1 - x/b]*EllipticF[ArcSin[Sqrt[x]/Sq
rt[a]], a/b])/(d*Sqrt[(a - x)*(b - x)*x]) + (2*Sqrt[a]*Sqrt[x]*Sqrt[1 - x/a
]*Sqrt[1 - x/b]*EllipticPi[(2*a*d)/(1 + a*d + b*d - Sqrt[a^2*d^2 + 2*a*d*(1
 - b*d) + (1 + b*d)^2]), ArcSin[Sqrt[x]/Sqrt[a]], a/b])/(d*Sqrt[(a - x)*(b
- x)*x]) + (2*Sqrt[a]*Sqrt[x]*Sqrt[1 - x/a]*Sqrt[1 - x/b]*EllipticPi[(2*a*d
)/(1 + a*d + b*d + Sqrt[a^2*d^2 + 2*a*d*(1 - b*d) + (1 + b*d)^2]), ArcSin[S
qrt[x]/Sqrt[a]], a/b])/(d*Sqrt[(a - x)*(b - x)*x])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{ab-x^2}{\sqrt{x(-a+x)(-b+x)}\left(abd-(1+ad+bd)x+dx^2\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{ab-x^2}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\left(abd-(1+ad+bd)\right.}}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int\left(-\frac{1}{d\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}}+\frac{}{d\sqrt{x}\sqrt{-}}\right.}{\sqrt{x(-a+x)(-b+x)}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int\frac{1}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}}\,dx}{d\sqrt{x(-a+x)(-b+x)}} + \frac{\left(\sqrt{x}\right.}{}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int\left(\frac{-1-ad-bd-\sqrt{1+2ad+}}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\left(-1-ad-bd-\sqrt{1}\right.}\right.}{}$$

$$= -\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{d\sqrt{(a-x)(b-x)x}} + \frac{\left(\left(-1-\right.\right.}{}$$

$$= -\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{d\sqrt{(a-x)(b-x)x}} - \frac{\left(2\left(-1-\right.\right.}{}$$

$$= -\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{d\sqrt{(a-x)(b-x)x}} - \frac{\left(2\left(-1-\right.\right.}{}$$

$$= -\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{d\sqrt{(a-x)(b-x)x}} - \frac{\left(2\left(-1-\right.\right.}{}$$

$$= -\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{d\sqrt{(a-x)(b-x)x}} + \frac{2\sqrt{a}\sqrt{x}}{}$$

Mathematica [C] time = 5.77, size = 210, normalized size = 5.53

$$\frac{2ix^{3/2}\sqrt{1-\frac{a}{x}}\sqrt{1-\frac{b}{x}}\left(-\Pi\left(\frac{2bd}{ad+bd-\sqrt{(ad+bd+1)^2-4abd^2}+1};i\sinh^{-1}\left(\frac{\sqrt{-a}}{\sqrt{x}}\right)|\frac{b}{a}\right)-\Pi\left(\frac{2bd}{ad+bd+\sqrt{(ad+bd+1)^2-4abd^2}+1};i\sinh^{-1}\left(\frac{\sqrt{-a}}{\sqrt{x}}\right)|\frac{b}{a}\right)+F\left(i\sinh^{-1}\left(\frac{\sqrt{-a}}{\sqrt{x}}\right)|\frac{b}{a}\right)\right)}{\sqrt{-a}\,d\sqrt{x(x-a)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b - x^2)/(Sqrt[x*(-a + x)*(-b + x)]*(a*b*d - (1 + a*d + b*d)*x
 + d*x^2)),x]
```

```
[Out] ((2*I)*Sqrt[1 - a/x]*Sqrt[1 - b/x]*x^(3/2)*(EllipticF[I*ArcSinh[Sqrt[-a]/Sq
rt[x]], b/a] - EllipticPi[(2*b*d)/(1 + a*d + b*d - Sqrt[-4*a*b*d^2 + (1 + a
*d + b*d)^2]), I*ArcSinh[Sqrt[-a]/Sqrt[x]], b/a] - EllipticPi[(2*b*d)/(1 +
a*d + b*d + Sqrt[-4*a*b*d^2 + (1 + a*d + b*d)^2]), I*ArcSinh[Sqrt[-a]/Sqrt[
x]], b/a]))/(Sqrt[-a]*d*Sqrt[x*(-a + x)*(-b + x)])
```

IntegrateAlgebraic [A] time = 0.20, size = 38, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{x}{\sqrt{d}\sqrt{x^2(-a-b)+abx+x^3}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b - x^2)/(Sqrt[x*(-a + x)*(-b + x)]*(a*b*d - (1 + a*d
 + b*d)*x + d*x^2)),x]
```

```
[Out] (2*ArcTanh[x/(Sqrt[d]*Sqrt[a*b*x + (-a - b)*x^2 + x^3])])/Sqrt[d]
```

fricas [B] time = 0.74, size = 299, normalized size = 7.87

$$\left[\frac{\log\left(\frac{a^2b^2d^2+d^2x^4-2\left((a+b)d^2-3\,d\right)x^3+\left(\left(a^2+4\,ab+b^2\right)d^2-6\,(a+b)d+1\right)x^2+4\left(abd+dx^2-((a+b)d-1)x\right)\sqrt{abx-(a+b)x^2+x^3}\,\sqrt{d}+2\left(3\,abd-\left(a^2b+ab^2\right)d^2\right)x}{a^2b^2d^2+d^2x^4-2\left((a+b)d^2+d\right)x^3+\left(\left(a^2+4\,ab+b^2\right)d^2+2\,(a+b)d+1\right)x^2-2\left(abd+\left(a^2b+ab^2\right)d^2\right)x}\right)}{2\sqrt{d}},\ -\frac{\sqrt{-d}\ \arctan\left(\frac{\left(abd+dx^2-((a+b)d-1)x\right)\sqrt{abx-(a+b)x^2+x^3}\,\sqrt{-d}}{2\left(abdx-(a+b)dx^2+dx^3\right)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a*b*d-(a*d+b*d+1)*x+d*x^2),x,
algorithm="fricas")
```

```
[Out] [1/2*log((a^2*b^2*d^2 + d^2*x^4 - 2*((a + b)*d^2 - 3*d)*x^3 + ((a^2 + 4*a*b
 + b^2)*d^2 - 6*(a + b)*d + 1)*x^2 + 4*(a*b*d + d*x^2 - ((a + b)*d - 1)*x)*
sqrt(a*b*x - (a + b)*x^2 + x^3)*sqrt(d) + 2*(3*a*b*d - (a^2*b + a*b^2)*d^2)
*x)/(a^2*b^2*d^2 + d^2*x^4 - 2*((a + b)*d^2 + d)*x^3 + ((a^2 + 4*a*b + b^2)
*d^2 + 2*(a + b)*d + 1)*x^2 - 2*(a*b*d + (a^2*b + a*b^2)*d^2)*x))/sqrt(d),
-sqrt(-d)*arctan(1/2*(a*b*d + d*x^2 - ((a + b)*d - 1)*x)*sqrt(a*b*x - (a +
b)*x^2 + x^3)*sqrt(-d)/(a*b*d*x - (a + b)*d*x^2 + d*x^3))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab-x^2}{\left(abd+dx^2-(ad+bd+1)x\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a*b*d-(a*d+b*d+1)*x+d*x^2),x,
algorithm="giac")
```

```
[Out] integrate((a*b - x^2)/((a*b*d + d*x^2 - (a*d + b*d + 1)*x)*sqrt((a - x)*(b
 - x)*x)), x)
```

maple [C] time = 0.06, size = 4007, normalized size = 105.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b-x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a*b*d-(a*d+b*d+1)*x+d*x^2),x)
```

```
[Out] 2/d*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x
^2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))-1/d*(-1/(a^2*d^2
-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)*a^3*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(
a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2/
d-1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x
)/a)^(1/2),a/(a-1/2/d*(a*d+b*d+1+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^
(1/2))),(a/(a-b))^(1/2))*d+2/(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2
)*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x
^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2/d-1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a
*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(a*d+b*d+1+(a^2*
d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))*b*d-2/(a^2*d^
2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/
(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2
/d-1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+
x)/a)^(1/2),a/(a-1/2/d*(a*d+b*d+1+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)
^(1/2))),(a/(a-b))^(1/2))-1/(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)
*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-
b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2/d-1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+
2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(a*d+b*d+1+(a^2*d^2
-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))*b^2*d-2/(a^2*d^2
-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-
b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2/d-
1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/
a)^(1/2),a/(a-1/2/d*(a*d+b*d+1+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1
/2))),(a/(a-b))^(1/2))*b-1/(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)*
a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b
*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2/d-1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2
*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(a*d+b*d+1+(a^2*d^2-
2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))/d-a^2*(1-1/a*x)^(
1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/
2)/(1/2*a-1/2*b-1/2/d-1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)
)*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(a*d+b*d+1+(a^2*d^2-2*a*b*d^2+b^2
*d^2+2*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))-a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+
1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1
/2/d-1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-
a+x)/a)^(1/2),a/(a-1/2/d*(a*d+b*d+1+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+
1)^(1/2))),(a/(a-b))^(1/2))*b-a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2
)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2/d-1/2/d*(a^2
*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a
/(a-1/2/d*(a*d+b*d+1+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))),(a/(
a-b))^(1/2))/d+1/(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)*a^3*(1-1/a
*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3
)^(1/2)/(1/2*a-1/2*b+1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-
1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-a*d-b*d+(a^2*d^2-2*a*b*d^2+b
^2*d^2+2*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))*d-2/(a^2*d^2-2*a*b*d^2+b
^2*d^2+2*a*d+2*b*d+1)^(1/2)*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2
)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b+1/2/d*(a^2*d^2-2
*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a
/(a+1/2*(-a*d-b*d+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1)/d),(a/
(a-b))^(1/2))*b*d+2/(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)*a^2*(1-
1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+
x^3)^(1/2)/(1/2*a-1/2*b+1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/
2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-a*d-b*d+(a^2*d^2-2*a*b*d^
2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))+1/(a^2*d^2-2*a*b*d^2+
b^2*d^2+2*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)
*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b+1/2/d*(a^2*d^2-2*
```

```
a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/
(a+1/2*(-a*d-b*d+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1)/d),(a/(
a-b))^(1/2))*b^2*d+2/(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)*a*(1-1
/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x
^3)^(1/2)/(1/2*a-1/2*b+1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2
)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-a*d-b*d+(a^2*d^2-2*a*b*d^2
+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))*b-a^2*(1-1/a*x)^(1/2)*
(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1
/2*a-1/2*b+1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1/2/d)*Ell
ipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-a*d-b*d+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a
*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))-a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a
-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b+1/2/d
*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)
/a)^(1/2),a/(a+1/2*(-a*d-b*d+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2
)-1)/d),(a/(a-b))^(1/2))*b-a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(
1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b+1/2/d*(a^2*d^2-2*a*
b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a
+1/2*(-a*d-b*d+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-
b))^(1/2))/d+1/(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^
(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1
/2)/(1/2*a-1/2*b+1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1/2/
d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-a*d-b*d+(a^2*d^2-2*a*b*d^2+b^2*d
^2+2*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))/d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a*b*d-(a*d+b*d+1)*x+d*x^2),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b*d+a*d+1)^2>0)', see `assume?` fo
r more details)Is (b*d+a*d+1)^2    -4*a*b*d^2 positive, negative or zero?
```

mupad [B] time = 0.35, size = 531, normalized size = 13.97

$$\frac{2\,b\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}}{d\,\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}+\frac{b\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(\frac{b}{b-\frac{a\,d+b\,d+\sqrt{a^2\,d^2-2\,a\,b\,d^2+2\,a\,d+b^2\,d^2+2\,b\,d+1}+1}{2\,d}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\left(a\,d+b\,d+\sqrt{a^2\,d^2-2\,a\,b\,d^2+2\,a\,d+b^2\,d^2+2\,b\,d+1}+1\right)}{d^2\,\left(b-\frac{a\,d+b\,d+\sqrt{a^2\,d^2-2\,a\,b\,d^2+2\,a\,d+b^2\,d^2+2\,b\,d+1}+1}{2\,d}\right)\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}+\frac{b\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(\frac{b}{b-\frac{a\,d+b\,d-\sqrt{a^2\,d^2-2\,a\,b\,d^2+2\,a\,d+b^2\,d^2+2\,b\,d+1}+1}{2\,d}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\left(a\,d+b\,d-\sqrt{a^2\,d^2-2\,a\,b\,d^2+2\,a\,d+b^2\,d^2+2\,b\,d+1}+1\right)}{d^2\,\left(b-\frac{a\,d+b\,d-\sqrt{a^2\,d^2-2\,a\,b\,d^2+2\,a\,d+b^2\,d^2+2\,b\,d+1}+1}{2\,d}\right)\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b - x^2)/((d*x^2 - x*(a*d + b*d + 1) + a*b*d)*(x*(a - x)*(b - x))^(1
/2)),x)
```

```
[Out] (2*b*ellipticF(asin(((b - x)/b)^(1/2)), -b/(a - b))*(x/b)^(1/2)*((b - x)/b)
^(1/2)*((a - x)/(a - b))^(1/2))/(d*(x^3 - x^2*(a + b) + a*b*x)^(1/2)) + (b*
(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*ellipticPi(b/(b - (a*
d + b*d + (2*a*d + 2*b*d + a^2*d^2 + b^2*d^2 - 2*a*b*d^2 + 1)^(1/2) + 1)/(2
*d)), asin(((b - x)/b)^(1/2)), -b/(a - b))*(a*d + b*d + (2*a*d + 2*b*d + a^
2*d^2 + b^2*d^2 - 2*a*b*d^2 + 1)^(1/2) + 1))/(d^2*(b - (a*d + b*d + (2*a*d
+ 2*b*d + a^2*d^2 + b^2*d^2 - 2*a*b*d^2 + 1)^(1/2) + 1)/(2*d))*(x^3 - x^2*(
a + b) + a*b*x)^(1/2)) + (b*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))
^(1/2)*ellipticPi(b/(b - (a*d + b*d - (2*a*d + 2*b*d + a^2*d^2 + b^2*d^2 -
2*a*b*d^2 + 1)^(1/2) + 1)/(2*d)), asin(((b - x)/b)^(1/2)), -b/(a - b))*(a*d
 + b*d - (2*a*d + 2*b*d + a^2*d^2 + b^2*d^2 - 2*a*b*d^2 + 1)^(1/2) + 1))/(d
^2*(b - (a*d + b*d - (2*a*d + 2*b*d + a^2*d^2 + b^2*d^2 - 2*a*b*d^2 + 1)^(1
/2) + 1)/(2*d))*(x^3 - x^2*(a + b) + a*b*x)^(1/2))
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-x**2)/(x*(-a+x)*(-b+x))**(1/2)/(a*b*d-(a*d+b*d+1)*x+d*x**2),
x)
```

```
[Out] Timed out
```

3.473 $\int \frac{-1+kx^2}{(1+kx^2)\sqrt{(1-x^2)(1-k^2x^2)}}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}}\right)}{-k-1}$$

Rubi [C] time = 0.97, antiderivative size = 113, normalized size of antiderivative = 2.97, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {6719, 6725, 419, 537}

$$\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}-\frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi\left(-k;\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + k*x^2)/((1 + k*x^2)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticF[ArcSin[x], k^2])/Sqrt[(1 - x^2)*
(1 - k^2*x^2)] - (2*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[-k, ArcSin[x
], k^2])/Sqrt[(1 - x^2)*(1 - k^2*x^2)]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+kx^2}{(1+kx^2)\sqrt{(1-x^2)(1-k^2x^2)}}\,dx = \frac{\left(\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \frac{-1+kx^2}{\sqrt{1-x^2}(1+kx^2)\sqrt{1-k^2x^2}}\,dx}{\sqrt{(1-x^2)(1-k^2x^2)}}$$
$$= \frac{\left(\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \left(\frac{1}{\sqrt{1-x^2}\sqrt{1-k^2x^2}} - \frac{2}{\sqrt{1-x^2}(1+kx^2)\sqrt{1-k^2x^2}}\right)dx}{\sqrt{(1-x^2)(1-k^2x^2)}}$$
$$= \frac{\left(\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \frac{1}{\sqrt{1-x^2}\sqrt{1-k^2x^2}}\,dx}{\sqrt{(1-x^2)(1-k^2x^2)}} - \frac{\left(2\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \frac{}{\sqrt{1-x^2}}}{\sqrt{(1-x^2)(1-k}}$$
$$= \frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}} - \frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi\left(-k;\sin^{-1}(x)\right.}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

Mathematica [C] time = 0.26, size = 63, normalized size = 1.66

$$\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\left(F\left(\sin^{-1}(x)|k^2\right)-2\Pi\left(-k;\sin^{-1}(x)|k^2\right)\right)}{\sqrt{(x^2-1)(k^2x^2-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + k*x^2)/((1 + k*x^2)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),x]
[Out] (Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*(EllipticF[ArcSin[x], k^2] - 2*EllipticPi[
-k, ArcSin[x], k^2]))/Sqrt[(-1 + x^2)*(-1 + k^2*x^2)]
```

IntegrateAlgebraic [A] time = 1.92, size = 38, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}}\right)}{-k-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + k*x^2)/((1 + k*x^2)*Sqrt[(1 - x^2)*(1 - k^2*x^2)])
,x]
[Out] ArcTan[((1 + k)*x)/Sqrt[1 + (-1 - k^2)*x^2 + k^2*x^4]]/(-1 - k)
```

fricas [A] time = 0.46, size = 37, normalized size = 0.97

$$\frac{\arctan\left(\frac{\sqrt{k^2x^4-(k^2+1)x^2+1}}{(k+1)x}\right)}{k+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^2-1)/(k*x^2+1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algorithm="f
ricas")
[Out] arctan(sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)/((k + 1)*x))/(k + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx^2-1}{(kx^2+1)\sqrt{(k^2x^2-1)(x^2-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^2-1)/(k*x^2+1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((k*x^2 - 1)/((k*x^2 + 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))), x)
```

maple [C] time = 0.07, size = 104, normalized size = 2.74

$$\frac{\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\,\mathrm{EllipticF}\left(x,k\right)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}-\frac{2\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\,\mathrm{EllipticPi}\left(x,-k,k\right)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^2-1)/(k*x^2+1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x)
```

```
[Out] (-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*EllipticF(x
,k)-2*(-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*Ellip
ticPi(x,-k,k)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx^2-1}{\left(kx^2+1\right)\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^2-1)/(k*x^2+1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((k*x^2 - 1)/((k*x^2 + 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{k\,x^2-1}{\left(k\,x^2+1\right)\,\sqrt{\left(x^2-1\right)\,\left(k^2\,x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^2 - 1)/((k*x^2 + 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)),x)
```

```
[Out] int((k*x^2 - 1)/((k*x^2 + 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx^2-1}{\sqrt{(x-1)\,(x+1)\,(kx-1)\,(kx+1)}\,\left(kx^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x**2-1)/(k*x**2+1)/((-x**2+1)*(-k**2*x**2+1))**(1/2),x)
```

```
[Out] Integral((k*x**2 - 1)/(sqrt((x - 1)*(x + 1)*(k*x - 1)*(k*x + 1))*(k*x**2 +
1)), x)
```

3.474 $\int \frac{1+kx^2}{(-1+kx^2)\sqrt{(1-x^2)(1-k^2x^2)}}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}}\right)}{1-k}$$

Rubi [C] time = 0.78, antiderivative size = 111, normalized size of antiderivative = 2.92, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {6719, 6725, 419, 537}

$$\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}-\frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi\left(k;\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + k*x^2)/((-1 + k*x^2)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticF[ArcSin[x], k^2])/Sqrt[(1 - x^2)*
(1 - k^2*x^2)] - (2*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[k, ArcSin[x]
, k^2])/Sqrt[(1 - x^2)*(1 - k^2*x^2)]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+kx^2}{\left(-1+kx^2\right)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}\,dx = \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1+kx^2}{\sqrt{1-x^2}\left(-1+kx^2\right)\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \left(\frac{1}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}} + \frac{2}{\sqrt{1-x^2}\left(-1+kx^2\right)\sqrt{1-k^2x^2}}\right)dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int}{\sqrt{\left(1-x^2\right)(}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(k;\sin^{-1}\right.}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

Mathematica [C] time = 0.22, size = 61, normalized size = 1.61

$$\frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\left(F\left(\sin^{-1}(x)|k^2\right)-2\Pi\left(k;\sin^{-1}(x)|k^2\right)\right)}{\sqrt{\left(x^2-1\right)\left(k^2x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + k*x^2)/((-1 + k*x^2)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*(EllipticF[ArcSin[x], k^2] - 2*EllipticPi[
k, ArcSin[x], k^2]))/Sqrt[(-1 + x^2)*(-1 + k^2*x^2)]
```

IntegrateAlgebraic [A] time = 1.84, size = 38, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{1-k}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + k*x^2)/((-1 + k*x^2)*Sqrt[(1 - x^2)*(1 - k^2*x^2)])
,x]
```

```
[Out] ArcTan[((-1 + k)*x)/Sqrt[1 + (-1 - k^2)*x^2 + k^2*x^4]]/(1 - k)
```

fricas [A] time = 0.47, size = 37, normalized size = 0.97

$$\frac{\arctan\left(\frac{\sqrt{k^2x^4-\left(k^2+1\right)x^2+1}}{(k-1)x}\right)}{k-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^2+1)/(k*x^2-1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algorithm="f
ricas")
```

```
[Out] arctan(sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)/((k - 1)*x))/(k - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx^2+1}{\left(kx^2-1\right)\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^2+1)/(k*x^2-1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((k*x^2 + 1)/((k*x^2 - 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))), x)
```

maple [C] time = 0.04, size = 102, normalized size = 2.68

$$\frac{\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\ \mathrm{EllipticF}\left(x,k\right)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}-\frac{2\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\ \mathrm{EllipticPi}\left(x,k,k\right)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^2+1)/(k*x^2-1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x)
```

```
[Out] (-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*EllipticF(x
,k)-2*(-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*Ellip
ticPi(x,k,k)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{kx^2+1}{\left(kx^2-1\right)\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^2+1)/(k*x^2-1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((k*x^2 + 1)/((k*x^2 - 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{k\,x^2+1}{\left(k\,x^2-1\right)\,\sqrt{\left(x^2-1\right)\,\left(k^2\,x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^2 + 1)/((k*x^2 - 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)),x)
```

```
[Out] int((k*x^2 + 1)/((k*x^2 - 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{kx^2+1}{\sqrt{\left(x-1\right)\left(x+1\right)\left(kx-1\right)\left(kx+1\right)}\,\left(kx^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x**2+1)/(k*x**2-1)/((-x**2+1)*(-k**2*x**2+1))**(1/2),x)
```

```
[Out] Integral((k*x**2 + 1)/(sqrt((x - 1)*(x + 1)*(k*x - 1)*(k*x + 1))*(k*x**2 -
1)), x)
```

3.475 $\int \frac{1-2k^2x+k^2x^2}{x\sqrt{(1-x)x(1-k^2x)}\left(-1+k^2x\right)}\,dx$

Optimal. Leaf size=38

$$-\frac{2\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{x\left(k^2x-1\right)}$$

Rubi [A] time = 0.77, antiderivative size = 26, normalized size of antiderivative = 0.68, number of steps used = 4, number of rules used = 4, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.085, Rules used = {6718, 21, 1614, 8}

$$\frac{2(1-x)}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*k^2*x + k^2*x^2)/(x*Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^2*x)),x]
```

```
[Out] (2*(1 - x))/Sqrt[(1 - x)*x*(1 - k^2*x)]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 1614

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rubi steps

$$\int \frac{1-2k^2x+k^2x^2}{x\sqrt{(1-x)x\left(1-k^2x\right)}\left(-1+k^2x\right)}\,dx = \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{1-2k^2x+k^2x^2}{\sqrt{1-x}\,x^{3/2}\sqrt{1-k^2x}\left(-1+k^2x\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$
$$= -\frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{1-2k^2x+k^2x^2}{\sqrt{1-x}\,x^{3/2}\left(1-k^2x\right)^{3/2}}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$
$$= \frac{2(1-x)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int 0\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$
$$= \frac{2(1-x)}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

Mathematica [A] time = 0.21, size = 21, normalized size = 0.55

$$-\frac{2(x-1)}{\sqrt{(x-1)x\left(k^2x-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*k^2*x + k^2*x^2)/(x*Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^2*
x)),x]
```

```
[Out] (-2*(-1 + x))/Sqrt[(-1 + x)*x*(-1 + k^2*x)]
```

IntegrateAlgebraic [A] time = 0.19, size = 38, normalized size = 1.00

$$-\frac{2\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{x\left(k^2x-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - 2*k^2*x + k^2*x^2)/(x*Sqrt[(1 - x)*x*(1 - k^2*x)]*(
-1 + k^2*x)),x]
```

```
[Out] (-2*Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3])/(x*(-1 + k^2*x))
```

fricas [A] time = 0.41, size = 36, normalized size = 0.95

$$-\frac{2\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}}{k^2x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-2*k^2*x+1)/x/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x-1),x, alg
orithm="fricas")
```

```
[Out] -2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)/(k^2*x^2 - x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^2-2k^2x+1}{\left(k^2x-1\right)\sqrt{\left(k^2x-1\right)(x-1)x}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-2*k^2*x+1)/x/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x-1),x, alg
orithm="giac")
```

```
[Out] integrate((k^2*x^2 - 2*k^2*x + 1)/((k^2*x - 1)*sqrt((k^2*x - 1)*(x - 1)*x)*
x), x)
```

maple [A] time = 0.01, size = 20, normalized size = 0.53

$$-\frac{2\left(-1+x\right)}{\sqrt{\left(-1+x\right)x\left(k^{2}x-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^2-2*k^2*x+1)/x/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x-1),x)
```

```
[Out] -2*(-1+x)/((-1+x)*x*(k^2*x-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^{2}x^{2}-2\,k^{2}x+1}{\left(k^{2}x-1\right)\sqrt{\left(k^{2}x-1\right)\left(x-1\right)x}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-2*k^2*x+1)/x/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x-1),x, alg
orithm="maxima")
```

```
[Out] integrate((k^2*x^2 - 2*k^2*x + 1)/((k^2*x - 1)*sqrt((k^2*x - 1)*(x - 1)*x)*
x), x)
```

mupad [B] time = 0.29, size = 28, normalized size = 0.74

$$-\frac{2\,\sqrt{x\,\left(k^2\,x-1\right)\,\left(x-1\right)}}{x\,\left(k^2\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^2 - 2*k^2*x + 1)/(x*(k^2*x - 1)*(x*(k^2*x - 1)*(x - 1))^(1/2)),x
)
```

```
[Out] -(2*(x*(k^2*x - 1)*(x - 1))^(1/2))/(x*(k^2*x - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^{2}x^{2}-2k^{2}x+1}{x\sqrt{x\left(x-1\right)\left(k^{2}x-1\right)}\left(k^{2}x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k**2*x**2-2*k**2*x+1)/x/((1-x)*x*(-k**2*x+1))**(1/2)/(k**2*x-1),
x)
```

```
[Out] Integral((k**2*x**2 - 2*k**2*x + 1)/(x*sqrt(x*(x - 1)*(k**2*x - 1))*(k**2*x
 - 1)), x)
```

3.476 $\int \frac{1}{x^7\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=38

$$\frac{1}{4}\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{\sqrt{x^3-1}\,\left(3x^3+2\right)}{12x^6}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 51, 63, 203}

$$\frac{\sqrt{x^3-1}}{4x^3}+\frac{1}{4}\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{\sqrt{x^3-1}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[-1 + x^3]),x]
```

```
[Out] Sqrt[-1 + x^3]/(6*x^6) + Sqrt[-1 + x^3]/(4*x^3) + ArcTan[Sqrt[-1 + x^3]]/4
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\sqrt{-1+x^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^3}\,dx, x, x^3\right)$$
$$= \frac{\sqrt{-1+x^3}}{6x^6} + \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx, x, x^3\right)$$
$$= \frac{\sqrt{-1+x^3}}{6x^6} + \frac{\sqrt{-1+x^3}}{4x^3} + \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^3\right)$$
$$= \frac{\sqrt{-1+x^3}}{6x^6} + \frac{\sqrt{-1+x^3}}{4x^3} + \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^3}\right)$$
$$= \frac{\sqrt{-1+x^3}}{6x^6} + \frac{\sqrt{-1+x^3}}{4x^3} + \frac{1}{4}\tan^{-1}\left(\sqrt{-1+x^3}\right)$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.74

$$\frac{2}{3}\sqrt{x^3-1}\ {}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[-1 + x^3]),x]
[Out] (2*Sqrt[-1 + x^3]*Hypergeometric2F1[1/2, 3, 3/2, 1 - x^3])/3
```

IntegrateAlgebraic [A] time = 0.00, size = 38, normalized size = 1.00

$$\frac{1}{4}\tan^{-1}\left(\sqrt{x^3-1}\right) + \frac{\sqrt{x^3-1}\left(3x^3+2\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*Sqrt[-1 + x^3]),x]
[Out] (Sqrt[-1 + x^3]*(2 + 3*x^3))/(12*x^6) + ArcTan[Sqrt[-1 + x^3]]/4
```

fricas [A] time = 0.40, size = 34, normalized size = 0.89

$$\frac{3\,x^6\arctan\left(\sqrt{x^3-1}\right) + \left(3\,x^3+2\right)\sqrt{x^3-1}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] 1/12*(3*x^6*arctan(sqrt(x^3 - 1)) + (3*x^3 + 2)*sqrt(x^3 - 1))/x^6
```

giac [A] time = 0.32, size = 35, normalized size = 0.92

$$\frac{3\left(x^3-1\right)^{\frac{3}{2}} + 5\sqrt{x^3-1}}{12\,x^6} + \frac{1}{4}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^3-1)^(1/2),x, algorithm="giac")
[Out] 1/12*(3*(x^3 - 1)^(3/2) + 5*sqrt(x^3 - 1))/x^6 + 1/4*arctan(sqrt(x^3 - 1))
```

maple [A] time = 0.02, size = 36, normalized size = 0.95

$$\frac{\sqrt{x^3-1}}{6x^6}+\frac{\sqrt{x^3-1}}{4x^3}+\frac{\arctan\left(\sqrt{x^3-1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^3-1)^(1/2),x)
```

```
[Out] 1/6*(x^3-1)^(1/2)/x^6+1/4*(x^3-1)^(1/2)/x^3+1/4*arctan((x^3-1)^(1/2))
```

maxima [A] time = 0.42, size = 48, normalized size = 1.26

$$\frac{3\left(x^3-1\right)^{\frac{3}{2}}+5\sqrt{x^3-1}}{12\left(2x^3+\left(x^3-1\right)^2-1\right)}+\frac{1}{4}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(x^3 - 1)^(3/2) + 5*sqrt(x^3 - 1))/(2*x^3 + (x^3 - 1)^2 - 1) + 1/4*
arctan(sqrt(x^3 - 1))
```

mupad [B] time = 0.20, size = 189, normalized size = 4.97

$$\frac{\sqrt{x^3-1}}{4x^3}+\frac{\sqrt{x^3-1}}{6x^6}-\frac{3\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^3 - 1)^(1/2)),x)
```

```
[Out] (x^3 - 1)^(1/2)/(4*x^3) + (x^3 - 1)^(1/2)/(6*x^6) - (3*((3^(1/2)*1i)/2 + 3/
2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2
)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(4*(((3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [B] time = 2.43, size = 138, normalized size = 3.63

$$\begin{cases} \frac{i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{4}-\frac{i}{4x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x^3}}}+\frac{i}{12x^{\frac{9}{2}}\sqrt{-1+\frac{1}{x^3}}}+\frac{i}{6x^{\frac{15}{2}}\sqrt{-1+\frac{1}{x^3}}} & \text{for}\ \frac{1}{|x^3|}>1 \\ -\frac{\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{4}+\frac{1}{4x^{\frac{3}{2}}\sqrt{1-\frac{1}{x^3}}}-\frac{1}{12x^{\frac{9}{2}}\sqrt{1-\frac{1}{x^3}}}-\frac{1}{6x^{\frac{15}{2}}\sqrt{1-\frac{1}{x^3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**3-1)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(x**(-3/2))/4 - I/(4*x**(3/2)*sqrt(-1 + x**(-3))) + I/(12
*x**(9/2)*sqrt(-1 + x**(-3))) + I/(6*x**(15/2)*sqrt(-1 + x**(-3))), 1/Abs(x
**3) > 1), (-asin(x**(-3/2))/4 + 1/(4*x**(3/2)*sqrt(1 - 1/x**3)) - 1/(12*x*
*(9/2)*sqrt(1 - 1/x**3)) - 1/(6*x**(15/2)*sqrt(1 - 1/x**3)), True))
```

3.477 $\int \frac{1}{x^7\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=38

$$\frac{\sqrt{x^3+1}\,(3x^3-2)}{12x^6}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 51, 63, 207}

$$\frac{\sqrt{x^3+1}}{4x^3}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{\sqrt{x^3+1}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[1 + x^3]),x]
```

```
[Out] -1/6*Sqrt[1 + x^3]/x^6 + Sqrt[1 + x^3]/(4*x^3) - ArcTanh[Sqrt[1 + x^3]]/4
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\sqrt{1+x^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+x}}\,dx,x,x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{6x^6}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx,x,x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{6x^6}+\frac{\sqrt{1+x^3}}{4x^3}+\frac{1}{8}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{6x^6}+\frac{\sqrt{1+x^3}}{4x^3}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^3}\right)\\ &= -\frac{\sqrt{1+x^3}}{6x^6}+\frac{\sqrt{1+x^3}}{4x^3}-\frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.68

$$-\frac{2}{3}\sqrt{x^3+1}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[1 + x^3]),x]
[Out] (-2*Sqrt[1 + x^3]*Hypergeometric2F1[1/2, 3, 3/2, 1 + x^3])/3
```

IntegrateAlgebraic [A] time = 0.00, size = 38, normalized size = 1.00

$$\frac{\sqrt{x^3+1}\,\left(3x^3-2\right)}{12x^6}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*Sqrt[1 + x^3]),x]
[Out] (Sqrt[1 + x^3]*(-2 + 3*x^3))/(12*x^6) - ArcTanh[Sqrt[1 + x^3]]/4
```

fricas [A] time = 0.40, size = 52, normalized size = 1.37

$$-\frac{3\,x^6\log\left(\sqrt{x^3+1}\,+1\right)-3\,x^6\log\left(\sqrt{x^3+1}\,-1\right)-2\left(3\,x^3-2\right)\sqrt{x^3+1}}{24\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] -1/24*(3*x^6*log(sqrt(x^3 + 1) + 1) - 3*x^6*log(sqrt(x^3 + 1) - 1) - 2*(3*x
^3 - 2)*sqrt(x^3 + 1))/x^6
```

giac [A] time = 0.31, size = 50, normalized size = 1.32

$$\frac{3\left(x^3+1\right)^{\frac{3}{2}}-5\,\sqrt{x^3+1}}{12\,x^6}-\frac{1}{8}\,\log\left(\sqrt{x^3+1}\,+1\right)+\frac{1}{8}\,\log\left(\left|\sqrt{x^3+1}\,-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^3+1)^(1/2),x, algorithm="giac")
[Out] 1/12*(3*(x^3 + 1)^(3/2) - 5*sqrt(x^3 + 1))/x^6 - 1/8*log(sqrt(x^3 + 1) + 1)
 + 1/8*log(abs(sqrt(x^3 + 1) - 1))
```

maple [A] time = 0.03, size = 36, normalized size = 0.95

$$-\frac{\sqrt{x^3+1}}{6x^6}+\frac{\sqrt{x^3+1}}{4x^3}-\frac{\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^3+1)^(1/2),x)
```

```
[Out] -1/6*(x^3+1)^(1/2)/x^6+1/4*(x^3+1)^(1/2)/x^3-1/4*arctanh((x^3+1)^(1/2))
```

maxima [B] time = 0.32, size = 64, normalized size = 1.68

$$-\frac{3\left(x^3+1\right)^{\frac{3}{2}}-5\sqrt{x^3+1}}{12\left(2x^3-\left(x^3+1\right)^2+1\right)}-\frac{1}{8}\log\left(\sqrt{x^3+1}+1\right)+\frac{1}{8}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*(x^3 + 1)^(3/2) - 5*sqrt(x^3 + 1))/(2*x^3 - (x^3 + 1)^2 + 1) - 1/8
*log(sqrt(x^3 + 1) + 1) + 1/8*log(sqrt(x^3 + 1) - 1)
```

mupad [B] time = 0.07, size = 189, normalized size = 4.97

$$\frac{\sqrt{x^3+1}}{4x^3}-\frac{\sqrt{x^3+1}}{6x^6}-\frac{3\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^3 + 1)^(1/2)),x)
```

```
[Out] (x^3 + 1)^(1/2)/(4*x^3) - (x^3 + 1)^(1/2)/(6*x^6) - (3*((3^(1/2)*1i)/2 + 3/
2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(4*(x^3 - x*((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*
((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [B] time = 2.49, size = 65, normalized size = 1.71

$$-\frac{\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{4}+\frac{1}{4x^{\frac{3}{2}}\sqrt{1+\frac{1}{x^3}}}+\frac{1}{12x^{\frac{9}{2}}\sqrt{1+\frac{1}{x^3}}}-\frac{1}{6x^{\frac{15}{2}}\sqrt{1+\frac{1}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**3+1)**(1/2),x)
```

```
[Out] -asinh(x**(-3/2))/4 + 1/(4*x**(3/2)*sqrt(1 + x**(-3))) + 1/(12*x**(9/2)*sqr
t(1 + x**(-3))) - 1/(6*x**(15/2)*sqrt(1 + x**(-3)))
```

3.478 $\int \frac{\sqrt{1+x^3}}{x^7} dx$

Optimal. Leaf size=38

$$\frac{1}{12}\tanh^{-1}\left(\sqrt{x^3+1}\right)+\frac{\sqrt{x^3+1}\left(-x^3-2\right)}{12x^6}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {266, 47, 51, 63, 207}

$$-\frac{\sqrt{x^3+1}}{12x^3}+\frac{1}{12}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{\sqrt{x^3+1}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^3]/x^7,x]
```

```
[Out] -1/6*Sqrt[1 + x^3]/x^6 - Sqrt[1 + x^3]/(12*x^3) + ArcTanh[Sqrt[1 + x^3]]/12
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^3}}{x^7}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^3}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{6x^6}+\frac{1}{12}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{6x^6}-\frac{\sqrt{1+x^3}}{12x^3}-\frac{1}{24}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{6x^6}-\frac{\sqrt{1+x^3}}{12x^3}-\frac{1}{12}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)\\ &= -\frac{\sqrt{1+x^3}}{6x^6}-\frac{\sqrt{1+x^3}}{12x^3}+\frac{1}{12}\tanh^{-1}\left(\sqrt{1+x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.68

$$-\frac{2}{9}\left(x^3+1\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^3]/x^7,x]
[Out] (-2*(1 + x^3)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + x^3])/9
```

IntegrateAlgebraic [A] time = 0.10, size = 38, normalized size = 1.00

$$\frac{1}{12}\tanh^{-1}\left(\sqrt{x^3+1}\right)+\frac{\sqrt{x^3+1}\left(-x^3-2\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^3]/x^7,x]
[Out] ((-2 - x^3)*Sqrt[1 + x^3])/(12*x^6) + ArcTanh[Sqrt[1 + x^3]]/12
```

fricas [A] time = 0.40, size = 49, normalized size = 1.29

$$\frac{x^6\log\left(\sqrt{x^3+1}+1\right)-x^6\log\left(\sqrt{x^3+1}-1\right)-2\left(x^3+2\right)\sqrt{x^3+1}}{24\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)/x^7,x, algorithm="fricas")
[Out] 1/24*(x^6*log(sqrt(x^3 + 1) + 1) - x^6*log(sqrt(x^3 + 1) - 1) - 2*(x^3 + 2)
*sqrt(x^3 + 1))/x^6
```

giac [B] time = 0.33, size = 78, normalized size = 2.05

$$-\frac{\sqrt{x^3+1}+\frac{1}{\sqrt{x^3+1}}}{12\left(\left(\sqrt{x^3+1}+\frac{1}{\sqrt{x^3+1}}\right)^2-4\right)}+\frac{1}{48}\log\left(\sqrt{x^3+1}+\frac{1}{\sqrt{x^3+1}}+2\right)-\frac{1}{48}\log\left(\left|\sqrt{x^3+1}+\frac{1}{\sqrt{x^3+1}}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(sqrt(x^3 + 1) + 1/sqrt(x^3 + 1))/((sqrt(x^3 + 1) + 1/sqrt(x^3 + 1))^
2 - 4) + 1/48*log(sqrt(x^3 + 1) + 1/sqrt(x^3 + 1) + 2) - 1/48*log(abs(sqrt(
x^3 + 1) + 1/sqrt(x^3 + 1) - 2))
```

maple [A] time = 0.02, size = 36, normalized size = 0.95

$$-\frac{\sqrt{x^3+1}}{6x^6}-\frac{\sqrt{x^3+1}}{12x^3}+\frac{\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(1/2)/x^7,x)
```

```
[Out] -1/6*(x^3+1)^(1/2)/x^6-1/12*(x^3+1)^(1/2)/x^3+1/12*arctanh((x^3+1)^(1/2))
```

maxima [B] time = 0.32, size = 60, normalized size = 1.58

$$\frac{\left(x^3+1\right)^{\frac{3}{2}}+\sqrt{x^3+1}}{12\left(2\,x^3-\left(x^3+1\right)^2+1\right)}+\frac{1}{24}\,\log\left(\sqrt{x^3+1}+1\right)-\frac{1}{24}\,\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/12*((x^3 + 1)^(3/2) + sqrt(x^3 + 1))/(2*x^3 - (x^3 + 1)^2 + 1) + 1/24*log
(sqrt(x^3 + 1) + 1) - 1/24*log(sqrt(x^3 + 1) - 1)
```

mupad [B] time = 0.06, size = 189, normalized size = 4.97

$$-\frac{\sqrt{x^3+1}}{12\,x^3}-\frac{\sqrt{x^3+1}}{6\,x^6}+\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\;\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)^(1/2)/x^7,x)
```

```
[Out] (((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))
^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
3/2)))/(4*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (x^3 + 1)^(1/2)/(6*x^
6) - (x^3 + 1)^(1/2)/(12*x^3)
```

sympy [B] time = 2.26, size = 65, normalized size = 1.71

$$\frac{\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{12}-\frac{1}{12x^{\frac{3}{2}}\sqrt{1+\frac{1}{x^3}}}-\frac{1}{4x^{\frac{9}{2}}\sqrt{1+\frac{1}{x^3}}}-\frac{1}{6x^{\frac{15}{2}}\sqrt{1+\frac{1}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(1/2)/x**7,x)
```

```
[Out] asinh(x**(-3/2))/12 - 1/(12*x**(3/2)*sqrt(1 + x**(-3))) - 1/(4*x**(9/2)*sqr
t(1 + x**(-3))) - 1/(6*x**(15/2)*sqrt(1 + x**(-3)))
```

3.479 $\int \frac{1+x^3}{x^7 \sqrt{-1+x^3}} \, dx$

Optimal. Leaf size=38

$$\frac{7}{12}\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{\sqrt{x^3-1}\,(7x^3+2)}{12x^6}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {446, 78, 51, 63, 203}

$$\frac{7\sqrt{x^3-1}}{12x^3}+\frac{7}{12}\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{\sqrt{x^3-1}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(x^7*Sqrt[-1 + x^3]),x]
```

```
[Out] Sqrt[-1 + x^3]/(6*x^6) + (7*Sqrt[-1 + x^3])/(12*x^3) + (7*ArcTan[Sqrt[-1 +
x^3]])/12
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^3}{x^7\sqrt{-1+x^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1+x}{\sqrt{-1+x}\,x^3}\,dx,x,x^3\right)\\ &= \frac{\sqrt{-1+x^3}}{6x^6}+\frac{7}{12}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx,x,x^3\right)\\ &= \frac{\sqrt{-1+x^3}}{6x^6}+\frac{7\sqrt{-1+x^3}}{12x^3}+\frac{7}{24}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx,x,x^3\right)\\ &= \frac{\sqrt{-1+x^3}}{6x^6}+\frac{7\sqrt{-1+x^3}}{12x^3}+\frac{7}{12}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x^3}\right)\\ &= \frac{\sqrt{-1+x^3}}{6x^6}+\frac{7\sqrt{-1+x^3}}{12x^3}+\frac{7}{12}\tan^{-1}\left(\sqrt{-1+x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 1.32

$$\frac{1}{12}\sqrt{x^3-1}\left(\frac{7\tanh^{-1}\left(\sqrt{1-x^3}\right)}{\sqrt{1-x^3}}+\frac{7x^3+2}{x^6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(x^7*Sqrt[-1 + x^3]),x]
[Out] (Sqrt[-1 + x^3]*((2 + 7*x^3)/x^6 + (7*ArcTanh[Sqrt[1 - x^3]])/Sqrt[1 - x^3]
))/12
```

IntegrateAlgebraic [A] time = 0.06, size = 38, normalized size = 1.00

$$\frac{7}{12}\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{\sqrt{x^3-1}\left(7x^3+2\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)/(x^7*Sqrt[-1 + x^3]),x]
[Out] (Sqrt[-1 + x^3]*(2 + 7*x^3))/(12*x^6) + (7*ArcTan[Sqrt[-1 + x^3]])/12
```

fricas [A] time = 0.39, size = 34, normalized size = 0.89

$$\frac{7\,x^6\arctan\left(\sqrt{x^3-1}\right)+\left(7\,x^3+2\right)\sqrt{x^3-1}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x^7/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] 1/12*(7*x^6*arctan(sqrt(x^3 - 1)) + (7*x^3 + 2)*sqrt(x^3 - 1))/x^6
```

giac [A] time = 0.31, size = 35, normalized size = 0.92

$$\frac{7\left(x^3-1\right)^{\frac{3}{2}}+9\sqrt{x^3-1}}{12\,x^6}+\frac{7}{12}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x^7/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(7*(x^3 - 1)^(3/2) + 9*sqrt(x^3 - 1))/x^6 + 7/12*arctan(sqrt(x^3 - 1))
```

maple [A] time = 0.01, size = 36, normalized size = 0.95

$$\frac{7\sqrt{x^3-1}}{12x^3}+\frac{7\arctan\left(\sqrt{x^3-1}\right)}{12}+\frac{\sqrt{x^3-1}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/x^7/(x^3-1)^(1/2),x)
```

```
[Out] 7/12*(x^3-1)^(1/2)/x^3+7/12*arctan((x^3-1)^(1/2))+1/6*(x^3-1)^(1/2)/x^6
```

maxima [A] time = 0.43, size = 60, normalized size = 1.58

$$\frac{3\left(x^3-1\right)^{\frac{3}{2}}+5\sqrt{x^3-1}}{12\left(2x^3+\left(x^3-1\right)^2-1\right)}+\frac{\sqrt{x^3-1}}{3x^3}+\frac{7}{12}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x^7/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(x^3 - 1)^(3/2) + 5*sqrt(x^3 - 1))/(2*x^3 + (x^3 - 1)^2 - 1) + 1/3*
sqrt(x^3 - 1)/x^3 + 7/12*arctan(sqrt(x^3 - 1))
```

mupad [B] time = 0.06, size = 189, normalized size = 4.97

$$\frac{7\sqrt{x^3-1}}{12x^3}+\frac{\sqrt{x^3-1}}{6x^6}-\frac{7\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/(x^7*(x^3 - 1)^(1/2)),x)
```

```
[Out] (7*(x^3 - 1)^(1/2))/(12*x^3) + (x^3 - 1)^(1/2)/(6*x^6) - (7*((3^(1/2)*1i)/2
 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3
^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(4*(((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [A] time = 45.16, size = 63, normalized size = 1.66

$$-\frac{1-\frac{1}{x^3-1}}{12\left(1+\frac{1}{x^3-1}\right)^2\sqrt{x^3-1}}-\frac{7\,\mathrm{atan}\left(\frac{1}{\sqrt{x^3-1}}\right)}{12}+\frac{2}{3\left(1+\frac{1}{x^3-1}\right)\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/x**7/(x**3-1)**(1/2),x)
```

```
[Out] -(1 - 1/(x**3 - 1))/(12*(1 + 1/(x**3 - 1))**2*sqrt(x**3 - 1)) - 7*atan(1/sq
rt(x**3 - 1))/12 + 2/(3*(1 + 1/(x**3 - 1))*sqrt(x**3 - 1))
```

3.480 $\int \frac{\sqrt{-1+x^3}\left(2+x^3\right)}{x^2\left(-2-4x^2+2x^3\right)}\,dx$

Optimal. Leaf size=38

$$\frac{\sqrt{x^3-1}}{x}-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^3-1}}\right)$$

Rubi [F] time = 0.69, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-1+x^3}\left(2+x^3\right)}{x^2\left(-2-4x^2+2x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-1 + x^3]*(2 + x^3))/(x^2*(-2 - 4*x^2 + 2*x^3)),x]
```

```
[Out] (3*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + Sqrt[-1 + x^3]/x - (3*3^(1/4)*Sqrt[2
 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSi
n[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(2*Sqrt[-((1 - x)/
(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (Sqrt[2]*3^(3/4)*(1 - x)*Sqrt[(1 +
x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[
3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
x^3]) - 2*Defer[Int][Sqrt[-1 + x^3]/(-1 - 2*x^2 + x^3), x] + (3*Defer[Int][
(x*Sqrt[-1 + x^3])/(-1 - 2*x^2 + x^3), x])/2
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x^3}\left(2+x^3\right)}{x^2\left(-2-4x^2+2x^3\right)}\,dx &= \int \left(-\frac{\sqrt{-1+x^3}}{x^2}+\frac{(-4+3x)\sqrt{-1+x^3}}{2\left(-1-2x^2+x^3\right)}\right)dx\\
&= \frac{1}{2}\int \frac{(-4+3x)\sqrt{-1+x^3}}{-1-2x^2+x^3}\,dx-\int \frac{\sqrt{-1+x^3}}{x^2}\,dx\\
&= \frac{\sqrt{-1+x^3}}{x}+\frac{1}{2}\int \left(-\frac{4\sqrt{-1+x^3}}{-1-2x^2+x^3}+\frac{3x\sqrt{-1+x^3}}{-1-2x^2+x^3}\right)dx-\frac{3}{2}\int \frac{x}{\sqrt{-1+x^3}}\,dx\\
&= \frac{\sqrt{-1+x^3}}{x}+\frac{3}{2}\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx+\frac{3}{2}\int \frac{x\sqrt{-1+x^3}}{-1-2x^2+x^3}\,dx-2\int \frac{\sqrt{-1+x^3}}{-1-2x^2+x^3}\,dx\,-\\
&= \frac{3\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{\sqrt{-1+x^3}}{x}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-\right.}{2\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}
\end{aligned}$$

Mathematica [C] time = 6.16, size = 1701, normalized size = 44.76

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[-1 + x^3]*(2 + x^3))/(x^2*(-2 - 4*x^2 + 2*x^3)),x]
```

```
[Out] Sqrt[-1 + x^3]/x + (2*Sqrt[(-1 + x)/(-1 - (-1)^(1/3))]*((-1)^(1/3) + x)*Sqr
t[(-(-1)^(2/3) + x)/(-(-1)^(1/3) - (-1)^(2/3))]*EllipticF[ArcSin[Sqrt[-(((-
1)^(2/3)*((-1)^(1/3) + x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)])/(Sqrt[((-1)^(1
/3) + x)/((-1)^(1/3) + (-1)^(2/3))]*Sqrt[-1 + x^3]) + (4*(-(-1)^(1/3) - (-1
```

```
)^(2/3))*Sqrt[(-1 + x)/(-1 - (-1)^(1/3))]*Sqrt[(((-1)^(2/3) - x)*((-1)^(1/3
) + x))/(-(-1)^(1/3) - (-1)^(2/3))^2]*EllipticPi[((-1)^(1/3) + (-1)^(2/3))/
((-1)^(1/3) + Root[-1 - 2*#1^2 + #1^3 & , 1, 0]), ArcSin[Sqrt[-(((-1)^(2/3)
*((-1)^(1/3) + x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)])/(Sqrt[-1 + x^3]*((-1)^
(1/3) + Root[-1 - 2*#1^2 + #1^3 & , 1, 0])*(Root[-1 - 2*#1^2 + #1^3 & , 1,
0] - Root[-1 - 2*#1^2 + #1^3 & , 2, 0])*(Root[-1 - 2*#1^2 + #1^3 & , 1, 0]
- Root[-1 - 2*#1^2 + #1^3 & , 3, 0])) + (2*(-(-1)^(1/3) - (-1)^(2/3))*Sqrt[
(-1 + x)/(-1 - (-1)^(1/3))]*Sqrt[(((-1)^(2/3) - x)*((-1)^(1/3) + x))/(-(-1)
^(1/3) - (-1)^(2/3))^2]*EllipticPi[((-1)^(1/3) + (-1)^(2/3))/((-1)^(1/3) +
Root[-1 - 2*#1^2 + #1^3 & , 1, 0]), ArcSin[Sqrt[-(((-1)^(2/3)*((-1)^(1/3) +
 x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)]*Root[-1 - 2*#1^2 + #1^3 & , 1, 0]^3)/
(Sqrt[-1 + x^3]*((-1)^(1/3) + Root[-1 - 2*#1^2 + #1^3 & , 1, 0])*(Root[-1 -
 2*#1^2 + #1^3 & , 1, 0] - Root[-1 - 2*#1^2 + #1^3 & , 2, 0])*(Root[-1 - 2*
#1^2 + #1^3 & , 1, 0] - Root[-1 - 2*#1^2 + #1^3 & , 3, 0])) + (4*(-(-1)^(1/
3) - (-1)^(2/3))*Sqrt[(-1 + x)/(-1 - (-1)^(1/3))]*Sqrt[(((-1)^(2/3) - x)*((
-1)^(1/3) + x))/(-(-1)^(1/3) - (-1)^(2/3))^2]*EllipticPi[((-1)^(1/3) + (-1)
^(2/3))/((-1)^(1/3) + Root[-1 - 2*#1^2 + #1^3 & , 2, 0]), ArcSin[Sqrt[-(((-
1)^(2/3)*((-1)^(1/3) + x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)])/(Sqrt[-1 + x^3
]*((-1)^(1/3) + Root[-1 - 2*#1^2 + #1^3 & , 2, 0])*(-Root[-1 - 2*#1^2 + #1^
3 & , 1, 0] + Root[-1 - 2*#1^2 + #1^3 & , 2, 0])*(Root[-1 - 2*#1^2 + #1^3 &
 , 2, 0] - Root[-1 - 2*#1^2 + #1^3 & , 3, 0])) + (2*(-(-1)^(1/3) - (-1)^(2/
3))*Sqrt[(-1 + x)/(-1 - (-1)^(1/3))]*Sqrt[(((-1)^(2/3) - x)*((-1)^(1/3) + x
))/(-(-1)^(1/3) - (-1)^(2/3))^2]*EllipticPi[((-1)^(1/3) + (-1)^(2/3))/((-1)
^(1/3) + Root[-1 - 2*#1^2 + #1^3 & , 2, 0]), ArcSin[Sqrt[-(((-1)^(2/3)*((-1
)^(1/3) + x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)]*Root[-1 - 2*#1^2 + #1^3 & ,
2, 0]^3)/(Sqrt[-1 + x^3]*((-1)^(1/3) + Root[-1 - 2*#1^2 + #1^3 & , 2, 0])*(
-Root[-1 - 2*#1^2 + #1^3 & , 1, 0] + Root[-1 - 2*#1^2 + #1^3 & , 2, 0])*(Ro
ot[-1 - 2*#1^2 + #1^3 & , 2, 0] - Root[-1 - 2*#1^2 + #1^3 & , 3, 0])) + (4*
(-(-1)^(1/3) - (-1)^(2/3))*Sqrt[(-1 + x)/(-1 - (-1)^(1/3))]*Sqrt[(((-1)^(2/
3) - x)*((-1)^(1/3) + x))/(-(-1)^(1/3) - (-1)^(2/3))^2]*EllipticPi[((-1)^(1
/3) + (-1)^(2/3))/((-1)^(1/3) + Root[-1 - 2*#1^2 + #1^3 & , 3, 0]), ArcSin[
Sqrt[-(((-1)^(2/3)*((-1)^(1/3) + x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)])/(Sqr
t[-1 + x^3]*((-1)^(1/3) + Root[-1 - 2*#1^2 + #1^3 & , 3, 0])*(-Root[-1 - 2*
#1^2 + #1^3 & , 1, 0] + Root[-1 - 2*#1^2 + #1^3 & , 3, 0])*(-Root[-1 - 2*#1
^2 + #1^3 & , 2, 0] + Root[-1 - 2*#1^2 + #1^3 & , 3, 0])) + (2*(-(-1)^(1/3)
 - (-1)^(2/3))*Sqrt[(-1 + x)/(-1 - (-1)^(1/3))]*Sqrt[(((-1)^(2/3) - x)*((-1
)^(1/3) + x))/(-(-1)^(1/3) - (-1)^(2/3))^2]*EllipticPi[((-1)^(1/3) + (-1)^(
2/3))/((-1)^(1/3) + Root[-1 - 2*#1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[-(((-1)
^(2/3)*((-1)^(1/3) + x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)]*Root[-1 - 2*#1^2
+ #1^3 & , 3, 0]^3)/(Sqrt[-1 + x^3]*((-1)^(1/3) + Root[-1 - 2*#1^2 + #1^3 &
 , 3, 0])*(-Root[-1 - 2*#1^2 + #1^3 & , 1, 0] + Root[-1 - 2*#1^2 + #1^3 & ,
 3, 0])*(-Root[-1 - 2*#1^2 + #1^3 & , 2, 0] + Root[-1 - 2*#1^2 + #1^3 & , 3
, 0]))
```

IntegrateAlgebraic [A] time = 0.56, size = 38, normalized size = 1.00

$$\frac{\sqrt{x^3-1}}{x}-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^3]*(2 + x^3))/(x^2*(-2 - 4*x^2 + 2*x^3)),x]
```

```
[Out] Sqrt[-1 + x^3]/x - Sqrt[2]*ArcTanh[(Sqrt[2]*x)/Sqrt[-1 + x^3]]
```

fricas [B] time = 0.51, size = 99, normalized size = 2.61

$$\frac{\sqrt{2}\,x\log\left(-\frac{x^6+12\,x^5+4\,x^4-2\,x^3-4\,\sqrt{2}\left(x^4+2\,x^3-x\right)\sqrt{x^3-1}-12\,x^2+1}{x^6-4\,x^5+4\,x^4-2\,x^3+4\,x^2+1}\right)+4\,\sqrt{x^3-1}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)*(x^3+2)/x^2/(2*x^3-4*x^2-2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*x*log(-(x^6 + 12*x^5 + 4*x^4 - 2*x^3 - 4*sqrt(2)*(x^4 + 2*x^3
- x)*sqrt(x^3 - 1) - 12*x^2 + 1)/(x^6 - 4*x^5 + 4*x^4 - 2*x^3 + 4*x^2 + 1))
 + 4*sqrt(x^3 - 1))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+2\right)\sqrt{x^3-1}}{2\left(x^3-2x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)*(x^3+2)/x^2/(2*x^3-4*x^2-2),x, algorithm="giac")
```

```
[Out] integrate(1/2*(x^3 + 2)*sqrt(x^3 - 1)/((x^3 - 2*x^2 - 1)*x^2), x)
```

maple [C] time = 0.12, size = 306, normalized size = 8.05

$$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{\sqrt{2}\left(\sum_{\_a=\operatorname{RootOf}\left(\_Z^3-2\_Z^2-1\right)}\frac{\_a\left(-\_a^2+\_a+1\right)\left(-3-i\sqrt{3}\right)\sqrt{\frac{-1+x}{-3-i\sqrt{3}}}\sqrt{\frac{1+2x-i\sqrt{3}}{3-i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}+2x+1}{3+i\sqrt{3}}}\operatorname{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{3\_a^2}{4}+\frac{3\_a}{4}+\frac{3}{4}-\frac{i\_a^2\sqrt{3}}{4}+\frac{i\sqrt{3}\,\_a}{4}+\frac{i\sqrt{3}}{4},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}\right)}{2}+\frac{\sqrt{x^3-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/2)*(x^3+2)/x^2/(2*x^3-4*x^2-2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+1/2*2^(1/2)*sum(_alpha*(-_alpha^
2+_alpha+1)*(-3-I*3^(1/2))*((-1+x)/(-3-I*3^(1/2)))^(1/2)*((1+2*x-I*3^(1/2))
/(3-I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x+1)/(I*3^(1/2)+3))^(1/2)/(x^3-1)^(1/2)
*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-3/4*_alpha^2+3/4*_alpha+3/
4-1/4*I*3^(1/2)*_alpha^2+1/4*I*3^(1/2)*_alpha+1/4*I*3^(1/2),((3/2+1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^3-2*_Z^2-1))+(x^3-1)^(1/
2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\int \frac{\left(x^3+2\right)\sqrt{x^3-1}}{\left(x^3-2x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)*(x^3+2)/x^2/(2*x^3-4*x^2-2),x, algorithm="maxima")
```

```
[Out] 1/2*integrate((x^3 + 2)*sqrt(x^3 - 1)/((x^3 - 2*x^2 - 1)*x^2), x)
```

mupad [B] time = 0.82, size = 56, normalized size = 1.47

$$\frac{\sqrt{x^3-1}}{x}+\frac{\sqrt{2}\,\ln\left(\frac{2\,x^2+x^3-2\,\sqrt{2}\,x\,\sqrt{x^3-1}-1}{-8\,x^3+16\,x^2+8}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3 - 1)^(1/2)*(x^3 + 2))/(x^2*(4*x^2 - 2*x^3 + 2)),x)
```

```
[Out] (x^3 - 1)^(1/2)/x + (2^(1/2)*log((2*x^2 + x^3 - 2*2^(1/2)*x*(x^3 - 1)^(1/2)
 - 1)/(16*x^2 - 8*x^3 + 8)))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{2\sqrt{x^3-1}}{x^5-2x^4-x^2}\,dx + \int \frac{x^3\sqrt{x^3-1}}{x^5-2x^4-x^2}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/2)*(x**3+2)/x**2/(2*x**3-4*x**2-2),x)
```

```
[Out] (Integral(2*sqrt(x**3 - 1)/(x**5 - 2*x**4 - x**2), x) + Integral(x**3*sqrt(
x**3 - 1)/(x**5 - 2*x**4 - x**2), x))/2
```

3.481 $\int x^5\sqrt{b+ax^3}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{ax^3+b}\left(3a^2x^6+abx^3-2b^2\right)}{45a^2}$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{2\left(ax^3+b\right)^{5/2}}{15a^2}-\frac{2b\left(ax^3+b\right)^{3/2}}{9a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[b + a*x^3],x]
```

```
[Out] (-2*b*(b + a*x^3)^(3/2))/(9*a^2) + (2*(b + a*x^3)^(5/2))/(15*a^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{b+ax^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int x\sqrt{b+ax}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(-\frac{b\sqrt{b+ax}}{a}+\frac{(b+ax)^{3/2}}{a}\right)dx,x,x^3\right)\\ &= -\frac{2b\left(b+ax^3\right)^{3/2}}{9a^2}+\frac{2\left(b+ax^3\right)^{5/2}}{15a^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.74

$$\frac{2\left(ax^3+b\right)^{3/2}\left(3ax^3-2b\right)}{45a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[b + a*x^3],x]
```

```
[Out] (2*(b + a*x^3)^(3/2)*(-2*b + 3*a*x^3))/(45*a^2)
```

IntegrateAlgebraic [A] time = 0.04, size = 38, normalized size = 1.00

$$\frac{2\sqrt{ax^3+b}\left(3a^2x^6+abx^3-2b^2\right)}{45a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[b + a*x^3],x]
[Out] (2*Sqrt[b + a*x^3]*(-2*b^2 + a*b*x^3 + 3*a^2*x^6))/(45*a^2)
```

fricas [A] time = 0.40, size = 34, normalized size = 0.89

$$\frac{2\left(3\,a^2x^6+abx^3-2\,b^2\right)\sqrt{ax^3+b}}{45\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a*x^3+b)^(1/2),x, algorithm="fricas")
[Out] 2/45*(3*a^2*x^6 + a*b*x^3 - 2*b^2)*sqrt(a*x^3 + b)/a^2
```

giac [A] time = 0.36, size = 29, normalized size = 0.76

$$\frac{2\left(3\left(ax^3+b\right)^{\frac{5}{2}}-5\left(ax^3+b\right)^{\frac{3}{2}}b\right)}{45\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a*x^3+b)^(1/2),x, algorithm="giac")
[Out] 2/45*(3*(a*x^3 + b)^(5/2) - 5*(a*x^3 + b)^(3/2)*b)/a^2
```

maple [A] time = 0.01, size = 25, normalized size = 0.66

$$\frac{2\left(a\,x^3+b\right)^{\frac{3}{2}}\left(3a\,x^3-2b\right)}{45a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a*x^3+b)^(1/2),x)
[Out] 2/45*(a*x^3+b)^(3/2)*(3*a*x^3-2*b)/a^2
```

maxima [A] time = 0.32, size = 30, normalized size = 0.79

$$\frac{2\left(ax^3+b\right)^{\frac{5}{2}}}{15\,a^2}-\frac{2\left(ax^3+b\right)^{\frac{3}{2}}b}{9\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a*x^3+b)^(1/2),x, algorithm="maxima")
[Out] 2/15*(a*x^3 + b)^(5/2)/a^2 - 2/9*(a*x^3 + b)^(3/2)*b/a^2
```

mupad [B] time = 0.33, size = 29, normalized size = 0.76

$$-\frac{10\,b\left(a\,x^3+b\right)^{3/2}-6\left(a\,x^3+b\right)^{5/2}}{45\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b + a*x^3)^(1/2),x)
[Out] -(10*b*(b + a*x^3)^(3/2) - 6*(b + a*x^3)^(5/2))/(45*a^2)
```

sympy [A] time = 0.47, size = 66, normalized size = 1.74

$$\begin{cases} \frac{2x^6\sqrt{ax^3+b}}{15} + \frac{2bx^3\sqrt{ax^3+b}}{45a} - \frac{4b^2\sqrt{ax^3+b}}{45a^2} & \text{for } a \neq 0 \\ \frac{\sqrt{b}\,x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a*x**3+b)**(1/2),x)
```

```
[Out] Piecewise((2*x**6*sqrt(a*x**3 + b)/15 + 2*b*x**3*sqrt(a*x**3 + b)/(45*a) -
4*b**2*sqrt(a*x**3 + b)/(45*a**2), Ne(a, 0)), (sqrt(b)*x**6/6, True))
```

3.482 $\int \frac{(-3+x^4)\sqrt[3]{1+x^4}(1+x^3+x^4)}{x^8}\,dx$

Optimal. Leaf size=38

$$\frac{3\sqrt[3]{x^4+1}\left(4x^8+7x^7+8x^4+7x^3+4\right)}{28x^7}$$

Rubi [A] time = 0.08, antiderivative size = 33, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1833, 1584, 446, 74, 1478, 449}

$$\frac{3\left(x^4+1\right)^{4/3}}{4x^4}+\frac{3\left(x^4+1\right)^{7/3}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + x^4)*(1 + x^4)^(1/3)*(1 + x^3 + x^4))/x^8,x]
```

```
[Out] (3*(1 + x^4)^(4/3))/(4*x^4) + (3*(1 + x^4)^(7/3))/(7*x^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1478

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n
_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p
]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
```

```
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-3+x^4\right)\sqrt[3]{1+x^4}\left(1+x^3+x^4\right)}{x^8}\,dx &= \int \left(\frac{\sqrt[3]{1+x^4}\left(-3x^2+x^6\right)}{x^7}+\frac{\sqrt[3]{1+x^4}\left(-3-2x^4+x^8\right)}{x^8}\right)dx\\
&= \int \frac{\sqrt[3]{1+x^4}\left(-3x^2+x^6\right)}{x^7}\,dx+\int \frac{\sqrt[3]{1+x^4}\left(-3-2x^4+x^8\right)}{x^8}\,dx\\
&= \int \frac{\left(-3+x^4\right)\sqrt[3]{1+x^4}}{x^5}\,dx+\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{4/3}}{x^8}\,dx\\
&= \frac{3\left(1+x^4\right)^{7/3}}{7x^7}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{(-3+x)\sqrt[3]{1+x}}{x^2}\,dx,x,x^4\right)\\
&= \frac{3\left(1+x^4\right)^{4/3}}{4x^4}+\frac{3\left(1+x^4\right)^{7/3}}{7x^7}
\end{aligned}
$$

Mathematica [C] time = 0.19, size = 169, normalized size = 4.45

$$
\frac{1}{16}\left(-9\left(x^4+1\right)^{4/3}{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};x^4+1\right)+16x\,{}_2F_1\left(-\frac{1}{3},\frac{1}{4};\frac{5}{4};-x^4\right)+12\sqrt[3]{x^4+1}+4\log\left(1-\sqrt[3]{x^4+1}\right)-2\log\left(\left(x^4+1\right)^{2/3}+\sqrt[3]{x^4+1}+1\right)-4\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}+1}{\sqrt{3}}\right)\right)+\frac{3\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{3};-\frac{3}{4};-x^4\right)}{7x^7}+\frac{2\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{3};\frac{1}{4};-x^4\right)}{3x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + x^4)*(1 + x^4)^(1/3)*(1 + x^3 + x^4))/x^8,x]
```

```
[Out] (3*Hypergeometric2F1[-7/4, -1/3, -3/4, -x^4])/(7*x^7) + (2*Hypergeometric2F
1[-3/4, -1/3, 1/4, -x^4])/(3*x^3) + (12*(1 + x^4)^(1/3) - 4*Sqrt[3]*ArcTan[
(1 + 2*(1 + x^4)^(1/3))/Sqrt[3]] + 16*x*Hypergeometric2F1[-1/3, 1/4, 5/4, -
x^4] - 9*(1 + x^4)^(4/3)*Hypergeometric2F1[4/3, 2, 7/3, 1 + x^4] + 4*Log[1
- (1 + x^4)^(1/3)] - 2*Log[1 + (1 + x^4)^(1/3) + (1 + x^4)^(2/3)])/16
```

IntegrateAlgebraic [A] time = 0.35, size = 28, normalized size = 0.74

$$
\frac{3\left(x^4+1\right)^{4/3}\left(4x^4+7x^3+4\right)}{28x^7}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 + x^4)^(1/3)*(1 + x^3 + x^4))/x^8,x]
```

```
[Out] (3*(1 + x^4)^(4/3)*(4 + 7*x^3 + 4*x^4))/(28*x^7)
```

fricas [A] time = 0.42, size = 34, normalized size = 0.89

$$
\frac{3\left(4\,x^8+7\,x^7+8\,x^4+7\,x^3+4\right)\left(x^4+1\right)^{\frac{1}{3}}}{28\,x^7}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(1/3)*(x^4+x^3+1)/x^8,x, algorithm="fricas")
```

```
[Out] 3/28*(4*x^8 + 7*x^7 + 8*x^4 + 7*x^3 + 4)*(x^4 + 1)^(1/3)/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3+1\right)\left(x^4+1\right)^{\frac{1}{3}}\left(x^4-3\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(1/3)*(x^4+x^3+1)/x^8,x, algorithm="giac")
```

```
[Out] integrate((x^4 + x^3 + 1)*(x^4 + 1)^(1/3)*(x^4 - 3)/x^8, x)
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$\frac{3\left(x^4+1\right)^{\frac{4}{3}}\left(4x^4+7x^3+4\right)}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4+1)^(1/3)*(x^4+x^3+1)/x^8,x)
```

```
[Out] 3/28*(x^4+1)^(4/3)*(4*x^4+7*x^3+4)/x^7
```

maxima [A] time = 0.46, size = 34, normalized size = 0.89

$$\frac{3\left(4\,x^8+7\,x^7+8\,x^4+7\,x^3+4\right)\left(x^4+1\right)^{\frac{1}{3}}}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(1/3)*(x^4+x^3+1)/x^8,x, algorithm="maxima")
```

```
[Out] 3/28*(4*x^8 + 7*x^7 + 8*x^4 + 7*x^3 + 4)*(x^4 + 1)^(1/3)/x^7
```

mupad [B] time = 0.45, size = 50, normalized size = 1.32

$$\left(\frac{3\,x}{7}+\frac{3}{4}\right)\,{\left(x^4+1\right)}^{1/3}+\frac{6\,{\left(x^4+1\right)}^{1/3}}{7\,x^3}+\frac{3\,{\left(x^4+1\right)}^{1/3}}{4\,x^4}+\frac{3\,{\left(x^4+1\right)}^{1/3}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/3)*(x^4 - 3)*(x^3 + x^4 + 1))/x^8,x)
```

```
[Out] ((3*x)/7 + 3/4)*(x^4 + 1)^(1/3) + (6*(x^4 + 1)^(1/3))/(7*x^3) + (3*(x^4 + 1
)^(1/3))/(4*x^4) + (3*(x^4 + 1)^(1/3))/(7*x^7)
```

sympy [C] time = 4.81, size = 178, normalized size = 4.68

$$-\frac{x^{\frac{4}{3}}\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{4\Gamma\left(\frac{2}{3}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}-\frac{\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{3}\\ \frac{1}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{2x^3\Gamma\left(\frac{1}{4}\right)}-\frac{3\Gamma\left(-\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{4},-\frac{1}{3}\\ -\frac{3}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}+\frac{3\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{4x^{\frac{8}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4+1)**(1/3)*(x**4+x**3+1)/x**8,x)
```

```
[Out] -x**(4/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), exp_polar(I*pi)/x**4)/(4*
gamma(2/3)) + x*gamma(1/4)*hyper((-1/3, 1/4), (5/4,), x**4*exp_polar(I*pi))
/(4*gamma(5/4)) - gamma(-3/4)*hyper((-3/4, -1/3), (1/4,), x**4*exp_polar(I*
pi))/(2*x**3*gamma(1/4)) - 3*gamma(-7/4)*hyper((-7/4, -1/3), (-3/4,), x**4*
exp_polar(I*pi))/(4*x**7*gamma(-3/4)) + 3*gamma(2/3)*hyper((-1/3, 2/3), (5/
3,), exp_polar(I*pi)/x**4)/(4*x**(8/3)*gamma(5/3))
```

3.483 $\int \frac{(-3+x^4)(1+x^4)^{2/3}(1+x^3+x^4)}{x^9} dx$

Optimal. Leaf size=38

$$\frac{3\left(x^4+1\right)^{2/3}\left(5x^8+8x^7+10x^4+8x^3+5\right)}{40x^8}$$

Rubi [A] time = 0.09, antiderivative size = 33, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1833, 1584, 449, 1474, 847, 74}

$$\frac{3\left(x^4+1\right)^{8/3}}{8x^8}+\frac{3\left(x^4+1\right)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + x^4)*(1 + x^4)^(2/3)*(1 + x^3 + x^4))/x^9,x]
```

```
[Out] (3*(1 + x^4)^(5/3))/(5*x^5) + (3*(1 + x^4)^(8/3))/(8*x^8)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 847

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d
+ (c*x)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g,
 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
&&  !IGtQ[n, 0]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
```

```
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}\left(1+x^3+x^4\right)}{x^9}\,dx &= \int \left(\frac{\left(1+x^4\right)^{2/3}\left(-3x^2+x^6\right)}{x^8}+\frac{\left(1+x^4\right)^{2/3}\left(-3-2x^4+x^8\right)}{x^9}\right)dx \\
&= \int \frac{\left(1+x^4\right)^{2/3}\left(-3x^2+x^6\right)}{x^8}\,dx+\int \frac{\left(1+x^4\right)^{2/3}\left(-3-2x^4+x^8\right)}{x^9}\,dx \\
&= \frac{1}{4}\operatorname{Subst}\left(\int \frac{(1+x)^{2/3}\left(-3-2x+x^2\right)}{x^3}\,dx,x,x^4\right)+\int \frac{\left(-3+x^4\right)\left(1\right.}{x^6} \\
&= \frac{3\left(1+x^4\right)^{5/3}}{5x^5}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{(-3+x)(1+x)^{5/3}}{x^3}\,dx,x,x^4\right) \\
&= \frac{3\left(1+x^4\right)^{5/3}}{5x^5}+\frac{3\left(1+x^4\right)^{8/3}}{8x^8}
\end{aligned}$$

Mathematica [C] time = 0.23, size = 158, normalized size = 4.16

$$\frac{24\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};-x^4\right)+x^4\left(x\left(-12\left(x^4+1\right)^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};x^4+1\right)+18\left(x^4+1\right)^{5/3}\,{}_2F_1\left(\frac{5}{3},3;\frac{8}{3};x^4+1\right)+5\left(3\left(\left(x^4+1\right)^{2/3}+\log\left(1-\sqrt[3]{x^4+1}\right)\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}+1}{\sqrt{3}}\right)-4\log(x)\right)\right)-40\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{4};\frac{3}{4};-x^4\right)\right)}{40x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + x^4)*(1 + x^4)^(2/3)*(1 + x^3 + x^4))/x^9,x]
```

```
[Out] (24*Hypergeometric2F1[-5/4, -2/3, -1/4, -x^4] + x^4*(-40*Hypergeometric2F1[
-2/3, -1/4, 3/4, -x^4] + x*(-12*(1 + x^4)^(5/3)*Hypergeometric2F1[5/3, 2, 8
/3, 1 + x^4] + 18*(1 + x^4)^(5/3)*Hypergeometric2F1[5/3, 3, 8/3, 1 + x^4] +
 5*(2*Sqrt[3]*ArcTan[(1 + 2*(1 + x^4)^(1/3))/Sqrt[3]] - 4*Log[x] + 3*((1 +
x^4)^(2/3) + Log[1 - (1 + x^4)^(1/3)])))))/(40*x^5)
```

IntegrateAlgebraic [A] time = 0.29, size = 28, normalized size = 0.74

$$\frac{3\left(x^4+1\right)^{5/3}\left(5x^4+8x^3+5\right)}{40x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 + x^4)^(2/3)*(1 + x^3 + x^4))/x^9,x]
```

```
[Out] (3*(1 + x^4)^(5/3)*(5 + 8*x^3 + 5*x^4))/(40*x^8)
```

fricas [A] time = 0.40, size = 34, normalized size = 0.89

$$\frac{3\left(5\,x^8+8\,x^7+10\,x^4+8\,x^3+5\right)\left(x^4+1\right)^{\frac{2}{3}}}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)*(x^4+x^3+1)/x^9,x, algorithm="fricas")
```

```
[Out] 3/40*(5*x^8 + 8*x^7 + 10*x^4 + 8*x^3 + 5)*(x^4 + 1)^(2/3)/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3+1\right)\left(x^4+1\right)^{\frac{2}{3}}\left(x^4-3\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)*(x^4+x^3+1)/x^9,x, algorithm="giac")
```

```
[Out] integrate((x^4 + x^3 + 1)*(x^4 + 1)^(2/3)*(x^4 - 3)/x^9, x)
```

maple [A] time = 0.01, size = 25, normalized size = 0.66

$$\frac{3\left(x^4+1\right)^{\frac{5}{3}}\left(5x^4+8x^3+5\right)}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4+1)^(2/3)*(x^4+x^3+1)/x^9,x)
```

```
[Out] 3/40*(x^4+1)^(5/3)*(5*x^4+8*x^3+5)/x^8
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{2\left(x^4+1\right)^{\frac{5}{3}}+\left(x^4+1\right)^{\frac{2}{3}}}{8\left(2x^4-\left(x^4+1\right)^2+1\right)}+\int \frac{\left(x^5+x^4-2x-3\right)\left(x^4+1\right)^{\frac{2}{3}}}{x^6}\,dx-\frac{1}{24}\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{12}\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)*(x^4+x^3+1)/x^9,x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 + 1)^(1/3) + 1)) - 1/8*(2*(x^4 + 1)
^(5/3) + (x^4 + 1)^(2/3))/(2*x^4 - (x^4 + 1)^2 + 1) + integrate((x^5 + x^4
- 2*x - 3)*(x^4 + 1)^(2/3)/x^6, x) - 1/24*log((x^4 + 1)^(2/3) + (x^4 + 1)^(
1/3) + 1) + 1/12*log((x^4 + 1)^(1/3) - 1)
```

mupad [B] time = 0.47, size = 58, normalized size = 1.53

$$\frac{3\left(x^4+1\right)^{2/3}}{8}+\frac{3\left(x^4+1\right)^{2/3}}{5\,x}+\frac{3\left(x^4+1\right)^{2/3}}{4\,x^4}+\frac{3\left(x^4+1\right)^{2/3}}{5\,x^5}+\frac{3\left(x^4+1\right)^{2/3}}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(2/3)*(x^4 - 3)*(x^3 + x^4 + 1))/x^9,x)
```

```
[Out] (3*(x^4 + 1)^(2/3))/8 + (3*(x^4 + 1)^(2/3))/(5*x) + (3*(x^4 + 1)^(2/3))/(4*
x^4) + (3*(x^4 + 1)^(2/3))/(5*x^5) + (3*(x^4 + 1)^(2/3))/(8*x^8)
```

sympy [C] time = 5.46, size = 180, normalized size = 4.74

$$-\frac{x^{\frac{8}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{4\Gamma\left(\frac{1}{3}\right)}+\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{3\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{2}{3}\\ -\frac{1}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}+\frac{\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{3\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{4x^{\frac{16}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4+1)**(2/3)*(x**4+x**3+1)/x**9,x)
```

```
[Out] -x**(8/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), exp_polar(I*pi)/x**4)/(4*
gamma(1/3)) + gamma(-1/4)*hyper((-2/3, -1/4), (3/4,), x**4*exp_polar(I*pi))
/(4*x*gamma(3/4)) - 3*gamma(-5/4)*hyper((-5/4, -2/3), (-1/4,), x**4*exp_pol
ar(I*pi))/(4*x**5*gamma(-1/4)) + gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_
polar(I*pi)/x**4)/(2*x**(4/3)*gamma(4/3)) + 3*gamma(4/3)*hyper((-2/3, 4/3),
 (7/3,), exp_polar(I*pi)/x**4)/(4*x**(16/3)*gamma(7/3))
```

3.484 $\int \frac{(-1+x^4)^{2/3}(3+x^4)(-2-x^3+2x^4)}{x^9}\,dx$

Optimal. Leaf size=38

$$\frac{3\left(x^4-1\right)^{2/3}\left(5x^8-4x^7-10x^4+4x^3+5\right)}{20x^8}$$

Rubi [A] time = 0.09, antiderivative size = 33, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1833, 1584, 449, 1474, 847, 74}

$$\frac{3\left(x^4-1\right)^{8/3}}{4x^8}-\frac{3\left(x^4-1\right)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4)^(2/3)*(3 + x^4)*(-2 - x^3 + 2*x^4))/x^9,x]

[Out] (-3*(-1 + x^4)^(5/3))/(5*x^5) + (3*(-1 + x^4)^(8/3))/(4*x^8)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 847

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d
+ (c*x)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g,
 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
&&  !IGtQ[n, 0]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
```

```
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)\left(-2-x^3+2x^4\right)}{x^9}\,dx &= \int \left(\frac{\left(-1+x^4\right)^{2/3}\left(-3x^2-x^6\right)}{x^8}+\frac{\left(-1+x^4\right)^{2/3}\left(-6+4x^4+2x^8\right)}{x^9}\right) dx\\
&= \int \frac{\left(-1+x^4\right)^{2/3}\left(-3x^2-x^6\right)}{x^8}\,dx+\int \frac{\left(-1+x^4\right)^{2/3}\left(-6+4x^4+2x^8\right)}{x^9}\\
&= \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{(-1+x)^{2/3}\left(-6+4x+2x^2\right)}{x^3}\,dx,x,x^4\right)+\int \frac{\left(-3-x^4\right)}{}\\
&= -\frac{3\left(-1+x^4\right)^{5/3}}{5x^5}+\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{(-1+x)^{5/3}(6+2x)}{x^3}\,dx,x,x^4\right)\\
&= -\frac{3\left(-1+x^4\right)^{5/3}}{5x^5}+\frac{3\left(-1+x^4\right)^{8/3}}{4x^8}
\end{aligned}
$$

Mathematica [C] time = 0.12, size = 155, normalized size = 4.08

$$\frac{\left(x^4-1\right)^{2/3}\left(x^4\left(20\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{4};\frac{3}{4};x^4\right)+3x\left(1-x^4\right)^{2/3}\left(-6x^4\,{}_2F_1\left(\frac{5}{3},3;\frac{8}{3};1-x^4\right)-5\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-x^4\right)+4\left(x^4-1\right)\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};1-x^4\right)+6\,{}_2F_1\left(\frac{5}{3},3;\frac{8}{3};1-x^4\right)+5\right)\right)+12\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};x^4\right)\right)}{20x^5\left(1-x^4\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)^(2/3)*(3 + x^4)*(-2 - x^3 + 2*x^4))/x^9,x]
```

```
[Out] ((-1 + x^4)^(2/3)*(12*Hypergeometric2F1[-5/4, -2/3, -1/4, x^4] + x^4*(20*Hy
pergeometric2F1[-2/3, -1/4, 3/4, x^4] + 3*x*(1 - x^4)^(2/3)*(5 - 5*Hypergeo
metric2F1[2/3, 1, 5/3, 1 - x^4] + 4*(-1 + x^4)*Hypergeometric2F1[5/3, 2, 8/
3, 1 - x^4] + 6*Hypergeometric2F1[5/3, 3, 8/3, 1 - x^4] - 6*x^4*Hypergeomet
ric2F1[5/3, 3, 8/3, 1 - x^4]))))/(20*x^5*(1 - x^4)^(2/3))
```

IntegrateAlgebraic [A] time = 0.31, size = 28, normalized size = 0.74

$$\frac{3\left(x^4-1\right)^{5/3}\left(5x^4-4x^3-5\right)}{20x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)^(2/3)*(3 + x^4)*(-2 - x^3 + 2*x^4))/x^9,x]
```

```
[Out] (3*(-1 + x^4)^(5/3)*(-5 - 4*x^3 + 5*x^4))/(20*x^8)
```

fricas [A] time = 0.41, size = 34, normalized size = 0.89

$$\frac{3\left(5\,x^8-4\,x^7-10\,x^4+4\,x^3+5\right)\left(x^4-1\right)^{\frac{2}{3}}}{20\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)*(2*x^4-x^3-2)/x^9,x, algorithm="fricas")
```

```
[Out] 3/20*(5*x^8 - 4*x^7 - 10*x^4 + 4*x^3 + 5)*(x^4 - 1)^(2/3)/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^4-x^3-2\right)\left(x^4+3\right)\left(x^4-1\right)^{\frac{2}{3}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)*(2*x^4-x^3-2)/x^9,x, algorithm="giac")
```

```
[Out] integrate((2*x^4 - x^3 - 2)*(x^4 + 3)*(x^4 - 1)^(2/3)/x^9, x)
```

maple [A] time = 0.01, size = 36, normalized size = 0.95

$$\frac{3\left(x^2+1\right)(-1+x)(1+x)\left(5x^4-4x^3-5\right)\left(x^4-1\right)^{\frac{2}{3}}}{20x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(2/3)*(x^4+3)*(2*x^4-x^3-2)/x^9,x)
```

```
[Out] 3/20*(x^2+1)*(-1+x)*(1+x)*(5*x^4-4*x^3-5)*(x^4-1)^(2/3)/x^8
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4-1\right)^{\frac{1}{3}}-1\right)\right)-\frac{2\left(x^4-1\right)^{\frac{5}{3}}-\left(x^4-1\right)^{\frac{2}{3}}}{4\left(2x^4+\left(x^4-1\right)^2-1\right)}+\int\frac{\left(2x^5-x^4+4x-3\right)\left(x^2+1\right)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}(x-1)^{\frac{2}{3}}}{x^6}\,dx-\frac{1}{12}\log\left(\left(x^4-1\right)^{\frac{2}{3}}-\left(x^4-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(\left(x^4-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)*(2*x^4-x^3-2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 - 1)^(1/3) - 1)) - 1/4*(2*(x^4 - 1)
^(5/3) - (x^4 - 1)^(2/3))/(2*x^4 + (x^4 - 1)^2 - 1) + integrate((2*x^5 - x^
4 + 4*x - 3)*(x^2 + 1)^(2/3)*(x + 1)^(2/3)*(x - 1)^(2/3)/x^6, x) - 1/12*log
((x^4 - 1)^(2/3) - (x^4 - 1)^(1/3) + 1) + 1/6*log((x^4 - 1)^(1/3) + 1)
```

mupad [B] time = 0.54, size = 58, normalized size = 1.53

$$\frac{3\left(x^4-1\right)^{2/3}}{4}-\frac{3\left(x^4-1\right)^{2/3}}{5x}-\frac{3\left(x^4-1\right)^{2/3}}{2x^4}+\frac{3\left(x^4-1\right)^{2/3}}{5x^5}+\frac{3\left(x^4-1\right)^{2/3}}{4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^4 - 1)^(2/3)*(x^4 + 3)*(x^3 - 2*x^4 + 2))/x^9,x)
```

```
[Out] (3*(x^4 - 1)^(2/3))/4 - (3*(x^4 - 1)^(2/3))/(5*x) - (3*(x^4 - 1)^(2/3))/(2*
x^4) + (3*(x^4 - 1)^(2/3))/(5*x^5) + (3*(x^4 - 1)^(2/3))/(4*x^8)
```

sympy [C] time = 5.95, size = 187, normalized size = 4.92

$$-\frac{x^{\frac{8}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\\frac{1}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^4}\right)}{2\Gamma\left(\frac{1}{3}\right)}+\frac{e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|x^4\right)}{4x\Gamma\left(\frac{3}{4}\right)}+\frac{3e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{2}{3}\\-\frac{1}{4}\end{matrix}\middle|x^4\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}-\frac{\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^4}\right)}{x^{\frac{4}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{3\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^4}\right)}{2x^{\frac{16}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(2/3)*(x**4+3)*(2*x**4-x**3-2)/x**9,x)
```

```
[Out] -x**(8/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), exp_polar(2*I*pi)/x**4)/(
2*gamma(1/3)) + exp(-I*pi/3)*gamma(-1/4)*hyper((-2/3, -1/4), (3/4,), x**4)/
(4*x*gamma(3/4)) + 3*exp(-I*pi/3)*gamma(-5/4)*hyper((-5/4, -2/3), (-1/4,),
x**4)/(4*x**5*gamma(-1/4)) - gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_pola
r(2*I*pi)/x**4)/(x**(4/3)*gamma(4/3)) + 3*gamma(4/3)*hyper((-2/3, 4/3), (7/
3,), exp_polar(2*I*pi)/x**4)/(2*x**(16/3)*gamma(7/3))
```

3.485 $$\int \frac{\left(-1+x^5\right)^{3/4}\left(4+x^5\right)\left(-1-x^4+x^5\right)}{x^{12}}\,dx$$

Optimal. Leaf size=38

$$\frac{4\left(x^5-1\right)^{3/4}\left(7x^{10}-11x^9-14x^5+11x^4+7\right)}{77x^{11}}$$

Rubi [B] time = 0.19, antiderivative size = 88, normalized size of antiderivative = 2.32, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1820, 1825, 1835, 1586, 449}

$$\frac{15\left(x^5-1\right)^{3/4}}{11x}-\frac{5\left(x^5-1\right)^{3/4}}{22x^6}-\frac{15\left(x^5-1\right)^{3/4}}{14x^2}+\frac{1}{154}\left(x^5-1\right)^{3/4}\left(\frac{56}{x^{11}}+\frac{88}{x^7}-\frac{77}{x^6}+\frac{77}{x^2}-\frac{154}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^5)^(3/4)*(4 + x^5)*(-1 - x^4 + x^5))/x^12,x]
```

```
[Out] ((56/x^11 + 88/x^7 - 77/x^6 + 77/x^2 - 154/x)*(-1 + x^5)^(3/4))/154 - (5*(-
1 + x^5)^(3/4))/(22*x^6) - (15*(-1 + x^5)^(3/4))/(14*x^2) + (15*(-1 + x^5)^
(3/4))/(11*x)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(-1+x^5\right)^{3/4}\left(4+x^5\right)\left(-1-x^4+x^5\right)}{x^{12}}\,dx = \frac{1}{154}\left(\frac{56}{x^{11}}+\frac{88}{x^7}-\frac{77}{x^6}+\frac{77}{x^2}-\frac{154}{x}\right)\left(-1+x^5\right)^{3/4}-\frac{15}{4}\int \frac{\frac{4}{11}+\frac{4x^4}{7}}{x^7 \cdots}$$

$$= \frac{1}{154}\left(\frac{56}{x^{11}}+\frac{88}{x^7}-\frac{77}{x^6}+\frac{77}{x^2}-\frac{154}{x}\right)\left(-1+x^5\right)^{3/4}-\frac{5\left(-1+x^5\right)^{3/4}}{22x^6} \cdots$$

$$= \frac{1}{154}\left(\frac{56}{x^{11}}+\frac{88}{x^7}-\frac{77}{x^6}+\frac{77}{x^2}-\frac{154}{x}\right)\left(-1+x^5\right)^{3/4}-\frac{5\left(-1+x^5\right)^{3/4}}{22x^6} \cdots$$

$$= \frac{1}{154}\left(\frac{56}{x^{11}}+\frac{88}{x^7}-\frac{77}{x^6}+\frac{77}{x^2}-\frac{154}{x}\right)\left(-1+x^5\right)^{3/4}-\frac{5\left(-1+x^5\right)^{3/4}}{22x^6} \cdots$$

$$= \frac{1}{154}\left(\frac{56}{x^{11}}+\frac{88}{x^7}-\frac{77}{x^6}+\frac{77}{x^2}-\frac{154}{x}\right)\left(-1+x^5\right)^{3/4}-\frac{5\left(-1+x^5\right)^{3/4}}{22x^6} \cdots$$

$$= \frac{1}{154}\left(\frac{56}{x^{11}}+\frac{88}{x^7}-\frac{77}{x^6}+\frac{77}{x^2}-\frac{154}{x}\right)\left(-1+x^5\right)^{3/4}-\frac{5\left(-1+x^5\right)^{3/4}}{22x^6} \cdots$$

$$= \frac{1}{154}\left(\frac{56}{x^{11}}+\frac{88}{x^7}-\frac{77}{x^6}+\frac{77}{x^2}-\frac{154}{x}\right)\left(-1+x^5\right)^{3/4}-\frac{5\left(-1+x^5\right)^{3/4}}{22x^6} \cdots$$

Mathematica [C] time = 0.07, size = 117, normalized size = 3.08

$$\frac{\left(x^5-1\right)^{3/4}\left(56\,{}_2F_1\left(-\frac{11}{5},-\frac{3}{4};-\frac{6}{5};x^5\right)+11x^4\left(8\,{}_2F_1\left(-\frac{7}{5},-\frac{3}{4};-\frac{2}{5};x^5\right)-7x\left(2x^5\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{5};\frac{4}{5};x^5\right)+{}_2F_1\left(-\frac{6}{5},-\frac{3}{4};-\frac{1}{5};x^5\right)-x^4\,{}_2F_1\left(-\frac{3}{4},-\frac{2}{5};\frac{3}{5};x^5\right)\right)\right)\right)}{154x^{11}\left(1-x^5\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^5)^(3/4)*(4 + x^5)*(-1 - x^4 + x^5))/x^12,x]
```

```
[Out] ((-1 + x^5)^(3/4)*(56*Hypergeometric2F1[-11/5, -3/4, -6/5, x^5] + 11*x^4*(8
*Hypergeometric2F1[-7/5, -3/4, -2/5, x^5] - 7*x*(Hypergeometric2F1[-6/5, -3
/4, -1/5, x^5] - x^4*Hypergeometric2F1[-3/4, -2/5, 3/5, x^5] + 2*x^5*Hyperg
eometric2F1[-3/4, -1/5, 4/5, x^5]))))/(154*x^11*(1 - x^5)^(3/4))
```

IntegrateAlgebraic [A] time = 3.12, size = 28, normalized size = 0.74

$$\frac{4\left(x^5-1\right)^{7/4}\left(7x^5-11x^4-7\right)}{77x^{11}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^5)^(3/4)*(4 + x^5)*(-1 - x^4 + x^5))/x^12,x]
```

```
[Out] (4*(-1 + x^5)^(7/4)*(-7 - 11*x^4 + 7*x^5))/(77*x^11)
```

fricas [A] time = 0.39, size = 34, normalized size = 0.89

$$\frac{4\left(7\,x^{10}-11\,x^9-14\,x^5+11\,x^4+7\right)\left(x^5-1\right)^{\frac{3}{4}}}{77\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(3/4)*(x^5+4)*(x^5-x^4-1)/x^12,x, algorithm="fricas")
```

```
[Out] 4/77*(7*x^10 - 11*x^9 - 14*x^5 + 11*x^4 + 7)*(x^5 - 1)^(3/4)/x^11
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{5}-x^{4}-1\right)\left(x^{5}+4\right)\left(x^{5}-1\right)^{\frac{3}{4}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(3/4)*(x^5+4)*(x^5-x^4-1)/x^12,x, algorithm="giac")
[Out] integrate((x^5 - x^4 - 1)*(x^5 + 4)*(x^5 - 1)^(3/4)/x^12, x)
```

maple [A] time = 0.01, size = 40, normalized size = 1.05

$$\frac{4\left(x^{5}-1\right)^{\frac{3}{4}}\left(7x^{5}-11x^{4}-7\right)\left(-1+x\right)\left(x^{4}+x^{3}+x^{2}+x+1\right)}{77x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)^(3/4)*(x^5+4)*(x^5-x^4-1)/x^12,x)
[Out] 4/77*(x^5-1)^(3/4)*(7*x^5-11*x^4-7)*(-1+x)*(x^4+x^3+x^2+x+1)/x^11
```

maxima [A] time = 0.48, size = 46, normalized size = 1.21

$$\frac{4\left(7\,x^{10}-11\,x^{9}-14\,x^{5}+11\,x^{4}+7\right)\left(x^{4}+x^{3}+x^{2}+x+1\right)^{\frac{3}{4}}(x-1)^{\frac{3}{4}}}{77\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(3/4)*(x^5+4)*(x^5-x^4-1)/x^12,x, algorithm="maxima")
[Out] 4/77*(7*x^10 - 11*x^9 - 14*x^5 + 11*x^4 + 7)*(x^4 + x^3 + x^2 + x + 1)^(3/4
)*(x - 1)^(3/4)/x^11
```

mupad [B] time = 0.83, size = 61, normalized size = 1.61

$$\frac{4\left(x^{5}-1\right)^{3/4}}{11\,x}-\frac{4\left(x^{5}-1\right)^{3/4}}{7\,x^{2}}-\frac{8\left(x^{5}-1\right)^{3/4}}{11\,x^{6}}+\frac{4\left(x^{5}-1\right)^{3/4}}{7\,x^{7}}+\frac{4\left(x^{5}-1\right)^{3/4}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^5 - 1)^(3/4)*(x^5 + 4)*(x^4 - x^5 + 1))/x^12,x)
[Out] (4*(x^5 - 1)^(3/4))/(11*x) - (4*(x^5 - 1)^(3/4))/(7*x^2) - (8*(x^5 - 1)^(3/
4))/(11*x^6) + (4*(x^5 - 1)^(3/4))/(7*x^7) + (4*(x^5 - 1)^(3/4))/(11*x^11)
```

sympy [C] time = 6.70, size = 199, normalized size = 5.24

$$-\frac{e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{1}{5}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{5}\\ \frac{4}{5}\end{matrix}\middle|x^{5}\right)}{5x\Gamma\left(\frac{4}{5}\right)}+\frac{e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{2}{5}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{2}{5}\\ \frac{3}{5}\end{matrix}\middle|x^{5}\right)}{5x^{2}\Gamma\left(\frac{3}{5}\right)}-\frac{3e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{6}{5}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{6}{5},-\frac{3}{4}\\ -\frac{1}{5}\end{matrix}\middle|x^{5}\right)}{5x^{6}\Gamma\left(-\frac{1}{5}\right)}+\frac{4e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{7}{5}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{5},-\frac{3}{4}\\ -\frac{2}{5}\end{matrix}\middle|x^{5}\right)}{5x^{7}\Gamma\left(-\frac{2}{5}\right)}+\frac{4e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{11}{5}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{11}{5},-\frac{3}{4}\\ -\frac{6}{5}\end{matrix}\middle|x^{5}\right)}{5x^{11}\Gamma\left(-\frac{6}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)**(3/4)*(x**5+4)*(x**5-x**4-1)/x**12,x)
[Out] -exp(-I*pi/4)*gamma(-1/5)*hyper((-3/4, -1/5), (4/5,), x**5)/(5*x*gamma(4/5)
) + exp(-I*pi/4)*gamma(-2/5)*hyper((-3/4, -2/5), (3/5,), x**5)/(5*x**2*gamm
a(3/5)) - 3*exp(-I*pi/4)*gamma(-6/5)*hyper((-6/5, -3/4), (-1/5,), x**5)/(5*
x**6*gamma(-1/5)) + 4*exp(-I*pi/4)*gamma(-7/5)*hyper((-7/5, -3/4), (-2/5,),
 x**5)/(5*x**7*gamma(-2/5)) + 4*exp(-I*pi/4)*gamma(-11/5)*hyper((-11/5, -3/
4), (-6/5,), x**5)/(5*x**11*gamma(-6/5))
```

3.486 $\int \frac{(1+x^5)^{2/3}(1+x^3+x^5)(-3+2x^5)}{x^9}\,dx$

Optimal. Leaf size=38

$$\frac{3(x^5+1)^{2/3}(5x^{10}+8x^8+10x^5+8x^3+5)}{40x^8}$$

Rubi [B] time = 0.17, antiderivative size = 89, normalized size of antiderivative = 2.34, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1826, 1835, 1586, 1584, 449}

$$-\frac{6(x^5+1)^{2/3}}{5x^5}-\frac{15(x^5+1)^{2/3}}{56x^8}+\frac{15(x^5+1)^{2/3}}{4x^3}+\frac{3(x^5+1)^{2/3}(35x^{11}+56x^9-280x^6+168x^4+60x)}{280x^9}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^5)^(2/3)*(1 + x^3 + x^5)*(-3 + 2*x^5))/x^9,x]
```

```
[Out] (-15*(1 + x^5)^(2/3))/(56*x^8) - (6*(1 + x^5)^(2/3))/(5*x^5) + (15*(1 + x^5
)^(2/3))/(4*x^3) + (3*(1 + x^5)^(2/3)*(60*x + 168*x^4 - 280*x^6 + 56*x^9 +
35*x^11))/(280*x^9)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(1+x^5\right)^{2/3}\left(1+x^3+x^5\right)\left(-3+2x^5\right)}{x^9}\,dx = \frac{3\left(1+x^5\right)^{2/3}\left(60x+168x^4-280x^6+56x^9+35x^{11}\right)}{280x^9} + \frac{10}{3}\int \frac{\frac{9}{14}+\ldots}{\ldots}$$

$$= -\frac{15\left(1+x^5\right)^{2/3}}{56x^8} + \frac{3\left(1+x^5\right)^{2/3}\left(60x+168x^4-280x^6+56x^9+35x^{1\ldots}\right)}{280x^9}$$

$$= -\frac{15\left(1+x^5\right)^{2/3}}{56x^8} + \frac{3\left(1+x^5\right)^{2/3}\left(60x+168x^4-280x^6+56x^9+35x^{1\ldots}\right)}{280x^9}$$

$$= -\frac{15\left(1+x^5\right)^{2/3}}{56x^8} + \frac{3\left(1+x^5\right)^{2/3}\left(60x+168x^4-280x^6+56x^9+35x^{1\ldots}\right)}{280x^9}$$

$$= -\frac{15\left(1+x^5\right)^{2/3}}{56x^8} - \frac{6\left(1+x^5\right)^{2/3}}{5x^5} + \frac{3\left(1+x^5\right)^{2/3}\left(60x+168x^4-280x^{\ldots}\right)}{280x^9}$$

$$= -\frac{15\left(1+x^5\right)^{2/3}}{56x^8} - \frac{6\left(1+x^5\right)^{2/3}}{5x^5} + \frac{3\left(1+x^5\right)^{2/3}\left(60x+168x^4-280x^{\ldots}\right)}{280x^9}$$

$$= -\frac{15\left(1+x^5\right)^{2/3}}{56x^8} - \frac{6\left(1+x^5\right)^{2/3}}{5x^5} + \frac{15\left(1+x^5\right)^{2/3}}{4x^3} + \frac{3\left(1+x^5\right)^{2/3}\left(60x\ldots\right)}{\ldots}$$

Mathematica [C] time = 0.27, size = 153, normalized size = 4.03

$$-\frac{9}{25}\left(x^5+1\right)^{5/3} {}_2F_1\left(\frac{5}{3},2;\frac{8}{3};x^5+1\right) + \frac{3\,{}_2F_1\left(-\frac{8}{5},-\frac{2}{3};-\frac{3}{5};-x^5\right)}{8x^8} + \frac{{}_2F_1\left(-\frac{2}{3},-\frac{3}{5};\frac{2}{5};-x^5\right)}{3x^3} + x^2\,{}_2F_1\left(-\frac{2}{3},\frac{2}{5};\frac{7}{5};-x^5\right) + \frac{1}{5}\left(3\left(x^5+1\right)^{2/3} + 3\log\left(1-\sqrt[3]{x^5+1}\right) + 2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^5+1}+1}{\sqrt{3}}\right) - 5\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^5)^(2/3)*(1 + x^3 + x^5)*(-3 + 2*x^5))/x^9,x]
```

```
[Out] (3*Hypergeometric2F1[-8/5, -2/3, -3/5, -x^5])/(8*x^8) + Hypergeometric2F1[-
2/3, -3/5, 2/5, -x^5]/(3*x^3) + x^2*Hypergeometric2F1[-2/3, 2/5, 7/5, -x^5]
 - (9*(1 + x^5)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 + x^5])/25 + (3*(1 +
 x^5)^(2/3) + 2*Sqrt[3]*ArcTan[(1 + 2*(1 + x^5)^(1/3))/Sqrt[3]] - 5*Log[x]
+ 3*Log[1 - (1 + x^5)^(1/3)])/5
```

IntegrateAlgebraic [A] time = 0.98, size = 28, normalized size = 0.74

$$\frac{3\left(x^5+1\right)^{5/3}\left(5x^5+8x^3+5\right)}{40x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^5)^(2/3)*(1 + x^3 + x^5)*(-3 + 2*x^5))/x^9,x]
```

```
[Out] (3*(1 + x^5)^(5/3)*(5 + 8*x^3 + 5*x^5))/(40*x^8)
```

fricas [A] time = 0.40, size = 34, normalized size = 0.89

$$\frac{3\left(5\,x^{10}+8\,x^8+10\,x^5+8\,x^3+5\right)\left(x^5+1\right)^{\frac{2}{3}}}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)*(x^5+x^3+1)*(2*x^5-3)/x^9,x, algorithm="fricas")
```

```
[Out] 3/40*(5*x^10 + 8*x^8 + 10*x^5 + 8*x^3 + 5)*(x^5 + 1)^(2/3)/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{5}-3\right)\left(x^{5}+x^{3}+1\right)\left(x^{5}+1\right)^{\frac{2}{3}}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)*(x^5+x^3+1)*(2*x^5-3)/x^9,x, algorithm="giac")
```

```
[Out] integrate((2*x^5 - 3)*(x^5 + x^3 + 1)*(x^5 + 1)^(2/3)/x^9, x)
```

maple [A] time = 0.00, size = 44, normalized size = 1.16

$$\frac{3\left(1+x\right)\left(x^{4}-x^{3}+x^{2}-x+1\right)\left(5x^{5}+8x^{3}+5\right)\left(x^{5}+1\right)^{\frac{2}{3}}}{40x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)^(2/3)*(x^5+x^3+1)*(2*x^5-3)/x^9,x)
```

```
[Out] 3/40*(1+x)*(x^4-x^3+x^2-x+1)*(5*x^5+8*x^3+5)*(x^5+1)^(2/3)/x^8
```

maxima [A] time = 0.51, size = 50, normalized size = 1.32

$$\frac{3\left(5\,x^{10}+8\,x^{8}+10\,x^{5}+8\,x^{3}+5\right)\left(x^{4}-x^{3}+x^{2}-x+1\right)^{\frac{2}{3}}\left(x+1\right)^{\frac{2}{3}}}{40\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)*(x^5+x^3+1)*(2*x^5-3)/x^9,x, algorithm="maxima")
```

```
[Out] 3/40*(5*x^10 + 8*x^8 + 10*x^5 + 8*x^3 + 5)*(x^4 - x^3 + x^2 - x + 1)^(2/3)*
(x + 1)^(2/3)/x^8
```

mupad [B] time = 0.38, size = 52, normalized size = 1.37

$${\left(x^5+1\right)}^{2/3}\,\left(\frac{3\,x^2}{8}+\frac{3}{5}\right)+\frac{3\,{\left(x^5+1\right)}^{2/3}}{4\,x^3}+\frac{3\,{\left(x^5+1\right)}^{2/3}}{5\,x^5}+\frac{3\,{\left(x^5+1\right)}^{2/3}}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 + 1)^(2/3)*(2*x^5 - 3)*(x^3 + x^5 + 1))/x^9,x)
```

```
[Out] (x^5 + 1)^(2/3)*((3*x^2)/8 + 3/5) + (3*(x^5 + 1)^(2/3))/(4*x^3) + (3*(x^5 +
 1)^(2/3))/(5*x^5) + (3*(x^5 + 1)^(2/3))/(8*x^8)
```

sympy [C] time = 5.60, size = 184, normalized size = 4.84

$$-\frac{2x^{\frac{10}{3}}\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^{5}}\right)}{5\Gamma\left(\frac{1}{3}\right)}+\frac{2x^{2}\Gamma\left(\frac{2}{5}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{2}{5}\\ \frac{7}{5}\end{matrix}\middle|x^{5}e^{i\pi}\right)}{5\Gamma\left(\frac{7}{5}\right)}-\frac{\Gamma\left(-\frac{3}{5}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},-\frac{3}{5}\\ \frac{2}{5}\end{matrix}\middle|x^{5}e^{i\pi}\right)}{5x^{3}\Gamma\left(\frac{2}{5}\right)}-\frac{3\Gamma\left(-\frac{8}{5}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{8}{5},-\frac{2}{3}\\ -\frac{3}{5}\end{matrix}\middle|x^{5}e^{i\pi}\right)}{5x^{8}\Gamma\left(-\frac{3}{5}\right)}+\frac{3\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^{5}}\right)}{5x^{\frac{5}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+1)**(2/3)*(x**5+x**3+1)*(2*x**5-3)/x**9,x)
```

```
[Out] -2*x**(10/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), exp_polar(I*pi)/x**5)/
(5*gamma(1/3)) + 2*x**2*gamma(2/5)*hyper((-2/3, 2/5), (7/5,), x**5*exp_pola
r(I*pi))/(5*gamma(7/5)) - gamma(-3/5)*hyper((-2/3, -3/5), (2/5,), x**5*exp_
polar(I*pi))/(5*x**3*gamma(2/5)) - 3*gamma(-8/5)*hyper((-8/5, -2/3), (-3/5,
), x**5*exp_polar(I*pi))/(5*x**8*gamma(-3/5)) + 3*gamma(1/3)*hyper((-2/3, 1
/3), (4/3,), exp_polar(I*pi)/x**5)/(5*x**(5/3)*gamma(4/3))
```

3.487 $\int \frac{(-1+x^5)^{2/3}(-1+x^3+x^5)(3+2x^5)}{x^9}\,dx$

Optimal. Leaf size=38

$$\frac{3(x^5-1)^{2/3}(5x^{10}+8x^8-10x^5-8x^3+5)}{40x^8}$$

Rubi [B] time = 0.17, antiderivative size = 89, normalized size of antiderivative = 2.34, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1826, 1835, 1586, 1584, 449}

$$\frac{6(x^5-1)^{2/3}}{5x^5}-\frac{15(x^5-1)^{2/3}}{56x^8}-\frac{15(x^5-1)^{2/3}}{4x^3}+\frac{3(x^5-1)^{2/3}(35x^{11}+56x^9+280x^6-168x^4+60x)}{280x^9}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^5)^(2/3)*(-1 + x^3 + x^5)*(3 + 2*x^5))/x^9,x]
```

```
[Out] (-15*(-1 + x^5)^(2/3))/(56*x^8) + (6*(-1 + x^5)^(2/3))/(5*x^5) - (15*(-1 +
x^5)^(2/3))/(4*x^3) + (3*(-1 + x^5)^(2/3)*(60*x - 168*x^4 + 280*x^6 + 56*x^
9 + 35*x^11))/(280*x^9)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(-1+x^5\right)^{2/3}\left(-1+x^3+x^5\right)\left(3+2x^5\right)}{x^9}\,dx = \frac{3\left(-1+x^5\right)^{2/3}\left(60x-168x^4+280x^6+56x^9+35x^{11}\right)}{280x^9} - \frac{10}{3}\int$$

$$= -\frac{15\left(-1+x^5\right)^{2/3}}{56x^8} + \frac{3\left(-1+x^5\right)^{2/3}\left(60x-168x^4+280x^6+56x^9\right.}{280x^9}$$

$$= -\frac{15\left(-1+x^5\right)^{2/3}}{56x^8} + \frac{3\left(-1+x^5\right)^{2/3}\left(60x-168x^4+280x^6+56x^9\right.}{280x^9}$$

$$= -\frac{15\left(-1+x^5\right)^{2/3}}{56x^8} + \frac{3\left(-1+x^5\right)^{2/3}\left(60x-168x^4+280x^6+56x^9\right.}{280x^9}$$

$$= -\frac{15\left(-1+x^5\right)^{2/3}}{56x^8} + \frac{6\left(-1+x^5\right)^{2/3}}{5x^5} + \frac{3\left(-1+x^5\right)^{2/3}\left(60x-168x\right.}{28}$$

$$= -\frac{15\left(-1+x^5\right)^{2/3}}{56x^8} + \frac{6\left(-1+x^5\right)^{2/3}}{5x^5} + \frac{3\left(-1+x^5\right)^{2/3}\left(60x-168x\right.}{28}$$

$$= -\frac{15\left(-1+x^5\right)^{2/3}}{56x^8} + \frac{6\left(-1+x^5\right)^{2/3}}{5x^5} - \frac{15\left(-1+x^5\right)^{2/3}}{4x^3} + 3\left(-1+\right.$$

Mathematica [C] time = 0.12, size = 139, normalized size = 3.66

$$\frac{\left(x^5-1\right)^{2/3}\left(225\,{}_2F_1\left(-\frac{8}{5},-\frac{2}{3};-\frac{3}{5};x^5\right)+8x^5\left(75x^5\,{}_2F_1\left(-\frac{2}{3},\frac{2}{5};\frac{7}{5};x^5\right)-25\,{}_2F_1\left(-\frac{2}{3},-\frac{3}{5};\frac{2}{5};x^5\right)+9\left(1-x^5\right)^{2/3}x^3\left(-5\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-x^5\right)+3\left(x^5-1\right)\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};1-x^5\right)+5\right)\right)\right)}{600x^8\left(1-x^5\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^5)^(2/3)*(-1 + x^3 + x^5)*(3 + 2*x^5))/x^9,x]
```

```
[Out] ((-1 + x^5)^(2/3)*(225*Hypergeometric2F1[-8/5, -2/3, -3/5, x^5] + 8*x^5*(-2
5*Hypergeometric2F1[-2/3, -3/5, 2/5, x^5] + 75*x^5*Hypergeometric2F1[-2/3,
2/5, 7/5, x^5] + 9*x^3*(1 - x^5)^(2/3)*(5 - 5*Hypergeometric2F1[2/3, 1, 5/3
, 1 - x^5] + 3*(-1 + x^5)*Hypergeometric2F1[5/3, 2, 8/3, 1 - x^5]))))/(600*
x^8*(1 - x^5)^(2/3))
```

IntegrateAlgebraic [A] time = 0.98, size = 28, normalized size = 0.74

$$\frac{3\left(x^5-1\right)^{5/3}\left(5x^5+8x^3-5\right)}{40x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^5)^(2/3)*(-1 + x^3 + x^5)*(3 + 2*x^5))/x^9,x]
```

```
[Out] (3*(-1 + x^5)^(5/3)*(-5 + 8*x^3 + 5*x^5))/(40*x^8)
```

fricas [A] time = 0.40, size = 34, normalized size = 0.89

$$\frac{3\left(5\,x^{10}+8\,x^8-10\,x^5-8\,x^3+5\right)\left(x^5-1\right)^{\frac{2}{3}}}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(2/3)*(x^5+x^3-1)*(2*x^5+3)/x^9,x, algorithm="fricas")
```

[Out] 3/40*(5*x^10 + 8*x^8 - 10*x^5 - 8*x^3 + 5)*(x^5 - 1)^(2/3)/x^8

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 x^{5}+3\right)\left(x^{5}+x^{3}-1\right)\left(x^{5}-1\right)^{\frac{2}{3}}}{x^{9}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5-1)^(2/3)*(x^5+x^3-1)*(2*x^5+3)/x^9,x, algorithm="giac")

[Out] integrate((2*x^5 + 3)*(x^5 + x^3 - 1)*(x^5 - 1)^(2/3)/x^9, x)

maple [A] time = 0.01, size = 40, normalized size = 1.05

$$\frac{3 \left(x^{4}+x^{3}+x^{2}+x +1\right) \left(-1+x \right) \left(5 x^{5}+8 x^{3}-5\right) \left(x^{5}-1\right)^{\frac{2}{3}}}{40 x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5-1)^(2/3)*(x^5+x^3-1)*(2*x^5+3)/x^9,x)

[Out] 3/40*(x^4+x^3+x^2+x+1)*(-1+x)*(5*x^5+8*x^3-5)*(x^5-1)^(2/3)/x^8

maxima [A] time = 0.51, size = 46, normalized size = 1.21

$$\frac{3 \left(5 x^{10}+8 x^{8}-10 x^{5}-8 x^{3}+5\right)\left(x^{4}+x^{3}+x^{2}+x+1\right)^{\frac{2}{3}} (x-1)^{\frac{2}{3}}}{40 x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5-1)^(2/3)*(x^5+x^3-1)*(2*x^5+3)/x^9,x, algorithm="maxima")

[Out] 3/40*(5*x^10 + 8*x^8 - 10*x^5 - 8*x^3 + 5)*(x^4 + x^3 + x^2 + x + 1)^(2/3)*
(x - 1)^(2/3)/x^8

mupad [B] time = 0.49, size = 52, normalized size = 1.37

$$\left(x^5-1\right)^{2/3}\,\left(\frac{3\,x^2}{8}+\frac{3}{5}\right)-\frac{3\,\left(x^5-1\right)^{2/3}}{4\,x^3}-\frac{3\,\left(x^5-1\right)^{2/3}}{5\,x^5}+\frac{3\,\left(x^5-1\right)^{2/3}}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^5 - 1)^(2/3)*(2*x^5 + 3)*(x^3 + x^5 - 1))/x^9,x)

[Out] (x^5 - 1)^(2/3)*((3*x^2)/8 + 3/5) - (3*(x^5 - 1)^(2/3))/(4*x^3) - (3*(x^5 -
 1)^(2/3))/(5*x^5) + (3*(x^5 - 1)^(2/3))/(8*x^8)

sympy [C] time = 5.71, size = 194, normalized size = 5.11

$$- \frac{2 x^{\frac{10}{3}} \Gamma\left(- \frac{2}{3}\right) {{}_{2}F_{1}\left(\begin{matrix} - \frac{2}{3}, - \frac{2}{3} \\ \frac{1}{3} \end{matrix}\middle| {\frac{e^{2 i \pi}}{x^{5}}} \right)}}{5 \Gamma\left(\frac{1}{3}\right)} + \frac{2 x^{2} e^{\frac{2 i \pi}{3}} \Gamma\left(\frac{2}{5}\right) {{}_{2}F_{1}\left(\begin{matrix} - \frac{2}{3}, \frac{2}{5} \\ \frac{7}{5} \end{matrix}\middle| {x^{5}} \right)}}{5 \Gamma\left(\frac{7}{5}\right)} - \frac{e^{- \frac{i \pi}{3}} \Gamma\left(- \frac{3}{5}\right) {{}_{2}F_{1}\left(\begin{matrix} - \frac{2}{3}, - \frac{3}{5} \\ \frac{2}{5} \end{matrix}\middle| {x^{5}} \right)}}{5 x^{3} \Gamma\left(\frac{2}{5}\right)} + \frac{3 e^{- \frac{i \pi}{3}} \Gamma\left(- \frac{8}{5}\right) {{}_{2}F_{1}\left(\begin{matrix} - \frac{8}{5}, - \frac{2}{3} \\ - \frac{3}{5} \end{matrix}\middle| {x^{5}} \right)}}{5 x^{8} \Gamma\left(- \frac{3}{5}\right)} - \frac{3 \Gamma\left(\frac{1}{3}\right) {{}_{2}F_{1}\left(\begin{matrix} - \frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| {\frac{e^{2 i \pi}}{x^{5}}} \right)}}{5 x^{\frac{5}{3}} \Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**5-1)**(2/3)*(x**5+x**3-1)*(2*x**5+3)/x**9,x)

[Out] -2*x**(10/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), exp_polar(2*I*pi)/x**5
)/(5*gamma(1/3)) + 2*x**2*exp(2*I*pi/3)*gamma(2/5)*hyper((-2/3, 2/5), (7/5,
), x**5)/(5*gamma(7/5)) - exp(-I*pi/3)*gamma(-3/5)*hyper((-2/3, -3/5), (2/5
,), x**5)/(5*x**3*gamma(2/5)) + 3*exp(-I*pi/3)*gamma(-8/5)*hyper((-8/5, -2/
3), (-3/5,), x**5)/(5*x**8*gamma(-3/5)) - 3*gamma(1/3)*hyper((-2/3, 1/3), (
4/3,), exp_polar(2*I*pi)/x**5)/(5*x**(5/3)*gamma(4/3))

3.488 $\int \frac{(-4+x^5)(1+x^5)^{3/4}(2-x^4+2x^5)}{x^{12}}\,dx$

Optimal. Leaf size=38

$$\frac{4\left(x^5+1\right)^{3/4}\left(14x^{10}-11x^9+28x^5-11x^4+14\right)}{77x^{11}}$$

Rubi [B] time = 0.19, antiderivative size = 88, normalized size of antiderivative = 2.32, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1820, 1825, 1835, 1586, 449}

$$\frac{30\left(x^5+1\right)^{3/4}}{11x}+\frac{5\left(x^5+1\right)^{3/4}}{11x^6}-\frac{15\left(x^5+1\right)^{3/4}}{14x^2}+\frac{1}{154}\left(x^5+1\right)^{3/4}\left(\frac{112}{x^{11}}-\frac{88}{x^7}+\frac{154}{x^6}+\frac{77}{x^2}-\frac{308}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-4 + x^5)*(1 + x^5)^(3/4)*(2 - x^4 + 2*x^5))/x^12,x]
```

```
[Out] ((112/x^11 - 88/x^7 + 154/x^6 + 77/x^2 - 308/x)*(1 + x^5)^(3/4))/154 + (5*(
1 + x^5)^(3/4))/(11*x^6) - (15*(1 + x^5)^(3/4))/(14*x^2) + (30*(1 + x^5)^(3
/4))/(11*x)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rule 1825

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Dist[b*n*p, Int[x^(m + n
)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b},
 x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1,
 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(-4+x^5\right)\left(1+x^5\right)^{3/4}\left(2-x^4+2x^5\right)}{x^{12}}\,dx = \frac{1}{154}\left(\frac{112}{x^{11}}-\frac{88}{x^7}+\frac{154}{x^6}+\frac{77}{x^2}-\frac{308}{x}\right)\left(1+x^5\right)^{3/4}-\frac{15}{4}\int \frac{\frac{8}{11}-\frac{4x^4}{7}+x}{x^7\sqrt[4]{1}}$$

$$= \frac{1}{154}\left(\frac{112}{x^{11}}-\frac{88}{x^7}+\frac{154}{x^6}+\frac{77}{x^2}-\frac{308}{x}\right)\left(1+x^5\right)^{3/4}+\frac{5\left(1+x^5\right)^{3/4}}{11x^6}+\frac{5}{16}$$

$$= \frac{1}{154}\left(\frac{112}{x^{11}}-\frac{88}{x^7}+\frac{154}{x^6}+\frac{77}{x^2}-\frac{308}{x}\right)\left(1+x^5\right)^{3/4}+\frac{5\left(1+x^5\right)^{3/4}}{11x^6}+\frac{5}{16}$$

$$= \frac{1}{154}\left(\frac{112}{x^{11}}-\frac{88}{x^7}+\frac{154}{x^6}+\frac{77}{x^2}-\frac{308}{x}\right)\left(1+x^5\right)^{3/4}+\frac{5\left(1+x^5\right)^{3/4}}{11x^6}+\frac{5}{16}$$

$$= \frac{1}{154}\left(\frac{112}{x^{11}}-\frac{88}{x^7}+\frac{154}{x^6}+\frac{77}{x^2}-\frac{308}{x}\right)\left(1+x^5\right)^{3/4}+\frac{5\left(1+x^5\right)^{3/4}}{11x^6}+\frac{5}{16}$$

$$= \frac{1}{154}\left(\frac{112}{x^{11}}-\frac{88}{x^7}+\frac{154}{x^6}+\frac{77}{x^2}-\frac{308}{x}\right)\left(1+x^5\right)^{3/4}+\frac{5\left(1+x^5\right)^{3/4}}{11x^6}-\frac{15}{}$$

$$= \frac{1}{154}\left(\frac{112}{x^{11}}-\frac{88}{x^7}+\frac{154}{x^6}+\frac{77}{x^2}-\frac{308}{x}\right)\left(1+x^5\right)^{3/4}+\frac{5\left(1+x^5\right)^{3/4}}{11x^6}-\frac{15}{}$$

Mathematica [C] time = 0.04, size = 106, normalized size = 2.79

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{5};\frac{4}{5};-x^5\right)}{x}+\frac{8\,{}_2F_1\left(-\frac{11}{5},-\frac{3}{4};-\frac{6}{5};-x^5\right)}{11x^{11}}-\frac{4\,{}_2F_1\left(-\frac{7}{5},-\frac{3}{4};-\frac{2}{5};-x^5\right)}{7x^7}+\frac{{}_2F_1\left(-\frac{6}{5},-\frac{3}{4};-\frac{1}{5};-x^5\right)}{x^6}+\frac{{}_2F_1\left(-\frac{3}{4},-\frac{2}{5};\frac{3}{5};-x^5\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-4 + x^5)*(1 + x^5)^(3/4)*(2 - x^4 + 2*x^5))/x^12,x]
```

```
[Out] (8*Hypergeometric2F1[-11/5, -3/4, -6/5, -x^5])/(11*x^11) - (4*Hypergeometri
c2F1[-7/5, -3/4, -2/5, -x^5])/(7*x^7) + Hypergeometric2F1[-6/5, -3/4, -1/5,
 -x^5]/x^6 + Hypergeometric2F1[-3/4, -2/5, 3/5, -x^5]/(2*x^2) - (2*Hypergeo
metric2F1[-3/4, -1/5, 4/5, -x^5])/x
```

IntegrateAlgebraic [A] time = 3.16, size = 28, normalized size = 0.74

$$\frac{4\left(x^5+1\right)^{7/4}\left(14x^5-11x^4+14\right)}{77x^{11}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + x^5)*(1 + x^5)^(3/4)*(2 - x^4 + 2*x^5))/x^12,x]
```

```
[Out] (4*(1 + x^5)^(7/4)*(14 - 11*x^4 + 14*x^5))/(77*x^11)
```

fricas [A] time = 0.41, size = 34, normalized size = 0.89

$$\frac{4\left(14\,x^{10}-11\,x^9+28\,x^5-11\,x^4+14\right)\left(x^5+1\right)^{\frac{3}{4}}}{77\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-4)*(x^5+1)^(3/4)*(2*x^5-x^4+2)/x^12,x, algorithm="fricas")
```

```
[Out] 4/77*(14*x^10 - 11*x^9 + 28*x^5 - 11*x^4 + 14)*(x^5 + 1)^(3/4)/x^11
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 x^5 - x^4 + 2\right)\left(x^5+1\right)^{\frac{3}{4}}\left(x^5-4\right)}{x^{12}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-4)*(x^5+1)^(3/4)*(2*x^5-x^4+2)/x^12,x, algorithm="giac")
[Out] integrate((2*x^5 - x^4 + 2)*(x^5 + 1)^(3/4)*(x^5 - 4)/x^12, x)
```

maple [A] time = 0.01, size = 44, normalized size = 1.16

$$\frac{4\,(1+x)\left(x^4-x^3+x^2-x+1\right)\left(14x^5-11x^4+14\right)\left(x^5+1\right)^{\frac{3}{4}}}{77x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-4)*(x^5+1)^(3/4)*(2*x^5-x^4+2)/x^12,x)
[Out] 4/77*(1+x)*(x^4-x^3+x^2-x+1)*(14*x^5-11*x^4+14)*(x^5+1)^(3/4)/x^11
```

maxima [A] time = 0.52, size = 50, normalized size = 1.32

$$\frac{4\left(14\,x^{10}-11\,x^9+28\,x^5-11\,x^4+14\right)\left(x^4-x^3+x^2-x+1\right)^{\frac{3}{4}}(x+1)^{\frac{3}{4}}}{77\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-4)*(x^5+1)^(3/4)*(2*x^5-x^4+2)/x^12,x, algorithm="maxima")
[Out] 4/77*(14*x^10 - 11*x^9 + 28*x^5 - 11*x^4 + 14)*(x^4 - x^3 + x^2 - x + 1)^(3
/4)*(x + 1)^(3/4)/x^11
```

mupad [B] time = 0.70, size = 61, normalized size = 1.61

$$\frac{8\left(x^5+1\right)^{3/4}}{11\,x}-\frac{4\left(x^5+1\right)^{3/4}}{7\,x^2}+\frac{16\left(x^5+1\right)^{3/4}}{11\,x^6}-\frac{4\left(x^5+1\right)^{3/4}}{7\,x^7}+\frac{8\left(x^5+1\right)^{3/4}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 + 1)^(3/4)*(x^5 - 4)*(2*x^5 - x^4 + 2))/x^12,x)
[Out] (8*(x^5 + 1)^(3/4))/(11*x) - (4*(x^5 + 1)^(3/4))/(7*x^2) + (16*(x^5 + 1)^(3
/4))/(11*x^6) - (4*(x^5 + 1)^(3/4))/(7*x^7) + (8*(x^5 + 1)^(3/4))/(11*x^11)
```

sympy [C] time = 6.57, size = 192, normalized size = 5.05

$$\frac{2\Gamma\left(-\frac{1}{5}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{5}\\ \frac{4}{5}\end{matrix}\middle|x^5e^{i\pi}\right)}{5x\Gamma\left(\frac{4}{5}\right)}-\frac{\Gamma\left(-\frac{2}{5}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{2}{5}\\ \frac{3}{5}\end{matrix}\middle|x^5e^{i\pi}\right)}{5x^2\Gamma\left(\frac{3}{5}\right)}-\frac{6\Gamma\left(-\frac{6}{5}\right){}_2F_1\left(\begin{matrix}-\frac{6}{5},-\frac{3}{4}\\ -\frac{1}{5}\end{matrix}\middle|x^5e^{i\pi}\right)}{5x^6\Gamma\left(-\frac{1}{5}\right)}+\frac{4\Gamma\left(-\frac{7}{5}\right){}_2F_1\left(\begin{matrix}-\frac{7}{5},-\frac{3}{4}\\ -\frac{2}{5}\end{matrix}\middle|x^5e^{i\pi}\right)}{5x^7\Gamma\left(-\frac{2}{5}\right)}-\frac{8\Gamma\left(-\frac{11}{5}\right){}_2F_1\left(\begin{matrix}-\frac{11}{5},-\frac{3}{4}\\ -\frac{6}{5}\end{matrix}\middle|x^5e^{i\pi}\right)}{5x^{11}\Gamma\left(-\frac{6}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-4)*(x**5+1)**(3/4)*(2*x**5-x**4+2)/x**12,x)
[Out] 2*gamma(-1/5)*hyper((-3/4, -1/5), (4/5,), x**5*exp_polar(I*pi))/(5*x*gamma(
4/5)) - gamma(-2/5)*hyper((-3/4, -2/5), (3/5,), x**5*exp_polar(I*pi))/(5*x*
*2*gamma(3/5)) - 6*gamma(-6/5)*hyper((-6/5, -3/4), (-1/5,), x**5*exp_polar(
I*pi))/(5*x**6*gamma(-1/5)) + 4*gamma(-7/5)*hyper((-7/5, -3/4), (-2/5,), x*
*5*exp_polar(I*pi))/(5*x**7*gamma(-2/5)) - 8*gamma(-11/5)*hyper((-11/5, -3/
4), (-6/5,), x**5*exp_polar(I*pi))/(5*x**11*gamma(-6/5))
```

3.489 $\int \frac{(1+x^5)^{2/3}(-3+2x^5)(4+3x^3+4x^5)}{x^9}\,dx$

Optimal. Leaf size=38

$$\frac{3\left(x^5+1\right)^{2/3}\left(5x^{10}+6x^8+10x^5+6x^3+5\right)}{10x^8}$$

Rubi [B] time = 0.17, antiderivative size = 87, normalized size of antiderivative = 2.29, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1826, 1835, 1586, 1584, 449}

$$-\frac{18\left(x^5+1\right)^{2/3}}{5x^5}-\frac{15\left(x^5+1\right)^{2/3}}{14x^8}+\frac{15\left(x^5+1\right)^{2/3}}{x^3}+\frac{3\left(x^5+1\right)^{2/3}\left(35x^{11}+42x^9-280x^6+126x^4+60x\right)}{70x^9}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^5)^(2/3)*(-3 + 2*x^5)*(4 + 3*x^3 + 4*x^5))/x^9,x]
```

```
[Out] (-15*(1 + x^5)^(2/3))/(14*x^8) - (18*(1 + x^5)^(2/3))/(5*x^5) + (15*(1 + x^
5)^(2/3))/x^3 + (3*(1 + x^5)^(2/3)*(60*x + 126*x^4 - 280*x^6 + 42*x^9 + 35*
x^11))/(70*x^9)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1826

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> M
odule[{q = Expon[Pq, x], i}, Simp[(c*x)^m*(a + b*x^n)^p*Sum[(Coeff[Pq, x, i
]*x^(i + 1))/(m + n*p + i + 1), {i, 0, q}], x] + Dist[a*n*p, Int[(c*x)^m*(a
 + b*x^n)^(p - 1)*Sum[(Coeff[Pq, x, i]*x^i)/(m + n*p + i + 1), {i, 0, q}],
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[(n - 1)/2, 0] &&
GtQ[p, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(1+x^5\right)^{2/3}\left(-3+2x^5\right)\left(4+3x^3+4x^5\right)}{x^9}\,dx = \frac{3\left(1+x^5\right)^{2/3}\left(60x+126x^4-280x^6+42x^9+35x^{11}\right)}{70x^9}+\frac{10}{3}\int$$

$$= -\frac{15\left(1+x^5\right)^{2/3}}{14x^8}+\frac{3\left(1+x^5\right)^{2/3}\left(60x+126x^4-280x^6+42x^9+\right.}{70x^9}$$

$$= -\frac{15\left(1+x^5\right)^{2/3}}{14x^8}+\frac{3\left(1+x^5\right)^{2/3}\left(60x+126x^4-280x^6+42x^9+\right.}{70x^9}$$

$$= -\frac{15\left(1+x^5\right)^{2/3}}{14x^8}+\frac{3\left(1+x^5\right)^{2/3}\left(60x+126x^4-280x^6+42x^9+\right.}{70x^9}$$

$$= -\frac{15\left(1+x^5\right)^{2/3}}{14x^8}-\frac{18\left(1+x^5\right)^{2/3}}{5x^5}+\frac{3\left(1+x^5\right)^{2/3}\left(60x+126x^4-\right.}{70x^9}$$

$$= -\frac{15\left(1+x^5\right)^{2/3}}{14x^8}-\frac{18\left(1+x^5\right)^{2/3}}{5x^5}+\frac{3\left(1+x^5\right)^{2/3}\left(60x+126x^4-\right.}{70x^9}$$

$$= -\frac{15\left(1+x^5\right)^{2/3}}{14x^8}-\frac{18\left(1+x^5\right)^{2/3}}{5x^5}+\frac{15\left(1+x^5\right)^{2/3}}{x^3}+\frac{3\left(1+x^5\right)^{2}}{}$$

Mathematica [C] time = 0.20, size = 154, normalized size = 4.05

$$-\frac{27}{25}\left(x^5+1\right)^{5/3}{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};x^5+1\right)+\frac{3\,{}_2F_1\left(-\frac{8}{5},-\frac{2}{3};-\frac{3}{5};-x^5\right)}{2x^8}+\frac{4\,{}_2F_1\left(-\frac{2}{3},-\frac{3}{5};\frac{2}{5};-x^5\right)}{3x^3}+4x^2\,{}_2F_1\left(-\frac{2}{3},\frac{2}{5};\frac{7}{5};-x^5\right)+\frac{3}{5}\left(3\left(x^5+1\right)^{2/3}+3\log\left(1-\sqrt[3]{x^5+1}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^5+1}+1}{\sqrt{3}}\right)-5\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^5)^(2/3)*(-3 + 2*x^5)*(4 + 3*x^3 + 4*x^5))/x^9,x]
```

```
[Out] (3*Hypergeometric2F1[-8/5, -2/3, -3/5, -x^5])/(2*x^8) + (4*Hypergeometric2F
1[-2/3, -3/5, 2/5, -x^5])/(3*x^3) + 4*x^2*Hypergeometric2F1[-2/3, 2/5, 7/5,
 -x^5] - (27*(1 + x^5)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 + x^5])/25 +
(3*(3*(1 + x^5)^(2/3) + 2*Sqrt[3]*ArcTan[(1 + 2*(1 + x^5)^(1/3))/Sqrt[3]] -
 5*Log[x] + 3*Log[1 - (1 + x^5)^(1/3)]))/5
```

IntegrateAlgebraic [A] time = 1.15, size = 28, normalized size = 0.74

$$\frac{3\left(x^5+1\right)^{5/3}\left(5x^5+6x^3+5\right)}{10x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^5)^(2/3)*(-3 + 2*x^5)*(4 + 3*x^3 + 4*x^5))/x^9,x
]
```

```
[Out] (3*(1 + x^5)^(5/3)*(5 + 6*x^3 + 5*x^5))/(10*x^8)
```

fricas [A] time = 0.40, size = 34, normalized size = 0.89

$$\frac{3\left(5\,x^{10}+6\,x^{8}+10\,x^{5}+6\,x^{3}+5\right)\left(x^{5}+1\right)^{\frac{2}{3}}}{10\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)*(2*x^5-3)*(4*x^5+3*x^3+4)/x^9,x, algorithm="fricas"
)
```

```
[Out] 3/10*(5*x^10 + 6*x^8 + 10*x^5 + 6*x^3 + 5)*(x^5 + 1)^(2/3)/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^{5}+3\,x^{3}+4\right)\left(2\,x^{5}-3\right)\left(x^{5}+1\right)^{\frac{2}{3}}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)*(2*x^5-3)*(4*x^5+3*x^3+4)/x^9,x, algorithm="giac")
```

```
[Out] integrate((4*x^5 + 3*x^3 + 4)*(2*x^5 - 3)*(x^5 + 1)^(2/3)/x^9, x)
```

maple [A] time = 0.01, size = 44, normalized size = 1.16

$$\frac{3\,(1+x)\left(x^{4}-x^{3}+x^{2}-x+1\right)\left(5x^{5}+6x^{3}+5\right)\left(x^{5}+1\right)^{\frac{2}{3}}}{10x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)^(2/3)*(2*x^5-3)*(4*x^5+3*x^3+4)/x^9,x)
```

```
[Out] 3/10*(1+x)*(x^4-x^3+x^2-x+1)*(5*x^5+6*x^3+5)*(x^5+1)^(2/3)/x^8
```

maxima [A] time = 0.50, size = 50, normalized size = 1.32

$$\frac{3\left(5\,x^{10}+6\,x^{8}+10\,x^{5}+6\,x^{3}+5\right)\left(x^{4}-x^{3}+x^{2}-x+1\right)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}}{10\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)*(2*x^5-3)*(4*x^5+3*x^3+4)/x^9,x, algorithm="maxima"
)
```

```
[Out] 3/10*(5*x^10 + 6*x^8 + 10*x^5 + 6*x^3 + 5)*(x^4 - x^3 + x^2 - x + 1)^(2/3)*
(x + 1)^(2/3)/x^8
```

mupad [B] time = 0.49, size = 52, normalized size = 1.37

$$\left(x^{5}+1\right)^{2/3}\left(\frac{3\,x^{2}}{2}+\frac{9}{5}\right)+\frac{3\left(x^{5}+1\right)^{2/3}}{x^{3}}+\frac{9\left(x^{5}+1\right)^{2/3}}{5\,x^{5}}+\frac{3\left(x^{5}+1\right)^{2/3}}{2\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 + 1)^(2/3)*(2*x^5 - 3)*(3*x^3 + 4*x^5 + 4))/x^9,x)
```

```
[Out] (x^5 + 1)^(2/3)*((3*x^2)/2 + 9/5) + (3*(x^5 + 1)^(2/3))/x^3 + (9*(x^5 + 1)^
(2/3))/(5*x^5) + (3*(x^5 + 1)^(2/3))/(2*x^8)
```

sympy [C] time = 5.64, size = 185, normalized size = 4.87

$$-\frac{6x^{\frac{10}{3}}\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\right|\frac{e^{i\pi}}{x^{5}}\right)}{5\Gamma\left(\frac{1}{3}\right)}+\frac{8x^{2}\Gamma\left(\frac{2}{5}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},\frac{2}{5}\\ \frac{7}{5}\end{matrix}\right|x^{5}e^{i\pi}\right)}{5\Gamma\left(\frac{7}{5}\right)}-\frac{4\Gamma\left(-\frac{3}{5}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},-\frac{3}{5}\\ \frac{2}{5}\end{matrix}\right|x^{5}e^{i\pi}\right)}{5x^{3}\Gamma\left(\frac{2}{5}\right)}-\frac{12\Gamma\left(-\frac{8}{5}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{8}{5},-\frac{2}{3}\\ -\frac{3}{5}\end{matrix}\right|x^{5}e^{i\pi}\right)}{5x^{8}\Gamma\left(-\frac{3}{5}\right)}+\frac{9\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{e^{i\pi}}{x^{5}}\right)}{5x^{\frac{5}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+1)**(2/3)*(2*x**5-3)*(4*x**5+3*x**3+4)/x**9,x)
```

```
[Out] -6*x**(10/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), exp_polar(I*pi)/x**5)/
(5*gamma(1/3)) + 8*x**2*gamma(2/5)*hyper((-2/3, 2/5), (7/5,), x**5*exp_pola
```

```
r(I*pi))/(5*gamma(7/5)) - 4*gamma(-3/5)*hyper((-2/3, -3/5), (2/5,), x**5*ex
p_polar(I*pi))/(5*x**3*gamma(2/5)) - 12*gamma(-8/5)*hyper((-8/5, -2/3), (-3
/5,), x**5*exp_polar(I*pi))/(5*x**8*gamma(-3/5)) + 9*gamma(1/3)*hyper((-2/3
, 1/3), (4/3,), exp_polar(I*pi)/x**5)/(5*x**(5/3)*gamma(4/3))
```

3.490 $\int \frac{1}{x^{13}\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=38

$$\frac{1}{8}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}\,\left(3x^6+2\right)}{24x^{12}}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 51, 63, 203}

$$\frac{\sqrt{x^6-1}}{8x^6}+\frac{1}{8}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*Sqrt[-1 + x^6]),x]
```

```
[Out] Sqrt[-1 + x^6]/(12*x^12) + Sqrt[-1 + x^6]/(8*x^6) + ArcTan[Sqrt[-1 + x^6]]/
8
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^{13}\sqrt{-1+x^6}}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^3}\,dx, x, x^6\right)$$
$$= \frac{\sqrt{-1+x^6}}{12x^{12}} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx, x, x^6\right)$$
$$= \frac{\sqrt{-1+x^6}}{12x^{12}} + \frac{\sqrt{-1+x^6}}{8x^6} + \frac{1}{16}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right)$$
$$= \frac{\sqrt{-1+x^6}}{12x^{12}} + \frac{\sqrt{-1+x^6}}{8x^6} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^6}\right)$$
$$= \frac{\sqrt{-1+x^6}}{12x^{12}} + \frac{\sqrt{-1+x^6}}{8x^6} + \frac{1}{8}\tan^{-1}\left(\sqrt{-1+x^6}\right)$$

Mathematica [C] time = 0.00, size = 28, normalized size = 0.74

$$\frac{1}{3}\sqrt{x^6-1}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^13*Sqrt[-1 + x^6]),x]
[Out] (Sqrt[-1 + x^6]*Hypergeometric2F1[1/2, 3, 3/2, 1 - x^6])/3
```

IntegrateAlgebraic [A] time = 0.05, size = 38, normalized size = 1.00

$$\frac{1}{8}\tan^{-1}\left(\sqrt{x^6-1}\right) + \frac{\sqrt{x^6-1}\left(3x^6+2\right)}{24x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^13*Sqrt[-1 + x^6]),x]
[Out] (Sqrt[-1 + x^6]*(2 + 3*x^6))/(24*x^12) + ArcTan[Sqrt[-1 + x^6]]/8
```

fricas [A] time = 0.40, size = 34, normalized size = 0.89

$$\frac{3\,x^{12}\arctan\left(\sqrt{x^6-1}\right) + \left(3\,x^6+2\right)\sqrt{x^6-1}}{24\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^6-1)^(1/2),x, algorithm="fricas")
[Out] 1/24*(3*x^12*arctan(sqrt(x^6 - 1)) + (3*x^6 + 2)*sqrt(x^6 - 1))/x^12
```

giac [A] time = 0.25, size = 35, normalized size = 0.92

$$\frac{3\left(x^6-1\right)^{\frac{3}{2}} + 5\sqrt{x^6-1}}{24\,x^{12}} + \frac{1}{8}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^6-1)^(1/2),x, algorithm="giac")
[Out] 1/24*(3*(x^6 - 1)^(3/2) + 5*sqrt(x^6 - 1))/x^12 + 1/8*arctan(sqrt(x^6 - 1))
```

maple [A] time = 0.03, size = 32, normalized size = 0.84

$$\frac{3x^{12}-x^{6}-2}{24x^{12}\sqrt{x^{6}-1}}-\frac{\arcsin\left(\frac{1}{x^{3}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(x^6-1)^(1/2),x)
```

```
[Out] 1/24*(3*x^12-x^6-2)/x^12/(x^6-1)^(1/2)-1/8*arcsin(1/x^3)
```

maxima [A] time = 0.44, size = 48, normalized size = 1.26

$$\frac{3\left(x^{6}-1\right)^{\frac{3}{2}}+5\sqrt{x^{6}-1}}{24\left(2x^{6}+\left(x^{6}-1\right)^{2}-1\right)}+\frac{1}{8}\arctan\left(\sqrt{x^{6}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^6-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*(3*(x^6 - 1)^(3/2) + 5*sqrt(x^6 - 1))/(2*x^6 + (x^6 - 1)^2 - 1) + 1/8*
arctan(sqrt(x^6 - 1))
```

mupad [B] time = 0.44, size = 35, normalized size = 0.92

$$\frac{\mathrm{atan}\left(\sqrt{x^{6}-1}\right)}{8}+\frac{\sqrt{x^{6}-1}}{8x^{6}}+\frac{\sqrt{x^{6}-1}}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^13*(x^6 - 1)^(1/2)),x)
```

```
[Out] atan((x^6 - 1)^(1/2))/8 + (x^6 - 1)^(1/2)/(8*x^6) + (x^6 - 1)^(1/2)/(12*x^1
2)
```

sympy [A] time = 2.82, size = 124, normalized size = 3.26

$$\begin{cases}\frac{i\operatorname{acosh}\left(\frac{1}{x^{3}}\right)}{8}-\frac{i}{8x^{3}\sqrt{-1+\frac{1}{x^{6}}}}+\frac{i}{24x^{9}\sqrt{-1+\frac{1}{x^{6}}}}+\frac{i}{12x^{15}\sqrt{-1+\frac{1}{x^{6}}}} & \text{for}\ \frac{1}{|x^{6}|}>1\\ -\frac{\operatorname{asin}\left(\frac{1}{x^{3}}\right)}{8}+\frac{1}{8x^{3}\sqrt{1-\frac{1}{x^{6}}}}-\frac{1}{24x^{9}\sqrt{1-\frac{1}{x^{6}}}}-\frac{1}{12x^{15}\sqrt{1-\frac{1}{x^{6}}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(x**6-1)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(x**(-3))/8 - I/(8*x**3*sqrt(-1 + x**(-6))) + I/(24*x**9*
sqrt(-1 + x**(-6))) + I/(12*x**15*sqrt(-1 + x**(-6))), 1/Abs(x**6) > 1), (-
asin(x**(-3))/8 + 1/(8*x**3*sqrt(1 - 1/x**6)) - 1/(24*x**9*sqrt(1 - 1/x**6)
) - 1/(12*x**15*sqrt(1 - 1/x**6)), True))
```

3.491 $\int \frac{-1+x^6}{x^{13}\sqrt{1+x^6}}\,dx$

Optimal. Leaf size=38

$$\frac{7}{24}\tanh^{-1}\left(\sqrt{x^6+1}\right)+\frac{\sqrt{x^6+1}\left(2-7x^6\right)}{24x^{12}}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {446, 78, 51, 63, 207}

$$-\frac{7\sqrt{x^6+1}}{24x^6}+\frac{7}{24}\tanh^{-1}\left(\sqrt{x^6+1}\right)+\frac{\sqrt{x^6+1}}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)/(x^13*Sqrt[1 + x^6]),x]
```

```
[Out] Sqrt[1 + x^6]/(12*x^12) - (7*Sqrt[1 + x^6])/(24*x^6) + (7*ArcTanh[Sqrt[1 +
x^6]])/24
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^6}{x^{13}\sqrt{1+x^6}}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+x}{x^3\sqrt{1+x}}\,dx,x,x^6\right)\\ &= \frac{\sqrt{1+x^6}}{12x^{12}}+\frac{7}{24}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx,x,x^6\right)\\ &= \frac{\sqrt{1+x^6}}{12x^{12}}-\frac{7\sqrt{1+x^6}}{24x^6}-\frac{7}{48}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^6\right)\\ &= \frac{\sqrt{1+x^6}}{12x^{12}}-\frac{7\sqrt{1+x^6}}{24x^6}-\frac{7}{24}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^6}\right)\\ &= \frac{\sqrt{1+x^6}}{12x^{12}}-\frac{7\sqrt{1+x^6}}{24x^6}+\frac{7}{24}\tanh^{-1}\left(\sqrt{1+x^6}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.97

$$\frac{1}{24}\left(7\tanh^{-1}\left(\sqrt{x^6+1}\right)+\frac{\sqrt{x^6+1}\left(2-7x^6\right)}{x^{12}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)/(x^13*Sqrt[1 + x^6]),x]
[Out] (((2 - 7*x^6)*Sqrt[1 + x^6])/x^12 + 7*ArcTanh[Sqrt[1 + x^6]])/24
```

IntegrateAlgebraic [A] time = 0.08, size = 38, normalized size = 1.00

$$\frac{7}{24}\tanh^{-1}\left(\sqrt{x^6+1}\right)+\frac{\sqrt{x^6+1}\left(2-7x^6\right)}{24x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^6)/(x^13*Sqrt[1 + x^6]),x]
[Out] ((2 - 7*x^6)*Sqrt[1 + x^6])/(24*x^12) + (7*ArcTanh[Sqrt[1 + x^6]])/24
```

fricas [A] time = 0.39, size = 52, normalized size = 1.37

$$\frac{7\,x^{12}\log\left(\sqrt{x^6+1}+1\right)-7\,x^{12}\log\left(\sqrt{x^6+1}-1\right)-2\left(7\,x^6-2\right)\sqrt{x^6+1}}{48\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/x^13/(x^6+1)^(1/2),x, algorithm="fricas")
[Out] 1/48*(7*x^12*log(sqrt(x^6 + 1) + 1) - 7*x^12*log(sqrt(x^6 + 1) - 1) - 2*(7*
x^6 - 2)*sqrt(x^6 + 1))/x^12
```

giac [A] time = 0.32, size = 49, normalized size = 1.29

$$-\frac{7\left(x^6+1\right)^{\frac{3}{2}}-9\sqrt{x^6+1}}{24\,x^{12}}+\frac{7}{48}\log\left(\sqrt{x^6+1}+1\right)-\frac{7}{48}\log\left(\sqrt{x^6+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/x^13/(x^6+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(7*(x^6 + 1)^(3/2) - 9*sqrt(x^6 + 1))/x^12 + 7/48*log(sqrt(x^6 + 1) +
 1) - 7/48*log(sqrt(x^6 + 1) - 1)
```

maple [A] time = 0.04, size = 44, normalized size = 1.16

$$-\frac{7x^{12}+5x^{6}-2}{24x^{12}\sqrt{x^{6}+1}}-\frac{7\ln\left(\frac{\sqrt{x^{6}+1}-1}{\sqrt{x^{6}}}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)/x^13/(x^6+1)^(1/2),x)
```

```
[Out] -1/24*(7*x^12+5*x^6-2)/x^12/(x^6+1)^(1/2)-7/24*ln(((x^6+1)^(1/2)-1)/(x^6)^(
1/2))
```

maxima [B] time = 0.44, size = 76, normalized size = 2.00

$$\frac{3\left(x^{6}+1\right)^{\frac{3}{2}}-5\sqrt{x^{6}+1}}{24\left(2x^{6}-\left(x^{6}+1\right)^{2}+1\right)}-\frac{\sqrt{x^{6}+1}}{6x^{6}}+\frac{7}{48}\log\left(\sqrt{x^{6}+1}+1\right)-\frac{7}{48}\log\left(\sqrt{x^{6}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/x^13/(x^6+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*(3*(x^6 + 1)^(3/2) - 5*sqrt(x^6 + 1))/(2*x^6 - (x^6 + 1)^2 + 1) - 1/6*
sqrt(x^6 + 1)/x^6 + 7/48*log(sqrt(x^6 + 1) + 1) - 7/48*log(sqrt(x^6 + 1) -
1)
```

mupad [B] time = 0.85, size = 47, normalized size = 1.24

$$\frac{7\,\mathrm{atanh}\left(\sqrt{x^{6}+1}\right)}{24}-\frac{\sqrt{x^{6}+1}}{6x^{6}}+\frac{5\sqrt{x^{6}+1}}{24x^{12}}-\frac{\left(x^{6}+1\right)^{3/2}}{8x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)/(x^13*(x^6 + 1)^(1/2)),x)
```

```
[Out] (7*atanh((x^6 + 1)^(1/2)))/24 - (x^6 + 1)^(1/2)/(6*x^6) + (5*(x^6 + 1)^(1/2
))/(24*x^12) - (x^6 + 1)^(3/2)/(8*x^12)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)/x**13/(x**6+1)**(1/2),x)
```

```
[Out] Timed out
```

3.492 $\int \frac{\sqrt{1+x^6}}{x^{13}} \, dx$

Optimal. Leaf size=38

$$\frac{1}{24} \tanh^{-1}\left(\sqrt{x^6+1}\right)+\frac{\sqrt{x^6+1}\,\left(-x^6-2\right)}{24x^{12}}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {266, 47, 51, 63, 207}

$$-\frac{\sqrt{x^6+1}}{24x^6}+\frac{1}{24} \tanh^{-1}\left(\sqrt{x^6+1}\right)-\frac{\sqrt{x^6+1}}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^6]/x^13,x]
```

```
[Out] -1/12*Sqrt[1 + x^6]/x^12 - Sqrt[1 + x^6]/(24*x^6) + ArcTanh[Sqrt[1 + x^6]]/
24
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{1+x^6}}{x^{13}} \, dx = \frac{1}{6} \text{Subst}\left(\int \frac{\sqrt{1+x}}{x^3} \, dx, x, x^6\right)$$
$$= -\frac{\sqrt{1+x^6}}{12x^{12}} + \frac{1}{24} \text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}} \, dx, x, x^6\right)$$
$$= -\frac{\sqrt{1+x^6}}{12x^{12}} - \frac{\sqrt{1+x^6}}{24x^6} - \frac{1}{48} \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}} \, dx, x, x^6\right)$$
$$= -\frac{\sqrt{1+x^6}}{12x^{12}} - \frac{\sqrt{1+x^6}}{24x^6} - \frac{1}{24} \text{Subst}\left(\int \frac{1}{-1+x^2} \, dx, x, \sqrt{1+x^6}\right)$$
$$= -\frac{\sqrt{1+x^6}}{12x^{12}} - \frac{\sqrt{1+x^6}}{24x^6} + \frac{1}{24} \tanh^{-1}\left(\sqrt{1+x^6}\right)$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.68

$$-\frac{1}{9}\left(x^6+1\right)^{3/2} {}_2F_1\left(\frac{3}{2},3;\frac{5}{2};x^6+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^6]/x^13,x]
[Out] -1/9*((1 + x^6)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + x^6])
```

IntegrateAlgebraic [A] time = 0.09, size = 38, normalized size = 1.00

$$\frac{1}{24}\tanh^{-1}\left(\sqrt{x^6+1}\right) + \frac{\sqrt{x^6+1}\left(-x^6-2\right)}{24x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^6]/x^13,x]
[Out] ((-2 - x^6)*Sqrt[1 + x^6])/(24*x^12) + ArcTanh[Sqrt[1 + x^6]]/24
```

fricas [A] time = 0.39, size = 49, normalized size = 1.29

$$\frac{x^{12}\log\left(\sqrt{x^6+1}+1\right) - x^{12}\log\left(\sqrt{x^6+1}-1\right) - 2\left(x^6+2\right)\sqrt{x^6+1}}{48\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/2)/x^13,x, algorithm="fricas")
[Out] 1/48*(x^12*log(sqrt(x^6 + 1) + 1) - x^12*log(sqrt(x^6 + 1) - 1) - 2*(x^6 +
2)*sqrt(x^6 + 1))/x^12
```

giac [B] time = 0.42, size = 77, normalized size = 2.03

$$-\frac{\sqrt{x^6+1}+\frac{1}{\sqrt{x^6+1}}}{24\left(\left(\sqrt{x^6+1}+\frac{1}{\sqrt{x^6+1}}\right)^2-4\right)} + \frac{1}{96}\log\left(\sqrt{x^6+1}+\frac{1}{\sqrt{x^6+1}}+2\right) - \frac{1}{96}\log\left(\sqrt{x^6+1}+\frac{1}{\sqrt{x^6+1}}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(x^6 + 1) + 1/sqrt(x^6 + 1))/((sqrt(x^6 + 1) + 1/sqrt(x^6 + 1))^
2 - 4) + 1/96*log(sqrt(x^6 + 1) + 1/sqrt(x^6 + 1) + 2) - 1/96*log(sqrt(x^6
+ 1) + 1/sqrt(x^6 + 1) - 2)
```

maple [A] time = 0.03, size = 42, normalized size = 1.11

$$-\frac{x^{12}+3x^{6}+2}{24x^{12}\sqrt{x^{6}+1}}-\frac{\ln\left(\frac{\sqrt{x^{6}+1}-1}{\sqrt{x^{6}}}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)^(1/2)/x^13,x)
```

```
[Out] -1/24*(x^12+3*x^6+2)/x^12/(x^6+1)^(1/2)-1/24*ln(((x^6+1)^(1/2)-1)/(x^6)^(1/
2))
```

maxima [B] time = 0.36, size = 60, normalized size = 1.58

$$\frac{\left(x^{6}+1\right)^{\frac{3}{2}}+\sqrt{x^{6}+1}}{24\left(2\,x^{6}-\left(x^{6}+1\right)^{2}+1\right)}+\frac{1}{48}\,\log\left(\sqrt{x^{6}+1}+1\right)-\frac{1}{48}\,\log\left(\sqrt{x^{6}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/2)/x^13,x, algorithm="maxima")
```

```
[Out] 1/24*((x^6 + 1)^(3/2) + sqrt(x^6 + 1))/(2*x^6 - (x^6 + 1)^2 + 1) + 1/48*log
(sqrt(x^6 + 1) + 1) - 1/48*log(sqrt(x^6 + 1) - 1)
```

mupad [B] time = 0.45, size = 49, normalized size = 1.29

$$\frac{\mathrm{atanh}\left(\sqrt{x^{6}+1}\right)}{24}+\frac{\frac{\sqrt{x^{6}+1}}{24}+\frac{\left(x^{6}+1\right)^{3/2}}{24}}{2\,x^{6}-\left(x^{6}+1\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)^(1/2)/x^13,x)
```

```
[Out] atanh((x^6 + 1)^(1/2))/24 + ((x^6 + 1)^(1/2)/24 + (x^6 + 1)^(3/2)/24)/(2*x^
6 - (x^6 + 1)^2 + 1)
```

sympy [A] time = 2.64, size = 58, normalized size = 1.53

$$\frac{\mathrm{asinh}\left(\frac{1}{x^{3}}\right)}{24}-\frac{1}{24x^{3}\sqrt{1+\frac{1}{x^{6}}}}-\frac{1}{8x^{9}\sqrt{1+\frac{1}{x^{6}}}}-\frac{1}{12x^{15}\sqrt{1+\frac{1}{x^{6}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)**(1/2)/x**13,x)
```

```
[Out] asinh(x**(-3))/24 - 1/(24*x**3*sqrt(1 + x**(-6))) - 1/(8*x**9*sqrt(1 + x**(
-6))) - 1/(12*x**15*sqrt(1 + x**(-6)))
```

3.493 $\int \frac{1+x^6}{x^{13}\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=38

$$\frac{7}{24}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}\,(7x^6+2)}{24x^{12}}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {446, 78, 51, 63, 203}

$$\frac{7\sqrt{x^6-1}}{24x^6}+\frac{7}{24}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)/(x^13*Sqrt[-1 + x^6]),x]
```

```
[Out] Sqrt[-1 + x^6]/(12*x^12) + (7*Sqrt[-1 + x^6])/(24*x^6) + (7*ArcTan[Sqrt[-1
+ x^6]])/24
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^6}{x^{13}\sqrt{-1+x^6}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1+x}{\sqrt{-1+x}\,x^3}\,dx,x,x^6\right)\\ &= \frac{\sqrt{-1+x^6}}{12x^{12}}+\frac{7}{24}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx,x,x^6\right)\\ &= \frac{\sqrt{-1+x^6}}{12x^{12}}+\frac{7\sqrt{-1+x^6}}{24x^6}+\frac{7}{48}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx,x,x^6\right)\\ &= \frac{\sqrt{-1+x^6}}{12x^{12}}+\frac{7\sqrt{-1+x^6}}{24x^6}+\frac{7}{24}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x^6}\right)\\ &= \frac{\sqrt{-1+x^6}}{12x^{12}}+\frac{7\sqrt{-1+x^6}}{24x^6}+\frac{7}{24}\tan^{-1}\left(\sqrt{-1+x^6}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 1.32

$$\frac{1}{24}\sqrt{x^6-1}\left(\frac{7\tanh^{-1}\left(\sqrt{1-x^6}\right)}{\sqrt{1-x^6}}+\frac{7x^6+2}{x^{12}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)/(x^13*Sqrt[-1 + x^6]),x]
```

```
[Out] (Sqrt[-1 + x^6]*((2 + 7*x^6)/x^12 + (7*ArcTanh[Sqrt[1 - x^6]])/Sqrt[1 - x^6
]))/24
```

IntegrateAlgebraic [A] time = 0.05, size = 38, normalized size = 1.00

$$\frac{7}{24}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}\left(7x^6+2\right)}{24x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^6)/(x^13*Sqrt[-1 + x^6]),x]
```

```
[Out] (Sqrt[-1 + x^6]*(2 + 7*x^6))/(24*x^12) + (7*ArcTan[Sqrt[-1 + x^6]])/24
```

fricas [A] time = 0.38, size = 34, normalized size = 0.89

$$\frac{7\,x^{12}\arctan\left(\sqrt{x^6-1}\right)+\left(7\,x^6+2\right)\sqrt{x^6-1}}{24\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x^13/(x^6-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(7*x^12*arctan(sqrt(x^6 - 1)) + (7*x^6 + 2)*sqrt(x^6 - 1))/x^12
```

giac [A] time = 0.28, size = 35, normalized size = 0.92

$$\frac{7\left(x^6-1\right)^{\frac{3}{2}}+9\,\sqrt{x^6-1}}{24\,x^{12}}+\frac{7}{24}\,\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x^13/(x^6-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(7*(x^6 - 1)^(3/2) + 9*sqrt(x^6 - 1))/x^12 + 7/24*arctan(sqrt(x^6 - 1)
)
```

maple [A] time = 0.03, size = 32, normalized size = 0.84

$$\frac{7x^{12}-5x^{6}-2}{24x^{12}\sqrt{x^{6}-1}}-\frac{7\arcsin\left(\frac{1}{x^{3}}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/x^13/(x^6-1)^(1/2),x)
```

```
[Out] 1/24*(7*x^12-5*x^6-2)/x^12/(x^6-1)^(1/2)-7/24*arcsin(1/x^3)
```

maxima [A] time = 0.61, size = 60, normalized size = 1.58

$$\frac{3\left(x^{6}-1\right)^{\frac{3}{2}}+5\sqrt{x^{6}-1}}{24\left(2x^{6}+\left(x^{6}-1\right)^{2}-1\right)}+\frac{\sqrt{x^{6}-1}}{6x^{6}}+\frac{7}{24}\arctan\left(\sqrt{x^{6}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x^13/(x^6-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*(3*(x^6 - 1)^(3/2) + 5*sqrt(x^6 - 1))/(2*x^6 + (x^6 - 1)^2 - 1) + 1/6*
sqrt(x^6 - 1)/x^6 + 7/24*arctan(sqrt(x^6 - 1))
```

mupad [B] time = 0.58, size = 35, normalized size = 0.92

$$\frac{7\,\mathrm{atan}\left(\sqrt{x^{6}-1}\right)}{24}+\frac{7\sqrt{x^{6}-1}}{24\,x^{6}}+\frac{\sqrt{x^{6}-1}}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)/(x^13*(x^6 - 1)^(1/2)),x)
```

```
[Out] (7*atan((x^6 - 1)^(1/2)))/24 + (7*(x^6 - 1)^(1/2))/(24*x^6) + (x^6 - 1)^(1/
2)/(12*x^12)
```

sympy [A] time = 138.36, size = 63, normalized size = 1.66

$$-\frac{1-\frac{1}{x^{6}-1}}{24\left(1+\frac{1}{x^{6}-1}\right)^{2}\sqrt{x^{6}-1}}-\frac{7\,\mathrm{atan}\left(\frac{1}{\sqrt{x^{6}-1}}\right)}{24}+\frac{1}{3\left(1+\frac{1}{x^{6}-1}\right)\sqrt{x^{6}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/x**13/(x**6-1)**(1/2),x)
```

```
[Out] -(1 - 1/(x**6 - 1))/(24*(1 + 1/(x**6 - 1))**2*sqrt(x**6 - 1)) - 7*atan(1/sq
rt(x**6 - 1))/24 + 1/(3*(1 + 1/(x**6 - 1))*sqrt(x**6 - 1))
```

3.494 $$\int \frac{\sqrt[3]{-1+x^6}\left(1+x^6\right)\left(-1+x^3+x^6\right)}{x^8}\,dx$$

Optimal. Leaf size=38

$$\frac{\sqrt[3]{x^6-1}\left(4x^{12}+7x^9-8x^6-7x^3+4\right)}{28x^7}$$

Rubi [A] time = 0.13, antiderivative size = 49, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1835, 1586, 1584, 449}

$$\frac{\left(x^6-1\right)^{4/3}}{7x}-\frac{\left(x^6-1\right)^{4/3}}{7x^7}+\frac{\left(x^6-1\right)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^6)^(1/3)*(1 + x^6)*(-1 + x^3 + x^6))/x^8,x]
```

```
[Out] -1/7*(-1 + x^6)^(4/3)/x^7 + (-1 + x^6)^(4/3)/(4*x^4) + (-1 + x^6)^(4/3)/(7*
x)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\sqrt[3]{-1+x^6}\left(1+x^6\right)\left(-1+x^3+x^6\right)}{x^8}\,dx = -\frac{\left(-1+x^6\right)^{4/3}}{7x^7}+\frac{1}{14}\int \frac{\sqrt[3]{-1+x^6}\left(14x^2+2x^5+14x^8+14x^{11}\right)}{x^7}\,dx$$
$$= -\frac{\left(-1+x^6\right)^{4/3}}{7x^7}+\frac{1}{14}\int \frac{\sqrt[3]{-1+x^6}\left(14x+2x^4+14x^7+14x^{10}\right)}{x^6}\,dx$$
$$= -\frac{\left(-1+x^6\right)^{4/3}}{7x^7}+\frac{1}{14}\int \frac{\sqrt[3]{-1+x^6}\left(14+2x^3+14x^6+14x^9\right)}{x^5}\,dx$$
$$= -\frac{\left(-1+x^6\right)^{4/3}}{7x^7}+\frac{\left(-1+x^6\right)^{4/3}}{4x^4}+\frac{1}{112}\int \frac{\sqrt[3]{-1+x^6}\left(16x^2+112x^8\right)}{x^4}\,dx$$
$$= -\frac{\left(-1+x^6\right)^{4/3}}{7x^7}+\frac{\left(-1+x^6\right)^{4/3}}{4x^4}+\frac{1}{112}\int \frac{\sqrt[3]{-1+x^6}\left(16+112x^6\right)}{x^2}\,dx$$
$$= -\frac{\left(-1+x^6\right)^{4/3}}{7x^7}+\frac{\left(-1+x^6\right)^{4/3}}{4x^4}+\frac{\left(-1+x^6\right)^{4/3}}{7x}$$

Mathematica [C] time = 0.05, size = 100, normalized size = 2.63

$$\frac{\sqrt[3]{x^6-1}\left(20\,{}_2F_1\left(-\frac{7}{6},-\frac{1}{3};-\frac{1}{6};x^6\right)+7x^3\left(10x^6\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};x^6\right)-5\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{1}{3};x^6\right)+4x^9\,{}_2F_1\left(-\frac{1}{3},\frac{5}{6};\frac{11}{6};x^6\right)\right)\right)}{140x^7\sqrt[3]{1-x^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^6)^(1/3)*(1 + x^6)*(-1 + x^3 + x^6))/x^8,x]
```

```
[Out] ((-1 + x^6)^(1/3)*(20*Hypergeometric2F1[-7/6, -1/3, -1/6, x^6] + 7*x^3*(-5*
Hypergeometric2F1[-2/3, -1/3, 1/3, x^6] + 10*x^6*Hypergeometric2F1[-1/3, 1/
3, 4/3, x^6] + 4*x^9*Hypergeometric2F1[-1/3, 5/6, 11/6, x^6])))/(140*x^7*(1
 - x^6)^(1/3))
```

IntegrateAlgebraic [A] time = 0.84, size = 28, normalized size = 0.74

$$\frac{\left(x^6-1\right)^{4/3}\left(4x^6+7x^3-4\right)}{28x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^6)^(1/3)*(1 + x^6)*(-1 + x^3 + x^6))/x^8,x]
```

```
[Out] ((-1 + x^6)^(4/3)*(-4 + 7*x^3 + 4*x^6))/(28*x^7)
```

fricas [A] time = 0.40, size = 34, normalized size = 0.89

$$\frac{\left(4\,x^{12}+7\,x^9-8\,x^6-7\,x^3+4\right)\left(x^6-1\right)^{\frac{1}{3}}}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(x^6+1)*(x^6+x^3-1)/x^8,x, algorithm="fricas")
```

```
[Out] 1/28*(4*x^12 + 7*x^9 - 8*x^6 - 7*x^3 + 4)*(x^6 - 1)^(1/3)/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^3-1\right)\left(x^6+1\right)\left(x^6-1\right)^{\frac{1}{3}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(x^6+1)*(x^6+x^3-1)/x^8,x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^3 - 1)*(x^6 + 1)*(x^6 - 1)^(1/3)/x^8, x)
```

maple [A] time = 0.01, size = 45, normalized size = 1.18

$$\frac{\left(x^{6}-1\right)^{\frac{1}{3}} \left(4 x^{6}+7 x^{3}-4\right) \left(-1+x \right) \left(1+x \right) \left(x^{2}+x +1\right) \left(x^{2}-x +1\right)}{28 x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/3)*(x^6+1)*(x^6+x^3-1)/x^8,x)
```

```
[Out] 1/28*(x^6-1)^(1/3)*(4*x^6+7*x^3-4)*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)/x^7
```

maxima [A] time = 0.82, size = 55, normalized size = 1.45

$$\frac{\left(4\, x^{12} + 7\, x^{9} - 8\, x^{6} - 7\, x^{3} + 4\right) {\left(x^{2} + x + 1\right)}^{\frac{1}{3}} {\left(x^{2} - x + 1\right)}^{\frac{1}{3}} {\left(x + 1\right)}^{\frac{1}{3}} {\left(x - 1\right)}^{\frac{1}{3}}}{28\, x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(x^6+1)*(x^6+x^3-1)/x^8,x, algorithm="maxima")
```

```
[Out] 1/28*(4*x^12 + 7*x^9 - 8*x^6 - 7*x^3 + 4)*(x^2 + x + 1)^(1/3)*(x^2 - x + 1)
^(1/3)*(x + 1)^(1/3)*(x - 1)^(1/3)/x^7
```

mupad [B] time = 0.46, size = 56, normalized size = 1.47

$$\left(x^6-1\right)^{1/3}\,\left(\frac{x^5}{7}+\frac{x^2}{4}\right)-\frac{2\,\left(x^6-1\right)^{1/3}}{7\,x}-\frac{\left(x^6-1\right)^{1/3}}{4\,x^4}+\frac{\left(x^6-1\right)^{1/3}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/3)*(x^6 + 1)*(x^3 + x^6 - 1))/x^8,x)
```

```
[Out] (x^6 - 1)^(1/3)*(x^2/4 + x^5/7) - (2*(x^6 - 1)^(1/3))/(7*x) - (x^6 - 1)^(1/
3)/(4*x^4) + (x^6 - 1)^(1/3)/(7*x^7)
```

sympy [C] time = 4.08, size = 148, normalized size = 3.89

$$\frac{x^{5} e^{\frac{i \pi}{3}} \Gamma\left(\frac{5}{6}\right) {{}_{2}F_{1}\left(\begin{matrix} - \frac{1}{3}, \frac{5}{6} \\ \frac{11}{6} \end{matrix}\middle| {x^{6}} \right)}}{6 \Gamma\left(\frac{11}{6}\right)} + \frac{x^{2} e^{\frac{i \pi}{3}} \Gamma\left(\frac{1}{3}\right) {{}_{2}F_{1}\left(\begin{matrix} - \frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| {x^{6}} \right)}}{6 \Gamma\left(\frac{4}{3}\right)} - \frac{e^{- \frac{2 i \pi}{3}} \Gamma\left(- \frac{2}{3}\right) {{}_{2}F_{1}\left(\begin{matrix} - \frac{2}{3}, - \frac{1}{3} \\ \frac{1}{3} \end{matrix}\middle| {x^{6}} \right)}}{6 x^{4} \Gamma\left(\frac{1}{3}\right)} + \frac{e^{- \frac{2 i \pi}{3}} \Gamma\left(- \frac{7}{6}\right) {{}_{2}F_{1}\left(\begin{matrix} - \frac{7}{6}, - \frac{1}{3} \\ - \frac{1}{6} \end{matrix}\middle| {x^{6}} \right)}}{6 x^{7} \Gamma\left(- \frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/3)*(x**6+1)*(x**6+x**3-1)/x**8,x)
```

```
[Out] x**5*exp(I*pi/3)*gamma(5/6)*hyper((-1/3, 5/6), (11/6,), x**6)/(6*gamma(11/6
)) + x**2*exp(I*pi/3)*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), x**6)/(6*gamma(
4/3)) - exp(-2*I*pi/3)*gamma(-2/3)*hyper((-2/3, -1/3), (1/3,), x**6)/(6*x**
4*gamma(1/3)) + exp(-2*I*pi/3)*gamma(-7/6)*hyper((-7/6, -1/3), (-1/6,), x**
6)/(6*x**7*gamma(-1/6))
```

3.495 $\int \frac{(-1+x^6)^{3/4}(2+x^6)(-1-x^4+x^6)}{x^{12}}\,dx$

Optimal. Leaf size=38

$$\frac{2\left(x^6-1\right)^{3/4}\left(7x^{12}-11x^{10}-14x^6+11x^4+7\right)}{77x^{11}}$$

Rubi [A] time = 0.09, antiderivative size = 33, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1833, 1584, 449, 1478}

$$\frac{2\left(x^6-1\right)^{11/4}}{11x^{11}}-\frac{2\left(x^6-1\right)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^6)^(3/4)*(2 + x^6)*(-1 - x^4 + x^6))/x^12,x]
```

```
[Out] (-2*(-1 + x^6)^(7/4))/(7*x^7) + (2*(-1 + x^6)^(11/4))/(11*x^11)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1478

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n
_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p
]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{\left(-1+x^6\right)^{3/4}\left(2+x^6\right)\left(-1-x^4+x^6\right)}{x^{12}}\,dx=\int \left(\frac{\left(-1+x^6\right)^{3/4}\left(-2x^3-x^9\right)}{x^{11}}+\frac{\left(-1+x^6\right)^{3/4}\left(-2+x^6+x^{12}\right)}{x^{12}}\right)dx$$
$$=\int \frac{\left(-1+x^6\right)^{3/4}\left(-2x^3-x^9\right)}{x^{11}}\,dx+\int \frac{\left(-1+x^6\right)^{3/4}\left(-2+x^6+x^{12}\right)}{x^{12}}\,dx$$
$$=\int \frac{\left(-2-x^6\right)\left(-1+x^6\right)^{3/4}}{x^8}\,dx+\int \frac{\left(-1+x^6\right)^{7/4}\left(2+x^6\right)}{x^{12}}\,dx$$
$$=-\frac{2\left(-1+x^6\right)^{7/4}}{7x^7}+\frac{2\left(-1+x^6\right)^{11/4}}{11x^{11}}$$

Mathematica [C] time = 0.07, size = 118, normalized size = 3.11

$$\frac{\left(x^6-1\right)^{3/4}\left(70\,{}_2F_1\left(-\frac{11}{6},-\frac{3}{4};-\frac{5}{6};x^6\right)+11x^4\left(10\,{}_2F_1\left(-\frac{7}{6},-\frac{3}{4};-\frac{1}{6};x^6\right)+35x^6\left({}_2F_1\left(-\frac{3}{4},-\frac{1}{6};\frac{5}{6};x^6\right)+x^2\,{}_2F_1\left(-\frac{3}{4},\frac{1}{6};\frac{7}{6};x^6\right)\right)-7x^2\,{}_2F_1\left(-\frac{5}{6},-\frac{3}{4};\frac{1}{6};x^6\right)\right)\right)}{385x^{11}\left(1-x^6\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^6)^(3/4)*(2 + x^6)*(-1 - x^4 + x^6))/x^12,x]
```

```
[Out] ((-1 + x^6)^(3/4)*(70*Hypergeometric2F1[-11/6, -3/4, -5/6, x^6] + 11*x^4*(1
0*Hypergeometric2F1[-7/6, -3/4, -1/6, x^6] - 7*x^2*Hypergeometric2F1[-5/6,
-3/4, 1/6, x^6] + 35*x^6*(Hypergeometric2F1[-3/4, -1/6, 5/6, x^6] + x^2*Hyp
ergeometric2F1[-3/4, 1/6, 7/6, x^6]))))/(385*x^11*(1 - x^6)^(3/4))
```

IntegrateAlgebraic [A] time = 3.04, size = 28, normalized size = 0.74

$$\frac{2\left(x^6-1\right)^{7/4}\left(7x^6-11x^4-7\right)}{77x^{11}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^6)^(3/4)*(2 + x^6)*(-1 - x^4 + x^6))/x^12,x]
```

```
[Out] (2*(-1 + x^6)^(7/4)*(-7 - 11*x^4 + 7*x^6))/(77*x^11)
```

fricas [A] time = 0.43, size = 34, normalized size = 0.89

$$\frac{2\left(7\,x^{12}-11\,x^{10}-14\,x^{6}+11\,x^{4}+7\right)\left(x^{6}-1\right)^{\frac{3}{4}}}{77\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(3/4)*(x^6+2)*(x^6-x^4-1)/x^12,x, algorithm="fricas")
```

```
[Out] 2/77*(7*x^12 - 11*x^10 - 14*x^6 + 11*x^4 + 7)*(x^6 - 1)^(3/4)/x^11
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}-x^{4}-1\right)\left(x^{6}+2\right)\left(x^{6}-1\right)^{\frac{3}{4}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(3/4)*(x^6+2)*(x^6-x^4-1)/x^12,x, algorithm="giac")
```

```
[Out] integrate((x^6 - x^4 - 1)*(x^6 + 2)*(x^6 - 1)^(3/4)/x^12, x)
```

maple [A] time = 0.01, size = 45, normalized size = 1.18

$$\frac{2 (-1+x) (1+x) \left(x^{2}+x+1\right) \left(x^{2}-x+1\right) \left(7 x^{6}-11 x^{4}-7\right) \left(x^{6}-1\right)^{\frac{3}{4}}}{77 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(3/4)*(x^6+2)*(x^6-x^4-1)/x^12,x)
[Out] 2/77*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)*(7*x^6-11*x^4-7)*(x^6-1)^(3/4)/x^11
```

maxima [A] time = 0.77, size = 55, normalized size = 1.45

$$\frac{2 \left(7 x^{12}-11 x^{10}-14 x^{6}+11 x^{4}+7\right) \left(x^{2}+x+1\right)^{\frac{3}{4}} \left(x^{2}-x+1\right)^{\frac{3}{4}} (x+1)^{\frac{3}{4}} (x-1)^{\frac{3}{4}}}{77 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(3/4)*(x^6+2)*(x^6-x^4-1)/x^12,x, algorithm="maxima")
[Out] 2/77*(7*x^12 - 11*x^10 - 14*x^6 + 11*x^4 + 7)*(x^2 + x + 1)^(3/4)*(x^2 - x
+ 1)^(3/4)*(x + 1)^(3/4)*(x - 1)^(3/4)/x^11
```

mupad [B] time = 0.59, size = 59, normalized size = 1.55

$$\frac{2 x \left(x^{6}-1\right)^{3/4}}{11}-\frac{2 \left(x^{6}-1\right)^{3/4}}{7 x}-\frac{4 \left(x^{6}-1\right)^{3/4}}{11 x^{5}}+\frac{2 \left(x^{6}-1\right)^{3/4}}{7 x^{7}}+\frac{2 \left(x^{6}-1\right)^{3/4}}{11 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^6 - 1)^(3/4)*(x^6 + 2)*(x^4 - x^6 + 1))/x^12,x)
[Out] (2*x*(x^6 - 1)^(3/4))/11 - (2*(x^6 - 1)^(3/4))/(7*x) - (4*(x^6 - 1)^(3/4))/
(11*x^5) + (2*(x^6 - 1)^(3/4))/(7*x^7) + (2*(x^6 - 1)^(3/4))/(11*x^11)
```

sympy [C] time = 6.77, size = 187, normalized size = 4.92

$$\frac{x e^{\frac{3 i \pi}{4}} \Gamma\left(\frac{1}{6}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, \frac{1}{6} \\ \frac{7}{6} \end{matrix}\middle| {x^{6}} \right)}{6 \Gamma\left(\frac{7}{6}\right)} + \frac{e^{-\frac{i \pi}{4}} \Gamma\left(-\frac{1}{6}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{6} \\ \frac{5}{6} \end{matrix}\middle| {x^{6}} \right)}{6 x \Gamma\left(\frac{5}{6}\right)} - \frac{e^{-\frac{i \pi}{4}} \Gamma\left(-\frac{5}{6}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{6}, -\frac{3}{4} \\ \frac{1}{6} \end{matrix}\middle| {x^{6}} \right)}{6 x^{5} \Gamma\left(\frac{1}{6}\right)} + \frac{e^{-\frac{i \pi}{4}} \Gamma\left(-\frac{7}{6}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{7}{6}, -\frac{3}{4} \\ -\frac{1}{6} \end{matrix}\middle| {x^{6}} \right)}{3 x^{7} \Gamma\left(-\frac{1}{6}\right)} + \frac{e^{-\frac{i \pi}{4}} \Gamma\left(-\frac{11}{6}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{11}{6}, -\frac{3}{4} \\ -\frac{5}{6} \end{matrix}\middle| {x^{6}} \right)}{3 x^{11} \Gamma\left(-\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(3/4)*(x**6+2)*(x**6-x**4-1)/x**12,x)
[Out] x*exp(3*I*pi/4)*gamma(1/6)*hyper((-3/4, 1/6), (7/6,), x**6)/(6*gamma(7/6))
+ exp(-I*pi/4)*gamma(-1/6)*hyper((-3/4, -1/6), (5/6,), x**6)/(6*x*gamma(5/6
)) - exp(-I*pi/4)*gamma(-5/6)*hyper((-5/6, -3/4), (1/6,), x**6)/(6*x**5*gam
ma(1/6)) + exp(-I*pi/4)*gamma(-7/6)*hyper((-7/6, -3/4), (-1/6,), x**6)/(3*x
**7*gamma(-1/6)) + exp(-I*pi/4)*gamma(-11/6)*hyper((-11/6, -3/4), (-5/6,),
x**6)/(3*x**11*gamma(-5/6))
```

3.496 $\int \frac{(-2+x^6)(1+x^6)^{3/4}(1-x^4+x^6)}{x^{12}} \, dx$

Optimal. Leaf size=38

$$\frac{2(x^6+1)^{3/4}(7x^{12}-11x^{10}+14x^6-11x^4+7)}{77x^{11}}$$

Rubi [A] time = 0.09, antiderivative size = 33, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1833, 1584, 449, 1478}

$$\frac{2(x^6+1)^{11/4}}{11x^{11}} - \frac{2(x^6+1)^{7/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((-2 + x^6)*(1 + x^6)^(3/4)*(1 - x^4 + x^6))/x^12,x]
```

```
[Out] (-2*(1 + x^6)^(7/4))/(7*x^7) + (2*(1 + x^6)^(11/4))/(11*x^11)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1478

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n
_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p
]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(-2+x^6)(1+x^6)^{3/4}(1-x^4+x^6)}{x^{12}}\,dx = \int \left(\frac{(1+x^6)^{3/4}(2x^3-x^9)}{x^{11}} + \frac{(1+x^6)^{3/4}(-2-x^6+x^{12})}{x^{12}}\right)dx$$
$$= \int \frac{(1+x^6)^{3/4}(2x^3-x^9)}{x^{11}}\,dx + \int \frac{(1+x^6)^{3/4}(-2-x^6+x^{12})}{x^{12}}\,dx$$
$$= \int \frac{(2-x^6)(1+x^6)^{3/4}}{x^8}\,dx + \int \frac{(-2+x^6)(1+x^6)^{7/4}}{x^{12}}\,dx$$
$$= -\frac{2(1+x^6)^{7/4}}{7x^7} + \frac{2(1+x^6)^{11/4}}{11x^{11}}$$

Mathematica [C] time = 0.03, size = 103, normalized size = 2.71

$$\frac{{}_2F_1\left(-\frac{3}{4},-\frac{1}{6};\frac{5}{6};-x^6\right)}{x} + x\,{}_2F_1\left(-\frac{3}{4},\frac{1}{6};\frac{7}{6};-x^6\right) + \frac{2\,{}_2F_1\left(-\frac{11}{6},-\frac{3}{4};-\frac{5}{6};-x^6\right)}{11x^{11}} - \frac{2\,{}_2F_1\left(-\frac{7}{6},-\frac{3}{4};-\frac{1}{6};-x^6\right)}{7x^7} + \frac{{}_2F_1\left(-\frac{5}{6},-\frac{3}{4};\frac{1}{6};-x^6\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-2 + x^6)*(1 + x^6)^(3/4)*(1 - x^4 + x^6))/x^12,x]
```

```
[Out] (2*Hypergeometric2F1[-11/6, -3/4, -5/6, -x^6])/(11*x^11) - (2*Hypergeometri
c2F1[-7/6, -3/4, -1/6, -x^6])/(7*x^7) + Hypergeometric2F1[-5/6, -3/4, 1/6,
-x^6]/(5*x^5) + Hypergeometric2F1[-3/4, -1/6, 5/6, -x^6]/x + x*Hypergeometr
ic2F1[-3/4, 1/6, 7/6, -x^6]
```

IntegrateAlgebraic [A] time = 2.61, size = 28, normalized size = 0.74

$$\frac{2(x^6+1)^{7/4}(7x^6-11x^4+7)}{77x^{11}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^6)*(1 + x^6)^(3/4)*(1 - x^4 + x^6))/x^12,x]
```

```
[Out] (2*(1 + x^6)^(7/4)*(7 - 11*x^4 + 7*x^6))/(77*x^11)
```

fricas [A] time = 0.41, size = 34, normalized size = 0.89

$$\frac{2\,(7\,x^{12}-11\,x^{10}+14\,x^6-11\,x^4+7)(x^6+1)^{\frac{3}{4}}}{77\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6+1)^(3/4)*(x^6-x^4+1)/x^12,x, algorithm="fricas")
```

```
[Out] 2/77*(7*x^12 - 11*x^10 + 14*x^6 - 11*x^4 + 7)*(x^6 + 1)^(3/4)/x^11
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^6-x^4+1)(x^6+1)^{\frac{3}{4}}(x^6-2)}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6+1)^(3/4)*(x^6-x^4+1)/x^12,x, algorithm="giac")
```

```
[Out] integrate((x^6 - x^4 + 1)*(x^6 + 1)^(3/4)*(x^6 - 2)/x^12, x)
```

maple [A] time = 0.01, size = 40, normalized size = 1.05

$$\frac{2\left(x^{2}+1\right)\left(x^{4}-x^{2}+1\right)\left(7x^{6}-11x^{4}+7\right)\left(x^{6}+1\right)^{\frac{3}{4}}}{77x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-2)*(x^6+1)^(3/4)*(x^6-x^4+1)/x^12,x)
```

```
[Out] 2/77*(x^2+1)*(x^4-x^2+1)*(7*x^6-11*x^4+7)*(x^6+1)^(3/4)/x^11
```

maxima [A] time = 0.77, size = 46, normalized size = 1.21

$$\frac{2\left(7\,x^{12}-11\,x^{10}+14\,x^{6}-11\,x^{4}+7\right)\left(x^{4}-x^{2}+1\right)^{\frac{3}{4}}\left(x^{2}+1\right)^{\frac{3}{4}}}{77\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6+1)^(3/4)*(x^6-x^4+1)/x^12,x, algorithm="maxima")
```

```
[Out] 2/77*(7*x^12 - 11*x^10 + 14*x^6 - 11*x^4 + 7)*(x^4 - x^2 + 1)^(3/4)*(x^2 +
1)^(3/4)/x^11
```

mupad [B] time = 0.72, size = 59, normalized size = 1.55

$$\frac{2\,x\,\left(x^{6}+1\right)^{3/4}}{11}-\frac{2\,\left(x^{6}+1\right)^{3/4}}{7\,x}+\frac{4\,\left(x^{6}+1\right)^{3/4}}{11\,x^{5}}-\frac{2\,\left(x^{6}+1\right)^{3/4}}{7\,x^{7}}+\frac{2\,\left(x^{6}+1\right)^{3/4}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 + 1)^(3/4)*(x^6 - 2)*(x^6 - x^4 + 1))/x^12,x)
```

```
[Out] (2*x*(x^6 + 1)^(3/4))/11 - (2*(x^6 + 1)^(3/4))/(7*x) + (4*(x^6 + 1)^(3/4))/
(11*x^5) - (2*(x^6 + 1)^(3/4))/(7*x^7) + (2*(x^6 + 1)^(3/4))/(11*x^11)
```

sympy [C] time = 6.68, size = 177, normalized size = 4.66

$$\frac{x\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|x^{6}e^{i\pi}\right)}{6\Gamma\left(\frac{7}{6}\right)}-\frac{\Gamma\left(-\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|x^{6}e^{i\pi}\right)}{6x\Gamma\left(\frac{5}{6}\right)}-\frac{\Gamma\left(-\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{6},-\frac{3}{4}\\ \frac{1}{6}\end{matrix}\middle|x^{6}e^{i\pi}\right)}{6x^{5}\Gamma\left(\frac{1}{6}\right)}+\frac{\Gamma\left(-\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{7}{6},-\frac{3}{4}\\ -\frac{1}{6}\end{matrix}\middle|x^{6}e^{i\pi}\right)}{3x^{7}\Gamma\left(-\frac{1}{6}\right)}-\frac{\Gamma\left(-\frac{11}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{11}{6},-\frac{3}{4}\\ -\frac{5}{6}\end{matrix}\middle|x^{6}e^{i\pi}\right)}{3x^{11}\Gamma\left(-\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-2)*(x**6+1)**(3/4)*(x**6-x**4+1)/x**12,x)
```

```
[Out] x*gamma(1/6)*hyper((-3/4, 1/6), (7/6,), x**6*exp_polar(I*pi))/(6*gamma(7/6)
) - gamma(-1/6)*hyper((-3/4, -1/6), (5/6,), x**6*exp_polar(I*pi))/(6*x*gamm
a(5/6)) - gamma(-5/6)*hyper((-5/6, -3/4), (1/6,), x**6*exp_polar(I*pi))/(6*
x**5*gamma(1/6)) + gamma(-7/6)*hyper((-7/6, -3/4), (-1/6,), x**6*exp_polar(
I*pi))/(3*x**7*gamma(-1/6)) - gamma(-11/6)*hyper((-11/6, -3/4), (-5/6,), x*
*6*exp_polar(I*pi))/(3*x**11*gamma(-5/6))
```

3.497 $\int \frac{-2x+3x^2}{\sqrt{5-4x^2+4x^3+x^4-2x^5+x^6}}\,dx$

Optimal. Leaf size=38

$$\log\left(x^3-x^2+\sqrt{x^6-2x^5+x^4+4x^3-4x^2+5}+2\right)$$

Rubi [F] time = 0.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2x+3x^2}{\sqrt{5-4x^2+4x^3+x^4-2x^5+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*x + 3*x^2)/Sqrt[5 - 4*x^2 + 4*x^3 + x^4 - 2*x^5 + x^6],x]
```

```
[Out] -2*Defer[Int][x/Sqrt[5 - 4*x^2 + 4*x^3 + x^4 - 2*x^5 + x^6], x] + 3*Defer[I
nt][x^2/Sqrt[5 - 4*x^2 + 4*x^3 + x^4 - 2*x^5 + x^6], x]
```

Rubi steps

$$\begin{aligned}\int \frac{-2x+3x^2}{\sqrt{5-4x^2+4x^3+x^4-2x^5+x^6}}\,dx &= \int \frac{x(-2+3x)}{\sqrt{5-4x^2+4x^3+x^4-2x^5+x^6}}\,dx\\ &= \int \left(-\frac{2x}{\sqrt{5-4x^2+4x^3+x^4-2x^5+x^6}}+\frac{3x^2}{\sqrt{5-4x^2+4x^3+x^4-2x^5}}\right.\\ &= -\left(2\int \frac{x}{\sqrt{5-4x^2+4x^3+x^4-2x^5+x^6}}\,dx\right)+3\int \frac{x^2}{\sqrt{5-4x^2+4x^3}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 11, normalized size = 0.29

$$\sinh^{-1}\left(x^3-x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*x + 3*x^2)/Sqrt[5 - 4*x^2 + 4*x^3 + x^4 - 2*x^5 + x^6],x]
```

```
[Out] ArcSinh[2 - x^2 + x^3]
```

IntegrateAlgebraic [A] time = 0.19, size = 38, normalized size = 1.00

$$\log\left(x^3-x^2+\sqrt{x^6-2x^5+x^4+4x^3-4x^2+5}+2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*x + 3*x^2)/Sqrt[5 - 4*x^2 + 4*x^3 + x^4 - 2*x^5 + x^
6],x]
```

```
[Out] Log[2 - x^2 + x^3 + Sqrt[5 - 4*x^2 + 4*x^3 + x^4 - 2*x^5 + x^6]]
```

fricas [A] time = 0.44, size = 36, normalized size = 0.95

$$\log\left(x^3-x^2+\sqrt{x^6-2\,x^5+x^4+4\,x^3-4\,x^2+5}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-2*x)/(x^6-2*x^5+x^4+4*x^3-4*x^2+5)^(1/2),x, algorithm="fri
cas")
```

```
[Out] log(x^3 - x^2 + sqrt(x^6 - 2*x^5 + x^4 + 4*x^3 - 4*x^2 + 5) + 2)
```

giac [A] time = 0.69, size = 38, normalized size = 1.00

$$-\log\left(-x^3 + x^2 + \sqrt{4x^3 + \left(x^3 - x^2\right)^2 - 4x^2 + 5} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-2*x)/(x^6-2*x^5+x^4+4*x^3-4*x^2+5)^(1/2),x, algorithm="gia
c")
```

```
[Out] -log(-x^3 + x^2 + sqrt(4*x^3 + (x^3 - x^2)^2 - 4*x^2 + 5) - 2)
```

maple [A] time = 0.20, size = 39, normalized size = 1.03

$$-\ln\left(-x^3 + x^2 + \sqrt{x^6 - 2x^5 + x^4 + 4x^3 - 4x^2 + 5} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-2*x)/(x^6-2*x^5+x^4+4*x^3-4*x^2+5)^(1/2),x)
```

```
[Out] -ln(-x^3+x^2+(x^6-2*x^5+x^4+4*x^3-4*x^2+5)^(1/2)-2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2 - 2x}{\sqrt{x^6 - 2x^5 + x^4 + 4x^3 - 4x^2 + 5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-2*x)/(x^6-2*x^5+x^4+4*x^3-4*x^2+5)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((3*x^2 - 2*x)/sqrt(x^6 - 2*x^5 + x^4 + 4*x^3 - 4*x^2 + 5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{2x - 3x^2}{\sqrt{x^6 - 2x^5 + x^4 + 4x^3 - 4x^2 + 5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 3*x^2)/(4*x^3 - 4*x^2 + x^4 - 2*x^5 + x^6 + 5)^(1/2),x)
```

```
[Out] int(-(2*x - 3*x^2)/(4*x^3 - 4*x^2 + x^4 - 2*x^5 + x^6 + 5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(3x - 2)}{\sqrt{x^6 - 2x^5 + x^4 + 4x^3 - 4x^2 + 5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-2*x)/(x**6-2*x**5+x**4+4*x**3-4*x**2+5)**(1/2),x)
```

```
[Out] Integral(x*(3*x - 2)/sqrt(x**6 - 2*x**5 + x**4 + 4*x**3 - 4*x**2 + 5), x)
```

3.498 $\int \frac{-2+5x^6}{x\sqrt{-1+x^6}\left(2+x^6\right)}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{x^6-1}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {573, 156, 63, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{x^6-1}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 5*x^6)/(x*Sqrt[-1 + x^6]*(2 + x^6)),x]
```

```
[Out] -1/3*ArcTan[Sqrt[-1 + x^6]] + (2*ArcTan[Sqrt[-1 + x^6]/Sqrt[3]])/Sqrt[3]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rubi steps

$$\int \frac{-2+5x^6}{x\sqrt{-1+x^6}\left(2+x^6\right)}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{-2+5x}{\sqrt{-1+x}\,x(2+x)}\,dx, x, x^6\right)$$
$$= -\left(\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right)\right) + \text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,(2+x)}\,dx, x, x^6\right)$$
$$= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^6}\right)\right) + 2\,\text{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, \sqrt{-1+x^6}\right)$$
$$= -\frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right) + \frac{2\tan^{-1}\left(\frac{\sqrt{-1+x^6}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{x^6-1}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 5*x^6)/(x*Sqrt[-1 + x^6]*(2 + x^6)),x]
[Out] -1/3*ArcTan[Sqrt[-1 + x^6]] + (2*ArcTan[Sqrt[-1 + x^6]/Sqrt[3]])/Sqrt[3]
```

IntegrateAlgebraic [A] time = 0.05, size = 38, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{x^6-1}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + 5*x^6)/(x*Sqrt[-1 + x^6]*(2 + x^6)),x]
[Out] -1/3*ArcTan[Sqrt[-1 + x^6]] + (2*ArcTan[Sqrt[-1 + x^6]/Sqrt[3]])/Sqrt[3]
```

fricas [A] time = 0.41, size = 29, normalized size = 0.76

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^6-1}\right) - \frac{1}{3}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6-2)/x/(x^6-1)^(1/2)/(x^6+2),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(x^6 - 1)) - 1/3*arctan(sqrt(x^6 - 1))
```

giac [A] time = 0.36, size = 29, normalized size = 0.76

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^6-1}\right) - \frac{1}{3}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6-2)/x/(x^6-1)^(1/2)/(x^6+2),x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(x^6 - 1)) - 1/3*arctan(sqrt(x^6 - 1))
```

maple [C] time = 1.01, size = 76, normalized size = 2.00

$$\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{x^6-1}}{x^3}\right)}{3}-\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+3\right)x^6+6\sqrt{x^6-1}-4\,\mathrm{RootOf}\left(\_Z^2+3\right)}{x^6+2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6-2)/x/(x^6-1)^(1/2)/(x^6+2),x)
```

```
[Out] 1/3*RootOf(_Z^2+1)*ln((-RootOf(_Z^2+1)+(x^6-1)^(1/2))/x^3)-1/3*RootOf(_Z^2+
3)*ln((RootOf(_Z^2+3)*x^6+6*(x^6-1)^(1/2)-4*RootOf(_Z^2+3))/(x^6+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,x^6-2}{\left(x^6+2\right)\sqrt{x^6-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6-2)/x/(x^6-1)^(1/2)/(x^6+2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^6 - 2)/((x^6 + 2)*sqrt(x^6 - 1)*x), x)
```

mupad [B] time = 0.51, size = 29, normalized size = 0.76

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\sqrt{x^6-1}}{3}\right)}{3}-\frac{\mathrm{atan}\left(\sqrt{x^6-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6 - 2)/(x*(x^6 - 1)^(1/2)*(x^6 + 2)),x)
```

```
[Out] (2*3^(1/2)*atan((3^(1/2)*(x^6 - 1)^(1/2))/3))/3 - atan((x^6 - 1)^(1/2))/3
```

sympy [A] time = 19.84, size = 36, normalized size = 0.95

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\sqrt{x^6-1}}{3}\right)}{3}-\frac{\mathrm{atan}\left(\sqrt{x^6-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6-2)/x/(x**6-1)**(1/2)/(x**6+2),x)
```

```
[Out] 2*sqrt(3)*atan(sqrt(3)*sqrt(x**6 - 1)/3)/3 - atan(sqrt(x**6 - 1))/3
```

3.499 $\int \frac{-1+10x^6}{x\sqrt{-1+x^6}\left(-1+4x^6\right)}\,dx$

Optimal. Leaf size=38

$$\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt{x^6-1}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {573, 156, 63, 203}

$$\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt{x^6-1}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 10*x^6)/(x*Sqrt[-1 + x^6]*(-1 + 4*x^6)),x]
```

```
[Out] ArcTan[Sqrt[-1 + x^6]]/3 + ArcTan[(2*Sqrt[-1 + x^6])/Sqrt[3]]/Sqrt[3]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rubi steps

$$\int \frac{-1+10x^6}{x\sqrt{-1+x^6}\left(-1+4x^6\right)}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{-1+10x}{\sqrt{-1+x}\,x(-1+4x)}\,dx, x, x^6\right)$$
$$= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right) + \text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,(-1+4x)}\,dx, x, x^6\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^6}\right) + 2\,\text{Subst}\left(\int \frac{1}{3+4x^2}\,dx, x, \sqrt{-1+x^6}\right)$$
$$= \frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right) + \frac{\tan^{-1}\left(\frac{2\sqrt{-1+x^6}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Mathematica [B] time = 0.06, size = 77, normalized size = 2.03

$$\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right) - \frac{\tan^{-1}\left(\frac{2-x^3}{\sqrt{3}\sqrt{x^6-1}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{x^3+2}{\sqrt{3}\sqrt{x^6-1}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 10*x^6)/(x*Sqrt[-1 + x^6]*(-1 + 4*x^6)),x]
[Out] -1/2*ArcTan[(2 - x^3)/(Sqrt[3]*Sqrt[-1 + x^6])]/Sqrt[3] - ArcTan[(2 + x^3)/
(Sqrt[3]*Sqrt[-1 + x^6])]/(2*Sqrt[3]) + ArcTan[Sqrt[-1 + x^6]]/3
```

IntegrateAlgebraic [A] time = 0.05, size = 38, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right) + \frac{\tan^{-1}\left(\frac{2\sqrt{x^6-1}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 10*x^6)/(x*Sqrt[-1 + x^6]*(-1 + 4*x^6)),x]
[Out] ArcTan[Sqrt[-1 + x^6]]/3 + ArcTan[(2*Sqrt[-1 + x^6])/Sqrt[3]]/Sqrt[3]
```

fricas [A] time = 0.40, size = 29, normalized size = 0.76

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x^6-1}\right) + \frac{1}{3}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*x^6-1)/x/(x^6-1)^(1/2)/(4*x^6-1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x^6 - 1)) + 1/3*arctan(sqrt(x^6 - 1))
```

giac [A] time = 0.31, size = 29, normalized size = 0.76

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x^6-1}\right) + \frac{1}{3}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*x^6-1)/x/(x^6-1)^(1/2)/(4*x^6-1),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x^6 - 1)) + 1/3*arctan(sqrt(x^6 - 1))
```

maple [C] time = 1.04, size = 86, normalized size = 2.26

$$\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{x^6-1}+\mathrm{RootOf}\left(\_Z^2+1\right)}{x^3}\right)}{3}-\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(\frac{4\,\mathrm{RootOf}\left(\_Z^2+3\right)x^6+12\sqrt{x^6-1}-7\,\mathrm{RootOf}\left(\_Z^2+3\right)}{\left(2x^3-1\right)\left(2x^3+1\right)}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((10*x^6-1)/x/(x^6-1)^(1/2)/(4*x^6-1),x)
```

```
[Out] 1/3*RootOf(_Z^2+1)*ln(((x^6-1)^(1/2)+RootOf(_Z^2+1))/x^3)-1/6*RootOf(_Z^2+3
)*ln((4*RootOf(_Z^2+3)*x^6+12*(x^6-1)^(1/2)-7*RootOf(_Z^2+3))/(2*x^3-1)/(2*
x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{10\,x^6-1}{\left(4\,x^6-1\right)\sqrt{x^6-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*x^6-1)/x/(x^6-1)^(1/2)/(4*x^6-1),x, algorithm="maxima")
```

```
[Out] integrate((10*x^6 - 1)/((4*x^6 - 1)*sqrt(x^6 - 1)*x), x)
```

mupad [B] time = 0.49, size = 29, normalized size = 0.76

$$\frac{\mathrm{atan}\left(\sqrt{x^6-1}\right)}{3}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,\sqrt{x^6-1}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((10*x^6 - 1)/(x*(x^6 - 1)^(1/2)*(4*x^6 - 1)),x)
```

```
[Out] atan((x^6 - 1)^(1/2))/3 + (3^(1/2)*atan((2*3^(1/2)*(x^6 - 1)^(1/2))/3))/3
```

sympy [A] time = 22.74, size = 36, normalized size = 0.95

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,\sqrt{x^6-1}}{3}\right)}{3}+\frac{\mathrm{atan}\left(\sqrt{x^6-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*x**6-1)/x/(x**6-1)**(1/2)/(4*x**6-1),x)
```

```
[Out] sqrt(3)*atan(2*sqrt(3)*sqrt(x**6 - 1)/3)/3 + atan(sqrt(x**6 - 1))/3
```

3.500 $\int \frac{2+x+x^2}{x^2\left(1+x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=39

$$-\frac{2\sqrt[4]{x^2+1}}{x}-\tan^{-1}\left(\sqrt[4]{x^2+1}\right)-\tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Rubi [A] time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1807, 266, 63, 212, 206, 203}

$$-\frac{2\sqrt[4]{x^2+1}}{x}-\tan^{-1}\left(\sqrt[4]{x^2+1}\right)-\tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + x^2)/(x^2*(1 + x^2)^(3/4)),x]
```

```
[Out] (-2*(1 + x^2)^(1/4))/x - ArcTan[(1 + x^2)^(1/4)] - ArcTanh[(1 + x^2)^(1/4)]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
```

```
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x+x^2}{x^2\left(1+x^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{1+x^2}}{x}+\int \frac{1}{x\left(1+x^2\right)^{3/4}}\,dx \\
&= -\frac{2\sqrt[4]{1+x^2}}{x}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{3/4}}\,dx,x,x^2\right) \\
&= -\frac{2\sqrt[4]{1+x^2}}{x}+2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\sqrt[4]{1+x^2}\right) \\
&= -\frac{2\sqrt[4]{1+x^2}}{x}-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[4]{1+x^2}\right)-\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt[4]{1+x^2}\right) \\
&= -\frac{2\sqrt[4]{1+x^2}}{x}-\tan^{-1}\left(\sqrt[4]{1+x^2}\right)-\tanh^{-1}\left(\sqrt[4]{1+x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.00

$$-\frac{2\sqrt[4]{x^2+1}}{x}-\tan^{-1}\left(\sqrt[4]{x^2+1}\right)-\tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + x^2)/(x^2*(1 + x^2)^(3/4)),x]
[Out] (-2*(1 + x^2)^(1/4))/x - ArcTan[(1 + x^2)^(1/4)] - ArcTanh[(1 + x^2)^(1/4)]
```

IntegrateAlgebraic [A] time = 15.59, size = 39, normalized size = 1.00

$$-\frac{2\sqrt[4]{x^2+1}}{x}-\tan^{-1}\left(\sqrt[4]{x^2+1}\right)-\tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x + x^2)/(x^2*(1 + x^2)^(3/4)),x]
[Out] (-2*(1 + x^2)^(1/4))/x - ArcTan[(1 + x^2)^(1/4)] - ArcTanh[(1 + x^2)^(1/4)]
```

fricas [B] time = 1.09, size = 77, normalized size = 1.97

$$\frac{x\arctan\left(\frac{2\left(\left(x^2+1\right)^{\frac{3}{4}}+\left(x^2+1\right)^{\frac{1}{4}}\right)}{x^2}\right)+x\log\left(\frac{x^2-2\left(x^2+1\right)^{\frac{3}{4}}+2\sqrt{x^2+1}-2\left(x^2+1\right)^{\frac{1}{4}}+2}{x^2}\right)-4\left(x^2+1\right)^{\frac{1}{4}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+2)/x^2/(x^2+1)^(3/4),x, algorithm="fricas")
[Out] 1/2*(x*arctan(2*((x^2 + 1)^(3/4) + (x^2 + 1)^(1/4))/x^2) + x*log((x^2 - 2*(
x^2 + 1)^(3/4) + 2*sqrt(x^2 + 1) - 2*(x^2 + 1)^(1/4) + 2)/x^2) - 4*(x^2 + 1
)^(1/4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+x+2}{(x^2+1)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+2)/x^2/(x^2+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((x^2 + x + 2)/((x^2 + 1)^(3/4)*x^2), x)
```

maple [C] time = 0.19, size = 56, normalized size = 1.44

$$-\frac{2\left(x^2+1\right)^{\frac{1}{4}}}{x}+\frac{-\frac{3\Gamma\left(\frac{3}{4}\right)x^2\,\mathrm{hypergeom}\left(\left[1,1,\frac{7}{4}\right],[2,2],-x^2\right)}{4}+\left(-3\ln(2)+\frac{\pi}{2}+2\ln(x)\right)\Gamma\left(\frac{3}{4}\right)}{2\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+2)/x^2/(x^2+1)^(3/4),x)
```

```
[Out] -2*(x^2+1)^(1/4)/x+1/2/GAMMA(3/4)*(-3/4*GAMMA(3/4)*x^2*hypergeom([1,1,7/4],
[2,2],-x^2)+(-3*ln(2)+1/2*Pi+2*ln(x))*GAMMA(3/4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+x+2}{(x^2+1)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+2)/x^2/(x^2+1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + x + 2)/((x^2 + 1)^(3/4)*x^2), x)
```

mupad [B] time = 0.55, size = 48, normalized size = 1.23

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ -x^2\right)-\mathrm{atanh}\left(\left(x^2+1\right)^{1/4}\right)-\mathrm{atan}\left(\left(x^2+1\right)^{1/4}\right)-\frac{2\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{3}{4};\ \frac{1}{2};\ -x^2\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 2)/(x^2*(x^2 + 1)^(3/4)),x)
```

```
[Out] x*hypergeom([1/2, 3/4], 3/2, -x^2) - atanh((x^2 + 1)^(1/4)) - atan((x^2 + 1
)^(1/4)) - (2*hypergeom([-1/2, 3/4], 1/2, -x^2))/x
```

sympy [C] time = 3.05, size = 68, normalized size = 1.74

$$x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\,x^2e^{i\pi}\right)-\frac{2\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{1}{2}\end{matrix}\middle|\,x^2e^{i\pi}\right)}{x}-\frac{\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\,\frac{e^{i\pi}}{x^2}\right)}{2x^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+2)/x**2/(x**2+1)**(3/4),x)
```

```
[Out] x*hyper((1/2, 3/4), (3/2,), x**2*exp_polar(I*pi)) - 2*hyper((-1/2, 3/4), (1
/2,), x**2*exp_polar(I*pi))/x - gamma(3/4)*hyper((3/4, 3/4), (7/4,), exp_po
lar(I*pi)/x**2)/(2*x**(3/2)*gamma(7/4))
```

3.501 $\int \frac{-3+2x}{\sqrt[4]{-x+x^2}\left(1-x+x^3\right)}\,dx$

Optimal. Leaf size=39

$$2\tan^{-1}\left(\frac{\sqrt[4]{x^2-x}}{x}\right)-2\tanh^{-1}\left(\frac{\left(x^2-x\right)^{3/4}}{x-1}\right)$$

Rubi [F] time = 1.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3+2x}{\sqrt[4]{-x+x^2}\left(1-x+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3 + 2*x)/((-x + x^2)^(1/4)*(1 - x + x^3)),x]
```

```
[Out] (-12*(-1 + x)^(1/4)*x^(1/4)*Defer[Subst][Defer[Int][x^2/((-1 + x^4)^(1/4)*(
1 - x^4 + x^12)), x], x, x^(1/4)])/(-x + x^2)^(1/4) + (8*(-1 + x)^(1/4)*x^(
1/4)*Defer[Subst][Defer[Int][x^6/((-1 + x^4)^(1/4)*(1 - x^4 + x^12)), x], x
, x^(1/4)])/(-x + x^2)^(1/4)
```

Rubi steps

$$\begin{aligned}\int \frac{-3+2x}{\sqrt[4]{-x+x^2}\left(1-x+x^3\right)}\,dx &= \frac{\left(\sqrt[4]{-1+x}\sqrt[4]{x}\right)\int \frac{-3+2x}{\sqrt[4]{-1+x}\sqrt[4]{x}\left(1-x+x^3\right)}\,dx}{\sqrt[4]{-x+x^2}}\\ &= \frac{\left(4\sqrt[4]{-1+x}\sqrt[4]{x}\right)\text{Subst}\left(\int \frac{x^2\left(-3+2x^4\right)}{\sqrt[4]{-1+x^4}\left(1-x^4+x^{12}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-x+x^2}}\\ &= \frac{\left(4\sqrt[4]{-1+x}\sqrt[4]{x}\right)\text{Subst}\left(\int \left(-\frac{3x^2}{\sqrt[4]{-1+x^4}\left(1-x^4+x^{12}\right)}+\frac{2x^6}{\sqrt[4]{-1+x^4}\left(1-x^4+x^{12}\right)}\right)dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-x+x^2}}\\ &= \frac{\left(8\sqrt[4]{-1+x}\sqrt[4]{x}\right)\text{Subst}\left(\int \frac{x^6}{\sqrt[4]{-1+x^4}\left(1-x^4+x^{12}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-x+x^2}}-\frac{\left(12\sqrt[4]{-1+x}\sqrt[4]{x}\right)\text{Subst}\left(\right.}{\sqrt[4]{}}\end{aligned}$$

Mathematica [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{-3+2x}{\sqrt[4]{-x+x^2}\left(1-x+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3 + 2*x)/((-x + x^2)^(1/4)*(1 - x + x^3)),x]
```

```
[Out] Integrate[(-3 + 2*x)/((-x + x^2)^(1/4)*(1 - x + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.12, size = 39, normalized size = 1.00

$$2\tan^{-1}\left(\frac{\sqrt[4]{x^2-x}}{x}\right)-2\tanh^{-1}\left(\frac{\left(x^2-x\right)^{3/4}}{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 + 2*x)/((-x + x^2)^(1/4)*(1 - x + x^3)),x]
[Out] 2*ArcTan[(-x + x^2)^(1/4)/x] - 2*ArcTanh[(-x + x^2)^(3/4)/(-1 + x)]
```

fricas [B] time = 7.52, size = 93, normalized size = 2.38

$$\arctan\left(\frac{2\left(\left(x^2-x\right)^{\frac{1}{4}}x^2+\left(x^2-x\right)^{\frac{3}{4}}\right)}{x^3-x+1}\right)+\log\left(-\frac{x^3-2\left(x^2-x\right)^{\frac{1}{4}}x^2+2\sqrt{x^2-x}\,x+x-2\left(x^2-x\right)^{\frac{3}{4}}-1}{x^3-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(x^2-x)^(1/4)/(x^3-x+1),x, algorithm="fricas")
[Out] arctan(2*((x^2 - x)^(1/4)*x^2 + (x^2 - x)^(3/4))/(x^3 - x + 1)) + log(-(x^3
 - 2*(x^2 - x)^(1/4)*x^2 + 2*sqrt(x^2 - x)*x + x - 2*(x^2 - x)^(3/4) - 1)/(
x^3 - x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x-3}{\left(x^3-x+1\right)\left(x^2-x\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(x^2-x)^(1/4)/(x^3-x+1),x, algorithm="giac")
[Out] integrate((2*x - 3)/((x^3 - x + 1)*(x^2 - x)^(1/4)), x)
```

maple [C] time = 1.86, size = 152, normalized size = 3.90

$$\ln\left(-\frac{2\left(x^2-x\right)^{\frac{3}{4}}-2x\sqrt{x^2-x}+2\left(x^2-x\right)^{\frac{1}{4}}x^2-x^3-x+1}{x^3-x+1}\right)+\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{x^2-x}\,x-\mathrm{RootOf}\left(\_Z^2+1\right)x^3+2\left(x^2-x\right)^{\frac{3}{4}}-2\left(x^2-x\right)^{\frac{1}{4}}x^2-\mathrm{RootOf}\left(\_Z^2+1\right)x+\mathrm{RootOf}\left(\_Z^2+1\right)}{x^3-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*x)/(x^2-x)^(1/4)/(x^3-x+1),x)
[Out] ln(-(2*(x^2-x)^(3/4)-2*x*(x^2-x)^(1/2)+2*(x^2-x)^(1/4)*x^2-x^3-x+1)/(x^3-x+
1))+RootOf(_Z^2+1)*ln(-(2*RootOf(_Z^2+1)*(x^2-x)^(1/2)*x-RootOf(_Z^2+1)*x^3
+2*(x^2-x)^(3/4)-2*(x^2-x)^(1/4)*x^2-RootOf(_Z^2+1)*x+RootOf(_Z^2+1))/(x^3-
x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x-3}{\left(x^3-x+1\right)\left(x^2-x\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(x^2-x)^(1/4)/(x^3-x+1),x, algorithm="maxima")
[Out] integrate((2*x - 3)/((x^3 - x + 1)*(x^2 - x)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{2\,x-3}{{\left(x^2-x\right)}^{1/4}\,\left(x^3-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 3)/((x^2 - x)^(1/4)*(x^3 - x + 1)),x)
```

```
[Out] int((2*x - 3)/((x^2 - x)^(1/4)*(x^3 - x + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(x**2-x)**(1/4)/(x**3-x+1),x)
```

```
[Out] Timed out
```

3.502 $\int \frac{(-1+x^2)\sqrt{x+x^3}}{(1+x^2)(1+x+x^2)^2}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{x^3+x}}{x^2+x+1}-\tan^{-1}\left(\frac{\sqrt{x^3+x}}{x^2+1}\right)$$

Rubi [C] time = 9.42, antiderivative size = 2062, normalized size of antiderivative = 52.87, number of steps used = 196, number of rules used = 18, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2056, 6733, 6742, 1727, 1742, 12, 1248, 725, 206, 1715, 1196, 1709, 220, 1707, 1725, 1217, 2, 6728}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^2)*Sqrt[x + x^3])/((1 + x^2)*(1 + x + x^2)^2),x]
```

```
[Out] (2*Sqrt[x + x^3])/(3*(1 + I*Sqrt[3])*(1 - I*Sqrt[3] - 2*Sqrt[x])*Sqrt[x]) +
 ((I + Sqrt[3])*Sqrt[x + x^3])/(3*(I - Sqrt[3])*(1 - I*Sqrt[3] - 2*Sqrt[x])
*Sqrt[x]) + (2*Sqrt[x + x^3])/(3*(1 - I*Sqrt[3])*(1 + I*Sqrt[3] - 2*Sqrt[x]
)*Sqrt[x]) + ((I - Sqrt[3])*Sqrt[x + x^3])/(3*(I + Sqrt[3])*(1 + I*Sqrt[3]
- 2*Sqrt[x])*Sqrt[x]) - (2*Sqrt[x + x^3])/(3*(1 + I*Sqrt[3])*(1 - I*Sqrt[3]
 + 2*Sqrt[x])*Sqrt[x]) - ((I + Sqrt[3])*Sqrt[x + x^3])/(3*(I - Sqrt[3])*(1
- I*Sqrt[3] + 2*Sqrt[x])*Sqrt[x]) - (2*Sqrt[x + x^3])/(3*(1 - I*Sqrt[3])*(1
 + I*Sqrt[3] + 2*Sqrt[x])*Sqrt[x]) - ((I - Sqrt[3])*Sqrt[x + x^3])/(3*(I +
Sqrt[3])*(1 + I*Sqrt[3] + 2*Sqrt[x])*Sqrt[x]) + (2*Sqrt[x + x^3])/(3*(1 - I
*Sqrt[3])*(1 + x)) + (2*Sqrt[x + x^3])/(3*(1 + I*Sqrt[3])*(1 + x)) + ((I -
Sqrt[3])*Sqrt[x + x^3])/(3*(I + Sqrt[3])*(1 + x)) + ((I + Sqrt[3])*Sqrt[x +
 x^3])/(3*(I - Sqrt[3])*(1 + x)) - (2*Sqrt[x + x^3]*ArcTan[Sqrt[x]/Sqrt[1 +
 x^2]])/(3*Sqrt[x]*Sqrt[1 + x^2]) + (Sqrt[x + x^3]*ArcTan[Sqrt[x]/Sqrt[1 +
x^2]])/(Sqrt[3]*(I - Sqrt[3])*Sqrt[x]*Sqrt[1 + x^2]) - ((1 - I*Sqrt[3])*Sqr
t[x + x^3]*ArcTan[Sqrt[x]/Sqrt[1 + x^2]])/(6*Sqrt[x]*Sqrt[1 + x^2]) - ((1 +
 I*Sqrt[3])*Sqrt[x + x^3]*ArcTan[Sqrt[x]/Sqrt[1 + x^2]])/(6*Sqrt[x]*Sqrt[1
+ x^2]) - (Sqrt[x + x^3]*ArcTan[Sqrt[x]/Sqrt[1 + x^2]])/(Sqrt[3]*(I + Sqrt[
3])*Sqrt[x]*Sqrt[1 + x^2]) - ((I - Sqrt[3])*Sqrt[x + x^3]*ArcTan[Sqrt[x]/Sq
rt[1 + x^2]])/(2*(I + Sqrt[3])*Sqrt[x]*Sqrt[1 + x^2]) - ((I + Sqrt[3])*Sqrt
[x + x^3]*ArcTan[Sqrt[x]/Sqrt[1 + x^2]])/(2*(I - Sqrt[3])*Sqrt[x]*Sqrt[1 +
x^2]) - (Sqrt[x + x^3]*ArcTanh[(2 - (1 - I*Sqrt[3])*x)/(Sqrt[2*(1 - I*Sqrt[
3])]*Sqrt[1 + x^2])])/(2*(I + Sqrt[3])*Sqrt[x]*Sqrt[1 + x^2]) + (Sqrt[x + x
^3]*ArcTanh[(4 + (1 + I*Sqrt[3])^2*x)/(2*Sqrt[2*(1 - I*Sqrt[3])]*Sqrt[1 + x
^2])])/(2*(I + Sqrt[3])*Sqrt[x]*Sqrt[1 + x^2]) - (2*(1 + x)*Sqrt[(1 + x^2)/
(1 + x)^2]*Sqrt[x + x^3]*EllipticE[2*ArcTan[Sqrt[x]], 1/2])/(3*(1 - I*Sqrt[
3])*Sqrt[x]*(1 + x^2)) - (2*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*Sqrt[x + x^3]
*EllipticE[2*ArcTan[Sqrt[x]], 1/2])/(3*(1 + I*Sqrt[3])*Sqrt[x]*(1 + x^2)) -
 ((I - Sqrt[3])*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*Sqrt[x + x^3]*EllipticE[2
*ArcTan[Sqrt[x]], 1/2])/(3*(I + Sqrt[3])*Sqrt[x]*(1 + x^2)) - ((I + Sqrt[3]
)*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*Sqrt[x + x^3]*EllipticE[2*ArcTan[Sqrt[x
]], 1/2])/(3*(I - Sqrt[3])*Sqrt[x]*(1 + x^2)) + ((1 + x)*Sqrt[(1 + x^2)/(1
+ x)^2]*Sqrt[x + x^3]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/(3*Sqrt[x]*(1 + x^
2)) + ((1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*Sqrt[x + x^3]*EllipticF[2*ArcTan[S
qrt[x]], 1/2])/(3*(1 - I*Sqrt[3])*Sqrt[x]*(1 + x^2)) - ((1 - I*Sqrt[3])*(1
+ x)*Sqrt[(1 + x^2)/(1 + x)^2]*Sqrt[x + x^3]*EllipticF[2*ArcTan[Sqrt[x]], 1
/2])/(4*Sqrt[x]*(1 + x^2)) + ((1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*Sqrt[x + x^
3]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/(3*(1 + I*Sqrt[3])*Sqrt[x]*(1 + x^2))
 - ((1 + I*Sqrt[3])*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*Sqrt[x + x^3]*Ellipti
cF[2*ArcTan[Sqrt[x]], 1/2])/(4*Sqrt[x]*(1 + x^2)) + ((3 - I*Sqrt[3])*(1 + x
)*Sqrt[(1 + x^2)/(1 + x)^2]*Sqrt[x + x^3]*EllipticPi[1/4, 2*ArcTan[Sqrt[x]]
, 1/2])/(24*Sqrt[x]*(1 + x^2)) + ((3 + I*Sqrt[3])*(1 + x)*Sqrt[(1 + x^2)/(1
 + x)^2]*Sqrt[x + x^3]*EllipticPi[1/4, 2*ArcTan[Sqrt[x]], 1/2])/(24*Sqrt[x]
```

```
*(1 + x^2)) + ((I - Sqrt[3])*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*Sqrt[x + x^3
]*EllipticPi[1/4, 2*ArcTan[Sqrt[x]], 1/2])/(4*(I + Sqrt[3])*Sqrt[x]*(1 + x^
2)) + ((I + Sqrt[3])*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*Sqrt[x + x^3]*Ellipt
icPi[1/4, 2*ArcTan[Sqrt[x]], 1/2])/(4*(I - Sqrt[3])*Sqrt[x]*(1 + x^2))
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
```

```
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1709

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[(a*(B*d - A*e)*(e
+ d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1715

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[d,
 Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Dist[e, Int[x/((d^2 - e^
2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Rule 1727

```
Int[((d_) + (e_.)*(x_))^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[(
e^3*(d + e*x)^(q + 1)*Sqrt[a + c*x^4])/((q + 1)*(c*d^4 + a*e^4)), x] + Dist
[c/((q + 1)*(c*d^4 + a*e^4)), Int[((d + e*x)^(q + 1)*Simp[d^3*(q + 1) - d^2
*e*(q + 1)*x + d*e^2*(q + 1)*x^2 - e^3*(q + 3)*x^3, x])/Sqrt[a + c*x^4], x]
, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^4 + a*e^4, 0] && ILtQ[q, -1]
```

Rule 1742

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(-1+x^2)\sqrt{x+x^3}}{(1+x^2)(1+x+x^2)^2}\,dx = \frac{\sqrt{x+x^3}\int \frac{\sqrt{x}(-1+x^2)}{\sqrt{1+x^2}(1+x+x^2)^2}\,dx}{\sqrt{x}\sqrt{1+x^2}}$$

$$= \frac{\left(2\sqrt{x+x^3}\right)\text{Subst}\left(\int \frac{x^2(-1+x^4)}{\sqrt{1+x^4}(1+x^2+x^4)^2}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{1+x^2}}$$

$$= \frac{\left(2\sqrt{x+x^3}\right)\text{Subst}\left(\int \left(\frac{-1-x}{4(1-x+x^2)^2\sqrt{1+x^4}} + \frac{1+x}{4(1-x+x^2)\sqrt{1+x^4}} + \frac{-1+x}{4(1+x+x^2)^2\sqrt{1+x^4}} + \frac{\ldots}{4(1+x\ldots}\right.\right.}{\sqrt{x}\sqrt{1+x^2}}$$

$$= \frac{\sqrt{x+x^3}\,\text{Subst}\left(\int \frac{-1-x}{(1-x+x^2)^2\sqrt{1+x^4}}\,dx, x, \sqrt{x}\right)}{2\sqrt{x}\sqrt{1+x^2}} + \frac{\sqrt{x+x^3}\,\text{Subst}\left(\int \frac{1+x}{(1-x+x^2)\sqrt{1+x^4}}\,d\ldots\right.}{2\sqrt{x}\sqrt{1+x^2}}$$

= rest of steps removed due to Latex formating problem

Mathematica [C] time = 1.03, size = 117, normalized size = 3.00

$$\sqrt{x^3+x}\left(\frac{1}{x^2+x+1}+\frac{\sqrt[4]{-1}\sqrt{\frac{1}{x^2}+1}\sqrt{x}\left(-F\left(i\sinh^{-1}\left(\frac{\sqrt[4]{-1}}{\sqrt{x}}\right)\middle|-1\right)+\Pi\left(-\sqrt[6]{-1};i\sinh^{-1}\left(\frac{\sqrt[4]{-1}}{\sqrt{x}}\right)\middle|-1\right)+\Pi\left(-(-1)^{5/6};i\sinh^{-1}\left(\frac{\sqrt[4]{-1}}{\sqrt{x}}\right)\middle|-1\right)\right)}{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)*Sqrt[x + x^3])/((1 + x^2)*(1 + x + x^2)^2),x]
```

```
[Out] Sqrt[x + x^3]*((1 + x + x^2)^(-1) + ((-1)^(1/4)*Sqrt[1 + x^(-2)]*Sqrt[x]*(-
EllipticF[I*ArcSinh[(-1)^(1/4)/Sqrt[x]], -1] + EllipticPi[-(-1)^(1/6), I*Ar
cSinh[(-1)^(1/4)/Sqrt[x]], -1] + EllipticPi[-(-1)^(5/6), I*ArcSinh[(-1)^(1/
4)/Sqrt[x]], -1]))/(1 + x^2))
```

IntegrateAlgebraic [A] time = 0.27, size = 39, normalized size = 1.00

$$\frac{\sqrt{x^3+x}}{x^2+x+1}-\tan^{-1}\left(\frac{\sqrt{x^3+x}}{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*Sqrt[x + x^3])/((1 + x^2)*(1 + x + x^2)^2),x
]
```

```
[Out] Sqrt[x + x^3]/(1 + x + x^2) - ArcTan[Sqrt[x + x^3]/(1 + x^2)]
```

fricas [A] time = 0.42, size = 45, normalized size = 1.15

$$\frac{\left(x^2+x+1\right)\arctan\left(\frac{x^2-x+1}{2\sqrt{x^3+x}}\right)+2\sqrt{x^3+x}}{2\left(x^2+x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^3+x)^(1/2)/(x^2+1)/(x^2+x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + x + 1)*arctan(1/2*(x^2 - x + 1)/sqrt(x^3 + x)) + 2*sqrt(x^3 + x
))/(x^2 + x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^3+x}\left(x^2-1\right)}{\left(x^2+x+1\right)^2\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^3+x)^(1/2)/(x^2+1)/(x^2+x+1)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^3 + x)*(x^2 - 1)/((x^2 + x + 1)^2*(x^2 + 1)), x)
```

maple [C] time = 0.08, size = 410, normalized size = 10.51

$$\frac{\sqrt{x^3+x}}{x^2+x+1}+\frac{i\sqrt{-i(i+x)}\,\sqrt{2}\,\sqrt{i(-i+x)}\,\sqrt{ix}\,\operatorname{EllipticF}\left(\sqrt{-i(i+x)},\frac{\sqrt{2}}{2}\right)}{2\sqrt{x^3+x}}-\frac{i\left(-\frac{3}{4}+\frac{i\sqrt{3}}{12}\right)\sqrt{-i(i+x)}\,\sqrt{2}\,\sqrt{i(-i+x)}\,\sqrt{ix}\,\left(i\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)+1+i\right)\operatorname{EllipticPi}\left(\sqrt{-i(i+x)},\frac{1}{2}-i+\frac{i\sqrt{3}}{2},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3+x}}-\frac{i\left(-\frac{3}{4}-\frac{i\sqrt{3}}{12}\right)\sqrt{-i(i+x)}\,\sqrt{2}\,\sqrt{i(-i+x)}\,\sqrt{ix}\,\left(i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+1+i\right)\operatorname{EllipticPi}\left(\sqrt{-i(i+x)},\frac{1}{2}-i-\frac{i\sqrt{3}}{2},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3+x}}-\frac{2i\left(\frac{1}{2}-\frac{i\sqrt{3}}{6}\right)\sqrt{-i(i+x)}\,\sqrt{2}\,\sqrt{i(-i+x)}\,\sqrt{ix}\,\left(i\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)+1+i\right)\operatorname{EllipticPi}\left(\sqrt{-i(i+x)},\frac{1}{2}-i+\frac{i\sqrt{3}}{2},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3+x}}-\frac{2i\left(\frac{i\sqrt{3}}{6}+\frac{1}{2}\right)\sqrt{-i(i+x)}\,\sqrt{2}\,\sqrt{i(-i+x)}\,\sqrt{ix}\,\left(i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+1+i\right)\operatorname{EllipticPi}\left(\sqrt{-i(i+x)},\frac{1}{2}-i-\frac{i\sqrt{3}}{2},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(x^3+x)^(1/2)/(x^2+1)/(x^2+x+1)^2,x)
```

```
[Out] (x^3+x)^(1/2)/(x^2+x+1)+1/2*I*(-I*(I+x))^(1/2)*2^(1/2)*(I*(-I+x))^(1/2)*(I*
x)^(1/2)/(x^3+x)^(1/2)*EllipticF((-I*(I+x))^(1/2),1/2*2^(1/2))-I*(-3/4+1/12
*I*3^(1/2))*(-I*(I+x))^(1/2)*2^(1/2)*(I*(-I+x))^(1/2)*(I*x)^(1/2)/(x^3+x)^(
1/2)*(I*(-1/2+1/2*I*3^(1/2))+1+I)*EllipticPi((-I*(I+x))^(1/2),1/2-I+1/2*I*3
^(1/2),1/2*2^(1/2))-I*(-3/4-1/12*I*3^(1/2))*(-I*(I+x))^(1/2)*2^(1/2)*(I*(-I
+x))^(1/2)*(I*x)^(1/2)/(x^3+x)^(1/2)*(I*(-1/2-1/2*I*3^(1/2))+1+I)*EllipticP
i((-I*(I+x))^(1/2),1/2-I-1/2*I*3^(1/2),1/2*2^(1/2))-2*I*(1/2-1/6*I*3^(1/2))
*(-I*(I+x))^(1/2)*2^(1/2)*(I*(-I+x))^(1/2)*(I*x)^(1/2)/(x^3+x)^(1/2)*(I*(-1
/2+1/2*I*3^(1/2))+1+I)*EllipticPi((-I*(I+x))^(1/2),1/2-I+1/2*I*3^(1/2),1/2*
2^(1/2))-2*I*(1/6*I*3^(1/2)+1/2)*(-I*(I+x))^(1/2)*2^(1/2)*(I*(-I+x))^(1/2)*
(I*x)^(1/2)/(x^3+x)^(1/2)*(I*(-1/2-1/2*I*3^(1/2))+1+I)*EllipticPi((-I*(I+x)
)^(1/2),1/2-I-1/2*I*3^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^3+x}\left(x^2-1\right)}{\left(x^2+x+1\right)^2\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^3+x)^(1/2)/(x^2+1)/(x^2+x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^3 + x)*(x^2 - 1)/((x^2 + x + 1)^2*(x^2 + 1)), x)
```

mupad [B] time = 0.73, size = 49, normalized size = 1.26

$$\frac{\sqrt{x^3+x}}{x^2+x+1}-\frac{\ln\left(x^2+x+1\right)\,1\mathrm{i}}{2}+\frac{\ln\left(x^2-x+1+\sqrt{x^3+x}\,2\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*(x + x^3)^(1/2))/((x^2 + 1)*(x + x^2 + 1)^2),x)
```

```
[Out] (log((x + x^3)^(1/2)*2i - x + x^2 + 1)*1i)/2 - (log(x + x^2 + 1)*1i)/2 + (x
 + x^3)^(1/2)/(x + x^2 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(x^2+1\right)}\,(x-1)\,(x+1)}{\left(x^2+1\right)\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**3+x)**(1/2)/(x**2+1)/(x**2+x+1)**2,x)
```

```
[Out] Integral(sqrt(x*(x**2 + 1))*(x - 1)*(x + 1)/((x**2 + 1)*(x**2 + x + 1)**2),
 x)
```

3.503 $\int \frac{(-2+x^3)\sqrt{2+x^2+2x^3}}{(1+x^3)(1+x^2+x^3)}\,dx$

Optimal. Leaf size=39

$$-2\tan^{-1}\left(\frac{x}{\sqrt{2x^3+x^2+2}}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt{2x^3+x^2+2}}\right)$$

Rubi [F] time = 74.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+x^3)\sqrt{2+x^2+2x^3}}{(1+x^3)(1+x^2+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + x^3)*Sqrt[2 + x^2 + 2*x^3])/((1 + x^3)*(1 + x^2 + x^3)),x]
```

```
[Out] (-2*Sqrt[2 + x^2 + 2*x^3])/3 + ((1 - I*Sqrt[3])*Sqrt[2 + x^2 + 2*x^3])/3 +
((1 + I*Sqrt[3])*Sqrt[2 + x^2 + 2*x^3])/3 - ((5*I)*Sqrt[2]*Sqrt[2 + x^2 + 2
*x^3]*EllipticE[ArcSin[((109 - 6*Sqrt[330])^(1/6)*Sqrt[I*(2 - (1 + I*Sqrt[3
])/(109 - 6*Sqrt[330])^(1/3) - (1 - I*Sqrt[3])*(109 - 6*Sqrt[330])^(1/3) +
12*x)])/(3^(1/4)*Sqrt[2*(1 - (109 - 6*Sqrt[330])^(2/3))])], (-2*Sqrt[3]*(1
- (109 - 6*Sqrt[330])^(2/3)))/(3*I - Sqrt[3] + (3*I + Sqrt[3])*(109 - 6*Sqr
t[330])^(2/3))])/((109 - 6*Sqrt[330])^(1/6)*Sqrt[(1 + (109 - 6*Sqrt[330])^(
-1/3) + (109 - 6*Sqrt[330])^(1/3) + 6*x)/(3 + I*Sqrt[3] + (3 - I*Sqrt[3])*(
109 - 6*Sqrt[330])^(2/3))]*Sqrt[-1 + (109 - 6*Sqrt[330])^(-2/3) + (109 - 6*
Sqrt[330])^(2/3) - ((109 - 6*Sqrt[330])^(-1/3) + (109 - 6*Sqrt[330])^(1/3))
*(1 + 6*x) + (1 + 6*x)^2]) - ((13*I - Sqrt[3])*Sqrt[2 + x^2 + 2*x^3]*Ellipt
icE[ArcSin[((109 - 6*Sqrt[330])^(1/6)*Sqrt[I*(2 - (1 + I*Sqrt[3])/(109 - 6*
Sqrt[330])^(1/3) + I*(I + Sqrt[3])*(109 - 6*Sqrt[330])^(1/3) + 12*x)])/(3^(
1/4)*Sqrt[2*(1 - (109 - 6*Sqrt[330])^(2/3))])], (-2*Sqrt[3]*(1 - (109 - 6*S
qrt[330])^(2/3)))/(3*I - Sqrt[3] + (3*I + Sqrt[3])*(109 - 6*Sqrt[330])^(2/3
))])/(Sqrt[2]*(109 - 6*Sqrt[330])^(1/6)*Sqrt[(1 + (109 - 6*Sqrt[330])^(-1/3
) + (109 - 6*Sqrt[330])^(1/3) + 6*x)/(3 + I*Sqrt[3] + (3 - I*Sqrt[3])*(109
- 6*Sqrt[330])^(2/3))]*Sqrt[-1 + (109 - 6*Sqrt[330])^(-2/3) + (109 - 6*Sqrt
[330])^(2/3) - ((1 + (109 - 6*Sqrt[330])^(2/3))*(1 + 6*x))/(109 - 6*Sqrt[33
0])^(1/3) + (1 + 6*x)^2]) - ((13*I + Sqrt[3])*Sqrt[2 + x^2 + 2*x^3]*Ellipti
cE[ArcSin[((109 - 6*Sqrt[330])^(1/6)*Sqrt[I*(2 - (1 + I*Sqrt[3])/(109 - 6*S
qrt[330])^(1/3) + I*(I + Sqrt[3])*(109 - 6*Sqrt[330])^(1/3) + 12*x)])/(3^(1
/4)*Sqrt[2*(1 - (109 - 6*Sqrt[330])^(2/3))])], (-2*Sqrt[3]*(1 - (109 - 6*Sq
rt[330])^(2/3)))/(3*I - Sqrt[3] + (3*I + Sqrt[3])*(109 - 6*Sqrt[330])^(2/3)
)])/(Sqrt[2]*(109 - 6*Sqrt[330])^(1/6)*Sqrt[(1 + (109 - 6*Sqrt[330])^(-1/3)
 + (109 - 6*Sqrt[330])^(1/3) + 6*x)/(3 + I*Sqrt[3] + (3 - I*Sqrt[3])*(109 -
 6*Sqrt[330])^(2/3))]*Sqrt[-1 + (109 - 6*Sqrt[330])^(-2/3) + (109 - 6*Sqrt[
330])^(2/3) - ((1 + (109 - 6*Sqrt[330])^(2/3))*(1 + 6*x))/(109 - 6*Sqrt[330
])^(1/3) + (1 + 6*x)^2]) + ((46*I)*Sqrt[2]*(109 - 6*Sqrt[330])^(1/6)*Sqrt[(
1 + (109 - 6*Sqrt[330])^(-1/3) + (109 - 6*Sqrt[330])^(1/3) + 6*x)/(3 + I*Sq
rt[3] + (3 - I*Sqrt[3])*(109 - 6*Sqrt[330])^(2/3))]*Sqrt[2 + x^2 + 2*x^3]*E
llipticF[ArcSin[((109 - 6*Sqrt[330])^(1/6)*Sqrt[I*(2 - (1 + I*Sqrt[3])/(109
 - 6*Sqrt[330])^(1/3) - (1 - I*Sqrt[3])*(109 - 6*Sqrt[330])^(1/3) + 12*x)])
/(3^(1/4)*Sqrt[2*(1 - (109 - 6*Sqrt[330])^(2/3))])], (-2*Sqrt[3]*(1 - (109
- 6*Sqrt[330])^(2/3)))/(3*I - Sqrt[3] + (3*I + Sqrt[3])*(109 - 6*Sqrt[330])
^(2/3))])/((1 + (109 - 6*Sqrt[330])^(-1/3) + (109 - 6*Sqrt[330])^(1/3) + 6*
x)*Sqrt[-1 + (109 - 6*Sqrt[330])^(-2/3) + (109 - 6*Sqrt[330])^(2/3) - ((109
 - 6*Sqrt[330])^(-1/3) + (109 - 6*Sqrt[330])^(1/3))*(1 + 6*x) + (1 + 6*x)^2
]) + ((10*I)*Sqrt[2]*(1 + (109 - 6*Sqrt[330])^(2/3))*Sqrt[(1 + (109 - 6*Sqr
t[330])^(-1/3) + (109 - 6*Sqrt[330])^(1/3) + 6*x)/(3 + I*Sqrt[3] + (3 - I*S
```

```
qrt[3])*(109 - 6*Sqrt[330])^(2/3))]*Sqrt[2 + x^2 + 2*x^3]*EllipticF[ArcSin[
((109 - 6*Sqrt[330])^(1/6)*Sqrt[I*(2 - (1 + I*Sqrt[3])/(109 - 6*Sqrt[330])^
(1/3) - (1 - I*Sqrt[3])*(109 - 6*Sqrt[330])^(1/3) + 12*x)])/(3^(1/4)*Sqrt[2
*(1 - (109 - 6*Sqrt[330])^(2/3))])], (-2*Sqrt[3]*(1 - (109 - 6*Sqrt[330])^(
2/3)))/(3*I - Sqrt[3] + (3*I + Sqrt[3])*(109 - 6*Sqrt[330])^(2/3))])/((109
- 6*Sqrt[330])^(1/6)*(1 + (109 - 6*Sqrt[330])^(-1/3) + (109 - 6*Sqrt[330])^
(1/3) + 6*x)*Sqrt[-1 + (109 - 6*Sqrt[330])^(-2/3) + (109 - 6*Sqrt[330])^(2/
3) - ((109 - 6*Sqrt[330])^(-1/3) + (109 - 6*Sqrt[330])^(1/3))*(1 + 6*x) + (
1 + 6*x)^2]) + (Sqrt[2]*(13*I - 24*Sqrt[3])*(1 + I*Sqrt[3])*(109 - 6*Sqrt[3
30])^(1/6)*Sqrt[(1 + (109 - 6*Sqrt[330])^(-1/3) + (109 - 6*Sqrt[330])^(1/3)
 + 6*x)/(3 + I*Sqrt[3] + (3 - I*Sqrt[3])*(109 - 6*Sqrt[330])^(2/3))]*Sqrt[2
 + x^2 + 2*x^3]*EllipticF[ArcSin[((109 - 6*Sqrt[330])^(1/6)*Sqrt[I*(2 - (1
+ I*Sqrt[3])/(109 - 6*Sqrt[330])^(1/3) + I*(I + Sqrt[3])*(109 - 6*Sqrt[330]
)^(1/3) + 12*x)])/(3^(1/4)*Sqrt[2*(1 - (109 - 6*Sqrt[330])^(2/3))])], (-2*S
qrt[3]*(1 - (109 - 6*Sqrt[330])^(2/3)))/(3*I - Sqrt[3] + (3*I + Sqrt[3])*(1
09 - 6*Sqrt[330])^(2/3))])/((1 + (109 - 6*Sqrt[330])^(-1/3) + (109 - 6*Sqrt
[330])^(1/3) + 6*x)*Sqrt[-1 + (109 - 6*Sqrt[330])^(-2/3) + (109 - 6*Sqrt[33
0])^(2/3) - ((1 + (109 - 6*Sqrt[330])^(2/3))*(1 + 6*x))/(109 - 6*Sqrt[330])
^(1/3) + (1 + 6*x)^2]) + (Sqrt[2]*(13 - (24*I)*Sqrt[3])*(I + Sqrt[3])*(109
- 6*Sqrt[330])^(1/6)*Sqrt[(1 + (109 - 6*Sqrt[330])^(-1/3) + (109 - 6*Sqrt[3
30])^(1/3) + 6*x)/(3 + I*Sqrt[3] + (3 - I*Sqrt[3])*(109 - 6*Sqrt[330])^(2/3
))]*Sqrt[2 + x^2 + 2*x^3]*EllipticF[ArcSin[((109 - 6*Sqrt[330])^(1/6)*Sqrt[
I*(2 - (1 + I*Sqrt[3])/(109 - 6*Sqrt[330])^(1/3) + I*(I + Sqrt[3])*(109 - 6
*Sqrt[330])^(1/3) + 12*x)])/(3^(1/4)*Sqrt[2*(1 - (109 - 6*Sqrt[330])^(2/3))
])], (-2*Sqrt[3]*(1 - (109 - 6*Sqrt[330])^(2/3)))/(3*I - Sqrt[3] + (3*I + S
qrt[3])*(109 - 6*Sqrt[330])^(2/3))])/((1 + (109 - 6*Sqrt[330])^(-1/3) + (10
9 - 6*Sqrt[330])^(1/3) + 6*x)*Sqrt[-1 + (109 - 6*Sqrt[330])^(-2/3) + (109 -
 6*Sqrt[330])^(2/3) - ((1 + (109 - 6*Sqrt[330])^(2/3))*(1 + 6*x))/(109 - 6*
Sqrt[330])^(1/3) + (1 + 6*x)^2]) + (Sqrt[2]*(4 + (3*I)*Sqrt[3])*(I + Sqrt[3
])*(1 + (109 - 6*Sqrt[330])^(2/3))*Sqrt[(1 + (109 - 6*Sqrt[330])^(-1/3) + (
109 - 6*Sqrt[330])^(1/3) + 6*x)/(3 + I*Sqrt[3] + (3 - I*Sqrt[3])*(109 - 6*S
qrt[330])^(2/3))]*Sqrt[2 + x^2 + 2*x^3]*EllipticF[ArcSin[((109 - 6*Sqrt[330
])^(1/6)*Sqrt[I*(2 - (1 + I*Sqrt[3])/(109 - 6*Sqrt[330])^(1/3) + I*(I + Sqr
t[3])*(109 - 6*Sqrt[330])^(1/3) + 12*x)])/(3^(1/4)*Sqrt[2*(1 - (109 - 6*Sqr
t[330])^(2/3))])], (-2*Sqrt[3]*(1 - (109 - 6*Sqrt[330])^(2/3)))/(3*I - Sqrt
[3] + (3*I + Sqrt[3])*(109 - 6*Sqrt[330])^(2/3))])/((109 - 6*Sqrt[330])^(1/
6)*(1 + (109 - 6*Sqrt[330])^(-1/3) + (109 - 6*Sqrt[330])^(1/3) + 6*x)*Sqrt[
-1 + (109 - 6*Sqrt[330])^(-2/3) + (109 - 6*Sqrt[330])^(2/3) - ((1 + (109 -
6*Sqrt[330])^(2/3))*(1 + 6*x))/(109 - 6*Sqrt[330])^(1/3) + (1 + 6*x)^2]) +
(Sqrt[2]*(1 + I*Sqrt[3])*(4*I + 3*Sqrt[3])*(1 + (109 - 6*Sqrt[330])^(2/3))*
Sqrt[(1 + (109 - 6*Sqrt[330])^(-1/3) + (109 - 6*Sqrt[330])^(1/3) + 6*x)/(3
+ I*Sqrt[3] + (3 - I*Sqrt[3])*(109 - 6*Sqrt[330])^(2/3))]*Sqrt[2 + x^2 + 2*
x^3]*EllipticF[ArcSin[((109 - 6*Sqrt[330])^(1/6)*Sqrt[I*(2 - (1 + I*Sqrt[3]
)/(109 - 6*Sqrt[330])^(1/3) + I*(I + Sqrt[3])*(109 - 6*Sqrt[330])^(1/3) + 1
2*x)])/(3^(1/4)*Sqrt[2*(1 - (109 - 6*Sqrt[330])^(2/3))])], (-2*Sqrt[3]*(1 -
 (109 - 6*Sqrt[330])^(2/3)))/(3*I - Sqrt[3] + (3*I + Sqrt[3])*(109 - 6*Sqrt
[330])^(2/3))])/((109 - 6*Sqrt[330])^(1/6)*(1 + (109 - 6*Sqrt[330])^(-1/3)
+ (109 - 6*Sqrt[330])^(1/3) + 6*x)*Sqrt[-1 + (109 - 6*Sqrt[330])^(-2/3) + (
109 - 6*Sqrt[330])^(2/3) - ((1 + (109 - 6*Sqrt[330])^(2/3))*(1 + 6*x))/(109
 - 6*Sqrt[330])^(1/3) + (1 + 6*x)^2]) - (27*(1 - I*Sqrt[3])^2*Sqrt[2*(109 -
 6*Sqrt[330])*(3 - I*Sqrt[3] + (3 + I*Sqrt[3])*(109 - 6*Sqrt[330])^(2/3))]*
Sqrt[2 + (I*(I + Sqrt[3]))/(109 - 6*Sqrt[330])^(1/3) - (1 + I*Sqrt[3])*(109
 - 6*Sqrt[330])^(1/3) + 12*x]*Sqrt[2 - (1 + I*Sqrt[3])/(109 - 6*Sqrt[330])^
(1/3) + I*(I + Sqrt[3])*(109 - 6*Sqrt[330])^(1/3) + 12*x]*Sqrt[2 + x^2 + 2*
x^3]*Sqrt[1 - (2*(1 + (109 - 6*Sqrt[330])^(2/3) + (109 - 6*Sqrt[330])^(1/3)
*(1 + 6*x)))/(3 + I*Sqrt[3] + (3 - I*Sqrt[3])*(109 - 6*Sqrt[330])^(2/3))]*S
qrt[1 - (2*(1 + (109 - 6*Sqrt[330])^(2/3) + (109 - 6*Sqrt[330])^(1/3)*(1 +
6*x)))/(3 - I*Sqrt[3] + (3 + I*Sqrt[3])*(109 - 6*Sqrt[330])^(2/3))]*Ellipti
cPi[(3*I + Sqrt[3] + (3*I - Sqrt[3])*(109 - 6*Sqrt[330])^(2/3))/(2*(I + ((8
```

```
*I - 6*Sqrt[3])*(109 - 6*Sqrt[330])^(1/3))/2 + I*(109 - 6*Sqrt[330])^(2/3))
), ArcSin[(Sqrt[2]*(109 - 6*Sqrt[330])^(1/6)*Sqrt[1 + (109 - 6*Sqrt[330])^(
-1/3) + (109 - 6*Sqrt[330])^(1/3) + 6*x])/Sqrt[3 - I*Sqrt[3] + (3 + I*Sqrt[
3])*(109 - 6*Sqrt[330])^(2/3)]], (3*I + Sqrt[3] + (3*I - Sqrt[3])*(109 - 6*
Sqrt[330])^(2/3))/(3*I - Sqrt[3] + (3*I + Sqrt[3])*(109 - 6*Sqrt[330])^(2/3
))])/((1 + (4 + (3*I)*Sqrt[3])*(109 - 6*Sqrt[330])^(1/3) + (109 - 6*Sqrt[33
0])^(2/3))*Sqrt[1 + (109 - 6*Sqrt[330])^(-1/3) + (109 - 6*Sqrt[330])^(1/3)
+ 6*x]*Sqrt[-1 + I*Sqrt[3] - (1 + I*Sqrt[3])*(109 - 6*Sqrt[330])^(2/3) + 2*
(109 - 6*Sqrt[330])^(1/3)*(1 + 6*x)]*Sqrt[-1 - I*Sqrt[3] + I*(I + Sqrt[3])*
(109 - 6*Sqrt[330])^(2/3) + 2*(109 - 6*Sqrt[330])^(1/3)*(1 + 6*x)]*(1 - (10
9 - 6*Sqrt[330])^(-2/3) - (109 - 6*Sqrt[330])^(2/3) + ((109 - 6*Sqrt[330])^
(-1/3) + (109 - 6*Sqrt[330])^(1/3))*(1 + 6*x) - (1 + 6*x)^2)) - (108*Sqrt[2
*(109 - 6*Sqrt[330])*(3 - I*Sqrt[3] + (3 + I*Sqrt[3])*(109 - 6*Sqrt[330])^(
2/3))]*Sqrt[2 + (I*(I + Sqrt[3]))/(109 - 6*Sqrt[330])^(1/3) - (1 + I*Sqrt[3
])*(109 - 6*Sqrt[330])^(1/3) + 12*x]*Sqrt[2 - (1 + I*Sqrt[3])/(109 - 6*Sqrt
[330])^(1/3) + I*(I + Sqrt[3])*(109 - 6*Sqrt[330])^(1/3) + 12*x]*Sqrt[2 + x
^2 + 2*x^3]*Sqrt[1 - (2*(1 + (109 - 6*Sqrt[330])^(2/3) + (109 - 6*Sqrt[330]
)^(1/3)*(1 + 6*x)))/(3 + I*Sqrt[3] + (3 - I*Sqrt[3])*(109 - 6*Sqrt[330])^(2
/3))]*Sqrt[1 - (2*(1 + (109 - 6*Sqrt[330])^(2/3) + (109 - 6*Sqrt[330])^(1/3
)*(1 + 6*x)))/(3 - I*Sqrt[3] + (3 + I*Sqrt[3])*(109 - 6*Sqrt[330])^(2/3))]*
EllipticPi[(3 - I*Sqrt[3] + (3 + I*Sqrt[3])*(109 - 6*Sqrt[330])^(2/3))/(2*(
1 - 5*(109 - 6*Sqrt[330])^(1/3) + (109 - 6*Sqrt[330])^(2/3))), ArcSin[(Sqrt
[2]*(109 - 6*Sqrt[330])^(1/6)*Sqrt[1 + (109 - 6*Sqrt[330])^(-1/3) + (109 -
6*Sqrt[330])^(1/3) + 6*x])/Sqrt[3 - I*Sqrt[3] + (3 + I*Sqrt[3])*(109 - 6*Sq
rt[330])^(2/3)]], (3*I + Sqrt[3] + (3*I - Sqrt[3])*(109 - 6*Sqrt[330])^(2/3
))/(3*I - Sqrt[3] + (3*I + Sqrt[3])*(109 - 6*Sqrt[330])^(2/3))])/((1 - 5*(1
09 - 6*Sqrt[330])^(1/3) + (109 - 6*Sqrt[330])^(2/3))*Sqrt[1 + (109 - 6*Sqrt
[330])^(-1/3) + (109 - 6*Sqrt[330])^(1/3) + 6*x]*Sqrt[-1 + I*Sqrt[3] - (1 +
 I*Sqrt[3])*(109 - 6*Sqrt[330])^(2/3) + 2*(109 - 6*Sqrt[330])^(1/3)*(1 + 6*
x)]*Sqrt[-1 - I*Sqrt[3] + I*(I + Sqrt[3])*(109 - 6*Sqrt[330])^(2/3) + 2*(10
9 - 6*Sqrt[330])^(1/3)*(1 + 6*x)]*(1 - (109 - 6*Sqrt[330])^(-2/3) - (109 -
6*Sqrt[330])^(2/3) + ((109 - 6*Sqrt[330])^(-1/3) + (109 - 6*Sqrt[330])^(1/3
))*(1 + 6*x) - (1 + 6*x)^2)) - (27*(1 + I*Sqrt[3])^2*Sqrt[2*(109 - 6*Sqrt[3
30])*(3 - I*Sqrt[3] + (3 + I*Sqrt[3])*(109 - 6*Sqrt[330])^(2/3))]*Sqrt[2 +
(I*(I + Sqrt[3]))/(109 - 6*Sqrt[330])^(1/3) - (1 + I*Sqrt[3])*(109 - 6*Sqrt
[330])^(1/3) + 12*x]*Sqrt[2 - (1 + I*Sqrt[3])/(109 - 6*Sqrt[330])^(1/3) + I
*(I + Sqrt[3])*(109 - 6*Sqrt[330])^(1/3) + 12*x]*Sqrt[2 + x^2 + 2*x^3]*Sqrt
[1 - (2*(1 + (109 - 6*Sqrt[330])^(2/3) + (109 - 6*Sqrt[330])^(1/3)*(1 + 6*x
)))/(3 + I*Sqrt[3] + (3 - I*Sqrt[3])*(109 - 6*Sqrt[330])^(2/3))]*Sqrt[1 - (
2*(1 + (109 - 6*Sqrt[330])^(2/3) + (109 - 6*Sqrt[330])^(1/3)*(1 + 6*x)))/(3
 - I*Sqrt[3] + (3 + I*Sqrt[3])*(109 - 6*Sqrt[330])^(2/3))]*EllipticPi[(3 -
I*Sqrt[3] + (3 + I*Sqrt[3])*(109 - 6*Sqrt[330])^(2/3))/(2*(1 + (4 - (3*I)*S
qrt[3])*(109 - 6*Sqrt[330])^(1/3) + (109 - 6*Sqrt[330])^(2/3))), ArcSin[(Sq
rt[2]*(109 - 6*Sqrt[330])^(1/6)*Sqrt[1 + (109 - 6*Sqrt[330])^(-1/3) + (109
- 6*Sqrt[330])^(1/3) + 6*x])/Sqrt[3 - I*Sqrt[3] + (3 + I*Sqrt[3])*(109 - 6*
Sqrt[330])^(2/3)]], (3*I + Sqrt[3] + (3*I - Sqrt[3])*(109 - 6*Sqrt[330])^(2
/3))/(3*I - Sqrt[3] + (3*I + Sqrt[3])*(109 - 6*Sqrt[330])^(2/3))])/((1 + (4
 - (3*I)*Sqrt[3])*(109 - 6*Sqrt[330])^(1/3) + (109 - 6*Sqrt[330])^(2/3))*Sq
rt[1 + (109 - 6*Sqrt[330])^(-1/3) + (109 - 6*Sqrt[330])^(1/3) + 6*x]*Sqrt[-
1 + I*Sqrt[3] - (1 + I*Sqrt[3])*(109 - 6*Sqrt[330])^(2/3) + 2*(109 - 6*Sqrt
[330])^(1/3)*(1 + 6*x)]*Sqrt[-1 - I*Sqrt[3] + I*(I + Sqrt[3])*(109 - 6*Sqrt
[330])^(2/3) + 2*(109 - 6*Sqrt[330])^(1/3)*(1 + 6*x)]*(1 - (109 - 6*Sqrt[33
0])^(-2/3) - (109 - 6*Sqrt[330])^(2/3) + ((109 - 6*Sqrt[330])^(-1/3) + (109
 - 6*Sqrt[330])^(1/3))*(1 + 6*x) - (1 + 6*x)^2)) - 2*Defer[Int][Sqrt[2 + x^
2 + 2*x^3]/(1 + x^2 + x^3), x] - 3*Defer[Int][(x*Sqrt[2 + x^2 + 2*x^3])/(1
+ x^2 + x^3), x]
```

Rubi steps

$$\int \frac{\left(-2+x^3\right)\sqrt{2+x^2+2x^3}}{\left(1+x^3\right)\left(1+x^2+x^3\right)}dx = \int \left(\frac{\sqrt{2+x^2+2x^3}}{-1-x}+\frac{(1+x)\sqrt{2+x^2+2x^3}}{1-x+x^2}+\frac{(-2-3x)\sqrt{2+x^2+2x^3}}{1+x^2+x^3}\right)dx$$
$$= \int \frac{\sqrt{2+x^2+2x^3}}{-1-x}dx+\int \frac{(1+x)\sqrt{2+x^2+2x^3}}{1-x+x^2}dx+\int \frac{(-2-3x)\sqrt{2+x^2+}}{1+x^2+x^3}$$
$$= \int \left(\frac{\left(1-i\sqrt{3}\right)\sqrt{2+x^2+2x^3}}{-1-i\sqrt{3}+2x}+\frac{\left(1+i\sqrt{3}\right)\sqrt{2+x^2+2x^3}}{-1+i\sqrt{3}+2x}\right)dx+\int \left(-\frac{2\sqrt{2+}}{1+x}\right.$$

= rest of steps removed due to Latex formating problem

Mathematica [C] time = 6.34, size = 10734, normalized size = 275.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-2 + x^3)*Sqrt[2 + x^2 + 2*x^3])/((1 + x^3)*(1 + x^2 + x^3)),x]
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.38, size = 39, normalized size = 1.00

$$-2\tan^{-1}\left(\frac{x}{\sqrt{2x^3+x^2+2}}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt{2x^3+x^2+2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^3)*Sqrt[2 + x^2 + 2*x^3])/((1 + x^3)*(1 + x^2 +
 x^3)),x]
[Out] -2*ArcTan[x/Sqrt[2 + x^2 + 2*x^3]] - 2*ArcTanh[x/Sqrt[2 + x^2 + 2*x^3]]
```

fricas [A] time = 0.42, size = 66, normalized size = 1.69

$$\arctan\left(\frac{\sqrt{2\,x^3+x^2+2}\left(x^3+1\right)}{2\,x^4+x^3+2\,x}\right)+\log\left(\frac{x^3+x^2-\sqrt{2\,x^3+x^2+2}\,x+1}{x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(2*x^3+x^2+2)^(1/2)/(x^3+1)/(x^3+x^2+1),x, algorithm="fri
cas")
[Out] arctan(sqrt(2*x^3 + x^2 + 2)*(x^3 + 1)/(2*x^4 + x^3 + 2*x)) + log((x^3 + x^
2 - sqrt(2*x^3 + x^2 + 2)*x + 1)/(x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^3+x^2+2}\left(x^3-2\right)}{\left(x^3+x^2+1\right)\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(2*x^3+x^2+2)^(1/2)/(x^3+1)/(x^3+x^2+1),x, algorithm="gia
c")
[Out] integrate(sqrt(2*x^3 + x^2 + 2)*(x^3 - 2)/((x^3 + x^2 + 1)*(x^3 + 1)), x)
```

maple [C] time = 3.44, size = 4717, normalized size = 120.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2)*(2*x^3+x^2+2)^(1/2)/(x^3+1)/(x^3+x^2+1),x)
```

```
[Out] 4/3*I*3^(1/2)*(-1/6*(109+6*330^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3))*(I
*(x-1/12*(109+6*330^(1/2))^(1/3)-1/12/(109+6*330^(1/2))^(1/3)+1/6+1/2*I*3^(
1/2)*(-1/6*(109+6*330^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3)))*3^(1/2)/(1
/6*(109+6*330^(1/2))^(1/3)-1/6/(109+6*330^(1/2))^(1/3)))^(1/2)*((x+1/6*(109
+6*330^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3)+1/6)/(1/4*(109+6*330^(1/2))
^(1/3)+1/4/(109+6*330^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/6*(109+6*330^(1/2))^(1
/3)+1/6/(109+6*330^(1/2))^(1/3))))^(1/2)*(-I*(x-1/12*(109+6*330^(1/2))^(1/3
)-1/12/(109+6*330^(1/2))^(1/3)+1/6-1/2*I*3^(1/2)*(-1/6*(109+6*330^(1/2))^(1
/3)+1/6/(109+6*330^(1/2))^(1/3)))*3^(1/2)/(1/6*(109+6*330^(1/2))^(1/3)-1/6/
(109+6*330^(1/2))^(1/3)))^(1/2)/(2*x^3+x^2+2)^(1/2)*EllipticF(1/3*3^(1/2)*(
I*(x-1/12*(109+6*330^(1/2))^(1/3)-1/12/(109+6*330^(1/2))^(1/3)+1/6+1/2*I*3^
(1/2)*(-1/6*(109+6*330^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3)))*3^(1/2)/(
1/6*(109+6*330^(1/2))^(1/3)-1/6/(109+6*330^(1/2))^(1/3)))^(1/2),(I*3^(1/2)*
(1/6*(109+6*330^(1/2))^(1/3)-1/6/(109+6*330^(1/2))^(1/3))/(1/4*(109+6*330^(
1/2))^(1/3)+1/4/(109+6*330^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/6*(109+6*330^(1/2
))^(1/3)+1/6/(109+6*330^(1/2))^(1/3))))^(1/2))+1/18*I*2^(1/2)*sum(_alpha*(-
(109+6*330^(1/2))^(1/3)+1/(109+6*330^(1/2))^(1/3))*(1/2*I*(12*x+2-(109+6*33
0^(1/2))^(1/3)-1/(109+6*330^(1/2))^(1/3)+I*3^(1/2)*(-(109+6*330^(1/2))^(1/3
)+1/(109+6*330^(1/2))^(1/3)))/((109+6*330^(1/2))^(1/3)-1/(109+6*330^(1/2))^
(1/3)))^(1/2)*((6*x+1+(109+6*330^(1/2))^(1/3)+1/(109+6*330^(1/2))^(1/3))/(3
*(109+6*330^(1/2))^(1/3)+3/(109+6*330^(1/2))^(1/3)-I*3^(1/2)*(-(109+6*330^(
1/2))^(1/3)+1/(109+6*330^(1/2))^(1/3))))^(1/2)*(-1/2*I*(12*x+2-(109+6*330^(
1/2))^(1/3)-1/(109+6*330^(1/2))^(1/3)-I*3^(1/2)*(-(109+6*330^(1/2))^(1/3)+1
/(109+6*330^(1/2))^(1/3)))/((109+6*330^(1/2))^(1/3)-1/(109+6*330^(1/2))^(1/
3)))^(1/2)/(2*x^3+x^2+2)^(1/2)*(12+18*(109+6*3^(1/2)*110^(1/2))^(1/3)-18*(1
09+6*3^(1/2)*110^(1/2))^(2/3)+127*I*3^(1/2)*_alpha*(109+6*3^(1/2)*110^(1/2)
)^(2/3)-21*I*110^(1/2)*_alpha*(109+6*3^(1/2)*110^(1/2))^(2/3)+17*I*3^(1/2)*
_alpha*(109+6*3^(1/2)*110^(1/2))^(1/3)-3*I*110^(1/2)*_alpha*(109+6*3^(1/2)*
110^(1/2))^(1/3)+109*I*3^(1/2)*_alpha^2*(109+6*3^(1/2)*110^(1/2))^(2/3)-18*
I*110^(1/2)*_alpha^2*(109+6*3^(1/2)*110^(1/2))^(2/3)-I*3^(1/2)*_alpha^2*(10
9+6*3^(1/2)*110^(1/2))^(1/3)-4*_alpha^2-4*_alpha+7*_alpha*110^(1/2)*3^(1/2)
*(109+6*3^(1/2)*110^(1/2))^(2/3)-_alpha*(109+6*3^(1/2)*110^(1/2))^(1/3)*110
^(1/2)*3^(1/2)+6*_alpha^2*110^(1/2)*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^(2/3)
+18*I*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^(1/3)+18*I*3^(1/2)*(109+6*3^(1/2)*1
10^(1/2))^(2/3)-3*I*110^(1/2)*(109+6*3^(1/2)*110^(1/2))^(2/3)-3*I*110^(1/2)
*(109+6*3^(1/2)*110^(1/2))^(1/3)-(109+6*3^(1/2)*110^(1/2))^(1/3)*110^(1/2)*
3^(1/2)+110^(1/2)*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^(2/3)-127*_alpha*(109+6
*3^(1/2)*110^(1/2))^(2/3)+17*_alpha*(109+6*3^(1/2)*110^(1/2))^(1/3)-109*_al
pha^2*(109+6*3^(1/2)*110^(1/2))^(2/3)-_alpha^2*(109+6*3^(1/2)*110^(1/2))^(1
/3))*EllipticPi(1/3*3^(1/2)*(I*(x-1/12*(109+6*330^(1/2))^(1/3)-1/12/(109+6*
330^(1/2))^(1/3)+1/6+1/2*I*3^(1/2)*(-1/6*(109+6*330^(1/2))^(1/3)+1/6/(109+6
*330^(1/2))^(1/3)))*3^(1/2)/(1/6*(109+6*330^(1/2))^(1/3)-1/6/(109+6*330^(1/
2))^(1/3)))^(1/2),3+5/6*I*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^(1/3)-215/6*I*3
^(1/2)*(109+6*3^(1/2)*110^(1/2))^(2/3)+71/12*I*(109+6*3^(1/2)*110^(1/2))^(2
/3)*110^(1/2)-1/12*I*(109+6*3^(1/2)*110^(1/2))^(1/3)*110^(1/2)-25/12*I*_alp
ha*(109+6*3^(1/2)*110^(1/2))^(2/3)*110^(1/2)+41/12*I*_alpha*3^(1/2)*(109+6*
3^(1/2)*110^(1/2))^(1/3)-7/12*I*_alpha*(109+6*3^(1/2)*110^(1/2))^(1/3)*110^
(1/2)+145/12*I*_alpha^2*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^(2/3)-2*I*_alpha^
2*(109+6*3^(1/2)*110^(1/2))^(2/3)*110^(1/2)+35/12*I*_alpha^2*3^(1/2)*(109+6
*3^(1/2)*110^(1/2))^(1/3)+1/6*I*110^(1/2)+3/2*(109+6*3^(1/2)*110^(1/2))^(1/
3)-3/2*(109+6*3^(1/2)*110^(1/2))^(2/3)-1/6*_alpha^2+17/6*_alpha+1/12*_alpha
*110^(1/2)*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^(2/3)-7/12*_alpha*(109+6*3^(1/
2)*110^(1/2))^(1/3)*110^(1/2)*3^(1/2)-1/2*(109+6*3^(1/2)*110^(1/2))^(1/3)*_
alpha^2*110^(1/2)*3^(1/2)-1/2*I*(109+6*3^(1/2)*110^(1/2))^(1/3)*_alpha^2*11
0^(1/2)+151/12*I*_alpha*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^(2/3)+1/6*I*_alph
a*110^(1/2)-1/12*(109+6*3^(1/2)*110^(1/2))^(1/3)*110^(1/2)*3^(1/2)+1/12*110
```

```
^(1/2)*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^(2/3)-17/12*_alpha*(109+6*3^(1/2)*
110^(1/2))^(2/3)+127/12*_alpha*(109+6*3^(1/2)*110^(1/2))^(1/3)+1/12*_alpha^
2*(109+6*3^(1/2)*110^(1/2))^(2/3)+109/12*_alpha^2*(109+6*3^(1/2)*110^(1/2))
^(1/3),(I*3^(1/2)*(1/6*(109+6*330^(1/2))^(1/3)-1/6/(109+6*330^(1/2))^(1/3))
/(1/4*(109+6*330^(1/2))^(1/3)+1/4/(109+6*330^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1
/6*(109+6*330^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3))))^(1/2)),_alpha=Roo
tOf(_Z^3+_Z^2+1))-2/3*I*3^(1/2)*(-1/6*(109+6*330^(1/2))^(1/3)+1/6/(109+6*33
0^(1/2))^(1/3))*(I*(x-1/12*(109+6*330^(1/2))^(1/3)-1/12/(109+6*330^(1/2))^(
1/3)+1/6+1/2*I*3^(1/2)*(-1/6*(109+6*330^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^
(1/3)))*3^(1/2)/(1/6*(109+6*330^(1/2))^(1/3)-1/6/(109+6*330^(1/2))^(1/3)))^
(1/2)*((x+1/6*(109+6*330^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3)+1/6)/(1/4
*(109+6*330^(1/2))^(1/3)+1/4/(109+6*330^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/6*(1
09+6*330^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3))))^(1/2)*(-I*(x-1/12*(109
+6*330^(1/2))^(1/3)-1/12/(109+6*330^(1/2))^(1/3)+1/6-1/2*I*3^(1/2)*(-1/6*(1
09+6*330^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3)))*3^(1/2)/(1/6*(109+6*330
^(1/2))^(1/3)-1/6/(109+6*330^(1/2))^(1/3)))^(1/2)/(2*x^3+x^2+2)^(1/2)/(1/12
*(109+6*330^(1/2))^(1/3)+1/12/(109+6*330^(1/2))^(1/3)+5/6-1/2*I*3^(1/2)*(-1
/6*(109+6*330^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3)))*EllipticPi(1/3*3^(
1/2)*(I*(x-1/12*(109+6*330^(1/2))^(1/3)-1/12/(109+6*330^(1/2))^(1/3)+1/6+1/
2*I*3^(1/2)*(-1/6*(109+6*330^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3)))*3^(
1/2)/(1/6*(109+6*330^(1/2))^(1/3)-1/6/(109+6*330^(1/2))^(1/3)))^(1/2),-I*3^
(1/2)*(-1/6*(109+6*330^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3))/(1/12*(109
+6*330^(1/2))^(1/3)+1/12/(109+6*330^(1/2))^(1/3)+5/6-1/2*I*3^(1/2)*(-1/6*(1
09+6*330^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3))),(I*3^(1/2)*(1/6*(109+6*
330^(1/2))^(1/3)-1/6/(109+6*330^(1/2))^(1/3))/(1/4*(109+6*330^(1/2))^(1/3)+
1/4/(109+6*330^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/6*(109+6*330^(1/2))^(1/3)+1/6
/(109+6*330^(1/2))^(1/3))))^(1/2))+2/3*I*(1/2+1/2*I*3^(1/2))*3^(1/2)*(-1/6*
(109+6*330^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3))*(I*(x-1/12*(109+6*330^
(1/2))^(1/3)-1/12/(109+6*330^(1/2))^(1/3)+1/6+1/2*I*3^(1/2)*(-1/6*(109+6*33
0^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3)))*3^(1/2)/(1/6*(109+6*330^(1/2))
^(1/3)-1/6/(109+6*330^(1/2))^(1/3)))^(1/2)*((x+1/6*(109+6*330^(1/2))^(1/3)+
1/6/(109+6*330^(1/2))^(1/3)+1/6)/(1/4*(109+6*330^(1/2))^(1/3)+1/4/(109+6*33
0^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/6*(109+6*330^(1/2))^(1/3)+1/6/(109+6*330^(
1/2))^(1/3))))^(1/2)*(-I*(x-1/12*(109+6*330^(1/2))^(1/3)-1/12/(109+6*330^(1
/2))^(1/3)+1/6-1/2*I*3^(1/2)*(-1/6*(109+6*330^(1/2))^(1/3)+1/6/(109+6*330^(
1/2))^(1/3)))*3^(1/2)/(1/6*(109+6*330^(1/2))^(1/3)-1/6/(109+6*330^(1/2))^(1
/3)))^(1/2)/(2*x^3+x^2+2)^(1/2)*(109/6*I*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^
(2/3)-7/6*(1/2+1/2*I*3^(1/2))*110^(1/2)*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^(
2/3)+1/2*I*(1/2+1/2*I*3^(1/2))*(109+6*3^(1/2)*110^(1/2))^(1/3)*110^(1/2)+12
7/6*(1/2+1/2*I*3^(1/2))*(109+6*3^(1/2)*110^(1/2))^(2/3)+7/2*I*(1/2+1/2*I*3^
(1/2))*(109+6*3^(1/2)*110^(1/2))^(2/3)*110^(1/2)+1/6*(1/2+1/2*I*3^(1/2))*(1
09+6*3^(1/2)*110^(1/2))^(1/3)*110^(1/2)*3^(1/2)-3*I*110^(1/2)*(109+6*3^(1/2
)*110^(1/2))^(2/3)-17/6*(1/2+1/2*I*3^(1/2))*(109+6*3^(1/2)*110^(1/2))^(1/3)
-7/3-1/6*I*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^(1/3)+110^(1/2)*3^(1/2)*(109+6
*3^(1/2)*110^(1/2))^(2/3)-17/6*I*(1/2+1/2*I*3^(1/2))*3^(1/2)*(109+6*3^(1/2)
*110^(1/2))^(1/3)+1/3*I*3^(1/2)-109/6*(109+6*3^(1/2)*110^(1/2))^(2/3)-127/6
*I*(1/2+1/2*I*3^(1/2))*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^(2/3)-1/6*(109+6*3
^(1/2)*110^(1/2))^(1/3))*EllipticPi(1/3*3^(1/2)*(I*(x-1/12*(109+6*330^(1/2)
)^(1/3)-1/12/(109+6*330^(1/2))^(1/3)+1/6+1/2*I*3^(1/2)*(-1/6*(109+6*330^(1/
2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3)))*3^(1/2)/(1/6*(109+6*330^(1/2))^(1/3
)-1/6/(109+6*330^(1/2))^(1/3)))^(1/2),-1/2*I*(109+6*3^(1/2)*110^(1/2))^(1/3
)*110^(1/2)-1/12*(1/2+1/2*I*3^(1/2))*110^(1/2)*3^(1/2)*(109+6*3^(1/2)*110^(
1/2))^(2/3)+7/12*I*(1/2+1/2*I*3^(1/2))*(109+6*3^(1/2)*110^(1/2))^(1/3)*110^
(1/2)+17/12*(1/2+1/2*I*3^(1/2))*(109+6*3^(1/2)*110^(1/2))^(2/3)-1/6*I*(1/2+
1/2*I*3^(1/2))*110^(1/2)+7/12*(1/2+1/2*I*3^(1/2))*(109+6*3^(1/2)*110^(1/2))
^(1/3)*110^(1/2)*3^(1/2)+31/12*I*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^(1/3)-12
7/12*(1/2+1/2*I*3^(1/2))*(109+6*3^(1/2)*110^(1/2))^(1/3)-41/12*I*(1/2+1/2*I
*3^(1/2))*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^(1/3)-19/12-151/12*I*(1/2+1/2*I
*3^(1/2))*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^(2/3)+581/12*I*3^(1/2)*(109+6*3
```

```
^(1/2)*110^(1/2))^(2/3)+25/12*I*(1/2+1/2*I*3^(1/2))*(109+6*3^(1/2)*110^(1/2
))^(2/3)*110^(1/2)+1/12*(109+6*3^(1/2)*110^(1/2))^(2/3)-8*I*(109+6*3^(1/2)*
110^(1/2))^(2/3)*110^(1/2)-1/2*(109+6*3^(1/2)*110^(1/2))^(1/3)*110^(1/2)*3^
(1/2)-17/12*I*3^(1/2)+109/12*(109+6*3^(1/2)*110^(1/2))^(1/3),(I*3^(1/2)*(1/
6*(109+6*330^(1/2))^(1/3)-1/6/(109+6*330^(1/2))^(1/3))/(1/4*(109+6*330^(1/2
))^(1/3)+1/4/(109+6*330^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/6*(109+6*330^(1/2))^
(1/3)+1/6/(109+6*330^(1/2))^(1/3))))^(1/2))+2/3*I*(1/2-1/2*I*3^(1/2))*3^(1/
2)*(-1/6*(109+6*330^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3))*(I*(x-1/12*(1
09+6*330^(1/2))^(1/3)-1/12/(109+6*330^(1/2))^(1/3)+1/6+1/2*I*3^(1/2)*(-1/6*
(109+6*330^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3)))*3^(1/2)/(1/6*(109+6*3
30^(1/2))^(1/3)-1/6/(109+6*330^(1/2))^(1/3)))^(1/2)*((x+1/6*(109+6*330^(1/2
))^(1/3)+1/6/(109+6*330^(1/2))^(1/3)+1/6)/(1/4*(109+6*330^(1/2))^(1/3)+1/4/
(109+6*330^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/6*(109+6*330^(1/2))^(1/3)+1/6/(10
9+6*330^(1/2))^(1/3))))^(1/2)*(-I*(x-1/12*(109+6*330^(1/2))^(1/3)-1/12/(109
+6*330^(1/2))^(1/3)+1/6-1/2*I*3^(1/2)*(-1/6*(109+6*330^(1/2))^(1/3)+1/6/(10
9+6*330^(1/2))^(1/3)))*3^(1/2)/(1/6*(109+6*330^(1/2))^(1/3)-1/6/(109+6*330^
(1/2))^(1/3)))^(1/2)/(2*x^3+x^2+2)^(1/2)*(109/6*I*3^(1/2)*(109+6*3^(1/2)*11
0^(1/2))^(2/3)-7/6*(1/2-1/2*I*3^(1/2))*110^(1/2)*3^(1/2)*(109+6*3^(1/2)*110
^(1/2))^(2/3)-3*I*110^(1/2)*(109+6*3^(1/2)*110^(1/2))^(2/3)+127/6*(1/2-1/2*
I*3^(1/2))*(109+6*3^(1/2)*110^(1/2))^(2/3)-1/6*I*3^(1/2)*(109+6*3^(1/2)*110
^(1/2))^(1/3)+1/6*(1/2-1/2*I*3^(1/2))*(109+6*3^(1/2)*110^(1/2))^(1/3)*110^(
1/2)*3^(1/2)-17/6*I*(1/2-1/2*I*3^(1/2))*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^(
1/3)-17/6*(1/2-1/2*I*3^(1/2))*(109+6*3^(1/2)*110^(1/2))^(1/3)-7/3+1/2*I*(1/
2-1/2*I*3^(1/2))*(109+6*3^(1/2)*110^(1/2))^(1/3)*110^(1/2)+110^(1/2)*3^(1/2
)*(109+6*3^(1/2)*110^(1/2))^(2/3)-127/6*I*(1/2-1/2*I*3^(1/2))*3^(1/2)*(109+
6*3^(1/2)*110^(1/2))^(2/3)+7/2*I*(1/2-1/2*I*3^(1/2))*(109+6*3^(1/2)*110^(1/
2))^(2/3)*110^(1/2)-109/6*(109+6*3^(1/2)*110^(1/2))^(2/3)-1/3*I*3^(1/2)-1/6
*(109+6*3^(1/2)*110^(1/2))^(1/3))*EllipticPi(1/3*3^(1/2)*(I*(x-1/12*(109+6*
330^(1/2))^(1/3)-1/12/(109+6*330^(1/2))^(1/3)+1/6+1/2*I*3^(1/2)*(-1/6*(109+
6*330^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3)))*3^(1/2)/(1/6*(109+6*330^(1
/2))^(1/3)-1/6/(109+6*330^(1/2))^(1/3)))^(1/2),-1/2*I*(109+6*3^(1/2)*110^(1
/2))^(1/3)*110^(1/2)-1/12*(1/2-1/2*I*3^(1/2))*110^(1/2)*3^(1/2)*(109+6*3^(1
/2)*110^(1/2))^(2/3)-1/6*I*(1/2-1/2*I*3^(1/2))*110^(1/2)+17/12*(1/2-1/2*I*3
^(1/2))*(109+6*3^(1/2)*110^(1/2))^(2/3)+31/12*I*3^(1/2)*(109+6*3^(1/2)*110^
(1/2))^(1/3)+7/12*(1/2-1/2*I*3^(1/2))*(109+6*3^(1/2)*110^(1/2))^(1/3)*110^(
1/2)*3^(1/2)-41/12*I*(1/2-1/2*I*3^(1/2))*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^
(1/3)-127/12*(1/2-1/2*I*3^(1/2))*(109+6*3^(1/2)*110^(1/2))^(1/3)+581/12*I*3
^(1/2)*(109+6*3^(1/2)*110^(1/2))^(2/3)-19/12-8*I*(109+6*3^(1/2)*110^(1/2))^
(2/3)*110^(1/2)+7/12*I*(1/2-1/2*I*3^(1/2))*(109+6*3^(1/2)*110^(1/2))^(1/3)*
110^(1/2)-151/12*I*(1/2-1/2*I*3^(1/2))*3^(1/2)*(109+6*3^(1/2)*110^(1/2))^(2
/3)+1/12*(109+6*3^(1/2)*110^(1/2))^(2/3)+25/12*I*(1/2-1/2*I*3^(1/2))*(109+6
*3^(1/2)*110^(1/2))^(2/3)*110^(1/2)-1/2*(109+6*3^(1/2)*110^(1/2))^(1/3)*110
^(1/2)*3^(1/2)+17/12*I*3^(1/2)+109/12*(109+6*3^(1/2)*110^(1/2))^(1/3),(I*3^
(1/2)*(1/6*(109+6*330^(1/2))^(1/3)-1/6/(109+6*330^(1/2))^(1/3))/(1/4*(109+6
*330^(1/2))^(1/3)+1/4/(109+6*330^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/6*(109+6*33
0^(1/2))^(1/3)+1/6/(109+6*330^(1/2))^(1/3))))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^3+x^2+2}\left(x^3-2\right)}{\left(x^3+x^2+1\right)\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(2*x^3+x^2+2)^(1/2)/(x^3+1)/(x^3+x^2+1),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(2*x^3 + x^2 + 2)*(x^3 - 2)/((x^3 + x^2 + 1)*(x^3 + 1)), x)
```

mupad [B] time = 1.60, size = 2611, normalized size = 66.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 2)*(x^2 + 2*x^3 + 2)^(1/2))/((x^3 + 1)*(x^2 + x^3 + 1)),x)
```

```
[Out] symsum(-(2*(-(x + (3^(1/2)*(1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3
)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))*1i)/2 - 1/(72*(109/216 - (
55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3
)/2 + 1/6)/(1/(24*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (3^(1/2)*(1
/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216
^(1/2))/216)^(1/3))*1i)/2 + (3*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))/
2))^(1/2)*((x + 1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) + (109/21
6 - (55^(1/2)*216^(1/2))/216)^(1/3) + 1/6)/(1/(24*(109/216 - (55^(1/2)*216^
(1/2))/216)^(1/3)) - (3^(1/2)*(1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(
1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))*1i)/2 + (3*(109/216 - (
55^(1/2)*216^(1/2))/216)^(1/3))/2))^(1/2)*(x^2/2 + x^3 + 1)^(1/2)*(1/(24*(1
09/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (3^(1/2)*(1/(36*(109/216 - (55^
(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))*
1i)/2 + (3*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))/2)*ellipticPi(-(1/(2
4*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (3^(1/2)*(1/(36*(109/216 -
(55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/
3))*1i)/2 + (3*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))/2)/(_X187 + (3^(
1/2)*(1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1
/2)*216^(1/2))/216)^(1/3))*1i)/2 - 1/(72*(109/216 - (55^(1/2)*216^(1/2))/21
6)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)/2 + 1/6), asin((-(x
+ (3^(1/2)*(1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 -
(55^(1/2)*216^(1/2))/216)^(1/3))*1i)/2 - 1/(72*(109/216 - (55^(1/2)*216^(1/
2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)/2 + 1/6)/(1/(2
4*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (3^(1/2)*(1/(36*(109/216 -
(55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/
3))*1i)/2 + (3*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))/2))^(1/2)), (3^(
1/2)*(1/(24*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (3^(1/2)*(1/(36*(
109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2)
)/216)^(1/3))*1i)/2 + (3*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))/2)*1i)
/(3*(1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/
2)*216^(1/2))/216)^(1/3))))*(-(3^(1/2)*((3^(1/2)*(1/(36*(109/216 - (55^(1/2
)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))*1i)/
2 - x + 1/(72*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) + (109/216 - (55^
(1/2)*216^(1/2))/216)^(1/3)/2 - 1/6)*1i)/(3*(1/(36*(109/216 - (55^(1/2)*216
^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))))^(1/2)*(
4*_X187^2 + 6*_X187^3 + _X187^5 + 6))/(((_X187^3 + 1)*(2*_X187 + 3*_X187^2)
 + 3*_X187^2*(_X187^2 + _X187^3 + 1))*(x^2 + 2*x^3 + 2)^(1/2)*(x^2/2 - x*((
(3^(1/2)*(1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (5
5^(1/2)*216^(1/2))/216)^(1/3))*1i)/2 - 1/(72*(109/216 - (55^(1/2)*216^(1/2)
)/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)/2 + 1/6)*((3^(1/
2)*(1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2
)*216^(1/2))/216)^(1/3))*1i)/2 + 1/(72*(109/216 - (55^(1/2)*216^(1/2))/216)
^(1/3)) + (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)/2 - 1/6) - (1/(36*(109
/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) + (109/216 - (55^(1/2)*216^(1/2))/2
16)^(1/3) + 1/6)*((3^(1/2)*(1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3
)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))*1i)/2 - 1/(72*(109/216 - (
55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3
)/2 + 1/6) + (1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) + (109/216
- (55^(1/2)*216^(1/2))/216)^(1/3) + 1/6)*((3^(1/2)*(1/(36*(109/216 - (55^(1
/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))*1i
)/2 + 1/(72*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) + (109/216 - (55^(1
/2)*216^(1/2))/216)^(1/3)/2 - 1/6)) + x^3 - (1/(36*(109/216 - (55^(1/2)*216
^(1/2))/216)^(1/3)) + (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3) + 1/6)*((3
^(1/2)*(1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^
```

```
(1/2)*216^(1/2))/216)^(1/3))*1i)/2 - 1/(72*(109/216 - (55^(1/2)*216^(1/2))/
216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)/2 + 1/6)*((3^(1/2)
*(1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*
216^(1/2))/216)^(1/3))*1i)/2 + 1/(72*(109/216 - (55^(1/2)*216^(1/2))/216)^(
1/3)) + (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)/2 - 1/6))^(1/2)*(_X187 +
 (3^(1/2)*(1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (
55^(1/2)*216^(1/2))/216)^(1/3))*1i)/2 - 1/(72*(109/216 - (55^(1/2)*216^(1/2
))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)/2 + 1/6)), _X18
7 in {-1, 1/2 - (3^(1/2)*1i)/2, (3^(1/2)*1i)/2 + 1/2} union root(z^3 + z^2
+ 1, z)) - (4*(-(x + (3^(1/2)*(1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(
1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))*1i)/2 - 1/(72*(109/216
- (55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(
1/3)/2 + 1/6)/(1/(24*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (3^(1/2)
*(1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*
216^(1/2))/216)^(1/3))*1i)/2 + (3*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3
))/2))^(1/2)*((x + 1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) + (109
/216 - (55^(1/2)*216^(1/2))/216)^(1/3) + 1/6)/(1/(24*(109/216 - (55^(1/2)*2
16^(1/2))/216)^(1/3)) - (3^(1/2)*(1/(36*(109/216 - (55^(1/2)*216^(1/2))/216
)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))*1i)/2 + (3*(109/216
- (55^(1/2)*216^(1/2))/216)^(1/3))/2))^(1/2)*ellipticF(asin((-(x + (3^(1/2)
*(1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*
216^(1/2))/216)^(1/3))*1i)/2 - 1/(72*(109/216 - (55^(1/2)*216^(1/2))/216)^(
1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)/2 + 1/6)/(1/(24*(109/216
 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (3^(1/2)*(1/(36*(109/216 - (55^(1/2)*
216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))*1i)/2
+ (3*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))/2))^(1/2)), (3^(1/2)*(1/(2
4*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (3^(1/2)*(1/(36*(109/216 -
(55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/
3))*1i)/2 + (3*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))/2)*1i)/(3*(1/(36
*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/
2))/216)^(1/3))))*(x^2/2 + x^3 + 1)^(1/2)*(1/(24*(109/216 - (55^(1/2)*216^(
1/2))/216)^(1/3)) - (3^(1/2)*(1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1
/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))*1i)/2 + (3*(109/216 - (5
5^(1/2)*216^(1/2))/216)^(1/3))/2)*(-(3^(1/2)*((3^(1/2)*(1/(36*(109/216 - (5
5^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)
)*1i)/2 - x + 1/(72*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) + (109/216
- (55^(1/2)*216^(1/2))/216)^(1/3)/2 - 1/6)*1i)/(3*(1/(36*(109/216 - (55^(1/
2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))))^(
1/2))/((x^2 + 2*x^3 + 2)^(1/2)*(x^2/2 - x*(((3^(1/2)*(1/(36*(109/216 - (55^
(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))*
1i)/2 - 1/(72*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^
(1/2)*216^(1/2))/216)^(1/3)/2 + 1/6)*((3^(1/2)*(1/(36*(109/216 - (55^(1/2)*
216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))*1i)/2
+ 1/(72*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) + (109/216 - (55^(1/2)*
216^(1/2))/216)^(1/3)/2 - 1/6) - (1/(36*(109/216 - (55^(1/2)*216^(1/2))/216
)^(1/3)) + (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3) + 1/6)*((3^(1/2)*(1/(
36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(
1/2))/216)^(1/3))*1i)/2 - 1/(72*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))
 - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)/2 + 1/6) + (1/(36*(109/216 -
(55^(1/2)*216^(1/2))/216)^(1/3)) + (109/216 - (55^(1/2)*216^(1/2))/216)^(1/
3) + 1/6)*((3^(1/2)*(1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (1
09/216 - (55^(1/2)*216^(1/2))/216)^(1/3))*1i)/2 + 1/(72*(109/216 - (55^(1/2
)*216^(1/2))/216)^(1/3)) + (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)/2 - 1
/6)) + x^3 - (1/(36*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) + (109/216
- (55^(1/2)*216^(1/2))/216)^(1/3) + 1/6)*((3^(1/2)*(1/(36*(109/216 - (55^(1
/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))*1i
)/2 - 1/(72*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) - (109/216 - (55^(1
/2)*216^(1/2))/216)^(1/3)/2 + 1/6)*((3^(1/2)*(1/(36*(109/216 - (55^(1/2)*21
6^(1/2))/216)^(1/3)) - (109/216 - (55^(1/2)*216^(1/2))/216)^(1/3))*1i)/2 +
```

```
1/(72*(109/216 - (55^(1/2)*216^(1/2))/216)^(1/3)) + (109/216 - (55^(1/2)*21
6^(1/2))/216)^(1/3)/2 - 1/6))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^3-2)\sqrt{2x^3+x^2+2}}{(x+1)(x^2-x+1)(x^3+x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2)*(2*x**3+x**2+2)**(1/2)/(x**3+1)/(x**3+x**2+1),x)
```

```
[Out] Integral((x**3 - 2)*sqrt(2*x**3 + x**2 + 2)/((x + 1)*(x**2 - x + 1)*(x**3 +
 x**2 + 1)), x)
```

3.504 $\int \frac{b+ax^2}{(-b+cx+ax^2)\sqrt{-bx+ax^3}}\,dx$

Optimal. Leaf size=39

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax^3-bx}}{ax^2-b}\right)}{\sqrt{c}}$$

Rubi [C] time = 1.99, antiderivative size = 254, normalized size of antiderivative = 6.51, number of steps used = 14, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2056, 6728, 329, 224, 221, 933, 168, 537}

$$-\frac{2\sqrt[4]{b}\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\,\Pi\left(-\frac{2\sqrt{a}\sqrt{b}}{c-\sqrt{c^2+4ab}};\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{ax^3-bx}}-\frac{2\sqrt[4]{b}\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\,\Pi\left(-\frac{2\sqrt{a}\sqrt{b}}{c+\sqrt{c^2+4ab}};\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{ax^3-bx}}+\frac{2\sqrt[4]{b}\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{ax^3-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^2)/((-b + c*x + a*x^2)*Sqrt[-(b*x) + a*x^3]),x]
```

```
[Out] (2*b^(1/4)*Sqrt[x]*Sqrt[1 - (a*x^2)/b]*EllipticF[ArcSin[(a^(1/4)*Sqrt[x])/b
^(1/4)], -1])/(a^(1/4)*Sqrt[-(b*x) + a*x^3]) - (2*b^(1/4)*Sqrt[x]*Sqrt[1 -
(a*x^2)/b]*EllipticPi[(-2*Sqrt[a]*Sqrt[b])/(c - Sqrt[4*a*b + c^2]), ArcSin[
(a^(1/4)*Sqrt[x])/b^(1/4)], -1])/(a^(1/4)*Sqrt[-(b*x) + a*x^3]) - (2*b^(1/4
)*Sqrt[x]*Sqrt[1 - (a*x^2)/b]*EllipticPi[(-2*Sqrt[a]*Sqrt[b])/(c + Sqrt[4*a
*b + c^2]), ArcSin[(a^(1/4)*Sqrt[x])/b^(1/4)], -1])/(a^(1/4)*Sqrt[-(b*x) +
a*x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
```

```
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{b+ax^2}{(-b+cx+ax^2)\sqrt{-bx+ax^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\int \frac{b+ax^2}{\sqrt{x}\sqrt{-b+ax^2}(-b+cx+ax^2)}\,dx}{\sqrt{-bx+ax^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{-b+ax^2}} + \frac{2b-cx}{\sqrt{x}\sqrt{-b+ax^2}(-b+cx+ax^2)}\right)dx}{\sqrt{-bx+ax^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-b+ax^2}}\,dx}{\sqrt{-bx+ax^3}} + \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\int \frac{2b-cx}{\sqrt{x}\sqrt{-b+ax^2}(-b+c}}{\sqrt{-bx+ax^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\int \left(\frac{-c+\sqrt{4ab+c^2}}{\sqrt{x}\left(c-\sqrt{4ab+c^2}+2ax\right)\sqrt{-b+ax^2}} + \frac{-c-\sqrt{4ab+c^2}}{\sqrt{x}\left(c+\sqrt{4ab+c^2}+2ax\right)\sqrt{-b+a}}\right.}{\sqrt{-bx+ax^3}}$$

$$= \frac{\left(\left(-c-\sqrt{4ab+c^2}\right)\sqrt{x}\sqrt{-b+ax^2}\right)\int \frac{1}{\sqrt{x}\left(c+\sqrt{4ab+c^2}+2ax\right)\sqrt{-b+ax^2}}\,dx}{\sqrt{-bx+ax^3}} + \frac{((}{}$$

$$= \frac{2\sqrt[4]{b}\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{-bx+ax^3}} + \frac{\left(\left(-c-\sqrt{4ab+c^2}\right)\sqrt{x}\sqrt{1-\right.}{}$$

$$= \frac{2\sqrt[4]{b}\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{-bx+ax^3}} - \frac{\left(2\left(-c-\sqrt{4ab+c^2}\right)\sqrt{x}\sqrt{1\right.}{}$$

$$= \frac{2\sqrt[4]{b}\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{-bx+ax^3}} - \frac{2\sqrt[4]{b}\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\,\Pi\left(-\frac{2\sqrt{a}}{c-\sqrt{4a}}\right.}{\sqrt[4]{a}\sqrt{-bx}}$$

Mathematica [C] time = 1.65, size = 204, normalized size = 5.23

$$-\frac{2ix^{3/2}\sqrt{1-\frac{b}{ax^2}}\left(-\Pi\left(\frac{2\sqrt{a}\sqrt{b}}{c-\sqrt{c^2+4ab}};i\sinh^{-1}\left(\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(\frac{2\sqrt{a}\sqrt{b}}{c+\sqrt{c^2+4ab}};i\sinh^{-1}\left(\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{x}}\right)\middle|-1\right)+F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{ax^3-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^2)/((-b + c*x + a*x^2)*Sqrt[-(b*x) + a*x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[1 - b/(a*x^2)]*x^(3/2)*(EllipticF[I*ArcSinh[Sqrt[-(Sqrt[b]/Sqr
t[a])]/Sqrt[x]], -1] - EllipticPi[(2*Sqrt[a]*Sqrt[b])/(c - Sqrt[4*a*b + c^2
]), I*ArcSinh[Sqrt[-(Sqrt[b]/Sqrt[a])]/Sqrt[x]], -1] - EllipticPi[(2*Sqrt[a
]*Sqrt[b])/(c + Sqrt[4*a*b + c^2]), I*ArcSinh[Sqrt[-(Sqrt[b]/Sqrt[a])]/Sqrt
[x]], -1]))/(Sqrt[-(Sqrt[b]/Sqrt[a])]*Sqrt[-(b*x) + a*x^3])
```

IntegrateAlgebraic [A] time = 0.34, size = 39, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax^3-bx}}{ax^2-b}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^2)/((-b + c*x + a*x^2)*Sqrt[-(b*x) + a*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[c]*Sqrt[-(b*x) + a*x^3])/(-b + a*x^2)])/Sqrt[c]
```

fricas [A] time = 0.48, size = 172, normalized size = 4.41

$$\left[-\frac{\sqrt{-c}\log\left(\frac{a^2x^4-6\,acx^3+6\,bcx-\left(2\,ab-c^2\right)x^2+b^2-4\,\sqrt{ax^3-bx}\left(ax^2-cx-b\right)\sqrt{-c}}{a^2x^4+2\,acx^3-2\,bcx-\left(2\,ab-c^2\right)x^2+b^2}\right)}{2\,c},\ \frac{\arctan\left(\frac{\sqrt{ax^3-bx}\left(ax^2-cx-b\right)\sqrt{c}}{2\left(acx^3-bcx\right)}\right)}{\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a*x^2+c*x-b)/(a*x^3-b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-c)*log((a^2*x^4 - 6*a*c*x^3 + 6*b*c*x - (2*a*b - c^2)*x^2 + b^2
 - 4*sqrt(a*x^3 - b*x)*(a*x^2 - c*x - b)*sqrt(-c))/(a^2*x^4 + 2*a*c*x^3 - 2
*b*c*x - (2*a*b - c^2)*x^2 + b^2))/c, arctan(1/2*sqrt(a*x^3 - b*x)*(a*x^2 -
 c*x - b)*sqrt(c)/(a*c*x^3 - b*c*x))/sqrt(c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2+b}{\sqrt{ax^3-bx}\left(ax^2+cx-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a*x^2+c*x-b)/(a*x^3-b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^2 + b)/(sqrt(a*x^3 - b*x)*(a*x^2 + c*x - b)), x)
```

maple [C] time = 0.08, size = 1106, normalized size = 28.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)/(a*x^2+c*x-b)/(a*x^3-b*x)^(1/2),x)
```

```
[Out] 1/a*(a*b)^(1/2)*((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2)*(-2*(x-1/a*(a*b)^
(1/2))*a/(a*b)^(1/2))^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)*Elli
pticF(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/2/(4*a*b+c^2
)^(1/2)/a^2*(a*b)^(1/2)*(x*a/(a*b)^(1/2)+1)^(1/2)*(-2*x*a/(a*b)^(1/2)+2)^(1
/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)/(-1/a*(a*b)^(1/2)+1/2*c/a-1/
2/a*(4*a*b+c^2)^(1/2))*EllipticPi(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2)
,-1/a*(a*b)^(1/2)/(-1/a*(a*b)^(1/2)-1/2/a*(-c+(4*a*b+c^2)^(1/2))),1/2*2^(1/
2))*c^2-1/2/a^2*(a*b)^(1/2)*(x*a/(a*b)^(1/2)+1)^(1/2)*(-2*x*a/(a*b)^(1/2)+2
)^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)/(-1/a*(a*b)^(1/2)+1/2*c/
a-1/2/a*(4*a*b+c^2)^(1/2))*EllipticPi(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(
1/2),-1/a*(a*b)^(1/2)/(-1/a*(a*b)^(1/2)-1/2/a*(-c+(4*a*b+c^2)^(1/2))),1/2*2
^(1/2))*c+2/(4*a*b+c^2)^(1/2)/a*(a*b)^(1/2)*(x*a/(a*b)^(1/2)+1)^(1/2)*(-2*x
*a/(a*b)^(1/2)+2)^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)/(-1/a*(a
*b)^(1/2)+1/2*c/a-1/2/a*(4*a*b+c^2)^(1/2))*EllipticPi(((x+1/a*(a*b)^(1/2))*
a/(a*b)^(1/2))^(1/2),-1/a*(a*b)^(1/2)/(-1/a*(a*b)^(1/2)-1/2/a*(-c+(4*a*b+c^
2)^(1/2))),1/2*2^(1/2))*b-1/2/(4*a*b+c^2)^(1/2)/a^2*(a*b)^(1/2)*(x*a/(a*b)^
(1/2)+1)^(1/2)*(-2*x*a/(a*b)^(1/2)+2)^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3
-b*x)^(1/2)/(-1/a*(a*b)^(1/2)+1/2*c/a+1/2/a*(4*a*b+c^2)^(1/2))*EllipticPi((
(x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2),-1/a*(a*b)^(1/2)/(-1/a*(a*b)^(1/2)
+1/2*(c+(4*a*b+c^2)^(1/2))/a),1/2*2^(1/2))*c^2-1/2/a^2*(a*b)^(1/2)*(x*a/(a*
b)^(1/2)+1)^(1/2)*(-2*x*a/(a*b)^(1/2)+2)^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*
x^3-b*x)^(1/2)/(-1/a*(a*b)^(1/2)+1/2*c/a+1/2/a*(4*a*b+c^2)^(1/2))*EllipticP
i(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2),-1/a*(a*b)^(1/2)/(-1/a*(a*b)^(1
/2)+1/2*(c+(4*a*b+c^2)^(1/2))/a),1/2*2^(1/2))*c-2/(4*a*b+c^2)^(1/2)/a*(a*b)
```

```
^(1/2)*(x*a/(a*b)^(1/2)+1)^(1/2)*(-2*x*a/(a*b)^(1/2)+2)^(1/2)*(-x*a/(a*b)^(
1/2))^(1/2)/(a*x^3-b*x)^(1/2)/(-1/a*(a*b)^(1/2)+1/2*c/a+1/2/a*(4*a*b+c^2)^(
1/2))*EllipticPi(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2),-1/a*(a*b)^(1/2)
/(-1/a*(a*b)^(1/2)+1/2*(c+(4*a*b+c^2)^(1/2))/a),1/2*2^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2+b}{\sqrt{ax^3-bx}\left(ax^2+cx-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a*x^2+c*x-b)/(a*x^3-b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 + b)/(sqrt(a*x^3 - b*x)*(a*x^2 + c*x - b)), x)
```

mupad [B] time = 2.19, size = 51, normalized size = 1.31

$$\frac{\ln\left(\frac{b+c\,x-a\,x^2-\sqrt{c}\,\sqrt{a\,x^3-b\,x}\,2\mathrm{i}}{a\,x^2+c\,x-b}\right)\,1\mathrm{i}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^2)/((a*x^3 - b*x)^(1/2)*(c*x - b + a*x^2)),x)
```

```
[Out] (log((b + c*x - a*x^2 - c^(1/2)*(a*x^3 - b*x)^(1/2)*2i)/(c*x - b + a*x^2))*
1i)/c^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2+b}{\sqrt{x\left(ax^2-b\right)}\left(ax^2-b+cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b)/(a*x**2+c*x-b)/(a*x**3-b*x)**(1/2),x)
```

```
[Out] Integral((a*x**2 + b)/(sqrt(x*(a*x**2 - b))*(a*x**2 - b + c*x)), x)
```

3.505 $\int x\sqrt{-1+x^4}\,dx$

Optimal. Leaf size=39

$$\frac{1}{4}x^2\sqrt{x^4-1}-\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{x^4-1}}{x^2+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {275, 195, 217, 206}

$$\frac{1}{4}x^2\sqrt{x^4-1}-\frac{1}{4}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[-1 + x^4],x]
```

```
[Out] (x^2*Sqrt[-1 + x^4])/4 - ArcTanh[x^2/Sqrt[-1 + x^4]]/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{-1+x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int\sqrt{-1+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{4}x^2\sqrt{-1+x^4}-\frac{1}{4}\text{Subst}\left(\int\frac{1}{\sqrt{-1+x^2}}\,dx,x,x^2\right)\\
&= \frac{1}{4}x^2\sqrt{-1+x^4}-\frac{1}{4}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x^2}{\sqrt{-1+x^4}}\right)\\
&= \frac{1}{4}x^2\sqrt{-1+x^4}-\frac{1}{4}\tanh^{-1}\left(\frac{x^2}{\sqrt{-1+x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.08

$$\frac{\left(x^4-1\right)\left(\sin^{-1}\left(x^2\right)+\sqrt{1-x^4}\,x^2\right)}{4\sqrt{-\left(x^4-1\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[-1 + x^4],x]
[Out] ((-1 + x^4)*(x^2*Sqrt[1 - x^4] + ArcSin[x^2]))/(4*Sqrt[-(-1 + x^4)^2])
```

IntegrateAlgebraic [A] time = 0.16, size = 39, normalized size = 1.00

$$\frac{1}{4}x^2\sqrt{x^4-1}-\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{x^4-1}}{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[-1 + x^4],x]
[Out] (x^2*Sqrt[-1 + x^4])/4 - ArcTanh[Sqrt[-1 + x^4]/(1 + x^2)]/2
```

fricas [A] time = 0.40, size = 29, normalized size = 0.74

$$\frac{1}{4}\sqrt{x^4-1}\,x^2+\frac{1}{4}\log\left(-x^2+\sqrt{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4-1)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(x^4 - 1)*x^2 + 1/4*log(-x^2 + sqrt(x^4 - 1))
```

giac [A] time = 0.30, size = 29, normalized size = 0.74

$$\frac{1}{4}\sqrt{x^4-1}\,x^2+\frac{1}{4}\log\left(x^2-\sqrt{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4-1)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(x^4 - 1)*x^2 + 1/4*log(x^2 - sqrt(x^4 - 1))
```

maple [A] time = 0.01, size = 28, normalized size = 0.72

$$\frac{x^2\sqrt{x^4-1}}{4}-\frac{\ln\left(x^2+\sqrt{x^4-1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^4-1)^(1/2),x)
[Out] 1/4*x^2*(x^4-1)^(1/2)-1/4*ln(x^2+(x^4-1)^(1/2))
```

maxima [A] time = 0.32, size = 58, normalized size = 1.49

$$-\frac{\sqrt{x^4-1}}{4\,x^2\left(\frac{x^4-1}{x^4}-1\right)}-\frac{1}{8}\log\left(\frac{\sqrt{x^4-1}}{x^2}+1\right)+\frac{1}{8}\log\left(\frac{\sqrt{x^4-1}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(x^4 - 1)/(x^2*((x^4 - 1)/x^4 - 1)) - 1/8*log(sqrt(x^4 - 1)/x^2 +
1) + 1/8*log(sqrt(x^4 - 1)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x\sqrt{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^4 - 1)^(1/2),x)
```

```
[Out] int(x*(x^4 - 1)^(1/2), x)
```

sympy [A] time = 1.28, size = 60, normalized size = 1.54

$$\begin{cases} \frac{x^6}{4\sqrt{x^4-1}} - \frac{x^2}{4\sqrt{x^4-1}} - \frac{\operatorname{acosh}\left(x^2\right)}{4} & \text{for}\ \left|x^4\right| > 1 \\ \frac{ix^2\sqrt{1-x^4}}{4} + \frac{i\operatorname{asin}\left(x^2\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**4-1)**(1/2),x)
```

```
[Out] Piecewise((x**6/(4*sqrt(x**4 - 1)) - x**2/(4*sqrt(x**4 - 1)) - acosh(x**2)/
4, Abs(x**4) > 1), (I*x**2*sqrt(1 - x**4)/4 + I*asin(x**2)/4, True))
```

3.506 $\int \frac{\sqrt{-x+x^4}}{x^3} \, dx$

Optimal. Leaf size=39

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)-\frac{2\sqrt{x^4-x}}{3x^2}$$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2020, 2029, 206}

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)-\frac{2\sqrt{x^4-x}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-x + x^4]/x^3,x]
```

```
[Out] (-2*Sqrt[-x + x^4])/(3*x^2) + (2*ArcTanh[x^2/Sqrt[-x + x^4]])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-x+x^4}}{x^3} \, dx &= -\frac{2\sqrt{-x+x^4}}{3x^2}+\int \frac{x}{\sqrt{-x+x^4}} \, dx\\
&= -\frac{2\sqrt{-x+x^4}}{3x^2}+\frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x^2} \, dx,x,\frac{x^2}{\sqrt{-x+x^4}}\right)\\
&= -\frac{2\sqrt{-x+x^4}}{3x^2}+\frac{2}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{-x+x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.36

$$-\frac{2\sqrt{x\left(x^3-1\right)}\left(x^{3/2}\sin^{-1}\left(x^{3/2}\right)+\sqrt{1-x^3}\right)}{3x^2\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x + x^4]/x^3,x]
```

```
[Out] (-2*Sqrt[x*(-1 + x^3)]*(Sqrt[1 - x^3] + x^(3/2)*ArcSin[x^(3/2)]))/(3*x^2*Sq
rt[1 - x^3])
```

IntegrateAlgebraic [A] time = 0.37, size = 39, normalized size = 1.00

$$\frac{2}{3}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)-\frac{2\sqrt{x^4-x}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-x + x^4]/x^3,x]
```

```
[Out] (-2*Sqrt[-x + x^4])/(3*x^2) + (2*ArcTanh[x^2/Sqrt[-x + x^4]])/3
```

fricas [A] time = 0.44, size = 41, normalized size = 1.05

$$\frac{x^2\log\left(-2\,x^3-2\,\sqrt{x^4-x}\,x+1\right)-2\,\sqrt{x^4-x}}{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/3*(x^2*log(-2*x^3 - 2*sqrt(x^4 - x)*x + 1) - 2*sqrt(x^4 - x))/x^2
```

giac [A] time = 0.26, size = 41, normalized size = 1.05

$$-\frac{2}{3}\sqrt{-\frac{1}{x^3}+1}+\frac{1}{3}\log\left(\sqrt{-\frac{1}{x^3}+1}+1\right)-\frac{1}{3}\log\left(\left|\sqrt{-\frac{1}{x^3}+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -2/3*sqrt(-1/x^3 + 1) + 1/3*log(sqrt(-1/x^3 + 1) + 1) - 1/3*log(abs(sqrt(-1
/x^3 + 1) - 1))
```

maple [C] time = 0.04, size = 303, normalized size = 7.77

$$-\frac{2\sqrt{x^4-x}}{3x^2}+\frac{2\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}}\,(-1+x)^2\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(-1+x)}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)-\mathrm{EllipticPi}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\frac{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(-1+x)\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x)^(1/2)/x^3,x)
```

```
[Out] -2/3*(x^4-x)^(1/2)/x^2+2*(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+
1/2*I*3^(1/2))/(-1+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^
(1/2))/(-1+x))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1
/2)/(-3/2+1/2*I*3^(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/
2)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^
(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2
*I*3^(1/2)))^(1/2))-EllipticPi(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))
/(-1+x))^(1/2),(-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)),((3/2+1/2*I*3^(1/2
))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^4-x}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 - x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{x^4-x}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x)^(1/2)/x^3,x)
```

```
[Out] int((x^4 - x)^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x(x-1)\left(x^2+x+1\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x*(x - 1)*(x**2 + x + 1))/x**3, x)
```

3.507 $\int x^3\sqrt{x+x^4}\,dx$

Optimal. Leaf size=39

$$\frac{1}{12}\sqrt{x^4+x}\left(2x^4+x\right)-\frac{1}{12}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2021, 2024, 2029, 206}

$$\frac{1}{6}\sqrt{x^4+x}\,x^4+\frac{1}{12}\sqrt{x^4+x}\,x-\frac{1}{12}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[x + x^4],x]
```

```
[Out] (x*Sqrt[x + x^4])/12 + (x^4*Sqrt[x + x^4])/6 - ArcTanh[x^2/Sqrt[x + x^4]]/1
2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{x+x^4}\,dx &= \frac{1}{6}x^4\sqrt{x+x^4}+\frac{1}{4}\int \frac{x^4}{\sqrt{x+x^4}}\,dx\\ &= \frac{1}{12}x\sqrt{x+x^4}+\frac{1}{6}x^4\sqrt{x+x^4}-\frac{1}{8}\int \frac{x}{\sqrt{x+x^4}}\,dx\\ &= \frac{1}{12}x\sqrt{x+x^4}+\frac{1}{6}x^4\sqrt{x+x^4}-\frac{1}{12}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x^2}{\sqrt{x+x^4}}\right)\\ &= \frac{1}{12}x\sqrt{x+x^4}+\frac{1}{6}x^4\sqrt{x+x^4}-\frac{1}{12}\tanh^{-1}\left(\frac{x^2}{\sqrt{x+x^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 1.23

$$\frac{\sqrt{x^4+x}\left(2x^{9/2}+x^{3/2}-\frac{\sinh^{-1}\left(x^{3/2}\right)}{\sqrt{x^3+1}}\right)}{12\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[x + x^4],x]
```

```
[Out] (Sqrt[x + x^4]*(x^(3/2) + 2*x^(9/2) - ArcSinh[x^(3/2)]/Sqrt[1 + x^3]))/(12*
Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.41, size = 39, normalized size = 1.00

$$\frac{1}{12}\sqrt{x^4+x}\left(2x^4+x\right)-\frac{1}{12}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[x + x^4],x]
```

```
[Out] (Sqrt[x + x^4]*(x + 2*x^4))/12 - ArcTanh[x^2/Sqrt[x + x^4]]/12
```

fricas [A] time = 0.43, size = 37, normalized size = 0.95

$$\frac{1}{12}\left(2\,x^4+x\right)\sqrt{x^4+x}+\frac{1}{24}\,\log\left(2\,x^3-2\,\sqrt{x^4+x}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(2*x^4 + x)*sqrt(x^4 + x) + 1/24*log(2*x^3 - 2*sqrt(x^4 + x)*x + 1)
```

giac [A] time = 0.34, size = 43, normalized size = 1.10

$$\frac{1}{12}\,\sqrt{x^4+x}\left(2\,x^3+1\right)x-\frac{1}{24}\,\log\left(\sqrt{\frac{1}{x^3}+1}+1\right)+\frac{1}{24}\,\log\left(\left|\sqrt{\frac{1}{x^3}+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(x^4 + x)*(2*x^3 + 1)*x - 1/24*log(sqrt(1/x^3 + 1) + 1) + 1/24*log
(abs(sqrt(1/x^3 + 1) - 1))
```

maple [C] time = 0.04, size = 313, normalized size = 8.03

$$\frac{x^4\sqrt{x^4+x}}{6}+\frac{x\sqrt{x^4+x}}{12}+\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\,(1+x)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(1+x)}}\sqrt{-\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\left(-\mathrm{EllipticF}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)+\mathrm{EllipticPi}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\frac{\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{4\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(1+x)\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4+x)^(1/2),x)
```

```
[Out] 1/6*x^4*(x^4+x)^(1/2)+1/12*x*(x^4+x)^(1/2)+1/4*(-1/2-1/2*I*3^(1/2))*((3/2+1
/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(
1/2))/(1/2-1/2*I*3^(1/2))/(1+x))^(1/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3
^(1/2))/(1+x))^(1/2)/(3/2+1/2*I*3^(1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-
1/2-1/2*I*3^(1/2)))^(1/2)*(-EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(
1/2))/(1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+EllipticPi(((3/2+1/2*I*3^(1/2))*x/(1/2
+1/2*I*3^(1/2))/(1+x))^(1/2),(1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)),((-3/2
+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/
2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+x}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^3\,\sqrt{x^4+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x + x^4)^(1/2),x)
```

```
[Out] int(x^3*(x + x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{x\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**4+x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(x*(x + 1)*(x**2 - x + 1)), x)
```

3.508 $\int \frac{1+x}{\sqrt{16+18x+13x^2+4x^3+x^4}}\,dx$

Optimal. Leaf size=39

$$\frac{1}{2}\log\left(2x^2+2\sqrt{x^4+4x^3+13x^2+18x+16}+4x+9\right)$$

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1680, 1107, 621, 206}

$$\frac{1}{2}\tanh^{-1}\left(\frac{2(x+1)^2+7}{2\sqrt{(x+1)^4+7(x+1)^2+8}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/Sqrt[16 + 18*x + 13*x^2 + 4*x^3 + x^4],x]
```

```
[Out] ArcTanh[(7 + 2*(1 + x)^2)/(2*Sqrt[8 + 7*(1 + x)^2 + (1 + x)^4])]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{\sqrt{16+18x+13x^2+4x^3+x^4}}\,dx &= \text{Subst}\left(\int \frac{x}{\sqrt{8+7x^2+x^4}}\,dx,x,1+x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{8+7x+x^2}}\,dx,x,(1+x)^2\right)\\ &= \text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{7+2(1+x)^2}{\sqrt{8+7(1+x)^2+(1+x)^4}}\right)\\ &= \frac{1}{2}\tanh^{-1}\left(\frac{7+2(1+x)^2}{2\sqrt{8+7(1+x)^2+(1+x)^4}}\right)\end{aligned}$$

Mathematica [C] time = 6.08, size = 2092, normalized size = 53.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x)/Sqrt[16 + 18*x + 13*x^2 + 4*x^3 + x^4],x]
```

```
[Out] (2*((-2 - I*Sqrt[2*(7 - Sqrt[17])])/2 + (2 - I*Sqrt[2*(7 + Sqrt[17])])/2)*(
(2 - I*Sqrt[2*(7 - Sqrt[17])])/2 + x)^2*Sqrt[(((2 - I*Sqrt[2*(7 - Sqrt[17])
])/2 + (-2 + I*Sqrt[2*(7 + Sqrt[17])])/2)*((2 + I*Sqrt[2*(7 - Sqrt[17])])/2
 + x))/(((2 + I*Sqrt[2*(7 - Sqrt[17])])/2 + (-2 + I*Sqrt[2*(7 + Sqrt[17])])
/2)*((2 - I*Sqrt[2*(7 - Sqrt[17])])/2 + x))]*Sqrt[(((-2 + I*Sqrt[2*(7 - Sqr
t[17])])/2 + (2 + I*Sqrt[2*(7 - Sqrt[17])])/2)*((2 - I*Sqrt[2*(7 + Sqrt[17]
)])/2 + x))/(((2 + I*Sqrt[2*(7 - Sqrt[17])])/2 + (-2 + I*Sqrt[2*(7 + Sqrt[1
7])])/2)*((2 - I*Sqrt[2*(7 - Sqrt[17])])/2 + x))]*Sqrt[(((-2 + I*Sqrt[2*(7
- Sqrt[17])])/2 + (2 + I*Sqrt[2*(7 - Sqrt[17])])/2)*((2 + I*Sqrt[2*(7 + Sqr
t[17])])/2 + x))/(((2 + I*Sqrt[2*(7 - Sqrt[17])])/2 + (-2 - I*Sqrt[2*(7 + S
qrt[17])])/2)*((2 - I*Sqrt[2*(7 - Sqrt[17])])/2 + x))]*EllipticF[ArcSin[Sqr
t[(((2 - I*Sqrt[2*(7 - Sqrt[17])])/2 + (-2 + I*Sqrt[2*(7 + Sqrt[17])])/2)*(
(2 + I*Sqrt[2*(7 - Sqrt[17])])/2 + x))/(((2 + I*Sqrt[2*(7 - Sqrt[17])])/2 +
 (-2 + I*Sqrt[2*(7 + Sqrt[17])])/2)*((2 - I*Sqrt[2*(7 - Sqrt[17])])/2 + x))
]], (((-2 - I*Sqrt[2*(7 - Sqrt[17])])/2 + (2 - I*Sqrt[2*(7 + Sqrt[17])])/2)
*((-2 + I*Sqrt[2*(7 - Sqrt[17])])/2 + (2 + I*Sqrt[2*(7 + Sqrt[17])])/2))/((
(-2 + I*Sqrt[2*(7 - Sqrt[17])])/2 + (2 - I*Sqrt[2*(7 + Sqrt[17])])/2)*((-2
- I*Sqrt[2*(7 - Sqrt[17])])/2 + (2 + I*Sqrt[2*(7 + Sqrt[17])])/2))])/(((-2
+ I*Sqrt[2*(7 - Sqrt[17])])/2 + (2 + I*Sqrt[2*(7 - Sqrt[17])])/2)*((2 - I*S
qrt[2*(7 - Sqrt[17])])/2 + (-2 + I*Sqrt[2*(7 + Sqrt[17])])/2)*Sqrt[16 + 18*
x + 13*x^2 + 4*x^3 + x^4]) + (2*Sqrt[(2*(7 - Sqrt[17]))/(Sqrt[7 - Sqrt[17]]
 + Sqrt[7 + Sqrt[17]])]*((-2 - I*Sqrt[2*(7 - Sqrt[17])])/2 + (2 - I*Sqrt[2*
(7 + Sqrt[17])])/2)*Sqrt[((Sqrt[7 - Sqrt[17]] - Sqrt[7 + Sqrt[17]])*(-2*I +
 Sqrt[2*(7 - Sqrt[17])] - (2*I)*x))/(2*I + Sqrt[2*(7 - Sqrt[17])] + (2*I)*x
)]*((2 - I*Sqrt[2*(7 - Sqrt[17])])/2 + x)^2*Sqrt[(I*((2 - I*Sqrt[2*(7 + Sqr
t[17])])/2 + x))/(((2 + I*Sqrt[2*(7 - Sqrt[17])])/2 + (-2 + I*Sqrt[2*(7 + S
qrt[17])])/2)*((2 - I*Sqrt[2*(7 - Sqrt[17])])/2 + x))]*Sqrt[(I*((2 + I*Sqrt
[2*(7 + Sqrt[17])])/2 + x))/(((2 + I*Sqrt[2*(7 - Sqrt[17])])/2 + (-2 - I*Sq
rt[2*(7 + Sqrt[17])])/2)*((2 - I*Sqrt[2*(7 - Sqrt[17])])/2 + x))]*(((2 - I*
Sqrt[2*(7 - Sqrt[17])])*EllipticF[ArcSin[Sqrt[((Sqrt[7 - Sqrt[17]] - Sqrt[7
 + Sqrt[17]])*(-2*I + Sqrt[2*(7 - Sqrt[17])] - (2*I)*x))/((Sqrt[7 - Sqrt[17
]] + Sqrt[7 + Sqrt[17]])*(2*I + Sqrt[2*(7 - Sqrt[17])] + (2*I)*x))]], (Sqrt
[7 - Sqrt[17]] + Sqrt[7 + Sqrt[17]])^2/(Sqrt[7 - Sqrt[17]] - Sqrt[7 + Sqrt[
17]])^2])/2 + I*Sqrt[2*(7 - Sqrt[17])]*EllipticPi[((2 + I*Sqrt[2*(7 - Sqrt[
17])])/2 + (-2 + I*Sqrt[2*(7 + Sqrt[17])])/2)/((2 - I*Sqrt[2*(7 - Sqrt[17])
])/2 + (-2 + I*Sqrt[2*(7 + Sqrt[17])])/2), ArcSin[Sqrt[((Sqrt[7 - Sqrt[17]]
 - Sqrt[7 + Sqrt[17]])*(-2*I + Sqrt[2*(7 - Sqrt[17])] - (2*I)*x))/((Sqrt[7
- Sqrt[17]] + Sqrt[7 + Sqrt[17]])*(2*I + Sqrt[2*(7 - Sqrt[17])] + (2*I)*x))
]], (Sqrt[7 - Sqrt[17]] + Sqrt[7 + Sqrt[17]])^2/(Sqrt[7 - Sqrt[17]] - Sqrt[
7 + Sqrt[17]])^2]))/(((-2 + I*Sqrt[2*(7 - Sqrt[17])])/2 + (2 + I*Sqrt[2*(7
- Sqrt[17])])/2)*((-2 + I*Sqrt[2*(7 - Sqrt[17])])/2 + (2 - I*Sqrt[2*(7 + Sq
rt[17])])/2)*Sqrt[16 + 18*x + 13*x^2 + 4*x^3 + x^4])
```

IntegrateAlgebraic [A] time = 0.19, size = 39, normalized size = 1.00

$$\frac{1}{2}\log\left(2x^2+2\sqrt{x^4+4x^3+13x^2+18x+16}+4x+9\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/Sqrt[16 + 18*x + 13*x^2 + 4*x^3 + x^4],x]
```

```
[Out] Log[9 + 4*x + 2*x^2 + 2*Sqrt[16 + 18*x + 13*x^2 + 4*x^3 + x^4]]/2
```

fricas [A] time = 0.42, size = 35, normalized size = 0.90

$$\frac{1}{2}\log\left(2x^2+4x+2\sqrt{x^4+4x^3+13x^2+18x+16}+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^4+4*x^3+13*x^2+18*x+16)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(2*x^2 + 4*x + 2*sqrt(x^4 + 4*x^3 + 13*x^2 + 18*x + 16) + 9)
```

giac [A] time = 0.40, size = 46, normalized size = 1.18

$$-\frac{1}{2}\log\left(\sqrt{2}\left(\sqrt{2}\left(x^2+2x\right)-2\sqrt{\frac{1}{2}\left(x^2+2x\right)^2+\frac{9}{2}x^2+9x+8}\right)+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^4+4*x^3+13*x^2+18*x+16)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(sqrt(2)*(sqrt(2)*(x^2 + 2*x) - 2*sqrt(1/2*(x^2 + 2*x)^2 + 9/2*x^2
+ 9*x + 8)) + 9)
```

maple [C] time = 0.53, size = 1422, normalized size = 36.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^4+4*x^3+13*x^2+18*x+16)^(1/2),x)
```

```
[Out] -2*I*(-1/2*I*(14+2*17^(1/2))^(1/2)-1/2*I*(14-2*17^(1/2))^(1/2))*((1/2*I*(14
-2*17^(1/2))^(1/2)-1/2*I*(14+2*17^(1/2))^(1/2))*(x+1+1/2*I*(14+2*17^(1/2))^
(1/2))/(1/2*I*(14-2*17^(1/2))^(1/2)+1/2*I*(14+2*17^(1/2))^(1/2))/(x+1-1/2*I
*(14+2*17^(1/2))^(1/2)))^(1/2)*(x+1-1/2*I*(14+2*17^(1/2))^(1/2))^2*(I*(14+2
*17^(1/2))^(1/2)*(x+1+1/2*I*(14-2*17^(1/2))^(1/2))/(-1/2*I*(14-2*17^(1/2))^
(1/2)+1/2*I*(14+2*17^(1/2))^(1/2))/(x+1-1/2*I*(14+2*17^(1/2))^(1/2)))^(1/2)
*(I*(14+2*17^(1/2))^(1/2)*(x+1-1/2*I*(14-2*17^(1/2))^(1/2))/(1/2*I*(14-2*17
^(1/2))^(1/2)+1/2*I*(14+2*17^(1/2))^(1/2))/(x+1-1/2*I*(14+2*17^(1/2))^(1/2)
))^(1/2)/(1/2*I*(14-2*17^(1/2))^(1/2)-1/2*I*(14+2*17^(1/2))^(1/2))/(14+2*17
^(1/2))^(1/2)/((x+1+1/2*I*(14+2*17^(1/2))^(1/2))*(x+1-1/2*I*(14+2*17^(1/2))
^(1/2))*(x+1+1/2*I*(14-2*17^(1/2))^(1/2))*(x+1-1/2*I*(14-2*17^(1/2))^(1/2))
)^(1/2)*((-1+1/2*I*(14+2*17^(1/2))^(1/2))*EllipticF(((1/2*I*(14-2*17^(1/2))
^(1/2)-1/2*I*(14+2*17^(1/2))^(1/2))*(x+1+1/2*I*(14+2*17^(1/2))^(1/2))/(1/2*
I*(14-2*17^(1/2))^(1/2)+1/2*I*(14+2*17^(1/2))^(1/2))/(x+1-1/2*I*(14+2*17^(1
/2))^(1/2)))^(1/2),((1/2*I*(14-2*17^(1/2))^(1/2)+1/2*I*(14+2*17^(1/2))^(1/2
))*(-1/2*I*(14+2*17^(1/2))^(1/2)-1/2*I*(14-2*17^(1/2))^(1/2))/(1/2*I*(14-2*
17^(1/2))^(1/2)-1/2*I*(14+2*17^(1/2))^(1/2))/(-1/2*I*(14-2*17^(1/2))^(1/2)+
1/2*I*(14+2*17^(1/2))^(1/2)))^(1/2))-I*(14+2*17^(1/2))^(1/2)*EllipticPi(((1
/2*I*(14-2*17^(1/2))^(1/2)-1/2*I*(14+2*17^(1/2))^(1/2))*(x+1+1/2*I*(14+2*17
^(1/2))^(1/2))/(1/2*I*(14-2*17^(1/2))^(1/2)+1/2*I*(14+2*17^(1/2))^(1/2))/(x
+1-1/2*I*(14+2*17^(1/2))^(1/2)))^(1/2),(1/2*I*(14-2*17^(1/2))^(1/2)+1/2*I*(
14+2*17^(1/2))^(1/2))/(1/2*I*(14-2*17^(1/2))^(1/2)-1/2*I*(14+2*17^(1/2))^(1
/2)),((1/2*I*(14-2*17^(1/2))^(1/2)+1/2*I*(14+2*17^(1/2))^(1/2))*(-1/2*I*(14
+2*17^(1/2))^(1/2)-1/2*I*(14-2*17^(1/2))^(1/2))/(1/2*I*(14-2*17^(1/2))^(1/2
)-1/2*I*(14+2*17^(1/2))^(1/2))/(-1/2*I*(14-2*17^(1/2))^(1/2)+1/2*I*(14+2*17
^(1/2))^(1/2)))^(1/2)))-2*I*(-1/2*I*(14+2*17^(1/2))^(1/2)-1/2*I*(14-2*17^(1
/2))^(1/2))*((1/2*I*(14-2*17^(1/2))^(1/2)-1/2*I*(14+2*17^(1/2))^(1/2))*(x+1
+1/2*I*(14+2*17^(1/2))^(1/2))/(1/2*I*(14-2*17^(1/2))^(1/2)+1/2*I*(14+2*17^(
1/2))^(1/2))/(x+1-1/2*I*(14+2*17^(1/2))^(1/2)))^(1/2)*(x+1-1/2*I*(14+2*17^(
1/2))^(1/2))^2*(I*(14+2*17^(1/2))^(1/2)*(x+1+1/2*I*(14-2*17^(1/2))^(1/2))/(
-1/2*I*(14-2*17^(1/2))^(1/2)+1/2*I*(14+2*17^(1/2))^(1/2))/(x+1-1/2*I*(14+2*
```

```
17^(1/2))^(1/2)))^(1/2)*(I*(14+2*17^(1/2))^(1/2)*(x+1-1/2*I*(14-2*17^(1/2))
^(1/2))/(1/2*I*(14-2*17^(1/2))^(1/2)+1/2*I*(14+2*17^(1/2))^(1/2))/(x+1-1/2*
I*(14+2*17^(1/2))^(1/2)))^(1/2)/(1/2*I*(14-2*17^(1/2))^(1/2)-1/2*I*(14+2*17
^(1/2))^(1/2))/(14+2*17^(1/2))^(1/2)/((x+1+1/2*I*(14+2*17^(1/2))^(1/2))*(x+
1-1/2*I*(14+2*17^(1/2))^(1/2))*(x+1+1/2*I*(14-2*17^(1/2))^(1/2))*(x+1-1/2*I
*(14-2*17^(1/2))^(1/2)))^(1/2)*EllipticF(((1/2*I*(14-2*17^(1/2))^(1/2)-1/2*
I*(14+2*17^(1/2))^(1/2))*(x+1+1/2*I*(14+2*17^(1/2))^(1/2))/(1/2*I*(14-2*17^
(1/2))^(1/2)+1/2*I*(14+2*17^(1/2))^(1/2))/(x+1-1/2*I*(14+2*17^(1/2))^(1/2))
)^(1/2),((1/2*I*(14-2*17^(1/2))^(1/2)+1/2*I*(14+2*17^(1/2))^(1/2))*(-1/2*I*
(14+2*17^(1/2))^(1/2)-1/2*I*(14-2*17^(1/2))^(1/2))/(1/2*I*(14-2*17^(1/2))^(
1/2)-1/2*I*(14+2*17^(1/2))^(1/2))/(-1/2*I*(14-2*17^(1/2))^(1/2)+1/2*I*(14+2
*17^(1/2))^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^4+4\,x^3+13\,x^2+18\,x+16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^4+4*x^3+13*x^2+18*x+16)^(1/2),x, algorithm="maxima")
[Out] integrate((x + 1)/sqrt(x^4 + 4*x^3 + 13*x^2 + 18*x + 16), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x+1}{\sqrt{x^4+4\,x^3+13\,x^2+18\,x+16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(18*x + 13*x^2 + 4*x^3 + x^4 + 16)^(1/2),x)
[Out] int((x + 1)/(18*x + 13*x^2 + 4*x^3 + x^4 + 16)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^4+4x^3+13x^2+18x+16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**4+4*x**3+13*x**2+18*x+16)**(1/2),x)
[Out] Integral((x + 1)/sqrt(x**4 + 4*x**3 + 13*x**2 + 18*x + 16), x)
```

3.509 $\int \frac{\sqrt{-1+x^5}\left(2+3x^5\right)}{x^2\left(-1-ax^2+x^5\right)}\,dx$

Optimal. Leaf size=39

$$\frac{2\sqrt{x^5-1}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^5-1}}\right)$$

Rubi [F] time = 0.79, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-1+x^5}\left(2+3x^5\right)}{x^2\left(-1-ax^2+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-1 + x^5]*(2 + 3*x^5))/(x^2*(-1 - a*x^2 + x^5)),x]
```

```
[Out] (2*Sqrt[-1 + x^5])/x - (5*x^4*Sqrt[1 - x^5]*Hypergeometric2F1[1/2, 4/5, 9/5
, x^5])/(4*Sqrt[-1 + x^5]) + 2*a*Defer[Int][Sqrt[-1 + x^5]/(1 + a*x^2 - x^5
), x] + 5*Defer[Int][(x^3*Sqrt[-1 + x^5])/(-1 - a*x^2 + x^5), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x^5}\left(2+3x^5\right)}{x^2\left(-1-ax^2+x^5\right)}\,dx &= \int \left(-\frac{2\sqrt{-1+x^5}}{x^2}+\frac{\left(2a-5x^3\right)\sqrt{-1+x^5}}{1+ax^2-x^5}\right)dx\\
&= -\left(2\int \frac{\sqrt{-1+x^5}}{x^2}\,dx\right)+\int \frac{\left(2a-5x^3\right)\sqrt{-1+x^5}}{1+ax^2-x^5}\,dx\\
&= \frac{2\sqrt{-1+x^5}}{x}-5\int \frac{x^3}{\sqrt{-1+x^5}}\,dx+\int \left(\frac{2a\sqrt{-1+x^5}}{1+ax^2-x^5}+\frac{5x^3\sqrt{-1+x^5}}{-1-ax^2+x^5}\right)dx\\
&= \frac{2\sqrt{-1+x^5}}{x}+5\int \frac{x^3\sqrt{-1+x^5}}{-1-ax^2+x^5}\,dx+(2a)\int \frac{\sqrt{-1+x^5}}{1+ax^2-x^5}\,dx-\frac{\left(5\sqrt{1-x^5}\right)\int -}{\sqrt{-1+x^5}}\\
&= \frac{2\sqrt{-1+x^5}}{x}-\frac{5x^4\sqrt{1-x^5}\,{}_2F_1\left(\frac{1}{2},\frac{4}{5};\frac{9}{5};x^5\right)}{4\sqrt{-1+x^5}}+5\int \frac{x^3\sqrt{-1+x^5}}{-1-ax^2+x^5}\,dx+(2a)\int \frac{\sqrt{}}{1+}
\end{aligned}$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-1+x^5}\left(2+3x^5\right)}{x^2\left(-1-ax^2+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[-1 + x^5]*(2 + 3*x^5))/(x^2*(-1 - a*x^2 + x^5)),x]
```

```
[Out] Integrate[(Sqrt[-1 + x^5]*(2 + 3*x^5))/(x^2*(-1 - a*x^2 + x^5)), x]
```

IntegrateAlgebraic [A] time = 1.10, size = 39, normalized size = 1.00

$$\frac{2\sqrt{x^5-1}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^5-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^5]*(2 + 3*x^5))/(x^2*(-1 - a*x^2 + x^5)),x]
```

```
[Out] (2*Sqrt[-1 + x^5])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[-1 + x^5]]
```

fricas [A] time = 2.93, size = 157, normalized size = 4.03

$$\left[\frac{\sqrt{a}\,x\log\left(-\frac{x^{10}+6\,ax^7+a^2x^4-2\,x^5-6\,ax^2-4\,(x^6+ax^3-x)\sqrt{x^5-1}\,\sqrt{a}+1}{x^{10}-2\,ax^7+a^2x^4-2\,x^5+2\,ax^2+1}\right)+4\,\sqrt{x^5-1}}{2\,x}, \frac{\sqrt{-a}\,x\arctan\left(\frac{2\,\sqrt{x^5-1}\,\sqrt{-a}\,x}{x^5+ax^2-1}\right)+2\,\sqrt{x^5-1}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(1/2)*(3*x^5+2)/x^2/(x^5-a*x^2-1),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*x*log(-(x^10 + 6*a*x^7 + a^2*x^4 - 2*x^5 - 6*a*x^2 - 4*(x^6 +
 a*x^3 - x)*sqrt(x^5 - 1)*sqrt(a) + 1)/(x^10 - 2*a*x^7 + a^2*x^4 - 2*x^5 +
2*a*x^2 + 1)) + 4*sqrt(x^5 - 1))/x, (sqrt(-a)*x*arctan(2*sqrt(x^5 - 1)*sqrt
(-a)*x/(x^5 + a*x^2 - 1)) + 2*sqrt(x^5 - 1))/x]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3\,x^5+2)\sqrt{x^5-1}}{(x^5-ax^2-1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(1/2)*(3*x^5+2)/x^2/(x^5-a*x^2-1),x, algorithm="giac")
```

```
[Out] integrate((3*x^5 + 2)*sqrt(x^5 - 1)/((x^5 - a*x^2 - 1)*x^2), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^5-1}\,\left(3x^5+2\right)}{x^2\left(x^5-a\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)^(1/2)*(3*x^5+2)/x^2/(x^5-a*x^2-1),x)
```

```
[Out] int((x^5-1)^(1/2)*(3*x^5+2)/x^2/(x^5-a*x^2-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3\,x^5+2)\sqrt{x^5-1}}{(x^5-ax^2-1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(1/2)*(3*x^5+2)/x^2/(x^5-a*x^2-1),x, algorithm="maxima")
```

```
[Out] integrate((3*x^5 + 2)*sqrt(x^5 - 1)/((x^5 - a*x^2 - 1)*x^2), x)
```

mupad [B] time = 0.89, size = 56, normalized size = 1.44

$$\sqrt{a}\,\ln\left(\frac{a\,x^2+x^5-2\,\sqrt{a}\,x\,\sqrt{x^5-1}-1}{-x^5+a\,x^2+1}\right)+\frac{2\,\sqrt{x^5-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^5 - 1)^(1/2)*(3*x^5 + 2))/(x^2*(a*x^2 - x^5 + 1)),x)
```

```
[Out] a^(1/2)*log((a*x^2 + x^5 - 2*a^(1/2)*x*(x^5 - 1)^(1/2) - 1)/(a*x^2 - x^5 +
1)) + (2*(x^5 - 1)^(1/2))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)\left(x^4+x^3+x^2+x+1\right)}\left(3x^5+2\right)}{x^2\left(-ax^2+x^5-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)**(1/2)*(3*x**5+2)/x**2/(x**5-a*x**2-1),x)
```

```
[Out] Integral(sqrt((x - 1)*(x**4 + x**3 + x**2 + x + 1))*(3*x**5 + 2)/(x**2*(-a*
x**2 + x**5 - 1)), x)
```

3.510 $\int x\sqrt{-x+x^6}\,dx$

Optimal. Leaf size=39

$$\frac{1}{5}x^2\sqrt{x^6-x}-\frac{1}{5}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-x}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2021, 2029, 206}

$$\frac{1}{5}x^2\sqrt{x^6-x}-\frac{1}{5}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[-x + x^6],x]
```

```
[Out] (x^2*Sqrt[-x + x^6])/5 - ArcTanh[x^3/Sqrt[-x + x^6]]/5
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{-x+x^6}\,dx &= \frac{1}{5}x^2\sqrt{-x+x^6}-\frac{1}{2}\int\frac{x^2}{\sqrt{-x+x^6}}\,dx\\
&= \frac{1}{5}x^2\sqrt{-x+x^6}-\frac{1}{5}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x^3}{\sqrt{-x+x^6}}\right)\\
&= \frac{1}{5}x^2\sqrt{-x+x^6}-\frac{1}{5}\tanh^{-1}\left(\frac{x^3}{\sqrt{-x+x^6}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 1.13

$$\frac{\sqrt{x\left(x^5-1\right)}\left(x^{5/2}+\frac{\sin^{-1}\left(x^{5/2}\right)}{\sqrt{1-x^5}}\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[-x + x^6],x]
[Out] (Sqrt[x*(-1 + x^5)]*(x^(5/2) + ArcSin[x^(5/2)]/Sqrt[1 - x^5]))/(5*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.29, size = 39, normalized size = 1.00

$$\frac{1}{5}x^2\sqrt{x^6-x}-\frac{1}{5}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[-x + x^6],x]
[Out] (x^2*Sqrt[-x + x^6])/5 - ArcTanh[x^3/Sqrt[-x + x^6]]/5
```

fricas [A] time = 0.44, size = 39, normalized size = 1.00

$$\frac{1}{5}\sqrt{x^6-x}\,x^2+\frac{1}{10}\log\left(-2\,x^5+2\,\sqrt{x^6-x}\,x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^6-x)^(1/2),x, algorithm="fricas")
[Out] 1/5*sqrt(x^6 - x)*x^2 + 1/10*log(-2*x^5 + 2*sqrt(x^6 - x)*x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^6-x}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^6-x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^6 - x)*x, x)
```

maple [C] time = 0.21, size = 45, normalized size = 1.15

$$\frac{x^3\left(x^5-1\right)}{5\sqrt{x\left(x^5-1\right)}}-\frac{\sqrt{-\mathrm{signum}\left(x^5-1\right)}\,\arcsin\left(x^{\frac{5}{2}}\right)}{5\sqrt{\mathrm{signum}\left(x^5-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^6-x)^(1/2),x)
[Out] 1/5*x^3*(x^5-1)/(x*(x^5-1))^(1/2)-1/5/signum(x^5-1)^(1/2)*(-signum(x^5-1))^
(1/2)*arcsin(x^(5/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^6-x}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^6-x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^6 - x)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x\,\sqrt{x^6-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^6 - x)^(1/2),x)
```

```
[Out] int(x*(x^6 - x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x\,(x-1)\left(x^4+x^3+x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**6-x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x*(x - 1)*(x**4 + x**3 + x**2 + x + 1)), x)
```

3.511 $\int \frac{(1+x^5)(-1+4x^5)}{x(1-ax+x^5)\sqrt{x+x^6}}\,dx$

Optimal. Leaf size=39

$$\frac{2\sqrt{x^6+x}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^6+x}}\right)$$

Rubi [F] time = 1.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+x^5)(-1+4x^5)}{x(1-ax+x^5)\sqrt{x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^5)*(-1 + 4*x^5))/(x*(1 - a*x + x^5)*Sqrt[x + x^6]),x]
```

```
[Out] (2*(1 + x^5))/Sqrt[x + x^6] - (10*x^5*Sqrt[1 + x^5]*Hypergeometric2F1[1/2,
9/10, 19/10, -x^5])/(9*Sqrt[x + x^6]) + (2*a*Sqrt[x]*Sqrt[1 + x^5]*Defer[Su
bst][Defer[Int][Sqrt[1 + x^10]/(-1 + a*x^2 - x^10), x], x, Sqrt[x]])/Sqrt[x
 + x^6] + (10*Sqrt[x]*Sqrt[1 + x^5]*Defer[Subst][Defer[Int][(x^8*Sqrt[1 + x
^10])/(1 - a*x^2 + x^10), x], x, Sqrt[x]])/Sqrt[x + x^6]
```

Rubi steps

$$\int \frac{(1+x^5)(-1+4x^5)}{x(1-ax+x^5)\sqrt{x+x^6}}\,dx=\frac{\left(\sqrt{x}\sqrt{1+x^5}\right)\int \frac{\sqrt{1+x^5}\,(-1+4x^5)}{x^{3/2}(1-ax+x^5)}\,dx}{\sqrt{x+x^6}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^{10}}\,(-1+4x^{10})}{x^2(1-ax^2+x^{10})}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^6}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \left(-\frac{\sqrt{1+x^{10}}}{x^2}+\frac{(a-5x^8)\sqrt{1+x^{10}}}{-1+ax^2-x^{10}}\right)dx,x,\sqrt{x}\right)}{\sqrt{x+x^6}}$$

$$=-\frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^{10}}}{x^2}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^6}}+\frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{(a-\ldots}{-\ldots}\right.}{\sqrt{x+x^6}}$$

$$=\frac{2(1+x^5)}{\sqrt{x+x^6}}+\frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \left(\frac{a\sqrt{1+x^{10}}}{-1+ax^2-x^{10}}+\frac{5x^8\sqrt{1+x^{10}}}{1-ax^2+x^{10}}\right)dx,x,\sqrt{x}\right)}{\sqrt{x+x^6}}-\ldots$$

$$=\frac{2(1+x^5)}{\sqrt{x+x^6}}-\frac{10x^5\sqrt{1+x^5}\,{}_2F_1\left(\frac{1}{2},\frac{9}{10};\frac{19}{10};-x^5\right)}{9\sqrt{x+x^6}}+\frac{\left(10\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{x\ldots}{1\ldots}\right.}{\sqrt{x+x^6}}$$

Mathematica [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{(1+x^5)(-1+4x^5)}{x(1-ax+x^5)\sqrt{x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^5)*(-1 + 4*x^5))/(x*(1 - a*x + x^5)*Sqrt[x + x^6]),x]
```

```
[Out] Integrate[((1 + x^5)*(-1 + 4*x^5))/(x*(1 - a*x + x^5)*Sqrt[x + x^6]), x]
```

IntegrateAlgebraic [A] time = 0.66, size = 39, normalized size = 1.00

$$\frac{2\sqrt{x^6+x}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^6+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^5)*(-1 + 4*x^5))/(x*(1 - a*x + x^5)*Sqrt[x + x^6
]),x]
```

```
[Out] (2*Sqrt[x + x^6])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[x + x^6]]
```

fricas [A] time = 3.87, size = 146, normalized size = 3.74

$$\left[\frac{\sqrt{a}\,x\log\left(-\frac{x^{10}+6\,ax^{6}+2\,x^{5}+a^{2}x^{2}-4\,\sqrt{x^{6}+x}\left(x^{5}+ax+1\right)\sqrt{a}+6\,ax+1}{x^{10}-2\,ax^{6}+2\,x^{5}+a^{2}x^{2}-2\,ax+1}\right)+4\,\sqrt{x^{6}+x}}{2\,x},\ \frac{\sqrt{-a}\,x\arctan\left(\frac{2\,\sqrt{x^{6}+x}\,\sqrt{-a}}{x^{5}+ax+1}\right)+2\,\sqrt{x^{6}+x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)*(4*x^5-1)/x/(x^5-a*x+1)/(x^6+x)^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/2*(sqrt(a)*x*log(-(x^10 + 6*a*x^6 + 2*x^5 + a^2*x^2 - 4*sqrt(x^6 + x)*(x
^5 + a*x + 1)*sqrt(a) + 6*a*x + 1)/(x^10 - 2*a*x^6 + 2*x^5 + a^2*x^2 - 2*a*
x + 1)) + 4*sqrt(x^6 + x))/x, (sqrt(-a)*x*arctan(2*sqrt(x^6 + x)*sqrt(-a)/(
x^5 + a*x + 1)) + 2*sqrt(x^6 + x))/x]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^5-1\right)\left(x^5+1\right)}{\sqrt{x^6+x}\,\left(x^5-ax+1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)*(4*x^5-1)/x/(x^5-a*x+1)/(x^6+x)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((4*x^5 - 1)*(x^5 + 1)/(sqrt(x^6 + x)*(x^5 - a*x + 1)*x), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5+1\right)\left(4x^5-1\right)}{x\left(x^5-ax+1\right)\sqrt{x^6+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)*(4*x^5-1)/x/(x^5-a*x+1)/(x^6+x)^(1/2),x)
```

```
[Out] int((x^5+1)*(4*x^5-1)/x/(x^5-a*x+1)/(x^6+x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^5-1\right)\left(x^5+1\right)}{\sqrt{x^6+x}\,\left(x^5-ax+1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)*(4*x^5-1)/x/(x^5-a*x+1)/(x^6+x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((4*x^5 - 1)*(x^5 + 1)/(sqrt(x^6 + x)*(x^5 - a*x + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(x^5+1\right)\,\left(4\,x^5-1\right)}{x\,\sqrt{x^6+x}\,\left(x^5-a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 + 1)*(4*x^5 - 1))/(x*(x + x^6)^(1/2)*(x^5 - a*x + 1)),x)
```

```
[Out] int(((x^5 + 1)*(4*x^5 - 1))/(x*(x + x^6)^(1/2)*(x^5 - a*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)\left(4x^5-1\right)\left(x^4-x^3+x^2-x+1\right)}{x\sqrt{x\,(x+1)\left(x^4-x^3+x^2-x+1\right)}\left(-ax+x^5+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+1)*(4*x**5-1)/x/(x**5-a*x+1)/(x**6+x)**(1/2),x)
```

```
[Out] Integral((x + 1)*(4*x**5 - 1)*(x**4 - x**3 + x**2 - x + 1)/(x*sqrt(x*(x + 1
)*(x**4 - x**3 + x**2 - x + 1))*(-a*x + x**5 + 1)), x)
```

3.512 $\int \frac{1}{\sqrt{x-\sqrt{-1+x^2}}} \, dx$

Optimal. Leaf size=39

$$\sqrt{x-\sqrt{x^2-1}}+\frac{1}{3\left(x-\sqrt{x^2-1}\right)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2117, 14}

$$\sqrt{x-\sqrt{x^2-1}}+\frac{1}{3\left(x-\sqrt{x^2-1}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x - Sqrt[-1 + x^2]],x]
```

```
[Out] 1/(3*(x - Sqrt[-1 + x^2])^(3/2)) + Sqrt[x - Sqrt[-1 + x^2]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x-\sqrt{-1+x^2}}} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{-1+x^2}{x^{5/2}} \, dx, x, x-\sqrt{-1+x^2}\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(-\frac{1}{x^{5/2}}+\frac{1}{\sqrt{x}}\right) \, dx, x, x-\sqrt{-1+x^2}\right)\\
&= \frac{1}{3\left(x-\sqrt{-1+x^2}\right)^{3/2}}+\sqrt{x-\sqrt{-1+x^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.00

$$\sqrt{x-\sqrt{x^2-1}}+\frac{1}{3\left(x-\sqrt{x^2-1}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x - Sqrt[-1 + x^2]],x]
```

```
[Out] 1/(3*(x - Sqrt[-1 + x^2])^(3/2)) + Sqrt[x - Sqrt[-1 + x^2]]
```

IntegrateAlgebraic [A] time = 0.07, size = 39, normalized size = 1.00

$$\sqrt{x-\sqrt{x^2-1}}+\frac{1}{3\left(x-\sqrt{x^2-1}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[x - Sqrt[-1 + x^2]],x]
[Out] 1/(3*(x - Sqrt[-1 + x^2])^(3/2)) + Sqrt[x - Sqrt[-1 + x^2]]
```

fricas [A] time = 0.39, size = 29, normalized size = 0.74

$$\frac{2}{3}\left(x^2+\sqrt{x^2-1}\,x+1\right)\sqrt{x-\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x^2-1)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2/3*(x^2 + sqrt(x^2 - 1)*x + 1)*sqrt(x - sqrt(x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-\sqrt{x^2-1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x^2-1)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(x - sqrt(x^2 - 1)), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-\sqrt{x^2-1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(x^2-1)^(1/2))^(1/2),x)
[Out] int(1/(x-(x^2-1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-\sqrt{x^2-1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x^2-1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(x - sqrt(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x-\sqrt{x^2-1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (x^2 - 1)^(1/2))^(1/2),x)
[Out] int(1/(x - (x^2 - 1)^(1/2))^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-\sqrt{x^2-1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x**2-1)**(1/2))**(1/2),x)
[Out] Integral(1/sqrt(x - sqrt(x**2 - 1)), x)
```

3.513 $\int \frac{1+x^2}{\left(-1+2x+x^2\right)\sqrt{-x-x^2+x^3}}\,dx$

Optimal. Leaf size=40

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)}{\sqrt{3}}$$

Rubi [C] time = 1.28, antiderivative size = 387, normalized size of antiderivative = 9.68, number of steps used = 15, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2056, 6728, 716, 1098, 934, 168, 538, 537}

$$\frac{\sqrt{x}\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}\sqrt{\frac{\left(1+\sqrt{5}\right)x+2}{\left(1-\sqrt{5}\right)x+2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{\left(1-\sqrt{5}\right)x+2}}\sqrt{x^3-x^2-x}}-\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(\frac{1}{2}\left(1-\sqrt{2}\right)\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\sqrt{x^3-x^2-x}}-\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(\frac{1}{2}\left(1+\sqrt{2}\right)\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\sqrt{x^3-x^2-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^2)/((-1 + 2*x + x^2)*Sqrt[-x - x^2 + x^3]),x]
```

```
[Out] (Sqrt[x]*Sqrt[-2 - (1 - Sqrt[5])*x]*Sqrt[(2 + (1 + Sqrt[5])*x)/(2 + (1 - Sq
rt[5])*x)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*Sqrt[x])/Sqrt[-2 - (1 - Sqrt[5
])*x]], (5 - Sqrt[5])/10])/(5^(1/4)*Sqrt[(2 + (1 - Sqrt[5])*x)^(-1)]*Sqrt[-
x - x^2 + x^3]) - (Sqrt[3 + Sqrt[5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[
1 - (2*x)/(1 + Sqrt[5])]*EllipticPi[((1 - Sqrt[2])*(1 + Sqrt[5]))/2, ArcSin
[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]], (-3 - Sqrt[5])/2])/Sqrt[-x - x^2 + x^3] -
(Sqrt[3 + Sqrt[5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1 + Sqr
t[5])]*EllipticPi[((1 + Sqrt[2])*(1 + Sqrt[5]))/2, ArcSin[Sqrt[2/(1 + Sqrt[
5])]*Sqrt[x]], (-3 - Sqrt[5])/2])/Sqrt[-x - x^2 + x^3]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 716

```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2,
Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^2}{\left(-1+2x+x^2\right)\sqrt{-x-x^2+x^3}}\,dx &= \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{1+x^2}{\sqrt{x}\sqrt{-1-x+x^2}\left(-1+2x+x^2\right)}\,dx}{\sqrt{-x-x^2+x^3}} \\
&= \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{-1-x+x^2}}+\frac{2(1-x)}{\sqrt{x}\sqrt{-1-x+x^2}\left(-1+2x+x^2\right)}\right)dx}{\sqrt{-x-x^2+x^3}} \\
&= \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-1-x+x^2}}\,dx}{\sqrt{-x-x^2+x^3}} + \frac{\left(2\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{}{\sqrt{x}\sqrt{-1}}}{\sqrt{-x-x^2+x}} \\
&= \frac{\left(2\sqrt{x}\sqrt{-1-x+x^2}\right)\int \left(\frac{-1+\sqrt{2}}{\sqrt{x}\left(2-2\sqrt{2}+2x\right)\sqrt{-1-x+x^2}}+\frac{-1-\sqrt{2}}{\sqrt{x}\left(2+2\sqrt{2}+2x\right)\sqrt{-1-x+x}}\right.}{\sqrt{-x-x^2+x^3}} \\
&= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right.\right.}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} \\
&= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right.\right.}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} \\
&= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right.\right.}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} \\
&= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right.\right.}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} \\
&= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right.\right.}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}}
\end{aligned}
$$

Mathematica [C] time = 0.97, size = 215, normalized size = 5.38

$$
-\frac{2i\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{-\frac{1}{x^2}-\frac{1}{x}+1}\,x^{3/2}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{2}{1+\sqrt{5}}}}{\sqrt{x}}\right)|-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-\Pi\left(-\frac{1}{2}\left(-1+\sqrt{2}\right)\left(1+\sqrt{5}\right);i\sinh^{-1}\left(\frac{\sqrt{\frac{2}{1+\sqrt{5}}}}{\sqrt{x}}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)-\Pi\left(\frac{1}{2}\left(1+\sqrt{2}\right)\left(1+\sqrt{5}\right);i\sinh^{-1}\left(\frac{\sqrt{\frac{2}{1+\sqrt{5}}}}{\sqrt{x}}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)\right)}{\sqrt{x\left(x^2-x-1\right)}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2)/((-1 + 2*x + x^2)*Sqrt[-x - x^2 + x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 - x^(-2) - x^(-1)]*x^(3/2)*(EllipticF
[I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], -3/2 - Sqrt[5]/2] - EllipticPi[-
1/2*((-1 + Sqrt[2])*(1 + Sqrt[5])), I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]
], (-3 - Sqrt[5])/2] - EllipticPi[((1 + Sqrt[2])*(1 + Sqrt[5]))/2, I*ArcSin
```

```
h[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], (-3 - Sqrt[5])/2]))/Sqrt[x*(-1 - x + x^2)
]
```

IntegrateAlgebraic [A] time = 0.10, size = 40, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 + 2*x + x^2)*Sqrt[-x - x^2 + x^3]),x]
[Out] (-2*ArcTan[(Sqrt[3]*Sqrt[-x - x^2 + x^3])/(-1 - x + x^2)])/Sqrt[3]
```

fricas [A] time = 0.43, size = 33, normalized size = 0.82

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x^2-4x-1\right)}{6\sqrt{x^3-x^2-x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+2*x-1)/(x^3-x^2-x)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/6*sqrt(3)*(x^2 - 4*x - 1)/sqrt(x^3 - x^2 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+1}{\sqrt{x^3-x^2-x}\left(x^2+2x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+2*x-1)/(x^3-x^2-x)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 + 1)/(sqrt(x^3 - x^2 - x)*(x^2 + 2*x - 1)), x)
```

maple [C] time = 0.07, size = 1393, normalized size = 34.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2+2*x-1)/(x^3-x^2-x)^(1/2),x)
[Out] 2/5*(1/2*5^(1/2)-1/2)*((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x-
1/2-1/2*5^(1/2))*5^(1/2))^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1
/2)*EllipticF(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),1/5*5^(1/2)*((1
/2*5^(1/2)-1/2)*5^(1/2))^(1/2))+1/5*2^(1/2)*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5
^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+
25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(3/2-1/2*5^(1/2)
-2^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2
*5^(1/2))/(3/2-1/2*5^(1/2)-2^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))
^(1/2))*5^(1/2)-1/5*2^(1/2)*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/
(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/
(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(3/2-1/2*5^(1/2)-2^(1/2))*Ellipt
icPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(3/2-1
/2*5^(1/2)-2^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))-1/5*(x/(
1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)
*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^
2-x)^(1/2)/(3/2-1/2*5^(1/2)-2^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5
^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(3/2-1/2*5^(1/2)-2^(1/2)),1/5*5^(1/2)*
```

```
((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))*5^(1/2)+1/5*(x/(1/2*5^(1/2)-1/2)-1/2/(1/
2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/
2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(3/2-1/2*5^(1
/2)-2^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-
1/2*5^(1/2))/(3/2-1/2*5^(1/2)-2^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/
2))^(1/2))-1/5*2^(1/2)*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*
5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*
5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(3/2+2^(1/2)-1/2*5^(1/2))*EllipticPi(
((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(3/2+2^(1/2
)-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))*5^(1/2)+1/5*2
^(1/2)*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(
1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1
/2)/(x^3-x^2-x)^(1/2)/(3/2+2^(1/2)-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1
/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(3/2+2^(1/2)-1/2*5^(1/2)),1
/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))-1/5*(x/(1/2*5^(1/2)-1/2)-1/2/
(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^
(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(3/2+2^(1/
2)-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1
/2-1/2*5^(1/2))/(3/2+2^(1/2)-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^
(1/2))^(1/2))*5^(1/2)+1/5*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1
/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1
/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(3/2+2^(1/2)-1/2*5^(1/2))*Elliptic
Pi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(3/2+2^(
1/2)-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x^3-x^2-x}\left(x^2+2\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+2*x-1)/(x^3-x^2-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(sqrt(x^3 - x^2 - x)*(x^2 + 2*x - 1)), x)
```

mupad [B] time = 0.15, size = 223, normalized size = 5.58

$$-\frac{\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\left(\sqrt{5}+1\right)\,\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\left(\Pi\left(\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\sqrt{2}-1};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)+\Pi\left(-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\sqrt{2}+1};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)\right)}{\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\,\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((2*x + x^2 - 1)*(x^3 - x^2 - x)^(1/2)),x)
```

```
[Out] -((x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/2 - 1/2)/(5^(1/2)/2 - 1/2))^(1/
2)*(5^(1/2) + 1)*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 + 1/2))^(1/2)*(ellipticP
i((5^(1/2)/2 + 1/2)/(2^(1/2) - 1), asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(
1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2)) - ellipticF(asin((x/(5^(1/2)/2 + 1/2))^(1/
2)), -(5^(1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2)) + ellipticPi(-(5^(1/2)/2 + 1/2)/
(2^(1/2) + 1), asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/2 + 1/2)/(5^(1/
2)/2 - 1/2))))/(x^3 - x^2 - x*(5^(1/2)/2 - 1/2)*(5^(1/2)/2 + 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x\left(x^2-x-1\right)}\left(x^2+2x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2+2*x-1)/(x**3-x**2-x)**(1/2),x)
[Out] Integral((x**2 + 1)/(sqrt(x*(x**2 - x - 1))*(x**2 + 2*x - 1)), x)
```

3.514 $\int \frac{(2+x^3)\sqrt{-1+x^2+x^3}}{(-1+x^3)^2}\,dx$

Optimal. Leaf size=40

$$-\frac{\sqrt{x^3+x^2-1}\,x}{x^3-1}-\tanh^{-1}\left(\frac{x}{\sqrt{x^3+x^2-1}}\right)$$

Rubi [F] time = 180.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$Aborted

Verification is not applicable to the result.

```
[In] Int[((2 + x^3)*Sqrt[-1 + x^2 + x^3])/(-1 + x^3)^2,x]
```

```
[Out] $Aborted
```

Rubi steps

Aborted

Mathematica [C] time = 2.95, size = 1451, normalized size = 36.28

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + x^3)*Sqrt[-1 + x^2 + x^3])/(-1 + x^3)^2,x]
```

```
[Out] (-((x*(-1 + x^2 + x^3))/(-1 + x^3)) + (EllipticF[ArcSin[Sqrt[(-x + Root[-1
+ #1^2 + #1^3 & , 3, 0])/(-Root[-1 + #1^2 + #1^3 & , 2, 0] + Root[-1 + #1^2
 + #1^3 & , 3, 0])]], (Root[-1 + #1^2 + #1^3 & , 2, 0] - Root[-1 + #1^2 + #
1^3 & , 3, 0])/(Root[-1 + #1^2 + #1^3 & , 1, 0] - Root[-1 + #1^2 + #1^3 & ,
 3, 0])]*(x - Root[-1 + #1^2 + #1^3 & , 3, 0])*Sqrt[(-x + Root[-1 + #1^2 +
#1^3 & , 1, 0])/(Root[-1 + #1^2 + #1^3 & , 1, 0] - Root[-1 + #1^2 + #1^3 &
, 3, 0])]*Sqrt[(-x + Root[-1 + #1^2 + #1^3 & , 2, 0])/(Root[-1 + #1^2 + #1^
3 & , 2, 0] - Root[-1 + #1^2 + #1^3 & , 3, 0])])/Sqrt[(x - Root[-1 + #1^2 +
 #1^3 & , 3, 0])/(Root[-1 + #1^2 + #1^3 & , 2, 0] - Root[-1 + #1^2 + #1^3 &
 , 3, 0])] - (EllipticPi[(-Root[-1 + #1^2 + #1^3 & , 2, 0] + Root[-1 + #1^2
 + #1^3 & , 3, 0])/(-1 + Root[-1 + #1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[(-x
+ Root[-1 + #1^2 + #1^3 & , 3, 0])/(-Root[-1 + #1^2 + #1^3 & , 2, 0] + Root
[-1 + #1^2 + #1^3 & , 3, 0])]], (Root[-1 + #1^2 + #1^3 & , 2, 0] - Root[-1
+ #1^2 + #1^3 & , 3, 0])/(Root[-1 + #1^2 + #1^3 & , 1, 0] - Root[-1 + #1^2
+ #1^3 & , 3, 0])]*Sqrt[(-x + Root[-1 + #1^2 + #1^3 & , 1, 0])/(Root[-1 + #
1^2 + #1^3 & , 1, 0] - Root[-1 + #1^2 + #1^3 & , 3, 0])]*Sqrt[-(((x - Root[
-1 + #1^2 + #1^3 & , 2, 0])*(x - Root[-1 + #1^2 + #1^3 & , 3, 0]))/(Root[-1
 + #1^2 + #1^3 & , 2, 0] - Root[-1 + #1^2 + #1^3 & , 3, 0])^2)]*(-Root[-1 +
 #1^2 + #1^3 & , 2, 0] + Root[-1 + #1^2 + #1^3 & , 3, 0]))/(-1 + Root[-1 +
#1^2 + #1^3 & , 3, 0]) + (3*(-1)^(2/3)*EllipticPi[(-Root[-1 + #1^2 + #1^3 &
 , 2, 0] + Root[-1 + #1^2 + #1^3 & , 3, 0])/((-1)^(1/3) + Root[-1 + #1^2 +
#1^3 & , 3, 0]), ArcSin[Sqrt[(-x + Root[-1 + #1^2 + #1^3 & , 3, 0])/(-Root[
-1 + #1^2 + #1^3 & , 2, 0] + Root[-1 + #1^2 + #1^3 & , 3, 0])]], (Root[-1 +
 #1^2 + #1^3 & , 2, 0] - Root[-1 + #1^2 + #1^3 & , 3, 0])/(Root[-1 + #1^2 +
 #1^3 & , 1, 0] - Root[-1 + #1^2 + #1^3 & , 3, 0])]*Sqrt[(-x + Root[-1 + #1
^2 + #1^3 & , 1, 0])/(Root[-1 + #1^2 + #1^3 & , 1, 0] - Root[-1 + #1^2 + #1
^3 & , 3, 0])]*Sqrt[-(((x - Root[-1 + #1^2 + #1^3 & , 2, 0])*(x - Root[-1 +
 #1^2 + #1^3 & , 3, 0]))/(Root[-1 + #1^2 + #1^3 & , 2, 0] - Root[-1 + #1^2
+ #1^3 & , 3, 0])^2)]*(-Root[-1 + #1^2 + #1^3 & , 2, 0] + Root[-1 + #1^2 +
```

```
#1^3 & , 3, 0]))/((1 + (-1)^(1/3))^2*((-1)^(1/3) + Root[-1 + #1^2 + #1^3 &
, 3, 0])) - (2*(-1)^(2/3)*EllipticPi[(Root[-1 + #1^2 + #1^3 & , 2, 0] - Roo
t[-1 + #1^2 + #1^3 & , 3, 0])/((-1)^(2/3) - Root[-1 + #1^2 + #1^3 & , 3, 0]
), ArcSin[Sqrt[(-x + Root[-1 + #1^2 + #1^3 & , 3, 0])/(-Root[-1 + #1^2 + #1
^3 & , 2, 0] + Root[-1 + #1^2 + #1^3 & , 3, 0])]], (Root[-1 + #1^2 + #1^3 &
 , 2, 0] - Root[-1 + #1^2 + #1^3 & , 3, 0])/(Root[-1 + #1^2 + #1^3 & , 1, 0
] - Root[-1 + #1^2 + #1^3 & , 3, 0])]*Sqrt[(-x + Root[-1 + #1^2 + #1^3 & ,
1, 0])/(Root[-1 + #1^2 + #1^3 & , 1, 0] - Root[-1 + #1^2 + #1^3 & , 3, 0])]
*Sqrt[-(((x - Root[-1 + #1^2 + #1^3 & , 2, 0])*(x - Root[-1 + #1^2 + #1^3 &
 , 3, 0]))/(Root[-1 + #1^2 + #1^3 & , 2, 0] - Root[-1 + #1^2 + #1^3 & , 3,
0])^2)]*(-Root[-1 + #1^2 + #1^3 & , 2, 0] + Root[-1 + #1^2 + #1^3 & , 3, 0]
))/(1 - I*Sqrt[3] + 2*Root[-1 + #1^2 + #1^3 & , 3, 0]))/Sqrt[-1 + x^2 + x^3
]
```

IntegrateAlgebraic [A] time = 0.32, size = 40, normalized size = 1.00

$$-\frac{\sqrt{x^3+x^2-1}\,x}{x^3-1}-\tanh^{-1}\left(\frac{x}{\sqrt{x^3+x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^3)*Sqrt[-1 + x^2 + x^3])/(-1 + x^3)^2,x]
[Out] -((x*Sqrt[-1 + x^2 + x^3])/(-1 + x^3)) - ArcTanh[x/Sqrt[-1 + x^2 + x^3]]
```

fricas [A] time = 0.42, size = 61, normalized size = 1.52

$$\frac{\left(x^3-1\right)\log\left(\frac{x^3+2\,x^2-2\,\sqrt{x^3+x^2-1}\,x-1}{x^3-1}\right)-2\,\sqrt{x^3+x^2-1}\,x}{2\,\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^3+x^2-1)^(1/2)/(x^3-1)^2,x, algorithm="fricas")
[Out] 1/2*((x^3 - 1)*log((x^3 + 2*x^2 - 2*sqrt(x^3 + x^2 - 1)*x - 1)/(x^3 - 1)) -
 2*sqrt(x^3 + x^2 - 1)*x)/(x^3 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^3+x^2-1}\left(x^3+2\right)}{\left(x^3-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^3+x^2-1)^(1/2)/(x^3-1)^2,x, algorithm="giac")
[Out] integrate(sqrt(x^3 + x^2 - 1)*(x^3 + 2)/(x^3 - 1)^2, x)
```

maple [C] time = 1.36, size = 5693, normalized size = 142.32

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2)*(x^3+x^2-1)^(1/2)/(x^3-1)^2,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^3+x^2-1}\left(x^3+2\right)}{\left(x^3-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^3+x^2-1)^(1/2)/(x^3-1)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^3 + x^2 - 1)*(x^3 + 2)/(x^3 - 1)^2, x)
```

mupad [B] time = 0.70, size = 57, normalized size = 1.42

$$\frac{\ln\left(\frac{2\,x\,\sqrt{x^3+x^2-1}-2\,x^2-x^3+1}{x^3-1}\right)}{2}-\frac{x\,\sqrt{x^3+x^2-1}}{x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 2)*(x^2 + x^3 - 1)^(1/2))/(x^3 - 1)^2,x)
```

```
[Out] log((2*x*(x^2 + x^3 - 1)^(1/2) - 2*x^2 - x^3 + 1)/(x^3 - 1))/2 - (x*(x^2 +
x^3 - 1)^(1/2))/(x^3 - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+2\right)\sqrt{x^3+x^2-1}}{(x-1)^2\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2)*(x**3+x**2-1)**(1/2)/(x**3-1)**2,x)
```

```
[Out] Integral((x**3 + 2)*sqrt(x**3 + x**2 - 1)/((x - 1)**2*(x**2 + x + 1)**2), x
)
```

3.515 $\int \frac{2+x^2}{(-2+x^2)\sqrt{-2x+2x^2+x^3}} \, dx$

Optimal. Leaf size=40

$$-\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x^3+2x^2-2x}}{x^2+2x-2}\right)$$

Rubi [C] time = 1.05, antiderivative size = 355, normalized size of antiderivative = 8.88, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2056, 6725, 716, 1098, 934, 168, 538, 537}

$$\frac{\sqrt{x}\sqrt{\frac{2-(1-\sqrt{3})x}{2-(1+\sqrt{3})x}}\sqrt{(1+\sqrt{3})x-2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}\sqrt{x}}{\sqrt{(1+\sqrt{3})x-2}}\right)|\frac{1}{6}(3+\sqrt{3})\right)}{\sqrt[4]{3}\sqrt{\frac{1}{2-(1+\sqrt{3})x}}\sqrt{x^3+2x^2-2x}}-\frac{2\sqrt{2-\sqrt{3}}\sqrt{x}\sqrt{x+\sqrt{3}+1}\sqrt{\frac{x}{1-\sqrt{3}}+1}\,\Pi\left(-\frac{1-\sqrt{3}}{\sqrt{2}};\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{-1+\sqrt{3}}}\right)|-2+\sqrt{3}\right)}{\sqrt{x^3+2x^2-2x}}-\frac{2\sqrt{2-\sqrt{3}}\sqrt{x}\sqrt{x+\sqrt{3}+1}\sqrt{\frac{x}{1-\sqrt{3}}+1}\,\Pi\left(\frac{1-\sqrt{3}}{\sqrt{2}};\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{-1+\sqrt{3}}}\right)|-2+\sqrt{3}\right)}{\sqrt{x^3+2x^2-2x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(2 + x^2)/((-2 + x^2)*Sqrt[-2*x + 2*x^2 + x^3]),x]
```

```
[Out] (Sqrt[x]*Sqrt[(2 - (1 - Sqrt[3])*x)/(2 - (1 + Sqrt[3])*x)]*Sqrt[-2 + (1 + S
qrt[3])*x]*EllipticF[ArcSin[(Sqrt[2]*3^(1/4)*Sqrt[x])/Sqrt[-2 + (1 + Sqrt[3
])*x]], (3 + Sqrt[3])/6])/(3^(1/4)*Sqrt[(2 - (1 + Sqrt[3])*x)^(-1)]*Sqrt[-2
*x + 2*x^2 + x^3]) - (2*Sqrt[2 - Sqrt[3]]*Sqrt[x]*Sqrt[1 + Sqrt[3] + x]*Sqr
t[1 + x/(1 - Sqrt[3])]*EllipticPi[-((1 - Sqrt[3])/Sqrt[2]), ArcSin[Sqrt[x]/
Sqrt[-1 + Sqrt[3]]], -2 + Sqrt[3]])/Sqrt[-2*x + 2*x^2 + x^3] - (2*Sqrt[2 -
Sqrt[3]]*Sqrt[x]*Sqrt[1 + Sqrt[3] + x]*Sqrt[1 + x/(1 - Sqrt[3])]*EllipticPi
[(1 - Sqrt[3])/Sqrt[2], ArcSin[Sqrt[x]/Sqrt[-1 + Sqrt[3]]], -2 + Sqrt[3]])/
Sqrt[-2*x + 2*x^2 + x^3]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 716

```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2,
Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{2+x^2}{\left(-2+x^2\right)\sqrt{-2x+2x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{-2+2x+x^2}\right)\int \frac{2+x^2}{\sqrt{x}\left(-2+x^2\right)\sqrt{-2+2x+x^2}}\,dx}{\sqrt{-2x+2x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-2+2x+x^2}\right)\int\left(\frac{1}{\sqrt{x}\sqrt{-2+2x+x^2}}+\frac{4}{\sqrt{x}\left(-2+x^2\right)\sqrt{-2+2x+x^2}}\right)dx}{\sqrt{-2x+2x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-2+2x+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-2+2x+x^2}}\,dx}{\sqrt{-2x+2x^2+x^3}} + \frac{\left(4\sqrt{x}\sqrt{-2+2x+x^2}\right)\int \frac{}{\sqrt{x}\left(-2+x\right.}}{\sqrt{-2x+2x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-2+2x+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-2+2x^2+x^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{-2x+2x^2+x^3}} + \frac{\left(4\sqrt{x}\sqrt{-2+2x+\right.}{}$$

$$= \frac{\sqrt{x}\sqrt{\frac{2-\left(1-\sqrt{3}\right)x}{2-\left(1+\sqrt{3}\right)x}}\sqrt{-2+\left(1+\sqrt{3}\right)x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}\sqrt{x}}{\sqrt{-2+\left(1+\sqrt{3}\right)x}}\right)|\frac{1}{6}\left(3+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{1}{2-\left(1+\sqrt{3}\right)x}}\sqrt{-2x+2x^2+x^3}} -$$

$$= \frac{\sqrt{x}\sqrt{\frac{2-\left(1-\sqrt{3}\right)x}{2-\left(1+\sqrt{3}\right)x}}\sqrt{-2+\left(1+\sqrt{3}\right)x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}\sqrt{x}}{\sqrt{-2+\left(1+\sqrt{3}\right)x}}\right)|\frac{1}{6}\left(3+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{1}{2-\left(1+\sqrt{3}\right)x}}\sqrt{-2x+2x^2+x^3}} -$$

$$= \frac{\sqrt{x}\sqrt{\frac{2-\left(1-\sqrt{3}\right)x}{2-\left(1+\sqrt{3}\right)x}}\sqrt{-2+\left(1+\sqrt{3}\right)x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}\sqrt{x}}{\sqrt{-2+\left(1+\sqrt{3}\right)x}}\right)|\frac{1}{6}\left(3+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{1}{2-\left(1+\sqrt{3}\right)x}}\sqrt{-2x+2x^2+x^3}} +$$

$$= \frac{\sqrt{x}\sqrt{\frac{2-\left(1-\sqrt{3}\right)x}{2-\left(1+\sqrt{3}\right)x}}\sqrt{-2+\left(1+\sqrt{3}\right)x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}\sqrt{x}}{\sqrt{-2+\left(1+\sqrt{3}\right)x}}\right)|\frac{1}{6}\left(3+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{1}{2-\left(1+\sqrt{3}\right)x}}\sqrt{-2x+2x^2+x^3}} +$$

$$= \frac{\sqrt{x}\sqrt{\frac{2-\left(1-\sqrt{3}\right)x}{2-\left(1+\sqrt{3}\right)x}}\sqrt{-2+\left(1+\sqrt{3}\right)x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}\sqrt{x}}{\sqrt{-2+\left(1+\sqrt{3}\right)x}}\right)|\frac{1}{6}\left(3+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{1}{2-\left(1+\sqrt{3}\right)x}}\sqrt{-2x+2x^2+x^3}} -$$

Mathematica [C] time = 0.71, size = 173, normalized size = 4.32

$$-\frac{2i\sqrt{-\frac{2}{x^2}+\frac{2}{x}+1}\,x^{3/2}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{1+\sqrt{3}}}{\sqrt{x}}\right)|-2+\sqrt{3}\right)-\Pi\left(-\sqrt{\frac{3}{2}}+\frac{1}{\sqrt{2}};i\sinh^{-1}\left(\frac{\sqrt{1+\sqrt{3}}}{\sqrt{x}}\right)|-2+\sqrt{3}\right)-\Pi\left(\frac{-1+\sqrt{3}}{\sqrt{2}};i\sinh^{-1}\left(\frac{\sqrt{1+\sqrt{3}}}{\sqrt{x}}\right)|-2+\sqrt{3}\right)\right)}{\sqrt{1+\sqrt{3}}\sqrt{x\left(x^2+2x-2\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + x^2)/((-2 + x^2)*Sqrt[-2*x + 2*x^2 + x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[1 - 2/x^2 + 2/x]*x^(3/2)*(EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[3]
]/Sqrt[x]], -2 + Sqrt[3]] - EllipticPi[-Sqrt[3/2] + 1/Sqrt[2], I*ArcSinh[Sq
rt[1 + Sqrt[3]]/Sqrt[x]], -2 + Sqrt[3]] - EllipticPi[(-1 + Sqrt[3])/Sqrt[2]
, I*ArcSinh[Sqrt[1 + Sqrt[3]]/Sqrt[x]], -2 + Sqrt[3]]))/(Sqrt[1 + Sqrt[3]]*
Sqrt[x*(-2 + 2*x + x^2)])
```

IntegrateAlgebraic [A] time = 0.11, size = 40, normalized size = 1.00

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x^3+2x^2-2x}}{x^2+2x-2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x^2)/((-2 + x^2)*Sqrt[-2*x + 2*x^2 + x^3]),x]
[Out] -(Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[-2*x + 2*x^2 + x^3])/(-2 + 2*x + x^2)])
```

fricas [A] time = 0.41, size = 64, normalized size = 1.60

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^4+16x^3-4\sqrt{2}\sqrt{x^3+2x^2-2x}\left(x^2+4x-2\right)+28x^2-32x+4}{x^4-4x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(x^2-2)/(x^3+2*x^2-2*x)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((x^4 + 16*x^3 - 4*sqrt(2)*sqrt(x^3 + 2*x^2 - 2*x)*(x^2 + 4*
x - 2) + 28*x^2 - 32*x + 4)/(x^4 - 4*x^2 + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+2}{\sqrt{x^3+2x^2-2x}\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(x^2-2)/(x^3+2*x^2-2*x)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 + 2)/(sqrt(x^3 + 2*x^2 - 2*x)*(x^2 - 2)), x)
```

maple [C] time = 0.08, size = 677, normalized size = 16.92

$$\frac{\left(1+\sqrt{3}\right)\sqrt{\frac{x+1+\sqrt{3}}{1+\sqrt{3}}}\,\sqrt{-6\left(x-\sqrt{3}+1\right)\sqrt{3}}\,\sqrt{-\frac{x}{1+\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1+\sqrt{3}}{1+\sqrt{3}}},\frac{\sqrt{6}\sqrt{\left(1+\sqrt{3}\right)\sqrt{3}}}{6}\right)}{3\sqrt{x^3+2x^2-2x}}+\frac{\sqrt{2}\sqrt{\frac{x}{1+\sqrt{3}}+\frac{1}{1+\sqrt{3}}+\frac{\sqrt{3}}{1+\sqrt{3}}}\,\sqrt{-6x\sqrt{3}+18-6\sqrt{3}}\,\sqrt{-\frac{x}{1+\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+1+\sqrt{3}}{1+\sqrt{3}}},\frac{-1-\sqrt{3}}{-1-\sqrt{3}-\sqrt{2}},\frac{\sqrt{6}\sqrt{\left(1+\sqrt{3}\right)\sqrt{3}}}{6}\right)}{3\sqrt{x^3+2x^2-2x}\left(-1-\sqrt{3}-\sqrt{2}\right)}+\frac{\sqrt{2}\sqrt{\frac{x}{1+\sqrt{3}}+\frac{1}{1+\sqrt{3}}+\frac{\sqrt{3}}{1+\sqrt{3}}}\,\sqrt{-6x\sqrt{3}+18-6\sqrt{3}}\,\sqrt{-\frac{x}{1+\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+1+\sqrt{3}}{1+\sqrt{3}}},\frac{-1-\sqrt{3}}{-1-\sqrt{3}-\sqrt{2}},\frac{\sqrt{6}\sqrt{\left(1+\sqrt{3}\right)\sqrt{3}}}{6}\right)\sqrt{3}}{3\sqrt{x^3+2x^2-2x}\left(-1-\sqrt{3}-\sqrt{2}\right)}-\frac{\sqrt{2}\sqrt{\frac{x}{1+\sqrt{3}}+\frac{1}{1+\sqrt{3}}+\frac{\sqrt{3}}{1+\sqrt{3}}}\,\sqrt{-6x\sqrt{3}+18-6\sqrt{3}}\,\sqrt{-\frac{x}{1+\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+1+\sqrt{3}}{1+\sqrt{3}}},\frac{-1-\sqrt{3}}{-1-\sqrt{3}+\sqrt{2}},\frac{\sqrt{6}\sqrt{\left(1+\sqrt{3}\right)\sqrt{3}}}{6}\right)}{3\sqrt{x^3+2x^2-2x}\left(-1-\sqrt{3}+\sqrt{2}\right)}-\frac{\sqrt{2}\sqrt{\frac{x}{1+\sqrt{3}}+\frac{1}{1+\sqrt{3}}+\frac{\sqrt{3}}{1+\sqrt{3}}}\,\sqrt{-6x\sqrt{3}+18-6\sqrt{3}}\,\sqrt{-\frac{x}{1+\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+1+\sqrt{3}}{1+\sqrt{3}}},\frac{-1-\sqrt{3}}{-1-\sqrt{3}+\sqrt{2}},\frac{\sqrt{6}\sqrt{\left(1+\sqrt{3}\right)\sqrt{3}}}{6}\right)\sqrt{3}}{3\sqrt{x^3+2x^2-2x}\left(-1-\sqrt{3}+\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)/(x^2-2)/(x^3+2*x^2-2*x)^(1/2),x)
[Out] 1/3*(1+3^(1/2))*((x+1+3^(1/2))/(1+3^(1/2)))^(1/2)*(-6*(x-3^(1/2)+1)*3^(1/2)
)^(1/2)*(-x/(1+3^(1/2)))^(1/2)/(x^3+2*x^2-2*x)^(1/2)*EllipticF(((x+1+3^(1/2
))/(1+3^(1/2)))^(1/2),1/6*6^(1/2)*((1+3^(1/2))*3^(1/2))^(1/2))+1/3*2^(1/2)*
(x/(1+3^(1/2))+1/(1+3^(1/2))+1/(1+3^(1/2))*3^(1/2))^(1/2)*(-6*x*3^(1/2)+18-
6*3^(1/2))^(1/2)*(-x/(1+3^(1/2)))^(1/2)/(x^3+2*x^2-2*x)^(1/2)/(-1-3^(1/2)-2
^(1/2))*EllipticPi(((x+1+3^(1/2))/(1+3^(1/2)))^(1/2),(-1-3^(1/2))/(-1-3^(1/
2)-2^(1/2)),1/6*6^(1/2)*((1+3^(1/2))*3^(1/2))^(1/2))+1/3*2^(1/2)*(x/(1+3^(1
/2))+1/(1+3^(1/2))+1/(1+3^(1/2))*3^(1/2))^(1/2)*(-6*x*3^(1/2)+18-6*3^(1/2))
^(1/2)*(-x/(1+3^(1/2)))^(1/2)/(x^3+2*x^2-2*x)^(1/2)/(-1-3^(1/2)-2^(1/2))*El
lipticPi(((x+1+3^(1/2))/(1+3^(1/2)))^(1/2),(-1-3^(1/2))/(-1-3^(1/2)-2^(1/2)
),1/6*6^(1/2)*((1+3^(1/2))*3^(1/2))^(1/2))*3^(1/2)-1/3*2^(1/2)*(x/(1+3^(1/2
))+1/(1+3^(1/2))+1/(1+3^(1/2))*3^(1/2))^(1/2)*(-6*x*3^(1/2)+18-6*3^(1/2))^(
1/2)*(-x/(1+3^(1/2)))^(1/2)/(x^3+2*x^2-2*x)^(1/2)/(-1-3^(1/2)+2^(1/2))*Elli
pticPi(((x+1+3^(1/2))/(1+3^(1/2)))^(1/2),(-1-3^(1/2))/(-1-3^(1/2)+2^(1/2)),
1/6*6^(1/2)*((1+3^(1/2))*3^(1/2))^(1/2))-1/3*2^(1/2)*(x/(1+3^(1/2))+1/(1+3^
(1/2))+1/(1+3^(1/2))*3^(1/2))^(1/2)*(-6*x*3^(1/2)+18-6*3^(1/2))^(1/2)*(-x/(
1+3^(1/2)))^(1/2)/(x^3+2*x^2-2*x)^(1/2)/(-1-3^(1/2)+2^(1/2))*EllipticPi(((x
+1+3^(1/2))/(1+3^(1/2)))^(1/2),(-1-3^(1/2))/(-1-3^(1/2)+2^(1/2)),1/6*6^(1/2
)*((1+3^(1/2))*3^(1/2))^(1/2))*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2}{\sqrt{x^3+2\,x^2-2\,x}\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(x^2-2)/(x^3+2*x^2-2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 2)/(sqrt(x^3 + 2*x^2 - 2*x)*(x^2 - 2)), x)
```

mupad [B] time = 0.15, size = 227, normalized size = 5.68

$$-\frac{2\sqrt{x}\,\sqrt{\frac{1}{\sqrt{3}+1}}\,\Pi\left(\sqrt{2}\,\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right);\mathrm{asin}\left(\sqrt{\frac{x}{\sqrt{3}-1}}\right)\middle|-\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)\sqrt{x+\sqrt{3}+1}\,\sqrt{\sqrt{3}-x-1}+2\sqrt{x}\,\sqrt{\frac{1}{\sqrt{3}+1}}\,\Pi\left(-\sqrt{2}\,\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right);\mathrm{asin}\left(\sqrt{\frac{x}{\sqrt{3}-1}}\right)\middle|-\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)\sqrt{x+\sqrt{3}+1}\,\sqrt{\sqrt{3}-x-1}-2\sqrt{x}\,\sqrt{\frac{1}{\sqrt{3}+1}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x}{\sqrt{3}-1}}\right)\middle|-\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)\sqrt{x+\sqrt{3}+1}\,\sqrt{\sqrt{3}-x-1}}{\sqrt{x^3+2\,x^2-\left(\sqrt{3}-1\right)\,\left(\sqrt{3}+1\right)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 2)/((x^2 - 2)*(2*x^2 - 2*x + x^3)^(1/2)),x)
```

```
[Out] -(2*x^(1/2)*(1/(3^(1/2) + 1))^(1/2)*ellipticPi(2^(1/2)*(3^(1/2)/2 - 1/2), a
sin((x/(3^(1/2) - 1))^(1/2)), -(3^(1/2) - 1)/(3^(1/2) + 1))*(x + 3^(1/2) +
1)^(1/2)*(3^(1/2) - x - 1)^(1/2) + 2*x^(1/2)*(1/(3^(1/2) + 1))^(1/2)*ellipt
icPi(-2^(1/2)*(3^(1/2)/2 - 1/2), asin((x/(3^(1/2) - 1))^(1/2)), -(3^(1/2) -
 1)/(3^(1/2) + 1))*(x + 3^(1/2) + 1)^(1/2)*(3^(1/2) - x - 1)^(1/2) - 2*x^(1
/2)*(1/(3^(1/2) + 1))^(1/2)*ellipticF(asin((x/(3^(1/2) - 1))^(1/2)), -(3^(1
/2) - 1)/(3^(1/2) + 1))*(x + 3^(1/2) + 1)^(1/2)*(3^(1/2) - x - 1)^(1/2))/(2
*x^2 + x^3 - x*(3^(1/2) - 1)*(3^(1/2) + 1))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2}{\sqrt{x\left(x^2+2x-2\right)}\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)/(x**2-2)/(x**3+2*x**2-2*x)**(1/2),x)
```

```
[Out] Integral((x**2 + 2)/(sqrt(x*(x**2 + 2*x - 2))*(x**2 - 2)), x)
```

3.516 $\int \frac{2+x}{(-1+x)\sqrt{-1+3x+ax^2+x^3}}\,dx$

Optimal. Leaf size=40

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-a-3}\,x}{\sqrt{ax^2+x^3+3x-1}}\right)}{\sqrt{-a-3}}$$

Rubi [C] time = 137.42, antiderivative size = 5379, normalized size of antiderivative = 134.48, number of steps used = 13, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {6742, 2067, 2066, 718, 419, 2081, 2080, 934, 169, 538, 537}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(2 + x)/((-1 + x)*Sqrt[-1 + 3*x + a*x^2 + x^3]),x]
```

```
[Out] (2^(1/3)*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a
 - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) + 4*a^2*(27 + 27
*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) - 2^(1/3)*(27
+ 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(4/3)]*Sqrt[(-18
*2^(1/3) + 2*2^(1/3)*a^2 - 2*a*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a
)^2*(-15 + 4*a))])^(1/3) + (54 + 54*a - 4*a^3 + 6*Sqrt[3]*Sqrt[-((3 + a)^2*
(-15 + 4*a))])^(2/3) - 6*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-
15 + 4*a))])^(1/3)*x)/(-54 + 6*a^2 + 3*2^(1/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[
3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) + 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3
) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt
[-((3 + a)^2*(-15 + 4*a))])^(2/3) + 4*a^2*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sq
rt[-((3 + a)^2*(-15 + 4*a))])^(2/3) - 2^(1/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[3
]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(4/3)])]*Sqrt[-((2*(9 - a^2) + (2*2^(2/3)
*(9 - a^2)^2)/(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2
/3) + 2^(1/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2
/3) + 18*(a/3 + x)^2 - (2^(1/3)*(18 - 2*a^2 - 2^(1/3)*(27 + 27*a - 2*a^3 +
3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3))*(a + 3*x))/(27 + 27*a - 2*a^3
+ 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(1/3))/((18 - 2*a^2 - 2^(1/3)*(27 +
 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3))^2/(18*2^(1/3)*
(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3)) - (2*(2*(
9 - a^2) + (2*2^(2/3)*(9 - a^2)^2)/(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15
- 4*a)*(3 + a)^2])^(2/3) + 2^(1/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15
- 4*a)*(3 + a)^2])^(2/3)))/9))]*EllipticF[ArcSin[Sqrt[(-18*2^(1/3) + 2*2^(1
/3)*a^2 + 4*a*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))]
)^(1/3) + (54 + 54*a - 4*a^3 + 6*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2
/3) + Sqrt[6]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 +
 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) + 4*a^2*(27
 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) - 2^(1/3)
*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(4/3)] + 12
*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(1/3)*x)/Sq
rt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*a^3 +
3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) + 4*a^2*(27 + 27*a - 2*a^3
+ 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) - 2^(1/3)*(27 + 27*a - 2*
a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(4/3)]]/(2^(3/4)*3^(1/4))],
 (-2*2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 3
6*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4*a^2*
(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/3)*
(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)])/(54 - 6*
a^2 - 3*2^(1/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^
(2/3) - 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4
- 36*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4*a
^2*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/
```

```
3)*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)])])/(3*
Sqrt[3]*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(1/3
)*Sqrt[-1 + 3*x + a*x^2 + x^3]) - (3*2^(2/3)*Sqrt[54 - 6*a^2 - 3*2^(1/3)*(2
7 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) + 2^(1/6
)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*
a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4*a^2*(27 + 27*a
 - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/3)*(27 + 27*a
 - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)]]*Sqrt[2*a + (2^(1/3)
*(18 - 2*a^2 - 2^(1/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 +
a)^2])^(2/3)))/(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(
1/3) + 6*x]*Sqrt[(2^(1/3)*(-18 + 2*a^2 + 2^(1/3)*(27 + 27*a - 2*a^3 + 3*Sqr
t[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3)))/(27 + 27*a - 2*a^3 + 3*Sqrt[3]
*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(1/3) - (Sqrt[6]*Sqrt[-162*2^(2/3) + 36*2^
(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a
)^2*(-15 + 4*a))])^(2/3) + 4*a^2*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 +
 a)^2*(-15 + 4*a))])^(2/3) - 2^(1/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-(
(3 + a)^2*(-15 + 4*a))])^(4/3)])/(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 +
 a)^2*(-15 + 4*a))])^(1/3) + 4*(a + 3*x)]*Sqrt[(2^(1/3)*(-18 + 2*a^2 + 2^(1
/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3)))/
(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(1/3) + (Sqr
t[6]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2
*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) + 4*a^2*(27 + 27*a -
 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) - 2^(1/3)*(27 + 27
*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(4/3)])/(27 + 27*a -
 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(1/3) + 4*(a + 3*x)]*Sqr
t[1 + (2*(18 - 2*a^2 - 2^(1/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a
)^2*(-15 + 4*a))])^(2/3) + 2^(2/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3
 + a)^2*(-15 + 4*a))])^(1/3)*(a + 3*x)))/(-54 + 6*a^2 + 3*2^(1/3)*(27 + 27*
a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) - 2^(1/6)*Sqrt[
3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*a
^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4*a^2*(27 + 27*a - 2*a^3
 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/3)*(27 + 27*a - 2*a^3
 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)])]*Sqrt[1 + (2*(18 - 2*a^2 -
 2^(1/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/
3) + 2^(2/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])
^(1/3)*(a + 3*x)))/(-54 + 6*a^2 + 3*2^(1/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*
Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) + 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) +
 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(1
5 - 4*a)*(3 + a)^2])^(2/3) + 4*a^2*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15
- 4*a)*(3 + a)^2])^(2/3) - 2^(1/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15
- 4*a)*(3 + a)^2])^(4/3)])]*EllipticPi[(54 - 6*a^2 - 3*2^(1/3)*(27 + 27*a -
 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 2^(1/6)*Sqrt[3]*Sqrt
[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*a^3 + 3*
Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4*a^2*(27 + 27*a - 2*a^3 + 3*Sq
rt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/3)*(27 + 27*a - 2*a^3 + 3*Sq
rt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)])/(2^(2/3)*(27 + 27*a - 2*a^3 + 3*S
qrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(1/3)*(2*(3 + a) + (2^(1/3)*(18 - 2*a^2
- 2^(1/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3))
)/(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(1/3))), ArcSi
n[(2^(1/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(
1/6)*Sqrt[2*a + (2^(1/3)*(18 - 2*a^2 - 2^(1/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[
3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3)))/(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[
(15 - 4*a)*(3 + a)^2])^(1/3) + 6*x])/Sqrt[54 - 6*a^2 - 3*2^(1/3)*(27 + 27*a
 - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) + 2^(1/6)*Sqrt[3
]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*a^
3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4*a^2*(27 + 27*a - 2*a^3
+ 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/3)*(27 + 27*a - 2*a^3
+ 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)]]], (54 - 6*a^2 - 3*2^(1/3)*(
27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 2^(1/6)*S
```

```
qrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a -
 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4*a^2*(27 + 27*a - 2
*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/3)*(27 + 27*a - 2
*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)])/(54 - 6*a^2 - 3*2^(1/3
)*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/6
)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*
a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4*a^2*(27 + 27*a
- 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/3)*(27 + 27*a
- 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)]])]/((27 + 27*a - 2*a
^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(1/6)*(2*(3 + a) + (2^(1/3)*
(18 - 2*a^2 - 2^(1/3)*(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a
)^2])^(2/3)))/(27 + 27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(1
/3))*Sqrt[-1 + 3*x + a*x^2 + x^3]*Sqrt[(-18*2^(1/3) + 2*2^(1/3)*a^2 + (54 +
 54*a - 4*a^3 + 6*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) - Sqrt[6]*S
qrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*a^3 +
 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4*a^2*(27 + 27*a - 2*a^3 + 3
*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/3)*(27 + 27*a - 2*a^3 + 3
*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)] + 4*(27 + 27*a - 2*a^3 + 3*Sqrt
[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(1/3)*(a + 3*x))/(27 + 27*a - 2*a^3 + 3
*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(1/3)]*Sqrt[(-18*2^(1/3) + 2*2^(1/
3)*a^2 + (54 + 54*a - 4*a^3 + 6*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/
3) + Sqrt[6]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 +
27*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4*a^2*(27 + 27
*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/3)*(27 + 27
*a - 2*a^3 + 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)] + 4*(27 + 27*a -
2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(1/3)*(a + 3*x))/(27 + 27
*a - 2*a^3 + 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(1/3)])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 2066

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> With[{r = Rt[-9*
a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}, Dist[(a + b*x + d*x^3)^p
/(Simp[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d*x, x]^p*Simp[(b*d)/3 + (12^(1/
3)*b^2*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*((2^(1/3)*b*d)/(3^(1/3)*r) - r/1
8^(1/3))*x + d^2*x^2, x]^p), Int[Simp[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d
*x, x]^p*Simp[(b*d)/3 + (12^(1/3)*b^2*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*(
(2^(1/3)*b*d)/(3^(1/3)*r) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ
[{a, b, d, p}, x] && NeQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 2080

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Dist[(a + b*x + d*x^3)^p/(Simp[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d*x, x
]^p*Simp[(b*d)/3 + (12^(1/3)*b^2*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*((2^(1
/3)*b*d)/(3^(1/3)*r) - r/18^(1/3))*x + d^2*x^2, x]^p), Int[(e + f*x)^m*Simp
[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d*x, x]^p*Simp[(b*d)/3 + (12^(1/3)*b^2
*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*((2^(1/3)*b*d)/(3^(1/3)*r) - r/18^(1/3
))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*b
^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 2081

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
```

]

Rubi steps

$$\int \frac{2+x}{(-1+x)\sqrt{-1+3x+ax^2+x^3}}\,dx = \int \left(\frac{1}{\sqrt{-1+3x+ax^2+x^3}}+\frac{3}{(-1+x)\sqrt{-1+3x+ax^2+x^3}}\right)dx$$
$$= 3\int \frac{1}{(-1+x)\sqrt{-1+3x+ax^2+x^3}}\,dx+\int \frac{1}{\sqrt{-1+3x+ax^2+x^3}}\,dx$$
$$= 3\,\text{Subst}\left(\int \frac{1}{\left(\frac{1}{3}(-3-a)+x\right)\sqrt{\frac{1}{27}\left(-27-27a+2a^3\right)+\frac{1}{3}\left(9-a^2\right)x+x^3}}\right.$$

= rest of steps removed due to Latex formating problem

Mathematica [C] time = 1.05, size = 1015, normalized size = 25.38

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + x)/((-1 + x)*Sqrt[-1 + 3*x + a*x^2 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x + Root[3 + a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 1])/(R
oot[3 + a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 1] - Root[3 + a + (6 + 2
*a)*#1 + (3 + a)*#1^2 + #1^3 & , 3])]*((3*EllipticPi[1 - Root[3 + a + (6 +
2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 2]/Root[3 + a + (6 + 2*a)*#1 + (3 + a)*#1
^2 + #1^3 & , 3], ArcSin[Sqrt[(1 - x + Root[3 + a + (6 + 2*a)*#1 + (3 + a)*
#1^2 + #1^3 & , 3])/(-Root[3 + a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 2
] + Root[3 + a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 3])]], (Root[3 + a
+ (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 2] - Root[3 + a + (6 + 2*a)*#1 + (
3 + a)*#1^2 + #1^3 & , 3])/(Root[3 + a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3
 & , 1] - Root[3 + a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 3])]*Sqrt[-((
(-1 + x - Root[3 + a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 2])*(-1 + x -
 Root[3 + a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 3]))/(Root[3 + a + (6
+ 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 2] - Root[3 + a + (6 + 2*a)*#1 + (3 + a
)*#1^2 + #1^3 & , 3])^2)]*(Root[3 + a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3
& , 2] - Root[3 + a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 3]))/Root[3 +
a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 3] + (EllipticF[ArcSin[Sqrt[(1 -
 x + Root[3 + a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 3])/(-Root[3 + a +
 (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 2] + Root[3 + a + (6 + 2*a)*#1 + (3
 + a)*#1^2 + #1^3 & , 3])]], (Root[3 + a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1
^3 & , 2] - Root[3 + a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 3])/(Root[3
 + a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 1] - Root[3 + a + (6 + 2*a)*#
1 + (3 + a)*#1^2 + #1^3 & , 3])]*(-1 + x - Root[3 + a + (6 + 2*a)*#1 + (3 +
 a)*#1^2 + #1^3 & , 3])*Sqrt[(1 - x + Root[3 + a + (6 + 2*a)*#1 + (3 + a)*#
1^2 + #1^3 & , 2])/(Root[3 + a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 2]
- Root[3 + a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 3])])/Sqrt[(1 - x + R
oot[3 + a + (6 + 2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 3])/(-Root[3 + a + (6 +
2*a)*#1 + (3 + a)*#1^2 + #1^3 & , 2] + Root[3 + a + (6 + 2*a)*#1 + (3 + a)*
#1^2 + #1^3 & , 3])]))/Sqrt[-1 + 3*x + a*x^2 + x^3]
```

IntegrateAlgebraic [A] time = 0.13, size = 40, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-a-3}\,x}{\sqrt{ax^2+x^3+3x-1}}\right)}{\sqrt{-a-3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)/((-1 + x)*Sqrt[-1 + 3*x + a*x^2 + x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 - a]*x)/Sqrt[-1 + 3*x + a*x^2 + x^3]])/Sqrt[-3 - a]
```

fricas [B] time = 0.44, size = 225, normalized size = 5.62

$$\left[\frac{\log\left(\frac{2\,(4\,a+9)x^5+x^6+\left(8\,a^2+24\,a+15\right)x^4+4\,(6\,a+13)x^3-(8\,a+9)x^2-4\left((2\,a+3)x^3+x^4+3\,x^2-x\right)\sqrt{ax^2+x^3+3\,x-1}\,\sqrt{a+3}-6\,x+1}{x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1}\right)}{2\,\sqrt{a+3}},\ \frac{\sqrt{-a-3}\,\arctan\left(\frac{\left((2\,a+3)x^2+x^3+3\,x-1\right)\sqrt{ax^2+x^3+3\,x-1}\,\sqrt{-a-3}}{2\left((a+3)x^4+\left(a^2+3\,a\right)x^3+3\,(a+3)x^2-(a+3)x\right)}\right)}{a+3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-1+x)/(a*x^2+x^3+3*x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*(4*a + 9)*x^5 + x^6 + (8*a^2 + 24*a + 15)*x^4 + 4*(6*a + 13)*x^
3 - (8*a + 9)*x^2 - 4*((2*a + 3)*x^3 + x^4 + 3*x^2 - x)*sqrt(a*x^2 + x^3 +
3*x - 1)*sqrt(a + 3) - 6*x + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6
*x + 1))/sqrt(a + 3), sqrt(-a - 3)*arctan(1/2*((2*a + 3)*x^2 + x^3 + 3*x -
1)*sqrt(a*x^2 + x^3 + 3*x - 1)*sqrt(-a - 3)/((a + 3)*x^4 + (a^2 + 3*a)*x^3
+ 3*(a + 3)*x^2 - (a + 3)*x))/(a + 3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\sqrt{ax^2+x^3+3\,x-1}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-1+x)/(a*x^2+x^3+3*x-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 2)/(sqrt(a*x^2 + x^3 + 3*x - 1)*(x - 1)), x)
```

maple [C] time = 0.68, size = 3006, normalized size = 75.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-1+x)/(a*x^2+x^3+3*x-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1
/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/
3))*(-I*(x+1/12*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)
-3*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+
1/3*a+1/2*I*3^(1/2)*(1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/
2))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2
))^(1/3)))*3^(1/2)/(1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2
))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2)
)^(1/3)))^(1/2)*((x-1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2
))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2)
)^(1/3)+1/3*a)/(-1/4*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^
(1/3)+9*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(
1/3)-1/2*I*3^(1/2)*(1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2
))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2)
)^(1/3))))^(1/2)*(I*(x+1/12*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^
(1/2))^(1/3)-3*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(
1/2))^(1/3)+1/3*a-1/2*I*3^(1/2)*(1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+16
2*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162
*a+405)^(1/2))^(1/3)))*3^(1/2)/(1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162
*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*
a+405)^(1/2))^(1/3)))^(1/2)/(a*x^2+x^3+3*x-1)^(1/2)*EllipticF(1/3*3^(1/2)*(
-I*(x+1/12*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)-3*(1
-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+1/3*a
```

```
+1/2*I*3^(1/2)*(1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(
1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1
/3)))*3^(1/2)/(1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1
/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/
3)))^(1/2),(-I*3^(1/2)*(1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^
(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(
1/2))^(1/3))/(-1/4*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1
/3)+9*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/
3)-1/2*I*3^(1/2)*(1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))
^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^
(1/3))))^(1/2))+2*I*3^(1/2)*(1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+
405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+4
05)^(1/2))^(1/3))*(-I*(x+1/12*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405
)^(1/2))^(1/3)-3*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)
^(1/2))^(1/3)+1/3*a+1/2*I*3^(1/2)*(1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+
162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+1
62*a+405)^(1/2))^(1/3)))*3^(1/2)/(1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+1
62*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+16
2*a+405)^(1/2))^(1/3)))^(1/2)*((x-1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+1
62*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+16
2*a+405)^(1/2))^(1/3)+1/3*a)/(-1/4*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*
a+405)^(1/2))^(1/3)+9*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a
+405)^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+1
62*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+16
2*a+405)^(1/2))^(1/3))))^(1/2)*(I*(x+1/12*(108*a+108-8*a^3+12*(-12*a^3-27*a
^2+162*a+405)^(1/2))^(1/3)-3*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^
2+162*a+405)^(1/2))^(1/3)+1/3*a-1/2*I*3^(1/2)*(1/6*(108*a+108-8*a^3+12*(-12
*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*
a^3-27*a^2+162*a+405)^(1/2))^(1/3)))*3^(1/2)/(1/6*(108*a+108-8*a^3+12*(-12*
a^3-27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a
^3-27*a^2+162*a+405)^(1/2))^(1/3)))^(1/2)/(a*x^2+x^3+3*x-1)^(1/2)/(-1/12*(1
08*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+3*(1-1/9*a^2)/(10
8*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)-1/3*a-1/2*I*3^(1/2
)*(1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9
*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3))-1)*Ellip
ticPi(1/3*3^(1/2)*(-I*(x+1/12*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405
)^(1/2))^(1/3)-3*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+162*a+405)
^(1/2))^(1/3)+1/3*a+1/2*I*3^(1/2)*(1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+
162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+1
62*a+405)^(1/2))^(1/3)))*3^(1/2)/(1/6*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+1
62*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+16
2*a+405)^(1/2))^(1/3)))^(1/2),-I*3^(1/2)*(1/6*(108*a+108-8*a^3+12*(-12*a^3-
27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-2
7*a^2+162*a+405)^(1/2))^(1/3))/(-1/12*(108*a+108-8*a^3+12*(-12*a^3-27*a^2+1
62*a+405)^(1/2))^(1/3)+3*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a^2+16
2*a+405)^(1/2))^(1/3)-1/3*a-1/2*I*3^(1/2)*(1/6*(108*a+108-8*a^3+12*(-12*a^3
-27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-
27*a^2+162*a+405)^(1/2))^(1/3))-1),(-I*3^(1/2)*(1/6*(108*a+108-8*a^3+12*(-1
2*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12
*a^3-27*a^2+162*a+405)^(1/2))^(1/3))/(-1/4*(108*a+108-8*a^3+12*(-12*a^3-27*
a^2+162*a+405)^(1/2))^(1/3)+9*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-27*a
^2+162*a+405)^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/6*(108*a+108-8*a^3+12*(-12*a^3-
27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a+108-8*a^3+12*(-12*a^3-2
7*a^2+162*a+405)^(1/2))^(1/3))))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\sqrt{ax^2+x^3+3x-1}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-1+x)/(a*x^2+x^3+3*x-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 2)/(sqrt(a*x^2 + x^3 + 3*x - 1)*(x - 1)), x)
```

mupad [B] time = 0.71, size = 62, normalized size = 1.55

$$\frac{\ln\left(\frac{\left(\sqrt{x^3+a\,x^2+3\,x-1}+x\,\sqrt{a+3}\right)\left(\sqrt{x^3+a\,x^2+3\,x-1}-x\,\sqrt{a+3}\right)^3}{(x-1)^6}\right)}{\sqrt{a+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((x - 1)*(3*x + a*x^2 + x^3 - 1)^(1/2)),x)
```

```
[Out] log((((3*x + a*x^2 + x^3 - 1)^(1/2) + x*(a + 3)^(1/2))*((3*x + a*x^2 + x^3
- 1)^(1/2) - x*(a + 3)^(1/2))^3)/(x - 1)^6)/(a + 3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{(x-1)\sqrt{ax^2+x^3+3x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-1+x)/(a*x**2+x**3+3*x-1)**(1/2),x)
```

```
[Out] Integral((x + 2)/((x - 1)*sqrt(a*x**2 + x**3 + 3*x - 1)), x)
```

3.517 $\int \frac{-2+x}{(1+x)\sqrt{1+3x+ax^2+x^3}}\,dx$

Optimal. Leaf size=40

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3-a}\,x}{\sqrt{ax^2+x^3+3x+1}}\right)}{\sqrt{3-a}}$$

Rubi [C] time = 131.81, antiderivative size = 5437, normalized size of antiderivative = 135.92, number of steps used = 13, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {6742, 2067, 2066, 718, 419, 2081, 2080, 934, 169, 538, 537}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(-2 + x)/((1 + x)*Sqrt[1 + 3*x + a*x^2 + x^3]),x]
```

```
[Out] (2^(1/3)*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a
 + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(2/3) + 4*a^2*(27 - 27*a
+ 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(2/3) - 2^(1/3)*(27 - 27*a
 + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(4/3)]*Sqrt[(-18*2^(1/3)
+ 2*2^(1/3)*a^2 + (54 - 54*a + 4*a^3 - 6*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)
])^(2/3) + 2*a*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^
(1/3) + 6*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(1/3)
*x)/(-54 + 6*a^2 + 3*2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2
*(15 + 4*a)])^(2/3) - 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 -
2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a
)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)
])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a
)])^(4/3)])]*Sqrt[-((2*(9 - a^2) + 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*S
qrt[(3 - a)^2*(15 + 4*a)])^(2/3) + (2*(9 - a^2)^2)/((-27*a)/2 + a^3 + (3*(9
 - Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)]))/2)^(2/3) + 18*(a/3 + x)^2 + (2^(1/3
)*(18 - 2*a^2 - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 +
 4*a)])^(2/3))*(a + 3*x))/(-27*a + 2*a^3 + 3*(9 - Sqrt[3]*Sqrt[(3 - a)^2*(1
5 + 4*a)]))^(1/3))/((18 - 2*a^2 - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sq
rt[(3 - a)^2*(15 + 4*a)])^(2/3))^2/(18*2^(1/3)*(-27*a + 2*a^3 + 3*(9 - Sqrt
[3]*Sqrt[(3 - a)^2*(15 + 4*a)]))^(2/3)) - (2*(2*(9 - a^2) + 2^(1/3)*(27 - 2
7*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) + (2*(9 - a^2)^2)
/((-27*a)/2 + a^3 + (3*(9 - Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)]))/2)^(2/3)))
/9))]*EllipticF[ArcSin[Sqrt[(18*2^(1/3) - 2*2^(1/3)*a^2 - (54 - 54*a + 4*a^
3 - 6*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(2/3) + 4*a*(27 - 27*a + 2*a^3 -
 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(1/3) + Sqrt[6]*Sqrt[-162*2^(2/3) +
 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-
3 + a)^2*(15 + 4*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3
 + a)^2*(15 + 4*a)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-
3 + a)^2*(15 + 4*a)])^(4/3)] + 12*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 +
 a)^2*(15 + 4*a)])^(1/3)*x)/Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*
a^4 - 36*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(2/3)
+ 4*a^2*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(2/3) -
 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(4/3)]
]/(2^(3/4)*3^(1/4))], (2*2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2
 - 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4
*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a
)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a
)])^(4/3)])/(54 - 6*a^2 - 3*2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3
- a)^2*(15 + 4*a)])^(2/3) + 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*
a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(15
+ 4*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 +
4*a)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 +
```

4*a)])^(4/3)])])/(3*Sqrt[3]*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*
(15 + 4*a)])^(1/3)*Sqrt[1 + 3*x + a*x^2 + x^3]) - (6*2^(1/6)*Sqrt[3]*Sqrt[-
54 + 6*a^2 + 3*2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 +
4*a)])^(2/3) + 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/
3)*a^4 - 36*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3
) + 4*a^2*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3)
- 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(4/3)]
]*Sqrt[a/3 + (-18 + 2*a^2 + 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3
 + a)^2*(15 + 4*a)])^(2/3))/(3*2^(2/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[
(-3 + a)^2*(15 + 4*a)])^(1/3)) + x]*Sqrt[-((Sqrt[6]*Sqrt[-162*2^(2/3) + 36*
2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 +
a)^2*(15 + 4*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a
)^2*(15 + 4*a)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 +
a)^2*(15 + 4*a)])^(4/3)])/(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(1
5 + 4*a)])^(1/3)) - (-18 + 2*a^2 + 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*S
qrt[(-3 + a)^2*(15 + 4*a)])^(2/3))/((-27*a)/2 + a^3 - (3*(-9 + Sqrt[3]*Sqrt
[(-3 + a)^2*(15 + 4*a)]))/2)^(1/3) + 4*(a + 3*x)]*Sqrt[(Sqrt[6]*Sqrt[-162*2
^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 - 3*Sqrt[3]
Sqrt[(-3 + a)^2(15 + 4*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*
Sqrt[(-3 + a)^2*(15 + 4*a)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]
Sqrt[(-3 + a)^2(15 + 4*a)])^(4/3)])/(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(
-3 + a)^2*(15 + 4*a)])^(1/3) - (-18 + 2*a^2 + 2^(1/3)*(27 - 27*a + 2*a^3 -
3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(2/3))/((-27*a)/2 + a^3 - (3*(-9 + S
qrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)]))/2)^(1/3) + 4*(a + 3*x)]*Sqrt[1 - (2*(1
8 - 2*a^2 - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*
a)])^(2/3) - 2^(2/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4
a)])^(1/3)(a + 3*x)))/(54 - 6*a^2 - 3*2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt
[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(2/3) + 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3)
+ 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(
-3 + a)^2*(15 + 4*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-
3 + a)^2*(15 + 4*a)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(
-3 + a)^2*(15 + 4*a)])^(4/3)])]*Sqrt[1 + (2*(18 - 2*a^2 - 2^(1/3)*(27 - 27*
a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(2/3) - 2^(2/3)*(27 - 27
*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(1/3)*(a + 3*x)))/(-54
+ 6*a^2 + 3*2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*
a)])^(2/3) + 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)
a^4 - 36(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(2/3)
 + 4*a^2*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(2/3)
- 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(4/3)
])]*EllipticPi[(54 - 6*a^2 - 3*2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[
(3 - a)^2*(15 + 4*a)])^(2/3) - 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/
3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(
15 + 4*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(15
 + 4*a)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(15
 + 4*a)])^(4/3)])/(2*(18 - 2*a^2 + 3*2^(2/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]
Sqrt[(3 - a)^2(15 + 4*a)])^(1/3) - 2^(2/3)*a*(27 - 27*a + 2*a^3 - 3*Sqrt[
3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(1/3) - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[
3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3))), ArcSin[(2^(1/3)*(27 - 27*a + 2*a^3
- 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(1/6)*Sqrt[(-18 + 2*a^2 + 2^(1/3)*(
27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(2/3))/((-27*a)/
2 + a^3 - (3*(-9 + Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)]))/2)^(1/3) + 2*(a +
3*x)])/Sqrt[-54 + 6*a^2 + 3*2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3
- a)^2*(15 + 4*a)])^(2/3) + 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*
a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(15
+ 4*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 +
4*a)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 +
4*a)])^(4/3)]]], (54 - 6*a^2 - 3*2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqr
t[(3 - a)^2*(15 + 4*a)])^(2/3) - 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(
2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2

```
*(15 + 4*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(
15 + 4*a)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^2*(
15 + 4*a)])^(4/3)])/(54 - 6*a^2 - 3*2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*
Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) + 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*
2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a
)^2*(15 + 4*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^
2*(15 + 4*a)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(3 - a)^
2*(15 + 4*a)])^(4/3)]])/((-27*a + 2*a^3 + 3*(9 - Sqrt[3]*Sqrt[(3 - a)^2*(1
5 + 4*a)]))^(1/6)*(6 - 2*a + (18 - 2*a^2 - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*S
qrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3))/((-27*a)/2 + a^3 + (3*(9 - Sqrt[3
]*Sqrt[(3 - a)^2*(15 + 4*a)]))/2)^(1/3))*Sqrt[1 + 3*x + a*x^2 + x^3]*Sqrt[(
18*2^(1/3) - 2*2^(1/3)*a^2 - (54 - 54*a + 4*a^3 - 6*Sqrt[3]*Sqrt[(-3 + a)^2
*(15 + 4*a)])^(2/3) - Sqrt[6]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3
)*a^4 - 36*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(2/3
) + 4*a^2*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(2/3)
 - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(4/3
)] + 4*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(1/3)*(a
 + 3*x))/(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(1/3)]
*Sqrt[(18*2^(1/3) - 2*2^(1/3)*a^2 - (54 - 54*a + 4*a^3 - 6*Sqrt[3]*Sqrt[(-3
 + a)^2*(15 + 4*a)])^(2/3) + Sqrt[6]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2
*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)
])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)]
)^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)
])^(4/3)] + 4*(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(
1/3)*(a + 3*x))/(27 - 27*a + 2*a^3 - 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])
^(1/3)])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 2066

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> With[{r = Rt[-9*
a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}, Dist[(a + b*x + d*x^3)^p
/(Simp[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d*x, x]^p*Simp[(b*d)/3 + (12^(1/
3)*b^2*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*((2^(1/3)*b*d)/(3^(1/3)*r) - r/1
8^(1/3))*x + d^2*x^2, x]^p), Int[Simp[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d
*x, x]^p*Simp[(b*d)/3 + (12^(1/3)*b^2*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*(
(2^(1/3)*b*d)/(3^(1/3)*r) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ
[{a, b, d, p}, x] && NeQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 2080

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Dist[(a + b*x + d*x^3)^p/(Simp[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d*x, x
]^p*Simp[(b*d)/3 + (12^(1/3)*b^2*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*((2^(1
/3)*b*d)/(3^(1/3)*r) - r/18^(1/3))*x + d^2*x^2, x]^p), Int[(e + f*x)^m*Simp
[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d*x, x]^p*Simp[(b*d)/3 + (12^(1/3)*b^2
*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*((2^(1/3)*b*d)/(3^(1/3)*r) - r/18^(1/3
))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*b
^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 2081

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
```

]

Rubi steps

$$\int \frac{-2+x}{(1+x)\sqrt{1+3x+ax^2+x^3}}\,dx = \int \left(\frac{1}{\sqrt{1+3x+ax^2+x^3}} - \frac{3}{(1+x)\sqrt{1+3x+ax^2+x^3}}\right) dx$$
$$= -\left(3\int \frac{1}{(1+x)\sqrt{1+3x+ax^2+x^3}}\,dx\right) + \int \frac{1}{\sqrt{1+3x+ax^2+x^3}}\,dx$$
$$= -\left(3\,\text{Subst}\left(\int \frac{1}{\left(\frac{3-a}{3}+x\right)\sqrt{\frac{1}{27}\left(27-27a+2a^3\right)+\frac{1}{3}\left(9-a^2\right)x+x^3}}\,dx, x,\right.\right.$$
$$= \text{rest of steps removed due to Latex formating problem}$$

Mathematica [C] time = 0.76, size = 824, normalized size = 20.60

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x)/((1 + x)*Sqrt[1 + 3*x + a*x^2 + x^3]),x]
```

```
[Out] (2*Sqrt[-(((1 + x - Root[-3 + a + (6 - 2*a)*#1 + (-3 + a)*#1^2 + #1^3 & , 2
])*(1 + x - Root[-3 + a + (6 - 2*a)*#1 + (-3 + a)*#1^2 + #1^3 & , 3]))/(Roo
t[-3 + a + (6 - 2*a)*#1 + (-3 + a)*#1^2 + #1^3 & , 2] - Root[-3 + a + (6 -
2*a)*#1 + (-3 + a)*#1^2 + #1^3 & , 3])^2)]*(Root[-3 + a + (6 - 2*a)*#1 + (-
3 + a)*#1^2 + #1^3 & , 2] - Root[-3 + a + (6 - 2*a)*#1 + (-3 + a)*#1^2 + #1
^3 & , 3])*Sqrt[(1 + x - Root[-3 + a + (6 - 2*a)*#1 + (-3 + a)*#1^2 + #1^3
& , 1])/(-Root[-3 + a + (6 - 2*a)*#1 + (-3 + a)*#1^2 + #1^3 & , 1] + Root[-
3 + a + (6 - 2*a)*#1 + (-3 + a)*#1^2 + #1^3 & , 3])]*(-3*EllipticPi[1 - Roo
t[-3 + a + (6 - 2*a)*#1 + (-3 + a)*#1^2 + #1^3 & , 2]/Root[-3 + a + (6 - 2*
a)*#1 + (-3 + a)*#1^2 + #1^3 & , 3], ArcSin[Sqrt[(1 + x - Root[-3 + a + (6
- 2*a)*#1 + (-3 + a)*#1^2 + #1^3 & , 3])/(Root[-3 + a + (6 - 2*a)*#1 + (-3
+ a)*#1^2 + #1^3 & , 2] - Root[-3 + a + (6 - 2*a)*#1 + (-3 + a)*#1^2 + #1^3
 & , 3])]], (Root[-3 + a + (6 - 2*a)*#1 + (-3 + a)*#1^2 + #1^3 & , 2] - Roo
t[-3 + a + (6 - 2*a)*#1 + (-3 + a)*#1^2 + #1^3 & , 3])/(Root[-3 + a + (6 -
2*a)*#1 + (-3 + a)*#1^2 + #1^3 & , 1] - Root[-3 + a + (6 - 2*a)*#1 + (-3 +
a)*#1^2 + #1^3 & , 3])] + EllipticF[ArcSin[Sqrt[(1 + x - Root[-3 + a + (6 -
 2*a)*#1 + (-3 + a)*#1^2 + #1^3 & , 3])/(Root[-3 + a + (6 - 2*a)*#1 + (-3 +
 a)*#1^2 + #1^3 & , 2] - Root[-3 + a + (6 - 2*a)*#1 + (-3 + a)*#1^2 + #1^3
& , 3])]], (Root[-3 + a + (6 - 2*a)*#1 + (-3 + a)*#1^2 + #1^3 & , 2] - Root
[-3 + a + (6 - 2*a)*#1 + (-3 + a)*#1^2 + #1^3 & , 3])/(Root[-3 + a + (6 - 2
*a)*#1 + (-3 + a)*#1^2 + #1^3 & , 1] - Root[-3 + a + (6 - 2*a)*#1 + (-3 + a
)*#1^2 + #1^3 & , 3])]*Root[-3 + a + (6 - 2*a)*#1 + (-3 + a)*#1^2 + #1^3 &
, 3]))/(Sqrt[1 + 3*x + a*x^2 + x^3]*Root[-3 + a + (6 - 2*a)*#1 + (-3 + a)*#
1^2 + #1^3 & , 3])
```

IntegrateAlgebraic [A] time = 0.13, size = 40, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3-a}\,x}{\sqrt{ax^2+x^3+3x+1}}\right)}{\sqrt{3-a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x)/((1 + x)*Sqrt[1 + 3*x + a*x^2 + x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3 - a]*x)/Sqrt[1 + 3*x + a*x^2 + x^3]])/Sqrt[3 - a]
```

fricas [B] time = 0.45, size = 221, normalized size = 5.52

$$\left[\frac{\log\left(\frac{2\,(4\,a-9)x^5+x^6+\left(8\,a^2-24\,a+15\right)x^4+4\,(6\,a-13)x^3+(8\,a-9)x^2-4\,\left((2\,a-3)x^3+x^4+3\,x^2+x\right)\sqrt{ax^2+x^3+3\,x+1}\,\sqrt{a-3}+6\,x+1}{x^6+6\,x^5+15\,x^4+20\,x^3+15\,x^2+6\,x+1}\right)}{2\,\sqrt{a-3}}, \frac{\sqrt{-a+3}\,\arctan\left(\frac{\left((2\,a-3)x^2+x^3+3\,x+1\right)\sqrt{ax^2+x^3+3\,x+1}\,\sqrt{-a+3}}{2\left((a-3)x^4+\left(a^2-3\,a\right)x^3+3\,(a-3)x^2+(a-3)x\right)}\right)}{a-3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(1+x)/(a*x^2+x^3+3*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*(4*a - 9)*x^5 + x^6 + (8*a^2 - 24*a + 15)*x^4 + 4*(6*a - 13)*x^
3 + (8*a - 9)*x^2 - 4*((2*a - 3)*x^3 + x^4 + 3*x^2 + x)*sqrt(a*x^2 + x^3 +
3*x + 1)*sqrt(a - 3) + 6*x + 1)/(x^6 + 6*x^5 + 15*x^4 + 20*x^3 + 15*x^2 + 6
*x + 1))/sqrt(a - 3), sqrt(-a + 3)*arctan(1/2*((2*a - 3)*x^2 + x^3 + 3*x +
1)*sqrt(a*x^2 + x^3 + 3*x + 1)*sqrt(-a + 3)/((a - 3)*x^4 + (a^2 - 3*a)*x^3
+ 3*(a - 3)*x^2 + (a - 3)*x))/(a - 3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-2}{\sqrt{ax^2+x^3+3\,x+1}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(1+x)/(a*x^2+x^3+3*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - 2)/(sqrt(a*x^2 + x^3 + 3*x + 1)*(x + 1)), x)
```

maple [C] time = 0.75, size = 3006, normalized size = 75.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+x)/(1+x)/(a*x^2+x^3+3*x+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(1/6*(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/
3)+6*(1-1/9*a^2)/(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)
)*(-I*(x+1/12*(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)-3*
(1-1/9*a^2)/(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+1/3*
a+1/2*I*3^(1/2)*(1/6*(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(
1/3)+6*(1-1/9*a^2)/(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/
3)))*3^(1/2)/(1/6*(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3
)+6*(1-1/9*a^2)/(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3))
)^(1/2)*((x-1/6*(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+
6*(1-1/9*a^2)/(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+1/
3*a)/(-1/4*(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+9*(1-
1/9*a^2)/(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)-1/2*I*3
^(1/2)*(1/6*(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1
-1/9*a^2)/(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3))))^(1/
2)*(I*(x+1/12*(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)-3*
(1-1/9*a^2)/(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+1/3*
a-1/2*I*3^(1/2)*(1/6*(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(
1/3)+6*(1-1/9*a^2)/(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/
3)))*3^(1/2)/(1/6*(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3
)+6*(1-1/9*a^2)/(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3))
)^(1/2)/(a*x^2+x^3+3*x+1)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/12*(108*a-10
8-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)-3*(1-1/9*a^2)/(108*a-108-
8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+1/3*a+1/2*I*3^(1/2)*(1/6*(1
08*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108
*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)))*3^(1/2)/(1/6*(108*
a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a-
```

```
108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)))^(1/2),(-I*3^(1/2)*(1/
6*(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/
(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3))/(-1/4*(108*a-10
8-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+9*(1-1/9*a^2)/(108*a-108-
8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/6*(108*a-1
08-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a-108
-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3))))^(1/2))-2*I*3^(1/2)*(1/6
*(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(
108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3))*(-I*(x+1/12*(108
*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)-3*(1-1/9*a^2)/(108*a
-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+1/3*a+1/2*I*3^(1/2)*(1
/6*(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)
/(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)))*3^(1/2)/(1/6*
(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(1
08*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)))^(1/2)*((x-1/6*(1
08*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108
*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+1/3*a)/(-1/4*(108*a-
108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+9*(1-1/9*a^2)/(108*a-10
8-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/6*(108*a
-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a-1
08-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3))))^(1/2)*(I*(x+1/12*(108
*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)-3*(1-1/9*a^2)/(108*a
-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+1/3*a-1/2*I*3^(1/2)*(1
/6*(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)
/(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)))*3^(1/2)/(1/6*
(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(1
08*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)))^(1/2)/(a*x^2+x^3
+3*x+1)^(1/2)/(-1/12*(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(
1/3)+3*(1-1/9*a^2)/(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/
3)-1/3*a-1/2*I*3^(1/2)*(1/6*(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(
1/2))^(1/3)+6*(1-1/9*a^2)/(108*a-108-8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/
2))^(1/3))+1)*EllipticPi(1/3*3^(1/2)*(-I*(x+1/12*(108*a-108-8*a^3+12*(12*a^
3-27*a^2-162*a+405)^(1/2))^(1/3)-3*(1-1/9*a^2)/(108*a-108-8*a^3+12*(12*a^3-
27*a^2-162*a+405)^(1/2))^(1/3)+1/3*a+1/2*I*3^(1/2)*(1/6*(108*a-108-8*a^3+12
*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a-108-8*a^3+12*(
12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)))*3^(1/2)/(1/6*(108*a-108-8*a^3+12*(1
2*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a-108-8*a^3+12*(12*
a^3-27*a^2-162*a+405)^(1/2))^(1/3)))^(1/2),-I*3^(1/2)*(1/6*(108*a-108-8*a^3
+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a-108-8*a^3+1
2*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3))/(-1/12*(108*a-108-8*a^3+12*(12*a^
3-27*a^2-162*a+405)^(1/2))^(1/3)+3*(1-1/9*a^2)/(108*a-108-8*a^3+12*(12*a^3-
27*a^2-162*a+405)^(1/2))^(1/3)-1/3*a-1/2*I*3^(1/2)*(1/6*(108*a-108-8*a^3+12
*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a-108-8*a^3+12*(
12*a^3-27*a^2-162*a+405)^(1/2))^(1/3))+1),(-I*3^(1/2)*(1/6*(108*a-108-8*a^3
+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a-108-8*a^3+1
2*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3))/(-1/4*(108*a-108-8*a^3+12*(12*a^3
-27*a^2-162*a+405)^(1/2))^(1/3)+9*(1-1/9*a^2)/(108*a-108-8*a^3+12*(12*a^3-2
7*a^2-162*a+405)^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/6*(108*a-108-8*a^3+12*(12*a^
3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(108*a-108-8*a^3+12*(12*a^3-
27*a^2-162*a+405)^(1/2))^(1/3))))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-2}{\sqrt{ax^2+x^3+3x+1}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(1+x)/(a*x^2+x^3+3*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - 2)/(sqrt(a*x^2 + x^3 + 3*x + 1)*(x + 1)), x)
```

mupad [B] time = 0.69, size = 62, normalized size = 1.55

$$\frac{\ln\left(\frac{\left(\sqrt{x^3+a\,x^2+3\,x+1}+x\,\sqrt{a-3}\right)\left(\sqrt{x^3+a\,x^2+3\,x+1}-x\,\sqrt{a-3}\right)^3}{(x+1)^6}\right)}{\sqrt{a-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2)/((x + 1)*(3*x + a*x^2 + x^3 + 1)^(1/2)),x)
```

```
[Out] log((((3*x + a*x^2 + x^3 + 1)^(1/2) + x*(a - 3)^(1/2))*((3*x + a*x^2 + x^3
+ 1)^(1/2) - x*(a - 3)^(1/2))^3)/(x + 1)^6)/(a - 3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-2}{(x+1)\sqrt{ax^2+x^3+3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(1+x)/(a*x**2+x**3+3*x+1)**(1/2),x)
```

```
[Out] Integral((x - 2)/((x + 1)*sqrt(a*x**2 + x**3 + 3*x + 1)), x)
```

3.518 $\int \frac{x\left(3ab-2(a+b)x+x^2\right)}{\sqrt{x(-a+x)(-b+x)}\left(-abd+(a+b)dx-dx^2+x^3\right)}\,dx$

Optimal. Leaf size=40

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{x^2(-a-b)+abx+x^3}}{x^2}\right)}{\sqrt{d}}$$

Rubi [F] time = 16.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\left(3ab-2(a+b)x+x^2\right)}{\sqrt{x(-a+x)(-b+x)}\left(-abd+(a+b)dx-dx^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(3*a*b - 2*(a + b)*x + x^2))/(Sqrt[x*(-a + x)*(-b + x)]*(-(a*b*d) +
(a + b)*d*x - d*x^2 + x^3)),x]
```

```
[Out] (2*Sqrt[b]*Sqrt[x]*Sqrt[1 - x/a]*Sqrt[1 - x/b]*EllipticF[ArcSin[Sqrt[x]/Sqr
t[b]], b/a])/Sqrt[(a - x)*(b - x)*x] - (2*(3*a*b - a*d - b*d)*Sqrt[x]*Sqrt[
-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][x^2/(Sqrt[-a + x^2]*Sqrt[-b +
x^2]*(a*b*d - a*(1 + b/a)*d*x^2 + d*x^4 - x^6)), x], x, Sqrt[x]])/Sqrt[(a -
 x)*(b - x)*x] + (2*(2*a + 2*b - d)*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer
[Subst][Defer[Int][x^4/(Sqrt[-a + x^2]*Sqrt[-b + x^2]*(a*b*d - a*(1 + b/a)*
d*x^2 + d*x^4 - x^6)), x], x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x] + (2*a*b*d*
Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][1/(Sqrt[-a + x^2]
*Sqrt[-b + x^2]*(-(a*b*d) + a*(1 + b/a)*d*x^2 - d*x^4 + x^6)), x], x, Sqrt[
x]])/Sqrt[(a - x)*(b - x)*x]
```

Rubi steps

$$\int \frac{x\left(3ab-2(a+b)x+x^2\right)}{\sqrt{x(-a+x)(-b+x)}\left(-abd+(a+b)dx-dx^2+x^3\right)}dx = \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{\sqrt{x}\left(3ab-2(a+b)x+x^2\right)}{\sqrt{-a+x}\sqrt{-b+x}\left(-abd+(a+b)dx-d\right.}}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{x^2\left(3ab-2(a\right.}{\sqrt{-a+x^2}\sqrt{-b+x^2}\left(-ab\right.}\right.}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}\left(\int\left(\frac{1}{\sqrt{-a+x^2}\sqrt{-b+x^2}}+\right.\right.}{\sqrt{x(-a+x)(\cdot}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^2}\sqrt{-b+x^2}}dx,\right.}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}\left(\int\left(\frac{(-3ab+}{\sqrt{-a+x^2}\sqrt{-b+x^2}\left(ab\right.}\right.\right.}{}$$

$$= \frac{\left(2(2a+2b-d)\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt{-a\cdot}}\right.}{\sqrt{x(-a+x)(-b}}$$

$$= \frac{2\sqrt{b}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{b}}\right)|\frac{b}{a}\right)}{\sqrt{(a-x)(b-x)x}} + \frac{(2(2a+}{}$$

Mathematica [C] time = 5.45, size = 2258, normalized size = 56.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(3*a*b - 2*(a + b)*x + x^2))/(Sqrt[x*(-a + x)*(-b + x)]*(-(a*b
*d) + (a + b)*d*x - d*x^2 + x^3)),x]
```

```
[Out] ((2*I)*x*Sqrt[1 - x/b]*(2*a^2*d*EllipticPi[a/Root[-(a*b*d) + (a*d + b*d)*#1
 - d*#1^2 + #1^3 & , 2], I*ArcSinh[Sqrt[-(x/a)]], a/b] + a*b*d*EllipticPi[a
/Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 2], I*ArcSinh[Sqrt[-(x/
a)]], a/b] + 2*b^2*d*EllipticPi[a/Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 +
 #1^3 & , 2], I*ArcSinh[Sqrt[-(x/a)]], a/b] + 3*a*b*d*EllipticPi[a/Root[-(a
*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 3], I*ArcSinh[Sqrt[-(x/a)]], a/b
] + 3*a*b*(EllipticPi[a/Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & ,
2], I*ArcSinh[Sqrt[-(x/a)]], a/b] - 2*EllipticPi[a/Root[-(a*b*d) + (a*d + b
*d)*#1 - d*#1^2 + #1^3 & , 3], I*ArcSinh[Sqrt[-(x/a)]], a/b])*Root[-(a*b*d)
 + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 2] - EllipticPi[a/Root[-(a*b*d) + (a*
d + b*d)*#1 - d*#1^2 + #1^3 & , 2], I*ArcSinh[Sqrt[-(x/a)]], a/b]*Root[-(a*
b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 1]*Root[-(a*b*d) + (a*d + b*d)*#1
 - d*#1^2 + #1^3 & , 2]^2 + 3*a*b*(2*EllipticPi[a/Root[-(a*b*d) + (a*d + b*
d)*#1 - d*#1^2 + #1^3 & , 2], I*ArcSinh[Sqrt[-(x/a)]], a/b] - EllipticPi[a/
Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 3], I*ArcSinh[Sqrt[-(x/a
)]], a/b])*Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 3] - 2*(a + b
)*(EllipticPi[a/Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 2], I*Ar
cSinh[Sqrt[-(x/a)]], a/b] + EllipticPi[a/Root[-(a*b*d) + (a*d + b*d)*#1 - d
*#1^2 + #1^3 & , 3], I*ArcSinh[Sqrt[-(x/a)]], a/b])*Root[-(a*b*d) + (a*d +
```

```
b*d)*#1 - d*#1^2 + #1^3 & , 1]*Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1
^3 & , 3] - 2*(a + b)*(2*EllipticPi[a/Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1
^2 + #1^3 & , 2], I*ArcSinh[Sqrt[-(x/a)]], a/b] - EllipticPi[a/Root[-(a*b*d
) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 3], I*ArcSinh[Sqrt[-(x/a)]], a/b])*R
oot[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 2]*Root[-(a*b*d) + (a*d +
 b*d)*#1 - d*#1^2 + #1^3 & , 3] + EllipticPi[a/Root[-(a*b*d) + (a*d + b*d)*
#1 - d*#1^2 + #1^3 & , 2], I*ArcSinh[Sqrt[-(x/a)]], a/b]*Root[-(a*b*d) + (a
*d + b*d)*#1 - d*#1^2 + #1^3 & , 2]^2*Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1
^2 + #1^3 & , 3] + EllipticPi[a/Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #
1^3 & , 3], I*ArcSinh[Sqrt[-(x/a)]], a/b]*Root[-(a*b*d) + (a*d + b*d)*#1 -
d*#1^2 + #1^3 & , 1]*Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 3]^
2 - EllipticPi[a/Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 3], I*A
rcSinh[Sqrt[-(x/a)]], a/b]*Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 &
 , 2]*Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 3]^2 + EllipticF[I
*ArcSinh[Sqrt[-(x/a)]], a/b]*(-(d*(-3*a*b + a*d + b*d)) + 2*Root[-(a*b*d) +
 (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 1]*Root[-(a*b*d) + (a*d + b*d)*#1 - d*#
1^2 + #1^3 & , 2]^2 + 2*Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & ,
1]^2*Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 3] + 2*Root[-(a*b*d
) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 2]*Root[-(a*b*d) + (a*d + b*d)*#1 -
d*#1^2 + #1^3 & , 3]^2) + EllipticPi[a/Root[-(a*b*d) + (a*d + b*d)*#1 - d*#
1^2 + #1^3 & , 1], I*ArcSinh[Sqrt[-(x/a)]], a/b]*(d*(-2*a^2 + a*(-7*b + d)
+ b*(-2*b + d)) - d*Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 2]^2
 + Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 2]^3 + (-3*a*b + 4*(a
 + b)*Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 1] - 2*Root[-(a*b*
d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 1]^2)*Root[-(a*b*d) + (a*d + b*d)*#
1 - d*#1^2 + #1^3 & , 3] - Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 &
 , 1]*Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 3]^2 + Root[-(a*b*
d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 2]*(3*a*b + 2*(a + b)*Root[-(a*b*d)
 + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 3] - Root[-(a*b*d) + (a*d + b*d)*#1 -
 d*#1^2 + #1^3 & , 3]^2))))/(Sqrt[x/(-a + x)]*Sqrt[x*(-a + x)*(-b + x)]*(Ro
ot[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 1] - Root[-(a*b*d) + (a*d
+ b*d)*#1 - d*#1^2 + #1^3 & , 2])*(Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2
+ #1^3 & , 1] - Root[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 3])*(Roo
t[-(a*b*d) + (a*d + b*d)*#1 - d*#1^2 + #1^3 & , 2] - Root[-(a*b*d) + (a*d +
 b*d)*#1 - d*#1^2 + #1^3 & , 3]))
```

IntegrateAlgebraic [A] time = 0.55, size = 40, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{x^2(-a-b)+abx+x^3}}{x^2}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(3*a*b - 2*(a + b)*x + x^2))/(Sqrt[x*(-a + x)*(-b + x
)]*(-(a*b*d) + (a + b)*d*x - d*x^2 + x^3)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[d]*Sqrt[a*b*x + (-a - b)*x^2 + x^3])/x^2])/Sqrt[d]
```

fricas [B] time = 0.79, size = 312, normalized size = 7.80

$$\left[\frac{\log\left(\frac{a^2b^2d^2+6\,dx^5+x^6+\left(a^2+4\,ab+b^2\right)d^2x^2-\left(6\,(a+b)d-d^2\right)x^4-2\,\left(a^2b+ab^2\right)d^2x+2\,\left(3\,abd-(a+b)d^2\right)x^3-4\,\left(abdx-(a+b)dx^2+dx^3+x^4\right)\sqrt{abx-(a+b)x^2+x^3}\,\sqrt{d}}{a^2b^2d^2-2\,dx^5+x^6+\left(a^2+4\,ab+b^2\right)d^2x^2+\left(2\,(a+b)d+d^2\right)x^4-2\,\left(a^2b+ab^2\right)d^2x-2\,\left(abd+(a+b)d^2\right)x^3}\right)}{2\,\sqrt{d}},\ \frac{\sqrt{-d}\arctan\left(\frac{\left(abd-(a+b)dx+dx^2+x^3\right)\sqrt{abx-(a+b)x^2+x^3}\,\sqrt{-d}}{2\,\left(abdx^2-(a+b)dx^3+dx^4\right)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*a*b-2*(a+b)*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(-a*b*d+(a+b)*d*x
-d*x^2+x^3),x, algorithm="fricas")
```

```
[Out] [1/2*log((a^2*b^2*d^2 + 6*d*x^5 + x^6 + (a^2 + 4*a*b + b^2)*d^2*x^2 - (6*(a
 + b)*d - d^2)*x^4 - 2*(a^2*b + a*b^2)*d^2*x + 2*(3*a*b*d - (a + b)*d^2)*x^
```

```
3 - 4*(a*b*d*x - (a + b)*d*x^2 + d*x^3 + x^4)*sqrt(a*b*x - (a + b)*x^2 + x^
3)*sqrt(d))/(a^2*b^2*d^2 - 2*d*x^5 + x^6 + (a^2 + 4*a*b + b^2)*d^2*x^2 + (2
*(a + b)*d + d^2)*x^4 - 2*(a^2*b + a*b^2)*d^2*x - 2*(a*b*d + (a + b)*d^2)*x
^3))/sqrt(d), sqrt(-d)*arctan(1/2*(a*b*d - (a + b)*d*x + d*x^2 + x^3)*sqrt(
a*b*x - (a + b)*x^2 + x^3)*sqrt(-d)/(a*b*d*x^2 - (a + b)*d*x^3 + d*x^4))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(3\,ab - 2\,(a+b)x + x^2\right)x}{\left(abd - (a+b)dx + dx^2 - x^3\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*a*b-2*(a+b)*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(-a*b*d+(a+b)*d*x
-d*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate(-(3*a*b - 2*(a + b)*x + x^2)*x/((a*b*d - (a + b)*d*x + d*x^2 - x^
3)*sqrt((a - x)*(b - x)*x)), x)
```

maple [C] time = 0.06, size = 291, normalized size = 7.28

$$-\frac{2a\sqrt{-\frac{-a+x}{a}}\,\sqrt{\frac{-b+x}{a-b}}\,\sqrt{\frac{x}{a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{-a+x}{a}},\sqrt{\frac{a}{a-b}}\right)}{\sqrt{abx-a\,x^2-b\,x^2+x^3}}+\frac{2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^3-d\,\_Z^2+(ad+bd)\_Z-abd\right)}\frac{\left(-2\_\alpha^2a-2\_\alpha^2b+\_\alpha^2d+3\_\alpha ab-\_\alpha ad-\_\alpha bd+abd\right)\left(\_\alpha^2+\_\alpha a-\_\alpha d+a^2+bd\right)\sqrt{-\frac{-a+x}{a}}\,\sqrt{\frac{-b+x}{a-b}}\,\sqrt{\frac{x}{a}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-a+x}{a}},\frac{\_\alpha^2+\_\alpha a-\_\alpha d+a^2+bd}{a^2},\sqrt{\frac{a}{a-b}}\right)}{\left(-3\_\alpha^2+2\_\alpha d-ad-bd\right)\sqrt{x\left(ab-ax-bx+x^2\right)}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*a*b-2*(a+b)*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(-a*b*d+(a+b)*d*x-d*x^2
+x^3),x)
```

```
[Out] -2*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^
2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))+2/a^2*sum((-2*_al
pha^2*a-2*_alpha^2*b+_alpha^2*d+3*_alpha*a*b-_alpha*a*d-_alpha*b*d+a*b*d)/(
-3*_alpha^2+2*_alpha*d-a*d-b*d)*(_alpha^2+_alpha*a-_alpha*d+a^2+b*d)*(-(-a+
x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(x*(a*b-a*x-b*x+x^2))^(1/2)*
EllipticPi((-(-a+x)/a)^(1/2),(_alpha^2+_alpha*a-_alpha*d+a^2+b*d)/a^2,(a/(a
-b))^(1/2)),_alpha=RootOf(_Z^3-d*_Z^2+(a*d+b*d)*_Z-a*b*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(3\,ab - 2\,(a+b)x + x^2\right)x}{\left(abd - (a+b)dx + dx^2 - x^3\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*a*b-2*(a+b)*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(-a*b*d+(a+b)*d*x
-d*x^2+x^3),x, algorithm="maxima")
```

```
[Out] -integrate((3*a*b - 2*(a + b)*x + x^2)*x/((a*b*d - (a + b)*d*x + d*x^2 - x^
3)*sqrt((a - x)*(b - x)*x)), x)
```

mupad [B] time = 0.75, size = 457, normalized size = 11.42

$$\left(\sum_{k=1}^{3}\left(-\frac{2\,b\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(-\frac{b}{\mathrm{root}\left(z^3-d\,z^2+d\,z\,(a+b)-a\,b\,d,z,k\right)-b};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\left(2\,a\,\mathrm{root}\left(z^3-d\,z^2+d\,z\,(a+b)-a\,b\,d,z,k\right)^2+2\,b\,\mathrm{root}\left(z^3-d\,z^2+d\,z\,(a+b)-a\,b\,d,z,k\right)^2-d\,\mathrm{root}\left(z^3-d\,z^2+d\,z\,(a+b)-a\,b\,d,z,k\right)^2-3\,a\,b\,\mathrm{root}\left(z^3-d\,z^2+d\,z\,(a+b)-a\,b\,d,z,k\right)+a\,d\,\mathrm{root}\left(z^3-d\,z^2+d\,z\,(a+b)-a\,b\,d,z,k\right)+b\,d\,\mathrm{root}\left(z^3-d\,z^2+d\,z\,(a+b)-a\,b\,d,z,k\right)-a\,b\,d\right)}{\left(\mathrm{root}\left(z^3-d\,z^2+d\,z\,(a+b)-a\,b\,d,z,k\right)-b\right)\sqrt{x\,(a-x)\,(b-x)}\,\left(3\,\mathrm{root}\left(z^3-d\,z^2+d\,z\,(a+b)-a\,b\,d,z,k\right)^2-2\,d\,\mathrm{root}\left(z^3-d\,z^2+d\,z\,(a+b)-a\,b\,d,z,k\right)+a\,d+b\,d\right)}\right)\right)-\frac{2\,b\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}}{\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(3*a*b + x^2 - 2*x*(a + b)))/((x*(a - x)*(b - x))^(1/2)*(d*x^2 - x^
3 - d*x*(a + b) + a*b*d)),x)
```

```
[Out] symsum(-(2*b*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*elliptic
Pi(-b/(root(z^3 - d*z^2 + d*z*(a + b) - a*b*d, z, k) - b), asin(((b - x)/b)
```

```
^(1/2)), -b/(a - b))*(2*a*root(z^3 - d*z^2 + d*z*(a + b) - a*b*d, z, k)^2 +
 2*b*root(z^3 - d*z^2 + d*z*(a + b) - a*b*d, z, k)^2 - d*root(z^3 - d*z^2 +
 d*z*(a + b) - a*b*d, z, k)^2 - 3*a*b*root(z^3 - d*z^2 + d*z*(a + b) - a*b*
d, z, k) + a*d*root(z^3 - d*z^2 + d*z*(a + b) - a*b*d, z, k) + b*d*root(z^3
 - d*z^2 + d*z*(a + b) - a*b*d, z, k) - a*b*d))/((root(z^3 - d*z^2 + d*z*(a
 + b) - a*b*d, z, k) - b)*(x*(a - x)*(b - x))^(1/2)*(a*d + b*d + 3*root(z^3
 - d*z^2 + d*z*(a + b) - a*b*d, z, k)^2 - 2*d*root(z^3 - d*z^2 + d*z*(a + b
) - a*b*d, z, k))), k, 1, 3) - (2*b*ellipticF(asin(((b - x)/b)^(1/2)), -b/(
a - b))*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2))/(x^3 - x^2*(
a + b) + a*b*x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*a*b-2*(a+b)*x+x**2)/(x*(-a+x)*(-b+x))**(1/2)/(-a*b*d+(a+b)*d
*x-d*x**2+x**3),x)
```

```
[Out] Timed out
```

3.519 $\int \frac{3abx-2(a+b)x^2+x^3}{\sqrt{x(-a+x)(-b+x)}\left(-ab+(a+b)x-x^2+dx^3\right)}\,dx$

Optimal. Leaf size=40

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^2(-a-b)+abx+x^3}}{\sqrt{d}\,x^2}\right)}{\sqrt{d}}$$

Rubi [F] time = 11.96, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3abx-2(a+b)x^2+x^3}{\sqrt{x(-a+x)(-b+x)}\left(-ab+(a+b)x-x^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3*a*b*x - 2*(a + b)*x^2 + x^3)/(Sqrt[x*(-a + x)*(-b + x)]*(-(a*b) + (a
 + b)*x - x^2 + d*x^3)),x]
```

```
[Out] (2*Sqrt[b]*Sqrt[x]*Sqrt[1 - x/a]*Sqrt[1 - x/b]*EllipticF[ArcSin[Sqrt[x]/Sqr
t[b]], b/a])/(d*Sqrt[(a - x)*(b - x)*x]) + (2*(a + b - 3*a*b*d)*Sqrt[x]*Sqr
t[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][x^2/(Sqrt[-a + x^2]*Sqrt[-b
+ x^2]*(a*b - a*(1 + b/a)*x^2 + x^4 - d*x^6)), x], x, Sqrt[x]])/(d*Sqrt[(a
- x)*(b - x)*x]) - (2*(1 - 2*a*d - 2*b*d)*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]
*Defer[Subst][Defer[Int][x^4/(Sqrt[-a + x^2]*Sqrt[-b + x^2]*(a*b - a*(1 + b
/a)*x^2 + x^4 - d*x^6)), x], x, Sqrt[x]])/(d*Sqrt[(a - x)*(b - x)*x]) + (2*
a*b*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][1/(Sqrt[-a +
x^2]*Sqrt[-b + x^2]*(-(a*b) + a*(1 + b/a)*x^2 - x^4 + d*x^6)), x], x, Sqrt[
x]])/(d*Sqrt[(a - x)*(b - x)*x])
```

Rubi steps

$$\int \frac{3abx-2(a+b)x^2+x^3}{\sqrt{x(-a+x)(-b+x)}\left(-ab+(a+b)x-x^2+dx^3\right)}\,dx = \int \frac{x\left(3ab-2(a+b)x+x^2\right)}{\sqrt{x(-a+x)(-b+x)}\left(-ab+(a+b)x-x^2+dx^3\right)}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{\sqrt{x}\left(3ab-2(a+b)x+x^2\right)}{\sqrt{-a+x}\sqrt{-b+x}\left(-ab+(a+b)x-x^2+\right.}}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\mathrm{Subst}\left(\int \frac{x^2\left(3ab-2(a\right.}{\sqrt{-a+x^2}\sqrt{-b+x^2}\left(-a\right.}\right.}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\mathrm{Subst}\left(\int\left(\frac{1}{d\sqrt{-a+x^2}\sqrt{-b+x^2}}\right.\right.}{\sqrt{x(-a+x)(}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^2}\sqrt{-b+x^2}}\,dx\right.}{d\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\mathrm{Subst}\left(\int\left(\frac{(b+a(1}{\sqrt{-a+x^2}\sqrt{-b+x^2}\left(a\right.}\right.\right.}{}$$

$$= \frac{\left(2ab\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{-a+x^2}\sqrt{-b+x^2}\left(\right.}\right.}{d\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{2\sqrt{b}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{b}}\right)|\frac{b}{a}\right)}{d\sqrt{(a-x)(b-x)x}} + \frac{\left(2ab\sqrt{x}\right.}{}$$

Mathematica [C] time = 4.30, size = 1418, normalized size = 35.45

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3*a*b*x - 2*(a + b)*x^2 + x^3)/(Sqrt[x*(-a + x)*(-b + x)]*(-(a*b
) + (a + b)*x - x^2 + d*x^3)),x]
```

```
[Out] ((-2*I)*x*Sqrt[1 - x/b]*(EllipticF[I*ArcSinh[Sqrt[-(x/a)]], a/b]*(a + b - 3
*a*b*d - 2*d^2*Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 1]*Root[-(a*b)
+ (a + b)*#1 - #1^2 + d*#1^3 & , 2]^2 - 2*d^2*Root[-(a*b) + (a + b)*#1 - #1
^2 + d*#1^3 & , 1]^2*Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 3] - 2*d^
2*Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 2]*Root[-(a*b) + (a + b)*#1
- #1^2 + d*#1^3 & , 3]^2) - EllipticPi[a/Root[-(a*b) + (a + b)*#1 - #1^2 +
d*#1^3 & , 1], I*ArcSinh[Sqrt[-(x/a)]], a/b]*(a + b - 2*a^2*d - 7*a*b*d - 2
*b^2*d - d*Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 2]^2 + d^2*Root[-(a
*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 2]^3 + d^2*(-3*a*b + 4*(a + b)*Root[-(
a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 1] - 2*Root[-(a*b) + (a + b)*#1 - #1^
2 + d*#1^3 & , 1]^2)*Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 3] - d^2*
Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 1]*Root[-(a*b) + (a + b)*#1 -
#1^2 + d*#1^3 & , 3]^2 + d^2*Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 2
]*(3*a*b + 2*(a + b)*Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 3] - Root
[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 3]^2)) - d*(EllipticPi[a/Root[-(a*
b) + (a + b)*#1 - #1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[-(x/a)]], a/b]*(2*a^
2 + a*b + 2*b^2 + 6*a*b*d*Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 3] -
```

```
 2*(a + b)*d*Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 1]*Root[-(a*b) +
(a + b)*#1 - #1^2 + d*#1^3 & , 3] + d*Root[-(a*b) + (a + b)*#1 - #1^2 + d*#
1^3 & , 2]^2*(-Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 1] + Root[-(a*b
) + (a + b)*#1 - #1^2 + d*#1^3 & , 3]) + d*Root[-(a*b) + (a + b)*#1 - #1^2
+ d*#1^3 & , 2]*(3*a*b - 4*(a + b)*Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3
 & , 3])) + EllipticPi[a/Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 3], I
*ArcSinh[Sqrt[-(x/a)]], a/b]*(3*a*b - 3*a*b*d*Root[-(a*b) + (a + b)*#1 - #1
^2 + d*#1^3 & , 3] - 2*(a + b)*d*Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 &
 , 1]*Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 3] + d*Root[-(a*b) + (a
+ b)*#1 - #1^2 + d*#1^3 & , 1]*Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & ,
 3]^2 - d*Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 2]*(6*a*b - 2*(a + b
)*Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 3] + Root[-(a*b) + (a + b)*#
1 - #1^2 + d*#1^3 & , 3]^2)))))/(d^3*Sqrt[x/(-a + x)]*Sqrt[x*(-a + x)*(-b +
 x)]*(Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 1] - Root[-(a*b) + (a +
b)*#1 - #1^2 + d*#1^3 & , 2])*(Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & ,
 1] - Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & , 3])*(Root[-(a*b) + (a +
b)*#1 - #1^2 + d*#1^3 & , 2] - Root[-(a*b) + (a + b)*#1 - #1^2 + d*#1^3 & ,
 3]))
```

IntegrateAlgebraic [A] time = 0.52, size = 40, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^2(-a-b)+abx+x^3}}{\sqrt{d}\,x^2}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*a*b*x - 2*(a + b)*x^2 + x^3)/(Sqrt[x*(-a + x)*(-b + x
)]*(-(a*b) + (a + b)*x - x^2 + d*x^3)),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a*b*x + (-a - b)*x^2 + x^3]/(Sqrt[d]*x^2)])/Sqrt[d]
```

fricas [B] time = 1.76, size = 285, normalized size = 7.12

$$\left[\frac{\log\left(\frac{d^2x^6+6\,dx^5-(6\,(a+b)d-1)x^4+a^2b^2+2\,(3\,abd-a-b)x^3+\left(a^2+4\,ab+b^2\right)x^2-4\,\left(dx^4+abx-(a+b)x^2+x^3\right)\sqrt{abx-(a+b)x^2+x^3}\,\sqrt{d}-2\,\left(a^2b+ab^2\right)x}{d^2x^6-2\,dx^5+(2\,(a+b)d+1)x^4+a^2b^2-2\,(abd+a+b)x^3+\left(a^2+4\,ab+b^2\right)x^2-2\,\left(a^2b+ab^2\right)x}\right)}{2\,\sqrt{d}},\ \frac{\sqrt{-d}\,\arctan\left(\frac{\left(dx^3+ab-(a+b)x+x^2\right)\sqrt{abx-(a+b)x^2+x^3}\,\sqrt{-d}}{2\,\left(abdx^2-(a+b)dx^3+dx^4\right)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*b*x-2*(a+b)*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a*b+(a+b)*x-x
^2+d*x^3),x, algorithm="fricas")
```

```
[Out] [1/2*log((d^2*x^6 + 6*d*x^5 - (6*(a + b)*d - 1)*x^4 + a^2*b^2 + 2*(3*a*b*d
- a - b)*x^3 + (a^2 + 4*a*b + b^2)*x^2 - 4*(d*x^4 + a*b*x - (a + b)*x^2 + x
^3)*sqrt(a*b*x - (a + b)*x^2 + x^3)*sqrt(d) - 2*(a^2*b + a*b^2)*x)/(d^2*x^6
 - 2*d*x^5 + (2*(a + b)*d + 1)*x^4 + a^2*b^2 - 2*(a*b*d + a + b)*x^3 + (a^2
 + 4*a*b + b^2)*x^2 - 2*(a^2*b + a*b^2)*x))/sqrt(d), sqrt(-d)*arctan(1/2*(d
*x^3 + a*b - (a + b)*x + x^2)*sqrt(a*b*x - (a + b)*x^2 + x^3)*sqrt(-d)/(a*b
*d*x^2 - (a + b)*d*x^3 + d*x^4))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,abx - 2\,(a+b)x^2 + x^3}{\left(dx^3 - ab + (a+b)x - x^2\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*b*x-2*(a+b)*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a*b+(a+b)*x-x
^2+d*x^3),x, algorithm="giac")
```

```
[Out] integrate((3*a*b*x - 2*(a + b)*x^2 + x^3)/((d*x^3 - a*b + (a + b)*x - x^2)*
sqrt((a - x)*(b - x)*x)), x)
```

maple [C] time = 0.05, size = 296, normalized size = 7.40

$$-\frac{2a\sqrt{-\frac{-a+x}{a}}\,\sqrt{\frac{-b+x}{a-b}}\,\sqrt{\frac{x}{a}}\,\operatorname{EllipticF}\left(\sqrt{-\frac{-a+x}{a}},\sqrt{\frac{a}{a-b}}\right)}{d\sqrt{abx-a\,x^2-b\,x^2+x^3}}+\frac{2\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\,\_Z^3-\_Z^2+(a+b)\_Z-ab\right)}\frac{\left(-2\_\alpha^2ad-2\_\alpha^2bd+3\_\alpha abd+\_\alpha^2-\_\alpha a-\_\alpha b+ab\right)\left(\_\alpha^2d+\_\alpha ad+a^2d-\_\alpha+b\right)\sqrt{-\frac{-a+x}{a}}\,\sqrt{\frac{-b+x}{a-b}}\,\sqrt{\frac{x}{a}}\,\operatorname{EllipticPi}\left(\sqrt{-\frac{-a+x}{a}},\frac{\_\alpha^2d+\_\alpha ad+a^2d-\_\alpha+b}{a^2d},\sqrt{\frac{a}{a-b}}\right)}{\left(-3\_\alpha^2d+2\_\alpha-a-b\right)\sqrt{x\left(ab-ax-bx+x^2\right)}}\right)}{d^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a*b*x-2*(a+b)*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a*b+(a+b)*x-x^2+d*x
^3),x)
```

```
[Out] -2/d*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*
x^2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))+2/d^2/a^2*sum((
-2*_alpha^2*a*d-2*_alpha^2*b*d+3*_alpha*a*b*d+_alpha^2-_alpha*a-_alpha*b+a*
b)/(-3*_alpha^2*d+2*_alpha-a-b)*(_alpha^2*d+_alpha*a*d+a^2*d-_alpha+b)*(-(-
a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(x*(a*b-a*x-b*x+x^2))^(1/2
)*EllipticPi((-(-a+x)/a)^(1/2),(_alpha^2*d+_alpha*a*d+a^2*d-_alpha+b)/a^2/d
,(a/(a-b))^(1/2)),_alpha=RootOf(d*_Z^3-_Z^2+(a+b)*_Z-a*b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,abx-2\,(a+b)x^2+x^3}{\left(dx^3-ab+(a+b)x-x^2\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*b*x-2*(a+b)*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a*b+(a+b)*x-x
^2+d*x^3),x, algorithm="maxima")
```

```
[Out] integrate((3*a*b*x - 2*(a + b)*x^2 + x^3)/((d*x^3 - a*b + (a + b)*x - x^2)*
sqrt((a - x)*(b - x)*x)), x)
```

mupad [B] time = 3.31, size = 69, normalized size = 1.72

$$\frac{\ln\left(\frac{a\,b-a\,x-b\,x+d\,x^3+x^2-2\,\sqrt{d}\,x\,\sqrt{x\,(a-x)\,(b-x)}}{a\,x-a\,b+b\,x+d\,x^3-x^2}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 2*x^2*(a + b) + 3*a*b*x)/((x*(a - x)*(b - x))^(1/2)*(a*b - d*x^
3 + x^2 - x*(a + b))),x)
```

```
[Out] log((a*b - a*x - b*x + d*x^3 + x^2 - 2*d^(1/2)*x*(x*(a - x)*(b - x))^(1/2))
/(a*x - a*b + b*x + d*x^3 - x^2))/d^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*b*x-2*(a+b)*x**2+x**3)/(x*(-a+x)*(-b+x))**(1/2)/(-a*b+(a+b)*
x-x**2+d*x**3),x)
```

```
[Out] Timed out
```

3.520 $\int \frac{(-1+x^2)\sqrt{1+2x^2+x^4}}{(1+x^2)(1+x^4)}\,dx$

Optimal. Leaf size=40

$$-\frac{\sqrt{\left(x^2+1\right)^2}\tanh^{-1}\left(\frac{\sqrt{2}x}{x^2+1}\right)}{\sqrt{2}\left(x^2+1\right)}$$

Rubi [C] time = 0.66, antiderivative size = 74, normalized size of antiderivative = 1.85, number of steps used = 12, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {6725, 1147, 8, 1148, 388, 206, 203}

$$\frac{\sqrt{x^4+2x^2+1}\tanh^{-1}\left((-1)^{3/4}x\right)}{\sqrt{2}\left(x^2+1\right)}+\frac{i\sqrt{x^4+2x^2+1}\tan^{-1}\left((-1)^{3/4}x\right)}{\sqrt{2}\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^2)*Sqrt[1 + 2*x^2 + x^4])/((1 + x^2)*(1 + x^4)),x]
```

```
[Out] (I*Sqrt[1 + 2*x^2 + x^4]*ArcTan[(-1)^(3/4)*x])/(Sqrt[2]*(1 + x^2)) + (Sqrt[
1 + 2*x^2 + x^4]*ArcTanh[(-1)^(3/4)*x])/(Sqrt[2]*(1 + x^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1147

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^p/(d + e*x^2)^(2*p), Int[(d + e*x^2)^
(q + 2*p), x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
 && !IntegerQ[p] && EqQ[2*c*d - b*e, 0]
```

Rule 1148

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^
2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ
[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^2)\sqrt{1+2x^2+x^4}}{(1+x^2)(1+x^4)}\,dx &= \int \left(\frac{\sqrt{1+2x^2+x^4}}{-1-x^2}+\frac{x^2\sqrt{1+2x^2+x^4}}{1+x^4}\right)dx\\
&= \int \frac{\sqrt{1+2x^2+x^4}}{-1-x^2}\,dx+\int \frac{x^2\sqrt{1+2x^2+x^4}}{1+x^4}\,dx\\
&= \frac{\sqrt{1+2x^2+x^4}\int 1\,dx}{-1-x^2}+\int \left(-\frac{\sqrt{1+2x^2+x^4}}{2(i-x^2)}+\frac{\sqrt{1+2x^2+x^4}}{2(i+x^2)}\right)dx\\
&= -\frac{x\sqrt{1+2x^2+x^4}}{1+x^2}-\frac{1}{2}\int \frac{\sqrt{1+2x^2+x^4}}{i-x^2}\,dx+\frac{1}{2}\int \frac{\sqrt{1+2x^2+x^4}}{i+x^2}\,dx\\
&= -\frac{x\sqrt{1+2x^2+x^4}}{1+x^2}-\frac{\sqrt{1+2x^2+x^4}\int \frac{1+x^2}{i-x^2}\,dx}{2(1+x^2)}+\frac{\sqrt{1+2x^2+x^4}\int \frac{1+x^2}{i+x^2}\,dx}{2(1+x^2)}\\
&= \frac{\left(\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{1+2x^2+x^4}\right)\int \frac{1}{i+x^2}\,dx}{1+x^2}-\frac{\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{1+2x^2+x^4}\right)\int \frac{1}{i-x^2}\,dx}{1+x^2}\\
&= \frac{i\sqrt{1+2x^2+x^4}\tan^{-1}\left((-1)^{3/4}x\right)}{\sqrt{2}(1+x^2)}+\frac{\sqrt{1+2x^2+x^4}\tanh^{-1}\left((-1)^{3/4}x\right)}{\sqrt{2}(1+x^2)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.45

$$\frac{\sqrt{(x^2+1)^2}\left(\log\left(-x^2+\sqrt{2}x-1\right)-\log\left(x^2+\sqrt{2}x+1\right)\right)}{2\sqrt{2}(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)*Sqrt[1 + 2*x^2 + x^4])/((1 + x^2)*(1 + x^4)),x]
[Out] (Sqrt[(1 + x^2)^2]*(Log[-1 + Sqrt[2]*x - x^2] - Log[1 + Sqrt[2]*x + x^2]))/
(2*Sqrt[2]*(1 + x^2))
```

IntegrateAlgebraic [A] time = 0.23, size = 29, normalized size = 0.72

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+2x^2+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*Sqrt[1 + 2*x^2 + x^4])/((1 + x^2)*(1 + x^4))
,x]
[Out] -(ArcTanh[(Sqrt[2]*x)/Sqrt[1 + 2*x^2 + x^4]]/Sqrt[2])
```

fricas [A] time = 0.41, size = 34, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^4+4x^2-2\sqrt{2}(x^3+x)+1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*((x^2+1)^2)^(1/2)/(x^2+1)/(x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^4 + 4*x^2 - 2*sqrt(2)*(x^3 + x) + 1)/(x^4 + 1))
```

giac [A] time = 0.27, size = 34, normalized size = 0.85

$$-\frac{1}{4}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{4}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*((x^2+1)^2)^(1/2)/(x^2+1)/(x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/4*sqrt(2)*log(x^2 - sqrt(2)*x + 1
)
```

maple [B] time = 0.02, size = 79, normalized size = 1.98

$$-\frac{\sqrt{\left(x^2+1\right)^2}\,\sqrt{2}\,\left(\ln\left(-\frac{x^2+\sqrt{2}\,x+1}{\sqrt{2}\,x-x^2-1}\right)-\ln\left(-\frac{\sqrt{2}\,x-x^2-1}{x^2+\sqrt{2}\,x+1}\right)\right)}{8\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*((x^2+1)^2)^(1/2)/(x^2+1)/(x^4+1),x)
```

```
[Out] -1/8*((x^2+1)^2)^(1/2)*2^(1/2)*(ln(-(x^2+2^(1/2)*x+1)/(2^(1/2)*x-x^2-1))-ln
(-(2^(1/2)*x-x^2-1)/(x^2+2^(1/2)*x+1)))/(x^2+1)
```

maxima [A] time = 0.57, size = 34, normalized size = 0.85

$$-\frac{1}{4}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{4}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*((x^2+1)^2)^(1/2)/(x^2+1)/(x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/4*sqrt(2)*log(x^2 - sqrt(2)*x + 1
)
```

mupad [B] time = 0.09, size = 18, normalized size = 0.45

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{x^2+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*((x^2 + 1)^2)^(1/2))/((x^2 + 1)*(x^4 + 1)),x)
```

```
[Out] -(2^(1/2)*atanh((2^(1/2)*x)/(x^2 + 1)))/2
```

sympy [A] time = 0.12, size = 49, normalized size = 1.22

$$\frac{\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{\left(x^2+1\right)^2}\right)}{4}-\frac{\sqrt{2}\log\left(\sqrt{2}x+\sqrt{\left(x^2+1\right)^2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*((x**2+1)**2)**(1/2)/(x**2+1)/(x**4+1),x)
```

```
[Out] sqrt(2)*log(-sqrt(2)*x + sqrt((x**2 + 1)**2))/4 - sqrt(2)*log(sqrt(2)*x + s
qrt((x**2 + 1)**2))/4
```

3.521 $\int \frac{-1+x}{\sqrt{-7+4x+14x^2-12x^3+x^4}}\,dx$

Optimal. Leaf size=40

$$\log(x-1)-\log\left(-x^2+\sqrt{x^4-12x^3+14x^2+4x-7}+6x-5\right)$$

Rubi [F] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+x}{\sqrt{-7+4x+14x^2-12x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x)/Sqrt[-7 + 4*x + 14*x^2 - 12*x^3 + x^4],x]
```

```
[Out] -Defer[Int][1/Sqrt[-7 + 4*x + 14*x^2 - 12*x^3 + x^4], x] + Defer[Int][x/Sqr
t[-7 + 4*x + 14*x^2 - 12*x^3 + x^4], x]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x}{\sqrt{-7+4x+14x^2-12x^3+x^4}}\,dx &= \int \left(-\frac{1}{\sqrt{-7+4x+14x^2-12x^3+x^4}}+\frac{x}{\sqrt{-7+4x+14x^2-12x^3+x^4}}\right)\\ &= -\int \frac{1}{\sqrt{-7+4x+14x^2-12x^3+x^4}}\,dx+\int \frac{x}{\sqrt{-7+4x+14x^2-12x^3+}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.35

$$\frac{(x-1)\sqrt{x^2-10x-7}\,\log\left(-\sqrt{x^2-10x-7}-x+5\right)}{\sqrt{(x-1)^2\left(x^2-10x-7\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/Sqrt[-7 + 4*x + 14*x^2 - 12*x^3 + x^4],x]
```

```
[Out] ((-1 + x)*Sqrt[-7 - 10*x + x^2]*Log[5 - x - Sqrt[-7 - 10*x + x^2]])/Sqrt[(-
1 + x)^2*(-7 - 10*x + x^2)]
```

IntegrateAlgebraic [A] time = 0.15, size = 40, normalized size = 1.00

$$\log(x-1)-\log\left(-x^2+\sqrt{x^4-12x^3+14x^2+4x-7}+6x-5\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/Sqrt[-7 + 4*x + 14*x^2 - 12*x^3 + x^4],x]
```

```
[Out] Log[-1 + x] - Log[-5 + 6*x - x^2 + Sqrt[-7 + 4*x + 14*x^2 - 12*x^3 + x^4]]
```

fricas [A] time = 0.40, size = 40, normalized size = 1.00

$$-\log\left(-\frac{x^2-6\,x-\sqrt{x^4-12\,x^3+14\,x^2+4\,x-7}+5}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^4-12*x^3+14*x^2+4*x-7)^(1/2),x, algorithm="fricas")
[Out] -log(-(x^2 - 6*x - sqrt(x^4 - 12*x^3 + 14*x^2 + 4*x - 7) + 5)/(x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^4-12*x^3+14*x^2+4*x-7)^(1/2),x, algorithm="giac")
[Out] undef
```

maple [A] time = 0.00, size = 49, normalized size = 1.22

$$\frac{(-1+x)\sqrt{x^2-10x-7}\ \ln\left(x-5+\sqrt{x^2-10x-7}\right)}{\sqrt{x^4-12x^3+14x^2+4x-7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(x^4-12*x^3+14*x^2+4*x-7)^(1/2),x)
[Out] 1/(x^4-12*x^3+14*x^2+4*x-7)^(1/2)*(-1+x)*(x^2-10*x-7)^(1/2)*ln(x-5+(x^2-10*
x-7)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x^4-12\,x^3+14\,x^2+4\,x-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^4-12*x^3+14*x^2+4*x-7)^(1/2),x, algorithm="maxima")
[Out] integrate((x - 1)/sqrt(x^4 - 12*x^3 + 14*x^2 + 4*x - 7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x-1}{\sqrt{x^4-12\,x^3+14\,x^2+4\,x-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(4*x + 14*x^2 - 12*x^3 + x^4 - 7)^(1/2),x)
[Out] int((x - 1)/(4*x + 14*x^2 - 12*x^3 + x^4 - 7)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{(x-1)^2\left(x^2-10x-7\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**4-12*x**3+14*x**2+4*x-7)**(1/2),x)
[Out] Integral((x - 1)/sqrt((x - 1)**2*(x**2 - 10*x - 7)), x)
```

3.522 $\int \frac{1-2k^2x^2+k^2x^4}{x^2\sqrt{(1-x^2)(1-k^2x^2)}\,(-1+k^2x^2)}\,dx$

Optimal. Leaf size=40

$$-\frac{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}}{x\left(k^2x^2-1\right)}$$

Rubi [C] time = 1.71, antiderivative size = 451, normalized size of antiderivative = 11.28, number of steps used = 14, number of rules used = 10, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6719, 21, 6742, 414, 424, 472, 583, 524, 419, 471}

$$\frac{k^2x\left(1-x^2\right)}{\left(1-k^2\right)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}+\frac{k^2\left(1-x^2\right)}{\left(1-k^2\right)x\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}+\frac{\left(1-2k^2\right)\left(1-x^2\right)\left(1-k^2x^2\right)}{\left(1-k^2\right)x\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}-\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}+\frac{2k^2\sqrt{1-x^2}\sqrt{1-k^2x^2}E\left(\sin^{-1}(x)|k^2\right)}{\left(1-k^2\right)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}-\frac{k\sqrt{1-x^2}\sqrt{1-k^2x^2}E\left(\sin^{-1}(kx)|\frac{1}{k^2}\right)}{\left(1-k^2\right)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}+\frac{\left(1-2k^2\right)\sqrt{1-x^2}\sqrt{1-k^2x^2}E\left(\sin^{-1}(x)|k^2\right)}{\left(1-k^2\right)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}-\frac{2k^4x\left(1-x^2\right)}{\left(1-k^2\right)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*k^2*x^2 + k^2*x^4)/(x^2*Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(-1 + k^2*
x^2)),x]
```

```
[Out] (k^2*(1 - x^2))/((1 - k^2)*x*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) + (k^2*x*(1 - x
^2))/((1 - k^2)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) - (2*k^4*x*(1 - x^2))/((1 -
k^2)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) + ((1 - 2*k^2)*(1 - x^2)*(1 - k^2*x^2))
/((1 - k^2)*x*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) + (2*k^2*Sqrt[1 - x^2]*Sqrt[1
- k^2*x^2]*EllipticE[ArcSin[x], k^2])/((1 - k^2)*Sqrt[(1 - x^2)*(1 - k^2*x^
2)]) + ((1 - 2*k^2)*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticE[ArcSin[x], k^
2])/((1 - k^2)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) - (k*Sqrt[1 - x^2]*Sqrt[1 - k
^2*x^2]*EllipticE[ArcSin[k*x], k^(-2)])/((1 - k^2)*Sqrt[(1 - x^2)*(1 - k^2*
x^2)]) - (Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticF[ArcSin[x], k^2])/Sqrt[(
1 - x^2)*(1 - k^2*x^2)]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1-2k^2x^2+k^2x^4}{x^2\sqrt{(1-x^2)(1-k^2x^2)}\,(-1+k^2x^2)}\,dx = \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1-2k^2x^2+k^2x^4}{x^2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,(-1+k^2x^2)}\,dx}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

$$= -\frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1-2k^2x^2+k^2x^4}{x^2\sqrt{1-x^2}\,(1-k^2x^2)^{3/2}}\,dx}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

$$= -\frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \left(-\frac{2k^2}{\sqrt{1-x^2}\,(1-k^2x^2)^{3/2}} + \frac{1}{x^2\sqrt{1-x^2}\,(1-k^2x^2)^{3/2}} + \right.}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

$$= -\frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1}{x^2\sqrt{1-x^2}\,(1-k^2x^2)^{3/2}}\,dx}{\sqrt{(1-x^2)(1-k^2x^2)}} - \frac{\left(k^2\sqrt{1-x^2}\,\sqrt{1-\right.}{\sqrt{(1-}}$$

$$= \frac{k^2(1-x^2)}{(1-k^2)\,x\sqrt{(1-x^2)(1-k^2x^2)}} + \frac{k^2x(1-x^2)}{(1-k^2)\sqrt{(1-x^2)(1-k^2x^2)}}$$

$$= \frac{k^2(1-x^2)}{(1-k^2)\,x\sqrt{(1-x^2)(1-k^2x^2)}} + \frac{k^2x(1-x^2)}{(1-k^2)\sqrt{(1-x^2)(1-k^2x^2)}}$$

$$= \frac{k^2(1-x^2)}{(1-k^2)\,x\sqrt{(1-x^2)(1-k^2x^2)}} + \frac{k^2x(1-x^2)}{(1-k^2)\sqrt{(1-x^2)(1-k^2x^2)}}$$

$$= \frac{k^2(1-x^2)}{(1-k^2)\,x\sqrt{(1-x^2)(1-k^2x^2)}} + \frac{k^2x(1-x^2)}{(1-k^2)\sqrt{(1-x^2)(1-k^2x^2)}}$$

Mathematica [A] time = 0.14, size = 30, normalized size = 0.75

$$\frac{1-x^2}{x\sqrt{(x^2-1)(k^2x^2-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*k^2*x^2 + k^2*x^4)/(x^2*Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(-1
+ k^2*x^2)),x]
```

```
[Out] (1 - x^2)/(x*Sqrt[(-1 + x^2)*(-1 + k^2*x^2)])
```

IntegrateAlgebraic [A] time = 5.67, size = 40, normalized size = 1.00

$$-\frac{\sqrt{k^2x^4+(-k^2-1)\,x^2+1}}{x\,(k^2x^2-1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - 2*k^2*x^2 + k^2*x^4)/(x^2*Sqrt[(1 - x^2)*(1 - k^2*x
^2)]*(-1 + k^2*x^2)),x]
```

```
[Out] -(Sqrt[1 + (-1 - k^2)*x^2 + k^2*x^4]/(x*(-1 + k^2*x^2)))
```

fricas [A] time = 0.41, size = 36, normalized size = 0.90

$$-\frac{\sqrt{k^2x^4-(k^2+1)x^2+1}}{k^2x^3-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^4-2*k^2*x^2+1)/x^2/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^2-1),x, algorithm="fricas")
```

```
[Out] -sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)/(k^2*x^3 - x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^4-2k^2x^2+1}{(k^2x^2-1)\sqrt{(k^2x^2-1)(x^2-1)}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^4-2*k^2*x^2+1)/x^2/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^2-1),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^4 - 2*k^2*x^2 + 1)/((k^2*x^2 - 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))*x^2), x)
```

maple [A] time = 0.01, size = 29, normalized size = 0.72

$$-\frac{(-1+x)(1+x)}{\sqrt{(x^2-1)(k^2x^2-1)}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^4-2*k^2*x^2+1)/x^2/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^2-1),x)
```

```
[Out] -(-1+x)*(1+x)/((x^2-1)*(k^2*x^2-1))^(1/2)/x
```

maxima [A] time = 0.70, size = 34, normalized size = 0.85

$$-\frac{x^2-1}{\sqrt{kx+1}\,\sqrt{kx-1}\,\sqrt{x+1}\,\sqrt{x-1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^4-2*k^2*x^2+1)/x^2/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^2-1),x, algorithm="maxima")
```

```
[Out] -(x^2 - 1)/(sqrt(k*x + 1)*sqrt(k*x - 1)*sqrt(x + 1)*sqrt(x - 1)*x)
```

mupad [B] time = 0.33, size = 33, normalized size = 0.82

$$-\frac{\sqrt{(x^2-1)\,(k^2\,x^2-1)}}{x\,(k^2\,x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^4 - 2*k^2*x^2 + 1)/(x^2*(k^2*x^2 - 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)),x)
```

```
[Out] -((x^2 - 1)*(k^2*x^2 - 1))^(1/2)/(x*(k^2*x^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^4 - 2k^2x^2 + 1}{x^2\sqrt{(x-1)(x+1)(kx-1)(kx+1)}\ (kx-1)(kx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k**2*x**4-2*k**2*x**2+1)/x**2/((-x**2+1)*(-k**2*x**2+1))**(1/2)/
(k**2*x**2-1),x)
```

```
[Out] Integral((k**2*x**4 - 2*k**2*x**2 + 1)/(x**2*sqrt((x - 1)*(x + 1)*(k*x - 1)
*(k*x + 1))*(k*x - 1)*(k*x + 1)), x)
```

3.523 $\int \frac{-2+x^6}{x\sqrt{-1+x^6}\left(2+x^6\right)}\,dx$

Optimal. Leaf size=40

$$\frac{2\tan^{-1}\left(\frac{\sqrt{x^6-1}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {573, 156, 63, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{x^6-1}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^6)/(x*Sqrt[-1 + x^6]*(2 + x^6)),x]
```

```
[Out] -1/3*ArcTan[Sqrt[-1 + x^6]] + (2*ArcTan[Sqrt[-1 + x^6]/Sqrt[3]])/(3*Sqrt[3]
)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rubi steps

$$\int \frac{-2+x^6}{x\sqrt{-1+x^6}\,(2+x^6)}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{-2+x}{\sqrt{-1+x}\,x(2+x)}\,dx, x, x^6\right)$$
$$= -\left(\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right)\right) + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,(2+x)}\,dx, x, x^6\right)$$
$$= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^6}\right)\right) + \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, \sqrt{-1+x^6}\right)$$
$$= -\frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right) + \frac{2\tan^{-1}\left(\frac{\sqrt{-1+x^6}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{x^6-1}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^6)/(x*Sqrt[-1 + x^6]*(2 + x^6)),x]
[Out] -1/3*ArcTan[Sqrt[-1 + x^6]] + (2*ArcTan[Sqrt[-1 + x^6]/Sqrt[3]])/(3*Sqrt[3]
)
```

IntegrateAlgebraic [A] time = 0.04, size = 40, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{x^6-1}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x^6)/(x*Sqrt[-1 + x^6]*(2 + x^6)),x]
[Out] -1/3*ArcTan[Sqrt[-1 + x^6]] + (2*ArcTan[Sqrt[-1 + x^6]/Sqrt[3]])/(3*Sqrt[3]
)
```

fricas [A] time = 0.42, size = 29, normalized size = 0.72

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^6-1}\right) - \frac{1}{3}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)/x/(x^6-1)^(1/2)/(x^6+2),x, algorithm="fricas")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(x^6 - 1)) - 1/3*arctan(sqrt(x^6 - 1))
```

giac [A] time = 0.27, size = 29, normalized size = 0.72

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^6-1}\right) - \frac{1}{3}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)/x/(x^6-1)^(1/2)/(x^6+2),x, algorithm="giac")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(x^6 - 1)) - 1/3*arctan(sqrt(x^6 - 1))
```

maple [C] time = 0.95, size = 76, normalized size = 1.90

$$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^2+1\right)+\sqrt{x^6-1}}{x^3}\right)}{3}-\frac{\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+3\right)x^6+6\sqrt{x^6-1}-4\operatorname{RootOf}\left(\_Z^2+3\right)}{x^6+2}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-2)/x/(x^6-1)^(1/2)/(x^6+2),x)
```

```
[Out] 1/3*RootOf(_Z^2+1)*ln((-RootOf(_Z^2+1)+(x^6-1)^(1/2))/x^3)-1/9*RootOf(_Z^2+
3)*ln((RootOf(_Z^2+3)*x^6+6*(x^6-1)^(1/2)-4*RootOf(_Z^2+3))/(x^6+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6-2}{\left(x^6+2\right)\sqrt{x^6-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)/x/(x^6-1)^(1/2)/(x^6+2),x, algorithm="maxima")
```

```
[Out] integrate((x^6 - 2)/((x^6 + 2)*sqrt(x^6 - 1)*x), x)
```

mupad [B] time = 0.39, size = 29, normalized size = 0.72

$$\frac{2\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\,\sqrt{x^6-1}}{3}\right)}{9}-\frac{\operatorname{atan}\left(\sqrt{x^6-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 2)/(x*(x^6 - 1)^(1/2)*(x^6 + 2)),x)
```

```
[Out] (2*3^(1/2)*atan((3^(1/2)*(x^6 - 1)^(1/2))/3))/9 - atan((x^6 - 1)^(1/2))/3
```

sympy [A] time = 19.09, size = 36, normalized size = 0.90

$$\frac{2\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\,\sqrt{x^6-1}}{3}\right)}{9}-\frac{\operatorname{atan}\left(\sqrt{x^6-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-2)/x/(x**6-1)**(1/2)/(x**6+2),x)
```

```
[Out] 2*sqrt(3)*atan(sqrt(3)*sqrt(x**6 - 1)/3)/9 - atan(sqrt(x**6 - 1))/3
```

3.524 $\int \frac{(2+x^3)(1+x^3+x^6)}{x\sqrt{1+x^3}} \, dx$

Optimal. Leaf size=40

$$\frac{2}{45}\sqrt{x^3+1}\left(3x^6+11x^3+23\right)-\frac{4}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Rubi [A] time = 0.10, antiderivative size = 54, normalized size of antiderivative = 1.35, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1821, 1612, 63, 207}

$$\frac{2}{15}\left(x^3+1\right)^{5/2}+\frac{2}{9}\left(x^3+1\right)^{3/2}+\frac{2\sqrt{x^3+1}}{3}-\frac{4}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x^3)*(1 + x^3 + x^6))/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[1 + x^3])/3 + (2*(1 + x^3)^(3/2))/9 + (2*(1 + x^3)^(5/2))/15 - (4*A
rcTanh[Sqrt[1 + x^3]])/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+x^3)(1+x^3+x^6)}{x\sqrt{1+x^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(2+x)(1+x+x^2)}{x\sqrt{1+x}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{\sqrt{1+x}} + \frac{2}{x\sqrt{1+x}} + \sqrt{1+x} + (1+x)^{3/2}\right)dx, x, x^3\right)\\ &= \frac{2\sqrt{1+x^3}}{3} + \frac{2}{9}\left(1+x^3\right)^{3/2} + \frac{2}{15}\left(1+x^3\right)^{5/2} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)\\ &= \frac{2\sqrt{1+x^3}}{3} + \frac{2}{9}\left(1+x^3\right)^{3/2} + \frac{2}{15}\left(1+x^3\right)^{5/2} + \frac{4}{3}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)\\ &= \frac{2\sqrt{1+x^3}}{3} + \frac{2}{9}\left(1+x^3\right)^{3/2} + \frac{2}{15}\left(1+x^3\right)^{5/2} - \frac{4}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 40, normalized size = 1.00

$$\frac{2}{45}\sqrt{x^3+1}\left(3x^6+11x^3+23\right) - \frac{4}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x^3)*(1 + x^3 + x^6))/(x*Sqrt[1 + x^3]),x]
[Out] (2*Sqrt[1 + x^3]*(23 + 11*x^3 + 3*x^6))/45 - (4*ArcTanh[Sqrt[1 + x^3]])/3
```

IntegrateAlgebraic [A] time = 0.03, size = 40, normalized size = 1.00

$$\frac{2}{45}\sqrt{x^3+1}\left(3x^6+11x^3+23\right) - \frac{4}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^3)*(1 + x^3 + x^6))/(x*Sqrt[1 + x^3]),x]
[Out] (2*Sqrt[1 + x^3]*(23 + 11*x^3 + 3*x^6))/45 - (4*ArcTanh[Sqrt[1 + x^3]])/3
```

fricas [A] time = 0.40, size = 46, normalized size = 1.15

$$\frac{2}{45}\left(3\,x^6+11\,x^3+23\right)\sqrt{x^3+1} - \frac{2}{3}\,\log\left(\sqrt{x^3+1}+1\right) + \frac{2}{3}\,\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^6+x^3+1)/x/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] 2/45*(3*x^6 + 11*x^3 + 23)*sqrt(x^3 + 1) - 2/3*log(sqrt(x^3 + 1) + 1) + 2/3
*log(sqrt(x^3 + 1) - 1)
```

giac [A] time = 0.34, size = 53, normalized size = 1.32

$$\frac{2}{15}\left(x^3+1\right)^{\frac{5}{2}} + \frac{2}{9}\left(x^3+1\right)^{\frac{3}{2}} + \frac{2}{3}\sqrt{x^3+1} - \frac{2}{3}\,\log\left(\sqrt{x^3+1}+1\right) + \frac{2}{3}\,\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^6+x^3+1)/x/(x^3+1)^(1/2),x, algorithm="giac")
[Out] 2/15*(x^3 + 1)^(5/2) + 2/9*(x^3 + 1)^(3/2) + 2/3*sqrt(x^3 + 1) - 2/3*log(sq
rt(x^3 + 1) + 1) + 2/3*log(abs(sqrt(x^3 + 1) - 1))
```

maple [A] time = 0.04, size = 45, normalized size = 1.12

$$\frac{2x^6\sqrt{x^3+1}}{15}+\frac{22x^3\sqrt{x^3+1}}{45}+\frac{46\sqrt{x^3+1}}{45}-\frac{4\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2)*(x^6+x^3+1)/x/(x^3+1)^(1/2),x)
```

```
[Out] 2/15*x^6*(x^3+1)^(1/2)+22/45*x^3*(x^3+1)^(1/2)+46/45*(x^3+1)^(1/2)-4/3*arct
anh((x^3+1)^(1/2))
```

maxima [A] time = 0.46, size = 52, normalized size = 1.30

$$\frac{2}{15}\left(x^3+1\right)^{\frac{5}{2}}+\frac{2}{9}\left(x^3+1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{x^3+1}-\frac{2}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{2}{3}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^6+x^3+1)/x/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(x^3 + 1)^(5/2) + 2/9*(x^3 + 1)^(3/2) + 2/3*sqrt(x^3 + 1) - 2/3*log(sq
rt(x^3 + 1) + 1) + 2/3*log(sqrt(x^3 + 1) - 1)
```

mupad [B] time = 0.20, size = 198, normalized size = 4.95

$$\frac{46\sqrt{x^3+1}}{45}+\frac{22\,x^3\sqrt{x^3+1}}{45}+\frac{2\,x^6\sqrt{x^3+1}}{15}-\frac{4\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 2)*(x^3 + x^6 + 1))/(x*(x^3 + 1)^(1/2)),x)
```

```
[Out] (46*(x^3 + 1)^(1/2))/45 + (22*x^3*(x^3 + 1)^(1/2))/45 + (2*x^6*(x^3 + 1)^(1
/2))/15 - (4*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2
- x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, a
sin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1
/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
+ 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [A] time = 27.31, size = 68, normalized size = 1.70

$$\frac{2\left(x^3+1\right)^{\frac{5}{2}}}{15}+\frac{2\left(x^3+1\right)^{\frac{3}{2}}}{9}+\frac{2\sqrt{x^3+1}}{3}+\frac{2\log\left(-1+\frac{1}{\sqrt{x^3+1}}\right)}{3}-\frac{2\log\left(1+\frac{1}{\sqrt{x^3+1}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2)*(x**6+x**3+1)/x/(x**3+1)**(1/2),x)
```

```
[Out] 2*(x**3 + 1)**(5/2)/15 + 2*(x**3 + 1)**(3/2)/9 + 2*sqrt(x**3 + 1)/3 + 2*log
(-1 + 1/sqrt(x**3 + 1))/3 - 2*log(1 + 1/sqrt(x**3 + 1))/3
```

3.525 $\int \frac{4x+3x^2}{\sqrt{-5+4x^2+2x^3+4x^4+4x^5+x^6}}\,dx$

Optimal. Leaf size=40

$$\log\left(x^3+2x^2+\sqrt{x^6+4x^5+4x^4+2x^3+4x^2-5}+1\right)$$

Rubi [F] time = 0.42, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{4x+3x^2}{\sqrt{-5+4x^2+2x^3+4x^4+4x^5+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(4*x + 3*x^2)/Sqrt[-5 + 4*x^2 + 2*x^3 + 4*x^4 + 4*x^5 + x^6],x]
```

```
[Out] 4*Defer[Int][x/Sqrt[-5 + 4*x^2 + 2*x^3 + 4*x^4 + 4*x^5 + x^6], x] + 3*Defer
[Int][x^2/Sqrt[-5 + 4*x^2 + 2*x^3 + 4*x^4 + 4*x^5 + x^6], x]
```

Rubi steps

$$\begin{aligned}\int \frac{4x+3x^2}{\sqrt{-5+4x^2+2x^3+4x^4+4x^5+x^6}}\,dx &= \int \frac{x(4+3x)}{\sqrt{-5+4x^2+2x^3+4x^4+4x^5+x^6}}\,dx\\ &= \int \left(\frac{4x}{\sqrt{-5+4x^2+2x^3+4x^4+4x^5+x^6}}+\frac{3x^2}{\sqrt{-5+4x^2+2x^3+4x^4+}}\right.\\ &= 3\int \frac{x^2}{\sqrt{-5+4x^2+2x^3+4x^4+4x^5+x^6}}\,dx+4\int \frac{}{\sqrt{-5+4x^2+2x^3}}\end{aligned}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{4x+3x^2}{\sqrt{-5+4x^2+2x^3+4x^4+4x^5+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(4*x + 3*x^2)/Sqrt[-5 + 4*x^2 + 2*x^3 + 4*x^4 + 4*x^5 + x^6],x]
```

```
[Out] Integrate[(4*x + 3*x^2)/Sqrt[-5 + 4*x^2 + 2*x^3 + 4*x^4 + 4*x^5 + x^6], x]
```

IntegrateAlgebraic [A] time = 0.17, size = 40, normalized size = 1.00

$$\log\left(x^3+2x^2+\sqrt{x^6+4x^5+4x^4+2x^3+4x^2-5}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4*x + 3*x^2)/Sqrt[-5 + 4*x^2 + 2*x^3 + 4*x^4 + 4*x^5 +
x^6],x]
```

```
[Out] Log[1 + 2*x^2 + x^3 + Sqrt[-5 + 4*x^2 + 2*x^3 + 4*x^4 + 4*x^5 + x^6]]
```

fricas [A] time = 0.46, size = 38, normalized size = 0.95

$$\log\left(x^3+2\,x^2+\sqrt{x^6+4\,x^5+4\,x^4+2\,x^3+4\,x^2-5}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x)/(x^6+4*x^5+4*x^4+2*x^3+4*x^2-5)^(1/2),x, algorithm="f
ricas")
```

```
[Out] log(x^3 + 2*x^2 + sqrt(x^6 + 4*x^5 + 4*x^4 + 2*x^3 + 4*x^2 - 5) + 1)
```

giac [A] time = 0.35, size = 41, normalized size = 1.02

$$-\log\left(\left|-x^3-2x^2+\sqrt{2x^3+\left(x^3+2x^2\right)^2+4x^2-5}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x)/(x^6+4*x^5+4*x^4+2*x^3+4*x^2-5)^(1/2),x, algorithm="g
iac")
```

```
[Out] -log(abs(-x^3 - 2*x^2 + sqrt(2*x^3 + (x^3 + 2*x^2)^2 + 4*x^2 - 5) - 1))
```

maple [A] time = 0.20, size = 39, normalized size = 0.98

$$\ln\left(1+2x^2+x^3+\sqrt{x^6+4x^5+4x^4+2x^3+4x^2-5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+4*x)/(x^6+4*x^5+4*x^4+2*x^3+4*x^2-5)^(1/2),x)
```

```
[Out] ln(1+2*x^2+x^3+(x^6+4*x^5+4*x^4+2*x^3+4*x^2-5)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+4x}{\sqrt{x^6+4x^5+4x^4+2x^3+4x^2-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x)/(x^6+4*x^5+4*x^4+2*x^3+4*x^2-5)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((3*x^2 + 4*x)/sqrt(x^6 + 4*x^5 + 4*x^4 + 2*x^3 + 4*x^2 - 5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{3x^2+4x}{\sqrt{x^6+4x^5+4x^4+2x^3+4x^2-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 3*x^2)/(4*x^2 + 2*x^3 + 4*x^4 + 4*x^5 + x^6 - 5)^(1/2),x)
```

```
[Out] int((4*x + 3*x^2)/(4*x^2 + 2*x^3 + 4*x^4 + 4*x^5 + x^6 - 5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(3x+4)}{\sqrt{x^6+4x^5+4x^4+2x^3+4x^2-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+4*x)/(x**6+4*x**5+4*x**4+2*x**3+4*x**2-5)**(1/2),x)
```

```
[Out] Integral(x*(3*x + 4)/sqrt(x**6 + 4*x**5 + 4*x**4 + 2*x**3 + 4*x**2 - 5), x)
```

3.526 $\int \frac{x^2\sqrt{q+px^5}\left(-2q+3px^5\right)}{bx^6+a\left(q+px^5\right)^3}\,dx$

Optimal. Leaf size=40

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}\left(px^5+q\right)^{3/2}}\right)}{3\sqrt{a}\,\sqrt{b}}$$

Rubi [A] time = 0.67, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6714, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\left(px^5+q\right)^{3/2}}{\sqrt{b}\,x^3}\right)}{3\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(b*x^6 + a*(q + p*x^5)^3),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*(q + p*x^5)^(3/2))/(Sqrt[b]*x^3)])/(3*Sqrt[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 6714

```
Int[(u_)*(v_)^(r_.)*(w_)^(s_.)*((a_.)*(v_)^(p_.) + (b_.)*(w_)^(q_.))^(m_.),
 x_Symbol] :> With[{c = Simplify[u/(p*w*D[v, x] - q*v*D[w, x])]}, -Dist[(c*
q)/(s + 1), Subst[Int[(a + b*x^(q/(s + 1)))^m, x], x, v^(m*p + r + 1)*w^(s
+ 1)], x] /; FreeQ[c, x]] /; FreeQ[{a, b, m, p, q, r, s}, x] && EqQ[p*(s +
1) + q*(m*p + r + 1), 0] && NeQ[s, -1] && IntegerQ[q/(s + 1)] && IntegerQ[m
]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{q+px^5}\left(-2q+3px^5\right)}{bx^6+a\left(q+px^5\right)^3}\,dx &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \frac{\left(q+px^5\right)^{3/2}}{x^3}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\left(q+px^5\right)^{3/2}}{\sqrt{b}\,x^3}\right)}{3\sqrt{a}\,\sqrt{b}}\end{aligned}$$

Mathematica [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{q+px^5}\left(-2q+3px^5\right)}{bx^6+a\left(q+px^5\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(b*x^6 + a*(q + p*x^5)^3),
x]
```

```
[Out] Integrate[(x^2*Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(b*x^6 + a*(q + p*x^5)^3),
 x]
```

IntegrateAlgebraic [A] time = 3.12, size = 40, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}\left(px^5+q\right)^{3/2}}\right)}{3\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(b*x^6 + a*(q + p
*x^5)^3),x]
```

```
[Out] (-2*ArcTan[(Sqrt[b]*x^3)/(Sqrt[a]*(q + p*x^5)^(3/2))])/(3*Sqrt[a]*Sqrt[b])
```

fricas [B] time = 1.16, size = 465, normalized size = 11.62

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{a^2p^6x^{30}+6a^2p^5qx^{25}+15a^2p^4q^2x^{20}-6abp^3x^{21}+20a^2p^3q^3x^{15}-18abp^2qx^{16}+15a^2p^2q^4x^{10}-18abpq^2x^{11}+b^2x^{12}+6a^2pq^5x^5-6abq^3x^6+a^2q^6-4\left(ap^4x^{23}+4ap^3qx^{18}+6ap^2q^2x^{13}-bpx^{14}+4apq^3x^8-bqx^9+aq^4x^3\right)\sqrt{px^5+q}\sqrt{-ab}}{a^2p^6x^{30}+6a^2p^5qx^{25}+15a^2p^4q^2x^{20}+2abp^3x^{21}+20a^2p^3q^3x^{15}+6abp^2qx^{16}+15a^2p^2q^4x^{10}+6abpq^2x^{11}+b^2x^{12}+6a^2pq^5x^5+2abq^3x^6+a^2q^6}\right)}{6ab}, \frac{\sqrt{ab}\arctan\left(\frac{\left(ap^3x^{15}+3ap^2qx^{10}+3apq^2x^5-bx^6+aq^3\right)\sqrt{px^5+q}\sqrt{ab}}{2\left(abp^2x^{13}+2abpqx^8+abq^2x^3\right)}\right)}{3ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(p*x^5+q)^(1/2)*(3*p*x^5-2*q)/(b*x^6+a*(p*x^5+q)^3),x, algori
thm="fricas")
```

```
[Out] [-1/6*sqrt(-a*b)*log((a^2*p^6*x^30 + 6*a^2*p^5*q*x^25 + 15*a^2*p^4*q^2*x^20
 - 6*a*b*p^3*x^21 + 20*a^2*p^3*q^3*x^15 - 18*a*b*p^2*q*x^16 + 15*a^2*p^2*q^
4*x^10 - 18*a*b*p*q^2*x^11 + b^2*x^12 + 6*a^2*p*q^5*x^5 - 6*a*b*q^3*x^6 + a
^2*q^6 - 4*(a*p^4*x^23 + 4*a*p^3*q*x^18 + 6*a*p^2*q^2*x^13 - b*p*x^14 + 4*a
*p*q^3*x^8 - b*q*x^9 + a*q^4*x^3)*sqrt(p*x^5 + q)*sqrt(-a*b))/(a^2*p^6*x^30
 + 6*a^2*p^5*q*x^25 + 15*a^2*p^4*q^2*x^20 + 2*a*b*p^3*x^21 + 20*a^2*p^3*q^3
*x^15 + 6*a*b*p^2*q*x^16 + 15*a^2*p^2*q^4*x^10 + 6*a*b*p*q^2*x^11 + b^2*x^1
2 + 6*a^2*p*q^5*x^5 + 2*a*b*q^3*x^6 + a^2*q^6))/(a*b), 1/3*sqrt(a*b)*arctan
(1/2*(a*p^3*x^15 + 3*a*p^2*q*x^10 + 3*a*p*q^2*x^5 - b*x^6 + a*q^3)*sqrt(p*x
^5 + q)*sqrt(a*b)/(a*b*p^2*x^13 + 2*a*b*p*q*x^8 + a*b*q^2*x^3))/(a*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,px^5-2\,q\right)\sqrt{px^5+q}\,x^2}{bx^6+\left(px^5+q\right)^3a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(p*x^5+q)^(1/2)*(3*p*x^5-2*q)/(b*x^6+a*(p*x^5+q)^3),x, algori
thm="giac")
```

```
[Out] integrate((3*p*x^5 - 2*q)*sqrt(p*x^5 + q)*x^2/(b*x^6 + (p*x^5 + q)^3*a), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{p\,x^5+q}\,\left(3p\,x^5-2q\right)}{b\,x^6+a\left(p\,x^5+q\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(p*x^5+q)^(1/2)*(3*p*x^5-2*q)/(b*x^6+a*(p*x^5+q)^3),x)
```

```
[Out] int(x^2*(p*x^5+q)^(1/2)*(3*p*x^5-2*q)/(b*x^6+a*(p*x^5+q)^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,px^5 - 2\,q\right)\sqrt{px^5 + q}\,x^2}{bx^6 + \left(px^5 + q\right)^3 a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(p*x^5+q)^(1/2)*(3*p*x^5-2*q)/(b*x^6+a*(p*x^5+q)^3),x, algori
thm="maxima")
```

```
[Out] integrate((3*p*x^5 - 2*q)*sqrt(p*x^5 + q)*x^2/(b*x^6 + (p*x^5 + q)^3*a), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(q + p*x^5)^(1/2)*(2*q - 3*p*x^5))/(a*(q + p*x^5)^3 + b*x^6),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{px^5 + q}\,\left(3px^5 - 2q\right)}{ap^3x^{15} + 3ap^2qx^{10} + 3apq^2x^5 + aq^3 + bx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(p*x**5+q)**(1/2)*(3*p*x**5-2*q)/(b*x**6+a*(p*x**5+q)**3),x)
```

```
[Out] Integral(x**2*sqrt(p*x**5 + q)*(3*p*x**5 - 2*q)/(a*p**3*x**15 + 3*a*p**2*q*
x**10 + 3*a*p*q**2*x**5 + a*q**3 + b*x**6), x)
```

3.527 $\int \frac{x^8}{\sqrt{-b+ax^3}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{ax^3-b}\left(3a^2x^6+4abx^3+8b^2\right)}{45a^3}$$

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.59, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {266, 43}

$$\frac{2b^2\sqrt{ax^3-b}}{3a^3}+\frac{2\left(ax^3-b\right)^{5/2}}{15a^3}+\frac{4b\left(ax^3-b\right)^{3/2}}{9a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[-b + a*x^3],x]
```

```
[Out] (2*b^2*Sqrt[-b + a*x^3])/(3*a^3) + (4*b*(-b + a*x^3)^(3/2))/(9*a^3) + (2*(-
b + a*x^3)^(5/2))/(15*a^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\sqrt{-b+ax^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{-b+ax}}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{b^2}{a^2\sqrt{-b+ax}}+\frac{2b\sqrt{-b+ax}}{a^2}+\frac{(-b+ax)^{3/2}}{a^2}\right)dx,x,x^3\right)\\
&= \frac{2b^2\sqrt{-b+ax^3}}{3a^3}+\frac{4b\left(-b+ax^3\right)^{3/2}}{9a^3}+\frac{2\left(-b+ax^3\right)^{5/2}}{15a^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.00

$$\frac{2\sqrt{ax^3-b}\left(3a^2x^6+4abx^3+8b^2\right)}{45a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[-b + a*x^3],x]
```

```
[Out] (2*Sqrt[-b + a*x^3]*(8*b^2 + 4*a*b*x^3 + 3*a^2*x^6))/(45*a^3)
```

IntegrateAlgebraic [A] time = 0.03, size = 41, normalized size = 1.00

$$\frac{2\sqrt{ax^3-b}\left(3a^2x^6+4abx^3+8b^2\right)}{45a^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/Sqrt[-b + a*x^3],x]
[Out] (2*Sqrt[-b + a*x^3]*(8*b^2 + 4*a*b*x^3 + 3*a^2*x^6))/(45*a^3)
```

fricas [A] time = 0.41, size = 37, normalized size = 0.90

$$\frac{2\left(3\,a^2x^6+4\,abx^3+8\,b^2\right)\sqrt{ax^3-b}}{45\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a*x^3-b)^(1/2),x, algorithm="fricas")
[Out] 2/45*(3*a^2*x^6 + 4*a*b*x^3 + 8*b^2)*sqrt(a*x^3 - b)/a^3
```

giac [A] time = 0.30, size = 53, normalized size = 1.29

$$\frac{2\sqrt{ax^3-b}\,b^2}{3\,a^3}+\frac{2\left(3\left(ax^3-b\right)^{\frac{5}{2}}+10\left(ax^3-b\right)^{\frac{3}{2}}b\right)}{45\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a*x^3-b)^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt(a*x^3 - b)*b^2/a^3 + 2/45*(3*(a*x^3 - b)^(5/2) + 10*(a*x^3 - b)^(3
/2)*b)/a^3
```

maple [A] time = 0.01, size = 38, normalized size = 0.93

$$\frac{2\sqrt{a\,x^3-b}\;\left(3a^2x^6+4ab\,x^3+8b^2\right)}{45a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*x^3-b)^(1/2),x)
[Out] 2/45*(a*x^3-b)^(1/2)*(3*a^2*x^6+4*a*b*x^3+8*b^2)/a^3
```

maxima [A] time = 0.46, size = 53, normalized size = 1.29

$$\frac{2\left(ax^3-b\right)^{\frac{5}{2}}}{15\,a^3}+\frac{4\left(ax^3-b\right)^{\frac{3}{2}}b}{9\,a^3}+\frac{2\sqrt{ax^3-b}\,b^2}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a*x^3-b)^(1/2),x, algorithm="maxima")
[Out] 2/15*(a*x^3 - b)^(5/2)/a^3 + 4/9*(a*x^3 - b)^(3/2)*b/a^3 + 2/3*sqrt(a*x^3 -
 b)*b^2/a^3
```

mupad [B] time = 0.62, size = 37, normalized size = 0.90

$$\frac{2\sqrt{a\,x^3-b}\;\left(3\,a^2\,x^6+4\,a\,b\,x^3+8\,b^2\right)}{45\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*x^3 - b)^(1/2),x)
[Out] (2*(a*x^3 - b)^(1/2)*(8*b^2 + 3*a^2*x^6 + 4*a*b*x^3))/(45*a^3)
```

sympy [A] time = 1.25, size = 71, normalized size = 1.73

$$\begin{cases} \frac{2x^6\sqrt{ax^3-b}}{15a} + \frac{8bx^3\sqrt{ax^3-b}}{45a^2} + \frac{16b^2\sqrt{ax^3-b}}{45a^3} & \text{for } a \neq 0 \\ \frac{x^9}{9\sqrt{-b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(a*x**3-b)**(1/2),x)
```

```
[Out] Piecewise((2*x**6*sqrt(a*x**3 - b)/(15*a) + 8*b*x**3*sqrt(a*x**3 - b)/(45*a
**2) + 16*b**2*sqrt(a*x**3 - b)/(45*a**3), Ne(a, 0)), (x**9/(9*sqrt(-b)), T
rue))
```

3.528 $\int x^5 \sqrt{-b+ax^3}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{ax^3-b}\left(3a^2x^6-abx^3-2b^2\right)}{45a^2}$$

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {266, 43}

$$\frac{2\left(ax^3-b\right)^{5/2}}{15a^2}+\frac{2b\left(ax^3-b\right)^{3/2}}{9a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[-b + a*x^3],x]
```

```
[Out] (2*b*(-b + a*x^3)^(3/2))/(9*a^2) + (2*(-b + a*x^3)^(5/2))/(15*a^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{-b+ax^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int x\sqrt{-b+ax}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(\frac{b\sqrt{-b+ax}}{a}+\frac{(-b+ax)^{3/2}}{a}\right)dx,x,x^3\right)\\ &= \frac{2b\left(-b+ax^3\right)^{3/2}}{9a^2}+\frac{2\left(-b+ax^3\right)^{5/2}}{15a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.73

$$\frac{2\left(ax^3-b\right)^{3/2}\left(3ax^3+2b\right)}{45a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[-b + a*x^3],x]
```

```
[Out] (2*(-b + a*x^3)^(3/2)*(2*b + 3*a*x^3))/(45*a^2)
```

IntegrateAlgebraic [A] time = 0.03, size = 41, normalized size = 1.00

$$\frac{2\sqrt{ax^3-b}\left(3a^2x^6-abx^3-2b^2\right)}{45a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[-b + a*x^3],x]
[Out] (2*Sqrt[-b + a*x^3]*(-2*b^2 - a*b*x^3 + 3*a^2*x^6))/(45*a^2)
```

fricas [A] time = 0.39, size = 37, normalized size = 0.90

$$\frac{2\left(3\,a^{2}x^{6}-abx^{3}-2\,b^{2}\right)\sqrt{ax^{3}-b}}{45\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a*x^3-b)^(1/2),x, algorithm="fricas")
[Out] 2/45*(3*a^2*x^6 - a*b*x^3 - 2*b^2)*sqrt(a*x^3 - b)/a^2
```

giac [A] time = 0.34, size = 33, normalized size = 0.80

$$\frac{2\left(3\left(ax^{3}-b\right)^{\frac{5}{2}}+5\left(ax^{3}-b\right)^{\frac{3}{2}}b\right)}{45\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a*x^3-b)^(1/2),x, algorithm="giac")
[Out] 2/45*(3*(a*x^3 - b)^(5/2) + 5*(a*x^3 - b)^(3/2)*b)/a^2
```

maple [A] time = 0.00, size = 27, normalized size = 0.66

$$\frac{2\left(a\,x^{3}-b\right)^{\frac{3}{2}}\left(3a\,x^{3}+2b\right)}{45a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a*x^3-b)^(1/2),x)
[Out] 2/45*(a*x^3-b)^(3/2)*(3*a*x^3+2*b)/a^2
```

maxima [A] time = 0.40, size = 34, normalized size = 0.83

$$\frac{2\left(ax^{3}-b\right)^{\frac{5}{2}}}{15\,a^{2}}+\frac{2\left(ax^{3}-b\right)^{\frac{3}{2}}b}{9\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a*x^3-b)^(1/2),x, algorithm="maxima")
[Out] 2/15*(a*x^3 - b)^(5/2)/a^2 + 2/9*(a*x^3 - b)^(3/2)*b/a^2
```

mupad [B] time = 0.55, size = 33, normalized size = 0.80

$$\frac{6\left(a\,x^{3}-b\right)^{5/2}+10\,b\left(a\,x^{3}-b\right)^{3/2}}{45\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a*x^3 - b)^(1/2),x)
[Out] (6*(a*x^3 - b)^(5/2) + 10*b*(a*x^3 - b)^(3/2))/(45*a^2)
```

sympy [**A**] time = 0.48, size = 68, normalized size = 1.66

$$\begin{cases} \frac{2x^6\sqrt{ax^3-b}}{15} - \frac{2bx^3\sqrt{ax^3-b}}{45a} - \frac{4b^2\sqrt{ax^3-b}}{45a^2} & \text{for } a \neq 0 \\ \frac{x^6\sqrt{-b}}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a*x**3-b)**(1/2),x)
```

```
[Out] Piecewise((2*x**6*sqrt(a*x**3 - b)/15 - 2*b*x**3*sqrt(a*x**3 - b)/(45*a) -
4*b**2*sqrt(a*x**3 - b)/(45*a**2), Ne(a, 0)), (x**6*sqrt(-b)/6, True))
```

3.529 $\int \frac{2b+ax^3}{\sqrt{-b+ax^3}\left(-2b-3x^2+2ax^3\right)}\,dx$

Optimal. Leaf size=41

$$\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x\sqrt{ax^3-b}}{b-ax^3}\right)$$

Rubi [F] time = 1.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2b+ax^3}{\sqrt{-b+ax^3}\left(-2b-3x^2+2ax^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2*b + a*x^3)/(Sqrt[-b + a*x^3]*(-2*b - 3*x^2 + 2*a*x^3)),x]
```

```
[Out] -((Sqrt[2 - Sqrt[3]]*(b^(1/3) - a^(1/3)*x)*Sqrt[(b^(2/3) + a^(1/3)*b^(1/3)*
x + a^(2/3)*x^2)/((1 - Sqrt[3])*b^(1/3) - a^(1/3)*x)^2]*EllipticF[ArcSin[((
1 + Sqrt[3])*b^(1/3) - a^(1/3)*x)/((1 - Sqrt[3])*b^(1/3) - a^(1/3)*x)], -7
+ 4*Sqrt[3]])/(3^(1/4)*a^(1/3)*Sqrt[-((b^(1/3)*(b^(1/3) - a^(1/3)*x))/((1 -
 Sqrt[3])*b^(1/3) - a^(1/3)*x)^2)]*Sqrt[-b + a*x^3])) - 3*b*Defer[Int][1/((
2*b + 3*x^2 - 2*a*x^3)*Sqrt[-b + a*x^3]), x] + (3*Defer[Int][x^2/(Sqrt[-b +
 a*x^3]*(-2*b - 3*x^2 + 2*a*x^3)), x])/2
```

Rubi steps

$$\begin{aligned}
\int \frac{2b+ax^3}{\sqrt{-b+ax^3}\left(-2b-3x^2+2ax^3\right)}\,dx &= \int \left(\frac{1}{2\sqrt{-b+ax^3}}+\frac{3\left(2b+x^2\right)}{2\sqrt{-b+ax^3}\left(-2b-3x^2+2ax^3\right)}\right)dx\\
&= \frac{1}{2}\int \frac{1}{\sqrt{-b+ax^3}}\,dx+\frac{3}{2}\int \frac{2b+x^2}{\sqrt{-b+ax^3}\left(-2b-3x^2+2ax^3\right)}\,dx\\
&= -\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)\sqrt{\frac{b^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\sqrt[3]{a}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}-\sqrt[3]{a}x}{\left(1-\sqrt{3}\right)\sqrt[3]{b}-\sqrt[3]{a}x}\right)\right.}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{-\frac{\sqrt[3]{b}\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\sqrt[3]{a}x\right)^2}}\sqrt{-b+ax^3}}\\
&= -\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)\sqrt{\frac{b^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\sqrt[3]{a}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}-\sqrt[3]{a}x}{\left(1-\sqrt{3}\right)\sqrt[3]{b}-\sqrt[3]{a}x}\right)\right.}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{-\frac{\sqrt[3]{b}\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\sqrt[3]{a}x\right)^2}}\sqrt{-b+ax^3}}
\end{aligned}$$

Mathematica [C] time = 6.33, size = 2865, normalized size = 69.88

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*b + a*x^3)/(Sqrt[-b + a*x^3]*(-2*b - 3*x^2 + 2*a*x^3)),x]
```

```
[Out] (Sqrt[(-(b^(1/3)/a^(1/3)) + x)/(-(b^(1/3)/a^(1/3)) - ((-1)^(1/3)*b^(1/3))/a
^(1/3))]*(((-1)^(1/3)*b^(1/3))/a^(1/3) + x)*Sqrt[(-(((-1)^(2/3)*b^(1/3))/a^
```

```
(1/3)) + x)/(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3)
)]*EllipticF[ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) + a^(1/3)*x)/(((-1)^(1/3) + (-
1)^(2/3))*b^(1/3))]], (-1)^(1/3)])/(Sqrt[(((-1)^(1/3)*b^(1/3))/a^(1/3) + x)
/(((-1)^(1/3)*b^(1/3))/a^(1/3) + ((-1)^(2/3)*b^(1/3))/a^(1/3))]*Sqrt[-b + a
*x^3]) + (2*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3)
)*b*Sqrt[(-(b^(1/3)/a^(1/3)) + x)/(-(b^(1/3)/a^(1/3)) - ((-1)^(1/3)*b^(1/3)
)/a^(1/3))]*Sqrt[((((-1)^(2/3)*b^(1/3))/a^(1/3) - x)*(((-1)^(1/3)*b^(1/3))/
a^(1/3) + x))/(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/
3))^2]*EllipticPi[((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(
1/3) + a^(1/3)*Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 1]), ArcSin[Sqrt[((-1)^(1/
3)*b^(1/3) + a^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)])
/(a*Sqrt[-b + a*x^3]*(((-1)^(1/3)*b^(1/3))/a^(1/3) + Root[-2*b - 3*#1^2 + 2
*a*#1^3 & , 1])*(Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 1] - Root[-2*b - 3*#1^2
+ 2*a*#1^3 & , 2])*(Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 1] - Root[-2*b - 3*#1
^2 + 2*a*#1^3 & , 3])) + ((-(((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^
(1/3))/a^(1/3))*Sqrt[(-(b^(1/3)/a^(1/3)) + x)/(-(b^(1/3)/a^(1/3)) - ((-1)^(
1/3)*b^(1/3))/a^(1/3))]*Sqrt[((((-1)^(2/3)*b^(1/3))/a^(1/3) - x)*(((-1)^(1/
3)*b^(1/3))/a^(1/3) + x))/(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^
(1/3))/a^(1/3))^2]*EllipticPi[((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-
1)^(1/3)*b^(1/3) + a^(1/3)*Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 1]), ArcSin[Sq
rt[((-1)^(1/3)*b^(1/3) + a^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]],
(-1)^(1/3)]*Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 1]^3)/(Sqrt[-b + a*x^3]*(((-1
)^(1/3)*b^(1/3))/a^(1/3) + Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 1])*(Root[-2*b
 - 3*#1^2 + 2*a*#1^3 & , 1] - Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 2])*(Root[-
2*b - 3*#1^2 + 2*a*#1^3 & , 1] - Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 3])) + (
2*(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))*b*Sqrt[(
-(b^(1/3)/a^(1/3)) + x)/(-(b^(1/3)/a^(1/3)) - ((-1)^(1/3)*b^(1/3))/a^(1/3))
]*Sqrt[((((-1)^(2/3)*b^(1/3))/a^(1/3) - x)*(((-1)^(1/3)*b^(1/3))/a^(1/3) +
x))/(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*Ell
ipticPi[((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) + a^(
1/3)*Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 2]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3)
 + a^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)])/(a*Sqrt[-
b + a*x^3]*(((-1)^(1/3)*b^(1/3))/a^(1/3) + Root[-2*b - 3*#1^2 + 2*a*#1^3 &
, 2])*(-Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 1] + Root[-2*b - 3*#1^2 + 2*a*#1^
3 & , 2])*(Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 2] - Root[-2*b - 3*#1^2 + 2*a*
#1^3 & , 3])) + ((-(((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^
(1/3))*Sqrt[(-(b^(1/3)/a^(1/3)) + x)/(-(b^(1/3)/a^(1/3)) - ((-1)^(1/3)*b^(1
/3))/a^(1/3))]*Sqrt[((((-1)^(2/3)*b^(1/3))/a^(1/3) - x)*(((-1)^(1/3)*b^(1/3
))/a^(1/3) + x))/(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^
(1/3))^2]*EllipticPi[((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*
b^(1/3) + a^(1/3)*Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 2]), ArcSin[Sqrt[((-1)^
(1/3)*b^(1/3) + a^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3
)]*Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 2]^3)/(Sqrt[-b + a*x^3]*(((-1)^(1/3)*b
^(1/3))/a^(1/3) + Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 2])*(-Root[-2*b - 3*#1^
2 + 2*a*#1^3 & , 1] + Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 2])*(Root[-2*b - 3*
#1^2 + 2*a*#1^3 & , 2] - Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 3])) + (2*(-(((-
1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))*b*Sqrt[(-(b^(1/3
)/a^(1/3)) + x)/(-(b^(1/3)/a^(1/3)) - ((-1)^(1/3)*b^(1/3))/a^(1/3))]*Sqrt[(
(((-1)^(2/3)*b^(1/3))/a^(1/3) - x)*(((-1)^(1/3)*b^(1/3))/a^(1/3) + x))/(-((
(-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))^2]*EllipticPi[
((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3) + a^(1/3)*Roo
t[-2*b - 3*#1^2 + 2*a*#1^3 & , 3]), ArcSin[Sqrt[((-1)^(1/3)*b^(1/3) + a^(1/
3)*x)/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)])/(a*Sqrt[-b + a*x^
3]*(((-1)^(1/3)*b^(1/3))/a^(1/3) + Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 3])*(-
Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 1] + Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 3]
)*(-Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 2] + Root[-2*b - 3*#1^2 + 2*a*#1^3 &
, 3])) + ((-(((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))*
Sqrt[(-(b^(1/3)/a^(1/3)) + x)/(-(b^(1/3)/a^(1/3)) - ((-1)^(1/3)*b^(1/3))/a^
(1/3))]*Sqrt[((((-1)^(2/3)*b^(1/3))/a^(1/3) - x)*(((-1)^(1/3)*b^(1/3))/a^(1
```

```
/3) + x))/(-(((-1)^(1/3)*b^(1/3))/a^(1/3)) - ((-1)^(2/3)*b^(1/3))/a^(1/3))^
2]*EllipticPi[((-1)^(1/3)*b^(1/3) + (-1)^(2/3)*b^(1/3))/((-1)^(1/3)*b^(1/3)
 + a^(1/3)*Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 3]), ArcSin[Sqrt[((-1)^(1/3)*b
^(1/3) + a^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3))*b^(1/3))]], (-1)^(1/3)]*Root
[-2*b - 3*#1^2 + 2*a*#1^3 & , 3]^3)/(Sqrt[-b + a*x^3]*(((-1)^(1/3)*b^(1/3))
/a^(1/3) + Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 3])*(-Root[-2*b - 3*#1^2 + 2*a
*#1^3 & , 1] + Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 3])*(-Root[-2*b - 3*#1^2 +
 2*a*#1^3 & , 2] + Root[-2*b - 3*#1^2 + 2*a*#1^3 & , 3]))
```

IntegrateAlgebraic [A] time = 0.59, size = 41, normalized size = 1.00

$$\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x\sqrt{ax^3-b}}{b-ax^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*b + a*x^3)/(Sqrt[-b + a*x^3]*(-2*b - 3*x^2 + 2*a*x^3)
),x]
```

```
[Out] Sqrt[2/3]*ArcTanh[(Sqrt[3/2]*x*Sqrt[-b + a*x^3])/(b - a*x^3)]
```

fricas [B] time = 0.44, size = 123, normalized size = 3.00

$$\frac{1}{12}\sqrt{3}\sqrt{2}\log\left(\frac{4a^2x^6+36ax^5-8abx^3+9x^4-36bx^2-4\sqrt{3}\sqrt{2}\left(2ax^4+3x^3-2bx\right)\sqrt{ax^3-b}+4b^2}{4a^2x^6-12ax^5-8abx^3+9x^4+12bx^2+4b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b)/(a*x^3-b)^(1/2)/(2*a*x^3-3*x^2-2*b),x, algorithm="fri
cas")
```

```
[Out] 1/12*sqrt(3)*sqrt(2)*log((4*a^2*x^6 + 36*a*x^5 - 8*a*b*x^3 + 9*x^4 - 36*b*x
^2 - 4*sqrt(3)*sqrt(2)*(2*a*x^4 + 3*x^3 - 2*b*x)*sqrt(a*x^3 - b) + 4*b^2)/(
4*a^2*x^6 - 12*a*x^5 - 8*a*b*x^3 + 9*x^4 + 12*b*x^2 + 4*b^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3+2b}{\left(2ax^3-3x^2-2b\right)\sqrt{ax^3-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b)/(a*x^3-b)^(1/2)/(2*a*x^3-3*x^2-2*b),x, algorithm="gia
c")
```

```
[Out] integrate((a*x^3 + 2*b)/((2*a*x^3 - 3*x^2 - 2*b)*sqrt(a*x^3 - b)), x)
```

maple [C] time = 0.18, size = 790, normalized size = 19.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+2*b)/(a*x^3-b)^(1/2)/(2*a*x^3-3*x^2-2*b),x)
```

```
[Out] 1/3*I*3^(1/2)/a*(a^2*b)^(1/3)*(-I*(x+1/2/a*(a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(a
^2*b)^(1/3))*3^(1/2)*a/(a^2*b)^(1/3))^(1/2)*((x-1/a*(a^2*b)^(1/3))/(-3/2/a*
(a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(a^2*b)^(1/3)))^(1/2)*(I*(x+1/2/a*(a^2*b)^(1/
3)-1/2*I*3^(1/2)/a*(a^2*b)^(1/3))*3^(1/2)*a/(a^2*b)^(1/3))^(1/2)/(a*x^3-b)^
(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/a*(a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(a^2
*b)^(1/3))*3^(1/2)*a/(a^2*b)^(1/3))^(1/2),(-I*3^(1/2)/a*(a^2*b)^(1/3)/(-3/2
/a*(a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(a^2*b)^(1/3)))^(1/2))+1/12*I/a^2/b*2^(1/2
```

```
)*sum((-_alpha^2-2*b)/_alpha/(_alpha*a-1)*(a^2*b)^(1/3)*(-1/2*I*a*(2*x+1/a*
(I*3^(1/2)*(a^2*b)^(1/3)+(a^2*b)^(1/3)))/(a^2*b)^(1/3))^(1/2)*(a*(x-1/a*(a^
2*b)^(1/3))/(-3*(a^2*b)^(1/3)-I*3^(1/2)*(a^2*b)^(1/3)))^(1/2)*(1/2*I*a*(2*x
+1/a*(-I*3^(1/2)*(a^2*b)^(1/3)+(a^2*b)^(1/3)))/(a^2*b)^(1/3))^(1/2)/(a*x^3-
b)^(1/2)*(-2*I*(a^2*b)^(1/3)*3^(1/2)*_alpha^2*a^2+2*I*(a^2*b)^(2/3)*3^(1/2)
*_alpha*a+3*I*(a^2*b)^(1/3)*3^(1/2)*_alpha*a-2*(a^2*b)^(1/3)*_alpha^2*a^2-3
*I*(a^2*b)^(2/3)*3^(1/2)-2*_alpha*(a^2*b)^(2/3)*a+3*(a^2*b)^(1/3)*_alpha*a+
4*a^2*b+3*(a^2*b)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/a*(a^2*b)^(1/3)+
1/2*I*3^(1/2)/a*(a^2*b)^(1/3))*3^(1/2)*a/(a^2*b)^(1/3))^(1/2),-1/6/a*(2*I*(
a^2*b)^(2/3)*3^(1/2)*_alpha^2*a+2*I*3^(1/2)*_alpha*a^2*b-3*I*(a^2*b)^(2/3)*
3^(1/2)*_alpha-6*(a^2*b)^(2/3)*_alpha^2*a-4*I*(a^2*b)^(1/3)*3^(1/2)*a*b-3*I
*3^(1/2)*a*b+6*_alpha*a^2*b+9*(a^2*b)^(2/3)*_alpha-9*a*b)/b,(-I*3^(1/2)/a*(
a^2*b)^(1/3)/(-3/2/a*(a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(a^2*b)^(1/3)))^(1/2)),_
alpha=RootOf(2*_Z^3*a-3*_Z^2-2*b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 + 2\,b}{\left(2\,ax^3 - 3\,x^2 - 2\,b\right)\sqrt{ax^3 - b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b)/(a*x^3-b)^(1/2)/(2*a*x^3-3*x^2-2*b),x, algorithm="max
ima")
```

```
[Out] integrate((a*x^3 + 2*b)/((2*a*x^3 - 3*x^2 - 2*b)*sqrt(a*x^3 - b)), x)
```

mupad [B] time = 1.64, size = 56, normalized size = 1.37

$$\frac{\sqrt{6}\,\ln\left(\frac{2\,b-2\,a\,x^3-3\,x^2+2\,\sqrt{6}\,x\,\sqrt{a\,x^3-b}}{-2\,a\,x^3+3\,x^2+2\,b}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*b + a*x^3)/((a*x^3 - b)^(1/2)*(2*b - 2*a*x^3 + 3*x^2)),x)
```

```
[Out] (6^(1/2)*log((2*b - 2*a*x^3 - 3*x^2 + 2*6^(1/2)*x*(a*x^3 - b)^(1/2))/(2*b -
 2*a*x^3 + 3*x^2)))/6
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+2*b)/(a*x**3-b)**(1/2)/(2*a*x**3-3*x**2-2*b),x)
```

```
[Out] Timed out
```

3.530 $\int \frac{(-1+x^4)\sqrt{1+x^4}\,(1+x^2+x^4)}{x^4(1-x^2+x^4)}\,dx$

Optimal. Leaf size=41

$$\frac{\sqrt{x^4+1}\,\left(x^4+6x^2+1\right)}{3x^3}-2\tanh^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)$$

Rubi [C] time = 1.19, antiderivative size = 334, normalized size of antiderivative = 8.15, number of steps used = 26, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6728, 195, 220, 277, 305, 1196, 1209, 1198, 1217, 1707}

$$\frac{1}{3}\sqrt{x^4+1}\,x+\frac{2\sqrt{x^4+1}}{x}-2\tanh^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)+\frac{\sqrt{x^4+1}}{3x^3}-\frac{\left(\sqrt{3}+i\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{\left(\sqrt{3}+3i\right)\sqrt{x^4+1}}+\frac{\left(3+i\sqrt{3}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{x^4+1}}+\frac{\left(3-i\sqrt{3}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{\left(-\sqrt{3}+i\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{\left(-\sqrt{3}+3i\right)\sqrt{x^4+1}}-\frac{2\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4)*Sqrt[1 + x^4]*(1 + x^2 + x^4))/(x^4*(1 - x^2 + x^4)),x]
```

```
[Out] Sqrt[1 + x^4]/(3*x^3) + (2*Sqrt[1 + x^4])/x + (x*Sqrt[1 + x^4])/3 - 2*ArcTa
nh[x/Sqrt[1 + x^4]] - (2*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*
ArcTan[x], 1/2])/Sqrt[1 + x^4] - ((I - Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1
 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/((3*I - Sqrt[3])*Sqrt[1 + x^4]) + (
(3 - I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x]
, 1/2])/(2*Sqrt[1 + x^4]) + ((3 + I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 +
x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(2*Sqrt[1 + x^4]) - ((I + Sqrt[3])*(1
+ x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/((3*I + Sqr
t[3])*Sqrt[1 + x^4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
```

```
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(-1+x^4\right)\sqrt{1+x^4}\left(1+x^2+x^4\right)}{x^4\left(1-x^2+x^4\right)}\,dx = \int\left(\sqrt{1+x^4}-\frac{\sqrt{1+x^4}}{x^4}-\frac{2\sqrt{1+x^4}}{x^2}+\frac{2\left(-1+2x^2\right)\sqrt{1+x^4}}{1-x^2+x^4}\right)dx$$

$$= -\left(2\int\frac{\sqrt{1+x^4}}{x^2}\,dx\right)+2\int\frac{\left(-1+2x^2\right)\sqrt{1+x^4}}{1-x^2+x^4}\,dx+\int\sqrt{1+x^4}\,dx$$

$$= \frac{\sqrt{1+x^4}}{3x^3}+\frac{2\sqrt{1+x^4}}{x}+\frac{1}{3}x\sqrt{1+x^4}+2\int\left(\frac{2\sqrt{1+x^4}}{-1-i\sqrt{3}+2x^2}+\frac{2}{-1+\ldots}\right.$$

$$= \frac{\sqrt{1+x^4}}{3x^3}+\frac{2\sqrt{1+x^4}}{x}+\frac{1}{3}x\sqrt{1+x^4}-4\int\frac{1}{\sqrt{1+x^4}}\,dx+4\int\frac{1-\ldots}{\sqrt{1+\ldots}}$$

$$= \frac{\sqrt{1+x^4}}{3x^3}+\frac{2\sqrt{1+x^4}}{x}+\frac{1}{3}x\sqrt{1+x^4}-\frac{4x\sqrt{1+x^4}}{1+x^2}+\frac{4\left(1+x^2\right)\sqrt{\frac{1\ldots}{(1+\ldots}}}{\sqrt{\ldots}}$$

$$= \frac{\sqrt{1+x^4}}{3x^3}+\frac{2\sqrt{1+x^4}}{x}+\frac{1}{3}x\sqrt{1+x^4}-\frac{4x\sqrt{1+x^4}}{1+x^2}+\frac{4\left(1+x^2\right)\sqrt{\frac{1\ldots}{(1+\ldots}}}{\sqrt{\ldots}}$$

$$= \frac{\sqrt{1+x^4}}{3x^3}+\frac{2\sqrt{1+x^4}}{x}+\frac{1}{3}x\sqrt{1+x^4}-\frac{4x\sqrt{1+x^4}}{1+x^2}-2\tanh^{-1}\left(\frac{x}{\sqrt{1+\ldots}}\right.$$

Mathematica [C] time = 0.48, size = 145, normalized size = 3.54

$$\frac{x^8+6x^6+2x^4+6x^2-6\sqrt[4]{-1}\sqrt{x^4+1}\,x^3F\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)+6\sqrt[4]{-1}\sqrt{x^4+1}\,x^3\Pi\left(\sqrt[6]{-1};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)+6\sqrt[4]{-1}\sqrt{x^4+1}\,x^3\Pi\left((-1)^{5/6};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)+1}{3x^3\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)*Sqrt[1 + x^4]*(1 + x^2 + x^4))/(x^4*(1 - x^2 + x^4)),
x]
```

```
[Out] (1 + 6*x^2 + 2*x^4 + 6*x^6 + x^8 - 6*(-1)^(1/4)*x^3*Sqrt[1 + x^4]*EllipticF
[I*ArcSinh[(-1)^(1/4)*x], -1] + 6*(-1)^(1/4)*x^3*Sqrt[1 + x^4]*EllipticPi[(
-1)^(1/6), I*ArcSinh[(-1)^(1/4)*x], -1] + 6*(-1)^(1/4)*x^3*Sqrt[1 + x^4]*El
lipticPi[(-1)^(5/6), I*ArcSinh[(-1)^(1/4)*x], -1])/(3*x^3*Sqrt[1 + x^4])
```

IntegrateAlgebraic [A] time = 0.69, size = 41, normalized size = 1.00

$$\frac{\sqrt{x^4+1}\left(x^4+6x^2+1\right)}{3x^3}-2\tanh^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*Sqrt[1 + x^4]*(1 + x^2 + x^4))/(x^4*(1 - x^2
 + x^4)),x]
```

```
[Out] (Sqrt[1 + x^4]*(1 + 6*x^2 + x^4))/(3*x^3) - 2*ArcTanh[x/Sqrt[1 + x^4]]
```

fricas [A] time = 0.45, size = 62, normalized size = 1.51

$$\frac{3\,x^3\log\left(-\frac{x^4+x^2-2\,\sqrt{x^4+1}\,x+1}{x^4-x^2+1}\right)+\left(x^4+6\,x^2+1\right)\sqrt{x^4+1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/2)*(x^4+x^2+1)/x^4/(x^4-x^2+1),x, algorithm="f
ricas")
```

```
[Out] 1/3*(3*x^3*log(-(x^4 + x^2 - 2*sqrt(x^4 + 1)*x + 1)/(x^4 - x^2 + 1)) + (x^4
 + 6*x^2 + 1)*sqrt(x^4 + 1))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4 + x^2 + 1)\sqrt{x^4 + 1}\,(x^4 - 1)}{(x^4 - x^2 + 1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/2)*(x^4+x^2+1)/x^4/(x^4-x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate((x^4 + x^2 + 1)*sqrt(x^4 + 1)*(x^4 - 1)/((x^4 - x^2 + 1)*x^4), x)
```

maple [C] time = 0.08, size = 205, normalized size = 5.00

$$\frac{\sqrt{x^4+1}\,x}{3} + \frac{2\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right), i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}} + \frac{\sqrt{x^4+1}}{3x^3} + \frac{\left(\sum_{\_\alpha=\operatorname{RootOf}(\_Z^4-\_Z^2+1)} \_\alpha\left(-\frac{\operatorname{arctanh}\left(\frac{\_\alpha^2(-\_\alpha^2+x^2+1)}{\sqrt{\_\alpha^2}\,\sqrt{x^4+1}}\right)}{\sqrt{\_\alpha^2}} + \frac{2(-1)^{\frac{3}{4}}(-\_\alpha^3+\_\alpha)\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticPi}\left((-1)^{\frac{1}{4}}x, i\_\alpha^2-i, i\right)}{\sqrt{x^4+1}}\right)\right)}{2} + \frac{2\sqrt{x^4+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^4+1)^(1/2)*(x^4+x^2+1)/x^4/(x^4-x^2+1),x)
```

```
[Out] 1/3*(x^4+1)^(1/2)*x+2/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)
^(1/2)/(x^4+1)^(1/2)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+1/3*(x^4+1)
^(1/2)/x^3+1/2*sum(_alpha*(-1/(_alpha^2)^(1/2)*arctanh(_alpha^2*(-_alpha^2+
x^2+1)/(_alpha^2)^(1/2)/(x^4+1)^(1/2))+2*(-1)^(3/4)*(-_alpha^3+_alpha)*(1-I
*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I*_alpha^
2-I,I)),_alpha=RootOf(_Z^4-_Z^2+1))+2*(x^4+1)^(1/2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4 + x^2 + 1)\sqrt{x^4 + 1}\,(x^4 - 1)}{(x^4 - x^2 + 1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/2)*(x^4+x^2+1)/x^4/(x^4-x^2+1),x, algorithm="m
axima")
```

```
[Out] integrate((x^4 + x^2 + 1)*sqrt(x^4 + 1)*(x^4 - 1)/((x^4 - x^2 + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x^4 - 1)\,\sqrt{x^4 + 1}\,\left(x^4 + x^2 + 1\right)}{x^4\,\left(x^4 - x^2 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(x^4 + 1)^(1/2)*(x^2 + x^4 + 1))/(x^4*(x^4 - x^2 + 1)),x)
```

```
[Out] int(((x^4 - 1)*(x^4 + 1)^(1/2)*(x^2 + x^4 + 1))/(x^4*(x^4 - x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2+1\right)\sqrt{x^4+1}\,\left(x^2-x+1\right)\left(x^2+x+1\right)}{x^4\left(x^4-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**4+1)**(1/2)*(x**4+x**2+1)/x**4/(x**4-x**2+1),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)*sqrt(x**4 + 1)*(x**2 - x + 1)*(x**2 + x
 + 1)/(x**4*(x**4 - x**2 + 1)), x)
```

3.531 $\int \frac{4c+3bx+2ax^2}{\sqrt[4]{c+bx+ax^2}\left(-c-bx-ax^2+x^4\right)} dx$

Optimal. Leaf size=41

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^2+bx+c}}{x}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt[4]{ax^2+bx+c}}\right)$$

Rubi [F] time = 1.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{4c+3bx+2ax^2}{\sqrt[4]{c+bx+ax^2}\left(-c-bx-ax^2+x^4\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(4*c + 3*b*x + 2*a*x^2)/((c + b*x + a*x^2)^(1/4)*(-c - b*x - a*x^2 + x^
4)),x]
```

```
[Out] -4*c*Defer[Int][1/((c + b*x + a*x^2)^(1/4)*(c + b*x + a*x^2 - x^4)), x] - 3
*b*Defer[Int][x/((c + b*x + a*x^2)^(1/4)*(c + b*x + a*x^2 - x^4)), x] - 2*a
*Defer[Int][x^2/((c + b*x + a*x^2)^(1/4)*(c + b*x + a*x^2 - x^4)), x]
```

Rubi steps

$$\int \frac{4c+3bx+2ax^2}{\sqrt[4]{c+bx+ax^2}\left(-c-bx-ax^2+x^4\right)} dx = \int \left(-\frac{4c}{\sqrt[4]{c+bx+ax^2}\left(c+bx+ax^2-x^4\right)}-\frac{3bx}{\sqrt[4]{c+bx+ax^2}\left(c+bx+\right.}\right.$$

$$= -\left((2a)\int \frac{x^2}{\sqrt[4]{c+bx+ax^2}\left(c+bx+ax^2-x^4\right)} dx\right)-(3b)\int \frac{}{\sqrt[4]{c+bx}}$$

Mathematica [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{4c+3bx+2ax^2}{\sqrt[4]{c+bx+ax^2}\left(-c-bx-ax^2+x^4\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(4*c + 3*b*x + 2*a*x^2)/((c + b*x + a*x^2)^(1/4)*(-c - b*x - a*x^
2 + x^4)),x]
```

```
[Out] Integrate[(4*c + 3*b*x + 2*a*x^2)/((c + b*x + a*x^2)^(1/4)*(-c - b*x - a*x^
2 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.38, size = 41, normalized size = 1.00

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^2+bx+c}}{x}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt[4]{ax^2+bx+c}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4*c + 3*b*x + 2*a*x^2)/((c + b*x + a*x^2)^(1/4)*(-c - b
*x - a*x^2 + x^4)),x]
```

```
[Out] 2*ArcTan[(c + b*x + a*x^2)^(1/4)/x] - 2*ArcTanh[x/(c + b*x + a*x^2)^(1/4)]
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2+3*b*x+4*c)/(a*x^2+b*x+c)^(1/4)/(x^4-a*x^2-b*x-c),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2+3*b*x+4*c)/(a*x^2+b*x+c)^(1/4)/(x^4-a*x^2-b*x-c),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [**F**] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{2a\,x^{2}+3bx+4c}{\left(a\,x^{2}+bx+c\right)^{\frac{1}{4}}\left(x^{4}-a\,x^{2}-bx-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x^2+3*b*x+4*c)/(a*x^2+b*x+c)^(1/4)/(x^4-a*x^2-b*x-c),x)
```

```
[Out] int((2*a*x^2+3*b*x+4*c)/(a*x^2+b*x+c)^(1/4)/(x^4-a*x^2-b*x-c),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax^{2}+3\,bx+4\,c}{\left(x^{4}-ax^{2}-bx-c\right)\left(ax^{2}+bx+c\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2+3*b*x+4*c)/(a*x^2+b*x+c)^(1/4)/(x^4-a*x^2-b*x-c),x, algo
rithm="maxima")
```

```
[Out] integrate((2*a*x^2 + 3*b*x + 4*c)/((x^4 - a*x^2 - b*x - c)*(a*x^2 + b*x + c
)^(1/4)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{2\,a\,x^{2}+3\,b\,x+4\,c}{\left(a\,x^{2}+b\,x+c\right)^{1/4}\,\left(-x^{4}+a\,x^{2}+b\,x+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*c + 3*b*x + 2*a*x^2)/((c + b*x + a*x^2)^(1/4)*(c + b*x + a*x^2 - x^
4)),x)
```

```
[Out] int(-(4*c + 3*b*x + 2*a*x^2)/((c + b*x + a*x^2)^(1/4)*(c + b*x + a*x^2 - x^
4)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x**2+3*b*x+4*c)/(a*x**2+b*x+c)**(1/4)/(x**4-a*x**2-b*x-c),x)
```

```
[Out] Timed out
```

3.532 $\int \frac{(2+x^3)(1+x^3+x^6)}{x^4\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{x^3+1}\,(x^6+7x^3-3)}{9x^3}-\frac{4}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Rubi [A] time = 0.12, antiderivative size = 57, normalized size of antiderivative = 1.39, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1821, 1612, 51, 63, 207}

$$\frac{2}{9}\left(x^3+1\right)^{3/2}-\frac{2\sqrt{x^3+1}}{3x^3}+\frac{4\sqrt{x^3+1}}{3}-\frac{4}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x^3)*(1 + x^3 + x^6))/(x^4*Sqrt[1 + x^3]),x]
```

```
[Out] (4*Sqrt[1 + x^3])/3 - (2*Sqrt[1 + x^3])/(3*x^3) + (2*(1 + x^3)^(3/2))/9 - (
4*ArcTanh[Sqrt[1 + x^3]])/3
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(2+x^3)(1+x^3+x^6)}{x^4\sqrt{1+x^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(2+x)(1+x+x^2)}{x^2\sqrt{1+x}}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{2}{\sqrt{1+x}}+\frac{2}{x^2\sqrt{1+x}}+\frac{3}{x\sqrt{1+x}}+\sqrt{1+x}\right)dx,x,x^3\right)$$
$$= \frac{4\sqrt{1+x^3}}{3}+\frac{2}{9}(1+x^3)^{3/2}+\frac{2}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx,x,x^3\right)+\text{Subst}\left(\int \frac{1}{x\sqrt{1+}}\right.$$
$$= \frac{4\sqrt{1+x^3}}{3}-\frac{2\sqrt{1+x^3}}{3x^3}+\frac{2}{9}(1+x^3)^{3/2}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^3\right)+2\,\text{Subs}$$
$$= \frac{4\sqrt{1+x^3}}{3}-\frac{2\sqrt{1+x^3}}{3x^3}+\frac{2}{9}(1+x^3)^{3/2}-2\tanh^{-1}\left(\sqrt{1+x^3}\right)-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-1+}\right.$$
$$= \frac{4\sqrt{1+x^3}}{3}-\frac{2\sqrt{1+x^3}}{3x^3}+\frac{2}{9}(1+x^3)^{3/2}-\frac{4}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right)$$

Mathematica [A] time = 0.05, size = 40, normalized size = 0.98

$$\frac{2}{9}\left(\frac{\sqrt{x^3+1}\,(x^6+7x^3-3)}{x^3}-6\tanh^{-1}\left(\sqrt{x^3+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x^3)*(1 + x^3 + x^6))/(x^4*Sqrt[1 + x^3]),x]
[Out] (2*((Sqrt[1 + x^3]*(-3 + 7*x^3 + x^6))/x^3 - 6*ArcTanh[Sqrt[1 + x^3]]))/9
```

IntegrateAlgebraic [A] time = 0.05, size = 41, normalized size = 1.00

$$\frac{2\sqrt{x^3+1}\,(x^6+7x^3-3)}{9x^3}-\frac{4}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^3)*(1 + x^3 + x^6))/(x^4*Sqrt[1 + x^3]),x]
[Out] (2*Sqrt[1 + x^3]*(-3 + 7*x^3 + x^6))/(9*x^3) - (4*ArcTanh[Sqrt[1 + x^3]])/3
```

fricas [A] time = 0.41, size = 55, normalized size = 1.34

$$-\frac{2\left(3x^3\log\left(\sqrt{x^3+1}+1\right)-3x^3\log\left(\sqrt{x^3+1}-1\right)-(x^6+7x^3-3)\sqrt{x^3+1}\right)}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^6+x^3+1)/x^4/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] -2/9*(3*x^3*log(sqrt(x^3 + 1) + 1) - 3*x^3*log(sqrt(x^3 + 1) - 1) - (x^6 +
7*x^3 - 3)*sqrt(x^3 + 1))/x^3
```

giac [A] time = 0.40, size = 56, normalized size = 1.37

$$\frac{2}{9}(x^3+1)^{\frac{3}{2}}+\frac{4}{3}\sqrt{x^3+1}-\frac{2\sqrt{x^3+1}}{3x^3}-\frac{2}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{2}{3}\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^6+x^3+1)/x^4/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*(x^3 + 1)^(3/2) + 4/3*sqrt(x^3 + 1) - 2/3*sqrt(x^3 + 1)/x^3 - 2/3*log(s
qrt(x^3 + 1) + 1) + 2/3*log(abs(sqrt(x^3 + 1) - 1))
```

maple [A] time = 0.01, size = 45, normalized size = 1.10

$$\frac{2x^3\sqrt{x^3+1}}{9}+\frac{14\sqrt{x^3+1}}{9}-\frac{2\sqrt{x^3+1}}{3x^3}-\frac{4\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2)*(x^6+x^3+1)/x^4/(x^3+1)^(1/2),x)
```

```
[Out] 2/9*x^3*(x^3+1)^(1/2)+14/9*(x^3+1)^(1/2)-2/3*(x^3+1)^(1/2)/x^3-4/3*arctanh(
(x^3+1)^(1/2))
```

maxima [A] time = 0.67, size = 55, normalized size = 1.34

$$\frac{2}{9}\left(x^3+1\right)^{\frac{3}{2}}+\frac{4}{3}\sqrt{x^3+1}-\frac{2\sqrt{x^3+1}}{3x^3}-\frac{2}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{2}{3}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^6+x^3+1)/x^4/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(x^3 + 1)^(3/2) + 4/3*sqrt(x^3 + 1) - 2/3*sqrt(x^3 + 1)/x^3 - 2/3*log(s
qrt(x^3 + 1) + 1) + 2/3*log(sqrt(x^3 + 1) - 1)
```

mupad [B] time = 0.05, size = 198, normalized size = 4.83

$$\frac{14\sqrt{x^3+1}}{9}-\frac{2\sqrt{x^3+1}}{3x^3}+\frac{2x^3\sqrt{x^3+1}}{9}-\frac{4\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 2)*(x^3 + x^6 + 1))/(x^4*(x^3 + 1)^(1/2)),x)
```

```
[Out] (14*(x^3 + 1)^(1/2))/9 - (2*(x^3 + 1)^(1/2))/(3*x^3) + (2*x^3*(x^3 + 1)^(1/
2))/9 - (4*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 -
x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asi
n(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2
)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [B] time = 74.24, size = 83, normalized size = 2.02

$$\frac{2\left(x^3+1\right)^{\frac{3}{2}}}{9}+\frac{4\sqrt{x^3+1}}{3}+\frac{2\log\left(-1+\frac{1}{\sqrt{x^3+1}}\right)}{3}-\frac{2\log\left(1+\frac{1}{\sqrt{x^3+1}}\right)}{3}+\frac{1}{3\left(1+\frac{1}{\sqrt{x^3+1}}\right)}+\frac{1}{3\left(-1+\frac{1}{\sqrt{x^3+1}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2)*(x**6+x**3+1)/x**4/(x**3+1)**(1/2),x)
```

```
[Out] 2*(x**3 + 1)**(3/2)/9 + 4*sqrt(x**3 + 1)/3 + 2*log(-1 + 1/sqrt(x**3 + 1))/3
 - 2*log(1 + 1/sqrt(x**3 + 1))/3 + 1/(3*(1 + 1/sqrt(x**3 + 1))) + 1/(3*(-1
+ 1/sqrt(x**3 + 1)))
```

3.533 $\int \frac{\sqrt{-1+x^3}\left(-2+x^3+2x^6\right)}{x^{10}}\,dx$

Optimal. Leaf size=41

$$\frac{2}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)-\frac{2\sqrt{x^3-1}\left(3x^6+x^3-1\right)}{9x^9}$$

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.54, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1474, 897, 1257, 1157, 12, 288, 203}

$$-\frac{2\sqrt{x^3-1}}{3x^3}+\frac{2}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{2\sqrt{x^3-1}}{9x^9}-\frac{2\sqrt{x^3-1}}{9x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^3]*(-2 + x^3 + 2*x^6))/x^10,x]
```

```
[Out] (2*Sqrt[-1 + x^3])/(9*x^9) - (2*Sqrt[-1 + x^3])/(9*x^6) - (2*Sqrt[-1 + x^3]
)/(3*x^3) + (2*ArcTan[Sqrt[-1 + x^3]])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-1+x^3}\left(-2+x^3+2x^6\right)}{x^{10}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{-1+x}\left(-2+x+2x^2\right)}{x^4}\,dx, x, x^3\right)\\
&= \frac{2}{3}\,\text{Subst}\left(\int \frac{x^2\left(1+5x^2+2x^4\right)}{\left(1+x^2\right)^4}\,dx, x, \sqrt{-1+x^3}\right)\\
&= \frac{2\sqrt{-1+x^3}}{9x^9}-\frac{1}{9}\,\text{Subst}\left(\int \frac{2-18x^2-12x^4}{\left(1+x^2\right)^3}\,dx, x, \sqrt{-1+x^3}\right)\\
&= \frac{2\sqrt{-1+x^3}}{9x^9}-\frac{2\sqrt{-1+x^3}}{9x^6}+\frac{1}{36}\,\text{Subst}\left(\int \frac{48x^2}{\left(1+x^2\right)^2}\,dx, x, \sqrt{-1+x^3}\right)\\
&= \frac{2\sqrt{-1+x^3}}{9x^9}-\frac{2\sqrt{-1+x^3}}{9x^6}+\frac{4}{3}\,\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2}\,dx, x, \sqrt{-1+x^3}\right)\\
&= \frac{2\sqrt{-1+x^3}}{9x^9}-\frac{2\sqrt{-1+x^3}}{9x^6}-\frac{2\sqrt{-1+x^3}}{3x^3}+\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^3}\right)\\
&= \frac{2\sqrt{-1+x^3}}{9x^9}-\frac{2\sqrt{-1+x^3}}{9x^6}-\frac{2\sqrt{-1+x^3}}{3x^3}+\frac{2}{3}\tan^{-1}\left(\sqrt{-1+x^3}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 41, normalized size = 1.00

$$\frac{2}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)-\frac{2\sqrt{x^3-1}\left(3x^6+x^3-1\right)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^3]*(-2 + x^3 + 2*x^6))/x^10,x]
```

```
[Out] (-2*Sqrt[-1 + x^3]*(-1 + x^3 + 3*x^6))/(9*x^9) + (2*ArcTan[Sqrt[-1 + x^3]])
/3
```

IntegrateAlgebraic [A] time = 0.07, size = 41, normalized size = 1.00

$$\frac{2}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)-\frac{2\sqrt{x^3-1}\left(3x^6+x^3-1\right)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^3]*(-2 + x^3 + 2*x^6))/x^10,x]
```

```
[Out] (-2*Sqrt[-1 + x^3]*(-1 + x^3 + 3*x^6))/(9*x^9) + (2*ArcTan[Sqrt[-1 + x^3]])
/3
```

fricas [A] time = 0.43, size = 38, normalized size = 0.93

$$\frac{2\left(3\,x^{9}\arctan\left(\sqrt{x^{3}-1}\right)-\left(3\,x^{6}+x^{3}-1\right)\sqrt{x^{3}-1}\right)}{9\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)*(2*x^6+x^3-2)/x^10,x, algorithm="fricas")
```

```
[Out] 2/9*(3*x^9*arctan(sqrt(x^3 - 1)) - (3*x^6 + x^3 - 1)*sqrt(x^3 - 1))/x^9
```

giac [A] time = 0.64, size = 44, normalized size = 1.07

$$-\frac{2\left(3\left(x^{3}-1\right)^{\frac{5}{2}}+7\left(x^{3}-1\right)^{\frac{3}{2}}+3\sqrt{x^{3}-1}\right)}{9\,x^{9}}+\frac{2}{3}\arctan\left(\sqrt{x^{3}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)*(2*x^6+x^3-2)/x^10,x, algorithm="giac")
```

```
[Out] -2/9*(3*(x^3 - 1)^(5/2) + 7*(x^3 - 1)^(3/2) + 3*sqrt(x^3 - 1))/x^9 + 2/3*ar
ctan(sqrt(x^3 - 1))
```

maple [A] time = 0.04, size = 48, normalized size = 1.17

$$-\frac{2\sqrt{x^{3}-1}}{3x^{3}}+\frac{2\arctan\left(\sqrt{x^{3}-1}\right)}{3}-\frac{2\sqrt{x^{3}-1}}{9x^{6}}+\frac{2\sqrt{x^{3}-1}}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/2)*(2*x^6+x^3-2)/x^10,x)
```

```
[Out] -2/3*(x^3-1)^(1/2)/x^3+2/3*arctan((x^3-1)^(1/2))-2/9*(x^3-1)^(1/2)/x^6+2/9*
(x^3-1)^(1/2)/x^9
```

maxima [B] time = 0.68, size = 113, normalized size = 2.76

$$-\frac{3\left(x^{3}-1\right)^{\frac{5}{2}}+8\left(x^{3}-1\right)^{\frac{3}{2}}-3\sqrt{x^{3}-1}}{36\left(\left(x^{3}-1\right)^{3}+3\,x^{3}+3\left(x^{3}-1\right)^{2}-2\right)}+\frac{\left(x^{3}-1\right)^{\frac{3}{2}}-\sqrt{x^{3}-1}}{12\left(2\,x^{3}+\left(x^{3}-1\right)^{2}-1\right)}-\frac{2\sqrt{x^{3}-1}}{3\,x^{3}}+\frac{2}{3}\arctan\left(\sqrt{x^{3}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)*(2*x^6+x^3-2)/x^10,x, algorithm="maxima")
```

```
[Out] -1/36*(3*(x^3 - 1)^(5/2) + 8*(x^3 - 1)^(3/2) - 3*sqrt(x^3 - 1))/((x^3 - 1)^
3 + 3*x^3 + 3*(x^3 - 1)^2 - 2) + 1/12*((x^3 - 1)^(3/2) - sqrt(x^3 - 1))/(2*
x^3 + (x^3 - 1)^2 - 1) - 2/3*sqrt(x^3 - 1)/x^3 + 2/3*arctan(sqrt(x^3 - 1))
```

mupad [B] time = 0.22, size = 201, normalized size = 4.90

$$\frac{2\sqrt{x^{3}-1}}{9\,x^{9}}-\frac{2\sqrt{x^{3}-1}}{9\,x^{6}}-\frac{2\sqrt{x^{3}-1}}{3\,x^{3}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^{3}+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(1/2)*(x^3 + 2*x^6 - 2))/x^10,x)
```

```
[Out] (2*(x^3 - 1)^(1/2))/(9*x^9) - (2*(x^3 - 1)^(1/2))/(9*x^6) - (2*(x^3 - 1)^(1
/2))/(3*x^3) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(
1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 +
 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2
)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) -
 x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/2)*(2*x**6+x**3-2)/x**10,x)
```

```
[Out] Timed out
```

3.534 $\int \frac{-4+13x^6}{x\sqrt{-1+x^6}\left(-1+4x^6\right)}\,dx$

Optimal. Leaf size=41

$$\frac{4}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt{x^6-1}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {573, 156, 63, 203}

$$\frac{4}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt{x^6-1}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 13*x^6)/(x*Sqrt[-1 + x^6]*(-1 + 4*x^6)),x]
```

```
[Out] (4*ArcTan[Sqrt[-1 + x^6]])/3 - ArcTan[(2*Sqrt[-1 + x^6])/Sqrt[3]]/(2*Sqrt[3
])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rubi steps

$$\int \frac{-4+13x^6}{x\sqrt{-1+x^6}\left(-1+4x^6\right)}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{-4+13x}{\sqrt{-1+x}\,x(-1+4x)}\,dx, x, x^6\right)$$
$$= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,(-1+4x)}\,dx, x, x^6\right)\right) + \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x,\right.$$
$$= \frac{4}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^6}\right) - \text{Subst}\left(\int \frac{1}{3+4x^2}\,dx, x, \sqrt{-1+x^6}\right)$$
$$= \frac{4}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right) - \frac{\tan^{-1}\left(\frac{2\sqrt{-1+x^6}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Mathematica [A] time = 0.07, size = 73, normalized size = 1.78

$$\frac{1}{12}\left(16\tan^{-1}\left(\sqrt{x^6-1}\right) + \sqrt{3}\,\tan^{-1}\left(\frac{2-x^3}{\sqrt{3}\,\sqrt{x^6-1}}\right) + \sqrt{3}\,\tan^{-1}\left(\frac{x^3+2}{\sqrt{3}\,\sqrt{x^6-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 13*x^6)/(x*Sqrt[-1 + x^6]*(-1 + 4*x^6)),x]
[Out] (Sqrt[3]*ArcTan[(2 - x^3)/(Sqrt[3]*Sqrt[-1 + x^6])] + Sqrt[3]*ArcTan[(2 + x
^3)/(Sqrt[3]*Sqrt[-1 + x^6])] + 16*ArcTan[Sqrt[-1 + x^6]])/12
```

IntegrateAlgebraic [A] time = 0.04, size = 41, normalized size = 1.00

$$\frac{4}{3}\tan^{-1}\left(\sqrt{x^6-1}\right) - \frac{\tan^{-1}\left(\frac{2\sqrt{x^6-1}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-4 + 13*x^6)/(x*Sqrt[-1 + x^6]*(-1 + 4*x^6)),x]
[Out] (4*ArcTan[Sqrt[-1 + x^6]])/3 - ArcTan[(2*Sqrt[-1 + x^6])/Sqrt[3]]/(2*Sqrt[3
])
```

fricas [A] time = 0.43, size = 29, normalized size = 0.71

$$-\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{2}{3}\,\sqrt{3}\,\sqrt{x^6-1}\right) + \frac{4}{3}\,\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((13*x^6-4)/x/(x^6-1)^(1/2)/(4*x^6-1),x, algorithm="fricas")
[Out] -1/6*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x^6 - 1)) + 4/3*arctan(sqrt(x^6 - 1))
```

giac [A] time = 0.32, size = 29, normalized size = 0.71

$$-\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{2}{3}\,\sqrt{3}\,\sqrt{x^6-1}\right) + \frac{4}{3}\,\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((13*x^6-4)/x/(x^6-1)^(1/2)/(4*x^6-1),x, algorithm="giac")
[Out] -1/6*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x^6 - 1)) + 4/3*arctan(sqrt(x^6 - 1))
```

maple [C] time = 1.05, size = 88, normalized size = 2.15

$$-\frac{4\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^{2}+1\right)+\sqrt{x^{6}-1}}{x^{3}}\right)}{3}+\frac{\mathrm{RootOf}\left(\_Z^{2}+3\right)\ln\left(\frac{4\,\mathrm{RootOf}\left(\_Z^{2}+3\right)x^{6}+12\sqrt{x^{6}-1}-7\,\mathrm{RootOf}\left(\_Z^{2}+3\right)}{\left(2x^{3}-1\right)\left(2x^{3}+1\right)}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((13*x^6-4)/x/(x^6-1)^(1/2)/(4*x^6-1),x)
```

```
[Out] -4/3*RootOf(_Z^2+1)*ln((-RootOf(_Z^2+1)+(x^6-1)^(1/2))/x^3)+1/12*RootOf(_Z^
2+3)*ln((4*RootOf(_Z^2+3)*x^6+12*(x^6-1)^(1/2)-7*RootOf(_Z^2+3))/(2*x^3-1)/
(2*x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{13\,x^{6}-4}{\left(4\,x^{6}-1\right)\sqrt{x^{6}-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((13*x^6-4)/x/(x^6-1)^(1/2)/(4*x^6-1),x, algorithm="maxima")
```

```
[Out] integrate((13*x^6 - 4)/((4*x^6 - 1)*sqrt(x^6 - 1)*x), x)
```

mupad [B] time = 0.54, size = 29, normalized size = 0.71

$$\frac{4\,\mathrm{atan}\left(\sqrt{x^{6}-1}\right)}{3}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,\sqrt{x^{6}-1}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((13*x^6 - 4)/(x*(x^6 - 1)^(1/2)*(4*x^6 - 1)),x)
```

```
[Out] (4*atan((x^6 - 1)^(1/2)))/3 - (3^(1/2)*atan((2*3^(1/2)*(x^6 - 1)^(1/2))/3))
/6
```

sympy [A] time = 22.45, size = 37, normalized size = 0.90

$$-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,\sqrt{x^{6}-1}}{3}\right)}{6}+\frac{4\,\mathrm{atan}\left(\sqrt{x^{6}-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((13*x**6-4)/x/(x**6-1)**(1/2)/(4*x**6-1),x)
```

```
[Out] -sqrt(3)*atan(2*sqrt(3)*sqrt(x**6 - 1)/3)/6 + 4*atan(sqrt(x**6 - 1))/3
```

3.535 $\int \frac{-b+2ax^3}{(b-x+ax^3)\sqrt[4]{bx^3+ax^6}}\,dx$

Optimal. Leaf size=41

$$-2\tan^{-1}\left(\frac{x}{\sqrt[4]{ax^6+bx^3}}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt[4]{ax^6+bx^3}}\right)$$

Rubi [F] time = 1.71, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-b+2ax^3}{(b-x+ax^3)\sqrt[4]{bx^3+ax^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-b + 2*a*x^3)/((b - x + a*x^3)*(b*x^3 + a*x^6)^(1/4)),x]
```

```
[Out] (8*x*(1 + (a*x^3)/b)^(1/4)*Hypergeometric2F1[1/12, 1/4, 13/12, -((a*x^3)/b)
])/(b*x^3 + a*x^6)^(1/4) - (12*b*x^(3/4)*(b + a*x^3)^(1/4)*Defer[Subst][Def
er[Int][1/((b + a*x^12)^(1/4)*(b - x^4 + a*x^12)), x], x, x^(1/4)])/(b*x^3
+ a*x^6)^(1/4) + (8*x^(3/4)*(b + a*x^3)^(1/4)*Defer[Subst][Defer[Int][x^4/(
(b + a*x^12)^(1/4)*(b - x^4 + a*x^12)), x], x, x^(1/4)])/(b*x^3 + a*x^6)^(1
/4)
```

Rubi steps

$$\begin{aligned}
\int \frac{-b+2ax^3}{(b-x+ax^3)\sqrt[4]{bx^3+ax^6}}\,dx &= \frac{\left(x^{3/4}\sqrt[4]{b+ax^3}\right)\int \frac{-b+2ax^3}{x^{3/4}\sqrt[4]{b+ax^3}\,(b-x+ax^3)}\,dx}{\sqrt[4]{bx^3+ax^6}}\\
&= \frac{\left(4x^{3/4}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{-b+2ax^{12}}{\sqrt[4]{b+ax^{12}}\,(b-x^4+ax^{12})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx^3+ax^6}}\\
&= \frac{\left(4x^{3/4}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \left(\frac{2}{\sqrt[4]{b+ax^{12}}}-\frac{3b-2x^4}{\sqrt[4]{b+ax^{12}}\,(b-x^4+ax^{12})}\right)dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx^3+ax^6}}\\
&= -\frac{\left(4x^{3/4}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{3b-2x^4}{\sqrt[4]{b+ax^{12}}\,(b-x^4+ax^{12})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx^3+ax^6}}+\frac{\left(8x^{3/4}\sqrt[4]{b+ax\right.}{}\\
&= -\frac{\left(4x^{3/4}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \left(\frac{3b}{\sqrt[4]{b+ax^{12}}\,(b-x^4+ax^{12})}-\frac{2x^4}{\sqrt[4]{b+ax^{12}}\,(b-x^4+ax^{12})}\right)dx,x,\right.}{\sqrt[4]{bx^3+ax^6}}\\
&= \frac{8x\sqrt[4]{1+\frac{ax^3}{b}}\,{}_2F_1\left(\frac{1}{12},\frac{1}{4};\frac{13}{12};-\frac{ax^3}{b}\right)}{\sqrt[4]{bx^3+ax^6}}+\frac{\left(8x^{3/4}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{b+ax^{12}}\,(b-x^4\right.}{\sqrt[4]{bx^3+ax^6}}
\end{aligned}$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{-b+2ax^3}{(b-x+ax^3)\sqrt[4]{bx^3+ax^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-b + 2*a*x^3)/((b - x + a*x^3)*(b*x^3 + a*x^6)^(1/4)),x]
```

```
[Out] Integrate[(-b + 2*a*x^3)/((b - x + a*x^3)*(b*x^3 + a*x^6)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 2.73, size = 41, normalized size = 1.00

$$-2\tan^{-1}\left(\frac{x}{\sqrt[4]{ax^6+bx^3}}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt[4]{ax^6+bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + 2*a*x^3)/((b - x + a*x^3)*(b*x^3 + a*x^6)^(1/4)),x
]
```

```
[Out] -2*ArcTan[x/(b*x^3 + a*x^6)^(1/4)] - 2*ArcTanh[x/(b*x^3 + a*x^6)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^3-b)/(a*x^3+b-x)/(a*x^6+b*x^3)^(1/4),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax^3-b}{\left(ax^6+bx^3\right)^{\frac{1}{4}}\left(ax^3+b-x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^3-b)/(a*x^3+b-x)/(a*x^6+b*x^3)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((2*a*x^3 - b)/((a*x^6 + b*x^3)^(1/4)*(a*x^3 + b - x)), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{2a\,x^3-b}{\left(a\,x^3+b-x\right)\left(a\,x^6+b\,x^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x^3-b)/(a*x^3+b-x)/(a*x^6+b*x^3)^(1/4),x)
```

```
[Out] int((2*a*x^3-b)/(a*x^3+b-x)/(a*x^6+b*x^3)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax^3-b}{\left(ax^6+bx^3\right)^{\frac{1}{4}}\left(ax^3+b-x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^3-b)/(a*x^3+b-x)/(a*x^6+b*x^3)^(1/4),x, algorithm="maxima"
)
```

```
[Out] integrate((2*a*x^3 - b)/((a*x^6 + b*x^3)^(1/4)*(a*x^3 + b - x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{b-2\,a\,x^3}{{\left(a\,x^6+b\,x^3\right)}^{1/4}\,\left(a\,x^3-x+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - 2*a*x^3)/((a*x^6 + b*x^3)^(1/4)*(b - x + a*x^3)),x)
[Out] int(-(b - 2*a*x^3)/((a*x^6 + b*x^3)^(1/4)*(b - x + a*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x**3-b)/(a*x**3+b-x)/(a*x**6+b*x**3)**(1/4),x)
[Out] Timed out
```

3.536 $\int \frac{x^3(b+2ax^5)}{\sqrt[4]{bx+ax^6}\,(-1+bx^5+ax^{10})}\,dx$

Optimal. Leaf size=41

$$\frac{2}{5}\tan^{-1}\left(x\sqrt[4]{ax^6+bx}\right)-\frac{2}{5}\tanh^{-1}\left(x\sqrt[4]{ax^6+bx}\right)$$

Rubi [C] time = 1.44, antiderivative size = 187, normalized size of antiderivative = 4.56, number of steps used = 11, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2056, 6728, 466, 465, 511, 510}

$$\frac{8ax^4\sqrt[4]{\frac{ax^5}{b}+1}\,F_1\left(\frac{3}{4};1,\frac{1}{4};\frac{7}{4};-\frac{2ax^5}{b-\sqrt{b^2+4a}},-\frac{ax^5}{b}\right)}{15\left(b-\sqrt{4a+b^2}\right)\sqrt[4]{ax^6+bx}}+\frac{8ax^4\sqrt[4]{\frac{ax^5}{b}+1}\,F_1\left(\frac{3}{4};1,\frac{1}{4};\frac{7}{4};-\frac{2ax^5}{b+\sqrt{b^2+4a}},-\frac{ax^5}{b}\right)}{15\left(\sqrt{4a+b^2}+b\right)\sqrt[4]{ax^6+bx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^3*(b + 2*a*x^5))/((b*x + a*x^6)^(1/4)*(-1 + b*x^5 + a*x^10)),x]
```

```
[Out] (8*a*x^4*(1 + (a*x^5)/b)^(1/4)*AppellF1[3/4, 1, 1/4, 7/4, (-2*a*x^5)/(b - S
qrt[4*a + b^2]), -((a*x^5)/b)])/(15*(b - Sqrt[4*a + b^2])*(b*x + a*x^6)^(1/
4)) + (8*a*x^4*(1 + (a*x^5)/b)^(1/4)*AppellF1[3/4, 1, 1/4, 7/4, (-2*a*x^5)/
(b + Sqrt[4*a + b^2]), -((a*x^5)/b)])/(15*(b + Sqrt[4*a + b^2])*(b*x + a*x^
6)^(1/4))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3 \left(b+2ax^5\right)}{\sqrt[4]{bx+ax^6}\left(-1+bx^5+ax^{10}\right)}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^5}\right)\int \frac{x^{11/4}\left(b+2ax^5\right)}{\sqrt[4]{b+ax^5}\left(-1+bx^5+ax^{10}\right)}\,dx}{\sqrt[4]{bx+ax^6}}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^5}\right)\int \left(\frac{2ax^{11/4}}{\sqrt[4]{b+ax^5}\left(b-\sqrt{4a+b^2}+2ax^5\right)}+\frac{2ax^{11/4}}{\sqrt[4]{b+ax^5}\left(b+\sqrt{4a+b^2}+2ax^5\right)}\right)dx}{\sqrt[4]{bx+ax^6}}$$

$$= \frac{\left(2a\sqrt[4]{x}\sqrt[4]{b+ax^5}\right)\int \frac{x^{11/4}}{\sqrt[4]{b+ax^5}\left(b-\sqrt{4a+b^2}+2ax^5\right)}\,dx}{\sqrt[4]{bx+ax^6}} + \frac{\left(2a\sqrt[4]{x}\sqrt[4]{b+ax^5}\right)\int \frac{}{\sqrt[4]{}}}{\sqrt[4]{bx}}$$

$$= \frac{\left(8a\sqrt[4]{x}\sqrt[4]{b+ax^5}\right)\text{Subst}\left(\int \frac{x^{14}}{\sqrt[4]{b+ax^{20}}\left(b-\sqrt{4a+b^2}+2ax^{20}\right)}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^6}} + \frac{\left(8a\sqrt[4]{}\right.}{}$$

$$= \frac{\left(8a\sqrt[4]{x}\sqrt[4]{b+ax^5}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{b+ax^4}\left(b-\sqrt{4a+b^2}+2ax^4\right)}\,dx, x, x^{5/4}\right)}{5\sqrt[4]{bx+ax^6}} + \frac{\left(8a\sqrt[4]{x}\right.}{}$$

$$= \frac{\left(8a\sqrt[4]{x}\sqrt[4]{1+\frac{ax^5}{b}}\right)\text{Subst}\left(\int \frac{x^2}{\left(b-\sqrt{4a+b^2}+2ax^4\right)\sqrt[4]{1+\frac{ax^4}{b}}}\,dx, x, x^{5/4}\right)}{5\sqrt[4]{bx+ax^6}} + \frac{\left(8a\sqrt[4]{x}\right.}{}$$

$$= \frac{8ax^4\sqrt[4]{1+\frac{ax^5}{b}}\,F_1\left(\frac{3}{4};1,\frac{1}{4};\frac{7}{4};-\frac{2ax^5}{b-\sqrt{4a+b^2}},-\frac{ax^5}{b}\right)}{15\left(b-\sqrt{4a+b^2}\right)\sqrt[4]{bx+ax^6}} + \frac{8ax^4\sqrt[4]{1+\frac{ax^5}{b}}\,F_1\left(\frac{3}{4};1,\right.}{15\left(b+\sqrt{4a+\right.}}$$

Mathematica [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(b+2ax^5\right)}{\sqrt[4]{bx+ax^6}\left(-1+bx^5+ax^{10}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(b + 2*a*x^5))/((b*x + a*x^6)^(1/4)*(-1 + b*x^5 + a*x^10)),x
]
```

```
[Out] Integrate[(x^3*(b + 2*a*x^5))/((b*x + a*x^6)^(1/4)*(-1 + b*x^5 + a*x^10)),
x]
```

IntegrateAlgebraic [A] time = 19.86, size = 41, normalized size = 1.00

$$\frac{2}{5}\tan^{-1}\left(x\sqrt[4]{ax^6+bx}\right)-\frac{2}{5}\tanh^{-1}\left(x\sqrt[4]{ax^6+bx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(b + 2*a*x^5))/((b*x + a*x^6)^(1/4)*(-1 + b*x^5 + a
*x^10)),x]
```

```
[Out] (2*ArcTan[x*(b*x + a*x^6)^(1/4)])/5 - (2*ArcTanh[x*(b*x + a*x^6)^(1/4)])/5
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*a*x^5+b)/(a*x^6+b*x)^(1/4)/(a*x^10+b*x^5-1),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,ax^5+b\right)x^3}{\left(ax^{10}+bx^5-1\right)\left(ax^6+bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*a*x^5+b)/(a*x^6+b*x)^(1/4)/(a*x^10+b*x^5-1),x, algorithm="
giac")
```

```
[Out] integrate((2*a*x^5 + b)*x^3/((a*x^10 + b*x^5 - 1)*(a*x^6 + b*x)^(1/4)), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(2a\,x^5+b\right)}{\left(a\,x^6+bx\right)^{\frac{1}{4}}\left(a\,x^{10}+b\,x^5-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*a*x^5+b)/(a*x^6+b*x)^(1/4)/(a*x^10+b*x^5-1),x)
```

```
[Out] int(x^3*(2*a*x^5+b)/(a*x^6+b*x)^(1/4)/(a*x^10+b*x^5-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,ax^5+b\right)x^3}{\left(ax^{10}+bx^5-1\right)\left(ax^6+bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*a*x^5+b)/(a*x^6+b*x)^(1/4)/(a*x^10+b*x^5-1),x, algorithm="
maxima")
```

```
[Out] integrate((2*a*x^5 + b)*x^3/((a*x^10 + b*x^5 - 1)*(a*x^6 + b*x)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,\left(2\,a\,x^5+b\right)}{{\left(a\,x^6+b\,x\right)}^{1/4}\,\left(a\,x^{10}+b\,x^5-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(b + 2*a*x^5))/((b*x + a*x^6)^(1/4)*(a*x^10 + b*x^5 - 1)),x)
[Out] int((x^3*(b + 2*a*x^5))/((b*x + a*x^6)^(1/4)*(a*x^10 + b*x^5 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(2*a*x**5+b)/(a*x**6+b*x)**(1/4)/(a*x**10+b*x**5-1),x)
[Out] Timed out
```

3.537 $\int \frac{\sqrt[4]{1+x^3}}{x}\,dx$

Optimal. Leaf size=42

$$\frac{4}{3}\sqrt[4]{x^3+1}-\frac{2}{3}\tan^{-1}\left(\sqrt[4]{x^3+1}\right)-\frac{2}{3}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)$$

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 50, 63, 212, 206, 203}

$$\frac{4}{3}\sqrt[4]{x^3+1}-\frac{2}{3}\tan^{-1}\left(\sqrt[4]{x^3+1}\right)-\frac{2}{3}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)^(1/4)/x,x]
```

```
[Out] (4*(1 + x^3)^(1/4))/3 - (2*ArcTan[(1 + x^3)^(1/4)])/3 - (2*ArcTanh[(1 + x^3
)^(1/4)])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{1+x^3}}{x}\,dx &= \frac{1}{3}\mathrm{Subst}\left(\int \frac{\sqrt[4]{1+x}}{x}\,dx, x, x^3\right)\\
&= \frac{4}{3}\sqrt[4]{1+x^3} + \frac{1}{3}\mathrm{Subst}\left(\int \frac{1}{x(1+x)^{3/4}}\,dx, x, x^3\right)\\
&= \frac{4}{3}\sqrt[4]{1+x^3} + \frac{4}{3}\mathrm{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt[4]{1+x^3}\right)\\
&= \frac{4}{3}\sqrt[4]{1+x^3} - \frac{2}{3}\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1+x^3}\right) - \frac{2}{3}\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{1+x^3}\right)\\
&= \frac{4}{3}\sqrt[4]{1+x^3} - \frac{2}{3}\tan^{-1}\left(\sqrt[4]{1+x^3}\right) - \frac{2}{3}\tanh^{-1}\left(\sqrt[4]{1+x^3}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{4}{3}\sqrt[4]{x^3+1} - \frac{2}{3}\tan^{-1}\left(\sqrt[4]{x^3+1}\right) - \frac{2}{3}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)^(1/4)/x,x]
[Out] (4*(1 + x^3)^(1/4))/3 - (2*ArcTan[(1 + x^3)^(1/4)])/3 - (2*ArcTanh[(1 + x^3
)^(1/4)])/3
```

IntegrateAlgebraic [A] time = 0.03, size = 42, normalized size = 1.00

$$\frac{4}{3}\sqrt[4]{x^3+1} - \frac{2}{3}\tan^{-1}\left(\sqrt[4]{x^3+1}\right) - \frac{2}{3}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)^(1/4)/x,x]
[Out] (4*(1 + x^3)^(1/4))/3 - (2*ArcTan[(1 + x^3)^(1/4)])/3 - (2*ArcTanh[(1 + x^3
)^(1/4)])/3
```

fricas [A] time = 0.73, size = 44, normalized size = 1.05

$$\frac{4}{3}\left(x^3+1\right)^{\frac{1}{4}} - \frac{2}{3}\arctan\left(\left(x^3+1\right)^{\frac{1}{4}}\right) - \frac{1}{3}\log\left(\left(x^3+1\right)^{\frac{1}{4}}+1\right) + \frac{1}{3}\log\left(\left(x^3+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/4)/x,x, algorithm="fricas")
[Out] 4/3*(x^3 + 1)^(1/4) - 2/3*arctan((x^3 + 1)^(1/4)) - 1/3*log((x^3 + 1)^(1/4)
 + 1) + 1/3*log((x^3 + 1)^(1/4) - 1)
```

giac [A] time = 0.33, size = 45, normalized size = 1.07

$$\frac{4}{3}\left(x^3+1\right)^{\frac{1}{4}} - \frac{2}{3}\arctan\left(\left(x^3+1\right)^{\frac{1}{4}}\right) - \frac{1}{3}\log\left(\left(x^3+1\right)^{\frac{1}{4}}+1\right) + \frac{1}{3}\log\left(\left|\left(x^3+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/4)/x,x, algorithm="giac")
```

```
[Out] 4/3*(x^3 + 1)^(1/4) - 2/3*arctan((x^3 + 1)^(1/4)) - 1/3*log((x^3 + 1)^(1/4)
 + 1) + 1/3*log(abs((x^3 + 1)^(1/4) - 1))
```

maple [C] time = 0.22, size = 45, normalized size = 1.07

$$-\frac{-\Gamma\left(\frac{3}{4}\right)x^3\,\mathrm{hypergeom}\left(\left[\frac{3}{4},1,1\right],[2,2],-x^3\right)-4\left(4-3\ln(2)+\frac{\pi}{2}+3\ln(x)\right)\Gamma\left(\frac{3}{4}\right)}{12\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(1/4)/x,x)
```

```
[Out] -1/12/GAMMA(3/4)*(-GAMMA(3/4)*x^3*hypergeom([3/4,1,1],[2,2],-x^3)-4*(4-3*ln
(2)+1/2*Pi+3*ln(x))*GAMMA(3/4))
```

maxima [A] time = 0.59, size = 44, normalized size = 1.05

$$\frac{4}{3}\left(x^3+1\right)^{\frac{1}{4}}-\frac{2}{3}\arctan\left(\left(x^3+1\right)^{\frac{1}{4}}\right)-\frac{1}{3}\log\left(\left(x^3+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{3}\log\left(\left(x^3+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/4)/x,x, algorithm="maxima")
```

```
[Out] 4/3*(x^3 + 1)^(1/4) - 2/3*arctan((x^3 + 1)^(1/4)) - 1/3*log((x^3 + 1)^(1/4)
 + 1) + 1/3*log((x^3 + 1)^(1/4) - 1)
```

mupad [B] time = 0.28, size = 30, normalized size = 0.71

$$\frac{4\left(x^3+1\right)^{1/4}}{3}-\frac{2\,\mathrm{atanh}\left(\left(x^3+1\right)^{1/4}\right)}{3}-\frac{2\,\mathrm{atan}\left(\left(x^3+1\right)^{1/4}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)^(1/4)/x,x)
```

```
[Out] (4*(x^3 + 1)^(1/4))/3 - (2*atanh((x^3 + 1)^(1/4)))/3 - (2*atan((x^3 + 1)^(1
/4)))/3
```

sympy [C] time = 0.79, size = 37, normalized size = 0.88

$$-\frac{x^{\frac{3}{4}}\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{e^{i\pi}}{x^3}\right)}{3\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(1/4)/x,x)
```

```
[Out] -x**(3/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), exp_polar(I*pi)/x**3)/(3*
gamma(3/4))
```

3.538 $\int \frac{-2aq+3bpx^2+apx^3}{\sqrt{q+px^3}\left(b^2c+dq+2abcx+a^2cx^2+dpx^3\right)}\,dx$

Optimal. Leaf size=42

$$\frac{2\tan^{-1}\left(\frac{\sqrt{d}\sqrt{px^3+q}}{\sqrt{c}\,(ax+b)}\right)}{\sqrt{c}\sqrt{d}}$$

Rubi [F] time = 2.63, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2aq+3bpx^2+apx^3}{\sqrt{q+px^3}\left(b^2c+dq+2abcx+a^2cx^2+dpx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*a*q + 3*b*p*x^2 + a*p*x^3)/(Sqrt[q + p*x^3]*(b^2*c + d*q + 2*a*b*c*
x + a^2*c*x^2 + d*p*x^3)),x]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*a*(q^(1/3) + p^(1/3)*x)*Sqrt[(q^(2/3) - p^(1/3)*q^(1/3
)*x + p^(2/3)*x^2)/((1 + Sqrt[3])*q^(1/3) + p^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*q^(1/3) + p^(1/3)*x)/((1 + Sqrt[3])*q^(1/3) + p^(1/3)*x)], -
7 - 4*Sqrt[3]])/(3^(1/4)*d*p^(1/3)*Sqrt[(q^(1/3)*(q^(1/3) + p^(1/3)*x))/((1
 + Sqrt[3])*q^(1/3) + p^(1/3)*x)^2]*Sqrt[q + p*x^3]) - (a*(b^2*c + 3*d*q)*D
efer[Int][1/(Sqrt[q + p*x^3]*(b^2*c + d*q + 2*a*b*c*x + a^2*c*x^2 + d*p*x^3
)), x])/d - (2*a^2*b*c*Defer[Int][x/(Sqrt[q + p*x^3]*(b^2*c + d*q + 2*a*b*c
*x + a^2*c*x^2 + d*p*x^3)), x])/d - ((a^3*c - 3*b*d*p)*Defer[Int][x^2/(Sqrt
[q + p*x^3]*(b^2*c + d*q + 2*a*b*c*x + a^2*c*x^2 + d*p*x^3)), x])/d
```

Rubi steps

$$\int \frac{-2aq+3bpx^2+apx^3}{\sqrt{q+px^3}\left(b^2c+dq+2abcx+a^2cx^2+dpx^3\right)}\,dx = \int \left(\frac{a}{d\sqrt{q+px^3}} - \frac{a\left(b^2c+3dq\right)+2a^2bcx+\left(a^3c-3b\right.}{d\sqrt{q+px^3}\left(b^2c+dq+2abcx+a^2cx^2\right.}\right.$$

$$= -\frac{\int \frac{a\left(b^2c+3dq\right)+2a^2bcx+\left(a^3c-3bdp\right)x^2}{\sqrt{q+px^3}\left(b^2c+dq+2abcx+a^2cx^2+dpx^3\right)}\,dx}{d} + \frac{a\int \frac{1}{\sqrt{q+px^3}}\,dx}{d}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{q}+\sqrt[3]{p}\,x\right)\sqrt{\frac{q^{2/3}-\sqrt[3]{p}\sqrt[3]{q}\,x+p^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{q}+\sqrt[3]{p}\,x\right)^2}}\,F\left(\sin^{-1}\right.}{\sqrt[4]{3}\,d\sqrt[3]{p}\sqrt{\frac{\sqrt[3]{q}\left(\sqrt[3]{q}+\sqrt[3]{p}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{q}+\sqrt[3]{p}\,x\right)^2}}\sqrt{q}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{q}+\sqrt[3]{p}\,x\right)\sqrt{\frac{q^{2/3}-\sqrt[3]{p}\sqrt[3]{q}\,x+p^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{q}+\sqrt[3]{p}\,x\right)^2}}\,F\left(\sin^{-1}\right.}{\sqrt[4]{3}\,d\sqrt[3]{p}\sqrt{\frac{\sqrt[3]{q}\left(\sqrt[3]{q}+\sqrt[3]{p}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{q}+\sqrt[3]{p}\,x\right)^2}}\sqrt{q}}$$

Mathematica [C] time = 6.79, size = 5105, normalized size = 121.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-2*a*q + 3*b*p*x^2 + a*p*x^3)/(Sqrt[q + p*x^3]*(b^2*c + d*q + 2*
a*b*c*x + a^2*c*x^2 + d*p*x^3)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 6.84, size = 42, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{d}\sqrt{px^3+q}}{\sqrt{c}\,(ax+b)}\right)}{\sqrt{c}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*a*q + 3*b*p*x^2 + a*p*x^3)/(Sqrt[q + p*x^3]*(b^2*c +
 d*q + 2*a*b*c*x + a^2*c*x^2 + d*p*x^3)),x]
```

```
[Out] (2*ArcTan[(Sqrt[d]*Sqrt[q + p*x^3])/(Sqrt[c]*(b + a*x))])/(Sqrt[c]*Sqrt[d])
```

fricas [B] time = 2.55, size = 459, normalized size = 10.93

$$\left[-\frac{\sqrt{-cd}\log\left(-\frac{6\,a^2cdpx^5-d^2p^2x^6-b^4c^2+6\,b^2cdq-\left(a^4c^2-12\,abcdp\right)x^4-d^2q^2-2\left(2\,a^3bc^2-3\,b^2cdp+d^2pq\right)x^3-6\left(a^2b^2c^2-a^2cdq\right)x^2+4\left(adpx^4-3\,a^2bcx^2-b^3c-\left(a^3c-bdp\right)x^3+bdq-\left(3\,ab^2c-adq\right)x\right)\sqrt{px^3+q}\sqrt{-cd}-4\left(ab^3c^2-3\,abcdq\right)x}{2\,a^2cdpx^5+d^2p^2x^6+b^4c^2+2\,b^2cdq+\left(a^4c^2+4\,abcdp\right)x^4+d^2q^2+2\left(2\,a^3bc^2+b^2cdp+d^2pq\right)x^3+2\left(3\,a^2b^2c^2+a^2cdq\right)x^2+4\left(ab^3c^2+abcdq\right)x}\right)}{2\,cd},\ \frac{\sqrt{cd}\arctan\left(-\frac{\left(a^2cx^2-dpx^3+2\,abcx+b^2c-dq\right)\sqrt{px^3+q}\sqrt{cd}}{2\left(acdpx^4+bcdpx^3+acdqx+bcdq\right)}\right)}{cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*p*x^3+3*b*p*x^2-2*a*q)/(p*x^3+q)^(1/2)/(a^2*c*x^2+d*p*x^3+2*a*
b*c*x+b^2*c+d*q),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-c*d)*log(-(6*a^2*c*d*p*x^5 - d^2*p^2*x^6 - b^4*c^2 + 6*b^2*c*d*
q - (a^4*c^2 - 12*a*b*c*d*p)*x^4 - d^2*q^2 - 2*(2*a^3*b*c^2 - 3*b^2*c*d*p +
 d^2*p*q)*x^3 - 6*(a^2*b^2*c^2 - a^2*c*d*q)*x^2 + 4*(a*d*p*x^4 - 3*a^2*b*c*
x^2 - b^3*c - (a^3*c - b*d*p)*x^3 + b*d*q - (3*a*b^2*c - a*d*q)*x)*sqrt(p*x
^3 + q)*sqrt(-c*d) - 4*(a*b^3*c^2 - 3*a*b*c*d*q)*x)/(2*a^2*c*d*p*x^5 + d^2*
p^2*x^6 + b^4*c^2 + 2*b^2*c*d*q + (a^4*c^2 + 4*a*b*c*d*p)*x^4 + d^2*q^2 + 2
*(2*a^3*b*c^2 + b^2*c*d*p + d^2*p*q)*x^3 + 2*(3*a^2*b^2*c^2 + a^2*c*d*q)*x^
2 + 4*(a*b^3*c^2 + a*b*c*d*q)*x))/(c*d), sqrt(c*d)*arctan(-1/2*(a^2*c*x^2 -
 d*p*x^3 + 2*a*b*c*x + b^2*c - d*q)*sqrt(p*x^3 + q)*sqrt(c*d)/(a*c*d*p*x^4
+ b*c*d*p*x^3 + a*c*d*q*x + b*c*d*q))/(c*d)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*p*x^3+3*b*p*x^2-2*a*q)/(p*x^3+q)^(1/2)/(a^2*c*x^2+d*p*x^3+2*a*
b*c*x+b^2*c+d*q),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.82, size = 2264, normalized size = 53.90

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*p*x^3+3*b*p*x^2-2*a*q)/(p*x^3+q)^(1/2)/(a^2*c*x^2+d*p*x^3+2*a*b*c*x+
b^2*c+d*q),x)
```

```
[Out] -2/3*I*a/d*3^(1/2)/p*(-q*p^2)^(1/3)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I*3^(1/2
)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2)*((x-1/p*(-q*p^2)^(1/3))
/(-3/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2)*(-I*(x+1/2/p
*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(
```

```
1/2)/(p*x^3+q)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I
*3^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2),(I*3^(1/2)/p*(-q
*p^2)^(1/3)/(-3/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2))-
I/d/p^2/c*2^(1/2)*sum((-_alpha^2*a^3*c+3*_alpha^2*b*d*p-2*_alpha*a^2*b*c-a*
b^2*c-3*a*d*q)/(-3*_alpha^2*d*p-2*_alpha*a^2*c-2*a*b*c)/(a^6*q^2-2*a^3*b^3*
p*q+b^6*p^2)*(-q*p^2)^(1/3)*(1/2*I*p*(2*x+1/p*(-I*3^(1/2)*(-q*p^2)^(1/3)+(-
q*p^2)^(1/3)))/(-q*p^2)^(1/3))^(1/2)*(p*(x-1/p*(-q*p^2)^(1/3))/(-3*(-q*p^2)
^(1/3)+I*3^(1/2)*(-q*p^2)^(1/3)))^(1/2)*(-1/2*I*p*(2*x+1/p*(I*3^(1/2)*(-q*p
^2)^(1/3)+(-q*p^2)^(1/3)))/(-q*p^2)^(1/3))^(1/2)/(p*x^3+q)^(1/2)*(-(-q*p^2)
^(2/3)*a^6*c*q+(-q*p^2)^(2/3)*b^4*d*p^2+2*I*(-q*p^2)^(2/3)*3^(1/2)*a^3*b*d*
q*p+3*I*(-q*p^2)^(1/3)*3^(1/2)*a^2*b^2*d*q*p^2+2*p^2*(-2*_alpha^2*a^3*b*d*p
*q-_alpha^2*b^4*d*p^2-2*_alpha*a^5*b*c*q-_alpha*a^2*b^4*c*p+3*_alpha*a^2*b^
2*d*p*q-a^4*b^2*c*q-2*a*b^5*c*p-a^4*d*q^2-2*a*b^3*d*p*q)-I*(-q*p^2)^(1/3)*p
^3*3^(1/2)*_alpha*b^4*d+2*(-q*p^2)^(1/3)*_alpha*a^3*b*d*p^2*q+2*I*(-q*p^2)^
(1/3)*p^3*3^(1/2)*_alpha^2*a*b^3*d+I*(-q*p^2)^(1/3)*3^(1/2)*_alpha^2*a^4*d*
q*p^2+I*(-q*p^2)^(1/3)*3^(1/2)*_alpha*a^6*q*p*c+2*I*(-q*p^2)^(1/3)*3^(1/2)*
_alpha*a^3*b^3*p^2*c-I*(-q*p^2)^(2/3)*3^(1/2)*_alpha*a^4*d*q*p-2*I*(-q*p^2)
^(2/3)*3^(1/2)*_alpha*a*b^3*d*p^2+3*I*(-q*p^2)^(2/3)*3^(1/2)*_alpha^2*a^2*b
^2*d*p^2+3*I*(-q*p^2)^(2/3)*3^(1/2)*_alpha*a^4*b^2*p*c+3*(-q*p^2)^(2/3)*_al
pha^2*a^2*b^2*d*p^2+3*(-q*p^2)^(2/3)*_alpha*a^4*b^2*c*p-(-q*p^2)^(1/3)*_alp
ha^2*a^4*d*p^2*q-2*(-q*p^2)^(1/3)*_alpha^2*a*b^3*d*p^3-(-q*p^2)^(1/3)*_alph
a*a^6*c*p*q-2*(-q*p^2)^(1/3)*_alpha*a^3*b^3*c*p^2-(-q*p^2)^(2/3)*_alpha*a^4
*d*p*q-2*(-q*p^2)^(2/3)*_alpha*a*b^3*d*p^2+4*(-q*p^2)^(2/3)*a^3*b^3*c*p-3*(
-q*p^2)^(1/3)*a^2*b^4*c*p^2+2*(-q*p^2)^(2/3)*a^3*b*d*p*q-3*(-q*p^2)^(1/3)*a
^2*b^2*d*p^2*q+I*(-q*p^2)^(2/3)*3^(1/2)*b^4*d*p^2-I*(-q*p^2)^(2/3)*3^(1/2)*
a^6*q*c+(-q*p^2)^(1/3)*_alpha*b^4*d*p^3+3*I*(-q*p^2)^(1/3)*3^(1/2)*a^2*b^4*
p^2*c+4*I*(-q*p^2)^(2/3)*3^(1/2)*a^3*b^3*p*c-2*I*(-q*p^2)^(1/3)*3^(1/2)*_al
pha*a^3*b*d*q*p^2)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I*
3^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2),-1/2/p*(6*a^3*b*d
*q^2*p^2-3*a^6*q^2*p*c+6*(-q*p^2)^(2/3)*_alpha*a^3*b*d*p*q+I*(-q*p^2)^(2/3)
*3^(1/2)*_alpha*b^4*d*p^2+I*3^(1/2)*_alpha*a^4*d*p^2*q^2-I*(-q*p^2)^(2/3)*3
^(1/2)*_alpha*a^6*c*q-2*I*(-q*p^2)^(1/3)*3^(1/2)*_alpha^2*b^4*d*p^3-9*(-q*p
^2)^(2/3)*a^2*b^2*d*p*q+I*3^(1/2)*a^6*c*p*q^2-I*3^(1/2)*b^4*d*p^3*q-I*(-q*p
^2)^(2/3)*3^(1/2)*_alpha^2*a^4*d*p*q-2*I*(-q*p^2)^(2/3)*3^(1/2)*_alpha^2*a*
b^3*d*p^2-2*I*(-q*p^2)^(2/3)*3^(1/2)*_alpha*a^3*b^3*c*p-2*I*(-q*p^2)^(1/3)*
3^(1/2)*_alpha*a^2*b^4*c*p^2-3*I*3^(1/2)*_alpha^2*a^2*b^2*d*p^3*q-3*I*3^(1/
2)*_alpha*a^4*b^2*c*p^2*q+2*I*3^(1/2)*_alpha*a*b^3*d*p^3*q+3*p^3*b^4*d*q-2*
I*(-q*p^2)^(1/3)*3^(1/2)*a^4*b^2*c*p*q-3*I*(-q*p^2)^(2/3)*3^(1/2)*a^2*b^2*d
*p*q-4*I*(-q*p^2)^(1/3)*3^(1/2)*a*b^3*d*p^2*q+9*p^3*_alpha^2*a^2*b^2*d*q+9*
_alpha*a^4*b^2*p^2*q*c-6*p^3*_alpha*a*b^3*d*q-3*(-q*p^2)^(2/3)*a^4*d*_alpha
^2*p*q-6*(-q*p^2)^(2/3)*a*d*_alpha^2*b^3*p^2-6*(-q*p^2)^(2/3)*_alpha*a^3*b^
3*p*c+12*a^3*b^3*p^2*q*c-9*(-q*p^2)^(2/3)*a^2*b^4*p*c-3*_alpha*a^4*d*q^2*p^
2-3*(-q*p^2)^(2/3)*_alpha*a^6*q*c+3*(-q*p^2)^(2/3)*_alpha*b^4*d*p^2-3*I*(-q
*p^2)^(2/3)*3^(1/2)*a^2*b^4*c*p-4*I*(-q*p^2)^(1/3)*3^(1/2)*a*b^5*c*p^2-4*I*
3^(1/2)*a^3*b^3*c*p^2*q-2*I*(-q*p^2)^(1/3)*3^(1/2)*a^4*d*p*q^2-2*I*3^(1/2)*
a^3*b*d*p^2*q^2-4*I*(-q*p^2)^(1/3)*3^(1/2)*_alpha^2*a^3*b*d*p^2*q-4*I*(-q*p
^2)^(1/3)*3^(1/2)*_alpha*a^5*b*c*p*q+2*I*(-q*p^2)^(2/3)*3^(1/2)*_alpha*a^3*
b*d*p*q+6*I*(-q*p^2)^(1/3)*3^(1/2)*_alpha*a^2*b^2*d*p^2*q)/(a^6*q^2-2*a^3*b
^3*p*q+b^6*p^2)/c,(I*3^(1/2)/p*(-q*p^2)^(1/3)/(-3/2/p*(-q*p^2)^(1/3)+1/2*I*
3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d*p+_Z^2*a^2*c+2*_Z*a*
b*c+b^2*c+d*q))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{apx^3 + 3\,bpx^2 - 2\,aq}{\left(a^2cx^2 + dpx^3 + 2\,abcx + b^2c + dq\right)\sqrt{px^3 + q}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*p*x^3+3*b*p*x^2-2*a*q)/(p*x^3+q)^(1/2)/(a^2*c*x^2+d*p*x^3+2*a*
```

```
b*c*x+b^2*c+d*q),x, algorithm="maxima")
```

```
[Out] integrate((a*p*x^3 + 3*b*p*x^2 - 2*a*q)/((a^2*c*x^2 + d*p*x^3 + 2*a*b*c*x +
 b^2*c + d*q)*sqrt(p*x^3 + q)), x)
```

mupad [B] time = 10.90, size = 456, normalized size = 10.86

$$\frac{\ln\left(\frac{\left(a^3\,b\,c\,1\mathrm{i}-b^2\,d\,p\,1\mathrm{i}+a^4\,c\,x\,1\mathrm{i}+2\,a^3\,\sqrt{c}\,\sqrt{d}\,\sqrt{p\,x^3+q}-a^2\,d\,p\,x^2\,1\mathrm{i}+a\,b\,d\,p\,x\,1\mathrm{i}\right)\,\left(a^3\,b^3\,c^2\,1\mathrm{i}+a^6\,c^2\,x^3\,1\mathrm{i}+a^4\,b^2\,c^2\,x\,3\mathrm{i}+a^5\,b\,c^2\,x^2\,3\mathrm{i}-b^4\,c\,d\,p\,1\mathrm{i}+b^2\,d^2\,p\,\left(p\,x^3+q\right)\,1\mathrm{i}+a^2\,d^2\,p\,x^2\,\left(p\,x^3+q\right)\,1\mathrm{i}+a^3\,b\,c\,d\,q\,1\mathrm{i}+a^3\,b\,c\,d\,\left(p\,x^3+q\right)\,2\mathrm{i}+a^4\,c\,d\,q\,x\,1\mathrm{i}+a^4\,c\,d\,x\,\left(p\,x^3+q\right)\,2\mathrm{i}+2\,a^3\,\sqrt{c}\,d^{3/2}\,q\,\sqrt{p\,x^3+q}-2\,b^3\,\sqrt{c}\,d^{3/2}\,p\,\sqrt{p\,x^3+q}-a\,b^3\,c\,d\,p\,x\,1\mathrm{i}-a\,b\,d^2\,p\,x\,\left(p\,x^3+q\right)\,1\mathrm{i}\right)}{\left(c\,a^2\,x^2+2\,c\,a\,b\,x+c\,b^2+d\,p\,x^3+d\,q\right)\,\left(a^8\,c^2\,x^2+2\,a^7\,b\,c^2\,x+a^6\,b^2\,c^2+2\,a^6\,c\,d\,p\,x^3+4\,q\,a^6\,c\,d+a^4\,d^2\,p^2\,x^4-2\,a^3\,b^3\,c\,d\,p-2\,a^3\,b\,d^2\,p^2\,x^3+3\,a^2\,b^2\,d^2\,p^2\,x^2-2\,a\,b^3\,d^2\,p^2\,x+b^4\,d^2\,p^2\right)}\right)\,1\mathrm{i}}{\sqrt{c}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*p*x^3 - 2*a*q + 3*b*p*x^2)/((q + p*x^3)^(1/2)*(d*q + b^2*c + d*p*x^3
 + a^2*c*x^2 + 2*a*b*c*x)),x)
```

```
[Out] (log(((a^3*b*c*1i - b^2*d*p*1i + a^4*c*x*1i + 2*a^3*c^(1/2)*d^(1/2)*(q + p*
x^3)^(1/2) - a^2*d*p*x^2*1i + a*b*d*p*x*1i)*(a^3*b^3*c^2*1i + a^6*c^2*x^3*1
i + a^4*b^2*c^2*x*3i + a^5*b*c^2*x^2*3i - b^4*c*d*p*1i + b^2*d^2*p*(q + p*x
^3)*1i + a^2*d^2*p*x^2*(q + p*x^3)*1i + a^3*b*c*d*q*1i + a^3*b*c*d*(q + p*x
^3)*2i + a^4*c*d*q*x*1i + a^4*c*d*x*(q + p*x^3)*2i + 2*a^3*c^(1/2)*d^(3/2)*
q*(q + p*x^3)^(1/2) - 2*b^3*c^(1/2)*d^(3/2)*p*(q + p*x^3)^(1/2) - a*b^3*c*d
*p*x*1i - a*b*d^2*p*x*(q + p*x^3)*1i))/((d*q + b^2*c + d*p*x^3 + a^2*c*x^2
+ 2*a*b*c*x)*(a^6*b^2*c^2 + b^4*d^2*p^2 + a^8*c^2*x^2 + 4*a^6*c*d*q + a^4*d
^2*p^2*x^4 + 2*a^7*b*c^2*x - 2*a^3*b^3*c*d*p + 2*a^6*c*d*p*x^3 + 3*a^2*b^2*
d^2*p^2*x^2 - 2*a*b^3*d^2*p^2*x - 2*a^3*b*d^2*p^2*x^3)))*1i)/(c^(1/2)*d^(1/
2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*p*x**3+3*b*p*x**2-2*a*q)/(p*x**3+q)**(1/2)/(a**2*c*x**2+d*p*x*
*3+2*a*b*c*x+b**2*c+d*q),x)
```

```
[Out] Timed out
```

3.539 $\int \frac{(-1+x^3)\sqrt{-1+x^6}}{x^7(1+x^3)}\,dx$

Optimal. Leaf size=42

$$\frac{(1-4x^3)\sqrt{x^6-1}}{6x^6}-\tan^{-1}\left(\frac{x^3+1}{\sqrt{x^6-1}}\right)$$

Rubi [F] time = 0.68, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^3)\sqrt{-1+x^6}}{x^7(1+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^3)*Sqrt[-1 + x^6])/(x^7*(1 + x^3)),x]
```

```
[Out] (-2*Sqrt[-1 + x^6])/3 + Sqrt[-1 + x^6]/(6*x^6) - (2*Sqrt[-1 + x^6])/(3*x^3)
 + ArcTan[Sqrt[-1 + x^6]]/2 + (2*ArcTanh[x^3/Sqrt[-1 + x^6]])/3 + (2*Defer[
Int][Sqrt[-1 + x^6]/(1 + x), x])/3 + (4*Defer[Int][Sqrt[-1 + x^6]/(-1 - I*S
qrt[3] + 2*x), x])/3 + (4*Defer[Int][Sqrt[-1 + x^6]/(-1 + I*Sqrt[3] + 2*x),
 x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^3)\sqrt{-1+x^6}}{x^7(1+x^3)}\,dx &= \int \left(-\frac{\sqrt{-1+x^6}}{x^7}+\frac{2\sqrt{-1+x^6}}{x^4}-\frac{2\sqrt{-1+x^6}}{x}+\frac{2\sqrt{-1+x^6}}{3(1+x)}+\frac{2(-1+2x)\sqrt{-1+}}{3(1-x+x^2)}\right.\\
&=\frac{2}{3}\int \frac{\sqrt{-1+x^6}}{1+x}\,dx+\frac{2}{3}\int \frac{(-1+2x)\sqrt{-1+x^6}}{1-x+x^2}\,dx+2\int \frac{\sqrt{-1+x^6}}{x^4}\,dx-2\int \frac{\sqrt{-}}{}\\
&=-\left(\frac{1}{6}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x^2}\,dx,x,x^6\right)\right)-\frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx,x,x^6\right)+\frac{2}{3}\int \frac{\sqrt{-}}{}\\
&=-\frac{2}{3}\sqrt{-1+x^6}+\frac{\sqrt{-1+x^6}}{6x^6}-\frac{2\sqrt{-1+x^6}}{3x^3}-\frac{1}{12}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx,x,x^6\right)+\\
&=-\frac{2}{3}\sqrt{-1+x^6}+\frac{\sqrt{-1+x^6}}{6x^6}-\frac{2\sqrt{-1+x^6}}{3x^3}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x^6}\right)\\
&=-\frac{2}{3}\sqrt{-1+x^6}+\frac{\sqrt{-1+x^6}}{6x^6}-\frac{2\sqrt{-1+x^6}}{3x^3}+\frac{1}{2}\tan^{-1}\left(\sqrt{-1+x^6}\right)+\frac{2}{3}\tanh^{-1}\left(\frac{}{\sqrt{-}}\right.
\end{aligned}$$

Mathematica [A] time = 0.15, size = 37, normalized size = 0.88

$$\frac{1}{6}\left(3\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}\,(1-4x^3)}{x^6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)*Sqrt[-1 + x^6])/(x^7*(1 + x^3)),x]
```

```
[Out] (((1 - 4*x^3)*Sqrt[-1 + x^6])/x^6 + 3*ArcTan[Sqrt[-1 + x^6]])/6
```

IntegrateAlgebraic [A] time = 0.18, size = 44, normalized size = 1.05

$$\frac{(1-4x^3)\sqrt{x^6-1}}{6x^6}-\tan^{-1}\left(\frac{\sqrt{x^6-1}}{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)*Sqrt[-1 + x^6])/(x^7*(1 + x^3)),x]
[Out] ((1 - 4*x^3)*Sqrt[-1 + x^6])/(6*x^6) - ArcTan[Sqrt[-1 + x^6]/(-1 + x^3)]
```

fricas [A] time = 0.64, size = 46, normalized size = 1.10

$$\frac{6\,x^6\arctan\left(-x^3+\sqrt{x^6-1}\right)-4\,x^6-\sqrt{x^6-1}\left(4\,x^3-1\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x^7/(x^3+1),x, algorithm="fricas")
[Out] 1/6*(6*x^6*arctan(-x^3 + sqrt(x^6 - 1)) - 4*x^6 - sqrt(x^6 - 1)*(4*x^3 - 1)
)/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^6-1}\left(x^3-1\right)}{\left(x^3+1\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x^7/(x^3+1),x, algorithm="giac")
[Out] integrate(sqrt(x^6 - 1)*(x^3 - 1)/((x^3 + 1)*x^7), x)
```

maple [A] time = 0.04, size = 37, normalized size = 0.88

$$-\frac{4x^9-x^6-4x^3+1}{6x^6\sqrt{x^6-1}}-\frac{\arcsin\left(\frac{1}{x^3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)*(x^6-1)^(1/2)/x^7/(x^3+1),x)
[Out] -1/6*(4*x^9-x^6-4*x^3+1)/x^6/(x^6-1)^(1/2)-1/2*arcsin(1/x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^6-1}\left(x^3-1\right)}{\left(x^3+1\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x^7/(x^3+1),x, algorithm="maxima")
[Out] integrate(sqrt(x^6 - 1)*(x^3 - 1)/((x^3 + 1)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(x^3-1\right)\sqrt{x^6-1}}{x^7\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)*(x^6 - 1)^(1/2))/(x^7*(x^3 + 1)),x)
```

```
[Out] int(((x^3 - 1)*(x^6 - 1)^(1/2))/(x^7*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,(x-1)\left(x^2+x+1\right)}{x^7\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)*(x**6-1)**(1/2)/x**7/(x**3+1),x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1))*(x - 1)*(x**2
+ x + 1)/(x**7*(x + 1)*(x**2 - x + 1)), x)
```

3.540 $\int \frac{(1+x^3)\sqrt{-1+x^6}}{x^7(-1+x^3)}\,dx$

Optimal. Leaf size=42

$$\frac{(4x^3+1)\sqrt{x^6-1}}{6x^6}-\tan^{-1}\left(\frac{x^3+1}{\sqrt{x^6-1}}\right)$$

Rubi [F] time = 0.63, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+x^3)\sqrt{-1+x^6}}{x^7(-1+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^3)*Sqrt[-1 + x^6])/(x^7*(-1 + x^3)),x]
```

```
[Out] (-2*Sqrt[-1 + x^6])/3 + Sqrt[-1 + x^6]/(6*x^6) + (2*Sqrt[-1 + x^6])/(3*x^3)
 + ArcTan[Sqrt[-1 + x^6]]/2 - (2*ArcTanh[x^3/Sqrt[-1 + x^6]])/3 + (2*Defer[
Int][Sqrt[-1 + x^6]/(-1 + x), x])/3 + (4*Defer[Int][Sqrt[-1 + x^6]/(1 - I*S
qrt[3] + 2*x), x])/3 + (4*Defer[Int][Sqrt[-1 + x^6]/(1 + I*Sqrt[3] + 2*x),
x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^3)\sqrt{-1+x^6}}{x^7(-1+x^3)}\,dx &= \int \left(\frac{2\sqrt{-1+x^6}}{3(-1+x)}-\frac{\sqrt{-1+x^6}}{x^7}-\frac{2\sqrt{-1+x^6}}{x^4}-\frac{2\sqrt{-1+x^6}}{x}+\frac{2(1+2x)\sqrt{-1+x^6}}{3(1+x+x^2)}\right)dx\\
&= \frac{2}{3}\int \frac{\sqrt{-1+x^6}}{-1+x}\,dx+\frac{2}{3}\int \frac{(1+2x)\sqrt{-1+x^6}}{1+x+x^2}\,dx-2\int \frac{\sqrt{-1+x^6}}{x^4}\,dx-2\int \frac{\sqrt{-1+x^6}}{x}\\
&= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x^2}\,dx,x,x^6\right)\right)-\frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx,x,x^6\right)+\frac{2}{3}\int \frac{\sqrt{-1+}}{-1+}\\
&= -\frac{2}{3}\sqrt{-1+x^6}+\frac{\sqrt{-1+x^6}}{6x^6}+\frac{2\sqrt{-1+x^6}}{3x^3}-\frac{1}{12}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx,x,x^6\right)+\frac{1}{3}\text{Su}\\
&= -\frac{2}{3}\sqrt{-1+x^6}+\frac{\sqrt{-1+x^6}}{6x^6}+\frac{2\sqrt{-1+x^6}}{3x^3}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x^6}\right)+\frac{2}{3}\\
&= -\frac{2}{3}\sqrt{-1+x^6}+\frac{\sqrt{-1+x^6}}{6x^6}+\frac{2\sqrt{-1+x^6}}{3x^3}+\frac{1}{2}\tan^{-1}\left(\sqrt{-1+x^6}\right)-\frac{2}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+}}\right.
\end{aligned}$$

Mathematica [A] time = 0.12, size = 37, normalized size = 0.88

$$\frac{1}{6}\left(3\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}\,(4x^3+1)}{x^6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)*Sqrt[-1 + x^6])/(x^7*(-1 + x^3)),x]
```

```
[Out] (((1 + 4*x^3)*Sqrt[-1 + x^6])/x^6 + 3*ArcTan[Sqrt[-1 + x^6]])/6
```

IntegrateAlgebraic [A] time = 0.16, size = 44, normalized size = 1.05

$$\frac{(4x^3+1)\sqrt{x^6-1}}{6x^6} - \tan^{-1}\left(\frac{\sqrt{x^6-1}}{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)*Sqrt[-1 + x^6])/(x^7*(-1 + x^3)),x]
[Out] ((1 + 4*x^3)*Sqrt[-1 + x^6])/(6*x^6) - ArcTan[Sqrt[-1 + x^6]/(-1 + x^3)]
```

fricas [A] time = 0.81, size = 45, normalized size = 1.07

$$\frac{6\,x^6\arctan\left(-x^3+\sqrt{x^6-1}\right)+4\,x^6+\sqrt{x^6-1}\left(4\,x^3+1\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(x^6-1)^(1/2)/x^7/(x^3-1),x, algorithm="fricas")
[Out] 1/6*(6*x^6*arctan(-x^3 + sqrt(x^6 - 1)) + 4*x^6 + sqrt(x^6 - 1)*(4*x^3 + 1)
)/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^6-1}\left(x^3+1\right)}{\left(x^3-1\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(x^6-1)^(1/2)/x^7/(x^3-1),x, algorithm="giac")
[Out] integrate(sqrt(x^6 - 1)*(x^3 + 1)/((x^3 - 1)*x^7), x)
```

maple [A] time = 0.03, size = 35, normalized size = 0.83

$$\frac{4x^9+x^6-4x^3-1}{6x^6\sqrt{x^6-1}} - \frac{\arcsin\left(\frac{1}{x^3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)*(x^6-1)^(1/2)/x^7/(x^3-1),x)
[Out] 1/6*(4*x^9+x^6-4*x^3-1)/x^6/(x^6-1)^(1/2)-1/2*arcsin(1/x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^6-1}\left(x^3+1\right)}{\left(x^3-1\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(x^6-1)^(1/2)/x^7/(x^3-1),x, algorithm="maxima")
[Out] integrate(sqrt(x^6 - 1)*(x^3 + 1)/((x^3 - 1)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x^3+1\right)\sqrt{x^6-1}}{x^7\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)*(x^6 - 1)^(1/2))/(x^7*(x^3 - 1)),x)
```

```
[Out] int(((x^3 + 1)*(x^6 - 1)^(1/2))/(x^7*(x^3 - 1)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)(x^2-x+1)(x^2+x+1)}\,(x+1)(x^2-x+1)}{x^7(x-1)(x^2+x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)*(x**6-1)**(1/2)/x**7/(x**3-1),x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1))*(x + 1)*(x**2
- x + 1)/(x**7*(x - 1)*(x**2 + x + 1)), x)
```

3.541 $\int \frac{\sqrt[4]{1+x^6}}{x} dx$

Optimal. Leaf size=42

$$\frac{2}{3}\sqrt[4]{x^6+1} - \frac{1}{3}\tan^{-1}\left(\sqrt[4]{x^6+1}\right) - \frac{1}{3}\tanh^{-1}\left(\sqrt[4]{x^6+1}\right)$$

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 50, 63, 212, 206, 203}

$$\frac{2}{3}\sqrt[4]{x^6+1} - \frac{1}{3}\tan^{-1}\left(\sqrt[4]{x^6+1}\right) - \frac{1}{3}\tanh^{-1}\left(\sqrt[4]{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)^(1/4)/x,x]
```

```
[Out] (2*(1 + x^6)^(1/4))/3 - ArcTan[(1 + x^6)^(1/4)]/3 - ArcTanh[(1 + x^6)^(1/4)
]/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{1+x^6}}{x}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt[4]{1+x}}{x}\,dx, x, x^6\right)\\
&= \frac{2}{3}\sqrt[4]{1+x^6} + \frac{1}{6}\text{Subst}\left(\int \frac{1}{x(1+x)^{3/4}}\,dx, x, x^6\right)\\
&= \frac{2}{3}\sqrt[4]{1+x^6} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt[4]{1+x^6}\right)\\
&= \frac{2}{3}\sqrt[4]{1+x^6} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1+x^6}\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{1+x^6}\right)\\
&= \frac{2}{3}\sqrt[4]{1+x^6} - \frac{1}{3}\tan^{-1}\left(\sqrt[4]{1+x^6}\right) - \frac{1}{3}\tanh^{-1}\left(\sqrt[4]{1+x^6}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{2}{3}\sqrt[4]{x^6+1} - \frac{1}{3}\tan^{-1}\left(\sqrt[4]{x^6+1}\right) - \frac{1}{3}\tanh^{-1}\left(\sqrt[4]{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)^(1/4)/x,x]
[Out] (2*(1 + x^6)^(1/4))/3 - ArcTan[(1 + x^6)^(1/4)]/3 - ArcTanh[(1 + x^6)^(1/4)
]/3
```

IntegrateAlgebraic [A] time = 0.03, size = 42, normalized size = 1.00

$$\frac{2}{3}\sqrt[4]{x^6+1} - \frac{1}{3}\tan^{-1}\left(\sqrt[4]{x^6+1}\right) - \frac{1}{3}\tanh^{-1}\left(\sqrt[4]{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^6)^(1/4)/x,x]
[Out] (2*(1 + x^6)^(1/4))/3 - ArcTan[(1 + x^6)^(1/4)]/3 - ArcTanh[(1 + x^6)^(1/4)
]/3
```

fricas [A] time = 0.74, size = 44, normalized size = 1.05

$$\frac{2}{3}\left(x^6+1\right)^{\frac{1}{4}} - \frac{1}{3}\arctan\left(\left(x^6+1\right)^{\frac{1}{4}}\right) - \frac{1}{6}\log\left(\left(x^6+1\right)^{\frac{1}{4}}+1\right) + \frac{1}{6}\log\left(\left(x^6+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/4)/x,x, algorithm="fricas")
[Out] 2/3*(x^6 + 1)^(1/4) - 1/3*arctan((x^6 + 1)^(1/4)) - 1/6*log((x^6 + 1)^(1/4)
 + 1) + 1/6*log((x^6 + 1)^(1/4) - 1)
```

giac [A] time = 1.93, size = 44, normalized size = 1.05

$$\frac{2}{3}\left(x^6+1\right)^{\frac{1}{4}} - \frac{1}{3}\arctan\left(\left(x^6+1\right)^{\frac{1}{4}}\right) - \frac{1}{6}\log\left(\left(x^6+1\right)^{\frac{1}{4}}+1\right) + \frac{1}{6}\log\left(\left(x^6+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/4)/x,x, algorithm="giac")
```

```
[Out] 2/3*(x^6 + 1)^(1/4) - 1/3*arctan((x^6 + 1)^(1/4)) - 1/6*log((x^6 + 1)^(1/4)
 + 1) + 1/6*log((x^6 + 1)^(1/4) - 1)
```

maple [C] time = 0.22, size = 45, normalized size = 1.07

$$-\frac{-\Gamma\left(\frac{3}{4}\right)x^{6}\,\mathrm{hypergeom}\left(\left[\frac{3}{4},1,1\right],[2,2],-x^{6}\right)-4\left(4-3\ln(2)+\frac{\pi}{2}+6\ln(x)\right)\Gamma\left(\frac{3}{4}\right)}{24\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)^(1/4)/x,x)
```

```
[Out] -1/24/GAMMA(3/4)*(-GAMMA(3/4)*x^6*hypergeom([3/4,1,1],[2,2],-x^6)-4*(4-3*ln
(2)+1/2*Pi+6*ln(x))*GAMMA(3/4))
```

maxima [A] time = 0.56, size = 44, normalized size = 1.05

$$\frac{2}{3}\left(x^{6}+1\right)^{\frac{1}{4}}-\frac{1}{3}\arctan\left(\left(x^{6}+1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(x^{6}+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(x^{6}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/4)/x,x, algorithm="maxima")
```

```
[Out] 2/3*(x^6 + 1)^(1/4) - 1/3*arctan((x^6 + 1)^(1/4)) - 1/6*log((x^6 + 1)^(1/4)
 + 1) + 1/6*log((x^6 + 1)^(1/4) - 1)
```

mupad [B] time = 0.31, size = 30, normalized size = 0.71

$$\frac{2\left(x^{6}+1\right)^{1/4}}{3}-\frac{\mathrm{atanh}\left(\left(x^{6}+1\right)^{1/4}\right)}{3}-\frac{\mathrm{atan}\left(\left(x^{6}+1\right)^{1/4}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)^(1/4)/x,x)
```

```
[Out] (2*(x^6 + 1)^(1/4))/3 - atanh((x^6 + 1)^(1/4))/3 - atan((x^6 + 1)^(1/4))/3
```

sympy [C] time = 0.79, size = 37, normalized size = 0.88

$$-\frac{x^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{e^{i\pi}}{x^{6}}\right)}{6\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)**(1/4)/x,x)
```

```
[Out] -x**(3/2)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), exp_polar(I*pi)/x**6)/(6*
gamma(3/4))
```

3.542 $\int \frac{1+x^{12}}{x^4\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{x^6-1}\left(x^6+2\right)}{6x^3}+\frac{1}{3}\tanh^{-1}\left(\frac{x^3+1}{\sqrt{x^6-1}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1487, 451, 275, 217, 206}

$$\frac{1}{6}\sqrt{x^6-1}\,x^3+\frac{\sqrt{x^6-1}}{3x^3}+\frac{1}{6}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^12)/(x^4*Sqrt[-1 + x^6]),x]
```

```
[Out] Sqrt[-1 + x^6]/(3*x^3) + (x^3*Sqrt[-1 + x^6])/6 + ArcTanh[x^3/Sqrt[-1 + x^6
]]/6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1487

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 2*n*p - n + 1)*(d + e*x^n)^(
q + 1))/(e*f^(2*n*p - n + 1)*(m + 2*n*p + n*q + 1)), x] + Dist[1/(e*(m + 2*
n*p + n*q + 1)), Int[(f*x)^m*(d + e*x^n)^q*ExpandToSum[e*(m + 2*n*p + n*q +
 1)*((a + c*x^(2*n))^p - c^p*x^(2*n*p)) - d*c^p*(m + 2*n*p - n + 1)*x^(2*n*
p - n), x], x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IG
tQ[n, 0] && IGtQ[p, 0] && GtQ[2*n*p, n - 1] &&  !IntegerQ[q] && NeQ[m + 2*n
*p + n*q + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^{12}}{x^4\sqrt{-1+x^6}}\,dx &= \frac{1}{6}x^3\sqrt{-1+x^6}+\frac{1}{6}\int \frac{6+3x^6}{x^4\sqrt{-1+x^6}}\,dx\\ &= \frac{\sqrt{-1+x^6}}{3x^3}+\frac{1}{6}x^3\sqrt{-1+x^6}+\frac{1}{2}\int \frac{x^2}{\sqrt{-1+x^6}}\,dx\\ &= \frac{\sqrt{-1+x^6}}{3x^3}+\frac{1}{6}x^3\sqrt{-1+x^6}+\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx,x,x^3\right)\\ &= \frac{\sqrt{-1+x^6}}{3x^3}+\frac{1}{6}x^3\sqrt{-1+x^6}+\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x^3}{\sqrt{-1+x^6}}\right)\\ &= \frac{\sqrt{-1+x^6}}{3x^3}+\frac{1}{6}x^3\sqrt{-1+x^6}+\frac{1}{6}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.88

$$\frac{1}{6}\left(\frac{\sqrt{x^6-1}\,(x^6+2)}{x^3}+\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^12)/(x^4*Sqrt[-1 + x^6]),x]
[Out] ((Sqrt[-1 + x^6]*(2 + x^6))/x^3 + ArcTanh[x^3/Sqrt[-1 + x^6]])/6
```

IntegrateAlgebraic [A] time = 0.25, size = 42, normalized size = 1.00

$$\frac{\sqrt{x^6-1}\,(x^6+2)}{6x^3}+\frac{1}{3}\tanh^{-1}\left(\frac{x^3+1}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^12)/(x^4*Sqrt[-1 + x^6]),x]
[Out] (Sqrt[-1 + x^6]*(2 + x^6))/(6*x^3) + ArcTanh[(1 + x^3)/Sqrt[-1 + x^6]]/3
```

fricas [A] time = 0.83, size = 43, normalized size = 1.02

$$-\frac{x^3\log\left(-x^3+\sqrt{x^6-1}\right)-2\,x^3-(x^6+2)\sqrt{x^6-1}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^12+1)/x^4/(x^6-1)^(1/2),x, algorithm="fricas")
[Out] -1/6*(x^3*log(-x^3 + sqrt(x^6 - 1)) - 2*x^3 - (x^6 + 2)*sqrt(x^6 - 1))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}+1}{\sqrt{x^6-1}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^12+1)/x^4/(x^6-1)^(1/2),x, algorithm="giac")
[Out] integrate((x^12 + 1)/(sqrt(x^6 - 1)*x^4), x)
```

maple [C] time = 0.03, size = 46, normalized size = 1.10

$$\frac{x^{12}+x^{6}-2}{6x^{3}\sqrt{x^{6}-1}}+\frac{\sqrt{-\mathrm{signum}\left(x^{6}-1\right)}\,\arcsin\left(x^{3}\right)}{6\sqrt{\mathrm{signum}\left(x^{6}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^12+1)/x^4/(x^6-1)^(1/2),x)
```

```
[Out] 1/6*(x^12+x^6-2)/x^3/(x^6-1)^(1/2)+1/6/signum(x^6-1)^(1/2)*(-signum(x^6-1))
^(1/2)*arcsin(x^3)
```

maxima [B] time = 0.54, size = 70, normalized size = 1.67

$$\frac{\sqrt{x^{6}-1}}{3\,x^{3}}-\frac{\sqrt{x^{6}-1}}{6\,x^{3}\left(\frac{x^{6}-1}{x^{6}}-1\right)}+\frac{1}{12}\,\log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}+1\right)-\frac{1}{12}\,\log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^12+1)/x^4/(x^6-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^6 - 1)/x^3 - 1/6*sqrt(x^6 - 1)/(x^3*((x^6 - 1)/x^6 - 1)) + 1/12*
log(sqrt(x^6 - 1)/x^3 + 1) - 1/12*log(sqrt(x^6 - 1)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{12}+1}{x^{4}\,\sqrt{x^{6}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^12 + 1)/(x^4*(x^6 - 1)^(1/2)),x)
```

```
[Out] int((x^12 + 1)/(x^4*(x^6 - 1)^(1/2)), x)
```

sympy [C] time = 3.54, size = 92, normalized size = 2.19

$$\begin{cases} \frac{i\sqrt{-1+\frac{1}{x^{6}}}}{3} & \text{for}\: \frac{1}{\left|x^{6}\right|} > 1 \\ \frac{\sqrt{1-\frac{1}{x^{6}}}}{3} & \text{otherwise} \end{cases} + \begin{cases} \frac{x^{3}\sqrt{x^{6}-1}}{6}+\frac{\operatorname{acosh}\left(x^{3}\right)}{6} & \text{for}\: \left|x^{6}\right| > 1 \\ -\frac{ix^{9}}{6\sqrt{1-x^{6}}}+\frac{ix^{3}}{6\sqrt{1-x^{6}}}-\frac{i\operatorname{asin}\left(x^{3}\right)}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**12+1)/x**4/(x**6-1)**(1/2),x)
```

```
[Out] Piecewise((I*sqrt(-1 + x**(-6))/3, 1/Abs(x**6) > 1), (sqrt(1 - 1/x**6)/3, T
rue)) + Piecewise((x**3*sqrt(x**6 - 1)/6 + acosh(x**3)/6, Abs(x**6) > 1), (
-I*x**9/(6*sqrt(1 - x**6)) + I*x**3/(6*sqrt(1 - x**6)) - I*asin(x**3)/6, Tr
ue))
```

3.543 $\int \frac{1}{x^{10}\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=43

$$\frac{5}{24}\tanh^{-1}\left(\sqrt{x^3+1}\right)+\frac{\sqrt{x^3+1}\left(-15x^6+10x^3-8\right)}{72x^9}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.47, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 51, 63, 207}

$$-\frac{5\sqrt{x^3+1}}{24x^3}+\frac{5}{24}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{\sqrt{x^3+1}}{9x^9}+\frac{5\sqrt{x^3+1}}{36x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*Sqrt[1 + x^3]),x]
```

```
[Out] -1/9*Sqrt[1 + x^3]/x^9 + (5*Sqrt[1 + x^3])/(36*x^6) - (5*Sqrt[1 + x^3])/(24
*x^3) + (5*ArcTanh[Sqrt[1 + x^3]])/24
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}\sqrt{1+x^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^4\sqrt{1+x}}\,dx,x,x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{9x^9}-\frac{5}{18}\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+x}}\,dx,x,x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{9x^9}+\frac{5\sqrt{1+x^3}}{36x^6}+\frac{5}{24}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx,x,x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{9x^9}+\frac{5\sqrt{1+x^3}}{36x^6}-\frac{5\sqrt{1+x^3}}{24x^3}-\frac{5}{48}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{9x^9}+\frac{5\sqrt{1+x^3}}{36x^6}-\frac{5\sqrt{1+x^3}}{24x^3}-\frac{5}{24}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^3}\right)\\ &= -\frac{\sqrt{1+x^3}}{9x^9}+\frac{5\sqrt{1+x^3}}{36x^6}-\frac{5\sqrt{1+x^3}}{24x^3}+\frac{5}{24}\tanh^{-1}\left(\sqrt{1+x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 26, normalized size = 0.60

$$\frac{2}{3}\sqrt{x^3+1}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*Sqrt[1 + x^3]),x]
[Out] (2*Sqrt[1 + x^3]*Hypergeometric2F1[1/2, 4, 3/2, 1 + x^3])/3
```

IntegrateAlgebraic [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{5}{24}\tanh^{-1}\left(\sqrt{x^3+1}\right)+\frac{\sqrt{x^3+1}\left(-15x^6+10x^3-8\right)}{72x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^10*Sqrt[1 + x^3]),x]
[Out] (Sqrt[1 + x^3]*(-8 + 10*x^3 - 15*x^6))/(72*x^9) + (5*ArcTanh[Sqrt[1 + x^3]]
)/24
```

fricas [A] time = 0.71, size = 57, normalized size = 1.33

$$\frac{15\,x^9\log\left(\sqrt{x^3+1}+1\right)-15\,x^9\log\left(\sqrt{x^3+1}-1\right)-2\left(15\,x^6-10\,x^3+8\right)\sqrt{x^3+1}}{144\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] 1/144*(15*x^9*log(sqrt(x^3 + 1) + 1) - 15*x^9*log(sqrt(x^3 + 1) - 1) - 2*(1
5*x^6 - 10*x^3 + 8)*sqrt(x^3 + 1))/x^9
```

giac [A] time = 0.30, size = 59, normalized size = 1.37

$$-\frac{15\left(x^3+1\right)^{\frac{5}{2}}-40\left(x^3+1\right)^{\frac{3}{2}}+33\sqrt{x^3+1}}{72\,x^9}+\frac{5}{48}\log\left(\sqrt{x^3+1}+1\right)-\frac{5}{48}\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/72*(15*(x^3 + 1)^(5/2) - 40*(x^3 + 1)^(3/2) + 33*sqrt(x^3 + 1))/x^9 + 5/
48*log(sqrt(x^3 + 1) + 1) - 5/48*log(abs(sqrt(x^3 + 1) - 1))
```

maple [A] time = 0.03, size = 48, normalized size = 1.12

$$-\frac{\sqrt{x^3+1}}{9x^9}+\frac{5\sqrt{x^3+1}}{36x^6}-\frac{5\sqrt{x^3+1}}{24x^3}+\frac{5\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(x^3+1)^(1/2),x)
```

```
[Out] -1/9*(x^3+1)^(1/2)/x^9+5/36*(x^3+1)^(1/2)/x^6-5/24*(x^3+1)^(1/2)/x^3+5/24*a
rctanh((x^3+1)^(1/2))
```

maxima [B] time = 0.40, size = 80, normalized size = 1.86

$$-\frac{15\left(x^3+1\right)^{\frac{5}{2}}-40\left(x^3+1\right)^{\frac{3}{2}}+33\sqrt{x^3+1}}{72\left(\left(x^3+1\right)^3+3x^3-3\left(x^3+1\right)^2+2\right)}+\frac{5}{48}\log\left(\sqrt{x^3+1}+1\right)-\frac{5}{48}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/72*(15*(x^3 + 1)^(5/2) - 40*(x^3 + 1)^(3/2) + 33*sqrt(x^3 + 1))/((x^3 +
1)^3 + 3*x^3 - 3*(x^3 + 1)^2 + 2) + 5/48*log(sqrt(x^3 + 1) + 1) - 5/48*log(
sqrt(x^3 + 1) - 1)
```

mupad [B] time = 0.05, size = 201, normalized size = 4.67

$$\frac{5\sqrt{x^3+1}}{36\,x^6}-\frac{5\sqrt{x^3+1}}{24\,x^3}-\frac{\sqrt{x^3+1}}{9\,x^9}+\frac{5\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{8\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(x^3 + 1)^(1/2)),x)
```

```
[Out] (5*(x^3 + 1)^(1/2))/(36*x^6) - (5*(x^3 + 1)^(1/2))/(24*x^3) - (x^3 + 1)^(1/
2)/(9*x^9) + (5*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2
)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)
/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2
, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3
^(1/2)*1i)/2 - 3/2)))/(8*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [B] time = 3.93, size = 85, normalized size = 1.98

$$\frac{5\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{24}-\frac{5}{24x^{\frac{3}{2}}\sqrt{1+\frac{1}{x^3}}}-\frac{5}{72x^{\frac{9}{2}}\sqrt{1+\frac{1}{x^3}}}+\frac{1}{36x^{\frac{15}{2}}\sqrt{1+\frac{1}{x^3}}}-\frac{1}{9x^{\frac{21}{2}}\sqrt{1+\frac{1}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(x**3+1)**(1/2),x)
```

```
[Out] 5*asinh(x**(-3/2))/24 - 5/(24*x**(3/2)*sqrt(1 + x**(-3))) - 5/(72*x**(9/2)*
sqrt(1 + x**(-3))) + 1/(36*x**(15/2)*sqrt(1 + x**(-3))) - 1/(9*x**(21/2)*sq
rt(1 + x**(-3)))
```

3.544 $\int \frac{-1+x^3}{x^6 \sqrt[3]{x^2+x^3}} \, dx$

Optimal. Leaf size=43

$$-\frac{3\left(x^3+x^2\right)^{2/3}\left(19071x^5-12714x^4+10595x^3-3600x^2+3300x-3080\right)}{52360x^7}$$

Rubi [B] time = 0.32, antiderivative size = 109, normalized size of antiderivative = 2.53, number of steps used = 11, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2052, 2016, 2014}

$$-\frac{57213\left(x^3+x^2\right)^{2/3}}{52360x^2}+\frac{19071\left(x^3+x^2\right)^{2/3}}{26180x^3}+\frac{3\left(x^3+x^2\right)^{2/3}}{17x^7}-\frac{45\left(x^3+x^2\right)^{2/3}}{238x^6}+\frac{270\left(x^3+x^2\right)^{2/3}}{1309x^5}-\frac{6357\left(x^3+x^2\right)^{2/3}}{10472x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(x^6*(x^2 + x^3)^(1/3)),x]
```

```
[Out] (3*(x^2 + x^3)^(2/3))/(17*x^7) - (45*(x^2 + x^3)^(2/3))/(238*x^6) + (270*(x
^2 + x^3)^(2/3))/(1309*x^5) - (6357*(x^2 + x^3)^(2/3))/(10472*x^4) + (19071
*(x^2 + x^3)^(2/3))/(26180*x^3) - (57213*(x^2 + x^3)^(2/3))/(52360*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{-1+x^3}{x^6\sqrt[3]{x^2+x^3}}\,dx = \int \left(-\frac{1}{x^6\sqrt[3]{x^2+x^3}}+\frac{1}{x^3\sqrt[3]{x^2+x^3}}\right)dx$$
$$= -\int \frac{1}{x^6\sqrt[3]{x^2+x^3}}\,dx + \int \frac{1}{x^3\sqrt[3]{x^2+x^3}}\,dx$$
$$= \frac{3(x^2+x^3)^{2/3}}{17x^7} - \frac{3(x^2+x^3)^{2/3}}{8x^4} - \frac{3}{4}\int \frac{1}{x^2\sqrt[3]{x^2+x^3}}\,dx + \frac{15}{17}\int \frac{1}{x^5\sqrt[3]{x^2+x^3}}\,dx$$
$$= \frac{3(x^2+x^3)^{2/3}}{17x^7} - \frac{45(x^2+x^3)^{2/3}}{238x^6} - \frac{3(x^2+x^3)^{2/3}}{8x^4} + \frac{9(x^2+x^3)^{2/3}}{20x^3} + \frac{9}{20}\int \frac{1}{x\sqrt[3]{x^2+x^3}}\,dx$$
$$= \frac{3(x^2+x^3)^{2/3}}{17x^7} - \frac{45(x^2+x^3)^{2/3}}{238x^6} + \frac{270(x^2+x^3)^{2/3}}{1309x^5} - \frac{3(x^2+x^3)^{2/3}}{8x^4} + \frac{9(x^2+x^3)^{2/3}}{20x^3} - 2$$
$$= \frac{3(x^2+x^3)^{2/3}}{17x^7} - \frac{45(x^2+x^3)^{2/3}}{238x^6} + \frac{270(x^2+x^3)^{2/3}}{1309x^5} - \frac{6357(x^2+x^3)^{2/3}}{10472x^4} + \frac{9(x^2+x^3)^{2/3}}{20x^3}$$
$$= \frac{3(x^2+x^3)^{2/3}}{17x^7} - \frac{45(x^2+x^3)^{2/3}}{238x^6} + \frac{270(x^2+x^3)^{2/3}}{1309x^5} - \frac{6357(x^2+x^3)^{2/3}}{10472x^4} + \frac{19071(x^2+x^3}{26180x^3}$$
$$= \frac{3(x^2+x^3)^{2/3}}{17x^7} - \frac{45(x^2+x^3)^{2/3}}{238x^6} + \frac{270(x^2+x^3)^{2/3}}{1309x^5} - \frac{6357(x^2+x^3)^{2/3}}{10472x^4} + \frac{19071(x^2+x^3}{26180x^3}$$

Mathematica [A] time = 0.06, size = 43, normalized size = 1.00

$$-\frac{3\left(x^2(x+1)\right)^{2/3}\left(19071x^5-12714x^4+10595x^3-3600x^2+3300x-3080\right)}{52360x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(x^6*(x^2 + x^3)^(1/3)),x]
```

```
[Out] (-3*(x^2*(1 + x))^(2/3)*(-3080 + 3300*x - 3600*x^2 + 10595*x^3 - 12714*x^4
+ 19071*x^5))/(52360*x^7)
```

IntegrateAlgebraic [A] time = 0.28, size = 43, normalized size = 1.00

$$-\frac{3\left(x^3+x^2\right)^{2/3}\left(19071x^5-12714x^4+10595x^3-3600x^2+3300x-3080\right)}{52360x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)/(x^6*(x^2 + x^3)^(1/3)),x]
```

```
[Out] (-3*(x^2 + x^3)^(2/3)*(-3080 + 3300*x - 3600*x^2 + 10595*x^3 - 12714*x^4 +
19071*x^5))/(52360*x^7)
```

fricas [A] time = 0.65, size = 39, normalized size = 0.91

$$-\frac{3\left(19071\,x^5-12714\,x^4+10595\,x^3-3600\,x^2+3300\,x-3080\right)\left(x^3+x^2\right)^{\frac{2}{3}}}{52360\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/x^6/(x^3+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/52360*(19071*x^5 - 12714*x^4 + 10595*x^3 - 3600*x^2 + 3300*x - 3080)*(x^
3 + x^2)^(2/3)/x^7
```

giac [A] time = 0.45, size = 55, normalized size = 1.28

$$\frac{3}{17}\left(\frac{1}{x}+1\right)^{\frac{17}{3}}-\frac{15}{14}\left(\frac{1}{x}+1\right)^{\frac{14}{3}}+\frac{30}{11}\left(\frac{1}{x}+1\right)^{\frac{11}{3}}-\frac{33}{8}\left(\frac{1}{x}+1\right)^{\frac{8}{3}}+\frac{21}{5}\left(\frac{1}{x}+1\right)^{\frac{5}{3}}-3\left(\frac{1}{x}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/x^6/(x^3+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] 3/17*(1/x + 1)^(17/3) - 15/14*(1/x + 1)^(14/3) + 30/11*(1/x + 1)^(11/3) - 3
3/8*(1/x + 1)^(8/3) + 21/5*(1/x + 1)^(5/3) - 3*(1/x + 1)^(2/3)
```

maple [A] time = 0.01, size = 43, normalized size = 1.00

$$-\frac{3\left(19071x^{5}-12714x^{4}+10595x^{3}-3600x^{2}+3300x-3080\right)(1+x)}{52360\left(x^{3}+x^{2}\right)^{\frac{1}{3}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/x^6/(x^3+x^2)^(1/3),x)
```

```
[Out] -3/52360*(19071*x^5-12714*x^4+10595*x^3-3600*x^2+3300*x-3080)*(1+x)/(x^3+x^
2)^(1/3)/x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}-1}{\left(x^{3}+x^{2}\right)^{\frac{1}{3}}x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/x^6/(x^3+x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 1)/((x^3 + x^2)^(1/3)*x^6), x)
```

mupad [B] time = 0.30, size = 85, normalized size = 1.98

$$\frac{19071\left(x^{3}+x^{2}\right)^{2/3}}{26180\,x^{3}}-\frac{57213\left(x^{3}+x^{2}\right)^{2/3}}{52360\,x^{2}}-\frac{6357\left(x^{3}+x^{2}\right)^{2/3}}{10472\,x^{4}}+\frac{270\left(x^{3}+x^{2}\right)^{2/3}}{1309\,x^{5}}-\frac{45\left(x^{3}+x^{2}\right)^{2/3}}{238\,x^{6}}+\frac{3\left(x^{3}+x^{2}\right)^{2/3}}{17\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x^6*(x^2 + x^3)^(1/3)),x)
```

```
[Out] (19071*(x^2 + x^3)^(2/3))/(26180*x^3) - (57213*(x^2 + x^3)^(2/3))/(52360*x^
2) - (6357*(x^2 + x^3)^(2/3))/(10472*x^4) + (270*(x^2 + x^3)^(2/3))/(1309*x
^5) - (45*(x^2 + x^3)^(2/3))/(238*x^6) + (3*(x^2 + x^3)^(2/3))/(17*x^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\left(x^{2}+x+1\right)}{x^{6}\sqrt[3]{x^{2}(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/x**6/(x**3+x**2)**(1/3),x)
```

```
[Out] Integral((x - 1)*(x**2 + x + 1)/(x**6*(x**2*(x + 1))**(1/3)), x)
```

3.545 $\int \frac{(-2+x^3)\sqrt{1+x^3}(2+x^2+2x^3)}{x^4(1+x^2+x^3)}\,dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{x^3+1}\,(2x^3-3x^2+2)}{3x^3}-2\tan^{-1}\left(\frac{x}{\sqrt{x^3+1}}\right)$$

Rubi [F] time = 0.96, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+x^3)\sqrt{1+x^3}\,(2+x^2+2x^3)}{x^4\,(1+x^2+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + x^3)*Sqrt[1 + x^3]*(2 + x^2 + 2*x^3))/(x^4*(1 + x^2 + x^3)),x]
```

```
[Out] (4*Sqrt[1 + x^3])/3 + (4*Sqrt[1 + x^3])/(3*x^3) - (2*Sqrt[1 + x^3])/x + (6*
Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqr
t[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1
+ Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1
 + x^3]) + (2*Sqrt[2]*3^(3/4)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^
2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/
(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - 2*Defer[Int][Sqrt[1 + x
^3]/(1 + x^2 + x^3), x] - 3*Defer[Int][(x*Sqrt[1 + x^3])/(1 + x^2 + x^3), x
]
```

Rubi steps

$$\int \frac{(-2+x^3)\sqrt{1+x^3}\,(2+x^2+2x^3)}{x^4\,(1+x^2+x^3)}\,dx = \int \left(-\frac{4\sqrt{1+x^3}}{x^4}+\frac{2\sqrt{1+x^3}}{x^2}+\frac{2\sqrt{1+x^3}}{x}+\frac{(-2-3x)\sqrt{1+x^3}}{1+x^2+x^3}\right) dx$$

$$= 2\int \frac{\sqrt{1+x^3}}{x^2}\,dx + 2\int \frac{\sqrt{1+x^3}}{x}\,dx - 4\int \frac{\sqrt{1+x^3}}{x^4}\,dx + \int \frac{(-2-}{1+}$$

$$= -\frac{2\sqrt{1+x^3}}{x}+\frac{2}{3}\text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, x^3\right)-\frac{4}{3}\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\right.$$

$$= \frac{4\sqrt{1+x^3}}{3}+\frac{4\sqrt{1+x^3}}{3x^3}-\frac{2\sqrt{1+x^3}}{x}-2\int \frac{\sqrt{1+x^3}}{1+x^2+x^3}\,dx+3\int \frac{1}{}$$

$$= \frac{4\sqrt{1+x^3}}{3}+\frac{4\sqrt{1+x^3}}{3x^3}-\frac{2\sqrt{1+x^3}}{x}+\frac{6\sqrt{1+x^3}}{1+\sqrt{3}+x}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{}}}{}$$

Mathematica [C] time = 6.18, size = 1638, normalized size = 38.09

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-2 + x^3)*Sqrt[1 + x^3]*(2 + x^2 + 2*x^3))/(x^4*(1 + x^2 + x^3)
),x]
```

```
[Out] (4/3 + 4/(3*x^3) - 2/x)*Sqrt[1 + x^3] + (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(
-(-1)^(1/3) + x)*Sqrt[((-1)^(2/3) + x)/((-1)^(1/3) + (-1)^(2/3))]*EllipticF
[ArcSin[Sqrt[-(((-1)^(2/3)*((-1)^(1/3) - x))/(1 + (-1)^(1/3)))]], (-1)^(1/3
)])/(Sqrt[(-(-1)^(1/3) + x)/(-(-1)^(1/3) - (-1)^(2/3))]*Sqrt[1 + x^3]) - (4
*((-1)^(1/3) + (-1)^(2/3))*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[((-(-1)^(2/3
) - x)*(-(-1)^(1/3) + x))/((-1)^(1/3) + (-1)^(2/3))^2]*EllipticPi[((-1)^(1/
3) + (-1)^(2/3))/((-1)^(1/3) - Root[1 + #1^2 + #1^3 & , 1, 0]), ArcSin[Sqrt
[-(((-1)^(2/3)*((-1)^(1/3) - x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)])/(Sqrt[1
+ x^3]*(-(-1)^(1/3) + Root[1 + #1^2 + #1^3 & , 1, 0])*(Root[1 + #1^2 + #1^3
 & , 1, 0] - Root[1 + #1^2 + #1^3 & , 2, 0])*(Root[1 + #1^2 + #1^3 & , 1, 0
] - Root[1 + #1^2 + #1^3 & , 3, 0])) + (2*((-1)^(1/3) + (-1)^(2/3))*Sqrt[(1
 + x)/(1 + (-1)^(1/3))]*Sqrt[((-(-1)^(2/3) - x)*(-(-1)^(1/3) + x))/((-1)^(1
/3) + (-1)^(2/3))^2]*EllipticPi[((-1)^(1/3) + (-1)^(2/3))/((-1)^(1/3) - Roo
t[1 + #1^2 + #1^3 & , 1, 0]), ArcSin[Sqrt[-(((-1)^(2/3)*((-1)^(1/3) - x))/(
1 + (-1)^(1/3)))]], (-1)^(1/3)]*Root[1 + #1^2 + #1^3 & , 1, 0]^3)/(Sqrt[1 +
 x^3]*(-(-1)^(1/3) + Root[1 + #1^2 + #1^3 & , 1, 0])*(Root[1 + #1^2 + #1^3
& , 1, 0] - Root[1 + #1^2 + #1^3 & , 2, 0])*(Root[1 + #1^2 + #1^3 & , 1, 0]
 - Root[1 + #1^2 + #1^3 & , 3, 0])) - (4*((-1)^(1/3) + (-1)^(2/3))*Sqrt[(1
+ x)/(1 + (-1)^(1/3))]*Sqrt[((-(-1)^(2/3) - x)*(-(-1)^(1/3) + x))/((-1)^(1/
3) + (-1)^(2/3))^2]*EllipticPi[((-1)^(1/3) + (-1)^(2/3))/((-1)^(1/3) - Root
[1 + #1^2 + #1^3 & , 2, 0]), ArcSin[Sqrt[-(((-1)^(2/3)*((-1)^(1/3) - x))/(1
 + (-1)^(1/3)))]], (-1)^(1/3)])/(Sqrt[1 + x^3]*(-(-1)^(1/3) + Root[1 + #1^2
 + #1^3 & , 2, 0])*(-Root[1 + #1^2 + #1^3 & , 1, 0] + Root[1 + #1^2 + #1^3
& , 2, 0])*(Root[1 + #1^2 + #1^3 & , 2, 0] - Root[1 + #1^2 + #1^3 & , 3, 0]
)) + (2*((-1)^(1/3) + (-1)^(2/3))*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[((-(-
1)^(2/3) - x)*(-(-1)^(1/3) + x))/((-1)^(1/3) + (-1)^(2/3))^2]*EllipticPi[((
-1)^(1/3) + (-1)^(2/3))/((-1)^(1/3) - Root[1 + #1^2 + #1^3 & , 2, 0]), ArcS
in[Sqrt[-(((-1)^(2/3)*((-1)^(1/3) - x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)]*Ro
ot[1 + #1^2 + #1^3 & , 2, 0]^3)/(Sqrt[1 + x^3]*(-(-1)^(1/3) + Root[1 + #1^2
 + #1^3 & , 2, 0])*(-Root[1 + #1^2 + #1^3 & , 1, 0] + Root[1 + #1^2 + #1^3
& , 2, 0])*(Root[1 + #1^2 + #1^3 & , 2, 0] - Root[1 + #1^2 + #1^3 & , 3, 0]
)) - (4*((-1)^(1/3) + (-1)^(2/3))*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[((-(-
1)^(2/3) - x)*(-(-1)^(1/3) + x))/((-1)^(1/3) + (-1)^(2/3))^2]*EllipticPi[((
-1)^(1/3) + (-1)^(2/3))/((-1)^(1/3) - Root[1 + #1^2 + #1^3 & , 3, 0]), ArcS
in[Sqrt[-(((-1)^(2/3)*((-1)^(1/3) - x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)])/(
Sqrt[1 + x^3]*(-(-1)^(1/3) + Root[1 + #1^2 + #1^3 & , 3, 0])*(-Root[1 + #1^
2 + #1^3 & , 1, 0] + Root[1 + #1^2 + #1^3 & , 3, 0])*(-Root[1 + #1^2 + #1^3
 & , 2, 0] + Root[1 + #1^2 + #1^3 & , 3, 0])) + (2*((-1)^(1/3) + (-1)^(2/3)
)*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[((-(-1)^(2/3) - x)*(-(-1)^(1/3) + x))
/((-1)^(1/3) + (-1)^(2/3))^2]*EllipticPi[((-1)^(1/3) + (-1)^(2/3))/((-1)^(1
/3) - Root[1 + #1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[-(((-1)^(2/3)*((-1)^(1/3
) - x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)]*Root[1 + #1^2 + #1^3 & , 3, 0]^3)/
(Sqrt[1 + x^3]*(-(-1)^(1/3) + Root[1 + #1^2 + #1^3 & , 3, 0])*(-Root[1 + #1
^2 + #1^3 & , 1, 0] + Root[1 + #1^2 + #1^3 & , 3, 0])*(-Root[1 + #1^2 + #1^
3 & , 2, 0] + Root[1 + #1^2 + #1^3 & , 3, 0]))
```

IntegrateAlgebraic [A] time = 0.60, size = 43, normalized size = 1.00

$$\frac{2\sqrt{x^3+1}\left(2x^3-3x^2+2\right)}{3x^3}-2\tan^{-1}\left(\frac{x}{\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^3)*Sqrt[1 + x^3]*(2 + x^2 + 2*x^3))/(x^4*(1 + x
^2 + x^3)),x]
```

```
[Out] (2*Sqrt[1 + x^3]*(2 - 3*x^2 + 2*x^3))/(3*x^3) - 2*ArcTan[x/Sqrt[1 + x^3]]
```

fricas [A] time = 0.47, size = 55, normalized size = 1.28

$$-\frac{3\,x^3\arctan\left(\frac{2\,\sqrt{x^3+1}\,x}{x^3-x^2+1}\right)-2\left(2\,x^3-3\,x^2+2\right)\sqrt{x^3+1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3+1)^(1/2)*(2*x^3+x^2+2)/x^4/(x^3+x^2+1),x, algorithm=
"fricas")
```

```
[Out] -1/3*(3*x^3*arctan(2*sqrt(x^3 + 1)*x/(x^3 - x^2 + 1)) - 2*(2*x^3 - 3*x^2 +
2)*sqrt(x^3 + 1))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^3+x^2+2\right)\sqrt{x^3+1}\left(x^3-2\right)}{\left(x^3+x^2+1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3+1)^(1/2)*(2*x^3+x^2+2)/x^4/(x^3+x^2+1),x, algorithm=
"giac")
```

```
[Out] integrate((2*x^3 + x^2 + 2)*sqrt(x^3 + 1)*(x^3 - 2)/((x^3 + x^2 + 1)*x^4),
x)
```

maple [C] time = 0.08, size = 302, normalized size = 7.02

$$\frac{4\sqrt{x^3+1}}{3x^3}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^3+\_Z^2+1\right)}\frac{\_\alpha^3\left(3-i\sqrt{3}\right)\sqrt{\frac{1+x}{3-i\sqrt{3}}}\,\sqrt{\frac{-1+2x-i\sqrt{3}}{-3-i\sqrt{3}}}\,\sqrt{\frac{-1+2x+i\sqrt{3}}{-3+i\sqrt{3}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{3\_\alpha^2}{2}+\frac{i\_\alpha^2\sqrt{3}}{2},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}\right)-\frac{2\sqrt{x^3+1}}{x}+\frac{4\sqrt{x^3+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2)*(x^3+1)^(1/2)*(2*x^3+x^2+2)/x^4/(x^3+x^2+1),x)
```

```
[Out] 4/3*(x^3+1)^(1/2)/x^3+2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/
2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2)
)/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(
1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2^(1/2)*sum
(_alpha^3*(3-I*3^(1/2))*((1+x)/(3-I*3^(1/2)))^(1/2)*((-1+2*x-I*3^(1/2))/(-3
-I*3^(1/2)))^(1/2)*((-1+2*x+I*3^(1/2))/(-3+I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*
EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-3/2*_alpha^2+1/2*I*_alpha^2*3
^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^
3+_Z^2+1))-2*(x^3+1)^(1/2)/x+4/3*(x^3+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^3+x^2+2\right)\sqrt{x^3+1}\left(x^3-2\right)}{\left(x^3+x^2+1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3+1)^(1/2)*(2*x^3+x^2+2)/x^4/(x^3+x^2+1),x, algorithm=
"maxima")
```

```
[Out] integrate((2*x^3 + x^2 + 2)*sqrt(x^3 + 1)*(x^3 - 2)/((x^3 + x^2 + 1)*x^4),
x)
```

mupad [B] time = 1.31, size = 70, normalized size = 1.63

$$\frac{4\,\sqrt{x^3+1}}{3}-\frac{2\,\sqrt{x^3+1}}{x}+\frac{4\,\sqrt{x^3+1}}{3\,x^3}+\ln\left(\frac{x^3-x^2+1+x\,\sqrt{x^3+1}\,2{}\mathrm{i}}{x^3+x^2+1}\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)^(1/2)*(x^3 - 2)*(x^2 + 2*x^3 + 2))/(x^4*(x^2 + x^3 + 1)),x)
```

```
[Out] log((x*(x^3 + 1)^(1/2)*2i - x^2 + x^3 + 1)/(x^2 + x^3 + 1))*1i + (4*(x^3 +
1)^(1/2))/3 - (2*(x^3 + 1)^(1/2))/x + (4*(x^3 + 1)^(1/2))/(3*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2)*(x**3+1)**(1/2)*(2*x**3+x**2+2)/x**4/(x**3+x**2+1),x)
```

```
[Out] Timed out
```

3.546 $\int \frac{(-1+x^4)\sqrt{1+x^2+x^4}}{(1+x^4)(1-x^2+x^4)}\,dx$

Optimal. Leaf size=43

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+x^2+1}}\right)$$

Rubi [C] time = 1.43, antiderivative size = 350, normalized size of antiderivative = 8.14, number of steps used = 34, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {6725, 1208, 1197, 1103, 1195, 1216, 1706, 6728}

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+x^2+1}}\right)-\frac{\left(\sqrt{3}+i\right)(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\left(\sqrt{3}+3i\right)\sqrt{x^4+x^2+1}}+\frac{\left(5+i\sqrt{3}\right)(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{\left(5-i\sqrt{3}\right)(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{\left(-\sqrt{3}+i\right)(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\left(-\sqrt{3}+3i\right)\sqrt{x^4+x^2+1}}-\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4)*Sqrt[1 + x^2 + x^4])/((1 + x^4)*(1 - x^2 + x^4)),x]
```

```
[Out] ArcTanh[x/Sqrt[1 + x^2 + x^4]] - Sqrt[2]*ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^2 +
 x^4]] - (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[
x], 1/4])/(2*Sqrt[1 + x^2 + x^4]) - ((I - Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^2
+ x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/((3*I - Sqrt[3])*Sqrt[1 +
x^2 + x^4]) + ((5 - I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*
EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4]) + ((5 + I*Sqrt[3])*(1
+ x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sq
rt[1 + x^2 + x^4]) - ((I + Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2
)^2]*EllipticF[2*ArcTan[x], 1/4])/((3*I + Sqrt[3])*Sqrt[1 + x^2 + x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(-1+x^4\right)\sqrt{1+x^2+x^4}}{\left(1+x^4\right)\left(1-x^2+x^4\right)}\,dx = \int \left(-\frac{2x^2\sqrt{1+x^2+x^4}}{1+x^4}+\frac{\left(-1+2x^2\right)\sqrt{1+x^2+x^4}}{1-x^2+x^4}\right)dx$$

$$= -\left(2\int \frac{x^2\sqrt{1+x^2+x^4}}{1+x^4}\,dx\right)+\int \frac{\left(-1+2x^2\right)\sqrt{1+x^2+x^4}}{1-x^2+x^4}\,dx$$

$$= -\left(2\int \left(-\frac{\sqrt{1+x^2+x^4}}{2\left(i-x^2\right)}+\frac{\sqrt{1+x^2+x^4}}{2\left(i+x^2\right)}\right)dx\right)+\int \left(\frac{2\sqrt{1+x^2+x^4}}{-1-i\sqrt{3}+2x^2}+\frac{2\sqrt{1+\cdots}}{-1+i\sqrt{\cdots}}\right.$$

$$= 2\int \frac{\sqrt{1+x^2+x^4}}{-1-i\sqrt{3}+2x^2}\,dx+2\int \frac{\sqrt{1+x^2+x^4}}{-1+i\sqrt{3}+2x^2}\,dx+\int \frac{\sqrt{1+x^2+x^4}}{i-x^2}\,dx-\int \cdots$$

$$= i\int \frac{1}{\left(i-x^2\right)\sqrt{1+x^2+x^4}}\,dx+i\int \frac{1}{\left(i+x^2\right)\sqrt{1+x^2+x^4}}\,dx-\frac{1}{2}\int \frac{-3-i\sqrt{3}\cdots}{\sqrt{1+x^2+\cdots}}$$

$$= (-2+i)\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx+\left(-\frac{1}{2}+\frac{i}{2}\right)\int \frac{1+x^2}{\left(i+x^2\right)\sqrt{1+x^2+x^4}}\,dx+\left(\frac{1}{2}-\frac{i}{2}\right)\cdots$$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)+\frac{\sqrt{\frac{3}{2}}\left(1-i\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^2+x^4}}\right)}{3i-\sqrt{3}}+\frac{\sqrt{\frac{3}{2}}\left(i-\sqrt{3}\right)\tan\cdots}{3-\cdots}$$

Mathematica [C] time = 0.79, size = 284, normalized size = 6.60

$$\frac{\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(\left((-1)^{2/3}-1\right)F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-2\left(\sqrt[3]{-1}-2\right)\Pi\left(-1;i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-4(-1)^{2/3}\Pi\left(-(-1)^{2/3};i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+2\sqrt[3]{-1}\Pi\left(-(-1)^{2/3};i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+(-1)^{2/3}\Pi\left(-(-1)^{5/6};i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-\Pi\left(-(-1)^{5/6};i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+(-1)^{2/3}\Pi\left((-1)^{5/6};i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-\Pi\left((-1)^{5/6};i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)*Sqrt[1 + x^2 + x^4])/((1 + x^4)*(1 - x^2 + x^4)),x]
```

```
[Out] (Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*((-1 + (-1)^(2/3))*Ellip
ticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - 2*(-2 + (-1)^(1/3))*EllipticPi[
-1, I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2*(-1)^(1/3)*EllipticPi[-(-1)^(2
/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - 4*(-1)^(2/3)*EllipticPi[-(-1)^(
2/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - EllipticPi[-(-1)^(5/6), I*ArcS
inh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(2/3)*EllipticPi[-(-1)^(5/6), I*ArcSi
nh[(-1)^(5/6)*x], (-1)^(2/3)] - EllipticPi[(-1)^(5/6), I*ArcSinh[(-1)^(5/6)
*x], (-1)^(2/3)] + (-1)^(2/3)*EllipticPi[(-1)^(5/6), I*ArcSinh[(-1)^(5/6)*x
], (-1)^(2/3)]))/((1 + (-1)^(1/3))*Sqrt[1 + x^2 + x^4])
```

IntegrateAlgebraic [A] time = 0.36, size = 43, normalized size = 1.00

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*Sqrt[1 + x^2 + x^4])/((1 + x^4)*(1 - x^2 + x
^4)),x]
```

```
[Out] ArcTanh[x/Sqrt[1 + x^2 + x^4]] - Sqrt[2]*ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^2 +
 x^4]]
```

fricas [B] time = 0.46, size = 111, normalized size = 2.58

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{x^8+14x^6+19x^4-4\sqrt{2}\left(x^5+3x^3+x\right)\sqrt{x^4+x^2+1}+14x^2+1}{x^8-2x^6+3x^4-2x^2+1}\right)+\frac{1}{2}\log\left(-\frac{x^4+2x^2+2\sqrt{x^4+x^2+1}x+1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+x^2+1)^(1/2)/(x^4+1)/(x^4-x^2+1),x, algorithm="frica
s")
```

```
[Out] 1/4*sqrt(2)*log(-(x^8 + 14*x^6 + 19*x^4 - 4*sqrt(2)*(x^5 + 3*x^3 + x)*sqrt(
x^4 + x^2 + 1) + 14*x^2 + 1)/(x^8 - 2*x^6 + 3*x^4 - 2*x^2 + 1)) + 1/2*log(-
(x^4 + 2*x^2 + 2*sqrt(x^4 + x^2 + 1)*x + 1)/(x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+x^2+1}\left(x^4-1\right)}{\left(x^4-x^2+1\right)\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+x^2+1)^(1/2)/(x^4+1)/(x^4-x^2+1),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^4 - 1)/((x^4 - x^2 + 1)*(x^4 + 1)), x)
```

maple [C] time = 0.27, size = 436, normalized size = 10.14

$$\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}}-\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4+1\right)}\_\alpha\left(-\frac{\operatorname{arctanh}\left(\frac{\left(2\_\alpha^2+1\right)\left(-3\_\alpha^2+5x^2+4\right)}{10\sqrt{\_\alpha^2}\sqrt{x^4+x^2+1}}\right)}{\sqrt{\_\alpha^2}}+\frac{\sqrt{2}\_\alpha^3\sqrt{x^2+2-ix^2\sqrt{3}}\sqrt{x^2+2+ix^2\sqrt{3}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}x,\frac{\_\alpha^2}{2}+\frac{i\_\alpha^2\sqrt{3}}{2},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{i\sqrt{3}-1}\sqrt{x^4+x^2+1}}\right)\right)}{4}+\frac{\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4-\_Z^2+1\right)}\_\alpha\left(-\frac{\operatorname{arctanh}\left(\frac{\left(2\_\alpha^2+1\right)\left(-3\_\alpha^2+7x^2+8\right)\sqrt{2}}{28\sqrt{\_\alpha^2}\sqrt{x^4+x^2+1}}\right)}{\sqrt{\_\alpha^2}}-\frac{2\left(-\_\alpha^3+\_\alpha\right)\sqrt{x^2+2-ix^2\sqrt{3}}\sqrt{x^2+2+ix^2\sqrt{3}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}x,\frac{i\_\alpha^2\sqrt{3}}{2}+\frac{\_\alpha^2}{2}-\frac{1}{2}-\frac{i\sqrt{3}}{2},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{i\sqrt{3}-1}\sqrt{x^4+x^2+1}}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^4+x^2+1)^(1/2)/(x^4+1)/(x^4-x^2+1),x)
```

```
[Out] 2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*
3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2
),1/2*(-2+2*I*3^(1/2))^(1/2))-1/4*sum(_alpha*(-1/(_alpha^2)^(1/2)*arctanh(1
/10*(2*_alpha^2+1)*(-3*_alpha^2+5*x^2+4)/(_alpha^2)^(1/2)/(x^4+x^2+1)^(1/2)
)+2^(1/2)*_alpha^3/(I*3^(1/2)-1)^(1/2)*(x^2+2-I*3^(1/2)*x^2)^(1/2)*(x^2+2+I
*3^(1/2)*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)
*x,1/2*_alpha^2+1/2*I*_alpha^2*3^(1/2),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2
*I*3^(1/2))^(1/2))),_alpha=RootOf(_Z^4+1))+1/4*2^(1/2)*sum(_alpha*(-1/(_alp
ha^2)^(1/2)*arctanh(1/28*(2*_alpha^2+1)*(-3*_alpha^2+7*x^2+8)*2^(1/2)/(_alp
ha^2)^(1/2)/(x^4+x^2+1)^(1/2))-2*(-_alpha^3+_alpha)/(I*3^(1/2)-1)^(1/2)*(x^
2+2-I*3^(1/2)*x^2)^(1/2)*(x^2+2+I*3^(1/2)*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*Elli
pticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,1/2*I*_alpha^2*3^(1/2)+1/2*_alpha^2-1/2
-1/2*I*3^(1/2),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))),_alp
ha=RootOf(_Z^4-_Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+x^2+1}\left(x^4-1\right)}{\left(x^4-x^2+1\right)\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+x^2+1)^(1/2)/(x^4+1)/(x^4-x^2+1),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^4 - 1)/((x^4 - x^2 + 1)*(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x^4-1\right)\,\sqrt{x^4+x^2+1}}{\left(x^4+1\right)\,\left(x^4-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(x^2 + x^4 + 1)^(1/2))/((x^4 + 1)*(x^4 - x^2 + 1)),x)
```

```
[Out] int(((x^4 - 1)*(x^2 + x^4 + 1)^(1/2))/((x^4 + 1)*(x^4 - x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}\,(x-1)\,(x+1)\left(x^2+1\right)}{\left(x^4+1\right)\left(x^4-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**4+x**2+1)**(1/2)/(x**4+1)/(x**4-x**2+1),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x - 1)*(x + 1)*(x**2 + 1)/((x
**4 + 1)*(x**4 - x**2 + 1)), x)
```

3.547 $\int \frac{\sqrt{1+x+x^2+x^4}\left(-2-x+2x^4\right)}{\left(1+x+x^4\right)^2}\,dx$

Optimal. Leaf size=43

$$-\frac{\sqrt{x^4+x^2+x+1}\,x}{x^4+x+1}-\tanh^{-1}\left(\frac{x}{\sqrt{x^4+x^2+x+1}}\right)$$

Rubi [F] time = 0.59, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+x+x^2+x^4}\left(-2-x+2x^4\right)}{\left(1+x+x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[1 + x + x^2 + x^4]*(-2 - x + 2*x^4))/(1 + x + x^4)^2,x]
```

```
[Out] -4*Defer[Int][Sqrt[1 + x + x^2 + x^4]/(1 + x + x^4)^2, x] - 3*Defer[Int][(x
*Sqrt[1 + x + x^2 + x^4])/(1 + x + x^4)^2, x] + 2*Defer[Int][Sqrt[1 + x + x
^2 + x^4]/(1 + x + x^4), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x+x^2+x^4}\left(-2-x+2x^4\right)}{\left(1+x+x^4\right)^2}\,dx &= \int \left(\frac{(-4-3x)\sqrt{1+x+x^2+x^4}}{\left(1+x+x^4\right)^2}+\frac{2\sqrt{1+x+x^2+x^4}}{1+x+x^4}\right)dx\\
&= 2\int \frac{\sqrt{1+x+x^2+x^4}}{1+x+x^4}\,dx+\int \frac{(-4-3x)\sqrt{1+x+x^2+x^4}}{\left(1+x+x^4\right)^2}\,dx\\
&= 2\int \frac{\sqrt{1+x+x^2+x^4}}{1+x+x^4}\,dx+\int \left(-\frac{4\sqrt{1+x+x^2+x^4}}{\left(1+x+x^4\right)^2}-\frac{3x\sqrt{1+x+}}{\left(1+x+\right.}\right.\\
&= 2\int \frac{\sqrt{1+x+x^2+x^4}}{1+x+x^4}\,dx-3\int \frac{x\sqrt{1+x+x^2+x^4}}{\left(1+x+x^4\right)^2}\,dx-4\int \frac{\sqrt{1+}}{(1}
\end{aligned}$$

Mathematica [C] time = 6.21, size = 17667, normalized size = 410.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 + x + x^2 + x^4]*(-2 - x + 2*x^4))/(1 + x + x^4)^2,x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.14, size = 43, normalized size = 1.00

$$-\frac{\sqrt{x^4+x^2+x+1}\,x}{x^4+x+1}-\tanh^{-1}\left(\frac{x}{\sqrt{x^4+x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x + x^2 + x^4]*(-2 - x + 2*x^4))/(1 + x + x^4)
^2,x]
```

```
[Out] -((x*Sqrt[1 + x + x^2 + x^4])/(1 + x + x^4)) - ArcTanh[x/Sqrt[1 + x + x^2 +
 x^4]]
```

fricas [A] time = 0.43, size = 67, normalized size = 1.56

$$\frac{\left(x^4+x+1\right)\log\left(\frac{x^4+2\,x^2-2\,\sqrt{x^4+x^2+x+1}\,x+x+1}{x^4+x+1}\right)-2\,\sqrt{x^4+x^2+x+1}\,x}{2\,\left(x^4+x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+x+1)^(1/2)*(2*x^4-x-2)/(x^4+x+1)^2,x, algorithm="fricas"
)
```

```
[Out] 1/2*((x^4 + x + 1)*log((x^4 + 2*x^2 - 2*sqrt(x^4 + x^2 + x + 1)*x + x + 1)/
(x^4 + x + 1)) - 2*sqrt(x^4 + x^2 + x + 1)*x)/(x^4 + x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2\,x^4-x-2\right)\sqrt{x^4+x^2+x+1}}{\left(x^4+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+x+1)^(1/2)*(2*x^4-x-2)/(x^4+x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((2*x^4 - x - 2)*sqrt(x^4 + x^2 + x + 1)/(x^4 + x + 1)^2, x)
```

maple [C] time = 0.48, size = 9105, normalized size = 211.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+x+1)^(1/2)*(2*x^4-x-2)/(x^4+x+1)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2\,x^4-x-2\right)\sqrt{x^4+x^2+x+1}}{\left(x^4+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+x+1)^(1/2)*(2*x^4-x-2)/(x^4+x+1)^2,x, algorithm="maxima"
)
```

```
[Out] integrate((2*x^4 - x - 2)*sqrt(x^4 + x^2 + x + 1)/(x^4 + x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int-\frac{\left(-2\,x^4+x+2\right)\,\sqrt{x^4+x^2+x+1}}{\left(x^4+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x - 2*x^4 + 2)*(x + x^2 + x^4 + 1)^(1/2))/(x + x^4 + 1)^2,x)
```

```
[Out] int(-((x - 2*x^4 + 2)*(x + x^2 + x^4 + 1)^(1/2))/(x + x^4 + 1)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+x+1)**(1/2)*(2*x**4-x-2)/(x**4+x+1)**2,x)
```

```
[Out] Timed out
```

3.548 $\int \frac{(-1+x^4)(1+3x^4)}{x(-1-ax+x^4)\sqrt{-x+x^5}}\,dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{x^5-x}}{x}-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^5-x}}\right)$$

Rubi [F] time = 1.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^4)(1+3x^4)}{x(-1-ax+x^4)\sqrt{-x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^4)*(1 + 3*x^4))/(x*(-1 - a*x + x^4)*Sqrt[-x + x^5]),x]
```

```
[Out] (-2*(1 - x^4))/Sqrt[-x + x^5] - (8*x^4*Sqrt[1 - x^4]*Hypergeometric2F1[1/2,
 7/8, 15/8, x^4])/(7*Sqrt[-x + x^5]) + (2*a*Sqrt[x]*Sqrt[-1 + x^4]*Defer[Su
bst][Defer[Int][Sqrt[-1 + x^8]/(1 + a*x^2 - x^8), x], x, Sqrt[x]])/Sqrt[-x
+ x^5] + (8*Sqrt[x]*Sqrt[-1 + x^4]*Defer[Subst][Defer[Int][(x^6*Sqrt[-1 + x
^8])/(-1 - a*x^2 + x^8), x], x, Sqrt[x]])/Sqrt[-x + x^5]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^4)(1+3x^4)}{x(-1-ax+x^4)\sqrt{-x+x^5}}\,dx &= \frac{\left(\sqrt{x}\sqrt{-1+x^4}\right)\int \frac{\sqrt{-1+x^4}(1+3x^4)}{x^{3/2}(-1-ax+x^4)}\,dx}{\sqrt{-x+x^5}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \frac{\sqrt{-1+x^8}(1+3x^8)}{x^2(-1-ax^2+x^8)}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^5}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \left(-\frac{\sqrt{-1+x^8}}{x^2}+\frac{(a-4x^6)\sqrt{-1+x^8}}{1+ax^2-x^8}\right)dx,x,\sqrt{x}\right)}{\sqrt{-x+x^5}}\\
&= -\frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \frac{\sqrt{-1+x^8}}{x^2}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^5}}+\frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int\right.}{\sqrt{-x}}\\
&= -\frac{2(1-x^4)}{\sqrt{-x+x^5}}+\frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \left(\frac{a\sqrt{-1+x^8}}{1+ax^2-x^8}+\frac{4x^6\sqrt{-1+x^8}}{-1-ax^2+x^8}\right)dx,x,\sqrt{x}\right)}{\sqrt{-x+x^5}}\\
&= -\frac{2(1-x^4)}{\sqrt{-x+x^5}}-\frac{\left(8\sqrt{x}\sqrt{1-x^4}\right)\text{Subst}\left(\int \frac{x^6}{\sqrt{1-x^8}}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^5}}+\frac{\left(8\sqrt{x}\sqrt{-1+x^4}\right.}{}\\
&= -\frac{2(1-x^4)}{\sqrt{-x+x^5}}-\frac{8x^4\sqrt{1-x^4}\,{}_2F_1\left(\frac{1}{2},\frac{7}{8};\frac{15}{8};x^4\right)}{7\sqrt{-x+x^5}}+\frac{\left(8\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \frac{x^6}{-1}\right.}{\sqrt{-x+x^5}}
\end{aligned}$$

Mathematica [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^4)(1+3x^4)}{x(-1-ax+x^4)\sqrt{-x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^4)*(1 + 3*x^4))/(x*(-1 - a*x + x^4)*Sqrt[-x + x^5]),x]
```

```
[Out] Integrate[((-1 + x^4)*(1 + 3*x^4))/(x*(-1 - a*x + x^4)*Sqrt[-x + x^5]), x]
```

IntegrateAlgebraic [A] time = 0.37, size = 43, normalized size = 1.00

$$\frac{2\sqrt{x^5-x}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^5-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(1 + 3*x^4))/(x*(-1 - a*x + x^4)*Sqrt[-x + x
^5]),x]
```

```
[Out] (2*Sqrt[-x + x^5])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[-x + x^5]]
```

fricas [A] time = 2.40, size = 154, normalized size = 3.58

$$\left[\frac{\sqrt{a}\,x\log\left(-\frac{x^8+6\,ax^5+a^2x^2-2\,x^4-4\,\sqrt{x^5-x}\left(x^4+ax-1\right)\sqrt{a}-6\,ax+1}{x^8-2\,ax^5+a^2x^2-2\,x^4+2\,ax+1}\right)+4\,\sqrt{x^5-x}}{2\,x},\ \frac{\sqrt{-a}\,x\arctan\left(\frac{2\,\sqrt{x^5-x}\,\sqrt{-a}}{x^4+ax-1}\right)+2\,\sqrt{x^5-x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(3*x^4+1)/x/(x^4-a*x-1)/(x^5-x)^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/2*(sqrt(a)*x*log(-(x^8 + 6*a*x^5 + a^2*x^2 - 2*x^4 - 4*sqrt(x^5 - x)*(x^
4 + a*x - 1)*sqrt(a) - 6*a*x + 1)/(x^8 - 2*a*x^5 + a^2*x^2 - 2*x^4 + 2*a*x
+ 1)) + 4*sqrt(x^5 - x))/x, (sqrt(-a)*x*arctan(2*sqrt(x^5 - x)*sqrt(-a)/(x^
4 + a*x - 1)) + 2*sqrt(x^5 - x))/x]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^4+1\right)\left(x^4-1\right)}{\sqrt{x^5-x}\left(x^4-ax-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(3*x^4+1)/x/(x^4-a*x-1)/(x^5-x)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((3*x^4 + 1)*(x^4 - 1)/(sqrt(x^5 - x)*(x^4 - a*x - 1)*x), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)\left(3x^4+1\right)}{x\left(x^4-ax-1\right)\sqrt{x^5-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(3*x^4+1)/x/(x^4-a*x-1)/(x^5-x)^(1/2),x)
```

```
[Out] int((x^4-1)*(3*x^4+1)/x/(x^4-a*x-1)/(x^5-x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^4+1\right)\left(x^4-1\right)}{\sqrt{x^5-x}\left(x^4-ax-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(3*x^4+1)/x/(x^4-a*x-1)/(x^5-x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((3*x^4 + 1)*(x^4 - 1)/(sqrt(x^5 - x)*(x^4 - a*x - 1)*x), x)
```

mupad [B] time = 0.57, size = 55, normalized size = 1.28

$$\sqrt{a}\ \ln\left(\frac{a\,x-2\,\sqrt{a}\ \sqrt{x^5-x}\ +x^4-1}{-x^4+a\,x+1}\right)+\frac{2\,\sqrt{x^5-x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^4 - 1)*(3*x^4 + 1))/(x*(x^5 - x)^(1/2)*(a*x - x^4 + 1)),x)
```

```
[Out] a^(1/2)*log((a*x - 2*a^(1/2)*(x^5 - x)^(1/2) + x^4 - 1)/(a*x - x^4 + 1)) +
(2*(x^5 - x)^(1/2))/x
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(3*x**4+1)/x/(x**4-a*x-1)/(x**5-x)**(1/2),x)
```

```
[Out] Timed out
```

3.549 $\int \frac{1}{x^{19}\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=43

$$\frac{5}{48}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}\,\left(15x^{12}+10x^6+8\right)}{144x^{18}}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.47, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 51, 63, 203}

$$\frac{5\sqrt{x^6-1}}{48x^6}+\frac{5}{48}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}}{18x^{18}}+\frac{5\sqrt{x^6-1}}{72x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^19*Sqrt[-1 + x^6]),x]
```

```
[Out] Sqrt[-1 + x^6]/(18*x^18) + (5*Sqrt[-1 + x^6])/(72*x^12) + (5*Sqrt[-1 + x^6]
)/(48*x^6) + (5*ArcTan[Sqrt[-1 + x^6]])/48
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{19}\sqrt{-1+x^6}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^4}\,dx,x,x^6\right)\\
&= \frac{\sqrt{-1+x^6}}{18x^{18}}+\frac{5}{36}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^3}\,dx,x,x^6\right)\\
&= \frac{\sqrt{-1+x^6}}{18x^{18}}+\frac{5\sqrt{-1+x^6}}{72x^{12}}+\frac{5}{48}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx,x,x^6\right)\\
&= \frac{\sqrt{-1+x^6}}{18x^{18}}+\frac{5\sqrt{-1+x^6}}{72x^{12}}+\frac{5\sqrt{-1+x^6}}{48x^6}+\frac{5}{96}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx,x,x^6\right)\\
&= \frac{\sqrt{-1+x^6}}{18x^{18}}+\frac{5\sqrt{-1+x^6}}{72x^{12}}+\frac{5\sqrt{-1+x^6}}{48x^6}+\frac{5}{48}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x^6}\right)\\
&= \frac{\sqrt{-1+x^6}}{18x^{18}}+\frac{5\sqrt{-1+x^6}}{72x^{12}}+\frac{5\sqrt{-1+x^6}}{48x^6}+\frac{5}{48}\tan^{-1}\left(\sqrt{-1+x^6}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.65

$$\frac{1}{3}\sqrt{x^6-1}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^19*Sqrt[-1 + x^6]),x]
```

```
[Out] (Sqrt[-1 + x^6]*Hypergeometric2F1[1/2, 4, 3/2, 1 - x^6])/3
```

IntegrateAlgebraic [A] time = 0.04, size = 43, normalized size = 1.00

$$\frac{5}{48}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}\left(15x^{12}+10x^6+8\right)}{144x^{18}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^19*Sqrt[-1 + x^6]),x]
```

```
[Out] (Sqrt[-1 + x^6]*(8 + 10*x^6 + 15*x^12))/(144*x^18) + (5*ArcTan[Sqrt[-1 + x^
6]])/48
```

fricas [A] time = 0.38, size = 39, normalized size = 0.91

$$\frac{15\,x^{18}\arctan\left(\sqrt{x^6-1}\right)+\left(15\,x^{12}+10\,x^6+8\right)\sqrt{x^6-1}}{144\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^19/(x^6-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/144*(15*x^18*arctan(sqrt(x^6 - 1)) + (15*x^12 + 10*x^6 + 8)*sqrt(x^6 - 1)
)/x^18
```

giac [A] time = 0.36, size = 44, normalized size = 1.02

$$\frac{15\left(x^6-1\right)^{\frac{5}{2}}+40\left(x^6-1\right)^{\frac{3}{2}}+33\sqrt{x^6-1}}{144\,x^{18}}+\frac{5}{48}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^19/(x^6-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/144*(15*(x^6 - 1)^(5/2) + 40*(x^6 - 1)^(3/2) + 33*sqrt(x^6 - 1))/x^18 + 5
/48*arctan(sqrt(x^6 - 1))
```

maple [A] time = 0.04, size = 37, normalized size = 0.86

$$\frac{15x^{18}-5x^{12}-2x^{6}-8}{144x^{18}\sqrt{x^{6}-1}}-\frac{5\arcsin\left(\frac{1}{x^{3}}\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^19/(x^6-1)^(1/2),x)
```

```
[Out] 1/144*(15*x^18-5*x^12-2*x^6-8)/x^18/(x^6-1)^(1/2)-5/48*arcsin(1/x^3)
```

maxima [A] time = 0.42, size = 66, normalized size = 1.53

$$\frac{15\left(x^{6}-1\right)^{\frac{5}{2}}+40\left(x^{6}-1\right)^{\frac{3}{2}}+33\sqrt{x^{6}-1}}{144\left(3x^{6}+\left(x^{6}-1\right)^{3}+3\left(x^{6}-1\right)^{2}-2\right)}+\frac{5}{48}\arctan\left(\sqrt{x^{6}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^19/(x^6-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/144*(15*(x^6 - 1)^(5/2) + 40*(x^6 - 1)^(3/2) + 33*sqrt(x^6 - 1))/(3*x^6 +
 (x^6 - 1)^3 + 3*(x^6 - 1)^2 - 2) + 5/48*arctan(sqrt(x^6 - 1))
```

mupad [B] time = 0.59, size = 47, normalized size = 1.09

$$\frac{5\,\mathrm{atan}\left(\sqrt{x^{6}-1}\right)}{48}+\frac{11\sqrt{x^{6}-1}}{48\,x^{18}}+\frac{5\left(x^{6}-1\right)^{3/2}}{18\,x^{18}}+\frac{5\left(x^{6}-1\right)^{5/2}}{48\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^19*(x^6 - 1)^(1/2)),x)
```

```
[Out] (5*atan((x^6 - 1)^(1/2)))/48 + (11*(x^6 - 1)^(1/2))/(48*x^18) + (5*(x^6 - 1
)^(3/2))/(18*x^18) + (5*(x^6 - 1)^(5/2))/(48*x^18)
```

sympy [B] time = 4.77, size = 165, normalized size = 3.84

$$\begin{cases}\frac{5i\,\mathrm{acosh}\left(\frac{1}{x^{3}}\right)}{48}-\frac{5i}{48x^{3}\sqrt{-1+\frac{1}{x^{6}}}}+\frac{5i}{144x^{9}\sqrt{-1+\frac{1}{x^{6}}}}+\frac{i}{72x^{15}\sqrt{-1+\frac{1}{x^{6}}}}+\frac{i}{18x^{21}\sqrt{-1+\frac{1}{x^{6}}}} & \text{for}\ \frac{1}{|x^{6}|}>1\\ -\frac{5\,\mathrm{asin}\left(\frac{1}{x^{3}}\right)}{48}+\frac{5}{48x^{3}\sqrt{1-\frac{1}{x^{6}}}}-\frac{5}{144x^{9}\sqrt{1-\frac{1}{x^{6}}}}-\frac{1}{72x^{15}\sqrt{1-\frac{1}{x^{6}}}}-\frac{1}{18x^{21}\sqrt{1-\frac{1}{x^{6}}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**19/(x**6-1)**(1/2),x)
```

```
[Out] Piecewise((5*I*acosh(x**(-3))/48 - 5*I/(48*x**3*sqrt(-1 + x**(-6))) + 5*I/(
144*x**9*sqrt(-1 + x**(-6))) + I/(72*x**15*sqrt(-1 + x**(-6))) + I/(18*x**2
1*sqrt(-1 + x**(-6))), 1/Abs(x**6) > 1), (-5*asin(x**(-3))/48 + 5/(48*x**3*
sqrt(1 - 1/x**6)) - 5/(144*x**9*sqrt(1 - 1/x**6)) - 1/(72*x**15*sqrt(1 - 1/
x**6)) - 1/(18*x**21*sqrt(1 - 1/x**6)), True))
```

3.550 $\int \frac{x^{14}}{\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=43

$$\frac{1}{8}\log\left(\sqrt{x^6-1}+x^3\right)+\frac{1}{24}\sqrt{x^6-1}\left(2x^9+3x^3\right)$$

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {275, 321, 217, 206}

$$\frac{1}{12}\sqrt{x^6-1}\,x^9+\frac{1}{8}\sqrt{x^6-1}\,x^3+\frac{1}{8}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^14/Sqrt[-1 + x^6],x]
```

```
[Out] (x^3*Sqrt[-1 + x^6])/8 + (x^9*Sqrt[-1 + x^6])/12 + ArcTanh[x^3/Sqrt[-1 + x^
6]]/8
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{14}}{\sqrt{-1+x^6}}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{-1+x^2}}\,dx, x, x^3\right)\\ &= \frac{1}{12}x^9\sqrt{-1+x^6} + \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{-1+x^2}}\,dx, x, x^3\right)\\ &= \frac{1}{8}x^3\sqrt{-1+x^6} + \frac{1}{12}x^9\sqrt{-1+x^6} + \frac{1}{8}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, x^3\right)\\ &= \frac{1}{8}x^3\sqrt{-1+x^6} + \frac{1}{12}x^9\sqrt{-1+x^6} + \frac{1}{8}\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^3}{\sqrt{-1+x^6}}\right)\\ &= \frac{1}{8}x^3\sqrt{-1+x^6} + \frac{1}{12}x^9\sqrt{-1+x^6} + \frac{1}{8}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.95

$$\frac{1}{24}\left(\sqrt{x^6-1}\,\left(2x^6+3\right)x^3+3\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/Sqrt[-1 + x^6],x]
[Out] (x^3*Sqrt[-1 + x^6]*(3 + 2*x^6) + 3*ArcTanh[x^3/Sqrt[-1 + x^6]])/24
```

IntegrateAlgebraic [A] time = 0.18, size = 43, normalized size = 1.00

$$\frac{1}{8}\log\left(\sqrt{x^6-1}+x^3\right)+\frac{1}{24}\sqrt{x^6-1}\,\left(2x^9+3x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^14/Sqrt[-1 + x^6],x]
[Out] (Sqrt[-1 + x^6]*(3*x^3 + 2*x^9))/24 + Log[x^3 + Sqrt[-1 + x^6]]/8
```

fricas [A] time = 0.40, size = 37, normalized size = 0.86

$$\frac{1}{24}\left(2\,x^9+3\,x^3\right)\sqrt{x^6-1}-\frac{1}{8}\,\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^6-1)^(1/2),x, algorithm="fricas")
[Out] 1/24*(2*x^9 + 3*x^3)*sqrt(x^6 - 1) - 1/8*log(-x^3 + sqrt(x^6 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt{x^6-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^6-1)^(1/2),x, algorithm="giac")
[Out] integrate(x^14/sqrt(x^6 - 1), x)
```

maple [C] time = 0.23, size = 45, normalized size = 1.05

$$\frac{x^3\left(2x^6+3\right)\sqrt{x^6-1}}{24}+\frac{\sqrt{-\mathrm{signum}\left(x^6-1\right)}\,\arcsin\left(x^3\right)}{8\sqrt{\mathrm{signum}\left(x^6-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(x^6-1)^(1/2),x)
```

```
[Out] 1/24*x^3*(2*x^6+3)*(x^6-1)^(1/2)+1/8/signum(x^6-1)^(1/2)*(-signum(x^6-1))^(
1/2)*arcsin(x^3)
```

maxima [B] time = 0.33, size = 86, normalized size = 2.00

$$-\frac{\frac{5\sqrt{x^6-1}}{x^3}-\frac{3\left(x^6-1\right)^{\frac{3}{2}}}{x^9}}{24\left(\frac{2\left(x^6-1\right)}{x^6}-\frac{\left(x^6-1\right)^2}{x^{12}}-1\right)}+\frac{1}{16}\log\left(\frac{\sqrt{x^6-1}}{x^3}+1\right)-\frac{1}{16}\log\left(\frac{\sqrt{x^6-1}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^6-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*(5*sqrt(x^6 - 1)/x^3 - 3*(x^6 - 1)^(3/2)/x^9)/(2*(x^6 - 1)/x^6 - (x^6
 - 1)^2/x^12 - 1) + 1/16*log(sqrt(x^6 - 1)/x^3 + 1) - 1/16*log(sqrt(x^6 - 1
)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{14}}{\sqrt{x^6-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(x^6 - 1)^(1/2),x)
```

```
[Out] int(x^14/(x^6 - 1)^(1/2), x)
```

sympy [A] time = 2.74, size = 104, normalized size = 2.42

$$\begin{cases} \frac{x^{15}}{12\sqrt{x^6-1}}+\frac{x^9}{24\sqrt{x^6-1}}-\frac{x^3}{8\sqrt{x^6-1}}+\frac{\operatorname{acosh}\left(x^3\right)}{8} & \text{for}\ \left|x^6\right|>1 \\ -\frac{ix^{15}}{12\sqrt{1-x^6}}-\frac{ix^9}{24\sqrt{1-x^6}}+\frac{ix^3}{8\sqrt{1-x^6}}-\frac{i\operatorname{asin}\left(x^3\right)}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(x**6-1)**(1/2),x)
```

```
[Out] Piecewise((x**15/(12*sqrt(x**6 - 1)) + x**9/(24*sqrt(x**6 - 1)) - x**3/(8*s
qrt(x**6 - 1)) + acosh(x**3)/8, Abs(x**6) > 1), (-I*x**15/(12*sqrt(1 - x**6
)) - I*x**9/(24*sqrt(1 - x**6)) + I*x**3/(8*sqrt(1 - x**6)) - I*asin(x**3)/
8, True))
```

3.551 $\int \frac{\sqrt{-1+x^6}}{x^{19}} dx$

Optimal. Leaf size=43

$$\frac{1}{48}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}\,(3x^{12}+2x^6-8)}{144x^{18}}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.47, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {266, 47, 51, 63, 203}

$$\frac{\sqrt{x^6-1}}{48x^6}+\frac{1}{48}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{\sqrt{x^6-1}}{18x^{18}}+\frac{\sqrt{x^6-1}}{72x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^6]/x^19,x]
```

```
[Out] -1/18*Sqrt[-1 + x^6]/x^18 + Sqrt[-1 + x^6]/(72*x^12) + Sqrt[-1 + x^6]/(48*x
^6) + ArcTan[Sqrt[-1 + x^6]]/48
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x^6}}{x^{19}}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x^4}\,dx,x,x^6\right)\\
&= -\frac{\sqrt{-1+x^6}}{18x^{18}}+\frac{1}{36}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^3}\,dx,x,x^6\right)\\
&= -\frac{\sqrt{-1+x^6}}{18x^{18}}+\frac{\sqrt{-1+x^6}}{72x^{12}}+\frac{1}{48}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx,x,x^6\right)\\
&= -\frac{\sqrt{-1+x^6}}{18x^{18}}+\frac{\sqrt{-1+x^6}}{72x^{12}}+\frac{\sqrt{-1+x^6}}{48x^6}+\frac{1}{96}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx,x,x^6\right)\\
&= -\frac{\sqrt{-1+x^6}}{18x^{18}}+\frac{\sqrt{-1+x^6}}{72x^{12}}+\frac{\sqrt{-1+x^6}}{48x^6}+\frac{1}{48}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x^6}\right)\\
&= -\frac{\sqrt{-1+x^6}}{18x^{18}}+\frac{\sqrt{-1+x^6}}{72x^{12}}+\frac{\sqrt{-1+x^6}}{48x^6}+\frac{1}{48}\tan^{-1}\left(\sqrt{-1+x^6}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.65

$$\frac{1}{9}\left(x^6-1\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^6]/x^19,x]
```

```
[Out] ((-1 + x^6)^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, 1 - x^6])/9
```

IntegrateAlgebraic [A] time = 0.04, size = 43, normalized size = 1.00

$$\frac{1}{48}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}\left(3x^{12}+2x^6-8\right)}{144x^{18}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x^6]/x^19,x]
```

```
[Out] (Sqrt[-1 + x^6]*(-8 + 2*x^6 + 3*x^12))/(144*x^18) + ArcTan[Sqrt[-1 + x^6]]/
48
```

fricas [A] time = 0.39, size = 39, normalized size = 0.91

$$\frac{3\,x^{18}\arctan\left(\sqrt{x^6-1}\right)+\left(3\,x^{12}+2\,x^6-8\right)\sqrt{x^6-1}}{144\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^19,x, algorithm="fricas")
```

```
[Out] 1/144*(3*x^18*arctan(sqrt(x^6 - 1)) + (3*x^12 + 2*x^6 - 8)*sqrt(x^6 - 1))/x
^18
```

giac [A] time = 0.34, size = 44, normalized size = 1.02

$$\frac{3\left(x^6-1\right)^{\frac{5}{2}}+8\left(x^6-1\right)^{\frac{3}{2}}-3\sqrt{x^6-1}}{144\,x^{18}}+\frac{1}{48}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^19,x, algorithm="giac")
```

```
[Out] 1/144*(3*(x^6 - 1)^(5/2) + 8*(x^6 - 1)^(3/2) - 3*sqrt(x^6 - 1))/x^18 + 1/48
*arctan(sqrt(x^6 - 1))
```

maple [A] time = 0.03, size = 37, normalized size = 0.86

$$\frac{3x^{18}-x^{12}-10x^6+8}{144x^{18}\sqrt{x^6-1}}-\frac{\arcsin\left(\frac{1}{x^3}\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/2)/x^19,x)
```

```
[Out] 1/144*(3*x^18-x^12-10*x^6+8)/x^18/(x^6-1)^(1/2)-1/48*arcsin(1/x^3)
```

maxima [A] time = 0.49, size = 66, normalized size = 1.53

$$\frac{3\left(x^6-1\right)^{\frac{5}{2}}+8\left(x^6-1\right)^{\frac{3}{2}}-3\sqrt{x^6-1}}{144\left(3x^6+\left(x^6-1\right)^3+3\left(x^6-1\right)^2-2\right)}+\frac{1}{48}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)/x^19,x, algorithm="maxima")
```

```
[Out] 1/144*(3*(x^6 - 1)^(5/2) + 8*(x^6 - 1)^(3/2) - 3*sqrt(x^6 - 1))/(3*x^6 + (x
^6 - 1)^3 + 3*(x^6 - 1)^2 - 2) + 1/48*arctan(sqrt(x^6 - 1))
```

mupad [B] time = 0.55, size = 47, normalized size = 1.09

$$\frac{\operatorname{atan}\left(\sqrt{x^6-1}\right)}{48}-\frac{\sqrt{x^6-1}}{48\,x^{18}}+\frac{\left(x^6-1\right)^{3/2}}{18\,x^{18}}+\frac{\left(x^6-1\right)^{5/2}}{48\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)^(1/2)/x^19,x)
```

```
[Out] atan((x^6 - 1)^(1/2))/48 - (x^6 - 1)^(1/2)/(48*x^18) + (x^6 - 1)^(3/2)/(18*
x^18) + (x^6 - 1)^(5/2)/(48*x^18)
```

sympy [B] time = 4.33, size = 160, normalized size = 3.72

$$\begin{cases} \frac{i\operatorname{acosh}\left(\frac{1}{x^3}\right)}{48}-\frac{i}{48x^3\sqrt{-1+\frac{1}{x^6}}}+\frac{i}{144x^9\sqrt{-1+\frac{1}{x^6}}}+\frac{5i}{72x^{15}\sqrt{-1+\frac{1}{x^6}}}-\frac{i}{18x^{21}\sqrt{-1+\frac{1}{x^6}}} & \text{for}\ \frac{1}{|x^6|}>1 \\ -\frac{\operatorname{asin}\left(\frac{1}{x^3}\right)}{48}+\frac{1}{48x^3\sqrt{1-\frac{1}{x^6}}}-\frac{1}{144x^9\sqrt{1-\frac{1}{x^6}}}-\frac{5}{72x^{15}\sqrt{1-\frac{1}{x^6}}}+\frac{1}{18x^{21}\sqrt{1-\frac{1}{x^6}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/2)/x**19,x)
```

```
[Out] Piecewise((I*acosh(x**(-3))/48 - I/(48*x**3*sqrt(-1 + x**(-6))) + I/(144*x*
*9*sqrt(-1 + x**(-6))) + 5*I/(72*x**15*sqrt(-1 + x**(-6))) - I/(18*x**21*sq
rt(-1 + x**(-6))), 1/Abs(x**6) > 1), (-asin(x**(-3))/48 + 1/(48*x**3*sqrt(1
 - 1/x**6)) - 1/(144*x**9*sqrt(1 - 1/x**6)) - 5/(72*x**15*sqrt(1 - 1/x**6))
 + 1/(18*x**21*sqrt(1 - 1/x**6)), True))
```

3.552 $\int x^8\sqrt{-1+x^6}\,dx$

Optimal. Leaf size=43

$$\frac{1}{24}\sqrt{x^6-1}\left(2x^9-x^3\right)-\frac{1}{24}\log\left(\sqrt{x^6-1}+x^3\right)$$

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {275, 279, 321, 217, 206}

$$\frac{1}{12}\sqrt{x^6-1}\,x^9-\frac{1}{24}\sqrt{x^6-1}\,x^3-\frac{1}{24}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Sqrt[-1 + x^6],x]
```

```
[Out] -1/24*(x^3*Sqrt[-1 + x^6]) + (x^9*Sqrt[-1 + x^6])/12 - ArcTanh[x^3/Sqrt[-1
+ x^6]]/24
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int x^8\sqrt{-1+x^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int x^2\sqrt{-1+x^2}\,dx,x,x^3\right)\\
&= \frac{1}{12}x^9\sqrt{-1+x^6}-\frac{1}{12}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x^2}}\,dx,x,x^3\right)\\
&= -\frac{1}{24}x^3\sqrt{-1+x^6}+\frac{1}{12}x^9\sqrt{-1+x^6}-\frac{1}{24}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx,x,x^3\right)\\
&= -\frac{1}{24}x^3\sqrt{-1+x^6}+\frac{1}{12}x^9\sqrt{-1+x^6}-\frac{1}{24}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x^3}{\sqrt{-1+x^6}}\right)\\
&= -\frac{1}{24}x^3\sqrt{-1+x^6}+\frac{1}{12}x^9\sqrt{-1+x^6}-\frac{1}{24}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.14

$$
\frac{\left(x^6-1\right)\left(\sin^{-1}\left(x^3\right)+\sqrt{1-x^6}\left(2x^6-1\right)x^3\right)}{24\sqrt{-\left(x^6-1\right)^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[-1 + x^6],x]
[Out] ((-1 + x^6)*(x^3*Sqrt[1 - x^6]*(-1 + 2*x^6) + ArcSin[x^3]))/(24*Sqrt[-(-1 +
 x^6)^2])
```

IntegrateAlgebraic [A] time = 0.11, size = 43, normalized size = 1.00

$$
\frac{1}{24}\sqrt{x^6-1}\left(2x^9-x^3\right)-\frac{1}{24}\log\left(\sqrt{x^6-1}+x^3\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8*Sqrt[-1 + x^6],x]
[Out] (Sqrt[-1 + x^6]*(-x^3 + 2*x^9))/24 - Log[x^3 + Sqrt[-1 + x^6]]/24
```

fricas [A] time = 0.39, size = 37, normalized size = 0.86

$$
\frac{1}{24}\left(2\,x^9-x^3\right)\sqrt{x^6-1}+\frac{1}{24}\,\log\left(-x^3+\sqrt{x^6-1}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(x^6-1)^(1/2),x, algorithm="fricas")
[Out] 1/24*(2*x^9 - x^3)*sqrt(x^6 - 1) + 1/24*log(-x^3 + sqrt(x^6 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \sqrt{x^6-1}\,x^8\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(x^6-1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^6 - 1)*x^8, x)
```

maple [C] time = 0.22, size = 45, normalized size = 1.05

$$\frac{x^{3} \left(2 x^{6}-1\right) \sqrt{x^{6}-1}}{24}-\frac{\sqrt{-\mathrm{signum}\left(x^{6}-1\right)}\, \arcsin \left(x^{3}\right)}{24 \sqrt{\mathrm{signum}\left(x^{6}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(x^6-1)^(1/2),x)
```

```
[Out] 1/24*x^3*(2*x^6-1)*(x^6-1)^(1/2)-1/24/signum(x^6-1)^(1/2)*(-signum(x^6-1))^
(1/2)*arcsin(x^3)
```

maxima [B] time = 0.34, size = 84, normalized size = 1.95

$$-\frac{\frac{\sqrt{x^{6}-1}}{x^{3}}+\frac{\left(x^{6}-1\right)^{\frac{3}{2}}}{x^{9}}}{24 \left(\frac{2 \left(x^{6}-1\right)}{x^{6}}-\frac{\left(x^{6}-1\right)^{2}}{x^{12}}-1\right)}-\frac{1}{48} \log \left(\frac{\sqrt{x^{6}-1}}{x^{3}}+1\right)+\frac{1}{48} \log \left(\frac{\sqrt{x^{6}-1}}{x^{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(x^6-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*(sqrt(x^6 - 1)/x^3 + (x^6 - 1)^(3/2)/x^9)/(2*(x^6 - 1)/x^6 - (x^6 - 1
)^2/x^12 - 1) - 1/48*log(sqrt(x^6 - 1)/x^3 + 1) + 1/48*log(sqrt(x^6 - 1)/x^
3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{8} \sqrt{x^{6}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(x^6 - 1)^(1/2),x)
```

```
[Out] int(x^8*(x^6 - 1)^(1/2), x)
```

sympy [A] time = 2.26, size = 104, normalized size = 2.42

$$\begin{cases} \frac{x^{15}}{12 \sqrt{x^{6}-1}}-\frac{x^{9}}{8 \sqrt{x^{6}-1}}+\frac{x^{3}}{24 \sqrt{x^{6}-1}}-\frac{\operatorname{acosh}\left(x^{3}\right)}{24} & \text{for}\ \left|x^{6}\right|>1 \\ -\frac{i x^{15}}{12 \sqrt{1-x^{6}}}+\frac{i x^{9}}{8 \sqrt{1-x^{6}}}-\frac{i x^{3}}{24 \sqrt{1-x^{6}}}+\frac{i \operatorname{asin}\left(x^{3}\right)}{24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(x**6-1)**(1/2),x)
```

```
[Out] Piecewise((x**15/(12*sqrt(x**6 - 1)) - x**9/(8*sqrt(x**6 - 1)) + x**3/(24*s
qrt(x**6 - 1)) - acosh(x**3)/24, Abs(x**6) > 1), (-I*x**15/(12*sqrt(1 - x**
6)) + I*x**9/(8*sqrt(1 - x**6)) - I*x**3/(24*sqrt(1 - x**6)) + I*asin(x**3)
/24, True))
```

3.553 $\int \frac{-1+x^6}{\sqrt{1+x^4}\left(1+x^6\right)} \, dx$

Optimal. Leaf size=43

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{3\sqrt{2}}-\frac{2}{3}\tanh^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)$$

Rubi [C] time = 0.68, antiderivative size = 216, normalized size of antiderivative = 5.02, number of steps used = 17, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6725, 220, 2073, 1211, 1699, 203, 6728, 1217, 1707}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{3\sqrt{2}}-\frac{2}{3}\tanh^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)-\frac{\left(\sqrt{3}+i\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\left(\sqrt{3}+3i\right)\sqrt{x^4+1}}-\frac{\left(-\sqrt{3}+i\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\left(-\sqrt{3}+3i\right)\sqrt{x^4+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)/(Sqrt[1 + x^4]*(1 + x^6)),x]
```

```
[Out] -1/3*ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2] - (2*ArcTanh[x/Sqrt[1 + x^4]
])/3 + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/
(3*Sqrt[1 + x^4]) - ((I - Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*El
lipticF[2*ArcTan[x], 1/2])/(3*(3*I - Sqrt[3])*Sqrt[1 + x^4]) - ((I + Sqrt[3
])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(3*(3
*I + Sqrt[3])*Sqrt[1 + x^4])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-1+x^6}{\sqrt{1+x^4}\left(1+x^6\right)}\,dx = \int \left(\frac{1}{\sqrt{1+x^4}} - \frac{2}{\sqrt{1+x^4}\left(1+x^6\right)}\right) dx$$

$$= -\left(2\int \frac{1}{\sqrt{1+x^4}\left(1+x^6\right)}\,dx\right) + \int \frac{1}{\sqrt{1+x^4}}\,dx$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} - 2\int\left(\frac{1}{3\left(1+x^2\right)\sqrt{1+x^4}} + \frac{2-x^2}{3\sqrt{1+x^4}\left(1-x^2 \right.}\right.$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} - \frac{2}{3}\int \frac{1}{\left(1+x^2\right)\sqrt{1+x^4}}\,dx - \frac{2}{3}\int \frac{2-x}{\sqrt{1+x^4}\left(1-\right.}$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} - \frac{1}{3}\int \frac{1}{\sqrt{1+x^4}}\,dx - \frac{1}{3}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^4}}\,dx -$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{1+x^4}} - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right) + \frac{1}{3}\left(2\left(1\right.\right.$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{3\sqrt{2}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{1+x^4}} - \frac{\left(2\left(i-\sqrt{3}\right)\right)\int \frac{1}{\sqrt{1+x^4}}\,dx}{3\left(3i-\sqrt{3}\right)} +$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{3\sqrt{2}} - \frac{2}{3}\tanh^{-1}\left(\frac{x}{\sqrt{1+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{1+x^4}} - \frac{\left(i-\right.}{}$$

Mathematica [C] time = 0.35, size = 125, normalized size = 2.91

$$\frac{(-1)^{5/12}\left(2\left(\Pi\left(-i;i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)|-1\right)+\Pi\left(\frac{i}{2}-\frac{\sqrt{3}}{2};i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)|-1\right)+\Pi\left(\frac{1}{2}\left(i+\sqrt{3}\right);i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)|-1\right)\right)-3F\left(i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)|-1\right)\right)}{\sqrt{3}\left(1+\sqrt[3]{-1}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + x^6)/(Sqrt[1 + x^4]*(1 + x^6)),x]
```

```
[Out] ((-1)^(5/12)*(-3*EllipticF[I*ArcSinh[((1 + I)*x)/Sqrt[2]], -1] + 2*(Ellipti
cPi[-I, I*ArcSinh[((1 + I)*x)/Sqrt[2]], -1] + EllipticPi[I/2 - Sqrt[3]/2, I
*ArcSinh[((1 + I)*x)/Sqrt[2]], -1] + EllipticPi[(I + Sqrt[3])/2, I*ArcSinh[
((1 + I)*x)/Sqrt[2]], -1])))/(Sqrt[3]*(1 + (-1)^(1/3)))
```

IntegrateAlgebraic [A] time = 0.70, size = 43, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{3\sqrt{2}} - \frac{2}{3}\tanh^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^6)/(Sqrt[1 + x^4]*(1 + x^6)),x]
```

[Out] -1/3*ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2] - (2*ArcTanh[x/Sqrt[1 + x^4]])/3

fricas [A] time = 0.45, size = 53, normalized size = 1.23

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)+\frac{1}{3}\log\left(\frac{x^4+x^2-2\sqrt{x^4+1}\,x+1}{x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^4+1)^(1/2)/(x^6+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) + 1/3*log((x^4 + x^2 - 2*sqrt(
x^4 + 1)*x + 1)/(x^4 - x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6-1}{\left(x^6+1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^4+1)^(1/2)/(x^6+1),x, algorithm="giac")
```

```
[Out] integrate((x^6 - 1)/((x^6 + 1)*sqrt(x^4 + 1)), x)
```

maple [C] time = 0.03, size = 220, normalized size = 5.12

$$\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4-\_Z^2+1\right)}\_\alpha\left(-\frac{\mathrm{arctanh}\left(\frac{\_\alpha^2\left(-\_\alpha^2+x^2+1\right)}{\sqrt{\_\alpha^2}\,\sqrt{x^4+1}}\right)}{\sqrt{\_\alpha^2}}+\frac{2(-1)^{\frac{3}{4}}\left(-\_\alpha^3+\_\alpha\right)\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x,i\_\alpha^2-i,i\right)}{\sqrt{x^4+1}}\right)\right)}{6}+\frac{2\,(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x,i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{3\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)/(x^4+1)^(1/2)/(x^6+1),x)
```

```
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+1/6*sum(_alpha*(-1/(_alpha^2)^(
1/2)*arctanh(_alpha^2*(-_alpha^2+x^2+1)/(_alpha^2)^(1/2)/(x^4+1)^(1/2))+2*(
-1)^(3/4)*(-_alpha^3+_alpha)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*
EllipticPi((-1)^(1/4)*x,I*_alpha^2-I,I)),_alpha=RootOf(_Z^4-_Z^2+1))+2/3*(-
1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4
)*x,I,(-I)^(1/2)/(-1)^(1/4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6-1}{\left(x^6+1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^4+1)^(1/2)/(x^6+1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 - 1)/((x^6 + 1)*sqrt(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^6-1}{\sqrt{x^4+1}\,\left(x^6+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)/((x^4 + 1)^(1/2)*(x^6 + 1)),x)
```

```
[Out] int((x^6 - 1)/((x^4 + 1)^(1/2)*(x^6 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}{\left(x^2+1\right)\sqrt{x^4+1}\left(x^4-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)/(x**4+1)**(1/2)/(x**6+1),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1)/((x**2 + 1)*sqrt(x**
4 + 1)*(x**4 - x**2 + 1)), x)
```

3.554 $\int \frac{x^{14}}{\sqrt{1+x^6}} \, dx$

Optimal. Leaf size=43

$$\frac{1}{8}\log\left(\sqrt{x^6+1}+x^3\right)+\frac{1}{24}\sqrt{x^6+1}\left(2x^9-3x^3\right)$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {275, 321, 215}

$$\frac{1}{8}\sinh^{-1}\left(x^3\right)+\frac{1}{12}\sqrt{x^6+1}\,x^9-\frac{1}{8}\sqrt{x^6+1}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^14/Sqrt[1 + x^6],x]
```

```
[Out] -1/8*(x^3*Sqrt[1 + x^6]) + (x^9*Sqrt[1 + x^6])/12 + ArcSinh[x^3]/8
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{14}}{\sqrt{1+x^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}}\,dx,x,x^3\right)\\
&= \frac{1}{12}x^9\sqrt{1+x^6}-\frac{1}{4}\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx,x,x^3\right)\\
&= -\frac{1}{8}x^3\sqrt{1+x^6}+\frac{1}{12}x^9\sqrt{1+x^6}+\frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,x^3\right)\\
&= -\frac{1}{8}x^3\sqrt{1+x^6}+\frac{1}{12}x^9\sqrt{1+x^6}+\frac{1}{8}\sinh^{-1}\left(x^3\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.72

$$\frac{1}{24}\left(3\sinh^{-1}\left(x^3\right)+\sqrt{x^6+1}\left(2x^6-3\right)x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/Sqrt[1 + x^6],x]
[Out] (x^3*Sqrt[1 + x^6]*(-3 + 2*x^6) + 3*ArcSinh[x^3])/24
```

IntegrateAlgebraic [A] time = 0.16, size = 43, normalized size = 1.00

$$\frac{1}{8}\log\left(\sqrt{x^6+1}+x^3\right)+\frac{1}{24}\sqrt{x^6+1}\left(2x^9-3x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^14/Sqrt[1 + x^6],x]
[Out] (Sqrt[1 + x^6]*(-3*x^3 + 2*x^9))/24 + Log[x^3 + Sqrt[1 + x^6]]/8
```

fricas [A] time = 0.39, size = 37, normalized size = 0.86

$$\frac{1}{24}\left(2x^9-3x^3\right)\sqrt{x^6+1}-\frac{1}{8}\log\left(-x^3+\sqrt{x^6+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^6+1)^(1/2),x, algorithm="fricas")
[Out] 1/24*(2*x^9 - 3*x^3)*sqrt(x^6 + 1) - 1/8*log(-x^3 + sqrt(x^6 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt{x^6+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^6+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^14/sqrt(x^6 + 1), x)
```

maple [A] time = 0.20, size = 27, normalized size = 0.63

$$\frac{x^3\left(2x^6-3\right)\sqrt{x^6+1}}{24}+\frac{\operatorname{arcsinh}\left(x^3\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(x^6+1)^(1/2),x)
[Out] 1/24*x^3*(2*x^6-3)*(x^6+1)^(1/2)+1/8*arcsinh(x^3)
```

maxima [B] time = 0.32, size = 86, normalized size = 2.00

$$-\frac{\frac{5\sqrt{x^6+1}}{x^3}-\frac{3\left(x^6+1\right)^{\frac{3}{2}}}{x^9}}{24\left(\frac{2\left(x^6+1\right)}{x^6}-\frac{\left(x^6+1\right)^2}{x^{12}}-1\right)}+\frac{1}{16}\log\left(\frac{\sqrt{x^6+1}}{x^3}+1\right)-\frac{1}{16}\log\left(\frac{\sqrt{x^6+1}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^6+1)^(1/2),x, algorithm="maxima")
[Out] -1/24*(5*sqrt(x^6 + 1)/x^3 - 3*(x^6 + 1)^(3/2)/x^9)/(2*(x^6 + 1)/x^6 - (x^6
 + 1)^2/x^12 - 1) + 1/16*log(sqrt(x^6 + 1)/x^3 + 1) - 1/16*log(sqrt(x^6 + 1
)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{14}}{\sqrt{x^6+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(x^6 + 1)^(1/2),x)
```

```
[Out] int(x^14/(x^6 + 1)^(1/2), x)
```

sympy [A] time = 2.74, size = 46, normalized size = 1.07

$$\frac{x^{15}}{12\sqrt{x^6+1}} - \frac{x^9}{24\sqrt{x^6+1}} - \frac{x^3}{8\sqrt{x^6+1}} + \frac{\operatorname{asinh}\left(x^3\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(x**6+1)**(1/2),x)
```

```
[Out] x**15/(12*sqrt(x**6 + 1)) - x**9/(24*sqrt(x**6 + 1)) - x**3/(8*sqrt(x**6 +
1)) + asinh(x**3)/8
```

3.555 $\int \frac{-1+x^6}{x^{19}\sqrt{1+x^6}}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{x^6+1}\left(33x^{12}-22x^6+8\right)}{144x^{18}}-\frac{11}{48}\tanh^{-1}\left(\sqrt{x^6+1}\right)$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.47, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {446, 78, 51, 63, 207}

$$\frac{11\sqrt{x^6+1}}{48x^6}-\frac{11}{48}\tanh^{-1}\left(\sqrt{x^6+1}\right)+\frac{\sqrt{x^6+1}}{18x^{18}}-\frac{11\sqrt{x^6+1}}{72x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)/(x^19*Sqrt[1 + x^6]),x]
```

```
[Out] Sqrt[1 + x^6]/(18*x^18) - (11*Sqrt[1 + x^6])/(72*x^12) + (11*Sqrt[1 + x^6])
/(48*x^6) - (11*ArcTanh[Sqrt[1 + x^6]])/48
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x^6}{x^{19}\sqrt{1+x^6}}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+x}{x^4\sqrt{1+x}}\,dx,x,x^6\right)\\
&= \frac{\sqrt{1+x^6}}{18x^{18}}+\frac{11}{36}\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{1+x}}\,dx,x,x^6\right)\\
&= \frac{\sqrt{1+x^6}}{18x^{18}}-\frac{11\sqrt{1+x^6}}{72x^{12}}-\frac{11}{48}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx,x,x^6\right)\\
&= \frac{\sqrt{1+x^6}}{18x^{18}}-\frac{11\sqrt{1+x^6}}{72x^{12}}+\frac{11\sqrt{1+x^6}}{48x^6}+\frac{11}{96}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^6\right)\\
&= \frac{\sqrt{1+x^6}}{18x^{18}}-\frac{11\sqrt{1+x^6}}{72x^{12}}+\frac{11\sqrt{1+x^6}}{48x^6}+\frac{11}{48}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^6}\right)\\
&= \frac{\sqrt{1+x^6}}{18x^{18}}-\frac{11\sqrt{1+x^6}}{72x^{12}}+\frac{11\sqrt{1+x^6}}{48x^6}-\frac{11}{48}\tanh^{-1}\left(\sqrt{1+x^6}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.84

$$
\frac{\sqrt{x^6+1}\left(1-11x^{18}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};x^6+1\right)\right)}{18x^{18}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)/(x^19*Sqrt[1 + x^6]),x]
```

```
[Out] (Sqrt[1 + x^6]*(1 - 11*x^18*Hypergeometric2F1[1/2, 3, 3/2, 1 + x^6]))/(18*x
^18)
```

IntegrateAlgebraic [A] time = 0.06, size = 43, normalized size = 1.00

$$
\frac{\sqrt{x^6+1}\left(33x^{12}-22x^6+8\right)}{144x^{18}}-\frac{11}{48}\tanh^{-1}\left(\sqrt{x^6+1}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^6)/(x^19*Sqrt[1 + x^6]),x]
```

```
[Out] (Sqrt[1 + x^6]*(8 - 22*x^6 + 33*x^12))/(144*x^18) - (11*ArcTanh[Sqrt[1 + x^
6]])/48
```

fricas [A] time = 0.40, size = 57, normalized size = 1.33

$$
-\frac{33\,x^{18}\log\left(\sqrt{x^6+1}+1\right)-33\,x^{18}\log\left(\sqrt{x^6+1}-1\right)-2\left(33\,x^{12}-22\,x^6+8\right)\sqrt{x^6+1}}{288\,x^{18}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/x^19/(x^6+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/288*(33*x^18*log(sqrt(x^6 + 1) + 1) - 33*x^18*log(sqrt(x^6 + 1) - 1) - 2
*(33*x^12 - 22*x^6 + 8)*sqrt(x^6 + 1))/x^18
```

giac [A] time = 0.32, size = 58, normalized size = 1.35

$$
\frac{33\left(x^6+1\right)^{\frac{5}{2}}-88\left(x^6+1\right)^{\frac{3}{2}}+63\sqrt{x^6+1}}{144\,x^{18}}-\frac{11}{96}\log\left(\sqrt{x^6+1}+1\right)+\frac{11}{96}\log\left(\sqrt{x^6+1}-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/x^19/(x^6+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/144*(33*(x^6 + 1)^(5/2) - 88*(x^6 + 1)^(3/2) + 63*sqrt(x^6 + 1))/x^18 - 1
1/96*log(sqrt(x^6 + 1) + 1) + 11/96*log(sqrt(x^6 + 1) - 1)
```

maple [A] time = 0.05, size = 49, normalized size = 1.14

$$\frac{33x^{18}+11x^{12}-14x^{6}+8}{144x^{18}\sqrt{x^{6}+1}}+\frac{11\ln\left(\frac{\sqrt{x^{6}+1}-1}{\sqrt{x^{6}}}\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)/x^19/(x^6+1)^(1/2),x)
```

```
[Out] 1/144*(33*x^18+11*x^12-14*x^6+8)/x^18/(x^6+1)^(1/2)+11/48*ln(((x^6+1)^(1/2)
-1)/(x^6)^(1/2))
```

maxima [B] time = 0.49, size = 119, normalized size = 2.77

$$\frac{15\left(x^{6}+1\right)^{\frac{5}{2}}-40\left(x^{6}+1\right)^{\frac{3}{2}}+33\sqrt{x^{6}+1}}{144\left(3x^{6}+\left(x^{6}+1\right)^{3}-3\left(x^{6}+1\right)^{2}+2\right)}-\frac{3\left(x^{6}+1\right)^{\frac{3}{2}}-5\sqrt{x^{6}+1}}{24\left(2x^{6}-\left(x^{6}+1\right)^{2}+1\right)}-\frac{11}{96}\log\left(\sqrt{x^{6}+1}+1\right)+\frac{11}{96}\log\left(\sqrt{x^{6}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/x^19/(x^6+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/144*(15*(x^6 + 1)^(5/2) - 40*(x^6 + 1)^(3/2) + 33*sqrt(x^6 + 1))/(3*x^6 +
 (x^6 + 1)^3 - 3*(x^6 + 1)^2 + 2) - 1/24*(3*(x^6 + 1)^(3/2) - 5*sqrt(x^6 +
1))/(2*x^6 - (x^6 + 1)^2 + 1) - 11/96*log(sqrt(x^6 + 1) + 1) + 11/96*log(sq
rt(x^6 + 1) - 1)
```

mupad [B] time = 1.01, size = 85, normalized size = 1.98

$$\frac{\frac{5\sqrt{x^{6}+1}}{24}-\frac{\left(x^{6}+1\right)^{3/2}}{8}}{2x^{6}-\left(x^{6}+1\right)^{2}+1}-\frac{11\,\mathrm{atanh}\left(\sqrt{x^{6}+1}\right)}{48}+\frac{11\sqrt{x^{6}+1}}{48x^{18}}-\frac{5\left(x^{6}+1\right)^{3/2}}{18x^{18}}+\frac{5\left(x^{6}+1\right)^{5/2}}{48x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)/(x^19*(x^6 + 1)^(1/2)),x)
```

```
[Out] ((5*(x^6 + 1)^(1/2))/24 - (x^6 + 1)^(3/2)/8)/(2*x^6 - (x^6 + 1)^2 + 1) - (1
1*atanh((x^6 + 1)^(1/2)))/48 + (11*(x^6 + 1)^(1/2))/(48*x^18) - (5*(x^6 + 1
)^(3/2))/(18*x^18) + (5*(x^6 + 1)^(5/2))/(48*x^18)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)/x**19/(x**6+1)**(1/2),x)
```

```
[Out] Timed out
```

3.556 $\int \frac{1+x^6}{\sqrt{1+x^4}\left(1-x^6\right)}\,dx$

Optimal. Leaf size=43

$$\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{3\sqrt{2}}$$

Rubi [C] time = 1.55, antiderivative size = 354, normalized size of antiderivative = 8.23, number of steps used = 41, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {6725, 220, 2073, 1211, 1699, 207, 6728, 1725, 1217, 1707, 1248, 725, 206}

$$\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{3\sqrt{2}}+\frac{\left(1+i\sqrt{3}\right)\tanh^{-1}\left(\frac{2-\left(1-i\sqrt{3}\right)x^2}{\sqrt{2\left(1-i\sqrt{3}\right)}\sqrt{x^4+1}}\right)}{6\sqrt{2\left(1-i\sqrt{3}\right)}}-\frac{\left(1+i\sqrt{3}\right)\tanh^{-1}\left(\frac{4+\left(1+i\sqrt{3}\right)^2x^2}{2\sqrt{2\left(1-i\sqrt{3}\right)}\sqrt{x^4+1}}\right)}{6\sqrt{2\left(1-i\sqrt{3}\right)}}+\frac{\left(1+i\sqrt{3}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\sqrt{x^4+1}}+\frac{\left(1-i\sqrt{3}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\sqrt{x^4+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)/(Sqrt[1 + x^4]*(1 - x^6)),x]
```

```
[Out] (2*ArcTan[x/Sqrt[1 + x^4]])/3 + ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(3*Sqrt[
2]) + ((1 + I*Sqrt[3])*ArcTanh[(2 - (1 - I*Sqrt[3])*x^2)/(Sqrt[2*(1 - I*Sqr
t[3])]*Sqrt[1 + x^4])])/(6*Sqrt[2*(1 - I*Sqrt[3])]) - ((1 + I*Sqrt[3])*ArcT
anh[(4 + (1 + I*Sqrt[3])^2*x^2)/(2*Sqrt[2*(1 - I*Sqrt[3])]*Sqrt[1 + x^4])])
/(6*Sqrt[2*(1 - I*Sqrt[3])]) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*Ellip
ticF[2*ArcTan[x], 1/2])/(3*Sqrt[1 + x^4]) + ((1 - I*Sqrt[3])*(1 + x^2)*Sqrt
[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(6*Sqrt[1 + x^4]) + ((
1 + I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x],
 1/2])/(6*Sqrt[1 + x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
```

```
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[d,
 Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Dist[e, Int[x/((d^2 - e^
2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^6}{\sqrt{1+x^4}\left(1-x^6\right)}\,dx &= \int \left(-\frac{1}{\sqrt{1+x^4}} + \frac{2}{\sqrt{1+x^4}\left(1-x^6\right)}\right) dx \\
&= 2\int \frac{1}{\sqrt{1+x^4}\left(1-x^6\right)}\,dx - \int \frac{1}{\sqrt{1+x^4}}\,dx \\
&= -\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} + 2\int \left(-\frac{1}{3\left(-1+x^2\right)\sqrt{1+x^4}} + \frac{2-x}{6\left(1-x+x^2\right)\sqrt{1}} \right. \\
&= -\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} + \frac{1}{3}\int \frac{2-x}{\left(1-x+x^2\right)\sqrt{1+x^4}}\,dx + \frac{1}{3}\int \frac{2-}{\left(1+x+x\right.} \\
&= -\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} + \frac{1}{3}\int \frac{1}{\sqrt{1+x^4}}\,dx + \frac{1}{3}\int \frac{-1-x^2}{\left(-1+x^2\right)\sqrt{1+x^4}}\,dx \\
&= -\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{1+x^4}} - \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right) + \frac{1}{3}\left(-1\right. \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{3\sqrt{2}} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{1+x^4}} - \frac{1}{3}\left(2\left(1-i\sqrt{3}\right)\right)\int \frac{}{\left(\left(-1-i\sqrt{3}\right.\right.} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{3\sqrt{2}} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{1+x^4}} - \frac{1}{3}\left(1-i\sqrt{3}\right)\operatorname{Subst}\left(\int \frac{}{\left(\left(1-i\right.\right.}\right. \\
&= \frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt{1+x^4}}\right) + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{3\sqrt{2}} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{1+x^4}} + \frac{\left(1+i\sqrt{3}\right.}{} \\
&= \frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt{1+x^4}}\right) + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{3\sqrt{2}} + \frac{\left(1+i\sqrt{3}\right)\tanh^{-1}\left(\frac{2-\left(1-i\sqrt{3}\right)x^2}{\sqrt{2\left(1-i\sqrt{3}\right)}\sqrt{1+x^4}}\right)}{6\sqrt{2\left(1-i\sqrt{3}\right)}} - \frac{\left(1+i\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.27, size = 114, normalized size = 2.65

$$\frac{1}{3}\sqrt[4]{-1}\left(3F\left(i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)\middle|-1\right)-2\left(\Pi\left(i;i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)\middle|-1\right)+\Pi\left(-\frac{i}{2}-\frac{\sqrt{3}}{2};i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)\middle|-1\right)+\Pi\left(\frac{1}{2}\left(-i+\sqrt{3}\right);i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)\middle|-1\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)/(Sqrt[1 + x^4]*(1 - x^6)),x]
```

```
[Out] ((-1)^(1/4)*(3*EllipticF[I*ArcSinh[((1 + I)*x)/Sqrt[2]], -1] - 2*(EllipticP
i[I, I*ArcSinh[((1 + I)*x)/Sqrt[2]], -1] + EllipticPi[-1/2*I - Sqrt[3]/2, I
*ArcSinh[((1 + I)*x)/Sqrt[2]], -1] + EllipticPi[(-I + Sqrt[3])/2, I*ArcSinh
[((1 + I)*x)/Sqrt[2]], -1])))/3
```

IntegrateAlgebraic [A] time = 0.68, size = 43, normalized size = 1.00

$$\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^6)/(Sqrt[1 + x^4]*(1 - x^6)),x]
```

```
[Out] (2*ArcTan[x/Sqrt[1 + x^4]])/3 + ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(3*Sqrt[
2])
```

fricas [B] time = 0.45, size = 68, normalized size = 1.58

$$\frac{1}{12}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\sqrt{x^4+1}\,x+2\,x^2+1}{x^4-2\,x^2+1}\right)+\frac{1}{3}\arctan\left(\frac{2\sqrt{x^4+1}\,x}{x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^4+1)^(1/2)/(-x^6+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*log((x^4 + 2*sqrt(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2
 + 1)) + 1/3*arctan(2*sqrt(x^4 + 1)*x/(x^4 - x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^6+1}{\left(x^6-1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^4+1)^(1/2)/(-x^6+1),x, algorithm="giac")
```

```
[Out] integrate(-(x^6 + 1)/((x^6 - 1)*sqrt(x^4 + 1)), x)
```

maple [C] time = 0.07, size = 572, normalized size = 13.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/(x^4+1)^(1/2)/(-x^6+1),x)
```

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-2/3*(-1)^(3/4)*(1-I*x^2)^(1/2)
*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(
1/4))-1/3*(-1/2+1/2*I*3^(1/2))*(1/2/(1/2+1/2*I*3^(1/2))^(1/2)*arctanh((1/2+
1/2*I*3^(1/2))^(1/2)*(x^2-1/2+1/2*I*3^(1/2))/(x^4+1)^(1/2))+(-1)^(3/4)*(-1/
2-1/2*I*3^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((
-1)^(1/4)*x,-I*(-1/2+1/2*I*3^(1/2)),I))-1/3*(-1/2-1/2*I*3^(1/2))*(1/2/(1/2-
1/2*I*3^(1/2))^(1/2)*arctanh((1/2-1/2*I*3^(1/2))^(1/2)*(x^2-1/2-1/2*I*3^(1/
2))/(x^4+1)^(1/2))+(-1)^(3/4)*(-1/2+1/2*I*3^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2
)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I*(-1/2-1/2*I*3^(1/2)),I))-1
/3*(1/2+1/2*I*3^(1/2))*(-1/2/(1/2-1/2*I*3^(1/2))^(1/2)*arctanh((-1/2+1/2*I*
3^(1/2))*(x^2-1/2-1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))^(1/2)/(x^4+1)^(1/2))+(
-1)^(3/4)*(1/2-1/2*I*3^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticPi((-1)^(1/4)*x,I*(1/2+1/2*I*3^(1/2)),I))-1/3*(1/2-1/2*I*3^(1/2))*
(-1/2/(1/2+1/2*I*3^(1/2))^(1/2)*arctanh((-1/2-1/2*I*3^(1/2))*(x^2-1/2+1/2*I
*3^(1/2))/(1/2+1/2*I*3^(1/2))^(1/2)/(x^4+1)^(1/2))+(-1)^(3/4)*(1/2+1/2*I*3^
```

```
(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*
x,I*(1/2-1/2*I*3^(1/2)),I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^6+1}{(x^6-1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^4+1)^(1/2)/(-x^6+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^6 + 1)/((x^6 - 1)*sqrt(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x^6+1}{\sqrt{x^4+1}\,(x^6-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^6 + 1)/((x^4 + 1)^(1/2)*(x^6 - 1)),x)
```

```
[Out] int(-(x^6 + 1)/((x^4 + 1)^(1/2)*(x^6 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^6}{x^6\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx-\int \frac{1}{x^6\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/(x**4+1)**(1/2)/(-x**6+1),x)
```

```
[Out] -Integral(x**6/(x**6*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x) - Integral(1/(x**
6*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x)
```

3.557 $\int \frac{1+x^6}{\sqrt{1+x^4}\left(-1+x^6\right)}\,dx$

Optimal. Leaf size=43

$$-\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{3\sqrt{2}}$$

Rubi [C] time = 1.32, antiderivative size = 354, normalized size of antiderivative = 8.23, number of steps used = 41, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {6725, 220, 2073, 1211, 1699, 207, 6728, 1725, 1217, 1707, 1248, 725, 206}

$$-\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{3\sqrt{2}}-\frac{\left(1+i\sqrt{3}\right)\tanh^{-1}\left(\frac{2-\left(1-i\sqrt{3}\right)x^2}{\sqrt{2\left(1-i\sqrt{3}\right)}\sqrt{x^4+1}}\right)}{6\sqrt{2\left(1-i\sqrt{3}\right)}}+\frac{\left(1+i\sqrt{3}\right)\tanh^{-1}\left(\frac{4+\left(1+i\sqrt{3}\right)^2x^2}{2\sqrt{2\left(1-i\sqrt{3}\right)}\sqrt{x^4+1}}\right)}{6\sqrt{2\left(1-i\sqrt{3}\right)}}-\frac{\left(1+i\sqrt{3}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\sqrt{x^4+1}}-\frac{\left(1-i\sqrt{3}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\sqrt{x^4+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)/(Sqrt[1 + x^4]*(-1 + x^6)),x]
```

```
[Out] (-2*ArcTan[x/Sqrt[1 + x^4]])/3 - ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(3*Sqrt
[2]) - ((1 + I*Sqrt[3])*ArcTanh[(2 - (1 - I*Sqrt[3])*x^2)/(Sqrt[2*(1 - I*Sq
rt[3])]*Sqrt[1 + x^4])])/(6*Sqrt[2*(1 - I*Sqrt[3])]) + ((1 + I*Sqrt[3])*Arc
Tanh[(4 + (1 + I*Sqrt[3])^2*x^2)/(2*Sqrt[2*(1 - I*Sqrt[3])]*Sqrt[1 + x^4])]
)/(6*Sqrt[2*(1 - I*Sqrt[3])]) + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*Elli
pticF[2*ArcTan[x], 1/2])/(3*Sqrt[1 + x^4]) - ((1 - I*Sqrt[3])*(1 + x^2)*Sqr
t[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(6*Sqrt[1 + x^4]) - (
(1 + I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x]
, 1/2])/(6*Sqrt[1 + x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
```

```
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[d,
 Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Dist[e, Int[x/((d^2 - e^
2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^6}{\sqrt{1+x^4}\left(-1+x^6\right)}\,dx &= \int \left(\frac{1}{\sqrt{1+x^4}}+\frac{2}{\sqrt{1+x^4}\left(-1+x^6\right)}\right)dx \\
&= 2\int \frac{1}{\sqrt{1+x^4}\left(-1+x^6\right)}\,dx+\int \frac{1}{\sqrt{1+x^4}}\,dx \\
&= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}}+2\int \left(\frac{1}{3\left(-1+x^2\right)\sqrt{1+x^4}}+\frac{-2+x}{6\left(1-x+x^2\right)\sqrt{\cdots}}\right. \\
&= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}}+\frac{1}{3}\int \frac{-2+x}{\left(1-x+x^2\right)\sqrt{1+x^4}}\,dx+\frac{1}{3}\int \frac{\cdots}{\left(1+x\cdots\right.} \\
&= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}}-\frac{1}{3}\int \frac{1}{\sqrt{1+x^4}}\,dx-\frac{1}{3}\int \frac{-1-x^2}{\left(-1+x^2\right)\sqrt{1+x^4}}\,d\cdots \\
&= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{1+x^4}}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx,x,\frac{x}{\sqrt{1+x^4}}\right)+\frac{1}{3}\left(\cdots\right. \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{3\sqrt{2}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{1+x^4}}+\frac{1}{3}\left(2\left(1-i\sqrt{3}\right)\right)\int \frac{\cdots}{\left(\left(-1\cdots\right.\right.} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{3\sqrt{2}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{1+x^4}}+\frac{1}{3}\left(1-i\sqrt{3}\right)\text{Subst}\left(\int \cdots\right. \\
&= -\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt{1+x^4}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{3\sqrt{2}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{1+x^4}}+\frac{\left(i\cdots\right.}{\cdots} \\
&= -\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt{1+x^4}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{3\sqrt{2}}-\frac{\left(1+i\sqrt{3}\right)\tanh^{-1}\left(\frac{2-\left(1-i\sqrt{3}\right)x^2}{\sqrt{2\left(1-i\sqrt{3}\right)}\sqrt{1+x^4}}\right)}{6\sqrt{2\left(1-i\sqrt{3}\right)}}+\cdots
\end{aligned}
$$

Mathematica [C] time = 0.18, size = 114, normalized size = 2.65

$$
\frac{1}{3}\sqrt[4]{-1}\left(2\left(\Pi\left(i;i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)\middle|-1\right)+\Pi\left(-\frac{i}{2}-\frac{\sqrt{3}}{2};i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)\middle|-1\right)+\Pi\left(\frac{1}{2}\left(-i+\sqrt{3}\right);i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)\middle|-1\right)\right)-3F\left(i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)\middle|-1\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)/(Sqrt[1 + x^4]*(-1 + x^6)),x]
```

```
[Out] ((-1)^(1/4)*(-3*EllipticF[I*ArcSinh[((1 + I)*x)/Sqrt[2]], -1] + 2*(Elliptic
Pi[I, I*ArcSinh[((1 + I)*x)/Sqrt[2]], -1] + EllipticPi[-1/2*I - Sqrt[3]/2,
I*ArcSinh[((1 + I)*x)/Sqrt[2]], -1] + EllipticPi[(-I + Sqrt[3])/2, I*ArcSin
h[((1 + I)*x)/Sqrt[2]], -1])))/3
```

IntegrateAlgebraic [A] time = 0.71, size = 43, normalized size = 1.00

$$-\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^6)/(Sqrt[1 + x^4]*(-1 + x^6)),x]
```

```
[Out] (-2*ArcTan[x/Sqrt[1 + x^4]])/3 - ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(3*Sqrt
[2])
```

fricas [B] time = 0.47, size = 68, normalized size = 1.58

$$\frac{1}{12}\sqrt{2}\log\left(\frac{x^4-2\sqrt{2}\sqrt{x^4+1}\,x+2x^2+1}{x^4-2x^2+1}\right)-\frac{1}{3}\arctan\left(\frac{2\sqrt{x^4+1}\,x}{x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^4+1)^(1/2)/(x^6-1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*log((x^4 - 2*sqrt(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2
 + 1)) - 1/3*arctan(2*sqrt(x^4 + 1)*x/(x^4 - x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+1}{\left(x^6-1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^4+1)^(1/2)/(x^6-1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 1)/((x^6 - 1)*sqrt(x^4 + 1)), x)
```

maple [C] time = 0.01, size = 571, normalized size = 13.28

$$\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{2(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x,-i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{3\sqrt{x^4+1}}+\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{\mathrm{arctanh}\left(\frac{\sqrt{\frac{1}{2}+\frac{i\sqrt{3}}{2}}\left(x^2-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{\sqrt{x^4+1}}\right)}{2\sqrt{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}+\frac{(-1)^{\frac{3}{4}}\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x,-i\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right),i\right)}{\sqrt{x^4+1}}\right)}{3}+\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(\frac{\mathrm{arctanh}\left(\frac{\sqrt{\frac{1}{2}-\frac{i\sqrt{3}}{2}}\left(x^2-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\sqrt{x^4+1}}\right)}{2\sqrt{\frac{1}{2}-\frac{i\sqrt{3}}{2}}}+\frac{(-1)^{\frac{3}{4}}\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x,-i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right),i\right)}{\sqrt{x^4+1}}\right)}{3}+\frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{\mathrm{arctanh}\left(\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x^2-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\sqrt{\frac{1}{2}-\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+1}}\right)}{2\sqrt{\frac{1}{2}-\frac{i\sqrt{3}}{2}}}+\frac{(-1)^{\frac{3}{4}}\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x,i\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right),i\right)}{\sqrt{x^4+1}}\right)}{3}+\frac{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(-\frac{\mathrm{arctanh}\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(x^2-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{\sqrt{\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+1}}\right)}{2\sqrt{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}+\frac{(-1)^{\frac{3}{4}}\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x,i\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right),i\right)}{\sqrt{x^4+1}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/(x^4+1)^(1/2)/(x^6-1),x)
```

```
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+2/3*(-1)^(3/4)*(1-I*x^2)^(1/2)*
(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1
/4))+1/3*(-1/2+1/2*I*3^(1/2))*(1/2/(1/2+1/2*I*3^(1/2))^(1/2)*arctanh((1/2+1
/2*I*3^(1/2))^(1/2)*(x^2-1/2+1/2*I*3^(1/2))/(x^4+1)^(1/2))+(-1)^(3/4)*(-1/2
-1/2*I*3^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-
1)^(1/4)*x,-I*(-1/2+1/2*I*3^(1/2)),I))+1/3*(-1/2-1/2*I*3^(1/2))*(1/2/(1/2-1
/2*I*3^(1/2))^(1/2)*arctanh((1/2-1/2*I*3^(1/2))^(1/2)*(x^2-1/2-1/2*I*3^(1/2
))/(x^4+1)^(1/2))+(-1)^(3/4)*(-1/2+1/2*I*3^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)
^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I*(-1/2-1/2*I*3^(1/2)),I))+1/
3*(1/2+1/2*I*3^(1/2))*(-1/2/(1/2-1/2*I*3^(1/2))^(1/2)*arctanh((-1/2+1/2*I*3
^(1/2))*(x^2-1/2-1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))^(1/2)/(x^4+1)^(1/2))+(-
1)^(3/4)*(1/2-1/2*I*3^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*
EllipticPi((-1)^(1/4)*x,I*(1/2+1/2*I*3^(1/2)),I))+1/3*(1/2-1/2*I*3^(1/2))*(
-1/2/(1/2+1/2*I*3^(1/2))^(1/2)*arctanh((-1/2-1/2*I*3^(1/2))*(x^2-1/2+1/2*I*
3^(1/2))/(1/2+1/2*I*3^(1/2))^(1/2)/(x^4+1)^(1/2))+(-1)^(3/4)*(1/2+1/2*I*3^(
```

```
1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x
,I*(1/2-1/2*I*3^(1/2)),I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+1}{(x^6-1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^4+1)^(1/2)/(x^6-1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + 1)/((x^6 - 1)*sqrt(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^6+1}{\sqrt{x^4+1}\,\left(x^6-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)/((x^4 + 1)^(1/2)*(x^6 - 1)),x)
```

```
[Out] int((x^6 + 1)/((x^4 + 1)^(1/2)*(x^6 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)\left(x^4-x^2+1\right)}{(x-1)\,(x+1)\,\sqrt{x^4+1}\,\left(x^2-x+1\right)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/(x**4+1)**(1/2)/(x**6-1),x)
```

```
[Out] Integral((x**2 + 1)*(x**4 - x**2 + 1)/((x - 1)*(x + 1)*sqrt(x**4 + 1)*(x**2
 - x + 1)*(x**2 + x + 1)), x)
```

3.558 $\int \frac{(-1+x^5)(1+4x^5)}{x(-1-ax+x^5)\sqrt{-x+x^6}}\,dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{x^6-x}}{x}-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^6-x}}\right)$$

Rubi [F] time = 1.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^5)(1+4x^5)}{x(-1-ax+x^5)\sqrt{-x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^5)*(1 + 4*x^5))/(x*(-1 - a*x + x^5)*Sqrt[-x + x^6]),x]
```

```
[Out] (-2*(1 - x^5))/Sqrt[-x + x^6] - (10*x^5*Sqrt[1 - x^5]*Hypergeometric2F1[1/2
, 9/10, 19/10, x^5])/(9*Sqrt[-x + x^6]) + (2*a*Sqrt[x]*Sqrt[-1 + x^5]*Defer
[Subst][Defer[Int][Sqrt[-1 + x^10]/(1 + a*x^2 - x^10), x], x, Sqrt[x]])/Sqr
t[-x + x^6] + (10*Sqrt[x]*Sqrt[-1 + x^5]*Defer[Subst][Defer[Int][(x^8*Sqrt[
-1 + x^10])/(-1 - a*x^2 + x^10), x], x, Sqrt[x]])/Sqrt[-x + x^6]
```

Rubi steps

$$\int \frac{(-1+x^5)(1+4x^5)}{x(-1-ax+x^5)\sqrt{-x+x^6}}\,dx=\frac{\left(\sqrt{x}\sqrt{-1+x^5}\right)\int \frac{\sqrt{-1+x^5}(1+4x^5)}{x^{3/2}(-1-ax+x^5)}\,dx}{\sqrt{-x+x^6}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{\sqrt{-1+x^{10}}(1+4x^{10})}{x^2(-1-ax^2+x^{10})}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \left(-\frac{\sqrt{-1+x^{10}}}{x^2}+\frac{(a-5x^8)\sqrt{-1+x^{10}}}{1+ax^2-x^{10}}\right)dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}}$$

$$=-\frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{\sqrt{-1+x^{10}}}{x^2}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}}+\frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int\right.}{\sqrt{-x}}$$

$$=-\frac{2(1-x^5)}{\sqrt{-x+x^6}}+\frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \left(\frac{a\sqrt{-1+x^{10}}}{1+ax^2-x^{10}}+\frac{5x^8\sqrt{-1+x^{10}}}{-1-ax^2+x^{10}}\right)dx,x,\sqrt{x}\right.}{\sqrt{-x+x^6}}$$

$$=-\frac{2(1-x^5)}{\sqrt{-x+x^6}}-\frac{\left(10\sqrt{x}\sqrt{1-x^5}\right)\text{Subst}\left(\int \frac{x^8}{\sqrt{1-x^{10}}}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}}+\frac{\left(10\sqrt{x}\sqrt{-1+\right.}{}$$

$$=-\frac{2(1-x^5)}{\sqrt{-x+x^6}}-\frac{10x^5\sqrt{1-x^5}\,{}_2F_1\left(\frac{1}{2},\frac{9}{10};\frac{19}{10};x^5\right)}{9\sqrt{-x+x^6}}+\frac{\left(10\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int\right.}{\sqrt{-x+x}}$$

Mathematica [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^5)(1+4x^5)}{x(-1-ax+x^5)\sqrt{-x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^5)*(1 + 4*x^5))/(x*(-1 - a*x + x^5)*Sqrt[-x + x^6]),x]
```

```
[Out] Integrate[((-1 + x^5)*(1 + 4*x^5))/(x*(-1 - a*x + x^5)*Sqrt[-x + x^6]), x]
```

IntegrateAlgebraic [A] time = 0.59, size = 43, normalized size = 1.00

$$\frac{2\sqrt{x^6-x}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{x^6-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^5)*(1 + 4*x^5))/(x*(-1 - a*x + x^5)*Sqrt[-x + x
^6]),x]
```

```
[Out] (2*Sqrt[-x + x^6])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[-x + x^6]]
```

fricas [A] time = 3.89, size = 154, normalized size = 3.58

$$\left[\frac{\sqrt{a}\,x\log\left(-\frac{x^{10}+6\,ax^6-2\,x^5+a^2x^2-4\,\sqrt{x^6-x}\left(x^5+ax-1\right)\sqrt{a}-6\,ax+1}{x^{10}-2\,ax^6-2\,x^5+a^2x^2+2\,ax+1}\right)+4\,\sqrt{x^6-x}}{2\,x},\ \frac{\sqrt{-a}\,x\arctan\left(\frac{2\,\sqrt{x^6-x}\,\sqrt{-a}}{x^5+ax-1}\right)+2\,\sqrt{x^6-x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)*(4*x^5+1)/x/(x^5-a*x-1)/(x^6-x)^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/2*(sqrt(a)*x*log(-(x^10 + 6*a*x^6 - 2*x^5 + a^2*x^2 - 4*sqrt(x^6 - x)*(x
^5 + a*x - 1)*sqrt(a) - 6*a*x + 1)/(x^10 - 2*a*x^6 - 2*x^5 + a^2*x^2 + 2*a*
x + 1)) + 4*sqrt(x^6 - x))/x, (sqrt(-a)*x*arctan(2*sqrt(x^6 - x)*sqrt(-a)/(
x^5 + a*x - 1)) + 2*sqrt(x^6 - x))/x]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(4\,x^5+1\right)\left(x^5-1\right)}{\sqrt{x^6-x}\left(x^5-ax-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)*(4*x^5+1)/x/(x^5-a*x-1)/(x^6-x)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((4*x^5 + 1)*(x^5 - 1)/(sqrt(x^6 - x)*(x^5 - a*x - 1)*x), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int\frac{\left(x^5-1\right)\left(4x^5+1\right)}{x\left(x^5-ax-1\right)\sqrt{x^6-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)*(4*x^5+1)/x/(x^5-a*x-1)/(x^6-x)^(1/2),x)
```

```
[Out] int((x^5-1)*(4*x^5+1)/x/(x^5-a*x-1)/(x^6-x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(4\,x^5+1\right)\left(x^5-1\right)}{\sqrt{x^6-x}\left(x^5-ax-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)*(4*x^5+1)/x/(x^5-a*x-1)/(x^6-x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((4*x^5 + 1)*(x^5 - 1)/(sqrt(x^6 - x)*(x^5 - a*x - 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\left(x^5-1\right)\,\left(4\,x^5+1\right)}{x\,\sqrt{x^6-x}\,\left(-x^5+a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^5 - 1)*(4*x^5 + 1))/(x*(x^6 - x)^(1/2)*(a*x - x^5 + 1)),x)
```

```
[Out] -int(((x^5 - 1)*(4*x^5 + 1))/(x*(x^6 - x)^(1/2)*(a*x - x^5 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\left(4x^5+1\right)\left(x^4+x^3+x^2+x+1\right)}{x\sqrt{x(x-1)\left(x^4+x^3+x^2+x+1\right)}\left(-ax+x^5-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)*(4*x**5+1)/x/(x**5-a*x-1)/(x**6-x)**(1/2),x)
```

```
[Out] Integral((x - 1)*(4*x**5 + 1)*(x**4 + x**3 + x**2 + x + 1)/(x*sqrt(x*(x - 1
)*(x**4 + x**3 + x**2 + x + 1))*(-a*x + x**5 - 1)), x)
```

3.559 $\int \frac{(2+x^3)(1+x^3+x^6)}{x^7\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{x^3+1}\,(4x^6-3x^3-2)}{6x^6}-\frac{3}{2}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Rubi [A] time = 0.11, antiderivative size = 60, normalized size of antiderivative = 1.40, number of steps used = 12, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1821, 1612, 51, 63, 207}

$$-\frac{\sqrt{x^3+1}}{2x^3}+\frac{2\sqrt{x^3+1}}{3}-\frac{3}{2}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{\sqrt{x^3+1}}{3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x^3)*(1 + x^3 + x^6))/(x^7*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[1 + x^3])/3 - Sqrt[1 + x^3]/(3*x^6) - Sqrt[1 + x^3]/(2*x^3) - (3*Ar
cTanh[Sqrt[1 + x^3]])/2
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(2+x^3)(1+x^3+x^6)}{x^7\sqrt{1+x^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(2+x)(1+x+x^2)}{x^3\sqrt{1+x}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{1+x}}+\frac{2}{x^3\sqrt{1+x}}+\frac{3}{x^2\sqrt{1+x}}+\frac{3}{x\sqrt{1+x}}\right)dx, x, x^3\right)$$
$$= \frac{2\sqrt{1+x^3}}{3}+\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+x}}\,dx, x, x^3\right)+\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, x^3\right)+\text{S}$$
$$= \frac{2\sqrt{1+x^3}}{3}-\frac{\sqrt{1+x^3}}{3x^6}-\frac{\sqrt{1+x^3}}{x^3}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, x^3\right)-\frac{1}{2}\,\text{Subst}\Big($$
$$= \frac{2\sqrt{1+x^3}}{3}-\frac{\sqrt{1+x^3}}{3x^6}-\frac{\sqrt{1+x^3}}{2x^3}-2\tanh^{-1}\left(\sqrt{1+x^3}\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,d\right.$$
$$= \frac{2\sqrt{1+x^3}}{3}-\frac{\sqrt{1+x^3}}{3x^6}-\frac{\sqrt{1+x^3}}{2x^3}-\tanh^{-1}\left(\sqrt{1+x^3}\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,\right.$$
$$= \frac{2\sqrt{1+x^3}}{3}-\frac{\sqrt{1+x^3}}{3x^6}-\frac{\sqrt{1+x^3}}{2x^3}-\frac{3}{2}\tanh^{-1}\left(\sqrt{1+x^3}\right)$$

Mathematica [C] time = 0.06, size = 61, normalized size = 1.42

$$\frac{1}{3}\left(-4\sqrt{x^3+1}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};x^3+1\right)+\frac{\sqrt{x^3+1}\,(2x^3-3)}{x^3}-3\tanh^{-1}\left(\sqrt{x^3+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x^3)*(1 + x^3 + x^6))/(x^7*Sqrt[1 + x^3]),x]
```

```
[Out] ((Sqrt[1 + x^3]*(-3 + 2*x^3))/x^3 - 3*ArcTanh[Sqrt[1 + x^3]] - 4*Sqrt[1 + x
^3]*Hypergeometric2F1[1/2, 3, 3/2, 1 + x^3])/3
```

IntegrateAlgebraic [A] time = 0.05, size = 43, normalized size = 1.00

$$\frac{\sqrt{x^3+1}\,(4x^6-3x^3-2)}{6x^6}-\frac{3}{2}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^3)*(1 + x^3 + x^6))/(x^7*Sqrt[1 + x^3]),x]
```

```
[Out] (Sqrt[1 + x^3]*(-2 - 3*x^3 + 4*x^6))/(6*x^6) - (3*ArcTanh[Sqrt[1 + x^3]])/2
```

fricas [A] time = 0.40, size = 57, normalized size = 1.33

$$-\frac{9\,x^6\log\left(\sqrt{x^3+1}+1\right)-9\,x^6\log\left(\sqrt{x^3+1}-1\right)-2\,(4\,x^6-3\,x^3-2)\sqrt{x^3+1}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^6+x^3+1)/x^7/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(9*x^6*log(sqrt(x^3 + 1) + 1) - 9*x^6*log(sqrt(x^3 + 1) - 1) - 2*(4*x
^6 - 3*x^3 - 2)*sqrt(x^3 + 1))/x^6
```

giac [A] time = 0.34, size = 59, normalized size = 1.37

$$\frac{2}{3}\sqrt{x^3+1}-\frac{3\,(x^3+1)^{\frac{3}{2}}-\sqrt{x^3+1}}{6\,x^6}-\frac{3}{4}\log\left(\sqrt{x^3+1}+1\right)+\frac{3}{4}\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^6+x^3+1)/x^7/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(x^3 + 1) - 1/6*(3*(x^3 + 1)^(3/2) - sqrt(x^3 + 1))/x^6 - 3/4*log(s
qrt(x^3 + 1) + 1) + 3/4*log(abs(sqrt(x^3 + 1) - 1))
```

maple [A] time = 0.01, size = 45, normalized size = 1.05

$$\frac{2\sqrt{x^3+1}}{3}-\frac{\sqrt{x^3+1}}{2x^3}-\frac{3\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{2}-\frac{\sqrt{x^3+1}}{3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2)*(x^6+x^3+1)/x^7/(x^3+1)^(1/2),x)
```

```
[Out] 2/3*(x^3+1)^(1/2)-1/2*(x^3+1)^(1/2)/x^3-3/2*arctanh((x^3+1)^(1/2))-1/3*(x^3
+1)^(1/2)/x^6
```

maxima [B] time = 0.50, size = 85, normalized size = 1.98

$$\frac{2}{3}\sqrt{x^3+1}-\frac{3\left(x^3+1\right)^{\frac{3}{2}}-5\sqrt{x^3+1}}{6\left(2x^3-\left(x^3+1\right)^2+1\right)}-\frac{\sqrt{x^3+1}}{x^3}-\frac{3}{4}\log\left(\sqrt{x^3+1}+1\right)+\frac{3}{4}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^6+x^3+1)/x^7/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(x^3 + 1) - 1/6*(3*(x^3 + 1)^(3/2) - 5*sqrt(x^3 + 1))/(2*x^3 - (x^3
 + 1)^2 + 1) - sqrt(x^3 + 1)/x^3 - 3/4*log(sqrt(x^3 + 1) + 1) + 3/4*log(sqr
t(x^3 + 1) - 1)
```

mupad [B] time = 0.24, size = 198, normalized size = 4.60

$$\frac{2\sqrt{x^3+1}}{3}-\frac{\sqrt{x^3+1}}{2x^3}-\frac{\sqrt{x^3+1}}{3x^6}-\frac{9\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 2)*(x^3 + x^6 + 1))/(x^7*(x^3 + 1)^(1/2)),x)
```

```
[Out] (2*(x^3 + 1)^(1/2))/3 - (x^3 + 1)^(1/2)/(2*x^3) - (x^3 + 1)^(1/2)/(3*x^6) -
 (9*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x +
 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
 - 3/2)))/(2*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) -
 ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [B] time = 151.83, size = 105, normalized size = 2.44

$$\frac{2\sqrt{x^3+1}}{3}+\frac{3\log\left(-1+\frac{1}{\sqrt{x^3+1}}\right)}{4}-\frac{3\log\left(1+\frac{1}{\sqrt{x^3+1}}\right)}{4}+\frac{1}{12\left(1+\frac{1}{\sqrt{x^3+1}}\right)}+\frac{1}{12\left(1+\frac{1}{\sqrt{x^3+1}}\right)^2}+\frac{1}{12\left(-1+\frac{1}{\sqrt{x^3+1}}\right)}-\frac{1}{12\left(-1+\frac{1}{\sqrt{x^3+1}}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2)*(x**6+x**3+1)/x**7/(x**3+1)**(1/2),x)
```

```
[Out] 2*sqrt(x**3 + 1)/3 + 3*log(-1 + 1/sqrt(x**3 + 1))/4 - 3*log(1 + 1/sqrt(x**3
 + 1))/4 + 1/(12*(1 + 1/sqrt(x**3 + 1))) + 1/(12*(1 + 1/sqrt(x**3 + 1))**2)
 + 1/(12*(-1 + 1/sqrt(x**3 + 1))) - 1/(12*(-1 + 1/sqrt(x**3 + 1))**2)
```

3.560 $\int \frac{-x+4x^6}{(1+x^5)(a-x+ax^5)\sqrt{x+x^6}}\,dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{x^6+x}}{x^5+1}-2\sqrt{a}\tanh^{-1}\left(\frac{x}{\sqrt{a}\sqrt{x^6+x}}\right)$$

Rubi [F] time = 1.75, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-x+4x^6}{(1+x^5)(a-x+ax^5)\sqrt{x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-x + 4*x^6)/((1 + x^5)*(a - x + a*x^5)*Sqrt[x + x^6]),x]
```

```
[Out] (8*x^2)/(5*a*Sqrt[x + x^6]) + (16*x^2*Sqrt[1 + x^5]*Hypergeometric2F1[3/10,
 1/2, 13/10, -x^5])/(15*a*Sqrt[x + x^6]) - (10*Sqrt[x]*Sqrt[1 + x^5]*Defer[
Subst][Defer[Int][x^2/((1 + x^10)^(3/2)*(a - x^2 + a*x^10)), x], x, Sqrt[x]
])/Sqrt[x + x^6] + (8*Sqrt[x]*Sqrt[1 + x^5]*Defer[Subst][Defer[Int][x^4/((1
 + x^10)^(3/2)*(a - x^2 + a*x^10)), x], x, Sqrt[x]])/(a*Sqrt[x + x^6])
```

Rubi steps

$$\begin{aligned}
\int \frac{-x+4x^6}{(1+x^5)(a-x+ax^5)\sqrt{x+x^6}}\,dx &= \int \frac{x(-1+4x^5)}{(1+x^5)(a-x+ax^5)\sqrt{x+x^6}}\,dx\\
&= \frac{\left(\sqrt{x}\sqrt{1+x^5}\right)\int \frac{\sqrt{x}(-1+4x^5)}{(1+x^5)^{3/2}(a-x+ax^5)}\,dx}{\sqrt{x+x^6}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{x^2(-1+4x^{10})}{(1+x^{10})^{3/2}(a-x^2+ax^{10})}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^6}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \left(\frac{4x^2}{a(1+x^{10})^{3/2}}+\frac{x^2(-5a+4x^2)}{a(1+x^{10})^{3/2}(a-x^2+ax^{10})}\right)dx,x,\sqrt{x}\right)}{\sqrt{x+x^6}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{x^2(-5a+4x^2)}{(1+x^{10})^{3/2}(a-x^2+ax^{10})}\,dx,x,\sqrt{x}\right)}{a\sqrt{x+x^6}}+\frac{\left(8\sqrt{x}\sqrt{1+x^5}\right.}{}\\
&= \frac{8x^2}{5a\sqrt{x+x^6}}+\frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int\left(-\frac{5ax^2}{(1+x^{10})^{3/2}(a-x^2+ax^{10})}+\frac{4}{(1+x^{10})^{3/2}(}\right.\right.}{a\sqrt{x+x^6}}\\
&= \frac{8x^2}{5a\sqrt{x+x^6}}+\frac{16x^2\sqrt{1+x^5}\,{}_2F_1\left(\frac{3}{10},\frac{1}{2};\frac{13}{10};-x^5\right)}{15a\sqrt{x+x^6}}-\frac{\left(10\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}}{}
\end{aligned}$$

Mathematica [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{-x+4x^6}{(1+x^5)(a-x+ax^5)\sqrt{x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-x + 4*x^6)/((1 + x^5)*(a - x + a*x^5)*Sqrt[x + x^6]),x]
```

```
[Out] Integrate[(-x + 4*x^6)/((1 + x^5)*(a - x + a*x^5)*Sqrt[x + x^6]), x]
```

IntegrateAlgebraic [A] time = 2.70, size = 43, normalized size = 1.00

$$\frac{2\sqrt{x^6+x}}{x^5+1}-2\sqrt{a}\tanh^{-1}\left(\frac{x}{\sqrt{a}\sqrt{x^6+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x + 4*x^6)/((1 + x^5)*(a - x + a*x^5)*Sqrt[x + x^6]),x
]
```

```
[Out] (2*Sqrt[x + x^6])/(1 + x^5) - 2*Sqrt[a]*ArcTanh[x/(Sqrt[a]*Sqrt[x + x^6])]
```

fricas [A] time = 0.57, size = 172, normalized size = 4.00

$$\left[\frac{\left(x^5+1\right)\sqrt{a}\log\left(-\frac{a^2x^{10}+2a^2x^5+6ax^6-4\left(ax^5+a+x\right)\sqrt{x^6+x}\sqrt{a}+a^2+6ax+x^2}{a^2x^{10}+2a^2x^5-2ax^6+a^2-2ax+x^2}\right)+4\sqrt{x^6+x}}{2\left(x^5+1\right)}, \frac{\left(x^5+1\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{x^6+x}\sqrt{-a}}{ax^5+a+x}\right)+2\sqrt{x^6+x}}{x^5+1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6-x)/(x^5+1)/(a*x^5+a-x)/(x^6+x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/2*((x^5 + 1)*sqrt(a)*log(-(a^2*x^10 + 2*a^2*x^5 + 6*a*x^6 - 4*(a*x^5 + a
 + x)*sqrt(x^6 + x)*sqrt(a) + a^2 + 6*a*x + x^2)/(a^2*x^10 + 2*a^2*x^5 - 2*
a*x^6 + a^2 - 2*a*x + x^2)) + 4*sqrt(x^6 + x))/(x^5 + 1), ((x^5 + 1)*sqrt(-
a)*arctan(2*sqrt(x^6 + x)*sqrt(-a)/(a*x^5 + a + x)) + 2*sqrt(x^6 + x))/(x^5
 + 1)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6-x)/(x^5+1)/(a*x^5+a-x)/(x^6+x)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{4x^6-x}{\left(x^5+1\right)\left(a\,x^5+a-x\right)\sqrt{x^6+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^6-x)/(x^5+1)/(a*x^5+a-x)/(x^6+x)^(1/2),x)
```

```
[Out] int((4*x^6-x)/(x^5+1)/(a*x^5+a-x)/(x^6+x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x^6-x}{\left(ax^5+a-x\right)\sqrt{x^6+x}\left(x^5+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6-x)/(x^5+1)/(a*x^5+a-x)/(x^6+x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((4*x^6 - x)/((a*x^5 + a - x)*sqrt(x^6 + x)*(x^5 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x-4x^6}{(x^5+1)\sqrt{x^6+x}\,(a x^5-x+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 4*x^6)/((x^5 + 1)*(x + x^6)^(1/2)*(a - x + a*x^5)),x)
```

```
[Out] int(-(x - 4*x^6)/((x^5 + 1)*(x + x^6)^(1/2)*(a - x + a*x^5)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**6-x)/(x**5+1)/(a*x**5+a-x)/(x**6+x)**(1/2),x)
```

```
[Out] Timed out
```

3.561 $\int \frac{2+x^3+x^6}{x\sqrt[4]{1+x^6}\left(-4+5x^3-4x^6+x^9\right)}\,dx$

Optimal. Leaf size=43

$$\frac{1}{3}\tan^{-1}\left(\frac{1-x^3}{\sqrt[4]{x^6+1}}\right)-\frac{1}{3}\tanh^{-1}\left(\frac{x^3-1}{\sqrt[4]{x^6+1}}\right)$$

Rubi [F] time = 1.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2+x^3+x^6}{x\sqrt[4]{1+x^6}\left(-4+5x^3-4x^6+x^9\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2 + x^3 + x^6)/(x*(1 + x^6)^(1/4)*(-4 + 5*x^3 - 4*x^6 + x^9)),x]
```

```
[Out] -1/6*ArcTan[(1 + x^6)^(1/4)] + ArcTanh[(1 + x^6)^(1/4)]/6 + (7*Defer[Subst]
[Defer[Int][1/((1 + x^2)^(1/4)*(-4 + 5*x - 4*x^2 + x^3)), x], x, x^3])/6 -
Defer[Subst][Defer[Int][x/((1 + x^2)^(1/4)*(-4 + 5*x - 4*x^2 + x^3)), x], x
, x^3]/3 + Defer[Subst][Defer[Int][x^2/((1 + x^2)^(1/4)*(-4 + 5*x - 4*x^2 +
 x^3)), x], x, x^3]/6
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x^3+x^6}{x\sqrt[4]{1+x^6}\left(-4+5x^3-4x^6+x^9\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{2+x+x^2}{x\sqrt[4]{1+x^2}\left(-4+5x-4x^2+x^3\right)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{1}{2x\sqrt[4]{1+x^2}}+\frac{7-2x+x^2}{2\sqrt[4]{1+x^2}\left(-4+5x-4x^2+x^3\right)}\right)dx,x\right.\\
&= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1+x^2}}\,dx,x,x^3\right)\right)+\frac{1}{6}\text{Subst}\left(\int \frac{7-2x}{\sqrt[4]{1+x^2}\left(-4+\right.}\right.\\
&= -\left(\frac{1}{12}\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1+x}}\,dx,x,x^6\right)\right)+\frac{1}{6}\text{Subst}\left(\int\left(\frac{}{\sqrt[4]{1+x^2}\left(-4+\right.}\right.\right.\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{1+x^2}\left(-4+5x-4x^2+x^3\right)}\,dx,x,x^3\right)-\frac{1}{3}\text{Subst}\left(\int\right.\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[4]{1+x^6}\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt[4]{1+}\right.\\
&= -\frac{1}{6}\tan^{-1}\left(\sqrt[4]{1+x^6}\right)+\frac{1}{6}\tanh^{-1}\left(\sqrt[4]{1+x^6}\right)+\frac{1}{6}\text{Subst}\left(\int \frac{}{\sqrt[4]{1+x^2}}\right.
\end{aligned}$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{2+x^3+x^6}{x\sqrt[4]{1+x^6}\left(-4+5x^3-4x^6+x^9\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2 + x^3 + x^6)/(x*(1 + x^6)^(1/4)*(-4 + 5*x^3 - 4*x^6 + x^9)),x]
```

```
[Out] Integrate[(2 + x^3 + x^6)/(x*(1 + x^6)^(1/4)*(-4 + 5*x^3 - 4*x^6 + x^9)), x
]
```

IntegrateAlgebraic [A] time = 15.63, size = 43, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\frac{1-x^3}{\sqrt[4]{x^6+1}}\right)-\frac{1}{3}\tanh^{-1}\left(\frac{x^3-1}{\sqrt[4]{x^6+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x^3 + x^6)/(x*(1 + x^6)^(1/4)*(-4 + 5*x^3 - 4*x^6 +
 x^9)),x]
```

```
[Out] ArcTan[(1 - x^3)/(1 + x^6)^(1/4)]/3 - ArcTanh[(-1 + x^3)/(1 + x^6)^(1/4)]/3
```

fricas [B] time = 12.49, size = 167, normalized size = 3.88

$$\frac{1}{6}\arctan\left(\frac{2\left(\left(x^6+1\right)^{\frac{3}{4}}\left(x^3-1\right)+\left(x^9-3x^6+3x^3-1\right)\left(x^6+1\right)^{\frac{1}{4}}\right)}{x^{12}-4x^9+5x^6-4x^3}\right)+\frac{1}{6}\log\left(-\frac{x^{12}-4x^9+7x^6-4x^3-2\left(x^6+1\right)^{\frac{3}{4}}\left(x^3-1\right)+2\left(x^6-2x^3+1\right)\sqrt{x^6+1}-2\left(x^9-3x^6+3x^3-1\right)\left(x^6+1\right)^{\frac{1}{4}}+2}{x^{12}-4x^9+5x^6-4x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+2)/x/(x^6+1)^(1/4)/(x^9-4*x^6+5*x^3-4),x, algorithm="fri
cas")
```

```
[Out] 1/6*arctan(2*((x^6 + 1)^(3/4)*(x^3 - 1) + (x^9 - 3*x^6 + 3*x^3 - 1)*(x^6 +
1)^(1/4))/(x^12 - 4*x^9 + 5*x^6 - 4*x^3)) + 1/6*log(-(x^12 - 4*x^9 + 7*x^6
- 4*x^3 - 2*(x^6 + 1)^(3/4)*(x^3 - 1) + 2*(x^6 - 2*x^3 + 1)*sqrt(x^6 + 1) -
 2*(x^9 - 3*x^6 + 3*x^3 - 1)*(x^6 + 1)^(1/4) + 2)/(x^12 - 4*x^9 + 5*x^6 - 4
*x^3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+x^3+2}{\left(x^9-4x^6+5x^3-4\right)\left(x^6+1\right)^{\frac{1}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+2)/x/(x^6+1)^(1/4)/(x^9-4*x^6+5*x^3-4),x, algorithm="gia
c")
```

```
[Out] integrate((x^6 + x^3 + 2)/((x^9 - 4*x^6 + 5*x^3 - 4)*(x^6 + 1)^(1/4)*x), x)
```

maple [C] time = 112.80, size = 346, normalized size = 8.05

$$-\frac{\ln\left(-\frac{x^{12}+2\left(x^6+1\right)^{\frac{1}{4}}x^9-4x^9+2\sqrt{x^6+1}\,x^6-6\left(x^6+1\right)^{\frac{1}{4}}x^6+2\left(x^6+1\right)^{\frac{3}{4}}x^3+7x^6-4x^3\sqrt{x^6+1}+6\left(x^6+1\right)^{\frac{1}{4}}x^3-2\left(x^6+1\right)^{\frac{3}{4}}-4x^3+2\sqrt{x^6+1}-2\left(x^6+1\right)^{\frac{1}{4}}+2}{\left(x^9-4x^6+5x^3-4\right)x^3}\right)}{6}-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x^{12}-2\left(x^6+1\right)^{\frac{1}{4}}x^9-4\operatorname{RootOf}\left(\_Z^2+1\right)x^9-2\sqrt{x^6+1}\,\operatorname{RootOf}\left(\_Z^2+1\right)x^6+6\left(x^6+1\right)^{\frac{1}{4}}x^6+7\operatorname{RootOf}\left(\_Z^2+1\right)x^6+2\left(x^6+1\right)^{\frac{3}{4}}x^3+4\sqrt{x^6+1}\,\operatorname{RootOf}\left(\_Z^2+1\right)x^3-6\left(x^6+1\right)^{\frac{1}{4}}x^3-4\operatorname{RootOf}\left(\_Z^2+1\right)x^3-2\left(x^6+1\right)^{\frac{3}{4}}-2\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{x^6+1}+2\left(x^6+1\right)^{\frac{1}{4}}+2\operatorname{RootOf}\left(\_Z^2+1\right)}{\left(x^9-4x^6+5x^3-4\right)x^3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+x^3+2)/x/(x^6+1)^(1/4)/(x^9-4*x^6+5*x^3-4),x)
```

```
[Out] -1/6*ln(-(x^12+2*(x^6+1)^(1/4)*x^9-4*x^9+2*(x^6+1)^(1/2)*x^6-6*(x^6+1)^(1/4
)*x^6+2*(x^6+1)^(3/4)*x^3+7*x^6-4*x^3*(x^6+1)^(1/2)+6*(x^6+1)^(1/4)*x^3-2*(
x^6+1)^(3/4)-4*x^3+2*(x^6+1)^(1/2)-2*(x^6+1)^(1/4)+2)/(x^9-4*x^6+5*x^3-4)/x
^3)-1/6*RootOf(_Z^2+1)*ln(-(RootOf(_Z^2+1)*x^12-2*(x^6+1)^(1/4)*x^9-4*RootO
f(_Z^2+1)*x^9-2*(x^6+1)^(1/2)*RootOf(_Z^2+1)*x^6+6*(x^6+1)^(1/4)*x^6+7*Root
Of(_Z^2+1)*x^6+2*(x^6+1)^(3/4)*x^3+4*(x^6+1)^(1/2)*RootOf(_Z^2+1)*x^3-6*(x^
6+1)^(1/4)*x^3-4*RootOf(_Z^2+1)*x^3-2*(x^6+1)^(3/4)-2*RootOf(_Z^2+1)*(x^6+1
)^(1/2)+2*(x^6+1)^(1/4)+2*RootOf(_Z^2+1))/(x^9-4*x^6+5*x^3-4)/x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 + x^3 + 2}{\left(x^9 - 4\,x^6 + 5\,x^3 - 4\right)\left(x^6 + 1\right)^{\frac{1}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+2)/x/(x^6+1)^(1/4)/(x^9-4*x^6+5*x^3-4),x, algorithm="max
ima")
```

```
[Out] integrate((x^6 + x^3 + 2)/((x^9 - 4*x^6 + 5*x^3 - 4)*(x^6 + 1)^(1/4)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^6 + x^3 + 2}{x\,\left(x^6 + 1\right)^{1/4}\,\left(x^9 - 4\,x^6 + 5\,x^3 - 4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + x^6 + 2)/(x*(x^6 + 1)^(1/4)*(5*x^3 - 4*x^6 + x^9 - 4)),x)
```

```
[Out] int((x^3 + x^6 + 2)/(x*(x^6 + 1)^(1/4)*(5*x^3 - 4*x^6 + x^9 - 4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+x**3+2)/x/(x**6+1)**(1/4)/(x**9-4*x**6+5*x**3-4),x)
```

```
[Out] Timed out
```

3.562

$$\int \sqrt[5]{243-5265x+47250x^2-225810x^3+615255x^4-954733x^5+82}$$

Optimal. Leaf size=43

$$\frac{x\sqrt[5]{\left(4x^2-13x+3\right)^5}\left(8x^2-39x+18\right)}{6\left(4x^2-13x+3\right)}$$

Rubi [B] time = 0.06, antiderivative size = 102, normalized size of antiderivative = 2.37, number of steps used = 3, number of rules used = 2, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {6688, 6720}

$$-\frac{13\sqrt[5]{\left(4x^2-13x+3\right)^5}x^2}{2\left(4x^2-13x+3\right)}+\frac{3\sqrt[5]{\left(4x^2-13x+3\right)^5}x}{4x^2-13x+3}+\frac{4\sqrt[5]{\left(4x^2-13x+3\right)^5}x^3}{3\left(4x^2-13x+3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(243 - 5265*x + 47250*x^2 - 225810*x^3 + 615255*x^4 - 954733*x^5 + 8203
40*x^6 - 401440*x^7 + 112000*x^8 - 16640*x^9 + 1024*x^10)^(1/5),x]
```

```
[Out] (3*x*((3 - 13*x + 4*x^2)^5)^(1/5))/(3 - 13*x + 4*x^2) - (13*x^2*((3 - 13*x
+ 4*x^2)^5)^(1/5))/(2*(3 - 13*x + 4*x^2)) + (4*x^3*((3 - 13*x + 4*x^2)^5)^(
1/5))/(3*(3 - 13*x + 4*x^2))
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \sqrt[5]{243-5265x+47250x^2-225810x^3+615255x^4-954733x^5+820340x^6-401440x^7+112000x^8-16640x^9}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{x\sqrt[5]{\left(4x^2-13x+3\right)^5}\left(8x^2-39x+18\right)}{6\left(4x^2-13x+3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(243 - 5265*x + 47250*x^2 - 225810*x^3 + 615255*x^4 - 954733*x^5
+ 820340*x^6 - 401440*x^7 + 112000*x^8 - 16640*x^9 + 1024*x^10)^(1/5),x]
```

```
[Out] (x*((3 - 13*x + 4*x^2)^5)^(1/5)*(18 - 39*x + 8*x^2))/(6*(3 - 13*x + 4*x^2))
```

IntegrateAlgebraic [A] time = 19.10, size = 43, normalized size = 1.00

$$\frac{x\sqrt[5]{\left(4x^2-13x+3\right)^5}\,\left(8x^2-39x+18\right)}{6\left(4x^2-13x+3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(243 - 5265*x + 47250*x^2 - 225810*x^3 + 615255*x^4 - 95
4733*x^5 + 820340*x^6 - 401440*x^7 + 112000*x^8 - 16640*x^9 + 1024*x^10)^(1
/5),x]
```

```
[Out] (x*((3 - 13*x + 4*x^2)^5)^(1/5)*(18 - 39*x + 8*x^2))/(6*(3 - 13*x + 4*x^2))
```

fricas [A] time = 0.37, size = 14, normalized size = 0.33

$$\frac{4}{3}\,x^3-\frac{13}{2}\,x^2+3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1024*x^10-16640*x^9+112000*x^8-401440*x^7+820340*x^6-954733*x^5+
615255*x^4-225810*x^3+47250*x^2-5265*x+243)^(1/5),x, algorithm="fricas")
```

```
[Out] 4/3*x^3 - 13/2*x^2 + 3*x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1024\,x^{10}-16640\,x^9+112000\,x^8-401440\,x^7+820340\,x^6-954733\,x^5+615255\,x^4-225810\,x^3+47250\,x^2-5265\,x+243\right)^{\frac{1}{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1024*x^10-16640*x^9+112000*x^8-401440*x^7+820340*x^6-954733*x^5+
615255*x^4-225810*x^3+47250*x^2-5265*x+243)^(1/5),x, algorithm="giac")
```

```
[Out] integrate((1024*x^10 - 16640*x^9 + 112000*x^8 - 401440*x^7 + 820340*x^6 - 9
54733*x^5 + 615255*x^4 - 225810*x^3 + 47250*x^2 - 5265*x + 243)^(1/5), x)
```

maple [B] time = 0.01, size = 78, normalized size = 1.81

$$\frac{x\left(8x^2-39x+18\right)\left(1024x^{10}-16640x^9+112000x^8-401440x^7+820340x^6-954733x^5+615255x^4-225810x^3+47250x^2-5265x+243\right)^{\frac{1}{5}}}{6\left(-1+4x\right)\left(-3+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1024*x^10-16640*x^9+112000*x^8-401440*x^7+820340*x^6-954733*x^5+615255
*x^4-225810*x^3+47250*x^2-5265*x+243)^(1/5),x)
```

```
[Out] 1/6*x*(8*x^2-39*x+18)*(1024*x^10-16640*x^9+112000*x^8-401440*x^7+820340*x^6
-954733*x^5+615255*x^4-225810*x^3+47250*x^2-5265*x+243)^(1/5)/(-1+4*x)/(-3+
x)
```

maxima [A] time = 0.35, size = 14, normalized size = 0.33

$$\frac{4}{3}\,x^3-\frac{13}{2}\,x^2+3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1024*x^10-16640*x^9+112000*x^8-401440*x^7+820340*x^6-954733*x^5+
615255*x^4-225810*x^3+47250*x^2-5265*x+243)^(1/5),x, algorithm="maxima")
```

```
[Out] 4/3*x^3 - 13/2*x^2 + 3*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(1024\,x^{10}-16640\,x^9+112000\,x^8-401440\,x^7+820340\,x^6-954733\,x^5+615255\,x^4-225810\,x^3+47250\,x^2-5265\,x+243\right)^{1/5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((47250*x^2 - 5265*x - 225810*x^3 + 615255*x^4 - 954733*x^5 + 820340*x^6
 - 401440*x^7 + 112000*x^8 - 16640*x^9 + 1024*x^10 + 243)^(1/5),x)
```

```
[Out] int((47250*x^2 - 5265*x - 225810*x^3 + 615255*x^4 - 954733*x^5 + 820340*x^6
 - 401440*x^7 + 112000*x^8 - 16640*x^9 + 1024*x^10 + 243)^(1/5), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[5]{1024x^{10}-16640x^9+112000x^8-401440x^7+820340x^6-954733x^5+615255x^4-225810x^3+47250x^2-5265x+243}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1024*x**10-16640*x**9+112000*x**8-401440*x**7+820340*x**6-954733
*x**5+615255*x**4-225810*x**3+47250*x**2-5265*x+243)**(1/5),x)
```

```
[Out] Integral((1024*x**10 - 16640*x**9 + 112000*x**8 - 401440*x**7 + 820340*x**6
 - 954733*x**5 + 615255*x**4 - 225810*x**3 + 47250*x**2 - 5265*x + 243)**(1
/5), x)
```

3.563 $\int \frac{-bx^3+2ax^8}{\sqrt[4]{-bx+ax^6}\left(-1-bx^5+ax^{10}\right)}\,dx$

Optimal. Leaf size=43

$$\frac{2}{5}\tan^{-1}\left(x\sqrt[4]{ax^6-bx}\right)-\frac{2}{5}\tanh^{-1}\left(x\sqrt[4]{ax^6-bx}\right)$$

Rubi [C] time = 1.48, antiderivative size = 189, normalized size of antiderivative = 4.40, number of steps used = 12, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {1593, 2056, 6728, 466, 465, 511, 510}

$$-\frac{8ax^4\sqrt[4]{1-\frac{ax^5}{b}}F_1\left(\frac{3}{4};1,\frac{1}{4};\frac{7}{4};\frac{2ax^5}{b-\sqrt{b^2+4a}},\frac{ax^5}{b}\right)}{15\left(b-\sqrt{4a+b^2}\right)\sqrt[4]{ax^6-bx}}-\frac{8ax^4\sqrt[4]{1-\frac{ax^5}{b}}F_1\left(\frac{3}{4};1,\frac{1}{4};\frac{7}{4};\frac{2ax^5}{b+\sqrt{b^2+4a}},\frac{ax^5}{b}\right)}{15\left(\sqrt{4a+b^2}+b\right)\sqrt[4]{ax^6-bx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-(b*x^3) + 2*a*x^8)/((-(b*x) + a*x^6)^(1/4)*(-1 - b*x^5 + a*x^10)),x]
```

```
[Out] (-8*a*x^4*(1 - (a*x^5)/b)^(1/4)*AppellF1[3/4, 1, 1/4, 7/4, (2*a*x^5)/(b - S
qrt[4*a + b^2]), (a*x^5)/b])/(15*(b - Sqrt[4*a + b^2])*(-(b*x) + a*x^6)^(1/
4)) - (8*a*x^4*(1 - (a*x^5)/b)^(1/4)*AppellF1[3/4, 1, 1/4, 7/4, (2*a*x^5)/(
b + Sqrt[4*a + b^2]), (a*x^5)/b])/(15*(b + Sqrt[4*a + b^2])*(-(b*x) + a*x^6
)^(1/4))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-bx^3+2ax^8}{\sqrt[4]{-bx+ax^6}\left(-1-bx^5+ax^{10}\right)}\,dx=\int \frac{x^3\left(-b+2ax^5\right)}{\sqrt[4]{-bx+ax^6}\left(-1-bx^5+ax^{10}\right)}\,dx$$

$$=\frac{\left(\sqrt[4]{x}\sqrt[4]{-b+ax^5}\right)\int \frac{x^{11/4}\left(-b+2ax^5\right)}{\sqrt[4]{-b+ax^5}\left(-1-bx^5+ax^{10}\right)}\,dx}{\sqrt[4]{-bx+ax^6}}$$

$$=\frac{\left(\sqrt[4]{x}\sqrt[4]{-b+ax^5}\right)\int \left(\frac{2ax^{11/4}}{\sqrt[4]{-b+ax^5}\left(-b-\sqrt{4a+b^2}+2ax^5\right)}+\frac{2ax^{11/4}}{\sqrt[4]{-b+ax^5}\left(-b+\sqrt{4a+b^2}+2ax^5\right)}\right)}{\sqrt[4]{-bx+ax^6}}$$

$$=\frac{\left(2a\sqrt[4]{x}\sqrt[4]{-b+ax^5}\right)\int \frac{x^{11/4}}{\sqrt[4]{-b+ax^5}\left(-b-\sqrt{4a+b^2}+2ax^5\right)}\,dx}{\sqrt[4]{-bx+ax^6}}+\frac{\left(2a\sqrt[4]{x}\sqrt[4]{-b+ax^5}\right)}{\sqrt[4]{}}$$

$$=\frac{\left(8a\sqrt[4]{x}\sqrt[4]{-b+ax^5}\right)\text{Subst}\left(\int \frac{x^{14}}{\sqrt[4]{-b+ax^{20}}\left(-b-\sqrt{4a+b^2}+2ax^{20}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^6}}+\frac{(8a}{}$$

$$=\frac{\left(8a\sqrt[4]{x}\sqrt[4]{-b+ax^5}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{-b+ax^4}\left(-b-\sqrt{4a+b^2}+2ax^4\right)}\,dx,x,x^{5/4}\right)}{5\sqrt[4]{-bx+ax^6}}+\frac{(8a\sqrt[4]{}}{}$$

$$=\frac{\left(8a\sqrt[4]{x}\sqrt[4]{1-\frac{ax^5}{b}}\right)\text{Subst}\left(\int \frac{x^2}{\left(-b-\sqrt{4a+b^2}+2ax^4\right)\sqrt[4]{1-\frac{ax^4}{b}}}\,dx,x,x^{5/4}\right)}{5\sqrt[4]{-bx+ax^6}}+\frac{(8a\sqrt[4]{x}}{}$$

$$=-\frac{8ax^4\sqrt[4]{1-\frac{ax^5}{b}}\,F_1\left(\frac{3}{4};1,\frac{1}{4};\frac{7}{4};\frac{2ax^5}{b-\sqrt{4a+b^2}},\frac{ax^5}{b}\right)}{15\left(b-\sqrt{4a+b^2}\right)\sqrt[4]{-bx+ax^6}}-\frac{8ax^4\sqrt[4]{1-\frac{ax^5}{b}}\,F_1\left(\frac{3}{4};1,\frac{1}{4};\frac{7}{4}\right.}{15\left(b+\sqrt{4a+b^2}\right)\sqrt[4]{}}$$

Mathematica [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{-bx^3+2ax^8}{\sqrt[4]{-bx+ax^6}\left(-1-bx^5+ax^{10}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-(b*x^3) + 2*a*x^8)/((-(b*x) + a*x^6)^(1/4)*(-1 - b*x^5 + a*x^10
)),x]
```

```
[Out] Integrate[(-(b*x^3) + 2*a*x^8)/((-(b*x) + a*x^6)^(1/4)*(-1 - b*x^5 + a*x^10
)), x]
```

IntegrateAlgebraic [A] time = 20.01, size = 43, normalized size = 1.00

$$\frac{2}{5}\tan^{-1}\left(x\sqrt[4]{ax^6-bx}\right)-\frac{2}{5}\tanh^{-1}\left(x\sqrt[4]{ax^6-bx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(b*x^3) + 2*a*x^8)/((-(b*x) + a*x^6)^(1/4)*(-1 - b*x^5
 + a*x^10)),x]
```

```
[Out] (2*ArcTan[x*(-(b*x) + a*x^6)^(1/4)])/5 - (2*ArcTanh[x*(-(b*x) + a*x^6)^(1/4
)])/5
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^8-b*x^3)/(a*x^6-b*x)^(1/4)/(a*x^10-b*x^5-1),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax^8-bx^3}{\left(ax^{10}-bx^5-1\right)\left(ax^6-bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^8-b*x^3)/(a*x^6-b*x)^(1/4)/(a*x^10-b*x^5-1),x, algorithm="
giac")
```

```
[Out] integrate((2*a*x^8 - b*x^3)/((a*x^10 - b*x^5 - 1)*(a*x^6 - b*x)^(1/4)), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{2a\,x^8-b\,x^3}{\left(a\,x^6-bx\right)^{\frac{1}{4}}\left(a\,x^{10}-b\,x^5-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x^8-b*x^3)/(a*x^6-b*x)^(1/4)/(a*x^10-b*x^5-1),x)
```

```
[Out] int((2*a*x^8-b*x^3)/(a*x^6-b*x)^(1/4)/(a*x^10-b*x^5-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax^8-bx^3}{\left(ax^{10}-bx^5-1\right)\left(ax^6-bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^8-b*x^3)/(a*x^6-b*x)^(1/4)/(a*x^10-b*x^5-1),x, algorithm="
maxima")
```

```
[Out] integrate((2*a*x^8 - b*x^3)/((a*x^10 - b*x^5 - 1)*(a*x^6 - b*x)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{2\,a\,x^8-b\,x^3}{{\left(a\,x^6-b\,x\right)}^{1/4}\,\left(-a\,x^{10}+b\,x^5+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*a*x^8 - b*x^3)/((a*x^6 - b*x)^(1/4)*(b*x^5 - a*x^10 + 1)),x)
```

```
[Out] int(-(2*a*x^8 - b*x^3)/((a*x^6 - b*x)^(1/4)*(b*x^5 - a*x^10 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x**8-b*x**3)/(a*x**6-b*x)**(1/4)/(a*x**10-b*x**5-1),x)
```

```
[Out] Timed out
```

3.564 $\int \frac{x^5(-7b+10ax^3)}{\sqrt[4]{-bx^3+ax^6}\left(-1-bx^7+ax^{10}\right)}\,dx$

Optimal. Leaf size=43

$$2\tan^{-1}\left(x\sqrt[4]{ax^6-bx^3}\right)-2\tanh^{-1}\left(x\sqrt[4]{ax^6-bx^3}\right)$$

Rubi [F] time = 2.79, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^5\left(-7b+10ax^3\right)}{\sqrt[4]{-bx^3+ax^6}\left(-1-bx^7+ax^{10}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^5*(-7*b + 10*a*x^3))/((-(b*x^3) + a*x^6)^(1/4)*(-1 - b*x^7 + a*x^10)
),x]
```

```
[Out] (28*b*x^(3/4)*(-b + a*x^3)^(1/4)*Defer[Subst][Defer[Int][x^20/((-b + a*x^12
)^(1/4)*(1 + b*x^28 - a*x^40)), x], x, x^(1/4)])/(-(b*x^3) + a*x^6)^(1/4) +
 (40*a*x^(3/4)*(-b + a*x^3)^(1/4)*Defer[Subst][Defer[Int][x^32/((-b + a*x^1
2)^(1/4)*(-1 - b*x^28 + a*x^40)), x], x, x^(1/4)])/(-(b*x^3) + a*x^6)^(1/4)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(-7b+10ax^3\right)}{\sqrt[4]{-bx^3+ax^6}\left(-1-bx^7+ax^{10}\right)}\,dx &= \frac{\left(x^{3/4}\sqrt[4]{-b+ax^3}\right)\int \frac{x^{17/4}\left(-7b+10ax^3\right)}{\sqrt[4]{-b+ax^3}\left(-1-bx^7+ax^{10}\right)}\,dx}{\sqrt[4]{-bx^3+ax^6}}\\
&= \frac{\left(4x^{3/4}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{x^{20}\left(-7b+10ax^{12}\right)}{\sqrt[4]{-b+ax^{12}}\left(-1-bx^{28}+ax^{40}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx^3+ax^6}}\\
&= \frac{\left(4x^{3/4}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \left(\frac{7bx^{20}}{\sqrt[4]{-b+ax^{12}}\left(1+bx^{28}-ax^{40}\right)}+\frac{10ax^{32}}{\sqrt[4]{-b+ax^{12}}\left(-1-bx^{28}\right.}\right.\right.}{\sqrt[4]{-bx^3+ax^6}}\\
&= \frac{\left(40ax^{3/4}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{x^{32}}{\sqrt[4]{-b+ax^{12}}\left(-1-bx^{28}+ax^{40}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx^3+ax^6}}+\frac{(2}{}
\end{aligned}$$

Mathematica [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(-7b+10ax^3\right)}{\sqrt[4]{-bx^3+ax^6}\left(-1-bx^7+ax^{10}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^5*(-7*b + 10*a*x^3))/((-(b*x^3) + a*x^6)^(1/4)*(-1 - b*x^7 + a
*x^10)),x]
```

```
[Out] Integrate[(x^5*(-7*b + 10*a*x^3))/((-(b*x^3) + a*x^6)^(1/4)*(-1 - b*x^7 + a
*x^10)), x]
```

IntegrateAlgebraic [A] time = 18.50, size = 43, normalized size = 1.00

$$2\tan^{-1}\left(x\sqrt[4]{ax^6-bx^3}\right)-2\tanh^{-1}\left(x\sqrt[4]{ax^6-bx^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(-7*b + 10*a*x^3))/((-(b*x^3) + a*x^6)^(1/4)*(-1 -
b*x^7 + a*x^10)),x]
```

```
[Out] 2*ArcTan[x*(-(b*x^3) + a*x^6)^(1/4)] - 2*ArcTanh[x*(-(b*x^3) + a*x^6)^(1/4)
]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(10*a*x^3-7*b)/(a*x^6-b*x^3)^(1/4)/(a*x^10-b*x^7-1),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(10\,ax^3-7\,b\right)x^5}{\left(ax^{10}-bx^7-1\right)\left(ax^6-bx^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(10*a*x^3-7*b)/(a*x^6-b*x^3)^(1/4)/(a*x^10-b*x^7-1),x, algori
thm="giac")
```

```
[Out] integrate((10*a*x^3 - 7*b)*x^5/((a*x^10 - b*x^7 - 1)*(a*x^6 - b*x^3)^(1/4))
, x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(10a\,x^3-7b\right)}{\left(a\,x^6-b\,x^3\right)^{\frac{1}{4}}\left(a\,x^{10}-b\,x^7-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(10*a*x^3-7*b)/(a*x^6-b*x^3)^(1/4)/(a*x^10-b*x^7-1),x)
```

```
[Out] int(x^5*(10*a*x^3-7*b)/(a*x^6-b*x^3)^(1/4)/(a*x^10-b*x^7-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(10\,ax^3-7\,b\right)x^5}{\left(ax^{10}-bx^7-1\right)\left(ax^6-bx^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(10*a*x^3-7*b)/(a*x^6-b*x^3)^(1/4)/(a*x^10-b*x^7-1),x, algori
thm="maxima")
```

```
[Out] integrate((10*a*x^3 - 7*b)*x^5/((a*x^10 - b*x^7 - 1)*(a*x^6 - b*x^3)^(1/4))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^5\,\left(7\,b-10\,a\,x^3\right)}{{\left(a\,x^6-b\,x^3\right)}^{1/4}\,\left(-a\,x^{10}+b\,x^7+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(7*b - 10*a*x^3))/((a*x^6 - b*x^3)^(1/4)*(b*x^7 - a*x^10 + 1)),x)
[Out] int((x^5*(7*b - 10*a*x^3))/((a*x^6 - b*x^3)^(1/4)*(b*x^7 - a*x^10 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \left(10 a x^{3} - 7 b\right)}{\sqrt[4]{x^{3} \left(a x^{3} - b\right)} \left(a x^{10} - b x^{7} - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(10*a*x**3-7*b)/(a*x**6-b*x**3)**(1/4)/(a*x**10-b*x**7-1),x)
[Out] Integral(x**5*(10*a*x**3 - 7*b)/((x**3*(a*x**3 - b))**(1/4)*(a*x**10 - b*x*
*7 - 1)), x)
```

3.565 $\int \frac{1}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$

Optimal. Leaf size=43

$$\frac{2b\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{ax}$$

Rubi [F] time = 1.77, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx=\int \frac{1}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$$

Mathematica [A] time = 0.65, size = 72, normalized size = 1.67

$$\frac{2\left(bx\sqrt{\frac{a(ax^2-1)}{b^2}}+ax^2-1\right)}{\sqrt{\frac{a(ax^2-1)}{b^2}}\sqrt{x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]),x]
```

```
[Out] (2*(-1 + a*x^2 + b*x*Sqrt[(a*(-1 + a*x^2))/b^2]))/(Sqrt[(a*(-1 + a*x^2))/b^2]*Sqrt[x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])])
```

IntegrateAlgebraic [A] time = 2.89, size = 43, normalized size = 1.00

$$\frac{2b\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]),x]
```

```
[Out] (2*b*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]])/(a*x)
```

fricas [A] time = 0.44, size = 37, normalized size = 0.86

$$\frac{2\sqrt{ax^2 + bx\sqrt{\frac{a^2x^2-a}{b^2}}}\,b}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a/b^2+a^2*x^2/b^2)^(1/2)/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1
/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*b/(a*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\,bx}\,\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a/b^2+a^2*x^2/b^2)^(1/2)/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1
/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)*sqrt(a^2*x^2/b^2 -
 a/b^2)*x), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}\,\sqrt{a\,x^2 + bx\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a/b^2+a^2*x^2/b^2)^(1/2)/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(
1/2),x)
```

```
[Out] int(1/x/(-a/b^2+a^2*x^2/b^2)^(1/2)/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(
1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\,bx}\,\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a/b^2+a^2*x^2/b^2)^(1/2)/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1
/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)*sqrt(a^2*x^2/b^2 -
 a/b^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\sqrt{a\,x^2 + b\,x\sqrt{\frac{a^2\,x^2}{b^2} - \frac{a}{b^2}}}\,\sqrt{\frac{a^2\,x^2}{b^2} - \frac{a}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)*((a^2*x^2)/b^2 -
 a/b^2)^(1/2)),x)
```

```
[Out] int(1/(x*(a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)*((a^2*x^2)/b^2 -
 a/b^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x\left(ax+b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)}\sqrt{\frac{a(ax^2-1)}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a/b**2+a**2*x**2/b**2)**(1/2)/(a*x**2+b*x*(-a/b**2+a**2*x**
2/b**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))*sqrt(a*(a*x**
2 - 1)/b**2)), x)
```

3.566 $\int \frac{ab-2ax+x^2}{\sqrt{x(-a+x)(-b+x)}\left(ad-(b+d)x+x^2\right)}\,dx$

Optimal. Leaf size=44

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{x^2(-a-b)+abx+x^3}}{\sqrt{d}\,(a-x)}\right)}{\sqrt{d}}$$

Rubi [C] time = 6.25, antiderivative size = 377, normalized size of antiderivative = 8.57, number of steps used = 15, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {6718, 6728, 117, 116, 169, 538, 537}

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\left(\sqrt{-4ad+b^2+2bd+d^2}+2a-b-d\right)\Pi\left(\frac{2a}{b+d-\sqrt{b^2+2db+d^2-4ad}};\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{\left(-\sqrt{-4ad+b^2+2bd+d^2}+b+d\right)\sqrt{x(a-x)(b-x)}}+\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\left(-\sqrt{-4ad+b^2+2bd+d^2}+2a-b-d\right)\Pi\left(\frac{2a}{b+d+\sqrt{b^2+2db+d^2-4ad}};\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{\left(\sqrt{-4ad+b^2+2bd+d^2}+b+d\right)\sqrt{x(a-x)(b-x)}}+\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{\sqrt{x(a-x)(b-x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b - 2*a*x + x^2)/(Sqrt[x*(-a + x)*(-b + x)]*(a*d - (b + d)*x + x^2))
,x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 - x/a]*Sqrt[1 - x/b]*EllipticF[ArcSin[Sqrt[x]/Sqr
t[a]], a/b])/Sqrt[(a - x)*(b - x)*x] + (2*Sqrt[a]*(2*a - b - d + Sqrt[b^2 -
 4*a*d + 2*b*d + d^2])*Sqrt[x]*Sqrt[1 - x/a]*Sqrt[1 - x/b]*EllipticPi[(2*a)
/(b + d - Sqrt[b^2 - 4*a*d + 2*b*d + d^2]), ArcSin[Sqrt[x]/Sqrt[a]], a/b])/
((b + d - Sqrt[b^2 - 4*a*d + 2*b*d + d^2])*Sqrt[(a - x)*(b - x)*x]) + (2*Sq
rt[a]*(2*a - b - d - Sqrt[b^2 - 4*a*d + 2*b*d + d^2])*Sqrt[x]*Sqrt[1 - x/a]
*Sqrt[1 - x/b]*EllipticPi[(2*a)/(b + d + Sqrt[b^2 - 4*a*d + 2*b*d + d^2]),
ArcSin[Sqrt[x]/Sqrt[a]], a/b])/((b + d + Sqrt[b^2 - 4*a*d + 2*b*d + d^2])*S
qrt[(a - x)*(b - x)*x])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !FreeQ[w, x] &&  !Fr
eeQ[z, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{ab-2ax+x^2}{\sqrt{x(-a+x)(-b+x)}\left(ad-(b+d)x+x^2\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{ab-2ax+x^2}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\left(ad-(b+d)x+x^2\right)}\,dx}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int\left(\frac{1}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}} + \frac{a(b-d)-(2\cdots}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+x\cdots}}\right.}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{1}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}}\,dx}{\sqrt{x(-a+x)(-b+x)}} + \frac{\left(\sqrt{x}\sqrt{-a+\cdots\right.}{\cdots}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int\left(\frac{-2a+b+d+\sqrt{b^2-4ad+2bd+d^2}}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\left(-b-d-\sqrt{b^2-4ad+2bd+\cdots}\right.}\right.}{\sqrt{x(-a+x)(-\cdots}}$$

$$= \frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{\sqrt{(a-x)(b-x)x}} + \frac{\left(\left(-2a+b+d+\cdots\right.\right.}{\cdots}$$

$$= \frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{\sqrt{(a-x)(b-x)x}} - \frac{\left(2\left(-2a+b+\cdots\right.\right.}{\cdots}$$

$$= \frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{\sqrt{(a-x)(b-x)x}} - \frac{\left(2\left(-2a+b+\cdots\right.\right.}{\cdots}$$

$$= \frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{\sqrt{(a-x)(b-x)x}} - \frac{\left(2\left(-2a+b+\cdots\right.\right.}{\cdots}$$

$$= \frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{\sqrt{(a-x)(b-x)x}} + \frac{2\sqrt{a}\left(2a-b-\cdots\right.}{\cdots}$$

Mathematica [C] time = 4.81, size = 346, normalized size = 7.86

$$-\frac{iax^{3/2}\sqrt{1-\frac{a}{x}}\sqrt{1-\frac{b}{x}}\left(\left(-b\left(\sqrt{-4ad+b^2+2bd+d^2}-2d\right)+d\left(\sqrt{-4ad+b^2+2bd+d^2}-4a+d\right)+b^2\right)\Pi\left(\frac{2d}{b+d-\sqrt{b^2+2db+d^2-4ad}};i\sinh^{-1}\left(\frac{\sqrt{-a}}{\sqrt{x}}\right)|\frac{b}{a}\right)-\left(b\left(\sqrt{-4ad+b^2+2bd+d^2}+2d\right)+d\left(-\sqrt{-4ad+b^2+2bd+d^2}-4a+d\right)+b^2\right)\Pi\left(\frac{2d}{b+d+\sqrt{b^2+2db+d^2-4ad}};i\sinh^{-1}\left(\frac{\sqrt{-a}}{\sqrt{x}}\right)|\frac{b}{a}\right)+2b\sqrt{d(d-4a)+b^2+2bd}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-a}}{\sqrt{x}}\right)|\frac{b}{a}\right)\right)}{(-a)^{3/2}d\sqrt{d(d-4a)+b^2+2bd}\sqrt{x(x-a)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b - 2*a*x + x^2)/(Sqrt[x*(-a + x)*(-b + x)]*(a*d - (b + d)*x +
 x^2)),x]
```

```
[Out] ((-I)*a*Sqrt[1 - a/x]*Sqrt[1 - b/x]*x^(3/2)*(2*b*Sqrt[b^2 + 2*b*d + d*(-4*a
 + d)]*EllipticF[I*ArcSinh[Sqrt[-a]/Sqrt[x]], b/a] + (b^2 - b*(-2*d + Sqrt[
b^2 - 4*a*d + 2*b*d + d^2]) + d*(-4*a + d + Sqrt[b^2 - 4*a*d + 2*b*d + d^2]
))*EllipticPi[(2*d)/(b + d - Sqrt[b^2 - 4*a*d + 2*b*d + d^2]), I*ArcSinh[Sq
rt[-a]/Sqrt[x]], b/a] - (b^2 + d*(-4*a + d - Sqrt[b^2 - 4*a*d + 2*b*d + d^2
]) + b*(2*d + Sqrt[b^2 - 4*a*d + 2*b*d + d^2]))*EllipticPi[(2*d)/(b + d + S
qrt[b^2 - 4*a*d + 2*b*d + d^2]), I*ArcSinh[Sqrt[-a]/Sqrt[x]], b/a]))/((-a)^
(3/2)*d*Sqrt[b^2 + 2*b*d + d*(-4*a + d)]*Sqrt[x*(-a + x)*(-b + x)])
```

IntegrateAlgebraic [A] time = 0.35, size = 44, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{x^2(-a-b)+abx+x^3}}{\sqrt{d}\,(a-x)}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b - 2*a*x + x^2)/(Sqrt[x*(-a + x)*(-b + x)]*(a*d - (b
 + d)*x + x^2)),x]
```

```
[Out] (2*ArcTanh[Sqrt[a*b*x + (-a - b)*x^2 + x^3]/(Sqrt[d]*(a - x))])/Sqrt[d]
```

fricas [B] time = 0.64, size = 224, normalized size = 5.09

$$\left[\frac{\log\left(\frac{a^2d^2-2\,(b-3\,d)x^3+x^4+\left(b^2-6\,(a+b)d+d^2\right)x^2+4\,\sqrt{abx-(a+b)x^2+x^3}\,\left(ad+(b-d)x-x^2\right)\sqrt{d}+2\left(3\,abd-ad^2\right)x}{a^2d^2-2\,(b+d)x^3+x^4+\left(b^2+2\,(a+b)d+d^2\right)x^2-2\left(abd+ad^2\right)x}\right)}{2\,\sqrt{d}},\ \frac{\sqrt{-d}\,\arctan\left(-\frac{\sqrt{abx-(a+b)x^2+x^3}\,\left(ad+(b-d)x-x^2\right)\sqrt{-d}}{2\left(abdx-(a+b)dx^2+dx^3\right)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a*d-(b+d)*x+x^2),x, algo
rithm="fricas")
```

```
[Out] [1/2*log((a^2*d^2 - 2*(b - 3*d)*x^3 + x^4 + (b^2 - 6*(a + b)*d + d^2)*x^2 +
 4*sqrt(a*b*x - (a + b)*x^2 + x^3)*(a*d + (b - d)*x - x^2)*sqrt(d) + 2*(3*a
*b*d - a*d^2)*x)/(a^2*d^2 - 2*(b + d)*x^3 + x^4 + (b^2 + 2*(a + b)*d + d^2)
*x^2 - 2*(a*b*d + a*d^2)*x))/sqrt(d), sqrt(-d)*arctan(-1/2*sqrt(a*b*x - (a
+ b)*x^2 + x^3)*(a*d + (b - d)*x - x^2)*sqrt(-d)/(a*b*d*x - (a + b)*d*x^2 +
 d*x^3))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab - 2\,ax + x^2}{\sqrt{(a-x)(b-x)x}\,\left(ad - (b+d)x + x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a*d-(b+d)*x+x^2),x, algo
rithm="giac")
```

```
[Out] integrate((a*b - 2*a*x + x^2)/(sqrt((a - x)*(b - x)*x)*(a*d - (b + d)*x + x
^2)), x)
```

maple [C] time = 0.06, size = 2321, normalized size = 52.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a*d-(b+d)*x+x^2),x)
```

```
[Out] -2*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^
2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))+4/(-4*a*d+b^2+2*b
*d+d^2)^(1/2)*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2
)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(
1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d
^2)^(1/2)),(a/(a-b))^(1/2))*d+2*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^
(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2*d-1/2*(-4*a*
d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2*b-1/2*d-1/2*(
-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))-1/(-4*a*d+b^2+2*b*d+d^2)^(1/2
)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2
-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*Elliptic
```

```
Pi((-(-a+x)/a)^(1/2),a/(a-1/2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/
(a-b))^(1/2))*b^2-2/(-4*a*d+b^2+2*b*d+d^2)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b
)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1
/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/
2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*b*d-a*(1-1/a*x
)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^
(1/2)/(a-1/2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/
a)^(1/2),a/(a-1/2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2)
)*b-1/(-4*a*d+b^2+2*b*d+d^2)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)
^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2*d-1/2*(-4*a
*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2*b-1/2*d-1/2*
(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*d^2-a*(1-1/a*x)^(1/2)*(-1/(a
-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b
-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-
1/2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*d-4/(-4*a*d+
b^2+2*b*d+d^2)^(1/2)*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*
x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d
+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+
2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*d+2*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a
-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2*d+1/2
*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2*b-1/2*
d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))+1/(-4*a*d+b^2+2*b*d+d^
2)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*
x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*E
llipticPi((-(-a+x)/a)^(1/2),a/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/
2)),(a/(a-b))^(1/2))*b^2+2/(-4*a*d+b^2+2*b*d+d^2)^(1/2)*a*(1-1/a*x)^(1/2)*(
-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-
1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),
a/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*b*d-a*(
1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^
2+x^3)^(1/2)/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-
(-a+x)/a)^(1/2),a/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b)
)^(1/2))*b+1/(-4*a*d+b^2+2*b*d+d^2)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(
a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2*d+1/
2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2*b-1/2
*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*d^2-a*(1-1/a*x)^(1/2)
*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(
a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2
),a/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a*d-(b+d)*x+x^2),x, algo
rithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((d+b)^2-4*a*d>0)', see `assume?` fo
r more details)Is (d+b)^2-4*a*d    positive, negative or zero?
```

mupad [B] time = 0.37, size = 465, normalized size = 10.57

$$\frac{2\,b\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(\frac{b}{\frac{b}{2}-\frac{d}{2}+\frac{\sqrt{b^2+2\,b\,d+d^2-4\,a\,d}}{2}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\,\left(\left(\frac{b}{2}+\frac{d}{2}-\frac{\sqrt{b^2+2\,b\,d+d^2-4\,a\,d}}{2}\right)\,\left(b-2\,a+d\right)+a\,b-a\,d\right)}{\sqrt{x^3+\left(-a-b\right)\,x^2+a\,b\,x}\,\left(\frac{b}{2}-\frac{d}{2}+\frac{\sqrt{b^2+2\,b\,d+d^2-4\,a\,d}}{2}\right)\,\sqrt{b^2+2\,b\,d+d^2-4\,a\,d}}-\frac{2\,b\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}}{\sqrt{x^3+\left(-a-b\right)\,x^2+a\,b\,x}}+\frac{2\,b\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(-\frac{b}{\frac{d}{2}-\frac{b}{2}+\frac{\sqrt{b^2+2\,b\,d+d^2-4\,a\,d}}{2}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\,\left(\left(\frac{b}{2}+\frac{d}{2}+\frac{\sqrt{b^2+2\,b\,d+d^2-4\,a\,d}}{2}\right)\,\left(b-2\,a+d\right)+a\,b-a\,d\right)}{\sqrt{x^3+\left(-a-b\right)\,x^2+a\,b\,x}\,\left(\frac{d}{2}-\frac{b}{2}+\frac{\sqrt{b^2+2\,b\,d+d^2-4\,a\,d}}{2}\right)\,\sqrt{b^2+2\,b\,d+d^2-4\,a\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b - 2*a*x + x^2)/((x*(a - x)*(b - x))^(1/2)*(a*d + x^2 - x*(b + d)))
,x)
```

```
[Out] (2*b*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*ellipticPi(b/(b/
2 - d/2 + (2*b*d - 4*a*d + b^2 + d^2)^(1/2)/2), asin(((b - x)/b)^(1/2)), -b
/(a - b))*((b/2 + d/2 - (2*b*d - 4*a*d + b^2 + d^2)^(1/2)/2)*(b - 2*a + d)
+ a*b - a*d))/((x^3 - x^2*(a + b) + a*b*x)^(1/2)*(b/2 - d/2 + (2*b*d - 4*a*
d + b^2 + d^2)^(1/2)/2)*(2*b*d - 4*a*d + b^2 + d^2)^(1/2)) - (2*b*ellipticF
(asin(((b - x)/b)^(1/2)), -b/(a - b))*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x
)/(a - b))^(1/2))/(x^3 - x^2*(a + b) + a*b*x)^(1/2) + (2*b*(x/b)^(1/2)*((b
- x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*ellipticPi(-b/(d/2 - b/2 + (2*b*d - 4
*a*d + b^2 + d^2)^(1/2)/2), asin(((b - x)/b)^(1/2)), -b/(a - b))*((b/2 + d/
2 + (2*b*d - 4*a*d + b^2 + d^2)^(1/2)/2)*(b - 2*a + d) + a*b - a*d))/((x^3
- x^2*(a + b) + a*b*x)^(1/2)*(d/2 - b/2 + (2*b*d - 4*a*d + b^2 + d^2)^(1/2)
/2)*(2*b*d - 4*a*d + b^2 + d^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*a*x+x**2)/(x*(-a+x)*(-b+x))**(1/2)/(a*d-(b+d)*x+x**2),x)
```

```
[Out] Timed out
```

3.567 $\int \frac{ab-2ax+x^2}{\sqrt{x(-a+x)(-b+x)}\left(a-(1+bd)x+dx^2\right)}\,dx$

Optimal. Leaf size=44

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{x^2(-a-b)+abx+x^3}}{a-x}\right)}{\sqrt{d}}$$

Rubi [C] time = 7.29, antiderivative size = 376, normalized size of antiderivative = 8.55, number of steps used = 15, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {6718, 6728, 117, 116, 169, 538, 537}

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{d\sqrt{x(a-x)(b-x)}}-\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\left(-\sqrt{(bd+1)^2-4ad}-2ad+bd+1\right)\Pi\left(\frac{2ad}{bd-\sqrt{(bd+1)^2-4ad}+1};\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{d\left(-\sqrt{(bd+1)^2-4ad}+bd+1\right)\sqrt{x(a-x)(b-x)}}-\frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\left(\sqrt{(bd+1)^2-4ad}-2ad+bd+1\right)\Pi\left(\frac{2ad}{bd+\sqrt{(bd+1)^2-4ad}+1};\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{d\left(\sqrt{(bd+1)^2-4ad}+bd+1\right)\sqrt{x(a-x)(b-x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b - 2*a*x + x^2)/(Sqrt[x*(-a + x)*(-b + x)]*(a - (1 + b*d)*x + d*x^2
)),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 - x/a]*Sqrt[1 - x/b]*EllipticF[ArcSin[Sqrt[x]/Sqr
t[a]], a/b])/(d*Sqrt[(a - x)*(b - x)*x]) - (2*Sqrt[a]*(1 - 2*a*d + b*d - Sq
rt[-4*a*d + (1 + b*d)^2])*Sqrt[x]*Sqrt[1 - x/a]*Sqrt[1 - x/b]*EllipticPi[(2
*a*d)/(1 + b*d - Sqrt[-4*a*d + (1 + b*d)^2]), ArcSin[Sqrt[x]/Sqrt[a]], a/b]
)/(d*(1 + b*d - Sqrt[-4*a*d + (1 + b*d)^2])*Sqrt[(a - x)*(b - x)*x]) - (2*S
qrt[a]*(1 - 2*a*d + b*d + Sqrt[-4*a*d + (1 + b*d)^2])*Sqrt[x]*Sqrt[1 - x/a]
*Sqrt[1 - x/b]*EllipticPi[(2*a*d)/(1 + b*d + Sqrt[-4*a*d + (1 + b*d)^2]), A
rcSin[Sqrt[x]/Sqrt[a]], a/b])/(d*(1 + b*d + Sqrt[-4*a*d + (1 + b*d)^2])*Sqr
t[(a - x)*(b - x)*x])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !FreeQ[w, x] &&  !Fr
eeQ[z, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{ab-2ax+x^2}{\sqrt{x(-a+x)(-b+x)}\left(a-(1+bd)x+dx^2\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{ab-2ax+x^2}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\left(a-(1+bd)x+dx^2\right)}\,dx}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int\left(\frac{1}{d\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}} - \frac{a-abd\cdots}{d\sqrt{x}\sqrt{-a+x}\sqrt{-\cdots}}\right.}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{1}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}}\,dx}{d\sqrt{x(-a+x)(-b+x)}} - \frac{\left(\sqrt{x}\sqrt{-a+\cdots}\right.}{\cdots}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int\left(\frac{-1+2ad-bd-\sqrt{-4ad+(1+b\cdots}}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\left(-1-bd-\sqrt{1-4ad+2\cdots}\right.}\right.}{d\sqrt{x(-a\cdots}}$$

$$= \frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{d\sqrt{(a-x)(b-x)x}} - \frac{\left(\left(-1+2ad-\cdots\right.\right.}{\cdots}$$

$$= \frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{d\sqrt{(a-x)(b-x)x}} + \frac{\left(2\left(-1+2ad\cdots\right.\right.}{\cdots}$$

$$= \frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{d\sqrt{(a-x)(b-x)x}} + \frac{\left(2\left(-1+2ad\cdots\right.\right.}{\cdots}$$

$$= \frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{d\sqrt{(a-x)(b-x)x}} + \frac{\left(2\left(-1+2ad\cdots\right.\right.}{\cdots}$$

$$= \frac{2\sqrt{a}\sqrt{x}\sqrt{1-\frac{x}{a}}\sqrt{1-\frac{x}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)|\frac{a}{b}\right)}{d\sqrt{(a-x)(b-x)x}} - \frac{2\sqrt{a}\left(1-2ad\cdots\right.}{\cdots}$$

Mathematica [C] time = 7.22, size = 238, normalized size = 5.41

$$\frac{2i(a-x)^{3/2}\sqrt{\frac{x}{x-a}}\sqrt{\frac{b-x}{a-x}}\left(-\Pi\left(\frac{2(a-b)d}{2ad-bd+\sqrt{(bd+1)^2-4ad}-1};i\sinh^{-1}\left(\frac{\sqrt{-a}}{\sqrt{a-x}}\right)|1-\frac{b}{a}\right)-\Pi\left(-\frac{2(a-b)d}{-2ad+bd+\sqrt{(bd+1)^2-4ad}+1};i\sinh^{-1}\left(\frac{\sqrt{-a}}{\sqrt{a-x}}\right)|1-\frac{b}{a}\right)+F\left(i\sinh^{-1}\left(\frac{\sqrt{-a}}{\sqrt{a-x}}\right)|1-\frac{b}{a}\right)\right)}{\sqrt{-a}\,d\sqrt{x(x-a)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b - 2*a*x + x^2)/(Sqrt[x*(-a + x)*(-b + x)]*(a - (1 + b*d)*x +
 d*x^2)),x]
```

```
[Out] ((2*I)*(a - x)^(3/2)*Sqrt[(b - x)/(a - x)]*Sqrt[x/(-a + x)]*(EllipticF[I*Ar
cSinh[Sqrt[-a]/Sqrt[a - x]], 1 - b/a] - EllipticPi[(2*(a - b)*d)/(-1 + 2*a*
d - b*d + Sqrt[-4*a*d + (1 + b*d)^2]), I*ArcSinh[Sqrt[-a]/Sqrt[a - x]], 1 -
 b/a] - EllipticPi[(-2*(a - b)*d)/(1 - 2*a*d + b*d + Sqrt[-4*a*d + (1 + b*d
)^2]), I*ArcSinh[Sqrt[-a]/Sqrt[a - x]], 1 - b/a]))/(Sqrt[-a]*d*Sqrt[x*(-a +
 x)*(-b + x)])
```

IntegrateAlgebraic [A] time = 0.32, size = 44, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{x^2(-a-b)+abx+x^3}}{a-x}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b - 2*a*x + x^2)/(Sqrt[x*(-a + x)*(-b + x)]*(a - (1 +
 b*d)*x + d*x^2)),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a*b*x + (-a - b)*x^2 + x^3])/(a - x)])/Sqrt[d]
```

fricas [B] time = 0.77, size = 231, normalized size = 5.25

$$\left[\frac{\log\left(\frac{d^{2} x^{4}-2\left(b d^{2}-3 d\right) x^{3}+\left(b^{2} d^{2}-6(a+b) d+1\right) x^{2}+a^{2}-4 \sqrt{a b x-(a+b) x^{2}+x^{3}}\left(d x^{2}-(b d-1) x-a\right) \sqrt{d}+2(3 a b d-a) x}{d^{2} x^{4}-2\left(b d^{2}+d\right) x^{3}+\left(b^{2} d^{2}+2(a+b) d+1\right) x^{2}+a^{2}-2(a b d+a) x}\right)}{2 \sqrt{d}}, \frac{\sqrt{-d} \arctan\left(\frac{\sqrt{a b x-(a+b) x^{2}+x^{3}}\left(d x^{2}-(b d-1) x-a\right) \sqrt{-d}}{2\left(a b d x-(a+b) d x^{2}+d x^{3}\right)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a-(b*d+1)*x+d*x^2),x, al
gorithm="fricas")
```

```
[Out] [1/2*log((d^2*x^4 - 2*(b*d^2 - 3*d)*x^3 + (b^2*d^2 - 6*(a + b)*d + 1)*x^2 +
 a^2 - 4*sqrt(a*b*x - (a + b)*x^2 + x^3)*(d*x^2 - (b*d - 1)*x - a)*sqrt(d)
+ 2*(3*a*b*d - a)*x)/(d^2*x^4 - 2*(b*d^2 + d)*x^3 + (b^2*d^2 + 2*(a + b)*d
+ 1)*x^2 + a^2 - 2*(a*b*d + a)*x))/sqrt(d), sqrt(-d)*arctan(1/2*sqrt(a*b*x
- (a + b)*x^2 + x^3)*(d*x^2 - (b*d - 1)*x - a)*sqrt(-d)/(a*b*d*x - (a + b)*
d*x^2 + d*x^3))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab - 2\,ax + x^2}{\sqrt{(a-x)(b-x)x}\left(dx^2 - (bd+1)x + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a-(b*d+1)*x+d*x^2),x, al
gorithm="giac")
```

```
[Out] integrate((a*b - 2*a*x + x^2)/(sqrt((a - x)*(b - x)*x)*(d*x^2 - (b*d + 1)*x
 + a)), x)
```

maple [C] time = 0.08, size = 2508, normalized size = 57.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a-(b*d+1)*x+d*x^2),x)
```

```
[Out] -2/d*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*
x^2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))+1/d*(-1/(b^2*d^
2-4*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*
x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2/d-1/2/d*(b^2*d^2-4*a*d+
2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(b*d+1+(b^2*d^2-4*a
*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))*b^2*d-2/(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*
a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b
*x^2+x^3)^(1/2)/(a-1/2*b-1/2/d-1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2))*Ellipti
cPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(b*d+1+(b^2*d^2-4*a*d+2*b*d+1)^(1/2))),(a/
(a-b))^(1/2))*b+4/(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*a^2*(1-1/a*x)^(1/2)*(-1/(a-
b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-
1/2/d-1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(
a-1/2/d*(b*d+1+(b^2*d^2-4*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))-1/(b^2*d^2-
4*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)
^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2/d-1/2/d*(b^2*d^2-4*a*d+2*
b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(b*d+1+(b^2*d^2-4*a*d
+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))/d+2*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a
```

```
-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2/d-1/2
/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(
b*d+1+(b^2*d^2-4*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))-a*(1-1/a*x)^(1/2)*(-
1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1
/2*b-1/2/d-1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2
),a/(a-1/2/d*(b*d+1+(b^2*d^2-4*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))*b-a*(1
-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2
+x^3)^(1/2)/(a-1/2*b-1/2/d-1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2))*EllipticPi(
(-(-a+x)/a)^(1/2),a/(a-1/2/d*(b*d+1+(b^2*d^2-4*a*d+2*b*d+1)^(1/2))),(a/(a-b
))^(1/2))/d+1/(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1
/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b+1/2/d*
(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*
(-b*d+(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))*b^2*d+2/(b^2*d^2
-4*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x
)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b+1/2/d*(b^2*d^2-4*a*d+2*b*d+1
)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-b*d+(b^2*d^2-4*a*d+2
*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))*b+2*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/
(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b+1/2/d*(
b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(
-b*d+(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))-a*(1-1/a*x)^(1/2)
*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(
a-1/2*b+1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(
1/2),a/(a+1/2*(-b*d+(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))*b-
a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b
*x^2+x^3)^(1/2)/(a-1/2*b+1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1/2/d)*Ellipti
cPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-b*d+(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1)/d),(a
/(a-b))^(1/2))/d-4/(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*a^2*(1-1/a*x)^(1/2)*(-1/(a
-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b
+1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/
(a+1/2*(-b*d+(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))+1/(b^2*d^
2-4*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*
x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b+1/2/d*(b^2*d^2-4*a*d+2*b*d+
1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-b*d+(b^2*d^2-4*a*d+
2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))/d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a-(b*d+1)*x+d*x^2),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b*d+1)^2-4*a*d>0)', see `assume?`
for more details)Is (b*d+1)^2-4*a*d    positive, negative or zero?
```

mupad [B] time = 0.38, size = 437, normalized size = 9.93

$$\frac{b\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(\frac{b}{b-\frac{b\,d-\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}+1}{2\,d}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\left(2\,a\,d-b\,d+\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}-1\right)}{d^2\left(b-\frac{b\,d-\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}+1}{2\,d}\right)\sqrt{x^3+\left(-a-b\right)\,x^2+a\,b\,x}}-\frac{2\,b\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}}{d\,\sqrt{x^3+\left(-a-b\right)\,x^2+a\,b\,x}}-\frac{b\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(\frac{b}{b-\frac{b\,d+\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}+1}{2\,d}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\left(b\,d-2\,a\,d+\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}+1\right)}{d^2\left(b-\frac{b\,d+\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}+1}{2\,d}\right)\sqrt{x^3+\left(-a-b\right)\,x^2+a\,b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b - 2*a*x + x^2)/((x*(a - x)*(b - x))^(1/2)*(a - x*(b*d + 1) + d*x^2
)),x)
```

```
[Out] (b*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*ellipticPi(b/(b -
(b*d - (2*b*d - 4*a*d + b^2*d^2 + 1)^(1/2) + 1)/(2*d)), asin(((b - x)/b)^(1
/2)), -b/(a - b))*(2*a*d - b*d + (2*b*d - 4*a*d + b^2*d^2 + 1)^(1/2) - 1))/
```

```
(d^2*(b - (b*d - (2*b*d - 4*a*d + b^2*d^2 + 1)^(1/2) + 1)/(2*d))*(x^3 - x^2
*(a + b) + a*b*x)^(1/2)) - (2*b*ellipticF(asin(((b - x)/b)^(1/2)), -b/(a -
b))*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2))/(d*(x^3 - x^2*(a
 + b) + a*b*x)^(1/2)) - (b*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^
(1/2)*ellipticPi(b/(b - (b*d + (2*b*d - 4*a*d + b^2*d^2 + 1)^(1/2) + 1)/(2*
d)), asin(((b - x)/b)^(1/2)), -b/(a - b))*(b*d - 2*a*d + (2*b*d - 4*a*d + b
^2*d^2 + 1)^(1/2) + 1))/(d^2*(b - (b*d + (2*b*d - 4*a*d + b^2*d^2 + 1)^(1/2
) + 1)/(2*d))*(x^3 - x^2*(a + b) + a*b*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*a*x+x**2)/(x*(-a+x)*(-b+x))**(1/2)/(a-(b*d+1)*x+d*x**2),x)
[Out] Timed out
```

3.568 $\int \frac{(-2+x^3)\sqrt{1-x^2+x^3}}{(1+x^3)^2}\,dx$

Optimal. Leaf size=44

$$-\frac{\sqrt{x^3-x^2+1}\,x}{x^3+1}-\tan^{-1}\left(\frac{x}{\sqrt{x^3-x^2+1}}\right)$$

Rubi [F] time = 180.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$Aborted

Verification is not applicable to the result.

```
[In] Int[((-2 + x^3)*Sqrt[1 - x^2 + x^3])/(1 + x^3)^2,x]
```

```
[Out] $Aborted
```

Rubi steps

Aborted

Mathematica [C] time = 3.13, size = 1609, normalized size = 36.57

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-2 + x^3)*Sqrt[1 - x^2 + x^3])/(1 + x^3)^2,x]
```

```
[Out] (-((x - x^3 + x^4)/(1 + x^3)) + (EllipticF[ArcSin[Sqrt[(-x + Root[1 - #1^2
+ #1^3 & , 3, 0])/(-Root[1 - #1^2 + #1^3 & , 2, 0] + Root[1 - #1^2 + #1^3 &
 , 3, 0])]], (Root[1 - #1^2 + #1^3 & , 2, 0] - Root[1 - #1^2 + #1^3 & , 3,
0])/(Root[1 - #1^2 + #1^3 & , 1, 0] - Root[1 - #1^2 + #1^3 & , 3, 0])]*(x -
 Root[1 - #1^2 + #1^3 & , 3, 0])*Sqrt[(-x + Root[1 - #1^2 + #1^3 & , 1, 0])
/(Root[1 - #1^2 + #1^3 & , 1, 0] - Root[1 - #1^2 + #1^3 & , 3, 0])]*Sqrt[(-
x + Root[1 - #1^2 + #1^3 & , 2, 0])/(Root[1 - #1^2 + #1^3 & , 2, 0] - Root[
1 - #1^2 + #1^3 & , 3, 0])])/Sqrt[(x - Root[1 - #1^2 + #1^3 & , 3, 0])/(Roo
t[1 - #1^2 + #1^3 & , 2, 0] - Root[1 - #1^2 + #1^3 & , 3, 0])] + (3*(-1)^(2
/3)*EllipticPi[(Root[1 - #1^2 + #1^3 & , 2, 0] - Root[1 - #1^2 + #1^3 & , 3
, 0])/((-1)^(1/3) - Root[1 - #1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[(-x + Root
[1 - #1^2 + #1^3 & , 3, 0])/(-Root[1 - #1^2 + #1^3 & , 2, 0] + Root[1 - #1^
2 + #1^3 & , 3, 0])]], (Root[1 - #1^2 + #1^3 & , 2, 0] - Root[1 - #1^2 + #1
^3 & , 3, 0])/(Root[1 - #1^2 + #1^3 & , 1, 0] - Root[1 - #1^2 + #1^3 & , 3,
 0])]*Sqrt[(-x + Root[1 - #1^2 + #1^3 & , 1, 0])/(Root[1 - #1^2 + #1^3 & ,
1, 0] - Root[1 - #1^2 + #1^3 & , 3, 0])]*Sqrt[-(((x - Root[1 - #1^2 + #1^3
& , 2, 0])*(x - Root[1 - #1^2 + #1^3 & , 3, 0]))/(Root[1 - #1^2 + #1^3 & ,
2, 0] - Root[1 - #1^2 + #1^3 & , 3, 0])^2)]*(-Root[1 - #1^2 + #1^3 & , 2, 0
] + Root[1 - #1^2 + #1^3 & , 3, 0]))/((1 + (-1)^(1/3))^2*((-1)^(1/3) - Root
[1 - #1^2 + #1^3 & , 3, 0])) + (EllipticPi[(-Root[1 - #1^2 + #1^3 & , 2, 0]
 + Root[1 - #1^2 + #1^3 & , 3, 0])/(1 + Root[1 - #1^2 + #1^3 & , 3, 0]), Ar
cSin[Sqrt[(-x + Root[1 - #1^2 + #1^3 & , 3, 0])/(-Root[1 - #1^2 + #1^3 & ,
2, 0] + Root[1 - #1^2 + #1^3 & , 3, 0])]], (Root[1 - #1^2 + #1^3 & , 2, 0]
- Root[1 - #1^2 + #1^3 & , 3, 0])/(Root[1 - #1^2 + #1^3 & , 1, 0] - Root[1
- #1^2 + #1^3 & , 3, 0])]*Sqrt[(-x + Root[1 - #1^2 + #1^3 & , 1, 0])/(Root[
1 - #1^2 + #1^3 & , 1, 0] - Root[1 - #1^2 + #1^3 & , 3, 0])]*Sqrt[-(((x - R
oot[1 - #1^2 + #1^3 & , 2, 0])*(x - Root[1 - #1^2 + #1^3 & , 3, 0]))/(Root[
1 - #1^2 + #1^3 & , 2, 0] - Root[1 - #1^2 + #1^3 & , 3, 0])^2)]*(-Root[1 -
#1^2 + #1^3 & , 2, 0] + Root[1 - #1^2 + #1^3 & , 3, 0]))/(1 + Root[1 - #1^2
```

```
+ #1^3 & , 3, 0]) + (2*(-1)^(2/3)*EllipticPi[(-Root[1 - #1^2 + #1^3 & , 2,
0] + Root[1 - #1^2 + #1^3 & , 3, 0])/((-1)^(2/3) + Root[1 - #1^2 + #1^3 &
, 3, 0]), ArcSin[Sqrt[(-x + Root[1 - #1^2 + #1^3 & , 3, 0])/(-Root[1 - #1^2
+ #1^3 & , 2, 0] + Root[1 - #1^2 + #1^3 & , 3, 0])]], (Root[1 - #1^2 + #1^
3 & , 2, 0] - Root[1 - #1^2 + #1^3 & , 3, 0])/(Root[1 - #1^2 + #1^3 & , 1,
0] - Root[1 - #1^2 + #1^3 & , 3, 0])]*Sqrt[(-x + Root[1 - #1^2 + #1^3 & , 1
, 0])/(Root[1 - #1^2 + #1^3 & , 1, 0] - Root[1 - #1^2 + #1^3 & , 3, 0])]*Sq
rt[-(((x - Root[1 - #1^2 + #1^3 & , 2, 0])*(x - Root[1 - #1^2 + #1^3 & , 3,
0]))/(Root[1 - #1^2 + #1^3 & , 2, 0] - Root[1 - #1^2 + #1^3 & , 3, 0])^2)]
*(-Root[1 - #1^2 + #1^3 & , 2, 0] + Root[1 - #1^2 + #1^3 & , 3, 0]))/(-1 +
I*Sqrt[3] + 2*Root[1 - #1^2 + #1^3 & , 3, 0]))/Sqrt[1 - x^2 + x^3]
```

IntegrateAlgebraic [A] time = 0.30, size = 44, normalized size = 1.00

$$-\frac{\sqrt{x^3-x^2+1}\,x}{x^3+1}-\tan^{-1}\left(\frac{x}{\sqrt{x^3-x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^3)*Sqrt[1 - x^2 + x^3])/(1 + x^3)^2,x]
```

```
[Out] -((x*Sqrt[1 - x^2 + x^3])/(1 + x^3)) - ArcTan[x/Sqrt[1 - x^2 + x^3]]
```

fricas [A] time = 0.40, size = 68, normalized size = 1.55

$$\frac{\left(x^3+1\right)\arctan\left(\frac{\sqrt{x^3-x^2+1}\left(x^3-2x^2+1\right)}{2\left(x^4-x^3+x\right)}\right)-2\sqrt{x^3-x^2+1}\,x}{2\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3-x^2+1)^(1/2)/(x^3+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^3 + 1)*arctan(1/2*sqrt(x^3 - x^2 + 1)*(x^3 - 2*x^2 + 1)/(x^4 - x^3
+ x)) - 2*sqrt(x^3 - x^2 + 1)*x)/(x^3 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^3-x^2+1}\left(x^3-2\right)}{\left(x^3+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3-x^2+1)^(1/2)/(x^3+1)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^3 - x^2 + 1)*(x^3 - 2)/(x^3 + 1)^2, x)
```

maple [C] time = 0.68, size = 5687, normalized size = 129.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2)*(x^3-x^2+1)^(1/2)/(x^3+1)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^3-x^2+1}\left(x^3-2\right)}{\left(x^3+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3-x^2+1)^(1/2)/(x^3+1)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^3 - x^2 + 1)*(x^3 - 2)/(x^3 + 1)^2, x)
```

mupad [B] time = 1.13, size = 61, normalized size = 1.39

$$-\frac{x\sqrt{x^3-x^2+1}}{x^3+1}+\frac{\ln\left(\frac{x^3-2x^2+1+x\sqrt{x^3-x^2+1}\,2\mathrm{i}}{x^3+1}\right)1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 2)*(x^3 - x^2 + 1)^(1/2))/(x^3 + 1)^2,x)
```

```
[Out] (log((x*(x^3 - x^2 + 1)^(1/2)*2i - 2*x^2 + x^3 + 1)/(x^3 + 1))*1i)/2 - (x*(
x^3 - x^2 + 1)^(1/2))/(x^3 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^3-2\right)\sqrt{x^3-x^2+1}}{\left(x+1\right)^2\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2)*(x**3-x**2+1)**(1/2)/(x**3+1)**2,x)
```

```
[Out] Integral((x**3 - 2)*sqrt(x**3 - x**2 + 1)/((x + 1)**2*(x**2 - x + 1)**2), x
)
```

3.569 $\int \frac{a^2b-a(2a-b)x-(-a+2b)x^2+x^3}{\sqrt{x(-a+x)(-b+x)}\left(-a^3+\left(3a^2+bd\right)x-(3a+d)x^2+x^3\right)}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{x^2(-a-b)+abx+x^3}}{(a-x)^2}\right)}{\sqrt{d}}$$

Rubi [F] time = 7.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a^2b-a(2a-b)x-(-a+2b)x^2+x^3}{\sqrt{x(-a+x)(-b+x)}\left(-a^3+\left(3a^2+bd\right)x-(3a+d)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a^2*b - a*(2*a - b)*x - (-a + 2*b)*x^2 + x^3)/(Sqrt[x*(-a + x)*(-b + x
)]*(-a^3 + (3*a^2 + b*d)*x - (3*a + d)*x^2 + x^3)),x]
```

```
[Out] (2*a*b*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][Sqrt[-a +
x^2]/(Sqrt[-b + x^2]*(a^3 - 3*a^2*(1 + (b*d)/(3*a^2))*x^2 + 3*a*(1 + d/(3*a
))*x^4 - x^6)), x], x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x] - (4*(a - b)*Sqrt[
x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][(x^2*Sqrt[-a + x^2])/(
Sqrt[-b + x^2]*(a^3 - 3*a^2*(1 + (b*d)/(3*a^2))*x^2 + 3*a*(1 + d/(3*a))*x^4
 - x^6)), x], x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x] + (2*Sqrt[x]*Sqrt[-a + x
]*Sqrt[-b + x]*Defer[Subst][Defer[Int][(x^4*Sqrt[-a + x^2])/(Sqrt[-b + x^2]
*(-a^3 + 3*a^2*(1 + (b*d)/(3*a^2))*x^2 - 3*a*(1 + d/(3*a))*x^4 + x^6)), x],
 x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x]
```

Rubi steps

$$\int \frac{a^2b-a(2a-b)x-(-a+2b)x^2+x^3}{\sqrt{x(-a+x)(-b+x)}\left(-a^3+\left(3a^2+bd\right)x-(3a+d)x^2+x^3\right)}\,dx = \frac{\left(\sqrt{x}\,\sqrt{-a+x}\,\sqrt{-b+x}\right)\int \frac{a^2b-a(2a-b}{\sqrt{x}\,\sqrt{-a+x}\,\sqrt{-b+x}\,(-a}}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(\sqrt{x}\,\sqrt{-a+x}\,\sqrt{-b+x}\right)\int \frac{\sqrt{-a+x}\,(-ab+(}{\sqrt{x}\,\sqrt{-b+x}\,(-a^3+(3a^2+}}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt{-a+x}\,\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{\sqrt{-a-}}{\sqrt{-b+x^2}\,(-}\right.}{\sqrt{x(-a+x)(-b}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt{-a+x}\,\sqrt{-b+x}\right)\text{Subst}\left(\int\left(\frac{}{\sqrt{-b+x^2}\,(}\right.\right.}{}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt{-a+x}\,\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt{-b+x^2}\,(-}\right.}{\sqrt{x(-a+x)(}}$$

Mathematica [C] time = 4.99, size = 2730, normalized size = 62.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^2*b - a*(2*a - b)*x - (-a + 2*b)*x^2 + x^3)/(Sqrt[x*(-a + x)*(
-b + x)]*(-a^3 + (3*a^2 + b*d)*x - (3*a + d)*x^2 + x^3)),x]
```

```
[Out] ((-2*I)*(a - x)*Sqrt[(-b + x)/(a - b)]*(-5*a^2*d*EllipticPi[a/Root[a^2*d -
a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 2], I*ArcSinh[Sqrt[-1 + x/a]]
, a/(a - b)] + 5*a*b*d*EllipticPi[a/Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1
+ d*#1^2 + #1^3 & , 2], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] - 2*b^2*d*Ell
ipticPi[a/Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 2], I*
ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] - 3*a^2*d*EllipticPi[a/Root[a^2*d - a*b
*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 3], I*ArcSinh[Sqrt[-1 + x/a]], a
/(a - b)] + 3*a*b*d*EllipticPi[a/Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d
*#1^2 + #1^3 & , 3], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] + 3*a*(a - b)*(E
llipticPi[a/Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 2],
I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] - 2*EllipticPi[a/Root[a^2*d - a*b*d +
 (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 3], I*ArcSinh[Sqrt[-1 + x/a]], a/(a
- b)])*Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 2] - Elli
pticPi[a/Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 2], I*A
rcSinh[Sqrt[-1 + x/a]], a/(a - b)]*Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 +
 d*#1^2 + #1^3 & , 1]*Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^
3 & , 2]^2 + 3*a*(a - b)*(2*EllipticPi[a/Root[a^2*d - a*b*d + (-2*a*d + b*d
)*#1 + d*#1^2 + #1^3 & , 2], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] - Ellipt
icPi[a/Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 3], I*Arc
Sinh[Sqrt[-1 + x/a]], a/(a - b)])*Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 +
d*#1^2 + #1^3 & , 3] - 2*(2*a - b)*(EllipticPi[a/Root[a^2*d - a*b*d + (-2*a
*d + b*d)*#1 + d*#1^2 + #1^3 & , 2], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)]
+ EllipticPi[a/Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 3
], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)])*Root[a^2*d - a*b*d + (-2*a*d + b*
d)*#1 + d*#1^2 + #1^3 & , 1]*Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^
2 + #1^3 & , 3] - 2*(2*a - b)*(2*EllipticPi[a/Root[a^2*d - a*b*d + (-2*a*d
+ b*d)*#1 + d*#1^2 + #1^3 & , 2], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] - E
llipticPi[a/Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 3],
I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)])*Root[a^2*d - a*b*d + (-2*a*d + b*d)*
#1 + d*#1^2 + #1^3 & , 2]*Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 +
 #1^3 & , 3] + EllipticPi[a/Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2
 + #1^3 & , 2], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)]*Root[a^2*d - a*b*d +
(-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 2]^2*Root[a^2*d - a*b*d + (-2*a*d + b
*d)*#1 + d*#1^2 + #1^3 & , 3] + EllipticPi[a/Root[a^2*d - a*b*d + (-2*a*d +
 b*d)*#1 + d*#1^2 + #1^3 & , 3], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)]*Root
[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 1]*Root[a^2*d - a*b*
d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 3]^2 - EllipticPi[a/Root[a^2*d -
a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 3], I*ArcSinh[Sqrt[-1 + x/a]]
, a/(a - b)]*Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 2]*
Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 3]^2 + EllipticF
[I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)]*(d*(-3*a^2 + 3*a*b - 2*a*d + b*d) +
2*Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 1]*Root[a^2*d
- a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 2]^2 + 2*Root[a^2*d - a*b*d
 + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 1]^2*Root[a^2*d - a*b*d + (-2*a*d
+ b*d)*#1 + d*#1^2 + #1^3 & , 3] + 2*Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1
 + d*#1^2 + #1^3 & , 2]*Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #
1^3 & , 3]^2) + EllipticPi[a/Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^
2 + #1^3 & , 1], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)]*(d*(11*a^2 - 11*a*b
+ 2*b^2 + 2*a*d - b*d) + d*Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2
+ #1^3 & , 2]^2 + Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 &
, 2]^3 + (3*a*(-a + b) + (8*a - 4*b)*Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1
 + d*#1^2 + #1^3 & , 1] - 2*Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2
 + #1^3 & , 1]^2)*Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 &
, 3] - Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 1]*Root[a
^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 3]^2 + Root[a^2*d - a*
b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 2]*(3*a*(a - b) + (4*a - 2*b)*R
oot[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 3] - Root[a^2*d -
```

```
 a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 3]^2))))/(Sqrt[1 - a/x]*Sqrt
[x*(-a + x)*(-b + x)]*(Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1
^3 & , 1] - Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 2])*
(Root[a^2*d - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 1] - Root[a^2*d
 - a*b*d + (-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 3])*(Root[a^2*d - a*b*d +
(-2*a*d + b*d)*#1 + d*#1^2 + #1^3 & , 2] - Root[a^2*d - a*b*d + (-2*a*d + b
*d)*#1 + d*#1^2 + #1^3 & , 3]))
```

IntegrateAlgebraic [A] time = 0.64, size = 44, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{x^2(-a-b)+abx+x^3}}{(a-x)^2}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2*b - a*(2*a - b)*x - (-a + 2*b)*x^2 + x^3)/(Sqrt[x*(
-a + x)*(-b + x)]*(-a^3 + (3*a^2 + b*d)*x - (3*a + d)*x^2 + x^3)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[d]*Sqrt[a*b*x + (-a - b)*x^2 + x^3])/(a - x)^2])/Sqrt[d]
```

fricas [B] time = 0.81, size = 407, normalized size = 9.25

$$\left[\frac{\log\left(\frac{a^6-6(a-d)x^5+x^6+\left(15a^2-6(3a+b)d+d^2\right)x^4-2\left(10a^3+bd^2-9\left(a^2+ab\right)d\right)x^3+\left(15a^4+b^2d^2-6\left(a^3+3a^2b\right)d\right)x^2-4\left(a^4-(4a-d)x^3+x^4+\left(6a^2-(a+b)d\right)x^2-\left(4a^3-abd\right)x\right)\sqrt{abx-(a+b)x^2+x^3}\sqrt{d}-6\left(a^5-a^3bd\right)x}{a^6-2(3a+d)x^5+x^6+\left(15a^2+2(3a+b)d+d^2\right)x^4-2\left(10a^3+bd^2+3\left(a^2+ab\right)d\right)x^3+\left(15a^4+b^2d^2+2\left(a^3+3a^2b\right)d\right)x^2-2\left(3a^5+a^3bd\right)x}\right)}{2\sqrt{d}}, \frac{\sqrt{-d}\arctan\left(\frac{\left(a^3+(3a-d)x^2-x^3-\left(3a^2-bd\right)x\right)\sqrt{abx-(a+b)x^2+x^3}\sqrt{-d}}{2\left(a^2bdx+(2a+b)dx^3-dx^4-\left(a^2+2ab\right)dx^2\right)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*b-a*(2*a-b)*x-(-a+2*b)*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a^
3+(3*a^2+b*d)*x-(3*a+d)*x^2+x^3),x, algorithm="fricas")
```

```
[Out] [1/2*log((a^6 - 6*(a - d)*x^5 + x^6 + (15*a^2 - 6*(3*a + b)*d + d^2)*x^4 -
2*(10*a^3 + b*d^2 - 9*(a^2 + a*b)*d)*x^3 + (15*a^4 + b^2*d^2 - 6*(a^3 + 3*a
^2*b)*d)*x^2 - 4*(a^4 - (4*a - d)*x^3 + x^4 + (6*a^2 - (a + b)*d)*x^2 - (4*
a^3 - a*b*d)*x)*sqrt(a*b*x - (a + b)*x^2 + x^3)*sqrt(d) - 6*(a^5 - a^3*b*d)
*x)/(a^6 - 2*(3*a + d)*x^5 + x^6 + (15*a^2 + 2*(3*a + b)*d + d^2)*x^4 - 2*(
10*a^3 + b*d^2 + 3*(a^2 + a*b)*d)*x^3 + (15*a^4 + b^2*d^2 + 2*(a^3 + 3*a^2*
b)*d)*x^2 - 2*(3*a^5 + a^3*b*d)*x))/sqrt(d), sqrt(-d)*arctan(1/2*(a^3 + (3*
a - d)*x^2 - x^3 - (3*a^2 - b*d)*x)*sqrt(a*b*x - (a + b)*x^2 + x^3)*sqrt(-d
)/(a^2*b*d*x + (2*a + b)*d*x^3 - d*x^4 - (a^2 + 2*a*b)*d*x^2))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{a^2b-(2a-b)ax+(a-2b)x^2+x^3}{\left(a^3+(3a+d)x^2-x^3-\left(3a^2+bd\right)x\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*b-a*(2*a-b)*x-(-a+2*b)*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a^
3+(3*a^2+b*d)*x-(3*a+d)*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*b - (2*a - b)*a*x + (a - 2*b)*x^2 + x^3)/((a^3 + (3*a + d)*
x^2 - x^3 - (3*a^2 + b*d)*x)*sqrt((a - x)*(b - x)*x)), x)
```

maple [C] time = 0.06, size = 336, normalized size = 7.64

$$-\frac{2a\sqrt{-\frac{-a+x}{a}}\sqrt{\frac{-b+x}{a-b}}\sqrt{\frac{x}{a}}\,\operatorname{EllipticF}\left(\sqrt{-\frac{-a+x}{a}},\sqrt{\frac{a}{a-b}}\right)}{\sqrt{abx-a\,x^2-b\,x^2+x^3}}+\frac{2\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^3+(-3a-d)\_Z^2+\left(3a^2+bd\right)\_Z-a^3\right)}\frac{\left(-4\_\alpha^2a+2\_\alpha^2b-\_\alpha^2d+5\_\alpha a^2-\_\alpha ab+\_\alpha bd-a^3-a^2b\right)\left(\_\alpha^2-2\_\alpha a-\_\alpha d+a^2-ad+bd\right)\sqrt{-\frac{-a+x}{a}}\sqrt{\frac{-b+x}{a-b}}\sqrt{\frac{x}{a}}\,\operatorname{EllipticPi}\left(\sqrt{-\frac{-a+x}{a}},-\frac{\_\alpha^2-2\_\alpha a-\_\alpha d+a^2-ad+bd}{(a-b)d},\sqrt{\frac{a}{a-b}}\right)}{\left(-3\_\alpha^2+6\_\alpha a+2\_\alpha d-3a^2-bd\right)(a-b)\sqrt{x\left(ab-ax-bx+x^2\right)}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*b-a*(2*a-b)*x-(-a+2*b)*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a^3+(3*a
^2+b*d)*x-(3*a+d)*x^2+x^3),x)
```

[Out] -2*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))+2/d*sum((-4*_alpha^2*a+2*_alpha^2*b-_alpha^2*d+5*_alpha*a^2-_alpha*a*b+_alpha*b*d-a^3-a^2*b)/(-3*_alpha^2+6*_alpha*a+2*_alpha*d-3*a^2-b*d)*(_alpha^2-2*_alpha*a-_alpha*d+a^2-a*d+b*d)/(a-b)*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(x*(a*b-a*x-b*x+x^2))^(1/2)*EllipticPi((-(-a+x)/a)^(1/2),-(_alpha^2-2*_alpha*a-_alpha*d+a^2-a*d+b*d)/(a-b)/d,(a/(a-b))^(1/2)),_alpha=RootOf(_Z^3+(-3*a-d)*_Z^2+(3*a^2+b*d)*_Z-a^3))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2 b - (2\,a - b)ax + (a - 2\,b)x^2 + x^3}{\left(a^3 + (3\,a + d)x^2 - x^3 - \left(3\,a^2 + bd\right)x\right)\sqrt{(a - x)(b - x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2*b-a*(2*a-b)*x-(-a+2*b)*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a^3+(3*a^2+b*d)*x-(3*a+d)*x^2+x^3),x, algorithm="maxima")

[Out] -integrate((a^2*b - (2*a - b)*a*x + (a - 2*b)*x^2 + x^3)/((a^3 + (3*a + d)*x^2 - x^3 - (3*a^2 + b*d)*x)*sqrt((a - x)*(b - x)*x)), x)

mupad [B] time = 0.69, size = 589, normalized size = 13.39

$$\left(\sum_{k=1}^{3} \frac{2\,b\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(-\frac{b}{\operatorname{root}\left(z^3-z^2\,(3\,a+d)+z\,\left(b\,d+3\,a^2\right)-a^3,z,k\right)-b};\operatorname{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\left(a^2\,b+a^3+4\,a\,\operatorname{root}\left(z^3-z^2\,(3\,a+d)+z\,\left(b\,d+3\,a^2\right)-a^3,z,k\right)^2-5\,a^2\,\operatorname{root}\left(z^3-z^2\,(3\,a+d)+z\,\left(b\,d+3\,a^2\right)-a^3,z,k\right)-2\,b\,\operatorname{root}\left(z^3-z^2\,(3\,a+d)+z\,\left(b\,d+3\,a^2\right)-a^3,z,k\right)^2+d\,\operatorname{root}\left(z^3-z^2\,(3\,a+d)+z\,\left(b\,d+3\,a^2\right)-a^3,z,k\right)^2+a\,b\,\operatorname{root}\left(z^3-z^2\,(3\,a+d)+z\,\left(b\,d+3\,a^2\right)-a^3,z,k\right)-b\,d\,\operatorname{root}\left(z^3-z^2\,(3\,a+d)+z\,\left(b\,d+3\,a^2\right)-a^3,z,k\right)\right)}{\left(\operatorname{root}\left(z^3-z^2\,(3\,a+d)+z\,\left(b\,d+3\,a^2\right)-a^3,z,k\right)-b\right)\sqrt{x\,(a-x)\,(b-x)}\,\left(3\,a^2-6\,a\,\operatorname{root}\left(z^3-z^2\,(3\,a+d)+z\,\left(b\,d+3\,a^2\right)-a^3,z,k\right)+3\,\operatorname{root}\left(z^3-z^2\,(3\,a+d)+z\,\left(b\,d+3\,a^2\right)-a^3,z,k\right)^2-2\,d\,\operatorname{root}\left(z^3-z^2\,(3\,a+d)+z\,\left(b\,d+3\,a^2\right)-a^3,z,k\right)+b\,d\right)}\right)-\frac{2\,b\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}}{\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*b + x^2*(a - 2*b) + x^3 - a*x*(2*a - b))/((x*(a - x)*(b - x))^(1/2)*(x*(b*d + 3*a^2) - x^2*(3*a + d) - a^3 + x^3)),x)

[Out] symsum((2*b*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*ellipticPi(-b/(root(z^3 - z^2*(3*a + d) + z*(b*d + 3*a^2) - a^3, z, k) - b), asin(((b - x)/b)^(1/2)), -b/(a - b))*(a^2*b + a^3 + 4*a*root(z^3 - z^2*(3*a + d) + z*(b*d + 3*a^2) - a^3, z, k)^2 - 5*a^2*root(z^3 - z^2*(3*a + d) + z*(b*d + 3*a^2) - a^3, z, k) - 2*b*root(z^3 - z^2*(3*a + d) + z*(b*d + 3*a^2) - a^3, z, k)^2 + d*root(z^3 - z^2*(3*a + d) + z*(b*d + 3*a^2) - a^3, z, k)^2 + a*b*root(z^3 - z^2*(3*a + d) + z*(b*d + 3*a^2) - a^3, z, k) - b*d*root(z^3 - z^2*(3*a + d) + z*(b*d + 3*a^2) - a^3, z, k)))/((root(z^3 - z^2*(3*a + d) + z*(b*d + 3*a^2) - a^3, z, k) - b)*(x*(a - x)*(b - x))^(1/2)*(b*d + 3*root(z^3 - z^2*(3*a + d) + z*(b*d + 3*a^2) - a^3, z, k)^2 - 6*a*root(z^3 - z^2*(3*a + d) + z*(b*d + 3*a^2) - a^3, z, k) - 2*d*root(z^3 - z^2*(3*a + d) + z*(b*d + 3*a^2) - a^3, z, k) + 3*a^2)), k, 1, 3) - (2*b*ellipticF(asin(((b - x)/b)^(1/2)), -b/(a - b))*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2))/(x^3 - x^2*(a + b) + a*b*x)^(1/2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2*b-a*(2*a-b)*x-(-a+2*b)*x**2+x**3)/(x*(-a+x)*(-b+x))**(1/2)/(-a**3+(3*a**2+b*d)*x-(3*a+d)*x**2+x**3),x)

[Out] Timed out

3.570 $\int \frac{a^2b-a(2a-b)x-(-a+2b)x^2+x^3}{\sqrt{x(-a+x)(-b+x)}\left(-a^3d+\left(b+3a^2d\right)x-(1+3ad)x^2+dx^3\right)}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^2(-a-b)+abx+x^3}}{\sqrt{d}\,(a-x)^2}\right)}{\sqrt{d}}$$

Rubi [F] time = 7.75, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a^2b-a(2a-b)x-(-a+2b)x^2+x^3}{\sqrt{x(-a+x)(-b+x)}\left(-a^3d+\left(b+3a^2d\right)x-(1+3ad)x^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a^2*b - a*(2*a - b)*x - (-a + 2*b)*x^2 + x^3)/(Sqrt[x*(-a + x)*(-b + x
)]*(-(a^3*d) + (b + 3*a^2*d)*x - (1 + 3*a*d)*x^2 + d*x^3)),x]
```

```
[Out] (2*a*b*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][Sqrt[-a +
x^2]/(Sqrt[-b + x^2]*(a^3*d - b*(1 + (3*a^2*d)/b)*x^2 + (1 + 3*a*d)*x^4 - d
*x^6)), x], x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x] - (4*(a - b)*Sqrt[x]*Sqrt[
-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][(x^2*Sqrt[-a + x^2])/(Sqrt[-b
+ x^2]*(a^3*d - b*(1 + (3*a^2*d)/b)*x^2 + (1 + 3*a*d)*x^4 - d*x^6)), x], x,
 Sqrt[x]])/Sqrt[(a - x)*(b - x)*x] + (2*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*D
efer[Subst][Defer[Int][(x^4*Sqrt[-a + x^2])/(Sqrt[-b + x^2]*(-(a^3*d) + b*(
1 + (3*a^2*d)/b)*x^2 - (1 + 3*a*d)*x^4 + d*x^6)), x], x, Sqrt[x]])/Sqrt[(a
- x)*(b - x)*x]
```

Rubi steps

$$\int \frac{a^2b-a(2a-b)x-(-a+2b)x^2+x^3}{\sqrt{x(-a+x)(-b+x)}\left(-a^3d+\left(b+3a^2d\right)x-(1+3ad)x^2+dx^3\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{a^2b-}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+}}}{\sqrt{x(-a+x)(-}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{\sqrt{-a+x}(}{\sqrt{x}\sqrt{-b+x}\left(-a^3d+\right.}}{\sqrt{x(-a+x)(-b+}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt{-b+}}\right.}{\sqrt{x(-a+}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}\left(\int\left(\frac{}{\sqrt{-b}}\right.\right.}{}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt{-b+}}\right.}{\sqrt{x(-a+}}$$

Mathematica [C] time = 5.18, size = 1571, normalized size = 35.70

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^2*b - a*(2*a - b)*x - (-a + 2*b)*x^2 + x^3)/(Sqrt[x*(-a + x)*(
-b + x)]*(-(a^3*d) + (b + 3*a^2*d)*x - (1 + 3*a*d)*x^2 + d*x^3)),x]
```

```
[Out] ((2*I)*(a - x)*Sqrt[(-b + x)/(a - b)]*(EllipticF[I*ArcSinh[Sqrt[-1 + x/a]],
 a/(a - b)]*(2*a - b + 3*a^2*d - 3*a*b*d - 2*d^2*Root[a^2 - a*b + (-2*a + b
)*#1 + #1^2 + d*#1^3 & , 1]*Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3
& , 2]^2 - 2*d^2*Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 1]^2*Ro
ot[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 3] - 2*d^2*Root[a^2 - a*b
+ (-2*a + b)*#1 + #1^2 + d*#1^3 & , 2]*Root[a^2 - a*b + (-2*a + b)*#1 + #1^
2 + d*#1^3 & , 3]^2) - EllipticPi[a/Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 +
 d*#1^3 & , 1], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)]*(2*a - b + 11*a^2*d -
 11*a*b*d + 2*b^2*d + d*Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & ,
2]^2 + d^2*Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 2]^3 + d^2*(3
*a*(-a + b) + (8*a - 4*b)*Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 &
, 1] - 2*Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 1]^2)*Root[a^2
- a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 3] - d^2*Root[a^2 - a*b + (-2*a +
 b)*#1 + #1^2 + d*#1^3 & , 1]*Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^
3 & , 3]^2 + d^2*Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 2]*(3*a
*(a - b) + (4*a - 2*b)*Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 3
] - Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 3]^2)) + d*(Elliptic
Pi[a/Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[
-1 + x/a]], a/(a - b)]*(5*a^2 - 5*a*b + 2*b^2 + d*Root[a^2 - a*b + (-2*a +
b)*#1 + #1^2 + d*#1^3 & , 2]^2*(Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#
1^3 & , 1] - Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 3]) - 2*d*(
3*a*(a - b) + (-2*a + b)*Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & ,
 1])*Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 3] + d*Root[a^2 - a
*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 2]*(3*a*(-a + b) + (8*a - 4*b)*Root[
a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 3])) + EllipticPi[a/Root[a^2
- a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[-1 + x/a]], a/
(a - b)]*(3*a*(a - b) + d*(3*a*(a - b) + (4*a - 2*b)*Root[a^2 - a*b + (-2*a
 + b)*#1 + #1^2 + d*#1^3 & , 1])*Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*
#1^3 & , 3] - d*Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 1]*Root[
a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 3]^2 + d*Root[a^2 - a*b + (-2
*a + b)*#1 + #1^2 + d*#1^3 & , 2]*(6*a*(a - b) + (-4*a + 2*b)*Root[a^2 - a*
b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 3] + Root[a^2 - a*b + (-2*a + b)*#1 +
 #1^2 + d*#1^3 & , 3]^2)))))/(d^3*Sqrt[1 - a/x]*Sqrt[x*(-a + x)*(-b + x)]*(
Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 1] - Root[a^2 - a*b + (-
2*a + b)*#1 + #1^2 + d*#1^3 & , 2])*(Root[a^2 - a*b + (-2*a + b)*#1 + #1^2
+ d*#1^3 & , 1] - Root[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 3])*(R
oot[a^2 - a*b + (-2*a + b)*#1 + #1^2 + d*#1^3 & , 2] - Root[a^2 - a*b + (-2
*a + b)*#1 + #1^2 + d*#1^3 & , 3]))
```

IntegrateAlgebraic [A] time = 0.67, size = 44, normalized size = 1.00

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{x^2(-a-b)+abx+x^3}}{\sqrt{d}\,(a-x)^2}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2*b - a*(2*a - b)*x - (-a + 2*b)*x^2 + x^3)/(Sqrt[x*(
-a + x)*(-b + x)]*(-(a^3*d) + (b + 3*a^2*d)*x - (1 + 3*a*d)*x^2 + d*x^3)),x
]
```

```
[Out] (-2*ArcTanh[Sqrt[a*b*x + (-a - b)*x^2 + x^3]/(Sqrt[d]*(a - x)^2)])/Sqrt[d]
```

fricas [B] time = 1.15, size = 441, normalized size = 10.02

$$\left[\frac{\log\left(\frac{a^6d^2+d^2x^6-6\left(ad^2-d\right)x^5+\left(15a^2d^2-6(3a+b)d+1\right)x^4-2\left(10a^3d^2-9\left(a^2+ab\right)d+b\right)x^3+\left(15a^4d^2+b^2-6\left(a^3+3a^2b\right)d\right)x^2-4\left(a^4d+dx^4-(4ad-1)x^3+\left(6a^2d-a-b\right)x^2-\left(4a^3d-ab\right)x\right)\sqrt{abx-(a+b)x^2+x^3}\,\sqrt{d}-6\left(a^5d^2-a^3bd\right)x}{a^6d^2+d^2x^6-2\left(3ad^2+d\right)x^5+\left(15a^2d^2+2(3a+b)d+1\right)x^4-2\left(10a^3d^2+3\left(a^2+ab\right)d+b\right)x^3+\left(15a^4d^2+b^2+2\left(a^3+3a^2b\right)d\right)x^2-2\left(3a^5d^2+a^3bd\right)x}\right)}{2\sqrt{d}}, \frac{\sqrt{-d}\arctan\left(\frac{\left(a^3d-dx^3+(3ad-1)x^2-\left(3a^2d-b\right)x\right)\sqrt{abx-(a+b)x^2+x^3}\,\sqrt{-d}}{2\left(a^2bdx+(2a+b)dx^3-dx^4-\left(a^2+2ab\right)dx^2\right)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*b-a*(2*a-b)*x-(-a+2*b)*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a^
3*d+(3*a^2*d+b)*x-(3*a*d+1)*x^2+d*x^3),x, algorithm="fricas")
```

```
[Out] [1/2*log((a^6*d^2 + d^2*x^6 - 6*(a*d^2 - d)*x^5 + (15*a^2*d^2 - 6*(3*a + b)
*d + 1)*x^4 - 2*(10*a^3*d^2 - 9*(a^2 + a*b)*d + b)*x^3 + (15*a^4*d^2 + b^2
- 6*(a^3 + 3*a^2*b)*d)*x^2 - 4*(a^4*d + d*x^4 - (4*a*d - 1)*x^3 + (6*a^2*d
- a - b)*x^2 - (4*a^3*d - a*b)*x)*sqrt(a*b*x - (a + b)*x^2 + x^3)*sqrt(d) -
 6*(a^5*d^2 - a^3*b*d)*x)/(a^6*d^2 + d^2*x^6 - 2*(3*a*d^2 + d)*x^5 + (15*a^
2*d^2 + 2*(3*a + b)*d + 1)*x^4 - 2*(10*a^3*d^2 + 3*(a^2 + a*b)*d + b)*x^3 +
 (15*a^4*d^2 + b^2 + 2*(a^3 + 3*a^2*b)*d)*x^2 - 2*(3*a^5*d^2 + a^3*b*d)*x))
/sqrt(d), sqrt(-d)*arctan(1/2*(a^3*d - d*x^3 + (3*a*d - 1)*x^2 - (3*a^2*d -
 b)*x)*sqrt(a*b*x - (a + b)*x^2 + x^3)*sqrt(-d)/(a^2*b*d*x + (2*a + b)*d*x^
3 - d*x^4 - (a^2 + 2*a*b)*d*x^2))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{a^2 b - (2\,a - b)ax + (a - 2\,b)x^2 + x^3}{\left(a^3 d - dx^3 + (3\,ad + 1)x^2 - \left(3\,a^2 d + b\right)x\right)\sqrt{(a - x)(b - x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*b-a*(2*a-b)*x-(-a+2*b)*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a^
3*d+(3*a^2*d+b)*x-(3*a*d+1)*x^2+d*x^3),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*b - (2*a - b)*a*x + (a - 2*b)*x^2 + x^3)/((a^3*d - d*x^3 +
(3*a*d + 1)*x^2 - (3*a^2*d + b)*x)*sqrt((a - x)*(b - x)*x)), x)
```

maple [C] time = 0.06, size = 344, normalized size = 7.82

$$-\frac{2a\sqrt{-\frac{-a+x}{a}}\,\sqrt{\frac{-b+x}{a-b}}\,\sqrt{\frac{x}{a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{-a+x}{a}},\sqrt{\frac{a}{a-b}}\right)}{d\sqrt{abx-a\,x^2-b\,x^2+x^3}}+\frac{2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\,\_Z^3+(-3ad-1)\_Z^2+\left(3a^2d+b\right)\_Z-a^3d\right)}\frac{\left(-4\_\alpha^2ad+2\_\alpha^2bd+5\_\alpha\,a^2d-\_\alpha abd-a^3d-a^2bd-\_\alpha^2+\_\alpha b\right)\left(\_\alpha^2d-2\_\alpha da+a^2d-\_\alpha-a+b\right)\sqrt{-\frac{-a+x}{a}}\,\sqrt{\frac{-b+x}{a-b}}\,\sqrt{\frac{x}{a}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-a+x}{a}},-\frac{\_\alpha^2d-2\_\alpha da+a^2d-\_\alpha-a+b}{a-b},\sqrt{\frac{a}{a-b}}\right)}{\left(-3\_\alpha^2d+6\_\alpha da-3a^2d+2\_\alpha-b\right)(a-b)\sqrt{x\left(ab-ax-bx+x^2\right)}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*b-a*(2*a-b)*x-(-a+2*b)*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a^3*d+(3
*a^2*d+b)*x-(3*a*d+1)*x^2+d*x^3),x)
```

```
[Out] -2/d*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*
x^2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))+2/d*sum((-4*_al
pha^2*a*d+2*_alpha^2*b*d+5*_alpha*a^2*d-_alpha*a*b*d-a^3*d-a^2*b*d-_alpha^2
+_alpha*b)/(-3*_alpha^2*d+6*_alpha*a*d-3*a^2*d+2*_alpha-b)*(_alpha^2*d-2*_a
lpha*a*d+a^2*d-_alpha-a+b)/(a-b)*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/
a*x)^(1/2)/(x*(a*b-a*x-b*x+x^2))^(1/2)*EllipticPi((-(-a+x)/a)^(1/2),-(_alph
a^2*d-2*_alpha*a*d+a^2*d-_alpha-a+b)/(a-b),(a/(a-b))^(1/2)),_alpha=RootOf(d
*_Z^3+(-3*a*d-1)*_Z^2+(3*a^2*d+b)*_Z-a^3*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2 b - (2\,a - b)ax + (a - 2\,b)x^2 + x^3}{\left(a^3 d - dx^3 + (3\,ad + 1)x^2 - \left(3\,a^2 d + b\right)x\right)\sqrt{(a - x)(b - x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*b-a*(2*a-b)*x-(-a+2*b)*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a^
3*d+(3*a^2*d+b)*x-(3*a*d+1)*x^2+d*x^3),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*b - (2*a - b)*a*x + (a - 2*b)*x^2 + x^3)/((a^3*d - d*x^3 +
(3*a*d + 1)*x^2 - (3*a^2*d + b)*x)*sqrt((a - x)*(b - x)*x)), x)
```

mupad [B] time = 5.92, size = 368, normalized size = 8.36

$$\frac{\ln\left(\frac{\left(a-b+x+a^2\,d-2\,\sqrt{d}\,\sqrt{x\,(a-x)\,(b-x)}+d\,x^2-2\,a\,d\,x\right)\left(a\,x^2-a^4\,d-2\,b\,x^2+b^2\,x-2\,d\,x^4+x^3-a^5\,d^2+d^2\,x^5+2\,a^2\,\sqrt{d}\,\sqrt{x\,(a-x)\,(b-x)}-3\,a^2\,d\,x^2-5\,a\,d^2\,x^4+5\,a^4\,d^2\,x-a\,b\,x+10\,a^2\,d^2\,x^3-10\,a^3\,d^2\,x^2+a^3\,b\,d+4\,a\,d\,x^3+2\,a^3\,d\,x+2\,b\,d\,x^3-2\,a\,b\,\sqrt{d}\,\sqrt{x\,(a-x)\,(b-x)}-3\,a\,b\,d\,x^2\right)}{\left(-d\,a^3+3\,d\,a^2\,x-3\,d\,a\,x^2+d\,x^3-x^2+b\,x\right)\left(a^4\,d^2-4\,a^3\,d^2\,x+2\,a^3\,d-2\,a^2\,b\,d+6\,a^2\,d^2\,x^2-2\,a^2\,d\,x+a^2-2\,a\,b-4\,a\,d^2\,x^3+2\,a\,d\,x^2+2\,a\,x+b^2+2\,b\,d\,x^2-2\,b\,x+d^2\,x^4-2\,d\,x^3+x^2\right)}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*b + x^2*(a - 2*b) + x^3 - a*x*(2*a - b))/((x*(a - x)*(b - x))^(1/2
)*(x*(b + 3*a^2*d) - a^3*d + d*x^3 - x^2*(3*a*d + 1))),x)
```

```
[Out] log(((a - b + x + a^2*d - 2*d^(1/2)*(x*(a - x)*(b - x))^(1/2) + d*x^2 - 2*a
*d*x)*(a*x^2 - a^4*d - 2*b*x^2 + b^2*x - 2*d*x^4 + x^3 - a^5*d^2 + d^2*x^5
+ 2*a^2*d^(1/2)*(x*(a - x)*(b - x))^(1/2) - 3*a^2*d*x^2 - 5*a*d^2*x^4 + 5*a
^4*d^2*x - a*b*x + 10*a^2*d^2*x^3 - 10*a^3*d^2*x^2 + a^3*b*d + 4*a*d*x^3 +
2*a^3*d*x + 2*b*d*x^3 - 2*a*b*d^(1/2)*(x*(a - x)*(b - x))^(1/2) - 3*a*b*d*x
^2))/((b*x - a^3*d + d*x^3 - x^2 - 3*a*d*x^2 + 3*a^2*d*x)*(2*a*x - 2*a*b -
2*b*x + 2*a^3*d - 2*d*x^3 + a^2 + b^2 + x^2 + a^4*d^2 + d^2*x^4 - 4*a*d^2*x
^3 - 4*a^3*d^2*x + 6*a^2*d^2*x^2 - 2*a^2*b*d + 2*a*d*x^2 - 2*a^2*d*x + 2*b*
d*x^2)))/d^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*b-a*(2*a-b)*x-(-a+2*b)*x**2+x**3)/(x*(-a+x)*(-b+x))**(1/2)/
(-a**3*d+(3*a**2*d+b)*x-(3*a*d+1)*x**2+d*x**3),x)
```

```
[Out] Timed out
```

3.571 $\displaystyle\int \frac{-3x+2x^2}{(-2+2x+x^3)\sqrt{-2x+2x^2+3x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{3x^4+2x^2-2x}}{3x^3+2x-2}\right)}{\sqrt{2}}$$

Rubi [F] time = 1.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3x+2x^2}{(-2+2x+x^3)\sqrt{-2x+2x^2+3x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3*x + 2*x^2)/((-2 + 2*x + x^3)*Sqrt[-2*x + 2*x^2 + 3*x^4]),x]
```

```
[Out] (-6*Sqrt[x]*Sqrt[-2 + 2*x + 3*x^3]*Defer[Subst][Defer[Int][x^2/((-2 + 2*x^2
 + x^6)*Sqrt[-2 + 2*x^2 + 3*x^6]), x], x, Sqrt[x]])/Sqrt[-2*x + 2*x^2 + 3*x
^4] + (4*Sqrt[x]*Sqrt[-2 + 2*x + 3*x^3]*Defer[Subst][Defer[Int][x^4/((-2 +
2*x^2 + x^6)*Sqrt[-2 + 2*x^2 + 3*x^6]), x], x, Sqrt[x]])/Sqrt[-2*x + 2*x^2
+ 3*x^4]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3x+2x^2}{(-2+2x+x^3)\sqrt{-2x+2x^2+3x^4}}\,dx &= \int \frac{x(-3+2x)}{(-2+2x+x^3)\sqrt{-2x+2x^2+3x^4}}\,dx\\
&= \frac{\left(\sqrt{x}\sqrt{-2+2x+3x^3}\right)\int \frac{\sqrt{x}(-3+2x)}{(-2+2x+x^3)\sqrt{-2+2x+3x^3}}\,dx}{\sqrt{-2x+2x^2+3x^4}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-2+2x+3x^3}\right)\text{Subst}\left(\int \frac{x^2(-3+2x^2)}{(-2+2x^2+x^6)\sqrt{-2+2x^2+3x^6}}\,dx,x,\sqrt{x}\right)}{\sqrt{-2x+2x^2+3x^4}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-2+2x+3x^3}\right)\text{Subst}\left(\int \left(-\frac{3x^2}{(-2+2x^2+x^6)\sqrt{-2+2x^2+3x^6}}+\frac{}{(-2+2x^2+}\right.\right.}{\sqrt{-2x+2x^2+3x^4}}\\
&= \frac{\left(4\sqrt{x}\sqrt{-2+2x+3x^3}\right)\text{Subst}\left(\int \frac{x^4}{(-2+2x^2+x^6)\sqrt{-2+2x^2+3x^6}}\,dx,x,\sqrt{x}\right)}{\sqrt{-2x+2x^2+3x^4}} -
\end{aligned}$$

Mathematica [C] time = 4.60, size = 4253, normalized size = 96.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3*x + 2*x^2)/((-2 + 2*x + x^3)*Sqrt[-2*x + 2*x^2 + 3*x^4]),x]
```

```
[Out] (2*(x - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])^2*(3*(Root[-2 + 2*#1 + #1^3 & ,
2, 0] - Root[-2 + 2*#1 + #1^3 & , 3, 0])*(Root[-2 + 2*#1 + #1^3 & , 2, 0] -
 Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*(Root[-2 + 2*#1 + #1^3 & , 3, 0] - Root
[-2 + 2*#1 + 3*#1^3 & , 1, 0])*(EllipticF[ArcSin[Sqrt[(x*(-Root[-2 + 2*#1 +
```

```
 3*#1^3 & , 1, 0] + Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))/((x - Root[-2 + 2*#
1 + 3*#1^3 & , 1, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])]], ((Root[-2 + 2*#
1 + 3*#1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 2, 0])*Root[-2 + 2*#1 +
3*#1^3 & , 3, 0])/(Root[-2 + 2*#1 + 3*#1^3 & , 2, 0]*(Root[-2 + 2*#1 + 3*#1
^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))]*Root[-2 + 2*#1 + #1^3 &
 , 1, 0] - EllipticPi[-(((Root[-2 + 2*#1 + #1^3 & , 1, 0] - Root[-2 + 2*#1
+ 3*#1^3 & , 1, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])/(Root[-2 + 2*#1 + #1
^3 & , 1, 0]*(Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 &
 , 3, 0]))), ArcSin[Sqrt[(x*(-Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] + Root[-2 +
 2*#1 + 3*#1^3 & , 3, 0]))/((x - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*Root[-2
 + 2*#1 + 3*#1^3 & , 3, 0])]], ((Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-
2 + 2*#1 + 3*#1^3 & , 2, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])/(Root[-2 +
2*#1 + 3*#1^3 & , 2, 0]*(Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-2 + 2*#1
 + 3*#1^3 & , 3, 0]))]*Root[-2 + 2*#1 + 3*#1^3 & , 1, 0]) - 2*Root[-2 + 2*#
1 + #1^3 & , 1, 0]*(Root[-2 + 2*#1 + #1^3 & , 2, 0] - Root[-2 + 2*#1 + #1^3
 & , 3, 0])*(Root[-2 + 2*#1 + #1^3 & , 2, 0] - Root[-2 + 2*#1 + 3*#1^3 & ,
1, 0])*(Root[-2 + 2*#1 + #1^3 & , 3, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0]
)*(EllipticF[ArcSin[Sqrt[(x*(-Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] + Root[-2 +
 2*#1 + 3*#1^3 & , 3, 0]))/((x - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*Root[-2
 + 2*#1 + 3*#1^3 & , 3, 0])]], ((Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-
2 + 2*#1 + 3*#1^3 & , 2, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])/(Root[-2 +
2*#1 + 3*#1^3 & , 2, 0]*(Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-2 + 2*#1
 + 3*#1^3 & , 3, 0]))]*Root[-2 + 2*#1 + #1^3 & , 1, 0] - EllipticPi[-(((Roo
t[-2 + 2*#1 + #1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*Root[-2 +
 2*#1 + 3*#1^3 & , 3, 0])/(Root[-2 + 2*#1 + #1^3 & , 1, 0]*(Root[-2 + 2*#1
+ 3*#1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))), ArcSin[Sqrt[(x*(
-Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] + Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))/((
x - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])]]
, ((Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 2, 0])*
Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])/(Root[-2 + 2*#1 + 3*#1^3 & , 2, 0]*(Root
[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))]*Root[-
2 + 2*#1 + 3*#1^3 & , 1, 0]) + 2*Root[-2 + 2*#1 + #1^3 & , 2, 0]*(Root[-2 +
 2*#1 + #1^3 & , 1, 0] - Root[-2 + 2*#1 + #1^3 & , 3, 0])*(Root[-2 + 2*#1 +
 #1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*(Root[-2 + 2*#1 + #1^3
 & , 3, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*(EllipticF[ArcSin[Sqrt[(x*(
-Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] + Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))/((
x - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])]]
, ((Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 2, 0])*
Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])/(Root[-2 + 2*#1 + 3*#1^3 & , 2, 0]*(Root
[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))]*Root[-
2 + 2*#1 + #1^3 & , 2, 0] - EllipticPi[-(((Root[-2 + 2*#1 + #1^3 & , 2, 0]
- Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])/(Ro
ot[-2 + 2*#1 + #1^3 & , 2, 0]*(Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-2
+ 2*#1 + 3*#1^3 & , 3, 0]))), ArcSin[Sqrt[(x*(-Root[-2 + 2*#1 + 3*#1^3 & ,
1, 0] + Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))/((x - Root[-2 + 2*#1 + 3*#1^3 &
 , 1, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])]], ((Root[-2 + 2*#1 + 3*#1^3 &
 , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 2, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 3
, 0])/(Root[-2 + 2*#1 + 3*#1^3 & , 2, 0]*(Root[-2 + 2*#1 + 3*#1^3 & , 1, 0]
 - Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))]*Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])
+ 3*(Root[-2 + 2*#1 + #1^3 & , 1, 0] - Root[-2 + 2*#1 + #1^3 & , 3, 0])*(Ro
ot[-2 + 2*#1 + #1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*(Root[-2
 + 2*#1 + #1^3 & , 3, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*(-(EllipticF[
ArcSin[Sqrt[(x*(-Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] + Root[-2 + 2*#1 + 3*#1^
3 & , 3, 0]))/((x - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*Root[-2 + 2*#1 + 3*#
1^3 & , 3, 0])]], ((Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*
#1^3 & , 2, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])/(Root[-2 + 2*#1 + 3*#1^3
 & , 2, 0]*(Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & ,
 3, 0]))]*Root[-2 + 2*#1 + #1^3 & , 2, 0]) + EllipticPi[-(((Root[-2 + 2*#1
+ #1^3 & , 2, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*Root[-2 + 2*#1 + 3*#1
```

```
^3 & , 3, 0])/(Root[-2 + 2*#1 + #1^3 & , 2, 0]*(Root[-2 + 2*#1 + 3*#1^3 & ,
 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))), ArcSin[Sqrt[(x*(-Root[-2 + 2
*#1 + 3*#1^3 & , 1, 0] + Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))/((x - Root[-2
+ 2*#1 + 3*#1^3 & , 1, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])]], ((Root[-2
+ 2*#1 + 3*#1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 2, 0])*Root[-2 + 2*
#1 + 3*#1^3 & , 3, 0])/(Root[-2 + 2*#1 + 3*#1^3 & , 2, 0]*(Root[-2 + 2*#1 +
 3*#1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))]*Root[-2 + 2*#1 + 3
*#1^3 & , 1, 0]) + 3*(Root[-2 + 2*#1 + #1^3 & , 1, 0] - Root[-2 + 2*#1 + #1
^3 & , 2, 0])*(Root[-2 + 2*#1 + #1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 &
, 1, 0])*(Root[-2 + 2*#1 + #1^3 & , 2, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 1,
0])*(EllipticF[ArcSin[Sqrt[(x*(-Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] + Root[-2
 + 2*#1 + 3*#1^3 & , 3, 0]))/((x - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*Root[
-2 + 2*#1 + 3*#1^3 & , 3, 0])]], ((Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root
[-2 + 2*#1 + 3*#1^3 & , 2, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])/(Root[-2
+ 2*#1 + 3*#1^3 & , 2, 0]*(Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-2 + 2*
#1 + 3*#1^3 & , 3, 0]))]*Root[-2 + 2*#1 + #1^3 & , 3, 0] - EllipticPi[-(((R
oot[-2 + 2*#1 + #1^3 & , 3, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*Root[-2
 + 2*#1 + 3*#1^3 & , 3, 0])/(Root[-2 + 2*#1 + #1^3 & , 3, 0]*(Root[-2 + 2*#
1 + 3*#1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))), ArcSin[Sqrt[(x
*(-Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] + Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))/
((x - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])
]], ((Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 2, 0]
)*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])/(Root[-2 + 2*#1 + 3*#1^3 & , 2, 0]*(Ro
ot[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))]*Root
[-2 + 2*#1 + 3*#1^3 & , 1, 0]) - 2*(Root[-2 + 2*#1 + #1^3 & , 1, 0] - Root[
-2 + 2*#1 + #1^3 & , 2, 0])*Root[-2 + 2*#1 + #1^3 & , 3, 0]*(Root[-2 + 2*#1
 + #1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*(Root[-2 + 2*#1 + #1
^3 & , 2, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*(EllipticF[ArcSin[Sqrt[(x
*(-Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] + Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))/
((x - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])
]], ((Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 2, 0]
)*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])/(Root[-2 + 2*#1 + 3*#1^3 & , 2, 0]*(Ro
ot[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))]*Root
[-2 + 2*#1 + #1^3 & , 3, 0] - EllipticPi[-(((Root[-2 + 2*#1 + #1^3 & , 3, 0
] - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])/(
Root[-2 + 2*#1 + #1^3 & , 3, 0]*(Root[-2 + 2*#1 + 3*#1^3 & , 1, 0] - Root[-
2 + 2*#1 + 3*#1^3 & , 3, 0]))), ArcSin[Sqrt[(x*(-Root[-2 + 2*#1 + 3*#1^3 &
, 1, 0] + Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))/((x - Root[-2 + 2*#1 + 3*#1^3
 & , 1, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])]], ((Root[-2 + 2*#1 + 3*#1^3
 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 2, 0])*Root[-2 + 2*#1 + 3*#1^3 & ,
 3, 0])/(Root[-2 + 2*#1 + 3*#1^3 & , 2, 0]*(Root[-2 + 2*#1 + 3*#1^3 & , 1,
0] - Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))]*Root[-2 + 2*#1 + 3*#1^3 & , 1, 0]
))*Sqrt[(x - Root[-2 + 2*#1 + 3*#1^3 & , 2, 0])/((x - Root[-2 + 2*#1 + 3*#1
^3 & , 1, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 2, 0])]*Sqrt[(x - Root[-2 + 2*#1
+ 3*#1^3 & , 3, 0])/((x - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*Root[-2 + 2*#1
 + 3*#1^3 & , 3, 0])]*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]*Sqrt[(x*(-Root[-2 +
 2*#1 + 3*#1^3 & , 1, 0] + Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))/((x - Root[-
2 + 2*#1 + 3*#1^3 & , 1, 0])*Root[-2 + 2*#1 + 3*#1^3 & , 3, 0])])/(Sqrt[x*(
-2 + 2*x + 3*x^3)]*(Root[-2 + 2*#1 + #1^3 & , 1, 0] - Root[-2 + 2*#1 + #1^3
 & , 2, 0])*(Root[-2 + 2*#1 + #1^3 & , 1, 0] - Root[-2 + 2*#1 + #1^3 & , 3,
 0])*(Root[-2 + 2*#1 + #1^3 & , 2, 0] - Root[-2 + 2*#1 + #1^3 & , 3, 0])*(R
oot[-2 + 2*#1 + #1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*(Root[-
2 + 2*#1 + #1^3 & , 2, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*(Root[-2 + 2
*#1 + #1^3 & , 3, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 1, 0])*(Root[-2 + 2*#1 +
 3*#1^3 & , 1, 0] - Root[-2 + 2*#1 + 3*#1^3 & , 3, 0]))
```

IntegrateAlgebraic [A] time = 0.41, size = 44, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{3x^4+2x^2-2x}}{3x^3+2x-2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*x + 2*x^2)/((-2 + 2*x + x^3)*Sqrt[-2*x + 2*x^2 + 3*x
^4]),x]
```

```
[Out] ArcTanh[(Sqrt[2]*x*Sqrt[-2*x + 2*x^2 + 3*x^4])/(-2 + 2*x + 3*x^3)]/Sqrt[2]
```

fricas [B] time = 0.45, size = 93, normalized size = 2.11

$$\frac{1}{8}\,\sqrt{2}\,\log\left(-\frac{49\,x^6+36\,x^4-36\,x^3+4\,\sqrt{2}\left(5\,x^4+2\,x^2-2\,x\right)\sqrt{3\,x^4+2\,x^2-2\,x}+4\,x^2-8\,x+4}{x^6+4\,x^4-4\,x^3+4\,x^2-8\,x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-3*x)/(x^3+2*x-2)/(3*x^4+2*x^2-2*x)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/8*sqrt(2)*log(-(49*x^6 + 36*x^4 - 36*x^3 + 4*sqrt(2)*(5*x^4 + 2*x^2 - 2*x
)*sqrt(3*x^4 + 2*x^2 - 2*x) + 4*x^2 - 8*x + 4)/(x^6 + 4*x^4 - 4*x^3 + 4*x^2
 - 8*x + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2-3\,x}{\sqrt{3\,x^4+2\,x^2-2\,x}\left(x^3+2\,x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-3*x)/(x^3+2*x-2)/(3*x^4+2*x^2-2*x)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((2*x^2 - 3*x)/(sqrt(3*x^4 + 2*x^2 - 2*x)*(x^3 + 2*x - 2)), x)
```

maple [C] time = 1.88, size = 1659, normalized size = 37.70

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-3*x)/(x^3+2*x-2)/(3*x^4+2*x^2-2*x)^(1/2),x)
```

```
[Out] -1/864*108^(1/2)*sum(_alpha^2*((9+89^(1/2))^(1/3)-2/(9+89^(1/2))^(1/3)+I*3^
(1/2)*((9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)))*(x*(-3*(9+89^(1/2))^(1/3)+
6/(9+89^(1/2))^(1/3)-I*3^(1/2)*((9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)))/(
-(9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)-I*3^(1/2)*((9+89^(1/2))^(1/3)+2/(9
+89^(1/2))^(1/3)))/(3*x-(9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)))^(1/2)*(3*
x-(9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3))^2*(((9+89^(1/2))^(1/3)-2/(9+89^(
1/2))^(1/3))*(6*x+(9+89^(1/2))^(1/3)-2/(9+89^(1/2))^(1/3)-I*3^(1/2)*((9+89^
(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)))/(-(9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/
3)+I*3^(1/2)*((9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)))/(3*x-(9+89^(1/2))^(
1/3)+2/(9+89^(1/2))^(1/3)))^(1/2)*(((9+89^(1/2))^(1/3)-2/(9+89^(1/2))^(1/3)
)*(6*x+(9+89^(1/2))^(1/3)-2/(9+89^(1/2))^(1/3)+I*3^(1/2)*((9+89^(1/2))^(1/3
)+2/(9+89^(1/2))^(1/3)))/(-(9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)-I*3^(1/2
)*((9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)))/(3*x-(9+89^(1/2))^(1/3)+2/(9+8
9^(1/2))^(1/3)))^(1/2)/(-3*(9+89^(1/2))^(1/3)+6/(9+89^(1/2))^(1/3)-I*3^(1/2
```

```
)*((9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)))/((9+89^(1/2))^(1/3)-2/(9+89^(1
/2))^(1/3))/(x*(3*x-(9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3))*(6*x+(9+89^(1/
2))^(1/3)-2/(9+89^(1/2))^(1/3)-I*3^(1/2)*((9+89^(1/2))^(1/3)+2/(9+89^(1/2))
^(1/3)))*(6*x+(9+89^(1/2))^(1/3)-2/(9+89^(1/2))^(1/3)+I*3^(1/2)*((9+89^(1/2
))^(1/3)+2/(9+89^(1/2))^(1/3))))^(1/2)*(-44*_alpha^2-8*_alpha-96-31*_alpha^
2*(9+89^(1/2))^(2/3)+3*_alpha^2*(9+89^(1/2))^(2/3)*89^(1/2)+6*_alpha^2*(9+8
9^(1/2))^(1/3)-2*_alpha^2*(9+89^(1/2))^(1/3)*89^(1/2)-2*_alpha*(9+89^(1/2))
^(1/3)*89^(1/2)+6*_alpha*(9+89^(1/2))^(1/3)-31*_alpha*(9+89^(1/2))^(2/3)+3*
_alpha*(9+89^(1/2))^(2/3)*89^(1/2)+30*(9+89^(1/2))^(1/3)-6*(9+89^(1/2))^(1/
3)*89^(1/2)-66*(9+89^(1/2))^(2/3)+6*(9+89^(1/2))^(2/3)*89^(1/2))*(6*Ellipti
cF(((-1/2*(9+89^(1/2))^(1/3)+1/(9+89^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(9+89^
(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/3)))*x/(-1/6*(9+89^(1/2))^(1/3)+1/3/(9+89^
(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(9+89^(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/3)))
/(x-1/3*(9+89^(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/3)))^(1/2),((1/2*(9+89^(1/2)
)^(1/3)-1/(9+89^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(9+89^(1/2))^(1/3)+2/3/(9+8
9^(1/2))^(1/3)))*(1/6*(9+89^(1/2))^(1/3)-1/3/(9+89^(1/2))^(1/3)+1/2*I*3^(1/
2)*(1/3*(9+89^(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/3)))/(1/6*(9+89^(1/2))^(1/3)
-1/3/(9+89^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(9+89^(1/2))^(1/3)+2/3/(9+89^(1/
2))^(1/3)))/(1/2*(9+89^(1/2))^(1/3)-1/(9+89^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/3
*(9+89^(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/3))))^(1/2))-(_alpha^2+2)*((9+89^(1
/2))^(1/3)-2/(9+89^(1/2))^(1/3))*EllipticPi(((-1/2*(9+89^(1/2))^(1/3)+1/(9+
89^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(9+89^(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/3
)))*x/(-1/6*(9+89^(1/2))^(1/3)+1/3/(9+89^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(9
+89^(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/3)))/(x-1/3*(9+89^(1/2))^(1/3)+2/3/(9+
89^(1/2))^(1/3)))^(1/2),1/2-1/356*(9+89^(1/2))^(1/3)*89^(1/2)+29/1068*(9+89
^(1/2))^(2/3)*89^(1/2)-1/712*I*3^(1/2)*(9+89^(1/2))^(1/3)*89^(1/2)*_alpha^2
+1/16*I*3^(1/2)*(9+89^(1/2))^(2/3)*_alpha^2+31/1602*I*3^(1/2)*89^(1/2)-1/72
*I*(9+89^(1/2))^(1/3)*3^(1/2)*_alpha^2-1/4*(9+89^(1/2))^(2/3)-1/36*I*(9+89^
(1/2))^(1/3)*3^(1/2)-3/16*_alpha^2*(9+89^(1/2))^(2/3)-1/12*(9+89^(1/2))^(1/
3)+1/801*I*3^(1/2)*89^(1/2)*_alpha^2+1/12*I*(9+89^(1/2))^(2/3)*3^(1/2)-29/3
204*I*(9+89^(1/2))^(2/3)*89^(1/2)*3^(1/2)-3/712*_alpha^2*(9+89^(1/2))^(1/3)
*89^(1/2)-1/24*_alpha^2*(9+89^(1/2))^(1/3)+85/4272*_alpha^2*(9+89^(1/2))^(2
/3)*89^(1/2)-1/1068*I*3^(1/2)*(9+89^(1/2))^(1/3)*89^(1/2)-85/12816*I*3^(1/2
)*(9+89^(1/2))^(2/3)*89^(1/2)*_alpha^2,((1/2*(9+89^(1/2))^(1/3)-1/(9+89^(1/
2))^(1/3)-1/2*I*3^(1/2)*(1/3*(9+89^(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/3)))*(1
/6*(9+89^(1/2))^(1/3)-1/3/(9+89^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/3*(9+89^(1/2)
)^(1/3)+2/3/(9+89^(1/2))^(1/3)))/(1/6*(9+89^(1/2))^(1/3)-1/3/(9+89^(1/2))^(
1/3)-1/2*I*3^(1/2)*(1/3*(9+89^(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/3)))/(1/2*(9
+89^(1/2))^(1/3)-1/(9+89^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/3*(9+89^(1/2))^(1/3)
+2/3/(9+89^(1/2))^(1/3))))^(1/2))),_alpha=RootOf(_Z^3+2*_Z-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2 - 3x}{\sqrt{3x^4 + 2x^2 - 2x}\left(x^3 + 2x - 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-3*x)/(x^3+2*x-2)/(3*x^4+2*x^2-2*x)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((2*x^2 - 3*x)/(sqrt(3*x^4 + 2*x^2 - 2*x)*(x^3 + 2*x - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{3x - 2x^2}{\left(x^3 + 2x - 2\right)\sqrt{3x^4 + 2x^2 - 2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - 2*x^2)/((2*x + x^3 - 2)*(2*x^2 - 2*x + 3*x^4)^(1/2)),x)
```

```
[Out] -int((3*x - 2*x^2)/((2*x + x^3 - 2)*(2*x^2 - 2*x + 3*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(2x-3)}{\sqrt{x\left(3x^3+2x-2\right)}\left(x^3+2x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-3*x)/(x**3+2*x-2)/(3*x**4+2*x**2-2*x)**(1/2),x)
[Out] Integral(x*(2*x - 3)/(sqrt(x*(3*x**3 + 2*x - 2))*(x**3 + 2*x - 2)), x)
```

3.572 $\int \frac{4aqx-3bpx^2+apx^4}{\sqrt{q+px^3}\left(b^2c+dq+2abcx^2+dpx^3+a^2cx^4\right)}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{px^3+q}}{\sqrt{c}\left(ax^2+b\right)}\right)}{\sqrt{c}\,\sqrt{d}}$$

Rubi [F] time = 4.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{4aqx-3bpx^2+apx^4}{\sqrt{q+px^3}\left(b^2c+dq+2abcx^2+dpx^3+a^2cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(4*a*q*x - 3*b*p*x^2 + a*p*x^4)/(Sqrt[q + p*x^3]*(b^2*c + d*q + 2*a*b*c
*x^2 + d*p*x^3 + a^2*c*x^4)),x]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*p^(2/3)*(q^(1/3) + p^(1/3)*x)*Sqrt[(q^(2/3) - p^(1/3)*
q^(1/3)*x + p^(2/3)*x^2)/((1 + Sqrt[3])*q^(1/3) + p^(1/3)*x)^2]*EllipticF[A
rcSin[((1 - Sqrt[3])*q^(1/3) + p^(1/3)*x)/((1 + Sqrt[3])*q^(1/3) + p^(1/3)*
x)], -7 - 4*Sqrt[3]])/(3^(1/4)*a*c*Sqrt[(q^(1/3)*(q^(1/3) + p^(1/3)*x))/((1
 + Sqrt[3])*q^(1/3) + p^(1/3)*x)^2]*Sqrt[q + p*x^3]) - (p*(b^2*c + d*q)*Def
er[Int][1/(Sqrt[q + p*x^3]*(b^2*c + d*q + 2*a*b*c*x^2 + d*p*x^3 + a^2*c*x^4
)), x])/(a*c) + 4*a*q*Defer[Int][x/(Sqrt[q + p*x^3]*(b^2*c + d*q + 2*a*b*c*
x^2 + d*p*x^3 + a^2*c*x^4)), x] - 5*b*p*Defer[Int][x^2/(Sqrt[q + p*x^3]*(b^
2*c + d*q + 2*a*b*c*x^2 + d*p*x^3 + a^2*c*x^4)), x] - (d*p^2*Defer[Int][x^3
/(Sqrt[q + p*x^3]*(b^2*c + d*q + 2*a*b*c*x^2 + d*p*x^3 + a^2*c*x^4)), x])/(
a*c)
```

Rubi steps

$$\int \frac{4aqx-3bpx^2+apx^4}{\sqrt{q+px^3}\left(b^2c+dq+2abcx^2+dpx^3+a^2cx^4\right)}\,dx = \int \frac{x\left(4aq-3bpx+apx^3\right)}{\sqrt{q+px^3}\left(b^2c+dq+2abcx^2+dpx^3+a^2cx^4\right)}\,dx$$

$$= \int \left(\frac{p}{ac\sqrt{q+px^3}} - \frac{p\left(b^2c+dq\right)-4a^2cqx+5abcpx^2+}{ac\sqrt{q+px^3}\left(b^2c+dq+2abcx^2+dpx\right.}\right.$$

$$= -\frac{\int \frac{p\left(b^2c+dq\right)-4a^2cqx+5abcpx^2+dp^2x^3}{\sqrt{q+px^3}\left(b^2c+dq+2abcx^2+dpx^3+a^2cx^4\right)}\,dx}{ac} + \frac{p\int \frac{1}{\sqrt{q+px^3}}\,dx}{ac}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,p^{2/3}\left(\sqrt[3]{q}+\sqrt[3]{p}\,x\right)\sqrt{\frac{q^{2/3}-\sqrt[3]{p}\,\sqrt[3]{q}\,x+p^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{q}+\sqrt[3]{p}\,x\right)^2}}\,F\left(\sin^{-1}\right.}{\sqrt[4]{3}\,ac\sqrt{\frac{\sqrt[3]{q}\left(\sqrt[3]{q}+\sqrt[3]{p}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{q}+\sqrt[3]{p}\,x\right)^2}}\,\sqrt{q+}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,p^{2/3}\left(\sqrt[3]{q}+\sqrt[3]{p}\,x\right)\sqrt{\frac{q^{2/3}-\sqrt[3]{p}\,\sqrt[3]{q}\,x+p^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{q}+\sqrt[3]{p}\,x\right)^2}}\,F\left(\sin^{-1}\right.}{\sqrt[4]{3}\,ac\sqrt{\frac{\sqrt[3]{q}\left(\sqrt[3]{q}+\sqrt[3]{p}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{q}+\sqrt[3]{p}\,x\right)^2}}\,\sqrt{q+}}$$

Mathematica [C] time = 7.09, size = 8005, normalized size = 181.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(4*a*q*x - 3*b*p*x^2 + a*p*x^4)/(Sqrt[q + p*x^3]*(b^2*c + d*q + 2
*a*b*c*x^2 + d*p*x^3 + a^2*c*x^4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 18.16, size = 44, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{d}\sqrt{px^3+q}}{\sqrt{c}\left(ax^2+b\right)}\right)}{\sqrt{c}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4*a*q*x - 3*b*p*x^2 + a*p*x^4)/(Sqrt[q + p*x^3]*(b^2*c
+ d*q + 2*a*b*c*x^2 + d*p*x^3 + a^2*c*x^4)),x]
```

```
[Out] (-2*ArcTan[(Sqrt[d]*Sqrt[q + p*x^3])/(Sqrt[c]*(b + a*x^2))])/(Sqrt[c]*Sqrt[
d])
```

fricas [B] time = 3.86, size = 471, normalized size = 10.70

$$\left[-\frac{\sqrt{-cd}\log\left(\frac{a^4c^2x^8-6\,a^2cdpx^7-12\,abcdpx^5+\left(4\,a^3bc^2+d^2p^2\right)x^6+b^4c^2-6\,b^2cdq+6\left(a^2b^2c^2-a^2cdq\right)x^4+d^2q^2-2\left(3\,b^2cdp-d^2pq\right)x^3+4\left(ab^3c^2-3\,abcdq\right)x^2-4\left(a^3cx^6+3\,a^2bcx^4-adpx^5-bdpx^3+b^3c-bdq+\left(3\,ab^2c-adq\right)x^2\right)\sqrt{px^3+q}\sqrt{-cd}}{a^4c^2x^8+2\,a^2cdpx^7+4\,abcdpx^5+\left(4\,a^3bc^2+d^2p^2\right)x^6+b^4c^2+2\,b^2cdq+2\left(3\,a^2b^2c^2+a^2cdq\right)x^4+d^2q^2+2\left(b^2cdp+d^2pq\right)x^3+4\left(ab^3c^2+abcdq\right)x^2}\right)}{2\,cd},\ \frac{\sqrt{cd}\arctan\left(\frac{\left(a^2cx^4+2\,abcx^2-dpx^3+b^2c-dq\right)\sqrt{px^3+q}\sqrt{cd}}{2\left(acdpx^5+bcdpx^3+acdqx^2+bcdq\right)}\right)}{cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*p*x^4-3*b*p*x^2+4*a*q*x)/(p*x^3+q)^(1/2)/(a^2*c*x^4+2*a*b*c*x^
2+d*p*x^3+b^2*c+d*q),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-c*d)*log((a^4*c^2*x^8 - 6*a^2*c*d*p*x^7 - 12*a*b*c*d*p*x^5 + (4
*a^3*b*c^2 + d^2*p^2)*x^6 + b^4*c^2 - 6*b^2*c*d*q + 6*(a^2*b^2*c^2 - a^2*c*
d*q)*x^4 + d^2*q^2 - 2*(3*b^2*c*d*p - d^2*p*q)*x^3 + 4*(a*b^3*c^2 - 3*a*b*c
*d*q)*x^2 - 4*(a^3*c*x^6 + 3*a^2*b*c*x^4 - a*d*p*x^5 - b*d*p*x^3 + b^3*c -
b*d*q + (3*a*b^2*c - a*d*q)*x^2)*sqrt(p*x^3 + q)*sqrt(-c*d))/(a^4*c^2*x^8 +
 2*a^2*c*d*p*x^7 + 4*a*b*c*d*p*x^5 + (4*a^3*b*c^2 + d^2*p^2)*x^6 + b^4*c^2
+ 2*b^2*c*d*q + 2*(3*a^2*b^2*c^2 + a^2*c*d*q)*x^4 + d^2*q^2 + 2*(b^2*c*d*p
+ d^2*p*q)*x^3 + 4*(a*b^3*c^2 + a*b*c*d*q)*x^2))/(c*d), sqrt(c*d)*arctan(1/
2*(a^2*c*x^4 + 2*a*b*c*x^2 - d*p*x^3 + b^2*c - d*q)*sqrt(p*x^3 + q)*sqrt(c*
d)/(a*c*d*p*x^5 + b*c*d*p*x^3 + a*c*d*q*x^2 + b*c*d*q))/(c*d)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*p*x^4-3*b*p*x^2+4*a*q*x)/(p*x^3+q)^(1/2)/(a^2*c*x^4+2*a*b*c*x^
2+d*p*x^3+b^2*c+d*q),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.72, size = 3173, normalized size = 72.11

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*p*x^4-3*b*p*x^2+4*a*q*x)/(p*x^3+q)^(1/2)/(a^2*c*x^4+2*a*b*c*x^2+d*p*
x^3+b^2*c+d*q),x)
```

```
[Out] -2/3*I/a/c*3^(1/2)*(-q*p^2)^(1/3)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I*3^(1/2)/
p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2)*((x-1/p*(-q*p^2)^(1/3))/(
```

-3/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2)*(-I*(x+1/2/p*(
-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/
2)/(p*x^3+q)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I*3
^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2),(I*3^(1/2)/p*(-q*p
^2)^(1/3)/(-3/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2))+I/
a/c^2/p^2*2^(1/2)*sum((-_alpha^3*d*p^2-5*_alpha^2*a*b*c*p+4*_alpha*a^2*c*q-
b^2*c*p-d*p*q)/_alpha/(4*_alpha^2*a^2*c+3*_alpha*d*p+4*a*b*c)/(a^6*q^4+2*a^
3*b^3*p^2*q^2+b^6*p^4)*(-q*p^2)^(1/3)*(1/2*I*p*(2*x+1/p*(-I*3^(1/2)*(-q*p^2
)^(1/3)+(-q*p^2)^(1/3)))/(-q*p^2)^(1/3))^(1/2)*(p*(x-1/p*(-q*p^2)^(1/3))/(-
3*(-q*p^2)^(1/3)+I*3^(1/2)*(-q*p^2)^(1/3)))^(1/2)*(-1/2*I*p*(2*x+1/p*(I*3^(
1/2)*(-q*p^2)^(1/3)+(-q*p^2)^(1/3)))/(-q*p^2)^(1/3))^(1/2)/(p*x^3+q)^(1/2)*
((-q*p^2)^(2/3)*b^4*d*p^4-(-q*p^2)^(2/3)*a^6*c*q^3+2*p^2*(2*_alpha^3*a^5*b*
c*p*q^2-_alpha^3*a^2*b^4*c*p^3+_alpha^2*a^6*c*q^3-2*_alpha^2*a^3*b^3*c*p^2*
q+2*_alpha^2*a^3*b*d*p^2*q^2-_alpha^2*b^4*d*p^4+_alpha*a^4*b^2*c*p*q^2-2*_a
lpha*a*b^5*c*p^3+_alpha*a^4*d*p*q^3-2*_alpha*a*b^3*d*p^3*q-3*a^2*b^4*c*p^2*
q-3*a^2*b^2*d*p^2*q^2)+I*(-q*p^2)^(1/3)*3^(1/2)*a^4*d*q^3*p^2-2*I*(-q*p^2)^
(1/3)*p^4*3^(1/2)*a*b^5*c+(-q*p^2)^(1/3)*_alpha^2*a^2*b^4*c*p^4+(-q*p^2)^(2
/3)*_alpha^2*a^4*d*p^2*q^2-2*(-q*p^2)^(2/3)*_alpha^2*a*b^3*d*p^4-3*(-q*p^2)
^(2/3)*_alpha*a^2*b^4*c*p^3-(-q*p^2)^(1/3)*_alpha*a^6*c*p*q^3+(-q*p^2)^(2/3
)*_alpha^3*a^6*c*p*q^2-2*(-q*p^2)^(2/3)*_alpha^3*a^3*b^3*c*p^3+I*(-q*p^2)^(
1/3)*3^(1/2)*a^4*b^2*q^2*p^2*c-4*I*(-q*p^2)^(2/3)*3^(1/2)*a^3*b^3*q*p^2*c-2
I(-q*p^2)^(2/3)*3^(1/2)*a^3*b*d*q^2*p^2-2*I*(-q*p^2)^(1/3)*p^4*3^(1/2)*a*
b^3*d*q+2*I*(-q*p^2)^(1/3)*3^(1/2)*_alpha^2*a^5*b*q^2*p^2*c+3*I*(-q*p^2)^(1
/3)*p^4*3^(1/2)*_alpha^2*a^2*b^2*d*q+4*I*(-q*p^2)^(1/3)*p^3*3^(1/2)*_alpha*
a^3*b^3*q*c-3*I*(-q*p^2)^(2/3)*p^3*3^(1/2)*_alpha*a^2*b^2*d*q+2*I*(-q*p^2)^
(1/3)*p^3*3^(1/2)*_alpha*a^3*b*d*q^2+3*I*(-q*p^2)^(1/3)*p^3*3^(1/2)*_alpha^
3*a^4*b^2*q*c-3*I*(-q*p^2)^(2/3)*3^(1/2)*_alpha^2*a^4*b^2*q*p^2*c+2*(-q*p^2
)^(1/3)*a*b^5*c*p^4-(-q*p^2)^(1/3)*a^4*d*p^2*q^3-4*(-q*p^2)^(2/3)*a^3*b^3*c
*p^2*q-(-q*p^2)^(1/3)*a^4*b^2*c*p^2*q^2-2*(-q*p^2)^(2/3)*a^3*b*d*p^2*q^2+2*
(-q*p^2)^(1/3)*a*b^3*d*p^4*q+I*(-q*p^2)^(2/3)*p^4*3^(1/2)*b^4*d-I*(-q*p^2)^
(2/3)*3^(1/2)*a^6*q^3*c+I*(-q*p^2)^(1/3)*3^(1/2)*_alpha*a^6*q^3*p*c-2*I*(-q
*p^2)^(2/3)*p^3*3^(1/2)*_alpha^3*a^3*b^3*c-I*(-q*p^2)^(1/3)*p^4*3^(1/2)*_al
pha^2*a^2*b^4*c-2*I*(-q*p^2)^(2/3)*p^4*3^(1/2)*_alpha^2*a*b^3*d-3*I*(-q*p^2
)^(2/3)*p^3*3^(1/2)*_alpha*a^2*b^4*c+I*(-q*p^2)^(2/3)*3^(1/2)*_alpha^3*a^6*
q^2*p*c+I*(-q*p^2)^(2/3)*3^(1/2)*_alpha^2*a^4*d*q^2*p^2+(-q*p^2)^(1/3)*_alp
ha*b^4*d*p^5-I*(-q*p^2)^(1/3)*p^5*3^(1/2)*_alpha*b^4*d-3*(-q*p^2)^(1/3)*_al
pha^3*a^4*b^2*c*p^3*q-3*(-q*p^2)^(2/3)*_alpha^2*a^4*b^2*c*p^2*q-2*(-q*p^2)^
(1/3)*_alpha^2*a^5*b*c*p^2*q^2-3*(-q*p^2)^(1/3)*_alpha^2*a^2*b^2*d*p^4*q-4*
(-q*p^2)^(1/3)*_alpha*a^3*b^3*c*p^3*q-3*(-q*p^2)^(2/3)*_alpha*a^2*b^2*d*p^3
q-2(-q*p^2)^(1/3)*_alpha*a^3*b*d*p^3*q^2)*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2/p*(-q*p^2)^(1/3)-1/2*I*3^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3)
)^(1/2),1/2/p*(12*a^3*b^3*p^3*q^2*c-3*p^5*b^4*d*q+6*p^3*a^3*b*d*q^3+3*a^6*q
^4*p*c-3*p^3*(-q*p^2)^(2/3)*_alpha^2*a^2*b^4*c+6*p^4*_alpha^3*a^3*b^3*q*c+9
*p^3*_alpha^2*a^4*b^2*q^2*c+6*p^5*_alpha^2*a*b^3*d*q+9*p^4*_alpha*a^2*b^4*q
*c+9*p^4*_alpha*a^2*b^2*d*q^2-6*p^3*(-q*p^2)^(2/3)*a*b^5*c+3*q^3*(-q*p^2)^(
2/3)*a^4*d*p+3*q^2*(-q*p^2)^(2/3)*a^4*b^2*p*c-6*p^3*(-q*p^2)^(2/3)*a*b^3*d*
q+I*3^(1/2)*b^4*d*p^5*q-I*3^(1/2)*a^6*c*p*q^4+I*(-q*p^2)^(2/3)*3^(1/2)*a^4*
b^2*c*p*q^2+6*I*(-q*p^2)^(1/3)*3^(1/2)*a^2*b^4*c*p^3*q-2*I*(-q*p^2)^(2/3)*3
^(1/2)*a*b^3*d*p^3*q+6*I*(-q*p^2)^(1/3)*3^(1/2)*a^2*b^2*d*p^3*q^2+3*I*(-q*p
^2)^(2/3)*3^(1/2)*_alpha^3*a^4*b^2*c*p^2*q-4*I*(-q*p^2)^(1/3)*3^(1/2)*_alph
a^3*a^5*b*c*p^2*q^2+2*I*(-q*p^2)^(2/3)*3^(1/2)*_alpha^2*a^5*b*c*p*q^2+4*I*(
-q*p^2)^(1/3)*3^(1/2)*_alpha^2*a^3*b^3*c*p^3*q+3*I*(-q*p^2)^(2/3)*3^(1/2)*_
alpha^2*a^2*b^2*d*p^3*q+4*I*(-q*p^2)^(2/3)*3^(1/2)*_alpha*a^3*b^3*c*p^2*q-4
I(-q*p^2)^(1/3)*3^(1/2)*_alpha^2*a^3*b*d*p^3*q^2-2*I*(-q*p^2)^(1/3)*3^(1/
2)*_alpha*a^4*b^2*c*p^2*q^2+2*I*(-q*p^2)^(2/3)*3^(1/2)*_alpha*a^3*b*d*p^2*q
^2+4*I*(-q*p^2)^(1/3)*3^(1/2)*_alpha*a*b^3*d*p^4*q+2*I*(-q*p^2)^(1/3)*3^(1/
2)*_alpha^3*a^2*b^4*c*p^4-2*I*3^(1/2)*_alpha^3*a^3*b^3*c*p^4*q-I*(-q*p^2)^(
2/3)*3^(1/2)*_alpha^2*a^2*b^4*c*p^3-2*I*(-q*p^2)^(1/3)*3^(1/2)*_alpha^2*a^6
*c*p*q^3-3*I*3^(1/2)*_alpha^2*a^4*b^2*c*p^3*q^2+4*I*(-q*p^2)^(1/3)*3^(1/2)*

```
_alpha*a*b^5*c*p^4-2*I*3^(1/2)*_alpha^2*a*b^3*d*p^5*q-3*I*3^(1/2)*_alpha*a^
2*b^4*c*p^4*q-2*I*(-q*p^2)^(1/3)*3^(1/2)*_alpha*a^4*d*p^2*q^3-3*I*3^(1/2)*_
alpha*a^2*b^2*d*p^4*q^2+3*q^3*(-q*p^2)^(2/3)*_alpha*a^6*c-3*p^4*(-q*p^2)^(2
/3)*_alpha*b^4*d-3*_alpha^3*a^6*q^3*p^2*c-3*p^3*_alpha^2*a^4*d*q^3+9*(-q*p^
2)^(2/3)*_alpha^3*a^4*b^2*p^2*q*c+6*q^2*(-q*p^2)^(2/3)*_alpha^2*a^5*b*p*c+9
*p^3*(-q*p^2)^(2/3)*_alpha^2*a^2*b^2*d*q+12*(-q*p^2)^(2/3)*_alpha*a^3*b^3*p
^2*q*c+6*q^2*(-q*p^2)^(2/3)*_alpha*a^3*b*d*p^2+I*3^(1/2)*_alpha^3*a^6*c*p^2
*q^3+I*(-q*p^2)^(2/3)*3^(1/2)*_alpha*a^6*c*q^3+I*3^(1/2)*_alpha^2*a^4*d*p^3
*q^3+2*I*(-q*p^2)^(1/3)*3^(1/2)*_alpha^2*b^4*d*p^5-I*(-q*p^2)^(2/3)*3^(1/2)
*_alpha*b^4*d*p^4+I*(-q*p^2)^(2/3)*3^(1/2)*a^4*d*p*q^3-2*I*(-q*p^2)^(2/3)*3
^(1/2)*a*b^5*c*p^3-4*I*3^(1/2)*a^3*b^3*c*p^3*q^2-2*I*3^(1/2)*a^3*b*d*p^3*q^
3)/c/(a^6*q^4+2*a^3*b^3*p^2*q^2+b^6*p^4),(I*3^(1/2)/p*(-q*p^2)^(1/3)/(-3/2/
p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^4
*a^2*c+_Z^3*d*p+2*_Z^2*a*b*c+b^2*c+d*q))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{apx^4 - 3\,bpx^2 + 4\,aqx}{\left(a^2cx^4 + 2\,abcx^2 + dpx^3 + b^2c + dq\right)\sqrt{px^3 + q}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*p*x^4-3*b*p*x^2+4*a*q*x)/(p*x^3+q)^(1/2)/(a^2*c*x^4+2*a*b*c*x^
2+d*p*x^3+b^2*c+d*q),x, algorithm="maxima")
```

```
[Out] integrate((a*p*x^4 - 3*b*p*x^2 + 4*a*q*x)/((a^2*c*x^4 + 2*a*b*c*x^2 + d*p*x
^3 + b^2*c + d*q)*sqrt(p*x^3 + q)), x)
```

mupad [B] time = 80.21, size = 1058, normalized size = 24.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*p*x^4 - 3*b*p*x^2 + 4*a*q*x)/((q + p*x^3)^(1/2)*(d*q + b^2*c + d*p*x
^3 + a^2*c*x^4 + 2*a*b*c*x^2)),x)
```

```
[Out] (log(((a^2*c*x^2*1i + a*b*c*1i - d*p*x*1i - 2*a*c^(1/2)*d^(1/2)*(q + p*x^3)
^(1/2))*(a^4*b^2*c^2*q*1i + a^6*c^2*q*x^4*1i - b^2*d^2*p^3*x*1i + a^4*c*d*q
^2*4i + a*b^3*c*d*p^2*1i + a^3*b^3*c^2*p*x*1i + a^5*b*c^2*p*x^5*1i + a*b*d^
2*p^3*x^3*1i + a^5*b*c^2*q*x^2*2i + a^4*b^2*c^2*p*x^3*2i + a^2*d^2*p^2*q*x^
2*1i - a^2*b^2*c*d*p^2*x^2*1i + a^4*c*d*p*q*x^3*2i + 2*a*b^2*c^(1/2)*d^(3/2
)*p^2*(q + p*x^3)^(1/2) + a^3*b*c*d*p^2*x^4*2i + a^3*b*c*d*p*q*x*2i)*(a^3*b
^3*c^2*(q + p*x^3)*3i - a^3*b^3*c^2*q*2i + a^6*c^2*q*x^6*1i - b^2*d^2*p^2*(
q + p*x^3)*1i + b^4*c*d*p^2*1i + a^3*b*c*d*q^2*1i + a^3*b*c*d*(q + p*x^3)^2
*2i - 2*a^3*c^(1/2)*d^(3/2)*q^2*(q + p*x^3)^(1/2) - 2*b^3*c^(1/2)*d^(3/2)*p
^2*(q + p*x^3)^(1/2) + a^2*b^4*c^2*p*x*1i + a^5*b*c^2*q*x^4*2i + a^4*c*d*q^
2*x^2*1i + a^5*b*c^2*x^4*(q + p*x^3)*1i + a^4*b^2*c^2*x^2*(q + p*x^3)*3i +
a*b*d^2*p^2*x^2*(q + p*x^3)*1i + a^4*c*d*q*x^2*(q + p*x^3)*2i + a^2*d^2*p*q
*x*(q + p*x^3)*1i + a*b^3*c*d*p^2*x^2*1i + a^2*b^2*c*d*p*x*(q + p*x^3)*2i))
/((d*q + b^2*c + d*p*x^3 + a^2*c*x^4 + 2*a*b*c*x^2)*(a^2*b^2*c^2 + a^4*c^2*
x^4 + d^2*p^2*x^2 + 2*a^3*b*c^2*x^2 + 4*a^2*c*d*q + 2*a^2*c*d*p*x^3 - 2*a*b
*c*d*p*x)*(b^4*d^2*p^4 + a^6*b^2*c^2*q^2 + a^8*c^2*q^2*x^4 + 4*a^6*c*d*q^3
+ a^4*b^4*c^2*p^2*x^2 + 2*a^5*b^3*c^2*p^2*x^4 + a^6*b^2*c^2*p^2*x^6 + a^2*b
^2*d^2*p^4*x^4 + a^4*d^2*p^2*q^2*x^2 - 2*a*b^3*d^2*p^4*x^2 + 2*a^7*b*c^2*q^
2*x^2 + 2*a^4*b^2*c*d*p^3*x^5 + 4*a^6*b^2*c^2*p*q*x^3 - 2*a^2*b^2*d^2*p^3*q
*x + 2*a^3*b*d^2*p^3*q*x^3 + 2*a^3*b^3*c*d*p^2*q + 2*a^2*b^4*c*d*p^3*x + 2*
a^5*b^3*c^2*p*q*x + 2*a^7*b*c^2*p*q*x^5 + 2*a^6*c*d*p*q^2*x^3 + 6*a^5*b*c*d
*p*q^2*x + 4*a^5*b*c*d*p^2*q*x^4 + 2*a^4*b^2*c*d*p^2*q*x^2)))*1i)/(c^(1/2)*
d^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*p*x**4-3*b*p*x**2+4*a*q*x)/(p*x**3+q)**(1/2)/(a**2*c*x**4+2*a*
b*c*x**2+d*p*x**3+b**2*c+d*q),x)
```

```
[Out] Timed out
```

3.573 $\int \frac{1+x^{12}}{x^{10}\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=44

$$\frac{\sqrt{x^6-1}\,(2x^6+1)}{9x^9}+\frac{2}{3}\tanh^{-1}\left(\frac{x^3+1}{\sqrt{x^6-1}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1489, 271, 264, 275, 217, 206}

$$\frac{\sqrt{x^6-1}}{9x^9}+\frac{2\sqrt{x^6-1}}{9x^3}+\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^12)/(x^10*Sqrt[-1 + x^6]),x]
```

```
[Out] Sqrt[-1 + x^6]/(9*x^9) + (2*Sqrt[-1 + x^6])/(9*x^3) + ArcTanh[x^3/Sqrt[-1 +
 x^6]]/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1489

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IG
tQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^{12}}{x^{10}\sqrt{-1+x^6}}\,dx &= \int \left(\frac{1}{x^{10}\sqrt{-1+x^6}}+\frac{x^2}{\sqrt{-1+x^6}}\right)dx\\
&= \int \frac{1}{x^{10}\sqrt{-1+x^6}}\,dx + \int \frac{x^2}{\sqrt{-1+x^6}}\,dx\\
&= \frac{\sqrt{-1+x^6}}{9x^9} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, x^3\right) + \frac{2}{3}\int \frac{1}{x^4\sqrt{-1+x^6}}\,dx\\
&= \frac{\sqrt{-1+x^6}}{9x^9} + \frac{2\sqrt{-1+x^6}}{9x^3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^3}{\sqrt{-1+x^6}}\right)\\
&= \frac{\sqrt{-1+x^6}}{9x^9} + \frac{2\sqrt{-1+x^6}}{9x^3} + \frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.93

$$\frac{1}{9}\left(\frac{\sqrt{x^6-1}\,(2x^6+1)}{x^9}+3\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^12)/(x^10*Sqrt[-1 + x^6]),x]
```

```
[Out] ((Sqrt[-1 + x^6]*(1 + 2*x^6))/x^9 + 3*ArcTanh[x^3/Sqrt[-1 + x^6]])/9
```

IntegrateAlgebraic [A] time = 0.30, size = 44, normalized size = 1.00

$$\frac{\sqrt{x^6-1}\,(2x^6+1)}{9x^9}+\frac{2}{3}\tanh^{-1}\left(\frac{x^3+1}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^12)/(x^10*Sqrt[-1 + x^6]),x]
```

```
[Out] (Sqrt[-1 + x^6]*(1 + 2*x^6))/(9*x^9) + (2*ArcTanh[(1 + x^3)/Sqrt[-1 + x^6]]
)/3
```

fricas [A] time = 0.39, size = 46, normalized size = 1.05

$$-\frac{3\,x^9\log\left(-x^3+\sqrt{x^6-1}\right)-2\,x^9-\left(2\,x^6+1\right)\sqrt{x^6-1}}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^12+1)/x^10/(x^6-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(3*x^9*log(-x^3 + sqrt(x^6 - 1)) - 2*x^9 - (2*x^6 + 1)*sqrt(x^6 - 1))/
x^9
```

giac [A] time = 0.42, size = 59, normalized size = 1.34

$$-\frac{2\left(-\frac{1}{x^6}+1\right)^{\frac{3}{2}}-6\sqrt{-\frac{1}{x^6}+1}-3\,\log\left(\sqrt{-\frac{1}{x^6}+1}+1\right)+3\,\log\left(-\sqrt{-\frac{1}{x^6}+1}+1\right)}{18\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^12+1)/x^10/(x^6-1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/18*(2*(-1/x^6 + 1)^(3/2) - 6*sqrt(-1/x^6 + 1) - 3*log(sqrt(-1/x^6 + 1) +
 1) + 3*log(-sqrt(-1/x^6 + 1) + 1))/sgn(x)
```

maple [C] time = 0.23, size = 50, normalized size = 1.14

$$\frac{2x^{12}-x^{6}-1}{9x^{9}\sqrt{x^{6}-1}}+\frac{\sqrt{-\mathrm{signum}\left(x^{6}-1\right)}\ \arcsin\left(x^{3}\right)}{3\sqrt{\mathrm{signum}\left(x^{6}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^12+1)/x^10/(x^6-1)^(1/2),x)
```

```
[Out] 1/9*(2*x^12-x^6-1)/x^9/(x^6-1)^(1/2)+1/3/signum(x^6-1)^(1/2)*(-signum(x^6-1
))^(1/2)*arcsin(x^3)
```

maxima [A] time = 0.45, size = 57, normalized size = 1.30

$$\frac{\sqrt{x^{6}-1}}{3\,x^{3}}-\frac{\left(x^{6}-1\right)^{\frac{3}{2}}}{9\,x^{9}}+\frac{1}{6}\,\log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}+1\right)-\frac{1}{6}\,\log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^12+1)/x^10/(x^6-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^6 - 1)/x^3 - 1/9*(x^6 - 1)^(3/2)/x^9 + 1/6*log(sqrt(x^6 - 1)/x^3
 + 1) - 1/6*log(sqrt(x^6 - 1)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{12}+1}{x^{10}\,\sqrt{x^{6}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^12 + 1)/(x^10*(x^6 - 1)^(1/2)),x)
```

```
[Out] int((x^12 + 1)/(x^10*(x^6 - 1)^(1/2)), x)
```

sympy [A] time = 2.62, size = 37, normalized size = 0.84

$$\frac{\begin{cases}\frac{\sqrt{x^{6}-1}}{x^{3}}-\frac{\left(x^{6}-1\right)^{\frac{3}{2}}}{3x^{9}} & \text{for}\: x > -1 \wedge x < 1\end{cases}}{3}+\frac{\operatorname{acosh}\left(x^{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**12+1)/x**10/(x**6-1)**(1/2),x)
```

```
[Out] Piecewise((sqrt(x**6 - 1)/x**3 - (x**6 - 1)**(3/2)/(3*x**9), (x > -1) & (x
< 1)))/3 + acosh(x**3)/3
```

3.574 $\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt[10]{1+5x^4+10x^8+10x^{12}+5x^{16}+x^{20}}}\,dx$

Optimal. Leaf size=44

$$-\frac{\left(\left(x^4+1\right)^5\right)^{9/10}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{\sqrt{2}\left(x^4+1\right)^{9/2}}$$

Rubi [A] time = 0.40, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6688, 6720, 1699, 203}

$$-\frac{\sqrt{x^4+1}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{\sqrt{2}\sqrt[10]{\left(x^4+1\right)^5}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/((1 + x^2)*(1 + 5*x^4 + 10*x^8 + 10*x^12 + 5*x^16 + x^20)^(1
/10)),x]
```

```
[Out] -((Sqrt[1 + x^4]*ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]])/(Sqrt[2]*((1 + x^4)^5)^
(1/10)))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt[10]{1+5x^4+10x^8+10x^{12}+5x^{16}+x^{20}}}\,dx = \int \frac{-1+x^2}{\left(1+x^2\right)\sqrt[10]{\left(1+x^4\right)^5}}\,dx$$
$$= \frac{\sqrt{1+x^4}\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt{1+x^4}}\,dx}{\sqrt[10]{\left(1+x^4\right)^5}}$$
$$= -\frac{\sqrt{1+x^4}\,\mathrm{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)}{\sqrt[10]{\left(1+x^4\right)^5}}$$
$$= -\frac{\sqrt{1+x^4}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}\sqrt[10]{\left(1+x^4\right)^5}}$$

Mathematica [C] time = 0.11, size = 61, normalized size = 1.39

$$-\frac{\sqrt[4]{-1}\sqrt{x^4+1}\left(F\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)-2\Pi\left(-i;i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)\right)}{\sqrt[10]{\left(x^4+1\right)^5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*(1 + 5*x^4 + 10*x^8 + 10*x^12 + 5*x^16 + x^
20)^(1/10)),x]
```

```
[Out] -(((-1)^(1/4)*Sqrt[1 + x^4]*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - 2*Ell
ipticPi[-I, I*ArcSinh[(-1)^(1/4)*x], -1]))/((1 + x^4)^5)^(1/10))
```

IntegrateAlgebraic [A] time = 20.39, size = 44, normalized size = 1.00

$$-\frac{\left(\left(x^4+1\right)^5\right)^{9/10}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}\left(x^4+1\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*(1 + 5*x^4 + 10*x^8 + 10*x^12 + 5*
x^16 + x^20)^(1/10)),x]
```

```
[Out] -((((1 + x^4)^5)^(9/10)*ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]])/(Sqrt[2]*(1 + x^
4)^(9/2)))
```

fricas [A] time = 0.43, size = 45, normalized size = 1.02

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^{20}+5x^{16}+10x^{12}+10x^8+5x^4+1\right)^{\frac{1}{10}}x}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^20+5*x^16+10*x^12+10*x^8+5*x^4+1)^(1/10),x, al
gorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*(x^20 + 5*x^16 + 10*x^12 + 10*x^8 + 5*x^4 + 1)^
(1/10)*x/(x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^{20}+5\,x^{16}+10\,x^{12}+10\,x^8+5\,x^4+1\right)^{\frac{1}{10}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^20+5*x^16+10*x^12+10*x^8+5*x^4+1)^(1/10),x, al
gorithm="giac")
```

```
[Out] integrate((x^2 - 1)/((x^20 + 5*x^16 + 10*x^12 + 10*x^8 + 5*x^4 + 1)^(1/10)*
(x^2 + 1)), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^2+1\right)\left(x^{20}+5x^{16}+10x^{12}+10x^8+5x^4+1\right)^{\frac{1}{10}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^20+5*x^16+10*x^12+10*x^8+5*x^4+1)^(1/10),x)
```

```
[Out] int((x^2-1)/(x^2+1)/(x^20+5*x^16+10*x^12+10*x^8+5*x^4+1)^(1/10),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^{20}+5\,x^{16}+10\,x^{12}+10\,x^8+5\,x^4+1\right)^{\frac{1}{10}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^20+5*x^16+10*x^12+10*x^8+5*x^4+1)^(1/10),x, al
gorithm="maxima")
```

```
[Out] integrate((x^2 - 1)/((x^20 + 5*x^16 + 10*x^12 + 10*x^8 + 5*x^4 + 1)^(1/10)*
(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2-1}{\left(x^2+1\right)\,{\left(x^{20}+5\,x^{16}+10\,x^{12}+10\,x^8+5\,x^4+1\right)}^{1/10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + 1)*(5*x^4 + 10*x^8 + 10*x^12 + 5*x^16 + x^20 + 1)^(1/
10)),x)
```

```
[Out] int((x^2 - 1)/((x^2 + 1)*(5*x^4 + 10*x^8 + 10*x^12 + 5*x^16 + x^20 + 1)^(1/
10)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)}{\left(x^2+1\right)\,\sqrt[10]{\left(x^4+1\right)^5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**20+5*x**16+10*x**12+10*x**8+5*x**4+1)**(1/1
0),x)
```

```
[Out] Integral((x - 1)*(x + 1)/((x**2 + 1)*((x**4 + 1)**5)**(1/10)), x)
```

3.575 $\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b^2+ax^2}\,dx$

Optimal. Leaf size=44

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Rubi [F] time = 0.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b^2+ax^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2),x]
```

```
[Out] Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b - Sqrt[-a]*x), x]/(2*b) + Defer[I
nt][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b + Sqrt[-a]*x), x]/(2*b)
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b^2+ax^2}\,dx &= \int \left(\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{2b\left(b-\sqrt{-a}\,x\right)}+\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{2b\left(b+\sqrt{-a}\,x\right)}\right)dx\\ &= \frac{\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b-\sqrt{-a}\,x}\,dx}{2b}+\frac{\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b+\sqrt{-a}\,x}\,dx}{2b}\end{aligned}$$

Mathematica [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b^2+ax^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2),x]
```

```
[Out] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2), x]
```

IntegrateAlgebraic [A] time = 0.13, size = 44, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*x)/(Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/(Sqrt[a]*Sqrt
[b])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{ax^2+b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2),x, algorithm="giac")
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 + b^2), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{a\,x^2+b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2),x)
[Out] int((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{ax^2+b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2),x, algorithm="maxima")
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 + b^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b+\sqrt{b^2+a\,x^2}}}{b^2+a\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(a*x^2 + b^2),x)
[Out] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(a*x^2 + b^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{ax^2+b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x**2+b**2)**(1/2))**(1/2)/(a*x**2+b**2),x)
[Out] Integral(sqrt(b + sqrt(a*x**2 + b**2))/(a*x**2 + b**2), x)
```

3.576 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}} \, dx$

Optimal. Leaf size=44

$$\frac{\log \left(\sqrt{x^4+1}+x^2+\sqrt{2} \sqrt{\sqrt{x^4+1}+x^2}\, x\right)}{\sqrt{2}}$$

Rubi [A] time = 0.06, antiderivative size = 31, normalized size of antiderivative = 0.70, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2132, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\, x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}} \, dx &= \text{Subst}\left(\int \frac{1}{1-2 x^2} \, dx, x, \frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\, x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\, x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

IntegrateAlgebraic [A] time = 0.20, size = 44, normalized size = 1.00

$$\frac{\log\left(\sqrt{x^4+1}+x^2+\sqrt{2}\sqrt{\sqrt{x^4+1}+x^2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
[Out] Log[x^2 + Sqrt[1 + x^4] + Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

fricas [A] time = 0.60, size = 60, normalized size = 1.36

$$\frac{1}{4}\sqrt{2}\log\left(4\,x^4+4\,\sqrt{x^4+1}\,x^2+2\left(\sqrt{2}\,x^3+\sqrt{2}\,\sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt
(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2),x)
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2), x)
```

sympy [A] time = 0.99, size = 15, normalized size = 0.34

$$\frac{G_{3,3}^{2,2}\left(\begin{matrix} 1,1 & \frac{1}{2} \\ \frac{1}{4},\frac{3}{4} & 0 \end{matrix} \middle| x^4\right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
[Out] meijerg(((1, 1), (1/2,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

3.577 $\int \frac{1}{x^3(1+x^2)^{3/4}}\,dx$

Optimal. Leaf size=45

$$-\frac{\sqrt[4]{x^2+1}}{2x^2}+\frac{3}{4}\tan^{-1}\left(\sqrt[4]{x^2+1}\right)+\frac{3}{4}\tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 51, 63, 212, 206, 203}

$$-\frac{\sqrt[4]{x^2+1}}{2x^2}+\frac{3}{4}\tan^{-1}\left(\sqrt[4]{x^2+1}\right)+\frac{3}{4}\tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x^2)^(3/4)),x]
```

```
[Out] -1/2*(1 + x^2)^(1/4)/x^2 + (3*ArcTan[(1 + x^2)^(1/4)])/4 + (3*ArcTanh[(1 +
x^2)^(1/4)])/4
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(1+x^2\right)^{3/4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(1+x)^{3/4}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt[4]{1+x^2}}{2x^2}-\frac{3}{8}\text{Subst}\left(\int \frac{1}{x(1+x)^{3/4}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt[4]{1+x^2}}{2x^2}-\frac{3}{2}\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\sqrt[4]{1+x^2}\right)\\
&= -\frac{\sqrt[4]{1+x^2}}{2x^2}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[4]{1+x^2}\right)+\frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt[4]{1+x^2}\right)\\
&= -\frac{\sqrt[4]{1+x^2}}{2x^2}+\frac{3}{4}\tan^{-1}\left(\sqrt[4]{1+x^2}\right)+\frac{3}{4}\tanh^{-1}\left(\sqrt[4]{1+x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.00, size = 24, normalized size = 0.53

$$2\sqrt[4]{x^2+1}\,{}_2F_1\left(\frac{1}{4},2;\frac{5}{4};x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x^2)^(3/4)),x]
```

```
[Out] 2*(1 + x^2)^(1/4)*Hypergeometric2F1[1/4, 2, 5/4, 1 + x^2]
```

IntegrateAlgebraic [A] time = 0.06, size = 45, normalized size = 1.00

$$-\frac{\sqrt[4]{x^2+1}}{2x^2}+\frac{3}{4}\tan^{-1}\left(\sqrt[4]{x^2+1}\right)+\frac{3}{4}\tanh^{-1}\left(\sqrt[4]{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(1 + x^2)^(3/4)),x]
```

```
[Out] -1/2*(1 + x^2)^(1/4)/x^2 + (3*ArcTan[(1 + x^2)^(1/4)])/4 + (3*ArcTanh[(1 +
x^2)^(1/4)])/4
```

fricas [A] time = 0.40, size = 58, normalized size = 1.29

$$\frac{6\,x^2\arctan\left(\left(x^2+1\right)^{\frac{1}{4}}\right)+3\,x^2\log\left(\left(x^2+1\right)^{\frac{1}{4}}+1\right)-3\,x^2\log\left(\left(x^2+1\right)^{\frac{1}{4}}-1\right)-4\left(x^2+1\right)^{\frac{1}{4}}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+1)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/8*(6*x^2*arctan((x^2 + 1)^(1/4)) + 3*x^2*log((x^2 + 1)^(1/4) + 1) - 3*x^2
*log((x^2 + 1)^(1/4) - 1) - 4*(x^2 + 1)^(1/4))/x^2
```

giac [A] time = 0.34, size = 47, normalized size = 1.04

$$-\frac{\left(x^2+1\right)^{\frac{1}{4}}}{2\,x^2}+\frac{3}{4}\arctan\left(\left(x^2+1\right)^{\frac{1}{4}}\right)+\frac{3}{8}\log\left(\left(x^2+1\right)^{\frac{1}{4}}+1\right)-\frac{3}{8}\log\left(\left(x^2+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+1)^(3/4),x, algorithm="giac")
```

```
[Out] -1/2*(x^2 + 1)^(1/4)/x^2 + 3/4*arctan((x^2 + 1)^(1/4)) + 3/8*log((x^2 + 1)^
(1/4) + 1) - 3/8*log((x^2 + 1)^(1/4) - 1)
```

maple [C] time = 0.21, size = 56, normalized size = 1.24

$$-\frac{\left(x^2+1\right)^{\frac{1}{4}}}{2x^2}-\frac{3\left(-\frac{3\Gamma\left(\frac{3}{4}\right)x^2\,\mathrm{hypergeom}\left(\left[1,1,\frac{7}{4}\right],[2,2],-x^2\right)}{4}+\left(-3\ln(2)+\frac{\pi}{2}+2\ln(x)\right)\Gamma\left(\frac{3}{4}\right)\right)}{8\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^2+1)^(3/4),x)
```

```
[Out] -1/2*(x^2+1)^(1/4)/x^2-3/8/GAMMA(3/4)*(-3/4*GAMMA(3/4)*x^2*hypergeom([1,1,7
/4],[2,2],-x^2)+(-3*ln(2)+1/2*Pi+2*ln(x))*GAMMA(3/4))
```

maxima [A] time = 0.45, size = 47, normalized size = 1.04

$$-\frac{\left(x^2+1\right)^{\frac{1}{4}}}{2\,x^2}+\frac{3}{4}\,\arctan\left(\left(x^2+1\right)^{\frac{1}{4}}\right)+\frac{3}{8}\,\log\left(\left(x^2+1\right)^{\frac{1}{4}}+1\right)-\frac{3}{8}\,\log\left(\left(x^2+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+1)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 + 1)^(1/4)/x^2 + 3/4*arctan((x^2 + 1)^(1/4)) + 3/8*log((x^2 + 1)^
(1/4) + 1) - 3/8*log((x^2 + 1)^(1/4) - 1)
```

mupad [B] time = 0.56, size = 33, normalized size = 0.73

$$\frac{3\,\mathrm{atan}\left(\left(x^2+1\right)^{1/4}\right)}{4}+\frac{3\,\mathrm{atanh}\left(\left(x^2+1\right)^{1/4}\right)}{4}-\frac{\left(x^2+1\right)^{1/4}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^2 + 1)^(3/4)),x)
```

```
[Out] (3*atan((x^2 + 1)^(1/4)))/4 + (3*atanh((x^2 + 1)^(1/4)))/4 - (x^2 + 1)^(1/4
)/(2*x^2)
```

sympy [C] time = 0.92, size = 32, normalized size = 0.71

$$-\frac{\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^2}\right)}{2x^{\frac{7}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**2+1)**(3/4),x)
```

```
[Out] -gamma(7/4)*hyper((3/4, 7/4), (11/4,), exp_polar(I*pi)/x**2)/(2*x**(7/2)*ga
mma(11/4))
```

3.578 $\int \frac{1}{x^4 \sqrt[4]{1+x^3}} \, dx$

Optimal. Leaf size=45

$$-\frac{(x^3+1)^{3/4}}{3x^3} - \frac{1}{6}\tan^{-1}\left(\sqrt[4]{x^3+1}\right) + \frac{1}{6}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 51, 63, 298, 203, 206}

$$-\frac{(x^3+1)^{3/4}}{3x^3} - \frac{1}{6}\tan^{-1}\left(\sqrt[4]{x^3+1}\right) + \frac{1}{6}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + x^3)^(1/4)),x]
```

```
[Out] -1/3*(1 + x^3)^(3/4)/x^3 - ArcTan[(1 + x^3)^(1/4)]/6 + ArcTanh[(1 + x^3)^(1
/4)]/6
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\sqrt[4]{1+x^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt[4]{1+x}}\,dx,x,x^3\right)\\
&= -\frac{\left(1+x^3\right)^{3/4}}{3x^3}-\frac{1}{12}\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1+x}}\,dx,x,x^3\right)\\
&= -\frac{\left(1+x^3\right)^{3/4}}{3x^3}-\frac{1}{3}\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\sqrt[4]{1+x^3}\right)\\
&= -\frac{\left(1+x^3\right)^{3/4}}{3x^3}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[4]{1+x^3}\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt[4]{1+x^3}\right)\\
&= -\frac{\left(1+x^3\right)^{3/4}}{3x^3}-\frac{1}{6}\tan^{-1}\left(\sqrt[4]{1+x^3}\right)+\frac{1}{6}\tanh^{-1}\left(\sqrt[4]{1+x^3}\right)
\end{aligned}$$

Mathematica [C] time = 0.00, size = 26, normalized size = 0.58

$$\frac{4}{9}\left(x^3+1\right)^{3/4}{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + x^3)^(1/4)),x]
[Out] (4*(1 + x^3)^(3/4)*Hypergeometric2F1[3/4, 2, 7/4, 1 + x^3])/9
```

IntegrateAlgebraic [A] time = 0.05, size = 45, normalized size = 1.00

$$-\frac{\left(x^3+1\right)^{3/4}}{3x^3}-\frac{1}{6}\tan^{-1}\left(\sqrt[4]{x^3+1}\right)+\frac{1}{6}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(1 + x^3)^(1/4)),x]
[Out] -1/3*(1 + x^3)^(3/4)/x^3 - ArcTan[(1 + x^3)^(1/4)]/6 + ArcTanh[(1 + x^3)^(1
/4)]/6
```

fricas [A] time = 0.42, size = 57, normalized size = 1.27

$$-\frac{2\,x^3\arctan\left(\left(x^3+1\right)^{\frac{1}{4}}\right)-x^3\log\left(\left(x^3+1\right)^{\frac{1}{4}}+1\right)+x^3\log\left(\left(x^3+1\right)^{\frac{1}{4}}-1\right)+4\left(x^3+1\right)^{\frac{3}{4}}}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^3+1)^(1/4),x, algorithm="fricas")
[Out] -1/12*(2*x^3*arctan((x^3 + 1)^(1/4)) - x^3*log((x^3 + 1)^(1/4) + 1) + x^3*l
og((x^3 + 1)^(1/4) - 1) + 4*(x^3 + 1)^(3/4))/x^3
```

giac [A] time = 0.17, size = 48, normalized size = 1.07

$$-\frac{\left(x^3+1\right)^{\frac{3}{4}}}{3\,x^3}-\frac{1}{6}\arctan\left(\left(x^3+1\right)^{\frac{1}{4}}\right)+\frac{1}{12}\log\left(\left(x^3+1\right)^{\frac{1}{4}}+1\right)-\frac{1}{12}\log\left(\left|\left(x^3+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^3+1)^(1/4),x, algorithm="giac")
```

```
[Out] -1/3*(x^3 + 1)^(3/4)/x^3 - 1/6*arctan((x^3 + 1)^(1/4)) + 1/12*log((x^3 + 1)
^(1/4) + 1) - 1/12*log(abs((x^3 + 1)^(1/4) - 1))
```

maple [C] time = 0.23, size = 72, normalized size = 1.60

$$-\frac{\left(x^{3}+1\right)^{\frac{3}{4}}}{3x^{3}}-\frac{\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)\left(-\frac{\pi\sqrt{2}\,x^{3}\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{4}\right],[2,2],-x^{3}\right)}{4\Gamma\left(\frac{3}{4}\right)}+\frac{\left(-3\ln(2)-\frac{\pi}{2}+3\ln(x)\right)\pi\sqrt{2}}{\Gamma\left(\frac{3}{4}\right)}\right)}{24\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^3+1)^(1/4),x)
```

```
[Out] -1/3*(x^3+1)^(3/4)/x^3-1/24/Pi*2^(1/2)*GAMMA(3/4)*(-1/4*Pi*2^(1/2)/GAMMA(3/
4)*x^3*hypergeom([1,1,5/4],[2,2],-x^3)+(-3*ln(2)-1/2*Pi+3*ln(x))*Pi*2^(1/2)
/GAMMA(3/4))
```

maxima [A] time = 0.44, size = 47, normalized size = 1.04

$$-\frac{\left(x^{3}+1\right)^{\frac{3}{4}}}{3\,x^{3}}-\frac{1}{6}\,\arctan\left(\left(x^{3}+1\right)^{\frac{1}{4}}\right)+\frac{1}{12}\,\log\left(\left(x^{3}+1\right)^{\frac{1}{4}}+1\right)-\frac{1}{12}\,\log\left(\left(x^{3}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^3+1)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/3*(x^3 + 1)^(3/4)/x^3 - 1/6*arctan((x^3 + 1)^(1/4)) + 1/12*log((x^3 + 1)
^(1/4) + 1) - 1/12*log((x^3 + 1)^(1/4) - 1)
```

mupad [B] time = 0.58, size = 33, normalized size = 0.73

$$\frac{\mathrm{atanh}\left(\left(x^{3}+1\right)^{1/4}\right)}{6}-\frac{\mathrm{atan}\left(\left(x^{3}+1\right)^{1/4}\right)}{6}-\frac{\left(x^{3}+1\right)^{3/4}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^3 + 1)^(1/4)),x)
```

```
[Out] atanh((x^3 + 1)^(1/4))/6 - atan((x^3 + 1)^(1/4))/6 - (x^3 + 1)^(3/4)/(3*x^3
)
```

sympy [C] time = 0.90, size = 32, normalized size = 0.71

$$-\frac{\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{e^{i\pi}}{x^{3}}\right)}{3x^{\frac{15}{4}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**3+1)**(1/4),x)
```

```
[Out] -gamma(5/4)*hyper((1/4, 5/4), (9/4,), exp_polar(I*pi)/x**3)/(3*x**(15/4)*ga
mma(9/4))
```

3.579 $\int \frac{\sqrt[4]{1+x^3}}{x^4}\,dx$

Optimal. Leaf size=45

$$-\frac{\sqrt[4]{x^3+1}}{3x^3}-\frac{1}{6}\tan^{-1}\left(\sqrt[4]{x^3+1}\right)-\frac{1}{6}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 47, 63, 212, 206, 203}

$$-\frac{\sqrt[4]{x^3+1}}{3x^3}-\frac{1}{6}\tan^{-1}\left(\sqrt[4]{x^3+1}\right)-\frac{1}{6}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)^(1/4)/x^4,x]
```

```
[Out] -1/3*(1 + x^3)^(1/4)/x^3 - ArcTan[(1 + x^3)^(1/4)]/6 - ArcTanh[(1 + x^3)^(1
/4)]/6
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{1+x^3}}{x^4}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt[4]{1+x}}{x^2}\,dx, x, x^3\right)\\ &= -\frac{\sqrt[4]{1+x^3}}{3x^3}+\frac{1}{12}\operatorname{Subst}\left(\int \frac{1}{x(1+x)^{3/4}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt[4]{1+x^3}}{3x^3}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt[4]{1+x^3}\right)\\ &= -\frac{\sqrt[4]{1+x^3}}{3x^3}-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1+x^3}\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{1+x^3}\right)\\ &= -\frac{\sqrt[4]{1+x^3}}{3x^3}-\frac{1}{6}\tan^{-1}\left(\sqrt[4]{1+x^3}\right)-\frac{1}{6}\tanh^{-1}\left(\sqrt[4]{1+x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.58

$$\frac{4}{15}\left(x^3+1\right)^{5/4}{}_2F_1\left(\frac{5}{4},2;\frac{9}{4};x^3+1\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^3)^(1/4)/x^4,x]

[Out] (4*(1 + x^3)^(5/4)*Hypergeometric2F1[5/4, 2, 9/4, 1 + x^3])/15

IntegrateAlgebraic [A] time = 0.04, size = 45, normalized size = 1.00

$$-\frac{\sqrt[4]{x^3+1}}{3x^3}-\frac{1}{6}\tan^{-1}\left(\sqrt[4]{x^3+1}\right)-\frac{1}{6}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + x^3)^(1/4)/x^4,x]

[Out] -1/3*(1 + x^3)^(1/4)/x^3 - ArcTan[(1 + x^3)^(1/4)]/6 - ArcTanh[(1 + x^3)^(1/4)]/6

fricas [A] time = 0.40, size = 57, normalized size = 1.27

$$-\frac{2\,x^3\arctan\left(\left(x^3+1\right)^{\frac{1}{4}}\right)+x^3\log\left(\left(x^3+1\right)^{\frac{1}{4}}+1\right)-x^3\log\left(\left(x^3+1\right)^{\frac{1}{4}}-1\right)+4\left(x^3+1\right)^{\frac{1}{4}}}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)^(1/4)/x^4,x, algorithm="fricas")

[Out] -1/12*(2*x^3*arctan((x^3 + 1)^(1/4)) + x^3*log((x^3 + 1)^(1/4) + 1) - x^3*log((x^3 + 1)^(1/4) - 1) + 4*(x^3 + 1)^(1/4))/x^3

giac [A] time = 0.22, size = 48, normalized size = 1.07

$$-\frac{\left(x^3+1\right)^{\frac{1}{4}}}{3\,x^3}-\frac{1}{6}\arctan\left(\left(x^3+1\right)^{\frac{1}{4}}\right)-\frac{1}{12}\log\left(\left(x^3+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{12}\log\left(\left|\left(x^3+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/4)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(x^3 + 1)^(1/4)/x^3 - 1/6*arctan((x^3 + 1)^(1/4)) - 1/12*log((x^3 + 1)
^(1/4) + 1) + 1/12*log(abs((x^3 + 1)^(1/4) - 1))
```

maple [C] time = 0.30, size = 56, normalized size = 1.24

$$-\frac{\left(x^{3}+1\right)^{\frac{1}{4}}}{3x^{3}}+\frac{-\frac{3\Gamma\left(\frac{3}{4}\right)x^{3}\,\mathrm{hypergeom}\left(\left[1,1,\frac{7}{4}\right],[2,2],-x^{3}\right)}{4}+\left(-3\ln(2)+\frac{\pi}{2}+3\ln(x)\right)\Gamma\left(\frac{3}{4}\right)}{12\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(1/4)/x^4,x)
```

```
[Out] -1/3*(x^3+1)^(1/4)/x^3+1/12/GAMMA(3/4)*(-3/4*GAMMA(3/4)*x^3*hypergeom([1,1,
7/4],[2,2],-x^3)+(-3*ln(2)+1/2*Pi+3*ln(x))*GAMMA(3/4))
```

maxima [A] time = 0.52, size = 47, normalized size = 1.04

$$-\frac{\left(x^{3}+1\right)^{\frac{1}{4}}}{3\,x^{3}}-\frac{1}{6}\,\arctan\left(\left(x^{3}+1\right)^{\frac{1}{4}}\right)-\frac{1}{12}\,\log\left(\left(x^{3}+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{12}\,\log\left(\left(x^{3}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/4)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(x^3 + 1)^(1/4)/x^3 - 1/6*arctan((x^3 + 1)^(1/4)) - 1/12*log((x^3 + 1)
^(1/4) + 1) + 1/12*log((x^3 + 1)^(1/4) - 1)
```

mupad [B] time = 0.54, size = 33, normalized size = 0.73

$$-\frac{\mathrm{atan}\left(\left(x^{3}+1\right)^{1/4}\right)}{6}-\frac{\mathrm{atanh}\left(\left(x^{3}+1\right)^{1/4}\right)}{6}-\frac{\left(x^{3}+1\right)^{1/4}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)^(1/4)/x^4,x)
```

```
[Out] - atan((x^3 + 1)^(1/4))/6 - atanh((x^3 + 1)^(1/4))/6 - (x^3 + 1)^(1/4)/(3*x
^3)
```

sympy [C] time = 0.90, size = 34, normalized size = 0.76

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{e^{i\pi}}{x^{3}}\right)}{3x^{\frac{9}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(1/4)/x**4,x)
```

```
[Out] -gamma(3/4)*hyper((-1/4, 3/4), (7/4,), exp_polar(I*pi)/x**3)/(3*x**(9/4)*ga
mma(7/4))
```

3.580 $\int x^3\sqrt{-x+x^4}\,dx$

Optimal. Leaf size=45

$$\frac{1}{12}\sqrt{x^4-x}\left(2x^4-x\right)-\frac{1}{12}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)$$

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2021, 2024, 2029, 206}

$$\frac{1}{6}\sqrt{x^4-x}\,x^4-\frac{1}{12}\sqrt{x^4-x}\,x-\frac{1}{12}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[-x + x^4],x]
```

```
[Out] -1/12*(x*Sqrt[-x + x^4]) + (x^4*Sqrt[-x + x^4])/6 - ArcTanh[x^2/Sqrt[-x + x
^4]]/12
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{-x+x^4}\,dx &= \frac{1}{6}x^4\sqrt{-x+x^4} - \frac{1}{4}\int \frac{x^4}{\sqrt{-x+x^4}}\,dx \\
&= -\frac{1}{12}x\sqrt{-x+x^4} + \frac{1}{6}x^4\sqrt{-x+x^4} - \frac{1}{8}\int \frac{x}{\sqrt{-x+x^4}}\,dx \\
&= -\frac{1}{12}x\sqrt{-x+x^4} + \frac{1}{6}x^4\sqrt{-x+x^4} - \frac{1}{12}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^2}{\sqrt{-x+x^4}}\right) \\
&= -\frac{1}{12}x\sqrt{-x+x^4} + \frac{1}{6}x^4\sqrt{-x+x^4} - \frac{1}{12}\tanh^{-1}\left(\frac{x^2}{\sqrt{-x+x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 1.38

$$\frac{\sqrt{x(x^3-1)}\left(\sin^{-1}\left(x^{3/2}\right)+\sqrt{1-x^3}\left(2x^3-1\right)x^{3/2}\right)}{12\sqrt{x}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[-x + x^4],x]
[Out] (Sqrt[x*(-1 + x^3)]*(x^(3/2)*Sqrt[1 - x^3]*(-1 + 2*x^3) + ArcSin[x^(3/2)]))
/(12*Sqrt[x]*Sqrt[1 - x^3])
```

IntegrateAlgebraic [A] time = 0.41, size = 45, normalized size = 1.00

$$\frac{1}{12}\sqrt{x^4-x}\left(2x^4-x\right)-\frac{1}{12}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[-x + x^4],x]
[Out] (Sqrt[-x + x^4]*(-x + 2*x^4))/12 - ArcTanh[x^2/Sqrt[-x + x^4]]/12
```

fricas [A] time = 0.45, size = 43, normalized size = 0.96

$$\frac{1}{12}\left(2\,x^4-x\right)\sqrt{x^4-x}+\frac{1}{24}\,\log\left(2\,x^3-2\,\sqrt{x^4-x}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4-x)^(1/2),x, algorithm="fricas")
[Out] 1/12*(2*x^4 - x)*sqrt(x^4 - x) + 1/24*log(2*x^3 - 2*sqrt(x^4 - x)*x - 1)
```

giac [A] time = 0.20, size = 49, normalized size = 1.09

$$\frac{1}{12}\,\sqrt{x^4-x}\left(2\,x^3-1\right)x-\frac{1}{24}\,\log\left(\sqrt{-\frac{1}{x^3}+1}+1\right)+\frac{1}{24}\,\log\left(\left|\sqrt{-\frac{1}{x^3}+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4-x)^(1/2),x, algorithm="giac")
[Out] 1/12*sqrt(x^4 - x)*(2*x^3 - 1)*x - 1/24*log(sqrt(-1/x^3 + 1) + 1) + 1/24*lo
g(abs(sqrt(-1/x^3 + 1) - 1))
```

maple [C] time = 0.37, size = 315, normalized size = 7.00

$$\frac{x^{4} \sqrt{x^{4}-x}}{6}-\frac{x \sqrt{x^{4}-x}}{12}-\frac{\left(\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{\left(-\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) x}{\left(-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \left(-1+x\right)}}\, \left(-1+x\right)^{2} \sqrt{\frac{x+\frac{1}{2}+\frac{i \sqrt{3}}{2}}{\left(-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \left(-1+x\right)}}\, \sqrt{\frac{x+\frac{1}{2}-\frac{i \sqrt{3}}{2}}{\left(-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \left(-1+x\right)}}\, \left(\operatorname{EllipticF}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) x}{\left(-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \left(-1+x\right)}}, \sqrt{\frac{\left(\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) \left(\frac{1}{2}-\frac{i \sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \left(\frac{3}{2}-\frac{i \sqrt{3}}{2}\right)}}\right)-\operatorname{EllipticPi}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) x}{\left(-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \left(-1+x\right)}}, \frac{-\frac{1}{2}+\frac{i \sqrt{3}}{2}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{\left(\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) \left(\frac{1}{2}-\frac{i \sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \left(\frac{3}{2}-\frac{i \sqrt{3}}{2}\right)}}\right)\right)}{4 \left(-\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{x \left(-1+x\right) \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4-x)^(1/2),x)
```

```
[Out] 1/6*x^4*(x^4-x)^(1/2)-1/12*x*(x^4-x)^(1/2)-1/4*(1/2-1/2*I*3^(1/2))*((-3/2+1
/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3
^(1/2))/(-1/2-1/2*I*3^(1/2))/(-1+x))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2
*I*3^(1/2))/(-1+x))^(1/2)/(-3/2+1/2*I*3^(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/
2))*(x+1/2-1/2*I*3^(1/2)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1
/2*I*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-EllipticPi(((-3/2+1/2*I*3^(1/2))*
x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),(-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
2)),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I
*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{4}-x}\, x^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 - x)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{3}\, \sqrt{x^{4}-x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4 - x)^(1/2),x)
```

```
[Out] int(x^3*(x^4 - x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} \sqrt{x \left(x-1\right) \left(x^{2}+x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**4-x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(x*(x - 1)*(x**2 + x + 1)), x)
```

3.581 $\int \frac{(1+x^4)(-1+3x^4)}{x(1-ax+x^4)\sqrt{x+x^5}}\,dx$

Optimal. Leaf size=45

$$\frac{2\sqrt{x^5+x}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x^5+x}}{x^4+1}\right)$$

Rubi [F] time = 1.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+x^4)(-1+3x^4)}{x(1-ax+x^4)\sqrt{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^4)*(-1 + 3*x^4))/(x*(1 - a*x + x^4)*Sqrt[x + x^5]),x]
```

```
[Out] (2*(1 + x^4))/Sqrt[x + x^5] - (8*x^4*Sqrt[1 + x^4]*Hypergeometric2F1[1/2, 7
/8, 15/8, -x^4])/(7*Sqrt[x + x^5]) + (2*a*Sqrt[x]*Sqrt[1 + x^4]*Defer[Subst
][Defer[Int][Sqrt[1 + x^8]/(-1 + a*x^2 - x^8), x], x, Sqrt[x]])/Sqrt[x + x^
5] + (8*Sqrt[x]*Sqrt[1 + x^4]*Defer[Subst][Defer[Int][(x^6*Sqrt[1 + x^8])/(
1 - a*x^2 + x^8), x], x, Sqrt[x]])/Sqrt[x + x^5]
```

Rubi steps

$$\int \frac{(1+x^4)(-1+3x^4)}{x(1-ax+x^4)\sqrt{x+x^5}}\,dx=\frac{\left(\sqrt{x}\sqrt{1+x^4}\right)\int \frac{\sqrt{1+x^4}(-1+3x^4)}{x^{3/2}(1-ax+x^4)}\,dx}{\sqrt{x+x^5}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^8}(-1+3x^8)}{x^2(1-ax^2+x^8)}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^5}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \left(-\frac{\sqrt{1+x^8}}{x^2}+\frac{(a-4x^6)\sqrt{1+x^8}}{-1+ax^2-x^8}\right)dx,x,\sqrt{x}\right)}{\sqrt{x+x^5}}$$

$$=-\frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^8}}{x^2}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^5}}+\frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \frac{(a-4x^6)}{-1+ax}\right.}{\sqrt{x+x^5}}$$

$$=\frac{2(1+x^4)}{\sqrt{x+x^5}}+\frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \left(\frac{a\sqrt{1+x^8}}{-1+ax^2-x^8}+\frac{4x^6\sqrt{1+x^8}}{1-ax^2+x^8}\right)dx,x,\sqrt{x}\right)}{\sqrt{x+x^5}}-\frac{(8\sqrt{}}{}$$

$$=\frac{2(1+x^4)}{\sqrt{x+x^5}}-\frac{8x^4\sqrt{1+x^4}\,{}_2F_1\left(\frac{1}{2},\frac{7}{8};\frac{15}{8};-x^4\right)}{7\sqrt{x+x^5}}+\frac{\left(8\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \frac{x^6\sqrt{1+x^8}}{1-ax^2+x^8}\right.}{\sqrt{x+x^5}}$$

Mathematica [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(1+x^4)(-1+3x^4)}{x(1-ax+x^4)\sqrt{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^4)*(-1 + 3*x^4))/(x*(1 - a*x + x^4)*Sqrt[x + x^5]),x]
[Out] Integrate[((1 + x^4)*(-1 + 3*x^4))/(x*(1 - a*x + x^4)*Sqrt[x + x^5]), x]
```

IntegrateAlgebraic [A] time = 0.37, size = 45, normalized size = 1.00

$$\frac{2\sqrt{x^5+x}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x^5+x}}{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^4)*(-1 + 3*x^4))/(x*(1 - a*x + x^4)*Sqrt[x + x^5
]),x]
[Out] (2*Sqrt[x + x^5])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[x + x^5])/(1 + x^4)]
```

fricas [A] time = 1.65, size = 146, normalized size = 3.24

$$\left[\frac{\sqrt{a}\,x\log\left(-\frac{x^8+6\,ax^5+a^2x^2+2\,x^4-4\,\sqrt{x^5+x}\left(x^4+ax+1\right)\sqrt{a}+6\,ax+1}{x^8-2\,ax^5+a^2x^2+2\,x^4-2\,ax+1}\right)+4\,\sqrt{x^5+x}}{2\,x},\ \frac{\sqrt{-a}\,x\arctan\left(\frac{2\,\sqrt{x^5+x}\,\sqrt{-a}}{x^4+ax+1}\right)+2\,\sqrt{x^5+x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(3*x^4-1)/x/(x^4-a*x+1)/(x^5+x)^(1/2),x, algorithm="frica
s")
[Out] [1/2*(sqrt(a)*x*log(-(x^8 + 6*a*x^5 + a^2*x^2 + 2*x^4 - 4*sqrt(x^5 + x)*(x^
4 + a*x + 1)*sqrt(a) + 6*a*x + 1)/(x^8 - 2*a*x^5 + a^2*x^2 + 2*x^4 - 2*a*x
+ 1)) + 4*sqrt(x^5 + x))/x, (sqrt(-a)*x*arctan(2*sqrt(x^5 + x)*sqrt(-a)/(x^
4 + a*x + 1)) + 2*sqrt(x^5 + x))/x]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^4-1\right)\left(x^4+1\right)}{\sqrt{x^5+x}\left(x^4-ax+1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(3*x^4-1)/x/(x^4-a*x+1)/(x^5+x)^(1/2),x, algorithm="giac"
)
[Out] integrate((3*x^4 - 1)*(x^4 + 1)/(sqrt(x^5 + x)*(x^4 - a*x + 1)*x), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)\left(3x^4-1\right)}{x\left(x^4-ax+1\right)\sqrt{x^5+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)*(3*x^4-1)/x/(x^4-a*x+1)/(x^5+x)^(1/2),x)
[Out] int((x^4+1)*(3*x^4-1)/x/(x^4-a*x+1)/(x^5+x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^4-1\right)\left(x^4+1\right)}{\sqrt{x^5+x}\left(x^4-ax+1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(3*x^4-1)/x/(x^4-a*x+1)/(x^5+x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((3*x^4 - 1)*(x^4 + 1)/(sqrt(x^5 + x)*(x^4 - a*x + 1)*x), x)
```

mupad [B] time = 0.71, size = 50, normalized size = 1.11

$$\frac{2\sqrt{x^5+x}}{x}+\sqrt{a}\,\ln\left(\frac{a\,x-2\sqrt{a}\,\sqrt{x^5+x}+x^4+1}{x^4-a\,x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)*(3*x^4 - 1))/(x*(x + x^5)^(1/2)*(x^4 - a*x + 1)),x)
```

```
[Out] (2*(x + x^5)^(1/2))/x + a^(1/2)*log((a*x - 2*a^(1/2)*(x + x^5)^(1/2) + x^4
+ 1)/(x^4 - a*x + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)*(3*x**4-1)/x/(x**4-a*x+1)/(x**5+x)**(1/2),x)
```

```
[Out] Timed out
```

3.582 $\int \frac{\sqrt{1+x^2+x^5}\left(-2+3x^5\right)}{\left(1+x^5\right)\left(1-x^2+x^5\right)}\,dx$

Optimal. Leaf size=45

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^5+x^2+1}}\right)-2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^5+x^2+1}}\right)$$

Rubi [F] time = 1.64, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+x^2+x^5}\left(-2+3x^5\right)}{\left(1+x^5\right)\left(1-x^2+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[1 + x^2 + x^5]*(-2 + 3*x^5))/((1 + x^5)*(1 - x^2 + x^5)),x]
```

```
[Out] Defer[Int][Sqrt[1 + x^2 + x^5]/(1 + x), x] + Defer[Int][Sqrt[1 + x^2 + x^5]
/(-1 + x - x^2 + x^3 - x^4), x] + 2*Defer[Int][(x*Sqrt[1 + x^2 + x^5])/(1 -
 x + x^2 - x^3 + x^4), x] - 3*Defer[Int][(x^2*Sqrt[1 + x^2 + x^5])/(1 - x +
 x^2 - x^3 + x^4), x] - Defer[Int][(x^3*Sqrt[1 + x^2 + x^5])/(1 - x + x^2 -
 x^3 + x^4), x] - 2*Defer[Int][Sqrt[1 + x^2 + x^5]/(1 - x^2 + x^5), x] + 5*
Defer[Int][(x^3*Sqrt[1 + x^2 + x^5])/(1 - x^2 + x^5), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x^2+x^5}\left(-2+3x^5\right)}{\left(1+x^5\right)\left(1-x^2+x^5\right)}\,dx &= \int\left(\frac{\sqrt{1+x^2+x^5}}{1+x}+\frac{\left(-1+2x-3x^2-x^3\right)\sqrt{1+x^2+x^5}}{1-x+x^2-x^3+x^4}+\frac{\left(-2+5x^3\right)\sqrt{1+\right.}{1-x^2+x}\right.\\
&= \int \frac{\sqrt{1+x^2+x^5}}{1+x}\,dx+\int \frac{\left(-1+2x-3x^2-x^3\right)\sqrt{1+x^2+x^5}}{1-x+x^2-x^3+x^4}\,dx+\int \frac{(-2+}{}\\
&= \int \frac{\sqrt{1+x^2+x^5}}{1+x}\,dx+\int\left(\frac{\sqrt{1+x^2+x^5}}{-1+x-x^2+x^3-x^4}+\frac{2x\sqrt{1+x^2+x^5}}{1-x+x^2-x^3+x^4}-\frac{}{1}\right.\\
&= 2\int \frac{x\sqrt{1+x^2+x^5}}{1-x+x^2-x^3+x^4}\,dx-2\int \frac{\sqrt{1+x^2+x^5}}{1-x^2+x^5}\,dx-3\int \frac{x^2\sqrt{1+x^2+x}}{1-x+x^2-x^3}
\end{aligned}$$

Mathematica [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+x^2+x^5}\left(-2+3x^5\right)}{\left(1+x^5\right)\left(1-x^2+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[1 + x^2 + x^5]*(-2 + 3*x^5))/((1 + x^5)*(1 - x^2 + x^5)),x]
```

```
[Out] Integrate[(Sqrt[1 + x^2 + x^5]*(-2 + 3*x^5))/((1 + x^5)*(1 - x^2 + x^5)), x
]
```

IntegrateAlgebraic [A] time = 2.66, size = 45, normalized size = 1.00

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^5+x^2+1}}\right)-2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^5+x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x^2 + x^5]*(-2 + 3*x^5))/((1 + x^5)*(1 - x^2 +
 x^5)),x]
```

```
[Out] 2*ArcTanh[x/Sqrt[1 + x^2 + x^5]] - 2*Sqrt[2]*ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x
^2 + x^5]]
```

fricas [B] time = 0.44, size = 115, normalized size = 2.56

$$\frac{1}{2}\sqrt{2}\log\left(\frac{x^{10}+14x^{7}+2x^{5}+17x^{4}-4\sqrt{2}\left(x^{6}+3x^{3}+x\right)\sqrt{x^{5}+x^{2}+1}+14x^{2}+1}{x^{10}-2x^{7}+2x^{5}+x^{4}-2x^{2}+1}\right)+\log\left(\frac{x^{5}+2x^{2}+2\sqrt{x^{5}+x^{2}+1}\,x+1}{x^{5}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x^2+1)^(1/2)*(3*x^5-2)/(x^5+1)/(x^5-x^2+1),x, algorithm="fri
cas")
```

```
[Out] 1/2*sqrt(2)*log((x^10 + 14*x^7 + 2*x^5 + 17*x^4 - 4*sqrt(2)*(x^6 + 3*x^3 +
x)*sqrt(x^5 + x^2 + 1) + 14*x^2 + 1)/(x^10 - 2*x^7 + 2*x^5 + x^4 - 2*x^2 +
1)) + log((x^5 + 2*x^2 + 2*sqrt(x^5 + x^2 + 1)*x + 1)/(x^5 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^{5}-2\right)\sqrt{x^{5}+x^{2}+1}}{\left(x^{5}-x^{2}+1\right)\left(x^{5}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x^2+1)^(1/2)*(3*x^5-2)/(x^5+1)/(x^5-x^2+1),x, algorithm="gia
c")
```

```
[Out] integrate((3*x^5 - 2)*sqrt(x^5 + x^2 + 1)/((x^5 - x^2 + 1)*(x^5 + 1)), x)
```

maple [C] time = 0.84, size = 121, normalized size = 2.69

$$-\ln\left(-\frac{-x^{5}+2\sqrt{x^{5}+x^{2}+1}\,x-2x^{2}-1}{(1+x)\left(x^{4}-x^{3}+x^{2}-x+1\right)}\right)+\mathrm{RootOf}\left(\_Z^{2}-2\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^{2}-2\right)x^{5}-3\,\mathrm{RootOf}\left(\_Z^{2}-2\right)x^{2}+4\sqrt{x^{5}+x^{2}+1}\,x-\mathrm{RootOf}\left(\_Z^{2}-2\right)}{x^{5}-x^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+x^2+1)^(1/2)*(3*x^5-2)/(x^5+1)/(x^5-x^2+1),x)
```

```
[Out] -ln(-(-x^5+2*(x^5+x^2+1)^(1/2)*x-2*x^2-1)/(1+x)/(x^4-x^3+x^2-x+1))+RootOf(_
Z^2-2)*ln(-(-RootOf(_Z^2-2)*x^5-3*RootOf(_Z^2-2)*x^2+4*(x^5+x^2+1)^(1/2)*x-
RootOf(_Z^2-2))/(x^5-x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^{5}-2\right)\sqrt{x^{5}+x^{2}+1}}{\left(x^{5}-x^{2}+1\right)\left(x^{5}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x^2+1)^(1/2)*(3*x^5-2)/(x^5+1)/(x^5-x^2+1),x, algorithm="max
ima")
```

```
[Out] integrate((3*x^5 - 2)*sqrt(x^5 + x^2 + 1)/((x^5 - x^2 + 1)*(x^5 + 1)), x)
```

mupad [B] time = 3.55, size = 77, normalized size = 1.71

$$\ln\left(\frac{2x\sqrt{x^{5}+x^{2}+1}+2x^{2}+x^{5}+1}{x^{5}+1}\right)+\sqrt{2}\,\ln\left(\frac{3x^{2}+x^{5}-2\sqrt{2}\,x\sqrt{x^{5}+x^{2}+1}+1}{x^{5}-x^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^5 - 2)*(x^2 + x^5 + 1)^(1/2))/((x^5 + 1)*(x^5 - x^2 + 1)),x)
```

```
[Out] log((2*x*(x^2 + x^5 + 1)^(1/2) + 2*x^2 + x^5 + 1)/(x^5 + 1)) + 2^(1/2)*log(
(3*x^2 + x^5 - 2*2^(1/2)*x*(x^2 + x^5 + 1)^(1/2) + 1)/(x^5 - x^2 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+x**2+1)**(1/2)*(3*x**5-2)/(x**5+1)/(x**5-x**2+1),x)
```

```
[Out] Timed out
```

3.583 $\int \frac{\sqrt[4]{1+x^6}}{x^7} \, dx$

Optimal. Leaf size=45

$$-\frac{\sqrt[4]{x^6+1}}{6x^6}-\frac{1}{12}\tan^{-1}\left(\sqrt[4]{x^6+1}\right)-\frac{1}{12}\tanh^{-1}\left(\sqrt[4]{x^6+1}\right)$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 47, 63, 212, 206, 203}

$$-\frac{\sqrt[4]{x^6+1}}{6x^6}-\frac{1}{12}\tan^{-1}\left(\sqrt[4]{x^6+1}\right)-\frac{1}{12}\tanh^{-1}\left(\sqrt[4]{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)^(1/4)/x^7,x]
```

```
[Out] -1/6*(1 + x^6)^(1/4)/x^6 - ArcTan[(1 + x^6)^(1/4)]/12 - ArcTanh[(1 + x^6)^(
1/4)]/12
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{1+x^6}}{x^7}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{\sqrt[4]{1+x}}{x^2}\,dx, x, x^6\right)\\ &= -\frac{\sqrt[4]{1+x^6}}{6x^6}+\frac{1}{24}\operatorname{Subst}\left(\int \frac{1}{x(1+x)^{3/4}}\,dx, x, x^6\right)\\ &= -\frac{\sqrt[4]{1+x^6}}{6x^6}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt[4]{1+x^6}\right)\\ &= -\frac{\sqrt[4]{1+x^6}}{6x^6}-\frac{1}{12}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1+x^6}\right)-\frac{1}{12}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{1+x^6}\right)\\ &= -\frac{\sqrt[4]{1+x^6}}{6x^6}-\frac{1}{12}\tan^{-1}\left(\sqrt[4]{1+x^6}\right)-\frac{1}{12}\tanh^{-1}\left(\sqrt[4]{1+x^6}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.58

$$\frac{2}{15}\left(x^6+1\right)^{5/4}\,{}_2F_1\left(\frac{5}{4},2;\frac{9}{4};x^6+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)^(1/4)/x^7,x]
[Out] (2*(1 + x^6)^(5/4)*Hypergeometric2F1[5/4, 2, 9/4, 1 + x^6])/15
```

IntegrateAlgebraic [A] time = 0.04, size = 45, normalized size = 1.00

$$-\frac{\sqrt[4]{x^6+1}}{6x^6}-\frac{1}{12}\tan^{-1}\left(\sqrt[4]{x^6+1}\right)-\frac{1}{12}\tanh^{-1}\left(\sqrt[4]{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^6)^(1/4)/x^7,x]
[Out] -1/6*(1 + x^6)^(1/4)/x^6 - ArcTan[(1 + x^6)^(1/4)]/12 - ArcTanh[(1 + x^6)^(
1/4)]/12
```

fricas [A] time = 0.41, size = 57, normalized size = 1.27

$$-\frac{2\,x^6\arctan\left(\left(x^6+1\right)^{\frac{1}{4}}\right)+x^6\log\left(\left(x^6+1\right)^{\frac{1}{4}}+1\right)-x^6\log\left(\left(x^6+1\right)^{\frac{1}{4}}-1\right)+4\left(x^6+1\right)^{\frac{1}{4}}}{24\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/4)/x^7,x, algorithm="fricas")
[Out] -1/24*(2*x^6*arctan((x^6 + 1)^(1/4)) + x^6*log((x^6 + 1)^(1/4) + 1) - x^6*l
og((x^6 + 1)^(1/4) - 1) + 4*(x^6 + 1)^(1/4))/x^6
```

giac [A] time = 0.21, size = 47, normalized size = 1.04

$$-\frac{\left(x^6+1\right)^{\frac{1}{4}}}{6\,x^6}-\frac{1}{12}\arctan\left(\left(x^6+1\right)^{\frac{1}{4}}\right)-\frac{1}{24}\log\left(\left(x^6+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{24}\log\left(\left(x^6+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/4)/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(x^6 + 1)^(1/4)/x^6 - 1/12*arctan((x^6 + 1)^(1/4)) - 1/24*log((x^6 + 1
)^(1/4) + 1) + 1/24*log((x^6 + 1)^(1/4) - 1)
```

maple [C] time = 0.29, size = 56, normalized size = 1.24

$$-\frac{\left(x^6+1\right)^{\frac{1}{4}}}{6x^6}+\frac{-\frac{3\Gamma\left(\frac{3}{4}\right)x^6\,\mathrm{hypergeom}\left(\left[1,1,\frac{7}{4}\right],[2,2],-x^6\right)}{4}+\left(-3\ln(2)+\frac{\pi}{2}+6\ln(x)\right)\Gamma\left(\frac{3}{4}\right)}{24\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)^(1/4)/x^7,x)
```

```
[Out] -1/6*(x^6+1)^(1/4)/x^6+1/24/GAMMA(3/4)*(-3/4*GAMMA(3/4)*x^6*hypergeom([1,1,
7/4],[2,2],-x^6)+(-3*ln(2)+1/2*Pi+6*ln(x))*GAMMA(3/4))
```

maxima [A] time = 0.45, size = 47, normalized size = 1.04

$$-\frac{\left(x^6+1\right)^{\frac{1}{4}}}{6\,x^6}-\frac{1}{12}\,\arctan\left(\left(x^6+1\right)^{\frac{1}{4}}\right)-\frac{1}{24}\,\log\left(\left(x^6+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{24}\,\log\left(\left(x^6+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(1/4)/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(x^6 + 1)^(1/4)/x^6 - 1/12*arctan((x^6 + 1)^(1/4)) - 1/24*log((x^6 + 1
)^(1/4) + 1) + 1/24*log((x^6 + 1)^(1/4) - 1)
```

mupad [B] time = 0.60, size = 33, normalized size = 0.73

$$-\frac{\mathrm{atan}\left(\left(x^6+1\right)^{1/4}\right)}{12}-\frac{\mathrm{atanh}\left(\left(x^6+1\right)^{1/4}\right)}{12}-\frac{\left(x^6+1\right)^{1/4}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)^(1/4)/x^7,x)
```

```
[Out] - atan((x^6 + 1)^(1/4))/12 - atanh((x^6 + 1)^(1/4))/12 - (x^6 + 1)^(1/4)/(6
*x^6)
```

sympy [C] time = 1.05, size = 34, normalized size = 0.76

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{e^{i\pi}}{x^6}\right)}{6x^{\frac{9}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)**(1/4)/x**7,x)
```

```
[Out] -gamma(3/4)*hyper((-1/4, 3/4), (7/4,), exp_polar(I*pi)/x**6)/(6*x**(9/2)*ga
mma(7/4))
```

3.584 $\int \frac{(-1+x)^2(-10-8x+5x^2+5x^3)}{\left(\frac{1+x}{-2+x^2}\right)^{3/4}(-2+x^2)(-3+7x-11x^2+4x^3+4x^4-4x^5+x^6)}\,dx$

Optimal. Leaf size=45

$$2\tan^{-1}\left(\frac{x-1}{\sqrt[4]{\frac{x+1}{x^2-2}}}\right)-2\tanh^{-1}\left(\frac{x-1}{\sqrt[4]{\frac{x+1}{x^2-2}}}\right)$$

Rubi [F] time = 14.35, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x)^2\left(-10-8x+5x^2+5x^3\right)}{\left(\frac{1+x}{-2+x^2}\right)^{3/4}\left(-2+x^2\right)\left(-3+7x-11x^2+4x^3+4x^4-4x^5+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x)^2*(-10 - 8*x + 5*x^2 + 5*x^3))/(((1 + x)/(-2 + x^2))^(3/4)*(-
2 + x^2)*(-3 + 7*x - 11*x^2 + 4*x^3 + 4*x^4 - 4*x^5 + x^6)),x]
```

```
[Out] (16*(1 + x)^(3/4)*Defer[Subst][Defer[Int][1/((-1 - 2*x^4 + x^8)^(1/4)*(16 +
 x^4 - 56*x^8 + 72*x^12 - 39*x^16 + 10*x^20 - x^24)), x], x, (1 + x)^(1/4)]
)/((-((1 + x)/(2 - x^2)))^(3/4)*(-2 + x^2)^(3/4)) - (16*(1 + x)^(3/4)*Defer
[Subst][Defer[Int][1/((-1 - 2*x^4 + x^8)^(1/4)*(-16 - x^4 + 56*x^8 - 72*x^1
2 + 39*x^16 - 10*x^20 + x^24)), x], x, (1 + x)^(1/4)])/((-((1 + x)/(2 - x^2
)))^(3/4)*(-2 + x^2)^(3/4)) - (16*(1 + x)^(3/4)*Defer[Subst][Defer[Int][x^4
/((-1 - 2*x^4 + x^8)^(1/4)*(-16 - x^4 + 56*x^8 - 72*x^12 + 39*x^16 - 10*x^2
0 + x^24)), x], x, (1 + x)^(1/4)])/((-((1 + x)/(2 - x^2)))^(3/4)*(-2 + x^2)
^(3/4)) - (120*(1 + x)^(3/4)*Defer[Subst][Defer[Int][x^8/((-1 - 2*x^4 + x^8
)^(1/4)*(-16 - x^4 + 56*x^8 - 72*x^12 + 39*x^16 - 10*x^20 + x^24)), x], x,
(1 + x)^(1/4)])/((-((1 + x)/(2 - x^2)))^(3/4)*(-2 + x^2)^(3/4)) + (228*(1 +
 x)^(3/4)*Defer[Subst][Defer[Int][x^12/((-1 - 2*x^4 + x^8)^(1/4)*(-16 - x^4
 + 56*x^8 - 72*x^12 + 39*x^16 - 10*x^20 + x^24)), x], x, (1 + x)^(1/4)])/((
-((1 + x)/(2 - x^2)))^(3/4)*(-2 + x^2)^(3/4)) - (120*(1 + x)^(3/4)*Defer[Su
bst][Defer[Int][x^16/((-1 - 2*x^4 + x^8)^(1/4)*(-16 - x^4 + 56*x^8 - 72*x^1
2 + 39*x^16 - 10*x^20 + x^24)), x], x, (1 + x)^(1/4)])/((-((1 + x)/(2 - x^2
)))^(3/4)*(-2 + x^2)^(3/4)) + (20*(1 + x)^(3/4)*Defer[Subst][Defer[Int][x^2
0/((-1 - 2*x^4 + x^8)^(1/4)*(-16 - x^4 + 56*x^8 - 72*x^12 + 39*x^16 - 10*x^
20 + x^24)), x], x, (1 + x)^(1/4)])/((-((1 + x)/(2 - x^2)))^(3/4)*(-2 + x^2
)^(3/4))
```

Rubi steps

$$\int \frac{(-1+x)^2\left(-10-8x+5x^2+5x^3\right)}{\left(\frac{1+x}{-2+x^2}\right)^{3/4}\left(-2+x^2\right)\left(-3+7x-11x^2+4x^3+4x^4-4x^5+x^6\right)}\,dx = \frac{(1+x)^{3/4}\int \frac{(-1+x)^2\left(-10-8x+5x^2+5x^3\right)}{(1+x)^{3/4}\sqrt[4]{-2+x^2}\left(-3+7x-11x^2+4x^3+4\right.}}{\left(\frac{1+x}{-2+x^2}\right)^{3/4}\left(-2+x^2\right)^{3/4}}$$

$$= \frac{(1+x)^{3/4}\int\left(-\frac{10}{(1+x)^{3/4}\sqrt[4]{-2+x^2}\left(-3+7x-11x^2+4x^3\right.}\right.}{}$$

$$= -\frac{\left(5(1+x)^{3/4}\right)\int \frac{x^4}{(1+x)^{3/4}\sqrt[4]{-2+x^2}\left(-3+7x-11x^2+\right.}}{\left(\frac{1+x}{-2+x^2}\right)^{3/4}\left(-2+x^2\right)^{3/4}}$$

$$= -\frac{\left(20(1+x)^{3/4}\right)\mathrm{Subst}\left(\int \frac{}{\sqrt[4]{-1-2x^4+x^8}\left(-16-x\right.}\right.}{\left(\frac{1+x}{-2+x^2}\right)^{3/4}}$$

$$= -\frac{\left(20(1+x)^{3/4}\right)\mathrm{Subst}\left(\int\left(\frac{}{\sqrt[4]{-1-2x^4+x^8}\left(-16-\right.}\right.\right.}{}$$

$$= \frac{\left(20(1+x)^{3/4}\right)\mathrm{Subst}\left(\int \frac{}{\sqrt[4]{-1-2x^4+x^8}\left(16+x^4-\right.}\right.}{\left(\frac{1+x}{-2+x^2}\right)^{3/4}\left(-\right.}$$

Mathematica [F] time = 2.34, size = 0, normalized size = 0.00

$$\int \frac{(-1+x)^2\left(-10-8x+5x^2+5x^3\right)}{\left(\frac{1+x}{-2+x^2}\right)^{3/4}\left(-2+x^2\right)\left(-3+7x-11x^2+4x^3+4x^4-4x^5+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x)^2*(-10 - 8*x + 5*x^2 + 5*x^3))/(((1 + x)/(-2 + x^2))^(3
/4)*(-2 + x^2)*(-3 + 7*x - 11*x^2 + 4*x^3 + 4*x^4 - 4*x^5 + x^6)),x]
```

```
[Out] Integrate[((-1 + x)^2*(-10 - 8*x + 5*x^2 + 5*x^3))/(((1 + x)/(-2 + x^2))^(3
/4)*(-2 + x^2)*(-3 + 7*x - 11*x^2 + 4*x^3 + 4*x^4 - 4*x^5 + x^6)), x]
```

IntegrateAlgebraic [A] time = 4.50, size = 45, normalized size = 1.00

$$2\tan^{-1}\left(\frac{x-1}{\sqrt[4]{\frac{x+1}{x^2-2}}}\right)-2\tanh^{-1}\left(\frac{x-1}{\sqrt[4]{\frac{x+1}{x^2-2}}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)^2*(-10 - 8*x + 5*x^2 + 5*x^3))/(((1 + x)/(-2 +
 x^2))^(3/4)*(-2 + x^2)*(-3 + 7*x - 11*x^2 + 4*x^3 + 4*x^4 - 4*x^5 + x^6)),
x]
```

```
[Out] 2*ArcTan[(-1 + x)/((1 + x)/(-2 + x^2))^(1/4)] - 2*ArcTanh[(-1 + x)/((1 + x)
/(-2 + x^2))^(1/4)]
```

fricas [B] time = 46.52, size = 256, normalized size = 5.69

$$-\arctan\left(\frac{2\left((x^3-x^2-2x+2)\left(\frac{x+1}{x^2-2}\right)^{\frac{3}{4}}+(x^5-3x^4+x^3+5x^2-6x+2)\left(\frac{x+1}{x^2-2}\right)^{\frac{1}{4}}\right)}{x^6-4x^5+4x^4+4x^3-11x^2+7x-3}\right)+\log\left(\frac{x^6-4x^5+4x^4+4x^3-11x^2-2(x^3-x^2-2x+2)\left(\frac{x+1}{x^2-2}\right)^{\frac{3}{4}}+2(x^4-2x^3-x^2+4x-2)\sqrt{\frac{x+1}{x^2-2}}-2(x^5-3x^4+x^3+5x^2-6x+2)\left(\frac{x+1}{x^2-2}\right)^{\frac{1}{4}}+9x-1}{x^6-4x^5+4x^4+4x^3-11x^2+7x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^2*(5*x^3+5*x^2-8*x-10)/((1+x)/(x^2-2))^(3/4)/(x^2-2)/(x^6-
4*x^5+4*x^4+4*x^3-11*x^2+7*x-3),x, algorithm="fricas")
```

```
[Out] -arctan(2*((x^3 - x^2 - 2*x + 2)*((x + 1)/(x^2 - 2))^(3/4) + (x^5 - 3*x^4 +
 x^3 + 5*x^2 - 6*x + 2)*((x + 1)/(x^2 - 2))^(1/4))/(x^6 - 4*x^5 + 4*x^4 + 4
*x^3 - 11*x^2 + 7*x - 3)) + log((x^6 - 4*x^5 + 4*x^4 + 4*x^3 - 11*x^2 - 2*(
x^3 - x^2 - 2*x + 2)*((x + 1)/(x^2 - 2))^(3/4) + 2*(x^4 - 2*x^3 - x^2 + 4*x
 - 2)*sqrt((x + 1)/(x^2 - 2)) - 2*(x^5 - 3*x^4 + x^3 + 5*x^2 - 6*x + 2)*((x
 + 1)/(x^2 - 2))^(1/4) + 9*x - 1)/(x^6 - 4*x^5 + 4*x^4 + 4*x^3 - 11*x^2 + 7
*x - 3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^3+5x^2-8x-10)(x-1)^2}{(x^6-4x^5+4x^4+4x^3-11x^2+7x-3)(x^2-2)\left(\frac{x+1}{x^2-2}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^2*(5*x^3+5*x^2-8*x-10)/((1+x)/(x^2-2))^(3/4)/(x^2-2)/(x^6-
4*x^5+4*x^4+4*x^3-11*x^2+7*x-3),x, algorithm="giac")
```

```
[Out] integrate((5*x^3 + 5*x^2 - 8*x - 10)*(x - 1)^2/((x^6 - 4*x^5 + 4*x^4 + 4*x^
3 - 11*x^2 + 7*x - 3)*(x^2 - 2)*((x + 1)/(x^2 - 2))^(3/4)), x)
```

maple [C] time = 5.13, size = 805, normalized size = 17.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^2*(5*x^3+5*x^2-8*x-10)/((1+x)/(x^2-2))^(3/4)/(x^2-2)/(x^6-4*x^5+
4*x^4+4*x^3-11*x^2+7*x-3),x)
```

```
[Out] -ln(-(2*(-(-1-x)/(x^2-2))^(3/4)*x^3+2*(-(-1-x)/(x^2-2))^(1/2)*x^4+2*(-(-1-x
)/(x^2-2))^(1/4)*x^5+x^6-2*(-(-1-x)/(x^2-2))^(3/4)*x^2-4*(-(-1-x)/(x^2-2))^
(1/2)*x^3-6*(-(-1-x)/(x^2-2))^(1/4)*x^4-4*x^5-4*(-(-1-x)/(x^2-2))^(3/4)*x-2
*(-(-1-x)/(x^2-2))^(1/2)*x^2+2*(-(-1-x)/(x^2-2))^(1/4)*x^3+4*x^4+4*(-(-1-x)
/(x^2-2))^(3/4)+8*(-(-1-x)/(x^2-2))^(1/2)*x+10*(-(-1-x)/(x^2-2))^(1/4)*x^2+
4*x^3-4*(-(-1-x)/(x^2-2))^(1/2)-12*(-(-1-x)/(x^2-2))^(1/4)*x-11*x^2+4*(-(-1
-x)/(x^2-2))^(1/4)+9*x-1)/(x^6-4*x^5+4*x^4+4*x^3-11*x^2+7*x-3))+RootOf(_Z^2
+1)*ln((2*(-(-1-x)/(x^2-2))^(1/2)*RootOf(_Z^2+1)*x^4-RootOf(_Z^2+1)*x^6-4*(
-(-1-x)/(x^2-2))^(1/2)*RootOf(_Z^2+1)*x^3+4*RootOf(_Z^2+1)*x^5-2*(-(-1-x)/(
x^2-2))^(3/4)*x^3+2*(-(-1-x)/(x^2-2))^(1/4)*x^5-2*(-(-1-x)/(x^2-2))^(1/2)*R
ootOf(_Z^2+1)*x^2-4*RootOf(_Z^2+1)*x^4+2*(-(-1-x)/(x^2-2))^(3/4)*x^2-6*(-(-
1-x)/(x^2-2))^(1/4)*x^4+8*(-(-1-x)/(x^2-2))^(1/2)*RootOf(_Z^2+1)*x-4*RootOf
(_Z^2+1)*x^3+4*(-(-1-x)/(x^2-2))^(3/4)*x+2*(-(-1-x)/(x^2-2))^(1/4)*x^3-4*(-
(-1-x)/(x^2-2))^(1/2)*RootOf(_Z^2+1)+11*RootOf(_Z^2+1)*x^2-4*(-(-1-x)/(x^2-
2))^(3/4)+10*(-(-1-x)/(x^2-2))^(1/4)*x^2-9*RootOf(_Z^2+1)*x-12*(-(-1-x)/(x^
2-2))^(1/4)*x+RootOf(_Z^2+1)+4*(-(-1-x)/(x^2-2))^(1/4))/(x^6-4*x^5+4*x^4+4*
x^3-11*x^2+7*x-3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^3 + 5x^2 - 8x - 10)(x-1)^2}{(x^6 - 4x^5 + 4x^4 + 4x^3 - 11x^2 + 7x - 3)(x^2 - 2)\left(\frac{x+1}{x^2-2}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^2*(5*x^3+5*x^2-8*x-10)/((1+x)/(x^2-2))^(3/4)/(x^2-2)/(x^6-
4*x^5+4*x^4+4*x^3-11*x^2+7*x-3),x, algorithm="maxima")
```

```
[Out] integrate((5*x^3 + 5*x^2 - 8*x - 10)*(x - 1)^2/((x^6 - 4*x^5 + 4*x^4 + 4*x^
3 - 11*x^2 + 7*x - 3)*(x^2 - 2)*((x + 1)/(x^2 - 2))^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{(x-1)^2\,(-5x^3 - 5x^2 + 8x + 10)}{(x^2-2)\left(\frac{x+1}{x^2-2}\right)^{3/4}(x^6 - 4x^5 + 4x^4 + 4x^3 - 11x^2 + 7x - 3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x - 1)^2*(8*x - 5*x^2 - 5*x^3 + 10))/((x^2 - 2)*((x + 1)/(x^2 - 2))^
(3/4)*(7*x - 11*x^2 + 4*x^3 + 4*x^4 - 4*x^5 + x^6 - 3)),x)
```

```
[Out] int(-((x - 1)^2*(8*x - 5*x^2 - 5*x^3 + 10))/((x^2 - 2)*((x + 1)/(x^2 - 2))^
(3/4)*(7*x - 11*x^2 + 4*x^3 + 4*x^4 - 4*x^5 + x^6 - 3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**2*(5*x**3+5*x**2-8*x-10)/((1+x)/(x**2-2))**(3/4)/(x**2-2)
/(x**6-4*x**5+4*x**4+4*x**3-11*x**2+7*x-3),x)
```

```
[Out] Timed out
```

3.585 $\int \frac{x^2(10b+9ax)}{\sqrt[4]{bx^2+ax^3}\left(-b-ax+x^{10}\right)}\,dx$

Optimal. Leaf size=45

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^3+bx^2}}{x^3}\right)-2\tanh^{-1}\left(\frac{x^3}{\sqrt[4]{ax^3+bx^2}}\right)$$

Rubi [F] time = 1.98, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2(10b+9ax)}{\sqrt[4]{bx^2+ax^3}\left(-b-ax+x^{10}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(10*b + 9*a*x))/((b*x^2 + a*x^3)^(1/4)*(-b - a*x + x^10)),x]
```

```
[Out] (-20*b*Sqrt[x]*(b + a*x)^(1/4)*Defer[Subst][Defer[Int][x^4/((b + a*x^2)^(1/
4)*(b + a*x^2 - x^20)), x], x, Sqrt[x]])/(b*x^2 + a*x^3)^(1/4) - (18*a*Sqrt
[x]*(b + a*x)^(1/4)*Defer[Subst][Defer[Int][x^6/((b + a*x^2)^(1/4)*(b + a*x
^2 - x^20)), x], x, Sqrt[x]])/(b*x^2 + a*x^3)^(1/4)
```

Rubi steps

$$\int \frac{x^2(10b+9ax)}{\sqrt[4]{bx^2+ax^3}\left(-b-ax+x^{10}\right)}\,dx=\frac{\left(\sqrt{x}\sqrt[4]{b+ax}\right)\int \frac{x^{3/2}(10b+9ax)}{\sqrt[4]{b+ax}\left(-b-ax+x^{10}\right)}\,dx}{\sqrt[4]{bx^2+ax^3}}$$

$$=\frac{\left(2\sqrt{x}\sqrt[4]{b+ax}\right)\text{Subst}\left(\int \frac{x^4\left(10b+9ax^2\right)}{\sqrt[4]{b+ax^2}\left(-b-ax^2+x^{20}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{bx^2+ax^3}}$$

$$=\frac{\left(2\sqrt{x}\sqrt[4]{b+ax}\right)\text{Subst}\left(\int \left(-\frac{10bx^4}{\sqrt[4]{b+ax^2}\left(b+ax^2-x^{20}\right)}-\frac{9ax^6}{\sqrt[4]{b+ax^2}\left(b+ax^2-x^{20}\right)}\right)dx,x,$$

$$=-\frac{\left(18a\sqrt{x}\sqrt[4]{b+ax}\right)\text{Subst}\left(\int \frac{x^6}{\sqrt[4]{b+ax^2}\left(b+ax^2-x^{20}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{bx^2+ax^3}}-\frac{\left(20b\sqrt{x}\sqrt[4]{b}\right.}{}$$

Mathematica [F] time = 1.96, size = 0, normalized size = 0.00

$$\int \frac{x^2(10b+9ax)}{\sqrt[4]{bx^2+ax^3}\left(-b-ax+x^{10}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(10*b + 9*a*x))/((b*x^2 + a*x^3)^(1/4)*(-b - a*x + x^10)),x]
```

```
[Out] Integrate[(x^2*(10*b + 9*a*x))/((b*x^2 + a*x^3)^(1/4)*(-b - a*x + x^10)), x
]
```

IntegrateAlgebraic [A] time = 2.04, size = 45, normalized size = 1.00

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^3+bx^2}}{x^3}\right)-2\tanh^{-1}\left(\frac{x^3}{\sqrt[4]{ax^3+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(10*b + 9*a*x))/((b*x^2 + a*x^3)^(1/4)*(-b - a*x +
x^10)),x]
```

```
[Out] 2*ArcTan[(b*x^2 + a*x^3)^(1/4)/x^3] - 2*ArcTanh[x^3/(b*x^2 + a*x^3)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(9*a*x+10*b)/(a*x^3+b*x^2)^(1/4)/(x^10-a*x-b),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(9\,ax + 10\,b)x^2}{\left(x^{10} - ax - b\right)\left(ax^3 + bx^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(9*a*x+10*b)/(a*x^3+b*x^2)^(1/4)/(x^10-a*x-b),x, algorithm="g
iac")
```

```
[Out] integrate((9*a*x + 10*b)*x^2/((x^10 - a*x - b)*(a*x^3 + b*x^2)^(1/4)), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{x^2\,(9ax + 10b)}{\left(a\,x^3 + b\,x^2\right)^{\frac{1}{4}}\left(x^{10} - ax - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(9*a*x+10*b)/(a*x^3+b*x^2)^(1/4)/(x^10-a*x-b),x)
```

```
[Out] int(x^2*(9*a*x+10*b)/(a*x^3+b*x^2)^(1/4)/(x^10-a*x-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(9\,ax + 10\,b)x^2}{\left(x^{10} - ax - b\right)\left(ax^3 + bx^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(9*a*x+10*b)/(a*x^3+b*x^2)^(1/4)/(x^10-a*x-b),x, algorithm="m
axima")
```

```
[Out] integrate((9*a*x + 10*b)*x^2/((x^10 - a*x - b)*(a*x^3 + b*x^2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x^2\,\left(10\,b + 9\,a\,x\right)}{{\left(a\,x^3 + b\,x^2\right)}^{1/4}\,\left(-x^{10} + a\,x + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(10*b + 9*a*x))/((a*x^3 + b*x^2)^(1/4)*(b + a*x - x^10)),x)
```

```
[Out] int(-(x^2*(10*b + 9*a*x))/((a*x^3 + b*x^2)^(1/4)*(b + a*x - x^10)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (9ax + 10b)}{\sqrt[4]{x^2 (ax + b)} \left(-ax - b + x^{10}\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(9*a*x+10*b)/(a*x**3+b*x**2)**(1/4)/(x**10-a*x-b),x)
```

```
[Out] Integral(x**2*(9*a*x + 10*b)/((x**2*(a*x + b))**(1/4)*(-a*x - b + x**10)),
x)
```

3.586 $\int \frac{\sqrt{x^2+x\sqrt{-1+x^2}}}{x\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=45

$$-\sqrt{2}\log\left(\sqrt{x^2-1}-\sqrt{2}\sqrt{x^2+\sqrt{x^2-1}\,x}+x\right)$$

Rubi [A] time = 0.18, antiderivative size = 18, normalized size of antiderivative = 0.40, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2130, 215}

$$\sqrt{2}\sinh^{-1}\left(\sqrt{x^2-1}+x\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + x*Sqrt[-1 + x^2]]/(x*Sqrt[-1 + x^2]),x]
```

```
[Out] Sqrt[2]*ArcSinh[x + Sqrt[-1 + x^2]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2130

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x^2+x\sqrt{-1+x^2}}}{x\sqrt{-1+x^2}}\,dx &= \sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x+\sqrt{-1+x^2}\right)\\ &= \sqrt{2}\sinh^{-1}\left(x+\sqrt{-1+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.06, size = 70, normalized size = 1.56

$$\frac{\sqrt{2}\sqrt{x\left(\sqrt{x^2-1}+x\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\sqrt{x^2-1}+x}}\right)}{\sqrt{x}\sqrt{\sqrt{x^2-1}+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + x*Sqrt[-1 + x^2]]/(x*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[2]*Sqrt[x*(x + Sqrt[-1 + x^2])]*ArcTanh[(Sqrt[2]*Sqrt[x])/Sqrt[x + Sq
rt[-1 + x^2]]])/(Sqrt[x]*Sqrt[x + Sqrt[-1 + x^2]])
```

IntegrateAlgebraic [A] time = 1.43, size = 45, normalized size = 1.00

$$-\sqrt{2}\log\left(\sqrt{x^2-1}-\sqrt{2}\sqrt{x^2+\sqrt{x^2-1}\,x}+x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2 + x*Sqrt[-1 + x^2]]/(x*Sqrt[-1 + x^2]),x]
```

```
[Out] -(Sqrt[2]*Log[x + Sqrt[-1 + x^2] - Sqrt[2]*Sqrt[x^2 + x*Sqrt[-1 + x^2]]])
```

fricas [A] time = 0.73, size = 57, normalized size = 1.27

$$\frac{1}{2}\sqrt{2}\log\left(-4x^2 - 2\sqrt{x^2 + \sqrt{x^2-1}\,x}\left(\sqrt{2}x + \sqrt{2}\sqrt{x^2-1}\right) - 4\sqrt{x^2-1}\,x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x*(x^2-1)^(1/2))^(1/2)/x/(x^2-1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*sqrt(2)*log(-4*x^2 - 2*sqrt(x^2 + sqrt(x^2 - 1)*x)*(sqrt(2)*x + sqrt(2)
*sqrt(x^2 - 1)) - 4*sqrt(x^2 - 1)*x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^2-1}\,x}}{\sqrt{x^2-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x*(x^2-1)^(1/2))^(1/2)/x/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^2 - 1)*x)/(sqrt(x^2 - 1)*x), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + x\sqrt{x^2-1}}}{x\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x*(x^2-1)^(1/2))^(1/2)/x/(x^2-1)^(1/2),x)
```

```
[Out] int((x^2+x*(x^2-1)^(1/2))^(1/2)/x/(x^2-1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^2-1}\,x}}{\sqrt{x^2-1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x*(x^2-1)^(1/2))^(1/2)/x/(x^2-1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^2 - 1)*x)/(sqrt(x^2 - 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x\sqrt{x^2-1} + x^2}}{x\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^2 - 1)^(1/2) + x^2)^(1/2)/(x*(x^2 - 1)^(1/2)),x)
```

```
[Out] int((x*(x^2 - 1)^(1/2) + x^2)^(1/2)/(x*(x^2 - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(x+\sqrt{x^2-1}\right)}}{x\sqrt{(x-1)\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x*(x**2-1)**(1/2))**(1/2)/x/(x**2-1)**(1/2),x)
[Out] Integral(sqrt(x*(x + sqrt(x**2 - 1)))/(x*sqrt((x - 1)*(x + 1))), x)
```

3.587 $\int x^6\sqrt{x+x^4}\,dx$

Optimal. Leaf size=46

$$\frac{1}{72}\sqrt{x^4+x}\left(8x^7+2x^4-3x\right)+\frac{1}{24}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)$$

Rubi [A] time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.41, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2021, 2024, 2029, 206}

$$\frac{1}{36}\sqrt{x^4+x}\,x^4-\frac{1}{24}\sqrt{x^4+x}\,x+\frac{1}{9}\sqrt{x^4+x}\,x^7+\frac{1}{24}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*Sqrt[x + x^4],x]
```

```
[Out] -1/24*(x*Sqrt[x + x^4]) + (x^4*Sqrt[x + x^4])/36 + (x^7*Sqrt[x + x^4])/9 +
ArcTanh[x^2/Sqrt[x + x^4]]/24
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int x^6\sqrt{x+x^4}\,dx = \frac{1}{9}x^7\sqrt{x+x^4} + \frac{1}{6}\int \frac{x^7}{\sqrt{x+x^4}}\,dx$$
$$= \frac{1}{36}x^4\sqrt{x+x^4} + \frac{1}{9}x^7\sqrt{x+x^4} - \frac{1}{8}\int \frac{x^4}{\sqrt{x+x^4}}\,dx$$
$$= -\frac{1}{24}x\sqrt{x+x^4} + \frac{1}{36}x^4\sqrt{x+x^4} + \frac{1}{9}x^7\sqrt{x+x^4} + \frac{1}{16}\int \frac{x}{\sqrt{x+x^4}}\,dx$$
$$= -\frac{1}{24}x\sqrt{x+x^4} + \frac{1}{36}x^4\sqrt{x+x^4} + \frac{1}{9}x^7\sqrt{x+x^4} + \frac{1}{24}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^2}{\sqrt{x+x^4}}\right)$$
$$= -\frac{1}{24}x\sqrt{x+x^4} + \frac{1}{36}x^4\sqrt{x+x^4} + \frac{1}{9}x^7\sqrt{x+x^4} + \frac{1}{24}\tanh^{-1}\left(\frac{x^2}{\sqrt{x+x^4}}\right)$$

Mathematica [A] time = 0.04, size = 54, normalized size = 1.17

$$\frac{\sqrt{x^4+x}\left(\frac{3\sinh^{-1}\left(x^{3/2}\right)}{\sqrt{x^3+1}} + \left(8x^6+2x^3-3\right)x^{3/2}\right)}{72\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*Sqrt[x + x^4],x]
```

```
[Out] (Sqrt[x + x^4]*(x^(3/2)*(-3 + 2*x^3 + 8*x^6) + (3*ArcSinh[x^(3/2)])/Sqrt[1
+ x^3]))/(72*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.46, size = 46, normalized size = 1.00

$$\frac{1}{72}\sqrt{x^4+x}\left(8x^7+2x^4-3x\right) + \frac{1}{24}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6*Sqrt[x + x^4],x]
```

```
[Out] (Sqrt[x + x^4]*(-3*x + 2*x^4 + 8*x^7))/72 + ArcTanh[x^2/Sqrt[x + x^4]]/24
```

fricas [A] time = 0.44, size = 44, normalized size = 0.96

$$\frac{1}{72}\left(8x^7+2x^4-3x\right)\sqrt{x^4+x} + \frac{1}{48}\log\left(-2x^3-2\sqrt{x^4+x}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^4+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/72*(8*x^7 + 2*x^4 - 3*x)*sqrt(x^4 + x) + 1/48*log(-2*x^3 - 2*sqrt(x^4 + x
)*x - 1)
```

giac [A] time = 0.20, size = 50, normalized size = 1.09

$$\frac{1}{72}\left(2\left(4x^3+1\right)x^3-3\right)\sqrt{x^4+x}\,x + \frac{1}{48}\log\left(\sqrt{\frac{1}{x^3}+1}+1\right) - \frac{1}{48}\log\left(\left|\sqrt{\frac{1}{x^3}+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^4+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/72*(2*(4*x^3 + 1)*x^3 - 3)*sqrt(x^4 + x)*x + 1/48*log(sqrt(1/x^3 + 1) + 1
) - 1/48*log(abs(sqrt(1/x^3 + 1) - 1))
```

maple [C] time = 0.34, size = 325, normalized size = 7.07

$$\frac{x^{7} \sqrt{x^{4}+x}}{9}+\frac{x^{4} \sqrt{x^{4}+x}}{36}-\frac{x \sqrt{x^{4}+x}}{24}-\frac{\left(-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{\left(\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) x}{\left(\frac{1}{2}+\frac{i \sqrt{3}}{2}\right)(1+x)}}\, (1+x)^{2} \sqrt{-\frac{x-\frac{1}{2}+\frac{i \sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i \sqrt{3}}{2}\right)(1+x)}}\, \sqrt{-\frac{x-\frac{1}{2}-\frac{i \sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i \sqrt{3}}{2}\right)(1+x)}}\, \left(-\operatorname{EllipticF}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) x}{\left(\frac{1}{2}+\frac{i \sqrt{3}}{2}\right)(1+x)}}, \sqrt{\frac{\left(-\frac{3}{2}+\frac{i \sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i \sqrt{3}}{2}\right)}}\right)+\operatorname{EllipticPi}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) x}{\left(\frac{1}{2}+\frac{i \sqrt{3}}{2}\right)(1+x)}}, \frac{\frac{1}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{\left(-\frac{3}{2}+\frac{i \sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i \sqrt{3}}{2}\right)}}\right)\right)}{8\left(\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{x (1+x)\left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^4+x)^(1/2),x)
```

```
[Out] 1/9*x^7*(x^4+x)^(1/2)+1/36*x^4*(x^4+x)^(1/2)-1/24*x*(x^4+x)^(1/2)-1/8*(-1/2
-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)*(1+
x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))/(1+x))^(1/2)*(-(x-1/2-1/2*
I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(3/2+1/2*I*3^(1/2))/(x*(1+x)*(x
-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1/2)*(-EllipticF(((3/2+1/2*I*3^
(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I
*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+EllipticPi(((3/
2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),(1/2+1/2*I*3^(1/2))/(3/
2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{4}+x}\, x^{6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^4+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x)*x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{6} \sqrt{x^{4}+x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x + x^4)^(1/2),x)
```

```
[Out] int(x^6*(x + x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{6} \sqrt{x (x+1)\left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(x**4+x)**(1/2),x)
```

```
[Out] Integral(x**6*sqrt(x*(x + 1)*(x**2 - x + 1)), x)
```

3.588 $\int \frac{(-1+x)(1+x)^3}{\left(1+x^2\right)^2 \sqrt{1+x^2+x^4}} \, dx$

Optimal. Leaf size=46

$$\frac{2\sqrt{x^4+x^2+1}}{x^2+1} - 2\tan^{-1}\left(\frac{x}{x^2+\sqrt{x^4+x^2+1}+1}\right)$$

Rubi [A] time = 0.20, antiderivative size = 39, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1687, 1586, 1698, 203, 1685, 802}

$$\frac{2\sqrt{x^4+x^2+1}}{x^2+1} - \tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)*(1 + x)^3)/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] (2*Sqrt[1 + x^2 + x^4])/(1 + x^2) - ArcTan[x/Sqrt[1 + x^2 + x^4]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 802

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] /; FreeQ[{a, b
, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && EqQ[b*(e*f + d*g) - 2*(c*d*f +
a*e*g), 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1685

```
Int[(Px_)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(Px /. x -> Sqrt[x])*(d + e*x)
^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
&& PolyQ[Px, x^2]
```

Rule 1687

```
Int[(Pr_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Module[{r = Expon[Pr, x], k}, Int[Sum[Coeff[Pr, x, 2*k]*
x^(2*k), {k, 0, r/2}]*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[C
oeff[Pr, x, 2*k + 1]*x^(2*k), {k, 0, (r - 1)/2}]*(d + e*x^2)^q*(a + b*x^2 +
 c*x^4)^p, x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Pr, x] &&  !Poly
Q[Pr, x^2]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x)(1+x)^3}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx &= \int \frac{x\left(-2+2x^2\right)}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx + \int \frac{-1+x^4}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{-2+2x}{(1+x)^2\sqrt{1+x+x^2}}\,dx, x, x^2\right) + \int \frac{-1+x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx\\
&= \frac{2\sqrt{1+x^2+x^4}}{1+x^2} - \operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right)\\
&= \frac{2\sqrt{1+x^2+x^4}}{1+x^2} - \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)
\end{aligned}$$

Mathematica [C] time = 5.04, size = 709, normalized size = 15.41

$$\frac{2\left(\frac{x^4+x^2+1}{x^2+1}-\frac{\sqrt[6]{-1}\sqrt{\frac{(\sqrt{3}-i)(x-\sqrt[3]{-1}+1)(2x-i\sqrt{3}-1)}{(x+(-1)^{2/3}+1)^2}}\sqrt{\frac{2\sqrt{3}x-\sqrt{3}+3i}{2(\sqrt{3}+i)x+4i}}\left(x+(-1)^{2/3}+1\right)^2F\left(\sin^{-1}\left(\sqrt{\frac{i\sqrt{3}x+x+2}{x+(-1)^{2/3}+1}}\right)\middle|\frac{1}{4}\right)}{2\sqrt{2}\sqrt[4]{3}}+\frac{\sqrt[6]{-1}\sqrt{\frac{-2\sqrt{3}x+\sqrt{3}+3i}{-2ix+\sqrt{3}-i}}\sqrt{\frac{i\sqrt{3}x+x+2}{x+(-1)^{2/3}+1}}\sqrt{\frac{2\sqrt{3}x-\sqrt{3}+3i}{(\sqrt{3}+i)x+2i}}\left(x+(-1)^{2/3}+1\right)^2\left(((1+2i)-i\sqrt{3})F\left(\sin^{-1}\left(\sqrt{\frac{2(-i+\sqrt{3})x-4i}{-2ix+\sqrt{3}-i}}\right)\middle|\frac{1}{4}\right)+2i\sqrt{3}\Pi\left(\frac{(1+2i)+i\sqrt{3}}{(4+2i)-2\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{2(-i+\sqrt{3})x-4i}{-2ix+\sqrt{3}-i}}\right)\middle|\frac{1}{4}\right)\right)}{4\sqrt{6}}-\frac{(-1)^{2/3}\sqrt{\frac{-2\sqrt{3}x+\sqrt{3}+3i}{-2ix+\sqrt{3}-i}}\sqrt{\frac{i\sqrt{3}x+x+2}{x+(-1)^{2/3}+1}}\sqrt{\frac{2\sqrt{3}x-\sqrt{3}+3i}{(\sqrt{3}+i)x+2i}}\left(x+(-1)^{2/3}+1\right)^2\left((\sqrt{3}+(2+i))F\left(\sin^{-1}\left(\sqrt{\frac{2(-i+\sqrt{3})x-4i}{-2ix+\sqrt{3}-i}}\right)\middle|\frac{1}{4}\right)-2\sqrt{3}\Pi\left(\frac{(1-2i)+i\sqrt{3}}{(4-2i)+2\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{2(-i+\sqrt{3})x-4i}{-2ix+\sqrt{3}-i}}\right)\middle|\frac{1}{4}\right)\right)}{4\sqrt{6}}\right)}{\sqrt{x^4+x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x)*(1 + x)^3)/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] (2*((1 + x^2 + x^4)/(1 + x^2) - ((-1)^(1/6)*(1 + (-1)^(2/3) + x)^2*Sqrt[((-
I + Sqrt[3])*(1 - (-1)^(1/3) + x)*(-1 - I*Sqrt[3] + 2*x))/(1 + (-1)^(2/3) +
 x)^2]*Sqrt[(3*I - Sqrt[3] + 2*Sqrt[3]*x)/(4*I + 2*(I + Sqrt[3])*x)]*Ellipt
icF[ArcSin[Sqrt[(2 + x + I*Sqrt[3]*x)/(1 + (-1)^(2/3) + x)]], 1/4])/(2*Sqrt
[2]*3^(1/4)) + ((-1)^(1/6)*(1 + (-1)^(2/3) + x)^2*Sqrt[(3*I + Sqrt[3] - 2*S
qrt[3]*x)/(-I + Sqrt[3] - (2*I)*x)]*Sqrt[(2 + x + I*Sqrt[3]*x)/(1 + (-1)^(2
/3) + x)]*Sqrt[(3*I - Sqrt[3] + 2*Sqrt[3]*x)/(2*I + (I + Sqrt[3])*x)]*(((1
+ 2*I) - I*Sqrt[3])*EllipticF[ArcSin[Sqrt[(-4*I + 2*(-I + Sqrt[3])*x)/(-I +
 Sqrt[3] - (2*I)*x)]], 1/4] + (2*I)*Sqrt[3]*EllipticPi[((1 + 2*I) + I*Sqrt[
3])/((4 + 2*I) - 2*Sqrt[3]), ArcSin[Sqrt[(-4*I + 2*(-I + Sqrt[3])*x)/(-I +
Sqrt[3] - (2*I)*x)]], 1/4]))/(4*Sqrt[6]) - ((-1)^(2/3)*(1 + (-1)^(2/3) + x)
^2*Sqrt[(3*I + Sqrt[3] - 2*Sqrt[3]*x)/(-I + Sqrt[3] - (2*I)*x)]*Sqrt[(2 + x
 + I*Sqrt[3]*x)/(1 + (-1)^(2/3) + x)]*Sqrt[(3*I - Sqrt[3] + 2*Sqrt[3]*x)/(2
*I + (I + Sqrt[3])*x)]*(((2 + I) + Sqrt[3])*EllipticF[ArcSin[Sqrt[(-4*I + 2
*(-I + Sqrt[3])*x)/(-I + Sqrt[3] - (2*I)*x)]], 1/4] - 2*Sqrt[3]*EllipticPi[
((1 - 2*I) + I*Sqrt[3])/((4 - 2*I) + 2*Sqrt[3]), ArcSin[Sqrt[(-4*I + 2*(-I
+ Sqrt[3])*x)/(-I + Sqrt[3] - (2*I)*x)]], 1/4]))/(4*Sqrt[6])))/Sqrt[1 + x^2
 + x^4]
```

IntegrateAlgebraic [A] time = 0.66, size = 46, normalized size = 1.00

$$\frac{2\sqrt{x^4+x^2+1}}{x^2+1} - 2\tan^{-1}\left(\frac{x}{x^2+\sqrt{x^4+x^2+1}+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)*(1 + x)^3)/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x
]
```

```
[Out] (2*Sqrt[1 + x^2 + x^4])/(1 + x^2) - 2*ArcTan[x/(1 + x^2 + Sqrt[1 + x^2 + x^
4])]
```

fricas [A] time = 0.44, size = 41, normalized size = 0.89

$$-\frac{\left(x^2+1\right)\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-2\sqrt{x^4+x^2+1}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(1+x)^3/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -((x^2 + 1)*arctan(x/sqrt(x^4 + x^2 + 1)) - 2*sqrt(x^4 + x^2 + 1))/(x^2 + 1
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x+1)^3(x-1)}{\sqrt{x^4+x^2+1}\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(1+x)^3/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)^3*(x - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

maple [C] time = 0.07, size = 266, normalized size = 5.78

$$\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\operatorname{arctanh}\left(\frac{x^2}{2\sqrt{x^4+x^2+1}}-\frac{1}{2\sqrt{x^4+x^2+1}}\right)-\frac{2\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\operatorname{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x,-\frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{\left(x^2+1\right)^2-x^2}}{x^2+1}+\operatorname{arctanh}\left(\frac{-x^2+1}{2\sqrt{\left(x^2+1\right)^2-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)*(1+x)^3/(x^2+1)^2/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*
3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2
),1/2*(-2+2*I*3^(1/2))^(1/2))+arctanh(1/2/(x^4+x^2+1)^(1/2)*x^2-1/2/(x^4+x^
2+1)^(1/2))-2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2
)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/
2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1
/2+1/2*I*3^(1/2))^(1/2))+2/(x^2+1)*((x^2+1)^2-x^2)^(1/2)+arctanh(1/2*(-x^2+
1)/((x^2+1)^2-x^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x+1)^3(x-1)}{\sqrt{x^4+x^2+1}\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(1+x)^3/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)^3*(x - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(x-1)\,(x+1)^3}{\left(x^2+1\right)^2\,\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)*(x + 1)^3)/((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2)),x)
```

```
[Out] int(((x - 1)*(x + 1)^3)/((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)^3}{\sqrt{(x^2-x+1)(x^2+x+1)}\,(x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(1+x)**3/(x**2+1)**2/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)**3/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)
**2), x)
```

3.589 $\int \frac{-1+x^2}{(1+x^2)\sqrt{1-x-x^2-x^3+x^4}}\,dx$

Optimal. Leaf size=46

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}x}{x^2-\sqrt{x^4-x^3-x^2-x+1}+1}\right)}{\sqrt{3}}$$

Rubi [F] time = 0.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{1-x-x^2-x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[1 - x - x^2 - x^3 + x^4]),x]
```

```
[Out] Defer[Int][1/Sqrt[1 - x - x^2 - x^3 + x^4], x] - I*Defer[Int][1/((I - x)*Sq
rt[1 - x - x^2 - x^3 + x^4]), x] - I*Defer[Int][1/((I + x)*Sqrt[1 - x - x^2
 - x^3 + x^4]), x]
```

Rubi steps

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{1-x-x^2-x^3+x^4}}\,dx = \int \left(\frac{1}{\sqrt{1-x-x^2-x^3+x^4}} - \frac{2}{(1+x^2)\sqrt{1-x-x^2-x^3+x^4}}\right) dx$$
$$= -\left(2\int \frac{1}{(1+x^2)\sqrt{1-x-x^2-x^3+x^4}}\,dx\right) + \int \frac{1}{\sqrt{1-x-x^2-x^3+x^4}}\,d$$
$$= -\left(2\int \left(\frac{i}{2(i-x)\sqrt{1-x-x^2-x^3+x^4}} + \frac{i}{2(i+x)\sqrt{1-x-x^2-x^3+x^4}}\right)\right.$$
$$= -\left(i\int \frac{1}{(i-x)\sqrt{1-x-x^2-x^3+x^4}}\,dx\right) - i\int \frac{1}{(i+x)\sqrt{1-x-x^2-x^3+}}$$

Mathematica [C] time = 3.22, size = 3173, normalized size = 68.98

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[1 - x - x^2 - x^3 + x^4]),x]
```

```
[Out] (2*(1 + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*x)^2*Sqrt[(-x + Root[1 - #1
- #1^2 - #1^3 + #1^4 & , 3, 0])/((1 + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])] -
4*x)*(-Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0] + Root[1 - #1 - #1^2 - #1
^3 + #1^4 & , 3, 0]))]*Sqrt[-(((x - Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1,
 0])*(1 + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*Root[1 - #1 - #1^2 - #1^3
+ #1^4 & , 4, 0]))/((1 + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*x)*(Root[1
- #1 - #1^2 - #1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2 - #1^3 + #1^4 & ,
4, 0])))]*Sqrt[(-x + Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0])/((1 + Sqrt
[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*x)*(-Root[1 - #1 - #1^2 - #1^3 + #1^4 &
, 1, 0] + Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0]))]*(((-4*I)*((Elliptic
Pi[(((1 - 4*I) + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])])*(Root[1 - #1 - #1^2 -
#1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0]))/((-I +
```

```
Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0])*(1 + Sqrt[13] + Sqrt[2*(-1 + S
qrt[13])] - 4*Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0])), ArcSin[Sqrt[-((
(x - Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0])*(1 + Sqrt[13] + Sqrt[2*(-1
 + Sqrt[13])] - 4*Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0]))/((1 + Sqrt[1
3] + Sqrt[2*(-1 + Sqrt[13])] - 4*x)*(Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1
, 0] - Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0])))]], -(((1 + Sqrt[13] +
Sqrt[2*(-1 + Sqrt[13])] - 4*Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 3, 0])*(Ro
ot[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2 - #1^3 + #1^4
 & , 4, 0]))/((-Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0] + Root[1 - #1 -
#1^2 - #1^3 + #1^4 & , 3, 0])*(1 + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*R
oot[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0])))]*(1 + Sqrt[13] + Sqrt[2*(-1 +
Sqrt[13])] - 4*Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0]))/4 + EllipticF[A
rcSin[Sqrt[-(((x - Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0])*(1 + Sqrt[13
] + Sqrt[2*(-1 + Sqrt[13])] - 4*Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0])
)/((1 + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*x)*(Root[1 - #1 - #1^2 - #1^
3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0])))]], -(((1
 + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*Root[1 - #1 - #1^2 - #1^3 + #1^4
& , 3, 0])*(Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2
 - #1^3 + #1^4 & , 4, 0]))/((-Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0] +
Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 3, 0])*(1 + Sqrt[13] + Sqrt[2*(-1 + Sq
rt[13])] - 4*Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0])))]*(-I + Root[1 -
#1 - #1^2 - #1^3 + #1^4 & , 1, 0]))*(Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1
, 0] - Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0]))/(((1 - 4*I) + Sqrt[13]
+ Sqrt[2*(-1 + Sqrt[13])])*(-I + Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0]
)) + ((4*I)*((EllipticPi[(((1 + 4*I) + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])])*
(Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2 - #1^3 + #
1^4 & , 4, 0]))/((I + Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0])*(1 + Sqrt
[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4,
0])), ArcSin[Sqrt[-(((x - Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0])*(1 +
Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*Root[1 - #1 - #1^2 - #1^3 + #1^4 & ,
 4, 0]))/((1 + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*x)*(Root[1 - #1 - #1^
2 - #1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0])))]]
, -(((1 + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*Root[1 - #1 - #1^2 - #1^3
+ #1^4 & , 3, 0])*(Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0] - Root[1 - #1
 - #1^2 - #1^3 + #1^4 & , 4, 0]))/((-Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1
, 0] + Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 3, 0])*(1 + Sqrt[13] + Sqrt[2*(
-1 + Sqrt[13])] - 4*Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0])))]*(1 + Sqr
t[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1,
 0]))/4 + EllipticF[ArcSin[Sqrt[-(((x - Root[1 - #1 - #1^2 - #1^3 + #1^4 &
, 1, 0])*(1 + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*Root[1 - #1 - #1^2 - #
1^3 + #1^4 & , 4, 0]))/((1 + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*x)*(Roo
t[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2 - #1^3 + #1^4
& , 4, 0])))]], -(((1 + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*Root[1 - #1
- #1^2 - #1^3 + #1^4 & , 3, 0])*(Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0]
 - Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0]))/((-Root[1 - #1 - #1^2 - #1^
3 + #1^4 & , 1, 0] + Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 3, 0])*(1 + Sqrt[
13] + Sqrt[2*(-1 + Sqrt[13])] - 4*Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0
])))]*(I + Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0]))*(Root[1 - #1 - #1^2
 - #1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0]))/(((
1 + 4*I) + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])])*(I + Root[1 - #1 - #1^2 - #1
^3 + #1^4 & , 1, 0])) + EllipticF[ArcSin[Sqrt[-(((x - Root[1 - #1 - #1^2 -
#1^3 + #1^4 & , 1, 0])*(1 + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*Root[1 -
 #1 - #1^2 - #1^3 + #1^4 & , 4, 0]))/((1 + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13]
)] - 4*x)*(Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2
- #1^3 + #1^4 & , 4, 0])))]], ((1 + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*
Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 3, 0])*(Root[1 - #1 - #1^2 - #1^3 + #1
^4 & , 1, 0] - Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0]))/((Root[1 - #1 -
 #1^2 - #1^3 + #1^4 & , 1, 0] - Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 3, 0])
*(1 + Sqrt[13] + Sqrt[2*(-1 + Sqrt[13])] - 4*Root[1 - #1 - #1^2 - #1^3 + #1
```

```
^4 & , 4, 0]))]*(-Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 1, 0] + Root[1 - #1
- #1^2 - #1^3 + #1^4 & , 4, 0])))/(Sqrt[1 - x - x^2 - x^3 + x^4]*(1 + Sqrt[
13] + Sqrt[2*(-1 + Sqrt[13])] - 4*Root[1 - #1 - #1^2 - #1^3 + #1^4 & , 4, 0
]))
```

IntegrateAlgebraic [A] time = 0.20, size = 46, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}x}{x^2-\sqrt{x^4-x^3-x^2-x+1}+1}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*Sqrt[1 - x - x^2 - x^3 + x^4]),x]
[Out] (2*ArcTan[(Sqrt[3]*x)/(1 + x^2 - Sqrt[1 - x - x^2 - x^3 + x^4])])/Sqrt[3]
```

fricas [A] time = 0.43, size = 39, normalized size = 0.85

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x^2+6x+1\right)}{6\sqrt{x^4-x^3-x^2-x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^4-x^3-x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*(x^2 + 6*x + 1)/sqrt(x^4 - x^3 - x^2 - x +
1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^4-x^3-x^2-x+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^4-x^3-x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 - 1)/(sqrt(x^4 - x^3 - x^2 - x + 1)*(x^2 + 1)), x)
```

maple [C] time = 0.92, size = 100440, normalized size = 2183.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^4-x^3-x^2-x+1)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^4-x^3-x^2-x+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^4-x^3-x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 - 1)/(sqrt(x^4 - x^3 - x^2 - x + 1)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2-1}{\left(x^2+1\right)\sqrt{x^4-x^3-x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + 1)*(x^4 - x^2 - x^3 - x + 1)^(1/2)),x)
[Out] int((x^2 - 1)/((x^2 + 1)*(x^4 - x^2 - x^3 - x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)}{\left(x^2+1\right)\sqrt{x^4-x^3-x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**4-x**3-x**2-x+1)**(1/2),x)
[Out] Integral((x - 1)*(x + 1)/((x**2 + 1)*sqrt(x**4 - x**3 - x**2 - x + 1)), x)
```

3.590 $\int \frac{-1-2x+3x^2}{\sqrt{-3-2x-x^2+4x^3-x^4-2x^5+x^6}}\,dx$

Optimal. Leaf size=46

$$-\log\left(-x^3+x^2+\sqrt{x^6-2x^5-x^4+4x^3-x^2-2x-3}+x-1\right)$$

Rubi [F] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1-2x+3x^2}{\sqrt{-3-2x-x^2+4x^3-x^4-2x^5+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 - 2*x + 3*x^2)/Sqrt[-3 - 2*x - x^2 + 4*x^3 - x^4 - 2*x^5 + x^6],x]
```

```
[Out] -Defer[Int][1/Sqrt[-3 - 2*x - x^2 + 4*x^3 - x^4 - 2*x^5 + x^6], x] - 2*Defe
r[Int][x/Sqrt[-3 - 2*x - x^2 + 4*x^3 - x^4 - 2*x^5 + x^6], x] + 3*Defer[Int
][x^2/Sqrt[-3 - 2*x - x^2 + 4*x^3 - x^4 - 2*x^5 + x^6], x]
```

Rubi steps

$$\int \frac{-1-2x+3x^2}{\sqrt{-3-2x-x^2+4x^3-x^4-2x^5+x^6}}\,dx = \int \left(-\frac{1}{\sqrt{-3-2x-x^2+4x^3-x^4-2x^5+x^6}}-\frac{2x}{\sqrt{-3-2x-x^2+4x\ldots}}\right.$$
$$= -\left(2\int \frac{x}{\sqrt{-3-2x-x^2+4x^3-x^4-2x^5+x^6}}\,dx\right)+3\int \frac{\ldots}{\sqrt{-3-2\ldots}}$$

Mathematica [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{-1-2x+3x^2}{\sqrt{-3-2x-x^2+4x^3-x^4-2x^5+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 - 2*x + 3*x^2)/Sqrt[-3 - 2*x - x^2 + 4*x^3 - x^4 - 2*x^5 + x^
6],x]
```

```
[Out] Integrate[(-1 - 2*x + 3*x^2)/Sqrt[-3 - 2*x - x^2 + 4*x^3 - x^4 - 2*x^5 + x^
6], x]
```

IntegrateAlgebraic [A] time = 4.07, size = 46, normalized size = 1.00

$$-\log\left(-x^3+x^2+\sqrt{x^6-2x^5-x^4+4x^3-x^2-2x-3}+x-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 2*x + 3*x^2)/Sqrt[-3 - 2*x - x^2 + 4*x^3 - x^4 - 2
*x^5 + x^6],x]
```

```
[Out] -Log[-1 + x + x^2 - x^3 + Sqrt[-3 - 2*x - x^2 + 4*x^3 - x^4 - 2*x^5 + x^6]]
```

fricas [A] time = 0.44, size = 44, normalized size = 0.96

$$\log\left(-x^3+x^2+x-\sqrt{x^6-2\,x^5-x^4+4\,x^3-x^2-2\,x-3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-2*x-1)/(x^6-2*x^5-x^4+4*x^3-x^2-2*x-3)^(1/2),x, algorithm=
"fricas")
```

```
[Out] log(-x^3 + x^2 + x - sqrt(x^6 - 2*x^5 - x^4 + 4*x^3 - x^2 - 2*x - 3) - 1)
```

giac [A] time = 0.23, size = 46, normalized size = 1.00

$$-\log\left(\left|-x^3+x^2+x+\sqrt{2x^3+\left(x^3-x^2-x\right)^2-2x^2-2x-3}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-2*x-1)/(x^6-2*x^5-x^4+4*x^3-x^2-2*x-3)^(1/2),x, algorithm=
"giac")
```

```
[Out] -log(abs(-x^3 + x^2 + x + sqrt(2*x^3 + (x^3 - x^2 - x)^2 - 2*x^2 - 2*x - 3)
 - 1))
```

maple [A] time = 0.29, size = 45, normalized size = 0.98

$$\ln\left(-x^3+x^2-\sqrt{x^6-2x^5-x^4+4x^3-x^2-2x-3}+x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-2*x-1)/(x^6-2*x^5-x^4+4*x^3-x^2-2*x-3)^(1/2),x)
```

```
[Out] ln(-x^3+x^2-(x^6-2*x^5-x^4+4*x^3-x^2-2*x-3)^(1/2)+x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2-2x-1}{\sqrt{x^6-2x^5-x^4+4x^3-x^2-2x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-2*x-1)/(x^6-2*x^5-x^4+4*x^3-x^2-2*x-3)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((3*x^2 - 2*x - 1)/sqrt(x^6 - 2*x^5 - x^4 + 4*x^3 - x^2 - 2*x - 3)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{-3x^2+2x+1}{\sqrt{x^6-2x^5-x^4+4x^3-x^2-2x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 3*x^2 + 1)/(4*x^3 - x^2 - 2*x - x^4 - 2*x^5 + x^6 - 3)^(1/2),x)
```

```
[Out] int(-(2*x - 3*x^2 + 1)/(4*x^3 - x^2 - 2*x - x^4 - 2*x^5 + x^6 - 3)^(1/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(3x+1)}{\sqrt{\left(x^3-x^2-x-1\right)\left(x^3-x^2-x+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-2*x-1)/(x**6-2*x**5-x**4+4*x**3-x**2-2*x-3)**(1/2),x)
```

```
[Out] Integral((x - 1)*(3*x + 1)/sqrt((x**3 - x**2 - x - 1)*(x**3 - x**2 - x + 3)
), x)
```

3.591 $\int \frac{-x+x^2}{\sqrt{-2x+4x^2-2x^3+x^4-2x^5+x^6}}\,dx$

Optimal. Leaf size=46

$$\frac{2}{3}\tanh^{-1}\left(\frac{(x-1)\left(x^3-2\right)}{x\sqrt{x^6-2x^5+x^4-2x^3+4x^2-2x}}\right)$$

Rubi [B] time = 0.61, antiderivative size = 113, normalized size of antiderivative = 2.46, number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {1593, 2056, 6688, 6719, 329, 275, 217, 206}

$$-\frac{2(1-x)\sqrt{x}\sqrt{x^3-2}\sqrt{x^5-2x^4+x^3-2x^2+4x-2}\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{x^3-2}}\right)}{3\sqrt{-(1-x)^2\left(2-x^3\right)}\sqrt{x^6-2x^5+x^4-2x^3+4x^2-2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^2)/Sqrt[-2*x + 4*x^2 - 2*x^3 + x^4 - 2*x^5 + x^6],x]
```

```
[Out] (-2*(1 - x)*Sqrt[x]*Sqrt[-2 + x^3]*Sqrt[-2 + 4*x - 2*x^2 + x^3 - 2*x^4 + x^
5]*ArcTanh[x^(3/2)/Sqrt[-2 + x^3]])/(3*Sqrt[-((1 - x)^2*(2 - x^3))]*Sqrt[-2
*x + 4*x^2 - 2*x^3 + x^4 - 2*x^5 + x^6])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
```

```
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{-x+x^2}{\sqrt{-2x+4x^2-2x^3+x^4-2x^5+x^6}}\,dx = \int \frac{(-1+x)x}{\sqrt{-2x+4x^2-2x^3+x^4-2x^5+x^6}}\,dx$$

$$= \frac{\left(\sqrt{x}\sqrt{-2+4x-2x^2+x^3-2x^4+x^5}\right)\int \frac{(-1+x)\sqrt{x}}{\sqrt{-2+4x-2x^2+x^3-2x^4+x^5}}\,dx}{\sqrt{-2x+4x^2-2x^3+x^4-2x^5+x^6}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-2+4x-2x^2+x^3-2x^4+x^5}\right)\int \frac{(-1+x)\sqrt{x}}{\sqrt{(-1+x)^2(-2+x^3)}}\,dx}{\sqrt{-2x+4x^2-2x^3+x^4-2x^5+x^6}}$$

$$= \frac{\left((-1+x)\sqrt{x}\sqrt{-2+x^3}\sqrt{-2+4x-2x^2+x^3-2x^4+x^5}\right)\int \frac{\sqrt{x}}{\sqrt{-2+x}}}{\sqrt{(-1+x)^2\left(-2+x^3\right)}\sqrt{-2x+4x^2-2x^3+x^4-2x^5+x^6}}$$

$$= \frac{\left(2(-1+x)\sqrt{x}\sqrt{-2+x^3}\sqrt{-2+4x-2x^2+x^3-2x^4+x^5}\right)\text{Subst}(}{\sqrt{(-1+x)^2\left(-2+x^3\right)}\sqrt{-2x+4x^2-2x^3+x^4-2x}}$$

$$= \frac{\left(2(-1+x)\sqrt{x}\sqrt{-2+x^3}\sqrt{-2+4x-2x^2+x^3-2x^4+x^5}\right)\text{Subst}(}{3\sqrt{(-1+x)^2\left(-2+x^3\right)}\sqrt{-2x+4x^2-2x^3+x^4-2x}}$$

$$= \frac{\left(2(-1+x)\sqrt{x}\sqrt{-2+x^3}\sqrt{-2+4x-2x^2+x^3-2x^4+x^5}\right)\text{Subst}(}{3\sqrt{(-1+x)^2\left(-2+x^3\right)}\sqrt{-2x+4x^2-2x^3+x^4-2x}}$$

$$= -\frac{2(1-x)\sqrt{x}\sqrt{-2+x^3}\sqrt{-2+4x-2x^2+x^3-2x^4+x^5}\tanh^{-1}(}{3\sqrt{-(1-x)^2\left(2-x^3\right)}\sqrt{-2x+4x^2-2x^3+x^4-2x^5+x^6}}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.15

$$\frac{2(x-1)\sqrt{x}\sqrt{x^3-2}\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{x^3-2}}\right)}{3\sqrt{(x-1)^2x\left(x^3-2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^2)/Sqrt[-2*x + 4*x^2 - 2*x^3 + x^4 - 2*x^5 + x^6],x]
```

```
[Out] (2*(-1 + x)*Sqrt[x]*Sqrt[-2 + x^3]*ArcTanh[x^(3/2)/Sqrt[-2 + x^3]])/(3*Sqrt
[(-1 + x)^2*x*(-2 + x^3)])
```

IntegrateAlgebraic [A] time = 0.41, size = 46, normalized size = 1.00

$$\frac{2}{3}\tanh^{-1}\left(\frac{(x-1)\left(x^3-2\right)}{x\sqrt{x^6-2x^5+x^4-2x^3+4x^2-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x + x^2)/Sqrt[-2*x + 4*x^2 - 2*x^3 + x^4 - 2*x^5 + x^6
],x]
```

```
[Out] (2*ArcTanh[((-1 + x)*(-2 + x^3))/(x*Sqrt[-2*x + 4*x^2 - 2*x^3 + x^4 - 2*x^5
 + x^6])])/3
```

fricas [A] time = 0.43, size = 52, normalized size = 1.13

$$\frac{1}{3}\log\left(-\frac{x^4-x^3+\sqrt{x^6-2\,x^5+x^4-2\,x^3+4\,x^2-2\,x}\,x-x+1}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)/(x^6-2*x^5+x^4-2*x^3+4*x^2-2*x)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/3*log(-(x^4 - x^3 + sqrt(x^6 - 2*x^5 + x^4 - 2*x^3 + 4*x^2 - 2*x)*x - x +
 1)/(x - 1))
```

giac [A] time = 0.17, size = 42, normalized size = 0.91

$$\frac{\log\left(\sqrt{-\frac{2}{x^3}+1}+1\right)-\log\left(\left|\sqrt{-\frac{2}{x^3}+1}-1\right|\right)}{3\,\mathrm{sgn}\left(\frac{1}{x^3}-\frac{1}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)/(x^6-2*x^5+x^4-2*x^3+4*x^2-2*x)^(1/2),x, algorithm="giac"
)
```

```
[Out] 1/3*(log(sqrt(-2/x^3 + 1) + 1) - log(abs(sqrt(-2/x^3 + 1) - 1)))/sgn(1/x^3
- 1/x^4)
```

maple [C] time = 0.55, size = 390, normalized size = 8.48

$$-\frac{4(-1+x)\sqrt{x\left(x^3-2\right)}\left(1+i\sqrt{3}\right)\sqrt{-\frac{\left(3+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)\left(-x+2^{\frac{1}{3}}\right)}}\left(-x+2^{\frac{1}{3}}\right)^2\sqrt{\frac{i\sqrt{3}\,2^{\frac{1}{3}}-2^{\frac{1}{3}}-2x}{\left(i\sqrt{3}-1\right)\left(-x+2^{\frac{1}{3}}\right)}}\sqrt{\frac{i\sqrt{3}\,2^{\frac{1}{3}}+2^{\frac{1}{3}}+2x}{\left(1+i\sqrt{3}\right)\left(-x+2^{\frac{1}{3}}\right)}}\left(\mathrm{EllipticF}\left(\sqrt{-\frac{\left(3+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)\left(-x+2^{\frac{1}{3}}\right)}},\sqrt{\frac{\left(-3+i\sqrt{3}\right)\left(1+i\sqrt{3}\right)}{\left(i\sqrt{3}-1\right)\left(3+i\sqrt{3}\right)}}\right)-\mathrm{EllipticPi}\left(\sqrt{-\frac{\left(3+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)\left(-x+2^{\frac{1}{3}}\right)}},\frac{1+i\sqrt{3}}{3+i\sqrt{3}},\sqrt{\frac{\left(-3+i\sqrt{3}\right)\left(1+i\sqrt{3}\right)}{\left(i\sqrt{3}-1\right)\left(3+i\sqrt{3}\right)}}\right)\right)}{\sqrt{x^6-2x^5+x^4-2x^3+4x^2-2x}\left(3+i\sqrt{3}\right)\sqrt{x\left(-x+2^{\frac{1}{3}}\right)\left(i\sqrt{3}\,2^{\frac{1}{3}}-2^{\frac{1}{3}}-2x\right)\left(i\sqrt{3}\,2^{\frac{1}{3}}+2^{\frac{1}{3}}+2x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x)/(x^6-2*x^5+x^4-2*x^3+4*x^2-2*x)^(1/2),x)
```

```
[Out] -4*(-1+x)*(x*(x^3-2))^(1/2)*(1+I*3^(1/2))*(-(I*3^(1/2)+3)*x/(1+I*3^(1/2))/(
-x+2^(1/3)))^(1/2)*(-x+2^(1/3))^2*((I*3^(1/2)*2^(1/3)-2^(1/3)-2*x)/(I*3^(1/
2)-1)/(-x+2^(1/3)))^(1/2)*((I*3^(1/2)*2^(1/3)+2^(1/3)+2*x)/(1+I*3^(1/2))/(-
x+2^(1/3)))^(1/2)*(EllipticF((-(I*3^(1/2)+3)*x/(1+I*3^(1/2))/(-x+2^(1/3)))^
(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(I*3^(1/2)+3))^(1/2))-Ell
ipticPi((-(I*3^(1/2)+3)*x/(1+I*3^(1/2))/(-x+2^(1/3)))^(1/2),(1+I*3^(1/2))/(
I*3^(1/2)+3),((-3+I*3^(1/2))*(1+I*3^(1/2))/(I*3^(1/2)-1)/(I*3^(1/2)+3))^(1/
2)))/(x^6-2*x^5+x^4-2*x^3+4*x^2-2*x)^(1/2)/(I*3^(1/2)+3)/(x*(-x+2^(1/3))*(I
*3^(1/2)*2^(1/3)-2^(1/3)-2*x)*(I*3^(1/2)*2^(1/3)+2^(1/3)+2*x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - x}{\sqrt{x^6 - 2\,x^5 + x^4 - 2\,x^3 + 4\,x^2 - 2\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)/(x^6-2*x^5+x^4-2*x^3+4*x^2-2*x)^(1/2),x, algorithm="maxim
a")
[Out] integrate((x^2 - x)/sqrt(x^6 - 2*x^5 + x^4 - 2*x^3 + 4*x^2 - 2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x - x^2}{\sqrt{x^6 - 2\,x^5 + x^4 - 2\,x^3 + 4\,x^2 - 2\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - x^2)/(4*x^2 - 2*x - 2*x^3 + x^4 - 2*x^5 + x^6)^(1/2),x)
[Out] int(-(x - x^2)/(4*x^2 - 2*x - 2*x^3 + x^4 - 2*x^5 + x^6)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(x-1)}{\sqrt{x\,(x-1)^2\left(x^3-2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x)/(x**6-2*x**5+x**4-2*x**3+4*x**2-2*x)**(1/2),x)
[Out] Integral(x*(x - 1)/sqrt(x*(x - 1)**2*(x**3 - 2)), x)
```

3.592 $\int \frac{\sqrt[4]{-1+x^4}}{x^2}\,dx$

Optimal. Leaf size=47

$$-\frac{\sqrt[4]{x^4-1}}{x}-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {277, 331, 298, 203, 206}

$$-\frac{\sqrt[4]{x^4-1}}{x}-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)^(1/4)/x^2,x]
```

```
[Out] -((-1 + x^4)^(1/4)/x) - ArcTan[x/(-1 + x^4)^(1/4)]/2 + ArcTanh[x/(-1 + x^4)
^(1/4)]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{-1+x^4}}{x^2}\,dx &= -\frac{\sqrt[4]{-1+x^4}}{x}+\int \frac{x^2}{\left(-1+x^4\right)^{3/4}}\,dx\\ &= -\frac{\sqrt[4]{-1+x^4}}{x}+\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)\\ &= -\frac{\sqrt[4]{-1+x^4}}{x}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)\\ &= -\frac{\sqrt[4]{-1+x^4}}{x}-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.81

$$-\frac{\sqrt[4]{x^4-1}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};x^4\right)}{x\sqrt[4]{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)^(1/4)/x^2,x]
```

```
[Out] -(((-1 + x^4)^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, x^4])/(x*(1 - x^4)^(
1/4)))
```

IntegrateAlgebraic [A] time = 0.17, size = 47, normalized size = 1.00

$$-\frac{\sqrt[4]{x^4-1}}{x}-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)^(1/4)/x^2,x]
```

```
[Out] -((-1 + x^4)^(1/4)/x) - ArcTan[x/(-1 + x^4)^(1/4)]/2 + ArcTanh[x/(-1 + x^4)
^(1/4)]/2
```

fricas [B] time = 2.22, size = 85, normalized size = 1.81

$$\frac{x\arctan\left(2\left(x^4-1\right)^{\frac{1}{4}}x^3+2\left(x^4-1\right)^{\frac{3}{4}}x\right)+x\log\left(2x^4+2\left(x^4-1\right)^{\frac{1}{4}}x^3+2\sqrt{x^4-1}\,x^2+2\left(x^4-1\right)^{\frac{3}{4}}x-1\right)-4\left(x^4-1\right)^{\frac{1}{4}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(x*arctan(2*(x^4 - 1)^(1/4)*x^3 + 2*(x^4 - 1)^(3/4)*x) + x*log(2*x^4 +
2*(x^4 - 1)^(1/4)*x^3 + 2*sqrt(x^4 - 1)*x^2 + 2*(x^4 - 1)^(3/4)*x - 1) - 4*
(x^4 - 1)^(1/4))/x
```

giac [A] time = 0.21, size = 59, normalized size = 1.26

$$\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}-\frac{1}{2}\arctan\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{4}\log\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}+1\right)+\frac{1}{4}\log\left(-\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] (x^4 - 1)^(1/4)/x - 1/2*arctan((x^4 - 1)^(1/4)/x) - 1/4*log((x^4 - 1)^(1/4)
/x + 1) + 1/4*log(-(x^4 - 1)^(1/4)/x + 1)
```

maple [C] time = 0.31, size = 46, normalized size = 0.98

$$-\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}+\frac{\left(-\mathrm{signum}\left(x^4-1\right)\right)^{\frac{3}{4}} x^3 \,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],x^4\right)}{3\mathrm{signum}\left(x^4-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(1/4)/x^2,x)
```

```
[Out] -(x^4-1)^(1/4)/x+1/3/signum(x^4-1)^(3/4)*(-signum(x^4-1))^(3/4)*x^3*hyperge
om([3/4,3/4],[7/4],x^4)
```

maxima [A] time = 0.53, size = 59, normalized size = 1.26

$$-\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}+\frac{1}{2}\arctan\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{4}\log\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}+1\right)-\frac{1}{4}\log\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] -(x^4 - 1)^(1/4)/x + 1/2*arctan((x^4 - 1)^(1/4)/x) + 1/4*log((x^4 - 1)^(1/4
)/x + 1) - 1/4*log((x^4 - 1)^(1/4)/x - 1)
```

mupad [B] time = 0.68, size = 29, normalized size = 0.62

$$-\frac{\left(x^4-1\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ x^4\right)}{x\left(1-x^4\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)^(1/4)/x^2,x)
```

```
[Out] -((x^4 - 1)^(1/4)*hypergeom([-1/4, -1/4], 3/4, x^4))/(x*(1 - x^4)^(1/4))
```

sympy [C] time = 0.83, size = 34, normalized size = 0.72

$$\frac{e^{\frac{i\pi}{4}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle| x^4\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(1/4)/x**2,x)
```

```
[Out] exp(I*pi/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), x**4)/(4*x*gamma(3/4))
```

3.593 $\int \frac{\left(-1+x^4\right)\sqrt[4]{1+x^4}}{x}\,dx$

Optimal. Leaf size=47

$$\frac{1}{5}\sqrt[4]{x^4+1}\left(x^4-4\right)+\frac{1}{2}\tan^{-1}\left(\sqrt[4]{x^4+1}\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {446, 80, 50, 63, 212, 206, 203}

$$\frac{1}{5}\left(x^4+1\right)^{5/4}-\sqrt[4]{x^4+1}+\frac{1}{2}\tan^{-1}\left(\sqrt[4]{x^4+1}\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4)*(1 + x^4)^(1/4))/x,x]
```

```
[Out] -(1 + x^4)^(1/4) + (1 + x^4)^(5/4)/5 + ArcTan[(1 + x^4)^(1/4)]/2 + ArcTanh[
(1 + x^4)^(1/4)]/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-1+x^4\right)\sqrt[4]{1+x^4}}{x}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{(-1+x)\sqrt[4]{1+x}}{x}\,dx, x, x^4\right)\\
&= \frac{1}{5}\left(1+x^4\right)^{5/4}-\frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt[4]{1+x}}{x}\,dx, x, x^4\right)\\
&= -\sqrt[4]{1+x^4}+\frac{1}{5}\left(1+x^4\right)^{5/4}-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{3/4}}\,dx, x, x^4\right)\\
&= -\sqrt[4]{1+x^4}+\frac{1}{5}\left(1+x^4\right)^{5/4}-\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt[4]{1+x^4}\right)\\
&= -\sqrt[4]{1+x^4}+\frac{1}{5}\left(1+x^4\right)^{5/4}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1+x^4}\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^2}\right.\\
&= -\sqrt[4]{1+x^4}+\frac{1}{5}\left(1+x^4\right)^{5/4}+\frac{1}{2}\tan^{-1}\left(\sqrt[4]{1+x^4}\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{1+x^4}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.96

$$\frac{1}{10}\left(2\sqrt[4]{x^4+1}\left(x^4-4\right)+5\tan^{-1}\left(\sqrt[4]{x^4+1}\right)+5\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)*(1 + x^4)^(1/4))/x,x]
```

```
[Out] (2*(-4 + x^4)*(1 + x^4)^(1/4) + 5*ArcTan[(1 + x^4)^(1/4)] + 5*ArcTanh[(1 +
x^4)^(1/4)])/10
```

IntegrateAlgebraic [A] time = 0.07, size = 47, normalized size = 1.00

$$\frac{1}{5}\sqrt[4]{x^4+1}\left(x^4-4\right)+\frac{1}{2}\tan^{-1}\left(\sqrt[4]{x^4+1}\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(1 + x^4)^(1/4))/x,x]
```

```
[Out] ((-4 + x^4)*(1 + x^4)^(1/4))/5 + ArcTan[(1 + x^4)^(1/4)]/2 + ArcTanh[(1 + x
^4)^(1/4)]/2
```

fricas [A] time = 0.40, size = 49, normalized size = 1.04

$$\frac{1}{5}\left(x^4+1\right)^{\frac{1}{4}}\left(x^4-4\right)+\frac{1}{2}\arctan\left(\left(x^4+1\right)^{\frac{1}{4}}\right)+\frac{1}{4}\log\left(\left(x^4+1\right)^{\frac{1}{4}}+1\right)-\frac{1}{4}\log\left(\left(x^4+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/4)/x,x, algorithm="fricas")
```

```
[Out] 1/5*(x^4 + 1)^(1/4)*(x^4 - 4) + 1/2*arctan((x^4 + 1)^(1/4)) + 1/4*log((x^4
+ 1)^(1/4) + 1) - 1/4*log((x^4 + 1)^(1/4) - 1)
```

giac [A] time = 0.29, size = 53, normalized size = 1.13

$$\frac{1}{5}\left(x^4+1\right)^{\frac{5}{4}}-\left(x^4+1\right)^{\frac{1}{4}}+\frac{1}{2}\arctan\left(\left(x^4+1\right)^{\frac{1}{4}}\right)+\frac{1}{4}\log\left(\left(x^4+1\right)^{\frac{1}{4}}+1\right)-\frac{1}{4}\log\left(\left(x^4+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/4)/x,x, algorithm="giac")
```

```
[Out] 1/5*(x^4 + 1)^(5/4) - (x^4 + 1)^(1/4) + 1/2*arctan((x^4 + 1)^(1/4)) + 1/4*l
og((x^4 + 1)^(1/4) + 1) - 1/4*log((x^4 + 1)^(1/4) - 1)
```

maple [C] time = 0.31, size = 62, normalized size = 1.32

$$\frac{-\Gamma\left(\frac{3}{4}\right)x^4\,\mathrm{hypergeom}\left(\left[\frac{3}{4},1,1\right],[2,2],-x^4\right)-4\left(4-3\ln(2)+\frac{\pi}{2}+4\ln(x)\right)\Gamma\left(\frac{3}{4}\right)}{16\Gamma\left(\frac{3}{4}\right)}+\frac{x^4\,\mathrm{hypergeom}\left(\left[-\frac{1}{4},1\right],[2],-x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^4+1)^(1/4)/x,x)
```

```
[Out] 1/16/GAMMA(3/4)*(-GAMMA(3/4)*x^4*hypergeom([3/4,1,1],[2,2],-x^4)-4*(4-3*ln(
2)+1/2*Pi+4*ln(x))*GAMMA(3/4))+1/4*x^4*hypergeom([-1/4,1],[2],-x^4)
```

maxima [A] time = 0.49, size = 53, normalized size = 1.13

$$\frac{1}{5}\left(x^4+1\right)^{\frac{5}{4}}-\left(x^4+1\right)^{\frac{1}{4}}+\frac{1}{2}\arctan\left(\left(x^4+1\right)^{\frac{1}{4}}\right)+\frac{1}{4}\log\left(\left(x^4+1\right)^{\frac{1}{4}}+1\right)-\frac{1}{4}\log\left(\left(x^4+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/4)/x,x, algorithm="maxima")
```

```
[Out] 1/5*(x^4 + 1)^(5/4) - (x^4 + 1)^(1/4) + 1/2*arctan((x^4 + 1)^(1/4)) + 1/4*l
og((x^4 + 1)^(1/4) + 1) - 1/4*log((x^4 + 1)^(1/4) - 1)
```

mupad [B] time = 0.73, size = 39, normalized size = 0.83

$$\frac{\mathrm{atan}\left(\left(x^4+1\right)^{1/4}\right)}{2}+\frac{\mathrm{atanh}\left(\left(x^4+1\right)^{1/4}\right)}{2}-\left(x^4+1\right)^{1/4}+\frac{\left(x^4+1\right)^{5/4}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(x^4 + 1)^(1/4))/x,x)
```

```
[Out] atan((x^4 + 1)^(1/4))/2 + atanh((x^4 + 1)^(1/4))/2 - (x^4 + 1)^(1/4) + (x^4
 + 1)^(5/4)/5
```

sympy [A] time = 37.10, size = 56, normalized size = 1.19

$$\frac{\left(x^4+1\right)^{\frac{5}{4}}}{5}-\sqrt[4]{x^4+1}-\frac{\log\left(\sqrt[4]{x^4+1}-1\right)}{4}+\frac{\log\left(\sqrt[4]{x^4+1}+1\right)}{4}+\frac{\mathrm{atan}\left(\sqrt[4]{x^4+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**4+1)**(1/4)/x,x)
```

```
[Out] (x**4 + 1)**(5/4)/5 - (x**4 + 1)**(1/4) - log((x**4 + 1)**(1/4) - 1)/4 + lo
g((x**4 + 1)**(1/4) + 1)/4 + atan((x**4 + 1)**(1/4))/2
```

3.594 $\int \frac{1}{\sqrt{3+4x+x^4}}\,dx$

Optimal. Leaf size=47

$$\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{6}\,x+\sqrt{6}}{-\sqrt{x^4+4x+3}+x^2+2x+1}\right)$$

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{3+4x+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/Sqrt[3 + 4*x + x^4],x]
```

```
[Out] Defer[Int][1/Sqrt[3 + 4*x + x^4], x]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+4x+x^4}}\,dx=\int \frac{1}{\sqrt{3+4x+x^4}}\,dx$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.21

$$\frac{(x+1)\sqrt{x^2-2x+3}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}(x-2)}{\sqrt{x^2-2x+3}}\right)}{\sqrt{6}\sqrt{x^4+4x+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 4*x + x^4],x]
```

```
[Out] ((1 + x)*Sqrt[3 - 2*x + x^2]*ArcTanh[(Sqrt[2/3]*(-2 + x))/Sqrt[3 - 2*x + x^
2]])/(Sqrt[6]*Sqrt[3 + 4*x + x^4])
```

IntegrateAlgebraic [A] time = 0.28, size = 47, normalized size = 1.00

$$\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{6}\,x+\sqrt{6}}{-\sqrt{x^4+4x+3}+x^2+2x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[3 + 4*x + x^4],x]
```

```
[Out] Sqrt[2/3]*ArcTanh[(Sqrt[6] + Sqrt[6]*x)/(1 + 2*x + x^2 - Sqrt[3 + 4*x + x^4
])]
```

fricas [A] time = 0.38, size = 66, normalized size = 1.40

$$\frac{1}{6}\sqrt{3}\sqrt{2}\log\left(-\frac{\sqrt{3}\sqrt{2}\left(x^2-x-2\right)+2\,x^2+\sqrt{x^4+4\,x+3}\left(\sqrt{3}\sqrt{2}+3\right)-2\,x-4}{x^2+2\,x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2)*log(-(sqrt(3)*sqrt(2)*(x^2 - x - 2) + 2*x^2 + sqrt(x^4
+ 4*x + 3)*(sqrt(3)*sqrt(2) + 3) - 2*x - 4)/(x^2 + 2*x + 1))
```

giac [A] time = 0.48, size = 61, normalized size = 1.30

$$\frac{\sqrt{6}\log\left(\frac{\left|-2x-2\sqrt{6}+2\sqrt{x^2-2x+3}-2\right|}{\left|-2x+2\sqrt{6}+2\sqrt{x^2-2x+3}-2\right|}\right)}{6\,\mathrm{sgn}\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(6)*log(abs(-2*x - 2*sqrt(6) + 2*sqrt(x^2 - 2*x + 3) - 2)/abs(-2*x
+ 2*sqrt(6) + 2*sqrt(x^2 - 2*x + 3) - 2))/sgn(x + 1)
```

maple [A] time = 0.01, size = 48, normalized size = 1.02

$$\frac{(1+x)\sqrt{x^2-2x+3}\,\sqrt{6}\,\operatorname{arctanh}\left(\frac{(-2+x)\sqrt{6}}{3\sqrt{x^2-2x+3}}\right)}{6\sqrt{x^4+4x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+4*x+3)^(1/2),x)
```

```
[Out] 1/6/(x^4+4*x+3)^(1/2)*(1+x)*(x^2-2*x+3)^(1/2)*6^(1/2)*arctanh(1/3*(-2+x)*6^
(1/2)/(x^2-2*x+3)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+4\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^4 + 4*x + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^4+4\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(4*x + x^4 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+4x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+4*x+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**4 + 4*x + 3), x)
```

3.595 $\int \frac{(1+x^4)\sqrt{-1+2x^2+x^4}}{(-1+x^4)(-1+x^2+x^4)}\,dx$

Optimal. Leaf size=47

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^4+2x^2-1}}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+2x^2-1}}\right)$$

Rubi [C] time = 2.61, antiderivative size = 1670, normalized size of antiderivative = 35.53, number of steps used = 52, number of rules used = 13, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.351, Rules used = {6725, 1208, 1187, 1098, 1184, 1214, 1456, 540, 421, 419, 538, 537, 6728}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[((1 + x^4)*Sqrt[-1 + 2*x^2 + x^4])/((-1 + x^4)*(-1 + x^2 + x^4)),x]
```

```
[Out] (Sqrt[2]*Sqrt[1 + Sqrt[2] + x^2]*Sqrt[1 - (1 + Sqrt[2])*x^2]*EllipticF[ArcS
in[Sqrt[1 + Sqrt[2]]*x], -3 + 2*Sqrt[2]])/Sqrt[-1 + 2*x^2 + x^4] + (Sqrt[2*
(3 - 2*Sqrt[2])]*Sqrt[1 + Sqrt[2] + x^2]*Sqrt[1 - (1 + Sqrt[2])*x^2]*Ellipt
icF[ArcSin[Sqrt[1 + Sqrt[2]]*x], -3 + 2*Sqrt[2]])/Sqrt[-1 + 2*x^2 + x^4] +
(Sqrt[3 - 2*Sqrt[2]]*(1 + Sqrt[5])*Sqrt[1 + Sqrt[2] + x^2]*Sqrt[1 - (1 + Sq
rt[2])*x^2]*EllipticF[ArcSin[Sqrt[1 + Sqrt[2]]*x], -3 + 2*Sqrt[2]])/((1 - 2
*Sqrt[2] - Sqrt[5])*Sqrt[-1 + 2*x^2 + x^4]) + (Sqrt[3 - 2*Sqrt[2]]*(1 - Sqr
t[5])*Sqrt[1 + Sqrt[2] + x^2]*Sqrt[1 - (1 + Sqrt[2])*x^2]*EllipticF[ArcSin[
Sqrt[1 + Sqrt[2]]*x], -3 + 2*Sqrt[2]])/((1 - 2*Sqrt[2] + Sqrt[5])*Sqrt[-1 +
 2*x^2 + x^4]) - (2^(1/4)*Sqrt[(1 - (1 - Sqrt[2])*x^2)/(1 - (1 + Sqrt[2])*x
^2)]*Sqrt[-1 + (1 + Sqrt[2])*x^2]*EllipticF[ArcSin[(2^(3/4)*x)/Sqrt[-1 + (1
 + Sqrt[2])*x^2]], (2 + Sqrt[2])/4])/((2 - Sqrt[2])*Sqrt[(1 - (1 + Sqrt[2])
*x^2)^(-1)]*Sqrt[-1 + 2*x^2 + x^4]) + ((1 + Sqrt[2])*Sqrt[(1 - (1 - Sqrt[2]
)*x^2)/(1 - (1 + Sqrt[2])*x^2)]*Sqrt[-1 + (1 + Sqrt[2])*x^2]*EllipticF[ArcS
in[(2^(3/4)*x)/Sqrt[-1 + (1 + Sqrt[2])*x^2]], (2 + Sqrt[2])/4])/(2^(1/4)*Sq
rt[(1 - (1 + Sqrt[2])*x^2)^(-1)]*Sqrt[-1 + 2*x^2 + x^4]) - ((1 + 2*Sqrt[2]
- Sqrt[5])*Sqrt[(1 - (1 - Sqrt[2])*x^2)/(1 - (1 + Sqrt[2])*x^2)]*Sqrt[-1 +
(1 + Sqrt[2])*x^2]*EllipticF[ArcSin[(2^(3/4)*x)/Sqrt[-1 + (1 + Sqrt[2])*x^2
]], (2 + Sqrt[2])/4])/(2*2^(3/4)*Sqrt[(1 - (1 + Sqrt[2])*x^2)^(-1)]*Sqrt[-1
 + 2*x^2 + x^4]) - ((1 + Sqrt[5])*Sqrt[(1 - (1 - Sqrt[2])*x^2)/(1 - (1 + Sq
rt[2])*x^2)]*Sqrt[-1 + (1 + Sqrt[2])*x^2]*EllipticF[ArcSin[(2^(3/4)*x)/Sqrt
[-1 + (1 + Sqrt[2])*x^2]], (2 + Sqrt[2])/4])/(2^(3/4)*(1 - 2*Sqrt[2] - Sqrt
[5])*Sqrt[(1 - (1 + Sqrt[2])*x^2)^(-1)]*Sqrt[-1 + 2*x^2 + x^4]) - ((1 - Sqr
t[5])*Sqrt[(1 - (1 - Sqrt[2])*x^2)/(1 - (1 + Sqrt[2])*x^2)]*Sqrt[-1 + (1 +
Sqrt[2])*x^2]*EllipticF[ArcSin[(2^(3/4)*x)/Sqrt[-1 + (1 + Sqrt[2])*x^2]], (
2 + Sqrt[2])/4])/(2^(3/4)*(1 - 2*Sqrt[2] + Sqrt[5])*Sqrt[(1 - (1 + Sqrt[2])
*x^2)^(-1)]*Sqrt[-1 + 2*x^2 + x^4]) - ((1 + 2*Sqrt[2] + Sqrt[5])*Sqrt[(1 -
(1 - Sqrt[2])*x^2)/(1 - (1 + Sqrt[2])*x^2)]*Sqrt[-1 + (1 + Sqrt[2])*x^2]*El
lipticF[ArcSin[(2^(3/4)*x)/Sqrt[-1 + (1 + Sqrt[2])*x^2]], (2 + Sqrt[2])/4])
/(2*2^(3/4)*Sqrt[(1 - (1 + Sqrt[2])*x^2)^(-1)]*Sqrt[-1 + 2*x^2 + x^4]) - (2
*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 + Sqrt[2] + x^2]*Sqrt[1 - (1 + Sqrt[2])*x^2]*El
lipticPi[1 - Sqrt[2], ArcSin[Sqrt[1 + Sqrt[2]]*x], -3 + 2*Sqrt[2]])/Sqrt[-1
 + 2*x^2 + x^4] - (2*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 + Sqrt[2] + x^2]*Sqrt[1 - (
1 + Sqrt[2])*x^2]*EllipticPi[-1 + Sqrt[2], ArcSin[Sqrt[1 + Sqrt[2]]*x], -3
+ 2*Sqrt[2]])/Sqrt[-1 + 2*x^2 + x^4] + (Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 + Sqrt[2
] + x^2]*Sqrt[1 - (1 + Sqrt[2])*x^2]*EllipticPi[(2*(1 - Sqrt[2]))/(1 - Sqrt
[5]), ArcSin[Sqrt[1 + Sqrt[2]]*x], -3 + 2*Sqrt[2]])/Sqrt[-1 + 2*x^2 + x^4]
+ (Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 + Sqrt[2] + x^2]*Sqrt[1 - (1 + Sqrt[2])*x^2]*
EllipticPi[(2*(1 - Sqrt[2]))/(1 + Sqrt[5]), ArcSin[Sqrt[1 + Sqrt[2]]*x], -3
 + 2*Sqrt[2]])/Sqrt[-1 + 2*x^2 + x^4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 540

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x
] + Dist[(b*c - a*d)/b, Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e*x*(b + q + 2*c*x^2))/(2*c*Sqrt
[a + b*x^2 + c*x^4]), x] - Simp[(e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q
)*x^2)]*Sqrt[(2*a + (b + q)*x^2)/q]*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*
x^2)/(2*q)]], (b + q)/(2*q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b
 + q)*x^2)]), x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1187

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*d - e*(b - q))/(2*c), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e/(2*c), Int[(b - q + 2*c*x^2)/Sqrt[
a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(1+x^4)\sqrt{-1+2x^2+x^4}}{(-1+x^4)(-1+x^2+x^4)}\,dx = \int \left(\frac{\sqrt{-1+2x^2+x^4}}{-1+x^2} + \frac{\sqrt{-1+2x^2+x^4}}{1+x^2} + \frac{(-1-2x^2)\sqrt{-1+2x^2+x^4}}{-1+x^2+x^4}\right) dx$$

$$= \int \frac{\sqrt{-1+2x^2+x^4}}{-1+x^2}\,dx + \int \frac{\sqrt{-1+2x^2+x^4}}{1+x^2}\,dx + \int \frac{(-1-2x^2)\sqrt{-1+2x^2}}{-1+x^2+x^4}$$

$$= 2\int \frac{1}{(-1+x^2)\sqrt{-1+2x^2+x^4}}\,dx - 2\int \frac{1}{(1+x^2)\sqrt{-1+2x^2+x^4}}\,dx - \int$$

$$= 2\left(\frac{1}{2}\int \frac{2-2\sqrt{2}+2x^2}{\sqrt{-1+2x^2+x^4}}\,dx\right) - 2\int \frac{\sqrt{-1+2x^2+x^4}}{1-\sqrt{5}+2x^2}\,dx - 2\int \frac{\sqrt{-1+2x^2+}}{1+\sqrt{5}+2}$$

$$= 2\left(\frac{x(1+\sqrt{2}+x^2)}{\sqrt{-1+2x^2+x^4}} - \frac{2^{3/4}\sqrt{\frac{1-(1-\sqrt{2})x^2}{1-(1+\sqrt{2})x^2}}\sqrt{-1+(1+\sqrt{2})x^2}\,E\left(\sin^{-1}\left(\frac{2^3}{\sqrt{-1+(1}}\right.\right.}{\sqrt{\frac{1}{1-(1+\sqrt{2})x^2}}\sqrt{-1+2x^2+x^4}}\right.$$

$$= 2\left(\frac{x(1+\sqrt{2}+x^2)}{\sqrt{-1+2x^2+x^4}} - \frac{2^{3/4}\sqrt{\frac{1-(1-\sqrt{2})x^2}{1-(1+\sqrt{2})x^2}}\sqrt{-1+(1+\sqrt{2})x^2}\,E\left(\sin^{-1}\left(\frac{2^3}{\sqrt{-1+(1}}\right.\right.}{\sqrt{\frac{1}{1-(1+\sqrt{2})x^2}}\sqrt{-1+2x^2+x^4}}\right.$$

$$= -\frac{\sqrt[4]{2}\sqrt{\frac{1-(1-\sqrt{2})x^2}{1-(1+\sqrt{2})x^2}}\sqrt{-1+(1+\sqrt{2})x^2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}x}{\sqrt{-1+(1+\sqrt{2})x^2}}\right)\middle|\frac{1}{4}(2+\sqrt{2})\right.}{(2-\sqrt{2})\sqrt{\frac{1}{1-(1+\sqrt{2})x^2}}\sqrt{-1+2x^2+x^4}}$$

$$= \frac{\sqrt{2}\sqrt{1+\sqrt{2}+x^2}\sqrt{1+\frac{x^2}{1-\sqrt{2}}}\,F\left(\sin^{-1}\left(\sqrt{1+\sqrt{2}}\,x\right)\middle|-3+2\sqrt{2}\right)}{\sqrt{-1+2x^2+x^4}} + \frac{\sqrt{2(3}}{}$$

$$= \frac{\sqrt{2}\sqrt{1+\sqrt{2}+x^2}\sqrt{1+\frac{x^2}{1-\sqrt{2}}}\,F\left(\sin^{-1}\left(\sqrt{1+\sqrt{2}}\,x\right)\middle|-3+2\sqrt{2}\right)}{\sqrt{-1+2x^2+x^4}} + \frac{\sqrt{2(3}}{}$$

$$= \frac{\sqrt{2}\sqrt{1+\sqrt{2}+x^2}\sqrt{1+\frac{x^2}{1-\sqrt{2}}}\,F\left(\sin^{-1}\left(\sqrt{1+\sqrt{2}}\,x\right)\middle|-3+2\sqrt{2}\right)}{\sqrt{-1+2x^2+x^4}} + \frac{\sqrt{2(3}}{}$$

Mathematica [C] time = 5.04, size = 1280, normalized size = 27.23

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + x^4)*Sqrt[-1 + 2*x^2 + x^4])/((-1 + x^4)*(-1 + x^2 + x^4)),
x]
```

```
[Out] ((-1 + x)*(1 + x)*(1 + x^2)*(-1 + x^2 + x^4)*((-I)*Sqrt[1 + Sqrt[2]]*Sqrt[1
 - 2*x^2 - x^4]*EllipticF[I*ArcSinh[x/Sqrt[1 + Sqrt[2]]], -3 - 2*Sqrt[2]] +
```

```
(2*I)*Sqrt[1 + Sqrt[2]]*Sqrt[1 - 2*x^2 - x^4]*EllipticPi[1 + Sqrt[2], I*Ar
cSinh[x/Sqrt[1 + Sqrt[2]]], -3 - 2*Sqrt[2]] + ((3*I)*Sqrt[1 + Sqrt[2]]*Sqrt
[1 - 2*x^2 - x^4]*EllipticPi[(-2*(1 + Sqrt[2]))/(-1 + Sqrt[5]), I*ArcSinh[x
/Sqrt[1 + Sqrt[2]]], -3 - 2*Sqrt[2]])/(-3 + Sqrt[5]) - (I*Sqrt[5*(1 + Sqrt[
2])]*Sqrt[1 - 2*x^2 - x^4]*EllipticPi[(-2*(1 + Sqrt[2]))/(-1 + Sqrt[5]), I*
ArcSinh[x/Sqrt[1 + Sqrt[2]]], -3 - 2*Sqrt[2]])/(-3 + Sqrt[5]) - ((3*I)*Sqrt
[1 + Sqrt[2]]*Sqrt[1 - 2*x^2 - x^4]*EllipticPi[(2*(1 + Sqrt[2]))/(1 + Sqrt[
5]), I*ArcSinh[x/Sqrt[1 + Sqrt[2]]], -3 - 2*Sqrt[2]])/(3 + Sqrt[5]) - (I*Sq
rt[5*(1 + Sqrt[2])]*Sqrt[1 - 2*x^2 - x^4]*EllipticPi[(2*(1 + Sqrt[2]))/(1 +
 Sqrt[5]), I*ArcSinh[x/Sqrt[1 + Sqrt[2]]], -3 - 2*Sqrt[2]])/(3 + Sqrt[5]) -
 (4*(Sqrt[-1 + Sqrt[2]] + I*Sqrt[1 + Sqrt[2]])*(Sqrt[-1 + Sqrt[2]] - x)^2*S
qrt[-((I*Sqrt[1 + Sqrt[2]] + x)/((-1 - I) + Sqrt[2] - Sqrt[-1 + Sqrt[2]]*x
+ I*Sqrt[1 + Sqrt[2]]*x))]*Sqrt[(I*Sqrt[1 + Sqrt[2]] - x)/((-1 + I) + Sqrt[
2] - (Sqrt[-1 + Sqrt[2]] + I*Sqrt[1 + Sqrt[2]])*x)]*Sqrt[((-1 - I) + Sqrt[2
] + (Sqrt[-1 + Sqrt[2]] - I*Sqrt[1 + Sqrt[2]])*x)/((-1 + I) + Sqrt[2] - (Sq
rt[-1 + Sqrt[2]] + I*Sqrt[1 + Sqrt[2]])*x)]*(-EllipticF[ArcSin[Sqrt[((-1 -
I) + Sqrt[2] + (Sqrt[-1 + Sqrt[2]] - I*Sqrt[1 + Sqrt[2]])*x)/((-1 + I) + Sq
rt[2] - (Sqrt[-1 + Sqrt[2]] + I*Sqrt[1 + Sqrt[2]])*x)]], -I] + Sqrt[-1 + Sq
rt[2]]*(EllipticPi[((-1 + Sqrt[-1 + Sqrt[2]])*(Sqrt[-1 + Sqrt[2]] + I*Sqrt[
1 + Sqrt[2]]))/((1 + Sqrt[-1 + Sqrt[2]])*(Sqrt[-1 + Sqrt[2]] - I*Sqrt[1 + S
qrt[2]])), ArcSin[Sqrt[((-1 - I) + Sqrt[2] + Sqrt[-1 + Sqrt[2]]*x - I*Sqrt[
1 + Sqrt[2]]*x)/((-1 + I) + Sqrt[2] - (Sqrt[-1 + Sqrt[2]] + I*Sqrt[1 + Sqrt
[2]])*x)]], -I] - EllipticPi[((1 + Sqrt[-1 + Sqrt[2]])*(Sqrt[-1 + Sqrt[2]]
+ I*Sqrt[1 + Sqrt[2]]))/((-1 + Sqrt[-1 + Sqrt[2]])*(Sqrt[-1 + Sqrt[2]] - I*
Sqrt[1 + Sqrt[2]])), ArcSin[Sqrt[((-1 - I) + Sqrt[2] + Sqrt[-1 + Sqrt[2]]*x
 - I*Sqrt[1 + Sqrt[2]]*x)/((-1 + I) + Sqrt[2] - (Sqrt[-1 + Sqrt[2]] + I*Sqr
t[1 + Sqrt[2]])*x)]], -I])))/((-2 + Sqrt[2])*(Sqrt[-1 + Sqrt[2]] - I*Sqrt[1
 + Sqrt[2]]))))/(Sqrt[-1 + 2*x^2 + x^4]*(1 - x^2 - 2*x^4 + x^6 + x^8))
```

IntegrateAlgebraic [A] time = 0.45, size = 47, normalized size = 1.00

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^4+2x^2-1}}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+2x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^4)*Sqrt[-1 + 2*x^2 + x^4])/((-1 + x^4)*(-1 + x^2
 + x^4)),x]
```

```
[Out] ArcTanh[x/Sqrt[-1 + 2*x^2 + x^4]] - Sqrt[2]*ArcTanh[(Sqrt[2]*x)/Sqrt[-1 + 2
*x^2 + x^4]]
```

fricas [B] time = 0.45, size = 109, normalized size = 2.32

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{x^8+16x^6+30x^4-4\sqrt{2}\left(x^5+4x^3-x\right)\sqrt{x^4+2x^2-1}-16x^2+1}{x^8-2x^4+1}\right)+\frac{1}{2}\log\left(\frac{x^4+3x^2+2\sqrt{x^4+2x^2-1}\,x-1}{x^4+x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^4+2*x^2-1)^(1/2)/(x^4-1)/(x^4+x^2-1),x, algorithm="fri
cas")
```

```
[Out] 1/4*sqrt(2)*log(-(x^8 + 16*x^6 + 30*x^4 - 4*sqrt(2)*(x^5 + 4*x^3 - x)*sqrt(
x^4 + 2*x^2 - 1) - 16*x^2 + 1)/(x^8 - 2*x^4 + 1)) + 1/2*log((x^4 + 3*x^2 +
2*sqrt(x^4 + 2*x^2 - 1)*x - 1)/(x^4 + x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+2x^2-1}\left(x^4+1\right)}{\left(x^4+x^2-1\right)\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^4+2*x^2-1)^(1/2)/(x^4-1)/(x^4+x^2-1),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(x^4 + 2*x^2 - 1)*(x^4 + 1)/((x^4 + x^2 - 1)*(x^4 - 1)), x)
```

maple [C] time = 0.34, size = 712, normalized size = 15.15

$$-\frac{2\sqrt{1-\left(1-\sqrt{2}\right)x^{2}}\,\sqrt{1-\left(1+\sqrt{2}\right)x^{2}}\,\left(\mathrm{EllipticF}\left(x\sqrt{1-\sqrt{2}}, i+i\sqrt{2}\right)-\mathrm{EllipticE}\left(x\sqrt{1-\sqrt{2}}, i+i\sqrt{2}\right)\right)}{\sqrt{1-\sqrt{2}}\,\sqrt{x^{4}+2x^{2}-1}\,\left(2+2\sqrt{2}\right)}-\frac{2\sqrt{1-\left(1-\sqrt{2}\right)x^{2}}\,\sqrt{1-\left(1+\sqrt{2}\right)x^{2}}\,\mathrm{EllipticPi}\left(x\sqrt{1-\sqrt{2}}, \frac{1}{1-\sqrt{2}}, \frac{\sqrt{1+\sqrt{2}}}{\sqrt{1-\sqrt{2}}}\right)}{\sqrt{1-\sqrt{2}}\,\sqrt{x^{4}+2x^{2}-1}}+\frac{\sqrt{\sqrt{2}\,x^{2}-x^{2}+1}\,\sqrt{-\sqrt{2}\,x^{2}-x^{2}+1}\,\mathrm{EllipticF}\left(x\sqrt{1-\sqrt{2}}, i+i\sqrt{2}\right)}{\sqrt{1-\sqrt{2}}\,\sqrt{x^{4}+2x^{2}-1}}+\frac{2\sqrt{\sqrt{2}\,x^{2}-x^{2}+1}\,\sqrt{-\sqrt{2}\,x^{2}-x^{2}+1}\,\mathrm{EllipticF}\left(x\sqrt{1-\sqrt{2}}, i+i\sqrt{2}\right)}{\sqrt{1-\sqrt{2}}\,\sqrt{x^{4}+2x^{2}-1}\,\left(2+2\sqrt{2}\right)}-\frac{2\sqrt{\sqrt{2}\,x^{2}-x^{2}+1}\,\sqrt{-\sqrt{2}\,x^{2}-x^{2}+1}\,\mathrm{EllipticE}\left(x\sqrt{1-\sqrt{2}}, i+i\sqrt{2}\right)}{\sqrt{1-\sqrt{2}}\,\sqrt{x^{4}+2x^{2}-1}\,\left(2+2\sqrt{2}\right)}-\frac{2\sqrt{\sqrt{2}\,x^{2}-x^{2}+1}\,\sqrt{-\sqrt{2}\,x^{2}-x^{2}+1}\,\mathrm{EllipticPi}\left(x\sqrt{1-\sqrt{2}}, -\frac{1}{1-\sqrt{2}}, \frac{\sqrt{1+\sqrt{2}}}{\sqrt{1-\sqrt{2}}}\right)}{\sqrt{1-\sqrt{2}}\,\sqrt{x^{4}+2x^{2}-1}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}+\_Z^{2}-1\right)}\_\alpha\left(-\frac{\mathrm{arctanh}\left(\frac{\left(\_\alpha^{2}+1\right)\left(-2\_\alpha^{2}+x^{2}+1\right)}{\sqrt{\_\alpha^{2}}\,\sqrt{x^{4}+2x^{2}-1}}\right)}{\sqrt{\_\alpha^{2}}}-\frac{2\left(\_\alpha^{3}+\_\alpha\right)\sqrt{\sqrt{2}\,x^{2}-x^{2}+1}\,\sqrt{-\sqrt{2}\,x^{2}-x^{2}+1}\,\mathrm{EllipticPi}\left(x\sqrt{1-\sqrt{2}}, -\_\alpha^{2}\sqrt{2}-\_\alpha^{2}-\sqrt{2}-1, \frac{\sqrt{1+\sqrt{2}}}{\sqrt{1-\sqrt{2}}}\right)}{\sqrt{1-\sqrt{2}}\,\sqrt{x^{4}+2x^{2}-1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)*(x^4+2*x^2-1)^(1/2)/(x^4-1)/(x^4+x^2-1),x)
```

```
[Out] -2/(1-2^(1/2))^(1/2)*(1-(1-2^(1/2))*x^2)^(1/2)*(1-(1+2^(1/2))*x^2)^(1/2)/(x
^4+2*x^2-1)^(1/2)/(2+2*2^(1/2))*(EllipticF(x*(1-2^(1/2))^(1/2),I+I*2^(1/2))
-EllipticE(x*(1-2^(1/2))^(1/2),I+I*2^(1/2)))-2/(1-2^(1/2))^(1/2)*(1-(1-2^(1
/2))*x^2)^(1/2)*(1-(1+2^(1/2))*x^2)^(1/2)/(x^4+2*x^2-1)^(1/2)*EllipticPi(x*
(1-2^(1/2))^(1/2),1/(1-2^(1/2)),(1+2^(1/2))^(1/2)/(1-2^(1/2))^(1/2))+1/(1-2
^(1/2))^(1/2)*(2^(1/2)*x^2-x^2+1)^(1/2)*(-2^(1/2)*x^2-x^2+1)^(1/2)/(x^4+2*x
^2-1)^(1/2)*EllipticF(x*(1-2^(1/2))^(1/2),I+I*2^(1/2))+2/(1-2^(1/2))^(1/2)*
(2^(1/2)*x^2-x^2+1)^(1/2)*(-2^(1/2)*x^2-x^2+1)^(1/2)/(x^4+2*x^2-1)^(1/2)/(2
+2*2^(1/2))*EllipticF(x*(1-2^(1/2))^(1/2),I+I*2^(1/2))-2/(1-2^(1/2))^(1/2)*
(2^(1/2)*x^2-x^2+1)^(1/2)*(-2^(1/2)*x^2-x^2+1)^(1/2)/(x^4+2*x^2-1)^(1/2)/(2
+2*2^(1/2))*EllipticE(x*(1-2^(1/2))^(1/2),I+I*2^(1/2))-2/(1-2^(1/2))^(1/2)*
(2^(1/2)*x^2-x^2+1)^(1/2)*(-2^(1/2)*x^2-x^2+1)^(1/2)/(x^4+2*x^2-1)^(1/2)*El
lipticPi(x*(1-2^(1/2))^(1/2),-1/(1-2^(1/2)),(1+2^(1/2))^(1/2)/(1-2^(1/2))^(
1/2))-1/4*sum(_alpha*(-1/(_alpha^2)^(1/2)*arctanh((_alpha^2+1)*(-2*_alpha^2
+x^2+1)/(_alpha^2)^(1/2)/(x^4+2*x^2-1)^(1/2))-2*(_alpha^3+_alpha)/(1-2^(1/2
))^(1/2)*(2^(1/2)*x^2-x^2+1)^(1/2)*(-2^(1/2)*x^2-x^2+1)^(1/2)/(x^4+2*x^2-1)
^(1/2)*EllipticPi(x*(1-2^(1/2))^(1/2),-_alpha^2*2^(1/2)-_alpha^2-2^(1/2)-1,
(1+2^(1/2))^(1/2)/(1-2^(1/2))^(1/2))),_alpha=RootOf(_Z^4+_Z^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{4}+2\,x^{2}-1}\,\left(x^{4}+1\right)}{\left(x^{4}+x^{2}-1\right)\left(x^{4}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^4+2*x^2-1)^(1/2)/(x^4-1)/(x^4+x^2-1),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x^4 + 2*x^2 - 1)*(x^4 + 1)/((x^4 + x^2 - 1)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x^{4}+1\right)\,\sqrt{x^{4}+2\,x^{2}-1}}{\left(x^{4}-1\right)\,\left(x^{4}+x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)*(2*x^2 + x^4 - 1)^(1/2))/((x^4 - 1)*(x^2 + x^4 - 1)),x)
```

```
[Out] int(((x^4 + 1)*(2*x^2 + x^4 - 1)^(1/2))/((x^4 - 1)*(x^2 + x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}+1\right)\sqrt{x^{4}+2x^{2}-1}}{\left(x-1\right)\left(x+1\right)\left(x^{2}+1\right)\left(x^{4}+x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)*(x**4+2*x**2-1)**(1/2)/(x**4-1)/(x**4+x**2-1),x)
```

```
[Out] Integral((x**4 + 1)*sqrt(x**4 + 2*x**2 - 1)/((x - 1)*(x + 1)*(x**2 + 1)*(x*
*4 + x**2 - 1)), x)
```

3.596 $\int \frac{\sqrt{-1+x-x^2+x^4}\,(2-x+2x^4)}{(-1+x+x^4)^2}\,dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{x^4-x^2+x-1}\,x}{x^4+x-1}-\tan^{-1}\left(\frac{x}{\sqrt{x^4-x^2+x-1}}\right)$$

Rubi [F] time = 0.60, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-1+x-x^2+x^4}\,(2-x+2x^4)}{(-1+x+x^4)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-1 + x - x^2 + x^4]*(2 - x + 2*x^4))/(-1 + x + x^4)^2,x]
```

```
[Out] 4*Defer[Int][Sqrt[-1 + x - x^2 + x^4]/(-1 + x + x^4)^2, x] - 3*Defer[Int][(
x*Sqrt[-1 + x - x^2 + x^4])/(-1 + x + x^4)^2, x] + 2*Defer[Int][Sqrt[-1 + x
 - x^2 + x^4]/(-1 + x + x^4), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x-x^2+x^4}\,(2-x+2x^4)}{(-1+x+x^4)^2}\,dx &= \int \left(\frac{(4-3x)\sqrt{-1+x-x^2+x^4}}{(-1+x+x^4)^2}+\frac{2\sqrt{-1+x-x^2+x^4}}{-1+x+x^4}\right)dx\\
&= 2\int \frac{\sqrt{-1+x-x^2+x^4}}{-1+x+x^4}\,dx+\int \frac{(4-3x)\sqrt{-1+x-x^2+x^4}}{(-1+x+x^4)^2}\,dx\\
&= 2\int \frac{\sqrt{-1+x-x^2+x^4}}{-1+x+x^4}\,dx+\int \left(\frac{4\sqrt{-1+x-x^2+x^4}}{(-1+x+x^4)^2}-\frac{3x\sqrt{-1+x}}{(-1+x}\right.\\
&= 2\int \frac{\sqrt{-1+x-x^2+x^4}}{-1+x+x^4}\,dx-3\int \frac{x\sqrt{-1+x-x^2+x^4}}{(-1+x+x^4)^2}\,dx+4\int \frac{\sqrt{-}}{(}
\end{aligned}$$

Mathematica [C] time = 6.32, size = 12187, normalized size = 259.30

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[-1 + x - x^2 + x^4]*(2 - x + 2*x^4))/(-1 + x + x^4)^2,x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.17, size = 47, normalized size = 1.00

$$-\frac{\sqrt{x^4-x^2+x-1}\,x}{x^4+x-1}-\tan^{-1}\left(\frac{x}{\sqrt{x^4-x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x - x^2 + x^4]*(2 - x + 2*x^4))/(-1 + x + x^4
)^2,x]
```

```
[Out] -((x*Sqrt[-1 + x - x^2 + x^4])/(-1 + x + x^4)) - ArcTan[x/Sqrt[-1 + x - x^2
 + x^4]]
```

fricas [A] time = 0.47, size = 64, normalized size = 1.36

$$-\frac{\left(x^4+x-1\right)\arctan\left(\frac{2\sqrt{x^4-x^2+x-1}\,x}{x^4-2\,x^2+x-1}\right)+2\sqrt{x^4-x^2+x-1}\,x}{2\left(x^4+x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2+x-1)^(1/2)*(2*x^4-x+2)/(x^4+x-1)^2,x, algorithm="fricas"
)
```

```
[Out] -1/2*((x^4 + x - 1)*arctan(2*sqrt(x^4 - x^2 + x - 1)*x/(x^4 - 2*x^2 + x - 1
)) + 2*sqrt(x^4 - x^2 + x - 1)*x)/(x^4 + x - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2\,x^4-x+2\right)\sqrt{x^4-x^2+x-1}}{\left(x^4+x-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2+x-1)^(1/2)*(2*x^4-x+2)/(x^4+x-1)^2,x, algorithm="giac")
```

```
[Out] integrate((2*x^4 - x + 2)*sqrt(x^4 - x^2 + x - 1)/(x^4 + x - 1)^2, x)
```

maple [C] time = 13.22, size = 6119, normalized size = 130.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^2+x-1)^(1/2)*(2*x^4-x+2)/(x^4+x-1)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2\,x^4-x+2\right)\sqrt{x^4-x^2+x-1}}{\left(x^4+x-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2+x-1)^(1/2)*(2*x^4-x+2)/(x^4+x-1)^2,x, algorithm="maxima"
)
```

```
[Out] integrate((2*x^4 - x + 2)*sqrt(x^4 - x^2 + x - 1)/(x^4 + x - 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(2\,x^4-x+2\right)\,\sqrt{x^4-x^2+x-1}}{\left(x^4+x-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^4 - x + 2)*(x - x^2 + x^4 - 1)^(1/2))/(x + x^4 - 1)^2,x)
```

```
[Out] int(((2*x^4 - x + 2)*(x - x^2 + x^4 - 1)^(1/2))/(x + x^4 - 1)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**2+x-1)**(1/2)*(2*x**4-x+2)/(x**4+x-1)**2,x)
[Out] Timed out
```

3.597 $$\int \frac{x+3x^5}{(-1+x^4)(-a-x+ax^4)\sqrt{-x+x^5}}\,dx$$

Optimal. Leaf size=47

$$\frac{2\sqrt{x^5-x}}{x^4-1}-2\sqrt{a}\tanh^{-1}\left(\frac{x}{\sqrt{a}\sqrt{x^5-x}}\right)$$

Rubi [F] time = 2.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x+3x^5}{(-1+x^4)(-a-x+ax^4)\sqrt{-x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x + 3*x^5)/((-1 + x^4)*(-a - x + a*x^4)*Sqrt[-x + x^5]),x]
```

```
[Out] (-3*x^2)/(2*a*Sqrt[-x + x^5]) + (3*x^2*Sqrt[-(((-1)^(3/4)*(1 + (-1)^(1/4)*x
)^2)/x)]*Sqrt[(I*(1 - x^4))/x^2]*EllipticF[ArcSin[Sqrt[((-1)^(3/4)*(Sqrt[2]
 - 2*(-1)^(1/4)*x + I*Sqrt[2]*x^2))/x]/2], -2*(1 - Sqrt[2])])/(4*Sqrt[2 + S
qrt[2]]*a*(1 + (-1)^(1/4)*x)*Sqrt[-x + x^5]) + (3*x^2*Sqrt[((-1)^(3/4)*(1 -
 (-1)^(1/4)*x)^2)/x]*Sqrt[(I*(1 - x^4))/x^2]*EllipticF[ArcSin[Sqrt[-(((-1)^
(3/4)*(Sqrt[2] + 2*(-1)^(1/4)*x + I*Sqrt[2]*x^2))/x)]/2], -2*(1 - Sqrt[2])]
)/(4*Sqrt[2 + Sqrt[2]]*a*(1 - (-1)^(1/4)*x)*Sqrt[-x + x^5]) + (8*Sqrt[x]*Sq
rt[-1 + x^4]*Defer[Subst][Defer[Int][x^2/((-1 + x^8)^(3/2)*(-a - x^2 + a*x^
8)), x], x, Sqrt[x]])/Sqrt[-x + x^5] + (6*Sqrt[x]*Sqrt[-1 + x^4]*Defer[Subs
t][Defer[Int][x^4/((-1 + x^8)^(3/2)*(-a - x^2 + a*x^8)), x], x, Sqrt[x]])/(
a*Sqrt[-x + x^5])
```

Rubi steps

$$\int \frac{x+3x^5}{\left(-1+x^4\right)\left(-a-x+ax^4\right)\sqrt{-x+x^5}}\,dx = \int \frac{x\left(1+3x^4\right)}{\left(-1+x^4\right)\left(-a-x+ax^4\right)\sqrt{-x+x^5}}\,dx$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^4}\right)\int \frac{\sqrt{x}\left(1+3x^4\right)}{\left(-1+x^4\right)^{3/2}\left(-a-x+ax^4\right)}\,dx}{\sqrt{-x+x^5}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \frac{x^2\left(1+3x^8\right)}{\left(-1+x^8\right)^{3/2}\left(-a-x^2+ax^8\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{-x+x^5}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int\left(\frac{3x^2}{a\left(-1+x^8\right)^{3/2}}+\frac{x^2\left(4a+3x^2\right)}{a\left(-1+x^8\right)^{3/2}\left(-a-x^2+ax^8\right)}\right) dx \ldots}{\sqrt{-x+x^5}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \frac{x^2\left(4a+3x^2\right)}{\left(-1+x^8\right)^{3/2}\left(-a-x^2+ax^8\right)}\,dx, x, \sqrt{x}\right)}{a\sqrt{-x+x^5}} + \frac{\left(6\sqrt{\ldots}\right.}{\ldots}$$

$$= -\frac{3x^2}{2a\sqrt{-x+x^5}} - \frac{\left(3\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x^8}}\,dx, x, \sqrt{x}\right)}{2a\sqrt{-x+x^5}} + \ldots$$

$$= -\frac{3x^2}{2a\sqrt{-x+x^5}} + \frac{\left(8\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \frac{x^2}{\left(-1+x^8\right)^{3/2}\left(-a-x^2+ax^8\right)}\,d\ldots\right.}{\sqrt{-x+x^5}}$$

$$= -\frac{3x^2}{2a\sqrt{-x+x^5}} + \frac{3x^2\sqrt{-\frac{(-1)^{3/4}\left(1+\sqrt[4]{-1}\,x\right)^2}{x}}\sqrt{\frac{i\left(1-x^4\right)}{x^2}}\,F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{\frac{(-\ldots}{\ldots}}\right.\right.}{4\sqrt{2+\sqrt{2}}\,a\left(1+\sqrt[4]{-1}\ldots\right.}$$

Mathematica [F] time = 1.49, size = 0, normalized size = 0.00

$$\int \frac{x+3x^5}{\left(-1+x^4\right)\left(-a-x+ax^4\right)\sqrt{-x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x + 3*x^5)/((-1 + x^4)*(-a - x + a*x^4)*Sqrt[-x + x^5]),x]
[Out] Integrate[(x + 3*x^5)/((-1 + x^4)*(-a - x + a*x^4)*Sqrt[-x + x^5]), x]
```

IntegrateAlgebraic [A] time = 2.33, size = 47, normalized size = 1.00

$$\frac{2\sqrt{x^5-x}}{x^4-1} - 2\sqrt{a}\,\tanh^{-1}\left(\frac{x}{\sqrt{a}\sqrt{x^5-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + 3*x^5)/((-1 + x^4)*(-a - x + a*x^4)*Sqrt[-x + x^5])
,x]
[Out] (2*Sqrt[-x + x^5])/(-1 + x^4) - 2*Sqrt[a]*ArcTanh[x/(Sqrt[a]*Sqrt[-x + x^5]
)]
```

fricas [A] time = 0.46, size = 193, normalized size = 4.11

$$\left[\frac{\left(x^{4}-1\right)\sqrt{a}\log\left(\frac{a^{2}x^{8}-2\,a^{2}x^{4}+6\,ax^{5}-4\,\left(ax^{4}-a+x\right)\sqrt{x^{5}-x}\,\sqrt{a}+a^{2}-6\,ax+x^{2}}{a^{2}x^{8}-2\,a^{2}x^{4}-2\,ax^{5}+a^{2}+2\,ax+x^{2}}\right)+4\,\sqrt{x^{5}-x}}{2\,\left(x^{4}-1\right)},\ \frac{\left(x^{4}-1\right)\sqrt{-a}\arctan\left(\frac{\left(ax^{4}-a+x\right)\sqrt{x^{5}-x}\,\sqrt{-a}}{2\,\left(ax^{5}-ax\right)}\right)+2\,\sqrt{x^{5}-x}}{x^{4}-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+x)/(x^4-1)/(a*x^4-a-x)/(x^5-x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/2*((x^4 - 1)*sqrt(a)*log((a^2*x^8 - 2*a^2*x^4 + 6*a*x^5 - 4*(a*x^4 - a +
 x)*sqrt(x^5 - x)*sqrt(a) + a^2 - 6*a*x + x^2)/(a^2*x^8 - 2*a^2*x^4 - 2*a*x
^5 + a^2 + 2*a*x + x^2)) + 4*sqrt(x^5 - x))/(x^4 - 1), ((x^4 - 1)*sqrt(-a)*
arctan(1/2*(a*x^4 - a + x)*sqrt(x^5 - x)*sqrt(-a)/(a*x^5 - a*x)) + 2*sqrt(x
^5 - x))/(x^4 - 1)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^{5}+x}{\left(ax^{4}-a-x\right)\sqrt{x^{5}-x}\,\left(x^{4}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+x)/(x^4-1)/(a*x^4-a-x)/(x^5-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^5 + x)/((a*x^4 - a - x)*sqrt(x^5 - x)*(x^4 - 1)), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{3x^{5}+x}{\left(x^{4}-1\right)\left(a\,x^{4}-a-x\right)\sqrt{x^{5}-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^5+x)/(x^4-1)/(a*x^4-a-x)/(x^5-x)^(1/2),x)
```

```
[Out] int((3*x^5+x)/(x^4-1)/(a*x^4-a-x)/(x^5-x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^{5}+x}{\left(ax^{4}-a-x\right)\sqrt{x^{5}-x}\,\left(x^{4}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+x)/(x^4-1)/(a*x^4-a-x)/(x^5-x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((3*x^5 + x)/((a*x^4 - a - x)*sqrt(x^5 - x)*(x^4 - 1)), x)
```

mupad [B] time = 0.79, size = 61, normalized size = 1.30

$$\frac{2\,\sqrt{x^{5}-x}}{x^{4}-1}+\sqrt{a}\,\ln\left(\frac{a-x+2\,\sqrt{a}\,\sqrt{x^{5}-x}-a\,x^{4}}{-a\,x^{4}+x+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3*x^5)/((x^5 - x)^(1/2)*(x^4 - 1)*(a + x - a*x^4)),x)
```

```
[Out] (2*(x^5 - x)^(1/2))/(x^4 - 1) + a^(1/2)*log((a - x + 2*a^(1/2)*(x^5 - x)^(1
/2) - a*x^4)/(a + x - a*x^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**5+x)/(x**4-1)/(a*x**4-a-x)/(x**5-x)**(1/2),x)
[Out] Timed out
```

3.598 $\int \frac{\sqrt{1+x^2+x^6}(-1+2x^6)}{(1+x^6)(2-x^2+2x^6)}\,dx$

Optimal. Leaf size=47

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^6+x^2+1}}\right)-\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt{x^6+x^2+1}}\right)$$

Rubi [F] time = 1.93, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+x^2+x^6}\left(-1+2x^6\right)}{\left(1+x^6\right)\left(2-x^2+2x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[1 + x^2 + x^6]*(-1 + 2*x^6))/((1 + x^6)*(2 - x^2 + 2*x^6)),x]
```

```
[Out] (-1/2*I)*Defer[Int][Sqrt[1 + x^2 + x^6]/(I - x), x] - (I/2)*Defer[Int][Sqrt
[1 + x^2 + x^6]/(I + x), x] + Defer[Int][Sqrt[1 + x^2 + x^6]/(Sqrt[1 - I*Sq
rt[3]] - Sqrt[2]*x), x]/Sqrt[1 - I*Sqrt[3]] + Defer[Int][Sqrt[1 + x^2 + x^6
]/(Sqrt[1 + I*Sqrt[3]] - Sqrt[2]*x), x]/Sqrt[1 + I*Sqrt[3]] + Defer[Int][Sq
rt[1 + x^2 + x^6]/(Sqrt[1 - I*Sqrt[3]] + Sqrt[2]*x), x]/Sqrt[1 - I*Sqrt[3]]
 + Defer[Int][Sqrt[1 + x^2 + x^6]/(Sqrt[1 + I*Sqrt[3]] + Sqrt[2]*x), x]/Sqr
t[1 + I*Sqrt[3]] + Defer[Int][Sqrt[1 + x^2 + x^6]/(-2 + x^2 - 2*x^6), x] +
6*Defer[Int][(x^4*Sqrt[1 + x^2 + x^6])/(2 - x^2 + 2*x^6), x]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2+x^6}\left(-1+2x^6\right)}{\left(1+x^6\right)\left(2-x^2+2x^6\right)}\,dx = \int \left(\frac{\sqrt{1+x^2+x^6}}{-1-x^2}+\frac{\left(1-2x^2\right)\sqrt{1+x^2+x^6}}{1-x^2+x^4}+\frac{\left(-1+6x^4\right)\sqrt{1+x^2+x^6}}{2-x^2+2x^6}\right)dx$$

$$= \int \frac{\sqrt{1+x^2+x^6}}{-1-x^2}\,dx+\int \frac{\left(1-2x^2\right)\sqrt{1+x^2+x^6}}{1-x^2+x^4}\,dx+\int \frac{\left(-1+6x^4\right)\sqrt{1+x^2-}}{2-x^2+2x^6}$$

$$= \int \left(-\frac{i\sqrt{1+x^2+x^6}}{2(i-x)}-\frac{i\sqrt{1+x^2+x^6}}{2(i+x)}\right)dx+\int \left(-\frac{2\sqrt{1+x^2+x^6}}{-1-i\sqrt{3}+2x^2}-\frac{2\sqrt{1+x^2}}{-1+i\sqrt{3}}\right.$$

$$= -\left(\frac{1}{2}i\int \frac{\sqrt{1+x^2+x^6}}{i-x}\,dx\right)-\frac{1}{2}i\int \frac{\sqrt{1+x^2+x^6}}{i+x}\,dx-2\int \frac{\sqrt{1+x^2+x^6}}{-1-i\sqrt{3}+2x^2}\,dx$$

$$= -\left(\frac{1}{2}i\int \frac{\sqrt{1+x^2+x^6}}{i-x}\,dx\right)-\frac{1}{2}i\int \frac{\sqrt{1+x^2+x^6}}{i+x}\,dx-2\int \left(\frac{\sqrt{1-i\sqrt{3}}\sqrt{1}}{2\left(-1+i\sqrt{3}\right)\left(\sqrt{1}\right.}\right.$$

$$= -\left(\frac{1}{2}i\int \frac{\sqrt{1+x^2+x^6}}{i-x}\,dx\right)-\frac{1}{2}i\int \frac{\sqrt{1+x^2+x^6}}{i+x}\,dx+6\int \frac{x^4\sqrt{1+x^2+x^6}}{2-x^2+2x^6}\,dx$$

Mathematica [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+x^2+x^6}\left(-1+2x^6\right)}{\left(1+x^6\right)\left(2-x^2+2x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[1 + x^2 + x^6]*(-1 + 2*x^6))/((1 + x^6)*(2 - x^2 + 2*x^6)),
x]
```

```
[Out] Integrate[(Sqrt[1 + x^2 + x^6]*(-1 + 2*x^6))/((1 + x^6)*(2 - x^2 + 2*x^6)),
 x]
```

IntegrateAlgebraic [A] time = 2.75, size = 47, normalized size = 1.00

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^6+x^2+1}}\right)-\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt{x^6+x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x^2 + x^6]*(-1 + 2*x^6))/((1 + x^6)*(2 - x^2 +
 2*x^6)),x]
```

```
[Out] ArcTanh[x/Sqrt[1 + x^2 + x^6]] - Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)/Sqrt[1 + x
^2 + x^6]]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^2+1)^(1/2)*(2*x^6-1)/(x^6+1)/(2*x^6-x^2+2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^6-1\right)\sqrt{x^6+x^2+1}}{\left(2\,x^6-x^2+2\right)\left(x^6+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^2+1)^(1/2)*(2*x^6-1)/(x^6+1)/(2*x^6-x^2+2),x, algorithm="g
iac")
```

```
[Out] integrate((2*x^6 - 1)*sqrt(x^6 + x^2 + 1)/((2*x^6 - x^2 + 2)*(x^6 + 1)), x)
```

maple [C] time = 0.66, size = 120, normalized size = 2.55

$$-\frac{\ln\left(-\frac{-x^6+2\sqrt{x^6+x^2+1}\,x-2x^2-1}{\left(x^2+1\right)\left(x^4-x^2+1\right)}\right)}{2}+\frac{\mathrm{RootOf}\left(\_Z^2-6\right)\ln\left(-\frac{-2\,\mathrm{RootOf}\left(\_Z^2-6\right)x^6-5\,\mathrm{RootOf}\left(\_Z^2-6\right)x^2+12\sqrt{x^6+x^2+1}\,x-2\,\mathrm{RootOf}\left(\_Z^2-6\right)}{2x^6-x^2+2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+x^2+1)^(1/2)*(2*x^6-1)/(x^6+1)/(2*x^6-x^2+2),x)
```

```
[Out] -1/2*ln(-(-x^6+2*(x^6+x^2+1)^(1/2)*x-2*x^2-1)/(x^2+1)/(x^4-x^2+1))+1/4*Root
Of(_Z^2-6)*ln(-(-2*RootOf(_Z^2-6)*x^6-5*RootOf(_Z^2-6)*x^2+12*(x^6+x^2+1)^(
1/2)*x-2*RootOf(_Z^2-6))/(2*x^6-x^2+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^6-1\right)\sqrt{x^6+x^2+1}}{\left(2\,x^6-x^2+2\right)\left(x^6+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^2+1)^(1/2)*(2*x^6-1)/(x^6+1)/(2*x^6-x^2+2),x, algorithm="m
axima")
```

```
[Out] integrate((2*x^6 - 1)*sqrt(x^6 + x^2 + 1)/((2*x^6 - x^2 + 2)*(x^6 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(2\,x^6-1\right)\,\sqrt{x^6+x^2+1}}{\left(x^6+1\right)\,\left(2\,x^6-x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^6 - 1)*(x^2 + x^6 + 1)^(1/2))/((x^6 + 1)*(2*x^6 - x^2 + 2)),x)
```

```
[Out] int(((2*x^6 - 1)*(x^2 + x^6 + 1)^(1/2))/((x^6 + 1)*(2*x^6 - x^2 + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+x**2+1)**(1/2)*(2*x**6-1)/(x**6+1)/(2*x**6-x**2+2),x)
```

```
[Out] Timed out
```

3.599 $\int \frac{x+4x^6}{(-1+x^5)(-a-x+ax^5)\sqrt{-x+x^6}}\,dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{x^6-x}}{x^5-1}-2\sqrt{a}\tanh^{-1}\left(\frac{x}{\sqrt{a}\sqrt{x^6-x}}\right)$$

Rubi [F] time = 1.95, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x+4x^6}{(-1+x^5)(-a-x+ax^5)\sqrt{-x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x + 4*x^6)/((-1 + x^5)*(-a - x + a*x^5)*Sqrt[-x + x^6]),x]
```

```
[Out] (-8*x^2)/(5*a*Sqrt[-x + x^6]) - (16*x^2*Sqrt[1 - x^5]*Hypergeometric2F1[3/1
0, 1/2, 13/10, x^5])/(15*a*Sqrt[-x + x^6]) + (10*Sqrt[x]*Sqrt[-1 + x^5]*Def
er[Subst][Defer[Int][x^2/((-1 + x^10)^(3/2)*(-a - x^2 + a*x^10)), x], x, Sq
rt[x]])/Sqrt[-x + x^6] + (8*Sqrt[x]*Sqrt[-1 + x^5]*Defer[Subst][Defer[Int][
x^4/((-1 + x^10)^(3/2)*(-a - x^2 + a*x^10)), x], x, Sqrt[x]])/(a*Sqrt[-x +
x^6])
```

Rubi steps

$$\begin{aligned}
\int \frac{x+4x^6}{(-1+x^5)(-a-x+ax^5)\sqrt{-x+x^6}}\,dx &= \int \frac{x(1+4x^5)}{(-1+x^5)(-a-x+ax^5)\sqrt{-x+x^6}}\,dx\\
&= \frac{\left(\sqrt{x}\sqrt{-1+x^5}\right)\int \frac{\sqrt{x}(1+4x^5)}{(-1+x^5)^{3/2}(-a-x+ax^5)}\,dx}{\sqrt{-x+x^6}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{x^2(1+4x^{10})}{(-1+x^{10})^{3/2}(-a-x^2+ax^{10})}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \left(\frac{4x^2}{a(-1+x^{10})^{3/2}}+\frac{x^2(5a+4x^2)}{a(-1+x^{10})^{3/2}(-a-x^2+ax^{10})}\right)\ldots\right.}{\sqrt{-x+x^6}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{x^2(5a+4x^2)}{(-1+x^{10})^{3/2}(-a-x^2+ax^{10})}\,dx,x,\sqrt{x}\right)}{a\sqrt{-x+x^6}}+\frac{(8\ldots}{\ldots}\\
&= -\frac{8x^2}{5a\sqrt{-x+x^6}}+\frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \left(\frac{5ax^2}{(-1+x^{10})^{3/2}(-a-x^2+ax^{10}}\ldots\right.\right.}{a\sqrt{-x+x^6}}\\
&= -\frac{8x^2}{5a\sqrt{-x+x^6}}-\frac{\left(16\sqrt{x}\sqrt{1-x^5}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^{10}}}\,dx,x,\sqrt{x}\right)}{5a\sqrt{-x+x^6}}+\ldots\\
&= -\frac{8x^2}{5a\sqrt{-x+x^6}}-\frac{16x^2\sqrt{1-x^5}\,{}_2F_1\left(\frac{3}{10},\frac{1}{2};\frac{13}{10};x^5\right)}{15a\sqrt{-x+x^6}}+\frac{\left(10\sqrt{x}\sqrt{-1+\ldots\right.}{\ldots}
\end{aligned}$$

Mathematica [F] time = 1.51, size = 0, normalized size = 0.00

$$\int \frac{x+4x^6}{(-1+x^5)(-a-x+ax^5)\sqrt{-x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x + 4*x^6)/((-1 + x^5)*(-a - x + a*x^5)*Sqrt[-x + x^6]),x]
```

```
[Out] Integrate[(x + 4*x^6)/((-1 + x^5)*(-a - x + a*x^5)*Sqrt[-x + x^6]), x]
```

IntegrateAlgebraic [A] time = 2.72, size = 47, normalized size = 1.00

$$\frac{2\sqrt{x^6-x}}{x^5-1}-2\sqrt{a}\tanh^{-1}\left(\frac{x}{\sqrt{a}\sqrt{x^6-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + 4*x^6)/((-1 + x^5)*(-a - x + a*x^5)*Sqrt[-x + x^6])
,x]
```

```
[Out] (2*Sqrt[-x + x^6])/(-1 + x^5) - 2*Sqrt[a]*ArcTanh[x/(Sqrt[a]*Sqrt[-x + x^6]
)]
```

fricas [A] time = 0.54, size = 184, normalized size = 3.91

$$\left[\frac{(x^5-1)\sqrt{a}\log\left(-\frac{a^2x^{10}-2a^2x^5+6ax^6-4(ax^5-a+x)\sqrt{x^6-x}\sqrt{a}+a^2-6ax+x^2}{a^2x^{10}-2a^2x^5-2ax^6+a^2+2ax+x^2}\right)+4\sqrt{x^6-x}}{2(x^5-1)}, \frac{(x^5-1)\sqrt{-a}\arctan\left(\frac{2\sqrt{x^6-x}\sqrt{-a}}{ax^5-a+x}\right)+2\sqrt{x^6-x}}{x^5-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6+x)/(x^5-1)/(a*x^5-a-x)/(x^6-x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/2*((x^5 - 1)*sqrt(a)*log(-(a^2*x^10 - 2*a^2*x^5 + 6*a*x^6 - 4*(a*x^5 - a
 + x)*sqrt(x^6 - x)*sqrt(a) + a^2 - 6*a*x + x^2)/(a^2*x^10 - 2*a^2*x^5 - 2*
a*x^6 + a^2 + 2*a*x + x^2)) + 4*sqrt(x^6 - x))/(x^5 - 1), ((x^5 - 1)*sqrt(-
a)*arctan(2*sqrt(x^6 - x)*sqrt(-a)/(a*x^5 - a + x)) + 2*sqrt(x^6 - x))/(x^5
 - 1)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6+x)/(x^5-1)/(a*x^5-a-x)/(x^6-x)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{4x^6+x}{(x^5-1)(a\,x^5-a-x)\sqrt{x^6-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^6+x)/(x^5-1)/(a*x^5-a-x)/(x^6-x)^(1/2),x)
```

```
[Out] int((4*x^6+x)/(x^5-1)/(a*x^5-a-x)/(x^6-x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^6 + x}{(ax^5 - a - x)\sqrt{x^6 - x}\,(x^5 - 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6+x)/(x^5-1)/(a*x^5-a-x)/(x^6-x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((4*x^6 + x)/((a*x^5 - a - x)*sqrt(x^6 - x)*(x^5 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{4x^6 + x}{\sqrt{x^6 - x}\,(x^5 - 1)\,(-a\,x^5 + x + a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 4*x^6)/((x^6 - x)^(1/2)*(x^5 - 1)*(a + x - a*x^5)),x)
```

```
[Out] -int((x + 4*x^6)/((x^6 - x)^(1/2)*(x^5 - 1)*(a + x - a*x^5)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**6+x)/(x**5-1)/(a*x**5-a-x)/(x**6-x)**(1/2),x)
```

```
[Out] Timed out
```

3.600 $\int \frac{\left(-1+6x^4\right)\sqrt{x+2x^5}}{\left(1+2x^4\right)\left(1-x^2+4x^4+4x^8\right)}\,dx$

Optimal. Leaf size=47

$$\tan^{-1}\left(\frac{\sqrt{2x^5+x}}{2x^4+1}\right)-\tanh^{-1}\left(\frac{\sqrt{2x^5+x}}{2x^4+1}\right)$$

Rubi [F] time = 1.80, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1+6x^4\right)\sqrt{x+2x^5}}{\left(1+2x^4\right)\left(1-x^2+4x^4+4x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + 6*x^4)*Sqrt[x + 2*x^5])/((1 + 2*x^4)*(1 - x^2 + 4*x^4 + 4*x^8)),
x]
```

```
[Out] (-4*Sqrt[x + 2*x^5]*Defer[Subst][Defer[Int][1/(Sqrt[1 + 2*x^8]*(1 - x^2 + 2
*x^8)), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + 2*x^4]) + (3*Sqrt[x + 2*x^5]*Def
er[Subst][Defer[Int][x^2/(Sqrt[1 + 2*x^8]*(1 - x^2 + 2*x^8)), x], x, Sqrt[x
]])/(Sqrt[x]*Sqrt[1 + 2*x^4]) + (4*Sqrt[x + 2*x^5]*Defer[Subst][Defer[Int][
1/(Sqrt[1 + 2*x^8]*(1 + x^2 + 2*x^8)), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + 2
*x^4]) + (3*Sqrt[x + 2*x^5]*Defer[Subst][Defer[Int][x^2/(Sqrt[1 + 2*x^8]*(1
 + x^2 + 2*x^8)), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + 2*x^4])
```

Rubi steps

$$\int \frac{\left(-1+6x^4\right)\sqrt{x+2x^5}}{\left(1+2x^4\right)\left(1-x^2+4x^4+4x^8\right)}\,dx = \frac{\sqrt{x+2x^5}\int \frac{\sqrt{x}\left(-1+6x^4\right)}{\sqrt{1+2x^4}\left(1-x^2+4x^4+4x^8\right)}\,dx}{\sqrt{x}\sqrt{1+2x^4}}$$

$$= \frac{\left(2\sqrt{x+2x^5}\right)\text{Subst}\left(\int \frac{x^2\left(-1+6x^8\right)}{\sqrt{1+2x^8}\left(1-x^4+4x^8+4x^{16}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1+2x^4}}$$

$$= \frac{\left(2\sqrt{x+2x^5}\right)\text{Subst}\left(\int \left(\frac{-4+3x^2}{2\sqrt{1+2x^8}\left(1-x^2+2x^8\right)}+\frac{4+3x^2}{2\sqrt{1+2x^8}\left(1+x^2+2x^8\right)}\right)dx,x,\sqrt{x}\right.}{\sqrt{x}\sqrt{1+2x^4}}$$

$$= \frac{\sqrt{x+2x^5}\,\text{Subst}\left(\int \frac{-4+3x^2}{\sqrt{1+2x^8}\left(1-x^2+2x^8\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1+2x^4}}+\frac{\sqrt{x+2x^5}\,\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.}{\sqrt{x}\sqrt{}}$$

$$= \frac{\sqrt{x+2x^5}\,\text{Subst}\left(\int \left(-\frac{4}{\sqrt{1+2x^8}\left(1-x^2+2x^8\right)}+\frac{3x^2}{\sqrt{1+2x^8}\left(1-x^2+2x^8\right)}\right)dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1+2x^4}}+$$

$$= \frac{\left(3\sqrt{x+2x^5}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+2x^8}\left(1-x^2+2x^8\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1+2x^4}}+\frac{\left(3\sqrt{x+2x^5}\right)\text{Subst}}{}$$

Mathematica [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+6x^4\right)\sqrt{x+2x^5}}{\left(1+2x^4\right)\left(1-x^2+4x^4+4x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + 6*x^4)*Sqrt[x + 2*x^5])/((1 + 2*x^4)*(1 - x^2 + 4*x^4 + 4*
x^8)),x]
```

```
[Out] Integrate[((-1 + 6*x^4)*Sqrt[x + 2*x^5])/((1 + 2*x^4)*(1 - x^2 + 4*x^4 + 4*
x^8)), x]
```

IntegrateAlgebraic [A] time = 0.52, size = 47, normalized size = 1.00

$$\tan^{-1}\left(\frac{\sqrt{2x^5+x}}{2x^4+1}\right)-\tanh^{-1}\left(\frac{\sqrt{2x^5+x}}{2x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + 6*x^4)*Sqrt[x + 2*x^5])/((1 + 2*x^4)*(1 - x^2 + 4
*x^4 + 4*x^8)),x]
```

```
[Out] ArcTan[Sqrt[x + 2*x^5]/(1 + 2*x^4)] - ArcTanh[Sqrt[x + 2*x^5]/(1 + 2*x^4)]
```

fricas [A] time = 0.43, size = 60, normalized size = 1.28

$$-\frac{1}{2}\arctan\left(\frac{2x^4-x+1}{2\sqrt{2x^5+x}}\right)+\frac{1}{2}\log\left(\frac{2x^4+x-2\sqrt{2x^5+x}+1}{2x^4-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^4-1)*(2*x^5+x)^(1/2)/(2*x^4+1)/(4*x^8+4*x^4-x^2+1),x, algori
thm="fricas")
```

```
[Out] -1/2*arctan(1/2*(2*x^4 - x + 1)/sqrt(2*x^5 + x)) + 1/2*log((2*x^4 + x - 2*s
qrt(2*x^5 + x) + 1)/(2*x^4 - x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{2x^5+x}\left(6x^4-1\right)}{\left(4x^8+4x^4-x^2+1\right)\left(2x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^4-1)*(2*x^5+x)^(1/2)/(2*x^4+1)/(4*x^8+4*x^4-x^2+1),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(2*x^5 + x)*(6*x^4 - 1)/((4*x^8 + 4*x^4 - x^2 + 1)*(2*x^4 + 1
)), x)
```

maple [C] time = 0.98, size = 97, normalized size = 2.06

$$\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)x^4-\mathrm{RootOf}\left(\_Z^2+1\right)x+2\sqrt{2x^5+x}+\mathrm{RootOf}\left(\_Z^2+1\right)}{2x^4+x+1}\right)}{2}-\frac{\ln\left(-\frac{2x^4+2\sqrt{2x^5+x}+x+1}{2x^4-x+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x^4-1)*(2*x^5+x)^(1/2)/(2*x^4+1)/(4*x^8+4*x^4-x^2+1),x)
```

```
[Out] 1/2*RootOf(_Z^2+1)*ln(-(2*RootOf(_Z^2+1)*x^4-RootOf(_Z^2+1)*x+2*(2*x^5+x)^(
1/2)+RootOf(_Z^2+1))/(2*x^4+x+1))-1/2*ln(-(2*x^4+2*(2*x^5+x)^(1/2)+x+1)/(2*
x^4-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^5+x}\,\left(6\,x^4-1\right)}{\left(4\,x^8+4\,x^4-x^2+1\right)\left(2\,x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^4-1)*(2*x^5+x)^(1/2)/(2*x^4+1)/(4*x^8+4*x^4-x^2+1),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(2*x^5 + x)*(6*x^4 - 1)/((4*x^8 + 4*x^4 - x^2 + 1)*(2*x^4 + 1
)), x)
```

mupad [B] time = 1.34, size = 72, normalized size = 1.53

$$\frac{\ln\left(\frac{x}{2}-\sqrt{2\,x^5+x}+x^4+\frac{1}{2}\right)}{2}-\frac{\ln\left(2\,x^4-x+1\right)}{2}+\frac{\ln\left(x^4-\frac{x}{2}+\frac{1}{2}-\sqrt{2\,x^5+x}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}-\frac{\ln\left(2\,x^4+x+1\right)\,1{}\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 2*x^5)^(1/2)*(6*x^4 - 1))/((2*x^4 + 1)*(4*x^4 - x^2 + 4*x^8 + 1))
,x)
```

```
[Out] log(x/2 - (x + 2*x^5)^(1/2) + x^4 + 1/2)/2 + (log(x^4 - (x + 2*x^5)^(1/2)*1
i - x/2 + 1/2)*1i)/2 - (log(x + 2*x^4 + 1)*1i)/2 - log(2*x^4 - x + 1)/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x**4-1)*(2*x**5+x)**(1/2)/(2*x**4+1)/(4*x**8+4*x**4-x**2+1),x)
```

```
[Out] Timed out
```

3.601 $\int \sqrt{1+\sqrt{1+x}}\, dx$

Optimal. Leaf size=47

$$\frac{4}{15}\sqrt{\sqrt{x+1}+1}\,(3x+1)+\frac{4}{15}\sqrt{x+1}\sqrt{\sqrt{x+1}+1}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {247, 190, 43}

$$\frac{4}{5}\left(\sqrt{x+1}+1\right)^{5/2}-\frac{4}{3}\left(\sqrt{x+1}+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 + x]],x]
```

```
[Out] (-4*(1 + Sqrt[1 + x])^(3/2))/3 + (4*(1 + Sqrt[1 + x])^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1+\sqrt{1+x}}\, dx &= \text{Subst}\left(\int \sqrt{1+\sqrt{x}}\, dx, x, 1+x\right)\\
&= 2\,\text{Subst}\left(\int x\sqrt{1+x}\, dx, x, \sqrt{1+x}\right)\\
&= 2\,\text{Subst}\left(\int \left(-\sqrt{1+x}+(1+x)^{3/2}\right) dx, x, \sqrt{1+x}\right)\\
&= -\frac{4}{3}\left(1+\sqrt{1+x}\right)^{3/2}+\frac{4}{5}\left(1+\sqrt{1+x}\right)^{5/2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.60

$$\frac{4}{15}\left(\sqrt{x+1}+1\right)^{3/2}\left(3\sqrt{x+1}-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + x]],x]
[Out] (4*(1 + Sqrt[1 + x])^(3/2)*(-2 + 3*Sqrt[1 + x]))/15
```

IntegrateAlgebraic [A] time = 0.02, size = 28, normalized size = 0.60

$$\frac{4}{15}\left(\sqrt{x+1}+1\right)^{3/2}\left(3\sqrt{x+1}-2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 + x]],x]
[Out] (4*(1 + Sqrt[1 + x])^(3/2)*(-2 + 3*Sqrt[1 + x]))/15
```

fricas [A] time = 0.38, size = 21, normalized size = 0.45

$$\frac{4}{15}\left(3x+\sqrt{x+1}+1\right)\sqrt{\sqrt{x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 4/15*(3*x + sqrt(x + 1) + 1)*sqrt(sqrt(x + 1) + 1)
```

giac [C] time = 0.23, size = 23, normalized size = 0.49

$$\frac{4}{5}\left(\sqrt{x+1}+1\right)^{\frac{5}{2}}-\frac{4}{3}\left(\sqrt{x+1}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+x)^(1/2))^(1/2),x, algorithm="giac")
[Out] 4/5*(sqrt(x + 1) + 1)^(5/2) - 4/3*(sqrt(x + 1) + 1)^(3/2)
```

maple [A] time = 0.01, size = 24, normalized size = 0.51

$$\frac{4\left(1+\sqrt{1+x}\right)^{\frac{5}{2}}}{5}-\frac{4\left(1+\sqrt{1+x}\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(1+x)^(1/2))^(1/2),x)
[Out] 4/5*(1+(1+x)^(1/2))^(5/2)-4/3*(1+(1+x)^(1/2))^(3/2)
```

maxima [C] time = 0.32, size = 23, normalized size = 0.49

$$\frac{4}{5}\left(\sqrt{x+1}+1\right)^{\frac{5}{2}}-\frac{4}{3}\left(\sqrt{x+1}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] 4/5*(sqrt(x + 1) + 1)^(5/2) - 4/3*(sqrt(x + 1) + 1)^(3/2)
```

mupad [B] time = 0.52, size = 16, normalized size = 0.34

$$(x+1)\ {}_2\mathrm{F}_1\left(-\frac{1}{2},2;\ 3;\ -\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] (x + 1)*hypergeom([-1/2, 2], 3, -(x + 1)^(1/2))
```

sympy [B] time = 0.80, size = 184, normalized size = 3.91

$$\frac{12(x+1)^{\frac{7}{2}}\sqrt{\sqrt{x+1}+1}}{15(x+1)^{\frac{5}{2}}+15(x+1)^{2}}-\frac{4(x+1)^{\frac{5}{2}}\sqrt{\sqrt{x+1}+1}}{15(x+1)^{\frac{5}{2}}+15(x+1)^{2}}+\frac{8(x+1)^{\frac{5}{2}}}{15(x+1)^{\frac{5}{2}}+15(x+1)^{2}}+\frac{16(x+1)^{3}\sqrt{\sqrt{x+1}+1}}{15(x+1)^{\frac{5}{2}}+15(x+1)^{2}}-\frac{8(x+1)^{2}\sqrt{\sqrt{x+1}+1}}{15(x+1)^{\frac{5}{2}}+15(x+1)^{2}}+\frac{8(x+1)^{2}}{15(x+1)^{\frac{5}{2}}+15(x+1)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+x)**(1/2))**(1/2),x)
```

```
[Out] 12*(x + 1)**(7/2)*sqrt(sqrt(x + 1) + 1)/(15*(x + 1)**(5/2) + 15*(x + 1)**2)
 - 4*(x + 1)**(5/2)*sqrt(sqrt(x + 1) + 1)/(15*(x + 1)**(5/2) + 15*(x + 1)**
2) + 8*(x + 1)**(5/2)/(15*(x + 1)**(5/2) + 15*(x + 1)**2) + 16*(x + 1)**3*s
qrt(sqrt(x + 1) + 1)/(15*(x + 1)**(5/2) + 15*(x + 1)**2) - 8*(x + 1)**2*sqr
t(sqrt(x + 1) + 1)/(15*(x + 1)**(5/2) + 15*(x + 1)**2) + 8*(x + 1)**2/(15*(
x + 1)**(5/2) + 15*(x + 1)**2)
```

3.602 $\int \frac{\sqrt{1+\sqrt{1+x^2}}}{x} dx$

Optimal. Leaf size=47

$$2\sqrt{\sqrt{x^2+1}+1}-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+1}}{\sqrt{2}}\right)$$

Rubi [A] time = 0.11, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {371, 1398, 783, 80, 63, 207}

$$2\sqrt{\sqrt{x^2+1}+1}-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 + x^2]]/x,x]
```

```
[Out] 2*Sqrt[1 + Sqrt[1 + x^2]] - Sqrt[2]*ArcTanh[Sqrt[1 + Sqrt[1 + x^2]]/Sqrt[2]
]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 783

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+\sqrt{1+x^2}}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1+\sqrt{1+x}}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1+\sqrt{x}}}{-1+x}\,dx, x, 1+x^2\right)\\
&= \text{Subst}\left(\int \frac{x\sqrt{1+x}}{-1+x^2}\,dx, x, \sqrt{1+x^2}\right)\\
&= \text{Subst}\left(\int \frac{x}{(-1+x)\sqrt{1+x}}\,dx, x, \sqrt{1+x^2}\right)\\
&= 2\sqrt{1+\sqrt{1+x^2}} + \text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{1+x}}\,dx, x, \sqrt{1+x^2}\right)\\
&= 2\sqrt{1+\sqrt{1+x^2}} + 2\,\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{1+\sqrt{1+x^2}}\right)\\
&= 2\sqrt{1+\sqrt{1+x^2}} - \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{1+x^2}}}{\sqrt{2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 47, normalized size = 1.00

$$2\sqrt{\sqrt{x^2+1}+1} - \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + x^2]]/x,x]
```

```
[Out] 2*Sqrt[1 + Sqrt[1 + x^2]] - Sqrt[2]*ArcTanh[Sqrt[1 + Sqrt[1 + x^2]]/Sqrt[2]
]
```

IntegrateAlgebraic [A] time = 0.11, size = 47, normalized size = 1.00

$$2\sqrt{\sqrt{x^2+1}+1} - \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 + x^2]]/x,x]
```

```
[Out] 2*Sqrt[1 + Sqrt[1 + x^2]] - Sqrt[2]*ArcTanh[Sqrt[1 + Sqrt[1 + x^2]]/Sqrt[2]
]
```

fricas [A] time = 0.82, size = 67, normalized size = 1.43

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{x^2-2\left(\sqrt{2}\sqrt{x^2+1}+\sqrt{2}\right)\sqrt{\sqrt{x^2+1}+1}+4\sqrt{x^2+1}+4}{x^2}\right)+2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(x^2 - 2*(sqrt(2)*sqrt(x^2 + 1) + sqrt(2))*sqrt(sqrt(x^2 +
 1) + 1) + 4*sqrt(x^2 + 1) + 4)/x^2) + 2*sqrt(sqrt(x^2 + 1) + 1)
```

giac [A] time = 0.43, size = 56, normalized size = 1.19

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{\sqrt{x^2+1}+1}}{\sqrt{2}+\sqrt{\sqrt{x^2+1}+1}}\right)+2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - sqrt(sqrt(x^2 + 1) + 1))/(sqrt(2) + sqrt(sqrt(x
^2 + 1) + 1))) + 2*sqrt(sqrt(x^2 + 1) + 1)
```

maple [C] time = 0.08, size = 51, normalized size = 1.09

$$-\frac{-\frac{\sqrt{\pi}\,\sqrt{2}\,x^2\,\mathrm{hypergeom}\left(\left[\frac{3}{4},1,1,\frac{5}{4}\right],\left[\frac{3}{2},2,2\right],-x^2\right)}{2}-4\left(-4\ln(2)+4+2\ln(x)\right)\sqrt{\pi}\,\sqrt{2}}{8\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x^2+1)^(1/2))^(1/2)/x,x)
```

```
[Out] -1/8/Pi^(1/2)*(-1/2*Pi^(1/2)*2^(1/2)*x^2*hypergeom([3/4,1,1,5/4],[3/2,2,2],
-x^2)-4*(-4*ln(2)+4+2*ln(x))*Pi^(1/2)*2^(1/2))
```

maxima [A] time = 0.45, size = 56, normalized size = 1.19

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{\sqrt{x^2+1}+1}}{\sqrt{2}+\sqrt{\sqrt{x^2+1}+1}}\right)+2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - sqrt(sqrt(x^2 + 1) + 1))/(sqrt(2) + sqrt(sqrt(x
^2 + 1) + 1))) + 2*sqrt(sqrt(x^2 + 1) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)^(1/2) + 1)^(1/2)/x,x)
```

```
[Out] int(((x^2 + 1)^(1/2) + 1)^(1/2)/x, x)
```

sympy [C] time = 1.14, size = 49, normalized size = 1.04

$$\frac{x^2\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_4F_3\left(\begin{matrix}\frac{3}{4},1,1,\frac{5}{4}\\ \frac{3}{2},2,2\end{matrix}\middle|x^2e^{i\pi}\right)}{4\pi}+\frac{\log\left(x^2\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x**2+1)**(1/2))**(1/2)/x,x)
```

```
[Out] x**2*gamma(3/4)*gamma(5/4)*hyper((3/4, 1, 1, 5/4), (3/2, 2, 2), x**2*exp_po
lar(I*pi))/(4*pi) + log(x**2)*gamma(1/4)*gamma(3/4)/(2*pi)
```

3.603 $\int \frac{1}{1+\sqrt[4]{9-6x+x^2}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{(x-3)^2}\left(2\sqrt[4]{x^2-6x+9}-2\log\left(\sqrt[4]{x^2-6x+9}+1\right)\right)}{x-3}$$

Rubi [B] time = 0.20, antiderivative size = 215, normalized size of antiderivative = 4.57, number of steps used = 26, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6742, 646, 50, 63, 204, 207, 43}

$$-\frac{2\left(x^2-6x+9\right)^{3/4}}{3-x}+\frac{\sqrt{x^2-6x+9}\log(2-x)}{2(3-x)}+\frac{\sqrt{x^2-6x+9}\log(4-x)}{2(3-x)}-\frac{\left(x^2-6x+9\right)^{3/4}\tan^{-1}\left(\sqrt{x-3}\right)}{(x-3)^{3/2}}+\frac{\sqrt[4]{x^2-6x+9}\tan^{-1}\left(\sqrt{x-3}\right)}{\sqrt{x-3}}-\frac{\left(x^2-6x+9\right)^{3/4}\tanh^{-1}\left(\sqrt{x-3}\right)}{(x-3)^{3/2}}-\frac{\sqrt[4]{x^2-6x+9}\tanh^{-1}\left(\sqrt{x-3}\right)}{\sqrt{x-3}}+\frac{1}{2}\log(2-x)-\frac{1}{2}\log(4-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (9 - 6*x + x^2)^(1/4))^(-1),x]
```

```
[Out] (-2*(9 - 6*x + x^2)^(3/4))/(3 - x) + ((9 - 6*x + x^2)^(1/4)*ArcTan[Sqrt[-3
+ x]])/Sqrt[-3 + x] - ((9 - 6*x + x^2)^(3/4)*ArcTan[Sqrt[-3 + x]])/(-3 + x)
^(3/2) - ((9 - 6*x + x^2)^(1/4)*ArcTanh[Sqrt[-3 + x]])/Sqrt[-3 + x] - ((9 -
 6*x + x^2)^(3/4)*ArcTanh[Sqrt[-3 + x]])/(-3 + x)^(3/2) + Log[2 - x]/2 + (S
qrt[9 - 6*x + x^2]*Log[2 - x])/(2*(3 - x)) - Log[4 - x]/2 + (Sqrt[9 - 6*x +
 x^2]*Log[4 - x])/(2*(3 - x))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+\sqrt[4]{9-6x+x^2}}\,dx &= \int \left(-\frac{1}{2(-4+x)}+\frac{1}{2(-2+x)}+\frac{\sqrt[4]{9-6x+x^2}}{2(2-x)}+\frac{\sqrt[4]{9-6x+x^2}}{2(-4+x)}+\frac{\sqrt{9-6x+x^2}}{2(4-x)}+\cdots\right.\\
&= \frac{1}{2}\log(2-x)-\frac{1}{2}\log(4-x)+\frac{1}{2}\int \frac{\sqrt[4]{9-6x+x^2}}{2-x}\,dx+\frac{1}{2}\int \frac{\sqrt[4]{9-6x+x^2}}{-4+x}\,dx+\frac{1}{2}\cdots\\
&= \frac{1}{2}\log(2-x)-\frac{1}{2}\log(4-x)+\frac{\sqrt[4]{9-6x+x^2}\int \frac{\sqrt{-3+x}}{2-x}\,dx}{2\sqrt{-3+x}}+\frac{\sqrt[4]{9-6x+x^2}\int \frac{\sqrt{-3+x}}{-4+x}\cdots}{2\sqrt{-3+x}}\\
&= \frac{1}{2}\log(2-x)-\frac{1}{2}\log(4-x)-\frac{\sqrt[4]{9-6x+x^2}\int \frac{1}{(2-x)\sqrt{-3+x}}\,dx}{2\sqrt{-3+x}}+\frac{\sqrt[4]{9-6x+x^2}\int \frac{\cdots}{(-4\cdots}}{2\sqrt{-3+\cdots}}\\
&= -\frac{2\left(9-6x+x^2\right)^{3/4}}{3-x}+\frac{1}{2}\log(2-x)+\frac{\sqrt{9-6x+x^2}\,\log(2-x)}{2(3-x)}-\frac{1}{2}\log(4-x)+\cdots\\
&= -\frac{2\left(9-6x+x^2\right)^{3/4}}{3-x}+\frac{\sqrt[4]{9-6x+x^2}\,\tan^{-1}\left(\sqrt{-3+x}\right)}{\sqrt{-3+x}}-\frac{\sqrt[4]{9-6x+x^2}\,\tanh^{-1}\left(\sqrt{\cdots}\right.}{\sqrt{-3+x}}\\
&= -\frac{2\left(9-6x+x^2\right)^{3/4}}{3-x}+\frac{\sqrt[4]{9-6x+x^2}\,\tan^{-1}\left(\sqrt{-3+x}\right)}{\sqrt{-3+x}}-\frac{\left(9-6x+x^2\right)^{3/4}\tan^{-1}\left(\sqrt{\cdots}\right.}{(-3+x)^{3/2}}
\end{aligned}$$

Mathematica [B] time = 0.24, size = 138, normalized size = 2.94

$$\frac{\sqrt{x-3}\left(4\left((x-3)^2\right)^{3/4}-\left(\left(-x+\sqrt{(x-3)^2}+3\right)\log(2-x)\right)-\left(x+\sqrt{(x-3)^2}-3\right)\log(4-x)\right)-2\left(-x+\sqrt{(x-3)^2}+3\right)\sqrt[4]{(x-3)^2}\,\tan^{-1}\left(\sqrt{x-3}\right)-2\sqrt[4]{(x-3)^2}\left(x+\sqrt{(x-3)^2}-3\right)\tanh^{-1}\left(\sqrt{x-3}\right)}{2(x-3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (9 - 6*x + x^2)^(1/4))^(-1),x]
```

```
[Out] (-2*(3 + Sqrt[(-3 + x)^2] - x)*((-3 + x)^2)^(1/4)*ArcTan[Sqrt[-3 + x]] - 2*
((-3 + x)^2)^(1/4)*(-3 + Sqrt[(-3 + x)^2] + x)*ArcTanh[Sqrt[-3 + x]] + Sqrt
[-3 + x]*(4*((-3 + x)^2)^(3/4) - (3 + Sqrt[(-3 + x)^2] - x)*Log[2 - x] - (-
3 + Sqrt[(-3 + x)^2] + x)*Log[4 - x]))/(2*(-3 + x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.11, size = 47, normalized size = 1.00

$$\frac{\sqrt{(x-3)^2}\left(2\sqrt[4]{x^2-6x+9}-2\log\left(\sqrt[4]{x^2-6x+9}+1\right)\right)}{x-3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + (9 - 6*x + x^2)^(1/4))^(-1),x]
```

```
[Out] (Sqrt[(-3 + x)^2]*(2*(9 - 6*x + x^2)^(1/4) - 2*Log[1 + (9 - 6*x + x^2)^(1/4
)]))/(-3 + x)
```

fricas [A] time = 0.39, size = 28, normalized size = 0.60

$$2\left(x^{2}-6\,x+9\right)^{\frac{1}{4}}-2\,\log\left(\left(x^{2}-6\,x+9\right)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^2-6*x+9)^(1/4)),x, algorithm="fricas")
```

```
[Out] 2*(x^2 - 6*x + 9)^(1/4) - 2*log((x^2 - 6*x + 9)^(1/4) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{2}-6\,x+9\right)^{\frac{1}{4}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^2-6*x+9)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - 6*x + 9)^(1/4) + 1), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{1+\left(x^{2}-6x+9\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(x^2-6*x+9)^(1/4)),x)
```

```
[Out] int(1/(1+(x^2-6*x+9)^(1/4)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{2}-6\,x+9\right)^{\frac{1}{4}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^2-6*x+9)^(1/4)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - 6*x + 9)^(1/4) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(x^2-6\,x+9\right)}^{1/4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 6*x + 9)^(1/4) + 1),x)
```

```
[Out] int(1/((x^2 - 6*x + 9)^(1/4) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^2 - 6x + 9} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x**2-6*x+9)**(1/4)),x)
```

```
[Out] Integral(1/((x**2 - 6*x + 9)**(1/4) + 1), x)
```

3.604 $\int \frac{x^2}{(-2+x^2)(-1+x^2)^{3/4}}\,dx$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 + x^2)*(-1 + x^2)^(3/4)),x]
```

```
[Out] ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))]/Sqrt[2] - ArcTanh[x/(Sqrt[2]*(-1 + x^2
)^(1/4))]/Sqrt[2]
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2+x^2\right)\left(-1+x^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{\sqrt{2}}$$

Mathematica [C] time = 0.04, size = 48, normalized size = 1.00

$$-\frac{x^3\left(1-x^2\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};x^2,\frac{x^2}{2}\right)}{6\left(x^2-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2 + x^2)*(-1 + x^2)^(3/4)),x]
```

```
[Out] -1/6*(x^3*(1 - x^2)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, x^2, x^2/2])/(-1 + x^2
)^(3/4)
```

IntegrateAlgebraic [A] time = 2.17, size = 48, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-2 + x^2)*(-1 + x^2)^(3/4)),x]
```

```
[Out] ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))]/Sqrt[2] - ArcTanh[x/(Sqrt[2]*(-1 + x^2
)^(1/4))]/Sqrt[2]
```

fricas [B] time = 0.40, size = 91, normalized size = 1.90

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{4}\sqrt{2}\log\left(-\frac{x^4-2\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}x^3+4\sqrt{x^2-1}\,x^2-4\sqrt{2}\left(x^2-1\right)^{\frac{3}{4}}x+4x^2-4}{x^4-4x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2-2)/(x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*(x^2 - 1)^(1/4)/x) + 1/4*sqrt(2)*log(-(x^4 - 2*
sqrt(2)*(x^2 - 1)^(1/4)*x^3 + 4*sqrt(x^2 - 1)*x^2 - 4*sqrt(2)*(x^2 - 1)^(3/
4)*x + 4*x^2 - 4)/(x^4 - 4*x^2 + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2-1\right)^{\frac{3}{4}}\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2-2)/(x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 - 1)^(3/4)*(x^2 - 2)), x)
```

maple [C] time = 1.96, size = 121, normalized size = 2.52

$$\frac{\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{\left(x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2-2\right)-x\sqrt{x^2-1}+\mathrm{RootOf}\left(\_Z^2-2\right)\left(x^2-1\right)^{\frac{1}{4}}-x}{x^2-2}\right)}{2}+\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{\left(x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+2\right)+x\sqrt{x^2-1}-\mathrm{RootOf}\left(\_Z^2+2\right)\left(x^2-1\right)^{\frac{1}{4}}-x}{x^2-2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2-2)/(x^2-1)^(3/4),x)
```

```
[Out] 1/2*RootOf(_Z^2-2)*ln(-((x^2-1)^(3/4)*RootOf(_Z^2-2)-x*(x^2-1)^(1/2)+RootOf
(_Z^2-2)*(x^2-1)^(1/4)-x)/(x^2-2))+1/2*RootOf(_Z^2+2)*ln(((x^2-1)^(3/4)*Roo
tOf(_Z^2+2)+x*(x^2-1)^(1/2)-RootOf(_Z^2+2)*(x^2-1)^(1/4)-x)/(x^2-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2-1\right)^{\frac{3}{4}}\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2-2)/(x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 - 1)^(3/4)*(x^2 - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{{\left(x^2-1\right)}^{3/4}\,\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 - 1)^(3/4)*(x^2 - 2)),x)
```

```
[Out] int(x^2/((x^2 - 1)^(3/4)*(x^2 - 2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left((x-1)(x+1)\right)^{\frac{3}{4}}\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2-2)/(x**2-1)**(3/4),x)
```

```
[Out] Integral(x**2/(((x - 1)*(x + 1))**(3/4)*(x**2 - 2)), x)
```

3.605 $\int \frac{(-1+x^2)\sqrt{1+x^4}}{(1+x^2)^3}\,dx$

Optimal. Leaf size=48

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}-\frac{\sqrt{x^4+1}\,x}{2\left(x^2+1\right)^2}$$

Rubi [A] time = 0.36, antiderivative size = 72, normalized size of antiderivative = 1.50, number of steps used = 20, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1721, 220, 1224, 1697, 1713, 1196, 1701, 1699, 203, 1211}

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)+\frac{3\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}-\frac{\sqrt{x^4+1}\,x}{2\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^2)*Sqrt[1 + x^4])/(1 + x^2)^3,x]
```

```
[Out] -1/2*(x*Sqrt[1 + x^4])/(1 + x^2)^2 + (3*ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]])/
(2*Sqrt[2]) - Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1224

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> -Sim
p[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2*d*(q + 1)*(c*d^2 + a*e^2))
, x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[
a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2 + c*e^2*(2*q + 5)*x
^4, x])/Sqrt[a + c*x^4], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1697

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, -Simp[((C*d^2 - B*d*e + A*e^2)*x*(d + e*x^2)^(q + 1)*Sqrt[a + c*x^4])/(2
*d*(q + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x^2)^(q + 1)*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*c*d^2*(
q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C*d^2 - B*d*e + A*e^
2)*(2*q + 5)*x^4, x])/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] &&
 PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[
q, -1]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 1701

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + c*x^4], x], x] - Di
st[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^2 -
a*e^2, 0] && NeQ[B*d + A*e, 0]
```

Rule 1713

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, -D
ist[C/e^2, Int[(d - e*x^2)/Sqrt[a + c*x^4], x], x] + Dist[1/e^2, Int[(C*d^2
 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c
, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^2 - a
*e^2, 0]
```

Rule 1721

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Rubi steps

$$\int \frac{(-1+x^2)\sqrt{1+x^4}}{(1+x^2)^3}\,dx = \int \left(\frac{1}{\sqrt{1+x^4}} - \frac{4}{(1+x^2)^3\sqrt{1+x^4}} + \frac{6}{(1+x^2)^2\sqrt{1+x^4}} - \frac{4}{(1+x^2)\sqrt{1+x^4}}\right) dx$$

$$= -\left(4\int \frac{1}{(1+x^2)^3\sqrt{1+x^4}}\,dx\right) - 4\int \frac{1}{(1+x^2)\sqrt{1+x^4}}\,dx + 6\int \frac{1}{(1+x^2)^2\sqrt{1+x$$

$$= -\frac{x\sqrt{1+x^4}}{2(1+x^2)^2} + \frac{3x\sqrt{1+x^4}}{2(1+x^2)} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{1+x^4}} + \frac{1}{2}\int \frac{-7+4x}{(1+x^2)^2}$$

$$= -\frac{x\sqrt{1+x^4}}{2(1+x^2)^2} - \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{1+x^4}} - \frac{1}{8}\int \frac{16-20x^2-12x^4}{(1+x^2)\sqrt{1+x^4}}\,dx + \frac{3}{2}$$

$$= -\frac{x\sqrt{1+x^4}}{2(1+x^2)^2} - \frac{3x\sqrt{1+x^4}}{2(1+x^2)} - \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right) + \frac{3(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,E\left(2\tan\right.}{2\sqrt{1+x^4}}$$

$$= -\frac{x\sqrt{1+x^4}}{2(1+x^2)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{\sqrt{2}} - \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right) - \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\,\mathrm{t}\right.}{2\sqrt{1+x^4}}$$

$$= -\frac{x\sqrt{1+x^4}}{2(1+x^2)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{\sqrt{2}} - \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right) - \frac{3}{2}\mathrm{Subst}\left(\int \frac{1}{1+2x^2}\,dx\right.$$

$$= -\frac{x\sqrt{1+x^4}}{2(1+x^2)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}} - \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)$$

Mathematica [C] time = 0.25, size = 68, normalized size = 1.42

$$-\frac{\sqrt{x^4+1}\,x}{2(x^2+1)^2} - \frac{1}{2}\sqrt[4]{-1}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right) + \sqrt[4]{-1}\,\Pi\left(-i;i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)*Sqrt[1 + x^4])/(1 + x^2)^3,x]
```

```
[Out] -1/2*(x*Sqrt[1 + x^4])/(1 + x^2)^2 - ((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(
1/4)*x], -1])/2 + (-1)^(1/4)*EllipticPi[-I, I*ArcSinh[(-1)^(1/4)*x], -1]
```

IntegrateAlgebraic [A] time = 0.43, size = 48, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}} - \frac{\sqrt{x^4+1}\,x}{2(x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*Sqrt[1 + x^4])/(1 + x^2)^3,x]
```

```
[Out] -1/2*(x*Sqrt[1 + x^4])/(1 + x^2)^2 - ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*S
qrt[2])
```

fricas [A] time = 0.42, size = 52, normalized size = 1.08

$$-\frac{\sqrt{2}\left(x^4+2x^2+1\right)\arctan\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)+2\sqrt{x^4+1}\,x}{4\left(x^4+2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+1)^(1/2)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) + 2*sqrt(x^
4 + 1)*x)/(x^4 + 2*x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^4+1}\left(x^2-1\right)}{\left(x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+1)^(1/2)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 1)*(x^2 - 1)/(x^2 + 1)^3, x)
```

maple [C] time = 0.06, size = 331, normalized size = 6.90

$$-\frac{x\sqrt{x^4+1}}{2\left(x^2+1\right)^2}+\frac{i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x,i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^4+1}}+\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\mathrm{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(x^4+1)^(1/2)/(x^2+1)^3,x)
```

```
[Out] -1/2*x*(x^4+1)^(1/2)/(x^2+1)^2+1/2*I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^
(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2))
,I)-1/2*I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+
1)^(1/2)*EllipticE(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+(-1)^(3/4)*(1-I*x^2)^(1
/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)
^(1/4))+1/2/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^
4+1)^(1/2)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-1/2*I/(1/2*2^(1/2)+1/
2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*(EllipticF(x*(1/
2*2^(1/2)+1/2*I*2^(1/2)),I)-EllipticE(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^4+1}\left(x^2-1\right)}{\left(x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+1)^(1/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 1)*(x^2 - 1)/(x^2 + 1)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(x^2-1\right)\sqrt{x^4+1}}{\left(x^2+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*(x^4 + 1)^(1/2))/(x^2 + 1)^3,x)
[Out] int(((x^2 - 1)*(x^4 + 1)^(1/2))/(x^2 + 1)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\sqrt{x^4+1}}{(x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**4+1)**(1/2)/(x**2+1)**3,x)
[Out] Integral((x - 1)*(x + 1)*sqrt(x**4 + 1)/(x**2 + 1)**3, x)
```

3.606 $\int \frac{77-46x+5x^2}{(-23+82x-23x^2)\sqrt{-60+83x-21x^2-3x^3+x^4}}\,dx$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{2\sqrt{42}\sqrt{x^4-3x^3-21x^2+83x-60}}{19x^2-86x+103}\right)}{\sqrt{42}}$$

Rubi [F] time = 0.77, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{77-46x+5x^2}{(-23+82x-23x^2)\sqrt{-60+83x-21x^2-3x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(77 - 46*x + 5*x^2)/((-23 + 82*x - 23*x^2)*Sqrt[-60 + 83*x - 21*x^2 - 3
*x^3 + x^4]),x]
```

```
[Out] (-5*Defer[Int][1/Sqrt[-60 + 83*x - 21*x^2 - 3*x^3 + x^4], x])/23 - (24*(27
+ 10*Sqrt[2])*Defer[Int][1/((82 - 48*Sqrt[2] - 46*x)*Sqrt[-60 + 83*x - 21*x
^2 - 3*x^3 + x^4]), x])/23 - (24*(27 - 10*Sqrt[2])*Defer[Int][1/((82 + 48*S
qrt[2] - 46*x)*Sqrt[-60 + 83*x - 21*x^2 - 3*x^3 + x^4]), x])/23
```

Rubi steps

$$\int \frac{77-46x+5x^2}{(-23+82x-23x^2)\sqrt{-60+83x-21x^2-3x^3+x^4}}\,dx = \int \left(-\frac{5}{23\sqrt{-60+83x-21x^2-3x^3+x^4}} + \frac{}{23(-23}\right.$$

$$= -\left(\frac{5}{23}\int \frac{1}{\sqrt{-60+83x-21x^2-3x^3+x^4}}\,dx\right) + \frac{72}{23}$$

$$= -\left(\frac{5}{23}\int \frac{1}{\sqrt{-60+83x-21x^2-3x^3+x^4}}\,dx\right) + \frac{72}{23}$$

$$= -\left(\frac{5}{23}\int \frac{1}{\sqrt{-60+83x-21x^2-3x^3+x^4}}\,dx\right) - \frac{1}{23}($$

Mathematica [C] time = 0.46, size = 196, normalized size = 4.08

$$\frac{\sqrt{\frac{x-4}{x-1}}\sqrt{\frac{x-3}{x-1}}(x-1)^2\sqrt{\frac{x+5}{x-1}}\left(-7F\left(\sin^{-1}\left(\frac{\sqrt{\frac{x+5}{x-1}}}{\sqrt{3}}\right)|\frac{3}{4}\right)+\left(5+4\sqrt{2}\right)\Pi\left(\frac{3}{14}-\frac{3\sqrt{2}}{7};\sin^{-1}\left(\frac{\sqrt{\frac{x+5}{x-1}}}{\sqrt{3}}\right)|\frac{3}{4}\right)+\left(5-4\sqrt{2}\right)\Pi\left(\frac{3}{14}+\frac{3\sqrt{2}}{7};\sin^{-1}\left(\frac{\sqrt{\frac{x+5}{x-1}}}{\sqrt{3}}\right)|\frac{3}{4}\right)\right)}{7\sqrt{6}\sqrt{x^4-3x^3-21x^2+83x-60}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(77 - 46*x + 5*x^2)/((-23 + 82*x - 23*x^2)*Sqrt[-60 + 83*x - 21*x
^2 - 3*x^3 + x^4]),x]
```

```
[Out] (Sqrt[(-4 + x)/(-1 + x)]*Sqrt[(-3 + x)/(-1 + x)]*(-1 + x)^2*Sqrt[(5 + x)/(-
1 + x)]*(-7*EllipticF[ArcSin[Sqrt[(5 + x)/(-1 + x)]/Sqrt[3]], 3/4] + (5 + 4
*Sqrt[2])*EllipticPi[3/14 - (3*Sqrt[2])/7, ArcSin[Sqrt[(5 + x)/(-1 + x)]/Sq
rt[3]], 3/4] + (5 - 4*Sqrt[2])*EllipticPi[3/14 + (3*Sqrt[2])/7, ArcSin[Sqrt
[(5 + x)/(-1 + x)]/Sqrt[3]], 3/4]))/(7*Sqrt[6]*Sqrt[-60 + 83*x - 21*x^2 - 3
*x^3 + x^4])
```

IntegrateAlgebraic [A] time = 0.24, size = 48, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2\sqrt{42}\sqrt{x^4-3x^3-21x^2+83x-60}}{19x^2-86x+103}\right)}{\sqrt{42}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(77 - 46*x + 5*x^2)/((-23 + 82*x - 23*x^2)*Sqrt[-60 + 83
*x - 21*x^2 - 3*x^3 + x^4]),x]
```

```
[Out] ArcTan[(2*Sqrt[42]*Sqrt[-60 + 83*x - 21*x^2 - 3*x^3 + x^4])/(103 - 86*x + 1
9*x^2)]/Sqrt[42]
```

fricas [A] time = 0.46, size = 49, normalized size = 1.02

$$\frac{1}{42}\sqrt{21}\sqrt{2}\arctan\left(\frac{2\sqrt{21}\sqrt{2}\sqrt{x^4-3x^3-21x^2+83x-60}}{19x^2-86x+103}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2-46*x+77)/(-23*x^2+82*x-23)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/42*sqrt(21)*sqrt(2)*arctan(2*sqrt(21)*sqrt(2)*sqrt(x^4 - 3*x^3 - 21*x^2 +
 83*x - 60)/(19*x^2 - 86*x + 103))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{5x^2-46x+77}{\sqrt{x^4-3x^3-21x^2+83x-60}\left(23x^2-82x+23\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2-46*x+77)/(-23*x^2+82*x-23)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2
),x, algorithm="giac")
```

```
[Out] integrate(-(5*x^2 - 46*x + 77)/(sqrt(x^4 - 3*x^3 - 21*x^2 + 83*x - 60)*(23*
x^2 - 82*x + 23)), x)
```

maple [C] time = 0.12, size = 3042, normalized size = 63.38

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2-46*x+77)/(-23*x^2+82*x-23)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2),x)
```

```
[Out] -324/529*(x/(-1+x)+5/(-1+x))^(1/2)*(-3/(-1+x)+x/(-1+x))^(1/2)*6^(1/2)*(x/(-
1+x)-4/(-1+x))^(1/2)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2)/(-18/23-24/23*2^(1/2)
)/(-156/23-24/23*2^(1/2))*EllipticPi(1/3*3^(1/2)*((5+x)/(-1+x))^(1/2),3*(-1
8/23-24/23*2^(1/2))/(-156/23-24/23*2^(1/2)),1/2*3^(1/2))-54/529*(x/(-1+x)+5
/(-1+x))^(1/2)*(-3/(-1+x)+x/(-1+x))^(1/2)*6^(1/2)*(x/(-1+x)-4/(-1+x))^(1/2)
/(x^4-3*x^3-21*x^2+83*x-60)^(1/2)/(-18/23-24/23*2^(1/2))*x^2*EllipticF(1/3*
3^(1/2)*((5+x)/(-1+x))^(1/2),1/2*3^(1/2))+108/529*(x/(-1+x)+5/(-1+x))^(1/2)
*(-3/(-1+x)+x/(-1+x))^(1/2)*6^(1/2)*(x/(-1+x)-4/(-1+x))^(1/2)/(x^4-3*x^3-21
*x^2+83*x-60)^(1/2)/(-18/23-24/23*2^(1/2))*x*EllipticF(1/3*3^(1/2)*((5+x)/(
-1+x))^(1/2),1/2*3^(1/2))-324/529*(x/(-1+x)+5/(-1+x))^(1/2)*(-3/(-1+x)+x/(-
1+x))^(1/2)*6^(1/2)*(x/(-1+x)-4/(-1+x))^(1/2)/(x^4-3*x^3-21*x^2+83*x-60)^(1
/2)/(-18/23+24/23*2^(1/2))/(-156/23+24/23*2^(1/2))*EllipticPi(1/3*3^(1/2)*(
(5+x)/(-1+x))^(1/2),3*(-18/23+24/23*2^(1/2))/(-156/23+24/23*2^(1/2)),1/2*3^
```

```
(1/2))+5/138*((5+x)/(-1+x))^(1/2)*(-1+x)^2*((-3+x)/(-1+x))^(1/2)*6^(1/2)*((
x-4)/(-1+x))^(1/2)/((5+x)*(-1+x)*(-3+x)*(x-4))^(1/2)*EllipticF(1/3*3^(1/2)*
((5+x)/(-1+x))^(1/2),1/2*3^(1/2))-54/529*(x/(-1+x)+5/(-1+x))^(1/2)*(-3/(-1+
x)+x/(-1+x))^(1/2)*6^(1/2)*(x/(-1+x)-4/(-1+x))^(1/2)/(x^4-3*x^3-21*x^2+83*x
-60)^(1/2)/(-18/23-24/23*2^(1/2))*EllipticF(1/3*3^(1/2)*((5+x)/(-1+x))^(1/2
),1/2*3^(1/2))-54/529*(x/(-1+x)+5/(-1+x))^(1/2)*(-3/(-1+x)+x/(-1+x))^(1/2)*
6^(1/2)*(x/(-1+x)-4/(-1+x))^(1/2)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2)/(-18/23+
24/23*2^(1/2))*EllipticF(1/3*3^(1/2)*((5+x)/(-1+x))^(1/2),1/2*3^(1/2))-54/5
29*(x/(-1+x)+5/(-1+x))^(1/2)*(-3/(-1+x)+x/(-1+x))^(1/2)*6^(1/2)*(x/(-1+x)-4
/(-1+x))^(1/2)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2)/(-18/23+24/23*2^(1/2))*x^2*
EllipticF(1/3*3^(1/2)*((5+x)/(-1+x))^(1/2),1/2*3^(1/2))+108/529*(x/(-1+x)+5
/(-1+x))^(1/2)*(-3/(-1+x)+x/(-1+x))^(1/2)*6^(1/2)*(x/(-1+x)-4/(-1+x))^(1/2)
/(x^4-3*x^3-21*x^2+83*x-60)^(1/2)/(-18/23+24/23*2^(1/2))*x*EllipticF(1/3*3^
(1/2)*((5+x)/(-1+x))^(1/2),1/2*3^(1/2))+20/529*2^(1/2)*(x/(-1+x)+5/(-1+x))^
(1/2)*(-3/(-1+x)+x/(-1+x))^(1/2)*6^(1/2)*(x/(-1+x)-4/(-1+x))^(1/2)/(x^4-3*x
^3-21*x^2+83*x-60)^(1/2)/(-18/23-24/23*2^(1/2))*EllipticF(1/3*3^(1/2)*((5+x
)/(-1+x))^(1/2),1/2*3^(1/2))-20/529*2^(1/2)*(x/(-1+x)+5/(-1+x))^(1/2)*(-3/(
-1+x)+x/(-1+x))^(1/2)*6^(1/2)*(x/(-1+x)-4/(-1+x))^(1/2)/(x^4-3*x^3-21*x^2+8
3*x-60)^(1/2)/(-18/23+24/23*2^(1/2))*EllipticF(1/3*3^(1/2)*((5+x)/(-1+x))^(
1/2),1/2*3^(1/2))-120/529*2^(1/2)*(x/(-1+x)+5/(-1+x))^(1/2)*(-3/(-1+x)+x/(-
1+x))^(1/2)*6^(1/2)*(x/(-1+x)-4/(-1+x))^(1/2)/(x^4-3*x^3-21*x^2+83*x-60)^(1
/2)/(-18/23+24/23*2^(1/2))*x^2/(-156/23+24/23*2^(1/2))*EllipticPi(1/3*3^(1/
2)*((5+x)/(-1+x))^(1/2),3*(-18/23+24/23*2^(1/2))/(-156/23+24/23*2^(1/2)),1/
2*3^(1/2))+240/529*2^(1/2)*(x/(-1+x)+5/(-1+x))^(1/2)*(-3/(-1+x)+x/(-1+x))^(
1/2)*6^(1/2)*(x/(-1+x)-4/(-1+x))^(1/2)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2)/(-1
8/23+24/23*2^(1/2))*x/(-156/23+24/23*2^(1/2))*EllipticPi(1/3*3^(1/2)*((5+x)
/(-1+x))^(1/2),3*(-18/23+24/23*2^(1/2))/(-156/23+24/23*2^(1/2)),1/2*3^(1/2)
)+120/529*2^(1/2)*(x/(-1+x)+5/(-1+x))^(1/2)*(-3/(-1+x)+x/(-1+x))^(1/2)*6^(1
/2)*(x/(-1+x)-4/(-1+x))^(1/2)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2)/(-18/23-24/2
3*2^(1/2))*x^2/(-156/23-24/23*2^(1/2))*EllipticPi(1/3*3^(1/2)*((5+x)/(-1+x)
)^(1/2),3*(-18/23-24/23*2^(1/2))/(-156/23-24/23*2^(1/2)),1/2*3^(1/2))-240/5
29*2^(1/2)*(x/(-1+x)+5/(-1+x))^(1/2)*(-3/(-1+x)+x/(-1+x))^(1/2)*6^(1/2)*(x/
(-1+x)-4/(-1+x))^(1/2)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2)/(-18/23-24/23*2^(1/
2))*x/(-156/23-24/23*2^(1/2))*EllipticPi(1/3*3^(1/2)*((5+x)/(-1+x))^(1/2),3
*(-18/23-24/23*2^(1/2))/(-156/23-24/23*2^(1/2)),1/2*3^(1/2))+20/529*2^(1/2)
*(x/(-1+x)+5/(-1+x))^(1/2)*(-3/(-1+x)+x/(-1+x))^(1/2)*6^(1/2)*(x/(-1+x)-4/(
-1+x))^(1/2)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2)/(-18/23-24/23*2^(1/2))*x^2*El
lipticF(1/3*3^(1/2)*((5+x)/(-1+x))^(1/2),1/2*3^(1/2))-40/529*2^(1/2)*(x/(-1
+x)+5/(-1+x))^(1/2)*(-3/(-1+x)+x/(-1+x))^(1/2)*6^(1/2)*(x/(-1+x)-4/(-1+x))^
(1/2)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2)/(-18/23-24/23*2^(1/2))*x*EllipticF(1
/3*3^(1/2)*((5+x)/(-1+x))^(1/2),1/2*3^(1/2))+120/529*2^(1/2)*(x/(-1+x)+5/(-
1+x))^(1/2)*(-3/(-1+x)+x/(-1+x))^(1/2)*6^(1/2)*(x/(-1+x)-4/(-1+x))^(1/2)/(x
^4-3*x^3-21*x^2+83*x-60)^(1/2)/(-18/23-24/23*2^(1/2))/(-156/23-24/23*2^(1/2
))*EllipticPi(1/3*3^(1/2)*((5+x)/(-1+x))^(1/2),3*(-18/23-24/23*2^(1/2))/(-1
56/23-24/23*2^(1/2)),1/2*3^(1/2))-324/529*(x/(-1+x)+5/(-1+x))^(1/2)*(-3/(-1
+x)+x/(-1+x))^(1/2)*6^(1/2)*(x/(-1+x)-4/(-1+x))^(1/2)/(x^4-3*x^3-21*x^2+83*
x-60)^(1/2)/(-18/23-24/23*2^(1/2))*x^2/(-156/23-24/23*2^(1/2))*EllipticPi(1
/3*3^(1/2)*((5+x)/(-1+x))^(1/2),3*(-18/23-24/23*2^(1/2))/(-156/23-24/23*2^(
1/2)),1/2*3^(1/2))+648/529*(x/(-1+x)+5/(-1+x))^(1/2)*(-3/(-1+x)+x/(-1+x))^(
1/2)*6^(1/2)*(x/(-1+x)-4/(-1+x))^(1/2)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2)/(-1
8/23-24/23*2^(1/2))*x/(-156/23-24/23*2^(1/2))*EllipticPi(1/3*3^(1/2)*((5+x)
/(-1+x))^(1/2),3*(-18/23-24/23*2^(1/2))/(-156/23-24/23*2^(1/2)),1/2*3^(1/2)
)-20/529*2^(1/2)*(x/(-1+x)+5/(-1+x))^(1/2)*(-3/(-1+x)+x/(-1+x))^(1/2)*6^(1/
2)*(x/(-1+x)-4/(-1+x))^(1/2)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2)/(-18/23+24/23
*2^(1/2))*x^2*EllipticF(1/3*3^(1/2)*((5+x)/(-1+x))^(1/2),1/2*3^(1/2))+40/52
9*2^(1/2)*(x/(-1+x)+5/(-1+x))^(1/2)*(-3/(-1+x)+x/(-1+x))^(1/2)*6^(1/2)*(x/(
-1+x)-4/(-1+x))^(1/2)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2)/(-18/23+24/23*2^(1/2
))*x*EllipticF(1/3*3^(1/2)*((5+x)/(-1+x))^(1/2),1/2*3^(1/2))-120/529*2^(1/2
)*(x/(-1+x)+5/(-1+x))^(1/2)*(-3/(-1+x)+x/(-1+x))^(1/2)*6^(1/2)*(x/(-1+x)-4/
```

```
(-1+x))^(1/2)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2)/(-18/23+24/23*2^(1/2))/(-156
/23+24/23*2^(1/2))*EllipticPi(1/3*3^(1/2)*((5+x)/(-1+x))^(1/2),3*(-18/23+24
/23*2^(1/2))/(-156/23+24/23*2^(1/2)),1/2*3^(1/2))-324/529*(x/(-1+x)+5/(-1+x
))^(1/2)*(-3/(-1+x)+x/(-1+x))^(1/2)*6^(1/2)*(x/(-1+x)-4/(-1+x))^(1/2)/(x^4-
3*x^3-21*x^2+83*x-60)^(1/2)/(-18/23+24/23*2^(1/2))*x^2/(-156/23+24/23*2^(1/
2))*EllipticPi(1/3*3^(1/2)*((5+x)/(-1+x))^(1/2),3*(-18/23+24/23*2^(1/2))/(-
156/23+24/23*2^(1/2)),1/2*3^(1/2))+648/529*(x/(-1+x)+5/(-1+x))^(1/2)*(-3/(-
1+x)+x/(-1+x))^(1/2)*6^(1/2)*(x/(-1+x)-4/(-1+x))^(1/2)/(x^4-3*x^3-21*x^2+83
*x-60)^(1/2)/(-18/23+24/23*2^(1/2))*x/(-156/23+24/23*2^(1/2))*EllipticPi(1/
3*3^(1/2)*((5+x)/(-1+x))^(1/2),3*(-18/23+24/23*2^(1/2))/(-156/23+24/23*2^(1
/2)),1/2*3^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{5\,x^2-46\,x+77}{\sqrt{x^4-3\,x^3-21\,x^2+83\,x-60}\,\left(23\,x^2-82\,x+23\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2-46*x+77)/(-23*x^2+82*x-23)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2
),x, algorithm="maxima")
```

```
[Out] -integrate((5*x^2 - 46*x + 77)/(sqrt(x^4 - 3*x^3 - 21*x^2 + 83*x - 60)*(23*
x^2 - 82*x + 23)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{5\,x^2-46\,x+77}{\left(23\,x^2-82\,x+23\right)\,\sqrt{x^4-3\,x^3-21\,x^2+83\,x-60}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(5*x^2 - 46*x + 77)/((23*x^2 - 82*x + 23)*(83*x - 21*x^2 - 3*x^3 + x^4
 - 60)^(1/2)),x)
```

```
[Out] int(-(5*x^2 - 46*x + 77)/((23*x^2 - 82*x + 23)*(83*x - 21*x^2 - 3*x^3 + x^4
 - 60)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{46x}{23x^2\sqrt{x^4-3x^3-21x^2+83x-60}-82x\sqrt{x^4-3x^3-21x^2+83x-60}+23\sqrt{x^4-3x^3-21x^2+83x-60}}\right)dx-\int \frac{5x^2}{23x^2\sqrt{x^4-3x^3-21x^2+83x-60}-82x\sqrt{x^4-3x^3-21x^2+83x-60}+23\sqrt{x^4-3x^3-21x^2+83x-60}}\,dx-\int \frac{77}{23x^2\sqrt{x^4-3x^3-21x^2+83x-60}-82x\sqrt{x^4-3x^3-21x^2+83x-60}+23\sqrt{x^4-3x^3-21x^2+83x-60}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2-46*x+77)/(-23*x**2+82*x-23)/(x**4-3*x**3-21*x**2+83*x-60)
**(1/2),x)
```

```
[Out] -Integral(-46*x/(23*x**2*sqrt(x**4 - 3*x**3 - 21*x**2 + 83*x - 60) - 82*x*s
qrt(x**4 - 3*x**3 - 21*x**2 + 83*x - 60) + 23*sqrt(x**4 - 3*x**3 - 21*x**2
+ 83*x - 60)), x) - Integral(5*x**2/(23*x**2*sqrt(x**4 - 3*x**3 - 21*x**2 +
 83*x - 60) - 82*x*sqrt(x**4 - 3*x**3 - 21*x**2 + 83*x - 60) + 23*sqrt(x**4
 - 3*x**3 - 21*x**2 + 83*x - 60)), x) - Integral(77/(23*x**2*sqrt(x**4 - 3*
x**3 - 21*x**2 + 83*x - 60) - 82*x*sqrt(x**4 - 3*x**3 - 21*x**2 + 83*x - 60
) + 23*sqrt(x**4 - 3*x**3 - 21*x**2 + 83*x - 60)), x)
```

3.607 $\int \frac{(-1+x)\sqrt[4]{x^3+x^4}}{x(1+x)}\,dx$

Optimal. Leaf size=48

$$\sqrt[4]{x^4+x^3}+\frac{7}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)-\frac{7}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)$$

Rubi [A] time = 0.16, antiderivative size = 94, normalized size of antiderivative = 1.96, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2056, 80, 63, 331, 298, 203, 206}

$$\sqrt[4]{x^4+x^3}+\frac{7\sqrt[4]{x^4+x^3}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{2x^{3/4}\sqrt[4]{x+1}}-\frac{7\sqrt[4]{x^4+x^3}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{2x^{3/4}\sqrt[4]{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)*(x^3 + x^4)^(1/4))/(x*(1 + x)),x]
```

```
[Out] (x^3 + x^4)^(1/4) + (7*(x^3 + x^4)^(1/4)*ArcTan[x^(1/4)/(1 + x)^(1/4)])/(2*
x^(3/4)*(1 + x)^(1/4)) - (7*(x^3 + x^4)^(1/4)*ArcTanh[x^(1/4)/(1 + x)^(1/4)
])/(2*x^(3/4)*(1 + x)^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-1+x)\sqrt[4]{x^3+x^4}}{x(1+x)}\,dx &= \frac{\sqrt[4]{x^3+x^4}\int \frac{-1+x}{\sqrt[4]{x}(1+x)^{3/4}}\,dx}{x^{3/4}\sqrt[4]{1+x}}\\
&= \sqrt[4]{x^3+x^4}-\frac{\left(7\sqrt[4]{x^3+x^4}\right)\int \frac{1}{\sqrt[4]{x}(1+x)^{3/4}}\,dx}{4x^{3/4}\sqrt[4]{1+x}}\\
&= \sqrt[4]{x^3+x^4}-\frac{\left(7\sqrt[4]{x^3+x^4}\right)\operatorname{Subst}\left(\int \frac{x^2}{\left(1+x^4\right)^{3/4}}\,dx,x,\sqrt[4]{x}\right)}{x^{3/4}\sqrt[4]{1+x}}\\
&= \sqrt[4]{x^3+x^4}-\frac{\left(7\sqrt[4]{x^3+x^4}\right)\operatorname{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}}\\
&= \sqrt[4]{x^3+x^4}-\frac{\left(7\sqrt[4]{x^3+x^4}\right)\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{2x^{3/4}\sqrt[4]{1+x}}+\frac{\left(7\sqrt[4]{x^3+x^4}\right)\operatorname{Subst}\left(\int \frac{1}{1+x}\right.}{2x^{3/4}\sqrt[4]{1+x}}\\
&= \sqrt[4]{x^3+x^4}+\frac{7\sqrt[4]{x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{2x^{3/4}\sqrt[4]{1+x}}-\frac{7\sqrt[4]{x^3+x^4}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{2x^{3/4}\sqrt[4]{1+x}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.94

$$\frac{x^3\left(-7(x+1)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-x\right)+3x+3\right)}{3\left(x^3(x+1)\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)*(x^3 + x^4)^(1/4))/(x*(1 + x)),x]
```

```
[Out] (x^3*(3 + 3*x - 7*(1 + x)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, -x]))/(3*(
x^3*(1 + x))^(3/4))
```

IntegrateAlgebraic [A] time = 0.28, size = 48, normalized size = 1.00

$$\sqrt[4]{x^4+x^3}+\frac{7}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)-\frac{7}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)*(x^3 + x^4)^(1/4))/(x*(1 + x)),x]
```

```
[Out] (x^3 + x^4)^(1/4) + (7*ArcTan[x/(x^3 + x^4)^(1/4)])/2 - (7*ArcTanh[x/(x^3 +
 x^4)^(1/4)])/2
```

fricas [A] time = 0.40, size = 65, normalized size = 1.35

$$\left(x^4+x^3\right)^{\frac{1}{4}}-\frac{7}{2}\arctan\left(\frac{\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)-\frac{7}{4}\log\left(\frac{x+\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{7}{4}\log\left(-\frac{x-\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(x^4+x^3)^(1/4)/x/(1+x),x, algorithm="fricas")
```

```
[Out] (x^4 + x^3)^(1/4) - 7/2*arctan((x^4 + x^3)^(1/4)/x) - 7/4*log((x + (x^4 + x
^3)^(1/4))/x) + 7/4*log(-(x - (x^4 + x^3)^(1/4))/x)
```

giac [A] time = 0.39, size = 45, normalized size = 0.94

$$x\left(\frac{1}{x}+1\right)^{\frac{1}{4}}-\frac{7}{2}\arctan\left(\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right)-\frac{7}{4}\log\left(\left(\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right)+\frac{7}{4}\log\left(\left|\left(\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(x^4+x^3)^(1/4)/x/(1+x),x, algorithm="giac")
```

```
[Out] x*(1/x + 1)^(1/4) - 7/2*arctan((1/x + 1)^(1/4)) - 7/4*log((1/x + 1)^(1/4) +
 1) + 7/4*log(abs((1/x + 1)^(1/4) - 1))
```

maple [C] time = 0.59, size = 366, normalized size = 7.62

$$\left(x^3(1+x)\right)^{\frac{1}{4}}+\frac{\left(-\frac{7\ln\left(\frac{2\left(x^4+3x^3+3x^2+x\right)^{\frac{3}{4}}+2\sqrt{x^4+3x^3+3x^2+x}\,x+2\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}x^2+2x^3+2\sqrt{x^4+3x^3+3x^2+x}+4\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}x+5x^2+2\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}+4x+1}{(1+x)^2}\right)}{4}-\frac{7\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-2\sqrt{x^4+3x^3+3x^2+x}\,\mathrm{RootOf}\left(\_Z^2+1\right)x+2\,\mathrm{RootOf}\left(\_Z^2+1\right)x^3+2\left(x^4+3x^3+3x^2+x\right)^{\frac{3}{4}}-2\sqrt{x^4+3x^3+3x^2+x}\,\mathrm{RootOf}\left(\_Z^2+1\right)-2\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}x^2+5\,\mathrm{RootOf}\left(\_Z^2+1\right)x^2-4\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}x+4\,\mathrm{RootOf}\left(\_Z^2+1\right)x-2\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}+\mathrm{RootOf}\left(\_Z^2+1\right)}{(1+x)^2}\right)}{4}\right)\left(x^3(1+x)\right)^{\frac{1}{4}}\left(x(1+x)^3\right)^{\frac{1}{4}}}{x(1+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)*(x^4+x^3)^(1/4)/x/(1+x),x)
```

```
[Out] (x^3*(1+x))^(1/4)+(-7/4*ln((2*(x^4+3*x^3+3*x^2+x)^(3/4)+2*(x^4+3*x^3+3*x^2+
x)^(1/2)*x+2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2+2*x^3+2*(x^4+3*x^3+3*x^2+x)^(1/2
)+4*(x^4+3*x^3+3*x^2+x)^(1/4)*x+5*x^2+2*(x^4+3*x^3+3*x^2+x)^(1/4)+4*x+1)/(1
+x)^2)-7/4*RootOf(_Z^2+1)*ln((-2*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^2+1)*x
+2*RootOf(_Z^2+1)*x^3+2*(x^4+3*x^3+3*x^2+x)^(3/4)-2*(x^4+3*x^3+3*x^2+x)^(1/
2)*RootOf(_Z^2+1)-2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2+5*RootOf(_Z^2+1)*x^2-4*(x
^4+3*x^3+3*x^2+x)^(1/4)*x+4*RootOf(_Z^2+1)*x-2*(x^4+3*x^3+3*x^2+x)^(1/4)+Ro
otOf(_Z^2+1))/(1+x)^2))*(x^3*(1+x))^(1/4)/x/(1+x)*(x*(1+x)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^4+x^3\right)^{\frac{1}{4}}(x-1)}{(x+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(x^4+x^3)^(1/4)/x/(1+x),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + x^3)^(1/4)*(x - 1)/((x + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(x^4+x^3\right)^{1/4}\,(x-1)}{x\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + x^4)^(1/4)*(x - 1))/(x*(x + 1)),x)
```

```
[Out] int(((x^3 + x^4)^(1/4)*(x - 1))/(x*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3 (x+1)}\,(x-1)}{x (x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(x**4+x**3)**(1/4)/x/(1+x),x)
```

```
[Out] Integral((x**3*(x + 1))**(1/4)*(x - 1)/(x*(x + 1)), x)
```

3.608 $\int \frac{(-1+x^4)\sqrt[4]{x^3+x^4}}{x^8}\,dx$

Optimal. Leaf size=48

$$\frac{4\sqrt[4]{x^4+x^3}\,(22748x^6-5687x^5-39955x^4+960x^3-780x^2+663x+13923)}{348075x^7}$$

Rubi [B] time = 0.26, antiderivative size = 109, normalized size of antiderivative = 2.27, number of steps used = 10, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2052, 2016, 2014}

$$\frac{4(x^4+x^3)^{5/4}}{25x^{10}}-\frac{16(x^4+x^3)^{5/4}}{105x^9}+\frac{256(x^4+x^3)^{5/4}}{1785x^8}-\frac{1024(x^4+x^3)^{5/4}}{7735x^7}-\frac{22748(x^4+x^3)^{5/4}}{69615x^6}+\frac{90992(x^4+x^3)^{5/4}}{348075x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4)*(x^3 + x^4)^(1/4))/x^8,x]
```

```
[Out] (4*(x^3 + x^4)^(5/4))/(25*x^10) - (16*(x^3 + x^4)^(5/4))/(105*x^9) + (256*(
x^3 + x^4)^(5/4))/(1785*x^8) - (1024*(x^3 + x^4)^(5/4))/(7735*x^7) - (22748
*(x^3 + x^4)^(5/4))/(69615*x^6) + (90992*(x^3 + x^4)^(5/4))/(348075*x^5)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{(-1+x^4)\sqrt[4]{x^3+x^4}}{x^8}\,dx = \int\left(-\frac{\sqrt[4]{x^3+x^4}}{x^8}+\frac{\sqrt[4]{x^3+x^4}}{x^4}\right)dx$$
$$= -\int \frac{\sqrt[4]{x^3+x^4}}{x^8}\,dx + \int \frac{\sqrt[4]{x^3+x^4}}{x^4}\,dx$$
$$= \frac{4(x^3+x^4)^{5/4}}{25x^{10}} - \frac{4(x^3+x^4)^{5/4}}{9x^6} - \frac{4}{9}\int \frac{\sqrt[4]{x^3+x^4}}{x^3}\,dx + \frac{4}{5}\int \frac{\sqrt[4]{x^3+x^4}}{x^7}\,dx$$
$$= \frac{4(x^3+x^4)^{5/4}}{25x^{10}} - \frac{16(x^3+x^4)^{5/4}}{105x^9} - \frac{4(x^3+x^4)^{5/4}}{9x^6} + \frac{16(x^3+x^4)^{5/4}}{45x^5} - \frac{64}{105}\int \frac{\sqrt[4]{x^3}}{x}$$
$$= \frac{4(x^3+x^4)^{5/4}}{25x^{10}} - \frac{16(x^3+x^4)^{5/4}}{105x^9} + \frac{256(x^3+x^4)^{5/4}}{1785x^8} - \frac{4(x^3+x^4)^{5/4}}{9x^6} + \frac{16(x^3+x^4}{45x^5}$$
$$= \frac{4(x^3+x^4)^{5/4}}{25x^{10}} - \frac{16(x^3+x^4)^{5/4}}{105x^9} + \frac{256(x^3+x^4)^{5/4}}{1785x^8} - \frac{1024(x^3+x^4)^{5/4}}{7735x^7} - \frac{4(x^3+}{9x}$$
$$= \frac{4(x^3+x^4)^{5/4}}{25x^{10}} - \frac{16(x^3+x^4)^{5/4}}{105x^9} + \frac{256(x^3+x^4)^{5/4}}{1785x^8} - \frac{1024(x^3+x^4)^{5/4}}{7735x^7} - \frac{22748(}{69}$$
$$= \frac{4(x^3+x^4)^{5/4}}{25x^{10}} - \frac{16(x^3+x^4)^{5/4}}{105x^9} + \frac{256(x^3+x^4)^{5/4}}{1785x^8} - \frac{1024(x^3+x^4)^{5/4}}{7735x^7} - \frac{22748(}{69}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 0.79

$$\frac{4(x^3(x+1))^{9/4}(22748x^4-51183x^3+39663x^2-27183x+13923)}{348075x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)*(x^3 + x^4)^(1/4))/x^8,x]
```

```
[Out] (4*(x^3*(1 + x))^(9/4)*(13923 - 27183*x + 39663*x^2 - 51183*x^3 + 22748*x^4
))/(348075*x^13)
```

IntegrateAlgebraic [A] time = 0.37, size = 48, normalized size = 1.00

$$\frac{4\sqrt[4]{x^4+x^3}\,(22748x^6-5687x^5-39955x^4+960x^3-780x^2+663x+13923)}{348075x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(x^3 + x^4)^(1/4))/x^8,x]
```

```
[Out] (4*(x^3 + x^4)^(1/4)*(13923 + 663*x - 780*x^2 + 960*x^3 - 39955*x^4 - 5687*
x^5 + 22748*x^6))/(348075*x^7)
```

fricas [A] time = 0.38, size = 44, normalized size = 0.92

$$\frac{4\left(22748\,x^6-5687\,x^5-39955\,x^4+960\,x^3-780\,x^2+663\,x+13923\right)\left(x^4+x^3\right)^{\frac{1}{4}}}{348075\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+x^3)^(1/4)/x^8,x, algorithm="fricas")
```

```
[Out] 4/348075*(22748*x^6 - 5687*x^5 - 39955*x^4 + 960*x^3 - 780*x^2 + 663*x + 13
923)*(x^4 + x^3)^(1/4)/x^7
```

giac [A] time = 0.36, size = 46, normalized size = 0.96

$$\frac{4}{25}\left(\frac{1}{x}+1\right)^{\frac{25}{4}}-\frac{20}{21}\left(\frac{1}{x}+1\right)^{\frac{21}{4}}+\frac{40}{17}\left(\frac{1}{x}+1\right)^{\frac{17}{4}}-\frac{40}{13}\left(\frac{1}{x}+1\right)^{\frac{13}{4}}+\frac{16}{9}\left(\frac{1}{x}+1\right)^{\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+x^3)^(1/4)/x^8,x, algorithm="giac")
```

```
[Out] 4/25*(1/x + 1)^(25/4) - 20/21*(1/x + 1)^(21/4) + 40/17*(1/x + 1)^(17/4) - 4
0/13*(1/x + 1)^(13/4) + 16/9*(1/x + 1)^(9/4)
```

maple [A] time = 0.01, size = 40, normalized size = 0.83

$$\frac{4\,(1+x)^{2}\left(22748x^{4}-51183x^{3}+39663x^{2}-27183x+13923\right)\left(x^{4}+x^{3}\right)^{\frac{1}{4}}}{348075x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^4+x^3)^(1/4)/x^8,x)
```

```
[Out] 4/348075*(1+x)^2*(22748*x^4-51183*x^3+39663*x^2-27183*x+13923)*(x^4+x^3)^(1
/4)/x^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^{4}+x^{3}\right)^{\frac{1}{4}}\left(x^{4}-1\right)}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+x^3)^(1/4)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + x^3)^(1/4)*(x^4 - 1)/x^8, x)
```

mupad [B] time = 1.10, size = 99, normalized size = 2.06

$$\frac{90992\left(x^{4}+x^{3}\right)^{1/4}}{348075\,x}-\frac{22748\left(x^{4}+x^{3}\right)^{1/4}}{348075\,x^{2}}-\frac{31964\left(x^{4}+x^{3}\right)^{1/4}}{69615\,x^{3}}+\frac{256\left(x^{4}+x^{3}\right)^{1/4}}{23205\,x^{4}}-\frac{16\left(x^{4}+x^{3}\right)^{1/4}}{1785\,x^{5}}+\frac{4\left(x^{4}+x^{3}\right)^{1/4}}{525\,x^{6}}+\frac{4\left(x^{4}+x^{3}\right)^{1/4}}{25\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + x^4)^(1/4)*(x^4 - 1))/x^8,x)
```

```
[Out] (90992*(x^3 + x^4)^(1/4))/(348075*x) - (22748*(x^3 + x^4)^(1/4))/(348075*x^
2) - (31964*(x^3 + x^4)^(1/4))/(69615*x^3) + (256*(x^3 + x^4)^(1/4))/(23205
*x^4) - (16*(x^3 + x^4)^(1/4))/(1785*x^5) + (4*(x^3 + x^4)^(1/4))/(525*x^6)
 + (4*(x^3 + x^4)^(1/4))/(25*x^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[4]{x^{3}\,(x+1)}\;(x-1)\,(x+1)\left(x^{2}+1\right)}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**4+x**3)**(1/4)/x**8,x)
```

```
[Out] Integral((x**3*(x + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)/x**8, x)
```

3.609 $$\int \frac{-1+4x-4x^2+4x^4}{\sqrt{\frac{1-2x^2}{1+2x^2}}\left(1+2x^2\right)\left(-1-4x+12x^2-8x^3+4x^4\right)}\,dx$$

Optimal. Leaf size=48

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}x-\sqrt{\frac{2}{3}}}{\sqrt{\frac{1-2x^2}{2x^2+1}}}\right)}{\sqrt{6}}$$

Rubi [F] time = 2.58, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+4x-4x^2+4x^4}{\sqrt{\frac{1-2x^2}{1+2x^2}}\left(1+2x^2\right)\left(-1-4x+12x^2-8x^3+4x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 4*x - 4*x^2 + 4*x^4)/(Sqrt[(1 - 2*x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*(
-1 - 4*x + 12*x^2 - 8*x^3 + 4*x^4)),x]
```

```
[Out] (Sqrt[1 - 2*x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1])/(Sqrt[2]*Sqrt[(1 - 2*x^2
)/(1 + 2*x^2)]*Sqrt[1 + 2*x^2]) + (8*Sqrt[1 - 2*x^2]*Defer[Int][x/(Sqrt[1 -
 4*x^4]*(-1 - 4*x + 12*x^2 - 8*x^3 + 4*x^4)), x])/(Sqrt[(1 - 2*x^2)/(1 + 2*
x^2)]*Sqrt[1 + 2*x^2]) - (16*Sqrt[1 - 2*x^2]*Defer[Int][x^2/(Sqrt[1 - 4*x^4
]*(-1 - 4*x + 12*x^2 - 8*x^3 + 4*x^4)), x])/(Sqrt[(1 - 2*x^2)/(1 + 2*x^2)]*
Sqrt[1 + 2*x^2]) + (8*Sqrt[1 - 2*x^2]*Defer[Int][x^3/(Sqrt[1 - 4*x^4]*(-1 -
 4*x + 12*x^2 - 8*x^3 + 4*x^4)), x])/(Sqrt[(1 - 2*x^2)/(1 + 2*x^2)]*Sqrt[1
+ 2*x^2])
```

Rubi steps

$$\int \frac{-1+4x-4x^2+4x^4}{\sqrt{\frac{1-2x^2}{1+2x^2}}\left(1+2x^2\right)\left(-1-4x+12x^2-8x^3+4x^4\right)}\,dx = \frac{\sqrt{1-2x^2}\int \frac{-1+4x-4x^2+4x^4}{\sqrt{1-2x^2}\sqrt{1+2x^2}\left(-1-4x+12x^2-8x^3+4x^4\right)}\,dx}{\sqrt{\frac{1-2x^2}{1+2x^2}}\sqrt{1+2x^2}}$$

$$= \frac{\sqrt{1-2x^2}\int \left(\frac{1}{\sqrt{1-2x^2}\sqrt{1+2x^2}} + \frac{8x\left(1-2x+x^2\right)}{\sqrt{1-2x^2}\sqrt{1+2x^2}\left(-1-4x+12x^2-8\right.}\right.}{\sqrt{\frac{1-2x^2}{1+2x^2}}\sqrt{1+2x^2}}$$

$$= \frac{\sqrt{1-2x^2}\int \frac{1}{\sqrt{1-2x^2}\sqrt{1+2x^2}}\,dx}{\sqrt{\frac{1-2x^2}{1+2x^2}}\sqrt{1+2x^2}} + \frac{\left(8\sqrt{1-2x^2}\right)\int \frac{}{\sqrt{1-2x^2}\sqrt{}}}{\sqrt{\frac{1-2x^2}{1+2x^2}}}$$

$$= \frac{\sqrt{1-2x^2}\int \frac{1}{\sqrt{1-4x^4}}\,dx}{\sqrt{\frac{1-2x^2}{1+2x^2}}\sqrt{1+2x^2}} + \frac{\left(8\sqrt{1-2x^2}\right)\int \frac{(-1}{\sqrt{1-2x^2}\sqrt{1+2x^2}\left(-\right.}}{\sqrt{\frac{1-2x^2}{1+2x^2}}\sqrt{1+}}$$

$$= \frac{\sqrt{1-2x^2}\,F\left(\sin^{-1}\left(\sqrt{2}\,x\right)\middle|-1\right)}{\sqrt{2}\sqrt{\frac{1-2x^2}{1+2x^2}}\sqrt{1+2x^2}} + \frac{\left(8\sqrt{1-2x^2}\right)\int \frac{}{\sqrt{1-4x^4}}}{\sqrt{\frac{1-2x^2}{1+2x^2}}}$$

$$= \frac{\sqrt{1-2x^2}\,F\left(\sin^{-1}\left(\sqrt{2}\,x\right)\middle|-1\right)}{\sqrt{2}\sqrt{\frac{1-2x^2}{1+2x^2}}\sqrt{1+2x^2}} + \frac{\left(8\sqrt{1-2x^2}\right)\int \left(\frac{}{\sqrt{1-4x}}\right.}{}$$

$$= \frac{\sqrt{1-2x^2}\,F\left(\sin^{-1}\left(\sqrt{2}\,x\right)\middle|-1\right)}{\sqrt{2}\sqrt{\frac{1-2x^2}{1+2x^2}}\sqrt{1+2x^2}} + \frac{\left(8\sqrt{1-2x^2}\right)\int \frac{}{\sqrt{1-4x^4}}}{\sqrt{\frac{1-2x^2}{1+2x^2}}}$$

Mathematica [C] time = 13.04, size = 10080, normalized size = 210.00

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 4*x - 4*x^2 + 4*x^4)/(Sqrt[(1 - 2*x^2)/(1 + 2*x^2)]*(1 + 2*
x^2)*(-1 - 4*x + 12*x^2 - 8*x^3 + 4*x^4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.37, size = 48, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,x-\sqrt{\frac{2}{3}}}{\sqrt{\frac{1-2x^2}{2x^2+1}}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 4*x - 4*x^2 + 4*x^4)/(Sqrt[(1 - 2*x^2)/(1 + 2*x^2)
]*(1 + 2*x^2)*(-1 - 4*x + 12*x^2 - 8*x^3 + 4*x^4)),x]
```

```
[Out] ArcTanh[(-Sqrt[2/3] + Sqrt[2/3]*x)/Sqrt[(1 - 2*x^2)/(1 + 2*x^2)]]/Sqrt[6]
```

fricas [B] time = 0.49, size = 150, normalized size = 3.12

$$\frac{1}{24}\sqrt{6}\log\left(-\frac{16x^8-64x^7-32x^6+160x^5+8x^4-80x^3+40x^2+4\sqrt{6}\left(8x^7-24x^6+20x^5-20x^4+26x^3-14x^2+9x-5\right)\sqrt{-\frac{2x^2-1}{2x^2+1}}-88x+49}{16x^8-64x^7+160x^6-224x^5+200x^4-80x^3-8x^2+8x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-4*x^2+4*x-1)/((-2*x^2+1)/(2*x^2+1))^(1/2)/(2*x^2+1)/(4*x^4
-8*x^3+12*x^2-4*x-1),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(6)*log(-(16*x^8 - 64*x^7 - 32*x^6 + 160*x^5 + 8*x^4 - 80*x^3 + 40
*x^2 + 4*sqrt(6)*(8*x^7 - 24*x^6 + 20*x^5 - 20*x^4 + 26*x^3 - 14*x^2 + 9*x
- 5)*sqrt(-(2*x^2 - 1)/(2*x^2 + 1)) - 88*x + 49)/(16*x^8 - 64*x^7 + 160*x^6
 - 224*x^5 + 200*x^4 - 80*x^3 - 8*x^2 + 8*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x^4-4\,x^2+4\,x-1}{\left(4\,x^4-8\,x^3+12\,x^2-4\,x-1\right)\left(2\,x^2+1\right)\sqrt{-\frac{2\,x^2-1}{2\,x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-4*x^2+4*x-1)/((-2*x^2+1)/(2*x^2+1))^(1/2)/(2*x^2+1)/(4*x^4
-8*x^3+12*x^2-4*x-1),x, algorithm="giac")
```

```
[Out] integrate((4*x^4 - 4*x^2 + 4*x - 1)/((4*x^4 - 8*x^3 + 12*x^2 - 4*x - 1)*(2*
x^2 + 1)*sqrt(-(2*x^2 - 1)/(2*x^2 + 1))), x)
```

maple [C] time = 0.12, size = 505, normalized size = 10.52

$$-\frac{\left(2x^2-1\right)\left(24\sqrt{2}\,\sqrt{-2x^2+1}\,\sqrt{2x^2+1}\,\mathrm{EllipticF}\left(\sqrt{2}\,x,i\right)+\left(\sum_{\_a=\mathrm{RootOf}\left(4\_Z^4-8\_Z^3+12\_Z^2-4\_Z-1\right)}\frac{\left(2\_a^3-2\_a^2+\_a-1\right)\left(16\sqrt{2}\,\sqrt{-2\_a^3+3\_a^2-\_a}\,\sqrt{-2x^2+1}\,\sqrt{2x^2+1}\,\mathrm{EllipticPi}\left(\sqrt{2}\,x,-8\_a^3+18\_a^2-28\_a+14,\frac{\sqrt{-2}\,\sqrt{2}}{2}\right)\_a^3-32\sqrt{2}\,\sqrt{-2\_a^3+3\_a^2-\_a}\,\sqrt{-2x^2+1}\,\sqrt{2x^2+1}\,\mathrm{EllipticPi}\left(\sqrt{2}\,x,-8\_a^3+18\_a^2-28\_a+14,\frac{\sqrt{-2}\,\sqrt{2}}{2}\right)\_a^2+48\sqrt{2}\,\sqrt{-2\_a^3+3\_a^2-\_a}\,\sqrt{-2x^2+1}\,\sqrt{2x^2+1}\,\mathrm{EllipticPi}\left(\sqrt{2}\,x,-8\_a^3+18\_a^2-28\_a+14,\frac{\sqrt{-2}\,\sqrt{2}}{2}\right)\_a-16\sqrt{2}\,\sqrt{-2x^2+1}\,\sqrt{2x^2+1}\,\mathrm{EllipticPi}\left(\sqrt{2}\,x,-8\_a^3+18\_a^2-28\_a+14,\frac{\sqrt{-2}\,\sqrt{2}}{2}\right)\sqrt{-2\_a^3+3\_a^2-\_a}-\sqrt{4}\,\mathrm{arctanh}\left(\frac{2\_a^2\left(4\_a^3-9\_a^2+x^2+14\_a-7\right)}{\sqrt{-2\_a^3+3\_a^2-\_a}\,\sqrt{-4x^4+1}}\right)\sqrt{-4x^4+1}\right)}{\sqrt{-2\_a^3+3\_a^2-\_a}\,\sqrt{-4x^4+1}}\right)\sqrt{-4x^4+1}\right)}{48\sqrt{-\frac{2x^2-1}{2x^2+1}}\,\sqrt{-\left(2x^2+1\right)\left(2x^2-1\right)}\,\sqrt{-4x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-4*x^2+4*x-1)/((-2*x^2+1)/(2*x^2+1))^(1/2)/(2*x^2+1)/(4*x^4-8*x^3
+12*x^2-4*x-1),x)
```

```
[Out] -1/48*(2*x^2-1)*(24*2^(1/2)*(-2*x^2+1)^(1/2)*(2*x^2+1)^(1/2)*EllipticF(2^(1
/2)*x,I)+sum((2*_alpha^3-2*_alpha^2+_alpha-1)*(16*2^(1/2)*(-2*_alpha^3+3*_a
lpha^2-_alpha)^(1/2)*(-2*x^2+1)^(1/2)*(2*x^2+1)^(1/2)*EllipticPi(2^(1/2)*x,
-8*_alpha^3+18*_alpha^2-28*_alpha+14,1/2*(-2)^(1/2)*2^(1/2))*_alpha^3-32*2^
(1/2)*(-2*_alpha^3+3*_alpha^2-_alpha)^(1/2)*(-2*x^2+1)^(1/2)*(2*x^2+1)^(1/2
)*EllipticPi(2^(1/2)*x,-8*_alpha^3+18*_alpha^2-28*_alpha+14,1/2*(-2)^(1/2)*
2^(1/2))*_alpha^2+48*2^(1/2)*(-2*_alpha^3+3*_alpha^2-_alpha)^(1/2)*(-2*x^2+
1)^(1/2)*(2*x^2+1)^(1/2)*EllipticPi(2^(1/2)*x,-8*_alpha^3+18*_alpha^2-28*_a
lpha+14,1/2*(-2)^(1/2)*2^(1/2))*_alpha-16*2^(1/2)*(-2*x^2+1)^(1/2)*(2*x^2+1
)^(1/2)*EllipticPi(2^(1/2)*x,-8*_alpha^3+18*_alpha^2-28*_alpha+14,1/2*(-2)^
(1/2)*2^(1/2))*(-2*_alpha^3+3*_alpha^2-_alpha)^(1/2)-4^(1/2)*arctanh(2*_alp
ha^2*(4*_alpha^3-9*_alpha^2+x^2+14*_alpha-7)/(-2*_alpha^3+3*_alpha^2-_alpha
)^(1/2)/(-4*x^4+1)^(1/2))*(-4*x^4+1)^(1/2))/(-2*_alpha^3+3*_alpha^2-_alpha)
^(1/2)/(-4*x^4+1)^(1/2),_alpha=RootOf(4*_Z^4-8*_Z^3+12*_Z^2-4*_Z-1))*(-4*x^
4+1)^(1/2))/(-(2*x^2-1)/(2*x^2+1))^(1/2)/(-(2*x^2+1)*(2*x^2-1))^(1/2)/(-4*x
^4+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x^4-4\,x^2+4\,x-1}{\left(4\,x^4-8\,x^3+12\,x^2-4\,x-1\right)\left(2\,x^2+1\right)\sqrt{-\frac{2\,x^2-1}{2\,x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-4*x^2+4*x-1)/((-2*x^2+1)/(2*x^2+1))^(1/2)/(2*x^2+1)/(4*x^4
-8*x^3+12*x^2-4*x-1),x, algorithm="maxima")
```

```
[Out] integrate((4*x^4 - 4*x^2 + 4*x - 1)/((4*x^4 - 8*x^3 + 12*x^2 - 4*x - 1)*(2*
x^2 + 1)*sqrt(-(2*x^2 - 1)/(2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{4x^4-4x^2+4x-1}{\left(2x^2+1\right)\sqrt{-\frac{2x^2-1}{2x^2+1}}\,\left(-4x^4+8x^3-12x^2+4x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x - 4*x^2 + 4*x^4 - 1)/((2*x^2 + 1)*(-(2*x^2 - 1)/(2*x^2 + 1))^(1/2
)*(4*x - 12*x^2 + 8*x^3 - 4*x^4 + 1)),x)
```

```
[Out] -int((4*x - 4*x^2 + 4*x^4 - 1)/((2*x^2 + 1)*(-(2*x^2 - 1)/(2*x^2 + 1))^(1/2
)*(4*x - 12*x^2 + 8*x^3 - 4*x^4 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-4*x**2+4*x-1)/((-2*x**2+1)/(2*x**2+1))**(1/2)/(2*x**2+1)/
(4*x**4-8*x**3+12*x**2-4*x-1),x)
```

```
[Out] Timed out
```

3.610 $\int \frac{(-3+x^4)(1-2x^3+x^4)(1-x^3+x^4)}{x^6(1+x^4)\sqrt[4]{x+x^5}}\,dx$

Optimal. Leaf size=48

$$\frac{4\left(x^5+x\right)^{3/4}\left(x^8-7x^7-14x^6+2x^4-7x^3+1\right)}{7x^6\left(x^4+1\right)}$$

Rubi [C] time = 0.57, antiderivative size = 208, normalized size of antiderivative = 4.33, number of steps used = 18, number of rules used = 9, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2056, 1833, 1585, 1478, 449, 1835, 1586, 1844, 364}

$$\frac{8\sqrt[4]{x^4+1}\,x^5\,{}_2F_1\left(\frac{19}{16},\frac{5}{4};\frac{35}{16};-x^4\right)}{19\sqrt[4]{x^5+x}}-\frac{8\sqrt[4]{x^4+1}\,x\,{}_2F_1\left(\frac{3}{16},\frac{5}{4};\frac{19}{16};-x^4\right)}{\sqrt[4]{x^5+x}}+\frac{4\sqrt[4]{x^4+1}\,x^7\,{}_2F_1\left(\frac{5}{4},\frac{27}{16};\frac{43}{16};-x^4\right)}{27\sqrt[4]{x^5+x}}+\frac{4\sqrt[4]{x^4+1}\,x^3\,{}_2F_1\left(\frac{11}{16},\frac{5}{4};\frac{27}{16};-x^4\right)}{7\sqrt[4]{x^5+x}}+\frac{8}{7\sqrt[4]{x^5+x}\,x}+\frac{4}{7\sqrt[4]{x^5+x}\,x^5}-\frac{4\left(x^4+1\right)}{\sqrt[4]{x^5+x}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + x^4)*(1 - 2*x^3 + x^4)*(1 - x^3 + x^4))/(x^6*(1 + x^4)*(x + x^5)
^(1/4)),x]
```

```
[Out] 4/(7*x^5*(x + x^5)^(1/4)) + 8/(7*x*(x + x^5)^(1/4)) - (4*(1 + x^4))/(x^2*(x
 + x^5)^(1/4)) - (8*x*(1 + x^4)^(1/4)*Hypergeometric2F1[3/16, 5/4, 19/16, -
x^4])/(x + x^5)^(1/4) + (4*x^3*(1 + x^4)^(1/4)*Hypergeometric2F1[11/16, 5/4
, 27/16, -x^4])/(7*(x + x^5)^(1/4)) + (8*x^5*(1 + x^4)^(1/4)*Hypergeometric
2F1[19/16, 5/4, 35/16, -x^4])/(19*(x + x^5)^(1/4)) + (4*x^7*(1 + x^4)^(1/4)
*Hypergeometric2F1[5/4, 27/16, 43/16, -x^4])/(27*(x + x^5)^(1/4))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1478

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n
_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p
]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\left(-3+x^4\right)\left(1-2x^3+x^4\right)\left(1-x^3+x^4\right)}{x^6\left(1+x^4\right)\sqrt[4]{x+x^5}}\,dx = \frac{\left(\sqrt[4]{x}\,\sqrt[4]{1+x^4}\right)\int \frac{\left(-3+x^4\right)\left(1-2x^3+x^4\right)\left(1-x^3+x^4\right)}{x^{25/4}\left(1+x^4\right)^{5/4}}\,dx}{\sqrt[4]{x+x^5}}$$

$$= \frac{\left(\sqrt[4]{x}\,\sqrt[4]{1+x^4}\right)\int \left(\frac{9x^2+6x^6-3x^{10}}{x^{21/4}\left(1+x^4\right)^{5/4}} + \frac{-3-5x^4-6x^6-x^8+2x^{10}+x^{12}}{x^{25/4}\left(1+x^4\right)^{5/4}}\right)dx}{\sqrt[4]{x+x^5}}$$

$$= \frac{\left(\sqrt[4]{x}\,\sqrt[4]{1+x^4}\right)\int \frac{9x^2+6x^6-3x^{10}}{x^{21/4}\left(1+x^4\right)^{5/4}}\,dx}{\sqrt[4]{x+x^5}} + \frac{\left(\sqrt[4]{x}\,\sqrt[4]{1+x^4}\right)\int \frac{-3-5x^4-6x^6-\cdots}{x^{25/4}\left(1-\cdots\right.}}{\sqrt[4]{x+x^5}}$$

$$= \frac{4}{7x^5\sqrt[4]{x+x^5}} - \frac{\left(2\sqrt[4]{x}\,\sqrt[4]{1+x^4}\right)\int \frac{15x^3+63x^5+\frac{21x^7}{2}-21x^9-\frac{21x^{11}}{2}}{x^{21/4}\left(1+x^4\right)^{5/4}}\,dx}{21\sqrt[4]{x+x^5}} + \cdots$$

$$= \frac{4}{7x^5\sqrt[4]{x+x^5}} - \frac{\left(2\sqrt[4]{x}\,\sqrt[4]{1+x^4}\right)\int \frac{15x^2+63x^4+\frac{21x^6}{2}-21x^8-\frac{21x^{10}}{2}}{x^{17/4}\left(1+x^4\right)^{5/4}}\,dx}{21\sqrt[4]{x+x^5}} + \cdots$$

$$= \frac{4}{7x^5\sqrt[4]{x+x^5}} - \frac{4\left(1+x^4\right)}{x^2\sqrt[4]{x+x^5}} - \frac{\left(2\sqrt[4]{x}\,\sqrt[4]{1+x^4}\right)\int \frac{15x+63x^3+\frac{21x^5}{2}-2\cdots}{x^{13/4}\left(1+x^4\right)^{\cdots}}}{21\sqrt[4]{x+x^5}}$$

$$= \frac{4}{7x^5\sqrt[4]{x+x^5}} - \frac{4\left(1+x^4\right)}{x^2\sqrt[4]{x+x^5}} - \frac{\left(2\sqrt[4]{x}\,\sqrt[4]{1+x^4}\right)\int \frac{15+63x^2+\frac{21x^4}{2}-21\cdots}{x^{9/4}\left(1+x^4\right)^{5/\cdots}}}{21\sqrt[4]{x+x^5}}$$

$$= \frac{4}{7x^5\sqrt[4]{x+x^5}} + \frac{8}{7x\sqrt[4]{x+x^5}} - \frac{4\left(1+x^4\right)}{x^2\sqrt[4]{x+x^5}} + \frac{\left(4\sqrt[4]{x}\,\sqrt[4]{1+x^4}\right)\int \cdots}{105\cdots}$$

$$= \frac{4}{7x^5\sqrt[4]{x+x^5}} + \frac{8}{7x\sqrt[4]{x+x^5}} - \frac{4\left(1+x^4\right)}{x^2\sqrt[4]{x+x^5}} + \frac{\left(4\sqrt[4]{x}\,\sqrt[4]{1+x^4}\right)\int \cdots}{105\cdots}$$

$$= \frac{4}{7x^5\sqrt[4]{x+x^5}} + \frac{8}{7x\sqrt[4]{x+x^5}} - \frac{4\left(1+x^4\right)}{x^2\sqrt[4]{x+x^5}} + \frac{\left(4\sqrt[4]{x}\,\sqrt[4]{1+x^4}\right)\int \left(\cdots\right.}{\cdots}$$

$$= \frac{4}{7x^5\sqrt[4]{x+x^5}} + \frac{8}{7x\sqrt[4]{x+x^5}} - \frac{4\left(1+x^4\right)}{x^2\sqrt[4]{x+x^5}} + \frac{\left(\sqrt[4]{x}\,\sqrt[4]{1+x^4}\right)\int \frac{\cdots}{(1\cdots}}{\sqrt[4]{x+x^5}}$$

$$= \frac{4}{7x^5\sqrt[4]{x+x^5}} + \frac{8}{7x\sqrt[4]{x+x^5}} - \frac{4\left(1+x^4\right)}{x^2\sqrt[4]{x+x^5}} - \frac{8x\sqrt[4]{1+x^4}\;{}_2F_1\left(\frac{3}{16},\cdots\right.}{\sqrt[4]{x+x^5}\cdots}$$

Mathematica [C] time = 0.16, size = 203, normalized size = 4.23

$$\frac{4\sqrt[4]{x^4+1}\left(129789\,{}_2F_1\left(-\frac{21}{16},\frac{5}{4};-\frac{5}{16};-x^4\right)+x^3\left(778734x^4\,{}_2F_1\left(\frac{7}{16},\frac{5}{4};\frac{23}{16};-x^4\right)+908523x\,{}_2F_1\left(-\frac{5}{16},\frac{5}{4};\frac{11}{16};-x^4\right)-908523\,{}_2F_1\left(-\frac{9}{16},\frac{5}{4};\frac{7}{16};-x^4\right)+33649x^9\,{}_2F_1\left(\frac{5}{4},\frac{27}{16};\frac{43}{16};-x^4\right)-118503x^8\,{}_2F_1\left(\frac{5}{4},\frac{23}{16};\frac{39}{16};-x^4\right)+95634x^7\,{}_2F_1\left(\frac{19}{16},\frac{5}{4};\frac{35}{16};-x^4\right)-82593x^5\,{}_2F_1\left(\frac{11}{16},\frac{5}{4};\frac{27}{16};-x^4\right)-1817046x^3\,{}_2F_1\left(\frac{3}{16},\frac{5}{4};\frac{19}{16};-x^4\right)\right)\right)}{908523x^5\sqrt[4]{x^5+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + x^4)*(1 - 2*x^3 + x^4)*(1 - x^3 + x^4))/(x^6*(1 + x^4)*(x
+ x^5)^(1/4)),x]
```

```
[Out] (4*(1 + x^4)^(1/4)*(129789*Hypergeometric2F1[-21/16, 5/4, -5/16, -x^4] + x^
3*(-908523*Hypergeometric2F1[-9/16, 5/4, 7/16, -x^4] + 908523*x*Hypergeomet
```

```
ric2F1[-5/16, 5/4, 11/16, -x^4] - 1817046*x^3*Hypergeometric2F1[3/16, 5/4,
19/16, -x^4] + 778734*x^4*Hypergeometric2F1[7/16, 5/4, 23/16, -x^4] - 82593
*x^5*Hypergeometric2F1[11/16, 5/4, 27/16, -x^4] + 95634*x^7*Hypergeometric2
F1[19/16, 5/4, 35/16, -x^4] - 118503*x^8*Hypergeometric2F1[5/4, 23/16, 39/1
6, -x^4] + 33649*x^9*Hypergeometric2F1[5/4, 27/16, 43/16, -x^4])))/(908523*
x^5*(x + x^5)^(1/4))
```

IntegrateAlgebraic [A] time = 2.65, size = 48, normalized size = 1.00

$$\frac{4\left(x^5+x\right)^{3/4}\left(x^8-7x^7-14x^6+2x^4-7x^3+1\right)}{7x^6\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 - 2*x^3 + x^4)*(1 - x^3 + x^4))/(x^6*(1 +
 x^4)*(x + x^5)^(1/4)),x]
```

```
[Out] (4*(x + x^5)^(3/4)*(1 - 7*x^3 + 2*x^4 - 14*x^6 - 7*x^7 + x^8))/(7*x^6*(1 +
x^4))
```

fricas [A] time = 0.40, size = 43, normalized size = 0.90

$$\frac{4\left(x^8-7x^7-14x^6+2x^4-7x^3+1\right)\left(x^5+x\right)^{\frac{3}{4}}}{7\left(x^{10}+x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4-2*x^3+1)*(x^4-x^3+1)/x^6/(x^4+1)/(x^5+x)^(1/4),x, al
gorithm="fricas")
```

```
[Out] 4/7*(x^8 - 7*x^7 - 14*x^6 + 2*x^4 - 7*x^3 + 1)*(x^5 + x)^(3/4)/(x^10 + x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^3+1\right)\left(x^4-2x^3+1\right)\left(x^4-3\right)}{\left(x^5+x\right)^{\frac{1}{4}}\left(x^4+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4-2*x^3+1)*(x^4-x^3+1)/x^6/(x^4+1)/(x^5+x)^(1/4),x, al
gorithm="giac")
```

```
[Out] integrate((x^4 - x^3 + 1)*(x^4 - 2*x^3 + 1)*(x^4 - 3)/((x^5 + x)^(1/4)*(x^4
 + 1)*x^6), x)
```

maple [A] time = 0.01, size = 38, normalized size = 0.79

$$\frac{\frac{4}{7}x^8-4x^7-8x^6+\frac{8}{7}x^4-4x^3+\frac{4}{7}}{\left(x^5+x\right)^{\frac{1}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4-2*x^3+1)*(x^4-x^3+1)/x^6/(x^4+1)/(x^5+x)^(1/4),x)
```

```
[Out] 4/7*(x^8-7*x^7-14*x^6+2*x^4-7*x^3+1)/(x^5+x)^(1/4)/x^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^3+1\right)\left(x^4-2x^3+1\right)\left(x^4-3\right)}{\left(x^5+x\right)^{\frac{1}{4}}\left(x^4+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4-2*x^3+1)*(x^4-x^3+1)/x^6/(x^4+1)/(x^5+x)^(1/4),x, al
gorithm="maxima")
```

```
[Out] integrate((x^4 - x^3 + 1)*(x^4 - 2*x^3 + 1)*(x^4 - 3)/((x^5 + x)^(1/4)*(x^4
 + 1)*x^6), x)
```

mupad [B] time = 0.66, size = 53, normalized size = 1.10

$$\frac{4\left(x^5+x\right)^{3/4}}{7\,x^2}-\frac{8\left(x^5+x\right)^{3/4}}{x^4+1}-\frac{4\left(x^5+x\right)^{3/4}}{x^3}+\frac{4\left(x^5+x\right)^{3/4}}{7\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 3)*(x^4 - x^3 + 1)*(x^4 - 2*x^3 + 1))/(x^6*(x^4 + 1)*(x + x^5)^
(1/4)),x)
```

```
[Out] (4*(x + x^5)^(3/4))/(7*x^2) - (8*(x + x^5)^(3/4))/(x^4 + 1) - (4*(x + x^5)^
(3/4))/x^3 + (4*(x + x^5)^(3/4))/(7*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\left(x^4-3\right)\left(x^4-x^3+1\right)\left(x^3-x^2-x-1\right)}{x^6\sqrt[4]{x\left(x^4+1\right)}\,\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4-2*x**3+1)*(x**4-x**3+1)/x**6/(x**4+1)/(x**5+x)**(1
/4),x)
```

```
[Out] Integral((x - 1)*(x**4 - 3)*(x**4 - x**3 + 1)*(x**3 - x**2 - x - 1)/(x**6*(
x*(x**4 + 1))**(1/4)*(x**4 + 1)), x)
```

3.611 $\int \frac{x^{20}}{\sqrt{-1+x^6}}\,dx$

Optimal. Leaf size=48

$$\frac{5}{48}\log\left(\sqrt{x^6-1}+x^3\right)+\frac{1}{144}\sqrt{x^6-1}\left(8x^{15}+10x^9+15x^3\right)$$

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {275, 321, 217, 206}

$$\frac{1}{18}\sqrt{x^6-1}\,x^{15}+\frac{5}{72}\sqrt{x^6-1}\,x^9+\frac{5}{48}\sqrt{x^6-1}\,x^3+\frac{5}{48}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^20/Sqrt[-1 + x^6],x]
```

```
[Out] (5*x^3*Sqrt[-1 + x^6])/48 + (5*x^9*Sqrt[-1 + x^6])/72 + (x^15*Sqrt[-1 + x^6
])/18 + (5*ArcTanh[x^3/Sqrt[-1 + x^6]])/48
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{20}}{\sqrt{-1+x^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^6}{\sqrt{-1+x^2}}\,dx,x,x^3\right)\\ &= \frac{1}{18}x^{15}\sqrt{-1+x^6}+\frac{5}{18}\text{Subst}\left(\int \frac{x^4}{\sqrt{-1+x^2}}\,dx,x,x^3\right)\\ &= \frac{5}{72}x^9\sqrt{-1+x^6}+\frac{1}{18}x^{15}\sqrt{-1+x^6}+\frac{5}{24}\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x^2}}\,dx,x,x^3\right)\\ &= \frac{5}{48}x^3\sqrt{-1+x^6}+\frac{5}{72}x^9\sqrt{-1+x^6}+\frac{1}{18}x^{15}\sqrt{-1+x^6}+\frac{5}{48}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx,x,x^3\right)\\ &= \frac{5}{48}x^3\sqrt{-1+x^6}+\frac{5}{72}x^9\sqrt{-1+x^6}+\frac{1}{18}x^{15}\sqrt{-1+x^6}+\frac{5}{48}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x^3}{\sqrt{-1+x^6}}\right)\\ &= \frac{5}{48}x^3\sqrt{-1+x^6}+\frac{5}{72}x^9\sqrt{-1+x^6}+\frac{1}{18}x^{15}\sqrt{-1+x^6}+\frac{5}{48}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.96

$$\frac{1}{144}\left(15\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)+\sqrt{x^6-1}\left(8x^{12}+10x^6+15\right)x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^20/Sqrt[-1 + x^6],x]
[Out] (x^3*Sqrt[-1 + x^6]*(15 + 10*x^6 + 8*x^12) + 15*ArcTanh[x^3/Sqrt[-1 + x^6]]
)/144
```

IntegrateAlgebraic [A] time = 0.20, size = 48, normalized size = 1.00

$$\frac{5}{48}\log\left(\sqrt{x^6-1}+x^3\right)+\frac{1}{144}\sqrt{x^6-1}\left(8x^{15}+10x^9+15x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^20/Sqrt[-1 + x^6],x]
[Out] (Sqrt[-1 + x^6]*(15*x^3 + 10*x^9 + 8*x^15))/144 + (5*Log[x^3 + Sqrt[-1 + x^
6]])/48
```

fricas [A] time = 0.38, size = 42, normalized size = 0.88

$$\frac{1}{144}\left(8x^{15}+10x^9+15x^3\right)\sqrt{x^6-1}-\frac{5}{48}\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20/(x^6-1)^(1/2),x, algorithm="fricas")
[Out] 1/144*(8*x^15 + 10*x^9 + 15*x^3)*sqrt(x^6 - 1) - 5/48*log(-x^3 + sqrt(x^6 -
 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{20}}{\sqrt{x^6-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20/(x^6-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^20/sqrt(x^6 - 1), x)
```

maple [C] time = 0.31, size = 50, normalized size = 1.04

$$\frac{x^{3} \left(8 x^{12}+10 x^{6}+15\right) \sqrt{x^{6}-1}}{144}+\frac{5 \sqrt{-\mathrm{signum}\left(x^{6}-1\right)}\, \arcsin \left(x^{3}\right)}{48 \sqrt{\mathrm{signum}\left(x^{6}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^20/(x^6-1)^(1/2),x)
```

```
[Out] 1/144*x^3*(8*x^12+10*x^6+15)*(x^6-1)^(1/2)+5/48/signum(x^6-1)^(1/2)*(-signu
m(x^6-1))^(1/2)*arcsin(x^3)
```

maxima [B] time = 0.32, size = 109, normalized size = 2.27

$$-\frac{\frac{33 \sqrt{x^{6}-1}}{x^{3}}-\frac{40 \left(x^{6}-1\right)^{\frac{3}{2}}}{x^{9}}+\frac{15 \left(x^{6}-1\right)^{\frac{5}{2}}}{x^{15}}}{144 \left(\frac{3 \left(x^{6}-1\right)}{x^{6}}-\frac{3 \left(x^{6}-1\right)^{2}}{x^{12}}+\frac{\left(x^{6}-1\right)^{3}}{x^{18}}-1\right)}+\frac{5}{96} \log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}+1\right)-\frac{5}{96} \log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20/(x^6-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/144*(33*sqrt(x^6 - 1)/x^3 - 40*(x^6 - 1)^(3/2)/x^9 + 15*(x^6 - 1)^(5/2)/
x^15)/(3*(x^6 - 1)/x^6 - 3*(x^6 - 1)^2/x^12 + (x^6 - 1)^3/x^18 - 1) + 5/96*
log(sqrt(x^6 - 1)/x^3 + 1) - 5/96*log(sqrt(x^6 - 1)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{20}}{\sqrt{x^{6}-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^20/(x^6 - 1)^(1/2),x)
```

```
[Out] int(x^20/(x^6 - 1)^(1/2), x)
```

sympy [A] time = 4.68, size = 143, normalized size = 2.98

$$\begin{cases} \frac{x^{21}}{18 \sqrt{x^{6}-1}}+\frac{x^{15}}{72 \sqrt{x^{6}-1}}+\frac{5 x^{9}}{144 \sqrt{x^{6}-1}}-\frac{5 x^{3}}{48 \sqrt{x^{6}-1}}+\frac{5 \operatorname{acosh}\left(x^{3}\right)}{48} & \text{for}\: \left|x^{6}\right|>1 \\ -\frac{i x^{21}}{18 \sqrt{1-x^{6}}}-\frac{i x^{15}}{72 \sqrt{1-x^{6}}}-\frac{5 i x^{9}}{144 \sqrt{1-x^{6}}}+\frac{5 i x^{3}}{48 \sqrt{1-x^{6}}}-\frac{5 i \operatorname{asin}\left(x^{3}\right)}{48} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**20/(x**6-1)**(1/2),x)
```

```
[Out] Piecewise((x**21/(18*sqrt(x**6 - 1)) + x**15/(72*sqrt(x**6 - 1)) + 5*x**9/(
144*sqrt(x**6 - 1)) - 5*x**3/(48*sqrt(x**6 - 1)) + 5*acosh(x**3)/48, Abs(x*
*6) > 1), (-I*x**21/(18*sqrt(1 - x**6)) - I*x**15/(72*sqrt(1 - x**6)) - 5*I
*x**9/(144*sqrt(1 - x**6)) + 5*I*x**3/(48*sqrt(1 - x**6)) - 5*I*asin(x**3)/
48, True))
```

3.612 $\int x^{14}\sqrt{-1+x^6}\,dx$

Optimal. Leaf size=48

$$\frac{1}{144}\sqrt{x^6-1}\left(8x^{15}-2x^9-3x^3\right)-\frac{1}{48}\log\left(\sqrt{x^6-1}+x^3\right)$$

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {275, 279, 321, 217, 206}

$$\frac{1}{18}\sqrt{x^6-1}\,x^{15}-\frac{1}{72}\sqrt{x^6-1}\,x^9-\frac{1}{48}\sqrt{x^6-1}\,x^3-\frac{1}{48}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^14*Sqrt[-1 + x^6],x]
```

```
[Out] -1/48*(x^3*Sqrt[-1 + x^6]) - (x^9*Sqrt[-1 + x^6])/72 + (x^15*Sqrt[-1 + x^6]
)/18 - ArcTanh[x^3/Sqrt[-1 + x^6]]/48
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^{14}\sqrt{-1+x^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int x^4\sqrt{-1+x^2}\,dx,x,x^3\right)\\
&= \frac{1}{18}x^{15}\sqrt{-1+x^6}-\frac{1}{18}\,\text{Subst}\left(\int\frac{x^4}{\sqrt{-1+x^2}}\,dx,x,x^3\right)\\
&= -\frac{1}{72}x^9\sqrt{-1+x^6}+\frac{1}{18}x^{15}\sqrt{-1+x^6}-\frac{1}{24}\,\text{Subst}\left(\int\frac{x^2}{\sqrt{-1+x^2}}\,dx,x,x^3\right)\\
&= -\frac{1}{48}x^3\sqrt{-1+x^6}-\frac{1}{72}x^9\sqrt{-1+x^6}+\frac{1}{18}x^{15}\sqrt{-1+x^6}-\frac{1}{48}\,\text{Subst}\left(\int\frac{1}{\sqrt{-1+x^2}}\,dx,x,x^3\right)\\
&= -\frac{1}{48}x^3\sqrt{-1+x^6}-\frac{1}{72}x^9\sqrt{-1+x^6}+\frac{1}{18}x^{15}\sqrt{-1+x^6}-\frac{1}{48}\,\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x^3}{\sqrt{-1+x^6}}\right)\\
&= -\frac{1}{48}x^3\sqrt{-1+x^6}-\frac{1}{72}x^9\sqrt{-1+x^6}+\frac{1}{18}x^{15}\sqrt{-1+x^6}-\frac{1}{48}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 1.17

$$\frac{(x^6-1)\left(3\sin^{-1}(x^3)+\sqrt{1-x^6}\,(8x^{12}-2x^6-3)\,x^3\right)}{144\sqrt{-(x^6-1)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*Sqrt[-1 + x^6],x]
```

```
[Out] ((-1 + x^6)*(x^3*Sqrt[1 - x^6]*(-3 - 2*x^6 + 8*x^12) + 3*ArcSin[x^3]))/(144
*Sqrt[-(-1 + x^6)^2])
```

IntegrateAlgebraic [A] time = 0.16, size = 48, normalized size = 1.00

$$\frac{1}{144}\sqrt{x^6-1}\,(8x^{15}-2x^9-3x^3)-\frac{1}{48}\log\left(\sqrt{x^6-1}+x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^14*Sqrt[-1 + x^6],x]
```

```
[Out] (Sqrt[-1 + x^6]*(-3*x^3 - 2*x^9 + 8*x^15))/144 - Log[x^3 + Sqrt[-1 + x^6]]/
48
```

fricas [A] time = 0.39, size = 42, normalized size = 0.88

$$\frac{1}{144}\left(8\,x^{15}-2\,x^9-3\,x^3\right)\sqrt{x^6-1}+\frac{1}{48}\,\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(x^6-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/144*(8*x^15 - 2*x^9 - 3*x^3)*sqrt(x^6 - 1) + 1/48*log(-x^3 + sqrt(x^6 - 1
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^6-1}\,x^{14}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(x^6-1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^6 - 1)*x^14, x)
```

maple [C] time = 0.31, size = 50, normalized size = 1.04

$$\frac{x^{3}\left(8x^{12}-2x^{6}-3\right)\sqrt{x^{6}-1}}{144}-\frac{\sqrt{-\mathrm{signum}\left(x^{6}-1\right)}\,\arcsin\left(x^{3}\right)}{48\sqrt{\mathrm{signum}\left(x^{6}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(x^6-1)^(1/2),x)
[Out] 1/144*x^3*(8*x^12-2*x^6-3)*(x^6-1)^(1/2)-1/48/signum(x^6-1)^(1/2)*(-signum(
x^6-1))^(1/2)*arcsin(x^3)
```

maxima [B] time = 0.32, size = 109, normalized size = 2.27

$$-\frac{\frac{3\sqrt{x^{6}-1}}{x^{3}}+\frac{8\left(x^{6}-1\right)^{\frac{3}{2}}}{x^{9}}-\frac{3\left(x^{6}-1\right)^{\frac{5}{2}}}{x^{15}}}{144\left(\frac{3\left(x^{6}-1\right)}{x^{6}}-\frac{3\left(x^{6}-1\right)^{2}}{x^{12}}+\frac{\left(x^{6}-1\right)^{3}}{x^{18}}-1\right)}-\frac{1}{96}\log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}+1\right)+\frac{1}{96}\log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(x^6-1)^(1/2),x, algorithm="maxima")
[Out] -1/144*(3*sqrt(x^6 - 1)/x^3 + 8*(x^6 - 1)^(3/2)/x^9 - 3*(x^6 - 1)^(5/2)/x^1
5)/(3*(x^6 - 1)/x^6 - 3*(x^6 - 1)^2/x^12 + (x^6 - 1)^3/x^18 - 1) - 1/96*log
(sqrt(x^6 - 1)/x^3 + 1) + 1/96*log(sqrt(x^6 - 1)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{14}\sqrt{x^{6}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(x^6 - 1)^(1/2),x)
[Out] int(x^14*(x^6 - 1)^(1/2), x)
```

sympy [A] time = 3.84, size = 136, normalized size = 2.83

$$\begin{cases}\frac{x^{21}}{18\sqrt{x^{6}-1}}-\frac{5x^{15}}{72\sqrt{x^{6}-1}}-\frac{x^{9}}{144\sqrt{x^{6}-1}}+\frac{x^{3}}{48\sqrt{x^{6}-1}}-\frac{\operatorname{acosh}\left(x^{3}\right)}{48} & \text{for}\ \left|x^{6}\right|>1\\ -\frac{ix^{21}}{18\sqrt{1-x^{6}}}+\frac{5ix^{15}}{72\sqrt{1-x^{6}}}+\frac{ix^{9}}{144\sqrt{1-x^{6}}}-\frac{ix^{3}}{48\sqrt{1-x^{6}}}+\frac{i\operatorname{asin}\left(x^{3}\right)}{48} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(x**6-1)**(1/2),x)
[Out] Piecewise((x**21/(18*sqrt(x**6 - 1)) - 5*x**15/(72*sqrt(x**6 - 1)) - x**9/(
144*sqrt(x**6 - 1)) + x**3/(48*sqrt(x**6 - 1)) - acosh(x**3)/48, Abs(x**6)
> 1), (-I*x**21/(18*sqrt(1 - x**6)) + 5*I*x**15/(72*sqrt(1 - x**6)) + I*x**
9/(144*sqrt(1 - x**6)) - I*x**3/(48*sqrt(1 - x**6)) + I*asin(x**3)/48, True
))
```

3.613 $\int \frac{x^{20}}{\sqrt{1+x^6}}\,dx$

Optimal. Leaf size=48

$$\frac{1}{144}\sqrt{x^6+1}\left(8x^{15}-10x^9+15x^3\right)-\frac{5}{48}\log\left(\sqrt{x^6+1}+x^3\right)$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {275, 321, 215}

$$-\frac{5}{48}\sinh^{-1}\left(x^3\right)+\frac{1}{18}\sqrt{x^6+1}\,x^{15}-\frac{5}{72}\sqrt{x^6+1}\,x^9+\frac{5}{48}\sqrt{x^6+1}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^20/Sqrt[1 + x^6],x]
```

```
[Out] (5*x^3*Sqrt[1 + x^6])/48 - (5*x^9*Sqrt[1 + x^6])/72 + (x^15*Sqrt[1 + x^6])/
18 - (5*ArcSinh[x^3])/48
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{20}}{\sqrt{1+x^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^6}{\sqrt{1+x^2}}\,dx,x,x^3\right)\\
&= \frac{1}{18}x^{15}\sqrt{1+x^6}-\frac{5}{18}\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}}\,dx,x,x^3\right)\\
&= -\frac{5}{72}x^9\sqrt{1+x^6}+\frac{1}{18}x^{15}\sqrt{1+x^6}+\frac{5}{24}\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx,x,x^3\right)\\
&= \frac{5}{48}x^3\sqrt{1+x^6}-\frac{5}{72}x^9\sqrt{1+x^6}+\frac{1}{18}x^{15}\sqrt{1+x^6}-\frac{5}{48}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,x^3\right)\\
&= \frac{5}{48}x^3\sqrt{1+x^6}-\frac{5}{72}x^9\sqrt{1+x^6}+\frac{1}{18}x^{15}\sqrt{1+x^6}-\frac{5}{48}\sinh^{-1}\left(x^3\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.75

$$\frac{1}{144}\left(x^3\sqrt{x^6+1}\left(8x^{12}-10x^6+15\right)-15\sinh^{-1}\left(x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^20/Sqrt[1 + x^6],x]
[Out] (x^3*Sqrt[1 + x^6]*(15 - 10*x^6 + 8*x^12) - 15*ArcSinh[x^3])/144
```

IntegrateAlgebraic [A] time = 0.20, size = 48, normalized size = 1.00

$$\frac{1}{144}\sqrt{x^6+1}\left(8x^{15}-10x^9+15x^3\right)-\frac{5}{48}\log\left(\sqrt{x^6+1}+x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^20/Sqrt[1 + x^6],x]
[Out] (Sqrt[1 + x^6]*(15*x^3 - 10*x^9 + 8*x^15))/144 - (5*Log[x^3 + Sqrt[1 + x^6]
])/48
```

fricas [A] time = 0.38, size = 42, normalized size = 0.88

$$\frac{1}{144}\left(8\,x^{15}-10\,x^9+15\,x^3\right)\sqrt{x^6+1}+\frac{5}{48}\log\left(-x^3+\sqrt{x^6+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20/(x^6+1)^(1/2),x, algorithm="fricas")
[Out] 1/144*(8*x^15 - 10*x^9 + 15*x^3)*sqrt(x^6 + 1) + 5/48*log(-x^3 + sqrt(x^6 +
 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{20}}{\sqrt{x^6+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20/(x^6+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^20/sqrt(x^6 + 1), x)
```

maple [A] time = 0.29, size = 32, normalized size = 0.67

$$\frac{x^3\left(8x^{12}-10x^6+15\right)\sqrt{x^6+1}}{144}-\frac{5\,\mathrm{arcsinh}\left(x^3\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^20/(x^6+1)^(1/2),x)
[Out] 1/144*x^3*(8*x^12-10*x^6+15)*(x^6+1)^(1/2)-5/48*arcsinh(x^3)
```

maxima [B] time = 0.33, size = 109, normalized size = 2.27

$$\frac{\frac{33\sqrt{x^6+1}}{x^3}-\frac{40\left(x^6+1\right)^{\frac{3}{2}}}{x^9}+\frac{15\left(x^6+1\right)^{\frac{5}{2}}}{x^{15}}}{144\left(\frac{3\left(x^6+1\right)}{x^6}-\frac{3\left(x^6+1\right)^2}{x^{12}}+\frac{\left(x^6+1\right)^3}{x^{18}}-1\right)}-\frac{5}{96}\log\left(\frac{\sqrt{x^6+1}}{x^3}+1\right)+\frac{5}{96}\log\left(\frac{\sqrt{x^6+1}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20/(x^6+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/144*(33*sqrt(x^6 + 1)/x^3 - 40*(x^6 + 1)^(3/2)/x^9 + 15*(x^6 + 1)^(5/2)/x
^15)/(3*(x^6 + 1)/x^6 - 3*(x^6 + 1)^2/x^12 + (x^6 + 1)^3/x^18 - 1) - 5/96*l
og(sqrt(x^6 + 1)/x^3 + 1) + 5/96*log(sqrt(x^6 + 1)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{20}}{\sqrt{x^6+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^20/(x^6 + 1)^(1/2),x)
```

```
[Out] int(x^20/(x^6 + 1)^(1/2), x)
```

sympy [A] time = 4.72, size = 65, normalized size = 1.35

$$\frac{x^{21}}{18\sqrt{x^6+1}} - \frac{x^{15}}{72\sqrt{x^6+1}} + \frac{5x^9}{144\sqrt{x^6+1}} + \frac{5x^3}{48\sqrt{x^6+1}} - \frac{5\operatorname{asinh}\left(x^3\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**20/(x**6+1)**(1/2),x)
```

```
[Out] x**21/(18*sqrt(x**6 + 1)) - x**15/(72*sqrt(x**6 + 1)) + 5*x**9/(144*sqrt(x*
*6 + 1)) + 5*x**3/(48*sqrt(x**6 + 1)) - 5*asinh(x**3)/48
```

3.614 $\int \frac{x-4x^6}{\sqrt{x+x^6}\left(1-ax^2+2x^5+x^{10}\right)}\,dx$

Optimal. Leaf size=48

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{x^6+x}}\right)}{a^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{x^6+x}}\right)}{a^{3/4}}$$

Rubi [F] time = 1.51, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x-4x^6}{\sqrt{x+x^6}\left(1-ax^2+2x^5+x^{10}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x - 4*x^6)/(Sqrt[x + x^6]*(1 - a*x^2 + 2*x^5 + x^10)),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 + x^5]*Defer[Subst][Defer[Int][x^2/(Sqrt[1 + x^10]*(1 - a
*x^4 + 2*x^10 + x^20)), x], x, Sqrt[x]])/Sqrt[x + x^6] - (8*Sqrt[x]*Sqrt[1
+ x^5]*Defer[Subst][Defer[Int][x^12/(Sqrt[1 + x^10]*(1 - a*x^4 + 2*x^10 + x
^20)), x], x, Sqrt[x]])/Sqrt[x + x^6]
```

Rubi steps

$$\begin{aligned}\int \frac{x-4x^6}{\sqrt{x+x^6}\left(1-ax^2+2x^5+x^{10}\right)}\,dx &= \int \frac{x\left(1-4x^5\right)}{\sqrt{x+x^6}\left(1-ax^2+2x^5+x^{10}\right)}\,dx\\ &= \frac{\left(\sqrt{x}\sqrt{1+x^5}\right)\int \frac{\sqrt{x}\left(1-4x^5\right)}{\sqrt{1+x^5}\left(1-ax^2+2x^5+x^{10}\right)}\,dx}{\sqrt{x+x^6}}\\ &= \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{x^2\left(1-4x^{10}\right)}{\sqrt{1+x^{10}}\left(1-ax^4+2x^{10}+x^{20}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^6}}\\ &= \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \left(\frac{x^2}{\sqrt{1+x^{10}}\left(1-ax^4+2x^{10}+x^{20}\right)}-\frac{4x^{12}}{\sqrt{1+x^{10}}\left(1-ax^4+2x^{10}+x^{2}\right.}\right.\right.}{\sqrt{x+x^6}}\\ &= \frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^{10}}\left(1-ax^4+2x^{10}+x^{20}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^6}}-\frac{\left(8\sqrt{x}\sqrt{1\right.}{}\end{aligned}$$

Mathematica [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{x-4x^6}{\sqrt{x+x^6}\left(1-ax^2+2x^5+x^{10}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x - 4*x^6)/(Sqrt[x + x^6]*(1 - a*x^2 + 2*x^5 + x^10)),x]
```

```
[Out] Integrate[(x - 4*x^6)/(Sqrt[x + x^6]*(1 - a*x^2 + 2*x^5 + x^10)), x]
```

IntegrateAlgebraic [A] time = 2.74, size = 48, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{x^6+x}}\right)}{a^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{x^6+x}}\right)}{a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x - 4*x^6)/(Sqrt[x + x^6]*(1 - a*x^2 + 2*x^5 + x^10)),x
]
```

```
[Out] -(ArcTan[(a^(1/4)*x)/Sqrt[x + x^6]]/a^(3/4)) + ArcTanh[(a^(1/4)*x)/Sqrt[x +
 x^6]]/a^(3/4)
```

fricas [B] time = 1.13, size = 321, normalized size = 6.69

$$-\left(\frac{1}{a^3}\right)^{\frac{1}{4}} \arctan\left(\frac{2\sqrt{x^6+x}\left(a^3\left(\frac{1}{a^3}\right)^{\frac{3}{4}}x+\left(ax^5+a\right)\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\right)+\left(2\left(a^3x^6+a^3x\right)\left(\frac{1}{a^3}\right)^{\frac{3}{4}}+\left(ax^{10}+2\,ax^5+a^2x^2+a\right)\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\right)\sqrt{a\sqrt{\frac{1}{a^3}}}}{x^{10}+2\,x^5-ax^2+1}\right)+\frac{1}{4}\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(-\frac{\left(a^2x^{10}+2\,a^2x^5+a^3x^2+a^2\right)\left(\frac{1}{a^3}\right)^{\frac{3}{4}}+2\sqrt{x^6+x}\left(x^5+a^2\sqrt{\frac{1}{a^3}}\,x+1\right)+2\left(ax^6+ax\right)\left(\frac{1}{a^3}\right)^{\frac{1}{4}}}{2\left(x^{10}+2\,x^5-ax^2+1\right)}\right)-\frac{1}{4}\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{\left(a^2x^{10}+2\,a^2x^5+a^3x^2+a^2\right)\left(\frac{1}{a^3}\right)^{\frac{3}{4}}-2\sqrt{x^6+x}\left(x^5+a^2\sqrt{\frac{1}{a^3}}\,x+1\right)+2\left(ax^6+ax\right)\left(\frac{1}{a^3}\right)^{\frac{1}{4}}}{2\left(x^{10}+2\,x^5-ax^2+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^6+x)/(x^6+x)^(1/2)/(x^10+2*x^5-a*x^2+1),x, algorithm="frica
s")
```

```
[Out] -(a^(-3))^(1/4)*arctan((2*sqrt(x^6 + x)*(a^3*(a^(-3))^(3/4)*x + (a*x^5 + a)
*(a^(-3))^(1/4)) + (2*(a^3*x^6 + a^3*x)*(a^(-3))^(3/4) + (a*x^10 + 2*a*x^5
+ a^2*x^2 + a)*(a^(-3))^(1/4))*sqrt(a*sqrt(a^(-3))))/(x^10 + 2*x^5 - a*x^2
+ 1)) + 1/4*(a^(-3))^(1/4)*log(-1/2*((a^2*x^10 + 2*a^2*x^5 + a^3*x^2 + a^2)
*(a^(-3))^(3/4) + 2*sqrt(x^6 + x)*(x^5 + a^2*sqrt(a^(-3))*x + 1) + 2*(a*x^6
 + a*x)*(a^(-3))^(1/4))/(x^10 + 2*x^5 - a*x^2 + 1)) - 1/4*(a^(-3))^(1/4)*lo
g(1/2*((a^2*x^10 + 2*a^2*x^5 + a^3*x^2 + a^2)*(a^(-3))^(3/4) - 2*sqrt(x^6 +
 x)*(x^5 + a^2*sqrt(a^(-3))*x + 1) + 2*(a*x^6 + a*x)*(a^(-3))^(1/4))/(x^10
+ 2*x^5 - a*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{4\,x^6 - x}{\left(x^{10} + 2\,x^5 - ax^2 + 1\right)\sqrt{x^6 + x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^6+x)/(x^6+x)^(1/2)/(x^10+2*x^5-a*x^2+1),x, algorithm="giac"
)
```

```
[Out] integrate(-(4*x^6 - x)/((x^10 + 2*x^5 - a*x^2 + 1)*sqrt(x^6 + x)), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{-4x^6 + x}{\sqrt{x^6 + x}\,\left(x^{10} + 2x^5 - a\,x^2 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^6+x)/(x^6+x)^(1/2)/(x^10+2*x^5-a*x^2+1),x)
```

```
[Out] int((-4*x^6+x)/(x^6+x)^(1/2)/(x^10+2*x^5-a*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{4\,x^6 - x}{\left(x^{10} + 2\,x^5 - ax^2 + 1\right)\sqrt{x^6 + x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^6+x)/(x^6+x)^(1/2)/(x^10+2*x^5-a*x^2+1),x, algorithm="maxim
a")
```

```
[Out] -integrate((4*x^6 - x)/((x^10 + 2*x^5 - a*x^2 + 1)*sqrt(x^6 + x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x-4\,x^6}{\sqrt{x^6+x}\,\left(x^{10}+2\,x^5-a\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 4*x^6)/((x + x^6)^(1/2)*(2*x^5 - a*x^2 + x^10 + 1)),x)
```

```
[Out] int((x - 4*x^6)/((x + x^6)^(1/2)*(2*x^5 - a*x^2 + x^10 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{x}{-ax^2\sqrt{x^6+x}+x^{10}\sqrt{x^6+x}+2x^5\sqrt{x^6+x}+\sqrt{x^6+x}}\right)dx-\int \frac{4x^6}{-ax^2\sqrt{x^6+x}+x^{10}\sqrt{x^6+x}+2x^5\sqrt{x^6+x}+\sqrt{x^6+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**6+x)/(x**6+x)**(1/2)/(x**10+2*x**5-a*x**2+1),x)
```

```
[Out] -Integral(-x/(-a*x**2*sqrt(x**6 + x) + x**10*sqrt(x**6 + x) + 2*x**5*sqrt(x
**6 + x) + sqrt(x**6 + x)), x) - Integral(4*x**6/(-a*x**2*sqrt(x**6 + x) +
x**10*sqrt(x**6 + x) + 2*x**5*sqrt(x**6 + x) + sqrt(x**6 + x)), x)
```

3.615 $\int \frac{-x+4x^6}{\sqrt{x+x^6}\left(a-x^2+2ax^5+ax^{10}\right)}\,dx$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{x^6+x}}\right)}{\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{x^6+x}}\right)}{\sqrt[4]{a}}$$

Rubi [F] time = 1.60, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-x+4x^6}{\sqrt{x+x^6}\left(a-x^2+2ax^5+ax^{10}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-x + 4*x^6)/(Sqrt[x + x^6]*(a - x^2 + 2*a*x^5 + a*x^10)),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[1 + x^5]*Defer[Subst][Defer[Int][x^2/(Sqrt[1 + x^10]*(a -
x^4 + 2*a*x^10 + a*x^20)), x], x, Sqrt[x]])/Sqrt[x + x^6] + (8*Sqrt[x]*Sqrt
[1 + x^5]*Defer[Subst][Defer[Int][x^12/(Sqrt[1 + x^10]*(a - x^4 + 2*a*x^10
+ a*x^20)), x], x, Sqrt[x]])/Sqrt[x + x^6]
```

Rubi steps

$$\int \frac{-x+4x^6}{\sqrt{x+x^6}\left(a-x^2+2ax^5+ax^{10}\right)}\,dx=\int \frac{x\left(-1+4x^5\right)}{\sqrt{x+x^6}\left(a-x^2+2ax^5+ax^{10}\right)}\,dx$$

$$=\frac{\left(\sqrt{x}\sqrt{1+x^5}\right)\int \frac{\sqrt{x}\left(-1+4x^5\right)}{\sqrt{1+x^5}\left(a-x^2+2ax^5+ax^{10}\right)}\,dx}{\sqrt{x+x^6}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{x^2\left(-1+4x^{10}\right)}{\sqrt{1+x^{10}}\left(a-x^4+2ax^{10}+ax^{20}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^6}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \left(-\frac{x^2}{\sqrt{1+x^{10}}\left(a-x^4+2ax^{10}+ax^{20}\right)}+\frac{4x^{12}}{\sqrt{1+x^{10}}\left(a-x^4+2ax^{10}+a\right.}\right.\right.}{\sqrt{x+x^6}}$$

$$=-\frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^{10}}\left(a-x^4+2ax^{10}+ax^{20}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^6}}+\frac{\left(8\sqrt{x}\sqrt{\right.}}{}$$

Mathematica [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{-x+4x^6}{\sqrt{x+x^6}\left(a-x^2+2ax^5+ax^{10}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-x + 4*x^6)/(Sqrt[x + x^6]*(a - x^2 + 2*a*x^5 + a*x^10)),x]
```

```
[Out] Integrate[(-x + 4*x^6)/(Sqrt[x + x^6]*(a - x^2 + 2*a*x^5 + a*x^10)), x]
```

IntegrateAlgebraic [A] time = 2.75, size = 48, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{a}\,\sqrt{x^6+x}}\right)}{\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{a}\,\sqrt{x^6+x}}\right)}{\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x + 4*x^6)/(Sqrt[x + x^6]*(a - x^2 + 2*a*x^5 + a*x^10)
),x]
```

```
[Out] ArcTan[x/(a^(1/4)*Sqrt[x + x^6])]/a^(1/4) - ArcTanh[x/(a^(1/4)*Sqrt[x + x^6
])]/a^(1/4)
```

fricas [B] time = 1.15, size = 264, normalized size = 5.50

$$\frac{\arctan\left(\frac{2\,\sqrt{x^6+x}\left(a^{\frac{1}{4}}x+\frac{ax^5+a}{a^{\frac{1}{4}}}\right)+\left(\frac{a^2x^{10}+2\,a^2x^5+ax^2+a^2}{a^{\frac{1}{4}}}+\frac{2\left(a^2x^6+a^2x\right)}{a^{\frac{3}{4}}}\right)\sqrt{\frac{1}{a^{\frac{3}{2}}}}}{ax^{10}+2\,ax^5-x^2+a}\right)}{a^{\frac{1}{4}}}-\frac{\log\left(-\frac{2\,\sqrt{x^6+x}\left(x^5+\frac{x}{\sqrt{a}}+1\right)+\frac{2\left(x^6+x\right)}{a^{\frac{1}{4}}}+\frac{ax^{10}+2\,ax^5+x^2+a}{a^{\frac{3}{4}}}}{2\left(ax^{10}+2\,ax^5-x^2+a\right)}\right)}{4\,a^{\frac{1}{4}}}+\frac{\log\left(-\frac{2\,\sqrt{x^6+x}\left(x^5+\frac{x}{\sqrt{a}}+1\right)-\frac{2\left(x^6+x\right)}{a^{\frac{1}{4}}}-\frac{ax^{10}+2\,ax^5+x^2+a}{a^{\frac{3}{4}}}}{2\left(ax^{10}+2\,ax^5-x^2+a\right)}\right)}{4\,a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6-x)/(x^6+x)^(1/2)/(a*x^10+2*a*x^5-x^2+a),x, algorithm="fric
as")
```

```
[Out] arctan((2*sqrt(x^6 + x)*(a^(1/4)*x + (a*x^5 + a)/a^(1/4)) + ((a^2*x^10 + 2*
a^2*x^5 + a*x^2 + a^2)/a^(1/4) + 2*(a^2*x^6 + a^2*x)/a^(3/4))*sqrt(a^(-3/2)
))/(a*x^10 + 2*a*x^5 - x^2 + a))/a^(1/4) - 1/4*log(-1/2*(2*sqrt(x^6 + x)*(x
^5 + x/sqrt(a) + 1) + 2*(x^6 + x)/a^(1/4) + (a*x^10 + 2*a*x^5 + x^2 + a)/a^
(3/4))/(a*x^10 + 2*a*x^5 - x^2 + a))/a^(1/4) + 1/4*log(-1/2*(2*sqrt(x^6 + x
)*(x^5 + x/sqrt(a) + 1) - 2*(x^6 + x)/a^(1/4) - (a*x^10 + 2*a*x^5 + x^2 + a
)/a^(3/4))/(a*x^10 + 2*a*x^5 - x^2 + a))/a^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x^6-x}{\left(ax^{10}+2\,ax^5-x^2+a\right)\sqrt{x^6+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6-x)/(x^6+x)^(1/2)/(a*x^10+2*a*x^5-x^2+a),x, algorithm="giac
")
```

```
[Out] integrate((4*x^6 - x)/((a*x^10 + 2*a*x^5 - x^2 + a)*sqrt(x^6 + x)), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{4x^6-x}{\sqrt{x^6+x}\,\left(a\,x^{10}+2a\,x^5-x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^6-x)/(x^6+x)^(1/2)/(a*x^10+2*a*x^5-x^2+a),x)
```

```
[Out] int((4*x^6-x)/(x^6+x)^(1/2)/(a*x^10+2*a*x^5-x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x^6-x}{\left(ax^{10}+2\,ax^5-x^2+a\right)\sqrt{x^6+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6-x)/(x^6+x)^(1/2)/(a*x^10+2*a*x^5-x^2+a),x, algorithm="maxi
ma")
```

```
[Out] integrate((4*x^6 - x)/((a*x^10 + 2*a*x^5 - x^2 + a)*sqrt(x^6 + x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x-4\,x^6}{\sqrt{x^6+x}\,\left(a\,x^{10}+2\,a\,x^5-x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 4*x^6)/((x + x^6)^(1/2)*(a + 2*a*x^5 + a*x^10 - x^2)),x)
```

```
[Out] -int((x - 4*x^6)/((x + x^6)^(1/2)*(a + 2*a*x^5 + a*x^10 - x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(4x^5-1\right)}{\sqrt{x\,(x+1)\left(x^4-x^3+x^2-x+1\right)}\,\left(ax^{10}+2ax^5+a-x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**6-x)/(x**6+x)**(1/2)/(a*x**10+2*a*x**5-x**2+a),x)
```

```
[Out] Integral(x*(4*x**5 - 1)/(sqrt(x*(x + 1)*(x**4 - x**3 + x**2 - x + 1))*(a*x*
*10 + 2*a*x**5 + a - x**2)), x)
```

3.616 $\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=48

$$4\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}-4\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)$$

Rubi [F] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{1+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[1 + x^2],x]
[Out] Defer[Int][Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[1 + x^2], x]
```

Rubi steps

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{1+x^2}}\,dx=\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{1+x^2}}\,dx$$

Mathematica [A] time = 0.05, size = 48, normalized size = 1.00

$$4\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}-\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[1 + x^2],x]
[Out] 4*(Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] - ArcTanh[Sqrt[1 + Sqrt[x + Sqrt[1 + x
^2]]]])
```

IntegrateAlgebraic [A] time = 0.14, size = 48, normalized size = 1.00

$$4\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}-4\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[1 + x^2],x]
[Out] 4*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] - 4*ArcTanh[Sqrt[1 + Sqrt[x + Sqrt[1 +
x^2]]]]
```

fricas [A] time = 0.40, size = 58, normalized size = 1.21

$$4\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}-2\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}+1\right)+2\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 4*sqrt(sqrt(x + sqrt(x^2 + 1)) + 1) - 2*log(sqrt(sqrt(x + sqrt(x^2 + 1)) +
1) + 1) + 2*log(sqrt(sqrt(x + sqrt(x^2 + 1)) + 1) - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] int((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(sqrt(x + sqrt(x^2 + 1)) + 1)/sqrt(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2)/(x^2 + 1)^(1/2),x)
```

```
[Out] int(((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2)/(x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x+(x**2+1)**(1/2))**(1/2))**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x + sqrt(x**2 + 1)) + 1)/sqrt(x**2 + 1), x)
```

3.617 $\int \frac{-3b+2ax}{\sqrt[4]{-bx+ax^2}\left(b-ax+x^3\right)}\,dx$

Optimal. Leaf size=49

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^2-bx}}{x}\right)-2\tanh^{-1}\left(\frac{\left(ax^2-bx\right)^{3/4}}{ax-b}\right)$$

Rubi [F] time = 2.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3b+2ax}{\sqrt[4]{-bx+ax^2}\left(b-ax+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3*b + 2*a*x)/((-(b*x) + a*x^2)^(1/4)*(b - a*x + x^3)),x]
```

```
[Out] (-8*a*x^(1/4)*(-b + a*x)^(1/4)*Defer[Subst][Defer[Int][x^6/((-b + a*x^4)^(1
/4)*(-b + a*x^4 - x^12)), x], x, x^(1/4)])/(-(b*x) + a*x^2)^(1/4) - (12*b*x
^(1/4)*(-b + a*x)^(1/4)*Defer[Subst][Defer[Int][x^2/((-b + a*x^4)^(1/4)*(b
- a*x^4 + x^12)), x], x, x^(1/4)])/(-(b*x) + a*x^2)^(1/4)
```

Rubi steps

$$\begin{aligned}
\int \frac{-3b+2ax}{\sqrt[4]{-bx+ax^2}\left(b-ax+x^3\right)}\,dx &= \frac{\left(\sqrt[4]{x}\sqrt[4]{-b+ax}\right)\int \frac{-3b+2ax}{\sqrt[4]{x}\sqrt[4]{-b+ax}\left(b-ax+x^3\right)}\,dx}{\sqrt[4]{-bx+ax^2}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax}\right)\text{Subst}\left(\int \frac{x^2\left(-3b+2ax^4\right)}{\sqrt[4]{-b+ax^4}\left(b-ax^4+x^{12}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^2}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax}\right)\text{Subst}\left(\int \left(-\frac{2ax^6}{\sqrt[4]{-b+ax^4}\left(-b+ax^4-x^{12}\right)}-\frac{3bx^2}{\sqrt[4]{-b+ax^4}\left(b-ax^4+x^{12}\right)}\right)dx,\right.}{\sqrt[4]{-bx+ax^2}}\\
&= -\frac{\left(8a\sqrt[4]{x}\sqrt[4]{-b+ax}\right)\text{Subst}\left(\int \frac{x^6}{\sqrt[4]{-b+ax^4}\left(-b+ax^4-x^{12}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^2}}-\frac{\left(12b\sqrt[4]{x}\sqrt[4]{-\right.}{}
\end{aligned}$$

Mathematica [F] time = 1.48, size = 0, normalized size = 0.00

$$\int \frac{-3b+2ax}{\sqrt[4]{-bx+ax^2}\left(b-ax+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3*b + 2*a*x)/((-(b*x) + a*x^2)^(1/4)*(b - a*x + x^3)),x]
```

```
[Out] Integrate[(-3*b + 2*a*x)/((-(b*x) + a*x^2)^(1/4)*(b - a*x + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.23, size = 49, normalized size = 1.00

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^2-bx}}{x}\right)-2\tanh^{-1}\left(\frac{\left(ax^2-bx\right)^{3/4}}{ax-b}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*b + 2*a*x)/((-(b*x) + a*x^2)^(1/4)*(b - a*x + x^3)),
x]
```

```
[Out] 2*ArcTan[(-(b*x) + a*x^2)^(1/4)/x] - 2*ArcTanh[(-(b*x) + a*x^2)^(3/4)/(-b +
 a*x)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-3*b)/(a*x^2-b*x)^(1/4)/(x^3-a*x+b),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax - 3\,b}{\left(ax^2 - bx\right)^{\frac{1}{4}}\left(x^3 - ax + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-3*b)/(a*x^2-b*x)^(1/4)/(x^3-a*x+b),x, algorithm="giac")
```

```
[Out] integrate((2*a*x - 3*b)/((a*x^2 - b*x)^(1/4)*(x^3 - a*x + b)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{2ax - 3b}{\left(a\,x^2 - bx\right)^{\frac{1}{4}}\left(x^3 - ax + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x-3*b)/(a*x^2-b*x)^(1/4)/(x^3-a*x+b),x)
```

```
[Out] int((2*a*x-3*b)/(a*x^2-b*x)^(1/4)/(x^3-a*x+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax - 3\,b}{\left(ax^2 - bx\right)^{\frac{1}{4}}\left(x^3 - ax + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-3*b)/(a*x^2-b*x)^(1/4)/(x^3-a*x+b),x, algorithm="maxima")
```

```
[Out] integrate((2*a*x - 3*b)/((a*x^2 - b*x)^(1/4)*(x^3 - a*x + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{3\,b - 2\,a\,x}{\left(a\,x^2 - b\,x\right)^{1/4}\,\left(x^3 - a\,x + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*b - 2*a*x)/((a*x^2 - b*x)^(1/4)*(b - a*x + x^3)),x)
```

```
[Out] int(-(3*b - 2*a*x)/((a*x^2 - b*x)^(1/4)*(b - a*x + x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-3*b)/(a*x**2-b*x)**(1/4)/(x**3-a*x+b),x)
```

```
[Out] Timed out
```

3.618 $\int \frac{(-4+x^3)\sqrt{2-x^2+x^3}}{(2+x^3)(2+x^2+x^3)}\,dx$

Optimal. Leaf size=49

$$2\tan^{-1}\left(\frac{x}{\sqrt{x^3-x^2+2}}\right)-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^3-x^2+2}}\right)$$

Rubi [F] time = 51.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-4+x^3)\sqrt{2-x^2+x^3}}{(2+x^3)(2+x^2+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-4 + x^3)*Sqrt[2 - x^2 + x^3])/((2 + x^3)*(2 + x^2 + x^3)),x]
```

```
[Out] -1/3*((-2)^(2/3)*Sqrt[2 - x^2 + x^3]) - (2^(2/3)*Sqrt[2 - x^2 + x^3])/3 + (
(-1)^(1/3)*2^(2/3)*Sqrt[2 - x^2 + x^3])/3 + ((1 - 3*(-2)^(1/3))*(-2*(26 + 1
5*Sqrt[3]))^(1/6)*Sqrt[2 - x^2 + x^3]*EllipticE[ArcSin[((26 - 15*Sqrt[3])^(
1/6)*Sqrt[(-I)*(2 + (26 - 15*Sqrt[3])^(1/3)*(1 - I*Sqrt[3]) + (1 + I*Sqrt[3
])/(26 - 15*Sqrt[3])^(1/3) - 6*x)])/(3^(1/4)*Sqrt[2*(1 - (26 - 15*Sqrt[3])^
(2/3))])], (-2*Sqrt[3]*(1 - (26 - 15*Sqrt[3])^(2/3)))/(3*I - Sqrt[3] + (26
- 15*Sqrt[3])^(2/3)*(3*I + Sqrt[3]))])/(Sqrt[-((1 - (26 - 15*Sqrt[3])^(-1/3
) - (26 - 15*Sqrt[3])^(1/3) - 3*x)/(3 + I*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)
*(3 - I*Sqrt[3])))]*Sqrt[-1 + (26 - 15*Sqrt[3])^(-2/3) + (26 - 15*Sqrt[3])^
(2/3) + ((1 + (26 - 15*Sqrt[3])^(2/3))*(1 - 3*x))/(26 - 15*Sqrt[3])^(1/3) +
 (-1 + 3*x)^2]) + (I*((-1)^(1/3) - 3*2^(1/3))*(2*(26 + 15*Sqrt[3]))^(1/6)*S
qrt[2 - x^2 + x^3]*EllipticE[ArcSin[((26 - 15*Sqrt[3])^(1/6)*Sqrt[(-I)*(2 +
 (26 - 15*Sqrt[3])^(1/3)*(1 - I*Sqrt[3]) + (1 + I*Sqrt[3])/(26 - 15*Sqrt[3]
)^(1/3) - 6*x)])/(3^(1/4)*Sqrt[2*(1 - (26 - 15*Sqrt[3])^(2/3))])], (-2*Sqrt
[3]*(1 - (26 - 15*Sqrt[3])^(2/3)))/(3*I - Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)
*(3*I + Sqrt[3]))])/(Sqrt[-((1 - (26 - 15*Sqrt[3])^(-1/3) - (26 - 15*Sqrt[3
])^(1/3) - 3*x)/(3 + I*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3 - I*Sqrt[3])))]
*Sqrt[-1 + (26 - 15*Sqrt[3])^(-2/3) + (26 - 15*Sqrt[3])^(2/3) + ((1 + (26 -
 15*Sqrt[3])^(2/3))*(1 - 3*x))/(26 - 15*Sqrt[3])^(1/3) + (-1 + 3*x)^2]) - (
I*(1 + 3*2^(1/3))*(2*(26 + 15*Sqrt[3]))^(1/6)*Sqrt[2 - x^2 + x^3]*EllipticE
[ArcSin[((26 - 15*Sqrt[3])^(1/6)*Sqrt[(-I)*(2 + (26 - 15*Sqrt[3])^(1/3)*(1
- I*Sqrt[3]) + (1 + I*Sqrt[3])/(26 - 15*Sqrt[3])^(1/3) - 6*x)])/(3^(1/4)*Sq
rt[2*(1 - (26 - 15*Sqrt[3])^(2/3))])], (-2*Sqrt[3]*(1 - (26 - 15*Sqrt[3])^(
2/3)))/(3*I - Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3*I + Sqrt[3]))])/(Sqrt[-(
(1 - (26 - 15*Sqrt[3])^(-1/3) - (26 - 15*Sqrt[3])^(1/3) - 3*x)/(3 + I*Sqrt[
3] + (26 - 15*Sqrt[3])^(2/3)*(3 - I*Sqrt[3])))]*Sqrt[-1 + (26 - 15*Sqrt[3])
^(-2/3) + (26 - 15*Sqrt[3])^(2/3) + ((1 + (26 - 15*Sqrt[3])^(2/3))*(1 - 3*x
))/(26 - 15*Sqrt[3])^(1/3) + (-1 + 3*x)^2]) - (2*(1 + 6*(-2)^(1/3) - 9*(-2)
^(2/3))*(-2*(26 - 15*Sqrt[3]))^(1/6)*Sqrt[-((1 - (26 - 15*Sqrt[3])^(-1/3) -
 (26 - 15*Sqrt[3])^(1/3) - 3*x)/(3 + I*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3
 - I*Sqrt[3])))]*Sqrt[2 - x^2 + x^3]*EllipticF[ArcSin[((26 - 15*Sqrt[3])^(1
/6)*Sqrt[(-I)*(2 + (26 - 15*Sqrt[3])^(1/3)*(1 - I*Sqrt[3]) + (1 + I*Sqrt[3]
)/(26 - 15*Sqrt[3])^(1/3) - 6*x)])/(3^(1/4)*Sqrt[2*(1 - (26 - 15*Sqrt[3])^(
2/3))])], (-2*Sqrt[3]*(1 - (26 - 15*Sqrt[3])^(2/3)))/(3*I - Sqrt[3] + (26 -
 15*Sqrt[3])^(2/3)*(3*I + Sqrt[3]))])/((1 - (26 - 15*Sqrt[3])^(-1/3) - (26
- 15*Sqrt[3])^(1/3) - 3*x)*Sqrt[-1 + (26 - 15*Sqrt[3])^(-2/3) + (26 - 15*Sq
rt[3])^(2/3) + ((1 + (26 - 15*Sqrt[3])^(2/3))*(1 - 3*x))/(26 - 15*Sqrt[3])^
(1/3) + (-1 + 3*x)^2]) + ((2*I)*(1 - 6*2^(1/3) - 9*2^(2/3))*(2*(26 - 15*Sqr
t[3]))^(1/6)*Sqrt[-((1 - (26 - 15*Sqrt[3])^(-1/3) - (26 - 15*Sqrt[3])^(1/3)
```

- 3*x)/(3 + I*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3 - I*Sqrt[3])))]*Sqrt[2 - x^2 + x^3]*EllipticF[ArcSin[((26 - 15*Sqrt[3])^(1/6)*Sqrt[(-I)*(2 + (26 - 15*Sqrt[3])^(1/3)*(1 - I*Sqrt[3]) + (1 + I*Sqrt[3])/(26 - 15*Sqrt[3])^(1/3) - 6*x)])/(3^(1/4)*Sqrt[2*(1 - (26 - 15*Sqrt[3])^(2/3))])], (-2*Sqrt[3]*(1 - (26 - 15*Sqrt[3])^(2/3)))/(3*I - Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3*I + Sqrt[3]))])/((1 - (26 - 15*Sqrt[3])^(-1/3) - (26 - 15*Sqrt[3])^(1/3) - 3*x)*Sqrt[-1 + (26 - 15*Sqrt[3])^(-2/3) + (26 - 15*Sqrt[3])^(2/3) + ((1 + (26 - 15*Sqrt[3])^(2/3))*(1 - 3*x))/(26 - 15*Sqrt[3])^(1/3) + (-1 + 3*x)^2]) - (2*(-1)^(5/6)*(1 - 6*(-1)^(2/3)*2^(1/3) + 9*(-1)^(1/3)*2^(2/3))*(2*(26 - 15*Sqrt[3]))^(1/6)*Sqrt[-((1 - (26 - 15*Sqrt[3])^(-1/3) - (26 - 15*Sqrt[3])^(1/3) - 3*x)/(3 + I*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3 - I*Sqrt[3])))]*Sqrt[2 - x^2 + x^3]*EllipticF[ArcSin[((26 - 15*Sqrt[3])^(1/6)*Sqrt[(-I)*(2 + (26 - 15*Sqrt[3])^(1/3)*(1 - I*Sqrt[3]) + (1 + I*Sqrt[3])/(26 - 15*Sqrt[3])^(1/3) - 6*x)])/(3^(1/4)*Sqrt[2*(1 - (26 - 15*Sqrt[3])^(2/3))])], (-2*Sqrt[3]*(1 - (26 - 15*Sqrt[3])^(2/3)))/(3*I - Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3*I + Sqrt[3]))])/((1 - (26 - 15*Sqrt[3])^(-1/3) - (26 - 15*Sqrt[3])^(1/3) - 3*x)*Sqrt[-1 + (26 - 15*Sqrt[3])^(-2/3) + (26 - 15*Sqrt[3])^(2/3) + ((1 + (26 - 15*Sqrt[3])^(2/3))*(1 - 3*x))/(26 - 15*Sqrt[3])^(1/3) + (-1 + 3*x)^2]) + (2*(1 - 3*(-2)^(1/3))*(-2*(26 + 15*Sqrt[3]))^(1/6)*(1 + (26 - 15*Sqrt[3])^(2/3))*Sqrt[-((1 - (26 - 15*Sqrt[3])^(-1/3) - (26 - 15*Sqrt[3])^(1/3) - 3*x)/(3 + I*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3 - I*Sqrt[3])))]*Sqrt[2 - x^2 + x^3]*EllipticF[ArcSin[((26 - 15*Sqrt[3])^(1/6)*Sqrt[(-I)*(2 + (26 - 15*Sqrt[3])^(1/3)*(1 - I*Sqrt[3]) + (1 + I*Sqrt[3])/(26 - 15*Sqrt[3])^(1/3) - 6*x)])/(3^(1/4)*Sqrt[2*(1 - (26 - 15*Sqrt[3])^(2/3))])], (-2*Sqrt[3]*(1 - (26 - 15*Sqrt[3])^(2/3)))/(3*I - Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3*I + Sqrt[3]))])/((1 - (26 - 15*Sqrt[3])^(-1/3) - (26 - 15*Sqrt[3])^(1/3) - 3*x)*Sqrt[-1 + (26 - 15*Sqrt[3])^(-2/3) + (26 - 15*Sqrt[3])^(2/3) + ((1 + (26 - 15*Sqrt[3])^(2/3))*(1 - 3*x))/(26 - 15*Sqrt[3])^(1/3) + (-1 + 3*x)^2]) + ((2*I)*((-1)^(1/3) - 3*2^(1/3))*(2*(26 + 15*Sqrt[3]))^(1/6)*(1 + (26 - 15*Sqrt[3])^(2/3))*Sqrt[-((1 - (26 - 15*Sqrt[3])^(-1/3) - (26 - 15*Sqrt[3])^(1/3) - 3*x)/(3 + I*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3 - I*Sqrt[3])))]*Sqrt[2 - x^2 + x^3]*EllipticF[ArcSin[((26 - 15*Sqrt[3])^(1/6)*Sqrt[(-I)*(2 + (26 - 15*Sqrt[3])^(1/3)*(1 - I*Sqrt[3]) + (1 + I*Sqrt[3])/(26 - 15*Sqrt[3])^(1/3) - 6*x)])/(3^(1/4)*Sqrt[2*(1 - (26 - 15*Sqrt[3])^(2/3))])], (-2*Sqrt[3]*(1 - (26 - 15*Sqrt[3])^(2/3)))/(3*I - Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3*I + Sqrt[3]))])/((1 - (26 - 15*Sqrt[3])^(-1/3) - (26 - 15*Sqrt[3])^(1/3) - 3*x)*Sqrt[-1 + (26 - 15*Sqrt[3])^(-2/3) + (26 - 15*Sqrt[3])^(2/3) + ((1 + (26 - 15*Sqrt[3])^(2/3))*(1 - 3*x))/(26 - 15*Sqrt[3])^(1/3) + (-1 + 3*x)^2]) - ((2*I)*(1 + 3*2^(1/3))*(2*(26 + 15*Sqrt[3]))^(1/6)*(1 + (26 - 15*Sqrt[3])^(2/3))*Sqrt[-((1 - (26 - 15*Sqrt[3])^(-1/3) - (26 - 15*Sqrt[3])^(1/3) - 3*x)/(3 + I*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3 - I*Sqrt[3])))]*Sqrt[2 - x^2 + x^3]*EllipticF[ArcSin[((26 - 15*Sqrt[3])^(1/6)*Sqrt[(-I)*(2 + (26 - 15*Sqrt[3])^(1/3)*(1 - I*Sqrt[3]) + (1 + I*Sqrt[3])/(26 - 15*Sqrt[3])^(1/3) - 6*x)])/(3^(1/4)*Sqrt[2*(1 - (26 - 15*Sqrt[3])^(2/3))])], (-2*Sqrt[3]*(1 - (26 - 15*Sqrt[3])^(2/3)))/(3*I - Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3*I + Sqrt[3]))])/((1 - (26 - 15*Sqrt[3])^(-1/3) - (26 - 15*Sqrt[3])^(1/3) - 3*x)*Sqrt[-1 + (26 - 15*Sqrt[3])^(-2/3) + (26 - 15*Sqrt[3])^(2/3) + ((1 + (26 - 15*Sqrt[3])^(2/3))*(1 - 3*x))/(26 - 15*Sqrt[3])^(1/3) + (-1 + 3*x)^2]) + (27*2^(5/6)*Sqrt[(26 - 15*Sqrt[3])*(3 - I*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3 + I*Sqrt[3]))]*Sqrt[1 - (2*(1 + (26 - 15*Sqrt[3])^(2/3) - (26 - 15*Sqrt[3])^(1/3)*(1 - 3*x)))/(3 + I*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3 - I*Sqrt[3]))]*Sqrt[1 - (2*(1 + (26 - 15*Sqrt[3])^(2/3) - (26 - 15*Sqrt[3])^(1/3)*(1 - 3*x)))/(3 - I*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3 + I*Sqrt[3]))]*Sqrt[-2 - (26 - 15*Sqrt[3])^(1/3)*(1 + I*Sqrt[3]) + (I*(I + Sqrt[3]))/(26 - 15*Sqrt[3])^(1/3) + 6*x]*Sqrt[-2 - (1 + I*Sqrt[3])/(26 - 15*Sqrt[3])^(1/3) + I*(26 - 15*Sqrt[3])^(1/3)*(I + Sqrt[3]) + 6*x]*Sqrt[2 - x^2 + x^3]*EllipticPi[(3 - I*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3 + I*Sqrt[3]))/(2*(1 - 3*(52 - 30*Sqrt[3])^(1/3) - (26 - 15*Sqrt[3])^(1/3) + (26 - 15*Sqrt[3])^(2/3))), ArcSin[(Sqrt[2]*(26 - 15*Sqrt[3])^(1/6)*Sqrt[-1 + (26 - 15*Sqrt[3])^(-1/3) + (26

```
 - 15*Sqrt[3])^(1/3) + 3*x])/Sqrt[3 - I*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(
3 + I*Sqrt[3])]], (3*I + Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3*I - Sqrt[3]))
/(3*I - Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3*I + Sqrt[3]))])/((1 - 3*(52 -
30*Sqrt[3])^(1/3) - (26 - 15*Sqrt[3])^(1/3) + (26 - 15*Sqrt[3])^(2/3))*Sqrt
[-1 + I*Sqrt[3] - (26 - 15*Sqrt[3])^(2/3)*(1 + I*Sqrt[3]) - 2*(26 - 15*Sqrt
[3])^(1/3)*(1 - 3*x)]*Sqrt[-1 - I*Sqrt[3] + I*(26 - 15*Sqrt[3])^(2/3)*(I +
Sqrt[3]) - 2*(26 - 15*Sqrt[3])^(1/3)*(1 - 3*x)]*(1 - (26 - 15*Sqrt[3])^(-2/
3) - (26 - 15*Sqrt[3])^(2/3) - ((1 + (26 - 15*Sqrt[3])^(2/3))*(1 - 3*x))/(2
6 - 15*Sqrt[3])^(1/3) - (1 - 3*x)^2)*Sqrt[-1 + (26 - 15*Sqrt[3])^(-1/3) + (
26 - 15*Sqrt[3])^(1/3) + 3*x]) + (27*2^(5/6)*(26 - 15*Sqrt[3])^(1/6)*Sqrt[3
 - I*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3 + I*Sqrt[3])]*Sqrt[1 - (2*(1 + (2
6 - 15*Sqrt[3])^(2/3) - (26 - 15*Sqrt[3])^(1/3)*(1 - 3*x)))/(3 + I*Sqrt[3]
+ (26 - 15*Sqrt[3])^(2/3)*(3 - I*Sqrt[3]))]*Sqrt[1 - (2*(1 + (26 - 15*Sqrt[
3])^(2/3) - (26 - 15*Sqrt[3])^(1/3)*(1 - 3*x)))/(3 - I*Sqrt[3] + (26 - 15*S
qrt[3])^(2/3)*(3 + I*Sqrt[3]))]*Sqrt[-2 - (26 - 15*Sqrt[3])^(1/3)*(1 + I*Sq
rt[3]) + (I*(I + Sqrt[3]))/(26 - 15*Sqrt[3])^(1/3) + 6*x]*Sqrt[-2 - (1 + I*
Sqrt[3])/(26 - 15*Sqrt[3])^(1/3) + I*(26 - 15*Sqrt[3])^(1/3)*(I + Sqrt[3])
+ 6*x]*Sqrt[2 - x^2 + x^3]*EllipticPi[-1/2*((-26 - 15*Sqrt[3])^(1/3)*(3 - I
*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3 + I*Sqrt[3])))/((-1)^(1/3) - 3*2^(1/3
) - (-26 - 15*Sqrt[3])^(1/3) - (-26 + 15*Sqrt[3])^(1/3)), ArcSin[(Sqrt[2]*(
26 - 15*Sqrt[3])^(1/6)*Sqrt[-1 + (26 - 15*Sqrt[3])^(-1/3) + (26 - 15*Sqrt[3
])^(1/3) + 3*x])/Sqrt[3 - I*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3 + I*Sqrt[3
])]], (3*I + Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3*I - Sqrt[3]))/(3*I - Sqrt
[3] + (26 - 15*Sqrt[3])^(2/3)*(3*I + Sqrt[3]))])/(((-1)^(1/3) - 3*2^(1/3) -
 (-26 - 15*Sqrt[3])^(1/3) - (-26 + 15*Sqrt[3])^(1/3))*Sqrt[-1 + I*Sqrt[3] -
 (26 - 15*Sqrt[3])^(2/3)*(1 + I*Sqrt[3]) - 2*(26 - 15*Sqrt[3])^(1/3)*(1 - 3
*x)]*Sqrt[-1 - I*Sqrt[3] + I*(26 - 15*Sqrt[3])^(2/3)*(I + Sqrt[3]) - 2*(26
- 15*Sqrt[3])^(1/3)*(1 - 3*x)]*(1 - (26 - 15*Sqrt[3])^(-2/3) - (26 - 15*Sqr
t[3])^(2/3) - ((1 + (26 - 15*Sqrt[3])^(2/3))*(1 - 3*x))/(26 - 15*Sqrt[3])^(
1/3) - (1 - 3*x)^2)*Sqrt[-1 + (26 - 15*Sqrt[3])^(-1/3) + (26 - 15*Sqrt[3])^
(1/3) + 3*x]) + (27*2^(5/6)*Sqrt[(26 - 15*Sqrt[3])*(3 - I*Sqrt[3] + (26 - 1
5*Sqrt[3])^(2/3)*(3 + I*Sqrt[3]))]*Sqrt[1 - (2*(1 + (26 - 15*Sqrt[3])^(2/3)
 - (26 - 15*Sqrt[3])^(1/3)*(1 - 3*x)))/(3 + I*Sqrt[3] + (26 - 15*Sqrt[3])^(
2/3)*(3 - I*Sqrt[3]))]*Sqrt[1 - (2*(1 + (26 - 15*Sqrt[3])^(2/3) - (26 - 15*
Sqrt[3])^(1/3)*(1 - 3*x)))/(3 - I*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3 + I*
Sqrt[3]))]*Sqrt[-2 - (26 - 15*Sqrt[3])^(1/3)*(1 + I*Sqrt[3]) + (I*(I + Sqrt
[3]))/(26 - 15*Sqrt[3])^(1/3) + 6*x]*Sqrt[-2 - (1 + I*Sqrt[3])/(26 - 15*Sqr
t[3])^(1/3) + I*(26 - 15*Sqrt[3])^(1/3)*(I + Sqrt[3]) + 6*x]*Sqrt[2 - x^2 +
 x^3]*EllipticPi[((-1)^(1/6)*(3*I + Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3*I
- Sqrt[3])))/(2*((-1)^(2/3) - 3*(52 - 30*Sqrt[3])^(1/3) - (-1)^(2/3)*(26 -
15*Sqrt[3])^(1/3) + (-26 + 15*Sqrt[3])^(2/3))), ArcSin[(Sqrt[2]*(26 - 15*Sq
rt[3])^(1/6)*Sqrt[-1 + (26 - 15*Sqrt[3])^(-1/3) + (26 - 15*Sqrt[3])^(1/3) +
 3*x])/Sqrt[3 - I*Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3 + I*Sqrt[3])]], (3*I
 + Sqrt[3] + (26 - 15*Sqrt[3])^(2/3)*(3*I - Sqrt[3]))/(3*I - Sqrt[3] + (26
- 15*Sqrt[3])^(2/3)*(3*I + Sqrt[3]))])/(((-1)^(2/3) - 3*(52 - 30*Sqrt[3])^(
1/3) - (-1)^(2/3)*(26 - 15*Sqrt[3])^(1/3) + (-26 + 15*Sqrt[3])^(2/3))*Sqrt[
-1 + I*Sqrt[3] - (26 - 15*Sqrt[3])^(2/3)*(1 + I*Sqrt[3]) - 2*(26 - 15*Sqrt[
3])^(1/3)*(1 - 3*x)]*Sqrt[-1 - I*Sqrt[3] + I*(26 - 15*Sqrt[3])^(2/3)*(I + S
qrt[3]) - 2*(26 - 15*Sqrt[3])^(1/3)*(1 - 3*x)]*(1 - (26 - 15*Sqrt[3])^(-2/3
) - (26 - 15*Sqrt[3])^(2/3) - ((1 + (26 - 15*Sqrt[3])^(2/3))*(1 - 3*x))/(26
 - 15*Sqrt[3])^(1/3) - (1 - 3*x)^2)*Sqrt[-1 + (26 - 15*Sqrt[3])^(-1/3) + (2
6 - 15*Sqrt[3])^(1/3) + 3*x]) - 2*Defer[Int][Sqrt[2 - x^2 + x^3]/(2 + x^2 +
 x^3), x] - 3*Defer[Int][(x*Sqrt[2 - x^2 + x^3])/(2 + x^2 + x^3), x]
```

Rubi steps

$$\int \frac{\left(-4+x^3\right)\sqrt{2-x^2+x^3}}{\left(2+x^3\right)\left(2+x^2+x^3\right)}\,dx = \int \left(\frac{3x\sqrt{2-x^2+x^3}}{2+x^3}+\frac{(-2-3x)\sqrt{2-x^2+x^3}}{2+x^2+x^3}\right)dx$$
$$= 3\int \frac{x\sqrt{2-x^2+x^3}}{2+x^3}\,dx+\int \frac{(-2-3x)\sqrt{2-x^2+x^3}}{2+x^2+x^3}\,dx$$
$$= 3\int \left(-\frac{\sqrt{2-x^2+x^3}}{3\sqrt[3]{2}\left(\sqrt[3]{2}+x\right)}-\frac{(-1)^{2/3}\sqrt{2-x^2+x^3}}{3\sqrt[3]{2}\left(\sqrt[3]{2}-\sqrt[3]{-1}\,x\right)}+\frac{\sqrt[3]{-\frac{1}{2}}\sqrt{2-x^2+x^3}}{3\left(\sqrt[3]{2}+(-1)^{2/3}x\right)}\right)dx+\int$$
$$= -\left(2\int \frac{\sqrt{2-x^2+x^3}}{2+x^2+x^3}\,dx\right)-3\int \frac{x\sqrt{2-x^2+x^3}}{2+x^2+x^3}\,dx+\sqrt[3]{-\frac{1}{2}}\int \frac{\sqrt{2-x^2+x^3}}{\sqrt[3]{2}+(-1)^{2/3}}$$
= rest of steps removed due to Latex formating problem

Mathematica [C] time = 6.40, size = 5117, normalized size = 104.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-4 + x^3)*Sqrt[2 - x^2 + x^3])/((2 + x^3)*(2 + x^2 + x^3)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.39, size = 49, normalized size = 1.00

$$2\tan^{-1}\left(\frac{x}{\sqrt{x^3-x^2+2}}\right)-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^3-x^2+2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + x^3)*Sqrt[2 - x^2 + x^3])/((2 + x^3)*(2 + x^2 + x
^3)),x]
```

```
[Out] 2*ArcTan[x/Sqrt[2 - x^2 + x^3]] - 2*Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[2 - x^2
 + x^3]]
```

fricas [B] time = 0.41, size = 88, normalized size = 1.80

$$\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x^3-x^2+2}\left(x^3-3x^2+2\right)}{4\left(x^4-x^3+2x\right)}\right)-\arctan\left(\frac{\sqrt{x^3-x^2+2}\left(x^3-2x^2+2\right)}{2\left(x^4-x^3+2x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^3-x^2+2)^(1/2)/(x^3+2)/(x^3+x^2+2),x, algorithm="frica
s")
```

```
[Out] sqrt(2)*arctan(1/4*sqrt(2)*sqrt(x^3 - x^2 + 2)*(x^3 - 3*x^2 + 2)/(x^4 - x^3
 + 2*x)) - arctan(1/2*sqrt(x^3 - x^2 + 2)*(x^3 - 2*x^2 + 2)/(x^4 - x^3 + 2*
x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^3-x^2+2}\left(x^3-4\right)}{\left(x^3+x^2+2\right)\left(x^3+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^3-x^2+2)^(1/2)/(x^3+2)/(x^3+x^2+2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(x^3 - x^2 + 2)*(x^3 - 4)/((x^3 + x^2 + 2)*(x^3 + 2)), x)
```

maple [C] time = 0.10, size = 278, normalized size = 5.67

$$\frac{(4-2i)\sqrt{\left(\frac{2}{5}+\frac{i}{5}\right)(1+x)}\,\sqrt{\left(-\frac{2}{5}+\frac{i}{5}\right)(x-1-i)}\,\sqrt{\left(-\frac{2}{5}-\frac{i}{5}\right)(x-1+i)}\,\mathrm{EllipticF}\left(\sqrt{\left(\frac{2}{5}+\frac{i}{5}\right)(1+x)},\frac{2\sqrt{5}}{5}-\frac{i\sqrt{5}}{5}\right)}{\sqrt{x^3-x^2+2}}+(-4+2i)\left(\sum_{\_\alpha=\mathrm{RootOf}(\_Z^3+2)}\frac{\_\alpha\left(\_\alpha^2-\_\alpha+1\right)\sqrt{\left(\frac{2}{5}+\frac{i}{5}\right)(1+x)}\,\sqrt{\left(-\frac{2}{5}+\frac{i}{5}\right)(x-1-i)}\,\sqrt{\left(-\frac{2}{5}-\frac{i}{5}\right)(x-1+i)}\,\mathrm{EllipticPi}\left(\sqrt{\left(\frac{2}{5}+\frac{i}{5}\right)(1+x)},i\_\alpha^2-2\_\alpha^2-i\_\alpha+2\_\alpha+i-2,\frac{2\sqrt{5}}{5}-\frac{i\sqrt{5}}{5}\right)}{\sqrt{x^3-x^2+2}}\right)+(4-2i)\left(\sum_{\_\alpha=\mathrm{RootOf}(\_Z^3+\_Z^2+2)}\frac{\_\alpha^3\sqrt{\left(\frac{2}{5}+\frac{i}{5}\right)(1+x)}\,\sqrt{\left(-\frac{2}{5}+\frac{i}{5}\right)(x-1-i)}\,\sqrt{\left(-\frac{2}{5}-\frac{i}{5}\right)(x-1+i)}\,\mathrm{EllipticPi}\left(\sqrt{\left(\frac{2}{5}+\frac{i}{5}\right)(1+x)},\frac{1}{2}i\_\alpha^2-\_\alpha^2,\frac{2\sqrt{5}}{5}-\frac{i\sqrt{5}}{5}\right)}{\sqrt{x^3-x^2+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-4)*(x^3-x^2+2)^(1/2)/(x^3+2)/(x^3+x^2+2),x)
```

```
[Out] (4-2*I)*((2/5+1/5*I)*(1+x))^(1/2)*((-2/5+1/5*I)*(x-1-I))^(1/2)*((-2/5-1/5*I
)*(x-1+I))^(1/2)/(x^3-x^2+2)^(1/2)*EllipticF(((2/5+1/5*I)*(1+x))^(1/2),2/5*
5^(1/2)-1/5*I*5^(1/2))+(-4+2*I)*sum(_alpha*(_alpha^2-_alpha+1)*((2/5+1/5*I)
*(1+x))^(1/2)*((-2/5+1/5*I)*(x-1-I))^(1/2)*((-2/5-1/5*I)*(x-1+I))^(1/2)/(x^
3-x^2+2)^(1/2)*EllipticPi(((2/5+1/5*I)*(1+x))^(1/2),I*_alpha^2-2*_alpha^2-I
*_alpha+2*_alpha-2+I,2/5*5^(1/2)-1/5*I*5^(1/2)),_alpha=RootOf(_Z^3+2))+(4-2
*I)*sum(_alpha^3*((2/5+1/5*I)*(1+x))^(1/2)*((-2/5+1/5*I)*(x-1-I))^(1/2)*((-
2/5-1/5*I)*(x-1+I))^(1/2)/(x^3-x^2+2)^(1/2)*EllipticPi(((2/5+1/5*I)*(1+x))^
(1/2),1/2*I*_alpha^2-_alpha^2,2/5*5^(1/2)-1/5*I*5^(1/2)),_alpha=RootOf(_Z^3
+_Z^2+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^3-x^2+2}\left(x^3-4\right)}{\left(x^3+x^2+2\right)\left(x^3+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^3-x^2+2)^(1/2)/(x^3+2)/(x^3+x^2+2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(x^3 - x^2 + 2)*(x^3 - 4)/((x^3 + x^2 + 2)*(x^3 + 2)), x)
```

mupad [B] time = 0.60, size = 235, normalized size = 4.80

$$\left(\sum_{\mathrm{X264}\in\left\{-2^{1/3},2^{1/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right),-2^{1/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right\}\cup\mathrm{root}(z^3+z^2+2,z)}\frac{\sqrt{5}\,\sqrt{x\,(2-\mathrm{i})+2-\mathrm{i}}\,\sqrt{3+x\,(-2+1\mathrm{i})+1\mathrm{i}}\,\sqrt{3+x\,(-2-\mathrm{i})-\mathrm{i}}\,\Pi\left(\frac{2+1\mathrm{i}}{\mathrm{X264}+1};\mathrm{asin}\left(\frac{\sqrt{5}\,\sqrt{x\,(2-\mathrm{i})+2-\mathrm{i}}}{5}\right)\middle|\frac{3}{5}+\frac{4}{5}\mathrm{i}\right)\left(2\,\mathrm{X264}^5+6\,\mathrm{X264}^3-2\,\mathrm{X264}^2+12\right)\left(\frac{4}{25}+\frac{2}{25}\mathrm{i}\right)}{\mathrm{X264}\,\left(\mathrm{X264}+1\right)\,\sqrt{x^3-x^2+2}\,\left(6\,\mathrm{X264}^4+5\,\mathrm{X264}^3+12\,\mathrm{X264}+4\right)}\right)+\frac{\sqrt{x\left(\frac{2}{5}-\frac{1}{5}\mathrm{i}\right)+\frac{2}{5}-\frac{1}{5}\mathrm{i}}\,\sqrt{\frac{3}{5}+x\left(-\frac{2}{5}+\frac{1}{5}\mathrm{i}\right)+\frac{1}{5}\mathrm{i}}\,\sqrt{\frac{3}{5}+x\left(-\frac{2}{5}-\frac{1}{5}\mathrm{i}\right)-\frac{1}{5}\mathrm{i}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{x\left(\frac{2}{5}-\frac{1}{5}\mathrm{i}\right)+\frac{2}{5}-\frac{1}{5}\mathrm{i}}\right)\middle|\frac{3}{5}+\frac{4}{5}\mathrm{i}\right)(4+2\mathrm{i})}{\sqrt{x^3-x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 4)*(x^3 - x^2 + 2)^(1/2))/((x^3 + 2)*(x^2 + x^3 + 2)),x)
```

```
[Out] symsum((5^(1/2)*(x*(2 - 1i) + (2 - 1i))^(1/2)*((3 + 1i) - x*(2 - 1i))^(1/2)
*((3 - 1i) - x*(2 + 1i))^(1/2)*ellipticPi((2 + 1i)/(_X264 + 1), asin((5^(1/
2)*(x*(2 - 1i) + (2 - 1i))^(1/2))/5), 3/5 + 4i/5)*(6*_X264^3 - 2*_X264^2 +
2*_X264^5 + 12)*(4/25 + 2i/25))/(_X264*(_X264 + 1)*(x^3 - x^2 + 2)^(1/2)*(1
2*_X264 + 5*_X264^3 + 6*_X264^4 + 4)), _X264 in {-2^(1/3), 2^(1/3)*((3^(1/2
)*1i)/2 + 1/2), -2^(1/3)*((3^(1/2)*1i)/2 - 1/2)} union root(z^3 + z^2 + 2,
z)) + ((x*(2/5 - 1i/5) + (2/5 - 1i/5))^(1/2)*((3/5 + 1i/5) - x*(2/5 - 1i/5)
)^(1/2)*((3/5 - 1i/5) - x*(2/5 + 1i/5))^(1/2)*ellipticF(asin((x*(2/5 - 1i/5
) + (2/5 - 1i/5))^(1/2)), 3/5 + 4i/5)*(4 + 2i))/(x^3 - x^2 + 2)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-4)*(x**3-x**2+2)**(1/2)/(x**3+2)/(x**3+x**2+2),x)
```

```
[Out] Timed out
```

3.619 $\int x^2 \sqrt[4]{-1+x^4}\,dx$

Optimal. Leaf size=49

$$\frac{1}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{8}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{4}\sqrt[4]{x^4-1}\,x^3$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {279, 331, 298, 203, 206}

$$\frac{1}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{8}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{4}\sqrt[4]{x^4-1}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(-1 + x^4)^(1/4),x]
```

```
[Out] (x^3*(-1 + x^4)^(1/4))/4 + ArcTan[x/(-1 + x^4)^(1/4)]/8 - ArcTanh[x/(-1 + x
^4)^(1/4)]/8
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int x^2\sqrt[4]{-1+x^4}\,dx = \frac{1}{4}x^3\sqrt[4]{-1+x^4} - \frac{1}{4}\int \frac{x^2}{\left(-1+x^4\right)^{3/4}}\,dx$$
$$= \frac{1}{4}x^3\sqrt[4]{-1+x^4} - \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)$$
$$= \frac{1}{4}x^3\sqrt[4]{-1+x^4} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right) + \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)$$
$$= \frac{1}{4}x^3\sqrt[4]{-1+x^4} + \frac{1}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right) - \frac{1}{8}\tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.82

$$\frac{x^3\sqrt[4]{x^4-1}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};x^4\right)}{3\sqrt[4]{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(-1 + x^4)^(1/4),x]
```

```
[Out] (x^3*(-1 + x^4)^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, x^4])/(3*(1 - x^4)^
(1/4))
```

IntegrateAlgebraic [A] time = 0.18, size = 49, normalized size = 1.00

$$\frac{1}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right) - \frac{1}{8}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right) + \frac{1}{4}\sqrt[4]{x^4-1}\,x^3$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(-1 + x^4)^(1/4),x]
```

```
[Out] (x^3*(-1 + x^4)^(1/4))/4 + ArcTan[x/(-1 + x^4)^(1/4)]/8 - ArcTanh[x/(-1 + x
^4)^(1/4)]/8
```

fricas [A] time = 0.41, size = 62, normalized size = 1.27

$$\frac{1}{4}\left(x^4-1\right)^{\frac{1}{4}}x^3 - \frac{1}{8}\arctan\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}\right) - \frac{1}{16}\log\left(\frac{x+\left(x^4-1\right)^{\frac{1}{4}}}{x}\right) + \frac{1}{16}\log\left(-\frac{x-\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/4*(x^4 - 1)^(1/4)*x^3 - 1/8*arctan((x^4 - 1)^(1/4)/x) - 1/16*log((x + (x^
4 - 1)^(1/4))/x) + 1/16*log(-(x - (x^4 - 1)^(1/4))/x)
```

giac [A] time = 0.19, size = 60, normalized size = 1.22

$$-\frac{1}{4}\left(x^4-1\right)^{\frac{1}{4}}x^3 + \frac{1}{8}\arctan\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}\right) + \frac{1}{16}\log\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}+1\right) - \frac{1}{16}\log\left(-\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4-1)^(1/4),x, algorithm="giac")
```

```
[Out] -1/4*(x^4 - 1)^(1/4)*x^3 + 1/8*arctan((x^4 - 1)^(1/4)/x) + 1/16*log((x^4 -
1)^(1/4)/x + 1) - 1/16*log(-(x^4 - 1)^(1/4)/x + 1)
```

maple [C] time = 0.30, size = 46, normalized size = 0.94

$$\frac{x^3\left(x^4-1\right)^{\frac{1}{4}}}{4}-\frac{\left(-\mathrm{signum}\left(x^4-1\right)\right)^{\frac{3}{4}}x^3\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],x^4\right)}{12\mathrm{signum}\left(x^4-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4-1)^(1/4),x)
```

```
[Out] 1/4*x^3*(x^4-1)^(1/4)-1/12/signum(x^4-1)^(3/4)*(-signum(x^4-1))^(3/4)*x^3*h
ypergeom([3/4,3/4],[7/4],x^4)
```

maxima [A] time = 0.41, size = 72, normalized size = 1.47

$$-\frac{\left(x^4-1\right)^{\frac{1}{4}}}{4\,x\left(\frac{x^4-1}{x^4}-1\right)}-\frac{1}{8}\arctan\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{16}\log\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}+1\right)+\frac{1}{16}\log\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4-1)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/4*(x^4 - 1)^(1/4)/(x*((x^4 - 1)/x^4 - 1)) - 1/8*arctan((x^4 - 1)^(1/4)/x
) - 1/16*log((x^4 - 1)^(1/4)/x + 1) + 1/16*log((x^4 - 1)^(1/4)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\left(x^4-1\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4 - 1)^(1/4),x)
```

```
[Out] int(x^2*(x^4 - 1)^(1/4), x)
```

sympy [C] time = 0.87, size = 36, normalized size = 0.73

$$-\frac{x^3e^{-\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**4-1)**(1/4),x)
```

```
[Out] -x**3*exp(-3*I*pi/4)*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), x**4)/(4*gamma(7
/4))
```

3.620 $\int x^2 \sqrt[4]{1+x^4}\, dx$

Optimal. Leaf size=49

$$-\frac{1}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{8}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{4}\sqrt[4]{x^4+1}\,x^3$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {279, 331, 298, 203, 206}

$$-\frac{1}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{8}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{4}\sqrt[4]{x^4+1}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 + x^4)^(1/4),x]
```

```
[Out] (x^3*(1 + x^4)^(1/4))/4 - ArcTan[x/(1 + x^4)^(1/4)]/8 + ArcTanh[x/(1 + x^4)
^(1/4)]/8
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int x^2\sqrt[4]{1+x^4}\,dx = \frac{1}{4}x^3\sqrt[4]{1+x^4}+\frac{1}{4}\int\frac{x^2}{\left(1+x^4\right)^{3/4}}\,dx$$
$$=\frac{1}{4}x^3\sqrt[4]{1+x^4}+\frac{1}{4}\text{Subst}\left(\int\frac{x^2}{1-x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)$$
$$=\frac{1}{4}x^3\sqrt[4]{1+x^4}+\frac{1}{8}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{1}{8}\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)$$
$$=\frac{1}{4}x^3\sqrt[4]{1+x^4}-\frac{1}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{8}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.45

$$\frac{1}{3}x^3\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 + x^4)^(1/4),x]
```

```
[Out] (x^3*Hypergeometric2F1[-1/4, 3/4, 7/4, -x^4])/3
```

IntegrateAlgebraic [A] time = 0.19, size = 49, normalized size = 1.00

$$-\frac{1}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{8}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{4}\sqrt[4]{x^4+1}\,x^3$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(1 + x^4)^(1/4),x]
```

```
[Out] (x^3*(1 + x^4)^(1/4))/4 - ArcTan[x/(1 + x^4)^(1/4)]/8 + ArcTanh[x/(1 + x^4)
^(1/4)]/8
```

fricas [A] time = 0.40, size = 62, normalized size = 1.27

$$\frac{1}{4}\left(x^4+1\right)^{\frac{1}{4}}x^3+\frac{1}{8}\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{16}\log\left(\frac{x+\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{16}\log\left(-\frac{x-\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4+1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/4*(x^4 + 1)^(1/4)*x^3 + 1/8*arctan((x^4 + 1)^(1/4)/x) + 1/16*log((x + (x^
4 + 1)^(1/4))/x) - 1/16*log(-(x - (x^4 + 1)^(1/4))/x)
```

giac [A] time = 0.51, size = 59, normalized size = 1.20

$$\frac{1}{4}\left(x^4+1\right)^{\frac{1}{4}}x^3+\frac{1}{8}\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{16}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}+1\right)-\frac{1}{16}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4+1)^(1/4),x, algorithm="giac")
```

```
[Out] 1/4*(x^4 + 1)^(1/4)*x^3 + 1/8*arctan((x^4 + 1)^(1/4)/x) + 1/16*log((x^4 + 1
)^(1/4)/x + 1) - 1/16*log((x^4 + 1)^(1/4)/x - 1)
```

maple [C] time = 0.30, size = 30, normalized size = 0.61

$$\frac{x^{3}\left(x^{4}+1\right)^{\frac{1}{4}}}{4}+\frac{x^{3}\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],-x^{4}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4+1)^(1/4),x)
```

```
[Out] 1/4*x^3*(x^4+1)^(1/4)+1/12*x^3*hypergeom([3/4,3/4],[7/4],-x^4)
```

maxima [A] time = 0.42, size = 72, normalized size = 1.47

$$\frac{\left(x^{4}+1\right)^{\frac{1}{4}}}{4\,x\left(\frac{x^{4}+1}{x^{4}}-1\right)}+\frac{1}{8}\,\arctan\left(\frac{\left(x^{4}+1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{16}\,\log\left(\frac{\left(x^{4}+1\right)^{\frac{1}{4}}}{x}+1\right)-\frac{1}{16}\,\log\left(\frac{\left(x^{4}+1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4+1)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/4*(x^4 + 1)^(1/4)/(x*((x^4 + 1)/x^4 - 1)) + 1/8*arctan((x^4 + 1)^(1/4)/x)
 + 1/16*log((x^4 + 1)^(1/4)/x + 1) - 1/16*log((x^4 + 1)^(1/4)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{2}\left(x^{4}+1\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4 + 1)^(1/4),x)
```

```
[Out] int(x^2*(x^4 + 1)^(1/4), x)
```

sympy [C] time = 0.82, size = 31, normalized size = 0.63

$$\frac{x^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|x^{4}e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**4+1)**(1/4),x)
```

```
[Out] x**3*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), x**4*exp_polar(I*pi))/(4*gamma(7
/4))
```

3.621 $\int \frac{(-b+ax^3)\sqrt{x+x^4}}{x^3}\,dx$

Optimal. Leaf size=49

$$\frac{1}{3}(a-2b)\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)+\frac{\sqrt{x^4+x}\,(ax^3+2b)}{3x^2}$$

Rubi [A] time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2038, 2004, 2029, 206}

$$\frac{1}{3}x\sqrt{x^4+x}\,(a-2b)+\frac{1}{3}(a-2b)\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)+\frac{2b\left(x^4+x\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a*x^3)*Sqrt[x + x^4])/x^3,x]
```

```
[Out] ((a - 2*b)*x*Sqrt[x + x^4])/3 + (2*b*(x + x^4)^(3/2))/(3*x^3) + ((a - 2*b)*
ArcTanh[x^2/Sqrt[x + x^4]])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(-b+ax^3)\sqrt{x+x^4}}{x^3}\,dx = \frac{2b\left(x+x^4\right)^{3/2}}{3x^3} - (-a+2b)\int \sqrt{x+x^4}\,dx$$
$$= \frac{1}{3}(a-2b)x\sqrt{x+x^4} + \frac{2b\left(x+x^4\right)^{3/2}}{3x^3} - \frac{1}{2}(-a+2b)\int \frac{x}{\sqrt{x+x^4}}\,dx$$
$$= \frac{1}{3}(a-2b)x\sqrt{x+x^4} + \frac{2b\left(x+x^4\right)^{3/2}}{3x^3} - \frac{1}{3}(-a+2b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^2}{\sqrt{x+x^4}}\right)$$
$$= \frac{1}{3}(a-2b)x\sqrt{x+x^4} + \frac{2b\left(x+x^4\right)^{3/2}}{3x^3} + \frac{1}{3}(a-2b)\tanh^{-1}\left(\frac{x^2}{\sqrt{x+x^4}}\right)$$

Mathematica [A] time = 0.03, size = 62, normalized size = 1.27

$$\frac{\sqrt{x^4+x}\left(x^{3/2}(a-2b)\sinh^{-1}\left(x^{3/2}\right)+\sqrt{x^3+1}\left(ax^3+2b\right)\right)}{3x^2\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + a*x^3)*Sqrt[x + x^4])/x^3,x]
```

```
[Out] (Sqrt[x + x^4]*(Sqrt[1 + x^3]*(2*b + a*x^3) + (a - 2*b)*x^(3/2)*ArcSinh[x^(
3/2)]))/(3*x^2*Sqrt[1 + x^3])
```

IntegrateAlgebraic [A] time = 0.48, size = 49, normalized size = 1.00

$$\frac{1}{3}(a-2b)\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)+\frac{\sqrt{x^4+x}\left(ax^3+2b\right)}{3x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^3)*Sqrt[x + x^4])/x^3,x]
```

```
[Out] ((2*b + a*x^3)*Sqrt[x + x^4])/(3*x^2) + ((a - 2*b)*ArcTanh[x^2/Sqrt[x + x^4
]])/3
```

fricas [A] time = 0.43, size = 51, normalized size = 1.04

$$-\frac{(a-2\,b)x^2\log\left(2\,x^3-2\,\sqrt{x^4+x}\,x+1\right)-2\left(ax^3+2\,b\right)\sqrt{x^4+x}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(x^4+x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/6*((a - 2*b)*x^2*log(2*x^3 - 2*sqrt(x^4 + x)*x + 1) - 2*(a*x^3 + 2*b)*sq
rt(x^4 + x))/x^2
```

giac [A] time = 0.35, size = 57, normalized size = 1.16

$$\frac{1}{3}\sqrt{x^4+x}\,ax+\frac{1}{6}\,(a-2\,b)\log\left(\sqrt{\frac{1}{x^3}+1}+1\right)-\frac{1}{6}\,(a-2\,b)\log\left(\left|\sqrt{\frac{1}{x^3}+1}-1\right|\right)+\frac{2}{3}\,b\sqrt{\frac{1}{x^3}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(x^4+x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(x^4 + x)*a*x + 1/6*(a - 2*b)*log(sqrt(1/x^3 + 1) + 1) - 1/6*(a - 2
*b)*log(abs(sqrt(1/x^3 + 1) - 1)) + 2/3*b*sqrt(1/x^3 + 1)
```

maple [C] time = 0.02, size = 609, normalized size = 12.43

$$a\left(\frac{x\sqrt{x^4+x}}{3}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\,(1+x)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(1+x)}}\sqrt{-\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\left(-\mathrm{EllipticF}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)+\mathrm{EllipticPi}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\frac{\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(1+x)\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)-b\left(-\frac{2\sqrt{x^4+x}}{3x^2}-\frac{2\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\,(1+x)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(1+x)}}\sqrt{-\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\left(-\mathrm{EllipticF}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)+\mathrm{EllipticPi}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\frac{\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(1+x)\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b)*(x^4+x)^(1/2)/x^3,x)
```

```
[Out] a*(1/3*x*(x^4+x)^(1/2)-(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1/2
*I*3^(1/2))/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2)
)/(1+x))^(1/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(3/
2+1/2*I*3^(1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1/2
)*(-EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-3/
2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1
/2)))^(1/2))+EllipticPi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(
1/2),(1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-1/
2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))-b*(-2/3*(x
^4+x)^(1/2)/x^2-2*(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^
(1/2))/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))/(1+
x))^(1/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(3/2+1/2
*I*3^(1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1/2)*(-E
llipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-3/2+1/2
*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2))+EllipticPi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),
(1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3
^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^3-b\right)\sqrt{x^4+x}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(x^4+x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x^3 - b)*sqrt(x^4 + x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\left(b-a\,x^3\right)\,\sqrt{x^4+x}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b - a*x^3)*(x + x^4)^(1/2))/x^3,x)
```

```
[Out] -int(((b - a*x^3)*(x + x^4)^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\,(x+1)\left(x^2-x+1\right)}\,\left(ax^3-b\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b)*(x**4+x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x*(x + 1)*(x**2 - x + 1))*(a*x**3 - b)/x**3, x)
```

3.622 $\int \frac{-4b+ax^3}{\sqrt[4]{-b+ax^3}\left(b-ax^3+x^4\right)}\,dx$

Optimal. Leaf size=49

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^3-b}}{x}\right)+2\tanh^{-1}\left(\frac{x\left(ax^3-b\right)^{3/4}}{b-ax^3}\right)$$

Rubi [F] time = 0.84, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-4b+ax^3}{\sqrt[4]{-b+ax^3}\left(b-ax^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-4*b + a*x^3)/((-b + a*x^3)^(1/4)*(b - a*x^3 + x^4)),x]
```

```
[Out] -(a*Defer[Int][x^3/((-b + a*x^3)^(1/4)*(-b + a*x^3 - x^4)), x]) - 4*b*Defer
[Int][1/((-b + a*x^3)^(1/4)*(b - a*x^3 + x^4)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{-4b+ax^3}{\sqrt[4]{-b+ax^3}\left(b-ax^3+x^4\right)}\,dx &= \int \left(-\frac{ax^3}{\sqrt[4]{-b+ax^3}\left(-b+ax^3-x^4\right)}-\frac{4b}{\sqrt[4]{-b+ax^3}\left(b-ax^3+x^4\right)}\right)dx\\ &= -\left(a\int \frac{x^3}{\sqrt[4]{-b+ax^3}\left(-b+ax^3-x^4\right)}\,dx\right)-(4b)\int \frac{1}{\sqrt[4]{-b+ax^3}\left(b-ax^3+x^4\right)}\,d\end{aligned}$$

Mathematica [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{-4b+ax^3}{\sqrt[4]{-b+ax^3}\left(b-ax^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-4*b + a*x^3)/((-b + a*x^3)^(1/4)*(b - a*x^3 + x^4)),x]
```

```
[Out] Integrate[(-4*b + a*x^3)/((-b + a*x^3)^(1/4)*(b - a*x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.64, size = 49, normalized size = 1.00

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^3-b}}{x}\right)+2\tanh^{-1}\left(\frac{x\left(ax^3-b\right)^{3/4}}{b-ax^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-4*b + a*x^3)/((-b + a*x^3)^(1/4)*(b - a*x^3 + x^4)),x]
```

```
[Out] 2*ArcTan[(-b + a*x^3)^(1/4)/x] + 2*ArcTanh[(x*(-b + a*x^3)^(3/4))/(b - a*x^
3)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-4*b)/(a*x^3-b)^(1/4)/(-a*x^3+x^4+b),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ax^3 - 4\,b}{\left(ax^3 - x^4 - b\right)\left(ax^3 - b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-4*b)/(a*x^3-b)^(1/4)/(-a*x^3+x^4+b),x, algorithm="giac")
[Out] integrate(-(a*x^3 - 4*b)/((a*x^3 - x^4 - b)*(a*x^3 - b)^(1/4)), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{a\,x^3 - 4b}{\left(a\,x^3 - b\right)^{\frac{1}{4}}\left(-a\,x^3 + x^4 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-4*b)/(a*x^3-b)^(1/4)/(-a*x^3+x^4+b),x)
[Out] int((a*x^3-4*b)/(a*x^3-b)^(1/4)/(-a*x^3+x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax^3 - 4\,b}{\left(ax^3 - x^4 - b\right)\left(ax^3 - b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-4*b)/(a*x^3-b)^(1/4)/(-a*x^3+x^4+b),x, algorithm="maxima")
[Out] -integrate((a*x^3 - 4*b)/((a*x^3 - x^4 - b)*(a*x^3 - b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{4\,b - a\,x^3}{\left(a\,x^3 - b\right)^{1/4}\,\left(x^4 - a\,x^3 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*b - a*x^3)/((a*x^3 - b)^(1/4)*(b - a*x^3 + x^4)),x)
[Out] int(-(4*b - a*x^3)/((a*x^3 - b)^(1/4)*(b - a*x^3 + x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-4*b)/(a*x**3-b)**(1/4)/(-a*x**3+x**4+b),x)
[Out] Timed out
```

3.623 $\int \frac{-x+3x^5}{(1+x^4)(a-x+ax^4)\sqrt{x+x^5}}\,dx$

Optimal. Leaf size=49

$$\frac{2\sqrt{x^5+x}}{x^4+1}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{x^5+x}}{\sqrt{a}\left(x^4+1\right)}\right)$$

Rubi [F] time = 2.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-x+3x^5}{\left(1+x^4\right)\left(a-x+ax^4\right)\sqrt{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-x + 3*x^5)/((1 + x^4)*(a - x + a*x^4)*Sqrt[x + x^5]),x]
```

```
[Out] (3*x^2)/(2*a*Sqrt[x + x^5]) - (3*x^2*Sqrt[(1 + x)^2/x]*Sqrt[-((1 + x^4)/x^2
)]*EllipticF[ArcSin[Sqrt[-((Sqrt[2] - 2*x + Sqrt[2]*x^2)/x)]/2], -2*(1 - Sq
rt[2])])/(4*Sqrt[2 + Sqrt[2]]*a*(1 + x)*Sqrt[x + x^5]) - (3*Sqrt[-((1 - x)^
2/x)]*x^2*Sqrt[-((1 + x^4)/x^2)]*EllipticF[ArcSin[Sqrt[(Sqrt[2] + 2*x + Sqr
t[2]*x^2)/x]/2], -2*(1 - Sqrt[2])])/(4*Sqrt[2 + Sqrt[2]]*a*(1 - x)*Sqrt[x +
 x^5]) - (8*Sqrt[x]*Sqrt[1 + x^4]*Defer[Subst][Defer[Int][x^2/((1 + x^8)^(3
/2)*(a - x^2 + a*x^8)), x], x, Sqrt[x]])/Sqrt[x + x^5] + (6*Sqrt[x]*Sqrt[1
+ x^4]*Defer[Subst][Defer[Int][x^4/((1 + x^8)^(3/2)*(a - x^2 + a*x^8)), x],
 x, Sqrt[x]])/(a*Sqrt[x + x^5])
```

Rubi steps

$$\int \frac{-x+3x^5}{\left(1+x^4\right)\left(a-x+ax^4\right)\sqrt{x+x^5}}\,dx = \int \frac{x\left(-1+3x^4\right)}{\left(1+x^4\right)\left(a-x+ax^4\right)\sqrt{x+x^5}}\,dx$$
$$= \frac{\left(\sqrt{x}\sqrt{1+x^4}\right)\int \frac{\sqrt{x}\left(-1+3x^4\right)}{\left(1+x^4\right)^{3/2}\left(a-x+ax^4\right)}\,dx}{\sqrt{x+x^5}}$$
$$= \frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\mathrm{Subst}\left(\int \frac{x^2\left(-1+3x^8\right)}{\left(1+x^8\right)^{3/2}\left(a-x^2+ax^8\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{x+x^5}}$$
$$= \frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\mathrm{Subst}\left(\int \left(\frac{3x^2}{a\left(1+x^8\right)^{3/2}} + \frac{x^2\left(-4a+3x^2\right)}{a\left(1+x^8\right)^{3/2}\left(a-x^2+ax^8\right)}\right)dx, x, \sqrt{x}\right)}{\sqrt{x+x^5}}$$
$$= \frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\mathrm{Subst}\left(\int \frac{x^2\left(-4a+3x^2\right)}{\left(1+x^8\right)^{3/2}\left(a-x^2+ax^8\right)}\,dx, x, \sqrt{x}\right)}{a\sqrt{x+x^5}} + \frac{\left(6\sqrt{x}\sqrt{1+x\right.}{}}{}$$
$$= \frac{3x^2}{2a\sqrt{x+x^5}} + \frac{\left(3\sqrt{x}\sqrt{1+x^4}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1+x^8}}\,dx, x, \sqrt{x}\right)}{2a\sqrt{x+x^5}} + \frac{\left(2\sqrt{x}\sqrt{1\right.}{}}{}$$
$$= \frac{3x^2}{2a\sqrt{x+x^5}} - \frac{\left(8\sqrt{x}\sqrt{1+x^4}\right)\mathrm{Subst}\left(\int \frac{x^2}{\left(1+x^8\right)^{3/2}\left(a-x^2+ax^8\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{x+x^5}} -$$
$$= \frac{3x^2}{2a\sqrt{x+x^5}} - \frac{3x^2\sqrt{\frac{(1+x)^2}{x}}\sqrt{-\frac{1+x^4}{x^2}}\,F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}-2x+\sqrt{2}x^2}{x}}\right)\middle| -2\left(1\right.\right.}{4\sqrt{2+\sqrt{2}}\,a(1+x)\sqrt{x+x^5}}$$

Mathematica [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{-x+3x^5}{\left(1+x^4\right)\left(a-x+ax^4\right)\sqrt{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-x + 3*x^5)/((1 + x^4)*(a - x + a*x^4)*Sqrt[x + x^5]),x]
[Out] Integrate[(-x + 3*x^5)/((1 + x^4)*(a - x + a*x^4)*Sqrt[x + x^5]), x]
```

IntegrateAlgebraic [A] time = 2.21, size = 49, normalized size = 1.00

$$\frac{2\sqrt{x^5+x}}{x^4+1} - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{x^5+x}}{\sqrt{a}\left(x^4+1\right)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x + 3*x^5)/((1 + x^4)*(a - x + a*x^4)*Sqrt[x + x^5]),x
]
[Out] (2*Sqrt[x + x^5])/(1 + x^4) - 2*Sqrt[a]*ArcTanh[Sqrt[x + x^5]/(Sqrt[a]*(1 +
 x^4))]
```

fricas [A] time = 0.46, size = 180, normalized size = 3.67

$$\left[\frac{\left(x^4+1\right)\sqrt{a}\log\left(\frac{a^2x^8+2a^2x^4+6ax^5-4\left(ax^4+a+x\right)\sqrt{x^5+x}\sqrt{a}+a^2+6ax+x^2}{a^2x^8+2a^2x^4-2ax^5+a^2-2ax+x^2}\right)+4\sqrt{x^5+x}}{2\left(x^4+1\right)}, \frac{\left(x^4+1\right)\sqrt{-a}\arctan\left(\frac{\left(ax^4+a+x\right)\sqrt{x^5+x}\sqrt{-a}}{2\left(ax^5+ax\right)}\right)+2\sqrt{x^5+x}}{x^4+1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-x)/(x^4+1)/(a*x^4+a-x)/(x^5+x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/2*((x^4 + 1)*sqrt(a)*log((a^2*x^8 + 2*a^2*x^4 + 6*a*x^5 - 4*(a*x^4 + a +
 x)*sqrt(x^5 + x)*sqrt(a) + a^2 + 6*a*x + x^2)/(a^2*x^8 + 2*a^2*x^4 - 2*a*x
^5 + a^2 - 2*a*x + x^2)) + 4*sqrt(x^5 + x))/(x^4 + 1), ((x^4 + 1)*sqrt(-a)*
arctan(1/2*(a*x^4 + a + x)*sqrt(x^5 + x)*sqrt(-a)/(a*x^5 + a*x)) + 2*sqrt(x
^5 + x))/(x^4 + 1)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^5-x}{\left(ax^4+a-x\right)\sqrt{x^5+x}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-x)/(x^4+1)/(a*x^4+a-x)/(x^5+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^5 - x)/((a*x^4 + a - x)*sqrt(x^5 + x)*(x^4 + 1)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{3x^5-x}{\left(x^4+1\right)\left(a\,x^4+a-x\right)\sqrt{x^5+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^5-x)/(x^4+1)/(a*x^4+a-x)/(x^5+x)^(1/2),x)
```

```
[Out] int((3*x^5-x)/(x^4+1)/(a*x^4+a-x)/(x^5+x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^5-x}{\left(ax^4+a-x\right)\sqrt{x^5+x}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-x)/(x^4+1)/(a*x^4+a-x)/(x^5+x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((3*x^5 - x)/((a*x^4 + a - x)*sqrt(x^5 + x)*(x^4 + 1)), x)
```

mupad [B] time = 0.77, size = 55, normalized size = 1.12

$$\frac{2\sqrt{x^5+x}}{x^4+1}+\sqrt{a}\,\ln\left(\frac{a+x-2\sqrt{a}\,\sqrt{x^5+x}+a\,x^4}{a\,x^4-x+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 3*x^5)/((x^4 + 1)*(x + x^5)^(1/2)*(a - x + a*x^4)),x)
```

```
[Out] (2*(x + x^5)^(1/2))/(x^4 + 1) + a^(1/2)*log((a + x - 2*a^(1/2)*(x + x^5)^(1
/2) + a*x^4)/(a - x + a*x^4))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**5-x)/(x**4+1)/(a*x**4+a-x)/(x**5+x)**(1/2),x)
[Out] Timed out
```

3.624 $\int \frac{(-1+x^3)\sqrt{-1+x^6}}{x^{10}}\,dx$

Optimal. Leaf size=49

$$\frac{(-2x^6-3x^3+2)\sqrt{x^6-1}}{18x^9}-\frac{1}{3}\tan^{-1}\left(\frac{x^3+1}{\sqrt{x^6-1}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1475, 807, 266, 47, 63, 203}

$$-\frac{\sqrt{x^6-1}}{6x^6}+\frac{1}{6}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{(x^6-1)^{3/2}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)*Sqrt[-1 + x^6])/x^10,x]
```

```
[Out] -1/6*Sqrt[-1 + x^6]/x^6 - (-1 + x^6)^(3/2)/(9*x^9) + ArcTan[Sqrt[-1 + x^6]]
/6
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^3\right)\sqrt{-1+x^6}}{x^{10}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(-1+x)\sqrt{-1+x^2}}{x^4}\,dx, x, x^3\right)\\
&= -\frac{\left(-1+x^6\right)^{3/2}}{9x^9}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{-1+x^2}}{x^3}\,dx, x, x^3\right)\\
&= -\frac{\left(-1+x^6\right)^{3/2}}{9x^9}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{\sqrt{-1+x}}{x^2}\,dx, x, x^6\right)\\
&= -\frac{\sqrt{-1+x^6}}{6x^6}-\frac{\left(-1+x^6\right)^{3/2}}{9x^9}+\frac{1}{12}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right)\\
&= -\frac{\sqrt{-1+x^6}}{6x^6}-\frac{\left(-1+x^6\right)^{3/2}}{9x^9}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^6}\right)\\
&= -\frac{\sqrt{-1+x^6}}{6x^6}-\frac{\left(-1+x^6\right)^{3/2}}{9x^9}+\frac{1}{6}\tan^{-1}\left(\sqrt{-1+x^6}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.33

$$-\frac{x^6+\sqrt{1-x^6}\,x^6\tanh^{-1}\left(\sqrt{1-x^6}\right)-1}{6x^6\sqrt{x^6-1}}-\frac{\left(x^6-1\right)^{3/2}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)*Sqrt[-1 + x^6])/x^10,x]
[Out] -1/9*(-1 + x^6)^(3/2)/x^9 - (-1 + x^6 + x^6*Sqrt[1 - x^6]*ArcTanh[Sqrt[1 -
x^6]])/(6*x^6*Sqrt[-1 + x^6])
```

IntegrateAlgebraic [A] time = 0.20, size = 51, normalized size = 1.04

$$\frac{\left(-2x^6-3x^3+2\right)\sqrt{x^6-1}}{18x^9}-\frac{1}{3}\tan^{-1}\left(\frac{\sqrt{x^6-1}}{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)*Sqrt[-1 + x^6])/x^10,x]
[Out] ((2 - 3*x^3 - 2*x^6)*Sqrt[-1 + x^6])/(18*x^9) - ArcTan[Sqrt[-1 + x^6]/(-1 +
 x^3)]/3
```

fricas [A] time = 0.42, size = 51, normalized size = 1.04

$$\frac{6\,x^9\arctan\left(-x^3+\sqrt{x^6-1}\right)-2\,x^9-\left(2\,x^6+3\,x^3-2\right)\sqrt{x^6-1}}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(6*x^9*arctan(-x^3 + sqrt(x^6 - 1)) - 2*x^9 - (2*x^6 + 3*x^3 - 2)*sqrt
(x^6 - 1))/x^9
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^6-1}\,(x^3-1)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^6 - 1)*(x^3 - 1)/x^10, x)
```

maple [A] time = 0.05, size = 42, normalized size = 0.86

$$-\frac{2x^{12}+3x^9-4x^6-3x^3+2}{18x^9\sqrt{x^6-1}}-\frac{\arcsin\left(\frac{1}{x^3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)*(x^6-1)^(1/2)/x^10,x)
```

```
[Out] -1/18*(2*x^12+3*x^9-4*x^6-3*x^3+2)/x^9/(x^6-1)^(1/2)-1/6*arcsin(1/x^3)
```

maxima [A] time = 0.53, size = 35, normalized size = 0.71

$$-\frac{\sqrt{x^6-1}}{6\,x^6}-\frac{\left(x^6-1\right)^{\frac{3}{2}}}{9\,x^9}+\frac{1}{6}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(x^6 - 1)/x^6 - 1/9*(x^6 - 1)^(3/2)/x^9 + 1/6*arctan(sqrt(x^6 - 1)
)
```

mupad [B] time = 1.10, size = 35, normalized size = 0.71

$$\frac{\mathrm{atan}\left(\sqrt{x^6-1}\right)}{6}-\frac{\sqrt{x^6-1}}{6\,x^6}-\frac{\left(x^6-1\right)^{3/2}}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)*(x^6 - 1)^(1/2))/x^10,x)
```

```
[Out] atan((x^6 - 1)^(1/2))/6 - (x^6 - 1)^(1/2)/(6*x^6) - (x^6 - 1)^(3/2)/(9*x^9)
```

sympy [A] time = 4.25, size = 49, normalized size = 1.00

$$-\frac{\begin{cases}\frac{\left(x^6-1\right)^{\frac{3}{2}}}{3x^9} & \text{for}\: x > -1 \wedge x < 1\end{cases}}{3}+\frac{\begin{cases}\frac{\mathrm{acos}\left(\frac{1}{x^3}\right)}{2}-\frac{\sqrt{1-\frac{1}{x^6}}}{2x^3} & \text{for}\: x > -1 \wedge x < 1\end{cases}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)*(x**6-1)**(1/2)/x**10,x)
```

```
[Out] -Piecewise(((x**6 - 1)**(3/2)/(3*x**9), (x > -1) & (x < 1)))/3 + Piecewise(
(acos(x**(-3))/2 - sqrt(1 - 1/x**6)/(2*x**3), (x > -1) & (x < 1)))/3
```

3.625 $\int \frac{x(6b+5ax)}{\sqrt[4]{bx^2+ax^3}\left(-b-ax+x^6\right)}\,dx$

Optimal. Leaf size=49

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^3+bx^2}}{x^2}\right)-2\tanh^{-1}\left(\frac{\left(ax^3+bx^2\right)^{3/4}}{ax+b}\right)$$

Rubi [F] time = 1.84, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(6b+5ax)}{\sqrt[4]{bx^2+ax^3}\left(-b-ax+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(6*b + 5*a*x))/((b*x^2 + a*x^3)^(1/4)*(-b - a*x + x^6)),x]
```

```
[Out] (-12*b*Sqrt[x]*(b + a*x)^(1/4)*Defer[Subst][Defer[Int][x^2/((b + a*x^2)^(1/
4)*(b + a*x^2 - x^12)), x], x, Sqrt[x]])/(b*x^2 + a*x^3)^(1/4) - (10*a*Sqrt
[x]*(b + a*x)^(1/4)*Defer[Subst][Defer[Int][x^4/((b + a*x^2)^(1/4)*(b + a*x
^2 - x^12)), x], x, Sqrt[x]])/(b*x^2 + a*x^3)^(1/4)
```

Rubi steps

$$\int \frac{x(6b+5ax)}{\sqrt[4]{bx^2+ax^3}\left(-b-ax+x^6\right)}\,dx=\frac{\left(\sqrt{x}\sqrt[4]{b+ax}\right)\int \frac{\sqrt{x}(6b+5ax)}{\sqrt[4]{b+ax}\left(-b-ax+x^6\right)}\,dx}{\sqrt[4]{bx^2+ax^3}}$$

$$=\frac{\left(2\sqrt{x}\sqrt[4]{b+ax}\right)\text{Subst}\left(\int \frac{x^2\left(6b+5ax^2\right)}{\sqrt[4]{b+ax^2}\left(-b-ax^2+x^{12}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{bx^2+ax^3}}$$

$$=\frac{\left(2\sqrt{x}\sqrt[4]{b+ax}\right)\text{Subst}\left(\int\left(-\frac{6bx^2}{\sqrt[4]{b+ax^2}\left(b+ax^2-x^{12}\right)}-\frac{5ax^4}{\sqrt[4]{b+ax^2}\left(b+ax^2-x^{12}\right)}\right)dx,x,\ldots}{\sqrt[4]{bx^2+ax^3}}$$

$$=-\frac{\left(10a\sqrt{x}\sqrt[4]{b+ax}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{b+ax^2}\left(b+ax^2-x^{12}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{bx^2+ax^3}}-\frac{\left(12b\sqrt{x}\sqrt[4]{b+\ldots}\right.}{\ldots}$$

Mathematica [C] time = 17.68, size = 57707, normalized size = 1177.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(6*b + 5*a*x))/((b*x^2 + a*x^3)^(1/4)*(-b - a*x + x^6)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.67, size = 49, normalized size = 1.00

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^3+bx^2}}{x^2}\right)-2\tanh^{-1}\left(\frac{\left(ax^3+bx^2\right)^{3/4}}{ax+b}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(6*b + 5*a*x))/((b*x^2 + a*x^3)^(1/4)*(-b - a*x + x^6
)),x]
```

```
[Out] 2*ArcTan[(b*x^2 + a*x^3)^(1/4)/x^2] - 2*ArcTanh[(b*x^2 + a*x^3)^(3/4)/(b +
a*x)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*a*x+6*b)/(a*x^3+b*x^2)^(1/4)/(x^6-a*x-b),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,ax + 6\,b)x}{\left(x^6 - ax - b\right)\left(ax^3 + bx^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*a*x+6*b)/(a*x^3+b*x^2)^(1/4)/(x^6-a*x-b),x, algorithm="giac"
)
```

```
[Out] integrate((5*a*x + 6*b)*x/((x^6 - a*x - b)*(a*x^3 + b*x^2)^(1/4)), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x\,(5ax + 6b)}{\left(a\,x^3 + b\,x^2\right)^{\frac{1}{4}}\left(x^6 - ax - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(5*a*x+6*b)/(a*x^3+b*x^2)^(1/4)/(x^6-a*x-b),x)
```

```
[Out] int(x*(5*a*x+6*b)/(a*x^3+b*x^2)^(1/4)/(x^6-a*x-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,ax + 6\,b)x}{\left(x^6 - ax - b\right)\left(ax^3 + bx^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*a*x+6*b)/(a*x^3+b*x^2)^(1/4)/(x^6-a*x-b),x, algorithm="maxim
a")
```

```
[Out] integrate((5*a*x + 6*b)*x/((x^6 - a*x - b)*(a*x^3 + b*x^2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x\,\left(6\,b + 5\,a\,x\right)}{{\left(a\,x^3 + b\,x^2\right)}^{1/4}\,\left(-x^6 + a\,x + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(6*b + 5*a*x))/((a*x^3 + b*x^2)^(1/4)*(b + a*x - x^6)),x)
[Out] int(-(x*(6*b + 5*a*x))/((a*x^3 + b*x^2)^(1/4)*(b + a*x - x^6)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(5ax+6b)}{\sqrt[4]{x^2(ax+b)}\left(-ax-b+x^6\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*a*x+6*b)/(a*x**3+b*x**2)**(1/4)/(x**6-a*x-b),x)
[Out] Integral(x*(5*a*x + 6*b)/((x**2*(a*x + b))**(1/4)*(-a*x - b + x**6)), x)
```

3.626 $\int \frac{\sqrt{-1+x^5}\left(2+3x^5\right)}{1-ax^4-2x^5+x^{10}}\,dx$

Optimal. Leaf size=49

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{x^5-1}}\right)}{\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{x^5-1}}\right)}{\sqrt[4]{a}}$$

Rubi [F] time = 0.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-1+x^5}\left(2+3x^5\right)}{1-ax^4-2x^5+x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-1 + x^5]*(2 + 3*x^5))/(1 - a*x^4 - 2*x^5 + x^10),x]
```

```
[Out] -2*Defer[Int][Sqrt[-1 + x^5]/(-1 + a*x^4 + 2*x^5 - x^10), x] + 3*Defer[Int]
[(x^5*Sqrt[-1 + x^5])/(1 - a*x^4 - 2*x^5 + x^10), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+x^5}\left(2+3x^5\right)}{1-ax^4-2x^5+x^{10}}\,dx &= \int \left(-\frac{2\sqrt{-1+x^5}}{-1+ax^4+2x^5-x^{10}}+\frac{3x^5\sqrt{-1+x^5}}{1-ax^4-2x^5+x^{10}}\right)dx\\ &= -\left(2\int \frac{\sqrt{-1+x^5}}{-1+ax^4+2x^5-x^{10}}\,dx\right)+3\int \frac{x^5\sqrt{-1+x^5}}{1-ax^4-2x^5+x^{10}}\,dx\end{aligned}$$

Mathematica [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-1+x^5}\left(2+3x^5\right)}{1-ax^4-2x^5+x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[-1 + x^5]*(2 + 3*x^5))/(1 - a*x^4 - 2*x^5 + x^10),x]
```

```
[Out] Integrate[(Sqrt[-1 + x^5]*(2 + 3*x^5))/(1 - a*x^4 - 2*x^5 + x^10), x]
```

IntegrateAlgebraic [A] time = 4.14, size = 49, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{x^5-1}}\right)}{\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{x^5-1}}\right)}{\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^5]*(2 + 3*x^5))/(1 - a*x^4 - 2*x^5 + x^10),
x]
```

```
[Out] -(ArcTan[(a^(1/4)*x)/Sqrt[-1 + x^5]]/a^(1/4)) - ArcTanh[(a^(1/4)*x)/Sqrt[-1
 + x^5]]/a^(1/4)
```

fricas [B] time = 1.00, size = 195, normalized size = 3.98

$$-\frac{\arctan\left(\frac{a^{\frac{1}{4}}x}{\sqrt{x^5-1}}\right)}{a^{\frac{1}{4}}}-\frac{\log\left(\frac{x^{10}+ax^4-2\,x^5+2\,\sqrt{x^5-1}\left(a^{\frac{3}{4}}x^3+\frac{ax^6-ax}{a^{\frac{3}{4}}}\right)+\frac{2\left(ax^7-ax^2\right)}{\sqrt{a}}+1}{x^{10}-ax^4-2\,x^5+1}\right)}{4\,a^{\frac{1}{4}}}+\frac{\log\left(\frac{x^{10}+ax^4-2\,x^5-2\,\sqrt{x^5-1}\left(a^{\frac{3}{4}}x^3+\frac{ax^6-ax}{a^{\frac{3}{4}}}\right)+\frac{2\left(ax^7-ax^2\right)}{\sqrt{a}}+1}{x^{10}-ax^4-2\,x^5+1}\right)}{4\,a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(1/2)*(3*x^5+2)/(x^10-a*x^4-2*x^5+1),x, algorithm="fricas
")
```

```
[Out] -arctan(a^(1/4)*x/sqrt(x^5 - 1))/a^(1/4) - 1/4*log((x^10 + a*x^4 - 2*x^5 +
2*sqrt(x^5 - 1)*(a^(3/4)*x^3 + (a*x^6 - a*x)/a^(3/4)) + 2*(a*x^7 - a*x^2)/s
qrt(a) + 1)/(x^10 - a*x^4 - 2*x^5 + 1))/a^(1/4) + 1/4*log((x^10 + a*x^4 - 2
*x^5 - 2*sqrt(x^5 - 1)*(a^(3/4)*x^3 + (a*x^6 - a*x)/a^(3/4)) + 2*(a*x^7 - a
*x^2)/sqrt(a) + 1)/(x^10 - a*x^4 - 2*x^5 + 1))/a^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^5+2\right)\sqrt{x^5-1}}{x^{10}-ax^4-2\,x^5+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(1/2)*(3*x^5+2)/(x^10-a*x^4-2*x^5+1),x, algorithm="giac")
```

```
[Out] integrate((3*x^5 + 2)*sqrt(x^5 - 1)/(x^10 - a*x^4 - 2*x^5 + 1), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^5-1}\,\left(3x^5+2\right)}{x^{10}-a\,x^4-2x^5+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)^(1/2)*(3*x^5+2)/(x^10-a*x^4-2*x^5+1),x)
```

```
[Out] int((x^5-1)^(1/2)*(3*x^5+2)/(x^10-a*x^4-2*x^5+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^5+2\right)\sqrt{x^5-1}}{x^{10}-ax^4-2\,x^5+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(1/2)*(3*x^5+2)/(x^10-a*x^4-2*x^5+1),x, algorithm="maxima
")
```

```
[Out] integrate((3*x^5 + 2)*sqrt(x^5 - 1)/(x^10 - a*x^4 - 2*x^5 + 1), x)
```

mupad [B] time = 8.02, size = 98, normalized size = 2.00

$$\frac{\ln\left(\frac{x^5+\sqrt{a}\,x^2-2\,a^{1/4}\,x\,\sqrt{x^5-1}-1}{\sqrt{a}\,x^2-x^5+1}\right)}{2\,a^{1/4}}+\frac{\ln\left(\frac{x^5-\sqrt{a}\,x^2-1+a^{1/4}\,x\,\sqrt{x^5-1}\,2{}\mathrm{i}}{x^5+\sqrt{a}\,x^2-1}\right)\,1{}\mathrm{i}}{2\,a^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^5 - 1)^(1/2)*(3*x^5 + 2))/(a*x^4 + 2*x^5 - x^10 - 1),x)
```

```
[Out] log((x^5 + a^(1/2)*x^2 - 2*a^(1/4)*x*(x^5 - 1)^(1/2) - 1)/(a^(1/2)*x^2 - x^
5 + 1))/(2*a^(1/4)) + (log((x^5 - a^(1/2)*x^2 + a^(1/4)*x*(x^5 - 1)^(1/2)*2
i - 1)/(x^5 + a^(1/2)*x^2 - 1))*1i)/(2*a^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)\left(x^4+x^3+x^2+x+1\right)}\,\left(3x^5+2\right)}{-ax^4+x^{10}-2x^5+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)**(1/2)*(3*x**5+2)/(x**10-a*x**4-2*x**5+1),x)
```

```
[Out] Integral(sqrt((x - 1)*(x**4 + x**3 + x**2 + x + 1))*(3*x**5 + 2)/(-a*x**4 +
 x**10 - 2*x**5 + 1), x)
```

3.627 $\int \frac{\sqrt{-1+x^5}\left(2+3x^5\right)}{a-x^4-2ax^5+ax^{10}}\,dx$

Optimal. Leaf size=49

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{x^5-1}}\right)}{a^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{x^5-1}}\right)}{a^{3/4}}$$

Rubi [F] time = 0.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-1+x^5}\left(2+3x^5\right)}{a-x^4-2ax^5+ax^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-1 + x^5]*(2 + 3*x^5))/(a - x^4 - 2*a*x^5 + a*x^10),x]
```

```
[Out] 2*Defer[Int][Sqrt[-1 + x^5]/(a - x^4 - 2*a*x^5 + a*x^10), x] + 3*Defer[Int]
[(x^5*Sqrt[-1 + x^5])/(a - x^4 - 2*a*x^5 + a*x^10), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+x^5}\left(2+3x^5\right)}{a-x^4-2ax^5+ax^{10}}\,dx &= \int \left(\frac{2\sqrt{-1+x^5}}{a-x^4-2ax^5+ax^{10}}+\frac{3x^5\sqrt{-1+x^5}}{a-x^4-2ax^5+ax^{10}}\right)dx\\ &= 2\int \frac{\sqrt{-1+x^5}}{a-x^4-2ax^5+ax^{10}}\,dx+3\int \frac{x^5\sqrt{-1+x^5}}{a-x^4-2ax^5+ax^{10}}\,dx\end{aligned}$$

Mathematica [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-1+x^5}\left(2+3x^5\right)}{a-x^4-2ax^5+ax^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[-1 + x^5]*(2 + 3*x^5))/(a - x^4 - 2*a*x^5 + a*x^10),x]
```

```
[Out] Integrate[(Sqrt[-1 + x^5]*(2 + 3*x^5))/(a - x^4 - 2*a*x^5 + a*x^10), x]
```

IntegrateAlgebraic [A] time = 7.57, size = 49, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{x^5-1}}\right)}{a^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{x^5-1}}\right)}{a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^5]*(2 + 3*x^5))/(a - x^4 - 2*a*x^5 + a*x^10
),x]
```

```
[Out] -(ArcTan[x/(a^(1/4)*Sqrt[-1 + x^5])]/a^(3/4)) - ArcTanh[x/(a^(1/4)*Sqrt[-1
+ x^5])]/a^(3/4)
```

fricas [B] time = 0.96, size = 242, normalized size = 4.94

$$-\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2\left(\frac{1}{a^3}\right)^{\frac{3}{4}}x}{\sqrt{x^5-1}}\right)-\frac{1}{4}\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{ax^{10}-2\,ax^5+x^4+2\sqrt{x^5-1}\left(a\left(\frac{1}{a^3}\right)^{\frac{1}{4}}x^3+\left(a^3x^6-a^3x\right)\left(\frac{1}{a^3}\right)^{\frac{3}{4}}\right)+2\left(a^2x^7-a^2x^2\right)\sqrt{\frac{1}{a^3}}+a}{ax^{10}-2\,ax^5-x^4+a}\right)+\frac{1}{4}\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{ax^{10}-2\,ax^5+x^4-2\sqrt{x^5-1}\left(a\left(\frac{1}{a^3}\right)^{\frac{1}{4}}x^3+\left(a^3x^6-a^3x\right)\left(\frac{1}{a^3}\right)^{\frac{3}{4}}\right)+2\left(a^2x^7-a^2x^2\right)\sqrt{\frac{1}{a^3}}+a}{ax^{10}-2\,ax^5-x^4+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(1/2)*(3*x^5+2)/(a*x^10-2*a*x^5-x^4+a),x, algorithm="fric
as")
```

```
[Out] -(a^(-3))^(1/4)*arctan(a^2*(a^(-3))^(3/4)*x/sqrt(x^5 - 1)) - 1/4*(a^(-3))^(
1/4)*log((a*x^10 - 2*a*x^5 + x^4 + 2*sqrt(x^5 - 1)*(a*(a^(-3))^(1/4)*x^3 +
(a^3*x^6 - a^3*x)*(a^(-3))^(3/4)) + 2*(a^2*x^7 - a^2*x^2)*sqrt(a^(-3)) + a)
/(a*x^10 - 2*a*x^5 - x^4 + a)) + 1/4*(a^(-3))^(1/4)*log((a*x^10 - 2*a*x^5 +
 x^4 - 2*sqrt(x^5 - 1)*(a*(a^(-3))^(1/4)*x^3 + (a^3*x^6 - a^3*x)*(a^(-3))^(
3/4)) + 2*(a^2*x^7 - a^2*x^2)*sqrt(a^(-3)) + a)/(a*x^10 - 2*a*x^5 - x^4 + a
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^{5}+2\right)\sqrt{x^{5}-1}}{ax^{10}-2\,ax^{5}-x^{4}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(1/2)*(3*x^5+2)/(a*x^10-2*a*x^5-x^4+a),x, algorithm="giac
")
```

```
[Out] integrate((3*x^5 + 2)*sqrt(x^5 - 1)/(a*x^10 - 2*a*x^5 - x^4 + a), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{5}-1}\,\left(3x^{5}+2\right)}{a\,x^{10}-2a\,x^{5}-x^{4}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)^(1/2)*(3*x^5+2)/(a*x^10-2*a*x^5-x^4+a),x)
```

```
[Out] int((x^5-1)^(1/2)*(3*x^5+2)/(a*x^10-2*a*x^5-x^4+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^{5}+2\right)\sqrt{x^{5}-1}}{ax^{10}-2\,ax^{5}-x^{4}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(1/2)*(3*x^5+2)/(a*x^10-2*a*x^5-x^4+a),x, algorithm="maxi
ma")
```

```
[Out] integrate((3*x^5 + 2)*sqrt(x^5 - 1)/(a*x^10 - 2*a*x^5 - x^4 + a), x)
```

mupad [B] time = 14.91, size = 311, normalized size = 6.35

$$\frac{\ln\left(\frac{\left(x\,\sqrt{a^3}+a^2\,x^4-2\,a\,\sqrt{x^5-1}\,{\left(a^3\right)}^{1/4}\right)\,\left(2\,x^6\,\sqrt{a^3}-a\,x^3-3\,x\,\sqrt{a^3}+a^2\,x^4-a^2\,x^9+2\,a\,\sqrt{x^5-1}\,{\left(a^3\right)}^{1/4}\right)}{\left(x^2\,\sqrt{a^3}+a^2-a^2\,x^5\right)\,\left(4\,\sqrt{a^3}-2\,x^5\,\sqrt{a^3}+a\,x^2+a^2\,x^8\right)}\right)}{2\,{\left(a^3\right)}^{1/4}}+\frac{\ln\left(\frac{\left(2\,a\,\sqrt{x^5-1}\,{\left(a^3\right)}^{1/4}+x\,\sqrt{a^3}\,1{}\mathrm{i}-a^2\,x^4\,1{}\mathrm{i}\right)\,\left(-2\,a\,\sqrt{x^5-1}\,{\left(a^3\right)}^{1/4}+x^6\,\sqrt{a^3}\,2{}\mathrm{i}+a\,x^3\,1{}\mathrm{i}-x\,\sqrt{a^3}\,3{}\mathrm{i}-a^2\,x^4\,1{}\mathrm{i}+a^2\,x^9\,1{}\mathrm{i}\right)}{\left(x^2\,\sqrt{a^3}-a^2+a^2\,x^5\right)\,\left(2\,x^5\,\sqrt{a^3}-4\,\sqrt{a^3}+a\,x^2+a^2\,x^8\right)}\right)\,1{}\mathrm{i}}{2\,{\left(a^3\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 - 1)^(1/2)*(3*x^5 + 2))/(a - 2*a*x^5 + a*x^10 - x^4),x)
```

```
[Out] (log(((x*(a^3)^(1/2)*1i - a^2*x^4*1i + 2*a*(x^5 - 1)^(1/2)*(a^3)^(1/4))*(x^
6*(a^3)^(1/2)*2i + a*x^3*1i - x*(a^3)^(1/2)*3i - a^2*x^4*1i + a^2*x^9*1i -
2*a*(x^5 - 1)^(1/2)*(a^3)^(1/4)))/((x^2*(a^3)^(1/2) - a^2 + a^2*x^5)*(2*x^5
*(a^3)^(1/2) - 4*(a^3)^(1/2) + a*x^2 + a^2*x^8)))*1i)/(2*(a^3)^(1/4)) + log
(((x*(a^3)^(1/2) + a^2*x^4 - 2*a*(x^5 - 1)^(1/2)*(a^3)^(1/4))*(2*x^6*(a^3)^
```

```
(1/2) - a*x^3 - 3*x*(a^3)^(1/2) + a^2*x^4 - a^2*x^9 + 2*a*(x^5 - 1)^(1/2)*(
a^3)^(1/4)))/((x^2*(a^3)^(1/2) + a^2 - a^2*x^5)*(4*(a^3)^(1/2) - 2*x^5*(a^3
)^(1/2) + a*x^2 + a^2*x^8)))/(2*(a^3)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)\left(x^4+x^3+x^2+x+1\right)}\left(3x^5+2\right)}{ax^{10}-2ax^5+a-x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)**(1/2)*(3*x**5+2)/(a*x**10-2*a*x**5-x**4+a),x)
```

```
[Out] Integral(sqrt((x - 1)*(x**4 + x**3 + x**2 + x + 1))*(3*x**5 + 2)/(a*x**10 -
 2*a*x**5 + a - x**4), x)
```

3.628 $\int \frac{\sqrt{1+x^5}\left(-2+3x^5\right)}{a-x^4+2ax^5+ax^{10}}\,dx$

Optimal. Leaf size=49

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{x^5+1}}\right)}{a^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{x^5+1}}\right)}{a^{3/4}}$$

Rubi [F] time = 0.55, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+x^5}\left(-2+3x^5\right)}{a-x^4+2ax^5+ax^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[1 + x^5]*(-2 + 3*x^5))/(a - x^4 + 2*a*x^5 + a*x^10),x]
```

```
[Out] -2*Defer[Int][Sqrt[1 + x^5]/(a - x^4 + 2*a*x^5 + a*x^10), x] + 3*Defer[Int]
[(x^5*Sqrt[1 + x^5])/(a - x^4 + 2*a*x^5 + a*x^10), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^5}\left(-2+3x^5\right)}{a-x^4+2ax^5+ax^{10}}\,dx &= \int \left(-\frac{2\sqrt{1+x^5}}{a-x^4+2ax^5+ax^{10}}+\frac{3x^5\sqrt{1+x^5}}{a-x^4+2ax^5+ax^{10}}\right)dx\\ &= -\left(2\int \frac{\sqrt{1+x^5}}{a-x^4+2ax^5+ax^{10}}\,dx\right)+3\int \frac{x^5\sqrt{1+x^5}}{a-x^4+2ax^5+ax^{10}}\,dx\end{aligned}$$

Mathematica [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+x^5}\left(-2+3x^5\right)}{a-x^4+2ax^5+ax^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[1 + x^5]*(-2 + 3*x^5))/(a - x^4 + 2*a*x^5 + a*x^10),x]
```

```
[Out] Integrate[(Sqrt[1 + x^5]*(-2 + 3*x^5))/(a - x^4 + 2*a*x^5 + a*x^10), x]
```

IntegrateAlgebraic [A] time = 7.54, size = 49, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{x^5+1}}\right)}{a^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{x^5+1}}\right)}{a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x^5]*(-2 + 3*x^5))/(a - x^4 + 2*a*x^5 + a*x^10
),x]
```

```
[Out] -(ArcTan[x/(a^(1/4)*Sqrt[1 + x^5])]/a^(3/4)) - ArcTanh[x/(a^(1/4)*Sqrt[1 +
x^5])]/a^(3/4)
```

fricas [B] time = 0.89, size = 238, normalized size = 4.86

$$-\frac{1}{a^3}^{\frac{1}{4}}\arctan\left(\frac{a^2\frac{1}{a^3}^{\frac{3}{4}}x}{\sqrt{x^5+1}}\right)-\frac{1}{4}\frac{1}{a^3}^{\frac{1}{4}}\log\left(\frac{ax^{10}+2\,ax^5+x^4+2\sqrt{x^5+1}\left(a\frac{1}{a^3}^{\frac{1}{4}}x^3+\left(a^3x^6+a^3x\right)\frac{1}{a^3}^{\frac{3}{4}}\right)+2\left(a^2x^7+a^2x^2\right)\sqrt{\frac{1}{a^3}}+a}{ax^{10}+2\,ax^5-x^4+a}\right)+\frac{1}{4}\frac{1}{a^3}^{\frac{1}{4}}\log\left(\frac{ax^{10}+2\,ax^5+x^4-2\sqrt{x^5+1}\left(a\frac{1}{a^3}^{\frac{1}{4}}x^3+\left(a^3x^6+a^3x\right)\frac{1}{a^3}^{\frac{3}{4}}\right)+2\left(a^2x^7+a^2x^2\right)\sqrt{\frac{1}{a^3}}+a}{ax^{10}+2\,ax^5-x^4+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(1/2)*(3*x^5-2)/(a*x^10+2*a*x^5-x^4+a),x, algorithm="fric
as")
```

```
[Out] -(a^(-3))^(1/4)*arctan(a^2*(a^(-3))^(3/4)*x/sqrt(x^5 + 1)) - 1/4*(a^(-3))^(
1/4)*log((a*x^10 + 2*a*x^5 + x^4 + 2*sqrt(x^5 + 1)*(a*(a^(-3))^(1/4)*x^3 +
(a^3*x^6 + a^3*x)*(a^(-3))^(3/4)) + 2*(a^2*x^7 + a^2*x^2)*sqrt(a^(-3)) + a)
/(a*x^10 + 2*a*x^5 - x^4 + a)) + 1/4*(a^(-3))^(1/4)*log((a*x^10 + 2*a*x^5 +
 x^4 - 2*sqrt(x^5 + 1)*(a*(a^(-3))^(1/4)*x^3 + (a^3*x^6 + a^3*x)*(a^(-3))^(
3/4)) + 2*(a^2*x^7 + a^2*x^2)*sqrt(a^(-3)) + a)/(a*x^10 + 2*a*x^5 - x^4 + a
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3 x^{5}-2\right) \sqrt{x^{5}+1}}{a x^{10}+2 a x^{5}-x^{4}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(1/2)*(3*x^5-2)/(a*x^10+2*a*x^5-x^4+a),x, algorithm="giac
")
```

```
[Out] integrate((3*x^5 - 2)*sqrt(x^5 + 1)/(a*x^10 + 2*a*x^5 - x^4 + a), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{5}+1}\,\left(3 x^{5}-2\right)}{a\,x^{10}+2 a\,x^{5}-x^{4}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)^(1/2)*(3*x^5-2)/(a*x^10+2*a*x^5-x^4+a),x)
```

```
[Out] int((x^5+1)^(1/2)*(3*x^5-2)/(a*x^10+2*a*x^5-x^4+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3 x^{5}-2\right) \sqrt{x^{5}+1}}{a x^{10}+2 a x^{5}-x^{4}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(1/2)*(3*x^5-2)/(a*x^10+2*a*x^5-x^4+a),x, algorithm="maxi
ma")
```

```
[Out] integrate((3*x^5 - 2)*sqrt(x^5 + 1)/(a*x^10 + 2*a*x^5 - x^4 + a), x)
```

mupad [B] time = 14.59, size = 309, normalized size = 6.31

$$\frac{\ln\left(\frac{\left(x\,\sqrt{a^3}+a^2\,x^4-2\,a\,\sqrt{x^5+1}\,{\left(a^3\right)}^{1/4}\right)\left(a\,x^3-2\,x^6\,\sqrt{a^3}-3\,x\,\sqrt{a^3}+a^2\,x^4+a^2\,x^9+2\,a\,\sqrt{x^5+1}\,{\left(a^3\right)}^{1/4}\right)}{\left(a^2-x^2\,\sqrt{a^3}+a^2\,x^5\right)\left(4\,\sqrt{a^3}+2\,x^5\,\sqrt{a^3}-a\,x^2-a^2\,x^8\right)}\right)}{2\,{\left(a^3\right)}^{1/4}}+\frac{\ln\left(\frac{\left(2\,a\,\sqrt{x^5+1}\,{\left(a^3\right)}^{1/4}+x\,\sqrt{a^3}\,1{}\mathrm{i}-a^2\,x^4\,1{}\mathrm{i}\right)\left(2\,a\,\sqrt{x^5+1}\,{\left(a^3\right)}^{1/4}+x^6\,\sqrt{a^3}\,2{}\mathrm{i}+a\,x^3\,1{}\mathrm{i}+x\,\sqrt{a^3}\,3{}\mathrm{i}+a^2\,x^4\,1{}\mathrm{i}+a^2\,x^9\,1{}\mathrm{i}\right)}{\left(x^2\,\sqrt{a^3}+a^2+a^2\,x^5\right)\left(4\,\sqrt{a^3}+2\,x^5\,\sqrt{a^3}+a\,x^2+a^2\,x^8\right)}\right)\,1{}\mathrm{i}}{2\,{\left(a^3\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 + 1)^(1/2)*(3*x^5 - 2))/(a + 2*a*x^5 + a*x^10 - x^4),x)
```

```
[Out] log(((x*(a^3)^(1/2) + a^2*x^4 - 2*a*(x^5 + 1)^(1/2)*(a^3)^(1/4))*(a*x^3 - 2
*x^6*(a^3)^(1/2) - 3*x*(a^3)^(1/2) + a^2*x^4 + a^2*x^9 + 2*a*(x^5 + 1)^(1/2
)*(a^3)^(1/4)))/((a^2 - x^2*(a^3)^(1/2) + a^2*x^5)*(4*(a^3)^(1/2) + 2*x^5*(
a^3)^(1/2) - a*x^2 - a^2*x^8)))/(2*(a^3)^(1/4)) + (log(((x*(a^3)^(1/2)*1i -
 a^2*x^4*1i + 2*a*(x^5 + 1)^(1/2)*(a^3)^(1/4))*(x^6*(a^3)^(1/2)*2i + a*x^3*
```

```
1i + x*(a^3)^(1/2)*3i + a^2*x^4*1i + a^2*x^9*1i + 2*a*(x^5 + 1)^(1/2)*(a^3)
^(1/4)))/((x^2*(a^3)^(1/2) + a^2 + a^2*x^5)*(4*(a^3)^(1/2) + 2*x^5*(a^3)^(1
/2) + a*x^2 + a^2*x^8)))*1i)/(2*(a^3)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}\,\left(3x^5-2\right)}{ax^{10}+2ax^5+a-x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+1)**(1/2)*(3*x**5-2)/(a*x**10+2*a*x**5-x**4+a),x)
```

```
[Out] Integral(sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1))*(3*x**5 - 2)/(a*x**10 +
 2*a*x**5 + a - x**4), x)
```

3.629 $\int \frac{1}{\sqrt{1+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=49

$$\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}\right)$$

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{1+\sqrt{1+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/Sqrt[1 + Sqrt[1 + x^2]],x]
[Out] Defer[Int][1/Sqrt[1 + Sqrt[1 + x^2]], x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\sqrt{1+x^2}}}\,dx=\int \frac{1}{\sqrt{1+\sqrt{1+x^2}}}\,dx$$

Mathematica [C] time = 0.22, size = 106, normalized size = 2.16

$$\frac{\sqrt{\sqrt{x^2+1}+1}\left(-2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}-\frac{\sqrt{x^2+1}}{2}\right)+4\sqrt{x^2+1}-\sqrt{2}\sqrt{\sqrt{x^2+1}-1}\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}-1}}{\sqrt{2}}\right)-2\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Sqrt[1 + x^2]],x]
[Out] (Sqrt[1 + Sqrt[1 + x^2]]*(-2 + 4*Sqrt[1 + x^2] - Sqrt[2]*Sqrt[-1 + Sqrt[1 +
 x^2]]*ArcTan[Sqrt[-1 + Sqrt[1 + x^2]]/Sqrt[2]] - 2*Hypergeometric2F1[-1/2,
 1, 1/2, 1/2 - Sqrt[1 + x^2]/2]))/(2*x)
```

IntegrateAlgebraic [A] time = 0.11, size = 49, normalized size = 1.00

$$\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[1 + Sqrt[1 + x^2]],x]
[Out] (2*x)/Sqrt[1 + Sqrt[1 + x^2]] - Sqrt[2]*ArcTan[x/(Sqrt[2]*Sqrt[1 + Sqrt[1 +
 x^2]])]
```

fricas [A] time = 1.87, size = 51, normalized size = 1.04

$$\frac{\sqrt{2}\,x\arctan\left(\frac{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}{x}\right)+2\sqrt{\sqrt{x^2+1}+1}\left(\sqrt{x^2+1}-1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*x*arctan(sqrt(2)*sqrt(sqrt(x^2 + 1) + 1)/x) + 2*sqrt(sqrt(x^2 + 1)
 + 1)*(sqrt(x^2 + 1) - 1))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(sqrt(x^2 + 1) + 1), x)
```

maple [C] time = 0.03, size = 20, normalized size = 0.41

$$\frac{\sqrt{2}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2},\frac{3}{2}\right],-x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*x*hypergeom([1/4,1/2,3/4],[3/2,3/2],-x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(sqrt(x^2 + 1) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int(1/((x^2 + 1)^(1/2) + 1)^(1/2), x)
```

sympy [C] time = 0.73, size = 32, normalized size = 0.65

$$\frac{x\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_{3}F_{2}\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2},\frac{3}{4}\\\frac{3}{2},\frac{3}{2}\end{matrix}\right|x^2e^{i\pi}\right)}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] x*gamma(1/4)*gamma(3/4)*hyper((1/4, 1/2, 3/4), (3/2, 3/2), x**2*exp_polar(I
*pi))/(2*pi)
```

3.630 $\int x^6\sqrt{-x+x^4}\,dx$

Optimal. Leaf size=50

$$\frac{1}{72}\sqrt{x^4-x}\left(8x^7-2x^4-3x\right)-\frac{1}{24}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)$$

Rubi [A] time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.46, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2021, 2024, 2029, 206}

$$-\frac{1}{36}\sqrt{x^4-x}\,x^4-\frac{1}{24}\sqrt{x^4-x}\,x+\frac{1}{9}\sqrt{x^4-x}\,x^7-\frac{1}{24}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*Sqrt[-x + x^4],x]
```

```
[Out] -1/24*(x*Sqrt[-x + x^4]) - (x^4*Sqrt[-x + x^4])/36 + (x^7*Sqrt[-x + x^4])/9
 - ArcTanh[x^2/Sqrt[-x + x^4]]/24
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int x^6\sqrt{-x+x^4}\,dx &= \frac{1}{9}x^7\sqrt{-x+x^4} - \frac{1}{6}\int \frac{x^7}{\sqrt{-x+x^4}}\,dx \\
&= -\frac{1}{36}x^4\sqrt{-x+x^4} + \frac{1}{9}x^7\sqrt{-x+x^4} - \frac{1}{8}\int \frac{x^4}{\sqrt{-x+x^4}}\,dx \\
&= -\frac{1}{24}x\sqrt{-x+x^4} - \frac{1}{36}x^4\sqrt{-x+x^4} + \frac{1}{9}x^7\sqrt{-x+x^4} - \frac{1}{16}\int \frac{x}{\sqrt{-x+x^4}}\,dx \\
&= -\frac{1}{24}x\sqrt{-x+x^4} - \frac{1}{36}x^4\sqrt{-x+x^4} + \frac{1}{9}x^7\sqrt{-x+x^4} - \frac{1}{24}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^2}{\sqrt{-x+x^4}}\right) \\
&= -\frac{1}{24}x\sqrt{-x+x^4} - \frac{1}{36}x^4\sqrt{-x+x^4} + \frac{1}{9}x^7\sqrt{-x+x^4} - \frac{1}{24}\tanh^{-1}\left(\frac{x^2}{\sqrt{-x+x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 1.16

$$\frac{\sqrt{x\left(x^3-1\right)}\left(\frac{3\sin^{-1}\left(x^{3/2}\right)}{\sqrt{1-x^3}}+\left(8x^6-2x^3-3\right)x^{3/2}\right)}{72\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*Sqrt[-x + x^4],x]
```

```
[Out] (Sqrt[x*(-1 + x^3)]*(x^(3/2)*(-3 - 2*x^3 + 8*x^6) + (3*ArcSin[x^(3/2)])/Sqr
t[1 - x^3]))/(72*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.44, size = 50, normalized size = 1.00

$$\frac{1}{72}\sqrt{x^4-x}\left(8x^7-2x^4-3x\right)-\frac{1}{24}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6*Sqrt[-x + x^4],x]
```

```
[Out] (Sqrt[-x + x^4]*(-3*x - 2*x^4 + 8*x^7))/72 - ArcTanh[x^2/Sqrt[-x + x^4]]/24
```

fricas [A] time = 0.66, size = 48, normalized size = 0.96

$$\frac{1}{72}\left(8\,x^7-2\,x^4-3\,x\right)\sqrt{x^4-x}+\frac{1}{48}\,\log\left(2\,x^3-2\,\sqrt{x^4-x}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^4-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/72*(8*x^7 - 2*x^4 - 3*x)*sqrt(x^4 - x) + 1/48*log(2*x^3 - 2*sqrt(x^4 - x)
*x - 1)
```

giac [A] time = 0.20, size = 56, normalized size = 1.12

$$\frac{1}{72}\left(2\left(4\,x^3-1\right)x^3-3\right)\sqrt{x^4-x}\,x-\frac{1}{48}\,\log\left(\sqrt{-\frac{1}{x^3}+1}+1\right)+\frac{1}{48}\,\log\left(\left|\sqrt{-\frac{1}{x^3}+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^4-x)^(1/2),x, algorithm="giac")
```

[Out] 1/72*(2*(4*x^3 - 1)*x^3 - 3)*sqrt(x^4 - x)*x - 1/48*log(sqrt(-1/x^3 + 1) + 1) + 1/48*log(abs(sqrt(-1/x^3 + 1) - 1))

maple [C] time = 0.19, size = 329, normalized size = 6.58

$$\frac{x^{7}\sqrt{x^{4}-x}}{9}-\frac{x^{4}\sqrt{x^{4}-x}}{36}-\frac{x\sqrt{x^{4}-x}}{24}-\frac{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}}\,(-1+x)^{2}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(-1+x)}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)-\mathrm{EllipticPi}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\frac{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{8\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(-1+x)\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(x^4-x)^(1/2),x)

[Out] 1/9*x^7*(x^4-x)^(1/2)-1/36*x^4*(x^4-x)^(1/2)-1/24*x*(x^4-x)^(1/2)-1/8*(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2))/(-1+x))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2)/(-3/2+1/2*I*3^(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/2)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-EllipticPi(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),(-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4 - x}\, x^6\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(x^4-x)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x^4 - x)*x^6, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^6\, \sqrt{x^4 - x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(x^4 - x)^(1/2),x)

[Out] int(x^6*(x^4 - x)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{6} \sqrt{x \left(x - 1\right) \left(x^{2} + x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(x**4-x)**(1/2),x)

[Out] Integral(x**6*sqrt(x*(x - 1)*(x**2 + x + 1)), x)

3.631 $\int \frac{(-1+x^2)\sqrt[4]{-x^3+x^4}}{x^8}\,dx$

Optimal. Leaf size=50

$$\frac{4\sqrt[4]{x^4-x^3}\left(5248x^6+1312x^5+820x^4+615x^3-21255x^2-663x+13923\right)}{348075x^7}$$

Rubi [B] time = 0.31, antiderivative size = 121, normalized size of antiderivative = 2.42, number of steps used = 12, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2052, 2016, 2014}

$$-\frac{4\left(x^4-x^3\right)^{5/4}}{25x^{10}}-\frac{16\left(x^4-x^3\right)^{5/4}}{105x^9}+\frac{164\left(x^4-x^3\right)^{5/4}}{1785x^8}+\frac{656\left(x^4-x^3\right)^{5/4}}{7735x^7}+\frac{5248\left(x^4-x^3\right)^{5/4}}{69615x^6}+\frac{20992\left(x^4-x^3\right)^{5/4}}{348075x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^2)*(-x^3 + x^4)^(1/4))/x^8,x]
```

```
[Out] (-4*(-x^3 + x^4)^(5/4))/(25*x^10) - (16*(-x^3 + x^4)^(5/4))/(105*x^9) + (16
4*(-x^3 + x^4)^(5/4))/(1785*x^8) + (656*(-x^3 + x^4)^(5/4))/(7735*x^7) + (5
248*(-x^3 + x^4)^(5/4))/(69615*x^6) + (20992*(-x^3 + x^4)^(5/4))/(348075*x^
5)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-1+x^2\right)\sqrt[4]{-x^3+x^4}}{x^8}\,dx &= \int \left(-\frac{\sqrt[4]{-x^3+x^4}}{x^8}+\frac{\sqrt[4]{-x^3+x^4}}{x^6}\right)dx \\
&= -\int \frac{\sqrt[4]{-x^3+x^4}}{x^8}\,dx+\int \frac{\sqrt[4]{-x^3+x^4}}{x^6}\,dx \\
&= -\frac{4\left(-x^3+x^4\right)^{5/4}}{25x^{10}}+\frac{4\left(-x^3+x^4\right)^{5/4}}{17x^8}+\frac{12}{17}\int \frac{\sqrt[4]{-x^3+x^4}}{x^5}\,dx-\frac{4}{5}\int \frac{\sqrt[4]{-x^3+x^4}}{x^7} \\
&= -\frac{4\left(-x^3+x^4\right)^{5/4}}{25x^{10}}-\frac{16\left(-x^3+x^4\right)^{5/4}}{105x^9}+\frac{4\left(-x^3+x^4\right)^{5/4}}{17x^8}+\frac{48\left(-x^3+x^4\right)^{5/4}}{221x^7}+ \\
&= -\frac{4\left(-x^3+x^4\right)^{5/4}}{25x^{10}}-\frac{16\left(-x^3+x^4\right)^{5/4}}{105x^9}+\frac{164\left(-x^3+x^4\right)^{5/4}}{1785x^8}+\frac{48\left(-x^3+x^4\right)^{5/4}}{221x^7}+ \\
&= -\frac{4\left(-x^3+x^4\right)^{5/4}}{25x^{10}}-\frac{16\left(-x^3+x^4\right)^{5/4}}{105x^9}+\frac{164\left(-x^3+x^4\right)^{5/4}}{1785x^8}+\frac{656\left(-x^3+x^4\right)^{5/4}}{7735x^7}+ \\
&= -\frac{4\left(-x^3+x^4\right)^{5/4}}{25x^{10}}-\frac{16\left(-x^3+x^4\right)^{5/4}}{105x^9}+\frac{164\left(-x^3+x^4\right)^{5/4}}{1785x^8}+\frac{656\left(-x^3+x^4\right)^{5/4}}{7735x^7}+ \\
&= -\frac{4\left(-x^3+x^4\right)^{5/4}}{25x^{10}}-\frac{16\left(-x^3+x^4\right)^{5/4}}{105x^9}+\frac{164\left(-x^3+x^4\right)^{5/4}}{1785x^8}+\frac{656\left(-x^3+x^4\right)^{5/4}}{7735x^7}+
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.76

$$\frac{4\left((x-1)x^3\right)^{9/4}\left(5248x^4+11808x^3+19188x^2+27183x+13923\right)}{348075x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)*(-x^3 + x^4)^(1/4))/x^8,x]
```

```
[Out] (4*((-1 + x)*x^3)^(9/4)*(13923 + 27183*x + 19188*x^2 + 11808*x^3 + 5248*x^4
))/(348075*x^13)
```

IntegrateAlgebraic [A] time = 0.28, size = 50, normalized size = 1.00

$$\frac{4\sqrt[4]{x^4-x^3}\left(5248x^6+1312x^5+820x^4+615x^3-21255x^2-663x+13923\right)}{348075x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*(-x^3 + x^4)^(1/4))/x^8,x]
```

```
[Out] (4*(-x^3 + x^4)^(1/4)*(13923 - 663*x - 21255*x^2 + 615*x^3 + 820*x^4 + 1312
*x^5 + 5248*x^6))/(348075*x^7)
```

fricas [A] time = 0.38, size = 46, normalized size = 0.92

$$\frac{4\left(5248\,x^6+1312\,x^5+820\,x^4+615\,x^3-21255\,x^2-663\,x+13923\right)\left(x^4-x^3\right)^{\frac{1}{4}}}{348075\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4-x^3)^(1/4)/x^8,x, algorithm="fricas")
```

```
[Out] 4/348075*(5248*x^6 + 1312*x^5 + 820*x^4 + 615*x^3 - 21255*x^2 - 663*x + 139
23)*(x^4 - x^3)^(1/4)/x^7
```

giac [A] time = 0.21, size = 91, normalized size = 1.82

$$-\frac{4}{25}\left(\frac{1}{x}-1\right)^{6}\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-\frac{20}{21}\left(\frac{1}{x}-1\right)^{5}\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-\frac{36}{17}\left(\frac{1}{x}-1\right)^{4}\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-\frac{28}{13}\left(\frac{1}{x}-1\right)^{3}\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-\frac{8}{9}\left(\frac{1}{x}-1\right)^{2}\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4-x^3)^(1/4)/x^8,x, algorithm="giac")
```

```
[Out] -4/25*(1/x - 1)^6*(-1/x + 1)^(1/4) - 20/21*(1/x - 1)^5*(-1/x + 1)^(1/4) - 3
6/17*(1/x - 1)^4*(-1/x + 1)^(1/4) - 28/13*(1/x - 1)^3*(-1/x + 1)^(1/4) - 8/
9*(1/x - 1)^2*(-1/x + 1)^(1/4)
```

maple [A] time = 0.00, size = 42, normalized size = 0.84

$$\frac{4\left(x^{4}-x^{3}\right)^{\frac{1}{4}}(-1+x)^{2}\left(5248x^{4}+11808x^{3}+19188x^{2}+27183x+13923\right)}{348075x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(x^4-x^3)^(1/4)/x^8,x)
```

```
[Out] 4/348075*(x^4-x^3)^(1/4)*(-1+x)^2*(5248*x^4+11808*x^3+19188*x^2+27183*x+139
23)/x^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^{4}-x^{3}\right)^{\frac{1}{4}}\left(x^{2}-1\right)}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4-x^3)^(1/4)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^3)^(1/4)*(x^2 - 1)/x^8, x)
```

mupad [B] time = 1.23, size = 113, normalized size = 2.26

$$\frac{20992\left(x^{4}-x^{3}\right)^{1/4}}{348075\,x}+\frac{5248\left(x^{4}-x^{3}\right)^{1/4}}{348075\,x^{2}}+\frac{656\left(x^{4}-x^{3}\right)^{1/4}}{69615\,x^{3}}+\frac{164\left(x^{4}-x^{3}\right)^{1/4}}{23205\,x^{4}}-\frac{436\left(x^{4}-x^{3}\right)^{1/4}}{1785\,x^{5}}-\frac{4\left(x^{4}-x^{3}\right)^{1/4}}{525\,x^{6}}+\frac{4\left(x^{4}-x^{3}\right)^{1/4}}{25\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*(x^4 - x^3)^(1/4))/x^8,x)
```

```
[Out] (20992*(x^4 - x^3)^(1/4))/(348075*x) + (5248*(x^4 - x^3)^(1/4))/(348075*x^2
) + (656*(x^4 - x^3)^(1/4))/(69615*x^3) + (164*(x^4 - x^3)^(1/4))/(23205*x^
4) - (436*(x^4 - x^3)^(1/4))/(1785*x^5) - (4*(x^4 - x^3)^(1/4))/(525*x^6) +
 (4*(x^4 - x^3)^(1/4))/(25*x^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[4]{x^{3}(x-1)}\,(x-1)(x+1)}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**4-x**3)**(1/4)/x**8,x)
```

```
[Out] Integral((x**3*(x - 1))**(1/4)*(x - 1)*(x + 1)/x**8, x)
```

3.632 $\int \frac{(-1+x^3)\sqrt{-1+x^6}}{x(1+x^3)}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{x^6-1}}{3}-\frac{2}{3}\log\left(\sqrt{x^6-1}+x^3\right)+\frac{2}{3}\tan^{-1}\left(\sqrt{x^6-1}+x^3\right)$$

Rubi [F] time = 0.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1+x^3\right)\sqrt{-1+x^6}}{x\left(1+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^3)*Sqrt[-1 + x^6])/(x*(1 + x^3)),x]
```

```
[Out] -1/3*Sqrt[-1 + x^6] + ArcTan[Sqrt[-1 + x^6]]/3 + (2*Defer[Int][Sqrt[-1 + x^
6]/(1 + x), x])/3 + (4*Defer[Int][Sqrt[-1 + x^6]/(-1 - I*Sqrt[3] + 2*x), x]
)/3 + (4*Defer[Int][Sqrt[-1 + x^6]/(-1 + I*Sqrt[3] + 2*x), x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^3\right)\sqrt{-1+x^6}}{x\left(1+x^3\right)}\,dx &= \int \left(-\frac{\sqrt{-1+x^6}}{x}+\frac{2\sqrt{-1+x^6}}{3(1+x)}+\frac{2(-1+2x)\sqrt{-1+x^6}}{3\left(1-x+x^2\right)}\right)dx\\
&= \frac{2}{3}\int \frac{\sqrt{-1+x^6}}{1+x}\,dx+\frac{2}{3}\int \frac{(-1+2x)\sqrt{-1+x^6}}{1-x+x^2}\,dx-\int \frac{\sqrt{-1+x^6}}{x}\,dx\\
&= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx,x,x^6\right)\right)+\frac{2}{3}\int \frac{\sqrt{-1+x^6}}{1+x}\,dx+\frac{2}{3}\int \left(\frac{2\sqrt{-1+x^6}}{-1-i\sqrt{3}+2x}\right.\\
&= -\frac{1}{3}\sqrt{-1+x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx,x,x^6\right)+\frac{2}{3}\int \frac{\sqrt{-1+x^6}}{1+x}\,dx+\frac{4}{3}\int \frac{}{-1}\\
&= -\frac{1}{3}\sqrt{-1+x^6}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x^6}\right)+\frac{2}{3}\int \frac{\sqrt{-1+x^6}}{1+x}\,dx+\frac{4}{3}\int\\
&= -\frac{1}{3}\sqrt{-1+x^6}+\frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right)+\frac{2}{3}\int \frac{\sqrt{-1+x^6}}{1+x}\,dx+\frac{4}{3}\int \frac{\sqrt{-1+x^6}}{-1-i\sqrt{3}+2x}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 55, normalized size = 1.10

$$\frac{1}{3}\left(\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{x^6-2\sqrt{x^6-1}\log\left(\sqrt{x^6-1}+x^3\right)-1}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)*Sqrt[-1 + x^6])/(x*(1 + x^3)),x]
```

```
[Out] (ArcTan[Sqrt[-1 + x^6]] + (-1 + x^6 - 2*Sqrt[-1 + x^6]*Log[x^3 + Sqrt[-1 +
x^6]])/Sqrt[-1 + x^6])/3
```

IntegrateAlgebraic [A] time = 0.17, size = 54, normalized size = 1.08

$$\frac{\sqrt{x^6-1}}{3}+\frac{2}{3}\log\left(\sqrt{x^6-1}-x^3\right)-\frac{2}{3}\tan^{-1}\left(x^3-\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)*Sqrt[-1 + x^6])/(x*(1 + x^3)),x]
```

```
[Out] Sqrt[-1 + x^6]/3 - (2*ArcTan[x^3 - Sqrt[-1 + x^6]])/3 + (2*Log[-x^3 + Sqrt[
-1 + x^6]])/3
```

fricas [A] time = 0.41, size = 42, normalized size = 0.84

$$\frac{1}{3}\sqrt{x^6-1}+\frac{2}{3}\arctan\left(-x^3+\sqrt{x^6-1}\right)+\frac{2}{3}\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(x^6 - 1) + 2/3*arctan(-x^3 + sqrt(x^6 - 1)) + 2/3*log(-x^3 + sqrt(
x^6 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^6-1}\left(x^3-1\right)}{\left(x^3+1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^6 - 1)*(x^3 - 1)/((x^3 + 1)*x), x)
```

maple [C] time = 0.53, size = 52, normalized size = 1.04

$$\frac{\sqrt{x^6-1}}{3}-\frac{2\ln\left(x^3+\sqrt{x^6-1}\right)}{3}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{x^6-1}+\mathrm{RootOf}\left(\_Z^2+1\right)}{x^3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)*(x^6-1)^(1/2)/x/(x^3+1),x)
```

```
[Out] 1/3*(x^6-1)^(1/2)-2/3*ln(x^3+(x^6-1)^(1/2))+1/3*RootOf(_Z^2+1)*ln(((x^6-1)^
(1/2)+RootOf(_Z^2+1))/x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^6-1}\left(x^3-1\right)}{\left(x^3+1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^6 - 1)*(x^3 - 1)/((x^3 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x^3-1\right)\sqrt{x^6-1}}{x\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)*(x^6 - 1)^(1/2))/(x*(x^3 + 1)),x)
```

```
[Out] int(((x^3 - 1)*(x^6 - 1)^(1/2))/(x*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,(x-1)\left(x^2+x+1\right)}{x\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)*(x**6-1)**(1/2)/x/(x**3+1),x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1))*(x - 1)*(x**2
+ x + 1)/(x*(x + 1)*(x**2 - x + 1)), x)
```

3.633 $\int \frac{(1+x^3)\sqrt{-1+x^6}}{x(-1+x^3)}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{x^6-1}}{3}+\frac{2}{3}\log\left(\sqrt{x^6-1}+x^3\right)+\frac{2}{3}\tan^{-1}\left(\sqrt{x^6-1}+x^3\right)$$

Rubi [F] time = 0.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+x^3)\sqrt{-1+x^6}}{x(-1+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^3)*Sqrt[-1 + x^6])/(x*(-1 + x^3)),x]
```

```
[Out] -1/3*Sqrt[-1 + x^6] + ArcTan[Sqrt[-1 + x^6]]/3 + (2*Defer[Int][Sqrt[-1 + x^
6]/(-1 + x), x])/3 + (4*Defer[Int][Sqrt[-1 + x^6]/(1 - I*Sqrt[3] + 2*x), x]
)/3 + (4*Defer[Int][Sqrt[-1 + x^6]/(1 + I*Sqrt[3] + 2*x), x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^3)\sqrt{-1+x^6}}{x(-1+x^3)}\,dx &= \int \left(\frac{2\sqrt{-1+x^6}}{3(-1+x)}-\frac{\sqrt{-1+x^6}}{x}+\frac{2(1+2x)\sqrt{-1+x^6}}{3(1+x+x^2)}\right)dx\\
&= \frac{2}{3}\int \frac{\sqrt{-1+x^6}}{-1+x}\,dx+\frac{2}{3}\int \frac{(1+2x)\sqrt{-1+x^6}}{1+x+x^2}\,dx-\int \frac{\sqrt{-1+x^6}}{x}\,dx\\
&= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx,x,x^6\right)\right)+\frac{2}{3}\int \frac{\sqrt{-1+x^6}}{-1+x}\,dx+\frac{2}{3}\int \left(\frac{2\sqrt{-1+x^6}}{1-i\sqrt{3}+2x}+\frac{2}{1-}\right.\\
&= -\frac{1}{3}\sqrt{-1+x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx,x,x^6\right)+\frac{2}{3}\int \frac{\sqrt{-1+x^6}}{-1+x}\,dx+\frac{4}{3}\int \frac{\sqrt{-1}}{1-i\sqrt{}}\\
&= -\frac{1}{3}\sqrt{-1+x^6}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x^6}\right)+\frac{2}{3}\int \frac{\sqrt{-1+x^6}}{-1+x}\,dx+\frac{4}{3}\int \frac{\sqrt{}}{1-}\\
&= -\frac{1}{3}\sqrt{-1+x^6}+\frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right)+\frac{2}{3}\int \frac{\sqrt{-1+x^6}}{-1+x}\,dx+\frac{4}{3}\int \frac{\sqrt{-1+x^6}}{1-i\sqrt{3}+2x}\,dx+
\end{aligned}$$

Mathematica [A] time = 0.13, size = 55, normalized size = 1.10

$$\frac{1}{3}\left(\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{x^6+2\sqrt{x^6-1}\log\left(\sqrt{x^6-1}+x^3\right)-1}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)*Sqrt[-1 + x^6])/(x*(-1 + x^3)),x]
```

```
[Out] (ArcTan[Sqrt[-1 + x^6]] + (-1 + x^6 + 2*Sqrt[-1 + x^6]*Log[x^3 + Sqrt[-1 +
x^6]])/Sqrt[-1 + x^6])/3
```

IntegrateAlgebraic [A] time = 0.14, size = 54, normalized size = 1.08

$$\frac{\sqrt{x^6-1}}{3}-\frac{2}{3}\log\left(\sqrt{x^6-1}-x^3\right)-\frac{2}{3}\tan^{-1}\left(x^3-\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)*Sqrt[-1 + x^6])/(x*(-1 + x^3)),x]
```

```
[Out] Sqrt[-1 + x^6]/3 - (2*ArcTan[x^3 - Sqrt[-1 + x^6]])/3 - (2*Log[-x^3 + Sqrt[
-1 + x^6]])/3
```

fricas [A] time = 0.38, size = 42, normalized size = 0.84

$$\frac{1}{3}\sqrt{x^6-1}+\frac{2}{3}\arctan\left(-x^3+\sqrt{x^6-1}\right)-\frac{2}{3}\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(x^6-1)^(1/2)/x/(x^3-1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(x^6 - 1) + 2/3*arctan(-x^3 + sqrt(x^6 - 1)) - 2/3*log(-x^3 + sqrt(
x^6 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^6-1}\left(x^3+1\right)}{\left(x^3-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(x^6-1)^(1/2)/x/(x^3-1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^6 - 1)*(x^3 + 1)/((x^3 - 1)*x), x)
```

maple [C] time = 0.56, size = 54, normalized size = 1.08

$$\frac{\sqrt{x^6-1}}{3}+\frac{2\ln\left(x^3+\sqrt{x^6-1}\right)}{3}-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^2+1\right)+\sqrt{x^6-1}}{x^3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)*(x^6-1)^(1/2)/x/(x^3-1),x)
```

```
[Out] 1/3*(x^6-1)^(1/2)+2/3*ln(x^3+(x^6-1)^(1/2))-1/3*RootOf(_Z^2+1)*ln((-RootOf(
_Z^2+1)+(x^6-1)^(1/2))/x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^6-1}\left(x^3+1\right)}{\left(x^3-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(x^6-1)^(1/2)/x/(x^3-1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^6 - 1)*(x^3 + 1)/((x^3 - 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x^3+1\right)\sqrt{x^6-1}}{x\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)*(x^6 - 1)^(1/2))/(x*(x^3 - 1)),x)
```

```
[Out] int(((x^3 + 1)*(x^6 - 1)^(1/2))/(x*(x^3 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}{x\,(x-1)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)*(x**6-1)**(1/2)/x/(x**3-1),x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1))*(x + 1)*(x**2
- x + 1)/(x*(x - 1)*(x**2 + x + 1)), x)
```

3.634 $\int \sqrt{1+\sqrt{1+x^2}}\, dx$

Optimal. Leaf size=50

$$\frac{2\sqrt{x^2+1}\,x}{3\sqrt{\sqrt{x^2+1}+1}}+\frac{4x}{3\sqrt{\sqrt{x^2+1}+1}}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 0.82, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2129}

$$\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}+\frac{2x^3}{3\left(\sqrt{x^2+1}+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 + x^2]],x]
[Out] (2*x^3)/(3*(1 + Sqrt[1 + x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 + x^2]]
```

Rule 2129

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[(2*b^2*
d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt
[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{1+\sqrt{1+x^2}}\, dx=\frac{2x^3}{3\left(1+\sqrt{1+x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1+x^2}}}$$

Mathematica [A] time = 0.06, size = 44, normalized size = 0.88

$$\frac{2\left(\sqrt{x^2+1}-1\right)\sqrt{\sqrt{x^2+1}+1}\left(\sqrt{x^2+1}+2\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + x^2]],x]
[Out] (2*(-1 + Sqrt[1 + x^2])*Sqrt[1 + Sqrt[1 + x^2]]*(2 + Sqrt[1 + x^2]))/(3*x)
```

IntegrateAlgebraic [A] time = 0.00, size = 50, normalized size = 1.00

$$\frac{2\sqrt{x^2+1}\,x}{3\sqrt{\sqrt{x^2+1}+1}}+\frac{4x}{3\sqrt{\sqrt{x^2+1}+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 + x^2]],x]
[Out] (4*x)/(3*Sqrt[1 + Sqrt[1 + x^2]]) + (2*x*Sqrt[1 + x^2])/(3*Sqrt[1 + Sqrt[1
+ x^2]])
```

fricas [A] time = 1.20, size = 28, normalized size = 0.56

$$\frac{2\left(x^2+\sqrt{x^2+1}-1\right)\sqrt{\sqrt{x^2+1}+1}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2/3*(x^2 + sqrt(x^2 + 1) - 1)*sqrt(sqrt(x^2 + 1) + 1)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

maple [C] time = 0.02, size = 55, normalized size = 1.10

$$-\frac{-\frac{32\sqrt{\pi}\,\sqrt{2}\,x^3\cosh\left(\frac{3\,\mathrm{arcsinh}(x)}{2}\right)}{3}-\frac{8\sqrt{\pi}\,\sqrt{2}\left(-\frac{4}{3}x^4-\frac{2}{3}x^2+\frac{2}{3}\right)\sinh\left(\frac{3\,\mathrm{arcsinh}(x)}{2}\right)}{\sqrt{x^2+1}}}{8\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x^2+1)^(1/2))^(1/2),x)
[Out] -1/8/Pi^(1/2)*(-32/3*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(x))-8*Pi^(1/2)*2
^(1/2)*(-4/3*x^4-2/3*x^2+2/3)*sinh(3/2*arcsinh(x))/(x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)^(1/2) + 1)^(1/2),x)
[Out] int(((x^2 + 1)^(1/2) + 1)^(1/2), x)
```

sympy [B] time = 0.97, size = 197, normalized size = 3.94

$$-\frac{\sqrt{2}\,x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}-\frac{3\sqrt{2}\,x\sqrt{x^2+1}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}-\frac{3\sqrt{2}\,x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] -sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2 +
 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*sqrt(x**2 + 1)*gam
ma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*pi*
sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(
x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1))
```

3.635 $\int \frac{\sqrt{1+\sqrt{1+x^2}}}{x^2}\,dx$

Optimal. Leaf size=50

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}\right)}{\sqrt{2}}-\frac{\sqrt{\sqrt{x^2+1}+1}}{x}$$

Rubi [F] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+\sqrt{1+x^2}}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + Sqrt[1 + x^2]]/x^2,x]
```

```
[Out] Defer[Int][Sqrt[1 + Sqrt[1 + x^2]]/x^2, x]
```

Rubi steps

$$\int \frac{\sqrt{1+\sqrt{1+x^2}}}{x^2}\,dx=\int \frac{\sqrt{1+\sqrt{1+x^2}}}{x^2}\,dx$$

Mathematica [A] time = 0.12, size = 78, normalized size = 1.56

$$\frac{\left(\sqrt{x^2+1}-1\right)\left(\sqrt{x^2+1}+1\right)^{3/2}\left(\sqrt{2}\sqrt{\sqrt{x^2+1}-1}\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}-1}}{\sqrt{2}}\right)-2\right)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + x^2]]/x^2,x]
```

```
[Out] ((-1 + Sqrt[1 + x^2])*(1 + Sqrt[1 + x^2])^(3/2)*(-2 + Sqrt[2]*Sqrt[-1 + Sqr
t[1 + x^2]]*ArcTan[Sqrt[-1 + Sqrt[1 + x^2]]/Sqrt[2]]))/(2*x^3)
```

IntegrateAlgebraic [A] time = 0.12, size = 50, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}\right)}{\sqrt{2}}-\frac{\sqrt{\sqrt{x^2+1}+1}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 + x^2]]/x^2,x]
```

```
[Out] -(Sqrt[1 + Sqrt[1 + x^2]]/x) + ArcTan[x/(Sqrt[2]*Sqrt[1 + Sqrt[1 + x^2]])]/
Sqrt[2]
```

fricas [A] time = 2.75, size = 43, normalized size = 0.86

$$-\frac{\sqrt{2}\,x\arctan\left(\frac{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}{x}\right)+2\sqrt{\sqrt{x^2+1}+1}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*x*arctan(sqrt(2)*sqrt(sqrt(x^2 + 1) + 1)/x) + 2*sqrt(sqrt(x^2
 + 1) + 1))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1)/x^2, x)
```

maple [C] time = 0.03, size = 22, normalized size = 0.44

$$-\frac{\sqrt{2}\ \mathrm{hypergeom}\left(\left[-\frac{1}{2},-\frac{1}{4},\frac{1}{4}\right],\left[\frac{1}{2},\frac{1}{2}\right],-x^2\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x^2+1)^(1/2))^(1/2)/x^2,x)
```

```
[Out] -2^(1/2)/x*hypergeom([-1/2,-1/4,1/4],[1/2,1/2],-x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)^(1/2) + 1)^(1/2)/x^2,x)
```

```
[Out] int(((x^2 + 1)^(1/2) + 1)^(1/2)/x^2, x)
```

sympy [C] time = 0.88, size = 37, normalized size = 0.74

$$\frac{\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_{3}F_{2}\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{1}{2}\end{matrix}\right|x^2e^{i\pi}\right)}{4\pi x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x**2+1)**(1/2))**(1/2)/x**2,x)
```

```
[Out] gamma(-1/4)*gamma(1/4)*hyper((-1/2, -1/4, 1/4), (1/2, 1/2), x**2*exp_polar(
I*pi))/(4*pi*x)
```

3.636 $\int \frac{1}{x\left(b+ax^2\right)^{3/4}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{b^{3/4}}$$

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 63, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b + a*x^2)^(3/4)),x]
```

```
[Out] -(ArcTan[(b + a*x^2)^(1/4)/b^(1/4)]/b^(3/4)) - ArcTanh[(b + a*x^2)^(1/4)/b^
(1/4)]/b^(3/4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(b+ax^2\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(b+ax)^{3/4}}\,dx, x, x^2\right)$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^2}\right)}{a}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^2}\right)}{\sqrt{b}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx, x, \sqrt[4]{b+ax^2}\right)}{\sqrt{b}}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b+ax^2}}{\sqrt[4]{b}}\right)}{b^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^2}}{\sqrt[4]{b}}\right)}{b^{3/4}}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.86

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b + a*x^2)^(3/4)),x]
```

```
[Out] -((ArcTan[(b + a*x^2)^(1/4)/b^(1/4)] + ArcTanh[(b + a*x^2)^(1/4)/b^(1/4)])/
b^(3/4))
```

IntegrateAlgebraic [A] time = 0.08, size = 51, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{b^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(b + a*x^2)^(3/4)),x]
```

```
[Out] -(ArcTan[(b + a*x^2)^(1/4)/b^(1/4)]/b^(3/4)) - ArcTanh[(b + a*x^2)^(1/4)/b^
(1/4)]/b^(3/4)
```

fricas [B] time = 0.41, size = 110, normalized size = 2.16

$$2\left(\frac{1}{b^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^2\sqrt{\frac{1}{b^3}}+\sqrt{ax^2+b}}\,b^2\left(\frac{1}{b^3}\right)^{\frac{3}{4}} - \left(ax^2+b\right)^{\frac{1}{4}}b^2\left(\frac{1}{b^3}\right)^{\frac{3}{4}}\right) - \frac{1}{2}\left(\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(b\left(\frac{1}{b^3}\right)^{\frac{1}{4}}+\left(ax^2+b\right)^{\frac{1}{4}}\right) + \frac{1}{2}\left(\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(-b\left(\frac{1}{b^3}\right)^{\frac{1}{4}}+\left(ax^2+b\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^2+b)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*(b^(-3))^(1/4)*arctan(sqrt(b^2*sqrt(b^(-3)) + sqrt(a*x^2 + b))*b^2*(b^(-3
))^(3/4) - (a*x^2 + b)^(1/4)*b^2*(b^(-3))^(3/4)) - 1/2*(b^(-3))^(1/4)*log(b
*(b^(-3))^(1/4) + (a*x^2 + b)^(1/4)) + 1/2*(b^(-3))^(1/4)*log(-b*(b^(-3))^(
1/4) + (a*x^2 + b)^(1/4))
```

giac [B] time = 0.50, size = 186, normalized size = 3.65

$$-\frac{\sqrt{2}\,(-b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-b)^{\frac{1}{4}}+2\left(ax^2+b\right)^{\frac{1}{4}}\right)}{2\,(-b)^{\frac{1}{4}}}\right)}{2\,b} - \frac{\sqrt{2}\,(-b)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-b)^{\frac{1}{4}}-2\left(ax^2+b\right)^{\frac{1}{4}}\right)}{2\,(-b)^{\frac{1}{4}}}\right)}{2\,b} - \frac{\sqrt{2}\,(-b)^{\frac{1}{4}}\log\left(\sqrt{2}\left(ax^2+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^2+b}+\sqrt{-b}\right)}{4\,b} + \frac{\sqrt{2}\,(-b)^{\frac{1}{4}}\log\left(-\sqrt{2}\left(ax^2+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^2+b}+\sqrt{-b}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^2+b)^(3/4),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(-b)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^2 +
 b)^(1/4))/(-b)^(1/4))/b - 1/2*sqrt(2)*(-b)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(-b)^(1/4) - 2*(a*x^2 + b)^(1/4))/(-b)^(1/4))/b - 1/4*sqrt(2)*(-b)^(1/4
)*log(sqrt(2)*(a*x^2 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^2 + b) + sqrt(-b))/b
+ 1/4*sqrt(2)*(-b)^(1/4)*log(-sqrt(2)*(a*x^2 + b)^(1/4)*(-b)^(1/4) + sqrt(a
*x^2 + b) + sqrt(-b))/b
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a\,x^{2}+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^2+b)^(3/4),x)
```

```
[Out] int(1/x/(a*x^2+b)^(3/4),x)
```

maxima [A] time = 0.43, size = 57, normalized size = 1.12

$$-\frac{\arctan\left(\frac{\left(ax^{2}+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{\log\left(\frac{\left(ax^{2}+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^{2}+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{2\,b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^2+b)^(3/4),x, algorithm="maxima")
```

```
[Out] -arctan((a*x^2 + b)^(1/4)/b^(1/4))/b^(3/4) + 1/2*log(((a*x^2 + b)^(1/4) - b
^(1/4))/((a*x^2 + b)^(1/4) + b^(1/4)))/b^(3/4)
```

mupad [B] time = 0.70, size = 34, normalized size = 0.67

$$-\frac{\mathrm{atan}\left(\frac{\left(a\,x^{2}+b\right)^{1/4}}{b^{1/4}}\right)+\mathrm{atanh}\left(\frac{\left(a\,x^{2}+b\right)^{1/4}}{b^{1/4}}\right)}{b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b + a*x^2)^(3/4)),x)
```

```
[Out] -(atan((b + a*x^2)^(1/4)/b^(1/4)) + atanh((b + a*x^2)^(1/4)/b^(1/4)))/b^(3/
4)
```

sympy [C] time = 0.93, size = 41, normalized size = 0.80

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^{2}}\right)}{2a^{\frac{3}{4}}x^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**2+b)**(3/4),x)
```

```
[Out] -gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*exp_polar(I*pi)/(a*x**2))/(2*a**(3/
4)*x**(3/2)*gamma(7/4))
```

3.637 $\int \frac{-7+x}{(-11+5x)\sqrt{-60+83x-21x^2-3x^3+x^4}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{6\sqrt{6}\sqrt{x^4-3x^3-21x^2+83x-60}}{29x^2-106x+41}\right)}{3\sqrt{6}}$$

Rubi [F] time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-7+x}{(-11+5x)\sqrt{-60+83x-21x^2-3x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-7 + x)/((-11 + 5*x)*Sqrt[-60 + 83*x - 21*x^2 - 3*x^3 + x^4]),x]
```

```
[Out] Defer[Int][1/Sqrt[-60 + 83*x - 21*x^2 - 3*x^3 + x^4], x]/5 - (24*Defer[Int]
[1/((-11 + 5*x)*Sqrt[-60 + 83*x - 21*x^2 - 3*x^3 + x^4]), x])/5
```

Rubi steps

$$\int \frac{-7+x}{(-11+5x)\sqrt{-60+83x-21x^2-3x^3+x^4}}\,dx = \int \left(\frac{1}{5\sqrt{-60+83x-21x^2-3x^3+x^4}} - \frac{}{5(-11+5x)\sqrt{-60}}\right.$$
$$= \frac{1}{5}\int \frac{1}{\sqrt{-60+83x-21x^2-3x^3+x^4}}\,dx - \frac{24}{5}\int \frac{}{(-11+5x}$$

Mathematica [C] time = 0.24, size = 131, normalized size = 2.57

$$-\frac{\sqrt{\frac{x-4}{x-1}}\sqrt{\frac{x-3}{x-1}}(x-1)^2\sqrt{\frac{x+5}{x-1}}\left(3F\left(\sin^{-1}\left(\frac{\sqrt{\frac{x+5}{x-1}}}{\sqrt{3}}\right)\middle|\frac{3}{4}\right)-2\Pi\left(\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{\frac{x+5}{x-1}}}{\sqrt{3}}\right)\middle|\frac{3}{4}\right)\right)}{3\sqrt{6}\sqrt{x^4-3x^3-21x^2+83x-60}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-7 + x)/((-11 + 5*x)*Sqrt[-60 + 83*x - 21*x^2 - 3*x^3 + x^4]),x]
```

```
[Out] -1/3*(Sqrt[(-4 + x)/(-1 + x)]*Sqrt[(-3 + x)/(-1 + x)]*(-1 + x)^2*Sqrt[(5 +
x)/(-1 + x)]*(3*EllipticF[ArcSin[Sqrt[(5 + x)/(-1 + x)]/Sqrt[3]], 3/4] - 2*
EllipticPi[1/2, ArcSin[Sqrt[(5 + x)/(-1 + x)]/Sqrt[3]], 3/4]))/(Sqrt[6]*Sqr
t[-60 + 83*x - 21*x^2 - 3*x^3 + x^4])
```

IntegrateAlgebraic [A] time = 0.13, size = 51, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{6\sqrt{6}\sqrt{x^4-3x^3-21x^2+83x-60}}{29x^2-106x+41}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-7 + x)/((-11 + 5*x)*Sqrt[-60 + 83*x - 21*x^2 - 3*x^3 +
 x^4]),x]
```

```
[Out] -1/3*ArcTanh[(6*Sqrt[6]*Sqrt[-60 + 83*x - 21*x^2 - 3*x^3 + x^4])/(41 - 106*
x + 29*x^2)]/Sqrt[6]
```

fricas [B] time = 0.44, size = 91, normalized size = 1.78

$$\frac{1}{36}\sqrt{3}\sqrt{2}\log\left(-\frac{1057x^4-6796x^3-12\sqrt{3}\sqrt{2}\sqrt{x^4-3x^3-21x^2+83x-60}\left(29x^2-106x+41\right)+9078x^2+9236x-11279}{625x^4-5500x^3+18150x^2-26620x+14641}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7+x)/(-11+5*x)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/36*sqrt(3)*sqrt(2)*log(-(1057*x^4 - 6796*x^3 - 12*sqrt(3)*sqrt(2)*sqrt(x^
4 - 3*x^3 - 21*x^2 + 83*x - 60)*(29*x^2 - 106*x + 41) + 9078*x^2 + 9236*x -
 11279)/(625*x^4 - 5500*x^3 + 18150*x^2 - 26620*x + 14641))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-7}{\sqrt{x^4-3x^3-21x^2+83x-60}\,(5x-11)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7+x)/(-11+5*x)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((x - 7)/(sqrt(x^4 - 3*x^3 - 21*x^2 + 83*x - 60)*(5*x - 11)), x)
```

maple [C] time = 0.03, size = 188, normalized size = 3.69

$$-\frac{\sqrt{\frac{5+x}{-1+x}}\,(-1+x)^2\sqrt{\frac{-3+x}{-1+x}}\,\sqrt{6}\,\sqrt{\frac{x-4}{-1+x}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{\frac{5+x}{-1+x}}}{3},\frac{\sqrt{3}}{2}\right)}{30\sqrt{(5+x)(-1+x)(-3+x)(x-4)}}-\frac{2\sqrt{\frac{5+x}{-1+x}}\,(-1+x)^2\sqrt{\frac{-3+x}{-1+x}}\,\sqrt{6}\,\sqrt{\frac{x-4}{-1+x}}\left(\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{\frac{5+x}{-1+x}}}{3},\frac{\sqrt{3}}{2}\right)-\frac{5\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{5+x}{-1+x}}}{3},\frac{1}{2},\frac{\sqrt{3}}{2}\right)}{6}\right)}{15\sqrt{(5+x)(-1+x)(-3+x)(x-4)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7+x)/(-11+5*x)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2),x)
```

```
[Out] -1/30*((5+x)/(-1+x))^(1/2)*(-1+x)^2*((-3+x)/(-1+x))^(1/2)*6^(1/2)*((x-4)/(-
1+x))^(1/2)/((5+x)*(-1+x)*(-3+x)*(x-4))^(1/2)*EllipticF(1/3*3^(1/2)*((5+x)/
(-1+x))^(1/2),1/2*3^(1/2))-2/15*((5+x)/(-1+x))^(1/2)*(-1+x)^2*((-3+x)/(-1+x
))^(1/2)*6^(1/2)*((x-4)/(-1+x))^(1/2)/((5+x)*(-1+x)*(-3+x)*(x-4))^(1/2)*(El
lipticF(1/3*3^(1/2)*((5+x)/(-1+x))^(1/2),1/2*3^(1/2))-5/6*EllipticPi(1/3*3^
(1/2)*((5+x)/(-1+x))^(1/2),1/2,1/2*3^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-7}{\sqrt{x^4-3x^3-21x^2+83x-60}\,(5x-11)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7+x)/(-11+5*x)/(x^4-3*x^3-21*x^2+83*x-60)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((x - 7)/(sqrt(x^4 - 3*x^3 - 21*x^2 + 83*x - 60)*(5*x - 11)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x-7}{(5x-11)\,\sqrt{x^4-3x^3-21x^2+83x-60}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 7)/((5*x - 11)*(83*x - 21*x^2 - 3*x^3 + x^4 - 60)^(1/2)),x)
[Out] int((x - 7)/((5*x - 11)*(83*x - 21*x^2 - 3*x^3 + x^4 - 60)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-7}{\sqrt{(x-4)(x-3)(x-1)(x+5)}\,(5x-11)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7+x)/(-11+5*x)/(x**4-3*x**3-21*x**2+83*x-60)**(1/2),x)
[Out] Integral((x - 7)/(sqrt((x - 4)*(x - 3)*(x - 1)*(x + 5))*(5*x - 11)), x)
```

3.638 $\int \frac{(-2+x^6)\sqrt{-1+x^6}}{x(2+x^6)}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{x^6-1}}{3}+\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{2\tan^{-1}\left(\frac{\sqrt{x^6-1}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {573, 154, 156, 63, 203}

$$\frac{\sqrt{x^6-1}}{3}+\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{2\tan^{-1}\left(\frac{\sqrt{x^6-1}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((-2 + x^6)*Sqrt[-1 + x^6])/(x*(2 + x^6)),x]
```

```
[Out] Sqrt[-1 + x^6]/3 + ArcTan[Sqrt[-1 + x^6]]/3 - (2*ArcTan[Sqrt[-1 + x^6]/Sqrt
[3]])/Sqrt[3]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
```

```
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rubi steps

$$\begin{aligned}\int \frac{(-2+x^6)\sqrt{-1+x^6}}{x(2+x^6)}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{(-2+x)\sqrt{-1+x}}{x(2+x)}\,dx, x, x^6\right)\\ &= \frac{1}{3}\sqrt{-1+x^6}+\frac{1}{3}\text{Subst}\left(\int \frac{1-\frac{5x}{2}}{\sqrt{-1+x}\,x(2+x)}\,dx, x, x^6\right)\\ &= \frac{1}{3}\sqrt{-1+x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right)-\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,(2+x)}\,dx,\right.\\ &= \frac{1}{3}\sqrt{-1+x^6}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^6}\right)-2\,\text{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, \right.\\ &= \frac{1}{3}\sqrt{-1+x^6}+\frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right)-\frac{2\tan^{-1}\left(\frac{\sqrt{-1+x^6}}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 47, normalized size = 0.92

$$\frac{1}{3}\left(\sqrt{x^6-1}+\tan^{-1}\left(\sqrt{x^6-1}\right)-2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{x^6-1}}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-2 + x^6)*Sqrt[-1 + x^6])/(x*(2 + x^6)),x]
```

```
[Out] (Sqrt[-1 + x^6] + ArcTan[Sqrt[-1 + x^6]] - 2*Sqrt[3]*ArcTan[Sqrt[-1 + x^6]/
Sqrt[3]])/3
```

IntegrateAlgebraic [A] time = 0.04, size = 51, normalized size = 1.00

$$\frac{\sqrt{x^6-1}}{3}+\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{2\tan^{-1}\left(\frac{\sqrt{x^6-1}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^6)*Sqrt[-1 + x^6])/(x*(2 + x^6)),x]
```

```
[Out] Sqrt[-1 + x^6]/3 + ArcTan[Sqrt[-1 + x^6]]/3 - (2*ArcTan[Sqrt[-1 + x^6]/Sqrt
[3]])/Sqrt[3]
```

fricas [A] time = 0.39, size = 38, normalized size = 0.75

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^6-1}\right)+\frac{1}{3}\sqrt{x^6-1}+\frac{1}{3}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-1)^(1/2)/x/(x^6+2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(x^6 - 1)) + 1/3*sqrt(x^6 - 1) + 1/3*ar
ctan(sqrt(x^6 - 1))
```

giac [A] time = 0.18, size = 38, normalized size = 0.75

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^6-1}\right)+\frac{1}{3}\sqrt{x^6-1}+\frac{1}{3}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-1)^(1/2)/x/(x^6+2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(x^6 - 1)) + 1/3*sqrt(x^6 - 1) + 1/3*ar
ctan(sqrt(x^6 - 1))
```

maple [C] time = 1.00, size = 83, normalized size = 1.63

$$\frac{\sqrt{x^6-1}}{3}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{x^6-1}+\mathrm{RootOf}\left(\_Z^2+1\right)}{x^3}\right)}{3}+\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+3\right)x^6+6\sqrt{x^6-1}-4\,\mathrm{RootOf}\left(\_Z^2+3\right)}{x^6+2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-2)*(x^6-1)^(1/2)/x/(x^6+2),x)
```

```
[Out] 1/3*(x^6-1)^(1/2)+1/3*RootOf(_Z^2+1)*ln(((x^6-1)^(1/2)+RootOf(_Z^2+1))/x^3)
+1/3*RootOf(_Z^2+3)*ln((RootOf(_Z^2+3)*x^6+6*(x^6-1)^(1/2)-4*RootOf(_Z^2+3)
)/(x^6+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^6-1}\left(x^6-2\right)}{\left(x^6+2\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-1)^(1/2)/x/(x^6+2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^6 - 1)*(x^6 - 2)/((x^6 + 2)*x), x)
```

mupad [B] time = 0.86, size = 38, normalized size = 0.75

$$\frac{\mathrm{atan}\left(\sqrt{x^6-1}\right)}{3}-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\sqrt{x^6-1}}{3}\right)}{3}+\frac{\sqrt{x^6-1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/2)*(x^6 - 2))/(x*(x^6 + 2)),x)
```

```
[Out] atan((x^6 - 1)^(1/2))/3 - (2*3^(1/2)*atan((3^(1/2)*(x^6 - 1)^(1/2))/3))/3 +
 (x^6 - 1)^(1/2)/3
```

sympy [A] time = 19.83, size = 46, normalized size = 0.90

$$\frac{\sqrt{x^6-1}}{3}-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\sqrt{x^6-1}}{3}\right)}{3}+\frac{\mathrm{atan}\left(\sqrt{x^6-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-2)*(x**6-1)**(1/2)/x/(x**6+2),x)
```

```
[Out] sqrt(x**6 - 1)/3 - 2*sqrt(3)*atan(sqrt(3)*sqrt(x**6 - 1)/3)/3 + atan(sqrt(x
**6 - 1))/3
```

3.639 $\int \frac{\sqrt{-1-x^2+x^6}(1+2x^6)}{(-1+x^6)(-2+x^2+2x^6)}\,dx$

Optimal. Leaf size=51

$$\tan^{-1}\left(\frac{x}{\sqrt{x^6-x^2-1}}\right)-\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt{x^6-x^2-1}}\right)$$

Rubi [F] time = 1.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-1-x^2+x^6}\left(1+2x^6\right)}{\left(-1+x^6\right)\left(-2+x^2+2x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-1 - x^2 + x^6]*(1 + 2*x^6))/((-1 + x^6)*(-2 + x^2 + 2*x^6)),x]
```

```
[Out] Defer[Int][Sqrt[-1 - x^2 + x^6]/(-1 + x), x]/2 - Defer[Int][Sqrt[-1 - x^2 +
 x^6]/(1 + x), x]/2 + ((1 - I*Sqrt[3])*Defer[Int][Sqrt[-1 - x^2 + x^6]/(-1
- I*Sqrt[3] + 2*x), x])/2 - ((1 + I*Sqrt[3])*Defer[Int][Sqrt[-1 - x^2 + x^6
]/(1 - I*Sqrt[3] + 2*x), x])/2 + ((1 + I*Sqrt[3])*Defer[Int][Sqrt[-1 - x^2
+ x^6]/(-1 + I*Sqrt[3] + 2*x), x])/2 - ((1 - I*Sqrt[3])*Defer[Int][Sqrt[-1
- x^2 + x^6]/(1 + I*Sqrt[3] + 2*x), x])/2 + Defer[Int][Sqrt[-1 - x^2 + x^6]
/(2 - x^2 - 2*x^6), x] - 6*Defer[Int][(x^4*Sqrt[-1 - x^2 + x^6])/(-2 + x^2
+ 2*x^6), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1-x^2+x^6}\left(1+2x^6\right)}{\left(-1+x^6\right)\left(-2+x^2+2x^6\right)}\,dx &= \int \left(\frac{\sqrt{-1-x^2+x^6}}{-1+x^2}+\frac{(1+x)\sqrt{-1-x^2+x^6}}{2\left(1-x+x^2\right)}+\frac{(1-x)\sqrt{-1-x^2+x^6}}{2\left(1+x+x^2\right)}+\frac{(-\right.\\
&= \frac{1}{2}\int \frac{(1+x)\sqrt{-1-x^2+x^6}}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{(1-x)\sqrt{-1-x^2+x^6}}{1+x+x^2}\,dx+\int \frac{\sqrt{-1}}{-}\\
&= \frac{1}{2}\int \left(\frac{\left(1-i\sqrt{3}\right)\sqrt{-1-x^2+x^6}}{-1-i\sqrt{3}+2x}+\frac{\left(1+i\sqrt{3}\right)\sqrt{-1-x^2+x^6}}{-1+i\sqrt{3}+2x}\right)dx+\frac{1}{2}\int \Bigg(\\
&= \frac{1}{2}\int \frac{\sqrt{-1-x^2+x^6}}{-1+x}\,dx-\frac{1}{2}\int \frac{\sqrt{-1-x^2+x^6}}{1+x}\,dx-6\int \frac{x^4\sqrt{-1-x^2+x^6}}{-2+x^2+2x^6}
\end{aligned}$$

Mathematica [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-1-x^2+x^6}\left(1+2x^6\right)}{\left(-1+x^6\right)\left(-2+x^2+2x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[-1 - x^2 + x^6]*(1 + 2*x^6))/((-1 + x^6)*(-2 + x^2 + 2*x^6)
),x]
```

```
[Out] Integrate[(Sqrt[-1 - x^2 + x^6]*(1 + 2*x^6))/((-1 + x^6)*(-2 + x^2 + 2*x^6)
), x]
```

IntegrateAlgebraic [A] time = 2.53, size = 51, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt{x^6-x^2-1}}\right)-\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt{x^6-x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 - x^2 + x^6]*(1 + 2*x^6))/((-1 + x^6)*(-2 + x^2
 + 2*x^6)),x]
```

```
[Out] ArcTan[x/Sqrt[-1 - x^2 + x^6]] - Sqrt[3/2]*ArcTan[(Sqrt[3/2]*x)/Sqrt[-1 - x
^2 + x^6]]
```

fricas [A] time = 0.51, size = 75, normalized size = 1.47

$$-\frac{1}{4}\sqrt{3}\sqrt{2}\arctan\left(\frac{2\sqrt{3}\sqrt{2}\sqrt{x^6-x^2-1}\,x}{2x^6-5x^2-2}\right)+\frac{1}{2}\arctan\left(\frac{2\sqrt{x^6-x^2-1}\,x}{x^6-2x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2-1)^(1/2)*(2*x^6+1)/(x^6-1)/(2*x^6+x^2-2),x, algorithm="f
ricas")
```

```
[Out] -1/4*sqrt(3)*sqrt(2)*arctan(2*sqrt(3)*sqrt(2)*sqrt(x^6 - x^2 - 1)*x/(2*x^6
- 5*x^2 - 2)) + 1/2*arctan(2*sqrt(x^6 - x^2 - 1)*x/(x^6 - 2*x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2x^6+1\right)\sqrt{x^6-x^2-1}}{\left(2x^6+x^2-2\right)\left(x^6-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2-1)^(1/2)*(2*x^6+1)/(x^6-1)/(2*x^6+x^2-2),x, algorithm="g
iac")
```

```
[Out] integrate((2*x^6 + 1)*sqrt(x^6 - x^2 - 1)/((2*x^6 + x^2 - 2)*(x^6 - 1)), x)
```

maple [C] time = 0.65, size = 155, normalized size = 3.04

$$-\frac{\operatorname{RootOf}\left(\_Z^2+6\right)\ln\left(-\frac{2\operatorname{RootOf}\left(\_Z^2+6\right)x^6-5\operatorname{RootOf}\left(\_Z^2+6\right)x^2+12\sqrt{x^6-x^2-1}\,x-2\operatorname{RootOf}\left(\_Z^2+6\right)}{2x^6+x^2-2}\right)}{4}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x^6-2\operatorname{RootOf}\left(\_Z^2+1\right)x^2+2\sqrt{x^6-x^2-1}\,x-\operatorname{RootOf}\left(\_Z^2+1\right)}{(-1+x)(1+x)\left(x^2+x+1\right)\left(x^2-x+1\right)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-x^2-1)^(1/2)*(2*x^6+1)/(x^6-1)/(2*x^6+x^2-2),x)
```

```
[Out] -1/4*RootOf(_Z^2+6)*ln(-(2*RootOf(_Z^2+6)*x^6-5*RootOf(_Z^2+6)*x^2+12*(x^6-
x^2-1)^(1/2)*x-2*RootOf(_Z^2+6))/(2*x^6+x^2-2))+1/2*RootOf(_Z^2+1)*ln(-(Roo
tOf(_Z^2+1)*x^6-2*RootOf(_Z^2+1)*x^2+2*(x^6-x^2-1)^(1/2)*x-RootOf(_Z^2+1))/
(-1+x)/(1+x)/(x^2+x+1)/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2x^6+1\right)\sqrt{x^6-x^2-1}}{\left(2x^6+x^2-2\right)\left(x^6-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2-1)^(1/2)*(2*x^6+1)/(x^6-1)/(2*x^6+x^2-2),x, algorithm="m
axima")
```

```
[Out] integrate((2*x^6 + 1)*sqrt(x^6 - x^2 - 1)/((2*x^6 + x^2 - 2)*(x^6 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(2\,x^6+1\right)\,\sqrt{x^6-x^2-1}}{\left(x^6-1\right)\,\left(2\,x^6+x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^6 + 1)*(x^6 - x^2 - 1)^(1/2))/((x^6 - 1)*(x^2 + 2*x^6 - 2)),x)
[Out] int(((2*x^6 + 1)*(x^6 - x^2 - 1)^(1/2))/((x^6 - 1)*(x^2 + 2*x^6 - 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-x**2-1)**(1/2)*(2*x**6+1)/(x**6-1)/(2*x**6+x**2-2),x)
[Out] Timed out
```

3.640 $\int \frac{x(-8b+5ax^3)}{\sqrt[4]{-b+ax^3}\,(b-ax^3+x^8)}\,dx$

Optimal. Leaf size=51

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^3-b}}{x^2}\right)+2\tanh^{-1}\left(\frac{x^2\left(ax^3-b\right)^{3/4}}{b-ax^3}\right)$$

Rubi [F] time = 1.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\left(-8b+5ax^3\right)}{\sqrt[4]{-b+ax^3}\left(b-ax^3+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-8*b + 5*a*x^3))/((-b + a*x^3)^(1/4)*(b - a*x^3 + x^8)),x]
```

```
[Out] -5*a*Defer[Int][x^4/((-b + a*x^3)^(1/4)*(-b + a*x^3 - x^8)), x] - 8*b*Defer
[Int][x/((-b + a*x^3)^(1/4)*(b - a*x^3 + x^8)), x]
```

Rubi steps

$$\int \frac{x\left(-8b+5ax^3\right)}{\sqrt[4]{-b+ax^3}\left(b-ax^3+x^8\right)}\,dx=\int \left(-\frac{5ax^4}{\sqrt[4]{-b+ax^3}\left(-b+ax^3-x^8\right)}-\frac{8bx}{\sqrt[4]{-b+ax^3}\left(b-ax^3+x^8\right)}\right)dx$$

$$=-\left((5a)\int \frac{x^4}{\sqrt[4]{-b+ax^3}\left(-b+ax^3-x^8\right)}\,dx\right)-(8b)\int \frac{x}{\sqrt[4]{-b+ax^3}\left(b-ax^3+x^8\right.}$$

Mathematica [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x\left(-8b+5ax^3\right)}{\sqrt[4]{-b+ax^3}\left(b-ax^3+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(-8*b + 5*a*x^3))/((-b + a*x^3)^(1/4)*(b - a*x^3 + x^8)),x]
```

```
[Out] Integrate[(x*(-8*b + 5*a*x^3))/((-b + a*x^3)^(1/4)*(b - a*x^3 + x^8)), x]
```

IntegrateAlgebraic [A] time = 0.90, size = 51, normalized size = 1.00

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^3-b}}{x^2}\right)+2\tanh^{-1}\left(\frac{x^2\left(ax^3-b\right)^{3/4}}{b-ax^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-8*b + 5*a*x^3))/((-b + a*x^3)^(1/4)*(b - a*x^3 + x^
8)),x]
```

```
[Out] 2*ArcTan[(-b + a*x^3)^(1/4)/x^2] + 2*ArcTanh[(x^2*(-b + a*x^3)^(3/4))/(b -
a*x^3)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*a*x^3-8*b)/(a*x^3-b)^(1/4)/(x^8-a*x^3+b),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,ax^{3}-8\,b\right)x}{\left(x^{8}-ax^{3}+b\right)\left(ax^{3}-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*a*x^3-8*b)/(a*x^3-b)^(1/4)/(x^8-a*x^3+b),x, algorithm="giac"
)
```

```
[Out] integrate((5*a*x^3 - 8*b)*x/((x^8 - a*x^3 + b)*(a*x^3 - b)^(1/4)), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x\left(5a\,x^{3}-8b\right)}{\left(a\,x^{3}-b\right)^{\frac{1}{4}}\left(x^{8}-a\,x^{3}+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(5*a*x^3-8*b)/(a*x^3-b)^(1/4)/(x^8-a*x^3+b),x)
```

```
[Out] int(x*(5*a*x^3-8*b)/(a*x^3-b)^(1/4)/(x^8-a*x^3+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,ax^{3}-8\,b\right)x}{\left(x^{8}-ax^{3}+b\right)\left(ax^{3}-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*a*x^3-8*b)/(a*x^3-b)^(1/4)/(x^8-a*x^3+b),x, algorithm="maxim
a")
```

```
[Out] integrate((5*a*x^3 - 8*b)*x/((x^8 - a*x^3 + b)*(a*x^3 - b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x\,\left(8\,b-5\,a\,x^3\right)}{{\left(a\,x^3-b\right)}^{1/4}\,\left(x^8-a\,x^3+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(8*b - 5*a*x^3))/((a*x^3 - b)^(1/4)*(b - a*x^3 + x^8)),x)
```

```
[Out] int(-(x*(8*b - 5*a*x^3))/((a*x^3 - b)^(1/4)*(b - a*x^3 + x^8)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(5ax^3-8b\right)}{\sqrt[4]{ax^3-b}\left(-ax^3+b+x^8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*a*x**3-8*b)/(a*x**3-b)**(1/4)/(x**8-a*x**3+b),x)
```

```
[Out] Integral(x*(5*a*x**3 - 8*b)/((a*x**3 - b)**(1/4)*(-a*x**3 + b + x**8)), x)
```

3.641 $\displaystyle\int \frac{2+16x-x^2-9x^3}{\sqrt[4]{\frac{1+x}{-2+x^2}}\,(-2+x^2)(-3+2x+7x^2-7x^3-9x^4+9x^5+5x^6-5x^7-x^8+x^9)}\,dx$

Optimal. Leaf size=51

$$2\tanh^{-1}\left(\frac{x^2-1}{\sqrt[4]{\frac{x+1}{x^2-2}}}\right)-2\tan^{-1}\left(\frac{\sqrt[4]{\frac{x+1}{x^2-2}}}{x^2-1}\right)$$

Rubi [F] time = 9.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2+16x-x^2-9x^3}{\sqrt[4]{\frac{1+x}{-2+x^2}}\,(-2+x^2)\,(-3+2x+7x^2-7x^3-9x^4+9x^5+5x^6-5x^7-x^8+x^9)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2 + 16*x - x^2 - 9*x^3)/((((1 + x)/(-2 + x^2))^(1/4)*(-2 + x^2)*(-3 + 2
*x + 7*x^2 - 7*x^3 - 9*x^4 + 9*x^5 + 5*x^6 - 5*x^7 - x^8 + x^9)),x]
```

```
[Out] (-24*(-((1 + x)/(2 - x^2)))^(3/4)*(-2 + x^2)^(3/4)*Defer[Subst][Defer[Int][
x^2/((-1 - 2*x^4 + x^8)^(3/4)*(-1 - 16*x^12 + 56*x^20 - 72*x^24 + 39*x^28 -
 10*x^32 + x^36)), x], x, (1 + x)^(1/4)])/(1 + x)^(3/4) - (36*(-((1 + x)/(2
 - x^2)))^(3/4)*(-2 + x^2)^(3/4)*Defer[Subst][Defer[Int][x^6/((-1 - 2*x^4 +
 x^8)^(3/4)*(-1 - 16*x^12 + 56*x^20 - 72*x^24 + 39*x^28 - 10*x^32 + x^36)),
 x], x, (1 + x)^(1/4)])/(1 + x)^(3/4) + (104*(-((1 + x)/(2 - x^2)))^(3/4)*(
-2 + x^2)^(3/4)*Defer[Subst][Defer[Int][x^10/((-1 - 2*x^4 + x^8)^(3/4)*(-1
- 16*x^12 + 56*x^20 - 72*x^24 + 39*x^28 - 10*x^32 + x^36)), x], x, (1 + x)^
(1/4)])/(1 + x)^(3/4) - (36*(-((1 + x)/(2 - x^2)))^(3/4)*(-2 + x^2)^(3/4)*D
efer[Subst][Defer[Int][x^14/((-1 - 2*x^4 + x^8)^(3/4)*(-1 - 16*x^12 + 56*x^
20 - 72*x^24 + 39*x^28 - 10*x^32 + x^36)), x], x, (1 + x)^(1/4)])/(1 + x)^(
3/4)
```

Rubi steps

$$\int \frac{2+16x-x^2-9x^3}{\sqrt[4]{\frac{1+x}{-2+x^2}}\left(-2+x^2\right)\left(-3+2x+7x^2-7x^3-9x^4+9x^5+5x^6-5x^7-x^8+x^9\right)}\,dx = \int \frac{\left(\frac{1+x}{-2+x^2}\right)^{3/4}\left(-2-16x\right.}{3+x-9x^2+16x^4-14x}$$

$$= \frac{\left(\left(\frac{1+x}{-2+x^2}\right)^{3/4}\left(-2+x^2\right)^{3/4}\right)\int}{}$$

$$= \frac{\left(\left(\frac{1+x}{-2+x^2}\right)^{3/4}\left(-2+x^2\right)^{3/4}\right)\int}{}$$

$$= \frac{\left(\left(\frac{1+x}{-2+x^2}\right)^{3/4}\left(-2+x^2\right)^{3/4}\right)\int}{}$$

$$= -\frac{\left(\left(\frac{1+x}{-2+x^2}\right)^{3/4}\left(-2+x^2\right)^{3/4}\right)}{}$$

$$= -\frac{\left(4\left(\frac{1+x}{-2+x^2}\right)^{3/4}\left(-2+x^2\right)^{3/4}\right.}{}$$

$$= -\frac{\left(4\left(\frac{1+x}{-2+x^2}\right)^{3/4}\left(-2+x^2\right)^{3/4}\right.}{}$$

$$= -\frac{\left(4\left(\frac{1+x}{-2+x^2}\right)^{3/4}\left(-2+x^2\right)^{3/4}\right.}{}$$

Mathematica [F] time = 2.08, size = 0, normalized size = 0.00

$$\int \frac{2+16x-x^2-9x^3}{\sqrt[4]{\frac{1+x}{-2+x^2}}\left(-2+x^2\right)\left(-3+2x+7x^2-7x^3-9x^4+9x^5+5x^6-5x^7-x^8+x^9\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2 + 16*x - x^2 - 9*x^3)/(((1 + x)/(-2 + x^2))^(1/4)*(-2 + x^2)*(
-3 + 2*x + 7*x^2 - 7*x^3 - 9*x^4 + 9*x^5 + 5*x^6 - 5*x^7 - x^8 + x^9)),x]
```

```
[Out] Integrate[(2 + 16*x - x^2 - 9*x^3)/(((1 + x)/(-2 + x^2))^(1/4)*(-2 + x^2)*(
-3 + 2*x + 7*x^2 - 7*x^3 - 9*x^4 + 9*x^5 + 5*x^6 - 5*x^7 - x^8 + x^9)), x]
```

IntegrateAlgebraic [A] time = 0.20, size = 51, normalized size = 1.00

$$2\tanh^{-1}\left(\frac{x^2-1}{\sqrt[4]{\frac{x+1}{x^2-2}}}\right)-2\tan^{-1}\left(\frac{\sqrt[4]{\frac{x+1}{x^2-2}}}{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 16*x - x^2 - 9*x^3)/(((1 + x)/(-2 + x^2))^(1/4)*(-2
 + x^2)*(-3 + 2*x + 7*x^2 - 7*x^3 - 9*x^4 + 9*x^5 + 5*x^6 - 5*x^7 - x^8 + x
^9)),x]
```

```
[Out] -2*ArcTan[((1 + x)/(-2 + x^2))^(1/4)/(-1 + x^2)] + 2*ArcTanh[(-1 + x^2)/((1
 + x)/(-2 + x^2))^(1/4)]
```

fricas [B] time = 97.32, size = 331, normalized size = 6.49

$$-\arctan\left(\frac{2\left((x^3-x^2-2x+2)\left(\frac{x+1}{x^2-2}\right)^{\frac{3}{4}}+(x^7-x^6-4x^5+4x^4+5x^3-5x^2-2x+2)\left(\frac{x+1}{x^2-2}\right)^{\frac{1}{4}}\right)}{x^9-x^8-5x^7+5x^6+9x^5-9x^4-7x^3+7x^2+2x-3}\right)+\log\left(-\frac{x^9-x^8-5x^7+5x^6+9x^5-9x^4-7x^3+7x^2+2(x^3-x^2-2x+2)\left(\frac{x+1}{x^2-2}\right)^{\frac{3}{4}}+2(x^5-x^4-3x^3+3x^2+2x-2)\sqrt{\frac{x+1}{x^2-2}}+2(x^7-x^6-4x^5+4x^4+5x^3-5x^2-2x+2)\left(\frac{x+1}{x^2-2}\right)^{\frac{1}{4}}+2x-1}{x^9-x^8-5x^7+5x^6+9x^5-9x^4-7x^3+7x^2+2x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9*x^3-x^2+16*x+2)/((1+x)/(x^2-2))^(1/4)/(x^2-2)/(x^9-x^8-5*x^7+
5*x^6+9*x^5-9*x^4-7*x^3+7*x^2+2*x-3),x, algorithm="fricas")
```

```
[Out] -arctan(2*((x^3 - x^2 - 2*x + 2)*((x + 1)/(x^2 - 2))^(3/4) + (x^7 - x^6 - 4
*x^5 + 4*x^4 + 5*x^3 - 5*x^2 - 2*x + 2)*((x + 1)/(x^2 - 2))^(1/4))/(x^9 - x
^8 - 5*x^7 + 5*x^6 + 9*x^5 - 9*x^4 - 7*x^3 + 7*x^2 + 2*x - 3)) + log(-(x^9
- x^8 - 5*x^7 + 5*x^6 + 9*x^5 - 9*x^4 - 7*x^3 + 7*x^2 + 2*(x^3 - x^2 - 2*x
+ 2)*((x + 1)/(x^2 - 2))^(3/4) + 2*(x^5 - x^4 - 3*x^3 + 3*x^2 + 2*x - 2)*sq
rt((x + 1)/(x^2 - 2)) + 2*(x^7 - x^6 - 4*x^5 + 4*x^4 + 5*x^3 - 5*x^2 - 2*x
+ 2)*((x + 1)/(x^2 - 2))^(1/4) + 2*x - 1)/(x^9 - x^8 - 5*x^7 + 5*x^6 + 9*x^
5 - 9*x^4 - 7*x^3 + 7*x^2 + 2*x - 3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{9\,x^3+x^2-16\,x-2}{\left(x^9-x^8-5\,x^7+5\,x^6+9\,x^5-9\,x^4-7\,x^3+7\,x^2+2\,x-3\right)\left(x^2-2\right)\left(\frac{x+1}{x^2-2}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9*x^3-x^2+16*x+2)/((1+x)/(x^2-2))^(1/4)/(x^2-2)/(x^9-x^8-5*x^7+
5*x^6+9*x^5-9*x^4-7*x^3+7*x^2+2*x-3),x, algorithm="giac")
```

```
[Out] integrate(-(9*x^3 + x^2 - 16*x - 2)/((x^9 - x^8 - 5*x^7 + 5*x^6 + 9*x^5 - 9
*x^4 - 7*x^3 + 7*x^2 + 2*x - 3)*(x^2 - 2)*((x + 1)/(x^2 - 2))^(1/4)), x)
```

maple [C] time = 10.75, size = 1016, normalized size = 19.92

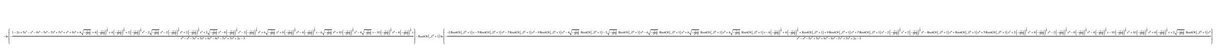

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-9*x^3-x^2+16*x+2)/((1+x)/(x^2-2))^(1/4)/(x^2-2)/(x^9-x^8-5*x^7+5*x^6+
9*x^5-9*x^4-7*x^3+7*x^2+2*x-3),x)
```

```
[Out] -ln(-(1-2*x+5*x^7-x^9-9*x^5-5*x^6-7*x^2+7*x^3+x^8+9*x^4+4*(-(-1-x)/(x^2-2))
^(1/2)+4*(-(-1-x)/(x^2-2))^(1/4)+4*(-(-1-x)/(x^2-2))^(3/4)+2*(-(-1-x)/(x^2-
2))^(1/4)*x^7-2*(-(-1-x)/(x^2-2))^(1/2)*x^5-2*(-(-1-x)/(x^2-2))^(1/4)*x^6+2
*(-(-1-x)/(x^2-2))^(3/4)*x^3+2*(-(-1-x)/(x^2-2))^(1/2)*x^4-8*(-(-1-x)/(x^2-
2))^(1/4)*x^5-2*(-(-1-x)/(x^2-2))^(3/4)*x^2+6*(-(-1-x)/(x^2-2))^(1/2)*x^3+8
*(-(-1-x)/(x^2-2))^(1/4)*x^4-4*(-(-1-x)/(x^2-2))^(3/4)*x-6*(-(-1-x)/(x^2-2)
)^(1/2)*x^2+10*(-(-1-x)/(x^2-2))^(1/4)*x^3-4*(-(-1-x)/(x^2-2))^(1/2)*x-10*(
-(-1-x)/(x^2-2))^(1/4)*x^2-4*(-(-1-x)/(x^2-2))^(1/4)*x)/(x^9-x^8-5*x^7+5*x^
6+9*x^5-9*x^4-7*x^3+7*x^2+2*x-3))-RootOf(_Z^2+1)*ln(-(-RootOf(_Z^2+1)*x^9+R
ootOf(_Z^2+1)*x^8+5*RootOf(_Z^2+1)*x^7+9*RootOf(_Z^2+1)*x^4-5*RootOf(_Z^2+1
)*x^6-7*RootOf(_Z^2+1)*x^2-9*RootOf(_Z^2+1)*x^5+7*RootOf(_Z^2+1)*x^3-4*(-(-
1-x)/(x^2-2))^(1/4)+4*(-(-1-x)/(x^2-2))^(3/4)+RootOf(_Z^2+1)-2*RootOf(_Z^2+
1)*x-2*(-(-1-x)/(x^2-2))^(1/4)*x^7+2*(-(-1-x)/(x^2-2))^(1/4)*x^6-4*(-(-1-x)
/(x^2-2))^(1/2)*RootOf(_Z^2+1)+2*(-(-1-x)/(x^2-2))^(3/4)*x^3+8*(-(-1-x)/(x^
2-2))^(1/4)*x^5-2*(-(-1-x)/(x^2-2))^(3/4)*x^2-8*(-(-1-x)/(x^2-2))^(1/4)*x^4
-4*(-(-1-x)/(x^2-2))^(3/4)*x-10*(-(-1-x)/(x^2-2))^(1/4)*x^3+10*(-(-1-x)/(x^
2-2))^(1/4)*x^2+4*(-(-1-x)/(x^2-2))^(1/4)*x-2*(-(-1-x)/(x^2-2))^(1/2)*RootO
```

```
f(_Z^2+1)*x^4-6*(-(-1-x)/(x^2-2))^(1/2)*RootOf(_Z^2+1)*x^3+6*(-(-1-x)/(x^2-
2))^(1/2)*RootOf(_Z^2+1)*x^2+4*(-(-1-x)/(x^2-2))^(1/2)*RootOf(_Z^2+1)*x+2*(
-(-1-x)/(x^2-2))^(1/2)*RootOf(_Z^2+1)*x^5)/(x^9-x^8-5*x^7+5*x^6+9*x^5-9*x^4
-7*x^3+7*x^2+2*x-3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{9\,x^{3}+x^{2}-16\,x-2}{\left(x^{9}-x^{8}-5\,x^{7}+5\,x^{6}+9\,x^{5}-9\,x^{4}-7\,x^{3}+7\,x^{2}+2\,x-3\right)\left(x^{2}-2\right)\left(\frac{x+1}{x^{2}-2}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9*x^3-x^2+16*x+2)/((1+x)/(x^2-2))^(1/4)/(x^2-2)/(x^9-x^8-5*x^7+
5*x^6+9*x^5-9*x^4-7*x^3+7*x^2+2*x-3),x, algorithm="maxima")
```

```
[Out] -integrate((9*x^3 + x^2 - 16*x - 2)/((x^9 - x^8 - 5*x^7 + 5*x^6 + 9*x^5 - 9
*x^4 - 7*x^3 + 7*x^2 + 2*x - 3)*(x^2 - 2)*((x + 1)/(x^2 - 2))^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{-9\,x^{3}-x^{2}+16\,x+2}{\left(x^{2}-2\right)\left(\frac{x+1}{x^{2}-2}\right)^{1/4}\left(x^{9}-x^{8}-5\,x^{7}+5\,x^{6}+9\,x^{5}-9\,x^{4}-7\,x^{3}+7\,x^{2}+2\,x-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((16*x - x^2 - 9*x^3 + 2)/((x^2 - 2)*((x + 1)/(x^2 - 2))^(1/4)*(2*x + 7*
x^2 - 7*x^3 - 9*x^4 + 9*x^5 + 5*x^6 - 5*x^7 - x^8 + x^9 - 3)),x)
```

```
[Out] int((16*x - x^2 - 9*x^3 + 2)/((x^2 - 2)*((x + 1)/(x^2 - 2))^(1/4)*(2*x + 7*
x^2 - 7*x^3 - 9*x^4 + 9*x^5 + 5*x^6 - 5*x^7 - x^8 + x^9 - 3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9*x**3-x**2+16*x+2)/((1+x)/(x**2-2))**(1/4)/(x**2-2)/(x**9-x**8
-5*x**7+5*x**6+9*x**5-9*x**4-7*x**3+7*x**2+2*x-3),x)
```

```
[Out] Timed out
```

3.642 $\int \frac{-1+7x^8}{(1+x^8)\sqrt{3-x+x^2+6x^8-x^9+3x^{16}}}\,dx$

Optimal. Leaf size=51

$$2\tanh^{-1}\left(\frac{x}{\sqrt{3}\,x^8-\sqrt{3x^{16}-x^9+6x^8+x^2-x+3}+\sqrt{3}}\right)$$

Rubi [F] time = 2.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+7x^8}{(1+x^8)\sqrt{3-x+x^2+6x^8-x^9+3x^{16}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 7*x^8)/((1 + x^8)*Sqrt[3 - x + x^2 + 6*x^8 - x^9 + 3*x^16]),x]
```

```
[Out] 7*Defer[Int][1/Sqrt[3 - x + x^2 + 6*x^8 - x^9 + 3*x^16], x] - (-1)^(1/8)*De
fer[Int][1/(((-1)^(1/8) - x)*Sqrt[3 - x + x^2 + 6*x^8 - x^9 + 3*x^16]), x]
- (-1)^(3/8)*Defer[Int][1/(((-1)^(3/8) - x)*Sqrt[3 - x + x^2 + 6*x^8 - x^9
+ 3*x^16]), x] + (-1)^(5/8)*Defer[Int][1/((-(-1)^(5/8) - x)*Sqrt[3 - x + x^
2 + 6*x^8 - x^9 + 3*x^16]), x] + (-1)^(7/8)*Defer[Int][1/((-(-1)^(7/8) - x)
*Sqrt[3 - x + x^2 + 6*x^8 - x^9 + 3*x^16]), x] - (-1)^(1/8)*Defer[Int][1/((
(-1)^(1/8) + x)*Sqrt[3 - x + x^2 + 6*x^8 - x^9 + 3*x^16]), x] - (-1)^(3/8)*
Defer[Int][1/(((-1)^(3/8) + x)*Sqrt[3 - x + x^2 + 6*x^8 - x^9 + 3*x^16]), x
] + (-1)^(5/8)*Defer[Int][1/((-(-1)^(5/8) + x)*Sqrt[3 - x + x^2 + 6*x^8 - x
^9 + 3*x^16]), x] + (-1)^(7/8)*Defer[Int][1/((-(-1)^(7/8) + x)*Sqrt[3 - x +
 x^2 + 6*x^8 - x^9 + 3*x^16]), x]
```

Rubi steps

$$\int \frac{-1+7x^8}{(1+x^8)\sqrt{3-x+x^2+6x^8-x^9+3x^{16}}}\,dx = \int \left(\frac{7}{\sqrt{3-x+x^2+6x^8-x^9+3x^{16}}}-\frac{8}{(1+x^8)\sqrt{3-x+x^2+\cdots}}\right.$$

$$= 7\int \frac{1}{\sqrt{3-x+x^2+6x^8-x^9+3x^{16}}}\,dx-8\int \frac{}{(1+x^8)\sqrt{3-\cdots}}$$

$$= 7\int \frac{1}{\sqrt{3-x+x^2+6x^8-x^9+3x^{16}}}\,dx-8\int \left(\frac{}{2(i-x^4)\sqrt{3-\cdots}}\right.$$

$$= -\left(4i\int \frac{1}{(i-x^4)\sqrt{3-x+x^2+6x^8-x^9+3x^{16}}}\,dx\right)-4i\int \frac{}{(\cdots}$$

$$= -\left(4i\int \left(-\frac{(-1)^{3/4}}{2\left(\sqrt[4]{-1}-x^2\right)\sqrt{3-x+x^2+6x^8-x^9+3x^{16}}}-\frac{}{2(\cdots}\right.\right.$$

$$= 7\int \frac{1}{\sqrt{3-x+x^2+6x^8-x^9+3x^{16}}}\,dx-\left(2\sqrt[4]{-1}\right)\int \frac{}{\left(\sqrt[4]{-1}-\cdots\right.}$$

$$= 7\int \frac{1}{\sqrt{3-x+x^2+6x^8-x^9+3x^{16}}}\,dx-\left(2\sqrt[4]{-1}\right)\int \left(-\frac{}{2\left(\sqrt[8]{\cdots}\right.}\right.$$

$$= 7\int \frac{1}{\sqrt{3-x+x^2+6x^8-x^9+3x^{16}}}\,dx-\sqrt[8]{-1}\int \frac{}{\left(\sqrt[8]{-1}-x\right)\cdots}$$

Mathematica [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{-1+7x^8}{\left(1+x^8\right)\sqrt{3-x+x^2+6x^8-x^9+3x^{16}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + 7*x^8)/((1 + x^8)*Sqrt[3 - x + x^2 + 6*x^8 - x^9 + 3*x^16])
,x]
```

```
[Out] Integrate[(-1 + 7*x^8)/((1 + x^8)*Sqrt[3 - x + x^2 + 6*x^8 - x^9 + 3*x^16])
, x]
```

IntegrateAlgebraic [A] time = 0.57, size = 51, normalized size = 1.00

$$2\tanh^{-1}\left(\frac{x}{\sqrt{3}\,x^8-\sqrt{3x^{16}-x^9+6x^8+x^2-x+3}+\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 7*x^8)/((1 + x^8)*Sqrt[3 - x + x^2 + 6*x^8 - x^9 +
 3*x^16]),x]
```

```
[Out] 2*ArcTanh[x/(Sqrt[3] + Sqrt[3]*x^8 - Sqrt[3 - x + x^2 + 6*x^8 - x^9 + 3*x^1
6])]
```

fricas [A] time = 1.24, size = 45, normalized size = 0.88

$$\log\left(-\frac{x^8-2\,x+2\,\sqrt{3\,x^{16}-x^9+6\,x^8+x^2-x+3}+1}{x^8+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^8-1)/(x^8+1)/(3*x^16-x^9+6*x^8+x^2-x+3)^(1/2),x, algorithm="
fricas")
```

```
[Out] log(-(x^8 - 2*x + 2*sqrt(3*x^16 - x^9 + 6*x^8 + x^2 - x + 3) + 1)/(x^8 + 1)
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{7\,x^8-1}{\sqrt{3\,x^{16}-x^9+6\,x^8+x^2-x+3}\left(x^8+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^8-1)/(x^8+1)/(3*x^16-x^9+6*x^8+x^2-x+3)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((7*x^8 - 1)/(sqrt(3*x^16 - x^9 + 6*x^8 + x^2 - x + 3)*(x^8 + 1)),
 x)
```

maple [A] time = 0.33, size = 50, normalized size = 0.98

$$-\ln\left(-\frac{-x^8+2\sqrt{3x^{16}-x^9+6x^8+x^2-x+3}+2x-1}{x^8+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7*x^8-1)/(x^8+1)/(3*x^16-x^9+6*x^8+x^2-x+3)^(1/2),x)
```

```
[Out] -ln(-(-x^8+2*(3*x^16-x^9+6*x^8+x^2-x+3)^(1/2)+2*x-1)/(x^8+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{7\,x^8-1}{\sqrt{3\,x^{16}-x^9+6\,x^8+x^2-x+3}\left(x^8+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^8-1)/(x^8+1)/(3*x^16-x^9+6*x^8+x^2-x+3)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((7*x^8 - 1)/(sqrt(3*x^16 - x^9 + 6*x^8 + x^2 - x + 3)*(x^8 + 1)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{7\,x^8-1}{\left(x^8+1\right)\,\sqrt{3\,x^{16}-x^9+6\,x^8+x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7*x^8 - 1)/((x^8 + 1)*(x^2 - x + 6*x^8 - x^9 + 3*x^16 + 3)^(1/2)),x)
```

```
[Out] int((7*x^8 - 1)/((x^8 + 1)*(x^2 - x + 6*x^8 - x^9 + 3*x^16 + 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{7x^8-1}{\left(x^8+1\right)\sqrt{3x^{16}-x^9+6x^8+x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x**8-1)/(x**8+1)/(3*x**16-x**9+6*x**8+x**2-x+3)**(1/2),x)
```

```
[Out] Integral((7*x**8 - 1)/((x**8 + 1)*sqrt(3*x**16 - x**9 + 6*x**8 + x**2 - x +
 3)), x)
```

3.643 $\int \frac{\sqrt{1+\sqrt{1+x^2}}}{x^2\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=51

$$-\frac{\sqrt{\sqrt{x^2+1}+1}}{x}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}\right)}{\sqrt{2}}$$

Rubi [F] time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+\sqrt{1+x^2}}}{x^2\sqrt{1+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + Sqrt[1 + x^2]]/(x^2*Sqrt[1 + x^2]),x]
```

```
[Out] Defer[Int][Sqrt[1 + Sqrt[1 + x^2]]/(x^2*Sqrt[1 + x^2]), x]
```

Rubi steps

$$\int \frac{\sqrt{1+\sqrt{1+x^2}}}{x^2\sqrt{1+x^2}}\,dx=\int \frac{\sqrt{1+\sqrt{1+x^2}}}{x^2\sqrt{1+x^2}}\,dx$$

Mathematica [C] time = 0.11, size = 45, normalized size = 0.88

$$-\frac{\sqrt{\sqrt{x^2+1}+1}\;{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}-\frac{\sqrt{x^2+1}}{2}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + x^2]]/(x^2*Sqrt[1 + x^2]),x]
```

```
[Out] -((Sqrt[1 + Sqrt[1 + x^2]]*Hypergeometric2F1[-1/2, 1, 1/2, 1/2 - Sqrt[1 + x
^2]/2])/x)
```

IntegrateAlgebraic [A] time = 0.09, size = 51, normalized size = 1.00

$$-\frac{\sqrt{\sqrt{x^2+1}+1}}{x}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 + x^2]]/(x^2*Sqrt[1 + x^2]),x]
```

```
[Out] -(Sqrt[1 + Sqrt[1 + x^2]]/x) - ArcTan[x/(Sqrt[2]*Sqrt[1 + Sqrt[1 + x^2]])]/
Sqrt[2]
```

fricas [A] time = 1.35, size = 43, normalized size = 0.84

$$\frac{\sqrt{2}\,x\arctan\left(\frac{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}{x}\right)-2\sqrt{\sqrt{x^2+1}+1}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/x^2/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*x*arctan(sqrt(2)*sqrt(sqrt(x^2 + 1) + 1)/x) - 2*sqrt(sqrt(x^2
+ 1) + 1))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{\sqrt{x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/x^2/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1)/(sqrt(x^2 + 1)*x^2), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\sqrt{x^2+1}}}{x^2\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x^2+1)^(1/2))^(1/2)/x^2/(x^2+1)^(1/2),x)
```

```
[Out] int((1+(x^2+1)^(1/2))^(1/2)/x^2/(x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{\sqrt{x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/x^2/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1)/(sqrt(x^2 + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{x^2\,\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)^(1/2) + 1)^(1/2)/(x^2*(x^2 + 1)^(1/2)),x)
```

```
[Out] int(((x^2 + 1)^(1/2) + 1)^(1/2)/(x^2*(x^2 + 1)^(1/2)), x)
```

sympy [C] time = 1.19, size = 34, normalized size = 0.67

$$-\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_{3}F_{2}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{1}{2}\end{matrix}\middle| x^{2}e^{i\pi}\right)}{\pi x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x**2+1)**(1/2))**(1/2)/x**2/(x**2+1)**(1/2),x)
```

```
[Out] -gamma(1/4)*gamma(3/4)*hyper((-1/2, 1/4, 3/4), (1/2, 1/2), x**2*exp_polar(I
*pi))/(pi*x)
```

3.644 $\int \frac{x}{x+\sqrt{1+\sqrt{1+x}}}\,dx$

Optimal. Leaf size=51

$$x-4\sqrt{\sqrt{x+1}+1}+\frac{8\tanh^{-1}\left(\frac{2\sqrt{\sqrt{x+1}+1}}{\sqrt{5}}+\frac{1}{\sqrt{5}}\right)}{\sqrt{5}}$$

Rubi [A] time = 0.31, antiderivative size = 67, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1628, 618, 206}

$$\left(\sqrt{x+1}+1\right)^2-4\sqrt{\sqrt{x+1}+1}-2\sqrt{x+1}+\frac{8\tanh^{-1}\left(\frac{2\sqrt{\sqrt{x+1}+1}+1}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(x + Sqrt[1 + Sqrt[1 + x]]),x]
```

```
[Out] -2*Sqrt[1 + x] - 4*Sqrt[1 + Sqrt[1 + x]] + (1 + Sqrt[1 + x])^2 + (8*ArcTanh
[(1 + 2*Sqrt[1 + Sqrt[1 + x]])/Sqrt[5]])/Sqrt[5]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x}{x+\sqrt{1+\sqrt{1+x}}}\,dx = 2\,\text{Subst}\left(\int \frac{x\left(-1+x^2\right)}{-1+x^2+\sqrt{1+x}}\,dx, x, \sqrt{1+x}\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x^2(1+x)\left(-2+x^2\right)}{-1+x+x^2}\,dx, x, \sqrt{1+\sqrt{1+x}}\right)$$
$$= 4\,\text{Subst}\left(\int \left(-1-x+x^3-\frac{1}{-1+x+x^2}\right)dx, x, \sqrt{1+\sqrt{1+x}}\right)$$
$$= -2\sqrt{1+x} - 4\sqrt{1+\sqrt{1+x}} + \left(1+\sqrt{1+x}\right)^2 - 4\,\text{Subst}\left(\int \frac{1}{-1+x+x^2}\,dx, x, \sqrt{1+\sqrt{1+x}}\right)$$
$$= -2\sqrt{1+x} - 4\sqrt{1+\sqrt{1+x}} + \left(1+\sqrt{1+x}\right)^2 + 8\,\text{Subst}\left(\int \frac{1}{5-x^2}\,dx, x, 1+2\sqrt{1+\sqrt{1+x}}\right)$$
$$= -2\sqrt{1+x} - 4\sqrt{1+\sqrt{1+x}} + \left(1+\sqrt{1+x}\right)^2 + \frac{8\tanh^{-1}\left(\frac{1+2\sqrt{1+\sqrt{1+x}}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Mathematica [A] time = 0.08, size = 48, normalized size = 0.94

$$x-4\sqrt{\sqrt{x+1}+1}+\frac{8\tanh^{-1}\left(\frac{2\sqrt{\sqrt{x+1}+1}+1}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x + Sqrt[1 + Sqrt[1 + x]]),x]
```

```
[Out] x - 4*Sqrt[1 + Sqrt[1 + x]] + (8*ArcTanh[(1 + 2*Sqrt[1 + Sqrt[1 + x]])/Sqrt
[5]])/Sqrt[5]
```

IntegrateAlgebraic [A] time = 0.09, size = 52, normalized size = 1.02

$$x-4\sqrt{\sqrt{x+1}+1}+\frac{8\tanh^{-1}\left(\frac{2\sqrt{\sqrt{x+1}+1}}{\sqrt{5}}+\frac{1}{\sqrt{5}}\right)}{\sqrt{5}}+1$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(x + Sqrt[1 + Sqrt[1 + x]]),x]
```

```
[Out] 1 + x - 4*Sqrt[1 + Sqrt[1 + x]] + (8*ArcTanh[1/Sqrt[5] + (2*Sqrt[1 + Sqrt[1
 + x]])/Sqrt[5]])/Sqrt[5]
```

fricas [B] time = 0.39, size = 106, normalized size = 2.08

$$\frac{4}{5}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}(3x+1)-\left(\sqrt{5}(x+2)-5x\right)\sqrt{x+1}+\left(\sqrt{5}(x+2)+\left(\sqrt{5}(2x-1)-5\right)\sqrt{x+1}-5x\right)\sqrt{\sqrt{x+1}+1}+3x+3}{x^2-x-1}\right)+x-4\sqrt{\sqrt{x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(1+(1+x)^(1/2))^(1/2)),x, algorithm="fricas")
```

```
[Out] 4/5*sqrt(5)*log((2*x^2 - sqrt(5)*(3*x + 1) - (sqrt(5)*(x + 2) - 5*x)*sqrt(x
 + 1) + (sqrt(5)*(x + 2) + (sqrt(5)*(2*x - 1) - 5)*sqrt(x + 1) - 5*x)*sqrt(
sqrt(x + 1) + 1) + 3*x + 3)/(x^2 - x - 1)) + x - 4*sqrt(sqrt(x + 1) + 1)
```

giac [A] time = 0.18, size = 71, normalized size = 1.39

$$\left(\sqrt{x+1}+1\right)^2-\frac{4}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{\sqrt{x+1}+1}-1}{\sqrt{5}+2\sqrt{\sqrt{x+1}+1}+1}\right)-2\sqrt{x+1}-4\sqrt{\sqrt{x+1}+1}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(1+(1+x)^(1/2))^(1/2)),x, algorithm="giac")
```

```
[Out] (sqrt(x + 1) + 1)^2 - 4/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(sqrt(x + 1) + 1) -
 1)/(sqrt(5) + 2*sqrt(sqrt(x + 1) + 1) + 1)) - 2*sqrt(x + 1) - 4*sqrt(sqrt(
x + 1) + 1) - 2
```

maple [A] time = 0.01, size = 54, normalized size = 1.06

$$\left(1+\sqrt{1+x}\right)^2-2\sqrt{1+x}-2-4\sqrt{1+\sqrt{1+x}}+\frac{8\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(1+2\sqrt{1+\sqrt{1+x}}\right)\sqrt{5}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+(1+(1+x)^(1/2))^(1/2)),x)
```

```
[Out] (1+(1+x)^(1/2))^2-2*(1+x)^(1/2)-2-4*(1+(1+x)^(1/2))^(1/2)+8/5*5^(1/2)*arcta
nh(1/5*(1+2*(1+(1+x)^(1/2))^(1/2))*5^(1/2))
```

maxima [A] time = 0.43, size = 71, normalized size = 1.39

$$\left(\sqrt{x+1}+1\right)^2-\frac{4}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{\sqrt{x+1}+1}-1}{\sqrt{5}+2\sqrt{\sqrt{x+1}+1}+1}\right)-2\sqrt{x+1}-4\sqrt{\sqrt{x+1}+1}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(1+(1+x)^(1/2))^(1/2)),x, algorithm="maxima")
```

```
[Out] (sqrt(x + 1) + 1)^2 - 4/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(sqrt(x + 1) + 1) -
 1)/(sqrt(5) + 2*sqrt(sqrt(x + 1) + 1) + 1)) - 2*sqrt(x + 1) - 4*sqrt(sqrt(
x + 1) + 1) - 2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{x+\sqrt{\sqrt{x+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + ((x + 1)^(1/2) + 1)^(1/2)),x)
```

```
[Out] int(x/(x + ((x + 1)^(1/2) + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x+\sqrt{\sqrt{x+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(1+(1+x)**(1/2))**(1/2)),x)
```

```
[Out] Integral(x/(x + sqrt(sqrt(x + 1) + 1)), x)
```

3.645 $\int \frac{a+2x}{\sqrt[4]{b+ax+x^2}\left(-1+2b+2ax+2x^2\right)}\,dx$

Optimal. Leaf size=52

$$\sqrt[4]{2}\tan^{-1}\left(\sqrt[4]{2}\sqrt[4]{ax+b+x^2}\right)-\sqrt[4]{2}\tanh^{-1}\left(\sqrt[4]{2}\sqrt[4]{ax+b+x^2}\right)$$

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+2x}{\sqrt[4]{b+ax+x^2}\left(-1+2b+2ax+2x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + 2*x)/((b + a*x + x^2)^(1/4)*(-1 + 2*b + 2*a*x + 2*x^2)),x]
```

```
[Out] Defer[Int][(a + 2*x)/((b + a*x + x^2)^(1/4)*(-1 + 2*b + 2*a*x + 2*x^2)), x]
```

Rubi steps

$$\int \frac{a+2x}{\sqrt[4]{b+ax+x^2}\left(-1+2b+2ax+2x^2\right)}\,dx=\int \frac{a+2x}{\sqrt[4]{b+ax+x^2}\left(-1+2b+2ax+2x^2\right)}\,dx$$

Mathematica [A] time = 0.28, size = 45, normalized size = 0.87

$$\sqrt[4]{2}\left(\tan^{-1}\left(\sqrt[4]{2}\sqrt[4]{x(a+x)+b}\right)-\tanh^{-1}\left(\sqrt[4]{2}\sqrt[4]{x(a+x)+b}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 2*x)/((b + a*x + x^2)^(1/4)*(-1 + 2*b + 2*a*x + 2*x^2)),x]
```

```
[Out] 2^(1/4)*(ArcTan[2^(1/4)*(b + x*(a + x))^(1/4)] - ArcTanh[2^(1/4)*(b + x*(a
+ x))^(1/4)])
```

IntegrateAlgebraic [A] time = 0.14, size = 52, normalized size = 1.00

$$\sqrt[4]{2}\tan^{-1}\left(\sqrt[4]{2}\sqrt[4]{ax+b+x^2}\right)-\sqrt[4]{2}\tanh^{-1}\left(\sqrt[4]{2}\sqrt[4]{ax+b+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + 2*x)/((b + a*x + x^2)^(1/4)*(-1 + 2*b + 2*a*x + 2*x
^2)),x]
```

```
[Out] 2^(1/4)*ArcTan[2^(1/4)*(b + a*x + x^2)^(1/4)] - 2^(1/4)*ArcTanh[2^(1/4)*(b
+ a*x + x^2)^(1/4)]
```

fricas [B] time = 0.41, size = 94, normalized size = 1.81

$$-\frac{1}{2}\cdot 8^{\frac{3}{4}}\arctan\left(\frac{1}{8}\cdot 8^{\frac{3}{4}}\sqrt{2\sqrt{2}+4\sqrt{ax+x^2+b}}-\frac{1}{4}\cdot 8^{\frac{3}{4}}\left(ax+x^2+b\right)^{\frac{1}{4}}\right)-\frac{1}{8}\cdot 8^{\frac{3}{4}}\log\left(8^{\frac{1}{4}}+2\left(ax+x^2+b\right)^{\frac{1}{4}}\right)+\frac{1}{8}\cdot 8^{\frac{3}{4}}\log\left(-8^{\frac{1}{4}}+2\left(ax+x^2+b\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*x)/(a*x+x^2+b)^(1/4)/(2*a*x+2*x^2+2*b-1),x, algorithm="frica
s")
```

```
[Out] -1/2*8^(3/4)*arctan(1/8*8^(3/4)*sqrt(2*sqrt(2) + 4*sqrt(a*x + x^2 + b)) - 1
/4*8^(3/4)*(a*x + x^2 + b)^(1/4)) - 1/8*8^(3/4)*log(8^(1/4) + 2*(a*x + x^2
+ b)^(1/4)) + 1/8*8^(3/4)*log(-8^(1/4) + 2*(a*x + x^2 + b)^(1/4))
```

giac [A] time = 0.30, size = 65, normalized size = 1.25

$$\frac{1}{4} \cdot 8^{\frac{3}{4}} \arctan\left(2 \left(\frac{1}{2}\right)^{\frac{3}{4}} \left(ax + x^2 + b\right)^{\frac{1}{4}}\right) - \frac{1}{8} \cdot 8^{\frac{3}{4}} \log\left(\left(\frac{1}{2}\right)^{\frac{1}{4}} + \left(ax + x^2 + b\right)^{\frac{1}{4}}\right) + \frac{1}{8} \cdot 8^{\frac{3}{4}} \log\left(\left|-\left(\frac{1}{2}\right)^{\frac{1}{4}} + \left(ax + x^2 + b\right)^{\frac{1}{4}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*x)/(a*x+x^2+b)^(1/4)/(2*a*x+2*x^2+2*b-1),x, algorithm="giac"
)
```

```
[Out] 1/4*8^(3/4)*arctan(2*(1/2)^(3/4)*(a*x + x^2 + b)^(1/4)) - 1/8*8^(3/4)*log((
1/2)^(1/4) + (a*x + x^2 + b)^(1/4)) + 1/8*8^(3/4)*log(abs(-(1/2)^(1/4) + (a
*x + x^2 + b)^(1/4)))
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{a + 2x}{\left(ax + x^2 + b\right)^{\frac{1}{4}} \left(2ax + 2x^2 + 2b - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+2*x)/(a*x+x^2+b)^(1/4)/(2*a*x+2*x^2+2*b-1),x)
```

```
[Out] int((a+2*x)/(a*x+x^2+b)^(1/4)/(2*a*x+2*x^2+2*b-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + 2\,x}{\left(2\,ax + 2\,x^2 + 2\,b - 1\right)\left(ax + x^2 + b\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*x)/(a*x+x^2+b)^(1/4)/(2*a*x+2*x^2+2*b-1),x, algorithm="maxim
a")
```

```
[Out] integrate((a + 2*x)/((2*a*x + 2*x^2 + 2*b - 1)*(a*x + x^2 + b)^(1/4)), x)
```

mupad [B] time = 0.99, size = 39, normalized size = 0.75

$$2^{1/4} \left(\mathrm{atan}\left(\left(2\,x^2 + 2\,a\,x + 2\,b\right)^{1/4}\right) - \mathrm{atanh}\left(\left(2\,x^2 + 2\,a\,x + 2\,b\right)^{1/4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 2*x)/((b + a*x + x^2)^(1/4)*(2*b + 2*a*x + 2*x^2 - 1)),x)
```

```
[Out] 2^(1/4)*(atan((2*b + 2*a*x + 2*x^2)^(1/4)) - atanh((2*b + 2*a*x + 2*x^2)^(1
/4)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + 2x}{\sqrt[4]{ax + b + x^2} \left(2ax + 2b + 2x^2 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*x)/(a*x+x**2+b)**(1/4)/(2*a*x+2*x**2+2*b-1),x)
```

```
[Out] Integral((a + 2*x)/((a*x + b + x**2)**(1/4)*(2*a*x + 2*b + 2*x**2 - 1)), x)
```

3.646 $\int \frac{1}{x^7 \sqrt[4]{1+x^3}} \, dx$

Optimal. Leaf size=52

$$\frac{5}{48} \tan^{-1}\left(\sqrt[4]{x^3+1}\right) - \frac{5}{48} \tanh^{-1}\left(\sqrt[4]{x^3+1}\right) + \frac{\left(x^3+1\right)^{3/4} \left(5x^3-4\right)}{24x^6}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {266, 51, 63, 298, 203, 206}

$$\frac{5\left(x^3+1\right)^{3/4}}{24x^3} + \frac{5}{48} \tan^{-1}\left(\sqrt[4]{x^3+1}\right) - \frac{5}{48} \tanh^{-1}\left(\sqrt[4]{x^3+1}\right) - \frac{\left(x^3+1\right)^{3/4}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + x^3)^(1/4)),x]
```

```
[Out] -1/6*(1 + x^3)^(3/4)/x^6 + (5*(1 + x^3)^(3/4))/(24*x^3) + (5*ArcTan[(1 + x^
3)^(1/4)])/48 - (5*ArcTanh[(1 + x^3)^(1/4)])/48
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\sqrt[4]{1+x^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt[4]{1+x}}\,dx, x, x^3\right)\\
&= -\frac{\left(1+x^3\right)^{3/4}}{6x^6} - \frac{5}{24}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt[4]{1+x}}\,dx, x, x^3\right)\\
&= -\frac{\left(1+x^3\right)^{3/4}}{6x^6} + \frac{5\left(1+x^3\right)^{3/4}}{24x^3} + \frac{5}{96}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{1+x}}\,dx, x, x^3\right)\\
&= -\frac{\left(1+x^3\right)^{3/4}}{6x^6} + \frac{5\left(1+x^3\right)^{3/4}}{24x^3} + \frac{5}{24}\operatorname{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt[4]{1+x^3}\right)\\
&= -\frac{\left(1+x^3\right)^{3/4}}{6x^6} + \frac{5\left(1+x^3\right)^{3/4}}{24x^3} - \frac{5}{48}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1+x^3}\right) + \frac{5}{48}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\right.\\
&= -\frac{\left(1+x^3\right)^{3/4}}{6x^6} + \frac{5\left(1+x^3\right)^{3/4}}{24x^3} + \frac{5}{48}\tan^{-1}\left(\sqrt[4]{1+x^3}\right) - \frac{5}{48}\tanh^{-1}\left(\sqrt[4]{1+x^3}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.50

$$-\frac{4}{9}\left(x^3+1\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + x^3)^(1/4)),x]
```

```
[Out] (-4*(1 + x^3)^(3/4)*Hypergeometric2F1[3/4, 3, 7/4, 1 + x^3])/9
```

IntegrateAlgebraic [A] time = 0.07, size = 52, normalized size = 1.00

$$\frac{5}{48}\tan^{-1}\left(\sqrt[4]{x^3+1}\right) - \frac{5}{48}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right) + \frac{\left(x^3+1\right)^{3/4}\left(5x^3-4\right)}{24x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(1 + x^3)^(1/4)),x]
```

```
[Out] ((1 + x^3)^(3/4)*(-4 + 5*x^3))/(24*x^6) + (5*ArcTan[(1 + x^3)^(1/4)])/48 -
(5*ArcTanh[(1 + x^3)^(1/4)])/48
```

fricas [A] time = 0.40, size = 65, normalized size = 1.25

$$\frac{10\,x^6\arctan\left(\left(x^3+1\right)^{\frac{1}{4}}\right) - 5\,x^6\log\left(\left(x^3+1\right)^{\frac{1}{4}}+1\right) + 5\,x^6\log\left(\left(x^3+1\right)^{\frac{1}{4}}-1\right) + 4\left(5\,x^3-4\right)\left(x^3+1\right)^{\frac{3}{4}}}{96\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^3+1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/96*(10*x^6*arctan((x^3 + 1)^(1/4)) - 5*x^6*log((x^3 + 1)^(1/4) + 1) + 5*x
^6*log((x^3 + 1)^(1/4) - 1) + 4*(5*x^3 - 4)*(x^3 + 1)^(3/4))/x^6
```

giac [A] time = 0.22, size = 60, normalized size = 1.15

$$\frac{5\left(x^3+1\right)^{\frac{7}{4}}-9\left(x^3+1\right)^{\frac{3}{4}}}{24\,x^6}+\frac{5}{48}\arctan\left(\left(x^3+1\right)^{\frac{1}{4}}\right)-\frac{5}{96}\log\left(\left(x^3+1\right)^{\frac{1}{4}}+1\right)+\frac{5}{96}\log\left(\left|\left(x^3+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^3+1)^(1/4),x, algorithm="giac")
```

```
[Out] 1/24*(5*(x^3 + 1)^(7/4) - 9*(x^3 + 1)^(3/4))/x^6 + 5/48*arctan((x^3 + 1)^(1
/4)) - 5/96*log((x^3 + 1)^(1/4) + 1) + 5/96*log(abs((x^3 + 1)^(1/4) - 1))
```

maple [C] time = 0.14, size = 82, normalized size = 1.58

$$\frac{5x^6+x^3-4}{24x^6\left(x^3+1\right)^{\frac{1}{4}}}+\frac{5\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)\left(-\frac{\pi\sqrt{2}\,x^3\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{4}\right],[2,2],-x^3\right)}{4\Gamma\left(\frac{3}{4}\right)}+\frac{\left(-3\ln(2)-\frac{\pi}{2}+3\ln(x)\right)\pi\sqrt{2}}{\Gamma\left(\frac{3}{4}\right)}\right)}{192\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^3+1)^(1/4),x)
```

```
[Out] 1/24*(5*x^6+x^3-4)/x^6/(x^3+1)^(1/4)+5/192/Pi*2^(1/2)*GAMMA(3/4)*(-1/4*Pi*2
^(1/2)/GAMMA(3/4)*x^3*hypergeom([1,1,5/4],[2,2],-x^3)+(-3*ln(2)-1/2*Pi+3*ln
(x))*Pi*2^(1/2)/GAMMA(3/4))
```

maxima [A] time = 0.42, size = 74, normalized size = 1.42

$$-\frac{5\left(x^3+1\right)^{\frac{7}{4}}-9\left(x^3+1\right)^{\frac{3}{4}}}{24\left(2\,x^3-\left(x^3+1\right)^2+1\right)}+\frac{5}{48}\arctan\left(\left(x^3+1\right)^{\frac{1}{4}}\right)-\frac{5}{96}\log\left(\left(x^3+1\right)^{\frac{1}{4}}+1\right)+\frac{5}{96}\log\left(\left(x^3+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^3+1)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/24*(5*(x^3 + 1)^(7/4) - 9*(x^3 + 1)^(3/4))/(2*x^3 - (x^3 + 1)^2 + 1) + 5
/48*arctan((x^3 + 1)^(1/4)) - 5/96*log((x^3 + 1)^(1/4) + 1) + 5/96*log((x^3
 + 1)^(1/4) - 1)
```

mupad [B] time = 0.79, size = 45, normalized size = 0.87

$$\frac{5\,\mathrm{atan}\left(\left(x^3+1\right)^{1/4}\right)}{48}-\frac{5\,\mathrm{atanh}\left(\left(x^3+1\right)^{1/4}\right)}{48}-\frac{3\left(x^3+1\right)^{3/4}}{8\,x^6}+\frac{5\left(x^3+1\right)^{7/4}}{24\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^3 + 1)^(1/4)),x)
```

```
[Out] (5*atan((x^3 + 1)^(1/4)))/48 - (5*atanh((x^3 + 1)^(1/4)))/48 - (3*(x^3 + 1)
^(3/4))/(8*x^6) + (5*(x^3 + 1)^(7/4))/(24*x^6)
```

sympy [C] time = 1.16, size = 32, normalized size = 0.62

$$-\frac{\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^3}\right)}{3x^{\frac{27}{4}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**3+1)**(1/4),x)
```

```
[Out] -gamma(9/4)*hyper((1/4, 9/4), (13/4,), exp_polar(I*pi)/x**3)/(3*x**(27/4)*g
amma(13/4))
```

3.647 $\int \frac{\sqrt[4]{1+x^3}}{x^7} \, dx$

Optimal. Leaf size=52

$$\frac{1}{16}\tan^{-1}\left(\sqrt[4]{x^3+1}\right)+\frac{1}{16}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)+\frac{\sqrt[4]{x^3+1}\,\left(-x^3-4\right)}{24x^6}$$

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {266, 47, 51, 63, 212, 206, 203}

$$-\frac{\sqrt[4]{x^3+1}}{24x^3}+\frac{1}{16}\tan^{-1}\left(\sqrt[4]{x^3+1}\right)+\frac{1}{16}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)-\frac{\sqrt[4]{x^3+1}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)^(1/4)/x^7,x]
```

```
[Out] -1/6*(1 + x^3)^(1/4)/x^6 - (1 + x^3)^(1/4)/(24*x^3) + ArcTan[(1 + x^3)^(1/4
)]/16 + ArcTanh[(1 + x^3)^(1/4)]/16
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{1+x^3}}{x^7}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[4]{1+x}}{x^3}\,dx, x, x^3\right)\\
&= -\frac{\sqrt[4]{1+x^3}}{6x^6} + \frac{1}{24}\,\text{Subst}\left(\int \frac{1}{x^2(1+x)^{3/4}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt[4]{1+x^3}}{6x^6} - \frac{\sqrt[4]{1+x^3}}{24x^3} - \frac{1}{32}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{3/4}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt[4]{1+x^3}}{6x^6} - \frac{\sqrt[4]{1+x^3}}{24x^3} - \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt[4]{1+x^3}\right)\\
&= -\frac{\sqrt[4]{1+x^3}}{6x^6} - \frac{\sqrt[4]{1+x^3}}{24x^3} + \frac{1}{16}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1+x^3}\right) + \frac{1}{16}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{}\right.\\
&= -\frac{\sqrt[4]{1+x^3}}{6x^6} - \frac{\sqrt[4]{1+x^3}}{24x^3} + \frac{1}{16}\tan^{-1}\left(\sqrt[4]{1+x^3}\right) + \frac{1}{16}\tanh^{-1}\left(\sqrt[4]{1+x^3}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.50

$$-\frac{4}{15}\left(x^3+1\right)^{5/4}\,{}_2F_1\left(\frac{5}{4},3;\frac{9}{4};x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)^(1/4)/x^7,x]
[Out] (-4*(1 + x^3)^(5/4)*Hypergeometric2F1[5/4, 3, 9/4, 1 + x^3])/15
```

IntegrateAlgebraic [A] time = 0.08, size = 52, normalized size = 1.00

$$\frac{1}{16}\tan^{-1}\left(\sqrt[4]{x^3+1}\right)+\frac{1}{16}\tanh^{-1}\left(\sqrt[4]{x^3+1}\right)+\frac{\sqrt[4]{x^3+1}\,\left(-x^3-4\right)}{24x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)^(1/4)/x^7,x]
[Out] ((-4 - x^3)*(1 + x^3)^(1/4))/(24*x^6) + ArcTan[(1 + x^3)^(1/4)]/16 + ArcTan
h[(1 + x^3)^(1/4)]/16
```

fricas [A] time = 0.39, size = 63, normalized size = 1.21

$$\frac{6\,x^6\arctan\left(\left(x^3+1\right)^{\frac{1}{4}}\right)+3\,x^6\log\left(\left(x^3+1\right)^{\frac{1}{4}}+1\right)-3\,x^6\log\left(\left(x^3+1\right)^{\frac{1}{4}}-1\right)-4\left(x^3+4\right)\left(x^3+1\right)^{\frac{1}{4}}}{96\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/4)/x^7,x, algorithm="fricas")
```

```
[Out] 1/96*(6*x^6*arctan((x^3 + 1)^(1/4)) + 3*x^6*log((x^3 + 1)^(1/4) + 1) - 3*x^
6*log((x^3 + 1)^(1/4) - 1) - 4*(x^3 + 4)*(x^3 + 1)^(1/4))/x^6
```

giac [A] time = 0.20, size = 58, normalized size = 1.12

$$-\frac{\left(x^3+1\right)^{\frac{5}{4}}+3\left(x^3+1\right)^{\frac{1}{4}}}{24\,x^6}+\frac{1}{16}\,\arctan\left(\left(x^3+1\right)^{\frac{1}{4}}\right)+\frac{1}{32}\,\log\left(\left(x^3+1\right)^{\frac{1}{4}}+1\right)-\frac{1}{32}\,\log\left(\left|\left(x^3+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/4)/x^7,x, algorithm="giac")
```

```
[Out] -1/24*((x^3 + 1)^(5/4) + 3*(x^3 + 1)^(1/4))/x^6 + 1/16*arctan((x^3 + 1)^(1/
4)) + 1/32*log((x^3 + 1)^(1/4) + 1) - 1/32*log(abs((x^3 + 1)^(1/4) - 1))
```

maple [C] time = 0.14, size = 66, normalized size = 1.27

$$-\frac{x^6+5x^3+4}{24x^6\left(x^3+1\right)^{\frac{3}{4}}}-\frac{-\frac{3\Gamma\left(\frac{3}{4}\right)x^3\,\mathrm{hypergeom}\left(\left[1,1,\frac{7}{4}\right],[2,2],-x^3\right)}{4}+\left(-3\ln(2)+\frac{\pi}{2}+3\ln(x)\right)\Gamma\left(\frac{3}{4}\right)}{32\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(1/4)/x^7,x)
```

```
[Out] -1/24*(x^6+5*x^3+4)/x^6/(x^3+1)^(3/4)-1/32/GAMMA(3/4)*(-3/4*GAMMA(3/4)*x^3*
hypergeom([1,1,7/4],[2,2],-x^3)+(-3*ln(2)+1/2*Pi+3*ln(x))*GAMMA(3/4))
```

maxima [A] time = 0.42, size = 72, normalized size = 1.38

$$\frac{\left(x^3+1\right)^{\frac{5}{4}}+3\left(x^3+1\right)^{\frac{1}{4}}}{24\left(2\,x^3-\left(x^3+1\right)^2+1\right)}+\frac{1}{16}\,\arctan\left(\left(x^3+1\right)^{\frac{1}{4}}\right)+\frac{1}{32}\,\log\left(\left(x^3+1\right)^{\frac{1}{4}}+1\right)-\frac{1}{32}\,\log\left(\left(x^3+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/4)/x^7,x, algorithm="maxima")
```

```
[Out] 1/24*((x^3 + 1)^(5/4) + 3*(x^3 + 1)^(1/4))/(2*x^3 - (x^3 + 1)^2 + 1) + 1/16
*arctan((x^3 + 1)^(1/4)) + 1/32*log((x^3 + 1)^(1/4) + 1) - 1/32*log((x^3 +
1)^(1/4) - 1)
```

mupad [B] time = 0.69, size = 45, normalized size = 0.87

$$\frac{\mathrm{atan}\left(\left(x^3+1\right)^{1/4}\right)}{16}+\frac{\mathrm{atanh}\left(\left(x^3+1\right)^{1/4}\right)}{16}-\frac{\left(x^3+1\right)^{1/4}}{8\,x^6}-\frac{\left(x^3+1\right)^{5/4}}{24\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)^(1/4)/x^7,x)
```

```
[Out] atan((x^3 + 1)^(1/4))/16 + atanh((x^3 + 1)^(1/4))/16 - (x^3 + 1)^(1/4)/(8*x
^6) - (x^3 + 1)^(5/4)/(24*x^6)
```

sympy [C] time = 1.16, size = 34, normalized size = 0.65

$$-\frac{\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{e^{i\pi}}{x^{3}}\right)}{3x^{\frac{21}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(1/4)/x**7,x)
```

```
[Out] -gamma(7/4)*hyper((-1/4, 7/4), (11/4,), exp_polar(I*pi)/x**3)/(3*x**(21/4)*
gamma(11/4))
```

3.648 $\int \frac{\sqrt{b+ax^3}\left(2b+ax^3\right)}{x}\,dx$

Optimal. Leaf size=52

$$\frac{2}{9}\sqrt{ax^3+b}\left(ax^3+7b\right)-\frac{4}{3}b^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 80, 50, 63, 208}

$$-\frac{4}{3}b^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)+\frac{4}{3}b\sqrt{ax^3+b}+\frac{2}{9}\left(ax^3+b\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b + a*x^3]*(2*b + a*x^3))/x,x]
```

```
[Out] (4*b*Sqrt[b + a*x^3])/3 + (2*(b + a*x^3)^(3/2))/9 - (4*b^(3/2)*ArcTanh[Sqrt
[b + a*x^3]/Sqrt[b]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{b+ax^3}\left(2b+ax^3\right)}{x}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{b+ax}\,(2b+ax)}{x}\,dx, x, x^3\right)\\
&= \frac{2}{9}\left(b+ax^3\right)^{3/2}+\frac{1}{3}(2b)\,\text{Subst}\left(\int \frac{\sqrt{b+ax}}{x}\,dx, x, x^3\right)\\
&= \frac{4}{3}b\sqrt{b+ax^3}+\frac{2}{9}\left(b+ax^3\right)^{3/2}+\frac{1}{3}\left(2b^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{b+ax}}\,dx, x, x^3\right)\\
&= \frac{4}{3}b\sqrt{b+ax^3}+\frac{2}{9}\left(b+ax^3\right)^{3/2}+\frac{\left(4b^2\right)\text{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^2}{a}}\,dx, x, \sqrt{b+ax^3}\right)}{3a}\\
&= \frac{4}{3}b\sqrt{b+ax^3}+\frac{2}{9}\left(b+ax^3\right)^{3/2}-\frac{4}{3}b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b+ax^3}}{\sqrt{b}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.98

$$\frac{2}{9}\left(\sqrt{ax^3+b}\left(ax^3+7b\right)-6b^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b + a*x^3]*(2*b + a*x^3))/x,x]
```

```
[Out] (2*(Sqrt[b + a*x^3]*(7*b + a*x^3) - 6*b^(3/2)*ArcTanh[Sqrt[b + a*x^3]/Sqrt[
b]]))/9
```

IntegrateAlgebraic [A] time = 0.05, size = 52, normalized size = 1.00

$$\frac{2}{9}\sqrt{ax^3+b}\left(ax^3+7b\right)-\frac{4}{3}b^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[b + a*x^3]*(2*b + a*x^3))/x,x]
```

```
[Out] (2*Sqrt[b + a*x^3]*(7*b + a*x^3))/9 - (4*b^(3/2)*ArcTanh[Sqrt[b + a*x^3]/Sq
rt[b]])/3
```

fricas [A] time = 0.40, size = 103, normalized size = 1.98

$$\left[\frac{2}{3}\,b^{\frac{3}{2}}\log\left(\frac{ax^3-2\,\sqrt{ax^3+b}\,\sqrt{b}+2\,b}{x^3}\right)+\frac{2}{9}\left(ax^3+7\,b\right)\sqrt{ax^3+b},\ \frac{4}{3}\,\sqrt{-b}\,b\arctan\left(\frac{\sqrt{ax^3+b}\,\sqrt{-b}}{b}\right)+\frac{2}{9}\left(ax^3+7\,b\right)\sqrt{ax^3+b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)^(1/2)*(a*x^3+2*b)/x,x, algorithm="fricas")
```

```
[Out] [2/3*b^(3/2)*log((a*x^3 - 2*sqrt(a*x^3 + b)*sqrt(b) + 2*b)/x^3) + 2/9*(a*x^
3 + 7*b)*sqrt(a*x^3 + b), 4/3*sqrt(-b)*b*arctan(sqrt(a*x^3 + b)*sqrt(-b)/b)
 + 2/9*(a*x^3 + 7*b)*sqrt(a*x^3 + b)]
```

giac [A] time = 0.14, size = 50, normalized size = 0.96

$$\frac{4\,b^2\arctan\left(\frac{\sqrt{ax^3+b}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}}+\frac{2}{9}\left(ax^3+b\right)^{\frac{3}{2}}+\frac{4}{3}\,\sqrt{ax^3+b}\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)^(1/2)*(a*x^3+2*b)/x,x, algorithm="giac")
```

```
[Out] 4/3*b^2*arctan(sqrt(a*x^3 + b)/sqrt(-b))/sqrt(-b) + 2/9*(a*x^3 + b)^(3/2) +
 4/3*sqrt(a*x^3 + b)*b
```

maple [A] time = 0.05, size = 47, normalized size = 0.90

$$\frac{2\left(a\,x^{3}+b\right)^{\frac{3}{2}}}{9}+2b\left(\frac{2\sqrt{a\,x^{3}+b}}{3}-\frac{2\sqrt{b}\ \operatorname{arctanh}\left(\frac{\sqrt{a\,x^{3}+b}}{\sqrt{b}}\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b)^(1/2)*(a*x^3+2*b)/x,x)
```

```
[Out] 2/9*(a*x^3+b)^(3/2)+2*b*(2/3*(a*x^3+b)^(1/2)-2/3*b^(1/2)*arctanh((a*x^3+b)^
(1/2)/b^(1/2)))
```

maxima [A] time = 0.42, size = 63, normalized size = 1.21

$$\frac{2}{3}\left(\sqrt{b}\log\left(\frac{\sqrt{ax^{3}+b}-\sqrt{b}}{\sqrt{ax^{3}+b}+\sqrt{b}}\right)+2\sqrt{ax^{3}+b}\right)b+\frac{2}{9}\left(ax^{3}+b\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)^(1/2)*(a*x^3+2*b)/x,x, algorithm="maxima")
```

```
[Out] 2/3*(sqrt(b)*log((sqrt(a*x^3 + b) - sqrt(b))/(sqrt(a*x^3 + b) + sqrt(b))) +
 2*sqrt(a*x^3 + b))*b + 2/9*(a*x^3 + b)^(3/2)
```

mupad [B] time = 0.70, size = 68, normalized size = 1.31

$$\frac{2\,b^{3/2}\,\ln\left(\frac{\left(\sqrt{a\,x^3+b}-\sqrt{b}\right)^3\,\left(\sqrt{a\,x^3+b}+\sqrt{b}\right)}{x^6}\right)}{3}+\frac{14\,b\,\sqrt{a\,x^3+b}}{9}+\frac{2\,a\,x^3\,\sqrt{a\,x^3+b}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a*x^3)^(1/2)*(2*b + a*x^3))/x,x)
```

```
[Out] (2*b^(3/2)*log((((b + a*x^3)^(1/2) - b^(1/2))^3*((b + a*x^3)^(1/2) + b^(1/2
)))/x^6))/3 + (14*b*(b + a*x^3)^(1/2))/9 + (2*a*x^3*(b + a*x^3)^(1/2))/9
```

sympy [A] time = 20.74, size = 78, normalized size = 1.50

$$-\frac{a\left(\begin{cases}-\sqrt{b}\,x^{3} & \text{for}\ a=0\\ -\frac{2\left(ax^{3}+b\right)^{\frac{3}{2}}}{3a} & \text{otherwise}\end{cases}\right)}{3}-\frac{2b\left(-\frac{2b\operatorname{atan}\left(\frac{\sqrt{ax^{3}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-2\sqrt{ax^{3}+b}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b)**(1/2)*(a*x**3+2*b)/x,x)
```

```
[Out] -a*Piecewise((-sqrt(b)*x**3, Eq(a, 0)), (-2*(a*x**3 + b)**(3/2)/(3*a), True
))/3 - 2*b*(-2*b*atan(sqrt(a*x**3 + b)/sqrt(-b))/sqrt(-b) - 2*sqrt(a*x**3 +
 b))/3
```

3.649 $\int \frac{\left(1+x^3\right)\sqrt{-1+x^6}}{x^{13}\left(-1+x^3\right)}\,dx$

Optimal. Leaf size=52

$$\frac{7}{12}\tan^{-1}\left(\sqrt{x^6-1}+x^3\right)+\frac{\sqrt{x^6-1}\left(32x^9+21x^6+16x^3+6\right)}{72x^{12}}$$

Rubi [F] time = 0.62, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+x^3\right)\sqrt{-1+x^6}}{x^{13}\left(-1+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^3)*Sqrt[-1 + x^6])/(x^13*(-1 + x^3)),x]
```

```
[Out] (-2*Sqrt[-1 + x^6])/3 + Sqrt[-1 + x^6]/(12*x^12) + (7*Sqrt[-1 + x^6])/(24*x
^6) + (2*Sqrt[-1 + x^6])/(3*x^3) - (2*(-1 + x^6)^(3/2))/(9*x^9) + (7*ArcTan
[Sqrt[-1 + x^6]])/24 - (2*ArcTanh[x^3/Sqrt[-1 + x^6]])/3 + (2*Defer[Int][Sq
rt[-1 + x^6]/(-1 + x), x])/3 + (4*Defer[Int][Sqrt[-1 + x^6]/(1 - I*Sqrt[3]
+ 2*x), x])/3 + (4*Defer[Int][Sqrt[-1 + x^6]/(1 + I*Sqrt[3] + 2*x), x])/3
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(1+x^3\right)\sqrt{-1+x^6}}{x^{13}\left(-1+x^3\right)}\,dx &= \int \left(\frac{2\sqrt{-1+x^6}}{3(-1+x)}-\frac{\sqrt{-1+x^6}}{x^{13}}-\frac{2\sqrt{-1+x^6}}{x^{10}}-\frac{2\sqrt{-1+x^6}}{x^7}-\frac{2\sqrt{-1+x^6}}{x^4}-\frac{2\sqrt{-1+}}{x}\right.\\
&= \frac{2}{3}\int \frac{\sqrt{-1+x^6}}{-1+x}\,dx+\frac{2}{3}\int \frac{(1+2x)\sqrt{-1+x^6}}{1+x+x^2}\,dx-2\int \frac{\sqrt{-1+x^6}}{x^{10}}\,dx-2\int \frac{\sqrt{-1+}}{x^7}\\
&= -\frac{2\left(-1+x^6\right)^{3/2}}{9x^9}-\frac{1}{6}\,\mathrm{Subst}\left(\int \frac{\sqrt{-1+x}}{x^3}\,dx,x,x^6\right)-\frac{1}{3}\,\mathrm{Subst}\left(\int \frac{\sqrt{-1+x}}{x^2}\,dx,x,x\right.\\
&= -\frac{2}{3}\sqrt{-1+x^6}+\frac{\sqrt{-1+x^6}}{12x^{12}}+\frac{\sqrt{-1+x^6}}{3x^6}+\frac{2\sqrt{-1+x^6}}{3x^3}-\frac{2\left(-1+x^6\right)^{3/2}}{9x^9}-\frac{1}{24}\,\mathrm{Subs}\\
&= -\frac{2}{3}\sqrt{-1+x^6}+\frac{\sqrt{-1+x^6}}{12x^{12}}+\frac{7\sqrt{-1+x^6}}{24x^6}+\frac{2\sqrt{-1+x^6}}{3x^3}-\frac{2\left(-1+x^6\right)^{3/2}}{9x^9}-\frac{1}{48}\,\mathrm{Sub}\\
&= -\frac{2}{3}\sqrt{-1+x^6}+\frac{\sqrt{-1+x^6}}{12x^{12}}+\frac{7\sqrt{-1+x^6}}{24x^6}+\frac{2\sqrt{-1+x^6}}{3x^3}-\frac{2\left(-1+x^6\right)^{3/2}}{9x^9}+\frac{1}{3}\tan^{-}\\
&= -\frac{2}{3}\sqrt{-1+x^6}+\frac{\sqrt{-1+x^6}}{12x^{12}}+\frac{7\sqrt{-1+x^6}}{24x^6}+\frac{2\sqrt{-1+x^6}}{3x^3}-\frac{2\left(-1+x^6\right)^{3/2}}{9x^9}+\frac{7}{24}\tan
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 47, normalized size = 0.90

$$\frac{1}{72}\left(21\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}\left(32x^9+21x^6+16x^3+6\right)}{x^{12}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)*Sqrt[-1 + x^6])/(x^13*(-1 + x^3)),x]
```

```
[Out] ((Sqrt[-1 + x^6]*(6 + 16*x^3 + 21*x^6 + 32*x^9))/x^12 + 21*ArcTan[Sqrt[-1 +
 x^6]])/72
```

IntegrateAlgebraic [A] time = 0.23, size = 54, normalized size = 1.04

$$\frac{\sqrt{x^6-1}\left(32x^9+21x^6+16x^3+6\right)}{72x^{12}}-\frac{7}{12}\tan^{-1}\left(x^3-\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)*Sqrt[-1 + x^6])/(x^13*(-1 + x^3)),x]
```

```
[Out] (Sqrt[-1 + x^6]*(6 + 16*x^3 + 21*x^6 + 32*x^9))/(72*x^12) - (7*ArcTan[x^3 -
 Sqrt[-1 + x^6]])/12
```

fricas [A] time = 0.39, size = 55, normalized size = 1.06

$$\frac{42\,x^{12}\arctan\left(-x^3+\sqrt{x^6-1}\right)+32\,x^{12}+\left(32\,x^9+21\,x^6+16\,x^3+6\right)\sqrt{x^6-1}}{72\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(x^6-1)^(1/2)/x^13/(x^3-1),x, algorithm="fricas")
```

```
[Out] 1/72*(42*x^12*arctan(-x^3 + sqrt(x^6 - 1)) + 32*x^12 + (32*x^9 + 21*x^6 + 1
6*x^3 + 6)*sqrt(x^6 - 1))/x^12
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^6-1}\left(x^3+1\right)}{\left(x^3-1\right)x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(x^6-1)^(1/2)/x^13/(x^3-1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^6 - 1)*(x^3 + 1)/((x^3 - 1)*x^13), x)
```

maple [A] time = 0.06, size = 47, normalized size = 0.90

$$\frac{32x^{15}+21x^{12}-16x^9-15x^6-16x^3-6}{72x^{12}\sqrt{x^6-1}}-\frac{7\arcsin\left(\frac{1}{x^3}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)*(x^6-1)^(1/2)/x^13/(x^3-1),x)
```

```
[Out] 1/72*(32*x^15+21*x^12-16*x^9-15*x^6-16*x^3-6)/x^12/(x^6-1)^(1/2)-7/24*arcsi
n(1/x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^6-1}\left(x^3+1\right)}{\left(x^3-1\right)x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(x^6-1)^(1/2)/x^13/(x^3-1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^6 - 1)*(x^3 + 1)/((x^3 - 1)*x^13), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x^3+1\right)\sqrt{x^6-1}}{x^{13}\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)*(x^6 - 1)^(1/2))/(x^13*(x^3 - 1)),x)
```

```
[Out] int(((x^3 + 1)*(x^6 - 1)^(1/2))/(x^13*(x^3 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}{x^{13}(x-1)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)*(x**6-1)**(1/2)/x**13/(x**3-1),x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1))*(x + 1)*(x**2
- x + 1)/(x**13*(x - 1)*(x**2 + x + 1)), x)
```

3.650 $\int \frac{\sqrt{x+x^4}\left(-b+ax^6\right)}{x^6}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{x^4+x}\left(3ax^6+2bx^3+2b\right)}{9x^5}+\frac{1}{3}a\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)$$

Rubi [C] time = 0.30, antiderivative size = 163, normalized size of antiderivative = 3.13, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2048, 2052, 2014, 2020, 2011, 329, 225}

$$\frac{a\left(x^4+x\right)^{3/2}}{3x^3}-\frac{2a\sqrt{x^4+x}}{5x^3}+\frac{3^{3/4}ax(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^4+x}}+\frac{2b\left(x^4+x\right)^{3/2}}{9x^6}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[x + x^4]*(-b + a*x^6))/x^6,x]
```

```
[Out] (-2*a*Sqrt[x + x^4])/(5*x^3) + (2*b*(x + x^4)^(3/2))/(9*x^6) + (a*(x + x^4)
^(3/2))/(3*x^3) + (3^(3/4)*a*x*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3]
)*x)^2]*EllipticF[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 +
 Sqrt[3])/4])/(5*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[x + x^4])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
```

```
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2048

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Int[(c*x)^
m*ExpandToSum[Pq - Pqq*x^q - (a*Pqq*(m + q - n + 1)*x^(q - n))/(b*(m + q +
n*p + 1)), x]*(a*x^j + b*x^n)^p, x] + Simp[(Pqq*(c*x)^(m + q - n + 1)*(a*x^
j + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /; GtQ[q, n -
1] && NeQ[m + q + n*p + 1, 0] && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*
n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] &&  !IntegerQ[p] && IGtQ
[j, 0] && IGtQ[n, 0] && LtQ[j, n]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\sqrt{x+x^4}\left(-b+ax^6\right)}{x^6}\,dx = \frac{a\left(x+x^4\right)^{3/2}}{3x^3}+\int \frac{\left(-b+ax^2\right)\sqrt{x+x^4}}{x^6}\,dx$$
$$= \frac{a\left(x+x^4\right)^{3/2}}{3x^3}+\int \left(-\frac{b\sqrt{x+x^4}}{x^6}+\frac{a\sqrt{x+x^4}}{x^4}\right)dx$$
$$= \frac{a\left(x+x^4\right)^{3/2}}{3x^3}+a\int \frac{\sqrt{x+x^4}}{x^4}\,dx-b\int \frac{\sqrt{x+x^4}}{x^6}\,dx$$
$$= -\frac{2a\sqrt{x+x^4}}{5x^3}+\frac{2b\left(x+x^4\right)^{3/2}}{9x^6}+\frac{a\left(x+x^4\right)^{3/2}}{3x^3}+\frac{1}{5}(3a)\int \frac{1}{\sqrt{x+x^4}}\,dx$$
$$= -\frac{2a\sqrt{x+x^4}}{5x^3}+\frac{2b\left(x+x^4\right)^{3/2}}{9x^6}+\frac{a\left(x+x^4\right)^{3/2}}{3x^3}+\frac{\left(3a\sqrt{x}\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{x}\sqrt{1+x^3}}\,dx}{5\sqrt{x+x^4}}$$
$$= -\frac{2a\sqrt{x+x^4}}{5x^3}+\frac{2b\left(x+x^4\right)^{3/2}}{9x^6}+\frac{a\left(x+x^4\right)^{3/2}}{3x^3}+\frac{\left(6a\sqrt{x}\sqrt{1+x^3}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^6}}\right.}{5\sqrt{x+x^4}}$$
$$= -\frac{2a\sqrt{x+x^4}}{5x^3}+\frac{2b\left(x+x^4\right)^{3/2}}{9x^6}+\frac{a\left(x+x^4\right)^{3/2}}{3x^3}+\frac{3^{3/4}ax(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(c\right.}{5\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)\right.}}}$$

Mathematica [A] time = 0.07, size = 65, normalized size = 1.25

$$\frac{\sqrt{x^4+x}\left(\sqrt{x^3+1}\left(3ax^6+2b\left(x^3+1\right)\right)+3ax^{9/2}\sinh^{-1}\left(x^{3/2}\right)\right)}{9x^5\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x + x^4]*(-b + a*x^6))/x^6,x]
```

```
[Out] (Sqrt[x + x^4]*(Sqrt[1 + x^3]*(3*a*x^6 + 2*b*(1 + x^3)) + 3*a*x^(9/2)*ArcSi
nh[x^(3/2)]))/(9*x^5*Sqrt[1 + x^3])
```

IntegrateAlgebraic [A] time = 0.50, size = 52, normalized size = 1.00

$$\frac{\sqrt{x^4+x}\,\left(3ax^6+2bx^3+2b\right)}{9x^5}+\frac{1}{3}a\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x + x^4]*(-b + a*x^6))/x^6,x]
```

```
[Out] (Sqrt[x + x^4]*(2*b + 2*b*x^3 + 3*a*x^6))/(9*x^5) + (a*ArcTanh[x^2/Sqrt[x +
 x^4]])/3
```

fricas [A] time = 0.43, size = 55, normalized size = 1.06

$$\frac{3\,ax^5\log\left(-2\,x^3-2\,\sqrt{x^4+x}\,x-1\right)+2\left(3\,ax^6+2\,bx^3+2\,b\right)\sqrt{x^4+x}}{18\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/2)*(a*x^6-b)/x^6,x, algorithm="fricas")
```

```
[Out] 1/18*(3*a*x^5*log(-2*x^3 - 2*sqrt(x^4 + x)*x - 1) + 2*(3*a*x^6 + 2*b*x^3 +
2*b)*sqrt(x^4 + x))/x^5
```

giac [A] time = 0.40, size = 49, normalized size = 0.94

$$\frac{1}{3}\sqrt{x^4+x}\,ax+\frac{2}{9}\,b\left(\frac{1}{x^3}+1\right)^{\frac{3}{2}}+\frac{1}{6}\,a\log\left(\sqrt{\frac{1}{x^3}+1}+1\right)-\frac{1}{6}\,a\log\left(\left|\sqrt{\frac{1}{x^3}+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/2)*(a*x^6-b)/x^6,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(x^4 + x)*a*x + 2/9*b*(1/x^3 + 1)^(3/2) + 1/6*a*log(sqrt(1/x^3 + 1)
 + 1) - 1/6*a*log(abs(sqrt(1/x^3 + 1) - 1))
```

maple [C] time = 0.19, size = 332, normalized size = 6.38

$$a\left(\frac{x\sqrt{x^4+x}}{3}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\,(1+x)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(1+x)}}\,\sqrt{-\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\left(-\mathrm{EllipticF}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)+\mathrm{EllipticPi}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\frac{\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(1+x)\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)-b\left(-\frac{2\sqrt{x^4+x}}{9x^5}-\frac{2\sqrt{x^4+x}}{9x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x)^(1/2)*(a*x^6-b)/x^6,x)
```

```
[Out] a*(1/3*x*(x^4+x)^(1/2)-(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1/2
*I*3^(1/2))/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2)
)/(1+x))^(1/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(3/
2+1/2*I*3^(1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1/2
)*(-EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-3/
2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1
/2)))^(1/2))+EllipticPi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(
1/2),(1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-1/
2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))-b*(-2/9*(x
^4+x)^(1/2)/x^5-2/9*(x^4+x)^(1/2)/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^6-b\right)\sqrt{x^4+x}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/2)*(a*x^6-b)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((a*x^6 - b)*sqrt(x^4 + x)/x^6, x)
```

mupad [B] time = 0.99, size = 47, normalized size = 0.90

$$\frac{2\,b\,\left(x^3+1\right)\,\sqrt{x^4+x}}{9\,x^5}+\frac{2\,a\,x\,\sqrt{x^4+x}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{2};\ \frac{3}{2};\ -x^3\right)}{3\,\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b - a*x^6)*(x + x^4)^(1/2))/x^6,x)
```

```
[Out] (2*b*(x^3 + 1)*(x + x^4)^(1/2))/(9*x^5) + (2*a*x*(x + x^4)^(1/2)*hypergeom(
[-1/2, 1/2], 3/2, -x^3))/(3*(x^3 + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(x+1\right)\left(x^2-x+1\right)}\,\left(ax^6-b\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x)**(1/2)*(a*x**6-b)/x**6,x)
```

```
[Out] Integral(sqrt(x*(x + 1)*(x**2 - x + 1))*(a*x**6 - b)/x**6, x)
```

3.651 $\int \frac{-7bx+5ax^3}{\sqrt[4]{-bx+ax^3}\left(b-ax^2+x^7\right)}\,dx$

Optimal. Leaf size=52

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^3-bx}}{x^2}\right)-2\tanh^{-1}\left(\frac{x\left(ax^3-bx\right)^{3/4}}{ax^2-b}\right)$$

Rubi [F] time = 2.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-7bx+5ax^3}{\sqrt[4]{-bx+ax^3}\left(b-ax^2+x^7\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-7*b*x + 5*a*x^3)/((-(b*x) + a*x^3)^(1/4)*(b - a*x^2 + x^7)),x]
```

```
[Out] (-20*a*x^(1/4)*(-b + a*x^2)^(1/4)*Defer[Subst][Defer[Int][x^14/((-b + a*x^8
)^(1/4)*(-b + a*x^8 - x^28)), x], x, x^(1/4)])/(-(b*x) + a*x^3)^(1/4) - (28
*b*x^(1/4)*(-b + a*x^2)^(1/4)*Defer[Subst][Defer[Int][x^6/((-b + a*x^8)^(1/
4)*(b - a*x^8 + x^28)), x], x, x^(1/4)])/(-(b*x) + a*x^3)^(1/4)
```

Rubi steps

$$\begin{aligned}
\int \frac{-7bx+5ax^3}{\sqrt[4]{-bx+ax^3}\left(b-ax^2+x^7\right)}\,dx &= \int \frac{x\left(-7b+5ax^2\right)}{\sqrt[4]{-bx+ax^3}\left(b-ax^2+x^7\right)}\,dx\\
&= \frac{\left(\sqrt[4]{x}\sqrt[4]{-b+ax^2}\right)\int \frac{x^{3/4}\left(-7b+5ax^2\right)}{\sqrt[4]{-b+ax^2}\left(b-ax^2+x^7\right)}\,dx}{\sqrt[4]{-bx+ax^3}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^2}\right)\text{Subst}\left(\int \frac{x^6\left(-7b+5ax^8\right)}{\sqrt[4]{-b+ax^8}\left(b-ax^8+x^{28}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^3}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^2}\right)\text{Subst}\left(\int \left(-\frac{5ax^{14}}{\sqrt[4]{-b+ax^8}\left(-b+ax^8-x^{28}\right)}-\frac{7bx^6}{\sqrt[4]{-b+ax^8}\left(b-ax^8+x^{28}\right)}\right)dx,\right.}{\sqrt[4]{-bx+ax^3}}\\
&= -\frac{\left(20a\sqrt[4]{x}\sqrt[4]{-b+ax^2}\right)\text{Subst}\left(\int \frac{x^{14}}{\sqrt[4]{-b+ax^8}\left(-b+ax^8-x^{28}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^3}}-\frac{\left(28b\sqrt[4]{x}\sqrt[4]{\right.}{}
\end{aligned}$$

Mathematica [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{-7bx+5ax^3}{\sqrt[4]{-bx+ax^3}\left(b-ax^2+x^7\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-7*b*x + 5*a*x^3)/((-(b*x) + a*x^3)^(1/4)*(b - a*x^2 + x^7)),x]
```

```
[Out] Integrate[(-7*b*x + 5*a*x^3)/((-(b*x) + a*x^3)^(1/4)*(b - a*x^2 + x^7)), x]
```

IntegrateAlgebraic [A] time = 2.27, size = 52, normalized size = 1.00

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^3-bx}}{x^2}\right)-2\tanh^{-1}\left(\frac{x\left(ax^3-bx\right)^{3/4}}{ax^2-b}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-7*b*x + 5*a*x^3)/((-(b*x) + a*x^3)^(1/4)*(b - a*x^2 +
x^7)),x]
```

```
[Out] 2*ArcTan[(-(b*x) + a*x^3)^(1/4)/x^2] - 2*ArcTanh[(x*(-(b*x) + a*x^3)^(3/4))
/(-b + a*x^2)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*a*x^3-7*b*x)/(a*x^3-b*x)^(1/4)/(x^7-a*x^2+b),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,ax^3-7\,bx}{\left(x^7-ax^2+b\right)\left(ax^3-bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*a*x^3-7*b*x)/(a*x^3-b*x)^(1/4)/(x^7-a*x^2+b),x, algorithm="gia
c")
```

```
[Out] integrate((5*a*x^3 - 7*b*x)/((x^7 - a*x^2 + b)*(a*x^3 - b*x)^(1/4)), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{5a\,x^3-7bx}{\left(a\,x^3-bx\right)^{\frac{1}{4}}\left(x^7-a\,x^2+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*a*x^3-7*b*x)/(a*x^3-b*x)^(1/4)/(x^7-a*x^2+b),x)
```

```
[Out] int((5*a*x^3-7*b*x)/(a*x^3-b*x)^(1/4)/(x^7-a*x^2+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,ax^3-7\,bx}{\left(x^7-ax^2+b\right)\left(ax^3-bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*a*x^3-7*b*x)/(a*x^3-b*x)^(1/4)/(x^7-a*x^2+b),x, algorithm="max
ima")
```

```
[Out] integrate((5*a*x^3 - 7*b*x)/((x^7 - a*x^2 + b)*(a*x^3 - b*x)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{7\,b\,x-5\,a\,x^3}{{\left(a\,x^3-b\,x\right)}^{1/4}\,\left(x^7-a\,x^2+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(7*b*x - 5*a*x^3)/((a*x^3 - b*x)^(1/4)*(b - a*x^2 + x^7)),x)
```

```
[Out] -int((7*b*x - 5*a*x^3)/((a*x^3 - b*x)^(1/4)*(b - a*x^2 + x^7)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(5 a x^{2} - 7 b\right)}{\sqrt[4]{x \left(a x^{2} - b\right)} \left(- a x^{2} + b + x^{7}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*a*x**3-7*b*x)/(a*x**3-b*x)**(1/4)/(x**7-a*x**2+b),x)
```

```
[Out] Integral(x*(5*a*x**2 - 7*b)/((x*(a*x**2 - b))**(1/4)*(-a*x**2 + b + x**7)),
 x)
```

3.652 $\int \frac{x+3x^5}{\sqrt{-x+x^5}\left(1-ax^2-2x^4+x^8\right)}\,dx$

Optimal. Leaf size=52

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{x^5-x}}\right)}{a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{x^5-x}}\right)}{a^{3/4}}$$

Rubi [F] time = 1.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x+3x^5}{\sqrt{-x+x^5}\left(1-ax^2-2x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x + 3*x^5)/(Sqrt[-x + x^5]*(1 - a*x^2 - 2*x^4 + x^8)),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[-1 + x^4]*Defer[Subst][Defer[Int][x^2/(Sqrt[-1 + x^8]*(1 -
a*x^4 - 2*x^8 + x^16)), x], x, Sqrt[x]])/Sqrt[-x + x^5] + (6*Sqrt[x]*Sqrt[-
1 + x^4]*Defer[Subst][Defer[Int][x^10/(Sqrt[-1 + x^8]*(1 - a*x^4 - 2*x^8 +
x^16)), x], x, Sqrt[x]])/Sqrt[-x + x^5]
```

Rubi steps

$$\begin{aligned}
\int \frac{x+3x^5}{\sqrt{-x+x^5}\left(1-ax^2-2x^4+x^8\right)}\,dx &= \int \frac{x\left(1+3x^4\right)}{\sqrt{-x+x^5}\left(1-ax^2-2x^4+x^8\right)}\,dx\\
&= \frac{\left(\sqrt{x}\sqrt{-1+x^4}\right)\int \frac{\sqrt{x}\left(1+3x^4\right)}{\sqrt{-1+x^4}\left(1-ax^2-2x^4+x^8\right)}\,dx}{\sqrt{-x+x^5}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \frac{x^2\left(1+3x^8\right)}{\sqrt{-1+x^8}\left(1-ax^4-2x^8+x^{16}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^5}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \left(\frac{x^2}{\sqrt{-1+x^8}\left(1-ax^4-2x^8+x^{16}\right)}+\frac{3x^{10}}{\sqrt{-1+x^8}\left(1-ax^4-2x^8+\right.}\right.\right.}{\sqrt{-x+x^5}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x^8}\left(1-ax^4-2x^8+x^{16}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^5}}+\frac{\left(6\sqrt{x}\right.}{}
\end{aligned}$$

Mathematica [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{x+3x^5}{\sqrt{-x+x^5}\left(1-ax^2-2x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x + 3*x^5)/(Sqrt[-x + x^5]*(1 - a*x^2 - 2*x^4 + x^8)),x]
```

```
[Out] Integrate[(x + 3*x^5)/(Sqrt[-x + x^5]*(1 - a*x^2 - 2*x^4 + x^8)), x]
```

IntegrateAlgebraic [A] time = 2.03, size = 52, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{x^5-x}}\right)}{a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{x^5-x}}\right)}{a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + 3*x^5)/(Sqrt[-x + x^5]*(1 - a*x^2 - 2*x^4 + x^8)),x
]
```

```
[Out] ArcTan[(a^(1/4)*x)/Sqrt[-x + x^5]]/a^(3/4) - ArcTanh[(a^(1/4)*x)/Sqrt[-x +
x^5]]/a^(3/4)
```

fricas [B] time = 0.53, size = 231, normalized size = 4.44

$$\left(\frac{1}{a^3}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{x^5-x}\, a \left(\frac{1}{a^3}\right)^{\frac{1}{4}}}{x^4-1}\right) - \frac{1}{4}\left(\frac{1}{a^3}\right)^{\frac{1}{4}} \log\left(\frac{x^8-2x^4+ax^2+2\sqrt{x^5-x}\left(a^3\left(\frac{1}{a^3}\right)^{\frac{3}{4}}x+\left(ax^4-a\right)\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\right)+2\left(a^2x^5-a^2x\right)\sqrt{\frac{1}{a^3}}+1}{x^8-2x^4-ax^2+1}\right) + \frac{1}{4}\left(\frac{1}{a^3}\right)^{\frac{1}{4}} \log\left(\frac{x^8-2x^4+ax^2-2\sqrt{x^5-x}\left(a^3\left(\frac{1}{a^3}\right)^{\frac{3}{4}}x+\left(ax^4-a\right)\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\right)+2\left(a^2x^5-a^2x\right)\sqrt{\frac{1}{a^3}}+1}{x^8-2x^4-ax^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+x)/(x^5-x)^(1/2)/(x^8-2*x^4-a*x^2+1),x, algorithm="fricas"
)
```

```
[Out] (a^(-3))^(1/4)*arctan(sqrt(x^5 - x)*a*(a^(-3))^(1/4)/(x^4 - 1)) - 1/4*(a^(-
3))^(1/4)*log((x^8 - 2*x^4 + a*x^2 + 2*sqrt(x^5 - x)*(a^3*(a^(-3))^(3/4)*x
+ (a*x^4 - a)*(a^(-3))^(1/4)) + 2*(a^2*x^5 - a^2*x)*sqrt(a^(-3)) + 1)/(x^8
- 2*x^4 - a*x^2 + 1)) + 1/4*(a^(-3))^(1/4)*log((x^8 - 2*x^4 + a*x^2 - 2*sqr
t(x^5 - x)*(a^3*(a^(-3))^(3/4)*x + (a*x^4 - a)*(a^(-3))^(1/4)) + 2*(a^2*x^5
 - a^2*x)*sqrt(a^(-3)) + 1)/(x^8 - 2*x^4 - a*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^5+x}{\left(x^8-2\,x^4-ax^2+1\right)\sqrt{x^5-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+x)/(x^5-x)^(1/2)/(x^8-2*x^4-a*x^2+1),x, algorithm="giac")
```

```
[Out] integrate((3*x^5 + x)/((x^8 - 2*x^4 - a*x^2 + 1)*sqrt(x^5 - x)), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{3x^5+x}{\sqrt{x^5-x}\,\left(x^8-2x^4-a\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^5+x)/(x^5-x)^(1/2)/(x^8-2*x^4-a*x^2+1),x)
```

```
[Out] int((3*x^5+x)/(x^5-x)^(1/2)/(x^8-2*x^4-a*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^5+x}{\left(x^8-2\,x^4-ax^2+1\right)\sqrt{x^5-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+x)/(x^5-x)^(1/2)/(x^8-2*x^4-a*x^2+1),x, algorithm="maxima"
)
```

```
[Out] integrate((3*x^5 + x)/((x^8 - 2*x^4 - a*x^2 + 1)*sqrt(x^5 - x)), x)
```

mupad [B] time = 4.01, size = 134, normalized size = 2.58

$$\frac{\ln\left(\frac{a^2+2\,\sqrt{x^5-x}\,{\left(a^3\right)}^{3/4}-a^2\,x^4-a\,x\,\sqrt{a^3}}{a-a\,x^4+x\,\sqrt{a^3}}\right)}{2\,{\left(a^3\right)}^{1/4}}+\frac{\ln\left(\frac{a^2\,1{}\mathrm{i}-2\,\sqrt{x^5-x}\,{\left(a^3\right)}^{3/4}-a^2\,x^4\,1{}\mathrm{i}+a\,x\,\sqrt{a^3}\,1{}\mathrm{i}}{a\,x^4-a+x\,\sqrt{a^3}}\right)\,1{}\mathrm{i}}{2\,{\left(a^3\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3*x^5)/((x^5 - x)^(1/2)*(a*x^2 + 2*x^4 - x^8 - 1)),x)
```

```
[Out] (log((a^2*1i - 2*(x^5 - x)^(1/2)*(a^3)^(3/4) - a^2*x^4*1i + a*x*(a^3)^(1/2)
*1i)/(a*x^4 - a + x*(a^3)^(1/2)))*1i)/(2*(a^3)^(1/4)) + log((a^2 + 2*(x^5 -
 x)^(1/2)*(a^3)^(3/4) - a^2*x^4 - a*x*(a^3)^(1/2))/(a - a*x^4 + x*(a^3)^(1/
2)))/(2*(a^3)^(1/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**5+x)/(x**5-x)**(1/2)/(x**8-2*x**4-a*x**2+1),x)
```

```
[Out] Timed out
```

3.653 $\int \frac{x+3x^5}{\sqrt{-x+x^5}\left(a-x^2-2ax^4+ax^8\right)}\,dx$

Optimal. Leaf size=52

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{x^5-x}}\right)}{\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{x^5-x}}\right)}{\sqrt[4]{a}}$$

Rubi [F] time = 1.51, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x+3x^5}{\sqrt{-x+x^5}\left(a-x^2-2ax^4+ax^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x + 3*x^5)/(Sqrt[-x + x^5]*(a - x^2 - 2*a*x^4 + a*x^8)),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[-1 + x^4]*Defer[Subst][Defer[Int][x^2/(Sqrt[-1 + x^8]*(a -
x^4 - 2*a*x^8 + a*x^16)), x], x, Sqrt[x]])/Sqrt[-x + x^5] + (6*Sqrt[x]*Sqrt
[-1 + x^4]*Defer[Subst][Defer[Int][x^10/(Sqrt[-1 + x^8]*(a - x^4 - 2*a*x^8
+ a*x^16)), x], x, Sqrt[x]])/Sqrt[-x + x^5]
```

Rubi steps

$$\int \frac{x+3x^5}{\sqrt{-x+x^5}\left(a-x^2-2ax^4+ax^8\right)}\,dx=\int \frac{x\left(1+3x^4\right)}{\sqrt{-x+x^5}\left(a-x^2-2ax^4+ax^8\right)}\,dx$$

$$=\frac{\left(\sqrt{x}\sqrt{-1+x^4}\right)\int \frac{\sqrt{x}\left(1+3x^4\right)}{\sqrt{-1+x^4}\left(a-x^2-2ax^4+ax^8\right)}\,dx}{\sqrt{-x+x^5}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \frac{x^2\left(1+3x^8\right)}{\sqrt{-1+x^8}\left(a-x^4-2ax^8+ax^{16}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^5}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \left(\frac{x^2}{\sqrt{-1+x^8}\left(a-x^4-2ax^8+ax^{16}\right)}+\frac{3x^{10}}{\sqrt{-1+x^8}\left(a-x^4-2ax^8+a\right.}\right.\right.}{\sqrt{-x+x^5}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{-1+x^4}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x^8}\left(a-x^4-2ax^8+ax^{16}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^5}}+\frac{\left(6\sqrt{x}\sqrt{-\right.}{}$$

Mathematica [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{x+3x^5}{\sqrt{-x+x^5}\left(a-x^2-2ax^4+ax^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x + 3*x^5)/(Sqrt[-x + x^5]*(a - x^2 - 2*a*x^4 + a*x^8)),x]
```

```
[Out] Integrate[(x + 3*x^5)/(Sqrt[-x + x^5]*(a - x^2 - 2*a*x^4 + a*x^8)), x]
```

IntegrateAlgebraic [A] time = 2.67, size = 52, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{x^5-x}}\right)}{\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{x^5-x}}\right)}{\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + 3*x^5)/(Sqrt[-x + x^5]*(a - x^2 - 2*a*x^4 + a*x^8))
,x]
```

```
[Out] ArcTan[x/(a^(1/4)*Sqrt[-x + x^5])]/a^(1/4) - ArcTanh[x/(a^(1/4)*Sqrt[-x + x
^5])]/a^(1/4)
```

fricas [B] time = 0.52, size = 202, normalized size = 3.88

$$\frac{\arctan\left(\frac{\sqrt{x^5-x}}{(x^4-1)a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}-\frac{\log\left(\frac{ax^8-2\,ax^4+x^2+2\,\sqrt{x^5-x}\left(a^{\frac{1}{4}}x+\frac{ax^4-a}{a^{\frac{1}{4}}}\right)+a+\frac{2\,(ax^5-ax)}{\sqrt{a}}}{ax^8-2\,ax^4-x^2+a}\right)}{4\,a^{\frac{1}{4}}}+\frac{\log\left(\frac{ax^8-2\,ax^4+x^2-2\,\sqrt{x^5-x}\left(a^{\frac{1}{4}}x+\frac{ax^4-a}{a^{\frac{1}{4}}}\right)+a+\frac{2\,(ax^5-ax)}{\sqrt{a}}}{ax^8-2\,ax^4-x^2+a}\right)}{4\,a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+x)/(x^5-x)^(1/2)/(a*x^8-2*a*x^4-x^2+a),x, algorithm="frica
s")
```

```
[Out] arctan(sqrt(x^5 - x)/((x^4 - 1)*a^(1/4)))/a^(1/4) - 1/4*log((a*x^8 - 2*a*x^
4 + x^2 + 2*sqrt(x^5 - x)*(a^(1/4)*x + (a*x^4 - a)/a^(1/4)) + a + 2*(a*x^5
- a*x)/sqrt(a))/(a*x^8 - 2*a*x^4 - x^2 + a))/a^(1/4) + 1/4*log((a*x^8 - 2*a
*x^4 + x^2 - 2*sqrt(x^5 - x)*(a^(1/4)*x + (a*x^4 - a)/a^(1/4)) + a + 2*(a*x
^5 - a*x)/sqrt(a))/(a*x^8 - 2*a*x^4 - x^2 + a))/a^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^5+x}{\left(ax^8-2\,ax^4-x^2+a\right)\sqrt{x^5-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+x)/(x^5-x)^(1/2)/(a*x^8-2*a*x^4-x^2+a),x, algorithm="giac"
)
```

```
[Out] integrate((3*x^5 + x)/((a*x^8 - 2*a*x^4 - x^2 + a)*sqrt(x^5 - x)), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{3x^5+x}{\sqrt{x^5-x}\,\left(a\,x^8-2a\,x^4-x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^5+x)/(x^5-x)^(1/2)/(a*x^8-2*a*x^4-x^2+a),x)
```

```
[Out] int((3*x^5+x)/(x^5-x)^(1/2)/(a*x^8-2*a*x^4-x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^5+x}{\left(ax^8-2\,ax^4-x^2+a\right)\sqrt{x^5-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+x)/(x^5-x)^(1/2)/(a*x^8-2*a*x^4-x^2+a),x, algorithm="maxim
a")
```

```
[Out] integrate((3*x^5 + x)/((a*x^8 - 2*a*x^4 - x^2 + a)*sqrt(x^5 - x)), x)
```

mupad [B] time = 3.56, size = 103, normalized size = 1.98

$$\frac{\ln\left(\frac{x-2\,a^{1/4}\,\sqrt{x^5-x}-\sqrt{a}+\sqrt{a}\,x^4}{x+\sqrt{a}-\sqrt{a}\,x^4}\right)}{2\,a^{1/4}}+\frac{\ln\left(\frac{x+\sqrt{a}-\sqrt{a}\,x^4+a^{1/4}\,\sqrt{x^5-x}\,2\mathrm{i}}{x-\sqrt{a}+\sqrt{a}\,x^4}\right)\,1\mathrm{i}}{2\,a^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^5)/((x^5 - x)^(1/2)*(a - 2*a*x^4 + a*x^8 - x^2)),x)
```

```
[Out] log((x - 2*a^(1/4)*(x^5 - x)^(1/2) - a^(1/2) + a^(1/2)*x^4)/(x + a^(1/2) -
a^(1/2)*x^4))/(2*a^(1/4)) + (log((x + a^(1/4)*(x^5 - x)^(1/2)*2i + a^(1/2)
- a^(1/2)*x^4)/(x - a^(1/2) + a^(1/2)*x^4))*1i)/(2*a^(1/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**5+x)/(x**5-x)**(1/2)/(a*x**8-2*a*x**4-x**2+a),x)
```

```
[Out] Timed out
```

3.654 $\int \frac{x+4x^6}{\sqrt{-x+x^6}\,(1-ax^2-2x^5+x^{10})}\,dx$

Optimal. Leaf size=52

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{x^6-x}}\right)}{a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{x^6-x}}\right)}{a^{3/4}}$$

Rubi [F] time = 1.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x+4x^6}{\sqrt{-x+x^6}\,(1-ax^2-2x^5+x^{10})}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x + 4*x^6)/(Sqrt[-x + x^6]*(1 - a*x^2 - 2*x^5 + x^10)),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[-1 + x^5]*Defer[Subst][Defer[Int][x^2/(Sqrt[-1 + x^10]*(1 -
 a*x^4 - 2*x^10 + x^20)), x], x, Sqrt[x]])/Sqrt[-x + x^6] + (8*Sqrt[x]*Sqrt
[-1 + x^5]*Defer[Subst][Defer[Int][x^12/(Sqrt[-1 + x^10]*(1 - a*x^4 - 2*x^1
0 + x^20)), x], x, Sqrt[x]])/Sqrt[-x + x^6]
```

Rubi steps

$$\begin{aligned}
\int \frac{x+4x^6}{\sqrt{-x+x^6}\,(1-ax^2-2x^5+x^{10})}\,dx &= \int \frac{x\left(1+4x^5\right)}{\sqrt{-x+x^6}\,(1-ax^2-2x^5+x^{10})}\,dx\\
&= \frac{\left(\sqrt{x}\sqrt{-1+x^5}\right)\int \frac{\sqrt{x}\left(1+4x^5\right)}{\sqrt{-1+x^5}\,(1-ax^2-2x^5+x^{10})}\,dx}{\sqrt{-x+x^6}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\operatorname{Subst}\left(\int \frac{x^2\left(1+4x^{10}\right)}{\sqrt{-1+x^{10}}\,(1-ax^4-2x^{10}+x^{20})}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\operatorname{Subst}\left(\int \left(\frac{x^2}{\sqrt{-1+x^{10}}\,(1-ax^4-2x^{10}+x^{20})}+\frac{4x^{12}}{\sqrt{-1+x^{10}}\,(1-ax^4-2\ldots}\right.\right.}{\sqrt{-x+x^6}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{-1+x^{10}}\,(1-ax^4-2x^{10}+x^{20})}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}}+\frac{\left(8\sqrt{x}\ldots\right.}{\ldots}
\end{aligned}$$

Mathematica [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{x+4x^6}{\sqrt{-x+x^6}\,(1-ax^2-2x^5+x^{10})}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x + 4*x^6)/(Sqrt[-x + x^6]*(1 - a*x^2 - 2*x^5 + x^10)),x]
```

```
[Out] Integrate[(x + 4*x^6)/(Sqrt[-x + x^6]*(1 - a*x^2 - 2*x^5 + x^10)), x]
```

IntegrateAlgebraic [A] time = 2.73, size = 52, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{x^6-x}}\right)}{a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{x^6-x}}\right)}{a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + 4*x^6)/(Sqrt[-x + x^6]*(1 - a*x^2 - 2*x^5 + x^10)),
x]
```

```
[Out] ArcTan[(a^(1/4)*x)/Sqrt[-x + x^6]]/a^(3/4) - ArcTanh[(a^(1/4)*x)/Sqrt[-x +
x^6]]/a^(3/4)
```

fricas [B] time = 1.13, size = 334, normalized size = 6.42

$$-\left(\frac{1}{a^3}\right)^{\frac{1}{4}} \arctan\left(-\frac{2\sqrt{x^6-x}\left(a^3\left(\frac{1}{a^3}\right)^{\frac{3}{4}}x+\left(ax^5-a\right)\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\right)-\left(2\left(a^3x^6-a^3x\right)\left(\frac{1}{a^3}\right)^{\frac{3}{4}}+\left(ax^{10}-2\,ax^5+a^2x^2+a\right)\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\right)\sqrt{a\sqrt{\frac{1}{a^3}}}}{x^{10}-2\,x^5-ax^2+1}\right)-\frac{1}{4}\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{\left(a^2x^{10}-2\,a^2x^5+a^3x^2+a^2\right)\left(\frac{1}{a^3}\right)^{\frac{3}{4}}+2\sqrt{x^6-x}\left(x^5+a^2\sqrt{\frac{1}{a^3}}x-1\right)+2\left(ax^6-ax\right)\left(\frac{1}{a^3}\right)^{\frac{1}{4}}}{2\left(x^{10}-2\,x^5-ax^2+1\right)}\right)+\frac{1}{4}\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(-\frac{\left(a^2x^{10}-2\,a^2x^5+a^3x^2+a^2\right)\left(\frac{1}{a^3}\right)^{\frac{3}{4}}-2\sqrt{x^6-x}\left(x^5+a^2\sqrt{\frac{1}{a^3}}x-1\right)+2\left(ax^6-ax\right)\left(\frac{1}{a^3}\right)^{\frac{1}{4}}}{2\left(x^{10}-2\,x^5-ax^2+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6+x)/(x^6-x)^(1/2)/(x^10-2*x^5-a*x^2+1),x, algorithm="fricas
")
```

```
[Out] -(a^(-3))^(1/4)*arctan(-(2*sqrt(x^6 - x)*(a^3*(a^(-3))^(3/4)*x + (a*x^5 - a
)*(a^(-3))^(1/4)) - (2*(a^3*x^6 - a^3*x)*(a^(-3))^(3/4) + (a*x^10 - 2*a*x^5
 + a^2*x^2 + a)*(a^(-3))^(1/4))*sqrt(a*sqrt(a^(-3))))/(x^10 - 2*x^5 - a*x^2
 + 1)) - 1/4*(a^(-3))^(1/4)*log(1/2*((a^2*x^10 - 2*a^2*x^5 + a^3*x^2 + a^2)
*(a^(-3))^(3/4) + 2*sqrt(x^6 - x)*(x^5 + a^2*sqrt(a^(-3))*x - 1) + 2*(a*x^6
 - a*x)*(a^(-3))^(1/4))/(x^10 - 2*x^5 - a*x^2 + 1)) + 1/4*(a^(-3))^(1/4)*lo
g(-1/2*((a^2*x^10 - 2*a^2*x^5 + a^3*x^2 + a^2)*(a^(-3))^(3/4) - 2*sqrt(x^6
- x)*(x^5 + a^2*sqrt(a^(-3))*x - 1) + 2*(a*x^6 - a*x)*(a^(-3))^(1/4))/(x^10
 - 2*x^5 - a*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x^6+x}{\left(x^{10}-2\,x^5-ax^2+1\right)\sqrt{x^6-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6+x)/(x^6-x)^(1/2)/(x^10-2*x^5-a*x^2+1),x, algorithm="giac")
```

```
[Out] integrate((4*x^6 + x)/((x^10 - 2*x^5 - a*x^2 + 1)*sqrt(x^6 - x)), x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{4x^6+x}{\sqrt{x^6-x}\,\left(x^{10}-2x^5-a\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^6+x)/(x^6-x)^(1/2)/(x^10-2*x^5-a*x^2+1),x)
```

```
[Out] int((4*x^6+x)/(x^6-x)^(1/2)/(x^10-2*x^5-a*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x^6+x}{\left(x^{10}-2\,x^5-ax^2+1\right)\sqrt{x^6-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6+x)/(x^6-x)^(1/2)/(x^10-2*x^5-a*x^2+1),x, algorithm="maxima
")
```

```
[Out] integrate((4*x^6 + x)/((x^10 - 2*x^5 - a*x^2 + 1)*sqrt(x^6 - x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{4\,x^6+x}{\sqrt{x^6-x}\,\left(-x^{10}+2\,x^5+a\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 4*x^6)/((x^6 - x)^(1/2)*(a*x^2 + 2*x^5 - x^10 - 1)),x)
```

```
[Out] int(-(x + 4*x^6)/((x^6 - x)^(1/2)*(a*x^2 + 2*x^5 - x^10 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(4x^5+1\right)}{\sqrt{x\left(x-1\right)\left(x^4+x^3+x^2+x+1\right)}\left(-ax^2+x^{10}-2x^5+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**6+x)/(x**6-x)**(1/2)/(x**10-2*x**5-a*x**2+1),x)
```

```
[Out] Integral(x*(4*x**5 + 1)/(sqrt(x*(x - 1)*(x**4 + x**3 + x**2 + x + 1))*(-a*x
**2 + x**10 - 2*x**5 + 1)), x)
```

3.655 $\int \frac{x+4x^6}{\sqrt{-x+x^6}\left(a-x^2-2ax^5+ax^{10}\right)} \, dx$

Optimal. Leaf size=52

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{x^6-x}}\right)}{\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{a}\sqrt{x^6-x}}\right)}{\sqrt[4]{a}}$$

Rubi [F] time = 1.55, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x+4x^6}{\sqrt{-x+x^6}\left(a-x^2-2ax^5+ax^{10}\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x + 4*x^6)/(Sqrt[-x + x^6]*(a - x^2 - 2*a*x^5 + a*x^10)),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[-1 + x^5]*Defer[Subst][Defer[Int][x^2/(Sqrt[-1 + x^10]*(a -
 x^4 - 2*a*x^10 + a*x^20)), x], x, Sqrt[x]])/Sqrt[-x + x^6] + (8*Sqrt[x]*Sq
rt[-1 + x^5]*Defer[Subst][Defer[Int][x^12/(Sqrt[-1 + x^10]*(a - x^4 - 2*a*x
^10 + a*x^20)), x], x, Sqrt[x]])/Sqrt[-x + x^6]
```

Rubi steps

$$\begin{aligned}
\int \frac{x+4x^6}{\sqrt{-x+x^6}\left(a-x^2-2ax^5+ax^{10}\right)} \, dx &= \int \frac{x\left(1+4x^5\right)}{\sqrt{-x+x^6}\left(a-x^2-2ax^5+ax^{10}\right)} \, dx \\
&= \frac{\left(\sqrt{x}\sqrt{-1+x^5}\right)\int \frac{\sqrt{x}\left(1+4x^5\right)}{\sqrt{-1+x^5}\left(a-x^2-2ax^5+ax^{10}\right)} \, dx}{\sqrt{-x+x^6}} \\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{x^2\left(1+4x^{10}\right)}{\sqrt{-1+x^{10}}\left(a-x^4-2ax^{10}+ax^{20}\right)} \, dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}} \\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \left(\frac{x^2}{\sqrt{-1+x^{10}}\left(a-x^4-2ax^{10}+ax^{20}\right)}+\frac{4x^{12}}{\sqrt{-1+x^{10}}\left(a-x^4-2ax\right.}\right.\right.}{\sqrt{-x+x^6}} \\
&= \frac{\left(2\sqrt{x}\sqrt{-1+x^5}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x^{10}}\left(a-x^4-2ax^{10}+ax^{20}\right)} \, dx,x,\sqrt{x}\right)}{\sqrt{-x+x^6}}+\frac{\left(8\sqrt{x}\right.}{}
\end{aligned}$$

Mathematica [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{x+4x^6}{\sqrt{-x+x^6}\left(a-x^2-2ax^5+ax^{10}\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x + 4*x^6)/(Sqrt[-x + x^6]*(a - x^2 - 2*a*x^5 + a*x^10)),x]
```

```
[Out] Integrate[(x + 4*x^6)/(Sqrt[-x + x^6]*(a - x^2 - 2*a*x^5 + a*x^10)), x]
```

IntegrateAlgebraic [A] time = 2.75, size = 52, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{a}\,\sqrt{x^6-x}}\right)}{\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{a}\,\sqrt{x^6-x}}\right)}{\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + 4*x^6)/(Sqrt[-x + x^6]*(a - x^2 - 2*a*x^5 + a*x^10)
),x]
```

```
[Out] ArcTan[x/(a^(1/4)*Sqrt[-x + x^6])]/a^(1/4) - ArcTanh[x/(a^(1/4)*Sqrt[-x + x
^6])]/a^(1/4)
```

fricas [B] time = 1.22, size = 280, normalized size = 5.38

$$-\frac{\arctan\left(-\frac{2\,\sqrt{x^6-x}\left(a^{\frac{1}{4}}x+\frac{ax^5-a}{a^{\frac{1}{4}}}\right)-\left(\frac{a^2x^{10}-2\,a^2x^5+ax^2+a^2}{a^{\frac{1}{4}}}+\frac{2\left(a^2x^6-a^2x\right)}{a^{\frac{3}{4}}}\right)\sqrt{\frac{1}{a^{\frac{3}{2}}}}}{ax^{10}-2\,ax^5-x^2+a}\right)}{a^{\frac{1}{4}}}-\frac{\log\left(\frac{2\,\sqrt{x^6-x}\left(x^5+\frac{x}{\sqrt{a}}-1\right)+\frac{2\left(x^6-x\right)}{a^{\frac{1}{4}}}+\frac{ax^{10}-2\,ax^5+x^2+a}{a^{\frac{3}{4}}}}{2\left(ax^{10}-2\,ax^5-x^2+a\right)}\right)}{4\,a^{\frac{1}{4}}}+\frac{\log\left(\frac{2\,\sqrt{x^6-x}\left(x^5+\frac{x}{\sqrt{a}}-1\right)-\frac{2\left(x^6-x\right)}{a^{\frac{1}{4}}}-\frac{ax^{10}-2\,ax^5+x^2+a}{a^{\frac{3}{4}}}}{2\left(ax^{10}-2\,ax^5-x^2+a\right)}\right)}{4\,a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6+x)/(x^6-x)^(1/2)/(a*x^10-2*a*x^5-x^2+a),x, algorithm="fric
as")
```

```
[Out] -arctan(-(2*sqrt(x^6 - x)*(a^(1/4)*x + (a*x^5 - a)/a^(1/4)) - ((a^2*x^10 -
2*a^2*x^5 + a*x^2 + a^2)/a^(1/4) + 2*(a^2*x^6 - a^2*x)/a^(3/4))*sqrt(a^(-3/
2)))/(a*x^10 - 2*a*x^5 - x^2 + a))/a^(1/4) - 1/4*log(1/2*(2*sqrt(x^6 - x)*(
x^5 + x/sqrt(a) - 1) + 2*(x^6 - x)/a^(1/4) + (a*x^10 - 2*a*x^5 + x^2 + a)/a
^(3/4))/(a*x^10 - 2*a*x^5 - x^2 + a))/a^(1/4) + 1/4*log(1/2*(2*sqrt(x^6 - x
)*(x^5 + x/sqrt(a) - 1) - 2*(x^6 - x)/a^(1/4) - (a*x^10 - 2*a*x^5 + x^2 + a
)/a^(3/4))/(a*x^10 - 2*a*x^5 - x^2 + a))/a^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x^6+x}{\left(ax^{10}-2\,ax^5-x^2+a\right)\sqrt{x^6-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6+x)/(x^6-x)^(1/2)/(a*x^10-2*a*x^5-x^2+a),x, algorithm="giac
")
```

```
[Out] integrate((4*x^6 + x)/((a*x^10 - 2*a*x^5 - x^2 + a)*sqrt(x^6 - x)), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{4x^6+x}{\sqrt{x^6-x}\,\left(a\,x^{10}-2a\,x^5-x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^6+x)/(x^6-x)^(1/2)/(a*x^10-2*a*x^5-x^2+a),x)
```

```
[Out] int((4*x^6+x)/(x^6-x)^(1/2)/(a*x^10-2*a*x^5-x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x^6+x}{\left(ax^{10}-2\,ax^5-x^2+a\right)\sqrt{x^6-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6+x)/(x^6-x)^(1/2)/(a*x^10-2*a*x^5-x^2+a),x, algorithm="maxi
ma")
```

```
[Out] integrate((4*x^6 + x)/((a*x^10 - 2*a*x^5 - x^2 + a)*sqrt(x^6 - x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{4\,x^6+x}{\sqrt{x^6-x}\,\left(a\,x^{10}-2\,a\,x^5-x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4*x^6)/((x^6 - x)^(1/2)*(a - 2*a*x^5 + a*x^10 - x^2)),x)
```

```
[Out] int((x + 4*x^6)/((x^6 - x)^(1/2)*(a - 2*a*x^5 + a*x^10 - x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(4x^5+1\right)}{\sqrt{x\left(x-1\right)\left(x^4+x^3+x^2+x+1\right)}\left(ax^{10}-2ax^5+a-x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**6+x)/(x**6-x)**(1/2)/(a*x**10-2*a*x**5-x**2+a),x)
```

```
[Out] Integral(x*(4*x**5 + 1)/(sqrt(x*(x - 1)*(x**4 + x**3 + x**2 + x + 1))*(a*x*
*10 - 2*a*x**5 + a - x**2)), x)
```

3.656 $\int \frac{1}{(-5+2x)^2 \sqrt[4]{4-4x+x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\left((x-2)^2\right)^{3/4}\left(\frac{\sqrt{x-2}}{5-2x}+\frac{\tanh^{-1}\left(\sqrt{2}\sqrt{x-2}\right)}{\sqrt{2}}\right)}{(x-2)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {646, 51, 63, 207}

$$\frac{\sqrt{x-2}\tanh^{-1}\left(\sqrt{2}\sqrt{x-2}\right)}{\sqrt{2}\sqrt[4]{x^2-4x+4}}-\frac{2-x}{(5-2x)\sqrt[4]{x^2-4x+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-5 + 2*x)^2*(4 - 4*x + x^2)^(1/4)),x]
```

```
[Out] -((2 - x)/((5 - 2*x)*(4 - 4*x + x^2)^(1/4))) + (Sqrt[-2 + x]*ArcTanh[Sqrt[2
]*Sqrt[-2 + x]])/(Sqrt[2]*(4 - 4*x + x^2)^(1/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rubi steps

$$\int \frac{1}{(-5+2x)^2\sqrt[4]{4-4x+x^2}}\,dx = \frac{\sqrt{-2+x}\int \frac{1}{\sqrt{-2+x}(-5+2x)^2}\,dx}{\sqrt[4]{4-4x+x^2}}$$
$$= -\frac{2-x}{(5-2x)\sqrt[4]{4-4x+x^2}} - \frac{\sqrt{-2+x}\int \frac{1}{\sqrt{-2+x}(-5+2x)}\,dx}{2\sqrt[4]{4-4x+x^2}}$$
$$= -\frac{2-x}{(5-2x)\sqrt[4]{4-4x+x^2}} - \frac{\sqrt{-2+x}\,\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sqrt{-2+x}\right)}{\sqrt[4]{4-4x+x^2}}$$
$$= -\frac{2-x}{(5-2x)\sqrt[4]{4-4x+x^2}} + \frac{\sqrt{-2+x}\tanh^{-1}\left(\sqrt{2}\sqrt{-2+x}\right)}{\sqrt{2}\sqrt[4]{4-4x+x^2}}$$

Mathematica [**A**] time = 0.02, size = 59, normalized size = 1.11

$$\frac{x-2}{(1-2(x-2))\sqrt[4]{(x-2)^2}} + \frac{\sqrt{x-2}\tanh^{-1}\left(\sqrt{2}\sqrt{x-2}\right)}{\sqrt{2}\sqrt[4]{(x-2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-5 + 2*x)^2*(4 - 4*x + x^2)^(1/4)),x]
```

```
[Out] (-2 + x)/((1 - 2*(-2 + x))*((-2 + x)^2)^(1/4)) + (Sqrt[-2 + x]*ArcTanh[Sqrt
[2]*Sqrt[-2 + x]])/(Sqrt[2]*((-2 + x)^2)^(1/4))
```

IntegrateAlgebraic [**A**] time = 4.93, size = 56, normalized size = 1.06

$$\frac{\left((x-2)^2\right)^{3/4}\left(\frac{\tanh^{-1}\left(\sqrt{2}\sqrt{x-2}\right)}{\sqrt{2}} - \frac{\sqrt{x-2}}{2(x-2)-1}\right)}{(x-2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-5 + 2*x)^2*(4 - 4*x + x^2)^(1/4)),x]
```

```
[Out] (((-2 + x)^2)^(3/4)*(-(Sqrt[-2 + x]/(-1 + 2*(-2 + x))) + ArcTanh[Sqrt[2]*Sq
rt[-2 + x]]/Sqrt[2]))/(-2 + x)^(3/2)
```

fricas [**A**] time = 0.40, size = 60, normalized size = 1.13

$$\frac{\sqrt{2}\,(2\,x-5)\log\left(\frac{2\,x+2\,\sqrt{2}\left(x^2-4\,x+4\right)^{\frac{1}{4}}-3}{2\,x-5}\right) - 4\left(x^2-4\,x+4\right)^{\frac{1}{4}}}{4\,(2\,x-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+2*x)^2/(x^2-4*x+4)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(2*x - 5)*log((2*x + 2*sqrt(2)*(x^2 - 4*x + 4)^(1/4) - 3)/(2*x
 - 5)) - 4*(x^2 - 4*x + 4)^(1/4))/(2*x - 5)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-4\,x+4\right)^{\frac{1}{4}}(2\,x-5)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+2*x)^2/(x^2-4*x+4)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - 4*x + 4)^(1/4)*(2*x - 5)^2), x)
```

maple [A] time = 0.04, size = 48, normalized size = 0.91

$$-\frac{-2+x}{(-5+2x)\left((-2+x)^2\right)^{\frac{1}{4}}}+\frac{\operatorname{arctanh}\left(\sqrt{2}\,\sqrt{-2+x}\right)\sqrt{2}\,\sqrt{-2+x}}{2\left((-2+x)^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5+2*x)^2/(x^2-4*x+4)^(1/4),x)
```

```
[Out] -(-2+x)/(-5+2*x)/((-2+x)^2)^(1/4)+1/2*arctanh(2^(1/2)*(-2+x)^(1/2))*2^(1/2)
/((-2+x)^2)^(1/4)*(-2+x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-4\,x+4\right)^{\frac{1}{4}}(2\,x-5)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+2*x)^2/(x^2-4*x+4)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - 4*x + 4)^(1/4)*(2*x - 5)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(2\,x-5)^2\left(x^2-4\,x+4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x - 5)^2*(x^2 - 4*x + 4)^(1/4)),x)
```

```
[Out] int(1/((2*x - 5)^2*(x^2 - 4*x + 4)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x-5)^2\sqrt[4]{(x-2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+2*x)**2/(x**2-4*x+4)**(1/4),x)
```

```
[Out] Integral(1/((2*x - 5)**2*((x - 2)**2)**(1/4)), x)
```

3.657 $\int \frac{(-2+x^3)\sqrt{1+x^3}(2-x^2+2x^3)}{x^4(1-3x^2+x^3)}\,dx$

Optimal. Leaf size=53

$$\frac{2\sqrt{x^3+1}\,(2x^3+15x^2+2)}{3x^3}-10\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\,x}{\sqrt{x^3+1}}\right)$$

Rubi [F] time = 0.99, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+x^3)\sqrt{1+x^3}\,(2-x^2+2x^3)}{x^4\,(1-3x^2+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + x^3)*Sqrt[1 + x^3]*(2 - x^2 + 2*x^3))/(x^4*(1 - 3*x^2 + x^3)),x]
```

```
[Out] (4*Sqrt[1 + x^3])/3 + (4*Sqrt[1 + x^3])/(3*x^3) + (10*Sqrt[1 + x^3])/x - (3
0*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) + (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqr
t[1 + x^3]) - (10*Sqrt[2]*3^(3/4)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] +
 x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3
]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - 30*Defer[Int][Sqrt[
1 + x^3]/(1 - 3*x^2 + x^3), x] + 15*Defer[Int][(x*Sqrt[1 + x^3])/(1 - 3*x^2
 + x^3), x]
```

Rubi steps

$$\int \frac{(-2+x^3)\sqrt{1+x^3}\,(2-x^2+2x^3)}{x^4\,(1-3x^2+x^3)}\,dx = \int \left(-\frac{4\sqrt{1+x^3}}{x^4}-\frac{10\sqrt{1+x^3}}{x^2}+\frac{2\sqrt{1+x^3}}{x}+\frac{15(-2+x)\sqrt{1+x^3}}{1-3x^2+x^3}\right) dx$$

$$= 2\int \frac{\sqrt{1+x^3}}{x}\,dx-4\int \frac{\sqrt{1+x^3}}{x^4}\,dx-10\int \frac{\sqrt{1+x^3}}{x^2}\,dx+15\int \frac{(-2+}{1-}$$

$$= \frac{10\sqrt{1+x^3}}{x}+\frac{2}{3}\text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx,x,x^3\right)-\frac{4}{3}\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx\right.$$

$$= \frac{4\sqrt{1+x^3}}{3}+\frac{4\sqrt{1+x^3}}{3x^3}+\frac{10\sqrt{1+x^3}}{x}-15\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx+15\int \frac{}{1}$$

$$= \frac{4\sqrt{1+x^3}}{3}+\frac{4\sqrt{1+x^3}}{3x^3}+\frac{10\sqrt{1+x^3}}{x}-\frac{30\sqrt{1+x^3}}{1+\sqrt{3}+x}+\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{}$$

Mathematica [C] time = 6.15, size = 1719, normalized size = 32.43

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-2 + x^3)*Sqrt[1 + x^3]*(2 - x^2 + 2*x^3))/(x^4*(1 - 3*x^2 + x^
3)),x]
```

```
[Out] (4/3 + 4/(3*x^3) + 10/x)*Sqrt[1 + x^3] + 15*((2*Sqrt[(1 + x)/(1 + (-1)^(1/3
))]*(-(-1)^(1/3) + x)*Sqrt[((-1)^(2/3) + x)/((-1)^(1/3) + (-1)^(2/3))]*Elli
pticF[ArcSin[Sqrt[-(((-1)^(2/3)*((-1)^(1/3) - x))/(1 + (-1)^(1/3)))]], (-1)
^(1/3)])/(Sqrt[(-(-1)^(1/3) + x)/(-(-1)^(1/3) - (-1)^(2/3))]*Sqrt[1 + x^3])
 - (4*((-1)^(1/3) + (-1)^(2/3))*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[((-(-1)
^(2/3) - x)*(-(-1)^(1/3) + x))/((-1)^(1/3) + (-1)^(2/3))^2]*EllipticPi[((-1
)^(1/3) + (-1)^(2/3))/((-1)^(1/3) - Root[1 - 3*#1^2 + #1^3 & , 1, 0]), ArcS
in[Sqrt[-(((-1)^(2/3)*((-1)^(1/3) - x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)])/(
Sqrt[1 + x^3]*(-(-1)^(1/3) + Root[1 - 3*#1^2 + #1^3 & , 1, 0])*(Root[1 - 3*
#1^2 + #1^3 & , 1, 0] - Root[1 - 3*#1^2 + #1^3 & , 2, 0])*(Root[1 - 3*#1^2
+ #1^3 & , 1, 0] - Root[1 - 3*#1^2 + #1^3 & , 3, 0])) + (2*((-1)^(1/3) + (-
1)^(2/3))*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[((-(-1)^(2/3) - x)*(-(-1)^(1/
3) + x))/((-1)^(1/3) + (-1)^(2/3))^2]*EllipticPi[((-1)^(1/3) + (-1)^(2/3))/
((-1)^(1/3) - Root[1 - 3*#1^2 + #1^3 & , 1, 0]), ArcSin[Sqrt[-(((-1)^(2/3)*
((-1)^(1/3) - x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)]*Root[1 - 3*#1^2 + #1^3 &
 , 1, 0]^3)/(Sqrt[1 + x^3]*(-(-1)^(1/3) + Root[1 - 3*#1^2 + #1^3 & , 1, 0])
*(Root[1 - 3*#1^2 + #1^3 & , 1, 0] - Root[1 - 3*#1^2 + #1^3 & , 2, 0])*(Roo
t[1 - 3*#1^2 + #1^3 & , 1, 0] - Root[1 - 3*#1^2 + #1^3 & , 3, 0])) - (4*((-
1)^(1/3) + (-1)^(2/3))*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[((-(-1)^(2/3) -
x)*(-(-1)^(1/3) + x))/((-1)^(1/3) + (-1)^(2/3))^2]*EllipticPi[((-1)^(1/3) +
 (-1)^(2/3))/((-1)^(1/3) - Root[1 - 3*#1^2 + #1^3 & , 2, 0]), ArcSin[Sqrt[-
(((-1)^(2/3)*((-1)^(1/3) - x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)])/(Sqrt[1 +
x^3]*(-(-1)^(1/3) + Root[1 - 3*#1^2 + #1^3 & , 2, 0])*(-Root[1 - 3*#1^2 + #
1^3 & , 1, 0] + Root[1 - 3*#1^2 + #1^3 & , 2, 0])*(Root[1 - 3*#1^2 + #1^3 &
 , 2, 0] - Root[1 - 3*#1^2 + #1^3 & , 3, 0])) + (2*((-1)^(1/3) + (-1)^(2/3)
)*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[((-(-1)^(2/3) - x)*(-(-1)^(1/3) + x))
/((-1)^(1/3) + (-1)^(2/3))^2]*EllipticPi[((-1)^(1/3) + (-1)^(2/3))/((-1)^(1
/3) - Root[1 - 3*#1^2 + #1^3 & , 2, 0]), ArcSin[Sqrt[-(((-1)^(2/3)*((-1)^(1
/3) - x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)]*Root[1 - 3*#1^2 + #1^3 & , 2, 0]
^3)/(Sqrt[1 + x^3]*(-(-1)^(1/3) + Root[1 - 3*#1^2 + #1^3 & , 2, 0])*(-Root[
1 - 3*#1^2 + #1^3 & , 1, 0] + Root[1 - 3*#1^2 + #1^3 & , 2, 0])*(Root[1 - 3
*#1^2 + #1^3 & , 2, 0] - Root[1 - 3*#1^2 + #1^3 & , 3, 0])) - (4*((-1)^(1/3
) + (-1)^(2/3))*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[((-(-1)^(2/3) - x)*(-(-
1)^(1/3) + x))/((-1)^(1/3) + (-1)^(2/3))^2]*EllipticPi[((-1)^(1/3) + (-1)^(
2/3))/((-1)^(1/3) - Root[1 - 3*#1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[-(((-1)^
(2/3)*((-1)^(1/3) - x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)])/(Sqrt[1 + x^3]*(-
(-1)^(1/3) + Root[1 - 3*#1^2 + #1^3 & , 3, 0])*(-Root[1 - 3*#1^2 + #1^3 & ,
 1, 0] + Root[1 - 3*#1^2 + #1^3 & , 3, 0])*(-Root[1 - 3*#1^2 + #1^3 & , 2,
0] + Root[1 - 3*#1^2 + #1^3 & , 3, 0])) + (2*((-1)^(1/3) + (-1)^(2/3))*Sqrt
[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[((-(-1)^(2/3) - x)*(-(-1)^(1/3) + x))/((-1)
^(1/3) + (-1)^(2/3))^2]*EllipticPi[((-1)^(1/3) + (-1)^(2/3))/((-1)^(1/3) -
Root[1 - 3*#1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[-(((-1)^(2/3)*((-1)^(1/3) -
x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)]*Root[1 - 3*#1^2 + #1^3 & , 3, 0]^3)/(S
qrt[1 + x^3]*(-(-1)^(1/3) + Root[1 - 3*#1^2 + #1^3 & , 3, 0])*(-Root[1 - 3*
#1^2 + #1^3 & , 1, 0] + Root[1 - 3*#1^2 + #1^3 & , 3, 0])*(-Root[1 - 3*#1^2
 + #1^3 & , 2, 0] + Root[1 - 3*#1^2 + #1^3 & , 3, 0])))
```

IntegrateAlgebraic [A] time = 0.73, size = 53, normalized size = 1.00

$$\frac{2\sqrt{x^3+1}\left(2x^3+15x^2+2\right)}{3x^3}-10\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\,x}{\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^3)*Sqrt[1 + x^3]*(2 - x^2 + 2*x^3))/(x^4*(1 - 3
*x^2 + x^3)),x]
```

```
[Out] (2*Sqrt[1 + x^3]*(2 + 15*x^2 + 2*x^3))/(3*x^3) - 10*Sqrt[3]*ArcTanh[(Sqrt[3
]*x)/Sqrt[1 + x^3]]
```

fricas [B] time = 0.47, size = 112, normalized size = 2.11

$$\frac{15 \sqrt{3} x^{3} \log \left(-\frac{x^{6}+18 x^{5}+9 x^{4}+2 x^{3}-4 \sqrt{3} \left(x^{4}+3 x^{3}+x\right) \sqrt{x^{3}+1}+18 x^{2}+1}{x^{6}-6 x^{5}+9 x^{4}+2 x^{3}-6 x^{2}+1}\right)+4 \left(2 x^{3}+15 x^{2}+2\right) \sqrt{x^{3}+1}}{6 x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3+1)^(1/2)*(2*x^3-x^2+2)/x^4/(x^3-3*x^2+1),x, algorith
m="fricas")
```

```
[Out] 1/6*(15*sqrt(3)*x^3*log(-(x^6 + 18*x^5 + 9*x^4 + 2*x^3 - 4*sqrt(3)*(x^4 + 3
*x^3 + x)*sqrt(x^3 + 1) + 18*x^2 + 1)/(x^6 - 6*x^5 + 9*x^4 + 2*x^3 - 6*x^2
+ 1)) + 4*(2*x^3 + 15*x^2 + 2)*sqrt(x^3 + 1))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 x^{3}-x^{2}+2\right) \sqrt{x^{3}+1} \left(x^{3}-2\right)}{\left(x^{3}-3 x^{2}+1\right) x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3+1)^(1/2)*(2*x^3-x^2+2)/x^4/(x^3-3*x^2+1),x, algorith
m="giac")
```

```
[Out] integrate((2*x^3 - x^2 + 2)*sqrt(x^3 + 1)*(x^3 - 2)/((x^3 - 3*x^2 + 1)*x^4)
, x)
```

maple [C] time = 0.24, size = 330, normalized size = 6.23

$$\frac{4 \sqrt{x^{3}+1}}{3 x^{3}}+\frac{30 \left(\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{1+x}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x-\frac{1}{2}-\frac{i \sqrt{3}}{2}}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x-\frac{1}{2}+\frac{i \sqrt{3}}{2}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}+5 \sqrt{2}\, \left(\sum_{\_\alpha =\mathit{RootOf}\left(\_Z^{3}-3 \_Z^{2}+1\right)}\frac{\_\alpha \left(-\_\alpha^{2}+4 \_\alpha -4\right) \left(3-i \sqrt{3}\right) \sqrt{\frac{1+x}{3-i \sqrt{3}}}\, \sqrt{\frac{-1+2 x-i \sqrt{3}}{-3-i \sqrt{3}}}\, \sqrt{\frac{-1+2 x+i \sqrt{3}}{-3+i \sqrt{3}}}\, \mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \frac{\_\alpha^{2}}{2}-2 \_\alpha +2-\frac{i \_\alpha^{2} \sqrt{3}}{6}+\frac{2 i \sqrt{3}\, \_\alpha}{3}-\frac{2 i \sqrt{3}}{3}, \sqrt{\frac{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}\right)+\frac{10 \sqrt{x^{3}+1}}{x}+\frac{4 \sqrt{x^{3}+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2)*(x^3+1)^(1/2)*(2*x^3-x^2+2)/x^4/(x^3-3*x^2+1),x)
```

```
[Out] 4/3*(x^3+1)^(1/2)/x^3+30*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1
/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2
))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^
(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+5*2^(1/2)*
sum(_alpha*(-_alpha^2+4*_alpha-4)*(3-I*3^(1/2))*((1+x)/(3-I*3^(1/2)))^(1/2)
*((-1+2*x-I*3^(1/2))/(-3-I*3^(1/2)))^(1/2)*((-1+2*x+I*3^(1/2))/(-3+I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2*_a
lpha^2-2*_alpha+2-1/6*I*3^(1/2)*_alpha^2+2/3*I*3^(1/2)*_alpha-2/3*I*3^(1/2)
,((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^3-3*_Z
^2+1))+10*(x^3+1)^(1/2)/x+4/3*(x^3+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 x^{3}-x^{2}+2\right) \sqrt{x^{3}+1} \left(x^{3}-2\right)}{\left(x^{3}-3 x^{2}+1\right) x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3+1)^(1/2)*(2*x^3-x^2+2)/x^4/(x^3-3*x^2+1),x, algorith
m="maxima")
```

```
[Out] integrate((2*x^3 - x^2 + 2)*sqrt(x^3 + 1)*(x^3 - 2)/((x^3 - 3*x^2 + 1)*x^4)
, x)
```

mupad [B]   time = 1.13, size = 76, normalized size = 1.43

$$5\sqrt{3}\ \ln\left(\frac{3x^2+x^3-2\sqrt{3}\,x\sqrt{x^3+1}+1}{x^3-3x^2+1}\right)+\frac{4\sqrt{x^3+1}}{3}+\frac{10\sqrt{x^3+1}}{x}+\frac{4\sqrt{x^3+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)^(1/2)*(x^3 - 2)*(2*x^3 - x^2 + 2))/(x^4*(x^3 - 3*x^2 + 1)),x
)
```

```
[Out] 5*3^(1/2)*log((3*x^2 + x^3 - 2*3^(1/2)*x*(x^3 + 1)^(1/2) + 1)/(x^3 - 3*x^2
+ 1)) + (4*(x^3 + 1)^(1/2))/3 + (10*(x^3 + 1)^(1/2))/x + (4*(x^3 + 1)^(1/2)
)/(3*x^3)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2)*(x**3+1)**(1/2)*(2*x**3-x**2+2)/x**4/(x**3-3*x**2+1),x)
```

```
[Out] Timed out
```

3.658 $\int \frac{(-b+ax^3)\sqrt{b+ax^3}}{x}\,dx$

Optimal. Leaf size=53

$$\frac{2}{3}b^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)-\frac{2}{9}\left(2b-ax^3\right)\sqrt{ax^3+b}$$

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 80, 50, 63, 208}

$$\frac{2}{3}b^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)-\frac{2}{3}b\sqrt{ax^3+b}+\frac{2}{9}\left(ax^3+b\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a*x^3)*Sqrt[b + a*x^3])/x,x]
```

```
[Out] (-2*b*Sqrt[b + a*x^3])/3 + (2*(b + a*x^3)^(3/2))/9 + (2*b^(3/2)*ArcTanh[Sqr
t[b + a*x^3]/Sqrt[b]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(-b+ax^3)\sqrt{b+ax^3}}{x}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(-b+ax)\sqrt{b+ax}}{x}\,dx, x, x^3\right)$$
$$= \frac{2}{9}\left(b+ax^3\right)^{3/2} - \frac{1}{3}b\operatorname{Subst}\left(\int \frac{\sqrt{b+ax}}{x}\,dx, x, x^3\right)$$
$$= -\frac{2}{3}b\sqrt{b+ax^3} + \frac{2}{9}\left(b+ax^3\right)^{3/2} - \frac{1}{3}b^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{b+ax}}\,dx, x, x^3\right)$$
$$= -\frac{2}{3}b\sqrt{b+ax^3} + \frac{2}{9}\left(b+ax^3\right)^{3/2} - \frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^2}{a}}\,dx, x, \sqrt{b+ax^3}\right)}{3a}$$
$$= -\frac{2}{3}b\sqrt{b+ax^3} + \frac{2}{9}\left(b+ax^3\right)^{3/2} + \frac{2}{3}b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b+ax^3}}{\sqrt{b}}\right)$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.98

$$\frac{1}{9}\left(6b^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)+2\left(ax^3-2b\right)\sqrt{ax^3+b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + a*x^3)*Sqrt[b + a*x^3])/x,x]
[Out] (2*(-2*b + a*x^3)*Sqrt[b + a*x^3] + 6*b^(3/2)*ArcTanh[Sqrt[b + a*x^3]/Sqrt[
b]])/9
```

IntegrateAlgebraic [A] time = 0.08, size = 53, normalized size = 1.00

$$\frac{2}{3}b^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)-\frac{2}{9}\left(2b-ax^3\right)\sqrt{ax^3+b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^3)*Sqrt[b + a*x^3])/x,x]
[Out] (-2*(2*b - a*x^3)*Sqrt[b + a*x^3])/9 + (2*b^(3/2)*ArcTanh[Sqrt[b + a*x^3]/S
qrt[b]])/3
```

fricas [A] time = 0.44, size = 103, normalized size = 1.94

$$\left[\frac{1}{3}b^{\frac{3}{2}}\log\left(\frac{ax^3+2\sqrt{ax^3+b}\sqrt{b}+2b}{x^3}\right)+\frac{2}{9}\sqrt{ax^3+b}\left(ax^3-2b\right), -\frac{2}{3}\sqrt{-b}\,b\arctan\left(\frac{\sqrt{ax^3+b}\sqrt{-b}}{b}\right)+\frac{2}{9}\sqrt{ax^3+b}\left(ax^3-2b\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(a*x^3+b)^(1/2)/x,x, algorithm="fricas")
[Out] [1/3*b^(3/2)*log((a*x^3 + 2*sqrt(a*x^3 + b)*sqrt(b) + 2*b)/x^3) + 2/9*sqrt(
a*x^3 + b)*(a*x^3 - 2*b), -2/3*sqrt(-b)*b*arctan(sqrt(a*x^3 + b)*sqrt(-b)/b
) + 2/9*sqrt(a*x^3 + b)*(a*x^3 - 2*b)]
```

giac [A] time = 0.19, size = 50, normalized size = 0.94

$$-\frac{2\,b^2\arctan\left(\frac{\sqrt{ax^3+b}}{\sqrt{-b}}\right)}{3\sqrt{-b}}+\frac{2}{9}\left(ax^3+b\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{ax^3+b}\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(a*x^3+b)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2/3*b^2*arctan(sqrt(a*x^3 + b)/sqrt(-b))/sqrt(-b) + 2/9*(a*x^3 + b)^(3/2)
- 2/3*sqrt(a*x^3 + b)*b
```

maple [A] time = 0.01, size = 47, normalized size = 0.89

$$\frac{2\left(a\,x^{3}+b\right)^{\frac{3}{2}}}{9}-b\left(\frac{2\sqrt{a\,x^{3}+b}}{3}-\frac{2\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{3}+b}}{\sqrt{b}}\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b)*(a*x^3+b)^(1/2)/x,x)
```

```
[Out] 2/9*(a*x^3+b)^(3/2)-b*(2/3*(a*x^3+b)^(1/2)-2/3*b^(1/2)*arctanh((a*x^3+b)^(1
/2)/b^(1/2)))
```

maxima [A] time = 0.67, size = 63, normalized size = 1.19

$$-\frac{1}{3}\left(\sqrt{b}\log\left(\frac{\sqrt{ax^{3}+b}-\sqrt{b}}{\sqrt{ax^{3}+b}+\sqrt{b}}\right)+2\sqrt{ax^{3}+b}\right)b+\frac{2}{9}\left(ax^{3}+b\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(a*x^3+b)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -1/3*(sqrt(b)*log((sqrt(a*x^3 + b) - sqrt(b))/(sqrt(a*x^3 + b) + sqrt(b)))
+ 2*sqrt(a*x^3 + b))*b + 2/9*(a*x^3 + b)^(3/2)
```

mupad [B] time = 0.71, size = 68, normalized size = 1.28

$$\frac{b^{3/2}\,\ln\left(\frac{\left(\sqrt{a\,x^3+b}-\sqrt{b}\right)\left(\sqrt{a\,x^3+b}+\sqrt{b}\right)^3}{x^6}\right)}{3}-\frac{4\,b\,\sqrt{a\,x^3+b}}{9}+\frac{2\,a\,x^3\,\sqrt{a\,x^3+b}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b + a*x^3)^(1/2)*(b - a*x^3))/x,x)
```

```
[Out] (b^(3/2)*log((((b + a*x^3)^(1/2) - b^(1/2))*((b + a*x^3)^(1/2) + b^(1/2))^3
)/x^6))/3 - (4*b*(b + a*x^3)^(1/2))/9 + (2*a*x^3*(b + a*x^3)^(1/2))/9
```

sympy [A] time = 21.16, size = 75, normalized size = 1.42

$$-\frac{a\left(\begin{cases}-\sqrt{b}\,x^{3} & \text{for}\: a = 0\\-\frac{2\left(ax^{3}+b\right)^{\frac{3}{2}}}{3a} & \text{otherwise}\end{cases}\right)}{3}+\frac{b\left(-\frac{2b\operatorname{atan}\left(\frac{\sqrt{ax^{3}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-2\sqrt{ax^{3}+b}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b)*(a*x**3+b)**(1/2)/x,x)
```

```
[Out] -a*Piecewise((-sqrt(b)*x**3, Eq(a, 0)), (-2*(a*x**3 + b)**(3/2)/(3*a), True
))/3 + b*(-2*b*atan(sqrt(a*x**3 + b)/sqrt(-b))/sqrt(-b) - 2*sqrt(a*x**3 + b
))/3
```

3.659 $\int \frac{x^2}{\left(-1+x^4\right)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{4\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.11, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {490, 1211, 220, 1699, 203, 206}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{4\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-1 + x^4)*Sqrt[1 + x^4]),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(4*Sqrt[2]) - ArcTanh[(Sqrt[2]*x)/Sqrt[1
+ x^4]]/(4*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
```

```
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(-1+x^4\right)\sqrt{1+x^4}}\,dx &= -\left(\frac{1}{2}\int \frac{1}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx\right)+\frac{1}{2}\int \frac{1}{\left(1+x^2\right)\sqrt{1+x^4}}\,dx\\ &= \frac{1}{4}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^4}}\,dx-\frac{1}{4}\int \frac{1+x^2}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx\\ &= -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\frac{x}{\sqrt{1+x^4}}\right)\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{4\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 26, normalized size = 0.49

$$-\frac{1}{3}x^3F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-x^4,x^4\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-1 + x^4)*Sqrt[1 + x^4]),x]
[Out] -1/3*(x^3*AppellF1[3/4, 1/2, 1, 7/4, -x^4, x^4])
```

IntegrateAlgebraic [A] time = 0.33, size = 53, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{4\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-1 + x^4)*Sqrt[1 + x^4]),x]
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(4*Sqrt[2]) - ArcTanh[(Sqrt[2]*x)/Sqrt[1
+ x^4]]/(4*Sqrt[2])
```

fricas [A] time = 0.50, size = 61, normalized size = 1.15

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)+\frac{1}{16}\sqrt{2}\log\left(\frac{x^4-2\sqrt{2}\sqrt{x^4+1}\,x+2x^2+1}{x^4-2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-1)/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] 1/8*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) + 1/16*sqrt(2)*log((x^4 - 2*sqr
t(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^4+1}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-1)/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(x^4 + 1)*(x^4 - 1)), x)
```

maple [C] time = 0.02, size = 102, normalized size = 1.92

$$\frac{(-1)^{\frac{3}{4}} \sqrt{-i x^2+1}\, \sqrt{i x^2+1}\, \mathrm{EllipticPi}\left((-1)^{\frac{1}{4}} x, -i, -\sqrt{-i}\, (-1)^{\frac{3}{4}}\right)}{2\sqrt{x^4+1}} - \frac{(-1)^{\frac{3}{4}} \sqrt{-i x^2+1}\, \sqrt{i x^2+1}\, \mathrm{EllipticPi}\left((-1)^{\frac{1}{4}} x, i, -\sqrt{-i}\, (-1)^{\frac{3}{4}}\right)}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4-1)/(x^4+1)^(1/2),x)
[Out] 1/2*(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1
)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/4))-1/2*(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2
)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^4+1}\left(x^4-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-1)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(x^4 + 1)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(x^4-1\right)\sqrt{x^4+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^4 - 1)*(x^4 + 1)^(1/2)),x)
[Out] int(x^2/((x^4 - 1)*(x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x-1)(x+1)\left(x^2+1\right)\sqrt{x^4+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4-1)/(x**4+1)**(1/2),x)
[Out] Integral(x**2/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(x**4 + 1)), x)
```

3.660 $\int \frac{\sqrt{1+x^4}}{-1+x^4}\,dx$

Optimal. Leaf size=53

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {404, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^4]/(-1 + x^4),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2] - ArcTanh[(Sqrt[2]*x)/Sqrt[1
 + x^4]]/(2*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 404

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[a/c,
 Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x^4}}{-1+x^4}\,dx &= -\text{Subst}\left(\int \frac{1}{1-4x^4}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.09, size = 108, normalized size = 2.04

$$\frac{5x\sqrt{x^4+1}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-x^4,x^4\right)}{\left(x^4-1\right)\left(2x^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-x^4,x^4\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-x^4,x^4\right)\right)+5F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-x^4,x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 + x^4]/(-1 + x^4),x]
```

```
[Out] (5*x*Sqrt[1 + x^4]*AppellF1[1/4, -1/2, 1, 5/4, -x^4, x^4])/((-1 + x^4)*(5*A
ppellF1[1/4, -1/2, 1, 5/4, -x^4, x^4] + 2*x^4*(2*AppellF1[5/4, -1/2, 2, 9/4
, -x^4, x^4] + AppellF1[5/4, 1/2, 1, 9/4, -x^4, x^4])))
```

IntegrateAlgebraic [A] time = 0.25, size = 53, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^4]/(-1 + x^4),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2] - ArcTanh[(Sqrt[2]*x)/Sqrt[1
 + x^4]]/(2*Sqrt[2])
```

fricas [A] time = 0.47, size = 61, normalized size = 1.15

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)+\frac{1}{8}\sqrt{2}\log\left(\frac{x^4-2\sqrt{2}\sqrt{x^4+1}x+2x^2+1}{x^4-2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(x^4-1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) + 1/8*sqrt(2)*log((x^4 - 2*sqr
t(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(x^4-1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 1)/(x^4 - 1), x)
```

maple [C] time = 0.03, size = 362, normalized size = 6.83

$$\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x,-i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^4+1}}-\frac{i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\mathrm{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x,i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(1/2)/(x^4-1),x)
```

```
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+1/2*I/(1/2*2^(1/2)+1/2*I*2^(1/2
))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticE(x*(1/2*2^(1/2)+1
/2*I*2^(1/2)),I)+(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*E
```

```
llipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/4))-1/2*I/(1/2*2^(1/2)+1/2*I*2
^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticF(x*(1/2*2^(1
/2)+1/2*I*2^(1/2)),I)+1/2*I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+
I*x^2)^(1/2)/(x^4+1)^(1/2)*(EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-Elli
pticE(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I))+(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2
)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(x^4-1),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 1)/(x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)^(1/2)/(x^4 - 1),x)
[Out] int((x^4 + 1)^(1/2)/(x^4 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}}{(x-1)(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(1/2)/(x**4-1),x)
[Out] Integral(sqrt(x**4 + 1)/((x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.661 $\int \left(b+ax^3\right)\sqrt{x+x^4}\,dx$

Optimal. Leaf size=53

$$\frac{1}{12}\sqrt{x^4+x}\left(2ax^4+ax+4bx\right)+\frac{1}{12}(4b-a)\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)$$

Rubi [A] time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.62, number of steps used = 9, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2053, 2004, 2029, 206, 2021, 2024}

$$\frac{1}{6}a\sqrt{x^4+x}\,x^4+\frac{1}{12}a\sqrt{x^4+x}\,x-\frac{1}{12}a\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)+\frac{1}{3}b\sqrt{x^4+x}\,x+\frac{1}{3}b\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^3)*Sqrt[x + x^4],x]
```

```
[Out] (a*x*Sqrt[x + x^4])/12 + (b*x*Sqrt[x + x^4])/3 + (a*x^4*Sqrt[x + x^4])/6 -
(a*ArcTanh[x^2/Sqrt[x + x^4]])/12 + (b*ArcTanh[x^2/Sqrt[x + x^4]])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2053

```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Expan
dIntegrand[Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && (Po
lyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IntegerQ[p] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int\left(b+ax^3\right)\sqrt{x+x^4}\,dx &= \int\left(b\sqrt{x+x^4}+ax^3\sqrt{x+x^4}\right)dx\\ &= a\int x^3\sqrt{x+x^4}\,dx+b\int\sqrt{x+x^4}\,dx\\ &= \frac{1}{3}bx\sqrt{x+x^4}+\frac{1}{6}ax^4\sqrt{x+x^4}+\frac{1}{4}a\int\frac{x^4}{\sqrt{x+x^4}}\,dx+\frac{1}{2}b\int\frac{x}{\sqrt{x+x^4}}\,dx\\ &= \frac{1}{12}ax\sqrt{x+x^4}+\frac{1}{3}bx\sqrt{x+x^4}+\frac{1}{6}ax^4\sqrt{x+x^4}-\frac{1}{8}a\int\frac{x}{\sqrt{x+x^4}}\,dx+\frac{1}{3}b\,\mathrm{Subst}\left(\int\frac{}{1}\right.\\ &= \frac{1}{12}ax\sqrt{x+x^4}+\frac{1}{3}bx\sqrt{x+x^4}+\frac{1}{6}ax^4\sqrt{x+x^4}+\frac{1}{3}b\tanh^{-1}\left(\frac{x^2}{\sqrt{x+x^4}}\right)-\frac{1}{12}a\,\mathrm{Subst}\left(\right.\\ &= \frac{1}{12}ax\sqrt{x+x^4}+\frac{1}{3}bx\sqrt{x+x^4}+\frac{1}{6}ax^4\sqrt{x+x^4}-\frac{1}{12}a\tanh^{-1}\left(\frac{x^2}{\sqrt{x+x^4}}\right)+\frac{1}{3}b\tanh^{-1}\end{aligned}$$

Mathematica [A] time = 0.06, size = 58, normalized size = 1.09

$$\frac{\sqrt{x^4+x}\left(x^{3/2}\left(2ax^3+a+4b\right)-\frac{(a-4b)\sinh^{-1}\left(x^{3/2}\right)}{\sqrt{x^3+1}}\right)}{12\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^3)*Sqrt[x + x^4],x]
```

```
[Out] (Sqrt[x + x^4]*(x^(3/2)*(a + 4*b + 2*a*x^3) - ((a - 4*b)*ArcSinh[x^(3/2)])/
Sqrt[1 + x^3]))/(12*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.49, size = 53, normalized size = 1.00

$$\frac{1}{12}\sqrt{x^4+x}\left(2ax^4+ax+4bx\right)+\frac{1}{12}(4b-a)\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^3)*Sqrt[x + x^4],x]
```

```
[Out] (Sqrt[x + x^4]*(a*x + 4*b*x + 2*a*x^4))/12 + ((-a + 4*b)*ArcTanh[x^2/Sqrt[x
 + x^4]])/12
```

fricas [A] time = 0.45, size = 49, normalized size = 0.92

$$-\frac{1}{24}(a-4b)\log\left(-2x^3-2\sqrt{x^4+x}\,x-1\right)+\frac{1}{12}\left(2ax^4+(a+4b)x\right)\sqrt{x^4+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)*(x^4+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(a - 4*b)*log(-2*x^3 - 2*sqrt(x^4 + x)*x - 1) + 1/12*(2*a*x^4 + (a +
4*b)*x)*sqrt(x^4 + x)
```

giac [A] time = 0.28, size = 53, normalized size = 1.00

$$\frac{1}{12}\left(2\,ax^3+a+4\,b\right)\sqrt{x^4+x}\,x-\frac{1}{24}\,(a-4\,b)\left(\log\left(\sqrt{\frac{1}{x^3}+1}+1\right)-\log\left(\left|\sqrt{\frac{1}{x^3}+1}-1\right|\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)*(x^4+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*a*x^3 + a + 4*b)*sqrt(x^4 + x)*x - 1/24*(a - 4*b)*(log(sqrt(1/x^3 +
 1) + 1) - log(abs(sqrt(1/x^3 + 1) - 1)))
```

maple [C] time = 0.01, size = 618, normalized size = 11.66

$$a\left(\frac{x^4\sqrt{x^4+x}}{6}+\frac{x\sqrt{x^4+x}}{12}+\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\,(1+x)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(1+x)}}\sqrt{-\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\left(-\mathrm{EllipticF}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)+\mathrm{EllipticPi}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\frac{\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{4\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(1+x)\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)+b\left(\frac{x\sqrt{x^4+x}}{3}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\,(1+x)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(1+x)}}\sqrt{-\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\left(-\mathrm{EllipticF}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)+\mathrm{EllipticPi}\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}},\frac{\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(1+x)\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b)*(x^4+x)^(1/2),x)
```

```
[Out] a*(1/6*x^4*(x^4+x)^(1/2)+1/12*x*(x^4+x)^(1/2)+1/4*(-1/2-1/2*I*3^(1/2))*((3/
2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*
3^(1/2))/(1/2-1/2*I*3^(1/2))/(1+x))^(1/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*
I*3^(1/2))/(1+x))^(1/2)/(3/2+1/2*I*3^(1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*
(x-1/2-1/2*I*3^(1/2)))^(1/2)*(-EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*
3^(1/2))/(1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*
I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+EllipticPi(((3/2+1/2*I*3^(1/2))*x/(
1/2+1/2*I*3^(1/2))/(1+x))^(1/2),(1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)),((-
3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2))))+b*(1/3*x*(x^4+x)^(1/2)-(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^
(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(
1/2-1/2*I*3^(1/2))/(1+x))^(1/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))
/(1+x))^(1/2)/(3/2+1/2*I*3^(1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2
*I*3^(1/2)))^(1/2)*(-EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(
1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))
/(-3/2-1/2*I*3^(1/2)))^(1/2))+EllipticPi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*
3^(1/2))/(1+x))^(1/2),(1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*
3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1
/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax^3+b\right)\sqrt{x^4+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)*(x^4+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^3 + b)*sqrt(x^4 + x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(a\,x^3+b\right)\,\sqrt{x^4+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^3)*(x + x^4)^(1/2),x)
```

```
[Out] int((b + a*x^3)*(x + x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x\left(x+1\right)\left(x^{2}-x+1\right)}\,\left(ax^{3}+b\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b)*(x**4+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(x + 1)*(x**2 - x + 1))*(a*x**3 + b), x)
```

3.662 $\int \sqrt[4]{x^2+x^4}\,dx$

Optimal. Leaf size=53

$$\frac{1}{2}\sqrt[4]{x^4+x^2}\,x-\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^2}}\right)+\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^2}}\right)$$

Rubi [B] time = 0.06, antiderivative size = 107, normalized size of antiderivative = 2.02, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {2004, 2032, 329, 331, 298, 203, 206}

$$\frac{1}{2}\sqrt[4]{x^4+x^2}\,x-\frac{\left(x^2+1\right)^{3/4}x^{3/2}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2+1}}\right)}{4\left(x^4+x^2\right)^{3/4}}+\frac{\left(x^2+1\right)^{3/4}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2+1}}\right)}{4\left(x^4+x^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2 + x^4)^(1/4),x]
```

```
[Out] (x*(x^2 + x^4)^(1/4))/2 - (x^(3/2)*(1 + x^2)^(3/4)*ArcTan[Sqrt[x]/(1 + x^2)
^(1/4)])/(4*(x^2 + x^4)^(3/4)) + (x^(3/2)*(1 + x^2)^(3/4)*ArcTanh[Sqrt[x]/(
1 + x^2)^(1/4)])/(4*(x^2 + x^4)^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
```

```
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt[4]{x^2+x^4}\,dx &= \frac{1}{2}x\sqrt[4]{x^2+x^4}+\frac{1}{4}\int \frac{x^2}{\left(x^2+x^4\right)^{3/4}}\,dx \\
&= \frac{1}{2}x\sqrt[4]{x^2+x^4}+\frac{\left(x^{3/2}\left(1+x^2\right)^{3/4}\right)\int \frac{\sqrt{x}}{\left(1+x^2\right)^{3/4}}\,dx}{4\left(x^2+x^4\right)^{3/4}} \\
&= \frac{1}{2}x\sqrt[4]{x^2+x^4}+\frac{\left(x^{3/2}\left(1+x^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{\left(1+x^4\right)^{3/4}}\,dx,x,\sqrt{x}\right)}{2\left(x^2+x^4\right)^{3/4}} \\
&= \frac{1}{2}x\sqrt[4]{x^2+x^4}+\frac{\left(x^{3/2}\left(1+x^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{2\left(x^2+x^4\right)^{3/4}} \\
&= \frac{1}{2}x\sqrt[4]{x^2+x^4}+\frac{\left(x^{3/2}\left(1+x^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{4\left(x^2+x^4\right)^{3/4}}-\frac{\left(x^{3/2}\left(1+x^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\right.}{4\left(x^2+x^4\right)^{3/4}} \\
&= \frac{1}{2}x\sqrt[4]{x^2+x^4}-\frac{x^{3/2}\left(1+x^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{4\left(x^2+x^4\right)^{3/4}}+\frac{x^{3/2}\left(1+x^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{4\left(x^2+x^4\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.75

$$\frac{2x\sqrt[4]{x^4+x^2}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-x^2\right)}{3\sqrt[4]{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2 + x^4)^(1/4),x]
```

```
[Out] (2*x*(x^2 + x^4)^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, -x^2])/(3*(1 + x^2
)^(1/4))
```

IntegrateAlgebraic [A] time = 0.16, size = 53, normalized size = 1.00

$$\frac{1}{2}\sqrt[4]{x^4+x^2}\,x-\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^2}}\right)+\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2 + x^4)^(1/4),x]
```

[Out] (x*(x^2 + x^4)^(1/4))/2 - ArcTan[x/(x^2 + x^4)^(1/4)]/4 + ArcTanh[x/(x^2 + x^4)^(1/4)]/4

fricas [B] time = 0.99, size = 95, normalized size = 1.79

$$\frac{1}{2}\left(x^4+x^2\right)^{\frac{1}{4}}x-\frac{1}{8}\arctan\left(\frac{2\left(\left(x^4+x^2\right)^{\frac{1}{4}}x^2+\left(x^4+x^2\right)^{\frac{3}{4}}\right)}{x}\right)+\frac{1}{8}\log\left(\frac{2\,x^3+2\left(x^4+x^2\right)^{\frac{1}{4}}x^2+2\sqrt{x^4+x^2}\,x+x+2\left(x^4+x^2\right)^{\frac{3}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2)^(1/4),x, algorithm="fricas")

[Out] 1/2*(x^4 + x^2)^(1/4)*x - 1/8*arctan(2*((x^4 + x^2)^(1/4)*x^2 + (x^4 + x^2)^(3/4))/x) + 1/8*log((2*x^3 + 2*(x^4 + x^2)^(1/4)*x^2 + 2*sqrt(x^4 + x^2)*x + x + 2*(x^4 + x^2)^(3/4))/x)

giac [A] time = 0.15, size = 47, normalized size = 0.89

$$\frac{1}{2}\,x^2\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}+\frac{1}{4}\arctan\left(\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)+\frac{1}{8}\log\left(\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}+1\right)-\frac{1}{8}\log\left(\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2)^(1/4),x, algorithm="giac")

[Out] 1/2*x^2*(1/x^2 + 1)^(1/4) + 1/4*arctan((1/x^2 + 1)^(1/4)) + 1/8*log((1/x^2 + 1)^(1/4) + 1) - 1/8*log((1/x^2 + 1)^(1/4) - 1)

maple [C] time = 0.15, size = 17, normalized size = 0.32

$$\frac{2x^{\frac{3}{2}}\,\mathrm{hypergeom}\left(\left[-\frac{1}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],-x^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+x^2)^(1/4),x)

[Out] 2/3*x^(3/2)*hypergeom([-1/4,3/4],[7/4],-x^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^4+x^2\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2)^(1/4),x, algorithm="maxima")

[Out] integrate((x^4 + x^2)^(1/4), x)

mupad [B] time = 0.57, size = 29, normalized size = 0.55

$$\frac{2\,x\left(x^4+x^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -x^2\right)}{3\left(x^2+1\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + x^4)^(1/4),x)

```
[Out] (2*x*(x^2 + x^4)^(1/4)*hypergeom([-1/4, 3/4], 7/4, -x^2))/(3*(x^2 + 1)^(1/4
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{x^4 + x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2)**(1/4),x)
```

```
[Out] Integral((x**4 + x**2)**(1/4), x)
```

3.663 $\int \frac{(-1+x^4)\sqrt{1+x^4}}{(1+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=53

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt{x^4+1}}\right)}{2\sqrt{3}}-\frac{\sqrt{x^4+1}\,x}{2\left(x^4+3x^2+1\right)}$$

Rubi [C] time = 2.88, antiderivative size = 1165, normalized size of antiderivative = 21.98, number of steps used = 80, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 1227, 1198, 220, 1196, 1217, 1707, 1209, 6728}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4)*Sqrt[1 + x^4])/(1 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-3*x*Sqrt[1 + x^4])/(5*(1 + x^2)) + (2*x*Sqrt[1 + x^4])/(5*(3 - Sqrt[5])*(
1 + x^2)) + (2*x*Sqrt[1 + x^4])/(5*(3 + Sqrt[5])*(1 + x^2)) + (3*x*Sqrt[1 +
 x^4])/(5*(3 - Sqrt[5] + 2*x^2)) - (4*x*Sqrt[1 + x^4])/(5*(3 - Sqrt[5])*(3
- Sqrt[5] + 2*x^2)) + (3*x*Sqrt[1 + x^4])/(5*(3 + Sqrt[5] + 2*x^2)) - (4*x*
Sqrt[1 + x^4])/(5*(3 + Sqrt[5])*(3 + Sqrt[5] + 2*x^2)) - (Sqrt[3]*(11 - 5*S
qrt[5])*ArcTan[(Sqrt[3]*x)/Sqrt[1 + x^4]])/(4*(25 - 11*Sqrt[5])) + ((2 - Sq
rt[5])*ArcTan[(Sqrt[3]*x)/Sqrt[1 + x^4]])/(Sqrt[3]*(25 - 11*Sqrt[5])) + ((2
 + Sqrt[5])*ArcTan[(Sqrt[3]*x)/Sqrt[1 + x^4]])/(Sqrt[3]*(25 + 11*Sqrt[5]))
- (Sqrt[3]*(2 + Sqrt[5])*(3 + Sqrt[5])*ArcTan[(Sqrt[3]*x)/Sqrt[1 + x^4]])/(
4*(25 + 11*Sqrt[5])) + (3*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticE[2
*ArcTan[x], 1/2])/(5*Sqrt[1 + x^4]) - (2*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)
^2]*EllipticE[2*ArcTan[x], 1/2])/(5*(3 - Sqrt[5])*Sqrt[1 + x^4]) - (2*(1 +
x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/2])/(5*(3 + Sqrt[
5])*Sqrt[1 + x^4]) + (3*(1 - Sqrt[5])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]
*EllipticF[2*ArcTan[x], 1/2])/(40*Sqrt[1 + x^4]) - ((1 - Sqrt[5])*(1 + x^2)
*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(10*(3 - Sqrt[5])
*Sqrt[1 + x^4]) - (3*(5 - Sqrt[5])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*El
lipticF[2*ArcTan[x], 1/2])/(40*Sqrt[1 + x^4]) + ((5 - Sqrt[5])*(1 + x^2)*Sq
rt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(10*(3 - Sqrt[5])*Sq
rt[1 + x^4]) + (3*(1 + Sqrt[5])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*Ellip
ticF[2*ArcTan[x], 1/2])/(40*Sqrt[1 + x^4]) - ((1 + Sqrt[5])*(1 + x^2)*Sqrt[
(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(10*(3 + Sqrt[5])*Sqrt[
1 + x^4]) - (3*(5 + Sqrt[5])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*Elliptic
F[2*ArcTan[x], 1/2])/(40*Sqrt[1 + x^4]) + ((5 + Sqrt[5])*(1 + x^2)*Sqrt[(1
+ x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(10*(3 + Sqrt[5])*Sqrt[1 +
 x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticPi[-1/4, 2*ArcTan
[x], 1/2])/(4*Sqrt[1 + x^4]) - ((5 - 2*Sqrt[5])*(1 + x^2)*Sqrt[(1 + x^4)/(1
 + x^2)^2]*EllipticPi[-1/4, 2*ArcTan[x], 1/2])/(2*(25 - 11*Sqrt[5])*Sqrt[1
+ x^4]) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticPi[-1/4, 2*ArcTan[
x], 1/2])/(2*(3 + Sqrt[5])*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
```

```
1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1227

```
Int[Sqrt[(a_) + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Symbol] :> Simp[(x
*Sqrt[a + c*x^4])/(2*d*(d + e*x^2)), x] + (Dist[c/(2*d*e^2), Int[(d - e*x^2
)/Sqrt[a + c*x^4], x], x] - Dist[(c*d^2 - a*e^2)/(2*d*e^2), Int[1/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]) /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^
2, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(-1+x^4)\sqrt{1+x^4}}{(1+3x^2+x^4)^2}\,dx = \int \left(\frac{(-2-3x^2)\sqrt{1+x^4}}{(1+3x^2+x^4)^2}+\frac{\sqrt{1+x^4}}{1+3x^2+x^4}\right)dx$$

$$= \int \frac{(-2-3x^2)\sqrt{1+x^4}}{(1+3x^2+x^4)^2}\,dx + \int \frac{\sqrt{1+x^4}}{1+3x^2+x^4}\,dx$$

$$= \int \left(-\frac{2\sqrt{1+x^4}}{\sqrt{5}\,(-3+\sqrt{5}-2x^2)}-\frac{2\sqrt{1+x^4}}{\sqrt{5}\,(3+\sqrt{5}+2x^2)}\right)dx + \int \left(-\frac{2\sqrt{1+x^4}}{(1+3x^2+x^4)^2} - \right.$$

$$= -\left(2\int \frac{\sqrt{1+x^4}}{(1+3x^2+x^4)^2}\,dx\right) - 3\int \frac{x^2\sqrt{1+x^4}}{(1+3x^2+x^4)^2}\,dx - \frac{2\int \frac{\sqrt{1+x^4}}{-3+\sqrt{5}-2x^2}\,dx}{\sqrt{5}} - \frac{2\int \frac{}{3+}}{}$$

$$= -\left(2\int \left(\frac{4\sqrt{1+x^4}}{5(-3+\sqrt{5}-2x^2)^2}+\frac{4\sqrt{1+x^4}}{5\sqrt{5}\,(-3+\sqrt{5}-2x^2)}+\frac{4\sqrt{1+x^4}}{5(3+\sqrt{5}+2x^2)^2}+\frac{}{5\sqrt{}}\right.\right.$$

$$= -\left(\frac{8}{5}\int \frac{\sqrt{1+x^4}}{(-3+\sqrt{5}-2x^2)^2}\,dx\right) - \frac{8}{5}\int \frac{\sqrt{1+x^4}}{(3+\sqrt{5}+2x^2)^2}\,dx - \frac{8\int \frac{\sqrt{1+x^4}}{-3+\sqrt{5}-2x^2}\,dx}{5\sqrt{5}} - \frac{8}{}$$

$$= \frac{3x\sqrt{1+x^4}}{5(3-\sqrt{5}+2x^2)} - \frac{4x\sqrt{1+x^4}}{5(3-\sqrt{5})(3-\sqrt{5}+2x^2)} + \frac{3x\sqrt{1+x^4}}{5(3+\sqrt{5}+2x^2)} - \frac{4x}{5(3+\sqrt{5}}$$

$$= \frac{3x\sqrt{1+x^4}}{5(3-\sqrt{5}+2x^2)} - \frac{4x\sqrt{1+x^4}}{5(3-\sqrt{5})(3-\sqrt{5}+2x^2)} + \frac{3x\sqrt{1+x^4}}{5(3+\sqrt{5}+2x^2)} - \frac{4x}{5(3+\sqrt{5}}$$

$$= \frac{2x\sqrt{1+x^4}}{5(3-\sqrt{5})(1+x^2)} + \frac{2x\sqrt{1+x^4}}{5(3+\sqrt{5})(1+x^2)} + \frac{3x\sqrt{1+x^4}}{5(3-\sqrt{5}+2x^2)} - \frac{4x\sqrt{1+}}{5(3-\sqrt{5})(3-}$$

Mathematica [C] time = 1.37, size = 257, normalized size = 4.85

$$-\frac{x^5-\sqrt[4]{-1}\sqrt{x^4+1}\,x^4\Pi\left(-\frac{2i}{3+\sqrt{5}};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)-\sqrt[4]{-1}\sqrt{x^4+1}\,\Pi\left(-\frac{2i}{3+\sqrt{5}};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)+\sqrt[4]{-1}\sqrt{x^4+1}\,(x^4+3x^2+1)F\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)-3\sqrt[4]{-1}\sqrt{x^4+1}\,x^2\Pi\left(-\frac{2i}{3+\sqrt{5}};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)-\sqrt[4]{-1}\sqrt{x^4+1}\,(x^4+3x^2+1)\Pi\left(\frac{2i}{-3+\sqrt{5}};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)+x}{2\sqrt{x^4+1}\,(x^4+3x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)*Sqrt[1 + x^4])/(1 + 3*x^2 + x^4)^2,x]
```

```
[Out] -1/2*(x + x^5 + (-1)^(1/4)*Sqrt[1 + x^4]*(1 + 3*x^2 + x^4)*EllipticF[I*ArcS
inh[(-1)^(1/4)*x], -1] - (-1)^(1/4)*Sqrt[1 + x^4]*(1 + 3*x^2 + x^4)*Ellipti
cPi[(2*I)/(-3 + Sqrt[5]), I*ArcSinh[(-1)^(1/4)*x], -1] - (-1)^(1/4)*Sqrt[1
+ x^4]*EllipticPi[(-2*I)/(3 + Sqrt[5]), I*ArcSinh[(-1)^(1/4)*x], -1] - 3*(-
1)^(1/4)*x^2*Sqrt[1 + x^4]*EllipticPi[(-2*I)/(3 + Sqrt[5]), I*ArcSinh[(-1)^
(1/4)*x], -1] - (-1)^(1/4)*x^4*Sqrt[1 + x^4]*EllipticPi[(-2*I)/(3 + Sqrt[5]
), I*ArcSinh[(-1)^(1/4)*x], -1])/(Sqrt[1 + x^4]*(1 + 3*x^2 + x^4))
```

IntegrateAlgebraic [A] time = 0.66, size = 53, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{\sqrt{x^4+1}}\right)}{2\sqrt{3}} - \frac{\sqrt{x^4+1}\,x}{2\left(x^4+3x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*Sqrt[1 + x^4])/(1 + 3*x^2 + x^4)^2,x]
```

```
[Out] -1/2*(x*Sqrt[1 + x^4])/(1 + 3*x^2 + x^4) - ArcTan[(Sqrt[3]*x)/Sqrt[1 + x^4]
]/(2*Sqrt[3])
```

fricas [A] time = 0.47, size = 65, normalized size = 1.23

$$-\frac{\sqrt{3}\left(x^4+3x^2+1\right)\arctan\left(\frac{2\sqrt{3}\sqrt{x^4+1}\,x}{x^4-3x^2+1}\right)+6\sqrt{x^4+1}\,x}{12\left(x^4+3x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/2)/(x^4+3*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(3)*(x^4 + 3*x^2 + 1)*arctan(2*sqrt(3)*sqrt(x^4 + 1)*x/(x^4 - 3*
x^2 + 1)) + 6*sqrt(x^4 + 1)*x)/(x^4 + 3*x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}\left(x^4-1\right)}{\left(x^4+3x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/2)/(x^4+3*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 1)*(x^4 - 1)/(x^4 + 3*x^2 + 1)^2, x)
```

maple [C] time = 0.10, size = 334, normalized size = 6.30

$$-\frac{x\sqrt{x^4+1}}{2\left(x^4+3x^2+1\right)}+\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4+3\_Z^2+1\right)}\_\alpha\left(12\_\alpha^2+23\right)\left(-\frac{\sqrt{3}\,\mathrm{arctanh}\left(\frac{\_\alpha^2\left(-\_\alpha^2+x^2-3\right)}{\sqrt{-3\_\alpha^2}\,\sqrt{x^4+1}}\right)}{\sqrt{-\_\alpha^2}}+\frac{6(-1)^{\frac{3}{4}}\left(-\_\alpha^3-3\_\alpha\right)\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x,i\_\alpha^2+3i,i\right)}{\sqrt{x^4+1}}\right)\right)}{120}-\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4+3\_Z^2+1\right)}\_\alpha\left(2\_\alpha^2+3\right)\left(-\frac{\sqrt{3}\,\mathrm{arctanh}\left(\frac{\_\alpha^2\left(-\_\alpha^2+x^2-3\right)}{\sqrt{-3\_\alpha^2}\,\sqrt{x^4+1}}\right)}{\sqrt{-\_\alpha^2}}+\frac{6(-1)^{\frac{3}{4}}\left(-\_\alpha^3-3\_\alpha\right)\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x,i\_\alpha^2+3i,i\right)}{\sqrt{x^4+1}}\right)\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^4+1)^(1/2)/(x^4+3*x^2+1)^2,x)
```

```
[Out] -1/2*x*(x^4+1)^(1/2)/(x^4+3*x^2+1)+1/2/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2
)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2
)),I)+1/120*sum(_alpha*(12*_alpha^2+23)*(-3^(1/2)/(-_alpha^2)^(1/2)*arctanh
(_alpha^2*(-_alpha^2+x^2-3)/(-3*_alpha^2)^(1/2)/(x^4+1)^(1/2))+6*(-1)^(3/4)
*(-_alpha^3-3*_alpha)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*Ellipti
cPi((-1)^(1/4)*x,I*_alpha^2+3*I,I)),_alpha=RootOf(_Z^4+3*_Z^2+1))-1/20*sum(
_alpha*(2*_alpha^2+3)*(-3^(1/2)/(-_alpha^2)^(1/2)*arctanh(_alpha^2*(-_alpha
^2+x^2-3)/(-3*_alpha^2)^(1/2)/(x^4+1)^(1/2))+6*(-1)^(3/4)*(-_alpha^3-3*_alp
ha)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I
*_alpha^2+3*I,I)),_alpha=RootOf(_Z^4+3*_Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}\left(x^4-1\right)}{\left(x^4+3x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/2)/(x^4+3*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 1)*(x^4 - 1)/(x^4 + 3*x^2 + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x^4-1\right)\,\sqrt{x^4+1}}{{\left(x^4+3\,x^2+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(x^4 + 1)^(1/2))/(3*x^2 + x^4 + 1)^2,x)
[Out] int(((x^4 - 1)*(x^4 + 1)^(1/2))/(3*x^2 + x^4 + 1)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**4+1)**(1/2)/(x**4+3*x**2+1)**2,x)
[Out] Timed out
```

3.664 $\int \frac{1}{x^8 \sqrt[4]{x^3+x^4}} \, dx$

Optimal. Leaf size=53

$$\frac{4 \left(x^4+x^3\right)^{3/4} \left(262144 x^7-196608 x^6+172032 x^5-157696 x^4+147840 x^3-140448 x^2+134596 x-129789\right)}{4023459 x^{10}}$$

Rubi [B] time = 0.22, antiderivative size = 145, normalized size of antiderivative = 2.74, number of steps used = 8, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2016, 2014}

$$\frac{1048576 \left(x^4+x^3\right)^{3/4}}{4023459 x^3}-\frac{262144 \left(x^4+x^3\right)^{3/4}}{1341153 x^4}-\frac{4 \left(x^4+x^3\right)^{3/4}}{31 x^{10}}+\frac{112 \left(x^4+x^3\right)^{3/4}}{837 x^9}-\frac{896 \left(x^4+x^3\right)^{3/4}}{6417 x^8}+\frac{17920 \left(x^4+x^3\right)^{3/4}}{121923 x^7}-\frac{57344 \left(x^4+x^3\right)^{3/4}}{365769 x^6}+\frac{229376 \left(x^4+x^3\right)^{3/4}}{1341153 x^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(x^3 + x^4)^(1/4)),x]
```

```
[Out] (-4*(x^3 + x^4)^(3/4))/(31*x^10) + (112*(x^3 + x^4)^(3/4))/(837*x^9) - (896
*(x^3 + x^4)^(3/4))/(6417*x^8) + (17920*(x^3 + x^4)^(3/4))/(121923*x^7) - (
57344*(x^3 + x^4)^(3/4))/(365769*x^6) + (229376*(x^3 + x^4)^(3/4))/(1341153
*x^5) - (262144*(x^3 + x^4)^(3/4))/(1341153*x^4) + (1048576*(x^3 + x^4)^(3/
4))/(4023459*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^8\sqrt[4]{x^3+x^4}}\,dx = -\frac{4\left(x^3+x^4\right)^{3/4}}{31x^{10}} - \frac{28}{31}\int \frac{1}{x^7\sqrt[4]{x^3+x^4}}\,dx$$

$$= -\frac{4\left(x^3+x^4\right)^{3/4}}{31x^{10}} + \frac{112\left(x^3+x^4\right)^{3/4}}{837x^9} + \frac{224}{279}\int \frac{1}{x^6\sqrt[4]{x^3+x^4}}\,dx$$

$$= -\frac{4\left(x^3+x^4\right)^{3/4}}{31x^{10}} + \frac{112\left(x^3+x^4\right)^{3/4}}{837x^9} - \frac{896\left(x^3+x^4\right)^{3/4}}{6417x^8} - \frac{4480\int \frac{1}{x^5\sqrt[4]{x^3+x^4}}\,dx}{6417}$$

$$= -\frac{4\left(x^3+x^4\right)^{3/4}}{31x^{10}} + \frac{112\left(x^3+x^4\right)^{3/4}}{837x^9} - \frac{896\left(x^3+x^4\right)^{3/4}}{6417x^8} + \frac{17920\left(x^3+x^4\right)^{3/4}}{121923x^7} + \frac{71680\int \frac{}{x^4}}{1219}$$

$$= -\frac{4\left(x^3+x^4\right)^{3/4}}{31x^{10}} + \frac{112\left(x^3+x^4\right)^{3/4}}{837x^9} - \frac{896\left(x^3+x^4\right)^{3/4}}{6417x^8} + \frac{17920\left(x^3+x^4\right)^{3/4}}{121923x^7} - \frac{57344\left(x^3\right.}{36576}$$

$$= -\frac{4\left(x^3+x^4\right)^{3/4}}{31x^{10}} + \frac{112\left(x^3+x^4\right)^{3/4}}{837x^9} - \frac{896\left(x^3+x^4\right)^{3/4}}{6417x^8} + \frac{17920\left(x^3+x^4\right)^{3/4}}{121923x^7} - \frac{57344\left(x^3\right.}{36576}$$

$$= -\frac{4\left(x^3+x^4\right)^{3/4}}{31x^{10}} + \frac{112\left(x^3+x^4\right)^{3/4}}{837x^9} - \frac{896\left(x^3+x^4\right)^{3/4}}{6417x^8} + \frac{17920\left(x^3+x^4\right)^{3/4}}{121923x^7} - \frac{57344\left(x^3\right.}{36576}$$

$$= -\frac{4\left(x^3+x^4\right)^{3/4}}{31x^{10}} + \frac{112\left(x^3+x^4\right)^{3/4}}{837x^9} - \frac{896\left(x^3+x^4\right)^{3/4}}{6417x^8} + \frac{17920\left(x^3+x^4\right)^{3/4}}{121923x^7} - \frac{57344\left(x^3\right.}{36576}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.00

$$\frac{4\left(x^3(x+1)\right)^{3/4}\left(262144x^7-196608x^6+172032x^5-157696x^4+147840x^3-140448x^2+134596x-129789\right)}{4023459x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(x^3 + x^4)^(1/4)),x]
```

```
[Out] (4*(x^3*(1 + x))^(3/4)*(-129789 + 134596*x - 140448*x^2 + 147840*x^3 - 1576
96*x^4 + 172032*x^5 - 196608*x^6 + 262144*x^7))/(4023459*x^10)
```

IntegrateAlgebraic [A] time = 0.33, size = 53, normalized size = 1.00

$$\frac{4\left(x^4+x^3\right)^{3/4}\left(262144x^7-196608x^6+172032x^5-157696x^4+147840x^3-140448x^2+134596x-129789\right)}{4023459x^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^8*(x^3 + x^4)^(1/4)),x]
```

```
[Out] (4*(x^3 + x^4)^(3/4)*(-129789 + 134596*x - 140448*x^2 + 147840*x^3 - 157696
*x^4 + 172032*x^5 - 196608*x^6 + 262144*x^7))/(4023459*x^10)
```

fricas [A] time = 0.39, size = 49, normalized size = 0.92

$$\frac{4\left(262144\,x^7-196608\,x^6+172032\,x^5-157696\,x^4+147840\,x^3-140448\,x^2+134596\,x-129789\right)\left(x^4+x^3\right)^{\frac{3}{4}}}{4023459\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^4+x^3)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/4023459*(262144*x^7 - 196608*x^6 + 172032*x^5 - 157696*x^4 + 147840*x^3 -
 140448*x^2 + 134596*x - 129789)*(x^4 + x^3)^(3/4)/x^10
```

giac [A] time = 0.35, size = 73, normalized size = 1.38

$$-\frac{4}{31}\left(\frac{1}{x}+1\right)^{\frac{31}{4}}+\frac{28}{27}\left(\frac{1}{x}+1\right)^{\frac{27}{4}}-\frac{84}{23}\left(\frac{1}{x}+1\right)^{\frac{23}{4}}+\frac{140}{19}\left(\frac{1}{x}+1\right)^{\frac{19}{4}}-\frac{28}{3}\left(\frac{1}{x}+1\right)^{\frac{15}{4}}+\frac{84}{11}\left(\frac{1}{x}+1\right)^{\frac{11}{4}}-4\left(\frac{1}{x}+1\right)^{\frac{7}{4}}+\frac{4}{3}\left(\frac{1}{x}+1\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^4+x^3)^(1/4),x, algorithm="giac")
```

```
[Out] -4/31*(1/x + 1)^(31/4) + 28/27*(1/x + 1)^(27/4) - 84/23*(1/x + 1)^(23/4) +
140/19*(1/x + 1)^(19/4) - 28/3*(1/x + 1)^(15/4) + 84/11*(1/x + 1)^(11/4) -
4*(1/x + 1)^(7/4) + 4/3*(1/x + 1)^(3/4)
```

maple [A] time = 0.00, size = 53, normalized size = 1.00

$$\frac{4\left(1+x\right)\left(262144x^{7}-196608x^{6}+172032x^{5}-157696x^{4}+147840x^{3}-140448x^{2}+134596x-129789\right)}{4023459x^{7}\left(x^{4}+x^{3}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^4+x^3)^(1/4),x)
```

```
[Out] 4/4023459*(1+x)*(262144*x^7-196608*x^6+172032*x^5-157696*x^4+147840*x^3-140
448*x^2+134596*x-129789)/x^7/(x^4+x^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{4}+x^{3}\right)^{\frac{1}{4}}x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^4+x^3)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + x^3)^(1/4)*x^8), x)
```

mupad [B] time = 0.58, size = 113, normalized size = 2.13

$$\frac{1048576\left(x^{4}+x^{3}\right)^{3/4}}{4023459\,x^{3}}-\frac{262144\left(x^{4}+x^{3}\right)^{3/4}}{1341153\,x^{4}}+\frac{229376\left(x^{4}+x^{3}\right)^{3/4}}{1341153\,x^{5}}-\frac{57344\left(x^{4}+x^{3}\right)^{3/4}}{365769\,x^{6}}+\frac{17920\left(x^{4}+x^{3}\right)^{3/4}}{121923\,x^{7}}-\frac{896\left(x^{4}+x^{3}\right)^{3/4}}{6417\,x^{8}}+\frac{112\left(x^{4}+x^{3}\right)^{3/4}}{837\,x^{9}}-\frac{4\left(x^{4}+x^{3}\right)^{3/4}}{31\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^3 + x^4)^(1/4)),x)
```

```
[Out] (1048576*(x^3 + x^4)^(3/4))/(4023459*x^3) - (262144*(x^3 + x^4)^(3/4))/(134
1153*x^4) + (229376*(x^3 + x^4)^(3/4))/(1341153*x^5) - (57344*(x^3 + x^4)^(
3/4))/(365769*x^6) + (17920*(x^3 + x^4)^(3/4))/(121923*x^7) - (896*(x^3 + x
^4)^(3/4))/(6417*x^8) + (112*(x^3 + x^4)^(3/4))/(837*x^9) - (4*(x^3 + x^4)^
(3/4))/(31*x^10)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{8}\sqrt[4]{x^{3}\left(x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**4+x**3)**(1/4),x)
```

```
[Out] Integral(1/(x**8*(x**3*(x + 1))**(1/4)), x)
```

3.665 $\int \frac{1+4x}{\sqrt{1-2x+3x^2+2x^3+x^4}}\,dx$

Optimal. Leaf size=53

$$-\log\left(-x^4-3x^3-5x^2+\left(x^2+2x+2\right)\sqrt{x^4+2x^3+3x^2-2x+1}-2x\right)$$

Rubi [F] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+4x}{\sqrt{1-2x+3x^2+2x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + 4*x)/Sqrt[1 - 2*x + 3*x^2 + 2*x^3 + x^4],x]
```

```
[Out] Defer[Int][1/Sqrt[1 - 2*x + 3*x^2 + 2*x^3 + x^4], x] + 4*Defer[Int][x/Sqrt[
1 - 2*x + 3*x^2 + 2*x^3 + x^4], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+4x}{\sqrt{1-2x+3x^2+2x^3+x^4}}\,dx &= \int \left(\frac{1}{\sqrt{1-2x+3x^2+2x^3+x^4}}+\frac{4x}{\sqrt{1-2x+3x^2+2x^3+x^4}}\right)dx\\ &= 4\int \frac{x}{\sqrt{1-2x+3x^2+2x^3+x^4}}\,dx+\int \frac{1}{\sqrt{1-2x+3x^2+2x^3+x^4}}\,dx\end{aligned}$$

Mathematica [C] time = 1.39, size = 484, normalized size = 9.13

$$\frac{2\left(-4i-\sqrt{1-4i}-\sqrt{1+4i}\right)\left(x+\frac{1}{2}\left((1-2i)+\sqrt{1-4i}\right)\right)^2\sqrt{\frac{\left(-4i+\sqrt{1-4i}-\sqrt{1+4i}\right)\left(-2x+\sqrt{1+4i}-(1+2i)\right)}{\sqrt{1+4i}\left(2x+\sqrt{1-4i}+(1-2i)\right)}}\sqrt{\frac{\sqrt{1-4i}\left(-4i+\sqrt{1-4i}-\sqrt{1+4i}\right)\left(-2x+\sqrt{1-4i}-(1-2i)\right)\left(2x+\sqrt{1+4i}+(1+2i)\right)}{\left(4i+\sqrt{1-4i}+\sqrt{1+4i}\right)^2\left(2x+\sqrt{1-4i}+(1-2i)\right)^2}}\left(\left((1-4i)+2\sqrt{1-4i}\right)F\left(\sin^{-1}\left(\sqrt{2}\sqrt{\frac{\sqrt{1-4i}\left(2x+\sqrt{1+4i}+(1+2i)\right)}{\left(4i+\sqrt{1-4i}+\sqrt{1+4i}\right)\left(2x+\sqrt{1-4i}+(1-2i)\right)}}\right)|\frac{1}{2}+\frac{9}{2\sqrt{17}}\right)+2\left(4i-\sqrt{1-4i}+\sqrt{1+4i}\right)\Pi\left(\frac{4i+\sqrt{1-4i}+\sqrt{1+4i}}{2\sqrt{1-4i}};\sin^{-1}\left(\sqrt{2}\sqrt{\frac{\sqrt{1-4i}\left(2x+\sqrt{1+4i}+(1+2i)\right)}{\left(4i+\sqrt{1-4i}+\sqrt{1+4i}\right)\left(2x+\sqrt{1-4i}+(1-2i)\right)}}\right)|\frac{1}{2}+\frac{9}{2\sqrt{17}}\right)\right)}{\sqrt{1-4i}\left(-4i+\sqrt{1-4i}-\sqrt{1+4i}\right)\sqrt{x^4+2x^3+3x^2-2x+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + 4*x)/Sqrt[1 - 2*x + 3*x^2 + 2*x^3 + x^4],x]
```

```
[Out] (2*(-4*I - Sqrt[1 - 4*I] - Sqrt[1 + 4*I])*(((1 - 2*I) + Sqrt[1 - 4*I])/2 +
x)^2*Sqrt[((-4*I + Sqrt[1 - 4*I] - Sqrt[1 + 4*I])*((-1 - 2*I) + Sqrt[1 + 4*
I] - 2*x))/(Sqrt[1 + 4*I]*((1 - 2*I) + Sqrt[1 - 4*I] + 2*x))]*Sqrt[(Sqrt[1
- 4*I]*(-4*I + Sqrt[1 - 4*I] - Sqrt[1 + 4*I])*((-1 + 2*I) + Sqrt[1 - 4*I] -
 2*x)*((1 + 2*I) + Sqrt[1 + 4*I] + 2*x))/((4*I + Sqrt[1 - 4*I] + Sqrt[1 + 4
*I])^2*((1 - 2*I) + Sqrt[1 - 4*I] + 2*x)^2)]*(((1 - 4*I) + 2*Sqrt[1 - 4*I])
*EllipticF[ArcSin[Sqrt[2]*Sqrt[(Sqrt[1 - 4*I]*((1 + 2*I) + Sqrt[1 + 4*I] +
2*x))/((4*I + Sqrt[1 - 4*I] + Sqrt[1 + 4*I])*((1 - 2*I) + Sqrt[1 - 4*I] + 2
*x))]], 1/2 + 9/(2*Sqrt[17])] + 2*(4*I - Sqrt[1 - 4*I] + Sqrt[1 + 4*I])*Ell
ipticPi[(4*I + Sqrt[1 - 4*I] + Sqrt[1 + 4*I])/(2*Sqrt[1 - 4*I]), ArcSin[Sqr
t[2]*Sqrt[(Sqrt[1 - 4*I]*((1 + 2*I) + Sqrt[1 + 4*I] + 2*x))/((4*I + Sqrt[1
- 4*I] + Sqrt[1 + 4*I])*((1 - 2*I) + Sqrt[1 - 4*I] + 2*x))]], 1/2 + 9/(2*Sq
rt[17])]))/(Sqrt[1 - 4*I]*(-4*I + Sqrt[1 - 4*I] - Sqrt[1 + 4*I])*Sqrt[1 - 2
*x + 3*x^2 + 2*x^3 + x^4])
```

IntegrateAlgebraic [A] time = 5.07, size = 53, normalized size = 1.00

$$-\log\left(-x^4-3x^3-5x^2+\left(x^2+2x+2\right)\sqrt{x^4+2x^3+3x^2-2x+1}-2x\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 4*x)/Sqrt[1 - 2*x + 3*x^2 + 2*x^3 + x^4],x]

[Out] -Log[-2*x - 5*x^2 - 3*x^3 - x^4 + (2 + 2*x + x^2)*Sqrt[1 - 2*x + 3*x^2 + 2*x^3 + x^4]]

fricas [A] time = 0.44, size = 47, normalized size = 0.89

$$\log\left(x^{4}+3\,x^{3}+5\,x^{2}+\sqrt{x^{4}+2\,x^{3}+3\,x^{2}-2\,x+1}\left(x^{2}+2\,x+2\right)+2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+4*x)/(x^4+2*x^3+3*x^2-2*x+1)^(1/2),x, algorithm="fricas")

[Out] log(x^4 + 3*x^3 + 5*x^2 + sqrt(x^4 + 2*x^3 + 3*x^2 - 2*x + 1)*(x^2 + 2*x + 2) + 2*x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x+1}{\sqrt{x^{4}+2\,x^{3}+3\,x^{2}-2\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+4*x)/(x^4+2*x^3+3*x^2-2*x+1)^(1/2),x, algorithm="giac")

[Out] integrate((4*x + 1)/sqrt(x^4 + 2*x^3 + 3*x^2 - 2*x + 1), x)

maple [C] time = 1.14, size = 2700, normalized size = 50.94

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+4*x)/(x^4+2*x^3+3*x^2-2*x+1)^(1/2),x)

[Out] 2*(-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)+RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))*((RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))/(RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))/(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)))^(1/2)*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2))^2*((RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=3))/(RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=3)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))/(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)))^(1/2)*((RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4))/(RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))/(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)))^(1/2)/(RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2))/(RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))/((x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=3))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)))^(1/2)*EllipticF(((RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))/(RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))/(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)))^(1/2),((RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=3))*(-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)+RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))/(-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=3)+RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))/(RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)))^(1/2))+8*(-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)+RootOf(_Z^4+2*_Z^3+3*_Z^2

```
-2*_Z+1,index=1))*((RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)-RootOf(_Z^4+2
*_Z^3+3*_Z^2-2*_Z+1,index=2))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))
/(RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+
1,index=1))/(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)))^(1/2)*(x-RootOf(
_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2))^2*((RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,in
dex=2)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))*(x-RootOf(_Z^4+2*_Z^3+3*_
Z^2-2*_Z+1,index=3))/(RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=3)-RootOf(_Z^4
+2*_Z^3+3*_Z^2-2*_Z+1,index=1))/(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2
)))^(1/2)*((RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)-RootOf(_Z^4+2*_Z^3+3*
_Z^2-2*_Z+1,index=1))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4))/(RootOf
(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=
1))/(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)))^(1/2)/(RootOf(_Z^4+2*_Z^
3+3*_Z^2-2*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2))/(RootOf
(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=
1))/((x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))*(x-RootOf(_Z^4+2*_Z^3+3*
_Z^2-2*_Z+1,index=2))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=3))*(x-Root
Of(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)))^(1/2)*(RootOf(_Z^4+2*_Z^3+3*_Z^2-2*
_Z+1,index=2)*EllipticF(((RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)-RootOf(
_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,ind
ex=1))/(RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2
-2*_Z+1,index=1))/(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)))^(1/2),((Ro
otOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,in
dex=3))*(-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)+RootOf(_Z^4+2*_Z^3+3*_Z
^2-2*_Z+1,index=1))/(-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=3)+RootOf(_Z^4
+2*_Z^3+3*_Z^2-2*_Z+1,index=1))/(RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)-
RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)))^(1/2))+(-RootOf(_Z^4+2*_Z^3+3*_
Z^2-2*_Z+1,index=2)+RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))*EllipticPi((
(RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1
,index=2))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))/(RootOf(_Z^4+2*_Z^
3+3*_Z^2-2*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))/(x-Root
Of(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)))^(1/2),(RootOf(_Z^4+2*_Z^3+3*_Z^2-2*
_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=1))/(RootOf(_Z^4+2*_Z^
3+3*_Z^2-2*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)),((RootO
f(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index
=3))*(-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)+RootOf(_Z^4+2*_Z^3+3*_Z^2-
2*_Z+1,index=1))/(-RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=3)+RootOf(_Z^4+2*
_Z^3+3*_Z^2-2*_Z+1,index=1))/(RootOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=2)-Roo
tOf(_Z^4+2*_Z^3+3*_Z^2-2*_Z+1,index=4)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x+1}{\sqrt{x^4+2\,x^3+3\,x^2-2\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(x^4+2*x^3+3*x^2-2*x+1)^(1/2),x, algorithm="maxima")
[Out] integrate((4*x + 1)/sqrt(x^4 + 2*x^3 + 3*x^2 - 2*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{4\,x+1}{\sqrt{x^4+2\,x^3+3\,x^2-2\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 1)/(3*x^2 - 2*x + 2*x^3 + x^4 + 1)^(1/2),x)
[Out] int((4*x + 1)/(3*x^2 - 2*x + 2*x^3 + x^4 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x+1}{\sqrt{x^4+2x^3+3x^2-2x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(x**4+2*x**3+3*x**2-2*x+1)**(1/2),x)
```

```
[Out] Integral((4*x + 1)/sqrt(x**4 + 2*x**3 + 3*x**2 - 2*x + 1), x)
```

3.666 $\int \frac{1}{\sqrt[4]{-1+x^4}\left(1+3x^4\right)}\,dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {377, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x^4)^(1/4)*(1 + 3*x^4)),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/(-1 + x^4)^(1/4)]/(2*Sqrt[2]) + ArcTanh[(Sqrt[2]*x)/(-1
+ x^4)^(1/4)]/(2*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{-1+x^4}\left(1+3x^4\right)}\,dx &= \text{Subst}\left(\int \frac{1}{1-4x^4}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{-1+x^4}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{-1+x^4}}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.83

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4-1}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x^4)^(1/4)*(1 + 3*x^4)),x]
```

```
[Out] (ArcTan[(Sqrt[2]*x)/(-1 + x^4)^(1/4)] + ArcTanh[(Sqrt[2]*x)/(-1 + x^4)^(1/4
)])/(2*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.23, size = 53, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x^4)^(1/4)*(1 + 3*x^4)),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/(-1 + x^4)^(1/4)]/(2*Sqrt[2]) + ArcTanh[(Sqrt[2]*x)/(-1
+ x^4)^(1/4)]/(2*Sqrt[2])
```

fricas [B] time = 7.58, size = 144, normalized size = 2.72

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{2\left(2\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}x^3+\sqrt{2}\left(x^4-1\right)^{\frac{3}{4}}x\right)}{3x^4+1}\right)+\frac{1}{16}\sqrt{2}\log\left(-\frac{73x^8-58x^4+4\sqrt{2}\left(13x^5-x\right)\left(x^4-1\right)^{\frac{3}{4}}+8\sqrt{2}\left(7x^7-3x^3\right)\left(x^4-1\right)^{\frac{1}{4}}+16\left(5x^6-x^2\right)\sqrt{x^4-1}+1}{9x^8+6x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)^(1/4)/(3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(2)*arctan(2*(2*sqrt(2)*(x^4 - 1)^(1/4)*x^3 + sqrt(2)*(x^4 - 1)^(3
/4)*x)/(3*x^4 + 1)) + 1/16*sqrt(2)*log(-(73*x^8 - 58*x^4 + 4*sqrt(2)*(13*x^
5 - x)*(x^4 - 1)^(3/4) + 8*sqrt(2)*(7*x^7 - 3*x^3)*(x^4 - 1)^(1/4) + 16*(5*
x^6 - x^2)*sqrt(x^4 - 1) + 1)/(9*x^8 + 6*x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^4+1\right)\left(x^4-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)^(1/4)/(3*x^4+1),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^4 + 1)*(x^4 - 1)^(1/4)), x)
```

maple [C] time = 1.29, size = 158, normalized size = 2.98

$$\frac{\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{4\,\mathrm{RootOf}\left(\_Z^2-2\right)\sqrt{x^4-1}\,x^2+5\,\mathrm{RootOf}\left(\_Z^2-2\right)x^4+4\left(x^4-1\right)^{\frac{3}{4}}x+8x^3\left(x^4-1\right)^{\frac{1}{4}}-\mathrm{RootOf}\left(\_Z^2-2\right)}{3x^4+1}\right)}{8}-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{4\,\mathrm{RootOf}\left(\_Z^2+2\right)\sqrt{x^4-1}\,x^2-5\,\mathrm{RootOf}\left(\_Z^2+2\right)x^4+4\left(x^4-1\right)^{\frac{3}{4}}x-8x^3\left(x^4-1\right)^{\frac{1}{4}}+\mathrm{RootOf}\left(\_Z^2+2\right)}{3x^4+1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-1)^(1/4)/(3*x^4+1),x)
```

```
[Out] 1/8*RootOf(_Z^2-2)*ln((4*RootOf(_Z^2-2)*(x^4-1)^(1/2)*x^2+5*RootOf(_Z^2-2)*
x^4+4*(x^4-1)^(3/4)*x+8*x^3*(x^4-1)^(1/4)-RootOf(_Z^2-2))/(3*x^4+1))-1/8*Ro
```

```
otOf(_Z^2+2)*ln((4*RootOf(_Z^2+2)*(x^4-1)^(1/2)*x^2-5*RootOf(_Z^2+2)*x^4+4*
(x^4-1)^(3/4)*x-8*x^3*(x^4-1)^(1/4)+RootOf(_Z^2+2))/(3*x^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^4+1\right)\left(x^4-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)^(1/4)/(3*x^4+1),x, algorithm="maxima")
[Out] integrate(1/((3*x^4 + 1)*(x^4 - 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(x^4-1\right)^{1/4}\,\left(3\,x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^4 - 1)^(1/4)*(3*x^4 + 1)),x)
[Out] int(1/((x^4 - 1)^(1/4)*(3*x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{(x-1)\,(x+1)\left(x^2+1\right)}\,\left(3x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-1)**(1/4)/(3*x**4+1),x)
[Out] Integral(1/(((x - 1)*(x + 1)*(x**2 + 1))**(1/4)*(3*x**4 + 1)), x)
```

3.667 $\int \frac{-3b+ax^4}{\left(b-x^3+ax^4\right)\sqrt[4]{bx+ax^5}}\,dx$

Optimal. Leaf size=53

$$-2\tan^{-1}\left(\frac{\left(ax^5+bx\right)^{3/4}}{ax^4+b}\right)-2\tanh^{-1}\left(\frac{\left(ax^5+bx\right)^{3/4}}{ax^4+b}\right)$$

Rubi [F] time = 2.29, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3b+ax^4}{\left(b-x^3+ax^4\right)\sqrt[4]{bx+ax^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3*b + a*x^4)/((b - x^3 + a*x^4)*(b*x + a*x^5)^(1/4)),x]
```

```
[Out] (4*x*(1 + (a*x^4)/b)^(1/4)*Hypergeometric2F1[3/16, 1/4, 19/16, -((a*x^4)/b)
])/(3*(b*x + a*x^5)^(1/4)) - (16*b*x^(1/4)*(b + a*x^4)^(1/4)*Defer[Subst][D
efer[Int][x^2/((b + a*x^16)^(1/4)*(b - x^12 + a*x^16)), x], x, x^(1/4)])/(b
*x + a*x^5)^(1/4) + (4*x^(1/4)*(b + a*x^4)^(1/4)*Defer[Subst][Defer[Int][x^
14/((b + a*x^16)^(1/4)*(b - x^12 + a*x^16)), x], x, x^(1/4)])/(b*x + a*x^5)
^(1/4)
```

Rubi steps

$$\int \frac{-3b+ax^4}{\left(b-x^3+ax^4\right)\sqrt[4]{bx+ax^5}}\,dx=\frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^4}\right)\int \frac{-3b+ax^4}{\sqrt[4]{x}\sqrt[4]{b+ax^4}\left(b-x^3+ax^4\right)}\,dx}{\sqrt[4]{bx+ax^5}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^4}\right)\operatorname{Subst}\left(\int \frac{x^2\left(-3b+ax^{16}\right)}{\sqrt[4]{b+ax^{16}}\left(b-x^{12}+ax^{16}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^5}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^4}\right)\operatorname{Subst}\left(\int \left(\frac{x^2}{\sqrt[4]{b+ax^{16}}}+\frac{x^2\left(-4b+x^{12}\right)}{\sqrt[4]{b+ax^{16}}\left(b-x^{12}+ax^{16}\right)}\right)dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^5}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^4}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt[4]{b+ax^{16}}}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^5}}+\frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^4}\right)\operatorname{Subst}\left(\int \frac{}{\sqrt[4]{}}\right.}{\sqrt[4]{bx+}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^4}\right)\operatorname{Subst}\left(\int \left(-\frac{4bx^2}{\sqrt[4]{b+ax^{16}}\left(b-x^{12}+ax^{16}\right)}+\frac{x^{14}}{\sqrt[4]{b+ax^{16}}\left(b-x^{12}+ax^{16}\right)}\right)dx,x,\right.}{\sqrt[4]{bx+ax^5}}$$

$$=\frac{4x\sqrt[4]{1+\frac{ax^4}{b}}\,{}_2F_1\left(\frac{3}{16},\frac{1}{4};\frac{19}{16};-\frac{ax^4}{b}\right)}{3\sqrt[4]{bx+ax^5}}+\frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^4}\right)\operatorname{Subst}\left(\int \frac{x^{14}}{\sqrt[4]{b+ax^{16}}\left(b-x^{12}+a\right.}\right.}{\sqrt[4]{bx+ax^5}}$$

Mathematica [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{-3b+ax^4}{\left(b-x^3+ax^4\right)\sqrt[4]{bx+ax^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3*b + a*x^4)/((b - x^3 + a*x^4)*(b*x + a*x^5)^(1/4)),x]
```

```
[Out] Integrate[(-3*b + a*x^4)/((b - x^3 + a*x^4)*(b*x + a*x^5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 1.16, size = 53, normalized size = 1.00

$$-2\tan^{-1}\left(\frac{\left(ax^5+bx\right)^{3/4}}{ax^4+b}\right)-2\tanh^{-1}\left(\frac{\left(ax^5+bx\right)^{3/4}}{ax^4+b}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*b + a*x^4)/((b - x^3 + a*x^4)*(b*x + a*x^5)^(1/4)),x
]
```

```
[Out] -2*ArcTan[(b*x + a*x^5)^(3/4)/(b + a*x^4)] - 2*ArcTanh[(b*x + a*x^5)^(3/4)/
(b + a*x^4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-3*b)/(a*x^4-x^3+b)/(a*x^5+b*x)^(1/4),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4-3\,b}{\left(ax^5+bx\right)^{\frac{1}{4}}\left(ax^4-x^3+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-3*b)/(a*x^4-x^3+b)/(a*x^5+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^4 - 3*b)/((a*x^5 + b*x)^(1/4)*(a*x^4 - x^3 + b)), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{a\,x^4-3b}{\left(a\,x^4-x^3+b\right)\left(a\,x^5+bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-3*b)/(a*x^4-x^3+b)/(a*x^5+b*x)^(1/4),x)
```

```
[Out] int((a*x^4-3*b)/(a*x^4-x^3+b)/(a*x^5+b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4-3\,b}{\left(ax^5+bx\right)^{\frac{1}{4}}\left(ax^4-x^3+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-3*b)/(a*x^4-x^3+b)/(a*x^5+b*x)^(1/4),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^4 - 3*b)/((a*x^5 + b*x)^(1/4)*(a*x^4 - x^3 + b)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{3\,b-a\,x^4}{{\left(a\,x^5+b\,x\right)}^{1/4}\,\left(a\,x^4-x^3+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*b - a*x^4)/((b*x + a*x^5)^(1/4)*(b + a*x^4 - x^3)),x)
[Out] int(-(3*b - a*x^4)/((b*x + a*x^5)^(1/4)*(b + a*x^4 - x^3)), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-3*b)/(a*x**4-x**3+b)/(a*x**5+b*x)**(1/4),x)
[Out] Timed out
```

3.668 $\int x^{20}\sqrt{-1+x^6}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{x^6-1}\left(48x^{21}-8x^{15}-10x^9-15x^3\right)}{1152}-\frac{5}{384}\log\left(\sqrt{x^6-1}+x^3\right)$$

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.57, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {275, 279, 321, 217, 206}

$$\frac{1}{24}\sqrt{x^6-1}\,x^{21}-\frac{1}{144}\sqrt{x^6-1}\,x^{15}-\frac{5}{576}\sqrt{x^6-1}\,x^9-\frac{5}{384}\sqrt{x^6-1}\,x^3-\frac{5}{384}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^20*Sqrt[-1 + x^6],x]
```

```
[Out] (-5*x^3*Sqrt[-1 + x^6])/384 - (5*x^9*Sqrt[-1 + x^6])/576 - (x^15*Sqrt[-1 +
x^6])/144 + (x^21*Sqrt[-1 + x^6])/24 - (5*ArcTanh[x^3/Sqrt[-1 + x^6]])/384
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^{20}\sqrt{-1+x^6}\,dx = \frac{1}{3}\,\text{Subst}\left(\int x^6\sqrt{-1+x^2}\,dx,x,x^3\right)$$
$$= \frac{1}{24}x^{21}\sqrt{-1+x^6} - \frac{1}{24}\,\text{Subst}\left(\int \frac{x^6}{\sqrt{-1+x^2}}\,dx,x,x^3\right)$$
$$= -\frac{1}{144}x^{15}\sqrt{-1+x^6} + \frac{1}{24}x^{21}\sqrt{-1+x^6} - \frac{5}{144}\,\text{Subst}\left(\int \frac{x^4}{\sqrt{-1+x^2}}\,dx,x,x^3\right)$$
$$= -\frac{5}{576}x^9\sqrt{-1+x^6} - \frac{1}{144}x^{15}\sqrt{-1+x^6} + \frac{1}{24}x^{21}\sqrt{-1+x^6} - \frac{5}{192}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x^2}}\,dx,x,\right.$$
$$= -\frac{5}{384}x^3\sqrt{-1+x^6} - \frac{5}{576}x^9\sqrt{-1+x^6} - \frac{1}{144}x^{15}\sqrt{-1+x^6} + \frac{1}{24}x^{21}\sqrt{-1+x^6} - \frac{5}{384}\,\text{Subst}\left(\right.$$
$$= -\frac{5}{384}x^3\sqrt{-1+x^6} - \frac{5}{576}x^9\sqrt{-1+x^6} - \frac{1}{144}x^{15}\sqrt{-1+x^6} + \frac{1}{24}x^{21}\sqrt{-1+x^6} - \frac{5}{384}\,\text{Subst}\left(\right.$$
$$= -\frac{5}{384}x^3\sqrt{-1+x^6} - \frac{5}{576}x^9\sqrt{-1+x^6} - \frac{1}{144}x^{15}\sqrt{-1+x^6} + \frac{1}{24}x^{21}\sqrt{-1+x^6} - \frac{5}{384}\tanh^{-1}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.15

$$\frac{\left(x^6-1\right)\left(15\sin^{-1}\left(x^3\right)+\sqrt{1-x^6}\left(48x^{18}-8x^{12}-10x^6-15\right)x^3\right)}{1152\sqrt{-\left(x^6-1\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^20*Sqrt[-1 + x^6],x]
```

```
[Out] ((-1 + x^6)*(x^3*Sqrt[1 - x^6]*(-15 - 10*x^6 - 8*x^12 + 48*x^18) + 15*ArcSi
n[x^3]))/(1152*Sqrt[-(-1 + x^6)^2])
```

IntegrateAlgebraic [A] time = 0.18, size = 53, normalized size = 1.00

$$\frac{\sqrt{x^6-1}\left(48x^{21}-8x^{15}-10x^9-15x^3\right)}{1152}-\frac{5}{384}\log\left(\sqrt{x^6-1}+x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^20*Sqrt[-1 + x^6],x]
```

```
[Out] (Sqrt[-1 + x^6]*(-15*x^3 - 10*x^9 - 8*x^15 + 48*x^21))/1152 - (5*Log[x^3 +
Sqrt[-1 + x^6]])/384
```

fricas [A] time = 0.40, size = 47, normalized size = 0.89

$$\frac{1}{1152}\left(48\,x^{21}-8\,x^{15}-10\,x^9-15\,x^3\right)\sqrt{x^6-1}+\frac{5}{384}\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20*(x^6-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1152*(48*x^21 - 8*x^15 - 10*x^9 - 15*x^3)*sqrt(x^6 - 1) + 5/384*log(-x^3
+ sqrt(x^6 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^6-1}\,x^{20}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20*(x^6-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^6 - 1)*x^20, x)
```

maple [C] time = 0.16, size = 55, normalized size = 1.04

$$\frac{x^{3} \left(48 x^{18}-8 x^{12}-10 x^{6}-15\right) \sqrt{x^{6}-1}}{1152}-\frac{5 \sqrt{-\operatorname{signum}\left(x^{6}-1\right)}\, \arcsin \left(x^{3}\right)}{384 \sqrt{\operatorname{signum}\left(x^{6}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^20*(x^6-1)^(1/2),x)
```

```
[Out] 1/1152*x^3*(48*x^18-8*x^12-10*x^6-15)*(x^6-1)^(1/2)-5/384/signum(x^6-1)^(1/
2)*(-signum(x^6-1))^(1/2)*arcsin(x^3)
```

maxima [B] time = 0.31, size = 134, normalized size = 2.53

$$-\frac{\frac{15 \sqrt{x^{6}-1}}{x^{3}}+\frac{73 \left(x^{6}-1\right)^{\frac{3}{2}}}{x^{9}}-\frac{55 \left(x^{6}-1\right)^{\frac{5}{2}}}{x^{15}}+\frac{15 \left(x^{6}-1\right)^{\frac{7}{2}}}{x^{21}}}{1152 \left(\frac{4 \left(x^{6}-1\right)}{x^{6}}-\frac{6 \left(x^{6}-1\right)^{2}}{x^{12}}+\frac{4 \left(x^{6}-1\right)^{3}}{x^{18}}-\frac{\left(x^{6}-1\right)^{4}}{x^{24}}-1\right)}-\frac{5}{768} \log \left(\frac{\sqrt{x^{6}-1}}{x^{3}}+1\right)+\frac{5}{768} \log \left(\frac{\sqrt{x^{6}-1}}{x^{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20*(x^6-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/1152*(15*sqrt(x^6 - 1)/x^3 + 73*(x^6 - 1)^(3/2)/x^9 - 55*(x^6 - 1)^(5/2)
/x^15 + 15*(x^6 - 1)^(7/2)/x^21)/(4*(x^6 - 1)/x^6 - 6*(x^6 - 1)^2/x^12 + 4*
(x^6 - 1)^3/x^18 - (x^6 - 1)^4/x^24 - 1) - 5/768*log(sqrt(x^6 - 1)/x^3 + 1)
 + 5/768*log(sqrt(x^6 - 1)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{20} \sqrt{x^{6}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^20*(x^6 - 1)^(1/2),x)
```

```
[Out] int(x^20*(x^6 - 1)^(1/2), x)
```

sympy [A] time = 6.23, size = 175, normalized size = 3.30

$$\begin{cases} \frac{x^{27}}{24 \sqrt{x^{6}-1}}-\frac{7 x^{21}}{144 \sqrt{x^{6}-1}}-\frac{x^{15}}{576 \sqrt{x^{6}-1}}-\frac{5 x^{9}}{1152 \sqrt{x^{6}-1}}+\frac{5 x^{3}}{384 \sqrt{x^{6}-1}}-\frac{5 \operatorname{acosh}\left(x^{3}\right)}{384} & \text{for}\ \left|x^{6}\right|>1 \\ -\frac{i x^{27}}{24 \sqrt{1-x^{6}}}+\frac{7 i x^{21}}{144 \sqrt{1-x^{6}}}+\frac{i x^{15}}{576 \sqrt{1-x^{6}}}+\frac{5 i x^{9}}{1152 \sqrt{1-x^{6}}}-\frac{5 i x^{3}}{384 \sqrt{1-x^{6}}}+\frac{5 i \operatorname{asin}\left(x^{3}\right)}{384} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**20*(x**6-1)**(1/2),x)
```

```
[Out] Piecewise((x**27/(24*sqrt(x**6 - 1)) - 7*x**21/(144*sqrt(x**6 - 1)) - x**15
/(576*sqrt(x**6 - 1)) - 5*x**9/(1152*sqrt(x**6 - 1)) + 5*x**3/(384*sqrt(x**
6 - 1)) - 5*acosh(x**3)/384, Abs(x**6) > 1), (-I*x**27/(24*sqrt(1 - x**6))
+ 7*I*x**21/(144*sqrt(1 - x**6)) + I*x**15/(576*sqrt(1 - x**6)) + 5*I*x**9/
(1152*sqrt(1 - x**6)) - 5*I*x**3/(384*sqrt(1 - x**6)) + 5*I*asin(x**3)/384,
 True))
```

3.669 $\int \frac{(-1+x^3)\sqrt{-1+x^6}}{x^4(1+x^3)}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{x^6-1}}{3x^3}+\frac{1}{3}\log\left(\sqrt{x^6-1}+x^3\right)-\frac{4}{3}\tan^{-1}\left(\sqrt{x^6-1}+x^3\right)$$

Rubi [F] time = 0.60, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^3)\sqrt{-1+x^6}}{x^4(1+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^3)*Sqrt[-1 + x^6])/(x^4*(1 + x^3)),x]
```

```
[Out] (2*Sqrt[-1 + x^6])/3 + Sqrt[-1 + x^6]/(3*x^3) - (2*ArcTan[Sqrt[-1 + x^6]])/
3 - ArcTanh[x^3/Sqrt[-1 + x^6]]/3 - (2*Defer[Int][Sqrt[-1 + x^6]/(1 + x), x
])/3 - (4*Defer[Int][Sqrt[-1 + x^6]/(-1 - I*Sqrt[3] + 2*x), x])/3 - (4*Defe
r[Int][Sqrt[-1 + x^6]/(-1 + I*Sqrt[3] + 2*x), x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^3)\sqrt{-1+x^6}}{x^4(1+x^3)}\,dx &= \int \left(-\frac{\sqrt{-1+x^6}}{x^4}+\frac{2\sqrt{-1+x^6}}{x}-\frac{2\sqrt{-1+x^6}}{3(1+x)}-\frac{2(-1+2x)\sqrt{-1+x^6}}{3\left(1-x+x^2\right)}\right)dx\\
&= -\left(\frac{2}{3}\int \frac{\sqrt{-1+x^6}}{1+x}\,dx\right)-\frac{2}{3}\int \frac{(-1+2x)\sqrt{-1+x^6}}{1-x+x^2}\,dx+2\int \frac{\sqrt{-1+x^6}}{x}\,dx-\int \frac{\sqrt{-1}}{}\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx,x,x^6\right)-\frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{-1+x^2}}{x^2}\,dx,x,x^3\right)-\frac{2}{3}\int \frac{\sqrt{-1+x}}{1+x}\\
&= \frac{2}{3}\sqrt{-1+x^6}+\frac{\sqrt{-1+x^6}}{3x^3}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx,x,x^6\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{-1+}}\right.\\
&= \frac{2}{3}\sqrt{-1+x^6}+\frac{\sqrt{-1+x^6}}{3x^3}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x^3}{\sqrt{-1+x^6}}\right)-\frac{2}{3}\int \frac{\sqrt{-1+x^6}}{1+x}\\
&= \frac{2}{3}\sqrt{-1+x^6}+\frac{\sqrt{-1+x^6}}{3x^3}-\frac{2}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right)-\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)-\frac{2}{3}\int \frac{\sqrt{-}}{}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 44, normalized size = 0.83

$$\frac{1}{3}\left(-2\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}}{x^3}+\log\left(\sqrt{x^6-1}+x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)*Sqrt[-1 + x^6])/(x^4*(1 + x^3)),x]
```

```
[Out] (Sqrt[-1 + x^6]/x^3 - 2*ArcTan[Sqrt[-1 + x^6]] + Log[x^3 + Sqrt[-1 + x^6]])
/3
```

IntegrateAlgebraic [A] time = 0.19, size = 57, normalized size = 1.08

$$\frac{\sqrt{x^6-1}}{3x^3}-\frac{1}{3}\log\left(\sqrt{x^6-1}-x^3\right)+\frac{4}{3}\tan^{-1}\left(x^3-\sqrt{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)*Sqrt[-1 + x^6])/(x^4*(1 + x^3)),x]
```

```
[Out] Sqrt[-1 + x^6]/(3*x^3) + (4*ArcTan[x^3 - Sqrt[-1 + x^6]])/3 - Log[-x^3 + Sq
rt[-1 + x^6]]/3
```

fricas [A] time = 0.42, size = 57, normalized size = 1.08

$$-\frac{4\,x^3\arctan\left(-x^3+\sqrt{x^6-1}\right)+x^3\log\left(-x^3+\sqrt{x^6-1}\right)-x^3-\sqrt{x^6-1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x^4/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*(4*x^3*arctan(-x^3 + sqrt(x^6 - 1)) + x^3*log(-x^3 + sqrt(x^6 - 1)) -
x^3 - sqrt(x^6 - 1))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^6-1}\left(x^3-1\right)}{\left(x^3+1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x^4/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^6 - 1)*(x^3 - 1)/((x^3 + 1)*x^4), x)
```

maple [C] time = 0.16, size = 98, normalized size = 1.85

$$\frac{\sqrt{x^6-1}}{3x^3}+\frac{\sqrt{-\mathrm{signum}\left(x^6-1\right)}\,\arcsin\left(x^3\right)}{3\sqrt{\mathrm{signum}\left(x^6-1\right)}}-\frac{\sqrt{-\mathrm{signum}\left(x^6-1\right)}\,\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^6+1}}{2}\right)+\left(-2\ln(2)+6\ln(x)+i\pi\right)\sqrt{\pi}\right)}{3\sqrt{\pi}\,\sqrt{\mathrm{signum}\left(x^6-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)*(x^6-1)^(1/2)/x^4/(x^3+1),x)
```

```
[Out] 1/3*(x^6-1)^(1/2)/x^3+1/3/signum(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*arcsin
(x^3)-1/3/Pi^(1/2)/signum(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*(-2*Pi^(1/2)*
ln(1/2+1/2*(-x^6+1)^(1/2))+(-2*ln(2)+6*ln(x)+I*Pi)*Pi^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^6-1}\left(x^3-1\right)}{\left(x^3+1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x^4/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^6 - 1)*(x^3 - 1)/((x^3 + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x^3-1\right)\sqrt{x^6-1}}{x^4\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)*(x^6 - 1)^(1/2))/(x^4*(x^3 + 1)),x)
```

```
[Out] int(((x^3 - 1)*(x^6 - 1)^(1/2))/(x^4*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,(x-1)\left(x^2+x+1\right)}{x^4(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)*(x**6-1)**(1/2)/x**4/(x**3+1),x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1))*(x - 1)*(x**2
+ x + 1)/(x**4*(x + 1)*(x**2 - x + 1)), x)
```

3.670 $\int \frac{1+x^6}{\sqrt{1-x^2+x^4}\,(1-x^6)}\,dx$

Optimal. Leaf size=53

$$\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4-x^2+1}}\right)+\frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{x^4-x^2+1}}\right)$$

Rubi [C] time = 2.15, antiderivative size = 228, normalized size of antiderivative = 4.30, number of steps used = 95, number of rules used = 19, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.655, Rules used = {1586, 6725, 1728, 1208, 1139, 1103, 1195, 1210, 1698, 206, 1247, 734, 843, 619, 215, 724, 1197, 1216, 1706}

$$\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4-x^2+1}}\right)+\frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{x^4-x^2+1}}\right)+\frac{\left(1+(-1)^{2/3}\right)\left(x^2+1\right)\sqrt{\frac{x^4-x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{3}{4}\right)}{6\sqrt{x^4-x^2+1}}+\frac{\left(1-\sqrt[3]{-1}\right)\left(x^2+1\right)\sqrt{\frac{x^4-x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{3}{4}\right)}{6\sqrt{x^4-x^2+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4-x^2+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{3}{4}\right)}{6\sqrt{x^4-x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)/(Sqrt[1 - x^2 + x^4]*(1 - x^6)),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2 + x^4]])/3 + ArcTanh[x/Sqrt[1 - x^
2 + x^4]]/3 - ((1 + x^2)*Sqrt[(1 - x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcT
an[x], 3/4])/(6*Sqrt[1 - x^2 + x^4]) + ((1 - (-1)^(1/3))*(1 + x^2)*Sqrt[(1
- x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 3/4])/(6*Sqrt[1 - x^2 + x^
4]) + ((1 + (-1)^(2/3))*(1 + x^2)*Sqrt[(1 - x^2 + x^4)/(1 + x^2)^2]*Ellipti
cF[2*ArcTan[x], 3/4])/(6*Sqrt[1 - x^2 + x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1210

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
```

```
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbo
l] :> Dist[d, Int[(a + b*x^2 + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, I
nt[(x*(a + b*x^2 + c*x^4)^p)/(d^2 - e^2*x^2), x], x] /; FreeQ[{a, b, c, d,
e}, x] && IntegerQ[p + 1/2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^6}{\sqrt{1-x^2+x^4}\left(1-x^6\right)}\,dx = \int \frac{\left(1+x^2\right)\sqrt{1-x^2+x^4}}{1-x^6}\,dx$$

$$= \int \left(\frac{\sqrt{1-x^2+x^4}}{3(1-x)} + \frac{\sqrt{1-x^2+x^4}}{3(1+x)} + \frac{\left(1-\sqrt[3]{-1}\right)\sqrt{1-x^2+x^4}}{6\left(1-\sqrt[3]{-1}\,x\right)} + \frac{\left(1-\sqrt[3]{-1}\right)\sqrt{1-}}{6\left(1+\sqrt[3]{-1}\right.}\right.$$

$$= \frac{1}{3}\int \frac{\sqrt{1-x^2+x^4}}{1-x}\,dx + \frac{1}{3}\int \frac{\sqrt{1-x^2+x^4}}{1+x}\,dx + \frac{1}{6}\left(1-\sqrt[3]{-1}\right)\int \frac{\sqrt{1-x^2+x^4}}{1-\sqrt[3]{-1}\,x}\,dx$$

$$= 2\left(\frac{1}{3}\int \frac{\sqrt{1-x^2+x^4}}{1-x^2}\,dx\right) + 2\left(\frac{1}{6}\left(1-\sqrt[3]{-1}\right)\int \frac{\sqrt{1-x^2+x^4}}{1-(-1)^{2/3}x^2}\,dx\right) + 2\left(\frac{1}{6}\left(1+(-1)\right.\right.$$

$$= 2\left(-\left(\frac{1}{3}\int \frac{x^2}{\sqrt{1-x^2+x^4}}\,dx\right) + \frac{1}{3}\int \frac{1}{\left(1-x^2\right)\sqrt{1-x^2+x^4}}\,dx\right) + 2\left(\frac{1}{3}\int \frac{}{\left(1+\sqrt[3]{-}\right.}\right.$$

$$= 2\left(\frac{1}{6}\int \frac{1}{\sqrt{1-x^2+x^4}}\,dx + \frac{1}{6}\int \frac{1+x^2}{\left(1-x^2\right)\sqrt{1-x^2+x^4}}\,dx - \frac{1}{3}\int \frac{1}{\sqrt{1-x^2+x^4}}\,dx\right.$$

$$= 2\left(\frac{x\sqrt{1-x^2+x^4}}{6\left(1+x^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2+x^4}}\right)}{6\sqrt{2}} - \frac{\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{3}{4}\right)}{6\sqrt{1-x^2+x^4}} + \frac{(1}{}\right.$$

$$= 2\left(-\frac{x\sqrt{1-x^2+x^4}}{3\left(1+x^2\right)} + \frac{1}{6}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1-x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x\right.}{3\sqrt{1-x^2+x^4}}\right.$$

Mathematica [C] time = 1.80, size = 542, normalized size = 10.23

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^6)/(Sqrt[1 - x^2 + x^4]*(1 - x^6)),x]
```

```
[Out] (9*(-1)^(1/6)*Sqrt[1 - (-1)^(1/3)*x^2]*Sqrt[1 + (-1)^(2/3)*x^2]*EllipticF[I
*ArcSinh[(-1)^(1/3)*x], (-1)^(2/3)] - ((18*I)*Sqrt[1 - (-1)^(1/3)*x^2]*Sqrt
[1 + (-1)^(2/3)*x^2]*EllipticPi[-1, I*ArcSinh[(-1)^(1/3)*x], (-1)^(2/3)])/(
1 + (-1)^(1/3))^2 - 6*(-1)^(1/6)*Sqrt[1 - (-1)^(1/3)*x^2]*Sqrt[1 + (-1)^(2/
3)*x^2]*EllipticPi[-(-1)^(2/3), I*ArcSinh[(-1)^(1/3)*x], (-1)^(2/3)] - 4*(-
1)^(1/3)*Sqrt[3]*(1 + (-1)^(2/3))*((-1)^(1/6) - x)^2*Sqrt[((-1)^(5/6) + x)/
((-1 + (-1)^(2/3))*((-1)^(1/6) - x))]*Sqrt[(1 + I*Sqrt[3] + (2*I)*x)/(2 + I
*x - Sqrt[3]*x)]*Sqrt[((2*I)*Sqrt[3] + (3*I + Sqrt[3])*x)/(-2 + (-I + Sqrt[
3])*x)]*(-EllipticF[ArcSin[Sqrt[((2*I)*Sqrt[3] + (3*I + Sqrt[3])*x)/(-2 + (
-I + Sqrt[3])*x)]], -1/3] + (-1)^(1/6)*(EllipticPi[(-2*(1 + (-1)^(2/3)))/((
3 + 6*I) + (2 + 3*I)*Sqrt[3]), ArcSin[Sqrt[((2*I)*Sqrt[3] + (3*I + Sqrt[3])
*x)/(-2 + (-I + Sqrt[3])*x)]], -1/3] - EllipticPi[((2*I)*(1 + (-1)^(2/3)))/
((-6 - 3*I) + (3 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((2*I)*Sqrt[3] + (3*I + Sqrt[
3])*x)/(-2 + (-I + Sqrt[3])*x)]], -1/3])))/(9*Sqrt[1 - x^2 + x^4])
```

IntegrateAlgebraic [A] time = 0.30, size = 53, normalized size = 1.00

$$\frac{1}{3}\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4-x^2+1}}\right) + \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{x^4-x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^6)/(Sqrt[1 - x^2 + x^4]*(1 - x^6)),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2 + x^4]])/3 + ArcTanh[x/Sqrt[1 - x^
2 + x^4]]/3
```

fricas [A] time = 0.47, size = 62, normalized size = 1.17

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{2\sqrt{2}\sqrt{x^4-x^2+1}\,x}{x^4-3x^2+1}\right)+\frac{1}{3}\log\left(\frac{x+\sqrt{x^4-x^2+1}}{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^4-x^2+1)^(1/2)/(-x^6+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(2)*arctan(2*sqrt(2)*sqrt(x^4 - x^2 + 1)*x/(x^4 - 3*x^2 + 1)) + 1/3
*log((x + sqrt(x^4 - x^2 + 1))/(x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^6+1}{(x^6-1)\sqrt{x^4-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^4-x^2+1)^(1/2)/(-x^6+1),x, algorithm="giac")
```

```
[Out] integrate(-(x^6 + 1)/((x^6 - 1)*sqrt(x^4 - x^2 + 1)), x)
```

maple [C] time = 0.16, size = 944, normalized size = 17.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/(x^4-x^2+1)^(1/2)/(-x^6+1),x)
```

```
[Out] -2/(2+2*I*3^(1/2))^(1/2)*(1-(1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(1/2-1/2*I*3^
(1/2))*x^2)^(1/2)/(x^4-x^2+1)^(1/2)*EllipticF(1/2*x*(2+2*I*3^(1/2))^(1/2),1
/2*(-2-2*I*3^(1/2))^(1/2))+2/3/(1/2+1/2*I*3^(1/2))^(1/2)*(1-(1/2+1/2*I*3^(1
/2))*x^2)^(1/2)*(1-(1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4-x^2+1)^(1/2)*Ellipti
cPi((1/2+1/2*I*3^(1/2))^(1/2)*x,1/(1/2+1/2*I*3^(1/2)),(1/2-1/2*I*3^(1/2))^(
1/2)/(1/2+1/2*I*3^(1/2))^(1/2))-1/3*(-1/2+1/2*I*3^(1/2))*(1/2/(1+I*3^(1/2))
^(1/2)*arctanh(1/14*(2+I*3^(1/2))*(7*x^2-13/2+3/2*I*3^(1/2))/(1+I*3^(1/2))^
(1/2)/(x^4-x^2+1)^(1/2))-1/(1/2+1/2*I*3^(1/2))^(1/2)*(-1/2-1/2*I*3^(1/2))*(
1-(1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4-x^2
+1)^(1/2)*EllipticPi((1/2+1/2*I*3^(1/2))^(1/2)*x,-1/2*I*3^(1/2)*(-1/2+1/2*I
*3^(1/2))-1/4+1/4*I*3^(1/2),(1/2-1/2*I*3^(1/2))^(1/2)/(1/2+1/2*I*3^(1/2))^(
1/2)))-1/3*(-1/2-1/2*I*3^(1/2))*(1/2/(1-I*3^(1/2))^(1/2)*arctanh(1/14*(2-I*
3^(1/2))*(7*x^2-13/2-3/2*I*3^(1/2))/(1-I*3^(1/2))^(1/2)/(x^4-x^2+1)^(1/2))-
1/(1/2+1/2*I*3^(1/2))^(1/2)*(-1/2+1/2*I*3^(1/2))*(1-(1/2+1/2*I*3^(1/2))*x^2
)^(1/2)*(1-(1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4-x^2+1)^(1/2)*EllipticPi((1/2
+1/2*I*3^(1/2))^(1/2)*x,-1/2*I*3^(1/2)*(-1/2-1/2*I*3^(1/2))-1/4-1/4*I*3^(1/
2),(1/2-1/2*I*3^(1/2))^(1/2)/(1/2+1/2*I*3^(1/2))^(1/2)))-1/3*(1/2+1/2*I*3^(
1/2))*(-1/2/(1-I*3^(1/2))^(1/2)*arctanh(1/14*(-2+I*3^(1/2))*(7*x^2-13/2-3/2
*I*3^(1/2))/(1-I*3^(1/2))^(1/2)/(x^4-x^2+1)^(1/2))-1/(1/2+1/2*I*3^(1/2))^(1
/2)*(1/2-1/2*I*3^(1/2))*(1-(1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(1/2-1/2*I*3^(
1/2))*x^2)^(1/2)/(x^4-x^2+1)^(1/2)*EllipticPi((1/2+1/2*I*3^(1/2))^(1/2)*x,1
/2*I*3^(1/2)*(1/2+1/2*I*3^(1/2))-1/4-1/4*I*3^(1/2),(1/2-1/2*I*3^(1/2))^(1/2
)/(1/2+1/2*I*3^(1/2))^(1/2)))-1/3*(1/2-1/2*I*3^(1/2))*(-1/2/(1+I*3^(1/2))^(
1/2)*arctanh(1/14*(-2-I*3^(1/2))*(7*x^2-13/2+3/2*I*3^(1/2))/(1+I*3^(1/2))^(
1/2)/(x^4-x^2+1)^(1/2))-(1/2+1/2*I*3^(1/2))^(1/2)*(1-(1/2+1/2*I*3^(1/2))*x^
```

```
2)^(1/2)*(1-(1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4-x^2+1)^(1/2)*EllipticPi((1/
2+1/2*I*3^(1/2))^(1/2)*x,1/2*I*3^(1/2)*(1/2-1/2*I*3^(1/2))-1/4+1/4*I*3^(1/2
),(1/2-1/2*I*3^(1/2))^(1/2)/(1/2+1/2*I*3^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^6+1}{\left(x^6-1\right)\sqrt{x^4-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^4-x^2+1)^(1/2)/(-x^6+1),x, algorithm="maxima")
[Out] -integrate((x^6 + 1)/((x^6 - 1)*sqrt(x^4 - x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x^6+1}{\left(x^6-1\right)\,\sqrt{x^4-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^6 + 1)/((x^6 - 1)*(x^4 - x^2 + 1)^(1/2)),x)
[Out] int(-(x^6 + 1)/((x^6 - 1)*(x^4 - x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x^4-x^2+1}}{x^6-1}\,dx-\int \frac{x^2\sqrt{x^4-x^2+1}}{x^6-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/(x**4-x**2+1)**(1/2)/(-x**6+1),x)
[Out] -Integral(sqrt(x**4 - x**2 + 1)/(x**6 - 1), x) - Integral(x**2*sqrt(x**4 -
x**2 + 1)/(x**6 - 1), x)
```

3.671 $\int \frac{\left(-2+x^6\right)\sqrt{-1+x^6}}{x^7\left(2+x^6\right)}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{x^6-1}}{6x^6}-\frac{1}{2}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\tan^{-1}\left(\frac{\sqrt{x^6-1}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {573, 149, 156, 63, 203}

$$\frac{\sqrt{x^6-1}}{6x^6}-\frac{1}{2}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\tan^{-1}\left(\frac{\sqrt{x^6-1}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((-2 + x^6)*Sqrt[-1 + x^6])/(x^7*(2 + x^6)),x]
```

```
[Out] Sqrt[-1 + x^6]/(6*x^6) - ArcTan[Sqrt[-1 + x^6]]/2 + ArcTan[Sqrt[-1 + x^6]/S
qrt[3]]/Sqrt[3]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
```

```
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-2+x^6\right)\sqrt{-1+x^6}}{x^7\left(2+x^6\right)}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{(-2+x)\sqrt{-1+x}}{x^2(2+x)}\,dx,x,x^6\right)\\ &= \frac{\sqrt{-1+x^6}}{6x^6}+\frac{1}{12}\operatorname{Subst}\left(\int \frac{-6+3x}{\sqrt{-1+x}\,x(2+x)}\,dx,x,x^6\right)\\ &= \frac{\sqrt{-1+x^6}}{6x^6}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx,x,x^6\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,(2+x)}\,dx,x\right.\\ &= \frac{\sqrt{-1+x^6}}{6x^6}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x^6}\right)+\operatorname{Subst}\left(\int \frac{1}{3+x^2}\,dx,x,\sqrt{-1+\right.\\ &= \frac{\sqrt{-1+x^6}}{6x^6}-\frac{1}{2}\tan^{-1}\left(\sqrt{-1+x^6}\right)+\frac{\tan^{-1}\left(\frac{\sqrt{-1+x^6}}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 96, normalized size = 1.81

$$\frac{1}{6}\left(-\frac{1}{x^6\sqrt{x^6-1}}+\frac{1}{\sqrt{x^6-1}}-2\tan^{-1}\left(\sqrt{x^6-1}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{x^6-1}}{\sqrt{3}}\right)+\frac{\sqrt{1-x^6}\tanh^{-1}\left(\sqrt{1-x^6}\right)}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-2 + x^6)*Sqrt[-1 + x^6])/(x^7*(2 + x^6)),x]
```

```
[Out] (1/Sqrt[-1 + x^6] - 1/(x^6*Sqrt[-1 + x^6]) - 2*ArcTan[Sqrt[-1 + x^6]] + 2*S
qrt[3]*ArcTan[Sqrt[-1 + x^6]/Sqrt[3]] + (Sqrt[1 - x^6]*ArcTanh[Sqrt[1 - x^6
]])/Sqrt[-1 + x^6])/6
```

IntegrateAlgebraic [A] time = 0.06, size = 53, normalized size = 1.00

$$\frac{\sqrt{x^6-1}}{6x^6}-\frac{1}{2}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\tan^{-1}\left(\frac{\sqrt{x^6-1}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^6)*Sqrt[-1 + x^6])/(x^7*(2 + x^6)),x]
```

```
[Out] Sqrt[-1 + x^6]/(6*x^6) - ArcTan[Sqrt[-1 + x^6]]/2 + ArcTan[Sqrt[-1 + x^6]/S
qrt[3]]/Sqrt[3]
```

fricas [A] time = 0.41, size = 47, normalized size = 0.89

$$\frac{2\sqrt{3}\,x^6\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^6-1}\right)-3\,x^6\arctan\left(\sqrt{x^6-1}\right)+\sqrt{x^6-1}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-1)^(1/2)/x^7/(x^6+2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*x^6*arctan(1/3*sqrt(3)*sqrt(x^6 - 1)) - 3*x^6*arctan(sqrt(x^
6 - 1)) + sqrt(x^6 - 1))/x^6
```

giac [A] time = 0.14, size = 41, normalized size = 0.77

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}}{6x^6}-\frac{1}{2}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-1)^(1/2)/x^7/(x^6+2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(x^6 - 1)) + 1/6*sqrt(x^6 - 1)/x^6 - 1/2
*arctan(sqrt(x^6 - 1))
```

maple [C] time = 1.03, size = 90, normalized size = 1.70

$$\frac{\sqrt{x^6-1}}{6x^6}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)-\sqrt{x^6-1}}{x^3}\right)}{2}+\frac{\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+3\right)x^6-4\operatorname{RootOf}\left(\_Z^2+3\right)-6\sqrt{x^6-1}}{x^6+2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-2)*(x^6-1)^(1/2)/x^7/(x^6+2),x)
```

```
[Out] 1/6*(x^6-1)^(1/2)/x^6+1/2*RootOf(_Z^2+1)*ln(-(RootOf(_Z^2+1)-(x^6-1)^(1/2))
/x^3)+1/6*RootOf(_Z^2+3)*ln(-(RootOf(_Z^2+3)*x^6-4*RootOf(_Z^2+3)-6*(x^6-1)
^(1/2))/(x^6+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^6-1}\left(x^6-2\right)}{\left(x^6+2\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-1)^(1/2)/x^7/(x^6+2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^6 - 1)*(x^6 - 2)/((x^6 + 2)*x^7), x)
```

mupad [B] time = 1.02, size = 41, normalized size = 0.77

$$\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{x^6-1}}{3}\right)}{3}-\frac{\operatorname{atan}\left(\sqrt{x^6-1}\right)}{2}+\frac{\sqrt{x^6-1}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/2)*(x^6 - 2))/(x^7*(x^6 + 2)),x)
```

```
[Out] (3^(1/2)*atan((3^(1/2)*(x^6 - 1)^(1/2))/3))/3 - atan((x^6 - 1)^(1/2))/2 + (
x^6 - 1)^(1/2)/(6*x^6)
```

sympy [A] time = 26.20, size = 48, normalized size = 0.91

$$\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{x^6-1}}{3}\right)}{3}-\frac{\operatorname{atan}\left(\sqrt{x^6-1}\right)}{2}+\frac{\sqrt{x^6-1}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-2)*(x**6-1)**(1/2)/x**7/(x**6+2),x)
```

```
[Out] sqrt(3)*atan(sqrt(3)*sqrt(x**6 - 1)/3)/3 - atan(sqrt(x**6 - 1))/2 + sqrt(x*
*6 - 1)/(6*x**6)
```

3.672 $\int \frac{x(-6b+5ax)}{\sqrt[4]{-bx^2+ax^3}\left(b-ax+x^6\right)}\,dx$

Optimal. Leaf size=53

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^3-bx^2}}{x^2}\right)-2\tanh^{-1}\left(\frac{\left(ax^3-bx^2\right)^{3/4}}{ax-b}\right)$$

Rubi [F] time = 1.99, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(-6b+5ax)}{\sqrt[4]{-bx^2+ax^3}\left(b-ax+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-6*b + 5*a*x))/((-(b*x^2) + a*x^3)^(1/4)*(b - a*x + x^6)),x]
```

```
[Out] (-10*a*Sqrt[x]*(-b + a*x)^(1/4)*Defer[Subst][Defer[Int][x^4/((-b + a*x^2)^(
1/4)*(-b + a*x^2 - x^12)), x], x, Sqrt[x]])/(-(b*x^2) + a*x^3)^(1/4) - (12*
b*Sqrt[x]*(-b + a*x)^(1/4)*Defer[Subst][Defer[Int][x^2/((-b + a*x^2)^(1/4)*
(b - a*x^2 + x^12)), x], x, Sqrt[x]])/(-(b*x^2) + a*x^3)^(1/4)
```

Rubi steps

$$\begin{aligned}
\int \frac{x(-6b+5ax)}{\sqrt[4]{-bx^2+ax^3}\left(b-ax+x^6\right)}\,dx &= \frac{\left(\sqrt{x}\sqrt[4]{-b+ax}\right)\int \frac{\sqrt{x}(-6b+5ax)}{\sqrt[4]{-b+ax}\left(b-ax+x^6\right)}\,dx}{\sqrt[4]{-bx^2+ax^3}}\\
&= \frac{\left(2\sqrt{x}\sqrt[4]{-b+ax}\right)\text{Subst}\left(\int \frac{x^2\left(-6b+5ax^2\right)}{\sqrt[4]{-b+ax^2}\left(b-ax^2+x^{12}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{-bx^2+ax^3}}\\
&= \frac{\left(2\sqrt{x}\sqrt[4]{-b+ax}\right)\text{Subst}\left(\int\left(-\frac{5ax^4}{\sqrt[4]{-b+ax^2}\left(-b+ax^2-x^{12}\right)}-\frac{6bx^2}{\sqrt[4]{-b+ax^2}\left(b-ax^2+x^{12}\right)}\right)dx,x\right.}{\sqrt[4]{-bx^2+ax^3}}\\
&= -\frac{\left(10a\sqrt{x}\sqrt[4]{-b+ax}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{-b+ax^2}\left(-b+ax^2-x^{12}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{-bx^2+ax^3}}-\frac{\left(12b\sqrt{x}\sqrt[4]{-\right.}{}
\end{aligned}$$

Mathematica [C] time = 18.23, size = 58426, normalized size = 1102.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(-6*b + 5*a*x))/((-(b*x^2) + a*x^3)^(1/4)*(b - a*x + x^6)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.70, size = 53, normalized size = 1.00

$$2\tan^{-1}\left(\frac{\sqrt[4]{ax^3-bx^2}}{x^2}\right)-2\tanh^{-1}\left(\frac{\left(ax^3-bx^2\right)^{3/4}}{ax-b}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-6*b + 5*a*x))/((-(b*x^2) + a*x^3)^(1/4)*(b - a*x +
x^6)),x]
```

```
[Out] 2*ArcTan[(-(b*x^2) + a*x^3)^(1/4)/x^2] - 2*ArcTanh[(-(b*x^2) + a*x^3)^(3/4)
/(-b + a*x)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*a*x-6*b)/(a*x^3-b*x^2)^(1/4)/(x^6-a*x+b),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,ax - 6\,b)x}{\left(x^6 - ax + b\right)\left(ax^3 - bx^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*a*x-6*b)/(a*x^3-b*x^2)^(1/4)/(x^6-a*x+b),x, algorithm="giac"
)
```

```
[Out] integrate((5*a*x - 6*b)*x/((x^6 - a*x + b)*(a*x^3 - b*x^2)^(1/4)), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x\,(5ax - 6b)}{\left(a\,x^3 - b\,x^2\right)^{\frac{1}{4}}\left(x^6 - ax + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(5*a*x-6*b)/(a*x^3-b*x^2)^(1/4)/(x^6-a*x+b),x)
```

```
[Out] int(x*(5*a*x-6*b)/(a*x^3-b*x^2)^(1/4)/(x^6-a*x+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,ax - 6\,b)x}{\left(x^6 - ax + b\right)\left(ax^3 - bx^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*a*x-6*b)/(a*x^3-b*x^2)^(1/4)/(x^6-a*x+b),x, algorithm="maxim
a")
```

```
[Out] integrate((5*a*x - 6*b)*x/((x^6 - a*x + b)*(a*x^3 - b*x^2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x\,\left(6\,b - 5\,a\,x\right)}{{\left(a\,x^3 - b\,x^2\right)}^{1/4}\,\left(x^6 - a\,x + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(6*b - 5*a*x))/((a*x^3 - b*x^2)^(1/4)*(b - a*x + x^6)),x)
[Out] -int((x*(6*b - 5*a*x))/((a*x^3 - b*x^2)^(1/4)*(b - a*x + x^6)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(5ax-6b)}{\sqrt[4]{x^2\,(ax-b)}\,\left(-ax+b+x^6\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*a*x-6*b)/(a*x**3-b*x**2)**(1/4)/(x**6-a*x+b),x)
[Out] Integral(x*(5*a*x - 6*b)/((x**2*(a*x - b))**(1/4)*(-a*x + b + x**6)), x)
```

3.673 $\int \frac{\sqrt{1+x^3}\left(2+2x^3+x^6\right)}{x\left(-1+x^6\right)}\,dx$

Optimal. Leaf size=53

$$\frac{2\sqrt{x^3+1}}{3}+\frac{4}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{5}{3}\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x^3+1}}{\sqrt{2}}\right)$$

Rubi [C] time = 3.54, antiderivative size = 1215, normalized size of antiderivative = 22.92, number of steps used = 33, number of rules used = 16, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {1586, 6725, 2136, 218, 2142, 2113, 537, 571, 93, 206, 266, 63, 207, 261, 6728, 205}

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[1 + x^3]*(2 + 2*x^3 + x^6))/(x*(-1 + x^6)),x]
```

```
[Out] (2*Sqrt[1 + x^3])/3 + (((5*I)/3)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] +
x)^2]*ArcTan[(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[((3 - 6*I) -
 (2 - 3*I)*Sqrt[3])/((4 + 6*I) - (2 + 4*I)*Sqrt[3])]*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2])])/(Sqrt[2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^
3]) + (10*Sqrt[((6 - 3*I) - (3 - 2*I)*Sqrt[3])/((-6 - 4*I) + (4 + 2*I)*Sqrt
[3])]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(3^(1/4)*Sqrt[
(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[((6 - 3*I) - (3 - 2*I)*Sqrt[3])/((-6 -
4*I) + (4 + 2*I)*Sqrt[3])]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(3^(3
/4)*(3*I - Sqrt[3])*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (5*(
1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTanh[Sqrt[(1 + x)/(1 + Sq
rt[3] + x)^2]/(Sqrt[2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(3*Sqrt[2
]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (4*ArcTanh[Sqrt[1 + x^
3]])/3 - (10*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcS
in[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[2
 + Sqrt[3]]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (20*Sqrt[2 +
 Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[
(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*(1 + (2 -
 I)*Sqrt[3])*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (20*Sqrt[2
+ Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*(1 + (2
+ I)*Sqrt[3])*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (40*Sqrt[2
 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[-((I
 + (1 + 2*I)*Sqrt[3])^2/(1 - (2 + I)*Sqrt[3])^2), ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*(7 + I*Sqrt[3])*Sqrt[(1 + x)/
(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (40*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1
 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[-((1 + (2 + I)*Sqrt[3])^2/(I -
(1 + 2*I)*Sqrt[3])^2), ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*
Sqrt[3]])/(3^(3/4)*(7 - I*Sqrt[3])*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1
 + x^3]) - (20*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] +
x)^2]*EllipticPi[97 + 56*Sqrt[3], ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x
)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^
3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2136

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, -Dist[q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x^3}\left(2+2x^3+x^6\right)}{x\left(-1+x^6\right)}\,dx = \int \frac{2+2x^3+x^6}{x\left(-1+x^3\right)\sqrt{1+x^3}}\,dx$$

$$= \int\left(\frac{5}{3(-1+x)\sqrt{1+x^3}} - \frac{2}{x\sqrt{1+x^3}} + \frac{x^2}{\sqrt{1+x^3}} + \frac{5(1+2x)}{3\left(1+x+x^2\right)\sqrt{1+x^3}}\right)dx$$

$$= \frac{5}{3}\int \frac{1}{(-1+x)\sqrt{1+x^3}}\,dx + \frac{5}{3}\int \frac{1+2x}{\left(1+x+x^2\right)\sqrt{1+x^3}}\,dx - 2\int \frac{1}{x\sqrt{1+x^3}}\,dx +$$

$$= \frac{2\sqrt{1+x^3}}{3} - \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right) + \frac{5}{3}\int\left(\frac{2}{\left(1-i\sqrt{3}+2x\right)\sqrt{1+x^3}} + \right.$$

$$= \frac{2\sqrt{1+x^3}}{3} - \frac{10(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{4}{3}\,\text{Subst}\left(\int\right.$$

$$= \frac{2\sqrt{1+x^3}}{3} + \frac{4}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right) - \frac{10(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-\right.}{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\sqrt{1+x^3}}{3} + \frac{4}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right) - \frac{10(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-\right.}{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\sqrt{1+x^3}}{3} + \frac{4}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right) - \frac{10(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-\right.}{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\sqrt{1+x^3}}{3} - \frac{5(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{3\sqrt{2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right.$$

$$= \frac{2\sqrt{1+x^3}}{3} - \frac{5(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{3\sqrt{2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right.$$

$$= \frac{2\sqrt{1+x^3}}{3} + \frac{5i(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{2-\sqrt{3}}\sqrt{\frac{3i+\sqrt{3}}{(-4-6i)+(2+4i)\sqrt{3}}}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{3\sqrt{2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + -$$

Mathematica [C] time = 0.20, size = 85, normalized size = 1.60

$$\frac{-4\sqrt{\frac{1}{x^3}+1}\,F_1\left(\frac{3}{2};\frac{1}{2},1;\frac{5}{2};-\frac{1}{x^3},\frac{1}{x^3}\right)-9\sqrt{x^3+1}\,x^6F_1\left(1;\frac{1}{2},1;2;-x^3,x^3\right)+6\left(x^6+x^3\right)}{9x^3\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 + x^3]*(2 + 2*x^3 + x^6))/(x*(-1 + x^6)),x]
```

```
[Out] (6*(x^3 + x^6) - 9*x^6*Sqrt[1 + x^3]*AppellF1[1, 1/2, 1, 2, -x^3, x^3] - 4*
Sqrt[1 + x^(-3)]*AppellF1[3/2, 1/2, 1, 5/2, -x^(-3), x^(-3)])/(9*x^3*Sqrt[1
 + x^3])
```

IntegrateAlgebraic [A] time = 0.07, size = 53, normalized size = 1.00

$$\frac{2\sqrt{x^3+1}}{3}+\frac{4}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{5}{3}\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x^3+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x^3]*(2 + 2*x^3 + x^6))/(x*(-1 + x^6)),x]
```

```
[Out] (2*Sqrt[1 + x^3])/3 + (4*ArcTanh[Sqrt[1 + x^3]])/3 - (5*Sqrt[2]*ArcTanh[Sqr
t[1 + x^3]/Sqrt[2]])/3
```

fricas [A] time = 0.42, size = 65, normalized size = 1.23

$$\frac{5}{6}\sqrt{2}\log\left(\frac{x^3-2\sqrt{2}\sqrt{x^3+1}+3}{x^3-1}\right)+\frac{2}{3}\sqrt{x^3+1}+\frac{2}{3}\log\left(\sqrt{x^3+1}+1\right)-\frac{2}{3}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)*(x^6+2*x^3+2)/x/(x^6-1),x, algorithm="fricas")
```

```
[Out] 5/6*sqrt(2)*log((x^3 - 2*sqrt(2)*sqrt(x^3 + 1) + 3)/(x^3 - 1)) + 2/3*sqrt(x
^3 + 1) + 2/3*log(sqrt(x^3 + 1) + 1) - 2/3*log(sqrt(x^3 + 1) - 1)
```

giac [A] time = 0.14, size = 72, normalized size = 1.36

$$\frac{5}{6}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{x^3+1}\right|}{2\left(\sqrt{2}+\sqrt{x^3+1}\right)}\right)+\frac{2}{3}\sqrt{x^3+1}+\frac{2}{3}\log\left(\sqrt{x^3+1}+1\right)-\frac{2}{3}\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)*(x^6+2*x^3+2)/x/(x^6-1),x, algorithm="giac")
```

```
[Out] 5/6*sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(x^3 + 1))/(sqrt(2) + sqrt(x^3 +
 1))) + 2/3*sqrt(x^3 + 1) + 2/3*log(sqrt(x^3 + 1) + 1) - 2/3*log(abs(sqrt(x
^3 + 1) - 1))
```

maple [C] time = 0.26, size = 434, normalized size = 8.19

$$\frac{2\sqrt{x^3+1}}{3}-\frac{5\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{3}{4}-\frac{i\sqrt{3}}{4},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}+\frac{10\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{3}{4}-\frac{3i\sqrt{3}}{4}+\frac{i\sqrt{3}\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{2},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}+\frac{10\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{3}{4}+\frac{3i\sqrt{3}}{4}+\frac{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{2},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}+\frac{4\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(1/2)*(x^6+2*x^3+2)/x/(x^6-1),x)
```

```
[Out] 2/3*(x^3+1)^(1/2)-5/3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)
*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/
```

```
(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1
/2)))^(1/2),3/4-1/4*I*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(
1/2))+10/3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I
*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*(-1/2+1/2*I*3^(1/2))*EllipticPi(((1+x)/(3/2-
1/2*I*3^(1/2)))^(1/2),3/4-3/4*I*3^(1/2)+1/2*I*3^(1/2)*(-1/2+1/2*I*3^(1/2)),
((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+10/3*(3/2-1/2*I*3^(1/2))
*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/
2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)
*(-1/2-1/2*I*3^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),3/4+3/4*
I*3^(1/2)+1/2*I*3^(1/2)*(-1/2-1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/
2*I*3^(1/2)))^(1/2))+4/3*arctanh((x^3+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^6+2x^3+2)\sqrt{x^3+1}}{(x^6-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)*(x^6+2*x^3+2)/x/(x^6-1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + 2*x^3 + 2)*sqrt(x^3 + 1)/((x^6 - 1)*x), x)
```

mupad [B] time = 0.04, size = 775, normalized size = 14.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)^(1/2)*(2*x^3 + x^6 + 2))/(x*(x^6 - 1)),x)
```

```
[Out] ((2*x^3)/3 + 2/3)/(x^3 + 1)^(1/2) + (4*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2
)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*elliptic
Pi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3
^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2))^(1/2) - (5*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2
)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)
/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/4 + 3/4
, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3
^(1/2)*1i)/2 - 3/2)))/(3*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*((3
^(1/2)*1i)/2 + 3/2)*(3*((3^(1/2)*1i)/2 - 1/2)^3 + 2)*((x + (3^(1/2)*1i)/2 -
 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*
(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1
/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 + 1/2), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)
/2 + 1/2)*(4*((3^(1/2)*1i)/2 - 1/2)^3 - 1)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2
))^(1/2)) + (2*((3^(1/2)*1i)/2 + 3/2)*(3*((3^(1/2)*1i)/2 + 1/2)^3 - 2)*((x
+ (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 1/2), asin(((x + 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/
2)))/(((3^(1/2)*1i)/2 - 1/2)*(4*((3^(1/2)*1i)/2 + 1/2)^3 + 1)*(x^3 - x*(((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [A] time = 28.45, size = 102, normalized size = 1.92

$$\frac{2\sqrt{x^3+1}}{3}+\frac{10\left(\begin{cases}-\frac{\sqrt{2}\,\operatorname{acoth}\left(\frac{\sqrt{2}\,\sqrt{x^3+1}}{2}\right)}{2} & \text{for } x^3+1>2\\ -\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{x^3+1}}{2}\right)}{2} & \text{for } x^3+1<2\end{cases}\right)}{3}-\frac{2\log\left(\sqrt{x^3+1}-1\right)}{3}+\frac{2\log\left(\sqrt{x^3+1}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(1/2)*(x**6+2*x**3+2)/x/(x**6-1),x)
```

```
[Out] 2*sqrt(x**3 + 1)/3 + 10*Piecewise((-sqrt(2)*acoth(sqrt(2)*sqrt(x**3 + 1)/2)
/2, x**3 + 1 > 2), (-sqrt(2)*atanh(sqrt(2)*sqrt(x**3 + 1)/2)/2, x**3 + 1 <
2))/3 - 2*log(sqrt(x**3 + 1) - 1)/3 + 2*log(sqrt(x**3 + 1) + 1)/3
```

3.674 $\int \frac{(-1+x^4)\sqrt{1+x^4}}{1+x^8}\,dx$

Optimal. Leaf size=53

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt[4]{2}}$$

Rubi [C] time = 0.65, antiderivative size = 314, normalized size of antiderivative = 5.92, number of steps used = 20, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6725, 406, 220, 409, 1217, 1707}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt[4]{2}}+\frac{i\left(\sqrt{2}+(1+i)\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{8\sqrt{x^4+1}}+\frac{i\left(\sqrt{2}+(-1+i)\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{8\sqrt{x^4+1}}+\frac{\left((-1-i)-i\sqrt{2}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{8\sqrt{x^4+1}}-\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\left(1+(-1)^{3/4}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4)*Sqrt[1 + x^4])/(1 + x^8),x]
```

```
[Out] -1/2*ArcTan[(2^(1/4)*x)/Sqrt[1 + x^4]]/2^(1/4) - ArcTanh[(2^(1/4)*x)/Sqrt[1
 + x^4]]/(2*2^(1/4)) + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*A
rcTan[x], 1/2])/(2*Sqrt[1 + x^4]) - ((1/8 - I/8)*(1 + (-1)^(3/4))*(1 + x^2)
*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/Sqrt[1 + x^4] + (
((-1 - I) - I*Sqrt[2])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*Ar
cTan[x], 1/2])/(8*Sqrt[1 + x^4]) + ((I/8)*((-1 + I) + Sqrt[2])*(1 + x^2)*Sq
rt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/Sqrt[1 + x^4] + ((I/
8)*((1 + I) + Sqrt[2])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*Ar
cTan[x], 1/2])/Sqrt[1 + x^4]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
```

```
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^4\right)\sqrt{1+x^4}}{1+x^8}\,dx &= \int \left(-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{1+x^4}}{i-x^4}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{1+x^4}}{i+x^4}\right)dx\\
&= \left(-\frac{1}{2}-\frac{i}{2}\right)\int \frac{\sqrt{1+x^4}}{i-x^4}\,dx+\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{\sqrt{1+x^4}}{i+x^4}\,dx\\
&= -\left(i\int \frac{1}{\left(i-x^4\right)\sqrt{1+x^4}}\,dx\right)-i\int \frac{1}{\left(i+x^4\right)\sqrt{1+x^4}}\,dx+\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{\sqrt{1+x^4}}\,dx \cdots\\
&= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}}-\frac{1}{2}\int \frac{1}{\left(1-\sqrt[4]{-1}\,x^2\right)\sqrt{1+x^4}}\,dx-\frac{1}{2}\int \frac{\cdots}{\left(1+\sqrt[4]{\cdots}\right.\cdots}\\
&= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}}-\left(\left(\frac{1}{4}+\frac{i}{4}\right)\left(1+\sqrt[4]{-1}\right)\right)\int \frac{1}{\sqrt{1+x^4}}\,dx-\left(\left(-\frac{1}{4}\cdots\right.\right.\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt[4]{2}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}}-\frac{\left(\frac{1}{8}+\frac{i}{8}\cdots\right.}{\cdots}
\end{aligned}$$

Mathematica [C] time = 0.44, size = 106, normalized size = 2.00

$$\frac{1}{2}\sqrt[4]{-1}\left(-2F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)|-1\right)+\Pi\left(-\sqrt[4]{-1};i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)|-1\right)+\Pi\left(\sqrt[4]{-1};i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)|-1\right)+\Pi\left(-(-1)^{3/4};i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)|-1\right)+\Pi\left((-1)^{3/4};i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)*Sqrt[1 + x^4])/(1 + x^8),x]
```

```
[Out] ((-1)^(1/4)*(-2*EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] + EllipticPi[-(-1)^(
1/4), I*ArcSinh[(-1)^(1/4)*x], -1] + EllipticPi[(-1)^(1/4), I*ArcSinh[(-1)^
(1/4)*x], -1] + EllipticPi[-(-1)^(3/4), I*ArcSinh[(-1)^(1/4)*x], -1] + Elli
pticPi[(-1)^(3/4), I*ArcSinh[(-1)^(1/4)*x], -1]))/2
```

IntegrateAlgebraic [A] time = 0.28, size = 53, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{x^4+1}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{x^4+1}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*Sqrt[1 + x^4])/(1 + x^8),x]
```

```
[Out] -1/2*ArcTan[(2^(1/4)*x)/Sqrt[1 + x^4]]/2^(1/4) - ArcTanh[(2^(1/4)*x)/Sqrt[1
 + x^4]]/(2*2^(1/4))
```

fricas [B] time = 0.56, size = 199, normalized size = 3.75

$$-\frac{1}{4}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}\left(2\cdot 2^{\frac{3}{4}}\left(x^{6}+x^{2}\right)+2^{\frac{1}{4}}\left(x^{8}+4x^{4}+1\right)\right)+4\sqrt{x^{4}+1}\left(2^{\frac{3}{4}}x^{3}+2^{\frac{1}{4}}\left(x^{5}+x\right)\right)}{2\left(x^{8}+1\right)}\right)-\frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(-\frac{2^{\frac{3}{4}}\left(x^{8}+4x^{4}+1\right)+4\left(x^{5}+\sqrt{2}x^{3}+x\right)\sqrt{x^{4}+1}+4\cdot 2^{\frac{1}{4}}\left(x^{6}+x^{2}\right)}{x^{8}+1}\right)+\frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(\frac{2^{\frac{3}{4}}\left(x^{8}+4x^{4}+1\right)-4\left(x^{5}+\sqrt{2}x^{3}+x\right)\sqrt{x^{4}+1}+4\cdot 2^{\frac{1}{4}}\left(x^{6}+x^{2}\right)}{x^{8}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/2)/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/4*2^(3/4)*arctan(1/2*(2^(3/4)*(2*2^(3/4)*(x^6 + x^2) + 2^(1/4)*(x^8 + 4*
x^4 + 1)) + 4*sqrt(x^4 + 1)*(2^(3/4)*x^3 + 2^(1/4)*(x^5 + x)))/(x^8 + 1)) -
 1/16*2^(3/4)*log(-(2^(3/4)*(x^8 + 4*x^4 + 1) + 4*(x^5 + sqrt(2)*x^3 + x)*s
qrt(x^4 + 1) + 4*2^(1/4)*(x^6 + x^2))/(x^8 + 1)) + 1/16*2^(3/4)*log((2^(3/4
)*(x^8 + 4*x^4 + 1) - 4*(x^5 + sqrt(2)*x^3 + x)*sqrt(x^4 + 1) + 4*2^(1/4)*(
x^6 + x^2))/(x^8 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}\left(x^4-1\right)}{x^8+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/2)/(x^8+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 1)*(x^4 - 1)/(x^8 + 1), x)
```

maple [A] time = 0.03, size = 74, normalized size = 1.40

$$\frac{2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}\sqrt{x^4+1}}{2x}\right)}{4}-\frac{2^{\frac{3}{4}}\ln\left(\frac{\frac{\sqrt{2}\,\sqrt{x^4+1}}{2x}+\frac{2^{\frac{3}{4}}}{2}}{\frac{\sqrt{2}\,\sqrt{x^4+1}}{2x}-\frac{2^{\frac{3}{4}}}{2}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^4+1)^(1/2)/(x^8+1),x)
```

```
[Out] 1/4*2^(3/4)*arctan(1/2*2^(3/4)/x*(x^4+1)^(1/2))-1/8*2^(3/4)*ln((1/2*2^(1/2)
/x*(x^4+1)^(1/2)+1/2*2^(3/4))/(1/2*2^(1/2)/x*(x^4+1)^(1/2)-1/2*2^(3/4)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}\left(x^4-1\right)}{x^8+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/2)/(x^8+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 1)*(x^4 - 1)/(x^8 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x^4-1\right)\,\sqrt{x^4+1}}{x^8+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(x^4 + 1)^(1/2))/(x^8 + 1),x)
[Out] int(((x^4 - 1)*(x^4 + 1)^(1/2))/(x^8 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2+1\right)\sqrt{x^4+1}}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**4+1)**(1/2)/(x**8+1),x)
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)*sqrt(x**4 + 1)/(x**8 + 1), x)
```

3.675 $\int \frac{-1+x^8}{\sqrt{1+x^4}\left(1+x^8\right)}\,dx$

Optimal. Leaf size=53

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt[4]{2}}$$

Rubi [C] time = 0.54, antiderivative size = 314, normalized size of antiderivative = 5.92, number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1586, 6725, 406, 220, 409, 1217, 1707}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt[4]{2}}+\frac{i\left(\sqrt{2}+(1+i)\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{8\sqrt{x^4+1}}+\frac{i\left(\sqrt{2}+(-1+i)\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{8\sqrt{x^4+1}}+\frac{\left((-1-i)-i\sqrt{2}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{8\sqrt{x^4+1}}-\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\left(1+(-1)^{3/4}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^8)/(Sqrt[1 + x^4]*(1 + x^8)),x]
```

```
[Out] -1/2*ArcTan[(2^(1/4)*x)/Sqrt[1 + x^4]]/2^(1/4) - ArcTanh[(2^(1/4)*x)/Sqrt[1
 + x^4]]/(2*2^(1/4)) + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*A
rcTan[x], 1/2])/(2*Sqrt[1 + x^4]) - ((1/8 - I/8)*(1 + (-1)^(3/4))*(1 + x^2)
*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/Sqrt[1 + x^4] + (
((-1 - I) - I*Sqrt[2])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*Ar
cTan[x], 1/2])/(8*Sqrt[1 + x^4]) + ((I/8)*((-1 + I) + Sqrt[2])*(1 + x^2)*Sq
rt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/Sqrt[1 + x^4] + ((I/
8)*((1 + I) + Sqrt[2])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*Ar
cTan[x], 1/2])/Sqrt[1 + x^4]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+x^8}{\sqrt{1+x^4}\left(1+x^8\right)}\,dx=\int \frac{\left(-1+x^4\right)\sqrt{1+x^4}}{1+x^8}\,dx$$

$$=\int \left(-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{1+x^4}}{i-x^4}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{1+x^4}}{i+x^4}\right)dx$$

$$=\left(-\frac{1}{2}-\frac{i}{2}\right)\int \frac{\sqrt{1+x^4}}{i-x^4}\,dx+\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{\sqrt{1+x^4}}{i+x^4}\,dx$$

$$=-\left(i\int \frac{1}{\left(i-x^4\right)\sqrt{1+x^4}}\,dx\right)-i\int \frac{1}{\left(i+x^4\right)\sqrt{1+x^4}}\,dx+\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{\sqrt{1+x^4}}\,dx+$$

$$=\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}}-\frac{1}{2}\int \frac{1}{\left(1-\sqrt[4]{-1}\,x^2\right)\sqrt{1+x^4}}\,dx-\frac{1}{2}\int \frac{}{\left(1+\sqrt[4]{-}\right.}$$

$$=\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}}-\left(\left(\frac{1}{4}+\frac{i}{4}\right)\left(1+\sqrt[4]{-1}\right)\right)\int \frac{1}{\sqrt{1+x^4}}\,dx-\left(\left(-\frac{1}{4}\right.\right.$$

$$=-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt[4]{2}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}}-\frac{\left(\frac{1}{8}+\frac{i}{8}\right)}{}$$

Mathematica [C] time = 0.33, size = 106, normalized size = 2.00

$$\frac{1}{2}\sqrt[4]{-1}\left(-2F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)|-1\right)+\Pi\left(-\sqrt[4]{-1};i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)|-1\right)+\Pi\left(\sqrt[4]{-1};i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)|-1\right)+\Pi\left(-(-1)^{3/4};i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)|-1\right)+\Pi\left((-1)^{3/4};i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^8)/(Sqrt[1 + x^4]*(1 + x^8)),x]
```

```
[Out] ((-1)^(1/4)*(-2*EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] + EllipticPi[-(-1)^(
1/4), I*ArcSinh[(-1)^(1/4)*x], -1] + EllipticPi[(-1)^(1/4), I*ArcSinh[(-1)^
```

```
(1/4)*x], -1] + EllipticPi[-(-1)^(3/4), I*ArcSinh[(-1)^(1/4)*x], -1] + Elli
pticPi[(-1)^(3/4), I*ArcSinh[(-1)^(1/4)*x], -1]))/2
```

IntegrateAlgebraic [A] time = 0.28, size = 53, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{x^4+1}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{x^4+1}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^8)/(Sqrt[1 + x^4]*(1 + x^8)),x]
```

```
[Out] -1/2*ArcTan[(2^(1/4)*x)/Sqrt[1 + x^4]]/2^(1/4) - ArcTanh[(2^(1/4)*x)/Sqrt[1
 + x^4]]/(2*2^(1/4))
```

fricas [B] time = 0.54, size = 199, normalized size = 3.75

$$-\frac{1}{4}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}\left(2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)+2^{\frac{1}{4}}\left(x^8+4x^4+1\right)\right)+4\sqrt{x^4+1}\left(2^{\frac{3}{4}}x^3+2^{\frac{1}{4}}\left(x^5+x\right)\right)}{2\left(x^8+1\right)}\right)-\frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(-\frac{2^{\frac{3}{4}}\left(x^8+4x^4+1\right)+4\left(x^5+\sqrt{2}x^3+x\right)\sqrt{x^4+1}+4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)}{x^8+1}\right)+\frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(\frac{2^{\frac{3}{4}}\left(x^8+4x^4+1\right)-4\left(x^5+\sqrt{2}x^3+x\right)\sqrt{x^4+1}+4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)}{x^8+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/(x^4+1)^(1/2)/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/4*2^(3/4)*arctan(1/2*(2^(3/4)*(2*2^(3/4)*(x^6 + x^2) + 2^(1/4)*(x^8 + 4*
x^4 + 1)) + 4*sqrt(x^4 + 1)*(2^(3/4)*x^3 + 2^(1/4)*(x^5 + x)))/(x^8 + 1)) -
 1/16*2^(3/4)*log(-(2^(3/4)*(x^8 + 4*x^4 + 1) + 4*(x^5 + sqrt(2)*x^3 + x)*s
qrt(x^4 + 1) + 4*2^(1/4)*(x^6 + x^2))/(x^8 + 1)) + 1/16*2^(3/4)*log((2^(3/4
)*(x^8 + 4*x^4 + 1) - 4*(x^5 + sqrt(2)*x^3 + x)*sqrt(x^4 + 1) + 4*2^(1/4)*(
x^6 + x^2))/(x^8 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8-1}{\left(x^8+1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/(x^4+1)^(1/2)/(x^8+1),x, algorithm="giac")
```

```
[Out] integrate((x^8 - 1)/((x^8 + 1)*sqrt(x^4 + 1)), x)
```

maple [C] time = 0.06, size = 161, normalized size = 3.04

$$\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^8+1\right)}\_\alpha\left(-\frac{\operatorname{arctanh}\left(\frac{\_\alpha^2\left(-\_\alpha^6+x^2\right)}{\sqrt{\_\alpha^4+1}\,\sqrt{x^4+1}}\right)}{\sqrt{\_\alpha^4+1}}-\frac{2(-1)^{\frac{3}{4}}\_\alpha^7\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticPi}\left((-1)^{\frac{1}{4}}x,i\_\alpha^6,i\right)}{\sqrt{x^4+1}}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-1)/(x^4+1)^(1/2)/(x^8+1),x)
```

```
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+1/8*sum(_alpha*(-1/(_alpha^4+1)
^(1/2)*arctanh(_alpha^2*(-_alpha^6+x^2)/(_alpha^4+1)^(1/2)/(x^4+1)^(1/2))-2
*(-1)^(3/4)*_alpha^7*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*Elliptic
Pi((-1)^(1/4)*x,I*_alpha^6,I)),_alpha=RootOf(_Z^8+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8-1}{\left(x^8+1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/(x^4+1)^(1/2)/(x^8+1),x, algorithm="maxima")
```

```
[Out] integrate((x^8 - 1)/((x^8 + 1)*sqrt(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^8-1}{\sqrt{x^4+1}\,\left(x^8+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 - 1)/((x^4 + 1)^(1/2)*(x^8 + 1)),x)
```

```
[Out] int((x^8 - 1)/((x^4 + 1)^(1/2)*(x^8 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)\,\left(x^2+1\right)\sqrt{x^4+1}}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-1)/(x**4+1)**(1/2)/(x**8+1),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)*sqrt(x**4 + 1)/(x**8 + 1), x)
```

3.676 $\int \frac{\sqrt{-2+x^4}\left(2+x^4\right)}{4-6x^4+x^8}\,dx$

Optimal. Leaf size=53

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{x^4-2}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{x^4-2}}\right)}{2\sqrt[4]{2}}$$

Rubi [C] time = 1.43, antiderivative size = 647, normalized size of antiderivative = 12.21, number of steps used = 40, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {6728, 406, 223, 409, 1215, 1457, 540, 253, 538, 537}

$$\frac{\left(1+\sqrt{5}\right)\sqrt{\frac{x^2+\sqrt{2}}{\sqrt{2}-x^2}}\sqrt{\sqrt{2}x^2-2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}x}{\sqrt{\sqrt{2}x^2-2}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{2}\sqrt{\frac{1}{2-\sqrt{2}x^2}}\sqrt{x^4-2}}+\frac{\left(1-\sqrt{5}\right)\sqrt{\frac{x^2+\sqrt{2}}{\sqrt{2}-x^2}}\sqrt{\sqrt{2}x^2-2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}x}{\sqrt{\sqrt{2}x^2-2}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{2}\sqrt{\frac{1}{2-\sqrt{2}x^2}}\sqrt{x^4-2}}-\frac{\left(2+\sqrt{2\left(3+\sqrt{5}\right)}\right)\sqrt{\sqrt{2}-x^2}\sqrt{\sqrt{2}x^2+2}\,\Pi\left(-\sqrt{\frac{2}{3+\sqrt{5}}};\sin^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)|-1\right)}{4\left(\sqrt{2}+\sqrt{3+\sqrt{5}}\right)\sqrt{x^4-2}}-\frac{\left(2-\sqrt{2\left(3+\sqrt{5}\right)}\right)\sqrt{\sqrt{2}-x^2}\sqrt{\sqrt{2}x^2+2}\,\Pi\left(\sqrt{\frac{2}{3+\sqrt{5}}};\sin^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)|-1\right)}{4\left(\sqrt{2}-\sqrt{3+\sqrt{5}}\right)\sqrt{x^4-2}}-\frac{\left(\sqrt{2}+\sqrt{3+\sqrt{5}}\right)\sqrt{\sqrt{2}-x^2}\sqrt{\sqrt{2}x^2+2}\,\Pi\left(-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)};\sin^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)|-1\right)}{2\left(2+\sqrt{2\left(3+\sqrt{5}\right)}\right)\sqrt{x^4-2}}-\frac{\left(\sqrt{2}-\sqrt{3+\sqrt{5}}\right)\sqrt{\sqrt{2}-x^2}\sqrt{\sqrt{2}x^2+2}\,\Pi\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)};\sin^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)|-1\right)}{2\left(2-\sqrt{2\left(3+\sqrt{5}\right)}\right)\sqrt{x^4-2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[-2 + x^4]*(2 + x^4))/(4 - 6*x^4 + x^8),x]
```

```
[Out] ((1 - Sqrt[5])*Sqrt[(Sqrt[2] + x^2)/(Sqrt[2] - x^2)]*Sqrt[-2 + Sqrt[2]*x^2]
*EllipticF[ArcSin[(2^(3/4)*x)/Sqrt[-2 + Sqrt[2]*x^2]], 1/2])/(4*2^(1/4)*Sqr
t[(2 - Sqrt[2]*x^2)^(-1)]*Sqrt[-2 + x^4]) + ((1 + Sqrt[5])*Sqrt[(Sqrt[2] +
x^2)/(Sqrt[2] - x^2)]*Sqrt[-2 + Sqrt[2]*x^2]*EllipticF[ArcSin[(2^(3/4)*x)/S
qrt[-2 + Sqrt[2]*x^2]], 1/2])/(4*2^(1/4)*Sqrt[(2 - Sqrt[2]*x^2)^(-1)]*Sqrt[
-2 + x^4]) - ((2 + Sqrt[2*(3 + Sqrt[5])])*Sqrt[Sqrt[2] - x^2]*Sqrt[2 + Sqrt
[2]*x^2]*EllipticPi[-Sqrt[2/(3 + Sqrt[5])], ArcSin[x/2^(1/4)], -1])/(4*(Sqr
t[2] + Sqrt[3 + Sqrt[5]])*Sqrt[-2 + x^4]) - ((2 - Sqrt[2*(3 + Sqrt[5])])*Sq
rt[Sqrt[2] - x^2]*Sqrt[2 + Sqrt[2]*x^2]*EllipticPi[Sqrt[2/(3 + Sqrt[5])], A
rcSin[x/2^(1/4)], -1])/(4*(Sqrt[2] - Sqrt[3 + Sqrt[5]])*Sqrt[-2 + x^4]) - (
(Sqrt[2] + Sqrt[3 + Sqrt[5]])*Sqrt[Sqrt[2] - x^2]*Sqrt[2 + Sqrt[2]*x^2]*Ell
ipticPi[-Sqrt[(3 + Sqrt[5])/2], ArcSin[x/2^(1/4)], -1])/(2*(2 + Sqrt[2*(3 +
 Sqrt[5])])*Sqrt[-2 + x^4]) - ((Sqrt[2] - Sqrt[3 + Sqrt[5]])*Sqrt[Sqrt[2] -
 x^2]*Sqrt[2 + Sqrt[2]*x^2]*EllipticPi[Sqrt[(3 + Sqrt[5])/2], ArcSin[x/2^(1
/4)], -1])/(2*(2 - Sqrt[2*(3 + Sqrt[5])])*Sqrt[-2 + x^4])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[(a - q*x^2)/(a + q*x^2)]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqr
t[(a + q*x^2)/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[a + b*x^4]*Sqrt[a/(a + q*x^2)]),
 x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 540

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x
] + Dist[(b*c - a*d)/b, Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

Rule 1215

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[c/(c*d + e*q), Int[1/Sqrt[a + c*x^4], x], x] + Di
st[e/(c*d + e*q), Int[(q - c*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
 FreeQ[{a, c, d, e}, x] && GtQ[-(a*c), 0] &&  !LtQ[c, 0]
```

Rule 1457

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + c*x^(2*n))^FracPart[p]/((d +
e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p]), Int[(d + e*x^n)^(p + q)*
(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, n, p,
 q, r}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{-2+x^4}\left(2+x^4\right)}{4-6x^4+x^8}\,dx = \int \left(\frac{\left(1+\sqrt{5}\right)\sqrt{-2+x^4}}{-6-2\sqrt{5}+2x^4}+\frac{\left(1-\sqrt{5}\right)\sqrt{-2+x^4}}{-6+2\sqrt{5}+2x^4}\right)dx$$

$$= \left(1-\sqrt{5}\right)\int \frac{\sqrt{-2+x^4}}{-6+2\sqrt{5}+2x^4}\,dx+\left(1+\sqrt{5}\right)\int \frac{\sqrt{-2+x^4}}{-6-2\sqrt{5}+2x^4}\,dx$$

$$= \frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{1}{\sqrt{-2+x^4}}\,dx+\left(2\left(3-\sqrt{5}\right)\right)\int \frac{1}{\sqrt{-2+x^4}\left(-6+2\sqrt{5}+2x^4\right)}\,dx+\frac{1}{2}$$

$$= \frac{\left(1-\sqrt{5}\right)\sqrt{\frac{\sqrt{2}+x^2}{\sqrt{2}-x^2}}\sqrt{-2+\sqrt{2}x^2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}x}{\sqrt{-2+\sqrt{2}x^2}}\right)\middle|\frac{1}{2}\right)}{4\sqrt[4]{2}\sqrt{\frac{1}{2-\sqrt{2}x^2}}\sqrt{-2+x^4}}+\frac{\left(1+\sqrt{5}\right)\sqrt{\frac{\sqrt{2}+x^2}{\sqrt{2}-x^2}}\sqrt{-2}}{4\sqrt[4]{2}\sqrt{}}$$

$$= \frac{\left(1-\sqrt{5}\right)\sqrt{\frac{\sqrt{2}+x^2}{\sqrt{2}-x^2}}\sqrt{-2+\sqrt{2}x^2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}x}{\sqrt{-2+\sqrt{2}x^2}}\right)\middle|\frac{1}{2}\right)}{4\sqrt[4]{2}\sqrt{\frac{1}{2-\sqrt{2}x^2}}\sqrt{-2+x^4}}+\frac{\left(1+\sqrt{5}\right)\sqrt{\frac{\sqrt{2}+x^2}{\sqrt{2}-x^2}}\sqrt{-2}}{4\sqrt[4]{2}\sqrt{}}$$

$$= \frac{\left(1-\sqrt{5}\right)\sqrt{\frac{\sqrt{2}+x^2}{\sqrt{2}-x^2}}\sqrt{-2+\sqrt{2}x^2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}x}{\sqrt{-2+\sqrt{2}x^2}}\right)\middle|\frac{1}{2}\right)}{4\sqrt[4]{2}\sqrt{\frac{1}{2-\sqrt{2}x^2}}\sqrt{-2+x^4}}+\frac{\left(1+\sqrt{5}\right)\sqrt{\frac{\sqrt{2}+x^2}{\sqrt{2}-x^2}}\sqrt{-2}}{4\sqrt[4]{2}\sqrt{}}$$

$$= \frac{\left(1-\sqrt{5}\right)\sqrt{\frac{\sqrt{2}+x^2}{\sqrt{2}-x^2}}\sqrt{-2+\sqrt{2}x^2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}x}{\sqrt{-2+\sqrt{2}x^2}}\right)\middle|\frac{1}{2}\right)}{4\sqrt[4]{2}\sqrt{\frac{1}{2-\sqrt{2}x^2}}\sqrt{-2+x^4}}+\frac{\left(1+\sqrt{5}\right)\sqrt{\frac{\sqrt{2}+x^2}{\sqrt{2}-x^2}}\sqrt{-2}}{4\sqrt[4]{2}\sqrt{}}$$

$$= \frac{\left(1-\sqrt{5}\right)\sqrt{\frac{\sqrt{2}+x^2}{\sqrt{2}-x^2}}\sqrt{-2+\sqrt{2}x^2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}x}{\sqrt{-2+\sqrt{2}x^2}}\right)\middle|\frac{1}{2}\right)}{4\sqrt[4]{2}\sqrt{\frac{1}{2-\sqrt{2}x^2}}\sqrt{-2+x^4}}+\frac{\left(1+\sqrt{5}\right)\sqrt{\frac{\sqrt{2}+x^2}{\sqrt{2}-x^2}}\sqrt{-2}}{4\sqrt[4]{2}\sqrt{}}$$

$$= \frac{\left(1-\sqrt{5}\right)\sqrt{\frac{\sqrt{2}+x^2}{\sqrt{2}-x^2}}\sqrt{-2+\sqrt{2}x^2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}x}{\sqrt{-2+\sqrt{2}x^2}}\right)\middle|\frac{1}{2}\right)}{4\sqrt[4]{2}\sqrt{\frac{1}{2-\sqrt{2}x^2}}\sqrt{-2+x^4}}+\frac{\left(1+\sqrt{5}\right)\sqrt{\frac{\sqrt{2}+x^2}{\sqrt{2}-x^2}}\sqrt{-2}}{4\sqrt[4]{2}\sqrt{}}$$

Mathematica [C] time = 0.28, size = 154, normalized size = 2.91

$$\frac{\sqrt{2-x^4}\left(2F\left(\sin^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)\middle|-1\right)-\Pi\left(-\sqrt{\frac{2}{3+\sqrt{5}}};\sin^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)\middle|-1\right)-\Pi\left(\sqrt{\frac{2}{3+\sqrt{5}}};\sin^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)\middle|-1\right)-\Pi\left(-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)};\sin^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)\middle|-1\right)-\Pi\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)};\sin^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)\middle|-1\right)\right)}{2\sqrt[4]{2}\sqrt{x^4-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-2 + x^4]*(2 + x^4))/(4 - 6*x^4 + x^8),x]
```

```
[Out] (Sqrt[2 - x^4]*(2*EllipticF[ArcSin[x/2^(1/4)], -1] - EllipticPi[-Sqrt[2/(3
+ Sqrt[5])], ArcSin[x/2^(1/4)], -1] - EllipticPi[Sqrt[2/(3 + Sqrt[5])], Arc
Sin[x/2^(1/4)], -1] - EllipticPi[-Sqrt[(3 + Sqrt[5])/2], ArcSin[x/2^(1/4)],
 -1] - EllipticPi[Sqrt[(3 + Sqrt[5])/2], ArcSin[x/2^(1/4)], -1]))/(2*2^(1/4
)*Sqrt[-2 + x^4])
```

IntegrateAlgebraic [A] time = 0.34, size = 53, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{x^4-2}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{x^4-2}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-2 + x^4]*(2 + x^4))/(4 - 6*x^4 + x^8),x]
```

```
[Out] -1/2*ArcTan[(2^(1/4)*x)/Sqrt[-2 + x^4]]/2^(1/4) - ArcTanh[(2^(1/4)*x)/Sqrt[
-2 + x^4]]/(2*2^(1/4))
```

fricas [B] time = 0.57, size = 226, normalized size = 4.26

$$\frac{1}{4}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}\left(2\cdot 2^{\frac{3}{4}}\left(x^{6}-2x^{2}\right)+2^{\frac{1}{4}}\left(x^{8}-2x^{4}+4\right)\right)-4\sqrt{x^{4}-2}\left(2^{\frac{3}{4}}x^{3}+2^{\frac{1}{4}}\left(x^{5}-2x\right)\right)}{2\left(x^{8}-6x^{4}+4\right)}\right)-\frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(\frac{2^{\frac{3}{4}}\left(x^{8}-2x^{4}+4\right)+4\left(x^{5}+\sqrt{2}x^{3}-2x\right)\sqrt{x^{4}-2}+4\cdot 2^{\frac{1}{4}}\left(x^{6}-2x^{2}\right)}{x^{8}-6x^{4}+4}\right)+\frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(-\frac{2^{\frac{3}{4}}\left(x^{8}-2x^{4}+4\right)-4\left(x^{5}+\sqrt{2}x^{3}-2x\right)\sqrt{x^{4}-2}+4\cdot 2^{\frac{1}{4}}\left(x^{6}-2x^{2}\right)}{x^{8}-6x^{4}+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2)^(1/2)*(x^4+2)/(x^8-6*x^4+4),x, algorithm="fricas")
```

```
[Out] 1/4*2^(3/4)*arctan(1/2*(2^(3/4)*(2*2^(3/4)*(x^6 - 2*x^2) + 2^(1/4)*(x^8 - 2
*x^4 + 4)) - 4*sqrt(x^4 - 2)*(2^(3/4)*x^3 + 2^(1/4)*(x^5 - 2*x)))/(x^8 - 6*
x^4 + 4)) - 1/16*2^(3/4)*log((2^(3/4)*(x^8 - 2*x^4 + 4) + 4*(x^5 + sqrt(2)*
x^3 - 2*x)*sqrt(x^4 - 2) + 4*2^(1/4)*(x^6 - 2*x^2))/(x^8 - 6*x^4 + 4)) + 1/
16*2^(3/4)*log(-(2^(3/4)*(x^8 - 2*x^4 + 4) - 4*(x^5 + sqrt(2)*x^3 - 2*x)*sq
rt(x^4 - 2) + 4*2^(1/4)*(x^6 - 2*x^2))/(x^8 - 6*x^4 + 4))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2)^(1/2)*(x^4+2)/(x^8-6*x^4+4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [A] time = 0.04, size = 74, normalized size = 1.40

$$\frac{2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}\sqrt{x^4-2}}{2x}\right)}{4}-\frac{2^{\frac{3}{4}}\ln\left(\frac{\frac{\sqrt{x^4-2}\,\sqrt{2}}{2x}+\frac{2^{\frac{3}{4}}}{2}}{\frac{\sqrt{x^4-2}\,\sqrt{2}}{2x}-\frac{2^{\frac{3}{4}}}{2}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2)^(1/2)*(x^4+2)/(x^8-6*x^4+4),x)
```

```
[Out] 1/4*2^(3/4)*arctan(1/2*2^(3/4)/x*(x^4-2)^(1/2))-1/8*2^(3/4)*ln((1/2*(x^4-2)
^(1/2)*2^(1/2)/x+1/2*2^(3/4))/(1/2*(x^4-2)^(1/2)*2^(1/2)/x-1/2*2^(3/4)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+2\right)\sqrt{x^4-2}}{x^8-6\,x^4+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2)^(1/2)*(x^4+2)/(x^8-6*x^4+4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 2)*sqrt(x^4 - 2)/(x^8 - 6*x^4 + 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x^4-2}\,\left(x^4+2\right)}{x^8-6\,x^4+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 2)^(1/2)*(x^4 + 2))/(x^8 - 6*x^4 + 4),x)
[Out] int(((x^4 - 2)^(1/2)*(x^4 + 2))/(x^8 - 6*x^4 + 4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2)**(1/2)*(x**4+2)/(x**8-6*x**4+4),x)
[Out] Timed out
```

3.677 $\int \frac{1}{x^2\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=53

$$-\frac{1}{x\sqrt{\sqrt{x^2+1}+x}}+\tan^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)+\tanh^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)$$

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2119, 457, 329, 212, 206, 203}

$$-\frac{1}{x\sqrt{\sqrt{x^2+1}+x}}+\tan^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)+\tanh^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -(1/(x*Sqrt[x + Sqrt[1 + x^2]])) + ArcTan[Sqrt[x + Sqrt[1 + x^2]]] + ArcTan
h[Sqrt[x + Sqrt[1 + x^2]]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{x+\sqrt{1+x^2}}}\,dx = 2\,\text{Subst}\left(\int \frac{1+x^2}{\sqrt{x}\left(-1+x^2\right)^2}\,dx, x, x+\sqrt{1+x^2}\right)$$

$$= -\frac{1}{x\sqrt{x+\sqrt{1+x^2}}} - \text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx, x, x+\sqrt{1+x^2}\right)$$

$$= -\frac{1}{x\sqrt{x+\sqrt{1+x^2}}} - 2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{x+\sqrt{1+x^2}}\right)$$

$$= -\frac{1}{x\sqrt{x+\sqrt{1+x^2}}} + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x+\sqrt{1+x^2}}\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x,\right.$$

$$= -\frac{1}{x\sqrt{x+\sqrt{1+x^2}}} + \tan^{-1}\left(\sqrt{x+\sqrt{1+x^2}}\right) + \tanh^{-1}\left(\sqrt{x+\sqrt{1+x^2}}\right)$$

Mathematica [C] time = 14.56, size = 983, normalized size = 18.55

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/2*((-1 + (x + Sqrt[1 + x^2])^2)*(2*Sqrt[x + Sqrt[1 + x^2]] - 2*ArcTan[Sq
rt[x + Sqrt[1 + x^2]]] - 2*ArcTanh[Sqrt[x + Sqrt[1 + x^2]]]))/(x*(1 + x/Sqr
t[1 + x^2])*(x + Sqrt[1 + x^2])*(1 - (-1 + (x + Sqrt[1 + x^2])^2)/(2*(x + S
qrt[1 + x^2])^2))) + (159120*(x + Sqrt[1 + x^2])^(19/2)*(1 + x^2 + x*Sqrt[1
 + x^2])*((-3645 - 6769*(x + Sqrt[1 + x^2])^2 - 1483*(x + Sqrt[1 + x^2])^4
+ 681*(x + Sqrt[1 + x^2])^6 + 5*(729 + 1208*(x + Sqrt[1 + x^2])^2 + 102*(x
+ Sqrt[1 + x^2])^4 - 248*(x + Sqrt[1 + x^2])^6 + (x + Sqrt[1 + x^2])^8)*Hyp
ergeometric2F1[1/4, 1, 5/4, (x + Sqrt[1 + x^2])^2])/(640*(x + Sqrt[1 + x^2]
)^4) + (16*(x + Sqrt[1 + x^2] + (x + Sqrt[1 + x^2])^3)^2*HypergeometricPFQ[
{5/4, 2, 2, 2}, {1, 1, 17/4}, (x + Sqrt[1 + x^2])^2])/585))/(x*(1 + x/Sqrt[
1 + x^2])*(1 + (x + Sqrt[1 + x^2])^2)*(1989*(-140 + 6909*x^2 + 42462*x^4 +
90944*x^6 + 107808*x^8 + 54400*x^10 + 967*x*Sqrt[1 + x^2] + 20418*x^3*Sqrt[
1 + x^2] + 57440*x^5*Sqrt[1 + x^2] + 80608*x^7*Sqrt[1 + x^2] + 54400*x^9*Sq
rt[1 + x^2] + 10*x*(-971*x - 5590*x^3 - 12608*x^5 - 15296*x^7 - 7296*x^9 +
256*x^11 - 182*Sqrt[1 + x^2] - 2672*x^2*Sqrt[1 + x^2] - 7776*x^4*Sqrt[1 + x
^2] - 11552*x^6*Sqrt[1 + x^2] - 7424*x^8*Sqrt[1 + x^2] + 256*x^10*Sqrt[1 +
x^2])*Hypergeometric2F1[1/4, 1, 5/4, (x + Sqrt[1 + x^2])^2]) + 4352*x*(112*
x + 2916*x^3 + 22008*x^5 + 71936*x^7 + 115200*x^9 + 89088*x^11 + 26624*x^13
 + 9*Sqrt[1 + x^2] + 696*x^2*Sqrt[1 + x^2] + 8680*x^4*Sqrt[1 + x^2] + 39424
*x^6*Sqrt[1 + x^2] + 80640*x^8*Sqrt[1 + x^2] + 75776*x^10*Sqrt[1 + x^2] + 2
6624*x^12*Sqrt[1 + x^2])*HypergeometricPFQ[{5/4, 2, 2, 2}, {1, 1, 17/4}, (x
 + Sqrt[1 + x^2])^2] + 40960*x*(14*x + 462*x^3 + 4480*x^5 + 19392*x^7 + 435
20*x^9 + 52736*x^11 + 32768*x^13 + 8192*x^15 + Sqrt[1 + x^2] + 98*x^2*Sqrt[
```

```
1 + x^2] + 1568*x^4*Sqrt[1 + x^2] + 9408*x^6*Sqrt[1 + x^2] + 26880*x^8*Sqrt
[1 + x^2] + 39424*x^10*Sqrt[1 + x^2] + 28672*x^12*Sqrt[1 + x^2] + 8192*x^14
*Sqrt[1 + x^2])*HypergeometricPFQ[{9/4, 3, 3, 3}, {2, 2, 21/4}, (x + Sqrt[1
 + x^2])^2]))
```

IntegrateAlgebraic [A] time = 0.15, size = 53, normalized size = 1.00

$$-\frac{1}{x\sqrt{\sqrt{x^2+1}+x}}+\tan^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)+\tanh^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -(1/(x*Sqrt[x + Sqrt[1 + x^2]])) + ArcTan[Sqrt[x + Sqrt[1 + x^2]]] + ArcTan
h[Sqrt[x + Sqrt[1 + x^2]]]
```

fricas [A] time = 0.42, size = 78, normalized size = 1.47

$$\frac{2x\arctan\left(\sqrt{x+\sqrt{x^2+1}}\right)+x\log\left(\sqrt{x+\sqrt{x^2+1}}+1\right)-x\log\left(\sqrt{x+\sqrt{x^2+1}}-1\right)+2\sqrt{x+\sqrt{x^2+1}}\left(x-\sqrt{x^2+1}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x*arctan(sqrt(x + sqrt(x^2 + 1))) + x*log(sqrt(x + sqrt(x^2 + 1)) +
1) - x*log(sqrt(x + sqrt(x^2 + 1)) - 1) + 2*sqrt(x + sqrt(x^2 + 1))*(x - sq
rt(x^2 + 1)))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x+\sqrt{x^2+1}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x + sqrt(x^2 + 1))*x^2), x)
```

maple [C] time = 0.05, size = 22, normalized size = 0.42

$$-\frac{\sqrt{2}\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4},\frac{3}{4}\right],\left[\frac{3}{2},\frac{7}{4}\right],-\frac{1}{x^2}\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] -1/3*2^(1/2)/x^(3/2)*hypergeom([1/4,3/4,3/4],[3/2,7/4],-1/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x+\sqrt{x^2+1}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x + sqrt(x^2 + 1))*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2 \sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x + (x^2 + 1)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^2*(x + (x^2 + 1)^(1/2))^(1/2)), x)
```

sympy [C] time = 1.76, size = 44, normalized size = 0.83

$$-\frac{\Gamma\left(\frac{1}{4}\right)\Gamma^2\left(\frac{3}{4}\right){}_3F_2\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{3}{4}\\\frac{3}{2},\frac{7}{4}\end{matrix}\right|\frac{e^{i\pi}}{x^2}\right)}{4\pi x^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] -gamma(1/4)*gamma(3/4)**2*hyper((1/4, 3/4, 3/4), (3/2, 7/4), exp_polar(I*pi
)/x**2)/(4*pi*x**(3/2)*gamma(7/4))
```

3.678 $\int \frac{-1+\sqrt{k}x}{\left(1+\sqrt{k}x\right)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}\,dx$

Optimal. Leaf size=54

$$-\frac{2\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2+2\sqrt{k}x+1}\right)}{k-1}$$

Rubi [C] time = 1.39, antiderivative size = 199, normalized size of antiderivative = 3.69, number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {6719, 6742, 419, 2113, 537, 571, 93, 205}

$$-\frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\tan^{-1}\left(\frac{\sqrt{k}\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{(1-k)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}+\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}-\frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\Pi\left(k;\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[k]*x)/((1 + Sqrt[k]*x)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),x]
```

```
[Out] (-2*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*ArcTan[(Sqrt[k]*Sqrt[1 - x^2])/Sqrt[1 -
 k^2*x^2]])/((1 - k)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) + (Sqrt[1 - x^2]*Sqrt[1
 - k^2*x^2]*EllipticF[ArcSin[x], k^2])/Sqrt[(1 - x^2)*(1 - k^2*x^2)] - (2*S
qrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[k, ArcSin[x], k^2])/Sqrt[(1 - x^2
)*(1 - k^2*x^2)]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
```

```
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-1+\sqrt{k}\,x}{\left(1+\sqrt{k}\,x\right)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}\,dx = \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{-1+\sqrt{k}\,x}{\left(1+\sqrt{k}\,x\right)\sqrt{1-x^2}\,\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \left(\frac{1}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}}-\frac{2}{\left(1+\sqrt{k}\,x\right)\sqrt{1-x^2}\,\sqrt{1-k^2x^2}}\right)dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{}{(1+}}{\sqrt{\left(1-x^2\right)\left(1-\right.}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1}{\sqrt{1-x^2}\,(1-k}}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(k;\sin^{-1}(x)|\right.}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(k;\sin^{-1}(x)|\right.}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= -\frac{2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\tan^{-1}\left(\frac{\sqrt{k}\,\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{(1-k)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x\right.}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

Mathematica [C] time = 0.34, size = 209, normalized size = 3.87

$$\frac{\sqrt{k-1}\sqrt{-((k-1)k)}\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)-2\left(\sqrt{k}\sqrt{x^2-1}\sqrt{k^2x^2-1}\tanh^{-1}\left(\frac{\sqrt{-((k-1)k)}\sqrt{x^2-1}}{\sqrt{k-1}\sqrt{k^2x^2-1}}\right)+\sqrt{k-1}\sqrt{-((k-1)k)}\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi\left(k;\sin^{-1}(x)|k^2\right)\right)}{\sqrt{k-1}\sqrt{-((k-1)k)}\sqrt{(x^2-1)(k^2x^2-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[k]*x)/((1 + Sqrt[k]*x)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),
x]
```

```
[Out] (Sqrt[-1 + k]*Sqrt[-((-1 + k)*k)]*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticF
[ArcSin[x], k^2] - 2*(Sqrt[k]*Sqrt[-1 + x^2]*Sqrt[-1 + k^2*x^2]*ArcTanh[(Sq
rt[-((-1 + k)*k)]*Sqrt[-1 + x^2])/(Sqrt[-1 + k]*Sqrt[-1 + k^2*x^2])] + Sqrt
[-1 + k]*Sqrt[-((-1 + k)*k)]*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[k,
ArcSin[x], k^2]))/(Sqrt[-1 + k]*Sqrt[-((-1 + k)*k)]*Sqrt[(-1 + x^2)*(-1 + k
^2*x^2)])
```

IntegrateAlgebraic [A] time = 3.39, size = 54, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2+2\sqrt{k}\,x+1}\right)}{k-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + Sqrt[k]*x)/((1 + Sqrt[k]*x)*Sqrt[(1 - x^2)*(1 - k^
2*x^2)]),x]
```

```
[Out] (-2*ArcTan[((-1 + k)*x)/(1 + 2*Sqrt[k]*x + k*x^2 + Sqrt[1 + (-1 - k^2)*x^2
+ k^2*x^4])])/(-1 + k)
```

fricas [B] time = 0.64, size = 100, normalized size = 1.85

$$\frac{\arctan\left(-\frac{\sqrt{k^2x^4-\left(k^2+1\right)x^2+1}\left(\left(k^3+k^2-k-1\right)x-2\left(\left(k^2-k\right)x^2+k-1\right)\sqrt{k}\right)}{4\,k^3x^4-\left(k^4+4\,k^3-2\,k^2+4\,k+1\right)x^2+4\,k}\right)}{k-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+k^(1/2)*x)/(1+k^(1/2)*x)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, alg
orithm="fricas")
```

```
[Out] arctan(-sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)*((k^3 + k^2 - k - 1)*x - 2*((k^2
- k)*x^2 + k - 1)*sqrt(k))/(4*k^3*x^4 - (k^4 + 4*k^3 - 2*k^2 + 4*k + 1)*x^2
 + 4*k))/(k - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{k}\,x-1}{\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}\left(\sqrt{k}\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+k^(1/2)*x)/(1+k^(1/2)*x)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((sqrt(k)*x - 1)/(sqrt((k^2*x^2 - 1)*(x^2 - 1))*(sqrt(k)*x + 1)),
x)
```

maple [C] time = 0.13, size = 221, normalized size = 4.09

$$\frac{\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\ \operatorname{EllipticF}(x,k)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}+\frac{\ln\left(\frac{-2k^2+4k-2+\left(-k^3+2k^2-k\right)\left(x^2-\frac{1}{k}\right)+2\sqrt{-(-1+k)^2}\,\sqrt{\left(x^2-\frac{1}{k}\right)^2k^3+\left(-k^3+2k^2-k\right)\left(x^2-\frac{1}{k}\right)-k^2+2k-1}}{x^2-\frac{1}{k}}\right)}{\sqrt{-\left(-1+k\right)^2}}-\frac{2\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\ \operatorname{EllipticPi}(x,k,k)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+k^(1/2)*x)/(1+k^(1/2)*x)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x)
```

```
[Out] (-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*EllipticF(x
,k)+1/(-(-1+k)^2)^(1/2)*ln((-2*k^2+4*k-2+(-k^3+2*k^2-k)*(x^2-1/k)+2*(-(-1+k
)^2)^(1/2)*((x^2-1/k)^2*k^3+(-k^3+2*k^2-k)*(x^2-1/k)-k^2+2*k-1)^(1/2))/(x^2
-1/k))-2*(-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*El
lipticPi(x,k,k)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{k}\,x-1}{\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}\left(\sqrt{k}\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+k^(1/2)*x)/(1+k^(1/2)*x)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((sqrt(k)*x - 1)/(sqrt((k^2*x^2 - 1)*(x^2 - 1))*(sqrt(k)*x + 1)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{k}\ x-1}{\left(\sqrt{k}\ x+1\right)\sqrt{\left(x^2-1\right)\left(k^2\,x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^(1/2)*x - 1)/((k^(1/2)*x + 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)),x)
```

```
[Out] int((k^(1/2)*x - 1)/((k^(1/2)*x + 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{k}\,x-1}{\sqrt{(x-1)\,(x+1)\,(kx-1)\,(kx+1)}\left(\sqrt{k}\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+k**(1/2)*x)/(1+k**(1/2)*x)/((-x**2+1)*(-k**2*x**2+1))**(1/2),
x)
```

```
[Out] Integral((sqrt(k)*x - 1)/(sqrt((x - 1)*(x + 1)*(k*x - 1)*(k*x + 1))*(sqrt(k
)*x + 1)), x)
```

3.679 $\int \frac{1+\sqrt{k}x}{\left(-1+\sqrt{k}x\right)\sqrt{(1-x^2)(1-k^2x^2)}}\,dx$

Optimal. Leaf size=54

$$-\frac{2\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}+kx^2-2\sqrt{k}x+1}\right)}{k-1}$$

Rubi [C] time = 1.17, antiderivative size = 199, normalized size of antiderivative = 3.69, number of steps used = 9, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {6719, 6742, 419, 2113, 537, 571, 93, 205}

$$\frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\tan^{-1}\left(\frac{\sqrt{k}\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{(1-k)\sqrt{(1-x^2)(1-k^2x^2)}}+\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}-\frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\Pi\left(k;\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[k]*x)/((-1 + Sqrt[k]*x)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),x]
```

```
[Out] (2*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*ArcTan[(Sqrt[k]*Sqrt[1 - x^2])/Sqrt[1 -
k^2*x^2]])/((1 - k)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) + (Sqrt[1 - x^2]*Sqrt[1
- k^2*x^2]*EllipticF[ArcSin[x], k^2])/Sqrt[(1 - x^2)*(1 - k^2*x^2)] - (2*Sq
rt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[k, ArcSin[x], k^2])/Sqrt[(1 - x^2)
*(1 - k^2*x^2)]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
```

```
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1+\sqrt{k}x}{\left(-1+\sqrt{k}x\right)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}\,dx = \frac{\left(\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \frac{1+\sqrt{k}x}{\left(-1+\sqrt{k}x\right)\sqrt{1-x^2}\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \left(\frac{1}{\sqrt{1-x^2}\sqrt{1-k^2x^2}}+\frac{2}{\left(-1+\sqrt{k}x\right)\sqrt{1-x^2}\sqrt{1-k^2x^2}}\right)dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \frac{1}{\sqrt{1-x^2}\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(2\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \frac{}{(-}}{\sqrt{\left(1-x^2\right)(1}}$$

$$= \frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(2\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \frac{}{\sqrt{1-x^2}(1-}}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right.}}$$

$$= \frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi\left(k;\sin^{-1}(x\right.}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi\left(k;\sin^{-1}(x\right.}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\tan^{-1}\left(\frac{\sqrt{k}\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{(1-k)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x\right.}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

Mathematica [C] time = 0.24, size = 208, normalized size = 3.85

$$\frac{2\sqrt{k}\sqrt{x^2-1}\sqrt{k^2x^2-1}\tanh^{-1}\left(\frac{\sqrt{-((k-1)k)}\sqrt{x^2-1}}{\sqrt{k-1}\sqrt{k^2x^2-1}}\right)+\sqrt{k-1}\sqrt{-((k-1)k)}\sqrt{1-x^2}\sqrt{1-k^2x^2}F\left(\sin^{-1}(x)|k^2\right)-2\sqrt{k-1}\sqrt{-((k-1)k)}\sqrt{1-x^2}\sqrt{1-k^2x^2}\Pi\left(k;\sin^{-1}(x)|k^2\right)}{\sqrt{k-1}\sqrt{-((k-1)k)}\sqrt{\left(x^2-1\right)\left(k^2x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[k]*x)/((-1 + Sqrt[k]*x)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),
x]
```

```
[Out] (2*Sqrt[k]*Sqrt[-1 + x^2]*Sqrt[-1 + k^2*x^2]*ArcTanh[(Sqrt[-((-1 + k)*k)]*S
qrt[-1 + x^2])/(Sqrt[-1 + k]*Sqrt[-1 + k^2*x^2])] + Sqrt[-1 + k]*Sqrt[-((-1
 + k)*k)]*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticF[ArcSin[x], k^2] - 2*Sqr
t[-1 + k]*Sqrt[-((-1 + k)*k)]*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[k,
 ArcSin[x], k^2])/(Sqrt[-1 + k]*Sqrt[-((-1 + k)*k)]*Sqrt[(-1 + x^2)*(-1 + k
^2*x^2)])
```

IntegrateAlgebraic [A] time = 3.39, size = 54, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2-2\sqrt{k}x+1}\right)}{k-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[k]*x)/((-1 + Sqrt[k]*x)*Sqrt[(1 - x^2)*(1 - k^
2*x^2)]),x]
```

```
[Out] (-2*ArcTan[((-1 + k)*x)/(1 - 2*Sqrt[k]*x + k*x^2 + Sqrt[1 + (-1 - k^2)*x^2
+ k^2*x^4])])/(-1 + k)
```

fricas [B] time = 0.66, size = 100, normalized size = 1.85

$$-\frac{\arctan\left(\frac{\sqrt{k^2x^4-\left(k^2+1\right)x^2+1}\left(\left(k^3+k^2-k-1\right)x+2\left(\left(k^2-k\right)x^2+k-1\right)\sqrt{k}\right)}{4k^3x^4-\left(k^4+4k^3-2k^2+4k+1\right)x^2+4k}\right)}{k-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+k^(1/2)*x)/(-1+k^(1/2)*x)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -arctan(sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)*((k^3 + k^2 - k - 1)*x + 2*((k^2
- k)*x^2 + k - 1)*sqrt(k))/(4*k^3*x^4 - (k^4 + 4*k^3 - 2*k^2 + 4*k + 1)*x^2
 + 4*k))/(k - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{k}x+1}{\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}\left(\sqrt{k}x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+k^(1/2)*x)/(-1+k^(1/2)*x)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((sqrt(k)*x + 1)/(sqrt((k^2*x^2 - 1)*(x^2 - 1))*(sqrt(k)*x - 1)),
x)
```

maple [C] time = 0.10, size = 222, normalized size = 4.11

$$\frac{\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\operatorname{EllipticF}(x,k)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}-\frac{\ln\left(\frac{-2k^2+4k-2+\left(-k^3+2k^2-k\right)\left(x^2-\frac{1}{k}\right)+2\sqrt{-(-1+k)^2}\,\sqrt{\left(x^2-\frac{1}{k}\right)^2k^3+\left(-k^3+2k^2-k\right)\left(x^2-\frac{1}{k}\right)-k^2+2k-1}}{x^2-\frac{1}{k}}\right)}{\sqrt{-(-1+k)^2}}-\frac{2\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\operatorname{EllipticPi}(x,k,k)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+k^(1/2)*x)/(-1+k^(1/2)*x)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x)
```

```
[Out] (-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*EllipticF(x
,k)-1/(-(-1+k)^2)^(1/2)*ln((-2*k^2+4*k-2+(-k^3+2*k^2-k)*(x^2-1/k)+2*(-(-1+k
)^2)^(1/2)*((x^2-1/k)^2*k^3+(-k^3+2*k^2-k)*(x^2-1/k)-k^2+2*k-1)^(1/2))/(x^2
-1/k))-2*(-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*El
lipticPi(x,k,k)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{k}\,x+1}{\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}\left(\sqrt{k}\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+k^(1/2)*x)/(-1+k^(1/2)*x)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((sqrt(k)*x + 1)/(sqrt((k^2*x^2 - 1)*(x^2 - 1))*(sqrt(k)*x - 1)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{k}\;x+1}{\left(\sqrt{k}\;x-1\right)\,\sqrt{\left(x^2-1\right)\,\left(k^2\,x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^(1/2)*x + 1)/((k^(1/2)*x - 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)),x)
```

```
[Out] int((k^(1/2)*x + 1)/((k^(1/2)*x - 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{k}\,x+1}{\sqrt{(x-1)\,(x+1)\,(kx-1)\,(kx+1)}\,\left(\sqrt{k}\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+k**(1/2)*x)/(-1+k**(1/2)*x)/((-x**2+1)*(-k**2*x**2+1))**(1/2),
x)
```

```
[Out] Integral((sqrt(k)*x + 1)/(sqrt((x - 1)*(x + 1)*(k*x - 1)*(k*x + 1))*(sqrt(k
)*x - 1)), x)
```

3.680 $\int \frac{2+x}{(-1+x)\sqrt{-1+3x-ax^2+x^3}}\,dx$

Optimal. Leaf size=54

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a-3}\,x\sqrt{-ax^2+x^3+3x-1}}{ax^2-x^3-3x+1}\right)}{\sqrt{a-3}}$$

Rubi [C] time = 137.71, antiderivative size = 5514, normalized size of antiderivative = 102.11, number of steps used = 13, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6742, 2067, 2066, 718, 419, 2081, 2080, 934, 169, 538, 537}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(2 + x)/((-1 + x)*Sqrt[-1 + 3*x - a*x^2 + x^3]),x]
```

```
[Out] (2^(1/3)*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a
 + 2*a^3 + 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(2/3) + 4*a^2*(27 - 27*a
+ 2*a^3 + 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(2/3) - 2^(1/3)*(27 - 27*a
 + 2*a^3 + 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(4/3)]*Sqrt[(-18*2^(1/3)
+ 2*2^(1/3)*a^2 + 2*a*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 +
4*a)])^(1/3) + (54 - 54*a + 4*a^3 + 6*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^
(2/3) - 6*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(1/3)
*x)/(-54 + 6*a^2 + 3*2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(-3 + a)^2
*(15 + 4*a)])^(2/3) + 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 -
2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a
)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)
])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a
)])^(4/3)])]*Sqrt[-((2*(9 - a^2) + 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*S
qrt[(3 - a)^2*(15 + 4*a)])^(2/3) + (2*(9 - a^2)^2)/((-27*a)/2 + a^3 + (3*(9
 + Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)]))/2)^(2/3) + 18*(-1/3*a + x)^2 - (2^(
1/3)*(18 - 2*a^2 - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(1
5 + 4*a)])^(2/3))*(-a + 3*x))/(-27*a + 2*a^3 + 3*(9 + Sqrt[3]*Sqrt[(3 - a)^
2*(15 + 4*a)]))^(1/3))/((18 - 2*a^2 - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3
]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3))^2/(18*2^(1/3)*(-27*a + 2*a^3 + 3*(9 +
Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)]))^(2/3)) - (2*(2*(9 - a^2) + 2^(1/3)*(27
 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) + (2*(9 - a^2
)^2)/((-27*a)/2 + a^3 + (3*(9 + Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)]))/2)^(2/
3)))/9))]*EllipticF[ArcSin[Sqrt[(-18*2^(1/3) + 2*2^(1/3)*a^2 - 4*a*(27 - 27
*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(1/3) + (54 - 54*a + 4*
a^3 + 6*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(2/3) + Sqrt[6]*Sqrt[-162*2^(2
/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sq
rt[(-3 + a)^2*(15 + 4*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqr
t[(-3 + a)^2*(15 + 4*a)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sq
rt[(-3 + a)^2*(15 + 4*a)])^(4/3)] + 12*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[
(-3 + a)^2*(15 + 4*a)])^(1/3)*x)/Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(
2/3)*a^4 - 36*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(
2/3) + 4*a^2*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(2
/3) - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(-3 + a)^2*(15 + 4*a)])^(
4/3)]]/(2^(3/4)*3^(1/4))], (-2*2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/
3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(
15 + 4*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15
 + 4*a)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15
 + 4*a)])^(4/3)])/(54 - 6*a^2 - 3*2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sq
rt[(3 - a)^2*(15 + 4*a)])^(2/3) - 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^
(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^
2*(15 + 4*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*
(15 + 4*a)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*
```

(15 + 4*a)])^(4/3)])])/(3*Sqrt[3]*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(-3 +
a)^2*(15 + 4*a)])^(1/3)*Sqrt[-1 + 3*x - a*x^2 + x^3]) - (3*2^(2/3)*Sqrt[54
- 6*a^2 - 3*2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*
a)])^(2/3) + 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)
a^4 - 36(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3)
+ 4*a^2*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) -
2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(4/3)]]*
Sqrt[1 - (2*(18 - 2*a^2 - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 -
a)^2*(15 + 4*a)])^(2/3) - 2^(2/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 -
a)^2*(15 + 4*a)])^(1/3)*(a - 3*x)))/(54 - 6*a^2 - 3*2^(1/3)*(27 - 27*a + 2*
a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) - 2^(1/6)*Sqrt[3]*Sqrt[-1
62*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 + 3*Sqr
t[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 + 3*Sqrt[
3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[
3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(4/3)])]*Sqrt[1 - (2*(18 - 2*a^2 - 2^(1/3)*(
27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) - 2^(2/3)*(
27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(1/3)*(a - 3*x)))
/(54 - 6*a^2 - 3*2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15
+ 4*a)])^(2/3) + 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(
2/3)*a^4 - 36*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2
/3) + 4*a^2*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3
) - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(4/3
)])]*Sqrt[-((2^(1/3)*(18 - 2*a^2 - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*S
qrt[(3 - a)^2*(15 + 4*a)])^(2/3)) - Sqrt[6]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*
a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15
+ 4*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 +
4*a)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 +
4*a)])^(4/3)] + 4*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)]
)^(1/3)*(a - 3*x))/(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)
])^(1/3))]*Sqrt[-((2^(1/3)*(18 - 2*a^2 - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqr
t[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3)) + Sqrt[6]*Sqrt[-162*2^(2/3) + 36*2^
(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^
2*(15 + 4*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*
(15 + 4*a)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*
(15 + 4*a)])^(4/3)] + 4*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 +
4*a)])^(1/3)*(a - 3*x))/(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15
+ 4*a)])^(1/3))]*Sqrt[-2*a + (18 - 2*a^2 - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*S
qrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3))/((-27*a)/2 + a^3 + (3*(9 + Sqrt[3
]*Sqrt[(3 - a)^2*(15 + 4*a)]))/2)^(1/3) + 6*x]*EllipticPi[(54 - 6*a^2 - 3*2
^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) + 2
^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27
- 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) + 4*a^2*(27 -
27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) - 2^(1/3)*(27 -
27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(4/3)])/(2*(18 - 2*a^2
+ 3*2^(2/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(1/
3) - 2^(2/3)*a*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(
1/3) - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(
2/3))), ArcSin[(2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 +
4*a)])^(1/6)*Sqrt[-2*a + (18 - 2*a^2 - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt
[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3))/((-27*a)/2 + a^3 + (3*(9 + Sqrt[3]*S
qrt[(3 - a)^2*(15 + 4*a)]))/2)^(1/3) + 6*x])/Sqrt[54 - 6*a^2 - 3*2^(1/3)*(2
7 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) + 2^(1/6)*Sq
rt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a +
2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*
a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*
a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(4/3)]]], (54 - 6*a^2 - 3*2^(1/
3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) + 2^(1/
6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27
*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) + 4*a^2*(27 - 27*a

```
 + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) - 2^(1/3)*(27 - 27*a
 + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(4/3)])/(54 - 6*a^2 - 3*2^
(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) - 2^
(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 -
 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) + 4*a^2*(27 - 2
7*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) - 2^(1/3)*(27 - 2
7*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(4/3)]])/((-27*a + 2*a
^3 + 3*(9 + Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)]))^(1/6)*(6 - 2*a + (18 - 2*a
^2 - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(2/
3))/((-27*a)/2 + a^3 + (3*(9 + Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)]))/2)^(1/3
))*Sqrt[-((18*2^(1/3) - 2*2^(1/3)*a^2 - (54 - 54*a + 4*a^3 + 6*Sqrt[3]*Sqrt
[(3 - a)^2*(15 + 4*a)])^(2/3) - Sqrt[6]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2
- 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*
a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)
])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)
])^(4/3)] + 4*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(1
/3)*(a - 3*x))/(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)])^(
1/3))]*Sqrt[-((18*2^(1/3) - 2*2^(1/3)*a^2 - (54 - 54*a + 4*a^3 + 6*Sqrt[3]*
Sqrt[(3 - a)^2*(15 + 4*a)])^(2/3) + Sqrt[6]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*
a^2 - 2*2^(2/3)*a^4 - 36*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15
+ 4*a)])^(2/3) + 4*a^2*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 +
4*a)])^(2/3) - 2^(1/3)*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 +
4*a)])^(4/3)] + 4*(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)]
)^(1/3)*(a - 3*x))/(27 - 27*a + 2*a^3 + 3*Sqrt[3]*Sqrt[(3 - a)^2*(15 + 4*a)
])^(1/3))]*Sqrt[-1 + 3*x - a*x^2 + x^3])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
```

```
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 2066

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> With[{r = Rt[-9*
a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}, Dist[(a + b*x + d*x^3)^p
/(Simp[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d*x, x]^p*Simp[(b*d)/3 + (12^(1/
3)*b^2*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*((2^(1/3)*b*d)/(3^(1/3)*r) - r/1
8^(1/3))*x + d^2*x^2, x]^p), Int[Simp[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d
*x, x]^p*Simp[(b*d)/3 + (12^(1/3)*b^2*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*(
(2^(1/3)*b*d)/(3^(1/3)*r) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ
[{a, b, d, p}, x] && NeQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 2080

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Dist[(a + b*x + d*x^3)^p/(Simp[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d*x, x
]^p*Simp[(b*d)/3 + (12^(1/3)*b^2*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*((2^(1
/3)*b*d)/(3^(1/3)*r) - r/18^(1/3))*x + d^2*x^2, x]^p), Int[(e + f*x)^m*Simp
[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d*x, x]^p*Simp[(b*d)/3 + (12^(1/3)*b^2
*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*((2^(1/3)*b*d)/(3^(1/3)*r) - r/18^(1/3
))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*b
^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 2081

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{2+x}{(-1+x)\sqrt{-1+3x-ax^2+x^3}}\,dx = \int \left(\frac{1}{\sqrt{-1+3x-ax^2+x^3}}+\frac{3}{(-1+x)\sqrt{-1+3x-ax^2+x^3}}\right)dx$$
$$= 3\int \frac{1}{(-1+x)\sqrt{-1+3x-ax^2+x^3}}\,dx+\int \frac{1}{\sqrt{-1+3x-ax^2+x^3}}\,dx$$
$$= 3\,\text{Subst}\left(\int \frac{1}{\left(\frac{1}{3}(-3+a)+x\right)\sqrt{\frac{1}{27}\left(-27+27a-2a^3\right)+\frac{1}{3}\left(9-a^2\right)x+x^3}}\right.$$
= rest of steps removed due to Latex formating problem

Mathematica [C] time = 1.05, size = 1148, normalized size = 21.26

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + x)/((-1 + x)*Sqrt[-1 + 3*x - a*x^2 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x + Root[3 - a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 1])/(R
oot[3 - a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 1] - Root[3 - a + (6 - 2
*a)*#1 + (3 - a)*#1^2 + #1^3 & , 3])]*((3*EllipticPi[1 - Root[3 - a + (6 -
2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 2]/Root[3 - a + (6 - 2*a)*#1 + (3 - a)*#1
^2 + #1^3 & , 3], ArcSin[Sqrt[(1 - x + Root[3 - a + (6 - 2*a)*#1 + (3 - a)*
#1^2 + #1^3 & , 3])/(-Root[3 - a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 2
] + Root[3 - a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 3])]], (Root[3 - a
+ (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 2] - Root[3 - a + (6 - 2*a)*#1 + (
3 - a)*#1^2 + #1^3 & , 3])/(Root[3 - a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3
 & , 1] - Root[3 - a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 3])]*Sqrt[-((
(-1 + x - Root[3 - a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 2])*(-1 + x -
 Root[3 - a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 3]))/(Root[3 - a + (6
- 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 2] - Root[3 - a + (6 - 2*a)*#1 + (3 - a
)*#1^2 + #1^3 & , 3])^2)]*(Root[3 - a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3
& , 2] - Root[3 - a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 3]))/Root[3 -
a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 3] + (EllipticF[ArcSin[Sqrt[(1 -
 x + Root[3 - a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 3])/(-Root[3 - a +
 (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 2] + Root[3 - a + (6 - 2*a)*#1 + (3
 - a)*#1^2 + #1^3 & , 3])]], (Root[3 - a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1
^3 & , 2] - Root[3 - a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 3])/(Root[3
 - a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 1] - Root[3 - a + (6 - 2*a)*#
1 + (3 - a)*#1^2 + #1^3 & , 3])]*(-1 + x - Root[3 - a + (6 - 2*a)*#1 + (3 -
 a)*#1^2 + #1^3 & , 3])*Sqrt[(1 - x + Root[3 - a + (6 - 2*a)*#1 + (3 - a)*#
1^2 + #1^3 & , 2])/(Root[3 - a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 2]
- Root[3 - a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 3])])/Sqrt[(1 - x + R
oot[3 - a + (6 - 2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 3])/(-Root[3 - a + (6 -
2*a)*#1 + (3 - a)*#1^2 + #1^3 & , 2] + Root[3 - a + (6 - 2*a)*#1 + (3 - a)*
#1^2 + #1^3 & , 3])]))/Sqrt[-1 + 3*x - a*x^2 + x^3]
```

IntegrateAlgebraic [A] time = 0.13, size = 54, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a-3}\,x\sqrt{-ax^2+x^3+3x-1}}{ax^2-x^3-3x+1}\right)}{\sqrt{a-3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)/((-1 + x)*Sqrt[-1 + 3*x - a*x^2 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + a]*x*Sqrt[-1 + 3*x - a*x^2 + x^3])/(1 - 3*x + a*x^2 -
x^3)])/Sqrt[-3 + a]
```

fricas [A] time = 0.49, size = 234, normalized size = 4.33

$$\left[-\frac{\sqrt{-a+3}\log\left(-\frac{2\,(4\,a-9)x^5-x^6-\left(8\,a^2-24\,a+15\right)x^4+4\,(6\,a-13)x^3-(8\,a-9)x^2-4\left((2\,a-3)x^3-x^4-3\,x^2+x\right)\sqrt{-ax^2+x^3+3\,x-1}\,\sqrt{-a+3}+6\,x-1}{x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1}\right)}{2\,(a-3)},\ \frac{\arctan\left(-\frac{\left((2\,a-3)x^2-x^3-3\,x+1\right)\sqrt{-ax^2+x^3+3\,x-1}\,\sqrt{a-3}}{2\left((a-3)x^4-\left(a^2-3\,a\right)x^3+3\,(a-3)x^2-(a-3)x\right)}\right)}{\sqrt{a-3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-1+x)/(-a*x^2+x^3+3*x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a + 3)*log(-(2*(4*a - 9)*x^5 - x^6 - (8*a^2 - 24*a + 15)*x^4 +
4*(6*a - 13)*x^3 - (8*a - 9)*x^2 - 4*((2*a - 3)*x^3 - x^4 - 3*x^2 + x)*sqrt
(-a*x^2 + x^3 + 3*x - 1)*sqrt(-a + 3) + 6*x - 1)/(x^6 - 6*x^5 + 15*x^4 - 20
*x^3 + 15*x^2 - 6*x + 1))/(a - 3), arctan(-1/2*((2*a - 3)*x^2 - x^3 - 3*x +
 1)*sqrt(-a*x^2 + x^3 + 3*x - 1)*sqrt(a - 3)/((a - 3)*x^4 - (a^2 - 3*a)*x^3
 + 3*(a - 3)*x^2 - (a - 3)*x))/sqrt(a - 3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\sqrt{-ax^2+x^3+3\,x-1}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-1+x)/(-a*x^2+x^3+3*x-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 2)/(sqrt(-a*x^2 + x^3 + 3*x - 1)*(x - 1)), x)
```

maple [C] time = 0.70, size = 3008, normalized size = 55.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-1+x)/(-a*x^2+x^3+3*x-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1
/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/
3))*(-I*(x+1/12*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)
-3*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)-
1/3*a+1/2*I*3^(1/2)*(1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/
2))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2
))^(1/3)))*3^(1/2)/(1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2
))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2)
)^(1/3)))^(1/2)*((x-1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2
))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2)
)^(1/3)-1/3*a)/(-1/4*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^
(1/3)+9*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(
1/3)-1/2*I*3^(1/2)*(1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2
))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2)
)^(1/3))))^(1/2)*(I*(x+1/12*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^
(1/2))^(1/3)-3*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(
1/2))^(1/3)-1/3*a-1/2*I*3^(1/2)*(1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-16
2*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162
*a+405)^(1/2))^(1/3)))*3^(1/2)/(1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162
*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*
a+405)^(1/2))^(1/3)))^(1/2)/(-a*x^2+x^3+3*x-1)^(1/2)*EllipticF(1/3*3^(1/2)*
(-I*(x+1/12*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)-3*(
1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)-1/3*
```

```
a+1/2*I*3^(1/2)*(1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^
(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(
1/3)))*3^(1/2)/(1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(
1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1
/3)))^(1/2),(-I*3^(1/2)*(1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)
^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^
(1/2))^(1/3))/(-1/4*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(
1/3)+9*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1
/3)-1/2*I*3^(1/2)*(1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2)
)^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))
^(1/3))))^(1/2))+2*I*3^(1/2)*(1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a
+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+
405)^(1/2))^(1/3))*(-I*(x+1/12*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+40
5)^(1/2))^(1/3)-3*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405
)^(1/2))^(1/3)-1/3*a+1/2*I*3^(1/2)*(1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2
-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-
162*a+405)^(1/2))^(1/3)))*3^(1/2)/(1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-
162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-1
62*a+405)^(1/2))^(1/3)))^(1/2)*((x-1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-
162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-1
62*a+405)^(1/2))^(1/3)-1/3*a)/(-1/4*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162
*a+405)^(1/2))^(1/3)+9*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*
a+405)^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-
162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-1
62*a+405)^(1/2))^(1/3))))^(1/2)*(I*(x+1/12*(-108*a+108+8*a^3+12*(12*a^3-27*
a^2-162*a+405)^(1/2))^(1/3)-3*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a
^2-162*a+405)^(1/2))^(1/3)-1/3*a-1/2*I*3^(1/2)*(1/6*(-108*a+108+8*a^3+12*(1
2*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12
*a^3-27*a^2-162*a+405)^(1/2))^(1/3)))*3^(1/2)/(1/6*(-108*a+108+8*a^3+12*(12
*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*
a^3-27*a^2-162*a+405)^(1/2))^(1/3)))^(1/2)/(-a*x^2+x^3+3*x-1)^(1/2)/(-1/12*
(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+3*(1-1/9*a^2)/(
-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+1/3*a-1/2*I*3^(1
/2)*(1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1
/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3))-1)*Ell
ipticPi(1/3*3^(1/2)*(-I*(x+1/12*(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+4
05)^(1/2))^(1/3)-3*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-162*a+40
5)^(1/2))^(1/3)-1/3*a+1/2*I*3^(1/2)*(1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^
2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2
-162*a+405)^(1/2))^(1/3)))*3^(1/2)/(1/6*(-108*a+108+8*a^3+12*(12*a^3-27*a^2
-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-
162*a+405)^(1/2))^(1/3)))^(1/2),-I*3^(1/2)*(1/6*(-108*a+108+8*a^3+12*(12*a^
3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3
-27*a^2-162*a+405)^(1/2))^(1/3))/(-1/12*(-108*a+108+8*a^3+12*(12*a^3-27*a^2
-162*a+405)^(1/2))^(1/3)+3*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27*a^2-
162*a+405)^(1/2))^(1/3)+1/3*a-1/2*I*3^(1/2)*(1/6*(-108*a+108+8*a^3+12*(12*a
^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^
3-27*a^2-162*a+405)^(1/2))^(1/3))-1),(-I*3^(1/2)*(1/6*(-108*a+108+8*a^3+12*
(12*a^3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(
12*a^3-27*a^2-162*a+405)^(1/2))^(1/3))/(-1/4*(-108*a+108+8*a^3+12*(12*a^3-2
7*a^2-162*a+405)^(1/2))^(1/3)+9*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3-27
*a^2-162*a+405)^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/6*(-108*a+108+8*a^3+12*(12*a^
3-27*a^2-162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a+108+8*a^3+12*(12*a^3
-27*a^2-162*a+405)^(1/2))^(1/3))))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\sqrt{-ax^2+x^3+3x-1}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-1+x)/(-a*x^2+x^3+3*x-1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + 2)/(sqrt(-a*x^2 + x^3 + 3*x - 1)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x+2}{(x-1)\,\sqrt{x^3-a\,x^2+3\,x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((x - 1)*(3*x - a*x^2 + x^3 - 1)^(1/2)),x)
[Out] int((x + 2)/((x - 1)*(3*x - a*x^2 + x^3 - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{(x-1)\sqrt{-ax^2+x^3+3x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-1+x)/(-a*x**2+x**3+3*x-1)**(1/2),x)
[Out] Integral((x + 2)/((x - 1)*sqrt(-a*x**2 + x**3 + 3*x - 1)), x)
```

3.681 $\int \frac{-2+x}{(1+x)\sqrt{1+3x-ax^2+x^3}}\,dx$

Optimal. Leaf size=54

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a+3}\,x\sqrt{-ax^2+x^3+3x+1}}{ax^2-x^3-3x-1}\right)}{\sqrt{a+3}}$$

Rubi [C] time = 133.56, antiderivative size = 5395, normalized size of antiderivative = 99.91, number of steps used = 13, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6742, 2067, 2066, 718, 419, 2081, 2080, 934, 169, 538, 537}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(-2 + x)/((1 + x)*Sqrt[1 + 3*x - a*x^2 + x^3]),x]
```

```
[Out] (2^(1/3)*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a
 - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) + 4*a^2*(27 + 27
*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) - 2^(1/3)*(27
+ 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(4/3)]*Sqrt[(-18
*2^(1/3) + 2*2^(1/3)*a^2 + (54 + 54*a - 4*a^3 - 6*Sqrt[3]*Sqrt[-((3 + a)^2*
(-15 + 4*a))])^(2/3) - 2*a*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*
(-15 + 4*a))])^(1/3) + 6*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-
15 + 4*a))])^(1/3)*x)/(-54 + 6*a^2 + 3*2^(1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[
3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) - 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3
) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt
[-((3 + a)^2*(-15 + 4*a))])^(2/3) + 4*a^2*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sq
rt[-((3 + a)^2*(-15 + 4*a))])^(2/3) - 2^(1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3
]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(4/3)])]*Sqrt[-((2*(9 - a^2) + (2*2^(2/3)
*(9 - a^2)^2)/(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2
/3) + 2^(1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2
/3) + 18*(-1/3*a + x)^2 + (2^(1/3)*(18 - 2*a^2 - 2^(1/3)*(27 + 27*a - 2*a^3
 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3))*(-a + 3*x))/(27 + 27*a - 2*
a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(1/3))/((18 - 2*a^2 - 2^(1/3)*(
27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3))^2/(18*2^(1
/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3)) - (2*
(2*(9 - a^2) + (2*2^(2/3)*(9 - a^2)^2)/(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[
(15 - 4*a)*(3 + a)^2])^(2/3) + 2^(1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[
(15 - 4*a)*(3 + a)^2])^(2/3)))/9))]*EllipticF[ArcSin[Sqrt[(18*2^(1/3) - 2*2
^(1/3)*a^2 - (54 + 54*a - 4*a^3 - 6*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])
^(2/3) - 4*a*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])
^(1/3) + Sqrt[6]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(2
7 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) + 4*a^2*
(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) - 2^(1
/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(4/3)] +
 12*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(1/3)*x)
/Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*a^3
 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) + 4*a^2*(27 + 27*a - 2*a
^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) - 2^(1/3)*(27 + 27*a -
 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(4/3)]]/(2^(3/4)*3^(1/4)
)], (2*2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 -
 36*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4*a^
2*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/3
)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)])/(54 -
6*a^2 - 3*2^(1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2]
)^(2/3) + 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^
4 - 36*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4
*a^2*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(
```

```
1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)])])/(
3*Sqrt[3]*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(1
/3)*Sqrt[1 + 3*x - a*x^2 + x^3]) - (6*2^(1/6)*Sqrt[3]*Sqrt[-54 + 6*a^2 + 3*
2^(1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) +
2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27
 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4*a^2*(27 +
 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/3)*(27 +
 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)]]*Sqrt[1 - (2*(
-18 + 2*a^2 + 2^(1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15
+ 4*a))])^(2/3) - 2^(2/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(
-15 + 4*a))])^(1/3)*(a - 3*x)))/(-54 + 6*a^2 + 3*2^(1/3)*(27 + 27*a - 2*a^3
 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) - 2^(1/6)*Sqrt[3]*Sqrt[-
162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*a^3 - 3*Sq
rt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4*a^2*(27 + 27*a - 2*a^3 - 3*Sqrt
[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt
[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)])]*Sqrt[1 - (2*(-18 + 2*a^2 + 2^(1/3)
*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) - 2^(
2/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(1/3)*(
a - 3*x)))/(-54 + 6*a^2 + 3*2^(1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((
3 + a)^2*(-15 + 4*a))])^(2/3) + 2^(1/6)*Sqrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2
/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)
*(3 + a)^2])^(2/3) + 4*a^2*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(
3 + a)^2])^(2/3) - 2^(1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(
3 + a)^2])^(4/3)])]*Sqrt[(2^(1/3)*(18 - 2*a^2 - 2^(1/3)*(27 + 27*a - 2*a^3
 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3)))/(27 + 27*a - 2*a^3 - 3*Sqrt
[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(1/3) - (Sqrt[6]*Sqrt[-162*2^(2/3) + 36*2^(
2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)
^2*(-15 + 4*a))])^(2/3) + 4*a^2*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 +
a)^2*(-15 + 4*a))])^(2/3) - 2^(1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((
3 + a)^2*(-15 + 4*a))])^(4/3)])/(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 +
a)^2*(-15 + 4*a))])^(1/3) - 4*(a - 3*x)]*Sqrt[(2^(1/3)*(18 - 2*a^2 - 2^(1/3
)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3)))/(27 +
27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(1/3) + (Sqrt[6]*Sqrt[
-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*a^3 - 3*S
qrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) + 4*a^2*(27 + 27*a - 2*a^3 - 3
*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) - 2^(1/3)*(27 + 27*a - 2*a^3
 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(4/3)])/(27 + 27*a - 2*a^3 - 3
*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(1/3) - 4*(a - 3*x)]*Sqrt[-1/3*a +
 (-18 + 2*a^2 + 2^(1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 +
 a)^2])^(2/3))/(3*2^(2/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3
 + a)^2])^(1/3)) + x]*EllipticPi[(54 - 6*a^2 - 3*2^(1/3)*(27 + 27*a - 2*a^3
 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/6)*Sqrt[3]*Sqrt[-162*
2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*a^3 - 3*Sqrt[3
]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4*a^2*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*
Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*
Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)])/(2^(2/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]
*Sqrt[(15 - 4*a)*(3 + a)^2])^(1/3)*(6 + 2*a + (2^(1/3)*(18 - 2*a^2 - 2^(1/3
)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3)))/(27 +
27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(1/3))), ArcSin[(2^(5/
6)*Sqrt[3]*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(1/6)
*Sqrt[-1/3*a + (-18 + 2*a^2 + 2^(1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(
15 - 4*a)*(3 + a)^2])^(2/3))/(3*2^(2/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt
[(15 - 4*a)*(3 + a)^2])^(1/3)) + x])/Sqrt[-54 + 6*a^2 + 3*2^(1/3)*(27 + 27*
a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 2^(1/6)*Sqrt[3]*S
qrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*a^3 -
 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4*a^2*(27 + 27*a - 2*a^3 - 3
*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/3)*(27 + 27*a - 2*a^3 - 3
*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)]]], (54 - 6*a^2 - 3*2^(1/3)*(27
+ 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/6)*Sqrt
```

```
[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*
a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4*a^2*(27 + 27*a - 2*a^
3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/3)*(27 + 27*a - 2*a^
3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)])/(54 - 6*a^2 - 3*2^(1/3)*(
27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 2^(1/6)*S
qrt[3]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a -
 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) + 4*a^2*(27 + 27*a - 2
*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/3) - 2^(1/3)*(27 + 27*a - 2
*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4/3)])])/((27 + 27*a - 2*a^3
- 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(1/6)*(6 + 2*a + (2^(1/3)*(18 - 2*a
^2 - 2^(1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2/
3)))/(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(1/3))*Sqrt
[(18*2^(1/3) - 2*2^(1/3)*a^2 - (54 + 54*a - 4*a^3 - 6*Sqrt[3]*Sqrt[-((3 + a
)^2*(-15 + 4*a))])^(2/3) - Sqrt[6]*Sqrt[-162*2^(2/3) + 36*2^(2/3)*a^2 - 2*2
^(2/3)*a^4 - 36*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^
(2/3) + 4*a^2*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(2
/3) - 2^(1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(3 + a)^2])^(4
/3)] - 4*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))])^(1/
3)*(a - 3*x))/(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15 + 4*a))]
)^(1/3)]*Sqrt[(18*2^(1/3) - 2*2^(1/3)*a^2 - (54 + 54*a - 4*a^3 - 6*Sqrt[3]*
Sqrt[-((3 + a)^2*(-15 + 4*a))])^(2/3) + Sqrt[6]*Sqrt[-162*2^(2/3) + 36*2^(2
/3)*a^2 - 2*2^(2/3)*a^4 - 36*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)
*(3 + a)^2])^(2/3) + 4*a^2*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(
3 + a)^2])^(2/3) - 2^(1/3)*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[(15 - 4*a)*(
3 + a)^2])^(4/3)] - 4*(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*(-15
+ 4*a))])^(1/3)*(a - 3*x))/(27 + 27*a - 2*a^3 - 3*Sqrt[3]*Sqrt[-((3 + a)^2*
(-15 + 4*a))])^(1/3)]*Sqrt[1 + 3*x - a*x^2 + x^3])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 2066

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> With[{r = Rt[-9*
a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}, Dist[(a + b*x + d*x^3)^p
/(Simp[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d*x, x]^p*Simp[(b*d)/3 + (12^(1/
3)*b^2*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*((2^(1/3)*b*d)/(3^(1/3)*r) - r/1
8^(1/3))*x + d^2*x^2, x]^p), Int[Simp[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d
*x, x]^p*Simp[(b*d)/3 + (12^(1/3)*b^2*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*(
(2^(1/3)*b*d)/(3^(1/3)*r) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ
[{a, b, d, p}, x] && NeQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 2080

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Dist[(a + b*x + d*x^3)^p/(Simp[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d*x, x
]^p*Simp[(b*d)/3 + (12^(1/3)*b^2*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*((2^(1
/3)*b*d)/(3^(1/3)*r) - r/18^(1/3))*x + d^2*x^2, x]^p), Int[(e + f*x)^m*Simp
[(18^(1/3)*b*d)/(3*r) - r/18^(1/3) + d*x, x]^p*Simp[(b*d)/3 + (12^(1/3)*b^2
*d^2)/(3*r^2) + r^2/(3*12^(1/3)) - d*((2^(1/3)*b*d)/(3^(1/3)*r) - r/18^(1/3
))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*b
^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 2081

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
```

]

Rubi steps

$$\int \frac{-2+x}{(1+x)\sqrt{1+3x-ax^2+x^3}}\,dx = \int \left(\frac{1}{\sqrt{1+3x-ax^2+x^3}} - \frac{3}{(1+x)\sqrt{1+3x-ax^2+x^3}}\right)dx$$

$$= -\left(3\int \frac{1}{(1+x)\sqrt{1+3x-ax^2+x^3}}\,dx\right) + \int \frac{1}{\sqrt{1+3x-ax^2+x^3}}\,dx$$

$$= -\left(3\,\mathrm{Subst}\left(\int \frac{1}{\left(\frac{3+a}{3}+x\right)\sqrt{\frac{1}{27}\left(27+27a-2a^3\right)+\frac{1}{3}\left(9-a^2\right)x+x^3}}\,dx, x,\right.\right.$$

= rest of steps removed due to Latex formating problem

Mathematica [C] time = 0.80, size = 933, normalized size = 17.28

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x)/((1 + x)*Sqrt[1 + 3*x - a*x^2 + x^3]),x]
```

```
[Out] (2*Sqrt[-(((1 + x - Root[-3 - a + (6 + 2*a)*#1 + (-3 - a)*#1^2 + #1^3 & , 2
])*(1 + x - Root[-3 - a + (6 + 2*a)*#1 + (-3 - a)*#1^2 + #1^3 & , 3]))/(Roo
t[-3 - a + (6 + 2*a)*#1 + (-3 - a)*#1^2 + #1^3 & , 2] - Root[-3 - a + (6 +
2*a)*#1 + (-3 - a)*#1^2 + #1^3 & , 3])^2)]*(Root[-3 - a + (6 + 2*a)*#1 + (-
3 - a)*#1^2 + #1^3 & , 2] - Root[-3 - a + (6 + 2*a)*#1 + (-3 - a)*#1^2 + #1
^3 & , 3])*Sqrt[(1 + x - Root[-3 - a + (6 + 2*a)*#1 + (-3 - a)*#1^2 + #1^3
& , 1])/(-Root[-3 - a + (6 + 2*a)*#1 + (-3 - a)*#1^2 + #1^3 & , 1] + Root[-
3 - a + (6 + 2*a)*#1 + (-3 - a)*#1^2 + #1^3 & , 3])]*(-3*EllipticPi[1 - Roo
t[-3 - a + (6 + 2*a)*#1 + (-3 - a)*#1^2 + #1^3 & , 2]/Root[-3 - a + (6 + 2*
a)*#1 + (-3 - a)*#1^2 + #1^3 & , 3], ArcSin[Sqrt[(1 + x - Root[-3 - a + (6
+ 2*a)*#1 + (-3 - a)*#1^2 + #1^3 & , 3])/(Root[-3 - a + (6 + 2*a)*#1 + (-3
- a)*#1^2 + #1^3 & , 2] - Root[-3 - a + (6 + 2*a)*#1 + (-3 - a)*#1^2 + #1^3
 & , 3])]], (Root[-3 - a + (6 + 2*a)*#1 + (-3 - a)*#1^2 + #1^3 & , 2] - Roo
t[-3 - a + (6 + 2*a)*#1 + (-3 - a)*#1^2 + #1^3 & , 3])/(Root[-3 - a + (6 +
2*a)*#1 + (-3 - a)*#1^2 + #1^3 & , 1] - Root[-3 - a + (6 + 2*a)*#1 + (-3 -
a)*#1^2 + #1^3 & , 3])] + EllipticF[ArcSin[Sqrt[(1 + x - Root[-3 - a + (6 +
 2*a)*#1 + (-3 - a)*#1^2 + #1^3 & , 3])/(Root[-3 - a + (6 + 2*a)*#1 + (-3 -
 a)*#1^2 + #1^3 & , 2] - Root[-3 - a + (6 + 2*a)*#1 + (-3 - a)*#1^2 + #1^3
& , 3])]], (Root[-3 - a + (6 + 2*a)*#1 + (-3 - a)*#1^2 + #1^3 & , 2] - Root
[-3 - a + (6 + 2*a)*#1 + (-3 - a)*#1^2 + #1^3 & , 3])/(Root[-3 - a + (6 + 2
*a)*#1 + (-3 - a)*#1^2 + #1^3 & , 1] - Root[-3 - a + (6 + 2*a)*#1 + (-3 - a
)*#1^2 + #1^3 & , 3])]*Root[-3 - a + (6 + 2*a)*#1 + (-3 - a)*#1^2 + #1^3 &
, 3]))/(Sqrt[1 + 3*x - a*x^2 + x^3]*Root[-3 - a + (6 + 2*a)*#1 + (-3 - a)*#
1^2 + #1^3 & , 3])
```

IntegrateAlgebraic [A] time = 0.12, size = 54, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a+3}\,x\sqrt{-ax^2+x^3+3x+1}}{ax^2-x^3-3x-1}\right)}{\sqrt{a+3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x)/((1 + x)*Sqrt[1 + 3*x - a*x^2 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3 + a]*x*Sqrt[1 + 3*x - a*x^2 + x^3])/(-1 - 3*x + a*x^2 - x
^3)]])/Sqrt[3 + a]
```

fricas [A] time = 0.47, size = 234, normalized size = 4.33

$$\left[-\frac{\sqrt{-a-3}\log\left(-\frac{2\,(4\,a+9)x^5-x^6-\left(8\,a^2+24\,a+15\right)x^4+4\,(6\,a+13)x^3+(8\,a+9)x^2-4\left((2\,a+3)x^3-x^4-3\,x^2-x\right)\sqrt{-ax^2+x^3+3\,x+1}\,\sqrt{-a-3}-6\,x-1}{x^6+6\,x^5+15\,x^4+20\,x^3+15\,x^2+6\,x+1}\right)}{2\,(a+3)},\ \frac{\arctan\left(-\frac{\left((2\,a+3)x^2-x^3-3\,x-1\right)\sqrt{-ax^2+x^3+3\,x+1}\,\sqrt{a+3}}{2\left((a+3)x^4-\left(a^2+3\,a\right)x^3+3\,(a+3)x^2+(a+3)x\right)}\right)}{\sqrt{a+3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(1+x)/(-a*x^2+x^3+3*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a - 3)*log(-(2*(4*a + 9)*x^5 - x^6 - (8*a^2 + 24*a + 15)*x^4 +
4*(6*a + 13)*x^3 + (8*a + 9)*x^2 - 4*((2*a + 3)*x^3 - x^4 - 3*x^2 - x)*sqrt
(-a*x^2 + x^3 + 3*x + 1)*sqrt(-a - 3) - 6*x - 1)/(x^6 + 6*x^5 + 15*x^4 + 20
*x^3 + 15*x^2 + 6*x + 1))/(a + 3), arctan(-1/2*((2*a + 3)*x^2 - x^3 - 3*x -
 1)*sqrt(-a*x^2 + x^3 + 3*x + 1)*sqrt(a + 3)/((a + 3)*x^4 - (a^2 + 3*a)*x^3
 + 3*(a + 3)*x^2 + (a + 3)*x))/sqrt(a + 3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-2}{\sqrt{-ax^2+x^3+3\,x+1}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(1+x)/(-a*x^2+x^3+3*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - 2)/(sqrt(-a*x^2 + x^3 + 3*x + 1)*(x + 1)), x)
```

maple [C] time = 0.72, size = 3008, normalized size = 55.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+x)/(1+x)/(-a*x^2+x^3+3*x+1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(1/6*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(
1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(
1/3))*(-I*(x+1/12*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1
/3)-3*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1
/3)-1/3*a+1/2*I*3^(1/2)*(1/6*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405
)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405
)^(1/2))^(1/3)))*3^(1/2)/(1/6*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+40
5)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+40
5)^(1/2))^(1/3)))^(1/2)*((x-1/6*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+
405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+
405)^(1/2))^(1/3)-1/3*a)/(-1/4*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+4
05)^(1/2))^(1/3)+9*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+4
05)^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/6*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+16
2*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+16
2*a+405)^(1/2))^(1/3))))^(1/2)*(I*(x+1/12*(-108*a-108+8*a^3+12*(-12*a^3-27*
a^2+162*a+405)^(1/2))^(1/3)-3*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*
a^2+162*a+405)^(1/2))^(1/3)-1/3*a-1/2*I*3^(1/2)*(1/6*(-108*a-108+8*a^3+12*(
-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(
-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)))*3^(1/2)/(1/6*(-108*a-108+8*a^3+12*
(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*
(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)))^(1/2)/(-a*x^2+x^3+3*x+1)^(1/2)*El
lipticF(1/3*3^(1/2)*(-I*(x+1/12*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+
405)^(1/2))^(1/3)-3*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+
```

```
405)^(1/2))^(1/3)-1/3*a+1/2*I*3^(1/2)*(1/6*(-108*a-108+8*a^3+12*(-12*a^3-27
*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27
*a^2+162*a+405)^(1/2))^(1/3)))*3^(1/2)/(1/6*(-108*a-108+8*a^3+12*(-12*a^3-2
7*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-2
7*a^2+162*a+405)^(1/2))^(1/3)))^(1/2),(-I*3^(1/2)*(1/6*(-108*a-108+8*a^3+12
*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12
*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3))/(-1/4*(-108*a-108+8*a^3+12*(-12*a
^3-27*a^2+162*a+405)^(1/2))^(1/3)+9*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a
^3-27*a^2+162*a+405)^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/6*(-108*a-108+8*a^3+12*(
-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(
-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3))))^(1/2))-2*I*3^(1/2)*(1/6*(-108*a-1
08+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-1
08+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3))*(-I*(x+1/12*(-108*a-10
8+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)-3*(1-1/9*a^2)/(-108*a-10
8+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)-1/3*a+1/2*I*3^(1/2)*(1/6
*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)
/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)))*3^(1/2)/(1/
6*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2
)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)))^(1/2)*((x-
1/6*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a
^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)-1/3*a)/(-1
/4*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+9*(1-1/9*a^
2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)-1/2*I*3^(1/
2)*(1/6*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1
/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3))))^(1/
2)*(I*(x+1/12*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)-
3*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)-
1/3*a-1/2*I*3^(1/2)*(1/6*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1
/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1
/2))^(1/3)))*3^(1/2)/(1/6*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(
1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(
1/2))^(1/3)))^(1/2)/(-a*x^2+x^3+3*x+1)^(1/2)/(-1/12*(-108*a-108+8*a^3+12*(-
12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+3*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-
12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+1/3*a-1/2*I*3^(1/2)*(1/6*(-108*a-108+
8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+
8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3))+1)*EllipticPi(1/3*3^(1/2)
*(-I*(x+1/12*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)-3
*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/2))^(1/3)-1
/3*a+1/2*I*3^(1/2)*(1/6*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/
2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1/
2))^(1/3)))*3^(1/2)/(1/6*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1
/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405)^(1
/2))^(1/3)))^(1/2),-I*3^(1/2)*(1/6*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162
*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162
*a+405)^(1/2))^(1/3))/(-1/12*(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405
)^(1/2))^(1/3)+3*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^2+162*a+405
)^(1/2))^(1/3)+1/3*a-1/2*I*3^(1/2)*(1/6*(-108*a-108+8*a^3+12*(-12*a^3-27*a^
2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^
2+162*a+405)^(1/2))^(1/3))+1),(-I*3^(1/2)*(1/6*(-108*a-108+8*a^3+12*(-12*a^
3-27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^
3-27*a^2+162*a+405)^(1/2))^(1/3))/(-1/4*(-108*a-108+8*a^3+12*(-12*a^3-27*a^
2+162*a+405)^(1/2))^(1/3)+9*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-27*a^
2+162*a+405)^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/6*(-108*a-108+8*a^3+12*(-12*a^3-
27*a^2+162*a+405)^(1/2))^(1/3)+6*(1-1/9*a^2)/(-108*a-108+8*a^3+12*(-12*a^3-
27*a^2+162*a+405)^(1/2))^(1/3))))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-2}{\sqrt{-ax^2+x^3+3\,x+1}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(1+x)/(-a*x^2+x^3+3*x+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x - 2)/(sqrt(-a*x^2 + x^3 + 3*x + 1)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x-2}{(x+1)\,\sqrt{x^3-a\,x^2+3\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2)/((x + 1)*(3*x - a*x^2 + x^3 + 1)^(1/2)),x)
[Out] int((x - 2)/((x + 1)*(3*x - a*x^2 + x^3 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-2}{(x+1)\sqrt{-ax^2+x^3+3x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(1+x)/(-a*x**2+x**3+3*x+1)**(1/2),x)
[Out] Integral((x - 2)/((x + 1)*sqrt(-a*x**2 + x**3 + 3*x + 1)), x)
```

3.682 $\int \frac{(-1+x^4)\sqrt[4]{1+x^4}}{x^2}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt[4]{x^4+1}\,(x^4+4)}{4x}+\frac{3}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{3}{8}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {453, 279, 331, 298, 203, 206}

$$\frac{(x^4+1)^{5/4}}{x}+\frac{3}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{3}{8}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{3}{4}\sqrt[4]{x^4+1}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4)*(1 + x^4)^(1/4))/x^2,x]
```

```
[Out] (-3*x^3*(1 + x^4)^(1/4))/4 + (1 + x^4)^(5/4)/x + (3*ArcTan[x/(1 + x^4)^(1/4
)])/8 - (3*ArcTanh[x/(1 + x^4)^(1/4)])/8
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^4\right)\sqrt[4]{1+x^4}}{x^2}\,dx &= \frac{\left(1+x^4\right)^{5/4}}{x}-3\int x^2\sqrt[4]{1+x^4}\,dx\\
&= -\frac{3}{4}x^3\sqrt[4]{1+x^4}+\frac{\left(1+x^4\right)^{5/4}}{x}-\frac{3}{4}\int \frac{x^2}{\left(1+x^4\right)^{3/4}}\,dx\\
&= -\frac{3}{4}x^3\sqrt[4]{1+x^4}+\frac{\left(1+x^4\right)^{5/4}}{x}-\frac{3}{4}\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)\\
&= -\frac{3}{4}x^3\sqrt[4]{1+x^4}+\frac{\left(1+x^4\right)^{5/4}}{x}-\frac{3}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{3}{8}\text{Subst}\left(\int \frac{1}{1+
\right.\\
&= -\frac{3}{4}x^3\sqrt[4]{1+x^4}+\frac{\left(1+x^4\right)^{5/4}}{x}+\frac{3}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{3}{8}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.63

$$\frac{\left(x^4+1\right)^{5/4}}{x}-x^3\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)*(1 + x^4)^(1/4))/x^2,x]
```

```
[Out] (1 + x^4)^(5/4)/x - x^3*Hypergeometric2F1[-1/4, 3/4, 7/4, -x^4]
```

IntegrateAlgebraic [A] time = 0.19, size = 54, normalized size = 1.00

$$\frac{\sqrt[4]{x^4+1}\left(x^4+4\right)}{4x}+\frac{3}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{3}{8}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(1 + x^4)^(1/4))/x^2,x]
```

```
[Out] ((1 + x^4)^(1/4)*(4 + x^4))/(4*x) + (3*ArcTan[x/(1 + x^4)^(1/4)])/8 - (3*Ar
cTanh[x/(1 + x^4)^(1/4)])/8
```

fricas [B] time = 2.42, size = 92, normalized size = 1.70

$$\frac{3\,x\arctan\left(2\left(x^4+1\right)^{\frac{1}{4}}x^3+2\left(x^4+1\right)^{\frac{3}{4}}x\right)+3\,x\log\left(-2\,x^4+2\left(x^4+1\right)^{\frac{1}{4}}x^3-2\sqrt{x^4+1}\,x^2+2\left(x^4+1\right)^{\frac{3}{4}}x-1\right)+4\left(x^4+4\right)\left(x^4+1\right)^{\frac{1}{4}}}{16\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/16*(3*x*arctan(2*(x^4 + 1)^(1/4)*x^3 + 2*(x^4 + 1)^(3/4)*x) + 3*x*log(-2*
x^4 + 2*(x^4 + 1)^(1/4)*x^3 - 2*sqrt(x^4 + 1)*x^2 + 2*(x^4 + 1)^(3/4)*x - 1
) + 4*(x^4 + 4)*(x^4 + 1)^(1/4))/x
```

giac [A] time = 0.18, size = 70, normalized size = 1.30

$$\frac{1}{4}\left(x^4+1\right)^{\frac{1}{4}}x^3+\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}-\frac{3}{8}\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-\frac{3}{16}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}+1\right)+\frac{3}{16}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(x^4 + 1)^(1/4)*x^3 + (x^4 + 1)^(1/4)/x - 3/8*arctan((x^4 + 1)^(1/4)/x)
 - 3/16*log((x^4 + 1)^(1/4)/x + 1) + 3/16*log((x^4 + 1)^(1/4)/x - 1)
```

maple [C] time = 0.14, size = 40, normalized size = 0.74

$$\frac{x^8+5x^4+4}{4x\left(x^4+1\right)^{\frac{3}{4}}}-\frac{x^3\,\text{hypergeom}\left(\left[\frac{3}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],-x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^4+1)^(1/4)/x^2,x)
```

```
[Out] 1/4*(x^8+5*x^4+4)/x/(x^4+1)^(3/4)-1/4*x^3*hypergeom([3/4,3/4],[7/4],-x^4)
```

maxima [A] time = 0.42, size = 83, normalized size = 1.54

$$\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}+\frac{\left(x^4+1\right)^{\frac{1}{4}}}{4\,x\left(\frac{x^4+1}{x^4}-1\right)}-\frac{3}{8}\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-\frac{3}{16}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}+1\right)+\frac{3}{16}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] (x^4 + 1)^(1/4)/x + 1/4*(x^4 + 1)^(1/4)/(x*((x^4 + 1)/x^4 - 1)) - 3/8*arcta
n((x^4 + 1)^(1/4)/x) - 3/16*log((x^4 + 1)^(1/4)/x + 1) + 3/16*log((x^4 + 1)
^(1/4)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x^4-1\right)\left(x^4+1\right)^{1/4}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(x^4 + 1)^(1/4))/x^2,x)
```

```
[Out] int(((x^4 - 1)*(x^4 + 1)^(1/4))/x^2, x)
```

sympy [C] time = 2.46, size = 65, normalized size = 1.20

$$\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**4+1)**(1/4)/x**2,x)
```

```
[Out] x**3*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), x**4*exp_polar(I*pi))/(4*gamma(7
/4)) - gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), x**4*exp_polar(I*pi))/(4*x*g
amma(3/4))
```

3.683 $\int \frac{2b+ax^2}{\sqrt[4]{b+ax^2}\left(-2b-2ax^2+x^4\right)}\,dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{ax^2+b}}{x}\right)}{2^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{ax^2+b}}\right)}{2^{3/4}}$$

Rubi [C] time = 0.60, antiderivative size = 443, normalized size of antiderivative = 8.20, number of steps used = 10, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1692, 399, 490, 1218}

$$\frac{\sqrt[4]{b}\left(a-\sqrt{a^2+2b}\right)\sqrt{-\frac{ax^2}{b}}\,\Pi\left(-\frac{\sqrt{b}}{\sqrt{a^2-\sqrt{a^2+2b}\,a+b}};\sin^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)\middle|-1\right)}{2x\sqrt{-a\sqrt{a^2+2b}+a^2+b}}-\frac{\sqrt[4]{b}\left(a-\sqrt{a^2+2b}\right)\sqrt{-\frac{ax^2}{b}}\,\Pi\left(\frac{\sqrt{b}}{\sqrt{a^2-\sqrt{a^2+2b}\,a+b}};\sin^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)\middle|-1\right)}{2x\sqrt{-a\sqrt{a^2+2b}+a^2+b}}+\frac{\sqrt[4]{b}\left(\sqrt{a^2+2b}+a\right)\sqrt{-\frac{ax^2}{b}}\,\Pi\left(-\frac{\sqrt{b}}{\sqrt{a^2+\sqrt{a^2+2b}\,a+b}};\sin^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)\middle|-1\right)}{2x\sqrt{a\sqrt{a^2+2b}+a^2+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a^2+2b}+a\right)\sqrt{-\frac{ax^2}{b}}\,\Pi\left(\frac{\sqrt{b}}{\sqrt{a^2+\sqrt{a^2+2b}\,a+b}};\sin^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)\middle|-1\right)}{2x\sqrt{a\sqrt{a^2+2b}+a^2+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*b + a*x^2)/((b + a*x^2)^(1/4)*(-2*b - 2*a*x^2 + x^4)),x]
```

```
[Out] (b^(1/4)*(a - Sqrt[a^2 + 2*b])*Sqrt[-((a*x^2)/b)]*EllipticPi[-(Sqrt[b]/Sqrt
[a^2 + b - a*Sqrt[a^2 + 2*b]]), ArcSin[(b + a*x^2)^(1/4)/b^(1/4)], -1])/(2*
Sqrt[a^2 + b - a*Sqrt[a^2 + 2*b]]*x) - (b^(1/4)*(a - Sqrt[a^2 + 2*b])*Sqrt[
-((a*x^2)/b)]*EllipticPi[Sqrt[b]/Sqrt[a^2 + b - a*Sqrt[a^2 + 2*b]], ArcSin[
(b + a*x^2)^(1/4)/b^(1/4)], -1])/(2*Sqrt[a^2 + b - a*Sqrt[a^2 + 2*b]]*x) +
(b^(1/4)*(a + Sqrt[a^2 + 2*b])*Sqrt[-((a*x^2)/b)]*EllipticPi[-(Sqrt[b]/Sqrt
[a^2 + b + a*Sqrt[a^2 + 2*b]]), ArcSin[(b + a*x^2)^(1/4)/b^(1/4)], -1])/(2*
Sqrt[a^2 + b + a*Sqrt[a^2 + 2*b]]*x) - (b^(1/4)*(a + Sqrt[a^2 + 2*b])*Sqrt[
-((a*x^2)/b)]*EllipticPi[Sqrt[b]/Sqrt[a^2 + b + a*Sqrt[a^2 + 2*b]], ArcSin[
(b + a*x^2)^(1/4)/b^(1/4)], -1])/(2*Sqrt[a^2 + b + a*Sqrt[a^2 + 2*b]]*x)
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{2b+ax^2}{\sqrt[4]{b+ax^2}\,\left(-2b-2ax^2+x^4\right)}\,dx = \int \left(\frac{a+\sqrt{a^2+2b}}{\left(-2a-2\sqrt{a^2+2b}+2x^2\right)\sqrt[4]{b+ax^2}} + \frac{a-\sqrt{a^2+2b}}{\left(-2a+2\sqrt{a^2+2b}+2x^2\right)\sqrt[4]{}}\right.$$

$$= \left(a-\sqrt{a^2+2b}\right)\int \frac{1}{\left(-2a+2\sqrt{a^2+2b}+2x^2\right)\sqrt[4]{b+ax^2}}\,dx + \left(a+\sqrt{a^2+}\right.$$

$$= \frac{\left(2\left(a-\sqrt{a^2+2b}\right)\sqrt{-\frac{ax^2}{b}}\right)\text{Subst}\left(\int \frac{x^2}{\left(-2b+a\left(-2a+2\sqrt{a^2+2b}\right)+2x^4\right)\sqrt{1-\frac{x^4}{b}}}\,dx,\right.}{x}$$

$$= -\frac{\left(\left(a-\sqrt{a^2+2b}\right)\sqrt{-\frac{ax^2}{b}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a^2+b-a\sqrt{a^2+2b}}-x^2\right)\sqrt{1-\frac{x^4}{b}}}\,dx, x, \sqrt[4]{b}\right.}{2x}$$

$$= \frac{\sqrt[4]{b}\left(a-\sqrt{a^2+2b}\right)\sqrt{-\frac{ax^2}{b}}\,\Pi\left(-\frac{\sqrt{b}}{\sqrt{a^2+b-a\sqrt{a^2+2b}}};\sin^{-1}\left(\frac{\sqrt[4]{b+ax^2}}{\sqrt[4]{b}}\right)\middle|-1\right)}{2\sqrt{a^2+b-a\sqrt{a^2+2b}}\,x} -$$

Mathematica [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{2b+ax^2}{\sqrt[4]{b+ax^2}\,\left(-2b-2ax^2+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2*b + a*x^2)/((b + a*x^2)^(1/4)*(-2*b - 2*a*x^2 + x^4)),x]
[Out] Integrate[(2*b + a*x^2)/((b + a*x^2)^(1/4)*(-2*b - 2*a*x^2 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.22, size = 54, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{ax^2+b}}{x}\right)}{2^{3/4}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{ax^2+b}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*b + a*x^2)/((b + a*x^2)^(1/4)*(-2*b - 2*a*x^2 + x^4))
,x]
[Out] ArcTan[(2^(1/4)*(b + a*x^2)^(1/4))/x]/2^(3/4) - ArcTanh[x/(2^(1/4)*(b + a*x
^2)^(1/4))]/2^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b)/(a*x^2+b)^(1/4)/(x^4-2*a*x^2-2*b),x, algorithm="frica
s")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + 2b}{\left(x^4 - 2ax^2 - 2b\right)\left(ax^2 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b)/(a*x^2+b)^(1/4)/(x^4-2*a*x^2-2*b),x, algorithm="giac"
)
```

```
[Out] integrate((a*x^2 + 2*b)/((x^4 - 2*a*x^2 - 2*b)*(a*x^2 + b)^(1/4)), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2 + 2b}{\left(a\,x^2 + b\right)^{\frac{1}{4}}\left(x^4 - 2a\,x^2 - 2b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+2*b)/(a*x^2+b)^(1/4)/(x^4-2*a*x^2-2*b),x)
```

```
[Out] int((a*x^2+2*b)/(a*x^2+b)^(1/4)/(x^4-2*a*x^2-2*b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + 2b}{\left(x^4 - 2ax^2 - 2b\right)\left(ax^2 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b)/(a*x^2+b)^(1/4)/(x^4-2*a*x^2-2*b),x, algorithm="maxim
a")
```

```
[Out] integrate((a*x^2 + 2*b)/((x^4 - 2*a*x^2 - 2*b)*(a*x^2 + b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{a\,x^2 + 2\,b}{\left(a\,x^2 + b\right)^{1/4}\left(-x^4 + 2\,a\,x^2 + 2\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*b + a*x^2)/((b + a*x^2)^(1/4)*(2*b + 2*a*x^2 - x^4)),x)
```

```
[Out] int(-(2*b + a*x^2)/((b + a*x^2)^(1/4)*(2*b + 2*a*x^2 - x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + 2b}{\sqrt[4]{ax^2 + b}\,\left(-2ax^2 - 2b + x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+2*b)/(a*x**2+b)**(1/4)/(x**4-2*a*x**2-2*b),x)
```

```
[Out] Integral((a*x**2 + 2*b)/((a*x**2 + b)**(1/4)*(-2*a*x**2 - 2*b + x**4)), x)
```

3.684 $\int \frac{\sqrt[4]{-x^3+x^4}}{x}\,dx$

Optimal. Leaf size=54

$$\sqrt[4]{x^4-x^3}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)$$

Rubi [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.85, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2021, 2032, 63, 240, 212, 206, 203}

$$\sqrt[4]{x^4-x^3}-\frac{(x-1)^{3/4}x^{9/4}\tan^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{2\left(x^4-x^3\right)^{3/4}}-\frac{(x-1)^{3/4}x^{9/4}\tanh^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{2\left(x^4-x^3\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + x^4)^(1/4)/x,x]
```

```
[Out] (-x^3 + x^4)^(1/4) - ((-1 + x)^(3/4)*x^(9/4)*ArcTan[(-1 + x)^(1/4)/x^(1/4)]
)/(2*(-x^3 + x^4)^(3/4)) - ((-1 + x)^(3/4)*x^(9/4)*ArcTanh[(-1 + x)^(1/4)/x
^(1/4)])/(2*(-x^3 + x^4)^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{-x^3+x^4}}{x}\,dx &= \sqrt[4]{-x^3+x^4}-\frac{1}{4}\int \frac{x^2}{\left(-x^3+x^4\right)^{3/4}}\,dx \\
&= \sqrt[4]{-x^3+x^4}-\frac{\left((-1+x)^{3/4}x^{9/4}\right)\int \frac{1}{(-1+x)^{3/4}\sqrt[4]{x}}\,dx}{4\left(-x^3+x^4\right)^{3/4}} \\
&= \sqrt[4]{-x^3+x^4}-\frac{\left((-1+x)^{3/4}x^{9/4}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^4}}\,dx,x,\sqrt[4]{-1+x}\right)}{\left(-x^3+x^4\right)^{3/4}} \\
&= \sqrt[4]{-x^3+x^4}-\frac{\left((-1+x)^{3/4}x^{9/4}\right)\operatorname{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{\left(-x^3+x^4\right)^{3/4}} \\
&= \sqrt[4]{-x^3+x^4}-\frac{\left((-1+x)^{3/4}x^{9/4}\right)\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{2\left(-x^3+x^4\right)^{3/4}}-\frac{\left((-1+x)^{3/4}x^{9/4}\right)\operatorname{Subst}\left(\int \frac{1}{1+x^2}\right.}{2\left(-x^3+x^4\right)^{3/4}} \\
&= \sqrt[4]{-x^3+x^4}-\frac{(-1+x)^{3/4}x^{9/4}\tan^{-1}\left(\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{2\left(-x^3+x^4\right)^{3/4}}-\frac{(-1+x)^{3/4}x^{9/4}\tanh^{-1}\left(\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{2\left(-x^3+x^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.65

$$\frac{4\left((x-1)x^3\right)^{5/4}\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};1-x\right)}{5x^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + x^4)^(1/4)/x,x]
```

```
[Out] (4*((-1 + x)*x^3)^(5/4)*Hypergeometric2F1[1/4, 5/4, 9/4, 1 - x])/(5*x^(15/4
))
```

IntegrateAlgebraic [A] time = 0.23, size = 54, normalized size = 1.00

$$\sqrt[4]{x^4-x^3}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^3 + x^4)^(1/4)/x,x]
```

```
[Out] (-x^3 + x^4)^(1/4) + ArcTan[x/(-x^3 + x^4)^(1/4)]/2 - ArcTanh[x/(-x^3 + x^4
)^(1/4)]/2
```

fricas [A] time = 0.39, size = 73, normalized size = 1.35

$$\left(x^{4}-x^{3}\right)^{\frac{1}{4}}-\frac{1}{2}\,\arctan\left(\frac{\left(x^{4}-x^{3}\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{4}\,\log\left(\frac{x+\left(x^{4}-x^{3}\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{4}\,\log\left(-\frac{x-\left(x^{4}-x^{3}\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x,x, algorithm="fricas")
```

```
[Out] (x^4 - x^3)^(1/4) - 1/2*arctan((x^4 - x^3)^(1/4)/x) - 1/4*log((x + (x^4 - x
^3)^(1/4))/x) + 1/4*log(-(x - (x^4 - x^3)^(1/4))/x)
```

giac [A] time = 0.20, size = 54, normalized size = 1.00

$$-x\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+\frac{1}{2}\,\arctan\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)+\frac{1}{4}\,\log\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right)-\frac{1}{4}\,\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x,x, algorithm="giac")
```

```
[Out] -x*(-1/x + 1)^(1/4) + 1/2*arctan((-1/x + 1)^(1/4)) + 1/4*log((-1/x + 1)^(1/
4) + 1) - 1/4*log(abs((-1/x + 1)^(1/4) - 1))
```

maple [C] time = 0.47, size = 390, normalized size = 7.22

$$\left(x^{3}\left(-1+x\right)\right)^{\frac{1}{4}}+\frac{\left(\frac{\ln\left(\frac{2\left(x^{4}-3x^{3}+3x^{2}-x\right)^{\frac{3}{4}}-2\sqrt{x^{4}-3x^{3}+3x^{2}-x}\,x+2\left(x^{4}-3x^{3}+3x^{2}-x\right)^{\frac{1}{4}}x^{2}-2x^{3}+2\sqrt{x^{4}-3x^{3}+3x^{2}-x}-4\left(x^{4}-3x^{3}+3x^{2}-x\right)^{\frac{1}{4}}x+5x^{2}+2\left(x^{4}-3x^{3}+3x^{2}-x\right)^{\frac{1}{4}}-4x+1}{\left(-1+x\right)^{2}}\right)}{4}+\frac{\mathit{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{2\sqrt{x^{4}-3x^{3}+3x^{2}-x}\,\mathit{RootOf}\left(\_Z^{2}+1\right)x-2\,\mathit{RootOf}\left(\_Z^{2}+1\right)x^{3}+2\left(x^{4}-3x^{3}+3x^{2}-x\right)^{\frac{3}{4}}-2\sqrt{x^{4}-3x^{3}+3x^{2}-x}\,\mathit{RootOf}\left(\_Z^{2}+1\right)-2\left(x^{4}-3x^{3}+3x^{2}-x\right)^{\frac{1}{4}}x^{2}+5\,\mathit{RootOf}\left(\_Z^{2}+1\right)x^{2}+4\left(x^{4}-3x^{3}+3x^{2}-x\right)^{\frac{1}{4}}x-4\,\mathit{RootOf}\left(\_Z^{2}+1\right)x-2\left(x^{4}-3x^{3}+3x^{2}-x\right)^{\frac{1}{4}}+\mathit{RootOf}\left(\_Z^{2}+1\right)}{\left(-1+x\right)^{2}}\right)}{4}\right)\left(x^{3}\left(-1+x\right)\right)^{\frac{1}{4}}\left(x\left(-1+x\right)^{3}\right)^{\frac{1}{4}}}{\left(-1+x\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3)^(1/4)/x,x)
```

```
[Out] (x^3*(-1+x))^(1/4)+(1/4*ln((2*(x^4-3*x^3+3*x^2-x)^(3/4)-2*(x^4-3*x^3+3*x^2-
x)^(1/2)*x+2*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2-2*x^3+2*(x^4-3*x^3+3*x^2-x)^(1/2
)-4*(x^4-3*x^3+3*x^2-x)^(1/4)*x+5*x^2+2*(x^4-3*x^3+3*x^2-x)^(1/4)-4*x+1)/(-
1+x)^2)+1/4*RootOf(_Z^2+1)*ln((2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+1)*x
-2*RootOf(_Z^2+1)*x^3+2*(x^4-3*x^3+3*x^2-x)^(3/4)-2*(x^4-3*x^3+3*x^2-x)^(1/
2)*RootOf(_Z^2+1)-2*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+5*RootOf(_Z^2+1)*x^2+4*(x
^4-3*x^3+3*x^2-x)^(1/4)*x-4*RootOf(_Z^2+1)*x-2*(x^4-3*x^3+3*x^2-x)^(1/4)+Ro
otOf(_Z^2+1))/(-1+x)^2))*(x^3*(-1+x))^(1/4)/(-1+x)/x*(x*(-1+x)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}-x^{3}\right)^{\frac{1}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x,x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^3)^(1/4)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(x^4-x^3\right)}^{1/4}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x^3)^(1/4)/x,x)
[Out] int((x^4 - x^3)^(1/4)/x, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3 (x-1)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3)**(1/4)/x,x)
[Out] Integral((x**3*(x - 1))**(1/4)/x, x)
```

3.685 $\int \frac{(-1+x^3)\sqrt{-1+x^6}}{x^{13}}\,dx$

Optimal. Leaf size=54

$$\frac{1}{12}\tan^{-1}\left(\frac{x^3+1}{\sqrt{x^6-1}}\right)+\frac{\sqrt{x^6-1}\,(8x^9-3x^6-8x^3+6)}{72x^{12}}$$

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1475, 835, 807, 266, 47, 63, 203}

$$\frac{\sqrt{x^6-1}}{24x^6}-\frac{1}{24}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{(x^6-1)^{3/2}}{12x^{12}}+\frac{(x^6-1)^{3/2}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)*Sqrt[-1 + x^6])/x^13,x]
```

```
[Out] Sqrt[-1 + x^6]/(24*x^6) - (-1 + x^6)^(3/2)/(12*x^12) + (-1 + x^6)^(3/2)/(9*
x^9) - ArcTan[Sqrt[-1 + x^6]]/24
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^3\right)\sqrt{-1+x^6}}{x^{13}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(-1+x)\sqrt{-1+x^2}}{x^5}\,dx, x, x^3\right)\\
&= -\frac{\left(-1+x^6\right)^{3/2}}{12x^{12}}+\frac{1}{12}\operatorname{Subst}\left(\int \frac{(4-x)\sqrt{-1+x^2}}{x^4}\,dx, x, x^3\right)\\
&= -\frac{\left(-1+x^6\right)^{3/2}}{12x^{12}}+\frac{\left(-1+x^6\right)^{3/2}}{9x^9}-\frac{1}{12}\operatorname{Subst}\left(\int \frac{\sqrt{-1+x^2}}{x^3}\,dx, x, x^3\right)\\
&= -\frac{\left(-1+x^6\right)^{3/2}}{12x^{12}}+\frac{\left(-1+x^6\right)^{3/2}}{9x^9}-\frac{1}{24}\operatorname{Subst}\left(\int \frac{\sqrt{-1+x}}{x^2}\,dx, x, x^6\right)\\
&= \frac{\sqrt{-1+x^6}}{24x^6}-\frac{\left(-1+x^6\right)^{3/2}}{12x^{12}}+\frac{\left(-1+x^6\right)^{3/2}}{9x^9}-\frac{1}{48}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right)\\
&= \frac{\sqrt{-1+x^6}}{24x^6}-\frac{\left(-1+x^6\right)^{3/2}}{12x^{12}}+\frac{\left(-1+x^6\right)^{3/2}}{9x^9}-\frac{1}{24}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^6}\right)\\
&= \frac{\sqrt{-1+x^6}}{24x^6}-\frac{\left(-1+x^6\right)^{3/2}}{12x^{12}}+\frac{\left(-1+x^6\right)^{3/2}}{9x^9}-\frac{1}{24}\tan^{-1}\left(\sqrt{-1+x^6}\right)
\end{aligned}$$

Mathematica [C] time = 0.02, size = 37, normalized size = 0.69

$$-\frac{\left(x^6-1\right)^{3/2}\left(x^9\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};1-x^6\right)-1\right)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)*Sqrt[-1 + x^6])/x^13,x]
```

```
[Out] -1/9*((-1 + x^6)^(3/2)*(-1 + x^9*Hypergeometric2F1[3/2, 3, 5/2, 1 - x^6]))/
x^9
```

IntegrateAlgebraic [A] time = 0.18, size = 56, normalized size = 1.04

$$\frac{1}{12}\tan^{-1}\left(\frac{\sqrt{x^6-1}}{x^3-1}\right)+\frac{\sqrt{x^6-1}\left(8x^9-3x^6-8x^3+6\right)}{72x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)*Sqrt[-1 + x^6])/x^13,x]
```

```
[Out] (Sqrt[-1 + x^6]*(6 - 8*x^3 - 3*x^6 + 8*x^9))/(72*x^12) + ArcTan[Sqrt[-1 + x
^6]/(-1 + x^3)]/12
```

fricas [A] time = 0.39, size = 56, normalized size = 1.04

$$-\frac{6\,x^{12}\arctan\left(-x^{3}+\sqrt{x^{6}-1}\right)-8\,x^{12}-\left(8\,x^{9}-3\,x^{6}-8\,x^{3}+6\right)\sqrt{x^{6}-1}}{72\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/72*(6*x^12*arctan(-x^3 + sqrt(x^6 - 1)) - 8*x^12 - (8*x^9 - 3*x^6 - 8*x^
3 + 6)*sqrt(x^6 - 1))/x^12
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{6}-1}\left(x^{3}-1\right)}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x^13,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^6 - 1)*(x^3 - 1)/x^13, x)
```

maple [A] time = 0.05, size = 47, normalized size = 0.87

$$\frac{8x^{15}-3x^{12}-16x^{9}+9x^{6}+8x^{3}-6}{72x^{12}\sqrt{x^{6}-1}}+\frac{\arcsin\left(\frac{1}{x^{3}}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)*(x^6-1)^(1/2)/x^13,x)
```

```
[Out] 1/72*(8*x^15-3*x^12-16*x^9+9*x^6+8*x^3-6)/x^12/(x^6-1)^(1/2)+1/24*arcsin(1/
x^3)
```

maxima [A] time = 0.43, size = 58, normalized size = 1.07

$$-\frac{\left(x^{6}-1\right)^{\frac{3}{2}}-\sqrt{x^{6}-1}}{24\left(2\,x^{6}+\left(x^{6}-1\right)^{2}-1\right)}+\frac{\left(x^{6}-1\right)^{\frac{3}{2}}}{9\,x^{9}}-\frac{1}{24}\arctan\left(\sqrt{x^{6}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x^13,x, algorithm="maxima")
```

```
[Out] -1/24*((x^6 - 1)^(3/2) - sqrt(x^6 - 1))/(2*x^6 + (x^6 - 1)^2 - 1) + 1/9*(x^
6 - 1)^(3/2)/x^9 - 1/24*arctan(sqrt(x^6 - 1))
```

mupad [B] time = 1.36, size = 46, normalized size = 0.85

$$\frac{\frac{\sqrt{x^6-1}}{24}-\frac{{\left(x^6-1\right)}^{3/2}}{24}}{x^{12}}-\frac{\mathrm{atan}\left(\sqrt{x^6-1}\right)}{24}+\frac{{\left(x^6-1\right)}^{3/2}}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)*(x^6 - 1)^(1/2))/x^13,x)
```

```
[Out] ((x^6 - 1)^(1/2)/24 - (x^6 - 1)^(3/2)/24)/x^12 - atan((x^6 - 1)^(1/2))/24 +
 (x^6 - 1)^(3/2)/(9*x^9)
```

sympy [A] time = 4.82, size = 56, normalized size = 1.04

$$\frac{\begin{cases} \frac{\left(x^6-1\right)^{\frac{3}{2}}}{3x^9} & \text{for } x > -1 \wedge x < 1 \end{cases}}{3} - \frac{\begin{cases} \frac{\operatorname{acos}\left(\frac{1}{x^3}\right)}{8} - \frac{\left(-1+\frac{2}{x^6}\right)\sqrt{1-\frac{1}{x^6}}}{8x^3} & \text{for } x > -1 \wedge x < 1 \end{cases}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)*(x**6-1)**(1/2)/x**13,x)
```

```
[Out] Piecewise(((x**6 - 1)**(3/2)/(3*x**9), (x > -1) & (x < 1)))/3 - Piecewise((
acos(x**(-3))/8 - (-1 + 2/x**6)*sqrt(1 - 1/x**6)/(8*x**3), (x > -1) & (x <
1)))/3
```

3.686 $\int \frac{1}{\sqrt[8]{1+2x^4+x^8}}\,dx$

Optimal. Leaf size=54

$$\frac{\left(\left(x^4+1\right)^2\right)^{7/8}\left(\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)\right)}{\left(x^4+1\right)^{7/4}}$$

Rubi [A] time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.46, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1343, 240, 212, 206, 203}

$$\frac{\sqrt[4]{x^4+1}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[8]{x^8+2x^4+1}}+\frac{\sqrt[4]{x^4+1}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[8]{x^8+2x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^4 + x^8)^(-1/8),x]
```

```
[Out] ((1 + x^4)^(1/4)*ArcTan[x/(1 + x^4)^(1/4)])/(2*(1 + 2*x^4 + x^8)^(1/8)) + (
(1 + x^4)^(1/4)*ArcTanh[x/(1 + x^4)^(1/4)])/(2*(1 + 2*x^4 + x^8)^(1/8))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 1343

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[8]{1+2x^4+x^8}}\,dx = \frac{\sqrt[4]{2+2x^4}\int \frac{1}{\sqrt[4]{2+2x^4}}\,dx}{\sqrt[8]{1+2x^4+x^8}}$$
$$= \frac{\sqrt[4]{2+2x^4}\ \text{Subst}\left(\int \frac{1}{1-2x^4}\,dx, x, \frac{x}{\sqrt[4]{2+2x^4}}\right)}{\sqrt[8]{1+2x^4+x^8}}$$
$$= \frac{\sqrt[4]{2+2x^4}\ \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x^2}\,dx, x, \frac{x}{\sqrt[4]{2+2x^4}}\right)}{2\sqrt[8]{1+2x^4+x^8}} + \frac{\sqrt[4]{2+2x^4}\ \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x^2}\,dx, x, \frac{x}{\sqrt[4]{2+2x^4}}\right)}{2\sqrt[8]{1+2x^4+x^8}}$$
$$= \frac{\sqrt[4]{1+x^4}\ \tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[8]{1+2x^4+x^8}} + \frac{\sqrt[4]{1+x^4}\ \tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[8]{1+2x^4+x^8}}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 1.30

$$\frac{\sqrt[4]{x^4+1}\left(-\log\left(1-\frac{x}{\sqrt[4]{x^4+1}}\right)+\log\left(\frac{x}{\sqrt[4]{x^4+1}}+1\right)+2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)\right)}{4\sqrt[8]{\left(x^4+1\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^4 + x^8)^(-1/8),x]
```

```
[Out] ((1 + x^4)^(1/4)*(2*ArcTan[x/(1 + x^4)^(1/4)] - Log[1 - x/(1 + x^4)^(1/4)]
+ Log[1 + x/(1 + x^4)^(1/4)]))/(4*((1 + x^4)^2)^(1/8))
```

IntegrateAlgebraic [A] time = 16.24, size = 54, normalized size = 1.00

$$\frac{\left(\left(x^4+1\right)^2\right)^{7/8}\left(\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)\right)}{\left(x^4+1\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x^4 + x^8)^(-1/8),x]
```

```
[Out] (((1 + x^4)^2)^(7/8)*(ArcTan[x/(1 + x^4)^(1/4)]/2 + ArcTanh[x/(1 + x^4)^(1/
4)]/2))/(1 + x^4)^(7/4)
```

fricas [A] time = 0.39, size = 65, normalized size = 1.20

$$-\frac{1}{2}\arctan\left(\frac{\left(x^8+2x^4+1\right)^{\frac{1}{8}}}{x}\right)+\frac{1}{4}\log\left(\frac{x+\left(x^8+2x^4+1\right)^{\frac{1}{8}}}{x}\right)-\frac{1}{4}\log\left(-\frac{x-\left(x^8+2x^4+1\right)^{\frac{1}{8}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+2*x^4+1)^(1/8),x, algorithm="fricas")
```

```
[Out] -1/2*arctan((x^8 + 2*x^4 + 1)^(1/8)/x) + 1/4*log((x + (x^8 + 2*x^4 + 1)^(1/
8))/x) - 1/4*log(-(x - (x^8 + 2*x^4 + 1)^(1/8))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^8+2x^4+1\right)^{\frac{1}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+2*x^4+1)^(1/8),x, algorithm="giac")
```

```
[Out] integrate((x^8 + 2*x^4 + 1)^(-1/8), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{8}+2 x^{4}+1\right)^{\frac{1}{8}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+2*x^4+1)^(1/8),x)
```

```
[Out] int(1/(x^8+2*x^4+1)^(1/8),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{\left(x^{4}+1\right)^{\frac{1}{4}}}+\int \frac{x^{4}}{\left(x^{4}+1\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+2*x^4+1)^(1/8),x, algorithm="maxima")
```

```
[Out] x/(x^4 + 1)^(1/4) + integrate(x^4/(x^4 + 1)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(x^8+2\,x^4+1\right)}^{1/8}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 + x^8 + 1)^(1/8),x)
```

```
[Out] int(1/(2*x^4 + x^8 + 1)^(1/8), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[8]{x^{8}+2 x^{4}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+2*x**4+1)**(1/8),x)
```

```
[Out] Integral((x**8 + 2*x**4 + 1)**(-1/8), x)
```

3.687 $\int \frac{1}{x\sqrt{x+\sqrt{1+x^2}}} \, dx$

Optimal. Leaf size=54

$$\frac{2}{\sqrt{\sqrt{x^2+1}+x}}+2\tan^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)-2\tanh^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)$$

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2119, 453, 329, 298, 203, 206}

$$\frac{2}{\sqrt{\sqrt{x^2+1}+x}}+2\tan^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)-2\tanh^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] 2/Sqrt[x + Sqrt[1 + x^2]] + 2*ArcTan[Sqrt[x + Sqrt[1 + x^2]]] - 2*ArcTanh[S
qrt[x + Sqrt[1 + x^2]]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{x+\sqrt{1+x^2}}}\,dx &= \text{Subst}\left(\int \frac{1+x^2}{x^{3/2}\left(-1+x^2\right)}\,dx,x,x+\sqrt{1+x^2}\right)\\
&= \frac{2}{\sqrt{x+\sqrt{1+x^2}}}+2\,\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx,x,x+\sqrt{1+x^2}\right)\\
&= \frac{2}{\sqrt{x+\sqrt{1+x^2}}}+4\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\sqrt{x+\sqrt{1+x^2}}\right)\\
&= \frac{2}{\sqrt{x+\sqrt{1+x^2}}}-2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x+\sqrt{1+x^2}}\right)+2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,\right.\\
&= \frac{2}{\sqrt{x+\sqrt{1+x^2}}}+2\tan^{-1}\left(\sqrt{x+\sqrt{1+x^2}}\right)-2\tanh^{-1}\left(\sqrt{x+\sqrt{1+x^2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.92, size = 175, normalized size = 3.24

$$\frac{2}{3}\sqrt{\sqrt{x^2+1}+x}\left(-\frac{\sqrt{x^2+1}\left(\sqrt{x^2+1}+x\right)^4\left(\left(4x^2+4\sqrt{x^2+1}\,x+2\right){}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\left(x+\sqrt{x^2+1}\right)^2\right)-x^2-\sqrt{x^2+1}\,x-2\right)}{16x^6+28x^4+13x^2+5\sqrt{x^2+1}\,x+16\sqrt{x^2+1}\,x^5+20\sqrt{x^2+1}\,x^3+1}+\sqrt{x^2+1}-2x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] (2*Sqrt[x + Sqrt[1 + x^2]]*(-2*x + Sqrt[1 + x^2] - (Sqrt[1 + x^2]*(x + Sqrt
[1 + x^2])^4*(-2 - x^2 - x*Sqrt[1 + x^2] + (2 + 4*x^2 + 4*x*Sqrt[1 + x^2])*
Hypergeometric2F1[3/4, 1, 7/4, (x + Sqrt[1 + x^2])^2]))/(1 + 13*x^2 + 28*x^
4 + 16*x^6 + 5*x*Sqrt[1 + x^2] + 20*x^3*Sqrt[1 + x^2] + 16*x^5*Sqrt[1 + x^2
])))/3
```

IntegrateAlgebraic [A] time = 0.11, size = 54, normalized size = 1.00

$$\frac{2}{\sqrt{\sqrt{x^2+1}+x}}+2\tan^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)-2\tanh^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] 2/Sqrt[x + Sqrt[1 + x^2]] + 2*ArcTan[Sqrt[x + Sqrt[1 + x^2]]] - 2*ArcTanh[S
qrt[x + Sqrt[1 + x^2]]]
```

fricas [A] time = 0.40, size = 69, normalized size = 1.28

$$-2\sqrt{x+\sqrt{x^2+1}}\left(x-\sqrt{x^2+1}\right)+2\arctan\left(\sqrt{x+\sqrt{x^2+1}}\right)-\log\left(\sqrt{x+\sqrt{x^2+1}}+1\right)+\log\left(\sqrt{x+\sqrt{x^2+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(x + sqrt(x^2 + 1))*(x - sqrt(x^2 + 1)) + 2*arctan(sqrt(x + sqrt(x^2
 + 1))) - log(sqrt(x + sqrt(x^2 + 1)) + 1) + log(sqrt(x + sqrt(x^2 + 1)) -
1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+\sqrt{x^2+1}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x + sqrt(x^2 + 1))*x), x)
```

maple [C] time = 0.04, size = 22, normalized size = 0.41

$$-\frac{\sqrt{2}\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{4},\frac{3}{4}\right],\left[\frac{5}{4},\frac{3}{2}\right],-\frac{1}{x^2}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] -2^(1/2)/x^(1/2)*hypergeom([1/4,1/4,3/4],[5/4,3/2],-1/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+\sqrt{x^2+1}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x + sqrt(x^2 + 1))*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + (x^2 + 1)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x*(x + (x^2 + 1)^(1/2))^(1/2)), x)
```

sympy [C] time = 1.34, size = 44, normalized size = 0.81

$$-\frac{\Gamma^2\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_3F_2\left(\begin{matrix}\frac{1}{4},\frac{1}{4},\frac{3}{4}\\ \frac{5}{4},\frac{3}{2}\end{matrix}\middle|\frac{e^{i\pi}}{x^2}\right)}{4\pi\sqrt{x}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] -gamma(1/4)**2*gamma(3/4)*hyper((1/4, 1/4, 3/4), (5/4, 3/2), exp_polar(I*pi
)/x**2)/(4*pi*sqrt(x)*gamma(5/4))
```

3.688 $\displaystyle \int \frac{1-2x+k^2x^2}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(-1+2x+\left(-2+k^2\right)x^2\right)}\,dx$

Optimal. Leaf size=55

$$\frac{2\tan^{-1}\left(\frac{\sqrt{k^2-2}\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{k^2x-1}\right)}{\sqrt{k^2-2}}$$

Rubi [C] time = 5.07, antiderivative size = 396, normalized size of antiderivative = 7.20, number of steps used = 12, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6718, 6728, 115, 168, 538, 537}

$$-\frac{2k^2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\left(2-k^2\right)\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{2\sqrt{k^2-1}\left(k^2+2\sqrt{k^2-1}\right)\sqrt{1-x}\sqrt{x}\sqrt{\frac{k^2(1-x)}{1-k^2}+1}\,\Pi\left(\frac{2-k^2}{-k^2-\sqrt{k^2-1}+1};\sin^{-1}\left(\sqrt{1-x}\right)|-\frac{k^2}{1-k^2}\right)}{\left(2-k^2\right)\left(-k^2-\sqrt{k^2-1}+1\right)\sqrt{(1-x)x\left(1-k^2x\right)}}-\frac{2\sqrt{k^2-1}\left(k^2-2\sqrt{k^2-1}\right)\sqrt{1-x}\sqrt{x}\sqrt{\frac{k^2(1-x)}{1-k^2}+1}\,\Pi\left(\frac{2-k^2}{-k^2+\sqrt{k^2-1}+1};\sin^{-1}\left(\sqrt{1-x}\right)|-\frac{k^2}{1-k^2}\right)}{\left(2-k^2\right)\left(-k^2+\sqrt{k^2-1}+1\right)\sqrt{(1-x)x\left(1-k^2x\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + 2*x + (-2 + k^2)
*x^2)),x]
```

```
[Out] (-2*k^2*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticF[ArcSin[Sqrt[x]], k^2]
)/((2 - k^2)*Sqrt[(1 - x)*x*(1 - k^2*x)]) + (2*Sqrt[-1 + k^2]*(k^2 + 2*Sqrt
[-1 + k^2])*Sqrt[1 + (k^2*(1 - x))/(1 - k^2)]*Sqrt[1 - x]*Sqrt[x]*EllipticP
i[(2 - k^2)/(1 - k^2 - Sqrt[-1 + k^2]), ArcSin[Sqrt[1 - x]], -(k^2/(1 - k^2
))])/((2 - k^2)*(1 - k^2 - Sqrt[-1 + k^2])*Sqrt[(1 - x)*x*(1 - k^2*x)]) - (
2*Sqrt[-1 + k^2]*(k^2 - 2*Sqrt[-1 + k^2])*Sqrt[1 + (k^2*(1 - x))/(1 - k^2)]
*Sqrt[1 - x]*Sqrt[x]*EllipticPi[(2 - k^2)/(1 - k^2 + Sqrt[-1 + k^2]), ArcSi
n[Sqrt[1 - x]], -(k^2/(1 - k^2))])/((2 - k^2)*(1 - k^2 + Sqrt[-1 + k^2])*Sq
rt[(1 - x)*x*(1 - k^2*x)])
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1-2x+k^2x^2}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(-1+2x+\left(-2+k^2\right)x^2\right)}\,dx = \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{1-2x+k^2x^2}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\left(-1+2x+\left(-2+k^2\right)x^2\right)}}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int\left(-\frac{k^2}{\left(2-k^2\right)\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}} + \frac{}{\left(-2+k\right.}\right.}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= -\frac{\left(k^2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}}\,dx}{\left(2-k^2\right)\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\left(1\right.\right.}{}$$

$$= -\frac{2k^2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\left(2-k^2\right)\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\left(1-\right.\right.}{}$$

$$= -\frac{2k^2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\left(2-k^2\right)\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\left(1-\right.\right.}{}$$

$$= -\frac{2k^2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\left(2-k^2\right)\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(4\left(1-\right.\right.}{}$$

$$= -\frac{2k^2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\left(2-k^2\right)\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(4\left(1-\right.\right.}{}$$

$$= -\frac{2k^2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\left(2-k^2\right)\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{2\sqrt{-1+}}{}$$

Mathematica [C] time = 2.92, size = 202, normalized size = 3.67

$$\frac{2i\sqrt{\frac{1}{x-1}+1}\,(x-1)^{3/2}\sqrt{\frac{1-\frac{1}{k^2}}{x-1}+1}\left(\left(k^2-2\right)F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+\left(\sqrt{k^2-1}+1\right)\Pi\left(\frac{k^2-1}{k^2-\sqrt{k^2-1}-1};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)-\left(\sqrt{k^2-1}-1\right)\Pi\left(\frac{k^2-1}{k^2+\sqrt{k^2-1}-1};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)}{\left(k^2-2\right)\sqrt{(x-1)x\left(k^2x-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + 2*x + (-2
+ k^2)*x^2)),x]
```

```
[Out] ((2*I)*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (1 - k^(-2))/(-1 + x)]*(-1 + x)^(3/
2)*((-2 + k^2)*EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] + (1 + Sqrt
[-1 + k^2])*EllipticPi[(-1 + k^2)/(-1 + k^2 - Sqrt[-1 + k^2]), I*ArcSinh[1/
Sqrt[-1 + x]], 1 - k^(-2)] - (-1 + Sqrt[-1 + k^2])*EllipticPi[(-1 + k^2)/(-
1 + k^2 + Sqrt[-1 + k^2]), I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)]))/((-2 +
k^2)*Sqrt[(-1 + x)*x*(-1 + k^2*x)])
```

IntegrateAlgebraic [A] time = 0.26, size = 55, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{k^2-2}\,\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{k^2x-1}\right)}{\sqrt{k^2-2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - 2*x + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + 2
*x + (-2 + k^2)*x^2)),x]
```

```
[Out] (2*ArcTan[(Sqrt[-2 + k^2]*Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3])/(-1 + k^2*x)]
)/Sqrt[-2 + k^2]
```

fricas [A] time = 0.51, size = 269, normalized size = 4.89

$$\left[-\frac{\sqrt{-k^2+2}\,\log\left(\frac{\left(k^4-4k^2+4\right)x^4-4\left(2k^4-5k^2+2\right)x^3+2\left(4k^4-5k^2-4\right)x^2-4\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\left(\left(k^2-2\right)x^2-2\left(k^2-1\right)x+1\right)\sqrt{-k^2+2}-4\left(2k^2-3\right)x+1}{\left(k^4-4k^2+4\right)x^4+4\left(k^2-2\right)x^3-2\left(k^2-4\right)x^2-4x+1}\right)}{2\left(k^2-2\right)},\ \frac{\arctan\left(\frac{\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\left(\left(k^2-2\right)x^2-2\left(k^2-1\right)x+1\right)\sqrt{k^2-2}}{2\left(\left(k^4-2k^2\right)x^3-\left(k^4-k^2-2\right)x^2+\left(k^2-2\right)x\right)}\right)}{\sqrt{k^2-2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-2*x+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(-1+2*x+(k^2-2)*x^2),x
, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-k^2 + 2)*log(((k^4 - 4*k^2 + 4)*x^4 - 4*(2*k^4 - 5*k^2 + 2)*x^3
 + 2*(4*k^4 - 5*k^2 - 4)*x^2 - 4*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*((k^2 -
2)*x^2 - 2*(k^2 - 1)*x + 1)*sqrt(-k^2 + 2) - 4*(2*k^2 - 3)*x + 1)/((k^4 - 4
*k^2 + 4)*x^4 + 4*(k^2 - 2)*x^3 - 2*(k^2 - 4)*x^2 - 4*x + 1))/(k^2 - 2), ar
ctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*((k^2 - 2)*x^2 - 2*(k^2 - 1)*x +
 1)*sqrt(k^2 - 2)/((k^4 - 2*k^2)*x^3 - (k^4 - k^2 - 2)*x^2 + (k^2 - 2)*x))/
sqrt(k^2 - 2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^2-2x+1}{\sqrt{\left(k^2x-1\right)(x-1)x}\left(\left(k^2-2\right)x^2+2x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-2*x+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(-1+2*x+(k^2-2)*x^2),x
, algorithm="giac")
```

```
[Out] integrate((k^2*x^2 - 2*x + 1)/(sqrt((k^2*x - 1)*(x - 1)*x)*((k^2 - 2)*x^2 +
 2*x - 1)), x)
```

maple [C] time = 0.08, size = 2705, normalized size = 49.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^2-2*x+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(-1+2*x+(k^2-2)*x^2),x)
```

```
[Out] -2/(k^2-2)*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k
^2*x^3-k^2*x^2-x^2+x)^(1/2)*EllipticF((-(x-1/k^2)*k^2)^(1/2),(1/k^2/(1/k^2-
1))^(1/2))+2/(k^2-2)*(-4/(-2*k^2/(k^2-2)+2/(k^2-2)*k^2*(k^2-1)^(1/2)+4/(k^2
-2)-4/(k^2-2)*(k^2-1)^(1/2)+2)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))
^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/(k^2-2)-1/(k^2-
2)*(k^2-1)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-(-1+(k^2-1
)^(1/2))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2))/(k^2-2)+4/(-2*k^2/(k^2-2)+2/(k^2
-2)*k^2*(k^2-1)^(1/2)+4/(k^2-2)-4/(k^2-2)*(k^2-1)^(1/2)+2)/k^2*(-k^2*x+1)^(
1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x
)^(1/2)/(1/k^2+1/(k^2-2)-1/(k^2-2)*(k^2-1)^(1/2))*EllipticPi((-(x-1/k^2)*k^
2)^(1/2),1/k^2/(1/k^2-(-1+(k^2-1)^(1/2))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2))/
(k^2-2)+4/(-2*k^2/(k^2-2)+2/(k^2-2)*k^2*(k^2-1)^(1/2)+4/(k^2-2)-4/(k^2-2)*(
k^2-1)^(1/2)+2)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^
(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/(k^2-2)-1/(k^2-2)*(k^2-1)^(1/2
))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-(-1+(k^2-1)^(1/2))/(k^2-2
)),(1/k^2/(1/k^2-1))^(1/2))/(k^2-2)*(k^2-1)^(1/2)-4/(-2*k^2/(k^2-2)+2/(k^2-
2)*k^2*(k^2-1)^(1/2)+4/(k^2-2)-4/(k^2-2)*(k^2-1)^(1/2)+2)/k^2*(-k^2*x+1)^(1
/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)
^(1/2)/(1/k^2+1/(k^2-2)-1/(k^2-2)*(k^2-1)^(1/2))*EllipticPi((-(x-1/k^2)*k^2
)^(1/2),1/k^2/(1/k^2-(-1+(k^2-1)^(1/2))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2))/(
k^2-2)*(k^2-1)^(1/2)-2/(-2*k^2/(k^2-2)+2/(k^2-2)*k^2*(k^2-1)^(1/2)+4/(k^2-2
)-4/(k^2-2)*(k^2-1)^(1/2)+2)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(
1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/(k^2-2)-1/(k^2-2)
*(k^2-1)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-(-1+(k^2-1)^
(1/2))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2))+2/(-2*k^2/(k^2-2)+2/(k^2-2)*k^2*(k
^2-1)^(1/2)+4/(k^2-2)-4/(k^2-2)*(k^2-1)^(1/2)+2)/k^2*(-k^2*x+1)^(1/2)*(1/(1
/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1
/k^2+1/(k^2-2)-1/(k^2-2)*(k^2-1)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1
/k^2/(1/k^2-(-1+(k^2-1)^(1/2))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2))-4/(-2*k^2/
(k^2-2)-2/(k^2-2)*k^2*(k^2-1)^(1/2)+4/(k^2-2)+4/(k^2-2)*(k^2-1)^(1/2)+2)*(-
k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2
*x^2-x^2+x)^(1/2)/(1/k^2+1/(k^2-2)+1/(k^2-2)*(k^2-1)^(1/2))*EllipticPi((-(x
-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+(1+(k^2-1)^(1/2))/(k^2-2)),(1/k^2/(1/k^2-1)
)^(1/2))/(k^2-2)+4/(-2*k^2/(k^2-2)-2/(k^2-2)*k^2*(k^2-1)^(1/2)+4/(k^2-2)+4/
(k^2-2)*(k^2-1)^(1/2)+2)/k^2*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(
1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/(k^2-2)+1/(k^2-2)
*(k^2-1)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+(1+(k^2-1)^(
1/2))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2))/(k^2-2)-4/(-2*k^2/(k^2-2)-2/(k^2-2)
*k^2*(k^2-1)^(1/2)+4/(k^2-2)+4/(k^2-2)*(k^2-1)^(1/2)+2)*(-k^2*x+1)^(1/2)*(1
/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)
/(1/k^2+1/(k^2-2)+1/(k^2-2)*(k^2-1)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2
),1/k^2/(1/k^2+(1+(k^2-1)^(1/2))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2))/(k^2-2)*
(k^2-1)^(1/2)+4/(-2*k^2/(k^2-2)-2/(k^2-2)*k^2*(k^2-1)^(1/2)+4/(k^2-2)+4/(k^
2-2)*(k^2-1)^(1/2)+2)/k^2*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2
)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/(k^2-2)+1/(k^2-2)*(k
^2-1)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+(1+(k^2-1)^(1/2
))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2))/(k^2-2)*(k^2-1)^(1/2)-2/(-2*k^2/(k^2-2
)-2/(k^2-2)*k^2*(k^2-1)^(1/2)+4/(k^2-2)+4/(k^2-2)*(k^2-1)^(1/2)+2)*(-k^2*x+
1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x
^2+x)^(1/2)/(1/k^2+1/(k^2-2)+1/(k^2-2)*(k^2-1)^(1/2))*EllipticPi((-(x-1/k^2
)*k^2)^(1/2),1/k^2/(1/k^2+(1+(k^2-1)^(1/2))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2
))+2/(-2*k^2/(k^2-2)-2/(k^2-2)*k^2*(k^2-1)^(1/2)+4/(k^2-2)+4/(k^2-2)*(k^2-1
)^(1/2)+2)/k^2*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(
1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/(k^2-2)+1/(k^2-2)*(k^2-1)^(1/2)
)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+(1+(k^2-1)^(1/2))/(k^2-2))
,(1/k^2/(1/k^2-1))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-2*x+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(-1+2*x+(k^2-2)*x^2),x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(k-1>0)', see `assume?` for more det
ails)Is k-1 positive, negative or zero?
```

mupad [B] time = 3.10, size = 84, normalized size = 1.53

$$\frac{\ln\left(\frac{x\,2\mathrm{i}+k^2\,x^2\,1\mathrm{i}-k^2\,x\,2\mathrm{i}-x^2\,2\mathrm{i}-2\,\sqrt{k^2-2}\,\sqrt{x\,\left(k^2\,x-1\right)\,\left(x-1\right)}+1\mathrm{i}}{2\,k^2\,x^2-4\,x^2+4\,x-2}\right)\,1\mathrm{i}}{\sqrt{k^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^2 - 2*x + 1)/((2*x + x^2*(k^2 - 2) - 1)*(x*(k^2*x - 1)*(x - 1))^
(1/2)),x)
```

```
[Out] (log((x*2i + k^2*x^2*1i - k^2*x*2i - x^2*2i - 2*(k^2 - 2)^(1/2)*(x*(k^2*x -
 1)*(x - 1))^(1/2) + 1i)/(4*x + 2*k^2*x^2 - 4*x^2 - 2))*1i)/(k^2 - 2)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k**2*x**2-2*x+1)/((1-x)*x*(-k**2*x+1))**(1/2)/(-1+2*x+(k**2-2)*x
**2),x)
```

```
[Out] Timed out
```

3.689 $\int \frac{1}{x(b+ax^3)^{3/4}}\,dx$

Optimal. Leaf size=55

$$-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{3b^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{3b^{3/4}}$$

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 63, 212, 206, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{3b^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{3b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b + a*x^3)^(3/4)),x]
```

```
[Out] (-2*ArcTan[(b + a*x^3)^(1/4)/b^(1/4)])/(3*b^(3/4)) - (2*ArcTanh[(b + a*x^3)
^(1/4)/b^(1/4)])/(3*b^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(b+ax^3\right)^{3/4}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(b+ax)^{3/4}}\,dx,x,x^3\right)\\
&= \frac{4\,\text{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{b+ax^3}\right)}{3a}\\
&= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx,x,\sqrt[4]{b+ax^3}\right)}{3\sqrt{b}}-\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx,x,\sqrt[4]{b+ax^3}\right)}{3\sqrt{b}}\\
&= -\frac{2\tan^{-1}\left(\frac{\sqrt[4]{b+ax^3}}{\sqrt[4]{b}}\right)}{3b^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^3}}{\sqrt[4]{b}}\right)}{3b^{3/4}}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.84

$$
-\frac{2\left(\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)\right)}{3b^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b + a*x^3)^(3/4)),x]
```

```
[Out] (-2*(ArcTan[(b + a*x^3)^(1/4)/b^(1/4)] + ArcTanh[(b + a*x^3)^(1/4)/b^(1/4)]
))/(3*b^(3/4))
```

IntegrateAlgebraic [A] time = 0.07, size = 55, normalized size = 1.00

$$
-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{3b^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{3b^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(b + a*x^3)^(3/4)),x]
```

```
[Out] (-2*ArcTan[(b + a*x^3)^(1/4)/b^(1/4)])/(3*b^(3/4)) - (2*ArcTanh[(b + a*x^3)
^(1/4)/b^(1/4)])/(3*b^(3/4))
```

fricas [B] time = 0.42, size = 110, normalized size = 2.00

$$
\frac{4}{3}\left(\frac{1}{b^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^2\sqrt{\frac{1}{b^3}}+\sqrt{ax^3+b}}\,b^2\left(\frac{1}{b^3}\right)^{\frac{3}{4}}-\left(ax^3+b\right)^{\frac{1}{4}}b^2\left(\frac{1}{b^3}\right)^{\frac{3}{4}}\right)-\frac{1}{3}\left(\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(b\left(\frac{1}{b^3}\right)^{\frac{1}{4}}+\left(ax^3+b\right)^{\frac{1}{4}}\right)+\frac{1}{3}\left(\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(-b\left(\frac{1}{b^3}\right)^{\frac{1}{4}}+\left(ax^3+b\right)^{\frac{1}{4}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3+b)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/3*(b^(-3))^(1/4)*arctan(sqrt(b^2*sqrt(b^(-3)) + sqrt(a*x^3 + b))*b^2*(b^(
-3))^(3/4) - (a*x^3 + b)^(1/4)*b^2*(b^(-3))^(3/4)) - 1/3*(b^(-3))^(1/4)*log
(b*(b^(-3))^(1/4) + (a*x^3 + b)^(1/4)) + 1/3*(b^(-3))^(1/4)*log(-b*(b^(-3))
^(1/4) + (a*x^3 + b)^(1/4))
```

giac [B] time = 0.13, size = 186, normalized size = 3.38

$$
-\frac{\sqrt{2}\,(-b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-b)^{\frac{1}{4}}+2\left(ax^3+b\right)^{\frac{1}{4}}\right)}{2\,(-b)^{\frac{1}{4}}}\right)}{3\,b}-\frac{\sqrt{2}\,(-b)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-b)^{\frac{1}{4}}-2\left(ax^3+b\right)^{\frac{1}{4}}\right)}{2\,(-b)^{\frac{1}{4}}}\right)}{3\,b}-\frac{\sqrt{2}\,(-b)^{\frac{1}{4}}\log\left(\sqrt{2}\left(ax^3+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^3+b}+\sqrt{-b}\right)}{6\,b}+\frac{\sqrt{2}\,(-b)^{\frac{1}{4}}\log\left(-\sqrt{2}\left(ax^3+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^3+b}+\sqrt{-b}\right)}{6\,b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3+b)^(3/4),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*(-b)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^3 +
 b)^(1/4))/(-b)^(1/4))/b - 1/3*sqrt(2)*(-b)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(-b)^(1/4) - 2*(a*x^3 + b)^(1/4))/(-b)^(1/4))/b - 1/6*sqrt(2)*(-b)^(1/4
)*log(sqrt(2)*(a*x^3 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^3 + b) + sqrt(-b))/b
+ 1/6*sqrt(2)*(-b)^(1/4)*log(-sqrt(2)*(a*x^3 + b)^(1/4)*(-b)^(1/4) + sqrt(a
*x^3 + b) + sqrt(-b))/b
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{x \left(a\,x^{3}+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^3+b)^(3/4),x)
```

```
[Out] int(1/x/(a*x^3+b)^(3/4),x)
```

maxima [A] time = 0.41, size = 57, normalized size = 1.04

$$-\frac{2\,\arctan\left(\frac{\left(ax^{3}+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{3\,b^{\frac{3}{4}}}+\frac{\log\left(\frac{\left(ax^{3}+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^{3}+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{3\,b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3+b)^(3/4),x, algorithm="maxima")
```

```
[Out] -2/3*arctan((a*x^3 + b)^(1/4)/b^(1/4))/b^(3/4) + 1/3*log(((a*x^3 + b)^(1/4)
 - b^(1/4))/((a*x^3 + b)^(1/4) + b^(1/4)))/b^(3/4)
```

mupad [B] time = 0.71, size = 39, normalized size = 0.71

$$-\frac{2\,\mathrm{atan}\left(\frac{{\left(a\,x^3+b\right)}^{1/4}}{b^{1/4}}\right)}{3\,b^{3/4}}-\frac{2\,\mathrm{atanh}\left(\frac{{\left(a\,x^3+b\right)}^{1/4}}{b^{1/4}}\right)}{3\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b + a*x^3)^(3/4)),x)
```

```
[Out] - (2*atan((b + a*x^3)^(1/4)/b^(1/4)))/(3*b^(3/4)) - (2*atanh((b + a*x^3)^(1
/4)/b^(1/4)))/(3*b^(3/4))
```

sympy [C] time = 0.93, size = 41, normalized size = 0.75

$$-\frac{\Gamma\left(\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^{3}}\right)}{3a^{\frac{3}{4}}x^{\frac{9}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**3+b)**(3/4),x)
```

```
[Out] -gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*exp_polar(I*pi)/(a*x**3))/(3*a**(3/
4)*x**(9/4)*gamma(7/4))
```

3.690 $\int \frac{(-b+ax^3)\sqrt{b+ax^3}}{x^4} dx$

Optimal. Leaf size=55

$$\frac{\sqrt{ax^3+b}\,(2ax^3+b)}{3x^3} - \frac{1}{3}a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 78, 50, 63, 208}

$$\frac{(ax^3+b)^{3/2}}{3x^3} + \frac{1}{3}a\sqrt{ax^3+b} - \frac{1}{3}a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a*x^3)*Sqrt[b + a*x^3])/x^4,x]
```

```
[Out] (a*Sqrt[b + a*x^3])/3 + (b + a*x^3)^(3/2)/(3*x^3) - (a*Sqrt[b]*ArcTanh[Sqrt
[b + a*x^3]/Sqrt[b]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-b+ax^3\right)\sqrt{b+ax^3}}{x^4}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(-b+ax)\sqrt{b+ax}}{x^2}\,dx,x,x^3\right)\\ &= \frac{\left(b+ax^3\right)^{3/2}}{3x^3}+\frac{1}{6}a\,\text{Subst}\left(\int \frac{\sqrt{b+ax}}{x}\,dx,x,x^3\right)\\ &= \frac{1}{3}a\sqrt{b+ax^3}+\frac{\left(b+ax^3\right)^{3/2}}{3x^3}+\frac{1}{6}(ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{b+ax}}\,dx,x,x^3\right)\\ &= \frac{1}{3}a\sqrt{b+ax^3}+\frac{\left(b+ax^3\right)^{3/2}}{3x^3}+\frac{1}{3}b\,\text{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^2}{a}}\,dx,x,\sqrt{b+ax^3}\right)\\ &= \frac{1}{3}a\sqrt{b+ax^3}+\frac{\left(b+ax^3\right)^{3/2}}{3x^3}-\frac{1}{3}a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b+ax^3}}{\sqrt{b}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.98

$$\frac{1}{3}\left(\frac{\sqrt{ax^3+b}\left(2ax^3+b\right)}{x^3}-a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + a*x^3)*Sqrt[b + a*x^3])/x^4,x]
```

```
[Out] ((Sqrt[b + a*x^3]*(b + 2*a*x^3))/x^3 - a*Sqrt[b]*ArcTanh[Sqrt[b + a*x^3]/Sq
rt[b]])/3
```

IntegrateAlgebraic [A] time = 0.10, size = 55, normalized size = 1.00

$$\frac{\sqrt{ax^3+b}\left(2ax^3+b\right)}{3x^3}-\frac{1}{3}a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^3)*Sqrt[b + a*x^3])/x^4,x]
```

```
[Out] (Sqrt[b + a*x^3]*(b + 2*a*x^3))/(3*x^3) - (a*Sqrt[b]*ArcTanh[Sqrt[b + a*x^3
]/Sqrt[b]])/3
```

fricas [A] time = 0.43, size = 115, normalized size = 2.09

$$\left[\frac{a\sqrt{b}\,x^3\log\left(\frac{ax^3-2\sqrt{ax^3+b}\sqrt{b}+2b}{x^3}\right)+2\left(2\,ax^3+b\right)\sqrt{ax^3+b}}{6\,x^3},\ \frac{a\sqrt{-b}\,x^3\arctan\left(\frac{\sqrt{ax^3+b}\sqrt{-b}}{b}\right)+\left(2\,ax^3+b\right)\sqrt{ax^3+b}}{3\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(a*x^3+b)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(a*sqrt(b)*x^3*log((a*x^3 - 2*sqrt(a*x^3 + b)*sqrt(b) + 2*b)/x^3) + 2*
(2*a*x^3 + b)*sqrt(a*x^3 + b))/x^3, 1/3*(a*sqrt(-b)*x^3*arctan(sqrt(a*x^3 +
 b)*sqrt(-b)/b) + (2*a*x^3 + b)*sqrt(a*x^3 + b))/x^3]
```

giac [**A**] time = 0.19, size = 61, normalized size = 1.11

$$\frac{\frac{a^2 b \arctan\left(\frac{\sqrt{ax^3+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + 2\sqrt{ax^3+b}\,a^2 + \frac{\sqrt{ax^3+b}\,ab}{x^3}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(a*x^3+b)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(a^2*b*arctan(sqrt(a*x^3 + b)/sqrt(-b))/sqrt(-b) + 2*sqrt(a*x^3 + b)*a^
2 + sqrt(a*x^3 + b)*a*b/x^3)/a
```

maple [**A**] time = 0.05, size = 73, normalized size = 1.33

$$-b\left(-\frac{\sqrt{a\,x^3+b}}{3x^3}-\frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^3+b}}{\sqrt{b}}\right)}{3\sqrt{b}}\right)+a\left(\frac{2\sqrt{a\,x^3+b}}{3}-\frac{2\sqrt{b}\ \mathrm{arctanh}\left(\frac{\sqrt{a\,x^3+b}}{\sqrt{b}}\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b)*(a*x^3+b)^(1/2)/x^4,x)
```

```
[Out] -b*(-1/3*(a*x^3+b)^(1/2)/x^3-1/3*a*arctanh((a*x^3+b)^(1/2)/b^(1/2))/b^(1/2)
)+a*(2/3*(a*x^3+b)^(1/2)-2/3*b^(1/2)*arctanh((a*x^3+b)^(1/2)/b^(1/2)))
```

maxima [**B**] time = 0.43, size = 107, normalized size = 1.95

$$\frac{1}{3}\left(\sqrt{b}\,\log\left(\frac{\sqrt{ax^3+b}-\sqrt{b}}{\sqrt{ax^3+b}+\sqrt{b}}\right)+2\sqrt{ax^3+b}\right)a-\frac{1}{6}\left(\frac{a\log\left(\frac{\sqrt{ax^3+b}-\sqrt{b}}{\sqrt{ax^3+b}+\sqrt{b}}\right)}{\sqrt{b}}-\frac{2\sqrt{ax^3+b}}{x^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(a*x^3+b)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(b)*log((sqrt(a*x^3 + b) - sqrt(b))/(sqrt(a*x^3 + b) + sqrt(b))) +
 2*sqrt(a*x^3 + b))*a - 1/6*(a*log((sqrt(a*x^3 + b) - sqrt(b))/(sqrt(a*x^3
+ b) + sqrt(b)))/sqrt(b) - 2*sqrt(a*x^3 + b)/x^3)*b
```

mupad [**B**] time = 0.81, size = 69, normalized size = 1.25

$$\frac{2\,a\,\sqrt{a\,x^3+b}}{3}+\frac{b\,\sqrt{a\,x^3+b}}{3\,x^3}+\frac{a\,\sqrt{b}\ \ln\left(\frac{\left(\sqrt{a\,x^3+b}-\sqrt{b}\right)^3\,\left(\sqrt{a\,x^3+b}+\sqrt{b}\right)}{x^6}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b + a*x^3)^(1/2)*(b - a*x^3))/x^4,x)
```

```
[Out] (2*a*(b + a*x^3)^(1/2))/3 + (b*(b + a*x^3)^(1/2))/(3*x^3) + (a*b^(1/2)*log(
(((b + a*x^3)^(1/2) - b^(1/2))^3*((b + a*x^3)^(1/2) + b^(1/2)))/x^6))/6
```

sympy [**B**] time = 25.80, size = 102, normalized size = 1.85

$$\frac{2a^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{1+\frac{b}{ax^3}}}+\frac{\sqrt{a}\,b\sqrt{1+\frac{b}{ax^3}}}{3x^{\frac{3}{2}}}+\frac{2\sqrt{a}\,b}{3x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^3}}}-\frac{a\sqrt{b}\ \mathrm{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b)*(a*x**3+b)**(1/2)/x**4,x)
```

```
[Out] 2*a**(3/2)*x**(3/2)/(3*sqrt(1 + b/(a*x**3))) + sqrt(a)*b*sqrt(1 + b/(a*x**3
))/(3*x**(3/2)) + 2*sqrt(a)*b/(3*x**(3/2)*sqrt(1 + b/(a*x**3))) - a*sqrt(b)
*asinh(sqrt(b)/(sqrt(a)*x**(3/2)))/3
```

3.691 $\displaystyle\int \frac{1-2x+k^2x^2}{\left(-1+2x-2x^2+k^2x^2\right)\sqrt{x-x^2-k^2x^2+k^2x^3}}\,dx$

Optimal. Leaf size=55

$$\frac{2\tan^{-1}\left(\frac{\sqrt{k^2-2}\,\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{k^2x-1}\right)}{\sqrt{k^2-2}}$$

Rubi [C] time = 2.76, antiderivative size = 508, normalized size of antiderivative = 9.24, number of steps used = 17, number of rules used = 9, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.155, Rules used = {6, 2056, 6728, 716, 1103, 934, 168, 538, 537}

$$-\frac{k^{3/2}\sqrt{x}\,(kx+1)\sqrt{\frac{k^2x^2-\left(k^2+1\right)x+1}{(kx+1)^2}}\,F\left(2\tan^{-1}\left(\sqrt{k}\,\sqrt{x}\right)|\frac{(k+1)^2}{4k}\right)}{\left(2-k^2\right)\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}}-\frac{2\sqrt{-k^4+2k^2-1}\left(k^2+2\sqrt{k^2-1}\right)\sqrt{x}\sqrt{\frac{k^2(1-x)}{1-k^2}+1}\,\sqrt{k^2x-k^2}\,\Pi\left(-\frac{k^4-3k^2+2}{k^2\left(-k^2-\sqrt{k^2-1}+1\right)};\sin^{-1}\left(\frac{\sqrt{k^2x-k^2}}{\sqrt{1-k^2}}\right)|1-\frac{1}{k^2}\right)}{\left(2-k^2\right)\left(-k^2-\sqrt{k^2-1}+1\right)k^2\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}}+\frac{2\sqrt{-k^4+2k^2-1}\left(k^2-2\sqrt{k^2-1}\right)\sqrt{x}\sqrt{\frac{k^2(1-x)}{1-k^2}+1}\,\sqrt{k^2x-k^2}\,\Pi\left(-\frac{k^4-3k^2+2}{k^2\left(-k^2+\sqrt{k^2-1}+1\right)};\sin^{-1}\left(\frac{\sqrt{k^2x-k^2}}{\sqrt{1-k^2}}\right)|1-\frac{1}{k^2}\right)}{\left(2-k^2\right)\left(-k^2+\sqrt{k^2-1}+1\right)k^2\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - 2*x + k^2*x^2)/((-1 + 2*x - 2*x^2 + k^2*x^2)*Sqrt[x - x^2 - k^2*x^
2 + k^2*x^3]),x]
```

```
[Out] -((k^(3/2)*Sqrt[x]*(1 + k*x)*Sqrt[(1 - (1 + k^2)*x + k^2*x^2)/(1 + k*x)^2]*
EllipticF[2*ArcTan[Sqrt[k]*Sqrt[x]], (1 + k)^2/(4*k)])/((2 - k^2)*Sqrt[x -
(1 + k^2)*x^2 + k^2*x^3])) - (2*Sqrt[-1 + 2*k^2 - k^4]*(k^2 + 2*Sqrt[-1 + k
^2])*Sqrt[1 + (k^2*(1 - x))/(1 - k^2)]*Sqrt[x]*Sqrt[-k^2 + k^2*x]*EllipticP
i[-((2 - 3*k^2 + k^4)/(k^2*(1 - k^2 - Sqrt[-1 + k^2]))), ArcSin[Sqrt[-k^2 +
 k^2*x]/Sqrt[1 - k^2]], 1 - k^(-2)])/(k^2*(2 - k^2)*(1 - k^2 - Sqrt[-1 + k^
2])*Sqrt[x - (1 + k^2)*x^2 + k^2*x^3]) + (2*Sqrt[-1 + 2*k^2 - k^4]*(k^2 - 2
*Sqrt[-1 + k^2])*Sqrt[1 + (k^2*(1 - x))/(1 - k^2)]*Sqrt[x]*Sqrt[-k^2 + k^2*
x]*EllipticPi[-((2 - 3*k^2 + k^4)/(k^2*(1 - k^2 + Sqrt[-1 + k^2]))), ArcSin
[Sqrt[-k^2 + k^2*x]/Sqrt[1 - k^2]], 1 - k^(-2)])/(k^2*(2 - k^2)*(1 - k^2 +
Sqrt[-1 + k^2])*Sqrt[x - (1 + k^2)*x^2 + k^2*x^3])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 716

```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2,
Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1-2x+k^2x^2}{\left(-1+2x-2x^2+k^2x^2\right)\sqrt{x-x^2-k^2x^2+k^2x^3}}\,dx = \int \frac{1-2x+k^2x^2}{\left(-1+2x+\left(-2+k^2\right)x^2\right)\sqrt{x-x^2-k^2x^2+k^2x^3}}\,dx$$

$$= \int \frac{1-2x+k^2x^2}{\left(-1+2x+\left(-2+k^2\right)x^2\right)\sqrt{x+\left(-1-k^2\right)x^2+k^2x^3}}\,dx$$

$$= \frac{\left(\sqrt{x}\sqrt{1+\left(-1-k^2\right)x+k^2x^2}\right)\int \frac{1-2x+k^2x^2}{\sqrt{x}\sqrt{1+\left(-1-k^2\right)x+k^2x^2}\left(-1+2\right.}}{\sqrt{x+\left(-1-k^2\right)x^2+k^2x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+\left(-1-k^2\right)x+k^2x^2}\right)\int\left(-\frac{k^2}{\left(2-k^2\right)\sqrt{x}\sqrt{1+\left(-1-k^2\right)x+k}}\right.}{\sqrt{x+\left(-1-k^2\right)}}$$

$$= -\frac{\left(k^2\sqrt{x}\sqrt{1+\left(-1-k^2\right)x+k^2x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+\left(-1-k^2\right)x+k^2x^2}}\,d}{\left(2-k^2\right)\sqrt{x+\left(-1-k^2\right)x^2+k^2x^3}}$$

$$= -\frac{\left(2k^2\sqrt{x}\sqrt{1+\left(-1-k^2\right)x+k^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\left(-1-k^2\right)x}}\right.}{\left(2-k^2\right)\sqrt{x+\left(-1-k^2\right)x^2+k^2x^3}}$$

$$= -\frac{k^{3/2}\sqrt{x}\,(1+kx)\sqrt{\frac{1-\left(1+k^2\right)x+k^2x^2}{(1+kx)^2}}\,F\left(2\tan^{-1}\left(\sqrt{k}\sqrt{x}\right)|\frac{(1+k}{4k}\right.}{\left(2-k^2\right)\sqrt{x-\left(1+k^2\right)x^2+k^2x^3}}$$

$$= -\frac{k^{3/2}\sqrt{x}\,(1+kx)\sqrt{\frac{1-\left(1+k^2\right)x+k^2x^2}{(1+kx)^2}}\,F\left(2\tan^{-1}\left(\sqrt{k}\sqrt{x}\right)|\frac{(1+k}{4k}\right.}{\left(2-k^2\right)\sqrt{x-\left(1+k^2\right)x^2+k^2x^3}}$$

$$= -\frac{k^{3/2}\sqrt{x}\,(1+kx)\sqrt{\frac{1-\left(1+k^2\right)x+k^2x^2}{(1+kx)^2}}\,F\left(2\tan^{-1}\left(\sqrt{k}\sqrt{x}\right)|\frac{(1+k}{4k}\right.}{\left(2-k^2\right)\sqrt{x-\left(1+k^2\right)x^2+k^2x^3}}$$

$$= -\frac{k^{3/2}\sqrt{x}\,(1+kx)\sqrt{\frac{1-\left(1+k^2\right)x+k^2x^2}{(1+kx)^2}}\,F\left(2\tan^{-1}\left(\sqrt{k}\sqrt{x}\right)|\frac{(1+k}{4k}\right.}{\left(2-k^2\right)\sqrt{x-\left(1+k^2\right)x^2+k^2x^3}}$$

$$= -\frac{k^{3/2}\sqrt{x}\,(1+kx)\sqrt{\frac{1-\left(1+k^2\right)x+k^2x^2}{(1+kx)^2}}\,F\left(2\tan^{-1}\left(\sqrt{k}\sqrt{x}\right)|\frac{(1+k}{4k}\right.}{\left(2-k^2\right)\sqrt{x-\left(1+k^2\right)x^2+k^2x^3}}$$

Mathematica [C] time = 0.76, size = 202, normalized size = 3.67

$$\frac{2i\sqrt{\frac{1}{x-1}+1}\,(x-1)^{3/2}\sqrt{\frac{1-\frac{1}{k^2}}{x-1}+1}\left(\left(k^2-2\right)F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+\left(\sqrt{k^2-1}+1\right)\Pi\left(\frac{k^2-1}{k^2-\sqrt{k^2-1}-1};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)-\left(\sqrt{k^2-1}-1\right)\Pi\left(\frac{k^2-1}{k^2+\sqrt{k^2-1}-1};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)}{\left(k^2-2\right)\sqrt{(x-1)x\left(k^2x-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x + k^2*x^2)/((-1 + 2*x - 2*x^2 + k^2*x^2)*Sqrt[x - x^2 -
k^2*x^2 + k^2*x^3]),x]
```

```
[Out] ((2*I)*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (1 - k^(-2))/(-1 + x)]*(-1 + x)^(3/
2)*((-2 + k^2)*EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] + (1 + Sqrt
[-1 + k^2])*EllipticPi[(-1 + k^2)/(-1 + k^2 - Sqrt[-1 + k^2]), I*ArcSinh[1/
Sqrt[-1 + x]], 1 - k^(-2)] - (-1 + Sqrt[-1 + k^2])*EllipticPi[(-1 + k^2)/(-
1 + k^2 + Sqrt[-1 + k^2]), I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)]))/((-2 +
k^2)*Sqrt[(-1 + x)*x*(-1 + k^2*x)])
```

IntegrateAlgebraic [A] time = 0.21, size = 55, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{k^2-2}\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{k^2x-1}\right)}{\sqrt{k^2-2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - 2*x + k^2*x^2)/((-1 + 2*x - 2*x^2 + k^2*x^2)*Sqrt[x
 - x^2 - k^2*x^2 + k^2*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-2 + k^2]*Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3])/(-1 + k^2*x)]
)/Sqrt[-2 + k^2]
```

fricas [A] time = 0.51, size = 269, normalized size = 4.89

$$\left[-\frac{\sqrt{-k^2+2}\log\left(\frac{\left(k^4-4k^2+4\right)x^4-4\left(2k^4-5k^2+2\right)x^3+2\left(4k^4-5k^2-4\right)x^2-4\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\left(\left(k^2-2\right)x^2-2\left(k^2-1\right)x+1\right)\sqrt{-k^2+2}-4\left(2k^2-3\right)x+1}{\left(k^4-4k^2+4\right)x^4+4\left(k^2-2\right)x^3-2\left(k^2-4\right)x^2-4x+1}\right)}{2\left(k^2-2\right)}, \frac{\arctan\left(\frac{\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\left(\left(k^2-2\right)x^2-2\left(k^2-1\right)x+1\right)\sqrt{k^2-2}}{2\left(\left(k^4-2k^2\right)x^3-\left(k^4-k^2-2\right)x^2+\left(k^2-2\right)x\right)}\right)}{\sqrt{k^2-2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-2*x+1)/(k^2*x^2-2*x^2+2*x-1)/(k^2*x^3-k^2*x^2-x^2+x)^(1/
2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-k^2 + 2)*log(((k^4 - 4*k^2 + 4)*x^4 - 4*(2*k^4 - 5*k^2 + 2)*x^3
 + 2*(4*k^4 - 5*k^2 - 4)*x^2 - 4*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*((k^2 -
2)*x^2 - 2*(k^2 - 1)*x + 1)*sqrt(-k^2 + 2) - 4*(2*k^2 - 3)*x + 1)/((k^4 - 4
*k^2 + 4)*x^4 + 4*(k^2 - 2)*x^3 - 2*(k^2 - 4)*x^2 - 4*x + 1))/(k^2 - 2), ar
ctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*((k^2 - 2)*x^2 - 2*(k^2 - 1)*x +
 1)*sqrt(k^2 - 2)/((k^4 - 2*k^2)*x^3 - (k^4 - k^2 - 2)*x^2 + (k^2 - 2)*x))/
sqrt(k^2 - 2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^2 - 2\,x + 1}{\sqrt{k^2x^3 - k^2x^2 - x^2 + x}\left(k^2x^2 - 2\,x^2 + 2\,x - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-2*x+1)/(k^2*x^2-2*x^2+2*x-1)/(k^2*x^3-k^2*x^2-x^2+x)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^2 - 2*x + 1)/(sqrt(k^2*x^3 - k^2*x^2 - x^2 + x)*(k^2*x^2 -
 2*x^2 + 2*x - 1)), x)
```

maple [C] time = 0.04, size = 2705, normalized size = 49.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^2-2*x+1)/(k^2*x^2-2*x^2+2*x-1)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2),x)
```

```
[Out] -2/(k^2-2)*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k
^2*x^3-k^2*x^2-x^2+x)^(1/2)*EllipticF((-(x-1/k^2)*k^2)^(1/2),(1/k^2/(1/k^2-
1))^(1/2))+2/(k^2-2)*(-4/(-2*k^2/(k^2-2)+2/(k^2-2)*k^2*(k^2-1)^(1/2)+4/(k^2
-2)-4/(k^2-2)*(k^2-1)^(1/2)+2)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))
^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/(k^2-2)-1/(k^2-
2)*(k^2-1)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-(-1+(k^2-1
)^(1/2))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2))/(k^2-2)+4/(-2*k^2/(k^2-2)+2/(k^2
-2)*k^2*(k^2-1)^(1/2)+4/(k^2-2)-4/(k^2-2)*(k^2-1)^(1/2)+2)/k^2*(-k^2*x+1)^(
1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x
)^(1/2)/(1/k^2+1/(k^2-2)-1/(k^2-2)*(k^2-1)^(1/2))*EllipticPi((-(x-1/k^2)*k^
2)^(1/2),1/k^2/(1/k^2-(-1+(k^2-1)^(1/2))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2))/
(k^2-2)+4/(-2*k^2/(k^2-2)+2/(k^2-2)*k^2*(k^2-1)^(1/2)+4/(k^2-2)-4/(k^2-2)*(
k^2-1)^(1/2)+2)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^
(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/(k^2-2)-1/(k^2-2)*(k^2-1)^(1/2
))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-(-1+(k^2-1)^(1/2))/(k^2-2
)),(1/k^2/(1/k^2-1))^(1/2))/(k^2-2)*(k^2-1)^(1/2)-4/(-2*k^2/(k^2-2)+2/(k^2-
2)*k^2*(k^2-1)^(1/2)+4/(k^2-2)-4/(k^2-2)*(k^2-1)^(1/2)+2)/k^2*(-k^2*x+1)^(1
/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)
^(1/2)/(1/k^2+1/(k^2-2)-1/(k^2-2)*(k^2-1)^(1/2))*EllipticPi((-(x-1/k^2)*k^2
)^(1/2),1/k^2/(1/k^2-(-1+(k^2-1)^(1/2))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2))/(
k^2-2)*(k^2-1)^(1/2)-2/(-2*k^2/(k^2-2)+2/(k^2-2)*k^2*(k^2-1)^(1/2)+4/(k^2-2
)-4/(k^2-2)*(k^2-1)^(1/2)+2)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(
1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/(k^2-2)-1/(k^2-2)
*(k^2-1)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-(-1+(k^2-1)^
(1/2))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2))+2/(-2*k^2/(k^2-2)+2/(k^2-2)*k^2*(k
^2-1)^(1/2)+4/(k^2-2)-4/(k^2-2)*(k^2-1)^(1/2)+2)/k^2*(-k^2*x+1)^(1/2)*(1/(1
/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1
/k^2+1/(k^2-2)-1/(k^2-2)*(k^2-1)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1
/k^2/(1/k^2-(-1+(k^2-1)^(1/2))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2))-4/(-2*k^2/
(k^2-2)-2/(k^2-2)*k^2*(k^2-1)^(1/2)+4/(k^2-2)+4/(k^2-2)*(k^2-1)^(1/2)+2)*(-
k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2
*x^2-x^2+x)^(1/2)/(1/k^2+1/(k^2-2)+1/(k^2-2)*(k^2-1)^(1/2))*EllipticPi((-(x
-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+(1+(k^2-1)^(1/2))/(k^2-2)),(1/k^2/(1/k^2-1)
)^(1/2))/(k^2-2)+4/(-2*k^2/(k^2-2)-2/(k^2-2)*k^2*(k^2-1)^(1/2)+4/(k^2-2)+4/
(k^2-2)*(k^2-1)^(1/2)+2)/k^2*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(
1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/(k^2-2)+1/(k^2-2)
*(k^2-1)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+(1+(k^2-1)^(
1/2))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2))/(k^2-2)-4/(-2*k^2/(k^2-2)-2/(k^2-2)
*k^2*(k^2-1)^(1/2)+4/(k^2-2)+4/(k^2-2)*(k^2-1)^(1/2)+2)*(-k^2*x+1)^(1/2)*(1
/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)
/(1/k^2+1/(k^2-2)+1/(k^2-2)*(k^2-1)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2
),1/k^2/(1/k^2+(1+(k^2-1)^(1/2))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2))/(k^2-2)*
(k^2-1)^(1/2)+4/(-2*k^2/(k^2-2)-2/(k^2-2)*k^2*(k^2-1)^(1/2)+4/(k^2-2)+4/(k^
2-2)*(k^2-1)^(1/2)+2)/k^2*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2
)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/(k^2-2)+1/(k^2-2)*(k
^2-1)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+(1+(k^2-1)^(1/2
))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2))/(k^2-2)*(k^2-1)^(1/2)-2/(-2*k^2/(k^2-2
)-2/(k^2-2)*k^2*(k^2-1)^(1/2)+4/(k^2-2)+4/(k^2-2)*(k^2-1)^(1/2)+2)*(-k^2*x+
1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x
^2+x)^(1/2)/(1/k^2+1/(k^2-2)+1/(k^2-2)*(k^2-1)^(1/2))*EllipticPi((-(x-1/k^2
)*k^2)^(1/2),1/k^2/(1/k^2+(1+(k^2-1)^(1/2))/(k^2-2)),(1/k^2/(1/k^2-1))^(1/2
))+2/(-2*k^2/(k^2-2)-2/(k^2-2)*k^2*(k^2-1)^(1/2)+4/(k^2-2)+4/(k^2-2)*(k^2-1
)^(1/2)+2)/k^2*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(
1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/(k^2-2)+1/(k^2-2)*(k^2-1)^(1/2)
)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+(1+(k^2-1)^(1/2))/(k^2-2))
,(1/k^2/(1/k^2-1))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-2*x+1)/(k^2*x^2-2*x^2+2*x-1)/(k^2*x^3-k^2*x^2-x^2+x)^(1/
2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(k-1>0)', see `assume?` for more det
ails)Is k-1 positive, negative or zero?
```

mupad [B] time = 1.83, size = 84, normalized size = 1.53

$$\frac{\ln\left(\frac{x\,2\mathrm{i}+k^2\,x^2\,1\mathrm{i}-k^2\,x\,2\mathrm{i}-x^2\,2\mathrm{i}-2\,\sqrt{k^2-2}\,\sqrt{x\,\left(k^2\,x-1\right)\,\left(x-1\right)}+1\mathrm{i}}{2\,k^2\,x^2-4\,x^2+4\,x-2}\right)\,1\mathrm{i}}{\sqrt{k^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^2 - 2*x + 1)/((x - k^2*x^2 + k^2*x^3 - x^2)^(1/2)*(2*x + k^2*x^2
 - 2*x^2 - 1)),x)
```

```
[Out] (log((x*2i + k^2*x^2*1i - k^2*x*2i - x^2*2i - 2*(k^2 - 2)^(1/2)*(x*(k^2*x -
 1)*(x - 1))^(1/2) + 1i)/(4*x + 2*k^2*x^2 - 4*x^2 - 2))*1i)/(k^2 - 2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^2-2x+1}{\sqrt{x\left(x-1\right)\left(k^2x-1\right)}\left(k^2x^2-2x^2+2x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k**2*x**2-2*x+1)/(k**2*x**2-2*x**2+2*x-1)/(k**2*x**3-k**2*x**2-x
**2+x)**(1/2),x)
```

```
[Out] Integral((k**2*x**2 - 2*x + 1)/(sqrt(x*(x - 1)*(k**2*x - 1))*(k**2*x**2 - 2
*x**2 + 2*x - 1)), x)
```

3.692 $\int \frac{2+x}{x\sqrt[4]{1+x^4}}\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\tan^{-1}\left(\sqrt[4]{x^4+1}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)$$

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1833, 240, 212, 206, 203, 266, 63, 298}

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\tan^{-1}\left(\sqrt[4]{x^4+1}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(x*(1 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)]/2 + ArcTan[(1 + x^4)^(1/4)] + ArcTanh[x/(1 + x^4)
^(1/4)]/2 - ArcTanh[(1 + x^4)^(1/4)]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x}{x\sqrt[4]{1+x^4}}\,dx &= \int \left(\frac{1}{\sqrt[4]{1+x^4}}+\frac{2}{x\sqrt[4]{1+x^4}}\right) dx\\
&= 2\int \frac{1}{x\sqrt[4]{1+x^4}}\,dx+\int \frac{1}{\sqrt[4]{1+x^4}}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{1+x}}\,dx,x,x^4\right)+\operatorname{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)+2\operatorname{Subst}\left(\int \cdots\right.\\
&= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[4]{1+x^4}\right)+\operatorname{Subst}\left(\int \cdots\right.\\
&= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\tan^{-1}\left(\sqrt[4]{1+x^4}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\tanh^{-1}\left(\sqrt[4]{1+x^4}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\tan^{-1}\left(\sqrt[4]{x^4+1}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(x*(1 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)]/2 + ArcTan[(1 + x^4)^(1/4)] + ArcTanh[x/(1 + x^4)
^(1/4)]/2 - ArcTanh[(1 + x^4)^(1/4)]
```

IntegrateAlgebraic [A] time = 4.55, size = 55, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\tan^{-1}\left(\sqrt[4]{x^4+1}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)/(x*(1 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)]/2 + ArcTan[(1 + x^4)^(1/4)] + ArcTanh[x/(1 + x^4)
^(1/4)]/2 - ArcTanh[(1 + x^4)^(1/4)]
```

fricas [B] time = 9.52, size = 134, normalized size = 2.44

$$\frac{1}{4}\arctan\left(2\left(x^4+1\right)^{\frac{1}{4}}x^3+2\left(x^4+1\right)^{\frac{3}{4}}x\right)-\frac{1}{2}\arctan\left(\frac{2\left(\left(x^4+1\right)^{\frac{3}{4}}+\left(x^4+1\right)^{\frac{1}{4}}\right)}{x^4}\right)+\frac{1}{4}\log\left(2x^4+2\left(x^4+1\right)^{\frac{1}{4}}x^3+2\sqrt{x^4+1}\,x^2+2\left(x^4+1\right)^{\frac{3}{4}}x+1\right)+\frac{1}{2}\log\left(-\frac{x^4-2\left(x^4+1\right)^{\frac{3}{4}}+2\sqrt{x^4+1}-2\left(x^4+1\right)^{\frac{1}{4}}+2}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/x/(x^4+1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/4*arctan(2*(x^4 + 1)^(1/4)*x^3 + 2*(x^4 + 1)^(3/4)*x) - 1/2*arctan(2*((x^
4 + 1)^(3/4) + (x^4 + 1)^(1/4))/x^4) + 1/4*log(2*x^4 + 2*(x^4 + 1)^(1/4)*x^
3 + 2*sqrt(x^4 + 1)*x^2 + 2*(x^4 + 1)^(3/4)*x + 1) + 1/2*log(-(x^4 - 2*(x^4
 + 1)^(3/4) + 2*sqrt(x^4 + 1) - 2*(x^4 + 1)^(1/4) + 2)/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\left(x^4+1\right)^{\frac{1}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/x/(x^4+1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x + 2)/((x^4 + 1)^(1/4)*x), x)
```

maple [C] time = 0.25, size = 73, normalized size = 1.33

$$\frac{\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)\left(-\frac{\pi\sqrt{2}\,x^4\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{4}\right],[2,2],-x^4\right)}{4\Gamma\left(\frac{3}{4}\right)}+\frac{\left(-3\ln(2)-\frac{\pi}{2}+4\ln(x)\right)\pi\sqrt{2}}{\Gamma\left(\frac{3}{4}\right)}\right)}{4\pi}+x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{4}\right],\left[\frac{5}{4}\right],-x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/x/(x^4+1)^(1/4),x)
```

```
[Out] 1/4/Pi*2^(1/2)*GAMMA(3/4)*(-1/4*Pi*2^(1/2)/GAMMA(3/4)*x^4*hypergeom([1,1,5/
4],[2,2],-x^4)+(-3*ln(2)-1/2*Pi+4*ln(x))*Pi*2^(1/2)/GAMMA(3/4))+x*hypergeom
([1/4,1/4],[5/4],-x^4)
```

maxima [A] time = 0.43, size = 79, normalized size = 1.44

$$\arctan\left(\left(x^4+1\right)^{\frac{1}{4}}\right)-\frac{1}{2}\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{2}\log\left(\left(x^4+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{2}\log\left(\left(x^4+1\right)^{\frac{1}{4}}-1\right)+\frac{1}{4}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}+1\right)-\frac{1}{4}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/x/(x^4+1)^(1/4),x, algorithm="maxima")
```

```
[Out] arctan((x^4 + 1)^(1/4)) - 1/2*arctan((x^4 + 1)^(1/4)/x) - 1/2*log((x^4 + 1)
^(1/4) + 1) + 1/2*log((x^4 + 1)^(1/4) - 1) + 1/4*log((x^4 + 1)^(1/4)/x + 1)
 - 1/4*log((x^4 + 1)^(1/4)/x - 1)
```

mupad [B] time = 0.78, size = 31, normalized size = 0.56

$$\mathrm{atan}\left(\left(x^4+1\right)^{1/4}\right)-\mathrm{atanh}\left(\left(x^4+1\right)^{1/4}\right)+x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/(x*(x^4 + 1)^(1/4)),x)
```

```
[Out] atan((x^4 + 1)^(1/4)) - atanh((x^4 + 1)^(1/4)) + x*hypergeom([1/4, 1/4], 5/
4, -x^4)
```

sympy [C] time = 2.40, size = 56, normalized size = 1.02

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}-\frac{\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle| \frac{e^{i\pi}}{x^4}\right)}{2x\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/x/(x**4+1)**(1/4),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/4), (5/4,), x**4*exp_polar(I*pi))/(4*gamma(5/4))
 - gamma(1/4)*hyper((1/4, 1/4), (5/4,), exp_polar(I*pi)/x**4)/(2*x*gamma(5/
4))
```

3.693 $\int x^6 \sqrt[4]{1+x^4}\, dx$

Optimal. Leaf size=55

$$\frac{3}{64}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{3}{64}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{32}\sqrt[4]{x^4+1}\left(4x^7+x^3\right)$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = $\{279, 321, 331, 298, 203, 206\}$

$$\frac{3}{64}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{3}{64}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{8}\sqrt[4]{x^4+1}\,x^7+\frac{1}{32}\sqrt[4]{x^4+1}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(1 + x^4)^(1/4),x]
```

```
[Out] (x^3*(1 + x^4)^(1/4))/32 + (x^7*(1 + x^4)^(1/4))/8 + (3*ArcTan[x/(1 + x^4)^
(1/4)])/64 - (3*ArcTanh[x/(1 + x^4)^(1/4)])/64
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int x^6\sqrt[4]{1+x^4}\,dx &= \frac{1}{8}x^7\sqrt[4]{1+x^4}+\frac{1}{8}\int\frac{x^6}{\left(1+x^4\right)^{3/4}}\,dx\\
&= \frac{1}{32}x^3\sqrt[4]{1+x^4}+\frac{1}{8}x^7\sqrt[4]{1+x^4}-\frac{3}{32}\int\frac{x^2}{\left(1+x^4\right)^{3/4}}\,dx\\
&= \frac{1}{32}x^3\sqrt[4]{1+x^4}+\frac{1}{8}x^7\sqrt[4]{1+x^4}-\frac{3}{32}\mathrm{Subst}\left(\int\frac{x^2}{1-x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)\\
&= \frac{1}{32}x^3\sqrt[4]{1+x^4}+\frac{1}{8}x^7\sqrt[4]{1+x^4}-\frac{3}{64}\mathrm{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{3}{64}\mathrm{Subst}\left(\int\frac{1}{1+x}\right.\\
&= \frac{1}{32}x^3\sqrt[4]{1+x^4}+\frac{1}{8}x^7\sqrt[4]{1+x^4}+\frac{3}{64}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{3}{64}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.62

$$\frac{1}{8}x^3\left(\left(x^4+1\right)^{5/4}-{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(1 + x^4)^(1/4),x]
[Out] (x^3*((1 + x^4)^(5/4) - Hypergeometric2F1[-1/4, 3/4, 7/4, -x^4]))/8
```

IntegrateAlgebraic [A] time = 0.19, size = 55, normalized size = 1.00

$$\frac{3}{64}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{3}{64}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{32}\sqrt[4]{x^4+1}\left(4x^7+x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6*(1 + x^4)^(1/4),x]
[Out] ((1 + x^4)^(1/4)*(x^3 + 4*x^7))/32 + (3*ArcTan[x/(1 + x^4)^(1/4)])/64 - (3*
ArcTanh[x/(1 + x^4)^(1/4)])/64
```

fricas [A] time = 0.40, size = 68, normalized size = 1.24

$$\frac{1}{32}\left(4x^7+x^3\right)\left(x^4+1\right)^{\frac{1}{4}}-\frac{3}{64}\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-\frac{3}{128}\log\left(\frac{x+\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+\frac{3}{128}\log\left(-\frac{x-\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^4+1)^(1/4),x, algorithm="fricas")
[Out] 1/32*(4*x^7 + x^3)*(x^4 + 1)^(1/4) - 3/64*arctan((x^4 + 1)^(1/4)/x) - 3/128
*log((x + (x^4 + 1)^(1/4))/x) + 3/128*log(-(x - (x^4 + 1)^(1/4))/x)
```

giac [A] time = 0.17, size = 81, normalized size = 1.47

$$\frac{1}{32}x^8\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}\left(\frac{1}{x^4}+1\right)}{x}+\frac{3\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-\frac{3}{64}\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-\frac{3}{128}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}+1\right)+\frac{3}{128}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^4+1)^(1/4),x, algorithm="giac")
```

```
[Out] 1/32*x^8*((x^4 + 1)^(1/4)*(1/x^4 + 1)/x + 3*(x^4 + 1)^(1/4)/x) - 3/64*arcta
n((x^4 + 1)^(1/4)/x) - 3/128*log((x^4 + 1)^(1/4)/x + 1) + 3/128*log((x^4 +
1)^(1/4)/x - 1)
```

maple [C] time = 0.27, size = 37, normalized size = 0.67

$$\frac{x^{3}\left(4x^{4}+1\right)\left(x^{4}+1\right)^{\frac{1}{4}}}{32}-\frac{x^{3}\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],-x^{4}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^4+1)^(1/4),x)
```

```
[Out] 1/32*x^3*(4*x^4+1)*(x^4+1)^(1/4)-1/32*x^3*hypergeom([3/4,3/4],[7/4],-x^4)
```

maxima [B] time = 0.42, size = 99, normalized size = 1.80

$$-\frac{\frac{3\left(x^{4}+1\right)^{\frac{1}{4}}}{x}+\frac{\left(x^{4}+1\right)^{\frac{5}{4}}}{x^{5}}}{32\left(\frac{2\left(x^{4}+1\right)}{x^{4}}-\frac{\left(x^{4}+1\right)^{2}}{x^{8}}-1\right)}-\frac{3}{64}\arctan\left(\frac{\left(x^{4}+1\right)^{\frac{1}{4}}}{x}\right)-\frac{3}{128}\log\left(\frac{\left(x^{4}+1\right)^{\frac{1}{4}}}{x}+1\right)+\frac{3}{128}\log\left(\frac{\left(x^{4}+1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^4+1)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/32*(3*(x^4 + 1)^(1/4)/x + (x^4 + 1)^(5/4)/x^5)/(2*(x^4 + 1)/x^4 - (x^4 +
 1)^2/x^8 - 1) - 3/64*arctan((x^4 + 1)^(1/4)/x) - 3/128*log((x^4 + 1)^(1/4)
/x + 1) + 3/128*log((x^4 + 1)^(1/4)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{6}\left(x^{4}+1\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^4 + 1)^(1/4),x)
```

```
[Out] int(x^6*(x^4 + 1)^(1/4), x)
```

sympy [C] time = 1.04, size = 31, normalized size = 0.56

$$\frac{x^{7}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|x^{4}e^{i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(x**4+1)**(1/4),x)
```

```
[Out] x**7*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), x**4*exp_polar(I*pi))/(4*gamma(
11/4))
```

3.694 $\int \frac{\sqrt[4]{-x+x^4}}{x^2}\,dx$

Optimal. Leaf size=55

$$-\frac{4\sqrt[4]{x^4-x}}{3x}-\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x}}\right)+\frac{2}{3}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x}}\right)$$

Rubi [A] time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.98, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2020, 2032, 329, 275, 331, 298, 203, 206}

$$-\frac{4\sqrt[4]{x^4-x}}{3x}-\frac{2x^{3/4}\left(x^3-1\right)^{3/4}\tan^{-1}\left(\frac{x^{3/4}}{\sqrt[4]{x^3-1}}\right)}{3\left(x^4-x\right)^{3/4}}+\frac{2x^{3/4}\left(x^3-1\right)^{3/4}\tanh^{-1}\left(\frac{x^{3/4}}{\sqrt[4]{x^3-1}}\right)}{3\left(x^4-x\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^4)^(1/4)/x^2,x]
```

```
[Out] (-4*(-x + x^4)^(1/4))/(3*x) - (2*x^(3/4)*(-1 + x^3)^(3/4)*ArcTan[x^(3/4)/(-
1 + x^3)^(1/4)])/(3*(-x + x^4)^(3/4)) + (2*x^(3/4)*(-1 + x^3)^(3/4)*ArcTanh
[x^(3/4)/(-1 + x^3)^(1/4)])/(3*(-x + x^4)^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{-x+x^4}}{x^2}\,dx &= -\frac{4\sqrt[4]{-x+x^4}}{3x}+\int \frac{x^2}{\left(-x+x^4\right)^{3/4}}\,dx\\
&= -\frac{4\sqrt[4]{-x+x^4}}{3x}+\frac{\left(x^{3/4}\left(-1+x^3\right)^{3/4}\right)\int \frac{x^{5/4}}{\left(-1+x^3\right)^{3/4}}\,dx}{\left(-x+x^4\right)^{3/4}}\\
&= -\frac{4\sqrt[4]{-x+x^4}}{3x}+\frac{\left(4x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^8}{\left(-1+x^{12}\right)^{3/4}}\,dx,x,\sqrt[4]{x}\right)}{\left(-x+x^4\right)^{3/4}}\\
&= -\frac{4\sqrt[4]{-x+x^4}}{3x}+\frac{\left(4x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{\left(-1+x^4\right)^{3/4}}\,dx,x,x^{3/4}\right)}{3\left(-x+x^4\right)^{3/4}}\\
&= -\frac{4\sqrt[4]{-x+x^4}}{3x}+\frac{\left(4x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{-1+x^3}}\right)}{3\left(-x+x^4\right)^{3/4}}\\
&= -\frac{4\sqrt[4]{-x+x^4}}{3x}+\frac{\left(2x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{-1+x^3}}\right)}{3\left(-x+x^4\right)^{3/4}}-\frac{\left(2x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subs}}{3\left(-x+\right.}\\
&= -\frac{4\sqrt[4]{-x+x^4}}{3x}-\frac{2x^{3/4}\left(-1+x^3\right)^{3/4}\tan^{-1}\left(\frac{x^{3/4}}{\sqrt[4]{-1+x^3}}\right)}{3\left(-x+x^4\right)^{3/4}}+\frac{2x^{3/4}\left(-1+x^3\right)^{3/4}\tanh^{-1}\left(\frac{x^{3/4}}{\sqrt[4]{-1+x^3}}\right)}{3\left(-x+x^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.76

$$-\frac{4\sqrt[4]{x\left(x^3-1\right)}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};x^3\right)}{3x\sqrt[4]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^4)^(1/4)/x^2,x]
```

```
[Out] (-4*(x*(-1 + x^3))^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, x^3])/(3*x*(1 -
 x^3)^(1/4))
```

IntegrateAlgebraic [A] time = 0.22, size = 55, normalized size = 1.00

$$-\frac{4\sqrt[4]{x^4-x}}{3x}-\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x}}\right)+\frac{2}{3}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x + x^4)^(1/4)/x^2,x]
```

```
[Out] (-4*(-x + x^4)^(1/4))/(3*x) - (2*ArcTan[x/(-x + x^4)^(1/4)])/3 + (2*ArcTanh
[x/(-x + x^4)^(1/4)])/3
```

fricas [B] time = 1.21, size = 93, normalized size = 1.69

$$\frac{x\arctan\left(2\left(x^4-x\right)^{\frac{1}{4}}x^2+2\left(x^4-x\right)^{\frac{3}{4}}\right)+x\log\left(-2x^3-2\left(x^4-x\right)^{\frac{1}{4}}x^2-2\sqrt{x^4-x}\,x-2\left(x^4-x\right)^{\frac{3}{4}}+1\right)-4\left(x^4-x\right)^{\frac{1}{4}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(x*arctan(2*(x^4 - x)^(1/4)*x^2 + 2*(x^4 - x)^(3/4)) + x*log(-2*x^3 - 2
*(x^4 - x)^(1/4)*x^2 - 2*sqrt(x^4 - x)*x - 2*(x^4 - x)^(3/4) + 1) - 4*(x^4
- x)^(1/4))/x
```

giac [A] time = 0.24, size = 53, normalized size = 0.96

$$\frac{4}{3}\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}-\frac{2}{3}\arctan\left(\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}\right)-\frac{1}{3}\log\left(\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{3}\log\left(\left|\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 4/3*(-1/x^3 + 1)^(1/4) - 2/3*arctan((-1/x^3 + 1)^(1/4)) - 1/3*log((-1/x^3 +
 1)^(1/4) + 1) + 1/3*log(abs((-1/x^3 + 1)^(1/4) - 1))
```

maple [C] time = 3.78, size = 442, normalized size = 8.04

$$-\frac{4\left(x\left(x^3-1\right)\right)^{\frac{1}{4}}}{3x}+\frac{\left(\frac{\ln\left(\frac{2x^9+2\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}x^6-5x^6+2\sqrt{x^{12}-3x^9+3x^6-x^3}\,x^3-4\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}x^3+2\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{3}{4}}+4x^3-2\sqrt{x^{12}-3x^9+3x^6-x^3}+2\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}-1}{\left(-1+x\right)^2\left(x^2+x+1\right)^2}\right)}{3}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-2x^9-2\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^2+1\right)x^6+5x^6+2\sqrt{x^{12}-3x^9+3x^6-x^3}\,x^3+4\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^2+1\right)x^3+2\operatorname{RootOf}\left(\_Z^2+1\right)\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{3}{4}}-4x^3-2\sqrt{x^{12}-3x^9+3x^6-x^3}-2\operatorname{RootOf}\left(\_Z^2+1\right)\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}+1}{\left(-1+x\right)^2\left(x^2+x+1\right)^2}\right)}{3}\right)\left(x\left(x^3-1\right)\right)^{\frac{1}{4}}\left(x^3\left(x^3-1\right)^3\right)^{\frac{1}{4}}}{x\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x)^(1/4)/x^2,x)
```

```
[Out] -4/3*(x*(x^3-1))^(1/4)/x+(1/3*ln((2*x^9+2*(x^12-3*x^9+3*x^6-x^3)^(1/4)*x^6-
5*x^6+2*(x^12-3*x^9+3*x^6-x^3)^(1/2)*x^3-4*(x^12-3*x^9+3*x^6-x^3)^(1/4)*x^3
+2*(x^12-3*x^9+3*x^6-x^3)^(3/4)+4*x^3-2*(x^12-3*x^9+3*x^6-x^3)^(1/2)+2*(x^1
2-3*x^9+3*x^6-x^3)^(1/4)-1)/(-1+x)^2/(x^2+x+1)^2)+1/3*RootOf(_Z^2+1)*ln((-2
*x^9-2*(x^12-3*x^9+3*x^6-x^3)^(1/4)*RootOf(_Z^2+1)*x^6+5*x^6+2*(x^12-3*x^9+
3*x^6-x^3)^(1/2)*x^3+4*(x^12-3*x^9+3*x^6-x^3)^(1/4)*RootOf(_Z^2+1)*x^3+2*Ro
otOf(_Z^2+1)*(x^12-3*x^9+3*x^6-x^3)^(3/4)-4*x^3-2*(x^12-3*x^9+3*x^6-x^3)^(1
/2)-2*RootOf(_Z^2+1)*(x^12-3*x^9+3*x^6-x^3)^(1/4)+1)/(-1+x)^2/(x^2+x+1)^2))
*(x*(x^3-1))^(1/4)/x*(x^3*(x^3-1)^3)^(1/4)/(x^3-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x)^(1/4)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x^4-x\right)^{1/4}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x)^(1/4)/x^2,x)
```

```
[Out] int((x^4 - x)^(1/4)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x\left(x-1\right)\left(x^2+x+1\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x)**(1/4)/x**2,x)
```

```
[Out] Integral((x*(x - 1)*(x**2 + x + 1))**(1/4)/x**2, x)
```

3.695 $\int x\sqrt[4]{-x+x^4}\,dx$

Optimal. Leaf size=55

$$\frac{1}{6}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x}}\right)-\frac{1}{6}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x}}\right)+\frac{1}{3}\sqrt[4]{x^4-x}\,x^2$$

Rubi [A] time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.98, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {2021, 2032, 329, 275, 331, 298, 203, 206}

$$\frac{1}{3}\sqrt[4]{x^4-x}\,x^2+\frac{\left(x^3-1\right)^{3/4}x^{3/4}\tan^{-1}\left(\frac{x^{3/4}}{\sqrt[4]{x^3-1}}\right)}{6\left(x^4-x\right)^{3/4}}-\frac{\left(x^3-1\right)^{3/4}x^{3/4}\tanh^{-1}\left(\frac{x^{3/4}}{\sqrt[4]{x^3-1}}\right)}{6\left(x^4-x\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(-x + x^4)^(1/4),x]
```

```
[Out] (x^2*(-x + x^4)^(1/4))/3 + (x^(3/4)*(-1 + x^3)^(3/4)*ArcTan[x^(3/4)/(-1 + x
^3)^(1/4)])/(6*(-x + x^4)^(3/4)) - (x^(3/4)*(-1 + x^3)^(3/4)*ArcTanh[x^(3/4
)/(-1 + x^3)^(1/4)])/(6*(-x + x^4)^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt[4]{-x+x^4}\,dx &= \frac{1}{3}x^2\sqrt[4]{-x+x^4}-\frac{1}{4}\int\frac{x^2}{\left(-x+x^4\right)^{3/4}}\,dx\\
&= \frac{1}{3}x^2\sqrt[4]{-x+x^4}-\frac{\left(x^{3/4}\left(-1+x^3\right)^{3/4}\right)\int\frac{x^{5/4}}{\left(-1+x^3\right)^{3/4}}\,dx}{4\left(-x+x^4\right)^{3/4}}\\
&= \frac{1}{3}x^2\sqrt[4]{-x+x^4}-\frac{\left(x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subst}\left(\int\frac{x^8}{\left(-1+x^{12}\right)^{3/4}}\,dx,x,\sqrt[4]{x}\right)}{\left(-x+x^4\right)^{3/4}}\\
&= \frac{1}{3}x^2\sqrt[4]{-x+x^4}-\frac{\left(x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subst}\left(\int\frac{x^2}{\left(-1+x^4\right)^{3/4}}\,dx,x,x^{3/4}\right)}{3\left(-x+x^4\right)^{3/4}}\\
&= \frac{1}{3}x^2\sqrt[4]{-x+x^4}-\frac{\left(x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subst}\left(\int\frac{x^2}{1-x^4}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{-1+x^3}}\right)}{3\left(-x+x^4\right)^{3/4}}\\
&= \frac{1}{3}x^2\sqrt[4]{-x+x^4}-\frac{\left(x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{-1+x^3}}\right)}{6\left(-x+x^4\right)^{3/4}}+\frac{\left(x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subst}}{6\left(-x+\right.}\\
&= \frac{1}{3}x^2\sqrt[4]{-x+x^4}+\frac{x^{3/4}\left(-1+x^3\right)^{3/4}\tan^{-1}\left(\frac{x^{3/4}}{\sqrt[4]{-1+x^3}}\right)}{6\left(-x+x^4\right)^{3/4}}-\frac{x^{3/4}\left(-1+x^3\right)^{3/4}\tanh^{-1}\left(\frac{x^{3/4}}{\sqrt[4]{-1+x^3}}\right)}{6\left(-x+x^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.76

$$\frac{4x^2\sqrt[4]{x\left(x^3-1\right)}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};x^3\right)}{9\sqrt[4]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-x + x^4)^(1/4),x]
```

```
[Out] (4*x^2*(x*(-1 + x^3))^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, x^3])/(9*(1 -
 x^3)^(1/4))
```

IntegrateAlgebraic [A] time = 0.22, size = 55, normalized size = 1.00

$$\frac{1}{6}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x}}\right)-\frac{1}{6}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x}}\right)+\frac{1}{3}\sqrt[4]{x^4-x}\,x^2$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(-x + x^4)^(1/4),x]
```

```
[Out] (x^2*(-x + x^4)^(1/4))/3 + ArcTan[x/(-x + x^4)^(1/4)]/6 - ArcTanh[x/(-x + x
^4)^(1/4)]/6
```

fricas [B] time = 1.25, size = 91, normalized size = 1.65

$$\frac{1}{3}\left(x^4-x\right)^{\frac{1}{4}}x^2-\frac{1}{12}\arctan\left(2\left(x^4-x\right)^{\frac{1}{4}}x^2+2\left(x^4-x\right)^{\frac{3}{4}}\right)+\frac{1}{12}\log\left(2x^3-2\left(x^4-x\right)^{\frac{1}{4}}x^2+2\sqrt{x^4-x}\,x-2\left(x^4-x\right)^{\frac{3}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4-x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3*(x^4 - x)^(1/4)*x^2 - 1/12*arctan(2*(x^4 - x)^(1/4)*x^2 + 2*(x^4 - x)^(
3/4)) + 1/12*log(2*x^3 - 2*(x^4 - x)^(1/4)*x^2 + 2*sqrt(x^4 - x)*x - 2*(x^4
 - x)^(3/4) - 1)
```

giac [A] time = 0.51, size = 56, normalized size = 1.02

$$-\frac{1}{3}x^3\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}+\frac{1}{6}\arctan\left(\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}\right)+\frac{1}{12}\log\left(\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}+1\right)-\frac{1}{12}\log\left(\left|\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4-x)^(1/4),x, algorithm="giac")
```

```
[Out] -1/3*x^3*(-1/x^3 + 1)^(1/4) + 1/6*arctan((-1/x^3 + 1)^(1/4)) + 1/12*log((-1
/x^3 + 1)^(1/4) + 1) - 1/12*log(abs((-1/x^3 + 1)^(1/4) - 1))
```

maple [C] time = 3.74, size = 444, normalized size = 8.07

$$\frac{x^2\left(x\left(x^3-1\right)\right)^{\frac{1}{4}}}{3}+\frac{\left(\frac{\ln\left(-\frac{-2x^9+2\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}x^6+5x^6-2\sqrt{x^{12}-3x^9+3x^6-x^3}\,x^3-4\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}x^3+2\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{3}{4}}-4x^3+2\sqrt{x^{12}-3x^9+3x^6-x^3}+2\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}+1}{\left(-1+x\right)^2\left(x^2+x+1\right)^2}\right)}{12}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{2x^9-2\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^2+1\right)x^6-5x^6-2\sqrt{x^{12}-3x^9+3x^6-x^3}\,x^3+4\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^2+1\right)x^3+2\operatorname{RootOf}\left(\_Z^2+1\right)\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{3}{4}}+4x^3+2\sqrt{x^{12}-3x^9+3x^6-x^3}-2\operatorname{RootOf}\left(\_Z^2+1\right)\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}-1}{\left(-1+x\right)^2\left(x^2+x+1\right)^2}\right)}{12}\right)\left(x\left(x^3-1\right)\right)^{\frac{1}{4}}\left(x^3\left(x^3-1\right)^3\right)^{\frac{1}{4}}}{x\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^4-x)^(1/4),x)
```

```
[Out] 1/3*x^2*(x*(x^3-1))^(1/4)+(1/12*ln(-(-2*x^9+2*(x^12-3*x^9+3*x^6-x^3)^(1/4)*
x^6+5*x^6-2*(x^12-3*x^9+3*x^6-x^3)^(1/2)*x^3-4*(x^12-3*x^9+3*x^6-x^3)^(1/4)
*x^3+2*(x^12-3*x^9+3*x^6-x^3)^(3/4)-4*x^3+2*(x^12-3*x^9+3*x^6-x^3)^(1/2)+2*
(x^12-3*x^9+3*x^6-x^3)^(1/4)+1)/(-1+x)^2/(x^2+x+1)^2)+1/12*RootOf(_Z^2+1)*l
n(-(2*x^9-2*(x^12-3*x^9+3*x^6-x^3)^(1/4)*RootOf(_Z^2+1)*x^6-5*x^6-2*(x^12-3
*x^9+3*x^6-x^3)^(1/2)*x^3+4*(x^12-3*x^9+3*x^6-x^3)^(1/4)*RootOf(_Z^2+1)*x^3
+2*RootOf(_Z^2+1)*(x^12-3*x^9+3*x^6-x^3)^(3/4)+4*x^3+2*(x^12-3*x^9+3*x^6-x^
3)^(1/2)-2*RootOf(_Z^2+1)*(x^12-3*x^9+3*x^6-x^3)^(1/4)-1)/(-1+x)^2/(x^2+x+1
)^2))*(x*(x^3-1))^(1/4)/x*(x^3*(x^3-1)^3)^(1/4)/(x^3-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^4-x\right)^{\frac{1}{4}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4-x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x)^(1/4)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\left(x^4-x\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^4 - x)^(1/4),x)
```

```
[Out] int(x*(x^4 - x)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt[4]{x\left(x-1\right)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**4-x)**(1/4),x)
```

```
[Out] Integral(x*(x*(x - 1)*(x**2 + x + 1))**(1/4), x)
```

3.696 $\int \frac{(-b+ax^3)\sqrt{-x+x^4}}{x^3}\,dx$

Optimal. Leaf size=55

$$\frac{1}{3}(-a-2b)\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)+\frac{\sqrt{x^4-x}\,\left(ax^3+2b\right)}{3x^2}$$

Rubi [A] time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2038, 2004, 2029, 206}

$$\frac{1}{3}x\sqrt{x^4-x}\,(a+2b)-\frac{1}{3}(a+2b)\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)-\frac{2b\left(x^4-x\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a*x^3)*Sqrt[-x + x^4])/x^3,x]
```

```
[Out] ((a + 2*b)*x*Sqrt[-x + x^4])/3 - (2*b*(-x + x^4)^(3/2))/(3*x^3) - ((a + 2*b
)*ArcTanh[x^2/Sqrt[-x + x^4]])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(-b+ax^3)\sqrt{-x+x^4}}{x^3}\,dx = -\frac{2b\left(-x+x^4\right)^{3/2}}{3x^3} + (a+2b)\int \sqrt{-x+x^4}\,dx$$
$$= \frac{1}{3}(a+2b)x\sqrt{-x+x^4} - \frac{2b\left(-x+x^4\right)^{3/2}}{3x^3} + \frac{1}{2}(-a-2b)\int \frac{x}{\sqrt{-x+x^4}}\,dx$$
$$= \frac{1}{3}(a+2b)x\sqrt{-x+x^4} - \frac{2b\left(-x+x^4\right)^{3/2}}{3x^3} + \frac{1}{3}(-a-2b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-x}}\right.$$
$$= \frac{1}{3}(a+2b)x\sqrt{-x+x^4} - \frac{2b\left(-x+x^4\right)^{3/2}}{3x^3} - \frac{1}{3}(a+2b)\tanh^{-1}\left(\frac{x^2}{\sqrt{-x+x^4}}\right)$$

Mathematica [A] time = 0.04, size = 68, normalized size = 1.24

$$\frac{\sqrt{x\left(x^3-1\right)}\left(x^{3/2}(a+2b)\sin^{-1}\left(x^{3/2}\right)+\sqrt{1-x^3}\left(ax^3+2b\right)\right)}{3x^2\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + a*x^3)*Sqrt[-x + x^4])/x^3,x]
[Out] (Sqrt[x*(-1 + x^3)]*(Sqrt[1 - x^3]*(2*b + a*x^3) + (a + 2*b)*x^(3/2)*ArcSin
[x^(3/2)]))/(3*x^2*Sqrt[1 - x^3])
```

IntegrateAlgebraic [A] time = 0.47, size = 55, normalized size = 1.00

$$\frac{1}{3}(-a-2b)\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)+\frac{\sqrt{x^4-x}\left(ax^3+2b\right)}{3x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^3)*Sqrt[-x + x^4])/x^3,x]
[Out] ((2*b + a*x^3)*Sqrt[-x + x^4])/(3*x^2) + ((-a - 2*b)*ArcTanh[x^2/Sqrt[-x +
x^4]])/3
```

fricas [A] time = 0.43, size = 55, normalized size = 1.00

$$\frac{(a+2b)x^2\log\left(2x^3-2\sqrt{x^4-x}\,x-1\right)+2\left(ax^3+2b\right)\sqrt{x^4-x}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(x^4-x)^(1/2)/x^3,x, algorithm="fricas")
[Out] 1/6*((a + 2*b)*x^2*log(2*x^3 - 2*sqrt(x^4 - x)*x - 1) + 2*(a*x^3 + 2*b)*sqr
t(x^4 - x))/x^2
```

giac [A] time = 0.23, size = 65, normalized size = 1.18

$$\frac{1}{3}\sqrt{x^4-x}\,ax-\frac{1}{6}(a+2b)\log\left(\sqrt{-\frac{1}{x^3}+1}+1\right)+\frac{1}{6}(a+2b)\log\left(\left|\sqrt{-\frac{1}{x^3}+1}-1\right|\right)+\frac{2}{3}b\sqrt{-\frac{1}{x^3}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(x^4-x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(x^4 - x)*a*x - 1/6*(a + 2*b)*log(sqrt(-1/x^3 + 1) + 1) + 1/6*(a +
2*b)*log(abs(sqrt(-1/x^3 + 1) - 1)) + 2/3*b*sqrt(-1/x^3 + 1)
```

maple [C] time = 0.02, size = 609, normalized size = 11.07

$$a\left(\frac{x\sqrt{x^4-x}}{3}-\frac{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}}\,(-1+x)^2\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(-1+x)}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)-\mathrm{EllipticPi}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\frac{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(-1+x)\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)-b\left(-\frac{2\sqrt{x^4-x}}{3x^2}+\frac{2\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}}\,(-1+x)^2\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(-1+x)}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)-\mathrm{EllipticPi}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\frac{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(-1+x)\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b)*(x^4-x)^(1/2)/x^3,x)
```

```
[Out] a*(1/3*x*(x^4-x)^(1/2)-(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/
2*I*3^(1/2))/(-1+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1
/2))/(-1+x))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2
)/(-3/2+1/2*I*3^(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/2)
))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1
/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I
*3^(1/2)))^(1/2))-EllipticPi(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(
-1+x))^(1/2),(-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)),((3/2+1/2*I*3^(1/2))
*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))))-b*(-
2/3*(x^4-x)^(1/2)/x^2+2*(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+1
/2*I*3^(1/2))/(-1+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(
1/2))/(-1+x))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/
2)/(-3/2+1/2*I*3^(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/2
)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(
1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*
I*3^(1/2)))^(1/2))-EllipticPi(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/
(-1+x))^(1/2),(-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)),((3/2+1/2*I*3^(1/2)
)*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^3-b\right)\sqrt{x^4-x}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(x^4-x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x^3 - b)*sqrt(x^4 - x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\sqrt{x^4-x}\,\left(b-a\,x^3\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^4 - x)^(1/2)*(b - a*x^3))/x^3,x)
```

```
[Out] -int(((x^4 - x)^(1/2)*(b - a*x^3))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(x-1\right)\left(x^2+x+1\right)}\,\left(ax^3-b\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b)*(x**4-x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x*(x - 1)*(x**2 + x + 1))*(a*x**3 - b)/x**3, x)
```

3.697 $\int \frac{\sqrt[4]{-x^3+x^4}}{x^2}\,dx$

Optimal. Leaf size=55

$$-\frac{4\sqrt[4]{x^4-x^3}}{x}-2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)+2\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)$$

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.84, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2020, 2032, 63, 240, 212, 206, 203}

$$-\frac{4\sqrt[4]{x^4-x^3}}{x}+\frac{2(x-1)^{3/4}x^{9/4}\tan^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{\left(x^4-x^3\right)^{3/4}}+\frac{2(x-1)^{3/4}x^{9/4}\tanh^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{\left(x^4-x^3\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + x^4)^(1/4)/x^2,x]
```

```
[Out] (-4*(-x^3 + x^4)^(1/4))/x + (2*(-1 + x)^(3/4)*x^(9/4)*ArcTan[(-1 + x)^(1/4)
/x^(1/4)])/(-x^3 + x^4)^(3/4) + (2*(-1 + x)^(3/4)*x^(9/4)*ArcTanh[(-1 + x)^
(1/4)/x^(1/4)])/(-x^3 + x^4)^(3/4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{-x^3+x^4}}{x^2}\,dx &= -\frac{4\sqrt[4]{-x^3+x^4}}{x}+\int \frac{x^2}{\left(-x^3+x^4\right)^{3/4}}\,dx\\
&= -\frac{4\sqrt[4]{-x^3+x^4}}{x}+\frac{\left((-1+x)^{3/4}x^{9/4}\right)\int \frac{1}{(-1+x)^{3/4}\sqrt[4]{x}}\,dx}{\left(-x^3+x^4\right)^{3/4}}\\
&= -\frac{4\sqrt[4]{-x^3+x^4}}{x}+\frac{\left(4(-1+x)^{3/4}x^{9/4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^4}}\,dx,x,\sqrt[4]{-1+x}\right)}{\left(-x^3+x^4\right)^{3/4}}\\
&= -\frac{4\sqrt[4]{-x^3+x^4}}{x}+\frac{\left(4(-1+x)^{3/4}x^{9/4}\right)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{\left(-x^3+x^4\right)^{3/4}}\\
&= -\frac{4\sqrt[4]{-x^3+x^4}}{x}+\frac{\left(2(-1+x)^{3/4}x^{9/4}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{\left(-x^3+x^4\right)^{3/4}}+\frac{\left(2(-1+x)^{3/4}x^{9/4}\right)\text{Subs}}{\left(-x^3+\right.}\\
&= -\frac{4\sqrt[4]{-x^3+x^4}}{x}+\frac{2(-1+x)^{3/4}x^{9/4}\tan^{-1}\left(\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{\left(-x^3+x^4\right)^{3/4}}+\frac{2(-1+x)^{3/4}x^{9/4}\tanh^{-1}\left(\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{\left(-x^3+x^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.64

$$\frac{4\left((x-1)x^3\right)^{5/4}{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};1-x\right)}{5x^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + x^4)^(1/4)/x^2,x]
```

```
[Out] (4*((-1 + x)*x^3)^(5/4)*Hypergeometric2F1[5/4, 5/4, 9/4, 1 - x])/(5*x^(15/4
))
```

IntegrateAlgebraic [A] time = 0.22, size = 55, normalized size = 1.00

$$-\frac{4\sqrt[4]{x^4-x^3}}{x}-2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)+2\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^3 + x^4)^(1/4)/x^2,x]
```

```
[Out] (-4*(-x^3 + x^4)^(1/4))/x - 2*ArcTan[x/(-x^3 + x^4)^(1/4)] + 2*ArcTanh[x/(-
x^3 + x^4)^(1/4)]
```

fricas [A] time = 0.38, size = 81, normalized size = 1.47

$$\frac{2\,x\arctan\left(\frac{\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+x\log\left(\frac{x+\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)-x\log\left(-\frac{x-\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)-4\left(x^4-x^3\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] (2*x*arctan((x^4 - x^3)^(1/4)/x) + x*log((x + (x^4 - x^3)^(1/4))/x) - x*log
(-(x - (x^4 - x^3)^(1/4))/x) - 4*(x^4 - x^3)^(1/4))/x
```

giac [A] time = 0.19, size = 51, normalized size = 0.93

$$4\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-2\,\arctan\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)-\log\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right)+\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 4*(-1/x + 1)^(1/4) - 2*arctan((-1/x + 1)^(1/4)) - log((-1/x + 1)^(1/4) + 1)
 + log(abs((-1/x + 1)^(1/4) - 1))
```

maple [C] time = 0.41, size = 394, normalized size = 7.16

$$-\frac{4\left(x^3(-1+x)\right)^{\frac{1}{4}}}{x}+\frac{\left(\ln\left(\frac{2\left(x^4-3x^3+3x^2-x\right)^{\frac{3}{4}}+2\sqrt{x^4-3x^3+3x^2-x}\,x+2\left(x^4-3x^3+3x^2-x\right)^{\frac{1}{4}}x^2+2x^3-2\sqrt{x^4-3x^3+3x^2-x}-4\left(x^4-3x^3+3x^2-x\right)^{\frac{1}{4}}x-5x^2+2\left(x^4-3x^3+3x^2-x\right)^{\frac{1}{4}}+4x-1}{(-1+x)^2}\right)+\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-2\sqrt{x^4-3x^3+3x^2-x}\,\mathrm{RootOf}\left(\_Z^2+1\right)x+2\,\mathrm{RootOf}\left(\_Z^2+1\right)x^3+2\left(x^4-3x^3+3x^2-x\right)^{\frac{3}{4}}+2\sqrt{x^4-3x^3+3x^2-x}\,\mathrm{RootOf}\left(\_Z^2+1\right)-2\left(x^4-3x^3+3x^2-x\right)^{\frac{1}{4}}x^2-5\,\mathrm{RootOf}\left(\_Z^2+1\right)x^2+4\left(x^4-3x^3+3x^2-x\right)^{\frac{1}{4}}x+4\,\mathrm{RootOf}\left(\_Z^2+1\right)x-2\left(x^4-3x^3+3x^2-x\right)^{\frac{1}{4}}-\mathrm{RootOf}\left(\_Z^2+1\right)}{(-1+x)^2}\right)\right)\left(x^3(-1+x)\right)^{\frac{1}{4}}\left(x(-1+x)^3\right)^{\frac{1}{4}}}{(-1+x)\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3)^(1/4)/x^2,x)
```

```
[Out] -4*(x^3*(-1+x))^(1/4)/x+(ln((2*(x^4-3*x^3+3*x^2-x)^(3/4)+2*(x^4-3*x^3+3*x^2
-x)^(1/2)*x+2*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+2*x^3-2*(x^4-3*x^3+3*x^2-x)^(1/
2)-4*(x^4-3*x^3+3*x^2-x)^(1/4)*x-5*x^2+2*(x^4-3*x^3+3*x^2-x)^(1/4)+4*x-1)/(
-1+x)^2)+RootOf(_Z^2+1)*ln((-2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+1)*x+2
*RootOf(_Z^2+1)*x^3+2*(x^4-3*x^3+3*x^2-x)^(3/4)+2*(x^4-3*x^3+3*x^2-x)^(1/2)
*RootOf(_Z^2+1)-2*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2-5*RootOf(_Z^2+1)*x^2+4*(x^4
-3*x^3+3*x^2-x)^(1/4)*x+4*RootOf(_Z^2+1)*x-2*(x^4-3*x^3+3*x^2-x)^(1/4)-Root
Of(_Z^2+1))/(-1+x)^2))*(x^3*(-1+x))^(1/4)/(-1+x)/x*(x*(-1+x)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^3\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^3)^(1/4)/x^2, x)
```

mupad [B] time = 0.72, size = 29, normalized size = 0.53

$$-\frac{4\left(x^4-x^3\right)^{1/4}{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ x\right)}{x\,(1-x)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x^3)^(1/4)/x^2,x)
[Out] -(4*(x^4 - x^3)^(1/4)*hypergeom([-1/4, -1/4], 3/4, x))/(x*(1 - x)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[4]{x^3\,(x-1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3)**(1/4)/x**2,x)
[Out] Integral((x**3*(x - 1))**(1/4)/x**2, x)
```

3.698 $\int \frac{x}{\sqrt{-17+18x-11x^2+6x^3+x^4}}\,dx$

Optimal. Leaf size=55

$$-\frac{1}{4}\log\left(-2x^4-24x^3-68x^2+\left(2x^2+18x+34\right)\sqrt{x^4+6x^3-11x^2+18x-17}-11\right)$$

Rubi [F] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\sqrt{-17+18x-11x^2+6x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/Sqrt[-17 + 18*x - 11*x^2 + 6*x^3 + x^4],x]
```

```
[Out] Defer[Int][x/Sqrt[-17 + 18*x - 11*x^2 + 6*x^3 + x^4], x]
```

Rubi steps

$$\int \frac{x}{\sqrt{-17+18x-11x^2+6x^3+x^4}}\,dx=\int \frac{x}{\sqrt{-17+18x-11x^2+6x^3+x^4}}\,dx$$

Mathematica [C] time = 0.88, size = 1522, normalized size = 27.67

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-17 + 18*x - 11*x^2 + 6*x^3 + x^4],x]
```

```
[Out] (-2*(x - Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 1, 0])*(x - Root[-1
7 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 2, 0])*(EllipticPi[(Root[-17 + 18*#
1 - 11*#1^2 + 6*#1^3 + #1^4 & , 1, 0] - Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3
 + #1^4 & , 4, 0])/(Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 2, 0] -
Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 4, 0]), ArcSin[Sqrt[((x - Ro
ot[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 1, 0])*(Root[-17 + 18*#1 - 11*
#1^2 + 6*#1^3 + #1^4 & , 2, 0] - Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4
 & , 4, 0]))/((x - Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 2, 0])*(R
oot[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 1, 0] - Root[-17 + 18*#1 - 11
*#1^2 + 6*#1^3 + #1^4 & , 4, 0]))]], -(((Root[-17 + 18*#1 - 11*#1^2 + 6*#1^
3 + #1^4 & , 2, 0] - Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 3, 0])*
(Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 1, 0] - Root[-17 + 18*#1 -
11*#1^2 + 6*#1^3 + #1^4 & , 4, 0]))/((-Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3
+ #1^4 & , 1, 0] + Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 3, 0])*(R
oot[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 2, 0] - Root[-17 + 18*#1 - 11
*#1^2 + 6*#1^3 + #1^4 & , 4, 0])))]*(Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 +
#1^4 & , 1, 0] - Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 2, 0]) + El
lipticF[ArcSin[Sqrt[((x - Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 1,
 0])*(Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 2, 0] - Root[-17 + 18*
#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 4, 0]))/((x - Root[-17 + 18*#1 - 11*#1^2 +
 6*#1^3 + #1^4 & , 2, 0])*(Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 1
, 0] - Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 4, 0]))]], -(((Root[-
17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 2, 0] - Root[-17 + 18*#1 - 11*#1^2
 + 6*#1^3 + #1^4 & , 3, 0])*(Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & ,
 1, 0] - Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 4, 0]))/((-Root[-17
 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 1, 0] + Root[-17 + 18*#1 - 11*#1^2 +
```

```
 6*#1^3 + #1^4 & , 3, 0])*(Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 2
, 0] - Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 4, 0]))))]*Root[-17 +
18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 2, 0])*Sqrt[(x - Root[-17 + 18*#1 - 11*
#1^2 + 6*#1^3 + #1^4 & , 3, 0])/((x - Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 +
 #1^4 & , 2, 0])*(-Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 1, 0] + R
oot[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 3, 0]))]*Sqrt[(x - Root[-17 +
 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 4, 0])/((x - Root[-17 + 18*#1 - 11*#1^
2 + 6*#1^3 + #1^4 & , 2, 0])*(-Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 &
 , 1, 0] + Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 4, 0]))])/(Sqrt[-
17 + 18*x - 11*x^2 + 6*x^3 + x^4]*Sqrt[((x - Root[-17 + 18*#1 - 11*#1^2 + 6
*#1^3 + #1^4 & , 1, 0])*(Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 2,
0] - Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 4, 0]))/((x - Root[-17
+ 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 2, 0])*(Root[-17 + 18*#1 - 11*#1^2 +
6*#1^3 + #1^4 & , 1, 0] - Root[-17 + 18*#1 - 11*#1^2 + 6*#1^3 + #1^4 & , 4,
 0]))])
```

IntegrateAlgebraic [A] time = 4.71, size = 55, normalized size = 1.00

$$-\frac{1}{4}\log\left(-2x^4-24x^3-68x^2+\left(2x^2+18x+34\right)\sqrt{x^4+6x^3-11x^2+18x-17}-11\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[-17 + 18*x - 11*x^2 + 6*x^3 + x^4],x]
```

```
[Out] -1/4*Log[-11 - 68*x^2 - 24*x^3 - 2*x^4 + (34 + 18*x + 2*x^2)*Sqrt[-17 + 18*
x - 11*x^2 + 6*x^3 + x^4]]
```

fricas [A] time = 0.44, size = 50, normalized size = 0.91

$$\frac{1}{4}\log\left(2x^4+24x^3+68x^2+2\sqrt{x^4+6x^3-11x^2+18x-17}\left(x^2+9x+17\right)+11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+6*x^3-11*x^2+18*x-17)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*log(2*x^4 + 24*x^3 + 68*x^2 + 2*sqrt(x^4 + 6*x^3 - 11*x^2 + 18*x - 17)*
(x^2 + 9*x + 17) + 11)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{x^4+6x^3-11x^2+18x-17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+6*x^3-11*x^2+18*x-17)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(x^4 + 6*x^3 - 11*x^2 + 18*x - 17), x)
```

maple [C] time = 1.05, size = 1610, normalized size = 29.27

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+6*x^3-11*x^2+18*x-17)^(1/2),x)
```

```
[Out] 2*(RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=1)-RootOf(_Z^4+6*_Z^3-11*_Z^2+
18*_Z-17,index=4))*((RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=4)-RootOf(_Z
^4+6*_Z^3-11*_Z^2+18*_Z-17,index=2))*(x-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17
,index=1))/(RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=4)-RootOf(_Z^4+6*_Z^3
```

```
-11*_Z^2+18*_Z-17,index=1))/(x-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=2)
))^(1/2)*(x-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=2))^2*(-(RootOf(_Z^4+
6*_Z^3-11*_Z^2+18*_Z-17,index=2)-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=
1))*(x-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=3))/(-RootOf(_Z^4+6*_Z^3-1
1*_Z^2+18*_Z-17,index=3)+RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=1))/(x-R
ootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=2)))^(1/2)*((RootOf(_Z^4+6*_Z^3-11
*_Z^2+18*_Z-17,index=2)-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=1))*(x-Ro
otOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=4))/(RootOf(_Z^4+6*_Z^3-11*_Z^2+18*
_Z-17,index=4)-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=1))/(x-RootOf(_Z^4
+6*_Z^3-11*_Z^2+18*_Z-17,index=2)))^(1/2)/(RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z
-17,index=4)-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=2))/(RootOf(_Z^4+6*_
Z^3-11*_Z^2+18*_Z-17,index=2)-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=1))
/((x-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=1))*(x-RootOf(_Z^4+6*_Z^3-11
*_Z^2+18*_Z-17,index=2))*(x-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=3))*(
x-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=4)))^(1/2)*(RootOf(_Z^4+6*_Z^3-
11*_Z^2+18*_Z-17,index=2)*EllipticF(((RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,i
ndex=4)-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=2))*(x-RootOf(_Z^4+6*_Z^3
-11*_Z^2+18*_Z-17,index=1))/(RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=4)-R
ootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=1))/(x-RootOf(_Z^4+6*_Z^3-11*_Z^2+
18*_Z-17,index=2)))^(1/2),((RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=2)-Ro
otOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=3))*(RootOf(_Z^4+6*_Z^3-11*_Z^2+18*
_Z-17,index=1)-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=4))/(-RootOf(_Z^4+
6*_Z^3-11*_Z^2+18*_Z-17,index=3)+RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=
1))/(RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=2)-RootOf(_Z^4+6*_Z^3-11*_Z^
2+18*_Z-17,index=4)))^(1/2))+(RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=1)-
RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=2))*EllipticPi(((RootOf(_Z^4+6*_Z
^3-11*_Z^2+18*_Z-17,index=4)-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=2))*
(x-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=1))/(RootOf(_Z^4+6*_Z^3-11*_Z^
2+18*_Z-17,index=4)-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=1))/(x-RootOf
(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=2)))^(1/2),(RootOf(_Z^4+6*_Z^3-11*_Z^2+
18*_Z-17,index=4)-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=1))/(RootOf(_Z^
4+6*_Z^3-11*_Z^2+18*_Z-17,index=4)-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,inde
x=2)),((RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=2)-RootOf(_Z^4+6*_Z^3-11*
_Z^2+18*_Z-17,index=3))*(RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=1)-RootO
f(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=4))/(-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z
-17,index=3)+RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=1))/(RootOf(_Z^4+6*_
Z^3-11*_Z^2+18*_Z-17,index=2)-RootOf(_Z^4+6*_Z^3-11*_Z^2+18*_Z-17,index=4))
)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^4+6x^3-11x^2+18x-17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+6*x^3-11*x^2+18*x-17)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(x^4 + 6*x^3 - 11*x^2 + 18*x - 17), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sqrt{x^4+6x^3-11x^2+18x-17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(18*x - 11*x^2 + 6*x^3 + x^4 - 17)^(1/2),x)
[Out] int(x/(18*x - 11*x^2 + 6*x^3 + x^4 - 17)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^4+6x^3-11x^2+18x-17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+6*x**3-11*x**2+18*x-17)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**4 + 6*x**3 - 11*x**2 + 18*x - 17), x)
```

3.699 $\int \frac{\sqrt{-1-x-x^2+x^4}\,(2+x+2x^4)}{(-1-x+x^4)(-1-x+x^2+x^4)}\,dx$

Optimal. Leaf size=55

$$2\tan^{-1}\left(\frac{x}{\sqrt{x^4-x^2-x-1}}\right)-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4-x^2-x-1}}\right)$$

Rubi [F] time = 1.71, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-1-x-x^2+x^4}\,(2+x+2x^4)}{(-1-x+x^4)(-1-x+x^2+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-1 - x - x^2 + x^4]*(2 + x + 2*x^4))/((-1 - x + x^4)*(-1 - x + x^
2 + x^4)),x]
```

```
[Out] Defer[Int][Sqrt[-1 - x - x^2 + x^4]/(1 - x), x] + 2*Defer[Int][Sqrt[-1 - x
- x^2 + x^4]/(1 + 2*x + x^2 + x^3), x] - Defer[Int][(x*Sqrt[-1 - x - x^2 +
x^4])/(1 + 2*x + x^2 + x^3), x] + 2*Defer[Int][(x^2*Sqrt[-1 - x - x^2 + x^4
])/(1 + 2*x + x^2 + x^3), x] + Defer[Int][Sqrt[-1 - x - x^2 + x^4]/(-1 - x
+ x^4), x] + 4*Defer[Int][(x^2*Sqrt[-1 - x - x^2 + x^4])/(-1 - x + x^4), x]
 - Defer[Int][(x^3*Sqrt[-1 - x - x^2 + x^4])/(-1 - x + x^4), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1-x-x^2+x^4}\,(2+x+2x^4)}{(-1-x+x^4)(-1-x+x^2+x^4)}\,dx &= \int \left(\frac{\sqrt{-1-x-x^2+x^4}}{1-x}+\frac{(2-x+2x^2)\sqrt{-1-x-x^2+x^4}}{1+2x+x^2+x^3}+\frac{(1+4x^2\\
&= \int \frac{\sqrt{-1-x-x^2+x^4}}{1-x}\,dx+\int \frac{(2-x+2x^2)\sqrt{-1-x-x^2+x^4}}{1+2x+x^2+x^3}\,dx+\int\\
&= \int \frac{\sqrt{-1-x-x^2+x^4}}{1-x}\,dx+\int \left(\frac{2\sqrt{-1-x-x^2+x^4}}{1+2x+x^2+x^3}-\frac{x\sqrt{-1-x-x^2}}{1+2x+x^2+}\right.\\
&= 2\int \frac{\sqrt{-1-x-x^2+x^4}}{1+2x+x^2+x^3}\,dx+2\int \frac{x^2\sqrt{-1-x-x^2+x^4}}{1+2x+x^2+x^3}\,dx+4\int \frac{x^2\sqrt{}}{}
\end{aligned}$$

Mathematica [C] time = 6.66, size = 59389, normalized size = 1079.80

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[-1 - x - x^2 + x^4]*(2 + x + 2*x^4))/((-1 - x + x^4)*(-1 -
x + x^2 + x^4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.23, size = 55, normalized size = 1.00

$$2\tan^{-1}\left(\frac{x}{\sqrt{x^4-x^2-x-1}}\right)-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4-x^2-x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 - x - x^2 + x^4]*(2 + x + 2*x^4))/((-1 - x + x^
4)*(-1 - x + x^2 + x^4)),x]
```

```
[Out] 2*ArcTan[x/Sqrt[-1 - x - x^2 + x^4]] - 2*Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[-1
 - x - x^2 + x^4]]
```

fricas [A] time = 0.52, size = 77, normalized size = 1.40

$$-\sqrt{2}\arctan\left(\frac{2\sqrt{2}\sqrt{x^4-x^2-x-1}\,x}{x^4-3\,x^2-x-1}\right)+\arctan\left(\frac{2\sqrt{x^4-x^2-x-1}\,x}{x^4-2\,x^2-x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2-x-1)^(1/2)*(2*x^4+x+2)/(x^4-x-1)/(x^4+x^2-x-1),x, algori
thm="fricas")
```

```
[Out] -sqrt(2)*arctan(2*sqrt(2)*sqrt(x^4 - x^2 - x - 1)*x/(x^4 - 3*x^2 - x - 1))
+ arctan(2*sqrt(x^4 - x^2 - x - 1)*x/(x^4 - 2*x^2 - x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2\,x^4+x+2\right)\sqrt{x^4-x^2-x-1}}{\left(x^4+x^2-x-1\right)\left(x^4-x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2-x-1)^(1/2)*(2*x^4+x+2)/(x^4-x-1)/(x^4+x^2-x-1),x, algori
thm="giac")
```

```
[Out] integrate((2*x^4 + x + 2)*sqrt(x^4 - x^2 - x - 1)/((x^4 + x^2 - x - 1)*(x^4
 - x - 1)), x)
```

maple [C] time = 14.24, size = 7840, normalized size = 142.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^2-x-1)^(1/2)*(2*x^4+x+2)/(x^4-x-1)/(x^4+x^2-x-1),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2\,x^4+x+2\right)\sqrt{x^4-x^2-x-1}}{\left(x^4+x^2-x-1\right)\left(x^4-x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2-x-1)^(1/2)*(2*x^4+x+2)/(x^4-x-1)/(x^4+x^2-x-1),x, algori
thm="maxima")
```

```
[Out] integrate((2*x^4 + x + 2)*sqrt(x^4 - x^2 - x - 1)/((x^4 + x^2 - x - 1)*(x^4
 - x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(2\,x^4+x+2\right)\,\sqrt{x^4-x^2-x-1}}{\left(-x^4+x+1\right)\,\left(-x^4-x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 2*x^4 + 2)*(x^4 - x^2 - x - 1)^(1/2))/((x - x^4 + 1)*(x - x^2 - x
^4 + 1)),x)
```

```
[Out] int(((x + 2*x^4 + 2)*(x^4 - x^2 - x - 1)^(1/2))/((x - x^4 + 1)*(x - x^2 - x
^4 + 1)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**2-x-1)**(1/2)*(2*x**4+x+2)/(x**4-x-1)/(x**4+x**2-x-1),x)
```

```
[Out] Timed out
```

3.700 $\int \frac{1}{x\left(b+ax^4\right)^{3/4}}\,dx$

Optimal. Leaf size=55

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{2b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{2b^{3/4}}$$

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 63, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{2b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{2b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b + a*x^4)^(3/4)),x]
```

```
[Out] -1/2*ArcTan[(b + a*x^4)^(1/4)/b^(1/4)]/b^(3/4) - ArcTanh[(b + a*x^4)^(1/4)/
b^(1/4)]/(2*b^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(b+ax^4\right)^{3/4}}\,dx = \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{x(b+ax)^{3/4}}\,dx, x, x^4\right)$$
$$= \frac{\mathrm{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^4}\right)}{a}$$
$$= -\frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^4}\right)}{2\sqrt{b}} - \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx, x, \sqrt[4]{b+ax^4}\right)}{2\sqrt{b}}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b+ax^4}}{\sqrt[4]{b}}\right)}{2b^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^4}}{\sqrt[4]{b}}\right)}{2b^{3/4}}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.84

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{2b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b + a*x^4)^(3/4)),x]
```

```
[Out] -1/2*(ArcTan[(b + a*x^4)^(1/4)/b^(1/4)] + ArcTanh[(b + a*x^4)^(1/4)/b^(1/4)
])/b^(3/4)
```

IntegrateAlgebraic [A] time = 0.05, size = 55, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{2b^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{2b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(b + a*x^4)^(3/4)),x]
```

```
[Out] -1/2*ArcTan[(b + a*x^4)^(1/4)/b^(1/4)]/b^(3/4) - ArcTanh[(b + a*x^4)^(1/4)/
b^(1/4)]/(2*b^(3/4))
```

fricas [B] time = 0.41, size = 109, normalized size = 1.98

$$\frac{1}{b^3}^{\frac{1}{4}} \arctan\left(\sqrt{b^2\sqrt{\frac{1}{b^3}}+\sqrt{ax^4+b}}\, b^2\frac{1}{b^3}^{\frac{3}{4}} - \left(ax^4+b\right)^{\frac{1}{4}} b^2\frac{1}{b^3}^{\frac{3}{4}}\right) - \frac{1}{4}\frac{1}{b^3}^{\frac{1}{4}} \log\left(b\frac{1}{b^3}^{\frac{1}{4}} + \left(ax^4+b\right)^{\frac{1}{4}}\right) + \frac{1}{4}\frac{1}{b^3}^{\frac{1}{4}} \log\left(-b\frac{1}{b^3}^{\frac{1}{4}} + \left(ax^4+b\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+b)^(3/4),x, algorithm="fricas")
```

```
[Out] (b^(-3))^(1/4)*arctan(sqrt(b^2*sqrt(b^(-3)) + sqrt(a*x^4 + b))*b^2*(b^(-3))
^(3/4) - (a*x^4 + b)^(1/4)*b^2*(b^(-3))^(3/4)) - 1/4*(b^(-3))^(1/4)*log(b*(
b^(-3))^(1/4) + (a*x^4 + b)^(1/4)) + 1/4*(b^(-3))^(1/4)*log(-b*(b^(-3))^(1/
4) + (a*x^4 + b)^(1/4))
```

giac [B] time = 0.18, size = 186, normalized size = 3.38

$$-\frac{\sqrt{2}\,(-b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-b)^{\frac{1}{4}}+2\left(ax^4+b\right)^{\frac{1}{4}}\right)}{2\,(-b)^{\frac{1}{4}}}\right)}{4\,b} - \frac{\sqrt{2}\,(-b)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-b)^{\frac{1}{4}}-2\left(ax^4+b\right)^{\frac{1}{4}}\right)}{2\,(-b)^{\frac{1}{4}}}\right)}{4\,b} - \frac{\sqrt{2}\,(-b)^{\frac{1}{4}}\log\left(\sqrt{2}\left(ax^4+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^4+b}+\sqrt{-b}\right)}{8\,b} + \frac{\sqrt{2}\,(-b)^{\frac{1}{4}}\log\left(-\sqrt{2}\left(ax^4+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^4+b}+\sqrt{-b}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+b)^(3/4),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(-b)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^4 +
 b)^(1/4))/(-b)^(1/4))/b - 1/4*sqrt(2)*(-b)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(-b)^(1/4) - 2*(a*x^4 + b)^(1/4))/(-b)^(1/4))/b - 1/8*sqrt(2)*(-b)^(1/4
)*log(sqrt(2)*(a*x^4 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^4 + b) + sqrt(-b))/b
+ 1/8*sqrt(2)*(-b)^(1/4)*log(-sqrt(2)*(a*x^4 + b)^(1/4)*(-b)^(1/4) + sqrt(a
*x^4 + b) + sqrt(-b))/b
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a\,x^{4}+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^4+b)^(3/4),x)
```

```
[Out] int(1/x/(a*x^4+b)^(3/4),x)
```

maxima [A] time = 0.43, size = 57, normalized size = 1.04

$$-\frac{\arctan\left(\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{2\,b^{\frac{3}{4}}}+\frac{\log\left(\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^{4}+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{4\,b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+b)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/2*arctan((a*x^4 + b)^(1/4)/b^(1/4))/b^(3/4) + 1/4*log(((a*x^4 + b)^(1/4)
 - b^(1/4))/((a*x^4 + b)^(1/4) + b^(1/4)))/b^(3/4)
```

mupad [B] time = 0.70, size = 34, normalized size = 0.62

$$-\frac{\mathrm{atan}\left(\frac{\left(a\,x^{4}+b\right)^{1/4}}{b^{1/4}}\right)+\mathrm{atanh}\left(\frac{\left(a\,x^{4}+b\right)^{1/4}}{b^{1/4}}\right)}{2\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b + a*x^4)^(3/4)),x)
```

```
[Out] -(atan((b + a*x^4)^(1/4)/b^(1/4)) + atanh((b + a*x^4)^(1/4)/b^(1/4)))/(2*b^
(3/4))
```

sympy [C] time = 0.91, size = 39, normalized size = 0.71

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^{4}}\right)}{4a^{\frac{3}{4}}x^{3}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**4+b)**(3/4),x)
```

```
[Out] -gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*exp_polar(I*pi)/(a*x**4))/(4*a**(3/
4)*x**3*gamma(7/4))
```

3.701 $\int \frac{1}{x\sqrt[4]{b+ax^4}}\,dx$

Optimal. Leaf size=55

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}$$

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 63, 298, 203, 206}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b + a*x^4)^(1/4)),x]
```

```
[Out] ArcTan[(b + a*x^4)^(1/4)/b^(1/4)]/(2*b^(1/4)) - ArcTanh[(b + a*x^4)^(1/4)/b
^(1/4)]/(2*b^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[4]{b+ax^4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{b+ax}}\,dx, x, x^4\right)$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^4}\right)}{a}$$
$$= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^4}\right)\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx, x, \sqrt[4]{b+ax^4}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{b+ax^4}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^4}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 0.87

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right) - \tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b + a*x^4)^(1/4)),x]
```

```
[Out] (ArcTan[(b + a*x^4)^(1/4)/b^(1/4)] - ArcTanh[(b + a*x^4)^(1/4)/b^(1/4)])/(2
*b^(1/4))
```

IntegrateAlgebraic [A] time = 0.05, size = 55, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(b + a*x^4)^(1/4)),x]
```

```
[Out] ArcTan[(b + a*x^4)^(1/4)/b^(1/4)]/(2*b^(1/4)) - ArcTanh[(b + a*x^4)^(1/4)/b
^(1/4)]/(2*b^(1/4))
```

fricas [B] time = 0.41, size = 81, normalized size = 1.47

$$-\frac{\arctan\left(\frac{\sqrt{\sqrt{ax^4+b}+\sqrt{b}}}{b^{\frac{1}{4}}} - \frac{\left(ax^4+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}} - \frac{\log\left(\left(ax^4+b\right)^{\frac{1}{4}} + b^{\frac{1}{4}}\right)}{4\,b^{\frac{1}{4}}} + \frac{\log\left(\left(ax^4+b\right)^{\frac{1}{4}} - b^{\frac{1}{4}}\right)}{4\,b^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+b)^(1/4),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(sqrt(a*x^4 + b) + sqrt(b))/b^(1/4) - (a*x^4 + b)^(1/4)/b^(1/4)
)/b^(1/4) - 1/4*log((a*x^4 + b)^(1/4) + b^(1/4))/b^(1/4) + 1/4*log((a*x^4 +
 b)^(1/4) - b^(1/4))/b^(1/4)
```

giac [B] time = 0.17, size = 186, normalized size = 3.38

$$-\frac{\sqrt{2}\left(-b\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b\right)^{\frac{1}{4}}+2\left(ax^4+b\right)^{\frac{1}{4}}\right)}{2\left(-b\right)^{\frac{1}{4}}}\right)}{4\,b} - \frac{\sqrt{2}\left(-b\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b\right)^{\frac{1}{4}}-2\left(ax^4+b\right)^{\frac{1}{4}}\right)}{2\left(-b\right)^{\frac{1}{4}}}\right)}{4\,b} + \frac{\sqrt{2}\left(-b\right)^{\frac{3}{4}}\log\left(\sqrt{2}\left(ax^4+b\right)^{\frac{1}{4}}\left(-b\right)^{\frac{1}{4}}+\sqrt{ax^4+b}+\sqrt{-b}\right)}{8\,b} - \frac{\sqrt{2}\left(-b\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\left(ax^4+b\right)^{\frac{1}{4}}\left(-b\right)^{\frac{1}{4}}+\sqrt{ax^4+b}+\sqrt{-b}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+b)^(1/4),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(-b)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^4 +
 b)^(1/4))/(-b)^(1/4))/b - 1/4*sqrt(2)*(-b)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(-b)^(1/4) - 2*(a*x^4 + b)^(1/4))/(-b)^(1/4))/b + 1/8*sqrt(2)*(-b)^(3/4
)*log(sqrt(2)*(a*x^4 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^4 + b) + sqrt(-b))/b
- 1/8*sqrt(2)*(-b)^(3/4)*log(-sqrt(2)*(a*x^4 + b)^(1/4)*(-b)^(1/4) + sqrt(a
*x^4 + b) + sqrt(-b))/b
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a\,x^{4}+b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^4+b)^(1/4),x)
```

```
[Out] int(1/x/(a*x^4+b)^(1/4),x)
```

maxima [A] time = 0.43, size = 57, normalized size = 1.04

$$\frac{\arctan\left(\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{2\,b^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^{4}+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{4\,b^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+b)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/2*arctan((a*x^4 + b)^(1/4)/b^(1/4))/b^(1/4) + 1/4*log(((a*x^4 + b)^(1/4)
- b^(1/4))/((a*x^4 + b)^(1/4) + b^(1/4)))/b^(1/4)
```

mupad [B] time = 0.67, size = 36, normalized size = 0.65

$$\frac{\mathrm{atan}\left(\frac{\left(a\,x^{4}+b\right)^{1/4}}{b^{1/4}}\right)-\mathrm{atanh}\left(\frac{\left(a\,x^{4}+b\right)^{1/4}}{b^{1/4}}\right)}{2\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b + a*x^4)^(1/4)),x)
```

```
[Out] (atan((b + a*x^4)^(1/4)/b^(1/4)) - atanh((b + a*x^4)^(1/4)/b^(1/4)))/(2*b^(
1/4))
```

sympy [C] time = 0.87, size = 37, normalized size = 0.67

$$-\frac{\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{4}\\\frac{5}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^{4}}\right)}{4\sqrt[4]{a}\,x\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**4+b)**(1/4),x)
```

```
[Out] -gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*exp_polar(I*pi)/(a*x**4))/(4*a**(1/
4)*x*gamma(5/4))
```

3.702 $\int \frac{1}{x(b+ax^5)^{3/4}} dx$

Optimal. Leaf size=55

$$-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{5b^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{5b^{3/4}}$$

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 63, 212, 206, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{5b^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{5b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b + a*x^5)^(3/4)),x]
```

```
[Out] (-2*ArcTan[(b + a*x^5)^(1/4)/b^(1/4)])/(5*b^(3/4)) - (2*ArcTanh[(b + a*x^5)
^(1/4)/b^(1/4)])/(5*b^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(b+ax^5\right)^{3/4}}\,dx = \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x(b+ax)^{3/4}}\,dx, x, x^5\right)$$
$$= \frac{4\operatorname{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^5}\right)}{5a}$$
$$= -\frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^5}\right)}{5\sqrt{b}} - \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx, x, \sqrt[4]{b+ax^5}\right)}{5\sqrt{b}}$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[4]{b+ax^5}}{\sqrt[4]{b}}\right)}{5b^{3/4}} - \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^5}}{\sqrt[4]{b}}\right)}{5b^{3/4}}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.84

$$-\frac{2\left(\tan^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)\right)}{5b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b + a*x^5)^(3/4)),x]
```

```
[Out] (-2*(ArcTan[(b + a*x^5)^(1/4)/b^(1/4)] + ArcTanh[(b + a*x^5)^(1/4)/b^(1/4)]
))/(5*b^(3/4))
```

IntegrateAlgebraic [A] time = 0.05, size = 55, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{5b^{3/4}} - \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{5b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(b + a*x^5)^(3/4)),x]
```

```
[Out] (-2*ArcTan[(b + a*x^5)^(1/4)/b^(1/4)])/(5*b^(3/4)) - (2*ArcTanh[(b + a*x^5)
^(1/4)/b^(1/4)])/(5*b^(3/4))
```

fricas [B] time = 0.40, size = 110, normalized size = 2.00

$$\frac{4}{5}\frac{1}{b^3}^{\frac{1}{4}}\arctan\left(\sqrt{b^2\sqrt{\frac{1}{b^3}}+\sqrt{ax^5+b}}\,b^2\frac{1}{b^3}^{\frac{3}{4}} - \left(ax^5+b\right)^{\frac{1}{4}}b^2\frac{1}{b^3}^{\frac{3}{4}}\right) - \frac{1}{5}\frac{1}{b^3}^{\frac{1}{4}}\log\left(b\frac{1}{b^3}^{\frac{1}{4}}+\left(ax^5+b\right)^{\frac{1}{4}}\right) + \frac{1}{5}\frac{1}{b^3}^{\frac{1}{4}}\log\left(-b\frac{1}{b^3}^{\frac{1}{4}}+\left(ax^5+b\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^5+b)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/5*(b^(-3))^(1/4)*arctan(sqrt(b^2*sqrt(b^(-3)) + sqrt(a*x^5 + b))*b^2*(b^(
-3))^(3/4) - (a*x^5 + b)^(1/4)*b^2*(b^(-3))^(3/4)) - 1/5*(b^(-3))^(1/4)*log
(b*(b^(-3))^(1/4) + (a*x^5 + b)^(1/4)) + 1/5*(b^(-3))^(1/4)*log(-b*(b^(-3))
^(1/4) + (a*x^5 + b)^(1/4))
```

giac [B] time = 0.20, size = 186, normalized size = 3.38

$$-\frac{\sqrt{2}\left(-b\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b\right)^{\frac{1}{4}}+2\left(ax^5+b\right)^{\frac{1}{4}}\right)}{2\left(-b\right)^{\frac{1}{4}}}\right)}{5b} - \frac{\sqrt{2}\left(-b\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b\right)^{\frac{1}{4}}-2\left(ax^5+b\right)^{\frac{1}{4}}\right)}{2\left(-b\right)^{\frac{1}{4}}}\right)}{5b} - \frac{\sqrt{2}\left(-b\right)^{\frac{1}{4}}\log\left(\sqrt{2}\left(ax^5+b\right)^{\frac{1}{4}}\left(-b\right)^{\frac{1}{4}}+\sqrt{ax^5+b}+\sqrt{-b}\right)}{10b} + \frac{\sqrt{2}\left(-b\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\left(ax^5+b\right)^{\frac{1}{4}}\left(-b\right)^{\frac{1}{4}}+\sqrt{ax^5+b}+\sqrt{-b}\right)}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^5+b)^(3/4),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(2)*(-b)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^5 +
 b)^(1/4))/(-b)^(1/4))/b - 1/5*sqrt(2)*(-b)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(-b)^(1/4) - 2*(a*x^5 + b)^(1/4))/(-b)^(1/4))/b - 1/10*sqrt(2)*(-b)^(1/
4)*log(sqrt(2)*(a*x^5 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^5 + b) + sqrt(-b))/b
 + 1/10*sqrt(2)*(-b)^(1/4)*log(-sqrt(2)*(a*x^5 + b)^(1/4)*(-b)^(1/4) + sqrt
(a*x^5 + b) + sqrt(-b))/b
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a\,x^{5}+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^5+b)^(3/4),x)
```

```
[Out] int(1/x/(a*x^5+b)^(3/4),x)
```

maxima [A] time = 0.43, size = 57, normalized size = 1.04

$$-\frac{2\,\arctan\left(\frac{\left(ax^{5}+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{5\,b^{\frac{3}{4}}}+\frac{\log\left(\frac{\left(ax^{5}+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^{5}+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{5\,b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^5+b)^(3/4),x, algorithm="maxima")
```

```
[Out] -2/5*arctan((a*x^5 + b)^(1/4)/b^(1/4))/b^(3/4) + 1/5*log(((a*x^5 + b)^(1/4)
 - b^(1/4))/((a*x^5 + b)^(1/4) + b^(1/4)))/b^(3/4)
```

mupad [B] time = 0.71, size = 39, normalized size = 0.71

$$-\frac{2\,\mathrm{atan}\left(\frac{{\left(a\,x^5+b\right)}^{1/4}}{b^{1/4}}\right)}{5\,b^{3/4}}-\frac{2\,\mathrm{atanh}\left(\frac{{\left(a\,x^5+b\right)}^{1/4}}{b^{1/4}}\right)}{5\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b + a*x^5)^(3/4)),x)
```

```
[Out] - (2*atan((b + a*x^5)^(1/4)/b^(1/4)))/(5*b^(3/4)) - (2*atanh((b + a*x^5)^(1
/4)/b^(1/4)))/(5*b^(3/4))
```

sympy [C] time = 0.93, size = 41, normalized size = 0.75

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^{5}}\right)}{5a^{\frac{3}{4}}x^{\frac{15}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**5+b)**(3/4),x)
```

```
[Out] -gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*exp_polar(I*pi)/(a*x**5))/(5*a**(3/
4)*x**(15/4)*gamma(7/4))
```

3.703 $\int \frac{(1+2x^3)\sqrt{-1+x^6}}{x}\,dx$

Optimal. Leaf size=55

$$\frac{1}{3}\sqrt{x^6-1}\,\left(x^3+1\right)-\frac{1}{3}\log\left(\sqrt{x^6-1}\,+x^3\right)-\frac{2}{3}\tan^{-1}\left(\sqrt{x^6-1}\,+x^3\right)$$

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1475, 815, 844, 217, 206, 266, 63, 203}

$$-\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{1}{3}\sqrt{x^6-1}\,\left(x^3+1\right)-\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x^3)*Sqrt[-1 + x^6])/x,x]
```

```
[Out] ((1 + x^3)*Sqrt[-1 + x^6])/3 - ArcTan[Sqrt[-1 + x^6]]/3 - ArcTanh[x^3/Sqrt[
-1 + x^6]]/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
```

```
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(1+2x^3\right)\sqrt{-1+x^6}}{x}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(1+2x)\sqrt{-1+x^2}}{x}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\left(1+x^3\right)\sqrt{-1+x^6} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{-2-2x}{x\sqrt{-1+x^2}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\left(1+x^3\right)\sqrt{-1+x^6} - \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, x^3\right) - \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1+x}}\right.$$
$$= \frac{1}{3}\left(1+x^3\right)\sqrt{-1+x^6} - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right) - \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx\right.$$
$$= \frac{1}{3}\left(1+x^3\right)\sqrt{-1+x^6} - \frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right) - \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+}\right.$$
$$= \frac{1}{3}\left(1+x^3\right)\sqrt{-1+x^6} - \frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right) - \frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)$$

Mathematica [A] time = 0.06, size = 77, normalized size = 1.40

$$\frac{-\sqrt{-\left(x^6-1\right)^2}\tan^{-1}\left(\sqrt{x^6-1}\right)+\left(x^3-1\right)\sqrt{1-x^6}\left(x^3+1\right)^2+\left(x^6-1\right)\sin^{-1}\left(x^3\right)}{3\sqrt{-\left(x^6-1\right)^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + 2*x^3)*Sqrt[-1 + x^6])/x,x]
```

```
[Out] ((-1 + x^3)*(1 + x^3)^2*Sqrt[1 - x^6] + (-1 + x^6)*ArcSin[x^3] - Sqrt[-(-1
+ x^6)^2]*ArcTan[Sqrt[-1 + x^6]])/(3*Sqrt[-(-1 + x^6)^2])
```

IntegrateAlgebraic [A] time = 0.16, size = 63, normalized size = 1.15

$$\frac{1}{3}\sqrt{x^6-1}\left(x^3+1\right)+\frac{2}{3}\tan^{-1}\left(\frac{\sqrt{x^6-1}}{x^3-1}\right)-\frac{2}{3}\tanh^{-1}\left(\frac{\sqrt{x^6-1}}{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + 2*x^3)*Sqrt[-1 + x^6])/x,x]
```

```
[Out] ((1 + x^3)*Sqrt[-1 + x^6])/3 + (2*ArcTan[Sqrt[-1 + x^6]/(-1 + x^3)])/3 - (2
*ArcTanh[Sqrt[-1 + x^6]/(-1 + x^3)])/3
```

fricas [A] time = 0.39, size = 47, normalized size = 0.85

$$\frac{1}{3}\sqrt{x^6-1}\left(x^3+1\right)-\frac{2}{3}\arctan\left(-x^3+\sqrt{x^6-1}\right)+\frac{1}{3}\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)*(x^6-1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(x^6 - 1)*(x^3 + 1) - 2/3*arctan(-x^3 + sqrt(x^6 - 1)) + 1/3*log(-x
^3 + sqrt(x^6 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^6-1}\left(2\,x^3+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)*(x^6-1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^6 - 1)*(2*x^3 + 1)/x, x)
```

maple [C] time = 0.24, size = 136, normalized size = 2.47

$$-\frac{\sqrt{\mathrm{signum}\left(x^6-1\right)}\left(4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{-x^6+1}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^6+1}}{2}\right)-2\left(2-2\ln(2)+6\ln(x)+i\pi\right)\sqrt{\pi}\right)}{12\sqrt{\pi}\,\sqrt{-\mathrm{signum}\left(x^6-1\right)}}+\frac{i\sqrt{\mathrm{signum}\left(x^6-1\right)}\left(-2i\sqrt{\pi}\,x^3\sqrt{-x^6+1}-2i\sqrt{\pi}\,\arcsin\left(x^3\right)\right)}{6\sqrt{\pi}\,\sqrt{-\mathrm{signum}\left(x^6-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+1)*(x^6-1)^(1/2)/x,x)
```

```
[Out] -1/12/Pi^(1/2)*signum(x^6-1)^(1/2)/(-signum(x^6-1))^(1/2)*(4*Pi^(1/2)-4*Pi^
(1/2)*(-x^6+1)^(1/2)+4*Pi^(1/2)*ln(1/2+1/2*(-x^6+1)^(1/2))-2*(2-2*ln(2)+6*l
n(x)+I*Pi)*Pi^(1/2))+1/6*I/Pi^(1/2)*signum(x^6-1)^(1/2)/(-signum(x^6-1))^(1
/2)*(-2*I*Pi^(1/2)*x^3*(-x^6+1)^(1/2)-2*I*Pi^(1/2)*arcsin(x^3))
```

maxima [A] time = 0.43, size = 77, normalized size = 1.40

$$\frac{1}{3}\sqrt{x^6-1}-\frac{\sqrt{x^6-1}}{3\,x^3\left(\frac{x^6-1}{x^6}-1\right)}-\frac{1}{3}\arctan\left(\sqrt{x^6-1}\right)-\frac{1}{6}\log\left(\frac{\sqrt{x^6-1}}{x^3}+1\right)+\frac{1}{6}\log\left(\frac{\sqrt{x^6-1}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)*(x^6-1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^6 - 1) - 1/3*sqrt(x^6 - 1)/(x^3*((x^6 - 1)/x^6 - 1)) - 1/3*arcta
n(sqrt(x^6 - 1)) - 1/6*log(sqrt(x^6 - 1)/x^3 + 1) + 1/6*log(sqrt(x^6 - 1)/x
^3 - 1)
```

mupad [B] time = 1.12, size = 54, normalized size = 0.98

$$\frac{\sqrt{x^6-1}}{3}-\frac{\ln\left(\sqrt{x^6-1}+x^3\right)}{3}+\frac{x^3\,\sqrt{x^6-1}}{3}-\frac{\ln\left(\frac{\sqrt{x^6-1}+1{}\mathrm{i}}{x^3}\right)\,1{}\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/2)*(2*x^3 + 1))/x,x)
```

```
[Out] (x^6 - 1)^(1/2)/3 - (log(((x^6 - 1)^(1/2) + 1i)/x^3)*1i)/3 - log((x^6 - 1)^
(1/2) + x^3)/3 + (x^3*(x^6 - 1)^(1/2))/3
```

sympy [C] time = 10.48, size = 151, normalized size = 2.75

$$2\left(\begin{cases}\frac{x^9}{6\sqrt{x^6-1}}-\frac{x^3}{6\sqrt{x^6-1}}-\frac{\operatorname{acosh}\left(x^3\right)}{6} & \text{for }\left|x^6\right|>1\\ \frac{ix^3\sqrt{1-x^6}}{6}+\frac{i\operatorname{asin}\left(x^3\right)}{6} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{ix^3}{3\sqrt{-1+\frac{1}{x^6}}}-\frac{i\operatorname{acosh}\left(\frac{1}{x^3}\right)}{3}+\frac{i}{3x^3\sqrt{-1+\frac{1}{x^6}}} & \text{for }\frac{1}{|x^6|}>1\\ \frac{x^3}{3\sqrt{1-\frac{1}{x^6}}}+\frac{\operatorname{asin}\left(\frac{1}{x^3}\right)}{3}-\frac{1}{3x^3\sqrt{1-\frac{1}{x^6}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+1)*(x**6-1)**(1/2)/x,x)
```

```
[Out] 2*Piecewise((x**9/(6*sqrt(x**6 - 1)) - x**3/(6*sqrt(x**6 - 1)) - acosh(x**3
)/6, Abs(x**6) > 1), (I*x**3*sqrt(1 - x**6)/6 + I*asin(x**3)/6, True)) + Pi
ecewise((-I*x**3/(3*sqrt(-1 + x**(-6))) - I*acosh(x**(-3))/3 + I/(3*x**3*sq
rt(-1 + x**(-6))), 1/Abs(x**6) > 1), (x**3/(3*sqrt(1 - 1/x**6)) + asin(x**(
-3))/3 - 1/(3*x**3*sqrt(1 - 1/x**6)), True))
```

3.704 $\int \frac{1}{x\left(b+ax^6\right)^{3/4}}\,dx$

Optimal. Leaf size=55

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{3b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{3b^{3/4}}$$

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 63, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{3b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{3b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b + a*x^6)^(3/4)),x]
```

```
[Out] -1/3*ArcTan[(b + a*x^6)^(1/4)/b^(1/4)]/b^(3/4) - ArcTanh[(b + a*x^6)^(1/4)/
b^(1/4)]/(3*b^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(b+ax^6\right)^{3/4}}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x(b+ax)^{3/4}}\,dx, x, x^6\right)$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^6}\right)}{3a}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^6}\right)}{3\sqrt{b}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx, x, \sqrt[4]{b+ax^6}\right)}{3\sqrt{b}}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b+ax^6}}{\sqrt[4]{b}}\right)}{3b^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^6}}{\sqrt[4]{b}}\right)}{3b^{3/4}}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.84

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{3b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b + a*x^6)^(3/4)),x]
```

```
[Out] -1/3*(ArcTan[(b + a*x^6)^(1/4)/b^(1/4)] + ArcTanh[(b + a*x^6)^(1/4)/b^(1/4)
])/b^(3/4)
```

IntegrateAlgebraic [A] time = 0.05, size = 55, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{3b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{3b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(b + a*x^6)^(3/4)),x]
```

```
[Out] -1/3*ArcTan[(b + a*x^6)^(1/4)/b^(1/4)]/b^(3/4) - ArcTanh[(b + a*x^6)^(1/4)/
b^(1/4)]/(3*b^(3/4))
```

fricas [B] time = 0.40, size = 110, normalized size = 2.00

$$\frac{2}{3}\left(\frac{1}{b^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^2\sqrt{\frac{1}{b^3}}+\sqrt{ax^6+b}}\,b^2\left(\frac{1}{b^3}\right)^{\frac{3}{4}}-\left(ax^6+b\right)^{\frac{1}{4}}b^2\left(\frac{1}{b^3}\right)^{\frac{3}{4}}\right)-\frac{1}{6}\left(\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(b\left(\frac{1}{b^3}\right)^{\frac{1}{4}}+\left(ax^6+b\right)^{\frac{1}{4}}\right)+\frac{1}{6}\left(\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(-b\left(\frac{1}{b^3}\right)^{\frac{1}{4}}+\left(ax^6+b\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^6+b)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/3*(b^(-3))^(1/4)*arctan(sqrt(b^2*sqrt(b^(-3)) + sqrt(a*x^6 + b))*b^2*(b^(
-3))^(3/4) - (a*x^6 + b)^(1/4)*b^2*(b^(-3))^(3/4)) - 1/6*(b^(-3))^(1/4)*log
(b*(b^(-3))^(1/4) + (a*x^6 + b)^(1/4)) + 1/6*(b^(-3))^(1/4)*log(-b*(b^(-3))
^(1/4) + (a*x^6 + b)^(1/4))
```

giac [B] time = 0.14, size = 186, normalized size = 3.38

$$-\frac{\sqrt{2}\left(-b\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b\right)^{\frac{1}{4}}+2\left(ax^6+b\right)^{\frac{1}{4}}\right)}{2\left(-b\right)^{\frac{1}{4}}}\right)}{6b}-\frac{\sqrt{2}\left(-b\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b\right)^{\frac{1}{4}}-2\left(ax^6+b\right)^{\frac{1}{4}}\right)}{2\left(-b\right)^{\frac{1}{4}}}\right)}{6b}-\frac{\sqrt{2}\left(-b\right)^{\frac{1}{4}}\log\left(\sqrt{2}\left(ax^6+b\right)^{\frac{1}{4}}\left(-b\right)^{\frac{1}{4}}+\sqrt{ax^6+b}+\sqrt{-b}\right)}{12b}+\frac{\sqrt{2}\left(-b\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\left(ax^6+b\right)^{\frac{1}{4}}\left(-b\right)^{\frac{1}{4}}+\sqrt{ax^6+b}+\sqrt{-b}\right)}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^6+b)^(3/4),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*(-b)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^6 +
 b)^(1/4))/(-b)^(1/4))/b - 1/6*sqrt(2)*(-b)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(-b)^(1/4) - 2*(a*x^6 + b)^(1/4))/(-b)^(1/4))/b - 1/12*sqrt(2)*(-b)^(1/
4)*log(sqrt(2)*(a*x^6 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^6 + b) + sqrt(-b))/b
 + 1/12*sqrt(2)*(-b)^(1/4)*log(-sqrt(2)*(a*x^6 + b)^(1/4)*(-b)^(1/4) + sqrt
(a*x^6 + b) + sqrt(-b))/b
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{x \left(a\,x^{6}+b\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^6+b)^(3/4),x)
```

```
[Out] int(1/x/(a*x^6+b)^(3/4),x)
```

maxima [A] time = 0.42, size = 57, normalized size = 1.04

$$-\frac{\arctan\left(\frac{\left(ax^{6}+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{3\,b^{\frac{3}{4}}}+\frac{\log\left(\frac{\left(ax^{6}+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^{6}+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{6\,b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^6+b)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/3*arctan((a*x^6 + b)^(1/4)/b^(1/4))/b^(3/4) + 1/6*log(((a*x^6 + b)^(1/4)
 - b^(1/4))/((a*x^6 + b)^(1/4) + b^(1/4)))/b^(3/4)
```

mupad [B] time = 0.72, size = 34, normalized size = 0.62

$$-\frac{\mathrm{atan}\left(\frac{{\left(a\,x^6+b\right)}^{1/4}}{b^{1/4}}\right)+\mathrm{atanh}\left(\frac{{\left(a\,x^6+b\right)}^{1/4}}{b^{1/4}}\right)}{3\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b + a*x^6)^(3/4)),x)
```

```
[Out] -(atan((b + a*x^6)^(1/4)/b^(1/4)) + atanh((b + a*x^6)^(1/4)/b^(1/4)))/(3*b^
(3/4))
```

sympy [C] time = 0.92, size = 41, normalized size = 0.75

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^{6}}\right)}{6a^{\frac{3}{4}}x^{\frac{9}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**6+b)**(3/4),x)
```

```
[Out] -gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*exp_polar(I*pi)/(a*x**6))/(6*a**(3/
4)*x**(9/2)*gamma(7/4))
```

3.705 $\int \frac{1}{(-1+x)\sqrt{-\sqrt{x}+x}}\,dx$

Optimal. Leaf size=56

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x-\sqrt{x}}}{\sqrt{2}\sqrt{x}}\right)-\frac{2\sqrt{x-\sqrt{x}}}{\sqrt{x}-1}$$

Rubi [A] time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.41, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2056, 1549, 848, 94, 93, 206}

$$\frac{\sqrt{2}\sqrt{\sqrt{x}-1}\sqrt[4]{x}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x}}{\sqrt{\sqrt{x}-1}}\right)}{\sqrt{x-\sqrt{x}}}-\frac{2\sqrt{x}}{\sqrt{x-\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)*Sqrt[-Sqrt[x] + x]),x]
```

```
[Out] (-2*Sqrt[x])/Sqrt[-Sqrt[x] + x] + (Sqrt[2]*Sqrt[-1 + Sqrt[x]]*x^(1/4)*ArcTa
nh[(Sqrt[2]*x^(1/4))/Sqrt[-1 + Sqrt[x]]])/Sqrt[-Sqrt[x] + x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 1549

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g*(m + 1)
- 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{
```

```
a, c, d, e, m, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(-1+x)\sqrt{-\sqrt{x}+x}}\,dx &= \frac{\left(\sqrt{-1+\sqrt{x}}\,\sqrt[4]{x}\right)\int \frac{1}{\sqrt{-1+\sqrt{x}}\,(-1+x)\sqrt[4]{x}}\,dx}{\sqrt{-\sqrt{x}+x}}\\
&= \frac{\left(2\sqrt{-1+\sqrt{x}}\,\sqrt[4]{x}\right)\mathrm{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{-1+x}\left(-1+x^2\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{-\sqrt{x}+x}}\\
&= \frac{\left(2\sqrt{-1+\sqrt{x}}\,\sqrt[4]{x}\right)\mathrm{Subst}\left(\int \frac{\sqrt{x}}{(-1+x)^{3/2}(1+x)}\,dx,x,\sqrt{x}\right)}{\sqrt{-\sqrt{x}+x}}\\
&= -\frac{2\sqrt{x}}{\sqrt{-\sqrt{x}+x}}+\frac{\left(\sqrt{-1+\sqrt{x}}\,\sqrt[4]{x}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{x}\,(1+x)}\,dx,x,\sqrt{x}\right)}{\sqrt{-\sqrt{x}+x}}\\
&= -\frac{2\sqrt{x}}{\sqrt{-\sqrt{x}+x}}+\frac{\left(2\sqrt{-1+\sqrt{x}}\,\sqrt[4]{x}\right)\mathrm{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\frac{\sqrt[4]{x}}{\sqrt{-1+\sqrt{x}}}\right)}{\sqrt{-\sqrt{x}+x}}\\
&= -\frac{2\sqrt{x}}{\sqrt{-\sqrt{x}+x}}+\frac{\sqrt{2}\sqrt{-1+\sqrt{x}}\,\sqrt[4]{x}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x}}{\sqrt{-1+\sqrt{x}}}\right)}{\sqrt{-\sqrt{x}+x}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 67, normalized size = 1.20

$$\frac{\sqrt{2}\sqrt{\sqrt{x}-1}\,\sqrt[4]{x}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}}{\sqrt{2}\sqrt[4]{x}}\right)-2\sqrt{x}}{\sqrt{x-\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)*Sqrt[-Sqrt[x] + x]),x]
```

```
[Out] (-2*Sqrt[x] + Sqrt[2]*Sqrt[-1 + Sqrt[x]]*x^(1/4)*ArcTanh[Sqrt[-1 + Sqrt[x]]
/(Sqrt[2]*x^(1/4))])/Sqrt[-Sqrt[x] + x]
```

IntegrateAlgebraic [A] time = 0.22, size = 60, normalized size = 1.07

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x-\sqrt{x}}}{\sqrt{x}-1}\right)-\frac{2\sqrt{x-\sqrt{x}}}{\sqrt{x}-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x)*Sqrt[-Sqrt[x] + x]),x]
```

```
[Out] (-2*Sqrt[-Sqrt[x] + x])/(-1 + Sqrt[x]) + Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[-Sqr
t[x] + x])/(-1 + Sqrt[x])]
```

fricas [B] time = 1.97, size = 98, normalized size = 1.75

$$\frac{\sqrt{2}\,(x-1)\log\left(-\frac{17\,x^2+4\left(\sqrt{2}\,(3\,x+5)\,\sqrt{x}-\sqrt{2}\,(7\,x+1)\right)\sqrt{x-\sqrt{x}}-16\,(3\,x+1)\,\sqrt{x}+46\,x+1}{x^2-2\,x+1}\right)-8\,\sqrt{x-\sqrt{x}}\,\left(\sqrt{x}+1\right)}{4\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(-x^(1/2)+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(x - 1)*log(-(17*x^2 + 4*(sqrt(2)*(3*x + 5)*sqrt(x) - sqrt(2)*
(7*x + 1))*sqrt(x - sqrt(x)) - 16*(3*x + 1)*sqrt(x) + 46*x + 1)/(x^2 - 2*x
+ 1)) - 8*sqrt(x - sqrt(x))*(sqrt(x) + 1))/(x - 1)
```

giac [A] time = 0.51, size = 74, normalized size = 1.32

$$\frac{1}{2}\,\sqrt{2}\,\log\left(\frac{2\left(\sqrt{2}-\sqrt{x-\sqrt{x}}+\sqrt{x}+1\right)}{\left|2\,\sqrt{2}+2\,\sqrt{x-\sqrt{x}}-2\,\sqrt{x}-2\right|}\right)-\frac{2}{\sqrt{x-\sqrt{x}}-\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(-x^(1/2)+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(2*(sqrt(2) - sqrt(x - sqrt(x)) + sqrt(x) + 1)/abs(2*sqrt(2)
 + 2*sqrt(x - sqrt(x)) - 2*sqrt(x) - 2)) - 2/(sqrt(x - sqrt(x)) - sqrt(x) +
 1)
```

maple [B] time = 0.03, size = 164, normalized size = 2.93

$$-\frac{\sqrt{-\sqrt{x}+x}\,\left(2\sqrt{2}\,\sqrt{x}\,\operatorname{arctanh}\left(\frac{\left(-1+3\sqrt{x}\right)\sqrt{2}}{4\sqrt{-\sqrt{x}+x}}\right)-\sqrt{2}\,x\operatorname{arctanh}\left(\frac{\left(-1+3\sqrt{x}\right)\sqrt{2}}{4\sqrt{-\sqrt{x}+x}}\right)+4\left(-\sqrt{x}+x\right)^{\frac{3}{2}}-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-1+3\sqrt{x}\right)\sqrt{2}}{4\sqrt{-\sqrt{x}+x}}\right)+8\sqrt{x}\,\sqrt{-\sqrt{x}+x}-4x\sqrt{-\sqrt{x}+x}-4\sqrt{-\sqrt{x}+x}\right)}{2\sqrt{\sqrt{x}\,\left(-1+\sqrt{x}\right)}\,\left(-1+\sqrt{x}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)/(-x^(1/2)+x)^(1/2),x)
```

```
[Out] -1/2*(-x^(1/2)+x)^(1/2)*(2*2^(1/2)*x^(1/2)*arctanh(1/4*(-1+3*x^(1/2))*2^(1/
2)/(-x^(1/2)+x)^(1/2))-2^(1/2)*x*arctanh(1/4*(-1+3*x^(1/2))*2^(1/2)/(-x^(1/
2)+x)^(1/2))+4*(-x^(1/2)+x)^(3/2)-2^(1/2)*arctanh(1/4*(-1+3*x^(1/2))*2^(1/2
)/(-x^(1/2)+x)^(1/2))+8*x^(1/2)*(-x^(1/2)+x)^(1/2)-4*x*(-x^(1/2)+x)^(1/2)-4
*(-x^(1/2)+x)^(1/2))/(x^(1/2)*(-1+x^(1/2)))^(1/2)/(-1+x^(1/2))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-\sqrt{x}}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(-x^(1/2)+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x - sqrt(x))*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x-\sqrt{x}}\,\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - x^(1/2))^(1/2)*(x - 1)),x)
[Out] int(1/((x - x^(1/2))^(1/2)*(x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\sqrt{x}+x}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(-x**(1/2)+x)**(1/2),x)
[Out] Integral(1/(sqrt(-sqrt(x) + x)*(x - 1)), x)
```

3.706 $\int \frac{1}{(1+2x)\sqrt[4]{1+2x+2x^2}}\,dx$

Optimal. Leaf size=56

$$\frac{\tan^{-1}\left(\sqrt[4]{2}\sqrt[4]{2x^2+2x+1}\right)}{2^{3/4}}-\frac{\tanh^{-1}\left(\sqrt[4]{2}\sqrt[4]{2x^2+2x+1}\right)}{2^{3/4}}$$

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {694, 266, 63, 298, 203, 206}

$$\frac{\tan^{-1}\left(\sqrt[4]{(2x+1)^2+1}\right)}{2^{3/4}}-\frac{\tanh^{-1}\left(\sqrt[4]{(2x+1)^2+1}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + 2*x)*(1 + 2*x + 2*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(1 + (1 + 2*x)^2)^(1/4)]/2^(3/4) - ArcTanh[(1 + (1 + 2*x)^2)^(1/4)]/
2^(3/4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
```

```
Q[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(1+2x)\sqrt[4]{1+2x+2x^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{\frac{1}{2}+\frac{x^2}{2}}}\,dx, x, 1+2x\right)$$
$$= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{\frac{1}{2}+\frac{x}{2}}\,x}\,dx, x, (1+2x)^2\right)$$
$$= 2\operatorname{Subst}\left(\int \frac{x^2}{-1+2x^4}\,dx, x, \sqrt[4]{1+2x+2x^2}\right)$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x^2}\,dx, x, \sqrt[4]{1+2x+2x^2}\right)}{\sqrt{2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x^2}\,dx, x, \sqrt[4]{1+2x+2x^2}\right)}{\sqrt{2}}$$
$$= \frac{\tan^{-1}\left(\sqrt[4]{2}\sqrt[4]{1+2x+2x^2}\right)}{2^{3/4}} - \frac{\tanh^{-1}\left(\sqrt[4]{2}\sqrt[4]{1+2x+2x^2}\right)}{2^{3/4}}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.70

$$\frac{\tan^{-1}\left(\sqrt[4]{4x^2+4x+2}\right) - \tanh^{-1}\left(\sqrt[4]{4x^2+4x+2}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + 2*x)*(1 + 2*x + 2*x^2)^(1/4)),x]
[Out] (ArcTan[(2 + 4*x + 4*x^2)^(1/4)] - ArcTanh[(2 + 4*x + 4*x^2)^(1/4)])/2^(3/4
)
```

IntegrateAlgebraic [A] time = 0.11, size = 56, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt[4]{2}\sqrt[4]{2x^2+2x+1}\right)}{2^{3/4}} - \frac{\tanh^{-1}\left(\sqrt[4]{2}\sqrt[4]{2x^2+2x+1}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + 2*x)*(1 + 2*x + 2*x^2)^(1/4)),x]
[Out] ArcTan[2^(1/4)*(1 + 2*x + 2*x^2)^(1/4)]/2^(3/4) - ArcTanh[2^(1/4)*(1 + 2*x
+ 2*x^2)^(1/4)]/2^(3/4)
```

fricas [B] time = 0.44, size = 102, normalized size = 1.82

$$-\frac{1}{4}\cdot 8^{\frac{3}{4}}\arctan\left(\frac{1}{8}\cdot 8^{\frac{3}{4}}\sqrt{2\sqrt{2}+4\sqrt{2x^2+2x+1}} - \frac{1}{4}\cdot 8^{\frac{3}{4}}\left(2x^2+2x+1\right)^{\frac{1}{4}}\right) - \frac{1}{16}\cdot 8^{\frac{3}{4}}\log\left(8^{\frac{1}{4}}+2\left(2x^2+2x+1\right)^{\frac{1}{4}}\right) + \frac{1}{16}\cdot 8^{\frac{3}{4}}\log\left(-8^{\frac{1}{4}}+2\left(2x^2+2x+1\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(2*x^2+2*x+1)^(1/4),x, algorithm="fricas")
[Out] -1/4*8^(3/4)*arctan(1/8*8^(3/4)*sqrt(2*sqrt(2) + 4*sqrt(2*x^2 + 2*x + 1)) -
 1/4*8^(3/4)*(2*x^2 + 2*x + 1)^(1/4)) - 1/16*8^(3/4)*log(8^(1/4) + 2*(2*x^2
 + 2*x + 1)^(1/4)) + 1/16*8^(3/4)*log(-8^(1/4) + 2*(2*x^2 + 2*x + 1)^(1/4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^2+2\,x+1\right)^{\frac{1}{4}}(2\,x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(2*x^2+2*x+1)^(1/4),x, algorithm="giac")
[Out] integrate(1/((2*x^2 + 2*x + 1)^(1/4)*(2*x + 1)), x)
```

maple [F] time = 1.51, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+2x)\left(2x^2+2x+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)/(2*x^2+2*x+1)^(1/4),x)
[Out] int(1/(1+2*x)/(2*x^2+2*x+1)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^2+2\,x+1\right)^{\frac{1}{4}}(2\,x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(2*x^2+2*x+1)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((2*x^2 + 2*x + 1)^(1/4)*(2*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(2\,x+1)\,\left(2\,x^2+2\,x+1\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + 1)*(2*x + 2*x^2 + 1)^(1/4)),x)
[Out] int(1/((2*x + 1)*(2*x + 2*x^2 + 1)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x+1)\sqrt[4]{2x^2+2x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(2*x**2+2*x+1)**(1/4),x)
[Out] Integral(1/((2*x + 1)*(2*x**2 + 2*x + 1)**(1/4)), x)
```

3.707 $\int \frac{\sqrt{b+ax^3}\,(2b+ax^3)}{x^4}\,dx$

Optimal. Leaf size=56

$$\frac{2\left(ax^3-b\right)\sqrt{ax^3+b}}{3x^3}-\frac{4}{3}a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 50, 63, 208}

$$-\frac{2\left(ax^3+b\right)^{3/2}}{3x^3}+\frac{4}{3}a\sqrt{ax^3+b}-\frac{4}{3}a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b + a*x^3]*(2*b + a*x^3))/x^4,x]
```

```
[Out] (4*a*Sqrt[b + a*x^3])/3 - (2*(b + a*x^3)^(3/2))/(3*x^3) - (4*a*Sqrt[b]*ArcT
anh[Sqrt[b + a*x^3]/Sqrt[b]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b+ax^3}\left(2b+ax^3\right)}{x^4}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{b+ax}\,(2b+ax)}{x^2}\,dx, x, x^3\right)\\
&= -\frac{2\left(b+ax^3\right)^{3/2}}{3x^3} + \frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{\sqrt{b+ax}}{x}\,dx, x, x^3\right)\\
&= \frac{4}{3}a\sqrt{b+ax^3} - \frac{2\left(b+ax^3\right)^{3/2}}{3x^3} + \frac{1}{3}(2ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{b+ax}}\,dx, x, x^3\right)\\
&= \frac{4}{3}a\sqrt{b+ax^3} - \frac{2\left(b+ax^3\right)^{3/2}}{3x^3} + \frac{1}{3}(4b)\,\text{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^2}{a}}\,dx, x, \sqrt{b+ax^3}\right)\\
&= \frac{4}{3}a\sqrt{b+ax^3} - \frac{2\left(b+ax^3\right)^{3/2}}{3x^3} - \frac{4}{3}a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b+ax^3}}{\sqrt{b}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.98

$$-\frac{2\sqrt{ax^3+b}\left(b-ax^3\right)}{3x^3} - \frac{4}{3}a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b + a*x^3]*(2*b + a*x^3))/x^4,x]
```

```
[Out] (-2*(b - a*x^3)*Sqrt[b + a*x^3])/(3*x^3) - (4*a*Sqrt[b]*ArcTanh[Sqrt[b + a*
x^3]/Sqrt[b]])/3
```

IntegrateAlgebraic [A] time = 0.08, size = 56, normalized size = 1.00

$$\frac{2\left(ax^3-b\right)\sqrt{ax^3+b}}{3x^3} - \frac{4}{3}a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[b + a*x^3]*(2*b + a*x^3))/x^4,x]
```

```
[Out] (2*(-b + a*x^3)*Sqrt[b + a*x^3])/(3*x^3) - (4*a*Sqrt[b]*ArcTanh[Sqrt[b + a*
x^3]/Sqrt[b]])/3
```

fricas [A] time = 0.42, size = 117, normalized size = 2.09

$$\left[\frac{2\left(a\sqrt{b}\,x^3\log\left(\frac{ax^3-2\sqrt{ax^3+b}\,\sqrt{b}+2b}{x^3}\right)+\sqrt{ax^3+b}\left(ax^3-b\right)\right)}{3x^3}, \frac{2\left(2a\sqrt{-b}\,x^3\arctan\left(\frac{\sqrt{ax^3+b}\,\sqrt{-b}}{b}\right)+\sqrt{ax^3+b}\left(ax^3-b\right)\right)}{3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)^(1/2)*(a*x^3+2*b)/x^4,x, algorithm="fricas")
```

```
[Out] [2/3*(a*sqrt(b)*x^3*log((a*x^3 - 2*sqrt(a*x^3 + b)*sqrt(b) + 2*b)/x^3) + sq
rt(a*x^3 + b)*(a*x^3 - b))/x^3, 2/3*(2*a*sqrt(-b)*x^3*arctan(sqrt(a*x^3 + b
)*sqrt(-b)/b) + sqrt(a*x^3 + b)*(a*x^3 - b))/x^3]
```

giac [A] time = 0.19, size = 62, normalized size = 1.11

$$\frac{2\left(\frac{2\,a^2b\arctan\left(\frac{\sqrt{ax^3+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\sqrt{ax^3+b}\,a^2-\frac{\sqrt{ax^3+b}\,ab}{x^3}\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)^(1/2)*(a*x^3+2*b)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(2*a^2*b*arctan(sqrt(a*x^3 + b)/sqrt(-b))/sqrt(-b) + sqrt(a*x^3 + b)*a^
2 - sqrt(a*x^3 + b)*a*b/x^3)/a
```

maple [A] time = 0.02, size = 73, normalized size = 1.30

$$2b\left(-\frac{\sqrt{a\,x^3+b}}{3x^3}-\frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^3+b}}{\sqrt{b}}\right)}{3\sqrt{b}}\right)+a\left(\frac{2\sqrt{a\,x^3+b}}{3}-\frac{2\sqrt{b}\ \mathrm{arctanh}\left(\frac{\sqrt{a\,x^3+b}}{\sqrt{b}}\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b)^(1/2)*(a*x^3+2*b)/x^4,x)
```

```
[Out] 2*b*(-1/3*(a*x^3+b)^(1/2)/x^3-1/3*a*arctanh((a*x^3+b)^(1/2)/b^(1/2))/b^(1/2
))+a*(2/3*(a*x^3+b)^(1/2)-2/3*b^(1/2)*arctanh((a*x^3+b)^(1/2)/b^(1/2)))
```

maxima [B] time = 0.44, size = 107, normalized size = 1.91

$$\frac{1}{3}\left(\sqrt{b}\,\log\left(\frac{\sqrt{ax^3+b}-\sqrt{b}}{\sqrt{ax^3+b}+\sqrt{b}}\right)+2\sqrt{ax^3+b}\right)a+\frac{1}{3}\left(\frac{a\log\left(\frac{\sqrt{ax^3+b}-\sqrt{b}}{\sqrt{ax^3+b}+\sqrt{b}}\right)}{\sqrt{b}}-\frac{2\sqrt{ax^3+b}}{x^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)^(1/2)*(a*x^3+2*b)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(b)*log((sqrt(a*x^3 + b) - sqrt(b))/(sqrt(a*x^3 + b) + sqrt(b))) +
 2*sqrt(a*x^3 + b))*a + 1/3*(a*log((sqrt(a*x^3 + b) - sqrt(b))/(sqrt(a*x^3
+ b) + sqrt(b)))/sqrt(b) - 2*sqrt(a*x^3 + b)/x^3)*b
```

mupad [B] time = 0.78, size = 69, normalized size = 1.23

$$\frac{2\,a\,\sqrt{a\,x^3+b}}{3}-\frac{2\,b\,\sqrt{a\,x^3+b}}{3\,x^3}+\frac{2\,a\,\sqrt{b}\,\ln\left(\frac{\left(\sqrt{a\,x^3+b}-\sqrt{b}\right)^3\left(\sqrt{a\,x^3+b}+\sqrt{b}\right)}{x^6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a*x^3)^(1/2)*(2*b + a*x^3))/x^4,x)
```

```
[Out] (2*a*(b + a*x^3)^(1/2))/3 - (2*b*(b + a*x^3)^(1/2))/(3*x^3) + (2*a*b^(1/2)*
log((((b + a*x^3)^(1/2) - b^(1/2))^3*((b + a*x^3)^(1/2) + b^(1/2)))/x^6))/3
```

sympy [A] time = 26.74, size = 105, normalized size = 1.88

$$\frac{2a^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{1+\frac{b}{ax^3}}}-\frac{2\sqrt{a}\,b\sqrt{1+\frac{b}{ax^3}}}{3x^{\frac{3}{2}}}+\frac{2\sqrt{a}\,b}{3x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^3}}}-\frac{4a\sqrt{b}\ \mathrm{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b)**(1/2)*(a*x**3+2*b)/x**4,x)
```

```
[Out] 2*a**(3/2)*x**(3/2)/(3*sqrt(1 + b/(a*x**3))) - 2*sqrt(a)*b*sqrt(1 + b/(a*x*
*3))/(3*x**(3/2)) + 2*sqrt(a)*b/(3*x**(3/2)*sqrt(1 + b/(a*x**3))) - 4*a*sqr
t(b)*asinh(sqrt(b)/(sqrt(a)*x**(3/2)))/3
```

3.708 $\int \frac{(-2q+px^3)\sqrt{q+px^3}}{x^2(aq+bx^2+apx^3)}\,dx$

Optimal. Leaf size=56

$$\frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}\sqrt{px^3+q}}\right)}{a^{3/2}}+\frac{2\sqrt{px^3+q}}{ax}$$

Rubi [F] time = 1.55, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2q+px^3)\sqrt{q+px^3}}{x^2(aq+bx^2+apx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*q + p*x^3)*Sqrt[q + p*x^3])/(x^2*(a*q + b*x^2 + a*p*x^3)),x]
```

```
[Out] (2*Sqrt[q + p*x^3])/(a*x) - (6*p^(1/3)*Sqrt[q + p*x^3])/(a*((1 + Sqrt[3])*q
^(1/3) + p^(1/3)*x)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*p^(1/3)*q^(1/3)*(q^(1/3
) + p^(1/3)*x)*Sqrt[(q^(2/3) - p^(1/3)*q^(1/3)*x + p^(2/3)*x^2)/((1 + Sqrt[
3])*q^(1/3) + p^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*q^(1/3) + p^(1/
3)*x)/((1 + Sqrt[3])*q^(1/3) + p^(1/3)*x)], -7 - 4*Sqrt[3]])/(a*Sqrt[(q^(1/
3)*(q^(1/3) + p^(1/3)*x))/((1 + Sqrt[3])*q^(1/3) + p^(1/3)*x)^2]*Sqrt[q + p
*x^3]) - (2*Sqrt[2]*3^(3/4)*p^(1/3)*q^(1/3)*(q^(1/3) + p^(1/3)*x)*Sqrt[(q^(
2/3) - p^(1/3)*q^(1/3)*x + p^(2/3)*x^2)/((1 + Sqrt[3])*q^(1/3) + p^(1/3)*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*q^(1/3) + p^(1/3)*x)/((1 + Sqrt[3])*q^(
1/3) + p^(1/3)*x)], -7 - 4*Sqrt[3]])/(a*Sqrt[(q^(1/3)*(q^(1/3) + p^(1/3)*x)
)/((1 + Sqrt[3])*q^(1/3) + p^(1/3)*x)^2]*Sqrt[q + p*x^3]) + (2*b*Defer[Int]
[Sqrt[q + p*x^3]/(a*q + b*x^2 + a*p*x^3), x])/a + 3*p*Defer[Int][(x*Sqrt[q
+ p*x^3])/(a*q + b*x^2 + a*p*x^3), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-2q+px^3)\sqrt{q+px^3}}{x^2(aq+bx^2+apx^3)}\,dx &= \int \left(-\frac{2\sqrt{q+px^3}}{ax^2}+\frac{(2b+3apx)\sqrt{q+px^3}}{a(aq+bx^2+apx^3)}\right)dx\\
&= \frac{\int \frac{(2b+3apx)\sqrt{q+px^3}}{aq+bx^2+apx^3}\,dx}{a}-\frac{2\int \frac{\sqrt{q+px^3}}{x^2}\,dx}{a}\\
&= \frac{2\sqrt{q+px^3}}{ax}+\frac{\int \left(\frac{2b\sqrt{q+px^3}}{aq+bx^2+apx^3}+\frac{3apx\sqrt{q+px^3}}{aq+bx^2+apx^3}\right)dx}{a}-\frac{(3p)\int \frac{x}{\sqrt{q+px^3}}\,dx}{a}\\
&= \frac{2\sqrt{q+px^3}}{ax}+\frac{(2b)\int \frac{\sqrt{q+px^3}}{aq+bx^2+apx^3}\,dx}{a}-\frac{\left(3p^{2/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{q}+\sqrt[3]{p}\,x}{\sqrt{q+px^3}}\,dx}{a}+(3p)\int \frac{x\sqrt{\ }}{aq+\ }\\
&= \frac{2\sqrt{q+px^3}}{ax}-\frac{6\sqrt[3]{p}\sqrt{q+px^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{q}+\sqrt[3]{p}\,x\right)}+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{p}\sqrt[3]{q}\left(\sqrt[3]{q}+\sqrt[3]{p}\,x\right)\sqrt{\ }}{a\sqrt{\frac{\ }{((1+}}}
\end{aligned}$$

Mathematica [C] time = 6.29, size = 2888, normalized size = 51.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-2*q + p*x^3)*Sqrt[q + p*x^3])/(x^2*(a*q + b*x^2 + a*p*x^3)),x]
```

```
[Out] (2*Sqrt[q + p*x^3])/(a*x) + (b*((-2*Sqrt[(q^(1/3)/p^(1/3) + x)/(q^(1/3)/p^(
1/3) + ((-1)^(1/3)*q^(1/3))/p^(1/3))]*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + x)
*Sqrt[(((-1)^(2/3)*q^(1/3))/p^(1/3) + x)/(((-1)^(1/3)*q^(1/3))/p^(1/3) + ((
-1)^(2/3)*q^(1/3))/p^(1/3))]*EllipticF[ArcSin[Sqrt[((-1)^(1/3)*q^(1/3) - p^
(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3))*q^(1/3))]], (-1)^(1/3)])/(a*Sqrt[(-(((-
1)^(1/3)*q^(1/3))/p^(1/3)) + x)/(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) - ((-1)^(2
/3)*q^(1/3))/p^(1/3))]*Sqrt[q + p*x^3]) + (4*(((-1)^(1/3)*q^(1/3))/p^(1/3)
+ ((-1)^(2/3)*q^(1/3))/p^(1/3))*q*Sqrt[(q^(1/3)/p^(1/3) + x)/(q^(1/3)/p^(1/
3) + ((-1)^(1/3)*q^(1/3))/p^(1/3))]*Sqrt[((-(((-1)^(2/3)*q^(1/3))/p^(1/3))
- x)*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + x))/(((-1)^(1/3)*q^(1/3))/p^(1/3) +
 ((-1)^(2/3)*q^(1/3))/p^(1/3))^2]*EllipticPi[((-1)^(1/3)*q^(1/3) + (-1)^(2/
3)*q^(1/3))/((-1)^(1/3)*q^(1/3) - p^(1/3)*Root[a*q + b*#1^2 + a*p*#1^3 & ,
1]), ArcSin[Sqrt[((-1)^(1/3)*q^(1/3) - p^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3)
)*q^(1/3))]], (-1)^(1/3)])/(a*p*Sqrt[q + p*x^3]*(-(((-1)^(1/3)*q^(1/3))/p^(
1/3)) + Root[a*q + b*#1^2 + a*p*#1^3 & , 1])*(Root[a*q + b*#1^2 + a*p*#1^3
& , 1] - Root[a*q + b*#1^2 + a*p*#1^3 & , 2])*(Root[a*q + b*#1^2 + a*p*#1^3
 & , 1] - Root[a*q + b*#1^2 + a*p*#1^3 & , 3])) - (2*(((-1)^(1/3)*q^(1/3))/
p^(1/3) + ((-1)^(2/3)*q^(1/3))/p^(1/3))*Sqrt[(q^(1/3)/p^(1/3) + x)/(q^(1/3)
/p^(1/3) + ((-1)^(1/3)*q^(1/3))/p^(1/3))]*Sqrt[((-(((-1)^(2/3)*q^(1/3))/p^(
1/3)) - x)*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + x))/(((-1)^(1/3)*q^(1/3))/p^(
1/3) + ((-1)^(2/3)*q^(1/3))/p^(1/3))^2]*EllipticPi[((-1)^(1/3)*q^(1/3) + (-
1)^(2/3)*q^(1/3))/((-1)^(1/3)*q^(1/3) - p^(1/3)*Root[a*q + b*#1^2 + a*p*#1^
3 & , 1]), ArcSin[Sqrt[((-1)^(1/3)*q^(1/3) - p^(1/3)*x)/(((-1)^(1/3) + (-1)
^(2/3))*q^(1/3))]], (-1)^(1/3)]*Root[a*q + b*#1^2 + a*p*#1^3 & , 1]^3)/(a*S
qrt[q + p*x^3]*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + Root[a*q + b*#1^2 + a*p*#
1^3 & , 1])*(Root[a*q + b*#1^2 + a*p*#1^3 & , 1] - Root[a*q + b*#1^2 + a*p*
#1^3 & , 2])*(Root[a*q + b*#1^2 + a*p*#1^3 & , 1] - Root[a*q + b*#1^2 + a*p
*#1^3 & , 3])) + (4*(((-1)^(1/3)*q^(1/3))/p^(1/3) + ((-1)^(2/3)*q^(1/3))/p^
(1/3))*q*Sqrt[(q^(1/3)/p^(1/3) + x)/(q^(1/3)/p^(1/3) + ((-1)^(1/3)*q^(1/3))
/p^(1/3))]*Sqrt[((-(((-1)^(2/3)*q^(1/3))/p^(1/3)) - x)*(-(((-1)^(1/3)*q^(1/
3))/p^(1/3)) + x))/(((-1)^(1/3)*q^(1/3))/p^(1/3) + ((-1)^(2/3)*q^(1/3))/p^(
1/3))^2]*EllipticPi[((-1)^(1/3)*q^(1/3) + (-1)^(2/3)*q^(1/3))/((-1)^(1/3)*q
^(1/3) - p^(1/3)*Root[a*q + b*#1^2 + a*p*#1^3 & , 2]), ArcSin[Sqrt[((-1)^(1
/3)*q^(1/3) - p^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3))*q^(1/3))]], (-1)^(1/3)]
)/(a*p*Sqrt[q + p*x^3]*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + Root[a*q + b*#1^2
 + a*p*#1^3 & , 2])*(-Root[a*q + b*#1^2 + a*p*#1^3 & , 1] + Root[a*q + b*#1
^2 + a*p*#1^3 & , 2])*(Root[a*q + b*#1^2 + a*p*#1^3 & , 2] - Root[a*q + b*#
1^2 + a*p*#1^3 & , 3])) - (2*(((-1)^(1/3)*q^(1/3))/p^(1/3) + ((-1)^(2/3)*q^
(1/3))/p^(1/3))*Sqrt[(q^(1/3)/p^(1/3) + x)/(q^(1/3)/p^(1/3) + ((-1)^(1/3)*q
^(1/3))/p^(1/3))]*Sqrt[((-(((-1)^(2/3)*q^(1/3))/p^(1/3)) - x)*(-(((-1)^(1/3
)*q^(1/3))/p^(1/3)) + x))/(((-1)^(1/3)*q^(1/3))/p^(1/3) + ((-1)^(2/3)*q^(1/
3))/p^(1/3))^2]*EllipticPi[((-1)^(1/3)*q^(1/3) + (-1)^(2/3)*q^(1/3))/((-1)^
(1/3)*q^(1/3) - p^(1/3)*Root[a*q + b*#1^2 + a*p*#1^3 & , 2]), ArcSin[Sqrt[(
(-1)^(1/3)*q^(1/3) - p^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3))*q^(1/3))]], (-1)
^(1/3)]*Root[a*q + b*#1^2 + a*p*#1^3 & , 2]^3)/(a*Sqrt[q + p*x^3]*(-(((-1)^
(1/3)*q^(1/3))/p^(1/3)) + Root[a*q + b*#1^2 + a*p*#1^3 & , 2])*(-Root[a*q +
 b*#1^2 + a*p*#1^3 & , 1] + Root[a*q + b*#1^2 + a*p*#1^3 & , 2])*(Root[a*q
+ b*#1^2 + a*p*#1^3 & , 2] - Root[a*q + b*#1^2 + a*p*#1^3 & , 3])) + (4*(((
-1)^(1/3)*q^(1/3))/p^(1/3) + ((-1)^(2/3)*q^(1/3))/p^(1/3))*q*Sqrt[(q^(1/3)/
p^(1/3) + x)/(q^(1/3)/p^(1/3) + ((-1)^(1/3)*q^(1/3))/p^(1/3))]*Sqrt[((-(((-
1)^(2/3)*q^(1/3))/p^(1/3)) - x)*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + x))/(((-
1)^(1/3)*q^(1/3))/p^(1/3) + ((-1)^(2/3)*q^(1/3))/p^(1/3))^2]*EllipticPi[((-
1)^(1/3)*q^(1/3) + (-1)^(2/3)*q^(1/3))/((-1)^(1/3)*q^(1/3) - p^(1/3)*Root[a
*q + b*#1^2 + a*p*#1^3 & , 3]), ArcSin[Sqrt[((-1)^(1/3)*q^(1/3) - p^(1/3)*x
)/(((-1)^(1/3) + (-1)^(2/3))*q^(1/3))]], (-1)^(1/3)])/(a*p*Sqrt[q + p*x^3]*
(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + Root[a*q + b*#1^2 + a*p*#1^3 & , 3])*(-R
```

```
oot[a*q + b*#1^2 + a*p*#1^3 & , 1] + Root[a*q + b*#1^2 + a*p*#1^3 & , 3])*(
-Root[a*q + b*#1^2 + a*p*#1^3 & , 2] + Root[a*q + b*#1^2 + a*p*#1^3 & , 3])
) - (2*(((-1)^(1/3)*q^(1/3))/p^(1/3) + ((-1)^(2/3)*q^(1/3))/p^(1/3))*Sqrt[(
q^(1/3)/p^(1/3) + x)/(q^(1/3)/p^(1/3) + ((-1)^(1/3)*q^(1/3))/p^(1/3))]*Sqrt
[((-(((-1)^(2/3)*q^(1/3))/p^(1/3)) - x)*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) +
x))/(((-1)^(1/3)*q^(1/3))/p^(1/3) + ((-1)^(2/3)*q^(1/3))/p^(1/3))^2]*Ellipt
icPi[((-1)^(1/3)*q^(1/3) + (-1)^(2/3)*q^(1/3))/((-1)^(1/3)*q^(1/3) - p^(1/3
)*Root[a*q + b*#1^2 + a*p*#1^3 & , 3]), ArcSin[Sqrt[((-1)^(1/3)*q^(1/3) - p
^(1/3)*x)/(((-1)^(1/3) + (-1)^(2/3))*q^(1/3))]], (-1)^(1/3)]*Root[a*q + b*#
1^2 + a*p*#1^3 & , 3]^3)/(a*Sqrt[q + p*x^3]*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)
) + Root[a*q + b*#1^2 + a*p*#1^3 & , 3])*(-Root[a*q + b*#1^2 + a*p*#1^3 & ,
 1] + Root[a*q + b*#1^2 + a*p*#1^3 & , 3])*(-Root[a*q + b*#1^2 + a*p*#1^3 &
 , 2] + Root[a*q + b*#1^2 + a*p*#1^3 & , 3]))))/a
```

IntegrateAlgebraic [A] time = 0.55, size = 56, normalized size = 1.00

$$\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}\,\sqrt{px^3+q}}\right)}{a^{3/2}}+\frac{2\sqrt{px^3+q}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*q + p*x^3)*Sqrt[q + p*x^3])/(x^2*(a*q + b*x^2 + a*p
*x^3)),x]
```

```
[Out] (2*Sqrt[q + p*x^3])/(a*x) + (2*Sqrt[b]*ArcTan[(Sqrt[b]*x)/(Sqrt[a]*Sqrt[q +
 p*x^3])])/a^(3/2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p*x^3+q)^(1/2)/x^2/(a*p*x^3+b*x^2+a*q),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^3+q}\,\left(px^3-2\,q\right)}{\left(apx^3+bx^2+aq\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p*x^3+q)^(1/2)/x^2/(a*p*x^3+b*x^2+a*q),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(p*x^3 + q)*(p*x^3 - 2*q)/((a*p*x^3 + b*x^2 + a*q)*x^2), x)
```

maple [C] time = 0.25, size = 1747, normalized size = 31.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^3-2*q)*(p*x^3+q)^(1/2)/x^2/(a*p*x^3+b*x^2+a*q),x)
```

```
[Out] 1/a*(2/3*I*b/a*3^(1/2)/p*(-q*p^2)^(1/3)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I*3^
(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2)*((x-1/p*(-q*p^2)^(1
/3))/(-3/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2)*(-I*(x+1
/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3
```

```
))^(1/2)/(p*x^3+q)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1
/2*I*3^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2),(I*3^(1/2)/p
*(-q*p^2)^(1/3)/(-3/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/
2))-2*I*3^(1/2)*(-q*p^2)^(1/3)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I*3^(1/2)/p*(
-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2)*((x-1/p*(-q*p^2)^(1/3))/(-3/
2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2)*(-I*(x+1/2/p*(-q*
p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2)/
(p*x^3+q)^(1/2)*((-3/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I*3^(1/2)/p*(-q*p^2)^(1/3
))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2),(I*3^(1/2)/p*(-q*p^2)^(1/3)/(-3/2/p*(-q*
p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2))+1/p*(-q*p^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I*3^(1/2)/p*(-q*p^2)^(1/3))
*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2),(I*3^(1/2)/p*(-q*p^2)^(1/3)/(-3/2/p*(-q*p^
2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2)))-I/a/p^2/q*2^(1/2)*sum((_a
lpha^2*b+3*a*q)/_alpha/(3*_alpha*a*p+2*b)*(-q*p^2)^(1/3)*(1/2*I*p*(2*x+1/p*
(-I*3^(1/2)*(-q*p^2)^(1/3)+(-q*p^2)^(1/3)))/(-q*p^2)^(1/3))^(1/2)*(p*(x-1/p
*(-q*p^2)^(1/3))/(-3*(-q*p^2)^(1/3)+I*3^(1/2)*(-q*p^2)^(1/3)))^(1/2)*(-1/2*
I*p*(2*x+1/p*(I*3^(1/2)*(-q*p^2)^(1/3)+(-q*p^2)^(1/3)))/(-q*p^2)^(1/3))^(1/
2)/(p*x^3+q)^(1/2)*(-I*(-q*p^2)^(1/3)*3^(1/2)*_alpha^2*a*p^2+I*(-q*p^2)^(2/
3)*3^(1/2)*_alpha*a*p-I*(-q*p^2)^(1/3)*3^(1/2)*_alpha*p*b+(-q*p^2)^(1/3)*_a
lpha^2*a*p^2+I*(-q*p^2)^(2/3)*3^(1/2)*b+(-q*p^2)^(2/3)*_alpha*a*p+(-q*p^2)^
(1/3)*_alpha*b*p+2*a*p^2*q+(-q*p^2)^(2/3)*b)*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/p*(-q*p^2)^(1/3)-1/2*I*3^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3
))^(1/2),-1/2/p*(-I*(-q*p^2)^(2/3)*3^(1/2)*_alpha^2*a*p+I*3^(1/2)*_alpha*a*
p^2*q-I*(-q*p^2)^(2/3)*3^(1/2)*_alpha*b-3*_alpha^2*(-q*p^2)^(2/3)*a*p-2*I*(
-q*p^2)^(1/3)*3^(1/2)*a*p*q+I*3^(1/2)*b*p*q-3*_alpha*a*q*p^2-3*(-q*p^2)^(2/
3)*_alpha*b-3*b*p*q)/q/b,(I*3^(1/2)/p*(-q*p^2)^(1/3)/(-3/2/p*(-q*p^2)^(1/3)
+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*a*p+_Z^2*b+a*q)
))-2/a*(-(p*x^3+q)^(1/2)/x-I*3^(1/2)*(-q*p^2)^(1/3)*(I*(x+1/2/p*(-q*p^2)^(1
/3)-1/2*I*3^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2)*((x-1/p
*(-q*p^2)^(1/3))/(-3/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1
/2)*(-I*(x+1/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(
-q*p^2)^(1/3))^(1/2)/(p*x^3+q)^(1/2)*((-3/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/
p*(-q*p^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I*3^
(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2),(I*3^(1/2)/p*(-q*p^
2)^(1/3)/(-3/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2))+1/p
*(-q*p^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I*3^(1
/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2),(I*3^(1/2)/p*(-q*p^2)
^(1/3)/(-3/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^3+q}\left(px^3-2q\right)}{\left(apx^3+bx^2+aq\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p*x^3+q)^(1/2)/x^2/(a*p*x^3+b*x^2+a*q),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(p*x^3 + q)*(p*x^3 - 2*q)/((a*p*x^3 + b*x^2 + a*q)*x^2), x)
```

mupad [B] time = 5.99, size = 102, normalized size = 1.82

$$\frac{2\sqrt{p\,x^3+q}}{a\,x}+\frac{\sqrt{b}\;\ln\left(\frac{a^5\,b\,p^4\,x^2-a^6\,p^4\,\left(p\,x^3+q\right)+a^{11/2}\,\sqrt{b}\,p^4\,x\,\sqrt{p\,x^3+q}\,2{}\mathrm{i}}{4\,b^2\,q\,x^2+4\,a\,b\,q\,\left(p\,x^3+q\right)}\right)\,1{}\mathrm{i}}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q + p*x^3)^(1/2)*(2*q - p*x^3))/(x^2*(a*q + b*x^2 + a*p*x^3)),x)
```

```
[Out] (2*(q + p*x^3)^(1/2))/(a*x) + (b^(1/2)*log((a^5*b*p^4*x^2 - a^6*p^4*(q + p*
x^3) + a^(11/2)*b^(1/2)*p^4*x*(q + p*x^3)^(1/2)*2i)/(4*b^2*q*x^2 + 4*a*b*q*
(q + p*x^3)))*1i)/a^(3/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**3-2*q)*(p*x**3+q)**(1/2)/x**2/(a*p*x**3+b*x**2+a*q),x)
```

```
[Out] Timed out
```

3.709 $\int \frac{-1+2x}{\sqrt{-8+4x-3x^2-10x^3+x^4}} \, dx$

Optimal. Leaf size=56

$$-\frac{1}{2}\log\left(x^4-18x^3+89x^2+\left(-x^2+13x-38\right)\sqrt{x^4-10x^3-3x^2+4x-8}-76x-90\right)$$

Rubi [F] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+2x}{\sqrt{-8+4x-3x^2-10x^3+x^4}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 2*x)/Sqrt[-8 + 4*x - 3*x^2 - 10*x^3 + x^4],x]
```

```
[Out] -Defer[Int][1/Sqrt[-8 + 4*x - 3*x^2 - 10*x^3 + x^4], x] + 2*Defer[Int][x/Sq
rt[-8 + 4*x - 3*x^2 - 10*x^3 + x^4], x]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+2x}{\sqrt{-8+4x-3x^2-10x^3+x^4}} \, dx &= \int \left(-\frac{1}{\sqrt{-8+4x-3x^2-10x^3+x^4}}+\frac{2x}{\sqrt{-8+4x-3x^2-10x^3+x^4}}\right) dx\\ &= 2\int \frac{x}{\sqrt{-8+4x-3x^2-10x^3+x^4}} \, dx-\int \frac{1}{\sqrt{-8+4x-3x^2-10x^3+x^4}}\end{aligned}$$

Mathematica [C] time = 2.13, size = 1333, normalized size = 23.80

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 2*x)/Sqrt[-8 + 4*x - 3*x^2 - 10*x^3 + x^4],x]
```

```
[Out] (2*(5 + Sqrt[17] + 4*Sqrt[4 + Sqrt[17]] - 2*x)*(-((4 + Sqrt[17] + 4*Sqrt[4
+ Sqrt[17]])*EllipticF[ArcSin[Sqrt[((5 + Sqrt[17] - 4*Sqrt[4 + Sqrt[17]] -
2*x)*(5 + Sqrt[17] + 4*Sqrt[4 + Sqrt[17]] - 2*Root[-8 + 4*#1 - 3*#1^2 - 10*
#1^3 + #1^4 & , 4, 0]))/((5 + Sqrt[17] + 4*Sqrt[4 + Sqrt[17]] - 2*x)*(5 + S
qrt[17] - 4*Sqrt[4 + Sqrt[17]] - 2*Root[-8 + 4*#1 - 3*#1^2 - 10*#1^3 + #1^4
 & , 4, 0]))]], ((5 + Sqrt[17] + 4*Sqrt[4 + Sqrt[17]] - 2*Root[-8 + 4*#1 -
3*#1^2 - 10*#1^3 + #1^4 & , 3, 0])*(5 + Sqrt[17] - 4*Sqrt[4 + Sqrt[17]] - 2
*Root[-8 + 4*#1 - 3*#1^2 - 10*#1^3 + #1^4 & , 4, 0]))/((5 + Sqrt[17] - 4*Sq
rt[4 + Sqrt[17]] - 2*Root[-8 + 4*#1 - 3*#1^2 - 10*#1^3 + #1^4 & , 3, 0])*(5
 + Sqrt[17] + 4*Sqrt[4 + Sqrt[17]] - 2*Root[-8 + 4*#1 - 3*#1^2 - 10*#1^3 +
#1^4 & , 4, 0]))]) + 8*Sqrt[4 + Sqrt[17]]*EllipticPi[(5 + Sqrt[17] - 4*Sqrt
[4 + Sqrt[17]] - 2*Root[-8 + 4*#1 - 3*#1^2 - 10*#1^3 + #1^4 & , 4, 0])/(5 +
 Sqrt[17] + 4*Sqrt[4 + Sqrt[17]] - 2*Root[-8 + 4*#1 - 3*#1^2 - 10*#1^3 + #1
^4 & , 4, 0]), ArcSin[Sqrt[((5 + Sqrt[17] - 4*Sqrt[4 + Sqrt[17]] - 2*x)*(5
+ Sqrt[17] + 4*Sqrt[4 + Sqrt[17]] - 2*Root[-8 + 4*#1 - 3*#1^2 - 10*#1^3 + #
1^4 & , 4, 0]))/((5 + Sqrt[17] + 4*Sqrt[4 + Sqrt[17]] - 2*x)*(5 + Sqrt[17]
- 4*Sqrt[4 + Sqrt[17]] - 2*Root[-8 + 4*#1 - 3*#1^2 - 10*#1^3 + #1^4 & , 4,
0]))]], ((5 + Sqrt[17] + 4*Sqrt[4 + Sqrt[17]] - 2*Root[-8 + 4*#1 - 3*#1^2 -
 10*#1^3 + #1^4 & , 3, 0])*(5 + Sqrt[17] - 4*Sqrt[4 + Sqrt[17]] - 2*Root[-8
 + 4*#1 - 3*#1^2 - 10*#1^3 + #1^4 & , 4, 0]))/((5 + Sqrt[17] - 4*Sqrt[4 + S
qrt[17]] - 2*Root[-8 + 4*#1 - 3*#1^2 - 10*#1^3 + #1^4 & , 3, 0])*(5 + Sqrt[
17] + 4*Sqrt[4 + Sqrt[17]] - 2*Root[-8 + 4*#1 - 3*#1^2 - 10*#1^3 + #1^4 & ,
```

```
 4, 0]))])*Sqrt[(x - Root[-8 + 4*#1 - 3*#1^2 - 10*#1^3 + #1^4 & , 3, 0])/((
5 + Sqrt[17] + 4*Sqrt[4 + Sqrt[17]] - 2*x)*(5 + Sqrt[17] - 4*Sqrt[4 + Sqrt[
17]] - 2*Root[-8 + 4*#1 - 3*#1^2 - 10*#1^3 + #1^4 & , 3, 0]))]*Sqrt[((5 + S
qrt[17] - 4*Sqrt[4 + Sqrt[17]] - 2*x)*(5 + Sqrt[17] + 4*Sqrt[4 + Sqrt[17]]
- 2*Root[-8 + 4*#1 - 3*#1^2 - 10*#1^3 + #1^4 & , 4, 0]))/((5 + Sqrt[17] + 4
*Sqrt[4 + Sqrt[17]] - 2*x)*(5 + Sqrt[17] - 4*Sqrt[4 + Sqrt[17]] - 2*Root[-8
 + 4*#1 - 3*#1^2 - 10*#1^3 + #1^4 & , 4, 0]))]*(x - Root[-8 + 4*#1 - 3*#1^2
 - 10*#1^3 + #1^4 & , 4, 0]))/(Sqrt[-8 + 4*x - 3*x^2 - 10*x^3 + x^4]*(5 + S
qrt[17] + 4*Sqrt[4 + Sqrt[17]] - 2*Root[-8 + 4*#1 - 3*#1^2 - 10*#1^3 + #1^4
 & , 4, 0])*Sqrt[(x - Root[-8 + 4*#1 - 3*#1^2 - 10*#1^3 + #1^4 & , 4, 0])/(
(5 + Sqrt[17] + 4*Sqrt[4 + Sqrt[17]] - 2*x)*(5 + Sqrt[17] - 4*Sqrt[4 + Sqrt
[17]] - 2*Root[-8 + 4*#1 - 3*#1^2 - 10*#1^3 + #1^4 & , 4, 0]))])
```

IntegrateAlgebraic [A] time = 5.03, size = 56, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^4-18x^3+89x^2+\left(-x^2+13x-38\right)\sqrt{x^4-10x^3-3x^2+4x-8}-76x-90\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x)/Sqrt[-8 + 4*x - 3*x^2 - 10*x^3 + x^4],x]
```

```
[Out] -1/2*Log[-90 - 76*x + 89*x^2 - 18*x^3 + x^4 + (-38 + 13*x - x^2)*Sqrt[-8 +
4*x - 3*x^2 - 10*x^3 + x^4]]
```

fricas [A] time = 0.44, size = 50, normalized size = 0.89

$$\frac{1}{2}\log\left(x^4-18\,x^3+89\,x^2+\sqrt{x^4-10\,x^3-3\,x^2+4\,x-8}\left(x^2-13\,x+38\right)-76\,x-90\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-10*x^3-3*x^2+4*x-8)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^4 - 18*x^3 + 89*x^2 + sqrt(x^4 - 10*x^3 - 3*x^2 + 4*x - 8)*(x^2 -
 13*x + 38) - 76*x - 90)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2\,x-1}{\sqrt{x^4-10\,x^3-3\,x^2+4\,x-8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-10*x^3-3*x^2+4*x-8)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*x - 1)/sqrt(x^4 - 10*x^3 - 3*x^2 + 4*x - 8), x)
```

maple [C] time = 1.14, size = 2702, normalized size = 48.25

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x)/(x^4-10*x^3-3*x^2+4*x-8)^(1/2),x)
```

```
[Out] -2*(RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4
*_Z-8,index=4))*((RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)-RootOf(_Z^4-10
*_Z^3-3*_Z^2+4*_Z-8,index=2))*(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1)
)/(RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*
_Z-8,index=1))/(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2)))^(1/2)*(x-Roo
tOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2))^2*(-(RootOf(_Z^4-10*_Z^3-3*_Z^2+4*
_Z-8,index=2)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1))*(x-RootOf(_Z^4-10
```

```
*_Z^3-3*_Z^2+4*_Z-8,index=3))/(-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=3)+
RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1))/(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4
*_Z-8,index=2)))^(1/2)*((RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2)-RootOf(
_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1))*(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,i
ndex=4))/(RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)-RootOf(_Z^4-10*_Z^3-3*
_Z^2+4*_Z-8,index=1))/(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2)))^(1/2)
/(RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_
Z-8,index=2))/(RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2)-RootOf(_Z^4-10*_Z
^3-3*_Z^2+4*_Z-8,index=1))/((x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1))*
(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2))*(x-RootOf(_Z^4-10*_Z^3-3*_Z^
2+4*_Z-8,index=3))*(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)))^(1/2)*El
lipticF(((RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)-RootOf(_Z^4-10*_Z^3-3*
_Z^2+4*_Z-8,index=2))*(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1))/(RootO
f(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,ind
ex=1))/(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2)))^(1/2),((RootOf(_Z^4-
10*_Z^3-3*_Z^2+4*_Z-8,index=2)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=3))*
(RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z
-8,index=4))/(-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=3)+RootOf(_Z^4-10*_Z
^3-3*_Z^2+4*_Z-8,index=1))/(RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2)-Root
Of(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)))^(1/2))+4*(RootOf(_Z^4-10*_Z^3-3*_Z
^2+4*_Z-8,index=1)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4))*((RootOf(_Z^
4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2)
)*(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1))/(RootOf(_Z^4-10*_Z^3-3*_Z^
2+4*_Z-8,index=4)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1))/(x-RootOf(_Z^
4-10*_Z^3-3*_Z^2+4*_Z-8,index=2)))^(1/2)*(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z
-8,index=2))^2*(-(RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2)-RootOf(_Z^4-10
*_Z^3-3*_Z^2+4*_Z-8,index=1))*(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=3)
)/(-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=3)+RootOf(_Z^4-10*_Z^3-3*_Z^2+4
*_Z-8,index=1))/(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2)))^(1/2)*((Roo
tOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,i
ndex=1))*(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4))/(RootOf(_Z^4-10*_Z^
3-3*_Z^2+4*_Z-8,index=4)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1))/(x-Roo
tOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2)))^(1/2)/(RootOf(_Z^4-10*_Z^3-3*_Z^2
+4*_Z-8,index=4)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2))/(RootOf(_Z^4-1
0*_Z^3-3*_Z^2+4*_Z-8,index=2)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1))/(
(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1))*(x-RootOf(_Z^4-10*_Z^3-3*_Z^
2+4*_Z-8,index=2))*(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=3))*(x-RootOf
(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)))^(1/2)*(RootOf(_Z^4-10*_Z^3-3*_Z^2+4*
_Z-8,index=2)*EllipticF(((RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)-RootOf
(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2))*(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,
index=1))/(RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)-RootOf(_Z^4-10*_Z^3-3
*_Z^2+4*_Z-8,index=1))/(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2)))^(1/2
),((RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4
*_Z-8,index=3))*(RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1)-RootOf(_Z^4-10*
_Z^3-3*_Z^2+4*_Z-8,index=4))/(-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=3)+R
ootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1))/(RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z
-8,index=2)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)))^(1/2))+(RootOf(_Z^
4-10*_Z^3-3*_Z^2+4*_Z-8,index=1)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2)
)*EllipticPi(((RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)-RootOf(_Z^4-10*_Z
^3-3*_Z^2+4*_Z-8,index=2))*(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1))/(
RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-
8,index=1))/(x-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2)))^(1/2),(RootOf(_
Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=
1))/(RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)-RootOf(_Z^4-10*_Z^3-3*_Z^2+
4*_Z-8,index=2)),((RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=2)-RootOf(_Z^4-1
0*_Z^3-3*_Z^2+4*_Z-8,index=3))*(RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1)-
RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4))/(-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*
_Z-8,index=3)+RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=1))/(RootOf(_Z^4-10*_
Z^3-3*_Z^2+4*_Z-8,index=2)-RootOf(_Z^4-10*_Z^3-3*_Z^2+4*_Z-8,index=4)))^(1/
```

```
2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-1}{\sqrt{x^4-10x^3-3x^2+4x-8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x^4-10*x^3-3*x^2+4*x-8)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x - 1)/sqrt(x^4 - 10*x^3 - 3*x^2 + 4*x - 8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{2x-1}{\sqrt{x^4-10x^3-3x^2+4x-8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 1)/(4*x - 3*x^2 - 10*x^3 + x^4 - 8)^(1/2),x)
[Out] int((2*x - 1)/(4*x - 3*x^2 - 10*x^3 + x^4 - 8)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-1}{\sqrt{x^4-10x^3-3x^2+4x-8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(x**4-10*x**3-3*x**2+4*x-8)**(1/2),x)
[Out] Integral((2*x - 1)/sqrt(x**4 - 10*x**3 - 3*x**2 + 4*x - 8), x)
```

3.710 $\int \frac{-1+x}{\sqrt{4-16x+12x^2-8x^3+x^4}}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{4}\log\left(x^4-12x^3+44x^2+\left(-x^2+8x-14\right)\sqrt{x^4-8x^3+12x^2-16x+4}-56x+36\right)$$

Rubi [F] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+x}{\sqrt{4-16x+12x^2-8x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x)/Sqrt[4 - 16*x + 12*x^2 - 8*x^3 + x^4],x]
```

```
[Out] -Defer[Int][1/Sqrt[4 - 16*x + 12*x^2 - 8*x^3 + x^4], x] + Defer[Int][x/Sqrt
[4 - 16*x + 12*x^2 - 8*x^3 + x^4], x]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x}{\sqrt{4-16x+12x^2-8x^3+x^4}}\,dx &= \int \left(-\frac{1}{\sqrt{4-16x+12x^2-8x^3+x^4}}+\frac{x}{\sqrt{4-16x+12x^2-8x^3+x^4}}\right)dx\\ &= -\int \frac{1}{\sqrt{4-16x+12x^2-8x^3+x^4}}\,dx+\int \frac{x}{\sqrt{4-16x+12x^2-8x^3+x^4}}\,d\end{aligned}$$

Mathematica [C] time = 6.08, size = 2609, normalized size = 46.59

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/Sqrt[4 - 16*x + 12*x^2 - 8*x^3 + x^4],x]
```

```
[Out] (2*(x - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 2, 0])^2*(-(EllipticF[
ArcSin[Sqrt[((x - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 1, 0])*(Root
[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 2, 0] - Root[4 - 16*#1 + 12*#1^2 -
 8*#1^3 + #1^4 & , 4, 0]))/((x - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 &
 , 2, 0])*(Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 1, 0] - Root[4 - 16
*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 4, 0]))]], -(((Root[4 - 16*#1 + 12*#1^2 -
 8*#1^3 + #1^4 & , 2, 0] - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 3,
0])*(Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 1, 0] - Root[4 - 16*#1 +
12*#1^2 - 8*#1^3 + #1^4 & , 4, 0]))/((-Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 +
#1^4 & , 1, 0] + Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 3, 0])*(Root[
4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 2, 0] - Root[4 - 16*#1 + 12*#1^2 -
8*#1^3 + #1^4 & , 4, 0])))]*Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 2,
 0]) + EllipticPi[(-Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 1, 0] + Ro
ot[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 4, 0])/(-Root[4 - 16*#1 + 12*#1^
2 - 8*#1^3 + #1^4 & , 2, 0] + Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & ,
4, 0]), ArcSin[Sqrt[((x - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 1, 0
])*(Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 2, 0] - Root[4 - 16*#1 + 1
2*#1^2 - 8*#1^3 + #1^4 & , 4, 0]))/((x - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3
+ #1^4 & , 2, 0])*(Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 1, 0] - Roo
t[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 4, 0]))]], -(((Root[4 - 16*#1 + 1
2*#1^2 - 8*#1^3 + #1^4 & , 2, 0] - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4
```

```
 & , 3, 0])*(Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 1, 0] - Root[4 -
16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 4, 0]))/((-Root[4 - 16*#1 + 12*#1^2 - 8
*#1^3 + #1^4 & , 1, 0] + Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 3, 0]
)*(Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 2, 0] - Root[4 - 16*#1 + 12
*#1^2 - 8*#1^3 + #1^4 & , 4, 0])))]*(-Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #
1^4 & , 1, 0] + Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 2, 0]))*Sqrt[(
x - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 3, 0])/((x - Root[4 - 16*#
1 + 12*#1^2 - 8*#1^3 + #1^4 & , 2, 0])*(-Root[4 - 16*#1 + 12*#1^2 - 8*#1^3
+ #1^4 & , 1, 0] + Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 3, 0]))]*(R
oot[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 1, 0] - Root[4 - 16*#1 + 12*#1^
2 - 8*#1^3 + #1^4 & , 4, 0])*Sqrt[((x - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 +
 #1^4 & , 1, 0])*(Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 2, 0] - Root
[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 4, 0]))/((x - Root[4 - 16*#1 + 12*
#1^2 - 8*#1^3 + #1^4 & , 2, 0])*(Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 &
 , 1, 0] - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 4, 0]))]*Sqrt[(x -
Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 4, 0])/((x - Root[4 - 16*#1 +
12*#1^2 - 8*#1^3 + #1^4 & , 2, 0])*(-Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1
^4 & , 1, 0] + Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 4, 0]))])/(Sqrt
[4 - 16*x + 12*x^2 - 8*x^3 + x^4]*(Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4
 & , 2, 0] - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 4, 0])) - (2*Elli
pticF[ArcSin[Sqrt[((x - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 1, 0])
*(-Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 2, 0] + Root[4 - 16*#1 + 12
*#1^2 - 8*#1^3 + #1^4 & , 4, 0]))/((x - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 +
 #1^4 & , 2, 0])*(-Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 1, 0] + Roo
t[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 4, 0]))]], ((Root[4 - 16*#1 + 12*
#1^2 - 8*#1^3 + #1^4 & , 2, 0] - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 &
 , 3, 0])*(Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 1, 0] - Root[4 - 16
*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 4, 0]))/((Root[4 - 16*#1 + 12*#1^2 - 8*#1
^3 + #1^4 & , 1, 0] - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 3, 0])*(
Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 2, 0] - Root[4 - 16*#1 + 12*#1
^2 - 8*#1^3 + #1^4 & , 4, 0]))]*(x - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1
^4 & , 2, 0])^2*Sqrt[(x - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 3, 0
])/((x - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 2, 0])*(-Root[4 - 16*
#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 1, 0] + Root[4 - 16*#1 + 12*#1^2 - 8*#1^3
+ #1^4 & , 3, 0]))]*(Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 1, 0] - R
oot[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 4, 0])*Sqrt[(x - Root[4 - 16*#1
 + 12*#1^2 - 8*#1^3 + #1^4 & , 4, 0])/((x - Root[4 - 16*#1 + 12*#1^2 - 8*#1
^3 + #1^4 & , 2, 0])*(-Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 1, 0] +
 Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 4, 0]))]*Sqrt[((x - Root[4 -
16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 1, 0])*(-Root[4 - 16*#1 + 12*#1^2 - 8*#
1^3 + #1^4 & , 2, 0] + Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 4, 0]))
/((x - Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #1^4 & , 2, 0])*(-Root[4 - 16*#1
 + 12*#1^2 - 8*#1^3 + #1^4 & , 1, 0] + Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 +
#1^4 & , 4, 0]))])/(Sqrt[4 - 16*x + 12*x^2 - 8*x^3 + x^4]*(-Root[4 - 16*#1
+ 12*#1^2 - 8*#1^3 + #1^4 & , 2, 0] + Root[4 - 16*#1 + 12*#1^2 - 8*#1^3 + #
1^4 & , 4, 0]))
```

IntegrateAlgebraic [A] time = 4.79, size = 56, normalized size = 1.00

$$-\frac{1}{4}\log\left(x^4-12x^3+44x^2+\left(-x^2+8x-14\right)\sqrt{x^4-8x^3+12x^2-16x+4}-56x+36\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/Sqrt[4 - 16*x + 12*x^2 - 8*x^3 + x^4],x]
```

```
[Out] -1/4*Log[36 - 56*x + 44*x^2 - 12*x^3 + x^4 + (-14 + 8*x - x^2)*Sqrt[4 - 16*
x + 12*x^2 - 8*x^3 + x^4]]
```

fricas [A] time = 0.44, size = 53, normalized size = 0.95

$$\frac{1}{4}\log\left(-x^4+12x^3-44x^2-\sqrt{x^4-8x^3+12x^2-16x+4}\left(x^2-8x+14\right)+56x-36\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^4-8*x^3+12*x^2-16*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*log(-x^4 + 12*x^3 - 44*x^2 - sqrt(x^4 - 8*x^3 + 12*x^2 - 16*x + 4)*(x^2
 - 8*x + 14) + 56*x - 36)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x-1}{\sqrt{x^4-8x^3+12x^2-16x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^4-8*x^3+12*x^2-16*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/sqrt(x^4 - 8*x^3 + 12*x^2 - 16*x + 4), x)
```

maple [C] time = 1.16, size = 2700, normalized size = 48.21

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(x^4-8*x^3+12*x^2-16*x+4)^(1/2),x)
```

```
[Out] 2*(RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1)-RootOf(_Z^4-8*_Z^3+12*_Z^2-1
6*_Z+4,index=4))*((RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=4)-RootOf(_Z^4-
8*_Z^3+12*_Z^2-16*_Z+4,index=2))*(x-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,inde
x=1))/(RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=4)-RootOf(_Z^4-8*_Z^3+12*_Z
^2-16*_Z+4,index=1))/(x-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=2)))^(1/2)
*(x-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=2))^2*((RootOf(_Z^4-8*_Z^3+12*
_Z^2-16*_Z+4,index=2)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1))*(x-RootO
f(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=3))/(RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4
,index=3)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1))/(x-RootOf(_Z^4-8*_Z^
3+12*_Z^2-16*_Z+4,index=2)))^(1/2)*((RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,ind
ex=2)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1))*(x-RootOf(_Z^4-8*_Z^3+12
*_Z^2-16*_Z+4,index=4))/(RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=4)-RootOf
(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1))/(x-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+
4,index=2)))^(1/2)/(RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=4)-RootOf(_Z^4
-8*_Z^3+12*_Z^2-16*_Z+4,index=2))/(RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index
=2)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1))/((x-RootOf(_Z^4-8*_Z^3+12*
_Z^2-16*_Z+4,index=1))*(x-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=2))*(x-R
ootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=3))*(x-RootOf(_Z^4-8*_Z^3+12*_Z^2-1
6*_Z+4,index=4)))^(1/2)*(RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=2)*Ellipt
icF(((RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=4)-RootOf(_Z^4-8*_Z^3+12*_Z^
2-16*_Z+4,index=2))*(x-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1))/(RootOf
(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=4)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,in
dex=1))/(x-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=2)))^(1/2),((RootOf(_Z^
4-8*_Z^3+12*_Z^2-16*_Z+4,index=2)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=
3))*(RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1)-RootOf(_Z^4-8*_Z^3+12*_Z^2
-16*_Z+4,index=4))/(-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=3)+RootOf(_Z^
4-8*_Z^3+12*_Z^2-16*_Z+4,index=1))/(RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,inde
x=2)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=4)))^(1/2))+(RootOf(_Z^4-8*_Z
^3+12*_Z^2-16*_Z+4,index=1)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=2))*El
lipticPi(((RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=4)-RootOf(_Z^4-8*_Z^3+1
2*_Z^2-16*_Z+4,index=2))*(x-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1))/(R
```

```
ootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=4)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z
+4,index=1))/(x-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=2)))^(1/2),(RootOf
(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=4)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,in
dex=1))/(RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=4)-RootOf(_Z^4-8*_Z^3+12*
_Z^2-16*_Z+4,index=2)),((RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=2)-RootOf
(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=3))*(RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,
index=1)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=4))/(-RootOf(_Z^4-8*_Z^3+
12*_Z^2-16*_Z+4,index=3)+RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1))/(Root
Of(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=2)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,
index=4)))^(1/2)))-2*(RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1)-RootOf(_Z
^4-8*_Z^3+12*_Z^2-16*_Z+4,index=4))*((RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,in
dex=4)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=2))*(x-RootOf(_Z^4-8*_Z^3+1
2*_Z^2-16*_Z+4,index=1))/(RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=4)-RootO
f(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1))/(x-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z
+4,index=2)))^(1/2)*(x-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=2))^2*((Roo
tOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=2)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4
,index=1))*(x-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=3))/(RootOf(_Z^4-8*_
Z^3+12*_Z^2-16*_Z+4,index=3)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1))/(
x-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=2)))^(1/2)*((RootOf(_Z^4-8*_Z^3+
12*_Z^2-16*_Z+4,index=2)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1))*(x-Ro
otOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=4))/(RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_
Z+4,index=4)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1))/(x-RootOf(_Z^4-8*
_Z^3+12*_Z^2-16*_Z+4,index=2)))^(1/2)/(RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,i
ndex=4)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=2))/(RootOf(_Z^4-8*_Z^3+12
*_Z^2-16*_Z+4,index=2)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1))/((x-Roo
tOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1))*(x-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*
_Z+4,index=2))*(x-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=3))*(x-RootOf(_Z
^4-8*_Z^3+12*_Z^2-16*_Z+4,index=4)))^(1/2)*EllipticF(((RootOf(_Z^4-8*_Z^3+1
2*_Z^2-16*_Z+4,index=4)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=2))*(x-Roo
tOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1))/(RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z
+4,index=4)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=1))/(x-RootOf(_Z^4-8*_
Z^3+12*_Z^2-16*_Z+4,index=2)))^(1/2),((RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,i
ndex=2)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=3))*(RootOf(_Z^4-8*_Z^3+12
*_Z^2-16*_Z+4,index=1)-RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=4))/(-RootO
f(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=3)+RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,i
ndex=1))/(RootOf(_Z^4-8*_Z^3+12*_Z^2-16*_Z+4,index=2)-RootOf(_Z^4-8*_Z^3+12
*_Z^2-16*_Z+4,index=4)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x^4-8\,x^3+12\,x^2-16\,x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^4-8*x^3+12*x^2-16*x+4)^(1/2),x, algorithm="maxima")
[Out] integrate((x - 1)/sqrt(x^4 - 8*x^3 + 12*x^2 - 16*x + 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x-1}{\sqrt{x^4-8\,x^3+12\,x^2-16\,x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(12*x^2 - 16*x - 8*x^3 + x^4 + 4)^(1/2),x)
[Out] int((x - 1)/(12*x^2 - 16*x - 8*x^3 + x^4 + 4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x^4-8x^3+12x^2-16x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**4-8*x**3+12*x**2-16*x+4)**(1/2),x)
```

```
[Out] Integral((x - 1)/sqrt(x**4 - 8*x**3 + 12*x**2 - 16*x + 4), x)
```

3.711 $\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)}{x^2\left(aq+bx^2+apx^5\right)}\,dx$

Optimal. Leaf size=56

$$\frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}\,\sqrt{px^5+q}}\right)}{a^{3/2}}+\frac{2\sqrt{px^5+q}}{ax}$$

Rubi [F] time = 1.41, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)}{x^2\left(aq+bx^2+apx^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(x^2*(a*q + b*x^2 + a*p*x^5)),x]
```

```
[Out] (2*Sqrt[q + p*x^5])/(a*x) - (5*p*x^4*Sqrt[1 + (p*x^5)/q]*Hypergeometric2F1[
1/2, 4/5, 9/5, -((p*x^5)/q)])/(4*a*Sqrt[q + p*x^5]) + (2*b*Defer[Int][Sqrt[
q + p*x^5]/(a*q + b*x^2 + a*p*x^5), x])/a + 5*p*Defer[Int][(x^3*Sqrt[q + p*
x^5])/(a*q + b*x^2 + a*p*x^5), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)}{x^2\left(aq+bx^2+apx^5\right)}\,dx &= \int \left(-\frac{2\sqrt{q+px^5}}{ax^2}+\frac{\left(2b+5apx^3\right)\sqrt{q+px^5}}{a\left(aq+bx^2+apx^5\right)}\right)dx\\
&= \frac{\int \frac{\left(2b+5apx^3\right)\sqrt{q+px^5}}{aq+bx^2+apx^5}\,dx}{a}-\frac{2\int \frac{\sqrt{q+px^5}}{x^2}\,dx}{a}\\
&= \frac{2\sqrt{q+px^5}}{ax}+\frac{\int \left(\frac{2b\sqrt{q+px^5}}{aq+bx^2+apx^5}+\frac{5apx^3\sqrt{q+px^5}}{aq+bx^2+apx^5}\right)dx}{a}-\frac{(5p)\int \frac{x^3}{\sqrt{q+px^5}}\,dx}{a}\\
&= \frac{2\sqrt{q+px^5}}{ax}+\frac{(2b)\int \frac{\sqrt{q+px^5}}{aq+bx^2+apx^5}\,dx}{a}+(5p)\int \frac{x^3\sqrt{q+px^5}}{aq+bx^2+apx^5}\,dx-\frac{\left(5p\sqrt{1+\frac{px^5}{q}}\right.}{a\sqrt{q}}\\
&= \frac{2\sqrt{q+px^5}}{ax}-\frac{5px^4\sqrt{1+\frac{px^5}{q}}\,{}_2F_1\left(\frac{1}{2},\frac{4}{5};\frac{9}{5};-\frac{px^5}{q}\right)}{4a\sqrt{q+px^5}}+\frac{(2b)\int \frac{\sqrt{q+px^5}}{aq+bx^2+apx^5}\,dx}{a}+(5p)\int
\end{aligned}$$

Mathematica [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)}{x^2\left(aq+bx^2+apx^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(x^2*(a*q + b*x^2 + a*p*x^5)),
x]
```

```
[Out] Integrate[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(x^2*(a*q + b*x^2 + a*p*x^5)),
 x]
```

IntegrateAlgebraic [A] time = 0.82, size = 56, normalized size = 1.00

$$\frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}\,\sqrt{px^5+q}}\right)}{a^{3/2}}+\frac{2\sqrt{px^5+q}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(x^2*(a*q + b*x^2 + a
*p*x^5)),x]
```

```
[Out] (2*Sqrt[q + p*x^5])/(a*x) + (2*Sqrt[b]*ArcTan[(Sqrt[b]*x)/(Sqrt[a]*Sqrt[q +
 p*x^5])])/a^(3/2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^5+q)^(1/2)*(3*p*x^5-2*q)/x^2/(a*p*x^5+b*x^2+a*q),x, algorith
m="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,px^5-2\,q\right)\sqrt{px^5+q}}{\left(apx^5+bx^2+aq\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^5+q)^(1/2)*(3*p*x^5-2*q)/x^2/(a*p*x^5+b*x^2+a*q),x, algorith
m="giac")
```

```
[Out] integrate((3*p*x^5 - 2*q)*sqrt(p*x^5 + q)/((a*p*x^5 + b*x^2 + a*q)*x^2), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p\,x^5+q}\,\left(3p\,x^5-2q\right)}{x^2\left(ap\,x^5+b\,x^2+aq\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^5+q)^(1/2)*(3*p*x^5-2*q)/x^2/(a*p*x^5+b*x^2+a*q),x)
```

```
[Out] int((p*x^5+q)^(1/2)*(3*p*x^5-2*q)/x^2/(a*p*x^5+b*x^2+a*q),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,px^5-2\,q\right)\sqrt{px^5+q}}{\left(apx^5+bx^2+aq\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^5+q)^(1/2)*(3*p*x^5-2*q)/x^2/(a*p*x^5+b*x^2+a*q),x, algorith
m="maxima")
```

```
[Out] integrate((3*p*x^5 - 2*q)*sqrt(p*x^5 + q)/((a*p*x^5 + b*x^2 + a*q)*x^2), x)
```

mupad [B] time = 6.40, size = 102, normalized size = 1.82

$$\frac{2\sqrt{p\,x^5+q}}{a\,x}+\frac{\sqrt{b}\,\ln\left(\frac{a^5\,b\,p^4\,x^2-a^6\,p^4\,\left(p\,x^5+q\right)+a^{11/2}\,\sqrt{b}\,p^4\,x\,\sqrt{p\,x^5+q}\,2\mathrm{i}}{4\,b^2\,q\,x^2+4\,a\,b\,q\,\left(p\,x^5+q\right)}\right)\,1\mathrm{i}}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q + p*x^5)^(1/2)*(2*q - 3*p*x^5))/(x^2*(a*q + b*x^2 + a*p*x^5)),x)
```

```
[Out] (2*(q + p*x^5)^(1/2))/(a*x) + (b^(1/2)*log((a^5*b*p^4*x^2 - a^6*p^4*(q + p*
x^5) + a^(11/2)*b^(1/2)*p^4*x*(q + p*x^5)^(1/2)*2i)/(4*b^2*q*x^2 + 4*a*b*q*
(q + p*x^5)))*1i)/a^(3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^5+q}\,\left(3px^5-2q\right)}{x^2\left(apx^5+aq+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**5+q)**(1/2)*(3*p*x**5-2*q)/x**2/(a*p*x**5+b*x**2+a*q),x)
```

```
[Out] Integral(sqrt(p*x**5 + q)*(3*p*x**5 - 2*q)/(x**2*(a*p*x**5 + a*q + b*x**2))
, x)
```

3.712 $\int \frac{x^2(-1+4x^5)}{(1+x^5)^2(a-x+ax^5)\sqrt{x+x^6}}\,dx$

Optimal. Leaf size=56

$$\frac{2\sqrt{x^6+x}\left(3ax^5+3a+x\right)}{3\left(x^5+1\right)^2}-2a^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{a}\sqrt{x^6+x}}\right)$$

Rubi [F] time = 1.92, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(-1+4x^5\right)}{\left(1+x^5\right)^2\left(a-x+ax^5\right)\sqrt{x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(-1 + 4*x^5))/((1 + x^5)^2*(a - x + a*x^5)*Sqrt[x + x^6]),x]
```

```
[Out] (8*x^3)/(5*a*(1 + x^5)*Sqrt[x + x^6]) + (16*x^8)/(15*a*(1 + x^5)*Sqrt[x + x
^6]) - (10*Sqrt[x]*Sqrt[1 + x^5]*Defer[Subst][Defer[Int][x^4/((1 + x^10)^(5
/2)*(a - x^2 + a*x^10)), x], x, Sqrt[x]])/Sqrt[x + x^6] + (8*Sqrt[x]*Sqrt[1
 + x^5]*Defer[Subst][Defer[Int][x^6/((1 + x^10)^(5/2)*(a - x^2 + a*x^10)),
x], x, Sqrt[x]])/(a*Sqrt[x + x^6])
```

Rubi steps

$$\int \frac{x^2\left(-1+4x^5\right)}{\left(1+x^5\right)^2\left(a-x+ax^5\right)\sqrt{x+x^6}}\,dx=\frac{\left(\sqrt{x}\sqrt{1+x^5}\right)\int \frac{x^{3/2}\left(-1+4x^5\right)}{\left(1+x^5\right)^{5/2}\left(a-x+ax^5\right)}\,dx}{\sqrt{x+x^6}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{x^4\left(-1+4x^{10}\right)}{\left(1+x^{10}\right)^{5/2}\left(a-x^2+ax^{10}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^6}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int\left(\frac{4x^4}{a\left(1+x^{10}\right)^{5/2}}+\frac{x^4\left(-5a+4x^2\right)}{a\left(1+x^{10}\right)^{5/2}\left(a-x^2+ax^{10}\right)}\right)dx,x,\sqrt{x}\right.}{\sqrt{x+x^6}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int \frac{x^4\left(-5a+4x^2\right)}{\left(1+x^{10}\right)^{5/2}\left(a-x^2+ax^{10}\right)}\,dx,x,\sqrt{x}\right)}{a\sqrt{x+x^6}}+\frac{\left(8\sqrt{x}\sqrt{1+\right.}{}$$

$$=\frac{8x^3}{5a\left(1+x^5\right)\sqrt{x+x^6}}+\frac{\left(2\sqrt{x}\sqrt{1+x^5}\right)\text{Subst}\left(\int\left(-\frac{5ax^4}{\left(1+x^{10}\right)^{5/2}\left(a-x^2+ax^{10}\right.}\right.\right.}{a\sqrt{x+x^6}}$$

$$=\frac{8x^3}{5a\left(1+x^5\right)\sqrt{x+x^6}}+\frac{16x^8}{15a\left(1+x^5\right)\sqrt{x+x^6}}-\frac{\left(10\sqrt{x}\sqrt{1+x^5}\right)\text{Sub}}{}$$

Mathematica [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-1+4x^5\right)}{\left(1+x^5\right)^2\left(a-x+ax^5\right)\sqrt{x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(-1 + 4*x^5))/((1 + x^5)^2*(a - x + a*x^5)*Sqrt[x + x^6]),x]
```

```
[Out] Integrate[(x^2*(-1 + 4*x^5))/((1 + x^5)^2*(a - x + a*x^5)*Sqrt[x + x^6]), x
]
```

IntegrateAlgebraic [A] time = 2.73, size = 56, normalized size = 1.00

$$\frac{2\sqrt{x^6+x}\left(3ax^5+3a+x\right)}{3\left(x^5+1\right)^2}-2a^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{a}\sqrt{x^6+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(-1 + 4*x^5))/((1 + x^5)^2*(a - x + a*x^5)*Sqrt[x +
 x^6]),x]
```

```
[Out] (2*(3*a + x + 3*a*x^5)*Sqrt[x + x^6])/(3*(1 + x^5)^2) - 2*a^(3/2)*ArcTanh[x
/(Sqrt[a]*Sqrt[x + x^6])]
```

fricas [A] time = 0.56, size = 223, normalized size = 3.98

$$\left[\frac{3\left(ax^{10}+2ax^5+a\right)\sqrt{a}\log\left(-\frac{a^2x^{10}+2a^2x^5+6ax^6-4\left(ax^5+a+x\right)\sqrt{x^6+x}\sqrt{a}+a^2+6ax+x^2}{a^2x^{10}+2a^2x^5-2ax^6+a^2-2ax+x^2}\right)+4\left(3ax^5+3a+x\right)\sqrt{x^6+x}}{6\left(x^{10}+2x^5+1\right)},\ \frac{3\left(ax^{10}+2ax^5+a\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{x^6+x}\sqrt{-a}}{ax^5+a+x}\right)+2\left(3ax^5+3a+x\right)\sqrt{x^6+x}}{3\left(x^{10}+2x^5+1\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^5-1)/(x^5+1)^2/(a*x^5+a-x)/(x^6+x)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/6*(3*(a*x^10 + 2*a*x^5 + a)*sqrt(a)*log(-(a^2*x^10 + 2*a^2*x^5 + 6*a*x^6
 - 4*(a*x^5 + a + x)*sqrt(x^6 + x)*sqrt(a) + a^2 + 6*a*x + x^2)/(a^2*x^10 +
 2*a^2*x^5 - 2*a*x^6 + a^2 - 2*a*x + x^2)) + 4*(3*a*x^5 + 3*a + x)*sqrt(x^6
 + x))/(x^10 + 2*x^5 + 1), 1/3*(3*(a*x^10 + 2*a*x^5 + a)*sqrt(-a)*arctan(2*
sqrt(x^6 + x)*sqrt(-a)/(a*x^5 + a + x)) + 2*(3*a*x^5 + 3*a + x)*sqrt(x^6 +
x))/(x^10 + 2*x^5 + 1)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^5-1)/(x^5+1)^2/(a*x^5+a-x)/(x^6+x)^(1/2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(4x^5-1\right)}{\left(x^5+1\right)^2\left(a\,x^5+a-x\right)\sqrt{x^6+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^5-1)/(x^5+1)^2/(a*x^5+a-x)/(x^6+x)^(1/2),x)
```

```
[Out] int(x^2*(4*x^5-1)/(x^5+1)^2/(a*x^5+a-x)/(x^6+x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^5-1\right)x^2}{\left(ax^5+a-x\right)\sqrt{x^6+x}\left(x^5+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^5-1)/(x^5+1)^2/(a*x^5+a-x)/(x^6+x)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((4*x^5 - 1)*x^2/((a*x^5 + a - x)*sqrt(x^6 + x)*(x^5 + 1)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,\left(4\,x^5-1\right)}{{\left(x^5+1\right)}^2\,\sqrt{x^6+x}\,\left(a\,x^5-x+a\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(4*x^5 - 1))/((x^5 + 1)^2*(x + x^6)^(1/2)*(a - x + a*x^5)),x)
```

```
[Out] int((x^2*(4*x^5 - 1))/((x^5 + 1)^2*(x + x^6)^(1/2)*(a - x + a*x^5)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(4*x**5-1)/(x**5+1)**2/(a*x**5+a-x)/(x**6+x)**(1/2),x)
```

```
[Out] Timed out
```

3.713 $\int \frac{\sqrt{-x+x^4}\left(-b+ax^6\right)}{x^6} dx$

Optimal. Leaf size=56

$$\frac{\sqrt{x^4-x}\left(3ax^6-2bx^3+2b\right)}{9x^5}-\frac{1}{3}a\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)$$

Rubi [C] time = 0.32, antiderivative size = 178, normalized size of antiderivative = 3.18, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2048, 2052, 2014, 2020, 2011, 329, 225}

$$\frac{a\left(x^4-x\right)^{3/2}}{3x^3}+\frac{2a\sqrt{x^4-x}}{5x^3}-\frac{3^{3/4}a(1-x)x\sqrt{\frac{x^2+x+1}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x}{1-\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt{-\frac{(1-x)x}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{x^4-x}}-\frac{2b\left(x^4-x\right)^{3/2}}{9x^6}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[-x + x^4]*(-b + a*x^6))/x^6,x]
```

```
[Out] (2*a*Sqrt[-x + x^4])/(5*x^3) - (2*b*(-x + x^4)^(3/2))/(9*x^6) + (a*(-x + x^
4)^(3/2))/(3*x^3) - (3^(3/4)*a*(1 - x)*x*Sqrt[(1 + x + x^2)/(1 - (1 + Sqrt[
3])*x)^2]*EllipticF[ArcCos[(1 - (1 - Sqrt[3])*x)/(1 - (1 + Sqrt[3])*x)], (2
 + Sqrt[3])/4])/(5*Sqrt[-(((1 - x)*x)/(1 - (1 + Sqrt[3])*x)^2)]*Sqrt[-x + x
^4])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2048

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Int[(c*x)^
m*ExpandToSum[Pq - Pqq*x^q - (a*Pqq*(m + q - n + 1)*x^(q - n))/(b*(m + q +
n*p + 1)), x]*(a*x^j + b*x^n)^p, x] + Simp[(Pqq*(c*x)^(m + q - n + 1)*(a*x^
j + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /; GtQ[q, n -
1] && NeQ[m + q + n*p + 1, 0] && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*
n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] &&  !IntegerQ[p] && IGtQ
[j, 0] && IGtQ[n, 0] && LtQ[j, n]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-x+x^4}\left(-b+ax^6\right)}{x^6}\,dx &= \frac{a\left(-x+x^4\right)^{3/2}}{3x^3}+\int \frac{\left(-b-ax^2\right)\sqrt{-x+x^4}}{x^6}\,dx \\
&= \frac{a\left(-x+x^4\right)^{3/2}}{3x^3}+\int \left(-\frac{b\sqrt{-x+x^4}}{x^6}-\frac{a\sqrt{-x+x^4}}{x^4}\right)dx \\
&= \frac{a\left(-x+x^4\right)^{3/2}}{3x^3}-a\int \frac{\sqrt{-x+x^4}}{x^4}\,dx-b\int \frac{\sqrt{-x+x^4}}{x^6}\,dx \\
&= \frac{2a\sqrt{-x+x^4}}{5x^3}-\frac{2b\left(-x+x^4\right)^{3/2}}{9x^6}+\frac{a\left(-x+x^4\right)^{3/2}}{3x^3}-\frac{1}{5}(3a)\int \frac{1}{\sqrt{-x+x^4}}\,dx \\
&= \frac{2a\sqrt{-x+x^4}}{5x^3}-\frac{2b\left(-x+x^4\right)^{3/2}}{9x^6}+\frac{a\left(-x+x^4\right)^{3/2}}{3x^3}-\frac{\left(3a\sqrt{x}\sqrt{-1+x^3}\right)\int \frac{1}{\sqrt{x}\sqrt{-1+}}}{5\sqrt{-x+x^4}} \\
&= \frac{2a\sqrt{-x+x^4}}{5x^3}-\frac{2b\left(-x+x^4\right)^{3/2}}{9x^6}+\frac{a\left(-x+x^4\right)^{3/2}}{3x^3}-\frac{\left(6a\sqrt{x}\sqrt{-1+x^3}\right)\text{Subst}\left(\int\right.}{5\sqrt{-x+x}} \\
&= \frac{2a\sqrt{-x+x^4}}{5x^3}-\frac{2b\left(-x+x^4\right)^{3/2}}{9x^6}+\frac{a\left(-x+x^4\right)^{3/2}}{3x^3}-\frac{3^{3/4}a(1-x)x\sqrt{\frac{1+x+x^2}{\left(1-\left(1+\sqrt{3}\right)x\right)^2}}}{5\sqrt{-\frac{(1-}{\left(1-(1+\right.}}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 71, normalized size = 1.27

$$\frac{\sqrt{x\left(x^3-1\right)}\left(\sqrt{1-x^3}\left(3ax^6-2b\left(x^3-1\right)\right)+3ax^{9/2}\sin^{-1}\left(x^{3/2}\right)\right)}{9x^5\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-x + x^4]*(-b + a*x^6))/x^6,x]
```

```
[Out] (Sqrt[x*(-1 + x^3)]*(Sqrt[1 - x^3]*(3*a*x^6 - 2*b*(-1 + x^3)) + 3*a*x^(9/2)
*ArcSin[x^(3/2)]))/(9*x^5*Sqrt[1 - x^3])
```

IntegrateAlgebraic [A] time = 0.48, size = 56, normalized size = 1.00

$$\frac{\sqrt{x^4-x}\,\left(3ax^6-2bx^3+2b\right)}{9x^5}-\frac{1}{3}a\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-x + x^4]*(-b + a*x^6))/x^6,x]
```

```
[Out] (Sqrt[-x + x^4]*(2*b - 2*b*x^3 + 3*a*x^6))/(9*x^5) - (a*ArcTanh[x^2/Sqrt[-x
 + x^4]])/3
```

fricas [A] time = 0.44, size = 59, normalized size = 1.05

$$\frac{3\,ax^5\log\left(2\,x^3-2\,\sqrt{x^4-x}\,x-1\right)+2\,\left(3\,ax^6-2\,bx^3+2\,b\right)\sqrt{x^4-x}}{18\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)*(a*x^6-b)/x^6,x, algorithm="fricas")
```

```
[Out] 1/18*(3*a*x^5*log(2*x^3 - 2*sqrt(x^4 - x)*x - 1) + 2*(3*a*x^6 - 2*b*x^3 + 2
*b)*sqrt(x^4 - x))/x^5
```

giac [A] time = 0.38, size = 57, normalized size = 1.02

$$\frac{1}{3}\,\sqrt{x^4-x}\,ax-\frac{2}{9}\,b\left(-\frac{1}{x^3}+1\right)^{\frac{3}{2}}-\frac{1}{6}\,a\log\left(\sqrt{-\frac{1}{x^3}+1}+1\right)+\frac{1}{6}\,a\log\left(\left|\sqrt{-\frac{1}{x^3}+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)*(a*x^6-b)/x^6,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(x^4 - x)*a*x - 2/9*b*(-1/x^3 + 1)^(3/2) - 1/6*a*log(sqrt(-1/x^3 +
1) + 1) + 1/6*a*log(abs(sqrt(-1/x^3 + 1) - 1))
```

maple [C] time = 0.29, size = 336, normalized size = 6.00

$$a\left(\frac{x\sqrt{x^4-x}}{3}-\frac{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}}\,(-1+x)^2\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(-1+x)}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)-\mathrm{EllipticPi}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\frac{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(-1+x)\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)-b\left(-\frac{2\sqrt{x^4-x}}{9x^5}+\frac{2\sqrt{x^4-x}}{9x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x)^(1/2)*(a*x^6-b)/x^6,x)
```

```
[Out] a*(1/3*x*(x^4-x)^(1/2)-(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/
2*I*3^(1/2))/(-1+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1
/2))/(-1+x))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2
)/(-3/2+1/2*I*3^(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/2)
))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1
/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I
*3^(1/2)))^(1/2))-EllipticPi(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(
-1+x))^(1/2),(-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)),((3/2+1/2*I*3^(1/2))
*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))))-b*(-
2/9*(x^4-x)^(1/2)/x^5+2/9*(x^4-x)^(1/2)/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^6 - b)\sqrt{x^4 - x}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)*(a*x^6-b)/x^6,x, algorithm="maxima")
[Out] integrate((a*x^6 - b)*sqrt(x^4 - x)/x^6, x)
```

mupad [B] time = 1.05, size = 51, normalized size = 0.91

$$\frac{2\,a\,x\,\sqrt{x^4 - x}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2};\ \frac{3}{2};\ x^3\right)}{3\,\sqrt{1 - x^3}} - \frac{2\,b\,\sqrt{x^4 - x}\,\left(x^3 - 1\right)}{9\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^4 - x)^(1/2)*(b - a*x^6))/x^6,x)
[Out] (2*a*x*(x^4 - x)^(1/2)*hypergeom([-1/2, 1/2], 3/2, x^3))/(3*(1 - x^3)^(1/2)
) - (2*b*(x^4 - x)^(1/2)*(x^3 - 1))/(9*x^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x(x-1)\left(x^2 + x + 1\right)}\left(ax^6 - b\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x)**(1/2)*(a*x**6-b)/x**6,x)
[Out] Integral(sqrt(x*(x - 1)*(x**2 + x + 1))*(a*x**6 - b)/x**6, x)
```

3.714 $\int \frac{(-2q+px^6)\sqrt{q+px^6}}{x^3(aq+bx^4+apx^6)}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}\,\sqrt{px^6+q}}\right)}{a^{3/2}}+\frac{\sqrt{px^6+q}}{ax^2}$$

Rubi [F] time = 2.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2q+px^6)\sqrt{q+px^6}}{x^3(aq+bx^4+apx^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*q + p*x^6)*Sqrt[q + p*x^6])/(x^3*(a*q + b*x^4 + a*p*x^6)),x]
```

```
[Out] Sqrt[q + p*x^6]/(a*x^2) - (3*p^(1/3)*Sqrt[q + p*x^6])/(a*((1 + Sqrt[3])*q^(
1/3) + p^(1/3)*x^2)) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*p^(1/3)*q^(1/3)*(q^(1/3
) + p^(1/3)*x^2)*Sqrt[(q^(2/3) - p^(1/3)*q^(1/3)*x^2 + p^(2/3)*x^4)/((1 + S
qrt[3])*q^(1/3) + p^(1/3)*x^2)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*q^(1/3) +
 p^(1/3)*x^2)/((1 + Sqrt[3])*q^(1/3) + p^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(2*a
*Sqrt[(q^(1/3)*(q^(1/3) + p^(1/3)*x^2))/((1 + Sqrt[3])*q^(1/3) + p^(1/3)*x^
2)^2]*Sqrt[q + p*x^6]) - (Sqrt[2]*3^(3/4)*p^(1/3)*q^(1/3)*(q^(1/3) + p^(1/3
)*x^2)*Sqrt[(q^(2/3) - p^(1/3)*q^(1/3)*x^2 + p^(2/3)*x^4)/((1 + Sqrt[3])*q^
(1/3) + p^(1/3)*x^2)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*q^(1/3) + p^(1/3)*x
^2)/((1 + Sqrt[3])*q^(1/3) + p^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(a*Sqrt[(q^(1/
3)*(q^(1/3) + p^(1/3)*x^2))/((1 + Sqrt[3])*q^(1/3) + p^(1/3)*x^2)^2]*Sqrt[q
 + p*x^6]) + (b*Defer[Subst][Defer[Int][Sqrt[q + p*x^3]/(a*q + b*x^2 + a*p*
x^3), x], x, x^2])/a + (3*p*Defer[Subst][Defer[Int][(x*Sqrt[q + p*x^3])/(a*
q + b*x^2 + a*p*x^3), x], x, x^2])/2
```

Rubi steps

$$\int \frac{(-2q+px^6)\sqrt{q+px^6}}{x^3(aq+bx^4+apx^6)}\,dx=\int\left(-\frac{2\sqrt{q+px^6}}{ax^3}+\frac{x(2b+3apx^2)\sqrt{q+px^6}}{a(aq+bx^4+apx^6)}\right)dx$$

$$=\frac{\int \frac{x(2b+3apx^2)\sqrt{q+px^6}}{aq+bx^4+apx^6}\,dx}{a}-\frac{2\int \frac{\sqrt{q+px^6}}{x^3}\,dx}{a}$$

$$=\frac{\text{Subst}\left(\int \frac{(2b+3apx)\sqrt{q+px^3}}{aq+bx^2+apx^3}\,dx,x,x^2\right)}{2a}-\frac{\text{Subst}\left(\int \frac{\sqrt{q+px^3}}{x^2}\,dx,x,x^2\right)}{a}$$

$$=\frac{\sqrt{q+px^6}}{ax^2}+\frac{\text{Subst}\left(\int\left(\frac{2b\sqrt{q+px^3}}{aq+bx^2+apx^3}+\frac{3apx\sqrt{q+px^3}}{aq+bx^2+apx^3}\right)dx,x,x^2\right)}{2a}-\frac{(3p)\,\text{Subst}\left(\int \frac{x}{\sqrt{q+p}}\right.}{2a}$$

$$=\frac{\sqrt{q+px^6}}{ax^2}+\frac{b\,\text{Subst}\left(\int \frac{\sqrt{q+px^3}}{aq+bx^2+apx^3}\,dx,x,x^2\right)}{a}-\frac{(3p^{2/3})\,\text{Subst}\left(\int \frac{(1-\sqrt{3})\sqrt[3]{q}+\sqrt[3]{p}\,x}{\sqrt{q+px^3}}\,d\right.}{2a}$$

$$=\frac{\sqrt{q+px^6}}{ax^2}-\frac{3\sqrt[3]{p}\sqrt{q+px^6}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{q}+\sqrt[3]{p}\,x^2\right)}+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{p}\,\sqrt[3]{q}\left(\sqrt[3]{q}+\sqrt[3]{p}\,x^2\right)\sqrt{}}{2a\sqrt{\frac{}{((}}}$$

Mathematica [C] time = 6.79, size = 2948, normalized size = 52.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-2*q + p*x^6)*Sqrt[q + p*x^6])/(x^3*(a*q + b*x^4 + a*p*x^6)),x]
```

```
[Out] Sqrt[q + p*x^6]/(a*x^2) + (b*((-2*Sqrt[(q^(1/3)/p^(1/3) + x^2)/(q^(1/3)/p^(
1/3) + ((-1)^(1/3)*q^(1/3))/p^(1/3))]*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + x^
2)*Sqrt[(((-1)^(2/3)*q^(1/3))/p^(1/3) + x^2)/(((-1)^(1/3)*q^(1/3))/p^(1/3)
+ ((-1)^(2/3)*q^(1/3))/p^(1/3))]*EllipticF[ArcSin[Sqrt[((-1)^(1/3)*q^(1/3)
- p^(1/3)*x^2)/(((-1)^(1/3) + (-1)^(2/3))*q^(1/3))]], (-1)^(1/3)])/(a*Sqrt[
(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + x^2)/(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) -
((-1)^(2/3)*q^(1/3))/p^(1/3))]*Sqrt[q + p*x^6]) + (4*(((-1)^(1/3)*q^(1/3))/
p^(1/3) + ((-1)^(2/3)*q^(1/3))/p^(1/3))*q*Sqrt[(q^(1/3)/p^(1/3) + x^2)/(q^(
1/3)/p^(1/3) + ((-1)^(1/3)*q^(1/3))/p^(1/3))]*Sqrt[((-(((-1)^(2/3)*q^(1/3))
/p^(1/3)) - x^2)*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + x^2))/(((-1)^(1/3)*q^(1
/3))/p^(1/3) + ((-1)^(2/3)*q^(1/3))/p^(1/3))^2]*EllipticPi[((-1)^(1/3)*q^(1
/3) + (-1)^(2/3)*q^(1/3))/((-1)^(1/3)*q^(1/3) - p^(1/3)*Root[a*q + b*#1^2 +
 a*p*#1^3 & , 1]), ArcSin[Sqrt[((-1)^(1/3)*q^(1/3) - p^(1/3)*x^2)/(((-1)^(1
/3) + (-1)^(2/3))*q^(1/3))]], (-1)^(1/3)])/(a*p*Sqrt[q + p*x^6]*(-(((-1)^(1
/3)*q^(1/3))/p^(1/3)) + Root[a*q + b*#1^2 + a*p*#1^3 & , 1])*(Root[a*q + b*
#1^2 + a*p*#1^3 & , 1] - Root[a*q + b*#1^2 + a*p*#1^3 & , 2])*(Root[a*q + b
*#1^2 + a*p*#1^3 & , 1] - Root[a*q + b*#1^2 + a*p*#1^3 & , 3])) - (2*(((-1)
^(1/3)*q^(1/3))/p^(1/3) + ((-1)^(2/3)*q^(1/3))/p^(1/3))*Sqrt[(q^(1/3)/p^(1/
3) + x^2)/(q^(1/3)/p^(1/3) + ((-1)^(1/3)*q^(1/3))/p^(1/3))]*Sqrt[((-(((-1)^
(2/3)*q^(1/3))/p^(1/3)) - x^2)*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + x^2))/(((
-1)^(1/3)*q^(1/3))/p^(1/3) + ((-1)^(2/3)*q^(1/3))/p^(1/3))^2]*EllipticPi[((
-1)^(1/3)*q^(1/3) + (-1)^(2/3)*q^(1/3))/((-1)^(1/3)*q^(1/3) - p^(1/3)*Root[
a*q + b*#1^2 + a*p*#1^3 & , 1]), ArcSin[Sqrt[((-1)^(1/3)*q^(1/3) - p^(1/3)*
x^2)/(((-1)^(1/3) + (-1)^(2/3))*q^(1/3))]], (-1)^(1/3)]*Root[a*q + b*#1^2 +
 a*p*#1^3 & , 1]^3)/(a*Sqrt[q + p*x^6]*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + R
oot[a*q + b*#1^2 + a*p*#1^3 & , 1])*(Root[a*q + b*#1^2 + a*p*#1^3 & , 1] -
Root[a*q + b*#1^2 + a*p*#1^3 & , 2])*(Root[a*q + b*#1^2 + a*p*#1^3 & , 1] -
 Root[a*q + b*#1^2 + a*p*#1^3 & , 3])) + (4*(((-1)^(1/3)*q^(1/3))/p^(1/3) +
 ((-1)^(2/3)*q^(1/3))/p^(1/3))*q*Sqrt[(q^(1/3)/p^(1/3) + x^2)/(q^(1/3)/p^(1
/3) + ((-1)^(1/3)*q^(1/3))/p^(1/3))]*Sqrt[((-(((-1)^(2/3)*q^(1/3))/p^(1/3))
 - x^2)*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + x^2))/(((-1)^(1/3)*q^(1/3))/p^(1
/3) + ((-1)^(2/3)*q^(1/3))/p^(1/3))^2]*EllipticPi[((-1)^(1/3)*q^(1/3) + (-1
)^(2/3)*q^(1/3))/((-1)^(1/3)*q^(1/3) - p^(1/3)*Root[a*q + b*#1^2 + a*p*#1^3
 & , 2]), ArcSin[Sqrt[((-1)^(1/3)*q^(1/3) - p^(1/3)*x^2)/(((-1)^(1/3) + (-1
)^(2/3))*q^(1/3))]], (-1)^(1/3)])/(a*p*Sqrt[q + p*x^6]*(-(((-1)^(1/3)*q^(1/
3))/p^(1/3)) + Root[a*q + b*#1^2 + a*p*#1^3 & , 2])*(-Root[a*q + b*#1^2 + a
*p*#1^3 & , 1] + Root[a*q + b*#1^2 + a*p*#1^3 & , 2])*(Root[a*q + b*#1^2 +
a*p*#1^3 & , 2] - Root[a*q + b*#1^2 + a*p*#1^3 & , 3])) - (2*(((-1)^(1/3)*q
^(1/3))/p^(1/3) + ((-1)^(2/3)*q^(1/3))/p^(1/3))*Sqrt[(q^(1/3)/p^(1/3) + x^2
)/(q^(1/3)/p^(1/3) + ((-1)^(1/3)*q^(1/3))/p^(1/3))]*Sqrt[((-(((-1)^(2/3)*q^
(1/3))/p^(1/3)) - x^2)*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + x^2))/(((-1)^(1/3
)*q^(1/3))/p^(1/3) + ((-1)^(2/3)*q^(1/3))/p^(1/3))^2]*EllipticPi[((-1)^(1/3
)*q^(1/3) + (-1)^(2/3)*q^(1/3))/((-1)^(1/3)*q^(1/3) - p^(1/3)*Root[a*q + b*
#1^2 + a*p*#1^3 & , 2]), ArcSin[Sqrt[((-1)^(1/3)*q^(1/3) - p^(1/3)*x^2)/(((
-1)^(1/3) + (-1)^(2/3))*q^(1/3))]], (-1)^(1/3)]*Root[a*q + b*#1^2 + a*p*#1^
3 & , 2]^3)/(a*Sqrt[q + p*x^6]*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + Root[a*q
+ b*#1^2 + a*p*#1^3 & , 2])*(-Root[a*q + b*#1^2 + a*p*#1^3 & , 1] + Root[a*
q + b*#1^2 + a*p*#1^3 & , 2])*(Root[a*q + b*#1^2 + a*p*#1^3 & , 2] - Root[a
*q + b*#1^2 + a*p*#1^3 & , 3])) + (4*(((-1)^(1/3)*q^(1/3))/p^(1/3) + ((-1)^
(2/3)*q^(1/3))/p^(1/3))*q*Sqrt[(q^(1/3)/p^(1/3) + x^2)/(q^(1/3)/p^(1/3) + (
(-1)^(1/3)*q^(1/3))/p^(1/3))]*Sqrt[((-(((-1)^(2/3)*q^(1/3))/p^(1/3)) - x^2)
*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + x^2))/(((-1)^(1/3)*q^(1/3))/p^(1/3) + (
```

```
(-1)^(2/3)*q^(1/3))/p^(1/3))^2]*EllipticPi[((-1)^(1/3)*q^(1/3) + (-1)^(2/3)
*q^(1/3))/((-1)^(1/3)*q^(1/3) - p^(1/3)*Root[a*q + b*#1^2 + a*p*#1^3 & , 3]
), ArcSin[Sqrt[((-1)^(1/3)*q^(1/3) - p^(1/3)*x^2)/(((-1)^(1/3) + (-1)^(2/3)
)*q^(1/3))]], (-1)^(1/3)])/(a*p*Sqrt[q + p*x^6]*(-(((-1)^(1/3)*q^(1/3))/p^(
1/3)) + Root[a*q + b*#1^2 + a*p*#1^3 & , 3])*(-Root[a*q + b*#1^2 + a*p*#1^3
 & , 1] + Root[a*q + b*#1^2 + a*p*#1^3 & , 3])*(-Root[a*q + b*#1^2 + a*p*#1
^3 & , 2] + Root[a*q + b*#1^2 + a*p*#1^3 & , 3])) - (2*(((-1)^(1/3)*q^(1/3)
)/p^(1/3) + ((-1)^(2/3)*q^(1/3))/p^(1/3))*Sqrt[(q^(1/3)/p^(1/3) + x^2)/(q^(
1/3)/p^(1/3) + ((-1)^(1/3)*q^(1/3))/p^(1/3))]*Sqrt[((-(((-1)^(2/3)*q^(1/3))
/p^(1/3)) - x^2)*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + x^2))/(((-1)^(1/3)*q^(1
/3))/p^(1/3) + ((-1)^(2/3)*q^(1/3))/p^(1/3))^2]*EllipticPi[((-1)^(1/3)*q^(1
/3) + (-1)^(2/3)*q^(1/3))/((-1)^(1/3)*q^(1/3) - p^(1/3)*Root[a*q + b*#1^2 +
 a*p*#1^3 & , 3]), ArcSin[Sqrt[((-1)^(1/3)*q^(1/3) - p^(1/3)*x^2)/(((-1)^(1
/3) + (-1)^(2/3))*q^(1/3))]], (-1)^(1/3)]*Root[a*q + b*#1^2 + a*p*#1^3 & ,
3]^3)/(a*Sqrt[q + p*x^6]*(-(((-1)^(1/3)*q^(1/3))/p^(1/3)) + Root[a*q + b*#1
^2 + a*p*#1^3 & , 3])*(-Root[a*q + b*#1^2 + a*p*#1^3 & , 1] + Root[a*q + b*
#1^2 + a*p*#1^3 & , 3])*(-Root[a*q + b*#1^2 + a*p*#1^3 & , 2] + Root[a*q +
b*#1^2 + a*p*#1^3 & , 3]))))/(2*a)
```

IntegrateAlgebraic [A] time = 11.66, size = 56, normalized size = 1.00

$$\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}\,\sqrt{px^6+q}}\right)}{a^{3/2}}+\frac{\sqrt{px^6+q}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*q + p*x^6)*Sqrt[q + p*x^6])/(x^3*(a*q + b*x^4 + a*p
*x^6)),x]

[Out] Sqrt[q + p*x^6]/(a*x^2) + (Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/(Sqrt[a]*Sqrt[q + p
*x^6])])/a^(3/2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^6-2*q)*(p*x^6+q)^(1/2)/x^3/(a*p*x^6+b*x^4+a*q),x, algorithm=
"fricas")

[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^6+q}\,\left(px^6-2\,q\right)}{\left(apx^6+bx^4+aq\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^6-2*q)*(p*x^6+q)^(1/2)/x^3/(a*p*x^6+b*x^4+a*q),x, algorithm=
"giac")

[Out] integrate(sqrt(p*x^6 + q)*(p*x^6 - 2*q)/((a*p*x^6 + b*x^4 + a*q)*x^3), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(p\,x^6-2q\right)\sqrt{p\,x^6+q}}{x^3\left(ap\,x^6+b\,x^4+aq\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^6-2*q)*(p*x^6+q)^(1/2)/x^3/(a*p*x^6+b*x^4+a*q),x)
```

```
[Out] int((p*x^6-2*q)*(p*x^6+q)^(1/2)/x^3/(a*p*x^6+b*x^4+a*q),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^6+q}\left(px^6-2\,q\right)}{\left(apx^6+bx^4+aq\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^6-2*q)*(p*x^6+q)^(1/2)/x^3/(a*p*x^6+b*x^4+a*q),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(p*x^6 + q)*(p*x^6 - 2*q)/((a*p*x^6 + b*x^4 + a*q)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{\sqrt{p\,x^6+q}\,\left(2\,q-p\,x^6\right)}{x^3\,\left(a\,p\,x^6+b\,x^4+a\,q\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q + p*x^6)^(1/2)*(2*q - p*x^6))/(x^3*(a*q + b*x^4 + a*p*x^6)),x)
```

```
[Out] int(-((q + p*x^6)^(1/2)*(2*q - p*x^6))/(x^3*(a*q + b*x^4 + a*p*x^6)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**6-2*q)*(p*x**6+q)**(1/2)/x**3/(a*p*x**6+b*x**4+a*q),x)
```

```
[Out] Timed out
```

3.715 $\int \frac{2(-2q+px^6)\sqrt{q+px^6}\left(aq+bx^4+apx^6\right)}{x^{11}}\,dx$

Optimal. Leaf size=56

$$\frac{2\sqrt{px^6+q}\left(3ap^2x^{12}+6apqx^6+3aq^2+5bpx^{10}+5bqx^4\right)}{15x^{10}}$$

Rubi [A] time = 0.14, antiderivative size = 39, normalized size of antiderivative = 0.70, number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {12, 1833, 1584, 446, 74, 1478, 449}

$$\frac{2a\left(px^6+q\right)^{5/2}}{5x^{10}}+\frac{2b\left(px^6+q\right)^{3/2}}{3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(2*(-2*q + p*x^6)*Sqrt[q + p*x^6]*(a*q + b*x^4 + a*p*x^6))/x^11,x]
```

```
[Out] (2*b*(q + p*x^6)^(3/2))/(3*x^6) + (2*a*(q + p*x^6)^(5/2))/(5*x^10)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1478

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n
_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p
]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{2\left(-2q+px^6\right)\sqrt{q+px^6}\left(aq+bx^4+apx^6\right)}{x^{11}}\,dx = 2\int \frac{\left(-2q+px^6\right)\sqrt{q+px^6}\left(aq+bx^4+apx^6\right)}{x^{11}}\,dx$$
$$= 2\int \left(\frac{\sqrt{q+px^6}\left(-2bqx^3+bpx^9\right)}{x^{10}}+\frac{\sqrt{q+px^6}\left(-2aq^2-ap\right.}{x^{11}}\right.$$
$$= 2\int \frac{\sqrt{q+px^6}\left(-2bqx^3+bpx^9\right)}{x^{10}}\,dx + 2\int \frac{\sqrt{q+px^6}\left(-2a\right.}{}$$
$$= 2\int \frac{\left(q+px^6\right)^{3/2}\left(-2aq+apx^6\right)}{x^{11}}\,dx + 2\int \frac{\sqrt{q+px^6}\left(-2b\right.}{x^7}$$
$$= \frac{2a\left(q+px^6\right)^{5/2}}{5x^{10}} + \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{q+px}\,(-2bq+bpx)}{x^2}\,dx, x\right.$$
$$= \frac{2b\left(q+px^6\right)^{3/2}}{3x^6} + \frac{2a\left(q+px^6\right)^{5/2}}{5x^{10}}$$

Mathematica [C] time = 0.58, size = 248, normalized size = 4.43

$$\frac{5x^4\left(6ap^2x^8\left(px^6+q\right){}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{px^6}{q}\right)+3apqx^2\left(px^6+q\right){}_2F_1\left(-\frac{2}{3},-\frac{1}{2};\frac{1}{3};-\frac{px^6}{q}\right)-4bp\sqrt{q}x^6\sqrt{px^6+q}\sqrt{\frac{px^6}{q}+1}\tanh^{-1}\left(\frac{\sqrt{px^6+q}}{\sqrt{q}}\right)+4b\left(px^6+q\right)\left(\left(px^6+q\right)\sqrt{\frac{px^6}{q}+1}+px^6\tanh^{-1}\left(\sqrt{\frac{px^6}{q}+1}\right)\right)\right)+12aq^2\left(px^6+q\right){}_2F_1\left(-\frac{5}{3},-\frac{1}{2};-\frac{2}{3};-\frac{px^6}{q}\right)}{30x^{10}\sqrt{px^6+q}\sqrt{\frac{px^6}{q}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*(-2*q + p*x^6)*Sqrt[q + p*x^6]*(a*q + b*x^4 + a*p*x^6))/x^11,x
]
```

```
[Out] (12*a*q^2*(q + p*x^6)*Hypergeometric2F1[-5/3, -1/2, -2/3, -((p*x^6)/q)] + 5
*x^4*(-4*b*p*Sqrt[q]*x^6*Sqrt[q + p*x^6]*Sqrt[1 + (p*x^6)/q]*ArcTanh[Sqrt[q
 + p*x^6]/Sqrt[q]] + 4*b*(q + p*x^6)*((q + p*x^6)*Sqrt[1 + (p*x^6)/q] + p*x
^6*ArcTanh[Sqrt[1 + (p*x^6)/q]]) + 3*a*p*q*x^2*(q + p*x^6)*Hypergeometric2F
1[-2/3, -1/2, 1/3, -((p*x^6)/q)] + 6*a*p^2*x^8*(q + p*x^6)*Hypergeometric2F
1[-1/2, 1/3, 4/3, -((p*x^6)/q)]))/(30*x^10*Sqrt[q + p*x^6]*Sqrt[1 + (p*x^6)
/q])
```

IntegrateAlgebraic [A] time = 18.75, size = 56, normalized size = 1.00

$$\frac{2\sqrt{px^6+q}\left(3ap^2x^{12}+6apqx^6+3aq^2+5bpx^{10}+5bqx^4\right)}{15x^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*(-2*q + p*x^6)*Sqrt[q + p*x^6]*(a*q + b*x^4 + a*p*x^6
))/x^11,x]
```

```
[Out] (2*Sqrt[q + p*x^6]*(3*a*q^2 + 5*b*q*x^4 + 6*a*p*q*x^6 + 5*b*p*x^10 + 3*a*p^
2*x^12))/(15*x^10)
```

fricas [A] time = 0.43, size = 52, normalized size = 0.93

$$\frac{2\left(3\,ap^2x^{12}+5\,bpx^{10}+6\,apqx^6+5\,bqx^4+3\,aq^2\right)\sqrt{px^6+q}}{15\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*(p*x^6-2*q)*(p*x^6+q)^(1/2)*(a*p*x^6+b*x^4+a*q)/x^11,x, algorit
hm="fricas")
```

```
[Out] 2/15*(3*a*p^2*x^12 + 5*b*p*x^10 + 6*a*p*q*x^6 + 5*b*q*x^4 + 3*a*q^2)*sqrt(p
*x^6 + q)/x^10
```

giac [A] time = 0.66, size = 65, normalized size = 1.16

$$\frac{2}{15}\sqrt{px^6+q}\left(3\,ap^2x^2+5\,bp\right)+\frac{2}{15}\left(6\,apq+\frac{5\,bq+\frac{3\,aq^2}{x^4}}{x^2}\right)\sqrt{\frac{p}{x^2}+\frac{q}{x^8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*(p*x^6-2*q)*(p*x^6+q)^(1/2)*(a*p*x^6+b*x^4+a*q)/x^11,x, algorit
hm="giac")
```

```
[Out] 2/15*sqrt(p*x^6 + q)*(3*a*p^2*x^2 + 5*b*p) + 2/15*(6*a*p*q + (5*b*q + 3*a*q
^2/x^4)/x^2)*sqrt(p/x^2 + q/x^8)
```

maple [A] time = 0.01, size = 33, normalized size = 0.59

$$\frac{2\left(p\,x^6+q\right)^{\frac{3}{2}}\left(3ap\,x^6+5b\,x^4+3aq\right)}{15x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*(p*x^6-2*q)*(p*x^6+q)^(1/2)*(a*p*x^6+b*x^4+a*q)/x^11,x)
```

```
[Out] 2/15*(p*x^6+q)^(3/2)*(3*a*p*x^6+5*b*x^4+3*a*q)/x^10
```

maxima [A] time = 0.37, size = 52, normalized size = 0.93

$$\frac{2\left(3\,ap^2x^{12}+5\,bpx^{10}+6\,apqx^6+5\,bqx^4+3\,aq^2\right)\sqrt{px^6+q}}{15\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*(p*x^6-2*q)*(p*x^6+q)^(1/2)*(a*p*x^6+b*x^4+a*q)/x^11,x, algorit
hm="maxima")
```

```
[Out] 2/15*(3*a*p^2*x^12 + 5*b*p*x^10 + 6*a*p*q*x^6 + 5*b*q*x^4 + 3*a*q^2)*sqrt(p
*x^6 + q)/x^10
```

mupad [B] time = 1.69, size = 76, normalized size = 1.36

$$\sqrt{p\,x^6+q}\,\left(\frac{2\,a\,p^2\,x^2}{5}+\frac{2\,b\,p}{3}\right)+\frac{2\,a\,q^2\,\sqrt{p\,x^6+q}}{5\,x^{10}}+\frac{2\,b\,q\,\sqrt{p\,x^6+q}}{3\,x^6}+\frac{4\,a\,p\,q\,\sqrt{p\,x^6+q}}{5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*(q + p*x^6)^(1/2)*(2*q - p*x^6)*(a*q + b*x^4 + a*p*x^6))/x^11,x)
```

```
[Out] (q + p*x^6)^(1/2)*((2*b*p)/3 + (2*a*p^2*x^2)/5) + (2*a*q^2*(q + p*x^6)^(1/2
))/(5*x^10) + (2*b*q*(q + p*x^6)^(1/2))/(3*x^6) + (4*a*p*q*(q + p*x^6)^(1/2
))/(5*x^4)
```

sympy [C] time = 10.91, size = 223, normalized size = 3.98

$$\frac{ap^2\sqrt{q}x^2\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{px^6e^{i\pi}}{q}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{apq^{\frac{3}{2}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{px^6e^{i\pi}}{q}\right)}{3x^4\Gamma\left(\frac{1}{3}\right)}-\frac{2aq^{\frac{5}{2}}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\ -\frac{2}{3}\end{matrix}\middle|\frac{px^6e^{i\pi}}{q}\right)}{3x^{10}\Gamma\left(-\frac{2}{3}\right)}+\frac{2bp^{\frac{3}{2}}x^3}{3\sqrt{1+\frac{q}{px^6}}}+\frac{2b\sqrt{p}q\sqrt{1+\frac{q}{px^6}}}{3x^3}+\frac{2b\sqrt{p}q}{3x^3\sqrt{1+\frac{q}{px^6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*(p*x**6-2*q)*(p*x**6+q)**(1/2)*(a*p*x**6+b*x**4+a*q)/x**11,x)
```

```
[Out] a*p**2*sqrt(q)*x**2*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), p*x**6*exp_polar(
I*pi)/q)/(3*gamma(4/3)) - a*p*q**(3/2)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3
,), p*x**6*exp_polar(I*pi)/q)/(3*x**4*gamma(1/3)) - 2*a*q**(5/2)*gamma(-5/3
)*hyper((-5/3, -1/2), (-2/3,), p*x**6*exp_polar(I*pi)/q)/(3*x**10*gamma(-2/
3)) + 2*b*p**(3/2)*x**3/(3*sqrt(1 + q/(p*x**6))) + 2*b*sqrt(p)*q*sqrt(1 + q
/(p*x**6))/(3*x**3) + 2*b*sqrt(p)*q/(3*x**3*sqrt(1 + q/(p*x**6)))
```

3.716 $\int \frac{1}{x^3 \sqrt[8]{256-256x^2+96x^4-16x^6+x^8}} \, dx$

Optimal. Leaf size=56

$$\frac{\left(\left(x^2-4\right)^4\right)^{7/8} \left(\frac{\sqrt{x^2-4}}{8x^2}+\frac{1}{16}\tan^{-1}\left(\frac{\sqrt{x^2-4}}{2}\right)\right)}{\left(x^2-4\right)^{7/2}}$$

Rubi [A] time = 0.15, antiderivative size = 64, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6688, 6720, 266, 51, 63, 203}

$$\frac{\sqrt{x^2-4}\tan^{-1}\left(\frac{\sqrt{x^2-4}}{2}\right)}{16\sqrt[8]{\left(x^2-4\right)^4}}-\frac{4-x^2}{8x^2\sqrt[8]{\left(x^2-4\right)^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(256 - 256*x^2 + 96*x^4 - 16*x^6 + x^8)^(1/8)),x]
```

```
[Out] -1/8*(4 - x^2)/(x^2*((-4 + x^2)^4)^(1/8)) + (Sqrt[-4 + x^2]*ArcTan[Sqrt[-4
+ x^2]/2])/(16*((-4 + x^2)^4)^(1/8))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt[8]{256-256x^2+96x^4-16x^6+x^8}}\,dx &= \int \frac{1}{x^3\sqrt[8]{\left(-4+x^2\right)^4}}\,dx\\
&= \frac{\sqrt{-4+x^2}\int \frac{1}{x^3\sqrt{-4+x^2}}\,dx}{\sqrt[8]{\left(-4+x^2\right)^4}}\\
&= \frac{\sqrt{-4+x^2}\ \mathrm{Subst}\left(\int \frac{1}{\sqrt{-4+x}\,x^2}\,dx,x,x^2\right)}{2\sqrt[8]{\left(-4+x^2\right)^4}}\\
&= -\frac{4-x^2}{8x^2\sqrt[8]{\left(-4+x^2\right)^4}}+\frac{\sqrt{-4+x^2}\ \mathrm{Subst}\left(\int \frac{1}{\sqrt{-4+x}\,x}\,dx,x,x^2\right)}{16\sqrt[8]{\left(-4+x^2\right)^4}}\\
&= -\frac{4-x^2}{8x^2\sqrt[8]{\left(-4+x^2\right)^4}}+\frac{\sqrt{-4+x^2}\ \mathrm{Subst}\left(\int \frac{1}{4+x^2}\,dx,x,\sqrt{-4+x^2}\right)}{8\sqrt[8]{\left(-4+x^2\right)^4}}\\
&= -\frac{4-x^2}{8x^2\sqrt[8]{\left(-4+x^2\right)^4}}+\frac{\sqrt{-4+x^2}\tan^{-1}\left(\frac{1}{2}\sqrt{-4+x^2}\right)}{16\sqrt[8]{\left(-4+x^2\right)^4}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.93

$$\frac{\left(x^2-4\right)\left(\frac{2}{x^2}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{x^2}{4}}\right)}{\sqrt{4-x^2}}\right)}{16\sqrt[8]{\left(x^2-4\right)^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(256 - 256*x^2 + 96*x^4 - 16*x^6 + x^8)^(1/8)),x]
```

```
[Out] ((-4 + x^2)*(2/x^2 + ArcTanh[Sqrt[1 - x^2/4]]/Sqrt[4 - x^2]))/(16*((-4 + x^
2)^4)^(1/8))
```

IntegrateAlgebraic [A] time = 11.25, size = 56, normalized size = 1.00

$$\frac{\left(\left(x^2-4\right)^4\right)^{7/8}\left(\frac{\sqrt{x^2-4}}{8x^2}+\frac{1}{16}\tan^{-1}\left(\frac{\sqrt{x^2-4}}{2}\right)\right)}{\left(x^2-4\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(256 - 256*x^2 + 96*x^4 - 16*x^6 + x^8)^(1/8)),x]
```

```
[Out] (((-4 + x^2)^4)^(7/8)*(Sqrt[-4 + x^2]/(8*x^2) + ArcTan[Sqrt[-4 + x^2]/2]/16
))/(-4 + x^2)^(7/2)
```

fricas [A] time = 0.38, size = 61, normalized size = 1.09

$$\frac{x^2\arctan\left(-\frac{1}{2}\,x+\frac{1}{2}\left(x^8-16\,x^6+96\,x^4-256\,x^2+256\right)^{\frac{1}{8}}\right)+\left(x^8-16\,x^6+96\,x^4-256\,x^2+256\right)^{\frac{1}{8}}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-16*x^6+96*x^4-256*x^2+256)^(1/8),x, algorithm="fricas"
)
```

```
[Out] 1/8*(x^2*arctan(-1/2*x + 1/2*(x^8 - 16*x^6 + 96*x^4 - 256*x^2 + 256)^(1/8))
 + (x^8 - 16*x^6 + 96*x^4 - 256*x^2 + 256)^(1/8))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^8-16\,x^6+96\,x^4-256\,x^2+256\right)^{\frac{1}{8}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-16*x^6+96*x^4-256*x^2+256)^(1/8),x, algorithm="giac")
```

```
[Out] integrate(1/((x^8 - 16*x^6 + 96*x^4 - 256*x^2 + 256)^(1/8)*x^3), x)
```

maple [A] time = 0.04, size = 49, normalized size = 0.88

$$\frac{x^2-4}{8x^2\left(\left(x^2-4\right)^4\right)^{\frac{1}{8}}}-\frac{\arctan\left(\frac{2}{\sqrt{x^2-4}}\right)\sqrt{x^2-4}}{16\left(\left(x^2-4\right)^4\right)^{\frac{1}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8-16*x^6+96*x^4-256*x^2+256)^(1/8),x)
```

```
[Out] 1/8*(x^2-4)/x^2/((x^2-4)^4)^(1/8)-1/16*arctan(2/(x^2-4)^(1/2))/((x^2-4)^4)^
(1/8)*(x^2-4)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^8-16\,x^6+96\,x^4-256\,x^2+256\right)^{\frac{1}{8}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-16*x^6+96*x^4-256*x^2+256)^(1/8),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((x^8 - 16*x^6 + 96*x^4 - 256*x^2 + 256)^(1/8)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,\left(x^8-16\,x^6+96\,x^4-256\,x^2+256\right)^{1/8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(96*x^4 - 256*x^2 - 16*x^6 + x^8 + 256)^(1/8)),x)
```

```
[Out] int(1/(x^3*(96*x^4 - 256*x^2 - 16*x^6 + x^8 + 256)^(1/8)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt[8]{(x-2)^4 (x+2)^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8-16*x**6+96*x**4-256*x**2+256)**(1/8),x)
```

```
[Out] Integral(1/(x**3*((x - 2)**4*(x + 2)**4)**(1/8)), x)
```

3.717 $\int \frac{x^3(5b+8ax^3)}{\sqrt[4]{bx+ax^4}\,(-2+bx^5+ax^8)}\,dx$

Optimal. Leaf size=56

$$2^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{ax^4+bx}}{\sqrt[4]{2}}\right)-2^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{ax^4+bx}}{\sqrt[4]{2}}\right)$$

Rubi [F] time = 2.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3\left(5b+8ax^3\right)}{\sqrt[4]{bx+ax^4}\left(-2+bx^5+ax^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*(5*b + 8*a*x^3))/((b*x + a*x^4)^(1/4)*(-2 + b*x^5 + a*x^8)),x]
```

```
[Out] (20*b*x^(1/4)*(b + a*x^3)^(1/4)*Defer[Subst][Defer[Int][x^14/((b + a*x^12)^
(1/4)*(-2 + b*x^20 + a*x^32)), x], x, x^(1/4)])/(b*x + a*x^4)^(1/4) + (32*a
*x^(1/4)*(b + a*x^3)^(1/4)*Defer[Subst][Defer[Int][x^26/((b + a*x^12)^(1/4)
*(-2 + b*x^20 + a*x^32)), x], x, x^(1/4)])/(b*x + a*x^4)^(1/4)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(5b+8ax^3\right)}{\sqrt[4]{bx+ax^4}\left(-2+bx^5+ax^8\right)}\,dx &= \frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\int \frac{x^{11/4}(5b+8ax^3)}{\sqrt[4]{b+ax^3}\,(-2+bx^5+ax^8)}\,dx}{\sqrt[4]{bx+ax^4}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{x^{14}(5b+8ax^{12})}{\sqrt[4]{b+ax^{12}}\,(-2+bx^{20}+ax^{32})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^4}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \left(\frac{5bx^{14}}{\sqrt[4]{b+ax^{12}}\,(-2+bx^{20}+ax^{32})}+\frac{8ax^{26}}{\sqrt[4]{b+ax^{12}}\,(-2+bx^{20}+ax^{32})}\right)d}{\sqrt[4]{bx+ax^4}}\\
&= \frac{\left(32a\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{x^{26}}{\sqrt[4]{b+ax^{12}}\,(-2+bx^{20}+ax^{32})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^4}}+\frac{\left(20b\sqrt[4]{x}\sqrt[4]{\right.}{}
\end{aligned}$$

Mathematica [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(5b+8ax^3\right)}{\sqrt[4]{bx+ax^4}\left(-2+bx^5+ax^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(5*b + 8*a*x^3))/((b*x + a*x^4)^(1/4)*(-2 + b*x^5 + a*x^8)),
x]
```

```
[Out] Integrate[(x^3*(5*b + 8*a*x^3))/((b*x + a*x^4)^(1/4)*(-2 + b*x^5 + a*x^8)),
 x]
```

IntegrateAlgebraic [A] time = 12.80, size = 56, normalized size = 1.00

$$2^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{ax^4+bx}}{\sqrt[4]{2}}\right)-2^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{ax^4+bx}}{\sqrt[4]{2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(5*b + 8*a*x^3))/((b*x + a*x^4)^(1/4)*(-2 + b*x^5 +
 a*x^8)),x]
```

```
[Out] 2^(3/4)*ArcTan[(x*(b*x + a*x^4)^(1/4))/2^(1/4)] - 2^(3/4)*ArcTanh[(x*(b*x +
 a*x^4)^(1/4))/2^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(8*a*x^3+5*b)/(a*x^4+b*x)^(1/4)/(a*x^8+b*x^5-2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(8\,ax^3+5\,b)x^3}{(ax^8+bx^5-2)(ax^4+bx)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(8*a*x^3+5*b)/(a*x^4+b*x)^(1/4)/(a*x^8+b*x^5-2),x, algorithm=
"giac")
```

```
[Out] integrate((8*a*x^3 + 5*b)*x^3/((a*x^8 + b*x^5 - 2)*(a*x^4 + b*x)^(1/4)), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x^3(8a\,x^3+5b)}{(a\,x^4+bx)^{\frac{1}{4}}(a\,x^8+b\,x^5-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(8*a*x^3+5*b)/(a*x^4+b*x)^(1/4)/(a*x^8+b*x^5-2),x)
```

```
[Out] int(x^3*(8*a*x^3+5*b)/(a*x^4+b*x)^(1/4)/(a*x^8+b*x^5-2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(8\,ax^3+5\,b)x^3}{(ax^8+bx^5-2)(ax^4+bx)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(8*a*x^3+5*b)/(a*x^4+b*x)^(1/4)/(a*x^8+b*x^5-2),x, algorithm=
"maxima")
```

```
[Out] integrate((8*a*x^3 + 5*b)*x^3/((a*x^8 + b*x^5 - 2)*(a*x^4 + b*x)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,\left(8\,a\,x^3+5\,b\right)}{{\left(a\,x^4+b\,x\right)}^{1/4}\,\left(a\,x^8+b\,x^5-2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(5*b + 8*a*x^3))/((b*x + a*x^4)^(1/4)*(a*x^8 + b*x^5 - 2)),x)
```

```
[Out] int((x^3*(5*b + 8*a*x^3))/((b*x + a*x^4)^(1/4)*(a*x^8 + b*x^5 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(8 a x^{3} + 5 b\right)}{\sqrt[4]{x \left(a x^{3} + b\right)} \left(a x^{8} + b x^{5} - 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(8*a*x**3+5*b)/(a*x**4+b*x)**(1/4)/(a*x**8+b*x**5-2),x)
```

```
[Out] Integral(x**3*(8*a*x**3 + 5*b)/((x*(a*x**3 + b))**(1/4)*(a*x**8 + b*x**5 -
2)), x)
```

3.718 $\int \sqrt{1-x^2-y^4}\,dx$

Optimal. Leaf size=56

$$\frac{1}{2}x\sqrt{-x^2-y^4+1}-\frac{1}{2}i\left(y^4-1\right)\log\left(\sqrt{-x^2-y^4+1}-ix\right)$$

Rubi [A] time = 0.01, antiderivative size = 52, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {195, 217, 203}

$$\frac{1}{2}x\sqrt{-x^2-y^4+1}+\frac{1}{2}\left(1-y^4\right)\tan^{-1}\left(\frac{x}{\sqrt{-x^2-y^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2 - y^4],x]
```

```
[Out] (x*Sqrt[1 - x^2 - y^4])/2 + ((1 - y^4)*ArcTan[x/Sqrt[1 - x^2 - y^4]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x^2-y^4}\,dx &= \frac{1}{2}x\sqrt{1-x^2-y^4}+\frac{1}{2}\left(1-y^4\right)\int \frac{1}{\sqrt{1-x^2-y^4}}\,dx\\ &= \frac{1}{2}x\sqrt{1-x^2-y^4}+\frac{1}{2}\left(1-y^4\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt{1-x^2-y^4}}\right)\\ &= \frac{1}{2}x\sqrt{1-x^2-y^4}+\frac{1}{2}\left(1-y^4\right)\tan^{-1}\left(\frac{x}{\sqrt{1-x^2-y^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.88

$$\frac{1}{2}\left(x\sqrt{-x^2-y^4+1}-\left(y^4-1\right)\tan^{-1}\left(\frac{x}{\sqrt{-x^2-y^4+1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2 - y^4],x]
```

```
[Out] (x*Sqrt[1 - x^2 - y^4] - (-1 + y^4)*ArcTan[x/Sqrt[1 - x^2 - y^4]])/2
```

IntegrateAlgebraic [A] time = 0.06, size = 56, normalized size = 1.00

$$\frac{1}{2}x\sqrt{-x^2-y^4+1}-\frac{1}{2}i\left(y^4-1\right)\log\left(\sqrt{-x^2-y^4+1}-ix\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x^2 - y^4],x]
[Out] (x*Sqrt[1 - x^2 - y^4])/2 - (I/2)*(-1 + y^4)*Log[(-I)*x + Sqrt[1 - x^2 - y^
4]]
```

fricas [A] time = 0.42, size = 44, normalized size = 0.79

$$\frac{1}{2}\left(y^4-1\right)\arctan\left(\frac{\sqrt{-y^4-x^2+1}}{x}\right)+\frac{1}{2}\sqrt{-y^4-x^2+1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-y^4-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*(y^4 - 1)*arctan(sqrt(-y^4 - x^2 + 1)/x) + 1/2*sqrt(-y^4 - x^2 + 1)*x
```

giac [A] time = 0.20, size = 37, normalized size = 0.66

$$-\frac{1}{2}\left(y^4-1\right)\arcsin\left(\frac{x}{\sqrt{-y^4+1}}\right)+\frac{1}{2}\sqrt{-y^4-x^2+1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-y^4-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/2*(y^4 - 1)*arcsin(x/sqrt(-y^4 + 1)) + 1/2*sqrt(-y^4 - x^2 + 1)*x
```

maple [A] time = 0.01, size = 60, normalized size = 1.07

$$\frac{x\sqrt{-y^4-x^2+1}}{2}-\frac{\arctan\left(\frac{x}{\sqrt{-y^4-x^2+1}}\right)y^4}{2}+\frac{\arctan\left(\frac{x}{\sqrt{-y^4-x^2+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-y^4-x^2+1)^(1/2),x)
[Out] 1/2*x*(-y^4-x^2+1)^(1/2)-1/2*arctan(1/(-y^4-x^2+1)^(1/2)*x)*y^4+1/2*arctan(
1/(-y^4-x^2+1)^(1/2)*x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-y^4-x^2+1)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((y-1)*(y+1)>0)', see `assume?` for
more details)Is (y-1)*(y+1) positive or negative?
```

mupad [B] time = 0.10, size = 48, normalized size = 0.86

$$\frac{x\sqrt{-x^2-y^4+1}}{2}+\ln\left(\sqrt{-x^2-y^4+1}+x\,1\mathrm{i}\right)\left(\frac{y^4\,1\mathrm{i}}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - y^4 - x^2)^(1/2),x)
```

```
[Out] (x*(1 - y^4 - x^2)^(1/2))/2 + log(x*1i + (1 - y^4 - x^2)^(1/2))*((y^4*1i)/2
 - 1i/2)
```

sympy [B] time = 1.65, size = 745, normalized size = 13.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-y**4-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-2*I*x**3*sqrt(polar_lift(1 - y**4))/(4*y**4*sqrt(x**2/polar_lif
t(1 - y**4) - 1) - 4*sqrt(x**2/polar_lift(1 - y**4) - 1)) - 2*I*x*y**4*sqrt
(polar_lift(1 - y**4))/(4*y**4*sqrt(x**2/polar_lift(1 - y**4) - 1) - 4*sqrt
(x**2/polar_lift(1 - y**4) - 1)) + 2*I*x*sqrt(polar_lift(1 - y**4))/(4*y**4
*sqrt(x**2/polar_lift(1 - y**4) - 1) - 4*sqrt(x**2/polar_lift(1 - y**4) - 1
)) - 2*I*y**4*(1 - y**4)*sqrt(x**2/polar_lift(1 - y**4) - 1)*acosh(x/sqrt(p
olar_lift(1 - y**4)))/(4*y**4*sqrt(x**2/polar_lift(1 - y**4) - 1) - 4*sqrt(
x**2/polar_lift(1 - y**4) - 1)) + pi*y**4*(1 - y**4)*sqrt(x**2/polar_lift(1
 - y**4) - 1)/(4*y**4*sqrt(x**2/polar_lift(1 - y**4) - 1) - 4*sqrt(x**2/pol
ar_lift(1 - y**4) - 1)) + 2*I*(1 - y**4)*sqrt(x**2/polar_lift(1 - y**4) - 1
)*acosh(x/sqrt(polar_lift(1 - y**4)))/(4*y**4*sqrt(x**2/polar_lift(1 - y**4
) - 1) - 4*sqrt(x**2/polar_lift(1 - y**4) - 1)) - pi*(1 - y**4)*sqrt(x**2/p
olar_lift(1 - y**4) - 1)/(4*y**4*sqrt(x**2/polar_lift(1 - y**4) - 1) - 4*sq
rt(x**2/polar_lift(1 - y**4) - 1)), Abs(x**2/(y**4 - 1)) > 1), (x**3*sqrt(p
olar_lift(1 - y**4))/(2*y**4*sqrt(-x**2/polar_lift(1 - y**4) + 1) - 2*sqrt(
-x**2/polar_lift(1 - y**4) + 1)) + x*y**4*sqrt(polar_lift(1 - y**4))/(2*y**
4*sqrt(-x**2/polar_lift(1 - y**4) + 1) - 2*sqrt(-x**2/polar_lift(1 - y**4)
+ 1)) - x*sqrt(polar_lift(1 - y**4))/(2*y**4*sqrt(-x**2/polar_lift(1 - y**4
) + 1) - 2*sqrt(-x**2/polar_lift(1 - y**4) + 1)) + y**4*(1 - y**4)*sqrt(-x*
*2/polar_lift(1 - y**4) + 1)*asin(x/sqrt(polar_lift(1 - y**4)))/(2*y**4*sqr
t(-x**2/polar_lift(1 - y**4) + 1) - 2*sqrt(-x**2/polar_lift(1 - y**4) + 1))
 - (1 - y**4)*sqrt(-x**2/polar_lift(1 - y**4) + 1)*asin(x/sqrt(polar_lift(1
 - y**4)))/(2*y**4*sqrt(-x**2/polar_lift(1 - y**4) + 1) - 2*sqrt(-x**2/pola
r_lift(1 - y**4) + 1)), True))
```

3.719 $\displaystyle \int \frac{1-2k^2x+k^2x^2}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(-a-bx+\left(ak^2+bk^2\right)x^2\right)}\,dx$

Optimal. Leaf size=57

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{\sqrt{a}\,(x-1)}\right)}{\sqrt{a}\,\sqrt{a+b}}$$

Rubi [C] time = 8.51, antiderivative size = 338, normalized size of antiderivative = 5.93, number of steps used = 12, number of rules used = 6, integrand size = 60, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6718, 6728, 115, 168, 538, 537}

$$\frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{\frac{k^2(1-x)}{1-k^2}+1}\,\Pi\left(-\frac{2(a+b)k^2}{-2ak^2-2bk^2+b+\sqrt{b^2+4ak^2b+4a^2k^2}};\sin^{-1}\left(\sqrt{1-x}\right)|-\frac{k^2}{1-k^2}\right)}{(a+b)\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{\frac{k^2(1-x)}{1-k^2}+1}\,\Pi\left(\frac{2(a+b)k^2}{2ak^2-b\left(1-2k^2\right)+\sqrt{b^2+4ak^2b+4a^2k^2}};\sin^{-1}\left(\sqrt{1-x}\right)|-\frac{k^2}{1-k^2}\right)}{(a+b)\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{(a+b)\sqrt{(1-x)x\left(1-k^2x\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*k^2*x + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-a - b*x + (a*k^2
 + b*k^2)*x^2)),x]
```

```
[Out] (2*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticF[ArcSin[Sqrt[x]], k^2])/((a
 + b)*Sqrt[(1 - x)*x*(1 - k^2*x)]) + (2*Sqrt[1 + (k^2*(1 - x))/(1 - k^2)]*S
qrt[1 - x]*Sqrt[x]*EllipticPi[(-2*(a + b)*k^2)/(b - 2*a*k^2 - 2*b*k^2 + Sqr
t[b^2 + 4*a^2*k^2 + 4*a*b*k^2]), ArcSin[Sqrt[1 - x]], -(k^2/(1 - k^2))])/((
a + b)*Sqrt[(1 - x)*x*(1 - k^2*x)]) + (2*Sqrt[1 + (k^2*(1 - x))/(1 - k^2)]*
Sqrt[1 - x]*Sqrt[x]*EllipticPi[(2*(a + b)*k^2)/(2*a*k^2 - b*(1 - 2*k^2) + S
qrt[b^2 + 4*a^2*k^2 + 4*a*b*k^2]), ArcSin[Sqrt[1 - x]], -(k^2/(1 - k^2))])/
((a + b)*Sqrt[(1 - x)*x*(1 - k^2*x)])
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1-2k^2x+k^2x^2}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(-a-bx+\left(ak^2+bk^2\right)x^2\right)}\,dx = \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{1-2k^2x+k^2x^2}{\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\left(-a-bx+\left(ak^2+bk^2\right.\right.}}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int\left(\frac{1}{(a+b)\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}}+\frac{}{(a+b)\sqrt{}}\right.}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int\frac{1}{\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}}\,dx}{(a+b)\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{\left(\sqrt{1-x}\right.}{}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{(a+b)\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{\left(\sqrt{1-x}\sqrt{}\right.}{}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{(a+b)\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{\left(\left(b-2ak^2\right.\right.}{}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{(a+b)\sqrt{(1-x)x\left(1-k^2x\right)}}-\frac{\left(2\left(b-2a\right.\right.}{}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{(a+b)\sqrt{(1-x)x\left(1-k^2x\right)}}-\frac{\left(2\left(b-2a\right.\right.}{}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{(a+b)\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{2\sqrt{1+\frac{k^2(}{1-}}}{}$$

Mathematica [C] time = 4.22, size = 227, normalized size = 3.98

$$-\frac{2i\sqrt{\frac{1}{x-1}+1}\,(x-1)^{3/2}\sqrt{\frac{1-\frac{1}{k^2}}{x-1}+1}\left(-\Pi\left(-\frac{2(a+b)\left(k^2-1\right)}{-2ak^2-2bk^2+b+\sqrt{b^2+4ak^2b+4a^2k^2}};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)-\Pi\left(\frac{2(a+b)\left(k^2-1\right)}{2ak^2+2bk^2-b+\sqrt{b^2+4ak^2b+4a^2k^2}};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)}{(a+b)\sqrt{(x-1)x\left(k^2x-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*k^2*x + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-a - b*x +
(a*k^2 + b*k^2)*x^2)),x]
```

```
[Out] ((-2*I)*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (1 - k^(-2))/(-1 + x)]*(-1 + x)^(3
/2)*(EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] - EllipticPi[(-2*(a +
 b)*(-1 + k^2))/(b - 2*a*k^2 - 2*b*k^2 + Sqrt[b^2 + 4*a^2*k^2 + 4*a*b*k^2])
, I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] - EllipticPi[(2*(a + b)*(-1 + k^2)
)/(-b + 2*a*k^2 + 2*b*k^2 + Sqrt[b^2 + 4*a^2*k^2 + 4*a*b*k^2]), I*ArcSinh[1
/Sqrt[-1 + x]], 1 - k^(-2)]))/((a + b)*Sqrt[(-1 + x)*x*(-1 + k^2*x)])
```

IntegrateAlgebraic [A] time = 0.31, size = 57, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{\sqrt{a}\,(x-1)}\right)}{\sqrt{a}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - 2*k^2*x + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-a
 - b*x + (a*k^2 + b*k^2)*x^2)),x]
```

```
[Out] (2*ArcTan[(Sqrt[a + b]*Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3])/(Sqrt[a]*(-1 + x
))])/(Sqrt[a]*Sqrt[a + b])
```

fricas [B] time = 0.81, size = 345, normalized size = 6.05

$$\left[-\frac{\sqrt{-a^2-ab}\,\log\left(\frac{\left(a^2+2\,ab+b^2\right)k^4x^4-2\left(4\,a^2+5\,ab+b^2\right)k^2x^3+\left(6\left(a^2+ab\right)k^2+8\,a^2+8\,ab+b^2\right)x^2-4\left((a+b)k^2x^2-(2\,a+b)x+a\right)\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\,\sqrt{-a^2-ab}+a^2-2\left(4\,a^2+3\,ab\right)x}{\left(a^2+2\,ab+b^2\right)k^4x^4-2\left(ab+b^2\right)k^2x^3+2\,abx-\left(2\left(a^2+ab\right)k^2-b^2\right)x^2+a^2}\right)}{2\left(a^2+ab\right)},\ \frac{\arctan\left(\frac{\left((a+b)k^2x^2-(2\,a+b)x+a\right)\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\,\sqrt{a^2+ab}}{2\left(\left(a^2+ab\right)k^2x^3-\left(\left(a^2+ab\right)k^2+a^2+ab\right)x^2+\left(a^2+ab\right)x\right)}\right)}{\sqrt{a^2+ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-2*k^2*x+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(-a-b*x+(a*k^2+b*k
^2)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 - a*b)*log(((a^2 + 2*a*b + b^2)*k^4*x^4 - 2*(4*a^2 + 5*a*b
+ b^2)*k^2*x^3 + (6*(a^2 + a*b)*k^2 + 8*a^2 + 8*a*b + b^2)*x^2 - 4*((a + b)
*k^2*x^2 - (2*a + b)*x + a)*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*sqrt(-a^2 - a
*b) + a^2 - 2*(4*a^2 + 3*a*b)*x)/((a^2 + 2*a*b + b^2)*k^4*x^4 - 2*(a*b + b^
2)*k^2*x^3 + 2*a*b*x - (2*(a^2 + a*b)*k^2 - b^2)*x^2 + a^2))/(a^2 + a*b), a
rctan(1/2*((a + b)*k^2*x^2 - (2*a + b)*x + a)*sqrt(k^2*x^3 - (k^2 + 1)*x^2
+ x)*sqrt(a^2 + a*b)/((a^2 + a*b)*k^2*x^3 - ((a^2 + a*b)*k^2 + a^2 + a*b)*x
^2 + (a^2 + a*b)*x))/sqrt(a^2 + a*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^2-2\,k^2x+1}{\sqrt{\left(k^2x-1\right)(x-1)x}\,\left(\left(ak^2+bk^2\right)x^2-bx-a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-2*k^2*x+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(-a-b*x+(a*k^2+b*k
^2)*x^2),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^2 - 2*k^2*x + 1)/(sqrt((k^2*x - 1)*(x - 1)*x)*((a*k^2 + b*
k^2)*x^2 - b*x - a)), x)
```

maple [C] time = 0.08, size = 4588, normalized size = 80.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^2-2*k^2*x+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(-a-b*x+(a*k^2+b*k^2)*x^
2),x)
```

```
[Out] -2/(a+b)/k^2*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/
(k^2*x^3-k^2*x^2-x^2+x)^(1/2)*EllipticF((-(x-1/k^2)*k^2)^(1/2),(1/k^2/(1/k^
2-1))^(1/2))+1/(a+b)*(2/(1/(a+b)*a*b+1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2
)*a+1/(a+b)*b^2+1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*b-b)/k^2*(-k^2*x+1)
^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2
+x)^(1/2)/(1/k^2-1/2*b/k^2/(a+b)-1/2/k^2/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1
/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-1/2*(b+(4*a^2*k^2+4*a*b
*k^2+b^2)^(1/2))/k^2/(a+b)),(1/k^2/(1/k^2-1))^(1/2))/(a+b)*a*b+2/(1/(a+b)*a
*b+1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*a+1/(a+b)*b^2+1/(a+b)*(4*a^2*k^2
+4*a*b*k^2+b^2)^(1/2)*b-b)/k^2*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))
^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2-1/2*b/k^2/(a+b)-1
/2/k^2/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(
1/2),1/k^2/(1/k^2-1/2*(b+(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))/k^2/(a+b)),(1/k^2
/(1/k^2-1))^(1/2))/(a+b)*b^2-1/(1/(a+b)*a*b+1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^
2)^(1/2)*a+1/(a+b)*b^2+1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*b-b)/k^4*(-k
^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*
x^2-x^2+x)^(1/2)/(1/k^2-1/2*b/k^2/(a+b)-1/2/k^2/(a+b)*(4*a^2*k^2+4*a*b*k^2+
b^2)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-1/2*(b+(4*a^2*k^
2+4*a*b*k^2+b^2)^(1/2))/k^2/(a+b)),(1/k^2/(1/k^2-1))^(1/2))/(a+b)*b^2+2/(1/
(a+b)*a*b+1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*a+1/(a+b)*b^2+1/(a+b)*(4*
a^2*k^2+4*a*b*k^2+b^2)^(1/2)*b-b)/k^2*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/
k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2-1/2*b/k^2/
(a+b)-1/2/k^2/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))*EllipticPi((-(x-1/k^2)
*k^2)^(1/2),1/k^2/(1/k^2-1/2*(b+(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))/k^2/(a+b))
,(1/k^2/(1/k^2-1))^(1/2))/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*a+2/(1/(a+b
)*a*b+1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*a+1/(a+b)*b^2+1/(a+b)*(4*a^2*
k^2+4*a*b*k^2+b^2)^(1/2)*b-b)/k^2*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-
1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2-1/2*b/k^2/(a+b
)-1/2/k^2/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))*EllipticPi((-(x-1/k^2)*k^2
)^(1/2),1/k^2/(1/k^2-1/2*(b+(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))/k^2/(a+b)),(1/
k^2/(1/k^2-1))^(1/2))/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*b-1/(1/(a+b)*a*
b+1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*a+1/(a+b)*b^2+1/(a+b)*(4*a^2*k^2+
4*a*b*k^2+b^2)^(1/2)*b-b)/k^4*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^
(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2-1/2*b/k^2/(a+b)-1/
2/k^2/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1
/2),1/k^2/(1/k^2-1/2*(b+(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))/k^2/(a+b)),(1/k^2/
(1/k^2-1))^(1/2))/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*b-4/(1/(a+b)*a*b+1/
(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*a+1/(a+b)*b^2+1/(a+b)*(4*a^2*k^2+4*a*
b*k^2+b^2)^(1/2)*b-b)/k^2*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2
)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2-1/2*b/k^2/(a+b)-1/2/k^
2/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),
1/k^2/(1/k^2-1/2*(b+(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))/k^2/(a+b)),(1/k^2/(1/k
^2-1))^(1/2))*a-2/(1/(a+b)*a*b+1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*a+1/
(a+b)*b^2+1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*b-b)/k^2*(-k^2*x+1)^(1/2)
*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1
/2)/(1/k^2-1/2*b/k^2/(a+b)-1/2/k^2/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))*E
llipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-1/2*(b+(4*a^2*k^2+4*a*b*k^2+b
^2)^(1/2))/k^2/(a+b)),(1/k^2/(1/k^2-1))^(1/2))*b+2/(1/(a+b)*a*b-1/(a+b)*(4*
a^2*k^2+4*a*b*k^2+b^2)^(1/2)*a+1/(a+b)*b^2-1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2
)^(1/2)*b-b)/k^2*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)
^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2-1/2*b/k^2/(a+b)+1/2/k^2/(a+b)*(
4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/
k^2+1/2*(-b+(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))/k^2/(a+b)),(1/k^2/(1/k^2-1))^(
1/2))/(a+b)*a*b+2/(1/(a+b)*a*b-1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*a+1/
(a+b)*b^2-1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*b-b)/k^2*(-k^2*x+1)^(1/2)
```

```
*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1
/2)/(1/k^2-1/2*b/k^2/(a+b)+1/2/k^2/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))*E
llipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+1/2*(-b+(4*a^2*k^2+4*a*b*k^2+
b^2)^(1/2))/k^2/(a+b)),(1/k^2/(1/k^2-1))^(1/2))/(a+b)*b^2-1/(1/(a+b)*a*b-1/
(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*a+1/(a+b)*b^2-1/(a+b)*(4*a^2*k^2+4*a*
b*k^2+b^2)^(1/2)*b-b)/k^4*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2
)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2-1/2*b/k^2/(a+b)+1/2/k^
2/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),
1/k^2/(1/k^2+1/2*(-b+(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))/k^2/(a+b)),(1/k^2/(1/
k^2-1))^(1/2))/(a+b)*b^2-2/(1/(a+b)*a*b-1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(
1/2)*a+1/(a+b)*b^2-1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*b-b)/k^2*(-k^2*x
+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-
x^2+x)^(1/2)/(1/k^2-1/2*b/k^2/(a+b)+1/2/k^2/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)
^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+1/2*(-b+(4*a^2*k^2+4
*a*b*k^2+b^2)^(1/2))/k^2/(a+b)),(1/k^2/(1/k^2-1))^(1/2))/(a+b)*(4*a^2*k^2+4
*a*b*k^2+b^2)^(1/2)*a-2/(1/(a+b)*a*b-1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2
)*a+1/(a+b)*b^2-1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*b-b)/k^2*(-k^2*x+1)
^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2
+x)^(1/2)/(1/k^2-1/2*b/k^2/(a+b)+1/2/k^2/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1
/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+1/2*(-b+(4*a^2*k^2+4*a*
b*k^2+b^2)^(1/2))/k^2/(a+b)),(1/k^2/(1/k^2-1))^(1/2))/(a+b)*(4*a^2*k^2+4*a*
b*k^2+b^2)^(1/2)*b+1/(1/(a+b)*a*b-1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*a
+1/(a+b)*b^2-1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*b-b)/k^4*(-k^2*x+1)^(1
/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)
^(1/2)/(1/k^2-1/2*b/k^2/(a+b)+1/2/k^2/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)
)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+1/2*(-b+(4*a^2*k^2+4*a*b*k
^2+b^2)^(1/2))/k^2/(a+b)),(1/k^2/(1/k^2-1))^(1/2))/(a+b)*(4*a^2*k^2+4*a*b*k
^2+b^2)^(1/2)*b-4/(1/(a+b)*a*b-1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*a+1/
(a+b)*b^2-1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*b-b)/k^2*(-k^2*x+1)^(1/2)
*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1
/2)/(1/k^2-1/2*b/k^2/(a+b)+1/2/k^2/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))*E
llipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+1/2*(-b+(4*a^2*k^2+4*a*b*k^2+
b^2)^(1/2))/k^2/(a+b)),(1/k^2/(1/k^2-1))^(1/2))*a-2/(1/(a+b)*a*b-1/(a+b)*(4
*a^2*k^2+4*a*b*k^2+b^2)^(1/2)*a+1/(a+b)*b^2-1/(a+b)*(4*a^2*k^2+4*a*b*k^2+b^
2)^(1/2)*b-b)/k^2*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x
)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2-1/2*b/k^2/(a+b)+1/2/k^2/(a+b)*
(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1
/k^2+1/2*(-b+(4*a^2*k^2+4*a*b*k^2+b^2)^(1/2))/k^2/(a+b)),(1/k^2/(1/k^2-1))^
(1/2))*b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^2 - 2\,k^2x + 1}{\sqrt{\left(k^2x - 1\right)(x - 1)x}\left(\left(ak^2 + bk^2\right)x^2 - bx - a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-2*k^2*x+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(-a-b*x+(a*k^2+b*k
^2)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^2 - 2*k^2*x + 1)/(sqrt((k^2*x - 1)*(x - 1)*x)*((a*k^2 + b*
k^2)*x^2 - b*x - a)), x)
```

mupad [B] time = 3.84, size = 122, normalized size = 2.14

$$\frac{\ln\left(\frac{a\,\sqrt{a\,(a+b)}-2\,a\,x\,\sqrt{a\,(a+b)}-b\,x\,\sqrt{a\,(a+b)}+a\,k^2\,x^2\,\sqrt{a\,(a+b)}+b\,k^2\,x^2\,\sqrt{a\,(a+b)}+a\,(a+b)\,\sqrt{x\,\left(k^2\,x-1\right)\,(x-1)}\,2{}\mathrm{i}}{a+b\,x-a\,k^2\,x^2-b\,k^2\,x^2}\right)\,1{}\mathrm{i}}{\sqrt{a^2+b\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(k^2*x^2 - 2*k^2*x + 1)/((a + b*x - x^2*(a*k^2 + b*k^2))*(x*(k^2*x - 1
)*(x - 1))^(1/2)),x)
```

```
[Out] (log((a*(a*(a + b))^(1/2) - 2*a*x*(a*(a + b))^(1/2) - b*x*(a*(a + b))^(1/2)
 + a*(a + b)*(x*(k^2*x - 1)*(x - 1))^(1/2)*2i + a*k^2*x^2*(a*(a + b))^(1/2)
 + b*k^2*x^2*(a*(a + b))^(1/2))/(a + b*x - a*k^2*x^2 - b*k^2*x^2))*1i)/(a*b
 + a^2)^(1/2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k**2*x**2-2*k**2*x+1)/((1-x)*x*(-k**2*x+1))**(1/2)/(-a-b*x+(a*k*
*2+b*k**2)*x**2),x)
```

```
[Out] Timed out
```

3.720

$$\int \frac{abc-(a+b+c)x^2+2x^3}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(-abcd+(-1+abd+acd+bcd)x-(a+b+c)dx^2+dx^3\right)}\,dx$$

Optimal. Leaf size=57

$$-\frac{2\tanh^{-1}\left(\frac{x}{\sqrt{d}\sqrt{x^3(-a-b-c)+x^2(ab+ac+bc)-abcx+x^4}}\right)}{\sqrt{d}}$$

Rubi [F] time = 4.77, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{abc-(a+b+c)x^2+2x^3}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(-abcd+(-1+abd+acd+bcd)x-(a+b+c)dx^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*b*c - (a + b + c)*x^2 + 2*x^3)/(Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*(
-(a*b*c*d) + (-1 + a*b*d + a*c*d + b*c*d)*x - (a + b + c)*d*x^2 + d*x^3)),x
]
```

```
[Out] (2*Defer[Int][1/Sqrt[x*(-a + x)*(-b + x)*(-c + x)], x])/d - 3*a*b*c*Defer[I
nt][1/(Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*(a*b*c*d + (1 - b*c*d - a*(b + c)
*d)*x + (a + b + c)*d*x^2 - d*x^3)), x] - (2*(1 - b*c*d - a*(b + c)*d)*Defe
r[Int][x/(Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*(a*b*c*d + (1 - b*c*d - a*(b +
 c)*d)*x + (a + b + c)*d*x^2 - d*x^3)), x])/d - (a + b + c)*Defer[Int][x^2/
(Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*(a*b*c*d + (1 - b*c*d - a*(b + c)*d)*x
+ (a + b + c)*d*x^2 - d*x^3)), x]
```

Rubi steps

$$\int \frac{abc-(a+b+c)x^2+2x^3}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(-abcd+(-1+abd+acd+bcd)x-(a+b+c)dx^2+dx^3\right)}\,dx = \int \frac{}{\sqrt{x(-a+x)(-}}$$

$$= \int \left(\frac{}{d\sqrt{x(-a+x}} \right.$$

$$= -\frac{\int \frac{3a}{\sqrt{x(-a+x)(-b+x}}}{}$$

$$= -\frac{\int \left(\frac{}{\sqrt{x(-a+x)(-b+}} \right.}{}$$

$$= -\left((3abc) \int \frac{}{\sqrt{x(}} \right.$$

Mathematica [C] time = 13.03, size = 8060, normalized size = 141.40

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*b*c - (a + b + c)*x^2 + 2*x^3)/(Sqrt[x*(-a + x)*(-b + x)*(-c +
 x)]*(-(a*b*c*d) + (-1 + a*b*d + a*c*d + b*c*d)*x - (a + b + c)*d*x^2 + d*x
^3)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.82, size = 59, normalized size = 1.04

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{x^3(-a-b-c)+x^2(ab+ac+bc)-abcx+x^4}}{x}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b*c - (a + b + c)*x^2 + 2*x^3)/(Sqrt[x*(-a + x)*(-b +
 x)*(-c + x)]*(-(a*b*c*d) + (-1 + a*b*d + a*c*d + b*c*d)*x - (a + b + c)*d*
x^2 + d*x^3)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[d]*Sqrt[-(a*b*c*x) + (a*b + a*c + b*c)*x^2 + (-a - b - c)
*x^3 + x^4])/x])/Sqrt[d]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*c-(a+b+c)*x^2+2*x^3)/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)/(-a*b*c*
d+(a*b*d+a*c*d+b*c*d-1)*x-(a+b+c)*d*x^2+d*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{abc-(a+b+c)x^2+2\,x^3}{\left(abcd+(a+b+c)dx^2-dx^3-(abd+acd+bcd-1)x\right)\sqrt{-(a-x)(b-x)(c-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*c-(a+b+c)*x^2+2*x^3)/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)/(-a*b*c*
d+(a*b*d+a*c*d+b*c*d-1)*x-(a+b+c)*d*x^2+d*x^3),x, algorithm="giac")
```

```
[Out] integrate(-(a*b*c - (a + b + c)*x^2 + 2*x^3)/((a*b*c*d + (a + b + c)*d*x^2
- d*x^3 - (a*b*d + a*c*d + b*c*d - 1)*x)*sqrt(-(a - x)*(b - x)*(c - x)*x)),
 x)
```

maple [C] time = 0.12, size = 568, normalized size = 9.96

$$-\frac{4a\sqrt{\frac{(a-c)x}{a(-c+x)}}\,(-c+x)^2\sqrt{\frac{c(-b+x)}{b(-c+x)}}\,\sqrt{\frac{c(-a+x)}{a(-c+x)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{(a-c)x}{a(-c+x)}},\sqrt{\frac{(-b+c)a}{b(c-a)}}\right)}{d\,(a-c)\,c\sqrt{x\,(-a+x)\,(-b+x)\,(-c+x)}}+\frac{2a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\,\_Z^3+(-ad-bd-cd)\_Z^2+(abd+acd+bcd-1)\_Z-abcd\right)}\frac{\left(-\_\alpha^2ad-\_\alpha^2bd-\_\alpha^2cd+2\_\alpha abd+2\_\alpha acd+2\_\alpha bcd-3abcd-2\_\alpha\right)(-c+x)^2\left(d\,\_\alpha^2-d\,\_\alpha a-d\,\_\alpha b+abd-1\right)\sqrt{\frac{(a-c)x}{a(-c+x)}}\,\sqrt{\frac{c(-b+x)}{b(-c+x)}}\,\sqrt{\frac{c(-a+x)}{a(-c+x)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{(a-c)x}{a(-c+x)}},\sqrt{\frac{(-b+c)a}{b(c-a)}}\right)-\frac{\left(d\,\_\alpha^2-d\,\_\alpha a-d\,\_\alpha b-\_\alpha cd+abd+acd+bcd-1\right)\mathrm{EllipticPi}\left(\sqrt{\frac{(a-c)x}{a(-c+x)}},-\frac{d\,\_\alpha^2-d\,\_\alpha a-d\,\_\alpha b-\_\alpha cd+acd+bcd-1}{bd(a-c)},\sqrt{\frac{(-b+c)a}{b(c-a)}}\right)}{abd}\right)}{\left(-3d\,\_\alpha^2+2d\,\_\alpha a+2d\,\_\alpha b+2\_\alpha cd-abd-acd-bcd+1\right)(a-c)\sqrt{x(-a+x)(-b+x)(-c+x)}}\right)}{d\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b*c-(a+b+c)*x^2+2*x^3)/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)/(-a*b*c*d+(a*b
*d+a*c*d+b*c*d-1)*x-(a+b+c)*d*x^2+d*x^3),x)
```

```
[Out] -4/d*a*((a-c)*x/a/(-c+x))^(1/2)*(-c+x)^2*(c*(-b+x)/b/(-c+x))^(1/2)*(c*(-a+x
)/a/(-c+x))^(1/2)/(a-c)/c/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)*EllipticF(((a-c)*x
/a/(-c+x))^(1/2),((-b+c)*a/b/(c-a))^(1/2))+2/d*a/c^2*sum((-_alpha^2*a*d-_al
pha^2*b*d-_alpha^2*c*d+2*_alpha*a*b*d+2*_alpha*a*c*d+2*_alpha*b*c*d-3*a*b*c
*d-2*_alpha)/(-3*_alpha^2*d+2*_alpha*a*d+2*_alpha*b*d+2*_alpha*c*d-a*b*d-a*
c*d-b*c*d+1)*(-c+x)^2/(a-c)*(_alpha^2*d-_alpha*a*d-_alpha*b*d+a*b*d-1)*((a-
c)*x/a/(-c+x))^(1/2)*(c*(-b+x)/b/(-c+x))^(1/2)*(c*(-a+x)/a/(-c+x))^(1/2)/(x
*(-a+x)*(-b+x)*(-c+x))^(1/2)*(EllipticF(((a-c)*x/a/(-c+x))^(1/2),((-b+c)*a/
b/(c-a))^(1/2))-(_alpha^2*d-_alpha*a*d-_alpha*b*d-_alpha*c*d+a*b*d+a*c*d+b*
c*d-1)/a/b/d*EllipticPi(((a-c)*x/a/(-c+x))^(1/2),-(_alpha^2*d-_alpha*a*d-_a
```

```
lpha*b*d-_alpha*c*d+a*c*d+b*c*d-1)/b/d/(a-c),((-b+c)*a/b/(c-a))^(1/2))),_al
pha=RootOf(d*_Z^3+(-a*d-b*d-c*d)*_Z^2+(a*b*d+a*c*d+b*c*d-1)*_Z-a*b*c*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{abc-(a+b+c)x^2+2x^3}{\left(abcd+(a+b+c)dx^2-dx^3-(abd+acd+bcd-1)x\right)\sqrt{-(a-x)(b-x)(c-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*c-(a+b+c)*x^2+2*x^3)/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)/(-a*b*c*
d+(a*b*d+a*c*d+b*c*d-1)*x-(a+b+c)*d*x^2+d*x^3),x, algorithm="maxima")
```

```
[Out] -integrate((a*b*c - (a + b + c)*x^2 + 2*x^3)/((a*b*c*d + (a + b + c)*d*x^2
- d*x^3 - (a*b*d + a*c*d + b*c*d - 1)*x)*sqrt(-(a - x)*(b - x)*(c - x)*x)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{2\,x^3+(-a-b-c)\,x^2+a\,b\,c}{\sqrt{-x\,(a-x)\,(b-x)\,(c-x)}\,\left(d\,x^3-d\,(a+b+c)\,x^2+(a\,b\,d+a\,c\,d+b\,c\,d-1)\,x-a\,b\,c\,d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3 - x^2*(a + b + c) + a*b*c)/((-x*(a - x)*(b - x)*(c - x))^(1/2)*(
d*x^3 + x*(a*b*d + a*c*d + b*c*d - 1) - d*x^2*(a + b + c) - a*b*c*d)),x)
```

```
[Out] int((2*x^3 - x^2*(a + b + c) + a*b*c)/((-x*(a - x)*(b - x)*(c - x))^(1/2)*(
d*x^3 + x*(a*b*d + a*c*d + b*c*d - 1) - d*x^2*(a + b + c) - a*b*c*d)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*c-(a+b+c)*x**2+2*x**3)/(x*(-a+x)*(-b+x)*(-c+x))**(1/2)/(-a*b
*c*d+(a*b*d+a*c*d+b*c*d-1)*x-(a+b+c)*d*x**2+d*x**3),x)
```

```
[Out] Timed out
```

3.721 $\int x^6 \sqrt[4]{-1+x^4}\, dx$

Optimal. Leaf size=57

$$\frac{3}{64}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{3}{64}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{32}\sqrt[4]{x^4-1}\left(4x^7-x^3\right)$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {279, 321, 331, 298, 203, 206}

$$\frac{3}{64}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{3}{64}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{8}\sqrt[4]{x^4-1}\,x^7-\frac{1}{32}\sqrt[4]{x^4-1}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(-1 + x^4)^(1/4),x]
```

```
[Out] -1/32*(x^3*(-1 + x^4)^(1/4)) + (x^7*(-1 + x^4)^(1/4))/8 + (3*ArcTan[x/(-1 +
 x^4)^(1/4)])/64 - (3*ArcTanh[x/(-1 + x^4)^(1/4)])/64
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int x^6\sqrt[4]{-1+x^4}\,dx &= \frac{1}{8}x^7\sqrt[4]{-1+x^4}-\frac{1}{8}\int\frac{x^6}{\left(-1+x^4\right)^{3/4}}\,dx\\ &= -\frac{1}{32}x^3\sqrt[4]{-1+x^4}+\frac{1}{8}x^7\sqrt[4]{-1+x^4}-\frac{3}{32}\int\frac{x^2}{\left(-1+x^4\right)^{3/4}}\,dx\\ &= -\frac{1}{32}x^3\sqrt[4]{-1+x^4}+\frac{1}{8}x^7\sqrt[4]{-1+x^4}-\frac{3}{32}\,\text{Subst}\left(\int\frac{x^2}{1-x^4}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)\\ &= -\frac{1}{32}x^3\sqrt[4]{-1+x^4}+\frac{1}{8}x^7\sqrt[4]{-1+x^4}-\frac{3}{64}\,\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)+\frac{3}{64}\,\text{Subst}\left(\int\right.\\ &= -\frac{1}{32}x^3\sqrt[4]{-1+x^4}+\frac{1}{8}x^7\sqrt[4]{-1+x^4}+\frac{3}{64}\tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)-\frac{3}{64}\tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)\end{aligned}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.95

$$\frac{x^3\sqrt[4]{x^4-1}\left({}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};x^4\right)-\left(1-x^4\right)^{5/4}\right)}{8\sqrt[4]{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(-1 + x^4)^(1/4),x]
```

```
[Out] (x^3*(-1 + x^4)^(1/4)*(-(1 - x^4)^(5/4) + Hypergeometric2F1[-1/4, 3/4, 7/4,
 x^4]))/(8*(1 - x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.19, size = 57, normalized size = 1.00

$$\frac{3}{64}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{3}{64}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{32}\sqrt[4]{x^4-1}\left(4x^7-x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6*(-1 + x^4)^(1/4),x]
```

```
[Out] ((-1 + x^4)^(1/4)*(-x^3 + 4*x^7))/32 + (3*ArcTan[x/(-1 + x^4)^(1/4)])/64 -
(3*ArcTanh[x/(-1 + x^4)^(1/4)])/64
```

fricas [A] time = 0.41, size = 70, normalized size = 1.23

$$\frac{1}{32}\left(4x^7-x^3\right)\left(x^4-1\right)^{\frac{1}{4}}-\frac{3}{64}\arctan\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)-\frac{3}{128}\log\left(\frac{x+\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)+\frac{3}{128}\log\left(-\frac{x-\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^4-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/32*(4*x^7 - x^3)*(x^4 - 1)^(1/4) - 3/64*arctan((x^4 - 1)^(1/4)/x) - 3/128
*log((x + (x^4 - 1)^(1/4))/x) + 3/128*log(-(x - (x^4 - 1)^(1/4))/x)
```

giac [A] time = 0.30, size = 82, normalized size = 1.44

$$\frac{1}{32}\,x^{8}\left(\frac{\left(x^{4}-1\right)^{\frac{1}{4}}\left(\frac{1}{x^{4}}-1\right)}{x}-\frac{3\left(x^{4}-1\right)^{\frac{1}{4}}}{x}\right)+\frac{3}{64}\arctan\left(\frac{\left(x^{4}-1\right)^{\frac{1}{4}}}{x}\right)+\frac{3}{128}\log\left(\frac{\left(x^{4}-1\right)^{\frac{1}{4}}}{x}+1\right)-\frac{3}{128}\log\left(-\frac{\left(x^{4}-1\right)^{\frac{1}{4}}}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^4-1)^(1/4),x, algorithm="giac")
```

```
[Out] 1/32*x^8*((x^4 - 1)^(1/4)*(1/x^4 - 1)/x - 3*(x^4 - 1)^(1/4)/x) + 3/64*arcta
n((x^4 - 1)^(1/4)/x) + 3/128*log((x^4 - 1)^(1/4)/x + 1) - 3/128*log(-(x^4 -
 1)^(1/4)/x + 1)
```

maple [C] time = 0.25, size = 53, normalized size = 0.93

$$\frac{x^{3}\left(4x^{4}-1\right)\left(x^{4}-1\right)^{\frac{1}{4}}}{32}-\frac{\left(-\mathrm{signum}\left(x^{4}-1\right)\right)^{\frac{3}{4}}x^{3}\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],x^{4}\right)}{32\mathrm{signum}\left(x^{4}-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^4-1)^(1/4),x)
```

```
[Out] 1/32*x^3*(4*x^4-1)*(x^4-1)^(1/4)-1/32/signum(x^4-1)^(3/4)*(-signum(x^4-1))^
(3/4)*x^3*hypergeom([3/4,3/4],[7/4],x^4)
```

maxima [B] time = 0.43, size = 99, normalized size = 1.74

$$-\frac{\frac{3\left(x^{4}-1\right)^{\frac{1}{4}}}{x}+\frac{\left(x^{4}-1\right)^{\frac{5}{4}}}{x^{5}}}{32\left(\frac{2\left(x^{4}-1\right)}{x^{4}}-\frac{\left(x^{4}-1\right)^{2}}{x^{8}}-1\right)}-\frac{3}{64}\arctan\left(\frac{\left(x^{4}-1\right)^{\frac{1}{4}}}{x}\right)-\frac{3}{128}\log\left(\frac{\left(x^{4}-1\right)^{\frac{1}{4}}}{x}+1\right)+\frac{3}{128}\log\left(\frac{\left(x^{4}-1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^4-1)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/32*(3*(x^4 - 1)^(1/4)/x + (x^4 - 1)^(5/4)/x^5)/(2*(x^4 - 1)/x^4 - (x^4 -
 1)^2/x^8 - 1) - 3/64*arctan((x^4 - 1)^(1/4)/x) - 3/128*log((x^4 - 1)^(1/4)
/x + 1) + 3/128*log((x^4 - 1)^(1/4)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{6}\left(x^{4}-1\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^4 - 1)^(1/4),x)
```

```
[Out] int(x^6*(x^4 - 1)^(1/4), x)
```

sympy [C] time = 1.09, size = 32, normalized size = 0.56

$$\frac{x^{7}e^{\frac{i\pi}{4}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|x^{4}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(x**4-1)**(1/4),x)
```

```
[Out] x**7*exp(I*pi/4)*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), x**4)/(4*gamma(11/4
))
```

3.722 $\int \frac{(-1+2x)\sqrt[4]{x^3+x^4}}{x}\,dx$

Optimal. Leaf size=57

$$\frac{1}{4}\sqrt[4]{x^4+x^3}\,(4x-3)+\frac{7}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)-\frac{7}{8}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)$$

Rubi [A] time = 0.14, antiderivative size = 113, normalized size of antiderivative = 1.98, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2039, 2021, 2032, 63, 331, 298, 203, 206}

$$-\frac{7}{4}\sqrt[4]{x^4+x^3}+\frac{\left(x^4+x^3\right)^{5/4}}{x^3}+\frac{7(x+1)^{3/4}x^{9/4}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{8\left(x^4+x^3\right)^{3/4}}-\frac{7(x+1)^{3/4}x^{9/4}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{8\left(x^4+x^3\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + 2*x)*(x^3 + x^4)^(1/4))/x,x]
```

```
[Out] (-7*(x^3 + x^4)^(1/4))/4 + (x^3 + x^4)^(5/4)/x^3 + (7*x^(9/4)*(1 + x)^(3/4)
*ArcTan[x^(1/4)/(1 + x)^(1/4)])/(8*(x^3 + x^4)^(3/4)) - (7*x^(9/4)*(1 + x)^
(3/4)*ArcTanh[x^(1/4)/(1 + x)^(1/4)])/(8*(x^3 + x^4)^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+2x)\sqrt[4]{x^3+x^4}}{x}\,dx &= \frac{\left(x^3+x^4\right)^{5/4}}{x^3}-\frac{7}{4}\int \frac{\sqrt[4]{x^3+x^4}}{x}\,dx\\
&= -\frac{7}{4}\sqrt[4]{x^3+x^4}+\frac{\left(x^3+x^4\right)^{5/4}}{x^3}-\frac{7}{16}\int \frac{x^2}{\left(x^3+x^4\right)^{3/4}}\,dx\\
&= -\frac{7}{4}\sqrt[4]{x^3+x^4}+\frac{\left(x^3+x^4\right)^{5/4}}{x^3}-\frac{\left(7x^{9/4}(1+x)^{3/4}\right)\int \frac{1}{\sqrt[4]{x}\,(1+x)^{3/4}}\,dx}{16\left(x^3+x^4\right)^{3/4}}\\
&= -\frac{7}{4}\sqrt[4]{x^3+x^4}+\frac{\left(x^3+x^4\right)^{5/4}}{x^3}-\frac{\left(7x^{9/4}(1+x)^{3/4}\right)\mathrm{Subst}\left(\int \frac{x^2}{\left(1+x^4\right)^{3/4}}\,dx,x,\sqrt[4]{x}\right)}{4\left(x^3+x^4\right)^{3/4}}\\
&= -\frac{7}{4}\sqrt[4]{x^3+x^4}+\frac{\left(x^3+x^4\right)^{5/4}}{x^3}-\frac{\left(7x^{9/4}(1+x)^{3/4}\right)\mathrm{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{4\left(x^3+x^4\right)^{3/4}}\\
&= -\frac{7}{4}\sqrt[4]{x^3+x^4}+\frac{\left(x^3+x^4\right)^{5/4}}{x^3}-\frac{\left(7x^{9/4}(1+x)^{3/4}\right)\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{8\left(x^3+x^4\right)^{3/4}}+\frac{\left(7x^{9/4}(\right.}{}\\
&= -\frac{7}{4}\sqrt[4]{x^3+x^4}+\frac{\left(x^3+x^4\right)^{5/4}}{x^3}+\frac{7x^{9/4}(1+x)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{8\left(x^3+x^4\right)^{3/4}}-\frac{7x^{9/4}(1+x)^{3/4}\tanh^{-1}}{8\left(x^3+x^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.82

$$\frac{\sqrt[4]{x^3(x+1)}\left(3(x+1)^{5/4}-7\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-x\right)\right)}{3\sqrt[4]{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + 2*x)*(x^3 + x^4)^(1/4))/x,x]
```

```
[Out] ((x^3*(1 + x))^(1/4)*(3*(1 + x)^(5/4) - 7*Hypergeometric2F1[-1/4, 3/4, 7/4,
 -x]))/(3*(1 + x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.26, size = 57, normalized size = 1.00

$$\frac{1}{4}\sqrt[4]{x^4+x^3}\,(4x-3)+\frac{7}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)-\frac{7}{8}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + 2*x)*(x^3 + x^4)^(1/4))/x,x]
```

```
[Out] ((-3 + 4*x)*(x^3 + x^4)^(1/4))/4 + (7*ArcTan[x/(x^3 + x^4)^(1/4)])/8 - (7*A
rcTanh[x/(x^3 + x^4)^(1/4)])/8
```

fricas [A] time = 0.39, size = 72, normalized size = 1.26

$$\frac{1}{4}\left(x^4+x^3\right)^{\frac{1}{4}}(4x-3)-\frac{7}{8}\arctan\left(\frac{\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)-\frac{7}{16}\log\left(\frac{x+\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{7}{16}\log\left(-\frac{x-\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*(x^4+x^3)^(1/4)/x,x, algorithm="fricas")
```

```
[Out] 1/4*(x^4 + x^3)^(1/4)*(4*x - 3) - 7/8*arctan((x^4 + x^3)^(1/4)/x) - 7/16*lo
g((x + (x^4 + x^3)^(1/4))/x) + 7/16*log(-(x - (x^4 + x^3)^(1/4))/x)
```

giac [A] time = 0.58, size = 60, normalized size = 1.05

$$-\frac{1}{4}\left(3\left(\frac{1}{x}+1\right)^{\frac{5}{4}}-7\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right)x^2-\frac{7}{8}\arctan\left(\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right)-\frac{7}{16}\log\left(\left(\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right)+\frac{7}{16}\log\left(\left|\left(\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*(x^4+x^3)^(1/4)/x,x, algorithm="giac")
```

```
[Out] -1/4*(3*(1/x + 1)^(5/4) - 7*(1/x + 1)^(1/4))*x^2 - 7/8*arctan((1/x + 1)^(1/
4)) - 7/16*log((1/x + 1)^(1/4) + 1) + 7/16*log(abs((1/x + 1)^(1/4) - 1))
```

maple [C] time = 0.41, size = 373, normalized size = 6.54

$$\frac{(-3+4x)\left(x^3(1+x)\right)^{\frac{1}{4}}}{4}+\frac{\left(-\frac{7\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-2\sqrt{x^4+3x^3+3x^2+x}\,\mathrm{RootOf}\left(\_Z^2+1\right)x+2\,\mathrm{RootOf}\left(\_Z^2+1\right)x^3+2\left(x^4+3x^3+3x^2+x\right)^{\frac{3}{4}}-2\sqrt{x^4+3x^3+3x^2+x}\,\mathrm{RootOf}\left(\_Z^2+1\right)-2\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}x^2+5\,\mathrm{RootOf}\left(\_Z^2+1\right)x^2-4\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}x+4\,\mathrm{RootOf}\left(\_Z^2+1\right)x-2\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}+\mathrm{RootOf}\left(\_Z^2+1\right)}{(1+x)^2}\right)}{16}+\frac{7\ln\left(\frac{2\left(x^4+3x^3+3x^2+x\right)^{\frac{3}{4}}-2\sqrt{x^4+3x^3+3x^2+x}\,x+2\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}x^2-2x^3-2\sqrt{x^4+3x^3+3x^2+x}+4\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}x-5x^2+2\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}-4x-1}{(1+x)^2}\right)}{16}\right)\left(x^3(1+x)\right)^{\frac{1}{4}}\left(x(1+x)^3\right)^{\frac{1}{4}}}{x(1+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x)*(x^4+x^3)^(1/4)/x,x)
```

```
[Out] 1/4*(-3+4*x)*(x^3*(1+x))^(1/4)+(-7/16*RootOf(_Z^2+1)*ln((-2*(x^4+3*x^3+3*x^
2+x)^(1/2)*RootOf(_Z^2+1)*x+2*RootOf(_Z^2+1)*x^3+2*(x^4+3*x^3+3*x^2+x)^(3/4
)-2*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^2+1)-2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^
2+5*RootOf(_Z^2+1)*x^2-4*(x^4+3*x^3+3*x^2+x)^(1/4)*x+4*RootOf(_Z^2+1)*x-2*(
x^4+3*x^3+3*x^2+x)^(1/4)+RootOf(_Z^2+1))/(1+x)^2)+7/16*ln((2*(x^4+3*x^3+3*x
^2+x)^(3/4)-2*(x^4+3*x^3+3*x^2+x)^(1/2)*x+2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2-2
*x^3-2*(x^4+3*x^3+3*x^2+x)^(1/2)+4*(x^4+3*x^3+3*x^2+x)^(1/4)*x-5*x^2+2*(x^4
+3*x^3+3*x^2+x)^(1/4)-4*x-1)/(1+x)^2))*(x^3*(1+x))^(1/4)/x*(x*(1+x)^3)^(1/4
)/(1+x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3\right)^{\frac{1}{4}}(2\,x-1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*(x^4+x^3)^(1/4)/x,x, algorithm="maxima")
[Out] integrate((x^4 + x^3)^(1/4)*(2*x - 1)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x^4+x^3\right)^{1/4}\,(2\,x-1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + x^4)^(1/4)*(2*x - 1))/x,x)
[Out] int(((x^3 + x^4)^(1/4)*(2*x - 1))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\,(x+1)}\,(2x-1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*(x**4+x**3)**(1/4)/x,x)
[Out] Integral((x**3*(x + 1))**(1/4)*(2*x - 1)/x, x)
```

3.723 $\int \frac{\sqrt{-1+x^2-2x^3+x^4}\left(1-x^3+x^4\right)}{\left(-1-2x^3+x^4\right)\left(-2-x^2-4x^3+2x^4\right)}\,dx$

Optimal. Leaf size=57

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^4-2x^3+x^2-1}}\right)-\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt{x^4-2x^3+x^2-1}}\right)$$

Rubi [F] time = 1.75, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-1+x^2-2x^3+x^4}\left(1-x^3+x^4\right)}{\left(-1-2x^3+x^4\right)\left(-2-x^2-4x^3+2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-1 + x^2 - 2*x^3 + x^4]*(1 - x^3 + x^4))/((-1 - 2*x^3 + x^4)*(-2
- x^2 - 4*x^3 + 2*x^4)),x]
```

```
[Out] Defer[Int][Sqrt[-1 + x^2 - 2*x^3 + x^4]/(2 + x^2 + 4*x^3 - 2*x^4), x] + 3*D
efer[Int][(x*Sqrt[-1 + x^2 - 2*x^3 + x^4])/(-1 - 2*x^3 + x^4), x] - 2*Defer
[Int][(x^2*Sqrt[-1 + x^2 - 2*x^3 + x^4])/(-1 - 2*x^3 + x^4), x] - 6*Defer[I
nt][(x*Sqrt[-1 + x^2 - 2*x^3 + x^4])/(-2 - x^2 - 4*x^3 + 2*x^4), x] + 4*Def
er[Int][(x^2*Sqrt[-1 + x^2 - 2*x^3 + x^4])/(-2 - x^2 - 4*x^3 + 2*x^4), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x^2-2x^3+x^4}\left(1-x^3+x^4\right)}{\left(-1-2x^3+x^4\right)\left(-2-x^2-4x^3+2x^4\right)}\,dx &= \int\left(-\frac{x(-3+2x)\sqrt{-1+x^2-2x^3+x^4}}{-1-2x^3+x^4}+\frac{\left(-1-6x+4x^2\right)\sqrt{-1}}{-2-x^2-4x}\right. \\
&= -\int \frac{x(-3+2x)\sqrt{-1+x^2-2x^3+x^4}}{-1-2x^3+x^4}\,dx+\int \frac{\left(-1-6x+4x^2\right)}{-2-x^2} \\
&= -\int\left(-\frac{3x\sqrt{-1+x^2-2x^3+x^4}}{-1-2x^3+x^4}+\frac{2x^2\sqrt{-1+x^2-2x^3+x^4}}{-1-2x^3+x^4}\right) d \\
&= -\left(2\int \frac{x^2\sqrt{-1+x^2-2x^3+x^4}}{-1-2x^3+x^4}\,dx\right)+3\int \frac{x\sqrt{-1+x^2-2x^3+}}{-1-2x^3+x^4}
\end{aligned}$$

Mathematica [C] time = 6.68, size = 59573, normalized size = 1045.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[-1 + x^2 - 2*x^3 + x^4]*(1 - x^3 + x^4))/((-1 - 2*x^3 + x^4
)*(-2 - x^2 - 4*x^3 + 2*x^4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.43, size = 57, normalized size = 1.00

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^4-2x^3+x^2-1}}\right)-\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt{x^4-2x^3+x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^2 - 2*x^3 + x^4]*(1 - x^3 + x^4))/((-1 - 2*
x^3 + x^4)*(-2 - x^2 - 4*x^3 + 2*x^4)),x]
```

```
[Out] ArcTanh[x/Sqrt[-1 + x^2 - 2*x^3 + x^4]] - Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)/S
qrt[-1 + x^2 - 2*x^3 + x^4]]
```

fricas [B] time = 0.52, size = 180, normalized size = 3.16

$$\frac{1}{8}\sqrt{3}\sqrt{2}\log\left(-\frac{4x^8-16x^7+60x^6-88x^5+41x^4+16x^3-4\sqrt{3}\sqrt{2}\left(2x^5-4x^4+5x^3-2x\right)\sqrt{x^4-2x^3+x^2-1}-44x^2+4}{4x^8-16x^7+12x^6+8x^5-7x^4+16x^3+4x^2+4}\right)+\frac{1}{2}\log\left(-\frac{x^4-2x^3+2x^2+2\sqrt{x^4-2x^3+x^2-1}x-1}{x^4-2x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+x^2-1)^(1/2)*(x^4-x^3+1)/(x^4-2*x^3-1)/(2*x^4-4*x^3-x^
2-2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(3)*sqrt(2)*log(-(4*x^8 - 16*x^7 + 60*x^6 - 88*x^5 + 41*x^4 + 16*x^
3 - 4*sqrt(3)*sqrt(2)*(2*x^5 - 4*x^4 + 5*x^3 - 2*x)*sqrt(x^4 - 2*x^3 + x^2
- 1) - 44*x^2 + 4)/(4*x^8 - 16*x^7 + 12*x^6 + 8*x^5 - 7*x^4 + 16*x^3 + 4*x^
2 + 4)) + 1/2*log(-(x^4 - 2*x^3 + 2*x^2 + 2*sqrt(x^4 - 2*x^3 + x^2 - 1)*x -
 1)/(x^4 - 2*x^3 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^3+1\right)\sqrt{x^4-2x^3+x^2-1}}{\left(2x^4-4x^3-x^2-2\right)\left(x^4-2x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+x^2-1)^(1/2)*(x^4-x^3+1)/(x^4-2*x^3-1)/(2*x^4-4*x^3-x^
2-2),x, algorithm="giac")
```

```
[Out] integrate((x^4 - x^3 + 1)*sqrt(x^4 - 2*x^3 + x^2 - 1)/((2*x^4 - 4*x^3 - x^2
 - 2)*(x^4 - 2*x^3 - 1)), x)
```

maple [C] time = 1.02, size = 1574, normalized size = 27.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^3+x^2-1)^(1/2)*(x^4-x^3+1)/(x^4-2*x^3-1)/(2*x^4-4*x^3-x^2-2),x
)
```

```
[Out] -1/3*I*(-1/2*I*3^(1/2)-1/2*5^(1/2))*((1/2*5^(1/2)-1/2*I*3^(1/2))*(x-1/2+1/2
*I*3^(1/2))/(1/2*5^(1/2)+1/2*I*3^(1/2))/(x-1/2-1/2*I*3^(1/2)))^(1/2)*(x-1/2
-1/2*I*3^(1/2))^2*(I*3^(1/2)*(x-1/2+1/2*5^(1/2))/(-1/2*5^(1/2)+1/2*I*3^(1/2
))/(x-1/2-1/2*I*3^(1/2)))^(1/2)*(I*3^(1/2)*(x-1/2-1/2*5^(1/2))/(1/2*5^(1/2)
+1/2*I*3^(1/2))/(x-1/2-1/2*I*3^(1/2)))^(1/2)/(1/2*5^(1/2)-1/2*I*3^(1/2))*3^
(1/2)/((x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2))*(x-1/2+1/2*5^(1/2))*(x-1
/2-1/2*5^(1/2)))^(1/2)*EllipticF(((1/2*5^(1/2)-1/2*I*3^(1/2))*(x-1/2+1/2*I*
3^(1/2))/(1/2*5^(1/2)+1/2*I*3^(1/2))/(x-1/2-1/2*I*3^(1/2)))^(1/2),((1/2*5^(
1/2)+1/2*I*3^(1/2))*(-1/2*I*3^(1/2)-1/2*5^(1/2))/(1/2*5^(1/2)-1/2*I*3^(1/2)
)/(-1/2*5^(1/2)+1/2*I*3^(1/2)))^(1/2))-1/96*I*16^(1/2)*sum(_alpha*(-I*3^(1/
2)-5^(1/2))*((5^(1/2)-I*3^(1/2))*(-1+2*x+I*3^(1/2))/(5^(1/2)+I*3^(1/2))/(-1
+2*x-I*3^(1/2)))^(1/2)*(-1+2*x-I*3^(1/2))^2*(2*I*(2*x-1+5^(1/2))/(I*3^(1/2)
-5^(1/2))/(-1+2*x-I*3^(1/2)))^(1/2)*(2*I*(-1+2*x-5^(1/2))/(5^(1/2)+I*3^(1/2
))/(-1+2*x-I*3^(1/2)))^(1/2)/(5^(1/2)-I*3^(1/2))/((-1+2*x+I*3^(1/2))*(-1+2*
x-I*3^(1/2))*(2*x-1+5^(1/2))*(-1+2*x-5^(1/2)))^(1/2)*(2*_alpha^3-6*_alpha^2
-_alpha+7+I*3^(1/2)*(2*_alpha^3-2*_alpha^2-5*_alpha-3))*(6*EllipticF(((1/2*
5^(1/2)-1/2*I*3^(1/2))*(x-1/2+1/2*I*3^(1/2))/(1/2*5^(1/2)+1/2*I*3^(1/2))/(x
-1/2-1/2*I*3^(1/2)))^(1/2),((1/2*5^(1/2)+1/2*I*3^(1/2))*(-1/2*I*3^(1/2)-1/2
```

```
*5^(1/2))/(1/2*5^(1/2)-1/2*I*3^(1/2))/(-1/2*5^(1/2)+1/2*I*3^(1/2)))^(1/2))+
I*3^(1/2)*(2*_alpha^3-6*_alpha^2-_alpha+7+I*3^(1/2)*(-2*_alpha^3+2*_alpha^2
+5*_alpha+3))*EllipticPi(((1/2*5^(1/2)-1/2*I*3^(1/2))*(x-1/2+1/2*I*3^(1/2))
/(1/2*5^(1/2)+1/2*I*3^(1/2))/(x-1/2-1/2*I*3^(1/2)))^(1/2),-1/8-1/4*_alpha^2
+1/4*_alpha^3+1/4*I*5^(1/2)*3^(1/2)*_alpha^3-5/8*_alpha-1/24*I*3^(1/2)*_alp
ha-1/8*I*5^(1/2)*3^(1/2)+1/12*I*3^(1/2)*_alpha^3-1/4*5^(1/2)*_alpha^3+3/4*5
^(1/2)*_alpha^2+1/8*_alpha*5^(1/2)-1/4*I*3^(1/2)*_alpha^2+7/24*I*3^(1/2)-1/
4*I*5^(1/2)*3^(1/2)*_alpha^2-5/8*I*5^(1/2)*3^(1/2)*_alpha-7/8*5^(1/2),((1/2
*5^(1/2)+1/2*I*3^(1/2))*(-1/2*I*3^(1/2)-1/2*5^(1/2))/(1/2*5^(1/2)-1/2*I*3^(
1/2))/(-1/2*5^(1/2)+1/2*I*3^(1/2)))^(1/2))),_alpha=RootOf(2*_Z^4-4*_Z^3-_Z^
2-2))+1/16*I*16^(1/2)*sum(_alpha*(-I*3^(1/2)-5^(1/2))*((5^(1/2)-I*3^(1/2))*
(-1+2*x+I*3^(1/2))/(5^(1/2)+I*3^(1/2))/(-1+2*x-I*3^(1/2)))^(1/2)*(-1+2*x-I*
3^(1/2))^2*(2*I*(2*x-1+5^(1/2))/(I*3^(1/2)-5^(1/2))/(-1+2*x-I*3^(1/2)))^(1/
2)*(2*I*(-1+2*x-5^(1/2))/(5^(1/2)+I*3^(1/2))/(-1+2*x-I*3^(1/2)))^(1/2)/(5^(
1/2)-I*3^(1/2))/((-1+2*x+I*3^(1/2))*(-1+2*x-I*3^(1/2))*(2*x-1+5^(1/2))*(-1+
2*x-5^(1/2)))^(1/2)*(_alpha^3-3*_alpha^2+3+I*3^(1/2)*(_alpha^3-_alpha^2-2*_
alpha-1))*(2*EllipticF(((1/2*5^(1/2)-1/2*I*3^(1/2))*(x-1/2+1/2*I*3^(1/2))/(
1/2*5^(1/2)+1/2*I*3^(1/2))/(x-1/2-1/2*I*3^(1/2)))^(1/2),((1/2*5^(1/2)+1/2*I
*3^(1/2))*(-1/2*I*3^(1/2)-1/2*5^(1/2))/(1/2*5^(1/2)-1/2*I*3^(1/2))/(-1/2*5^
(1/2)+1/2*I*3^(1/2)))^(1/2))+I*3^(1/2)*(_alpha^3-3*_alpha^2+3+I*3^(1/2)*(-_
alpha^3+_alpha^2+2*_alpha+1))*EllipticPi(((1/2*5^(1/2)-1/2*I*3^(1/2))*(x-1/
2+1/2*I*3^(1/2))/(1/2*5^(1/2)+1/2*I*3^(1/2))/(x-1/2-1/2*I*3^(1/2)))^(1/2),1
/8*I*_alpha^3*3^(1/2)-9/8*5^(1/2)-3/8*5^(1/2)*_alpha^3+3/8*I*3^(1/2)+9/8*5^
(1/2)*_alpha^2-3/4*_alpha-3/8*I*_alpha^2*3^(1/2)+3/8*I*_alpha^3*5^(1/2)*3^(
1/2)-1/8*I*5^(1/2)*3^(1/2)-3/8*I*_alpha^2*5^(1/2)*3^(1/2)-3/4*I*_alpha*5^(1
/2)*3^(1/2)+3/8*_alpha^3-1/8-3/8*_alpha^2,((1/2*5^(1/2)+1/2*I*3^(1/2))*(-1/
2*I*3^(1/2)-1/2*5^(1/2))/(1/2*5^(1/2)-1/2*I*3^(1/2))/(-1/2*5^(1/2)+1/2*I*3^
(1/2)))^(1/2))),_alpha=RootOf(_Z^4-2*_Z^3-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4 - x^3 + 1\right)\sqrt{x^4 - 2\,x^3 + x^2 - 1}}{\left(2\,x^4 - 4\,x^3 - x^2 - 2\right)\left(x^4 - 2\,x^3 - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+x^2-1)^(1/2)*(x^4-x^3+1)/(x^4-2*x^3-1)/(2*x^4-4*x^3-x^
2-2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^3 + 1)*sqrt(x^4 - 2*x^3 + x^2 - 1)/((2*x^4 - 4*x^3 - x^2
 - 2)*(x^4 - 2*x^3 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x^4 - x^3 + 1\right)\,\sqrt{x^4 - 2\,x^3 + x^2 - 1}}{\left(-x^4 + 2\,x^3 + 1\right)\,\left(-2\,x^4 + 4\,x^3 + x^2 + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - x^3 + 1)*(x^2 - 2*x^3 + x^4 - 1)^(1/2))/((2*x^3 - x^4 + 1)*(x^2
 + 4*x^3 - 2*x^4 + 2)),x)
```

```
[Out] int(((x^4 - x^3 + 1)*(x^2 - 2*x^3 + x^4 - 1)^(1/2))/((2*x^3 - x^4 + 1)*(x^2
 + 4*x^3 - 2*x^4 + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**3+x**2-1)**(1/2)*(x**4-x**3+1)/(x**4-2*x**3-1)/(2*x**4
-4*x**3-x**2-2),x)
```

```
[Out] Timed out
```

3.724 $\int \frac{x^2}{\left(b+ax^4\right)^{3/4}} dx$

Optimal. Leaf size=57

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2a^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2a^{3/4}}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {331, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2a^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b + a*x^4)^(3/4),x]
```

```
[Out] -1/2*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)]/a^(3/4) + ArcTanh[(a^(1/4)*x)/(b
 + a*x^4)^(1/4)]/(2*a^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{x^2}{\left(b+ax^4\right)^{3/4}}\,dx = \text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{2\sqrt{a}} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{2\sqrt{a}}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{2a^{3/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{2a^{3/4}}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 0.88

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right) - \tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b + a*x^4)^(3/4),x]
```

```
[Out] (-ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)] + ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(
1/4)])/(2*a^(3/4))
```

IntegrateAlgebraic [A] time = 0.28, size = 57, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2a^{3/4}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(b + a*x^4)^(3/4),x]
```

```
[Out] -1/2*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)]/a^(3/4) + ArcTanh[(a^(1/4)*x)/(b
 + a*x^4)^(1/4)]/(2*a^(3/4))
```

fricas [B] time = 0.41, size = 134, normalized size = 2.35

$$-\frac{1}{a^3}^{\frac{1}{4}} \arctan\left(\frac{a^2 \frac{1}{a^3}^{\frac{3}{4}} x \sqrt{\frac{a^2 \sqrt{\frac{1}{a^3}} x^2 + \sqrt{ax^4+b}}{x^2}} - \left(ax^4+b\right)^{\frac{1}{4}} a^2 \frac{1}{a^3}^{\frac{3}{4}}}{x}\right) + \frac{1}{4} \frac{1}{a^3}^{\frac{1}{4}} \log\left(\frac{a \frac{1}{a^3}^{\frac{1}{4}} x + \left(ax^4+b\right)^{\frac{1}{4}}}{x}\right) - \frac{1}{4} \frac{1}{a^3}^{\frac{1}{4}} \log\left(-\frac{a \frac{1}{a^3}^{\frac{1}{4}} x - \left(ax^4+b\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+b)^(3/4),x, algorithm="fricas")
```

```
[Out] -(a^(-3))^(1/4)*arctan((a^2*(a^(-3))^(3/4)*x*sqrt((a^2*sqrt(a^(-3))*x^2 + s
qrt(a*x^4 + b))/x^2) - (a*x^4 + b)^(1/4)*a^2*(a^(-3))^(3/4))/x) + 1/4*(a^(-
3))^(1/4)*log((a*(a^(-3))^(1/4)*x + (a*x^4 + b)^(1/4))/x) - 1/4*(a^(-3))^(1
/4)*log(-(a*(a^(-3))^(1/4)*x - (a*x^4 + b)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax^4+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+b)^(3/4),x, algorithm="giac")
[Out] integrate(x^2/(a*x^4 + b)^(3/4), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a\,x^4+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^4+b)^(3/4),x)
[Out] int(x^2/(a*x^4+b)^(3/4),x)
```

maxima [A] time = 0.42, size = 68, normalized size = 1.19

$$\frac{\arctan\left(\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}\,x}\right)}{2\,a^{\frac{3}{4}}}-\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}\right)}{4\,a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+b)^(3/4),x, algorithm="maxima")
[Out] 1/2*arctan((a*x^4 + b)^(1/4)/(a^(1/4)*x))/a^(3/4) - 1/4*log(-(a^(1/4) - (a*
x^4 + b)^(1/4)/x)/(a^(1/4) + (a*x^4 + b)^(1/4)/x))/a^(3/4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(a\,x^4+b\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b + a*x^4)^(3/4),x)
[Out] int(x^2/(b + a*x^4)^(3/4), x)
```

sympy [C] time = 0.88, size = 37, normalized size = 0.65

$$\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{ax^4e^{i\pi}}{b}\right)}{4b^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**4+b)**(3/4),x)
[Out] x**3*gamma(3/4)*hyper((3/4, 3/4), (7/4,), a*x**4*exp_polar(I*pi)/b)/(4*b**(
3/4)*gamma(7/4))
```

3.725 $\int \frac{-b+ax^8}{\sqrt[4]{b+ax^8}\left(b-cx^4+ax^8\right)} \, dx$

Optimal. Leaf size=57

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^8+b}}\right)}{2\sqrt[4]{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^8+b}}\right)}{2\sqrt[4]{c}}$$

Rubi [C] time = 1.48, antiderivative size = 461, normalized size of antiderivative = 8.09, number of steps used = 18, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6728, 246, 245, 1438, 430, 429, 511, 510}

$$-\frac{x\sqrt[4]{\frac{ax^8}{b}+1}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};-\frac{2a^2x^8}{2ab-c\left(c-\sqrt{c^2-4ab}\right)},-\frac{ax^8}{b}\right)}{\sqrt[4]{ax^8+b}}-\frac{x\sqrt[4]{\frac{ax^8}{b}+1}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};-\frac{2a^2x^8}{2ab-c\left(c+\sqrt{c^2-4ab}\right)},-\frac{ax^8}{b}\right)}{\sqrt[4]{ax^8+b}}+\frac{ax^5\left(c-\sqrt{c^2-4ab}\right)\sqrt[4]{\frac{ax^8}{b}+1}\,F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};-\frac{2a^2x^8}{2ab-c\left(c-\sqrt{c^2-4ab}\right)},-\frac{ax^8}{b}\right)}{5\left(2ab-c\left(c-\sqrt{c^2-4ab}\right)\right)\sqrt[4]{ax^8+b}}+\frac{ax^5\left(\sqrt{c^2-4ab}+c\right)\sqrt[4]{\frac{ax^8}{b}+1}\,F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};-\frac{2a^2x^8}{2ab-c\left(c+\sqrt{c^2-4ab}\right)},-\frac{ax^8}{b}\right)}{5\left(2ab-c\left(\sqrt{c^2-4ab}+c\right)\right)\sqrt[4]{ax^8+b}}+\frac{x\sqrt[4]{\frac{ax^8}{b}+1}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{ax^8}{b}\right)}{\sqrt[4]{ax^8+b}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-b + a*x^8)/((b + a*x^8)^(1/4)*(b - c*x^4 + a*x^8)),x]
```

```
[Out] -((x*(1 + (a*x^8)/b)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, (-2*a^2*x^8)/(2*a*b -
 c*(c - Sqrt[-4*a*b + c^2])), -((a*x^8)/b)])/(b + a*x^8)^(1/4)) - (x*(1 + (
a*x^8)/b)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, (-2*a^2*x^8)/(2*a*b - c*(c + Sqr
t[-4*a*b + c^2])), -((a*x^8)/b)])/(b + a*x^8)^(1/4) + (a*(c - Sqrt[-4*a*b +
 c^2])*x^5*(1 + (a*x^8)/b)^(1/4)*AppellF1[5/8, 1, 1/4, 13/8, (-2*a^2*x^8)/(
2*a*b - c*(c - Sqrt[-4*a*b + c^2])), -((a*x^8)/b)])/(5*(2*a*b - c*(c - Sqrt
[-4*a*b + c^2]))*(b + a*x^8)^(1/4)) + (a*(c + Sqrt[-4*a*b + c^2])*x^5*(1 +
(a*x^8)/b)^(1/4)*AppellF1[5/8, 1, 1/4, 13/8, (-2*a^2*x^8)/(2*a*b - c*(c + S
qrt[-4*a*b + c^2])), -((a*x^8)/b)])/(5*(2*a*b - c*(c + Sqrt[-4*a*b + c^2]))
*(b + a*x^8)^(1/4)) + (x*(1 + (a*x^8)/b)^(1/4)*Hypergeometric2F1[1/8, 1/4,
9/8, -((a*x^8)/b)])/(b + a*x^8)^(1/4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-b+ax^8}{\sqrt[4]{b+ax^8}\left(b-cx^4+ax^8\right)}dx = \int \left(\frac{1}{\sqrt[4]{b+ax^8}} - \frac{2b-cx^4}{\sqrt[4]{b+ax^8}\left(b-cx^4+ax^8\right)}\right)dx$$

$$= \int \frac{1}{\sqrt[4]{b+ax^8}}dx - \int \frac{2b-cx^4}{\sqrt[4]{b+ax^8}\left(b-cx^4+ax^8\right)}dx$$

$$= \frac{\sqrt[4]{1+\frac{ax^8}{b}}\int \frac{1}{\sqrt[4]{1+\frac{ax^8}{b}}}dx}{\sqrt[4]{b+ax^8}} - \int \left(\frac{-c-\sqrt{-4ab+c^2}}{\left(-c-\sqrt{-4ab+c^2}+2ax^4\right)\sqrt[4]{b+ax^8}} + \frac{}{\left(-c+\sqrt{}\right.}\right.$$

$$= \frac{x\sqrt[4]{1+\frac{ax^8}{b}}\ {}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{ax^8}{b}\right)}{\sqrt[4]{b+ax^8}} - \left(-c-\sqrt{-4ab+c^2}\right)\int \frac{1}{\left(-c-\sqrt{-4ab+c^2}+2\right.}$$

$$= \frac{x\sqrt[4]{1+\frac{ax^8}{b}}\ {}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{ax^8}{b}\right)}{\sqrt[4]{b+ax^8}} - \left(-c-\sqrt{-4ab+c^2}\right)\int \left(\frac{-c-\sqrt{}}{2\sqrt[4]{b+ax^8}\left(-2ab+c\right.}\right.$$

$$= \frac{x\sqrt[4]{1+\frac{ax^8}{b}}\ {}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{ax^8}{b}\right)}{\sqrt[4]{b+ax^8}} + \left(a\left(c-\sqrt{-4ab+c^2}\right)\right)\int \frac{}{\sqrt[4]{b+ax^8}\left(2ab-c^2\right.}$$

$$= \frac{x\sqrt[4]{1+\frac{ax^8}{b}}\ {}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{ax^8}{b}\right)}{\sqrt[4]{b+ax^8}} + \frac{\left(a\left(c-\sqrt{-4ab+c^2}\right)\sqrt[4]{1+\frac{ax^8}{b}}\right)\int \frac{}{\left(2ab-c^2+c\sqrt{-}\right.}}{\sqrt[4]{b+ax^8}}$$

$$= -\frac{x\sqrt[4]{1+\frac{ax^8}{b}}\ F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};-\frac{2a^2x^8}{2ab-c\left(c-\sqrt{-4ab+c^2}\right)},-\frac{ax^8}{b}\right)}{\sqrt[4]{b+ax^8}} - \frac{x\sqrt[4]{1+\frac{ax^8}{b}}\ F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\right.}{\sqrt[4]{b-}}$$

Mathematica [F] time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{-b+ax^8}{\sqrt[4]{b+ax^8}\left(b-cx^4+ax^8\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-b + a*x^8)/((b + a*x^8)^(1/4)*(b - c*x^4 + a*x^8)),x]
[Out] Integrate[(-b + a*x^8)/((b + a*x^8)^(1/4)*(b - c*x^4 + a*x^8)), x]
```

IntegrateAlgebraic [A] time = 11.60, size = 57, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^8+b}}\right)}{2\sqrt[4]{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^8+b}}\right)}{2\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^8)/((b + a*x^8)^(1/4)*(b - c*x^4 + a*x^8)),x]
[Out] -1/2*ArcTan[(c^(1/4)*x)/(b + a*x^8)^(1/4)]/c^(1/4) - ArcTanh[(c^(1/4)*x)/(b
 + a*x^8)^(1/4)]/(2*c^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-b)/(a*x^8+b)^(1/4)/(a*x^8-c*x^4+b),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^8 - b}{\left(ax^8 - cx^4 + b\right)\left(ax^8 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-b)/(a*x^8+b)^(1/4)/(a*x^8-c*x^4+b),x, algorithm="giac")
[Out] integrate((a*x^8 - b)/((a*x^8 - c*x^4 + b)*(a*x^8 + b)^(1/4)), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{a\,x^8 - b}{\left(a\,x^8 + b\right)^{\frac{1}{4}}\left(a\,x^8 - c\,x^4 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^8-b)/(a*x^8+b)^(1/4)/(a*x^8-c*x^4+b),x)
[Out] int((a*x^8-b)/(a*x^8+b)^(1/4)/(a*x^8-c*x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^8 - b}{\left(ax^8 - cx^4 + b\right)\left(ax^8 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-b)/(a*x^8+b)^(1/4)/(a*x^8-c*x^4+b),x, algorithm="maxima")
[Out] integrate((a*x^8 - b)/((a*x^8 - c*x^4 + b)*(a*x^8 + b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{b - a\,x^8}{\left(a\,x^8 + b\right)^{1/4}\,\left(a\,x^8 - c\,x^4 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^8)/((b + a*x^8)^(1/4)*(b + a*x^8 - c*x^4)),x)
[Out] int(-(b - a*x^8)/((b + a*x^8)^(1/4)*(b + a*x^8 - c*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**8-b)/(a*x**8+b)**(1/4)/(a*x**8-c*x**4+b),x)
[Out] Timed out
```

3.726 $\displaystyle \int \frac{(-3+2x^5)(1+2x^5+x^6+x^{10})}{x^6(1-x^3+x^5)\sqrt[4]{x+x^6}}\,dx$

Optimal. Leaf size=57

$$-4\tan^{-1}\left(\frac{x}{\sqrt[4]{x^6+x}}\right)-4\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^6+x}}\right)+\frac{4\left(x^6+x\right)^{3/4}\left(3x^5+7x^3+3\right)}{21x^6}$$

Rubi [F] time = 2.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-3+2x^5\right)\left(1+2x^5+x^6+x^{10}\right)}{x^6\left(1-x^3+x^5\right)\sqrt[4]{x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + 2*x^5)*(1 + 2*x^5 + x^6 + x^10))/(x^6*(1 - x^3 + x^5)*(x + x^6)^
(1/4)),x]
```

```
[Out] (4*(1 + x^5)^(1/4)*Hypergeometric2F1[-21/20, 1/4, -1/20, -x^5])/(7*x^5*(x +
 x^6)^(1/4)) + (4*(1 + x^5)^(1/4)*Hypergeometric2F1[-9/20, 1/4, 11/20, -x^5
])/(3*x^2*(x + x^6)^(1/4)) + (4*(1 + x^5)^(1/4)*Hypergeometric2F1[-1/20, 1/
4, 19/20, -x^5])/(x + x^6)^(1/4) + (16*x*(1 + x^5)^(1/4)*Hypergeometric2F1[
3/20, 1/4, 23/20, -x^5])/(3*(x + x^6)^(1/4)) + (8*x^3*(1 + x^5)^(1/4)*Hyper
geometric2F1[1/4, 11/20, 31/20, -x^5])/(11*(x + x^6)^(1/4)) + (8*x^5*(1 + x
^5)^(1/4)*Hypergeometric2F1[1/4, 19/20, 39/20, -x^5])/(19*(x + x^6)^(1/4))
- (40*x^(1/4)*(1 + x^5)^(1/4)*Defer[Subst][Defer[Int][x^2/((1 + x^20)^(1/4)
*(1 - x^12 + x^20)), x], x, x^(1/4)])/(x + x^6)^(1/4) + (16*x^(1/4)*(1 + x^
5)^(1/4)*Defer[Subst][Defer[Int][x^14/((1 + x^20)^(1/4)*(1 - x^12 + x^20)),
 x], x, x^(1/4)])/(x + x^6)^(1/4)
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-3+2x^5\right)\left(1+2x^5+x^6+x^{10}\right)}{x^6\left(1-x^3+x^5\right)\sqrt[4]{x+x^6}}\,dx &= \frac{\left(\sqrt[4]{x}\sqrt[4]{1+x^5}\right)\int \frac{\left(-3+2x^5\right)\left(1+2x^5+x^6+x^{10}\right)}{x^{25/4}\sqrt[4]{1+x^5}\left(1-x^3+x^5\right)}\,dx}{\sqrt[4]{x+x^6}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x^5}\right)\operatorname{Subst}\left(\int \frac{\left(-3+2x^{20}\right)\left(1+2x^{20}+x^{24}+x^{40}\right)}{x^{22}\sqrt[4]{1+x^{20}}\left(1-x^{12}+x^{20}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{x+x^6}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x^5}\right)\operatorname{Subst}\left(\int\left(-\frac{3}{x^{22}\sqrt[4]{1+x^{20}}}-\frac{3}{x^{10}\sqrt[4]{1+x^{20}}}-\frac{1}{x^2\sqrt[4]{1+x^{20}}}+\frac{4x^2}{\sqrt[4]{1+x^{20}}}\right.\right.}{\sqrt[4]{x+x^6}}\\
&= -\frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x^5}\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt[4]{1+x^{20}}}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{x+x^6}}+\frac{\left(8\sqrt[4]{x}\sqrt[4]{1+x^5}\right)\operatorname{Subs}}{\sqrt[4]{x}}\\
&= \frac{4\sqrt[4]{1+x^5}\,{}_2F_1\left(-\frac{21}{20},\frac{1}{4};-\frac{1}{20};-x^5\right)}{7x^5\sqrt[4]{x+x^6}}+\frac{4\sqrt[4]{1+x^5}\,{}_2F_1\left(-\frac{9}{20},\frac{1}{4};\frac{11}{20};-x^5\right)}{3x^2\sqrt[4]{x+x^6}}+\frac{4}{}\\
&= \frac{4\sqrt[4]{1+x^5}\,{}_2F_1\left(-\frac{21}{20},\frac{1}{4};-\frac{1}{20};-x^5\right)}{7x^5\sqrt[4]{x+x^6}}+\frac{4\sqrt[4]{1+x^5}\,{}_2F_1\left(-\frac{9}{20},\frac{1}{4};\frac{11}{20};-x^5\right)}{3x^2\sqrt[4]{x+x^6}}+\frac{4}{}
\end{aligned}$$

Mathematica [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{(-3+2x^5)(1+2x^5+x^6+x^{10})}{x^6(1-x^3+x^5)\sqrt[4]{x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + 2*x^5)*(1 + 2*x^5 + x^6 + x^10))/(x^6*(1 - x^3 + x^5)*(x +
 x^6)^(1/4)),x]
```

```
[Out] Integrate[((-3 + 2*x^5)*(1 + 2*x^5 + x^6 + x^10))/(x^6*(1 - x^3 + x^5)*(x +
 x^6)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 2.76, size = 57, normalized size = 1.00

$$-4\tan^{-1}\left(\frac{x}{\sqrt[4]{x^6+x}}\right)-4\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^6+x}}\right)+\frac{4\left(x^6+x\right)^{3/4}\left(3x^5+7x^3+3\right)}{21x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + 2*x^5)*(1 + 2*x^5 + x^6 + x^10))/(x^6*(1 - x^3 +
x^5)*(x + x^6)^(1/4)),x]
```

```
[Out] (4*(3 + 7*x^3 + 3*x^5)*(x + x^6)^(3/4))/(21*x^6) - 4*ArcTan[x/(x + x^6)^(1/
4)] - 4*ArcTanh[x/(x + x^6)^(1/4)]
```

fricas [B] time = 75.97, size = 124, normalized size = 2.18

$$-\frac{2\left(21\,x^6\arctan\left(\frac{2\left(\left(x^6+x\right)^{\frac{1}{4}}x^2+\left(x^6+x\right)^{\frac{3}{4}}\right)}{x^5-x^3+1}\right)-21\,x^6\log\left(\frac{x^5+x^3-2\left(x^6+x\right)^{\frac{1}{4}}x^2+2\sqrt{x^6+x}\,x-2\left(x^6+x\right)^{\frac{3}{4}}+1}{x^5-x^3+1}\right)-2\left(x^6+x\right)^{\frac{3}{4}}\left(3\,x^5+7\,x^3+3\right)\right)}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^5-3)*(x^10+x^6+2*x^5+1)/x^6/(x^5-x^3+1)/(x^6+x)^(1/4),x, alg
orithm="fricas")
```

```
[Out] -2/21*(21*x^6*arctan(2*((x^6 + x)^(1/4)*x^2 + (x^6 + x)^(3/4))/(x^5 - x^3 +
 1)) - 21*x^6*log((x^5 + x^3 - 2*(x^6 + x)^(1/4)*x^2 + 2*sqrt(x^6 + x)*x -
2*(x^6 + x)^(3/4) + 1)/(x^5 - x^3 + 1)) - 2*(x^6 + x)^(3/4)*(3*x^5 + 7*x^3
+ 3))/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{10}+x^6+2\,x^5+1\right)\left(2\,x^5-3\right)}{\left(x^6+x\right)^{\frac{1}{4}}\left(x^5-x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^5-3)*(x^10+x^6+2*x^5+1)/x^6/(x^5-x^3+1)/(x^6+x)^(1/4),x, alg
orithm="giac")
```

```
[Out] integrate((x^10 + x^6 + 2*x^5 + 1)*(2*x^5 - 3)/((x^6 + x)^(1/4)*(x^5 - x^3
+ 1)*x^6), x)
```

maple [C] time = 4.39, size = 185, normalized size = 3.25

$$\frac{\frac{4}{7}x^{10}+\frac{8}{7}x^5+\frac{4}{3}x^8+\frac{4}{3}x^3+\frac{4}{7}}{x^5\left(x\left(x^5+1\right)\right)^{\frac{1}{4}}}-2\ln\left(-\frac{x^5+2\left(x^6+x\right)^{\frac{3}{4}}+2x\sqrt{x^6+x}+2\left(x^6+x\right)^{\frac{1}{4}}x^2+x^3+1}{x^5-x^3+1}\right)+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^2+1\right)x^5+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{x^6+x}\,x-\mathrm{RootOf}\left(\_Z^2+1\right)x^3+2\left(x^6+x\right)^{\frac{3}{4}}-2\left(x^6+x\right)^{\frac{1}{4}}x^2-\mathrm{RootOf}\left(\_Z^2+1\right)}{x^5-x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^5-3)*(x^10+x^6+2*x^5+1)/x^6/(x^5-x^3+1)/(x^6+x)^(1/4),x)
```

```
[Out] 4/21*(3*x^10+7*x^8+6*x^5+7*x^3+3)/x^5/(x*(x^5+1))^(1/4)-2*ln(-(x^5+2*(x^6+x
)^(3/4)+2*x*(x^6+x)^(1/2)+2*(x^6+x)^(1/4)*x^2+x^3+1)/(x^5-x^3+1))+2*RootOf(
_Z^2+1)*ln(-(-RootOf(_Z^2+1)*x^5+2*RootOf(_Z^2+1)*(x^6+x)^(1/2)*x-RootOf(_Z
^2+1)*x^3+2*(x^6+x)^(3/4)-2*(x^6+x)^(1/4)*x^2-RootOf(_Z^2+1))/(x^5-x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{10}+x^6+2\,x^5+1\right)\left(2\,x^5-3\right)}{\left(x^6+x\right)^{\frac{1}{4}}\left(x^5-x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^5-3)*(x^10+x^6+2*x^5+1)/x^6/(x^5-x^3+1)/(x^6+x)^(1/4),x, alg
orithm="maxima")
```

```
[Out] integrate((x^10 + x^6 + 2*x^5 + 1)*(2*x^5 - 3)/((x^6 + x)^(1/4)*(x^5 - x^3
+ 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(2\,x^5-3\right)\,\left(x^{10}+x^6+2\,x^5+1\right)}{x^6\,{\left(x^6+x\right)}^{1/4}\,\left(x^5-x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^5 - 3)*(2*x^5 + x^6 + x^10 + 1))/(x^6*(x + x^6)^(1/4)*(x^5 - x^3
+ 1)),x)
```

```
[Out] int(((2*x^5 - 3)*(2*x^5 + x^6 + x^10 + 1))/(x^6*(x + x^6)^(1/4)*(x^5 - x^3
+ 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^5-3\right)\left(x^{10}+x^6+2x^5+1\right)}{x^6\sqrt[4]{x\left(x+1\right)\left(x^4-x^3+x^2-x+1\right)}\left(x^5-x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**5-3)*(x**10+x**6+2*x**5+1)/x**6/(x**5-x**3+1)/(x**6+x)**(1/
4),x)
```

```
[Out] Integral((2*x**5 - 3)*(x**10 + x**6 + 2*x**5 + 1)/(x**6*(x*(x + 1)*(x**4 -
x**3 + x**2 - x + 1))**(1/4)*(x**5 - x**3 + 1)), x)
```

3.727 $\int \sqrt{x^2+\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=57

$$\frac{1}{2}\sqrt{\sqrt{x^4+1}+x^2}\,x+\frac{\tan^{-1}\left(\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}\right)}{2\sqrt{2}}$$

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{x^2+\sqrt{1+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

```
[Out] Defer[Int][Sqrt[x^2 + Sqrt[1 + x^4]], x]
```

Rubi steps

$$\int \sqrt{x^2+\sqrt{1+x^4}}\,dx=\int \sqrt{x^2+\sqrt{1+x^4}}\,dx$$

Mathematica [A] time = 0.18, size = 92, normalized size = 1.61

$$\frac{2\left(\sqrt{x^4+1}+x^2\right)x^2+\sqrt{2}\sqrt{x^2\left(\sqrt{x^4+1}+x^2\right)}\tan^{-1}\left(\sqrt{\left(\sqrt{x^4+1}+x^2\right)^2-1}\right)}{4x\sqrt{\sqrt{x^4+1}+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

```
[Out] (2*x^2*(x^2 + Sqrt[1 + x^4]) + Sqrt[2]*Sqrt[x^2*(x^2 + Sqrt[1 + x^4])]*ArcT
an[Sqrt[-1 + (x^2 + Sqrt[1 + x^4])^2]])/(4*x*Sqrt[x^2 + Sqrt[1 + x^4]])
```

IntegrateAlgebraic [A] time = 0.12, size = 57, normalized size = 1.00

$$\frac{1}{2}\sqrt{\sqrt{x^4+1}+x^2}\,x+\frac{\tan^{-1}\left(\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

```
[Out] (x*Sqrt[x^2 + Sqrt[1 + x^4]])/2 + ArcTan[Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]
]]/(2*Sqrt[2])
```

fricas [A] time = 0.76, size = 61, normalized size = 1.07

$$\frac{1}{2}\sqrt{x^2+\sqrt{x^4+1}}\,x-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\left(\sqrt{2}\,x^2-\sqrt{2}\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 + sqrt(x^4 + 1))*x - 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x^2 - sq
rt(2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2 + \sqrt{x^4 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1)), x)
```

maple [C] time = 0.03, size = 22, normalized size = 0.39

$$\frac{\sqrt{2}\, x^2 \,\text{hypergeom}\left(\left[-\frac{1}{2}, -\frac{1}{4}, \frac{1}{4}\right], \left[\frac{1}{2}, \frac{1}{2}\right], -\frac{1}{x^4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*x^2*hypergeom([-1/2,-1/4,1/4],[1/2,1/2],-1/x^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2 + \sqrt{x^4 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{x^4 + 1} + x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

```
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

sympy [A] time = 0.70, size = 17, normalized size = 0.30

$$-\frac{G_{3,3}^{2,2}\left(\begin{matrix} \frac{3}{2}, 1 & 1 \\ \frac{1}{4}, \frac{3}{4} & 0 \end{matrix} \middle| {x^{4}} \right)}{16\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] -meijerg(((3/2, 1), (1,)), ((1/4, 3/4), (0,)), x**4)/(16*sqrt(pi))
```

3.728 $\int \frac{-1-x+x^2}{(1+x^2)\sqrt{-x+x^3}}\,dx$

Optimal. Leaf size=58

$$-\frac{1}{4}\tan^{-1}\left(\frac{2\sqrt{x^3-x}}{x^2-2x-1}\right)-\frac{3}{4}\tanh^{-1}\left(\frac{\frac{x^2}{2}+x-\frac{1}{2}}{\sqrt{x^3-x}}\right)$$

Rubi [C] time = 0.72, antiderivative size = 182, normalized size of antiderivative = 3.14, number of steps used = 13, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2056, 6725, 329, 222, 933, 168, 537}

$$\frac{\sqrt{2}\sqrt{x-1}\sqrt{x}\sqrt{x+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{x-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^3-x}}+\frac{\left(\frac{3}{2}+\frac{i}{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{2}-\frac{i}{2};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^3-x}}+\frac{\left(\frac{3}{2}-\frac{i}{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{2}+\frac{i}{2};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^3-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 - x + x^2)/((1 + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] (Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]*Sqrt[1 + x]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[x]
)/Sqrt[-1 + x]], 1/2])/Sqrt[-x + x^3] + ((3/2 + I/2)*Sqrt[x]*Sqrt[1 - x^2]*
EllipticPi[1/2 - I/2, ArcSin[Sqrt[1 - x]], 1/2])/(Sqrt[2]*Sqrt[-x + x^3]) +
 ((3/2 - I/2)*Sqrt[x]*Sqrt[1 - x^2]*EllipticPi[1/2 + I/2, ArcSin[Sqrt[1 - x
]], 1/2])/(Sqrt[2]*Sqrt[-x + x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
```

```
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1-x+x^2}{\left(1+x^2\right)\sqrt{-x+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{-1-x+x^2}{\sqrt{x}\sqrt{-1+x^2}\left(1+x^2\right)}\,dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{-1+x^2}}-\frac{2+x}{\sqrt{x}\sqrt{-1+x^2}\left(1+x^2\right)}\right)dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-1+x^2}}\,dx}{\sqrt{-x+x^3}} - \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{2+x}{\sqrt{x}\sqrt{-1+x^2}\left(1+x^2\right)}\,dx}{\sqrt{-x+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \left(-\frac{\frac{1}{2}-i}{(i-x)\sqrt{x}\sqrt{-1+x^2}}+\frac{\frac{1}{2}+i}{\sqrt{x}(i+x)\sqrt{-1+x^2}}\right)dx}{\sqrt{-x+x^3}} + \frac{\left(2\sqrt{x}\sqrt{-1+x^2}\right)\text{Subst}\left(\right.}{\sqrt{-x+}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} - -\frac{\left(\left(\frac{1}{2}-i\right)\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1}{(i-x)\sqrt{x}\sqrt{-1}}}{\sqrt{-x+x^3}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} - -\frac{\left(\left(\frac{1}{2}-i\right)\sqrt{x}\sqrt{1-x^2}\right)\int \frac{1}{(i-x)\sqrt{1-x}\sqrt{x}}}{\sqrt{-x+x^3}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} - -\frac{\left((1+2i)\sqrt{x}\sqrt{1-x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-}}\right.}{\sqrt{-x+}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} + \frac{\left(\frac{3}{2}+\frac{i}{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{2}-\frac{i}{2};\sin^{-1}\left(\right.\right.}{\sqrt{2}\sqrt{-x+x^3}}$$

Mathematica [C] time = 0.28, size = 95, normalized size = 1.64

$$\frac{2x\sqrt{1-x^2}\left(x\left(3xF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};x^2,-x^2\right)-5F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};x^2,-x^2\right)\right)-15F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};x^2,-x^2\right)\right)}{15\sqrt{x\left(x^2-1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 - x + x^2)/((1 + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] (2*x*Sqrt[1 - x^2]*(-15*AppellF1[1/4, 1/2, 1, 5/4, x^2, -x^2] + x*(-5*Appel
lF1[3/4, 1/2, 1, 7/4, x^2, -x^2] + 3*x*AppellF1[5/4, 1/2, 1, 9/4, x^2, -x^2
])))/(15*Sqrt[x*(-1 + x^2)])
```

IntegrateAlgebraic [A] time = 0.27, size = 58, normalized size = 1.00

$$-\frac{1}{4}\tan^{-1}\left(\frac{2\sqrt{x^3-x}}{x^2-2x-1}\right)-\frac{3}{4}\tanh^{-1}\left(\frac{\frac{x^2}{2}+x-\frac{1}{2}}{\sqrt{x^3-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - x + x^2)/((1 + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] -1/4*ArcTan[(2*Sqrt[-x + x^3])/(-1 - 2*x + x^2)] - (3*ArcTanh[(-1/2 + x + x
^2/2)/Sqrt[-x + x^3]])/4
```

fricas [A] time = 0.46, size = 76, normalized size = 1.31

$$\frac{1}{4}\arctan\left(\frac{x^2-2x-1}{2\sqrt{x^3-x}}\right)+\frac{3}{8}\log\left(\frac{x^4+8x^3+2x^2-4\sqrt{x^3-x}\left(x^2+2x-1\right)-8x+1}{x^4+2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)/(x^2+1)/(x^3-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*arctan(1/2*(x^2 - 2*x - 1)/sqrt(x^3 - x)) + 3/8*log((x^4 + 8*x^3 + 2*x^
2 - 4*sqrt(x^3 - x)*(x^2 + 2*x - 1) - 8*x + 1)/(x^4 + 2*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2-x-1}{\sqrt{x^3-x}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)/(x^2+1)/(x^3-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - x - 1)/(sqrt(x^3 - x)*(x^2 + 1)), x)
```

maple [C] time = 0.03, size = 210, normalized size = 3.62

$$\frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticF}\left(\sqrt{1+x},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-x}}-\frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{1+x},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^3-x}}-\frac{3i\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{1+x},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^3-x}}-\frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{1+x},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^3-x}}+\frac{3i\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{1+x},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^3-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x-1)/(x^2+1)/(x^3-x)^(1/2),x)
```

```
[Out] (1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticF((1+x)^(1/2),1/
2*2^(1/2))-1/4*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticP
i((1+x)^(1/2),1/2-1/2*I,1/2*2^(1/2))-3/4*I*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(
1/2)/(x^3-x)^(1/2)*EllipticPi((1+x)^(1/2),1/2-1/2*I,1/2*2^(1/2))-1/4*(1+x)^
(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticPi((1+x)^(1/2),1/2+1/2
*I,1/2*2^(1/2))+3/4*I*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*El
lipticPi((1+x)^(1/2),1/2+1/2*I,1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2-x-1}{\sqrt{x^3-x}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)/(x^2+1)/(x^3-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x - 1)/(sqrt(x^3 - x)*(x^2 + 1)), x)
```

mupad [B] time = 0.08, size = 100, normalized size = 1.72

$$\frac{-2\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)+\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(-\mathrm{i};\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)(2-\mathrm{i})+\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(1\mathrm{i};\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)(2+1\mathrm{i})}{\sqrt{x^3-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - x^2 + 1)/((x^3 - x)^(1/2)*(x^2 + 1)),x)
```

```
[Out] ((-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticPi(-1i, asin((-x)^(1/2)), -
1)*(2 - 1i) - 2*(-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticF(asin((-x)^
(1/2)), -1) + (-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticPi(1i, asin((-
x)^(1/2)), -1)*(2 + 1i))/(x^3 - x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-x-1}{\sqrt{x(x-1)(x+1)}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x-1)/(x**2+1)/(x**3-x)**(1/2),x)
```

```
[Out] Integral((x**2 - x - 1)/(sqrt(x*(x - 1)*(x + 1))*(x**2 + 1)), x)
```

3.729 $\int \frac{-1+x+x^2}{(1+x^2)\sqrt{-x+x^3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3}{4}\tan^{-1}\left(\frac{2\sqrt{x^3-x}}{x^2-2x-1}\right)-\frac{1}{4}\tanh^{-1}\left(\frac{\frac{x^2}{2}+x-\frac{1}{2}}{\sqrt{x^3-x}}\right)$$

Rubi [C] time = 0.69, antiderivative size = 182, normalized size of antiderivative = 3.14, number of steps used = 13, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2056, 6725, 329, 222, 933, 168, 537}

$$\frac{\sqrt{2}\sqrt{x-1}\sqrt{x}\sqrt{x+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{x-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^3-x}}+\frac{\left(\frac{1}{2}+\frac{3i}{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{2}-\frac{i}{2};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^3-x}}+\frac{\left(\frac{1}{2}-\frac{3i}{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{2}+\frac{i}{2};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^3-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x + x^2)/((1 + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] (Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]*Sqrt[1 + x]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[x]
)/Sqrt[-1 + x]], 1/2])/Sqrt[-x + x^3] + ((1/2 + (3*I)/2)*Sqrt[x]*Sqrt[1 - x
^2]*EllipticPi[1/2 - I/2, ArcSin[Sqrt[1 - x]], 1/2])/(Sqrt[2]*Sqrt[-x + x^3
]) + ((1/2 - (3*I)/2)*Sqrt[x]*Sqrt[1 - x^2]*EllipticPi[1/2 + I/2, ArcSin[Sq
rt[1 - x]], 1/2])/(Sqrt[2]*Sqrt[-x + x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
```

```
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+x+x^2}{(1+x^2)\sqrt{-x+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{-1+x+x^2}{\sqrt{x}\sqrt{-1+x^2}(1+x^2)}\,dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{-1+x^2}} - \frac{2-x}{\sqrt{x}\sqrt{-1+x^2}(1+x^2)}\right)dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-1+x^2}}\,dx}{\sqrt{-x+x^3}} - \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{2-x}{\sqrt{x}\sqrt{-1+x^2}(1+x^2)}\,dx}{\sqrt{-x+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \left(\frac{\frac{1}{2}+i}{(i-x)\sqrt{x}\sqrt{-1+x^2}} - \frac{\frac{1}{2}-i}{\sqrt{x}(i+x)\sqrt{-1+x^2}}\right)dx}{\sqrt{-x+x^3}} + \frac{\left(2\sqrt{x}\sqrt{-1+x^2}\right)\text{Subst}\left(\int\right.}{\sqrt{-x+x}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} - -\frac{\left(\left(\frac{1}{2}-i\right)\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{x}(i+x)\sqrt{-1}}}{\sqrt{-x+x^3}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} - -\frac{\left(\left(\frac{1}{2}-i\right)\sqrt{x}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-x}\sqrt{x}(i+x)}}{\sqrt{-x+x^3}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} - -\frac{\left((1+2i)\sqrt{x}\sqrt{1-x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-}}\right.}{\sqrt{-x+}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} + \frac{\left(\frac{1}{2}+\frac{3i}{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{2}-\frac{i}{2};\sin^{-1}\left(\right.\right.}{\sqrt{2}\sqrt{-x+x^3}}$$

Mathematica [C] time = 0.19, size = 95, normalized size = 1.64

$$\frac{2x\sqrt{1-x^2}\left(x\left(5F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};x^2,-x^2\right)+3xF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};x^2,-x^2\right)\right)-15F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};x^2,-x^2\right)\right)}{15\sqrt{x(x^2-1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + x + x^2)/((1 + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] (2*x*Sqrt[1 - x^2]*(-15*AppellF1[1/4, 1/2, 1, 5/4, x^2, -x^2] + x*(5*Appell
F1[3/4, 1/2, 1, 7/4, x^2, -x^2] + 3*x*AppellF1[5/4, 1/2, 1, 9/4, x^2, -x^2]
)))/(15*Sqrt[x*(-1 + x^2)])
```

IntegrateAlgebraic [A] time = 0.26, size = 58, normalized size = 1.00

$$-\frac{3}{4}\tan^{-1}\left(\frac{2\sqrt{x^3-x}}{x^2-2x-1}\right)-\frac{1}{4}\tanh^{-1}\left(\frac{\frac{x^2}{2}+x-\frac{1}{2}}{\sqrt{x^3-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x + x^2)/((1 + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] (-3*ArcTan[(2*Sqrt[-x + x^3])/(-1 - 2*x + x^2)])/4 - ArcTanh[(-1/2 + x + x^
2/2)/Sqrt[-x + x^3]]/4
```

fricas [A] time = 0.48, size = 76, normalized size = 1.31

$$\frac{3}{4}\arctan\left(\frac{x^2-2x-1}{2\sqrt{x^3-x}}\right)+\frac{1}{8}\log\left(\frac{x^4+8x^3+2x^2-4\sqrt{x^3-x}\left(x^2+2x-1\right)-8x+1}{x^4+2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(x^2+1)/(x^3-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 3/4*arctan(1/2*(x^2 - 2*x - 1)/sqrt(x^3 - x)) + 1/8*log((x^4 + 8*x^3 + 2*x^
2 - 4*sqrt(x^3 - x)*(x^2 + 2*x - 1) - 8*x + 1)/(x^4 + 2*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+x-1}{\sqrt{x^3-x}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(x^2+1)/(x^3-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + x - 1)/(sqrt(x^3 - x)*(x^2 + 1)), x)
```

maple [C] time = 0.03, size = 210, normalized size = 3.62

$$\frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticF}\left(\sqrt{1+x},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-x}}-\frac{3\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{1+x},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^3-x}}-\frac{i\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{1+x},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^3-x}}-\frac{3\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{1+x},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^3-x}}+\frac{i\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{1+x},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^3-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-1)/(x^2+1)/(x^3-x)^(1/2),x)
```

```
[Out] (1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticF((1+x)^(1/2),1/
2*2^(1/2))-3/4*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticP
i((1+x)^(1/2),1/2-1/2*I,1/2*2^(1/2))-1/4*I*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(
1/2)/(x^3-x)^(1/2)*EllipticPi((1+x)^(1/2),1/2-1/2*I,1/2*2^(1/2))-3/4*(1+x)^
(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticPi((1+x)^(1/2),1/2+1/2
*I,1/2*2^(1/2))+1/4*I*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*El
lipticPi((1+x)^(1/2),1/2+1/2*I,1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+x-1}{\sqrt{x^3-x}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(x^2+1)/(x^3-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + x - 1)/(sqrt(x^3 - x)*(x^2 + 1)), x)
```

mupad [B] time = 0.56, size = 100, normalized size = 1.72

$$\frac{-2\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{-x}\right)\middle|-1\right)+\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(-\mathrm{i};\operatorname{asin}\left(\sqrt{-x}\right)\middle|-1\right)\,(2+1\mathrm{i})+\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(1\mathrm{i};\operatorname{asin}\left(\sqrt{-x}\right)\middle|-1\right)\,(2-\mathrm{i})}{\sqrt{x^3-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 - 1)/((x^3 - x)^(1/2)*(x^2 + 1)),x)
```

```
[Out] ((-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticPi(-1i, asin((-x)^(1/2)), -
1)*(2 + 1i) - 2*(-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticF(asin((-x)^
(1/2)), -1) + (-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticPi(1i, asin((-
x)^(1/2)), -1)*(2 - 1i))/(x^3 - x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+x-1}{\sqrt{x\,(x-1)\,(x+1)}\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x-1)/(x**2+1)/(x**3-x)**(1/2),x)
```

```
[Out] Integral((x**2 + x - 1)/(sqrt(x*(x - 1)*(x + 1))*(x**2 + 1)), x)
```

3.730 $\int \frac{-7+x+7x^2}{(1+x^2)\sqrt{-x+x^3}}\,dx$

Optimal. Leaf size=58

$$-\frac{15}{4}\tan^{-1}\left(\frac{2\sqrt{x^3-x}}{x^2-2x-1}\right)-\frac{13}{4}\tanh^{-1}\left(\frac{\frac{x^2}{2}+x-\frac{1}{2}}{\sqrt{x^3-x}}\right)$$

Rubi [C] time = 0.69, antiderivative size = 183, normalized size of antiderivative = 3.16, number of steps used = 13, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2056, 6725, 329, 222, 933, 168, 537}

$$\frac{7\sqrt{2}\sqrt{x-1}\sqrt{x}\sqrt{x+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{x-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^3-x}}+\frac{\left(\frac{13}{2}+\frac{15i}{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{2}-\frac{i}{2};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^3-x}}+\frac{\left(\frac{13}{2}-\frac{15i}{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{2}+\frac{i}{2};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^3-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-7 + x + 7*x^2)/((1 + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] (7*Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]*Sqrt[1 + x]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[
x])/Sqrt[-1 + x]], 1/2])/Sqrt[-x + x^3] + ((13/2 + (15*I)/2)*Sqrt[x]*Sqrt[1
 - x^2]*EllipticPi[1/2 - I/2, ArcSin[Sqrt[1 - x]], 1/2])/(Sqrt[2]*Sqrt[-x +
 x^3]) + ((13/2 - (15*I)/2)*Sqrt[x]*Sqrt[1 - x^2]*EllipticPi[1/2 + I/2, Arc
Sin[Sqrt[1 - x]], 1/2])/(Sqrt[2]*Sqrt[-x + x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
```

```
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-7+x+7x^2}{\left(1+x^2\right)\sqrt{-x+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{-7+x+7x^2}{\sqrt{x}\sqrt{-1+x^2}\left(1+x^2\right)}\,dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \left(\frac{7}{\sqrt{x}\sqrt{-1+x^2}}-\frac{14-x}{\sqrt{x}\sqrt{-1+x^2}\left(1+x^2\right)}\right)dx}{\sqrt{-x+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{14-x}{\sqrt{x}\sqrt{-1+x^2}\left(1+x^2\right)}\,dx}{\sqrt{-x+x^3}}+\frac{\left(7\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-1+x^2}}\,dx}{\sqrt{-x+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \left(\frac{\frac{1}{2}+7i}{(i-x)\sqrt{x}\sqrt{-1+x^2}}-\frac{\frac{1}{2}-7i}{\sqrt{x}\,(i+x)\sqrt{-1+x^2}}\right)dx}{\sqrt{-x+x^3}}+\frac{\left(14\sqrt{x}\sqrt{-1+x^2}\right)\text{Subst}\Big(}{\sqrt{-x+}}$$

$$= \frac{7\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}}--\frac{\left(\left(\frac{1}{2}-7i\right)\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{x}\,(i+x)}}{\sqrt{-x+x^3}}$$

$$= \frac{7\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}}--\frac{\left(\left(\frac{1}{2}-7i\right)\sqrt{x}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-x}\sqrt{x}\,(i+}}{\sqrt{-x+x^3}}$$

$$= \frac{7\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}}--\frac{\left((1+14i)\sqrt{x}\sqrt{1-x^2}\right)\text{Subst}\Big(\int}{\sqrt{-x}}$$

$$= \frac{7\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}}+\frac{\left(\frac{13}{2}+\frac{15i}{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{2}-\frac{i}{2};\sin\right.}{\sqrt{2}\sqrt{-x+x^3}}$$

Mathematica [C] time = 0.53, size = 95, normalized size = 1.64

$$\frac{2x\sqrt{1-x^2}\left(x\left(5F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};x^2,-x^2\right)+21xF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};x^2,-x^2\right)\right)-105F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};x^2,-x^2\right)\right)}{15\sqrt{x\left(x^2-1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-7 + x + 7*x^2)/((1 + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] (2*x*Sqrt[1 - x^2]*(-105*AppellF1[1/4, 1/2, 1, 5/4, x^2, -x^2] + x*(5*Appel
lF1[3/4, 1/2, 1, 7/4, x^2, -x^2] + 21*x*AppellF1[5/4, 1/2, 1, 9/4, x^2, -x^
2])))/(15*Sqrt[x*(-1 + x^2)])
```

IntegrateAlgebraic [A] time = 0.27, size = 58, normalized size = 1.00

$$-\frac{15}{4}\tan^{-1}\left(\frac{2\sqrt{x^3-x}}{x^2-2x-1}\right)-\frac{13}{4}\tanh^{-1}\left(\frac{\frac{x^2}{2}+x-\frac{1}{2}}{\sqrt{x^3-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-7 + x + 7*x^2)/((1 + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] (-15*ArcTan[(2*Sqrt[-x + x^3])/(-1 - 2*x + x^2)])/4 - (13*ArcTanh[(-1/2 + x
 + x^2/2)/Sqrt[-x + x^3]])/4
```

fricas [A] time = 0.47, size = 76, normalized size = 1.31

$$\frac{15}{4}\arctan\left(\frac{x^2-2x-1}{2\sqrt{x^3-x}}\right)+\frac{13}{8}\log\left(\frac{x^4+8x^3+2x^2-4\sqrt{x^3-x}\left(x^2+2x-1\right)-8x+1}{x^4+2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^2+x-7)/(x^2+1)/(x^3-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 15/4*arctan(1/2*(x^2 - 2*x - 1)/sqrt(x^3 - x)) + 13/8*log((x^4 + 8*x^3 + 2*
x^2 - 4*sqrt(x^3 - x)*(x^2 + 2*x - 1) - 8*x + 1)/(x^4 + 2*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{7x^2+x-7}{\sqrt{x^3-x}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^2+x-7)/(x^2+1)/(x^3-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((7*x^2 + x - 7)/(sqrt(x^3 - x)*(x^2 + 1)), x)
```

maple [C] time = 0.03, size = 211, normalized size = 3.64

$$\frac{7\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticF}\left(\sqrt{1+x},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-x}}-\frac{15\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{1+x},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^3-x}}-\frac{13i\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{1+x},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^3-x}}-\frac{15\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{1+x},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^3-x}}+\frac{13i\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{1+x},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^3-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7*x^2+x-7)/(x^2+1)/(x^3-x)^(1/2),x)
```

```
[Out] 7*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticF((1+x)^(1/2),
1/2*2^(1/2))-15/4*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*Ellipt
icPi((1+x)^(1/2),1/2-1/2*I,1/2*2^(1/2))-13/4*I*(1+x)^(1/2)*(2-2*x)^(1/2)*(-
x)^(1/2)/(x^3-x)^(1/2)*EllipticPi((1+x)^(1/2),1/2-1/2*I,1/2*2^(1/2))-15/4*(
1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticPi((1+x)^(1/2),1/
2+1/2*I,1/2*2^(1/2))+13/4*I*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1
/2)*EllipticPi((1+x)^(1/2),1/2+1/2*I,1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{7x^2+x-7}{\sqrt{x^3-x}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^2+x-7)/(x^2+1)/(x^3-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((7*x^2 + x - 7)/(sqrt(x^3 - x)*(x^2 + 1)), x)
```

mupad [B] time = 0.56, size = 100, normalized size = 1.72

$$\frac{-14\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)+\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(-\mathrm{i};\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)\,(14+1\mathrm{i})+\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(1\mathrm{i};\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)\,(14-\mathrm{i})}{\sqrt{x^3-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 7*x^2 - 7)/((x^3 - x)^(1/2)*(x^2 + 1)),x)
```

```
[Out] ((-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticPi(-1i, asin((-x)^(1/2)), -
1)*(14 + 1i) - 14*(-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticF(asin((-x
)^(1/2)), -1) + (-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticPi(1i, asin(
(-x)^(1/2)), -1)*(14 - 1i))/(x^3 - x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{7x^2+x-7}{\sqrt{x(x-1)(x+1)}\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x**2+x-7)/(x**2+1)/(x**3-x)**(1/2),x)
```

```
[Out] Integral((7*x**2 + x - 7)/(sqrt(x*(x - 1)*(x + 1))*(x**2 + 1)), x)
```

3.731 $\int \frac{1}{(-2+x)\sqrt[4]{-x^2+x^3}}\,dx$

Optimal. Leaf size=58

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^3-x^2}}{x}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^3-x^2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.16, antiderivative size = 102, normalized size of antiderivative = 1.76, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2056, 106, 490, 1211, 220, 1699, 203, 206}

$$\frac{\sqrt[4]{x-1}\sqrt{x}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x-1}}{\sqrt{x}}\right)}{\sqrt{2}\sqrt[4]{x^3-x^2}}-\frac{\sqrt[4]{x-1}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x-1}}{\sqrt{x}}\right)}{\sqrt{2}\sqrt[4]{x^3-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + x)*(-x^2 + x^3)^(1/4)),x]
```

```
[Out] ((-1 + x)^(1/4)*Sqrt[x]*ArcTan[(Sqrt[2]*(-1 + x)^(1/4))/Sqrt[x]])/(Sqrt[2]*
(-x^2 + x^3)^(1/4)) - ((-1 + x)^(1/4)*Sqrt[x]*ArcTanh[(Sqrt[2]*(-1 + x)^(1/
4))/Sqrt[x]])/(Sqrt[2]*(-x^2 + x^3)^(1/4))
```

Rule 106

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
1/4)), x_Symbol] :> Dist[-4, Subst[Int[x^2/((b*e - a*f - b*x^4)*Sqrt[c - (d
*e)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f
}, x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{(-2+x)\sqrt[4]{-x^2+x^3}}\,dx = \frac{\left(\sqrt[4]{-1+x}\sqrt{x}\right)\int \frac{1}{(-2+x)\sqrt[4]{-1+x}\sqrt{x}}\,dx}{\sqrt[4]{-x^2+x^3}}$$

$$= -\frac{\left(4\sqrt[4]{-1+x}\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{x^2}{(1-x^4)\sqrt{1+x^4}}\,dx, x, \sqrt[4]{-1+x}\right)}{\sqrt[4]{-x^2+x^3}}$$

$$= -\frac{\left(2\sqrt[4]{-1+x}\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{1+x^4}}\,dx, x, \sqrt[4]{-1+x}\right)}{\sqrt[4]{-x^2+x^3}} + \frac{\left(2\sqrt[4]{-1+x}\sqrt{x}\right)\operatorname{Subst}\left(\int\right.}{\sqrt[4]{-}}$$

$$= \frac{\left(\sqrt[4]{-1+x}\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx, x, \sqrt[4]{-1+x}\right)}{\sqrt[4]{-x^2+x^3}} - \frac{\left(\sqrt[4]{-1+x}\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{}{(1-}\right.}{\sqrt[4]{-x^2+}}$$

$$= -\frac{\left(\sqrt[4]{-1+x}\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\sqrt[4]{-1+x}}{\sqrt{x}}\right)}{\sqrt[4]{-x^2+x^3}} + \frac{\left(\sqrt[4]{-1+x}\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x\right.}{\sqrt[4]{-x^2+x^3}}$$

$$= \frac{\sqrt[4]{-1+x}\sqrt{x}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-1+x}}{\sqrt{x}}\right)}{\sqrt{2}\sqrt[4]{-x^2+x^3}} - \frac{\sqrt[4]{-1+x}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-1+x}}{\sqrt{x}}\right)}{\sqrt{2}\sqrt[4]{-x^2+x^3}}$$

Mathematica [C] time = 0.02, size = 40, normalized size = 0.69

$$-\frac{\sqrt[4]{1-x}\,xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x,\frac{x}{2}\right)}{\sqrt[4]{(x-1)x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 + x)*(-x^2 + x^3)^(1/4)),x]
[Out] -(((1 - x)^(1/4)*x*AppellF1[1/2, 1/4, 1, 3/2, x, x/2])/((-1 + x)*x^2)^(1/4)
)
```

IntegrateAlgebraic [A] time = 0.21, size = 58, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{x^3-x^2}}{x}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\,\sqrt[4]{x^3-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2 + x)*(-x^2 + x^3)^(1/4)),x]
```

```
[Out] ArcTan[(Sqrt[2]*(-x^2 + x^3)^(1/4))/x]/Sqrt[2] - ArcTanh[x/(Sqrt[2]*(-x^2 +
 x^3)^(1/4))]/Sqrt[2]
```

fricas [B] time = 2.87, size = 193, normalized size = 3.33

$$\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{2\left(\sqrt{2}\left(x^3-x^2\right)^{\frac{1}{4}}x^2+2\,\sqrt{2}\left(x^3-x^2\right)^{\frac{3}{4}}\right)}{x^3-4\,x^2+4\,x}\right)+\frac{1}{8}\,\sqrt{2}\log\left(-\frac{x^5+56\,x^4-40\,x^3-8\,\sqrt{2}\left(x^3-x^2\right)^{\frac{3}{4}}\left(3\,x^2+4\,x-4\right)-32\,x^2-4\,\sqrt{2}\left(x^4+12\,x^3-12\,x^2\right)\left(x^3-x^2\right)^{\frac{1}{4}}+16\left(x^3+4\,x^2-4\,x\right)\sqrt{x^3-x^2}+16\,x}{x^5-8\,x^4+24\,x^3-32\,x^2+16\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(x^3-x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(2*(sqrt(2)*(x^3 - x^2)^(1/4)*x^2 + 2*sqrt(2)*(x^3 - x^2)
^(3/4))/(x^3 - 4*x^2 + 4*x)) + 1/8*sqrt(2)*log(-(x^5 + 56*x^4 - 40*x^3 - 8*
sqrt(2)*(x^3 - x^2)^(3/4)*(3*x^2 + 4*x - 4) - 32*x^2 - 4*sqrt(2)*(x^4 + 12*
x^3 - 12*x^2)*(x^3 - x^2)^(1/4) + 16*(x^3 + 4*x^2 - 4*x)*sqrt(x^3 - x^2) +
16*x)/(x^5 - 8*x^4 + 24*x^3 - 32*x^2 + 16*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3-x^2\right)^{\frac{1}{4}}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(x^3-x^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 - x^2)^(1/4)*(x - 2)), x)
```

maple [C] time = 2.41, size = 198, normalized size = 3.41

$$-\frac{\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{4\,\mathrm{RootOf}\left(\_Z^2-2\right)\sqrt{x^3-x^2}\,x+\mathrm{RootOf}\left(\_Z^2-2\right)x^3+8\left(x^3-x^2\right)^{\frac{3}{4}}+4\left(x^3-x^2\right)^{\frac{1}{4}}x^2+4\,\mathrm{RootOf}\left(\_Z^2-2\right)x^2-4\,\mathrm{RootOf}\left(\_Z^2-2\right)x}{(-2+x)^2 x}\right)}{4}-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{-4\,\mathrm{RootOf}\left(\_Z^2+2\right)\sqrt{x^3-x^2}\,x+\mathrm{RootOf}\left(\_Z^2+2\right)x^3+8\left(x^3-x^2\right)^{\frac{3}{4}}-4\left(x^3-x^2\right)^{\frac{1}{4}}x^2+4\,\mathrm{RootOf}\left(\_Z^2+2\right)x^2-4\,\mathrm{RootOf}\left(\_Z^2+2\right)x}{(-2+x)^2 x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2+x)/(x^3-x^2)^(1/4),x)
```

```
[Out] -1/4*RootOf(_Z^2-2)*ln((4*RootOf(_Z^2-2)*(x^3-x^2)^(1/2)*x+RootOf(_Z^2-2)*x
^3+8*(x^3-x^2)^(3/4)+4*(x^3-x^2)^(1/4)*x^2+4*RootOf(_Z^2-2)*x^2-4*RootOf(_Z
^2-2)*x)/(-2+x)^2/x)-1/4*RootOf(_Z^2+2)*ln((-4*RootOf(_Z^2+2)*(x^3-x^2)^(1/
2)*x+RootOf(_Z^2+2)*x^3+8*(x^3-x^2)^(3/4)-4*(x^3-x^2)^(1/4)*x^2+4*RootOf(_Z
^2+2)*x^2-4*RootOf(_Z^2+2)*x)/(-2+x)^2/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3-x^2\right)^{\frac{1}{4}}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(x^3-x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 - x^2)^(1/4)*(x - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(x^3 - x^2\right)^{1/4} (x - 2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 - x^2)^(1/4)*(x - 2)),x)
[Out] int(1/((x^3 - x^2)^(1/4)*(x - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^2 (x - 1)}\,(x - 2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(x**3-x**2)**(1/4),x)
[Out] Integral(1/((x**2*(x - 1))**(1/4)*(x - 2)), x)
```

3.732 $\int \left(b+ax^3\right)\sqrt{-x+x^4}\,dx$

Optimal. Leaf size=58

$$\frac{1}{12}\sqrt{x^4-x}\left(2ax^4-ax+4bx\right)+\frac{1}{12}(-a-4b)\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)$$

Rubi [A] time = 0.12, antiderivative size = 96, normalized size of antiderivative = 1.66, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2053, 2004, 2029, 206, 2021, 2024}

$$\frac{1}{6}a\sqrt{x^4-x}\,x^4-\frac{1}{12}a\sqrt{x^4-x}\,x-\frac{1}{12}a\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)+\frac{1}{3}b\sqrt{x^4-x}\,x-\frac{1}{3}b\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^3)*Sqrt[-x + x^4],x]
```

```
[Out] -1/12*(a*x*Sqrt[-x + x^4]) + (b*x*Sqrt[-x + x^4])/3 + (a*x^4*Sqrt[-x + x^4]
)/6 - (a*ArcTanh[x^2/Sqrt[-x + x^4]])/12 - (b*ArcTanh[x^2/Sqrt[-x + x^4]])/
3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2053

```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Expan
dIntegrand[Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && (Po
lyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IntegerQ[p] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \left(b+ax^3\right)\sqrt{-x+x^4}\,dx &= \int \left(b\sqrt{-x+x^4}+ax^3\sqrt{-x+x^4}\right)dx \\
&= a\int x^3\sqrt{-x+x^4}\,dx+b\int \sqrt{-x+x^4}\,dx \\
&= \frac{1}{3}bx\sqrt{-x+x^4}+\frac{1}{6}ax^4\sqrt{-x+x^4}-\frac{1}{4}a\int \frac{x^4}{\sqrt{-x+x^4}}\,dx-\frac{1}{2}b\int \frac{x}{\sqrt{-x+x^4}}\,dx \\
&= -\frac{1}{12}ax\sqrt{-x+x^4}+\frac{1}{3}bx\sqrt{-x+x^4}+\frac{1}{6}ax^4\sqrt{-x+x^4}-\frac{1}{8}a\int \frac{x}{\sqrt{-x+x^4}}\,dx-\frac{1}{3}b\,\mathrm{Sub} \\
&= -\frac{1}{12}ax\sqrt{-x+x^4}+\frac{1}{3}bx\sqrt{-x+x^4}+\frac{1}{6}ax^4\sqrt{-x+x^4}-\frac{1}{3}b\tanh^{-1}\left(\frac{x^2}{\sqrt{-x+x^4}}\right)-\frac{1}{12} \\
&= -\frac{1}{12}ax\sqrt{-x+x^4}+\frac{1}{3}bx\sqrt{-x+x^4}+\frac{1}{6}ax^4\sqrt{-x+x^4}-\frac{1}{12}a\tanh^{-1}\left(\frac{x^2}{\sqrt{-x+x^4}}\right)-\frac{1}{3}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 63, normalized size = 1.09

$$\frac{\sqrt{x\left(x^3-1\right)}\left(x^{3/2}\left(a\left(2x^3-1\right)+4b\right)+\frac{(a+4b)\sin^{-1}\left(x^{3/2}\right)}{\sqrt{1-x^3}}\right)}{12\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^3)*Sqrt[-x + x^4],x]
```

```
[Out] (Sqrt[x*(-1 + x^3)]*(x^(3/2)*(4*b + a*(-1 + 2*x^3)) + ((a + 4*b)*ArcSin[x^(
3/2)])/Sqrt[1 - x^3]))/(12*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.44, size = 58, normalized size = 1.00

$$\frac{1}{12}\sqrt{x^4-x}\left(2ax^4-ax+4bx\right)+\frac{1}{12}(-a-4b)\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^3)*Sqrt[-x + x^4],x]
```

```
[Out] (Sqrt[-x + x^4]*(-(a*x) + 4*b*x + 2*a*x^4))/12 + ((-a - 4*b)*ArcTanh[x^2/Sq
rt[-x + x^4]])/12
```

fricas [A] time = 0.44, size = 54, normalized size = 0.93

$$\frac{1}{24}(a+4b)\log\left(2x^3-2\sqrt{x^4-x}\,x-1\right)+\frac{1}{12}\left(2ax^4-(a-4b)x\right)\sqrt{x^4-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)*(x^4-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(a + 4*b)*log(2*x^3 - 2*sqrt(x^4 - x)*x - 1) + 1/12*(2*a*x^4 - (a - 4*
b)*x)*sqrt(x^4 - x)
```

giac [A] time = 0.38, size = 61, normalized size = 1.05

$$\frac{1}{12}\left(2\,ax^{3}-a+4\,b\right)\sqrt{x^{4}-x}\,x-\frac{1}{24}\,(a+4\,b)\left(\log\left(\sqrt{-\frac{1}{x^{3}}+1}+1\right)-\log\left(\left|\sqrt{-\frac{1}{x^{3}}+1}-1\right|\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)*(x^4-x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*a*x^3 - a + 4*b)*sqrt(x^4 - x)*x - 1/24*(a + 4*b)*(log(sqrt(-1/x^3
+ 1) + 1) - log(abs(sqrt(-1/x^3 + 1) - 1)))
```

maple [C] time = 0.01, size = 620, normalized size = 10.69

$$a\left(\frac{x^{4}\sqrt{x^{4}-x}}{6}-\frac{x\sqrt{x^{4}-x}}{12}-\frac{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}}\,(-1+x)^{2}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(-1+x)}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)-\mathrm{EllipticPi}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\frac{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{4\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(-1+x)\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)+b\left(\frac{x\sqrt{x^{4}-x}}{3}-\frac{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}}\,(-1+x)^{2}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(-1+x)}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)-\mathrm{EllipticPi}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\frac{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)\right)}{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(-1+x)\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b)*(x^4-x)^(1/2),x)
```

```
[Out] a*(1/6*x^4*(x^4-x)^(1/2)-1/12*x*(x^4-x)^(1/2)-1/4*(1/2-1/2*I*3^(1/2))*((-3/
2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*
I*3^(1/2))/(-1/2-1/2*I*3^(1/2))/(-1+x))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+
1/2*I*3^(1/2))/(-1+x))^(1/2)/(-3/2+1/2*I*3^(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^
(1/2))*(x+1/2-1/2*I*3^(1/2)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/
2+1/2*I*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-EllipticPi(((-3/2+1/2*I*3^(1/2
))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),(-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^
(1/2)),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/
2*I*3^(1/2)))^(1/2))))+b*(1/3*x*(x^4-x)^(1/2)-(1/2-1/2*I*3^(1/2))*((-3/2+1/
2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^
(1/2))/(-1/2-1/2*I*3^(1/2))/(-1+x))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*
I*3^(1/2))/(-1+x))^(1/2)/(-3/2+1/2*I*3^(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2
))*(x+1/2-1/2*I*3^(1/2)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/
2*I*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-EllipticPi(((-3/2+1/2*I*3^(1/2))*x
/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),(-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*
3^(1/2)))^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax^{3}+b\right)\sqrt{x^{4}-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)*(x^4-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^3 + b)*sqrt(x^4 - x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{x^{4}-x}\,\left(a\,x^{3}+b\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x)^(1/2)*(b + a*x^3),x)
```

```
[Out] int((x^4 - x)^(1/2)*(b + a*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x (x-1) \left(x^2+x+1\right)} \left(a x^3+b\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b)*(x**4-x)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(x - 1)*(x**2 + x + 1))*(a*x**3 + b), x)
```

3.733 $\int \frac{-2-x+2x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=58

$$\frac{1}{2}\tanh^{-1}\left(\frac{2x^2}{x^4+2x^2+(x^2-1)\sqrt{x^4+x^2+1}+1}\right)-2\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Rubi [A] time = 0.10, antiderivative size = 46, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1687, 1698, 203, 1247, 724, 206}

$$\frac{1}{2}\tanh^{-1}\left(\frac{1-x^2}{2\sqrt{x^4+x^2+1}}\right)-2\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 - x + 2*x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] -2*ArcTan[x/Sqrt[1 + x^2 + x^4]] + ArcTanh[(1 - x^2)/(2*Sqrt[1 + x^2 + x^4]
)]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pr_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Module[{r = Expon[Pr, x], k}, Int[Sum[Coeff[Pr, x, 2*k]*
x^(2*k), {k, 0, r/2}]*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[C
oeff[Pr, x, 2*k + 1]*x^(2*k), {k, 0, (r - 1)/2}]*(d + e*x^2)^q*(a + b*x^2 +
 c*x^4)^p, x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Pr, x] &&  !Poly
Q[Pr, x^2]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
```

```
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2-x+2x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx &= -\int \frac{x}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx+\int \frac{-2+2x^2}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)\sqrt{1+x+x^2}}\,dx,x,x^2\right)\right)-2\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt{1+x^2}}\right.\\
&= -2\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)+\operatorname{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{1-x^2}{\sqrt{1+x^2+x^4}}\right)\\
&= -2\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{1-x^2}{2\sqrt{1+x^2+x^4}}\right)
\end{aligned}$$

Mathematica [C]   time = 0.21, size = 170, normalized size = 2.93

$$\frac{1}{2}\tanh^{-1}\left(\frac{1-x^2}{2\sqrt{x^4+x^2+1}}\right)+\frac{2(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)}{\sqrt{x^4+x^2+1}}-\frac{4(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\,\Pi\left(\sqrt[3]{-1};i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 - x + 2*x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ArcTanh[(1 - x^2)/(2*Sqrt[1 + x^2 + x^4])]/2 + (2*(-1)^(2/3)*Sqrt[1 + (-1)^
(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)
^(2/3)])/Sqrt[1 + x^2 + x^4] - (4*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[
1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2
/3)])/Sqrt[1 + x^2 + x^4]
```

IntegrateAlgebraic [A]   time = 0.53, size = 58, normalized size = 1.00

$$\frac{1}{2}\tanh^{-1}\left(\frac{2x^2}{x^4+2x^2+\left(x^2-1\right)\sqrt{x^4+x^2+1}+1}\right)-2\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 - x + 2*x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] -2*ArcTan[x/Sqrt[1 + x^2 + x^4]] + ArcTanh[(2*x^2)/(1 + 2*x^2 + x^4 + (-1 +
 x^2)*Sqrt[1 + x^2 + x^4])]/2
```

fricas [A]   time = 0.51, size = 63, normalized size = 1.09

$$2\arctan\left(\frac{\sqrt{x^4+x^2+1}}{x}\right)+\frac{1}{4}\log\left(\frac{5x^4+2x^2-4\sqrt{x^4+x^2+1}\left(x^2-1\right)+5}{x^4+2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x-2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x^4 + x^2 + 1)/x) + 1/4*log((5*x^4 + 2*x^2 - 4*sqrt(x^4 + x^2
 + 1)*(x^2 - 1) + 5)/(x^4 + 2*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2 - x - 2}{\sqrt{x^4 + x^2 + 1}\left(x^2 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x-2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 - x - 2)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

maple [C] time = 0.05, size = 219, normalized size = 3.78

$$\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}, \frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}} - \frac{\operatorname{arctanh}\left(\frac{x^2}{2\sqrt{x^4+x^2+1}} - \frac{1}{2\sqrt{x^4+x^2+1}}\right)}{2} - \frac{4\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{i x^2\sqrt{3}}{2}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x, -\frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}, \frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x-2)/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 4/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*
3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2
),1/2*(-2+2*I*3^(1/2))^(1/2))-1/2*arctanh(1/2/(x^4+x^2+1)^(1/2)*x^2-1/2/(x^
4+x^2+1)^(1/2))-4/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^
(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/
2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)
/(-1/2+1/2*I*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2 - x - 2}{\sqrt{x^4 + x^2 + 1}\left(x^2 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x-2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x - 2)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{-2\,x^2 + x + 2}{\left(x^2 + 1\right)\sqrt{x^4 + x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 2*x^2 + 2)/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

```
[Out] int(-(x - 2*x^2 + 2)/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2 - x - 2}{\sqrt{\left(x^2 - x + 1\right)\left(x^2 + x + 1\right)}\left(x^2 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x-2)/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((2*x**2 - x - 2)/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)),
 x)
```

3.734 $\int \frac{x^3(-5b+9ax^4)}{\sqrt[4]{-bx+ax^5}\,(-2-bx^5+ax^9)}\,dx$

Optimal. Leaf size=58

$$2^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{ax^5-bx}}{\sqrt[4]{2}}\right)-2^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{ax^5-bx}}{\sqrt[4]{2}}\right)$$

Rubi [F] time = 2.64, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3(-5b+9ax^4)}{\sqrt[4]{-bx+ax^5}\,(-2-bx^5+ax^9)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*(-5*b + 9*a*x^4))/((-(b*x) + a*x^5)^(1/4)*(-2 - b*x^5 + a*x^9)),x]
```

```
[Out] (20*b*x^(1/4)*(-b + a*x^4)^(1/4)*Defer[Subst][Defer[Int][x^14/((-b + a*x^16
)^(1/4)*(2 + b*x^20 - a*x^36)), x], x, x^(1/4)])/(-(b*x) + a*x^5)^(1/4) + (
36*a*x^(1/4)*(-b + a*x^4)^(1/4)*Defer[Subst][Defer[Int][x^30/((-b + a*x^16)
^(1/4)*(-2 - b*x^20 + a*x^36)), x], x, x^(1/4)])/(-(b*x) + a*x^5)^(1/4)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(-5b+9ax^4)}{\sqrt[4]{-bx+ax^5}\,(-2-bx^5+ax^9)}\,dx &= \frac{\left(\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\int \frac{x^{11/4}(-5b+9ax^4)}{\sqrt[4]{-b+ax^4}\,(-2-bx^5+ax^9)}\,dx}{\sqrt[4]{-bx+ax^5}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\text{Subst}\left(\int \frac{x^{14}(-5b+9ax^{16})}{\sqrt[4]{-b+ax^{16}}\,(-2-bx^{20}+ax^{36})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^5}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\text{Subst}\left(\int\left(\frac{5bx^{14}}{\sqrt[4]{-b+ax^{16}}\,(2+bx^{20}-ax^{36})}+\frac{9ax^{30}}{\sqrt[4]{-b+ax^{16}}\,(-2-bx^{20}+ax^{36})}\right.\right.}{\sqrt[4]{-bx+ax^5}}\\
&= \frac{\left(36a\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\text{Subst}\left(\int \frac{x^{30}}{\sqrt[4]{-b+ax^{16}}\,(-2-bx^{20}+ax^{36})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^5}}+\frac{\left(20b\sqrt[4]{\cdot}\right.}{}
\end{aligned}$$

Mathematica [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{x^3(-5b+9ax^4)}{\sqrt[4]{-bx+ax^5}\,(-2-bx^5+ax^9)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(-5*b + 9*a*x^4))/((-(b*x) + a*x^5)^(1/4)*(-2 - b*x^5 + a*x^
9)),x]
```

```
[Out] Integrate[(x^3*(-5*b + 9*a*x^4))/((-(b*x) + a*x^5)^(1/4)*(-2 - b*x^5 + a*x^
9)), x]
```

IntegrateAlgebraic [A] time = 15.12, size = 58, normalized size = 1.00

$$2^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{ax^5-bx}}{\sqrt[4]{2}}\right)-2^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{ax^5-bx}}{\sqrt[4]{2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(-5*b + 9*a*x^4))/((-(b*x) + a*x^5)^(1/4)*(-2 - b*x
^5 + a*x^9)),x]
```

```
[Out] 2^(3/4)*ArcTan[(x*(-(b*x) + a*x^5)^(1/4))/2^(1/4)] - 2^(3/4)*ArcTanh[(x*(-(
b*x) + a*x^5)^(1/4))/2^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(9*a*x^4-5*b)/(a*x^5-b*x)^(1/4)/(a*x^9-b*x^5-2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(9\,ax^4-5\,b)x^3}{(ax^9-bx^5-2)(ax^5-bx)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(9*a*x^4-5*b)/(a*x^5-b*x)^(1/4)/(a*x^9-b*x^5-2),x, algorithm=
"giac")
```

```
[Out] integrate((9*a*x^4 - 5*b)*x^3/((a*x^9 - b*x^5 - 2)*(a*x^5 - b*x)^(1/4)), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(9a\,x^4-5b\right)}{\left(a\,x^5-bx\right)^{\frac{1}{4}}\left(a\,x^9-b\,x^5-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(9*a*x^4-5*b)/(a*x^5-b*x)^(1/4)/(a*x^9-b*x^5-2),x)
```

```
[Out] int(x^3*(9*a*x^4-5*b)/(a*x^5-b*x)^(1/4)/(a*x^9-b*x^5-2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(9\,ax^4-5\,b)x^3}{(ax^9-bx^5-2)(ax^5-bx)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(9*a*x^4-5*b)/(a*x^5-b*x)^(1/4)/(a*x^9-b*x^5-2),x, algorithm=
"maxima")
```

```
[Out] integrate((9*a*x^4 - 5*b)*x^3/((a*x^9 - b*x^5 - 2)*(a*x^5 - b*x)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,\left(5\,b-9\,a\,x^4\right)}{{\left(a\,x^5-b\,x\right)}^{1/4}\,\left(-a\,x^9+b\,x^5+2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(5*b - 9*a*x^4))/((a*x^5 - b*x)^(1/4)*(b*x^5 - a*x^9 + 2)),x)
```

```
[Out] int((x^3*(5*b - 9*a*x^4))/((a*x^5 - b*x)^(1/4)*(b*x^5 - a*x^9 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(9 a x^{4} - 5 b\right)}{\sqrt[4]{x \left(a x^{4} - b\right)} \left(a x^{9} - b x^{5} - 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(9*a*x**4-5*b)/(a*x**5-b*x)**(1/4)/(a*x**9-b*x**5-2),x)
```

```
[Out] Integral(x**3*(9*a*x**4 - 5*b)/((x*(a*x**4 - b))**(1/4)*(a*x**9 - b*x**5 -
2)), x)
```

3.735 $\int \frac{1+x^2}{(-1+x^2)\sqrt{x+x^2+x^3}} \, dx$

Optimal. Leaf size=59

$$-\tan^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x^3+x^2+x}}{x^2+x+1}\right)}{\sqrt{3}}$$

Rubi [C] time = 1.01, antiderivative size = 322, normalized size of antiderivative = 5.46, number of steps used = 17, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2056, 6725, 716, 1103, 934, 169, 538, 537}

$$\frac{\sqrt{x}(x+1)\sqrt{\frac{x^2+x+1}{(x+1)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x^3+x^2+x}}-\frac{4\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\Pi\left(\frac{1}{2}\left(1-i\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}-\frac{4\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\Pi\left(\frac{1}{2}\left(-1+i\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^2)/((-1 + x^2)*Sqrt[x + x^2 + x^3]),x]
```

```
[Out] (Sqrt[x]*(1 + x)*Sqrt[(1 + x + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]],
 1/4])/Sqrt[x + x^2 + x^3] - (4*Sqrt[x]*Sqrt[1 + (2*x)/(1 - I*Sqrt[3])]*Sqr
t[1 + (2*x)/(1 + I*Sqrt[3])]*EllipticPi[(1 - I*Sqrt[3])/2, ArcSin[((1 - I*S
qrt[3])*Sqrt[x])/2], (I + Sqrt[3])/(I - Sqrt[3])])/((1 - I*Sqrt[3])*Sqrt[x
+ x^2 + x^3]) - (4*Sqrt[x]*Sqrt[1 + (2*x)/(1 - I*Sqrt[3])]*Sqrt[1 + (2*x)/(
1 + I*Sqrt[3])]*EllipticPi[(-1 + I*Sqrt[3])/2, ArcSin[((1 - I*Sqrt[3])*Sqrt
[x])/2], (I + Sqrt[3])/(I - Sqrt[3])])/((1 - I*Sqrt[3])*Sqrt[x + x^2 + x^3]
)
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 716

```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2,
Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt{x+x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\,\sqrt{1+x+x^2}\right)\int \frac{1+x^2}{\sqrt{x}\left(-1+x^2\right)\sqrt{1+x+x^2}}\,dx}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\,\sqrt{1+x+x^2}\right)\int \left(\frac{1}{\sqrt{x}\,\sqrt{1+x+x^2}}+\frac{2}{\sqrt{x}\left(-1+x^2\right)\sqrt{1+x+x^2}}\right)dx}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\,\sqrt{1+x+x^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{1+x+x^2}}\,dx}{\sqrt{x+x^2+x^3}} + \frac{\left(2\sqrt{x}\,\sqrt{1+x+x^2}\right)\int \frac{1}{\sqrt{x}\left(-1+x^2\right)\sqrt{1+x+x^2}}}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt{1+x+x^2}\right)\int \left(-\frac{1}{2(1-x)\sqrt{x}\,\sqrt{1+x+x^2}}-\frac{1}{2\sqrt{x}\,(1+x)\sqrt{1+x+x^2}}\right)dx}{\sqrt{x+x^2+x^3}} + \frac{\left(2\sqrt{x}\,\sqrt{1}\right.}{}$$

$$= \frac{\sqrt{x}\,(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} - \frac{\left(\sqrt{x}\,\sqrt{1+x+x^2}\right)\int \frac{1}{(1-x)\sqrt{x}\,\sqrt{1+x+x}}}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\sqrt{x}\,(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} - \frac{\left(\sqrt{x}\,\sqrt{1-i\sqrt{3}+2x}\,\sqrt{1+i\sqrt{3}+2}\right.}{\sqrt{x+}}$$

$$= \frac{\sqrt{x}\,(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} + \frac{\left(2\sqrt{x}\,\sqrt{1-i\sqrt{3}+2x}\,\sqrt{1+i\sqrt{3}+}\right.}{}$$

$$= \frac{\sqrt{x}\,(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} + \frac{\left(2\sqrt{x}\,\sqrt{1+i\sqrt{3}+2x}\,\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\right.}{}$$

$$= \frac{\sqrt{x}\,(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} + \frac{\left(2\sqrt{x}\,\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\,\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\right)\mathrm{S}}{\sqrt{}}$$

$$= \frac{\sqrt{x}\,(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} - \frac{4\sqrt{x}\,\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\,\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\,\Pi\left(\right.}{\left(1-i\sqrt{\right.}}$$

Mathematica [C] time = 2.39, size = 667, normalized size = 11.31

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2)/((-1 + x^2)*Sqrt[x + x^2 + x^3]),x]
```

```
[Out] (-2*(-1)^(2/3)*(-1 + x^(-2))*x^(7/2)*(1 + x^2)*(Sqrt[1 - (-1)^(2/3)/x]*Sqrt
[((-1)^(1/3) + x)/x]*EllipticF[I*ArcSinh[(-1)^(5/6)/Sqrt[x]], (-1)^(2/3)] +
 (Sqrt[(1 - (-1)^(1/3) + Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x
]))]*(-1 + (-1)^(1/3)*Sqrt[x])^2*Sqrt[(-1 + (-1)^(2/3)*Sqrt[x])/((1 + (-1)^
(1/3))*(-1 + (-1)^(1/3)*Sqrt[x]))]*Sqrt[-((1 + (-1)^(2/3)*Sqrt[x])/(-1 + (-
1)^(1/3) + Sqrt[x]))]*((1 + (-1)^(1/3))*EllipticF[ArcSin[Sqrt[(1 - (-1)^(1/
3) + Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x]))]], -3] - 2*(-1)^
(1/3)*EllipticPi[-1, ArcSin[Sqrt[(1 - (-1)^(1/3) + Sqrt[x])/((1 + (-1)^(1/3
```

```
))*(-1 + (-1)^(1/3)*Sqrt[x]))]], -3]))/((-1 + (-1)^(1/3))^2*x) - (Sqrt[(1 -
 (-1)^(1/3) + Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x]))]*(-1 +
(-1)^(1/3)*Sqrt[x])^2*Sqrt[(-1 + (-1)^(2/3)*Sqrt[x])/((1 + (-1)^(1/3))*(-1
+ (-1)^(1/3)*Sqrt[x]))]*Sqrt[-((1 + (-1)^(2/3)*Sqrt[x])/(-1 + (-1)^(1/3) +
Sqrt[x]))]*((-1 + (-1)^(1/3))*EllipticF[ArcSin[Sqrt[(1 - (-1)^(1/3) + Sqrt[
x])/((1 + (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x]))]], -3] - 2*(-1)^(1/3)*Elli
pticPi[3, ArcSin[Sqrt[(1 - (-1)^(1/3) + Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-
1)^(1/3)*Sqrt[x]))]], -3]))/((-1 + (-1)^(1/3))^2*x) - Sqrt[1 - (-1)^(2/3)/x
]*Sqrt[((-1)^(1/3) + x)/x]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)/Sqrt
[x]], (-1)^(2/3)]))/(Sqrt[x*(1 + x + x^2)]*(-1 + x^4))
```

IntegrateAlgebraic [A] time = 0.12, size = 59, normalized size = 1.00

$$-\tan^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x^3+x^2+x}}{x^2+x+1}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 + x^2)*Sqrt[x + x^2 + x^3]),x]
```

```
[Out] -ArcTan[Sqrt[x + x^2 + x^3]/(1 + x + x^2)] - ArcTanh[(Sqrt[3]*Sqrt[x + x^2
+ x^3])/(1 + x + x^2)]/Sqrt[3]
```

fricas [A] time = 0.46, size = 89, normalized size = 1.51

$$\frac{1}{12}\sqrt{3}\log\left(\frac{x^4+20x^3-4\sqrt{3}\sqrt{x^3+x^2+x}\left(x^2+4x+1\right)+30x^2+20x+1}{x^4-4x^3+6x^2-4x+1}\right)+\frac{1}{2}\arctan\left(\frac{x^2+1}{2\sqrt{x^3+x^2+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^3+x^2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log((x^4 + 20*x^3 - 4*sqrt(3)*sqrt(x^3 + x^2 + x)*(x^2 + 4*x +
 1) + 30*x^2 + 20*x + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)) + 1/2*arctan(1/2*
(x^2 + 1)/sqrt(x^3 + x^2 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+1}{\sqrt{x^3+x^2+x}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^3+x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/(sqrt(x^3 + x^2 + x)*(x^2 - 1)), x)
```

maple [C] time = 0.02, size = 420, normalized size = 7.12

$$\frac{2\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}},\frac{\sqrt{3}\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{3\sqrt{x^3+x^2+x}}+\frac{2\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}},\frac{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}},\frac{\sqrt{3}\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{3\sqrt{x^3+x^2+x}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}-\frac{2\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}},\frac{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{1}{2}-\frac{i\sqrt{3}}{2}},\frac{\sqrt{3}\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{3\sqrt{x^3+x^2+x}\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-1)/(x^3+x^2+x)^(1/2),x)
```

```
[Out] 2/3*(1/2+1/2*I*3^(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3
^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/
2)/(x^3+x^2+x)^(1/2)*EllipticF(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^
(1/2),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))+2/3*(1/2+1/2*I*3^
(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3^(1/2)*(I*(x+1/2-
```

```
1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1
/2)/(-3/2-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/
2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2))-2/3*(1/2+1/2*I*3^(1/2))*((x+1/2+1/2*I*3^(1/2)
)/(1/2+1/2*I*3^(1/2)))^(1/2)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2)/(1/2-1/2*I*3^(1/2))*Elli
pticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1/2*I*3^(1/2
))/(1/2-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x^3+x^2+x}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^3+x^2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(sqrt(x^3 + x^2 + x)*(x^2 - 1)), x)
```

mupad [B] time = 0.17, size = 223, normalized size = 3.78

$$-\frac{\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\left(\sqrt{3}+1i\right)\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+\Pi\left(\frac{1}{2}-\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+\Pi\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\right)1i}{\sqrt{x^3+x^2-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((x^2 - 1)*(x + x^2 + x^3)^(1/2)),x)
```

```
[Out] -((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/2
)*(3^(1/2) + 1i)*(ellipticPi(1/2 - (3^(1/2)*1i)/2, asin((x/((3^(1/2)*1i)/2
- 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)) - ellipticF
(asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*
1i)/2 + 1/2)) + ellipticPi((3^(1/2)*1i)/2 - 1/2, asin((x/((3^(1/2)*1i)/2 -
1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))*1i)/(x^2 + x
^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x\left(x^2+x+1\right)}\,(x-1)\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-1)/(x**3+x**2+x)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)/(sqrt(x*(x**2 + x + 1))*(x - 1)*(x + 1)), x)
```

3.736 $\int \frac{abc-(a+b+c)x^2+2x^3}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(-abc+(ab+ac+bc-d)x-(a+b+c)x^2+x^3\right)}\,dx$

Optimal. Leaf size=59

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^3(-a-b-c)+x^2(ab+ac+bc)-abcx+x^4}}{\sqrt{d}\,x}\right)}{\sqrt{d}}$$

Rubi [F] time = 4.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{abc-(a+b+c)x^2+2x^3}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(-abc+(ab+ac+bc-d)x-(a+b+c)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*b*c - (a + b + c)*x^2 + 2*x^3)/(Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*(
-(a*b*c) + (a*b + a*c + b*c - d)*x - (a + b + c)*x^2 + x^3)),x]
```

```
[Out] 2*Defer[Int][1/Sqrt[x*(-a + x)*(-b + x)*(-c + x)], x] - 3*a*b*c*Defer[Int][
1/(Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*(a*b*c - (b*c + a*(b + c) - d)*x + (a
 + b + c)*x^2 - x^3)), x] + 2*(b*c + a*(b + c) - d)*Defer[Int][x/(Sqrt[x*(-
a + x)*(-b + x)*(-c + x)]*(a*b*c - (b*c + a*(b + c) - d)*x + (a + b + c)*x^
2 - x^3)), x] - (a + b + c)*Defer[Int][x^2/(Sqrt[x*(-a + x)*(-b + x)*(-c +
x)]*(a*b*c - (b*c + a*(b + c) - d)*x + (a + b + c)*x^2 - x^3)), x]
```

Rubi steps

$$\int \frac{abc-(a+b+c)x^2+2x^3}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(-abc+(ab+ac+bc-d)x-(a+b+c)x^2+x^3\right)}\,dx = \int \frac{}{\sqrt{x(-a+x)(-b+x)(-}}$$

$$= \int \left(\frac{2}{\sqrt{x(-a+x)(-b+x)(-}}\right.$$

$$= 2\int \frac{1}{\sqrt{x(-a+x)(-b+x)(}}$$

$$= 2\int \frac{1}{\sqrt{x(-a+x)(-b+x)(}}$$

$$= 2\int \frac{1}{\sqrt{x(-a+x)(-b+x)(}}$$

Mathematica [C] time = 13.10, size = 6921, normalized size = 117.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*b*c - (a + b + c)*x^2 + 2*x^3)/(Sqrt[x*(-a + x)*(-b + x)*(-c +
 x)]*(-(a*b*c) + (a*b + a*c + b*c - d)*x - (a + b + c)*x^2 + x^3)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.63, size = 59, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^3(-a-b-c)+x^2(ab+ac+bc)-abcx+x^4}}{\sqrt{d}\,x}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b*c - (a + b + c)*x^2 + 2*x^3)/(Sqrt[x*(-a + x)*(-b +
 x)*(-c + x)]*(-(a*b*c) + (a*b + a*c + b*c - d)*x - (a + b + c)*x^2 + x^3))
,x]
```

```
[Out] (-2*ArcTanh[Sqrt[-(a*b*c*x) + (a*b + a*c + b*c)*x^2 + (-a - b - c)*x^3 + x^
4]/(Sqrt[d]*x)])/Sqrt[d]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*c-(a+b+c)*x^2+2*x^3)/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)/(-a*b*c+
(a*b+a*c+b*c-d)*x-(a+b+c)*x^2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{abc - (a+b+c)x^2 + 2x^3}{\sqrt{-(a-x)(b-x)(c-x)x}\left(abc + (a+b+c)x^2 - x^3 - (ab+ac+bc-d)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*c-(a+b+c)*x^2+2*x^3)/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)/(-a*b*c+
(a*b+a*c+b*c-d)*x-(a+b+c)*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate(-(a*b*c - (a + b + c)*x^2 + 2*x^3)/(sqrt(-(a - x)*(b - x)*(c - x)
*x)*(a*b*c + (a + b + c)*x^2 - x^3 - (a*b + a*c + b*c - d)*x)), x)
```

maple [C] time = 0.09, size = 525, normalized size = 8.90

$$-\frac{4a\sqrt{\frac{(a-c)x}{a(-c+x)}}\,(-c+x)^2\sqrt{\frac{c(-b+x)}{b(-c+x)}}\sqrt{\frac{c(-a+x)}{a(-c+x)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{(a-c)x}{a(-c+x)}},\sqrt{\frac{(-b+c)a}{b(c-a)}}\right)}{(a-c)\,c\sqrt{x(-a+x)(-b+x)(-c+x)}}+\frac{2a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^3+(-a-b-c)\_Z^2+(ab+ac+bc-d)\_Z-abc\right)}\frac{\left(-\_\alpha^2a-\_\alpha^2b-\_\alpha^2c+2\_\alpha ab+2\_\alpha ac+2\_\alpha bc-3abc-2\_\alpha d\right)(-c+x)^2\left(\_\alpha^2-\_\alpha a-\_\alpha b+ab-d\right)\sqrt{\frac{(a-c)x}{a(-c+x)}}\sqrt{\frac{c(-b+x)}{b(-c+x)}}\sqrt{\frac{c(-a+x)}{a(-c+x)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{(a-c)x}{a(-c+x)}},\sqrt{\frac{(-b+c)a}{b(c-a)}}\right)-\frac{\left(\_\alpha^2-\_\alpha a-\_\alpha b-\_\alpha c+ab+ac+bc-d\right)\mathrm{EllipticPi}\left(\sqrt{\frac{(a-c)x}{a(-c+x)}},-\frac{\_\alpha^2-\_\alpha a-\_\alpha b-\_\alpha c+ac+bc-d}{b(a-c)},\sqrt{\frac{(-b+c)a}{b(c-a)}}\right)}{ab}\right)}{\left(-3\_\alpha^2+2\_\alpha a+2\_\alpha b+2\_\alpha c-ab-ac-bc+d\right)(a-c)\sqrt{x(-a+x)(-b+x)(-c+x)}}\right)}{c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b*c-(a+b+c)*x^2+2*x^3)/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)/(-a*b*c+(a*b+a
*c+b*c-d)*x-(a+b+c)*x^2+x^3),x)
```

```
[Out] -4*a*((a-c)*x/a/(-c+x))^(1/2)*(-c+x)^2*(c*(-b+x)/b/(-c+x))^(1/2)*(c*(-a+x)/
a/(-c+x))^(1/2)/(a-c)/c/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)*EllipticF(((a-c)*x/a
/(-c+x))^(1/2),((-b+c)*a/b/(c-a))^(1/2))+2*a/c^2/d*sum((-_alpha^2*a-_alpha^
2*b-_alpha^2*c+2*_alpha*a*b+2*_alpha*a*c+2*_alpha*b*c-3*a*b*c-2*_alpha*d)/(
-3*_alpha^2+2*_alpha*a+2*_alpha*b+2*_alpha*c-a*b-a*c-b*c+d)*(-c+x)^2/(a-c)*
(_alpha^2-_alpha*a-_alpha*b+a*b-d)*((a-c)*x/a/(-c+x))^(1/2)*(c*(-b+x)/b/(-c
+x))^(1/2)*(c*(-a+x)/a/(-c+x))^(1/2)/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)*(Ellipt
icF(((a-c)*x/a/(-c+x))^(1/2),((-b+c)*a/b/(c-a))^(1/2))-(_alpha^2-_alpha*a-_
alpha*b-_alpha*c+a*b+a*c+b*c-d)/a/b*EllipticPi(((a-c)*x/a/(-c+x))^(1/2),-(_
alpha^2-_alpha*a-_alpha*b-_alpha*c+a*c+b*c-d)/b/(a-c),((-b+c)*a/b/(c-a))^(1
/2))),_alpha=RootOf(_Z^3+(-a-b-c)*_Z^2+(a*b+a*c+b*c-d)*_Z-a*b*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{abc - (a+b+c)x^2 + 2x^3}{\sqrt{-(a-x)(b-x)(c-x)x}\left(abc + (a+b+c)x^2 - x^3 - (ab+ac+bc-d)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*c-(a+b+c)*x^2+2*x^3)/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)/(-a*b*c+
(a*b+a*c+b*c-d)*x-(a+b+c)*x^2+x^3),x, algorithm="maxima")
```

```
[Out] -integrate((a*b*c - (a + b + c)*x^2 + 2*x^3)/(sqrt(-(a - x)*(b - x)*(c - x)
*x)*(a*b*c + (a + b + c)*x^2 - x^3 - (a*b + a*c + b*c - d)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{2\,x^3+(-a-b-c)\,x^2+a\,b\,c}{\sqrt{-x\,(a-x)\,(b-x)\,(c-x)}\,\left(x^3+(-a-b-c)\,x^2+(a\,b-d+a\,c+b\,c)\,x-a\,b\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3 - x^2*(a + b + c) + a*b*c)/((-x*(a - x)*(b - x)*(c - x))^(1/2)*(
x*(a*b - d + a*c + b*c) - x^2*(a + b + c) + x^3 - a*b*c)),x)
```

```
[Out] int((2*x^3 - x^2*(a + b + c) + a*b*c)/((-x*(a - x)*(b - x)*(c - x))^(1/2)*(
x*(a*b - d + a*c + b*c) - x^2*(a + b + c) + x^3 - a*b*c)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*c-(a+b+c)*x**2+2*x**3)/(x*(-a+x)*(-b+x)*(-c+x))**(1/2)/(-a*b
*c+(a*b+a*c+b*c-d)*x-(a+b+c)*x**2+x**3),x)
```

```
[Out] Timed out
```

3.737 $\int \frac{(-b+ax^3)\sqrt{b+ax^3}}{x^7}\,dx$

Optimal. Leaf size=59

$$\frac{(2b-3ax^3)\sqrt{ax^3+b}}{12x^6}-\frac{5a^2\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)}{12\sqrt{b}}$$

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 47, 63, 208}

$$-\frac{5a^2\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)}{12\sqrt{b}}-\frac{5a\sqrt{ax^3+b}}{12x^3}+\frac{(ax^3+b)^{3/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a*x^3)*Sqrt[b + a*x^3])/x^7,x]
```

```
[Out] (-5*a*Sqrt[b + a*x^3])/(12*x^3) + (b + a*x^3)^(3/2)/(6*x^6) - (5*a^2*ArcTan
h[Sqrt[b + a*x^3]/Sqrt[b]])/(12*Sqrt[b])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-b+ax^3\right)\sqrt{b+ax^3}}{x^7}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(-b+ax)\sqrt{b+ax}}{x^3}\,dx,x,x^3\right)\\
&= \frac{\left(b+ax^3\right)^{3/2}}{6x^6}+\frac{1}{12}(5a)\operatorname{Subst}\left(\int \frac{\sqrt{b+ax}}{x^2}\,dx,x,x^3\right)\\
&= -\frac{5a\sqrt{b+ax^3}}{12x^3}+\frac{\left(b+ax^3\right)^{3/2}}{6x^6}+\frac{1}{24}\left(5a^2\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{b+ax}}\,dx,x,x^3\right)\\
&= -\frac{5a\sqrt{b+ax^3}}{12x^3}+\frac{\left(b+ax^3\right)^{3/2}}{6x^6}+\frac{1}{12}(5a)\operatorname{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^2}{a}}\,dx,x,\sqrt{b+ax^3}\right)\\
&= -\frac{5a\sqrt{b+ax^3}}{12x^3}+\frac{\left(b+ax^3\right)^{3/2}}{6x^6}-\frac{5a^2\tanh^{-1}\left(\frac{\sqrt{b+ax^3}}{\sqrt{b}}\right)}{12\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 76, normalized size = 1.29

$$\frac{-5a^2x^6\sqrt{\frac{ax^3}{b}+1}\,\tanh^{-1}\left(\sqrt{\frac{ax^3}{b}+1}\right)-3a^2x^6-abx^3+2b^2}{12x^6\sqrt{ax^3+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + a*x^3)*Sqrt[b + a*x^3])/x^7,x]
```

```
[Out] (2*b^2 - a*b*x^3 - 3*a^2*x^6 - 5*a^2*x^6*Sqrt[1 + (a*x^3)/b]*ArcTanh[Sqrt[1
 + (a*x^3)/b]])/(12*x^6*Sqrt[b + a*x^3])
```

IntegrateAlgebraic [A] time = 0.11, size = 59, normalized size = 1.00

$$\frac{\left(2b-3ax^3\right)\sqrt{ax^3+b}}{12x^6}-\frac{5a^2\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)}{12\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^3)*Sqrt[b + a*x^3])/x^7,x]
```

```
[Out] ((2*b - 3*a*x^3)*Sqrt[b + a*x^3])/(12*x^6) - (5*a^2*ArcTanh[Sqrt[b + a*x^3]
/Sqrt[b]])/(12*Sqrt[b])
```

fricas [A] time = 0.41, size = 138, normalized size = 2.34

$$\left[\frac{5\,a^2\sqrt{b}\,x^6\log\left(\frac{ax^3-2\,\sqrt{ax^3+b}\,\sqrt{b}+2\,b}{x^3}\right)-2\left(3\,abx^3-2\,b^2\right)\sqrt{ax^3+b}}{24\,bx^6},\frac{5\,a^2\sqrt{-b}\,x^6\arctan\left(\frac{\sqrt{ax^3+b}\,\sqrt{-b}}{b}\right)-\left(3\,abx^3-2\,b^2\right)\sqrt{ax^3+b}}{12\,bx^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(a*x^3+b)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/24*(5*a^2*sqrt(b)*x^6*log((a*x^3 - 2*sqrt(a*x^3 + b)*sqrt(b) + 2*b)/x^3)
 - 2*(3*a*b*x^3 - 2*b^2)*sqrt(a*x^3 + b))/(b*x^6), 1/12*(5*a^2*sqrt(-b)*x^6
```

```
*arctan(sqrt(a*x^3 + b)*sqrt(-b)/b) - (3*a*b*x^3 - 2*b^2)*sqrt(a*x^3 + b))/
(b*x^6)]
```

giac [A] time = 0.47, size = 70, normalized size = 1.19

$$\frac{\frac{5\,a^3\arctan\left(\frac{\sqrt{ax^3+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{3\left(ax^3+b\right)^{\frac{3}{2}}a^3-5\sqrt{ax^3+b}\,a^3b}{a^2x^6}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(a*x^3+b)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*(5*a^3*arctan(sqrt(a*x^3 + b)/sqrt(-b))/sqrt(-b) - (3*(a*x^3 + b)^(3/2
)*a^3 - 5*sqrt(a*x^3 + b)*a^3*b)/(a^2*x^6))/a
```

maple [B] time = 0.06, size = 97, normalized size = 1.64

$$a\left(-\frac{\sqrt{a\,x^3+b}}{3x^3}-\frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^3+b}}{\sqrt{b}}\right)}{3\sqrt{b}}\right)-b\left(-\frac{\sqrt{a\,x^3+b}}{6x^6}-\frac{a\sqrt{a\,x^3+b}}{12b\,x^3}+\frac{a^2\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^3+b}}{\sqrt{b}}\right)}{12b^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b)*(a*x^3+b)^(1/2)/x^7,x)
```

```
[Out] a*(-1/3*(a*x^3+b)^(1/2)/x^3-1/3*a*arctanh((a*x^3+b)^(1/2)/b^(1/2))/b^(1/2))
-b*(-1/6*(a*x^3+b)^(1/2)/x^6-1/12*a/b*(a*x^3+b)^(1/2)/x^3+1/12*a^2/b^(3/2)*
arctanh((a*x^3+b)^(1/2)/b^(1/2)))
```

maxima [B] time = 0.42, size = 158, normalized size = 2.68

$$\frac{1}{6}\left(\frac{a\log\left(\frac{\sqrt{ax^3+b}-\sqrt{b}}{\sqrt{ax^3+b}+\sqrt{b}}\right)}{\sqrt{b}}-\frac{2\sqrt{ax^3+b}}{x^3}\right)a+\frac{1}{24}\left(\frac{a^2\log\left(\frac{\sqrt{ax^3+b}-\sqrt{b}}{\sqrt{ax^3+b}+\sqrt{b}}\right)}{b^{\frac{3}{2}}}+\frac{2\left(\left(ax^3+b\right)^{\frac{3}{2}}a^2+\sqrt{ax^3+b}\,a^2b\right)}{\left(ax^3+b\right)^2b-2\left(ax^3+b\right)b^2+b^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(a*x^3+b)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*(a*log((sqrt(a*x^3 + b) - sqrt(b))/(sqrt(a*x^3 + b) + sqrt(b)))/sqrt(b)
 - 2*sqrt(a*x^3 + b)/x^3)*a + 1/24*(a^2*log((sqrt(a*x^3 + b) - sqrt(b))/(sq
rt(a*x^3 + b) + sqrt(b)))/b^(3/2) + 2*((a*x^3 + b)^(3/2)*a^2 + sqrt(a*x^3 +
 b)*a^2*b)/((a*x^3 + b)^2*b - 2*(a*x^3 + b)*b^2 + b^3))*b
```

mupad [B] time = 0.96, size = 74, normalized size = 1.25

$$\frac{b\sqrt{a\,x^3+b}}{6\,x^6}-\frac{a\sqrt{a\,x^3+b}}{4\,x^3}+\frac{5\,a^2\,\ln\left(\frac{\left(\sqrt{a\,x^3+b}-\sqrt{b}\right)^3\left(\sqrt{a\,x^3+b}+\sqrt{b}\right)}{x^6}\right)}{24\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b + a*x^3)^(1/2)*(b - a*x^3))/x^7,x)
```

```
[Out] (b*(b + a*x^3)^(1/2))/(6*x^6) - (a*(b + a*x^3)^(1/2))/(4*x^3) + (5*a^2*log(
(((b + a*x^3)^(1/2) - b^(1/2))^3*((b + a*x^3)^(1/2) + b^(1/2)))/x^6))/(24*b
^(1/2))
```

sympy [B] time = 65.16, size = 128, normalized size = 2.17

$$-\frac{a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^3}}}{3x^{\frac{3}{2}}}+\frac{a^{\frac{3}{2}}}{12x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^3}}}+\frac{\sqrt{a}\,b}{4x^{\frac{9}{2}}\sqrt{1+\frac{b}{ax^3}}}-\frac{5a^2\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x^{\frac{3}{2}}}\right)}{12\sqrt{b}}+\frac{b^2}{6\sqrt{a}\,x^{\frac{15}{2}}\sqrt{1+\frac{b}{ax^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b)*(a*x**3+b)**(1/2)/x**7,x)
```

```
[Out] -a**(3/2)*sqrt(1 + b/(a*x**3))/(3*x**(3/2)) + a**(3/2)/(12*x**(3/2)*sqrt(1
+ b/(a*x**3))) + sqrt(a)*b/(4*x**(9/2)*sqrt(1 + b/(a*x**3))) - 5*a**2*asinh
(sqrt(b)/(sqrt(a)*x**(3/2)))/(12*sqrt(b)) + b**2/(6*sqrt(a)*x**(15/2)*sqrt(
1 + b/(a*x**3)))
```

3.738 $\int \frac{\sqrt{b+ax^3}\left(2b+ax^3\right)}{x^7}\,dx$

Optimal. Leaf size=59

$$\frac{\left(-3ax^3-2b\right)\sqrt{ax^3+b}}{6x^6}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)}{6\sqrt{b}}$$

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 47, 63, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)}{6\sqrt{b}}-\frac{a\sqrt{ax^3+b}}{6x^3}-\frac{\left(ax^3+b\right)^{3/2}}{3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b + a*x^3]*(2*b + a*x^3))/x^7,x]
```

```
[Out] -1/6*(a*Sqrt[b + a*x^3])/x^3 - (b + a*x^3)^(3/2)/(3*x^6) - (a^2*ArcTanh[Sqr
t[b + a*x^3]/Sqrt[b]])/(6*Sqrt[b])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b+ax^3}\,(2b+ax^3)}{x^7}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{b+ax}\,(2b+ax)}{x^3}\,dx,x,x^3\right)\\
&= -\frac{(b+ax^3)^{3/2}}{3x^6}+\frac{1}{6}a\,\text{Subst}\left(\int \frac{\sqrt{b+ax}}{x^2}\,dx,x,x^3\right)\\
&= -\frac{a\sqrt{b+ax^3}}{6x^3}-\frac{(b+ax^3)^{3/2}}{3x^6}+\frac{1}{12}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{b+ax}}\,dx,x,x^3\right)\\
&= -\frac{a\sqrt{b+ax^3}}{6x^3}-\frac{(b+ax^3)^{3/2}}{3x^6}+\frac{1}{6}a\,\text{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^2}{a}}\,dx,x,\sqrt{b+ax^3}\right)\\
&= -\frac{a\sqrt{b+ax^3}}{6x^3}-\frac{(b+ax^3)^{3/2}}{3x^6}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b+ax^3}}{\sqrt{b}}\right)}{6\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 1.27

$$-\frac{a^2x^6\sqrt{\frac{ax^3}{b}+1}\,\tanh^{-1}\left(\sqrt{\frac{ax^3}{b}+1}\right)+3a^2x^6+5abx^3+2b^2}{6x^6\sqrt{ax^3+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b + a*x^3]*(2*b + a*x^3))/x^7,x]
```

```
[Out] -1/6*(2*b^2 + 5*a*b*x^3 + 3*a^2*x^6 + a^2*x^6*Sqrt[1 + (a*x^3)/b]*ArcTanh[S
qrt[1 + (a*x^3)/b]])/(x^6*Sqrt[b + a*x^3])
```

IntegrateAlgebraic [A] time = 0.09, size = 59, normalized size = 1.00

$$\frac{(-3ax^3-2b)\sqrt{ax^3+b}}{6x^6}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)}{6\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[b + a*x^3]*(2*b + a*x^3))/x^7,x]
```

```
[Out] ((-2*b - 3*a*x^3)*Sqrt[b + a*x^3])/(6*x^6) - (a^2*ArcTanh[Sqrt[b + a*x^3]/S
qrt[b]])/(6*Sqrt[b])
```

fricas [A] time = 0.45, size = 136, normalized size = 2.31

$$\left[\frac{a^2\sqrt{b}\,x^6\log\left(\frac{ax^3-2\,\sqrt{ax^3+b}\,\sqrt{b}+2\,b}{x^3}\right)-2\,(3\,abx^3+2\,b^2)\sqrt{ax^3+b}}{12\,bx^6},\ \frac{a^2\sqrt{-b}\,x^6\arctan\left(\frac{\sqrt{ax^3+b}\,\sqrt{-b}}{b}\right)-(3\,abx^3+2\,b^2)\sqrt{ax^3+b}}{6\,bx^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)^(1/2)*(a*x^3+2*b)/x^7,x, algorithm="fricas")
```

```
[Out] [1/12*(a^2*sqrt(b)*x^6*log((a*x^3 - 2*sqrt(a*x^3 + b)*sqrt(b) + 2*b)/x^3) -
 2*(3*a*b*x^3 + 2*b^2)*sqrt(a*x^3 + b))/(b*x^6), 1/6*(a^2*sqrt(-b)*x^6*arct
```

```
an(sqrt(a*x^3 + b)*sqrt(-b)/b) - (3*a*b*x^3 + 2*b^2)*sqrt(a*x^3 + b))/(b*x^
6)]
```

giac [A] time = 0.28, size = 69, normalized size = 1.17

$$\frac{\frac{a^3\arctan\left(\frac{\sqrt{ax^3+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{3\left(ax^3+b\right)^{\frac{3}{2}}a^3-\sqrt{ax^3+b}\,a^3b}{a^2x^6}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)^(1/2)*(a*x^3+2*b)/x^7,x, algorithm="giac")
```

```
[Out] 1/6*(a^3*arctan(sqrt(a*x^3 + b)/sqrt(-b))/sqrt(-b) - (3*(a*x^3 + b)^(3/2)*a
^3 - sqrt(a*x^3 + b)*a^3*b)/(a^2*x^6))/a
```

maple [B] time = 0.01, size = 97, normalized size = 1.64

$$a\left(-\frac{\sqrt{a\,x^3+b}}{3x^3}-\frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^3+b}}{\sqrt{b}}\right)}{3\sqrt{b}}\right)+2b\left(-\frac{\sqrt{a\,x^3+b}}{6x^6}-\frac{a\sqrt{a\,x^3+b}}{12b\,x^3}+\frac{a^2\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^3+b}}{\sqrt{b}}\right)}{12b^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b)^(1/2)*(a*x^3+2*b)/x^7,x)
```

```
[Out] a*(-1/3*(a*x^3+b)^(1/2)/x^3-1/3*a*arctanh((a*x^3+b)^(1/2)/b^(1/2))/b^(1/2))
+2*b*(-1/6*(a*x^3+b)^(1/2)/x^6-1/12*a/b*(a*x^3+b)^(1/2)/x^3+1/12*a^2/b^(3/2
)*arctanh((a*x^3+b)^(1/2)/b^(1/2)))
```

maxima [B] time = 0.43, size = 158, normalized size = 2.68

$$\frac{1}{6}\left(\frac{a\log\left(\frac{\sqrt{ax^3+b}-\sqrt{b}}{\sqrt{ax^3+b}+\sqrt{b}}\right)}{\sqrt{b}}-\frac{2\sqrt{ax^3+b}}{x^3}\right)a-\frac{1}{12}\left(\frac{a^2\log\left(\frac{\sqrt{ax^3+b}-\sqrt{b}}{\sqrt{ax^3+b}+\sqrt{b}}\right)}{b^{\frac{3}{2}}}+\frac{2\left(\left(ax^3+b\right)^{\frac{3}{2}}a^2+\sqrt{ax^3+b}\,a^2b\right)}{\left(ax^3+b\right)^2b-2\left(ax^3+b\right)b^2+b^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)^(1/2)*(a*x^3+2*b)/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*(a*log((sqrt(a*x^3 + b) - sqrt(b))/(sqrt(a*x^3 + b) + sqrt(b)))/sqrt(b)
 - 2*sqrt(a*x^3 + b)/x^3)*a - 1/12*(a^2*log((sqrt(a*x^3 + b) - sqrt(b))/(sq
rt(a*x^3 + b) + sqrt(b)))/b^(3/2) + 2*((a*x^3 + b)^(3/2)*a^2 + sqrt(a*x^3 +
 b)*a^2*b)/((a*x^3 + b)^2*b - 2*(a*x^3 + b)*b^2 + b^3))*b
```

mupad [B] time = 0.93, size = 74, normalized size = 1.25

$$\frac{a^2\,\ln\left(\frac{\left(\sqrt{a\,x^3+b}-\sqrt{b}\right)^3\left(\sqrt{a\,x^3+b}+\sqrt{b}\right)}{x^6}\right)}{12\,\sqrt{b}}-\frac{b\,\sqrt{a\,x^3+b}}{3\,x^6}-\frac{a\,\sqrt{a\,x^3+b}}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a*x^3)^(1/2)*(2*b + a*x^3))/x^7,x)
```

```
[Out] (a^2*log((((b + a*x^3)^(1/2) - b^(1/2))^3*((b + a*x^3)^(1/2) + b^(1/2)))/x^
6))/(12*b^(1/2)) - (b*(b + a*x^3)^(1/2))/(3*x^6) - (a*(b + a*x^3)^(1/2))/(2
*x^3)
```

sympy [B] time = 68.22, size = 128, normalized size = 2.17

$$-\frac{a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^3}}}{3x^{\frac{3}{2}}}-\frac{a^{\frac{3}{2}}}{6x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^3}}}-\frac{\sqrt{a}\,b}{2x^{\frac{9}{2}}\sqrt{1+\frac{b}{ax^3}}}-\frac{a^2\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x^{\frac{3}{2}}}\right)}{6\sqrt{b}}-\frac{b^2}{3\sqrt{a}\,x^{\frac{15}{2}}\sqrt{1+\frac{b}{ax^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b)**(1/2)*(a*x**3+2*b)/x**7,x)
```

```
[Out] -a**(3/2)*sqrt(1 + b/(a*x**3))/(3*x**(3/2)) - a**(3/2)/(6*x**(3/2)*sqrt(1 +
 b/(a*x**3))) - sqrt(a)*b/(2*x**(9/2)*sqrt(1 + b/(a*x**3))) - a**2*asinh(sq
rt(b)/(sqrt(a)*x**(3/2)))/(6*sqrt(b)) - b**2/(3*sqrt(a)*x**(15/2)*sqrt(1 +
b/(a*x**3)))
```

3.739 $\int \frac{3abcx-2(ab+ac+bc)x^2+(a+b+c)x^3}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(abc-(ab+ac+bc)x+(a+b+c)x^2+(-1+d)x^3\right)}\,dx$

Optimal. Leaf size=59

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{x^3(-a-b-c)+x^2(ab+ac+bc)-abcx+x^4}}{\sqrt{d}\,x^2}\right)}{\sqrt{d}}$$

Rubi [F] time = 6.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3abcx-2(ab+ac+bc)x^2+(a+b+c)x^3}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(abc-(ab+ac+bc)x+(a+b+c)x^2+(-1+d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3*a*b*c*x - 2*(a*b + a*c + b*c)*x^2 + (a + b + c)*x^3)/(Sqrt[x*(-a + x
)*(-b + x)*(-c + x)]*(a*b*c - (a*b + a*c + b*c)*x + (a + b + c)*x^2 + (-1 +
 d)*x^3)),x]
```

```
[Out] -(((a + b + c)*Defer[Int][1/Sqrt[x*(-a + x)*(-b + x)*(-c + x)], x])/(1 - d)
) + (a*b*c*(a + b + c)*Defer[Int][1/(Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*(a*
b*c - (b*c + a*(b + c))*x + (a + b + c)*x^2 - (1 - d)*x^3)), x])/(1 - d) -
((a^2*(b + c) + b*c*(b + c) + a*(b^2 + c^2 + 3*b*c*d))*Defer[Int][x/(Sqrt[x
*(-a + x)*(-b + x)*(-c + x)]*(a*b*c - (b*c + a*(b + c))*x + (a + b + c)*x^2
 - (1 - d)*x^3)), x])/(1 - d) + ((a^2 + b^2 + c^2 + 2*b*c*d + 2*a*(b + c)*d
)*Defer[Int][x^2/(Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*(a*b*c - (b*c + a*(b +
 c))*x + (a + b + c)*x^2 - (1 - d)*x^3)), x])/(1 - d)
```

Rubi steps

$$\int \frac{3abcx-2(ab+ac+bc)x^2+(a+b+c)x^3}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(abc-(ab+ac+bc)x+(a+b+c)x^2+(-1+d)x^3\right)}\,dx = \int \frac{}{\sqrt{x(-a+x)(-b}}$$

$$= \int \frac{}{\sqrt{x(-a+x)(-b}}$$

$$= \int \left(-\frac{a}{(1-d)\sqrt{x(-a}}\right.$$

$$= -\frac{(a+b+c)\int \frac{}{\sqrt{x(-}}}{1-}$$

$$= -\frac{(a+b+c)\int \frac{}{\sqrt{x(-}}}{1-}$$

$$= -\frac{(a+b+c)\int \frac{}{\sqrt{x(-}}}{1-}$$

Mathematica [C] time = 12.64, size = 28348, normalized size = 480.47

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3*a*b*c*x - 2*(a*b + a*c + b*c)*x^2 + (a + b + c)*x^3)/(Sqrt[x*(
-a + x)*(-b + x)*(-c + x)]*(a*b*c - (a*b + a*c + b*c)*x + (a + b + c)*x^2 +
 (-1 + d)*x^3)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.75, size = 59, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{x^3(-a-b-c)+x^2(ab+ac+bc)-abcx+x^4}}{\sqrt{d}\,x^2}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*a*b*c*x - 2*(a*b + a*c + b*c)*x^2 + (a + b + c)*x^3)/
(Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*(a*b*c - (a*b + a*c + b*c)*x + (a + b +
 c)*x^2 + (-1 + d)*x^3)),x]
```

```
[Out] (2*ArcTanh[Sqrt[-(a*b*c*x) + (a*b + a*c + b*c)*x^2 + (-a - b - c)*x^3 + x^4
]/(Sqrt[d]*x^2)])/Sqrt[d]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*b*c*x-2*(a*b+a*c+b*c)*x^2+(a+b+c)*x^3)/(x*(-a+x)*(-b+x)*(-c+
x))^(1/2)/(a*b*c-(a*b+a*c+b*c)*x+(a+b+c)*x^2+(-1+d)*x^3),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [A] time = 4.22, size = 62, normalized size = 1.05

$$-\frac{2\arctan\left(\frac{\sqrt{-\frac{abc}{x^3}+\frac{ab}{x^2}+\frac{ac}{x^2}+\frac{bc}{x^2}-\frac{a}{x}-\frac{b}{x}-\frac{c}{x}+1}}{\sqrt{-d}}\right)}{\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*b*c*x-2*(a*b+a*c+b*c)*x^2+(a+b+c)*x^3)/(x*(-a+x)*(-b+x)*(-c+
x))^(1/2)/(a*b*c-(a*b+a*c+b*c)*x+(a+b+c)*x^2+(-1+d)*x^3),x, algorithm="giac
")
```

```
[Out] -2*arctan(sqrt(-a*b*c/x^3 + a*b/x^2 + a*c/x^2 + b*c/x^2 - a/x - b/x - c/x +
 1)/sqrt(-d))/sqrt(-d)
```

maple [C] time = 0.12, size = 628, normalized size = 10.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a*b*c*x-2*(a*b+a*c+b*c)*x^2+(a+b+c)*x^3)/(x*(-a+x)*(-b+x)*(-c+x))^(1
/2)/(a*b*c-(a*b+a*c+b*c)*x+(a+b+c)*x^2+(-1+d)*x^3),x)
```

```
[Out] -2*(a+b+c)/(-1+d)*a*((a-c)*x/a/(-c+x))^(1/2)*(-c+x)^2*(c*(-b+x)/b/(-c+x))^(
1/2)*(c*(-a+x)/a/(-c+x))^(1/2)/(a-c)/c/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)*Ellip
ticF(((a-c)*x/a/(-c+x))^(1/2),((-b+c)*a/b/(c-a))^(1/2))-2/(-1+d)*a/c^4/d*su
m((-2*_alpha^2*a*b*d-2*_alpha^2*a*c*d-2*_alpha^2*b*c*d+3*_alpha*a*b*c*d-_al
```

```
pha^2*a^2-_alpha^2*b^2-_alpha^2*c^2+_alpha*a^2*b+_alpha*a^2*c+_alpha*a*b^2+
_alpha*a*c^2+_alpha*b^2*c+_alpha*b*c^2-a^2*b*c-a*b^2*c-a*b*c^2)/(-3*_alpha^
2*d+3*_alpha^2-2*_alpha*a-2*_alpha*b-2*_alpha*c+a*b+a*c+b*c)*(-c+x)^2/(a-c)
*(-_alpha^2*d-_alpha*c*d-c^2*d+_alpha^2-_alpha*a-_alpha*b+a*b)*((a-c)*x/a/(
-c+x))^(1/2)*(c*(-b+x)/b/(-c+x))^(1/2)*(c*(-a+x)/a/(-c+x))^(1/2)/(x*(-a+x)*
(-b+x)*(-c+x))^(1/2)*(EllipticF(((a-c)*x/a/(-c+x))^(1/2),((-b+c)*a/b/(c-a))
^(1/2))-(-_alpha^2*d+_alpha^2-_alpha*a-_alpha*b-_alpha*c+a*b+a*c+b*c)/a/b*E
llipticPi(((a-c)*x/a/(-c+x))^(1/2),-(-_alpha^2*d+_alpha^2-_alpha*a-_alpha*b
-_alpha*c+a*c+b*c)/b/(a-c),((-b+c)*a/b/(c-a))^(1/2))),_alpha=RootOf((-1+d)*
_Z^3+(a+b+c)*_Z^2+(-a*b-a*c-b*c)*_Z+a*b*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,abcx + (a+b+c)x^3 - 2\,(ab+ac+bc)x^2}{\sqrt{-(a-x)(b-x)(c-x)x}\left((d-1)x^3 + abc + (a+b+c)x^2 - (ab+ac+bc)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*b*c*x-2*(a*b+a*c+b*c)*x^2+(a+b+c)*x^3)/(x*(-a+x)*(-b+x)*(-c+
x))^(1/2)/(a*b*c-(a*b+a*c+b*c)*x+(a+b+c)*x^2+(-1+d)*x^3),x, algorithm="maxi
ma")
```

```
[Out] integrate((3*a*b*c*x + (a + b + c)*x^3 - 2*(a*b + a*c + b*c)*x^2)/(sqrt(-(a
 - x)*(b - x)*(c - x)*x)*((d - 1)*x^3 + a*b*c + (a + b + c)*x^2 - (a*b + a*
c + b*c)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,\left(a+b+c\right)-2\,x^2\,\left(a\,b+a\,c+b\,c\right)+3\,a\,b\,c\,x}{\left(\left(d-1\right)\,x^3+\left(a+b+c\right)\,x^2+\left(-a\,b-a\,c-b\,c\right)\,x+a\,b\,c\right)\,\sqrt{-x\,\left(a-x\right)\,\left(b-x\right)\,\left(c-x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b + c) - 2*x^2*(a*b + a*c + b*c) + 3*a*b*c*x)/((x^2*(a + b +
c) - x*(a*b + a*c + b*c) + x^3*(d - 1) + a*b*c)*(-x*(a - x)*(b - x)*(c - x)
)^(1/2)),x)
```

```
[Out] int((x^3*(a + b + c) - 2*x^2*(a*b + a*c + b*c) + 3*a*b*c*x)/((x^2*(a + b +
c) - x*(a*b + a*c + b*c) + x^3*(d - 1) + a*b*c)*(-x*(a - x)*(b - x)*(c - x)
)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*b*c*x-2*(a*b+a*c+b*c)*x**2+(a+b+c)*x**3)/(x*(-a+x)*(-b+x)*(-
c+x))**(1/2)/(a*b*c-(a*b+a*c+b*c)*x+(a+b+c)*x**2+(-1+d)*x**3),x)
```

```
[Out] Timed out
```

3.740

$$\int \frac{x\left(3abc-2(ab+ac+bc)x+(a+b+c)x^2\right)}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(-abcd+(ab+ac+bc)dx-(a+b+c)dx^2+(-1+d)x^3\right)}\,dx$$

Optimal. Leaf size=59

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{x^3(-a-b-c)+x^2(ab+ac+bc)-abcx+x^4}}{x^2}\right)}{\sqrt{d}}$$

Rubi [F] time = 7.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\left(3abc-2(ab+ac+bc)x+(a+b+c)x^2\right)}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(-abcd+(ab+ac+bc)dx-(a+b+c)dx^2+(-1+d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(3*a*b*c - 2*(a*b + a*c + b*c)*x + (a + b + c)*x^2))/(Sqrt[x*(-a + x
)*(-b + x)*(-c + x)]*(-(a*b*c*d) + (a*b + a*c + b*c)*d*x - (a + b + c)*d*x^
2 + (-1 + d)*x^3)),x]
```

```
[Out] -(((a + b + c)*Defer[Int][1/Sqrt[x*(-a + x)*(-b + x)*(-c + x)], x])/(1 - d)
) + (a*b*c*(a + b + c)*d*Defer[Int][1/(Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*(
a*b*c*d - (b*c + a*(b + c))*d*x + (a + b + c)*d*x^2 + (1 - d)*x^3)), x])/(1
 - d) - ((a^2*(b + c)*d + b*c*(b + c)*d + a*(3*b*c + b^2*d + c^2*d))*Defer[
Int][x/(Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*(a*b*c*d - (b*c + a*(b + c))*d*x
 + (a + b + c)*d*x^2 + (1 - d)*x^3)), x])/(1 - d) + ((2*b*c + 2*a*(b + c) +
 a^2*d + b^2*d + c^2*d)*Defer[Int][x^2/(Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*
(a*b*c*d - (b*c + a*(b + c))*d*x + (a + b + c)*d*x^2 + (1 - d)*x^3)), x])/(
1 - d)
```

Rubi steps

$$\int \frac{x\left(3abc-2(ab+ac+bc)x+(a+b+c)x^2\right)}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(-abcd+(ab+ac+bc)dx-(a+b+c)dx^2+(-1+d)x^3\right)}\,dx = \int \frac{}{\sqrt{x(-a+x)(-}}$$

$$= \int \left(-\frac{}{(1-d)\sqrt{x(-}} \right.$$

$$= -\frac{(a+b+c)\int \frac{}{\sqrt{x}}}{}$$

$$= -\frac{(a+b+c)\int \frac{}{\sqrt{x}}}{}$$

$$= -\frac{(a+b+c)\int \frac{}{\sqrt{x}}}{}$$

Mathematica [C] time = 12.27, size = 32877, normalized size = 557.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(3*a*b*c - 2*(a*b + a*c + b*c)*x + (a + b + c)*x^2))/(Sqrt[x*(
-a + x)*(-b + x)*(-c + x)]*(-(a*b*c*d) + (a*b + a*c + b*c)*d*x - (a + b + c
)*d*x^2 + (-1 + d)*x^3)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.98, size = 59, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{x^3(-a-b-c)+x^2(ab+ac+bc)-abcx+x^4}}{x^2}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(3*a*b*c - 2*(a*b + a*c + b*c)*x + (a + b + c)*x^2))/
(Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*(-(a*b*c*d) + (a*b + a*c + b*c)*d*x - (
a + b + c)*d*x^2 + (-1 + d)*x^3)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[d]*Sqrt[-(a*b*c*x) + (a*b + a*c + b*c)*x^2 + (-a - b - c)
*x^3 + x^4])/x^2])/Sqrt[d]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*a*b*c-2*(a*b+a*c+b*c)*x+(a+b+c)*x^2)/(x*(-a+x)*(-b+x)*(-c+x)
)^(1/2)/(-a*b*c*d+(a*b+a*c+b*c)*d*x-(a+b+c)*d*x^2+(-1+d)*x^3),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [A] time = 4.03, size = 69, normalized size = 1.17

$$\frac{2\left|d\right|\arctan\left(\frac{\sqrt{-\frac{abc}{x^3}+\frac{ab}{x^2}+\frac{ac}{x^2}+\frac{bc}{x^2}-\frac{a}{x}-\frac{b}{x}-\frac{c}{x}+1}}{\sqrt{-\frac{1}{d}}}\right)}{\sqrt{-d}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*a*b*c-2*(a*b+a*c+b*c)*x+(a+b+c)*x^2)/(x*(-a+x)*(-b+x)*(-c+x)
)^(1/2)/(-a*b*c*d+(a*b+a*c+b*c)*d*x-(a+b+c)*d*x^2+(-1+d)*x^3),x, algorithm=
"giac")
```

```
[Out] 2*abs(d)*arctan(sqrt(-a*b*c/x^3 + a*b/x^2 + a*c/x^2 + b*c/x^2 - a/x - b/x -
 c/x + 1)/sqrt(-1/d))/(sqrt(-d)*d)
```

maple [C] time = 0.12, size = 674, normalized size = 11.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*a*b*c-2*(a*b+a*c+b*c)*x+(a+b+c)*x^2)/(x*(-a+x)*(-b+x)*(-c+x))^(1/2
)/(-a*b*c*d+(a*b+a*c+b*c)*d*x-(a+b+c)*d*x^2+(-1+d)*x^3),x)
```

```
[Out] -2*(a+b+c)/(-1+d)*a*((a-c)*x/a/(-c+x))^(1/2)*(-c+x)^2*(c*(-b+x)/b/(-c+x))^(
1/2)*(c*(-a+x)/a/(-c+x))^(1/2)/(a-c)/c/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)*Ellip
ticF(((a-c)*x/a/(-c+x))^(1/2),((-b+c)*a/b/(c-a))^(1/2))+2/(-1+d)*a/c^4*sum(
(-_alpha^2*a^2*d-_alpha^2*b^2*d-_alpha^2*c^2*d+_alpha*a^2*b*d+_alpha*a^2*c*
d+_alpha*a*b^2*d+_alpha*a*c^2*d+_alpha*b^2*c*d+_alpha*b*c^2*d-a^2*b*c*d-a*b
^2*c*d-a*b*c^2*d-2*_alpha^2*a*b-2*_alpha^2*a*c-2*_alpha^2*b*c+3*_alpha*a*b*
```

```
c)/(-3*_alpha^2*d+2*_alpha*a*d+2*_alpha*b*d+2*_alpha*c*d-a*b*d-a*c*d-b*c*d+
3*_alpha^2)*(-c+x)^2/(a-c)*(_alpha^2*d-_alpha*a*d-_alpha*b*d+a*b*d-_alpha^2
-_alpha*c-c^2)*((a-c)*x/a/(-c+x))^(1/2)*(c*(-b+x)/b/(-c+x))^(1/2)*(c*(-a+x)
/a/(-c+x))^(1/2)/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)*(EllipticF(((a-c)*x/a/(-c+x
))^(1/2),((-b+c)*a/b/(c-a))^(1/2))-(_alpha^2*d-_alpha*a*d-_alpha*b*d-_alpha
*c*d+a*b*d+a*c*d+b*c*d-_alpha^2)/a/b/d*EllipticPi(((a-c)*x/a/(-c+x))^(1/2),
-(_alpha^2*d-_alpha*a*d-_alpha*b*d-_alpha*c*d+a*c*d+b*c*d-_alpha^2)/b/d/(a-
c),((-b+c)*a/b/(c-a))^(1/2))),_alpha=RootOf((-1+d)*_Z^3+(-a*d-b*d-c*d)*_Z^2
+(a*b*d+a*c*d+b*c*d)*_Z-a*b*c*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(3\,abc + (a+b+c)x^2 - 2\,(ab+ac+bc)x\right)x}{\left(abcd + (a+b+c)dx^2 - (d-1)x^3 - (ab+ac+bc)dx\right)\sqrt{-(a-x)(b-x)(c-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*a*b*c-2*(a*b+a*c+b*c)*x+(a+b+c)*x^2)/(x*(-a+x)*(-b+x)*(-c+x)
)^(1/2)/(-a*b*c*d+(a*b+a*c+b*c)*d*x-(a+b+c)*d*x^2+(-1+d)*x^3),x, algorithm=
"maxima")
```

```
[Out] -integrate((3*a*b*c + (a + b + c)*x^2 - 2*(a*b + a*c + b*c)*x)*x/((a*b*c*d
+ (a + b + c)*d*x^2 - (d - 1)*x^3 - (a*b + a*c + b*c)*d*x)*sqrt(-(a - x)*(b
 - x)*(c - x)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\left(x^2\,\left(a+b+c\right)-2\,x\,\left(a\,b+a\,c+b\,c\right)+3\,a\,b\,c\right)}{\left(\left(d-1\right)\,x^3-d\,\left(a+b+c\right)\,x^2+d\,\left(a\,b+a\,c+b\,c\right)\,x-a\,b\,c\,d\right)\,\sqrt{-x\,\left(a-x\right)\,\left(b-x\right)\,\left(c-x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^2*(a + b + c) - 2*x*(a*b + a*c + b*c) + 3*a*b*c))/((x^3*(d - 1) +
 d*x*(a*b + a*c + b*c) - d*x^2*(a + b + c) - a*b*c*d)*(-x*(a - x)*(b - x)*(
c - x))^(1/2)),x)
```

```
[Out] int((x*(x^2*(a + b + c) - 2*x*(a*b + a*c + b*c) + 3*a*b*c))/((x^3*(d - 1) +
 d*x*(a*b + a*c + b*c) - d*x^2*(a + b + c) - a*b*c*d)*(-x*(a - x)*(b - x)*(
c - x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*a*b*c-2*(a*b+a*c+b*c)*x+(a+b+c)*x**2)/(x*(-a+x)*(-b+x)*(-c+x
))**(1/2)/(-a*b*c*d+(a*b+a*c+b*c)*d*x-(a+b+c)*d*x**2+(-1+d)*x**3),x)
```

```
[Out] Timed out
```

3.741 $\int \sqrt[4]{-x^2+x^4}\,dx$

Optimal. Leaf size=59

$$\frac{1}{2}\sqrt[4]{x^4-x^2}\,x+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)-\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.92, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2004, 2032, 329, 331, 298, 203, 206}

$$\frac{1}{2}\sqrt[4]{x^4-x^2}\,x+\frac{(x^2-1)^{3/4}x^{3/2}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{4(x^4-x^2)^{3/4}}-\frac{(x^2-1)^{3/4}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{4(x^4-x^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + x^4)^(1/4),x]
```

```
[Out] (x*(-x^2 + x^4)^(1/4))/2 + (x^(3/2)*(-1 + x^2)^(3/4)*ArcTan[Sqrt[x]/(-1 + x
^2)^(1/4)])/(4*(-x^2 + x^4)^(3/4)) - (x^(3/2)*(-1 + x^2)^(3/4)*ArcTanh[Sqrt
[x]/(-1 + x^2)^(1/4)])/(4*(-x^2 + x^4)^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
```

```
b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \sqrt[4]{-x^2+x^4}\,dx = \frac{1}{2}x\sqrt[4]{-x^2+x^4}-\frac{1}{4}\int \frac{x^2}{\left(-x^2+x^4\right)^{3/4}}\,dx$$

$$= \frac{1}{2}x\sqrt[4]{-x^2+x^4}-\frac{\left(x^{3/2}\left(-1+x^2\right)^{3/4}\right)\int \frac{\sqrt{x}}{\left(-1+x^2\right)^{3/4}}\,dx}{4\left(-x^2+x^4\right)^{3/4}}$$

$$= \frac{1}{2}x\sqrt[4]{-x^2+x^4}-\frac{\left(x^{3/2}\left(-1+x^2\right)^{3/4}\right)\operatorname{Subst}\left(\int \frac{x^2}{\left(-1+x^4\right)^{3/4}}\,dx,x,\sqrt{x}\right)}{2\left(-x^2+x^4\right)^{3/4}}$$

$$= \frac{1}{2}x\sqrt[4]{-x^2+x^4}-\frac{\left(x^{3/2}\left(-1+x^2\right)^{3/4}\right)\operatorname{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{2\left(-x^2+x^4\right)^{3/4}}$$

$$= \frac{1}{2}x\sqrt[4]{-x^2+x^4}-\frac{\left(x^{3/2}\left(-1+x^2\right)^{3/4}\right)\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{4\left(-x^2+x^4\right)^{3/4}}+\frac{\left(x^{3/2}\left(-1+x^2\right)^{3/4}\right)\operatorname{Subst}}{4\left(-x^2+\right.}$$

$$= \frac{1}{2}x\sqrt[4]{-x^2+x^4}+\frac{x^{3/2}\left(-1+x^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{4\left(-x^2+x^4\right)^{3/4}}-\frac{x^{3/2}\left(-1+x^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{4\left(-x^2+x^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.71

$$\frac{2x\sqrt[4]{x^2\left(x^2-1\right)}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};x^2\right)}{3\sqrt[4]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + x^4)^(1/4),x]
```

```
[Out] (2*x*(x^2*(-1 + x^2))^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, x^2])/(3*(1 -
 x^2)^(1/4))
```

IntegrateAlgebraic [A] time = 0.12, size = 59, normalized size = 1.00

$$\frac{1}{2}\sqrt[4]{x^4-x^2}\,x+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)-\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^2 + x^4)^(1/4),x]
```

```
[Out] (x*(-x^2 + x^4)^(1/4))/2 + ArcTan[x/(-x^2 + x^4)^(1/4)]/4 - ArcTanh[x/(-x^2
 + x^4)^(1/4)]/4
```

fricas [B] time = 2.18, size = 110, normalized size = 1.86

$$\frac{1}{2}\left(x^4-x^2\right)^{\frac{1}{4}}x-\frac{1}{8}\arctan\left(\frac{2\left(\left(x^4-x^2\right)^{\frac{1}{4}}x^2+\left(x^4-x^2\right)^{\frac{3}{4}}\right)}{x}\right)+\frac{1}{8}\log\left(-\frac{2x^3-2\left(x^4-x^2\right)^{\frac{1}{4}}x^2+2\sqrt{x^4-x^2}\,x-x-2\left(x^4-x^2\right)^{\frac{3}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*(x^4 - x^2)^(1/4)*x - 1/8*arctan(2*((x^4 - x^2)^(1/4)*x^2 + (x^4 - x^2)
^(3/4))/x) + 1/8*log(-(2*x^3 - 2*(x^4 - x^2)^(1/4)*x^2 + 2*sqrt(x^4 - x^2)*
x - x - 2*(x^4 - x^2)^(3/4))/x)
```

giac [A] time = 0.26, size = 57, normalized size = 0.97

$$-\frac{1}{2}x^2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}+\frac{1}{4}\arctan\left(\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)+\frac{1}{8}\log\left(\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}+1\right)-\frac{1}{8}\log\left(-\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2)^(1/4),x, algorithm="giac")
```

```
[Out] -1/2*x^2*(-1/x^2 + 1)^(1/4) + 1/4*arctan((-1/x^2 + 1)^(1/4)) + 1/8*log((-1/
x^2 + 1)^(1/4) + 1) - 1/8*log(-(-1/x^2 + 1)^(1/4) + 1)
```

maple [C] time = 0.25, size = 33, normalized size = 0.56

$$\frac{2\mathrm{signum}\left(x^2-1\right)^{\frac{1}{4}}x^{\frac{3}{2}}\,\mathrm{hypergeom}\left(\left[-\frac{1}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],x^2\right)}{3\left(-\mathrm{signum}\left(x^2-1\right)\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^2)^(1/4),x)
```

```
[Out] 2/3*signum(x^2-1)^(1/4)/(-signum(x^2-1))^(1/4)*x^(3/2)*hypergeom([-1/4,3/4]
,[7/4],x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^4-x^2\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^2)^(1/4), x)
```

mupad [B] time = 0.77, size = 31, normalized size = 0.53

$$\frac{2\,x\,\left(x^4-x^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ x^2\right)}{3\,\left(1-x^2\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x^2)^(1/4),x)
```

```
[Out] (2*x*(x^4 - x^2)^(1/4)*hypergeom([-1/4, 3/4], 7/4, x^2))/(3*(1 - x^2)^(1/4)
)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{x^4 - x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**2)**(1/4),x)
```

```
[Out] Integral((x**4 - x**2)**(1/4), x)
```

3.742 $\int x^2 \sqrt[4]{x^2+x^4}\, dx$

Optimal. Leaf size=59

$$\frac{3}{32}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^2}}\right)-\frac{3}{32}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^2}}\right)+\frac{1}{16}\sqrt[4]{x^4+x^2}\left(4x^3+x\right)$$

Rubi [B] time = 0.11, antiderivative size = 125, normalized size of antiderivative = 2.12, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2021, 2024, 2032, 329, 331, 298, 203, 206}

$$\frac{1}{16}\sqrt[4]{x^4+x^2}\,x+\frac{3\left(x^2+1\right)^{3/4}x^{3/2}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2+1}}\right)}{32\left(x^4+x^2\right)^{3/4}}-\frac{3\left(x^2+1\right)^{3/4}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2+1}}\right)}{32\left(x^4+x^2\right)^{3/4}}+\frac{1}{4}\sqrt[4]{x^4+x^2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(x^2 + x^4)^(1/4),x]
```

```
[Out] (x*(x^2 + x^4)^(1/4))/16 + (x^3*(x^2 + x^4)^(1/4))/4 + (3*x^(3/2)*(1 + x^2)
^(3/4)*ArcTan[Sqrt[x]/(1 + x^2)^(1/4)])/(32*(x^2 + x^4)^(3/4)) - (3*x^(3/2)
*(1 + x^2)^(3/4)*ArcTanh[Sqrt[x]/(1 + x^2)^(1/4)])/(32*(x^2 + x^4)^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
```

```
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int x^2\sqrt[4]{x^2+x^4}\,dx &= \frac{1}{4}x^3\sqrt[4]{x^2+x^4}+\frac{1}{8}\int\frac{x^4}{\left(x^2+x^4\right)^{3/4}}\,dx\\
&= \frac{1}{16}x\sqrt[4]{x^2+x^4}+\frac{1}{4}x^3\sqrt[4]{x^2+x^4}-\frac{3}{32}\int\frac{x^2}{\left(x^2+x^4\right)^{3/4}}\,dx\\
&= \frac{1}{16}x\sqrt[4]{x^2+x^4}+\frac{1}{4}x^3\sqrt[4]{x^2+x^4}-\frac{\left(3x^{3/2}\left(1+x^2\right)^{3/4}\right)\int\frac{\sqrt{x}}{\left(1+x^2\right)^{3/4}}\,dx}{32\left(x^2+x^4\right)^{3/4}}\\
&= \frac{1}{16}x\sqrt[4]{x^2+x^4}+\frac{1}{4}x^3\sqrt[4]{x^2+x^4}-\frac{\left(3x^{3/2}\left(1+x^2\right)^{3/4}\right)\text{Subst}\left(\int\frac{x^2}{\left(1+x^4\right)^{3/4}}\,dx,x,\sqrt{x}\right)}{16\left(x^2+x^4\right)^{3/4}}\\
&= \frac{1}{16}x\sqrt[4]{x^2+x^4}+\frac{1}{4}x^3\sqrt[4]{x^2+x^4}-\frac{\left(3x^{3/2}\left(1+x^2\right)^{3/4}\right)\text{Subst}\left(\int\frac{x^2}{1-x^4}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{16\left(x^2+x^4\right)^{3/4}}\\
&= \frac{1}{16}x\sqrt[4]{x^2+x^4}+\frac{1}{4}x^3\sqrt[4]{x^2+x^4}-\frac{\left(3x^{3/2}\left(1+x^2\right)^{3/4}\right)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{32\left(x^2+x^4\right)^{3/4}}+\frac{\left(3x^{3/2}\left(1\right.\right.}{}\\
&= \frac{1}{16}x\sqrt[4]{x^2+x^4}+\frac{1}{4}x^3\sqrt[4]{x^2+x^4}+\frac{3x^{3/2}\left(1+x^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{32\left(x^2+x^4\right)^{3/4}}-\frac{3x^{3/2}\left(1+x^2\right)^{3/4}\tanh^{-1}\left(\right.}{32\left(x^2+x^4\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.88

$$
\frac{x\sqrt[4]{x^4+x^2}\left(\left(x^2+1\right)^{5/4}-{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-x^2\right)\right)}{4\sqrt[4]{x^2+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(x^2 + x^4)^(1/4),x]
```

```
[Out] (x*(x^2 + x^4)^(1/4)*((1 + x^2)^(5/4) - Hypergeometric2F1[-1/4, 3/4, 7/4, -
x^2]))/(4*(1 + x^2)^(1/4))
```

IntegrateAlgebraic [A] time = 0.14, size = 59, normalized size = 1.00

$$\frac{3}{32}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^2}}\right)-\frac{3}{32}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^2}}\right)+\frac{1}{16}\sqrt[4]{x^4+x^2}\left(4x^3+x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(x^2 + x^4)^(1/4),x]
```

```
[Out] ((x + 4*x^3)*(x^2 + x^4)^(1/4))/16 + (3*ArcTan[x/(x^2 + x^4)^(1/4)])/32 - (
3*ArcTanh[x/(x^2 + x^4)^(1/4)])/32
```

fricas [B] time = 1.73, size = 102, normalized size = 1.73

$$\frac{1}{16}\left(x^4+x^2\right)^{\frac{1}{4}}\left(4x^3+x\right)+\frac{3}{64}\arctan\left(\frac{2\left(\left(x^4+x^2\right)^{\frac{1}{4}}x^2+\left(x^4+x^2\right)^{\frac{3}{4}}\right)}{x}\right)+\frac{3}{64}\log\left(-\frac{2x^3-2\left(x^4+x^2\right)^{\frac{1}{4}}x^2+2\sqrt{x^4+x^2}\,x+x-2\left(x^4+x^2\right)^{\frac{3}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/16*(x^4 + x^2)^(1/4)*(4*x^3 + x) + 3/64*arctan(2*((x^4 + x^2)^(1/4)*x^2 +
 (x^4 + x^2)^(3/4))/x) + 3/64*log(-(2*x^3 - 2*(x^4 + x^2)^(1/4)*x^2 + 2*sqr
t(x^4 + x^2)*x + x - 2*(x^4 + x^2)^(3/4))/x)
```

giac [A] time = 0.29, size = 57, normalized size = 0.97

$$\frac{1}{16}\left(\left(\frac{1}{x^2}+1\right)^{\frac{5}{4}}+3\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)x^4-\frac{3}{32}\arctan\left(\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)-\frac{3}{64}\log\left(\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}+1\right)+\frac{3}{64}\log\left(\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4+x^2)^(1/4),x, algorithm="giac")
```

```
[Out] 1/16*((1/x^2 + 1)^(5/4) + 3*(1/x^2 + 1)^(1/4))*x^4 - 3/32*arctan((1/x^2 + 1
)^(1/4)) - 3/64*log((1/x^2 + 1)^(1/4) + 1) + 3/64*log((1/x^2 + 1)^(1/4) - 1
)
```

maple [C] time = 2.14, size = 417, normalized size = 7.07

$$\frac{x\left(4x^2+1\right)\left(x^2\left(x^2+1\right)\right)^{\frac{1}{4}}}{16}+\frac{\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{-2\left(x^8+3x^6+3x^4+x^2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)x^4+2x^6+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\left(x^8+3x^6+3x^4+x^2\right)^{\frac{3}{4}}-2\sqrt{x^8+3x^6+3x^4+x^2}\,x^2-4\left(x^8+3x^6+3x^4+x^2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)x^2+5x^4-2\sqrt{x^8+3x^6+3x^4+x^2}-2\,\mathrm{RootOf}\left(\_Z^2+1\right)\left(x^8+3x^6+3x^4+x^2\right)^{\frac{1}{4}}+4x^2+1}{\left(x^2+1\right)^2}\right)}{64}+\frac{3\ln\left(-\frac{-2x^6+2\left(x^8+3x^6+3x^4+x^2\right)^{\frac{1}{4}}x^4-2\sqrt{x^8+3x^6+3x^4+x^2}\,x^2-5x^4+2\left(x^8+3x^6+3x^4+x^2\right)^{\frac{3}{4}}+4\left(x^8+3x^6+3x^4+x^2\right)^{\frac{1}{4}}x^2-2\sqrt{x^8+3x^6+3x^4+x^2}-4x^2+2\left(x^8+3x^6+3x^4+x^2\right)^{\frac{1}{4}}-1}{\left(x^2+1\right)^2}\right)}{64}\right)\left(x^2\left(x^2+1\right)\right)^{\frac{1}{4}}\left(x^2\left(x^2+1\right)^3\right)^{\frac{1}{4}}}{x\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4+x^2)^(1/4),x)
```

```
[Out] 1/16*x*(4*x^2+1)*(x^2*(x^2+1))^(1/4)+(3/64*RootOf(_Z^2+1)*ln(-(-2*(x^8+3*x^
6+3*x^4+x^2)^(1/4)*RootOf(_Z^2+1)*x^4+2*x^6+2*RootOf(_Z^2+1)*(x^8+3*x^6+3*x
^4+x^2)^(3/4)-2*(x^8+3*x^6+3*x^4+x^2)^(1/2)*x^2-4*(x^8+3*x^6+3*x^4+x^2)^(1/
4)*RootOf(_Z^2+1)*x^2+5*x^4-2*(x^8+3*x^6+3*x^4+x^2)^(1/2)-2*RootOf(_Z^2+1)*
(x^8+3*x^6+3*x^4+x^2)^(1/4)+4*x^2+1)/(x^2+1)^2)+3/64*ln(-(-2*x^6+2*(x^8+3*x
^6+3*x^4+x^2)^(1/4)*x^4-2*(x^8+3*x^6+3*x^4+x^2)^(1/2)*x^2-5*x^4+2*(x^8+3*x^
6+3*x^4+x^2)^(3/4)+4*(x^8+3*x^6+3*x^4+x^2)^(1/4)*x^2-2*(x^8+3*x^6+3*x^4+x^2
)^(1/2)-4*x^2+2*(x^8+3*x^6+3*x^4+x^2)^(1/4)-1)/(x^2+1)^2))/x*(x^2*(x^2+1))^
(1/4)*(x^2*(x^2+1)^3)^(1/4)/(x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4 + x^2\right)^{\frac{1}{4}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4+x^2)^(1/4),x, algorithm="maxima")
[Out] integrate((x^4 + x^2)^(1/4)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \left(x^4 + x^2\right)^{1/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2 + x^4)^(1/4),x)
[Out] int(x^2*(x^2 + x^4)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt[4]{x^2 \left(x^2 + 1\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**4+x**2)**(1/4),x)
[Out] Integral(x**2*(x**2*(x**2 + 1))**(1/4), x)
```

3.743 $\int \frac{(-1+2x^4)\sqrt{1+3x^2+2x^4}}{(1+2x^2+2x^4)^2}\,dx$

Optimal. Leaf size=59

$$-\frac{\sqrt{2x^4+3x^2+1}\,x}{2\left(2x^4+2x^2+1\right)}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2x^4+3x^2+1}}\right)$$

Rubi [C] time = 2.43, antiderivative size = 253, normalized size of antiderivative = 4.29, number of steps used = 90, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6742, 1226, 1189, 1100, 1136, 1214, 1456, 539, 1208, 6728}

$$-\frac{i\sqrt{2x^4+3x^2+1}\,x}{-4x^2-(2-2i)}-\frac{i\sqrt{2x^4+3x^2+1}\,x}{4x^2+(2+2i)}-\frac{\left(x^2+1\right)\sqrt{\frac{2x^2+1}{x^2+1}}F\left(\tan^{-1}(x)|-1\right)}{2\sqrt{2x^4+3x^2+1}}+\frac{i\left(x^2+1\right)\Pi\left(\frac{1}{2}-\frac{i}{2};\tan^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{\frac{x^2+1}{2x^2+1}}\sqrt{2x^4+3x^2+1}}-\frac{i\left(x^2+1\right)\Pi\left(\frac{1}{2}+\frac{i}{2};\tan^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{\frac{x^2+1}{2x^2+1}}\sqrt{2x^4+3x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + 2*x^4)*Sqrt[1 + 3*x^2 + 2*x^4])/(1 + 2*x^2 + 2*x^4)^2,x]
```

```
[Out] ((-I)*x*Sqrt[1 + 3*x^2 + 2*x^4])/((-2 + 2*I) - 4*x^2) - (I*x*Sqrt[1 + 3*x^2
 + 2*x^4])/((2 + 2*I) + 4*x^2) - ((1 + x^2)*Sqrt[(1 + 2*x^2)/(1 + x^2)]*Ell
ipticF[ArcTan[x], -1])/(2*Sqrt[1 + 3*x^2 + 2*x^4]) + ((I/2)*(1 + x^2)*Ellip
ticPi[1/2 - I/2, ArcTan[Sqrt[2]*x], 1/2])/(Sqrt[2]*Sqrt[(1 + x^2)/(1 + 2*x^
2)]*Sqrt[1 + 3*x^2 + 2*x^4]) - ((I/2)*(1 + x^2)*EllipticPi[1/2 + I/2, ArcTa
n[Sqrt[2]*x], 1/2])/(Sqrt[2]*Sqrt[(1 + x^2)/(1 + 2*x^2)]*Sqrt[1 + 3*x^2 + 2
*x^4])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1100

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b - q)*x^2)*Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]*EllipticF[ArcTan[Rt[(b - q)/(2*a), 2]*x], (-2*q)/(b - q)])/(2
*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1136

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b - q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]*EllipticE[ArcTan[Rt[(b - q)/(2*a), 2]*x], (-2*q
)/(b - q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1226

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(d + e*x^2)), x] + (Dist[c/(2
*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*
e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(-1+2x^4\right)\sqrt{1+3x^2+2x^4}}{\left(1+2x^2+2x^4\right)^2}\,dx = \int \left(-\frac{2\left(1+x^2\right)\sqrt{1+3x^2+2x^4}}{\left(1+2x^2+2x^4\right)^2} + \frac{\sqrt{1+3x^2+2x^4}}{1+2x^2+2x^4}\right)dx$$

$$= -\left(2\int \frac{\left(1+x^2\right)\sqrt{1+3x^2+2x^4}}{\left(1+2x^2+2x^4\right)^2}\,dx\right) + \int \frac{\sqrt{1+3x^2+2x^4}}{1+2x^2+2x^4}\,dx$$

$$= -\left(2\int \left(\frac{\sqrt{1+3x^2+2x^4}}{\left(1+2x^2+2x^4\right)^2} + \frac{x^2\sqrt{1+3x^2+2x^4}}{\left(1+2x^2+2x^4\right)^2}\right)dx\right) + \int \left(\frac{2i\sqrt{1+3x^2+}}{(-2+2i)-}\right.$$

$$= 2i\int \frac{\sqrt{1+3x^2+2x^4}}{(-2+2i)-4x^2}\,dx + 2i\int \frac{\sqrt{1+3x^2+2x^4}}{(2+2i)+4x^2}\,dx - 2\int \frac{\sqrt{1+3x^2+2}}{\left(1+2x^2+2x\right.}$$

$$= (-1-i)\int \frac{1}{\left((-2+2i)-4x^2\right)\sqrt{1+3x^2+2x^4}}\,dx - \frac{1}{8}i\int \frac{(-8+4i)-8x^2}{\sqrt{1+3x^2+2x^4}}$$

$$= \left(-\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{\sqrt{1+3x^2+2x^4}}\,dx + \left(-\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{\sqrt{1+3x^2+2x^4}}\,dx + \left(-\right.$$

$$= -\frac{ix\sqrt{1+3x^2+2x^4}}{(-2+2i)-4x^2} - \frac{ix\sqrt{1+3x^2+2x^4}}{(2+2i)+4x^2} - (-2-2i)\int \frac{1}{\left((-2+2i)-4x^2\right)}$$

$$= -\frac{ix\sqrt{1+3x^2+2x^4}}{(-2+2i)-4x^2} - \frac{ix\sqrt{1+3x^2+2x^4}}{(2+2i)+4x^2} + \frac{\left(1+x^2\right)\Pi\left(\frac{1}{2}-\frac{i}{2};\tan^{-1}\left(\sqrt{2}\right.\right.}{2\sqrt{2}\sqrt{\frac{1+x^2}{1+2x^2}}\sqrt{1+3x^2+2}}$$

$$= -\frac{x\left(1+2x^2\right)}{2\sqrt{1+3x^2+2x^4}} - \frac{ix\sqrt{1+3x^2+2x^4}}{(-2+2i)-4x^2} - \frac{ix\sqrt{1+3x^2+2x^4}}{(2+2i)+4x^2} + \frac{\left(1+x^2\right)}{2}$$

$$= -\frac{x\left(1+2x^2\right)}{2\sqrt{1+3x^2+2x^4}} - \frac{ix\sqrt{1+3x^2+2x^4}}{(-2+2i)-4x^2} - \frac{ix\sqrt{1+3x^2+2x^4}}{(2+2i)+4x^2} + \frac{\left(1+x^2\right)}{2}$$

Mathematica [C] time = 0.87, size = 199, normalized size = 3.37

$$\frac{-i\sqrt{2}\sqrt{x^2+1}\sqrt{2x^2+1}F\left(i\sinh^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)+i\sqrt{2}\sqrt{x^2+1}\sqrt{2x^2+1}\Pi\left(\frac{1}{2}-\frac{i}{2};i\sinh^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)+i\sqrt{2}\sqrt{x^2+1}\sqrt{2x^2+1}\Pi\left(\frac{1}{2}+\frac{i}{2};i\sinh^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)-\frac{2x\left(2x^4+3x^2+1\right)}{2x^4+2x^2+1}}{4\sqrt{2x^4+3x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + 2*x^4)*Sqrt[1 + 3*x^2 + 2*x^4])/(1 + 2*x^2 + 2*x^4)^2,x]
```

```
[Out] ((-2*x*(1 + 3*x^2 + 2*x^4))/(1 + 2*x^2 + 2*x^4) - I*Sqrt[2]*Sqrt[1 + x^2]*S
qrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], 1/2] + I*Sqrt[2]*Sqrt[1 + x^
2]*Sqrt[1 + 2*x^2]*EllipticPi[1/2 - I/2, I*ArcSinh[Sqrt[2]*x], 1/2] + I*Sqr
t[2]*Sqrt[1 + x^2]*Sqrt[1 + 2*x^2]*EllipticPi[1/2 + I/2, I*ArcSinh[Sqrt[2]*
x], 1/2])/(4*Sqrt[1 + 3*x^2 + 2*x^4])
```

IntegrateAlgebraic [A] time = 0.66, size = 59, normalized size = 1.00

$$-\frac{\sqrt{2x^4+3x^2+1}\,x}{2\left(2x^4+2x^2+1\right)} - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2x^4+3x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + 2*x^4)*Sqrt[1 + 3*x^2 + 2*x^4])/(1 + 2*x^2 + 2*x^
4)^2,x]
```

```
[Out] -1/2*(x*Sqrt[1 + 3*x^2 + 2*x^4])/(1 + 2*x^2 + 2*x^4) - ArcTanh[x/Sqrt[1 + 3
*x^2 + 2*x^4]]/2
```

fricas [A] time = 0.43, size = 92, normalized size = 1.56

$$\frac{\left(2\,x^{4}+2\,x^{2}+1\right)\log\left(\frac{2\,x^{4}+4\,x^{2}-2\,\sqrt{2\,x^{4}+3\,x^{2}+1}\,x+1}{2\,x^{4}+2\,x^{2}+1}\right)-2\,\sqrt{2\,x^{4}+3\,x^{2}+1}\,x}{4\,\left(2\,x^{4}+2\,x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)*(2*x^4+3*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^2,x, algorithm="f
ricas")
```

```
[Out] 1/4*((2*x^4 + 2*x^2 + 1)*log((2*x^4 + 4*x^2 - 2*sqrt(2*x^4 + 3*x^2 + 1)*x +
 1)/(2*x^4 + 2*x^2 + 1)) - 2*sqrt(2*x^4 + 3*x^2 + 1)*x)/(2*x^4 + 2*x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^{4}+3\,x^{2}+1}\,\left(2\,x^{4}-1\right)}{\left(2\,x^{4}+2\,x^{2}+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)*(2*x^4+3*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^2,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(2*x^4 + 3*x^2 + 1)*(2*x^4 - 1)/(2*x^4 + 2*x^2 + 1)^2, x)
```

maple [C] time = 0.06, size = 340, normalized size = 5.76

$$-\frac{i\sqrt{x^{2}+1}\,\sqrt{2x^{2}+1}\,\operatorname{EllipticF}\left(ix,\sqrt{2}\right)}{2\sqrt{2x^{4}+3x^{2}+1}}-\frac{\left(\sum_{\_\alpha=\operatorname{RootOf}\left(2\_Z^{4}+2\_Z^{2}+1\right)}\_\alpha\left(2\_\alpha^{2}+1\right)\left(-\frac{\operatorname{arctanh}\left(\frac{\left(4\_\alpha^{2}+3\right)\left(\_\alpha^{2}+5x^{2}+4\right)}{10\sqrt{\_\alpha^{2}}\,\sqrt{2x^{4}+3x^{2}+1}}\right)}{\sqrt{\_\alpha^{2}}}+\frac{4i\left(-\_\alpha^{3}-\_\alpha\right)\sqrt{x^{2}+1}\,\sqrt{2x^{2}+1}\,\operatorname{EllipticPi}\left(ix,2\_\alpha^{2}+2,i\sqrt{-2}\right)}{\sqrt{2x^{4}+3x^{2}+1}}\right)\right)}{8}-\frac{x\sqrt{2x^{4}+3x^{2}+1}}{2\left(2x^{4}+2x^{2}+1\right)}+\frac{\left(\sum_{\_\alpha=\operatorname{RootOf}\left(2\_Z^{4}+2\_Z^{2}+1\right)}\_\alpha\left(\_\alpha^{2}+1\right)\left(-\frac{\operatorname{arctanh}\left(\frac{\left(4\_\alpha^{2}+3\right)\left(\_\alpha^{2}+5x^{2}+4\right)}{10\sqrt{\_\alpha^{2}}\,\sqrt{2x^{4}+3x^{2}+1}}\right)}{\sqrt{\_\alpha^{2}}}+\frac{4i\left(-\_\alpha^{3}-\_\alpha\right)\sqrt{x^{2}+1}\,\sqrt{2x^{2}+1}\,\operatorname{EllipticPi}\left(ix,2\_\alpha^{2}+2,i\sqrt{-2}\right)}{\sqrt{2x^{4}+3x^{2}+1}}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-1)*(2*x^4+3*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^2,x)
```

```
[Out] -1/2*I*(x^2+1)^(1/2)*(2*x^2+1)^(1/2)/(2*x^4+3*x^2+1)^(1/2)*EllipticF(I*x,2^
(1/2))-1/8*sum(_alpha*(2*_alpha^2+1)*(-1/(_alpha^2)^(1/2)*arctanh(1/10*(4*_
alpha^2+3)*(_alpha^2+5*x^2+4)/(_alpha^2)^(1/2)/(2*x^4+3*x^2+1)^(1/2))+4*I*(
-_alpha^3-_alpha)*(x^2+1)^(1/2)*(2*x^2+1)^(1/2)/(2*x^4+3*x^2+1)^(1/2)*Ellip
ticPi(I*x,2*_alpha^2+2,I*(-2)^(1/2))),_alpha=RootOf(2*_Z^4+2*_Z^2+1))-1/2*x
*(2*x^4+3*x^2+1)^(1/2)/(2*x^4+2*x^2+1)+1/4*sum(_alpha*(_alpha^2+1)*(-1/(_al
pha^2)^(1/2)*arctanh(1/10*(4*_alpha^2+3)*(_alpha^2+5*x^2+4)/(_alpha^2)^(1/2
)/(2*x^4+3*x^2+1)^(1/2))+4*I*(-_alpha^3-_alpha)*(x^2+1)^(1/2)*(2*x^2+1)^(1/
2)/(2*x^4+3*x^2+1)^(1/2)*EllipticPi(I*x,2*_alpha^2+2,I*(-2)^(1/2))),_alpha=
RootOf(2*_Z^4+2*_Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^{4}+3\,x^{2}+1}\,\left(2\,x^{4}-1\right)}{\left(2\,x^{4}+2\,x^{2}+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)*(2*x^4+3*x^2+1)^(1/2)/(2*x^4+2*x^2+1)^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(2*x^4 + 3*x^2 + 1)*(2*x^4 - 1)/(2*x^4 + 2*x^2 + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(2x^4-1)\sqrt{2x^4+3x^2+1}}{(2x^4+2x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^4 - 1)*(3*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 2*x^4 + 1)^2,x)
```

```
[Out] int(((2*x^4 - 1)*(3*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 2*x^4 + 1)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-1)*(2*x**4+3*x**2+1)**(1/2)/(2*x**4+2*x**2+1)**2,x)
```

```
[Out] Timed out
```

3.744 $\int \frac{\sqrt{-2+x^2-2x^3+2x^4}\left(2-x^3+2x^4\right)}{\left(-1-x^3+x^4\right)\left(-2-x^2-2x^3+2x^4\right)}\,dx$

Optimal. Leaf size=59

$$2\tanh^{-1}\left(\frac{x}{\sqrt{2x^4-2x^3+x^2-2}}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{2x^4-2x^3+x^2-2}}\right)$$

Rubi [F] time = 1.86, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-2+x^2-2x^3+2x^4}\left(2-x^3+2x^4\right)}{\left(-1-x^3+x^4\right)\left(-2-x^2-2x^3+2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-2 + x^2 - 2*x^3 + 2*x^4]*(2 - x^3 + 2*x^4))/((-1 - x^3 + x^4)*(-
2 - x^2 - 2*x^3 + 2*x^4)),x]
```

```
[Out] 2*Defer[Int][Sqrt[-2 + x^2 - 2*x^3 + 2*x^4]/(2 + x^2 + 2*x^3 - 2*x^4), x] +
 3*Defer[Int][(x*Sqrt[-2 + x^2 - 2*x^3 + 2*x^4])/(-1 - x^3 + x^4), x] - 4*D
efer[Int][(x^2*Sqrt[-2 + x^2 - 2*x^3 + 2*x^4])/(-1 - x^3 + x^4), x] - 6*Def
er[Int][(x*Sqrt[-2 + x^2 - 2*x^3 + 2*x^4])/(-2 - x^2 - 2*x^3 + 2*x^4), x] +
 8*Defer[Int][(x^2*Sqrt[-2 + x^2 - 2*x^3 + 2*x^4])/(-2 - x^2 - 2*x^3 + 2*x^
4), x]
```

Rubi steps

$$\int \frac{\sqrt{-2+x^2-2x^3+2x^4}\left(2-x^3+2x^4\right)}{\left(-1-x^3+x^4\right)\left(-2-x^2-2x^3+2x^4\right)}\,dx = \int \left(-\frac{x(-3+4x)\sqrt{-2+x^2-2x^3+2x^4}}{-1-x^3+x^4}+\frac{2\left(-1-3x+4x^2\right)\sqrt{-2}}{-2-x^2-2x^3}\right.$$

$$= 2\int \frac{\left(-1-3x+4x^2\right)\sqrt{-2+x^2-2x^3+2x^4}}{-2-x^2-2x^3+2x^4}\,dx-\int \frac{x(-3+4x)\sqrt{}}{-1}$$

$$= 2\int \left(\frac{\sqrt{-2+x^2-2x^3+2x^4}}{2+x^2+2x^3-2x^4}-\frac{3x\sqrt{-2+x^2-2x^3+2x^4}}{-2-x^2-2x^3+2x^4}+\frac{4x^2\sqrt{}}{-2}\right.$$

$$= 2\int \frac{\sqrt{-2+x^2-2x^3+2x^4}}{2+x^2+2x^3-2x^4}\,dx+3\int \frac{x\sqrt{-2+x^2-2x^3+2x^4}}{-1-x^3+x^4}\,dx$$

Mathematica [C] time = 6.68, size = 109133, normalized size = 1849.71

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[-2 + x^2 - 2*x^3 + 2*x^4]*(2 - x^3 + 2*x^4))/((-1 - x^3 + x
^4)*(-2 - x^2 - 2*x^3 + 2*x^4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.23, size = 59, normalized size = 1.00

$$2\tanh^{-1}\left(\frac{x}{\sqrt{2x^4-2x^3+x^2-2}}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{2x^4-2x^3+x^2-2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-2 + x^2 - 2*x^3 + 2*x^4]*(2 - x^3 + 2*x^4))/((-1
- x^3 + x^4)*(-2 - x^2 - 2*x^3 + 2*x^4)),x]
```

```
[Out] 2*ArcTanh[x/Sqrt[-2 + x^2 - 2*x^3 + 2*x^4]] - 2*Sqrt[2]*ArcTanh[(Sqrt[2]*x)
/Sqrt[-2 + x^2 - 2*x^3 + 2*x^4]]
```

fricas [B] time = 0.54, size = 168, normalized size = 2.85

$$\frac{1}{2}\,\sqrt{2}\log\left(-\frac{4\,x^{8}-8\,x^{7}+32\,x^{6}-28\,x^{5}+9\,x^{4}+8\,x^{3}-4\,\sqrt{2}\left(2\,x^{5}-2\,x^{4}+3\,x^{3}-2\,x\right)\sqrt{2\,x^{4}-2\,x^{3}+x^{2}-2}-28\,x^{2}+4}{4\,x^{8}-8\,x^{7}+4\,x^{5}-7\,x^{4}+8\,x^{3}+4\,x^{2}+4}\right)+\log\left(-\frac{x^{4}-x^{3}+x^{2}+\sqrt{2\,x^{4}-2\,x^{3}+x^{2}-2}\,x-1}{x^{4}-x^{3}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x^3+x^2-2)^(1/2)*(2*x^4-x^3+2)/(x^4-x^3-1)/(2*x^4-2*x^3-
x^2-2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(4*x^8 - 8*x^7 + 32*x^6 - 28*x^5 + 9*x^4 + 8*x^3 - 4*sqrt(
2)*(2*x^5 - 2*x^4 + 3*x^3 - 2*x)*sqrt(2*x^4 - 2*x^3 + x^2 - 2) - 28*x^2 + 4
)/(4*x^8 - 8*x^7 + 4*x^5 - 7*x^4 + 8*x^3 + 4*x^2 + 4)) + log(-(x^4 - x^3 +
x^2 + sqrt(2*x^4 - 2*x^3 + x^2 - 2)*x - 1)/(x^4 - x^3 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{4}-x^{3}+2\right)\sqrt{2\,x^{4}-2\,x^{3}+x^{2}-2}}{\left(2\,x^{4}-2\,x^{3}-x^{2}-2\right)\left(x^{4}-x^{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x^3+x^2-2)^(1/2)*(2*x^4-x^3+2)/(x^4-x^3-1)/(2*x^4-2*x^3-
x^2-2),x, algorithm="giac")
```

```
[Out] integrate((2*x^4 - x^3 + 2)*sqrt(2*x^4 - 2*x^3 + x^2 - 2)/((2*x^4 - 2*x^3 -
 x^2 - 2)*(x^4 - x^3 - 1)), x)
```

maple [C] time = 9.23, size = 12512, normalized size = 212.07

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-2*x^3+x^2-2)^(1/2)*(2*x^4-x^3+2)/(x^4-x^3-1)/(2*x^4-2*x^3-x^2-2)
,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{4}-x^{3}+2\right)\sqrt{2\,x^{4}-2\,x^{3}+x^{2}-2}}{\left(2\,x^{4}-2\,x^{3}-x^{2}-2\right)\left(x^{4}-x^{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x^3+x^2-2)^(1/2)*(2*x^4-x^3+2)/(x^4-x^3-1)/(2*x^4-2*x^3-
x^2-2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^4 - x^3 + 2)*sqrt(2*x^4 - 2*x^3 + x^2 - 2)/((2*x^4 - 2*x^3 -
 x^2 - 2)*(x^4 - x^3 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(2\,x^4-x^3+2\right)\,\sqrt{2\,x^4-2\,x^3+x^2-2}}{\left(-x^4+x^3+1\right)\,\left(-2\,x^4+2\,x^3+x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^4 - x^3 + 2)*(x^2 - 2*x^3 + 2*x^4 - 2)^(1/2))/((x^3 - x^4 + 1)*(x
^2 + 2*x^3 - 2*x^4 + 2)),x)
```

```
[Out] int(((2*x^4 - x^3 + 2)*(x^2 - 2*x^3 + 2*x^4 - 2)^(1/2))/((x^3 - x^4 + 1)*(x
^2 + 2*x^3 - 2*x^4 + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-2*x**3+x**2-2)**(1/2)*(2*x**4-x**3+2)/(x**4-x**3-1)/(2*x*
*4-2*x**3-x**2-2),x)
```

```
[Out] Timed out
```

3.745 $\int \frac{-b+ax^2}{(b+ax^2)\sqrt{b^2+a^2x^4}}\,dx$

Optimal. Leaf size=59

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}x}{\sqrt{a^2x^4+b^2}+ax^2+b}\right)}{\sqrt{a}\sqrt{b}}$$

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 0.85, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1699, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}x}{\sqrt{a^2x^4+b^2}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^2)/((b + a*x^2)*Sqrt[b^2 + a^2*x^4]),x]
```

```
[Out] -(ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[b]*x)/Sqrt[b^2 + a^2*x^4]]/(Sqrt[2]*Sqrt[a]*
Sqrt[b]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-b+ax^2}{(b+ax^2)\sqrt{b^2+a^2x^4}}\,dx &= -\left(b\,\text{Subst}\left(\int \frac{1}{b+2ab^2x^2}\,dx, x, \frac{x}{\sqrt{b^2+a^2x^4}}\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}x}{\sqrt{b^2+a^2x^4}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}\end{aligned}$$

Mathematica [C] time = 0.22, size = 95, normalized size = 1.61

$$-\frac{i\sqrt{\frac{a^2x^4}{b^2}+1}\left(F\left(i\sinh^{-1}\left(\sqrt{\frac{ia}{b}}x\right)\middle|-1\right)-2\Pi\left(-i;i\sinh^{-1}\left(\sqrt{\frac{ia}{b}}x\right)\middle|-1\right)\right)}{\sqrt{\frac{ia}{b}}\sqrt{a^2x^4+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^2)/((b + a*x^2)*Sqrt[b^2 + a^2*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (a^2*x^4)/b^2]*(EllipticF[I*ArcSinh[Sqrt[(I*a)/b]*x], -1] -
2*EllipticPi[-I, I*ArcSinh[Sqrt[(I*a)/b]*x], -1]))/(Sqrt[(I*a)/b]*Sqrt[b^2
+ a^2*x^4])
```

IntegrateAlgebraic [A] time = 0.48, size = 50, normalized size = 0.85

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}x}{\sqrt{a^2x^4+b^2}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^2)/((b + a*x^2)*Sqrt[b^2 + a^2*x^4]),x]
```

```
[Out] -(ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[b]*x)/Sqrt[b^2 + a^2*x^4]]/(Sqrt[2]*Sqrt[a]*
Sqrt[b]))
```

fricas [A] time = 0.47, size = 132, normalized size = 2.24

$$\left[\frac{1}{4}\sqrt{2}\sqrt{-\frac{1}{ab}}\log\left(\frac{a^2x^4+2\sqrt{2}\sqrt{a^2x^4+b^2}\,abx\sqrt{-\frac{1}{ab}}-2\,abx^2+b^2}{a^2x^4+2\,abx^2+b^2}\right),\frac{1}{2}\sqrt{2}\sqrt{\frac{1}{ab}}\arctan\left(\frac{\sqrt{2}\sqrt{a^2x^4+b^2}\sqrt{\frac{1}{ab}}}{2\,x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(a*x^2+b)/(a^2*x^4+b^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*sqrt(-1/(a*b))*log((a^2*x^4 + 2*sqrt(2)*sqrt(a^2*x^4 + b^2)*a*
b*x*sqrt(-1/(a*b)) - 2*a*b*x^2 + b^2)/(a^2*x^4 + 2*a*b*x^2 + b^2)), 1/2*sqr
t(2)*sqrt(1/(a*b))*arctan(1/2*sqrt(2)*sqrt(a^2*x^4 + b^2)*sqrt(1/(a*b))/x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax^2-b}{\sqrt{a^2x^4+b^2}\left(ax^2+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(a*x^2+b)/(a^2*x^4+b^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^2 - b)/(sqrt(a^2*x^4 + b^2)*(a*x^2 + b)), x)
```

maple [C] time = 0.08, size = 152, normalized size = 2.58

$$\frac{\sqrt{1-\frac{iax^2}{b}}\sqrt{1+\frac{iax^2}{b}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{ia}{b}},i\right)}{\sqrt{\frac{ia}{b}}\sqrt{a^2x^4+b^2}}-\frac{2\sqrt{1-\frac{iax^2}{b}}\sqrt{1+\frac{iax^2}{b}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{ia}{b}},i,\frac{\sqrt{-\frac{ia}{b}}}{\sqrt{\frac{ia}{b}}}\right)}{\sqrt{\frac{ia}{b}}\sqrt{a^2x^4+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)/(a*x^2+b)/(a^2*x^4+b^2)^(1/2),x)
```

```
[Out] 1/(I*a/b)^(1/2)*(1-I*a/b*x^2)^(1/2)*(1+I*a/b*x^2)^(1/2)/(a^2*x^4+b^2)^(1/2)
*EllipticF(x*(I*a/b)^(1/2),I)-2/(I*a/b)^(1/2)*(1-I*a/b*x^2)^(1/2)*(1+I*a/b*
x^2)^(1/2)/(a^2*x^4+b^2)^(1/2)*EllipticPi(x*(I*a/b)^(1/2),I,(-I*a/b)^(1/2)/
(I*a/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax^2-b}{\sqrt{a^2x^4+b^2}\left(ax^2+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(a*x^2+b)/(a^2*x^4+b^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 - b)/(sqrt(a^2*x^4 + b^2)*(a*x^2 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{b - a\,x^2}{\sqrt{a^2\,x^4 + b^2}\,\left(a\,x^2 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^2)/((b^2 + a^2*x^4)^(1/2)*(b + a*x^2)),x)
[Out] int(-(b - a*x^2)/((b^2 + a^2*x^4)^(1/2)*(b + a*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - b}{\left(ax^2 + b\right)\sqrt{a^2x^4 + b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)/(a*x**2+b)/(a**2*x**4+b**2)**(1/2),x)
[Out] Integral((a*x**2 - b)/((a*x**2 + b)*sqrt(a**2*x**4 + b**2)), x)
```

3.746 $\int \frac{3b+ax^4}{(-b-x^3+ax^4)\sqrt[4]{-bx+ax^5}}\,dx$

Optimal. Leaf size=59

$$-2\tan^{-1}\left(\frac{(ax^5-bx)^{3/4}}{ax^4-b}\right)-2\tanh^{-1}\left(\frac{(ax^5-bx)^{3/4}}{ax^4-b}\right)$$

Rubi [F] time = 2.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3b+ax^4}{(-b-x^3+ax^4)\sqrt[4]{-bx+ax^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3*b + a*x^4)/((-b - x^3 + a*x^4)*(-(b*x) + a*x^5)^(1/4)),x]
```

```
[Out] (4*x*(1 - (a*x^4)/b)^(1/4)*Hypergeometric2F1[3/16, 1/4, 19/16, (a*x^4)/b])/
(3*(-(b*x) + a*x^5)^(1/4)) - (16*b*x^(1/4)*(-b + a*x^4)^(1/4)*Defer[Subst][
Defer[Int][x^2/((b + x^12 - a*x^16)*(-b + a*x^16)^(1/4)), x], x, x^(1/4)])/
(-(b*x) + a*x^5)^(1/4) + (4*x^(1/4)*(-b + a*x^4)^(1/4)*Defer[Subst][Defer[I
nt][x^14/((-b + a*x^16)^(1/4)*(-b - x^12 + a*x^16)), x], x, x^(1/4)])/(-(b*
x) + a*x^5)^(1/4)
```

Rubi steps

$$\int \frac{3b+ax^4}{(-b-x^3+ax^4)\sqrt[4]{-bx+ax^5}}\,dx=\frac{\left(\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\int\frac{3b+ax^4}{\sqrt[4]{x}\sqrt[4]{-b+ax^4}\,(-b-x^3+ax^4)}\,dx}{\sqrt[4]{-bx+ax^5}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\text{Subst}\left(\int\frac{x^2(3b+ax^{16})}{\sqrt[4]{-b+ax^{16}}\,(-b-x^{12}+ax^{16})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^5}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\text{Subst}\left(\int\left(\frac{x^2}{\sqrt[4]{-b+ax^{16}}}+\frac{x^2(4b+x^{12})}{\sqrt[4]{-b+ax^{16}}\,(-b-x^{12}+ax^{16})}\right)dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^5}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt[4]{-b+ax^{16}}}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^5}}+\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\text{Subs}}{}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\text{Subst}\left(\int\left(-\frac{4bx^2}{(b+x^{12}-ax^{16})\sqrt[4]{-b+ax^{16}}}+\frac{x^{14}}{\sqrt[4]{-b+ax^{16}}\,(-b-x^{12}+ax^{16})}\right)\right.}{\sqrt[4]{-bx+ax^5}}$$

$$=\frac{4x\sqrt[4]{1-\frac{ax^4}{b}}\;{}_2F_1\left(\frac{3}{16},\frac{1}{4};\frac{19}{16};\frac{ax^4}{b}\right)}{3\sqrt[4]{-bx+ax^5}}+\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\text{Subst}\left(\int\frac{x^{14}}{\sqrt[4]{-b+ax^{16}}\,(-b-\right.}{\sqrt[4]{-bx+ax^5}}$$

Mathematica [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{3b+ax^4}{(-b-x^3+ax^4)\sqrt[4]{-bx+ax^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3*b + a*x^4)/((-b - x^3 + a*x^4)*(-(b*x) + a*x^5)^(1/4)),x]
[Out] Integrate[(3*b + a*x^4)/((-b - x^3 + a*x^4)*(-(b*x) + a*x^5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 1.19, size = 59, normalized size = 1.00

$$-2\tan^{-1}\left(\frac{\left(ax^5-bx\right)^{3/4}}{ax^4-b}\right)-2\tanh^{-1}\left(\frac{\left(ax^5-bx\right)^{3/4}}{ax^4-b}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*b + a*x^4)/((-b - x^3 + a*x^4)*(-(b*x) + a*x^5)^(1/4)
),x]
[Out] -2*ArcTan[(-(b*x) + a*x^5)^(3/4)/(-b + a*x^4)] - 2*ArcTanh[(-(b*x) + a*x^5)
^(3/4)/(-b + a*x^4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+3*b)/(a*x^4-x^3-b)/(a*x^5-b*x)^(1/4),x, algorithm="fricas"
)
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4+3\,b}{\left(ax^5-bx\right)^{\frac{1}{4}}\left(ax^4-x^3-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+3*b)/(a*x^4-x^3-b)/(a*x^5-b*x)^(1/4),x, algorithm="giac")
[Out] integrate((a*x^4 + 3*b)/((a*x^5 - b*x)^(1/4)*(a*x^4 - x^3 - b)), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{a\,x^4+3b}{\left(a\,x^4-x^3-b\right)\left(a\,x^5-bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+3*b)/(a*x^4-x^3-b)/(a*x^5-b*x)^(1/4),x)
[Out] int((a*x^4+3*b)/(a*x^4-x^3-b)/(a*x^5-b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4+3\,b}{\left(ax^5-bx\right)^{\frac{1}{4}}\left(ax^4-x^3-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+3*b)/(a*x^4-x^3-b)/(a*x^5-b*x)^(1/4),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^4 + 3*b)/((a*x^5 - b*x)^(1/4)*(a*x^4 - x^3 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{a\,x^4+3\,b}{{\left(a\,x^5-b\,x\right)}^{1/4}\,\left(-a\,x^4+x^3+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*b + a*x^4)/((a*x^5 - b*x)^(1/4)*(b - a*x^4 + x^3)),x)
[Out] int(-(3*b + a*x^4)/((a*x^5 - b*x)^(1/4)*(b - a*x^4 + x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+3*b)/(a*x**4-x**3-b)/(a*x**5-b*x)**(1/4),x)
[Out] Timed out
```

3.747 $\int \frac{\left(-1+x^3\right)\sqrt{-1+x^6}}{x}\,dx$

Optimal. Leaf size=59

$$\frac{1}{6}\sqrt{x^6-1}\left(x^3-2\right)-\frac{2}{3}\tan^{-1}\left(\frac{x^3+1}{\sqrt{x^6-1}}\right)-\frac{1}{3}\tanh^{-1}\left(\frac{x^3+1}{\sqrt{x^6-1}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1475, 815, 844, 217, 206, 266, 63, 203}

$$\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{1}{6}\sqrt{x^6-1}\left(2-x^3\right)-\frac{1}{6}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)*Sqrt[-1 + x^6])/x,x]
```

```
[Out] -1/6*((2 - x^3)*Sqrt[-1 + x^6]) + ArcTan[Sqrt[-1 + x^6]]/3 - ArcTanh[x^3/Sq
rt[-1 + x^6]]/6
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
```

```
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^3\right)\sqrt{-1+x^6}}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(-1+x)\sqrt{-1+x^2}}{x}\,dx, x, x^3\right)\\
&= -\frac{1}{6}\left(2-x^3\right)\sqrt{-1+x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{2-x}{x\sqrt{-1+x^2}}\,dx, x, x^3\right)\\
&= -\frac{1}{6}\left(2-x^3\right)\sqrt{-1+x^6}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, x^3\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{-1+x^2}}\right.\\
&= -\frac{1}{6}\left(2-x^3\right)\sqrt{-1+x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,\right.\\
&= -\frac{1}{6}\left(2-x^3\right)\sqrt{-1+x^6}-\frac{1}{6}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\right.\\
&= -\frac{1}{6}\left(2-x^3\right)\sqrt{-1+x^6}+\frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right)-\frac{1}{6}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 1.36

$$\frac{2\sqrt{-\left(x^6-1\right)^2}\tan^{-1}\left(\sqrt{x^6-1}\right)+\left(x^6-1\right)\sin^{-1}\left(x^3\right)+\sqrt{1-x^6}\left(x^9-2x^6-x^3+2\right)}{6\sqrt{-\left(x^6-1\right)^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^3)*Sqrt[-1 + x^6])/x,x]
```

```
[Out] (Sqrt[1 - x^6]*(2 - x^3 - 2*x^6 + x^9) + (-1 + x^6)*ArcSin[x^3] + 2*Sqrt[-(
-1 + x^6)^2]*ArcTan[Sqrt[-1 + x^6]])/(6*Sqrt[-(-1 + x^6)^2])
```

IntegrateAlgebraic [A] time = 0.17, size = 63, normalized size = 1.07

$$\frac{1}{6}\sqrt{x^6-1}\left(x^3-2\right)-\frac{2}{3}\tan^{-1}\left(\frac{\sqrt{x^6-1}}{x^3-1}\right)-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x^6-1}}{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)*Sqrt[-1 + x^6])/x,x]
```

```
[Out] ((-2 + x^3)*Sqrt[-1 + x^6])/6 - (2*ArcTan[Sqrt[-1 + x^6]/(-1 + x^3)])/3 - A
rcTanh[Sqrt[-1 + x^6]/(-1 + x^3)]/3
```

fricas [A] time = 0.39, size = 47, normalized size = 0.80

$$\frac{1}{6}\sqrt{x^6-1}\left(x^3-2\right)+\frac{2}{3}\arctan\left(-x^3+\sqrt{x^6-1}\right)+\frac{1}{6}\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(x^6 - 1)*(x^3 - 2) + 2/3*arctan(-x^3 + sqrt(x^6 - 1)) + 1/6*log(-x
^3 + sqrt(x^6 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^6-1}\left(x^3-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^6 - 1)*(x^3 - 1)/x, x)
```

maple [C] time = 0.26, size = 136, normalized size = 2.31

$$\frac{\sqrt{\mathrm{signum}\left(x^6-1\right)}\left(4\sqrt{\pi}-4\sqrt{\pi}\sqrt{-x^6+1}+4\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{-x^6+1}}{2}\right)-2\left(2-2\ln(2)+6\ln(x)+i\pi\right)\sqrt{\pi}\right)}{12\sqrt{\pi}\sqrt{-\mathrm{signum}\left(x^6-1\right)}}+\frac{i\sqrt{\mathrm{signum}\left(x^6-1\right)}\left(-2i\sqrt{\pi}\,x^3\sqrt{-x^6+1}-2i\sqrt{\pi}\arcsin\left(x^3\right)\right)}{12\sqrt{\pi}\sqrt{-\mathrm{signum}\left(x^6-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)*(x^6-1)^(1/2)/x,x)
```

```
[Out] 1/12/Pi^(1/2)*signum(x^6-1)^(1/2)/(-signum(x^6-1))^(1/2)*(4*Pi^(1/2)-4*Pi^(
1/2)*(-x^6+1)^(1/2)+4*Pi^(1/2)*ln(1/2+1/2*(-x^6+1)^(1/2))-2*(2-2*ln(2)+6*ln
(x)+I*Pi)*Pi^(1/2))+1/12*I/Pi^(1/2)*signum(x^6-1)^(1/2)/(-signum(x^6-1))^(1
/2)*(-2*I*Pi^(1/2)*x^3*(-x^6+1)^(1/2)-2*I*Pi^(1/2)*arcsin(x^3))
```

maxima [A] time = 0.43, size = 77, normalized size = 1.31

$$-\frac{1}{3}\sqrt{x^6-1}-\frac{\sqrt{x^6-1}}{6\,x^3\left(\frac{x^6-1}{x^6}-1\right)}+\frac{1}{3}\arctan\left(\sqrt{x^6-1}\right)-\frac{1}{12}\log\left(\frac{\sqrt{x^6-1}}{x^3}+1\right)+\frac{1}{12}\log\left(\frac{\sqrt{x^6-1}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(x^6 - 1) - 1/6*sqrt(x^6 - 1)/(x^3*((x^6 - 1)/x^6 - 1)) + 1/3*arct
an(sqrt(x^6 - 1)) - 1/12*log(sqrt(x^6 - 1)/x^3 + 1) + 1/12*log(sqrt(x^6 - 1
)/x^3 - 1)
```

mupad [B] time = 1.04, size = 54, normalized size = 0.92

$$\frac{x^3\sqrt{x^6-1}}{6}-\frac{\sqrt{x^6-1}}{3}-\frac{\ln\left(\sqrt{x^6-1}+x^3\right)}{6}+\frac{\ln\left(\frac{\sqrt{x^6-1}+1\mathrm{i}}{x^3}\right)\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)*(x^6 - 1)^(1/2))/x,x)
```

```
[Out] (log(((x^6 - 1)^(1/2) + 1i)/x^3)*1i)/3 - log((x^6 - 1)^(1/2) + x^3)/6 - (x^
6 - 1)^(1/2)/3 + (x^3*(x^6 - 1)^(1/2))/6
```

sympy [C] time = 10.29, size = 150, normalized size = 2.54

$$\begin{cases} \frac{x^9}{6\sqrt{x^6-1}} - \frac{x^3}{6\sqrt{x^6-1}} - \frac{\operatorname{acosh}\left(x^3\right)}{6} & \text{for } \left|x^6\right| > 1 \\ \frac{ix^3\sqrt{1-x^6}}{6} + \frac{i\operatorname{asin}\left(x^3\right)}{6} & \text{otherwise} \end{cases} - \begin{cases} -\frac{ix^3}{3\sqrt{-1+\frac{1}{x^6}}} - \frac{i\operatorname{acosh}\left(\frac{1}{x^3}\right)}{3} + \frac{i}{3x^3\sqrt{-1+\frac{1}{x^6}}} & \text{for } \frac{1}{\left|x^6\right|} > 1 \\ \frac{x^3}{3\sqrt{1-\frac{1}{x^6}}} + \frac{\operatorname{asin}\left(\frac{1}{x^3}\right)}{3} - \frac{1}{3x^3\sqrt{1-\frac{1}{x^6}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)*(x**6-1)**(1/2)/x,x)
```

```
[Out] Piecewise((x**9/(6*sqrt(x**6 - 1)) - x**3/(6*sqrt(x**6 - 1)) - acosh(x**3)/
6, Abs(x**6) > 1), (I*x**3*sqrt(1 - x**6)/6 + I*asin(x**3)/6, True)) - Piec
ewise((-I*x**3/(3*sqrt(-1 + x**(-6))) - I*acosh(x**(-3))/3 + I/(3*x**3*sqrt
(-1 + x**(-6))), 1/Abs(x**6) > 1), (x**3/(3*sqrt(1 - 1/x**6)) + asin(x**(-3
))/3 - 1/(3*x**3*sqrt(1 - 1/x**6)), True))
```

3.748 $\int \frac{b+ax^8}{\sqrt[4]{b-ax^8}\left(-b+cx^4+ax^8\right)}\,dx$

Optimal. Leaf size=59

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{b-ax^8}}\right)}{2\sqrt[4]{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{b-ax^8}}\right)}{2\sqrt[4]{c}}$$

Rubi [C] time = 1.50, antiderivative size = 460, normalized size of antiderivative = 7.80, number of steps used = 18, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6728, 246, 245, 1438, 430, 429, 511, 510}

$$-\frac{x\sqrt[4]{1-\frac{ax^8}{b}}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\frac{2a^2x^8}{c^2-\sqrt{c^2+4ab}\,c+2ab},\frac{ax^8}{b}\right)}{\sqrt[4]{b-ax^8}}-\frac{x\sqrt[4]{1-\frac{ax^8}{b}}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\frac{2a^2x^8}{2ab+c\left(c+\sqrt{c^2+4ab}\right)},\frac{ax^8}{b}\right)}{\sqrt[4]{b-ax^8}}+\frac{ax^5\left(c-\sqrt{4ab+c^2}\right)\sqrt[4]{1-\frac{ax^8}{b}}\,F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};\frac{2a^2x^8}{c^2-\sqrt{c^2+4ab}\,c+2ab},\frac{ax^8}{b}\right)}{5\left(c\left(c-\sqrt{4ab+c^2}\right)+2ab\right)\sqrt[4]{b-ax^8}}+\frac{ax^5\left(\sqrt{4ab+c^2}+c\right)\sqrt[4]{1-\frac{ax^8}{b}}\,F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};\frac{2a^2x^8}{2ab+c\left(c+\sqrt{c^2+4ab}\right)},\frac{ax^8}{b}\right)}{5\left(c\left(\sqrt{4ab+c^2}+c\right)+2ab\right)\sqrt[4]{b-ax^8}}+\frac{x\sqrt[4]{1-\frac{ax^8}{b}}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};\frac{ax^8}{b}\right)}{\sqrt[4]{b-ax^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(b + a*x^8)/((b - a*x^8)^(1/4)*(-b + c*x^4 + a*x^8)),x]
```

```
[Out] -((x*(1 - (a*x^8)/b)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, (2*a^2*x^8)/(2*a*b +
c^2 - c*Sqrt[4*a*b + c^2]), (a*x^8)/b])/(b - a*x^8)^(1/4)) - (x*(1 - (a*x^8
)/b)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, (2*a^2*x^8)/(2*a*b + c*(c + Sqrt[4*a*
b + c^2])), (a*x^8)/b])/(b - a*x^8)^(1/4) + (a*(c - Sqrt[4*a*b + c^2])*x^5*
(1 - (a*x^8)/b)^(1/4)*AppellF1[5/8, 1, 1/4, 13/8, (2*a^2*x^8)/(2*a*b + c^2
- c*Sqrt[4*a*b + c^2]), (a*x^8)/b])/(5*(2*a*b + c*(c - Sqrt[4*a*b + c^2]))*
(b - a*x^8)^(1/4)) + (a*(c + Sqrt[4*a*b + c^2])*x^5*(1 - (a*x^8)/b)^(1/4)*A
ppellF1[5/8, 1, 1/4, 13/8, (2*a^2*x^8)/(2*a*b + c*(c + Sqrt[4*a*b + c^2])),
 (a*x^8)/b])/(5*(2*a*b + c*(c + Sqrt[4*a*b + c^2]))*(b - a*x^8)^(1/4)) + (x
*(1 - (a*x^8)/b)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, (a*x^8)/b])/(b - a*
x^8)^(1/4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{b+ax^8}{\sqrt[4]{b-ax^8}\left(-b+cx^4+ax^8\right)}\,dx = \int \left(\frac{1}{\sqrt[4]{b-ax^8}}+\frac{2b-cx^4}{\sqrt[4]{b-ax^8}\left(-b+cx^4+ax^8\right)}\right)dx$$

$$= \int \frac{1}{\sqrt[4]{b-ax^8}}\,dx+\int \frac{2b-cx^4}{\sqrt[4]{b-ax^8}\left(-b+cx^4+ax^8\right)}\,dx$$

$$= \frac{\sqrt[4]{1-\frac{ax^8}{b}}\int \frac{1}{\sqrt[4]{1-\frac{ax^8}{b}}}\,dx}{\sqrt[4]{b-ax^8}}+\int \left(\frac{-c+\sqrt{4ab+c^2}}{\left(c-\sqrt{4ab+c^2}+2ax^4\right)\sqrt[4]{b-ax^8}}+\frac{}{\left(c+\sqrt{}\right.}\right.$$

$$= \frac{x\sqrt[4]{1-\frac{ax^8}{b}}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};\frac{ax^8}{b}\right)}{\sqrt[4]{b-ax^8}}+\left(-c-\sqrt{4ab+c^2}\right)\int \frac{1}{\left(c+\sqrt{4ab+c^2}+2a\right.}$$

$$= \frac{x\sqrt[4]{1-\frac{ax^8}{b}}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};\frac{ax^8}{b}\right)}{\sqrt[4]{b-ax^8}}+\left(-c-\sqrt{4ab+c^2}\right)\int \left(\frac{c+\sqrt{}}{2\sqrt[4]{b-ax^8}\left(2ab+c^2\right.}\right.$$

$$= \frac{x\sqrt[4]{1-\frac{ax^8}{b}}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};\frac{ax^8}{b}\right)}{\sqrt[4]{b-ax^8}}-\left(a\left(c-\sqrt{4ab+c^2}\right)\right)\int \frac{}{\sqrt[4]{b-ax^8}\left(-2ab-\right.}$$

$$= \frac{x\sqrt[4]{1-\frac{ax^8}{b}}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};\frac{ax^8}{b}\right)}{\sqrt[4]{b-ax^8}}-\frac{\left(a\left(c-\sqrt{4ab+c^2}\right)\sqrt[4]{1-\frac{ax^8}{b}}\right)\int \frac{}{\left(-2ab-c^2+c\right.}}{\sqrt[4]{b-ax^8}}$$

$$= -\frac{x\sqrt[4]{1-\frac{ax^8}{b}}\;F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\frac{2a^2x^8}{2ab+c^2-c\sqrt{4ab+c^2}},\frac{ax^8}{b}\right)}{\sqrt[4]{b-ax^8}}-\frac{x\sqrt[4]{1-\frac{ax^8}{b}}\;F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\right.}{\sqrt[4]{b-a}}$$

Mathematica [F] time = 1.16, size = 0, normalized size = 0.00

$$\int \frac{b+ax^8}{\sqrt[4]{b-ax^8}\left(-b+cx^4+ax^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(b + a*x^8)/((b - a*x^8)^(1/4)*(-b + c*x^4 + a*x^8)),x]
```

```
[Out] Integrate[(b + a*x^8)/((b - a*x^8)^(1/4)*(-b + c*x^4 + a*x^8)), x]
```

IntegrateAlgebraic [A] time = 12.27, size = 59, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{b-ax^8}}\right)}{2\sqrt[4]{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{b-ax^8}}\right)}{2\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^8)/((b - a*x^8)^(1/4)*(-b + c*x^4 + a*x^8)),x]
```

```
[Out] -1/2*ArcTan[(c^(1/4)*x)/(b - a*x^8)^(1/4)]/c^(1/4) - ArcTanh[(c^(1/4)*x)/(b
 - a*x^8)^(1/4)]/(2*c^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+b)/(-a*x^8+b)^(1/4)/(a*x^8+c*x^4-b),x, algorithm="fricas")
[Out] Timed out
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^8 + b}{(ax^8 + cx^4 - b)(-ax^8 + b)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+b)/(-a*x^8+b)^(1/4)/(a*x^8+c*x^4-b),x, algorithm="giac")
[Out] integrate((a*x^8 + b)/((a*x^8 + c*x^4 - b)*(-a*x^8 + b)^(1/4)), x)
```

maple **[F]** time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{a\,x^8 + b}{(-a\,x^8 + b)^{\frac{1}{4}}\,(a\,x^8 + c\,x^4 - b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^8+b)/(-a*x^8+b)^(1/4)/(a*x^8+c*x^4-b),x)
[Out] int((a*x^8+b)/(-a*x^8+b)^(1/4)/(a*x^8+c*x^4-b),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^8 + b}{(ax^8 + cx^4 - b)(-ax^8 + b)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+b)/(-a*x^8+b)^(1/4)/(a*x^8+c*x^4-b),x, algorithm="maxima")
[Out] integrate((a*x^8 + b)/((a*x^8 + c*x^4 - b)*(-a*x^8 + b)^(1/4)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a\,x^8 + b}{(b - a\,x^8)^{1/4}\,(a\,x^8 + c\,x^4 - b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^8)/((b - a*x^8)^(1/4)*(a*x^8 - b + c*x^4)),x)
[Out] int((b + a*x^8)/((b - a*x^8)^(1/4)*(a*x^8 - b + c*x^4)), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**8+b)/(-a*x**8+b)**(1/4)/(a*x**8+c*x**4-b),x)
[Out] Timed out
```

3.749 $\int \frac{ab+ac-bc-2ax+x^2}{\sqrt{(-a+x)(-b+x)(-c+x)}\left(bc+ad-(b+c+d)x+x^2\right)}\,dx$

Optimal. Leaf size=60

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}}{\sqrt{d}\,(a-x)}\right)}{\sqrt{d}}$$

Rubi [C] time = 15.47, antiderivative size = 440, normalized size of antiderivative = 7.33, number of steps used = 16, number of rules used = 7, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {6718, 6728, 121, 120, 169, 538, 537}

$$-\frac{2\sqrt{b-a}\,\sqrt{x-a}\,\sqrt{-\frac{b-x}{a-b}}\,\sqrt{-\frac{c-x}{a-c}}\,\Pi\left(\frac{2(a-b)}{2a-b-c-d-\sqrt{b^2-2cb+2db+c^2+d^2-4ad+2cd}};\sin^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{b-a}}\right)|\frac{a-b}{a-c}\right)}{\sqrt{-((a-x)(b-x)(c-x))}}-\frac{2\sqrt{b-a}\,\sqrt{x-a}\,\sqrt{-\frac{b-x}{a-b}}\,\sqrt{-\frac{c-x}{a-c}}\,\Pi\left(\frac{2(a-b)}{2a-b-c-d+\sqrt{b^2-2cb+2db+c^2+d^2-4ad+2cd}};\sin^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{b-a}}\right)|\frac{a-b}{a-c}\right)}{\sqrt{-((a-x)(b-x)(c-x))}}+\frac{2\sqrt{b-a}\,\sqrt{x-a}\,\sqrt{-\frac{b-x}{a-b}}\,\sqrt{-\frac{c-x}{a-c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{b-a}}\right)|\frac{a-b}{a-c}\right)}{\sqrt{-((a-x)(b-x)(c-x))}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b + a*c - b*c - 2*a*x + x^2)/(Sqrt[(-a + x)*(-b + x)*(-c + x)]*(b*c
+ a*d - (b + c + d)*x + x^2)),x]
```

```
[Out] (2*Sqrt[-a + b]*Sqrt[-((b - x)/(a - b))]*Sqrt[-((c - x)/(a - c))]*Sqrt[-a +
 x]*EllipticF[ArcSin[Sqrt[-a + x]/Sqrt[-a + b]], (a - b)/(a - c)])/Sqrt[-((
a - x)*(b - x)*(c - x))] - (2*Sqrt[-a + b]*Sqrt[-((b - x)/(a - b))]*Sqrt[-(
(c - x)/(a - c))]*Sqrt[-a + x]*EllipticPi[(2*(a - b))/(2*a - b - c - d - Sq
rt[b^2 - 2*b*c + c^2 - 4*a*d + 2*b*d + 2*c*d + d^2]), ArcSin[Sqrt[-a + x]/S
qrt[-a + b]], (a - b)/(a - c)])/Sqrt[-((a - x)*(b - x)*(c - x))] - (2*Sqrt[
-a + b]*Sqrt[-((b - x)/(a - b))]*Sqrt[-((c - x)/(a - c))]*Sqrt[-a + x]*Elli
pticPi[(2*(a - b))/(2*a - b - c - d + Sqrt[b^2 - 2*b*c + c^2 - 4*a*d + 2*b*
d + 2*c*d + d^2]), ArcSin[Sqrt[-a + x]/Sqrt[-a + b]], (a - b)/(a - c)])/Sqr
t[-((a - x)*(b - x)*(c - x))]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
```

```
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{ab+ac-bc-2ax+x^2}{\sqrt{(-a+x)(-b+x)(-c+x)}\left(bc+ad-(b+c+d)x+x^2\right)}\,dx = \frac{\left(\sqrt{-a+x}\sqrt{-b+x}\sqrt{-c+x}\right)\int \frac{ab}{\sqrt{-a+x}\sqrt{-b+x}}}{\sqrt{(-a+x)(-b+x)(-c}}$$

$$= \frac{\left(\sqrt{-a+x}\sqrt{-b+x}\sqrt{-c+x}\right)\int\left(\frac{1}{\sqrt{-a+x}\sqrt{-b+}}\right.}{\sqrt{(-a+x)}}$$

$$= \frac{\left(\sqrt{-a+x}\sqrt{-b+x}\sqrt{-c+x}\right)\int \frac{1}{\sqrt{-a+x}\sqrt{-b+x}}}{\sqrt{(-a+x)(-b+x)(-c+x)}}$$

$$= \frac{\left(\sqrt{-a+x}\sqrt{-b+x}\sqrt{-c+x}\right)\int\left(\frac{-}{\sqrt{-a+x}\sqrt{-b+}}\right.}{}$$

$$= \frac{\left(\left(-2a+b+c+d-\sqrt{b^2-2bc+c^2-4ad+}\right.\right.}{}$$

$$= \frac{2\sqrt{-a+b}\sqrt{-\frac{b-x}{a-b}}\sqrt{-\frac{c-x}{a-c}}\sqrt{-a+x}\,F\left(\sin^{-1}\left(\right.\right.}{\sqrt{-((a-x)(b-x)(c-x))}}$$

$$= \frac{2\sqrt{-a+b}\sqrt{-\frac{b-x}{a-b}}\sqrt{-\frac{c-x}{a-c}}\sqrt{-a+x}\,F\left(\sin^{-1}\left(\right.\right.}{\sqrt{-((a-x)(b-x)(c-x))}}$$

$$= \frac{2\sqrt{-a+b}\sqrt{-\frac{b-x}{a-b}}\sqrt{-\frac{c-x}{a-c}}\sqrt{-a+x}\,F\left(\sin^{-1}\left(\right.\right.}{\sqrt{-((a-x)(b-x)(c-x))}}$$

$$= \frac{2\sqrt{-a+b}\sqrt{-\frac{b-x}{a-b}}\sqrt{-\frac{c-x}{a-c}}\sqrt{-a+x}\,F\left(\sin^{-1}\left(\right.\right.}{\sqrt{-((a-x)(b-x)(c-x))}}$$

Mathematica [C] time = 6.91, size = 289, normalized size = 4.82

$$-\frac{2i(x-a)^{3/2}\sqrt{\frac{b-x}{a-x}}\sqrt{\frac{c-x}{a-x}}\left(-\Pi\left(-\frac{2(a-c)}{-2a+b+c+d-\sqrt{b^2-2cb+2db+c^2+d^2-4ad+2cd}};i\sinh^{-1}\left(\frac{\sqrt{a-b}}{\sqrt{x-a}}\right)|\frac{a-c}{a-b}\right)-\Pi\left(-\frac{2(a-c)}{-2a+b+c+d+\sqrt{b^2-2cb+2db+c^2+d^2-4ad+2cd}};i\sinh^{-1}\left(\frac{\sqrt{a-b}}{\sqrt{x-a}}\right)|\frac{a-c}{a-b}\right)+F\left(i\sinh^{-1}\left(\frac{\sqrt{a-b}}{\sqrt{x-a}}\right)|\frac{a-c}{a-b}\right)\right)}{\sqrt{a-b}\sqrt{(x-a)(x-b)(x-c)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b + a*c - b*c - 2*a*x + x^2)/(Sqrt[(-a + x)*(-b + x)*(-c + x)]
*(b*c + a*d - (b + c + d)*x + x^2)),x]
```

```
[Out] ((-2*I)*Sqrt[(b - x)/(a - x)]*Sqrt[(c - x)/(a - x)]*(-a + x)^(3/2)*(Ellipti
cF[I*ArcSinh[Sqrt[a - b]/Sqrt[-a + x]], (a - c)/(a - b)] - EllipticPi[(-2*(
a - c))/(-2*a + b + c + d - Sqrt[b^2 - 2*b*c + c^2 - 4*a*d + 2*b*d + 2*c*d
+ d^2]), I*ArcSinh[Sqrt[a - b]/Sqrt[-a + x]], (a - c)/(a - b)] - EllipticPi
[(-2*(a - c))/(-2*a + b + c + d + Sqrt[b^2 - 2*b*c + c^2 - 4*a*d + 2*b*d +
2*c*d + d^2]), I*ArcSinh[Sqrt[a - b]/Sqrt[-a + x]], (a - c)/(a - b)]))/(Sqr
t[a - b]*Sqrt[(-a + x)*(-b + x)*(-c + x)])
```

IntegrateAlgebraic [A] time = 2.80, size = 60, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}}{\sqrt{d}\,(a-x)}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b + a*c - b*c - 2*a*x + x^2)/(Sqrt[(-a + x)*(-b + x)*
(-c + x)]*(b*c + a*d - (b + c + d)*x + x^2)),x]
```

```
[Out] (2*ArcTanh[Sqrt[-(a*b*c) + (a*b + a*c + b*c)*x + (-a - b - c)*x^2 + x^3]/(S
qrt[d]*(a - x))])/Sqrt[d]
```

fricas [B] time = 21.83, size = 349, normalized size = 5.82

$$\left[\frac{\log\left(\frac{b^2c^2-6\,abcd+a^2d^2-2\,(b+c-3\,d)x^3+x^4+\left(b^2+4\,bc+c^2-6\,(a+b+c)d+d^2\right)x^2-4\,\sqrt{-abc-(a+b+c)x^2+x^3+(ab+(a+b)c)x}\left(bc-ad-(b+c-d)x+x^2\right)\sqrt{d}-2\left(b^2c+bc^2+ad^2-3\,(ab+(a+b)c)d\right)x}{b^2c^2+2\,abcd+a^2d^2-2\,(b+c+d)x^3+x^4+\left(b^2+4\,bc+c^2+2\,(a+b+c)d+d^2\right)x^2-2\left(b^2c+bc^2+ad^2+(ab+(a+b)c)d\right)x}\right)}{2\,\sqrt{d}},\ \frac{\sqrt{-d}\arctan\left(-\frac{\sqrt{-abc-(a+b+c)x^2+x^3+(ab+(a+b)c)x}\left(bc-ad-(b+c-d)x+x^2\right)\sqrt{-d}}{2\left(abcd+(a+b+c)dx^2-dx^3-(ab+(a+b)c)dx\right)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b+a*c-2*a*x-b*c+x^2)/((-a+x)*(-b+x)*(-c+x))^(1/2)/(b*c+a*d-(b+
c+d)*x+x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b^2*c^2 - 6*a*b*c*d + a^2*d^2 - 2*(b + c - 3*d)*x^3 + x^4 + (b^2
+ 4*b*c + c^2 - 6*(a + b + c)*d + d^2)*x^2 - 4*sqrt(-a*b*c - (a + b + c)*x^
2 + x^3 + (a*b + (a + b)*c)*x)*(b*c - a*d - (b + c - d)*x + x^2)*sqrt(d) -
2*(b^2*c + b*c^2 + a*d^2 - 3*(a*b + (a + b)*c)*d)*x)/(b^2*c^2 + 2*a*b*c*d +
 a^2*d^2 - 2*(b + c + d)*x^3 + x^4 + (b^2 + 4*b*c + c^2 + 2*(a + b + c)*d +
 d^2)*x^2 - 2*(b^2*c + b*c^2 + a*d^2 + (a*b + (a + b)*c)*d)*x))/sqrt(d), sq
rt(-d)*arctan(-1/2*sqrt(-a*b*c - (a + b + c)*x^2 + x^3 + (a*b + (a + b)*c)*
x)*(b*c - a*d - (b + c - d)*x + x^2)*sqrt(-d)/(a*b*c*d + (a + b + c)*d*x^2
- d*x^3 - (a*b + (a + b)*c)*d*x))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab + ac - bc - 2\,ax + x^2}{\sqrt{-(a - x)(b - x)(c - x)}\left(bc + ad - (b + c + d)x + x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b+a*c-2*a*x-b*c+x^2)/((-a+x)*(-b+x)*(-c+x))^(1/2)/(b*c+a*d-(b+
c+d)*x+x^2),x, algorithm="giac")
```

```
[Out] integrate((a*b + a*c - b*c - 2*a*x + x^2)/(sqrt(-(a - x)*(b - x)*(c - x))*(
b*c + a*d - (b + c + d)*x + x^2)), x)
```

maple [C] time = 0.11, size = 10248, normalized size = 170.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b+a*c-2*a*x-b*c+x^2)/((-a+x)*(-b+x)*(-c+x))^(1/2)/(b*c+a*d-(b+c+d)*x
+x^2),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b+a*c-2*a*x-b*c+x^2)/((-a+x)*(-b+x)*(-c+x))^(1/2)/(b*c+a*d-(b+
c+d)*x+x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume((d+c+b)^2>0)', see `assume?` for mo
re details)Is (d+c+b)^2    -4*(a*d+b*c)    positive, negative or zero?
```

mupad [B] time = 0.82, size = 711, normalized size = 11.85

$$\frac{2\,(a-c)\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{c-x}{a-c}}\right)\middle|\frac{a-c}{b-c}\right)\sqrt{\frac{a-x}{a-c}}\,\sqrt{-\frac{c-x}{a-c}}\,\sqrt{\frac{b-x}{b-c}}}{\sqrt{x^3+(-a-b-c)\,x^2+(a\,b+a\,c+b\,c)\,x-a\,b\,c}}+\frac{2\,(a-c)\,\sqrt{\frac{a-x}{a-c}}\,\sqrt{-\frac{c-x}{a-c}}\,\sqrt{\frac{b-x}{b-c}}\,\Pi\left(\frac{a-c}{\frac{b}{2}-\frac{c}{2}+\frac{d}{2}-\frac{\sqrt{b^2-2bc+2bd+c^2+2cd+d^2-4ad}}{2}};\mathrm{asin}\left(\sqrt{-\frac{c-x}{a-c}}\right)\middle|\frac{a-c}{b-c}\right)\left(a\,b+a\,c-a\,d-2\,b\,c+(b-2\,a+c+d)\left(\frac{b}{2}+\frac{c}{2}+\frac{d}{2}-\frac{\sqrt{b^2-2bc+2bd+c^2+2cd+d^2-4ad}}{2}\right)\right)}{\sqrt{x^3+(-a-b-c)\,x^2+(a\,b+a\,c+b\,c)\,x-a\,b\,c}\,\left(\frac{b}{2}-\frac{c}{2}+\frac{d}{2}-\frac{\sqrt{b^2-2bc+2bd+c^2+2cd+d^2-4ad}}{2}\right)\sqrt{b^2-2\,b\,c+2\,b\,d+c^2+2\,c\,d+d^2-4\,a\,d}}-\frac{2\,(a-c)\,\sqrt{\frac{a-x}{a-c}}\,\sqrt{-\frac{c-x}{a-c}}\,\sqrt{\frac{b-x}{b-c}}\,\Pi\left(\frac{a-c}{\frac{b}{2}-\frac{c}{2}+\frac{d}{2}+\frac{\sqrt{b^2-2bc+2bd+c^2+2cd+d^2-4ad}}{2}};\mathrm{asin}\left(\sqrt{-\frac{c-x}{a-c}}\right)\middle|\frac{a-c}{b-c}\right)\left(a\,b+a\,c-a\,d-2\,b\,c+(b-2\,a+c+d)\left(\frac{b}{2}+\frac{c}{2}+\frac{d}{2}+\frac{\sqrt{b^2-2bc+2bd+c^2+2cd+d^2-4ad}}{2}\right)\right)}{\sqrt{x^3+(-a-b-c)\,x^2+(a\,b+a\,c+b\,c)\,x-a\,b\,c}\,\left(\frac{b}{2}-\frac{c}{2}+\frac{d}{2}+\frac{\sqrt{b^2-2bc+2bd+c^2+2cd+d^2-4ad}}{2}\right)\sqrt{b^2-2\,b\,c+2\,b\,d+c^2+2\,c\,d+d^2-4\,a\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b + a*c - b*c - 2*a*x + x^2)/((-(a - x)*(b - x)*(c - x))^(1/2)*(a*d
+ b*c - x*(b + c + d) + x^2)),x)
```

```
[Out] (2*(a - c)*ellipticF(asin((-(c - x)/(a - c))^(1/2)), (a - c)/(b - c))*((a -
 x)/(a - c))^(1/2)*(-(c - x)/(a - c))^(1/2)*((b - x)/(b - c))^(1/2))/(x*(a*
b + a*c + b*c) - x^2*(a + b + c) + x^3 - a*b*c)^(1/2) + (2*(a - c)*((a - x)
/(a - c))^(1/2)*(-(c - x)/(a - c))^(1/2)*((b - x)/(b - c))^(1/2)*ellipticPi
((a - c)/(b/2 - c/2 + d/2 - (2*b*d - 2*b*c - 4*a*d + 2*c*d + b^2 + c^2 + d^
2)^(1/2)/2), asin((-(c - x)/(a - c))^(1/2)), (a - c)/(b - c))*(a*b + a*c -
a*d - 2*b*c + (b - 2*a + c + d)*(b/2 + c/2 + d/2 - (2*b*d - 2*b*c - 4*a*d +
 2*c*d + b^2 + c^2 + d^2)^(1/2)/2)))/((x*(a*b + a*c + b*c) - x^2*(a + b + c
) + x^3 - a*b*c)^(1/2)*(b/2 - c/2 + d/2 - (2*b*d - 2*b*c - 4*a*d + 2*c*d +
b^2 + c^2 + d^2)^(1/2)/2)*(2*b*d - 2*b*c - 4*a*d + 2*c*d + b^2 + c^2 + d^2)
^(1/2)) - (2*(a - c)*((a - x)/(a - c))^(1/2)*(-(c - x)/(a - c))^(1/2)*((b -
 x)/(b - c))^(1/2)*ellipticPi((a - c)/(b/2 - c/2 + d/2 + (2*b*d - 2*b*c - 4
*a*d + 2*c*d + b^2 + c^2 + d^2)^(1/2)/2), asin((-(c - x)/(a - c))^(1/2)), (
a - c)/(b - c))*(a*b + a*c - a*d - 2*b*c + (b - 2*a + c + d)*(b/2 + c/2 + d
/2 + (2*b*d - 2*b*c - 4*a*d + 2*c*d + b^2 + c^2 + d^2)^(1/2)/2)))/((x*(a*b
+ a*c + b*c) - x^2*(a + b + c) + x^3 - a*b*c)^(1/2)*(b/2 - c/2 + d/2 + (2*b
*d - 2*b*c - 4*a*d + 2*c*d + b^2 + c^2 + d^2)^(1/2)/2)*(2*b*d - 2*b*c - 4*a
*d + 2*c*d + b^2 + c^2 + d^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b+a*c-2*a*x-b*c+x**2)/((-a+x)*(-b+x)*(-c+x))**(1/2)/(b*c+a*d-(
b+c+d)*x+x**2),x)
```

```
[Out] Timed out
```

3.750 $\int \frac{ab+ac-bc-2ax+x^2}{\sqrt{(-a+x)(-b+x)(-c+x)}\left(a+bcd-(1+bd+cd)x+dx^2\right)}\,dx$

Optimal. Leaf size=60

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}}{a-x}\right)}{\sqrt{d}}$$

Rubi [C] time = 19.38, antiderivative size = 455, normalized size of antiderivative = 7.58, number of steps used = 16, number of rules used = 7, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6718, 6728, 121, 120, 169, 538, 537}

$$-\frac{2\sqrt{b-a}\sqrt{x-a}\sqrt{-\frac{b-x}{a-b}}\sqrt{-\frac{c-x}{a-c}}\,\Pi\left(-\frac{2(a-b)d}{-2ad+bd+cd-\sqrt{b^2d^2-4ad+2b(1-cd)d+(cd+1)^2}+1};\sin^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{b-a}}\right)|\frac{a-b}{a-c}\right)}{d\sqrt{-((a-x)(b-x)(c-x))}}-\frac{2\sqrt{b-a}\sqrt{x-a}\sqrt{-\frac{b-x}{a-b}}\sqrt{-\frac{c-x}{a-c}}\,\Pi\left(-\frac{2(a-b)d}{-2ad+bd+cd+\sqrt{b^2d^2-4ad+2b(1-cd)d+(cd+1)^2}+1};\sin^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{b-a}}\right)|\frac{a-b}{a-c}\right)}{d\sqrt{-((a-x)(b-x)(c-x))}}+\frac{2\sqrt{b-a}\sqrt{x-a}\sqrt{-\frac{b-x}{a-b}}\sqrt{-\frac{c-x}{a-c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{b-a}}\right)|\frac{a-b}{a-c}\right)}{d\sqrt{-((a-x)(b-x)(c-x))}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b + a*c - b*c - 2*a*x + x^2)/(Sqrt[(-a + x)*(-b + x)*(-c + x)]*(a +
b*c*d - (1 + b*d + c*d)*x + d*x^2)),x]
```

```
[Out] (2*Sqrt[-a + b]*Sqrt[-((b - x)/(a - b))]*Sqrt[-((c - x)/(a - c))]*Sqrt[-a +
 x]*EllipticF[ArcSin[Sqrt[-a + x]/Sqrt[-a + b]], (a - b)/(a - c)])/(d*Sqrt[
-((a - x)*(b - x)*(c - x))]) - (2*Sqrt[-a + b]*Sqrt[-((b - x)/(a - b))]*Sqr
t[-((c - x)/(a - c))]*Sqrt[-a + x]*EllipticPi[(-2*(a - b)*d)/(1 - 2*a*d + b
*d + c*d - Sqrt[-4*a*d + b^2*d^2 + 2*b*d*(1 - c*d) + (1 + c*d)^2]), ArcSin[
Sqrt[-a + x]/Sqrt[-a + b]], (a - b)/(a - c)])/(d*Sqrt[-((a - x)*(b - x)*(c
- x))]) - (2*Sqrt[-a + b]*Sqrt[-((b - x)/(a - b))]*Sqrt[-((c - x)/(a - c))]
*Sqrt[-a + x]*EllipticPi[(-2*(a - b)*d)/(1 - 2*a*d + b*d + c*d + Sqrt[-4*a*
d + b^2*d^2 + 2*b*d*(1 - c*d) + (1 + c*d)^2]), ArcSin[Sqrt[-a + x]/Sqrt[-a
+ b]], (a - b)/(a - c)])/(d*Sqrt[-((a - x)*(b - x)*(c - x))])
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
```

```
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{ab+ac-bc-2ax+x^2}{\sqrt{(-a+x)(-b+x)(-c+x)}\left(a+bcd-(1+bd+cd)x+dx^2\right)}\,dx = \frac{\left(\sqrt{-a+x}\,\sqrt{-b+x}\,\sqrt{-c+x}\right)\int \frac{}{\sqrt{-a+x}\,\sqrt{-b+x}}}{\sqrt{(-a+x)(-b+x)(-}}$$

$$= \frac{\left(\sqrt{-a+x}\,\sqrt{-b+x}\,\sqrt{-c+x}\right)\int\left(\frac{1}{d\sqrt{-a+x}\,\sqrt{-b}}\right.}{\sqrt{(-a+}}$$

$$= \frac{\left(\sqrt{-a+x}\,\sqrt{-b+x}\,\sqrt{-c+x}\right)\int \frac{1}{\sqrt{-a+x}\,\sqrt{-b+x}}}{d\sqrt{(-a+x)(-b+x)(-c+x)}}$$

$$= -\frac{\left(\sqrt{-a+x}\,\sqrt{-b+x}\,\sqrt{-c+x}\right)\int\left(\frac{}{\sqrt{-a+x}\,\sqrt{-}}\right.}{}$$

$$= -\frac{\left(\left(-1+2ad-bd-cd-\sqrt{-4ad+b^2d^2+2}\right.\right.}{}$$

$$= \frac{2\sqrt{-a+b}\,\sqrt{-\frac{b-x}{a-b}}\,\sqrt{-\frac{c-x}{a-c}}\,\sqrt{-a+x}\,F\left(\sin^{-1}\right.}{d\sqrt{-((a-x)(b-x)(c-x))}}$$

$$= \frac{2\sqrt{-a+b}\,\sqrt{-\frac{b-x}{a-b}}\,\sqrt{-\frac{c-x}{a-c}}\,\sqrt{-a+x}\,F\left(\sin^{-1}\right.}{d\sqrt{-((a-x)(b-x)(c-x))}}$$

$$= \frac{2\sqrt{-a+b}\,\sqrt{-\frac{b-x}{a-b}}\,\sqrt{-\frac{c-x}{a-c}}\,\sqrt{-a+x}\,F\left(\sin^{-1}\right.}{d\sqrt{-((a-x)(b-x)(c-x))}}$$

$$= \frac{2\sqrt{-a+b}\,\sqrt{-\frac{b-x}{a-b}}\,\sqrt{-\frac{c-x}{a-c}}\,\sqrt{-a+x}\,F\left(\sin^{-1}\right.}{d\sqrt{-((a-x)(b-x)(c-x))}}$$

Mathematica [C] time = 8.96, size = 308, normalized size = 5.13

$$-\frac{2i(x-a)^{3/2}\sqrt{\frac{b-x}{a-x}}\sqrt{\frac{c-x}{a-x}}\left(-\Pi\left(\frac{2(a-c)d}{2ad-bd-cd+\sqrt{b^2d^2-4ad-2b(cd-1)d+(cd+1)^2}-1};i\sinh^{-1}\left(\frac{\sqrt{a-b}}{\sqrt{x-a}}\right)|\frac{a-c}{a-b}\right)-\Pi\left(-\frac{2(a-c)d}{-2ad+bd+cd+\sqrt{b^2d^2-4ad-2b(cd-1)d+(cd+1)^2}+1};i\sinh^{-1}\left(\frac{\sqrt{a-b}}{\sqrt{x-a}}\right)|\frac{a-c}{a-b}\right)+F\left(i\sinh^{-1}\left(\frac{\sqrt{a-b}}{\sqrt{x-a}}\right)|\frac{a-c}{a-b}\right)\right)}{d\sqrt{a-b}\sqrt{(x-a)(x-b)(x-c)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b + a*c - b*c - 2*a*x + x^2)/(Sqrt[(-a + x)*(-b + x)*(-c + x)]
*(a + b*c*d - (1 + b*d + c*d)*x + d*x^2)),x]
```

```
[Out] ((-2*I)*Sqrt[(b - x)/(a - x)]*Sqrt[(c - x)/(a - x)]*(-a + x)^(3/2)*(Ellipti
cF[I*ArcSinh[Sqrt[a - b]/Sqrt[-a + x]], (a - c)/(a - b)] - EllipticPi[(2*(a
 - c)*d)/(-1 + 2*a*d - b*d - c*d + Sqrt[-4*a*d + b^2*d^2 - 2*b*d*(-1 + c*d)
 + (1 + c*d)^2]), I*ArcSinh[Sqrt[a - b]/Sqrt[-a + x]], (a - c)/(a - b)] - E
llipticPi[(-2*(a - c)*d)/(1 - 2*a*d + b*d + c*d + Sqrt[-4*a*d + b^2*d^2 - 2
*b*d*(-1 + c*d) + (1 + c*d)^2]), I*ArcSinh[Sqrt[a - b]/Sqrt[-a + x]], (a -
c)/(a - b)]))/(Sqrt[a - b]*d*Sqrt[(-a + x)*(-b + x)*(-c + x)])
```

IntegrateAlgebraic [A] time = 2.81, size = 60, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}}{a-x}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b + a*c - b*c - 2*a*x + x^2)/(Sqrt[(-a + x)*(-b + x)*
(-c + x)]*(a + b*c*d - (1 + b*d + c*d)*x + d*x^2)),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[-(a*b*c) + (a*b + a*c + b*c)*x + (-a - b - c)*x^2
+ x^3])/(a - x)])/Sqrt[d]
```

fricas [B] time = 29.66, size = 379, normalized size = 6.32

$$\left[\frac{\log\left(\frac{b^2c^2d^2+d^2x^4-6\,abcd-2\left((b+c)d^2-3\,d\right)x^3+\left(\left(b^2+4\,bc+c^2\right)d^2-6\,(a+b+c)d+1\right)x^2+a^2-4\,\sqrt{-abc-(a+b+c)x^2+x^3+(ab+(a+b)c)x}\left(bcd+dx^2-((b+c)d-1)x-a\right)\sqrt{d}-2\left(\left(b^2c+bc^2\right)d^2-3\,(ab+(a+b)c)d+a\right)x}{b^2c^2d^2+d^2x^4+2\,abcd-2\left((b+c)d^2+d\right)x^3+\left(\left(b^2+4\,bc+c^2\right)d^2+2\,(a+b+c)d+1\right)x^2+a^2-2\left(\left(b^2c+bc^2\right)d^2+(ab+(a+b)c)d+a\right)x}\right)}{2\,\sqrt{d}}, \frac{\sqrt{-d}\arctan\left(-\frac{\sqrt{-abc-(a+b+c)x^2+x^3+(ab+(a+b)c)x}\left(bcd+dx^2-((b+c)d-1)x-a\right)\sqrt{-d}}{2\left(abcd+(a+b+c)dx^2-dx^3-(ab+(a+b)c)dx\right)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b+a*c-2*a*x-b*c+x^2)/((-a+x)*(-b+x)*(-c+x))^(1/2)/(a+b*c*d-(b*
d+c*d+1)*x+d*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b^2*c^2*d^2 + d^2*x^4 - 6*a*b*c*d - 2*((b + c)*d^2 - 3*d)*x^3 + (
(b^2 + 4*b*c + c^2)*d^2 - 6*(a + b + c)*d + 1)*x^2 + a^2 - 4*sqrt(-a*b*c -
(a + b + c)*x^2 + x^3 + (a*b + (a + b)*c)*x)*(b*c*d + d*x^2 - ((b + c)*d -
1)*x - a)*sqrt(d) - 2*((b^2*c + b*c^2)*d^2 - 3*(a*b + (a + b)*c)*d + a)*x)/
(b^2*c^2*d^2 + d^2*x^4 + 2*a*b*c*d - 2*((b + c)*d^2 + d)*x^3 + ((b^2 + 4*b*
c + c^2)*d^2 + 2*(a + b + c)*d + 1)*x^2 + a^2 - 2*((b^2*c + b*c^2)*d^2 + (a
*b + (a + b)*c)*d + a)*x))/sqrt(d), sqrt(-d)*arctan(-1/2*sqrt(-a*b*c - (a +
 b + c)*x^2 + x^3 + (a*b + (a + b)*c)*x)*(b*c*d + d*x^2 - ((b + c)*d - 1)*x
 - a)*sqrt(-d)/(a*b*c*d + (a + b + c)*d*x^2 - d*x^3 - (a*b + (a + b)*c)*d*x
))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab + ac - bc - 2\,ax + x^2}{\sqrt{-(a-x)(b-x)(c-x)}\left(bcd + dx^2 - (bd + cd + 1)x + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b+a*c-2*a*x-b*c+x^2)/((-a+x)*(-b+x)*(-c+x))^(1/2)/(a+b*c*d-(b*
d+c*d+1)*x+d*x^2),x, algorithm="giac")
```

```
[Out] integrate((a*b + a*c - b*c - 2*a*x + x^2)/(sqrt(-(a - x)*(b - x)*(c - x))*(
b*c*d + d*x^2 - (b*d + c*d + 1)*x + a)), x)
```

maple [C] time = 0.10, size = 11540, normalized size = 192.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b+a*c-2*a*x-b*c+x^2)/((-a+x)*(-b+x)*(-c+x))^(1/2)/(a+b*c*d-(b*d+c*d+
1)*x+d*x^2),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b+a*c-2*a*x-b*c+x^2)/((-a+x)*(-b+x)*(-c+x))^(1/2)/(a+b*c*d-(b*
d+c*d+1)*x+d*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume((c*d+b*d+1)^2>0)', see `assume?` fo
r more details)Is (c*d+b*d+1)^2    -4*d*(b*c*d+a)    positive, negative or
zero?
```

mupad [B] time = 0.79, size = 690, normalized size = 11.50

$$\frac{2\,(a-c)\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{c-x}{a-c}}\right)\middle|\frac{a-c}{b-c}\right)\sqrt{\frac{a-x}{a-c}}\,\sqrt{-\frac{c-x}{a-c}}\,\sqrt{\frac{b-x}{b-c}}}{d\,\sqrt{x^3+(-a-b-c)\,x^2+(a\,b+a\,c+b\,c)\,x-a\,b\,c}}+\frac{(a-c)\,\sqrt{\frac{a-x}{a-c}}\,\sqrt{-\frac{c-x}{a-c}}\,\sqrt{\frac{b-x}{b-c}}\,\Pi\left(-\frac{a-c}{c-\frac{b\,d+c\,d+\sqrt{b^2\,d^2-2\,b\,c\,d^2+2\,b\,d+c^2\,d^2+2\,c\,d-4\,a\,d+1}+1}{2\,d}};\mathrm{asin}\left(\sqrt{-\frac{c-x}{a-c}}\right)\middle|\frac{a-c}{b-c}\right)\left(b\,d-2\,a\,d+c\,d+\sqrt{b^2\,d^2-2\,b\,c\,d^2+2\,b\,d+c^2\,d^2+2\,c\,d-4\,a\,d+1}+1\right)}{d^2\,\left(c-\frac{b\,d+c\,d+\sqrt{b^2\,d^2-2\,b\,c\,d^2+2\,b\,d+c^2\,d^2+2\,c\,d-4\,a\,d+1}+1}{2\,d}\right)\,\sqrt{x^3+(-a-b-c)\,x^2+(a\,b+a\,c+b\,c)\,x-a\,b\,c}}+\frac{(a-c)\,\sqrt{\frac{a-x}{a-c}}\,\sqrt{-\frac{c-x}{a-c}}\,\sqrt{\frac{b-x}{b-c}}\,\Pi\left(-\frac{a-c}{c-\frac{b\,d+c\,d-\sqrt{b^2\,d^2-2\,b\,c\,d^2+2\,b\,d+c^2\,d^2+2\,c\,d-4\,a\,d+1}+1}{2\,d}};\mathrm{asin}\left(\sqrt{-\frac{c-x}{a-c}}\right)\middle|\frac{a-c}{b-c}\right)\left(b\,d-2\,a\,d+c\,d-\sqrt{b^2\,d^2-2\,b\,c\,d^2+2\,b\,d+c^2\,d^2+2\,c\,d-4\,a\,d+1}+1\right)}{d^2\,\left(c-\frac{b\,d+c\,d-\sqrt{b^2\,d^2-2\,b\,c\,d^2+2\,b\,d+c^2\,d^2+2\,c\,d-4\,a\,d+1}+1}{2\,d}\right)\,\sqrt{x^3+(-a-b-c)\,x^2+(a\,b+a\,c+b\,c)\,x-a\,b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b + a*c - b*c - 2*a*x + x^2)/((-(a - x)*(b - x)*(c - x))^(1/2)*(a -
x*(b*d + c*d + 1) + d*x^2 + b*c*d)),x)
```

```
[Out] (2*(a - c)*ellipticF(asin((-(c - x)/(a - c))^(1/2)), (a - c)/(b - c))*((a -
 x)/(a - c))^(1/2)*(-(c - x)/(a - c))^(1/2)*((b - x)/(b - c))^(1/2))/(d*(x*
(a*b + a*c + b*c) - x^2*(a + b + c) + x^3 - a*b*c)^(1/2)) + ((a - c)*((a -
x)/(a - c))^(1/2)*(-(c - x)/(a - c))^(1/2)*((b - x)/(b - c))^(1/2)*elliptic
Pi(-(a - c)/(c - (b*d + c*d + (2*b*d - 4*a*d + 2*c*d + b^2*d^2 + c^2*d^2 -
2*b*c*d^2 + 1)^(1/2) + 1)/(2*d)), asin((-(c - x)/(a - c))^(1/2)), (a - c)/(
b - c))*(b*d - 2*a*d + c*d + (2*b*d - 4*a*d + 2*c*d + b^2*d^2 + c^2*d^2 - 2
*b*c*d^2 + 1)^(1/2) + 1))/(d^2*(c - (b*d + c*d + (2*b*d - 4*a*d + 2*c*d + b
^2*d^2 + c^2*d^2 - 2*b*c*d^2 + 1)^(1/2) + 1)/(2*d))*(x*(a*b + a*c + b*c) -
x^2*(a + b + c) + x^3 - a*b*c)^(1/2)) + ((a - c)*((a - x)/(a - c))^(1/2)*(-
(c - x)/(a - c))^(1/2)*((b - x)/(b - c))^(1/2)*ellipticPi(-(a - c)/(c - (b*
d + c*d - (2*b*d - 4*a*d + 2*c*d + b^2*d^2 + c^2*d^2 - 2*b*c*d^2 + 1)^(1/2)
 + 1)/(2*d)), asin((-(c - x)/(a - c))^(1/2)), (a - c)/(b - c))*(b*d - 2*a*d
 + c*d - (2*b*d - 4*a*d + 2*c*d + b^2*d^2 + c^2*d^2 - 2*b*c*d^2 + 1)^(1/2)
+ 1))/(d^2*(c - (b*d + c*d - (2*b*d - 4*a*d + 2*c*d + b^2*d^2 + c^2*d^2 - 2
*b*c*d^2 + 1)^(1/2) + 1)/(2*d))*(x*(a*b + a*c + b*c) - x^2*(a + b + c) + x^
3 - a*b*c)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b+a*c-2*a*x-b*c+x**2)/((-a+x)*(-b+x)*(-c+x))**(1/2)/(a+b*c*d-(
b*d+c*d+1)*x+d*x**2),x)
```

```
[Out] Timed out
```

3.751 $\int \frac{-1+k^2x^2}{\sqrt{(1-x)x(1-k^2x)}\left(1+k^2x^2\right)}\,dx$

Optimal. Leaf size=60

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{k^2+1}\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{(x-1)\left(k^2x-1\right)}\right)}{\sqrt{k^2+1}}$$

Rubi [C] time = 2.35, antiderivative size = 262, normalized size of antiderivative = 4.37, number of steps used = 16, number of rules used = 9, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {6718, 6688, 6725, 714, 115, 934, 12, 168, 537}

$$\frac{2(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\,\Pi\left(-\frac{1}{\sqrt{-k^2}};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{\sqrt{-k^2}\sqrt{x-x^2}\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{2(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\,\Pi\left(\frac{1}{\sqrt{-k^2}};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{\sqrt{-k^2}\sqrt{x-x^2}\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(1 + k^2*x^2)),x]
```

```
[Out] (2*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticF[ArcSin[Sqrt[x]], k^2])/Sqr
t[(1 - x)*x*(1 - k^2*x)] + (2*(1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^2*x]*Elli
pticPi[-(1/Sqrt[-k^2]), ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/(Sqrt[-k^2]*S
qrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2]) + (2*(1 - x)*Sqrt[-x]*Sqrt[x]*Sqr
t[1 - k^2*x]*EllipticPi[1/Sqrt[-k^2], ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])
/(Sqrt[-k^2]*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 714

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c*x)/b]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0]
```

```
&& RationalQ[b]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+k^2x^2}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(1+k^2x^2\right)}\,dx = \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{-1+k^2x^2}{\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\left(1+k^2x^2\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{-1+k^2x^2}{\sqrt{1-k^2x}\sqrt{x-x^2}\left(1+k^2x^2\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \left(\frac{1}{\sqrt{1-k^2x}\sqrt{x-x^2}} - \frac{2}{\sqrt{1-k^2x}\sqrt{x-x^2}\left(1+k^2x^2\right)}\right)dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-k^2x}\sqrt{x-x^2}}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int}{\sqrt{(1-x)x\,(}}$$

$$= \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)}{}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int}{\sqrt{(1-x)x\,(}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{2}\sqrt{2-2x}\sqrt{1-x}\sqrt{-x}\right.}{\sqrt{(1}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{2-2x}\sqrt{1-x}\sqrt{-x}\sqrt{x}\right.}{\sqrt{(1-}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\sqrt{2-2x}\sqrt{1-x}\sqrt{-x}\sqrt{x}\right.}{}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{2(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}}{\sqrt{-k^2}\sqrt{(1-}}$$

Mathematica [C] time = 2.49, size = 165, normalized size = 2.75

$$\frac{2i\sqrt{\frac{1}{x-1}+1}\,(x-1)^{3/2}\sqrt{\frac{1-\frac{1}{k^2}}{x-1}+1}\left(\left(k^2-1\right)F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+(1-ik)\Pi\left(\frac{k-i}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+(1+ik)\Pi\left(\frac{k+i}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)}{\left(k^2+1\right)\sqrt{(x-1)x\left(k^2x-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(1 + k^2*x^2)),x]
```

```
[Out] ((2*I)*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (1 - k^(-2))/(-1 + x)]*(-1 + x)^(3/
2)*((-1 + k^2)*EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] + (1 - I*k)
*EllipticPi[(-I + k)/k, I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] + (1 + I*k)*
```

```
EllipticPi[(I + k)/k, I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)]))/((1 + k^2)*S
qrt[(-1 + x)*x*(-1 + k^2*x)])
```

IntegrateAlgebraic [A] time = 0.14, size = 60, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{k^2+1}\sqrt{k^2x^3+(-k^2-1)x^2+x}}{(x-1)(k^2x-1)}\right)}{\sqrt{k^2+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(1 + k^2*x^2
)),x]
```

```
[Out] (-2*ArcTan[(Sqrt[1 + k^2]*Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3])/((-1 + x)*(-1
 + k^2*x))])/Sqrt[1 + k^2]
```

fricas [A] time = 0.44, size = 92, normalized size = 1.53

$$\frac{\arctan\left(\frac{\sqrt{k^2x^3-(k^2+1)x^2+x}\,(k^2x^2-2\,(k^2+1)x+1)\sqrt{k^2+1}}{2\,((k^4+k^2)x^3-(k^4+2\,k^2+1)x^2+(k^2+1)x)}\right)}{\sqrt{k^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2+1),x, algorithm="
fricas")
```

```
[Out] arctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2 - 2*(k^2 + 1)*x + 1)*
sqrt(k^2 + 1)/((k^4 + k^2)*x^3 - (k^4 + 2*k^2 + 1)*x^2 + (k^2 + 1)*x))/sqrt
(k^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^2-1}{(k^2x^2+1)\sqrt{(k^2x-1)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2+1),x, algorithm="
giac")
```

```
[Out] integrate((k^2*x^2 - 1)/((k^2*x^2 + 1)*sqrt((k^2*x - 1)*(x - 1)*x)), x)
```

maple [C] time = 0.07, size = 341, normalized size = 5.68

$$-\frac{2\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\;\mathrm{EllipticF}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^2\sqrt{k^2x^3-k^2x^2-x^2+x}}-\frac{2i\sqrt{-k^2x+1}\,\sqrt{\frac{x}{\frac{1}{k^2}-1}-\frac{1}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\;\mathrm{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}-\frac{i}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^3\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}-\frac{i}{k}\right)}+\frac{2i\sqrt{-k^2x+1}\,\sqrt{\frac{x}{\frac{1}{k^2}-1}-\frac{1}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\;\mathrm{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}+\frac{i}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^3\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}+\frac{i}{k}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^2-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2+1),x)
```

```
[Out] -2/k^2*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x
^3-k^2*x^2-x^2+x)^(1/2)*EllipticF((-(x-1/k^2)*k^2)^(1/2),(1/k^2/(1/k^2-1))^
(1/2))-2*I/k^3*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(
1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2-I/k)*EllipticPi((-(x-1/k^2)*k^2)^
(1/2),1/k^2/(1/k^2-I/k),(1/k^2/(1/k^2-1))^(1/2))+2*I/k^3*(-k^2*x+1)^(1/2)*(
1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2
)/(1/k^2+I/k)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+I/k),(1/k^2/(1
/k^2-1))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^2 - 1}{\left(k^2x^2 + 1\right)\sqrt{\left(k^2x - 1\right)(x - 1)x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2+1),x, algorithm="
maxima")
```

```
[Out] integrate((k^2*x^2 - 1)/((k^2*x^2 + 1)*sqrt((k^2*x - 1)*(x - 1)*x)), x)
```

mupad [B] time = 2.76, size = 64, normalized size = 1.07

$$\frac{\ln\left(\frac{k^2\,x^2-2\,x\,\left(k^2+1\right)+1+\sqrt{k^2+1}\,\sqrt{x\,\left(k^2\,x-1\right)\,\left(x-1\right)}\,2\mathrm{i}}{k^2\,x^2+1}\right)\,1\mathrm{i}}{\sqrt{k^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^2 - 1)/((k^2*x^2 + 1)*(x*(k^2*x - 1)*(x - 1))^(1/2)),x)
```

```
[Out] (log((k^2*x^2 - 2*x*(k^2 + 1) + (k^2 + 1)^(1/2)*(x*(k^2*x - 1)*(x - 1))^(1/
2)*2i + 1)/(k^2*x^2 + 1))*1i)/(k^2 + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(kx - 1)(kx + 1)}{\sqrt{x(x - 1)\left(k^2x - 1\right)}\left(k^2x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k**2*x**2-1)/((1-x)*x*(-k**2*x+1))**(1/2)/(k**2*x**2+1),x)
```

```
[Out] Integral((k*x - 1)*(k*x + 1)/(sqrt(x*(x - 1)*(k**2*x - 1))*(k**2*x**2 + 1))
, x)
```

3.752 $\int \frac{a(ab+ac-3bc)+\left(-2a^2+ab+ac+3bc\right)x+(a-2b-2c)x^2+x^3}{\sqrt{(-a+x)(-b+x)(-c+x)}\left(-a^3-bcd+\left(3a^2+bd+cd\right)x-(3a+d)x^2+x^3\right)}\,dx$

Optimal. Leaf size=60

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}}{(a-x)^2}\right)}{\sqrt{d}}$$

Rubi [F] time = 48.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a(ab+ac-3bc)+\left(-2a^2+ab+ac+3bc\right)x+(a-2b-2c)x^2+x^3}{\sqrt{(-a+x)(-b+x)(-c+x)}\left(-a^3-bcd+\left(3a^2+bd+cd\right)x-(3a+d)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*(a*b + a*c - 3*b*c) + (-2*a^2 + a*b + a*c + 3*b*c)*x + (a - 2*b - 2*
c)*x^2 + x^3)/(Sqrt[(-a + x)*(-b + x)*(-c + x)]*(-a^3 - b*c*d + (3*a^2 + b*
d + c*d)*x - (3*a + d)*x^2 + x^3)),x]
```

```
[Out] (-2*Sqrt[a - c]*(b - x)*Sqrt[-a + x]*EllipticF[ArcTan[Sqrt[-a + x]/Sqrt[a -
 c]], -((b - c)/(a - b))])/((a - b)*Sqrt[((a - c)*(b - x))/((a - b)*(c - x)
)]*Sqrt[-((a - x)*(b - x)*(c - x))]) - (2*(a - b)*(a - c)*d*Sqrt[-a + x]*Sq
rt[-b + x]*Sqrt[-c + x]*Defer[Subst][Defer[Int][1/(Sqrt[a - b + x^2]*Sqrt[a
 - c + x^2]*(a^2*(1 + (b*c - a*(b + c))/a^2)*d + 2*a*(1 - (b + c)/(2*a))*d*
x^2 + d*x^4 - x^6)), x], x, Sqrt[-a + x]])/Sqrt[-((a - x)*(b - x)*(c - x))]
 - (2*(a^2 + 2*a*d - (b + c)*d)*Sqrt[-a + x]*Sqrt[-b + x]*Sqrt[-c + x]*Defe
r[Subst][Defer[Int][x^2/(Sqrt[a - b + x^2]*Sqrt[a - c + x^2]*(a^2*(1 + (b*c
 - a*(b + c))/a^2)*d + 2*a*(1 - (b + c)/(2*a))*d*x^2 + d*x^4 - x^6)), x], x
, Sqrt[-a + x]])/Sqrt[-((a - x)*(b - x)*(c - x))] - (2*(2*a + d)*Sqrt[-a +
x]*Sqrt[-b + x]*Sqrt[-c + x]*Defer[Subst][Defer[Int][x^4/(Sqrt[a - b + x^2]
*Sqrt[a - c + x^2]*(a^2*(1 + (b*c - a*(b + c))/a^2)*d + 2*a*(1 - (b + c)/(2
*a))*d*x^2 + d*x^4 - x^6)), x], x, Sqrt[-a + x]])/Sqrt[-((a - x)*(b - x)*(c
 - x))] + (4*a*(a - b - c)*Sqrt[-a + x]*Sqrt[-b + x]*Sqrt[-c + x]*Defer[Sub
st][Defer[Int][x^2/(Sqrt[a - b + x^2]*Sqrt[a - c + x^2]*(-(a^2*(1 + (b*c -
a*(b + c))/a^2)*d) - 2*a*(1 - (b + c)/(2*a))*d*x^2 - d*x^4 + x^6)), x], x,
Sqrt[-a + x]])/Sqrt[-((a - x)*(b - x)*(c - x))] + (4*(a - b - c)*Sqrt[-a +
x]*Sqrt[-b + x]*Sqrt[-c + x]*Defer[Subst][Defer[Int][x^4/(Sqrt[a - b + x^2]
*Sqrt[a - c + x^2]*(-(a^2*(1 + (b*c - a*(b + c))/a^2)*d) - 2*a*(1 - (b + c)
/(2*a))*d*x^2 - d*x^4 + x^6)), x], x, Sqrt[-a + x]])/Sqrt[-((a - x)*(b - x)
*(c - x))] + (2*(3*b*c - a*(b + c))*Sqrt[-a + x]*Sqrt[-b + x]*Sqrt[-c + x]*
Defer[Subst][Defer[Int][x^2/(Sqrt[a - b + x^2]*Sqrt[a - c + x^2]*(-(a^2*d)
+ c*d*x^2 - d*x^4 + x^6 + a*d*(b + c - 2*x^2) + b*d*(-c + x^2))), x], x, Sq
rt[-a + x]])/Sqrt[-((a - x)*(b - x)*(c - x))]
```

Rubi steps

$$\int \frac{a(ab+ac-3bc)+\left(-2a^2+ab+ac+3bc\right)x+(a-2b-2c)x^2+x^3}{\sqrt{(-a+x)(-b+x)(-c+x)}\left(-a^3-bcd+\left(3a^2+bd+cd\right)x-(3a+d)x^2+x^3\right)}\,dx = \frac{\left(\sqrt{-a+x}\sqrt{-b+x}\sqrt{-\right.}{}$$

$$= \frac{\left(\sqrt{-a+x}\sqrt{-b+x}\sqrt{-\right.}{}$$

$$= \frac{\left(\sqrt{-a+x}\sqrt{-b+x}\sqrt{-\right.}{}$$

$$= \frac{\left(\sqrt{-a+x}\sqrt{-b+x}\sqrt{-\right.}{}$$

$$= \frac{\left(\sqrt{-a+x}\sqrt{-b+x}\sqrt{-\right.}{}$$

$$= \frac{\left(2\sqrt{-a+x}\sqrt{-b+x}\sqrt{\right.}{}$$

$$= \frac{\left(2\sqrt{-a+x}\sqrt{-b+x}\sqrt{\right.}{}$$

$$= \frac{\left(2\sqrt{-a+x}\sqrt{-b+x}\sqrt{\right.}{}$$

$$= -\frac{2\sqrt{a-c}\,(b-x)\sqrt{-a+}}{(a-b)\sqrt{\frac{(a-c)(b-x)}{(a-b)(c-x)}}}$$

$$= -\frac{2\sqrt{a-c}\,(b-x)\sqrt{-a+}}{(a-b)\sqrt{\frac{(a-c)(b-x)}{(a-b)(c-x)}}}$$

Mathematica [C] time = 6.21, size = 3908, normalized size = 65.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*(a*b + a*c - 3*b*c) + (-2*a^2 + a*b + a*c + 3*b*c)*x + (a - 2*
b - 2*c)*x^2 + x^3)/(Sqrt[(-a + x)*(-b + x)*(-c + x)]*(-a^3 - b*c*d + (3*a^
2 + b*d + c*d)*x - (3*a + d)*x^2 + x^3)),x]
```

```
[Out] ((2*I)*Sqrt[(a - x)/(b - x)]*(b - x)*Sqrt[(-c + x)/(a - c)]*(3*(a - b)*(a -
 c)*d*(EllipticF[I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] - Elli
pticPi[(a - b)/Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1
 + d*#1^2 + #1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)
]) + (2*a - b - c)*d^2*(EllipticF[I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b
```

```
)/(a - c)] - EllipticPi[(a - b)/Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*
d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]]
, (a - b)/(a - c)]) - 2*(-2*a + b + c)^2*d*(EllipticPi[(a - b)/Root[a^2*d -
 a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 1], I*
ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] - EllipticPi[(a - b)/Root
[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 &
, 2], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)]) - 3*(a - b)*(a -
 c)*d*(EllipticPi[(a - b)/Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*
d + c*d)*#1 + d*#1^2 + #1^3 & , 2], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a -
 b)/(a - c)] - EllipticPi[(a - b)/Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*
a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 3], I*ArcSinh[Sqrt[(-a + x)/(a - b)
]], (a - b)/(a - c)]) - 3*(a - b)*(a - c)*(EllipticPi[(a - b)/Root[a^2*d -
a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 1], I*A
rcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] + EllipticPi[(a - b)/Root[
a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & ,
 2], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] - 2*EllipticPi[(a
- b)/Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2
+ #1^3 & , 3], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)])*Root[a^
2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 2
] + (-2*EllipticF[I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] + Ell
ipticPi[(a - b)/Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#
1 + d*#1^2 + #1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c
)] + EllipticPi[(a - b)/Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d
+ c*d)*#1 + d*#1^2 + #1^3 & , 2], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b
)/(a - c)])*Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 +
d*#1^2 + #1^3 & , 1]*Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c
*d)*#1 + d*#1^2 + #1^3 & , 2]^2 + 3*(a - b)*(a - c)*(EllipticPi[(a - b)/Roo
t[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 &
 , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] - 2*EllipticPi[(
a - b)/Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^
2 + #1^3 & , 2], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] + Elli
pticPi[(a - b)/Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1
 + d*#1^2 + #1^3 & , 3], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)
])*Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 +
#1^3 & , 3] - 2*(2*a - b - c)*(2*EllipticPi[(a - b)/Root[a^2*d - a*b*d - a*
c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 1], I*ArcSinh[Sqr
t[(-a + x)/(a - b)]], (a - b)/(a - c)] - EllipticPi[(a - b)/Root[a^2*d - a*
b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 2], I*Arc
Sinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] - EllipticPi[(a - b)/Root[a^
2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 3
], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)])*Root[a^2*d - a*b*d
- a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 1]*Root[a^2*d
 - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 3] -
 2*(EllipticF[I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] - Ellipti
cPi[(a - b)/Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 +
d*#1^2 + #1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)])*
Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^
3 & , 1]^2*Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d
*#1^2 + #1^3 & , 3] - 2*(2*a - b - c)*(EllipticPi[(a - b)/Root[a^2*d - a*b*
d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 1], I*ArcSi
nh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] - 2*EllipticPi[(a - b)/Root[a^
2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 2
], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] + EllipticPi[(a - b)
/Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1
^3 & , 3], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)])*Root[a^2*d
- a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 2]*Ro
ot[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3
& , 3] + (EllipticPi[(a - b)/Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d +
 b*d + c*d)*#1 + d*#1^2 + #1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (
```

```
a - b)/(a - c)] - EllipticPi[(a - b)/Root[a^2*d - a*b*d - a*c*d + b*c*d + (
-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 2], I*ArcSinh[Sqrt[(-a + x)/(a -
 b)]], (a - b)/(a - c)])*Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d
 + c*d)*#1 + d*#1^2 + #1^3 & , 2]^2*Root[a^2*d - a*b*d - a*c*d + b*c*d + (-
2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 3] + (EllipticPi[(a - b)/Root[a^2
*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 1]
, I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] - EllipticPi[(a - b)/
Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^
3 & , 3], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)])*Root[a^2*d -
 a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 1]*Roo
t[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 &
 , 3]^2 + (-2*EllipticF[I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)]
 + EllipticPi[(a - b)/Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d +
c*d)*#1 + d*#1^2 + #1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/
(a - c)] + EllipticPi[(a - b)/Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d
+ b*d + c*d)*#1 + d*#1^2 + #1^3 & , 3], I*ArcSinh[Sqrt[(-a + x)/(a - b)]],
(a - b)/(a - c)])*Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)
*#1 + d*#1^2 + #1^3 & , 2]*Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b
*d + c*d)*#1 + d*#1^2 + #1^3 & , 3]^2))/(Sqrt[(-a + x)*(-b + x)*(-c + x)]*(
Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^
3 & , 1] - Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d
*#1^2 + #1^3 & , 2])*(Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a*d + b*d +
c*d)*#1 + d*#1^2 + #1^3 & , 1] - Root[a^2*d - a*b*d - a*c*d + b*c*d + (-2*a
*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 3])*(Root[a^2*d - a*b*d - a*c*d + b*
c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 2] - Root[a^2*d - a*b*d -
 a*c*d + b*c*d + (-2*a*d + b*d + c*d)*#1 + d*#1^2 + #1^3 & , 3]))
```

IntegrateAlgebraic [A] time = 3.14, size = 60, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}}{(a-x)^2}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*(a*b + a*c - 3*b*c) + (-2*a^2 + a*b + a*c + 3*b*c)*x
+ (a - 2*b - 2*c)*x^2 + x^3)/(Sqrt[(-a + x)*(-b + x)*(-c + x)]*(-a^3 - b*c*
d + (3*a^2 + b*d + c*d)*x - (3*a + d)*x^2 + x^3)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[d]*Sqrt[-(a*b*c) + (a*b + a*c + b*c)*x + (-a - b - c)*x^2
 + x^3])/(a - x)^2])/Sqrt[d]
```

fricas [B] time = 47.54, size = 638, normalized size = 10.63

$$\left[\frac{\log\left(\frac{a^6-6a^3bcd+b^2c^2d^2-6(a-d)x^5+x^6+\left(15a^2-6(3a+b+c)d+d^2\right)x^4-2\left(10a^3+(b+c)d^2-3\left(3a^2+3ab+(3a+b)c\right)d\right)x^3+\left(15a^4+\left(b^2+4bc+c^2\right)d^2-6\left(a^3+3a^2b+3\left(a^2+ab\right)c\right)d\right)x^2-4\left(a^4-abcd-(4a-d)x^3+x^4+\left(6a^2-(a+b+c)d\right)x^2-\left(4a^3-(ab+(a+b)c)d\right)x\right)\sqrt{-abc-(a+b+c)x^2+x^3+(ab+(a+b)c)x}\,\sqrt{d}-2\left(3a^5+\left(b^2c+bc^2\right)d^2-3\left(a^3b+\left(a^3+3a^2b\right)c\right)d\right)x}{a^6+2a^3bcd+b^2c^2d^2-2(3a+d)x^5+x^6+\left(15a^2+2(3a+b+c)d+d^2\right)x^4-2\left(10a^3+(b+c)d^2+\left(3a^2+3ab+(3a+b)c\right)d\right)x^3+\left(15a^4+\left(b^2+4bc+c^2\right)d^2+2\left(a^3+3a^2b+3\left(a^2+ab\right)c\right)d\right)x^2-2\left(3a^5+\left(b^2c+bc^2\right)d^2+\left(a^3b+\left(a^3+3a^2b\right)c\right)d\right)x}\right)}{2\sqrt{d}},\ \frac{\sqrt{-d}\arctan\left(-\frac{\left(a^3-bcd+(3a-d)x^2-x^3-\left(3a^2-(b+c)d\right)x\right)\sqrt{-abc-(a+b+c)x^2+x^3+(ab+(a+b)c)x}\,\sqrt{-d}}{2\left(a^2bcd-(2a+b+c)dx^3+dx^4+\left(a^2+2ab+(2a+b)c\right)dx^2-\left(a^2b+\left(a^2+2ab\right)c\right)dx\right)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(a*b+a*c-3*b*c)+(-2*a^2+a*b+a*c+3*b*c)*x+(a-2*b-2*c)*x^2+x^3)/
((-a+x)*(-b+x)*(-c+x))^(1/2)/(-a^3-b*c*d+(3*a^2+b*d+c*d)*x-(3*a+d)*x^2+x^3)
,x, algorithm="fricas")
```

```
[Out] [1/2*log((a^6 - 6*a^3*b*c*d + b^2*c^2*d^2 - 6*(a - d)*x^5 + x^6 + (15*a^2 -
 6*(3*a + b + c)*d + d^2)*x^4 - 2*(10*a^3 + (b + c)*d^2 - 3*(3*a^2 + 3*a*b
+ (3*a + b)*c)*d)*x^3 + (15*a^4 + (b^2 + 4*b*c + c^2)*d^2 - 6*(a^3 + 3*a^2*
b + 3*(a^2 + a*b)*c)*d)*x^2 - 4*(a^4 - a*b*c*d - (4*a - d)*x^3 + x^4 + (6*a
^2 - (a + b + c)*d)*x^2 - (4*a^3 - (a*b + (a + b)*c)*d)*x)*sqrt(-a*b*c - (a
 + b + c)*x^2 + x^3 + (a*b + (a + b)*c)*x)*sqrt(d) - 2*(3*a^5 + (b^2*c + b*
c^2)*d^2 - 3*(a^3*b + (a^3 + 3*a^2*b)*c)*d)*x)/(a^6 + 2*a^3*b*c*d + b^2*c^2
*d^2 - 2*(3*a + d)*x^5 + x^6 + (15*a^2 + 2*(3*a + b + c)*d + d^2)*x^4 - 2*(
10*a^3 + (b + c)*d^2 + (3*a^2 + 3*a*b + (3*a + b)*c)*d)*x^3 + (15*a^4 + (b^
```

```
2 + 4*b*c + c^2)*d^2 + 2*(a^3 + 3*a^2*b + 3*(a^2 + a*b)*c)*d)*x^2 - 2*(3*a^
5 + (b^2*c + b*c^2)*d^2 + (a^3*b + (a^3 + 3*a^2*b)*c)*d)*x))/sqrt(d), sqrt(
-d)*arctan(-1/2*(a^3 - b*c*d + (3*a - d)*x^2 - x^3 - (3*a^2 - (b + c)*d)*x)
*sqrt(-a*b*c - (a + b + c)*x^2 + x^3 + (a*b + (a + b)*c)*x)*sqrt(-d)/(a^2*b
*c*d - (2*a + b + c)*d*x^3 + d*x^4 + (a^2 + 2*a*b + (2*a + b)*c)*d*x^2 - (a
^2*b + (a^2 + 2*a*b)*c)*d*x))/d]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(a*b+a*c-3*b*c)+(-2*a^2+a*b+a*c+3*b*c)*x+(a-2*b-2*c)*x^2+x^3)/
((-a+x)*(-b+x)*(-c+x))^(1/2)/(-a^3-b*c*d+(3*a^2+b*d+c*d)*x-(3*a+d)*x^2+x^3)
,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.10, size = 486, normalized size = 8.10

$$\frac{2(-a+b)\sqrt{\frac{-a+x}{-a+b}}\sqrt{\frac{-c+x}{a-c}}\sqrt{\frac{-b+x}{a-b}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-a+x}{-a+b}},\sqrt{\frac{a-b}{a-c}}\right)}{\sqrt{-abc+abx+acx-a x^2+bcx-b x^2-c x^2+x^3}}-\frac{2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^3+(-3a-d)\_Z^2+\left(3a^2+bd+cd\right)\_Z-a^3-bcd\right)}\frac{\left(-4\_\alpha^2a+2\_\alpha^2b+2\_\alpha^2c-d\_\alpha^2+5\_\alpha a^2-\_\alpha ab-\_\alpha ac-3\_\alpha bc+\_\alpha bd+\_\alpha cd-a^3-a^2b-a^2c+3abc-bcd\right)(-a+b)\left(\_\alpha^2-2\_\alpha a-\_\alpha d+a^2-ad+bd+cd\right)\sqrt{\frac{-a+x}{-a+b}}\sqrt{\frac{-c+x}{a-c}}\sqrt{\frac{-b+x}{a-b}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-a+x}{-a+b}},-\frac{\_\alpha^2-2\_\alpha a-\_\alpha d+a^2-ad+bd+cd}{(a-c)d},\sqrt{\frac{a-b}{a-c}}\right)}{\left(-3\_\alpha^2+6\_\alpha a+2\_\alpha d-3a^2-bd-cd\right)\left(a^2-ab-ac+bc\right)\sqrt{-abc+abx+acx-a x^2+bcx-b x^2-c x^2+x^3}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(a*b+a*c-3*b*c)+(-2*a^2+a*b+a*c+3*b*c)*x+(a-2*b-2*c)*x^2+x^3)/((-a+x
)*(-b+x)*(-c+x))^(1/2)/(-a^3-b*c*d+(3*a^2+b*d+c*d)*x-(3*a+d)*x^2+x^3),x)
```

```
[Out] 2*(-a+b)*((-a+x)/(-a+b))^(1/2)*((-c+x)/(a-c))^(1/2)*((-b+x)/(a-b))^(1/2)/(-
a*b*c+a*b*x+a*c*x-a*x^2+b*c*x-b*x^2-c*x^2+x^3)^(1/2)*EllipticF(((-a+x)/(-a+
b))^(1/2),((a-b)/(a-c))^(1/2))-2/d*sum((-4*_alpha^2*a+2*_alpha^2*b+2*_alpha
^2*c-_alpha^2*d+5*_alpha*a^2-_alpha*a*b-_alpha*a*c-3*_alpha*b*c+_alpha*b*d+
_alpha*c*d-a^3-a^2*b-a^2*c+3*a*b*c-b*c*d)/(-3*_alpha^2+6*_alpha*a+2*_alpha*
d-3*a^2-b*d-c*d)*(-a+b)*(_alpha^2-2*_alpha*a-_alpha*d+a^2-a*d+b*d+c*d)/(a^2
-a*b-a*c+b*c)*((-a+x)/(-a+b))^(1/2)*((-c+x)/(a-c))^(1/2)*((-b+x)/(a-b))^(1/
2)/(-a*b*c+a*b*x+a*c*x-a*x^2+b*c*x-b*x^2-c*x^2+x^3)^(1/2)*EllipticPi(((-a+x
)/(-a+b))^(1/2),-(_alpha^2-2*_alpha*a-_alpha*d+a^2-a*d+b*d+c*d)/(a-c)/d,((a
-b)/(a-c))^(1/2)),_alpha=RootOf(_Z^3+(-3*a-d)*_Z^2+(3*a^2+b*d+c*d)*_Z-a^3-b
*c*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(a-2\,b-2\,c)x^2+x^3+(ab+ac-3\,bc)a-\left(2\,a^2-ab-ac-3\,bc\right)x}{\left(a^3+bcd+(3\,a+d)x^2-x^3-\left(3\,a^2+bd+cd\right)x\right)\sqrt{-(a-x)(b-x)(c-x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(a*b+a*c-3*b*c)+(-2*a^2+a*b+a*c+3*b*c)*x+(a-2*b-2*c)*x^2+x^3)/
((-a+x)*(-b+x)*(-c+x))^(1/2)/(-a^3-b*c*d+(3*a^2+b*d+c*d)*x-(3*a+d)*x^2+x^3)
,x, algorithm="maxima")
```

```
[Out] -integrate(((a - 2*b - 2*c)*x^2 + x^3 + (a*b + a*c - 3*b*c)*a - (2*a^2 - a*
b - a*c - 3*b*c)*x)/((a^3 + b*c*d + (3*a + d)*x^2 - x^3 - (3*a^2 + b*d + c*
d)*x)*sqrt(-(a - x)*(b - x)*(c - x))), x)
```

mupad [B] time = 1.73, size = 946, normalized size = 15.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*(a*b + a*c - 3*b*c) + x*(a*b + a*c + 3*b*c - 2*a^2) + x^3 - x^2*(2*
b - a + 2*c))/((-(a - x)*(b - x)*(c - x))^(1/2)*(x^2*(3*a + d) - x*(b*d + c
*d + 3*a^2) + a^3 - x^3 + b*c*d)),x)
```

```
[Out] symsum(-(2*(a - c)*((a - x)/(a - c))^(1/2)*(-(c - x)/(a - c))^(1/2)*((b - x
)/(b - c))^(1/2)*ellipticPi((a - c)/(root(z^3 - z^2*(3*a + d) + z*(b*d + c*
d + 3*a^2) - b*c*d - a^3, z, k) - c), asin((-(c - x)/(a - c))^(1/2)), (a -
c)/(b - c))*(a^2*b + a^2*c + 4*a*root(z^3 - z^2*(3*a + d) + z*(b*d + c*d +
3*a^2) - b*c*d - a^3, z, k)^2 - 5*a^2*root(z^3 - z^2*(3*a + d) + z*(b*d + c
*d + 3*a^2) - b*c*d - a^3, z, k) - 2*b*root(z^3 - z^2*(3*a + d) + z*(b*d +
c*d + 3*a^2) - b*c*d - a^3, z, k)^2 - 2*c*root(z^3 - z^2*(3*a + d) + z*(b*d
 + c*d + 3*a^2) - b*c*d - a^3, z, k)^2 + d*root(z^3 - z^2*(3*a + d) + z*(b*
d + c*d + 3*a^2) - b*c*d - a^3, z, k)^2 + a^3 - 3*a*b*c + b*c*d + a*b*root(
z^3 - z^2*(3*a + d) + z*(b*d + c*d + 3*a^2) - b*c*d - a^3, z, k) + a*c*root
(z^3 - z^2*(3*a + d) + z*(b*d + c*d + 3*a^2) - b*c*d - a^3, z, k) + 3*b*c*r
oot(z^3 - z^2*(3*a + d) + z*(b*d + c*d + 3*a^2) - b*c*d - a^3, z, k) - b*d*
root(z^3 - z^2*(3*a + d) + z*(b*d + c*d + 3*a^2) - b*c*d - a^3, z, k) - c*d
*root(z^3 - z^2*(3*a + d) + z*(b*d + c*d + 3*a^2) - b*c*d - a^3, z, k)))/((
root(z^3 - z^2*(3*a + d) + z*(b*d + c*d + 3*a^2) - b*c*d - a^3, z, k) - c)*
(-(a - x)*(b - x)*(c - x))^(1/2)*(b*d + c*d - 6*a*root(z^3 - z^2*(3*a + d)
+ z*(b*d + c*d + 3*a^2) - b*c*d - a^3, z, k) - 2*d*root(z^3 - z^2*(3*a + d)
 + z*(b*d + c*d + 3*a^2) - b*c*d - a^3, z, k) + 3*a^2 + 3*root(z^3 - z^2*(3
*a + d) + z*(b*d + c*d + 3*a^2) - b*c*d - a^3, z, k)^2)), k, 1, 3) + (2*(a
- c)*ellipticF(asin((-(c - x)/(a - c))^(1/2)), (a - c)/(b - c))*((a - x)/(a
 - c))^(1/2)*(-(c - x)/(a - c))^(1/2)*((b - x)/(b - c))^(1/2))/(x*(a*b + a*
c + b*c) - x^2*(a + b + c) + x^3 - a*b*c)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(a*b+a*c-3*b*c)+(-2*a**2+a*b+a*c+3*b*c)*x+(a-2*b-2*c)*x**2+x**
3)/((-a+x)*(-b+x)*(-c+x))**(1/2)/(-a**3-b*c*d+(3*a**2+b*d+c*d)*x-(3*a+d)*x*
*2+x**3),x)
```

```
[Out] Timed out
```

3.753 $\int \frac{(-2c+ax^3)\sqrt{c+bx^2+ax^3}}{(c+ax^3)^2}\,dx$

Optimal. Leaf size=60

$$-\frac{x\sqrt{ax^3+bx^2+c}}{ax^3+c}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^3+bx^2+c}}\right)}{\sqrt{b}}$$

Rubi [F] time = 180.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$Aborted

Verification is not applicable to the result.

```
[In] Int[((-2*c + a*x^3)*Sqrt[c + b*x^2 + a*x^3])/(c + a*x^3)^2,x]
```

```
[Out] $Aborted
```

Rubi steps

Aborted

Mathematica [C] time = 4.09, size = 1788, normalized size = 29.80

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-2*c + a*x^3)*Sqrt[c + b*x^2 + a*x^3])/(c + a*x^3)^2,x]
```

```
[Out] (-((x*(c + x^2*(b + a*x)))/(c + a*x^3)) + (EllipticF[ArcSin[Sqrt[(-x + Root
[c + b*#1^2 + a*#1^3 & , 3])/(-Root[c + b*#1^2 + a*#1^3 & , 2] + Root[c + b
*#1^2 + a*#1^3 & , 3])]], (Root[c + b*#1^2 + a*#1^3 & , 2] - Root[c + b*#1^
2 + a*#1^3 & , 3])/(Root[c + b*#1^2 + a*#1^3 & , 1] - Root[c + b*#1^2 + a*#
1^3 & , 3])]*(x - Root[c + b*#1^2 + a*#1^3 & , 3])*Sqrt[(-x + Root[c + b*#1
^2 + a*#1^3 & , 1])/(Root[c + b*#1^2 + a*#1^3 & , 1] - Root[c + b*#1^2 + a*
#1^3 & , 3])]*Sqrt[(-x + Root[c + b*#1^2 + a*#1^3 & , 2])/(Root[c + b*#1^2
+ a*#1^3 & , 2] - Root[c + b*#1^2 + a*#1^3 & , 3])]])/Sqrt[(x - Root[c + b*#
1^2 + a*#1^3 & , 3])/(Root[c + b*#1^2 + a*#1^3 & , 2] - Root[c + b*#1^2 + a
*#1^3 & , 3])] + (3*(-1)^(2/3)*c^(1/3)*EllipticPi[(a^(1/3)*(-Root[c + b*#1^
2 + a*#1^3 & , 2] + Root[c + b*#1^2 + a*#1^3 & , 3]))/(-((-1)^(1/3)*c^(1/3)
) + a^(1/3)*Root[c + b*#1^2 + a*#1^3 & , 3]), ArcSin[Sqrt[(-x + Root[c + b*
#1^2 + a*#1^3 & , 3])/(-Root[c + b*#1^2 + a*#1^3 & , 2] + Root[c + b*#1^2 +
 a*#1^3 & , 3])]], (Root[c + b*#1^2 + a*#1^3 & , 2] - Root[c + b*#1^2 + a*#
1^3 & , 3])/(Root[c + b*#1^2 + a*#1^3 & , 1] - Root[c + b*#1^2 + a*#1^3 & ,
 3])]*Sqrt[(-x + Root[c + b*#1^2 + a*#1^3 & , 1])/(Root[c + b*#1^2 + a*#1^3
 & , 1] - Root[c + b*#1^2 + a*#1^3 & , 3])]*Sqrt[-(((x - Root[c + b*#1^2 +
a*#1^3 & , 2])*(x - Root[c + b*#1^2 + a*#1^3 & , 3]))/(Root[c + b*#1^2 + a*
#1^3 & , 2] - Root[c + b*#1^2 + a*#1^3 & , 3])^2)]*(-Root[c + b*#1^2 + a*#1
^3 & , 2] + Root[c + b*#1^2 + a*#1^3 & , 3]))/((1 + (-1)^(1/3))^2*((-1)^(1/
3)*c^(1/3) - a^(1/3)*Root[c + b*#1^2 + a*#1^3 & , 3])) + (c^(1/3)*EllipticP
i[(a^(1/3)*(-Root[c + b*#1^2 + a*#1^3 & , 2] + Root[c + b*#1^2 + a*#1^3 & ,
 3]))/(c^(1/3) + a^(1/3)*Root[c + b*#1^2 + a*#1^3 & , 3]), ArcSin[Sqrt[(-x
+ Root[c + b*#1^2 + a*#1^3 & , 3])/(-Root[c + b*#1^2 + a*#1^3 & , 2] + Root
[c + b*#1^2 + a*#1^3 & , 3])]], (Root[c + b*#1^2 + a*#1^3 & , 2] - Root[c +
 b*#1^2 + a*#1^3 & , 3])/(Root[c + b*#1^2 + a*#1^3 & , 1] - Root[c + b*#1^2
 + a*#1^3 & , 3])]*Sqrt[(-x + Root[c + b*#1^2 + a*#1^3 & , 1])/(Root[c + b*
#1^2 + a*#1^3 & , 1] - Root[c + b*#1^2 + a*#1^3 & , 3])]*Sqrt[-(((x - Root[
c + b*#1^2 + a*#1^3 & , 2])*(x - Root[c + b*#1^2 + a*#1^3 & , 3]))/(Root[c
```

```
+ b*#1^2 + a*#1^3 & , 2] - Root[c + b*#1^2 + a*#1^3 & , 3])^2)]*(-Root[c +
b*#1^2 + a*#1^3 & , 2] + Root[c + b*#1^2 + a*#1^3 & , 3]))/(c^(1/3) + a^(1/
3)*Root[c + b*#1^2 + a*#1^3 & , 3]) + (2*(-1)^(2/3)*c^(1/3)*EllipticPi[(a^(
1/3)*(-Root[c + b*#1^2 + a*#1^3 & , 2] + Root[c + b*#1^2 + a*#1^3 & , 3]))/
((-1)^(2/3)*c^(1/3) + a^(1/3)*Root[c + b*#1^2 + a*#1^3 & , 3]), ArcSin[Sqrt
[(-x + Root[c + b*#1^2 + a*#1^3 & , 3])/(-Root[c + b*#1^2 + a*#1^3 & , 2] +
 Root[c + b*#1^2 + a*#1^3 & , 3])]], (Root[c + b*#1^2 + a*#1^3 & , 2] - Roo
t[c + b*#1^2 + a*#1^3 & , 3])/(Root[c + b*#1^2 + a*#1^3 & , 1] - Root[c + b
*#1^2 + a*#1^3 & , 3])]*Sqrt[(-x + Root[c + b*#1^2 + a*#1^3 & , 1])/(Root[c
 + b*#1^2 + a*#1^3 & , 1] - Root[c + b*#1^2 + a*#1^3 & , 3])]*Sqrt[-(((x -
Root[c + b*#1^2 + a*#1^3 & , 2])*(x - Root[c + b*#1^2 + a*#1^3 & , 3]))/(Ro
ot[c + b*#1^2 + a*#1^3 & , 2] - Root[c + b*#1^2 + a*#1^3 & , 3])^2)]*(-Root
[c + b*#1^2 + a*#1^3 & , 2] + Root[c + b*#1^2 + a*#1^3 & , 3]))/(I*(I + Sqr
t[3])*c^(1/3) + 2*a^(1/3)*Root[c + b*#1^2 + a*#1^3 & , 3]))/Sqrt[c + x^2*(b
 + a*x)]
```

IntegrateAlgebraic [A] time = 0.50, size = 60, normalized size = 1.00

$$-\frac{x\sqrt{ax^3+bx^2+c}}{ax^3+c}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^3+bx^2+c}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*c + a*x^3)*Sqrt[c + b*x^2 + a*x^3])/(c + a*x^3)^2,x
]
```

```
[Out] -((x*Sqrt[c + b*x^2 + a*x^3])/(c + a*x^3)) - ArcTanh[(Sqrt[b]*x)/Sqrt[c + b
*x^2 + a*x^3]]/Sqrt[b]
```

fricas [A] time = 0.47, size = 244, normalized size = 4.07

$$\left[-\frac{4\sqrt{ax^3+bx^2+c}\,bx-\left(ax^3+c\right)\sqrt{b}\log\left(\frac{a^2x^6+8\,abx^5+8\,b^2x^4+2\,acx^3+8\,bcx^2-4\left(ax^4+2\,bx^3+cx\right)\sqrt{ax^3+bx^2+c}\,\sqrt{b}+c^2}{a^2x^6+2\,acx^3+c^2}\right)}{4\left(abx^3+bc\right)},\ -\frac{2\sqrt{ax^3+bx^2+c}\,bx-\left(ax^3+c\right)\sqrt{-b}\arctan\left(\frac{\left(ax^3+2\,bx^2+c\right)\sqrt{ax^3+bx^2+c}\,\sqrt{-b}}{2\left(abx^4+b^2x^3+bcx\right)}\right)}{2\left(abx^3+bc\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*c)*(a*x^3+b*x^2+c)^(1/2)/(a*x^3+c)^2,x, algorithm="frica
s")
```

```
[Out] [-1/4*(4*sqrt(a*x^3 + b*x^2 + c)*b*x - (a*x^3 + c)*sqrt(b)*log((a^2*x^6 + 8
*a*b*x^5 + 8*b^2*x^4 + 2*a*c*x^3 + 8*b*c*x^2 - 4*(a*x^4 + 2*b*x^3 + c*x)*sq
rt(a*x^3 + b*x^2 + c)*sqrt(b) + c^2)/(a^2*x^6 + 2*a*c*x^3 + c^2)))/(a*b*x^3
 + b*c), -1/2*(2*sqrt(a*x^3 + b*x^2 + c)*b*x - (a*x^3 + c)*sqrt(-b)*arctan(
1/2*(a*x^3 + 2*b*x^2 + c)*sqrt(a*x^3 + b*x^2 + c)*sqrt(-b)/(a*b*x^4 + b^2*x
^3 + b*c*x)))/(a*b*x^3 + b*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3+bx^2+c}\left(ax^3-2\,c\right)}{\left(ax^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*c)*(a*x^3+b*x^2+c)^(1/2)/(a*x^3+c)^2,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*x^3 + b*x^2 + c)*(a*x^3 - 2*c)/(a*x^3 + c)^2, x)
```

maple [C] time = 2.63, size = 10183, normalized size = 169.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-2*c)*(a*x^3+b*x^2+c)^(1/2)/(a*x^3+c)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3 + bx^2 + c}\,(ax^3 - 2c)}{(ax^3 + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*c)*(a*x^3+b*x^2+c)^(1/2)/(a*x^3+c)^2,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*x^3 + b*x^2 + c)*(a*x^3 - 2*c)/(a*x^3 + c)^2, x)
```

mupad [B] time = 1.47, size = 79, normalized size = 1.32

$$\frac{\ln\left(\frac{c+a\,x^3+2\,b\,x^2-2\,\sqrt{b}\,x\,\sqrt{a\,x^3+b\,x^2+c}}{c^2+a\,c\,x^3}\right)}{2\,\sqrt{b}} - \frac{x\,\sqrt{a\,x^3+b\,x^2+c}}{a\,x^3+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*c - a*x^3)*(c + a*x^3 + b*x^2)^(1/2))/(c + a*x^3)^2,x)
```

```
[Out] log((c + a*x^3 + 2*b*x^2 - 2*b^(1/2)*x*(c + a*x^3 + b*x^2)^(1/2))/(c^2 + a*
c*x^3))/(2*b^(1/2)) - (x*(c + a*x^3 + b*x^2)^(1/2))/(c + a*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-2*c)*(a*x**3+b*x**2+c)**(1/2)/(a*x**3+c)**2,x)
```

```
[Out] Timed out
```

3.754 $\int \frac{-b+a^2x^2}{\left(b+2abx+a^2x^2\right)\sqrt{bx+a^2x^3}}\,dx$

Optimal. Leaf size=60

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+bx}}{a^2x^2+b}\right)}{\sqrt{a}\sqrt{b}}$$

Rubi [C] time = 2.12, antiderivative size = 290, normalized size of antiderivative = 4.83, number of steps used = 13, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {2056, 6728, 329, 220, 933, 168, 537}

$$\frac{\sqrt{x}\left(ax+\sqrt{b}\right)\sqrt{\frac{a^2x^2+b}{\left(ax+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt[4]{b}\sqrt{a^2x^3+bx}}-\frac{2\sqrt[4]{-b}\sqrt{x}\sqrt{\frac{a^2x^2}{b}+1}\,\Pi\left(\frac{\sqrt{-b}}{\left(\sqrt{b-1}-\sqrt{b}\right)\sqrt{b}};\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt[4]{-b}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{a^2x^3+bx}}-\frac{2\sqrt[4]{-b}\sqrt{x}\sqrt{\frac{a^2x^2}{b}+1}\,\Pi\left(-\frac{\sqrt{-b}}{\left(\sqrt{b-1}+\sqrt{b}\right)\sqrt{b}};\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt[4]{-b}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{a^2x^3+bx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-b + a^2*x^2)/((b + 2*a*b*x + a^2*x^2)*Sqrt[b*x + a^2*x^3]),x]
```

```
[Out] (Sqrt[x]*(Sqrt[b] + a*x)*Sqrt[(b + a^2*x^2)/(Sqrt[b] + a*x)^2]*EllipticF[2*
ArcTan[(Sqrt[a]*Sqrt[x])/b^(1/4)], 1/2])/(Sqrt[a]*b^(1/4)*Sqrt[b*x + a^2*x^
3]) - (2*(-b)^(1/4)*Sqrt[x]*Sqrt[1 + (a^2*x^2)/b]*EllipticPi[Sqrt[-b]/((Sqr
t[-1 + b] - Sqrt[b])*Sqrt[b]), ArcSin[(Sqrt[a]*Sqrt[x])/(-b)^(1/4)], -1])/(
Sqrt[a]*Sqrt[b*x + a^2*x^3]) - (2*(-b)^(1/4)*Sqrt[x]*Sqrt[1 + (a^2*x^2)/b]*
EllipticPi[-(Sqrt[-b]/((Sqrt[-1 + b] + Sqrt[b])*Sqrt[b])), ArcSin[(Sqrt[a]*
Sqrt[x])/(-b)^(1/4)], -1])/(Sqrt[a]*Sqrt[b*x + a^2*x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
```

```
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-b+a^2x^2}{\left(b+2abx+a^2x^2\right)\sqrt{bx+a^2x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{b+a^2x^2}\right)\int \frac{-b+a^2x^2}{\sqrt{x}\sqrt{b+a^2x^2}\left(b+2abx+a^2x^2\right)}\,dx}{\sqrt{bx+a^2x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{b+a^2x^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{b+a^2x^2}}-\frac{2(b+abx)}{\sqrt{x}\sqrt{b+a^2x^2}\left(b+2abx+a^2x^2\right)}\right)dx}{\sqrt{bx+a^2x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{b+a^2x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+a^2x^2}}\,dx}{\sqrt{bx+a^2x^3}} - \frac{\left(2\sqrt{x}\sqrt{b+a^2x^2}\right)\int \frac{b+abx}{\sqrt{x}\sqrt{b+a^2x^2}\left(b+2abx\cdots\right.}}{\sqrt{bx+a^2x^3}}$$

$$= -\frac{\left(2\sqrt{x}\sqrt{b+a^2x^2}\right)\int \left(\frac{-a\sqrt{-1+b}\sqrt{b}+ab}{\sqrt{x}\left(-2a\sqrt{-1+b}\sqrt{b}+2ab+2a^2x\right)\sqrt{b+a^2x^2}}+\frac{a\sqrt{-1+\cdots}}{\sqrt{x}\left(2a\sqrt{-1+b}\sqrt{b}+\cdots\right.}\right.}{\sqrt{bx+a^2x^3}}$$

$$= \frac{\sqrt{x}\left(\sqrt{b}+ax\right)\sqrt{\frac{b+a^2x^2}{\left(\sqrt{b}+ax\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt[4]{b}\sqrt{bx+a^2x^3}} + \frac{\left(2a\left(\sqrt{-1+b}-\sqrt{b}\right)\cdots\right.}{\cdots}$$

$$= \frac{\sqrt{x}\left(\sqrt{b}+ax\right)\sqrt{\frac{b+a^2x^2}{\left(\sqrt{b}+ax\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt[4]{b}\sqrt{bx+a^2x^3}} + \frac{\left(2a\left(\sqrt{-1+b}-\sqrt{b}\right)\cdots\right.}{\cdots}$$

$$= \frac{\sqrt{x}\left(\sqrt{b}+ax\right)\sqrt{\frac{b+a^2x^2}{\left(\sqrt{b}+ax\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt[4]{b}\sqrt{bx+a^2x^3}} - \frac{\left(4a\left(\sqrt{-1+b}-\sqrt{b}\right)\cdots\right.}{\cdots}$$

$$= \frac{\sqrt{x}\left(\sqrt{b}+ax\right)\sqrt{\frac{b+a^2x^2}{\left(\sqrt{b}+ax\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt[4]{b}\sqrt{bx+a^2x^3}} - \frac{2\sqrt[4]{-b}\sqrt{x}\sqrt{1+\frac{a^2x^2}{b}}\,\Gamma\cdots}{\cdots}$$

Mathematica [C] time = 2.31, size = 204, normalized size = 3.40

$$-\frac{2ix^{3/2}\sqrt{\frac{b}{a^2x^2}+1}\left(-\Pi\left(-\frac{ia\sqrt{b}}{ab-\sqrt{a^2(b-1)b}};i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(-\frac{ia\sqrt{b}}{ab+\sqrt{a^2(b-1)b}};i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{a}}}{\sqrt{x}}\right)\middle|-1\right)+F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{a}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{\sqrt{\frac{i\sqrt{b}}{a}}\sqrt{x\left(a^2x^2+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a^2*x^2)/((b + 2*a*b*x + a^2*x^2)*Sqrt[b*x + a^2*x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[1 + b/(a^2*x^2)]*x^(3/2)*(EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])
/a]/Sqrt[x]], -1] - EllipticPi[((-I)*a*Sqrt[b])/(a*b - Sqrt[a^2*(-1 + b)*b]
), I*ArcSinh[Sqrt[(I*Sqrt[b])/a]/Sqrt[x]], -1] - EllipticPi[((-I)*a*Sqrt[b]
)/(a*b + Sqrt[a^2*(-1 + b)*b]), I*ArcSinh[Sqrt[(I*Sqrt[b])/a]/Sqrt[x]], -1]
))/(Sqrt[(I*Sqrt[b])/a]*Sqrt[x*(b + a^2*x^2)])
```

IntegrateAlgebraic [A] time = 0.34, size = 60, normalized size = 1.00

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+bx}}{a^2x^2+b}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a^2*x^2)/((b + 2*a*b*x + a^2*x^2)*Sqrt[b*x + a^2*x
^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[b*x + a^2*x^3])/(b + a^2*x^
2)])/(Sqrt[a]*Sqrt[b]))
```

fricas [A] time = 0.45, size = 219, normalized size = 3.65

$$\left[\frac{1}{4}\sqrt{2}\sqrt{-\frac{1}{ab}}\log\left(\frac{a^4x^4-12a^3bx^3-12ab^2x+2\left(2a^2b^2+a^2b\right)x^2+4\sqrt{2}\left(a^3bx^2-2a^2b^2x+ab^2\right)\sqrt{a^2x^3+bx}\sqrt{-\frac{1}{ab}}+b^2}{a^4x^4+4a^3bx^3+4ab^2x+2\left(2a^2b^2+a^2b\right)x^2+b^2}\right), \frac{1}{2}\sqrt{2}\sqrt{\frac{1}{ab}}\arctan\left(\frac{\sqrt{2}\left(a^2x^2-2abx+b\right)\sqrt{\frac{1}{ab}}}{4\sqrt{a^2x^3+bx}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)/(a^2*x^2+2*a*b*x+b)/(a^2*x^3+b*x)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/4*sqrt(2)*sqrt(-1/(a*b))*log((a^4*x^4 - 12*a^3*b*x^3 - 12*a*b^2*x + 2*(2
*a^2*b^2 + a^2*b)*x^2 + 4*sqrt(2)*(a^3*b*x^2 - 2*a^2*b^2*x + a*b^2)*sqrt(a^
2*x^3 + b*x)*sqrt(-1/(a*b)) + b^2)/(a^4*x^4 + 4*a^3*b*x^3 + 4*a*b^2*x + 2*(
2*a^2*b^2 + a^2*b)*x^2 + b^2)), 1/2*sqrt(2)*sqrt(1/(a*b))*arctan(1/4*sqrt(2
)*(a^2*x^2 - 2*a*b*x + b)*sqrt(1/(a*b))/sqrt(a^2*x^3 + b*x))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2x^2-b}{\sqrt{a^2x^3+bx}\left(a^2x^2+2\,abx+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)/(a^2*x^2+2*a*b*x+b)/(a^2*x^3+b*x)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a^2*x^2 - b)/(sqrt(a^2*x^3 + b*x)*(a^2*x^2 + 2*a*b*x + b)), x)
```

maple [C] time = 0.11, size = 1096, normalized size = 18.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b)/(a^2*x^2+2*a*b*x+b)/(a^2*x^3+b*x)^(1/2),x)
```

```
[Out] (-b)^(1/2)/a*((x+(-b)^(1/2)/a)/(-b)^(1/2)*a)^(1/2)*(-2*(x-(-b)^(1/2)/a)/(-b
)^(1/2)*a)^(1/2)*(-x/(-b)^(1/2)*a)^(1/2)/(a^2*x^3+b*x)^(1/2)*EllipticF(((x+
(-b)^(1/2)/a)/(-b)^(1/2)*a)^(1/2),1/2*2^(1/2))-2*b*(-1/2/a^2/(b^2-b)^(1/2)*
(-b)^(1/2)*(x/(-b)^(1/2)*a+1)^(1/2)*(-2*x/(-b)^(1/2)*a+2)^(1/2)*(-x/(-b)^(1
/2)*a)^(1/2)/(a^2*x^3+b*x)^(1/2)/(-(-b)^(1/2)/a+b/a-1/a*(b^2-b)^(1/2))*Elli
pticPi(((x+(-b)^(1/2)/a)/(-b)^(1/2)*a)^(1/2),-(-b)^(1/2)/a/(-(-b)^(1/2)/a-(
-b+(b^2-b)^(1/2))/a),1/2*2^(1/2))*b+1/2/a^2*(-b)^(1/2)*(x/(-b)^(1/2)*a+1)^(
1/2)*(-2*x/(-b)^(1/2)*a+2)^(1/2)*(-x/(-b)^(1/2)*a)^(1/2)/(a^2*x^3+b*x)^(1/2
)/(-(-b)^(1/2)/a+b/a-1/a*(b^2-b)^(1/2))*EllipticPi(((x+(-b)^(1/2)/a)/(-b)^(
1/2)*a)^(1/2),-(-b)^(1/2)/a/(-(-b)^(1/2)/a-(-b+(b^2-b)^(1/2))/a),1/2*2^(1/2
))+1/2/a^2/(b^2-b)^(1/2)*(-b)^(1/2)*(x/(-b)^(1/2)*a+1)^(1/2)*(-2*x/(-b)^(1/
2)*a+2)^(1/2)*(-x/(-b)^(1/2)*a)^(1/2)/(a^2*x^3+b*x)^(1/2)/(-(-b)^(1/2)/a+b/
a-1/a*(b^2-b)^(1/2))*EllipticPi(((x+(-b)^(1/2)/a)/(-b)^(1/2)*a)^(1/2),-(-b)
^(1/2)/a/(-(-b)^(1/2)/a-(-b+(b^2-b)^(1/2))/a),1/2*2^(1/2))+1/2/a^2/(b^2-b)^
(1/2)*(-b)^(1/2)*(x/(-b)^(1/2)*a+1)^(1/2)*(-2*x/(-b)^(1/2)*a+2)^(1/2)*(-x/(
-b)^(1/2)*a)^(1/2)/(a^2*x^3+b*x)^(1/2)/(-(-b)^(1/2)/a+b/a+1/a*(b^2-b)^(1/2)
)*EllipticPi(((x+(-b)^(1/2)/a)/(-b)^(1/2)*a)^(1/2),-(-b)^(1/2)/a/(-(-b)^(1/
2)/a-(-b-(b^2-b)^(1/2))/a),1/2*2^(1/2))*b+1/2/a^2*(-b)^(1/2)*(x/(-b)^(1/2)*
a+1)^(1/2)*(-2*x/(-b)^(1/2)*a+2)^(1/2)*(-x/(-b)^(1/2)*a)^(1/2)/(a^2*x^3+b*x
)^(1/2)/(-(-b)^(1/2)/a+b/a+1/a*(b^2-b)^(1/2))*EllipticPi(((x+(-b)^(1/2)/a)/
(-b)^(1/2)*a)^(1/2),-(-b)^(1/2)/a/(-(-b)^(1/2)/a-(-b-(b^2-b)^(1/2))/a),1/2*
2^(1/2))-1/2/a^2/(b^2-b)^(1/2)*(-b)^(1/2)*(x/(-b)^(1/2)*a+1)^(1/2)*(-2*x/(-
b)^(1/2)*a+2)^(1/2)*(-x/(-b)^(1/2)*a)^(1/2)/(a^2*x^3+b*x)^(1/2)/(-(-b)^(1/2
)/a+b/a+1/a*(b^2-b)^(1/2))*EllipticPi(((x+(-b)^(1/2)/a)/(-b)^(1/2)*a)^(1/2)
,-(-b)^(1/2)/a/(-(-b)^(1/2)/a-(-b-(b^2-b)^(1/2))/a),1/2*2^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)/(a^2*x^2+2*a*b*x+b)/(a^2*x^3+b*x)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-1>0)', see `assume?` for more det
ails)Is b-1 positive, negative or zero?
```

mupad [B] time = 3.28, size = 78, normalized size = 1.30

$$\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,b}{4}+\frac{\sqrt{2}\,a^2\,x^2}{4}-\frac{\sqrt{2}\,a\,b\,x}{2}+\sqrt{a}\,\sqrt{b}\,\sqrt{a^2\,x^3+b\,x}\,1\mathrm{i}}{a^2\,x^2+2\,b\,a\,x+b}\right)\,1\mathrm{i}}{2\,\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a^2*x^2)/((b*x + a^2*x^3)^(1/2)*(b + a^2*x^2 + 2*a*b*x)),x)
```

```
[Out] (2^(1/2)*log(((2^(1/2)*b)/4 + (2^(1/2)*a^2*x^2)/4 + a^(1/2)*b^(1/2)*(b*x +
a^2*x^3)^(1/2)*1i - (2^(1/2)*a*b*x)/2)/(b + a^2*x^2 + 2*a*b*x))*1i)/(2*a^(1
/2)*b^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2x^2 - b}{\sqrt{x\left(a^2x^2 + b\right)}\,\left(a^2x^2 + 2abx + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b)/(a**2*x**2+2*a*b*x+b)/(a**2*x**3+b*x)**(1/2),x)
```

```
[Out] Integral((a**2*x**2 - b)/(sqrt(x*(a**2*x**2 + b))*(a**2*x**2 + 2*a*b*x + b)
), x)
```

3.755 $\int \frac{a(ab+ac-3bc)+\left(-2a^2+ab+ac+3bc\right)x+(a-2b-2c)x^2+x^3}{\sqrt{(-a+x)(-b+x)(-c+x)}\left(-bc-a^3d+\left(b+c+3a^2d\right)x-(1+3ad)x^2+dx^3\right)}\,dx$

Optimal. Leaf size=60

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}}{\sqrt{d}\,(a-x)^2}\right)}{\sqrt{d}}$$

Rubi [F] time = 53.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a(ab+ac-3bc)+\left(-2a^2+ab+ac+3bc\right)x+(a-2b-2c)x^2+x^3}{\sqrt{(-a+x)(-b+x)(-c+x)}\left(-bc-a^3d+\left(b+c+3a^2d\right)x-(1+3ad)x^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*(a*b + a*c - 3*b*c) + (-2*a^2 + a*b + a*c + 3*b*c)*x + (a - 2*b - 2*
c)*x^2 + x^3)/(Sqrt[(-a + x)*(-b + x)*(-c + x)]*(-(b*c) - a^3*d + (b + c +
3*a^2*d)*x - (1 + 3*a*d)*x^2 + d*x^3)),x]
```

```
[Out] (-2*Sqrt[a - c]*(b - x)*Sqrt[-a + x]*EllipticF[ArcTan[Sqrt[-a + x]/Sqrt[a -
 c]], -((b - c)/(a - b))])/((a - b)*d*Sqrt[((a - c)*(b - x))/((a - b)*(c -
x))]*Sqrt[-((a - x)*(b - x)*(c - x))]) - (2*(a - b)*(a - c)*Sqrt[-a + x]*Sq
rt[-b + x]*Sqrt[-c + x]*Defer[Subst][Defer[Int][1/(Sqrt[a - b + x^2]*Sqrt[a
 - c + x^2]*(a^2*(1 + (b*c - a*(b + c))/a^2) + 2*a*(1 - (b + c)/(2*a))*x^2
+ x^4 - d*x^6)), x], x, Sqrt[-a + x]])/(d*Sqrt[-((a - x)*(b - x)*(c - x))])
 - (2*(2*a - b - c + a^2*d)*Sqrt[-a + x]*Sqrt[-b + x]*Sqrt[-c + x]*Defer[Su
bst][Defer[Int][x^2/(Sqrt[a - b + x^2]*Sqrt[a - c + x^2]*(a^2*(1 + (b*c - a
*(b + c))/a^2) + 2*a*(1 - (b + c)/(2*a))*x^2 + x^4 - d*x^6)), x], x, Sqrt[-
a + x]])/(d*Sqrt[-((a - x)*(b - x)*(c - x))]) - (2*(1 + 2*a*d)*Sqrt[-a + x]
*Sqrt[-b + x]*Sqrt[-c + x]*Defer[Subst][Defer[Int][x^4/(Sqrt[a - b + x^2]*S
qrt[a - c + x^2]*(a^2*(1 + (b*c - a*(b + c))/a^2) + 2*a*(1 - (b + c)/(2*a))
*x^2 + x^4 - d*x^6)), x], x, Sqrt[-a + x]])/(d*Sqrt[-((a - x)*(b - x)*(c -
x))]) + (4*a*(a - b - c)*Sqrt[-a + x]*Sqrt[-b + x]*Sqrt[-c + x]*Defer[Subst
][Defer[Int][x^2/(Sqrt[a - b + x^2]*Sqrt[a - c + x^2]*(-(a^2*(1 + (b*c - a*
(b + c))/a^2)) - 2*a*(1 - (b + c)/(2*a))*x^2 - x^4 + d*x^6)), x], x, Sqrt[-
a + x]])/Sqrt[-((a - x)*(b - x)*(c - x))] + (4*(a - b - c)*Sqrt[-a + x]*Sqr
t[-b + x]*Sqrt[-c + x]*Defer[Subst][Defer[Int][x^4/(Sqrt[a - b + x^2]*Sqrt[
a - c + x^2]*(-(a^2*(1 + (b*c - a*(b + c))/a^2)) - 2*a*(1 - (b + c)/(2*a))*
x^2 - x^4 + d*x^6)), x], x, Sqrt[-a + x]])/Sqrt[-((a - x)*(b - x)*(c - x))]
 + (2*(3*b*c - a*(b + c))*Sqrt[-a + x]*Sqrt[-b + x]*Sqrt[-c + x]*Defer[Subs
t][Defer[Int][x^2/(Sqrt[a - b + x^2]*Sqrt[a - c + x^2]*(-(a^2*(1 + (b*c)/a^
2)) + (b + c)*x^2 - x^4 + d*x^6 + a*(b + c - 2*x^2))), x], x, Sqrt[-a + x]]
)/Sqrt[-((a - x)*(b - x)*(c - x))]
```

Rubi steps

$$\int \frac{a(ab+ac-3bc)+\left(-2a^2+ab+ac+3bc\right)x+(a-2b-2c)x^2+x^3}{\sqrt{(-a+x)(-b+x)(-c+x)}\left(-bc-a^3d+\left(b+c+3a^2d\right)x-(1+3ad)x^2+dx^3\right)}\,dx=\frac{\left(\sqrt{-a+x}\sqrt{-b+x}\sqrt{-\cdots}\right.}{\cdots}$$

$$=\frac{\left(\sqrt{-a+x}\sqrt{-b+x}\sqrt{-\cdots}\right.}{\cdots}$$

$$=\frac{\left(\sqrt{-a+x}\sqrt{-b+x}\sqrt{-\cdots}\right.}{\cdots}$$

$$=\frac{\left(\sqrt{-a+x}\sqrt{-b+x}\sqrt{-\cdots}\right.}{\cdots}$$

$$=\frac{\left(\sqrt{-a+x}\sqrt{-b+x}\sqrt{-\cdots}\right.}{\cdots}$$

$$=\frac{\left(2\sqrt{-a+x}\sqrt{-b+x}\cdots\right.}{\cdots}$$

$$=\frac{\left(2\sqrt{-a+x}\sqrt{-b+x}\cdots\right.}{\cdots}$$

$$=\frac{\left(2\sqrt{-a+x}\sqrt{-b+x}\cdots\right.}{\cdots}$$

$$=\frac{\left(2\sqrt{-a+x}\sqrt{-b+x}\cdots\right.}{\cdots}$$

$$=\frac{\left(4(a-b-c)\sqrt{-a+x}\cdots\right.}{\cdots}$$

$$=-\frac{2\sqrt{a-c}\,(b-x)\sqrt{-a\cdots}}{(a-b)d\sqrt{\frac{(a-c)(b-\cdots)}{(a-b)(c-\cdots)}}\cdots}$$

$$=-\frac{2\sqrt{a-c}\,(b-x)\sqrt{-a\cdots}}{(a-b)d\sqrt{\frac{(a-c)(b-\cdots)}{(a-b)(c-\cdots)}}\cdots}$$

Mathematica [C] time = 4.95, size = 4752, normalized size = 79.20

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*(a*b + a*c - 3*b*c) + (-2*a^2 + a*b + a*c + 3*b*c)*x + (a - 2*
```

```
b - 2*c)*x^2 + x^3)/(Sqrt[(-a + x)*(-b + x)*(-c + x)]*(-(b*c) - a^3*d + (b
+ c + 3*a^2*d)*x - (1 + 3*a*d)*x^2 + d*x^3)),x]
```

```
[Out] ((2*I)*(-a + x)*Sqrt[(-c + x)/(a - c)]*(-2*a*EllipticF[I*ArcSinh[Sqrt[(-a +
 x)/(a - b)]], (a - b)/(a - c)] + b*EllipticF[I*ArcSinh[Sqrt[(-a + x)/(a -
b)]], (a - b)/(a - c)] + c*EllipticF[I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a
- b)/(a - c)] - 3*a^2*d*EllipticF[I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b
)/(a - c)] + 3*a*b*d*EllipticF[I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(
a - c)] + 3*a*c*d*EllipticF[I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a -
 c)] - 3*b*c*d*EllipticF[I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)
] + 2*a*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 +
 #1^2 + d*#1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)]
- b*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^
2 + d*#1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] - c*
EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 +
d*#1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] + 11*a^2
*d*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2
 + d*#1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] - 11*
a*b*d*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #
1^2 + d*#1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] +
2*b^2*d*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 +
 #1^2 + d*#1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)]
- 11*a*c*d*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#
1 + #1^2 + d*#1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c
)] + 7*b*c*d*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)
*#1 + #1^2 + d*#1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a -
 c)] + 2*c^2*d*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a + b +
c)*#1 + #1^2 + d*#1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a
 - c)] - 5*a^2*d*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a + b
+ c)*#1 + #1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/
(a - c)] + 5*a*b*d*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a +
b + c)*#1 + #1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b
)/(a - c)] - 2*b^2*d*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a
+ b + c)*#1 + #1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a -
 b)/(a - c)] + 5*a*c*d*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*
a + b + c)*#1 + #1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a
 - b)/(a - c)] - b*c*d*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*
a + b + c)*#1 + #1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a
 - b)/(a - c)] - 2*c^2*d*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-
2*a + b + c)*#1 + #1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[(-a + x)/(a - b)]],
(a - b)/(a - c)] - 3*a^2*d*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c +
(-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[(-a + x)/(a - b)]]
, (a - b)/(a - c)] + 3*a*b*d*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c
+ (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[(-a + x)/(a - b)
]], (a - b)/(a - c)] + 3*a*c*d*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*
c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[(-a + x)/(a -
b)]], (a - b)/(a - c)] - 3*b*c*d*EllipticPi[(a - b)/Root[a^2 - a*b - a*c +
b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[(-a + x)/(a
- b)]], (a - b)/(a - c)] + 3*(a - b)*(a - c)*d^2*(EllipticPi[(a - b)/Root[a
^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 1], I*ArcSinh[
Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] + EllipticPi[(a - b)/Root[a^2 - a
*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[(
-a + x)/(a - b)]], (a - b)/(a - c)] - 2*EllipticPi[(a - b)/Root[a^2 - a*b -
 a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[(-a +
 x)/(a - b)]], (a - b)/(a - c)])*Root[a^2 - a*b - a*c + b*c + (-2*a + b + c
)*#1 + #1^2 + d*#1^3 & , 2] + d^2*(2*EllipticF[I*ArcSinh[Sqrt[(-a + x)/(a -
 b)]], (a - b)/(a - c)] - EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (
-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]],
 (a - b)/(a - c)] - EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a +
 b + c)*#1 + #1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a -
```

```
b)/(a - c)])*Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3
 & , 1]*Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & ,
2]^2 - 3*(a - b)*(a - c)*d^2*(EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c
 + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b
)]], (a - b)/(a - c)] - 2*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (
-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[(-a + x)/(a - b)]],
 (a - b)/(a - c)] + EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a +
 b + c)*#1 + #1^2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a -
b)/(a - c)])*Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3
 & , 3] + 2*(2*a - b - c)*d^2*(2*EllipticPi[(a - b)/Root[a^2 - a*b - a*c +
b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a
- b)]], (a - b)/(a - c)] - EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c +
(-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[(-a + x)/(a - b)]]
, (a - b)/(a - c)] - EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a
+ b + c)*#1 + #1^2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a -
 b)/(a - c)])*Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^
3 & , 1]*Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & ,
 3] + 2*d^2*(EllipticF[I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)]
- EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2
+ d*#1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)])*Root[
a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 1]^2*Root[a^2
 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 3] + 2*(2*a - b
- c)*d^2*(EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1
 + #1^2 + d*#1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)
] - 2*EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #
1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] +
EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 +
d*#1^3 & , 3], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)])*Root[a^
2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 2]*Root[a^2 - a
*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 3] - d^2*(EllipticPi
[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & ,
 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] - EllipticPi[(a -
b)/Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 2], I
*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)])*Root[a^2 - a*b - a*c +
b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 2]^2*Root[a^2 - a*b - a*c + b*c
 + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 3] - d^2*(EllipticPi[(a - b)/Root[
a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 1], I*ArcSinh
[Sqrt[(-a + x)/(a - b)]], (a - b)/(a - c)] - EllipticPi[(a - b)/Root[a^2 -
a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[
(-a + x)/(a - b)]], (a - b)/(a - c)])*Root[a^2 - a*b - a*c + b*c + (-2*a +
b + c)*#1 + #1^2 + d*#1^3 & , 1]*Root[a^2 - a*b - a*c + b*c + (-2*a + b + c
)*#1 + #1^2 + d*#1^3 & , 3]^2 + d^2*(2*EllipticF[I*ArcSinh[Sqrt[(-a + x)/(a
 - b)]], (a - b)/(a - c)] - EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c +
 (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 1], I*ArcSinh[Sqrt[(-a + x)/(a - b)]
], (a - b)/(a - c)] - EllipticPi[(a - b)/Root[a^2 - a*b - a*c + b*c + (-2*a
 + b + c)*#1 + #1^2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[(-a + x)/(a - b)]], (a
- b)/(a - c)])*Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1
^3 & , 2]*Root[a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 &
, 3]^2))/(d^3*Sqrt[(a - x)/(b - x)]*Sqrt[(-a + x)*(-b + x)*(-c + x)]*(Root[
a^2 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 1] - Root[a^2
 - a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 2])*(Root[a^2 -
a*b - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 1] - Root[a^2 - a*b
 - a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 3])*(Root[a^2 - a*b -
a*c + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 2] - Root[a^2 - a*b - a*c
 + b*c + (-2*a + b + c)*#1 + #1^2 + d*#1^3 & , 3]))
```

IntegrateAlgebraic [A] time = 3.11, size = 60, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}}{\sqrt{d}\,(a-x)^2}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*(a*b + a*c - 3*b*c) + (-2*a^2 + a*b + a*c + 3*b*c)*x
+ (a - 2*b - 2*c)*x^2 + x^3)/(Sqrt[(-a + x)*(-b + x)*(-c + x)]*(-(b*c) - a^
3*d + (b + c + 3*a^2*d)*x - (1 + 3*a*d)*x^2 + d*x^3)),x]
```

```
[Out] (-2*ArcTanh[Sqrt[-(a*b*c) + (a*b + a*c + b*c)*x + (-a - b - c)*x^2 + x^3]/(
Sqrt[d]*(a - x)^2)])/Sqrt[d]
```

fricas [B] time = 86.73, size = 651, normalized size = 10.85

$$\left[\frac{\log\left(\frac{a^6d^2+d^2x^6-6a^3bcd-6(ad^2-d)x^5+(15a^2d^2-6(3a+b+c)d+1)x^4+b^2c^2-2(10a^3d^2-3(3a^2+3ab+(3a+b)c)d+b+c)x^3+(15a^4d^2+b^2+4bc+c^2-6(a^3+3a^2b+3(a^2+ab)c)d)x^2-4(a^4d+dx^4-(4ad-1)x^3-abc+(6a^2d-a-b-c)x^2-(4a^3d-ab-(a+b)c)x)\sqrt{-abc-(a+b+c)x^2+x^3+(ab+(a+b)c)x}\sqrt{d}-2(3a^5d^2+b^2c+bc^2-3(a^3b+(a^3+3a^2b)c)d)x}{a^6d^2+d^2x^6+2a^3bcd-2(3ad^2+d)x^5+(15a^2d^2+2(3a+b+c)d+1)x^4+b^2c^2-2(10a^3d^2+(3a^2+3ab+(3a+b)c)d+b+c)x^3+(15a^4d^2+b^2+4bc+c^2+2(a^3+3a^2b+3(a^2+ab)c)d)x^2-2(3a^5d^2+b^2c+bc^2+(a^3b+(a^3+3a^2b)c)d)x}\right)}{2\sqrt{d}},\ \frac{\sqrt{-d}\arctan\left(-\frac{(a^3d-dx^3+(3ad-1)x^2-bc-(3a^2d-b-c)x)\sqrt{-abc-(a+b+c)x^2+x^3+(ab+(a+b)c)x}\sqrt{-d}}{2(a^2bcd-(2a+b+c)dx^3+dx^4+(a^2+2ab+(2a+b)c)dx^2-(a^2b+(a^2+2ab)c)dx)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(a*b+a*c-3*b*c)+(-2*a^2+a*b+a*c+3*b*c)*x+(a-2*b-2*c)*x^2+x^3)/
((-a+x)*(-b+x)*(-c+x))^(1/2)/(-b*c-a^3*d+(3*a^2*d+b+c)*x-(3*a*d+1)*x^2+d*x^
3),x, algorithm="fricas")
```

```
[Out] [1/2*log((a^6*d^2 + d^2*x^6 - 6*a^3*b*c*d - 6*(a*d^2 - d)*x^5 + (15*a^2*d^2
 - 6*(3*a + b + c)*d + 1)*x^4 + b^2*c^2 - 2*(10*a^3*d^2 - 3*(3*a^2 + 3*a*b
+ (3*a + b)*c)*d + b + c)*x^3 + (15*a^4*d^2 + b^2 + 4*b*c + c^2 - 6*(a^3 +
3*a^2*b + 3*(a^2 + a*b)*c)*d)*x^2 - 4*(a^4*d + d*x^4 - (4*a*d - 1)*x^3 - a*
b*c + (6*a^2*d - a - b - c)*x^2 - (4*a^3*d - a*b - (a + b)*c)*x)*sqrt(-a*b*
c - (a + b + c)*x^2 + x^3 + (a*b + (a + b)*c)*x)*sqrt(d) - 2*(3*a^5*d^2 + b
^2*c + b*c^2 - 3*(a^3*b + (a^3 + 3*a^2*b)*c)*d)*x)/(a^6*d^2 + d^2*x^6 + 2*a
^3*b*c*d - 2*(3*a*d^2 + d)*x^5 + (15*a^2*d^2 + 2*(3*a + b + c)*d + 1)*x^4 +
 b^2*c^2 - 2*(10*a^3*d^2 + (3*a^2 + 3*a*b + (3*a + b)*c)*d + b + c)*x^3 + (
15*a^4*d^2 + b^2 + 4*b*c + c^2 + 2*(a^3 + 3*a^2*b + 3*(a^2 + a*b)*c)*d)*x^2
 - 2*(3*a^5*d^2 + b^2*c + b*c^2 + (a^3*b + (a^3 + 3*a^2*b)*c)*d)*x))/sqrt(d
), sqrt(-d)*arctan(-1/2*(a^3*d - d*x^3 + (3*a*d - 1)*x^2 - b*c - (3*a^2*d -
 b - c)*x)*sqrt(-a*b*c - (a + b + c)*x^2 + x^3 + (a*b + (a + b)*c)*x)*sqrt(
-d)/(a^2*b*c*d - (2*a + b + c)*d*x^3 + d*x^4 + (a^2 + 2*a*b + (2*a + b)*c)*
d*x^2 - (a^2*b + (a^2 + 2*a*b)*c)*d*x))/d]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(a*b+a*c-3*b*c)+(-2*a^2+a*b+a*c+3*b*c)*x+(a-2*b-2*c)*x^2+x^3)/
((-a+x)*(-b+x)*(-c+x))^(1/2)/(-b*c-a^3*d+(3*a^2*d+b+c)*x-(3*a*d+1)*x^2+d*x^
3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 27.35Not invertible Error:
Bad Argument Value
```

maple [C] time = 0.10, size = 489, normalized size = 8.15

$$\frac{2(-a+b)\sqrt{\frac{-a+x}{-a+b}}\sqrt{\frac{-c+x}{a-c}}\sqrt{\frac{-b+x}{a-b}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-a+x}{-a+b}},\sqrt{\frac{a-b}{a-c}}\right)}{d\sqrt{-abc+abx+acx-a\,x^2+bcx-b\,x^2-c\,x^2+x^3}}-\frac{2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+(-3ad-1)\_Z^2+(3a^2d+b+c)\_Z-a^3d-bc\right)}\frac{(-4\_\alpha^2ad+2\_\alpha^2bd+2\_\alpha^2cd+5\_\alpha a^2d-\_\alpha abd-\_\alpha acd-3\_\alpha bcd-a^3d-a^2bd-a^2cd+3abcd-\_\alpha^2+\_\alpha b+\_\alpha c-bc)(-a+b)\sqrt{\frac{-a+x}{-a+b}}\sqrt{\frac{-c+x}{a-c}}\sqrt{\frac{-b+x}{a-b}}\,(d\_\alpha^2-2\_\alpha ad+a^2d-\_\alpha-a+b+c)\,\mathrm{EllipticPi}\left(\sqrt{\frac{-a+x}{-a+b}},-\frac{d\_\alpha^2-2\_\alpha ad+a^2d-\_\alpha-a+b+c}{a-c},\sqrt{\frac{a-b}{a-c}}\right)}{(-3d\_\alpha^2+6\_\alpha ad-3a^2d+2\_\alpha-b-c)\sqrt{-abc+abx+acx-a\,x^2+bcx-b\,x^2-c\,x^2+x^3}\,(a^2-ab-ac+bc)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(a*b+a*c-3*b*c)+(-2*a^2+a*b+a*c+3*b*c)*x+(a-2*b-2*c)*x^2+x^3)/((-a+x
)*(-b+x)*(-c+x))^(1/2)/(-b*c-a^3*d+(3*a^2*d+b+c)*x-(3*a*d+1)*x^2+d*x^3),x)
```

```
[Out] 2/d*(-a+b)*((-a+x)/(-a+b))^(1/2)*((-c+x)/(a-c))^(1/2)*((-b+x)/(a-b))^(1/2)/
(-a*b*c+a*b*x+a*c*x-a*x^2+b*c*x-b*x^2-c*x^2+x^3)^(1/2)*EllipticF(((-a+x)/(-
a+b))^(1/2),((a-b)/(a-c))^(1/2))-2/d*sum((-4*_alpha^2*a*d+2*_alpha^2*b*d+2*
_alpha^2*c*d+5*_alpha*a^2*d-_alpha*a*b*d-_alpha*a*c*d-3*_alpha*b*c*d-a^3*d-
a^2*b*d-a^2*c*d+3*a*b*c*d-_alpha^2+_alpha*b+_alpha*c-b*c)/(-3*_alpha^2*d+6*
_alpha*a*d-3*a^2*d+2*_alpha-b-c)*(-a+b)*((-a+x)/(-a+b))^(1/2)*((-c+x)/(a-c)
)^(1/2)*((-b+x)/(a-b))^(1/2)/(-a*b*c+a*b*x+a*c*x-a*x^2+b*c*x-b*x^2-c*x^2+x^
3)^(1/2)*(_alpha^2*d-2*_alpha*a*d+a^2*d-_alpha-a+b+c)/(a^2-a*b-a*c+b*c)*Ell
ipticPi(((-a+x)/(-a+b))^(1/2),-(_alpha^2*d-2*_alpha*a*d+a^2*d-_alpha-a+b+c)
/(a-c),((a-b)/(a-c))^(1/2)),_alpha=RootOf(d*_Z^3+(-3*a*d-1)*_Z^2+(3*a^2*d+b
+c)*_Z-a^3*d-b*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(a-2\,b-2\,c)x^2+x^3+(ab+ac-3\,bc)a-\left(2\,a^2-ab-ac-3\,bc\right)x}{\left(a^3d-dx^3+(3\,ad+1)x^2+bc-\left(3\,a^2d+b+c\right)x\right)\sqrt{-(a-x)(b-x)(c-x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(a*b+a*c-3*b*c)+(-2*a^2+a*b+a*c+3*b*c)*x+(a-2*b-2*c)*x^2+x^3)/
((-a+x)*(-b+x)*(-c+x))^(1/2)/(-b*c-a^3*d+(3*a^2*d+b+c)*x-(3*a*d+1)*x^2+d*x^
3),x, algorithm="maxima")
```

```
[Out] -integrate(((a - 2*b - 2*c)*x^2 + x^3 + (a*b + a*c - 3*b*c)*a - (2*a^2 - a*
b - a*c - 3*b*c)*x)/((a^3*d - d*x^3 + (3*a*d + 1)*x^2 + b*c - (3*a^2*d + b
+ c)*x)*sqrt(-(a - x)*(b - x)*(c - x))), x)
```

mupad [B] time = 19.26, size = 569, normalized size = 9.48

$$\frac{\ln\left(\frac{\left(a-b-c+x+a^2\,d+d\,x^2-2\,\sqrt{d}\,\sqrt{-(a-x)\,(b-x)\,(c-x)}-2\,a\,d\,x\right)\left(b\,c^2+b^2\,c+a^4\,d-a\,x^2+2\,b\,x^2-b^2\,x+2\,c\,x^2-c^2\,x+2\,d\,x^4-x^3+a^5\,d^2-d^2\,x^5+3\,a^2\,d\,x^2+5\,a\,d^2\,x^4-5\,a^4\,d^2\,x-2\,a^2\,\sqrt{d}\,\sqrt{-(a-x)\,(b-x)\,(c-x)}-a\,b\,c+a\,b\,x+a\,c\,x-3\,b\,c\,x-10\,a^2\,d^2\,x^3+10\,a^3\,d^2\,x^2-a^3\,b\,d-a^3\,c\,d-4\,a\,d\,x^3-2\,a^3\,d\,x-2\,b\,d\,x^3-2\,c\,d\,x^3+3\,a^2\,b\,c\,d+3\,a\,b\,d\,x^2+3\,a\,c\,d\,x^2+3\,b\,c\,d\,x^2+2\,a\,b\,\sqrt{d}\,\sqrt{-(a-x)\,(b-x)\,(c-x)}+2\,a\,c\,\sqrt{d}\,\sqrt{-(a-x)\,(b-x)\,(c-x)}-2\,b\,c\,\sqrt{d}\,\sqrt{-(a-x)\,(b-x)\,(c-x)}-6\,a\,b\,c\,d\,x\right)}{\left(b\,c-b\,x-c\,x+a^3\,d-d\,x^3+x^2+3\,a\,d\,x^2-3\,a^2\,d\,x\right)\left(a^4\,d^2-4\,a^3\,d^2\,x+2\,a^3\,d-2\,a^2\,b\,d-2\,a^2\,c\,d+6\,a^2\,d^2\,x^2-2\,a^2\,d\,x+a^2+4\,a\,b\,c\,d-2\,a\,b-2\,a\,c-4\,a\,d^2\,x^3+2\,a\,d\,x^2+2\,a\,x+b^2-4\,b\,c\,d\,x+2\,b\,c+2\,b\,d\,x^2-2\,b\,x+c^2+2\,c\,d\,x^2-2\,c\,x+d^2\,x^4-2\,d\,x^3+x^2\right)}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*(a*b + a*c - 3*b*c) + x*(a*b + a*c + 3*b*c - 2*a^2) + x^3 - x^2*(2*
b - a + 2*c))/((-(a - x)*(b - x)*(c - x))^(1/2)*(b*c - x*(b + c + 3*a^2*d)
+ a^3*d - d*x^3 + x^2*(3*a*d + 1))),x)
```

```
[Out] log(((a - b - c + x + a^2*d + d*x^2 - 2*d^(1/2)*(-(a - x)*(b - x)*(c - x))^
(1/2) - 2*a*d*x)*(b*c^2 + b^2*c + a^4*d - a*x^2 + 2*b*x^2 - b^2*x + 2*c*x^2
 - c^2*x + 2*d*x^4 - x^3 + a^5*d^2 - d^2*x^5 + 3*a^2*d*x^2 + 5*a*d^2*x^4 -
5*a^4*d^2*x - 2*a^2*d^(1/2)*(-(a - x)*(b - x)*(c - x))^(1/2) - a*b*c + a*b*
x + a*c*x - 3*b*c*x - 10*a^2*d^2*x^3 + 10*a^3*d^2*x^2 - a^3*b*d - a^3*c*d -
 4*a*d*x^3 - 2*a^3*d*x - 2*b*d*x^3 - 2*c*d*x^3 + 3*a^2*b*c*d + 3*a*b*d*x^2
+ 3*a*c*d*x^2 + 3*b*c*d*x^2 + 2*a*b*d^(1/2)*(-(a - x)*(b - x)*(c - x))^(1/2
) + 2*a*c*d^(1/2)*(-(a - x)*(b - x)*(c - x))^(1/2) - 2*b*c*d^(1/2)*(-(a - x
)*(b - x)*(c - x))^(1/2) - 6*a*b*c*d*x))/((b*c - b*x - c*x + a^3*d - d*x^3
+ x^2 + 3*a*d*x^2 - 3*a^2*d*x)*(2*b*c - 2*a*c - 2*a*b + 2*a*x - 2*b*x - 2*c
*x + 2*a^3*d - 2*d*x^3 + a^2 + b^2 + c^2 + x^2 + a^4*d^2 + d^2*x^4 - 4*a*d^
2*x^3 - 4*a^3*d^2*x + 6*a^2*d^2*x^2 - 2*a^2*b*d - 2*a^2*c*d + 2*a*d*x^2 - 2
*a^2*d*x + 2*b*d*x^2 + 2*c*d*x^2 - 4*b*c*d*x + 4*a*b*c*d)))/d^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(a*b+a*c-3*b*c)+(-2*a**2+a*b+a*c+3*b*c)*x+(a-2*b-2*c)*x**2+x**
3)/((-a+x)*(-b+x)*(-c+x))**(1/2)/(-b*c-a**3*d+(3*a**2*d+b+c)*x-(3*a*d+1)*x*
*2+d*x**3),x)
```

```
[Out] Timed out
```

3.756 $\int \frac{(-1+x^2)\sqrt{1+x^4}}{x^5}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{x^4+1}\,(1-2x^2)}{4x^4}+\frac{1}{2}\log\left(\sqrt{x^4+1}+x^2\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}+x^2\right)$$

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 0.77, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1252, 811, 844, 215, 266, 63, 207}

$$\frac{1}{4}\tanh^{-1}\left(\sqrt{x^4+1}\right)+\frac{1}{2}\sinh^{-1}\left(x^2\right)+\frac{\sqrt{x^4+1}\,(1-2x^2)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^2)*Sqrt[1 + x^4])/x^5,x]
```

```
[Out] ((1 - 2*x^2)*Sqrt[1 + x^4])/(4*x^4) + ArcSinh[x^2]/2 + ArcTanh[Sqrt[1 + x^4
]]/4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^2)\sqrt{1+x^4}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)\sqrt{1+x^2}}{x^3}\,dx, x, x^2\right)\\
&= \frac{(1-2x^2)\sqrt{1+x^4}}{4x^4} - \frac{1}{8}\text{Subst}\left(\int \frac{2-4x}{x\sqrt{1+x^2}}\,dx, x, x^2\right)\\
&= \frac{(1-2x^2)\sqrt{1+x^4}}{4x^4} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x^2}}\,dx, x, x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x\right.\\
&= \frac{(1-2x^2)\sqrt{1+x^4}}{4x^4} + \frac{1}{2}\sinh^{-1}(x^2) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^4\right)\\
&= \frac{(1-2x^2)\sqrt{1+x^4}}{4x^4} + \frac{1}{2}\sinh^{-1}(x^2) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^4}\right)\\
&= \frac{(1-2x^2)\sqrt{1+x^4}}{4x^4} + \frac{1}{2}\sinh^{-1}(x^2) + \frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^4}\right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 41, normalized size = 0.68

$$\frac{1}{4}\left(\tanh^{-1}\left(\sqrt{x^4+1}\right) + 2\sinh^{-1}(x^2) + \frac{\sqrt{x^4+1}\,(1-2x^2)}{x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)*Sqrt[1 + x^4])/x^5,x]
```

```
[Out] (((1 - 2*x^2)*Sqrt[1 + x^4])/x^4 + 2*ArcSinh[x^2] + ArcTanh[Sqrt[1 + x^4]])
/4
```

IntegrateAlgebraic [A] time = 0.17, size = 64, normalized size = 1.07

$$\frac{\sqrt{x^4+1}\,(1-2x^2)}{4x^4} - \frac{1}{2}\log\left(\sqrt{x^4+1}-x^2\right) - \frac{1}{2}\tanh^{-1}\left(x^2-\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*Sqrt[1 + x^4])/x^5,x]
```

```
[Out] ((1 - 2*x^2)*Sqrt[1 + x^4])/(4*x^4) - ArcTanh[x^2 - Sqrt[1 + x^4]]/2 - Log[
-x^2 + Sqrt[1 + x^4]]/2
```

fricas [A] time = 0.42, size = 85, normalized size = 1.42

$$\frac{x^4\log\left(-x^2+\sqrt{x^4+1}+1\right) - 2x^4\log\left(-x^2+\sqrt{x^4+1}\right) - x^4\log\left(-x^2+\sqrt{x^4+1}-1\right) - 2x^4 - \sqrt{x^4+1}\,(2x^2-1)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+1)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(x^4*log(-x^2 + sqrt(x^4 + 1) + 1) - 2*x^4*log(-x^2 + sqrt(x^4 + 1)) -
x^4*log(-x^2 + sqrt(x^4 + 1) - 1) - 2*x^4 - sqrt(x^4 + 1)*(2*x^2 - 1))/x^4
```

giac [B] time = 0.71, size = 118, normalized size = 1.97

$$-\frac{\left(x^2-\sqrt{x^4+1}\right)^3-2\left(x^2-\sqrt{x^4+1}\right)^2+x^2-\sqrt{x^4+1}+2}{2\left(\left(x^2-\sqrt{x^4+1}\right)^2-1\right)^2}-\frac{1}{4}\log\left(x^2-\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(-x^2+\sqrt{x^4+1}+1\right)-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+1)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/2*((x^2 - sqrt(x^4 + 1))^3 - 2*(x^2 - sqrt(x^4 + 1))^2 + x^2 - sqrt(x^4
+ 1) + 2)/((x^2 - sqrt(x^4 + 1))^2 - 1)^2 - 1/4*log(x^2 - sqrt(x^4 + 1) + 1
) + 1/4*log(-x^2 + sqrt(x^4 + 1) + 1) - 1/2*log(-x^2 + sqrt(x^4 + 1))
```

maple [A] time = 0.03, size = 63, normalized size = 1.05

$$-\frac{\left(x^4+1\right)^{\frac{3}{2}}}{2x^2}+\frac{x^2\sqrt{x^4+1}}{2}+\frac{\operatorname{arcsinh}\left(x^2\right)}{2}+\frac{\left(x^4+1\right)^{\frac{3}{2}}}{4x^4}-\frac{\sqrt{x^4+1}}{4}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(x^4+1)^(1/2)/x^5,x)
```

```
[Out] -1/2/x^2*(x^4+1)^(3/2)+1/2*x^2*(x^4+1)^(1/2)+1/2*arcsinh(x^2)+1/4/x^4*(x^4+
1)^(3/2)-1/4*(x^4+1)^(1/2)+1/4*arctanh(1/(x^4+1)^(1/2))
```

maxima [A] time = 0.42, size = 81, normalized size = 1.35

$$-\frac{\sqrt{x^4+1}}{2\,x^2}+\frac{\sqrt{x^4+1}}{4\,x^4}+\frac{1}{8}\log\left(\sqrt{x^4+1}+1\right)-\frac{1}{8}\log\left(\sqrt{x^4+1}-1\right)+\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right)-\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+1)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(x^4 + 1)/x^2 + 1/4*sqrt(x^4 + 1)/x^4 + 1/8*log(sqrt(x^4 + 1) + 1)
 - 1/8*log(sqrt(x^4 + 1) - 1) + 1/4*log(sqrt(x^4 + 1)/x^2 + 1) - 1/4*log(sq
rt(x^4 + 1)/x^2 - 1)
```

mupad [B] time = 0.97, size = 45, normalized size = 0.75

$$\frac{\operatorname{asinh}\left(x^2\right)}{2}-\frac{\sqrt{x^4+1}}{2\,x^2}+\frac{\sqrt{x^4+1}}{4\,x^4}-\frac{\operatorname{atan}\left(\sqrt{x^4+1}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*(x^4 + 1)^(1/2))/x^5,x)
```

```
[Out] asinh(x^2)/2 - (atan((x^4 + 1)^(1/2)*1i)*1i)/4 - (x^4 + 1)^(1/2)/(2*x^2) +
(x^4 + 1)^(1/2)/(4*x^4)
```

sympy [A] time = 4.70, size = 58, normalized size = 0.97

$$-\frac{x^2}{2\sqrt{x^4+1}}+\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{4}+\frac{\operatorname{asinh}\left(x^2\right)}{2}+\frac{\sqrt{1+\frac{1}{x^4}}}{4x^2}-\frac{1}{2x^2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**4+1)**(1/2)/x**5,x)
```

```
[Out] -x**2/(2*sqrt(x**4 + 1)) + asinh(x**(-2))/4 + asinh(x**2)/2 + sqrt(1 + x**(
-4))/(4*x**2) - 1/(2*x**2*sqrt(x**4 + 1))
```

3.757 $\int \frac{(-1+x^2)\sqrt{1+x^4}}{x^3}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{x^4+1}\,(x^2+1)}{2x^2}-\frac{1}{2}\log\left(\sqrt{x^4+1}+x^2+1\right)+\tanh^{-1}\left(-2\sqrt{x^4+1}-2x^2+1\right)$$

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1252, 813, 844, 215, 266, 63, 207}

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)-\frac{1}{2}\sinh^{-1}\left(x^2\right)+\frac{\sqrt{x^4+1}\,(x^2+1)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^2)*Sqrt[1 + x^4])/x^3,x]
```

```
[Out] ((1 + x^2)*Sqrt[1 + x^4])/(2*x^2) - ArcSinh[x^2]/2 - ArcTanh[Sqrt[1 + x^4]]
/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^2)\sqrt{1+x^4}}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)\sqrt{1+x^2}}{x^2}\,dx,x,x^2\right)\\
&= \frac{(1+x^2)\sqrt{1+x^4}}{2x^2}-\frac{1}{4}\text{Subst}\left(\int \frac{-2+2x}{x\sqrt{1+x^2}}\,dx,x,x^2\right)\\
&= \frac{(1+x^2)\sqrt{1+x^4}}{2x^2}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x^2}}\,dx,x,x^2\right)\\
&= \frac{(1+x^2)\sqrt{1+x^4}}{2x^2}-\frac{1}{2}\sinh^{-1}(x^2)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^4\right)\\
&= \frac{(1+x^2)\sqrt{1+x^4}}{2x^2}-\frac{1}{2}\sinh^{-1}(x^2)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^4}\right)\\
&= \frac{(1+x^2)\sqrt{1+x^4}}{2x^2}-\frac{1}{2}\sinh^{-1}(x^2)-\frac{1}{2}\tanh^{-1}\left(\sqrt{1+x^4}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.68

$$\frac{1}{2}\left(-\tanh^{-1}\left(\sqrt{x^4+1}\right)-\sinh^{-1}(x^2)+\frac{\sqrt{x^4+1}\,(x^2+1)}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)*Sqrt[1 + x^4])/x^3,x]
[Out] (((1 + x^2)*Sqrt[1 + x^4])/x^2 - ArcSinh[x^2] - ArcTanh[Sqrt[1 + x^4]])/2
```

IntegrateAlgebraic [A] time = 0.14, size = 58, normalized size = 0.97

$$\frac{\sqrt{x^4+1}\,(x^2+1)}{2x^2}+\frac{1}{2}\log\left(\sqrt{x^4+1}-x^2\right)+\tanh^{-1}\left(x^2-\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*Sqrt[1 + x^4])/x^3,x]
[Out] ((1 + x^2)*Sqrt[1 + x^4])/(2*x^2) + ArcTanh[x^2 - Sqrt[1 + x^4]] + Log[-x^2
 + Sqrt[1 + x^4]]/2
```

fricas [A] time = 0.40, size = 73, normalized size = 1.22

$$\frac{x^2\log\left(2\,x^4+x^2-\sqrt{x^4+1}\,(2\,x^2+1)+1\right)-x^2\log\left(-x^2+\sqrt{x^4+1}+1\right)+x^2+\sqrt{x^4+1}\,(x^2+1)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(x^2*log(2*x^4 + x^2 - sqrt(x^4 + 1)*(2*x^2 + 1) + 1) - x^2*log(-x^2 +
sqrt(x^4 + 1) + 1) + x^2 + sqrt(x^4 + 1)*(x^2 + 1))/x^2
```

giac [A] time = 0.35, size = 81, normalized size = 1.35

$$\frac{1}{2}\sqrt{x^4+1}-\frac{1}{\left(x^2-\sqrt{x^4+1}\right)^2-1}+\frac{1}{2}\log\left(x^2-\sqrt{x^4+1}+1\right)-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}+1\right)+\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^4 + 1) - 1/((x^2 - sqrt(x^4 + 1))^2 - 1) + 1/2*log(x^2 - sqrt(x^
4 + 1) + 1) - 1/2*log(-x^2 + sqrt(x^4 + 1) + 1) + 1/2*log(-x^2 + sqrt(x^4 +
 1))
```

maple [A] time = 0.03, size = 51, normalized size = 0.85

$$\frac{\left(x^4+1\right)^{\frac{3}{2}}}{2x^2}-\frac{x^2\sqrt{x^4+1}}{2}-\frac{\operatorname{arcsinh}\left(x^2\right)}{2}+\frac{\sqrt{x^4+1}}{2}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(x^4+1)^(1/2)/x^3,x)
```

```
[Out] 1/2/x^2*(x^4+1)^(3/2)-1/2*x^2*(x^4+1)^(1/2)-1/2*arcsinh(x^2)+1/2*(x^4+1)^(1
/2)-1/2*arctanh(1/(x^4+1)^(1/2))
```

maxima [A] time = 0.42, size = 78, normalized size = 1.30

$$\frac{1}{2}\sqrt{x^4+1}+\frac{\sqrt{x^4+1}}{2\,x^2}-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)-\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right)+\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^4 + 1) + 1/2*sqrt(x^4 + 1)/x^2 - 1/4*log(sqrt(x^4 + 1) + 1) + 1/
4*log(sqrt(x^4 + 1) - 1) - 1/4*log(sqrt(x^4 + 1)/x^2 + 1) + 1/4*log(sqrt(x^
4 + 1)/x^2 - 1)
```

mupad [B] time = 1.02, size = 42, normalized size = 0.70

$$\frac{\sqrt{x^4+1}}{2}-\frac{\operatorname{asinh}\left(x^2\right)}{2}+\frac{\sqrt{x^4+1}}{2\,x^2}+\frac{\operatorname{atan}\left(\sqrt{x^4+1}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*(x^4 + 1)^(1/2))/x^3,x)
```

```
[Out] (atan((x^4 + 1)^(1/2)*1i)*1i)/2 - asinh(x^2)/2 + (x^4 + 1)^(1/2)/2 + (x^4 +
 1)^(1/2)/(2*x^2)
```

sympy [A] time = 4.90, size = 75, normalized size = 1.25

$$\frac{x^2}{2\sqrt{x^4+1}}+\frac{x^2}{2\sqrt{1+\frac{1}{x^4}}}-\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2}-\frac{\operatorname{asinh}\left(x^2\right)}{2}+\frac{1}{2x^2\sqrt{x^4+1}}+\frac{1}{2x^2\sqrt{1+\frac{1}{x^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**4+1)**(1/2)/x**3,x)
```

```
[Out] x**2/(2*sqrt(x**4 + 1)) + x**2/(2*sqrt(1 + x**(-4))) - asinh(x**(-2))/2 - a
sinh(x**2)/2 + 1/(2*x**2*sqrt(x**4 + 1)) + 1/(2*x**2*sqrt(1 + x**(-4)))
```

3.758 $\int \frac{(1+2x^3)\sqrt{-1+x^6}}{x^7}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{x^6-1}\,(-4x^3-1)}{6x^6}+\frac{2}{3}\log\left(\sqrt{x^6-1}+x^3\right)+\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}+x^3\right)$$

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1475, 811, 844, 217, 206, 266, 63, 203}

$$\frac{1}{6}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{\sqrt{x^6-1}\,(4x^3+1)}{6x^6}+\frac{2}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x^3)*Sqrt[-1 + x^6])/x^7,x]
```

```
[Out] -1/6*((1 + 4*x^3)*Sqrt[-1 + x^6])/x^6 + ArcTan[Sqrt[-1 + x^6]]/6 + (2*ArcTa
nh[x^3/Sqrt[-1 + x^6]])/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
```

```
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+2x^3\right)\sqrt{-1+x^6}}{x^7}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{(1+2x)\sqrt{-1+x^2}}{x^3}\,dx, x, x^3\right)\\
&= -\frac{\left(1+4x^3\right)\sqrt{-1+x^6}}{6x^6}+\frac{1}{12}\,\mathrm{Subst}\left(\int \frac{2+8x}{x\sqrt{-1+x^2}}\,dx, x, x^3\right)\\
&= -\frac{\left(1+4x^3\right)\sqrt{-1+x^6}}{6x^6}+\frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{-1+x^2}}\,dx, x, x^3\right)+\frac{2}{3}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\right.\\
&= -\frac{\left(1+4x^3\right)\sqrt{-1+x^6}}{6x^6}+\frac{1}{12}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right)+\frac{2}{3}\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,\right.\\
&= -\frac{\left(1+4x^3\right)\sqrt{-1+x^6}}{6x^6}+\frac{2}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)+\frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\right.\\
&= -\frac{\left(1+4x^3\right)\sqrt{-1+x^6}}{6x^6}+\frac{1}{6}\tan^{-1}\left(\sqrt{-1+x^6}\right)+\frac{2}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 71, normalized size = 1.18

$$-\frac{\left(x^6-1\right)\left(-x^6\tanh^{-1}\left(\sqrt{1-x^6}\right)+\left(4x^3+1\right)\sqrt{1-x^6}+4x^6\sin^{-1}\left(x^3\right)\right)}{6x^6\sqrt{-\left(x^6-1\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x^3)*Sqrt[-1 + x^6])/x^7,x]
```

```
[Out] -1/6*((-1 + x^6)*((1 + 4*x^3)*Sqrt[1 - x^6] + 4*x^6*ArcSin[x^3] - x^6*ArcTa
nh[Sqrt[1 - x^6]]))/(x^6*Sqrt[-(-1 + x^6)^2])
```

IntegrateAlgebraic [A] time = 0.17, size = 68, normalized size = 1.13

$$\frac{\sqrt{x^6-1}\left(-4x^3-1\right)}{6x^6}-\frac{1}{3}\tan^{-1}\left(\frac{\sqrt{x^6-1}}{x^3-1}\right)+\frac{4}{3}\tanh^{-1}\left(\frac{\sqrt{x^6-1}}{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + 2*x^3)*Sqrt[-1 + x^6])/x^7,x]
```

```
[Out] ((-1 - 4*x^3)*Sqrt[-1 + x^6])/(6*x^6) - ArcTan[Sqrt[-1 + x^6]/(-1 + x^3)]/3
 + (4*ArcTanh[Sqrt[-1 + x^6]/(-1 + x^3)])/3
```

fricas [A] time = 0.40, size = 65, normalized size = 1.08

$$\frac{2\,x^6 \arctan\left(-x^3+\sqrt{x^6-1}\right)-4\,x^6 \log\left(-x^3+\sqrt{x^6-1}\right)-4\,x^6-\sqrt{x^6-1}\left(4\,x^3+1\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)*(x^6-1)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^6*arctan(-x^3 + sqrt(x^6 - 1)) - 4*x^6*log(-x^3 + sqrt(x^6 - 1)) -
 4*x^6 - sqrt(x^6 - 1)*(4*x^3 + 1))/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^6-1}\left(2\,x^3+1\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)*(x^6-1)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^6 - 1)*(2*x^3 + 1)/x^7, x)
```

maple [C] time = 0.34, size = 113, normalized size = 1.88

$$-\frac{4x^9+x^6-4x^3-1}{6x^6\sqrt{x^6-1}}+\frac{\sqrt{-\mathrm{signum}\left(x^6-1\right)}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^6+1}}{2}\right)+\left(-2\ln(2)+6\ln(x)+i\pi\right)\sqrt{\pi}\right)}{12\sqrt{\pi}\,\sqrt{\mathrm{signum}\left(x^6-1\right)}}+\frac{2\sqrt{-\mathrm{signum}\left(x^6-1\right)}\,\arcsin\left(x^3\right)}{3\sqrt{\mathrm{signum}\left(x^6-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+1)*(x^6-1)^(1/2)/x^7,x)
```

```
[Out] -1/6*(4*x^9+x^6-4*x^3-1)/x^6/(x^6-1)^(1/2)+1/12/Pi^(1/2)/signum(x^6-1)^(1/2
)*(-signum(x^6-1))^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(-x^6+1)^(1/2))+(-2*ln(2)+
6*ln(x)+I*Pi)*Pi^(1/2))+2/3/signum(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*arcs
in(x^3)
```

maxima [A] time = 0.42, size = 67, normalized size = 1.12

$$-\frac{2\,\sqrt{x^6-1}}{3\,x^3}-\frac{\sqrt{x^6-1}}{6\,x^6}+\frac{1}{6}\,\arctan\left(\sqrt{x^6-1}\right)+\frac{1}{3}\,\log\left(\frac{\sqrt{x^6-1}}{x^3}+1\right)-\frac{1}{3}\,\log\left(\frac{\sqrt{x^6-1}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)*(x^6-1)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(x^6 - 1)/x^3 - 1/6*sqrt(x^6 - 1)/x^6 + 1/6*arctan(sqrt(x^6 - 1))
+ 1/3*log(sqrt(x^6 - 1)/x^3 + 1) - 1/3*log(sqrt(x^6 - 1)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x^6-1}\,\left(2\,x^3+1\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/2)*(2*x^3 + 1))/x^7,x)
```

```
[Out] int(((x^6 - 1)^(1/2)*(2*x^3 + 1))/x^7, x)
```

sympy [C] time = 5.30, size = 153, normalized size = 2.55

$$\begin{cases} \frac{i\operatorname{acosh}\left(\frac{1}{x^3}\right)}{6} + \frac{i}{6x^3\sqrt{-1+\frac{1}{x^6}}} - \frac{i}{6x^9\sqrt{-1+\frac{1}{x^6}}} & \text{for } \frac{1}{|x^6|} > 1 \\ -\frac{\operatorname{asin}\left(\frac{1}{x^3}\right)}{6} - \frac{\sqrt{1-\frac{1}{x^6}}}{6x^3} & \text{otherwise} \end{cases} + 2\left(\begin{cases} -\frac{x^3}{3\sqrt{x^6-1}} + \frac{\operatorname{acosh}\left(x^3\right)}{3} + \frac{1}{3x^3\sqrt{x^6-1}} & \text{for } \left|x^6\right| > 1 \\ \frac{ix^3}{3\sqrt{1-x^6}} - \frac{i\operatorname{asin}\left(x^3\right)}{3} - \frac{i}{3x^3\sqrt{1-x^6}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+1)*(x**6-1)**(1/2)/x**7,x)
```

```
[Out] Piecewise((I*acosh(x**(-3))/6 + I/(6*x**3*sqrt(-1 + x**(-6))) - I/(6*x**9*s
qrt(-1 + x**(-6))), 1/Abs(x**6) > 1), (-asin(x**(-3))/6 - sqrt(1 - 1/x**6)/
(6*x**3), True)) + 2*Piecewise((-x**3/(3*sqrt(x**6 - 1)) + acosh(x**3)/3 +
1/(3*x**3*sqrt(x**6 - 1)), Abs(x**6) > 1), (I*x**3/(3*sqrt(1 - x**6)) - I*a
sin(x**3)/3 - I/(3*x**3*sqrt(1 - x**6)), True))
```

3.759 $\int \frac{(1+2x^3)\sqrt{-1+x^6}}{x^4}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{x^6-1}\,(2x^3-1)}{3x^3}+\frac{1}{3}\log\left(\sqrt{x^6-1}+x^3\right)-\frac{4}{3}\tan^{-1}\left(\sqrt{x^6-1}+x^3\right)$$

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1475, 813, 844, 217, 206, 266, 63, 203}

$$-\frac{2}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{\sqrt{x^6-1}\,(1-2x^3)}{3x^3}+\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x^3)*Sqrt[-1 + x^6])/x^4,x]
```

```
[Out] -1/3*((1 - 2*x^3)*Sqrt[-1 + x^6])/x^3 - (2*ArcTan[Sqrt[-1 + x^6]])/3 + ArcT
anh[x^3/Sqrt[-1 + x^6]]/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
```

```
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+2x^3\right)\sqrt{-1+x^6}}{x^4}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(1+2x)\sqrt{-1+x^2}}{x^2}\,dx, x, x^3\right)\\
&= -\frac{\left(1-2x^3\right)\sqrt{-1+x^6}}{3x^3} - \frac{1}{6}\operatorname{Subst}\left(\int \frac{4-2x}{x\sqrt{-1+x^2}}\,dx, x, x^3\right)\\
&= -\frac{\left(1-2x^3\right)\sqrt{-1+x^6}}{3x^3} + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, x^3\right) - \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1+x^2}}\right.\\
&= -\frac{\left(1-2x^3\right)\sqrt{-1+x^6}}{3x^3} - \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right) + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x\right.\\
&= -\frac{\left(1-2x^3\right)\sqrt{-1+x^6}}{3x^3} + \frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right) - \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^6}\right.\\
&= -\frac{\left(1-2x^3\right)\sqrt{-1+x^6}}{3x^3} - \frac{2}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right) + \frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 63, normalized size = 1.05

$$\frac{1}{3}\left(-2\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}\left(2x^3-1\right)}{x^3}-\frac{\sqrt{x^6-1}\sin^{-1}\left(x^3\right)}{\sqrt{1-x^6}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x^3)*Sqrt[-1 + x^6])/x^4,x]
```

```
[Out] (((-1 + 2*x^3)*Sqrt[-1 + x^6])/x^3 - (Sqrt[-1 + x^6]*ArcSin[x^3])/Sqrt[1 -
x^6] - 2*ArcTan[Sqrt[-1 + x^6]])/3
```

IntegrateAlgebraic [A] time = 0.18, size = 68, normalized size = 1.13

$$\frac{\sqrt{x^6-1}\left(2x^3-1\right)}{3x^3}+\frac{4}{3}\tan^{-1}\left(\frac{\sqrt{x^6-1}}{x^3-1}\right)+\frac{2}{3}\tanh^{-1}\left(\frac{\sqrt{x^6-1}}{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + 2*x^3)*Sqrt[-1 + x^6])/x^4,x]
```

```
[Out] ((-1 + 2*x^3)*Sqrt[-1 + x^6])/(3*x^3) + (4*ArcTan[Sqrt[-1 + x^6]/(-1 + x^3)
])/3 + (2*ArcTanh[Sqrt[-1 + x^6]/(-1 + x^3)])/3
```

fricas [A] time = 0.40, size = 62, normalized size = 1.03

$$-\frac{4\,x^{3}\arctan\left(-x^{3}+\sqrt{x^{6}-1}\right)+x^{3}\log\left(-x^{3}+\sqrt{x^{6}-1}\right)+x^{3}-\sqrt{x^{6}-1}\left(2\,x^{3}-1\right)}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)*(x^6-1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(4*x^3*arctan(-x^3 + sqrt(x^6 - 1)) + x^3*log(-x^3 + sqrt(x^6 - 1)) +
x^3 - sqrt(x^6 - 1)*(2*x^3 - 1))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{6}-1}\left(2\,x^{3}+1\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)*(x^6-1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^6 - 1)*(2*x^3 + 1)/x^4, x)
```

maple [C] time = 0.34, size = 141, normalized size = 2.35

$$-\frac{\sqrt{x^{6}-1}}{3x^{3}}-\frac{\sqrt{-\mathrm{signum}\left(x^{6}-1\right)}\,\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^{6}+1}\right)}{3\sqrt{\pi}\,\sqrt{\mathrm{signum}\left(x^{6}-1\right)}}+\frac{\sqrt{-\mathrm{signum}\left(x^{6}-1\right)}\,\arcsin\left(x^{3}\right)}{3\sqrt{\mathrm{signum}\left(x^{6}-1\right)}}-\frac{\sqrt{-\mathrm{signum}\left(x^{6}-1\right)}\,\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{6}+1}}{2}\right)+\left(-2\ln(2)+6\ln(x)+i\pi\right)\sqrt{\pi}\right)}{3\sqrt{\pi}\,\sqrt{\mathrm{signum}\left(x^{6}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+1)*(x^6-1)^(1/2)/x^4,x)
```

```
[Out] -1/3*(x^6-1)^(1/2)/x^3-1/3/Pi^(1/2)/signum(x^6-1)^(1/2)*(-signum(x^6-1))^(1
/2)*(-2*Pi^(1/2)+2*Pi^(1/2)*(-x^6+1)^(1/2))+1/3/signum(x^6-1)^(1/2)*(-signu
m(x^6-1))^(1/2)*arcsin(x^3)-1/3/Pi^(1/2)/signum(x^6-1)^(1/2)*(-signum(x^6-1
))^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(-x^6+1)^(1/2))+(-2*ln(2)+6*ln(x)+I*Pi)*Pi
^(1/2))
```

maxima [A] time = 0.42, size = 64, normalized size = 1.07

$$\frac{2}{3}\sqrt{x^{6}-1}-\frac{\sqrt{x^{6}-1}}{3\,x^{3}}-\frac{2}{3}\arctan\left(\sqrt{x^{6}-1}\right)+\frac{1}{6}\log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}+1\right)-\frac{1}{6}\log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)*(x^6-1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(x^6 - 1) - 1/3*sqrt(x^6 - 1)/x^3 - 2/3*arctan(sqrt(x^6 - 1)) + 1/6
*log(sqrt(x^6 - 1)/x^3 + 1) - 1/6*log(sqrt(x^6 - 1)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x^{6}-1}\,\left(2\,x^{3}+1\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/2)*(2*x^3 + 1))/x^4,x)
```

```
[Out] int(((x^6 - 1)^(1/2)*(2*x^3 + 1))/x^4, x)
```

sympy [C] time = 5.22, size = 168, normalized size = 2.80

$$\begin{cases} -\frac{x^3}{3\sqrt{x^6-1}} + \frac{\operatorname{acosh}(x^3)}{3} + \frac{1}{3x^3\sqrt{x^6-1}} & \text{for } |x^6| > 1 \\ \frac{ix^3}{3\sqrt{1-x^6}} - \frac{i\operatorname{asin}(x^3)}{3} - \frac{i}{3x^3\sqrt{1-x^6}} & \text{otherwise} \end{cases} + 2\left(\begin{cases} -\frac{ix^3}{3\sqrt{-1+\frac{1}{x^6}}} - \frac{i\operatorname{acosh}\left(\frac{1}{x^3}\right)}{3} + \frac{i}{3x^3\sqrt{-1+\frac{1}{x^6}}} & \text{for } \frac{1}{|x^6|} > 1 \\ \frac{x^3}{3\sqrt{1-\frac{1}{x^6}}} + \frac{\operatorname{asin}\left(\frac{1}{x^3}\right)}{3} - \frac{1}{3x^3\sqrt{1-\frac{1}{x^6}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+1)*(x**6-1)**(1/2)/x**4,x)
```

```
[Out] Piecewise((-x**3/(3*sqrt(x**6 - 1)) + acosh(x**3)/3 + 1/(3*x**3*sqrt(x**6 -
 1)), Abs(x**6) > 1), (I*x**3/(3*sqrt(1 - x**6)) - I*asin(x**3)/3 - I/(3*x*
*3*sqrt(1 - x**6)), True)) + 2*Piecewise((-I*x**3/(3*sqrt(-1 + x**(-6))) -
I*acosh(x**(-3))/3 + I/(3*x**3*sqrt(-1 + x**(-6))), 1/Abs(x**6) > 1), (x**3
/(3*sqrt(1 - 1/x**6)) + asin(x**(-3))/3 - 1/(3*x**3*sqrt(1 - 1/x**6)), True
))
```

3.760 $\int \frac{x-3x^5}{\sqrt{x+x^5}\left(1-ax^2+2x^4+x^8\right)}\,dx$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x^5+x}}{x^4+1}\right)}{a^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x^5+x}}{x^4+1}\right)}{a^{3/4}}$$

Rubi [F] time = 1.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x-3x^5}{\sqrt{x+x^5}\left(1-ax^2+2x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x - 3*x^5)/(Sqrt[x + x^5]*(1 - a*x^2 + 2*x^4 + x^8)),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 + x^4]*Defer[Subst][Defer[Int][x^2/(Sqrt[1 + x^8]*(1 - a*
x^4 + 2*x^8 + x^16)), x], x, Sqrt[x]])/Sqrt[x + x^5] - (6*Sqrt[x]*Sqrt[1 +
x^4]*Defer[Subst][Defer[Int][x^10/(Sqrt[1 + x^8]*(1 - a*x^4 + 2*x^8 + x^16)
), x], x, Sqrt[x]])/Sqrt[x + x^5]
```

Rubi steps

$$\begin{aligned}
\int \frac{x-3x^5}{\sqrt{x+x^5}\left(1-ax^2+2x^4+x^8\right)}\,dx &= \int \frac{x\left(1-3x^4\right)}{\sqrt{x+x^5}\left(1-ax^2+2x^4+x^8\right)}\,dx\\
&= \frac{\left(\sqrt{x}\sqrt{1+x^4}\right)\int \frac{\sqrt{x}\left(1-3x^4\right)}{\sqrt{1+x^4}\left(1-ax^2+2x^4+x^8\right)}\,dx}{\sqrt{x+x^5}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \frac{x^2\left(1-3x^8\right)}{\sqrt{1+x^8}\left(1-ax^4+2x^8+x^{16}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^5}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int\left(\frac{x^2}{\sqrt{1+x^8}\left(1-ax^4+2x^8+x^{16}\right)}-\frac{3x^{10}}{\sqrt{1+x^8}\left(1-ax^4+2x^8+x^{16}\right)}\right)\right.}{\sqrt{x+x^5}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^8}\left(1-ax^4+2x^8+x^{16}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^5}}-\frac{\left(6\sqrt{x}\sqrt{1+\right.}{}
\end{aligned}$$

Mathematica [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{x-3x^5}{\sqrt{x+x^5}\left(1-ax^2+2x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x - 3*x^5)/(Sqrt[x + x^5]*(1 - a*x^2 + 2*x^4 + x^8)),x]
```

```
[Out] Integrate[(x - 3*x^5)/(Sqrt[x + x^5]*(1 - a*x^2 + 2*x^4 + x^8)), x]
```

IntegrateAlgebraic [A] time = 1.97, size = 60, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x^5+x}}{x^4+1}\right)}{a^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x^5+x}}{x^4+1}\right)}{a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x - 3*x^5)/(Sqrt[x + x^5]*(1 - a*x^2 + 2*x^4 + x^8)),x]
```

```
[Out] -(ArcTan[(a^(1/4)*Sqrt[x + x^5])/(1 + x^4)]/a^(3/4)) + ArcTanh[(a^(1/4)*Sqr
t[x + x^5])/(1 + x^4)]/a^(3/4)
```

fricas [B] time = 0.53, size = 220, normalized size = 3.67

$$-\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{x^5+x}\,a\left(\frac{1}{a^3}\right)^{\frac{1}{4}}}{x^4+1}\right)+\frac{1}{4}\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{x^8+2\,x^4+ax^2+2\,\sqrt{x^5+x}\left(a^3\left(\frac{1}{a^3}\right)^{\frac{3}{4}}x+\left(ax^4+a\right)\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\right)+2\left(a^2x^5+a^2x\right)\sqrt{\frac{1}{a^3}}+1}{x^8+2\,x^4-ax^2+1}\right)-\frac{1}{4}\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{x^8+2\,x^4+ax^2-2\,\sqrt{x^5+x}\left(a^3\left(\frac{1}{a^3}\right)^{\frac{3}{4}}x+\left(ax^4+a\right)\left(\frac{1}{a^3}\right)^{\frac{1}{4}}\right)+2\left(a^2x^5+a^2x\right)\sqrt{\frac{1}{a^3}}+1}{x^8+2\,x^4-ax^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^5+x)/(x^5+x)^(1/2)/(x^8+2*x^4-a*x^2+1),x, algorithm="fricas
")
```

```
[Out] -(a^(-3))^(1/4)*arctan(sqrt(x^5 + x)*a*(a^(-3))^(1/4)/(x^4 + 1)) + 1/4*(a^(
-3))^(1/4)*log((x^8 + 2*x^4 + a*x^2 + 2*sqrt(x^5 + x)*(a^3*(a^(-3))^(3/4)*x
 + (a*x^4 + a)*(a^(-3))^(1/4)) + 2*(a^2*x^5 + a^2*x)*sqrt(a^(-3)) + 1)/(x^8
 + 2*x^4 - a*x^2 + 1)) - 1/4*(a^(-3))^(1/4)*log((x^8 + 2*x^4 + a*x^2 - 2*sq
rt(x^5 + x)*(a^3*(a^(-3))^(3/4)*x + (a*x^4 + a)*(a^(-3))^(1/4)) + 2*(a^2*x^
5 + a^2*x)*sqrt(a^(-3)) + 1)/(x^8 + 2*x^4 - a*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{3\,x^5-x}{\left(x^8+2\,x^4-ax^2+1\right)\sqrt{x^5+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^5+x)/(x^5+x)^(1/2)/(x^8+2*x^4-a*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(3*x^5 - x)/((x^8 + 2*x^4 - a*x^2 + 1)*sqrt(x^5 + x)), x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{-3x^5+x}{\sqrt{x^5+x}\,\left(x^8+2x^4-a\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^5+x)/(x^5+x)^(1/2)/(x^8+2*x^4-a*x^2+1),x)
```

```
[Out] int((-3*x^5+x)/(x^5+x)^(1/2)/(x^8+2*x^4-a*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3\,x^5-x}{\left(x^8+2\,x^4-ax^2+1\right)\sqrt{x^5+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^5+x)/(x^5+x)^(1/2)/(x^8+2*x^4-a*x^2+1),x, algorithm="maxima
")
```

```
[Out] -integrate((3*x^5 - x)/((x^8 + 2*x^4 - a*x^2 + 1)*sqrt(x^5 + x)), x)
```

mupad [B] time = 4.76, size = 217, normalized size = 3.62

$$\frac{\ln\left(\frac{512\,\sqrt{x^5+x}\,{\left(a^3\right)}^{7/4}+256\,a^5-27\,a^7+256\,a^5\,x^4-27\,x\,{\left(a^3\right)}^{5/2}-27\,a^7\,x^4-54\,a^5\,\sqrt{x^5+x}\,{\left(a^3\right)}^{3/4}+256\,a^4\,x\,\sqrt{a^3}}{a+a\,x^4-x\,\sqrt{a^3}}\right)}{2\,{\left(a^3\right)}^{1/4}}+\frac{\ln\left(\frac{54\,a^6\,\sqrt{x^5+x}\,{\left(a^3\right)}^{3/4}-512\,a\,\sqrt{x^5+x}\,{\left(a^3\right)}^{7/4}+a^6\,256\mathrm{i}-a^8\,27\mathrm{i}+a^6\,x^4\,256\mathrm{i}-a^8\,x^4\,27\mathrm{i}-a^5\,x\,\sqrt{a^3}\,256\mathrm{i}+a^7\,x\,\sqrt{a^3}\,27\mathrm{i}}{a+a\,x^4+x\,\sqrt{a^3}}\right)\,1\mathrm{i}}{2\,{\left(a^3\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3*x^5)/((x + x^5)^(1/2)*(2*x^4 - a*x^2 + x^8 + 1)),x)
```

```
[Out] log((512*(x + x^5)^(1/2)*(a^3)^(7/4) + 256*a^5 - 27*a^7 + 256*a^5*x^4 - 27*
x*(a^3)^(5/2) - 27*a^7*x^4 - 54*a^5*(x + x^5)^(1/2)*(a^3)^(3/4) + 256*a^4*x
*(a^3)^(1/2))/(a + a*x^4 - x*(a^3)^(1/2)))/(2*(a^3)^(1/4)) + (log((a^6*256i
 - a^8*27i + a^6*x^4*256i - a^8*x^4*27i + 54*a^6*(x + x^5)^(1/2)*(a^3)^(3/4
) - a^5*x*(a^3)^(1/2)*256i + a^7*x*(a^3)^(1/2)*27i - 512*a*(x + x^5)^(1/2)*
(a^3)^(7/4))/(a + a*x^4 + x*(a^3)^(1/2)))*1i)/(2*(a^3)^(1/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**5+x)/(x**5+x)**(1/2)/(x**8+2*x**4-a*x**2+1),x)
```

```
[Out] Timed out
```

3.761 $\int \frac{\sqrt{1-2x^8}\left(-1+2x^8\right)\left(1+2x^8\right)}{x^7\left(-1+x^4+2x^8\right)}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{1-2x^8}\left(2x^8-3x^4-1\right)}{6x^6}-\frac{1}{2}\tanh^{-1}\left(\frac{x^2\sqrt{1-2x^8}}{2x^8-1}\right)$$

Rubi [C] time = 0.89, antiderivative size = 335, normalized size of antiderivative = 5.58, number of steps used = 47, number of rules used = 17, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.415, Rules used = {21, 6728, 275, 277, 195, 221, 279, 307, 1181, 424, 1491, 1209, 1177, 524, 248, 1213, 537}

$$\frac{1}{15}\sqrt[4]{2}\left(3+5\sqrt{2}\right)F\left(\sin^{-1}\left(\sqrt[4]{2}x^2\right)\middle|-1\right)-\frac{\left(1+\sqrt{2}\right)F\left(\sin^{-1}\left(\sqrt[4]{2}x^2\right)\middle|-1\right)}{2^{3/4}}+\frac{\left(1-\sqrt{2}\right)F\left(\sin^{-1}\left(\sqrt[4]{2}x^2\right)\middle|-1\right)}{2\sqrt[4]{2}}+\frac{1}{15}2^{3/4}\left(5-3\sqrt{2}\right)F\left(\sin^{-1}\left(\sqrt[4]{2}x^2\right)\middle|-1\right)-\frac{2}{3}2^{3/4}F\left(\sin^{-1}\left(\sqrt[4]{2}x^2\right)\middle|-1\right)+\frac{6}{5}\sqrt[4]{2}F\left(\sin^{-1}\left(\sqrt[4]{2}x^2\right)\middle|-1\right)+\frac{\Pi\left(-\frac{1}{\sqrt{2}};\sin^{-1}\left(\sqrt[4]{2}x^2\right)\middle|-1\right)}{2\sqrt[4]{2}}+\frac{\Pi\left(\sqrt{2};\sin^{-1}\left(\sqrt[4]{2}x^2\right)\middle|-1\right)}{2\sqrt[4]{2}}-\frac{6}{5}\sqrt{1-2x^8}x^6-\frac{\left(1-2x^8\right)^{3/2}}{6x^6}-\frac{2}{3}\sqrt{1-2x^8}x^2-\frac{\left(1-2x^8\right)^{3/2}}{2x^2}+\frac{1}{15}\left(5-3x^4\right)\sqrt{1-2x^8}x^2+\frac{1}{15}\left(6x^4+5\right)\sqrt{1-2x^8}x^2$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - 2*x^8]*(-1 + 2*x^8)*(1 + 2*x^8))/(x^7*(-1 + x^4 + 2*x^8)),x]
```

```
[Out] (-2*x^2*Sqrt[1 - 2*x^8])/3 - (6*x^6*Sqrt[1 - 2*x^8])/5 + (x^2*(5 - 3*x^4)*S
qrt[1 - 2*x^8])/15 + (x^2*(5 + 6*x^4)*Sqrt[1 - 2*x^8])/15 - (1 - 2*x^8)^(3/
2)/(6*x^6) - (1 - 2*x^8)^(3/2)/(2*x^2) + (6*2^(1/4)*EllipticF[ArcSin[2^(1/4
)*x^2], -1])/5 - (2*2^(3/4)*EllipticF[ArcSin[2^(1/4)*x^2], -1])/3 + (2^(3/4
)*(5 - 3*Sqrt[2])*EllipticF[ArcSin[2^(1/4)*x^2], -1])/15 + ((1 - Sqrt[2])*E
llipticF[ArcSin[2^(1/4)*x^2], -1])/(2*2^(1/4)) - ((1 + Sqrt[2])*EllipticF[A
rcSin[2^(1/4)*x^2], -1])/2^(3/4) + (2^(1/4)*(3 + 5*Sqrt[2])*EllipticF[ArcSi
n[2^(1/4)*x^2], -1])/15 + EllipticPi[-(1/Sqrt[2]), ArcSin[2^(1/4)*x^2], -1]
/(2*2^(1/4)) + EllipticPi[Sqrt[2], ArcSin[2^(1/4)*x^2], -1]/(2*2^(1/4))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 248

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 1491

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k
 - 1)*(d + e*x^(n/k))^q*(a + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /
; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-2x^8}\left(-1+2x^8\right)\left(1+2x^8\right)}{x^7\left(-1+x^4+2x^8\right)}\,dx = -\int \frac{\left(1-2x^8\right)^{3/2}\left(1+2x^8\right)}{x^7\left(-1+x^4+2x^8\right)}\,dx$$

$$= -\int \left(-\frac{\left(1-2x^8\right)^{3/2}}{x^7} - \frac{\left(1-2x^8\right)^{3/2}}{x^3} - \frac{x\left(1-2x^8\right)^{3/2}}{1+x^4} + \frac{4x\left(1-2x^8\right)^{3/2}}{-1+2x^4}\right)$$

$$= -\left(4\int \frac{x\left(1-2x^8\right)^{3/2}}{-1+2x^4}\,dx\right) + \int \frac{\left(1-2x^8\right)^{3/2}}{x^7}\,dx + \int \frac{\left(1-2x^8\right)^{3/2}}{x^3}\,dx +$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{\left(1-2x^4\right)^{3/2}}{x^4}\,dx, x, x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{\left(1-2x^4\right)^{3/2}}{x^2}\,dx, x, x^2\right.$$

$$= -\frac{\left(1-2x^8\right)^{3/2}}{6x^6} - \frac{\left(1-2x^8\right)^{3/2}}{2x^2} - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-2x^4}}{1+x^2}\,dx, x, x^2\right) - \frac{1}{2}\text{Su}$$

$$= -\frac{2}{3}x^2\sqrt{1-2x^8} - \frac{6}{5}x^6\sqrt{1-2x^8} + \frac{1}{15}x^2\left(5-3x^4\right)\sqrt{1-2x^8} + \frac{1}{15}x^2\left(5\right.$$

$$= -\frac{2}{3}x^2\sqrt{1-2x^8} - \frac{6}{5}x^6\sqrt{1-2x^8} + \frac{1}{15}x^2\left(5-3x^4\right)\sqrt{1-2x^8} + \frac{1}{15}x^2\left(5\right.$$

$$= -\frac{2}{3}x^2\sqrt{1-2x^8} - \frac{6}{5}x^6\sqrt{1-2x^8} + \frac{1}{15}x^2\left(5-3x^4\right)\sqrt{1-2x^8} + \frac{1}{15}x^2\left(5\right.$$

$$= -\frac{2}{3}x^2\sqrt{1-2x^8} - \frac{6}{5}x^6\sqrt{1-2x^8} + \frac{1}{15}x^2\left(5-3x^4\right)\sqrt{1-2x^8} + \frac{1}{15}x^2\left(5\right.$$

$$= -\frac{2}{3}x^2\sqrt{1-2x^8} - \frac{6}{5}x^6\sqrt{1-2x^8} + \frac{1}{15}x^2\left(5-3x^4\right)\sqrt{1-2x^8} + \frac{1}{15}x^2\left(5\right.$$

Mathematica [C] time = 0.30, size = 149, normalized size = 2.48

$$-\frac{6\sqrt{1-2x^8}\,x^8\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};2x^8\right)+8x^{16}-12x^{12}-8x^8+6x^4-3\,2^{3/4}\sqrt{1-2x^8}\,x^6\,\Pi\left(-\frac{1}{\sqrt{2}};\sin^{-1}\left(\sqrt[4]{2}x^2\right)\middle|-1\right)-3\,2^{3/4}\sqrt{1-2x^8}\,x^6\,\Pi\left(\sqrt{2};\sin^{-1}\left(\sqrt[4]{2}x^2\right)\middle|-1\right)+2}{12x^6\sqrt{1-2x^8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - 2*x^8]*(-1 + 2*x^8)*(1 + 2*x^8))/(x^7*(-1 + x^4 + 2*x^8
)),x]
```

```
[Out] -1/12*(2 + 6*x^4 - 8*x^8 - 12*x^12 + 8*x^16 - 3*2^(3/4)*x^6*Sqrt[1 - 2*x^8]
*EllipticPi[-(1/Sqrt[2]), ArcSin[2^(1/4)*x^2], -1] - 3*2^(3/4)*x^6*Sqrt[1 -
 2*x^8]*EllipticPi[Sqrt[2], ArcSin[2^(1/4)*x^2], -1] + 6*x^8*Sqrt[1 - 2*x^8
]*Hypergeometric2F1[1/4, 1/2, 5/4, 2*x^8])/(x^6*Sqrt[1 - 2*x^8])
```

IntegrateAlgebraic [A] time = 22.25, size = 60, normalized size = 1.00

$$\frac{\sqrt{1-2x^8}\left(2x^8-3x^4-1\right)}{6x^6} - \frac{1}{2}\tanh^{-1}\left(\frac{x^2\sqrt{1-2x^8}}{2x^8-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 - 2*x^8]*(-1 + 2*x^8)*(1 + 2*x^8))/(x^7*(-1 + x^
4 + 2*x^8)),x]
```

```
[Out] (Sqrt[1 - 2*x^8]*(-1 - 3*x^4 + 2*x^8))/(6*x^6) - ArcTanh[(x^2*Sqrt[1 - 2*x^
8])/(-1 + 2*x^8)]/2
```

fricas [A] time = 0.47, size = 75, normalized size = 1.25

$$\frac{3\,x^{6} \log\left(-\frac{2\,x^{8}-x^{4}-2\,\sqrt{-2\,x^{8}+1}\,x^{2}-1}{2\,x^{8}+x^{4}-1}\right) + 2\left(2\,x^{8}-3\,x^{4}-1\right)\sqrt{-2\,x^{8}+1}}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^8+1)^(1/2)*(2*x^8-1)*(2*x^8+1)/x^7/(2*x^8+x^4-1),x, algorit
hm="fricas")
```

```
[Out] 1/12*(3*x^6*log(-(2*x^8 - x^4 - 2*sqrt(-2*x^8 + 1)*x^2 - 1)/(2*x^8 + x^4 -
1)) + 2*(2*x^8 - 3*x^4 - 1)*sqrt(-2*x^8 + 1))/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{8}+1\right)\left(2\,x^{8}-1\right)\sqrt{-2\,x^{8}+1}}{\left(2\,x^{8}+x^{4}-1\right)x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^8+1)^(1/2)*(2*x^8-1)*(2*x^8+1)/x^7/(2*x^8+x^4-1),x, algorit
hm="giac")
```

```
[Out] integrate((2*x^8 + 1)*(2*x^8 - 1)*sqrt(-2*x^8 + 1)/((2*x^8 + x^4 - 1)*x^7),
 x)
```

maple [A] time = 0.54, size = 85, normalized size = 1.42

$$-\frac{4x^{16}-6x^{12}-4x^{8}+3x^{4}+1}{6x^{6}\sqrt{-2x^{8}+1}} - \frac{\ln\left(-\frac{2x^{8}-x^{4}+2\sqrt{-2x^{8}+1}\,x^{2}-1}{\left(2x^{4}-1\right)\left(x^{4}+1\right)}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^8+1)^(1/2)*(2*x^8-1)*(2*x^8+1)/x^7/(2*x^8+x^4-1),x)
```

```
[Out] -1/6*(4*x^16-6*x^12-4*x^8+3*x^4+1)/x^6/(-2*x^8+1)^(1/2)-1/4*ln(-(2*x^8-x^4+
2*(-2*x^8+1)^(1/2)*x^2-1)/(2*x^4-1)/(x^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{8}+1\right)\left(2\,x^{8}-1\right)\sqrt{-2\,x^{8}+1}}{\left(2\,x^{8}+x^{4}-1\right)x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^8+1)^(1/2)*(2*x^8-1)*(2*x^8+1)/x^7/(2*x^8+x^4-1),x, algorit
hm="maxima")
```

```
[Out] integrate((2*x^8 + 1)*(2*x^8 - 1)*sqrt(-2*x^8 + 1)/((2*x^8 + x^4 - 1)*x^7),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{\left(1-2\,x^{8}\right)^{3/2}\,\left(2\,x^{8}+1\right)}{x^{7}\,\left(2\,x^{8}+x^{4}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1 - 2*x^8)^(3/2)*(2*x^8 + 1))/(x^7*(x^4 + 2*x^8 - 1)),x)
```

```
[Out] int(-((1 - 2*x^8)^(3/2)*(2*x^8 + 1))/(x^7*(x^4 + 2*x^8 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-2x^8}\left(2x^8-1\right)\left(2x^8+1\right)}{x^7\left(x^4+1\right)\left(2x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**8+1)**(1/2)*(2*x**8-1)*(2*x**8+1)/x**7/(2*x**8+x**4-1),x)
```

```
[Out] Integral(sqrt(1 - 2*x**8)*(2*x**8 - 1)*(2*x**8 + 1)/(x**7*(x**4 + 1)*(2*x**
4 - 1)), x)
```

3.762 $\int \frac{-x+3x^5}{\sqrt{x+x^5}\left(a-x^2+2ax^4+ax^8\right)}\,dx$

Optimal. Leaf size=60

$$\frac{\tan^{-1}\left(\frac{\sqrt{x^5+x}}{\sqrt[4]{a}\left(x^4+1\right)}\right)}{\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^5+x}}{\sqrt[4]{a}\left(x^4+1\right)}\right)}{\sqrt[4]{a}}$$

Rubi [F] time = 1.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-x+3x^5}{\sqrt{x+x^5}\left(a-x^2+2ax^4+ax^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-x + 3*x^5)/(Sqrt[x + x^5]*(a - x^2 + 2*a*x^4 + a*x^8)),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[1 + x^4]*Defer[Subst][Defer[Int][x^2/(Sqrt[1 + x^8]*(a - x
^4 + 2*a*x^8 + a*x^16)), x], x, Sqrt[x]])/Sqrt[x + x^5] + (6*Sqrt[x]*Sqrt[1
 + x^4]*Defer[Subst][Defer[Int][x^10/(Sqrt[1 + x^8]*(a - x^4 + 2*a*x^8 + a*
x^16)), x], x, Sqrt[x]])/Sqrt[x + x^5]
```

Rubi steps

$$\begin{aligned}
\int \frac{-x+3x^5}{\sqrt{x+x^5}\left(a-x^2+2ax^4+ax^8\right)}\,dx &= \int \frac{x\left(-1+3x^4\right)}{\sqrt{x+x^5}\left(a-x^2+2ax^4+ax^8\right)}\,dx\\
&= \frac{\left(\sqrt{x}\sqrt{1+x^4}\right)\int \frac{\sqrt{x}\left(-1+3x^4\right)}{\sqrt{1+x^4}\left(a-x^2+2ax^4+ax^8\right)}\,dx}{\sqrt{x+x^5}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \frac{x^2\left(-1+3x^8\right)}{\sqrt{1+x^8}\left(a-x^4+2ax^8+ax^{16}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^5}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \left(-\frac{x^2}{\sqrt{1+x^8}\left(a-x^4+2ax^8+ax^{16}\right)}+\frac{3x^{10}}{\sqrt{1+x^8}\left(a-x^4+2ax^8+ax^{16}\right)}\right.\right.}{\sqrt{x+x^5}}\\
&= -\frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^8}\left(a-x^4+2ax^8+ax^{16}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^5}}+\frac{\left(6\sqrt{x}\sqrt{1+\right.}{}
\end{aligned}$$

Mathematica [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{-x+3x^5}{\sqrt{x+x^5}\left(a-x^2+2ax^4+ax^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-x + 3*x^5)/(Sqrt[x + x^5]*(a - x^2 + 2*a*x^4 + a*x^8)),x]
```

```
[Out] Integrate[(-x + 3*x^5)/(Sqrt[x + x^5]*(a - x^2 + 2*a*x^4 + a*x^8)), x]
```

IntegrateAlgebraic [A] time = 2.66, size = 60, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{x^5+x}}{\sqrt[4]{a}\left(x^4+1\right)}\right)}{\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^5+x}}{\sqrt[4]{a}\left(x^4+1\right)}\right)}{\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x + 3*x^5)/(Sqrt[x + x^5]*(a - x^2 + 2*a*x^4 + a*x^8))
,x]
```

```
[Out] ArcTan[Sqrt[x + x^5]/(a^(1/4)*(1 + x^4))]/a^(1/4) - ArcTanh[Sqrt[x + x^5]/(
a^(1/4)*(1 + x^4))]/a^(1/4)
```

fricas [B] time = 0.52, size = 190, normalized size = 3.17

$$\frac{\arctan\left(\frac{\sqrt{x^5+x}}{\left(x^4+1\right)a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}-\frac{\log\left(\frac{ax^8+2\,ax^4+x^2+2\,\sqrt{x^5+x}\left(a^{\frac{1}{4}}x+\frac{ax^4+a}{a^{\frac{1}{4}}}\right)+a+\frac{2\left(ax^5+ax\right)}{\sqrt{a}}}{ax^8+2\,ax^4-x^2+a}\right)}{4\,a^{\frac{1}{4}}}+\frac{\log\left(\frac{ax^8+2\,ax^4+x^2-2\,\sqrt{x^5+x}\left(a^{\frac{1}{4}}x+\frac{ax^4+a}{a^{\frac{1}{4}}}\right)+a+\frac{2\left(ax^5+ax\right)}{\sqrt{a}}}{ax^8+2\,ax^4-x^2+a}\right)}{4\,a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-x)/(x^5+x)^(1/2)/(a*x^8+2*a*x^4-x^2+a),x, algorithm="frica
s")
```

```
[Out] arctan(sqrt(x^5 + x)/((x^4 + 1)*a^(1/4)))/a^(1/4) - 1/4*log((a*x^8 + 2*a*x^
4 + x^2 + 2*sqrt(x^5 + x)*(a^(1/4)*x + (a*x^4 + a)/a^(1/4)) + a + 2*(a*x^5
+ a*x)/sqrt(a))/(a*x^8 + 2*a*x^4 - x^2 + a))/a^(1/4) + 1/4*log((a*x^8 + 2*a
*x^4 + x^2 - 2*sqrt(x^5 + x)*(a^(1/4)*x + (a*x^4 + a)/a^(1/4)) + a + 2*(a*x
^5 + a*x)/sqrt(a))/(a*x^8 + 2*a*x^4 - x^2 + a))/a^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^5-x}{\left(ax^8+2\,ax^4-x^2+a\right)\sqrt{x^5+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-x)/(x^5+x)^(1/2)/(a*x^8+2*a*x^4-x^2+a),x, algorithm="giac"
)
```

```
[Out] integrate((3*x^5 - x)/((a*x^8 + 2*a*x^4 - x^2 + a)*sqrt(x^5 + x)), x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{3x^5-x}{\sqrt{x^5+x}\,\left(a\,x^8+2a\,x^4-x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^5-x)/(x^5+x)^(1/2)/(a*x^8+2*a*x^4-x^2+a),x)
```

```
[Out] int((3*x^5-x)/(x^5+x)^(1/2)/(a*x^8+2*a*x^4-x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^5-x}{\left(ax^8+2\,ax^4-x^2+a\right)\sqrt{x^5+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-x)/(x^5+x)^(1/2)/(a*x^8+2*a*x^4-x^2+a),x, algorithm="maxim
a")
```

```
[Out] integrate((3*x^5 - x)/((a*x^8 + 2*a*x^4 - x^2 + a)*sqrt(x^5 + x)), x)
```

mupad [B] time = 3.43, size = 98, normalized size = 1.63

$$\frac{\ln\left(\frac{x-2\,a^{1/4}\,\sqrt{x^5+x}+\sqrt{a}+\sqrt{a}\,x^4}{\sqrt{a}-x+\sqrt{a}\,x^4}\right)}{2\,a^{1/4}}+\frac{\ln\left(\frac{x-\sqrt{a}-\sqrt{a}\,x^4+a^{1/4}\,\sqrt{x^5+x}\,2\mathrm{i}}{x+\sqrt{a}+\sqrt{a}\,x^4}\right)\,1\mathrm{i}}{2\,a^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 3*x^5)/((x + x^5)^(1/2)*(a + 2*a*x^4 + a*x^8 - x^2)),x)
```

```
[Out] log((x - 2*a^(1/4)*(x + x^5)^(1/2) + a^(1/2) + a^(1/2)*x^4)/(a^(1/2) - x +
a^(1/2)*x^4))/(2*a^(1/4)) + (log((x + a^(1/4)*(x + x^5)^(1/2)*2i - a^(1/2)
- a^(1/2)*x^4)/(x + a^(1/2) + a^(1/2)*x^4))*1i)/(2*a^(1/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**5-x)/(x**5+x)**(1/2)/(a*x**8+2*a*x**4-x**2+a),x)
```

```
[Out] Timed out
```

3.763 $\int \frac{x^5(7b+9ax^2)}{\sqrt[4]{bx^3+ax^5}\,(-2+bx^7+ax^9)}\,dx$

Optimal. Leaf size=60

$$2^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{ax^5+bx^3}}{\sqrt[4]{2}}\right)-2^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{ax^5+bx^3}}{\sqrt[4]{2}}\right)$$

Rubi [F] time = 2.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^5\left(7b+9ax^2\right)}{\sqrt[4]{bx^3+ax^5}\left(-2+bx^7+ax^9\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^5*(7*b + 9*a*x^2))/((b*x^3 + a*x^5)^(1/4)*(-2 + b*x^7 + a*x^9)),x]
```

```
[Out] (28*b*x^(3/4)*(b + a*x^2)^(1/4)*Defer[Subst][Defer[Int][x^20/((b + a*x^8)^(
1/4)*(-2 + b*x^28 + a*x^36)), x], x, x^(1/4)])/(b*x^3 + a*x^5)^(1/4) + (36*
a*x^(3/4)*(b + a*x^2)^(1/4)*Defer[Subst][Defer[Int][x^28/((b + a*x^8)^(1/4)
*(-2 + b*x^28 + a*x^36)), x], x, x^(1/4)])/(b*x^3 + a*x^5)^(1/4)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(7b+9ax^2\right)}{\sqrt[4]{bx^3+ax^5}\left(-2+bx^7+ax^9\right)}\,dx &= \frac{\left(x^{3/4}\sqrt[4]{b+ax^2}\right)\int \frac{x^{17/4}(7b+9ax^2)}{\sqrt[4]{b+ax^2}\,(-2+bx^7+ax^9)}\,dx}{\sqrt[4]{bx^3+ax^5}}\\
&= \frac{\left(4x^{3/4}\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{x^{20}(7b+9ax^8)}{\sqrt[4]{b+ax^8}\,(-2+bx^{28}+ax^{36})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx^3+ax^5}}\\
&= \frac{\left(4x^{3/4}\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \left(\frac{7bx^{20}}{\sqrt[4]{b+ax^8}\,(-2+bx^{28}+ax^{36})}+\frac{9ax^{28}}{\sqrt[4]{b+ax^8}\,(-2+bx^{28}+ax^{36})}\right.\right.}{\sqrt[4]{bx^3+ax^5}}\\
&= \frac{\left(36ax^{3/4}\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{x^{28}}{\sqrt[4]{b+ax^8}\,(-2+bx^{28}+ax^{36})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx^3+ax^5}}+\frac{\left(28bx^{3/}\right.}{}
\end{aligned}$$

Mathematica [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(7b+9ax^2\right)}{\sqrt[4]{bx^3+ax^5}\left(-2+bx^7+ax^9\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^5*(7*b + 9*a*x^2))/((b*x^3 + a*x^5)^(1/4)*(-2 + b*x^7 + a*x^9)
),x]
```

```
[Out] Integrate[(x^5*(7*b + 9*a*x^2))/((b*x^3 + a*x^5)^(1/4)*(-2 + b*x^7 + a*x^9)
), x]
```

IntegrateAlgebraic [A] time = 15.68, size = 60, normalized size = 1.00

$$2^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{ax^5+bx^3}}{\sqrt[4]{2}}\right)-2^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{ax^5+bx^3}}{\sqrt[4]{2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(7*b + 9*a*x^2))/((b*x^3 + a*x^5)^(1/4)*(-2 + b*x^7
 + a*x^9)),x]
```

```
[Out] 2^(3/4)*ArcTan[(x*(b*x^3 + a*x^5)^(1/4))/2^(1/4)] - 2^(3/4)*ArcTanh[(x*(b*x
^3 + a*x^5)^(1/4))/2^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(9*a*x^2+7*b)/(a*x^5+b*x^3)^(1/4)/(a*x^9+b*x^7-2),x, algorith
m="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(9\,ax^2+7\,b\right)x^5}{\left(ax^9+bx^7-2\right)\left(ax^5+bx^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(9*a*x^2+7*b)/(a*x^5+b*x^3)^(1/4)/(a*x^9+b*x^7-2),x, algorith
m="giac")
```

```
[Out] integrate((9*a*x^2 + 7*b)*x^5/((a*x^9 + b*x^7 - 2)*(a*x^5 + b*x^3)^(1/4)),
x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(9a\,x^2+7b\right)}{\left(a\,x^5+b\,x^3\right)^{\frac{1}{4}}\left(a\,x^9+b\,x^7-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(9*a*x^2+7*b)/(a*x^5+b*x^3)^(1/4)/(a*x^9+b*x^7-2),x)
```

```
[Out] int(x^5*(9*a*x^2+7*b)/(a*x^5+b*x^3)^(1/4)/(a*x^9+b*x^7-2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(9\,ax^2+7\,b\right)x^5}{\left(ax^9+bx^7-2\right)\left(ax^5+bx^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(9*a*x^2+7*b)/(a*x^5+b*x^3)^(1/4)/(a*x^9+b*x^7-2),x, algorith
m="maxima")
```

```
[Out] integrate((9*a*x^2 + 7*b)*x^5/((a*x^9 + b*x^7 - 2)*(a*x^5 + b*x^3)^(1/4)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^5\,\left(9\,a\,x^2+7\,b\right)}{{\left(a\,x^5+b\,x^3\right)}^{1/4}\,\left(a\,x^9+b\,x^7-2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(7*b + 9*a*x^2))/((a*x^5 + b*x^3)^(1/4)*(a*x^9 + b*x^7 - 2)),x)
```

```
[Out] int((x^5*(7*b + 9*a*x^2))/((a*x^5 + b*x^3)^(1/4)*(a*x^9 + b*x^7 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \left(9 a x^{2} + 7 b\right)}{\sqrt[4]{x^{3} \left(a x^{2} + b\right)} \left(a x^{9} + b x^{7} - 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(9*a*x**2+7*b)/(a*x**5+b*x**3)**(1/4)/(a*x**9+b*x**7-2),x)
```

```
[Out] Integral(x**5*(9*a*x**2 + 7*b)/((x**3*(a*x**2 + b))**(1/4)*(a*x**9 + b*x**7
 - 2)), x)
```

3.764

$$\int \frac{x^2}{(1+x^2)\sqrt[5]{243-5265x+47250x^2-225810x^3+615255x^4-954733x^5+820340x^6-401440x^7+112000x}}$$

Optimal. Leaf size=60

$$\frac{(x-3)(4x-1)\left(-\frac{13}{340}\log\left(x^2+1\right)+\frac{9}{110}\log(x-3)-\frac{1}{187}\log(4x-1)+\frac{1}{170}\tan^{-1}(x)\right)}{\sqrt[5]{\left(4x^2-13x+3\right)^5}}$$

Rubi [B] time = 0.95, antiderivative size = 141, normalized size of antiderivative = 2.35, number of steps used = 9, number of rules used = 8, integrand size = 65, $\frac{\text{number of rules}}{\text{integrand size}} = 0.123$, Rules used = {6688, 6720, 1075, 632, 31, 635, 203, 260}

$$-\frac{\left(4x^2-13x+3\right)\log(1-4x)}{187\sqrt[5]{\left(4x^2-13x+3\right)^5}}+\frac{9\left(4x^2-13x+3\right)\log(3-x)}{110\sqrt[5]{\left(4x^2-13x+3\right)^5}}-\frac{13\left(4x^2-13x+3\right)\log\left(x^2+1\right)}{340\sqrt[5]{\left(4x^2-13x+3\right)^5}}+\frac{\left(4x^2-13x+3\right)\tan^{-1}(x)}{170\sqrt[5]{\left(4x^2-13x+3\right)^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x^2)*(243 - 5265*x + 47250*x^2 - 225810*x^3 + 615255*x^4 - 95
4733*x^5 + 820340*x^6 - 401440*x^7 + 112000*x^8 - 16640*x^9 + 1024*x^10)^(1
/5)),x]
```

```
[Out] ((3 - 13*x + 4*x^2)*ArcTan[x])/(170*((3 - 13*x + 4*x^2)^5)^(1/5)) - ((3 - 1
3*x + 4*x^2)*Log[1 - 4*x])/(187*((3 - 13*x + 4*x^2)^5)^(1/5)) + (9*(3 - 13*
x + 4*x^2)*Log[3 - x])/(110*((3 - 13*x + 4*x^2)^5)^(1/5)) - (13*(3 - 13*x +
 4*x^2)*Log[1 + x^2])/(340*((3 - 13*x + 4*x^2)^5)^(1/5))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1075

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.
)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}
, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*(-(b*C
*d) + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*
f - a*C*d*f + a*A*f^2 - f*(-(b*C*d) + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{x^2}{(1+x^2)\sqrt[5]{243-5265x+47250x^2-225810x^3+615255x^4-954733x^5+820340x^6-401440x^7+112000x}}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.98

$$\frac{(4x^2-13x+3)\left(-143\log\left(x^2+1\right)-20\log(1-4x)+306\log(3-x)+22\tan^{-1}(x)\right)}{3740\sqrt[5]{(4x^2-13x+3)^5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x^2)*(243 - 5265*x + 47250*x^2 - 225810*x^3 + 615255*x^
4 - 954733*x^5 + 820340*x^6 - 401440*x^7 + 112000*x^8 - 16640*x^9 + 1024*x^
10)^(1/5)),x]
```

```
[Out] ((3 - 13*x + 4*x^2)*(22*ArcTan[x] - 20*Log[1 - 4*x] + 306*Log[3 - x] - 143*
Log[1 + x^2]))/(3740*((3 - 13*x + 4*x^2)^5)^(1/5))
```

IntegrateAlgebraic [A] time = 19.17, size = 60, normalized size = 1.00

$$\frac{(x-3)(4x-1)\left(-\frac{13}{340}\log\left(x^2+1\right)+\frac{9}{110}\log(x-3)-\frac{1}{187}\log(4x-1)+\frac{1}{170}\tan^{-1}(x)\right)}{\sqrt[5]{\left(4x^2-13x+3\right)^5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((1 + x^2)*(243 - 5265*x + 47250*x^2 - 225810*x^3 +
615255*x^4 - 954733*x^5 + 820340*x^6 - 401440*x^7 + 112000*x^8 - 16640*x^9
+ 1024*x^10)^(1/5)),x]
```

```
[Out] ((-3 + x)*(-1 + 4*x)*(ArcTan[x]/170 + (9*Log[-3 + x])/110 - Log[-1 + 4*x]/1
87 - (13*Log[1 + x^2])/340))/((3 - 13*x + 4*x^2)^5)^(1/5)
```

fricas [A] time = 0.41, size = 27, normalized size = 0.45

$$\frac{1}{170}\arctan(x)-\frac{13}{340}\log\left(x^2+1\right)-\frac{1}{187}\log\left(4x-1\right)+\frac{9}{110}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(1024*x^10-16640*x^9+112000*x^8-401440*x^7+820340*x^6
-954733*x^5+615255*x^4-225810*x^3+47250*x^2-5265*x+243)^(1/5),x, algorithm=
"fricas")
```

```
[Out] 1/170*arctan(x) - 13/340*log(x^2 + 1) - 1/187*log(4*x - 1) + 9/110*log(x -
3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(1024\,x^{10}-16640\,x^9+112000\,x^8-401440\,x^7+820340\,x^6-954733\,x^5+615255\,x^4-225810\,x^3+47250\,x^2-5265\,x+243\right)^{\frac{1}{5}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(1024*x^10-16640*x^9+112000*x^8-401440*x^7+820340*x^6
-954733*x^5+615255*x^4-225810*x^3+47250*x^2-5265*x+243)^(1/5),x, algorithm=
"giac")
```

```
[Out] integrate(x^2/((1024*x^10 - 16640*x^9 + 112000*x^8 - 401440*x^7 + 820340*x^
6 - 954733*x^5 + 615255*x^4 - 225810*x^3 + 47250*x^2 - 5265*x + 243)^(1/5)*
(x^2 + 1)), x)
```

maple [C] time = 0.05, size = 130, normalized size = 2.17

$$-\frac{\left(4x^2-13x+3\right)\ln\left(-1+4x\right)}{187\left(\left(4x^2-13x+3\right)^5\right)^{\frac{1}{5}}}+\frac{9\left(4x^2-13x+3\right)\ln\left(-3+x\right)}{110\left(\left(4x^2-13x+3\right)^5\right)^{\frac{1}{5}}}+\frac{\left(-\frac{13}{340}+\frac{i}{340}\right)\left(4x^2-13x+3\right)\ln\left(i+x\right)}{\left(\left(4x^2-13x+3\right)^5\right)^{\frac{1}{5}}}+\frac{\left(-\frac{13}{340}-\frac{i}{340}\right)\left(4x^2-13x+3\right)\ln\left(-i+x\right)}{\left(\left(4x^2-13x+3\right)^5\right)^{\frac{1}{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)/(1024*x^10-16640*x^9+112000*x^8-401440*x^7+820340*x^6-95473
3*x^5+615255*x^4-225810*x^3+47250*x^2-5265*x+243)^(1/5),x)
```

```
[Out] -1/187/((4*x^2-13*x+3)^5)^(1/5)*(4*x^2-13*x+3)*ln(-1+4*x)+9/110/((4*x^2-13*
x+3)^5)^(1/5)*(4*x^2-13*x+3)*ln(-3+x)+(-13/340+1/340*I)/((4*x^2-13*x+3)^5)^
(1/5)*(4*x^2-13*x+3)*ln(I+x)-(13/340+1/340*I)/((4*x^2-13*x+3)^5)^(1/5)*(4*x
^2-13*x+3)*ln(-I+x)
```

maxima [A] time = 0.41, size = 27, normalized size = 0.45

$$\frac{1}{170}\arctan(x)-\frac{13}{340}\log\left(x^2+1\right)-\frac{1}{187}\log\left(4x-1\right)+\frac{9}{110}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(1024*x^10-16640*x^9+112000*x^8-401440*x^7+820340*x^6
-954733*x^5+615255*x^4-225810*x^3+47250*x^2-5265*x+243)^(1/5),x, algorithm=
"maxima")
```

```
[Out] 1/170*arctan(x) - 13/340*log(x^2 + 1) - 1/187*log(4*x - 1) + 9/110*log(x -
3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(x^2+1\right)\left(1024\,x^{10}-16640\,x^9+112000\,x^8-401440\,x^7+820340\,x^6-954733\,x^5+615255\,x^4-225810\,x^3+47250\,x^2-5265\,x+243\right)^{1/5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 1)*(47250*x^2 - 5265*x - 225810*x^3 + 615255*x^4 - 954733*x
^5 + 820340*x^6 - 401440*x^7 + 112000*x^8 - 16640*x^9 + 1024*x^10 + 243)^(1
/5)),x)
```

```
[Out] int(x^2/((x^2 + 1)*(47250*x^2 - 5265*x - 225810*x^3 + 615255*x^4 - 954733*x
^5 + 820340*x^6 - 401440*x^7 + 112000*x^8 - 16640*x^9 + 1024*x^10 + 243)^(1
/5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[5]{(x-3)^5\,(4x-1)^5}\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)/(1024*x**10-16640*x**9+112000*x**8-401440*x**7+8203
40*x**6-954733*x**5+615255*x**4-225810*x**3+47250*x**2-5265*x+243)**(1/5),x
)
```

```
[Out] Integral(x**2/(((x - 3)**5*(4*x - 1)**5)**(1/5)*(x**2 + 1)), x)
```

3.765 $\int \frac{1}{(-1+x)\sqrt[4]{x+x^3}}\,dx$

Optimal. Leaf size=61

$$\frac{\tan^{-1}\left(\frac{2^{3/4}\sqrt[4]{x^3+x}}{x+1}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{2^{3/4}\sqrt[4]{x^3+x}}{x+1}\right)}{2\sqrt[4]{2}}$$

Rubi [C] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.43, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2056, 959, 466, 510}

$$-\frac{4\sqrt[4]{x^2+1}\,x^2F_1\left(\frac{7}{8};1,\frac{1}{4};\frac{15}{8};x^2,-x^2\right)}{7\sqrt[4]{x^3+x}}-\frac{4\sqrt[4]{x^2+1}\,xF_1\left(\frac{3}{8};1,\frac{1}{4};\frac{11}{8};x^2,-x^2\right)}{3\sqrt[4]{x^3+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((-1 + x)*(x + x^3)^(1/4)),x]
```

```
[Out] (-4*x*(1 + x^2)^(1/4)*AppellF1[3/8, 1, 1/4, 11/8, x^2, -x^2])/(3*(x + x^3)^
(1/4)) - (4*x^2*(1 + x^2)^(1/4)*AppellF1[7/8, 1, 1/4, 15/8, x^2, -x^2])/(7*
(x + x^3)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{(-1+x)\sqrt[4]{x+x^3}}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{1+x^2}\right)\int \frac{1}{(-1+x)\sqrt[4]{x}\sqrt[4]{1+x^2}}\,dx}{\sqrt[4]{x+x^3}}$$
$$= -\frac{\left(\sqrt[4]{x}\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt[4]{x}(1-x^2)\sqrt[4]{1+x^2}}\,dx}{\sqrt[4]{x+x^3}} - \frac{\left(\sqrt[4]{x}\sqrt[4]{1+x^2}\right)\int \frac{x^{3/4}}{(1-x^2)\sqrt[4]{1+x^2}}\,dx}{\sqrt[4]{x+x^3}}$$
$$= -\frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{x^2}{(1-x^8)\sqrt[4]{1+x^8}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{x+x^3}} - \frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{\cdots}{(1-x^8 \cdots}\right.}{\sqrt[4]{x+x^3}}$$
$$= -\frac{4x\sqrt[4]{1+x^2}\,F_1\left(\frac{3}{8};1,\frac{1}{4};\frac{11}{8};x^2,-x^2\right)}{3\sqrt[4]{x+x^3}} - \frac{4x^2\sqrt[4]{1+x^2}\,F_1\left(\frac{7}{8};1,\frac{1}{4};\frac{15}{8};x^2,-x^2\right)}{7\sqrt[4]{x+x^3}}$$

Mathematica [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{(-1+x)\sqrt[4]{x+x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((-1 + x)*(x + x^3)^(1/4)),x]
[Out] Integrate[1/((-1 + x)*(x + x^3)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.26, size = 61, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2^{3/4}\sqrt[4]{x^3+x}}{x+1}\right)}{2\sqrt[4]{2}} - \frac{\tanh^{-1}\left(\frac{2^{3/4}\sqrt[4]{x^3+x}}{x+1}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x)*(x + x^3)^(1/4)),x]
[Out] ArcTan[(2^(3/4)*(x + x^3)^(1/4))/(1 + x)]/(2*2^(1/4)) - ArcTanh[(2^(3/4)*(x
 + x^3)^(1/4))/(1 + x)]/(2*2^(1/4))
```

fricas [B] time = 5.84, size = 332, normalized size = 5.44

$$\frac{1}{4}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{4\cdot 2^{\frac{3}{4}}(x^3+3x^2+3x+1)(x^3+x)^{\frac{1}{4}}+16\cdot 2^{\frac{1}{4}}(x^3+x)^{\frac{3}{4}}(x+1)+2^{\frac{3}{4}}\left(4\cdot 2^{\frac{3}{4}}\sqrt{x^3+x}(x^2+2x+1)+2^{\frac{1}{4}}(x^4+12x^3+6x^2+12x+1)\right)}{2(x^4-4x^3+6x^2-4x+1)}\right) - \frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(\frac{2^{\frac{3}{4}}(x^4+12x^3+6x^2+12x+1)+4\sqrt{2}(x^3+3x^2+3x+1)(x^3+x)^{\frac{1}{4}}+8\cdot 2^{\frac{1}{4}}\sqrt{x^3+x}(x^2+2x+1)+16(x^3+x)^{\frac{3}{4}}(x+1)}{x^4-4x^3+6x^2-4x+1}\right) + \frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(-\frac{2^{\frac{3}{4}}(x^4+12x^3+6x^2+12x+1)-4\sqrt{2}(x^3+3x^2+3x+1)(x^3+x)^{\frac{1}{4}}+8\cdot 2^{\frac{1}{4}}\sqrt{x^3+x}(x^2+2x+1)-16(x^3+x)^{\frac{3}{4}}(x+1)}{x^4-4x^3+6x^2-4x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^3+x)^(1/4),x, algorithm="fricas")
[Out] 1/4*2^(3/4)*arctan(1/2*(4*2^(3/4)*(x^3 + 3*x^2 + 3*x + 1)*(x^3 + x)^(1/4) +
 16*2^(1/4)*(x^3 + x)^(3/4)*(x + 1) + 2^(3/4)*(4*2^(3/4)*sqrt(x^3 + x)*(x^2
 + 2*x + 1) + 2^(1/4)*(x^4 + 12*x^3 + 6*x^2 + 12*x + 1)))/(x^4 - 4*x^3 + 6*
x^2 - 4*x + 1)) - 1/16*2^(3/4)*log((2^(3/4)*(x^4 + 12*x^3 + 6*x^2 + 12*x +
1) + 4*sqrt(2)*(x^3 + 3*x^2 + 3*x + 1)*(x^3 + x)^(1/4) + 8*2^(1/4)*sqrt(x^3
 + x)*(x^2 + 2*x + 1) + 16*(x^3 + x)^(3/4)*(x + 1))/(x^4 - 4*x^3 + 6*x^2 -
4*x + 1)) + 1/16*2^(3/4)*log(-(2^(3/4)*(x^4 + 12*x^3 + 6*x^2 + 12*x + 1) -
4*sqrt(2)*(x^3 + 3*x^2 + 3*x + 1)*(x^3 + x)^(1/4) + 8*2^(1/4)*sqrt(x^3 + x)
*(x^2 + 2*x + 1) - 16*(x^3 + x)^(3/4)*(x + 1))/(x^4 - 4*x^3 + 6*x^2 - 4*x +
 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+x\right)^{\frac{1}{4}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^3+x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + x)^(1/4)*(x - 1)), x)
```

maple [C] time = 11.46, size = 513, normalized size = 8.41

$$\frac{\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)\ln\left(\frac{2\sqrt{x^3+x}\,\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)\operatorname{RootOf}\left(\_Z^4-8\right)^2x^2+4\sqrt{x^3+x}\,\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)\operatorname{RootOf}\left(\_Z^4-8\right)^2x-2\left(x^3+x\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4-8\right)^2x^3+2\sqrt{x^3+x}\,\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)\operatorname{RootOf}\left(\_Z^4-8\right)^2-6\left(x^3+x\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4-8\right)^2x^2-\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)x^4+16\left(x^3+x\right)^{\frac{3}{4}}x-6\left(x^3+x\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4-8\right)^2x-12\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)x^3+16\left(x^3+x\right)^{\frac{3}{4}}-2\left(x^3+x\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4-8\right)^2-6\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)x^2-12\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)x-\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)}{\left(-1+x\right)^4}\right)}{8}+\frac{\operatorname{RootOf}\left(\_Z^4-8\right)\ln\left(\frac{-2\sqrt{x^3+x}\,\operatorname{RootOf}\left(\_Z^4-8\right)^3x^2-4\sqrt{x^3+x}\,\operatorname{RootOf}\left(\_Z^4-8\right)^3x+2\left(x^3+x\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4-8\right)^2x^3-2\sqrt{x^3+x}\,\operatorname{RootOf}\left(\_Z^4-8\right)^3+6\left(x^3+x\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4-8\right)^2x^2-\operatorname{RootOf}\left(\_Z^4-8\right)x^4+16\left(x^3+x\right)^{\frac{3}{4}}x+6\left(x^3+x\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4-8\right)^2x-12\operatorname{RootOf}\left(\_Z^4-8\right)x^3+16\left(x^3+x\right)^{\frac{3}{4}}+2\left(x^3+x\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4-8\right)^2-6\operatorname{RootOf}\left(\_Z^4-8\right)x^2-12\operatorname{RootOf}\left(\_Z^4-8\right)x-\operatorname{RootOf}\left(\_Z^4-8\right)}{\left(-1+x\right)^4}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)/(x^3+x)^(1/4),x)
```

```
[Out] 1/8*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln((2*(x^3+x)^(1/2)*RootOf(_Z^2+RootOf(_Z
^4-8)^2)*RootOf(_Z^4-8)^2*x^2+4*(x^3+x)^(1/2)*RootOf(_Z^2+RootOf(_Z^4-8)^2)
*RootOf(_Z^4-8)^2*x-2*(x^3+x)^(1/4)*RootOf(_Z^4-8)^2*x^3+2*(x^3+x)^(1/2)*Ro
otOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2-6*(x^3+x)^(1/4)*RootOf(_Z^4-8)
^2*x^2-RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^4+16*(x^3+x)^(3/4)*x-6*(x^3+x)^(1/4)
*RootOf(_Z^4-8)^2*x-12*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^3+16*(x^3+x)^(3/4)-2
*(x^3+x)^(1/4)*RootOf(_Z^4-8)^2-6*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^2-12*Root
Of(_Z^2+RootOf(_Z^4-8)^2)*x-RootOf(_Z^2+RootOf(_Z^4-8)^2))/(-1+x)^4)+1/8*Ro
otOf(_Z^4-8)*ln((-2*(x^3+x)^(1/2)*RootOf(_Z^4-8)^3*x^2-4*(x^3+x)^(1/2)*Root
Of(_Z^4-8)^3*x+2*(x^3+x)^(1/4)*RootOf(_Z^4-8)^2*x^3-2*(x^3+x)^(1/2)*RootOf(
_Z^4-8)^3+6*(x^3+x)^(1/4)*RootOf(_Z^4-8)^2*x^2-RootOf(_Z^4-8)*x^4+16*(x^3+x
)^(3/4)*x+6*(x^3+x)^(1/4)*RootOf(_Z^4-8)^2*x-12*RootOf(_Z^4-8)*x^3+16*(x^3+
x)^(3/4)+2*(x^3+x)^(1/4)*RootOf(_Z^4-8)^2-6*RootOf(_Z^4-8)*x^2-12*RootOf(_Z
^4-8)*x-RootOf(_Z^4-8))/(-1+x)^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+x\right)^{\frac{1}{4}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^3+x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + x)^(1/4)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(x^3+x\right)}^{1/4}\,\left(x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + x^3)^(1/4)*(x - 1)),x)
```

```
[Out] int(1/((x + x^3)^(1/4)*(x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x\left(x^2+1\right)}\,\left(x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x**3+x)**(1/4),x)
```

```
[Out] Integral(1/((x*(x**2 + 1))**(1/4)*(x - 1)), x)
```

3.766 $\int \frac{-1+x^4}{\sqrt{x+x^3}\left(1+x^4\right)} \, dx$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x^3+x}}{x^2+1}\right)}{\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x^3+x}}{x^2+1}\right)}{\sqrt[4]{2}}$$

Rubi [C] time = 1.09, antiderivative size = 386, normalized size of antiderivative = 6.33, number of steps used = 23, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {2056, 1586, 6715, 6725, 406, 220, 409, 1217, 1707}

$$-\frac{\sqrt{x}\sqrt{x^2+1}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt{x^2+1}}\right)}{\sqrt[4]{2}\sqrt{x^3+x}}-\frac{\sqrt{x}\sqrt{x^2+1}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt{x^2+1}}\right)}{\sqrt[4]{2}\sqrt{x^3+x}}+\frac{i\left(\sqrt{2}+(1+i)\right)\sqrt{x}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{4\sqrt{x^3+x}}+\frac{i\left(\sqrt{2}+(-1+i)\right)\sqrt{x}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{4\sqrt{x^3+x}}+\frac{\left((-1-i)-i\sqrt{2}\right)\sqrt{x}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{4\sqrt{x^3+x}}-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\left(1+(-1)^{3/4}\right)\sqrt{x}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^3+x}}+\frac{\sqrt{x}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)/(Sqrt[x + x^3]*(1 + x^4)),x]
```

```
[Out] -((Sqrt[x]*Sqrt[1 + x^2]*ArcTan[(2^(1/4)*Sqrt[x])/Sqrt[1 + x^2]])/(2^(1/4)*
Sqrt[x + x^3])) - (Sqrt[x]*Sqrt[1 + x^2]*ArcTanh[(2^(1/4)*Sqrt[x])/Sqrt[1 +
 x^2]])/(2^(1/4)*Sqrt[x + x^3]) + (Sqrt[x]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2
]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/Sqrt[x + x^3] - ((1/4 - I/4)*(1 + (-1)
^(3/4))*Sqrt[x]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x
]], 1/2])/Sqrt[x + x^3] + (((-1 - I) - I*Sqrt[2])*Sqrt[x]*(1 + x)*Sqrt[(1 +
 x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/(4*Sqrt[x + x^3]) + ((I
/4)*((-1 + I) + Sqrt[2])*Sqrt[x]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*Elliptic
F[2*ArcTan[Sqrt[x]], 1/2])/Sqrt[x + x^3] + ((I/4)*((1 + I) + Sqrt[2])*Sqrt[
x]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/Sqr
t[x + x^3]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+x^4}{\sqrt{x+x^3}\left(1+x^4\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{-1+x^4}{\sqrt{x}\sqrt{1+x^2}\left(1+x^4\right)}\,dx}{\sqrt{x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{\left(-1+x^2\right)\sqrt{1+x^2}}{\sqrt{x}\left(1+x^4\right)}\,dx}{\sqrt{x+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{1+x^2}\right)\text{Subst}\left(\int \frac{\left(-1+x^4\right)\sqrt{1+x^4}}{1+x^8}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{1+x^2}\right)\text{Subst}\left(\int \left(-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{1+x^4}}{i-x^4}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{1+x^4}}{i+x^4}\right)dx,x,\sqrt{x}\right)}{\sqrt{x+x^3}}$$

$$= -\frac{\left((1+i)\sqrt{x}\sqrt{1+x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{i-x^4}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^3}} + \frac{\left((1-i)\sqrt{x}\sqrt{1+x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+}}{i+}\right.}{\sqrt{x+x^3}}$$

$$= -\frac{\left(2i\sqrt{x}\sqrt{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\left(i-x^4\right)\sqrt{1+x^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^3}} - \frac{\left(2i\sqrt{x}\sqrt{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\left(i+x^4\right)\sqrt{1}}\right.}{\sqrt{x+x^3}}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x+x^3}} - \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\sqrt[4]{-1}x^2\right)\sqrt{1+x^4}}\,dx,x\right.}{\sqrt{x+x^3}}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x+x^3}} - \frac{\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1-\sqrt[4]{-1}\right)\sqrt{x}\sqrt{1+x^2}\right)\text{Subs}}{\sqrt{x+x^3}}$$

$$= -\frac{\sqrt{x}\sqrt{1+x^2}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt{1+x^2}}\right)}{\sqrt[4]{2}\sqrt{x+x^3}} - \frac{\sqrt{x}\sqrt{1+x^2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt{1+x^2}}\right)}{\sqrt[4]{2}\sqrt{x+x^3}} + \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}F\left(2\right.}{\sqrt{x+x^3}}$$

Mathematica [C] time = 0.80, size = 146, normalized size = 2.39

$$\frac{\sqrt[4]{-1}\sqrt{\frac{1}{x^2}+1}\,x^{3/2}\left(-2F\left(i\sinh^{-1}\left(\frac{\sqrt[4]{-1}}{\sqrt{x}}\right)\middle|-1\right)+\Pi\left(-\sqrt[4]{-1};i\sinh^{-1}\left(\frac{\sqrt[4]{-1}}{\sqrt{x}}\right)\middle|-1\right)+\Pi\left(\sqrt[4]{-1};i\sinh^{-1}\left(\frac{\sqrt[4]{-1}}{\sqrt{x}}\right)\middle|-1\right)+\Pi\left(-(-1)^{3/4};i\sinh^{-1}\left(\frac{\sqrt[4]{-1}}{\sqrt{x}}\right)\middle|-1\right)+\Pi\left((-1)^{3/4};i\sinh^{-1}\left(\frac{\sqrt[4]{-1}}{\sqrt{x}}\right)\middle|-1\right)\right)}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)/(Sqrt[x + x^3]*(1 + x^4)),x]
```

```
[Out] ((-1)^(1/4)*Sqrt[1 + x^(-2)]*x^(3/2)*(-2*EllipticF[I*ArcSinh[(-1)^(1/4)/Sqr
t[x]], -1] + EllipticPi[-(-1)^(1/4), I*ArcSinh[(-1)^(1/4)/Sqrt[x]], -1] + E
llipticPi[(-1)^(1/4), I*ArcSinh[(-1)^(1/4)/Sqrt[x]], -1] + EllipticPi[-(-1)
^(3/4), I*ArcSinh[(-1)^(1/4)/Sqrt[x]], -1] + EllipticPi[(-1)^(3/4), I*ArcSi
nh[(-1)^(1/4)/Sqrt[x]], -1]))/Sqrt[x + x^3]
```

IntegrateAlgebraic [A] time = 0.28, size = 61, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x^3+x}}{x^2+1}\right)}{\sqrt[4]{2}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x^3+x}}{x^2+1}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/(Sqrt[x + x^3]*(1 + x^4)),x]
```

```
[Out] -(ArcTan[(2^(1/4)*Sqrt[x + x^3])/(1 + x^2)]/2^(1/4)) - ArcTanh[(2^(1/4)*Sqr
t[x + x^3])/(1 + x^2)]/2^(1/4)
```

fricas [B] time = 0.44, size = 141, normalized size = 2.31

$$-\frac{1}{2}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{1}{4}}\sqrt{x^3+x}}{x^2+1}\right)-\frac{1}{8}\cdot 2^{\frac{3}{4}}\log\left(\frac{x^4+4x^2+2\sqrt{2}\left(x^3+x\right)+2\sqrt{x^3+x}\left(2^{\frac{3}{4}}x+2^{\frac{1}{4}}\left(x^2+1\right)\right)+1}{x^4+1}\right)+\frac{1}{8}\cdot 2^{\frac{3}{4}}\log\left(\frac{x^4+4x^2+2\sqrt{2}\left(x^3+x\right)-2\sqrt{x^3+x}\left(2^{\frac{3}{4}}x+2^{\frac{1}{4}}\left(x^2+1\right)\right)+1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^3+x)^(1/2)/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*2^(3/4)*arctan(2^(1/4)*sqrt(x^3 + x)/(x^2 + 1)) - 1/8*2^(3/4)*log((x^4
 + 4*x^2 + 2*sqrt(2)*(x^3 + x) + 2*sqrt(x^3 + x)*(2^(3/4)*x + 2^(1/4)*(x^2
+ 1)) + 1)/(x^4 + 1)) + 1/8*2^(3/4)*log((x^4 + 4*x^2 + 2*sqrt(2)*(x^3 + x)
- 2*sqrt(x^3 + x)*(2^(3/4)*x + 2^(1/4)*(x^2 + 1)) + 1)/(x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\left(x^4+1\right)\sqrt{x^3+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^3+x)^(1/2)/(x^4+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 1)/((x^4 + 1)*sqrt(x^3 + x)), x)
```

maple [C] time = 0.08, size = 151, normalized size = 2.48

$$\frac{i\sqrt{-i\left(i+x\right)}\,\sqrt{2}\,\sqrt{i\left(-i+x\right)}\,\sqrt{ix}\,\operatorname{EllipticF}\left(\sqrt{-i\left(i+x\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3+x}}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4+1\right)}\frac{\_\alpha\left(\_\alpha^3-i\_\alpha^2-\_\alpha+i\right)\sqrt{-i(i+x)}\,\sqrt{i(-i+x)}\,\sqrt{ix}\,\operatorname{EllipticPi}\left(\sqrt{-i(i+x)},-\frac{1}{2}i\_\alpha^3-\frac{1}{2}\_\alpha^2+\frac{1}{2}i\_\alpha+\frac{1}{2},\frac{\sqrt{2}}{2}\right)}{\sqrt{x(x^2+1)}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/(x^3+x)^(1/2)/(x^4+1),x)
```

```
[Out] I*(-I*(I+x))^(1/2)*2^(1/2)*(I*(-I+x))^(1/2)*(I*x)^(1/2)/(x^3+x)^(1/2)*Ellip
ticF((-I*(I+x))^(1/2),1/2*2^(1/2))+1/4*I*2^(1/2)*sum(_alpha*(I+_alpha^3-I*_
alpha^2-_alpha)*(-I*(I+x))^(1/2)*(I*(-I+x))^(1/2)*(I*x)^(1/2)/(x*(x^2+1))^(
1/2)*EllipticPi((-I*(I+x))^(1/2),-1/2*I*_alpha^3-1/2*_alpha^2+1/2*I*_alpha+
1/2,1/2*2^(1/2)),_alpha=RootOf(_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\left(x^4+1\right)\sqrt{x^3+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^3+x)^(1/2)/(x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 1)/((x^4 + 1)*sqrt(x^3 + x)), x)
```

mupad [B] time = 0.59, size = 234, normalized size = 3.84

$$-\frac{\sqrt{1-x\,1\mathrm{i}}\,\sqrt{1+x\,1\mathrm{i}}\,\sqrt{-x\,1\mathrm{i}}\,\Pi\left(\sqrt{2}\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right);\mathrm{asin}\left(\sqrt{-x\,1\mathrm{i}}\right)\middle|-1\right)\,1\mathrm{i}}{\sqrt{x^3+x}}-\frac{\sqrt{1-x\,1\mathrm{i}}\,\sqrt{1+x\,1\mathrm{i}}\,\sqrt{-x\,1\mathrm{i}}\,\Pi\left(\sqrt{2}\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right);\mathrm{asin}\left(\sqrt{-x\,1\mathrm{i}}\right)\middle|-1\right)\,1\mathrm{i}}{\sqrt{x^3+x}}-\frac{\sqrt{1-x\,1\mathrm{i}}\,\sqrt{1+x\,1\mathrm{i}}\,\sqrt{-x\,1\mathrm{i}}\,\Pi\left(\sqrt{2}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right);\mathrm{asin}\left(\sqrt{-x\,1\mathrm{i}}\right)\middle|-1\right)\,1\mathrm{i}}{\sqrt{x^3+x}}-\frac{\sqrt{1-x\,1\mathrm{i}}\,\sqrt{1+x\,1\mathrm{i}}\,\sqrt{-x\,1\mathrm{i}}\,\Pi\left(\sqrt{2}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right);\mathrm{asin}\left(\sqrt{-x\,1\mathrm{i}}\right)\middle|-1\right)\,1\mathrm{i}}{\sqrt{x^3+x}}+\frac{\sqrt{1-x\,1\mathrm{i}}\,\sqrt{1+x\,1\mathrm{i}}\,\sqrt{-x\,1\mathrm{i}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-x\,1\mathrm{i}}\right)\middle|-1\right)\,2\mathrm{i}}{\sqrt{x^3+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/((x^4 + 1)*(x + x^3)^(1/2)),x)
```

```
[Out] ((1 - x*1i)^(1/2)*(x*1i + 1)^(1/2)*(-x*1i)^(1/2)*ellipticF(asin((-x*1i)^(1/
2)), -1)*2i)/(x + x^3)^(1/2) - ((1 - x*1i)^(1/2)*(x*1i + 1)^(1/2)*(-x*1i)^(
1/2)*ellipticPi(2^(1/2)*(- 1/2 + 1i/2), asin((-x*1i)^(1/2)), -1)*1i)/(x + x
^3)^(1/2) - ((1 - x*1i)^(1/2)*(x*1i + 1)^(1/2)*(-x*1i)^(1/2)*ellipticPi(2^(
1/2)*(1/2 - 1i/2), asin((-x*1i)^(1/2)), -1)*1i)/(x + x^3)^(1/2) - ((1 - x*1
i)^(1/2)*(x*1i + 1)^(1/2)*(-x*1i)^(1/2)*ellipticPi(2^(1/2)*(1/2 + 1i/2), as
in((-x*1i)^(1/2)), -1)*1i)/(x + x^3)^(1/2) - ((1 - x*1i)^(1/2)*(x*1i + 1)^(
1/2)*(-x*1i)^(1/2)*ellipticPi(2^(1/2)*(- 1/2 - 1i/2), asin((-x*1i)^(1/2)),
-1)*1i)/(x + x^3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2+1\right)}{\sqrt{x\left(x^2+1\right)}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/(x**3+x)**(1/2)/(x**4+1),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)/(sqrt(x*(x**2 + 1))*(x**4 + 1)), x)
```

3.767 $\int \frac{1+x^3}{\left(-1+x^3\right)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=61

$$-\frac{4}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}+x^2+x+1}\right)-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}+x^2-2x+1}\right)$$

Rubi [C] time = 1.03, antiderivative size = 384, normalized size of antiderivative = 6.30, number of steps used = 29, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {6725, 220, 2074, 1725, 1211, 1699, 206, 1248, 725, 6728, 1217, 1707}

$$-\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{3\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x^2+1}{\sqrt{2}\sqrt{x^4+1}}\right)}{3\sqrt{2}}+\frac{\left(1-i\sqrt{3}\right)\tanh^{-1}\left(\frac{2-\left(1+i\sqrt{3}\right)x^2}{\sqrt{2\left(1+i\sqrt{3}\right)}\sqrt{x^4+1}}\right)}{3\sqrt{2\left(1+i\sqrt{3}\right)}}+\frac{\left(1+i\sqrt{3}\right)\tanh^{-1}\left(\frac{4+\left(1+i\sqrt{3}\right)^2x^2}{2\sqrt{2\left(1-i\sqrt{3}\right)}\sqrt{x^4+1}}\right)}{3\sqrt{2\left(1-i\sqrt{3}\right)}}-\frac{\left(1+i\sqrt{3}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\sqrt{x^4+1}}-\frac{\left(1-i\sqrt{3}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\sqrt{x^4+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/((-1 + x^3)*Sqrt[1 + x^4]),x]
```

```
[Out] (-2*ArcTan[x/Sqrt[1 + x^4]])/3 - ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(3*Sqrt
[2]) - ArcTanh[(1 + x^2)/(Sqrt[2]*Sqrt[1 + x^4])]/(3*Sqrt[2]) + ((1 - I*Sqr
t[3])*ArcTanh[(2 - (1 + I*Sqrt[3])*x^2)/(Sqrt[2*(1 + I*Sqrt[3])]*Sqrt[1 + x
^4])])/(3*Sqrt[2*(1 + I*Sqrt[3])]) + ((1 + I*Sqrt[3])*ArcTanh[(4 + (1 + I*S
qrt[3])^2*x^2)/(2*Sqrt[2*(1 - I*Sqrt[3])]*Sqrt[1 + x^4])])/(3*Sqrt[2*(1 - I
*Sqrt[3])]) + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x],
 1/2])/(3*Sqrt[1 + x^4]) - ((1 - I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x
^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(6*Sqrt[1 + x^4]) - ((1 + I*Sqrt[3])*(1
 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(6*Sqrt[1
+ x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
```

```
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[d,
 Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Dist[e, Int[x/((d^2 - e^
2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1+x^3}{\left(-1+x^3\right)\sqrt{1+x^4}}\,dx = \int \left(\frac{1}{\sqrt{1+x^4}} + \frac{2}{\left(-1+x^3\right)\sqrt{1+x^4}}\right)dx$$

$$= 2\int \frac{1}{\left(-1+x^3\right)\sqrt{1+x^4}}\,dx + \int \frac{1}{\sqrt{1+x^4}}\,dx$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} + 2\int \left(\frac{1}{3(-1+x)\sqrt{1+x^4}} + \frac{-2-x}{3\left(1+x+x^2\right)\sqrt{1}}\right.$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} + \frac{2}{3}\int \frac{1}{(-1+x)\sqrt{1+x^4}}\,dx + \frac{2}{3}\int \frac{-2-}{\left(1+x+x^2\right.}$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} - \frac{2}{3}\int \frac{1}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx - \frac{2}{3}\int \frac{x}{\left(1-x^2\right)\sqrt{1}}$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} - \frac{1}{3}\int \frac{1}{\sqrt{1+x^4}}\,dx - \frac{1}{3}\int \frac{1+x^2}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{1+x^4}} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right) + \frac{1}{3}\text{Sul}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{3\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{1+x^2}{\sqrt{2}\sqrt{1+x^4}}\right)}{3\sqrt{2}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{1+x^4}} + \frac{1}{3}$$

$$= -\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt{1+x^4}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{3\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{1+x^2}{\sqrt{2}\sqrt{1+x^4}}\right)}{3\sqrt{2}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}}{3\sqrt{1}}$$

$$= -\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt{1+x^4}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{3\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{1+x^2}{\sqrt{2}\sqrt{1+x^4}}\right)}{3\sqrt{2}} + \frac{\left(1-i\sqrt{3}\right)\tanh^{-1}}{3\sqrt{2\left(1\right.}}$$

Mathematica [C] time = 0.65, size = 525, normalized size = 8.61

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^3)/((-1 + x^3)*Sqrt[1 + x^4]),x]
```

```
[Out] (-2*Sqrt[1 - (-1)^(1/3)]*ArcTanh[((-1)^(1/3) - x^2)/(Sqrt[1 + (-1)^(2/3)]*S
qrt[1 + x^4])] + 2*(-1)^(1/3)*Sqrt[1 - (-1)^(1/3)]*ArcTanh[((-1)^(1/3) - x^
2)/(Sqrt[1 + (-1)^(2/3)]*Sqrt[1 + x^4])] + 4*(-1)^(2/3)*Sqrt[1 - (-1)^(1/3)
]*ArcTanh[((-1)^(1/3) - x^2)/(Sqrt[1 + (-1)^(2/3)]*Sqrt[1 + x^4])] - 2*Sqrt
[2]*ArcTanh[(1 + x^2)/(Sqrt[2]*Sqrt[1 + x^4])] - (-1)^(1/3)*Sqrt[2]*ArcTanh
[(1 + x^2)/(Sqrt[2]*Sqrt[1 + x^4])] + (-1)^(2/3)*Sqrt[2]*ArcTanh[(1 + x^2)/
```

```
(Sqrt[2]*Sqrt[1 + x^4])] + 2*Sqrt[1 + (-1)^(2/3)]*ArcTanh[((-1)^(2/3) + x^2
)/(Sqrt[1 - (-1)^(1/3)]*Sqrt[1 + x^4])] + 4*(-1)^(1/3)*Sqrt[1 + (-1)^(2/3)]
*ArcTanh[((-1)^(2/3) + x^2)/(Sqrt[1 - (-1)^(1/3)]*Sqrt[1 + x^4])] + 2*(-1)^
(2/3)*Sqrt[1 + (-1)^(2/3)]*ArcTanh[((-1)^(2/3) + x^2)/(Sqrt[1 - (-1)^(1/3)]
*Sqrt[1 + x^4])] - 18*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] + 1
2*(-1)^(1/4)*EllipticPi[I, ArcSin[(-1)^(3/4)*x], -1] + 12*(-1)^(7/12)*Ellip
ticPi[-(-1)^(1/6), I*ArcSinh[(-1)^(1/4)*x], -1] - 12*(-1)^(11/12)*EllipticP
i[-(-1)^(1/6), I*ArcSinh[(-1)^(1/4)*x], -1] + 12*(-1)^(1/4)*EllipticPi[-(-1
)^(5/6), I*ArcSinh[(-1)^(1/4)*x], -1])/18
```

IntegrateAlgebraic [A] time = 1.08, size = 61, normalized size = 1.00

$$-\frac{4}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}+x^2+x+1}\right)-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}+x^2-2x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)/((-1 + x^3)*Sqrt[1 + x^4]),x]
```

```
[Out] (-4*ArcTan[x/(1 + x + x^2 + Sqrt[1 + x^4])])/3 - (Sqrt[2]*ArcTanh[(Sqrt[2]*
x)/(1 - 2*x + x^2 + Sqrt[1 + x^4])])/3
```

fricas [A] time = 0.46, size = 90, normalized size = 1.48

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{3x^4-4x^3-2\sqrt{2}\sqrt{x^4+1}\left(x^2-x+1\right)+6x^2-4x+3}{x^4-4x^3+6x^2-4x+1}\right)+\frac{2}{3}\arctan\left(\frac{\sqrt{x^4+1}}{x^2+2x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*log(-(3*x^4 - 4*x^3 - 2*sqrt(2)*sqrt(x^4 + 1)*(x^2 - x + 1) +
6*x^2 - 4*x + 3)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)) + 2/3*arctan(sqrt(x^4 + 1
)/(x^2 + 2*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3+1}{\sqrt{x^4+1}\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-1)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)/(sqrt(x^4 + 1)*(x^3 - 1)), x)
```

maple [C] time = 0.03, size = 358, normalized size = 5.87

$$\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4\sqrt{x^4+1}}\right)}{6}+\frac{2\left(-1\right)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left(\left(-1\right)^{\frac{1}{4}}x,-i,-\sqrt{-i}\,\left(-1\right)^{\frac{3}{4}}\right)}{3\sqrt{x^4+1}}+\frac{2\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{\mathrm{arctanh}\left(\frac{\sqrt{\frac{1}{2}+\frac{i\sqrt{3}}{2}}\left(x^2-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{\sqrt{x^4+1}}\right)}{2\sqrt{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}+\frac{\left(-1\right)^{\frac{3}{4}}\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left(\left(-1\right)^{\frac{1}{4}}x,-i\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right),i\right)}{\sqrt{x^4+1}}\right)}{3}+\frac{2\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(\frac{\mathrm{arctanh}\left(\frac{\sqrt{\frac{1}{2}-\frac{i\sqrt{3}}{2}}\left(x^2-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\sqrt{x^4+1}}\right)}{2\sqrt{\frac{1}{2}-\frac{i\sqrt{3}}{2}}}+\frac{\left(-1\right)^{\frac{3}{4}}\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left(\left(-1\right)^{\frac{1}{4}}x,-i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right),i\right)}{\sqrt{x^4+1}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^3-1)/(x^4+1)^(1/2),x)
```

```
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-1/6*2^(1/2)*arctanh(1/4*(2*x^2+
2)*2^(1/2)/(x^4+1)^(1/2))+2/3*(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x
^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/4))+2/3*(-1/2+1/2
*I*3^(1/2))*(1/2/(1/2+1/2*I*3^(1/2))^(1/2)*arctanh((1/2+1/2*I*3^(1/2))^(1/2
)*(x^2-1/2+1/2*I*3^(1/2))/(x^4+1)^(1/2))+(-1)^(3/4)*(-1/2-1/2*I*3^(1/2))*(1
```

```
-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I*(-1/
2+1/2*I*3^(1/2)),I))+2/3*(-1/2-1/2*I*3^(1/2))*(1/2/(1/2-1/2*I*3^(1/2))^(1/2
)*arctanh((1/2-1/2*I*3^(1/2))^(1/2)*(x^2-1/2-1/2*I*3^(1/2))/(x^4+1)^(1/2))+
(-1)^(3/4)*(-1/2+1/2*I*3^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/
2)*EllipticPi((-1)^(1/4)*x,-I*(-1/2-1/2*I*3^(1/2)),I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3+1}{\sqrt{x^4+1}\,(x^3-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-1)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x^3 + 1)/(sqrt(x^4 + 1)*(x^3 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3+1}{(x^3-1)\,\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/((x^3 - 1)*(x^4 + 1)^(1/2)),x)
[Out] int((x^3 + 1)/((x^3 - 1)*(x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)\,(x^2-x+1)}{(x-1)\,\sqrt{x^4+1}\,(x^2+x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**3-1)/(x**4+1)**(1/2),x)
[Out] Integral((x + 1)*(x**2 - x + 1)/((x - 1)*sqrt(x**4 + 1)*(x**2 + x + 1)), x)
```

3.768 $\int \frac{-1+x}{x\sqrt[4]{1+x^4}}\,dx$

Optimal. Leaf size=61

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\tan^{-1}\left(\sqrt[4]{x^4+1}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)$$

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1833, 240, 212, 206, 203, 266, 63, 298}

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\tan^{-1}\left(\sqrt[4]{x^4+1}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(x*(1 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)]/2 - ArcTan[(1 + x^4)^(1/4)]/2 + ArcTanh[x/(1 + x^
4)^(1/4)]/2 + ArcTanh[(1 + x^4)^(1/4)]/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x}{x\sqrt[4]{1+x^4}}\,dx &= \int\left(\frac{1}{\sqrt[4]{1+x^4}}-\frac{1}{x\sqrt[4]{1+x^4}}\right)dx\\
&= \int\frac{1}{\sqrt[4]{1+x^4}}\,dx-\int\frac{1}{x\sqrt[4]{1+x^4}}\,dx\\
&= -\left(\frac{1}{4}\text{Subst}\left(\int\frac{1}{x\sqrt[4]{1+x}}\,dx,x,x^4\right)\right)+\text{Subst}\left(\int\frac{1}{1-x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{2}\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)-\text{Subst}\left(\int\frac{x}{-1\ \cdots}\right.\\
&= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{2}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt[4]{1+x^4}\right)-\frac{1}{2}\text{Subs}\cdots\\
&= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{1}{2}\tan^{-1}\left(\sqrt[4]{1+x^4}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{1+x^4}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\tan^{-1}\left(\sqrt[4]{x^4+1}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(x*(1 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)]/2 - ArcTan[(1 + x^4)^(1/4)]/2 + ArcTanh[x/(1 + x^
4)^(1/4)]/2 + ArcTanh[(1 + x^4)^(1/4)]/2
```

IntegrateAlgebraic [A] time = 4.11, size = 61, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\tan^{-1}\left(\sqrt[4]{x^4+1}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/(x*(1 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)]/2 - ArcTan[(1 + x^4)^(1/4)]/2 + ArcTanh[x/(1 + x^
4)^(1/4)]/2 + ArcTanh[(1 + x^4)^(1/4)]/2
```

fricas [B] time = 4.50, size = 101, normalized size = 1.66

$$\frac{1}{2}\arctan\left(\frac{(x^4+1)^{\frac{3}{4}}(x+1)+(x^4+1)^{\frac{1}{4}}(x^2+x)}{x^4-x^2+1}\right)+\frac{1}{2}\log\left(-\frac{x^4+x^3+(x^4+1)^{\frac{3}{4}}(x+1)+\sqrt{x^4+1}\,(x^2+x+1)+(x^4+1)^{\frac{1}{4}}(x^3+x^2+x+1)+x+1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x/(x^4+1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(((x^4 + 1)^(3/4)*(x + 1) + (x^4 + 1)^(1/4)*(x^2 + x))/(x^4 - x^2
 + 1)) + 1/2*log(-(x^4 + x^3 + (x^4 + 1)^(3/4)*(x + 1) + sqrt(x^4 + 1)*(x^2
 + x + 1) + (x^4 + 1)^(1/4)*(x^3 + x^2 + x + 1) + x + 1)/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{(x^4+1)^{\frac{1}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x/(x^4+1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/((x^4 + 1)^(1/4)*x), x)
```

maple [C] time = 0.34, size = 73, normalized size = 1.20

$$-\frac{\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)\left(-\frac{\pi\sqrt{2}\,x^4\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{4}\right],[2,2],-x^4\right)}{4\Gamma\left(\frac{3}{4}\right)}+\frac{\left(-3\ln(2)-\frac{\pi}{2}+4\ln(x)\right)\pi\sqrt{2}}{\Gamma\left(\frac{3}{4}\right)}\right)}{8\pi}+x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{4}\right],\left[\frac{5}{4}\right],-x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/x/(x^4+1)^(1/4),x)
```

```
[Out] -1/8/Pi*2^(1/2)*GAMMA(3/4)*(-1/4*Pi*2^(1/2)/GAMMA(3/4)*x^4*hypergeom([1,1,5
/4],[2,2],-x^4)+(-3*ln(2)-1/2*Pi+4*ln(x))*Pi*2^(1/2)/GAMMA(3/4))+x*hypergeo
m([1/4,1/4],[5/4],-x^4)
```

maxima [A] time = 0.42, size = 81, normalized size = 1.33

$$-\frac{1}{2}\arctan\left((x^4+1)^{\frac{1}{4}}\right)-\frac{1}{2}\arctan\left(\frac{(x^4+1)^{\frac{1}{4}}}{x}\right)+\frac{1}{4}\log\left((x^4+1)^{\frac{1}{4}}+1\right)-\frac{1}{4}\log\left((x^4+1)^{\frac{1}{4}}-1\right)+\frac{1}{4}\log\left(\frac{(x^4+1)^{\frac{1}{4}}}{x}+1\right)-\frac{1}{4}\log\left(\frac{(x^4+1)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x/(x^4+1)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/2*arctan((x^4 + 1)^(1/4)) - 1/2*arctan((x^4 + 1)^(1/4)/x) + 1/4*log((x^4
 + 1)^(1/4) + 1) - 1/4*log((x^4 + 1)^(1/4) - 1) + 1/4*log((x^4 + 1)^(1/4)/x
 + 1) - 1/4*log((x^4 + 1)^(1/4)/x - 1)
```

mupad [B] time = 0.77, size = 33, normalized size = 0.54

$$\frac{\mathrm{atanh}\left((x^4+1)^{1/4}\right)}{2}-\frac{\mathrm{atan}\left((x^4+1)^{1/4}\right)}{2}+x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(x*(x^4 + 1)^(1/4)),x)
```

```
[Out] atanh((x^4 + 1)^(1/4))/2 - atan((x^4 + 1)^(1/4))/2 + x*hypergeom([1/4, 1/4]
, 5/4, -x^4)
```

sympy [C] time = 2.39, size = 56, normalized size = 0.92

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{4x\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x/(x**4+1)**(1/4),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/4), (5/4,), x**4*exp_polar(I*pi))/(4*gamma(5/4))
 + gamma(1/4)*hyper((1/4, 1/4), (5/4,), exp_polar(I*pi)/x**4)/(4*x*gamma(5/
4))
```

3.769 $\int \frac{x^2}{\left(-b+ax^4\right)^{3/4}} \, dx$

Optimal. Leaf size=61

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2a^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2a^{3/4}}$$

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {331, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2a^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-b + a*x^4)^(3/4),x]
```

```
[Out] -1/2*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)]/a^(3/4) + ArcTanh[(a^(1/4)*x)/(
-b + a*x^4)^(1/4)]/(2*a^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{x^2}{\left(-b+ax^4\right)^{3/4}}\,dx = \text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{2\sqrt{a}} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{2\sqrt{a}}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{2a^{3/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{2a^{3/4}}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 0.89

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right) - \tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-b + a*x^4)^(3/4),x]
[Out] (-ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)] + ArcTanh[(a^(1/4)*x)/(-b + a*x^4)
^(1/4)])/(2*a^(3/4))
```

IntegrateAlgebraic [A] time = 0.29, size = 61, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2a^{3/4}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(-b + a*x^4)^(3/4),x]
[Out] -1/2*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)]/a^(3/4) + ArcTanh[(a^(1/4)*x)/(
-b + a*x^4)^(1/4)]/(2*a^(3/4))
```

fricas [B] time = 0.44, size = 142, normalized size = 2.33

$$-\left(\frac{1}{a^3}\right)^{\frac{1}{4}} \arctan\left(\frac{a^2\left(\frac{1}{a^3}\right)^{\frac{3}{4}} x\sqrt{\frac{a^2\sqrt{\frac{1}{a^3}}\,x^2+\sqrt{ax^4-b}}{x^2}} - \left(ax^4-b\right)^{\frac{1}{4}} a^2\left(\frac{1}{a^3}\right)^{\frac{3}{4}}}{x}\right) + \frac{1}{4}\left(\frac{1}{a^3}\right)^{\frac{1}{4}} \log\left(\frac{a\left(\frac{1}{a^3}\right)^{\frac{1}{4}} x + \left(ax^4-b\right)^{\frac{1}{4}}}{x}\right) - \frac{1}{4}\left(\frac{1}{a^3}\right)^{\frac{1}{4}} \log\left(-\frac{a\left(\frac{1}{a^3}\right)^{\frac{1}{4}} x - \left(ax^4-b\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4-b)^(3/4),x, algorithm="fricas")
[Out] -(a^(-3))^(1/4)*arctan((a^2*(a^(-3))^(3/4)*x*sqrt((a^2*sqrt(a^(-3))*x^2 + s
qrt(a*x^4 - b))/x^2) - (a*x^4 - b)^(1/4)*a^2*(a^(-3))^(3/4))/x) + 1/4*(a^(-
3))^(1/4)*log((a*(a^(-3))^(1/4)*x + (a*x^4 - b)^(1/4))/x) - 1/4*(a^(-3))^(1
/4)*log(-(a*(a^(-3))^(1/4)*x - (a*x^4 - b)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax^4-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4-b)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(a*x^4 - b)^(3/4), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a\,x^4-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^4-b)^(3/4),x)
```

```
[Out] int(x^2/(a*x^4-b)^(3/4),x)
```

maxima [A] time = 0.40, size = 74, normalized size = 1.21

$$\frac{\arctan\left(\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{2\,a^{\frac{3}{4}}}-\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}}\right)}{4\,a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/2*arctan((a*x^4 - b)^(1/4)/(a^(1/4)*x))/a^(3/4) - 1/4*log(-(a^(1/4) - (a*
x^4 - b)^(1/4)/x)/(a^(1/4) + (a*x^4 - b)^(1/4)/x))/a^(3/4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{{\left(a\,x^4-b\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^4 - b)^(3/4),x)
```

```
[Out] int(x^2/(a*x^4 - b)^(3/4), x)
```

sympy [C] time = 0.93, size = 41, normalized size = 0.67

$$\frac{x^3 e^{-\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{ax^4}{b}\right)}{4b^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**4-b)**(3/4),x)
```

```
[Out] x**3*exp(-3*I*pi/4)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), a*x**4/b)/(4*b**(3
/4)*gamma(7/4))
```

3.770 $\int \frac{(6+x^4)\sqrt{-2x+x^4+x^5}}{(-2+x^4)(-2-x^3+x^4)}\,dx$

Optimal. Leaf size=61

$$2\tanh^{-1}\left(\frac{x^2}{\sqrt{x^5+x^4-2x}}\right)-2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{x^5+x^4-2x}}{x^4+x^3-2}\right)$$

Rubi [F] time = 7.95, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(6+x^4)\sqrt{-2x+x^4+x^5}}{(-2+x^4)(-2-x^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((6 + x^4)*Sqrt[-2*x + x^4 + x^5])/((-2 + x^4)*(-2 - x^3 + x^4)),x]
```

```
[Out] ((-I)*Sqrt[-2*x + x^4 + x^5]*Defer[Subst][Defer[Int][Sqrt[-2 + x^6 + x^8]/(
I - x), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[-2 + x^3 + x^4]) + (I*Sqrt[-2*x + x^
4 + x^5]*Defer[Subst][Defer[Int][Sqrt[-2 + x^6 + x^8]/(I*2^(1/8) - x), x],
x, Sqrt[x]])/(2^(3/8)*Sqrt[x]*Sqrt[-2 + x^3 + x^4]) + (Sqrt[-2*x + x^4 + x^
5]*Defer[Subst][Defer[Int][Sqrt[-2 + x^6 + x^8]/(2^(1/8) - x), x], x, Sqrt[
x]])/(2^(3/8)*Sqrt[x]*Sqrt[-2 + x^3 + x^4]) - ((1 + I)*Sqrt[-2*x + x^4 + x^
5]*Defer[Subst][Defer[Int][Sqrt[-2 + x^6 + x^8]/((-1)^(1/4)*2^(1/8) - x), x
], x, Sqrt[x]])/(2^(7/8)*Sqrt[x]*Sqrt[-2 + x^3 + x^4]) - ((1 - I)*Sqrt[-2*x
 + x^4 + x^5]*Defer[Subst][Defer[Int][Sqrt[-2 + x^6 + x^8]/(-((-1)^(3/4)*2^
(1/8)) - x), x], x, Sqrt[x]])/(2^(7/8)*Sqrt[x]*Sqrt[-2 + x^3 + x^4]) - (I*S
qrt[-2*x + x^4 + x^5]*Defer[Subst][Defer[Int][Sqrt[-2 + x^6 + x^8]/(I + x),
 x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[-2 + x^3 + x^4]) + (I*Sqrt[-2*x + x^4 + x^5
]*Defer[Subst][Defer[Int][Sqrt[-2 + x^6 + x^8]/(I*2^(1/8) + x), x], x, Sqrt
[x]])/(2^(3/8)*Sqrt[x]*Sqrt[-2 + x^3 + x^4]) + (Sqrt[-2*x + x^4 + x^5]*Defe
r[Subst][Defer[Int][Sqrt[-2 + x^6 + x^8]/(2^(1/8) + x), x], x, Sqrt[x]])/(2
^(3/8)*Sqrt[x]*Sqrt[-2 + x^3 + x^4]) - ((1 + I)*Sqrt[-2*x + x^4 + x^5]*Defe
r[Subst][Defer[Int][Sqrt[-2 + x^6 + x^8]/((-1)^(1/4)*2^(1/8) + x), x], x, S
qrt[x]])/(2^(7/8)*Sqrt[x]*Sqrt[-2 + x^3 + x^4]) - ((1 - I)*Sqrt[-2*x + x^4
+ x^5]*Defer[Subst][Defer[Int][Sqrt[-2 + x^6 + x^8]/(-((-1)^(3/4)*2^(1/8))
+ x), x], x, Sqrt[x]])/(2^(7/8)*Sqrt[x]*Sqrt[-2 + x^3 + x^4]) - (4*Sqrt[-2*
x + x^4 + x^5]*Defer[Subst][Defer[Int][Sqrt[-2 + x^6 + x^8]/(-2 + 2*x^2 - 2
*x^4 + x^6), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[-2 + x^3 + x^4]) + (2*Sqrt[-2*x
 + x^4 + x^5]*Defer[Subst][Defer[Int][(x^2*Sqrt[-2 + x^6 + x^8])/(-2 + 2*x^
2 - 2*x^4 + x^6), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[-2 + x^3 + x^4]) + (2*Sqrt
[-2*x + x^4 + x^5]*Defer[Subst][Defer[Int][(x^4*Sqrt[-2 + x^6 + x^8])/(-2 +
 2*x^2 - 2*x^4 + x^6), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[-2 + x^3 + x^4])
```

Rubi steps

$$\int \frac{(6+x^4)\sqrt{-2x+x^4+x^5}}{(-2+x^4)(-2-x^3+x^4)}\,dx = \frac{\sqrt{-2x+x^4+x^5}\int \frac{\sqrt{x}(6+x^4)\sqrt{-2+x^3+x^4}}{(-2+x^4)(-2-x^3+x^4)}\,dx}{\sqrt{x}\sqrt{-2+x^3+x^4}}$$

$$= \frac{\sqrt{-2x+x^4+x^5}\int \left(\frac{\sqrt{x}\sqrt{-2+x^3+x^4}}{1+x} + \frac{\sqrt{x}(-1+3x-x^2)\sqrt{-2+x^3+x^4}}{-2+2x-2x^2+x^3} - \frac{4x^{3/2}\sqrt{-2+x^3+x^4}}{-2+x^4}\right) d}{\sqrt{x}\sqrt{-2+x^3+x^4}}$$

$$= \frac{\sqrt{-2x+x^4+x^5}\int \frac{\sqrt{x}\sqrt{-2+x^3+x^4}}{1+x}\,dx}{\sqrt{x}\sqrt{-2+x^3+x^4}} + \frac{\sqrt{-2x+x^4+x^5}\int \frac{\sqrt{x}(-1+3x-x^2)\sqrt{-2+x^3+x}}{-2+2x-2x^2+x^3}}{\sqrt{x}\sqrt{-2+x^3+x^4}}$$

$$= \frac{\left(2\sqrt{-2x+x^4+x^5}\right)\text{Subst}\left(\int \frac{x^2\sqrt{-2+x^6+x^8}}{1+x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{-2+x^3+x^4}} + \frac{\left(2\sqrt{-2x+x^4+x^5}\right)\text{Su}}{}$$

$$= \frac{\left(2\sqrt{-2x+x^4+x^5}\right)\text{Subst}\left(\int \left(\sqrt{-2+x^6+x^8} - \frac{\sqrt{-2+x^6+x^8}}{1+x^2}\right) dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{-2+x^3+x^4}} + \frac{(2\sqrt{}}{}$$

$$= -\frac{\left(2\sqrt{-2x+x^4+x^5}\right)\text{Subst}\left(\int \frac{\sqrt{-2+x^6+x^8}}{1+x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{-2+x^3+x^4}} - \frac{\left(2\sqrt{-2x+x^4+x^5}\right)\text{Su}}{\sqrt{}}$$

$$= -\frac{\left(2\sqrt{-2x+x^4+x^5}\right)\text{Subst}\left(\int \left(\frac{i\sqrt{-2+x^6+x^8}}{2(i-x)} + \frac{i\sqrt{-2+x^6+x^8}}{2(i+x)}\right) dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{-2+x^3+x^4}} - \frac{(2\sqrt{-}}{}$$

$$= -\frac{\left(i\sqrt{-2x+x^4+x^5}\right)\text{Subst}\left(\int \frac{\sqrt{-2+x^6+x^8}}{i-x}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{-2+x^3+x^4}} - \frac{\left(i\sqrt{-2x+x^4+x^5}\right)\text{Sub}}{\sqrt{x}\sqrt{-}}$$

$$= -\frac{\left(i\sqrt{-2x+x^4+x^5}\right)\text{Subst}\left(\int \frac{\sqrt{-2+x^6+x^8}}{i-x}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{-2+x^3+x^4}} - \frac{\left(i\sqrt{-2x+x^4+x^5}\right)\text{Sub}}{\sqrt{x}\sqrt{-}}$$

$$= -\frac{\left(i\sqrt{-2x+x^4+x^5}\right)\text{Subst}\left(\int \frac{\sqrt{-2+x^6+x^8}}{i-x}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{-2+x^3+x^4}} - \frac{\left(i\sqrt{-2x+x^4+x^5}\right)\text{Sub}}{\sqrt{x}\sqrt{-}}$$

$$= -\frac{\left(i\sqrt{-2x+x^4+x^5}\right)\text{Subst}\left(\int \frac{\sqrt{-2+x^6+x^8}}{i-x}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{-2+x^3+x^4}} - \frac{\left(i\sqrt{-2x+x^4+x^5}\right)\text{Sub}}{\sqrt{x}\sqrt{-}}$$

$$= -\frac{\left(i\sqrt{-2x+x^4+x^5}\right)\text{Subst}\left(\int \frac{\sqrt{-2+x^6+x^8}}{i-x}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{-2+x^3+x^4}} - \frac{\left(i\sqrt{-2x+x^4+x^5}\right)\text{Sub}}{\sqrt{x}\sqrt{-}}$$

Mathematica [F] time = 1.94, size = 0, normalized size = 0.00

$$\int \frac{(6+x^4)\sqrt{-2x+x^4+x^5}}{(-2+x^4)(-2-x^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((6 + x^4)*Sqrt[-2*x + x^4 + x^5])/((-2 + x^4)*(-2 - x^3 + x^4)),
x]
```

```
[Out] Integrate[((6 + x^4)*Sqrt[-2*x + x^4 + x^5])/((-2 + x^4)*(-2 - x^3 + x^4)),
 x]
```

IntegrateAlgebraic [A] time = 0.17, size = 61, normalized size = 1.00

$$2\tanh^{-1}\left(\frac{x^2}{\sqrt{x^5+x^4-2x}}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{x^5+x^4-2x}}{x^4+x^3-2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((6 + x^4)*Sqrt[-2*x + x^4 + x^5])/((-2 + x^4)*(-2 - x^3
 + x^4)),x]
```

```
[Out] 2*ArcTanh[x^2/Sqrt[-2*x + x^4 + x^5]] - 2*Sqrt[2]*ArcTanh[(Sqrt[2]*x*Sqrt[-
2*x + x^4 + x^5])/(-2 + x^3 + x^4)]
```

fricas [B] time = 0.45, size = 121, normalized size = 1.98

$$\frac{1}{2}\sqrt{2}\log\left(\frac{x^8+14x^7+17x^6-4x^4-28x^3-4\sqrt{2}\left(x^5+3x^4-2x\right)\sqrt{x^5+x^4-2x}+4}{x^8-2x^7+x^6-4x^4+4x^3+4}\right)+\log\left(\frac{x^4+2x^3+2\sqrt{x^5+x^4-2x}\,x-2}{x^4-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6)*(x^5+x^4-2*x)^(1/2)/(x^4-2)/(x^4-x^3-2),x, algorithm="fri
cas")
```

```
[Out] 1/2*sqrt(2)*log((x^8 + 14*x^7 + 17*x^6 - 4*x^4 - 28*x^3 - 4*sqrt(2)*(x^5 +
3*x^4 - 2*x)*sqrt(x^5 + x^4 - 2*x) + 4)/(x^8 - 2*x^7 + x^6 - 4*x^4 + 4*x^3
+ 4)) + log((x^4 + 2*x^3 + 2*sqrt(x^5 + x^4 - 2*x)*x - 2)/(x^4 - 2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^5+x^4-2x}\left(x^4+6\right)}{\left(x^4-x^3-2\right)\left(x^4-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6)*(x^5+x^4-2*x)^(1/2)/(x^4-2)/(x^4-x^3-2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(x^5 + x^4 - 2*x)*(x^4 + 6)/((x^4 - x^3 - 2)*(x^4 - 2)), x)
```

maple [C] time = 0.97, size = 111, normalized size = 1.82

$$\ln\left(\frac{x^4+2x^3+2\sqrt{x^5+x^4-2x}\,x-2}{x^4-2}\right)+\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2-2\right)x^4+3\,\mathrm{RootOf}\left(\_Z^2-2\right)x^3-4\sqrt{x^5+x^4-2x}\,x-2\,\mathrm{RootOf}\left(\_Z^2-2\right)}{(1+x)\left(x^3-2x^2+2x-2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+6)*(x^5+x^4-2*x)^(1/2)/(x^4-2)/(x^4-x^3-2),x)
```

```
[Out] ln((x^4+2*x^3+2*(x^5+x^4-2*x)^(1/2)*x-2)/(x^4-2))+RootOf(_Z^2-2)*ln(-(RootO
f(_Z^2-2)*x^4+3*RootOf(_Z^2-2)*x^3-4*(x^5+x^4-2*x)^(1/2)*x-2*RootOf(_Z^2-2)
)/(1+x)/(x^3-2*x^2+2*x-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^5+x^4-2x}\left(x^4+6\right)}{\left(x^4-x^3-2\right)\left(x^4-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6)*(x^5+x^4-2*x)^(1/2)/(x^4-2)/(x^4-x^3-2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x^5 + x^4 - 2*x)*(x^4 + 6)/((x^4 - x^3 - 2)*(x^4 - 2)), x)
```

mupad [B] time = 3.44, size = 81, normalized size = 1.33

$$\ln\left(\frac{2\,x\,\sqrt{x\,\left(x^4+x^3-2\right)}+2\,x^3+x^4-2}{x^4-2}\right)+\sqrt{2}\,\ln\left(\frac{3\,x^3+x^4-2\,\sqrt{2}\,x\,\sqrt{x\,\left(x^4+x^3-2\right)}-2}{-x^4+x^3+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^4 + 6)*(x^4 - 2*x + x^5)^(1/2))/((x^4 - 2)*(x^3 - x^4 + 2)),x)
```

```
[Out] log((2*x*(x*(x^3 + x^4 - 2))^(1/2) + 2*x^3 + x^4 - 2)/(x^4 - 2)) + 2^(1/2)*
log((3*x^3 + x^4 - 2*2^(1/2)*x*(x*(x^3 + x^4 - 2))^(1/2) - 2)/(x^3 - x^4 +
2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+6)*(x**5+x**4-2*x)**(1/2)/(x**4-2)/(x**4-x**3-2),x)
```

```
[Out] Timed out
```

3.771 $\int \frac{\sqrt{-1+x^6}\left(-1+2x^6\right)^2}{x\left(-1+4x^6\right)}\,dx$

Optimal. Leaf size=61

$$\frac{1}{36}\sqrt{x^6-1}\left(4x^6-13\right)+\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt{x^6-1}}{\sqrt{3}}\right)}{8\sqrt{3}}$$

Rubi [A] time = 0.10, antiderivative size = 67, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {573, 178, 88, 63, 203}

$$\frac{1}{9}\left(x^6-1\right)^{3/2}-\frac{\sqrt{x^6-1}}{4}+\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt{x^6-1}}{\sqrt{3}}\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^6]*(-1 + 2*x^6)^2)/(x*(-1 + 4*x^6)),x]
```

```
[Out] -1/4*Sqrt[-1 + x^6] + (-1 + x^6)^(3/2)/9 + ArcTan[Sqrt[-1 + x^6]]/3 - ArcTa
n[(2*Sqrt[-1 + x^6])/Sqrt[3]]/(8*Sqrt[3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 178

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_))/(((a_.) + (b_.)*(
x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[
((e + f*x)^(p - 1)*(g + h*x)^q)/(a + b*x), x], x] - Dist[(d*e - c*f)/(b*c -
 a*d), Int[((e + f*x)^(p - 1)*(g + h*x)^q)/(c + d*x), x], x] /; FreeQ[{a, b
, c, d, e, f, g, h, q}, x] && LtQ[0, p, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
```

]]

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+x^6}\left(-1+2x^6\right)^2}{x\left(-1+4x^6\right)}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{\sqrt{-1+x}\,(-1+2x)^2}{x(-1+4x)}\,dx, x, x^6\right)\\
&= \frac{1}{6}\,\text{Subst}\left(\int \frac{(-1+2x)^2}{\sqrt{-1+x}\,x}\,dx, x, x^6\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{(-1+2x)^2}{\sqrt{-1+x}\,(-1+4x)}\,dx, x, x^6\right)\\
&= \frac{1}{6}\,\text{Subst}\left(\int \left(4\sqrt{-1+x} + \frac{1}{\sqrt{-1+x}\,x}\right) dx, x, x^6\right) - \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{4\sqrt{-1+x}} + \sqrt{-\right.\right.\\
&= -\frac{1}{4}\sqrt{-1+x^6} + \frac{1}{9}\left(-1+x^6\right)^{3/2} - \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,(-1+4x)}\,dx, x, x^6\right) + \frac{1}{6}\,\text{Su}\\
&= -\frac{1}{4}\sqrt{-1+x^6} + \frac{1}{9}\left(-1+x^6\right)^{3/2} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{3+4x^2}\,dx, x, \sqrt{-1+x^6}\right) + \frac{1}{3}\,\text{Subs}\\
&= -\frac{1}{4}\sqrt{-1+x^6} + \frac{1}{9}\left(-1+x^6\right)^{3/2} + \frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^6}\right) - \frac{\tan^{-1}\left(\frac{2\sqrt{-1+x^6}}{\sqrt{3}}\right)}{8\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 100, normalized size = 1.64

$$\frac{1}{144}\left(16\sqrt{x^6-1}\,x^6 - 52\sqrt{x^6-1} + 48\tan^{-1}\left(\sqrt{x^6-1}\right) + 3\sqrt{3}\,\tan^{-1}\left(\frac{2-x^3}{\sqrt{3}\sqrt{x^6-1}}\right) + 3\sqrt{3}\,\tan^{-1}\left(\frac{x^3+2}{\sqrt{3}\sqrt{x^6-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^6]*(-1 + 2*x^6)^2)/(x*(-1 + 4*x^6)),x]
```

```
[Out] (-52*Sqrt[-1 + x^6] + 16*x^6*Sqrt[-1 + x^6] + 3*Sqrt[3]*ArcTan[(2 - x^3)/(S
qrt[3]*Sqrt[-1 + x^6])] + 3*Sqrt[3]*ArcTan[(2 + x^3)/(Sqrt[3]*Sqrt[-1 + x^6
])] + 48*ArcTan[Sqrt[-1 + x^6]])/144
```

IntegrateAlgebraic [A] time = 0.05, size = 61, normalized size = 1.00

$$\frac{1}{36}\sqrt{x^6-1}\left(4x^6-13\right) + \frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right) - \frac{\tan^{-1}\left(\frac{2\sqrt{x^6-1}}{\sqrt{3}}\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^6]*(-1 + 2*x^6)^2)/(x*(-1 + 4*x^6)),x]
```

```
[Out] (Sqrt[-1 + x^6]*(-13 + 4*x^6))/36 + ArcTan[Sqrt[-1 + x^6]]/3 - ArcTan[(2*Sq
rt[-1 + x^6])/Sqrt[3]]/(8*Sqrt[3])
```

fricas [A] time = 0.42, size = 45, normalized size = 0.74

$$\frac{1}{36}\left(4\,x^6-13\right)\sqrt{x^6-1} - \frac{1}{24}\sqrt{3}\,\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x^6-1}\right) + \frac{1}{3}\,\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)*(2*x^6-1)^2/x/(4*x^6-1),x, algorithm="fricas")
```

```
[Out] 1/36*(4*x^6 - 13)*sqrt(x^6 - 1) - 1/24*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x^6
- 1)) + 1/3*arctan(sqrt(x^6 - 1))
```

giac [A] time = 0.28, size = 47, normalized size = 0.77

$$\frac{1}{9}\left(x^{6}-1\right)^{\frac{3}{2}}-\frac{1}{24}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x^{6}-1}\right)-\frac{1}{4}\sqrt{x^{6}-1}+\frac{1}{3}\arctan\left(\sqrt{x^{6}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)*(2*x^6-1)^2/x/(4*x^6-1),x, algorithm="giac")
```

```
[Out] 1/9*(x^6 - 1)^(3/2) - 1/24*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x^6 - 1)) - 1/4*
sqrt(x^6 - 1) + 1/3*arctan(sqrt(x^6 - 1))
```

maple [C] time = 1.46, size = 104, normalized size = 1.70

$$\left(\frac{x^{6}}{9}-\frac{13}{36}\right)\sqrt{x^{6}-1}-\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right)-\sqrt{x^{6}-1}}{x^{3}}\right)}{3}-\frac{\mathrm{RootOf}\left(\_Z^{2}+3\right)\ln\left(\frac{-4\,\mathrm{RootOf}\left(\_Z^{2}+3\right)x^{6}+12\sqrt{x^{6}-1}+7\,\mathrm{RootOf}\left(\_Z^{2}+3\right)}{\left(2x^{3}-1\right)\left(2x^{3}+1\right)}\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/2)*(2*x^6-1)^2/x/(4*x^6-1),x)
```

```
[Out] (1/9*x^6-13/36)*(x^6-1)^(1/2)-1/3*RootOf(_Z^2+1)*ln(-(RootOf(_Z^2+1)-(x^6-1
)^(1/2))/x^3)-1/48*RootOf(_Z^2+3)*ln((-4*RootOf(_Z^2+3)*x^6+12*(x^6-1)^(1/2
)+7*RootOf(_Z^2+3))/(2*x^3-1)/(2*x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2\,x^{6}-1\right)^{2}\sqrt{x^{6}-1}}{\left(4\,x^{6}-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)*(2*x^6-1)^2/x/(4*x^6-1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^6 - 1)^2*sqrt(x^6 - 1)/((4*x^6 - 1)*x), x)
```

mupad [B] time = 0.78, size = 47, normalized size = 0.77

$$\frac{\mathrm{atan}\left(\sqrt{x^{6}-1}\right)}{3}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,\sqrt{x^{6}-1}}{3}\right)}{24}-\frac{\sqrt{x^{6}-1}}{4}+\frac{\left(x^{6}-1\right)^{3/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/2)*(2*x^6 - 1)^2)/(x*(4*x^6 - 1)),x)
```

```
[Out] atan((x^6 - 1)^(1/2))/3 - (3^(1/2)*atan((2*3^(1/2)*(x^6 - 1)^(1/2))/3))/24
- (x^6 - 1)^(1/2)/4 + (x^6 - 1)^(3/2)/9
```

sympy [A] time = 43.42, size = 56, normalized size = 0.92

$$\frac{\left(x^{6}-1\right)^{\frac{3}{2}}}{9}-\frac{\sqrt{x^{6}-1}}{4}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,\sqrt{x^{6}-1}}{3}\right)}{24}+\frac{\mathrm{atan}\left(\sqrt{x^{6}-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/2)*(2*x**6-1)**2/x/(4*x**6-1),x)
```

```
[Out] (x**6 - 1)**(3/2)/9 - sqrt(x**6 - 1)/4 - sqrt(3)*atan(2*sqrt(3)*sqrt(x**6 -
 1)/3)/24 + atan(sqrt(x**6 - 1))/3
```

3.772 $\int \frac{-b+ax^4}{(b-2x^2+ax^4)\sqrt[4]{bx^2+ax^6}}\,dx$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{ax^6+bx^2}}\right)}{\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{ax^6+bx^2}}\right)}{\sqrt[4]{2}}$$

Rubi [C] time = 2.11, antiderivative size = 444, normalized size of antiderivative = 7.28, number of steps used = 20, number of rules used = 10, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.256, Rules used = {2056, 6715, 6728, 246, 245, 1438, 430, 429, 511, 510}

$$-\frac{2x\sqrt[4]{\frac{ax^4}{b}+1}F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\frac{a^2x^4}{-ab-2\sqrt{1-ab}+2},-\frac{ax^4}{b}\right)}{\sqrt[4]{ax^6+bx^2}}-\frac{2x\sqrt[4]{\frac{ax^4}{b}+1}F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\frac{a^2x^4}{-ab+2\sqrt{1-ab}+2},-\frac{ax^4}{b}\right)}{\sqrt[4]{ax^6+bx^2}}-\frac{2ax^3\left(1-\sqrt{1-ab}\right)\sqrt[4]{\frac{ax^4}{b}+1}F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};\frac{a^2x^4}{-ab-2\sqrt{1-ab}+2},-\frac{ax^4}{b}\right)}{5\left(-ab-2\sqrt{1-ab}+2\right)\sqrt[4]{ax^6+bx^2}}-\frac{2ax^3\left(\sqrt{1-ab}+1\right)\sqrt[4]{\frac{ax^4}{b}+1}F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};\frac{a^2x^4}{-ab+2\sqrt{1-ab}+2},-\frac{ax^4}{b}\right)}{5\left(-ab+2\sqrt{1-ab}+2\right)\sqrt[4]{ax^6+bx^2}}+\frac{2x\sqrt[4]{\frac{ax^4}{b}+1}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{ax^4}{b}\right)}{\sqrt[4]{ax^6+bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-b + a*x^4)/((b - 2*x^2 + a*x^4)*(b*x^2 + a*x^6)^(1/4)),x]
```

```
[Out] (-2*x*(1 + (a*x^4)/b)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, (a^2*x^4)/(2 - a*b -
 2*Sqrt[1 - a*b]), -((a*x^4)/b)])/(b*x^2 + a*x^6)^(1/4) - (2*x*(1 + (a*x^4)
/b)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, (a^2*x^4)/(2 - a*b + 2*Sqrt[1 - a*b]),
 -((a*x^4)/b)])/(b*x^2 + a*x^6)^(1/4) - (2*a*(1 - Sqrt[1 - a*b])*x^3*(1 + (
a*x^4)/b)^(1/4)*AppellF1[5/8, 1, 1/4, 13/8, (a^2*x^4)/(2 - a*b - 2*Sqrt[1 -
 a*b]), -((a*x^4)/b)])/(5*(2 - a*b - 2*Sqrt[1 - a*b])*(b*x^2 + a*x^6)^(1/4)
) - (2*a*(1 + Sqrt[1 - a*b])*x^3*(1 + (a*x^4)/b)^(1/4)*AppellF1[5/8, 1, 1/4
, 13/8, (a^2*x^4)/(2 - a*b + 2*Sqrt[1 - a*b]), -((a*x^4)/b)])/(5*(2 - a*b +
 2*Sqrt[1 - a*b])*(b*x^2 + a*x^6)^(1/4)) + (2*x*(1 + (a*x^4)/b)^(1/4)*Hyper
geometric2F1[1/8, 1/4, 9/8, -((a*x^4)/b)])/(b*x^2 + a*x^6)^(1/4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-b+ax^4}{\left(b-2x^2+ax^4\right)\sqrt[4]{bx^2+ax^6}}\,dx = \frac{\left(\sqrt{x}\,\sqrt[4]{b+ax^4}\right)\int \frac{-b+ax^4}{\sqrt{x}\,\sqrt[4]{b+ax^4}\left(b-2x^2+ax^4\right)}\,dx}{\sqrt[4]{bx^2+ax^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{b+ax^4}\right)\text{Subst}\left(\int \frac{-b+ax^8}{\sqrt[4]{b+ax^8}\left(b-2x^4+ax^8\right)}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{bx^2+ax^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{b+ax^4}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[4]{b+ax^8}} - \frac{2\left(b-x^4\right)}{\sqrt[4]{b+ax^8}\left(b-2x^4+ax^8\right)}\right)dx, x, \sqrt{x}\right)}{\sqrt[4]{bx^2+ax^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{b+ax^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{b+ax^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{bx^2+ax^6}} - \frac{\left(4\sqrt{x}\,\sqrt[4]{b+ax^4}\right)\text{Subst}\left(\int \cdots\right.}{\sqrt[4]{bx^2 \cdots}}$$

$$= -\frac{\left(4\sqrt{x}\,\sqrt[4]{b+ax^4}\right)\text{Subst}\left(\int \left(\frac{-1-\sqrt{1-ab}}{\left(-2-2\sqrt{1-ab}+2ax^4\right)\sqrt[4]{b+ax^8}} + \frac{-1+\sqrt{1-ab}}{\left(-2+2\sqrt{1-ab}+2ax^4\right)\sqrt[4]{b+a\cdots}}\right.\right.}{\sqrt[4]{bx^2+ax^6}}$$

$$= \frac{2x\sqrt[4]{1+\frac{ax^4}{b}}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{ax^4}{b}\right)}{\sqrt[4]{bx^2+ax^6}} - \frac{\left(4\left(-1-\sqrt{1-ab}\right)\sqrt{x}\,\sqrt[4]{b+ax^4}\right)\text{Subst}\left(\cdots\right.}{\sqrt[4]{bx^2+a\cdots}}$$

$$= \frac{2x\sqrt[4]{1+\frac{ax^4}{b}}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{ax^4}{b}\right)}{\sqrt[4]{bx^2+ax^6}} - \frac{\left(4\left(-1-\sqrt{1-ab}\right)\sqrt{x}\,\sqrt[4]{b+ax^4}\right)\text{Subst}\left(\cdots\right.}{\sqrt[4]{bx^2 \cdots}}$$

$$= \frac{2x\sqrt[4]{1+\frac{ax^4}{b}}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{ax^4}{b}\right)}{\sqrt[4]{bx^2+ax^6}} + \frac{\left(2a\left(-1-\sqrt{1-ab}\right)\sqrt{x}\,\sqrt[4]{b+ax^4}\right)\text{Subst}\left(\cdots\right.}{\sqrt[4]{bx^2+\cdots}}$$

$$= \frac{2x\sqrt[4]{1+\frac{ax^4}{b}}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{ax^4}{b}\right)}{\sqrt[4]{bx^2+ax^6}} + \frac{\left(2a\left(-1-\sqrt{1-ab}\right)\sqrt{x}\,\sqrt[4]{1+\frac{ax^4}{b}}\right)\text{Subst}\left(\cdots\right.}{\sqrt[4]{bx^2+\cdots}}$$

$$= -\frac{2x\sqrt[4]{1+\frac{ax^4}{b}}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\frac{a^2x^4}{2-ab-2\sqrt{1-ab}},-\frac{ax^4}{b}\right)}{\sqrt[4]{bx^2+ax^6}} - \frac{2x\sqrt[4]{1+\frac{ax^4}{b}}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\cdots\right.}{\sqrt[4]{bx^2+ax\cdots}}$$

Mathematica [F] time = 1.84, size = 0, normalized size = 0.00

$$\int \frac{-b+ax^4}{\left(b-2x^2+ax^4\right)\sqrt[4]{bx^2+ax^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-b + a*x^4)/((b - 2*x^2 + a*x^4)*(b*x^2 + a*x^6)^(1/4)),x]
[Out] Integrate[(-b + a*x^4)/((b - 2*x^2 + a*x^4)*(b*x^2 + a*x^6)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 1.42, size = 61, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{ax^6+bx^2}}\right)}{\sqrt[4]{2}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{ax^6+bx^2}}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^4)/((b - 2*x^2 + a*x^4)*(b*x^2 + a*x^6)^(1/4))
,x]
```

```
[Out] -(ArcTan[(2^(1/4)*x)/(b*x^2 + a*x^6)^(1/4)]/2^(1/4)) - ArcTanh[(2^(1/4)*x)/
(b*x^2 + a*x^6)^(1/4)]/2^(1/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)/(a*x^4-2*x^2+b)/(a*x^6+b*x^2)^(1/4),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4 - b}{\left(ax^6 + bx^2\right)^{\frac{1}{4}}\left(ax^4 - 2x^2 + b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)/(a*x^4-2*x^2+b)/(a*x^6+b*x^2)^(1/4),x, algorithm="giac"
)
```

```
[Out] integrate((a*x^4 - b)/((a*x^6 + b*x^2)^(1/4)*(a*x^4 - 2*x^2 + b)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{a\,x^4 - b}{\left(a\,x^4 - 2x^2 + b\right)\left(a\,x^6 + b\,x^2\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b)/(a*x^4-2*x^2+b)/(a*x^6+b*x^2)^(1/4),x)
```

```
[Out] int((a*x^4-b)/(a*x^4-2*x^2+b)/(a*x^6+b*x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4 - b}{\left(ax^6 + bx^2\right)^{\frac{1}{4}}\left(ax^4 - 2x^2 + b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)/(a*x^4-2*x^2+b)/(a*x^6+b*x^2)^(1/4),x, algorithm="maxim
a")
```

```
[Out] integrate((a*x^4 - b)/((a*x^6 + b*x^2)^(1/4)*(a*x^4 - 2*x^2 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{b - a\,x^4}{\left(a\,x^6 + b\,x^2\right)^{1/4}\,\left(a\,x^4 - 2\,x^2 + b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^4)/((a*x^6 + b*x^2)^(1/4)*(b + a*x^4 - 2*x^2)),x)
```

```
[Out] int(-(b - a*x^4)/((a*x^6 + b*x^2)^(1/4)*(b + a*x^4 - 2*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4 - b}{\sqrt[4]{x^2\left(ax^4 + b\right)}\left(ax^4 + b - 2x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b)/(a*x**4-2*x**2+b)/(a*x**6+b*x**2)**(1/4),x)
```

```
[Out] Integral((a*x**4 - b)/((x**2*(a*x**4 + b))**(1/4)*(a*x**4 + b - 2*x**2)), x
)
```

3.773 $\int \frac{(4+x^5)\sqrt{1-2x^5+x^8+x^{10}}}{x^9}\,dx$

Optimal. Leaf size=61

$$\frac{\sqrt{x^{10}+x^8-2x^5+1}\,(x^5-1)}{2x^8}+\frac{1}{2}\log\left(x^5+\sqrt{x^{10}+x^8-2x^5+1}-1\right)-2\log(x)$$

Rubi [F] time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(4+x^5)\sqrt{1-2x^5+x^8+x^{10}}}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((4 + x^5)*Sqrt[1 - 2*x^5 + x^8 + x^10])/x^9,x]
```

```
[Out] 4*Defer[Int][Sqrt[1 - 2*x^5 + x^8 + x^10]/x^9, x] + Defer[Int][Sqrt[1 - 2*x
^5 + x^8 + x^10]/x^4, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(4+x^5)\sqrt{1-2x^5+x^8+x^{10}}}{x^9}\,dx &= \int \left(\frac{4\sqrt{1-2x^5+x^8+x^{10}}}{x^9}+\frac{\sqrt{1-2x^5+x^8+x^{10}}}{x^4}\right)dx\\ &= 4\int \frac{\sqrt{1-2x^5+x^8+x^{10}}}{x^9}\,dx+\int \frac{\sqrt{1-2x^5+x^8+x^{10}}}{x^4}\,dx\end{aligned}$$

Mathematica [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(4+x^5)\sqrt{1-2x^5+x^8+x^{10}}}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((4 + x^5)*Sqrt[1 - 2*x^5 + x^8 + x^10])/x^9,x]
```

```
[Out] Integrate[((4 + x^5)*Sqrt[1 - 2*x^5 + x^8 + x^10])/x^9, x]
```

IntegrateAlgebraic [A] time = 0.31, size = 61, normalized size = 1.00

$$\frac{\sqrt{x^{10}+x^8-2x^5+1}\,(x^5-1)}{2x^8}+\frac{1}{2}\log\left(x^5+\sqrt{x^{10}+x^8-2x^5+1}-1\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((4 + x^5)*Sqrt[1 - 2*x^5 + x^8 + x^10])/x^9,x]
```

```
[Out] ((-1 + x^5)*Sqrt[1 - 2*x^5 + x^8 + x^10])/(2*x^8) - 2*Log[x] + Log[-1 + x^5
 + Sqrt[1 - 2*x^5 + x^8 + x^10]]/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+4)*(x^10+x^8-2*x^5+1)^(1/2)/x^9,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{10} + x^{8} - 2\,x^{5} + 1}\,\left(x^{5} + 4\right)}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+4)*(x^10+x^8-2*x^5+1)^(1/2)/x^9,x, algorithm="giac")

[Out] integrate(sqrt(x^10 + x^8 - 2*x^5 + 1)*(x^5 + 4)/x^9, x)

maple [A] time = 0.54, size = 73, normalized size = 1.20

$$\frac{x^{15} + x^{13} - 3x^{10} - x^{8} + 3x^{5} - 1}{2x^{8}\sqrt{x^{10} + x^{8} - 2x^{5} + 1}} + \frac{\ln\left(-\frac{-1+x^{5}+\sqrt{x^{10}+x^{8}-2x^{5}+1}}{x^{4}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5+4)*(x^10+x^8-2*x^5+1)^(1/2)/x^9,x)

[Out] 1/2*(x^15+x^13-3*x^10-x^8+3*x^5-1)/x^8/(x^10+x^8-2*x^5+1)^(1/2)+1/2*ln(-(-1+x^5+(x^10+x^8-2*x^5+1)^(1/2))/x^4)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{10} + x^{8} - 2\,x^{5} + 1}\,\left(x^{5} + 4\right)}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+4)*(x^10+x^8-2*x^5+1)^(1/2)/x^9,x, algorithm="maxima")

[Out] integrate(sqrt(x^10 + x^8 - 2*x^5 + 1)*(x^5 + 4)/x^9, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x^{5} + 4\right)\,\sqrt{x^{10} + x^{8} - 2\,x^{5} + 1}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^5 + 4)*(x^8 - 2*x^5 + x^10 + 1)^(1/2))/x^9,x)

[Out] int(((x^5 + 4)*(x^8 - 2*x^5 + x^10 + 1)^(1/2))/x^9, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{5} + 4\right)\sqrt{x^{10} + x^{8} - 2x^{5} + 1}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**5+4)*(x**10+x**8-2*x**5+1)**(1/2)/x**9,x)

[Out] Integral((x**5 + 4)*sqrt(x**10 + x**8 - 2*x**5 + 1)/x**9, x)

3.774 $\int \frac{1}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$

Optimal. Leaf size=61

$$\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{a}-\frac{b^2}{3a\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2117, 14}

$$\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{a}-\frac{b^2}{3a\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] -1/3*b^2/(a*(a*x + Sqrt[b^2 + a^2*x^2])^(3/2)) + Sqrt[a*x + Sqrt[b^2 + a^2*
x^2]]/a
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx &= \frac{\text{Subst}\left(\int \frac{b^2+x^2}{x^{5/2}}\,dx,x,ax+\sqrt{b^2+a^2x^2}\right)}{2a}\\
&= \frac{\text{Subst}\left(\int \left(\frac{b^2}{x^{5/2}}+\frac{1}{\sqrt{x}}\right)dx,x,ax+\sqrt{b^2+a^2x^2}\right)}{2a}\\
&= -\frac{b^2}{3a\left(ax+\sqrt{b^2+a^2x^2}\right)^{3/2}}+\frac{\sqrt{ax+\sqrt{b^2+a^2x^2}}}{a}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.93

$$\frac{2\left(3ax\left(\sqrt{a^2x^2+b^2}+ax\right)+b^2\right)}{3a\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] (2*(b^2 + 3*a*x*(a*x + Sqrt[b^2 + a^2*x^2])))/(3*a*(a*x + Sqrt[b^2 + a^2*x^
2])^(3/2))
```

IntegrateAlgebraic [A] time = 0.09, size = 61, normalized size = 1.00

$$\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{a}-\frac{b^2}{3a\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] -1/3*b^2/(a*(a*x + Sqrt[b^2 + a^2*x^2])^(3/2)) + Sqrt[a*x + Sqrt[b^2 + a^2*
x^2]]/a
```

fricas [A] time = 0.39, size = 57, normalized size = 0.93

$$-\frac{2\left(a^2x^2-\sqrt{a^2x^2+b^2}\,ax-b^2\right)\sqrt{ax+\sqrt{a^2x^2+b^2}}}{3\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(a^2*x^2 - sqrt(a^2*x^2 + b^2)*a*x - b^2)*sqrt(a*x + sqrt(a^2*x^2 + b^
2))/(a*b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*x + sqrt(a^2*x^2 + b^2)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x + sqrt(a^2*x^2 + b^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a\,x+\sqrt{a^2\,x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+(a**2*x**2+b**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + sqrt(a**2*x**2 + b**2)), x)
```

3.775 $\int x^{10} \sqrt[4]{-1+x^4}\, dx$

Optimal. Leaf size=62

$$\frac{7}{256}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{7}{256}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{384}\sqrt[4]{x^4-1}\left(32x^{11}-4x^7-7x^3\right)$$

Rubi [A] time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {279, 321, 331, 298, 203, 206}

$$\frac{7}{256}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{7}{256}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{12}\sqrt[4]{x^4-1}\,x^{11}-\frac{1}{96}\sqrt[4]{x^4-1}\,x^7-\frac{7}{384}\sqrt[4]{x^4-1}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(-1 + x^4)^(1/4),x]
```

```
[Out] (-7*x^3*(-1 + x^4)^(1/4))/384 - (x^7*(-1 + x^4)^(1/4))/96 + (x^11*(-1 + x^4
)^(1/4))/12 + (7*ArcTan[x/(-1 + x^4)^(1/4)])/256 - (7*ArcTanh[x/(-1 + x^4)^
(1/4)])/256
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int x^{10}\sqrt[4]{-1+x^4}\,dx &= \frac{1}{12}x^{11}\sqrt[4]{-1+x^4} - \frac{1}{12}\int\frac{x^{10}}{\left(-1+x^4\right)^{3/4}}\,dx\\ &= -\frac{1}{96}x^7\sqrt[4]{-1+x^4} + \frac{1}{12}x^{11}\sqrt[4]{-1+x^4} - \frac{7}{96}\int\frac{x^6}{\left(-1+x^4\right)^{3/4}}\,dx\\ &= -\frac{7}{384}x^3\sqrt[4]{-1+x^4} - \frac{1}{96}x^7\sqrt[4]{-1+x^4} + \frac{1}{12}x^{11}\sqrt[4]{-1+x^4} - \frac{7}{128}\int\frac{x^2}{\left(-1+x^4\right)^{3/4}}\,dx\\ &= -\frac{7}{384}x^3\sqrt[4]{-1+x^4} - \frac{1}{96}x^7\sqrt[4]{-1+x^4} + \frac{1}{12}x^{11}\sqrt[4]{-1+x^4} - \frac{7}{128}\operatorname{Subst}\left(\int\frac{x^2}{1-x^4}\,dx, x, \ldots\right.\\ &= -\frac{7}{384}x^3\sqrt[4]{-1+x^4} - \frac{1}{96}x^7\sqrt[4]{-1+x^4} + \frac{1}{12}x^{11}\sqrt[4]{-1+x^4} - \frac{7}{256}\operatorname{Subst}\left(\int\frac{1}{1-x^2}\,dx, x, \ldots\right.\\ &= -\frac{7}{384}x^3\sqrt[4]{-1+x^4} - \frac{1}{96}x^7\sqrt[4]{-1+x^4} + \frac{1}{12}x^{11}\sqrt[4]{-1+x^4} + \frac{7}{256}\tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right) - \frac{7}{256}\ldots\end{aligned}$$

Mathematica [C] time = 0.02, size = 67, normalized size = 1.08

$$\frac{x^3\sqrt[4]{x^4-1}\left(7\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};x^4\right)+\sqrt[4]{1-x^4}\left(8x^8-x^4-7\right)\right)}{96\sqrt[4]{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(-1 + x^4)^(1/4),x]
```

```
[Out] (x^3*(-1 + x^4)^(1/4)*((1 - x^4)^(1/4)*(-7 - x^4 + 8*x^8) + 7*Hypergeometri
c2F1[-1/4, 3/4, 7/4, x^4]))/(96*(1 - x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.19, size = 62, normalized size = 1.00

$$\frac{7}{256}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{7}{256}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{384}\sqrt[4]{x^4-1}\left(32x^{11}-4x^7-7x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^10*(-1 + x^4)^(1/4),x]
```

```
[Out] ((-1 + x^4)^(1/4)*(-7*x^3 - 4*x^7 + 32*x^11))/384 + (7*ArcTan[x/(-1 + x^4)^
(1/4)])/256 - (7*ArcTanh[x/(-1 + x^4)^(1/4)])/256
```

fricas [A] time = 0.41, size = 75, normalized size = 1.21

$$\frac{1}{384}\left(32\,x^{11}-4\,x^7-7\,x^3\right)\left(x^4-1\right)^{\frac{1}{4}}-\frac{7}{256}\arctan\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)-\frac{7}{512}\log\left(\frac{x+\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)+\frac{7}{512}\log\left(-\frac{x-\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(x^4-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/384*(32*x^11 - 4*x^7 - 7*x^3)*(x^4 - 1)^(1/4) - 7/256*arctan((x^4 - 1)^(1
/4)/x) - 7/512*log((x + (x^4 - 1)^(1/4))/x) + 7/512*log(-(x - (x^4 - 1)^(1/
4))/x)
```

giac [B] time = 0.29, size = 105, normalized size = 1.69

$$\frac{1}{384}\,x^{12}\left(\frac{18\left(x^4-1\right)^{\frac{1}{4}}\left(\frac{1}{x^4}-1\right)}{x}-\frac{21\left(x^4-1\right)^{\frac{1}{4}}}{x}+\frac{7\left(x^8-2\,x^4+1\right)\left(x^4-1\right)^{\frac{1}{4}}}{x^9}\right)+\frac{7}{256}\arctan\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)+\frac{7}{512}\log\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}+1\right)-\frac{7}{512}\log\left(-\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(x^4-1)^(1/4),x, algorithm="giac")
```

```
[Out] 1/384*x^12*(18*(x^4 - 1)^(1/4)*(1/x^4 - 1)/x - 21*(x^4 - 1)^(1/4)/x + 7*(x^
8 - 2*x^4 + 1)*(x^4 - 1)^(1/4)/x^9) + 7/256*arctan((x^4 - 1)^(1/4)/x) + 7/5
12*log((x^4 - 1)^(1/4)/x + 1) - 7/512*log(-(x^4 - 1)^(1/4)/x + 1)
```

maple [C] time = 0.34, size = 58, normalized size = 0.94

$$\frac{x^3\left(32x^8-4x^4-7\right)\left(x^4-1\right)^{\frac{1}{4}}}{384}-\frac{7\left(-\mathrm{signum}\left(x^4-1\right)\right)^{\frac{3}{4}}x^3\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],x^4\right)}{384\mathrm{signum}\left(x^4-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(x^4-1)^(1/4),x)
```

```
[Out] 1/384*x^3*(32*x^8-4*x^4-7)*(x^4-1)^(1/4)-7/384/signum(x^4-1)^(3/4)*(-signum
(x^4-1))^(3/4)*x^3*hypergeom([3/4,3/4],[7/4],x^4)
```

maxima [B] time = 0.40, size = 123, normalized size = 1.98

$$-\frac{\frac{21\left(x^4-1\right)^{\frac{1}{4}}}{x}+\frac{18\left(x^4-1\right)^{\frac{5}{4}}}{x^5}-\frac{7\left(x^4-1\right)^{\frac{9}{4}}}{x^9}}{384\left(\frac{3\left(x^4-1\right)}{x^4}-\frac{3\left(x^4-1\right)^2}{x^8}+\frac{\left(x^4-1\right)^3}{x^{12}}-1\right)}-\frac{7}{256}\arctan\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)-\frac{7}{512}\log\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}+1\right)+\frac{7}{512}\log\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(x^4-1)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/384*(21*(x^4 - 1)^(1/4)/x + 18*(x^4 - 1)^(5/4)/x^5 - 7*(x^4 - 1)^(9/4)/x
^9)/(3*(x^4 - 1)/x^4 - 3*(x^4 - 1)^2/x^8 + (x^4 - 1)^3/x^12 - 1) - 7/256*ar
ctan((x^4 - 1)^(1/4)/x) - 7/512*log((x^4 - 1)^(1/4)/x + 1) + 7/512*log((x^4
 - 1)^(1/4)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{10}\left(x^4-1\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(x^4 - 1)^(1/4),x)
```

```
[Out] int(x^10*(x^4 - 1)^(1/4), x)
```

sympy [C] time = 1.40, size = 36, normalized size = 0.58

$$-\frac{x^{11}e^{-\frac{3i\pi}{4}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|x^4\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(x**4-1)**(1/4),x)
```

```
[Out] -x**11*exp(-3*I*pi/4)*gamma(11/4)*hyper((-1/4, 11/4), (15/4,), x**4)/(4*gam
ma(15/4))
```

3.776 $\int x^{10} \sqrt[4]{1+x^4}\, dx$

Optimal. Leaf size=62

$$-\frac{7}{256}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{7}{256}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{384}\sqrt[4]{x^4+1}\left(32x^{11}+4x^7-7x^3\right)$$

Rubi [A] time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {279, 321, 331, 298, 203, 206}

$$-\frac{7}{256}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{7}{256}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{12}\sqrt[4]{x^4+1}\,x^{11}+\frac{1}{96}\sqrt[4]{x^4+1}\,x^7-\frac{7}{384}\sqrt[4]{x^4+1}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(1 + x^4)^(1/4),x]
```

```
[Out] (-7*x^3*(1 + x^4)^(1/4))/384 + (x^7*(1 + x^4)^(1/4))/96 + (x^11*(1 + x^4)^(
1/4))/12 - (7*ArcTan[x/(1 + x^4)^(1/4)])/256 + (7*ArcTanh[x/(1 + x^4)^(1/4)
])/256
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int x^{10}\sqrt[4]{1+x^4}\,dx &= \frac{1}{12}x^{11}\sqrt[4]{1+x^4}+\frac{1}{12}\int\frac{x^{10}}{\left(1+x^4\right)^{3/4}}\,dx\\
&= \frac{1}{96}x^7\sqrt[4]{1+x^4}+\frac{1}{12}x^{11}\sqrt[4]{1+x^4}-\frac{7}{96}\int\frac{x^6}{\left(1+x^4\right)^{3/4}}\,dx\\
&= -\frac{7}{384}x^3\sqrt[4]{1+x^4}+\frac{1}{96}x^7\sqrt[4]{1+x^4}+\frac{1}{12}x^{11}\sqrt[4]{1+x^4}+\frac{7}{128}\int\frac{x^2}{\left(1+x^4\right)^{3/4}}\,dx\\
&= -\frac{7}{384}x^3\sqrt[4]{1+x^4}+\frac{1}{96}x^7\sqrt[4]{1+x^4}+\frac{1}{12}x^{11}\sqrt[4]{1+x^4}+\frac{7}{128}\text{Subst}\left(\int\frac{x^2}{1-x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right.\\
&= -\frac{7}{384}x^3\sqrt[4]{1+x^4}+\frac{1}{96}x^7\sqrt[4]{1+x^4}+\frac{1}{12}x^{11}\sqrt[4]{1+x^4}+\frac{7}{256}\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right.\\
&= -\frac{7}{384}x^3\sqrt[4]{1+x^4}+\frac{1}{96}x^7\sqrt[4]{1+x^4}+\frac{1}{12}x^{11}\sqrt[4]{1+x^4}-\frac{7}{256}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{7}{256}\tanh^{-1}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 0.73

$$\frac{1}{96}x^3\left(7\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-x^4\right)+\sqrt[4]{x^4+1}\left(8x^8+x^4-7\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(1 + x^4)^(1/4),x]
```

```
[Out] (x^3*((1 + x^4)^(1/4)*(-7 + x^4 + 8*x^8) + 7*Hypergeometric2F1[-1/4, 3/4, 7
/4, -x^4]))/96
```

IntegrateAlgebraic [A] time = 0.19, size = 62, normalized size = 1.00

$$-\frac{7}{256}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{7}{256}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{384}\sqrt[4]{x^4+1}\left(32x^{11}+4x^7-7x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^10*(1 + x^4)^(1/4),x]
```

```
[Out] ((1 + x^4)^(1/4)*(-7*x^3 + 4*x^7 + 32*x^11))/384 - (7*ArcTan[x/(1 + x^4)^(1
/4)])/256 + (7*ArcTanh[x/(1 + x^4)^(1/4)])/256
```

fricas [A] time = 0.41, size = 75, normalized size = 1.21

$$\frac{1}{384}\left(32\,x^{11}+4\,x^7-7\,x^3\right)\left(x^4+1\right)^{\frac{1}{4}}+\frac{7}{256}\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+\frac{7}{512}\log\left(\frac{x+\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-\frac{7}{512}\log\left(-\frac{x-\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(x^4+1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/384*(32*x^11 + 4*x^7 - 7*x^3)*(x^4 + 1)^(1/4) + 7/256*arctan((x^4 + 1)^(1
/4)/x) + 7/512*log((x + (x^4 + 1)^(1/4))/x) - 7/512*log(-(x - (x^4 + 1)^(1/
4))/x)
```

giac [B] time = 0.27, size = 104, normalized size = 1.68

$$\frac{1}{384}\,x^{12}\left(\frac{18\,\left(x^4+1\right)^{\frac{1}{4}}\left(\frac{1}{x^4}+1\right)}{x}+\frac{21\,\left(x^4+1\right)^{\frac{1}{4}}}{x}-\frac{7\,\left(x^8+2\,x^4+1\right)\left(x^4+1\right)^{\frac{1}{4}}}{x^9}\right)+\frac{7}{256}\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+\frac{7}{512}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}+1\right)-\frac{7}{512}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(x^4+1)^(1/4),x, algorithm="giac")
```

```
[Out] 1/384*x^12*(18*(x^4 + 1)^(1/4)*(1/x^4 + 1)/x + 21*(x^4 + 1)^(1/4)/x - 7*(x^
8 + 2*x^4 + 1)*(x^4 + 1)^(1/4)/x^9) + 7/256*arctan((x^4 + 1)^(1/4)/x) + 7/5
12*log((x^4 + 1)^(1/4)/x + 1) - 7/512*log((x^4 + 1)^(1/4)/x - 1)
```

maple [C] time = 0.31, size = 42, normalized size = 0.68

$$\frac{x^3\left(32x^8+4x^4-7\right)\left(x^4+1\right)^{\frac{1}{4}}}{384}+\frac{7x^3\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],-x^4\right)}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(x^4+1)^(1/4),x)
```

```
[Out] 1/384*x^3*(32*x^8+4*x^4-7)*(x^4+1)^(1/4)+7/384*x^3*hypergeom([3/4,3/4],[7/4
],-x^4)
```

maxima [B] time = 0.41, size = 123, normalized size = 1.98

$$\frac{\frac{21\left(x^4+1\right)^{\frac{1}{4}}}{x}+\frac{18\left(x^4+1\right)^{\frac{5}{4}}}{x^5}-\frac{7\left(x^4+1\right)^{\frac{9}{4}}}{x^9}}{384\left(\frac{3\left(x^4+1\right)}{x^4}-\frac{3\left(x^4+1\right)^2}{x^8}+\frac{\left(x^4+1\right)^3}{x^{12}}-1\right)}+\frac{7}{256}\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+\frac{7}{512}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}+1\right)-\frac{7}{512}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(x^4+1)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/384*(21*(x^4 + 1)^(1/4)/x + 18*(x^4 + 1)^(5/4)/x^5 - 7*(x^4 + 1)^(9/4)/x^
9)/(3*(x^4 + 1)/x^4 - 3*(x^4 + 1)^2/x^8 + (x^4 + 1)^3/x^12 - 1) + 7/256*arc
tan((x^4 + 1)^(1/4)/x) + 7/512*log((x^4 + 1)^(1/4)/x + 1) - 7/512*log((x^4
+ 1)^(1/4)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{10}\left(x^4+1\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(x^4 + 1)^(1/4),x)
```

```
[Out] int(x^10*(x^4 + 1)^(1/4), x)
```

sympy [C] time = 1.36, size = 31, normalized size = 0.50

$$\frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(x**4+1)**(1/4),x)
```

```
[Out] x**11*gamma(11/4)*hyper((-1/4, 11/4), (15/4,), x**4*exp_polar(I*pi))/(4*gam
ma(15/4))
```

3.777 $\int \frac{1-x^2}{x\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

Optimal. Leaf size=62

$$\frac{\log\left(2\sqrt{a}\sqrt{ax^4+a+bx^3+bx+cx^2}-2ax^2-2a-bx\right)}{\sqrt{a}}-\frac{\log(x)}{\sqrt{a}}$$

Rubi [F] time = 0.68, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1-x^2}{x\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - x^2)/(x*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x] - Defer[Int][x/S
qrt[a + b*x + c*x^2 + b*x^3 + a*x^4], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{x\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx &= \int \left(\frac{1}{x\sqrt{a+bx+cx^2+bx^3+ax^4}}-\frac{x}{\sqrt{a+bx+cx^2+bx^3+ax^4}}\right)dx\\ &= \int \frac{1}{x\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx-\int \frac{x}{\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx\end{aligned}$$

Mathematica [C] time = 1.88, size = 2477, normalized size = 39.95

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(x*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] (2*(x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])^2*((Root[a + b*#1
+ c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^
4 & , 2])*(EllipticPi[(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2]*(Roo
t[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#
1^3 + a*#1^4 & , 4]))/(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1]*(Roo
t[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#
1^3 + a*#1^4 & , 4])), ArcSin[Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 +
a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a +
b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#
1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Roo
t[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^
2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3
])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^
2 + b*#1^3 + a*#1^4 & , 4]))/((-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
, 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*
#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
, 4])))] + EllipticPi[(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Ro
ot[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])/(Root[a + b*#1 + c*#1^2 + b*
#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]), Ar
cSin[Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a +
```

```
b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 +
a*#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Roo
t[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#
1^3 + a*#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2
] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^
2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4
]))/((-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*
#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
, 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*Root[a + b*#1 +
c*#1^2 + b*#1^3 + a*#1^4 & , 1]*Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
, 2]) + EllipticF[ArcSin[Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^
4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1
+ c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 +
 a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a +
 b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b
*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(R
oot[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b
*#1^3 + a*#1^4 & , 4]))/((-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1]
+ Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2
+ b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])
))]*Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1]*(-1 + Root[a + b*#1 + c
*#1^2 + b*#1^3 + a*#1^4 & , 2]^2))*Sqrt[((Root[a + b*#1 + c*#1^2 + b*#1^3 +
 a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(x - Root
[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3]))/((x - Root[a + b*#1 + c*#1^2
+ b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1]
- Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3]))]*(Root[a + b*#1 + c*#1^
2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4
])*Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*
#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*
#1^4 & , 2])*(x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])*(Root[a
+ b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3
+ a*#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])^2*
(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 +
 b*#1^3 + a*#1^4 & , 4])^2)])/(Sqrt[x*(b + c*x + b*x^2) + a*(1 + x^4)]*Root
[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1]*Root[a + b*#1 + c*#1^2 + b*#1^3
 + a*#1^4 & , 2]*(-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a
 + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3
 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))
```

IntegrateAlgebraic [A] time = 0.30, size = 62, normalized size = 1.00

$$\frac{\log\left(2\sqrt{a}\sqrt{ax^4+a+bx^3+bx+cx^2}-2ax^2-2a-bx\right)}{\sqrt{a}}-\frac{\log(x)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^2)/(x*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] -(Log[x]/Sqrt[a]) + Log[-2*a - b*x - 2*a*x^2 + 2*Sqrt[a]*Sqrt[a + b*x + c*x
^2 + b*x^3 + a*x^4]]/Sqrt[a]
```

fricas [A] time = 1.92, size = 148, normalized size = 2.39

$$\left[\frac{\log\left(\frac{8\,a^2x^4+8\,abx^3+8\,abx+\left(8\,a^2+b^2+4\,ac\right)x^2-4\,\sqrt{ax^4+bx^3+cx^2+bx+a}\,\left(2\,ax^2+bx+2\,a\right)\sqrt{a}+8\,a^2}{x^2}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{ax^4+bx^3+cx^2+bx+a}\,\sqrt{-a}}{2\,ax^2+bx+2\,a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/x/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a^2*x^4 + 8*a*b*x^3 + 8*a*b*x + (8*a^2 + b^2 + 4*a*c)*x^2 - 4*s
qrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(2*a*x^2 + b*x + 2*a)*sqrt(a) + 8*a^2)
/x^2)/sqrt(a), sqrt(-a)*arctan(2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt
(-a)/(2*a*x^2 + b*x + 2*a))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 - 1}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/x/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*x), x)
```

maple [C] time = 0.18, size = 3365, normalized size = 54.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/x/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -2*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*
c+_Z*b+a,index=1))*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^
4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,in
dex=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_
Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/
2)*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))^2*((RootOf(_Z^4*a+_Z^3*b
+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-Roo
tOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b
+a,index=3)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_
Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,i
ndex=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-RootOf(_Z^4*a+_Z^3*
b+_Z^2*c+_Z*b+a,index=4))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-Root
Of(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*
b+a,index=2)))^(1/2)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z
^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,ind
ex=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(a*x^4+b*x^3+c*x^2+b*x+a
)^(1/2)*(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)*EllipticF(((RootOf(_Z^
4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=
2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_
Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootO
f(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2),((RootOf(_Z^4*a+_Z^3*b+_Z^2*
c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))*(-RootOf(_Z^
4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=
1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3)+RootOf(_Z^4*a+_Z^3*b+_Z^2
*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z
^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2))+(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_
Z*b+a,index=2)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*EllipticPi(((Ro
otOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+
a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+
_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/
(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2),(RootOf(_Z^4*a+_Z^3*
b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(Root
Of(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,
index=2)),((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*
b+_Z^2*c+_Z*b+a,index=3))*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+Roo
tOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*
b+a,index=3)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_
```

```
Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^
(1/2)))+2*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*
b+_Z^2*c+_Z*b+a,index=1))*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-Roo
tOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z
*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_
Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2
)))^(1/2)*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))^2*((RootOf(_Z^4*a
+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))
*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))/(RootOf(_Z^4*a+_Z^3*b+_Z^2
*c+_Z*b+a,index=3)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_
Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_
Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-RootOf(_Z^4*
a+_Z^3*b+_Z^2*c+_Z*b+a,index=4))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=
4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^
2*c+_Z*b+a,index=2)))^(1/2)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-Ro
otOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*
b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(a*x^4+b*x^3+c*x^
2+b*x+a)^(1/2)/RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)*(EllipticF(((Roo
tOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a
,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_
Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(
x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2),((RootOf(_Z^4*a+_Z^3*
b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))*(-Roo
tOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a
,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3)+RootOf(_Z^4*a+_Z^3
*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+Ro
otOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2))+(RootOf(_Z^4*a+_Z^3*b+_Z
^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/RootOf(_Z
^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1)*EllipticPi(((RootOf(_Z^4*a+_Z^3*b+_Z^2*c
+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^
4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,inde
x=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_
Z^2*c+_Z*b+a,index=2)))^(1/2),RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)*(
RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*
b+a,index=1))/RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1)/(RootOf(_Z^4*a+_Z
^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)),((
RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*
b+a,index=3))*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_
Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3)
+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c
+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-1}{\sqrt{ax^4+bx^3+cx^2+bx+a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/x/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 - 1)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x^2-1}{x\,\sqrt{a\,x^4+b\,x^3+c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)),x)
```

```
[Out] int(-(x^2 - 1)/(x*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{1}{x\sqrt{ax^4+a+bx^3+bx+cx^2}}\right)dx-\int\frac{x}{\sqrt{ax^4+a+bx^3+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/x/(a*x**4+b*x**3+c*x**2+b*x+a)**(1/2),x)
```

```
[Out] -Integral(-1/(x*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2)), x) - Integral(x/
sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2), x)
```

3.778 $\int \frac{x^2(-1+3x^4)}{(1+x^4)^2(a-x+ax^4)\sqrt{x+x^5}}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{x^5+x}\,(3ax^4+3a+x)}{3(x^4+1)^2}-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{x^5+x}}{\sqrt{a}\,(x^4+1)}\right)$$

Rubi [F] time = 1.92, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2(-1+3x^4)}{(1+x^4)^2(a-x+ax^4)\sqrt{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(-1 + 3*x^4))/((1 + x^4)^2*(a - x + a*x^4)*Sqrt[x + x^5]),x]
```

```
[Out] (7*x^3)/(8*a*Sqrt[x + x^5]) + x^3/(2*a*(1 + x^4)*Sqrt[x + x^5]) - (7*x^3*Sq
rt[1 + x^4]*Hypergeometric2F1[1/2, 5/8, 13/8, -x^4])/(40*a*Sqrt[x + x^5]) -
 (8*Sqrt[x]*Sqrt[1 + x^4]*Defer[Subst][Defer[Int][x^4/((1 + x^8)^(5/2)*(a -
 x^2 + a*x^8)), x], x, Sqrt[x]])/Sqrt[x + x^5] + (6*Sqrt[x]*Sqrt[1 + x^4]*D
efer[Subst][Defer[Int][x^6/((1 + x^8)^(5/2)*(a - x^2 + a*x^8)), x], x, Sqrt
[x]])/(a*Sqrt[x + x^5])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(-1+3x^4)}{(1+x^4)^2(a-x+ax^4)\sqrt{x+x^5}}\,dx &= \frac{\left(\sqrt{x}\sqrt{1+x^4}\right)\int \frac{x^{3/2}(-1+3x^4)}{(1+x^4)^{5/2}(a-x+ax^4)}\,dx}{\sqrt{x+x^5}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \frac{x^4(-1+3x^8)}{(1+x^8)^{5/2}(a-x^2+ax^8)}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^5}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int\left(\frac{3x^4}{a(1+x^8)^{5/2}}+\frac{x^4(-4a+3x^2)}{a(1+x^8)^{5/2}(a-x^2+ax^8)}\right)dx,x,\sqrt{x}\right)}{\sqrt{x+x^5}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \frac{x^4(-4a+3x^2)}{(1+x^8)^{5/2}(a-x^2+ax^8)}\,dx,x,\sqrt{x}\right)}{a\sqrt{x+x^5}}+\frac{\left(6\sqrt{x}\sqrt{1+x^4}\right)\cdots}{\cdots}\\
&= \frac{x^3}{2a(1+x^4)\sqrt{x+x^5}}+\frac{\left(2\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int\left(-\frac{4ax^4}{(1+x^8)^{5/2}(a-x^2+ax^8)}+\cdots\right.\right.}{a\sqrt{x+x^5}}\\
&= \frac{7x^3}{8a\sqrt{x+x^5}}+\frac{x^3}{2a(1+x^4)\sqrt{x+x^5}}-\frac{\left(8\sqrt{x}\sqrt{1+x^4}\right)\text{Subst}\left(\int \frac{x\cdots}{(1+x^8)^{5/2}(\cdots}\right.}{\sqrt{x+x^5}}\\
&= \frac{7x^3}{8a\sqrt{x+x^5}}+\frac{x^3}{2a(1+x^4)\sqrt{x+x^5}}-\frac{7x^3\sqrt{1+x^4}\,{}_2F_1\left(\frac{1}{2},\frac{5}{8};\frac{13}{8};-x^4\right)}{40a\sqrt{x+x^5}}-\cdots
\end{aligned}$$

Mathematica [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-1+3x^4\right)}{\left(1+x^4\right)^2\left(a-x+ax^4\right)\sqrt{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(-1 + 3*x^4))/((1 + x^4)^2*(a - x + a*x^4)*Sqrt[x + x^5]),x]
```

```
[Out] Integrate[(x^2*(-1 + 3*x^4))/((1 + x^4)^2*(a - x + a*x^4)*Sqrt[x + x^5]), x
]
```

IntegrateAlgebraic [A] time = 2.67, size = 62, normalized size = 1.00

$$\frac{2\sqrt{x^5+x}\left(3ax^4+3a+x\right)}{3\left(x^4+1\right)^2}-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{x^5+x}}{\sqrt{a}\left(x^4+1\right)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(-1 + 3*x^4))/((1 + x^4)^2*(a - x + a*x^4)*Sqrt[x +
 x^5]),x]
```

```
[Out] (2*(3*a + x + 3*a*x^4)*Sqrt[x + x^5])/(3*(1 + x^4)^2) - 2*a^(3/2)*ArcTanh[S
qrt[x + x^5]/(Sqrt[a]*(1 + x^4))]
```

fricas [A] time = 0.75, size = 231, normalized size = 3.73

$$\left[\frac{3\left(ax^8+2ax^4+a\right)\sqrt{a}\log\left(\frac{a^2x^8+2a^2x^4+6ax^5-4\left(ax^4+a+x\right)\sqrt{x^5+x}\sqrt{a}+a^2+6ax+x^2}{a^2x^8+2a^2x^4-2ax^5+a^2-2ax+x^2}\right)+4\left(3ax^4+3a+x\right)\sqrt{x^5+x}}{6\left(x^8+2x^4+1\right)},\ \frac{3\left(ax^8+2ax^4+a\right)\sqrt{-a}\arctan\left(\frac{\left(ax^4+a+x\right)\sqrt{x^5+x}\sqrt{-a}}{2\left(ax^5+ax\right)}\right)+2\left(3ax^4+3a+x\right)\sqrt{x^5+x}}{3\left(x^8+2x^4+1\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^4-1)/(x^4+1)^2/(a*x^4+a-x)/(x^5+x)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/6*(3*(a*x^8 + 2*a*x^4 + a)*sqrt(a)*log((a^2*x^8 + 2*a^2*x^4 + 6*a*x^5 -
4*(a*x^4 + a + x)*sqrt(x^5 + x)*sqrt(a) + a^2 + 6*a*x + x^2)/(a^2*x^8 + 2*a
^2*x^4 - 2*a*x^5 + a^2 - 2*a*x + x^2)) + 4*(3*a*x^4 + 3*a + x)*sqrt(x^5 + x
))/(x^8 + 2*x^4 + 1), 1/3*(3*(a*x^8 + 2*a*x^4 + a)*sqrt(-a)*arctan(1/2*(a*x
^4 + a + x)*sqrt(x^5 + x)*sqrt(-a)/(a*x^5 + a*x)) + 2*(3*a*x^4 + 3*a + x)*s
qrt(x^5 + x))/(x^8 + 2*x^4 + 1)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^4-1\right)x^2}{\left(ax^4+a-x\right)\sqrt{x^5+x}\left(x^4+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^4-1)/(x^4+1)^2/(a*x^4+a-x)/(x^5+x)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((3*x^4 - 1)*x^2/((a*x^4 + a - x)*sqrt(x^5 + x)*(x^4 + 1)^2), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(3x^4-1\right)}{\left(x^4+1\right)^2\left(a\,x^4+a-x\right)\sqrt{x^5+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^4-1)/(x^4+1)^2/(a*x^4+a-x)/(x^5+x)^(1/2),x)
```

```
[Out] int(x^2*(3*x^4-1)/(x^4+1)^2/(a*x^4+a-x)/(x^5+x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^4-1\right)x^2}{\left(ax^4+a-x\right)\sqrt{x^5+x}\,\left(x^4+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^4-1)/(x^4+1)^2/(a*x^4+a-x)/(x^5+x)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((3*x^4 - 1)*x^2/((a*x^4 + a - x)*sqrt(x^5 + x)*(x^4 + 1)^2), x)
```

mupad [B] time = 0.93, size = 73, normalized size = 1.18

$$a^{3/2}\,\ln\left(\frac{a+x-2\,\sqrt{a}\,\sqrt{x^5+x}\,+a\,x^4}{a\,x^4-x+a}\right)+\frac{2\,a\,\sqrt{x^5+x}}{x^4+1}+\frac{2\,x\,\sqrt{x^5+x}}{3\,\left(x^4+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*x^4 - 1))/((x^4 + 1)^2*(x + x^5)^(1/2)*(a - x + a*x^4)),x)
```

```
[Out] a^(3/2)*log((a + x - 2*a^(1/2)*(x + x^5)^(1/2) + a*x^4)/(a - x + a*x^4)) +
(2*a*(x + x^5)^(1/2))/(x^4 + 1) + (2*x*(x + x^5)^(1/2))/(3*(x^4 + 1)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**4-1)/(x**4+1)**2/(a*x**4+a-x)/(x**5+x)**(1/2),x)
```

```
[Out] Timed out
```

3.779
$$\int \frac{\left(-3+x^4\right)\left(1-x^3+2x^4-x^6-x^7+x^8\right)}{x^6\left(1-x^3+x^4\right)\sqrt[4]{x+x^5}}\,dx$$

Optimal. Leaf size=62

$$2\tan^{-1}\left(\frac{\left(x^5+x\right)^{3/4}}{x^4+1}\right)+2\tanh^{-1}\left(\frac{\left(x^5+x\right)^{3/4}}{x^4+1}\right)+\frac{4\left(x^5+x\right)^{3/4}\left(x^4+1\right)}{7x^6}$$

Rubi [F] time = 2.71, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-3+x^4\right)\left(1-x^3+2x^4-x^6-x^7+x^8\right)}{x^6\left(1-x^3+x^4\right)\sqrt[4]{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + x^4)*(1 - x^3 + 2*x^4 - x^6 - x^7 + x^8))/(x^6*(1 - x^3 + x^4)*(
x + x^5)^(1/4)),x]
```

```
[Out] (4*(1 + x^4)^(1/4)*Hypergeometric2F1[-21/16, 1/4, -5/16, -x^4])/(7*x^5*(x +
 x^5)^(1/4)) + (8*(1 + x^4)^(1/4)*Hypergeometric2F1[-5/16, 1/4, 11/16, -x^4
])/(5*x*(x + x^5)^(1/4)) - (4*x*(1 + x^4)^(1/4)*Hypergeometric2F1[3/16, 1/4
, 19/16, -x^4])/(3*(x + x^5)^(1/4)) + (4*x^3*(1 + x^4)^(1/4)*Hypergeometric
2F1[1/4, 11/16, 27/16, -x^4])/(11*(x + x^5)^(1/4)) + (16*x^(1/4)*(1 + x^4)^
(1/4)*Defer[Subst][Defer[Int][x^2/((1 + x^16)^(1/4)*(1 - x^12 + x^16)), x],
 x, x^(1/4)])/(x + x^5)^(1/4) - (4*x^(1/4)*(1 + x^4)^(1/4)*Defer[Subst][Def
er[Int][x^14/((1 + x^16)^(1/4)*(1 - x^12 + x^16)), x], x, x^(1/4)])/(x + x^
5)^(1/4)
```

Rubi steps

$$\int \frac{\left(-3+x^4\right)\left(1-x^3+2x^4-x^6-x^7+x^8\right)}{x^6\left(1-x^3+x^4\right)\sqrt[4]{x+x^5}}\,dx=\frac{\left(\sqrt[4]{x}\sqrt[4]{1+x^4}\right)\int \frac{\left(-3+x^4\right)\left(1-x^3+2x^4-x^6-x^7+x^8\right)}{x^{25/4}\sqrt[4]{1+x^4}\left(1-x^3+x^4\right)}\,dx}{\sqrt[4]{x+x^5}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{\left(-3+x^{16}\right)\left(1-x^{12}+2x^{16}-x^{24}-x^{28}+x^{32}\right)}{x^{22}\sqrt[4]{1+x^{16}}\left(1-x^{12}+x^{16}\right)}\,dx,x,\right.}{\sqrt[4]{x+x^5}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int\left(-\frac{3}{x^{22}\sqrt[4]{1+x^{16}}}-\frac{2}{x^6\sqrt[4]{1+x^{16}}}-\frac{x^2}{\sqrt[4]{1+x^{16}}}+\right.\right.}{\sqrt[4]{x+x^5}}$$

$$=-\frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{1+x^{16}}}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{x+x^5}}+\frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x\right.}{}$$

$$=\frac{4\sqrt[4]{1+x^4}\,{}_2F_1\left(-\frac{21}{16},\frac{1}{4};-\frac{5}{16};-x^4\right)}{7x^5\sqrt[4]{x+x^5}}+\frac{8\sqrt[4]{1+x^4}\,{}_2F_1\left(-\frac{5}{16},\frac{1}{4};\frac{11}{16};\right.}{5x\sqrt[4]{x+x^5}}$$

$$=\frac{4\sqrt[4]{1+x^4}\,{}_2F_1\left(-\frac{21}{16},\frac{1}{4};-\frac{5}{16};-x^4\right)}{7x^5\sqrt[4]{x+x^5}}+\frac{8\sqrt[4]{1+x^4}\,{}_2F_1\left(-\frac{5}{16},\frac{1}{4};\frac{11}{16};\right.}{5x\sqrt[4]{x+x^5}}$$

Mathematica [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{(-3+x^4)(1-x^3+2x^4-x^6-x^7+x^8)}{x^6(1-x^3+x^4)\sqrt[4]{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + x^4)*(1 - x^3 + 2*x^4 - x^6 - x^7 + x^8))/(x^6*(1 - x^3 +
x^4)*(x + x^5)^(1/4)),x]
```

```
[Out] Integrate[((-3 + x^4)*(1 - x^3 + 2*x^4 - x^6 - x^7 + x^8))/(x^6*(1 - x^3 +
x^4)*(x + x^5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 2.67, size = 62, normalized size = 1.00

$$2\tan^{-1}\left(\frac{(x^5+x)^{3/4}}{x^4+1}\right)+2\tanh^{-1}\left(\frac{(x^5+x)^{3/4}}{x^4+1}\right)+\frac{4(x^5+x)^{3/4}(x^4+1)}{7x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 - x^3 + 2*x^4 - x^6 - x^7 + x^8))/(x^6*(1
 - x^3 + x^4)*(x + x^5)^(1/4)),x]
```

```
[Out] (4*(1 + x^4)*(x + x^5)^(3/4))/(7*x^6) + 2*ArcTan[(x + x^5)^(3/4)/(1 + x^4)]
 + 2*ArcTanh[(x + x^5)^(3/4)/(1 + x^4)]
```

fricas [B] time = 41.71, size = 118, normalized size = 1.90

$$\frac{7\,x^6\arctan\left(\frac{(x^5+x)^{\frac{3}{4}}x-(x^5+x)^{\frac{1}{4}}(x^4+1)}{2\,(x^5+x)}\right)+7\,x^6\log\left(-\frac{x^4+x^3+2\,(x^5+x)^{\frac{1}{4}}x^2+2\,\sqrt{x^5+x}\,x+2\,(x^5+x)^{\frac{3}{4}}+1}{x^4-x^3+1}\right)+4\,(x^5+x)^{\frac{3}{4}}(x^4+1)}{7\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^8-x^7-x^6+2*x^4-x^3+1)/x^6/(x^4-x^3+1)/(x^5+x)^(1/4),x
, algorithm="fricas")
```

```
[Out] 1/7*(7*x^6*arctan(1/2*((x^5 + x)^(3/4)*x - (x^5 + x)^(1/4)*(x^4 + 1))/(x^5
+ x)) + 7*x^6*log(-(x^4 + x^3 + 2*(x^5 + x)^(1/4)*x^2 + 2*sqrt(x^5 + x)*x +
 2*(x^5 + x)^(3/4) + 1)/(x^4 - x^3 + 1)) + 4*(x^5 + x)^(3/4)*(x^4 + 1))/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^8-x^7-x^6+2\,x^4-x^3+1)(x^4-3)}{(x^5+x)^{\frac{1}{4}}(x^4-x^3+1)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^8-x^7-x^6+2*x^4-x^3+1)/x^6/(x^4-x^3+1)/(x^5+x)^(1/4),x
, algorithm="giac")
```

```
[Out] integrate((x^8 - x^7 - x^6 + 2*x^4 - x^3 + 1)*(x^4 - 3)/((x^5 + x)^(1/4)*(x
^4 - x^3 + 1)*x^6), x)
```

maple [C] time = 3.69, size = 175, normalized size = 2.82

$$\frac{\frac{4}{7}x^8+\frac{8}{7}x^4+\frac{4}{7}}{x^5\left(x(x^4+1)\right)^{\frac{1}{4}}}-\ln\left(\frac{-x^4+2(x^5+x)^{\frac{3}{4}}-2\sqrt{x^5+x}\,x+2(x^5+x)^{\frac{1}{4}}x^2-x^3-1}{x^4-x^3+1}\right)+\operatorname{RootOf}(\_Z^2+1)\ln\left(-\frac{-\operatorname{RootOf}(\_Z^2+1)x^4+2\operatorname{RootOf}(\_Z^2+1)\sqrt{x^5+x}\,x-\operatorname{RootOf}(\_Z^2+1)x^3-2(x^5+x)^{\frac{3}{4}}+2(x^5+x)^{\frac{1}{4}}x^2-\operatorname{RootOf}(\_Z^2+1)}{x^4-x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^8-x^7-x^6+2*x^4-x^3+1)/x^6/(x^4-x^3+1)/(x^5+x)^(1/4),x)
```

```
[Out] 4/7*(x^8+2*x^4+1)/x^5/(x*(x^4+1))^(1/4)-ln((-x^4+2*(x^5+x)^(3/4)-2*(x^5+x)^
(1/2)*x+2*(x^5+x)^(1/4)*x^2-x^3-1)/(x^4-x^3+1))+RootOf(_Z^2+1)*ln(-(-RootOf
(_Z^2+1)*x^4+2*RootOf(_Z^2+1)*(x^5+x)^(1/2)*x-RootOf(_Z^2+1)*x^3-2*(x^5+x)^
(3/4)+2*(x^5+x)^(1/4)*x^2-RootOf(_Z^2+1))/(x^4-x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8 - x^7 - x^6 + 2\,x^4 - x^3 + 1\right)\left(x^4 - 3\right)}{\left(x^5 + x\right)^{\frac{1}{4}}\left(x^4 - x^3 + 1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^8-x^7-x^6+2*x^4-x^3+1)/x^6/(x^4-x^3+1)/(x^5+x)^(1/4),x
, algorithm="maxima")
```

```
[Out] integrate((x^8 - x^7 - x^6 + 2*x^4 - x^3 + 1)*(x^4 - 3)/((x^5 + x)^(1/4)*(x
^4 - x^3 + 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{\left(x^4-3\right)\,\left(-x^8+x^7+x^6-2\,x^4+x^3-1\right)}{x^6\,{\left(x^5+x\right)}^{1/4}\,\left(x^4-x^3+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^4 - 3)*(x^3 - 2*x^4 + x^6 + x^7 - x^8 - 1))/(x^6*(x + x^5)^(1/4)*(
x^4 - x^3 + 1)),x)
```

```
[Out] int(-((x^4 - 3)*(x^3 - 2*x^4 + x^6 + x^7 - x^8 - 1))/(x^6*(x + x^5)^(1/4)*(
x^4 - x^3 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**8-x**7-x**6+2*x**4-x**3+1)/x**6/(x**4-x**3+1)/(x**5+
x)**(1/4),x)
```

```
[Out] Timed out
```

3.780 $\int \frac{1+x^2}{\sqrt{1+\sqrt{1+x}}} \, dx$

Optimal. Leaf size=62

$$\frac{4\sqrt{x+1}\left(315x^2+40x+1091\right)\sqrt{\sqrt{x+1}+1}}{3465}-\frac{8\left(175x^2-4x+1187\right)\sqrt{\sqrt{x+1}+1}}{3465}$$

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.63, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1900, 1620}

$$\frac{4}{11}\left(\sqrt{x+1}+1\right)^{11/2}-\frac{20}{9}\left(\sqrt{x+1}+1\right)^{9/2}+\frac{32}{7}\left(\sqrt{x+1}+1\right)^{7/2}-\frac{16}{5}\left(\sqrt{x+1}+1\right)^{5/2}+\frac{4}{3}\left(\sqrt{x+1}+1\right)^{3/2}-4\sqrt{\sqrt{x+1}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/Sqrt[1 + Sqrt[1 + x]],x]
```

```
[Out] -4*Sqrt[1 + Sqrt[1 + x]] + (4*(1 + Sqrt[1 + x])^(3/2))/3 - (16*(1 + Sqrt[1
+ x])^(5/2))/5 + (32*(1 + Sqrt[1 + x])^(7/2))/7 - (20*(1 + Sqrt[1 + x])^(9/
2))/9 + (4*(1 + Sqrt[1 + x])^(11/2))/11
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1900

```
Int[(Px_)^(q_.)*((a_.) + (b_.)*((c_) + (d_.)*(x_))^(n_))^(p_), x_Symbol] :>
 With[{k = Denominator[n]}, Dist[k/d, Subst[Int[SimplifyIntegrand[x^(k - 1)
*(Px /. x -> x^k/d - c/d)^q*(a + b*x^(k*n))^p, x], x], x, (c + d*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, p}, x] && PolynomialQ[Px, x] && IntegerQ[q] && F
ractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{\sqrt{1+\sqrt{1+x}}} \, dx &= 2\,\text{Subst}\left(\int \frac{x\left(2-2x^2+x^4\right)}{\sqrt{1+x}} \, dx, x, \sqrt{1+x}\right)\\
&= 2\,\text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}-4(1+x)^{3/2}+8(1+x)^{5/2}-5(1+x)^{7/2}+(1+x)^{9/2}\right) dx,\right.\\
&= -4\sqrt{1+\sqrt{1+x}}+\frac{4}{3}\left(1+\sqrt{1+x}\right)^{3/2}-\frac{16}{5}\left(1+\sqrt{1+x}\right)^{5/2}+\frac{32}{7}\left(1+\sqrt{1+x}\right)^{7/2}-\frac{20}{9}\left(1\right.
\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.94

$$\frac{4\sqrt{\sqrt{x+1}+1}\left(35\left(9\sqrt{x+1}-10\right)x^2+8\left(5\sqrt{x+1}+1\right)x+1091\sqrt{x+1}-2374\right)}{3465}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/Sqrt[1 + Sqrt[1 + x]],x]
```

```
[Out] (4*Sqrt[1 + Sqrt[1 + x]]*(-2374 + 1091*Sqrt[1 + x] + 8*x*(1 + 5*Sqrt[1 + x]
) + 35*x^2*(-10 + 9*Sqrt[1 + x])))/3465
```

IntegrateAlgebraic [A] time = 0.03, size = 58, normalized size = 0.94

$$\frac{4\sqrt{\sqrt{x+1}+1}\left(315(x+1)^{5/2}-350(x+1)^2-590(x+1)^{3/2}+708(x+1)+1366\sqrt{x+1}-2732\right)}{3465}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/Sqrt[1 + Sqrt[1 + x]],x]
```

```
[Out] (4*Sqrt[1 + Sqrt[1 + x]]*(-2732 + 1366*Sqrt[1 + x] + 708*(1 + x) - 590*(1 +
 x)^(3/2) - 350*(1 + x)^2 + 315*(1 + x)^(5/2)))/3465
```

fricas [A] time = 0.39, size = 38, normalized size = 0.61

$$-\frac{4}{3465}\left(350\,x^2-\left(315\,x^2+40\,x+1091\right)\sqrt{x+1}-8\,x+2374\right)\sqrt{\sqrt{x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(1+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/3465*(350*x^2 - (315*x^2 + 40*x + 1091)*sqrt(x + 1) - 8*x + 2374)*sqrt(s
qrt(x + 1) + 1)
```

giac [A] time = 0.15, size = 67, normalized size = 1.08

$$\frac{4}{11}\left(\sqrt{x+1}+1\right)^{\frac{11}{2}}-\frac{20}{9}\left(\sqrt{x+1}+1\right)^{\frac{9}{2}}+\frac{32}{7}\left(\sqrt{x+1}+1\right)^{\frac{7}{2}}-\frac{16}{5}\left(\sqrt{x+1}+1\right)^{\frac{5}{2}}+\frac{4}{3}\left(\sqrt{x+1}+1\right)^{\frac{3}{2}}-4\sqrt{\sqrt{x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(1+(1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/11*(sqrt(x + 1) + 1)^(11/2) - 20/9*(sqrt(x + 1) + 1)^(9/2) + 32/7*(sqrt(x
 + 1) + 1)^(7/2) - 16/5*(sqrt(x + 1) + 1)^(5/2) + 4/3*(sqrt(x + 1) + 1)^(3/
2) - 4*sqrt(sqrt(x + 1) + 1)
```

maple [A] time = 0.00, size = 68, normalized size = 1.10

$$\frac{4\left(1+\sqrt{1+x}\right)^{\frac{11}{2}}}{11}-\frac{20\left(1+\sqrt{1+x}\right)^{\frac{9}{2}}}{9}+\frac{32\left(1+\sqrt{1+x}\right)^{\frac{7}{2}}}{7}-\frac{16\left(1+\sqrt{1+x}\right)^{\frac{5}{2}}}{5}+\frac{4\left(1+\sqrt{1+x}\right)^{\frac{3}{2}}}{3}-4\sqrt{1+\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(1+(1+x)^(1/2))^(1/2),x)
```

```
[Out] 4/11*(1+(1+x)^(1/2))^(11/2)-20/9*(1+(1+x)^(1/2))^(9/2)+32/7*(1+(1+x)^(1/2))
^(7/2)-16/5*(1+(1+x)^(1/2))^(5/2)+4/3*(1+(1+x)^(1/2))^(3/2)-4*(1+(1+x)^(1/2
))^(1/2)
```

maxima [A] time = 1.05, size = 67, normalized size = 1.08

$$\frac{4}{11}\left(\sqrt{x+1}+1\right)^{\frac{11}{2}}-\frac{20}{9}\left(\sqrt{x+1}+1\right)^{\frac{9}{2}}+\frac{32}{7}\left(\sqrt{x+1}+1\right)^{\frac{7}{2}}-\frac{16}{5}\left(\sqrt{x+1}+1\right)^{\frac{5}{2}}+\frac{4}{3}\left(\sqrt{x+1}+1\right)^{\frac{3}{2}}-4\sqrt{\sqrt{x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(1+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/11*(sqrt(x + 1) + 1)^(11/2) - 20/9*(sqrt(x + 1) + 1)^(9/2) + 32/7*(sqrt(x
 + 1) + 1)^(7/2) - 16/5*(sqrt(x + 1) + 1)^(5/2) + 4/3*(sqrt(x + 1) + 1)^(3/
2) - 4*sqrt(sqrt(x + 1) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2+1}{\sqrt{\sqrt{x+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((x + 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int((x^2 + 1)/((x + 1)^(1/2) + 1)^(1/2), x)
```

sympy [B] time = 5.29, size = 6166, normalized size = 99.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(1+(1+x)**(1/2))**(1/2),x)
```

```
[Out] 4900*x**(39/2)*(x + 1)**(3/4)*cos(11*atan(sqrt(x))/2)*gamma(1/4)*gamma(3/4)
/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*
x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23
/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 2
28690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) + 45632*x**(37
/2)*(x + 1)**(3/4)*cos(11*atan(sqrt(x))/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x
**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2) +
 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 27442
80*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi*x*
*(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) + 183352*x**(35/2)*(x + 1
)**(3/4)*cos(11*atan(sqrt(x))/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) +
 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2) + 1715175*p
i*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(
21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi*x**(15/2) +
41580*pi*x**(13/2) + 3465*pi*x**(11/2)) + 512*x**(35/2)*gamma(1/4)*gamma(3/
4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*p
i*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(
23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) +
 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) + 406048*x**
(33/2)*(x + 1)**(3/4)*cos(11*atan(sqrt(x))/2)*gamma(1/4)*gamma(3/4)/(3465*p
i*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2
) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 27
44280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi
*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) + 6144*x**(33/2)*gamma
(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31
/2) + 762300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3
201660*pi*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*
pi*x**(17/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)
) + 511148*x**(31/2)*(x + 1)**(3/4)*cos(11*atan(sqrt(x))/2)*gamma(1/4)*gamm
a(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 7623
00*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*
x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/
2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) + 33792*
x**(31/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 2
28690*pi*x**(31/2) + 762300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*p
i*x**(25/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(
19/2) + 762300*pi*x**(17/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 34
65*pi*x**(11/2)) + 299200*x**(29/2)*(x + 1)**(3/4)*cos(11*atan(sqrt(x))/2)*
gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x
**(31/2) + 762300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2
) + 3201660*pi*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 76
2300*pi*x**(17/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(
```

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11/2)) + 112640*x**(29/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*
pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2) + 1715175*pi*x**(2
7/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(21/2) +
 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi*x**(15/2) + 41580*p
i*x**(13/2) + 3465*pi*x**(11/2)) - 56496*x**(27/2)*(x + 1)**(3/4)*cos(11*at
an(sqrt(x))/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2
) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744
280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi
*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2)
 + 3465*pi*x**(11/2)) + 253440*x**(27/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x**
(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2) + 1
715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 2744280
*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi*x**(
15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) - 154176*x**(25/2)*(x + 1)*
*(3/4)*cos(11*atan(sqrt(x))/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 4
1580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2) + 1715175*pi*
x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(21
/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi*x**(15/2) + 41
580*pi*x**(13/2) + 3465*pi*x**(11/2)) + 405504*x**(25/2)*gamma(1/4)*gamma(3
/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*
pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**
(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2)
+ 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) + 61116*x**
(23/2)*(x + 1)**(3/4)*cos(11*atan(sqrt(x))/2)*gamma(1/4)*gamma(3/4)/(3465*p
i*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2
) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 27
44280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi
*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) + 473088*x**(23/2)*gam
ma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(
31/2) + 762300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) +
 3201660*pi*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 76230
0*pi*x**(17/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/
2)) + 252736*x**(21/2)*(x + 1)**(3/4)*cos(11*atan(sqrt(x))/2)*gamma(1/4)*ga
mma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 76
2300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*p
i*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(1
7/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) + 4055
04*x**(21/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2)
+ 228690*pi*x**(31/2) + 762300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 274428
0*pi*x**(25/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x
**(19/2) + 762300*pi*x**(17/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) +
 3465*pi*x**(11/2)) + 222200*x**(19/2)*(x + 1)**(3/4)*cos(11*atan(sqrt(x))/
2)*gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*p
i*x**(31/2) + 762300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(2
5/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) +
 762300*pi*x**(17/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x
**(11/2)) + 253440*x**(19/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 415
80*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2) + 1715175*pi*x*
*(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(21/2
) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi*x**(15/2) + 4158
0*pi*x**(13/2) + 3465*pi*x**(11/2)) + 93472*x**(17/2)*(x + 1)**(3/4)*cos(11
*atan(sqrt(x))/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(3
3/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2
744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(21/2) + 1715175
*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13
/2) + 3465*pi*x**(11/2)) + 112640*x**(17/2)*gamma(1/4)*gamma(3/4)/(3465*pi*
x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2)
+ 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 2744
280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi*x
```

(15/2) + 41580*pi*x(13/2) + 3465*pi*x**(11/2)) + 16372*x**(15/2)*(x + 1
)**(3/4)*cos(11*atan(sqrt(x))/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) +
 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2) + 1715175*p
i*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(
21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi*x**(15/2) +
41580*pi*x**(13/2) + 3465*pi*x**(11/2)) + 33792*x**(15/2)*gamma(1/4)*gamma(
3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300
*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x*
*(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2)
 + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) - 832*x**(
13/2)*(x + 1)**(3/4)*cos(11*atan(sqrt(x))/2)*gamma(1/4)*gamma(3/4)/(3465*pi
*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2)
 + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 274
4280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi*
x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) + 6144*x**(13/2)*gamma(
1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/
2) + 762300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 32
01660*pi*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*p
i*x**(17/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2))
 - 512*x**(11/2)*(x + 1)**(3/4)*cos(11*atan(sqrt(x))/2)*gamma(1/4)*gamma(3/
4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*p
i*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(
23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) +
 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) + 512*x**(11
/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*
pi*x**(31/2) + 762300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(
25/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2)
+ 762300*pi*x**(17/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*
x**(11/2)) + 6160*x**20*(x + 1)**(1/4)*sin(9*atan(sqrt(x))/2)*gamma(1/4)*ga
mma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 76
2300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*p
i*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(1
7/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) + 5139
2*x**19*(x + 1)**(1/4)*sin(9*atan(sqrt(x))/2)*gamma(1/4)*gamma(3/4)/(3465*p
i*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2
) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 27
44280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi
*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) + 170368*x**18*(x + 1)
(1/4)*sin(9*atan(sqrt(x))/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x(35/2) + 4
1580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2) + 1715175*pi*
x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(21
/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi*x**(15/2) + 41
580*pi*x**(13/2) + 3465*pi*x**(11/2)) + 241120*x**17*(x + 1)**(1/4)*sin(9*a
tan(sqrt(x))/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/
2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 274
4280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*p
i*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2
) + 3465*pi*x**(11/2)) - 60016*x**16*(x + 1)**(1/4)*sin(9*atan(sqrt(x))/2)*
gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x
(31/2) + 762300*pi*x(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2
) + 3201660*pi*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 76
2300*pi*x**(17/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(
11/2)) - 793760*x**15*(x + 1)**(1/4)*sin(9*atan(sqrt(x))/2)*gamma(1/4)*gamm
a(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 7623
00*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*
x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/
2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) - 134745
6*x**14*(x + 1)**(1/4)*sin(9*atan(sqrt(x))/2)*gamma(1/4)*gamma(3/4)/(3465*p
i*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2

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) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 27
44280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi
*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) - 1091904*x**13*(x + 1
)**(1/4)*sin(9*atan(sqrt(x))/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) +
41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2) + 1715175*pi
*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(2
1/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi*x**(15/2) + 4
1580*pi*x**(13/2) + 3465*pi*x**(11/2)) - 319440*x**12*(x + 1)**(1/4)*sin(9*
atan(sqrt(x))/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33
/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 27
44280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*
pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/
2) + 3465*pi*x**(11/2)) + 170368*x**11*(x + 1)**(1/4)*sin(9*atan(sqrt(x))/2
)*gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi
*x**(31/2) + 762300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25
/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) +
762300*pi*x**(17/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x*
*(11/2)) + 154880*x**10*(x + 1)**(1/4)*sin(9*atan(sqrt(x))/2)*gamma(1/4)*ga
mma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 76
2300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*p
i*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(1
7/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) + 352*
x**9*(x + 1)**(1/4)*sin(9*atan(sqrt(x))/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x
**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2) +
 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 27442
80*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi*x*
*(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) - 43472*x**8*(x + 1)**(1/
4)*sin(9*atan(sqrt(x))/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*
pi*x**(33/2) + 228690*pi*x**(31/2) + 762300*pi*x**(29/2) + 1715175*pi*x**(2
7/2) + 2744280*pi*x**(25/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(21/2) +
 1715175*pi*x**(19/2) + 762300*pi*x**(17/2) + 228690*pi*x**(15/2) + 41580*p
i*x**(13/2) + 3465*pi*x**(11/2)) - 19360*x**7*(x + 1)**(1/4)*sin(9*atan(sqr
t(x))/2)*gamma(1/4)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 22
8690*pi*x**(31/2) + 762300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi
*x**(25/2) + 3201660*pi*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(1
9/2) + 762300*pi*x**(17/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 346
5*pi*x**(11/2)) - 2816*x**6*(x + 1)**(1/4)*sin(9*atan(sqrt(x))/2)*gamma(1/4
)*gamma(3/4)/(3465*pi*x**(35/2) + 41580*pi*x**(33/2) + 228690*pi*x**(31/2)
+ 762300*pi*x**(29/2) + 1715175*pi*x**(27/2) + 2744280*pi*x**(25/2) + 32016
60*pi*x**(23/2) + 2744280*pi*x**(21/2) + 1715175*pi*x**(19/2) + 762300*pi*x
**(17/2) + 228690*pi*x**(15/2) + 41580*pi*x**(13/2) + 3465*pi*x**(11/2)) -
4*(x + 1)**(5/2)*sqrt(sqrt(x + 1) + 1)/(3*(x + 1)**(5/2) + 3*(x + 1)**2) +
8*(x + 1)**(5/2)/(3*(x + 1)**(5/2) + 3*(x + 1)**2) + 4*(x + 1)**3*sqrt(sqrt
(x + 1) + 1)/(3*(x + 1)**(5/2) + 3*(x + 1)**2) - 8*(x + 1)**2*sqrt(sqrt(x +
 1) + 1)/(3*(x + 1)**(5/2) + 3*(x + 1)**2) + 8*(x + 1)**2/(3*(x + 1)**(5/2)
 + 3*(x + 1)**2)
```

3.781 $\int \frac{-1+2x}{(1+x)\sqrt{-a^2x^2+(1+x)^6}}\,dx$

Optimal. Leaf size=62

$$-\frac{2\tan^{-1}\left(\frac{ax}{\sqrt{(15-a^2)x^2+x^6+6x^5+15x^4+20x^3+6x+1}+x^3+3x^2+3x+1}\right)}{a}$$

Rubi [F] time = 0.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+2x}{(1+x)\sqrt{-a^2x^2+(1+x)^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 2*x)/((1 + x)*Sqrt[-(a^2*x^2) + (1 + x)^6]),x]
[Out] 2*Defer[Int][1/Sqrt[-(a^2*x^2) + (1 + x)^6], x] - 3*Defer[Int][1/((1 + x)*S
qrt[-(a^2*x^2) + (1 + x)^6]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+2x}{(1+x)\sqrt{-a^2x^2+(1+x)^6}}\,dx &= \int \left(\frac{2}{\sqrt{-a^2x^2+(1+x)^6}}-\frac{3}{(1+x)\sqrt{-a^2x^2+(1+x)^6}}\right)dx\\ &= 2\int \frac{1}{\sqrt{-a^2x^2+(1+x)^6}}\,dx-3\int \frac{1}{(1+x)\sqrt{-a^2x^2+(1+x)^6}}\,dx\end{aligned}$$

Mathematica [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{-1+2x}{(1+x)\sqrt{-a^2x^2+(1+x)^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + 2*x)/((1 + x)*Sqrt[-(a^2*x^2) + (1 + x)^6]),x]
[Out] Integrate[(-1 + 2*x)/((1 + x)*Sqrt[-(a^2*x^2) + (1 + x)^6]), x]
```

IntegrateAlgebraic [A] time = 0.44, size = 62, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{ax}{\sqrt{(15-a^2)x^2+x^6+6x^5+15x^4+20x^3+6x+1}+x^3+3x^2+3x+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x)/((1 + x)*Sqrt[-(a^2*x^2) + (1 + x)^6]),x]
[Out] (-2*ArcTan[(a*x)/(1 + 3*x + 3*x^2 + x^3 + Sqrt[1 + 6*x + (15 - a^2)*x^2 + 2
0*x^3 + 15*x^4 + 6*x^5 + x^6])])/a
```

fricas [A] time = 0.49, size = 47, normalized size = 0.76

$$\frac{\arctan\left(\frac{\sqrt{x^6+6\,x^5+15\,x^4-(a^2-15)x^2+20\,x^3+6\,x+1}}{ax}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(1+x)/(-a^2*x^2+(1+x)^6)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(x^6 + 6*x^5 + 15*x^4 - (a^2 - 15)*x^2 + 20*x^3 + 6*x + 1)/(a*x)
)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-1}{\sqrt{(x+1)^6-a^2x^2}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(1+x)/(-a^2*x^2+(1+x)^6)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*x - 1)/(sqrt((x + 1)^6 - a^2*x^2)*(x + 1)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{-1+2x}{(1+x)\sqrt{-a^2x^2+(1+x)^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x)/(1+x)/(-a^2*x^2+(1+x)^6)^(1/2),x)
```

```
[Out] int((-1+2*x)/(1+x)/(-a^2*x^2+(1+x)^6)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-1}{\sqrt{(x+1)^6-a^2x^2}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(1+x)/(-a^2*x^2+(1+x)^6)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x - 1)/(sqrt((x + 1)^6 - a^2*x^2)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{2x-1}{\sqrt{(x+1)^6-a^2\,x^2}\,\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 1)/(((x + 1)^6 - a^2*x^2)^(1/2)*(x + 1)),x)
```

```
[Out] int((2*x - 1)/(((x + 1)^6 - a^2*x^2)^(1/2)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-1}{\sqrt{\left(-ax+x^3+3x^2+3x+1\right)\left(ax+x^3+3x^2+3x+1\right)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(1+x)/(-a**2*x**2+(1+x)**6)**(1/2),x)
```

```
[Out] Integral((2*x - 1)/(sqrt((-a*x + x**3 + 3*x**2 + 3*x + 1)*(a*x + x**3 + 3*x
**2 + 3*x + 1))*(x + 1)), x)
```

3.782 $\int \frac{x}{\sqrt{-b+a^2x^2}\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx$

Optimal. Leaf size=62

$$\frac{2\left(\sqrt{a^2x^2-b}+ax\right)^{3/4}}{3a^2}-\frac{2b}{5a^2\left(\sqrt{a^2x^2-b}+ax\right)^{5/4}}$$

Rubi [A] time = 0.25, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2120, 14}

$$\frac{2\left(\sqrt{a^2x^2-b}+ax\right)^{3/4}}{3a^2}-\frac{2b}{5a^2\left(\sqrt{a^2x^2-b}+ax\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4)),x]
```

```
[Out] (-2*b)/(5*a^2*(a*x + Sqrt[-b + a^2*x^2])^(5/4)) + (2*(a*x + Sqrt[-b + a^2*x
^2])^(3/4))/(3*a^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{-b+a^2x^2}\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx &= \frac{\text{Subst}\left(\int \frac{b+x^2}{x^{9/4}}\,dx, x, ax+\sqrt{-b+a^2x^2}\right)}{2a^2}\\
&= \frac{\text{Subst}\left(\int \left(\frac{b}{x^{9/4}}+\frac{1}{\sqrt[4]{x}}\right)dx, x, ax+\sqrt{-b+a^2x^2}\right)}{2a^2}\\
&= -\frac{2b}{5a^2\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}+\frac{2\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}{3a^2}
\end{aligned}$$

Mathematica [B] time = 2.44, size = 510, normalized size = 8.23

$$\frac{4\sqrt{a^2x^2-b}\left(20480a^{15}x^{15}-84992a^{13}bx^{13}+142592a^{11}b^2x^{11}-123392a^9b^3x^9+58080a^7b^4x^7-14308a^5b^5x^5+1593a^3b^6x^3-4b^7\sqrt{a^2x^2-b}+353a^2b^6x^2\sqrt{a^2x^2-b}+20480a^{14}x^{14}\sqrt{a^2x^2-b}-74752a^{12}bx^{12}\sqrt{a^2x^2-b}+107776a^{10}b^2x^{10}\sqrt{a^2x^2-b}-77568a^8b^3x^8\sqrt{a^2x^2-b}+28896a^6b^4x^6\sqrt{a^2x^2-b}-5180a^4b^5x^4\sqrt{a^2x^2-b}-53ab^7x\right)}{15a^2\left(\sqrt{a^2x^2-b}+ax\right)^{13/4}\left(ax\left(\sqrt{a^2x^2-b}+ax\right)-b\right)\left(ab^4x\left(9\sqrt{a^2x^2-b}+41ax\right)+256a^9x^9\left(\sqrt{a^2x^2-b}+ax\right)-64a^7bx^7\left(9\sqrt{a^2x^2-b}+11ax\right)+16a^5b^2x^5\left(27\sqrt{a^2x^2-b}+43ax\right)-40a^3b^3x^3\left(3\sqrt{a^2x^2-b}+7ax\right)-b^5\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4)),x]
```

```
[Out] (4*Sqrt[-b + a^2*x^2]*(-53*a*b^7*x + 1593*a^3*b^6*x^3 - 14308*a^5*b^5*x^5 +
 58080*a^7*b^4*x^7 - 123392*a^9*b^3*x^9 + 142592*a^11*b^2*x^11 - 84992*a^13
*b*x^13 + 20480*a^15*x^15 - 4*b^7*Sqrt[-b + a^2*x^2] + 353*a^2*b^6*x^2*Sqrt
[-b + a^2*x^2] - 5180*a^4*b^5*x^4*Sqrt[-b + a^2*x^2] + 28896*a^6*b^4*x^6*Sq
rt[-b + a^2*x^2] - 77568*a^8*b^3*x^8*Sqrt[-b + a^2*x^2] + 107776*a^10*b^2*x
^10*Sqrt[-b + a^2*x^2] - 74752*a^12*b*x^12*Sqrt[-b + a^2*x^2] + 20480*a^14*
x^14*Sqrt[-b + a^2*x^2]))/(15*a^2*(a*x + Sqrt[-b + a^2*x^2])^(13/4)*(-b + a
*x*(a*x + Sqrt[-b + a^2*x^2]))*(-b^5 + 256*a^9*x^9*(a*x + Sqrt[-b + a^2*x^2
]) - 40*a^3*b^3*x^3*(7*a*x + 3*Sqrt[-b + a^2*x^2]) - 64*a^7*b*x^7*(11*a*x +
 9*Sqrt[-b + a^2*x^2]) + a*b^4*x*(41*a*x + 9*Sqrt[-b + a^2*x^2]) + 16*a^5*b
^2*x^5*(43*a*x + 27*Sqrt[-b + a^2*x^2])))
```

IntegrateAlgebraic [A] time = 0.12, size = 62, normalized size = 1.00

$$\frac{2\left(\sqrt{a^2x^2-b}+ax\right)^{3/4}}{3a^2}-\frac{2b}{5a^2\left(\sqrt{a^2x^2-b}+ax\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4)),
x]
```

```
[Out] (-2*b)/(5*a^2*(a*x + Sqrt[-b + a^2*x^2])^(5/4)) + (2*(a*x + Sqrt[-b + a^2*x
^2])^(3/4))/(3*a^2)
```

fricas [A] time = 0.41, size = 56, normalized size = 0.90

$$-\frac{4\left(3\,a^2x^2-3\,\sqrt{a^2x^2-b}\,ax-4\,b\right)\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{3}{4}}}{15\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4),x, algorithm="f
ricas")
```

```
[Out] -4/15*(3*a^2*x^2 - 3*sqrt(a^2*x^2 - b)*a*x - 4*b)*(a*x + sqrt(a^2*x^2 - b))
^(3/4)/(a^2*b)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4),x, algorithm="g
iac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a^2x^2-b}\,\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4),x)
```

```
[Out] int(x/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a^2x^2-b}\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4),x, algorithm="m
axima")
```

```
[Out] integrate(x/(sqrt(a^2*x^2 - b)*(a*x + sqrt(a^2*x^2 - b))^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/4}\,\sqrt{a^2\,x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x + (a^2*x^2 - b)^(1/2))^(1/4)*(a^2*x^2 - b)^(1/2)),x)
```

```
[Out] int(x/((a*x + (a^2*x^2 - b)^(1/2))^(1/4)*(a^2*x^2 - b)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[4]{ax+\sqrt{a^2x^2-b}}\,\sqrt{a^2x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*x**2-b)**(1/2)/(a*x+(a**2*x**2-b)**(1/2))**(1/4),x)
```

```
[Out] Integral(x/((a*x + sqrt(a**2*x**2 - b))**(1/4)*sqrt(a**2*x**2 - b)), x)
```

3.783 $\int \sqrt{ax+\sqrt{b^2+a^2x^2}}\,dx$

Optimal. Leaf size=62

$$\frac{\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}{3a}-\frac{b^2}{a\sqrt{\sqrt{a^2x^2+b^2}+ax}}$$

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2117, 14}

$$\frac{\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}{3a}-\frac{b^2}{a\sqrt{\sqrt{a^2x^2+b^2}+ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] -(b^2/(a*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])) + (a*x + Sqrt[b^2 + a^2*x^2])^(3
/2)/(3*a)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{ax+\sqrt{b^2+a^2x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{b^2+x^2}{x^{3/2}}\,dx,x,ax+\sqrt{b^2+a^2x^2}\right)}{2a}\\
&= \frac{\text{Subst}\left(\int \left(\frac{b^2}{x^{3/2}}+\sqrt{x}\right)dx,x,ax+\sqrt{b^2+a^2x^2}\right)}{2a}\\
&= -\frac{b^2}{a\sqrt{ax+\sqrt{b^2+a^2x^2}}}+\frac{\left(ax+\sqrt{b^2+a^2x^2}\right)^{3/2}}{3a}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 59, normalized size = 0.95

$$\frac{2ax\left(\sqrt{a^2x^2+b^2}+ax\right)-2b^2}{3a\sqrt{\sqrt{a^2x^2+b^2}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
[Out] (-2*b^2 + 2*a*x*(a*x + Sqrt[b^2 + a^2*x^2]))/(3*a*Sqrt[a*x + Sqrt[b^2 + a^2
*x^2]])
```

IntegrateAlgebraic [A] time = 0.09, size = 62, normalized size = 1.00

$$\frac{\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}{3a}-\frac{b^2}{a\sqrt{\sqrt{a^2x^2+b^2}+ax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
[Out] -(b^2/(a*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])) + (a*x + Sqrt[b^2 + a^2*x^2])^(3
/2)/(3*a)
```

fricas [A] time = 0.39, size = 44, normalized size = 0.71

$$\frac{2\left(2\,ax-\sqrt{a^2x^2+b^2}\right)\sqrt{ax+\sqrt{a^2x^2+b^2}}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2/3*(2*a*x - sqrt(a^2*x^2 + b^2))*sqrt(a*x + sqrt(a^2*x^2 + b^2))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ax+\sqrt{a^2x^2+b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x + sqrt(a^2*x^2 + b^2)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int\sqrt{ax+\sqrt{a^2x^2+b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
[Out] int((a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ax+\sqrt{a^2x^2+b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x + sqrt(a^2*x^2 + b^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a\,x+\sqrt{a^2\,x^2+b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + (b^2 + a^2*x^2)^(1/2))^(1/2),x)
[Out] int((a*x + (b^2 + a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax+\sqrt{a^2x^2+b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a**2*x**2+b**2)**(1/2))**(1/2),x)
[Out] Integral(sqrt(a*x + sqrt(a**2*x**2 + b**2)), x)
```

3.784 $\int \frac{1}{\left(1+\sqrt{x}\right)\sqrt{-\sqrt{x}+x}}\,dx$

Optimal. Leaf size=63

$$4\tanh^{-1}\left(\frac{\sqrt{x-\sqrt{x}}}{\sqrt{x}-1}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x-\sqrt{x}}}{\sqrt{x}-1}\right)$$

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1397, 843, 620, 206, 724}

$$\sqrt{2}\tanh^{-1}\left(\frac{1-3\sqrt{x}}{2\sqrt{2}\sqrt{x-\sqrt{x}}}\right)+4\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x-\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + Sqrt[x])*Sqrt[-Sqrt[x] + x]),x]
```

```
[Out] Sqrt[2]*ArcTanh[(1 - 3*Sqrt[x])/(2*Sqrt[2]*Sqrt[-Sqrt[x] + x])] + 4*ArcTanh
[Sqrt[x]/Sqrt[-Sqrt[x] + x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1397

```
Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g
 - 1)*(d + e*x^(g*n))^q*(a + b*x^(g*n) + c*x^(2*g*n))^p, x], x, x^(1/g)], x
]] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt{x}\right)\sqrt{-\sqrt{x}+x}}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x}{(1+x)\sqrt{-x+x^2}}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-x+x^2}}\,dx, x, \sqrt{x}\right) - 2\,\mathrm{Subst}\left(\int \frac{1}{(1+x)\sqrt{-x+x^2}}\,dx, x, \sqrt{x}\right)$$
$$= 4\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{-\sqrt{x}+x}}\right) + 4\,\mathrm{Subst}\left(\int \frac{1}{8-x^2}\,dx, x, \frac{1-3\sqrt{x}}{\sqrt{-\sqrt{x}+x}}\right)$$
$$= \sqrt{2}\tanh^{-1}\left(\frac{1-3\sqrt{x}}{2\sqrt{2}\sqrt{-\sqrt{x}+x}}\right) + 4\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{-\sqrt{x}+x}}\right)$$

Mathematica [A] time = 0.06, size = 79, normalized size = 1.25

$$-\sqrt{2}\log\left(\sqrt{x}+1\right) + 2\log\left(-2\sqrt{x} - 2\sqrt{x-\sqrt{x}}+1\right) + \sqrt{2}\log\left(-3\sqrt{x} + 2\sqrt{2}\sqrt{x-\sqrt{x}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + Sqrt[x])*Sqrt[-Sqrt[x] + x]),x]
[Out] -(Sqrt[2]*Log[1 + Sqrt[x]]) + 2*Log[1 - 2*Sqrt[x] - 2*Sqrt[-Sqrt[x] + x]] +
 Sqrt[2]*Log[1 - 3*Sqrt[x] + 2*Sqrt[2]*Sqrt[-Sqrt[x] + x]]
```

IntegrateAlgebraic [A] time = 0.20, size = 63, normalized size = 1.00

$$4\tanh^{-1}\left(\frac{\sqrt{x-\sqrt{x}}}{\sqrt{x}-1}\right) - 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x-\sqrt{x}}}{\sqrt{x}-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + Sqrt[x])*Sqrt[-Sqrt[x] + x]),x]
[Out] 4*ArcTanh[Sqrt[-Sqrt[x] + x]/(-1 + Sqrt[x])] - 2*Sqrt[2]*ArcTanh[(Sqrt[2]*S
qrt[-Sqrt[x] + x])/(-1 + Sqrt[x])]
```

fricas [B] time = 1.53, size = 102, normalized size = 1.62

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{17x^2 - 4\left(\sqrt{2}(3x+5)\sqrt{x} - \sqrt{2}(7x+1)\right)\sqrt{x-\sqrt{x}} - 16(3x+1)\sqrt{x} + 46x + 1}{x^2-2x+1}\right) + \log\left(-4\sqrt{x-\sqrt{x}}\left(2\sqrt{x}-1\right) - 8x + 8\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2))/(-x^(1/2)+x)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*log(-(17*x^2 - 4*(sqrt(2)*(3*x + 5)*sqrt(x) - sqrt(2)*(7*x + 1)
)*sqrt(x - sqrt(x)) - 16*(3*x + 1)*sqrt(x) + 46*x + 1)/(x^2 - 2*x + 1)) + l
og(-4*sqrt(x - sqrt(x))*(2*sqrt(x) - 1) - 8*x + 8*sqrt(x) - 1)
```

giac [A] time = 0.86, size = 76, normalized size = 1.21

$$-\sqrt{2}\log\left(\frac{2\left(\sqrt{2} - \sqrt{x-\sqrt{x}} + \sqrt{x}+1\right)}{\left|2\sqrt{2} + 2\sqrt{x-\sqrt{x}} - 2\sqrt{x} - 2\right|}\right) - 2\log\left(\left|2\sqrt{x-\sqrt{x}} - 2\sqrt{x}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2))/(-x^(1/2)+x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*log(2*(sqrt(2) - sqrt(x - sqrt(x)) + sqrt(x) + 1)/abs(2*sqrt(2) +
2*sqrt(x - sqrt(x)) - 2*sqrt(x) - 2)) - 2*log(abs(2*sqrt(x - sqrt(x)) - 2*s
qrt(x) + 1))
```

maple [A] time = 0.02, size = 67, normalized size = 1.06

$$-\frac{\sqrt{-\sqrt{x}+x}\left(\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-1+3\sqrt{x}\right)\sqrt{2}}{4\sqrt{-\sqrt{x}+x}}\right)-2\ln\left(\sqrt{x}-\frac{1}{2}+\sqrt{-\sqrt{x}+x}\right)\right)}{\sqrt{\sqrt{x}\left(-1+\sqrt{x}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x^(1/2))/(-x^(1/2)+x)^(1/2),x)
```

```
[Out] -(-x^(1/2)+x)^(1/2)*(2^(1/2)*arctanh(1/4*(-1+3*x^(1/2))*2^(1/2)/(-x^(1/2)+x
)^(1/2))-2*ln(x^(1/2)-1/2+(-x^(1/2)+x)^(1/2)))/(x^(1/2)*(-1+x^(1/2)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x-\sqrt{x}}\left(\sqrt{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2))/(-x^(1/2)+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x - sqrt(x))*(sqrt(x) + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{x-\sqrt{x}}\,\left(\sqrt{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - x^(1/2))^(1/2)*(x^(1/2) + 1)),x)
```

```
[Out] int(1/((x - x^(1/2))^(1/2)*(x^(1/2) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-\sqrt{x}+x}\left(\sqrt{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**(1/2))/(-x**(1/2)+x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-sqrt(x) + x)*(sqrt(x) + 1)), x)
```

3.785 $\int \frac{x}{\left(-1+x^2\right)\sqrt{x+x^2+x^3}}\,dx$

Optimal. Leaf size=63

$$\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x^3+x^2+x}}{x^2+x+1}\right)}{2\sqrt{3}}$$

Rubi [C] time = 0.81, antiderivative size = 273, normalized size of antiderivative = 4.33, number of steps used = 19, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2056, 6725, 943, 716, 1103, 934, 169, 538, 537}

$$\frac{2\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\,\Pi\left(\frac{1}{2}\left(1-i\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}-\frac{2\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\,\Pi\left(\frac{1}{2}\left(-1+i\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x/((-1 + x^2)*Sqrt[x + x^2 + x^3]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 + (2*x)/(1 - I*Sqrt[3])]*Sqrt[1 + (2*x)/(1 + I*Sqrt[3])]*
EllipticPi[(1 - I*Sqrt[3])/2, ArcSin[((1 - I*Sqrt[3])*Sqrt[x])/2], (I + Sqr
t[3])/(I - Sqrt[3])])/((1 - I*Sqrt[3])*Sqrt[x + x^2 + x^3]) - (2*Sqrt[x]*Sq
rt[1 + (2*x)/(1 - I*Sqrt[3])]*Sqrt[1 + (2*x)/(1 + I*Sqrt[3])]*EllipticPi[(-
1 + I*Sqrt[3])/2, ArcSin[((1 - I*Sqrt[3])*Sqrt[x])/2], (I + Sqrt[3])/(I - S
qrt[3])])/((1 - I*Sqrt[3])*Sqrt[x + x^2 + x^3])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 716

```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2,
Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
```

```
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 943

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + b*x
+ c*x^2]), x], x] + Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt
[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x}{\left(-1+x^2\right)\sqrt{x+x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{\sqrt{x}}{\left(-1+x^2\right)\sqrt{1+x+x^2}}\,dx}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \left(-\frac{\sqrt{x}}{2(1-x)\sqrt{1+x+x^2}}-\frac{\sqrt{x}}{2(1+x)\sqrt{1+x+x^2}}\right)dx}{\sqrt{x+x^2+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{\sqrt{x}}{(1-x)\sqrt{1+x+x^2}}\,dx}{2\sqrt{x+x^2+x^3}} - \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{\sqrt{x}}{(1+x)\sqrt{1+x+x^2}}\,dx}{2\sqrt{x+x^2+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1}{(1-x)\sqrt{x}\sqrt{1+x+x^2}}\,dx}{2\sqrt{x+x^2+x^3}} + \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1}{\sqrt{x}(1+x)\sqrt{1+x+\cdots}}}{2\sqrt{x+x^2+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{1-i\sqrt{3}+2x}\sqrt{1+i\sqrt{3}+2x}\right)\int \frac{1}{(1-x)\sqrt{x}\sqrt{1-i\sqrt{3}+2x}\sqrt{1+i\sqrt{3}+2x}}\,dx}{2\sqrt{x+x^2+x^3}} + \cdots$$

$$= -\frac{\left(\sqrt{x}\sqrt{1-i\sqrt{3}+2x}\sqrt{1+i\sqrt{3}+2x}\right)\text{Subst}\left(\int \frac{1}{\left(-1-x^2\right)\sqrt{1-i\sqrt{3}+2x^2}\sqrt{1+i\sqrt{3}+2x\cdots}}\right.}{\sqrt{x+x^2+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{1+i\sqrt{3}+2x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\right)\text{Subst}\left(\int \frac{1}{\left(-1-x^2\right)\sqrt{1+i\sqrt{3}+2x^2}\sqrt{1+\frac{2x^2}{1-i\sqrt{3}}}}\,dx,\right.}{\sqrt{x+x^2+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\right)\text{Subst}\left(\int \frac{1}{\left(-1-x^2\right)\sqrt{1+\frac{2x^2}{1-i\sqrt{3}}}\sqrt{1+\frac{2x^2}{1+i\sqrt{3}}}}\,dx, x, \sqrt{\cdots}\right.}{\sqrt{x+x^2+x^3}}$$

$$= \frac{2\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\,\Pi\left(\frac{1}{2}\left(1-i\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\cdots\right.}{\left(1-i\sqrt{3}\right)\sqrt{x+x^2+x^3}}$$

Mathematica [C] time = 2.28, size = 627, normalized size = 9.95

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((-1 + x^2)*Sqrt[x + x^2 + x^3]),x]
```

```
[Out] ((-1)^(2/3)*Sqrt[x]*(I*Sqrt[3]*((-1)^(1/3) - Sqrt[x])^2*Sqrt[((-1)^(2/3) -
Sqrt[x])/((1 + (-1)^(1/3))*((-1)^(1/3) - Sqrt[x]))]*Sqrt[(1 - (-1 + (-1)^(1
/3))*Sqrt[x])/((1 + (-1)^(1/3))*((-1)^(1/3) - Sqrt[x]))]*Sqrt[-(((-1)^(2/3)
 + Sqrt[x])/(1 + (-1 + (-1)^(1/3))*Sqrt[x]))]*((1 + (-1)^(1/3))*EllipticF[A
rcSin[Sqrt[(1 - (-1 + (-1)^(1/3))*Sqrt[x])/((1 + (-1)^(1/3))*((-1)^(1/3) -
Sqrt[x]))]], -3] - 2*(-1)^(1/3)*EllipticPi[-1, ArcSin[Sqrt[(1 - (-1 + (-1)^
(1/3))*Sqrt[x])/((1 + (-1)^(1/3))*((-1)^(1/3) - Sqrt[x]))]], -3]) - I*Sqrt[
3]*((-1)^(1/3) - Sqrt[x])^2*Sqrt[((-1)^(2/3) - Sqrt[x])/((1 + (-1)^(1/3))*(
(-1)^(1/3) - Sqrt[x]))]*Sqrt[(1 - (-1 + (-1)^(1/3))*Sqrt[x])/((1 + (-1)^(1/
3))*((-1)^(1/3) - Sqrt[x]))]*Sqrt[-(((-1)^(2/3) + Sqrt[x])/(1 + (-1 + (-1)^
(1/3))*Sqrt[x]))]*((-1 + (-1)^(1/3))*EllipticF[ArcSin[Sqrt[(1 - (-1 + (-1)^
(1/3))*Sqrt[x])/((1 + (-1)^(1/3))*((-1)^(1/3) - Sqrt[x]))]], -3] - 2*(-1)^(
1/3)*EllipticPi[3, ArcSin[Sqrt[(1 - (-1 + (-1)^(1/3))*Sqrt[x])/((1 + (-1)^(
```

```
1/3))*((-1)^(1/3) - Sqrt[x]))]], -3]) + (-1)^(1/3)*(-1 + (-1)^(1/3))^2*(1 +
 (-1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x]*Sqrt[1 - (-1)^(2/3)*x]*EllipticPi[(-1)^
(1/3), I*ArcSinh[(-1)^(5/6)*Sqrt[x]], (-1)^(2/3)]))/((1 - (-1)^(2/3))*Sqrt[
x*(1 + x + x^2)])
```

IntegrateAlgebraic [A] time = 0.11, size = 63, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x^3+x^2+x}}{x^2+x+1}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((-1 + x^2)*Sqrt[x + x^2 + x^3]),x]
```

```
[Out] ArcTan[Sqrt[x + x^2 + x^3]/(1 + x + x^2)]/2 - ArcTanh[(Sqrt[3]*Sqrt[x + x^2
 + x^3])/(1 + x + x^2)]/(2*Sqrt[3])
```

fricas [A] time = 0.44, size = 89, normalized size = 1.41

$$\frac{1}{24}\sqrt{3}\log\left(\frac{x^4+20x^3-4\sqrt{3}\sqrt{x^3+x^2+x}\left(x^2+4x+1\right)+30x^2+20x+1}{x^4-4x^3+6x^2-4x+1}\right)-\frac{1}{4}\arctan\left(\frac{x^2+1}{2\sqrt{x^3+x^2+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1)/(x^3+x^2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(3)*log((x^4 + 20*x^3 - 4*sqrt(3)*sqrt(x^3 + x^2 + x)*(x^2 + 4*x +
 1) + 30*x^2 + 20*x + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)) - 1/4*arctan(1/2*
(x^2 + 1)/sqrt(x^3 + x^2 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{x^3+x^2+x}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1)/(x^3+x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 + x^2 + x)*(x^2 - 1)), x)
```

maple [C] time = 0.02, size = 300, normalized size = 4.76

$$\frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}},\frac{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}},\frac{\sqrt{3}\,\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{3\sqrt{x^3+x^2+x}\,\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}+\frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}},\frac{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{1}{2}-\frac{i\sqrt{3}}{2}},\frac{\sqrt{3}\,\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{3\sqrt{x^3+x^2+x}\,\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-1)/(x^3+x^2+x)^(1/2),x)
```

```
[Out] 1/3*(1/2+1/2*I*3^(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3
^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/
2)/(x^3+x^2+x)^(1/2)/(-3/2-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))
/(1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)),1/3*3
^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))+1/3*(1/2+1/2*I*3^(1/2))*((x+
1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2)/(1/2-1/
2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),(
-1/2-1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3 + x^2 + x}\left(x^2 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1)/(x^3+x^2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 + x^2 + x)*(x^2 - 1)), x)
```

mupad [B] time = 0.76, size = 187, normalized size = 2.97

$$\frac{\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(\sqrt{3}+1\mathrm{i}\right)\left(\Pi\left(\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)1\mathrm{i}}{2\sqrt{x^3+x^2-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 - 1)*(x + x^2 + x^3)^(1/2)),x)
```

```
[Out] ((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i
)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/2)
*(3^(1/2) + 1i)*(ellipticPi(1/2 - (3^(1/2)*1i)/2, asin((x/((3^(1/2)*1i)/2 -
 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)) - ellipticPi
((3^(1/2)*1i)/2 - 1/2, asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/2)*1
i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))*1i)/(2*(x^2 + x^3 - x*((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x\left(x^2 + x + 1\right)}\,(x - 1)\,(x + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-1)/(x**3+x**2+x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(x*(x**2 + x + 1))*(x - 1)*(x + 1)), x)
```

3.786 $\int \frac{-abc+2a(b+c)x-(3a+b+c)x^2+2x^3}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(ad+(bc-d)x-(b+c)x^2+x^3\right)}\,dx$

Optimal. Leaf size=63

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{x^3(-a-b-c)+x^2(ab+ac+bc)-abcx+x^4}}{\sqrt{d}\,(a-x)}\right)}{\sqrt{d}}$$

Rubi [F] time = 3.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-abc+2a(b+c)x-(3a+b+c)x^2+2x^3}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(ad+(bc-d)x-(b+c)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a*b*c) + 2*a*(b + c)*x - (3*a + b + c)*x^2 + 2*x^3)/(Sqrt[x*(-a + x)
*(-b + x)*(-c + x)]*(a*d + (b*c - d)*x - (b + c)*x^2 + x^3)),x]
```

```
[Out] 2*Defer[Int][1/Sqrt[x*(-a + x)*(-b + x)*(-c + x)], x] - a*(b*c + 2*d)*Defer
[Int][1/(Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*(a*d + (b*c - d)*x - (b + c)*x^
2 + x^3)), x] - 2*(b*c - a*(b + c) - d)*Defer[Int][x/(Sqrt[x*(-a + x)*(-b +
 x)*(-c + x)]*(a*d + (b*c - d)*x - (b + c)*x^2 + x^3)), x] - (3*a - b - c)*
Defer[Int][x^2/(Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*(a*d + (b*c - d)*x - (b
+ c)*x^2 + x^3)), x]
```

Rubi steps

$$\int \frac{-abc+2a(b+c)x-(3a+b+c)x^2+2x^3}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(ad+(bc-d)x-(b+c)x^2+x^3\right)}\,dx = \int\left(\frac{2}{\sqrt{x(-a+x)(-b+x)(-c+x)}} - \frac{}{\sqrt{x(-}}\right.$$
$$= 2\int \frac{1}{\sqrt{x(-a+x)(-b+x)(-c+x)}}\,dx - \int$$
$$= 2\int \frac{1}{\sqrt{x(-a+x)(-b+x)(-c+x)}}\,dx - \int$$
$$= 2\int \frac{1}{\sqrt{x(-a+x)(-b+x)(-c+x)}}\,dx - (3a$$

Mathematica [C] time = 13.33, size = 8886, normalized size = 141.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-(a*b*c) + 2*a*(b + c)*x - (3*a + b + c)*x^2 + 2*x^3)/(Sqrt[x*(-
a + x)*(-b + x)*(-c + x)]*(a*d + (b*c - d)*x - (b + c)*x^2 + x^3)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.71, size = 63, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{x^3(-a-b-c)+x^2(ab+ac+bc)-abcx+x^4}}{\sqrt{d}\,(a-x)}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*b*c) + 2*a*(b + c)*x - (3*a + b + c)*x^2 + 2*x^3)/(
Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*(a*d + (b*c - d)*x - (b + c)*x^2 + x^3))
,x]
```

```
[Out] (2*ArcTanh[Sqrt[-(a*b*c*x) + (a*b + a*c + b*c)*x^2 + (-a - b - c)*x^3 + x^4
]/(Sqrt[d]*(a - x))])/Sqrt[d]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*c+2*a*(b+c)*x-(3*a+b+c)*x^2+2*x^3)/(x*(-a+x)*(-b+x)*(-c+x))
^(1/2)/(a*d+(b*c-d)*x-(b+c)*x^2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{abc - 2\,a(b+c)x + (3\,a+b+c)x^2 - 2\,x^3}{\sqrt{-(a-x)(b-x)(c-x)x}\,\left((b+c)x^2 - x^3 - ad - (bc-d)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*c+2*a*(b+c)*x-(3*a+b+c)*x^2+2*x^3)/(x*(-a+x)*(-b+x)*(-c+x))
^(1/2)/(a*d+(b*c-d)*x-(b+c)*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate((a*b*c - 2*a*(b + c)*x + (3*a + b + c)*x^2 - 2*x^3)/(sqrt(-(a - x
)*(b - x)*(c - x)*x)*((b + c)*x^2 - x^3 - a*d - (b*c - d)*x)), x)
```

maple [C] time = 0.10, size = 489, normalized size = 7.76

$$-\frac{4a\sqrt{\frac{(a-c)x}{a(-c+x)}}\,(-c+x)^2\sqrt{\frac{c(-b+x)}{b(-c+x)}}\,\sqrt{\frac{c(-a+x)}{a(-c+x)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{(a-c)x}{a(-c+x)}},\sqrt{\frac{(-b+c)a}{b(c-a)}}\right)}{(a-c)\,c\sqrt{x(-a+x)(-b+x)(-c+x)}}-\frac{2a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^3+(-b-c)\_Z^2+(bc-d)\_Z+ad\right)}\frac{\left(-3\_\alpha^2a+\_\alpha^2b+\_\alpha^2c+2\_\alpha ab+2\_\alpha ac-2\_\alpha bc-abc+2\_\alpha d-2ad\right)(-c+x)^2\left(-\_\alpha^2+\_\alpha b+d\right)\sqrt{\frac{(a-c)x}{a(-c+x)}}\,\sqrt{\frac{c(-b+x)}{b(-c+x)}}\,\sqrt{\frac{c(-a+x)}{a(-c+x)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{(a-c)x}{a(-c+x)}},\sqrt{\frac{(-b+c)a}{b(c-a)}}\right)+\frac{c\left(\_\alpha^2-\_\alpha b-\_\alpha c+bc-d\right)\mathrm{EllipticPi}\left(\sqrt{\frac{(a-c)x}{a(-c+x)}},\frac{\_\alpha^2c-\_\alpha bc-\_\alpha c^2+bc^2+ad-cd}{d(a-c)},\sqrt{\frac{(-b+c)a}{b(c-a)}}\right)}{ad}\right)}{\left(-3\_\alpha^2+2\_\alpha b+2\_\alpha c-bc+d\right)(a-c)^2\sqrt{x(-a+x)(-b+x)(-c+x)}}\right)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*b*c+2*a*(b+c)*x-(3*a+b+c)*x^2+2*x^3)/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)
/(a*d+(b*c-d)*x-(b+c)*x^2+x^3),x)
```

```
[Out] -4*a*((a-c)*x/a/(-c+x))^(1/2)*(-c+x)^2*(c*(-b+x)/b/(-c+x))^(1/2)*(c*(-a+x)/
a/(-c+x))^(1/2)/(a-c)/c/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)*EllipticF(((a-c)*x/a
/(-c+x))^(1/2),((-b+c)*a/b/(c-a))^(1/2))-2*a/c/d*sum((-3*_alpha^2*a+_alpha^
2*b+_alpha^2*c+2*_alpha*a*b+2*_alpha*a*c-2*_alpha*b*c-a*b*c+2*_alpha*d-2*a*
d)/(-3*_alpha^2+2*_alpha*b+2*_alpha*c-b*c+d)*(-c+x)^2/(a-c)^2*(-_alpha^2+_a
lpha*b+d)*((a-c)*x/a/(-c+x))^(1/2)*(c*(-b+x)/b/(-c+x))^(1/2)*(c*(-a+x)/a/(-
c+x))^(1/2)/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)*(EllipticF(((a-c)*x/a/(-c+x))^(1
/2),((-b+c)*a/b/(c-a))^(1/2))+c*(_alpha^2-_alpha*b-_alpha*c+b*c-d)/a/d*Elli
pticPi(((a-c)*x/a/(-c+x))^(1/2),(_alpha^2*c-_alpha*b*c-_alpha*c^2+b*c^2+a*d
-c*d)/d/(a-c),((-b+c)*a/b/(c-a))^(1/2))),_alpha=RootOf(_Z^3+(-b-c)*_Z^2+(b*
c-d)*_Z+a*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{abc - 2\,a(b+c)x + (3\,a+b+c)x^2 - 2\,x^3}{\sqrt{-(a-x)(b-x)(c-x)x}\,\left((b+c)x^2 - x^3 - ad - (bc-d)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*c+2*a*(b+c)*x-(3*a+b+c)*x^2+2*x^3)/(x*(-a+x)*(-b+x)*(-c+x))
^(1/2)/(a*d+(b*c-d)*x-(b+c)*x^2+x^3),x, algorithm="maxima")
```

```
[Out] integrate((a*b*c - 2*a*(b + c)*x + (3*a + b + c)*x^2 - 2*x^3)/(sqrt(-(a - x
)*(b - x)*(c - x)*x)*((b + c)*x^2 - x^3 - a*d - (b*c - d)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{-2\,x^3+\left(3\,a+b+c\right)\,x^2-2\,a\,\left(b+c\right)\,x+a\,b\,c}{\left(x^3+\left(-b-c\right)\,x^2+\left(b\,c-d\right)\,x+a\,d\right)\,\sqrt{-x\,\left(a-x\right)\,\left(b-x\right)\,\left(c-x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(3*a + b + c) - 2*x^3 - 2*a*x*(b + c) + a*b*c)/((a*d - x^2*(b + c
) - x*(d - b*c) + x^3)*(-x*(a - x)*(b - x)*(c - x))^(1/2)),x)
```

```
[Out] -int((x^2*(3*a + b + c) - 2*x^3 - 2*a*x*(b + c) + a*b*c)/((a*d - x^2*(b + c
) - x*(d - b*c) + x^3)*(-x*(a - x)*(b - x)*(c - x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*c+2*a*(b+c)*x-(3*a+b+c)*x**2+2*x**3)/(x*(-a+x)*(-b+x)*(-c+x
))**(1/2)/(a*d+(b*c-d)*x-(b+c)*x**2+x**3),x)
```

```
[Out] Timed out
```

3.787 $\int \frac{3b+ax^3}{x\left(-b+ax^3\right)\sqrt{b+ax^3}}\,dx$

Optimal. Leaf size=63

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{2}\sqrt{b}}\right)}{3\sqrt{b}}$$

Rubi [A] time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {573, 156, 63, 208, 207}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{2}\sqrt{b}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*b + a*x^3)/(x*(-b + a*x^3)*Sqrt[b + a*x^3]),x]
```

```
[Out] (2*ArcTanh[Sqrt[b + a*x^3]/Sqrt[b]])/Sqrt[b] - (4*Sqrt[2]*ArcTanh[Sqrt[b +
a*x^3]/(Sqrt[2]*Sqrt[b])])/(3*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rubi steps

$$\int \frac{3b+ax^3}{x\left(-b+ax^3\right)\sqrt{b+ax^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{3b+ax}{x(-b+ax)\sqrt{b+ax}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}(4a)\operatorname{Subst}\left(\int \frac{1}{(-b+ax)\sqrt{b+ax}}\,dx, x, x^3\right) - \operatorname{Subst}\left(\int \frac{1}{x\sqrt{b+ax}}\,dx, x, x^3\right)$$
$$= \frac{8}{3}\operatorname{Subst}\left(\int \frac{1}{-2b+x^2}\,dx, x, \sqrt{b+ax^3}\right) - \frac{2\operatorname{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^2}{a}}\,dx, x, \sqrt{b+ax^3}\right)}{a}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b+ax^3}}{\sqrt{b}}\right)}{\sqrt{b}} - \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{b+ax^3}}{\sqrt{2}\sqrt{b}}\right)}{3\sqrt{b}}$$

Mathematica [A] time = 0.08, size = 60, normalized size = 0.95

$$\frac{2\left(3\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right) - 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{2}\sqrt{b}}\right)\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*b + a*x^3)/(x*(-b + a*x^3)*Sqrt[b + a*x^3]),x]
```

```
[Out] (2*(3*ArcTanh[Sqrt[b + a*x^3]/Sqrt[b]] - 2*Sqrt[2]*ArcTanh[Sqrt[b + a*x^3]/
(Sqrt[2]*Sqrt[b])]))/(3*Sqrt[b])
```

IntegrateAlgebraic [A] time = 0.06, size = 63, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{b}}\right)}{\sqrt{b}} - \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{ax^3+b}}{\sqrt{2}\sqrt{b}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*b + a*x^3)/(x*(-b + a*x^3)*Sqrt[b + a*x^3]),x]
```

```
[Out] (2*ArcTanh[Sqrt[b + a*x^3]/Sqrt[b]])/Sqrt[b] - (4*Sqrt[2]*ArcTanh[Sqrt[b +
a*x^3]/(Sqrt[2]*Sqrt[b])])/(3*Sqrt[b])
```

fricas [A] time = 0.43, size = 154, normalized size = 2.44

$$\left[\frac{2\sqrt{2}\sqrt{b}\log\left(\frac{ax^3-2\sqrt{2}\sqrt{ax^3+b}\sqrt{b}+3b}{ax^3-b}\right)+3\sqrt{b}\log\left(\frac{ax^3+2\sqrt{ax^3+b}\sqrt{b}+2b}{x^3}\right)}{3b}, \frac{2\left(2\sqrt{2}b\sqrt{-\frac{1}{b}}\arctan\left(\frac{\sqrt{2}b\sqrt{-\frac{1}{b}}}{\sqrt{ax^3+b}}\right)-3\sqrt{-b}\arctan\left(\frac{\sqrt{ax^3+b}\sqrt{-b}}{b}\right)\right)}{3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+3*b)/x/(a*x^3-b)/(a*x^3+b)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(2*sqrt(2)*sqrt(b)*log((a*x^3 - 2*sqrt(2)*sqrt(a*x^3 + b)*sqrt(b) + 3*
b)/(a*x^3 - b)) + 3*sqrt(b)*log((a*x^3 + 2*sqrt(a*x^3 + b)*sqrt(b) + 2*b)/x
^3))/b, 2/3*(2*sqrt(2)*b*sqrt(-1/b)*arctan(sqrt(2)*b*sqrt(-1/b)/sqrt(a*x^3
+ b)) - 3*sqrt(-b)*arctan(sqrt(a*x^3 + b)*sqrt(-b)/b))/b]
```

giac [A] time = 0.13, size = 54, normalized size = 0.86

$$\frac{4\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{ax^3+b}}{2\sqrt{-b}}\right)}{3\sqrt{-b}} - \frac{2\arctan\left(\frac{\sqrt{ax^3+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+3*b)/x/(a*x^3-b)/(a*x^3+b)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*x^3 + b)/sqrt(-b))/sqrt(-b) - 2*arcta
n(sqrt(a*x^3 + b)/sqrt(-b))/sqrt(-b)
```

maple [C] time = 0.12, size = 433, normalized size = 6.87

$$\frac{2i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(a\_Z^3-b\right)}\frac{\left(-a^2b\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ia\left(2x+\frac{-i\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}+\left(-a^2b\right)^{\frac{1}{3}}}{a}\right)}{\left(-a^2b\right)^{\frac{1}{3}}}}\sqrt{\frac{a\left(x-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{a}\right)}{-3\left(-a^2b\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}}}\sqrt{-\frac{ia\left(2x+\frac{i\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}+\left(-a^2b\right)^{\frac{1}{3}}}{a}\right)}{2\left(-a^2b\right)^{\frac{1}{3}}}}\left(i\left(-a^2b\right)^{\frac{1}{3}}\sqrt{3}\,\_\alpha a-i\left(-a^2b\right)^{\frac{2}{3}}\sqrt{3}+2\_\alpha^2a^2-\left(-a^2b\right)^{\frac{1}{3}}\_\alpha a-\left(-a^2b\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-a^2b\right)^{\frac{1}{3}}}{2a}-\frac{i\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}}{2a}\right)\sqrt{3}\,a}{\left(-a^2b\right)^{\frac{1}{3}}}}}{3},-\frac{2i\left(-a^2b\right)^{\frac{1}{3}}\sqrt{3}\,\_\alpha^2a-i\left(-a^2b\right)^{\frac{2}{3}}\sqrt{3}\,\_\alpha+i\sqrt{3}\,ab-3\left(-a^2b\right)^{\frac{2}{3}}\_\alpha-3ab}{4ab},\sqrt{\frac{i\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}}{a\left(-\frac{3\left(-a^2b\right)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}}{2a}\right)}}\right)}{2\sqrt{a\,x^3+b}}\right)}{3a^2b}+\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^3+b}}{\sqrt{b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+3*b)/x/(a*x^3-b)/(a*x^3+b)^(1/2),x)
```

```
[Out] 2/3*I/a^2/b*2^(1/2)*sum((-a^2*b)^(1/3)*(1/2*I*a*(2*x+1/a*(-I*3^(1/2)*(-a^2*
b)^(1/3)+(-a^2*b)^(1/3)))/(-a^2*b)^(1/3))^(1/2)*(a*(x-1/a*(-a^2*b)^(1/3))/(
-3*(-a^2*b)^(1/3)+I*3^(1/2)*(-a^2*b)^(1/3)))^(1/2)*(-1/2*I*a*(2*x+1/a*(I*3^
(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3)))/(-a^2*b)^(1/3))^(1/2)/(a*x^3+b)^(1/2)
*(I*(-a^2*b)^(1/3)*3^(1/2)*_alpha*a-I*(-a^2*b)^(2/3)*3^(1/2)+2*_alpha^2*a^2
-(-a^2*b)^(1/3)*_alpha*a-(-a^2*b)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/a
*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*3^(1/2)*a/(-a^2*b)^(1/3))^(
1/2),-1/4/a*(2*I*(-a^2*b)^(1/3)*3^(1/2)*_alpha^2*a-I*(-a^2*b)^(2/3)*3^(1/2)
*_alpha+I*3^(1/2)*a*b-3*(-a^2*b)^(2/3)*_alpha-3*a*b)/b,(I*3^(1/2)/a*(-a^2*b
)^(1/3)/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2)),_alp
ha=RootOf(_Z^3*a-b))+2*arctanh((a*x^3+b)^(1/2)/b^(1/2))/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 + 3\,b}{\sqrt{ax^3 + b}\left(ax^3 - b\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+3*b)/x/(a*x^3-b)/(a*x^3+b)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^3 + 3*b)/(sqrt(a*x^3 + b)*(a*x^3 - b)*x), x)
```

mupad [B] time = 1.32, size = 89, normalized size = 1.41

$$\frac{\ln\left(\frac{\left(\sqrt{a\,x^3+b}-\sqrt{b}\right)\left(\sqrt{a\,x^3+b}+\sqrt{b}\right)^3}{x^6}\right)}{\sqrt{b}}+\frac{2\,\sqrt{2}\,\ln\left(\frac{3\,\sqrt{2}\,b-4\,\sqrt{b}\,\sqrt{a\,x^3+b}+\sqrt{2}\,a\,x^3}{b-a\,x^3}\right)}{3\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*b + a*x^3)/(x*(b + a*x^3)^(1/2)*(b - a*x^3)),x)
```

```
[Out] log((((b + a*x^3)^(1/2) - b^(1/2))*((b + a*x^3)^(1/2) + b^(1/2))^3)/x^6)/b^
(1/2) + (2*2^(1/2)*log((3*2^(1/2)*b - 4*b^(1/2)*(b + a*x^3)^(1/2) + 2^(1/2)
*a*x^3)/(b - a*x^3)))/(3*b^(1/2))
```

sympy [A] time = 37.51, size = 66, normalized size = 1.05

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{ax^3+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\frac{4\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{ax^3+b}}{2\sqrt{-b}}\right)}{3\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+3*b)/x/(a*x**3-b)/(a*x**3+b)**(1/2),x)
```

```
[Out] -2*atan(sqrt(a*x**3 + b)/sqrt(-b))/sqrt(-b) + 4*sqrt(2)*atan(sqrt(2)*sqrt(a
*x**3 + b)/(2*sqrt(-b)))/(3*sqrt(-b))
```

3.788 $\int \frac{(2c-ax^3)\sqrt{c+bx^2+ax^3}}{(c+(-3+b)x^2+ax^3)(c+(-2+b)x^2+ax^3)}\,dx$

Optimal. Leaf size=63

$$2\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\,x}{\sqrt{ax^3+bx^2+c}}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{ax^3+bx^2+c}}\right)$$

Rubi [F] time = 3.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(2c-ax^3)\sqrt{c+bx^2+ax^3}}{(c+(-3+b)x^2+ax^3)(c+(-2+b)x^2+ax^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((2*c - a*x^3)*Sqrt[c + b*x^2 + a*x^3])/((c + (-3 + b)*x^2 + a*x^3)*(c
+ (-2 + b)*x^2 + a*x^3)),x]
```

```
[Out] 2*(2 - b)*Defer[Int][Sqrt[c + b*x^2 + a*x^3]/(-c + (2 - b)*x^2 - a*x^3), x]
 - 2*(3 - b)*Defer[Int][Sqrt[c + b*x^2 + a*x^3]/(-c + (3 - b)*x^2 - a*x^3),
 x] + 3*a*Defer[Int][(x*Sqrt[c + b*x^2 + a*x^3])/(-c + (3 - b)*x^2 - a*x^3)
, x] + 3*a*Defer[Int][(x*Sqrt[c + b*x^2 + a*x^3])/(c - (2 - b)*x^2 + a*x^3)
, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2c-ax^3)\sqrt{c+bx^2+ax^3}}{(c+(-3+b)x^2+ax^3)(c+(-2+b)x^2+ax^3)}\,dx &= \int \frac{(2c-ax^3)\sqrt{c+bx^2+ax^3}}{(c-(2-b)x^2+ax^3)(c-(3-b)x^2+ax^3)}\,dx \\
&= \int \left(\frac{(-4+2b+3ax)\sqrt{c+bx^2+ax^3}}{c-(2-b)x^2+ax^3}+\frac{(6-2b-3ax)\sqrt{}}{c-(3-b)}\right. \\
&= \int \frac{(-4+2b+3ax)\sqrt{c+bx^2+ax^3}}{c-(2-b)x^2+ax^3}\,dx+\int \frac{(6-2b-3}{c-(3} \\
&= \int \left(\frac{2\left(1-\frac{3}{b}\right)b\sqrt{c+bx^2+ax^3}}{-c+(3-b)x^2-ax^3}+\frac{3ax\sqrt{c+bx^2+ax^3}}{-c+(3-b)x^2-ax^3}\right. \\
&= (3a)\int \frac{x\sqrt{c+bx^2+ax^3}}{-c+(3-b)x^2-ax^3}\,dx+(3a)\int \frac{x\sqrt{c+bx^2+}}{c-(2-b)x^2}
\end{aligned}$$

Mathematica [C] time = 6.92, size = 21715, normalized size = 344.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2*c - a*x^3)*Sqrt[c + b*x^2 + a*x^3])/((c + (-3 + b)*x^2 + a*x^
3)*(c + (-2 + b)*x^2 + a*x^3)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 2.91, size = 63, normalized size = 1.00

$$2\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\,x}{\sqrt{ax^3+bx^2+c}}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{ax^3+bx^2+c}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2*c - a*x^3)*Sqrt[c + b*x^2 + a*x^3])/((c + (-3 + b)*x
^2 + a*x^3)*(c + (-2 + b)*x^2 + a*x^3)),x]
```

```
[Out] -2*Sqrt[2]*ArcTanh[(Sqrt[2]*x)/Sqrt[c + b*x^2 + a*x^3]] + 2*Sqrt[3]*ArcTanh
[(Sqrt[3]*x)/Sqrt[c + b*x^2 + a*x^3]]
```

fricas [B] time = 0.69, size = 293, normalized size = 4.65

$$\frac{1}{2}\sqrt{2}\log\left(\frac{a^2x^6+2(ab+6a)x^5+2acx^3+(b^2+12b+4)x^4+2(b+6)cx^2-4\sqrt{2}\left(ax^4+(b+2)x^3+cx\right)\sqrt{ax^3+bx^2+c}+c^2}{a^2x^6+2(ab-2a)x^5+2acx^3+\left(b^2-4b+4\right)x^4+2(b-2)cx^2+c^2}\right)+\frac{1}{2}\sqrt{3}\log\left(\frac{a^2x^6+2(ab+9a)x^5+2acx^3+\left(b^2+18b+9\right)x^4+2(b+9)cx^2+4\sqrt{3}\left(ax^4+(b+3)x^3+cx\right)\sqrt{ax^3+bx^2+c}+c^2}{a^2x^6+2(ab-3a)x^5+2acx^3+\left(b^2-6b+9\right)x^4+2(b-3)cx^2+c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^3+2*c)*(a*x^3+b*x^2+c)^(1/2)/(c+(-3+b)*x^2+a*x^3)/(c+(-2+b)
*x^2+a*x^3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((a^2*x^6 + 2*(a*b + 6*a)*x^5 + 2*a*c*x^3 + (b^2 + 12*b + 4)
*x^4 + 2*(b + 6)*c*x^2 - 4*sqrt(2)*(a*x^4 + (b + 2)*x^3 + c*x)*sqrt(a*x^3 +
 b*x^2 + c) + c^2)/(a^2*x^6 + 2*(a*b - 2*a)*x^5 + 2*a*c*x^3 + (b^2 - 4*b +
4)*x^4 + 2*(b - 2)*c*x^2 + c^2)) + 1/2*sqrt(3)*log((a^2*x^6 + 2*(a*b + 9*a)
*x^5 + 2*a*c*x^3 + (b^2 + 18*b + 9)*x^4 + 2*(b + 9)*c*x^2 + 4*sqrt(3)*(a*x^
4 + (b + 3)*x^3 + c*x)*sqrt(a*x^3 + b*x^2 + c) + c^2)/(a^2*x^6 + 2*(a*b - 3
*a)*x^5 + 2*a*c*x^3 + (b^2 - 6*b + 9)*x^4 + 2*(b - 3)*c*x^2 + c^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{ax^3+bx^2+c}\left(ax^3-2c\right)}{\left(ax^3+(b-2)x^2+c\right)\left(ax^3+(b-3)x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^3+2*c)*(a*x^3+b*x^2+c)^(1/2)/(c+(-3+b)*x^2+a*x^3)/(c+(-2+b)
*x^2+a*x^3),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(a*x^3 + b*x^2 + c)*(a*x^3 - 2*c)/((a*x^3 + (b - 2)*x^2 + c)
*(a*x^3 + (b - 3)*x^2 + c)), x)
```

maple [C] time = 5.56, size = 16170, normalized size = 256.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x^3+2*c)*(a*x^3+b*x^2+c)^(1/2)/(c+(-3+b)*x^2+a*x^3)/(c+(-2+b)*x^2+a
*x^3),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{ax^3+bx^2+c}\left(ax^3-2c\right)}{\left(ax^3+(b-2)x^2+c\right)\left(ax^3+(b-3)x^2+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^3+2*c)*(a*x^3+b*x^2+c)^(1/2)/(c+(-3+b)*x^2+a*x^3)/(c+(-2+b)
*x^2+a*x^3),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(a*x^3 + b*x^2 + c)*(a*x^3 - 2*c)/((a*x^3 + (b - 2)*x^2 + c)
*(a*x^3 + (b - 3)*x^2 + c)), x)
```

mupad [B] time = 34.98, size = 125, normalized size = 1.98

$$\sqrt{2}\ \ln\left(\frac{c+a\,x^3+b\,x^2+2\,x^2-2\,\sqrt{2}\,x\,\sqrt{a\,x^3+b\,x^2+c}}{c+a\,x^3+b\,x^2-2\,x^2}\right)+\sqrt{3}\ \ln\left(\frac{c+a\,x^3+b\,x^2+3\,x^2+2\,\sqrt{3}\,x\,\sqrt{a\,x^3+b\,x^2+c}}{c+a\,x^3+b\,x^2-3\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*c - a*x^3)*(c + a*x^3 + b*x^2)^(1/2))/((c + a*x^3 + x^2*(b - 2))*(c
 + a*x^3 + x^2*(b - 3))),x)
```

```
[Out] 2^(1/2)*log((c + a*x^3 + b*x^2 + 2*x^2 - 2*2^(1/2)*x*(c + a*x^3 + b*x^2)^(1
/2))/(c + a*x^3 + b*x^2 - 2*x^2)) + 3^(1/2)*log((c + a*x^3 + b*x^2 + 3*x^2
+ 2*3^(1/2)*x*(c + a*x^3 + b*x^2)^(1/2))/(c + a*x^3 + b*x^2 - 3*x^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x**3+2*c)*(a*x**3+b*x**2+c)**(1/2)/(c+(-3+b)*x**2+a*x**3)/(c+
(-2+b)*x**2+a*x**3),x)
```

```
[Out] Timed out
```

3.789 $$\int \frac{-abc+2a(b+c)x-(3a+b+c)x^2+2x^3}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(a+(-1+bcd)x-(b+c)dx^2+dx^3\right)}\,dx$$

Optimal. Leaf size=63

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{x^3(-a-b-c)+x^2(ab+ac+bc)-abcx+x^4}}{a-x}\right)}{\sqrt{d}}$$

Rubi [F] time = 4.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-abc+2a(b+c)x-(3a+b+c)x^2+2x^3}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(a+(-1+bcd)x-(b+c)dx^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a*b*c) + 2*a*(b + c)*x - (3*a + b + c)*x^2 + 2*x^3)/(Sqrt[x*(-a + x)
*(-b + x)*(-c + x)]*(a + (-1 + b*c*d)*x - (b + c)*d*x^2 + d*x^3)),x]
```

```
[Out] (2*Defer[Int][1/Sqrt[x*(-a + x)*(-b + x)*(-c + x)], x])/d - (a*(2 + b*c*d)*
Defer[Int][1/(Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*(a - (1 - b*c*d)*x - (b +
c)*d*x^2 + d*x^3)), x])/d + (2*(1 - b*c*d + a*(b + c)*d)*Defer[Int][x/(Sqrt
[x*(-a + x)*(-b + x)*(-c + x)]*(a - (1 - b*c*d)*x - (b + c)*d*x^2 + d*x^3))
, x])/d - (3*a - b - c)*Defer[Int][x^2/(Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*
(a - (1 - b*c*d)*x - (b + c)*d*x^2 + d*x^3)), x]
```

Rubi steps

$$\int \frac{-abc+2a(b+c)x-(3a+b+c)x^2+2x^3}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(a+(-1+bcd)x-(b+c)dx^2+dx^3\right)}\,dx = \int \frac{-abc+2a(b+c)x-(3a}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(a-(\right.}$$
$$= \int \left(\frac{2}{d\sqrt{x(-a+x)(-b+x)(-c+x)}} - \right.$$
$$= -\frac{\int \frac{a(2+bcd)-2(1-bcd+a(b+c)d)x+(3a-b-c)}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(a-(1-bcd)x-(b+c)d\right.}}{d}$$
$$= -\frac{\int \left(\frac{a(2+bcd)}{\sqrt{x(-a+x)(-b+x)(-c+x)}\left(a-(1-bcd)x-(b+c\right.}\right.}{}$$
$$= -\left((3a-b-c)\int \frac{}{\sqrt{x(-a+x)(-b+x)}}\right.$$

Mathematica [C] time = 13.32, size = 8822, normalized size = 140.03

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-(a*b*c) + 2*a*(b + c)*x - (3*a + b + c)*x^2 + 2*x^3)/(Sqrt[x*(-
a + x)*(-b + x)*(-c + x)]*(a + (-1 + b*c*d)*x - (b + c)*d*x^2 + d*x^3)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.54, size = 63, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{x^3(-a-b-c)+x^2(ab+ac+bc)-abcx+x^4}}{a-x}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*b*c) + 2*a*(b + c)*x - (3*a + b + c)*x^2 + 2*x^3)/(
Sqrt[x*(-a + x)*(-b + x)*(-c + x)]*(a + (-1 + b*c*d)*x - (b + c)*d*x^2 + d*
x^3)),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[-(a*b*c*x) + (a*b + a*c + b*c)*x^2 + (-a - b - c)*
x^3 + x^4])/(a - x)])/Sqrt[d]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*c+2*a*(b+c)*x-(3*a+b+c)*x^2+2*x^3)/(x*(-a+x)*(-b+x)*(-c+x))
^(1/2)/(a+(b*c*d-1)*x-(b+c)*d*x^2+d*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{abc-2\,a(b+c)x+(3\,a+b+c)x^2-2\,x^3}{\sqrt{-(a-x)(b-x)(c-x)x}\left((b+c)dx^2-dx^3-(bcd-1)x-a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*c+2*a*(b+c)*x-(3*a+b+c)*x^2+2*x^3)/(x*(-a+x)*(-b+x)*(-c+x))
^(1/2)/(a+(b*c*d-1)*x-(b+c)*d*x^2+d*x^3),x, algorithm="giac")
```

```
[Out] integrate((a*b*c - 2*a*(b + c)*x + (3*a + b + c)*x^2 - 2*x^3)/(sqrt(-(a - x
)*(b - x)*(c - x)*x)*((b + c)*d*x^2 - d*x^3 - (b*c*d - 1)*x - a)), x)
```

maple [C] time = 0.10, size = 502, normalized size = 7.97

$$-\frac{4a\sqrt{\frac{(a-c)x}{a(-c+x)}}\,(-c+x)^2\sqrt{\frac{c(-b+x)}{b(-c+x)}}\sqrt{\frac{c(-a+x)}{a(-c+x)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{(a-c)x}{a(-c+x)}},\sqrt{\frac{(-b+c)a}{b(c-a)}}\right)}{d(a-c)c\sqrt{x(-a+x)(-b+x)(-c+x)}}-\frac{2a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+(-bd-cd)\_Z^2+(bcd-1)\_Z+a\right)}\frac{\left(-3\_\alpha^2ad+\_\alpha^2bd+\_\alpha^2cd+2\_\alpha abd+2\_\alpha acd-2\_\alpha bcd-abcd+2\_\alpha-2a\right)(-c+x)^2\left(-d\_\alpha^2+\_\alpha bd+1\right)\sqrt{\frac{(a-c)x}{a(-c+x)}}\sqrt{\frac{c(-b+x)}{b(-c+x)}}\sqrt{\frac{c(-a+x)}{a(-c+x)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{(a-c)x}{a(-c+x)}},\sqrt{\frac{(-b+c)a}{b(c-a)}}\right)+\frac{c\left(d\_\alpha^2-\_\alpha bd-\_\alpha cd+bcd-1\right)\mathrm{EllipticPi}\left(\sqrt{\frac{(a-c)x}{a(-c+x)}},\frac{\_\alpha^2cd-\_\alpha bcd-\_\alpha c^2d+bc^2d+a-c}{a-c},\sqrt{\frac{(-b+c)a}{b(c-a)}}\right)}{a}\right)}{\left(-3d\_\alpha^2+2\_\alpha bd+2\_\alpha cd-bcd+1\right)(a-c)^2\sqrt{x(-a+x)(-b+x)(-c+x)}}\right)}{dc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*b*c+2*a*(b+c)*x-(3*a+b+c)*x^2+2*x^3)/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)
/(a+(b*c*d-1)*x-(b+c)*d*x^2+d*x^3),x)
```

```
[Out] -4/d*a*((a-c)*x/a/(-c+x))^(1/2)*(-c+x)^2*(c*(-b+x)/b/(-c+x))^(1/2)*(c*(-a+x
)/a/(-c+x))^(1/2)/(a-c)/c/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)*EllipticF(((a-c)*x
/a/(-c+x))^(1/2),((-b+c)*a/b/(c-a))^(1/2))-2/d*a/c*sum((-3*_alpha^2*a*d+_al
pha^2*b*d+_alpha^2*c*d+2*_alpha*a*b*d+2*_alpha*a*c*d-2*_alpha*b*c*d-a*b*c*d
+2*_alpha-2*a)/(-3*_alpha^2*d+2*_alpha*b*d+2*_alpha*c*d-b*c*d+1)*(-c+x)^2/(
a-c)^2*(-_alpha^2*d+_alpha*b*d+1)*((a-c)*x/a/(-c+x))^(1/2)*(c*(-b+x)/b/(-c+
x))^(1/2)*(c*(-a+x)/a/(-c+x))^(1/2)/(x*(-a+x)*(-b+x)*(-c+x))^(1/2)*(Ellipti
cF(((a-c)*x/a/(-c+x))^(1/2),((-b+c)*a/b/(c-a))^(1/2))+c*(_alpha^2*d-_alpha*
b*d-_alpha*c*d+b*c*d-1)/a*EllipticPi(((a-c)*x/a/(-c+x))^(1/2),(_alpha^2*c*d
-_alpha*b*c*d-_alpha*c^2*d+b*c^2*d+a-c)/(a-c),((-b+c)*a/b/(c-a))^(1/2))),_a
lpha=RootOf(d*_Z^3+(-b*d-c*d)*_Z^2+(b*c*d-1)*_Z+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{abc-2\,a(b+c)x+(3\,a+b+c)x^2-2\,x^3}{\sqrt{-(a-x)(b-x)(c-x)x}\left((b+c)dx^2-dx^3-(bcd-1)x-a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*c+2*a*(b+c)*x-(3*a+b+c)*x^2+2*x^3)/(x*(-a+x)*(-b+x)*(-c+x))
^(1/2)/(a+(b*c*d-1)*x-(b+c)*d*x^2+d*x^3),x, algorithm="maxima")
```

```
[Out] integrate((a*b*c - 2*a*(b + c)*x + (3*a + b + c)*x^2 - 2*x^3)/(sqrt(-(a - x
)*(b - x)*(c - x)*x)*((b + c)*d*x^2 - d*x^3 - (b*c*d - 1)*x - a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{-2\,x^3+(3\,a+b+c)\,x^2-2\,a\,(b+c)\,x+a\,b\,c}{\sqrt{-x\,(a-x)\,(b-x)\,(c-x)}\,\left(d\,x^3-d\,(b+c)\,x^2+(b\,c\,d-1)\,x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(3*a + b + c) - 2*x^3 - 2*a*x*(b + c) + a*b*c)/((-x*(a - x)*(b -
x)*(c - x))^(1/2)*(a + d*x^3 + x*(b*c*d - 1) - d*x^2*(b + c))),x)
```

```
[Out] -int((x^2*(3*a + b + c) - 2*x^3 - 2*a*x*(b + c) + a*b*c)/((-x*(a - x)*(b -
x)*(c - x))^(1/2)*(a + d*x^3 + x*(b*c*d - 1) - d*x^2*(b + c))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*c+2*a*(b+c)*x-(3*a+b+c)*x**2+2*x**3)/(x*(-a+x)*(-b+x)*(-c+x
))**(1/2)/(a+(b*c*d-1)*x-(b+c)*d*x**2+d*x**3),x)
```

```
[Out] Timed out
```

3.790 $\int \frac{-1+x^3}{(1+x^3)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=63

$$-\frac{4}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}+x^2-x+1}\right)-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}+x^2+2x+1}\right)$$

Rubi [C] time = 1.04, antiderivative size = 380, normalized size of antiderivative = 6.03, number of steps used = 29, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {6725, 220, 2074, 1725, 1211, 1699, 206, 1248, 725, 6728, 1217, 1707}

$$-\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{x^2+1}{\sqrt{2}\sqrt{x^4+1}}\right)}{3\sqrt{2}}-\frac{\left(1+i\sqrt{3}\right)\tanh^{-1}\left(\frac{2-\left(1-i\sqrt{3}\right)x^2}{\sqrt{2\left(1-i\sqrt{3}\right)}\sqrt{x^4+1}}\right)}{3\sqrt{2\left(1-i\sqrt{3}\right)}}-\frac{\left(1-i\sqrt{3}\right)\tanh^{-1}\left(\frac{2-\left(1+i\sqrt{3}\right)x^2}{\sqrt{2\left(1+i\sqrt{3}\right)}\sqrt{x^4+1}}\right)}{3\sqrt{2\left(1+i\sqrt{3}\right)}}-\frac{\left(1+i\sqrt{3}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\sqrt{x^4+1}}-\frac{\left(1-i\sqrt{3}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\sqrt{x^4+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/((1 + x^3)*Sqrt[1 + x^4]),x]
```

```
[Out] (-2*ArcTan[x/Sqrt[1 + x^4]])/3 - ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(3*Sqrt
[2]) + ArcTanh[(1 + x^2)/(Sqrt[2]*Sqrt[1 + x^4])]/(3*Sqrt[2]) - ((1 + I*Sqr
t[3])*ArcTanh[(2 - (1 - I*Sqrt[3])*x^2)/(Sqrt[2*(1 - I*Sqrt[3])]*Sqrt[1 + x
^4])])/(3*Sqrt[2*(1 - I*Sqrt[3])]) - ((1 - I*Sqrt[3])*ArcTanh[(2 - (1 + I*S
qrt[3])*x^2)/(Sqrt[2*(1 + I*Sqrt[3])]*Sqrt[1 + x^4])])/(3*Sqrt[2*(1 + I*Sqr
t[3])]) + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2
])/(3*Sqrt[1 + x^4]) - ((1 - I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^
2]*EllipticF[2*ArcTan[x], 1/2])/(6*Sqrt[1 + x^4]) - ((1 + I*Sqrt[3])*(1 + x
^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(6*Sqrt[1 + x^
4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
```

```
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[d,
 Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Dist[e, Int[x/((d^2 - e^
2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-1+x^3}{\left(1+x^3\right)\sqrt{1+x^4}}\,dx = \int \left(\frac{1}{\sqrt{1+x^4}} - \frac{2}{\left(1+x^3\right)\sqrt{1+x^4}}\right)dx$$

$$= -\left(2\int \frac{1}{\left(1+x^3\right)\sqrt{1+x^4}}\,dx\right) + \int \frac{1}{\sqrt{1+x^4}}\,dx$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{1+x^4}} - 2\int\left(\frac{1}{3(1+x)\sqrt{1+x^4}} + \frac{2-x}{3\left(1-x+x^2\right)\sqrt{1+x\ldots}}\right.$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{1+x^4}} - \frac{2}{3}\int \frac{1}{(1+x)\sqrt{1+x^4}}\,dx - \frac{2}{3}\int \frac{2-x}{\left(1-x+x^2\right)\sqrt{\ldots}}$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{1+x^4}} - \frac{2}{3}\int \frac{1}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx + \frac{2}{3}\int \frac{x}{\left(1-x^2\right)\sqrt{1+\ldots}}$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{1+x^4}} - \frac{1}{3}\int \frac{1}{\sqrt{1+x^4}}\,dx - \frac{1}{3}\int \frac{1+x^2}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx + \ldots$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{1+x^4}} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right) - \frac{1}{3}\text{Subs}\ldots$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{3\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{1+x^2}{\sqrt{2}\sqrt{1+x^4}}\right)}{3\sqrt{2}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{1+x^4}} - \frac{1}{3}(2\ldots$$

$$= -\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt{1+x^4}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{3\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{1+x^2}{\sqrt{2}\sqrt{1+x^4}}\right)}{3\sqrt{2}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\ldots}{3\sqrt{1+\ldots}}$$

$$= -\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt{1+x^4}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{3\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{1+x^2}{\sqrt{2}\sqrt{1+x^4}}\right)}{3\sqrt{2}} - \frac{\left(1+i\sqrt{3}\right)\tanh^{-1}\left(\ldots\right.}{3\sqrt{2\left(1-\ldots\right.}}$$

Mathematica [C] time = 0.72, size = 525, normalized size = 8.33

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + x^3)/((1 + x^3)*Sqrt[1 + x^4]),x]
```

```
[Out] (2*Sqrt[1 - (-1)^(1/3)]*ArcTanh[((-1)^(1/3) - x^2)/(Sqrt[1 + (-1)^(2/3)]*Sq
rt[1 + x^4])] - 2*(-1)^(1/3)*Sqrt[1 - (-1)^(1/3)]*ArcTanh[((-1)^(1/3) - x^2
)/(Sqrt[1 + (-1)^(2/3)]*Sqrt[1 + x^4])] - 4*(-1)^(2/3)*Sqrt[1 - (-1)^(1/3)]
*ArcTanh[((-1)^(1/3) - x^2)/(Sqrt[1 + (-1)^(2/3)]*Sqrt[1 + x^4])] + 2*Sqrt[
2]*ArcTanh[(1 + x^2)/(Sqrt[2]*Sqrt[1 + x^4])] + (-1)^(1/3)*Sqrt[2]*ArcTanh[
```

```
(1 + x^2)/(Sqrt[2]*Sqrt[1 + x^4])] - (-1)^(2/3)*Sqrt[2]*ArcTanh[(1 + x^2)/(
Sqrt[2]*Sqrt[1 + x^4])] - 2*Sqrt[1 + (-1)^(2/3)]*ArcTanh[((-1)^(2/3) + x^2)
/(Sqrt[1 - (-1)^(1/3)]*Sqrt[1 + x^4])] - 4*(-1)^(1/3)*Sqrt[1 + (-1)^(2/3)]*
ArcTanh[((-1)^(2/3) + x^2)/(Sqrt[1 - (-1)^(1/3)]*Sqrt[1 + x^4])] - 2*(-1)^(
2/3)*Sqrt[1 + (-1)^(2/3)]*ArcTanh[((-1)^(2/3) + x^2)/(Sqrt[1 - (-1)^(1/3)]*
Sqrt[1 + x^4])] - 18*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] + 12
*(-1)^(1/4)*EllipticPi[I, ArcSin[(-1)^(3/4)*x], -1] + 12*(-1)^(7/12)*Ellipt
icPi[-(-1)^(1/6), I*ArcSinh[(-1)^(1/4)*x], -1] - 12*(-1)^(11/12)*EllipticPi
[-(-1)^(1/6), I*ArcSinh[(-1)^(1/4)*x], -1] + 12*(-1)^(1/4)*EllipticPi[-(-1)
^(5/6), I*ArcSinh[(-1)^(1/4)*x], -1])/18
```

IntegrateAlgebraic [A] time = 1.05, size = 63, normalized size = 1.00

$$-\frac{4}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}+x^2-x+1}\right)-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}+x^2+2x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)/((1 + x^3)*Sqrt[1 + x^4]),x]
```

```
[Out] (-4*ArcTan[x/(1 - x + x^2 + Sqrt[1 + x^4])])/3 - (Sqrt[2]*ArcTanh[(Sqrt[2]*
x)/(1 + 2*x + x^2 + Sqrt[1 + x^4])])/3
```

fricas [A] time = 0.63, size = 88, normalized size = 1.40

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{3x^4+4x^3+2\sqrt{2}\sqrt{x^4+1}\left(x^2+x+1\right)+6x^2+4x+3}{x^4+4x^3+6x^2+4x+1}\right)-\frac{2}{3}\arctan\left(\frac{\sqrt{x^4+1}}{x^2-2x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^3+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*log(-(3*x^4 + 4*x^3 + 2*sqrt(2)*sqrt(x^4 + 1)*(x^2 + x + 1) +
6*x^2 + 4*x + 3)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1)) - 2/3*arctan(sqrt(x^4 + 1
)/(x^2 - 2*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3-1}{\sqrt{x^4+1}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^3+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 1)/(sqrt(x^4 + 1)*(x^3 + 1)), x)
```

maple [C] time = 0.02, size = 374, normalized size = 5.94

$$\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4\sqrt{x^4+1}}\right)}{6}+\frac{2\left(-1\right)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left(\left(-1\right)^{\frac{1}{4}}x,-i,-\sqrt{-i}\,\left(-1\right)^{\frac{3}{4}}\right)}{3\sqrt{x^4+1}}+\frac{2\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{\mathrm{arctanh}\left(\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x^2-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\sqrt{\frac{1}{2}-\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+1}}\right)}{2\sqrt{\frac{1}{2}-\frac{i\sqrt{3}}{2}}}+\frac{\left(-1\right)^{\frac{3}{4}}\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left(\left(-1\right)^{\frac{1}{4}}x,i\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right),i\right)}{\sqrt{x^4+1}}\right)}{3}+\frac{2\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(-\frac{\mathrm{arctanh}\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(x^2-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{\sqrt{\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+1}}\right)}{2\sqrt{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}+\frac{\left(-1\right)^{\frac{3}{4}}\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left(\left(-1\right)^{\frac{1}{4}}x,i\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right),i\right)}{\sqrt{x^4+1}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x^3+1)/(x^4+1)^(1/2),x)
```

```
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+1/6*2^(1/2)*arctanh(1/4*(2*x^2+
2)*2^(1/2)/(x^4+1)^(1/2))+2/3*(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x
^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/4))+2/3*(1/2+1/2*
I*3^(1/2))*(-1/2/(1/2-1/2*I*3^(1/2))^(1/2)*arctanh((-1/2+1/2*I*3^(1/2))*(x^
```

```
2-1/2-1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))^(1/2)/(x^4+1)^(1/2))+(-1)^(3/4)*(1
/2-1/2*I*3^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi(
(-1)^(1/4)*x,I*(1/2+1/2*I*3^(1/2)),I))+2/3*(1/2-1/2*I*3^(1/2))*(-1/2/(1/2+1
/2*I*3^(1/2))^(1/2)*arctanh((-1/2-1/2*I*3^(1/2))*(x^2-1/2+1/2*I*3^(1/2))/(1
/2+1/2*I*3^(1/2))^(1/2)/(x^4+1)^(1/2))+(-1)^(3/4)*(1/2+1/2*I*3^(1/2))*(1-I*
x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I*(1/2-1/2
*I*3^(1/2)),I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 - 1}{\sqrt{x^4 + 1}\left(x^3 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^3+1)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x^3 - 1)/(sqrt(x^4 + 1)*(x^3 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3 - 1}{\left(x^3 + 1\right)\sqrt{x^4 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/((x^3 + 1)*(x^4 + 1)^(1/2)),x)
[Out] int((x^3 - 1)/((x^3 + 1)*(x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x - 1)\left(x^2 + x + 1\right)}{(x + 1)\sqrt{x^4 + 1}\left(x^2 - x + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(x**3+1)/(x**4+1)**(1/2),x)
[Out] Integral((x - 1)*(x**2 + x + 1)/((x + 1)*sqrt(x**4 + 1)*(x**2 - x + 1)), x)
```

3.791 $\int x^4 \sqrt[4]{-x+x^4}\, dx$

Optimal. Leaf size=63

$$\frac{1}{16}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x}}\right)-\frac{1}{16}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x}}\right)+\frac{1}{24}\sqrt[4]{x^4-x}\left(4x^5-x^2\right)$$

Rubi [B] time = 0.12, antiderivative size = 127, normalized size of antiderivative = 2.02, number of steps used = 9, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2021, 2024, 2032, 329, 275, 331, 298, 203, 206}

$$\frac{1}{6}\sqrt[4]{x^4-x}\,x^5-\frac{1}{24}\sqrt[4]{x^4-x}\,x^2+\frac{\left(x^3-1\right)^{3/4}x^{3/4}\tan^{-1}\left(\frac{x^{3/4}}{\sqrt[4]{x^3-1}}\right)}{16\left(x^4-x\right)^{3/4}}-\frac{\left(x^3-1\right)^{3/4}x^{3/4}\tanh^{-1}\left(\frac{x^{3/4}}{\sqrt[4]{x^3-1}}\right)}{16\left(x^4-x\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(-x + x^4)^(1/4),x]
```

```
[Out] -1/24*(x^2*(-x + x^4)^(1/4)) + (x^5*(-x + x^4)^(1/4))/6 + (x^(3/4)*(-1 + x^
3)^(3/4)*ArcTan[x^(3/4)/(-1 + x^3)^(1/4)])/(16*(-x + x^4)^(3/4)) - (x^(3/4)
*(-1 + x^3)^(3/4)*ArcTanh[x^(3/4)/(-1 + x^3)^(1/4)])/(16*(-x + x^4)^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^4\sqrt[4]{-x+x^4}\,dx=\frac{1}{6}x^5\sqrt[4]{-x+x^4}-\frac{1}{8}\int\frac{x^5}{\left(-x+x^4\right)^{3/4}}\,dx$$

$$=-\frac{1}{24}x^2\sqrt[4]{-x+x^4}+\frac{1}{6}x^5\sqrt[4]{-x+x^4}-\frac{3}{32}\int\frac{x^2}{\left(-x+x^4\right)^{3/4}}\,dx$$

$$=-\frac{1}{24}x^2\sqrt[4]{-x+x^4}+\frac{1}{6}x^5\sqrt[4]{-x+x^4}-\frac{\left(3x^{3/4}\left(-1+x^3\right)^{3/4}\right)\int\frac{x^{5/4}}{\left(-1+x^3\right)^{3/4}}\,dx}{32\left(-x+x^4\right)^{3/4}}$$

$$=-\frac{1}{24}x^2\sqrt[4]{-x+x^4}+\frac{1}{6}x^5\sqrt[4]{-x+x^4}-\frac{\left(3x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subst}\left(\int\frac{x^8}{\left(-1+x^{12}\right)^{3/4}}\,dx,x,\sqrt[4]{x}\right)}{8\left(-x+x^4\right)^{3/4}}$$

$$=-\frac{1}{24}x^2\sqrt[4]{-x+x^4}+\frac{1}{6}x^5\sqrt[4]{-x+x^4}-\frac{\left(x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subst}\left(\int\frac{x^2}{\left(-1+x^4\right)^{3/4}}\,dx,x,x^{3/4}\right)}{8\left(-x+x^4\right)^{3/4}}$$

$$=-\frac{1}{24}x^2\sqrt[4]{-x+x^4}+\frac{1}{6}x^5\sqrt[4]{-x+x^4}-\frac{\left(x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subst}\left(\int\frac{x^2}{1-x^4}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{-1+x^3}}\right)}{8\left(-x+x^4\right)^{3/4}}$$

$$=-\frac{1}{24}x^2\sqrt[4]{-x+x^4}+\frac{1}{6}x^5\sqrt[4]{-x+x^4}-\frac{\left(x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{-1+x^3}}\right)}{16\left(-x+x^4\right)^{3/4}}+\frac{\left(x^{3/4}\right.}{}$$

$$=-\frac{1}{24}x^2\sqrt[4]{-x+x^4}+\frac{1}{6}x^5\sqrt[4]{-x+x^4}+\frac{x^{3/4}\left(-1+x^3\right)^{3/4}\tan^{-1}\left(\frac{x^{3/4}}{\sqrt[4]{-1+x^3}}\right)}{16\left(-x+x^4\right)^{3/4}}-\frac{x^{3/4}\left(-1+x^3\right)^{3/4}\tan}{16\left(-x+x\right.}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.89

$$\frac{x^2\sqrt[4]{x\left(x^3-1\right)}\left({}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};x^3\right)-\left(1-x^3\right)^{5/4}\right)}{6\sqrt[4]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(-x + x^4)^(1/4),x]
```

```
[Out] (x^2*(x*(-1 + x^3))^(1/4)*(-(1 - x^3)^(5/4) + Hypergeometric2F1[-1/4, 3/4,
7/4, x^3]))/(6*(1 - x^3)^(1/4))
```

IntegrateAlgebraic [A] time = 0.25, size = 63, normalized size = 1.00

$$\frac{1}{16}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x}}\right)-\frac{1}{16}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x}}\right)+\frac{1}{24}\sqrt[4]{x^4-x}\left(4x^5-x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(-x + x^4)^(1/4),x]
```

```
[Out] ((-x + x^4)^(1/4)*(-x^2 + 4*x^5))/24 + ArcTan[x/(-x + x^4)^(1/4)]/16 - ArcT
anh[x/(-x + x^4)^(1/4)]/16
```

fricas [A] time = 2.03, size = 99, normalized size = 1.57

$$\frac{1}{24}\left(4x^5-x^2\right)\left(x^4-x\right)^{\frac{1}{4}}-\frac{1}{32}\arctan\left(2\left(x^4-x\right)^{\frac{1}{4}}x^2+2\left(x^4-x\right)^{\frac{3}{4}}\right)+\frac{1}{32}\log\left(2x^3-2\left(x^4-x\right)^{\frac{1}{4}}x^2+2\sqrt{x^4-x}\,x-2\left(x^4-x\right)^{\frac{3}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^4-x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/24*(4*x^5 - x^2)*(x^4 - x)^(1/4) - 1/32*arctan(2*(x^4 - x)^(1/4)*x^2 + 2*
(x^4 - x)^(3/4)) + 1/32*log(2*x^3 - 2*(x^4 - x)^(1/4)*x^2 + 2*sqrt(x^4 - x)
*x - 2*(x^4 - x)^(3/4) - 1)
```

giac [A] time = 0.24, size = 68, normalized size = 1.08

$$-\frac{1}{24}\left(\left(-\frac{1}{x^3}+1\right)^{\frac{5}{4}}+3\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}\right)x^6+\frac{1}{16}\arctan\left(\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}\right)+\frac{1}{32}\log\left(\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}+1\right)-\frac{1}{32}\log\left(\left|\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^4-x)^(1/4),x, algorithm="giac")
```

```
[Out] -1/24*((-1/x^3 + 1)^(5/4) + 3*(-1/x^3 + 1)^(1/4))*x^6 + 1/16*arctan((-1/x^3
 + 1)^(1/4)) + 1/32*log((-1/x^3 + 1)^(1/4) + 1) - 1/32*log(abs((-1/x^3 + 1)
^(1/4) - 1))
```

maple [C] time = 4.01, size = 451, normalized size = 7.16

$$\frac{x^2\left(4x^3-1\right)\left(x\left(x^3-1\right)\right)^{\frac{1}{4}}}{24}+\frac{\left(\frac{\ln\left(-\frac{-2x^9+2\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}x^6+5x^6-2\sqrt{x^{12}-3x^9+3x^6-x^3}\,x^3-4\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}x^3+2\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{3}{4}}-4x^3+2\sqrt{x^{12}-3x^9+3x^6-x^3}+2\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}+1}{\left(-1+x\right)^2\left(x^2+x+1\right)^2}\right)}{32}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{2x^9-2\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^2+1\right)x^6-5x^6-2\sqrt{x^{12}-3x^9+3x^6-x^3}\,x^3+4\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^2+1\right)x^3+2\operatorname{RootOf}\left(\_Z^2+1\right)\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{3}{4}}+4x^3+2\sqrt{x^{12}-3x^9+3x^6-x^3}-2\operatorname{RootOf}\left(\_Z^2+1\right)\left(x^{12}-3x^9+3x^6-x^3\right)^{\frac{1}{4}}-1}{\left(-1+x\right)^2\left(x^2+x+1\right)^2}\right)}{32}\right)\left(x\left(x^3-1\right)\right)^{\frac{1}{4}}\left(x^3\left(x^3-1\right)^3\right)^{\frac{1}{4}}}{x\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^4-x)^(1/4),x)
```

```
[Out] 1/24*x^2*(4*x^3-1)*(x*(x^3-1))^(1/4)+(1/32*ln(-(-2*x^9+2*(x^12-3*x^9+3*x^6-
x^3)^(1/4)*x^6+5*x^6-2*(x^12-3*x^9+3*x^6-x^3)^(1/2)*x^3-4*(x^12-3*x^9+3*x^6
-x^3)^(1/4)*x^3+2*(x^12-3*x^9+3*x^6-x^3)^(3/4)-4*x^3+2*(x^12-3*x^9+3*x^6-x^
3)^(1/2)+2*(x^12-3*x^9+3*x^6-x^3)^(1/4)+1)/(-1+x)^2/(x^2+x+1)^2)+1/32*RootO
f(_Z^2+1)*ln(-(2*x^9-2*(x^12-3*x^9+3*x^6-x^3)^(1/4)*RootOf(_Z^2+1)*x^6-5*x^
6-2*(x^12-3*x^9+3*x^6-x^3)^(1/2)*x^3+4*(x^12-3*x^9+3*x^6-x^3)^(1/4)*RootOf(
_Z^2+1)*x^3+2*RootOf(_Z^2+1)*(x^12-3*x^9+3*x^6-x^3)^(3/4)+4*x^3+2*(x^12-3*x
^9+3*x^6-x^3)^(1/2)-2*RootOf(_Z^2+1)*(x^12-3*x^9+3*x^6-x^3)^(1/4)-1)/(-1+x)
^2/(x^2+x+1)^2))*(x*(x^3-1))^(1/4)/x*(x^3*(x^3-1)^3)^(1/4)/(x^3-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^4-x\right)^{\frac{1}{4}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^4-x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x)^(1/4)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^4\,{\left(x^4-x\right)}^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^4 - x)^(1/4),x)
```

```
[Out] int(x^4*(x^4 - x)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \sqrt[4]{x (x-1) \left(x^2+x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**4-x)**(1/4),x)
```

```
[Out] Integral(x**4*(x*(x - 1)*(x**2 + x + 1))**(1/4), x)
```

3.792 $\int \sqrt[4]{-x^3+x^4}\,dx$

Optimal. Leaf size=63

$$\frac{1}{8}\sqrt[4]{x^4-x^3}\,(4x-1)+\frac{3}{16}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)-\frac{3}{16}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)$$

Rubi [A] time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.94, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {2004, 2024, 2032, 63, 240, 212, 206, 203}

$$\frac{1}{2}\sqrt[4]{x^4-x^3}\,x-\frac{1}{8}\sqrt[4]{x^4-x^3}-\frac{3(x-1)^{3/4}x^{9/4}\tan^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{16\left(x^4-x^3\right)^{3/4}}-\frac{3(x-1)^{3/4}x^{9/4}\tanh^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{16\left(x^4-x^3\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + x^4)^(1/4),x]
```

```
[Out] -1/8*(-x^3 + x^4)^(1/4) + (x*(-x^3 + x^4)^(1/4))/2 - (3*(-1 + x)^(3/4)*x^(9
/4)*ArcTan[(-1 + x)^(1/4)/x^(1/4)])/(16*(-x^3 + x^4)^(3/4)) - (3*(-1 + x)^(
3/4)*x^(9/4)*ArcTanh[(-1 + x)^(1/4)/x^(1/4)])/(16*(-x^3 + x^4)^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt[4]{-x^3+x^4}\,dx &= \frac{1}{2}x\sqrt[4]{-x^3+x^4} - \frac{1}{8}\int \frac{x^3}{\left(-x^3+x^4\right)^{3/4}}\,dx\\
&= -\frac{1}{8}\sqrt[4]{-x^3+x^4} + \frac{1}{2}x\sqrt[4]{-x^3+x^4} - \frac{3}{32}\int \frac{x^2}{\left(-x^3+x^4\right)^{3/4}}\,dx\\
&= -\frac{1}{8}\sqrt[4]{-x^3+x^4} + \frac{1}{2}x\sqrt[4]{-x^3+x^4} - \frac{\left(3(-1+x)^{3/4}x^{9/4}\right)\int \frac{1}{(-1+x)^{3/4}\sqrt[4]{x}}\,dx}{32\left(-x^3+x^4\right)^{3/4}}\\
&= -\frac{1}{8}\sqrt[4]{-x^3+x^4} + \frac{1}{2}x\sqrt[4]{-x^3+x^4} - \frac{\left(3(-1+x)^{3/4}x^{9/4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^4}}\,dx,x,\sqrt[4]{-1+x}\right)}{8\left(-x^3+x^4\right)^{3/4}}\\
&= -\frac{1}{8}\sqrt[4]{-x^3+x^4} + \frac{1}{2}x\sqrt[4]{-x^3+x^4} - \frac{\left(3(-1+x)^{3/4}x^{9/4}\right)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{8\left(-x^3+x^4\right)^{3/4}}\\
&= -\frac{1}{8}\sqrt[4]{-x^3+x^4} + \frac{1}{2}x\sqrt[4]{-x^3+x^4} - \frac{\left(3(-1+x)^{3/4}x^{9/4}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{16\left(-x^3+x^4\right)^{3/4}} - \frac{\left(3(-1+x\right.}{}\\
&= -\frac{1}{8}\sqrt[4]{-x^3+x^4} + \frac{1}{2}x\sqrt[4]{-x^3+x^4} - \frac{3(-1+x)^{3/4}x^{9/4}\tan^{-1}\left(\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{16\left(-x^3+x^4\right)^{3/4}} - \frac{3(-1+x)^{3/4}x^{9/4}\tanh^{-1}\left(\right.}{16\left(-x^3+x^4\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.56

$$
\frac{4\left((x-1)x^3\right)^{5/4}\,{}_2F_1\left(-\frac{3}{4},\frac{5}{4};\frac{9}{4};1-x\right)}{5x^{15/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + x^4)^(1/4),x]
```

```
[Out] (4*((-1 + x)*x^3)^(5/4)*Hypergeometric2F1[-3/4, 5/4, 9/4, 1 - x])/(5*x^(15/
4))
```

IntegrateAlgebraic [A] time = 0.24, size = 63, normalized size = 1.00

$$\frac{1}{8}\sqrt[4]{x^4-x^3}\,(4x-1)+\frac{3}{16}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)-\frac{3}{16}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^3 + x^4)^(1/4),x]
```

```
[Out] ((-1 + 4*x)*(-x^3 + x^4)^(1/4))/8 + (3*ArcTan[x/(-x^3 + x^4)^(1/4)])/16 - (
3*ArcTanh[x/(-x^3 + x^4)^(1/4)])/16
```

fricas [A] time = 0.46, size = 80, normalized size = 1.27

$$\frac{1}{8}\left(x^4-x^3\right)^{\frac{1}{4}}(4\,x-1)-\frac{3}{16}\,\arctan\left(\frac{\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)-\frac{3}{32}\,\log\left(\frac{x+\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{3}{32}\,\log\left(-\frac{x-\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/8*(x^4 - x^3)^(1/4)*(4*x - 1) - 3/16*arctan((x^4 - x^3)^(1/4)/x) - 3/32*l
og((x + (x^4 - x^3)^(1/4))/x) + 3/32*log(-(x - (x^4 - x^3)^(1/4))/x)
```

giac [A] time = 0.15, size = 68, normalized size = 1.08

$$-\frac{1}{8}\left(\left(-\frac{1}{x}+1\right)^{\frac{5}{4}}+3\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)x^2+\frac{3}{16}\,\arctan\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)+\frac{3}{32}\,\log\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right)-\frac{3}{32}\,\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4),x, algorithm="giac")
```

```
[Out] -1/8*((-1/x + 1)^(5/4) + 3*(-1/x + 1)^(1/4))*x^2 + 3/16*arctan((-1/x + 1)^(
1/4)) + 3/32*log((-1/x + 1)^(1/4) + 1) - 3/32*log(abs((-1/x + 1)^(1/4) - 1)
)
```

maple [C] time = 0.33, size = 27, normalized size = 0.43

$$\frac{4\mathrm{signum}\left(-1+x\right)^{\frac{1}{4}}x^{\frac{7}{4}}\,\mathrm{hypergeom}\left(\left[-\frac{1}{4},\frac{7}{4}\right],\left[\frac{11}{4}\right],x\right)}{7\left(-\mathrm{signum}\left(-1+x\right)\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3)^(1/4),x)
```

```
[Out] 4/7*signum(-1+x)^(1/4)/(-signum(-1+x))^(1/4)*x^(7/4)*hypergeom([-1/4,7/4],[
11/4],x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^4-x^3\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^3)^(1/4), x)
```

mupad [B] time = 0.84, size = 27, normalized size = 0.43

$$\frac{4\,x\,{\left(x^4-x^3\right)}^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{7}{4};\ \frac{11}{4};\ x\right)}{7\,{\left(1-x\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x^3)^(1/4),x)
```

```
[Out] (4*x*(x^4 - x^3)^(1/4)*hypergeom([-1/4, 7/4], 11/4, x))/(7*(1 - x)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{x^4-x^3}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3)**(1/4),x)
```

```
[Out] Integral((x**4 - x**3)**(1/4), x)
```

3.793 $\int \frac{\sqrt{2-x^3-x^4}\left(4+x^3+2x^4\right)}{\left(-2-3x^2+x^3+x^4\right)\left(-2-x^2+x^3+x^4\right)}\,dx$

Optimal. Leaf size=63

$$-\tan^{-1}\left(\frac{x}{\sqrt{-x^4-x^3+2}}\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x\sqrt{-x^4-x^3+2}}{x^4+x^3-2}\right)$$

Rubi [F] time = 1.69, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{2-x^3-x^4}\left(4+x^3+2x^4\right)}{\left(-2-3x^2+x^3+x^4\right)\left(-2-x^2+x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[2 - x^3 - x^4]*(4 + x^3 + 2*x^4))/((-2 - 3*x^2 + x^3 + x^4)*(-2 -
 x^2 + x^3 + x^4)),x]
```

```
[Out] -3*Defer[Int][Sqrt[2 - x^3 - x^4]/(-2 - 3*x^2 + x^3 + x^4), x] + (3*Defer[I
nt][(x*Sqrt[2 - x^3 - x^4])/(-2 - 3*x^2 + x^3 + x^4), x])/2 + 2*Defer[Int][
(x^2*Sqrt[2 - x^3 - x^4])/(-2 - 3*x^2 + x^3 + x^4), x] + Defer[Int][Sqrt[2
- x^3 - x^4]/(-2 - x^2 + x^3 + x^4), x] - (3*Defer[Int][(x*Sqrt[2 - x^3 - x
^4])/(-2 - x^2 + x^3 + x^4), x])/2 - 2*Defer[Int][(x^2*Sqrt[2 - x^3 - x^4])
/(-2 - x^2 + x^3 + x^4), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2-x^3-x^4}\left(4+x^3+2x^4\right)}{\left(-2-3x^2+x^3+x^4\right)\left(-2-x^2+x^3+x^4\right)}\,dx &= \int \left(\frac{\left(-6+3x+4x^2\right)\sqrt{2-x^3-x^4}}{2\left(-2-3x^2+x^3+x^4\right)}+\frac{\left(2-3x-4x^2\right)\sqrt{2-x}}{2\left(-2-x^2+x^3+\right.}\right.\\
&= \frac{1}{2}\int \frac{\left(-6+3x+4x^2\right)\sqrt{2-x^3-x^4}}{-2-3x^2+x^3+x^4}\,dx+\frac{1}{2}\int \frac{\left(2-3x-4x^2\right.}{-2-x^2}\\
&= \frac{1}{2}\int \left(-\frac{6\sqrt{2-x^3-x^4}}{-2-3x^2+x^3+x^4}+\frac{3x\sqrt{2-x^3-x^4}}{-2-3x^2+x^3+x^4}+\frac{4x^2\sqrt{}}{-2-3}\right.\\
&= \frac{3}{2}\int \frac{x\sqrt{2-x^3-x^4}}{-2-3x^2+x^3+x^4}\,dx-\frac{3}{2}\int \frac{x\sqrt{2-x^3-x^4}}{-2-x^2+x^3+x^4}\,dx+
\end{aligned}$$

Mathematica [C] time = 6.63, size = 61074, normalized size = 969.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - x^3 - x^4]*(4 + x^3 + 2*x^4))/((-2 - 3*x^2 + x^3 + x^4)
*(-2 - x^2 + x^3 + x^4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 1.34, size = 63, normalized size = 1.00

$$-\tan^{-1}\left(\frac{x}{\sqrt{-x^4-x^3+2}}\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x\sqrt{-x^4-x^3+2}}{x^4+x^3-2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[2 - x^3 - x^4]*(4 + x^3 + 2*x^4))/((-2 - 3*x^2 + x
^3 + x^4)*(-2 - x^2 + x^3 + x^4)),x]
```

```
[Out] -ArcTan[x/Sqrt[2 - x^3 - x^4]] - Sqrt[3]*ArcTan[(Sqrt[3]*x*Sqrt[2 - x^3 - x
^4])/(-2 + x^3 + x^4)]
```

fricas [A] time = 0.62, size = 75, normalized size = 1.19

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{2\sqrt{3}\sqrt{-x^4-x^3+2}\,x}{x^4+x^3+3x^2-2}\right)+\frac{1}{2}\arctan\left(\frac{2\sqrt{-x^4-x^3+2}\,x}{x^4+x^3+x^2-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-x^3+2)^(1/2)*(2*x^4+x^3+4)/(x^4+x^3-3*x^2-2)/(x^4+x^3-x^2-2
),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan(2*sqrt(3)*sqrt(-x^4 - x^3 + 2)*x/(x^4 + x^3 + 3*x^2 - 2
)) + 1/2*arctan(2*sqrt(-x^4 - x^3 + 2)*x/(x^4 + x^3 + x^2 - 2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^4+x^3+4\right)\sqrt{-x^4-x^3+2}}{\left(x^4+x^3-x^2-2\right)\left(x^4+x^3-3x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-x^3+2)^(1/2)*(2*x^4+x^3+4)/(x^4+x^3-3*x^2-2)/(x^4+x^3-x^2-2
),x, algorithm="giac")
```

```
[Out] integrate((2*x^4 + x^3 + 4)*sqrt(-x^4 - x^3 + 2)/((x^4 + x^3 - x^2 - 2)*(x^
4 + x^3 - 3*x^2 - 2)), x)
```

maple [C] time = 4.17, size = 5959, normalized size = 94.59

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4-x^3+2)^(1/2)*(2*x^4+x^3+4)/(x^4+x^3-3*x^2-2)/(x^4+x^3-x^2-2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^4+x^3+4\right)\sqrt{-x^4-x^3+2}}{\left(x^4+x^3-x^2-2\right)\left(x^4+x^3-3x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-x^3+2)^(1/2)*(2*x^4+x^3+4)/(x^4+x^3-3*x^2-2)/(x^4+x^3-x^2-2
),x, algorithm="maxima")
```

```
[Out] integrate((2*x^4 + x^3 + 4)*sqrt(-x^4 - x^3 + 2)/((x^4 + x^3 - x^2 - 2)*(x^
4 + x^3 - 3*x^2 - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{-x^4-x^3+2}\,\left(2x^4+x^3+4\right)}{\left(-x^4-x^3+3x^2+2\right)\,\left(-x^4-x^3+x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - x^4 - x^3)^(1/2)*(x^3 + 2*x^4 + 4))/((3*x^2 - x^3 - x^4 + 2)*(x^2
 - x^3 - x^4 + 2)),x)
```

```
[Out] int(((2 - x^4 - x^3)^(1/2)*(x^3 + 2*x^4 + 4))/((3*x^2 - x^3 - x^4 + 2)*(x^2
 - x^3 - x^4 + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4-x**3+2)**(1/2)*(2*x**4+x**3+4)/(x**4+x**3-3*x**2-2)/(x**4+
x**3-x**2-2),x)
```

```
[Out] Timed out
```

3.794 $\int \frac{x^2(-4+x^6)}{\sqrt{-1+x^6}\,(2+x^6)}\,dx$

Optimal. Leaf size=63

$$\frac{1}{3}\log\left(\sqrt{x^6-1}+x^3\right)+\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{x^6}{\sqrt{6}}+\frac{\sqrt{x^6-1}\,x^3}{\sqrt{6}}+\sqrt{\frac{2}{3}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {575, 523, 217, 206, 377}

$$\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)-\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(-4 + x^6))/(Sqrt[-1 + x^6]*(2 + x^6)),x]
```

```
[Out] ArcTanh[x^3/Sqrt[-1 + x^6]]/3 - Sqrt[2/3]*ArcTanh[(Sqrt[3/2]*x^3)/Sqrt[-1 +
 x^6]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 575

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Di
st[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q*(e
+ f*x^(n/k))^r, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, f, p, q
, r}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2(-4+x^6)}{\sqrt{-1+x^6}\,(2+x^6)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{-4+x^2}{\sqrt{-1+x^2}\,(2+x^2)}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx,x,x^3\right) - 2\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}\,(2+x^2)}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x^3}{\sqrt{-1+x^6}}\right) - 2\,\text{Subst}\left(\int \frac{1}{2-3x^2}\,dx,x,\frac{x^3}{\sqrt{-1+x^6}}\right)$$
$$= \frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right) - \sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x^3}{\sqrt{-1+x^6}}\right)$$

Mathematica [A] time = 0.07, size = 47, normalized size = 0.75

$$\frac{1}{3}\left(\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right) - \sqrt{6}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x^3}{\sqrt{x^6-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(-4 + x^6))/(Sqrt[-1 + x^6]*(2 + x^6)),x]
```

```
[Out] (ArcTanh[x^3/Sqrt[-1 + x^6]] - Sqrt[6]*ArcTanh[(Sqrt[3/2]*x^3)/Sqrt[-1 + x^
6]])/3
```

IntegrateAlgebraic [A] time = 0.14, size = 67, normalized size = 1.06

$$-\frac{1}{3}\log\left(\sqrt{x^6-1}-x^3\right) - \sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{x^6}{\sqrt{6}} - \frac{\sqrt{x^6-1}\,x^3}{\sqrt{6}} + \sqrt{\frac{2}{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(-4 + x^6))/(Sqrt[-1 + x^6]*(2 + x^6)),x]
```

```
[Out] -(Sqrt[2/3]*ArcTanh[Sqrt[2/3] + x^6/Sqrt[6] - (x^3*Sqrt[-1 + x^6])/Sqrt[6]]
) - Log[-x^3 + Sqrt[-1 + x^6]]/3
```

fricas [A] time = 0.45, size = 82, normalized size = 1.30

$$\frac{1}{6}\sqrt{3}\sqrt{2}\log\left(\frac{25x^6-2\sqrt{3}\sqrt{2}\left(5x^6-2\right)-2\sqrt{x^6-1}\left(5\sqrt{3}\sqrt{2}x^3-12x^3\right)-10}{x^6+2}\right) - \frac{1}{3}\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^6-4)/(x^6-1)^(1/2)/(x^6+2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2)*log((25*x^6 - 2*sqrt(3)*sqrt(2)*(5*x^6 - 2) - 2*sqrt(x^
6 - 1)*(5*sqrt(3)*sqrt(2)*x^3 - 12*x^3) - 10)/(x^6 + 2)) - 1/3*log(-x^3 + s
qrt(x^6 - 1))
```

giac [A] time = 0.20, size = 72, normalized size = 1.14

$$\frac{1}{6}\sqrt{6}\log\left(\frac{\left(x^3-\sqrt{x^6-1}\right)^2-2\sqrt{6}+5}{\left(x^3-\sqrt{x^6-1}\right)^2+2\sqrt{6}+5}\right) - \frac{1}{6}\log\left(\left(x^3-\sqrt{x^6-1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^6-4)/(x^6-1)^(1/2)/(x^6+2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(6)*log(((x^3 - sqrt(x^6 - 1))^2 - 2*sqrt(6) + 5)/((x^3 - sqrt(x^6
- 1))^2 + 2*sqrt(6) + 5)) - 1/6*log((x^3 - sqrt(x^6 - 1))^2)
```

maple [C] time = 0.60, size = 65, normalized size = 1.03

$$\frac{\ln\left(x^3+\sqrt{x^6-1}\right)}{3}-\frac{\mathrm{RootOf}\left(\_Z^2-6\right)\ln\left(\frac{5\,\mathrm{RootOf}\left(\_Z^2-6\right)x^6+12x^3\sqrt{x^6-1}-2\,\mathrm{RootOf}\left(\_Z^2-6\right)}{x^6+2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^6-4)/(x^6-1)^(1/2)/(x^6+2),x)
```

```
[Out] 1/3*ln(x^3+(x^6-1)^(1/2))-1/6*RootOf(_Z^2-6)*ln((5*RootOf(_Z^2-6)*x^6+12*x^
3*(x^6-1)^(1/2)-2*RootOf(_Z^2-6))/(x^6+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6-4\right)x^2}{\left(x^6+2\right)\sqrt{x^6-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^6-4)/(x^6-1)^(1/2)/(x^6+2),x, algorithm="maxima")
```

```
[Out] integrate((x^6 - 4)*x^2/((x^6 + 2)*sqrt(x^6 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,\left(x^6-4\right)}{\sqrt{x^6-1}\,\left(x^6+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^6 - 4))/((x^6 - 1)^(1/2)*(x^6 + 2)),x)
```

```
[Out] int((x^2*(x^6 - 4))/((x^6 - 1)^(1/2)*(x^6 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(x^3-2\right)\left(x^3+2\right)}{\sqrt{(x-1)(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,\left(x^6+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**6-4)/(x**6-1)**(1/2)/(x**6+2),x)
```

```
[Out] Integral(x**2*(x**3 - 2)*(x**3 + 2)/(sqrt((x - 1)*(x + 1)*(x**2 - x + 1)*(x
**2 + x + 1))*(x**6 + 2)), x)
```

3.795 $\int \frac{(-1+x^5)(-1+x^3+x^5)(3+2x^5)}{x^6(-1-x^3+x^5)\sqrt[4]{-x+x^6}}\,dx$

Optimal. Leaf size=63

$$-4\tan^{-1}\left(\frac{x}{\sqrt[4]{x^6-x}}\right)-4\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^6-x}}\right)+\frac{4\left(x^6-x\right)^{3/4}\left(3x^5+14x^3-3\right)}{21x^6}$$

Rubi [F] time = 2.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^5)(-1+x^3+x^5)(3+2x^5)}{x^6(-1-x^3+x^5)\sqrt[4]{-x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^5)*(-1 + x^3 + x^5)*(3 + 2*x^5))/(x^6*(-1 - x^3 + x^5)*(-x + x
^6)^(1/4)),x]
```

```
[Out] (4*(1 - x^5)^(1/4)*Hypergeometric2F1[-21/20, -3/4, -1/20, x^5])/(7*x^5*(-x
+ x^6)^(1/4)) - (8*(1 - x^5)^(1/4)*Hypergeometric2F1[-3/4, -9/20, 11/20, x^
5])/(3*x^2*(-x + x^6)^(1/4)) + (8*(1 - x^5)^(1/4)*Hypergeometric2F1[-3/4, -
1/20, 19/20, x^5])/(-x + x^6)^(1/4) - (24*x^(1/4)*(-1 + x^5)^(1/4)*Defer[Su
bst][Defer[Int][(x^2*(-1 + x^20)^(3/4))/(-1 - x^12 + x^20), x], x, x^(1/4)]
)/(-x + x^6)^(1/4) + (40*x^(1/4)*(-1 + x^5)^(1/4)*Defer[Subst][Defer[Int][(
x^10*(-1 + x^20)^(3/4))/(-1 - x^12 + x^20), x], x, x^(1/4)])/(-x + x^6)^(1/
4)
```

Rubi steps

$$\int \frac{(-1+x^5)(-1+x^3+x^5)(3+2x^5)}{x^6(-1-x^3+x^5)\sqrt[4]{-x+x^6}}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{-1+x^5}\right)\int \frac{(-1+x^5)^{3/4}(-1+x^3+x^5)(3+2x^5)}{x^{25/4}(-1-x^3+x^5)}\,dx}{\sqrt[4]{-x+x^6}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-1+x^5}\right)\text{Subst}\left(\int \frac{(-1+x^{20})^{3/4}(-1+x^{12}+x^{20})(3+2x^{20})}{x^{22}(-1-x^{12}+x^{20})}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-x+x^6}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-1+x^5}\right)\text{Subst}\left(\int \left(\frac{3(-1+x^{20})^{3/4}}{x^{22}}-\frac{6(-1+x^{20})^{3/4}}{x^{10}}+\frac{2(-1+x^{20})^{3/4}}{x^2}\right.\right.}{\sqrt[4]{-x+x^6}}$$

$$= \frac{\left(8\sqrt[4]{x}\sqrt[4]{-1+x^5}\right)\text{Subst}\left(\int \frac{(-1+x^{20})^{3/4}}{x^2}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-x+x^6}}+\frac{\left(8\sqrt[4]{x}\sqrt[4]{-1+x\right.}{}$$

$$= \frac{\left(8\sqrt[4]{x}\sqrt[4]{-1+x^5}\right)\text{Subst}\left(\int \left(-\frac{3x^2(-1+x^{20})^{3/4}}{-1-x^{12}+x^{20}}+\frac{5x^{10}(-1+x^{20})^{3/4}}{-1-x^{12}+x^{20}}\right)dx, x,\right.}{\sqrt[4]{-x+x^6}}$$

$$= \frac{4\sqrt[4]{1-x^5}\,{}_2F_1\left(-\frac{21}{20},-\frac{3}{4};-\frac{1}{20};x^5\right)}{7x^5\sqrt[4]{-x+x^6}}-\frac{8\sqrt[4]{1-x^5}\,{}_2F_1\left(-\frac{3}{4},-\frac{9}{20};\frac{11}{20};x^5\right)}{3x^2\sqrt[4]{-x+x^6}}$$

Mathematica [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^5\right)\left(-1+x^3+x^5\right)\left(3+2x^5\right)}{x^6\left(-1-x^3+x^5\right)\sqrt[4]{-x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^5)*(-1 + x^3 + x^5)*(3 + 2*x^5))/(x^6*(-1 - x^3 + x^5)*(
-x + x^6)^(1/4)),x]
```

```
[Out] Integrate[((-1 + x^5)*(-1 + x^3 + x^5)*(3 + 2*x^5))/(x^6*(-1 - x^3 + x^5)*(
-x + x^6)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 2.74, size = 63, normalized size = 1.00

$$-4\tan^{-1}\left(\frac{x}{\sqrt[4]{x^6-x}}\right)-4\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^6-x}}\right)+\frac{4\left(x^6-x\right)^{3/4}\left(3x^5+14x^3-3\right)}{21x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^5)*(-1 + x^3 + x^5)*(3 + 2*x^5))/(x^6*(-1 - x^3
 + x^5)*(-x + x^6)^(1/4)),x]
```

```
[Out] (4*(-3 + 14*x^3 + 3*x^5)*(-x + x^6)^(3/4))/(21*x^6) - 4*ArcTan[x/(-x + x^6)
^(1/4)] - 4*ArcTanh[x/(-x + x^6)^(1/4)]
```

fricas [B] time = 79.49, size = 137, normalized size = 2.17

$$-\frac{2\left(21\,x^{6}\arctan\left(\frac{2\left(\left(x^{6}-x\right)^{\frac{1}{4}}x^{2}+\left(x^{6}-x\right)^{\frac{3}{4}}\right)}{x^{5}-x^{3}-1}\right)-21\,x^{6}\log\left(-\frac{x^{5}+x^{3}-2\left(x^{6}-x\right)^{\frac{1}{4}}x^{2}+2\,\sqrt{x^{6}-x}\,x-2\left(x^{6}-x\right)^{\frac{3}{4}}-1}{x^{5}-x^{3}-1}\right)-2\left(x^{6}-x\right)^{\frac{3}{4}}\left(3\,x^{5}+14\,x^{3}-3\right)\right)}{21\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)*(x^5+x^3-1)*(2*x^5+3)/x^6/(x^5-x^3-1)/(x^6-x)^(1/4),x, al
gorithm="fricas")
```

```
[Out] -2/21*(21*x^6*arctan(2*((x^6 - x)^(1/4)*x^2 + (x^6 - x)^(3/4))/(x^5 - x^3 -
 1)) - 21*x^6*log(-(x^5 + x^3 - 2*(x^6 - x)^(1/4)*x^2 + 2*sqrt(x^6 - x)*x -
 2*(x^6 - x)^(3/4) - 1)/(x^5 - x^3 - 1)) - 2*(x^6 - x)^(3/4)*(3*x^5 + 14*x^
3 - 3))/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{5}+3\right)\left(x^{5}+x^{3}-1\right)\left(x^{5}-1\right)}{\left(x^{6}-x\right)^{\frac{1}{4}}\left(x^{5}-x^{3}-1\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)*(x^5+x^3-1)*(2*x^5+3)/x^6/(x^5-x^3-1)/(x^6-x)^(1/4),x, al
gorithm="giac")
```

```
[Out] integrate((2*x^5 + 3)*(x^5 + x^3 - 1)*(x^5 - 1)/((x^6 - x)^(1/4)*(x^5 - x^3
 - 1)*x^6), x)
```

maple [C] time = 7.92, size = 197, normalized size = 3.13

$$\frac{\frac{4}{7}x^{10}-\frac{8}{7}x^{5}+\frac{8}{3}x^{8}-\frac{8}{3}x^{3}+\frac{4}{7}}{x^{5}\left(x\left(x^{5}-1\right)\right)^{\frac{1}{4}}}+2\ln\left(\frac{-x^{5}+2\left(x^{6}-x\right)^{\frac{3}{4}}-2x\sqrt{x^{6}-x}+2\left(x^{6}-x\right)^{\frac{1}{4}}x^{2}-x^{3}+1}{x^{5}-x^{3}-1}\right)+2\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^{2}+1\right)x^{5}+2\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{x^{6}-x}\,x-\mathrm{RootOf}\left(\_Z^{2}+1\right)x^{3}+2\left(x^{6}-x\right)^{\frac{3}{4}}-2\left(x^{6}-x\right)^{\frac{1}{4}}x^{2}+\mathrm{RootOf}\left(\_Z^{2}+1\right)}{x^{5}-x^{3}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)*(x^5+x^3-1)*(2*x^5+3)/x^6/(x^5-x^3-1)/(x^6-x)^(1/4),x)
```

```
[Out] 4/21*(3*x^10+14*x^8-6*x^5-14*x^3+3)/x^5/(x*(x^5-1))^(1/4)+2*ln((-x^5+2*(x^6
-x)^(3/4)-2*x*(x^6-x)^(1/2)+2*(x^6-x)^(1/4)*x^2-x^3+1)/(x^5-x^3-1))+2*RootO
f(_Z^2+1)*ln((-RootOf(_Z^2+1)*x^5+2*RootOf(_Z^2+1)*(x^6-x)^(1/2)*x-RootOf(_
Z^2+1)*x^3+2*(x^6-x)^(3/4)-2*(x^6-x)^(1/4)*x^2+RootOf(_Z^2+1))/(x^5-x^3-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^5+3\right)\left(x^5+x^3-1\right)\left(x^5-1\right)}{\left(x^6-x\right)^{\frac{1}{4}}\left(x^5-x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)*(x^5+x^3-1)*(2*x^5+3)/x^6/(x^5-x^3-1)/(x^6-x)^(1/4),x, al
gorithm="maxima")
```

```
[Out] integrate((2*x^5 + 3)*(x^5 + x^3 - 1)*(x^5 - 1)/((x^6 - x)^(1/4)*(x^5 - x^3
 - 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{\left(x^5-1\right)\,\left(2\,x^5+3\right)\,\left(x^5+x^3-1\right)}{x^6\,{\left(x^6-x\right)}^{1/4}\,\left(-x^5+x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^5 - 1)*(2*x^5 + 3)*(x^3 + x^5 - 1))/(x^6*(x^6 - x)^(1/4)*(x^3 - x^
5 + 1)),x)
```

```
[Out] int(-((x^5 - 1)*(2*x^5 + 3)*(x^3 + x^5 - 1))/(x^6*(x^6 - x)^(1/4)*(x^3 - x^
5 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-1\right)\left(2x^5+3\right)\left(x^5+x^3-1\right)\left(x^4+x^3+x^2+x+1\right)}{x^6\sqrt[4]{x\left(x-1\right)\left(x^4+x^3+x^2+x+1\right)}\left(x^5-x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)*(x**5+x**3-1)*(2*x**5+3)/x**6/(x**5-x**3-1)/(x**6-x)**(1
/4),x)
```

```
[Out] Integral((x - 1)*(2*x**5 + 3)*(x**5 + x**3 - 1)*(x**4 + x**3 + x**2 + x + 1
)/(x**6*(x*(x - 1)*(x**4 + x**3 + x**2 + x + 1))**(1/4)*(x**5 - x**3 - 1)),
 x)
```

3.796 $\int \frac{\sqrt{1+x^3}\left(2+2x^3+x^6\right)}{x^7\left(-1+x^6\right)}\,dx$

Optimal. Leaf size=63

$$\frac{5}{2}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{5}{3}\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{x^3+1}}{\sqrt{2}}\right)+\frac{\sqrt{x^3+1}\,\left(5x^3+2\right)}{6x^6}$$

Rubi [C] time = 3.49, antiderivative size = 1234, normalized size of antiderivative = 19.59, number of steps used = 41, number of rules used = 16, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {1586, 6725, 2136, 218, 2142, 2113, 537, 571, 93, 206, 266, 51, 63, 207, 6728, 205}

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[1 + x^3]*(2 + 2*x^3 + x^6))/(x^7*(-1 + x^6)),x]
```

```
[Out] Sqrt[1 + x^3]/(3*x^6) + (5*Sqrt[1 + x^3])/(6*x^3) + (((5*I)/3)*(1 + x)*Sqrt
[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[
3] + x)^2])/(Sqrt[((3 - 6*I) - (2 - 3*I)*Sqrt[3])/((4 + 6*I) - (2 + 4*I)*Sq
rt[3])]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[2]*Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (10*Sqrt[((6 - 3*I) - (3 - 2*I)*Sqrt[3]
)/((-6 - 4*I) + (4 + 2*I)*Sqrt[3])]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3]
 + x)^2]*ArcTan[(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[((6 - 3*I
) - (3 - 2*I)*Sqrt[3])/((-6 - 4*I) + (4 + 2*I)*Sqrt[3])]*Sqrt[(1 - x + x^2)
/(1 + Sqrt[3] + x)^2])])/(3^(3/4)*(3*I - Sqrt[3])*Sqrt[(1 + x)/(1 + Sqrt[3]
 + x)^2]*Sqrt[1 + x^3]) - (5*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2
]*ArcTanh[Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]/(Sqrt[2]*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2])])/(3*Sqrt[2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 +
x^3]) + (5*ArcTanh[Sqrt[1 + x^3]])/2 - (10*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 -
 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[2 + Sqrt[3]]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]
*Sqrt[1 + x^3]) - (20*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqr
t[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*
Sqrt[3]])/(3*3^(1/4)*(1 + (2 - I)*Sqrt[3])*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2
]*Sqrt[1 + x^3]) - (20*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sq
rt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4
*Sqrt[3]])/(3*3^(1/4)*(1 + (2 + I)*Sqrt[3])*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^
2]*Sqrt[1 + x^3]) + (40*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + S
qrt[3] + x)^2]*EllipticPi[-((I + (1 + 2*I)*Sqrt[3])^2/(1 - (2 + I)*Sqrt[3])
^2), ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)
*(7 + I*Sqrt[3])*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (40*Sqr
t[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[-
((1 + (2 + I)*Sqrt[3])^2/(I - (1 + 2*I)*Sqrt[3])^2), ArcSin[(1 - Sqrt[3] +
x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*(7 - I*Sqrt[3])*Sqrt[(1 +
x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (20*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt
[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[97 + 56*Sqrt[3], ArcSin[(1 -
 Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2136

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, -Dist[q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2142

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[((1 + Sqrt[3])*f)/e]}, Dist[(4*3^(1/4)*Sqrt[2
 - Sqrt[3]]*f*(1 + q*x)*Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2])/(q
*Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x^3}\left(2+2x^3+x^6\right)}{x^7\left(-1+x^6\right)}\,dx = \int \frac{2+2x^3+x^6}{x^7\left(-1+x^3\right)\sqrt{1+x^3}}\,dx$$

$$= \int \left(\frac{5}{3(-1+x)\sqrt{1+x^3}} - \frac{2}{x^7\sqrt{1+x^3}} - \frac{4}{x^4\sqrt{1+x^3}} - \frac{5}{x\sqrt{1+x^3}} + \frac{5(1 -}{3\left(1+x+\right.}\right.$$

$$= \frac{5}{3}\int \frac{1}{(-1+x)\sqrt{1+x^3}}\,dx + \frac{5}{3}\int \frac{1+2x}{\left(1+x+x^2\right)\sqrt{1+x^3}}\,dx - 2\int \frac{1}{x^7\sqrt{1+x^3}}\,d$$

$$= -\left(\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+x}}\,dx, x, x^3\right)\right) - \frac{4}{3}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, x^3\right) + \frac{5}{3}$$

$$= \frac{\sqrt{1+x^3}}{3x^6} + \frac{4\sqrt{1+x^3}}{3x^3} - \frac{10(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle| -7-4\sqrt{3}\right.}{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{\sqrt{1+x^3}}{3x^6} + \frac{5\sqrt{1+x^3}}{6x^3} + \frac{10}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right) - \frac{10(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin\right.}{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt{\frac{}{(1}}}$$

$$= \frac{\sqrt{1+x^3}}{3x^6} + \frac{5\sqrt{1+x^3}}{6x^3} + 2\tanh^{-1}\left(\sqrt{1+x^3}\right) - \frac{10(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\right.}{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt{\frac{}{(1+}}}$$

$$= \frac{\sqrt{1+x^3}}{3x^6} + \frac{5\sqrt{1+x^3}}{6x^3} + \frac{5}{2}\tanh^{-1}\left(\sqrt{1+x^3}\right) - \frac{10(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-}\right.}{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt{\frac{}{(1+}}}$$

$$= \frac{\sqrt{1+x^3}}{3x^6} + \frac{5\sqrt{1+x^3}}{6x^3} - \frac{5(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\tanh^{-1}\left(\frac{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{3\sqrt{2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{5}{2}\tan$$

$$= \frac{\sqrt{1+x^3}}{3x^6} + \frac{5\sqrt{1+x^3}}{6x^3} - \frac{5(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\tanh^{-1}\left(\frac{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{2}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{3\sqrt{2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{5}{2}\tan$$

$$= \frac{\sqrt{1+x^3}}{3x^6} + \frac{5\sqrt{1+x^3}}{6x^3} + \frac{5i(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{2-\sqrt{3}}\sqrt{\frac{3i+\sqrt{3}}{(-4-6i)+(2+4i)\sqrt{3}}}\sqrt{}}\right.}{3\sqrt{2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.23, size = 84, normalized size = 1.33

$$\frac{-5\sqrt{\frac{1}{x^3}+1}\,x^3F_1\left(\frac{3}{2};\frac{1}{2},1;\frac{5}{2};-\frac{1}{x^3},\frac{1}{x^3}\right)+5x^6+7x^3+2}{6x^6\sqrt{x^3+1}}-\frac{5}{12}x^3F_1\left(1;\frac{1}{2},1;2;-x^3,x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 + x^3]*(2 + 2*x^3 + x^6))/(x^7*(-1 + x^6)),x]
```

```
[Out] (-5*x^3*AppellF1[1, 1/2, 1, 2, -x^3, x^3])/12 + (2 + 7*x^3 + 5*x^6 - 5*Sqrt
[1 + x^(-3)]*x^3*AppellF1[3/2, 1/2, 1, 5/2, -x^(-3), x^(-3)])/(6*x^6*Sqrt[1
 + x^3])
```

IntegrateAlgebraic [A] time = 0.10, size = 63, normalized size = 1.00

$$\frac{5}{2}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{5}{3}\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x^3+1}}{\sqrt{2}}\right)+\frac{\sqrt{x^3+1}\left(5x^3+2\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x^3]*(2 + 2*x^3 + x^6))/(x^7*(-1 + x^6)),x]
```

```
[Out] (Sqrt[1 + x^3]*(2 + 5*x^3))/(6*x^6) + (5*ArcTanh[Sqrt[1 + x^3]])/2 - (5*Sqr
t[2]*ArcTanh[Sqrt[1 + x^3]/Sqrt[2]])/3
```

fricas [A] time = 0.45, size = 86, normalized size = 1.37

$$\frac{10\sqrt{2}x^6\log\left(\frac{x^3-2\sqrt{2}\sqrt{x^3+1}+3}{x^3-1}\right)+15x^6\log\left(\sqrt{x^3+1}+1\right)-15x^6\log\left(\sqrt{x^3+1}-1\right)+2\left(5x^3+2\right)\sqrt{x^3+1}}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)*(x^6+2*x^3+2)/x^7/(x^6-1),x, algorithm="fricas")
```

```
[Out] 1/12*(10*sqrt(2)*x^6*log((x^3 - 2*sqrt(2)*sqrt(x^3 + 1) + 3)/(x^3 - 1)) + 1
5*x^6*log(sqrt(x^3 + 1) + 1) - 15*x^6*log(sqrt(x^3 + 1) - 1) + 2*(5*x^3 + 2
)*sqrt(x^3 + 1))/x^6
```

giac [A] time = 0.19, size = 87, normalized size = 1.38

$$\frac{5}{6}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{x^3+1}\right|}{2\left(\sqrt{2}+\sqrt{x^3+1}\right)}\right)+\frac{5\left(x^3+1\right)^{\frac{3}{2}}-3\sqrt{x^3+1}}{6x^6}+\frac{5}{4}\log\left(\sqrt{x^3+1}+1\right)-\frac{5}{4}\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)*(x^6+2*x^3+2)/x^7/(x^6-1),x, algorithm="giac")
```

```
[Out] 5/6*sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(x^3 + 1))/(sqrt(2) + sqrt(x^3 +
 1))) + 1/6*(5*(x^3 + 1)^(3/2) - 3*sqrt(x^3 + 1))/x^6 + 5/4*log(sqrt(x^3 +
1) + 1) - 5/4*log(abs(sqrt(x^3 + 1) - 1))
```

maple [C] time = 0.04, size = 449, normalized size = 7.13

$$-\frac{5\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{3}{4}-\frac{i\sqrt{3}}{4},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}+\frac{5\sqrt{x^3+1}}{6x^3}+\frac{5\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{2}+\frac{10\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{3}{4}-\frac{3i\sqrt{3}}{4}+\frac{i\sqrt{3}\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{2},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}+\frac{10\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{3}{4}+\frac{3i\sqrt{3}}{4}+\frac{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{2},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^3+1}}+\frac{\sqrt{x^3+1}}{3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(1/2)*(x^6+2*x^3+2)/x^7/(x^6-1),x)
```

```
[Out] -5/3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),3/4-1
/4*I*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+5/6*(x^3+1)
^(1/2)/x^3+5/2*arctanh((x^3+1)^(1/2))+10/3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-
1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((
x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*(-1/2+1/2*I*
3^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),3/4-3/4*I*3^(1/2)+1/2
*I*3^(1/2)*(-1/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2))+10/3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2
*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*(-1/2-1/2*I*3^(1/2))*EllipticPi(((1+x)/(3/
2-1/2*I*3^(1/2)))^(1/2),3/4+3/4*I*3^(1/2)+1/2*I*3^(1/2)*(-1/2-1/2*I*3^(1/2)
),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+1/3*(x^3+1)^(1/2)/x^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+2x^3+2\right)\sqrt{x^3+1}}{\left(x^6-1\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)*(x^6+2*x^3+2)/x^7/(x^6-1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + 2*x^3 + 2)*sqrt(x^3 + 1)/((x^6 - 1)*x^7), x)
```

mupad [B] time = 0.05, size = 724, normalized size = 11.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)^(1/2)*(2*x^3 + x^6 + 2))/(x^7*(x^6 - 1)),x)
```

```
[Out] (5*(x^3 + 1)^(1/2))/(6*x^3) + (x^3 + 1)^(1/2)/(3*x^6) + (15*((3^(1/2)*1i)/2
 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(2*(x^3 -
 x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 -
1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (5*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1
/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipt
icPi((3^(1/2)*1i)/4 + 3/4, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -(
(3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(x^3 - x*(((3^(1/2)*1i)/2
 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
2 + 1/2))^(1/2)) - (10*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/(
(3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1
/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)
/2 + 3/2)/((3^(1/2)*1i)/2 + 1/2), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 +
1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) + (10*((3^(1/2)*1i)/2 + 3/2)
*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 1/2), asin(((x +
 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
 - 3/2)))/(3*((3^(1/2)*1i)/2 - 1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1
/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2
))
```

sympy [A] time = 135.61, size = 148, normalized size = 2.35

$$\frac{10\left(\begin{cases} -\frac{\sqrt{2}\operatorname{acoth}\left(\frac{\sqrt{2}\sqrt{x^3+1}}{2}\right)}{2} & \text{for } x^3+1>2 \\ -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{x^3+1}}{2}\right)}{2} & \text{for } x^3+1<2 \end{cases}\right)}{3} - \frac{5\log\left(\sqrt{x^3+1}-1\right)}{4} + \frac{5\log\left(\sqrt{x^3+1}+1\right)}{4} + \frac{5}{12\left(\sqrt{x^3+1}+1\right)} - \frac{1}{12\left(\sqrt{x^3+1}+1\right)^2} + \frac{5}{12\left(\sqrt{x^3+1}-1\right)} + \frac{1}{12\left(\sqrt{x^3+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(1/2)*(x**6+2*x**3+2)/x**7/(x**6-1),x)
```

```
[Out] 10*Piecewise((-sqrt(2)*acoth(sqrt(2)*sqrt(x**3 + 1)/2)/2, x**3 + 1 > 2), (-
sqrt(2)*atanh(sqrt(2)*sqrt(x**3 + 1)/2)/2, x**3 + 1 < 2))/3 - 5*log(sqrt(x*
*3 + 1) - 1)/4 + 5*log(sqrt(x**3 + 1) + 1)/4 + 5/(12*(sqrt(x**3 + 1) + 1))
- 1/(12*(sqrt(x**3 + 1) + 1)**2) + 5/(12*(sqrt(x**3 + 1) - 1)) + 1/(12*(sqr
t(x**3 + 1) - 1)**2)
```

3.797 $\int \frac{\sqrt{-1+x^2+x^5}\,(2+3x^5)}{1+x^4-2x^5+x^{10}}\,dx$

Optimal. Leaf size=63

$$-\sqrt{1+i}\tan^{-1}\left(\frac{\sqrt{-1-i}\,x}{\sqrt{x^5+x^2-1}}\right)-\sqrt{1-i}\tan^{-1}\left(\frac{\sqrt{-1+i}\,x}{\sqrt{x^5+x^2-1}}\right)$$

Rubi [F] time = 0.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-1+x^2+x^5}\,(2+3x^5)}{1+x^4-2x^5+x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-1 + x^2 + x^5]*(2 + 3*x^5))/(1 + x^4 - 2*x^5 + x^10),x]
```

```
[Out] 2*Defer[Int][Sqrt[-1 + x^2 + x^5]/(1 + x^4 - 2*x^5 + x^10), x] + 3*Defer[In
t][(x^5*Sqrt[-1 + x^2 + x^5])/(1 + x^4 - 2*x^5 + x^10), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+x^2+x^5}\,(2+3x^5)}{1+x^4-2x^5+x^{10}}\,dx &= \int \left(\frac{2\sqrt{-1+x^2+x^5}}{1+x^4-2x^5+x^{10}}+\frac{3x^5\sqrt{-1+x^2+x^5}}{1+x^4-2x^5+x^{10}}\right)dx\\ &= 2\int \frac{\sqrt{-1+x^2+x^5}}{1+x^4-2x^5+x^{10}}\,dx+3\int \frac{x^5\sqrt{-1+x^2+x^5}}{1+x^4-2x^5+x^{10}}\,dx\end{aligned}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-1+x^2+x^5}\,(2+3x^5)}{1+x^4-2x^5+x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[-1 + x^2 + x^5]*(2 + 3*x^5))/(1 + x^4 - 2*x^5 + x^10),x]
```

```
[Out] Integrate[(Sqrt[-1 + x^2 + x^5]*(2 + 3*x^5))/(1 + x^4 - 2*x^5 + x^10), x]
```

IntegrateAlgebraic [A] time = 2.85, size = 63, normalized size = 1.00

$$-\sqrt{1+i}\tan^{-1}\left(\frac{\sqrt{-1-i}\,x}{\sqrt{x^5+x^2-1}}\right)-\sqrt{1-i}\tan^{-1}\left(\frac{\sqrt{-1+i}\,x}{\sqrt{x^5+x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^2 + x^5]*(2 + 3*x^5))/(1 + x^4 - 2*x^5 + x^
10),x]
```

```
[Out] -(Sqrt[1 + I]*ArcTan[(Sqrt[-1 - I]*x)/Sqrt[-1 + x^2 + x^5]]) - Sqrt[1 - I]*
ArcTan[(Sqrt[-1 + I]*x)/Sqrt[-1 + x^2 + x^5]]
```

fricas [B] time = 1.57, size = 3547, normalized size = 56.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x^2-1)^(1/2)*(3*x^5+2)/(x^10-2*x^5+x^4+1),x, algorithm="fric
as")
```

```
[Out] 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log((x^10 + 4*x^7 - 2*x^5 +
5*x^4 + 2^(1/4)*sqrt(x^5 + x^2 - 1)*(2*x^3 + sqrt(2)*(x^6 + x^3 - x))*sqrt(
2*sqrt(2) + 4) - 4*x^2 + 4*sqrt(2)*(x^7 + x^4 - x^2) + 1)/(x^10 - 2*x^5 + x
^4 + 1)) - 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log((x^10 + 4*x^7
 - 2*x^5 + 5*x^4 - 2^(1/4)*sqrt(x^5 + x^2 - 1)*(2*x^3 + sqrt(2)*(x^6 + x^3
- x))*sqrt(2*sqrt(2) + 4) - 4*x^2 + 4*sqrt(2)*(x^7 + x^4 - x^2) + 1)/(x^10
- 2*x^5 + x^4 + 1)) + 1/4*2^(3/4)*sqrt(2*sqrt(2) + 4)*arctan(1/4*(4*x^26 -
4*x^23 - 20*x^21 - 56*x^20 + 16*x^18 - 56*x^17 + 40*x^16 + 168*x^15 + 4*x^1
4 - 24*x^13 + 112*x^12 - 28*x^11 - 168*x^10 - 4*x^9 + 16*x^8 - 56*x^7 + 20*
x^6 + 56*x^5 - 4*x^3 + sqrt(x^5 + x^2 - 1)*(2^(3/4)*(x^25 + 7*x^22 - 5*x^20
 + 10*x^19 - 28*x^17 - 6*x^16 + 10*x^15 - 30*x^14 - 7*x^13 + 42*x^12 + 12*x
^11 - 7*x^10 + 30*x^9 + 7*x^8 - 28*x^7 - 6*x^6 + 5*x^5 - 10*x^4 + 7*x^2 - s
qrt(2)*(3*x^22 + 10*x^19 - 12*x^17 + 4*x^16 - 30*x^14 - 6*x^13 + 18*x^12 -
8*x^11 + x^10 + 30*x^9 + 6*x^8 - 12*x^7 + 4*x^6 - 10*x^4 + 3*x^2) - 1) + 2*
2^(1/4)*(2*x^22 + 2*x^19 - 8*x^17 + 10*x^16 - 6*x^14 + 18*x^13 + 12*x^12 -
20*x^11 + 8*x^10 + 6*x^9 - 18*x^8 - 8*x^7 + 10*x^6 - 2*x^4 + 2*x^2 + sqrt(2
)*(x^22 - 4*x^19 - 4*x^17 - 12*x^16 + 12*x^14 - 12*x^13 + 6*x^12 + 24*x^11
- 5*x^10 - 12*x^9 + 12*x^8 - 4*x^7 - 12*x^6 + 4*x^4 + x^2)))*sqrt(2*sqrt(2)
 + 4) - 2*sqrt(2)*(2*x^26 + 6*x^23 - 10*x^21 + 4*x^20 - 24*x^18 + 4*x^17 +
20*x^16 - 12*x^15 + 2*x^14 + 36*x^13 - 8*x^12 - 22*x^11 + 12*x^10 - 2*x^9 -
 24*x^8 + 4*x^7 + 10*x^6 - 4*x^5 + 6*x^3 + sqrt(2)*(x^26 - x^23 - 5*x^21 -
14*x^20 + 4*x^18 - 14*x^17 + 10*x^16 + 42*x^15 + x^14 - 6*x^13 + 28*x^12 -
7*x^11 - 42*x^10 - x^9 + 4*x^8 - 14*x^7 + 5*x^6 + 14*x^5 - x^3 - x) - 2*x)
+ 4*sqrt(2)*(x^26 + 3*x^23 - 5*x^21 + 2*x^20 - 12*x^18 + 2*x^17 + 10*x^16 -
 6*x^15 + x^14 + 18*x^13 - 4*x^12 - 11*x^11 + 6*x^10 - x^9 - 12*x^8 + 2*x^7
 + 5*x^6 - 2*x^5 + 3*x^3 - x) + (24*x^23 + 16*x^20 - 96*x^18 - 48*x^17 - 48
*x^15 - 48*x^14 + 144*x^13 + 96*x^12 - 8*x^11 + 48*x^10 + 48*x^9 - 96*x^8 -
 48*x^7 - 16*x^5 + 24*x^3 - sqrt(x^5 + x^2 - 1)*(2^(3/4)*(x^25 + 7*x^22 - 5
*x^20 + 2*x^19 - 28*x^17 - 14*x^16 + 10*x^15 - 6*x^14 - 15*x^13 + 42*x^12 +
 28*x^11 - 15*x^10 + 6*x^9 + 15*x^8 - 28*x^7 - 14*x^6 + 5*x^5 - 2*x^4 + 7*x
^2 - sqrt(2)*(x^22 - 2*x^19 - 4*x^17 + 4*x^16 + 6*x^14 + 14*x^13 + 6*x^12 -
 8*x^11 + 3*x^10 - 6*x^9 - 14*x^8 - 4*x^7 + 4*x^6 + 2*x^4 + x^2) - 1) + 2*2
^(1/4)*(2*x^22 + 6*x^19 - 8*x^17 - 2*x^16 - 18*x^14 - 10*x^13 + 12*x^12 + 4
*x^11 - 4*x^10 + 18*x^9 + 10*x^8 - 8*x^7 - 2*x^6 - 6*x^4 + 2*x^2 + sqrt(2)*
(x^22 - 4*x^17 - 8*x^16 - 8*x^13 + 6*x^12 + 16*x^11 - x^10 + 8*x^8 - 4*x^7
- 8*x^6 + x^2)))*sqrt(2*sqrt(2) + 4) + 2*sqrt(2)*(2*x^26 + 10*x^23 - 10*x^2
1 + 4*x^20 - 40*x^18 - 28*x^17 + 20*x^16 - 12*x^15 - 30*x^14 + 60*x^13 + 56
*x^12 - 26*x^11 + 12*x^10 + 30*x^9 - 40*x^8 - 28*x^7 + 10*x^6 - 4*x^5 + 10*
x^3 + sqrt(2)*(x^26 + 3*x^23 - 5*x^21 - 2*x^20 - 12*x^18 - 18*x^17 + 10*x^1
6 + 6*x^15 - 19*x^14 + 18*x^13 + 36*x^12 - 15*x^11 - 6*x^10 + 19*x^9 - 12*x
^8 - 18*x^7 + 5*x^6 + 2*x^5 + 3*x^3 - x) - 2*x) + 16*sqrt(2)*(x^23 - 4*x^18
 - 4*x^17 - 4*x^14 + 6*x^13 + 8*x^12 - x^11 + 4*x^9 - 4*x^8 - 4*x^7 + x^3))
*sqrt((x^10 + 4*x^7 - 2*x^5 + 5*x^4 + 2^(1/4)*sqrt(x^5 + x^2 - 1)*(2*x^3 +
sqrt(2)*(x^6 + x^3 - x))*sqrt(2*sqrt(2) + 4) - 4*x^2 + 4*sqrt(2)*(x^7 + x^4
 - x^2) + 1)/(x^10 - 2*x^5 + x^4 + 1)) - 4*x)/(x^26 + 9*x^23 - 5*x^21 + 2*x
^20 - 36*x^18 - 30*x^17 + 10*x^16 - 6*x^15 - 31*x^14 + 54*x^13 + 60*x^12 -
17*x^11 + 6*x^10 + 31*x^9 - 36*x^8 - 30*x^7 + 5*x^6 - 2*x^5 + 9*x^3 - x)) +
 1/4*2^(3/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/4*(4*x^26 - 4*x^23 - 20*x^21 - 5
6*x^20 + 16*x^18 - 56*x^17 + 40*x^16 + 168*x^15 + 4*x^14 - 24*x^13 + 112*x^
12 - 28*x^11 - 168*x^10 - 4*x^9 + 16*x^8 - 56*x^7 + 20*x^6 + 56*x^5 - 4*x^3
 - sqrt(x^5 + x^2 - 1)*(2^(3/4)*(x^25 + 7*x^22 - 5*x^20 + 10*x^19 - 28*x^17
 - 6*x^16 + 10*x^15 - 30*x^14 - 7*x^13 + 42*x^12 + 12*x^11 - 7*x^10 + 30*x^
9 + 7*x^8 - 28*x^7 - 6*x^6 + 5*x^5 - 10*x^4 + 7*x^2 - sqrt(2)*(3*x^22 + 10*
x^19 - 12*x^17 + 4*x^16 - 30*x^14 - 6*x^13 + 18*x^12 - 8*x^11 + x^10 + 30*x
^9 + 6*x^8 - 12*x^7 + 4*x^6 - 10*x^4 + 3*x^2) - 1) + 2*2^(1/4)*(2*x^22 + 2*
x^19 - 8*x^17 + 10*x^16 - 6*x^14 + 18*x^13 + 12*x^12 - 20*x^11 + 8*x^10 + 6
*x^9 - 18*x^8 - 8*x^7 + 10*x^6 - 2*x^4 + 2*x^2 + sqrt(2)*(x^22 - 4*x^19 - 4
```

```
*x^17 - 12*x^16 + 12*x^14 - 12*x^13 + 6*x^12 + 24*x^11 - 5*x^10 - 12*x^9 +
12*x^8 - 4*x^7 - 12*x^6 + 4*x^4 + x^2)))*sqrt(2*sqrt(2) + 4) - 2*sqrt(2)*(2
*x^26 + 6*x^23 - 10*x^21 + 4*x^20 - 24*x^18 + 4*x^17 + 20*x^16 - 12*x^15 +
2*x^14 + 36*x^13 - 8*x^12 - 22*x^11 + 12*x^10 - 2*x^9 - 24*x^8 + 4*x^7 + 10
*x^6 - 4*x^5 + 6*x^3 + sqrt(2)*(x^26 - x^23 - 5*x^21 - 14*x^20 + 4*x^18 - 1
4*x^17 + 10*x^16 + 42*x^15 + x^14 - 6*x^13 + 28*x^12 - 7*x^11 - 42*x^10 - x
^9 + 4*x^8 - 14*x^7 + 5*x^6 + 14*x^5 - x^3 - x) - 2*x) + 4*sqrt(2)*(x^26 +
3*x^23 - 5*x^21 + 2*x^20 - 12*x^18 + 2*x^17 + 10*x^16 - 6*x^15 + x^14 + 18*
x^13 - 4*x^12 - 11*x^11 + 6*x^10 - x^9 - 12*x^8 + 2*x^7 + 5*x^6 - 2*x^5 + 3
*x^3 - x) + (24*x^23 + 16*x^20 - 96*x^18 - 48*x^17 - 48*x^15 - 48*x^14 + 14
4*x^13 + 96*x^12 - 8*x^11 + 48*x^10 + 48*x^9 - 96*x^8 - 48*x^7 - 16*x^5 + 2
4*x^3 + sqrt(x^5 + x^2 - 1)*(2^(3/4)*(x^25 + 7*x^22 - 5*x^20 + 2*x^19 - 28*
x^17 - 14*x^16 + 10*x^15 - 6*x^14 - 15*x^13 + 42*x^12 + 28*x^11 - 15*x^10 +
 6*x^9 + 15*x^8 - 28*x^7 - 14*x^6 + 5*x^5 - 2*x^4 + 7*x^2 - sqrt(2)*(x^22 -
 2*x^19 - 4*x^17 + 4*x^16 + 6*x^14 + 14*x^13 + 6*x^12 - 8*x^11 + 3*x^10 - 6
*x^9 - 14*x^8 - 4*x^7 + 4*x^6 + 2*x^4 + x^2) - 1) + 2*2^(1/4)*(2*x^22 + 6*x
^19 - 8*x^17 - 2*x^16 - 18*x^14 - 10*x^13 + 12*x^12 + 4*x^11 - 4*x^10 + 18*
x^9 + 10*x^8 - 8*x^7 - 2*x^6 - 6*x^4 + 2*x^2 + sqrt(2)*(x^22 - 4*x^17 - 8*x
^16 - 8*x^13 + 6*x^12 + 16*x^11 - x^10 + 8*x^8 - 4*x^7 - 8*x^6 + x^2)))*sqr
t(2*sqrt(2) + 4) + 2*sqrt(2)*(2*x^26 + 10*x^23 - 10*x^21 + 4*x^20 - 40*x^18
 - 28*x^17 + 20*x^16 - 12*x^15 - 30*x^14 + 60*x^13 + 56*x^12 - 26*x^11 + 12
*x^10 + 30*x^9 - 40*x^8 - 28*x^7 + 10*x^6 - 4*x^5 + 10*x^3 + sqrt(2)*(x^26
+ 3*x^23 - 5*x^21 - 2*x^20 - 12*x^18 - 18*x^17 + 10*x^16 + 6*x^15 - 19*x^14
 + 18*x^13 + 36*x^12 - 15*x^11 - 6*x^10 + 19*x^9 - 12*x^8 - 18*x^7 + 5*x^6
+ 2*x^5 + 3*x^3 - x) - 2*x) + 16*sqrt(2)*(x^23 - 4*x^18 - 4*x^17 - 4*x^14 +
 6*x^13 + 8*x^12 - x^11 + 4*x^9 - 4*x^8 - 4*x^7 + x^3))*sqrt((x^10 + 4*x^7
- 2*x^5 + 5*x^4 - 2^(1/4)*sqrt(x^5 + x^2 - 1)*(2*x^3 + sqrt(2)*(x^6 + x^3 -
 x))*sqrt(2*sqrt(2) + 4) - 4*x^2 + 4*sqrt(2)*(x^7 + x^4 - x^2) + 1)/(x^10 -
 2*x^5 + x^4 + 1)) - 4*x)/(x^26 + 9*x^23 - 5*x^21 + 2*x^20 - 36*x^18 - 30*x
^17 + 10*x^16 - 6*x^15 - 31*x^14 + 54*x^13 + 60*x^12 - 17*x^11 + 6*x^10 + 3
1*x^9 - 36*x^8 - 30*x^7 + 5*x^6 - 2*x^5 + 9*x^3 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^5+2\right)\sqrt{x^5+x^2-1}}{x^{10}-2\,x^5+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x^2-1)^(1/2)*(3*x^5+2)/(x^10-2*x^5+x^4+1),x, algorithm="giac
")
```

```
[Out] integrate((3*x^5 + 2)*sqrt(x^5 + x^2 - 1)/(x^10 - 2*x^5 + x^4 + 1), x)
```

maple [C] time = 4.23, size = 485, normalized size = 7.70

$$-\frac{\mathrm{RootOf}\left(\_Z^2+4\,\mathrm{RootOf}\left(8\,\_Z^4+4\,\_Z^2+1\right)^2+2\right)\ln\left(\frac{\mathrm{RootOf}\left(8\_Z^4+4\_Z^2+1\right)^2 x^5 \mathrm{RootOf}\left(\_Z^2+4\mathrm{RootOf}\left(8\_Z^4+4\_Z^2+1\right)^2+2\right)+4\mathrm{RootOf}\left(\_Z^2+4\mathrm{RootOf}\left(8\_Z^4+4\_Z^2+1\right)^2+2\right)\mathrm{RootOf}\left(8\_Z^4+4\_Z^2+1\right)^4 x^2+3\mathrm{RootOf}\left(\_Z^2+4\mathrm{RootOf}\left(8\_Z^4+4\_Z^2+1\right)^2+2\right)\mathrm{RootOf}\left(8\_Z^4+4\_Z^2+1\right)^2 x^2+4\sqrt{x^5+x^2-1}\,\mathrm{RootOf}\left(8\_Z^4+4\_Z^2+1\right)^2 x-\mathrm{RootOf}\left(8\_Z^4+4\_Z^2+1\right)^2\mathrm{RootOf}\left(\_Z^2+4\mathrm{RootOf}\left(8\_Z^4+4\_Z^2+1\right)^2+2\right)+\sqrt{x^5+x^2-1}\,x}{-x^5+4\mathrm{RootOf}\left(8\_Z^4+4\_Z^2+1\right)^2 x^2+x^2+1}\right)}{2}-\mathrm{RootOf}\left(8\,\_Z^4+4\,\_Z^2+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(8\,\_Z^4+4\,\_Z^2+1\right)^3 x^5-8\,\mathrm{RootOf}\left(8\,\_Z^4+4\,\_Z^2+1\right)^5 x^2+\mathrm{RootOf}\left(8\,\_Z^4+4\,\_Z^2+1\right) x^5-2\,\mathrm{RootOf}\left(8\,\_Z^4+4\,\_Z^2+1\right)^3 x^2+4\sqrt{x^5+x^2-1}\,\mathrm{RootOf}\left(8\,\_Z^4+4\,\_Z^2+1\right)^2 x-2\,\mathrm{RootOf}\left(8\,\_Z^4+4\,\_Z^2+1\right)^3+\mathrm{RootOf}\left(8\,\_Z^4+4\,\_Z^2+1\right) x^2+\sqrt{x^5+x^2-1}\,x-\mathrm{RootOf}\left(8\,\_Z^4+4\,\_Z^2+1\right)}{x^5+4\,\mathrm{RootOf}\left(8\,\_Z^4+4\,\_Z^2+1\right)^2 x^2+x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+x^2-1)^(1/2)*(3*x^5+2)/(x^10-2*x^5+x^4+1),x)
```

```
[Out] -1/2*RootOf(_Z^2+4*RootOf(8*_Z^4+4*_Z^2+1)^2+2)*ln((RootOf(8*_Z^4+4*_Z^2+1)
^2*x^5*RootOf(_Z^2+4*RootOf(8*_Z^4+4*_Z^2+1)^2+2)+4*RootOf(_Z^2+4*RootOf(8*
_Z^4+4*_Z^2+1)^2+2)*RootOf(8*_Z^4+4*_Z^2+1)^4*x^2+3*RootOf(_Z^2+4*RootOf(8*
_Z^4+4*_Z^2+1)^2+2)*RootOf(8*_Z^4+4*_Z^2+1)^2*x^2+4*(x^5+x^2-1)^(1/2)*RootO
f(8*_Z^4+4*_Z^2+1)^2*x-RootOf(8*_Z^4+4*_Z^2+1)^2*RootOf(_Z^2+4*RootOf(8*_Z^
4+4*_Z^2+1)^2+2)+(x^5+x^2-1)^(1/2)*x)/(-x^5+4*RootOf(8*_Z^4+4*_Z^2+1)^2*x^2
+x^2+1))-RootOf(8*_Z^4+4*_Z^2+1)*ln(-(2*RootOf(8*_Z^4+4*_Z^2+1)^3*x^5-8*Roo
tOf(8*_Z^4+4*_Z^2+1)^5*x^2+RootOf(8*_Z^4+4*_Z^2+1)*x^5-2*RootOf(8*_Z^4+4*_Z
^2+1)^3*x^2+4*(x^5+x^2-1)^(1/2)*RootOf(8*_Z^4+4*_Z^2+1)^2*x-2*RootOf(8*_Z^4
```

```
+4*_Z^2+1)^3+RootOf(8*_Z^4+4*_Z^2+1)*x^2+(x^5+x^2-1)^(1/2)*x-RootOf(8*_Z^4+
4*_Z^2+1))/(x^5+4*RootOf(8*_Z^4+4*_Z^2+1)^2*x^2+x^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^5+2\right)\sqrt{x^5+x^2-1}}{x^{10}-2\,x^5+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x^2-1)^(1/2)*(3*x^5+2)/(x^10-2*x^5+x^4+1),x, algorithm="maxi
ma")
```

```
[Out] integrate((3*x^5 + 2)*sqrt(x^5 + x^2 - 1)/(x^10 - 2*x^5 + x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(3\,x^5+2\right)\,\sqrt{x^5+x^2-1}}{x^{10}-2\,x^5+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^5 + 2)*(x^2 + x^5 - 1)^(1/2))/(x^4 - 2*x^5 + x^10 + 1),x)
```

```
[Out] int(((3*x^5 + 2)*(x^2 + x^5 - 1)^(1/2))/(x^4 - 2*x^5 + x^10 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^5+2\right)\sqrt{x^5+x^2-1}}{x^{10}-2x^5+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+x**2-1)**(1/2)*(3*x**5+2)/(x**10-2*x**5+x**4+1),x)
```

```
[Out] Integral((3*x**5 + 2)*sqrt(x**5 + x**2 - 1)/(x**10 - 2*x**5 + x**4 + 1), x)
```

3.798 $\int \frac{1}{x\sqrt{x+x^2}\sqrt{x^2+x\sqrt{x+x^2}}}\,dx$

Optimal. Leaf size=63

$$\frac{4\sqrt{x\left(\sqrt{x^2+x}+x\right)}\,(4x-3)}{15x^2}+\frac{16\sqrt{x^2+x}\sqrt{x\left(\sqrt{x^2+x}+x\right)}}{15x^2}$$

Rubi [F] time = 1.71, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt{x+x^2}\sqrt{x^2+x\sqrt{x+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[x + x^2]*Sqrt[x^2 + x*Sqrt[x + x^2]]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 + x]*Defer[Subst][Defer[Int][1/(x^2*Sqrt[1 + x^2]*Sqrt[x^
4 + x^2*Sqrt[x^2 + x^4]]), x], x, Sqrt[x]])/Sqrt[x + x^2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{x+x^2}\sqrt{x^2+x\sqrt{x+x^2}}}\,dx=\frac{\left(\sqrt{x}\sqrt{1+x}\right)\int \frac{1}{x^{3/2}\sqrt{1+x}\sqrt{x^2+x\sqrt{x+x^2}}}\,dx}{\sqrt{x+x^2}}$$
$$=\frac{\left(2\sqrt{x}\sqrt{1+x}\right)\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}\sqrt{x^4+x^2\sqrt{x^2+x^4}}}\,dx,x,\sqrt{x}\right)}{\sqrt{x+x^2}}$$

Mathematica [A] time = 0.59, size = 87, normalized size = 1.38

$$\frac{4\sqrt{x\left(x+\sqrt{x(x+1)}\right)}\left(x+\sqrt{x(x+1)}+1\right)\left(2x+2\sqrt{x(x+1)}+1\right)\left(4x+4\sqrt{x(x+1)}-3\right)}{15\sqrt{x(x+1)}\left(x+\sqrt{x(x+1)}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[x + x^2]*Sqrt[x^2 + x*Sqrt[x + x^2]]),x]
```

```
[Out] (4*Sqrt[x*(x + Sqrt[x*(1 + x)])]*(1 + x + Sqrt[x*(1 + x)])*(1 + 2*x + 2*Sqr
t[x*(1 + x)])*(-3 + 4*x + 4*Sqrt[x*(1 + x)]))/(15*Sqrt[x*(1 + x)]*(x + Sqrt
[x*(1 + x)])^3)
```

IntegrateAlgebraic [A] time = 3.85, size = 63, normalized size = 1.00

$$\frac{4\sqrt{x\left(\sqrt{x^2+x}+x\right)}\,(4x-3)}{15x^2}+\frac{16\sqrt{x^2+x}\sqrt{x\left(\sqrt{x^2+x}+x\right)}}{15x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[x + x^2]*Sqrt[x^2 + x*Sqrt[x + x^2]]),x]
```

```
[Out] (4*(-3 + 4*x)*Sqrt[x*(x + Sqrt[x + x^2])])/(15*x^2) + (16*Sqrt[x + x^2]*Sqr
t[x*(x + Sqrt[x + x^2])])/(15*x^2)
```

fricas [A] time = 0.45, size = 34, normalized size = 0.54

$$\frac{4\sqrt{x^2+\sqrt{x^2+x}\,x}\left(4x+4\sqrt{x^2+x}-3\right)}{15x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] 4/15*sqrt(x^2 + sqrt(x^2 + x)*x)*(4*x + 4*sqrt(x^2 + x) - 3)/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2+\sqrt{x^2+x}\,x}\,\sqrt{x^2+x}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(x^2 + sqrt(x^2 + x)*x)*sqrt(x^2 + x)*x), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^2+x}\,\sqrt{x^2+x\sqrt{x^2+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x)
```

```
[Out] int(1/x/(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2+\sqrt{x^2+x}\,x}\,\sqrt{x^2+x}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(x^2 + sqrt(x^2 + x)*x)*sqrt(x^2 + x)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\sqrt{x^2+x\sqrt{x^2+x}}\,\sqrt{x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 + x*(x + x^2)^(1/2))^(1/2)*(x + x^2)^(1/2)),x)
```

```
[Out] int(1/(x*(x^2 + x*(x + x^2)^(1/2))^(1/2)*(x + x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x\left(x+1\right)}\,\sqrt{x\left(x+\sqrt{x^2+x}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+x)**(1/2)/(x**2+x*(x**2+x)**(1/2))**(1/2),x)
[Out] Integral(1/(x*sqrt(x*(x + 1))*sqrt(x*(x + sqrt(x**2 + x)))), x)
```

3.799 $\int \sqrt{b+\sqrt{b^2+ax^2}}\,dx$

Optimal. Leaf size=63

$$\frac{4bx}{3\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{2x\sqrt{ax^2+b^2}}{3\sqrt{\sqrt{ax^2+b^2}+b}}$$

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 0.81, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2129}

$$\frac{2bx}{\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{2ax^3}{3\left(\sqrt{ax^2+b^2}+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] (2*a*x^3)/(3*(b + Sqrt[b^2 + a*x^2])^(3/2)) + (2*b*x)/Sqrt[b + Sqrt[b^2 + a
*x^2]]
```

Rule 2129

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[(2*b^2*
d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt
[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{b+\sqrt{b^2+ax^2}}\,dx=\frac{2ax^3}{3\left(b+\sqrt{b^2+ax^2}\right)^{3/2}}+\frac{2bx}{\sqrt{b+\sqrt{b^2+ax^2}}}$$

Mathematica [A] time = 0.10, size = 41, normalized size = 0.65

$$\frac{2x\left(\sqrt{ax^2+b^2}+2b\right)}{3\sqrt{\sqrt{ax^2+b^2}+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] (2*x*(2*b + Sqrt[b^2 + a*x^2]))/(3*Sqrt[b + Sqrt[b^2 + a*x^2]])
```

IntegrateAlgebraic [A] time = 0.12, size = 63, normalized size = 1.00

$$\frac{4bx}{3\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{2x\sqrt{ax^2+b^2}}{3\sqrt{\sqrt{ax^2+b^2}+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

[Out] (4*b*x)/(3*Sqrt[b + Sqrt[b^2 + a*x^2]]) + (2*x*Sqrt[b^2 + a*x^2])/(3*Sqrt[b + Sqrt[b^2 + a*x^2]])

fricas [A] time = 0.49, size = 47, normalized size = 0.75

$$\frac{2\left(ax^2 - b^2 + \sqrt{ax^2 + b^2}\,b\right)\sqrt{b + \sqrt{ax^2 + b^2}}}{3\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 2/3*(a*x^2 - b^2 + sqrt(a*x^2 + b^2)*b)*sqrt(b + sqrt(a*x^2 + b^2))/(a*x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b + \sqrt{ax^2 + b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2)), x)

maple [C] time = 0.04, size = 118, normalized size = 1.87

$$-\frac{\left(b^2\right)^{\frac{1}{4}}\left(-\frac{32\sqrt{\pi}\,\sqrt{2}\,x^3\sqrt{\frac{a}{b^2}}\,a\cosh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{x\sqrt{a}}{b}\right)}{2}\right)}{3b^2}-\frac{8\sqrt{\pi}\,\sqrt{2}\,\sqrt{\frac{a}{b^2}}\left(-\frac{4x^4a^2}{3b^4}-\frac{2x^2a}{3b^2}+\frac{2}{3}\right)\sinh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{x\sqrt{a}}{b}\right)}{2}\right)b}{\sqrt{a}\,\sqrt{\frac{x^2a}{b^2}+1}}\right)}{8\sqrt{\pi}\,\sqrt{\frac{a}{b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b+(a*x^2+b^2)^(1/2))^(1/2),x)

[Out] -1/8*(b^2)^(1/4)/Pi^(1/2)/(a/b^2)^(1/2)*(-32/3*Pi^(1/2)*2^(1/2)*x^3*(a/b^2)^(1/2)*a/b^2*cosh(3/2*arcsinh(x*a^(1/2)/b))-8*Pi^(1/2)*2^(1/2)*(a/b^2)^(1/2)*(-4/3*x^4*a^2/b^4-2/3*x^2*a/b^2+2/3)*sinh(3/2*arcsinh(x*a^(1/2)/b))/a^(1/2)*b/(x^2*a/b^2+1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b + \sqrt{ax^2 + b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b + \sqrt{b^2 + a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + (a*x^2 + b^2)^(1/2))^(1/2),x)

```
[Out] int((b + (a*x^2 + b^2)^(1/2))^(1/2), x)
```

sympy [B] time = 1.18, size = 286, normalized size = 4.54

$$-\frac{\sqrt{2}a\sqrt{b}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi b^2\sqrt{\frac{ax^2}{b^2}+1}\sqrt{\sqrt{\frac{ax^2}{b^2}+1}+1}+12\pi b^2\sqrt{\sqrt{\frac{ax^2}{b^2}+1}+1}}-\frac{3\sqrt{2}b^{\frac{5}{2}}x\sqrt{\frac{ax^2}{b^2}+1}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi b^2\sqrt{\frac{ax^2}{b^2}+1}\sqrt{\sqrt{\frac{ax^2}{b^2}+1}+1}+12\pi b^2\sqrt{\sqrt{\frac{ax^2}{b^2}+1}+1}}-\frac{3\sqrt{2}b^{\frac{5}{2}}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi b^2\sqrt{\frac{ax^2}{b^2}+1}\sqrt{\sqrt{\frac{ax^2}{b^2}+1}+1}+12\pi b^2\sqrt{\sqrt{\frac{ax^2}{b^2}+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x**2+b**2)**(1/2))**(1/2),x)
```

```
[Out] -sqrt(2)*a*sqrt(b)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*b**2*sqrt(a*x**2/b**2
 + 1)*sqrt(sqrt(a*x**2/b**2 + 1) + 1) + 12*pi*b**2*sqrt(sqrt(a*x**2/b**2 +
1) + 1)) - 3*sqrt(2)*b**(5/2)*x*sqrt(a*x**2/b**2 + 1)*gamma(-1/4)*gamma(1/4
)/(12*pi*b**2*sqrt(a*x**2/b**2 + 1)*sqrt(sqrt(a*x**2/b**2 + 1) + 1) + 12*pi
*b**2*sqrt(sqrt(a*x**2/b**2 + 1) + 1)) - 3*sqrt(2)*b**(5/2)*x*gamma(-1/4)*g
amma(1/4)/(12*pi*b**2*sqrt(a*x**2/b**2 + 1)*sqrt(sqrt(a*x**2/b**2 + 1) + 1)
 + 12*pi*b**2*sqrt(sqrt(a*x**2/b**2 + 1) + 1))
```

3.800 $\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=63

$$-\frac{2\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}}{\sqrt{\sqrt{x^2+1}+x}}-2\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)$$

Rubi [F] time = 0.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/(Sqrt[1 + x^2]*Sqrt[x + Sqrt[1 + x^2]
]),x]
```

```
[Out] Defer[Int][Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/(Sqrt[1 + x^2]*Sqrt[x + Sqrt[1
 + x^2]]), x]
```

Rubi steps

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}}\,dx=\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Mathematica [A] time = 0.11, size = 117, normalized size = 1.86

$$-\frac{2\left(\sqrt{x^2+1}+\sqrt{\sqrt{x^2+1}+x}+\left(\sqrt{x^2+1}+x\right)\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)+x\right)}{\left(\sqrt{x^2+1}+x\right)\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/(Sqrt[1 + x^2]*Sqrt[x + Sqrt[1
+ x^2]]),x]
```

```
[Out] (-2*(x + Sqrt[1 + x^2] + Sqrt[x + Sqrt[1 + x^2]] + (x + Sqrt[1 + x^2])*Sqrt
[1 + Sqrt[x + Sqrt[1 + x^2]]]*ArcTanh[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]]))/
((x + Sqrt[1 + x^2])*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])
```

IntegrateAlgebraic [A] time = 0.13, size = 63, normalized size = 1.00

$$-\frac{2\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}}{\sqrt{\sqrt{x^2+1}+x}}-2\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/(Sqrt[1 + x^2]*Sqrt[x
+ Sqrt[1 + x^2]]),x]
```

```
[Out] (-2*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/Sqrt[x + Sqrt[1 + x^2]] - 2*ArcTanh[
Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]]
```

fricas [A] time = 0.45, size = 78, normalized size = 1.24

$$2\sqrt{x+\sqrt{x^2+1}}\left(x-\sqrt{x^2+1}\right)\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}-\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}+1\right)+\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x^2+1)^(1/2)/(x+(x^2+1)^(1/2))
^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + sqrt(x^2 + 1))*(x - sqrt(x^2 + 1))*sqrt(sqrt(x + sqrt(x^2 + 1))
+ 1) - log(sqrt(sqrt(x + sqrt(x^2 + 1)) + 1) + 1) + log(sqrt(sqrt(x + sqrt(
x^2 + 1)) + 1) - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x^2+1)^(1/2)/(x+(x^2+1)^(1/2))
^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}}{\sqrt{x^2+1}\,\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x^2+1)^(1/2)/(x+(x^2+1)^(1/2))^(1/2)
,x)
```

```
[Out] int((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x^2+1)^(1/2)/(x+(x^2+1)^(1/2))^(1/2)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}}{\sqrt{x^2+1}\,\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x^2+1)^(1/2)/(x+(x^2+1)^(1/2))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x + sqrt(x^2 + 1)) + 1)/(sqrt(x^2 + 1)*sqrt(x + sqrt(x^
2 + 1))), x)
```

mupad [B] time = 1.49, size = 121, normalized size = 1.92

$$-\frac{2\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}}{\sqrt{x+\sqrt{x^2+1}}}-\frac{\ln\left(\sqrt{\frac{1}{x+\sqrt{x^2+1}}+\frac{1}{\sqrt{x+\sqrt{x^2+1}}}}+\frac{1}{\sqrt{x+\sqrt{x^2+1}}}+\frac{1}{2}\right)\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}}{\sqrt{x+\sqrt{x^2+1}}\sqrt{\frac{1}{x+\sqrt{x^2+1}}+\frac{1}{\sqrt{x+\sqrt{x^2+1}}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2)/((x^2 + 1)^(1/2)*(x + (x^2 + 1)
^(1/2))^(1/2)),x)
```

```
[Out] - (2*((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2))/(x + (x^2 + 1)^(1/2))^(1/2) -
 (log((1/(x + (x^2 + 1)^(1/2)) + 1/(x + (x^2 + 1)^(1/2))^(1/2))^(1/2) + 1/(
x + (x^2 + 1)^(1/2))^(1/2) + 1/2)*((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2))/
((x + (x^2 + 1)^(1/2))^(1/2)*(1/(x + (x^2 + 1)^(1/2)) + 1/(x + (x^2 + 1)^(1
/2))^(1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}}{\sqrt{x+\sqrt{x^2+1}}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x+(x**2+1)**(1/2))**(1/2))**(1/2)/(x**2+1)**(1/2)/(x+(x**2+1)
**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x + sqrt(x**2 + 1)) + 1)/(sqrt(x + sqrt(x**2 + 1))*sqrt(
x**2 + 1)), x)
```

3.801 $\int \frac{-b+ax}{(b+ax)\sqrt{b^2x+a^2x^3}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{\sqrt{a}\sqrt{b}}$$

Rubi [A] time = 0.75, antiderivative size = 91, normalized size of antiderivative = 1.42, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2056, 6733, 1699, 205}

$$-\frac{\sqrt{2}\sqrt{x}\sqrt{a^2x^2+b^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{x}}{\sqrt{a^2x^2+b^2}}\right)}{\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x)/((b + a*x)*Sqrt[b^2*x + a^2*x^3]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[x]*Sqrt[b^2 + a^2*x^2]*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt
[x])/Sqrt[b^2 + a^2*x^2]])/(Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{-b+ax}{(b+ax)\sqrt{b^2x+a^2x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{b^2+a^2x^2}\right)\int \frac{-b+ax}{\sqrt{x}(b+ax)\sqrt{b^2+a^2x^2}}\,dx}{\sqrt{b^2x+a^2x^3}}$$
$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{-b+ax^2}{(b+ax^2)\sqrt{b^2+a^2x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}}$$
$$= -\frac{\left(2b\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{1}{b+2ab^2x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b^2+a^2x^2}}\right)}{\sqrt{b^2x+a^2x^3}}$$
$$= -\frac{\sqrt{2}\sqrt{x}\sqrt{b^2+a^2x^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{x}}{\sqrt{b^2+a^2x^2}}\right)}{\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}}$$

Mathematica [C] time = 0.47, size = 110, normalized size = 1.72

$$-\frac{2ix^{3/2}\sqrt{\frac{b^2}{a^2x^2}+1}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)-2\Pi\left(-i;i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{\sqrt{\frac{ib}{a}}\sqrt{x\left(a^2x^2+b^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x)/((b + a*x)*Sqrt[b^2*x + a^2*x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[1 + b^2/(a^2*x^2)]*x^(3/2)*(EllipticF[I*ArcSinh[Sqrt[(I*b)/a]/
Sqrt[x]], -1] - 2*EllipticPi[-I, I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1]))/(S
qrt[(I*b)/a]*Sqrt[x*(b^2 + a^2*x^2)])
```

IntegrateAlgebraic [A] time = 0.27, size = 64, normalized size = 1.00

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x)/((b + a*x)*Sqrt[b^2*x + a^2*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])/(b^2 + a^
2*x^2)])/(Sqrt[a]*Sqrt[b]))
```

fricas [A] time = 0.49, size = 213, normalized size = 3.33

$$\left[\frac{1}{4}\sqrt{2}\sqrt{-\frac{1}{ab}}\log\left(\frac{a^4x^4-12a^3bx^3+6a^2b^2x^2-12ab^3x+b^4+4\sqrt{2}\left(a^3bx^2-2a^2b^2x+ab^3\right)\sqrt{a^2x^3+b^2x}\sqrt{-\frac{1}{ab}}}{a^4x^4+4a^3bx^3+6a^2b^2x^2+4ab^3x+b^4}\right), -\frac{1}{2}\sqrt{2}\sqrt{\frac{1}{ab}}\arctan\left(\frac{2\sqrt{2}\sqrt{a^2x^3+b^2x}\,ab\sqrt{\frac{1}{ab}}}{a^2x^2-2abx+b^2}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/(a*x+b)/(a^2*x^3+b^2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*sqrt(-1/(a*b))*log((a^4*x^4 - 12*a^3*b*x^3 + 6*a^2*b^2*x^2 - 1
2*a*b^3*x + b^4 + 4*sqrt(2)*(a^3*b*x^2 - 2*a^2*b^2*x + a*b^3)*sqrt(a^2*x^3
+ b^2*x)*sqrt(-1/(a*b)))/(a^4*x^4 + 4*a^3*b*x^3 + 6*a^2*b^2*x^2 + 4*a*b^3*x
 + b^4)), -1/2*sqrt(2)*sqrt(1/(a*b))*arctan(2*sqrt(2)*sqrt(a^2*x^3 + b^2*x)
*a*b*sqrt(1/(a*b))/(a^2*x^2 - 2*a*b*x + b^2))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-b}{\sqrt{a^2x^3+b^2x}\,(ax+b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/(a*x+b)/(a^2*x^3+b^2*x)^(1/2),x, algorithm="giac")
[Out] integrate((a*x - b)/(sqrt(a^2*x^3 + b^2*x)*(a*x + b)), x)
```

maple [C] time = 0.10, size = 231, normalized size = 3.61

$$\frac{ib\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{2}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\;\mathrm{EllipticF}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a^2x^3+b^2x}}-\frac{2ib^2\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{2}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\;\mathrm{EllipticPi}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},-\frac{ib}{a\left(-\frac{ib}{a}+\frac{b}{a}\right)},\frac{\sqrt{2}}{2}\right)}{a^2\sqrt{a^2x^3+b^2x}\,\left(-\frac{ib}{a}+\frac{b}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-b)/(a*x+b)/(a^2*x^3+b^2*x)^(1/2),x)
[Out] I*b/a*(-I*(x+I*b/a)/b*a)^(1/2)*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1
/2)/(a^2*x^3+b^2*x)^(1/2)*EllipticF((-I*(x+I*b/a)/b*a)^(1/2),1/2*2^(1/2))-2
*I*b^2/a^2*(-I*(x+I*b/a)/b*a)^(1/2)*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*
a)^(1/2)/(a^2*x^3+b^2*x)^(1/2)/(-I*b/a+b/a)*EllipticPi((-I*(x+I*b/a)/b*a)^(
1/2),-I*b/a/(-I*b/a+b/a),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-b}{\sqrt{a^2x^3+b^2x}\,(ax+b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/(a*x+b)/(a^2*x^3+b^2*x)^(1/2),x, algorithm="maxima")
[Out] integrate((a*x - b)/(sqrt(a^2*x^3 + b^2*x)*(a*x + b)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x)/((b^2*x + a^2*x^3)^(1/2)*(b + a*x)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-b}{\sqrt{x\left(a^2x^2+b^2\right)}\,(ax+b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/(a*x+b)/(a**2*x**3+b**2*x)**(1/2),x)
[Out] Integral((a*x - b)/(sqrt(x*(a**2*x**2 + b**2))*(a*x + b)), x)
```

3.802 $\int \frac{b+ax}{(-b+ax)\sqrt{b^2x+a^2x^3}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{\sqrt{a}\sqrt{b}}$$

Rubi [A] time = 0.68, antiderivative size = 91, normalized size of antiderivative = 1.42, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2056, 6733, 1699, 208}

$$-\frac{\sqrt{2}\sqrt{x}\sqrt{a^2x^2+b^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{x}}{\sqrt{a^2x^2+b^2}}\right)}{\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x)/((-b + a*x)*Sqrt[b^2*x + a^2*x^3]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[x]*Sqrt[b^2 + a^2*x^2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqr
t[x])/Sqrt[b^2 + a^2*x^2]])/(Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{b+ax}{(-b+ax)\sqrt{b^2x+a^2x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{b^2+a^2x^2}\right)\int \frac{b+ax}{\sqrt{x}(-b+ax)\sqrt{b^2+a^2x^2}}\,dx}{\sqrt{b^2x+a^2x^3}}$$
$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{b+ax^2}{\left(-b+ax^2\right)\sqrt{b^2+a^2x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}}$$
$$= \frac{\left(2b\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{1}{-b+2ab^2x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b^2+a^2x^2}}\right)}{\sqrt{b^2x+a^2x^3}}$$
$$= -\frac{\sqrt{2}\sqrt{x}\sqrt{b^2+a^2x^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{x}}{\sqrt{b^2+a^2x^2}}\right)}{\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}}$$

Mathematica [C] time = 0.40, size = 110, normalized size = 1.72

$$-\frac{2ix^{3/2}\sqrt{\frac{b^2}{a^2x^2}+1}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)-2\Pi\left(i;i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{\sqrt{\frac{ib}{a}}\sqrt{x\left(a^2x^2+b^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x)/((-b + a*x)*Sqrt[b^2*x + a^2*x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[1 + b^2/(a^2*x^2)]*x^(3/2)*(EllipticF[I*ArcSinh[Sqrt[(I*b)/a]/
Sqrt[x]], -1] - 2*EllipticPi[I, I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1]))/(Sq
rt[(I*b)/a]*Sqrt[x*(b^2 + a^2*x^2)])
```

IntegrateAlgebraic [A] time = 0.27, size = 64, normalized size = 1.00

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x)/((-b + a*x)*Sqrt[b^2*x + a^2*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])/(b^2 + a
^2*x^2)])/(Sqrt[a]*Sqrt[b]))
```

fricas [A] time = 0.51, size = 213, normalized size = 3.33

$$\left[\frac{1}{4}\sqrt{2}\sqrt{\frac{1}{ab}}\log\left(\frac{a^4x^4+12a^3bx^3+6a^2b^2x^2+12ab^3x+b^4-4\sqrt{2}\left(a^3bx^2+2a^2b^2x+ab^3\right)\sqrt{a^2x^3+b^2x}\sqrt{\frac{1}{ab}}}{a^4x^4-4a^3bx^3+6a^2b^2x^2-4ab^3x+b^4}\right), \frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{ab}}\arctan\left(\frac{2\sqrt{2}\sqrt{a^2x^3+b^2x}ab\sqrt{-\frac{1}{ab}}}{a^2x^2+2abx+b^2}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(a*x-b)/(a^2*x^3+b^2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*sqrt(1/(a*b))*log((a^4*x^4 + 12*a^3*b*x^3 + 6*a^2*b^2*x^2 + 12
*a*b^3*x + b^4 - 4*sqrt(2)*(a^3*b*x^2 + 2*a^2*b^2*x + a*b^3)*sqrt(a^2*x^3 +
 b^2*x)*sqrt(1/(a*b)))/(a^4*x^4 - 4*a^3*b*x^3 + 6*a^2*b^2*x^2 - 4*a*b^3*x +
 b^4)), 1/2*sqrt(2)*sqrt(-1/(a*b))*arctan(2*sqrt(2)*sqrt(a^2*x^3 + b^2*x)*a
*b*sqrt(-1/(a*b))/(a^2*x^2 + 2*a*b*x + b^2))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+b}{\sqrt{a^2x^3+b^2x}\,(ax-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(a*x-b)/(a^2*x^3+b^2*x)^(1/2),x, algorithm="giac")
[Out] integrate((a*x + b)/(sqrt(a^2*x^3 + b^2*x)*(a*x - b)), x)
```

maple [C] time = 0.04, size = 233, normalized size = 3.64

$$\frac{ib\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{2}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\operatorname{EllipticF}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a^2x^3+b^2x}}+\frac{2ib^2\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{2}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\operatorname{EllipticPi}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},-\frac{ib}{a\left(-\frac{ib}{a}-\frac{b}{a}\right)},\frac{\sqrt{2}}{2}\right)}{a^2\sqrt{a^2x^3+b^2x}\,\left(-\frac{ib}{a}-\frac{b}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)/(a*x-b)/(a^2*x^3+b^2*x)^(1/2),x)
[Out] I*b/a*(-I*(x+I*b/a)/b*a)^(1/2)*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1
/2)/(a^2*x^3+b^2*x)^(1/2)*EllipticF((-I*(x+I*b/a)/b*a)^(1/2),1/2*2^(1/2))+2
*I*b^2/a^2*(-I*(x+I*b/a)/b*a)^(1/2)*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*
a)^(1/2)/(a^2*x^3+b^2*x)^(1/2)/(-I*b/a-b/a)*EllipticPi((-I*(x+I*b/a)/b*a)^(
1/2),-I*b/a/(-I*b/a-b/a),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+b}{\sqrt{a^2x^3+b^2x}\,(ax-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(a*x-b)/(a^2*x^3+b^2*x)^(1/2),x, algorithm="maxima")
[Out] integrate((a*x + b)/(sqrt(a^2*x^3 + b^2*x)*(a*x - b)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x)/((b^2*x + a^2*x^3)^(1/2)*(b - a*x)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+b}{\sqrt{x\left(a^2x^2+b^2\right)}\,(ax-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(a*x-b)/(a**2*x**3+b**2*x)**(1/2),x)
[Out] Integral((a*x + b)/(sqrt(x*(a**2*x**2 + b**2))*(a*x - b)), x)
```

3.803 $\int \frac{-bx^2+ax^3}{(bx^2+ax^3)\sqrt{b^2x+a^2x^3}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{\sqrt{a}\sqrt{b}}$$

Rubi [A] time = 0.71, antiderivative size = 91, normalized size of antiderivative = 1.42, number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {1593, 1584, 2056, 6733, 1699, 205}

$$-\frac{\sqrt{2}\sqrt{x}\sqrt{a^2x^2+b^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{x}}{\sqrt{a^2x^2+b^2}}\right)}{\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-(b*x^2) + a*x^3)/((b*x^2 + a*x^3)*Sqrt[b^2*x + a^2*x^3]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[x]*Sqrt[b^2 + a^2*x^2]*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt
[x])/Sqrt[b^2 + a^2*x^2]])/(Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{-bx^2+ax^3}{\left(bx^2+ax^3\right)\sqrt{b^2x+a^2x^3}}\,dx &= \int \frac{x^2(-b+ax)}{\left(bx^2+ax^3\right)\sqrt{b^2x+a^2x^3}}\,dx\\ &= \int \frac{-b+ax}{(b+ax)\sqrt{b^2x+a^2x^3}}\,dx\\ &= \frac{\left(\sqrt{x}\,\sqrt{b^2+a^2x^2}\right)\int \frac{-b+ax}{\sqrt{x}\,(b+ax)\sqrt{b^2+a^2x^2}}\,dx}{\sqrt{b^2x+a^2x^3}}\\ &= \frac{\left(2\sqrt{x}\,\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{-b+ax^2}{\left(b+ax^2\right)\sqrt{b^2+a^2x^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}}\\ &= -\frac{\left(2b\sqrt{x}\,\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{1}{b+2ab^2x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{b^2+a^2x^2}}\right)}{\sqrt{b^2x+a^2x^3}}\\ &= -\frac{\sqrt{2}\,\sqrt{x}\,\sqrt{b^2+a^2x^2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,\sqrt{x}}{\sqrt{b^2+a^2x^2}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{b^2x+a^2x^3}}\end{aligned}$$

Mathematica [C] time = 0.14, size = 110, normalized size = 1.72

$$-\frac{2ix^{3/2}\sqrt{\frac{b^2}{a^2x^2}+1}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)-2\Pi\left(-i;i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{\sqrt{\frac{ib}{a}}\sqrt{x\left(a^2x^2+b^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-(b*x^2) + a*x^3)/((b*x^2 + a*x^3)*Sqrt[b^2*x + a^2*x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[1 + b^2/(a^2*x^2)]*x^(3/2)*(EllipticF[I*ArcSinh[Sqrt[(I*b)/a]/
Sqrt[x]], -1] - 2*EllipticPi[-I, I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1]))/(S
qrt[(I*b)/a]*Sqrt[x*(b^2 + a^2*x^2)])
```

IntegrateAlgebraic [A] time = 0.28, size = 64, normalized size = 1.00

$$-\frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(b*x^2) + a*x^3)/((b*x^2 + a*x^3)*Sqrt[b^2*x + a^2*x^3
]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])/(b^2 + a^
2*x^2)])/(Sqrt[a]*Sqrt[b]))
```

fricas [A] time = 0.51, size = 213, normalized size = 3.33

$$\left[\frac{1}{4}\,\sqrt{2}\sqrt{-\frac{1}{ab}}\log\left(\frac{a^4x^4-12\,a^3bx^3+6\,a^2b^2x^2-12\,ab^3x+b^4+4\,\sqrt{2}\left(a^3bx^2-2\,a^2b^2x+ab^3\right)\sqrt{a^2x^3+b^2x}\,\sqrt{-\frac{1}{ab}}}{a^4x^4+4\,a^3bx^3+6\,a^2b^2x^2+4\,ab^3x+b^4}\right),-\frac{1}{2}\,\sqrt{2}\,\sqrt{\frac{1}{ab}}\,\arctan\left(\frac{2\,\sqrt{2}\,\sqrt{a^2x^3+b^2x}\,ab\sqrt{\frac{1}{ab}}}{a^2x^2-2\,abx+b^2}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b*x^2)/(a*x^3+b*x^2)/(a^2*x^3+b^2*x)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/4*sqrt(2)*sqrt(-1/(a*b))*log((a^4*x^4 - 12*a^3*b*x^3 + 6*a^2*b^2*x^2 - 1
2*a*b^3*x + b^4 + 4*sqrt(2)*(a^3*b*x^2 - 2*a^2*b^2*x + a*b^3)*sqrt(a^2*x^3
+ b^2*x)*sqrt(-1/(a*b)))/(a^4*x^4 + 4*a^3*b*x^3 + 6*a^2*b^2*x^2 + 4*a*b^3*x
 + b^4)), -1/2*sqrt(2)*sqrt(1/(a*b))*arctan(2*sqrt(2)*sqrt(a^2*x^3 + b^2*x)
*a*b*sqrt(1/(a*b))/(a^2*x^2 - 2*a*b*x + b^2))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 - bx^2}{\sqrt{a^2x^3 + b^2x}\left(ax^3 + bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b*x^2)/(a*x^3+b*x^2)/(a^2*x^3+b^2*x)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((a*x^3 - b*x^2)/(sqrt(a^2*x^3 + b^2*x)*(a*x^3 + b*x^2)), x)
```

maple [C] time = 0.02, size = 231, normalized size = 3.61

$$\frac{ib\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{2}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\operatorname{EllipticF}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}, \frac{\sqrt{2}}{2}\right)}{a\sqrt{a^2x^3 + b^2x}} - \frac{2ib^2\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{2}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\operatorname{EllipticPi}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}, -\frac{ib}{a\left(-\frac{ib}{a}+\frac{b}{a}\right)}, \frac{\sqrt{2}}{2}\right)}{a^2\sqrt{a^2x^3 + b^2x}\,\left(-\frac{ib}{a}+\frac{b}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b*x^2)/(a*x^3+b*x^2)/(a^2*x^3+b^2*x)^(1/2),x)
```

```
[Out] I*b/a*(-I*(x+I*b/a)/b*a)^(1/2)*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1
/2)/(a^2*x^3+b^2*x)^(1/2)*EllipticF((-I*(x+I*b/a)/b*a)^(1/2),1/2*2^(1/2))-2
*I*b^2/a^2*(-I*(x+I*b/a)/b*a)^(1/2)*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*
a)^(1/2)/(a^2*x^3+b^2*x)^(1/2)/(-I*b/a+b/a)*EllipticPi((-I*(x+I*b/a)/b*a)^(
1/2),-I*b/a/(-I*b/a+b/a),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 - bx^2}{\sqrt{a^2x^3 + b^2x}\left(ax^3 + bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b*x^2)/(a*x^3+b*x^2)/(a^2*x^3+b^2*x)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((a*x^3 - b*x^2)/(sqrt(a^2*x^3 + b^2*x)*(a*x^3 + b*x^2)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 - b*x^2)/((a*x^3 + b*x^2)*(b^2*x + a^2*x^3)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax - b}{\sqrt{x\left(a^2x^2 + b^2\right)}\,(ax + b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b*x**2)/(a*x**3+b*x**2)/(a**2*x**3+b**2*x)**(1/2),x)
[Out] Integral((a*x - b)/(sqrt(x*(a**2*x**2 + b**2))*(a*x + b)), x)
```

3.804 $\int \frac{2+5x}{\sqrt{-12+20x+5x^2+2x^3+x^4}}\,dx$

Optimal. Leaf size=64

$$-\log \left(-x^5-3x^4-8x^3-24x^2+\left(x^3+2x^2+4x+8\right)\sqrt{x^4+2x^3+5x^2+20x-12}-16x-16\right)$$

Rubi [F] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2+5x}{\sqrt{-12+20x+5x^2+2x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2 + 5*x)/Sqrt[-12 + 20*x + 5*x^2 + 2*x^3 + x^4],x]
```

```
[Out] 2*Defer[Int][1/Sqrt[-12 + 20*x + 5*x^2 + 2*x^3 + x^4], x] + 5*Defer[Int][x/
Sqrt[-12 + 20*x + 5*x^2 + 2*x^3 + x^4], x]
```

Rubi steps

$$\begin{aligned}\int \frac{2+5x}{\sqrt{-12+20x+5x^2+2x^3+x^4}}\,dx &= \int \left(\frac{2}{\sqrt{-12+20x+5x^2+2x^3+x^4}}+\frac{5x}{\sqrt{-12+20x+5x^2+2x^3+x^4}}\right)dx\\ &= 2\int \frac{1}{\sqrt{-12+20x+5x^2+2x^3+x^4}}\,dx+5\int \frac{x}{\sqrt{-12+20x+5x^2+2x^3+}}\end{aligned}$$

Mathematica [C] time = 1.12, size = 1144, normalized size = 17.88

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x)/Sqrt[-12 + 20*x + 5*x^2 + 2*x^3 + x^4],x]
```

```
[Out] (-2*(3 + x)*(x - Root[-4 + 8*#1 - #1^2 + #1^3 & , 1, 0])*(2*EllipticF[ArcSi
n[Sqrt[((3 + x)*(-Root[-4 + 8*#1 - #1^2 + #1^3 & , 1, 0] + Root[-4 + 8*#1 -
 #1^2 + #1^3 & , 3, 0]))/((x - Root[-4 + 8*#1 - #1^2 + #1^3 & , 1, 0])*(3 +
 Root[-4 + 8*#1 - #1^2 + #1^3 & , 3, 0]))]], ((Root[-4 + 8*#1 - #1^2 + #1^3
 & , 1, 0] - Root[-4 + 8*#1 - #1^2 + #1^3 & , 2, 0])*(3 + Root[-4 + 8*#1 -
#1^2 + #1^3 & , 3, 0]))/((3 + Root[-4 + 8*#1 - #1^2 + #1^3 & , 2, 0])*(Root
[-4 + 8*#1 - #1^2 + #1^3 & , 1, 0] - Root[-4 + 8*#1 - #1^2 + #1^3 & , 3, 0]
))] + 5*EllipticF[ArcSin[Sqrt[-(((3 + x)*(Root[-4 + 8*#1 - #1^2 + #1^3 & ,
1, 0] - Root[-4 + 8*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[-4 + 8*#1 - #1^
2 + #1^3 & , 1, 0])*(3 + Root[-4 + 8*#1 - #1^2 + #1^3 & , 3, 0])))]], ((Roo
t[-4 + 8*#1 - #1^2 + #1^3 & , 1, 0] - Root[-4 + 8*#1 - #1^2 + #1^3 & , 2, 0
])*(3 + Root[-4 + 8*#1 - #1^2 + #1^3 & , 3, 0]))/((3 + Root[-4 + 8*#1 - #1^
2 + #1^3 & , 2, 0])*(Root[-4 + 8*#1 - #1^2 + #1^3 & , 1, 0] - Root[-4 + 8*#
1 - #1^2 + #1^3 & , 3, 0]))]*Root[-4 + 8*#1 - #1^2 + #1^3 & , 1, 0] - 5*Ell
ipticPi[(3 + Root[-4 + 8*#1 - #1^2 + #1^3 & , 3, 0])/(-Root[-4 + 8*#1 - #1^
2 + #1^3 & , 1, 0] + Root[-4 + 8*#1 - #1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[-
(((3 + x)*(Root[-4 + 8*#1 - #1^2 + #1^3 & , 1, 0] - Root[-4 + 8*#1 - #1^2 +
 #1^3 & , 3, 0]))/((x - Root[-4 + 8*#1 - #1^2 + #1^3 & , 1, 0])*(3 + Root[-
4 + 8*#1 - #1^2 + #1^3 & , 3, 0])))]], ((Root[-4 + 8*#1 - #1^2 + #1^3 & , 1
, 0] - Root[-4 + 8*#1 - #1^2 + #1^3 & , 2, 0])*(3 + Root[-4 + 8*#1 - #1^2 +
 #1^3 & , 3, 0]))/((3 + Root[-4 + 8*#1 - #1^2 + #1^3 & , 2, 0])*(Root[-4 +
8*#1 - #1^2 + #1^3 & , 1, 0] - Root[-4 + 8*#1 - #1^2 + #1^3 & , 3, 0]))]*(3
```

```
 + Root[-4 + 8*#1 - #1^2 + #1^3 & , 1, 0]))*Sqrt[(x - Root[-4 + 8*#1 - #1^2
 + #1^3 & , 2, 0])/((x - Root[-4 + 8*#1 - #1^2 + #1^3 & , 1, 0])*(3 + Root[
-4 + 8*#1 - #1^2 + #1^3 & , 2, 0]))]*Sqrt[(x - Root[-4 + 8*#1 - #1^2 + #1^3
 & , 3, 0])/((x - Root[-4 + 8*#1 - #1^2 + #1^3 & , 1, 0])*(3 + Root[-4 + 8*
#1 - #1^2 + #1^3 & , 3, 0]))])/(Sqrt[-12 + 20*x + 5*x^2 + 2*x^3 + x^4]*Sqrt
[-(((3 + x)*(Root[-4 + 8*#1 - #1^2 + #1^3 & , 1, 0] - Root[-4 + 8*#1 - #1^2
 + #1^3 & , 3, 0]))/((x - Root[-4 + 8*#1 - #1^2 + #1^3 & , 1, 0])*(3 + Root
[-4 + 8*#1 - #1^2 + #1^3 & , 3, 0])))])
```

IntegrateAlgebraic [A] time = 5.06, size = 64, normalized size = 1.00

$$-\log\left(-x^5-3x^4-8x^3-24x^2+\left(x^3+2x^2+4x+8\right)\sqrt{x^4+2x^3+5x^2+20x-12}-16x-16\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 5*x)/Sqrt[-12 + 20*x + 5*x^2 + 2*x^3 + x^4],x]
```

```
[Out] -Log[-16 - 16*x - 24*x^2 - 8*x^3 - 3*x^4 - x^5 + (8 + 4*x + 2*x^2 + x^3)*Sq
rt[-12 + 20*x + 5*x^2 + 2*x^3 + x^4]]
```

fricas [A] time = 0.50, size = 58, normalized size = 0.91

$$\log\left(x^5+3\,x^4+8\,x^3+24\,x^2+\sqrt{x^4+2\,x^3+5\,x^2+20\,x-12}\left(x^3+2\,x^2+4\,x+8\right)+16\,x+16\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+5*x)/(x^4+2*x^3+5*x^2+20*x-12)^(1/2),x, algorithm="fricas")
```

```
[Out] log(x^5 + 3*x^4 + 8*x^3 + 24*x^2 + sqrt(x^4 + 2*x^3 + 5*x^2 + 20*x - 12)*(x
^3 + 2*x^2 + 4*x + 8) + 16*x + 16)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,x+2}{\sqrt{x^4+2\,x^3+5\,x^2+20\,x-12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+5*x)/(x^4+2*x^3+5*x^2+20*x-12)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x + 2)/sqrt(x^4 + 2*x^3 + 5*x^2 + 20*x - 12), x)
```

maple [C] time = 1.30, size = 2769, normalized size = 43.27

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+5*x)/(x^4+2*x^3+5*x^2+20*x-12)^(1/2),x)
```

```
[Out] 4*(-10/3+1/6*(19+12*87^(1/2))^(1/3)-23/6/(19+12*87^(1/2))^(1/3)+1/2*I*3^(1/
2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))*((-1/2*(19+12*
87^(1/2))^(1/3)+23/2/(19+12*87^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(19+12*87^(1
/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))*(3+x)/(-1/6*(19+12*87^(1/2))^(1/3)
+23/6/(19+12*87^(1/2))^(1/3)+10/3-1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)
+23/3/(19+12*87^(1/2))^(1/3)))/(x-1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87
^(1/2))^(1/3)-1/3))^(1/2)*(x-1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2
))^(1/3)-1/3)^2*((1/3*(19+12*87^(1/2))^(1/3)-23/3/(19+12*87^(1/2))^(1/3)+10
/3)*(x+1/6*(19+12*87^(1/2))^(1/3)-23/6/(19+12*87^(1/2))^(1/3)-1/3-1/2*I*3^(
1/2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))/(-1/6*(19+12
*87^(1/2))^(1/3)+23/6/(19+12*87^(1/2))^(1/3)+10/3+1/2*I*3^(1/2)*(1/3*(19+12
```

```
*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))/(x-1/3*(19+12*87^(1/2))^(1/3
)+23/3/(19+12*87^(1/2))^(1/3)-1/3))^(1/2)*((1/3*(19+12*87^(1/2))^(1/3)-23/3
/(19+12*87^(1/2))^(1/3)+10/3)*(x+1/6*(19+12*87^(1/2))^(1/3)-23/6/(19+12*87^
(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(
1/2))^(1/3)))/(-1/6*(19+12*87^(1/2))^(1/3)+23/6/(19+12*87^(1/2))^(1/3)+10/3
-1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))/(x
-1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)-1/3))^(1/2)/(-1/2*(
19+12*87^(1/2))^(1/3)+23/2/(19+12*87^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(19+12
*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))/(1/3*(19+12*87^(1/2))^(1/3)-
23/3/(19+12*87^(1/2))^(1/3)+10/3)/((3+x)*(x-1/3*(19+12*87^(1/2))^(1/3)+23/3
/(19+12*87^(1/2))^(1/3)-1/3)*(x+1/6*(19+12*87^(1/2))^(1/3)-23/6/(19+12*87^(
1/2))^(1/3)-1/3-1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1
/2))^(1/3)))*(x+1/6*(19+12*87^(1/2))^(1/3)-23/6/(19+12*87^(1/2))^(1/3)-1/3+
1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3))))^(1
/2)*EllipticF(((-1/2*(19+12*87^(1/2))^(1/3)+23/2/(19+12*87^(1/2))^(1/3)-1/2
*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))*(3+x)/
(-1/6*(19+12*87^(1/2))^(1/3)+23/6/(19+12*87^(1/2))^(1/3)+10/3-1/2*I*3^(1/2)
*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))/(x-1/3*(19+12*87
^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)-1/3))^(1/2),((1/2*(19+12*87^(1/2)
)^(1/3)-23/2/(19+12*87^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/
3)+23/3/(19+12*87^(1/2))^(1/3)))*(-10/3+1/6*(19+12*87^(1/2))^(1/3)-23/6/(19
+12*87^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*8
7^(1/2))^(1/3)))/(-10/3+1/6*(19+12*87^(1/2))^(1/3)-23/6/(19+12*87^(1/2))^(1
/3)-1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))
/(1/2*(19+12*87^(1/2))^(1/3)-23/2/(19+12*87^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/3
*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3))))^(1/2))+10*(-10/3+1/6
*(19+12*87^(1/2))^(1/3)-23/6/(19+12*87^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/3*(19+
12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))*((-1/2*(19+12*87^(1/2))^(1
/3)+23/2/(19+12*87^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+2
3/3/(19+12*87^(1/2))^(1/3)))*(3+x)/(-1/6*(19+12*87^(1/2))^(1/3)+23/6/(19+12
*87^(1/2))^(1/3)+10/3-1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12
*87^(1/2))^(1/3)))/(x-1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3
)-1/3))^(1/2)*(x-1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)-1/3
)^2*((1/3*(19+12*87^(1/2))^(1/3)-23/3/(19+12*87^(1/2))^(1/3)+10/3)*(x+1/6*(
19+12*87^(1/2))^(1/3)-23/6/(19+12*87^(1/2))^(1/3)-1/3-1/2*I*3^(1/2)*(1/3*(1
9+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))/(-1/6*(19+12*87^(1/2))^(
1/3)+23/6/(19+12*87^(1/2))^(1/3)+10/3+1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(
1/3)+23/3/(19+12*87^(1/2))^(1/3)))/(x-1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+1
2*87^(1/2))^(1/3)-1/3))^(1/2)*((1/3*(19+12*87^(1/2))^(1/3)-23/3/(19+12*87^(
1/2))^(1/3)+10/3)*(x+1/6*(19+12*87^(1/2))^(1/3)-23/6/(19+12*87^(1/2))^(1/3)
-1/3+1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3))
)/(-1/6*(19+12*87^(1/2))^(1/3)+23/6/(19+12*87^(1/2))^(1/3)+10/3-1/2*I*3^(1/
2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))/(x-1/3*(19+12*
87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)-1/3))^(1/2)/(-1/2*(19+12*87^(1/
2))^(1/3)+23/2/(19+12*87^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(
1/3)+23/3/(19+12*87^(1/2))^(1/3)))/(1/3*(19+12*87^(1/2))^(1/3)-23/3/(19+12*
87^(1/2))^(1/3)+10/3)/((3+x)*(x-1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(
1/2))^(1/3)-1/3)*(x+1/6*(19+12*87^(1/2))^(1/3)-23/6/(19+12*87^(1/2))^(1/3)-
1/3-1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))
*(x+1/6*(19+12*87^(1/2))^(1/3)-23/6/(19+12*87^(1/2))^(1/3)-1/3+1/2*I*3^(1/2
)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3))))^(1/2)*((1/3*(1
9+12*87^(1/2))^(1/3)-23/3/(19+12*87^(1/2))^(1/3)+1/3)*EllipticF(((-1/2*(19+
12*87^(1/2))^(1/3)+23/2/(19+12*87^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(19+12*87
^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))*(3+x)/(-1/6*(19+12*87^(1/2))^(1
/3)+23/6/(19+12*87^(1/2))^(1/3)+10/3-1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1
/3)+23/3/(19+12*87^(1/2))^(1/3)))/(x-1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12
*87^(1/2))^(1/3)-1/3))^(1/2),((1/2*(19+12*87^(1/2))^(1/3)-23/2/(19+12*87^(1
/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^
(1/3)))*(-10/3+1/6*(19+12*87^(1/2))^(1/3)-23/6/(19+12*87^(1/2))^(1/3)+1/2*I
```

```
*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))/(-10/3+1
/6*(19+12*87^(1/2))^(1/3)-23/6/(19+12*87^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(1
9+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))/(1/2*(19+12*87^(1/2))^(1
/3)-23/2/(19+12*87^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+2
3/3/(19+12*87^(1/2))^(1/3))))^(1/2))+(-10/3-1/3*(19+12*87^(1/2))^(1/3)+23/3
/(19+12*87^(1/2))^(1/3))*EllipticPi(((-1/2*(19+12*87^(1/2))^(1/3)+23/2/(19+
12*87^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87
^(1/2))^(1/3)))*(3+x)/(-1/6*(19+12*87^(1/2))^(1/3)+23/6/(19+12*87^(1/2))^(1
/3)+10/3-1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1
/3)))/(x-1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)-1/3))^(1/2)
,(-1/6*(19+12*87^(1/2))^(1/3)+23/6/(19+12*87^(1/2))^(1/3)+10/3-1/2*I*3^(1/2
)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))/(-1/2*(19+12*87
^(1/2))^(1/3)+23/2/(19+12*87^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2
))^(1/3)+23/3/(19+12*87^(1/2))^(1/3))),((1/2*(19+12*87^(1/2))^(1/3)-23/2/(1
9+12*87^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*
87^(1/2))^(1/3)))*(-10/3+1/6*(19+12*87^(1/2))^(1/3)-23/6/(19+12*87^(1/2))^(
1/3)+1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3))
)/(-10/3+1/6*(19+12*87^(1/2))^(1/3)-23/6/(19+12*87^(1/2))^(1/3)-1/2*I*3^(1/
2)*(1/3*(19+12*87^(1/2))^(1/3)+23/3/(19+12*87^(1/2))^(1/3)))/(1/2*(19+12*87
^(1/2))^(1/3)-23/2/(19+12*87^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/3*(19+12*87^(1/2
))^(1/3)+23/3/(19+12*87^(1/2))^(1/3))))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,x+2}{\sqrt{x^4+2\,x^3+5\,x^2+20\,x-12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+5*x)/(x^4+2*x^3+5*x^2+20*x-12)^(1/2),x, algorithm="maxima")
[Out] integrate((5*x + 2)/sqrt(x^4 + 2*x^3 + 5*x^2 + 20*x - 12), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{5\,x+2}{\sqrt{x^4+2\,x^3+5\,x^2+20\,x-12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 2)/(20*x + 5*x^2 + 2*x^3 + x^4 - 12)^(1/2),x)
[Out] int((5*x + 2)/(20*x + 5*x^2 + 2*x^3 + x^4 - 12)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x+2}{\sqrt{(x+3)\left(x^3-x^2+8x-4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+5*x)/(x**4+2*x**3+5*x**2+20*x-12)**(1/2),x)
[Out] Integral((5*x + 2)/sqrt((x + 3)*(x**3 - x**2 + 8*x - 4)), x)
```

3.805 $\int \frac{\left(-1+x^3\right)\sqrt{-1+x^6}}{x^7}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{x^6-1}\left(1-2x^3\right)}{6x^6}+\frac{1}{3}\tan^{-1}\left(\frac{x^3+1}{\sqrt{x^6-1}}\right)+\frac{2}{3}\tanh^{-1}\left(\frac{x^3+1}{\sqrt{x^6-1}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1475, 811, 844, 217, 206, 266, 63, 203}

$$-\frac{1}{6}\tan^{-1}\left(\sqrt{x^6-1}\right)+\frac{\sqrt{x^6-1}\left(1-2x^3\right)}{6x^6}+\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)*Sqrt[-1 + x^6])/x^7,x]
```

```
[Out] ((1 - 2*x^3)*Sqrt[-1 + x^6])/(6*x^6) - ArcTan[Sqrt[-1 + x^6]]/6 + ArcTanh[x
^3/Sqrt[-1 + x^6]]/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
```

```
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^3\right)\sqrt{-1+x^6}}{x^7}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(-1+x)\sqrt{-1+x^2}}{x^3}\,dx, x, x^3\right)\\
&= \frac{\left(1-2x^3\right)\sqrt{-1+x^6}}{6x^6} + \frac{1}{12}\operatorname{Subst}\left(\int \frac{-2+4x}{x\sqrt{-1+x^2}}\,dx, x, x^3\right)\\
&= \frac{\left(1-2x^3\right)\sqrt{-1+x^6}}{6x^6} - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1+x^2}}\,dx, x, x^3\right) + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}}\right.\\
&= \frac{\left(1-2x^3\right)\sqrt{-1+x^6}}{6x^6} - \frac{1}{12}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^6\right) + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,d\right.\\
&= \frac{\left(1-2x^3\right)\sqrt{-1+x^6}}{6x^6} + \frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right) - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+}\right.\\
&= \frac{\left(1-2x^3\right)\sqrt{-1+x^6}}{6x^6} - \frac{1}{6}\tan^{-1}\left(\sqrt{-1+x^6}\right) + \frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 1.09

$$-\frac{\left(x^6-1\right)\left(x^6\tanh^{-1}\left(\sqrt{1-x^6}\right)+\left(2x^3-1\right)\sqrt{1-x^6}+2x^6\sin^{-1}\left(x^3\right)\right)}{6x^6\sqrt{-\left(x^6-1\right)^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^3)*Sqrt[-1 + x^6])/x^7,x]
```

```
[Out] -1/6*((-1 + x^6)*((-1 + 2*x^3)*Sqrt[1 - x^6] + 2*x^6*ArcSin[x^3] + x^6*ArcT
anh[Sqrt[1 - x^6]]))/(x^6*Sqrt[-(-1 + x^6)^2])
```

IntegrateAlgebraic [A] time = 0.20, size = 68, normalized size = 1.06

$$\frac{\sqrt{x^6-1}\left(1-2x^3\right)}{6x^6}+\frac{1}{3}\tan^{-1}\left(\frac{\sqrt{x^6-1}}{x^3-1}\right)+\frac{2}{3}\tanh^{-1}\left(\frac{\sqrt{x^6-1}}{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)*Sqrt[-1 + x^6])/x^7,x]
```

```
[Out] ((1 - 2*x^3)*Sqrt[-1 + x^6])/(6*x^6) + ArcTan[Sqrt[-1 + x^6]/(-1 + x^3)]/3
+ (2*ArcTanh[Sqrt[-1 + x^6]/(-1 + x^3)])/3
```

fricas [A] time = 0.44, size = 64, normalized size = 1.00

$$-\frac{2\,x^{6}\arctan\left(-x^{3}+\sqrt{x^{6}-1}\right)+2\,x^{6}\log\left(-x^{3}+\sqrt{x^{6}-1}\right)+2\,x^{6}+\sqrt{x^{6}-1}\left(2\,x^{3}-1\right)}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(2*x^6*arctan(-x^3 + sqrt(x^6 - 1)) + 2*x^6*log(-x^3 + sqrt(x^6 - 1))
+ 2*x^6 + sqrt(x^6 - 1)*(2*x^3 - 1))/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^{6}-1}\left(x^{3}-1\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^6 - 1)*(x^3 - 1)/x^7, x)
```

maple [C] time = 0.35, size = 115, normalized size = 1.80

$$-\frac{2x^{9}-x^{6}-2x^{3}+1}{6x^{6}\sqrt{x^{6}-1}}-\frac{\sqrt{-\mathrm{signum}\left(x^{6}-1\right)}\,\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{6}+1}}{2}\right)+\left(-2\ln(2)+6\ln(x)+i\pi\right)\sqrt{\pi}\right)}{12\sqrt{\pi}\,\sqrt{\mathrm{signum}\left(x^{6}-1\right)}}+\frac{\sqrt{-\mathrm{signum}\left(x^{6}-1\right)}\,\arcsin\left(x^{3}\right)}{3\sqrt{\mathrm{signum}\left(x^{6}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)*(x^6-1)^(1/2)/x^7,x)
```

```
[Out] -1/6*(2*x^9-x^6-2*x^3+1)/x^6/(x^6-1)^(1/2)-1/12/Pi^(1/2)/signum(x^6-1)^(1/2
)*(-signum(x^6-1))^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(-x^6+1)^(1/2))+(-2*ln(2)+
6*ln(x)+I*Pi)*Pi^(1/2))+1/3/signum(x^6-1)^(1/2)*(-signum(x^6-1))^(1/2)*arcs
in(x^3)
```

maxima [A] time = 0.43, size = 67, normalized size = 1.05

$$-\frac{\sqrt{x^{6}-1}}{3\,x^{3}}+\frac{\sqrt{x^{6}-1}}{6\,x^{6}}-\frac{1}{6}\,\arctan\left(\sqrt{x^{6}-1}\right)+\frac{1}{6}\,\log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}+1\right)-\frac{1}{6}\,\log\left(\frac{\sqrt{x^{6}-1}}{x^{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6-1)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(x^6 - 1)/x^3 + 1/6*sqrt(x^6 - 1)/x^6 - 1/6*arctan(sqrt(x^6 - 1))
+ 1/6*log(sqrt(x^6 - 1)/x^3 + 1) - 1/6*log(sqrt(x^6 - 1)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(x^{3}-1\right)\,\sqrt{x^{6}-1}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)*(x^6 - 1)^(1/2))/x^7,x)
```

```
[Out] int(((x^3 - 1)*(x^6 - 1)^(1/2))/x^7, x)
```

sympy [C] time = 5.62, size = 151, normalized size = 2.36

$$-\begin{cases} \frac{i\operatorname{acosh}\left(\frac{1}{x^3}\right)}{6} + \frac{i}{6x^3\sqrt{-1+\frac{1}{x^6}}} - \frac{i}{6x^9\sqrt{-1+\frac{1}{x^6}}} & \text{for } \frac{1}{|x^6|} > 1 \\ -\frac{\operatorname{asin}\left(\frac{1}{x^3}\right)}{6} - \frac{\sqrt{1-\frac{1}{x^6}}}{6x^3} & \text{otherwise} \end{cases} + \begin{cases} -\frac{x^3}{3\sqrt{x^6-1}} + \frac{\operatorname{acosh}\left(x^3\right)}{3} + \frac{1}{3x^3\sqrt{x^6-1}} & \text{for } \left|x^6\right| > 1 \\ \frac{ix^3}{3\sqrt{1-x^6}} - \frac{i\operatorname{asin}\left(x^3\right)}{3} - \frac{i}{3x^3\sqrt{1-x^6}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)*(x**6-1)**(1/2)/x**7,x)
```

```
[Out] -Piecewise((I*acosh(x**(-3))/6 + I/(6*x**3*sqrt(-1 + x**(-6))) - I/(6*x**9*
sqrt(-1 + x**(-6))), 1/Abs(x**6) > 1), (-asin(x**(-3))/6 - sqrt(1 - 1/x**6)
/(6*x**3), True)) + Piecewise((-x**3/(3*sqrt(x**6 - 1)) + acosh(x**3)/3 + 1
/(3*x**3*sqrt(x**6 - 1)), Abs(x**6) > 1), (I*x**3/(3*sqrt(1 - x**6)) - I*as
in(x**3)/3 - I/(3*x**3*sqrt(1 - x**6)), True))
```

3.806 $\int \frac{\sqrt{-1+x^6}\left(-1+2x^6\right)^2}{x^7\left(-1+4x^6\right)}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{x^6-1}\left(2x^6+1\right)}{6x^6}-\frac{1}{6}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt{x^6-1}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.10, antiderivative size = 70, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {573, 180, 47, 63, 203, 50}

$$\frac{\sqrt{x^6-1}}{6x^6}+\frac{\sqrt{x^6-1}}{3}-\frac{1}{6}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt{x^6-1}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^6]*(-1 + 2*x^6)^2)/(x^7*(-1 + 4*x^6)),x]
```

```
[Out] Sqrt[-1 + x^6]/3 + Sqrt[-1 + x^6]/(6*x^6) - ArcTan[Sqrt[-1 + x^6]]/6 - ArcT
an[(2*Sqrt[-1 + x^6])/Sqrt[3]]/(2*Sqrt[3])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+x^6}\left(-1+2x^6\right)^2}{x^7\left(-1+4x^6\right)}dx &= \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt{-1+x}\,(-1+2x)^2}{x^2(-1+4x)}dx, x, x^6\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \left(-\frac{\sqrt{-1+x}}{x^2}+\frac{4\sqrt{-1+x}}{-1+4x}\right)dx, x, x^6\right)\\ &= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x^2}dx, x, x^6\right)\right)+\frac{2}{3}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{-1+4x}dx, x, x^6\right)\\ &= \frac{1}{3}\sqrt{-1+x^6}+\frac{\sqrt{-1+x^6}}{6x^6}-\frac{1}{12}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}dx, x, x^6\right)-\frac{1}{2}\text{Subst}\left(\int -\right.\\ &= \frac{1}{3}\sqrt{-1+x^6}+\frac{\sqrt{-1+x^6}}{6x^6}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x^2}dx, x, \sqrt{-1+x^6}\right)-\text{Subst}\left(\int \frac{}{3}\right.\\ &= \frac{1}{3}\sqrt{-1+x^6}+\frac{\sqrt{-1+x^6}}{6x^6}-\frac{1}{6}\tan^{-1}\left(\sqrt{-1+x^6}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt{-1+x^6}}{\sqrt{3}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [B] time = 0.09, size = 135, normalized size = 2.11

$$\frac{4x^{12}-2x^6+2\sqrt{1-x^6}\,x^6\tanh^{-1}\left(\sqrt{1-x^6}\right)-\sqrt{3}\sqrt{x^6-1}\,x^6\tan^{-1}\left(\frac{x^3-2}{\sqrt{3}\sqrt{x^6-1}}\right)+\sqrt{3}\sqrt{x^6-1}\,x^6\tan^{-1}\left(\frac{x^3+2}{\sqrt{3}\sqrt{x^6-1}}\right)-2}{12x^6\sqrt{x^6-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^6]*(-1 + 2*x^6)^2)/(x^7*(-1 + 4*x^6)),x]
```

```
[Out] (-2 - 2*x^6 + 4*x^12 - Sqrt[3]*x^6*Sqrt[-1 + x^6]*ArcTan[(-2 + x^3)/(Sqrt[3
]*Sqrt[-1 + x^6])] + Sqrt[3]*x^6*Sqrt[-1 + x^6]*ArcTan[(2 + x^3)/(Sqrt[3]*S
qrt[-1 + x^6])] + 2*x^6*Sqrt[1 - x^6]*ArcTanh[Sqrt[1 - x^6]])/(12*x^6*Sqrt[
-1 + x^6])
```

IntegrateAlgebraic [A] time = 0.08, size = 64, normalized size = 1.00

$$\frac{\sqrt{x^6-1}\left(2x^6+1\right)}{6x^6}-\frac{1}{6}\tan^{-1}\left(\sqrt{x^6-1}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt{x^6-1}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^6]*(-1 + 2*x^6)^2)/(x^7*(-1 + 4*x^6)),x]
```

```
[Out] (Sqrt[-1 + x^6]*(1 + 2*x^6))/(6*x^6) - ArcTan[Sqrt[-1 + x^6]]/6 - ArcTan[(2
*Sqrt[-1 + x^6])/Sqrt[3]]/(2*Sqrt[3])
```

fricas [A] time = 0.44, size = 54, normalized size = 0.84

$$-\frac{\sqrt{3}\,x^6\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x^6-1}\right)+x^6\arctan\left(\sqrt{x^6-1}\right)-\left(2\,x^6+1\right)\sqrt{x^6-1}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)*(2*x^6-1)^2/x^7/(4*x^6-1),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(3)*x^6*arctan(2/3*sqrt(3)*sqrt(x^6 - 1)) + x^6*arctan(sqrt(x^6 -
 1)) - (2*x^6 + 1)*sqrt(x^6 - 1))/x^6
```

giac [A] time = 0.42, size = 50, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x^6-1}\right)+\frac{1}{3}\sqrt{x^6-1}+\frac{\sqrt{x^6-1}}{6\,x^6}-\frac{1}{6}\arctan\left(\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)*(2*x^6-1)^2/x^7/(4*x^6-1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x^6 - 1)) + 1/3*sqrt(x^6 - 1) + 1/6*sq
rt(x^6 - 1)/x^6 - 1/6*arctan(sqrt(x^6 - 1))
```

maple [C] time = 1.25, size = 108, normalized size = 1.69

$$\frac{\sqrt{x^6-1}}{6x^6}+\frac{\sqrt{x^6-1}}{3}-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{x^6-1}+\mathrm{RootOf}\left(\_Z^2+1\right)}{x^3}\right)}{6}-\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(-\frac{4\,\mathrm{RootOf}\left(\_Z^2+3\right)x^6-7\,\mathrm{RootOf}\left(\_Z^2+3\right)-12\sqrt{x^6-1}}{\left(2x^3-1\right)\left(2x^3+1\right)}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/2)*(2*x^6-1)^2/x^7/(4*x^6-1),x)
```

```
[Out] 1/6*(x^6-1)^(1/2)/x^6+1/3*(x^6-1)^(1/2)-1/6*RootOf(_Z^2+1)*ln(((x^6-1)^(1/2
)+RootOf(_Z^2+1))/x^3)-1/12*RootOf(_Z^2+3)*ln(-(4*RootOf(_Z^2+3)*x^6-7*Root
Of(_Z^2+3)-12*(x^6-1)^(1/2))/(2*x^3-1)/(2*x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2\,x^6-1\right)^2\sqrt{x^6-1}}{\left(4\,x^6-1\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)*(2*x^6-1)^2/x^7/(4*x^6-1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^6 - 1)^2*sqrt(x^6 - 1)/((4*x^6 - 1)*x^7), x)
```

mupad [B] time = 0.98, size = 50, normalized size = 0.78

$$\frac{\sqrt{x^6-1}}{3}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\sqrt{x^6-1}}{3}\right)}{6}-\frac{\mathrm{atan}\left(\sqrt{x^6-1}\right)}{6}+\frac{\sqrt{x^6-1}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/2)*(2*x^6 - 1)^2)/(x^7*(4*x^6 - 1)),x)
```

```
[Out] (x^6 - 1)^(1/2)/3 - (3^(1/2)*atan((2*3^(1/2)*(x^6 - 1)^(1/2))/3))/6 - atan(
(x^6 - 1)^(1/2))/6 + (x^6 - 1)^(1/2)/(6*x^6)
```

sympy [A] time = 50.60, size = 60, normalized size = 0.94

$$\frac{\sqrt{x^6-1}}{3}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x^6-1}}{3}\right)}{6}-\frac{\operatorname{atan}\left(\sqrt{x^6-1}\right)}{6}+\frac{\sqrt{x^6-1}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/2)*(2*x**6-1)**2/x**7/(4*x**6-1),x)
```

```
[Out] sqrt(x**6 - 1)/3 - sqrt(3)*atan(2*sqrt(3)*sqrt(x**6 - 1)/3)/6 - atan(sqrt(x
**6 - 1))/6 + sqrt(x**6 - 1)/(6*x**6)
```

3.807 $\int \frac{-2+x^4}{\sqrt[4]{1+x^4}\left(-1+x^4+2x^8\right)}\,dx$

Optimal. Leaf size=64

$$\frac{x}{\sqrt[4]{x^4+1}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{3}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{3}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{3}}$$

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1454, 527, 12, 377, 212, 206, 203}

$$\frac{x}{\sqrt[4]{x^4+1}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{3}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{3}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^4)/((1 + x^4)^(1/4)*(-1 + x^4 + 2*x^8)),x]
```

```
[Out] x/(1 + x^4)^(1/4) + ArcTan[(3^(1/4)*x)/(1 + x^4)^(1/4)]/(2*3^(1/4)) + ArcTa
nh[(3^(1/4)*x)/(1 + x^4)^(1/4)]/(2*3^(1/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1454

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(d + e*x^n)^(p +
 q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, g, n,
 q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*
e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2+x^4}{\sqrt[4]{1+x^4}\left(-1+x^4+2x^8\right)}\,dx &= \int \frac{-2+x^4}{\left(1+x^4\right)^{5/4}\left(-1+2x^4\right)}\,dx \\
&= \frac{x}{\sqrt[4]{1+x^4}}+\frac{1}{3}\int -\frac{3}{\sqrt[4]{1+x^4}\left(-1+2x^4\right)}\,dx \\
&= \frac{x}{\sqrt[4]{1+x^4}}-\int \frac{1}{\sqrt[4]{1+x^4}\left(-1+2x^4\right)}\,dx \\
&= \frac{x}{\sqrt[4]{1+x^4}}-\text{Subst}\left(\int \frac{1}{-1+3x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right) \\
&= \frac{x}{\sqrt[4]{1+x^4}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}\,x^2}\right. \\
&= \frac{x}{\sqrt[4]{1+x^4}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{3}}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 82, normalized size = 1.28

$$\frac{x}{\sqrt[4]{x^4+1}}+\frac{-\log\left(1-\frac{\sqrt[4]{3}\,x}{\sqrt[4]{x^4+1}}\right)+\log\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{x^4+1}}+1\right)+2\tan^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^4)/((1 + x^4)^(1/4)*(-1 + x^4 + 2*x^8)),x]
```

```
[Out] x/(1 + x^4)^(1/4) + (2*ArcTan[(3^(1/4)*x)/(1 + x^4)^(1/4)] - Log[1 - (3^(1/
4)*x)/(1 + x^4)^(1/4)] + Log[1 + (3^(1/4)*x)/(1 + x^4)^(1/4)])/(4*3^(1/4))
```

IntegrateAlgebraic [A] time = 0.33, size = 64, normalized size = 1.00

$$\frac{x}{\sqrt[4]{x^4+1}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x^4)/((1 + x^4)^(1/4)*(-1 + x^4 + 2*x^8)),x]
```

```
[Out] x/(1 + x^4)^(1/4) + ArcTan[(3^(1/4)*x)/(1 + x^4)^(1/4)]/(2*3^(1/4)) + ArcTa
nh[(3^(1/4)*x)/(1 + x^4)^(1/4)]/(2*3^(1/4))
```

fricas [B] time = 5.37, size = 248, normalized size = 3.88

$$-\frac{4\cdot 3^{\frac{3}{4}}\left(x^4+1\right)\arctan\left(\frac{6\cdot 3^{\frac{3}{4}}\left(x^4+1\right)^{\frac{1}{4}}x^3+6\cdot 3^{\frac{1}{4}}\left(x^4+1\right)^{\frac{3}{4}}x+3^{\frac{3}{4}}\left(2\cdot 3^{\frac{3}{4}}\sqrt{x^4+1}\,x^2+3^{\frac{1}{4}}\left(4x^4+1\right)\right)}{3\left(2x^4-1\right)}\right)-3^{\frac{3}{4}}\left(x^4+1\right)\log\left(\frac{6\sqrt{3}\left(x^4+1\right)^{\frac{1}{4}}x^3+6\cdot 3^{\frac{1}{4}}\sqrt{x^4+1}\,x^2+3^{\frac{3}{4}}\left(4x^4+1\right)+6\left(x^4+1\right)^{\frac{3}{4}}x}{2x^4-1}\right)+3^{\frac{3}{4}}\left(x^4+1\right)\log\left(\frac{6\sqrt{3}\left(x^4+1\right)^{\frac{1}{4}}x^3-6\cdot 3^{\frac{1}{4}}\sqrt{x^4+1}\,x^2-3^{\frac{3}{4}}\left(4x^4+1\right)+6\left(x^4+1\right)^{\frac{3}{4}}x}{2x^4-1}\right)-24\left(x^4+1\right)^{\frac{3}{4}}x}{24\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2)/(x^4+1)^(1/4)/(2*x^8+x^4-1),x, algorithm="fricas")
```

```
[Out] -1/24*(4*3^(3/4)*(x^4 + 1)*arctan(1/3*(6*3^(3/4)*(x^4 + 1)^(1/4)*x^3 + 6*3^
(1/4)*(x^4 + 1)^(3/4)*x + 3^(3/4)*(2*3^(3/4)*sqrt(x^4 + 1)*x^2 + 3^(1/4)*(4
*x^4 + 1)))/(2*x^4 - 1)) - 3^(3/4)*(x^4 + 1)*log((6*sqrt(3)*(x^4 + 1)^(1/4)
*x^3 + 6*3^(1/4)*sqrt(x^4 + 1)*x^2 + 3^(3/4)*(4*x^4 + 1) + 6*(x^4 + 1)^(3/4
)*x)/(2*x^4 - 1)) + 3^(3/4)*(x^4 + 1)*log((6*sqrt(3)*(x^4 + 1)^(1/4)*x^3 -
6*3^(1/4)*sqrt(x^4 + 1)*x^2 - 3^(3/4)*(4*x^4 + 1) + 6*(x^4 + 1)^(3/4)*x)/(2
*x^4 - 1)) - 24*(x^4 + 1)^(3/4)*x)/(x^4 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-2}{\left(2\,x^8+x^4-1\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2)/(x^4+1)^(1/4)/(2*x^8+x^4-1),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 2)/((2*x^8 + x^4 - 1)*(x^4 + 1)^(1/4)), x)
```

maple [C] time = 2.00, size = 225, normalized size = 3.52

$$\frac{x}{\left(x^4+1\right)^{\frac{1}{4}}}+\frac{\mathrm{RootOf}\left(\_Z^4-27\right)\ln\left(-\frac{2\sqrt{x^4+1}\,\mathrm{RootOf}\left(\_Z^4-27\right)^3x^2+6\left(x^4+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-27\right)^2x^3+12\,\mathrm{RootOf}\left(\_Z^4-27\right)x^4+18\left(x^4+1\right)^{\frac{3}{4}}x+3\,\mathrm{RootOf}\left(\_Z^4-27\right)}{2x^4-1}\right)}{12}+\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-27\right)^2\right)\ln\left(-\frac{-2\sqrt{x^4+1}\,\mathrm{RootOf}\left(\_Z^4-27\right)^2\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-27\right)^2\right)x^2-6\left(x^4+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-27\right)^2x^3+12\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-27\right)^2\right)x^4+18\left(x^4+1\right)^{\frac{3}{4}}x+3\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-27\right)^2\right)}{2x^4-1}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2)/(x^4+1)^(1/4)/(2*x^8+x^4-1),x)
```

```
[Out] x/(x^4+1)^(1/4)+1/12*RootOf(_Z^4-27)*ln(-(2*(x^4+1)^(1/2)*RootOf(_Z^4-27)^3
*x^2+6*(x^4+1)^(1/4)*RootOf(_Z^4-27)^2*x^3+12*RootOf(_Z^4-27)*x^4+18*(x^4+1
)^(3/4)*x+3*RootOf(_Z^4-27))/(2*x^4-1))+1/12*RootOf(_Z^2+RootOf(_Z^4-27)^2)
*ln(-(-2*(x^4+1)^(1/2)*RootOf(_Z^4-27)^2*RootOf(_Z^2+RootOf(_Z^4-27)^2)*x^2
-6*(x^4+1)^(1/4)*RootOf(_Z^4-27)^2*x^3+12*RootOf(_Z^2+RootOf(_Z^4-27)^2)*x^
4+18*(x^4+1)^(3/4)*x+3*RootOf(_Z^2+RootOf(_Z^4-27)^2))/(2*x^4-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-2}{\left(2\,x^8+x^4-1\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2)/(x^4+1)^(1/4)/(2*x^8+x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 2)/((2*x^8 + x^4 - 1)*(x^4 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4-2}{{\left(x^4+1\right)}^{1/4}\,\left(2\,x^8+x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 2)/((x^4 + 1)^(1/4)*(x^4 + 2*x^8 - 1)),x)
[Out] int((x^4 - 2)/((x^4 + 1)^(1/4)*(x^4 + 2*x^8 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 - 2}{\left(x^4 + 1\right)^{\frac{5}{4}} \left(2x^4 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2)/(x**4+1)**(1/4)/(2*x**8+x**4-1),x)
[Out] Integral((x**4 - 2)/((x**4 + 1)**(5/4)*(2*x**4 - 1)), x)
```

3.808 $\int \frac{-x^2+10x^8}{\sqrt{-1+x^6}\left(-1+4x^6\right)}\,dx$

Optimal. Leaf size=64

$$\frac{5}{6}\log\left(\sqrt{x^6-1}+x^3\right)-\frac{\tan^{-1}\left(-\frac{4x^6}{\sqrt{3}}-\frac{4\sqrt{x^6-1}\,x^3}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.09, antiderivative size = 47, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1593, 575, 523, 217, 206, 377, 204}

$$\frac{5}{6}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x^3}{\sqrt{x^6-1}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + 10*x^8)/(Sqrt[-1 + x^6]*(-1 + 4*x^6)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3]*x^3)/Sqrt[-1 + x^6]]/Sqrt[3] + (5*ArcTanh[x^3/Sqrt[-1
+ x^6]])/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 575

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Di
st[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q*(e
+ f*x^(n/k))^r, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, f, p, q
```

```
, r}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-x^2+10x^8}{\sqrt{-1+x^6}\left(-1+4x^6\right)}\,dx &= \int \frac{x^2\left(-1+10x^6\right)}{\sqrt{-1+x^6}\left(-1+4x^6\right)}\,dx\\
&= \frac{1}{3}\,\text{Subst}\left(\int \frac{-1+10x^2}{\sqrt{-1+x^2}\left(-1+4x^2\right)}\,dx, x, x^3\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}\left(-1+4x^2\right)}\,dx, x, x^3\right)+\frac{5}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, x^3\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1-3x^2}\,dx, x, \frac{x^3}{\sqrt{-1+x^6}}\right)+\frac{5}{6}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^3}{\sqrt{-1+x^6}}\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{3}x^3}{\sqrt{-1+x^6}}\right)}{2\sqrt{3}}+\frac{5}{6}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 47, normalized size = 0.73

$$\frac{5}{6}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x^3}{\sqrt{x^6-1}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + 10*x^8)/(Sqrt[-1 + x^6]*(-1 + 4*x^6)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3]*x^3)/Sqrt[-1 + x^6]]/Sqrt[3] + (5*ArcTanh[x^3/Sqrt[-1
+ x^6]])/6
```

IntegrateAlgebraic [A] time = 0.11, size = 66, normalized size = 1.03

$$\frac{\tan^{-1}\left(-\frac{4x^6}{\sqrt{3}}+\frac{4\sqrt{x^6-1}\,x^3}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{5}{6}\log\left(\sqrt{x^6-1}-x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^2 + 10*x^8)/(Sqrt[-1 + x^6]*(-1 + 4*x^6)),x]
```

```
[Out] ArcTan[1/Sqrt[3] - (4*x^6)/Sqrt[3] + (4*x^3*Sqrt[-1 + x^6])/Sqrt[3]]/(2*Sqr
t[3]) - (5*Log[-x^3 + Sqrt[-1 + x^6]])/6
```

fricas [A] time = 0.50, size = 51, normalized size = 0.80

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{4}{3}\sqrt{3}\sqrt{x^6-1}\,x^3-\frac{1}{3}\sqrt{3}\left(4x^6-1\right)\right)-\frac{5}{6}\log\left(-x^3+\sqrt{x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*x^8-x^2)/(x^6-1)^(1/2)/(4*x^6-1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(4/3*sqrt(3)*sqrt(x^6 - 1)*x^3 - 1/3*sqrt(3)*(4*x^6 - 1))
 - 5/6*log(-x^3 + sqrt(x^6 - 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*x^8-x^2)/(x^6-1)^(1/2)/(4*x^6-1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:rootof minimal polynomial must be unitary Er
ror: Bad Argument Valuerootof minimal polynomial must be unitary Error: Bad
 Argument Valuerootof minimal polynomial must be unitary Error: Bad Argumen
t Valuerootof minimal polynomial must be unitary Error: Bad Argument Valuer
ootof minimal polynomial must be unitary Error: Bad Argument Valuerootof mi
nimal polynomial must be unitary Error: Bad Argument Valuerootof minimal po
lynomial must be unitary Error: Bad Argument Valuerootof minimal polynomial
 must be unitary Error: Bad Argument Valuerootof minimal polynomial must be
 unitary Error: Bad Argument Valuerootof minimal polynomial must be unitary
 Error: Bad Argument ValueWarning, integration of abs or sign assumes const
ant sign by intervals (correct if the argument is real):Check [abs(t_nostep
)]Warning, integration of abs or sign assumes constant sign by intervals (c
orrect if the argument is real):Check [abs(x)]Unable to cancel step at 0 of
 -5/12*ln(-sqrt(-x^6+1)+1)+5/12*ln(sqrt(-x^6+1)+1)+1/2/sqrt(3)*atan(sqrt(-x
^6+1)/sqrt(3))-5/12*ln(-sqrt(-x^6+1)+1)-5/12*ln(sqrt(-x^6+1)+1)-1/2/sqrt(3)
*atan(sqrt(-x^6+1)/sqrt(3))Done
```

maple [C] time = 0.61, size = 77, normalized size = 1.20

$$-\frac{5\ln\left(x^{3}-\sqrt{x^{6}-1}\right)}{6}+\frac{\operatorname{RootOf}\left(\_Z^{2}+3\right)\ln\left(-\frac{2\operatorname{RootOf}\left(\_Z^{2}+3\right)x^{6}+6x^{3}\sqrt{x^{6}-1}+\operatorname{RootOf}\left(\_Z^{2}+3\right)}{\left(2x^{3}-1\right)\left(2x^{3}+1\right)}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((10*x^8-x^2)/(x^6-1)^(1/2)/(4*x^6-1),x)
```

```
[Out] -5/6*ln(x^3-(x^6-1)^(1/2))+1/12*RootOf(_Z^2+3)*ln(-(2*RootOf(_Z^2+3)*x^6+6*
x^3*(x^6-1)^(1/2)+RootOf(_Z^2+3))/(2*x^3-1)/(2*x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{10\,x^{8}-x^{2}}{\left(4\,x^{6}-1\right)\sqrt{x^{6}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*x^8-x^2)/(x^6-1)^(1/2)/(4*x^6-1),x, algorithm="maxima")
```

```
[Out] integrate((10*x^8 - x^2)/((4*x^6 - 1)*sqrt(x^6 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x^{2}-10\,x^{8}}{\sqrt{x^{6}-1}\,\left(4\,x^{6}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 10*x^8)/((x^6 - 1)^(1/2)*(4*x^6 - 1)),x)
```

```
[Out] int(-(x^2 - 10*x^8)/((x^6 - 1)^(1/2)*(4*x^6 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(10x^6 - 1\right)}{\sqrt{(x-1)(x+1)\left(x^2 - x + 1\right)\left(x^2 + x + 1\right)}\left(2x^3 - 1\right)\left(2x^3 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*x**8-x**2)/(x**6-1)**(1/2)/(4*x**6-1),x)
```

```
[Out] Integral(x**2*(10*x**6 - 1)/(sqrt((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x
+ 1))*(2*x**3 - 1)*(2*x**3 + 1)), x)
```

3.809 $\int \frac{1+x^2}{(-1+x^2)(1+2x^2)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{x}{3\sqrt{2x^2+1}}-\frac{2\tanh^{-1}\left(-\sqrt{\frac{2}{3}}x^2+\frac{\sqrt{2x^2+1}\,x}{\sqrt{3}}+\sqrt{\frac{2}{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {527, 12, 377, 207}

$$-\frac{x}{3\sqrt{2x^2+1}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{2x^2+1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/((-1 + x^2)*(1 + 2*x^2)^(3/2)),x]
```

```
[Out] -1/3*x/Sqrt[1 + 2*x^2] - (2*ArcTanh[(Sqrt[3]*x)/Sqrt[1 + 2*x^2]])/(3*Sqrt[3
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1+x^2}{\left(-1+x^2\right)\left(1+2x^2\right)^{3/2}}\,dx = -\frac{x}{3\sqrt{1+2x^2}}+\frac{1}{3}\int \frac{2}{\left(-1+x^2\right)\sqrt{1+2x^2}}\,dx$$
$$= -\frac{x}{3\sqrt{1+2x^2}}+\frac{2}{3}\int \frac{1}{\left(-1+x^2\right)\sqrt{1+2x^2}}\,dx$$
$$= -\frac{x}{3\sqrt{1+2x^2}}+\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-1+3x^2}\,dx, x, \frac{x}{\sqrt{1+2x^2}}\right)$$
$$= -\frac{x}{3\sqrt{1+2x^2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{1+2x^2}}\right)}{3\sqrt{3}}$$

Mathematica [C] time = 1.31, size = 159, normalized size = 2.45

$$\frac{x\left(\frac{72\left(2x^2+1\right)x^2\,{}_2F_1\left(2,2;\frac{7}{2};\frac{3x^2}{x^2-1}\right)}{\left(x^2-1\right)^2}+\frac{10\left(4x^2+3\right)\left(\sqrt{\frac{6x^2+3}{1-x^2}}\,x^2+\sqrt{\frac{x^2}{x^2-1}}\left(2x^2+1\right)\sin^{-1}\left(\sqrt{3}\sqrt{\frac{x^2}{x^2-1}}\right)\right)}{\sqrt{\frac{2x^2+1}{3-3x^2}}\,x^4}+45\right)}{45\sqrt{2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2)/((-1 + x^2)*(1 + 2*x^2)^(3/2)),x]
```

```
[Out] (x*(45 + (10*(3 + 4*x^2)*(x^2*Sqrt[(3 + 6*x^2)/(1 - x^2)] + Sqrt[x^2/(-1 +
x^2)]*(1 + 2*x^2)*ArcSin[Sqrt[3]*Sqrt[x^2/(-1 + x^2)]]))/(x^4*Sqrt[(1 + 2*x
^2)/(3 - 3*x^2)]) + (72*x^2*(1 + 2*x^2)*Hypergeometric2F1[2, 2, 7/2, (3*x^2
)/(-1 + x^2)])/(-1 + x^2)^2))/(45*Sqrt[1 + 2*x^2])
```

IntegrateAlgebraic [A] time = 0.22, size = 65, normalized size = 1.00

$$-\frac{x}{3\sqrt{2x^2+1}}-\frac{2\tanh^{-1}\left(-\sqrt{\frac{2}{3}}\,x^2+\frac{\sqrt{2x^2+1}\,x}{\sqrt{3}}+\sqrt{\frac{2}{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 + x^2)*(1 + 2*x^2)^(3/2)),x]
```

```
[Out] -1/3*x/Sqrt[1 + 2*x^2] - (2*ArcTanh[Sqrt[2/3] - Sqrt[2/3]*x^2 + (x*Sqrt[1 +
 2*x^2])/Sqrt[3]])/(3*Sqrt[3])
```

fricas [A] time = 0.47, size = 66, normalized size = 1.02

$$\frac{\sqrt{3}\left(2\,x^2+1\right)\log\left(\frac{2\,\sqrt{3}\,\sqrt{2\,x^2+1}\,x-5\,x^2-1}{x^2-1}\right)-3\,\sqrt{2\,x^2+1}\,x}{9\left(2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*(sqrt(3)*(2*x^2 + 1)*log((2*sqrt(3)*sqrt(2*x^2 + 1)*x - 5*x^2 - 1)/(x^2
 - 1)) - 3*sqrt(2*x^2 + 1)*x)/(2*x^2 + 1)
```

giac [A] time = 0.35, size = 83, normalized size = 1.28

$$-\frac{1}{18}\,\sqrt{6}\,\sqrt{2}\,\log\left(\frac{\left|2\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)^2-4\,\sqrt{6}-10\right|}{\left|2\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)^2+4\,\sqrt{6}-10\right|}\right)-\frac{x}{3\,\sqrt{2\,x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(6)*sqrt(2)*log(abs(2*(sqrt(2)*x - sqrt(2*x^2 + 1))^2 - 4*sqrt(6)
 - 10)/abs(2*(sqrt(2)*x - sqrt(2*x^2 + 1))^2 + 4*sqrt(6) - 10)) - 1/3*x/sqr
t(2*x^2 + 1)
```

maple [B] time = 0.02, size = 139, normalized size = 2.14

$$\frac{x}{\sqrt{2x^2+1}}+\frac{1}{3\sqrt{2\left(-1+x\right)^2+4x-1}}-\frac{2x}{3\sqrt{2\left(-1+x\right)^2+4x-1}}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(2+4x)\sqrt{3}}{6\sqrt{2(-1+x)^2+4x-1}}\right)}{9}-\frac{1}{3\sqrt{2\left(1+x\right)^2-4x-1}}-\frac{2x}{3\sqrt{2\left(1+x\right)^2-4x-1}}+\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{(2-4x)\sqrt{3}}{6\sqrt{2(1+x)^2-4x-1}}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-1)/(2*x^2+1)^(3/2),x)
```

```
[Out] x/(2*x^2+1)^(1/2)+1/3/(2*(-1+x)^2+4*x-1)^(1/2)-2/3*x/(2*(-1+x)^2+4*x-1)^(1/
2)-1/9*3^(1/2)*arctanh(1/6*(2+4*x)*3^(1/2)/(2*(-1+x)^2+4*x-1)^(1/2))-1/3/(2
*(1+x)^2-4*x-1)^(1/2)-2/3*x/(2*(1+x)^2-4*x-1)^(1/2)+1/9*3^(1/2)*arctanh(1/6
*(2-4*x)*3^(1/2)/(2*(1+x)^2-4*x-1)^(1/2))
```

maxima [A] time = 0.42, size = 80, normalized size = 1.23

$$-\frac{1}{9}\sqrt{3}\,\operatorname{arsinh}\left(\frac{2\sqrt{2}\,x}{|2\,x+2|}-\frac{\sqrt{2}}{|2\,x+2|}\right)-\frac{1}{9}\sqrt{3}\,\operatorname{arsinh}\left(\frac{2\sqrt{2}\,x}{|2\,x-2|}+\frac{\sqrt{2}}{|2\,x-2|}\right)-\frac{x}{3\sqrt{2\,x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arcsinh(2*sqrt(2)*x/abs(2*x + 2) - sqrt(2)/abs(2*x + 2)) - 1/9
*sqrt(3)*arcsinh(2*sqrt(2)*x/abs(2*x - 2) + sqrt(2)/abs(2*x - 2)) - 1/3*x/s
qrt(2*x^2 + 1)
```

mupad [B] time = 0.83, size = 111, normalized size = 1.71

$$\frac{\sqrt{3}\left(\ln\left(x-1\right)-\ln\left(x+\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{x^2+\frac{1}{2}}}{2}+\frac{1}{2}\right)\right)}{9}-\frac{\sqrt{3}\left(\ln\left(x+1\right)-\ln\left(x-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{x^2+\frac{1}{2}}}{2}-\frac{1}{2}\right)\right)}{9}-\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{12\left(x-\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)}-\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{12\left(x+\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((x^2 - 1)*(2*x^2 + 1)^(3/2)),x)
```

```
[Out] (3^(1/2)*(log(x - 1) - log(x + (2^(1/2)*3^(1/2)*(x^2 + 1/2)^(1/2))/2 + 1/2)
))/9 - (3^(1/2)*(log(x + 1) - log(x - (2^(1/2)*3^(1/2)*(x^2 + 1/2)^(1/2))/2
 - 1/2)))/9 - (2^(1/2)*(x^2 + 1/2)^(1/2))/(12*(x - (2^(1/2)*1i)/2)) - (2^(1
/2)*(x^2 + 1/2)^(1/2))/(12*(x + (2^(1/2)*1i)/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+1}{\left(x-1\right)\left(x+1\right)\left(2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-1)/(2*x**2+1)**(3/2),x)
```

```
[Out] Integral((x**2 + 1)/((x - 1)*(x + 1)*(2*x**2 + 1)**(3/2)), x)
```

3.810 $\int \frac{(b+ax^2)^{3/4}}{x}\,dx$

Optimal. Leaf size=65

$$b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)-b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)+\frac{2}{3}\left(ax^2+b\right)^{3/4}$$

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 63, 298, 203, 206}

$$b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)-b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)+\frac{2}{3}\left(ax^2+b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^2)^(3/4)/x,x]
```

```
[Out] (2*(b + a*x^2)^(3/4))/3 + b^(3/4)*ArcTan[(b + a*x^2)^(1/4)/b^(1/4)] - b^(3/
4)*ArcTanh[(b + a*x^2)^(1/4)/b^(1/4)]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(b+ax^2\right)^{3/4}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(b+ax)^{3/4}}{x}\,dx, x, x^2\right)\\ &= \frac{2}{3}\left(b+ax^2\right)^{3/4}+\frac{1}{2}b\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{b+ax}}\,dx, x, x^2\right)\\ &= \frac{2}{3}\left(b+ax^2\right)^{3/4}+\frac{(2b)\operatorname{Subst}\left(\int \frac{x^2}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^2}\right)}{a}\\ &= \frac{2}{3}\left(b+ax^2\right)^{3/4}-b\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^2}\right)+b\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx, x, \sqrt[4]{b+a\right.\\ &= \frac{2}{3}\left(b+ax^2\right)^{3/4}+b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b+ax^2}}{\sqrt[4]{b}}\right)-b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^2}}{\sqrt[4]{b}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.00

$$b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)-b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)+\frac{2}{3}\left(ax^2+b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^2)^(3/4)/x,x]
```

```
[Out] (2*(b + a*x^2)^(3/4))/3 + b^(3/4)*ArcTan[(b + a*x^2)^(1/4)/b^(1/4)] - b^(3/
4)*ArcTanh[(b + a*x^2)^(1/4)/b^(1/4)]
```

IntegrateAlgebraic [A] time = 0.07, size = 65, normalized size = 1.00

$$b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)-b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)+\frac{2}{3}\left(ax^2+b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^2)^(3/4)/x,x]
```

```
[Out] (2*(b + a*x^2)^(3/4))/3 + b^(3/4)*ArcTan[(b + a*x^2)^(1/4)/b^(1/4)] - b^(3/
4)*ArcTanh[(b + a*x^2)^(1/4)/b^(1/4)]
```

fricas [B] time = 0.46, size = 132, normalized size = 2.03

$$-2\left(b^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(b^3\right)^{\frac{1}{4}}\left(ax^2+b\right)^{\frac{1}{4}}b^2-\sqrt{\sqrt{ax^2+b}\,b^4+\sqrt{b^3}\,b^3}\left(b^3\right)^{\frac{1}{4}}}{b^3}\right)-\frac{1}{2}\left(b^3\right)^{\frac{1}{4}}\log\left(\left(ax^2+b\right)^{\frac{1}{4}}b^2+\left(b^3\right)^{\frac{3}{4}}\right)+\frac{1}{2}\left(b^3\right)^{\frac{1}{4}}\log\left(\left(ax^2+b\right)^{\frac{1}{4}}b^2-\left(b^3\right)^{\frac{3}{4}}\right)+\frac{2}{3}\left(ax^2+b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)^(3/4)/x,x, algorithm="fricas")
```

```
[Out] -2*(b^3)^(1/4)*arctan(-((b^3)^(1/4)*(a*x^2 + b)^(1/4)*b^2 - sqrt(sqrt(a*x^2
 + b)*b^4 + sqrt(b^3)*b^3)*(b^3)^(1/4))/b^3) - 1/2*(b^3)^(1/4)*log((a*x^2 +
```

```
 b)^(1/4)*b^2 + (b^3)^(3/4)) + 1/2*(b^3)^(1/4)*log((a*x^2 + b)^(1/4)*b^2 -
(b^3)^(3/4)) + 2/3*(a*x^2 + b)^(3/4)
```

giac [B] time = 0.36, size = 185, normalized size = 2.85

$$-\frac{1}{2}\sqrt{2}(-b)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}+2\left(ax^2+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)-\frac{1}{2}\sqrt{2}(-b)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}-2\left(ax^2+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)+\frac{1}{4}\sqrt{2}(-b)^{\frac{3}{4}}\log\left(\sqrt{2}\left(ax^2+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^2+b}+\sqrt{-b}\right)-\frac{1}{4}\sqrt{2}(-b)^{\frac{3}{4}}\log\left(-\sqrt{2}\left(ax^2+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^2+b}+\sqrt{-b}\right)+\frac{2}{3}\left(ax^2+b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)^(3/4)/x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(-b)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^2 +
 b)^(1/4))/(-b)^(1/4)) - 1/2*sqrt(2)*(-b)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(-b)^(1/4) - 2*(a*x^2 + b)^(1/4))/(-b)^(1/4)) + 1/4*sqrt(2)*(-b)^(3/4)*lo
g(sqrt(2)*(a*x^2 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^2 + b) + sqrt(-b)) - 1/4*
sqrt(2)*(-b)^(3/4)*log(-sqrt(2)*(a*x^2 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^2 +
 b) + sqrt(-b)) + 2/3*(a*x^2 + b)^(3/4)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^2+b\right)^{\frac{3}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)^(3/4)/x,x)
```

```
[Out] int((a*x^2+b)^(3/4)/x,x)
```

maxima [A] time = 0.41, size = 71, normalized size = 1.09

$$\frac{1}{2}\,b\left(\frac{2\,\arctan\left(\frac{\left(ax^2+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(ax^2+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^2+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}\right)+\frac{2}{3}\left(ax^2+b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)^(3/4)/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*(2*arctan((a*x^2 + b)^(1/4)/b^(1/4))/b^(1/4) + log(((a*x^2 + b)^(1/4)
 - b^(1/4))/((a*x^2 + b)^(1/4) + b^(1/4)))/b^(1/4)) + 2/3*(a*x^2 + b)^(3/4)
```

mupad [B] time = 0.81, size = 49, normalized size = 0.75

$$b^{3/4}\,\mathrm{atan}\left(\frac{{\left(a\,x^2+b\right)}^{1/4}}{b^{1/4}}\right)-b^{3/4}\,\mathrm{atanh}\left(\frac{{\left(a\,x^2+b\right)}^{1/4}}{b^{1/4}}\right)+\frac{2\,{\left(a\,x^2+b\right)}^{3/4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^2)^(3/4)/x,x)
```

```
[Out] b^(3/4)*atan((b + a*x^2)^(1/4)/b^(1/4)) - b^(3/4)*atanh((b + a*x^2)^(1/4)/b
^(1/4)) + (2*(b + a*x^2)^(3/4))/3
```

sympy [C] time = 1.06, size = 46, normalized size = 0.71

$$-\frac{a^{\frac{3}{4}}x^{\frac{3}{2}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^2}\right)}{2\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b)**(3/4)/x,x)
```

```
[Out] -a**(3/4)*x**(3/2)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), b*exp_polar(I*pi
)/(a*x**2))/(2*gamma(1/4))
```

3.811 $\int \frac{1-x+x^2}{(-1+x^2)\sqrt{x+x^3}}\,dx$

Optimal. Leaf size=65

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x^3+x}}{x^2+1}\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x^3+x}}{x^2+1}\right)}{2\sqrt{2}}$$

Rubi [C] time = 0.79, antiderivative size = 172, normalized size of antiderivative = 2.65, number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2056, 6725, 329, 220, 932, 168, 538, 537}

$$\frac{\sqrt{x}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^3+x}}+\frac{\left(\frac{3}{2}-\frac{3i}{2}\right)\sqrt{ix}\sqrt{x^2+1}\,\Pi\left(\frac{1}{2}-\frac{i}{2};\sin^{-1}\left(\sqrt{1-ix}\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^3+x}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{ix}\sqrt{x^2+1}\,\Pi\left(\frac{1}{2}+\frac{i}{2};\sin^{-1}\left(\sqrt{1-ix}\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^3+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x + x^2)/((-1 + x^2)*Sqrt[x + x^3]),x]
```

```
[Out] (Sqrt[x]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2
])/Sqrt[x + x^3] + ((3/2 - (3*I)/2)*Sqrt[I*x]*Sqrt[1 + x^2]*EllipticPi[1/2
- I/2, ArcSin[Sqrt[1 - I*x]], 1/2])/(Sqrt[2]*Sqrt[x + x^3]) - ((1/2 + I/2)*
Sqrt[I*x]*Sqrt[1 + x^2]*EllipticPi[1/2 + I/2, ArcSin[Sqrt[1 - I*x]], 1/2])/
(Sqrt[2]*Sqrt[x + x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] &&  !GtQ[c, 0]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1-x+x^2}{\left(-1+x^2\right)\sqrt{x+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{1-x+x^2}{\sqrt{x}\left(-1+x^2\right)\sqrt{1+x^2}}\,dx}{\sqrt{x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{1+x^2}}+\frac{2-x}{\sqrt{x}\left(-1+x^2\right)\sqrt{1+x^2}}\right)dx}{\sqrt{x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+x^2}}\,dx}{\sqrt{x+x^3}} + \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{2-x}{\sqrt{x}\left(-1+x^2\right)\sqrt{1+x^2}}\,dx}{\sqrt{x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \left(-\frac{1}{2(1-x)\sqrt{x}\sqrt{1+x^2}}-\frac{3}{2\sqrt{x}(1+x)\sqrt{1+x^2}}\right)dx}{\sqrt{x+x^3}} + \frac{\left(2\sqrt{x}\sqrt{1+x^2}\right)\text{Subst}\left(\int -\ldots\right.}{\sqrt{x+x^3}}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x+x^3}} - \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{1}{(1-x)\sqrt{x}\sqrt{1+x^2}}\,dx}{2\sqrt{x+x^3}} - \frac{\left(3\sqrt{x}\ldots\right.}{\ldots}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x+x^3}} - \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{1}{(1-x)\sqrt{1-ix}\sqrt{1+ix}\sqrt{x}}\,dx}{2\sqrt{x+x^3}} - \frac{\left(3\ldots\right.}{\ldots}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x+x^3}} + \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\sqrt{-i+ix^2}\left((-1+i)\ldots\right.}\right.}{\sqrt{x+x^3}}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x+x^3}} + \frac{\left(\sqrt{ix}\sqrt{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}\left((-1+i)+\ldots\right.}\right.}{\sqrt{x+x^3}}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x+x^3}} + \frac{\left(\frac{3}{2}-\frac{3i}{2}\right)\sqrt{ix}\sqrt{1+x^2}\,\Pi\left(\frac{1}{2}-\frac{i}{2};\sin^{-1}\left(\sqrt{1-\ldots}\right.\right.}{\sqrt{2}\sqrt{x+x^3}}$$

Mathematica [C] time = 0.55, size = 177, normalized size = 2.72

$$\frac{2\left(5\sqrt{\frac{1}{x^2}+1}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{1}{x^2},\frac{1}{x^2}\right)-\frac{3\sqrt{\frac{1}{x^2}+1}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{1}{x^2},\frac{1}{x^2}\right)}{x}-\frac{75x^5F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{1}{x^2},\frac{1}{x^2}\right)}{(x^2-1)\left(5x^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{1}{x^2},\frac{1}{x^2}\right)+4F_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{1}{x^2},\frac{1}{x^2}\right)-2F_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{1}{x^2},\frac{1}{x^2}\right)\right)}\right)}{15\sqrt{x^3+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x + x^2)/((-1 + x^2)*Sqrt[x + x^3]),x]
```

```
[Out] (2*(5*Sqrt[1 + x^(-2)]*AppellF1[3/4, 1/2, 1, 7/4, -x^(-2), x^(-2)] - (3*Sqr
t[1 + x^(-2)]*AppellF1[5/4, 1/2, 1, 9/4, -x^(-2), x^(-2)])/x - (75*x^5*Appe
llF1[1/4, 1/2, 1, 5/4, -x^(-2), x^(-2)])/((-1 + x^2)*(5*x^2*AppellF1[1/4, 1
/2, 1, 5/4, -x^(-2), x^(-2)] + 4*AppellF1[5/4, 1/2, 2, 9/4, -x^(-2), x^(-2)
] - 2*AppellF1[5/4, 3/2, 1, 9/4, -x^(-2), x^(-2)]))))/(15*Sqrt[x + x^3])
```

IntegrateAlgebraic [A] time = 0.28, size = 65, normalized size = 1.00

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x^3+x}}{x^2+1}\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x^3+x}}{x^2+1}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x + x^2)/((-1 + x^2)*Sqrt[x + x^3]),x]
```

```
[Out] (-3*ArcTan[(Sqrt[2]*Sqrt[x + x^3])/(1 + x^2)])/(2*Sqrt[2]) - ArcTanh[(Sqrt[
2]*Sqrt[x + x^3])/(1 + x^2)]/(2*Sqrt[2])
```

fricas [A] time = 0.47, size = 92, normalized size = 1.42

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2-2x+1\right)}{4\sqrt{x^3+x}}\right)+\frac{1}{16}\sqrt{2}\log\left(\frac{x^4+12x^3-4\sqrt{2}\sqrt{x^3+x}\left(x^2+2x+1\right)+6x^2+12x+1}{x^4-4x^3+6x^2-4x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2-1)/(x^3+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 3/8*sqrt(2)*arctan(1/4*sqrt(2)*(x^2 - 2*x + 1)/sqrt(x^3 + x)) + 1/16*sqrt(2
)*log((x^4 + 12*x^3 - 4*sqrt(2)*sqrt(x^3 + x)*(x^2 + 2*x + 1) + 6*x^2 + 12*
x + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-x+1}{\sqrt{x^3+x}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2-1)/(x^3+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)/(sqrt(x^3 + x)*(x^2 - 1)), x)
```

maple [C] time = 0.04, size = 166, normalized size = 2.55

$$\frac{i\sqrt{-i(i+x)}\,\sqrt{2}\,\sqrt{i(-i+x)}\,\sqrt{ix}\,\mathrm{EllipticF}\left(\sqrt{-i(i+x)},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3+x}}+\frac{\left(-\frac{1}{4}-\frac{i}{4}\right)\sqrt{-i(i+x)}\,\sqrt{2}\,\sqrt{i(-i+x)}\,\sqrt{ix}\,\mathrm{EllipticPi}\left(\sqrt{-i(i+x)},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3+x}}+\frac{\left(\frac{3}{4}-\frac{3i}{4}\right)\sqrt{-i(i+x)}\,\sqrt{2}\,\sqrt{i(-i+x)}\,\sqrt{ix}\,\mathrm{EllipticPi}\left(\sqrt{-i(i+x)},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+1)/(x^2-1)/(x^3+x)^(1/2),x)
```

```
[Out] I*(-I*(I+x))^(1/2)*2^(1/2)*(I*(-I+x))^(1/2)*(I*x)^(1/2)/(x^3+x)^(1/2)*Ellip
ticF((-I*(I+x))^(1/2),1/2*2^(1/2))-(1/4+1/4*I)*(-I*(I+x))^(1/2)*2^(1/2)*(I*
(-I+x))^(1/2)*(I*x)^(1/2)/(x^3+x)^(1/2)*EllipticPi((-I*(I+x))^(1/2),1/2+1/2
*I,1/2*2^(1/2))+(3/4-3/4*I)*(-I*(I+x))^(1/2)*2^(1/2)*(I*(-I+x))^(1/2)*(I*x)
^(1/2)/(x^3+x)^(1/2)*EllipticPi((-I*(I+x))^(1/2),1/2-1/2*I,1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - x + 1}{\sqrt{x^3 + x}\left(x^2 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2-1)/(x^3+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)/(sqrt(x^3 + x)*(x^2 - 1)), x)
```

mupad [B] time = 0.72, size = 116, normalized size = 1.78

$$-\frac{-\sqrt{1-x\,1\mathrm{i}}\,\sqrt{1+x\,1\mathrm{i}}\,\sqrt{-x\,1\mathrm{i}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-x\,1\mathrm{i}}\right)\middle|-1\right)\,2\mathrm{i}+\sqrt{1-x\,1\mathrm{i}}\,\sqrt{1+x\,1\mathrm{i}}\,\sqrt{-x\,1\mathrm{i}}\,\Pi\left(-\mathrm{i};\mathrm{asin}\left(\sqrt{-x\,1\mathrm{i}}\right)\middle|-1\right)\,3\mathrm{i}+\sqrt{1-x\,1\mathrm{i}}\,\sqrt{1+x\,1\mathrm{i}}\,\sqrt{-x\,1\mathrm{i}}\,\Pi\left(1\mathrm{i};\mathrm{asin}\left(\sqrt{-x\,1\mathrm{i}}\right)\middle|-1\right)\,1\mathrm{i}}{\sqrt{x^3+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x + 1)/((x^2 - 1)*(x + x^3)^(1/2)),x)
```

```
[Out] -((1 - x*1i)^(1/2)*(x*1i + 1)^(1/2)*(-x*1i)^(1/2)*ellipticPi(-1i, asin((-x*
1i)^(1/2)), -1)*3i - (1 - x*1i)^(1/2)*(x*1i + 1)^(1/2)*(-x*1i)^(1/2)*ellipt
icF(asin((-x*1i)^(1/2)), -1)*2i + (1 - x*1i)^(1/2)*(x*1i + 1)^(1/2)*(-x*1i)
^(1/2)*ellipticPi(1i, asin((-x*1i)^(1/2)), -1)*1i)/(x + x^3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - x + 1}{\sqrt{x\left(x^2 + 1\right)}\,(x - 1)\,(x + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+1)/(x**2-1)/(x**3+x)**(1/2),x)
```

```
[Out] Integral((x**2 - x + 1)/(sqrt(x*(x**2 + 1))*(x - 1)*(x + 1)), x)
```

3.812 $\int \frac{-1+2x^2}{\left(1+x^2\right)\sqrt{-1-x^2+x^4}}\,dx$

Optimal. Leaf size=65

$$-\frac{1}{2}i\log\left(\frac{-2ix^2+\sqrt{x^4-x^2-1}\,x-i}{-2ix^2-\sqrt{x^4-x^2-1}\,x-i}\right)$$

Rubi [C] time = 0.41, antiderivative size = 520, normalized size of antiderivative = 8.00, number of steps used = 10, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1710, 1098, 1214, 1456, 540, 421, 419, 538, 537}

$$\frac{3\left(1+\sqrt{5}\right)\sqrt{2x^2+\sqrt{5}-1}\sqrt{1-\frac{2x^2}{1+\sqrt{5}}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\sqrt{2}\left(3+\sqrt{5}\right)\sqrt{x^4-x^2-1}}-\frac{3\sqrt{-\left(\left(1-\sqrt{5}\right)x^2\right)-2}\sqrt{\frac{\left(1+\sqrt{5}\right)x^2+2}{\left(1-\sqrt{5}\right)x^2+2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}x}{\sqrt{-\left(\left(1-\sqrt{5}\right)x^2\right)-2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\left(3+\sqrt{5}\right)\sqrt{\frac{1}{\left(1-\sqrt{5}\right)x^2+2}}\sqrt{x^4-x^2-1}}+\frac{\sqrt{-\left(\left(1-\sqrt{5}\right)x^2\right)-2}\sqrt{\frac{\left(1+\sqrt{5}\right)x^2+2}{\left(1-\sqrt{5}\right)x^2+2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}x}{\sqrt{-\left(\left(1-\sqrt{5}\right)x^2\right)-2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{\left(1-\sqrt{5}\right)x^2+2}}\sqrt{x^4-x^2-1}}-\frac{3\sqrt{2}\left(2+\sqrt{5}\right)\sqrt{2x^2+\sqrt{5}-1}\sqrt{1-\frac{2x^2}{1+\sqrt{5}}}\Pi\left(\frac{1}{2}\left(-1-\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\left(3+\sqrt{5}\right)\sqrt{x^4-x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + 2*x^2)/((1 + x^2)*Sqrt[-1 - x^2 + x^4]),x]
```

```
[Out] (3*(1 + Sqrt[5])*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[5])]
*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/2]])/(Sqrt[2]*(3
+ Sqrt[5])*Sqrt[-1 - x^2 + x^4]) + (Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 +
(1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/
4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10]])/(5^(1/4)*Sqrt[(2 +
(1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) - (3*Sqrt[-2 - (1 - Sqrt[5])
*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSi
n[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10]])/(5^
(1/4)*(3 + Sqrt[5])*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]
) - (3*Sqrt[2]*(2 + Sqrt[5])*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1
 + Sqrt[5])]*EllipticPi[(-1 - Sqrt[5])/2, ArcSin[Sqrt[2/(1 + Sqrt[5])]*x],
(-3 - Sqrt[5])/2]])/((3 + Sqrt[5])*Sqrt[-1 - x^2 + x^4])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 540

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x
] + Dist[(b*c - a*d)/b, Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1710

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + b*x^2 + c*x^4], x],
 x] + Dist[(e*A - d*B)/e, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x],
x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{-1+2x^2}{\left(1+x^2\right)\sqrt{-1-x^2+x^4}}\,dx = 2\int \frac{1}{\sqrt{-1-x^2+x^4}}\,dx - 3\int \frac{1}{\left(1+x^2\right)\sqrt{-1-x^2+x^4}}\,dx$$

$$= \frac{\sqrt{-2-\left(1-\sqrt{5}\right)x^2}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x^2}{2+\left(1-\sqrt{5}\right)x^2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}x}{\sqrt{-2-\left(1-\sqrt{5}\right)x^2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x^2}}\sqrt{-1-x^2+x^4}} + \frac{3}{}$$

$$= \frac{\sqrt{-2-\left(1-\sqrt{5}\right)x^2}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x^2}{2+\left(1-\sqrt{5}\right)x^2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}x}{\sqrt{-2-\left(1-\sqrt{5}\right)x^2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x^2}}\sqrt{-1-x^2+x^4}} - \frac{3}{}$$

$$= \frac{\sqrt{-2-\left(1-\sqrt{5}\right)x^2}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x^2}{2+\left(1-\sqrt{5}\right)x^2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}x}{\sqrt{-2-\left(1-\sqrt{5}\right)x^2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x^2}}\sqrt{-1-x^2+x^4}} - \frac{3}{}$$

$$= \frac{\sqrt{-2-\left(1-\sqrt{5}\right)x^2}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x^2}{2+\left(1-\sqrt{5}\right)x^2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}x}{\sqrt{-2-\left(1-\sqrt{5}\right)x^2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x^2}}\sqrt{-1-x^2+x^4}} - \frac{3}{}$$

$$= \frac{3\left(1+\sqrt{5}\right)\sqrt{-1+\sqrt{5}+2x^2}\sqrt{1-\frac{2x^2}{1+\sqrt{5}}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\sqrt{2}\left(3+\sqrt{5}\right)\sqrt{-1-x^2+x^4}}$$

Mathematica [C] time = 0.20, size = 131, normalized size = 2.02

$$-\frac{i\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{-x^4+x^2+1}\left(2F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)|\frac{1}{2}\left(-3+\sqrt{5}\right)\right)-3\Pi\left(\frac{1}{2}\left(-1+\sqrt{5}\right);i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}x\right)|\frac{1}{2}\left(-3+\sqrt{5}\right)\right)\right)}{\sqrt{x^4-x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 2*x^2)/((1 + x^2)*Sqrt[-1 - x^2 + x^4]),x]
```

```
[Out] ((-I)*Sqrt[2/(1 + Sqrt[5])]*Sqrt[1 + x^2 - x^4]*(2*EllipticF[I*ArcSinh[Sqrt
[2/(-1 + Sqrt[5])]*x], (-3 + Sqrt[5])/2] - 3*EllipticPi[(-1 + Sqrt[5])/2, I
*ArcSinh[Sqrt[2/(-1 + Sqrt[5])]*x], (-3 + Sqrt[5])/2]))/Sqrt[-1 - x^2 + x^4
]
```

IntegrateAlgebraic [A] time = 3.75, size = 65, normalized size = 1.00

$$-\frac{1}{2}i\log\left(\frac{-2ix^2+\sqrt{x^4-x^2-1}\,x-i}{-2ix^2-\sqrt{x^4-x^2-1}\,x-i}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x^2)/((1 + x^2)*Sqrt[-1 - x^2 + x^4]),x]
```

```
[Out] (-1/2*I)*Log[(-I - (2*I)*x^2 + x*Sqrt[-1 - x^2 + x^4])/(-I - (2*I)*x^2 - x*
Sqrt[-1 - x^2 + x^4])]
```

fricas [A] time = 0.48, size = 41, normalized size = 0.63

$$-\frac{1}{2}\arctan\left(\frac{2\sqrt{x^4-x^2-1}\left(2x^3+x\right)}{x^6-5x^4-5x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(x^2+1)/(x^4-x^2-1)^(1/2),x, algorithm="fricas")
[Out] -1/2*arctan(2*sqrt(x^4 - x^2 - 1)*(2*x^3 + x)/(x^6 - 5*x^4 - 5*x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2-1}{\sqrt{x^4-x^2-1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(x^2+1)/(x^4-x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate((2*x^2 - 1)/(sqrt(x^4 - x^2 - 1)*(x^2 + 1)), x)
```

maple [C] time = 0.02, size = 178, normalized size = 2.74

$$\frac{4\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2-2\sqrt{5}}}{2},\frac{i\sqrt{5}}{2}-\frac{i}{2}\right)}{\sqrt{-2-2\sqrt{5}}\,\sqrt{x^4-x^2-1}}-\frac{3\sqrt{1+\frac{x^2}{2}+\frac{\sqrt{5}\,x^2}{2}}\,\sqrt{1-\frac{\sqrt{5}\,x^2}{2}+\frac{x^2}{2}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}-\frac{\sqrt{5}}{2}}\,x,-\frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}},\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}{\sqrt{-\frac{1}{2}-\frac{\sqrt{5}}{2}}}\right)}{\sqrt{-\frac{1}{2}-\frac{\sqrt{5}}{2}}\,\sqrt{x^4-x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-1)/(x^2+1)/(x^4-x^2-1)^(1/2),x)
[Out] 4/(-2-2*5^(1/2))^(1/2)*(1-(-1/2-1/2*5^(1/2))*x^2)^(1/2)*(1-(1/2*5^(1/2)-1/2
)*x^2)^(1/2)/(x^4-x^2-1)^(1/2)*EllipticF(1/2*x*(-2-2*5^(1/2))^(1/2),1/2*I*5
^(1/2)-1/2*I)-3/(-1/2-1/2*5^(1/2))^(1/2)*(1+1/2*x^2+1/2*5^(1/2)*x^2)^(1/2)*
(1-1/2*5^(1/2)*x^2+1/2*x^2)^(1/2)/(x^4-x^2-1)^(1/2)*EllipticPi((-1/2-1/2*5^
(1/2))^(1/2)*x,-1/(-1/2-1/2*5^(1/2)),(1/2*5^(1/2)-1/2)^(1/2)/(-1/2-1/2*5^(1
/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2-1}{\sqrt{x^4-x^2-1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(x^2+1)/(x^4-x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x^2 - 1)/(sqrt(x^4 - x^2 - 1)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{2x^2-1}{\left(x^2+1\right)\sqrt{x^4-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - 1)/((x^2 + 1)*(x^4 - x^2 - 1)^(1/2)),x)
[Out] int((2*x^2 - 1)/((x^2 + 1)*(x^4 - x^2 - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2-1}{(x^2+1)\sqrt{x^4-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-1)/(x**2+1)/(x**4-x**2-1)**(1/2),x)
```

```
[Out] Integral((2*x**2 - 1)/((x**2 + 1)*sqrt(x**4 - x**2 - 1)), x)
```

3.813 $\int \frac{-2b+cx^2}{(-b+cx^2)\sqrt[4]{-b+cx^2+ax^4}}\,dx$

Optimal. Leaf size=65

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b+cx^2}}\right)}{\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b+cx^2}}\right)}{\sqrt[4]{a}}$$

Rubi [F] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2b+cx^2}{(-b+cx^2)\sqrt[4]{-b+cx^2+ax^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*b + c*x^2)/((-b + c*x^2)*(-b + c*x^2 + a*x^4)^(1/4)),x]
```

```
[Out] Defer[Int][(-2*b + c*x^2)/((-b + c*x^2)*(-b + c*x^2 + a*x^4)^(1/4)), x]
```

Rubi steps

$$\int \frac{-2b+cx^2}{(-b+cx^2)\sqrt[4]{-b+cx^2+ax^4}}\,dx=\int \frac{-2b+cx^2}{(-b+cx^2)\sqrt[4]{-b+cx^2+ax^4}}\,dx$$

Mathematica [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{-2b+cx^2}{(-b+cx^2)\sqrt[4]{-b+cx^2+ax^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*b + c*x^2)/((-b + c*x^2)*(-b + c*x^2 + a*x^4)^(1/4)),x]
```

```
[Out] Integrate[(-2*b + c*x^2)/((-b + c*x^2)*(-b + c*x^2 + a*x^4)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.43, size = 65, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b+cx^2}}\right)}{\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b+cx^2}}\right)}{\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*b + c*x^2)/((-b + c*x^2)*(-b + c*x^2 + a*x^4)^(1/4))
,x]
```

```
[Out] ArcTan[(a^(1/4)*x)/(-b + c*x^2 + a*x^4)^(1/4)]/a^(1/4) + ArcTanh[(a^(1/4)*x
)/(-b + c*x^2 + a*x^4)^(1/4)]/a^(1/4)
```

fricas [B] time = 0.45, size = 135, normalized size = 2.08

$$\frac{2\arctan\left(\frac{\frac{x\sqrt{\frac{\sqrt{a}x^2+\sqrt{ax^4+cx^2-b}}{x^2}}}{a^{\frac{1}{4}}}-\frac{(ax^4+cx^2-b)^{\frac{1}{4}}}{a^{\frac{1}{4}}}}{x}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{a^{\frac{1}{4}}x+(ax^4+cx^2-b)^{\frac{1}{4}}}{x}\right)}{2a^{\frac{1}{4}}}-\frac{\log\left(-\frac{a^{\frac{1}{4}}x-(ax^4+cx^2-b)^{\frac{1}{4}}}{x}\right)}{2a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2-2*b)/(c*x^2-b)/(a*x^4+c*x^2-b)^(1/4),x, algorithm="fricas"
)
```

```
[Out] 2*arctan((x*sqrt((sqrt(a)*x^2 + sqrt(a*x^4 + c*x^2 - b))/x^2)/a^(1/4) - (a*
x^4 + c*x^2 - b)^(1/4)/a^(1/4))/x)/a^(1/4) + 1/2*log((a^(1/4)*x + (a*x^4 +
c*x^2 - b)^(1/4))/x)/a^(1/4) - 1/2*log(-(a^(1/4)*x - (a*x^4 + c*x^2 - b)^(1
/4))/x)/a^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^2 - 2\,b}{\left(ax^4 + cx^2 - b\right)^{\frac{1}{4}}\left(cx^2 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2-2*b)/(c*x^2-b)/(a*x^4+c*x^2-b)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 - 2*b)/((a*x^4 + c*x^2 - b)^(1/4)*(c*x^2 - b)), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{c\,x^2 - 2b}{\left(c\,x^2 - b\right)\left(a\,x^4 + c\,x^2 - b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2-2*b)/(c*x^2-b)/(a*x^4+c*x^2-b)^(1/4),x)
```

```
[Out] int((c*x^2-2*b)/(c*x^2-b)/(a*x^4+c*x^2-b)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^2 - 2\,b}{\left(ax^4 + cx^2 - b\right)^{\frac{1}{4}}\left(cx^2 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2-2*b)/(c*x^2-b)/(a*x^4+c*x^2-b)^(1/4),x, algorithm="maxima"
)
```

```
[Out] integrate((c*x^2 - 2*b)/((a*x^4 + c*x^2 - b)^(1/4)*(c*x^2 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{2\,b - c\,x^2}{\left(b - c\,x^2\right)\left(a\,x^4 + c\,x^2 - b\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b - c*x^2)/((b - c*x^2)*(a*x^4 - b + c*x^2)^(1/4)),x)
```

```
[Out] int((2*b - c*x^2)/((b - c*x^2)*(a*x^4 - b + c*x^2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-2b + cx^2}{\left(-b + cx^2\right)\sqrt[4]{ax^4 - b + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2-2*b)/(c*x**2-b)/(a*x**4+c*x**2-b)**(1/4),x)
```

```
[Out] Integral((-2*b + c*x**2)/((-b + c*x**2)*(a*x**4 - b + c*x**2)**(1/4)), x)
```

3.814 $\int \frac{-4b+ax^3}{(-b+ax^3)\sqrt[4]{b-ax^3+cx^4}}\,dx$

Optimal. Leaf size=65

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-ax^3+b+cx^4}}\right)}{\sqrt[4]{c}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-ax^3+b+cx^4}}\right)}{\sqrt[4]{c}}$$

Rubi [F] time = 1.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-4b+ax^3}{(-b+ax^3)\sqrt[4]{b-ax^3+cx^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-4*b + a*x^3)/((-b + a*x^3)*(b - a*x^3 + c*x^4)^(1/4)),x]
```

```
[Out] Defer[Int][(b - a*x^3 + c*x^4)^(-1/4), x] + b^(1/3)*Defer[Int][1/((b^(1/3)
- a^(1/3)*x)*(b - a*x^3 + c*x^4)^(1/4)), x] + b^(1/3)*Defer[Int][1/((b^(1/3
) + (-1)^(1/3)*a^(1/3)*x)*(b - a*x^3 + c*x^4)^(1/4)), x] + b^(1/3)*Defer[In
t][1/((b^(1/3) - (-1)^(2/3)*a^(1/3)*x)*(b - a*x^3 + c*x^4)^(1/4)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-4b+ax^3}{(-b+ax^3)\sqrt[4]{b-ax^3+cx^4}}\,dx &= \int \left(\frac{1}{\sqrt[4]{b-ax^3+cx^4}}-\frac{3b}{(-b+ax^3)\sqrt[4]{b-ax^3+cx^4}}\right)dx\\
&= -\left((3b)\int \frac{1}{(-b+ax^3)\sqrt[4]{b-ax^3+cx^4}}\,dx\right)+\int \frac{1}{\sqrt[4]{b-ax^3+cx^4}}\,dx\\
&= -\left((3b)\int \left(-\frac{1}{3b^{2/3}\left(\sqrt[3]{b}-\sqrt[3]{a}\,x\right)\sqrt[4]{b-ax^3+cx^4}}-\frac{1}{3b^{2/3}\left(\sqrt[3]{b}+\sqrt[3]{-1}\sqrt[3]{a}\,x\right)\right.\right.\\
&= \sqrt[3]{b}\int \frac{1}{\left(\sqrt[3]{b}-\sqrt[3]{a}\,x\right)\sqrt[4]{b-ax^3+cx^4}}\,dx+\sqrt[3]{b}\int \frac{1}{\left(\sqrt[3]{b}+\sqrt[3]{-1}\sqrt[3]{a}\,x\right)\sqrt[4]{b-\right.}
\end{aligned}$$

Mathematica [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{-4b+ax^3}{(-b+ax^3)\sqrt[4]{b-ax^3+cx^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-4*b + a*x^3)/((-b + a*x^3)*(b - a*x^3 + c*x^4)^(1/4)),x]
```

```
[Out] Integrate[(-4*b + a*x^3)/((-b + a*x^3)*(b - a*x^3 + c*x^4)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.80, size = 65, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-ax^3+b+cx^4}}\right)}{\sqrt[4]{c}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-ax^3+b+cx^4}}\right)}{\sqrt[4]{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-4*b + a*x^3)/((-b + a*x^3)*(b - a*x^3 + c*x^4)^(1/4)), x]

[Out] (2*ArcTan[(c^(1/4)*x)/(b - a*x^3 + c*x^4)^(1/4)])/c^(1/4) + (2*ArcTanh[(c^(1/4)*x)/(b - a*x^3 + c*x^4)^(1/4)])/c^(1/4)

fricas [B] time = 0.47, size = 130, normalized size = 2.00

$$\frac{4 \arctan\left(\frac{\frac{x\sqrt{\frac{\sqrt{c}x^2+\sqrt{cx^4-ax^3+b}}{x^2}}}{c^{\frac{1}{4}}}-\frac{\left(cx^4-ax^3+b\right)^{\frac{1}{4}}}{c^{\frac{1}{4}}}}{x}\right)}{c^{\frac{1}{4}}}+\frac{\log\left(\frac{c^{\frac{1}{4}}x+\left(cx^4-ax^3+b\right)^{\frac{1}{4}}}{x}\right)}{c^{\frac{1}{4}}}-\frac{\log\left(-\frac{c^{\frac{1}{4}}x-\left(cx^4-ax^3+b\right)^{\frac{1}{4}}}{x}\right)}{c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^3-4*b)/(a*x^3-b)/(c*x^4-a*x^3+b)^(1/4),x, algorithm="fricas")

[Out] 4*arctan((x*sqrt((sqrt(c)*x^2 + sqrt(c*x^4 - a*x^3 + b))/x^2)/c^(1/4) - (c*x^4 - a*x^3 + b)^(1/4)/c^(1/4))/x)/c^(1/4) + log((c^(1/4)*x + (c*x^4 - a*x^3 + b)^(1/4))/x)/c^(1/4) - log(-(c^(1/4)*x - (c*x^4 - a*x^3 + b)^(1/4))/x)/c^(1/4)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 - 4\,b}{\left(cx^4 - ax^3 + b\right)^{\frac{1}{4}}\left(ax^3 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^3-4*b)/(a*x^3-b)/(c*x^4-a*x^3+b)^(1/4),x, algorithm="giac")

[Out] integrate((a*x^3 - 4*b)/((c*x^4 - a*x^3 + b)^(1/4)*(a*x^3 - b)), x)

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{a\,x^3 - 4b}{\left(a\,x^3 - b\right)\left(c\,x^4 - a\,x^3 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^3-4*b)/(a*x^3-b)/(c*x^4-a*x^3+b)^(1/4),x)

[Out] int((a*x^3-4*b)/(a*x^3-b)/(c*x^4-a*x^3+b)^(1/4),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 - 4\,b}{\left(cx^4 - ax^3 + b\right)^{\frac{1}{4}}\left(ax^3 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^3-4*b)/(a*x^3-b)/(c*x^4-a*x^3+b)^(1/4),x, algorithm="maxima")

[Out] integrate((a*x^3 - 4*b)/((c*x^4 - a*x^3 + b)^(1/4)*(a*x^3 - b)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{4\,b-a\,x^3}{\left(b-a\,x^3\right)\,{\left(c\,x^4-a\,x^3+b\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*b - a*x^3)/((b - a*x^3)*(b - a*x^3 + c*x^4)^(1/4)),x)
[Out] int((4*b - a*x^3)/((b - a*x^3)*(b - a*x^3 + c*x^4)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3-4b}{\left(ax^3-b\right)\sqrt[4]{-ax^3+b+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-4*b)/(a*x**3-b)/(c*x**4-a*x**3+b)**(1/4),x)
[Out] Integral((a*x**3 - 4*b)/((a*x**3 - b)*(-a*x**3 + b + c*x**4)**(1/4)), x)
```

3.815 $\int \frac{\sqrt{1-x^4}\left(1+x^4\right)}{4-7x^4+4x^8}\,dx$

Optimal. Leaf size=65

$$\frac{1}{8}\tan^{-1}\left(\frac{x^4+\frac{x^2}{2}-1}{x\sqrt{1-x^4}}\right)-\frac{1}{8}\tanh^{-1}\left(\frac{x^4-\frac{x^2}{2}-1}{x\sqrt{1-x^4}}\right)$$

Rubi [C] time = 0.54, antiderivative size = 155, normalized size of antiderivative = 2.38, number of steps used = 16, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {6728, 406, 221, 409, 1213, 537}

$$-\frac{1}{8}\left(1+i\sqrt{15}\right)F\left(\sin^{-1}(x)|-1\right)-\frac{1}{8}\left(1-i\sqrt{15}\right)F\left(\sin^{-1}(x)|-1\right)+\frac{1}{8}\Pi\left(-\frac{2}{\sqrt{\frac{1}{2}\left(7-i\sqrt{15}\right)}};\sin^{-1}(x)|-1\right)+\frac{1}{8}\Pi\left(\frac{2}{\sqrt{\frac{1}{2}\left(7-i\sqrt{15}\right)}};\sin^{-1}(x)|-1\right)+\frac{1}{8}\Pi\left(-\frac{2}{\sqrt{\frac{1}{2}\left(7+i\sqrt{15}\right)}};\sin^{-1}(x)|-1\right)+\frac{1}{8}\Pi\left(\frac{2}{\sqrt{\frac{1}{2}\left(7+i\sqrt{15}\right)}};\sin^{-1}(x)|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x^4]*(1 + x^4))/(4 - 7*x^4 + 4*x^8),x]
```

```
[Out] -1/8*((1 - I*Sqrt[15])*EllipticF[ArcSin[x], -1]) - ((1 + I*Sqrt[15])*Ellipt
icF[ArcSin[x], -1])/8 + EllipticPi[-2/Sqrt[(7 - I*Sqrt[15])/2], ArcSin[x],
-1]/8 + EllipticPi[2/Sqrt[(7 - I*Sqrt[15])/2], ArcSin[x], -1]/8 + EllipticP
i[-2/Sqrt[(7 + I*Sqrt[15])/2], ArcSin[x], -1]/8 + EllipticPi[2/Sqrt[(7 + I*
Sqrt[15])/2], ArcSin[x], -1]/8
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-x^4}\left(1+x^4\right)}{4-7x^4+4x^8}\,dx = \int \left(\frac{\left(1-i\sqrt{15}\right)\sqrt{1-x^4}}{-7-i\sqrt{15}+8x^4}+\frac{\left(1+i\sqrt{15}\right)\sqrt{1-x^4}}{-7+i\sqrt{15}+8x^4}\right)dx$$
$$=\left(1-i\sqrt{15}\right)\int \frac{\sqrt{1-x^4}}{-7-i\sqrt{15}+8x^4}\,dx+\left(1+i\sqrt{15}\right)\int \frac{\sqrt{1-x^4}}{-7+i\sqrt{15}+8x^4}\,dx$$
$$=\frac{1}{8}\left(-1-i\sqrt{15}\right)\int \frac{1}{\sqrt{1-x^4}}\,dx+\frac{1}{4}\left(-7+i\sqrt{15}\right)\int \frac{1}{\sqrt{1-x^4}\left(-7+i\sqrt{15}+8x^4\right)}\,dx +$$
$$=-\frac{1}{8}\left(1-i\sqrt{15}\right)F\left(\sin^{-1}(x)\middle|-1\right)-\frac{1}{8}\left(1+i\sqrt{15}\right)F\left(\sin^{-1}(x)\middle|-1\right)+\frac{1}{8}\int \frac{}{\left(1-\frac{2x}{\sqrt{\frac{1}{2}(7-}}\right.}$$
$$=-\frac{1}{8}\left(1-i\sqrt{15}\right)F\left(\sin^{-1}(x)\middle|-1\right)-\frac{1}{8}\left(1+i\sqrt{15}\right)F\left(\sin^{-1}(x)\middle|-1\right)+\frac{1}{8}\int \frac{}{\sqrt{1-x^2}\,\sqrt{1}}$$
$$=-\frac{1}{8}\left(1-i\sqrt{15}\right)F\left(\sin^{-1}(x)\middle|-1\right)-\frac{1}{8}\left(1+i\sqrt{15}\right)F\left(\sin^{-1}(x)\middle|-1\right)+\frac{1}{8}\Pi\left(-\frac{2}{\sqrt{\frac{1}{2}(7-}}\right.$$

Mathematica [C] time = 0.63, size = 107, normalized size = 1.65

$$\frac{1}{8}\left(-2F\left(\sin^{-1}(x)\middle|-1\right)+\Pi\left(\frac{1}{\sqrt{\frac{7}{8}-\frac{i\sqrt{15}}{8}}};\sin^{-1}(x)\middle|-1\right)+\Pi\left(\frac{1}{\sqrt{\frac{7}{8}+\frac{i\sqrt{15}}{8}}};\sin^{-1}(x)\middle|-1\right)+\Pi\left(-\frac{2}{\sqrt{\frac{1}{2}\left(7-i\sqrt{15}\right)}};\sin^{-1}(x)\middle|-1\right)+\Pi\left(-\frac{2}{\sqrt{\frac{1}{2}\left(7+i\sqrt{15}\right)}};\sin^{-1}(x)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x^4]*(1 + x^4))/(4 - 7*x^4 + 4*x^8),x]
```

```
[Out] (-2*EllipticF[ArcSin[x], -1] + EllipticPi[1/Sqrt[7/8 - (I/8)*Sqrt[15]], Arc
Sin[x], -1] + EllipticPi[1/Sqrt[7/8 + (I/8)*Sqrt[15]], ArcSin[x], -1] + Ell
ipticPi[-2/Sqrt[(7 - I*Sqrt[15])/2], ArcSin[x], -1] + EllipticPi[-2/Sqrt[(7
 + I*Sqrt[15])/2], ArcSin[x], -1])/8
```

IntegrateAlgebraic [C] time = 0.26, size = 57, normalized size = 0.88

$$\left(\frac{1}{8}-\frac{i}{8}\right)\tan^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)x}{\sqrt{1-x^4}}\right)-\left(\frac{1}{8}+\frac{i}{8}\right)\tan^{-1}\left(\frac{(1+i)\sqrt{1-x^4}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 - x^4]*(1 + x^4))/(4 - 7*x^4 + 4*x^8),x]
```

```
[Out] (1/8 - I/8)*ArcTan[((1/2 + I/2)*x)/Sqrt[1 - x^4]] - (1/8 + I/8)*ArcTan[((1
+ I)*Sqrt[1 - x^4])/x]
```

fricas [B] time = 0.61, size = 130, normalized size = 2.00

$$-\frac{1}{8}\arctan\left(-\frac{4x^8-7x^4-4\left(2x^5+x^3-2x\right)\sqrt{-x^4+1}+4}{4x^8+8x^6-7x^4-8x^2+4}\right)+\frac{1}{16}\log\left(\frac{4x^8-8x^6-7x^4+8x^2-4\left(2x^5-x^3-2x\right)\sqrt{-x^4+1}+4}{4x^8-7x^4+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)*(x^4+1)/(4*x^8-7*x^4+4),x, algorithm="fricas")
```

```
[Out] -1/8*arctan(-(4*x^8 - 7*x^4 - 4*(2*x^5 + x^3 - 2*x)*sqrt(-x^4 + 1) + 4)/(4*
x^8 + 8*x^6 - 7*x^4 - 8*x^2 + 4)) + 1/16*log((4*x^8 - 8*x^6 - 7*x^4 + 8*x^2
 - 4*(2*x^5 - x^3 - 2*x)*sqrt(-x^4 + 1) + 4)/(4*x^8 - 7*x^4 + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)\sqrt{-x^4+1}}{4\,x^8-7\,x^4+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)*(x^4+1)/(4*x^8-7*x^4+4),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)*sqrt(-x^4 + 1)/(4*x^8 - 7*x^4 + 4), x)
```

maple [A] time = 0.04, size = 102, normalized size = 1.57

$$\frac{\arctan\left(-\frac{2\sqrt{-x^4+1}}{x}+1\right)}{8}-\frac{\ln\left(\frac{\frac{-x^4+1}{2x^2}-\frac{\sqrt{-x^4+1}}{2x}+\frac{1}{4}}{\frac{-x^4+1}{2x^2}+\frac{\sqrt{-x^4+1}}{2x}+\frac{1}{4}}\right)}{16}-\frac{\arctan\left(\frac{2\sqrt{-x^4+1}}{x}+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^(1/2)*(x^4+1)/(4*x^8-7*x^4+4),x)
```

```
[Out] 1/8*arctan(-2*(-x^4+1)^(1/2)/x+1)-1/16*ln((1/2*(-x^4+1)/x^2-1/2*(-x^4+1)^(1
/2)/x+1/4)/(1/2*(-x^4+1)/x^2+1/2*(-x^4+1)^(1/2)/x+1/4))-1/8*arctan(2*(-x^4+
1)^(1/2)/x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)\sqrt{-x^4+1}}{4\,x^8-7\,x^4+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)*(x^4+1)/(4*x^8-7*x^4+4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)*sqrt(-x^4 + 1)/(4*x^8 - 7*x^4 + 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{1-x^4}\,\left(x^4+1\right)}{4\,x^8-7\,x^4+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - x^4)^(1/2)*(x^4 + 1))/(4*x^8 - 7*x^4 + 4),x)
```

```
[Out] int(((1 - x^4)^(1/2)*(x^4 + 1))/(4*x^8 - 7*x^4 + 4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)\left(x^2+1\right)}\,\left(x^4+1\right)}{4x^8-7x^4+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(1/2)*(x**4+1)/(4*x**8-7*x**4+4),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))*(x**4 + 1)/(4*x**8 - 7*x**4 + 4)
, x)
```

3.816 $\int \frac{\sqrt{-b+ax^3}}{x\left(2b+ax^3\right)}\,dx$

Optimal. Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{3}\,\sqrt{b}}\right)}{\sqrt{3}\,\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3\sqrt{b}}$$

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {446, 83, 63, 205, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{3}\,\sqrt{b}}\right)}{\sqrt{3}\,\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-b + a*x^3]/(x*(2*b + a*x^3)),x]
```

```
[Out] -1/3*ArcTan[Sqrt[-b + a*x^3]/Sqrt[b]]/Sqrt[b] + ArcTan[Sqrt[-b + a*x^3]/(Sq
rt[3]*Sqrt[b])]/(Sqrt[3]*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{-b+ax^3}}{x\left(2b+ax^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{-b+ax}}{x(2b+ax)}\,dx,x,x^3\right)$$
$$= -\left(\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-b+ax}}\,dx,x,x^3\right)\right)+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{\sqrt{-b+ax}\,(2b+ax)}\,dx,x,x^3\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^2}{a}}\,dx,x,\sqrt{-b+ax^3}\right)}{3a}+\text{Subst}\left(\int \frac{1}{3b+x^2}\,dx,x,\sqrt{-b+ax^3}\right)$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-b+ax^3}}{\sqrt{b}}\right)}{3\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{-b+ax^3}}{\sqrt{3}\sqrt{b}}\right)}{\sqrt{3}\sqrt{b}}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.94

$$-\frac{\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{3}\sqrt{b}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-b + a*x^3]/(x*(2*b + a*x^3)),x]
```

```
[Out] -1/3*(ArcTan[Sqrt[-b + a*x^3]/Sqrt[b]] - Sqrt[3]*ArcTan[Sqrt[-b + a*x^3]/(S
qrt[3]*Sqrt[b])])/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.06, size = 66, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{3}\sqrt{b}}\right)}{\sqrt{3}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-b + a*x^3]/(x*(2*b + a*x^3)),x]
```

```
[Out] -1/3*ArcTan[Sqrt[-b + a*x^3]/Sqrt[b]]/Sqrt[b] + ArcTan[Sqrt[-b + a*x^3]/(Sq
rt[3]*Sqrt[b])]/(Sqrt[3]*Sqrt[b])
```

fricas [A] time = 0.46, size = 151, normalized size = 2.29

$$\left[-\frac{\sqrt{3}\sqrt{-b}\log\left(\frac{ax^3-2\sqrt{3}\sqrt{ax^3-b}\sqrt{-b}-4b}{ax^3+2b}\right)+\sqrt{-b}\log\left(\frac{ax^3+2\sqrt{ax^3-b}\sqrt{-b}-2b}{x^3}\right)}{6b},\ \frac{\sqrt{3}\sqrt{b}\arctan\left(\frac{\sqrt{3}\sqrt{ax^3-b}}{3\sqrt{b}}\right)-\sqrt{b}\arctan\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(1/2)/x/(a*x^3+2*b),x, algorithm="fricas")
```

```
[Out] [-1/6*(sqrt(3)*sqrt(-b)*log((a*x^3 - 2*sqrt(3)*sqrt(a*x^3 - b)*sqrt(-b) - 4
*b)/(a*x^3 + 2*b)) + sqrt(-b)*log((a*x^3 + 2*sqrt(a*x^3 - b)*sqrt(-b) - 2*b
)/x^3))/b, 1/3*(sqrt(3)*sqrt(b)*arctan(1/3*sqrt(3)*sqrt(a*x^3 - b)/sqrt(b))
 - sqrt(b)*arctan(sqrt(a*x^3 - b)/sqrt(b)))/b]
```

giac [A] time = 0.26, size = 50, normalized size = 0.76

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{ax^3-b}}{3\sqrt{b}}\right)}{3\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(1/2)/x/(a*x^3+2*b),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(a*x^3 - b)/sqrt(b))/sqrt(b) - 1/3*arcta
n(sqrt(a*x^3 - b)/sqrt(b))/sqrt(b)
```

maple [C] time = 0.14, size = 458, normalized size = 6.94

$$-\frac{a\left(\frac{2\sqrt{a x^3-b}}{3a}-\frac{i\sqrt{2}\sum_{\_\alpha=\mathrm{RootOf}(a\_Z^3+2b)}\frac{(a^2b)^{\frac{1}{3}}\sqrt{\frac{\frac{i}{2}a\left(2x+\frac{i\sqrt{3}(a^2b)^{\frac{1}{3}}+(a^2b)^{\frac{1}{3}}}{a}\right)}{(a^2b)^{\frac{1}{3}}}}\sqrt{\frac{a\left(x-\frac{(a^2b)^{\frac{1}{3}}}{a}\right)}{-3(a^2b)^{\frac{1}{3}}-i\sqrt{3}(a^2b)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{-\frac{i}{2}a\left(2x+\frac{-i\sqrt{3}(a^2b)^{\frac{1}{3}}+(a^2b)^{\frac{1}{3}}}{a}\right)}{(a^2b)^{\frac{1}{3}}}}\left(-i(a^2b)^{\frac{1}{3}}\sqrt{3}\,\_\alpha a+i(a^2b)^{\frac{2}{3}}\sqrt{3}+2\_\alpha^2a^2-(a^2b)^{\frac{1}{3}}\_\alpha a-(a^2b)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{-\frac{i\left(x+\frac{(a^2b)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}(a^2b)^{\frac{1}{3}}}{2a}\right)\sqrt{3}\,a}{(a^2b)^{\frac{1}{3}}}}}{3},\frac{-2i(a^2b)^{\frac{1}{3}}\sqrt{3}\,\_\alpha^2a+i(a^2b)^{\frac{2}{3}}\sqrt{3}\,\_\alpha+i\sqrt{3}\,ab-3(a^2b)^{\frac{2}{3}}\_\alpha+3ab}{6ab},\sqrt{-\frac{\frac{i\sqrt{3}(a^2b)^{\frac{1}{3}}}{a}}{-\frac{3(a^2b)^{\frac{1}{3}}}{2a}-\frac{i\sqrt{3}(a^2b)^{\frac{1}{3}}}{2a}}}\right)}{2\sqrt{a x^3-b}}}{3a^3}\right)}{2b}+\frac{\frac{2\sqrt{a x^3-b}}{3}+\frac{2b\,\mathrm{arctanh}\left(\frac{\sqrt{a x^3-b}}{\sqrt{-b}}\right)}{3\sqrt{-b}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b)^(1/2)/x/(a*x^3+2*b),x)
```

```
[Out] -1/2*a/b*(2/3*(a*x^3-b)^(1/2)/a-1/3*I/a^3*2^(1/2)*sum((a^2*b)^(1/3)*(-1/2*I
*a*(2*x+1/a*(I*3^(1/2)*(a^2*b)^(1/3)+(a^2*b)^(1/3)))/(a^2*b)^(1/3))^(1/2)*(
a*(x-1/a*(a^2*b)^(1/3))/(-3*(a^2*b)^(1/3)-I*3^(1/2)*(a^2*b)^(1/3)))^(1/2)*(
1/2*I*a*(2*x+1/a*(-I*3^(1/2)*(a^2*b)^(1/3)+(a^2*b)^(1/3)))/(a^2*b)^(1/3))^(
1/2)/(a*x^3-b)^(1/2)*(-I*(a^2*b)^(1/3)*3^(1/2)*_alpha*a+I*(a^2*b)^(2/3)*3^(
1/2)+2*_alpha^2*a^2-(a^2*b)^(1/3)*_alpha*a-(a^2*b)^(2/3))*EllipticPi(1/3*3^
(1/2)*(-I*(x+1/2/a*(a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(a^2*b)^(1/3))*3^(1/2)*a/(
a^2*b)^(1/3))^(1/2),1/6/a*(-2*I*(a^2*b)^(1/3)*3^(1/2)*_alpha^2*a+I*(a^2*b)^
(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(a^2*b)^(2/3)*_alpha+3*a*b)/b,(-I*3^(1
/2)/a*(a^2*b)^(1/3)/(-3/2/a*(a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(a^2*b)^(1/3)))^(
1/2)),_alpha=RootOf(_Z^3*a+2*b)))+1/2/b*(2/3*(a*x^3-b)^(1/2)+2/3*b*arctanh(
(a*x^3-b)^(1/2)/(-b)^(1/2))/(-b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a x^3-b}}{\left(a x^3+2\,b\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(1/2)/x/(a*x^3+2*b),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^3 - b)/((a*x^3 + 2*b)*x), x)
```

mupad [B] time = 3.50, size = 96, normalized size = 1.45

$$\frac{\ln\left(\frac{2\,b-a\,x^3+\sqrt{b}\,\sqrt{a\,x^3-b}\,2\mathrm{i}}{x^3}\right)\,1\mathrm{i}}{6\,\sqrt{b}}+\frac{\sqrt{3}\,\ln\left(\frac{\sqrt{3}\,b\,4\mathrm{i}+6\,\sqrt{b}\,\sqrt{a\,x^3-b}-\sqrt{3}\,a\,x^3\,1\mathrm{i}}{2\,a\,x^3+4\,b}\right)\,1\mathrm{i}}{6\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 - b)^(1/2)/(x*(2*b + a*x^3)),x)
```

```
[Out] (log((2*b + b^(1/2)*(a*x^3 - b)^(1/2)*2i - a*x^3)/x^3)*1i)/(6*b^(1/2)) + (3
^(1/2)*log((3^(1/2)*b*4i + 6*b^(1/2)*(a*x^3 - b)^(1/2) - 3^(1/2)*a*x^3*1i)/
(4*b + 2*a*x^3))*1i)/(6*b^(1/2))
```

sympy [A] time = 6.49, size = 63, normalized size = 0.95

$$\frac{2\left(-\frac{a\operatorname{atan}\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{6\sqrt{b}}+\frac{\sqrt{3}\,a\operatorname{atan}\left(\frac{\sqrt{3}\,\sqrt{ax^3-b}}{3\sqrt{b}}\right)}{6\sqrt{b}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b)**(1/2)/x/(a*x**3+2*b),x)
```

```
[Out] 2*(-a*atan(sqrt(a*x**3 - b)/sqrt(b))/(6*sqrt(b)) + sqrt(3)*a*atan(sqrt(3)*s
qrt(a*x**3 - b)/(3*sqrt(b)))/(6*sqrt(b)))/a
```

3.817 $\int \frac{-b+4ax^3}{x\sqrt{-b+ax^3}\,(2b+ax^3)}\,dx$

Optimal. Leaf size=66

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{3}\sqrt{b}}\right)}{\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3\sqrt{b}}$$

Rubi [A] time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {573, 156, 63, 205, 203}

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{3}\sqrt{b}}\right)}{\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + 4*a*x^3)/(x*Sqrt[-b + a*x^3]*(2*b + a*x^3)),x]
```

```
[Out] -1/3*ArcTan[Sqrt[-b + a*x^3]/Sqrt[b]]/Sqrt[b] + (Sqrt[3]*ArcTan[Sqrt[-b + a
*x^3]/(Sqrt[3]*Sqrt[b])])/Sqrt[b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rubi steps

$$\int \frac{-b+4ax^3}{x\sqrt{-b+ax^3}\left(2b+ax^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{-b+4ax}{x\sqrt{-b+ax}\,(2b+ax)}\,dx, x, x^3\right)$$
$$= -\left(\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-b+ax}}\,dx, x, x^3\right)\right) + \frac{1}{2}(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{-b+ax}\,(2b+ax)}\,dx\right.$$
$$= 3\,\text{Subst}\left(\int \frac{1}{3b+x^2}\,dx, x, \sqrt{-b+ax^3}\right) - \frac{\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^2}{a}}\,dx, x, \sqrt{-b+ax^3}\right)}{3a}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-b+ax^3}}{\sqrt{b}}\right)}{3\sqrt{b}} + \frac{\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{-b+ax^3}}{\sqrt{3}\,\sqrt{b}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.08, size = 62, normalized size = 0.94

$$-\frac{\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right) - 3\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{3}\,\sqrt{b}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + 4*a*x^3)/(x*Sqrt[-b + a*x^3]*(2*b + a*x^3)),x]
```

```
[Out] -1/3*(ArcTan[Sqrt[-b + a*x^3]/Sqrt[b]] - 3*Sqrt[3]*ArcTan[Sqrt[-b + a*x^3]/
(Sqrt[3]*Sqrt[b])])/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.06, size = 66, normalized size = 1.00

$$\frac{\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{3}\,\sqrt{b}}\right)}{\sqrt{b}} - \frac{\tan^{-1}\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + 4*a*x^3)/(x*Sqrt[-b + a*x^3]*(2*b + a*x^3)),x]
```

```
[Out] -1/3*ArcTan[Sqrt[-b + a*x^3]/Sqrt[b]]/Sqrt[b] + (Sqrt[3]*ArcTan[Sqrt[-b + a
*x^3]/(Sqrt[3]*Sqrt[b])])/Sqrt[b]
```

fricas [A] time = 0.45, size = 158, normalized size = 2.39

$$\left[\frac{3\,\sqrt{3}\,b\sqrt{-\frac{1}{b}}\,\log\left(\frac{ax^3+2\,\sqrt{3}\,\sqrt{ax^3-b}\,b\sqrt{-\frac{1}{b}}-4\,b}{ax^3+2\,b}\right) - \sqrt{-b}\,\log\left(\frac{ax^3+2\,\sqrt{ax^3-b}\,\sqrt{-b}-2\,b}{x^3}\right)}{6\,b}, -\frac{3\,\sqrt{3}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{b}}{\sqrt{ax^3-b}}\right) + \sqrt{b}\,\arctan\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*a*x^3-b)/x/(a*x^3-b)^(1/2)/(a*x^3+2*b),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(3)*b*sqrt(-1/b)*log((a*x^3 + 2*sqrt(3)*sqrt(a*x^3 - b)*b*sqrt(
-1/b) - 4*b)/(a*x^3 + 2*b)) - sqrt(-b)*log((a*x^3 + 2*sqrt(a*x^3 - b)*sqrt(
-b) - 2*b)/x^3))/b, -1/3*(3*sqrt(3)*sqrt(b)*arctan(sqrt(3)*sqrt(b)/sqrt(a*x
^3 - b)) + sqrt(b)*arctan(sqrt(a*x^3 - b)/sqrt(b)))/b]
```

giac [A] time = 1.00, size = 49, normalized size = 0.74

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{ax^3-b}}{3\,\sqrt{b}}\right)}{\sqrt{b}} - \frac{\arctan\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*a*x^3-b)/x/(a*x^3-b)^(1/2)/(a*x^3+2*b),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*sqrt(a*x^3 - b)/sqrt(b))/sqrt(b) - 1/3*arctan(sq
rt(a*x^3 - b)/sqrt(b))/sqrt(b)
```

maple [C] time = 0.08, size = 418, normalized size = 6.33

$$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(a\_Z^3+2b\right)}\frac{\left(a^2b\right)^{\frac{1}{3}}\sqrt{-\frac{ia\left(2x+\frac{i\sqrt{3}\left(a^2b\right)^{\frac{1}{3}}+\left(a^2b\right)^{\frac{1}{3}}}{a}\right)}{2\left(a^2b\right)^{\frac{1}{3}}}}\sqrt{\frac{a\left(x-\frac{\left(a^2b\right)^{\frac{1}{3}}}{a}\right)}{-3\left(a^2b\right)^{\frac{1}{3}}-i\sqrt{3}\left(a^2b\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ia\left(2x+\frac{-i\sqrt{3}\left(a^2b\right)^{\frac{1}{3}}+\left(a^2b\right)^{\frac{1}{3}}}{a}\right)}{\left(a^2b\right)^{\frac{1}{3}}}}\left(-i\left(a^2b\right)^{\frac{1}{3}}\sqrt{3}\,\_\alpha a+i\left(a^2b\right)^{\frac{2}{3}}\sqrt{3}+2\_\alpha^2a^2-\left(a^2b\right)^{\frac{1}{3}}\_\alpha a-\left(a^2b\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{-\frac{i\left(x+\frac{\left(a^2b\right)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\left(a^2b\right)^{\frac{1}{3}}}{2a}\right)\sqrt{3}\,a}{\left(a^2b\right)^{\frac{1}{3}}}}}{3},\frac{-2i\left(a^2b\right)^{\frac{1}{3}}\sqrt{3}\,\_\alpha^2a+i\left(a^2b\right)^{\frac{2}{3}}\sqrt{3}\,\_\alpha+i\sqrt{3}\,ab-3\left(a^2b\right)^{\frac{2}{3}}\_\alpha+3ab}{6ab},\sqrt{-\frac{i\sqrt{3}\left(a^2b\right)^{\frac{1}{3}}}{a\left(-\frac{3\left(a^2b\right)^{\frac{1}{3}}}{2a}-\frac{i\sqrt{3}\left(a^2b\right)^{\frac{1}{3}}}{2a}\right)}}\right)}{2\sqrt{ax^3-b}}\right)}{2a^2b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{ax^3-b}}{\sqrt{-b}}\right)}{3\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*a*x^3-b)/x/(a*x^3-b)^(1/2)/(a*x^3+2*b),x)
```

```
[Out] 1/2*I/a^2/b*2^(1/2)*sum((a^2*b)^(1/3)*(-1/2*I*a*(2*x+1/a*(I*3^(1/2)*(a^2*b)
^(1/3)+(a^2*b)^(1/3)))/(a^2*b)^(1/3))^(1/2)*(a*(x-1/a*(a^2*b)^(1/3))/(-3*(a
^2*b)^(1/3)-I*3^(1/2)*(a^2*b)^(1/3)))^(1/2)*(1/2*I*a*(2*x+1/a*(-I*3^(1/2)*(
a^2*b)^(1/3)+(a^2*b)^(1/3)))/(a^2*b)^(1/3))^(1/2)/(a*x^3-b)^(1/2)*(-I*(a^2*
b)^(1/3)*3^(1/2)*_alpha*a+I*(a^2*b)^(2/3)*3^(1/2)+2*_alpha^2*a^2-(a^2*b)^(1
/3)*_alpha*a-(a^2*b)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/a*(a^2*b)^(1/
3)+1/2*I*3^(1/2)/a*(a^2*b)^(1/3))*3^(1/2)*a/(a^2*b)^(1/3))^(1/2),1/6/a*(-2*
I*(a^2*b)^(1/3)*3^(1/2)*_alpha^2*a+I*(a^2*b)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)
*a*b-3*(a^2*b)^(2/3)*_alpha+3*a*b)/b,(-I*3^(1/2)/a*(a^2*b)^(1/3)/(-3/2/a*(a
^2*b)^(1/3)-1/2*I*3^(1/2)/a*(a^2*b)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*a+2*b
))+1/3*arctanh((a*x^3-b)^(1/2)/(-b)^(1/2))/(-b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,ax^3-b}{\left(ax^3+2\,b\right)\sqrt{ax^3-b}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*a*x^3-b)/x/(a*x^3-b)^(1/2)/(a*x^3+2*b),x, algorithm="maxima")
```

```
[Out] integrate((4*a*x^3 - b)/((a*x^3 + 2*b)*sqrt(a*x^3 - b)*x), x)
```

mupad [B] time = 2.95, size = 96, normalized size = 1.45

$$\frac{\ln\left(\frac{2\,b-a\,x^3+\sqrt{b}\,\sqrt{a\,x^3-b}\,2{}\mathrm{i}}{x^3}\right)\,1{}\mathrm{i}}{6\,\sqrt{b}}+\frac{\sqrt{3}\,\ln\left(\frac{\sqrt{3}\,b\,4{}\mathrm{i}+6\,\sqrt{b}\,\sqrt{a\,x^3-b}-\sqrt{3}\,a\,x^3\,1{}\mathrm{i}}{6\,a\,x^3+12\,b}\right)\,1{}\mathrm{i}}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - 4*a*x^3)/(x*(a*x^3 - b)^(1/2)*(2*b + a*x^3)),x)
```

```
[Out] (log((2*b + b^(1/2)*(a*x^3 - b)^(1/2)*2i - a*x^3)/x^3)*1i)/(6*b^(1/2)) + (3
^(1/2)*log((3^(1/2)*b*4i + 6*b^(1/2)*(a*x^3 - b)^(1/2) - 3^(1/2)*a*x^3*1i)/
(12*b + 6*a*x^3))*1i)/(2*b^(1/2))
```

sympy [A] time = 18.26, size = 56, normalized size = 0.85

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{ax^3-b}}{\sqrt{b}}\right)}{3\sqrt{b}}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\,\sqrt{ax^3-b}}{3\sqrt{b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*a*x**3-b)/x/(a*x**3-b)**(1/2)/(a*x**3+2*b),x)
```

```
[Out] -atan(sqrt(a*x**3 - b)/sqrt(b))/(3*sqrt(b)) + sqrt(3)*atan(sqrt(3)*sqrt(a*x
**3 - b)/(3*sqrt(b)))/sqrt(b)
```

3.818 $\int \frac{\sqrt{1+6x^2+x^4}}{(-1+x)(1+x)^3} \, dx$

Optimal. Leaf size=66

$$\frac{\sqrt{x^4+6x^2+1}}{4(x+1)^2}-\frac{\tanh^{-1}\left(\frac{2\sqrt{2}x}{x^2+\sqrt{x^4+6x^2+1}-2x+1}\right)}{2\sqrt{2}}$$

Rubi [F] time = 0.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+6x^2+x^4}}{(-1+x)(1+x)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + 6*x^2 + x^4]/((-1 + x)*(1 + x)^3),x]
```

```
[Out] (x*(3 + 2*Sqrt[2] + x^2))/(4*Sqrt[1 + 6*x^2 + x^4]) - ArcTanh[(2*Sqrt[2]*x)
/Sqrt[1 + 6*x^2 + x^4]]/(2*Sqrt[2]) - (Sqrt[3 + 2*Sqrt[2]]*Sqrt[(1 + (3 - 2
*Sqrt[2])*x^2)/(1 + (3 + 2*Sqrt[2])*x^2)]*(1 + (3 + 2*Sqrt[2])*x^2)*Ellipti
cE[ArcTan[Sqrt[3 + 2*Sqrt[2]]*x], -4*(4 - 3*Sqrt[2])])/(4*Sqrt[1 + 6*x^2 +
x^4]) + (3*Sqrt[(1 + (3 - 2*Sqrt[2])*x^2)/(1 + (3 + 2*Sqrt[2])*x^2)]*(1 + (
3 + 2*Sqrt[2])*x^2)*EllipticF[ArcTan[Sqrt[3 + 2*Sqrt[2]]*x], -4*(4 - 3*Sqrt
[2])])/(4*Sqrt[3 + 2*Sqrt[2]]*Sqrt[1 + 6*x^2 + x^4]) - Defer[Int][Sqrt[1 +
6*x^2 + x^4]/(1 + x)^3, x]/2 - Defer[Int][Sqrt[1 + 6*x^2 + x^4]/(1 + x)^2,
x]/4
```

Rubi steps

$$\int \frac{\sqrt{1+6x^2+x^4}}{(-1+x)(1+x)^3} \, dx = \int \left(-\frac{\sqrt{1+6x^2+x^4}}{2(1+x)^3}-\frac{\sqrt{1+6x^2+x^4}}{4(1+x)^2}+\frac{\sqrt{1+6x^2+x^4}}{4\left(-1+x^2\right)}\right) dx$$

$$= -\left(\frac{1}{4}\int \frac{\sqrt{1+6x^2+x^4}}{(1+x)^2} \, dx\right)+\frac{1}{4}\int \frac{\sqrt{1+6x^2+x^4}}{-1+x^2} \, dx-\frac{1}{2}\int \frac{\sqrt{1+6x^2+x^4}}{(1+x)^3} \, dx$$

$$= -\left(\frac{1}{4}\int \frac{-7-x^2}{\sqrt{1+6x^2+x^4}} \, dx\right)-\frac{1}{4}\int \frac{\sqrt{1+6x^2+x^4}}{(1+x)^2} \, dx-\frac{1}{2}\int \frac{\sqrt{1+6x^2+x^4}}{(1+x)^3} \, dx+2 \ldots$$

$$= \frac{1}{4}\int \frac{x^2}{\sqrt{1+6x^2+x^4}} \, dx-\frac{1}{4}\int \frac{\sqrt{1+6x^2+x^4}}{(1+x)^2} \, dx-\frac{1}{2}\int \frac{\sqrt{1+6x^2+x^4}}{(1+x)^3} \, dx+\frac{7}{4}\int \ldots$$

$$= \frac{x\left(3+2\sqrt{2}+x^2\right)}{4\sqrt{1+6x^2+x^4}}-\frac{\sqrt{3+2\sqrt{2}}\sqrt{\frac{1+\left(3-2\sqrt{2}\right)x^2}{1+\left(3+2\sqrt{2}\right)x^2}}\left(1+\left(3+2\sqrt{2}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{3+2\ldots}\right.\right.}{4\sqrt{1+6x^2+x^4}}$$

$$= \frac{x\left(3+2\sqrt{2}+x^2\right)}{4\sqrt{1+6x^2+x^4}}-\frac{\tanh^{-1}\left(\frac{2\sqrt{2}x}{\sqrt{1+6x^2+x^4}}\right)}{2\sqrt{2}}-\frac{\sqrt{3+2\sqrt{2}}\sqrt{\frac{1+\left(3-2\sqrt{2}\right)x^2}{1+\left(3+2\sqrt{2}\right)x^2}}\left(1+\left(3+2\sqrt{2}\ldots\right.\right.}{4\sqrt{1+\ldots}}$$

Mathematica [C] time = 2.69, size = 641, normalized size = 9.71

$$\frac{\frac{x^4+6x^2+1}{(x+1)^2}+\frac{8\sqrt{\frac{\sqrt{3+2\sqrt{2}}+ix}{\sqrt{3-2\sqrt{2}}+ix}}\sqrt{-\frac{x+i\sqrt{3+2\sqrt{2}}}{\sqrt{3+2\sqrt{2}}x-\sqrt{3-2\sqrt{2}}x-2i\sqrt{2}+2i}}\sqrt{\frac{-i\left(\sqrt{3-2\sqrt{2}}-\sqrt{3+2\sqrt{2}}\right)x-2\sqrt{2}+2}{\sqrt{3-2\sqrt{2}}+ix}}\left(\sqrt{3-2\sqrt{2}}+ix\right)^2\left(\left(\sqrt{3-2\sqrt{2}}-i\left(\sqrt{2}-1\right)\right)F\left(\sin^{-1}\left(\sqrt{\frac{i\left(\sqrt{3-2\sqrt{2}}-\sqrt{3+2\sqrt{2}}\right)x+2\left(-1+\sqrt{2}\right)}{2\left(-2+\sqrt{2}\right)-i\left(\sqrt{3-2\sqrt{2}}+\sqrt{3+2\sqrt{2}}\right)x}}\right)\middle|2\right)-2\sqrt{3-2\sqrt{2}}\,\Pi\left(\frac{\left(i+\sqrt{3-2\sqrt{2}}\right)\left(\sqrt{3-2\sqrt{2}}+\sqrt{3+2\sqrt{2}}\right)}{\left(-i+\sqrt{3-2\sqrt{2}}\right)\left(\sqrt{3-2\sqrt{2}}-\sqrt{3+2\sqrt{2}}\right)};\sin^{-1}\left(\sqrt{\frac{i\left(\sqrt{3-2\sqrt{2}}-\sqrt{3+2\sqrt{2}}\right)x+2\left(-1+\sqrt{2}\right)}{2\left(-2+\sqrt{2}\right)-i\left(\sqrt{3-2\sqrt{2}}+\sqrt{3+2\sqrt{2}}\right)x}}\right)\middle|2\right)\right)}{\left(\sqrt{3-2\sqrt{2}}-i\right)\left(\sqrt{3-2\sqrt{2}}+i\right)\left(\sqrt{3-2\sqrt{2}}-\sqrt{3+2\sqrt{2}}\right)}}{4\sqrt{x^4+6x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 + 6*x^2 + x^4]/((-1 + x)*(1 + x)^3),x]
```

```
[Out] ((1 + 6*x^2 + x^4)/(1 + x)^2 + (8*(Sqrt[3 - 2*Sqrt[2]] + I*x)^2*Sqrt[(Sqrt[
3 + 2*Sqrt[2]] + I*x)/(Sqrt[3 - 2*Sqrt[2]] + I*x)]*Sqrt[-((I*Sqrt[3 + 2*Sqr
t[2]] + x)/(2*I - (2*I)*Sqrt[2] - Sqrt[3 - 2*Sqrt[2]]*x + Sqrt[3 + 2*Sqrt[2
]]*x))]*Sqrt[(2 - 2*Sqrt[2] - I*(Sqrt[3 - 2*Sqrt[2]] - Sqrt[3 + 2*Sqrt[2]])
*x)/(Sqrt[3 - 2*Sqrt[2]] + I*x)]*((Sqrt[3 - 2*Sqrt[2]] - I*(-1 + Sqrt[2]))*
EllipticF[ArcSin[Sqrt[(2*(-1 + Sqrt[2]) + I*(Sqrt[3 - 2*Sqrt[2]] - Sqrt[3 +
 2*Sqrt[2]])*x)/(2*(-2 + Sqrt[2]) - I*(Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqr
t[2]])*x)]], 2] - 2*Sqrt[3 - 2*Sqrt[2]]*EllipticPi[((I + Sqrt[3 - 2*Sqrt[2]
])*(Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]]))/((-I + Sqrt[3 - 2*Sqrt[2]])
*(Sqrt[3 - 2*Sqrt[2]] - Sqrt[3 + 2*Sqrt[2]])), ArcSin[Sqrt[(2*(-1 + Sqrt[2]
) + I*(Sqrt[3 - 2*Sqrt[2]] - Sqrt[3 + 2*Sqrt[2]])*x)/(2*(-2 + Sqrt[2]) - I*
(Sqrt[3 - 2*Sqrt[2]] + Sqrt[3 + 2*Sqrt[2]])*x)]], 2]))/((-I + Sqrt[3 - 2*Sq
rt[2]])*(I + Sqrt[3 - 2*Sqrt[2]])*(Sqrt[3 - 2*Sqrt[2]] - Sqrt[3 + 2*Sqrt[2]
])))/(4*Sqrt[1 + 6*x^2 + x^4])
```

IntegrateAlgebraic [A] time = 0.63, size = 66, normalized size = 1.00

$$\frac{\sqrt{x^4+6x^2+1}}{4(x+1)^2}-\frac{\tanh^{-1}\left(\frac{2\sqrt{2}x}{x^2+\sqrt{x^4+6x^2+1}-2x+1}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + 6*x^2 + x^4]/((-1 + x)*(1 + x)^3),x]
```

```
[Out] Sqrt[1 + 6*x^2 + x^4]/(4*(1 + x)^2) - ArcTanh[(2*Sqrt[2]*x)/(1 - 2*x + x^2
+ Sqrt[1 + 6*x^2 + x^4])]/(2*Sqrt[2])
```

fricas [A] time = 0.48, size = 106, normalized size = 1.61

$$\frac{\sqrt{2}\left(x^2+2x+1\right)\log\left(\frac{3x^4+4x^3-2\sqrt{2}\sqrt{x^4+6x^2+1}\left(x^2+2x+1\right)+18x^2+4x+3}{x^4-4x^3+6x^2-4x+1}\right)+4\sqrt{x^4+6x^2+1}}{16\left(x^2+2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^2+1)^(1/2)/(-1+x)/(1+x)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(2)*(x^2 + 2*x + 1)*log((3*x^4 + 4*x^3 - 2*sqrt(2)*sqrt(x^4 + 6*x
^2 + 1)*(x^2 + 2*x + 1) + 18*x^2 + 4*x + 3)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
) + 4*sqrt(x^4 + 6*x^2 + 1))/(x^2 + 2*x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^4+6x^2+1}}{(x+1)^3(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^2+1)^(1/2)/(-1+x)/(1+x)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 6*x^2 + 1)/((x + 1)^3*(x - 1)), x)
```

maple [C] time = 0.18, size = 222, normalized size = 3.36

$$-\frac{\sqrt{1-\left(-3+2\sqrt{2}\right)x^2}\,\sqrt{1-\left(-3-2\sqrt{2}\right)x^2}\,\mathrm{EllipticPi}\left(\sqrt{-3+2\sqrt{2}}\,x,\frac{1}{-3+2\sqrt{2}},\frac{\sqrt{-3-2\sqrt{2}}}{\sqrt{-3+2\sqrt{2}}}\right)}{\sqrt{-3+2\sqrt{2}}\,\sqrt{x^4+6x^2+1}}-\frac{\sqrt{8}\,\mathrm{arctanh}\left(\frac{\left(8x^2+8\right)\sqrt{8}}{16\sqrt{x^4+6x^2+1}}\right)}{16}+\frac{\sqrt{x^4+6x^2+1}}{4\left(1+x\right)^2}+\frac{\sqrt{1-\left(-3+2\sqrt{2}\right)x^2}\,\sqrt{1-\left(-3-2\sqrt{2}\right)x^2}\,\mathrm{EllipticF}\left(\left(i\sqrt{2}-i\right)x,3+2\sqrt{2}\right)}{2\left(i\sqrt{2}-i\right)\sqrt{x^4+6x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+6*x^2+1)^(1/2)/(-1+x)/(1+x)^3,x)
```

```
[Out] -1/(-3+2*2^(1/2))^(1/2)*(1-(-3+2*2^(1/2))*x^2)^(1/2)*(1-(-3-2*2^(1/2))*x^2)
^(1/2)/(x^4+6*x^2+1)^(1/2)*EllipticPi((-3+2*2^(1/2))^(1/2)*x,1/(-3+2*2^(1/2
)),(-3-2*2^(1/2))^(1/2)/(-3+2*2^(1/2))^(1/2))-1/16*8^(1/2)*arctanh(1/16*(8*
x^2+8)*8^(1/2)/(x^4+6*x^2+1)^(1/2))+1/4*(x^4+6*x^2+1)^(1/2)/(1+x)^2+1/2/(I*
2^(1/2)-I)*(1-(-3+2*2^(1/2))*x^2)^(1/2)*(1-(-3-2*2^(1/2))*x^2)^(1/2)/(x^4+6
*x^2+1)^(1/2)*EllipticF((I*2^(1/2)-I)*x,3+2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+6\,x^2+1}}{(x+1)^3(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^2+1)^(1/2)/(-1+x)/(1+x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 6*x^2 + 1)/((x + 1)^3*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x^4+6\,x^2+1}}{(x-1)\,{(x+1)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x^2 + x^4 + 1)^(1/2)/((x - 1)*(x + 1)^3),x)
```

```
[Out] int((6*x^2 + x^4 + 1)^(1/2)/((x - 1)*(x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+6x^2+1}}{(x-1)\,(x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+6*x**2+1)**(1/2)/(-1+x)/(1+x)**3,x)
```

```
[Out] Integral(sqrt(x**4 + 6*x**2 + 1)/((x - 1)*(x + 1)**3), x)
```

3.819 $\int \frac{\sqrt[4]{b+ax^4}}{x}\,dx$

Optimal. Leaf size=66

$$\sqrt[4]{ax^4+b}-\frac{1}{2}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)-\frac{1}{2}\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 63, 212, 206, 203}

$$\sqrt[4]{ax^4+b}-\frac{1}{2}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)-\frac{1}{2}\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^4)^(1/4)/x,x]
```

```
[Out] (b + a*x^4)^(1/4) - (b^(1/4)*ArcTan[(b + a*x^4)^(1/4)/b^(1/4)])/2 - (b^(1/4
)*ArcTanh[(b + a*x^4)^(1/4)/b^(1/4)])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{b+ax^4}}{x}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{\sqrt[4]{b+ax}}{x}\,dx,x,x^4\right)\\
&= \sqrt[4]{b+ax^4}+\frac{1}{4}b\,\text{Subst}\left(\int \frac{1}{x(b+ax)^{3/4}}\,dx,x,x^4\right)\\
&= \sqrt[4]{b+ax^4}+\frac{b\,\text{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{b+ax^4}\right)}{a}\\
&= \sqrt[4]{b+ax^4}-\frac{1}{2}\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx,x,\sqrt[4]{b+ax^4}\right)-\frac{1}{2}\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx,x,\sqrt[4]{b+ax^4}\right)\\
&= \sqrt[4]{b+ax^4}-\frac{1}{2}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b+ax^4}}{\sqrt[4]{b}}\right)-\frac{1}{2}\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^4}}{\sqrt[4]{b}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 66, normalized size = 1.00

$$\sqrt[4]{ax^4+b}-\frac{1}{2}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)-\frac{1}{2}\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^4)^(1/4)/x,x]
```

```
[Out] (b + a*x^4)^(1/4) - (b^(1/4)*ArcTan[(b + a*x^4)^(1/4)/b^(1/4)])/2 - (b^(1/4
)*ArcTanh[(b + a*x^4)^(1/4)/b^(1/4)])/2
```

IntegrateAlgebraic [A] time = 0.07, size = 66, normalized size = 1.00

$$\sqrt[4]{ax^4+b}-\frac{1}{2}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)-\frac{1}{2}\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^4)^(1/4)/x,x]
```

```
[Out] (b + a*x^4)^(1/4) - (b^(1/4)*ArcTan[(b + a*x^4)^(1/4)/b^(1/4)])/2 - (b^(1/4
)*ArcTanh[(b + a*x^4)^(1/4)/b^(1/4)])/2
```

fricas [A] time = 0.46, size = 93, normalized size = 1.41

$$b^{\frac{1}{4}}\arctan\left(\frac{b^{\frac{3}{4}}\sqrt{\sqrt{ax^4+b}+\sqrt{b}}-\left(ax^4+b\right)^{\frac{1}{4}}b^{\frac{3}{4}}}{b}\right)-\frac{1}{4}b^{\frac{1}{4}}\log\left(\left(ax^4+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}\right)+\frac{1}{4}b^{\frac{1}{4}}\log\left(\left(ax^4+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}\right)+\left(ax^4+b\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(1/4)/x,x, algorithm="fricas")
```

```
[Out] b^(1/4)*arctan((b^(3/4)*sqrt(sqrt(a*x^4 + b) + sqrt(b)) - (a*x^4 + b)^(1/4)
*b^(3/4))/b) - 1/4*b^(1/4)*log((a*x^4 + b)^(1/4) + b^(1/4)) + 1/4*b^(1/4)*l
og((a*x^4 + b)^(1/4) - b^(1/4)) + (a*x^4 + b)^(1/4)
```

giac [B] time = 0.87, size = 183, normalized size = 2.77

$$-\frac{1}{4}\sqrt{2}(-b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}+2\left(ax^4+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)-\frac{1}{4}\sqrt{2}(-b)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}-2\left(ax^4+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)-\frac{1}{8}\sqrt{2}(-b)^{\frac{1}{4}}\log\left(\sqrt{2}\left(ax^4+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^4+b}+\sqrt{-b}\right)+\frac{1}{8}\sqrt{2}(-b)^{\frac{1}{4}}\log\left(-\sqrt{2}\left(ax^4+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^4+b}+\sqrt{-b}\right)+\left(ax^4+b\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(1/4)/x,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(-b)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^4 +
 b)^(1/4))/(-b)^(1/4)) - 1/4*sqrt(2)*(-b)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(-b)^(1/4) - 2*(a*x^4 + b)^(1/4))/(-b)^(1/4)) - 1/8*sqrt(2)*(-b)^(1/4)*lo
g(sqrt(2)*(a*x^4 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^4 + b) + sqrt(-b)) + 1/8*
sqrt(2)*(-b)^(1/4)*log(-sqrt(2)*(a*x^4 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^4 +
 b) + sqrt(-b)) + (a*x^4 + b)^(1/4)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{4}+b\right)^{\frac{1}{4}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b)^(1/4)/x,x)
```

```
[Out] int((a*x^4+b)^(1/4)/x,x)
```

maxima [A] time = 0.41, size = 66, normalized size = 1.00

$$-\frac{1}{2}\,b^{\frac{1}{4}}\arctan\left(\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)+\frac{1}{4}\,b^{\frac{1}{4}}\log\left(\frac{\left(ax^4+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^4+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)+\left(ax^4+b\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(1/4)/x,x, algorithm="maxima")
```

```
[Out] -1/2*b^(1/4)*arctan((a*x^4 + b)^(1/4)/b^(1/4)) + 1/4*b^(1/4)*log(((a*x^4 +
b)^(1/4) - b^(1/4))/((a*x^4 + b)^(1/4) + b^(1/4))) + (a*x^4 + b)^(1/4)
```

mupad [B] time = 0.82, size = 48, normalized size = 0.73

$$\left(a\,x^4+b\right)^{1/4}-\frac{b^{1/4}\,\mathrm{atanh}\left(\frac{{\left(a\,x^4+b\right)}^{1/4}}{b^{1/4}}\right)}{2}-\frac{b^{1/4}\,\mathrm{atan}\left(\frac{{\left(a\,x^4+b\right)}^{1/4}}{b^{1/4}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^4)^(1/4)/x,x)
```

```
[Out] (b + a*x^4)^(1/4) - (b^(1/4)*atanh((b + a*x^4)^(1/4)/b^(1/4)))/2 - (b^(1/4)
*atan((b + a*x^4)^(1/4)/b^(1/4)))/2
```

sympy [C] time = 0.93, size = 42, normalized size = 0.64

$$-\frac{\sqrt[4]{a}\,x\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^4}\right)}{4\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b)**(1/4)/x,x)
```

```
[Out] -a**(1/4)*x*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*exp_polar(I*pi)/(a*x*
*4))/(4*gamma(3/4))
```

3.820 $\int \sqrt{x^3+x^2\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=66

$$\frac{4(2x^2-1)\sqrt{x^2\left(\sqrt{x^2-1}+x\right)}}{15x}-\frac{2}{15}\sqrt{x^2-1}\,\sqrt{x^2\left(\sqrt{x^2-1}+x\right)}$$

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{x^3+x^2\sqrt{-1+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[x^3 + x^2*Sqrt[-1 + x^2]],x]
[Out] Defer[Int][Sqrt[x^3 + x^2*Sqrt[-1 + x^2]], x]
```

Rubi steps

$$\int \sqrt{x^3+x^2\sqrt{-1+x^2}}\,dx=\int \sqrt{x^3+x^2\sqrt{-1+x^2}}\,dx$$

Mathematica [A] time = 0.11, size = 89, normalized size = 1.35

$$\frac{2x\sqrt{x^2-1}\left(6x^4-6x^2-3\sqrt{x^2-1}\,x+6\sqrt{x^2-1}\,x^3+2\right)}{15\sqrt{x^2\left(\sqrt{x^2-1}+x\right)}\left(x^2+\sqrt{x^2-1}\,x-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^3 + x^2*Sqrt[-1 + x^2]],x]
[Out] (2*x*Sqrt[-1 + x^2]*(2 - 6*x^2 + 6*x^4 - 3*x*Sqrt[-1 + x^2] + 6*x^3*Sqrt[-1
 + x^2]))/(15*Sqrt[x^2*(x + Sqrt[-1 + x^2])]*(-1 + x^2 + x*Sqrt[-1 + x^2]))
```

IntegrateAlgebraic [A] time = 3.26, size = 66, normalized size = 1.00

$$\frac{4(2x^2-1)\sqrt{x^2\left(\sqrt{x^2-1}+x\right)}}{15x}-\frac{2}{15}\sqrt{x^2-1}\,\sqrt{x^2\left(\sqrt{x^2-1}+x\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^3 + x^2*Sqrt[-1 + x^2]],x]
[Out] (-2*Sqrt[-1 + x^2]*Sqrt[x^2*(x + Sqrt[-1 + x^2])])/15 + (4*(-1 + 2*x^2)*Sqr
t[x^2*(x + Sqrt[-1 + x^2])])/(15*x)
```

fricas [A] time = 0.70, size = 39, normalized size = 0.59

$$\frac{2\sqrt{x^3+\sqrt{x^2-1}\,x^2}\left(4x^2-\sqrt{x^2-1}\,x-2\right)}{15x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2*(x^2-1)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2/15*sqrt(x^3 + sqrt(x^2 - 1)*x^2)*(4*x^2 - sqrt(x^2 - 1)*x - 2)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^3+\sqrt{x^2-1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2*(x^2-1)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^3 + sqrt(x^2 - 1)*x^2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \sqrt{x^3+x^2\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2*(x^2-1)^(1/2))^(1/2),x)
[Out] int((x^3+x^2*(x^2-1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^3+\sqrt{x^2-1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2*(x^2-1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^3 + sqrt(x^2 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x^2\,\sqrt{x^2-1}+x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^2 - 1)^(1/2) + x^3)^(1/2),x)
[Out] int((x^2*(x^2 - 1)^(1/2) + x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^3+x^2\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2*(x**2-1)**(1/2))**(1/2),x)
[Out] Integral(sqrt(x**3 + x**2*sqrt(x**2 - 1)), x)
```

3.821 $\int \frac{(-a+x)(-b+x)(-ab+x^2)}{x\sqrt{x(-a+x)(-b+x)}\left(ab-(a+b+d)x+x^2\right)}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt{x^2(-a-b)+abx+x^3}}{x}-2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{x^2(-a-b)+abx+x^3}}\right)$$

Rubi [F] time = 7.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-a+x)(-b+x)\left(-ab+x^2\right)}{x\sqrt{x(-a+x)(-b+x)}\left(ab-(a+b+d)x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-a + x)*(-b + x)*(-(a*b) + x^2))/(x*Sqrt[x*(-a + x)*(-b + x)]*(a*b -
(a + b + d)*x + x^2)),x]
```

```
[Out] (-2*Sqrt[(a - x)*(b - x)*x])/x - (4*Sqrt[a]*Sqrt[(a - x)*(b - x)*x]*Sqrt[1
- x/a]*EllipticE[ArcSin[Sqrt[x]/Sqrt[a]], a/b])/((a - x)*Sqrt[x]*Sqrt[1 - x
/b]) - (2*Sqrt[a]*(a - b)*Sqrt[(a - x)*(b - x)*x]*Sqrt[1 - x/a]*Sqrt[1 - x/
b]*EllipticF[ArcSin[Sqrt[x]/Sqrt[a]], a/b])/((a - x)*(b - x)*Sqrt[x]) + ((a
 + b + d + Sqrt[a^2 - 2*a*(b - d) + (b + d)^2])*Sqrt[(a - x)*(b - x)*x]*Def
er[Int][(Sqrt[a - x]*Sqrt[b - x])/(x^(3/2)*(-a - b - d - Sqrt[a^2 - 2*a*b +
 b^2 + 2*a*d + 2*b*d + d^2] + 2*x)), x])/(Sqrt[a - x]*Sqrt[b - x]*Sqrt[x])
+ ((a + b + d - Sqrt[a^2 - 2*a*(b - d) + (b + d)^2])*Sqrt[(a - x)*(b - x)*x
]*Defer[Int][(Sqrt[a - x]*Sqrt[b - x])/(x^(3/2)*(-a - b - d + Sqrt[a^2 - 2*
a*b + b^2 + 2*a*d + 2*b*d + d^2] + 2*x)), x])/(Sqrt[a - x]*Sqrt[b - x]*Sqrt
[x])
```

Rubi steps

$$\int \frac{(-a+x)(-b+x)\left(-ab+x^2\right)}{x\sqrt{x(-a+x)(-b+x)}\left(ab-(a+b+d)x+x^2\right)}dx = \int \frac{\sqrt{(a-x)(b-x)x}\left(-ab+x^2\right)}{x^2\left(ab-(a+b+d)x+x^2\right)}dx$$

$$= \frac{\sqrt{(a-x)(b-x)x}\int \frac{\sqrt{a-x}\sqrt{b-x}\left(-ab+x^2\right)}{x^{3/2}\left(ab-(a+b+d)x+x^2\right)}dx}{\sqrt{a-x}\sqrt{b-x}\sqrt{x}}$$

$$= \frac{\sqrt{(a-x)(b-x)x}\int \left(\frac{\sqrt{a-x}\sqrt{b-x}}{x^{3/2}} - \frac{\sqrt{a-x}\sqrt{b-x}\,(2ab-(a+b+d)x}{x^{3/2}\left(ab+(-a-b-d)x+x^2\right)}\right.}{\sqrt{a-x}\sqrt{b-x}\sqrt{x}}$$

$$= \frac{\sqrt{(a-x)(b-x)x}\int \frac{\sqrt{a-x}\sqrt{b-x}}{x^{3/2}}dx}{\sqrt{a-x}\sqrt{b-x}\sqrt{x}} - \frac{\sqrt{(a-x)(b-x)x}\int}{\sqrt{a-}}$$

$$= -\frac{2\sqrt{(a-x)(b-x)x}}{x} - \frac{\sqrt{(a-x)(b-x)x}\int\left(\frac{\left(-a-b-d-\sqrt{a^2}\right.}{x^{3/2}\left(-a-b-\right.}\right.}{}$$

$$= -\frac{2\sqrt{(a-x)(b-x)x}}{x} - \frac{\left(2\sqrt{(a-x)(b-x)x}\right)\int \frac{\sqrt{b-x}}{\sqrt{a-x}\sqrt{x}}}{\sqrt{a-x}\sqrt{b-x}\sqrt{x}}$$

$$= -\frac{2\sqrt{(a-x)(b-x)x}}{x} - \frac{\left(\left(-a-b-d-\sqrt{a^2-2a(b-d)}\right.\right.}{}$$

$$= -\frac{2\sqrt{(a-x)(b-x)x}}{x} - \frac{4\sqrt{a}\sqrt{(a-x)(b-x)x}\sqrt{1-\frac{x}{a}}\,E}{(a-x)\sqrt{x}\sqrt{1-\frac{x}{b}}}$$

Mathematica [C] time = 8.50, size = 504, normalized size = 7.52

$$\frac{\sqrt{x(x-a)(x-b)}\left(\frac{i\left(-a\left(\sqrt{a^2-2a(b-d)+(b+d)^2}+2b-2d\right)+b\sqrt{a^2-2a(b-d)+(b+d)^2}-d\sqrt{a^2-2a(b-d)+(b+d)^2}+a^2+b^2+2bd+d^2\right)\sqrt{\frac{x-b}{a-b}}\,\Pi\left(\frac{2a}{a-b-d+\sqrt{(-a+b+d)^2+4ad}};i\sinh^{-1}\left(\sqrt{\frac{x}{a}-1}\right)|\frac{a}{a-b}\right)}{\sqrt{1-\frac{a}{x}}(b-x)\sqrt{a^2-2a(b-d)+(b+d)^2}} - \frac{i\left(a\left(\sqrt{a^2-2a(b-d)+(b+d)^2}-2b+2d\right)-b\sqrt{a^2-2a(b-d)+(b+d)^2}+d\sqrt{a^2-2a(b-d)+(b+d)^2}+a^2+b^2+2bd+d^2\right)\sqrt{\frac{x-b}{a-b}}\,\Pi\left(-\frac{2a}{-a+b+d+\sqrt{(-a+b+d)^2+4ad}};i\sinh^{-1}\left(\sqrt{\frac{x}{a}-1}\right)|\frac{a}{a-b}\right)}{\sqrt{1-\frac{a}{x}}(b-x)\sqrt{a^2-2a(b-d)+(b+d)^2}} - \frac{2id\sqrt{\frac{x-b}{a-b}}F\left(i\sinh^{-1}\left(\sqrt{\frac{x}{a}-1}\right)|\frac{a}{a-b}\right)}{\sqrt{1-\frac{a}{x}}(x-b)}+2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-a + x)*(-b + x)*(-(a*b) + x^2))/(x*Sqrt[x*(-a + x)*(-b + x)]*(
a*b - (a + b + d)*x + x^2)),x]
```

```
[Out] (Sqrt[x*(-a + x)*(-b + x)]*(2 - ((2*I)*d*Sqrt[(-b + x)/(a - b)]*EllipticF[I
*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)])/(Sqrt[1 - a/x]*(-b + x)) + (I*(a^2 +
b^2 + 2*b*d + d^2 + b*Sqrt[a^2 - 2*a*(b - d) + (b + d)^2] - d*Sqrt[a^2 - 2*
a*(b - d) + (b + d)^2] - a*(2*b - 2*d + Sqrt[a^2 - 2*a*(b - d) + (b + d)^2]
))*Sqrt[(-b + x)/(a - b)]*EllipticPi[(2*a)/(a - b - d + Sqrt[4*a*d + (-a +
b + d)^2]), I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)])/(Sqrt[a^2 - 2*a*(b - d)
+ (b + d)^2]*Sqrt[1 - a/x]*(b - x)) - (I*(a^2 + b^2 + 2*b*d + d^2 - b*Sqrt[
a^2 - 2*a*(b - d) + (b + d)^2] + d*Sqrt[a^2 - 2*a*(b - d) + (b + d)^2] + a*
(-2*b + 2*d + Sqrt[a^2 - 2*a*(b - d) + (b + d)^2]))*Sqrt[(-b + x)/(a - b)]*
EllipticPi[(-2*a)/(-a + b + d + Sqrt[4*a*d + (-a + b + d)^2]), I*ArcSinh[Sq
rt[-1 + x/a]], a/(a - b)])/(Sqrt[a^2 - 2*a*(b - d) + (b + d)^2]*Sqrt[1 - a/
x]*(b - x))))/x
```

IntegrateAlgebraic [A] time = 0.18, size = 67, normalized size = 1.00

$$\frac{2\sqrt{x^2(-a-b)+abx+x^3}}{x} - 2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{x^2(-a-b)+abx+x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-a + x)*(-b + x)*(-(a*b) + x^2))/(x*Sqrt[x*(-a + x)*(-
b + x)]*(a*b - (a + b + d)*x + x^2)),x]
```

```
[Out] (2*Sqrt[a*b*x + (-a - b)*x^2 + x^3])/x - 2*Sqrt[d]*ArcTanh[(Sqrt[d]*x)/Sqrt
[a*b*x + (-a - b)*x^2 + x^3]]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-a*b+x^2)/x/(x*(-a+x)*(-b+x))^(1/2)/(a*b-(a+b+d)*x
+x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(ab - x^2\right)(a - x)(b - x)}{\sqrt{(a - x)(b - x)x}\left(ab - (a + b + d)x + x^2\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-a*b+x^2)/x/(x*(-a+x)*(-b+x))^(1/2)/(a*b-(a+b+d)*x
+x^2),x, algorithm="giac")
```

```
[Out] integrate(-(a*b - x^2)*(a - x)*(b - x)/(sqrt((a - x)*(b - x)*x)*(a*b - (a +
 b + d)*x + x^2)*x), x)
```

maple [C] time = 0.06, size = 3791, normalized size = 56.58

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x)*(-b+x)*(-a*b+x^2)/x/(x*(-a+x)*(-b+x))^(1/2)/(a*b-(a+b+d)*x+x^2),
x)
```

```
[Out] -2*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^
2+x^3)^(1/2)*((a-b)*EllipticE((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))+b*Elliptic
F((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2)))-2*d*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b)
)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(
1/2),(a/(a-b))^(1/2))+d*(-1/(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)*a^3*(1-1/
a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^
3)^(1/2)/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*Elli
pticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*
d+d^2)^(1/2)),(a/(a-b))^(1/2))+2/(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)*a^2*
(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x
^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))
*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2
+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*b-2/(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/
2)*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*
x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)
^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*
a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*d-a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*
b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b
-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1
/2),a/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b
))^(1/2))-1/(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-
```

```
b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/
2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)
^(1/2),a/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(
a-b))^(1/2))*b^2-2/(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)*a*(1-1/a*x)^(1/2)*
(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1
/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-
a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2
)),(a/(a-b))^(1/2))*b*d-a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a
*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2
*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*
d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*b-1/(a^2-2*a*
b+2*a*d+b^2+2*b*d+d^2)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)
*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*
a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b
-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*d^2-a*(1
-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2
+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*E
llipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d-1/2*(a^2-2*a*b+2*a*d+b^2+2
*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*d+1/(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)
*a^3*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^
2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(
1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*
d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))-2/(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^
(1/2)*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x
-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d
^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b
+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*b+2/(a^2-2*a*b+2*a*d+b^2+2*b*
d+d^2)^(1/2)*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)
/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+
2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d+1/2*(a^
2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*d-a^2*(1-1/a*x)^(1/2)*
(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1
/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-
a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2
)),(a/(a-b))^(1/2))+1/(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)*a*(1-1/a*x)^(1/
2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)
/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((
-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(
1/2)),(a/(a-b))^(1/2))*b^2+2/(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)*a*(1-1/a
*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3
)^(1/2)/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*Ellip
ticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d
+d^2)^(1/2)),(a/(a-b))^(1/2))*b*d-a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^
(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d+1/2*(a
^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-
1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*b+1
/(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b
)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d+1
/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1
/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2)
)*d^2-a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a
*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2*a*d+b^2+2*b*d+d^2
)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(1/2*a-1/2*b-1/2*d+1/2*(a^2-2*a*b+2
*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*d)-a*b*(-2*(a*b-a*x-b*x+x^2)/a/
b/(x*(a*b-a*x-b*x+x^2))^(1/2)-2*((a+b)/a/b+(-a-b)/a/b)*a*(-(-a+x)/a)^(1/2)*
((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)*EllipticF(
(-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))-2/b*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/
2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)*((a-b)*EllipticE((-(-a+x)/a)
^(1/2),(a/(a-b))^(1/2))+b*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ab - x^2)(a - x)(b - x)}{\sqrt{(a - x)(b - x)x}\,(ab - (a + b + d)x + x^2)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-a*b+x^2)/x/(x*(-a+x)*(-b+x))^(1/2)/(a*b-(a+b+d)*x
+x^2),x, algorithm="maxima")
```

```
[Out] -integrate((a*b - x^2)*(a - x)*(b - x)/(sqrt((a - x)*(b - x)*x)*(a*b - (a +
 b + d)*x + x^2)*x), x)
```

mupad [B] time = 0.11, size = 722, normalized size = 10.78

$$\frac{b\,d\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(-\frac{b}{\frac{a}{2}-\frac{b}{2}+\frac{d}{2}-\frac{\sqrt{a^2-2ab+2ad+b^2+2bd+d^2}}{2}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\left(a+b+d-\sqrt{a^2-2ab+2ad+b^2+2bd+d^2}\right)}{\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}\,\left(\frac{a}{2}-\frac{b}{2}+\frac{d}{2}-\frac{\sqrt{a^2-2ab+2ad+b^2+2bd+d^2}}{2}\right)}-\frac{2\,a\,b\left(\frac{\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)-\frac{\sqrt{\frac{b-x}{a-b}+1}\,\sqrt{\frac{b-x}{b}}}{\sqrt{1-\frac{b-x}{b}}}}{\frac{b}{a-b}+1}-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\right)\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}}{\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}-\frac{2\,b\,d\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}}{\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}-\frac{2\,b\left(a\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)-(a-b)\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\right)\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}}{\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}+\frac{b\,d\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(-\frac{b}{\frac{a}{2}-\frac{b}{2}+\frac{d}{2}+\frac{\sqrt{a^2-2ab+2ad+b^2+2bd+d^2}}{2}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\left(a+b+d+\sqrt{a^2-2ab+2ad+b^2+2bd+d^2}\right)}{\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}\,\left(\frac{a}{2}-\frac{b}{2}+\frac{d}{2}+\frac{\sqrt{a^2-2ab+2ad+b^2+2bd+d^2}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a - x)*(b - x)*(a*b - x^2))/(x*(x*(a - x)*(b - x))^(1/2)*(a*b - x*(a
 + b + d) + x^2)),x)
```

```
[Out] (b*d*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*ellipticPi(-b/(a
/2 - b/2 + d/2 - (2*a*d - 2*a*b + 2*b*d + a^2 + b^2 + d^2)^(1/2)/2), asin((
(b - x)/b)^(1/2)), -b/(a - b))*(a + b + d - (2*a*d - 2*a*b + 2*b*d + a^2 +
b^2 + d^2)^(1/2)))/((x^3 - x^2*(a + b) + a*b*x)^(1/2)*(a/2 - b/2 + d/2 - (2
*a*d - 2*a*b + 2*b*d + a^2 + b^2 + d^2)^(1/2)/2)) - (2*a*b*((ellipticE(asin
(((b - x)/b)^(1/2)), -b/(a - b)) - (((b - x)/(a - b) + 1)^(1/2)*((b - x)/b)
^(1/2))/(1 - (b - x)/b)^(1/2))/(b/(a - b) + 1) - ellipticF(asin(((b - x)/b)
^(1/2)), -b/(a - b)))*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2)
)/(x^3 - x^2*(a + b) + a*b*x)^(1/2) - (2*b*d*ellipticF(asin(((b - x)/b)^(1/
2)), -b/(a - b))*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2))/(x^
3 - x^2*(a + b) + a*b*x)^(1/2) - (2*b*(a*ellipticF(asin(((b - x)/b)^(1/2)),
 -b/(a - b)) - (a - b)*ellipticE(asin(((b - x)/b)^(1/2)), -b/(a - b)))*(x/b
)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2))/(x^3 - x^2*(a + b) + a*b
*x)^(1/2) + (b*d*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*elli
pticPi(-b/(a/2 - b/2 + d/2 + (2*a*d - 2*a*b + 2*b*d + a^2 + b^2 + d^2)^(1/2
)/2), asin(((b - x)/b)^(1/2)), -b/(a - b))*(a + b + d + (2*a*d - 2*a*b + 2*
b*d + a^2 + b^2 + d^2)^(1/2)))/((x^3 - x^2*(a + b) + a*b*x)^(1/2)*(a/2 - b/
2 + d/2 + (2*a*d - 2*a*b + 2*b*d + a^2 + b^2 + d^2)^(1/2)/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-a*b+x**2)/x/(x*(-a+x)*(-b+x))**(1/2)/(a*b-(a+b+d)
*x+x**2),x)
```

```
[Out] Timed out
```

3.822 $\int \frac{\sqrt{-1+x^4}}{1+x^4} \, dx$

Optimal. Leaf size=67

$$\frac{1}{4}\tan^{-1}\left(\frac{\frac{x^4}{2}-x^2-\frac{1}{2}}{x\sqrt{x^4-1}}\right)-\frac{1}{4}\tanh^{-1}\left(\frac{\frac{x^4}{2}+x^2-\frac{1}{2}}{x\sqrt{x^4-1}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 0.67, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {405}

$$-\frac{1}{2}\tan^{-1}\left(\frac{x\left(1-x^2\right)}{\sqrt{x^4-1}}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x\left(x^2+1\right)}{\sqrt{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^4]/(1 + x^4),x]
```

```
[Out] -1/2*ArcTan[(x*(1 - x^2))/Sqrt[-1 + x^4]] - ArcTanh[(x*(1 + x^2))/Sqrt[-1 +
 x^4]]/2
```

Rule 405

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[-(a*b), 4]}, Simp[(a*ArcTan[(q*x*(a + q^2*x^2))/(a*Sqrt[a + b*x^4])])/(
2*c*q), x] + Simp[(a*ArcTanh[(q*x*(a - q^2*x^2))/(a*Sqrt[a + b*x^4])])/(2*c
*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && NegQ[a*b]
```

Rubi steps

$$\int \frac{\sqrt{-1+x^4}}{1+x^4} \, dx=-\frac{1}{2}\tan^{-1}\left(\frac{x\left(1-x^2\right)}{\sqrt{-1+x^4}}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x\left(1+x^2\right)}{\sqrt{-1+x^4}}\right)$$

Mathematica [C] time = 0.11, size = 108, normalized size = 1.61

$$-\frac{5x\sqrt{x^4-1}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};x^4,-x^4\right)}{\left(x^4+1\right)\left(2x^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};x^4,-x^4\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};x^4,-x^4\right)\right)-5F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};x^4,-x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-1 + x^4]/(1 + x^4),x]
```

```
[Out] (-5*x*Sqrt[-1 + x^4]*AppellF1[1/4, -1/2, 1, 5/4, x^4, -x^4])/((1 + x^4)*(-5
*AppellF1[1/4, -1/2, 1, 5/4, x^4, -x^4] + 2*x^4*(2*AppellF1[5/4, -1/2, 2, 9
/4, x^4, -x^4] + AppellF1[5/4, 1/2, 1, 9/4, x^4, -x^4])))
```

IntegrateAlgebraic [C] time = 0.19, size = 53, normalized size = 0.79

$$\left(\frac{1}{4}+\frac{i}{4}\right)\tan^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^4-1}}{x}\right)-\left(\frac{1}{4}-\frac{i}{4}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x^4]/(1 + x^4),x]
```

```
[Out] (-1/4 + I/4)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] + (1/4 + I/4)*ArcTan[((1/2
+ I/2)*Sqrt[-1 + x^4])/x]
```

fricas [A] time = 0.47, size = 51, normalized size = 0.76

$$\frac{1}{2}\arctan\left(\frac{\sqrt{x^4-1}\,x}{x^2+1}\right)+\frac{1}{4}\log\left(\frac{x^4+2\,x^2-2\,\sqrt{x^4-1}\,x-1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/2)/(x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(sqrt(x^4 - 1)*x/(x^2 + 1)) + 1/4*log((x^4 + 2*x^2 - 2*sqrt(x^4 -
 1)*x - 1)/(x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^4-1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/2)/(x^4+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 - 1)/(x^4 + 1), x)
```

maple [A] time = 0.03, size = 88, normalized size = 1.31

$$\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}+1\right)}{4}-\frac{\arctan\left(-\frac{\sqrt{x^4-1}}{x}+1\right)}{4}+\frac{\ln\left(\frac{\frac{x^4-1}{2x^2}-\frac{\sqrt{x^4-1}}{x}+1}{\frac{x^4-1}{2x^2}+\frac{\sqrt{x^4-1}}{x}+1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(1/2)/(x^4+1),x)
```

```
[Out] 1/4*arctan((x^4-1)^(1/2)/x+1)-1/4*arctan(-(x^4-1)^(1/2)/x+1)+1/8*ln((1/2*(x
^4-1)/x^2-(x^4-1)^(1/2)/x+1)/(1/2*(x^4-1)/x^2+(x^4-1)^(1/2)/x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^4-1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/2)/(x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 - 1)/(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{x^4-1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)^(1/2)/(x^4 + 1),x)
```

```
[Out] int((x^4 - 1)^(1/2)/(x^4 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)\left(x^2+1\right)}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(1/2)/(x**4+1),x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1)*(x**2 + 1))/(x**4 + 1), x)
```

3.823 $\int x^2 \sqrt[4]{-x^2+x^4}\,dx$

Optimal. Leaf size=67

$$\frac{3}{32}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)-\frac{3}{32}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)+\frac{1}{16}\sqrt[4]{x^4-x^2}\left(4x^3-x\right)$$

Rubi [A] time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.99, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {2021, 2024, 2032, 329, 331, 298, 203, 206}

$$-\frac{1}{16}\sqrt[4]{x^4-x^2}\,x+\frac{3\left(x^2-1\right)^{3/4}x^{3/2}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{32\left(x^4-x^2\right)^{3/4}}-\frac{3\left(x^2-1\right)^{3/4}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{32\left(x^4-x^2\right)^{3/4}}+\frac{1}{4}\sqrt[4]{x^4-x^2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(-x^2 + x^4)^(1/4),x]
```

```
[Out] -1/16*(x*(-x^2 + x^4)^(1/4)) + (x^3*(-x^2 + x^4)^(1/4))/4 + (3*x^(3/2)*(-1
+ x^2)^(3/4)*ArcTan[Sqrt[x]/(-1 + x^2)^(1/4)])/(32*(-x^2 + x^4)^(3/4)) - (3
*x^(3/2)*(-1 + x^2)^(3/4)*ArcTanh[Sqrt[x]/(-1 + x^2)^(1/4)])/(32*(-x^2 + x^
4)^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
```

```
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^2\sqrt[4]{-x^2+x^4}\,dx = \frac{1}{4}x^3\sqrt[4]{-x^2+x^4}-\frac{1}{8}\int\frac{x^4}{\left(-x^2+x^4\right)^{3/4}}\,dx$$
$$= -\frac{1}{16}x\sqrt[4]{-x^2+x^4}+\frac{1}{4}x^3\sqrt[4]{-x^2+x^4}-\frac{3}{32}\int\frac{x^2}{\left(-x^2+x^4\right)^{3/4}}\,dx$$
$$= -\frac{1}{16}x\sqrt[4]{-x^2+x^4}+\frac{1}{4}x^3\sqrt[4]{-x^2+x^4}-\frac{\left(3x^{3/2}\left(-1+x^2\right)^{3/4}\right)\int\frac{\sqrt{x}}{\left(-1+x^2\right)^{3/4}}\,dx}{32\left(-x^2+x^4\right)^{3/4}}$$
$$= -\frac{1}{16}x\sqrt[4]{-x^2+x^4}+\frac{1}{4}x^3\sqrt[4]{-x^2+x^4}-\frac{\left(3x^{3/2}\left(-1+x^2\right)^{3/4}\right)\text{Subst}\left(\int\frac{x^2}{\left(-1+x^4\right)^{3/4}}\,dx,x,\sqrt{x}\right)}{16\left(-x^2+x^4\right)^{3/4}}$$
$$= -\frac{1}{16}x\sqrt[4]{-x^2+x^4}+\frac{1}{4}x^3\sqrt[4]{-x^2+x^4}-\frac{\left(3x^{3/2}\left(-1+x^2\right)^{3/4}\right)\text{Subst}\left(\int\frac{x^2}{1-x^4}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{16\left(-x^2+x^4\right)^{3/4}}$$
$$= -\frac{1}{16}x\sqrt[4]{-x^2+x^4}+\frac{1}{4}x^3\sqrt[4]{-x^2+x^4}-\frac{\left(3x^{3/2}\left(-1+x^2\right)^{3/4}\right)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{32\left(-x^2+x^4\right)^{3/4}}$$
$$= -\frac{1}{16}x\sqrt[4]{-x^2+x^4}+\frac{1}{4}x^3\sqrt[4]{-x^2+x^4}+\frac{3x^{3/2}\left(-1+x^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{32\left(-x^2+x^4\right)^{3/4}}-\frac{3x^{3/2}\left(-1+x^2\right.}{32\left(\right.}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.84

$$\frac{x\sqrt[4]{x^2\left(x^2-1\right)}\left({}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};x^2\right)-\left(1-x^2\right)^{5/4}\right)}{4\sqrt[4]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(-x^2 + x^4)^(1/4),x]
```

```
[Out] (x*(x^2*(-1 + x^2))^(1/4)*(-(1 - x^2)^(5/4) + Hypergeometric2F1[-1/4, 3/4, 
7/4, x^2]))/(4*(1 - x^2)^(1/4))
```

IntegrateAlgebraic [A] time = 0.18, size = 67, normalized size = 1.00

$$\frac{3}{32}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)-\frac{3}{32}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)+\frac{1}{16}\sqrt[4]{x^4-x^2}\left(4x^3-x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(-x^2 + x^4)^(1/4),x]
```

```
[Out] ((-x + 4*x^3)*(-x^2 + x^4)^(1/4))/16 + (3*ArcTan[x/(-x^2 + x^4)^(1/4)])/32 
- (3*ArcTanh[x/(-x^2 + x^4)^(1/4)])/32
```

fricas [B] time = 1.24, size = 118, normalized size = 1.76

$$\frac{1}{16}\left(x^4-x^2\right)^{\frac{1}{4}}\left(4x^3-x\right)-\frac{3}{64}\arctan\left(\frac{2\left(\left(x^4-x^2\right)^{\frac{1}{4}}x^2+\left(x^4-x^2\right)^{\frac{3}{4}}\right)}{x}\right)+\frac{3}{64}\log\left(-\frac{2x^3-2\left(x^4-x^2\right)^{\frac{1}{4}}x^2+2\sqrt{x^4-x^2}\,x-x-2\left(x^4-x^2\right)^{\frac{3}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4-x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/16*(x^4 - x^2)^(1/4)*(4*x^3 - x) - 3/64*arctan(2*((x^4 - x^2)^(1/4)*x^2 + 
 (x^4 - x^2)^(3/4))/x) + 3/64*log(-(2*x^3 - 2*(x^4 - x^2)^(1/4)*x^2 + 2*sqr
t(x^4 - x^2)*x - x - 2*(x^4 - x^2)^(3/4))/x)
```

giac [A] time = 0.20, size = 69, normalized size = 1.03

$$-\frac{1}{16}\left(\left(-\frac{1}{x^2}+1\right)^{\frac{5}{4}}+3\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)x^4+\frac{3}{32}\arctan\left(\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)+\frac{3}{64}\log\left(\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}+1\right)-\frac{3}{64}\log\left(-\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4-x^2)^(1/4),x, algorithm="giac")
```

```
[Out] -1/16*((-1/x^2 + 1)^(5/4) + 3*(-1/x^2 + 1)^(1/4))*x^4 + 3/32*arctan((-1/x^2
 + 1)^(1/4)) + 3/64*log((-1/x^2 + 1)^(1/4) + 1) - 3/64*log(-(-1/x^2 + 1)^(1
/4) + 1)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^2\left(x^4-x^2\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4-x^2)^(1/4),x)
```

```
[Out] int(x^2*(x^4-x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^4-x^2\right)^{\frac{1}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4-x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^2)^(1/4)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(x^4 - x^2\right)^{1/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4 - x^2)^(1/4),x)
[Out] int(x^2*(x^4 - x^2)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt[4]{x^2 (x-1)(x+1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**4-x**2)**(1/4),x)
[Out] Integral(x**2*(x**2*(x - 1)*(x + 1))**(1/4), x)
```

3.824 $\int \frac{x}{\sqrt{1+4x+3x^2-2x^3+x^4}}\,dx$

Optimal. Leaf size=67

$$\frac{1}{3}\tanh^{-1}\left(\frac{(x+1)\sqrt{x^4-2x^3+3x^2+4x+1}}{x^3}\right)+\tanh^{-1}\left(\frac{\sqrt{x^4-2x^3+3x^2+4x+1}-2x-1}{x^2}\right)$$

Rubi [F] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\sqrt{1+4x+3x^2-2x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/Sqrt[1 + 4*x + 3*x^2 - 2*x^3 + x^4],x]
```

```
[Out] Defer[Int][x/Sqrt[1 + 4*x + 3*x^2 - 2*x^3 + x^4], x]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+4x+3x^2-2x^3+x^4}}\,dx=\int \frac{x}{\sqrt{1+4x+3x^2-2x^3+x^4}}\,dx$$

Mathematica [C] time = 2.25, size = 1105, normalized size = 16.49

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[1 + 4*x + 3*x^2 - 2*x^3 + x^4],x]
```

```
[Out] ((-1)^(1/4)*(1 + 2*(-3)^(1/4) + I*Sqrt[3] - 2*x)*(-I + (2*I)*(-3)^(1/4) + S
qrt[3] + (2*I)*x)*(-2*EllipticF[ArcSin[Sqrt[-(((-1)^(3/4)*(-1 + 2*(-3)^(1/4
) - I*Sqrt[3] + 2*x)*(1 + 2*(-3)^(1/4) + I*Sqrt[3] - 2*Root[1 + 4*#1 + 3*#1
^2 - 2*#1^3 + #1^4 & , 2, 0]))/(x - Root[1 + 4*#1 + 3*#1^2 - 2*#1^3 + #1^4
& , 2, 0]))]/(2*Sqrt[2]*3^(1/8))], (-8*(-3)^(1/4)*(Root[1 + 4*#1 + 3*#1^2 -
 2*#1^3 + #1^4 & , 2, 0] - Root[1 + 4*#1 + 3*#1^2 - 2*#1^3 + #1^4 & , 3, 0]
))/((1 + 2*(-3)^(1/4) + I*Sqrt[3] - 2*Root[1 + 4*#1 + 3*#1^2 - 2*#1^3 + #1^
4 & , 2, 0])*(-1 + 2*(-3)^(1/4) - I*Sqrt[3] + 2*Root[1 + 4*#1 + 3*#1^2 - 2*
#1^3 + #1^4 & , 3, 0]))]*Root[1 + 4*#1 + 3*#1^2 - 2*#1^3 + #1^4 & , 2, 0] +
 EllipticPi[(4*(-3)^(1/4))/(1 + 2*(-3)^(1/4) + I*Sqrt[3] - 2*Root[1 + 4*#1
+ 3*#1^2 - 2*#1^3 + #1^4 & , 2, 0]), ArcSin[Sqrt[-(((-1)^(3/4)*(-1 + 2*(-3)
^(1/4) - I*Sqrt[3] + 2*x)*(1 + 2*(-3)^(1/4) + I*Sqrt[3] - 2*Root[1 + 4*#1 +
 3*#1^2 - 2*#1^3 + #1^4 & , 2, 0]))/(x - Root[1 + 4*#1 + 3*#1^2 - 2*#1^3 +
#1^4 & , 2, 0]))]/(2*Sqrt[2]*3^(1/8))], (-8*(-3)^(1/4)*(Root[1 + 4*#1 + 3*#
1^2 - 2*#1^3 + #1^4 & , 2, 0] - Root[1 + 4*#1 + 3*#1^2 - 2*#1^3 + #1^4 & ,
3, 0]))/((1 + 2*(-3)^(1/4) + I*Sqrt[3] - 2*Root[1 + 4*#1 + 3*#1^2 - 2*#1^3
+ #1^4 & , 2, 0])*(-1 + 2*(-3)^(1/4) - I*Sqrt[3] + 2*Root[1 + 4*#1 + 3*#1^2
 - 2*#1^3 + #1^4 & , 3, 0]))]*(-1 + 2*(-3)^(1/4) - I*Sqrt[3] + 2*Root[1 + 4
*#1 + 3*#1^2 - 2*#1^3 + #1^4 & , 2, 0]))*Sqrt[((-1 + 2*(-3)^(1/4) - I*Sqrt[
3] + 2*Root[1 + 4*#1 + 3*#1^2 - 2*#1^3 + #1^4 & , 2, 0])*(x - Root[1 + 4*#1
 + 3*#1^2 - 2*#1^3 + #1^4 & , 3, 0]))/((x - Root[1 + 4*#1 + 3*#1^2 - 2*#1^3
 + #1^4 & , 2, 0])*(-1 + 2*(-3)^(1/4) - I*Sqrt[3] + 2*Root[1 + 4*#1 + 3*#1^
2 - 2*#1^3 + #1^4 & , 3, 0]))]])/(Sqrt[1 + 4*x + 3*x^2 - 2*x^3 + x^4]*Sqrt[(
(1 + 2*(-3)^(1/4) + I*Sqrt[3] - 2*x)*(-I + (2*I)*(-3)^(1/4) + Sqrt[3] + (2*
I)*x)*(1 + 2*(-3)^(1/4) + I*Sqrt[3] - 2*Root[1 + 4*#1 + 3*#1^2 - 2*#1^3 + #
```

```
1^4 & , 2, 0])*(-1 + 2*(-3)^(1/4) - I*Sqrt[3] + 2*Root[1 + 4*#1 + 3*#1^2 -
2*#1^3 + #1^4 & , 2, 0]))/(x - Root[1 + 4*#1 + 3*#1^2 - 2*#1^3 + #1^4 & , 2
, 0])^2])
```

IntegrateAlgebraic [A] time = 13.31, size = 67, normalized size = 1.00

$$\frac{1}{3}\tanh^{-1}\left(\frac{(x+1)\sqrt{x^4-2x^3+3x^2+4x+1}}{x^3}\right)+\tanh^{-1}\left(\frac{\sqrt{x^4-2x^3+3x^2+4x+1}-2x-1}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[1 + 4*x + 3*x^2 - 2*x^3 + x^4],x]
```

```
[Out] ArcTanh[((1 + x)*Sqrt[1 + 4*x + 3*x^2 - 2*x^3 + x^4])/x^3]/3 + ArcTanh[(-1
- 2*x + Sqrt[1 + 4*x + 3*x^2 - 2*x^3 + x^4])/x^2]
```

fricas [A] time = 0.46, size = 70, normalized size = 1.04

$$\frac{1}{6}\log\left(2x^6-12x^5+36x^4-56x^3+42x^2+2\sqrt{x^4-2x^3+3x^2+4x+1}\left(x^4-5x^3+12x^2-14x+7\right)-13\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-2*x^3+3*x^2+4*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*log(2*x^6 - 12*x^5 + 36*x^4 - 56*x^3 + 42*x^2 + 2*sqrt(x^4 - 2*x^3 + 3*
x^2 + 4*x + 1)*(x^4 - 5*x^3 + 12*x^2 - 14*x + 7) - 13)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{x^4-2x^3+3x^2+4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-2*x^3+3*x^2+4*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(x^4 - 2*x^3 + 3*x^2 + 4*x + 1), x)
```

maple [C] time = 0.97, size = 1609, normalized size = 24.01

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4-2*x^3+3*x^2+4*x+1)^(1/2),x)
```

```
[Out] 2*(-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=4)+RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_
Z+1,index=1))*((RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=4)-RootOf(_Z^4-2*_Z^
3+3*_Z^2+4*_Z+1,index=2))*(x-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=1))/(Ro
otOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=4)-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,in
dex=1))/(x-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=2)))^(1/2)*(x-RootOf(_Z^4
-2*_Z^3+3*_Z^2+4*_Z+1,index=2))^2*((RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=
2)-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=1))*(x-RootOf(_Z^4-2*_Z^3+3*_Z^2+
4*_Z+1,index=3))/(RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=3)-RootOf(_Z^4-2*_
Z^3+3*_Z^2+4*_Z+1,index=1))/(x-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=2)))^
(1/2)*((RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=2)-RootOf(_Z^4-2*_Z^3+3*_Z^2
+4*_Z+1,index=1))*(x-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=4))/(RootOf(_Z^
4-2*_Z^3+3*_Z^2+4*_Z+1,index=4)-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=1))/
(x-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=2)))^(1/2)/(RootOf(_Z^4-2*_Z^3+3*
_Z^2+4*_Z+1,index=4)-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=2))/(RootOf(_Z^
4-2*_Z^3+3*_Z^2+4*_Z+1,index=2)-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=1))/
((x-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=1))*(x-RootOf(_Z^4-2*_Z^3+3*_Z^2
```

```
+4*_Z+1,index=2))*(x-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=3))*(x-RootOf(_
Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=4)))^(1/2)*(RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1
,index=2)*EllipticF(((RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=4)-RootOf(_Z^4
-2*_Z^3+3*_Z^2+4*_Z+1,index=2))*(x-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=1
))/(RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=4)-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_
Z+1,index=1))/(x-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=2)))^(1/2),((RootOf
(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=2)-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=
3))*(-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=4)+RootOf(_Z^4-2*_Z^3+3*_Z^2+4
*_Z+1,index=1))/(-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=3)+RootOf(_Z^4-2*_
Z^3+3*_Z^2+4*_Z+1,index=1))/(RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=2)-Root
Of(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=4)))^(1/2))+(-RootOf(_Z^4-2*_Z^3+3*_Z^2+
4*_Z+1,index=2)+RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=1))*EllipticPi(((Roo
tOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=4)-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,ind
ex=2))*(x-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=1))/(RootOf(_Z^4-2*_Z^3+3*
_Z^2+4*_Z+1,index=4)-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=1))/(x-RootOf(_
Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=2)))^(1/2),(RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1
,index=4)-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=1))/(RootOf(_Z^4-2*_Z^3+3*
_Z^2+4*_Z+1,index=4)-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=2)),((RootOf(_Z
^4-2*_Z^3+3*_Z^2+4*_Z+1,index=2)-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=3))
*(-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=4)+RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z
+1,index=1))/(-RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=3)+RootOf(_Z^4-2*_Z^3
+3*_Z^2+4*_Z+1,index=1))/(RootOf(_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=2)-RootOf(
_Z^4-2*_Z^3+3*_Z^2+4*_Z+1,index=4)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^4 - 2\,x^3 + 3\,x^2 + 4\,x + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-2*x^3+3*x^2+4*x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(x^4 - 2*x^3 + 3*x^2 + 4*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{x^4 - 2\,x^3 + 3\,x^2 + 4\,x + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x + 3*x^2 - 2*x^3 + x^4 + 1)^(1/2),x)
[Out] int(x/(4*x + 3*x^2 - 2*x^3 + x^4 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^4 - 2x^3 + 3x^2 + 4x + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4-2*x**3+3*x**2+4*x+1)**(1/2),x)
[Out] Integral(x/sqrt(x**4 - 2*x**3 + 3*x**2 + 4*x + 1), x)
```

3.825 $\int \frac{x}{\sqrt{1-4x+3x^2+2x^3+x^4}}\,dx$

Optimal. Leaf size=67

$$\frac{1}{3}\tanh^{-1}\left(\frac{(x-1)\sqrt{x^4+2x^3+3x^2-4x+1}}{x^3}\right)+\tanh^{-1}\left(\frac{\sqrt{x^4+2x^3+3x^2-4x+1}+2x-1}{x^2}\right)$$

Rubi [F] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\sqrt{1-4x+3x^2+2x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/Sqrt[1 - 4*x + 3*x^2 + 2*x^3 + x^4],x]
```

```
[Out] Defer[Int][x/Sqrt[1 - 4*x + 3*x^2 + 2*x^3 + x^4], x]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-4x+3x^2+2x^3+x^4}}\,dx=\int \frac{x}{\sqrt{1-4x+3x^2+2x^3+x^4}}\,dx$$

Mathematica [C] time = 1.97, size = 1189, normalized size = 17.75

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[1 - 4*x + 3*x^2 + 2*x^3 + x^4],x]
```

```
[Out] ((-1)^(1/4)*(x - Root[1 - 4*#1 + 3*#1^2 + 2*#1^3 + #1^4 & , 2, 0])^2*Sqrt[(
(-1 + 2*(-3)^(1/4) - I*Sqrt[3] - 2*x)*(1 + 2*(-3)^(1/4) + I*Sqrt[3] + 2*x)*
(-I + (2*I)*(-3)^(1/4) + Sqrt[3] - (2*I)*Root[1 - 4*#1 + 3*#1^2 + 2*#1^3 +
#1^4 & , 2, 0])*(1 + 2*(-3)^(1/4) + I*Sqrt[3] + 2*Root[1 - 4*#1 + 3*#1^2 +
2*#1^3 + #1^4 & , 2, 0]))/(x - Root[1 - 4*#1 + 3*#1^2 + 2*#1^3 + #1^4 & , 2
, 0])^2]*(-2*EllipticF[ArcSin[Sqrt[((-1)^(3/4)*(1/2 + (-3)^(1/4) + (I/2)*Sq
rt[3] + x)*(1/2 - (-3)^(1/4) + (I/2)*Sqrt[3] + Root[1 - 4*#1 + 3*#1^2 + 2*#
1^3 + #1^4 & , 2, 0]))/(x - Root[1 - 4*#1 + 3*#1^2 + 2*#1^3 + #1^4 & , 2, 0
])]/(Sqrt[2]*3^(1/8))], (2*(-3)^(1/4)*(Root[1 - 4*#1 + 3*#1^2 + 2*#1^3 + #1
^4 & , 2, 0] - Root[1 - 4*#1 + 3*#1^2 + 2*#1^3 + #1^4 & , 3, 0]))/((1/2 - (
-3)^(1/4) + (I/2)*Sqrt[3] + Root[1 - 4*#1 + 3*#1^2 + 2*#1^3 + #1^4 & , 2, 0
])*(1/2 + (-3)^(1/4) + (I/2)*Sqrt[3] + Root[1 - 4*#1 + 3*#1^2 + 2*#1^3 + #1
^4 & , 3, 0]))]*Root[1 - 4*#1 + 3*#1^2 + 2*#1^3 + #1^4 & , 2, 0] + Elliptic
Pi[(4*(-3)^(1/4))/(-1 + 2*(-3)^(1/4) - I*Sqrt[3] - 2*Root[1 - 4*#1 + 3*#1^2
 + 2*#1^3 + #1^4 & , 2, 0]), ArcSin[Sqrt[((-1)^(3/4)*(1/2 + (-3)^(1/4) + (I
/2)*Sqrt[3] + x)*(1/2 - (-3)^(1/4) + (I/2)*Sqrt[3] + Root[1 - 4*#1 + 3*#1^2
 + 2*#1^3 + #1^4 & , 2, 0]))/(x - Root[1 - 4*#1 + 3*#1^2 + 2*#1^3 + #1^4 &
, 2, 0])]/(Sqrt[2]*3^(1/8))], (2*(-3)^(1/4)*(Root[1 - 4*#1 + 3*#1^2 + 2*#1^
3 + #1^4 & , 2, 0] - Root[1 - 4*#1 + 3*#1^2 + 2*#1^3 + #1^4 & , 3, 0]))/((1
/2 - (-3)^(1/4) + (I/2)*Sqrt[3] + Root[1 - 4*#1 + 3*#1^2 + 2*#1^3 + #1^4 &
, 2, 0])*(1/2 + (-3)^(1/4) + (I/2)*Sqrt[3] + Root[1 - 4*#1 + 3*#1^2 + 2*#1^
3 + #1^4 & , 3, 0]))]*(1 + 2*(-3)^(1/4) + I*Sqrt[3] + 2*Root[1 - 4*#1 + 3*#
1^2 + 2*#1^3 + #1^4 & , 2, 0]))*Sqrt[((1 + 2*(-3)^(1/4) + I*Sqrt[3] + 2*Roo
t[1 - 4*#1 + 3*#1^2 + 2*#1^3 + #1^4 & , 2, 0])*(x - Root[1 - 4*#1 + 3*#1^2
+ 2*#1^3 + #1^4 & , 3, 0]))/((x - Root[1 - 4*#1 + 3*#1^2 + 2*#1^3 + #1^4 &
```

```
, 2, 0])*(1 + 2*(-3)^(1/4) + I*Sqrt[3] + 2*Root[1 - 4*#1 + 3*#1^2 + 2*#1^3
+ #1^4 & , 3, 0]))])/(Sqrt[1 - 4*x + 3*x^2 + 2*x^3 + x^4]*(-1 + 2*(-3)^(1/4
) - I*Sqrt[3] - 2*Root[1 - 4*#1 + 3*#1^2 + 2*#1^3 + #1^4 & , 2, 0])*(1 + 2*
(-3)^(1/4) + I*Sqrt[3] + 2*Root[1 - 4*#1 + 3*#1^2 + 2*#1^3 + #1^4 & , 2, 0]
))
```

IntegrateAlgebraic [A] time = 13.19, size = 67, normalized size = 1.00

$$\frac{1}{3}\tanh^{-1}\left(\frac{(x-1)\sqrt{x^4+2x^3+3x^2-4x+1}}{x^3}\right)+\tanh^{-1}\left(\frac{\sqrt{x^4+2x^3+3x^2-4x+1}+2x-1}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[1 - 4*x + 3*x^2 + 2*x^3 + x^4],x]
```

```
[Out] ArcTanh[((-1 + x)*Sqrt[1 - 4*x + 3*x^2 + 2*x^3 + x^4])/x^3]/3 + ArcTanh[(-1
 + 2*x + Sqrt[1 - 4*x + 3*x^2 + 2*x^3 + x^4])/x^2]
```

fricas [A] time = 0.46, size = 70, normalized size = 1.04

$$\frac{1}{6}\log\left(2x^6+12x^5+36x^4+56x^3+42x^2+2\left(x^4+5x^3+12x^2+14x+7\right)\sqrt{x^4+2x^3+3x^2-4x+1}-13\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^3+3*x^2-4*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*log(2*x^6 + 12*x^5 + 36*x^4 + 56*x^3 + 42*x^2 + 2*(x^4 + 5*x^3 + 12*x^2
 + 14*x + 7)*sqrt(x^4 + 2*x^3 + 3*x^2 - 4*x + 1) - 13)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^4+2x^3+3x^2-4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^3+3*x^2-4*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(x^4 + 2*x^3 + 3*x^2 - 4*x + 1), x)
```

maple [C] time = 1.05, size = 1609, normalized size = 24.01

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+2*x^3+3*x^2-4*x+1)^(1/2),x)
```

```
[Out] 2*(RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=1)-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z
+1,index=4))*((RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3
+3*_Z^2-4*_Z+1,index=2))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=1))/(Roo
tOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,ind
ex=1))/(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=2)))^(1/2)*(x-RootOf(_Z^4+
2*_Z^3+3*_Z^2-4*_Z+1,index=2))^2*((RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=2
)-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=1))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-4
*_Z+1,index=3))/(RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=3)-RootOf(_Z^4+2*_Z
^3+3*_Z^2-4*_Z+1,index=1))/(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=2)))^(
1/2)*((RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=2)-RootOf(_Z^4+2*_Z^3+3*_Z^2-
4*_Z+1,index=1))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=4))/(RootOf(_Z^4
+2*_Z^3+3*_Z^2-4*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=1))/(
x-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=2)))^(1/2)/(RootOf(_Z^4+2*_Z^3+3*_
Z^2-4*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=2))/(RootOf(_Z^4
```

```
+2*_Z^3+3*_Z^2-4*_Z+1,index=2)-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=1))/(
(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=1))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-
4*_Z+1,index=2))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=3))*(x-RootOf(_Z
^4+2*_Z^3+3*_Z^2-4*_Z+1,index=4)))^(1/2)*(RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,
index=2)*EllipticF(((RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=4)-RootOf(_Z^4+
2*_Z^3+3*_Z^2-4*_Z+1,index=2))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=1)
)/(RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z
+1,index=1))/(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=2)))^(1/2),((RootOf(
_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=2)-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=3
))*(RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=1)-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_
Z+1,index=4))/(-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=3)+RootOf(_Z^4+2*_Z^
3+3*_Z^2-4*_Z+1,index=1))/(RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=2)-RootOf
(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=4)))^(1/2))+(RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_
Z+1,index=1)-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=2))*EllipticPi(((RootOf
(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=
2))*(x-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=1))/(RootOf(_Z^4+2*_Z^3+3*_Z^
2-4*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=1))/(x-RootOf(_Z^4
+2*_Z^3+3*_Z^2-4*_Z+1,index=2)))^(1/2),(RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,in
dex=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=1))/(RootOf(_Z^4+2*_Z^3+3*_Z^
2-4*_Z+1,index=4)-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=2)),((RootOf(_Z^4+
2*_Z^3+3*_Z^2-4*_Z+1,index=2)-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=3))*(R
ootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=1)-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,i
ndex=4))/(-RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=3)+RootOf(_Z^4+2*_Z^3+3*_
Z^2-4*_Z+1,index=1))/(RootOf(_Z^4+2*_Z^3+3*_Z^2-4*_Z+1,index=2)-RootOf(_Z^4
+2*_Z^3+3*_Z^2-4*_Z+1,index=4)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^4 + 2\,x^3 + 3\,x^2 - 4\,x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^3+3*x^2-4*x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(x^4 + 2*x^3 + 3*x^2 - 4*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{x^4 + 2\,x^3 + 3\,x^2 - 4\,x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^2 - 4*x + 2*x^3 + x^4 + 1)^(1/2),x)
[Out] int(x/(3*x^2 - 4*x + 2*x^3 + x^4 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^4 + 2x^3 + 3x^2 - 4x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+2*x**3+3*x**2-4*x+1)**(1/2),x)
[Out] Integral(x/sqrt(x**4 + 2*x**3 + 3*x**2 - 4*x + 1), x)
```

3.826 $\int \frac{3-3x^2+2x^4}{\sqrt[4]{-1+x^2}\left(2-3x^2+x^4\right)}\,dx$

Optimal. Leaf size=67

$$\frac{4x}{\sqrt[4]{x^2-1}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^2-1}}{x}\right)}{2\sqrt{2}}-\frac{5\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}$$

Rubi [A] time = 0.39, antiderivative size = 65, normalized size of antiderivative = 0.97, number of steps used = 20, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1688, 6725, 199, 230, 305, 220, 1196, 288, 403, 398}

$$\frac{4x}{\sqrt[4]{x^2-1}}-\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}-\frac{5\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 3*x^2 + 2*x^4)/((-1 + x^2)^(1/4)*(2 - 3*x^2 + x^4)),x]
```

```
[Out] (4*x)/(-1 + x^2)^(1/4) - (5*ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2
]) - (5*ArcTanh[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Free
Q[{a, b}, x] && NegQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rule 403

```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/(
b*c - a*d), Int[(a + b*x^2)^p, x], x] - Dist[d/(b*c - a*d), Int[(a + b*x^2)
^(p + 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && LtQ[p, -1] && EqQ[Denominator[p], 4] && (EqQ[p, -5/4] || EqQ[p, -7/4])
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1688

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[Px*(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; F
reeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a
*e^2, 0] && IntegerQ[p] && (PolyQ[Px, x^2] || MatchQ[Px, ((f_) + (g_.)*x^2)
^(r_.) /; FreeQ[{f, g, r}, x]])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{3-3x^2+2x^4}{\sqrt[4]{-1+x^2}\left(2-3x^2+x^4\right)}\,dx &= \int \frac{3-3x^2+2x^4}{\left(-2+x^2\right)\left(-1+x^2\right)^{5/4}}\,dx \\
&= \int \left(\frac{1}{\left(-1+x^2\right)^{5/4}} + \frac{2x^2}{\left(-1+x^2\right)^{5/4}} + \frac{5}{\left(-2+x^2\right)\left(-1+x^2\right)^{5/4}}\right)dx \\
&= 2\int \frac{x^2}{\left(-1+x^2\right)^{5/4}}\,dx + 5\int \frac{1}{\left(-2+x^2\right)\left(-1+x^2\right)^{5/4}}\,dx + \int \frac{1}{\left(-1+x^2\right)^{5/4}}\,dx \\
&= -\frac{6x}{\sqrt[4]{-1+x^2}} + 4\int \frac{1}{\sqrt[4]{-1+x^2}}\,dx - 5\int \frac{1}{\left(-1+x^2\right)^{5/4}}\,dx + 5\int \frac{1}{\left(-2+x^2\right)\sqrt[4]{-1} \cdots} \\
&= \frac{4x}{\sqrt[4]{-1+x^2}} - \frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} - \frac{5\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} - 5\int \frac{1}{\sqrt[4]{-1+x^2}}\,dx \cdots \\
&= \frac{4x}{\sqrt[4]{-1+x^2}} - \frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} - \frac{5\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{\left(2\sqrt{x^2}\right)\text{Subst}\left(\int \cdots\right.}{\cdots} \\
&= \frac{4x}{\sqrt[4]{-1+x^2}} + \frac{10x\sqrt[4]{-1+x^2}}{1+\sqrt{-1+x^2}} - \frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} - \frac{5\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} - \cdots \\
&= \frac{4x}{\sqrt[4]{-1+x^2}} - \frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} - \frac{5\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.25, size = 119, normalized size = 1.78

$$
\frac{2x\left(\frac{15F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}{\left(x^2-2\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};x^2,\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};x^2,\frac{x^2}{2}\right)\right)+6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)\right)}+2\right)}{\sqrt[4]{x^2-1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - 3*x^2 + 2*x^4)/((-1 + x^2)^(1/4)*(2 - 3*x^2 + x^4)),x]
```

```
[Out] (2*x*(2 + (15*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2])/((-2 + x^2)*(6*Appell
F1[1/2, 1/4, 1, 3/2, x^2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, x^2, x
^2/2] + AppellF1[3/2, 5/4, 1, 5/2, x^2, x^2/2])))))/(-1 + x^2)^(1/4)
```

IntegrateAlgebraic [A] time = 0.39, size = 67, normalized size = 1.00

$$
\frac{4x}{\sqrt[4]{x^2-1}} + \frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^2-1}}{x}\right)}{2\sqrt{2}} - \frac{5\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - 3*x^2 + 2*x^4)/((-1 + x^2)^(1/4)*(2 - 3*x^2 + x^4))
,x]
```

```
[Out] (4*x)/(-1 + x^2)^(1/4) + (5*ArcTan[(Sqrt[2]*(-1 + x^2)^(1/4))/x])/(2*Sqrt[2
]) - (5*ArcTanh[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2])
```

fricas [B] time = 0.41, size = 120, normalized size = 1.79

$$\frac{10\sqrt{2}\left(x^2-1\right)\arctan\left(\frac{\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}}{x}\right)+5\sqrt{2}\left(x^2-1\right)\log\left(-\frac{x^4-2\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}x^3+4\sqrt{x^2-1}\,x^2-4\sqrt{2}\left(x^2-1\right)^{\frac{3}{4}}x+4x^2-4}{x^4-4x^2+4}\right)+32\left(x^2-1\right)^{\frac{3}{4}}x}{8\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-3*x^2+3)/(x^2-1)^(1/4)/(x^4-3*x^2+2),x, algorithm="fricas"
)
```

```
[Out] 1/8*(10*sqrt(2)*(x^2 - 1)*arctan(sqrt(2)*(x^2 - 1)^(1/4)/x) + 5*sqrt(2)*(x^
2 - 1)*log(-(x^4 - 2*sqrt(2)*(x^2 - 1)^(1/4)*x^3 + 4*sqrt(x^2 - 1)*x^2 - 4*
sqrt(2)*(x^2 - 1)^(3/4)*x + 4*x^2 - 4)/(x^4 - 4*x^2 + 4)) + 32*(x^2 - 1)^(3
/4)*x)/(x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^4-3x^2+3}{\left(x^4-3x^2+2\right)\left(x^2-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-3*x^2+3)/(x^2-1)^(1/4)/(x^4-3*x^2+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^4 - 3*x^2 + 3)/((x^4 - 3*x^2 + 2)*(x^2 - 1)^(1/4)), x)
```

maple [C] time = 1.32, size = 127, normalized size = 1.90

$$\frac{4x}{\left(x^2-1\right)^{\frac{1}{4}}}+\frac{5\,\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{\left(x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+2\right)-x\sqrt{x^2-1}-\mathrm{RootOf}\left(\_Z^2+2\right)\left(x^2-1\right)^{\frac{1}{4}}+x}{x^2-2}\right)}{4}-\frac{5\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{\left(x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2-2\right)+x\sqrt{x^2-1}+\mathrm{RootOf}\left(\_Z^2-2\right)\left(x^2-1\right)^{\frac{1}{4}}+x}{x^2-2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-3*x^2+3)/(x^2-1)^(1/4)/(x^4-3*x^2+2),x)
```

```
[Out] 4*x/(x^2-1)^(1/4)+5/4*RootOf(_Z^2+2)*ln(-((x^2-1)^(3/4)*RootOf(_Z^2+2)-x*(x
^2-1)^(1/2)-RootOf(_Z^2+2)*(x^2-1)^(1/4)+x)/(x^2-2))-5/4*RootOf(_Z^2-2)*ln(
((x^2-1)^(3/4)*RootOf(_Z^2-2)+x*(x^2-1)^(1/2)+RootOf(_Z^2-2)*(x^2-1)^(1/4)+
x)/(x^2-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^4-3x^2+3}{\left(x^4-3x^2+2\right)\left(x^2-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-3*x^2+3)/(x^2-1)^(1/4)/(x^4-3*x^2+2),x, algorithm="maxima"
)
```

```
[Out] integrate((2*x^4 - 3*x^2 + 3)/((x^4 - 3*x^2 + 2)*(x^2 - 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2x^4-3x^2+3}{\left(x^2-1\right)^{1/4}\,\left(x^4-3x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4 - 3*x^2 + 3)/((x^2 - 1)^(1/4)*(x^4 - 3*x^2 + 2)),x)
```

```
[Out] int((2*x^4 - 3*x^2 + 3)/((x^2 - 1)^(1/4)*(x^4 - 3*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^4 - 3x^2 + 3}{\sqrt[4]{(x-1)(x+1)}\,(x-1)(x+1)\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-3*x**2+3)/(x**2-1)**(1/4)/(x**4-3*x**2+2),x)
```

```
[Out] Integral((2*x**4 - 3*x**2 + 3)/(((x - 1)*(x + 1))**(1/4)*(x - 1)*(x + 1)*(x
**2 - 2)), x)
```

3.827 $\int \frac{4b+x^3}{(b+x^3)\sqrt[4]{-b-x^3+ax^4}}\,dx$

Optimal. Leaf size=67

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b-x^3}}\right)}{\sqrt[4]{a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b-x^3}}\right)}{\sqrt[4]{a}}$$

Rubi [F] time = 0.92, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{4b+x^3}{(b+x^3)\sqrt[4]{-b-x^3+ax^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(4*b + x^3)/((b + x^3)*(-b - x^3 + a*x^4)^(1/4)),x]
```

```
[Out] Defer[Int][(-b - x^3 + a*x^4)^(-1/4), x] - b^(1/3)*Defer[Int][1/((-b^(1/3)
- x)*(-b - x^3 + a*x^4)^(1/4)), x] - b^(1/3)*Defer[Int][1/((-b^(1/3) + (-1)
^(1/3)*x)*(-b - x^3 + a*x^4)^(1/4)), x] - b^(1/3)*Defer[Int][1/((-b^(1/3) -
 (-1)^(2/3)*x)*(-b - x^3 + a*x^4)^(1/4)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{4b+x^3}{(b+x^3)\sqrt[4]{-b-x^3+ax^4}}\,dx &= \int \left(\frac{1}{\sqrt[4]{-b-x^3+ax^4}}+\frac{3b}{(b+x^3)\sqrt[4]{-b-x^3+ax^4}}\right)dx\\ &= (3b)\int \frac{1}{(b+x^3)\sqrt[4]{-b-x^3+ax^4}}\,dx+\int \frac{1}{\sqrt[4]{-b-x^3+ax^4}}\,dx\\ &= (3b)\int \left(-\frac{1}{3b^{2/3}\left(-\sqrt[3]{b}-x\right)\sqrt[4]{-b-x^3+ax^4}}-\frac{1}{3b^{2/3}\left(-\sqrt[3]{b}+\sqrt[3]{-1}\,x\right)\sqrt[4]{-b-x}}\right.\\ &= -\left(\sqrt[3]{b}\int \frac{1}{\left(-\sqrt[3]{b}-x\right)\sqrt[4]{-b-x^3+ax^4}}\,dx\right)-\sqrt[3]{b}\int \frac{1}{\left(-\sqrt[3]{b}+\sqrt[3]{-1}\,x\right)\sqrt[4]{-b-x}}\end{aligned}$$

Mathematica [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{4b+x^3}{(b+x^3)\sqrt[4]{-b-x^3+ax^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(4*b + x^3)/((b + x^3)*(-b - x^3 + a*x^4)^(1/4)),x]
```

```
[Out] Integrate[(4*b + x^3)/((b + x^3)*(-b - x^3 + a*x^4)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.48, size = 67, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b-x^3}}\right)}{\sqrt[4]{a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b-x^3}}\right)}{\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4*b + x^3)/((b + x^3)*(-b - x^3 + a*x^4)^(1/4)),x]
```

```
[Out] (2*ArcTan[(a^(1/4)*x)/(-b - x^3 + a*x^4)^(1/4)])/a^(1/4) + (2*ArcTanh[(a^(1
/4)*x)/(-b - x^3 + a*x^4)^(1/4)])/a^(1/4)
```

fricas [B] time = 0.65, size = 134, normalized size = 2.00

$$\frac{4 \arctan\left(\frac{\frac{x\sqrt{\frac{\sqrt{a}x^2+\sqrt{ax^4-x^3-b}}{x^2}}}{a^{\frac{1}{4}}}-\frac{\left(ax^4-x^3-b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}}{x}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{a^{\frac{1}{4}}x+\left(ax^4-x^3-b\right)^{\frac{1}{4}}}{x}\right)}{a^{\frac{1}{4}}}-\frac{\log\left(-\frac{a^{\frac{1}{4}}x-\left(ax^4-x^3-b\right)^{\frac{1}{4}}}{x}\right)}{a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*b)/(x^3+b)/(a*x^4-x^3-b)^(1/4),x, algorithm="fricas")
```

```
[Out] 4*arctan((x*sqrt((sqrt(a)*x^2 + sqrt(a*x^4 - x^3 - b))/x^2)/a^(1/4) - (a*x^
4 - x^3 - b)^(1/4)/a^(1/4))/x)/a^(1/4) + log((a^(1/4)*x + (a*x^4 - x^3 - b)
^(1/4))/x)/a^(1/4) - log(-(a^(1/4)*x - (a*x^4 - x^3 - b)^(1/4))/x)/a^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3+4b}{\left(ax^4-x^3-b\right)^{\frac{1}{4}}\left(x^3+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*b)/(x^3+b)/(a*x^4-x^3-b)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 4*b)/((a*x^4 - x^3 - b)^(1/4)*(x^3 + b)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^3+4b}{\left(x^3+b\right)\left(a\,x^4-x^3-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+4*b)/(x^3+b)/(a*x^4-x^3-b)^(1/4),x)
```

```
[Out] int((x^3+4*b)/(x^3+b)/(a*x^4-x^3-b)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3+4b}{\left(ax^4-x^3-b\right)^{\frac{1}{4}}\left(x^3+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*b)/(x^3+b)/(a*x^4-x^3-b)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 4*b)/((a*x^4 - x^3 - b)^(1/4)*(x^3 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3+4\,b}{\left(x^3+b\right)\,{\left(a\,x^4-x^3-b\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*b + x^3)/((b + x^3)*(a*x^4 - b - x^3)^(1/4)),x)
```

```
[Out] int((4*b + x^3)/((b + x^3)*(a*x^4 - b - x^3)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4b + x^3}{\left(b + x^3\right) \sqrt[4]{ax^4 - b - x^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+4*b)/(x**3+b)/(a*x**4-x**3-b)**(1/4),x)
```

```
[Out] Integral((4*b + x**3)/((b + x**3)*(a*x**4 - b - x**3)**(1/4)), x)
```

3.828 $\int \frac{4b+ax^5}{(-b+ax^5)\sqrt[4]{-b+cx^4+ax^5}}\,dx$

Optimal. Leaf size=67

$$-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^5-b+cx^4}}\right)}{\sqrt[4]{c}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^5-b+cx^4}}\right)}{\sqrt[4]{c}}$$

Rubi [F] time = 1.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{4b+ax^5}{(-b+ax^5)\sqrt[4]{-b+cx^4+ax^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(4*b + a*x^5)/((-b + a*x^5)*(-b + c*x^4 + a*x^5)^(1/4)),x]
```

```
[Out] Defer[Int][(-b + c*x^4 + a*x^5)^(-1/4), x] - b^(1/5)*Defer[Int][1/((b^(1/5)
 - a^(1/5)*x)*(-b + c*x^4 + a*x^5)^(1/4)), x] - b^(1/5)*Defer[Int][1/((b^(1
/5) + (-1)^(1/5)*a^(1/5)*x)*(-b + c*x^4 + a*x^5)^(1/4)), x] - b^(1/5)*Defer
[Int][1/((b^(1/5) - (-1)^(2/5)*a^(1/5)*x)*(-b + c*x^4 + a*x^5)^(1/4)), x] -
 b^(1/5)*Defer[Int][1/((b^(1/5) + (-1)^(3/5)*a^(1/5)*x)*(-b + c*x^4 + a*x^5
)^(1/4)), x] - b^(1/5)*Defer[Int][1/((b^(1/5) - (-1)^(4/5)*a^(1/5)*x)*(-b +
 c*x^4 + a*x^5)^(1/4)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{4b+ax^5}{(-b+ax^5)\sqrt[4]{-b+cx^4+ax^5}}\,dx &= \int \left(\frac{1}{\sqrt[4]{-b+cx^4+ax^5}}+\frac{5b}{(-b+ax^5)\sqrt[4]{-b+cx^4+ax^5}}\right)dx\\
&=(5b)\int \frac{1}{(-b+ax^5)\sqrt[4]{-b+cx^4+ax^5}}\,dx+\int \frac{1}{\sqrt[4]{-b+cx^4+ax^5}}\,dx\\
&=(5b)\int \left(-\frac{1}{5b^{4/5}\left(\sqrt[5]{b}-\sqrt[5]{a}\,x\right)\sqrt[4]{-b+cx^4+ax^5}}-\frac{1}{5b^{4/5}\left(\sqrt[5]{b}+\sqrt[5]{-1}\,\sqrt[5]{a}\,x\right)\sqrt[4]{\ldots}}\right.\\
&=-\left(\sqrt[5]{b}\int \frac{1}{\left(\sqrt[5]{b}-\sqrt[5]{a}\,x\right)\sqrt[4]{-b+cx^4+ax^5}}\,dx\right)-\sqrt[5]{b}\int \frac{1}{\left(\sqrt[5]{b}+\sqrt[5]{-1}\,\sqrt[5]{a}\,x\right)\sqrt[4]{\ldots}}
\end{aligned}$$

Mathematica [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{4b+ax^5}{(-b+ax^5)\sqrt[4]{-b+cx^4+ax^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(4*b + a*x^5)/((-b + a*x^5)*(-b + c*x^4 + a*x^5)^(1/4)),x]
```

```
[Out] Integrate[(4*b + a*x^5)/((-b + a*x^5)*(-b + c*x^4 + a*x^5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 3.08, size = 67, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^5-b+cx^4}}\right)}{\sqrt[4]{c}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^5-b+cx^4}}\right)}{\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4*b + a*x^5)/((-b + a*x^5)*(-b + c*x^4 + a*x^5)^(1/4)),
x]
```

```
[Out] (-2*ArcTan[(c^(1/4)*x)/(-b + c*x^4 + a*x^5)^(1/4)])/c^(1/4) - (2*ArcTanh[(c
^(1/4)*x)/(-b + c*x^4 + a*x^5)^(1/4)])/c^(1/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5+4*b)/(a*x^5-b)/(a*x^5+c*x^4-b)^(1/4),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^5 + 4\,b}{\left(ax^5 + cx^4 - b\right)^{\frac{1}{4}}\left(ax^5 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5+4*b)/(a*x^5-b)/(a*x^5+c*x^4-b)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^5 + 4*b)/((a*x^5 + c*x^4 - b)^(1/4)*(a*x^5 - b)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{a\,x^5 + 4b}{\left(a\,x^5 - b\right)\left(a\,x^5 + c\,x^4 - b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^5+4*b)/(a*x^5-b)/(a*x^5+c*x^4-b)^(1/4),x)
```

```
[Out] int((a*x^5+4*b)/(a*x^5-b)/(a*x^5+c*x^4-b)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^5 + 4\,b}{\left(ax^5 + cx^4 - b\right)^{\frac{1}{4}}\left(ax^5 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5+4*b)/(a*x^5-b)/(a*x^5+c*x^4-b)^(1/4),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^5 + 4*b)/((a*x^5 + c*x^4 - b)^(1/4)*(a*x^5 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a\,x^5 + 4\,b}{\left(b - a\,x^5\right)\,\left(a\,x^5 + c\,x^4 - b\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*b + a*x^5)/((b - a*x^5)*(a*x^5 - b + c*x^4)^(1/4)),x)
```

```
[Out] int(-(4*b + a*x^5)/((b - a*x^5)*(a*x^5 - b + c*x^4)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^5 + 4b}{\left(ax^5 - b\right) \sqrt[4]{ax^5 - b + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**5+4*b)/(a*x**5-b)/(a*x**5+c*x**4-b)**(1/4),x)
[Out] Integral((a*x**5 + 4*b)/((a*x**5 - b)*(a*x**5 - b + c*x**4)**(1/4)), x)
```

3.829 $\int \frac{\sqrt{1+x^2-2x^6}\left(1+4x^6\right)}{\left(-1-4x^2+2x^6\right)\left(-1-2x^2+2x^6\right)}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{-2x^6+x^2+1}}\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x\sqrt{-2x^6+x^2+1}}{2x^6-x^2-1}\right)$$

Rubi [F] time = 1.39, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+x^2-2x^6}\left(1+4x^6\right)}{\left(-1-4x^2+2x^6\right)\left(-1-2x^2+2x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[1 + x^2 - 2*x^6]*(1 + 4*x^6))/((-1 - 4*x^2 + 2*x^6)*(-1 - 2*x^2 +
 2*x^6)),x]
```

```
[Out] -2*Defer[Int][Sqrt[1 + x^2 - 2*x^6]/(-1 - 4*x^2 + 2*x^6), x] + 3*Defer[Int]
[(x^4*Sqrt[1 + x^2 - 2*x^6])/(-1 - 4*x^2 + 2*x^6), x] + Defer[Int][Sqrt[1 +
 x^2 - 2*x^6]/(-1 - 2*x^2 + 2*x^6), x] - 3*Defer[Int][(x^4*Sqrt[1 + x^2 - 2
*x^6])/(-1 - 2*x^2 + 2*x^6), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x^2-2x^6}\left(1+4x^6\right)}{\left(-1-4x^2+2x^6\right)\left(-1-2x^2+2x^6\right)}\,dx &= \int \left(\frac{\left(-2+3x^4\right)\sqrt{1+x^2-2x^6}}{-1-4x^2+2x^6}+\frac{\left(1-3x^4\right)\sqrt{1+x^2-2x^6}}{-1-2x^2+2x^6}\right)dx\\
&= \int \frac{\left(-2+3x^4\right)\sqrt{1+x^2-2x^6}}{-1-4x^2+2x^6}\,dx+\int \frac{\left(1-3x^4\right)\sqrt{1+x^2-2x^6}}{-1-2x^2+2x^6}\,dx\\
&= \int \left(-\frac{2\sqrt{1+x^2-2x^6}}{-1-4x^2+2x^6}+\frac{3x^4\sqrt{1+x^2-2x^6}}{-1-4x^2+2x^6}\right)dx+\int \left(\frac{\sqrt{1+x^2-2}}{-1-2x^2+}\right.\\
&= -\left(2\int \frac{\sqrt{1+x^2-2x^6}}{-1-4x^2+2x^6}\,dx\right)+3\int \frac{x^4\sqrt{1+x^2-2x^6}}{-1-4x^2+2x^6}\,dx-3\int \frac{x^4\sqrt{}}{-1}
\end{aligned}$$

Mathematica [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+x^2-2x^6}\left(1+4x^6\right)}{\left(-1-4x^2+2x^6\right)\left(-1-2x^2+2x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[1 + x^2 - 2*x^6]*(1 + 4*x^6))/((-1 - 4*x^2 + 2*x^6)*(-1 - 2
*x^2 + 2*x^6)),x]
```

```
[Out] Integrate[(Sqrt[1 + x^2 - 2*x^6]*(1 + 4*x^6))/((-1 - 4*x^2 + 2*x^6)*(-1 - 2
*x^2 + 2*x^6)), x]
```

IntegrateAlgebraic [A] time = 2.65, size = 67, normalized size = 1.00

$$-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{-2x^6+x^2+1}}\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x\sqrt{-2x^6+x^2+1}}{2x^6-x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x^2 - 2*x^6]*(1 + 4*x^6))/((-1 - 4*x^2 + 2*x^6
)*(-1 - 2*x^2 + 2*x^6)),x]
```

```
[Out] -1/2*ArcTan[x/Sqrt[1 + x^2 - 2*x^6]] - (Sqrt[3]*ArcTan[(Sqrt[3]*x*Sqrt[1 +
x^2 - 2*x^6])/(-1 - x^2 + 2*x^6)])/2
```

fricas [A] time = 0.64, size = 66, normalized size = 0.99

$$-\frac{1}{4}\sqrt{3}\arctan\left(\frac{2\sqrt{3}\sqrt{-2x^6+x^2+1}\,x}{2x^6+2x^2-1}\right)+\frac{1}{4}\arctan\left(\frac{2\sqrt{-2x^6+x^2+1}\,x}{2x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^6+x^2+1)^(1/2)*(4*x^6+1)/(2*x^6-4*x^2-1)/(2*x^6-2*x^2-1),x,
 algorithm="fricas")
```

```
[Out] -1/4*sqrt(3)*arctan(2*sqrt(3)*sqrt(-2*x^6 + x^2 + 1)*x/(2*x^6 + 2*x^2 - 1))
 + 1/4*arctan(2*sqrt(-2*x^6 + x^2 + 1)*x/(2*x^6 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(4x^6+1\right)\sqrt{-2x^6+x^2+1}}{\left(2x^6-2x^2-1\right)\left(2x^6-4x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^6+x^2+1)^(1/2)*(4*x^6+1)/(2*x^6-4*x^2-1)/(2*x^6-2*x^2-1),x,
 algorithm="giac")
```

```
[Out] integrate((4*x^6 + 1)*sqrt(-2*x^6 + x^2 + 1)/((2*x^6 - 2*x^2 - 1)*(2*x^6 -
4*x^2 - 1)), x)
```

maple [C] time = 0.72, size = 133, normalized size = 1.99

$$-\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+3\right)x^6+2\,\mathrm{RootOf}\left(\_Z^2+3\right)x^2+6\sqrt{-2x^6+x^2+1}\,x-\mathrm{RootOf}\left(\_Z^2+3\right)}{2x^6-4x^2-1}\right)}{4}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)x^6+2\sqrt{-2x^6+x^2+1}\,x-\mathrm{RootOf}\left(\_Z^2+1\right)}{2x^6-2x^2-1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^6+x^2+1)^(1/2)*(4*x^6+1)/(2*x^6-4*x^2-1)/(2*x^6-2*x^2-1),x)
```

```
[Out] -1/4*RootOf(_Z^2+3)*ln(-(2*RootOf(_Z^2+3)*x^6+2*RootOf(_Z^2+3)*x^2+6*(-2*x^
6+x^2+1)^(1/2)*x-RootOf(_Z^2+3))/(2*x^6-4*x^2-1))+1/4*RootOf(_Z^2+1)*ln(-(2
*RootOf(_Z^2+1)*x^6+2*(-2*x^6+x^2+1)^(1/2)*x-RootOf(_Z^2+1))/(2*x^6-2*x^2-1
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(4x^6+1\right)\sqrt{-2x^6+x^2+1}}{\left(2x^6-2x^2-1\right)\left(2x^6-4x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^6+x^2+1)^(1/2)*(4*x^6+1)/(2*x^6-4*x^2-1)/(2*x^6-2*x^2-1),x,
 algorithm="maxima")
```

```
[Out] integrate((4*x^6 + 1)*sqrt(-2*x^6 + x^2 + 1)/((2*x^6 - 2*x^2 - 1)*(2*x^6 -
4*x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(4x^6+1\right)\sqrt{-2x^6+x^2+1}}{\left(-2x^6+2x^2+1\right)\left(-2x^6+4x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((4*x^6 + 1)*(x^2 - 2*x^6 + 1)^(1/2))/((2*x^2 - 2*x^6 + 1)*(4*x^2 - 2*x
^6 + 1)),x)
```

```
[Out] int(((4*x^6 + 1)*(x^2 - 2*x^6 + 1)^(1/2))/((2*x^2 - 2*x^6 + 1)*(4*x^2 - 2*x
^6 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**6+x**2+1)**(1/2)*(4*x**6+1)/(2*x**6-4*x**2-1)/(2*x**6-2*x*
*2-1),x)
```

```
[Out] Timed out
```

3.830 $\int \frac{x^4}{\sqrt[4]{-1+x^4}\left(-1+x^8\right)}\,dx$

Optimal. Leaf size=67

$$-\frac{x}{2\sqrt[4]{x^4-1}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt[4]{2}}$$

Rubi [C] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 0.87, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1479, 511, 510}

$$-\frac{x^5\sqrt[4]{1-x^4}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};\frac{2x^4}{x^4+1}\right)}{5\sqrt[4]{x^4-1}\left(x^4+1\right)^{5/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^4/((-1 + x^4)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -1/5*(x^5*(1 - x^4)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, (2*x^4)/(1 + x^4
)])/((-1 + x^4)^(1/4)*(1 + x^4)^(5/4))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1479

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n
2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*(a/d + (c*x^n)/e)^
p, x] /; FreeQ[{a, c, d, e, f, q, m, n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2
 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{-1+x^4}\left(-1+x^8\right)}\,dx=\int \frac{x^4}{\left(-1+x^4\right)^{5/4}\left(1+x^4\right)}\,dx$$
$$=-\frac{\sqrt[4]{1-x^4}\int \frac{x^4}{\left(1-x^4\right)^{5/4}\left(1+x^4\right)}\,dx}{\sqrt[4]{-1+x^4}}$$
$$=-\frac{x^5\sqrt[4]{1-x^4}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};\frac{2x^4}{1+x^4}\right)}{5\sqrt[4]{-1+x^4}\left(1+x^4\right)^{5/4}}$$

Mathematica [A] time = 0.18, size = 91, normalized size = 1.36

$$\frac{-\log\left(1-\frac{\sqrt[4]{2}x}{\sqrt[4]{1-x^4}}\right)+\log\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1-x^4}}+1\right)+2\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1-x^4}}\right)}{8\sqrt[4]{2}}-\frac{x}{2\sqrt[4]{x^4-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((-1 + x^4)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -1/2*x/(-1 + x^4)^(1/4) + (2*ArcTan[(2^(1/4)*x)/(1 - x^4)^(1/4)] - Log[1 -
(2^(1/4)*x)/(1 - x^4)^(1/4)] + Log[1 + (2^(1/4)*x)/(1 - x^4)^(1/4)])/(8*2^(
1/4))
```

IntegrateAlgebraic [A] time = 0.32, size = 67, normalized size = 1.00

$$-\frac{x}{2\sqrt[4]{x^4-1}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((-1 + x^4)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -1/2*x/(-1 + x^4)^(1/4) + ArcTan[(2^(1/4)*x)/(-1 + x^4)^(1/4)]/(4*2^(1/4))
+ ArcTanh[(2^(1/4)*x)/(-1 + x^4)^(1/4)]/(4*2^(1/4))
```

fricas [B] time = 5.24, size = 242, normalized size = 3.61

$$-\frac{4\cdot 2^{\frac{3}{4}}\left(x^4-1\right)\arctan\left(\frac{4\cdot 2^{\frac{3}{4}}\left(x^4-1\right)^{\frac{1}{4}}x^3+4\cdot 2^{\frac{1}{4}}\left(x^4-1\right)^{\frac{3}{4}}x+2^{\frac{3}{4}}\left(2\cdot 2^{\frac{3}{4}}\sqrt{x^4-1}\,x^2+2^{\frac{1}{4}}\left(3x^4-1\right)\right)}{2\left(x^4+1\right)}\right)-2^{\frac{3}{4}}\left(x^4-1\right)\log\left(\frac{4\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}x^3+4\cdot 2^{\frac{1}{4}}\sqrt{x^4-1}\,x^2+2^{\frac{3}{4}}\left(3x^4-1\right)+4\left(x^4-1\right)^{\frac{3}{4}}x}{x^4+1}\right)+2^{\frac{3}{4}}\left(x^4-1\right)\log\left(\frac{4\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}x^3-4\cdot 2^{\frac{1}{4}}\sqrt{x^4-1}\,x^2-2^{\frac{3}{4}}\left(3x^4-1\right)+4\left(x^4-1\right)^{\frac{3}{4}}x}{x^4+1}\right)+16\left(x^4-1\right)^{\frac{3}{4}}x}{32\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1)^(1/4)/(x^8-1),x, algorithm="fricas")
```

```
[Out] -1/32*(4*2^(3/4)*(x^4 - 1)*arctan(1/2*(4*2^(3/4)*(x^4 - 1)^(1/4)*x^3 + 4*2^
(1/4)*(x^4 - 1)^(3/4)*x + 2^(3/4)*(2*2^(3/4)*sqrt(x^4 - 1)*x^2 + 2^(1/4)*(3
*x^4 - 1)))/(x^4 + 1)) - 2^(3/4)*(x^4 - 1)*log((4*sqrt(2)*(x^4 - 1)^(1/4)*x
^3 + 4*2^(1/4)*sqrt(x^4 - 1)*x^2 + 2^(3/4)*(3*x^4 - 1) + 4*(x^4 - 1)^(3/4)*
x)/(x^4 + 1)) + 2^(3/4)*(x^4 - 1)*log((4*sqrt(2)*(x^4 - 1)^(1/4)*x^3 - 4*2^
(1/4)*sqrt(x^4 - 1)*x^2 - 2^(3/4)*(3*x^4 - 1) + 4*(x^4 - 1)^(3/4)*x)/(x^4 +
 1)) + 16*(x^4 - 1)^(3/4)*x)/(x^4 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^8-1\right)\left(x^4-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1)^(1/4)/(x^8-1),x, algorithm="giac")
```

```
[Out] integrate(x^4/((x^8 - 1)*(x^4 - 1)^(1/4)), x)
```

maple [C] time = 1.65, size = 218, normalized size = 3.25

$$-\frac{x}{2\left(x^4-1\right)^{\frac{1}{4}}}-\frac{\mathrm{RootOf}\left(\_Z^4-8\right)\ln\left(-\frac{\sqrt{x^4-1}\,\mathrm{RootOf}\left(\_Z^4-8\right)^3x^2-2\left(x^4-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-8\right)^2x^3+3\,\mathrm{RootOf}\left(\_Z^4-8\right)x^4-4\left(x^4-1\right)^{\frac{3}{4}}x-\mathrm{RootOf}\left(\_Z^4-8\right)}{x^4+1}\right)}{16}+\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\ln\left(-\frac{\sqrt{x^4-1}\,\mathrm{RootOf}\left(\_Z^4-8\right)^2\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)x^2+2\left(x^4-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-8\right)^2x^3-3\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)x^4-4\left(x^4-1\right)^{\frac{3}{4}}x+\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)}{x^4+1}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4-1)^(1/4)/(x^8-1),x)
```

```
[Out] -1/2*x/(x^4-1)^(1/4)-1/16*RootOf(_Z^4-8)*ln(-((x^4-1)^(1/2)*RootOf(_Z^4-8)^
3*x^2-2*(x^4-1)^(1/4)*RootOf(_Z^4-8)^2*x^3+3*RootOf(_Z^4-8)*x^4-4*(x^4-1)^(
3/4)*x-RootOf(_Z^4-8))/(x^4+1))+1/16*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln(-((x^
4-1)^(1/2)*RootOf(_Z^4-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^2+2*(x^4-1)^(1/
4)*RootOf(_Z^4-8)^2*x^3-3*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^4-4*(x^4-1)^(3/4)
*x+RootOf(_Z^2+RootOf(_Z^4-8)^2))/(x^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^8-1\right)\left(x^4-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1)^(1/4)/(x^8-1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((x^8 - 1)*(x^4 - 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(x^4-1\right)^{1/4}\,\left(x^8-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((x^4 - 1)^(1/4)*(x^8 - 1)),x)
```

```
[Out] int(x^4/((x^4 - 1)^(1/4)*(x^8 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt[4]{\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}\,\left(x-1\right)\left(x+1\right)\left(x^2+1\right)\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4-1)**(1/4)/(x**8-1),x)
```

```
[Out] Integral(x**4/(((x - 1)*(x + 1)*(x**2 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 +
1)*(x**4 + 1)), x)
```

3.831 $\int \frac{1}{\sqrt[4]{1+x^4}\left(-1+x^8\right)} dx$

Optimal. Leaf size=67

$$-\frac{x}{2\sqrt[4]{x^4+1}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[4]{2}}$$

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1404, 382, 377, 212, 206, 203}

$$-\frac{x}{2\sqrt[4]{x^4+1}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^4)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -1/2*x/(1 + x^4)^(1/4) - ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(4*2^(1/4)) -
ArcTanh[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(4*2^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 1404

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{1+x^4}\left(-1+x^8\right)}\,dx &= \int \frac{1}{\left(-1+x^4\right)\left(1+x^4\right)^{5/4}}\,dx\\
&= -\frac{x}{2\sqrt[4]{1+x^4}}+\frac{1}{2}\int \frac{1}{\left(-1+x^4\right)\sqrt[4]{1+x^4}}\,dx\\
&= -\frac{x}{2\sqrt[4]{1+x^4}}+\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{-1+2x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)\\
&= -\frac{x}{2\sqrt[4]{1+x^4}}-\frac{1}{4}\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{1}{4}\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x^2}\,dx,x,\right.\\
&= -\frac{x}{2\sqrt[4]{1+x^4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{4\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{4\sqrt[4]{2}}\end{aligned}$$

Mathematica [A] time = 0.25, size = 85, normalized size = 1.27

$$-\frac{x}{2\sqrt[4]{x^4+1}}-\frac{-\log\left(1-\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)+\log\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}+1\right)+2\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{8\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^4)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -1/2*x/(1 + x^4)^(1/4) - (2*ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)] - Log[1 - (
2^(1/4)*x)/(1 + x^4)^(1/4)] + Log[1 + (2^(1/4)*x)/(1 + x^4)^(1/4)])/(8*2^(1
/4))
```

IntegrateAlgebraic [A] time = 0.28, size = 67, normalized size = 1.00

$$-\frac{x}{2\sqrt[4]{x^4+1}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x^4)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -1/2*x/(1 + x^4)^(1/4) - ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(4*2^(1/4)) -
ArcTanh[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(4*2^(1/4))
```

fricas [B] time = 5.34, size = 242, normalized size = 3.61

$$\frac{4\cdot 2^{\frac{3}{4}}\left(x^4+1\right)\arctan\left(\frac{4\cdot 2^{\frac{3}{4}}\left(x^4+1\right)^{\frac{1}{4}}x^3+4\cdot 2^{\frac{1}{4}}\left(x^4+1\right)^{\frac{3}{4}}x+2^{\frac{3}{4}}\left(2\cdot 2^{\frac{3}{4}}\sqrt{x^4+1}\,x^2+2^{\frac{1}{4}}\left(3x^4+1\right)\right)}{2\left(x^4-1\right)}\right)-2^{\frac{3}{4}}\left(x^4+1\right)\log\left(\frac{4\sqrt{2}\left(x^4+1\right)^{\frac{1}{4}}x^3+4\cdot 2^{\frac{1}{4}}\sqrt{x^4+1}\,x^2+2^{\frac{3}{4}}\left(3x^4+1\right)+4\left(x^4+1\right)^{\frac{3}{4}}x}{x^4-1}\right)+2^{\frac{3}{4}}\left(x^4+1\right)\log\left(\frac{4\sqrt{2}\left(x^4+1\right)^{\frac{1}{4}}x^3-4\cdot 2^{\frac{1}{4}}\sqrt{x^4+1}\,x^2-2^{\frac{3}{4}}\left(3x^4+1\right)+4\left(x^4+1\right)^{\frac{3}{4}}x}{x^4-1}\right)-16\left(x^4+1\right)^{\frac{3}{4}}x}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4)/(x^8-1),x, algorithm="fricas")
```

```
[Out] 1/32*(4*2^(3/4)*(x^4 + 1)*arctan(1/2*(4*2^(3/4)*(x^4 + 1)^(1/4)*x^3 + 4*2^(
1/4)*(x^4 + 1)^(3/4)*x + 2^(3/4)*(2*2^(3/4)*sqrt(x^4 + 1)*x^2 + 2^(1/4)*(3*
```

```
x^4 + 1)))/(x^4 - 1)) - 2^(3/4)*(x^4 + 1)*log((4*sqrt(2)*(x^4 + 1)^(1/4)*x^
3 + 4*2^(1/4)*sqrt(x^4 + 1)*x^2 + 2^(3/4)*(3*x^4 + 1) + 4*(x^4 + 1)^(3/4)*x
)/(x^4 - 1)) + 2^(3/4)*(x^4 + 1)*log((4*sqrt(2)*(x^4 + 1)^(1/4)*x^3 - 4*2^(
1/4)*sqrt(x^4 + 1)*x^2 - 2^(3/4)*(3*x^4 + 1) + 4*(x^4 + 1)^(3/4)*x)/(x^4 -
1)) - 16*(x^4 + 1)^(3/4)*x)/(x^4 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^8-1\right)\left(x^4+1\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4)/(x^8-1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^8 - 1)*(x^4 + 1)^(1/4)), x)
```

maple [C] time = 1.68, size = 236, normalized size = 3.52

$$-\frac{x}{2\left(x^4+1\right)^{\frac{1}{4}}}+\frac{\mathrm{RootOf}\left(\_Z^4-8\right)\ln\left(\frac{\sqrt{x^4+1}\,\mathrm{RootOf}\left(\_Z^4-8\right)^3 x^2-2\left(x^4+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-8\right)^2 x^3+3\,\mathrm{RootOf}\left(\_Z^4-8\right)x^4-4\left(x^4+1\right)^{\frac{3}{4}}x+\mathrm{RootOf}\left(\_Z^4-8\right)}{(-1+x)(1+x)\left(x^2+1\right)}\right)}{16}-\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\ln\left(\frac{\sqrt{x^4+1}\,\mathrm{RootOf}\left(\_Z^4-8\right)^2\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)x^2+2\left(x^4+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-8\right)^2 x^3-3\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)x^4-4\left(x^4+1\right)^{\frac{3}{4}}x-\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)}{(-1+x)(1+x)\left(x^2+1\right)}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1)^(1/4)/(x^8-1),x)
```

```
[Out] -1/2*x/(x^4+1)^(1/4)+1/16*RootOf(_Z^4-8)*ln(((x^4+1)^(1/2)*RootOf(_Z^4-8)^3
*x^2-2*(x^4+1)^(1/4)*RootOf(_Z^4-8)^2*x^3+3*RootOf(_Z^4-8)*x^4-4*(x^4+1)^(3
/4)*x+RootOf(_Z^4-8))/(-1+x)/(1+x)/(x^2+1))-1/16*RootOf(_Z^2+RootOf(_Z^4-8)
^2)*ln(((x^4+1)^(1/2)*RootOf(_Z^4-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^2+2*
(x^4+1)^(1/4)*RootOf(_Z^4-8)^2*x^3-3*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^4-4*(x
^4+1)^(3/4)*x-RootOf(_Z^2+RootOf(_Z^4-8)^2))/(-1+x)/(1+x)/(x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^8-1\right)\left(x^4+1\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4)/(x^8-1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^8 - 1)*(x^4 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(x^4+1\right)}^{1/4}\,\left(x^8-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^4 + 1)^(1/4)*(x^8 - 1)),x)
```

```
[Out] int(1/((x^4 + 1)^(1/4)*(x^8 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x-1)\,(x+1)\left(x^2+1\right)\left(x^4+1\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)**(1/4)/(x**8-1),x)
```

```
[Out] Integral(1/((x - 1)*(x + 1)*(x**2 + 1)*(x**4 + 1)**(5/4)), x)
```

3.832 $\int \frac{-1+2x^4}{\sqrt[4]{-1+x^4}\left(-1+x^8\right)}\,dx$

Optimal. Leaf size=67

$$-\frac{x}{2\sqrt[4]{x^4-1}}+\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt[4]{2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt[4]{2}}$$

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1455, 527, 12, 377, 212, 206, 203}

$$-\frac{x}{2\sqrt[4]{x^4-1}}+\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt[4]{2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x^4)/((-1 + x^4)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -1/2*x/(-1 + x^4)^(1/4) + (3*ArcTan[(2^(1/4)*x)/(-1 + x^4)^(1/4)])/(4*2^(1/
4)) + (3*ArcTanh[(2^(1/4)*x)/(-1 + x^4)^(1/4)])/(4*2^(1/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1455

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, n, q, r}, x] && EqQ[n2,
 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+2x^4}{\sqrt[4]{-1+x^4}\left(-1+x^8\right)}\,dx &= \int \frac{-1+2x^4}{\left(-1+x^4\right)^{5/4}\left(1+x^4\right)}\,dx\\
&= -\frac{x}{2\sqrt[4]{-1+x^4}}+\frac{1}{2}\int \frac{3}{\sqrt[4]{-1+x^4}\left(1+x^4\right)}\,dx\\
&= -\frac{x}{2\sqrt[4]{-1+x^4}}+\frac{3}{2}\int \frac{1}{\sqrt[4]{-1+x^4}\left(1+x^4\right)}\,dx\\
&= -\frac{x}{2\sqrt[4]{-1+x^4}}+\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{1-2x^4}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)\\
&= -\frac{x}{2\sqrt[4]{-1+x^4}}+\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)+\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x
\right.\\
&= -\frac{x}{2\sqrt[4]{-1+x^4}}+\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{-1+x^4}}\right)}{4\sqrt[4]{2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{-1+x^4}}\right)}{4\sqrt[4]{2}}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 91, normalized size = 1.36

$$\frac{3\left(-\log\left(1-\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1-x^4}}\right)+\log\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1-x^4}}+1\right)+2\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1-x^4}}\right)\right)}{8\sqrt[4]{2}}-\frac{x}{2\sqrt[4]{x^4-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 2*x^4)/((-1 + x^4)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -1/2*x/(-1 + x^4)^(1/4) + (3*(2*ArcTan[(2^(1/4)*x)/(1 - x^4)^(1/4)] - Log[1
 - (2^(1/4)*x)/(1 - x^4)^(1/4)] + Log[1 + (2^(1/4)*x)/(1 - x^4)^(1/4)]))/(8
*2^(1/4))
```

IntegrateAlgebraic [A] time = 0.32, size = 67, normalized size = 1.00

$$-\frac{x}{2\sqrt[4]{x^4-1}}+\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt[4]{2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x^4)/((-1 + x^4)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -1/2*x/(-1 + x^4)^(1/4) + (3*ArcTan[(2^(1/4)*x)/(-1 + x^4)^(1/4)])/(4*2^(1/
4)) + (3*ArcTanh[(2^(1/4)*x)/(-1 + x^4)^(1/4)])/(4*2^(1/4))
```

fricas [B] time = 5.83, size = 243, normalized size = 3.63

$$-\frac{12 \cdot 2^{\frac{3}{4}} \left(x^4-1\right) \arctan\left(\frac{4 \cdot 2^{\frac{3}{4}} \left(x^4-1\right)^{\frac{1}{4}} x^3+4 \cdot 2^{\frac{1}{4}} \left(x^4-1\right)^{\frac{3}{4}} x+2^{\frac{3}{4}} \left(2 \cdot 2^{\frac{3}{4}} \sqrt{x^4-1}\, x^2+2^{\frac{1}{4}} \left(3 x^4-1\right)\right)}{2 \left(x^4+1\right)}\right)-3 \cdot 2^{\frac{3}{4}} \left(x^4-1\right) \log\left(\frac{4 \sqrt{2} \left(x^4-1\right)^{\frac{1}{4}} x^3+4 \cdot 2^{\frac{1}{4}} \sqrt{x^4-1}\, x^2+2^{\frac{3}{4}} \left(3 x^4-1\right)+4 \left(x^4-1\right)^{\frac{3}{4}} x}{x^4+1}\right)+3 \cdot 2^{\frac{3}{4}} \left(x^4-1\right) \log\left(\frac{4 \sqrt{2} \left(x^4-1\right)^{\frac{1}{4}} x^3-4 \cdot 2^{\frac{1}{4}} \sqrt{x^4-1}\, x^2-2^{\frac{3}{4}} \left(3 x^4-1\right)+4 \left(x^4-1\right)^{\frac{3}{4}} x}{x^4+1}\right)+16 \left(x^4-1\right)^{\frac{3}{4}} x}{32 \left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)/(x^4-1)^(1/4)/(x^8-1),x, algorithm="fricas")
```

```
[Out] -1/32*(12*2^(3/4)*(x^4 - 1)*arctan(1/2*(4*2^(3/4)*(x^4 - 1)^(1/4)*x^3 + 4*2
^(1/4)*(x^4 - 1)^(3/4)*x + 2^(3/4)*(2*2^(3/4)*sqrt(x^4 - 1)*x^2 + 2^(1/4)*(
3*x^4 - 1)))/(x^4 + 1)) - 3*2^(3/4)*(x^4 - 1)*log((4*sqrt(2)*(x^4 - 1)^(1/4
)*x^3 + 4*2^(1/4)*sqrt(x^4 - 1)*x^2 + 2^(3/4)*(3*x^4 - 1) + 4*(x^4 - 1)^(3/
4)*x)/(x^4 + 1)) + 3*2^(3/4)*(x^4 - 1)*log((4*sqrt(2)*(x^4 - 1)^(1/4)*x^3 -
 4*2^(1/4)*sqrt(x^4 - 1)*x^2 - 2^(3/4)*(3*x^4 - 1) + 4*(x^4 - 1)^(3/4)*x)/(
x^4 + 1)) + 16*(x^4 - 1)^(3/4)*x)/(x^4 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 x^4-1}{\left(x^8-1\right)\left(x^4-1\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)/(x^4-1)^(1/4)/(x^8-1),x, algorithm="giac")
```

```
[Out] integrate((2*x^4 - 1)/((x^8 - 1)*(x^4 - 1)^(1/4)), x)
```

maple [C] time = 1.52, size = 217, normalized size = 3.24

$$-\frac{x}{2 \left(x^4-1\right)^{\frac{1}{4}}}-\frac{3 \operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right) \ln\left(\frac{\sqrt{x^4-1}\, \operatorname{RootOf}\left(\_Z^4-8\right)^2 \operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right) x^2-2 \left(x^4-1\right)^{\frac{1}{4}} \operatorname{RootOf}\left(\_Z^4-8\right)^2 x^3-3 \operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right) x^4+4 \left(x^4-1\right)^{\frac{3}{4}} x+\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)}{x^4+1}\right)}{16}-\frac{3 \operatorname{RootOf}\left(\_Z^4-8\right) \ln\left(-\frac{\sqrt{x^4-1}\, \operatorname{RootOf}\left(\_Z^4-8\right)^3 x^2-2 \left(x^4-1\right)^{\frac{1}{4}} \operatorname{RootOf}\left(\_Z^4-8\right)^2 x^3+3 \operatorname{RootOf}\left(\_Z^4-8\right) x^4-4 \left(x^4-1\right)^{\frac{3}{4}} x-\operatorname{RootOf}\left(\_Z^4-8\right)}{x^4+1}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-1)/(x^4-1)^(1/4)/(x^8-1),x)
```

```
[Out] -1/2*x/(x^4-1)^(1/4)-3/16*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln(((x^4-1)^(1/2)*R
ootOf(_Z^4-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^2-2*(x^4-1)^(1/4)*RootOf(_Z
^4-8)^2*x^3-3*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^4+4*(x^4-1)^(3/4)*x+RootOf(_Z
^2+RootOf(_Z^4-8)^2))/(x^4+1))-3/16*RootOf(_Z^4-8)*ln(-((x^4-1)^(1/2)*RootO
f(_Z^4-8)^3*x^2-2*(x^4-1)^(1/4)*RootOf(_Z^4-8)^2*x^3+3*RootOf(_Z^4-8)*x^4-4
*(x^4-1)^(3/4)*x-RootOf(_Z^4-8))/(x^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 x^4-1}{\left(x^8-1\right)\left(x^4-1\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)/(x^4-1)^(1/4)/(x^8-1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^4 - 1)/((x^8 - 1)*(x^4 - 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2 x^4-1}{{\left(x^4-1\right)}^{1/4}\,\left(x^8-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4 - 1)/((x^4 - 1)^(1/4)*(x^8 - 1)),x)
```

```
[Out] int((2*x^4 - 1)/((x^4 - 1)^(1/4)*(x^8 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^4-1}{\sqrt[4]{(x-1)(x+1)\left(x^2+1\right)}\,(x-1)(x+1)\left(x^2+1\right)\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-1)/(x**4-1)**(1/4)/(x**8-1),x)
```

```
[Out] Integral((2*x**4 - 1)/(((x - 1)*(x + 1)*(x**2 + 1))**(1/4)*(x - 1)*(x + 1)*
(x**2 + 1)*(x**4 + 1)), x)
```

3.833 $\int \frac{1+x^8}{\sqrt{1+x^4}\left(-1+x^8\right)}\,dx$

Optimal. Leaf size=67

$$-\frac{x}{2\sqrt{x^4+1}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{4\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.35, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6725, 220, 1404, 414, 523, 409, 1211, 1699, 206, 203}

$$-\frac{x}{2\sqrt{x^4+1}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{4\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^8)/(Sqrt[1 + x^4]*(-1 + x^8)),x]
```

```
[Out] -1/2*x/Sqrt[1 + x^4] - ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(4*Sqrt[2]) - ArcT
anh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(4*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1404

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^8}{\sqrt{1+x^4}\left(-1+x^8\right)}\,dx = \int \left(\frac{1}{\sqrt{1+x^4}} + \frac{2}{\sqrt{1+x^4}\left(-1+x^8\right)}\right)dx$$
$$= 2\int \frac{1}{\sqrt{1+x^4}\left(-1+x^8\right)}\,dx + \int \frac{1}{\sqrt{1+x^4}}\,dx$$
$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} + 2\int \frac{1}{\left(-1+x^4\right)\left(1+x^4\right)^{3/2}}\,dx$$
$$= -\frac{x}{2\sqrt{1+x^4}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} + \frac{1}{2}\int \frac{3-x^4}{\left(-1+x^4\right)\sqrt{1+x^4}}\,dx$$
$$= -\frac{x}{2\sqrt{1+x^4}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} - \frac{1}{2}\int \frac{1}{\sqrt{1+x^4}}\,dx + \int \frac{1}{\left(-1+x^4\right)\cdots}$$
$$= -\frac{x}{2\sqrt{1+x^4}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{1+x^4}} - \frac{1}{2}\int \frac{1}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx - \frac{1}{2}\int \cdots$$
$$= -\frac{x}{2\sqrt{1+x^4}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{1+x^4}} - 2\left(\frac{1}{4}\int \frac{1}{\sqrt{1+x^4}}\,dx\right) - \frac{1}{4}\int \frac{\cdots}{\left(1+\cdots\right.}$$
$$= -\frac{x}{2\sqrt{1+x^4}} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right) - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1-\cdots}}\right.$$
$$= -\frac{x}{2\sqrt{1+x^4}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{4\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{4\sqrt{2}}$$

Mathematica [C] time = 0.28, size = 119, normalized size = 1.78

$$\frac{x\left(\frac{5\left(x^4+1\right)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-x^4,x^4\right)}{\left(x^4-1\right)\left(2x^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-x^4,x^4\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-x^4,x^4\right)\right)+5F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-x^4,x^4\right)\right)}-1\right)}{2\sqrt{x^4+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^8)/(Sqrt[1 + x^4]*(-1 + x^8)),x]
```

```
[Out] (x*(-1 + (5*(1 + x^4)*AppellF1[1/4, -1/2, 1, 5/4, -x^4, x^4])/((-1 + x^4)*(
5*AppellF1[1/4, -1/2, 1, 5/4, -x^4, x^4] + 2*x^4*(2*AppellF1[5/4, -1/2, 2,
9/4, -x^4, x^4] + AppellF1[5/4, 1/2, 1, 9/4, -x^4, x^4])))))/(2*Sqrt[1 + x^
4])
```

IntegrateAlgebraic [A] time = 0.36, size = 67, normalized size = 1.00

$$-\frac{x}{2\sqrt{x^4+1}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{4\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^8)/(Sqrt[1 + x^4]*(-1 + x^8)),x]
```

```
[Out] -1/2*x/Sqrt[1 + x^4] - ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(4*Sqrt[2]) - ArcT
anh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(4*Sqrt[2])
```

fricas [A] time = 0.52, size = 90, normalized size = 1.34

$$-\frac{2\sqrt{2}\left(x^4+1\right)\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)-\sqrt{2}\left(x^4+1\right)\log\left(\frac{x^4-2\sqrt{2}\sqrt{x^4+1}x+2x^2+1}{x^4-2x^2+1}\right)+8\sqrt{x^4+1}x}{16\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)/(x^4+1)^(1/2)/(x^8-1),x, algorithm="fricas")
```

```
[Out] -1/16*(2*sqrt(2)*(x^4 + 1)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) - sqrt(2)*(x^4 +
 1)*log((x^4 - 2*sqrt(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1)) +
8*sqrt(x^4 + 1)*x)/(x^4 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8+1}{\left(x^8-1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)/(x^4+1)^(1/2)/(x^8-1),x, algorithm="giac")
```

```
[Out] integrate((x^8 + 1)/((x^8 - 1)*sqrt(x^4 + 1)), x)
```

maple [C] time = 0.04, size = 172, normalized size = 2.57

$$\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{x}{2\sqrt{x^4+1}}+\frac{(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticPi}\left((-1)^{\frac{1}{4}}x,-i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{2\sqrt{x^4+1}}+\frac{(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticPi}\left((-1)^{\frac{1}{4}}x,i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8+1)/(x^4+1)^(1/2)/(x^8-1),x)
```

```
[Out] 1/2/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/
2)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-1/2*x/(x^4+1)^(1/2)+1/2*(-1)^
(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x
,-I,(-I)^(1/2)/(-1)^(1/4))+1/2*(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(
x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8+1}{\left(x^8-1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)/(x^4+1)^(1/2)/(x^8-1),x, algorithm="maxima")
```

```
[Out] integrate((x^8 + 1)/((x^8 - 1)*sqrt(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8+1}{\sqrt{x^4+1}\,\left(x^8-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 + 1)/((x^4 + 1)^(1/2)*(x^8 - 1)),x)
```

```
[Out] int((x^8 + 1)/((x^4 + 1)^(1/2)*(x^8 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8+1}{(x-1)(x+1)\left(x^2+1\right)\left(x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8+1)/(x**4+1)**(1/2)/(x**8-1),x)
```

```
[Out] Integral((x**8 + 1)/((x - 1)*(x + 1)*(x**2 + 1)*(x**4 + 1)**(3/2)), x)
```

3.834 $\int \frac{-1+x^{12}}{\sqrt{1+x^4}\left(1+x^{12}\right)}\,dx$

Optimal. Leaf size=67

$$-\frac{x}{3\sqrt{x^4+1}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{3}x}{\sqrt{x^4+1}}\right)}{3\sqrt[4]{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{3}x}{\sqrt{x^4+1}}\right)}{3\sqrt[4]{3}}$$

Rubi [C] time = 3.22, antiderivative size = 1382, normalized size of antiderivative = 20.63, number of steps used = 25, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6725, 220, 2073, 414, 523, 409, 1217, 1707}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^12)/(Sqrt[1 + x^4]*(1 + x^12)),x]
```

```
[Out] (2*x)/(Sqrt[3]*(3*I - Sqrt[3])*Sqrt[1 + x^4]) - (2*x)/(Sqrt[3]*(3*I + Sqrt[
3])*Sqrt[1 + x^4]) + ((2*I)*2^(1/4)*ArcTan[(Sqrt[3 - I*Sqrt[3]]*x)/((2*(1 -
 I*Sqrt[3]))^(1/4)*Sqrt[1 + x^4])])/(Sqrt[3]*(1 - I*Sqrt[3])^(3/4)*(3 - I*S
qrt[3])^(3/2)) - ((2*I)*2^(1/4)*ArcTan[(Sqrt[-3 + I*Sqrt[3]]*x)/((2*(1 - I*
Sqrt[3]))^(1/4)*Sqrt[1 + x^4])])/(Sqrt[3]*(1 - I*Sqrt[3])^(3/4)*(-3 + I*Sqr
t[3])^(3/2)) + ((2*I)*2^(1/4)*ArcTan[(Sqrt[-3 - I*Sqrt[3]]*x)/((2*(1 + I*Sq
rt[3]))^(1/4)*Sqrt[1 + x^4])])/(Sqrt[3]*(-3 - I*Sqrt[3])^(3/2)*(1 + I*Sqrt[
3])^(3/4)) - ((2*I)*2^(1/4)*ArcTan[(Sqrt[3 + I*Sqrt[3]]*x)/((2*(1 + I*Sqrt[
3]))^(1/4)*Sqrt[1 + x^4])])/(Sqrt[3]*(1 + I*Sqrt[3])^(3/4)*(3 + I*Sqrt[3])^
(3/2)) + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2]
)/(2*Sqrt[1 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*Ar
cTan[x], 1/2])/(Sqrt[3]*(3*I - Sqrt[3])*Sqrt[1 + x^4]) - ((1/6 - I/6)*(1 -
Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])
/((I - Sqrt[3])*Sqrt[1 + x^4]) + ((1/6 + I/6)*(1 - Sqrt[3])*(1 + x^2)*Sqrt[
(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/((I + Sqrt[3])*Sqrt[1 +
 x^4]) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2]
)/(Sqrt[3]*(3*I + Sqrt[3])*Sqrt[1 + x^4]) + ((1 + 1/Sqrt[(1 - I*Sqrt[3])/2]
)*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(Sqrt[
3]*(3*I - Sqrt[3])*Sqrt[1 + x^4]) - ((1 + 1/Sqrt[(1 + I*Sqrt[3])/2])*(1 + x
^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(Sqrt[3]*(3*I
+ Sqrt[3])*Sqrt[1 + x^4]) + ((3 + Sqrt[3]*(3*I + (2*I)*Sqrt[2 - (2*I)*Sqrt[
3]]))*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticPi[-1/4*(Sqrt[2] - Sqrt
[1 - I*Sqrt[3]])^2/Sqrt[2*(1 - I*Sqrt[3])], 2*ArcTan[x], 1/2])/(12*(3 - I*S
qrt[3])*Sqrt[1 + x^4]) + ((3*I - Sqrt[3]*(3 - 2*Sqrt[2 - (2*I)*Sqrt[3]]))*(
1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticPi[(Sqrt[2] + Sqrt[1 - I*Sqrt[
3]])^2/(4*Sqrt[2*(1 - I*Sqrt[3])]), 2*ArcTan[x], 1/2])/(12*(3*I + Sqrt[3])*
Sqrt[1 + x^4]) + ((3*I + 3*Sqrt[3] + 2*Sqrt[6 + (6*I)*Sqrt[3]])*(1 + x^2)*S
qrt[(1 + x^4)/(1 + x^2)^2]*EllipticPi[-1/4*(Sqrt[2] - Sqrt[1 + I*Sqrt[3]])^
2/Sqrt[2*(1 + I*Sqrt[3])], 2*ArcTan[x], 1/2])/(12*(3*I - Sqrt[3])*Sqrt[1 +
x^4]) + ((3*I + Sqrt[3]*(3 - 2*Sqrt[2 + (2*I)*Sqrt[3]]))*(1 + x^2)*Sqrt[(1
+ x^4)/(1 + x^2)^2]*EllipticPi[(Sqrt[2] + Sqrt[1 + I*Sqrt[3]])^2/(4*Sqrt[2*
(1 + I*Sqrt[3])]), 2*ArcTan[x], 1/2])/(12*(3*I - Sqrt[3])*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
```

```
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+x^{12}}{\sqrt{1+x^4}\left(1+x^{12}\right)}\,dx = \int \left(\frac{1}{\sqrt{1+x^4}} - \frac{2}{\sqrt{1+x^4}\left(1+x^{12}\right)}\right) dx$$

$$= -\left(2\int \frac{1}{\sqrt{1+x^4}\left(1+x^{12}\right)}\,dx\right) + \int \frac{1}{\sqrt{1+x^4}}\,dx$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} - 2\int \left(\frac{2i}{\sqrt{3}\left(1+i\sqrt{3}-2x^4\right)\left(1+x^4\right)^{3/2}} + \frac{}{\sqrt{3}\,(}\right.$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} - \frac{(4i)\int \frac{1}{\left(1+i\sqrt{3}-2x^4\right)\left(1+x^4\right)^{3/2}}\,dx}{\sqrt{3}} - \frac{(4i)\int \frac{}{\left(1+x^4\right)^{3/2}(}}{\sqrt{}}$$

$$= \frac{2x}{\sqrt{3}\left(3i-\sqrt{3}\right)\sqrt{1+x^4}} - \frac{2x}{\sqrt{3}\left(3i+\sqrt{3}\right)\sqrt{1+x^4}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x\right.}{2\sqrt{1+x^4}}$$

$$= \frac{2x}{\sqrt{3}\left(3i-\sqrt{3}\right)\sqrt{1+x^4}} - \frac{2x}{\sqrt{3}\left(3i+\sqrt{3}\right)\sqrt{1+x^4}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x\right.}{2\sqrt{1+x^4}}$$

$$= \frac{2x}{\sqrt{3}\left(3i-\sqrt{3}\right)\sqrt{1+x^4}} - \frac{2x}{\sqrt{3}\left(3i+\sqrt{3}\right)\sqrt{1+x^4}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x\right.}{2\sqrt{1+x^4}}$$

$$= \frac{2x}{\sqrt{3}\left(3i-\sqrt{3}\right)\sqrt{1+x^4}} - \frac{2x}{\sqrt{3}\left(3i+\sqrt{3}\right)\sqrt{1+x^4}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x\right.}{2\sqrt{1+x^4}}$$

$$= \frac{2x}{\sqrt{3}\left(3i-\sqrt{3}\right)\sqrt{1+x^4}} - \frac{2x}{\sqrt{3}\left(3i+\sqrt{3}\right)\sqrt{1+x^4}} - \frac{\sqrt[4]{1-i\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{3-i\sqrt{3}}\,x}{\sqrt[4]{2\left(1-i\sqrt{3}\right)}\,\sqrt{}}\right.}{3\,2^{3/4}\sqrt{3-i\sqrt{3}}}$$

Mathematica [C] time = 0.81, size = 175, normalized size = 2.61

$$\frac{1}{3}\left(-\frac{(-1)^{2/3}x}{\sqrt{x^4+1}} + \frac{\sqrt[3]{-1}\,x}{\sqrt{x^4+1}} - \frac{2x}{\sqrt{x^4+1}} - 2\sqrt[4]{-1}F\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right)|-1\right) + \sqrt[4]{-1}\Pi\left(-\sqrt[3]{-1};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)|-1\right) + \sqrt[4]{-1}\Pi\left(\sqrt[3]{-1};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)|-1\right) + \sqrt[4]{-1}\Pi\left(-(-1)^{2/3};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)|-1\right) + \sqrt[4]{-1}\Pi\left((-1)^{2/3};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^12)/(Sqrt[1 + x^4]*(1 + x^12)),x]
```

```
[Out] ((-2*x)/Sqrt[1 + x^4] + ((-1)^(1/3)*x)/Sqrt[1 + x^4] - ((-1)^(2/3)*x)/Sqrt[
1 + x^4] - 2*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] + (-1)^(1/4)
*EllipticPi[-(-1)^(1/3), I*ArcSinh[(-1)^(1/4)*x], -1] + (-1)^(1/4)*Elliptic
Pi[(-1)^(1/3), I*ArcSinh[(-1)^(1/4)*x], -1] + (-1)^(1/4)*EllipticPi[-(-1)^(
2/3), I*ArcSinh[(-1)^(1/4)*x], -1] + (-1)^(1/4)*EllipticPi[(-1)^(2/3), I*Ar
cSinh[(-1)^(1/4)*x], -1])/3
```

IntegrateAlgebraic [A] time = 0.51, size = 67, normalized size = 1.00

$$-\frac{x}{3\sqrt{x^4+1}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{3}x}{\sqrt{x^4+1}}\right)}{3\sqrt[4]{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{3}x}{\sqrt{x^4+1}}\right)}{3\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^12)/(Sqrt[1 + x^4]*(1 + x^12)),x]
```

```
[Out] -1/3*x/Sqrt[1 + x^4] - ArcTan[(3^(1/4)*x)/Sqrt[1 + x^4]]/(3*3^(1/4)) - ArcT
anh[(3^(1/4)*x)/Sqrt[1 + x^4]]/(3*3^(1/4))
```

fricas [B] time = 0.63, size = 247, normalized size = 3.69

$$-\frac{4\cdot 3^{\frac{3}{4}}(x^4+1)\arctan\left(\frac{3^{\frac{3}{4}}\left(2\cdot 3^{\frac{3}{4}}(x^6+x^2)+3^{\frac{1}{4}}(x^8+5x^4+1)\right)+6\sqrt{x^4+1}\left(3^{\frac{3}{4}}x^3+3^{\frac{1}{4}}(x^5+x)\right)}{3(x^8-x^4+1)}\right)+3^{\frac{3}{4}}(x^4+1)\log\left(-\frac{3^{\frac{3}{4}}(x^8+5x^4+1)+6(x^5+\sqrt{3}x^3+x)\sqrt{x^4+1}+6\cdot 3^{\frac{1}{4}}(x^6+x^2)}{x^8-x^4+1}\right)-3^{\frac{3}{4}}(x^4+1)\log\left(\frac{3^{\frac{3}{4}}(x^8+5x^4+1)-6(x^5+\sqrt{3}x^3+x)\sqrt{x^4+1}+6\cdot 3^{\frac{1}{4}}(x^6+x^2)}{x^8-x^4+1}\right)+12\sqrt{x^4+1}\,x}{36(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^12-1)/(x^4+1)^(1/2)/(x^12+1),x, algorithm="fricas")
```

```
[Out] -1/36*(4*3^(3/4)*(x^4 + 1)*arctan(1/3*(3^(3/4)*(2*3^(3/4)*(x^6 + x^2) + 3^(
1/4)*(x^8 + 5*x^4 + 1)) + 6*sqrt(x^4 + 1)*(3^(3/4)*x^3 + 3^(1/4)*(x^5 + x))
)/(x^8 - x^4 + 1)) + 3^(3/4)*(x^4 + 1)*log(-(3^(3/4)*(x^8 + 5*x^4 + 1) + 6*
(x^5 + sqrt(3)*x^3 + x)*sqrt(x^4 + 1) + 6*3^(1/4)*(x^6 + x^2))/(x^8 - x^4 +
 1)) - 3^(3/4)*(x^4 + 1)*log((3^(3/4)*(x^8 + 5*x^4 + 1) - 6*(x^5 + sqrt(3)*
x^3 + x)*sqrt(x^4 + 1) + 6*3^(1/4)*(x^6 + x^2))/(x^8 - x^4 + 1)) + 12*sqrt(
x^4 + 1)*x)/(x^4 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}-1}{(x^{12}+1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^12-1)/(x^4+1)^(1/2)/(x^12+1),x, algorithm="giac")
```

```
[Out] integrate((x^12 - 1)/((x^12 + 1)*sqrt(x^4 + 1)), x)
```

maple [C] time = 0.10, size = 193, normalized size = 2.88

$$\frac{2\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{3\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{x}{3\sqrt{x^4+1}}+\frac{\sum_{\_\alpha=\mathrm{RootOf}(\_Z^8-\_Z^4+1)}\_\alpha\left(-\frac{\mathrm{arctanh}\left(\frac{\_\alpha^2(-\_\alpha^6+\_\alpha^2+x^2)}{\sqrt{\_\alpha^4+1}\,\sqrt{x^4+1}}\right)}{\sqrt{\_\alpha^4+1}}+\frac{2(-1)^{\frac{3}{4}}(-\_\alpha^7+\_\alpha^3)\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x,i\_\alpha^6-i\_\alpha^2,i\right)}{\sqrt{x^4+1}}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^12-1)/(x^4+1)^(1/2)/(x^12+1),x)
```

```
[Out] 2/3/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/
2)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-1/3*x/(x^4+1)^(1/2)+1/12*sum(
_alpha*(-1/(_alpha^4+1)^(1/2)*arctanh(_alpha^2*(-_alpha^6+_alpha^2+x^2)/(_a
lpha^4+1)^(1/2)/(x^4+1)^(1/2))+2*(-1)^(3/4)*(-_alpha^7+_alpha^3)*(1-I*x^2)^
(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I*_alpha^6-I*_a
lpha^2,I)),_alpha=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}-1}{(x^{12}+1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^12-1)/(x^4+1)^(1/2)/(x^12+1),x, algorithm="maxima")
```

```
[Out] integrate((x^12 - 1)/((x^12 + 1)*sqrt(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{12}-1}{\sqrt{x^4+1}\,\left(x^{12}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^12 - 1)/((x^4 + 1)^(1/2)*(x^12 + 1)),x)
```

```
[Out] int((x^12 - 1)/((x^4 + 1)^(1/2)*(x^12 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2+1\right)\left(x^2-x+1\right)\left(x^2+x+1\right)\left(x^4-x^2+1\right)}{\left(x^4+1\right)^{\frac{3}{2}}\left(x^8-x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**12-1)/(x**4+1)**(1/2)/(x**12+1),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)*(x**2 - x + 1)*(x**2 + x + 1)*(x**4 - x
**2 + 1)/((x**4 + 1)**(3/2)*(x**8 - x**4 + 1)), x)
```

3.835 $\int \frac{-1+x^2}{\sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=67

$$\frac{2\left(10x^4+55x^2+23\right)}{15\left(\sqrt{x^2+1}+x\right)^{5/2}}+\frac{4\sqrt{x^2+1}\left(x^3+5x\right)}{3\left(\sqrt{x^2+1}+x\right)^{5/2}}$$

Rubi [A] time = 0.80, antiderivative size = 56, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6742, 2122, 30, 2120, 270}

$$\frac{1}{6}\left(\sqrt{x^2+1}+x\right)^{3/2}+\frac{3}{\sqrt{\sqrt{x^2+1}+x}}-\frac{1}{10\left(\sqrt{x^2+1}+x\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(Sqrt[1 + x^2]*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/10*1/(x + Sqrt[1 + x^2])^(5/2) + 3/Sqrt[x + Sqrt[1 + x^2]] + (x + Sqrt[1
 + x^2])^(3/2)/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x^2}{\sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}}\,dx &= \int \left(-\frac{1}{\sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}}+\frac{x^2}{\sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}}\right)dx \\
&= -\int \frac{1}{\sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}}\,dx+\int \frac{x^2}{\sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}}\,dx \\
&= \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^{7/2}}\,dx,x,x+\sqrt{1+x^2}\right)-\mathrm{Subst}\left(\int \frac{1}{x^{3/2}}\,dx,x,x+\sqrt{1+x^2}\right) \\
&= \frac{2}{\sqrt{x+\sqrt{1+x^2}}}+\frac{1}{4}\,\mathrm{Subst}\left(\int \left(\frac{1}{x^{7/2}}-\frac{2}{x^{3/2}}+\sqrt{x}\right)dx,x,x+\sqrt{1+x^2}\right) \\
&= -\frac{1}{10\left(x+\sqrt{1+x^2}\right)^{5/2}}+\frac{3}{\sqrt{x+\sqrt{1+x^2}}}+\frac{1}{6}\left(x+\sqrt{1+x^2}\right)^{3/2}
\end{aligned}
$$

Mathematica [B] time = 1.32, size = 350, normalized size = 5.22

$$
\frac{2\left(x^2+1\right)\left(81920x^{18}+757760x^{16}+2336768x^{14}+3539200x^{12}+2978432x^{10}+1435488x^8+384048x^6+51357x^4+2660x^2+349\sqrt{x^2+1}\,x+81920\sqrt{x^2+1}\,x^{17}+716800\sqrt{x^2+1}\,x^{15}+1988608\sqrt{x^2+1}\,x^{13}+2629376\sqrt{x^2+1}\,x^{11}+1870720\sqrt{x^2+1}\,x^9+730272\sqrt{x^2+1}\,x^7+148176\sqrt{x^2+1}\,x^5+13347\sqrt{x^2+1}\,x^3+23\right)}{15\left(\sqrt{x^2+1}+x\right)^{5/2}\left(x^2+\sqrt{x^2+1}\,x+1\right)\left(16x^6+28x^4+13x^2+5\sqrt{x^2+1}\,x+16\sqrt{x^2+1}\,x^5+20\sqrt{x^2+1}\,x^3+1\right)\left(64x^8+144x^6+104x^4+25x^2+7\sqrt{x^2+1}\,x+64\sqrt{x^2+1}\,x^7+112\sqrt{x^2+1}\,x^5+56\sqrt{x^2+1}\,x^3+1\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(Sqrt[1 + x^2]*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] (2*(1 + x^2)*(23 + 2660*x^2 + 51357*x^4 + 384048*x^6 + 1435488*x^8 + 297843
2*x^10 + 3539200*x^12 + 2336768*x^14 + 757760*x^16 + 81920*x^18 + 349*x*Sqr
t[1 + x^2] + 13347*x^3*Sqrt[1 + x^2] + 148176*x^5*Sqrt[1 + x^2] + 730272*x^
7*Sqrt[1 + x^2] + 1870720*x^9*Sqrt[1 + x^2] + 2629376*x^11*Sqrt[1 + x^2] +
1988608*x^13*Sqrt[1 + x^2] + 716800*x^15*Sqrt[1 + x^2] + 81920*x^17*Sqrt[1
+ x^2]))/(15*(x + Sqrt[1 + x^2])^(5/2)*(1 + x^2 + x*Sqrt[1 + x^2])*(1 + 13*
x^2 + 28*x^4 + 16*x^6 + 5*x*Sqrt[1 + x^2] + 20*x^3*Sqrt[1 + x^2] + 16*x^5*S
qrt[1 + x^2])*(1 + 25*x^2 + 104*x^4 + 144*x^6 + 64*x^8 + 7*x*Sqrt[1 + x^2]
+ 56*x^3*Sqrt[1 + x^2] + 112*x^5*Sqrt[1 + x^2] + 64*x^7*Sqrt[1 + x^2]))
```

IntegrateAlgebraic [A] time = 0.08, size = 67, normalized size = 1.00

$$
\frac{2\left(10x^4+55x^2+23\right)}{15\left(\sqrt{x^2+1}+x\right)^{5/2}}+\frac{4\sqrt{x^2+1}\left(x^3+5x\right)}{3\left(\sqrt{x^2+1}+x\right)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/(Sqrt[1 + x^2]*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] (4*Sqrt[1 + x^2]*(5*x + x^3))/(3*(x + Sqrt[1 + x^2])^(5/2)) + (2*(23 + 55*x
^2 + 10*x^4))/(15*(x + Sqrt[1 + x^2])^(5/2))
```

fricas [A] time = 0.42, size = 38, normalized size = 0.57

$$
\frac{2}{15}\left(3\,x^3-\left(3\,x^2-23\right)\sqrt{x^2+1}-19\,x\right)\sqrt{x+\sqrt{x^2+1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)^(1/2)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] 2/15*(3*x^3 - (3*x^2 - 23)*sqrt(x^2 + 1) - 19*x)*sqrt(x + sqrt(x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^2+1}\,\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)^(1/2)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^2 + 1)*sqrt(x + sqrt(x^2 + 1))), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^2+1}\,\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)^(1/2)/(x+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int((x^2-1)/(x^2+1)^(1/2)/(x+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^2+1}\,\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)^(1/2)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^2 + 1)*sqrt(x + sqrt(x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2-1}{\sqrt{x^2+1}\,\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + 1)^(1/2)*(x + (x^2 + 1)^(1/2))^(1/2)),x)
```

```
[Out] int((x^2 - 1)/((x^2 + 1)^(1/2)*(x + (x^2 + 1)^(1/2))^(1/2)), x)
```

sympy [A] time = 0.58, size = 63, normalized size = 0.94

$$\frac{2x^2}{15\sqrt{x+\sqrt{x^2+1}}} + \frac{8x\sqrt{x^2+1}}{15\sqrt{x+\sqrt{x^2+1}}} + \frac{46}{15\sqrt{x+\sqrt{x^2+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)**(1/2)/(x+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] 2*x**2/(15*sqrt(x + sqrt(x**2 + 1))) + 8*x*sqrt(x**2 + 1)/(15*sqrt(x + sqrt
(x**2 + 1))) + 46/(15*sqrt(x + sqrt(x**2 + 1)))
```

3.836 $\int \frac{\sqrt{1+x^2}}{\sqrt{x+\sqrt{1+x^2}}} dx$

Optimal. Leaf size=67

$$\frac{2(10x^4-5x^2-7)}{15\left(\sqrt{x^2+1}+x\right)^{5/2}}+\frac{4\sqrt{x^2+1}\,(x^3-x)}{3\left(\sqrt{x^2+1}+x\right)^{5/2}}$$

Rubi [A] time = 0.11, antiderivative size = 56, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2122, 270}

$$\frac{1}{6}\left(\sqrt{x^2+1}+x\right)^{3/2}-\frac{1}{\sqrt{\sqrt{x^2+1}+x}}-\frac{1}{10\left(\sqrt{x^2+1}+x\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]/Sqrt[x + Sqrt[1 + x^2]],x]
```

```
[Out] -1/10*1/(x + Sqrt[1 + x^2])^(5/2) - 1/Sqrt[x + Sqrt[1 + x^2]] + (x + Sqrt[1
 + x^2])^(3/2)/6
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^2}}{\sqrt{x+\sqrt{1+x^2}}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^{7/2}}\,dx,x,x+\sqrt{1+x^2}\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{x^{7/2}}+\frac{2}{x^{3/2}}+\sqrt{x}\right)dx,x,x+\sqrt{1+x^2}\right)\\ &= -\frac{1}{10\left(x+\sqrt{1+x^2}\right)^{5/2}}-\frac{1}{\sqrt{x+\sqrt{1+x^2}}}+\frac{1}{6}\left(x+\sqrt{1+x^2}\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.76

$$\frac{5\left(\sqrt{x^2+1}+x\right)^4-30\left(\sqrt{x^2+1}+x\right)^2-3}{30\left(\sqrt{x^2+1}+x\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[x + Sqrt[1 + x^2]],x]
```

```
[Out] (-3 - 30*(x + Sqrt[1 + x^2])^2 + 5*(x + Sqrt[1 + x^2])^4)/(30*(x + Sqrt[1 +
 x^2])^(5/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 67, normalized size = 1.00

$$\frac{2\left(10x^4-5x^2-7\right)}{15\left(\sqrt{x^2+1}+x\right)^{5/2}}+\frac{4\sqrt{x^2+1}\left(x^3-x\right)}{3\left(\sqrt{x^2+1}+x\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^2]/Sqrt[x + Sqrt[1 + x^2]],x]
```

```
[Out] (4*Sqrt[1 + x^2]*(-x + x^3))/(3*(x + Sqrt[1 + x^2])^(5/2)) + (2*(-7 - 5*x^2
 + 10*x^4))/(15*(x + Sqrt[1 + x^2])^(5/2))
```

fricas [A] time = 0.43, size = 38, normalized size = 0.57

$$\frac{2}{15}\left(3x^3-\left(3x^2+7\right)\sqrt{x^2+1}+11x\right)\sqrt{x+\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^3 - (3*x^2 + 7)*sqrt(x^2 + 1) + 11*x)*sqrt(x + sqrt(x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2+1}}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(x + sqrt(x^2 + 1)), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2+1}}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(x+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int((x^2+1)^(1/2)/(x+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2+1}}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(x + sqrt(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^2+1}}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/2)/(x + (x^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int((x^2 + 1)^(1/2)/(x + (x^2 + 1)^(1/2))^(1/2), x)
```

sympy [A] time = 0.51, size = 63, normalized size = 0.94

$$\frac{2x^2}{15\sqrt{x+\sqrt{x^2+1}}}+\frac{8x\sqrt{x^2+1}}{15\sqrt{x+\sqrt{x^2+1}}}-\frac{14}{15\sqrt{x+\sqrt{x^2+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(x+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] 2*x**2/(15*sqrt(x + sqrt(x**2 + 1))) + 8*x*sqrt(x**2 + 1)/(15*sqrt(x + sqrt
(x**2 + 1))) - 14/(15*sqrt(x + sqrt(x**2 + 1)))
```

3.837 $\int x^7 \sqrt[4]{-x+x^4}\,dx$

Optimal. Leaf size=68

$$\frac{7}{192}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x}}\right)-\frac{7}{192}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x}}\right)+\frac{1}{288}\sqrt[4]{x^4-x}\left(32x^8-4x^5-7x^2\right)$$

Rubi [B] time = 0.18, antiderivative size = 145, normalized size of antiderivative = 2.13, number of steps used = 10, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2021, 2024, 2032, 329, 275, 331, 298, 203, 206}

$$\frac{1}{9}\sqrt[4]{x^4-x}\,x^8-\frac{1}{72}\sqrt[4]{x^4-x}\,x^5-\frac{7}{288}\sqrt[4]{x^4-x}\,x^2+\frac{7\left(x^3-1\right)^{3/4}x^{3/4}\tan^{-1}\left(\frac{x^{3/4}}{\sqrt[4]{x^3-1}}\right)}{192\left(x^4-x\right)^{3/4}}-\frac{7\left(x^3-1\right)^{3/4}x^{3/4}\tanh^{-1}\left(\frac{x^{3/4}}{\sqrt[4]{x^3-1}}\right)}{192\left(x^4-x\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(-x + x^4)^(1/4),x]
```

```
[Out] (-7*x^2*(-x + x^4)^(1/4))/288 - (x^5*(-x + x^4)^(1/4))/72 + (x^8*(-x + x^4)
^(1/4))/9 + (7*x^(3/4)*(-1 + x^3)^(3/4)*ArcTan[x^(3/4)/(-1 + x^3)^(1/4)])/(
192*(-x + x^4)^(3/4)) - (7*x^(3/4)*(-1 + x^3)^(3/4)*ArcTanh[x^(3/4)/(-1 + x
^3)^(1/4)])/(192*(-x + x^4)^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^7 \sqrt[4]{-x+x^4}\,dx = \frac{1}{9}x^8\sqrt[4]{-x+x^4} - \frac{1}{12}\int \frac{x^8}{\left(-x+x^4\right)^{3/4}}\,dx$$

$$= -\frac{1}{72}x^5\sqrt[4]{-x+x^4} + \frac{1}{9}x^8\sqrt[4]{-x+x^4} - \frac{7}{96}\int \frac{x^5}{\left(-x+x^4\right)^{3/4}}\,dx$$

$$= -\frac{7}{288}x^2\sqrt[4]{-x+x^4} - \frac{1}{72}x^5\sqrt[4]{-x+x^4} + \frac{1}{9}x^8\sqrt[4]{-x+x^4} - \frac{7}{128}\int \frac{x^2}{\left(-x+x^4\right)^{3/4}}\,dx$$

$$= -\frac{7}{288}x^2\sqrt[4]{-x+x^4} - \frac{1}{72}x^5\sqrt[4]{-x+x^4} + \frac{1}{9}x^8\sqrt[4]{-x+x^4} - \frac{\left(7x^{3/4}\left(-1+x^3\right)^{3/4}\right)\int \frac{x^{5/4}}{\left(-1+x^3\right)^{3/4}}\,dx}{128\left(-x+x^4\right)^{3/4}}$$

$$= -\frac{7}{288}x^2\sqrt[4]{-x+x^4} - \frac{1}{72}x^5\sqrt[4]{-x+x^4} + \frac{1}{9}x^8\sqrt[4]{-x+x^4} - \frac{\left(7x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^8}{\left(-1+x^{1}\right.}\right.}{32\left(-x+x^4\right)^{3/4}}$$

$$= -\frac{7}{288}x^2\sqrt[4]{-x+x^4} - \frac{1}{72}x^5\sqrt[4]{-x+x^4} + \frac{1}{9}x^8\sqrt[4]{-x+x^4} - \frac{\left(7x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{\left(-1+x^4\right.}\right.}{96\left(-x+x^4\right)^{3/4}}$$

$$= -\frac{7}{288}x^2\sqrt[4]{-x+x^4} - \frac{1}{72}x^5\sqrt[4]{-x+x^4} + \frac{1}{9}x^8\sqrt[4]{-x+x^4} - \frac{\left(7x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,d\right.}{96\left(-x+x^4\right)^{3/4}}$$

$$= -\frac{7}{288}x^2\sqrt[4]{-x+x^4} - \frac{1}{72}x^5\sqrt[4]{-x+x^4} + \frac{1}{9}x^8\sqrt[4]{-x+x^4} - \frac{\left(7x^{3/4}\left(-1+x^3\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,d\right.}{192\left(-x+x^4\right)^{3/4}}$$

$$= -\frac{7}{288}x^2\sqrt[4]{-x+x^4} - \frac{1}{72}x^5\sqrt[4]{-x+x^4} + \frac{1}{9}x^8\sqrt[4]{-x+x^4} + \frac{7x^{3/4}\left(-1+x^3\right)^{3/4}\tan^{-1}\left(\frac{x^{3/4}}{\sqrt[4]{-1+x^3}}\right)}{192\left(-x+x^4\right)^{3/4}} -$$

Mathematica [C] time = 0.03, size = 69, normalized size = 1.01

$$\frac{x^2\sqrt[4]{x\left(x^3-1\right)}\left(7\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};x^3\right)+\sqrt[4]{1-x^3}\left(8x^6-x^3-7\right)\right)}{72\sqrt[4]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(-x + x^4)^(1/4),x]
```

```
[Out] (x^2*(x*(-1 + x^3))^(1/4)*((1 - x^3)^(1/4)*(-7 - x^3 + 8*x^6) + 7*Hypergeom
etric2F1[-1/4, 3/4, 7/4, x^3]))/(72*(1 - x^3)^(1/4))
```

IntegrateAlgebraic [A] time = 0.27, size = 68, normalized size = 1.00

$$\frac{7}{192}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x}}\right)-\frac{7}{192}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x}}\right)+\frac{1}{288}\sqrt[4]{x^4-x}\left(32x^8-4x^5-7x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(-x + x^4)^(1/4),x]
```

```
[Out] ((-x + x^4)^(1/4)*(-7*x^2 - 4*x^5 + 32*x^8))/288 + (7*ArcTan[x/(-x + x^4)^(
1/4)])/192 - (7*ArcTanh[x/(-x + x^4)^(1/4)])/192
```

fricas [A] time = 2.31, size = 104, normalized size = 1.53

$$\frac{1}{288}\left(32\,x^{8}-4\,x^{5}-7\,x^{2}\right)\left(x^{4}-x\right)^{\frac{1}{4}}-\frac{7}{384}\,\arctan\left(2\,\left(x^{4}-x\right)^{\frac{1}{4}}x^{2}+2\,\left(x^{4}-x\right)^{\frac{3}{4}}\right)+\frac{7}{384}\,\log\left(2\,x^{3}-2\,\left(x^{4}-x\right)^{\frac{1}{4}}x^{2}+2\,\sqrt{x^{4}-x}\,x-2\,\left(x^{4}-x\right)^{\frac{3}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^4-x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/288*(32*x^8 - 4*x^5 - 7*x^2)*(x^4 - x)^(1/4) - 7/384*arctan(2*(x^4 - x)^(
1/4)*x^2 + 2*(x^4 - x)^(3/4)) + 7/384*log(2*x^3 - 2*(x^4 - x)^(1/4)*x^2 + 2
*sqrt(x^4 - x)*x - 2*(x^4 - x)^(3/4) - 1)
```

giac [A] time = 0.24, size = 88, normalized size = 1.29

$$\frac{1}{288}\left(7\left(\frac{1}{x^{3}}-1\right)^{2}\left(-\frac{1}{x^{3}}+1\right)^{\frac{1}{4}}-18\left(-\frac{1}{x^{3}}+1\right)^{\frac{5}{4}}-21\left(-\frac{1}{x^{3}}+1\right)^{\frac{1}{4}}\right)x^{9}+\frac{7}{192}\,\arctan\left(\left(-\frac{1}{x^{3}}+1\right)^{\frac{1}{4}}\right)+\frac{7}{384}\,\log\left(\left(-\frac{1}{x^{3}}+1\right)^{\frac{1}{4}}+1\right)-\frac{7}{384}\,\log\left(\left|\left(-\frac{1}{x^{3}}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^4-x)^(1/4),x, algorithm="giac")
```

```
[Out] 1/288*(7*(1/x^3 - 1)^2*(-1/x^3 + 1)^(1/4) - 18*(-1/x^3 + 1)^(5/4) - 21*(-1/
x^3 + 1)^(1/4))*x^9 + 7/192*arctan((-1/x^3 + 1)^(1/4)) + 7/384*log((-1/x^3
+ 1)^(1/4) + 1) - 7/384*log(abs((-1/x^3 + 1)^(1/4) - 1))
```

maple [C] time = 3.91, size = 454, normalized size = 6.68

$$\frac{x^{2}\left(32x^{6}-4x^{3}-7\right)\left(x\left(x^{3}-1\right)\right)^{\frac{1}{4}}}{288}+\frac{\left(-\frac{7\ln\left(\frac{2x^{9}+2\left(x^{12}-3x^{9}+3x^{6}-x^{3}\right)^{\frac{1}{4}}x^{6}-5x^{6}+2\sqrt{x^{12}-3x^{9}+3x^{6}-x^{3}}\,x^{3}-4\left(x^{12}-3x^{9}+3x^{6}-x^{3}\right)^{\frac{1}{4}}x^{3}+2\left(x^{12}-3x^{9}+3x^{6}-x^{3}\right)^{\frac{3}{4}}+4x^{3}-2\sqrt{x^{12}-3x^{9}+3x^{6}-x^{3}}+2\left(x^{12}-3x^{9}+3x^{6}-x^{3}\right)^{\frac{1}{4}}-1}{\left(-1+x\right)^{2}\left(x^{2}+x+1\right)^{2}}\right)}{384}-\frac{7\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{-2x^{9}-2\left(x^{12}-3x^{9}+3x^{6}-x^{3}\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{2}+1\right)x^{6}+5x^{6}+2\sqrt{x^{12}-3x^{9}+3x^{6}-x^{3}}\,x^{3}+4\left(x^{12}-3x^{9}+3x^{6}-x^{3}\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{2}+1\right)x^{3}+2\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\left(x^{12}-3x^{9}+3x^{6}-x^{3}\right)^{\frac{3}{4}}-4x^{3}-2\sqrt{x^{12}-3x^{9}+3x^{6}-x^{3}}-2\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\left(x^{12}-3x^{9}+3x^{6}-x^{3}\right)^{\frac{1}{4}}+1}{\left(-1+x\right)^{2}\left(x^{2}+x+1\right)^{2}}\right)}{384}\right)\left(x\left(x^{3}-1\right)\right)^{\frac{1}{4}}\left(x^{3}\left(x^{3}-1\right)^{3}\right)^{\frac{1}{4}}}{x\left(x^{3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(x^4-x)^(1/4),x)
```

```
[Out] 1/288*x^2*(32*x^6-4*x^3-7)*(x*(x^3-1))^(1/4)+(-7/384*ln((2*x^9+2*(x^12-3*x^
9+3*x^6-x^3)^(1/4)*x^6-5*x^6+2*(x^12-3*x^9+3*x^6-x^3)^(1/2)*x^3-4*(x^12-3*x
^9+3*x^6-x^3)^(1/4)*x^3+2*(x^12-3*x^9+3*x^6-x^3)^(3/4)+4*x^3-2*(x^12-3*x^9+
3*x^6-x^3)^(1/2)+2*(x^12-3*x^9+3*x^6-x^3)^(1/4)-1)/(-1+x)^2/(x^2+x+1)^2)-7/
384*RootOf(_Z^2+1)*ln((-2*x^9-2*(x^12-3*x^9+3*x^6-x^3)^(1/4)*RootOf(_Z^2+1)
*x^6+5*x^6+2*(x^12-3*x^9+3*x^6-x^3)^(1/2)*x^3+4*(x^12-3*x^9+3*x^6-x^3)^(1/4
)*RootOf(_Z^2+1)*x^3+2*RootOf(_Z^2+1)*(x^12-3*x^9+3*x^6-x^3)^(3/4)-4*x^3-2*
(x^12-3*x^9+3*x^6-x^3)^(1/2)-2*RootOf(_Z^2+1)*(x^12-3*x^9+3*x^6-x^3)^(1/4)+
1)/(-1+x)^2/(x^2+x+1)^2))/x*(x*(x^3-1))^(1/4)*(x^3*(x^3-1)^3)^(1/4)/(x^3-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^{4}-x\right)^{\frac{1}{4}}x^{7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^4-x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x)^(1/4)*x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^7\,{\left(x^4-x\right)}^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(x^4 - x)^(1/4),x)
```

```
[Out] int(x^7*(x^4 - x)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{7}\sqrt[4]{x\left(x-1\right)\left(x^{2}+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(x**4-x)**(1/4),x)
```

```
[Out] Integral(x**7*(x*(x - 1)*(x**2 + x + 1))**(1/4), x)
```

3.838 $\int \frac{(2+x^2)\sqrt[4]{-1-x^2+x^4}\,(1+x^2+x^4)}{x^6(1+x^2)}\,dx$

Optimal. Leaf size=68

$$-\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2-1}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2-1}}\right)-\frac{2\sqrt[4]{x^4-x^2-1}\,(4x^4+x^2+1)}{5x^5}$$

Rubi [F] time = 6.84, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(2+x^2)\sqrt[4]{-1-x^2+x^4}\,(1+x^2+x^4)}{x^6(1+x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((2 + x^2)*(-1 - x^2 + x^4)^(1/4)*(1 + x^2 + x^4))/(x^6*(1 + x^2)),x]
```

```
[Out] (-2*(-1 - x^2 + x^4)^(1/4)*AppellF1[-1/2, -1/4, -1/4, 1/2, (2*x^2)/(1 + Sqr
t[5]), (2*x^2)/(1 - Sqrt[5])])/(x*(1 - (2*x^2)/(1 - Sqrt[5]))^(1/4)*(1 - (2
*x^2)/(1 + Sqrt[5]))^(1/4)) - ((1 - (2*x^2)/(1 - Sqrt[5]))^(5/4)*(-1 - x^2
+ x^4)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, (-2*(x^2/(1 - Sqrt[5]) - x
^2/(1 + Sqrt[5])))/(1 - (2*x^2)/(1 - Sqrt[5]))])/(3*x^3*(1 - (2*x^2)/(1 + S
qrt[5]))^(1/4)) + (4*(1 - (2*x^2)/(1 - Sqrt[5]))*(-1 - x^2 + x^4)^(1/4)*((3
*(1 - Sqrt[5]) + (13 - 3*Sqrt[5])*x^2 + 2*(1 - Sqrt[5])*x^4)*Gamma[-1/4]*Hy
pergeometric2F1[-1/4, 1, -1/2, (-2*Sqrt[5]*x^2)/(2 + (1 - Sqrt[5])*x^2)] +
4*x^2*(5 - Sqrt[5] + 2*Sqrt[5]*x^2)*Gamma[3/4]*Hypergeometric2F1[3/4, 2, 1/
2, (-2*Sqrt[5]*x^2)/(2 + (1 - Sqrt[5])*x^2)]))/(15*(3 - Sqrt[5])*x^5*(1 + S
qrt[5] - 2*x^2)*Gamma[-1/4]) + Defer[Int][(-1 - x^2 + x^4)^(1/4)/(-1 - x^2)
, x]
```

Rubi steps

$$\int \frac{(2+x^2)\sqrt[4]{-1-x^2+x^4}\,(1+x^2+x^4)}{x^6(1+x^2)}\,dx = \int \left(\frac{2\sqrt[4]{-1-x^2+x^4}}{x^6}+\frac{\sqrt[4]{-1-x^2+x^4}}{x^4}+\frac{2\sqrt[4]{-1-x^2+x^4}}{x^2}+\cdots\right.$$

$$= 2\int \frac{\sqrt[4]{-1-x^2+x^4}}{x^6}\,dx+2\int \frac{\sqrt[4]{-1-x^2+x^4}}{x^2}\,dx+\int \frac{\sqrt[4]{-1-x^2+x^4}}{x^4}\cdots$$

$$= \frac{\sqrt[4]{-1-x^2+x^4}\int \frac{\sqrt[4]{1+\frac{2x^2}{-1-\sqrt{5}}}\sqrt[4]{1+\frac{2x^2}{-1+\sqrt{5}}}}{x^4}\,dx}{\sqrt[4]{1+\frac{2x^2}{-1-\sqrt{5}}}\sqrt[4]{1+\frac{2x^2}{-1+\sqrt{5}}}}+\frac{\left(2\sqrt[4]{-1-x^2+x^4}\right)\cdots}{\sqrt[4]{1+\frac{2x^2}{-1-\cdots}}\cdots}$$

$$= -\frac{2\sqrt[4]{-1-x^2+x^4}\,F_1\left(-\frac{1}{2};-\frac{1}{4},-\frac{1}{4};\frac{1}{2};\frac{2x^2}{1+\sqrt{5}},\frac{2x^2}{1-\sqrt{5}}\right)}{x\sqrt[4]{1-\frac{2x^2}{1-\sqrt{5}}}\sqrt[4]{1-\frac{2x^2}{1+\sqrt{5}}}}-\frac{\left(1-\frac{2x^2}{1-\sqrt{5}}\right)\cdots}{\cdots}$$

Mathematica [F] time = 4.94, size = 0, normalized size = 0.00

$$\int \frac{(2+x^2)\sqrt[4]{-1-x^2+x^4}\,(1+x^2+x^4)}{x^6(1+x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((2 + x^2)*(-1 - x^2 + x^4)^(1/4)*(1 + x^2 + x^4))/(x^6*(1 + x^2)
),x]
```

```
[Out] Integrate[((2 + x^2)*(-1 - x^2 + x^4)^(1/4)*(1 + x^2 + x^4))/(x^6*(1 + x^2)
), x]
```

IntegrateAlgebraic [A] time = 0.26, size = 68, normalized size = 1.00

$$-\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2-1}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2-1}}\right)-\frac{2\sqrt[4]{x^4-x^2-1}\left(4x^4+x^2+1\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^2)*(-1 - x^2 + x^4)^(1/4)*(1 + x^2 + x^4))/(x^6*
(1 + x^2)),x]
```

```
[Out] (-2*(-1 - x^2 + x^4)^(1/4)*(1 + x^2 + 4*x^4))/(5*x^5) - ArcTan[x/(-1 - x^2
+ x^4)^(1/4)] + ArcTanh[x/(-1 - x^2 + x^4)^(1/4)]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)*(x^4-x^2-1)^(1/4)*(x^4+x^2+1)/x^6/(x^2+1),x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^2+1\right)\left(x^4-x^2-1\right)^{\frac{1}{4}}\left(x^2+2\right)}{\left(x^2+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)*(x^4-x^2-1)^(1/4)*(x^4+x^2+1)/x^6/(x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate((x^4 + x^2 + 1)*(x^4 - x^2 - 1)^(1/4)*(x^2 + 2)/((x^2 + 1)*x^6),
x)
```

maple [C] time = 6.62, size = 700, normalized size = 10.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)*(x^4-x^2-1)^(1/4)*(x^4+x^2+1)/x^6/(x^2+1),x)
```

```
[Out] -2/5*(4*x^8-3*x^6-4*x^4-2*x^2-1)/x^5/(x^4-x^2-1)^(3/4)+(1/2*RootOf(_Z^2+1)*
ln(-(-2*x^12-2*RootOf(_Z^2+1)*(x^12-3*x^10+5*x^6-3*x^2-1)^(1/4)*x^9+5*x^10+
4*RootOf(_Z^2+1)*(x^12-3*x^10+5*x^6-3*x^2-1)^(1/4)*x^7+2*(x^12-3*x^10+5*x^6
-3*x^2-1)^(1/2)*x^6+x^8+2*RootOf(_Z^2+1)*(x^12-3*x^10+5*x^6-3*x^2-1)^(3/4)*
x^3+2*RootOf(_Z^2+1)*(x^12-3*x^10+5*x^6-3*x^2-1)^(1/4)*x^5-2*(x^12-3*x^10+5
*x^6-3*x^2-1)^(1/2)*x^4-7*x^6-4*RootOf(_Z^2+1)*(x^12-3*x^10+5*x^6-3*x^2-1)^
```

```
(1/4)*x^3-2*(x^12-3*x^10+5*x^6-3*x^2-1)^(1/2)*x^2-x^4-2*RootOf(_Z^2+1)*(x^1
2-3*x^10+5*x^6-3*x^2-1)^(1/4)*x+3*x^2+1)/(x^2+1)/(x^4-x^2-1)^2)-1/2*ln((-2*
x^12+2*(x^12-3*x^10+5*x^6-3*x^2-1)^(1/4)*x^9+5*x^10-2*(x^12-3*x^10+5*x^6-3*
x^2-1)^(1/2)*x^6-4*(x^12-3*x^10+5*x^6-3*x^2-1)^(1/4)*x^7+x^8+2*(x^12-3*x^10
+5*x^6-3*x^2-1)^(3/4)*x^3+2*(x^12-3*x^10+5*x^6-3*x^2-1)^(1/2)*x^4-2*(x^12-3
*x^10+5*x^6-3*x^2-1)^(1/4)*x^5-7*x^6+2*(x^12-3*x^10+5*x^6-3*x^2-1)^(1/2)*x^
2+4*(x^12-3*x^10+5*x^6-3*x^2-1)^(1/4)*x^3-x^4+2*(x^12-3*x^10+5*x^6-3*x^2-1)
^(1/4)*x+3*x^2+1)/(x^2+1)/(x^4-x^2-1)^2))/(x^4-x^2-1)^(3/4)*((x^4-x^2-1)^3)
^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^2+1\right)\left(x^4-x^2-1\right)^{\frac{1}{4}}\left(x^2+2\right)}{\left(x^2+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)*(x^4-x^2-1)^(1/4)*(x^4+x^2+1)/x^6/(x^2+1),x, algorithm="m
axima")
```

```
[Out] integrate((x^4 + x^2 + 1)*(x^4 - x^2 - 1)^(1/4)*(x^2 + 2)/((x^2 + 1)*x^6),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+2\right)\,\left(x^4+x^2+1\right)\,{\left(x^4-x^2-1\right)}^{1/4}}{x^6\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 2)*(x^2 + x^4 + 1)*(x^4 - x^2 - 1)^(1/4))/(x^6*(x^2 + 1)),x)
```

```
[Out] int(((x^2 + 2)*(x^2 + x^4 + 1)*(x^4 - x^2 - 1)^(1/4))/(x^6*(x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}+2\right)\left(x^{2}-x+1\right)\left(x^{2}+x+1\right)\sqrt[4]{x^{4}-x^{2}-1}}{x^{6}\left(x^{2}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)*(x**4-x**2-1)**(1/4)*(x**4+x**2+1)/x**6/(x**2+1),x)
```

```
[Out] Integral((x**2 + 2)*(x**2 - x + 1)*(x**2 + x + 1)*(x**4 - x**2 - 1)**(1/4)/
(x**6*(x**2 + 1)), x)
```

3.839 $\int \frac{\sqrt[4]{x^3+x^4}}{x^2(-1+x^2)}\,dx$

Optimal. Leaf size=68

$$\frac{4\sqrt[4]{x^4+x^3}}{x}+\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^3}}\right)-\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^3}}\right)$$

Rubi [A] time = 0.19, antiderivative size = 114, normalized size of antiderivative = 1.68, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2056, 848, 96, 93, 298, 203, 206}

$$\frac{4\sqrt[4]{x^4+x^3}}{x}+\frac{\sqrt[4]{2}\sqrt[4]{x^4+x^3}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{x^{3/4}\sqrt[4]{x+1}}-\frac{\sqrt[4]{2}\sqrt[4]{x^4+x^3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{x^{3/4}\sqrt[4]{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3 + x^4)^(1/4)/(x^2*(-1 + x^2)),x]
```

```
[Out] (4*(x^3 + x^4)^(1/4))/x + (2^(1/4)*(x^3 + x^4)^(1/4)*ArcTan[(2^(1/4)*x^(1/4
))/(1 + x)^(1/4)])/(x^(3/4)*(1 + x)^(1/4)) - (2^(1/4)*(x^3 + x^4)^(1/4)*Arc
Tanh[(2^(1/4)*x^(1/4))/(1 + x)^(1/4)])/(x^(3/4)*(1 + x)^(1/4))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{x^3+x^4}}{x^2\left(-1+x^2\right)}\,dx &= \frac{\sqrt[4]{x^3+x^4}\int \frac{\sqrt[4]{1+x}}{x^{5/4}\left(-1+x^2\right)}\,dx}{x^{3/4}\sqrt[4]{1+x}}\\
&= \frac{\sqrt[4]{x^3+x^4}\int \frac{1}{(-1+x)x^{5/4}(1+x)^{3/4}}\,dx}{x^{3/4}\sqrt[4]{1+x}}\\
&= \frac{4\sqrt[4]{x^3+x^4}}{x}+\frac{\sqrt[4]{x^3+x^4}\int \frac{1}{(-1+x)\sqrt[4]{x}\,(1+x)^{3/4}}\,dx}{x^{3/4}\sqrt[4]{1+x}}\\
&= \frac{4\sqrt[4]{x^3+x^4}}{x}+\frac{\left(4\sqrt[4]{x^3+x^4}\right)\mathrm{Subst}\left(\int \frac{x^2}{-1+2x^4}\,dx,x,\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}}\\
&= \frac{4\sqrt[4]{x^3+x^4}}{x}-\frac{\left(\sqrt{2}\sqrt[4]{x^3+x^4}\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x^2}\,dx,x,\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}}+\frac{\left(\sqrt{2}\sqrt[4]{x^3+x^4}\right)\mathrm{Subst}\left(\int\right.}{x^{3/4}\sqrt[4]{1+}}\\
&= \frac{4\sqrt[4]{x^3+x^4}}{x}+\frac{\sqrt[4]{2}\sqrt[4]{x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}}-\frac{\sqrt[4]{2}\sqrt[4]{x^3+x^4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.62

$$-\frac{4x^2\left(x\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{2x}{x+1}\right)-3(x+1)\right)}{3\left(x^3(x+1)\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3 + x^4)^(1/4)/(x^2*(-1 + x^2)),x]
```

```
[Out] (-4*x^2*(-3*(1 + x) + x*Hypergeometric2F1[3/4, 1, 7/4, (2*x)/(1 + x)]))/(3*
(x^3*(1 + x))^(3/4))
```

IntegrateAlgebraic [A] time = 0.34, size = 68, normalized size = 1.00

$$\frac{4\sqrt[4]{x^4+x^3}}{x}+\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^3}}\right)-\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3 + x^4)^(1/4)/(x^2*(-1 + x^2)),x]
```

```
[Out] (4*(x^3 + x^4)^(1/4))/x + 2^(1/4)*ArcTan[(2^(1/4)*x)/(x^3 + x^4)^(1/4)] - 2
^(1/4)*ArcTanh[(2^(1/4)*x)/(x^3 + x^4)^(1/4)]
```

fricas [B] time = 0.45, size = 128, normalized size = 1.88

$$\frac{4\cdot 8^{\frac{3}{4}} x \arctan\left(\frac{8^{\frac{1}{4}} x \sqrt{\frac{\sqrt{2}x^2+\sqrt{x^4+x^3}}{x^2}} - 8^{\frac{1}{4}}\left(x^4+x^3\right)^{\frac{1}{4}}}{2x}\right) - 8^{\frac{3}{4}} x \log\left(\frac{8^{\frac{3}{4}}x+4\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right) + 8^{\frac{3}{4}}x\log\left(-\frac{8^{\frac{3}{4}}x-4\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right) + 32\left(x^4+x^3\right)^{\frac{1}{4}}}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3)^(1/4)/x^2/(x^2-1),x, algorithm="fricas")
```

```
[Out] 1/8*(4*8^(3/4)*x*arctan(1/2*(8^(1/4)*x*sqrt((sqrt(2)*x^2 + sqrt(x^4 + x^3))
/x^2) - 8^(1/4)*(x^4 + x^3)^(1/4))/x) - 8^(3/4)*x*log((8^(3/4)*x + 4*(x^4 +
 x^3)^(1/4))/x) + 8^(3/4)*x*log(-(8^(3/4)*x - 4*(x^4 + x^3)^(1/4))/x) + 32*
(x^4 + x^3)^(1/4))/x
```

giac [A] time = 0.20, size = 65, normalized size = 0.96

$$-2^{\frac{1}{4}} \arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right) - \frac{1}{2}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right) + \frac{1}{2}\cdot 2^{\frac{1}{4}}\log\left(\left|-2^{\frac{1}{4}}+\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right|\right) + 4\left(\frac{1}{x}+1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3)^(1/4)/x^2/(x^2-1),x, algorithm="giac")
```

```
[Out] -2^(1/4)*arctan(1/2*2^(3/4)*(1/x + 1)^(1/4)) - 1/2*2^(1/4)*log(2^(1/4) + (1
/x + 1)^(1/4)) + 1/2*2^(1/4)*log(abs(-2^(1/4) + (1/x + 1)^(1/4))) + 4*(1/x
+ 1)^(1/4)
```

maple [C] time = 1.54, size = 541, normalized size = 7.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^3)^(1/4)/x^2/(x^2-1),x)
```

```
[Out] 4*(x^3*(1+x))^(1/4)/x+(-1/2*RootOf(_Z^4-2)*ln((2*(x^4+3*x^3+3*x^2+x)^(1/2)*
RootOf(_Z^4-2)^3*x+2*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^4-2)^3+2*(x^4+3*x^
3+3*x^2+x)^(1/4)*RootOf(_Z^4-2)^2*x^2+4*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(_Z
^4-2)^2*x+3*RootOf(_Z^4-2)*x^3+4*(x^4+3*x^3+3*x^2+x)^(3/4)+2*(x^4+3*x^3+3*x
^2+x)^(1/4)*RootOf(_Z^4-2)^2+7*RootOf(_Z^4-2)*x^2+5*RootOf(_Z^4-2)*x+RootOf
(_Z^4-2))/(-1+x)/(1+x)^2)+1/2*RootOf(_Z^2+RootOf(_Z^4-2)^2)*ln((2*(x^4+3*x^
3+3*x^2+x)^(1/2)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2*x+2*(x^4+3*
x^3+3*x^2+x)^(1/2)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2-2*(x^4+3*
x^3+3*x^2+x)^(1/4)*RootOf(_Z^4-2)^2*x^2-3*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^3
-4*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(_Z^4-2)^2*x-7*RootOf(_Z^2+RootOf(_Z^4-2
)^2)*x^2+4*(x^4+3*x^3+3*x^2+x)^(3/4)-2*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(_Z^
4-2)^2-5*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x-RootOf(_Z^2+RootOf(_Z^4-2)^2))/(-1
+x)/(1+x)^2))*(x^3*(1+x))^(1/4)/x*(x*(1+x)^3)^(1/4)/(1+x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3\right)^{\frac{1}{4}}}{\left(x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3)^(1/4)/x^2/(x^2-1),x, algorithm="maxima")
[Out] integrate((x^4 + x^3)^(1/4)/((x^2 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(x^4+x^3\right)^{1/4}}{x^2-x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + x^4)^(1/4)/(x^2*(x^2 - 1)),x)
[Out] -int((x^3 + x^4)^(1/4)/(x^2 - x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\,(x+1)}}{x^2\,(x-1)\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**3)**(1/4)/x**2/(x**2-1),x)
[Out] Integral((x**3*(x + 1))**(1/4)/(x**2*(x - 1)*(x + 1)), x)
```

3.840 $\int \frac{b+2ax^4}{x^2\left(b+ax^4\right)^{3/4}}\,dx$

Optimal. Leaf size=68

$$-\frac{\sqrt[4]{ax^4+b}}{x}-\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {451, 331, 298, 203, 206}

$$-\frac{\sqrt[4]{ax^4+b}}{x}-\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*a*x^4)/(x^2*(b + a*x^4)^(3/4)),x]
```

```
[Out] -((b + a*x^4)^(1/4)/x) - a^(1/4)*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)] + a^
(1/4)*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{b+2ax^4}{x^2\left(b+ax^4\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{b+ax^4}}{x}+(2a)\int \frac{x^2}{\left(b+ax^4\right)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{b+ax^4}}{x}+(2a)\,\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right) \\
&= -\frac{\sqrt[4]{b+ax^4}}{x}+\sqrt{a}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)-\sqrt{a}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{a}\,x^2}\,dx\right. \\
&= -\frac{\sqrt[4]{b+ax^4}}{x}-\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)+\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 68, normalized size = 1.00

$$
-\frac{\sqrt[4]{ax^4+b}}{x}-\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*a*x^4)/(x^2*(b + a*x^4)^(3/4)),x]
```

```
[Out] -((b + a*x^4)^(1/4)/x) - a^(1/4)*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)] + a^
(1/4)*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)]
```

IntegrateAlgebraic [A] time = 0.33, size = 68, normalized size = 1.00

$$
-\frac{\sqrt[4]{ax^4+b}}{x}-\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + 2*a*x^4)/(x^2*(b + a*x^4)^(3/4)),x]
```

```
[Out] -((b + a*x^4)^(1/4)/x) - a^(1/4)*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)] + a^
(1/4)*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^4+b)/x^2/(a*x^4+b)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{2\,ax^4+b}{\left(ax^4+b\right)^{\frac{3}{4}}x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^4+b)/x^2/(a*x^4+b)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((2*a*x^4 + b)/((a*x^4 + b)^(3/4)*x^2), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{2a\,x^{4}+b}{x^{2}\left(a\,x^{4}+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x^4+b)/x^2/(a*x^4+b)^(3/4),x)
[Out] int((2*a*x^4+b)/x^2/(a*x^4+b)^(3/4),x)
```

maxima [A] time = 0.41, size = 86, normalized size = 1.26

$$\frac{1}{2}\,a\left(\frac{2\,\arctan\left(\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{3}{4}}}-\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{3}{4}}}\right)-\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^4+b)/x^2/(a*x^4+b)^(3/4),x, algorithm="maxima")
[Out] 1/2*a*(2*arctan((a*x^4 + b)^(1/4)/(a^(1/4)*x))/a^(3/4) - log(-(a^(1/4) - (a
*x^4 + b)^(1/4)/x)/(a^(1/4) + (a*x^4 + b)^(1/4)/x))/a^(3/4)) - (a*x^4 + b)^
(1/4)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,a\,x^{4}+b}{x^{2}\,{\left(a\,x^{4}+b\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*a*x^4)/(x^2*(b + a*x^4)^(3/4)),x)
[Out] int((b + 2*a*x^4)/(x^2*(b + a*x^4)^(3/4)), x)
```

sympy [C] time = 1.98, size = 70, normalized size = 1.03

$$\frac{\sqrt[4]{a}\,\sqrt[4]{1+\frac{b}{ax^{4}}}\,\Gamma\left(-\frac{1}{4}\right)}{4\Gamma\left(\frac{3}{4}\right)}+\frac{ax^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{ax^{4}e^{i\pi}}{b}\right)}{2b^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x**4+b)/x**2/(a*x**4+b)**(3/4),x)
[Out] a**(1/4)*(1 + b/(a*x**4))**(1/4)*gamma(-1/4)/(4*gamma(3/4)) + a*x**3*gamma(
3/4)*hyper((3/4, 3/4), (7/4,), a*x**4*exp_polar(I*pi)/b)/(2*b**(3/4)*gamma(
7/4))
```

3.841 $\int \frac{(1+x^2)\sqrt[4]{x^2+x^6}}{x^2(-1+x^2)}\,dx$

Optimal. Leaf size=68

$$\frac{2\sqrt[4]{x^6+x^2}}{x}+\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^6+x^2}}\right)-\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^6+x^2}}\right)$$

Rubi [C] time = 0.76, antiderivative size = 147, normalized size of antiderivative = 2.16, number of steps used = 20, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2056, 6725, 277, 329, 364, 1312, 1336, 325, 1337, 466, 510}

$$-\frac{8\sqrt[4]{x^6+x^2}\,xF_1\left(\frac{3}{8};1,\frac{3}{4};\frac{11}{8};x^4,-x^4\right)}{3\sqrt[4]{x^4+1}}-\frac{8\sqrt[4]{x^6+x^2}\,x^3F_1\left(\frac{7}{8};1,\frac{3}{4};\frac{15}{8};x^4,-x^4\right)}{7\sqrt[4]{x^4+1}}+\frac{4\sqrt[4]{x^6+x^2}\,x\,{}_2F_1\left(\frac{3}{8},\frac{3}{4};\frac{11}{8};-x^4\right)}{3\sqrt[4]{x^4+1}}+\frac{2\sqrt[4]{x^6+x^2}}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((1 + x^2)*(x^2 + x^6)^(1/4))/(x^2*(-1 + x^2)),x]
```

```
[Out] (2*(x^2 + x^6)^(1/4))/x - (8*x*(x^2 + x^6)^(1/4)*AppellF1[3/8, 1, 3/4, 11/8
, x^4, -x^4])/(3*(1 + x^4)^(1/4)) - (8*x^3*(x^2 + x^6)^(1/4)*AppellF1[7/8,
1, 3/4, 15/8, x^4, -x^4])/(7*(1 + x^4)^(1/4)) + (4*x*(x^2 + x^6)^(1/4)*Hype
rgeometric2F1[3/8, 3/4, 11/8, -x^4])/(3*(1 + x^4)^(1/4))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
```

```
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1312

```
Int[(((f_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^4)^(p_.))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a + c*x^4)^(p - 1)
, x], x] - Dist[(c*d^2 + a*e^2)/(d*e*f^2), Int[((f*x)^(m + 2)*(a + c*x^4)^(
p - 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && Lt
Q[m, 0]
```

Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 1337

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^4)^p, (d
/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4))^(-q), x], x], x] /; FreeQ[{a, c
, d, e, f, m, p}, x] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(1+x^2\right)\sqrt[4]{x^2+x^6}}{x^2\left(-1+x^2\right)}\,dx = \frac{\sqrt[4]{x^2+x^6}\int \frac{\left(1+x^2\right)\sqrt[4]{1+x^4}}{x^{3/2}\left(-1+x^2\right)}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\sqrt[4]{x^2+x^6}\int \left(\frac{\sqrt[4]{1+x^4}}{x^{3/2}}+\frac{2\sqrt[4]{1+x^4}}{x^{3/2}\left(-1+x^2\right)}\right)dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\sqrt[4]{x^2+x^6}\int \frac{\sqrt[4]{1+x^4}}{x^{3/2}}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{\sqrt[4]{1+x^4}}{x^{3/2}\left(-1+x^2\right)}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= -\frac{2\sqrt[4]{x^2+x^6}}{x} + \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{x^{5/2}}{\left(1+x^4\right)^{3/4}}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} - \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{1-x^2}{x^{3/2}\left(1+x^4\right)^{3/4}}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(4\sqrt[4]{\right.}{}}{}$$

$$= -\frac{2\sqrt[4]{x^2+x^6}}{x} - \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \left(\frac{1}{x^{3/2}\left(1+x^4\right)^{3/4}}-\frac{\sqrt{x}}{\left(1+x^4\right)^{3/4}}\right)dx}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(4\sqrt[4]{x^2+x^6}\right)\int \left(\frac{}{(-1+}\right.}{}$$

$$= -\frac{2\sqrt[4]{x^2+x^6}}{x} + \frac{4x^3\sqrt[4]{x^2+x^6}\,{}_2F_1\left(\frac{3}{4},\frac{7}{8};\frac{15}{8};-x^4\right)}{7\sqrt[4]{1+x^4}} - \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{1}{x^{3/2}\left(1+x^4\right)^{3/4}}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} +$$

$$= \frac{2\sqrt[4]{x^2+x^6}}{x} + \frac{4x^3\sqrt[4]{x^2+x^6}\,{}_2F_1\left(\frac{3}{4},\frac{7}{8};\frac{15}{8};-x^4\right)}{7\sqrt[4]{1+x^4}} - \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{x^{5/2}}{\left(1+x^4\right)^{3/4}}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(4\sqrt[4]{\right.}{}}{}$$

$$= \frac{2\sqrt[4]{x^2+x^6}}{x} - \frac{8x\sqrt[4]{x^2+x^6}\,F_1\left(\frac{3}{8};1,\frac{3}{4};\frac{11}{8};x^4,-x^4\right)}{3\sqrt[4]{1+x^4}} - \frac{8x^3\sqrt[4]{x^2+x^6}\,F_1\left(\frac{7}{8};1,\frac{3}{4};\frac{15}{8};x^4,\right.}{7\sqrt[4]{1+x^4}}$$

$$= \frac{2\sqrt[4]{x^2+x^6}}{x} - \frac{8x\sqrt[4]{x^2+x^6}\,F_1\left(\frac{3}{8};1,\frac{3}{4};\frac{11}{8};x^4,-x^4\right)}{3\sqrt[4]{1+x^4}} - \frac{8x^3\sqrt[4]{x^2+x^6}\,F_1\left(\frac{7}{8};1,\frac{3}{4};\frac{15}{8};x^4,\right.}{7\sqrt[4]{1+x^4}}$$

Mathematica [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{\left(1+x^2\right)\sqrt[4]{x^2+x^6}}{x^2\left(-1+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^2)*(x^2 + x^6)^(1/4))/(x^2*(-1 + x^2)),x]
[Out] Integrate[((1 + x^2)*(x^2 + x^6)^(1/4))/(x^2*(-1 + x^2)), x]
```

IntegrateAlgebraic [A] time = 0.39, size = 68, normalized size = 1.00

$$\frac{2\sqrt[4]{x^6+x^2}}{x} + \sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^6+x^2}}\right) - \sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^6+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^2)*(x^2 + x^6)^(1/4))/(x^2*(-1 + x^2)),x]
[Out] (2*(x^2 + x^6)^(1/4))/x + 2^(1/4)*ArcTan[(2^(1/4)*x)/(x^2 + x^6)^(1/4)] - 2
^(1/4)*ArcTanh[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]
```

fricas [B] time = 12.94, size = 262, normalized size = 3.85

$$\frac{4\cdot 2^{\frac{1}{4}}x\arctan\left(\frac{4\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2+2^{\frac{3}{4}}\left(2\cdot 2^{\frac{3}{4}}\sqrt{x^6+x^2}\,x+2^{\frac{1}{4}}\left(x^5+2x^3+x\right)\right)+4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}}{2\left(x^5-2x^3+x\right)}\right)-2^{\frac{1}{4}}x\log\left(-\frac{4\sqrt{2}\left(x^6+x^2\right)^{\frac{1}{4}}x^2+2^{\frac{3}{4}}\left(x^5+2x^3+x\right)+4\cdot 2^{\frac{1}{4}}\sqrt{x^6+x^2}\,x+4\left(x^6+x^2\right)^{\frac{3}{4}}}{x^5-2x^3+x}\right)+2^{\frac{1}{4}}x\log\left(-\frac{4\sqrt{2}\left(x^6+x^2\right)^{\frac{1}{4}}x^2-2^{\frac{3}{4}}\left(x^5+2x^3+x\right)-4\cdot 2^{\frac{1}{4}}\sqrt{x^6+x^2}\,x+4\left(x^6+x^2\right)^{\frac{3}{4}}}{x^5-2x^3+x}\right)+8\left(x^6+x^2\right)^{\frac{1}{4}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^6+x^2)^(1/4)/x^2/(x^2-1),x, algorithm="fricas")
```

```
[Out] 1/4*(4*2^(1/4)*x*arctan(1/2*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + 2^(3/4)*(2*2
^(3/4)*sqrt(x^6 + x^2)*x + 2^(1/4)*(x^5 + 2*x^3 + x)) + 4*2^(1/4)*(x^6 + x^
2)^(3/4))/(x^5 - 2*x^3 + x)) - 2^(1/4)*x*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4)*
x^2 + 2^(3/4)*(x^5 + 2*x^3 + x) + 4*2^(1/4)*sqrt(x^6 + x^2)*x + 4*(x^6 + x^
2)^(3/4))/(x^5 - 2*x^3 + x)) + 2^(1/4)*x*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4)*
x^2 - 2^(3/4)*(x^5 + 2*x^3 + x) - 4*2^(1/4)*sqrt(x^6 + x^2)*x + 4*(x^6 + x^
2)^(3/4))/(x^5 - 2*x^3 + x)) + 8*(x^6 + x^2)^(1/4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^2+1\right)}{\left(x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^6+x^2)^(1/4)/x^2/(x^2-1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^2)^(1/4)*(x^2 + 1)/((x^2 - 1)*x^2), x)
```

maple [F] time = 5.24, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)\left(x^6+x^2\right)^{\frac{1}{4}}}{x^2\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^6+x^2)^(1/4)/x^2/(x^2-1),x)
```

```
[Out] int((x^2+1)*(x^6+x^2)^(1/4)/x^2/(x^2-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^2+1\right)}{\left(x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^6+x^2)^(1/4)/x^2/(x^2-1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + x^2)^(1/4)*(x^2 + 1)/((x^2 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(x^6+x^2\right)}^{1/4}\,\left(x^2+1\right)}{x^2\,\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + x^6)^(1/4)*(x^2 + 1))/(x^2*(x^2 - 1)),x)
```

```
[Out] int(((x^2 + x^6)^(1/4)*(x^2 + 1))/(x^2*(x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\left(x^4+1\right)}\left(x^2+1\right)}{x^2\left(x-1\right)\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**6+x**2)**(1/4)/x**2/(x**2-1),x)
```

```
[Out] Integral((x**2*(x**4 + 1))**(1/4)*(x**2 + 1)/(x**2*(x - 1)*(x + 1)), x)
```

3.842 $\int \frac{\sqrt{-1+x^3}\left(1-x^3+x^6\right)}{x^{10}\left(2+x^3\right)}\,dx$

Optimal. Leaf size=68

$$\frac{13}{24}\tan^{-1}\left(\sqrt{x^3-1}\right)-\frac{7\tan^{-1}\left(\frac{\sqrt{x^3-1}}{\sqrt{3}}\right)}{8\sqrt{3}}+\frac{\sqrt{x^3-1}\left(-12x^6+5x^3-2\right)}{36x^9}$$

Rubi [A] time = 0.51, antiderivative size = 88, normalized size of antiderivative = 1.29, number of steps used = 25, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6725, 266, 47, 51, 63, 203, 50, 444}

$$-\frac{\sqrt{x^3-1}}{3x^3}+\frac{13}{24}\tan^{-1}\left(\sqrt{x^3-1}\right)-\frac{7\tan^{-1}\left(\frac{\sqrt{x^3-1}}{\sqrt{3}}\right)}{8\sqrt{3}}-\frac{\sqrt{x^3-1}}{18x^9}+\frac{5\sqrt{x^3-1}}{36x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^3]*(1 - x^3 + x^6))/(x^10*(2 + x^3)),x]
```

```
[Out] -1/18*Sqrt[-1 + x^3]/x^9 + (5*Sqrt[-1 + x^3])/(36*x^6) - Sqrt[-1 + x^3]/(3*
x^3) + (13*ArcTan[Sqrt[-1 + x^3]])/24 - (7*ArcTan[Sqrt[-1 + x^3]/Sqrt[3]])/
(8*Sqrt[3])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x^3}\left(1-x^3+x^6\right)}{x^{10}\left(2+x^3\right)}\,dx &= \int \left(\frac{\sqrt{-1+x^3}}{2x^{10}}-\frac{3\sqrt{-1+x^3}}{4x^7}+\frac{7\sqrt{-1+x^3}}{8x^4}-\frac{7\sqrt{-1+x^3}}{16x}+\frac{7x^2\sqrt{-1+x^3}}{16\left(2+x^3\right)}\right) d\\
&= -\left(\frac{7}{16}\int \frac{\sqrt{-1+x^3}}{x}\,dx\right)+\frac{7}{16}\int \frac{x^2\sqrt{-1+x^3}}{2+x^3}\,dx+\frac{1}{2}\int \frac{\sqrt{-1+x^3}}{x^{10}}\,dx-\frac{3}{4}\int\\
&= -\left(\frac{7}{48}\operatorname{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx,x,x^3\right)\right)+\frac{7}{48}\operatorname{Subst}\left(\int \frac{\sqrt{-1+x}}{2+x}\,dx,x,x^3\right)+\frac{1}{6}\\
&= -\frac{\sqrt{-1+x^3}}{18x^9}+\frac{\sqrt{-1+x^3}}{8x^6}-\frac{7\sqrt{-1+x^3}}{24x^3}+\frac{1}{36}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^3}\,dx,x,x^3\right)\\
&= -\frac{\sqrt{-1+x^3}}{18x^9}+\frac{5\sqrt{-1+x^3}}{36x^6}-\frac{17\sqrt{-1+x^3}}{48x^3}+\frac{1}{48}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx,x,x\right.\\
&= -\frac{\sqrt{-1+x^3}}{18x^9}+\frac{5\sqrt{-1+x^3}}{36x^6}-\frac{\sqrt{-1+x^3}}{3x^3}+\frac{7}{12}\tan^{-1}\left(\sqrt{-1+x^3}\right)-\frac{7\tan^{-1}\left(\frac{\sqrt{}}{}\right.}{8\sqrt{3}}\\
&= -\frac{\sqrt{-1+x^3}}{18x^9}+\frac{5\sqrt{-1+x^3}}{36x^6}-\frac{\sqrt{-1+x^3}}{3x^3}+\frac{25}{48}\tan^{-1}\left(\sqrt{-1+x^3}\right)-\frac{7\tan^{-1}\left(\frac{\sqrt{}}{}\right.}{8\sqrt{3}}\\
&= -\frac{\sqrt{-1+x^3}}{18x^9}+\frac{5\sqrt{-1+x^3}}{36x^6}-\frac{\sqrt{-1+x^3}}{3x^3}+\frac{13}{24}\tan^{-1}\left(\sqrt{-1+x^3}\right)-\frac{7\tan^{-1}\left(\frac{\sqrt{}}{}\right.}{8\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.18, size = 146, normalized size = 2.15

$$\frac{-12\left(x^3-1\right)^2\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};1-x^3\right)+8\left(x^3-1\right)^2\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};1-x^3\right)+21\left(\frac{1}{x^3}+\sqrt{x^3-1}\tan^{-1}\left(\sqrt{x^3-1}\right)-\sqrt{3}\sqrt{x^3-1}\tan^{-1}\left(\frac{\sqrt{x^3-1}}{\sqrt{3}}\right)-\sqrt{1-x^3}\tanh^{-1}\left(\sqrt{1-x^3}\right)-1\right)}{72\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^3]*(1 - x^3 + x^6))/(x^10*(2 + x^3)),x]
```

```
[Out] (21*(-1 + x^(-3) + Sqrt[-1 + x^3]*ArcTan[Sqrt[-1 + x^3]] - Sqrt[3]*Sqrt[-1
+ x^3]*ArcTan[Sqrt[-1 + x^3]/Sqrt[3]] - Sqrt[1 - x^3]*ArcTanh[Sqrt[1 - x^3]
]) - 12*(-1 + x^3)^2*Hypergeometric2F1[3/2, 3, 5/2, 1 - x^3] + 8*(-1 + x^3)
^2*Hypergeometric2F1[3/2, 4, 5/2, 1 - x^3])/(72*Sqrt[-1 + x^3])
```

IntegrateAlgebraic [A] time = 0.12, size = 68, normalized size = 1.00

$$\frac{13}{24}\tan^{-1}\left(\sqrt{x^3-1}\right)-\frac{7\tan^{-1}\left(\frac{\sqrt{x^3-1}}{\sqrt{3}}\right)}{8\sqrt{3}}+\frac{\sqrt{x^3-1}\left(-12x^6+5x^3-2\right)}{36x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^3]*(1 - x^3 + x^6))/(x^10*(2 + x^3)),x]
```

```
[Out] (Sqrt[-1 + x^3]*(-2 + 5*x^3 - 12*x^6))/(36*x^9) + (13*ArcTan[Sqrt[-1 + x^3]
])/24 - (7*ArcTan[Sqrt[-1 + x^3]/Sqrt[3]])/(8*Sqrt[3])
```

fricas [A] time = 0.43, size = 61, normalized size = 0.90

$$-\frac{21\sqrt{3}\,x^9\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^3-1}\right)-39\,x^9\arctan\left(\sqrt{x^3-1}\right)+2\left(12\,x^6-5\,x^3+2\right)\sqrt{x^3-1}}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)*(x^6-x^3+1)/x^10/(x^3+2),x, algorithm="fricas")
```

```
[Out] -1/72*(21*sqrt(3)*x^9*arctan(1/3*sqrt(3)*sqrt(x^3 - 1)) - 39*x^9*arctan(sqr
t(x^3 - 1)) + 2*(12*x^6 - 5*x^3 + 2)*sqrt(x^3 - 1))/x^9
```

giac [A] time = 0.19, size = 62, normalized size = 0.91

$$-\frac{7}{24}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^3-1}\right)-\frac{12\left(x^3-1\right)^{\frac{5}{2}}+19\left(x^3-1\right)^{\frac{3}{2}}+9\sqrt{x^3-1}}{36\,x^9}+\frac{13}{24}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)*(x^6-x^3+1)/x^10/(x^3+2),x, algorithm="giac")
```

```
[Out] -7/24*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(x^3 - 1)) - 1/36*(12*(x^3 - 1)^(5/2)
+ 19*(x^3 - 1)^(3/2) + 9*sqrt(x^3 - 1))/x^9 + 13/24*arctan(sqrt(x^3 - 1))
```

maple [C] time = 0.41, size = 218, normalized size = 3.21

$$-\frac{7\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^3+2\right)}\frac{\left(\_\alpha^2+\_\alpha+1\right)\left(-3-i\sqrt{3}\right)\sqrt{\frac{-1+x}{-3-i\sqrt{3}}}\,\sqrt{\frac{1+2x-i\sqrt{3}}{3-i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}+2x+1}{3+i\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\_\alpha^2}{2}+\frac{\_\alpha}{2}+\frac{1}{2}+\frac{i\_\alpha^2\sqrt{3}}{6}+\frac{i\sqrt{3}\,\_\alpha}{6}+\frac{i\sqrt{3}}{6},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}\right)}{48}-\frac{\sqrt{x^3-1}}{3x^3}+\frac{13\arctan\left(\sqrt{x^3-1}\right)}{24}-\frac{\sqrt{x^3-1}}{18x^9}+\frac{5\sqrt{x^3-1}}{36x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/2)*(x^6-x^3+1)/x^10/(x^3+2),x)
```

```
[Out] -7/48*2^(1/2)*sum((_alpha^2+_alpha+1)*(-3-I*3^(1/2))*((-1+x)/(-3-I*3^(1/2))
)^(1/2)*((1+2*x-I*3^(1/2))/(3-I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x+1)/(I*3^(1/
2)+3))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1
/2*_alpha^2+1/2*_alpha+1/2+1/6*I*3^(1/2)*_alpha^2+1/6*I*_alpha*3^(1/2)+1/6*
I*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z
```

```
^3+2))-1/3*(x^3-1)^(1/2)/x^3+13/24*arctan((x^3-1)^(1/2))-1/18*(x^3-1)^(1/2)
/x^9+5/36*(x^3-1)^(1/2)/x^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6 - x^3 + 1\right)\sqrt{x^3 - 1}}{\left(x^3 + 2\right)x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)*(x^6-x^3+1)/x^10/(x^3+2),x, algorithm="maxima")
```

```
[Out] integrate((x^6 - x^3 + 1)*sqrt(x^3 - 1)/((x^3 + 2)*x^10), x)
```

mupad [B] time = 2.51, size = 107, normalized size = 1.57

$$\frac{5\sqrt{x^3-1}}{36\,x^6} - \frac{\sqrt{x^3-1}}{3\,x^3} - \frac{\sqrt{x^3-1}}{18\,x^9} + \frac{\ln\left(\frac{\left(\sqrt{x^3-1}-\mathrm{i}\right)\left(\sqrt{x^3-1}+1\mathrm{i}\right)^3}{x^6}\right)13\mathrm{i}}{48} + \frac{\sqrt{3}\,\ln\left(\frac{6\sqrt{x^3-1}-\sqrt{3}\,4\mathrm{i}+\sqrt{3}\,x^3\,1\mathrm{i}}{x^3+2}\right)7\mathrm{i}}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(1/2)*(x^6 - x^3 + 1))/(x^10*(x^3 + 2)),x)
```

```
[Out] (log((((x^3 - 1)^(1/2) - 1i)*((x^3 - 1)^(1/2) + 1i)^3)/x^6)*13i)/48 + (3^(1
/2)*log((3^(1/2)*x^3*1i - 3^(1/2)*4i + 6*(x^3 - 1)^(1/2))/(x^3 + 2))*7i)/48
 - (x^3 - 1)^(1/2)/(3*x^3) + (5*(x^3 - 1)^(1/2))/(36*x^6) - (x^3 - 1)^(1/2)
/(18*x^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/2)*(x**6-x**3+1)/x**10/(x**3+2),x)
```

```
[Out] Timed out
```

3.843 $\int \frac{\sqrt{x^2+x\sqrt{x+x^2}}}{x\sqrt{x+x^2}}\,dx$

Optimal. Leaf size=68

$$\frac{2\sqrt{2}\sqrt{\sqrt{x^2+x}-x}\sqrt{x\left(\sqrt{x^2+x}+x\right)}\tanh^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+x}-x}\right)}{x}$$

Rubi [F] time = 1.92, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{x^2+x\sqrt{x+x^2}}}{x\sqrt{x+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[x^2 + x*Sqrt[x + x^2]]/(x*Sqrt[x + x^2]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 + x]*Defer[Subst][Defer[Int][Sqrt[x^4 + x^2*Sqrt[x^2 + x^
4]]/(x^2*Sqrt[1 + x^2]), x], x, Sqrt[x]])/Sqrt[x + x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x^2+x\sqrt{x+x^2}}}{x\sqrt{x+x^2}}\,dx &= \frac{\left(\sqrt{x}\sqrt{1+x}\right)\int \frac{\sqrt{x^2+x\sqrt{x+x^2}}}{x^{3/2}\sqrt{1+x}}\,dx}{\sqrt{x+x^2}}\\
&= \frac{\left(2\sqrt{x}\sqrt{1+x}\right)\text{Subst}\left(\int \frac{\sqrt{x^4+x^2\sqrt{x^2+x^4}}}{x^2\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)}{\sqrt{x+x^2}}
\end{aligned}$$

Mathematica [A] time = 0.47, size = 107, normalized size = 1.57

$$\frac{2\sqrt{2}(x+1)\sqrt{x\left(x+\sqrt{x(x+1)}\right)}\sqrt{2x+2\sqrt{x(x+1)}+1}\sinh^{-1}\left(\sqrt{2}\sqrt{x+\sqrt{x(x+1)}}\right)}{\sqrt{x(x+1)}\sqrt{x+\sqrt{x(x+1)}}\left(x+\sqrt{x(x+1)}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + x*Sqrt[x + x^2]]/(x*Sqrt[x + x^2]),x]
```

```
[Out] (2*Sqrt[2]*(1 + x)*Sqrt[x*(x + Sqrt[x*(1 + x)])]*Sqrt[1 + 2*x + 2*Sqrt[x*(1
 + x)]]*ArcSinh[Sqrt[2]*Sqrt[x + Sqrt[x*(1 + x)]]])/(Sqrt[x*(1 + x)]*Sqrt[x
 + Sqrt[x*(1 + x)]]*(1 + x + Sqrt[x*(1 + x)]))
```

IntegrateAlgebraic [A] time = 3.93, size = 68, normalized size = 1.00

$$\frac{2\sqrt{2}\sqrt{\sqrt{x^2+x}-x}\sqrt{x\left(\sqrt{x^2+x}+x\right)}\tanh^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+x}-x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2 + x*Sqrt[x + x^2]]/(x*Sqrt[x + x^2]),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[-x + Sqrt[x + x^2]]*Sqrt[x*(x + Sqrt[x + x^2])]*ArcTanh[Sqr
t[2]*Sqrt[-x + Sqrt[x + x^2]]])/x
```

fricas [A] time = 0.44, size = 60, normalized size = 0.88

$$\sqrt{2}\log\left(\frac{4x^2 + 2\sqrt{x^2 + \sqrt{x^2 + x}\,x}\left(\sqrt{2}\,x + \sqrt{2}\sqrt{x^2 + x}\right) + 4\sqrt{x^2 + x}\,x + x}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x*(x^2+x)^(1/2))^(1/2)/x/(x^2+x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(2)*log((4*x^2 + 2*sqrt(x^2 + sqrt(x^2 + x)*x)*(sqrt(2)*x + sqrt(2)*sqr
t(x^2 + x)) + 4*sqrt(x^2 + x)*x + x)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^2 + x}\,x}}{\sqrt{x^2 + x}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x*(x^2+x)^(1/2))^(1/2)/x/(x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^2 + x)*x)/(sqrt(x^2 + x)*x), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + x\sqrt{x^2 + x}}}{x\sqrt{x^2 + x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x*(x^2+x)^(1/2))^(1/2)/x/(x^2+x)^(1/2),x)
```

```
[Out] int((x^2+x*(x^2+x)^(1/2))^(1/2)/x/(x^2+x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^2 + x}\,x}}{\sqrt{x^2 + x}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x*(x^2+x)^(1/2))^(1/2)/x/(x^2+x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^2 + x)*x)/(sqrt(x^2 + x)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^2 + x\sqrt{x^2 + x}}}{x\sqrt{x^2 + x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x*(x + x^2)^(1/2))^(1/2)/(x*(x + x^2)^(1/2)),x)
```

```
[Out] int((x^2 + x*(x + x^2)^(1/2))^(1/2)/(x*(x + x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(x+\sqrt{x^2+x}\right)}}{x\sqrt{x\left(x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x*(x**2+x)**(1/2))**(1/2)/x/(x**2+x)**(1/2),x)
[Out] Integral(sqrt(x*(x + sqrt(x**2 + x)))/(x*sqrt(x*(x + 1))), x)
```

3.844 $\int \frac{(-a+x)(-b+x)\left(3ab-2(a+b)x+x^2\right)}{x^2\sqrt{x(-a+x)(-b+x)}\left(-ab+(a+b)x-x^2+dx^3\right)}\,dx$

Optimal. Leaf size=69

$$\frac{2\sqrt{x^2(-a-b)+abx+x^3}}{x^2}-2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{x^2(-a-b)+abx+x^3}}{\sqrt{d}\,x^2}\right)$$

Rubi [F] time = 23.71, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-a+x)(-b+x)\left(3ab-2(a+b)x+x^2\right)}{x^2\sqrt{x(-a+x)(-b+x)}\left(-ab+(a+b)x-x^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-a + x)*(-b + x)*(3*a*b - 2*(a + b)*x + x^2))/(x^2*Sqrt[x*(-a + x)*(-
b + x)]*(-(a*b) + (a + b)*x - x^2 + d*x^3)),x]
```

```
[Out] (2*(a - x)*(b - x))/(x*Sqrt[(a - x)*(b - x)*x]) + (2*(a + b)*(a - x)*Sqrt[x
]*Sqrt[1 - x/b]*EllipticE[ArcSin[Sqrt[x]/Sqrt[b]], b/a])/(a*Sqrt[b]*Sqrt[(a
 - x)*(b - x)*x]*Sqrt[1 - x/a]) + (2*(a - b)*Sqrt[x]*Sqrt[1 - x/a]*Sqrt[1 -
 x/b]*EllipticF[ArcSin[Sqrt[x]/Sqrt[b]], b/a])/(Sqrt[b]*Sqrt[(a - x)*(b - x
)*x]) - (2*(a^2 - b^2)*Sqrt[x]*Sqrt[1 - x/a]*Sqrt[1 - x/b]*EllipticF[ArcSin
[Sqrt[x]/Sqrt[b]], b/a])/(a*Sqrt[b]*Sqrt[(a - x)*(b - x)*x]) + (2*(a + b -
3*a*b*d)*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][(x^2*Sqr
t[-a + x^2]*Sqrt[-b + x^2])/(a*b - a*(1 + b/a)*x^2 + x^4 - d*x^6), x], x, S
qrt[x]])/(a*b*Sqrt[(a - x)*(b - x)*x]) - (2*(a + b)*d*Sqrt[x]*Sqrt[-a + x]*
Sqrt[-b + x]*Defer[Subst][Defer[Int][(x^4*Sqrt[-a + x^2]*Sqrt[-b + x^2])/(a
*b - a*(1 + b/a)*x^2 + x^4 - d*x^6), x], x, Sqrt[x]])/(a*b*Sqrt[(a - x)*(b
- x)*x]) + (2*(a^2 + b^2)*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][De
fer[Int][(Sqrt[-a + x^2]*Sqrt[-b + x^2])/(-(a*b) + a*(1 + b/a)*x^2 - x^4 +
d*x^6), x], x, Sqrt[x]])/(a*b*Sqrt[(a - x)*(b - x)*x])
```

Rubi steps

$$\int \frac{(-a+x)(-b+x)\left(3ab-2(a+b)x+x^2\right)}{x^2\sqrt{x(-a+x)(-b+x)}\left(-ab+(a+b)x-x^2+dx^3\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{\sqrt{-a+x}\sqrt{-b+x}\left(3ab-2(a+b)x+x^2\right)}{x^{5/2}\left(-ab+(a+b)x-x^2+dx^3\right)}}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{\sqrt{-a+x^2}\sqrt{-b+x^2}\left(3ab\right.}{x^4\left(-ab+(a+b)x^2\right.}\right.}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}\left(\int\left(-\frac{3\sqrt{-a+x^2}\sqrt{-b+x^2}}{x^4}\right.\right.}{}$$

$$= -\frac{\left(6\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{\sqrt{-a+x^2}\sqrt{-b+x^2}}{x^4}\,d\right.}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{2(a+b)(a-x)(b-x)}{ab\sqrt{(a-x)(b-x)x}} + \frac{2(a-x)(b-x)}{x\sqrt{(a-x)(b-x)x}} - \frac{\left(4\sqrt{x}\right.}{}$$

$$= \frac{2(a-x)(b-x)}{x\sqrt{(a-x)(b-x)x}} + \frac{\left(4\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}}{ab\sqrt{x(-a+}}$$

$$= \frac{2(a-x)(b-x)}{x\sqrt{(a-x)(b-x)x}} + \frac{\left(2(a-b)\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)}{b\sqrt{x(-a}}$$

$$= \frac{2(a-x)(b-x)}{x\sqrt{(a-x)(b-x)x}} + \frac{\left(2\left(a^2+b^2\right)\sqrt{x}\sqrt{-a+x}\sqrt{-b+}\right.}{ab\sqrt{x}}$$

$$= \frac{2(a-x)(b-x)}{x\sqrt{(a-x)(b-x)x}} + \frac{4(a+b)(a-x)\sqrt{x}\sqrt{1-\frac{x}{b}}\,E\left(\text{si}\right.}{a\sqrt{b}\sqrt{(a-x)(b-x)x}\sqrt{}}$$

$$= \frac{2(a-x)(b-x)}{x\sqrt{(a-x)(b-x)x}} + \frac{2(a+b)(a-x)\sqrt{x}\sqrt{1-\frac{x}{b}}\,E\left(\text{si}\right.}{a\sqrt{b}\sqrt{(a-x)(b-x)x}\sqrt{}}$$

Mathematica [C] time = 9.72, size = 4112, normalized size = 59.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-a + x)*(-b + x)*(3*a*b - 2*(a + b)*x + x^2))/(x^2*Sqrt[x*(-a +
 x)*(-b + x)]*(-(a*b) + (a + b)*x - x^2 + d*x^3)),x]
```

```
[Out] (2*Sqrt[x*(-a + x)*(-b + x)]*(1 - (I*x*Sqrt[(-b + x)/(a - b)]*(3*a^6*d^2*El
lipticF[I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] - a^3*(-1 + 3*a*d)*(-a + b +
```

```
3*a^2*d)*EllipticF[I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] + a^3*(-a + b + 3*
a^2*d)*(-1 + 2*a*d + 2*b*d)*(EllipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2*d
)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 1], I*ArcSinh[Sqrt[-1 + x/a]], a/(a -
b)] - EllipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2
 + d*#1^3 & , 2], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)]) + a^3*(-1 + 3*a*d)
*(b + 4*a^2*d + a*(-1 + b*d))*(EllipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2
*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[-1 + x/a]], a/(a
- b)] - EllipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1
^2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)]) - 3*a^5*(2*a - b
)*d^2*EllipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2
 + d*#1^3 & , 3], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] + a^2*(2*a - b)*(-1
 + 3*a*d)*(-a + b + 3*a^2*d)*EllipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2*d
)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[-1 + x/a]], a/(a -
b)] - 2*a^2*(2*a - b)*d^2*EllipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2*d)*#
1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)]
*Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 1]*R
oot[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 2]^2 -
 a^2*(2*a - b)*d^2*(EllipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1
 - 3*a*d)*#1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] + Ell
ipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3
 & , 3], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)])*Root[-(a^3*d) + (-a + b + 3
*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 1]^2*Root[-(a^3*d) + (-a + b + 3
*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 3] + a^2*(2*a - b)*d^2*(Elliptic
Pi[a/Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & ,
1], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] - EllipticPi[a/Root[-(a^3*d) + (-
a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[-1 +
 x/a]], a/(a - b)])*Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^
2 + d*#1^3 & , 2]^2*Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^
2 + d*#1^3 & , 3] - d*(b + 4*a^2*d + a*(-1 + b*d))*(EllipticPi[a/Root[-(a^3
*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 1], I*ArcSinh[S
qrt[-1 + x/a]], a/(a - b)] + EllipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2*d
)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[-1 + x/a]], a/(a -
b)] - 2*EllipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1
^2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)])*Root[-(a^3*d) +
(-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 1]*Root[-(a^3*d) + (-
a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 2]^2*Root[-(a^3*d) + (-
a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 3] + 2*d^2*EllipticF[I*
ArcSinh[Sqrt[-1 + x/a]], a/(a - b)]*Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 +
 (1 - 3*a*d)*#1^2 + d*#1^3 & , 1]^2*Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 +
 (1 - 3*a*d)*#1^2 + d*#1^3 & , 2]^3*Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 +
 (1 - 3*a*d)*#1^2 + d*#1^3 & , 3] + a^2*(2*a - b)*d^2*(EllipticPi[a/Root[-(
a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 2], I*ArcSin
h[Sqrt[-1 + x/a]], a/(a - b)] - EllipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^
2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[-1 + x/a]], a/(a
 - b)])*Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 &
 , 1]*Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & ,
 3]^2 - a^2*(2*a - b)*d^2*(EllipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2*d)*
#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 1], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)
] + EllipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 +
 d*#1^3 & , 3], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)])*Root[-(a^3*d) + (-a
+ b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 2]*Root[-(a^3*d) + (-a +
b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 3]^2 + d*(b + 4*a^2*d + a*(
-1 + b*d))*(EllipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d
)*#1^2 + d*#1^3 & , 1], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] - 2*EllipticP
i[a/Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 2
], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] + EllipticPi[a/Root[-(a^3*d) + (-a
 + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[-1 +
x/a]], a/(a - b)])*Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2
 + d*#1^3 & , 1]*Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 +
```

```
d*#1^3 & , 2]*Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d
*#1^3 & , 3]^2 - d*(-1 + 2*a*d + 2*b*d)*(2*EllipticPi[a/Root[-(a^3*d) + (-a
 + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 1], I*ArcSinh[Sqrt[-1 +
x/a]], a/(a - b)] - EllipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1
 - 3*a*d)*#1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] - Ell
ipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3
 & , 3], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)])*Root[-(a^3*d) + (-a + b + 3
*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 1]^2*Root[-(a^3*d) + (-a + b + 3
*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 2]*Root[-(a^3*d) + (-a + b + 3*a
^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 3]^2 + 2*d^2*EllipticF[I*ArcSinh[S
qrt[-1 + x/a]], a/(a - b)]*Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a
*d)*#1^2 + d*#1^3 & , 1]^3*Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a
*d)*#1^2 + d*#1^3 & , 2]*Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d
)*#1^2 + d*#1^3 & , 3]^2 - d*(-1 + 2*a*d + 2*b*d)*(EllipticPi[a/Root[-(a^3*
d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 1], I*ArcSinh[Sq
rt[-1 + x/a]], a/(a - b)] - 2*EllipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2*
d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 2], I*ArcSinh[Sqrt[-1 + x/a]], a/(a -
 b)] + EllipticPi[a/Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^
2 + d*#1^3 & , 3], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)])*Root[-(a^3*d) + (
-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 1]*Root[-(a^3*d) + (-a
 + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 2]^2*Root[-(a^3*d) + (-a
 + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 3]^2 + 2*d^2*EllipticF[I
*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)]*Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1
+ (1 - 3*a*d)*#1^2 + d*#1^3 & , 1]*Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 +
(1 - 3*a*d)*#1^2 + d*#1^3 & , 2]^2*Root[-(a^3*d) + (-a + b + 3*a^2*d)*#1 +
(1 - 3*a*d)*#1^2 + d*#1^3 & , 3]^3))/(d^2*Sqrt[1 - a/x]*(b - x)*Root[-(a^3*
d) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 1]*(Root[-(a^3*d
) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 1] - Root[-(a^3*d
) + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 2])*Root[-(a^3*d)
 + (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 2]*(Root[-(a^3*d)
+ (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 1] - Root[-(a^3*d)
+ (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 3])*(Root[-(a^3*d)
+ (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 2] - Root[-(a^3*d)
+ (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 3])*Root[-(a^3*d) +
 (-a + b + 3*a^2*d)*#1 + (1 - 3*a*d)*#1^2 + d*#1^3 & , 3])))/x^2
```

IntegrateAlgebraic [A] time = 0.95, size = 69, normalized size = 1.00

$$\frac{2\sqrt{x^2(-a-b)+abx+x^3}}{x^2}-2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{x^2(-a-b)+abx+x^3}}{\sqrt{d}\,x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-a + x)*(-b + x)*(3*a*b - 2*(a + b)*x + x^2))/(x^2*Sqr
t[x*(-a + x)*(-b + x)]*(-(a*b) + (a + b)*x - x^2 + d*x^3)),x]
```

```
[Out] (2*Sqrt[a*b*x + (-a - b)*x^2 + x^3])/x^2 - 2*Sqrt[d]*ArcTanh[Sqrt[a*b*x + (
-a - b)*x^2 + x^3]/(Sqrt[d]*x^2)]
```

fricas [A] time = 1.80, size = 338, normalized size = 4.90

$$\left[\frac{\sqrt{d}\,x^2\log\left(\frac{d^2x^6+6\,dx^5-(6\,(a+b)d-1)x^4+a^2b^2+2\,(3\,abd-a-b)x^3+\left(a^2+4\,ab+b^2\right)x^2-4\,\left(dx^4+abx-(a+b)x^2+x^3\right)\sqrt{abx-(a+b)x^2+x^3}\,\sqrt{d}-2\left(a^2b+ab^2\right)x}{d^2x^6-2\,dx^5+(2\,(a+b)d+1)x^4+a^2b^2-2\,(abd+a+b)x^3+\left(a^2+4\,ab+b^2\right)x^2-2\left(a^2b+ab^2\right)x}\right)+4\sqrt{abx-(a+b)x^2+x^3}}{2\,x^2},\ \frac{\sqrt{-d}\,x^2\arctan\left(\frac{\left(dx^3+ab-(a+b)x+x^2\right)\sqrt{abx-(a+b)x^2+x^3}\,\sqrt{-d}}{2\left(abdx^2-(a+b)dx^3+dx^4\right)}\right)+2\sqrt{abx-(a+b)x^2+x^3}}{x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(3*a*b-2*(a+b)*x+x^2)/x^2/(x*(-a+x)*(-b+x))^(1/2)/(
-a*b+(a+b)*x-x^2+d*x^3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(d)*x^2*log((d^2*x^6 + 6*d*x^5 - (6*(a + b)*d - 1)*x^4 + a^2*b^2
+ 2*(3*a*b*d - a - b)*x^3 + (a^2 + 4*a*b + b^2)*x^2 - 4*(d*x^4 + a*b*x - (a
```

```
 + b)*x^2 + x^3)*sqrt(a*b*x - (a + b)*x^2 + x^3)*sqrt(d) - 2*(a^2*b + a*b^2
)*x)/(d^2*x^6 - 2*d*x^5 + (2*(a + b)*d + 1)*x^4 + a^2*b^2 - 2*(a*b*d + a +
b)*x^3 + (a^2 + 4*a*b + b^2)*x^2 - 2*(a^2*b + a*b^2)*x)) + 4*sqrt(a*b*x - (
a + b)*x^2 + x^3))/x^2, (sqrt(-d)*x^2*arctan(1/2*(d*x^3 + a*b - (a + b)*x +
 x^2)*sqrt(a*b*x - (a + b)*x^2 + x^3)*sqrt(-d)/(a*b*d*x^2 - (a + b)*d*x^3 +
 d*x^4)) + 2*sqrt(a*b*x - (a + b)*x^2 + x^3))/x^2]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,ab - 2\,(a+b)x + x^2\right)(a-x)(b-x)}{\left(dx^3 - ab + (a+b)x - x^2\right)\sqrt{(a-x)(b-x)x}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(3*a*b-2*(a+b)*x+x^2)/x^2/(x*(-a+x)*(-b+x))^(1/2)/(
-a*b+(a+b)*x-x^2+d*x^3),x, algorithm="giac")
```

```
[Out] integrate((3*a*b - 2*(a + b)*x + x^2)*(a - x)*(b - x)/((d*x^3 - a*b + (a +
b)*x - x^2)*sqrt((a - x)*(b - x)*x)*x^2), x)
```

maple [C] time = 0.06, size = 817, normalized size = 11.84

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x)*(-b+x)*(3*a*b-2*(a+b)*x+x^2)/x^2/(x*(-a+x)*(-b+x))^(1/2)/(-a*b+(
a+b)*x-x^2+d*x^3),x)
```

```
[Out] 2/a^2/d*sum((-2*_alpha^2*a*d-2*_alpha^2*b*d+3*_alpha*a*b*d+_alpha^2-_alpha*
a-_alpha*b+a*b)/(-3*_alpha^2*d+2*_alpha-a-b)*(_alpha^2*d+_alpha*a*d+a^2*d-_
alpha+b)*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(x*(a*b-a*x-b
*x+x^2))^(1/2)*EllipticPi((-(-a+x)/a)^(1/2),(_alpha^2*d+_alpha*a*d+a^2*d-_a
lpha+b)/a^2/d,(a/(a-b))^(1/2)),_alpha=RootOf(d*_Z^3-_Z^2+(a+b)*_Z-a*b))-3*a
*b*(-2/3/a/b*(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/x^2-4/3*(a*b-a*x-b*x+x^2)*(a+b)/
a^2/b^2/(x*(a*b-a*x-b*x+x^2))^(1/2)-2*(-1/3/a/b+2/3*(a+b)^2/a^2/b^2+2/3*(-a
-b)*(a+b)/a^2/b^2)*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(
a*b*x-a*x^2-b*x^2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))-4
/3*(a+b)/a/b^2*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-
a*x^2-b*x^2+x^3)^(1/2)*((a-b)*EllipticE((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))+
b*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))))+(2*a+2*b)*(-2*(a*b-a*x-b*x
+x^2)/a/b/(x*(a*b-a*x-b*x+x^2))^(1/2)-2*((a+b)/a/b+(-a-b)/a/b)*a*(-(-a+x)/a
)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)*El
lipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))-2/b*(-(-a+x)/a)^(1/2)*((-b+x)/(a
-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)*((a-b)*EllipticE((-(
-a+x)/a)^(1/2),(a/(a-b))^(1/2))+b*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/
2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,ab - 2\,(a+b)x + x^2\right)(a-x)(b-x)}{\left(dx^3 - ab + (a+b)x - x^2\right)\sqrt{(a-x)(b-x)x}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(3*a*b-2*(a+b)*x+x^2)/x^2/(x*(-a+x)*(-b+x))^(1/2)/(
-a*b+(a+b)*x-x^2+d*x^3),x, algorithm="maxima")
```

```
[Out] integrate((3*a*b - 2*(a + b)*x + x^2)*(a - x)*(b - x)/((d*x^3 - a*b + (a +
b)*x - x^2)*sqrt((a - x)*(b - x)*x)*x^2), x)
```

mupad [**B**] time = 4.61, size = 97, normalized size = 1.41

$$\sqrt{d}\ \ln\left(\frac{a\,b-a\,x-b\,x+d\,x^3+x^2-2\,\sqrt{d}\,x\,\sqrt{x\,\left(a-x\right)\,\left(b-x\right)}}{a\,x-a\,b+b\,x+d\,x^3-x^2}\right)+\frac{2\,\sqrt{x^3-b\,x^2-a\,x^2+a\,b\,x}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a - x)*(b - x)*(3*a*b + x^2 - 2*x*(a + b)))/(x^2*(x*(a - x)*(b - x))
^(1/2)*(a*b - d*x^3 + x^2 - x*(a + b))),x)
```

```
[Out] d^(1/2)*log((a*b - a*x - b*x + d*x^3 + x^2 - 2*d^(1/2)*x*(x*(a - x)*(b - x)
)^(1/2))/(a*x - a*b + b*x + d*x^3 - x^2)) + (2*(x^3 - b*x^2 - a*x^2 + a*b*x
)^(1/2))/x^2
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(3*a*b-2*(a+b)*x+x**2)/x**2/(x*(-a+x)*(-b+x))**(1/2
)/(-a*b+(a+b)*x-x**2+d*x**3),x)
```

```
[Out] Timed out
```

3.845 $\int \frac{1}{\left(1+x^3\right)\sqrt[4]{-x+x^4}}\,dx$

Optimal. Leaf size=69

$$\frac{1}{3}2^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\left(x^4-x\right)^{3/4}}{x^3-1}\right)+\frac{1}{3}2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\left(x^4-x\right)^{3/4}}{x^3-1}\right)$$

Rubi [A] time = 0.14, antiderivative size = 111, normalized size of antiderivative = 1.61, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2056, 466, 465, 377, 212, 206, 203}

$$\frac{2^{3/4}\sqrt[4]{x}\sqrt[4]{x^3-1}\tan^{-1}\left(\frac{\sqrt[4]{2}x^{3/4}}{\sqrt[4]{x^3-1}}\right)}{3\sqrt[4]{x^4-x}}+\frac{2^{3/4}\sqrt[4]{x}\sqrt[4]{x^3-1}\tanh^{-1}\left(\frac{\sqrt[4]{2}x^{3/4}}{\sqrt[4]{x^3-1}}\right)}{3\sqrt[4]{x^4-x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^3)*(-x + x^4)^(1/4)),x]
```

```
[Out] (2^(3/4)*x^(1/4)*(-1 + x^3)^(1/4)*ArcTan[(2^(1/4)*x^(3/4))/(-1 + x^3)^(1/4)
])/(3*(-x + x^4)^(1/4)) + (2^(3/4)*x^(1/4)*(-1 + x^3)^(1/4)*ArcTanh[(2^(1/4
)*x^(3/4))/(-1 + x^3)^(1/4)])/(3*(-x + x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
```

```
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{(1+x^3)\sqrt[4]{-x+x^4}}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\int \frac{1}{\sqrt[4]{x}\sqrt[4]{-1+x^3}(1+x^3)}\,dx}{\sqrt[4]{-x+x^4}}$$
$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{-1+x^{12}}(1+x^{12})}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-x+x^4}}$$
$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-1+x^4}(1+x^4)}\,dx, x, x^{3/4}\right)}{3\sqrt[4]{-x+x^4}}$$
$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\text{Subst}\left(\int \frac{1}{1-2x^4}\,dx, x, \frac{x^{3/4}}{\sqrt[4]{-1+x^3}}\right)}{3\sqrt[4]{-x+x^4}}$$
$$= \frac{\left(2\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x^2}\,dx, x, \frac{x^{3/4}}{\sqrt[4]{-1+x^3}}\right)}{3\sqrt[4]{-x+x^4}} + \frac{\left(2\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x^2}\right.}{3\sqrt[4]{-x+x^4}}$$
$$= \frac{2^{3/4}\sqrt[4]{x}\sqrt[4]{-1+x^3}\tan^{-1}\left(\frac{\sqrt[4]{2}x^{3/4}}{\sqrt[4]{-1+x^3}}\right)}{3\sqrt[4]{-x+x^4}} + \frac{2^{3/4}\sqrt[4]{x}\sqrt[4]{-1+x^3}\tanh^{-1}\left(\frac{\sqrt[4]{2}x^{3/4}}{\sqrt[4]{-1+x^3}}\right)}{3\sqrt[4]{-x+x^4}}$$

Mathematica [C] time = 0.01, size = 58, normalized size = 0.84

$$\frac{4x\sqrt[4]{1-x^3}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{2x^3}{x^3+1}\right)}{3\sqrt[4]{x(x^3-1)}\sqrt[4]{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + x^3)*(-x + x^4)^(1/4)),x]
```

```
[Out] (4*x*(1 - x^3)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (2*x^3)/(1 + x^3)])/(
3*(x*(-1 + x^3))^(1/4)*(1 + x^3)^(1/4))
```

IntegrateAlgebraic [A] time = 0.32, size = 69, normalized size = 1.00

$$\frac{1}{3}2^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\left(x^4-x\right)^{3/4}}{x^3-1}\right)+\frac{1}{3}2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\left(x^4-x\right)^{3/4}}{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x^3)*(-x + x^4)^(1/4)),x]
```

```
[Out] (2^(3/4)*ArcTan[(2^(1/4)*(-x + x^4)^(3/4))/(-1 + x^3)])/3 + (2^(3/4)*ArcTan
h[(2^(1/4)*(-x + x^4)^(3/4))/(-1 + x^3)])/3
```

fricas [B] time = 1.94, size = 218, normalized size = 3.16

$$-\frac{1}{3}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{4\cdot 2^{\frac{3}{4}}\left(x^4-x\right)^{\frac{1}{4}}x^2+2^{\frac{3}{4}}\left(2\cdot 2^{\frac{3}{4}}\sqrt{x^4-x}\,x+2^{\frac{1}{4}}\left(3x^3-1\right)\right)+4\cdot 2^{\frac{1}{4}}\left(x^4-x\right)^{\frac{3}{4}}}{2\left(x^3+1\right)}\right)+\frac{1}{12}\cdot 2^{\frac{3}{4}}\log\left(\frac{4\sqrt{2}\left(x^4-x\right)^{\frac{1}{4}}x^2+2^{\frac{3}{4}}\left(3x^3-1\right)+4\cdot 2^{\frac{1}{4}}\sqrt{x^4-x}\,x+4\left(x^4-x\right)^{\frac{3}{4}}}{x^3+1}\right)-\frac{1}{12}\cdot 2^{\frac{3}{4}}\log\left(\frac{4\sqrt{2}\left(x^4-x\right)^{\frac{1}{4}}x^2-2^{\frac{3}{4}}\left(3x^3-1\right)-4\cdot 2^{\frac{1}{4}}\sqrt{x^4-x}\,x+4\left(x^4-x\right)^{\frac{3}{4}}}{x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+1)/(x^4-x)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/3*2^(3/4)*arctan(1/2*(4*2^(3/4)*(x^4 - x)^(1/4)*x^2 + 2^(3/4)*(2*2^(3/4)
*sqrt(x^4 - x)*x + 2^(1/4)*(3*x^3 - 1)) + 4*2^(1/4)*(x^4 - x)^(3/4))/(x^3 +
 1)) + 1/12*2^(3/4)*log((4*sqrt(2)*(x^4 - x)^(1/4)*x^2 + 2^(3/4)*(3*x^3 - 1
) + 4*2^(1/4)*sqrt(x^4 - x)*x + 4*(x^4 - x)^(3/4))/(x^3 + 1)) - 1/12*2^(3/4
)*log((4*sqrt(2)*(x^4 - x)^(1/4)*x^2 - 2^(3/4)*(3*x^3 - 1) - 4*2^(1/4)*sqrt
(x^4 - x)*x + 4*(x^4 - x)^(3/4))/(x^3 + 1))
```

giac [A] time = 0.34, size = 62, normalized size = 0.90

$$\frac{1}{3}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}+\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}\right)+\frac{1}{6}\cdot 2^{\frac{3}{4}}\log\left(\left|-2^{\frac{1}{4}}+\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+1)/(x^4-x)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3*2^(3/4)*arctan(1/2*2^(3/4)*(-1/x^3 + 1)^(1/4)) - 1/6*2^(3/4)*log(2^(1/4
) + (-1/x^3 + 1)^(1/4)) + 1/6*2^(3/4)*log(abs(-2^(1/4) + (-1/x^3 + 1)^(1/4)
))
```

maple [C] time = 6.10, size = 229, normalized size = 3.32

$$-\frac{\mathrm{RootOf}\left(\_Z^4-8\right)\ln\left(-\frac{-\sqrt{x^4-x}\,\mathrm{RootOf}\left(\_Z^4-8\right)^3x+2\left(x^4-x\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-8\right)^2x^2-3\,\mathrm{RootOf}\left(\_Z^4-8\right)x^3+4\left(x^4-x\right)^{\frac{3}{4}}+\mathrm{RootOf}\left(\_Z^4-8\right)}{(1+x)\left(x^2-x+1\right)}\right)}{6}-\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\ln\left(-\frac{\sqrt{x^4-x}\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\mathrm{RootOf}\left(\_Z^4-8\right)^2x-2\left(x^4-x\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-8\right)^2x^2-3\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)x^3+4\left(x^4-x\right)^{\frac{3}{4}}+\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)}{(1+x)\left(x^2-x+1\right)}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+1)/(x^4-x)^(1/4),x)
```

```
[Out] -1/6*RootOf(_Z^4-8)*ln(-(-(x^4-x)^(1/2)*RootOf(_Z^4-8)^3*x+2*(x^4-x)^(1/4)*
RootOf(_Z^4-8)^2*x^2-3*RootOf(_Z^4-8)*x^3+4*(x^4-x)^(3/4)+RootOf(_Z^4-8))/(
1+x)/(x^2-x+1))-1/6*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln(-((x^4-x)^(1/2)*RootOf
(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x-2*(x^4-x)^(1/4)*RootOf(_Z^4-8)^2
*x^2-3*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^3+4*(x^4-x)^(3/4)+RootOf(_Z^2+RootOf
(_Z^4-8)^2))/(1+x)/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4-x\right)^{\frac{1}{4}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+1)/(x^4-x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 - x)^(1/4)*(x^3 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(x^4-x\right)}^{1/4}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^4 - x)^(1/4)*(x^3 + 1)),x)
```

```
[Out] int(1/((x^4 - x)^(1/4)*(x^3 + 1)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+1)/(x**4-x)**(1/4),x)
```

```
[Out] Integral(1/((x*(x - 1)*(x**2 + x + 1))**(1/4)*(x + 1)*(x**2 - x + 1)), x)
```

3.846 $\int \frac{x\sqrt{-x^2+x^4}}{-3+2x^2}\,dx$

Optimal. Leaf size=69

$$\frac{1}{4}\sqrt{x^4-x^2}+\tanh^{-1}\left(\frac{(x-1)x}{\sqrt{x^4-x^2}}\right)-\frac{1}{4}\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x^4-x^2}}{x^2}\right)$$

Rubi [A] time = 0.18, antiderivative size = 79, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2034, 734, 843, 620, 206, 724}

$$\frac{1}{4}\sqrt{x^4-x^2}+\frac{1}{2}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x^2}}\right)+\frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{3-4x^2}{2\sqrt{3}\sqrt{x^4-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[-x^2 + x^4])/(-3 + 2*x^2),x]
```

```
[Out] Sqrt[-x^2 + x^4]/4 + ArcTanh[x^2/Sqrt[-x^2 + x^4]]/2 + (Sqrt[3]*ArcTanh[(3
- 4*x^2)/(2*Sqrt[3]*Sqrt[-x^2 + x^4])])/8
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{-x^2+x^4}}{-3+2x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-x+x^2}}{-3+2x}\,dx, x, x^2\right)\\
&= \frac{1}{4}\sqrt{-x^2+x^4} - \frac{1}{8}\text{Subst}\left(\int \frac{3-4x}{(-3+2x)\sqrt{-x+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{4}\sqrt{-x^2+x^4} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{-x+x^2}}\,dx, x, x^2\right) + \frac{3}{8}\text{Subst}\left(\int \frac{1}{(-3+2x)\sqrt{-x+x^2}}\,dx, x, x\right.\\
&= \frac{1}{4}\sqrt{-x^2+x^4} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^2}{\sqrt{-x^2+x^4}}\right) - \frac{3}{4}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{-3+4}{\sqrt{-x^2+}}\right.\\
&= \frac{1}{4}\sqrt{-x^2+x^4} + \frac{1}{2}\tanh^{-1}\left(\frac{x^2}{\sqrt{-x^2+x^4}}\right) + \frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{3-4x^2}{2\sqrt{3}\sqrt{-x^2+x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 1.12

$$\frac{x\sqrt{x^2-1}\left(\sqrt{x^2-1}\,x+2\log\left(\sqrt{x^2-1}+x\right)-\sqrt{3}\tanh^{-1}\left(\frac{x}{\sqrt{3}\sqrt{x^2-1}}\right)\right)}{4\sqrt{x^2\left(x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[-x^2 + x^4])/(-3 + 2*x^2),x]
```

```
[Out] (x*Sqrt[-1 + x^2]*(x*Sqrt[-1 + x^2] - Sqrt[3]*ArcTanh[x/(Sqrt[3]*Sqrt[-1 +
x^2])] + 2*Log[x + Sqrt[-1 + x^2]]))/(4*Sqrt[x^2*(-1 + x^2)])
```

IntegrateAlgebraic [A] time = 0.25, size = 80, normalized size = 1.16

$$\frac{1}{4}\sqrt{x^4-x^2}+\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{x^4-x^2}}{x^2-1}\right)-\frac{1}{4}\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{x^4-x^2}}{\sqrt{3}\left(x^2-1\right)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[-x^2 + x^4])/(-3 + 2*x^2),x]
```

```
[Out] Sqrt[-x^2 + x^4]/4 + ArcTanh[Sqrt[-x^2 + x^4]/(-1 + x^2)]/2 - (Sqrt[3]*ArcT
anh[Sqrt[-x^2 + x^4]/(Sqrt[3]*(-1 + x^2))])/4
```

fricas [A] time = 0.43, size = 94, normalized size = 1.36

$$\frac{1}{8}\sqrt{3}\log\left(\frac{8x^2-\sqrt{3}\left(4x^2-3\right)-2\sqrt{x^4-x^2}\left(2\sqrt{3}-3\right)-6}{2x^2-3}\right)+\frac{1}{4}\sqrt{x^4-x^2}-\frac{1}{2}\log\left(-\frac{x^2-\sqrt{x^4-x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4-x^2)^(1/2)/(2*x^2-3),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(3)*log((8*x^2 - sqrt(3)*(4*x^2 - 3) - 2*sqrt(x^4 - x^2)*(2*sqrt(3)
 - 3) - 6)/(2*x^2 - 3)) + 1/4*sqrt(x^4 - x^2) - 1/2*log(-(x^2 - sqrt(x^4 -
x^2))/x)
```

giac [C] time = 0.49, size = 119, normalized size = 1.72

$$-\frac{1}{24}\sqrt{3}\left(-2i\sqrt{3}\pi - 3\log\left(-\frac{\sqrt{3}+3}{\sqrt{3}-3}\right)\right)\operatorname{sgn}(x) + \frac{1}{4}\sqrt{x^2-1}\,x\operatorname{sgn}(x) - \frac{1}{8}\sqrt{3}\log\left(\frac{\left|2\left(x-\sqrt{x^2-1}\right)^2 - 2\sqrt{3} - 4\right|}{\left|2\left(x-\sqrt{x^2-1}\right)^2 + 2\sqrt{3} - 4\right|}\right)\operatorname{sgn}(x) - \frac{1}{4}\log\left(\left(x-\sqrt{x^2-1}\right)^2\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4-x^2)^(1/2)/(2*x^2-3),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(3)*(-2*I*sqrt(3)*pi - 3*log(-(sqrt(3) + 3)/(sqrt(3) - 3)))*sgn(x
) + 1/4*sqrt(x^2 - 1)*x*sgn(x) - 1/8*sqrt(3)*log(abs(2*(x - sqrt(x^2 - 1))^
2 - 2*sqrt(3) - 4)/abs(2*(x - sqrt(x^2 - 1))^2 + 2*sqrt(3) - 4))*sgn(x) - 1
/4*log((x - sqrt(x^2 - 1))^2)*sgn(x)
```

maple [A] time = 0.07, size = 101, normalized size = 1.46

$$-\frac{\sqrt{x^4-x^2}\left(\sqrt{6}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(\sqrt{6}\,x+2\right)\sqrt{2}}{2\sqrt{x^2-1}}\right) + \sqrt{6}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(\sqrt{6}\,x-2\right)\sqrt{2}}{2\sqrt{x^2-1}}\right) - 8\ln\left(x+\sqrt{x^2-1}\right) - 4x\sqrt{x^2-1}\right)}{16x\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^4-x^2)^(1/2)/(2*x^2-3),x)
```

```
[Out] -1/16*(x^4-x^2)^(1/2)*(6^(1/2)*2^(1/2)*arctanh(1/2*(6^(1/2)*x+2)*2^(1/2)/(x
^2-1)^(1/2))+6^(1/2)*2^(1/2)*arctanh(1/2*(6^(1/2)*x-2)*2^(1/2)/(x^2-1)^(1/2
))-8*ln(x+(x^2-1)^(1/2))-4*x*(x^2-1)^(1/2))/x/(x^2-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4-x^2}\,x}{2\,x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4-x^2)^(1/2)/(2*x^2-3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 - x^2)*x/(2*x^2 - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{x^4-x^2}}{2\,x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^4 - x^2)^(1/2))/(2*x^2 - 3),x)
```

```
[Out] int((x*(x^4 - x^2)^(1/2))/(2*x^2 - 3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{x^2\,(x-1)\,(x+1)}}{2x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**4-x**2)**(1/2)/(2*x**2-3),x)
```

```
[Out] Integral(x*sqrt(x**2*(x - 1)*(x + 1))/(2*x**2 - 3), x)
```

3.847 $\int \frac{(-1+x^4)(1+x^4)\sqrt{-1-x^2+x^4}}{(-2-x^2+2x^4)^2(-2+x^2+2x^4)}\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{x^4-x^2-1}\,x}{16\,(2x^4-x^2-2)}-\frac{1}{16}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt{x^4-x^2-1}}\right)$$

Rubi [C] time = 13.50, antiderivative size = 6713, normalized size of antiderivative = 97.29, number of steps used = 156, number of rules used = 14, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.274, Rules used = {6728, 6742, 1226, 1187, 1098, 1184, 1214, 1456, 540, 421, 419, 538, 537, 1208}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^4)*(1 + x^4)*Sqrt[-1 - x^2 + x^4])/((-2 - x^2 + 2*x^4)^2*(-2 +
 x^2 + 2*x^4)),x]
```

```
[Out] (x*(1 - Sqrt[5] - 2*x^2))/(17*Sqrt[-1 - x^2 + x^4]) - ((17 - 2*Sqrt[17])*x*
(1 - Sqrt[5] - 2*x^2))/(544*Sqrt[-1 - x^2 + x^4]) - (x*(1 - Sqrt[5] - 2*x^2
))/(34*(1 - Sqrt[17])*Sqrt[-1 - x^2 + x^4]) - (x*(1 - Sqrt[5] - 2*x^2))/(34
*(1 + Sqrt[17])*Sqrt[-1 - x^2 + x^4]) - ((17 + 2*Sqrt[17])*x*(1 - Sqrt[5] -
 2*x^2))/(544*Sqrt[-1 - x^2 + x^4]) + (x*Sqrt[-1 - x^2 + x^4])/(68*(1 - Sqr
t[17] - 4*x^2)) + (4*x*Sqrt[-1 - x^2 + x^4])/(17*(1 - Sqrt[17])*(1 - Sqrt[1
7] - 4*x^2)) + (x*Sqrt[-1 - x^2 + x^4])/(68*(1 + Sqrt[17] - 4*x^2)) + (4*x*
Sqrt[-1 - x^2 + x^4])/(17*(1 + Sqrt[17])*(1 + Sqrt[17] - 4*x^2)) + (5^(1/4)
*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5
])*x^2)]*EllipticE[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]]
, (5 - Sqrt[5])/10])/(17*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 +
 x^4]) - (5^(1/4)*(17 - 2*Sqrt[17])*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 +
(1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticE[ArcSin[(Sqrt[2]*5^(1/
4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(544*Sqrt[(2 + (1 -
 Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) - (5^(1/4)*Sqrt[-2 - (1 - Sqrt[5
])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticE[Arc
Sin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(
34*(1 - Sqrt[17])*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4])
- (5^(1/4)*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (
1 - Sqrt[5])*x^2)]*EllipticE[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt
[5])*x^2]], (5 - Sqrt[5])/10])/(34*(1 + Sqrt[17])*Sqrt[(2 + (1 - Sqrt[5])*x
^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) - (5^(1/4)*(17 + 2*Sqrt[17])*Sqrt[-2 - (1 -
 Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*Ellipt
icE[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])
/10])/(544*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) + (3*(1
 + Sqrt[5])*(1 - Sqrt[17])*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1 +
 Sqrt[5])]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/2])/(6
4*Sqrt[2]*(3 + 2*Sqrt[5] - Sqrt[17])*Sqrt[-1 - x^2 + x^4]) - ((1 + Sqrt[5])
*(1 + Sqrt[17])*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[5])]*
EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/2])/(32*Sqrt[34]*
(1 + 2*Sqrt[5] - Sqrt[17])*Sqrt[-1 - x^2 + x^4]) + ((1 + Sqrt[5])*(1 + Sqrt
[17])*(2 + Sqrt[17])*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[
5])]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/2])/(64*Sqrt
[34]*(1 + 2*Sqrt[5] - Sqrt[17])*Sqrt[-1 - x^2 + x^4]) + ((1 + Sqrt[5])*(1 -
 Sqrt[17])*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[5])]*Ellip
ticF[ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/2])/(32*Sqrt[34]*(1 +
2*Sqrt[5] + Sqrt[17])*Sqrt[-1 - x^2 + x^4]) - ((1 + Sqrt[5])*(1 - Sqrt[17])
*(2 - Sqrt[17])*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[5])]*
EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/2])/(64*Sqrt[34]*
```

(1 + 2*Sqrt[5] + Sqrt[17])*Sqrt[-1 - x^2 + x^4]) + (3*(1 + Sqrt[5])*(1 + Sqrt[17])*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[5])]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/2])/(64*Sqrt[2]*(3 + 2*Sqrt[5] + Sqrt[17])*Sqrt[-1 - x^2 + x^4]) + ((1 + Sqrt[5])*(1 + Sqrt[17])*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[5])]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/2])/(8*Sqrt[34]*(8 - Sqrt[5] - Sqrt[85])*Sqrt[-1 - x^2 + x^4]) + ((1 + Sqrt[5])*(9 + Sqrt[17])*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[5])]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/2])/(64*Sqrt[34]*(8 - Sqrt[5] - Sqrt[85])*Sqrt[-1 - x^2 + x^4]) - ((1 + Sqrt[5])*(1 - Sqrt[17])*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[5])]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/2])/(8*Sqrt[34]*(8 - Sqrt[5] + Sqrt[85])*Sqrt[-1 - x^2 + x^4]) - ((1 + Sqrt[5])*(9 - Sqrt[17])*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[5])]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/2])/(64*Sqrt[34]*(8 - Sqrt[5] + Sqrt[85])*Sqrt[-1 - x^2 + x^4]) + ((3 - 2*Sqrt[5] - Sqrt[17])*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(128*5^(1/4)*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) - (3*(1 - Sqrt[17])*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(64*5^(1/4)*(3 + 2*Sqrt[5] - Sqrt[17])*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) + ((3 + 2*Sqrt[5] - Sqrt[17])*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(2176*5^(1/4)*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) + ((3 + 2*Sqrt[5] - Sqrt[17])*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(136*5^(1/4)*(1 - Sqrt[17])*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) + ((17 + Sqrt[17])*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(544*5^(1/4)*(1 + 2*Sqrt[5] - Sqrt[17])*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) - ((17 - 2*Sqrt[17])*(1 - 2*Sqrt[5] + Sqrt[17])*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(2176*5^(1/4)*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) + ((3 - 2*Sqrt[5] + Sqrt[17])*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(128*5^(1/4)*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) - ((51 - 19*Sqrt[17])*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(1088*5^(1/4)*(1 + 2*Sqrt[5] + Sqrt[17])*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) + ((17 - Sqrt[17])*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(544*5^(1/4)*(1 + 2*Sqrt[5] + Sqrt[17])*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) - (3*(1 + Sqrt[17])*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(64*5^(1/4)*(3 + 2*Sqrt[5] + Sqrt[17])*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) + ((3 + 2*Sqrt[5] + Sqrt[17])*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(2176*5^(1/4)*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) + ((3 + 2*Sqrt[5] + Sqrt[17])*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(136*5^(1/4)*(1 + Sq

```
rt[17])*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) - ((1 - 2*
Sqrt[5] - Sqrt[17])*(17 + 2*Sqrt[17])*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2
+ (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(
1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(2176*5^(1/4)*Sqr
t[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) - ((51 + 19*Sqrt[17])
*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5
])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]]
, (5 - Sqrt[5])/10])/(1088*5^(1/4)*(1 + 2*Sqrt[5] - Sqrt[17])*Sqrt[(2 + (1
- Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) - ((17 + Sqrt[17])*Sqrt[-2 - (1
 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*Elli
pticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5
])/10])/(136*5^(1/4)*(8 - Sqrt[5] - Sqrt[85])*Sqrt[(2 + (1 - Sqrt[5])*x^2)^
(-1)]*Sqrt[-1 - x^2 + x^4]) - ((17 + 9*Sqrt[17])*Sqrt[-2 - (1 - Sqrt[5])*x^
2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(
Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(1088*
5^(1/4)*(8 - Sqrt[5] - Sqrt[85])*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1
 - x^2 + x^4]) - ((17 - 9*Sqrt[17])*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 +
(1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/
4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(1088*5^(1/4)*(8 -
Sqrt[5] + Sqrt[85])*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]
) - ((17 - Sqrt[17])*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x
^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 -
 (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(136*5^(1/4)*(8 - Sqrt[5] + Sqrt[8
5])*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) - ((17 - Sqrt[
17]*(1 - 2*Sqrt[5]))*Sqrt[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x
^2)/(2 + (1 - Sqrt[5])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 -
 (1 - Sqrt[5])*x^2]], (5 - Sqrt[5])/10])/(1088*5^(1/4)*Sqrt[(2 + (1 - Sqrt[
5])*x^2)^(-1)]*Sqrt[-1 - x^2 + x^4]) - ((17 + Sqrt[17]*(1 - 2*Sqrt[5]))*Sqr
t[-2 - (1 - Sqrt[5])*x^2]*Sqrt[(2 + (1 + Sqrt[5])*x^2)/(2 + (1 - Sqrt[5])*x
^2)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[5])*x^2]], (5
 - Sqrt[5])/10])/(1088*5^(1/4)*Sqrt[(2 + (1 - Sqrt[5])*x^2)^(-1)]*Sqrt[-1 -
 x^2 + x^4]) - (3*(1 + Sqrt[5])*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)
/(1 + Sqrt[5])]*EllipticPi[(-2*(1 + Sqrt[5]))/(1 - Sqrt[17]), ArcSin[Sqrt[2
/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/2])/(64*Sqrt[2]*Sqrt[-1 - x^2 + x^4]) +
(3*(1 + Sqrt[5])*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[5])]
*EllipticPi[(2*(1 + Sqrt[5]))/(1 - Sqrt[17]), ArcSin[Sqrt[2/(1 + Sqrt[5])]*
x], (-3 - Sqrt[5])/2])/(64*Sqrt[34]*Sqrt[-1 - x^2 + x^4]) + ((1 + Sqrt[5])*
Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[5])]*EllipticPi[(2*(1
 + Sqrt[5]))/(1 - Sqrt[17]), ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5]
)/2])/(4*Sqrt[34]*(1 - Sqrt[17])*Sqrt[-1 - x^2 + x^4]) - ((1 + Sqrt[5])*(2
- Sqrt[17])*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[5])]*Elli
pticPi[(2*(1 + Sqrt[5]))/(1 - Sqrt[17]), ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (
-3 - Sqrt[5])/2])/(64*Sqrt[34]*Sqrt[-1 - x^2 + x^4]) - (3*(1 + Sqrt[5])*Sqr
t[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[5])]*EllipticPi[(-2*(1 +
 Sqrt[5]))/(1 + Sqrt[17]), ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/
2])/(64*Sqrt[2]*Sqrt[-1 - x^2 + x^4]) - (3*(1 + Sqrt[5])*Sqrt[-1 + Sqrt[5]
+ 2*x^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[5])]*EllipticPi[(2*(1 + Sqrt[5]))/(1 +
Sqrt[17]), ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/2])/(64*Sqrt[34]
*Sqrt[-1 - x^2 + x^4]) - ((1 + Sqrt[5])*Sqrt[-1 + Sqrt[5] + 2*x^2]*Sqrt[1 -
 (2*x^2)/(1 + Sqrt[5])]*EllipticPi[(2*(1 + Sqrt[5]))/(1 + Sqrt[17]), ArcSin
[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/2])/(4*Sqrt[34]*(1 + Sqrt[17])*Sq
rt[-1 - x^2 + x^4]) + ((1 + Sqrt[5])*(2 + Sqrt[17])*Sqrt[-1 + Sqrt[5] + 2*x
^2]*Sqrt[1 - (2*x^2)/(1 + Sqrt[5])]*EllipticPi[(2*(1 + Sqrt[5]))/(1 + Sqrt[
17]), ArcSin[Sqrt[2/(1 + Sqrt[5])]*x], (-3 - Sqrt[5])/2])/(64*Sqrt[34]*Sqrt
[-1 - x^2 + x^4])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 540

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x
] + Dist[(b*c - a*d)/b, Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e*x*(b + q + 2*c*x^2))/(2*c*Sqrt
[a + b*x^2 + c*x^4]), x] - Simp[(e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q
)*x^2)]*Sqrt[(2*a + (b + q)*x^2)/q]*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*
x^2)/(2*q)]], (b + q)/(2*q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b
 + q)*x^2)]), x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1187

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*d - e*(b - q))/(2*c), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e/(2*c), Int[(b - q + 2*c*x^2)/Sqrt[
a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1226

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(d + e*x^2)), x] + (Dist[c/(2
*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*
e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(-1+x^4\right)\left(1+x^4\right)\sqrt{-1-x^2+x^4}}{\left(-2-x^2+2x^4\right)^2\left(-2+x^2+2x^4\right)}\,dx = \int \left(\frac{\left(4+x^2\right)\sqrt{-1-x^2+x^4}}{8\left(-2-x^2+2x^4\right)^2} + \frac{\left(1+4x^2\right)\sqrt{-1-x^2+x^4}}{16\left(-2-x^2+2x^4\right)} + \frac{\left(-1-4x\right.}{16\left(\right.}\right.$$

$$= \frac{1}{16}\int \frac{\left(1+4x^2\right)\sqrt{-1-x^2+x^4}}{-2-x^2+2x^4}\,dx + \frac{1}{16}\int \frac{\left(-1-4x^2\right)\sqrt{-1-x^2+}}{-2+x^2+2x^4}$$

$$= \frac{1}{16}\int \left(\frac{\left(4+\frac{8}{\sqrt{17}}\right)\sqrt{-1-x^2+x^4}}{-1-\sqrt{17}+4x^2} + \frac{\left(4-\frac{8}{\sqrt{17}}\right)\sqrt{-1-x^2+x^4}}{-1+\sqrt{17}+4x^2}\right)dx$$

$$= \frac{1}{8}\int \frac{x^2\sqrt{-1-x^2+x^4}}{\left(-2-x^2+2x^4\right)^2}\,dx - \frac{1}{4}\int \frac{\sqrt{-1-x^2+x^4}}{1-\sqrt{17}+4x^2}\,dx - \frac{1}{4}\int \frac{\sqrt{-1-}}{1+\sqrt{17}}$$

= rest of steps removed due to Latex formating problem

Mathematica [C] time = 1.82, size = 477, normalized size = 6.91

$$\frac{-2\sqrt{1+\sqrt{5}}\,x^5+2\sqrt{1+\sqrt{5}}\,x^3-3i\sqrt{2}\sqrt{-x^4+x^2+1}\left(2x^4-x^2-2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)|\frac{1}{2}\left(-3+\sqrt{5}\right)\right)+6i\sqrt{2}\sqrt{-x^4+x^2+1}\,x^4\Pi\left(\frac{2\left(-1+\sqrt{5}\right)}{1+\sqrt{17}};i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)|\frac{1}{2}\left(-3+\sqrt{5}\right)\right)-3i\sqrt{2}\sqrt{-x^4+x^2+1}\,x^2\Pi\left(\frac{2\left(-1+\sqrt{5}\right)}{1+\sqrt{17}};i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)|\frac{1}{2}\left(-3+\sqrt{5}\right)\right)+3i\sqrt{2}\sqrt{-x^4+x^2+1}\left(2x^4-x^2-2\right)\Pi\left(-\frac{2\left(-1+\sqrt{5}\right)}{-1+\sqrt{17}};i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)|\frac{1}{2}\left(-3+\sqrt{5}\right)\right)-6i\sqrt{2}\sqrt{-x^4+x^2+1}\,\Pi\left(\frac{2\left(-1+\sqrt{5}\right)}{1+\sqrt{17}};i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)|\frac{1}{2}\left(-3+\sqrt{5}\right)\right)+2\sqrt{1+\sqrt{5}}\,x}{32\sqrt{1+\sqrt{5}}\sqrt{x^4-x^2-1}\left(2x^4-x^2-2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^4)*(1 + x^4)*Sqrt[-1 - x^2 + x^4])/((-2 - x^2 + 2*x^4)^2
*(-2 + x^2 + 2*x^4)),x]
```

```
[Out] (2*Sqrt[1 + Sqrt[5]]*x + 2*Sqrt[1 + Sqrt[5]]*x^3 - 2*Sqrt[1 + Sqrt[5]]*x^5
- (3*I)*Sqrt[2]*Sqrt[1 + x^2 - x^4]*(-2 - x^2 + 2*x^4)*EllipticF[I*ArcSinh[
Sqrt[2/(-1 + Sqrt[5])]*x], (-3 + Sqrt[5])/2] + (3*I)*Sqrt[2]*Sqrt[1 + x^2 -
 x^4]*(-2 - x^2 + 2*x^4)*EllipticPi[(-2*(-1 + Sqrt[5]))/(-1 + Sqrt[17]), I*
ArcSinh[Sqrt[2/(-1 + Sqrt[5])]*x], (-3 + Sqrt[5])/2] - (6*I)*Sqrt[2]*Sqrt[1
 + x^2 - x^4]*EllipticPi[(2*(-1 + Sqrt[5]))/(1 + Sqrt[17]), I*ArcSinh[Sqrt[
2/(-1 + Sqrt[5])]*x], (-3 + Sqrt[5])/2] - (3*I)*Sqrt[2]*x^2*Sqrt[1 + x^2 -
x^4]*EllipticPi[(2*(-1 + Sqrt[5]))/(1 + Sqrt[17]), I*ArcSinh[Sqrt[2/(-1 + S
qrt[5])]*x], (-3 + Sqrt[5])/2] + (6*I)*Sqrt[2]*x^4*Sqrt[1 + x^2 - x^4]*Elli
pticPi[(2*(-1 + Sqrt[5]))/(1 + Sqrt[17]), I*ArcSinh[Sqrt[2/(-1 + Sqrt[5])]*
x], (-3 + Sqrt[5])/2])/(32*Sqrt[1 + Sqrt[5]]*Sqrt[-1 - x^2 + x^4]*(-2 - x^2
 + 2*x^4))
```

IntegrateAlgebraic [A] time = 0.75, size = 69, normalized size = 1.00

$$-\frac{\sqrt{x^4-x^2-1}\,x}{16\left(2x^4-x^2-2\right)} - \frac{1}{16}\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt{x^4-x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(1 + x^4)*Sqrt[-1 - x^2 + x^4])/((-2 - x^2 +
 2*x^4)^2*(-2 + x^2 + 2*x^4)),x]
```

```
[Out] -1/16*(x*Sqrt[-1 - x^2 + x^4])/(-2 - x^2 + 2*x^4) - (Sqrt[3/2]*ArcTan[(Sqrt
[3/2]*x)/Sqrt[-1 - x^2 + x^4]])/16
```

fricas [A] time = 0.48, size = 87, normalized size = 1.26

$$-\frac{\sqrt{3}\sqrt{2}\left(2\,x^4-x^2-2\right)\arctan\left(\frac{2\sqrt{3}\sqrt{2}\sqrt{x^4-x^2-1}\,x}{2\,x^4-5\,x^2-2}\right)+4\sqrt{x^4-x^2-1}\,x}{64\left(2\,x^4-x^2-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)*(x^4-x^2-1)^(1/2)/(2*x^4-x^2-2)^2/(2*x^4+x^2-2),x
, algorithm="fricas")
```

```
[Out] -1/64*(sqrt(3)*sqrt(2)*(2*x^4 - x^2 - 2)*arctan(2*sqrt(3)*sqrt(2)*sqrt(x^4
- x^2 - 1)*x/(2*x^4 - 5*x^2 - 2)) + 4*sqrt(x^4 - x^2 - 1)*x)/(2*x^4 - x^2 -
 2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4 - x^2 - 1}\left(x^4 + 1\right)\left(x^4 - 1\right)}{\left(2\,x^4 + x^2 - 2\right)\left(2\,x^4 - x^2 - 2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)*(x^4-x^2-1)^(1/2)/(2*x^4-x^2-2)^2/(2*x^4+x^2-2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 - x^2 - 1)*(x^4 + 1)*(x^4 - 1)/((2*x^4 + x^2 - 2)*(2*x^4
 - x^2 - 2)^2), x)
```

maple [C] time = 0.25, size = 293, normalized size = 4.25

$$-\frac{x\sqrt{x^4-x^2-1}}{16\left(2x^4-x^2-2\right)}+\frac{3\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-2-2\sqrt{5}}}{2}, \frac{i\sqrt{5}}{2}-\frac{i}{2}\right)}{16\sqrt{-2-2\sqrt{5}}\,\sqrt{x^4-x^2-1}}+\frac{\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(2\_Z^4+\_Z^2-2\right)}\_\alpha\left(-\frac{2\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(2\_\alpha^2-1\right)\left(-5\_\alpha^2+2x^2-6\right)}{4\sqrt{-\frac{3\_\alpha^2}{2}}\,\sqrt{x^4-x^2-1}}\right)}{\sqrt{-\_\alpha^2}}-\frac{3\left(2\_\alpha^3+\_\alpha\right)\sqrt{x^2+2+\sqrt{5}\,x^2}\,\sqrt{x^2+2-\sqrt{5}\,x^2}\,\operatorname{EllipticPi}\left(\sqrt{-\frac{1}{2}-\frac{\sqrt{5}}{2}}\,x, \frac{\_\alpha^2}{2}-\frac{\sqrt{5}\,\_\alpha^2}{2}+\frac{1}{4}-\frac{\sqrt{5}}{4}, \frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}{\sqrt{-\frac{1}{2}-\frac{\sqrt{5}}{2}}}\right)}{\sqrt{-\sqrt{5}-1}\,\sqrt{x^4-x^2-1}}\right)\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^4+1)*(x^4-x^2-1)^(1/2)/(2*x^4-x^2-2)^2/(2*x^4+x^2-2),x)
```

```
[Out] -1/16*x*(x^4-x^2-1)^(1/2)/(2*x^4-x^2-2)+3/16/(-2-2*5^(1/2))^(1/2)*(1-(-1/2-
1/2*5^(1/2))*x^2)^(1/2)*(1-(1/2*5^(1/2)-1/2)*x^2)^(1/2)/(x^4-x^2-1)^(1/2)*E
llipticF(1/2*x*(-2-2*5^(1/2))^(1/2),1/2*I*5^(1/2)-1/2*I)+1/256*2^(1/2)*sum(
_alpha*(-2*3^(1/2)/(-_alpha^2)^(1/2)*arctanh(1/4*(2*_alpha^2-1)*(-5*_alpha^
2+2*x^2-6)/(-3/2*_alpha^2)^(1/2)/(x^4-x^2-1)^(1/2))-3*(2*_alpha^3+_alpha)/(
-5^(1/2)-1)^(1/2)*(x^2+2+5^(1/2)*x^2)^(1/2)*(x^2+2-5^(1/2)*x^2)^(1/2)/(x^4-
x^2-1)^(1/2)*EllipticPi((-1/2-1/2*5^(1/2))^(1/2)*x,1/2*_alpha^2-1/2*5^(1/2)
*_alpha^2+1/4-1/4*5^(1/2),(1/2*5^(1/2)-1/2)^(1/2)/(-1/2-1/2*5^(1/2))^(1/2))
),_alpha=RootOf(2*_Z^4+_Z^2-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4 - x^2 - 1}\left(x^4 + 1\right)\left(x^4 - 1\right)}{\left(2\,x^4 + x^2 - 2\right)\left(2\,x^4 - x^2 - 2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)*(x^4-x^2-1)^(1/2)/(2*x^4-x^2-2)^2/(2*x^4+x^2-2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 - x^2 - 1)*(x^4 + 1)*(x^4 - 1)/((2*x^4 + x^2 - 2)*(2*x^4
 - x^2 - 2)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-1\right)\,\left(x^4+1\right)\,\sqrt{x^4-x^2-1}}{{\left(-2\,x^4+x^2+2\right)}^2\,\left(2\,x^4+x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(x^4 + 1)*(x^4 - x^2 - 1)^(1/2))/((x^2 - 2*x^4 + 2)^2*(x^2 +
 2*x^4 - 2)),x)
```

```
[Out] int(((x^4 - 1)*(x^4 + 1)*(x^4 - x^2 - 1)^(1/2))/((x^2 - 2*x^4 + 2)^2*(x^2 +
 2*x^4 - 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**4+1)*(x**4-x**2-1)**(1/2)/(2*x**4-x**2-2)**2/(2*x**4
+x**2-2),x)
```

```
[Out] Timed out
```

3.848 $\int \frac{\left(-b+ax^2\right)\sqrt{b^2+a^2x^4}}{x^2\left(b+ax^2\right)}\,dx$

Optimal. Leaf size=69

$$\frac{\sqrt{a^2x^4+b^2}}{x}+\sqrt{2}\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\,x}{\sqrt{a^2x^4+b^2}}\right)$$

Rubi [A] time = 0.97, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {6725, 277, 305, 220, 1196, 1209, 1211, 1699, 205}

$$\frac{\sqrt{a^2x^4+b^2}}{x}+\sqrt{2}\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\,x}{\sqrt{a^2x^4+b^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a*x^2)*Sqrt[b^2 + a^2*x^4])/(x^2*(b + a*x^2)),x]
```

```
[Out] Sqrt[b^2 + a^2*x^4]/x + Sqrt[2]*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[2]*Sqrt[a]*Sqr
t[b]*x)/Sqrt[b^2 + a^2*x^4]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
```

```
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-b+ax^2\right)\sqrt{b^2+a^2x^4}}{x^2\left(b+ax^2\right)}\,dx &= \int \left(-\frac{\sqrt{b^2+a^2x^4}}{x^2}+\frac{2a\sqrt{b^2+a^2x^4}}{b+ax^2}\right)dx\\
&= (2a)\int \frac{\sqrt{b^2+a^2x^4}}{b+ax^2}\,dx-\int \frac{\sqrt{b^2+a^2x^4}}{x^2}\,dx\\
&= \frac{\sqrt{b^2+a^2x^4}}{x}-\frac{2\int \frac{a^2b-a^3x^2}{\sqrt{b^2+a^2x^4}}\,dx}{a}-\left(2a^2\right)\int \frac{x^2}{\sqrt{b^2+a^2x^4}}\,dx+\left(4ab^2\right)\int \frac{}{\left(b+ax\right.}\\
&= \frac{\sqrt{b^2+a^2x^4}}{x}+\frac{2ax\sqrt{b^2+a^2x^4}}{b+ax^2}-\frac{2\sqrt{a}\sqrt{b}\left(b+ax^2\right)\sqrt{\frac{b^2+a^2x^4}{\left(b+ax^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{b}}\right.\right.}{\sqrt{b^2+a^2x^4}}\\
&= \frac{\sqrt{b^2+a^2x^4}}{x}+\left(2ab^2\right)\text{Subst}\left(\int \frac{1}{b+2ab^2x^2}\,dx,x,\frac{x}{\sqrt{b^2+a^2x^4}}\right)\\
&= \frac{\sqrt{b^2+a^2x^4}}{x}+\sqrt{2}\sqrt{a}\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\,x}{\sqrt{b^2+a^2x^4}}\right)
\end{aligned}$$

Mathematica [C] time = 0.29, size = 147, normalized size = 2.13

$$\frac{\sqrt{\frac{ia}{b}}\left(a^2x^4+b^2\right)+2iabx\sqrt{\frac{a^2x^4}{b^2}+1}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{ia}{b}}\,x\right)\middle|-1\right)-4iabx\sqrt{\frac{a^2x^4}{b^2}+1}\,\Pi\left(-i;i\sinh^{-1}\left(\sqrt{\frac{ia}{b}}\,x\right)\middle|-1\right)}{x\sqrt{\frac{ia}{b}}\sqrt{a^2x^4+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + a*x^2)*Sqrt[b^2 + a^2*x^4])/(x^2*(b + a*x^2)),x]
```

```
[Out] (Sqrt[(I*a)/b]*(b^2 + a^2*x^4) + (2*I)*a*b*x*Sqrt[1 + (a^2*x^4)/b^2]*Ellipt
icF[I*ArcSinh[Sqrt[(I*a)/b]*x], -1] - (4*I)*a*b*x*Sqrt[1 + (a^2*x^4)/b^2]*E
```

```
llipticPi[-I, I*ArcSinh[Sqrt[(I*a)/b]*x], -1])/(Sqrt[(I*a)/b]*x*Sqrt[b^2 +
a^2*x^4])
```

IntegrateAlgebraic [A] time = 0.41, size = 69, normalized size = 1.00

$$\frac{\sqrt{a^2x^4+b^2}}{x}+\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,x}{\sqrt{a^2x^4+b^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^2)*Sqrt[b^2 + a^2*x^4])/(x^2*(b + a*x^2)),x]
```

```
[Out] Sqrt[b^2 + a^2*x^4]/x + Sqrt[2]*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[2]*Sqrt[a]*Sqr
t[b]*x)/Sqrt[b^2 + a^2*x^4]]
```

fricas [A] time = 1.79, size = 162, normalized size = 2.35

$$\left[\frac{\sqrt{2}\,\sqrt{-ab}\,x\log\left(\frac{a^2x^4-2\,abx^2-2\,\sqrt{2}\,\sqrt{a^2x^4+b^2}\,\sqrt{-ab}\,x+b^2}{a^2x^4+2\,abx^2+b^2}\right)+2\,\sqrt{a^2x^4+b^2}}{2\,x},\ -\frac{\sqrt{2}\,\sqrt{ab}\,x\arctan\left(\frac{\sqrt{2}\,\sqrt{a^2x^4+b^2}\,\sqrt{ab}}{2\,abx}\right)-\sqrt{a^2x^4+b^2}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(a^2*x^4+b^2)^(1/2)/x^2/(a*x^2+b),x, algorithm="fricas"
)
```

```
[Out] [1/2*(sqrt(2)*sqrt(-a*b)*x*log((a^2*x^4 - 2*a*b*x^2 - 2*sqrt(2)*sqrt(a^2*x^
4 + b^2)*sqrt(-a*b)*x + b^2)/(a^2*x^4 + 2*a*b*x^2 + b^2)) + 2*sqrt(a^2*x^4
+ b^2))/x, -(sqrt(2)*sqrt(a*b)*x*arctan(1/2*sqrt(2)*sqrt(a^2*x^4 + b^2)*sqr
t(a*b)/(a*b*x)) - sqrt(a^2*x^4 + b^2))/x]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a^2x^4+b^2}\left(ax^2-b\right)}{\left(ax^2+b\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(a^2*x^4+b^2)^(1/2)/x^2/(a*x^2+b),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*x^4 + b^2)*(a*x^2 - b)/((a*x^2 + b)*x^2), x)
```

maple [C] time = 0.06, size = 398, normalized size = 5.77

$$2a\left(-\frac{b\sqrt{1-\frac{iax^2}{b}}\,\sqrt{1+\frac{iax^2}{b}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{ia}{b}},i\right)}{\sqrt{\frac{ia}{b}}\,\sqrt{a^2x^4+b^2}}+\frac{ib\sqrt{1-\frac{iax^2}{b}}\,\sqrt{1+\frac{iax^2}{b}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{ia}{b}},i\right)}{\sqrt{\frac{ia}{b}}\,\sqrt{a^2x^4+b^2}}-\frac{ib\sqrt{1-\frac{iax^2}{b}}\,\sqrt{1+\frac{iax^2}{b}}\,\mathrm{EllipticE}\left(x\sqrt{\frac{ia}{b}},i\right)}{\sqrt{\frac{ia}{b}}\,\sqrt{a^2x^4+b^2}}+\frac{2b\sqrt{1-\frac{iax^2}{b}}\,\sqrt{1+\frac{iax^2}{b}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{ia}{b}},i,\frac{\sqrt{-\frac{ia}{b}}}{\sqrt{\frac{ia}{b}}}\right)}{\sqrt{\frac{ia}{b}}\,\sqrt{a^2x^4+b^2}}\right)+\frac{\sqrt{a^2x^4+b^2}}{x}-\frac{2iab\sqrt{1-\frac{iax^2}{b}}\,\sqrt{1+\frac{iax^2}{b}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{ia}{b}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{ia}{b}},i\right)\right)}{\sqrt{\frac{ia}{b}}\,\sqrt{a^2x^4+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)*(a^2*x^4+b^2)^(1/2)/x^2/(a*x^2+b),x)
```

```
[Out] 2*a*(-b/(I*a/b)^(1/2)*(1-I*a/b*x^2)^(1/2)*(1+I*a/b*x^2)^(1/2)/(a^2*x^4+b^2)
^(1/2)*EllipticF(x*(I*a/b)^(1/2),I)+I*b/(I*a/b)^(1/2)*(1-I*a/b*x^2)^(1/2)*(
1+I*a/b*x^2)^(1/2)/(a^2*x^4+b^2)^(1/2)*EllipticF(x*(I*a/b)^(1/2),I)-I*b/(I*
a/b)^(1/2)*(1-I*a/b*x^2)^(1/2)*(1+I*a/b*x^2)^(1/2)/(a^2*x^4+b^2)^(1/2)*Elli
pticE(x*(I*a/b)^(1/2),I)+2*b/(I*a/b)^(1/2)*(1-I*a/b*x^2)^(1/2)*(1+I*a/b*x^2
)^(1/2)/(a^2*x^4+b^2)^(1/2)*EllipticPi(x*(I*a/b)^(1/2),I,(-I*a/b)^(1/2)/(I*
a/b)^(1/2)))+(a^2*x^4+b^2)^(1/2)/x-2*I*a*b/(I*a/b)^(1/2)*(1-I*a/b*x^2)^(1/2
)*(1+I*a/b*x^2)^(1/2)/(a^2*x^4+b^2)^(1/2)*(EllipticF(x*(I*a/b)^(1/2),I)-Ell
ipticE(x*(I*a/b)^(1/2),I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^4+b^2}\left(ax^2-b\right)}{\left(ax^2+b\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(a^2*x^4+b^2)^(1/2)/x^2/(a*x^2+b),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a^2*x^4 + b^2)*(a*x^2 - b)/((a*x^2 + b)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{a^2\,x^4+b^2}\,\left(b-a\,x^2\right)}{x^2\,\left(a\,x^2+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b^2 + a^2*x^4)^(1/2)*(b - a*x^2))/(x^2*(b + a*x^2)),x)
```

```
[Out] int(-((b^2 + a^2*x^4)^(1/2)*(b - a*x^2))/(x^2*(b + a*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2-b\right)\sqrt{a^2x^4+b^2}}{x^2\left(ax^2+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)*(a**2*x**4+b**2)**(1/2)/x**2/(a*x**2+b),x)
```

```
[Out] Integral((a*x**2 - b)*sqrt(a**2*x**4 + b**2)/(x**2*(a*x**2 + b)), x)
```

3.849 $\int \frac{4b+ax^3}{(b+ax^3)\sqrt[4]{-b-ax^3+cx^4}}\,dx$

Optimal. Leaf size=69

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-ax^3-b+cx^4}}\right)}{\sqrt[4]{c}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-ax^3-b+cx^4}}\right)}{\sqrt[4]{c}}$$

Rubi [F] time = 1.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{4b+ax^3}{(b+ax^3)\sqrt[4]{-b-ax^3+cx^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(4*b + a*x^3)/((b + a*x^3)*(-b - a*x^3 + c*x^4)^(1/4)),x]
```

```
[Out] Defer[Int][(-b - a*x^3 + c*x^4)^(-1/4), x] - b^(1/3)*Defer[Int][1/((-b^(1/3
) - a^(1/3)*x)*(-b - a*x^3 + c*x^4)^(1/4)), x] - b^(1/3)*Defer[Int][1/((-b^
(1/3) + (-1)^(1/3)*a^(1/3)*x)*(-b - a*x^3 + c*x^4)^(1/4)), x] - b^(1/3)*Def
er[Int][1/((-b^(1/3) - (-1)^(2/3)*a^(1/3)*x)*(-b - a*x^3 + c*x^4)^(1/4)), x
]
```

Rubi steps

$$\begin{aligned}
\int \frac{4b+ax^3}{(b+ax^3)\sqrt[4]{-b-ax^3+cx^4}}\,dx &= \int \left(\frac{1}{\sqrt[4]{-b-ax^3+cx^4}}+\frac{3b}{(b+ax^3)\sqrt[4]{-b-ax^3+cx^4}}\right)dx\\
&= (3b)\int \frac{1}{(b+ax^3)\sqrt[4]{-b-ax^3+cx^4}}\,dx+\int \frac{1}{\sqrt[4]{-b-ax^3+cx^4}}\,dx\\
&= (3b)\int \left(-\frac{1}{3b^{2/3}\left(-\sqrt[3]{b}-\sqrt[3]{a}\,x\right)\sqrt[4]{-b-ax^3+cx^4}}-\frac{1}{3b^{2/3}\left(-\sqrt[3]{b}+\sqrt[3]{-1}\sqrt[3]{a}\,x\right)\cdots}\right.\\
&= -\left(\sqrt[3]{b}\int \frac{1}{\left(-\sqrt[3]{b}-\sqrt[3]{a}\,x\right)\sqrt[4]{-b-ax^3+cx^4}}\,dx\right)-\sqrt[3]{b}\int \frac{1}{\left(-\sqrt[3]{b}+\sqrt[3]{-1}\sqrt[3]{a}\,x\right)\cdots}
\end{aligned}$$

Mathematica [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{4b+ax^3}{(b+ax^3)\sqrt[4]{-b-ax^3+cx^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(4*b + a*x^3)/((b + a*x^3)*(-b - a*x^3 + c*x^4)^(1/4)),x]
```

```
[Out] Integrate[(4*b + a*x^3)/((b + a*x^3)*(-b - a*x^3 + c*x^4)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.82, size = 69, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-ax^3-b+cx^4}}\right)}{\sqrt[4]{c}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{-ax^3-b+cx^4}}\right)}{\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4*b + a*x^3)/((b + a*x^3)*(-b - a*x^3 + c*x^4)^(1/4)),x
]
```

```
[Out] (2*ArcTan[(c^(1/4)*x)/(-b - a*x^3 + c*x^4)^(1/4)])/c^(1/4) + (2*ArcTanh[(c^
(1/4)*x)/(-b - a*x^3 + c*x^4)^(1/4)])/c^(1/4)
```

fricas [B] time = 0.68, size = 138, normalized size = 2.00

$$\frac{4\arctan\left(\frac{\frac{x\sqrt{\frac{\sqrt{c}x^2+\sqrt{cx^4-ax^3-b}}{x^2}}}{c^{\frac{1}{4}}}-\frac{\left(cx^4-ax^3-b\right)^{\frac{1}{4}}}{c^{\frac{1}{4}}}}{x}\right)}{c^{\frac{1}{4}}}+\frac{\log\left(\frac{c^{\frac{1}{4}}x+\left(cx^4-ax^3-b\right)^{\frac{1}{4}}}{x}\right)}{c^{\frac{1}{4}}}-\frac{\log\left(-\frac{c^{\frac{1}{4}}x-\left(cx^4-ax^3-b\right)^{\frac{1}{4}}}{x}\right)}{c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+4*b)/(a*x^3+b)/(c*x^4-a*x^3-b)^(1/4),x, algorithm="fricas"
)
```

```
[Out] 4*arctan((x*sqrt((sqrt(c)*x^2 + sqrt(c*x^4 - a*x^3 - b))/x^2)/c^(1/4) - (c*
x^4 - a*x^3 - b)^(1/4)/c^(1/4))/x)/c^(1/4) + log((c^(1/4)*x + (c*x^4 - a*x^
3 - b)^(1/4))/x)/c^(1/4) - log(-(c^(1/4)*x - (c*x^4 - a*x^3 - b)^(1/4))/x)/
c^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3+4b}{\left(cx^4-ax^3-b\right)^{\frac{1}{4}}\left(ax^3+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+4*b)/(a*x^3+b)/(c*x^4-a*x^3-b)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^3 + 4*b)/((c*x^4 - a*x^3 - b)^(1/4)*(a*x^3 + b)), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{a\,x^3+4b}{\left(a\,x^3+b\right)\left(c\,x^4-a\,x^3-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+4*b)/(a*x^3+b)/(c*x^4-a*x^3-b)^(1/4),x)
```

```
[Out] int((a*x^3+4*b)/(a*x^3+b)/(c*x^4-a*x^3-b)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3+4b}{\left(cx^4-ax^3-b\right)^{\frac{1}{4}}\left(ax^3+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+4*b)/(a*x^3+b)/(c*x^4-a*x^3-b)^(1/4),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^3 + 4*b)/((c*x^4 - a*x^3 - b)^(1/4)*(a*x^3 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x^3+4\,b}{\left(a\,x^3+b\right)\,\left(c\,x^4-a\,x^3-b\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*b + a*x^3)/((b + a*x^3)*(c*x^4 - a*x^3 - b)^(1/4)),x)
[Out] int((4*b + a*x^3)/((b + a*x^3)*(c*x^4 - a*x^3 - b)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3+4b}{\left(ax^3+b\right)\sqrt[4]{-ax^3-b+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+4*b)/(a*x**3+b)/(c*x**4-a*x**3-b)**(1/4),x)
[Out] Integral((a*x**3 + 4*b)/((a*x**3 + b)*(-a*x**3 - b + c*x**4)**(1/4)), x)
```

3.850 $\int \frac{\left(-3+x^4\right)\left(1+x^4\right)\left(1+x^3+x^4\right)}{x^6\left(1-x^3+x^4\right)\sqrt[4]{x+x^5}}\,dx$

Optimal. Leaf size=69

$$-4\tan^{-1}\left(\frac{\left(x^5+x\right)^{3/4}}{x^4+1}\right)-4\tanh^{-1}\left(\frac{\left(x^5+x\right)^{3/4}}{x^4+1}\right)+\frac{4\left(x^5+x\right)^{3/4}\left(3x^4+14x^3+3\right)}{21x^6}$$

Rubi [F] time = 2.62, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)\left(1+x^3+x^4\right)}{x^6\left(1-x^3+x^4\right)\sqrt[4]{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + x^4)*(1 + x^4)*(1 + x^3 + x^4))/(x^6*(1 - x^3 + x^4)*(x + x^5)^(
1/4)),x]
```

```
[Out] (4*(1 + x^4)^(1/4)*Hypergeometric2F1[-21/16, -3/4, -5/16, -x^4])/(7*x^5*(x
+ x^5)^(1/4)) + (8*(1 + x^4)^(1/4)*Hypergeometric2F1[-3/4, -9/16, 7/16, -x^
4])/(3*x^2*(x + x^5)^(1/4)) - (4*(1 + x^4)^(1/4)*Hypergeometric2F1[-3/4, -5
/16, 11/16, -x^4])/(5*x*(x + x^5)^(1/4)) - (24*x^(1/4)*(1 + x^4)^(1/4)*Defe
r[Subst][Defer[Int][(x^2*(1 + x^16)^(3/4))/(1 - x^12 + x^16), x], x, x^(1/4
)])/(x + x^5)^(1/4) + (32*x^(1/4)*(1 + x^4)^(1/4)*Defer[Subst][Defer[Int][(
x^6*(1 + x^16)^(3/4))/(1 - x^12 + x^16), x], x, x^(1/4)])/(x + x^5)^(1/4)
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-3+x^4\right)\left(1+x^4\right)\left(1+x^3+x^4\right)}{x^6\left(1-x^3+x^4\right)\sqrt[4]{x+x^5}}\,dx &= \frac{\left(\sqrt[4]{x}\sqrt[4]{1+x^4}\right)\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{3/4}\left(1+x^3+x^4\right)}{x^{25/4}\left(1-x^3+x^4\right)}\,dx}{\sqrt[4]{x+x^5}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{\left(-3+x^{16}\right)\left(1+x^{16}\right)^{3/4}\left(1+x^{12}+x^{16}\right)}{x^{22}\left(1-x^{12}+x^{16}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{x+x^5}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int\left(-\frac{3\left(1+x^{16}\right)^{3/4}}{x^{22}}-\frac{6\left(1+x^{16}\right)^{3/4}}{x^{10}}+\frac{\left(1+x^{16}\right)^{3/4}}{x^6}+\frac{2x^2(-\ldots}{\ldots}\right.\right.}{\sqrt[4]{x+x^5}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{\left(1+x^{16}\right)^{3/4}}{x^6}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{x+x^5}}+\frac{\left(8\sqrt[4]{x}\sqrt[4]{1+x^4}\right)\text{Sub}\ldots}{\ldots}\\
&= \frac{4\sqrt[4]{1+x^4}\,{}_2F_1\left(-\frac{21}{16},-\frac{3}{4};-\frac{5}{16};-x^4\right)}{7x^5\sqrt[4]{x+x^5}}+\frac{8\sqrt[4]{1+x^4}\,{}_2F_1\left(-\frac{3}{4},-\frac{9}{16};\frac{7}{16};-x^4\right.}{3x^2\sqrt[4]{x+x^5}}\\
&= \frac{4\sqrt[4]{1+x^4}\,{}_2F_1\left(-\frac{21}{16},-\frac{3}{4};-\frac{5}{16};-x^4\right)}{7x^5\sqrt[4]{x+x^5}}+\frac{8\sqrt[4]{1+x^4}\,{}_2F_1\left(-\frac{3}{4},-\frac{9}{16};\frac{7}{16};-x^4\right.}{3x^2\sqrt[4]{x+x^5}}
\end{aligned}$$

Mathematica [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{(-3+x^4)(1+x^4)(1+x^3+x^4)}{x^6(1-x^3+x^4)\sqrt[4]{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + x^4)*(1 + x^4)*(1 + x^3 + x^4))/(x^6*(1 - x^3 + x^4)*(x +
x^5)^(1/4)),x]
```

```
[Out] Integrate[((-3 + x^4)*(1 + x^4)*(1 + x^3 + x^4))/(x^6*(1 - x^3 + x^4)*(x +
x^5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 2.67, size = 69, normalized size = 1.00

$$-4\tan^{-1}\left(\frac{(x^5+x)^{3/4}}{x^4+1}\right)-4\tanh^{-1}\left(\frac{(x^5+x)^{3/4}}{x^4+1}\right)+\frac{4(x^5+x)^{3/4}(3x^4+14x^3+3)}{21x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 + x^4)*(1 + x^3 + x^4))/(x^6*(1 - x^3 + x
^4)*(x + x^5)^(1/4)),x]
```

```
[Out] (4*(3 + 14*x^3 + 3*x^4)*(x + x^5)^(3/4))/(21*x^6) - 4*ArcTan[(x + x^5)^(3/4
)/(1 + x^4)] - 4*ArcTanh[(x + x^5)^(3/4)/(1 + x^4)]
```

fricas [B] time = 46.48, size = 125, normalized size = 1.81

$$-\frac{2\left(21\,x^6\arctan\left(\frac{(x^5+x)^{\frac{3}{4}}x-(x^5+x)^{\frac{1}{4}}(x^4+1)}{2\,(x^5+x)}\right)-21\,x^6\log\left(-\frac{x^4+x^3-2\,(x^5+x)^{\frac{1}{4}}x^2+2\,\sqrt{x^5+x}\,x-2\,(x^5+x)^{\frac{3}{4}}+1}{x^4-x^3+1}\right)-2\,(x^5+x)^{\frac{3}{4}}(3\,x^4+14\,x^3+3)\right)}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)*(x^4+x^3+1)/x^6/(x^4-x^3+1)/(x^5+x)^(1/4),x, algo
rithm="fricas")
```

```
[Out] -2/21*(21*x^6*arctan(1/2*((x^5 + x)^(3/4)*x - (x^5 + x)^(1/4)*(x^4 + 1))/(x
^5 + x)) - 21*x^6*log(-(x^4 + x^3 - 2*(x^5 + x)^(1/4)*x^2 + 2*sqrt(x^5 + x)
*x - 2*(x^5 + x)^(3/4) + 1)/(x^4 - x^3 + 1)) - 2*(x^5 + x)^(3/4)*(3*x^4 + 1
4*x^3 + 3))/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4+x^3+1)(x^4+1)(x^4-3)}{(x^5+x)^{\frac{1}{4}}(x^4-x^3+1)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)*(x^4+x^3+1)/x^6/(x^4-x^3+1)/(x^5+x)^(1/4),x, algo
rithm="giac")
```

```
[Out] integrate((x^4 + x^3 + 1)*(x^4 + 1)*(x^4 - 3)/((x^5 + x)^(1/4)*(x^4 - x^3 +
 1)*x^6), x)
```

maple [C] time = 3.55, size = 187, normalized size = 2.71

$$\frac{\frac{4}{7}x^8+\frac{8}{7}x^4+\frac{8}{3}x^7+\frac{8}{3}x^3+\frac{4}{7}}{x^5\left(x\left(x^4+1\right)\right)^{\frac{1}{4}}}+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2+1\right)x^4+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{x^5+x}\,x-\mathrm{RootOf}\left(\_Z^2+1\right)x^3+2\left(x^5+x\right)^{\frac{3}{4}}-2\left(x^5+x\right)^{\frac{1}{4}}x^2-\mathrm{RootOf}\left(\_Z^2+1\right)}{x^4-x^3+1}\right)+2\ln\left(\frac{-x^4+2\left(x^5+x\right)^{\frac{3}{4}}-2\sqrt{x^5+x}\,x+2\left(x^5+x\right)^{\frac{1}{4}}x^2-x^3-1}{x^4-x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4+1)*(x^4+x^3+1)/x^6/(x^4-x^3+1)/(x^5+x)^(1/4),x)
```

```
[Out] 4/21*(3*x^8+14*x^7+6*x^4+14*x^3+3)/x^5/(x*(x^4+1))^(1/4)+2*RootOf(_Z^2+1)*l
n((-RootOf(_Z^2+1)*x^4+2*RootOf(_Z^2+1)*(x^5+x)^(1/2)*x-RootOf(_Z^2+1)*x^3+
2*(x^5+x)^(3/4)-2*(x^5+x)^(1/4)*x^2-RootOf(_Z^2+1))/(x^4-x^3+1))+2*ln((-x^4
+2*(x^5+x)^(3/4)-2*(x^5+x)^(1/2)*x+2*(x^5+x)^(1/4)*x^2-x^3-1)/(x^4-x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3+1\right)\left(x^4+1\right)\left(x^4-3\right)}{\left(x^5+x\right)^{\frac{1}{4}}\left(x^4-x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)*(x^4+x^3+1)/x^6/(x^4-x^3+1)/(x^5+x)^(1/4),x, algo
rithm="maxima")
```

```
[Out] integrate((x^4 + x^3 + 1)*(x^4 + 1)*(x^4 - 3)/((x^5 + x)^(1/4)*(x^4 - x^3 +
 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4+1\right)\,\left(x^4-3\right)\,\left(x^4+x^3+1\right)}{x^6\,{\left(x^5+x\right)}^{1/4}\,\left(x^4-x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)*(x^4 - 3)*(x^3 + x^4 + 1))/(x^6*(x + x^5)^(1/4)*(x^4 - x^3 +
 1)),x)
```

```
[Out] int(((x^4 + 1)*(x^4 - 3)*(x^3 + x^4 + 1))/(x^6*(x + x^5)^(1/4)*(x^4 - x^3 +
 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4+1)*(x**4+x**3+1)/x**6/(x**4-x**3+1)/(x**5+x)**(1/4
),x)
```

```
[Out] Timed out
```

3.851 $\int \frac{(-3+x^4)(1+2x^4+x^6+x^8)}{x^6(1-x^3+x^4)\sqrt[4]{x+x^5}}\,dx$

Optimal. Leaf size=69

$$-4\tan^{-1}\left(\frac{(x^5+x)^{3/4}}{x^4+1}\right)-4\tanh^{-1}\left(\frac{(x^5+x)^{3/4}}{x^4+1}\right)+\frac{4(x^5+x)^{3/4}(3x^4+7x^3+3)}{21x^6}$$

Rubi [F] time = 2.65, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-3+x^4)(1+2x^4+x^6+x^8)}{x^6(1-x^3+x^4)\sqrt[4]{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + x^4)*(1 + 2*x^4 + x^6 + x^8))/(x^6*(1 - x^3 + x^4)*(x + x^5)^(1/
4)),x]
```

```
[Out] (4*(1 + x^4)^(1/4)*Hypergeometric2F1[-21/16, 1/4, -5/16, -x^4])/(7*x^5*(x +
 x^5)^(1/4)) + (4*(1 + x^4)^(1/4)*Hypergeometric2F1[-9/16, 1/4, 7/16, -x^4]
)/(3*x^2*(x + x^5)^(1/4)) + (8*(1 + x^4)^(1/4)*Hypergeometric2F1[-5/16, 1/4
, 11/16, -x^4])/(5*x*(x + x^5)^(1/4)) + (8*x*(1 + x^4)^(1/4)*Hypergeometric
2F1[3/16, 1/4, 19/16, -x^4])/(3*(x + x^5)^(1/4)) + (4*x^2*(1 + x^4)^(1/4)*H
ypergeometric2F1[1/4, 7/16, 23/16, -x^4])/(7*(x + x^5)^(1/4)) + (4*x^3*(1 +
 x^4)^(1/4)*Hypergeometric2F1[1/4, 11/16, 27/16, -x^4])/(11*(x + x^5)^(1/4)
) - (32*x^(1/4)*(1 + x^4)^(1/4)*Defer[Subst][Defer[Int][x^2/((1 + x^16)^(1/
4)*(1 - x^12 + x^16)), x], x, x^(1/4)])/(x + x^5)^(1/4) + (8*x^(1/4)*(1 + x
^4)^(1/4)*Defer[Subst][Defer[Int][x^14/((1 + x^16)^(1/4)*(1 - x^12 + x^16))
, x], x, x^(1/4)])/(x + x^5)^(1/4)
```

Rubi steps

$$\int \frac{(-3+x^4)(1+2x^4+x^6+x^8)}{x^6(1-x^3+x^4)\sqrt[4]{x+x^5}}\,dx=\frac{\left(\sqrt[4]{x}\sqrt[4]{1+x^4}\right)\int \frac{(-3+x^4)(1+2x^4+x^6+x^8)}{x^{25/4}\sqrt[4]{1+x^4}(1-x^3+x^4)}\,dx}{\sqrt[4]{x+x^5}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x^4}\right)\operatorname{Subst}\left(\int \frac{(-3+x^{16})(1+2x^{16}+x^{24}+x^{32})}{x^{22}\sqrt[4]{1+x^{16}}(1-x^{12}+x^{16})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{x+x^5}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x^4}\right)\operatorname{Subst}\left(\int\left(-\frac{3}{x^{22}\sqrt[4]{1+x^{16}}}-\frac{3}{x^{10}\sqrt[4]{1+x^{16}}}-\frac{2}{x^6\sqrt[4]{1+x^{16}}}+\frac{2x^2}{\sqrt[4]{1+x^{16}}}+\right.\right.}{\sqrt[4]{x+x^5}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x^4}\right)\operatorname{Subst}\left(\int \frac{x^6}{\sqrt[4]{1+x^{16}}}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{x+x^5}}+\frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x^4}\right)\operatorname{Subst}\left(\int\right.}{\sqrt[4]{x+x^5}}$$

$$=\frac{4\sqrt[4]{1+x^4}\,{}_2F_1\left(-\frac{21}{16},\frac{1}{4};-\frac{5}{16};-x^4\right)}{7x^5\sqrt[4]{x+x^5}}+\frac{4\sqrt[4]{1+x^4}\,{}_2F_1\left(-\frac{9}{16},\frac{1}{4};\frac{7}{16};-x^4\right)}{3x^2\sqrt[4]{x+x^5}}+\frac{8\sqrt[4]{1}}{}$$

$$=\frac{4\sqrt[4]{1+x^4}\,{}_2F_1\left(-\frac{21}{16},\frac{1}{4};-\frac{5}{16};-x^4\right)}{7x^5\sqrt[4]{x+x^5}}+\frac{4\sqrt[4]{1+x^4}\,{}_2F_1\left(-\frac{9}{16},\frac{1}{4};\frac{7}{16};-x^4\right)}{3x^2\sqrt[4]{x+x^5}}+\frac{8\sqrt[4]{1}}{}$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(-3+x^4\right)\left(1+2x^4+x^6+x^8\right)}{x^6\left(1-x^3+x^4\right)\sqrt[4]{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + x^4)*(1 + 2*x^4 + x^6 + x^8))/(x^6*(1 - x^3 + x^4)*(x + x^
5)^(1/4)),x]
```

```
[Out] Integrate[((-3 + x^4)*(1 + 2*x^4 + x^6 + x^8))/(x^6*(1 - x^3 + x^4)*(x + x^
5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 2.67, size = 69, normalized size = 1.00

$$-4\tan^{-1}\left(\frac{\left(x^5+x\right)^{3/4}}{x^4+1}\right)-4\tanh^{-1}\left(\frac{\left(x^5+x\right)^{3/4}}{x^4+1}\right)+\frac{4\left(x^5+x\right)^{3/4}\left(3x^4+7x^3+3\right)}{21x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 + 2*x^4 + x^6 + x^8))/(x^6*(1 - x^3 + x^4
)*(x + x^5)^(1/4)),x]
```

```
[Out] (4*(3 + 7*x^3 + 3*x^4)*(x + x^5)^(3/4))/(21*x^6) - 4*ArcTan[(x + x^5)^(3/4)
/(1 + x^4)] - 4*ArcTanh[(x + x^5)^(3/4)/(1 + x^4)]
```

fricas [B] time = 42.86, size = 125, normalized size = 1.81

$$-\frac{2\left(21\,x^6\arctan\left(\frac{\left(x^5+x\right)^{\frac{3}{4}}x-\left(x^5+x\right)^{\frac{1}{4}}\left(x^4+1\right)}{2\left(x^5+x\right)}\right)-21\,x^6\log\left(-\frac{x^4+x^3-2\left(x^5+x\right)^{\frac{1}{4}}x^2+2\,\sqrt{x^5+x}\,x-2\left(x^5+x\right)^{\frac{3}{4}}+1}{x^4-x^3+1}\right)-2\left(x^5+x\right)^{\frac{3}{4}}\left(3\,x^4+7\,x^3+3\right)\right)}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^8+x^6+2*x^4+1)/x^6/(x^4-x^3+1)/(x^5+x)^(1/4),x, algori
thm="fricas")
```

```
[Out] -2/21*(21*x^6*arctan(1/2*((x^5 + x)^(3/4)*x - (x^5 + x)^(1/4)*(x^4 + 1))/(x
^5 + x)) - 21*x^6*log(-(x^4 + x^3 - 2*(x^5 + x)^(1/4)*x^2 + 2*sqrt(x^5 + x)
*x - 2*(x^5 + x)^(3/4) + 1)/(x^4 - x^3 + 1)) - 2*(x^5 + x)^(3/4)*(3*x^4 + 7
*x^3 + 3))/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+x^6+2\,x^4+1\right)\left(x^4-3\right)}{\left(x^5+x\right)^{\frac{1}{4}}\left(x^4-x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^8+x^6+2*x^4+1)/x^6/(x^4-x^3+1)/(x^5+x)^(1/4),x, algori
thm="giac")
```

```
[Out] integrate((x^8 + x^6 + 2*x^4 + 1)*(x^4 - 3)/((x^5 + x)^(1/4)*(x^4 - x^3 + 1
)*x^6), x)
```

maple [C] time = 3.27, size = 183, normalized size = 2.65

$$\frac{\frac{4}{7}x^8+\frac{8}{7}x^4+\frac{4}{3}x^7+\frac{4}{3}x^3+\frac{4}{7}}{x^5\left(x\left(x^4+1\right)\right)^{\frac{1}{4}}}-2\ln\left(\frac{x^4+2\left(x^5+x\right)^{\frac{3}{4}}+2\sqrt{x^5+x}\,x+2\left(x^5+x\right)^{\frac{1}{4}}x^2+x^3+1}{x^4-x^3+1}\right)+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2+1\right)x^4+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{x^5+x}\,x-\mathrm{RootOf}\left(\_Z^2+1\right)x^3+2\left(x^5+x\right)^{\frac{3}{4}}-2\left(x^5+x\right)^{\frac{1}{4}}x^2-\mathrm{RootOf}\left(\_Z^2+1\right)}{x^4-x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^8+x^6+2*x^4+1)/x^6/(x^4-x^3+1)/(x^5+x)^(1/4),x)
```

```
[Out] 4/21*(3*x^8+7*x^7+6*x^4+7*x^3+3)/x^5/(x*(x^4+1))^(1/4)-2*ln((x^4+2*(x^5+x)^
(3/4)+2*(x^5+x)^(1/2)*x+2*(x^5+x)^(1/4)*x^2+x^3+1)/(x^4-x^3+1))+2*RootOf(_Z
^2+1)*ln((-RootOf(_Z^2+1)*x^4+2*RootOf(_Z^2+1)*(x^5+x)^(1/2)*x-RootOf(_Z^2+
1)*x^3+2*(x^5+x)^(3/4)-2*(x^5+x)^(1/4)*x^2-RootOf(_Z^2+1))/(x^4-x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+x^6+2\,x^4+1\right)\left(x^4-3\right)}{\left(x^5+x\right)^{\frac{1}{4}}\left(x^4-x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^8+x^6+2*x^4+1)/x^6/(x^4-x^3+1)/(x^5+x)^(1/4),x, algori
thm="maxima")
```

```
[Out] integrate((x^8 + x^6 + 2*x^4 + 1)*(x^4 - 3)/((x^5 + x)^(1/4)*(x^4 - x^3 + 1
)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-3\right)\,\left(x^8+x^6+2\,x^4+1\right)}{x^6\,{\left(x^5+x\right)}^{1/4}\,\left(x^4-x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 3)*(2*x^4 + x^6 + x^8 + 1))/(x^6*(x + x^5)^(1/4)*(x^4 - x^3 + 1
)),x)
```

```
[Out] int(((x^4 - 3)*(2*x^4 + x^6 + x^8 + 1))/(x^6*(x + x^5)^(1/4)*(x^4 - x^3 + 1
)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**8+x**6+2*x**4+1)/x**6/(x**4-x**3+1)/(x**5+x)**(1/4),
x)
```

```
[Out] Timed out
```

3.852 $\int \frac{(b+ax^3)^{3/4}}{x}\,dx$

Optimal. Leaf size=70

$$\frac{2}{3}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)-\frac{2}{3}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)+\frac{4}{9}\left(ax^3+b\right)^{3/4}$$

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 63, 298, 203, 206}

$$\frac{2}{3}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)-\frac{2}{3}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)+\frac{4}{9}\left(ax^3+b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^3)^(3/4)/x,x]
```

```
[Out] (4*(b + a*x^3)^(3/4))/9 + (2*b^(3/4)*ArcTan[(b + a*x^3)^(1/4)/b^(1/4)])/3 -
 (2*b^(3/4)*ArcTanh[(b + a*x^3)^(1/4)/b^(1/4)])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(b+ax^3\right)^{3/4}}{x}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(b+ax)^{3/4}}{x}\,dx, x, x^3\right)\\ &= \frac{4}{9}\left(b+ax^3\right)^{3/4}+\frac{1}{3}b\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{b+ax}}\,dx, x, x^3\right)\\ &= \frac{4}{9}\left(b+ax^3\right)^{3/4}+\frac{(4b)\,\text{Subst}\left(\int \frac{x^2}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^3}\right)}{3a}\\ &= \frac{4}{9}\left(b+ax^3\right)^{3/4}-\frac{1}{3}(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^3}\right)+\frac{1}{3}(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx,\right.\\ &= \frac{4}{9}\left(b+ax^3\right)^{3/4}+\frac{2}{3}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b+ax^3}}{\sqrt[4]{b}}\right)-\frac{2}{3}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^3}}{\sqrt[4]{b}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.00

$$\frac{2}{3}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)-\frac{2}{3}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)+\frac{4}{9}\left(ax^3+b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^3)^(3/4)/x,x]
```

```
[Out] (4*(b + a*x^3)^(3/4))/9 + (2*b^(3/4)*ArcTan[(b + a*x^3)^(1/4)/b^(1/4)])/3 -
 (2*b^(3/4)*ArcTanh[(b + a*x^3)^(1/4)/b^(1/4)])/3
```

IntegrateAlgebraic [A] time = 0.07, size = 70, normalized size = 1.00

$$\frac{2}{3}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)-\frac{2}{3}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)+\frac{4}{9}\left(ax^3+b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^3)^(3/4)/x,x]
```

```
[Out] (4*(b + a*x^3)^(3/4))/9 + (2*b^(3/4)*ArcTan[(b + a*x^3)^(1/4)/b^(1/4)])/3 -
 (2*b^(3/4)*ArcTanh[(b + a*x^3)^(1/4)/b^(1/4)])/3
```

fricas [B] time = 0.42, size = 132, normalized size = 1.89

$$-\frac{4}{3}\left(b^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^3+b\right)^{\frac{1}{4}}\left(b^3\right)^{\frac{1}{4}}b^2-\sqrt{\sqrt{ax^3+b}\,b^4+\sqrt{b^3}\,b^3}\left(b^3\right)^{\frac{1}{4}}}{b^3}\right)-\frac{1}{3}\left(b^3\right)^{\frac{1}{4}}\log\left(\left(ax^3+b\right)^{\frac{1}{4}}b^2+\left(b^3\right)^{\frac{3}{4}}\right)+\frac{1}{3}\left(b^3\right)^{\frac{1}{4}}\log\left(\left(ax^3+b\right)^{\frac{1}{4}}b^2-\left(b^3\right)^{\frac{3}{4}}\right)+\frac{4}{9}\left(ax^3+b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)^(3/4)/x,x, algorithm="fricas")
```

```
[Out] -4/3*(b^3)^(1/4)*arctan(-((a*x^3 + b)^(1/4)*(b^3)^(1/4)*b^2 - sqrt(sqrt(a*x
^3 + b)*b^4 + sqrt(b^3)*b^3)*(b^3)^(1/4))/b^3) - 1/3*(b^3)^(1/4)*log((a*x^3
```

```
 + b)^(1/4)*b^2 + (b^3)^(3/4)) + 1/3*(b^3)^(1/4)*log((a*x^3 + b)^(1/4)*b^2
- (b^3)^(3/4)) + 4/9*(a*x^3 + b)^(3/4)
```

giac [B] time = 0.75, size = 185, normalized size = 2.64

$$-\frac{1}{3}\sqrt{2}(-b)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}+2\left(ax^3+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)-\frac{1}{3}\sqrt{2}(-b)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}-2\left(ax^3+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)+\frac{1}{6}\sqrt{2}(-b)^{\frac{3}{4}}\log\left(\sqrt{2}\left(ax^3+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^3+b}+\sqrt{-b}\right)-\frac{1}{6}\sqrt{2}(-b)^{\frac{3}{4}}\log\left(-\sqrt{2}\left(ax^3+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^3+b}+\sqrt{-b}\right)+\frac{4}{9}\left(ax^3+b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)^(3/4)/x,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*(-b)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^3 +
 b)^(1/4))/(-b)^(1/4)) - 1/3*sqrt(2)*(-b)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(-b)^(1/4) - 2*(a*x^3 + b)^(1/4))/(-b)^(1/4)) + 1/6*sqrt(2)*(-b)^(3/4)*lo
g(sqrt(2)*(a*x^3 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^3 + b) + sqrt(-b)) - 1/6*
sqrt(2)*(-b)^(3/4)*log(-sqrt(2)*(a*x^3 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^3 +
 b) + sqrt(-b)) + 4/9*(a*x^3 + b)^(3/4)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^3+b\right)^{\frac{3}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b)^(3/4)/x,x)
```

```
[Out] int((a*x^3+b)^(3/4)/x,x)
```

maxima [A] time = 0.41, size = 71, normalized size = 1.01

$$\frac{1}{3}b\left(\frac{2\arctan\left(\frac{\left(ax^3+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(ax^3+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^3+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}\right)+\frac{4}{9}\left(ax^3+b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)^(3/4)/x,x, algorithm="maxima")
```

```
[Out] 1/3*b*(2*arctan((a*x^3 + b)^(1/4)/b^(1/4))/b^(1/4) + log(((a*x^3 + b)^(1/4)
 - b^(1/4))/((a*x^3 + b)^(1/4) + b^(1/4)))/b^(1/4)) + 4/9*(a*x^3 + b)^(3/4)
```

mupad [B] time = 0.84, size = 50, normalized size = 0.71

$$\frac{2\,b^{3/4}\,\mathrm{atan}\left(\frac{{\left(a\,x^3+b\right)}^{1/4}}{b^{1/4}}\right)}{3}-\frac{2\,b^{3/4}\,\mathrm{atanh}\left(\frac{{\left(a\,x^3+b\right)}^{1/4}}{b^{1/4}}\right)}{3}+\frac{4\,{\left(a\,x^3+b\right)}^{3/4}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^3)^(3/4)/x,x)
```

```
[Out] (2*b^(3/4)*atan((b + a*x^3)^(1/4)/b^(1/4)))/3 - (2*b^(3/4)*atanh((b + a*x^3
)^(1/4)/b^(1/4)))/3 + (4*(b + a*x^3)^(3/4))/9
```

sympy [C] time = 1.14, size = 46, normalized size = 0.66

$$-\frac{a^{\frac{3}{4}}x^{\frac{9}{4}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b)**(3/4)/x,x)
```

```
[Out] -a**(3/4)*x**(9/4)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), b*exp_polar(I*pi
)/(a*x**3))/(3*gamma(1/4))
```

3.853 $\int \frac{1+3x+3x^4}{x\sqrt[4]{1+x^4}}\,dx$

Optimal. Leaf size=70

$$\left(x^4+1\right)^{3/4}+\frac{3}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\tan^{-1}\left(\sqrt[4]{x^4+1}\right)+\frac{3}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)$$

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1833, 240, 212, 206, 203, 446, 80, 63, 298}

$$\left(x^4+1\right)^{3/4}+\frac{3}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\tan^{-1}\left(\sqrt[4]{x^4+1}\right)+\frac{3}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 3*x^4)/(x*(1 + x^4)^(1/4)),x]
```

```
[Out] (1 + x^4)^(3/4) + (3*ArcTan[x/(1 + x^4)^(1/4)])/2 + ArcTan[(1 + x^4)^(1/4)]
/2 + (3*ArcTanh[x/(1 + x^4)^(1/4)])/2 - ArcTanh[(1 + x^4)^(1/4)]/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
```

```
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+3x^4}{x\sqrt[4]{1+x^4}}\,dx &= \int \left(\frac{3}{\sqrt[4]{1+x^4}}+\frac{1+3x^4}{x\sqrt[4]{1+x^4}}\right)dx\\
&= 3\int \frac{1}{\sqrt[4]{1+x^4}}\,dx+\int \frac{1+3x^4}{x\sqrt[4]{1+x^4}}\,dx\\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{1+3x}{x\sqrt[4]{1+x}}\,dx,x,x^4\right)+3\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)\\
&= \left(1+x^4\right)^{3/4}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1+x}}\,dx,x,x^4\right)+\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{3}{2}\,\text{Su}\\
&= \left(1+x^4\right)^{3/4}+\frac{3}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{3}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\sqrt[4]{1+x}\right.\\
&= \left(1+x^4\right)^{3/4}+\frac{3}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{3}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[4]{1+x}\right.\\
&= \left(1+x^4\right)^{3/4}+\frac{3}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{2}\tan^{-1}\left(\sqrt[4]{1+x^4}\right)+\frac{3}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{1}{2}\tanh^{-1}\left(\right.
\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.94

$$\frac{1}{2}\left(2\left(x^4+1\right)^{3/4}+3\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\tan^{-1}\left(\sqrt[4]{x^4+1}\right)+3\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 3*x^4)/(x*(1 + x^4)^(1/4)),x]
```

```
[Out] (2*(1 + x^4)^(3/4) + 3*ArcTan[x/(1 + x^4)^(1/4)] + ArcTan[(1 + x^4)^(1/4)]
+ 3*ArcTanh[x/(1 + x^4)^(1/4)] - ArcTanh[(1 + x^4)^(1/4)])/2
```

IntegrateAlgebraic [A] time = 4.42, size = 70, normalized size = 1.00

$$\left(x^4+1\right)^{3/4}+\frac{3}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\tan^{-1}\left(\sqrt[4]{x^4+1}\right)+\frac{3}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 3*x + 3*x^4)/(x*(1 + x^4)^(1/4)),x]
```

```
[Out] (1 + x^4)^(3/4) + (3*ArcTan[x/(1 + x^4)^(1/4)])/2 + ArcTan[(1 + x^4)^(1/4)]
/2 + (3*ArcTanh[x/(1 + x^4)^(1/4)])/2 - ArcTanh[(1 + x^4)^(1/4)]/2
```

fricas [B] time = 10.51, size = 141, normalized size = 2.01

$$\left(x^4+1\right)^{\frac{3}{4}}+\frac{3}{4}\arctan\left(2\left(x^4+1\right)^{\frac{1}{4}}x^3+2\left(x^4+1\right)^{\frac{3}{4}}x\right)-\frac{1}{4}\arctan\left(\frac{2\left(\left(x^4+1\right)^{\frac{3}{4}}+\left(x^4+1\right)^{\frac{1}{4}}\right)}{x^4}\right)+\frac{3}{4}\log\left(2x^4+2\left(x^4+1\right)^{\frac{1}{4}}x^3+2\sqrt{x^4+1}\,x^2+2\left(x^4+1\right)^{\frac{3}{4}}x+1\right)+\frac{1}{4}\log\left(-\frac{x^4-2\left(x^4+1\right)^{\frac{3}{4}}+2\sqrt{x^4+1}-2\left(x^4+1\right)^{\frac{1}{4}}+2}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+3*x+1)/x/(x^4+1)^(1/4),x, algorithm="fricas")
```

```
[Out] (x^4 + 1)^(3/4) + 3/4*arctan(2*(x^4 + 1)^(1/4)*x^3 + 2*(x^4 + 1)^(3/4)*x) -
 1/4*arctan(2*((x^4 + 1)^(3/4) + (x^4 + 1)^(1/4))/x^4) + 3/4*log(2*x^4 + 2*
(x^4 + 1)^(1/4)*x^3 + 2*sqrt(x^4 + 1)*x^2 + 2*(x^4 + 1)^(3/4)*x + 1) + 1/4*
log(-(x^4 - 2*(x^4 + 1)^(3/4) + 2*sqrt(x^4 + 1) - 2*(x^4 + 1)^(1/4) + 2)/x^
4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^4+3\,x+1}{\left(x^4+1\right)^{\frac{1}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+3*x+1)/x/(x^4+1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((3*x^4 + 3*x + 1)/((x^4 + 1)^(1/4)*x), x)
```

maple [C] time = 0.31, size = 90, normalized size = 1.29

$$\frac{\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)\left(-\frac{\pi\sqrt{2}\,x^4\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{4}\right],[2,2],-x^4\right)}{4\Gamma\left(\frac{3}{4}\right)}+\frac{\left(-3\ln(2)-\frac{\pi}{2}+4\ln(x)\right)\pi\sqrt{2}}{\Gamma\left(\frac{3}{4}\right)}\right)}{8\pi}+\frac{3x^4\,\mathrm{hypergeom}\left(\left[\frac{1}{4},1\right],[2],-x^4\right)}{4}+3x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{4}\right],\left[\frac{5}{4}\right],-x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4+3*x+1)/x/(x^4+1)^(1/4),x)
```

```
[Out] 1/8/Pi*2^(1/2)*GAMMA(3/4)*(-1/4*Pi*2^(1/2)/GAMMA(3/4)*x^4*hypergeom([1,1,5/
4],[2,2],-x^4)+(-3*ln(2)-1/2*Pi+4*ln(x))*Pi*2^(1/2)/GAMMA(3/4))+3/4*x^4*hyp
ergeom([1/4,1],[2],-x^4)+3*x*hypergeom([1/4,1/4],[5/4],-x^4)
```

maxima [A] time = 0.40, size = 88, normalized size = 1.26

$$\left(x^4+1\right)^{\frac{3}{4}}+\frac{1}{2}\arctan\left(\left(x^4+1\right)^{\frac{1}{4}}\right)-\frac{3}{2}\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{4}\log\left(\left(x^4+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{4}\log\left(\left(x^4+1\right)^{\frac{1}{4}}-1\right)+\frac{3}{4}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}+1\right)-\frac{3}{4}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+3*x+1)/x/(x^4+1)^(1/4),x, algorithm="maxima")
```

```
[Out] (x^4 + 1)^(3/4) + 1/2*arctan((x^4 + 1)^(1/4)) - 3/2*arctan((x^4 + 1)^(1/4)/
x) - 1/4*log((x^4 + 1)^(1/4) + 1) + 1/4*log((x^4 + 1)^(1/4) - 1) + 3/4*log(
(x^4 + 1)^(1/4)/x + 1) - 3/4*log((x^4 + 1)^(1/4)/x - 1)
```

mupad [B] time = 1.03, size = 41, normalized size = 0.59

$$\frac{\operatorname{atan}\left(\left(x^4+1\right)^{1/4}\right)}{2}-\frac{\operatorname{atanh}\left(\left(x^4+1\right)^{1/4}\right)}{2}+3\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -x^4\right)+\left(x^4+1\right)^{3/4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 3*x^4 + 1)/(x*(x^4 + 1)^(1/4)),x)
```

```
[Out] atan((x^4 + 1)^(1/4))/2 - atanh((x^4 + 1)^(1/4))/2 + 3*x*hypergeom([1/4, 1/
4], 5/4, -x^4) + (x^4 + 1)^(3/4)
```

sympy [C] time = 3.07, size = 66, normalized size = 0.94

$$\frac{3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\left(x^4+1\right)^{\frac{3}{4}}-\frac{\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{4x\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4+3*x+1)/x/(x**4+1)**(1/4),x)
```

```
[Out] 3*x*gamma(1/4)*hyper((1/4, 1/4), (5/4,), x**4*exp_polar(I*pi))/(4*gamma(5/4
)) + (x**4 + 1)**(3/4) - gamma(1/4)*hyper((1/4, 1/4), (5/4,), exp_polar(I*p
i)/x**4)/(4*x*gamma(5/4))
```

3.854 $\int \frac{(b+ax^4)^{3/4}}{x}\,dx$

Optimal. Leaf size=70

$$\frac{1}{2}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)-\frac{1}{2}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)+\frac{1}{3}\left(ax^4+b\right)^{3/4}$$

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 63, 298, 203, 206}

$$\frac{1}{2}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)-\frac{1}{2}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)+\frac{1}{3}\left(ax^4+b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^4)^(3/4)/x,x]
```

```
[Out] (b + a*x^4)^(3/4)/3 + (b^(3/4)*ArcTan[(b + a*x^4)^(1/4)/b^(1/4)])/2 - (b^(3
/4)*ArcTanh[(b + a*x^4)^(1/4)/b^(1/4)])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(b+ax^4\right)^{3/4}}{x}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{(b+ax)^{3/4}}{x}\,dx, x, x^4\right)\\
&= \frac{1}{3}\left(b+ax^4\right)^{3/4}+\frac{1}{4}b\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{b+ax}}\,dx, x, x^4\right)\\
&= \frac{1}{3}\left(b+ax^4\right)^{3/4}+\frac{b\,\text{Subst}\left(\int \frac{x^2}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^4}\right)}{a}\\
&= \frac{1}{3}\left(b+ax^4\right)^{3/4}-\frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^4}\right)+\frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx, x, \sqrt[4]{b}\right.\\
&= \frac{1}{3}\left(b+ax^4\right)^{3/4}+\frac{1}{2}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b+ax^4}}{\sqrt[4]{b}}\right)-\frac{1}{2}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^4}}{\sqrt[4]{b}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.00

$$\frac{1}{2}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)-\frac{1}{2}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)+\frac{1}{3}\left(ax^4+b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^4)^(3/4)/x,x]
```

```
[Out] (b + a*x^4)^(3/4)/3 + (b^(3/4)*ArcTan[(b + a*x^4)^(1/4)/b^(1/4)])/2 - (b^(3
/4)*ArcTanh[(b + a*x^4)^(1/4)/b^(1/4)])/2
```

IntegrateAlgebraic [A] time = 0.07, size = 70, normalized size = 1.00

$$\frac{1}{2}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)-\frac{1}{2}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)+\frac{1}{3}\left(ax^4+b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^4)^(3/4)/x,x]
```

```
[Out] (b + a*x^4)^(3/4)/3 + (b^(3/4)*ArcTan[(b + a*x^4)^(1/4)/b^(1/4)])/2 - (b^(3
/4)*ArcTanh[(b + a*x^4)^(1/4)/b^(1/4)])/2
```

fricas [B] time = 0.42, size = 132, normalized size = 1.89

$$-\left(b^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^4+b\right)^{\frac{1}{4}}\left(b^3\right)^{\frac{1}{4}}b^2-\sqrt{\sqrt{ax^4+b}\,b^4+\sqrt{b^3}\,b^3}\left(b^3\right)^{\frac{1}{4}}}{b^3}\right)-\frac{1}{4}\left(b^3\right)^{\frac{1}{4}}\log\left(\left(ax^4+b\right)^{\frac{1}{4}}b^2+\left(b^3\right)^{\frac{3}{4}}\right)+\frac{1}{4}\left(b^3\right)^{\frac{1}{4}}\log\left(\left(ax^4+b\right)^{\frac{1}{4}}b^2-\left(b^3\right)^{\frac{3}{4}}\right)+\frac{1}{3}\left(ax^4+b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4)/x,x, algorithm="fricas")
```

```
[Out] -(b^3)^(1/4)*arctan(-((a*x^4 + b)^(1/4)*(b^3)^(1/4)*b^2 - sqrt(sqrt(a*x^4 +
 b)*b^4 + sqrt(b^3)*b^3)*(b^3)^(1/4))/b^3) - 1/4*(b^3)^(1/4)*log((a*x^4 + b
```

```
)^(1/4)*b^2 + (b^3)^(3/4)) + 1/4*(b^3)^(1/4)*log((a*x^4 + b)^(1/4)*b^2 - (b
^3)^(3/4)) + 1/3*(a*x^4 + b)^(3/4)
```

giac [B] time = 0.27, size = 185, normalized size = 2.64

$$-\frac{1}{4}\sqrt{2}\,(-b)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-b)^{\frac{1}{4}}+2\left(ax^{4}+b\right)^{\frac{1}{4}}\right)}{2\,(-b)^{\frac{1}{4}}}\right)-\frac{1}{4}\sqrt{2}\,(-b)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-b)^{\frac{1}{4}}-2\left(ax^{4}+b\right)^{\frac{1}{4}}\right)}{2\,(-b)^{\frac{1}{4}}}\right)+\frac{1}{8}\sqrt{2}\,(-b)^{\frac{3}{4}}\log\left(\sqrt{2}\left(ax^{4}+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^{4}+b}+\sqrt{-b}\right)-\frac{1}{8}\sqrt{2}\,(-b)^{\frac{3}{4}}\log\left(-\sqrt{2}\left(ax^{4}+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^{4}+b}+\sqrt{-b}\right)+\frac{1}{3}\left(ax^{4}+b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4)/x,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(-b)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^4 +
 b)^(1/4))/(-b)^(1/4)) - 1/4*sqrt(2)*(-b)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(-b)^(1/4) - 2*(a*x^4 + b)^(1/4))/(-b)^(1/4)) + 1/8*sqrt(2)*(-b)^(3/4)*lo
g(sqrt(2)*(a*x^4 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^4 + b) + sqrt(-b)) - 1/8*
sqrt(2)*(-b)^(3/4)*log(-sqrt(2)*(a*x^4 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^4 +
 b) + sqrt(-b)) + 1/3*(a*x^4 + b)^(3/4)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{4}+b\right)^{\frac{3}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b)^(3/4)/x,x)
```

```
[Out] int((a*x^4+b)^(3/4)/x,x)
```

maxima [A] time = 0.42, size = 71, normalized size = 1.01

$$\frac{1}{4}\,b\left(\frac{2\,\arctan\left(\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^{4}+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}\right)+\frac{1}{3}\left(ax^{4}+b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4)/x,x, algorithm="maxima")
```

```
[Out] 1/4*b*(2*arctan((a*x^4 + b)^(1/4)/b^(1/4))/b^(1/4) + log(((a*x^4 + b)^(1/4)
 - b^(1/4))/((a*x^4 + b)^(1/4) + b^(1/4)))/b^(1/4)) + 1/3*(a*x^4 + b)^(3/4)
```

mupad [B] time = 0.81, size = 50, normalized size = 0.71

$$\frac{b^{3/4}\,\mathrm{atan}\left(\frac{{\left(a\,x^4+b\right)}^{1/4}}{b^{1/4}}\right)}{2}-\frac{b^{3/4}\,\mathrm{atanh}\left(\frac{{\left(a\,x^4+b\right)}^{1/4}}{b^{1/4}}\right)}{2}+\frac{{\left(a\,x^4+b\right)}^{3/4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^4)^(3/4)/x,x)
```

```
[Out] (b^(3/4)*atan((b + a*x^4)^(1/4)/b^(1/4)))/2 - (b^(3/4)*atanh((b + a*x^4)^(1
/4)/b^(1/4)))/2 + (b + a*x^4)^(3/4)/3
```

sympy [C] time = 1.09, size = 44, normalized size = 0.63

$$-\frac{a^{\frac{3}{4}}x^3\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^4}\right)}{4\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b)**(3/4)/x,x)
```

```
[Out] -a**(3/4)*x**3*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), b*exp_polar(I*pi)/(a
*x**4))/(4*gamma(1/4))
```

3.855 $\int \frac{-1+x^2}{(1+x^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

Optimal. Leaf size=70

$$\frac{2\tan^{-1}\left(\frac{x\sqrt{2a-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2+\sqrt{a}}\right)}{\sqrt{2a-c}}$$

Rubi [F] time = 1.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] Defer[Int][1/Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4], x] - I*Defer[Int][1/((I
 - x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x] - I*Defer[Int][1/((I + x)*
Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x]
```

Rubi steps

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx = \int \left(\frac{1}{\sqrt{a+bx+cx^2+bx^3+ax^4}} - \frac{2}{(1+x^2)\sqrt{a+bx+cx^2+bx^3}}\right.$$
$$= -\left(2\int \frac{1}{(1+x^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx\right) + \int \frac{}{\sqrt{a+bx+}}$$
$$= -\left(2\int\left(\frac{i}{2(i-x)\sqrt{a+bx+cx^2+bx^3+ax^4}} + \frac{i}{2(i+x)\sqrt{a+bx+}}\right.\right.$$
$$= -\left(i\int \frac{1}{(i-x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx\right) - i\int \frac{}{(i+x)\sqrt{a+b}}$$

Mathematica [C] time = 2.17, size = 3600, normalized size = 51.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] (2*(x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])^2*Sqrt[((Root[a +
b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 +
a*#1^4 & , 2])*(x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3]))/((x -
 Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 +
 b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3]))
]*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2
 + b*#1^3 + a*#1^4 & , 4])*Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#
1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#
1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(x - Root[a + b*#1 + c*#1^2 + b*#1^3 +
 a*#1^4 & , 4])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a +
 b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*
```

```
#1^3 + a*#1^4 & , 2])^2*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] -
Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])^2)]*(((-I)*(EllipticF[ArcS
in[Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*
#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*
#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[
a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^
3 + a*#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2]
- Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2
+ b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])
)/((-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1
^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & ,
2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*(-I + Root[a + b*#
1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1]) + EllipticPi[((-I + Root[a + b*#1 + c*
#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
, 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((-I + Root[a + b*
#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*
#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])), ArcSin[Sqr
t[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c
*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
 , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*
#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*
#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root
[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1
^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((-R
oot[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b
*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - R
oot[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*(-Root[a + b*#1 + c*#1^2
 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2]
)))/((-I + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(-I + Root[a +
b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 +
a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])) + (I*(Ell
ipticF[ArcSin[Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(
Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 +
b*#1^3 + a*#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & ,
 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#
1^2 + b*#1^3 + a*#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#
1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#
1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#
1^4 & , 4]))/((-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a +
b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 +
a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*(I + Ro
ot[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1]) + EllipticPi[((I + Root[a +
b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 +
a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((I + Roo
t[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#
1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])), Ar
cSin[Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a +
b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 +
a*#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Roo
t[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#
1^3 + a*#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2
] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^
2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4
]))/((-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*
#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
, 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*(-Root[a + b*#1
+ c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^
4 & , 2])))/((I + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(I + Roo
t[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#
```

```
1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])) + E
llipticF[ArcSin[Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])
*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2
+ b*#1^3 + a*#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
 , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c
*#1^2 + b*#1^3 + a*#1^4 & , 4]))]], ((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#
1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#
1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#
1^4 & , 4]))/((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b
*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a
*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))]/(-Root[a
+ b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] + Root[a + b*#1 + c*#1^2 + b*#1^3
+ a*#1^4 & , 4])))/(Sqrt[x*(b + c*x + b*x^2) + a*(1 + x^4)]*(-Root[a + b*#1
 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1
^4 & , 2]))
```

IntegrateAlgebraic [A] time = 0.54, size = 70, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{x\sqrt{2a-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2+\sqrt{a}}\right)}{\sqrt{2a-c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x
^4]),x]
```

```
[Out] (2*ArcTan[(Sqrt[2*a - c]*x)/(Sqrt[a] + Sqrt[a]*x^2 - Sqrt[a + b*x + c*x^2 +
 b*x^3 + a*x^4])])/Sqrt[2*a - c]
```

fricas [B] time = 0.72, size = 291, normalized size = 4.16

$$\left[-\frac{\sqrt{-2\,a+c}\log\left(-\frac{\left(8\,a^2-b^2-4\,ac\right)x^4+8\,(2\,ab-bc)x^3-2\left(8\,a^2+b^2-12\,ac+4\,c^2\right)x^2+8\,a^2+4\,\sqrt{ax^4+bx^3+cx^2+bx+a}\left(bx^2-2\,(2\,a-c)x+b\right)\sqrt{-2\,a+c}-b^2-4\,ac+8\,(2\,ab-bc)x}{x^4+2\,x^2+1}\right)}{2\,(2\,a-c)}, -\frac{\arctan\left(-\frac{\sqrt{ax^4+bx^3+cx^2+bx+a}\left(bx^2-2\,(2\,a-c)x+b\right)\sqrt{2\,a-c}}{2\left(\left(2\,a^2-ac\right)x^4+(2\,ab-bc)x^3+\left(2\,ac-c^2\right)x^2+2\,a^2-ac+(2\,ab-bc)x\right)}\right)}{\sqrt{2\,a-c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [-1/2*sqrt(-2*a + c)*log(-((8*a^2 - b^2 - 4*a*c)*x^4 + 8*(2*a*b - b*c)*x^3
- 2*(8*a^2 + b^2 - 12*a*c + 4*c^2)*x^2 + 8*a^2 + 4*sqrt(a*x^4 + b*x^3 + c*x
^2 + b*x + a)*(b*x^2 - 2*(2*a - c)*x + b)*sqrt(-2*a + c) - b^2 - 4*a*c + 8*
(2*a*b - b*c)*x)/(x^4 + 2*x^2 + 1))/(2*a - c), -arctan(-1/2*sqrt(a*x^4 + b*
x^3 + c*x^2 + b*x + a)*(b*x^2 - 2*(2*a - c)*x + b)*sqrt(2*a - c)/((2*a^2 -
a*c)*x^4 + (2*a*b - b*c)*x^3 + (2*a*c - c^2)*x^2 + 2*a^2 - a*c + (2*a*b - b
*c)*x))/sqrt(2*a - c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{ax^4+bx^3+cx^2+bx+a}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((x^2 - 1)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(x^2 + 1)), x)
```

maple [C] time = 0.15, size = 79345, normalized size = 1133.50

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - 1}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a}\,(x^2 + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((x^2 - 1)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 - 1}{(x^2 + 1)\,\sqrt{a\,x^4 + b\,x^3 + c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)),x)
```

```
[Out] int((x^2 - 1)/((x^2 + 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x - 1)\,(x + 1)}{(x^2 + 1)\,\sqrt{ax^4 + a + bx^3 + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(a*x**4+b*x**3+c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)/((x**2 + 1)*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**
2)), x)
```

3.856 $\int \frac{(b+ax^2)\sqrt{b^2+a^2x^4}}{x^2(-b+ax^2)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{a^2x^4+b^2}}{x}-\sqrt{2}\sqrt{a}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\,x}{\sqrt{a^2x^4+b^2}}\right)$$

Rubi [A] time = 0.97, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {6725, 277, 305, 220, 1196, 1209, 1198, 1211, 1699, 208}

$$\frac{\sqrt{a^2x^4+b^2}}{x}-\sqrt{2}\sqrt{a}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\,x}{\sqrt{a^2x^4+b^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((b + a*x^2)*Sqrt[b^2 + a^2*x^4])/(x^2*(-b + a*x^2)),x]
```

```
[Out] Sqrt[b^2 + a^2*x^4]/x - Sqrt[2]*Sqrt[a]*Sqrt[b]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sq
rt[b]*x)/Sqrt[b^2 + a^2*x^4]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
```

```
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b+ax^2)\sqrt{b^2+a^2x^4}}{x^2(-b+ax^2)}\,dx &= \int \left(-\frac{\sqrt{b^2+a^2x^4}}{x^2}+\frac{2a\sqrt{b^2+a^2x^4}}{-b+ax^2}\right)dx\\
&= (2a)\int \frac{\sqrt{b^2+a^2x^4}}{-b+ax^2}\,dx-\int \frac{\sqrt{b^2+a^2x^4}}{x^2}\,dx\\
&= \frac{\sqrt{b^2+a^2x^4}}{x}-\frac{2\int \frac{-a^2b-a^3x^2}{\sqrt{b^2+a^2x^4}}\,dx}{a}-\left(2a^2\right)\int \frac{x^2}{\sqrt{b^2+a^2x^4}}\,dx+\left(4ab^2\right)\int \frac{1}{\left(-b+ax^2\right)\cdots}\\
&= \frac{\sqrt{b^2+a^2x^4}}{x}-2\left((2ab)\int \frac{1}{\sqrt{b^2+a^2x^4}}\,dx\right)-(2ab)\int \frac{-b-ax^2}{\left(-b+ax^2\right)\sqrt{b^2+a^2x^4}}\,dx+\cdots\\
&= \frac{\sqrt{b^2+a^2x^4}}{x}+\left(2ab^2\right)\text{Subst}\left(\int \frac{1}{-b+2ab^2x^2}\,dx,x,\frac{x}{\sqrt{b^2+a^2x^4}}\right)\\
&= \frac{\sqrt{b^2+a^2x^4}}{x}-\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,x}{\sqrt{b^2+a^2x^4}}\right)
\end{aligned}$$

Mathematica [C] time = 0.30, size = 147, normalized size = 2.10

$$\frac{\sqrt{\frac{ia}{b}}\left(a^2x^4+b^2\right)-2iabx\sqrt{\frac{a^2x^4}{b^2}+1}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{ia}{b}}\,x\right)\middle|-1\right)+4iabx\sqrt{\frac{a^2x^4}{b^2}+1}\,\Pi\left(i;i\sinh^{-1}\left(\sqrt{\frac{ia}{b}}\,x\right)\middle|-1\right)}{x\sqrt{\frac{ia}{b}}\,\sqrt{a^2x^4+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b + a*x^2)*Sqrt[b^2 + a^2*x^4])/(x^2*(-b + a*x^2)),x]
```

```
[Out] (Sqrt[(I*a)/b]*(b^2 + a^2*x^4) - (2*I)*a*b*x*Sqrt[1 + (a^2*x^4)/b^2]*Ellipt
icF[I*ArcSinh[Sqrt[(I*a)/b]*x], -1] + (4*I)*a*b*x*Sqrt[1 + (a^2*x^4)/b^2]*E
llipticPi[I, I*ArcSinh[Sqrt[(I*a)/b]*x], -1])/(Sqrt[(I*a)/b]*x*Sqrt[b^2 + a
^2*x^4])
```

IntegrateAlgebraic [A] time = 0.39, size = 70, normalized size = 1.00

$$\frac{\sqrt{a^2x^4+b^2}}{x}-\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,x}{\sqrt{a^2x^4+b^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b + a*x^2)*Sqrt[b^2 + a^2*x^4])/(x^2*(-b + a*x^2)),x]
```

```
[Out] Sqrt[b^2 + a^2*x^4]/x - Sqrt[2]*Sqrt[a]*Sqrt[b]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sq
rt[b]*x)/Sqrt[b^2 + a^2*x^4]]
```

fricas [A] time = 1.14, size = 159, normalized size = 2.27

$$\left[\frac{\sqrt{2}\,\sqrt{ab}\,x\log\left(\frac{a^2x^4+2\,abx^2-2\,\sqrt{2}\,\sqrt{a^2x^4+b^2}\,\sqrt{ab}\,x+b^2}{a^2x^4-2\,abx^2+b^2}\right)+2\,\sqrt{a^2x^4+b^2}}{2\,x},\ \frac{\sqrt{2}\,\sqrt{-ab}\,x\arctan\left(\frac{\sqrt{2}\,\sqrt{a^2x^4+b^2}\,\sqrt{-ab}}{2\,abx}\right)+\sqrt{a^2x^4+b^2}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)*(a^2*x^4+b^2)^(1/2)/x^2/(a*x^2-b),x, algorithm="fricas"
)
```

```
[Out] [1/2*(sqrt(2)*sqrt(a*b)*x*log((a^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(a^2*x^4
 + b^2)*sqrt(a*b)*x + b^2)/(a^2*x^4 - 2*a*b*x^2 + b^2)) + 2*sqrt(a^2*x^4 +
b^2))/x, (sqrt(2)*sqrt(-a*b)*x*arctan(1/2*sqrt(2)*sqrt(a^2*x^4 + b^2)*sqrt(
-a*b)/(a*b*x)) + sqrt(a^2*x^4 + b^2))/x]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^4+b^2}\left(ax^2+b\right)}{\left(ax^2-b\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)*(a^2*x^4+b^2)^(1/2)/x^2/(a*x^2-b),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*x^4 + b^2)*(a*x^2 + b)/((a*x^2 - b)*x^2), x)
```

maple [C] time = 0.05, size = 397, normalized size = 5.67

$$2a\left(\frac{b\sqrt{1-\frac{iax^2}{b}}\,\sqrt{1+\frac{iax^2}{b}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{ia}{b}},i\right)}{\sqrt{\frac{ia}{b}}\,\sqrt{a^2x^4+b^2}}+\frac{ib\sqrt{1-\frac{iax^2}{b}}\,\sqrt{1+\frac{iax^2}{b}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{ia}{b}},i\right)}{\sqrt{\frac{ia}{b}}\,\sqrt{a^2x^4+b^2}}-\frac{ib\sqrt{1-\frac{iax^2}{b}}\,\sqrt{1+\frac{iax^2}{b}}\,\mathrm{EllipticE}\left(x\sqrt{\frac{ia}{b}},i\right)}{\sqrt{\frac{ia}{b}}\,\sqrt{a^2x^4+b^2}}-\frac{2b\sqrt{1-\frac{iax^2}{b}}\,\sqrt{1+\frac{iax^2}{b}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{ia}{b}},-i,\frac{\sqrt{-\frac{ia}{b}}}{\sqrt{\frac{ia}{b}}}\right)}{\sqrt{\frac{ia}{b}}\,\sqrt{a^2x^4+b^2}}\right)+\frac{\sqrt{a^2x^4+b^2}}{x}-\frac{2iab\sqrt{1-\frac{iax^2}{b}}\,\sqrt{1+\frac{iax^2}{b}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{ia}{b}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{ia}{b}},i\right)\right)}{\sqrt{\frac{ia}{b}}\,\sqrt{a^2x^4+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)*(a^2*x^4+b^2)^(1/2)/x^2/(a*x^2-b),x)
```

```
[Out] 2*a*(b/(I*a/b)^(1/2)*(1-I*a/b*x^2)^(1/2)*(1+I*a/b*x^2)^(1/2)/(a^2*x^4+b^2)^
(1/2)*EllipticF(x*(I*a/b)^(1/2),I)+I*b/(I*a/b)^(1/2)*(1-I*a/b*x^2)^(1/2)*(1
+I*a/b*x^2)^(1/2)/(a^2*x^4+b^2)^(1/2)*EllipticF(x*(I*a/b)^(1/2),I)-I*b/(I*a
/b)^(1/2)*(1-I*a/b*x^2)^(1/2)*(1+I*a/b*x^2)^(1/2)/(a^2*x^4+b^2)^(1/2)*Ellip
ticE(x*(I*a/b)^(1/2),I)-2*b/(I*a/b)^(1/2)*(1-I*a/b*x^2)^(1/2)*(1+I*a/b*x^2)
```

```
^(1/2)/(a^2*x^4+b^2)^(1/2)*EllipticPi(x*(I*a/b)^(1/2),-I,(-I*a/b)^(1/2)/(I*
a/b)^(1/2)))+(a^2*x^4+b^2)^(1/2)/x-2*I*a*b/(I*a/b)^(1/2)*(1-I*a/b*x^2)^(1/2
)*(1+I*a/b*x^2)^(1/2)/(a^2*x^4+b^2)^(1/2)*(EllipticF(x*(I*a/b)^(1/2),I)-Ell
ipticE(x*(I*a/b)^(1/2),I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^4+b^2}\left(ax^2+b\right)}{\left(ax^2-b\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)*(a^2*x^4+b^2)^(1/2)/x^2/(a*x^2-b),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a^2*x^4 + b^2)*(a*x^2 + b)/((a*x^2 - b)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{a^2\,x^4+b^2}\,\left(a\,x^2+b\right)}{x^2\,\left(b-a\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b^2 + a^2*x^4)^(1/2)*(b + a*x^2))/(x^2*(b - a*x^2)),x)
```

```
[Out] -int(((b^2 + a^2*x^4)^(1/2)*(b + a*x^2))/(x^2*(b - a*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2+b\right)\sqrt{a^2x^4+b^2}}{x^2\left(ax^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b)*(a**2*x**4+b**2)**(1/2)/x**2/(a*x**2-b),x)
```

```
[Out] Integral((a*x**2 + b)*sqrt(a**2*x**4 + b**2)/(x**2*(a*x**2 - b)), x)
```

3.857 $\int \frac{(b+ax^5)^{3/4}}{x} dx$

Optimal. Leaf size=70

$$\frac{2}{5}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)-\frac{2}{5}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)+\frac{4}{15}\left(ax^5+b\right)^{3/4}$$

Rubi [A] time = 0.09, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 63, 298, 203, 206}

$$\frac{2}{5}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)-\frac{2}{5}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)+\frac{4}{15}\left(ax^5+b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^5)^(3/4)/x,x]
```

```
[Out] (4*(b + a*x^5)^(3/4))/15 + (2*b^(3/4)*ArcTan[(b + a*x^5)^(1/4)/b^(1/4)])/5
- (2*b^(3/4)*ArcTanh[(b + a*x^5)^(1/4)/b^(1/4)])/5
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(b+ax^5\right)^{3/4}}{x}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{(b+ax)^{3/4}}{x}\,dx, x, x^5\right)\\
&= \frac{4}{15}\left(b+ax^5\right)^{3/4}+\frac{1}{5}b\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{b+ax}}\,dx, x, x^5\right)\\
&= \frac{4}{15}\left(b+ax^5\right)^{3/4}+\frac{(4b)\,\text{Subst}\left(\int \frac{x^2}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^5}\right)}{5a}\\
&= \frac{4}{15}\left(b+ax^5\right)^{3/4}-\frac{1}{5}(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^5}\right)+\frac{1}{5}(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx\right.\\
&= \frac{4}{15}\left(b+ax^5\right)^{3/4}+\frac{2}{5}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b+ax^5}}{\sqrt[4]{b}}\right)-\frac{2}{5}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^5}}{\sqrt[4]{b}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.00

$$\frac{2}{5}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)-\frac{2}{5}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)+\frac{4}{15}\left(ax^5+b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^5)^(3/4)/x,x]
```

```
[Out] (4*(b + a*x^5)^(3/4))/15 + (2*b^(3/4)*ArcTan[(b + a*x^5)^(1/4)/b^(1/4)])/5
- (2*b^(3/4)*ArcTanh[(b + a*x^5)^(1/4)/b^(1/4)])/5
```

IntegrateAlgebraic [A] time = 0.06, size = 70, normalized size = 1.00

$$\frac{2}{5}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)-\frac{2}{5}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)+\frac{4}{15}\left(ax^5+b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^5)^(3/4)/x,x]
```

```
[Out] (4*(b + a*x^5)^(3/4))/15 + (2*b^(3/4)*ArcTan[(b + a*x^5)^(1/4)/b^(1/4)])/5
- (2*b^(3/4)*ArcTanh[(b + a*x^5)^(1/4)/b^(1/4)])/5
```

fricas [B] time = 0.42, size = 132, normalized size = 1.89

$$-\frac{4}{5}\left(b^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^5+b\right)^{\frac{1}{4}}\left(b^3\right)^{\frac{1}{4}}b^2-\sqrt{\sqrt{ax^5+b}\,b^4+\sqrt{b^3}\,b^3}\left(b^3\right)^{\frac{1}{4}}}{b^3}\right)-\frac{1}{5}\left(b^3\right)^{\frac{1}{4}}\log\left(\left(ax^5+b\right)^{\frac{1}{4}}b^2+\left(b^3\right)^{\frac{3}{4}}\right)+\frac{1}{5}\left(b^3\right)^{\frac{1}{4}}\log\left(\left(ax^5+b\right)^{\frac{1}{4}}b^2-\left(b^3\right)^{\frac{3}{4}}\right)+\frac{4}{15}\left(ax^5+b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5+b)^(3/4)/x,x, algorithm="fricas")
```

```
[Out] -4/5*(b^3)^(1/4)*arctan(-((a*x^5 + b)^(1/4)*(b^3)^(1/4)*b^2 - sqrt(sqrt(a*x
^5 + b)*b^4 + sqrt(b^3)*b^3)*(b^3)^(1/4))/b^3) - 1/5*(b^3)^(1/4)*log((a*x^5
```

```
 + b)^(1/4)*b^2 + (b^3)^(3/4)) + 1/5*(b^3)^(1/4)*log((a*x^5 + b)^(1/4)*b^2
- (b^3)^(3/4)) + 4/15*(a*x^5 + b)^(3/4)
```

giac [B] time = 0.27, size = 185, normalized size = 2.64

$$-\frac{1}{5}\sqrt{2}\left(-b\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-b\right)^{\frac{1}{4}}+2\left(ax^{5}+b\right)^{\frac{1}{4}}\right)}{2\left(-b\right)^{\frac{1}{4}}}\right)-\frac{1}{5}\sqrt{2}\left(-b\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-b\right)^{\frac{1}{4}}-2\left(ax^{5}+b\right)^{\frac{1}{4}}\right)}{2\left(-b\right)^{\frac{1}{4}}}\right)+\frac{1}{10}\sqrt{2}\left(-b\right)^{\frac{3}{4}}\log\left(\sqrt{2}\left(ax^{5}+b\right)^{\frac{1}{4}}\left(-b\right)^{\frac{1}{4}}+\sqrt{ax^{5}+b}+\sqrt{-b}\right)-\frac{1}{10}\sqrt{2}\left(-b\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\left(ax^{5}+b\right)^{\frac{1}{4}}\left(-b\right)^{\frac{1}{4}}+\sqrt{ax^{5}+b}+\sqrt{-b}\right)+\frac{4}{15}\left(ax^{5}+b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5+b)^(3/4)/x,x, algorithm="giac")
```

```
[Out] -1/5*sqrt(2)*(-b)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^5 +
 b)^(1/4))/(-b)^(1/4)) - 1/5*sqrt(2)*(-b)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(-b)^(1/4) - 2*(a*x^5 + b)^(1/4))/(-b)^(1/4)) + 1/10*sqrt(2)*(-b)^(3/4)*l
og(sqrt(2)*(a*x^5 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^5 + b) + sqrt(-b)) - 1/1
0*sqrt(2)*(-b)^(3/4)*log(-sqrt(2)*(a*x^5 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^5
 + b) + sqrt(-b)) + 4/15*(a*x^5 + b)^(3/4)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{5}+b\right)^{\frac{3}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^5+b)^(3/4)/x,x)
```

```
[Out] int((a*x^5+b)^(3/4)/x,x)
```

maxima [A] time = 0.44, size = 71, normalized size = 1.01

$$\frac{1}{5}\,b\left(\frac{2\,\arctan\left(\frac{\left(ax^{5}+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(ax^{5}+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^{5}+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}\right)+\frac{4}{15}\left(ax^{5}+b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5+b)^(3/4)/x,x, algorithm="maxima")
```

```
[Out] 1/5*b*(2*arctan((a*x^5 + b)^(1/4)/b^(1/4))/b^(1/4) + log(((a*x^5 + b)^(1/4)
 - b^(1/4))/((a*x^5 + b)^(1/4) + b^(1/4)))/b^(1/4)) + 4/15*(a*x^5 + b)^(3/4
)
```

mupad [B] time = 0.88, size = 50, normalized size = 0.71

$$\frac{2\,b^{3/4}\,\mathrm{atan}\left(\frac{{\left(a\,x^5+b\right)}^{1/4}}{b^{1/4}}\right)}{5}-\frac{2\,b^{3/4}\,\mathrm{atanh}\left(\frac{{\left(a\,x^5+b\right)}^{1/4}}{b^{1/4}}\right)}{5}+\frac{4\,{\left(a\,x^5+b\right)}^{3/4}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^5)^(3/4)/x,x)
```

```
[Out] (2*b^(3/4)*atan((b + a*x^5)^(1/4)/b^(1/4)))/5 - (2*b^(3/4)*atanh((b + a*x^5
)^(1/4)/b^(1/4)))/5 + (4*(b + a*x^5)^(3/4))/15
```

sympy [C] time = 1.06, size = 46, normalized size = 0.66

$$-\frac{a^{\frac{3}{4}}x^{\frac{15}{4}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^5}\right)}{5\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**5+b)**(3/4)/x,x)
```

```
[Out] -a**(3/4)*x**(15/4)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), b*exp_polar(I*p
i)/(a*x**5))/(5*gamma(1/4))
```

3.858 $\int \frac{(3+2x^5)\sqrt{x-2x^4-x^6}}{(-1+x^5)^2}\,dx$

Optimal. Leaf size=70

$$-\frac{\sqrt{-x^6-2x^4+x}\,x}{x^5-1}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt{-x^6-2x^4+x}}{x^5+2x^3-1}\right)}{\sqrt{2}}$$

Rubi [F] time = 5.72, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(3+2x^5)\sqrt{x-2x^4-x^6}}{(-1+x^5)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((3 + 2*x^5)*Sqrt[x - 2*x^4 - x^6])/(-1 + x^5)^2,x]
```

```
[Out] (Sqrt[x - 2*x^4 - x^6]*Defer[Subst][Defer[Int][Sqrt[1 - 2*x^6 - x^10]/(-1 +
 x)^2, x], x, Sqrt[x]])/(10*Sqrt[x]*Sqrt[1 - 2*x^3 - x^5]) - (3*Sqrt[x - 2*
x^4 - x^6]*Defer[Subst][Defer[Int][Sqrt[1 - 2*x^6 - x^10]/(-1 + x), x], x,
Sqrt[x]])/(10*Sqrt[x]*Sqrt[1 - 2*x^3 - x^5]) + (Sqrt[x - 2*x^4 - x^6]*Defer
[Subst][Defer[Int][Sqrt[1 - 2*x^6 - x^10]/(1 + x)^2, x], x, Sqrt[x]])/(10*S
qrt[x]*Sqrt[1 - 2*x^3 - x^5]) + (3*Sqrt[x - 2*x^4 - x^6]*Defer[Subst][Defer
[Int][Sqrt[1 - 2*x^6 - x^10]/(1 + x), x], x, Sqrt[x]])/(10*Sqrt[x]*Sqrt[1 -
 2*x^3 - x^5]) - (Sqrt[x - 2*x^4 - x^6]*Defer[Subst][Defer[Int][Sqrt[1 - 2*
x^6 - x^10]/(1 - x + x^2 - x^3 + x^4)^2, x], x, Sqrt[x]])/(2*Sqrt[x]*Sqrt[1
 - 2*x^3 - x^5]) + (3*Sqrt[x - 2*x^4 - x^6]*Defer[Subst][Defer[Int][(x*Sqrt
[1 - 2*x^6 - x^10])/(1 - x + x^2 - x^3 + x^4)^2, x], x, Sqrt[x]])/(2*Sqrt[x
]*Sqrt[1 - 2*x^3 - x^5]) - (Sqrt[x - 2*x^4 - x^6]*Defer[Subst][Defer[Int][(
x^2*Sqrt[1 - 2*x^6 - x^10])/(1 - x + x^2 - x^3 + x^4)^2, x], x, Sqrt[x]])/(
2*Sqrt[x]*Sqrt[1 - 2*x^3 - x^5]) + (Sqrt[x - 2*x^4 - x^6]*Defer[Subst][Defe
r[Int][Sqrt[1 - 2*x^6 - x^10]/(1 - x + x^2 - x^3 + x^4), x], x, Sqrt[x]])/(
10*Sqrt[x]*Sqrt[1 - 2*x^3 - x^5]) - (Sqrt[x - 2*x^4 - x^6]*Defer[Subst][Def
er[Int][(x*Sqrt[1 - 2*x^6 - x^10])/(1 - x + x^2 - x^3 + x^4), x], x, Sqrt[x
]])/(10*Sqrt[x]*Sqrt[1 - 2*x^3 - x^5]) + (Sqrt[x - 2*x^4 - x^6]*Defer[Subst
][Defer[Int][(x^2*Sqrt[1 - 2*x^6 - x^10])/(1 - x + x^2 - x^3 + x^4), x], x,
 Sqrt[x]])/(2*Sqrt[x]*Sqrt[1 - 2*x^3 - x^5]) - (3*Sqrt[x - 2*x^4 - x^6]*Def
er[Subst][Defer[Int][(x^3*Sqrt[1 - 2*x^6 - x^10])/(1 - x + x^2 - x^3 + x^4)
, x], x, Sqrt[x]])/(10*Sqrt[x]*Sqrt[1 - 2*x^3 - x^5]) - (Sqrt[x - 2*x^4 - x
^6]*Defer[Subst][Defer[Int][Sqrt[1 - 2*x^6 - x^10]/(1 + x + x^2 + x^3 + x^4
)^2, x], x, Sqrt[x]])/(2*Sqrt[x]*Sqrt[1 - 2*x^3 - x^5]) - (3*Sqrt[x - 2*x^4
 - x^6]*Defer[Subst][Defer[Int][(x*Sqrt[1 - 2*x^6 - x^10])/(1 + x + x^2 + x
^3 + x^4)^2, x], x, Sqrt[x]])/(2*Sqrt[x]*Sqrt[1 - 2*x^3 - x^5]) - (Sqrt[x -
 2*x^4 - x^6]*Defer[Subst][Defer[Int][(x^2*Sqrt[1 - 2*x^6 - x^10])/(1 + x +
 x^2 + x^3 + x^4)^2, x], x, Sqrt[x]])/(2*Sqrt[x]*Sqrt[1 - 2*x^3 - x^5]) + (
Sqrt[x - 2*x^4 - x^6]*Defer[Subst][Defer[Int][Sqrt[1 - 2*x^6 - x^10]/(1 + x
 + x^2 + x^3 + x^4), x], x, Sqrt[x]])/(10*Sqrt[x]*Sqrt[1 - 2*x^3 - x^5]) +
(Sqrt[x - 2*x^4 - x^6]*Defer[Subst][Defer[Int][(x*Sqrt[1 - 2*x^6 - x^10])/(
1 + x + x^2 + x^3 + x^4), x], x, Sqrt[x]])/(10*Sqrt[x]*Sqrt[1 - 2*x^3 - x^5
]) + (Sqrt[x - 2*x^4 - x^6]*Defer[Subst][Defer[Int][(x^2*Sqrt[1 - 2*x^6 - x
^10])/(1 + x + x^2 + x^3 + x^4), x], x, Sqrt[x]])/(2*Sqrt[x]*Sqrt[1 - 2*x^3
 - x^5]) + (3*Sqrt[x - 2*x^4 - x^6]*Defer[Subst][Defer[Int][(x^3*Sqrt[1 - 2
*x^6 - x^10])/(1 + x + x^2 + x^3 + x^4), x], x, Sqrt[x]])/(10*Sqrt[x]*Sqrt[
1 - 2*x^3 - x^5])
```

Rubi steps

$$\int \frac{\left(3+2x^5\right)\sqrt{x-2x^4-x^6}}{\left(-1+x^5\right)^2}\,dx = \frac{\sqrt{x-2x^4-x^6}\int \frac{\sqrt{x}\sqrt{1-2x^3-x^5}\left(3+2x^5\right)}{\left(-1+x^5\right)^2}\,dx}{\sqrt{x}\sqrt{1-2x^3-x^5}}$$

$$= \frac{\left(2\sqrt{x-2x^4-x^6}\right)\text{Subst}\left(\int \frac{x^2\sqrt{1-2x^6-x^{10}}\left(3+2x^{10}\right)}{\left(-1+x^{10}\right)^2}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{1-2x^3-x^5}}$$

$$= \frac{\left(2\sqrt{x-2x^4-x^6}\right)\text{Subst}\left(\int \left(\frac{\sqrt{1-2x^6-x^{10}}}{20(-1+x)^2} + \frac{\sqrt{1-2x^6-x^{10}}}{20(1+x)^2} - \frac{3\sqrt{1-2x^6-x^{10}}}{10\left(-1+x^2\right)} + \frac{\left(-1+3x-x^2 \right.}{4\left(1-x+x\right.}\right.\right.}{}$$

$$= \frac{\sqrt{x-2x^4-x^6}\,\text{Subst}\left(\int \frac{\sqrt{1-2x^6-x^{10}}}{(-1+x)^2}\,dx, x, \sqrt{x}\right)}{10\sqrt{x}\sqrt{1-2x^3-x^5}} + \frac{\sqrt{x-2x^4-x^6}\,\text{Subst}\left(\int \frac{\sqrt{1-2}}{(1}\right.}{10\sqrt{x}\sqrt{1-2x^3}}$$

$$= \frac{\sqrt{x-2x^4-x^6}\,\text{Subst}\left(\int \frac{\sqrt{1-2x^6-x^{10}}}{(-1+x)^2}\,dx, x, \sqrt{x}\right)}{10\sqrt{x}\sqrt{1-2x^3-x^5}} + \frac{\sqrt{x-2x^4-x^6}\,\text{Subst}\left(\int \frac{\sqrt{1-2}}{(1}\right.}{10\sqrt{x}\sqrt{1-2x^3}}$$

$$= \frac{\sqrt{x-2x^4-x^6}\,\text{Subst}\left(\int \frac{\sqrt{1-2x^6-x^{10}}}{(-1+x)^2}\,dx, x, \sqrt{x}\right)}{10\sqrt{x}\sqrt{1-2x^3-x^5}} + \frac{\sqrt{x-2x^4-x^6}\,\text{Subst}\left(\int \frac{\sqrt{1-2}}{(1}\right.}{10\sqrt{x}\sqrt{1-2x^3}}$$

Mathematica [F] time = 2.55, size = 0, normalized size = 0.00

$$\int \frac{\left(3+2x^5\right)\sqrt{x-2x^4-x^6}}{\left(-1+x^5\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((3 + 2*x^5)*Sqrt[x - 2*x^4 - x^6])/(-1 + x^5)^2,x]
[Out] Integrate[((3 + 2*x^5)*Sqrt[x - 2*x^4 - x^6])/(-1 + x^5)^2, x]
```

IntegrateAlgebraic [A] time = 0.25, size = 70, normalized size = 1.00

$$-\frac{\sqrt{-x^6-2x^4+x}\,x}{x^5-1} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt{-x^6-2x^4+x}}{x^5+2x^3-1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3 + 2*x^5)*Sqrt[x - 2*x^4 - x^6])/(-1 + x^5)^2,x]
[Out] -((x*Sqrt[x - 2*x^4 - x^6])/(-1 + x^5)) - ArcTan[(Sqrt[2]*x*Sqrt[x - 2*x^4
- x^6])/(-1 + 2*x^3 + x^5)]/Sqrt[2]
```

fricas [A] time = 0.45, size = 69, normalized size = 0.99

$$-\frac{\sqrt{2}\left(x^5-1\right)\arctan\left(\frac{2\sqrt{2}\sqrt{-x^6-2x^4+x}\,x}{x^5+4x^3-1}\right) + 4\sqrt{-x^6-2x^4+x}\,x}{4\left(x^5-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^5+3)*(-x^6-2*x^4+x)^(1/2)/(x^5-1)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*(x^5 - 1)*arctan(2*sqrt(2)*sqrt(-x^6 - 2*x^4 + x)*x/(x^5 + 4*
x^3 - 1)) + 4*sqrt(-x^6 - 2*x^4 + x)*x)/(x^5 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^6 - 2\,x^4 + x}\,\left(2\,x^5 + 3\right)}{\left(x^5 - 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^5+3)*(-x^6-2*x^4+x)^(1/2)/(x^5-1)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^6 - 2*x^4 + x)*(2*x^5 + 3)/(x^5 - 1)^2, x)
```

maple [C] time = 0.77, size = 115, normalized size = 1.64

$$\frac{x^2\left(x^5+2x^3-1\right)}{\left(x^5-1\right)\sqrt{-x\left(x^5+2x^3-1\right)}}+\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)x^5+4\operatorname{RootOf}\left(\_Z^2+2\right)x^3-4\sqrt{-x^6-2x^4+x}\,x-\operatorname{RootOf}\left(\_Z^2+2\right)}{(-1+x)\left(x^4+x^3+x^2+x+1\right)}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^5+3)*(-x^6-2*x^4+x)^(1/2)/(x^5-1)^2,x)
```

```
[Out] x^2*(x^5+2*x^3-1)/(x^5-1)/(-x*(x^5+2*x^3-1))^(1/2)+1/4*RootOf(_Z^2+2)*ln(-(
RootOf(_Z^2+2)*x^5+4*RootOf(_Z^2+2)*x^3-4*(-x^6-2*x^4+x)^(1/2)*x-RootOf(_Z^
2+2))/(-1+x)/(x^4+x^3+x^2+x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^6 - 2\,x^4 + x}\,\left(2\,x^5 + 3\right)}{\left(x^5 - 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^5+3)*(-x^6-2*x^4+x)^(1/2)/(x^5-1)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^6 - 2*x^4 + x)*(2*x^5 + 3)/(x^5 - 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^5 + 3\right)\,\sqrt{-x^6 - 2\,x^4 + x}}{\left(x^5 - 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^5 + 3)*(x - 2*x^4 - x^6)^(1/2))/(x^5 - 1)^2,x)
```

```
[Out] int(((2*x^5 + 3)*(x - 2*x^4 - x^6)^(1/2))/(x^5 - 1)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**5+3)*(-x**6-2*x**4+x)**(1/2)/(x**5-1)**2,x)
```

```
[Out] Timed out
```

3.859 $\int \frac{(-2+x^6)(1+x^6)\sqrt[4]{1-x^4+x^6}}{x^6(1-2x^4+x^6)}\,dx$

Optimal. Leaf size=70

$$2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^6-x^4+1}}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^6-x^4+1}}\right)+\frac{2\sqrt[4]{x^6-x^4+1}\,(x^6+9x^4+1)}{5x^5}$$

Rubi [F] time = 2.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+x^6)(1+x^6)\sqrt[4]{1-x^4+x^6}}{x^6(1-2x^4+x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + x^6)*(1 + x^6)*(1 - x^4 + x^6)^(1/4))/(x^6*(1 - 2*x^4 + x^6)),x]
```

```
[Out] Defer[Int][(1 - x^4 + x^6)^(1/4), x] + Defer[Int][(1 - x^4 + x^6)^(1/4)/(-1
 - x), x] + Defer[Int][(1 - x^4 + x^6)^(1/4)/(-1 + x), x] - 2*Defer[Int][(1
 - x^4 + x^6)^(1/4)/x^6, x] - 4*Defer[Int][(1 - x^4 + x^6)^(1/4)/x^2, x] +
((2*I)*Defer[Int][(1 - x^4 + x^6)^(1/4)/(I*Sqrt[-1 + Sqrt[5]] - Sqrt[2]*x),
 x])/Sqrt[-1 + Sqrt[5]] - (2*Defer[Int][(1 - x^4 + x^6)^(1/4)/(Sqrt[1 + Sqr
t[5]] - Sqrt[2]*x), x])/Sqrt[1 + Sqrt[5]] + ((2*I)*Defer[Int][(1 - x^4 + x^
6)^(1/4)/(I*Sqrt[-1 + Sqrt[5]] + Sqrt[2]*x), x])/Sqrt[-1 + Sqrt[5]] - (2*De
fer[Int][(1 - x^4 + x^6)^(1/4)/(Sqrt[1 + Sqrt[5]] + Sqrt[2]*x), x])/Sqrt[1
+ Sqrt[5]]
```

Rubi steps

$$\begin{aligned}\int \frac{(-2+x^6)(1+x^6)\sqrt[4]{1-x^4+x^6}}{x^6(1-2x^4+x^6)}\,dx &= \int \left(\sqrt[4]{1-x^4+x^6}+\frac{\sqrt[4]{1-x^4+x^6}}{-1-x}+\frac{\sqrt[4]{1-x^4+x^6}}{-1+x}-\frac{2\sqrt[4]{1-x^4+x^6}}{x^6}-\right.\\
&= -\left(2\int \frac{\sqrt[4]{1-x^4+x^6}}{x^6}\,dx\right)+2\int \frac{(-1+2x^2)\sqrt[4]{1-x^4+x^6}}{-1-x^2+x^4}\,dx-4\int \frac{\sqrt[4]{1}}{}\\
&= -\left(2\int \frac{\sqrt[4]{1-x^4+x^6}}{x^6}\,dx\right)+2\int \left(\frac{2\sqrt[4]{1-x^4+x^6}}{-1-\sqrt{5}+2x^2}+\frac{2\sqrt[4]{1-x^4+x^6}}{-1+\sqrt{5}+2x^2}\right)d\\
&= -\left(2\int \frac{\sqrt[4]{1-x^4+x^6}}{x^6}\,dx\right)-4\int \frac{\sqrt[4]{1-x^4+x^6}}{x^2}\,dx+4\int \frac{\sqrt[4]{1-x^4+x^6}}{-1-\sqrt{5}+2x}\\
&= -\left(2\int \frac{\sqrt[4]{1-x^4+x^6}}{x^6}\,dx\right)-4\int \frac{\sqrt[4]{1-x^4+x^6}}{x^2}\,dx+4\int \left(\frac{i\sqrt[4]{}}{2\sqrt{-1+\sqrt{5}}}\right.\\
&= -\left(2\int \frac{\sqrt[4]{1-x^4+x^6}}{x^6}\,dx\right)-4\int \frac{\sqrt[4]{1-x^4+x^6}}{x^2}\,dx+\frac{(2i)\int \frac{\sqrt[4]{1-x^4+x^6}}{i\sqrt{-1+\sqrt{5}}-\sqrt{2}}}{\sqrt{-1+\sqrt{5}}}\end{aligned}$$

Mathematica [F] time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{(-2+x^6)(1+x^6)\sqrt[4]{1-x^4+x^6}}{x^6(1-2x^4+x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + x^6)*(1 + x^6)*(1 - x^4 + x^6)^(1/4))/(x^6*(1 - 2*x^4 + x^
6)),x]
```

```
[Out] Integrate[((-2 + x^6)*(1 + x^6)*(1 - x^4 + x^6)^(1/4))/(x^6*(1 - 2*x^4 + x^
6)), x]
```

IntegrateAlgebraic [A] time = 2.60, size = 70, normalized size = 1.00

$$2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^6-x^4+1}}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^6-x^4+1}}\right)+\frac{2\sqrt[4]{x^6-x^4+1}\,\left(x^6+9x^4+1\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^6)*(1 + x^6)*(1 - x^4 + x^6)^(1/4))/(x^6*(1 - 2
*x^4 + x^6)),x]
```

```
[Out] (2*(1 - x^4 + x^6)^(1/4)*(1 + 9*x^4 + x^6))/(5*x^5) + 2*ArcTan[x/(1 - x^4 +
 x^6)^(1/4)] - 2*ArcTanh[x/(1 - x^4 + x^6)^(1/4)]
```

fricas [B] time = 160.79, size = 154, normalized size = 2.20

$$\frac{5\,x^5\arctan\left(\frac{2\left(\left(x^6-x^4+1\right)^{\frac{1}{4}}x^3+\left(x^6-x^4+1\right)^{\frac{3}{4}}x\right)}{x^6-2\,x^4+1}\right)+5\,x^5\log\left(\frac{x^6-2\left(x^6-x^4+1\right)^{\frac{1}{4}}x^3+2\,\sqrt{x^6-x^4+1}\,x^2-2\left(x^6-x^4+1\right)^{\frac{3}{4}}x+1}{x^6-2\,x^4+1}\right)+2\left(x^6+9\,x^4+1\right)\left(x^6-x^4+1\right)^{\frac{1}{4}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6+1)*(x^6-x^4+1)^(1/4)/x^6/(x^6-2*x^4+1),x, algorithm=
"fricas")
```

```
[Out] 1/5*(5*x^5*arctan(2*((x^6 - x^4 + 1)^(1/4)*x^3 + (x^6 - x^4 + 1)^(3/4)*x)/(
x^6 - 2*x^4 + 1)) + 5*x^5*log((x^6 - 2*(x^6 - x^4 + 1)^(1/4)*x^3 + 2*sqrt(x
^6 - x^4 + 1)*x^2 - 2*(x^6 - x^4 + 1)^(3/4)*x + 1)/(x^6 - 2*x^4 + 1)) + 2*(
x^6 + 9*x^4 + 1)*(x^6 - x^4 + 1)^(1/4))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6-x^4+1\right)^{\frac{1}{4}}\left(x^6+1\right)\left(x^6-2\right)}{\left(x^6-2\,x^4+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6+1)*(x^6-x^4+1)^(1/4)/x^6/(x^6-2*x^4+1),x, algorithm=
"giac")
```

```
[Out] integrate((x^6 - x^4 + 1)^(1/4)*(x^6 + 1)*(x^6 - 2)/((x^6 - 2*x^4 + 1)*x^6)
, x)
```

maple [C] time = 2.07, size = 1242, normalized size = 17.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-2)*(x^6+1)*(x^6-x^4+1)^(1/4)/x^6/(x^6-2*x^4+1),x)
```

```
[Out] 2/5*(x^12+8*x^10-9*x^8+2*x^6+8*x^4+1)/x^5/(x^6-x^4+1)^(3/4)+(-ln(-(x^18-2*x
^16+2*(x^18-3*x^16+3*x^14+2*x^12-6*x^10+3*x^8+3*x^6-3*x^4+1)^(1/4)*x^13+x^1
4-4*(x^18-3*x^16+3*x^14+2*x^12-6*x^10+3*x^8+3*x^6-3*x^4+1)^(1/4)*x^11+3*x^1
2+2*(x^18-3*x^16+3*x^14+2*x^12-6*x^10+3*x^8+3*x^6-3*x^4+1)^(1/2)*x^8+2*(x^1
```

```
8-3*x^16+3*x^14+2*x^12-6*x^10+3*x^8+3*x^6-3*x^4+1)^(1/4)*x^9-4*x^10-2*(x^18
-3*x^16+3*x^14+2*x^12-6*x^10+3*x^8+3*x^6-3*x^4+1)^(1/2)*x^6+4*(x^18-3*x^16+
3*x^14+2*x^12-6*x^10+3*x^8+3*x^6-3*x^4+1)^(1/4)*x^7+x^8+2*(x^18-3*x^16+3*x^
14+2*x^12-6*x^10+3*x^8+3*x^6-3*x^4+1)^(3/4)*x^3-4*(x^18-3*x^16+3*x^14+2*x^1
2-6*x^10+3*x^8+3*x^6-3*x^4+1)^(1/4)*x^5+3*x^6+2*(x^18-3*x^16+3*x^14+2*x^12-
6*x^10+3*x^8+3*x^6-3*x^4+1)^(1/2)*x^2-2*x^4+2*(x^18-3*x^16+3*x^14+2*x^12-6*
x^10+3*x^8+3*x^6-3*x^4+1)^(1/4)*x+1)/(x^6-x^4+1)^2/(-1+x)/(1+x)/(x^4-x^2-1)
)+RootOf(_Z^2+1)*ln(-(-RootOf(_Z^2+1)*x^18+2*RootOf(_Z^2+1)*x^16-RootOf(_Z^
2+1)*x^14-2*(x^18-3*x^16+3*x^14+2*x^12-6*x^10+3*x^8+3*x^6-3*x^4+1)^(1/4)*x^
13-3*RootOf(_Z^2+1)*x^12+4*(x^18-3*x^16+3*x^14+2*x^12-6*x^10+3*x^8+3*x^6-3*
x^4+1)^(1/4)*x^11+2*(x^18-3*x^16+3*x^14+2*x^12-6*x^10+3*x^8+3*x^6-3*x^4+1)^
(1/2)*RootOf(_Z^2+1)*x^8+4*RootOf(_Z^2+1)*x^10-2*(x^18-3*x^16+3*x^14+2*x^12
-6*x^10+3*x^8+3*x^6-3*x^4+1)^(1/4)*x^9-2*(x^18-3*x^16+3*x^14+2*x^12-6*x^10+
3*x^8+3*x^6-3*x^4+1)^(1/2)*RootOf(_Z^2+1)*x^6-RootOf(_Z^2+1)*x^8-4*(x^18-3*
x^16+3*x^14+2*x^12-6*x^10+3*x^8+3*x^6-3*x^4+1)^(1/4)*x^7-3*RootOf(_Z^2+1)*x
^6+2*(x^18-3*x^16+3*x^14+2*x^12-6*x^10+3*x^8+3*x^6-3*x^4+1)^(3/4)*x^3+4*(x^
18-3*x^16+3*x^14+2*x^12-6*x^10+3*x^8+3*x^6-3*x^4+1)^(1/4)*x^5+2*(x^18-3*x^1
6+3*x^14+2*x^12-6*x^10+3*x^8+3*x^6-3*x^4+1)^(1/2)*RootOf(_Z^2+1)*x^2+2*Root
Of(_Z^2+1)*x^4-2*(x^18-3*x^16+3*x^14+2*x^12-6*x^10+3*x^8+3*x^6-3*x^4+1)^(1/
4)*x-RootOf(_Z^2+1))/(x^6-x^4+1)^2/(-1+x)/(1+x)/(x^4-x^2-1)))/(x^6-x^4+1)^(
3/4)*((x^6-x^4+1)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6 - x^4 + 1\right)^{\frac{1}{4}}\left(x^6 + 1\right)\left(x^6 - 2\right)}{\left(x^6 - 2\,x^4 + 1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6+1)*(x^6-x^4+1)^(1/4)/x^6/(x^6-2*x^4+1),x, algorithm=
"maxima")
```

```
[Out] integrate((x^6 - x^4 + 1)^(1/4)*(x^6 + 1)*(x^6 - 2)/((x^6 - 2*x^4 + 1)*x^6)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6+1\right)\,\left(x^6-2\right)\,{\left(x^6-x^4+1\right)}^{1/4}}{x^6\,\left(x^6-2\,x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 + 1)*(x^6 - 2)*(x^6 - x^4 + 1)^(1/4))/(x^6*(x^6 - 2*x^4 + 1)),x)
```

```
[Out] int(((x^6 + 1)*(x^6 - 2)*(x^6 - x^4 + 1)^(1/4))/(x^6*(x^6 - 2*x^4 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-2)*(x**6+1)*(x**6-x**4+1)**(1/4)/x**6/(x**6-2*x**4+1),x)
```

```
[Out] Timed out
```

3.860 $\int \frac{\left(b+ax^6\right)^{3/4}}{x}\,dx$

Optimal. Leaf size=70

$$\frac{1}{3}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)-\frac{1}{3}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)+\frac{2}{9}\left(ax^6+b\right)^{3/4}$$

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 63, 298, 203, 206}

$$\frac{1}{3}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)-\frac{1}{3}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)+\frac{2}{9}\left(ax^6+b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^6)^(3/4)/x,x]
```

```
[Out] (2*(b + a*x^6)^(3/4))/9 + (b^(3/4)*ArcTan[(b + a*x^6)^(1/4)/b^(1/4)])/3 - (
b^(3/4)*ArcTanh[(b + a*x^6)^(1/4)/b^(1/4)])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(b+ax^6\right)^{3/4}}{x}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{(b+ax)^{3/4}}{x}\,dx, x, x^6\right)\\
&= \frac{2}{9}\left(b+ax^6\right)^{3/4}+\frac{1}{6}b\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{b+ax}}\,dx, x, x^6\right)\\
&= \frac{2}{9}\left(b+ax^6\right)^{3/4}+\frac{(2b)\operatorname{Subst}\left(\int \frac{x^2}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^6}\right)}{3a}\\
&= \frac{2}{9}\left(b+ax^6\right)^{3/4}-\frac{1}{3}b\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^6}\right)+\frac{1}{3}b\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx, x, \sqrt[4]{b}\right.\\
&= \frac{2}{9}\left(b+ax^6\right)^{3/4}+\frac{1}{3}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b+ax^6}}{\sqrt[4]{b}}\right)-\frac{1}{3}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^6}}{\sqrt[4]{b}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.00

$$\frac{1}{3}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)-\frac{1}{3}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)+\frac{2}{9}\left(ax^6+b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^6)^(3/4)/x,x]
```

```
[Out] (2*(b + a*x^6)^(3/4))/9 + (b^(3/4)*ArcTan[(b + a*x^6)^(1/4)/b^(1/4)])/3 - (
b^(3/4)*ArcTanh[(b + a*x^6)^(1/4)/b^(1/4)])/3
```

IntegrateAlgebraic [A] time = 0.07, size = 70, normalized size = 1.00

$$\frac{1}{3}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)-\frac{1}{3}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)+\frac{2}{9}\left(ax^6+b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^6)^(3/4)/x,x]
```

```
[Out] (2*(b + a*x^6)^(3/4))/9 + (b^(3/4)*ArcTan[(b + a*x^6)^(1/4)/b^(1/4)])/3 - (
b^(3/4)*ArcTanh[(b + a*x^6)^(1/4)/b^(1/4)])/3
```

fricas [B] time = 0.44, size = 132, normalized size = 1.89

$$-\frac{2}{3}\left(b^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^6+b\right)^{\frac{1}{4}}\left(b^3\right)^{\frac{1}{4}}b^2-\sqrt{\sqrt{ax^6+b}\,b^4+\sqrt{b^3}\,b^3}\left(b^3\right)^{\frac{1}{4}}}{b^3}\right)-\frac{1}{6}\left(b^3\right)^{\frac{1}{4}}\log\left(\left(ax^6+b\right)^{\frac{1}{4}}b^2+\left(b^3\right)^{\frac{3}{4}}\right)+\frac{1}{6}\left(b^3\right)^{\frac{1}{4}}\log\left(\left(ax^6+b\right)^{\frac{1}{4}}b^2-\left(b^3\right)^{\frac{3}{4}}\right)+\frac{2}{9}\left(ax^6+b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)^(3/4)/x,x, algorithm="fricas")
```

```
[Out] -2/3*(b^3)^(1/4)*arctan(-((a*x^6 + b)^(1/4)*(b^3)^(1/4)*b^2 - sqrt(sqrt(a*x
^6 + b)*b^4 + sqrt(b^3)*b^3)*(b^3)^(1/4))/b^3) - 1/6*(b^3)^(1/4)*log((a*x^6
```

```
 + b)^(1/4)*b^2 + (b^3)^(3/4)) + 1/6*(b^3)^(1/4)*log((a*x^6 + b)^(1/4)*b^2
- (b^3)^(3/4)) + 2/9*(a*x^6 + b)^(3/4)
```

giac [B] time = 0.28, size = 185, normalized size = 2.64

$$-\frac{1}{6}\,\sqrt{2}\,(-b)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-b)^{\frac{1}{4}}+2\left(ax^6+b\right)^{\frac{1}{4}}\right)}{2\,(-b)^{\frac{1}{4}}}\right)-\frac{1}{6}\,\sqrt{2}\,(-b)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-b)^{\frac{1}{4}}-2\left(ax^6+b\right)^{\frac{1}{4}}\right)}{2\,(-b)^{\frac{1}{4}}}\right)+\frac{1}{12}\,\sqrt{2}\,(-b)^{\frac{3}{4}}\log\left(\sqrt{2}\left(ax^6+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^6+b}+\sqrt{-b}\right)-\frac{1}{12}\,\sqrt{2}\,(-b)^{\frac{3}{4}}\log\left(-\sqrt{2}\left(ax^6+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^6+b}+\sqrt{-b}\right)+\frac{2}{9}\left(ax^6+b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)^(3/4)/x,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*(-b)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^6 +
 b)^(1/4))/(-b)^(1/4)) - 1/6*sqrt(2)*(-b)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(-b)^(1/4) - 2*(a*x^6 + b)^(1/4))/(-b)^(1/4)) + 1/12*sqrt(2)*(-b)^(3/4)*l
og(sqrt(2)*(a*x^6 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^6 + b) + sqrt(-b)) - 1/1
2*sqrt(2)*(-b)^(3/4)*log(-sqrt(2)*(a*x^6 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^6
 + b) + sqrt(-b)) + 2/9*(a*x^6 + b)^(3/4)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{6}+b\right)^{\frac{3}{4}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6+b)^(3/4)/x,x)
```

```
[Out] int((a*x^6+b)^(3/4)/x,x)
```

maxima [A] time = 0.40, size = 71, normalized size = 1.01

$$\frac{1}{6}\,b\left(\frac{2\,\arctan\left(\frac{\left(ax^6+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(ax^6+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^6+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}\right)+\frac{2}{9}\left(ax^6+b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)^(3/4)/x,x, algorithm="maxima")
```

```
[Out] 1/6*b*(2*arctan((a*x^6 + b)^(1/4)/b^(1/4))/b^(1/4) + log(((a*x^6 + b)^(1/4)
 - b^(1/4))/((a*x^6 + b)^(1/4) + b^(1/4)))/b^(1/4)) + 2/9*(a*x^6 + b)^(3/4)
```

mupad [B] time = 0.83, size = 50, normalized size = 0.71

$$\frac{b^{3/4}\,\mathrm{atan}\left(\frac{{\left(a\,x^6+b\right)}^{1/4}}{b^{1/4}}\right)}{3}-\frac{b^{3/4}\,\mathrm{atanh}\left(\frac{{\left(a\,x^6+b\right)}^{1/4}}{b^{1/4}}\right)}{3}+\frac{2\,{\left(a\,x^6+b\right)}^{3/4}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^6)^(3/4)/x,x)
```

```
[Out] (b^(3/4)*atan((b + a*x^6)^(1/4)/b^(1/4)))/3 - (b^(3/4)*atanh((b + a*x^6)^(1
/4)/b^(1/4)))/3 + (2*(b + a*x^6)^(3/4))/9
```

sympy [C] time = 1.05, size = 46, normalized size = 0.66

$$-\frac{a^{\frac{3}{4}}x^{\frac{9}{2}}\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^{6}}\right)}{6\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6+b)**(3/4)/x,x)
```

```
[Out] -a**(3/4)*x**(9/2)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), b*exp_polar(I*pi
)/(a*x**6))/(6*gamma(1/4))
```

3.861 $\int \frac{\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}} dx$

Optimal. Leaf size=70

$$\sqrt{x+1}\sqrt{x+\sqrt{x+1}} - \frac{3}{2}\sqrt{x+\sqrt{x+1}} - \frac{7}{4}\log\left(2\sqrt{x+1} - 2\sqrt{x+\sqrt{x+1}} + 1\right)$$

Rubi [A] time = 0.12, antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {742, 640, 621, 206}

$$\sqrt{x+1}\sqrt{x+\sqrt{x+1}} - \frac{3}{2}\sqrt{x+\sqrt{x+1}} + \frac{7}{4}\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/Sqrt[x + Sqrt[1 + x]],x]
```

```
[Out] (-3*Sqrt[x + Sqrt[1 + x]])/2 + Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]] + (7*ArcTa
nh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 742

Rubi steps

$$\int \frac{\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)$$
$$= \sqrt{1+x}\sqrt{x+\sqrt{1+x}} + \text{Subst}\left(\int \frac{1-\frac{3x}{2}}{\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)$$
$$= -\frac{3}{2}\sqrt{x+\sqrt{1+x}} + \sqrt{1+x}\sqrt{x+\sqrt{1+x}} + \frac{7}{4}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)$$
$$= -\frac{3}{2}\sqrt{x+\sqrt{1+x}} + \sqrt{1+x}\sqrt{x+\sqrt{1+x}} + \frac{7}{2}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{1+2\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right)$$
$$= -\frac{3}{2}\sqrt{x+\sqrt{1+x}} + \sqrt{1+x}\sqrt{x+\sqrt{1+x}} + \frac{7}{4}\tanh^{-1}\left(\frac{1+2\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)$$

Mathematica [A] time = 0.04, size = 62, normalized size = 0.89

$$\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}-3\right)+\frac{7}{4}\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/Sqrt[x + Sqrt[1 + x]],x]
```

```
[Out] (Sqrt[x + Sqrt[1 + x]]*(-3 + 2*Sqrt[1 + x]))/2 + (7*ArcTanh[(1 + 2*Sqrt[1 +
 x])/(2*Sqrt[x + Sqrt[1 + x]])])/4
```

IntegrateAlgebraic [A] time = 0.13, size = 60, normalized size = 0.86

$$\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}-3\right)-\frac{7}{4}\log\left(-2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]/Sqrt[x + Sqrt[1 + x]],x]
```

```
[Out] (Sqrt[x + Sqrt[1 + x]]*(-3 + 2*Sqrt[1 + x]))/2 - (7*Log[-1 - 2*Sqrt[1 + x]
+ 2*Sqrt[x + Sqrt[1 + x]]])/4
```

fricas [A] time = 0.83, size = 56, normalized size = 0.80

$$\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}-3\right)+\frac{7}{8}\log\left(4\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)+8x+8\sqrt{x+1}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) - 3) + 7/8*log(4*sqrt(x + sqrt(x +
 1))*(2*sqrt(x + 1) + 1) + 8*x + 8*sqrt(x + 1) + 5)
```

giac [A] time = 0.36, size = 44, normalized size = 0.63

$$\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}-3\right)-\frac{7}{4}\log\left(-2\sqrt{x+\sqrt{x+1}}+2\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) - 3) - 7/4*log(-2*sqrt(x + sqrt(x
+ 1)) + 2*sqrt(x + 1) + 1)
```

maple [A] time = 0.01, size = 47, normalized size = 0.67

$$\sqrt{1+x}\,\sqrt{x+\sqrt{1+x}}-\frac{3\sqrt{x+\sqrt{1+x}}}{2}+\frac{7\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x)
```

```
[Out] (1+x)^(1/2)*(x+(1+x)^(1/2))^(1/2)-3/2*(x+(1+x)^(1/2))^(1/2)+7/4*ln(1/2+(1+x
)^(1/2)+(x+(1+x)^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + 1)/sqrt(x + sqrt(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(x + (x + 1)^(1/2))^(1/2),x)
```

```
[Out] int((x + 1)^(1/2)/(x + (x + 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x + 1)/sqrt(x + sqrt(x + 1)), x)
```

3.862 $\int \frac{\sqrt{x+\sqrt{1+x}}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=70

$$\sqrt{x+1}\sqrt{x+\sqrt{x+1}}+\frac{1}{2}\sqrt{x+\sqrt{x+1}}+\frac{5}{4}\log\left(2\sqrt{x+1}-2\sqrt{x+\sqrt{x+1}}+1\right)$$

Rubi [A] time = 0.13, antiderivative size = 62, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {612, 621, 206}

$$\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)-\frac{5}{4}\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[x + Sqrt[1 + x]]*(1 + 2*Sqrt[1 + x]))/2 - (5*ArcTanh[(1 + 2*Sqrt[1 +
x])/(2*Sqrt[x + Sqrt[1 + x]])])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x+\sqrt{1+x}}}{\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int \sqrt{-1+x+x^2}\,dx, x, \sqrt{1+x}\right)\\
&= \frac{1}{2}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right)-\frac{5}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)\\
&= \frac{1}{2}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right)-\frac{5}{2}\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{1+2\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right)\\
&= \frac{1}{2}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right)-\frac{5}{4}\tanh^{-1}\left(\frac{1+2\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 0.91

$$2\left(\frac{1}{4}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)-\frac{5}{8}\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] 2*((Sqrt[x + Sqrt[1 + x]]*(1 + 2*Sqrt[1 + x]))/4 - (5*ArcTanh[(1 + 2*Sqrt[1
 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/8)
```

IntegrateAlgebraic [A] time = 0.10, size = 60, normalized size = 0.86

$$\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)+\frac{5}{4}\log\left(-2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x + Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[x + Sqrt[1 + x]]*(1 + 2*Sqrt[1 + x]))/2 + (5*Log[-1 - 2*Sqrt[1 + x] +
 2*Sqrt[x + Sqrt[1 + x]]])/4
```

fricas [A] time = 0.83, size = 56, normalized size = 0.80

$$\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)+\frac{5}{8}\log\left(4\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)-8x-8\sqrt{x+1}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) + 1) + 5/8*log(4*sqrt(x + sqrt(x +
 1))*(2*sqrt(x + 1) + 1) - 8*x - 8*sqrt(x + 1) - 5)
```

giac [A] time = 0.26, size = 44, normalized size = 0.63

$$\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)+\frac{5}{4}\log\left(-2\sqrt{x+\sqrt{x+1}}+2\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) + 1) + 5/4*log(-2*sqrt(x + sqrt(x
+ 1)) + 2*sqrt(x + 1) + 1)
```

maple [A] time = 0.00, size = 41, normalized size = 0.59

$$\frac{\left(1+2\sqrt{1+x}\right)\sqrt{x+\sqrt{1+x}}}{2}-\frac{5\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x)
```

```
[Out] 1/2*(1+2*(1+x)^(1/2))*(x+(1+x)^(1/2))^(1/2)-5/4*ln(1/2+(1+x)^(1/2)+(x+(1+x)
^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1))/sqrt(x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x + 1)^(1/2))^(1/2)/(x + 1)^(1/2),x)
```

```
[Out] int((x + (x + 1)^(1/2))^(1/2)/(x + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)**(1/2))**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x + sqrt(x + 1))/sqrt(x + 1), x)
```

3.863 $\int \frac{(-1+x^2)\sqrt{1+\sqrt{1+x^2}}}{1+x^2}\,dx$

Optimal. Leaf size=70

$$\frac{2\sqrt{x^2+1}\,x}{3\sqrt{\sqrt{x^2+1}+1}}+\frac{4x}{3\sqrt{\sqrt{x^2+1}+1}}-4\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^2+1}+1}}\right)$$

Rubi [F] time = 0.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^2)\sqrt{1+\sqrt{1+x^2}}}{1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^2)*Sqrt[1 + Sqrt[1 + x^2]])/(1 + x^2),x]
```

```
[Out] (2*x^3)/(3*(1 + Sqrt[1 + x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 + x^2]] - I*D
efer[Int][Sqrt[1 + Sqrt[1 + x^2]]/(I - x), x] - I*Defer[Int][Sqrt[1 + Sqrt[
1 + x^2]]/(I + x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^2)\sqrt{1+\sqrt{1+x^2}}}{1+x^2}\,dx &= \int \left(\sqrt{1+\sqrt{1+x^2}}-\frac{2\sqrt{1+\sqrt{1+x^2}}}{1+x^2}\right)dx\\
&= -\left(2\int \frac{\sqrt{1+\sqrt{1+x^2}}}{1+x^2}\,dx\right)+\int \sqrt{1+\sqrt{1+x^2}}\,dx\\
&= \frac{2x^3}{3\left(1+\sqrt{1+x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1+x^2}}}-2\int \left(\frac{i\sqrt{1+\sqrt{1+x^2}}}{2(i-x)}+\frac{i\sqrt{1+\sqrt{1-}}}{2(i+x}\right.\\
&= \frac{2x^3}{3\left(1+\sqrt{1+x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1+x^2}}}-i\int \frac{\sqrt{1+\sqrt{1+x^2}}}{i-x}\,dx-i\int \frac{\sqrt{1+}}{i}
\end{aligned}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^2)\sqrt{1+\sqrt{1+x^2}}}{1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^2)*Sqrt[1 + Sqrt[1 + x^2]])/(1 + x^2),x]
```

```
[Out] Integrate[((-1 + x^2)*Sqrt[1 + Sqrt[1 + x^2]])/(1 + x^2), x]
```

IntegrateAlgebraic [A] time = 0.10, size = 70, normalized size = 1.00

$$\frac{2\sqrt{x^2+1}\,x}{3\sqrt{\sqrt{x^2+1}+1}}+\frac{4x}{3\sqrt{\sqrt{x^2+1}+1}}-4\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^2+1}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*Sqrt[1 + Sqrt[1 + x^2]])/(1 + x^2),x]
```

```
[Out] (4*x)/(3*Sqrt[1 + Sqrt[1 + x^2]]) + (2*x*Sqrt[1 + x^2])/(3*Sqrt[1 + Sqrt[1
+ x^2]]) - 4*ArcTan[x/Sqrt[1 + Sqrt[1 + x^2]]]
```

fricas [A] time = 1.53, size = 87, normalized size = 1.24

$$\frac{3\,x\arctan\left(\frac{4\left(x^4-12\,x^2+\left(5\,x^2-3\right)\sqrt{x^2+1}+3\right)\sqrt{\sqrt{x^2+1}+1}}{x^5-46\,x^3+17\,x}\right)+2\left(x^2+\sqrt{x^2+1}-1\right)\sqrt{\sqrt{x^2+1}+1}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(1+(x^2+1)^(1/2))^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x*arctan(4*(x^4 - 12*x^2 + (5*x^2 - 3)*sqrt(x^2 + 1) + 3)*sqrt(sqrt(
x^2 + 1) + 1)/(x^5 - 46*x^3 + 17*x)) + 2*(x^2 + sqrt(x^2 + 1) - 1)*sqrt(sqr
t(x^2 + 1) + 1))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-1\right)\sqrt{\sqrt{x^2+1}+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(1+(x^2+1)^(1/2))^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)*sqrt(sqrt(x^2 + 1) + 1)/(x^2 + 1), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-1\right)\sqrt{1+\sqrt{x^2+1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(1+(x^2+1)^(1/2))^(1/2)/(x^2+1),x)
```

```
[Out] int((x^2-1)*(1+(x^2+1)^(1/2))^(1/2)/(x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-1\right)\sqrt{\sqrt{x^2+1}+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(1+(x^2+1)^(1/2))^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)*sqrt(sqrt(x^2 + 1) + 1)/(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2-1\right)\sqrt{\sqrt{x^2+1}+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*((x^2 + 1)^(1/2) + 1)^(1/2))/(x^2 + 1),x)
[Out] int(((x^2 - 1)*((x^2 + 1)^(1/2) + 1)^(1/2))/(x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\sqrt{\sqrt{x^2+1}+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(1+(x**2+1)**(1/2))**(1/2)/(x**2+1),x)
[Out] Integral((x - 1)*(x + 1)*sqrt(sqrt(x**2 + 1) + 1)/(x**2 + 1), x)
```

3.864 $\int \frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal. Leaf size=70

$$\frac{x}{2\sqrt{\sqrt{x^4+1}+x^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)}{\sqrt{2}}$$

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/Sqrt[x^2 + Sqrt[1 + x^4]],x]
[Out] Defer[Int][1/Sqrt[x^2 + Sqrt[1 + x^4]], x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Mathematica [C] time = 11.73, size = 1665, normalized size = 23.79

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[x^2 + Sqrt[1 + x^4]],x]
[Out] (Sqrt[x^2 + Sqrt[1 + x^4]]*((Sqrt[2]*Sqrt[1 + x^4]*Sqrt[x^2*(x^2 + Sqrt[1 +
 x^4])]*(-(x^4*(1 + 2*x^4 + 2*x^2*Sqrt[1 + x^4])))^(3/2)*(Sqrt[2]*Sqrt[-(x^
2*(x^2 + Sqrt[1 + x^4]))]*(1 - 2*x^4 - 2*x^2*Sqrt[1 + x^4]) + (x^2 + Sqrt[1
 + x^4])*ArcSin[x^2 + Sqrt[1 + x^4]]))/(1 + 13*x^4 + 28*x^8 + 16*x^12 + 5*x
^2*Sqrt[1 + x^4] + 20*x^6*Sqrt[1 + x^4] + 16*x^10*Sqrt[1 + x^4]) + (56*x^6*
(x^2 + Sqrt[1 + x^4])^3*(1 + x^4 + x^2*Sqrt[1 + x^4])*(5*(1 + 5*x^4 + 2*x^8
 + 4*x^2*Sqrt[1 + x^4] + 2*x^6*Sqrt[1 + x^4])*Hypergeometric2F1[-1/2, 1/2,
5/2, (x^2 + Sqrt[1 + x^4])^2] + 2*(1 + 5*x^4 + 4*x^8 + 3*x^2*Sqrt[1 + x^4]
+ 4*x^6*Sqrt[1 + x^4])*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt[1 + x^4
])^2] + 2*(1 + x^4)*(1 + 8*x^4 + 8*x^8 + 4*x^2*Sqrt[1 + x^4] + 8*x^6*Sqrt[1
 + x^4])*HypergeometricPFQ[{1/2, 3/2, 2}, {1, 7/2}, (x^2 + Sqrt[1 + x^4])^2
]))/(630*x^2*Hypergeometric2F1[-1/2, 1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] + 3
990*x^6*Hypergeometric2F1[-1/2, 1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] + 6720*x
^10*Hypergeometric2F1[-1/2, 1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] + 3360*x^14*
Hypergeometric2F1[-1/2, 1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] + 105*Sqrt[1 + x
^4]*Hypergeometric2F1[-1/2, 1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] + 1890*x^4*S
qrt[1 + x^4]*Hypergeometric2F1[-1/2, 1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] + 5
040*x^8*Sqrt[1 + x^4]*Hypergeometric2F1[-1/2, 1/2, 5/2, (x^2 + Sqrt[1 + x^4
])^2] + 3360*x^12*Sqrt[1 + x^4]*Hypergeometric2F1[-1/2, 1/2, 5/2, (x^2 + Sq
rt[1 + x^4])^2] + 140*x^2*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt[1 +
x^4])^2] + 952*x^6*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt[1 + x^4])^2
```

```
] + 1232*x^10*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt[1 + x^4])^2] - 4
48*x^14*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt[1 + x^4])^2] - 896*x^1
8*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt[1 + x^4])^2] + 21*Sqrt[1 + x
^4]*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt[1 + x^4])^2] + 448*x^4*Sqr
t[1 + x^4]*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt[1 + x^4])^2] + 1120
*x^8*Sqrt[1 + x^4]*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt[1 + x^4])^2
] - 896*x^16*Sqrt[1 + x^4]*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt[1 +
 x^4])^2] + 54*x^2*Hypergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt[1 + x^4])^2
] + 720*x^6*Hypergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt[1 + x^4])^2] + 259
2*x^10*Hypergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt[1 + x^4])^2] + 3456*x^1
4*Hypergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt[1 + x^4])^2] + 1536*x^18*Hyp
ergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt[1 + x^4])^2] + 6*Sqrt[1 + x^4]*Hy
pergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt[1 + x^4])^2] + 240*x^4*Sqrt[1 +
x^4]*Hypergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt[1 + x^4])^2] + 1440*x^8*S
qrt[1 + x^4]*Hypergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt[1 + x^4])^2] + 26
88*x^12*Sqrt[1 + x^4]*Hypergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt[1 + x^4]
)^2] + 1536*x^16*Sqrt[1 + x^4]*Hypergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt
[1 + x^4])^2] + 14*(26*x^2 + 328*x^6 + 1136*x^10 + 1472*x^14 + 640*x^18 + 3
*Sqrt[1 + x^4] + 112*x^4*Sqrt[1 + x^4] + 640*x^8*Sqrt[1 + x^4] + 1152*x^12*
Sqrt[1 + x^4] + 640*x^16*Sqrt[1 + x^4])*HypergeometricPFQ[{1/2, 3/2, 2}, {1
, 7/2}, (x^2 + Sqrt[1 + x^4])^2] + 12*(10*x^2 + 170*x^6 + 832*x^10 + 1696*x
^14 + 1536*x^18 + 512*x^22 + Sqrt[1 + x^4] + 50*x^4*Sqrt[1 + x^4] + 400*x^8
*Sqrt[1 + x^4] + 1120*x^12*Sqrt[1 + x^4] + 1280*x^16*Sqrt[1 + x^4] + 512*x^
20*Sqrt[1 + x^4])*HypergeometricPFQ[{3/2, 5/2, 3}, {2, 9/2}, (x^2 + Sqrt[1
+ x^4])^2])))/(16*x^7)
```

IntegrateAlgebraic [A] time = 0.32, size = 70, normalized size = 1.00

$$\frac{x}{2\sqrt{\sqrt{x^4+1}+x^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

```
[Out] x/(2*Sqrt[x^2 + Sqrt[1 + x^4]]) + ArcTanh[(Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^
4]])/(1 + x^2 + Sqrt[1 + x^4])]/Sqrt[2]
```

fricas [A] time = 0.69, size = 90, normalized size = 1.29

$$-\frac{1}{2}\left(x^3-\sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}}+\frac{1}{8}\sqrt{2}\log\left(4x^4+4\sqrt{x^4+1}\,x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x^3 - sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1/8*sqrt(2)*log(4*
x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*sqrt(
x^2 + sqrt(x^4 + 1)) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

maple [C] time = 0.07, size = 51, normalized size = 0.73

$$-\frac{-\frac{\sqrt{\pi}\,\sqrt{2}\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{4},\frac{7}{4}\right],\left[2,2,\frac{5}{2}\right],-\frac{1}{x^4}\right)}{4x^4}+2\left(-4\ln(2)-2-4\ln(x)\right)\sqrt{\pi}\,\sqrt{2}}{16\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+(x^4+1)^(1/2))^(1/2),x)
[Out] -1/16/Pi^(1/2)*(-1/4*Pi^(1/2)*2^(1/2)/x^4*hypergeom([1,1,5/4,7/4],[2,2,5/2]
,-1/x^4)+2*(-4*ln(2)-2-4*ln(x))*Pi^(1/2)*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^4 + 1)^(1/2) + x^2)^(1/2),x)
[Out] int(1/((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

sympy [A] time = 0.98, size = 15, normalized size = 0.21

$$\frac{G_{3,3}^{2,2}\left(\begin{matrix}1,1 & \frac{3}{2} \\ \frac{1}{4},\frac{3}{4} & 0\end{matrix}\middle|\,x^4\right)}{16\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+(x**4+1)**(1/2))**(1/2),x)
[Out] meijerg(((1, 1), (3/2,)), ((1/4, 3/4), (0,)), x**4)/(16*sqrt(pi))
```

3.865 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2} dx$

Optimal. Leaf size=70

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\frac{\sqrt{\sqrt{x^4+1}+x^2}}{x}$$

Rubi [F] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/x^2,x]
[Out] Defer[Int][Sqrt[x^2 + Sqrt[1 + x^4]]/x^2, x]
```

Rubi steps

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2} dx=\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{x^2} dx$$

Mathematica [A] time = 0.22, size = 82, normalized size = 1.17

$$\frac{\sqrt{2}\sqrt{-x^2\left(\sqrt{x^4+1}+x^2\right)}\sin^{-1}\left(\sqrt{x^4+1}+x^2\right)-2\left(\sqrt{x^4+1}+x^2\right)}{2x\sqrt{\sqrt{x^4+1}+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/x^2,x]
[Out] (-2*(x^2 + Sqrt[1 + x^4]) + Sqrt[2]*Sqrt[-(x^2*(x^2 + Sqrt[1 + x^4]))]*ArcS
in[x^2 + Sqrt[1 + x^4]])/(2*x*Sqrt[x^2 + Sqrt[1 + x^4]])
```

IntegrateAlgebraic [A] time = 0.32, size = 70, normalized size = 1.00

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\frac{\sqrt{\sqrt{x^4+1}+x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2 + Sqrt[1 + x^4]]/x^2,x]
[Out] -(Sqrt[x^2 + Sqrt[1 + x^4]]/x) + Sqrt[2]*ArcTanh[(Sqrt[2]*x*Sqrt[x^2 + Sqrt
[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])]
```

fricas [A] time = 0.82, size = 81, normalized size = 1.16

$$\frac{\sqrt{2}\,x\log\left(4\,x^4+4\,\sqrt{x^4+1}\,x^2+2\left(\sqrt{2}\,x^3+\sqrt{2}\,\sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)-4\,\sqrt{x^2+\sqrt{x^4+1}}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*x*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*s
qrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1) - 4*sqrt(x^2 + sqrt(x^4 + 1)
))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/x^2, x)
```

maple [C] time = 0.06, size = 51, normalized size = 0.73

$$\frac{-\frac{\sqrt{\pi}\,\sqrt{2}\,\mathrm{hypergeom}\left(\left[\frac{3}{4},1,1,\frac{5}{4}\right],\left[\frac{3}{2},2,2\right],-\frac{1}{x^4}\right)}{2x^4}-4\left(-4\ln(2)+4-4\ln(x)\right)\sqrt{\pi}\,\sqrt{2}}{16\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/x^2,x)
```

```
[Out] 1/16/Pi^(1/2)*(-1/2*Pi^(1/2)*2^(1/2)/x^4*hypergeom([3/4,1,1,5/4],[3/2,2,2],
-1/x^4)-4*(-4*ln(2)+4-4*ln(x))*Pi^(1/2)*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{x^4 + 1} + x^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x^2,x)
```

```
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/x^2, x)
```

sympy [C] time = 1.11, size = 53, normalized size = 0.76

$$-\frac{\log\left(\frac{1}{x^4}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{4\pi} - \frac{\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_4F_3\left(\begin{matrix}\frac{3}{4},1,1,\frac{5}{4}\\ \frac{3}{2},2,2\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{8\pi x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/x**2,x)
```

```
[Out] -log(x**(-4))*gamma(1/4)*gamma(3/4)/(4*pi) - gamma(3/4)*gamma(5/4)*hyper((3
/4, 1, 1, 5/4), (3/2, 2, 2), exp_polar(I*pi)/x**4)/(8*pi*x**4)
```

3.866 $\int \frac{1}{\left(\sqrt{-1+x}+2\sqrt{x}\right)^2\sqrt{-1+x}}\,dx$

Optimal. Leaf size=71

$$-\frac{2\sqrt{x-1}}{3(3x+1)}+\frac{4\sqrt{x}}{3(3x+1)}+\frac{8\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{3}}-\frac{\sqrt{x}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.22, antiderivative size = 82, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6742, 51, 63, 203, 47}

$$-\frac{2\sqrt{x-1}}{3(3x+1)}+\frac{4\sqrt{x}}{3(3x+1)}+\frac{4\tan^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{x-1}\right)}{3\sqrt{3}}-\frac{4\tan^{-1}\left(\sqrt{3}\sqrt{x}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((Sqrt[-1 + x] + 2*Sqrt[x])^2*Sqrt[-1 + x]),x]
```

```
[Out] (-2*Sqrt[-1 + x])/(3*(1 + 3*x)) + (4*Sqrt[x])/(3*(1 + 3*x)) + (4*ArcTan[(Sq
rt[3]*Sqrt[-1 + x])/2])/(3*Sqrt[3]) - (4*ArcTan[Sqrt[3]*Sqrt[x]])/(3*Sqrt[3
])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{-1+x}+2\sqrt{x}\right)^2\sqrt{-1+x}}\,dx=\int\left(-\frac{8}{3\sqrt{-1+x}\,(1+3x)^2}-\frac{4\sqrt{x}}{(1+3x)^2}+\frac{5}{3\sqrt{-1+x}\,(1+3x)}\right)dx$$
$$=\frac{5}{3}\int\frac{1}{\sqrt{-1+x}\,(1+3x)}\,dx-\frac{8}{3}\int\frac{1}{\sqrt{-1+x}\,(1+3x)^2}\,dx-4\int\frac{\sqrt{x}}{(1+3x)^2}$$
$$=-\frac{2\sqrt{-1+x}}{3(1+3x)}+\frac{4\sqrt{x}}{3(1+3x)}-\frac{1}{3}\int\frac{1}{\sqrt{-1+x}\,(1+3x)}\,dx-\frac{2}{3}\int\frac{1}{\sqrt{x}\,(1+3x}$$
$$=-\frac{2\sqrt{-1+x}}{3(1+3x)}+\frac{4\sqrt{x}}{3(1+3x)}+\frac{5\tan^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{-1+x}\right)}{3\sqrt{3}}-\frac{2}{3}\text{Subst}\left(\int\frac{1}{4+}\right.$$
$$=-\frac{2\sqrt{-1+x}}{3(1+3x)}+\frac{4\sqrt{x}}{3(1+3x)}+\frac{4\tan^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{-1+x}\right)}{3\sqrt{3}}-\frac{4\tan^{-1}\left(\sqrt{3}\sqrt{x}\right)}{3\sqrt{3}}$$

Mathematica [A] time = 0.10, size = 78, normalized size = 1.10

$$\frac{-6\sqrt{x-1}+12\sqrt{x}+4\sqrt{3}(3x+1)\tan^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{x-1}\right)-4\sqrt{3}(3x+1)\tan^{-1}\left(\sqrt{3}\sqrt{x}\right)}{27x+9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((Sqrt[-1 + x] + 2*Sqrt[x])^2*Sqrt[-1 + x]),x]
```

```
[Out] (-6*Sqrt[-1 + x] + 12*Sqrt[x] + 4*Sqrt[3]*(1 + 3*x)*ArcTan[(Sqrt[3]*Sqrt[-1
 + x])/2] - 4*Sqrt[3]*(1 + 3*x)*ArcTan[Sqrt[3]*Sqrt[x]])/(9 + 27*x)
```

IntegrateAlgebraic [A] time = 0.34, size = 71, normalized size = 1.00

$$-\frac{2\sqrt{x-1}}{3(3x+1)}+\frac{4\sqrt{x}}{3(3x+1)}+\frac{8\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{3}}-\frac{\sqrt{x}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((Sqrt[-1 + x] + 2*Sqrt[x])^2*Sqrt[-1 + x]),x]
```

```
[Out] (-2*Sqrt[-1 + x])/(3*(1 + 3*x)) + (4*Sqrt[x])/(3*(1 + 3*x)) + (8*ArcTan[Sqr
t[-1 + x]/Sqrt[3] - Sqrt[x]/Sqrt[3]])/(3*Sqrt[3])
```

fricas [A] time = 0.40, size = 61, normalized size = 0.86

$$\frac{2\left(2\sqrt{3}(3x+1)\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{x-1}\right)-2\sqrt{3}(3x+1)\arctan\left(\sqrt{3}\sqrt{x}\right)-3\sqrt{x-1}+6\sqrt{x}\right)}{9(3x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+2*x^(1/2))^2/(-1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(2*sqrt(3)*(3*x + 1)*arctan(1/2*sqrt(3)*sqrt(x - 1)) - 2*sqrt(3)*(3*x +
 1)*arctan(sqrt(3)*sqrt(x)) - 3*sqrt(x - 1) + 6*sqrt(x))/(3*x + 1)
```

giac [B] time = 0.62, size = 132, normalized size = 1.86

$$\frac{2}{9}\sqrt{3}\left(\pi-2\arctan\left(-\frac{\sqrt{3}\left(\left(\sqrt{x-1}-\sqrt{x}\right)^2+1\right)}{2\left(\sqrt{x-1}-\sqrt{x}\right)}\right)\right)+\frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{x-1}\right)-\frac{8\left(\sqrt{x-1}-\sqrt{x}+\frac{1}{\sqrt{x-1}-\sqrt{x}}\right)}{3\left(3\left(\sqrt{x-1}-\sqrt{x}+\frac{1}{\sqrt{x-1}-\sqrt{x}}\right)^2+4\right)}-\frac{2\sqrt{x-1}}{3(3x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+2*x^(1/2))^2/(-1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*(pi - 2*arctan(-1/2*sqrt(3)*((sqrt(x - 1) - sqrt(x))^2 + 1)/(sq
rt(x - 1) - sqrt(x)))) + 4/9*sqrt(3)*arctan(1/2*sqrt(3)*sqrt(x - 1)) - 8/3*
(sqrt(x - 1) - sqrt(x) + 1/(sqrt(x - 1) - sqrt(x)))/(3*(sqrt(x - 1) - sqrt(
x) + 1/(sqrt(x - 1) - sqrt(x)))^2 + 4) - 2/3*sqrt(x - 1)/(3*x + 1)
```

maple [A] time = 0.03, size = 67, normalized size = 0.94

$$-\frac{\sqrt{-1+x}}{4\,(1+3x)}+\frac{4\sqrt{3}\ \arctan\left(\frac{\sqrt{-1+x}\ \sqrt{3}}{2}\right)}{9}+\frac{4\sqrt{x}}{9\left(\frac{1}{3}+x\right)}-\frac{4\sqrt{3}\ \arctan\left(\sqrt{x}\ \sqrt{3}\right)}{9}-\frac{5\sqrt{-1+x}}{36\left(\frac{1}{3}+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)^(1/2)+2*x^(1/2))^2/(-1+x)^(1/2),x)
```

```
[Out] -1/4*(-1+x)^(1/2)/(1+3*x)+4/9*3^(1/2)*arctan(1/2*(-1+x)^(1/2)*3^(1/2))+4/9*
x^(1/2)/(1/3+x)-4/9*3^(1/2)*arctan(x^(1/2)*3^(1/2))-5/36*(-1+x)^(1/2)/(1/3+
x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-1}\left(\sqrt{x-1}\ +2\sqrt{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+2*x^(1/2))^2/(-1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x - 1)*(sqrt(x - 1) + 2*sqrt(x))^2), x)
```

mupad [B] time = 1.96, size = 96, normalized size = 1.35

$$\frac{4\sqrt{x}}{3\,(3\,x+1)}-\frac{2\sqrt{x-1}}{3\,(3\,x+1)}+\frac{\sqrt{3}\ \ln\left(\frac{12\sqrt{x-1}-\sqrt{3}\,x\,3\mathrm{i}+\sqrt{3}\,7\mathrm{i}}{3\,x+1}\right)2\mathrm{i}}{9}+\frac{\sqrt{3}\ \ln\left(\frac{\sqrt{3}-3\sqrt{3}\,x+\sqrt{x}\,6\mathrm{i}}{x\,3\mathrm{i}+1\mathrm{i}}\right)2\mathrm{i}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x - 1)^(1/2) + 2*x^(1/2))^2*(x - 1)^(1/2)),x)
```

```
[Out] (3^(1/2)*log((12*(x - 1)^(1/2) - 3^(1/2)*x*3i + 3^(1/2)*7i)/(3*x + 1))*2i)/
9 - (2*(x - 1)^(1/2))/(3*(3*x + 1)) + (3^(1/2)*log((3^(1/2) - 3*3^(1/2)*x +
 x^(1/2)*6i)/(x*3i + 1i))*2i)/9 + (4*x^(1/2))/(3*(3*x + 1))
```

sympy [A] time = 20.56, size = 12, normalized size = 0.17

$$\tilde{\infty}\left(-\frac{2x}{x-1}+\frac{2}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**(1/2)+2*x**(1/2))**2/(-1+x)**(1/2),x)
```

```
[Out] zoo*(-2*x/(x - 1) + 2/(x - 1))
```

3.867 $\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)^2}\,dx$

Optimal. Leaf size=71

$$-\frac{3\sqrt{x^2-1}\,x}{8\left(3x^2-4\right)}+\frac{5}{32}\log\left(-x^2+\sqrt{x^2-1}\,x+2\right)-\frac{5}{32}\log\left(-3x^2+3\sqrt{x^2-1}\,x+2\right)$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 0.61, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {382, 377, 207}

$$\frac{3\sqrt{x^2-1}\,x}{8\left(4-3x^2\right)}+\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

```
[Out] (3*x*Sqrt[-1 + x^2])/(8*(4 - 3*x^2)) + (5*ArcTanh[x/(2*Sqrt[-1 + x^2])])/16
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)^2}\,dx &= \frac{3x\sqrt{-1+x^2}}{8\left(4-3x^2\right)}-\frac{5}{8}\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)}\,dx\\
&= \frac{3x\sqrt{-1+x^2}}{8\left(4-3x^2\right)}-\frac{5}{8}\text{Subst}\left(\int \frac{1}{-4+x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right)\\
&= \frac{3x\sqrt{-1+x^2}}{8\left(4-3x^2\right)}+\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{-1+x^2}}\right)
\end{aligned}$$

Mathematica [C] time = 3.25, size = 167, normalized size = 2.35

$$-\frac{x\sqrt{x^2-1}\left(\frac{8x^2(x^2-1)\,{}_2F_1\left(2,3;\frac{7}{2};\frac{x^2}{4-3x^2}\right)}{45x^2-60}-\frac{x^2(2x^2-3)\sqrt{\frac{x^2-1}{3x^2-4}}\left(2\sqrt{\frac{x^2-x^4}{(4-3x^2)^2}}-\sin^{-1}\left(\sqrt{\frac{x^2}{4-3x^2}}\right)\right)}{4\left(\frac{x^2}{4-3x^2}\right)^{5/2}(x^2-1)}\right)}{16\left(1-\frac{3x^2}{4}\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

```
[Out] -1/16*(x*Sqrt[-1 + x^2]*(-1/4*(x^2*(-3 + 2*x^2)*Sqrt[(-1 + x^2)/(-4 + 3*x^2
)]*(2*Sqrt[(x^2 - x^4)/(4 - 3*x^2)^2] - ArcSin[Sqrt[x^2/(4 - 3*x^2)]]))/((x
^2/(4 - 3*x^2))^(5/2)*(-1 + x^2)) + (8*x^2*(-1 + x^2)*Hypergeometric2F1[2,
3, 7/2, x^2/(4 - 3*x^2)])/(-60 + 45*x^2)))/(1 - (3*x^2)/4)^2
```

IntegrateAlgebraic [A] time = 0.10, size = 71, normalized size = 1.00

$$-\frac{3\sqrt{x^2-1}\,x}{8\left(3x^2-4\right)}+\frac{5}{32}\log\left(-x^2+\sqrt{x^2-1}\,x+2\right)-\frac{5}{32}\log\left(-3x^2+3\sqrt{x^2-1}\,x+2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

```
[Out] (-3*x*Sqrt[-1 + x^2])/(8*(-4 + 3*x^2)) + (5*Log[2 - x^2 + x*Sqrt[-1 + x^2]]
)/32 - (5*Log[2 - 3*x^2 + 3*x*Sqrt[-1 + x^2]])/32
```

fricas [A] time = 0.41, size = 80, normalized size = 1.13

$$-\frac{12\,x^2+5\left(3\,x^2-4\right)\log\left(3\,x^2-3\,\sqrt{x^2-1}\,x-2\right)-5\left(3\,x^2-4\right)\log\left(x^2-\sqrt{x^2-1}\,x-2\right)+12\,\sqrt{x^2-1}\,x-16}{32\left(3\,x^2-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(3*x^2-4)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(12*x^2 + 5*(3*x^2 - 4)*log(3*x^2 - 3*sqrt(x^2 - 1)*x - 2) - 5*(3*x^2
 - 4)*log(x^2 - sqrt(x^2 - 1)*x - 2) + 12*sqrt(x^2 - 1)*x - 16)/(3*x^2 - 4)
```

giac [A] time = 0.32, size = 94, normalized size = 1.32

$$\frac{5\left(x-\sqrt{x^2-1}\right)^2-3}{4\left(3\left(x-\sqrt{x^2-1}\right)^4-10\left(x-\sqrt{x^2-1}\right)^2+3\right)}-\frac{5}{32}\log\left(\left|3\left(x-\sqrt{x^2-1}\right)^2-1\right|\right)+\frac{5}{32}\log\left(\left|\left(x-\sqrt{x^2-1}\right)^2-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(3*x^2-4)^2,x, algorithm="giac")
```

```
[Out] 1/4*(5*(x - sqrt(x^2 - 1))^2 - 3)/(3*(x - sqrt(x^2 - 1))^4 - 10*(x - sqrt(x
^2 - 1))^2 + 3) - 5/32*log(abs(3*(x - sqrt(x^2 - 1))^2 - 1)) + 5/32*log(abs
((x - sqrt(x^2 - 1))^2 - 3))
```

maple [B] time = 0.07, size = 172, normalized size = 2.42

$$-\frac{5\,\mathrm{arctanh}\left(\frac{3\left(\frac{2}{3}-\frac{4\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{2\sqrt{9\left(x+\frac{2\sqrt{3}}{3}\right)^2-12\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)+3}}\right)}{32}-\frac{\sqrt{\left(x-\frac{2\sqrt{3}}{3}\right)^2+\frac{4\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)}{3}+\frac{1}{3}}}{16\left(x-\frac{2\sqrt{3}}{3}\right)}+\frac{5\,\mathrm{arctanh}\left(\frac{3\left(\frac{2}{3}+\frac{4\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{2\sqrt{9\left(x-\frac{2\sqrt{3}}{3}\right)^2+12\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)+3}}\right)}{32}-\frac{\sqrt{\left(x+\frac{2\sqrt{3}}{3}\right)^2-\frac{4\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)}{3}+\frac{1}{3}}}{16\left(x+\frac{2\sqrt{3}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(3*x^2-4)^2,x)
```

```
[Out] -5/32*arctanh(3/2*(2/3-4/3*3^(1/2)*(x+2/3*3^(1/2)))*3^(1/2)/(9*(x+2/3*3^(1/
2))^2-12*3^(1/2)*(x+2/3*3^(1/2))+3)^(1/2))-1/16/(x-2/3*3^(1/2))*((x-2/3*3^(
1/2))^2+4/3*3^(1/2)*(x-2/3*3^(1/2))+1/3)^(1/2)+5/32*arctanh(3/2*(2/3+4/3*3^
(1/2)*(x-2/3*3^(1/2)))*3^(1/2)/(9*(x-2/3*3^(1/2))^2+12*3^(1/2)*(x-2/3*3^(1/
2))+3)^(1/2))-1/16/(x+2/3*3^(1/2))*((x+2/3*3^(1/2))^2-4/3*3^(1/2)*(x+2/3*3^
(1/2))+1/3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-4\right)^2\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(3*x^2-4)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 4)^2*sqrt(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{x^2-1}\,\left(3\,x^2-4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(3*x^2 - 4)^2),x)
```

```
[Out] int(1/((x^2 - 1)^(1/2)*(3*x^2 - 4)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\,\left(3x^2-4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(3*x**2-4)**2,x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(3*x**2 - 4)**2), x)
```

3.868 $\int \frac{1}{x\sqrt[3]{1+x^3}}\,dx$

Optimal. Leaf size=71

$$\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-1\right)-\frac{1}{6}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 0.72, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {266, 55, 618, 204, 31}

$$\frac{1}{2}\log\left(1-\sqrt[3]{x^3+1}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + 2*(1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 + x
^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt[3]{1+x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx, x, x^3\right)\\ &= -\frac{\log(x)}{2} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+x^3}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1+x^3}\right)\\ &= -\frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1+x^3}\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+x^3}\right)\\ &= \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1+x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.72

$$\frac{1}{2}\log\left(1-\sqrt[3]{x^3+1}\right) + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + 2*(1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 + x
^3)^(1/3)]/2
```

IntegrateAlgebraic [A] time = 0.05, size = 71, normalized size = 1.00

$$\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-1\right) - \frac{1}{6}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right) + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(1 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[1/Sqrt[3] + (2*(1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[-1 + (1 + x^3
)^(1/3)]/3 - Log[1 + (1 + x^3)^(1/3) + (1 + x^3)^(2/3)]/6
```

fricas [A] time = 0.41, size = 56, normalized size = 0.79

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right) - \frac{1}{6}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right) + \frac{1}{3}\log\left(\left(x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(x^3 + 1)^(1/3) + 1/3*sqrt(3)) - 1/6*log((x^
3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) + 1/3*log((x^3 + 1)^(1/3) - 1)
```

giac [A] time = 0.29, size = 55, normalized size = 0.77

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{1}{6}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right) + \frac{1}{3}\log\left(\left|\left(x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) - 1/6*log((x^3 + 1)
^(2/3) + (x^3 + 1)^(1/3) + 1) + 1/3*log(abs((x^3 + 1)^(1/3) - 1))
```

maple [C] time = 0.30, size = 63, normalized size = 0.89

$$\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^{3}\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],-x^{3}\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{6\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^3+1)^(1/3),x)
```

```
[Out] 1/6/Pi*3^(1/2)*GAMMA(2/3)*(-2/9*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1,1,4/
3],[2,2],-x^3)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x))*Pi*3^(1/2)/GAMMA(2/3
))
```

maxima [A] time = 0.47, size = 54, normalized size = 0.76

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^{3}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{6}\log\left(\left(x^{3}+1\right)^{\frac{2}{3}}+\left(x^{3}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(x^{3}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) - 1/6*log((x^3 + 1)
^(2/3) + (x^3 + 1)^(1/3) + 1) + 1/3*log((x^3 + 1)^(1/3) - 1)
```

mupad [B] time = 0.90, size = 74, normalized size = 1.04

$$\frac{\ln\left(\left(x^{3}+1\right)^{1/3}-1\right)}{3}+\ln\left(\left(x^{3}+1\right)^{1/3}-9\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(\left(x^{3}+1\right)^{1/3}-9\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^3 + 1)^(1/3)),x)
```

```
[Out] log((x^3 + 1)^(1/3) - 1)/3 + log((x^3 + 1)^(1/3) - 9*((3^(1/2)*1i)/6 - 1/6)
^2)*((3^(1/2)*1i)/6 - 1/6) - log((x^3 + 1)^(1/3) - 9*((3^(1/2)*1i)/6 + 1/6)
^2)*((3^(1/2)*1i)/6 + 1/6)
```

sympy [C] time = 0.75, size = 29, normalized size = 0.41

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{e^{i\pi}}{x^{3}}\right)}{3x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**3+1)**(1/3),x)
```

```
[Out] -gamma(1/3)*hyper((1/3, 1/3), (4/3,), exp_polar(I*pi)/x**3)/(3*x*gamma(4/3)
)
```

3.869 $\int \frac{x^2\left(3-2(1+k)x+kx^2\right)}{((1-x)x(1-kx))^{3/4}\left(-1+(1+k)x-kx^2+dx^3\right)}\,dx$

Optimal. Leaf size=71

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{kx^3+(-k-1)x^2+x}}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{kx^3+(-k-1)x^2+x}}\right)}{d^{3/4}}$$

Rubi [F] time = 18.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(3-2(1+k)x+kx^2\right)}{((1-x)x(1-kx))^{3/4}\left(-1+(1+k)x-kx^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(3 - 2*(1 + k)*x + k*x^2))/(((1 - x)*x*(1 - k*x))^(3/4)*(-1 + (1 +
 k)*x - k*x^2 + d*x^3)),x]
```

```
[Out] (4*(k^2 - 2*d*(1 + k))*(1 - x)^(3/4)*x*(1 - k*x)^(3/4)*AppellF1[1/4, 3/4, 3
/4, 5/4, x, k*x])/(d^2*((1 - x)*x*(1 - k*x))^(3/4)) + (4*k*(1 - x)^(3/4)*x^
2*(1 - k*x)^(3/4)*AppellF1[5/4, 3/4, 3/4, 9/4, x, k*x])/(5*d*((1 - x)*x*(1
- k*x))^(3/4)) - (4*(k^2 - 2*d*(1 + k))*(1 - x)^(3/4)*x^(3/4)*(1 - k*x)^(3/
4)*Defer[Subst][Defer[Int][1/((1 - x^4)^(3/4)*(1 - k*x^4)^(3/4)*(1 - (1 + k
)*x^4 + k*x^8 - d*x^12)), x], x, x^(1/4)])/(d^2*((1 - x)*x*(1 - k*x))^(3/4)
) + (4*(k^2*(1 + k) - d*(2 + 5*k + 2*k^2))*(1 - x)^(3/4)*x^(3/4)*(1 - k*x)^
(3/4)*Defer[Subst][Defer[Int][x^4/((1 - x^4)^(3/4)*(1 - k*x^4)^(3/4)*(1 - (
1 + k)*x^4 + k*x^8 - d*x^12)), x], x, x^(1/4)])/(d^2*((1 - x)*x*(1 - k*x))^
(3/4)) - (4*(3*d^2 + k^3 - 3*d*k*(1 + k))*(1 - x)^(3/4)*x^(3/4)*(1 - k*x)^(
3/4)*Defer[Subst][Defer[Int][x^8/((1 - x^4)^(3/4)*(1 - k*x^4)^(3/4)*(1 - (1
 + k)*x^4 + k*x^8 - d*x^12)), x], x, x^(1/4)])/(d^2*((1 - x)*x*(1 - k*x))^(
3/4))
```

Rubi steps

$$\int \frac{x^2\left(3-2(1+k)x+kx^2\right)}{((1-x)x(1-kx))^{3/4}\left(-1+(1+k)x-kx^2+dx^3\right)}dx = \frac{\left((1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\int \frac{x^{5/4}\left(3-2(1+k)x+kx^2\right)}{(1-x)^{3/4}(1-kx)^{3/4}\left(-1+(1+k)x-kx^2+d\right.}}{((1-x)x(1-kx))^{3/4}}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subst}\left(\int \frac{x^8\left(3-2(1+k)\right.}{\left(1-x^4\right)^{3/4}\left(1-kx^4\right)^{3/4}\left(-1+\right.}\right.}{((1-x)x(1-kx))^{3/4}}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subst}\left(\int\left(\frac{k^2-2d(1+k)}{d^2\left(1-x^4\right)^{3/4}\left(1-kx^4\right)^{3/4}}\right.\right.}{}$$

$$= -\frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subst}\left(\int \frac{-k^2+2d(1+k)-\left(2d+5dk\right.}{\left(1-x^4\right)^{3/4}\left(1-\right.}\right.}{d^2((1-x)x(1-k}$$

$$= \frac{4\left(k^2-2d(1+k)\right)(1-x)^{3/4}x(1-kx)^{3/4}F_1\left(\frac{1}{4};\frac{3}{4},\frac{3}{4};\frac{5}{4};x,k\right.}{d^2((1-x)x(1-kx))^{3/4}}$$

$$= \frac{4\left(k^2-2d(1+k)\right)(1-x)^{3/4}x(1-kx)^{3/4}F_1\left(\frac{1}{4};\frac{3}{4},\frac{3}{4};\frac{5}{4};x,k\right.}{d^2((1-x)x(1-kx))^{3/4}}$$

Mathematica [F] time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(3-2(1+k)x+kx^2\right)}{((1-x)x(1-kx))^{3/4}\left(-1+(1+k)x-kx^2+dx^3\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(3 - 2*(1 + k)*x + k*x^2))/(((1 - x)*x*(1 - k*x))^(3/4)*(-1
+ (1 + k)*x - k*x^2 + d*x^3)),x]
```

```
[Out] Integrate[(x^2*(3 - 2*(1 + k)*x + k*x^2))/(((1 - x)*x*(1 - k*x))^(3/4)*(-1
+ (1 + k)*x - k*x^2 + d*x^3)), x]
```

IntegrateAlgebraic [A] time = 3.08, size = 71, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{kx^3+(-k-1)x^2+x}}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{kx^3+(-k-1)x^2+x}}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(3 - 2*(1 + k)*x + k*x^2))/(((1 - x)*x*(1 - k*x))^(
3/4)*(-1 + (1 + k)*x - k*x^2 + d*x^3)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*x)/(x + (-1 - k)*x^2 + k*x^3)^(1/4)])/d^(3/4) - (2*ArcTa
nh[(d^(1/4)*x)/(x + (-1 - k)*x^2 + k*x^3)^(1/4)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3-2*(1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(3/4)/(-1+(1+k)*x-k*x^
2+d*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.42, size = 264, normalized size = 3.72

$$-\frac{\sqrt{2}\left(-d\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-d\right)^{\frac{1}{4}}+2\left(\frac{k}{x}-\frac{k}{x^{2}}-\frac{1}{x^{2}}+\frac{1}{x^{3}}\right)^{\frac{1}{4}}\right)}{2\left(-d\right)^{\frac{1}{4}}}\right)}{d}-\frac{\sqrt{2}\left(-d\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-d\right)^{\frac{1}{4}}-2\left(\frac{k}{x}-\frac{k}{x^{2}}-\frac{1}{x^{2}}+\frac{1}{x^{3}}\right)^{\frac{1}{4}}\right)}{2\left(-d\right)^{\frac{1}{4}}}\right)}{d}-\frac{\sqrt{2}\left(-d\right)^{\frac{1}{4}}\log\left(\sqrt{2}\left(-d\right)^{\frac{1}{4}}\left(\frac{k}{x}-\frac{k}{x^{2}}-\frac{1}{x^{2}}+\frac{1}{x^{3}}\right)^{\frac{1}{4}}+\sqrt{-d}+\sqrt{\frac{k}{x}-\frac{k}{x^{2}}-\frac{1}{x^{2}}+\frac{1}{x^{3}}}\right)}{2\,d}+\frac{\sqrt{2}\left(-d\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\left(-d\right)^{\frac{1}{4}}\left(\frac{k}{x}-\frac{k}{x^{2}}-\frac{1}{x^{2}}+\frac{1}{x^{3}}\right)^{\frac{1}{4}}+\sqrt{-d}+\sqrt{\frac{k}{x}-\frac{k}{x^{2}}-\frac{1}{x^{2}}+\frac{1}{x^{3}}}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3-2*(1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(3/4)/(-1+(1+k)*x-k*x^
2+d*x^3),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(-d)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-d)^(1/4) + 2*(k/x - k/x^2
 - 1/x^2 + 1/x^3)^(1/4))/(-d)^(1/4))/d - sqrt(2)*(-d)^(1/4)*arctan(-1/2*sqr
t(2)*(sqrt(2)*(-d)^(1/4) - 2*(k/x - k/x^2 - 1/x^2 + 1/x^3)^(1/4))/(-d)^(1/4
))/d - 1/2*sqrt(2)*(-d)^(1/4)*log(sqrt(2)*(-d)^(1/4)*(k/x - k/x^2 - 1/x^2 +
 1/x^3)^(1/4) + sqrt(-d) + sqrt(k/x - k/x^2 - 1/x^2 + 1/x^3))/d + 1/2*sqrt(
2)*(-d)^(1/4)*log(-sqrt(2)*(-d)^(1/4)*(k/x - k/x^2 - 1/x^2 + 1/x^3)^(1/4) +
 sqrt(-d) + sqrt(k/x - k/x^2 - 1/x^2 + 1/x^3))/d
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(3-2 \left(1+k \right) x +k \,x^{2}\right)}{\left(\left(1-x \right) x \left(-k x +1\right)\right)^{\frac{3}{4}} \left(-1+\left(1+k \right) x -k \,x^{2}+d \,x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3-2*(1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(3/4)/(-1+(1+k)*x-k*x^2+d*x^
3),x)
```

```
[Out] int(x^2*(3-2*(1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(3/4)/(-1+(1+k)*x-k*x^2+d*x^
3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(k x^{2} - 2 \, {\left(k + 1\right)} x + 3\right)} x^{2}}{{\left(d x^{3} - k x^{2} + {\left(k + 1\right)} x - 1\right)} \left({\left(k x - 1\right)} {\left(x - 1\right)} x\right)^{\frac{3}{4}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3-2*(1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(3/4)/(-1+(1+k)*x-k*x^
2+d*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k*x^2 - 2*(k + 1)*x + 3)*x^2/((d*x^3 - k*x^2 + (k + 1)*x - 1)*((
k*x - 1)*(x - 1)*x)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(k\,x^2-2\,x\,\left(k+1\right)+3\right)}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{3/4}\,\left(d\,x^3-k\,x^2+\left(k+1\right)\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(k*x^2 - 2*x*(k + 1) + 3))/((x*(k*x - 1)*(x - 1))^(3/4)*(d*x^3 + x
*(k + 1) - k*x^2 - 1)),x)
```

```
[Out] int((x^2*(k*x^2 - 2*x*(k + 1) + 3))/((x*(k*x - 1)*(x - 1))^(3/4)*(d*x^3 + x
*(k + 1) - k*x^2 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3-2*(1+k)*x+k*x**2)/((1-x)*x*(-k*x+1))**(3/4)/(-1+(1+k)*x-k
*x**2+d*x**3),x)
```

```
[Out] Timed out
```

3.870 $\int \frac{(-2b+ax^6)(b+ax^6)^{3/4}}{x^4(b-cx^4+ax^6)}\,dx$

Optimal. Leaf size=71

$$-c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^6+b}}\right)-c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^6+b}}\right)+\frac{2\left(ax^6+b\right)^{3/4}}{3x^3}$$

Rubi [F] time = 1.75, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-2b+ax^6\right)\left(b+ax^6\right)^{3/4}}{x^4\left(b-cx^4+ax^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*b + a*x^6)*(b + a*x^6)^(3/4))/(x^4*(b - c*x^4 + a*x^6)),x]
```

```
[Out] (-2*a*x^3)/(b + a*x^6)^(1/4) + (2*(b + a*x^6)^(3/4))/(3*x^3) + (2*Sqrt[a]*S
qrt[b]*(1 + (a*x^6)/b)^(1/4)*EllipticE[ArcTan[(Sqrt[a]*x^3)/Sqrt[b]]/2, 2])
/(b + a*x^6)^(1/4) + 2*c*Defer[Int][(b + a*x^6)^(3/4)/(-b + c*x^4 - a*x^6),
 x] + 3*a*Defer[Int][(x^2*(b + a*x^6)^(3/4))/(b - c*x^4 + a*x^6), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-2b+ax^6\right)\left(b+ax^6\right)^{3/4}}{x^4\left(b-cx^4+ax^6\right)}\,dx &= \int \left(-\frac{2\left(b+ax^6\right)^{3/4}}{x^4}+\frac{\left(2c-3ax^2\right)\left(b+ax^6\right)^{3/4}}{-b+cx^4-ax^6}\right)dx\\
&= -\left(2\int \frac{\left(b+ax^6\right)^{3/4}}{x^4}\,dx\right)+\int \frac{\left(2c-3ax^2\right)\left(b+ax^6\right)^{3/4}}{-b+cx^4-ax^6}\,dx\\
&= -\left(\frac{2}{3}\operatorname{Subst}\left(\int \frac{\left(b+ax^2\right)^{3/4}}{x^2}\,dx,x,x^3\right)\right)+\int \left(\frac{2c\left(b+ax^6\right)^{3/4}}{-b+cx^4-ax^6}+\frac{3ax^2\left(b+ax\right.}{b-cx^4+a}\right.\\
&= \frac{2\left(b+ax^6\right)^{3/4}}{3x^3}-a\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{b+ax^2}}\,dx,x,x^3\right)+(3a)\int \frac{x^2\left(b+ax^6\right)^{3/4}}{b-cx^4+ax^6}\,dx\\
&= \frac{2\left(b+ax^6\right)^{3/4}}{3x^3}+(3a)\int \frac{x^2\left(b+ax^6\right)^{3/4}}{b-cx^4+ax^6}\,dx+(2c)\int \frac{\left(b+ax^6\right)^{3/4}}{-b+cx^4-ax^6}\,dx-\frac{(a}{}\\
&= -\frac{2ax^3}{\sqrt[4]{b+ax^6}}+\frac{2\left(b+ax^6\right)^{3/4}}{3x^3}+(3a)\int \frac{x^2\left(b+ax^6\right)^{3/4}}{b-cx^4+ax^6}\,dx+(2c)\int \frac{\left(b+ax\right.}{-b+cx^4}\\
&= -\frac{2ax^3}{\sqrt[4]{b+ax^6}}+\frac{2\left(b+ax^6\right)^{3/4}}{3x^3}+\frac{2\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{ax^6}{b}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{a}x^3}{\sqrt{b}}\right)\middle|2\right)}{\sqrt[4]{b+ax^6}}+(3
\end{aligned}$$

Mathematica [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\left(-2b+ax^6\right)\left(b+ax^6\right)^{3/4}}{x^4\left(b-cx^4+ax^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2*b + a*x^6)*(b + a*x^6)^(3/4))/(x^4*(b - c*x^4 + a*x^6)),x]
```

```
[Out] Integrate[((-2*b + a*x^6)*(b + a*x^6)^(3/4))/(x^4*(b - c*x^4 + a*x^6)), x]
```

IntegrateAlgebraic [A] time = 3.11, size = 71, normalized size = 1.00

$$-c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^6+b}}\right)-c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^6+b}}\right)+\frac{2\left(ax^6+b\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*b + a*x^6)*(b + a*x^6)^(3/4))/(x^4*(b - c*x^4 + a*x
^6)),x]
```

```
[Out] (2*(b + a*x^6)^(3/4))/(3*x^3) - c^(3/4)*ArcTan[(c^(1/4)*x)/(b + a*x^6)^(1/4
)] - c^(3/4)*ArcTanh[(c^(1/4)*x)/(b + a*x^6)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-2*b)*(a*x^6+b)^(3/4)/x^4/(a*x^6-c*x^4+b),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^6+b\right)^{\frac{3}{4}}\left(ax^6-2\,b\right)}{\left(ax^6-cx^4+b\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-2*b)*(a*x^6+b)^(3/4)/x^4/(a*x^6-c*x^4+b),x, algorithm="gia
c")
```

```
[Out] integrate((a*x^6 + b)^(3/4)*(a*x^6 - 2*b)/((a*x^6 - c*x^4 + b)*x^4), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^6-2b\right)\left(a\,x^6+b\right)^{\frac{3}{4}}}{x^4\left(a\,x^6-c\,x^4+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6-2*b)*(a*x^6+b)^(3/4)/x^4/(a*x^6-c*x^4+b),x)
```

```
[Out] int((a*x^6-2*b)*(a*x^6+b)^(3/4)/x^4/(a*x^6-c*x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^6+b\right)^{\frac{3}{4}}\left(ax^6-2\,b\right)}{\left(ax^6-cx^4+b\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-2*b)*(a*x^6+b)^(3/4)/x^4/(a*x^6-c*x^4+b),x, algorithm="max
ima")
```

```
[Out] integrate((a*x^6 + b)^(3/4)*(a*x^6 - 2*b)/((a*x^6 - c*x^4 + b)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(a\,x^6+b\right)^{3/4}\,\left(2\,b-a\,x^6\right)}{x^4\,\left(a\,x^6-c\,x^4+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b + a*x^6)^(3/4)*(2*b - a*x^6))/(x^4*(b + a*x^6 - c*x^4)),x)
```

```
[Out] int(-((b + a*x^6)^(3/4)*(2*b - a*x^6))/(x^4*(b + a*x^6 - c*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6-2*b)*(a*x**6+b)**(3/4)/x**4/(a*x**6-c*x**4+b),x)
```

```
[Out] Timed out
```

3.871 $\int \frac{b+ax^8}{\left(-b+ax^8\right)\sqrt[4]{-b+cx^4+ax^8}}\,dx$

Optimal. Leaf size=71

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^8-b+cx^4}}\right)}{2\sqrt[4]{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^8-b+cx^4}}\right)}{2\sqrt[4]{c}}$$

Rubi [F] time = 0.95, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{b+ax^8}{\left(-b+ax^8\right)\sqrt[4]{-b+cx^4+ax^8}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(b + a*x^8)/((-b + a*x^8)*(-b + c*x^4 + a*x^8)^(1/4)),x]
```

```
[Out] (x*(1 + (2*a*x^4)/(c - Sqrt[4*a*b + c^2]))^(1/4)*(1 + (2*a*x^4)/(c + Sqrt[4
*a*b + c^2]))^(1/4)*AppellF1[1/4, 1/4, 1/4, 5/4, (-2*a*x^4)/(c - Sqrt[4*a*b
 + c^2]), (-2*a*x^4)/(c + Sqrt[4*a*b + c^2])])/(-b + c*x^4 + a*x^8)^(1/4) -
 Sqrt[b]*Defer[Int][1/((Sqrt[b] - Sqrt[a]*x^4)*(-b + c*x^4 + a*x^8)^(1/4)),
 x] - Sqrt[b]*Defer[Int][1/((Sqrt[b] + Sqrt[a]*x^4)*(-b + c*x^4 + a*x^8)^(1
/4)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{b+ax^8}{\left(-b+ax^8\right)\sqrt[4]{-b+cx^4+ax^8}}\,dx &= \int \left(\frac{1}{\sqrt[4]{-b+cx^4+ax^8}}+\frac{2b}{\left(-b+ax^8\right)\sqrt[4]{-b+cx^4+ax^8}}\right)dx\\
&=(2b)\int \frac{1}{\left(-b+ax^8\right)\sqrt[4]{-b+cx^4+ax^8}}\,dx+\int \frac{1}{\sqrt[4]{-b+cx^4+ax^8}}\,dx\\
&=(2b)\int \left(-\frac{1}{2\sqrt{b}\left(\sqrt{b}-\sqrt{a}\,x^4\right)\sqrt[4]{-b+cx^4+ax^8}}-\frac{1}{2\sqrt{b}\left(\sqrt{b}+\sqrt{a}\,x^4\right)\sqrt[4]{-b}}\right.\\
&=\frac{x\sqrt[4]{1+\frac{2ax^4}{c-\sqrt{4ab+c^2}}}\sqrt[4]{1+\frac{2ax^4}{c+\sqrt{4ab+c^2}}}F_1\left(\frac{1}{4};\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{2ax^4}{c-\sqrt{4ab+c^2}},-\frac{2ax^4}{c+\sqrt{4ab+c^2}}\right)}{\sqrt[4]{-b+cx^4+ax^8}}-\end{aligned}$$

Mathematica [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{b+ax^8}{\left(-b+ax^8\right)\sqrt[4]{-b+cx^4+ax^8}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(b + a*x^8)/((-b + a*x^8)*(-b + c*x^4 + a*x^8)^(1/4)),x]
```

```
[Out] Integrate[(b + a*x^8)/((-b + a*x^8)*(-b + c*x^4 + a*x^8)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 2.79, size = 71, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^8-b+cx^4}}\right)}{2\sqrt[4]{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^8-b+cx^4}}\right)}{2\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^8)/((-b + a*x^8)*(-b + c*x^4 + a*x^8)^(1/4)),x]
[Out] -1/2*ArcTan[(c^(1/4)*x)/(-b + c*x^4 + a*x^8)^(1/4)]/c^(1/4) - ArcTanh[(c^(1
/4)*x)/(-b + c*x^4 + a*x^8)^(1/4)]/(2*c^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+b)/(a*x^8-b)/(a*x^8+c*x^4-b)^(1/4),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^8 + b}{\left(ax^8 + cx^4 - b\right)^{\frac{1}{4}}\left(ax^8 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+b)/(a*x^8-b)/(a*x^8+c*x^4-b)^(1/4),x, algorithm="giac")
[Out] integrate((a*x^8 + b)/((a*x^8 + c*x^4 - b)^(1/4)*(a*x^8 - b)), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{a\,x^8 + b}{\left(a\,x^8 - b\right)\left(a\,x^8 + c\,x^4 - b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^8+b)/(a*x^8-b)/(a*x^8+c*x^4-b)^(1/4),x)
[Out] int((a*x^8+b)/(a*x^8-b)/(a*x^8+c*x^4-b)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^8 + b}{\left(ax^8 + cx^4 - b\right)^{\frac{1}{4}}\left(ax^8 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+b)/(a*x^8-b)/(a*x^8+c*x^4-b)^(1/4),x, algorithm="maxima")
[Out] integrate((a*x^8 + b)/((a*x^8 + c*x^4 - b)^(1/4)*(a*x^8 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a\,x^8 + b}{\left(b - a\,x^8\right)\left(a\,x^8 + c\,x^4 - b\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x^8)/((b - a*x^8)*(a*x^8 - b + c*x^4)^(1/4)),x)
[Out] int(-(b + a*x^8)/((b - a*x^8)*(a*x^8 - b + c*x^4)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^8 + b}{\left(ax^8 - b\right)\sqrt[4]{ax^8 - b + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**8+b)/(a*x**8-b)/(a*x**8+c*x**4-b)**(1/4),x)
[Out] Integral((a*x**8 + b)/((a*x**8 - b)*(a*x**8 - b + c*x**4)**(1/4)), x)
```

3.872 $\int \frac{\sqrt{1+x^5}\left(-2+3x^5\right)}{1+x^4+2x^5+x^{10}}\,dx$

Optimal. Leaf size=71

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x\sqrt{x^5+1}}{x^5-x^2+1}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt{x^5+1}}{x^5+x^2+1}\right)}{\sqrt{2}}$$

Rubi [F] time = 0.60, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+x^5}\left(-2+3x^5\right)}{1+x^4+2x^5+x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[1 + x^5]*(-2 + 3*x^5))/(1 + x^4 + 2*x^5 + x^10),x]
```

```
[Out] -2*Defer[Int][Sqrt[1 + x^5]/(1 + x^4 + 2*x^5 + x^10), x] + 3*Defer[Int][(x^
5*Sqrt[1 + x^5])/(1 + x^4 + 2*x^5 + x^10), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^5}\left(-2+3x^5\right)}{1+x^4+2x^5+x^{10}}\,dx &= \int \left(-\frac{2\sqrt{1+x^5}}{1+x^4+2x^5+x^{10}}+\frac{3x^5\sqrt{1+x^5}}{1+x^4+2x^5+x^{10}}\right)dx\\ &= -\left(2\int \frac{\sqrt{1+x^5}}{1+x^4+2x^5+x^{10}}\,dx\right)+3\int \frac{x^5\sqrt{1+x^5}}{1+x^4+2x^5+x^{10}}\,dx\end{aligned}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+x^5}\left(-2+3x^5\right)}{1+x^4+2x^5+x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[1 + x^5]*(-2 + 3*x^5))/(1 + x^4 + 2*x^5 + x^10),x]
```

```
[Out] Integrate[(Sqrt[1 + x^5]*(-2 + 3*x^5))/(1 + x^4 + 2*x^5 + x^10), x]
```

IntegrateAlgebraic [A] time = 4.62, size = 82, normalized size = 1.15

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x\sqrt{x^5+1}}{x^5-x^2+1}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^5}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{x\sqrt{x^5+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x^5]*(-2 + 3*x^5))/(1 + x^4 + 2*x^5 + x^10),x]
```

```
[Out] -(ArcTan[(Sqrt[2]*x*Sqrt[1 + x^5])/(1 - x^2 + x^5)]/Sqrt[2]) - ArcTanh[(1/S
qrt[2] + x^2/Sqrt[2] + x^5/Sqrt[2])/(x*Sqrt[1 + x^5])]/Sqrt[2]
```

fricas [B] time = 0.49, size = 387, normalized size = 5.45

$$-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(x^5-x^2+1\right)\sqrt{x^5+1}+\left(2x^6-\sqrt{2}\left(x^5+x^2+1\right)\sqrt{x^5+1}+2x\right)\sqrt{\frac{x^{10}+4x^7+2x^5+x^4+2\sqrt{2}\left(x^6+x^3+x\right)\sqrt{x^5+1}+4x^2+1}{x^{10}+2x^5+x^4+1}}}{2\left(x^6+x\right)}\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(x^5-x^2+1\right)\sqrt{x^5+1}-\left(2x^6+\sqrt{2}\left(x^5+x^2+1\right)\sqrt{x^5+1}+2x\right)\sqrt{\frac{x^{10}+4x^7+2x^5+x^4-2\sqrt{2}\left(x^6+x^3+x\right)\sqrt{x^5+1}+4x^2+1}{x^{10}+2x^5+x^4+1}}}{2\left(x^6+x\right)}\right)-\frac{1}{8}\sqrt{2}\log\left(\frac{x^{10}+4x^7+2x^5+x^4+2\sqrt{2}\left(x^6+x^3+x\right)\sqrt{x^5+1}+4x^2+1}{x^{10}+2x^5+x^4+1}\right)+\frac{1}{8}\sqrt{2}\log\left(\frac{x^{10}+4x^7+2x^5+x^4-2\sqrt{2}\left(x^6+x^3+x\right)\sqrt{x^5+1}+4x^2+1}{x^{10}+2x^5+x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(1/2)*(3*x^5-2)/(x^10+2*x^5+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(-1/2*(sqrt(2)*(x^5 - x^2 + 1)*sqrt(x^5 + 1) + (2*x^6 -
sqrt(2)*(x^5 + x^2 + 1)*sqrt(x^5 + 1) + 2*x)*sqrt((x^10 + 4*x^7 + 2*x^5 + x
^4 + 2*sqrt(2)*(x^6 + x^3 + x)*sqrt(x^5 + 1) + 4*x^2 + 1)/(x^10 + 2*x^5 + x
^4 + 1)))/(x^6 + x)) - 1/2*sqrt(2)*arctan(-1/2*(sqrt(2)*(x^5 - x^2 + 1)*sqr
t(x^5 + 1) - (2*x^6 + sqrt(2)*(x^5 + x^2 + 1)*sqrt(x^5 + 1) + 2*x)*sqrt((x^
10 + 4*x^7 + 2*x^5 + x^4 - 2*sqrt(2)*(x^6 + x^3 + x)*sqrt(x^5 + 1) + 4*x^2
+ 1)/(x^10 + 2*x^5 + x^4 + 1)))/(x^6 + x)) - 1/8*sqrt(2)*log((x^10 + 4*x^7
+ 2*x^5 + x^4 + 2*sqrt(2)*(x^6 + x^3 + x)*sqrt(x^5 + 1) + 4*x^2 + 1)/(x^10
+ 2*x^5 + x^4 + 1)) + 1/8*sqrt(2)*log((x^10 + 4*x^7 + 2*x^5 + x^4 - 2*sqrt(
2)*(x^6 + x^3 + x)*sqrt(x^5 + 1) + 4*x^2 + 1)/(x^10 + 2*x^5 + x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^5-2\right)\sqrt{x^5+1}}{x^{10}+2\,x^5+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(1/2)*(3*x^5-2)/(x^10+2*x^5+x^4+1),x, algorithm="giac")
```

```
[Out] integrate((3*x^5 - 2)*sqrt(x^5 + 1)/(x^10 + 2*x^5 + x^4 + 1), x)
```

maple [C] time = 1.98, size = 151, normalized size = 2.13

$$\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^3 \ln\left(\frac{-\mathrm{RootOf}\left(\_Z^4+1\right)^3 x^5-\mathrm{RootOf}\left(\_Z^4+1\right)^5 x^2-\mathrm{RootOf}\left(\_Z^4+1\right)^3+2\sqrt{x^5+1}\,x}{-x^5+\mathrm{RootOf}\left(\_Z^4+1\right)^2 x^2-1}\right)}{2}-\frac{\mathrm{RootOf}\left(\_Z^4+1\right) \ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+1\right) x^5-\mathrm{RootOf}\left(\_Z^4+1\right)^3 x^2+2\sqrt{x^5+1}\,x+\mathrm{RootOf}\left(\_Z^4+1\right)}{x^5+\mathrm{RootOf}\left(\_Z^4+1\right)^2 x^2+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)^(1/2)*(3*x^5-2)/(x^10+2*x^5+x^4+1),x)
```

```
[Out] 1/2*RootOf(_Z^4+1)^3*ln((-RootOf(_Z^4+1)^3*x^5-RootOf(_Z^4+1)^5*x^2-RootOf(
_Z^4+1)^3+2*(x^5+1)^(1/2)*x)/(-x^5+RootOf(_Z^4+1)^2*x^2-1))-1/2*RootOf(_Z^4
+1)*ln((RootOf(_Z^4+1)*x^5-RootOf(_Z^4+1)^3*x^2+2*(x^5+1)^(1/2)*x+RootOf(_Z
^4+1))/(x^5+RootOf(_Z^4+1)^2*x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^5-2\right)\sqrt{x^5+1}}{x^{10}+2\,x^5+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(1/2)*(3*x^5-2)/(x^10+2*x^5+x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((3*x^5 - 2)*sqrt(x^5 + 1)/(x^10 + 2*x^5 + x^4 + 1), x)
```

mupad [B] time = 12.79, size = 173, normalized size = 2.44

$$\frac{(-1)^{1/4}\ln\left(2\,x^5-x^4+x^{10}+1-x^2\,2\mathrm{i}-x^7\,2\mathrm{i}+\sqrt{2}\,x^3\sqrt{x^5+1}\,\left(1+1\mathrm{i}\right)+\sqrt{2}\,x\left(x^5+1\right)^{3/2}\left(-1+1\mathrm{i}\right)\right)}{2}-\frac{(-1)^{1/4}\ln\left(x^{10}+2\,x^5+x^4+1\right)}{2}+\sqrt{2}\,\ln\left(2\,x^5-x^4+x^{10}+1+x^2\,2\mathrm{i}+x^7\,2\mathrm{i}+\sqrt{2}\,x^3\sqrt{x^5+1}\,\left(1-\mathrm{i}\right)+\sqrt{2}\,x\left(x^5+1\right)^{3/2}\left(-1-\mathrm{i}\right)\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+\sqrt{2}\,\ln\left(x^{10}+2\,x^5+x^4+1\right)\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 + 1)^(1/2)*(3*x^5 - 2))/(x^4 + 2*x^5 + x^10 + 1),x)
```

```
[Out] 2^(1/2)*log(x^2*2i - x^4 + 2*x^5 + x^7*2i + x^10 + 2^(1/2)*x^3*(x^5 + 1)^(1
/2)*(1 - 1i) - 2^(1/2)*x*(x^5 + 1)^(3/2)*(1 + 1i) + 1)*(1/4 - 1i/4) + ((-1)
^(1/4)*log(2*x^5 - x^4 - x^2*2i - x^7*2i + x^10 + 2^(1/2)*x^3*(x^5 + 1)^(1/
2)*(1 + 1i) - 2^(1/2)*x*(x^5 + 1)^(3/2)*(1 - 1i) + 1))/2 - 2^(1/2)*log(x^4
```

```
+ 2*x^5 + x^10 + 1)*(1/4 - 1i/4) - ((-1)^(1/4)*log(x^4 + 2*x^5 + x^10 + 1))
/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x+1)\left(x^4-x^3+x^2-x+1\right)}\left(3x^5-2\right)}{x^{10}+2x^5+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+1)**(1/2)*(3*x**5-2)/(x**10+2*x**5+x**4+1),x)
```

```
[Out] Integral(sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1))*(3*x**5 - 2)/(x**10 + 2
*x**5 + x**4 + 1), x)
```

3.873 $\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2} dx$

Optimal. Leaf size=71

$$-\frac{4x}{3\left(3x^2-1\right)}+\frac{2\sqrt{x^2+1}}{3\left(3x^2-1\right)}-\frac{2\tanh^{-1}\left(\frac{x}{\sqrt{3}}-\frac{\sqrt{x^2+1}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6742, 199, 207, 444, 47, 63}

$$\frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{x^2+1}}{3\left(1-3x^2\right)}+\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+1}\right)}{3\sqrt{3}}-\frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + Sqrt[1 + x^2])^(-2),x]
```

```
[Out] (4*x)/(3*(1 - 3*x^2)) - (2*Sqrt[1 + x^2])/(3*(1 - 3*x^2)) - ArcTanh[Sqrt[3]
*x]/(3*Sqrt[3]) + ArcTanh[(Sqrt[3]*Sqrt[1 + x^2])/2]/(3*Sqrt[3])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx &= \int \left(\frac{8}{3\left(-1+3x^2\right)^2}-\frac{4x\sqrt{1+x^2}}{\left(-1+3x^2\right)^2}+\frac{5}{3\left(-1+3x^2\right)}\right)dx\\
&= \frac{5}{3}\int \frac{1}{-1+3x^2}\,dx+\frac{8}{3}\int \frac{1}{\left(-1+3x^2\right)^2}\,dx-4\int \frac{x\sqrt{1+x^2}}{\left(-1+3x^2\right)^2}\,dx\\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{5\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{4}{3}\int \frac{1}{-1+3x^2}\,dx-2\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{(-1+3x)^2}\,dx,x,x\right.\\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{1+x}\,(-1+3x)}\,dx,x,x^2\right.\\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-4+3x^2}\,dx,x,\sqrt{1+x^2}\right)\\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\,\sqrt{1+x^2}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 99, normalized size = 1.39

$$\frac{1}{9}\left(\frac{12x}{1-3x^2}-\frac{\frac{6x^2+6}{1-3x^2}+\sqrt{3}\,\sqrt{-x^2-1}\,\tan^{-1}\left(\frac{1}{2}\sqrt{3}\,\sqrt{-x^2-1}\right)}{\sqrt{x^2+1}}-\sqrt{3}\,\tanh^{-1}\left(\sqrt{3}\,x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + Sqrt[1 + x^2])^(-2),x]
```

```
[Out] ((12*x)/(1 - 3*x^2) - ((6 + 6*x^2)/(1 - 3*x^2) + Sqrt[3]*Sqrt[-1 - x^2]*Arc
Tan[(Sqrt[3]*Sqrt[-1 - x^2])/2])/Sqrt[1 + x^2] - Sqrt[3]*ArcTanh[Sqrt[3]*x]
)/9
```

IntegrateAlgebraic [A] time = 0.35, size = 71, normalized size = 1.00

$$-\frac{4x}{3\left(3x^2-1\right)}+\frac{2\sqrt{x^2+1}}{3\left(3x^2-1\right)}-\frac{2\tanh^{-1}\left(\frac{x}{\sqrt{3}}-\frac{\sqrt{x^2+1}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*x + Sqrt[1 + x^2])^(-2),x]
```

```
[Out] (-4*x)/(3*(-1 + 3*x^2)) + (2*Sqrt[1 + x^2])/(3*(-1 + 3*x^2)) - (2*ArcTanh[x
/Sqrt[3] - Sqrt[1 + x^2]/Sqrt[3]])/(3*Sqrt[3])
```

fricas [A] time = 0.41, size = 100, normalized size = 1.41

$$\frac{\sqrt{3}\left(3x^2-1\right)\log\left(\frac{3x^2-2\sqrt{3}x+1}{3x^2-1}\right)+\sqrt{3}\left(3x^2-1\right)\log\left(\frac{3x^2+4\sqrt{3}\sqrt{x^2+1}+7}{3x^2-1}\right)-24x+12\sqrt{x^2+1}}{18\left(3x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/18*(sqrt(3)*(3*x^2 - 1)*log((3*x^2 - 2*sqrt(3)*x + 1)/(3*x^2 - 1)) + sqrt
(3)*(3*x^2 - 1)*log((3*x^2 + 4*sqrt(3)*sqrt(x^2 + 1) + 7)/(3*x^2 - 1)) - 24
*x + 12*sqrt(x^2 + 1))/(3*x^2 - 1)
```

giac [B] time = 0.22, size = 177, normalized size = 2.49

$$\frac{1}{18}\sqrt{3}\log\left(\frac{\left|6x-2\sqrt{3}\right|}{\left|6x+2\sqrt{3}\right|}\right)-\frac{1}{18}\sqrt{3}\log\left(-\frac{\left|-6x-8\sqrt{3}+6\sqrt{x^2+1}-\frac{6}{x-\sqrt{x^2+1}}\right|}{2\left(3x-4\sqrt{3}-3\sqrt{x^2+1}+\frac{3}{x-\sqrt{x^2+1}}\right)}\right)-\frac{4\left(x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}\right)}{3\left(3\left(x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}\right)^2-16\right)}-\frac{4x}{3\left(3x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/18*sqrt(3)*log(abs(6*x - 2*sqrt(3))/abs(6*x + 2*sqrt(3))) - 1/18*sqrt(3)*
log(-1/2*abs(-6*x - 8*sqrt(3) + 6*sqrt(x^2 + 1) - 6/(x - sqrt(x^2 + 1)))/(3
*x - 4*sqrt(3) - 3*sqrt(x^2 + 1) + 3/(x - sqrt(x^2 + 1)))) - 4/3*(x - sqrt(
x^2 + 1) + 1/(x - sqrt(x^2 + 1)))/(3*(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 +
 1)))^2 - 16) - 4/3*x/(3*x^2 - 1)
```

maple [B] time = 0.07, size = 370, normalized size = 5.21

$$-\frac{x}{2\left(3x^2-1\right)}-\frac{\sqrt{3}\,\mathrm{arctanh}\left(x\sqrt{3}\right)}{9}-\frac{5x}{18\left(x^2-\frac{1}{3}\right)}-\sqrt{3}\left(-\frac{\left(\left(x-\frac{\sqrt{3}}{3}\right)^2+\frac{2\sqrt{3}\left(x-\frac{\sqrt{3}}{3}\right)}{3}+\frac{4}{3}\right)^{\frac{3}{2}}}{12\left(x-\frac{\sqrt{3}}{3}\right)}+\frac{\sqrt{3}\left(\frac{\sqrt{9\left(x-\frac{\sqrt{3}}{3}\right)^2+6\sqrt{3}\left(x-\frac{\sqrt{3}}{3}\right)+12}}{3}+\frac{\sqrt{3}\,\mathrm{arcsinh}(x)}{3}-\frac{2\sqrt{3}\,\mathrm{arctanh}\left(\frac{3\left(\frac{8}{3}+\frac{2\sqrt{3}\left(x-\frac{\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{4\sqrt{9\left(x-\frac{\sqrt{3}}{3}\right)^2+6\sqrt{3}\left(x-\frac{\sqrt{3}}{3}\right)+12}}\right)}{3}\right)}{36}+\frac{x\sqrt{\left(x-\frac{\sqrt{3}}{3}\right)^2+\frac{2\sqrt{3}\left(x-\frac{\sqrt{3}}{3}\right)}{3}+\frac{4}{3}}}{12}+\frac{\mathrm{arcsinh}(x)}{12}\right)+\sqrt{3}\left(-\frac{\left(\left(x+\frac{\sqrt{3}}{3}\right)^2-\frac{2\sqrt{3}\left(x+\frac{\sqrt{3}}{3}\right)}{3}+\frac{4}{3}\right)^{\frac{3}{2}}}{12\left(x+\frac{\sqrt{3}}{3}\right)}-\frac{\sqrt{3}\left(\frac{\sqrt{9\left(x+\frac{\sqrt{3}}{3}\right)^2-6\sqrt{3}\left(x+\frac{\sqrt{3}}{3}\right)+12}}{3}-\frac{\sqrt{3}\,\mathrm{arcsinh}(x)}{3}-\frac{2\sqrt{3}\,\mathrm{arctanh}\left(\frac{3\left(\frac{8}{3}-\frac{2\sqrt{3}\left(x+\frac{\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{4\sqrt{9\left(x+\frac{\sqrt{3}}{3}\right)^2-6\sqrt{3}\left(x+\frac{\sqrt{3}}{3}\right)+12}}\right)}{3}\right)}{36}+\frac{x\sqrt{\left(x+\frac{\sqrt{3}}{3}\right)^2-\frac{2\sqrt{3}\left(x+\frac{\sqrt{3}}{3}\right)}{3}+\frac{4}{3}}}{12}+\frac{\mathrm{arcsinh}(x)}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+(x^2+1)^(1/2))^2,x)
```

```
[Out] -1/2*x/(3*x^2-1)-1/9*3^(1/2)*arctanh(x*3^(1/2))-5/18*x/(x^2-1/3)-3^(1/2)*(-
1/12/(x-1/3*3^(1/2))*((x-1/3*3^(1/2))^2+2/3*3^(1/2)*(x-1/3*3^(1/2))+4/3)^(3
/2)+1/36*3^(1/2)*(1/3*(9*(x-1/3*3^(1/2))^2+6*3^(1/2)*(x-1/3*3^(1/2))+12)^(1
/2)+1/3*3^(1/2)*arcsinh(x)-2/3*3^(1/2)*arctanh(3/4*(8/3+2/3*3^(1/2)*(x-1/3*
3^(1/2)))*3^(1/2)/(9*(x-1/3*3^(1/2))^2+6*3^(1/2)*(x-1/3*3^(1/2))+12)^(1/2))
)+1/12*x*((x-1/3*3^(1/2))^2+2/3*3^(1/2)*(x-1/3*3^(1/2))+4/3)^(1/2)+1/12*arc
sinh(x))+3^(1/2)*(-1/12/(x+1/3*3^(1/2))*((x+1/3*3^(1/2))^2-2/3*3^(1/2)*(x+1
/3*3^(1/2))+4/3)^(3/2)-1/36*3^(1/2)*(1/3*(9*(x+1/3*3^(1/2))^2-6*3^(1/2)*(x+
1/3*3^(1/2))+12)^(1/2)-1/3*3^(1/2)*arcsinh(x)-2/3*3^(1/2)*arctanh(3/4*(8/3-
2/3*3^(1/2)*(x+1/3*3^(1/2)))*3^(1/2)/(9*(x+1/3*3^(1/2))^2-6*3^(1/2)*(x+1/3*
3^(1/2))+12)^(1/2)))+1/12*x*((x+1/3*3^(1/2))^2-2/3*3^(1/2)*(x+1/3*3^(1/2))+
4/3)^(1/2)+1/12*arcsinh(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x+\sqrt{x^2+1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((2*x + sqrt(x^2 + 1))^(-2), x)
```

mupad [B] time = 0.21, size = 204, normalized size = 2.87

$$\frac{\sqrt{3}\,\left(\ln\left(x-\frac{\sqrt{3}}{3}\right)-\ln\left(x+\sqrt{3}+2\,\sqrt{x^2+1}\right)\right)}{18}-\frac{4\,x}{9\,\left(x^2-\frac{1}{3}\right)}+\frac{\sqrt{3}\,\left(\ln\left(x+\frac{\sqrt{3}}{3}\right)-\ln\left(x-\sqrt{3}-2\,\sqrt{x^2+1}\right)\right)}{18}-\frac{\sqrt{3}\,\left(6\,\ln\left(x-\frac{\sqrt{3}}{3}\right)-6\,\ln\left(x+\sqrt{3}+2\,\sqrt{x^2+1}\right)\right)}{54}-\frac{\sqrt{3}\,\left(6\,\ln\left(x+\frac{\sqrt{3}}{3}\right)-6\,\ln\left(x-\sqrt{3}-2\,\sqrt{x^2+1}\right)\right)}{54}+\frac{\sqrt{3}\,\sqrt{x^2+1}}{9\,\left(x-\frac{\sqrt{3}}{3}\right)}-\frac{\sqrt{3}\,\sqrt{x^2+1}}{9\,\left(x+\frac{\sqrt{3}}{3}\right)}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,x\,1{}i\right)\,1{}i}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + (x^2 + 1)^(1/2))^2,x)
```

```
[Out] (3^(1/2)*(log(x - 3^(1/2)/3) - log(x + 3^(1/2) + 2*(x^2 + 1)^(1/2))))/18 +
(3^(1/2)*atan(3^(1/2)*x*1i)*1i)/9 - (4*x)/(9*(x^2 - 1/3)) + (3^(1/2)*(log(x
 + 3^(1/2)/3) - log(x - 3^(1/2) - 2*(x^2 + 1)^(1/2))))/18 - (3^(1/2)*(6*log
(x - 3^(1/2)/3) - 6*log(x + 3^(1/2) + 2*(x^2 + 1)^(1/2))))/54 - (3^(1/2)*(6
*log(x + 3^(1/2)/3) - 6*log(x - 3^(1/2) - 2*(x^2 + 1)^(1/2))))/54 + (3^(1/2
)*(x^2 + 1)^(1/2))/(9*(x - 3^(1/2)/3)) - (3^(1/2)*(x^2 + 1)^(1/2))/(9*(x +
3^(1/2)/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x+\sqrt{x^2+1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x**2+1)**(1/2))**2,x)
```

```
[Out] Integral((2*x + sqrt(x**2 + 1))**(-2), x)
```

3.874 $\int \frac{\sqrt{ax^2+x\sqrt{-b+a^2x^2}}}{x\sqrt{-b+a^2x^2}}\,dx$

Optimal. Leaf size=71

$$-\frac{\sqrt{2}\log\left(\sqrt{a^2x^2-b}-\sqrt{2}\sqrt{a}\sqrt{x\sqrt{a^2x^2-b}+ax^2}+ax\right)}{\sqrt{a}}$$

Rubi [F] time = 1.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{ax^2+x\sqrt{-b+a^2x^2}}}{x\sqrt{-b+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a*x^2 + x*Sqrt[-b + a^2*x^2]]/(x*Sqrt[-b + a^2*x^2]),x]
```

```
[Out] Defer[Int][Sqrt[a*x^2 + x*Sqrt[-b + a^2*x^2]]/(x*Sqrt[-b + a^2*x^2]), x]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+x\sqrt{-b+a^2x^2}}}{x\sqrt{-b+a^2x^2}}\,dx=\int \frac{\sqrt{ax^2+x\sqrt{-b+a^2x^2}}}{x\sqrt{-b+a^2x^2}}\,dx$$

Mathematica [A] time = 0.14, size = 104, normalized size = 1.46

$$\frac{\sqrt{2}\sqrt{x\left(\sqrt{a^2x^2-b}+ax\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{\sqrt{a^2x^2-b}+ax}}\right)}{\sqrt{a}\sqrt{x}\sqrt{\sqrt{a^2x^2-b}+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + x*Sqrt[-b + a^2*x^2]]/(x*Sqrt[-b + a^2*x^2]),x]
```

```
[Out] (Sqrt[2]*Sqrt[x*(a*x + Sqrt[-b + a^2*x^2])]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[x
])/Sqrt[a*x + Sqrt[-b + a^2*x^2]]])/(Sqrt[a]*Sqrt[x]*Sqrt[a*x + Sqrt[-b + a
^2*x^2]])
```

IntegrateAlgebraic [A] time = 2.04, size = 71, normalized size = 1.00

$$-\frac{\sqrt{2}\log\left(\sqrt{a^2x^2-b}-\sqrt{2}\sqrt{a}\sqrt{x\sqrt{a^2x^2-b}+ax^2}+ax\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^2 + x*Sqrt[-b + a^2*x^2]]/(x*Sqrt[-b + a^2*x^2]
),x]
```

```
[Out] -((Sqrt[2]*Log[a*x + Sqrt[-b + a^2*x^2] - Sqrt[2]*Sqrt[a]*Sqrt[a*x^2 + x*Sq
rt[-b + a^2*x^2]]])/Sqrt[a])
```

fricas [A] time = 18.86, size = 142, normalized size = 2.00

$$\left[\frac{\sqrt{2}\log\left(-4\,a^2x^2-4\,\sqrt{a^2x^2-b}\,ax-2\left(\sqrt{2}\,a^{\frac{3}{2}}x+\sqrt{2}\,\sqrt{a^2x^2-b}\,\sqrt{a}\right)\sqrt{ax^2+\sqrt{a^2x^2-b}\,x}+b\right)}{2\,\sqrt{a}},-\sqrt{2}\,\sqrt{-\frac{1}{a}}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{ax^2+\sqrt{a^2x^2-b}\,x}\,\sqrt{-\frac{1}{a}}}{2\,x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+x*(a^2*x^2-b)^(1/2))^(1/2)/x/(a^2*x^2-b)^(1/2),x, algorith
m="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-4*a^2*x^2 - 4*sqrt(a^2*x^2 - b)*a*x - 2*(sqrt(2)*a^(3/2)*
x + sqrt(2)*sqrt(a^2*x^2 - b)*sqrt(a))*sqrt(a*x^2 + sqrt(a^2*x^2 - b)*x) +
b)/sqrt(a), -sqrt(2)*sqrt(-1/a)*arctan(1/2*sqrt(2)*sqrt(a*x^2 + sqrt(a^2*x^
2 - b)*x)*sqrt(-1/a)/x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2+\sqrt{a^2x^2-b}\,x}}{\sqrt{a^2x^2-b}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+x*(a^2*x^2-b)^(1/2))^(1/2)/x/(a^2*x^2-b)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2 - b)*x)/(sqrt(a^2*x^2 - b)*x), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\,x^2+x\sqrt{a^2x^2-b}}}{x\sqrt{a^2x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+x*(a^2*x^2-b)^(1/2))^(1/2)/x/(a^2*x^2-b)^(1/2),x)
```

```
[Out] int((a*x^2+x*(a^2*x^2-b)^(1/2))^(1/2)/x/(a^2*x^2-b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2+\sqrt{a^2x^2-b}\,x}}{\sqrt{a^2x^2-b}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+x*(a^2*x^2-b)^(1/2))^(1/2)/x/(a^2*x^2-b)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2 - b)*x)/(sqrt(a^2*x^2 - b)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x\,\sqrt{a^2\,x^2-b}+a\,x^2}}{x\,\sqrt{a^2\,x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a^2*x^2 - b)^(1/2) + a*x^2)^(1/2)/(x*(a^2*x^2 - b)^(1/2)),x)
```

```
[Out] int((x*(a^2*x^2 - b)^(1/2) + a*x^2)^(1/2)/(x*(a^2*x^2 - b)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(ax+\sqrt{a^2x^2-b}\right)}}{x\sqrt{a^2x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+x*(a**2*x**2-b)**(1/2))**(1/2)/x/(a**2*x**2-b)**(1/2),x)
[Out] Integral(sqrt(x*(a*x + sqrt(a**2*x**2 - b)))/(x*sqrt(a**2*x**2 - b)), x)
```

3.875 $\int \frac{(-1+x^2)\sqrt{1+x^4}}{(1-x+x^2)(1+x+x^2)^2} \, dx$

Optimal. Leaf size=72

$$\frac{\sqrt{x^4+1}}{2(x^2+x+1)}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}+x^2-x+1}\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}+x^2+x+1}\right)$$

Rubi [C] time = 9.86, antiderivative size = 2595, normalized size of antiderivative = 36.04, number of steps used = 248, number of rules used = 20, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.606, Rules used = {6728, 1729, 1209, 1198, 220, 1196, 1217, 1707, 1248, 735, 844, 215, 725, 206, 6742, 2153, 1227, 733, 204, 1336}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^2)*Sqrt[1 + x^4])/((1 - x + x^2)*(1 + x + x^2)^2),x]
```

```
[Out] ((1 - I*Sqrt[3])*Sqrt[1 + x^4])/16 + ((1 + I*Sqrt[3])*Sqrt[1 + x^4])/16 - (
(3 - (5*I)*Sqrt[3])*Sqrt[1 + x^4])/48 - ((3 + (5*I)*Sqrt[3])*Sqrt[1 + x^4])
/48 + ((I - Sqrt[3])*Sqrt[1 + x^4])/(3*(I - Sqrt[3] + (2*I)*x^2)) + ((I + S
qrt[3])*Sqrt[1 + x^4])/(6*(I - Sqrt[3] + (2*I)*x^2)) - ((I/3)*x*Sqrt[1 + x^
4])/(I - Sqrt[3] + (2*I)*x^2) + ((I + Sqrt[3])*x*Sqrt[1 + x^4])/(3*(I - Sqr
t[3] + (2*I)*x^2)) + ((I - Sqrt[3])*Sqrt[1 + x^4])/(6*(I + Sqrt[3] + (2*I)*
x^2)) + ((I + Sqrt[3])*Sqrt[1 + x^4])/(3*(I + Sqrt[3] + (2*I)*x^2)) - ((I/3
)*x*Sqrt[1 + x^4])/(I + Sqrt[3] + (2*I)*x^2) + ((I - Sqrt[3])*x*Sqrt[1 + x^
4])/(3*(I + Sqrt[3] + (2*I)*x^2)) + (7*x*Sqrt[1 + x^4])/(6*(1 + x^2)) - ((1
 - I*Sqrt[3])*x*Sqrt[1 + x^4])/(3*(1 + x^2)) - ((1 + I*Sqrt[3])*x*Sqrt[1 +
x^4])/(3*(1 + x^2)) - ((3 - (2*I)*Sqrt[3])*x*Sqrt[1 + x^4])/(12*(1 + x^2))
 - ((3 + (2*I)*Sqrt[3])*x*Sqrt[1 + x^4])/(12*(1 + x^2)) - ((1 - I*Sqrt[3])*A
rcSinh[x^2])/4 + ((9 - I*Sqrt[3])*ArcSinh[x^2])/48 - (3*(1 + I*Sqrt[3])*Arc
Sinh[x^2])/16 - ((1 + I*Sqrt[3])^2*ArcSinh[x^2])/32 + ((9 + I*Sqrt[3])*ArcS
inh[x^2])/48 + (5*ArcTan[x/Sqrt[1 + x^4]])/12 - ((1 - I*Sqrt[3])*ArcTan[x/S
qrt[1 + x^4]])/12 - ((3 - I*Sqrt[3])*ArcTan[x/Sqrt[1 + x^4]])/12 - ((1 + I*
Sqrt[3])*ArcTan[x/Sqrt[1 + x^4]])/12 - ((3 + I*Sqrt[3])*ArcTan[x/Sqrt[1 + x
^4]])/12 - ((3 - (2*I)*Sqrt[3])*ArcTan[x/Sqrt[1 + x^4]])/24 - ((3 + (2*I)*S
qrt[3])*ArcTan[x/Sqrt[1 + x^4]])/24 - ArcTan[(2*I - (I - Sqrt[3])*x^2)/(Sqr
t[2*(1 + I*Sqrt[3])]*Sqrt[1 + x^4])]/3 - ((I + Sqrt[3])*ArcTan[(2*I - (I -
Sqrt[3])*x^2)/(Sqrt[2*(1 + I*Sqrt[3])]*Sqrt[1 + x^4])])/(6*(I - Sqrt[3])) +
 ArcTan[(2*I - (I + Sqrt[3])*x^2)/(Sqrt[2*(1 - I*Sqrt[3])]*Sqrt[1 + x^4])]/
3 + ((I - Sqrt[3])*ArcTan[(2*I - (I + Sqrt[3])*x^2)/(Sqrt[2*(1 - I*Sqrt[3])
]*Sqrt[1 + x^4])])/(6*(I + Sqrt[3])) + (I/8)*ArcTanh[(2 - (1 - I*Sqrt[3])*x
^2)/(Sqrt[2*(1 - I*Sqrt[3])]*Sqrt[1 + x^4])] - (I/8)*ArcTanh[(2 - (1 + I*Sq
rt[3])*x^2)/(Sqrt[2*(1 + I*Sqrt[3])]*Sqrt[1 + x^4])] - ((3*I - 2*Sqrt[3])*A
rcTanh[(2 - (1 + I*Sqrt[3])*x^2)/(Sqrt[2*(1 + I*Sqrt[3])]*Sqrt[1 + x^4])])/
24 + ((3*I + 2*Sqrt[3])*ArcTanh[(4 + (1 + I*Sqrt[3])^2*x^2)/(2*Sqrt[2*(1 -
I*Sqrt[3])]*Sqrt[1 + x^4])])/24 - (7*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*
EllipticE[2*ArcTan[x], 1/2])/(6*Sqrt[1 + x^4]) + ((1 - I*Sqrt[3])*(1 + x^2)
*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/2])/(3*Sqrt[1 + x^4])
 + ((1 + I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTa
n[x], 1/2])/(3*Sqrt[1 + x^4]) + ((3 - (2*I)*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^
4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/2])/(12*Sqrt[1 + x^4]) + ((3 + (2*
I)*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/
2])/(12*Sqrt[1 + x^4]) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2
*ArcTan[x], 1/2])/(Sqrt[3]*(3*I - Sqrt[3])*Sqrt[1 + x^4]) + ((1 - I*Sqrt[3]
)*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqr
t[1 + x^4]) - ((3 - I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*Ellipt
icF[2*ArcTan[x], 1/2])/(24*Sqrt[1 + x^4]) - ((9 - I*Sqrt[3])*(1 + x^2)*Sqrt
[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(24*Sqrt[1 + x^4]) + (
```

```
3*(1 + I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[
x], 1/2])/(8*Sqrt[1 + x^4]) - ((1 + I*Sqrt[3])^2*(1 + x^2)*Sqrt[(1 + x^4)/(
1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(16*Sqrt[1 + x^4]) - ((3 + I*Sqrt[
3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(24*
Sqrt[1 + x^4]) - ((9 + I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*Ell
ipticF[2*ArcTan[x], 1/2])/(24*Sqrt[1 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^4)/(1
 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(Sqrt[3]*(3*I + Sqrt[3])*Sqrt[1 + x
^4]) + ((3*I - Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*A
rcTan[x], 1/2])/(12*(3*I + Sqrt[3])*Sqrt[1 + x^4]) + ((3*I + Sqrt[3])*(1 +
x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(12*(3*I - Sq
rt[3])*Sqrt[1 + x^4]) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticPi[1
/4, 2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4]) + ((1 - I*Sqrt[3])*(1 + x^2)*Sqrt[
(1 + x^4)/(1 + x^2)^2]*EllipticPi[1/4, 2*ArcTan[x], 1/2])/(8*Sqrt[1 + x^4])
 + ((2 - I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticPi[1/4, 2
*ArcTan[x], 1/2])/(16*Sqrt[1 + x^4]) - ((3 - I*Sqrt[3])*(1 + x^2)*Sqrt[(1 +
 x^4)/(1 + x^2)^2]*EllipticPi[1/4, 2*ArcTan[x], 1/2])/(24*Sqrt[1 + x^4]) +
((1 + I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticPi[1/4, 2*Ar
cTan[x], 1/2])/(8*Sqrt[1 + x^4]) + ((2 + I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4
)/(1 + x^2)^2]*EllipticPi[1/4, 2*ArcTan[x], 1/2])/(16*Sqrt[1 + x^4]) - ((3
+ I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticPi[1/4, 2*ArcTan
[x], 1/2])/(24*Sqrt[1 + x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1227

```
Int[Sqrt[(a_) + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Symbol] :> Simp[(x
*Sqrt[a + c*x^4])/(2*d*(d + e*x^2)), x] + (Dist[c/(2*d*e^2), Int[(d - e*x^2
)/Sqrt[a + c*x^4], x], x] - Dist[(c*d^2 - a*e^2)/(2*d*e^2), Int[1/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]) /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^
2, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Dist[d, I
nt[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, Int[(x*(a + c*x^4)^p)/(d
^2 - e^2*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Rule 2153

```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^nn)^p, (c/(c^2 - d^2*x^(2*n)) - (d*x^n)/
(c^2 - d^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !
IntegerQ[p] && ILtQ[q, 0] && IGtQ[Log[2, nn/n], 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(-1+x^2)\sqrt{1+x^4}}{(1-x+x^2)(1+x+x^2)^2}\,dx = \int \left(\frac{(1+x)\sqrt{1+x^4}}{4(1-x+x^2)} + \frac{(-1-2x)\sqrt{1+x^4}}{2(1+x+x^2)^2} + \frac{(-3-x)\sqrt{1+x^4}}{4(1+x+x^2)}\right)dx$$

$$= \frac{1}{4}\int \frac{(1+x)\sqrt{1+x^4}}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{(-3-x)\sqrt{1+x^4}}{1+x+x^2}\,dx + \frac{1}{2}\int \frac{(-1-2x)\sqrt{1+x^4}}{(1+x+x^2)^2}$$

$$= \frac{1}{4}\int \left(\frac{(1-i\sqrt{3})\sqrt{1+x^4}}{-1-i\sqrt{3}+2x} + \frac{(1+i\sqrt{3})\sqrt{1+x^4}}{-1+i\sqrt{3}+2x}\right)dx + \frac{1}{4}\int \left(\frac{\left(-1+\frac{5i}{\sqrt{3}}\right)\sqrt{1+}}{1-i\sqrt{3}+2x}\right.$$

$$= -\left(\frac{1}{2}\int \frac{\sqrt{1+x^4}}{(1+x+x^2)^2}\,dx\right) + \frac{1}{4}(1-i\sqrt{3})\int \frac{\sqrt{1+x^4}}{-1-i\sqrt{3}+2x}\,dx + \frac{1}{4}(1+i\sqrt{3})$$

= rest of steps removed due to Latex formating problem

Mathematica [C] time = 1.44, size = 350, normalized size = 4.86

$$\frac{1}{8}\left(\frac{4\sqrt{x^4+1}}{x^2+x+1}-i\sqrt{6+6i\sqrt{3}}\tanh^{-1}\left(\frac{2+\left(-1-i\sqrt{3}\right)x^2}{\sqrt{2+2i\sqrt{3}}\sqrt{x^4+1}}\right)-\sqrt{2+2i\sqrt{3}}\tanh^{-1}\left(\frac{2+\left(-1-i\sqrt{3}\right)x^2}{\sqrt{2+2i\sqrt{3}}\sqrt{x^4+1}}\right)+i\sqrt{6-6i\sqrt{3}}\tanh^{-1}\left(\frac{2+i\left(\sqrt{3}+i\right)x^2}{\sqrt{2-2i\sqrt{3}}\sqrt{x^4+1}}\right)-\sqrt{2-2i\sqrt{3}}\tanh^{-1}\left(\frac{2+i\left(\sqrt{3}+i\right)x^2}{\sqrt{2-2i\sqrt{3}}\sqrt{x^4+1}}\right)+4\sqrt[4]{-1}\Pi\left(-\sqrt[6]{-1};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)+4\sqrt[4]{-1}\Pi\left(-(-1)^{5/6};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)\right)-\frac{1}{2}\sqrt[4]{-1}F\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)*Sqrt[1 + x^4])/((1 - x + x^2)*(1 + x + x^2)^2),x]
```

```
[Out] -1/2*((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1]) + ((4*Sqrt[1 + x^4
])/(1 + x + x^2) - Sqrt[2 + (2*I)*Sqrt[3]]*ArcTanh[(2 + (-1 - I*Sqrt[3])*x^
2)/(Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 + x^4])] - I*Sqrt[6 + (6*I)*Sqrt[3]]*Arc
Tanh[(2 + (-1 - I*Sqrt[3])*x^2)/(Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 + x^4])] -
Sqrt[2 - (2*I)*Sqrt[3]]*ArcTanh[(2 + I*(I + Sqrt[3])*x^2)/(Sqrt[2 - (2*I)*S
qrt[3]]*Sqrt[1 + x^4])] + I*Sqrt[6 - (6*I)*Sqrt[3]]*ArcTanh[(2 + I*(I + Sqr
t[3])*x^2)/(Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 + x^4])] + 4*(-1)^(1/4)*Elliptic
Pi[-(-1)^(1/6), I*ArcSinh[(-1)^(1/4)*x], -1] + 4*(-1)^(1/4)*EllipticPi[-(-1
)^(5/6), I*ArcSinh[(-1)^(1/4)*x], -1])/8
```

IntegrateAlgebraic [A] time = 1.62, size = 72, normalized size = 1.00

$$\frac{\sqrt{x^4+1}}{2\left(x^2+x+1\right)}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}+x^2-x+1}\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}+x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*Sqrt[1 + x^4])/((1 - x + x^2)*(1 + x + x^2)^
2),x]
```

```
[Out] Sqrt[1 + x^4]/(2*(1 + x + x^2)) + ArcTan[x/(1 - x + x^2 + Sqrt[1 + x^4])]/2
 - (3*ArcTan[x/(1 + x + x^2 + Sqrt[1 + x^4])])/2
```

fricas [A] time = 0.49, size = 73, normalized size = 1.01

$$\frac{3\left(x^2+x+1\right)\arctan\left(\frac{\sqrt{x^4+1}}{x^2+2x+1}\right)+\left(x^2+x+1\right)\arctan\left(\frac{\sqrt{x^4+1}}{x^2-2x+1}\right)+2\sqrt{x^4+1}}{4\left(x^2+x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+1)^(1/2)/(x^2-x+1)/(x^2+x+1)^2,x, algorithm="fricas"
)
```

```
[Out] 1/4*(3*(x^2 + x + 1)*arctan(sqrt(x^4 + 1)/(x^2 + 2*x + 1)) + (x^2 + x + 1)*
arctan(sqrt(x^4 + 1)/(x^2 - 2*x + 1)) + 2*sqrt(x^4 + 1))/(x^2 + x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}\left(x^2-1\right)}{\left(x^2+x+1\right)^2\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+1)^(1/2)/(x^2-x+1)/(x^2+x+1)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 1)*(x^2 - 1)/((x^2 + x + 1)^2*(x^2 - x + 1)), x)
```

maple [C] time = 0.07, size = 753, normalized size = 10.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(x^4+1)^(1/2)/(x^2-x+1)/(x^2+x+1)^2,x)
```

```
[Out] 1/2*(x^4+1)^(1/2)/(x^2+x+1)+1/2/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)
*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-1
/3*I*3^(1/2)*(1/2/(1/2-1/2*I*3^(1/2))^(1/2)*arctanh((1/2-1/2*I*3^(1/2))^(1/
2)*(x^2-1/2-1/2*I*3^(1/2))/(x^4+1)^(1/2))+(-1)^(3/4)*(-1/2+1/2*I*3^(1/2))*(
1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I*(-1
/2-1/2*I*3^(1/2)),I))+1/3*I*3^(1/2)*(1/2/(1/2+1/2*I*3^(1/2))^(1/2)*arctanh(
(1/2+1/2*I*3^(1/2))^(1/2)*(x^2-1/2+1/2*I*3^(1/2))/(x^4+1)^(1/2))+(-1)^(3/4)
*(-1/2-1/2*I*3^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*Ellipti
cPi((-1)^(1/4)*x,-I*(-1/2+1/2*I*3^(1/2)),I))-1/4*(3/2-1/6*I*3^(1/2))*(1/2/(
1/2+1/2*I*3^(1/2))^(1/2)*arctanh((1/2+1/2*I*3^(1/2))^(1/2)*(x^2-1/2+1/2*I*3
^(1/2))/(x^4+1)^(1/2))+(-1)^(3/4)*(-1/2-1/2*I*3^(1/2))*(1-I*x^2)^(1/2)*(1+I
*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I*(-1/2+1/2*I*3^(1/2)),I
))-1/4*(3/2+1/6*I*3^(1/2))*(1/2/(1/2-1/2*I*3^(1/2))^(1/2)*arctanh((1/2-1/2*
I*3^(1/2))^(1/2)*(x^2-1/2-1/2*I*3^(1/2))/(x^4+1)^(1/2))+(-1)^(3/4)*(-1/2+1/
2*I*3^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^
(1/4)*x,-I*(-1/2-1/2*I*3^(1/2)),I))-1/4*(1/2+1/2*I*3^(1/2))*(-1/2/(1/2-1/2*
I*3^(1/2))^(1/2)*arctanh((-1/2+1/2*I*3^(1/2))*(x^2-1/2-1/2*I*3^(1/2))/(1/2-
1/2*I*3^(1/2))^(1/2)/(x^4+1)^(1/2))+(-1)^(3/4)*(1/2-1/2*I*3^(1/2))*(1-I*x^2
)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I*(1/2+1/2*I*
3^(1/2)),I))-1/4*(1/2-1/2*I*3^(1/2))*(-1/2/(1/2+1/2*I*3^(1/2))^(1/2)*arctan
h((-1/2-1/2*I*3^(1/2))*(x^2-1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))^(1/2)/(x
^4+1)^(1/2))+(-1)^(3/4)*(1/2+1/2*I*3^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)
/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I*(1/2-1/2*I*3^(1/2)),I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}\left(x^2-1\right)}{\left(x^2+x+1\right)^2\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+1)^(1/2)/(x^2-x+1)/(x^2+x+1)^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(x^4 + 1)*(x^2 - 1)/((x^2 + x + 1)^2*(x^2 - x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2-1\right)\,\sqrt{x^4+1}}{\left(x^2-x+1\right)\,\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*(x^4 + 1)^(1/2))/((x^2 - x + 1)*(x + x^2 + 1)^2),x)
```

```
[Out] int(((x^2 - 1)*(x^4 + 1)^(1/2))/((x^2 - x + 1)*(x + x^2 + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)\,\sqrt{x^4+1}}{\left(x^2-x+1\right)\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**4+1)**(1/2)/(x**2-x+1)/(x**2+x+1)**2,x)
```

```
[Out] Integral((x - 1)*(x + 1)*sqrt(x**4 + 1)/((x**2 - x + 1)*(x**2 + x + 1)**2),
 x)
```

3.876 $\int x^4 \sqrt[4]{-x^2+x^4}\, dx$

Optimal. Leaf size=72

$$\frac{7}{128}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)-\frac{7}{128}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)+\frac{1}{192}\sqrt[4]{x^4-x^2}\left(32x^5-4x^3-7x\right)$$

Rubi [B] time = 0.20, antiderivative size = 153, normalized size of antiderivative = 2.12, number of steps used = 9, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {2021, 2024, 2032, 329, 331, 298, 203, 206}

$$-\frac{7}{192}\sqrt[4]{x^4-x^2}\,x+\frac{7\left(x^2-1\right)^{3/4}x^{3/2}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{128\left(x^4-x^2\right)^{3/4}}-\frac{7\left(x^2-1\right)^{3/4}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{128\left(x^4-x^2\right)^{3/4}}+\frac{1}{6}\sqrt[4]{x^4-x^2}\,x^5-\frac{1}{48}\sqrt[4]{x^4-x^2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(-x^2 + x^4)^(1/4),x]
```

```
[Out] (-7*x*(-x^2 + x^4)^(1/4))/192 - (x^3*(-x^2 + x^4)^(1/4))/48 + (x^5*(-x^2 +
x^4)^(1/4))/6 + (7*x^(3/2)*(-1 + x^2)^(3/4)*ArcTan[Sqrt[x]/(-1 + x^2)^(1/4)
])/(128*(-x^2 + x^4)^(3/4)) - (7*x^(3/2)*(-1 + x^2)^(3/4)*ArcTanh[Sqrt[x]/(
-1 + x^2)^(1/4)])/(128*(-x^2 + x^4)^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
```

```
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^4\sqrt[4]{-x^2+x^4}\,dx = \frac{1}{6}x^5\sqrt[4]{-x^2+x^4} - \frac{1}{12}\int \frac{x^6}{\left(-x^2+x^4\right)^{3/4}}\,dx$$

$$= -\frac{1}{48}x^3\sqrt[4]{-x^2+x^4} + \frac{1}{6}x^5\sqrt[4]{-x^2+x^4} - \frac{7}{96}\int \frac{x^4}{\left(-x^2+x^4\right)^{3/4}}\,dx$$

$$= -\frac{7}{192}x\sqrt[4]{-x^2+x^4} - \frac{1}{48}x^3\sqrt[4]{-x^2+x^4} + \frac{1}{6}x^5\sqrt[4]{-x^2+x^4} - \frac{7}{128}\int \frac{x^2}{\left(-x^2+x^4\right)^{3/4}}\,dx$$

$$= -\frac{7}{192}x\sqrt[4]{-x^2+x^4} - \frac{1}{48}x^3\sqrt[4]{-x^2+x^4} + \frac{1}{6}x^5\sqrt[4]{-x^2+x^4} - \frac{\left(7x^{3/2}\left(-1+x^2\right)^{3/4}\right)\int \frac{\sqrt{x}}{\left(-1+x^2\right)^{3/4}}\,d}{128\left(-x^2+x^4\right)^{3/4}}$$

$$= -\frac{7}{192}x\sqrt[4]{-x^2+x^4} - \frac{1}{48}x^3\sqrt[4]{-x^2+x^4} + \frac{1}{6}x^5\sqrt[4]{-x^2+x^4} - \frac{\left(7x^{3/2}\left(-1+x^2\right)^{3/4}\right)\mathrm{Subst}\left(\int \frac{}{(-1}\right.}{64\left(-x^2+x^4\right)^{3/4}}$$

$$= -\frac{7}{192}x\sqrt[4]{-x^2+x^4} - \frac{1}{48}x^3\sqrt[4]{-x^2+x^4} + \frac{1}{6}x^5\sqrt[4]{-x^2+x^4} - \frac{\left(7x^{3/2}\left(-1+x^2\right)^{3/4}\right)\mathrm{Subst}\left(\int \frac{x^2}{1-x}\right.}{64\left(-x^2+x^4\right)^{3/4}}$$

$$= -\frac{7}{192}x\sqrt[4]{-x^2+x^4} - \frac{1}{48}x^3\sqrt[4]{-x^2+x^4} + \frac{1}{6}x^5\sqrt[4]{-x^2+x^4} - \frac{\left(7x^{3/2}\left(-1+x^2\right)^{3/4}\right)\mathrm{Subst}\left(\int \frac{1}{1-x}\right.}{128\left(-x^2+x^4\right)^{3/4}}$$

$$= -\frac{7}{192}x\sqrt[4]{-x^2+x^4} - \frac{1}{48}x^3\sqrt[4]{-x^2+x^4} + \frac{1}{6}x^5\sqrt[4]{-x^2+x^4} + \frac{7x^{3/2}\left(-1+x^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right.}{128\left(-x^2+x^4\right)^{3/4}}$$

Mathematica [C] time = 0.03, size = 69, normalized size = 0.96

$$\frac{x\sqrt[4]{x^2\left(x^2-1\right)}\left(7\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};x^2\right)+\sqrt[4]{1-x^2}\left(8x^4-x^2-7\right)\right)}{48\sqrt[4]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(-x^2 + x^4)^(1/4),x]
```

```
[Out] (x*(x^2*(-1 + x^2))^(1/4)*((1 - x^2)^(1/4)*(-7 - x^2 + 8*x^4) + 7*Hypergeom
etric2F1[-1/4, 3/4, 7/4, x^2]))/(48*(1 - x^2)^(1/4))
```

IntegrateAlgebraic [A] time = 0.18, size = 72, normalized size = 1.00

$$\frac{7}{128}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)-\frac{7}{128}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)+\frac{1}{192}\sqrt[4]{x^4-x^2}\left(32x^5-4x^3-7x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(-x^2 + x^4)^(1/4),x]
```

```
[Out] ((-x^2 + x^4)^(1/4)*(-7*x - 4*x^3 + 32*x^5))/192 + (7*ArcTan[x/(-x^2 + x^4)
^(1/4)])/128 - (7*ArcTanh[x/(-x^2 + x^4)^(1/4)])/128
```

fricas [B] time = 1.40, size = 123, normalized size = 1.71

$$\frac{1}{192}\left(32x^5-4x^3-7x\right)\left(x^4-x^2\right)^{\frac{1}{4}}-\frac{7}{256}\arctan\left(\frac{2\left(\left(x^4-x^2\right)^{\frac{1}{4}}x^2+\left(x^4-x^2\right)^{\frac{3}{4}}\right)}{x}\right)+\frac{7}{256}\log\left(-\frac{2x^3-2\left(x^4-x^2\right)^{\frac{1}{4}}x^2+2\sqrt{x^4-x^2}\,x-x-2\left(x^4-x^2\right)^{\frac{3}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^4-x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/192*(32*x^5 - 4*x^3 - 7*x)*(x^4 - x^2)^(1/4) - 7/256*arctan(2*((x^4 - x^2
)^(1/4)*x^2 + (x^4 - x^2)^(3/4))/x) + 7/256*log(-(2*x^3 - 2*(x^4 - x^2)^(1/
4)*x^2 + 2*sqrt(x^4 - x^2)*x - x - 2*(x^4 - x^2)^(3/4))/x)
```

giac [A] time = 0.30, size = 89, normalized size = 1.24

$$\frac{1}{192}\left(7\left(\frac{1}{x^2}-1\right)^2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}-18\left(-\frac{1}{x^2}+1\right)^{\frac{5}{4}}-21\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)x^6+\frac{7}{128}\arctan\left(\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)+\frac{7}{256}\log\left(\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}+1\right)-\frac{7}{256}\log\left(-\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^4-x^2)^(1/4),x, algorithm="giac")
```

```
[Out] 1/192*(7*(1/x^2 - 1)^2*(-1/x^2 + 1)^(1/4) - 18*(-1/x^2 + 1)^(5/4) - 21*(-1/
x^2 + 1)^(1/4))*x^6 + 7/128*arctan((-1/x^2 + 1)^(1/4)) + 7/256*log((-1/x^2
+ 1)^(1/4) + 1) - 7/256*log(-(-1/x^2 + 1)^(1/4) + 1)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^4\left(x^4-x^2\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^4-x^2)^(1/4),x)
```

```
[Out] int(x^4*(x^4-x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^4-x^2\right)^{\frac{1}{4}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^4-x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^2)^(1/4)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(x^4 - x^2\right)^{1/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^4 - x^2)^(1/4),x)
```

```
[Out] int(x^4*(x^4 - x^2)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \sqrt[4]{x^2 (x-1)(x+1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**4-x**2)**(1/4),x)
```

```
[Out] Integral(x**4*(x**2*(x - 1)*(x + 1))**(1/4), x)
```

3.877 $\int \frac{x^4 \sqrt[4]{x^3+x^4}}{1+x} \, dx$

Optimal. Leaf size=72

$$\frac{4389 \tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)}{4096}-\frac{4389 \tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)}{4096}+\frac{\sqrt[4]{x^4+x^3}\left(2048x^4-2432x^3+3040x^2-4180x+7315\right)}{10240}$$

Rubi [B] time = 0.15, antiderivative size = 168, normalized size of antiderivative = 2.33, number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2056, 50, 63, 331, 298, 203, 206}

$$\frac{1}{5}\sqrt[4]{x^4+x^3}\,x^4-\frac{19}{80}\sqrt[4]{x^4+x^3}\,x^3-\frac{209}{512}\sqrt[4]{x^4+x^3}\,x+\frac{1463\sqrt[4]{x^4+x^3}}{2048}+\frac{4389\sqrt[4]{x^4+x^3}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{4096\sqrt[4]{x+1}\,x^{3/4}}-\frac{4389\sqrt[4]{x^4+x^3}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{4096\sqrt[4]{x+1}\,x^{3/4}}+\frac{19}{64}\sqrt[4]{x^4+x^3}\,x^2$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(x^3 + x^4)^(1/4))/(1 + x),x]
```

```
[Out] (1463*(x^3 + x^4)^(1/4))/2048 - (209*x*(x^3 + x^4)^(1/4))/512 + (19*x^2*(x^
3 + x^4)^(1/4))/64 - (19*x^3*(x^3 + x^4)^(1/4))/80 + (x^4*(x^3 + x^4)^(1/4)
)/5 + (4389*(x^3 + x^4)^(1/4)*ArcTan[x^(1/4)/(1 + x)^(1/4)])/(4096*x^(3/4)*
(1 + x)^(1/4)) - (4389*(x^3 + x^4)^(1/4)*ArcTanh[x^(1/4)/(1 + x)^(1/4)])/(4
096*x^(3/4)*(1 + x)^(1/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{x^4\sqrt[4]{x^3+x^4}}{1+x}\,dx = \frac{\sqrt[4]{x^3+x^4}\int \frac{x^{19/4}}{(1+x)^{3/4}}\,dx}{x^{3/4}\sqrt[4]{1+x}}$$

$$= \frac{1}{5}x^4\sqrt[4]{x^3+x^4} - \frac{\left(19\sqrt[4]{x^3+x^4}\right)\int \frac{x^{15/4}}{(1+x)^{3/4}}\,dx}{20x^{3/4}\sqrt[4]{1+x}}$$

$$= -\frac{19}{80}x^3\sqrt[4]{x^3+x^4} + \frac{1}{5}x^4\sqrt[4]{x^3+x^4} + \frac{\left(57\sqrt[4]{x^3+x^4}\right)\int \frac{x^{11/4}}{(1+x)^{3/4}}\,dx}{64x^{3/4}\sqrt[4]{1+x}}$$

$$= \frac{19}{64}x^2\sqrt[4]{x^3+x^4} - \frac{19}{80}x^3\sqrt[4]{x^3+x^4} + \frac{1}{5}x^4\sqrt[4]{x^3+x^4} - \frac{\left(209\sqrt[4]{x^3+x^4}\right)\int \frac{x^{7/4}}{(1+x)^{3/4}}\,dx}{256x^{3/4}\sqrt[4]{1+x}}$$

$$= -\frac{209}{512}x\sqrt[4]{x^3+x^4} + \frac{19}{64}x^2\sqrt[4]{x^3+x^4} - \frac{19}{80}x^3\sqrt[4]{x^3+x^4} + \frac{1}{5}x^4\sqrt[4]{x^3+x^4} + \frac{\left(1463\sqrt[4]{x^3+x^4}\right)\int \frac{}{(}}{2048x^{3/4}\sqrt[4]{1+}}$$

$$= \frac{1463\sqrt[4]{x^3+x^4}}{2048} - \frac{209}{512}x\sqrt[4]{x^3+x^4} + \frac{19}{64}x^2\sqrt[4]{x^3+x^4} - \frac{19}{80}x^3\sqrt[4]{x^3+x^4} + \frac{1}{5}x^4\sqrt[4]{x^3+x^4} - \frac{(438}{}$$

$$= \frac{1463\sqrt[4]{x^3+x^4}}{2048} - \frac{209}{512}x\sqrt[4]{x^3+x^4} + \frac{19}{64}x^2\sqrt[4]{x^3+x^4} - \frac{19}{80}x^3\sqrt[4]{x^3+x^4} + \frac{1}{5}x^4\sqrt[4]{x^3+x^4} - \frac{(438}{}$$

$$= \frac{1463\sqrt[4]{x^3+x^4}}{2048} - \frac{209}{512}x\sqrt[4]{x^3+x^4} + \frac{19}{64}x^2\sqrt[4]{x^3+x^4} - \frac{19}{80}x^3\sqrt[4]{x^3+x^4} + \frac{1}{5}x^4\sqrt[4]{x^3+x^4} - \frac{(438}{}$$

$$= \frac{1463\sqrt[4]{x^3+x^4}}{2048} - \frac{209}{512}x\sqrt[4]{x^3+x^4} + \frac{19}{64}x^2\sqrt[4]{x^3+x^4} - \frac{19}{80}x^3\sqrt[4]{x^3+x^4} + \frac{1}{5}x^4\sqrt[4]{x^3+x^4} - \frac{(438}{}$$

$$= \frac{1463\sqrt[4]{x^3+x^4}}{2048} - \frac{209}{512}x\sqrt[4]{x^3+x^4} + \frac{19}{64}x^2\sqrt[4]{x^3+x^4} - \frac{19}{80}x^3\sqrt[4]{x^3+x^4} + \frac{1}{5}x^4\sqrt[4]{x^3+x^4} + \frac{438}{}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.53

$$\frac{4x^8(x+1)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{23}{4};\frac{27}{4};-x\right)}{23\left(x^3(x+1)\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(x^3 + x^4)^(1/4))/(1 + x),x]
```

```
[Out] (4*x^8*(1 + x)^(3/4)*Hypergeometric2F1[3/4, 23/4, 27/4, -x])/(23*(x^3*(1 + x))^(3/4))
```

IntegrateAlgebraic [A] time = 0.35, size = 72, normalized size = 1.00

$$\frac{4389\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)}{4096}-\frac{4389\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)}{4096}+\frac{\sqrt[4]{x^4+x^3}\,\left(2048x^4-2432x^3+3040x^2-4180x+7315\right)}{10240}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(x^3 + x^4)^(1/4))/(1 + x),x]
```

```
[Out] ((x^3 + x^4)^(1/4)*(7315 - 4180*x + 3040*x^2 - 2432*x^3 + 2048*x^4))/10240 + (4389*ArcTan[x/(x^3 + x^4)^(1/4)])/4096 - (4389*ArcTanh[x/(x^3 + x^4)^(1/4)])/4096
```

fricas [A] time = 0.41, size = 87, normalized size = 1.21

$$\frac{1}{10240}\left(2048\,x^4-2432\,x^3+3040\,x^2-4180\,x+7315\right)\left(x^4+x^3\right)^{\frac{1}{4}}-\frac{4389}{4096}\arctan\left(\frac{\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)-\frac{4389}{8192}\log\left(\frac{x+\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{4389}{8192}\log\left(-\frac{x-\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^4+x^3)^(1/4)/(1+x),x, algorithm="fricas")
```

```
[Out] 1/10240*(2048*x^4 - 2432*x^3 + 3040*x^2 - 4180*x + 7315)*(x^4 + x^3)^(1/4) - 4389/4096*arctan((x^4 + x^3)^(1/4)/x) - 4389/8192*log((x + (x^4 + x^3)^(1/4))/x) + 4389/8192*log(-(x - (x^4 + x^3)^(1/4))/x)
```

giac [A] time = 0.32, size = 87, normalized size = 1.21

$$\frac{1}{10240}\left(7315\left(\frac{1}{x}+1\right)^{\frac{17}{4}}-33440\left(\frac{1}{x}+1\right)^{\frac{13}{4}}+59470\left(\frac{1}{x}+1\right)^{\frac{9}{4}}-50312\left(\frac{1}{x}+1\right)^{\frac{5}{4}}+19015\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right)x^5-\frac{4389}{4096}\arctan\left(\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right)-\frac{4389}{8192}\log\left(\left(\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right)+\frac{4389}{8192}\log\left(\left|\left(\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^4+x^3)^(1/4)/(1+x),x, algorithm="giac")
```

```
[Out] 1/10240*(7315*(1/x + 1)^(17/4) - 33440*(1/x + 1)^(13/4) + 59470*(1/x + 1)^(9/4) - 50312*(1/x + 1)^(5/4) + 19015*(1/x + 1)^(1/4))*x^5 - 4389/4096*arctan((1/x + 1)^(1/4)) - 4389/8192*log((1/x + 1)^(1/4) + 1) + 4389/8192*log(abs((1/x + 1)^(1/4) - 1))
```

maple [C] time = 0.42, size = 390, normalized size = 5.42

$$\frac{\left(2048x^4-2432x^3+3040x^2-4180x+7315\right)\left(x^3(1+x)\right)^{\frac{1}{4}}}{10240}+\frac{\left(-\frac{4389\ln\left(\frac{2\left(x^4+3x^3+3x^2+x\right)^{\frac{3}{4}}+2\sqrt{x^4+3x^3+3x^2+x}\,x+2\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}x^2+2x^3+2\sqrt{x^4+3x^3+3x^2+x}+4\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}x+5x^2+2\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}+4x+1}{(1+x)^2}\right)}{8192}+\frac{4389\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\sqrt{x^4+3x^3+3x^2+x}\,\mathrm{RootOf}\left(\_Z^2+1\right)x-2\,\mathrm{RootOf}\left(\_Z^2+1\right)x^3+2\left(x^4+3x^3+3x^2+x\right)^{\frac{3}{4}}+2\sqrt{x^4+3x^3+3x^2+x}\,\mathrm{RootOf}\left(\_Z^2+1\right)-2\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}x^2-5\,\mathrm{RootOf}\left(\_Z^2+1\right)x^2-4\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}x-4\,\mathrm{RootOf}\left(\_Z^2+1\right)x-2\left(x^4+3x^3+3x^2+x\right)^{\frac{1}{4}}-\mathrm{RootOf}\left(\_Z^2+1\right)}{(1+x)^2}\right)}{8192}\right)\left(x^3(1+x)\right)^{\frac{1}{4}}\left(x(1+x)^3\right)^{\frac{1}{4}}}{x(1+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^4+x^3)^(1/4)/(1+x),x)
```

```
[Out] 1/10240*(2048*x^4-2432*x^3+3040*x^2-4180*x+7315)*(x^3*(1+x))^(1/4)+(-4389/8192*ln((2*(x^4+3*x^3+3*x^2+x)^(3/4)+2*(x^4+3*x^3+3*x^2+x)^(1/2)*x+2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2+2*x^3+2*(x^4+3*x^3+3*x^2+x)^(1/2)+4*(x^4+3*x^3+3*x^2+x)^(1/4)*x+5*x^2+2*(x^4+3*x^3+3*x^2+x)^(1/4)+4*x+1)/(1+x)^2)+4389/8192*RootOf(_Z^2+1)*ln((2*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^2+1)*x-2*RootOf(_Z^2+1)*x^3+2*(x^4+3*x^3+3*x^2+x)^(3/4)+2*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^2+1)-2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2-5*RootOf(_Z^2+1)*x^2-4*(x^4+3*x^3+3*x^2+x)^(1/4)*x-4*RootOf(_Z^2+1)*x-2*(x^4+3*x^3+3*x^2+x)^(1/4)-RootOf(_Z^2+1))/(1+x)^2))*(x^3*(1+x))^(1/4)/x*(x*(1+x)^3)^(1/4)/(1+x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3\right)^{\frac{1}{4}}x^4}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^4+x^3)^(1/4)/(1+x),x, algorithm="maxima")
[Out] integrate((x^4 + x^3)^(1/4)*x^4/(x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\left(x^4+x^3\right)^{1/4}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x^3 + x^4)^(1/4))/(x + 1),x)
[Out] int((x^4*(x^3 + x^4)^(1/4))/(x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt[4]{x^3\,(x+1)}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**4+x**3)**(1/4)/(1+x),x)
[Out] Integral(x**4*(x**3*(x + 1))**(1/4)/(x + 1), x)
```

3.878 $\int \frac{(4b+ax^3)(-b-ax^3+x^4)}{x^4\sqrt[4]{b+ax^3}(-b-ax^3+2x^4)}\,dx$

Optimal. Leaf size=72

$$-\frac{4\left(ax^3+b\right)^{3/4}}{3x^3}-2^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{2}\,x}\right)+2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{ax^3+b}}\right)$$

Rubi [F] time = 3.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(4b+ax^3)(-b-ax^3+x^4)}{x^4\sqrt[4]{b+ax^3}\,(-b-ax^3+2x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((4*b + a*x^3)*(-b - a*x^3 + x^4))/(x^4*(b + a*x^3)^(1/4)*(-b - a*x^3 +
 2*x^4)),x]
```

```
[Out] (-4*(b + a*x^3)^(3/4))/(3*x^3) + 4*b*Defer[Int][1/((b + a*x^3)^(1/4)*(b + a
*x^3 - 2*x^4)), x] + a*Defer[Int][x^3/((b + a*x^3)^(1/4)*(b + a*x^3 - 2*x^4
)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(4b+ax^3)(-b-ax^3+x^4)}{x^4\sqrt[4]{b+ax^3}\,(-b-ax^3+2x^4)}\,dx &= \int\left(\frac{4b}{x^4\sqrt[4]{b+ax^3}}+\frac{a}{x\sqrt[4]{b+ax^3}}+\frac{4b+ax^3}{\sqrt[4]{b+ax^3}\,(b+ax^3-2x^4)}\right)dx\\
&= a\int\frac{1}{x\sqrt[4]{b+ax^3}}\,dx+(4b)\int\frac{1}{x^4\sqrt[4]{b+ax^3}}\,dx+\int\frac{4b+ax^3}{\sqrt[4]{b+ax^3}\,(b+ax^3-\\
&= \frac{1}{3}a\,\text{Subst}\left(\int\frac{1}{x\sqrt[4]{b+ax}}\,dx,x,x^3\right)+\frac{1}{3}(4b)\,\text{Subst}\left(\int\frac{1}{x^2\sqrt[4]{b+ax}}\,dx,x,\right.\\
&= -\frac{4\left(b+ax^3\right)^{3/4}}{3x^3}+\frac{4}{3}\text{Subst}\left(\int\frac{x^2}{-\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{b+ax^3}\right)-\frac{1}{3}a\,\text{Subst}\left(\int\right.\\
&= -\frac{4\left(b+ax^3\right)^{3/4}}{3x^3}-\frac{4}{3}\text{Subst}\left(\int\frac{x^2}{-\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{b+ax^3}\right)-\frac{1}{3}(2a)\,\text{Subst}\\
&= -\frac{4\left(b+ax^3\right)^{3/4}}{3x^3}+\frac{2a\tan^{-1}\left(\frac{\sqrt[4]{b+ax^3}}{\sqrt[4]{b}}\right)}{3\sqrt[4]{b}}-\frac{2a\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^3}}{\sqrt[4]{b}}\right)}{3\sqrt[4]{b}}+\frac{1}{3}(2a)\,\text{Su}\\
&= -\frac{4\left(b+ax^3\right)^{3/4}}{3x^3}+a\int\frac{x^3}{\sqrt[4]{b+ax^3}\,(b+ax^3-2x^4)}\,dx+(4b)\int\frac{}{\sqrt[4]{b+ax^3}}
\end{aligned}$$

Mathematica [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{(4b+ax^3)(-b-ax^3+x^4)}{x^4\sqrt[4]{b+ax^3}\,(-b-ax^3+2x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((4*b + a*x^3)*(-b - a*x^3 + x^4))/(x^4*(b + a*x^3)^(1/4)*(-b - a
*x^3 + 2*x^4)),x]
```

```
[Out] Integrate[((4*b + a*x^3)*(-b - a*x^3 + x^4))/(x^4*(b + a*x^3)^(1/4)*(-b - a
*x^3 + 2*x^4)), x]
```

IntegrateAlgebraic [A] time = 1.67, size = 72, normalized size = 1.00

$$-\frac{4\left(ax^3+b\right)^{3/4}}{3x^3}-2^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{2}x}\right)+2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{ax^3+b}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((4*b + a*x^3)*(-b - a*x^3 + x^4))/(x^4*(b + a*x^3)^(1/4
)*(-b - a*x^3 + 2*x^4)),x]
```

```
[Out] (-4*(b + a*x^3)^(3/4))/(3*x^3) - 2^(3/4)*ArcTan[(b + a*x^3)^(1/4)/(2^(1/4)*
x)] + 2^(3/4)*ArcTanh[(2^(1/4)*x)/(b + a*x^3)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+4*b)*(-a*x^3+x^4-b)/x^4/(a*x^3+b)^(1/4)/(-a*x^3+2*x^4-b),x
, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^3-x^4+b\right)\left(ax^3+4\,b\right)}{\left(ax^3-2\,x^4+b\right)\left(ax^3+b\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+4*b)*(-a*x^3+x^4-b)/x^4/(a*x^3+b)^(1/4)/(-a*x^3+2*x^4-b),x
, algorithm="giac")
```

```
[Out] integrate((a*x^3 - x^4 + b)*(a*x^3 + 4*b)/((a*x^3 - 2*x^4 + b)*(a*x^3 + b)^
(1/4)*x^4), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^3+4b\right)\left(-a\,x^3+x^4-b\right)}{x^4\left(a\,x^3+b\right)^{\frac{1}{4}}\left(-a\,x^3+2x^4-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+4*b)*(-a*x^3+x^4-b)/x^4/(a*x^3+b)^(1/4)/(-a*x^3+2*x^4-b),x)
```

```
[Out] int((a*x^3+4*b)*(-a*x^3+x^4-b)/x^4/(a*x^3+b)^(1/4)/(-a*x^3+2*x^4-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^3-x^4+b\right)\left(ax^3+4\,b\right)}{\left(ax^3-2\,x^4+b\right)\left(ax^3+b\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+4*b)*(-a*x^3+x^4-b)/x^4/(a*x^3+b)^(1/4)/(-a*x^3+2*x^4-b),x
, algorithm="maxima")
```

```
[Out] integrate((a*x^3 - x^4 + b)*(a*x^3 + 4*b)/((a*x^3 - 2*x^4 + b)*(a*x^3 + b)^
(1/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^3+4\,b\right)\,\left(-x^4+a\,x^3+b\right)}{x^4\,{\left(a\,x^3+b\right)}^{1/4}\,\left(-2\,x^4+a\,x^3+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((4*b + a*x^3)*(b + a*x^3 - x^4))/(x^4*(b + a*x^3)^(1/4)*(b + a*x^3 - 2
*x^4)),x)
```

```
[Out] int(((4*b + a*x^3)*(b + a*x^3 - x^4))/(x^4*(b + a*x^3)^(1/4)*(b + a*x^3 - 2
*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+4*b)*(-a*x**3+x**4-b)/x**4/(a*x**3+b)**(1/4)/(-a*x**3+2*x
**4-b),x)
```

```
[Out] Timed out
```

3.879 $\int \frac{1-x}{\sqrt{3+2x-5x^2-4x^3+x^4+2x^5+x^6}} \, dx$

Optimal. Leaf size=72

$$-\sqrt{2}\tanh^{-1}\left(\frac{(x+1)\left(\sqrt{2}x-\sqrt{2}\right)}{x^3+x^2-\sqrt{x^6+2x^5+x^4-4x^3-5x^2+2x+3}-x-1}\right)$$

Rubi [A] time = 0.19, antiderivative size = 67, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {6688, 6719, 1033, 724, 206, 688, 207}

$$-\frac{\left(1-x^2\right)\sqrt{x^2+2x+3}\tanh^{-1}\left(\frac{\sqrt{x^2+2x+3}}{\sqrt{2}}\right)}{\sqrt{2}\sqrt{\left(1-x^2\right)^2\left(x^2+2x+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/Sqrt[3 + 2*x - 5*x^2 - 4*x^3 + x^4 + 2*x^5 + x^6],x]
```

```
[Out] -(((1 - x^2)*Sqrt[3 + 2*x + x^2]*ArcTanh[Sqrt[3 + 2*x + x^2]/Sqrt[2]])/(Sqr
t[2]*Sqrt[(1 - x^2)^2*(3 + 2*x + x^2)]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x}{\sqrt{3+2x-5x^2-4x^3+x^4+2x^5+x^6}}\,dx &= \int \frac{1-x}{\sqrt{\left(-1+x^2\right)^2\left(3+2x+x^2\right)}}\,dx \\
&= \frac{\left(\left(-1+x^2\right)\sqrt{3+2x+x^2}\right)\int \frac{1-x}{\left(-1+x^2\right)\sqrt{3+2x+x^2}}\,dx}{\sqrt{\left(-1+x^2\right)^2\left(3+2x+x^2\right)}} \\
&= -\frac{\left(\left(-1+x^2\right)\sqrt{3+2x+x^2}\right)\int \frac{1}{(1+x)\sqrt{3+2x+x^2}}\,dx}{\sqrt{\left(-1+x^2\right)^2\left(3+2x+x^2\right)}} \\
&= -\frac{\left(4\left(-1+x^2\right)\sqrt{3+2x+x^2}\right)\mathrm{Subst}\left(\int \frac{1}{-8+4x^2}\,dx, x, \sqrt{3+2x+\right.}{\sqrt{\left(-1+x^2\right)^2\left(3+2x+x^2\right)}} \\
&= -\frac{\left(1-x^2\right)\sqrt{3+2x+x^2}\,\tanh^{-1}\left(\frac{\sqrt{3+2x+x^2}}{\sqrt{2}}\right)}{\sqrt{2}\sqrt{\left(1-x^2\right)^2\left(3+2x+x^2\right)}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 1.00

$$-\frac{\left(x^2-1\right)\sqrt{x^2+2x+3}\left(\log(x+1)-\log\left(\sqrt{2}\sqrt{x^2+2x+3}+2\right)\right)}{\sqrt{2}\sqrt{\left(x^2-1\right)^2\left(x^2+2x+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/Sqrt[3 + 2*x - 5*x^2 - 4*x^3 + x^4 + 2*x^5 + x^6],x]
```

```
[Out] -(((-1 + x^2)*Sqrt[3 + 2*x + x^2]*(Log[1 + x] - Log[2 + Sqrt[2]*Sqrt[3 + 2*
x + x^2]]))/(Sqrt[2]*Sqrt[(-1 + x^2)^2*(3 + 2*x + x^2)]))
```

IntegrateAlgebraic [A] time = 0.32, size = 72, normalized size = 1.00

$$-\sqrt{2}\tanh^{-1}\left(\frac{(x+1)\left(\sqrt{2}x-\sqrt{2}\right)}{x^3+x^2-\sqrt{x^6+2x^5+x^4-4x^3-5x^2+2x+3}-x-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)/Sqrt[3 + 2*x - 5*x^2 - 4*x^3 + x^4 + 2*x^5 + x^6
],x]
```

```
[Out] -(Sqrt[2]*ArcTanh[((1 + x)*(-Sqrt[2] + Sqrt[2]*x))/(-1 - x + x^2 + x^3 - Sq
rt[3 + 2*x - 5*x^2 - 4*x^3 + x^4 + 2*x^5 + x^6])])
```

fricas [A] time = 0.67, size = 58, normalized size = 0.81

$$\frac{1}{2}\,\sqrt{2}\,\log\left(\frac{\sqrt{2}\left(x^{2}-1\right)+\sqrt{x^{6}+2\,x^{5}+x^{4}-4\,x^{3}-5\,x^{2}+2\,x+3}}{x^{3}+x^{2}-x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^6+2*x^5+x^4-4*x^3-5*x^2+2*x+3)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*sqrt(2)*log((sqrt(2)*(x^2 - 1) + sqrt(x^6 + 2*x^5 + x^4 - 4*x^3 - 5*x^2
 + 2*x + 3))/(x^3 + x^2 - x - 1))
```

giac [A] time = 0.53, size = 90, normalized size = 1.25

$$-\frac{\sqrt{2}\log\left(-\frac{\left|-2\sqrt{3}-2\sqrt{2}+2\sqrt{\frac{2}{x}+\frac{3}{x^{2}}+1}-\frac{2\sqrt{3}}{x}\right|}{2\left(\sqrt{3}-\sqrt{2}-\sqrt{\frac{2}{x}+\frac{3}{x^{2}}+1}+\frac{\sqrt{3}}{x}\right)}\right)}{2\,\mathrm{sgn}\left(-\frac{1}{x^{3}}+\frac{1}{x^{5}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^6+2*x^5+x^4-4*x^3-5*x^2+2*x+3)^(1/2),x, algorithm="giac"
)
```

```
[Out] -1/2*sqrt(2)*log(-1/2*abs(-2*sqrt(3) - 2*sqrt(2) + 2*sqrt(2/x + 3/x^2 + 1)
- 2*sqrt(3)/x)/(sqrt(3) - sqrt(2) - sqrt(2/x + 3/x^2 + 1) + sqrt(3)/x))/sgn
(-1/x^3 + 1/x^5)
```

maple [A] time = 0.01, size = 64, normalized size = 0.89

$$\frac{\left(x^{2}-1\right)\sqrt{x^{2}+2x+3}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{x^{2}+2x+3}}\right)}{2\sqrt{x^{6}+2x^{5}+x^{4}-4x^{3}-5x^{2}+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(x^6+2*x^5+x^4-4*x^3-5*x^2+2*x+3)^(1/2),x)
```

```
[Out] 1/2/(x^6+2*x^5+x^4-4*x^3-5*x^2+2*x+3)^(1/2)*(x^2-1)*(x^2+2*x+3)^(1/2)*2^(1/
2)*arctanh(2^(1/2)/(x^2+2*x+3)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x-1}{\sqrt{x^{6}+2\,x^{5}+x^{4}-4\,x^{3}-5\,x^{2}+2\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^6+2*x^5+x^4-4*x^3-5*x^2+2*x+3)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate((x - 1)/sqrt(x^6 + 2*x^5 + x^4 - 4*x^3 - 5*x^2 + 2*x + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x-1}{\sqrt{x^{6}+2\,x^{5}+x^{4}-4\,x^{3}-5\,x^{2}+2\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/(2*x - 5*x^2 - 4*x^3 + x^4 + 2*x^5 + x^6 + 3)^(1/2),x)
[Out] int(-(x - 1)/(2*x - 5*x^2 - 4*x^3 + x^4 + 2*x^5 + x^6 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{x^6+2x^5+x^4-4x^3-5x^2+2x+3}}\,dx-\int \left(-\frac{1}{\sqrt{x^6+2x^5+x^4-4x^3-5x^2+2x+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x**6+2*x**5+x**4-4*x**3-5*x**2+2*x+3)**(1/2),x)
[Out] -Integral(x/sqrt(x**6 + 2*x**5 + x**4 - 4*x**3 - 5*x**2 + 2*x + 3), x) - In
tegral(-1/sqrt(x**6 + 2*x**5 + x**4 - 4*x**3 - 5*x**2 + 2*x + 3), x)
```

3.880 $\int \frac{(-1+3x^4)\sqrt{1+x^2+2x^4+x^8}}{(1-x+x^4)^2(1+x+x^4)}\,dx$

Optimal. Leaf size=72

$$-\frac{\sqrt{x^8+2x^4+x^2+1}}{2\left(x^4-x+1\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{x^4+\sqrt{x^8+2x^4+x^2+1}+x+1}\right)}{\sqrt{2}}$$

Rubi [F] time = 2.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+3x^4)\sqrt{1+x^2+2x^4+x^8}}{(1-x+x^4)^2(1+x+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + 3*x^4)*Sqrt[1 + x^2 + 2*x^4 + x^8])/((1 - x + x^4)^2*(1 + x + x^
4)),x]
```

```
[Out] Defer[Int][Sqrt[1 + x^2 + 2*x^4 + x^8]/(-1 - x - x^4), x]/4 + Defer[Int][Sq
rt[1 + x^2 + 2*x^4 + x^8]/(-1 + x - x^4), x]/4 - Defer[Int][Sqrt[1 + x^2 +
2*x^4 + x^8]/(1 - x + x^4)^2, x]/2 + 2*Defer[Int][(x^3*Sqrt[1 + x^2 + 2*x^4
 + x^8])/(1 - x + x^4)^2, x] - Defer[Int][(x^2*Sqrt[1 + x^2 + 2*x^4 + x^8])
/(1 - x + x^4), x] + Defer[Int][(x^3*Sqrt[1 + x^2 + 2*x^4 + x^8])/(1 - x +
x^4), x]/4 + Defer[Int][(x^2*Sqrt[1 + x^2 + 2*x^4 + x^8])/(1 + x + x^4), x]
 - Defer[Int][(x^3*Sqrt[1 + x^2 + 2*x^4 + x^8])/(1 + x + x^4), x]/4
```

Rubi steps

$$\int \frac{(-1+3x^4)\sqrt{1+x^2+2x^4+x^8}}{(1-x+x^4)^2(1+x+x^4)}\,dx = \int \left(\frac{(-1+4x^3)\sqrt{1+x^2+2x^4+x^8}}{2(1-x+x^4)^2}+\frac{(-1-4x^2+x^3)\sqrt{1+x^2+2x^4+x^8}}{4(1-x+x^4)}\right.$$

$$=\frac{1}{4}\int \frac{(-1-4x^2+x^3)\sqrt{1+x^2+2x^4+x^8}}{1-x+x^4}\,dx+\frac{1}{4}\int \frac{(-1+4x^2-x^3)\sqrt{1}}{1+x+}$$

$$=\frac{1}{4}\int \left(\frac{\sqrt{1+x^2+2x^4+x^8}}{-1+x-x^4}-\frac{4x^2\sqrt{1+x^2+2x^4+x^8}}{1-x+x^4}+\frac{x^3\sqrt{1+x^2+2x^4}}{1-x+x^4}\right.$$

$$=\frac{1}{4}\int \frac{\sqrt{1+x^2+2x^4+x^8}}{-1-x-x^4}\,dx+\frac{1}{4}\int \frac{\sqrt{1+x^2+2x^4+x^8}}{-1+x-x^4}\,dx+\frac{1}{4}\int \frac{x^3\sqrt{1}}{}$$

Mathematica [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(-1+3x^4)\sqrt{1+x^2+2x^4+x^8}}{(1-x+x^4)^2(1+x+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + 3*x^4)*Sqrt[1 + x^2 + 2*x^4 + x^8])/((1 - x + x^4)^2*(1 +
x + x^4)),x]
```

```
[Out] Integrate[((-1 + 3*x^4)*Sqrt[1 + x^2 + 2*x^4 + x^8])/((1 - x + x^4)^2*(1 +
x + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.42, size = 72, normalized size = 1.00

$$-\frac{\sqrt{x^8+2x^4+x^2+1}}{2\left(x^4-x+1\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{x^4+\sqrt{x^8+2x^4+x^2+1}+x+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + 3*x^4)*Sqrt[1 + x^2 + 2*x^4 + x^8])/((1 - x + x^4
)^2*(1 + x + x^4)),x]
```

```
[Out] -1/2*Sqrt[1 + x^2 + 2*x^4 + x^8]/(1 - x + x^4) - ArcTanh[(Sqrt[2]*x)/(1 + x
 + x^4 + Sqrt[1 + x^2 + 2*x^4 + x^8])]/Sqrt[2]
```

fricas [A] time = 0.95, size = 120, normalized size = 1.67

$$\frac{\sqrt{2}\left(x^4-x+1\right)\log\left(\frac{3x^8-2x^5+6x^4+2\sqrt{2}\sqrt{x^8+2x^4+x^2+1}\left(x^4-x+1\right)+3x^2-2x+3}{x^8+2x^5+2x^4+x^2+2x+1}\right)-4\sqrt{x^8+2x^4+x^2+1}}{8\left(x^4-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)*(x^8+2*x^4+x^2+1)^(1/2)/(x^4-x+1)^2/(x^4+x+1),x, algori
thm="fricas")
```

```
[Out] 1/8*(sqrt(2)*(x^4 - x + 1)*log((3*x^8 - 2*x^5 + 6*x^4 + 2*sqrt(2)*sqrt(x^8
+ 2*x^4 + x^2 + 1)*(x^4 - x + 1) + 3*x^2 - 2*x + 3)/(x^8 + 2*x^5 + 2*x^4 +
x^2 + 2*x + 1)) - 4*sqrt(x^8 + 2*x^4 + x^2 + 1))/(x^4 - x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^8+2x^4+x^2+1}\left(3x^4-1\right)}{\left(x^4+x+1\right)\left(x^4-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)*(x^8+2*x^4+x^2+1)^(1/2)/(x^4-x+1)^2/(x^4+x+1),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(x^8 + 2*x^4 + x^2 + 1)*(3*x^4 - 1)/((x^4 + x + 1)*(x^4 - x +
 1)^2), x)
```

maple [C] time = 0.43, size = 91, normalized size = 1.26

$$-\frac{\sqrt{x^8+2x^4+x^2+1}}{2\left(x^4-x+1\right)}+\frac{\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2-2\right)x^4-\mathrm{RootOf}\left(\_Z^2-2\right)x+\mathrm{RootOf}\left(\_Z^2-2\right)+2\sqrt{x^8+2x^4+x^2+1}}{x^4+x+1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4-1)*(x^8+2*x^4+x^2+1)^(1/2)/(x^4-x+1)^2/(x^4+x+1),x)
```

```
[Out] -1/2*(x^8+2*x^4+x^2+1)^(1/2)/(x^4-x+1)+1/4*RootOf(_Z^2-2)*ln(-(RootOf(_Z^2-
2)*x^4-RootOf(_Z^2-2)*x+RootOf(_Z^2-2)+2*(x^8+2*x^4+x^2+1)^(1/2))/(x^4+x+1)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^8+2x^4+x^2+1}\left(3x^4-1\right)}{\left(x^4+x+1\right)\left(x^4-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)*(x^8+2*x^4+x^2+1)^(1/2)/(x^4-x+1)^2/(x^4+x+1),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(x^8 + 2*x^4 + x^2 + 1)*(3*x^4 - 1)/((x^4 + x + 1)*(x^4 - x +
 1)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^4-1\right)\,\sqrt{x^8+2\,x^4+x^2+1}}{{\left(x^4-x+1\right)}^2\,\left(x^4+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^4 - 1)*(x^2 + 2*x^4 + x^8 + 1)^(1/2))/((x^4 - x + 1)^2*(x + x^4 +
 1)),x)
```

```
[Out] int(((3*x^4 - 1)*(x^2 + 2*x^4 + x^8 + 1)^(1/2))/((x^4 - x + 1)^2*(x + x^4 +
 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4-1)*(x**8+2*x**4+x**2+1)**(1/2)/(x**4-x+1)**2/(x**4+x+1),x
)
```

```
[Out] Timed out
```

3.881 $\int \sqrt{c+bx+ax^2}\,dx$

Optimal. Leaf size=73

$$\frac{\left(b^2-4ac\right)\log\left(-2\sqrt{a}\,\sqrt{ax^2+bx+c}+2ax+b\right)}{8a^{3/2}}+\frac{(2ax+b)\sqrt{ax^2+bx+c}}{4a}$$

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 621, 206}

$$\frac{(2ax+b)\sqrt{ax^2+bx+c}}{4a}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\,\sqrt{ax^2+bx+c}}\right)}{8a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + b*x + a*x^2],x]
```

```
[Out] ((b + 2*a*x)*Sqrt[c + b*x + a*x^2])/(4*a) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*a
*x)/(2*Sqrt[a]*Sqrt[c + b*x + a*x^2])])/(8*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{c+bx+ax^2}\,dx &= \frac{(b+2ax)\sqrt{c+bx+ax^2}}{4a}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{c+bx+ax^2}}\,dx}{8a}\\
&= \frac{(b+2ax)\sqrt{c+bx+ax^2}}{4a}-\frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{b+2ax}{\sqrt{c+bx+ax^2}}\right)}{4a}\\
&= \frac{(b+2ax)\sqrt{c+bx+ax^2}}{4a}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2ax}{2\sqrt{a}\,\sqrt{c+bx+ax^2}}\right)}{8a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 71, normalized size = 0.97

$$\frac{(2ax+b)\sqrt{x(ax+b)+c}}{4a}-\frac{\left(b^2-4ac\right)\log\left(2\sqrt{a}\,\sqrt{x(ax+b)+c}+2ax+b\right)}{8a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + b*x + a*x^2],x]
```

```
[Out] ((b + 2*a*x)*Sqrt[c + x*(b + a*x)])/(4*a) - ((b^2 - 4*a*c)*Log[b + 2*a*x +
2*Sqrt[a]*Sqrt[c + x*(b + a*x)]])/(8*a^(3/2))
```

IntegrateAlgebraic [A] time = 0.21, size = 77, normalized size = 1.05

$$\frac{\left(b^2-4ac\right)\log\left(-2a^{3/2}\sqrt{ax^2+bx+c}+2a^2x+ab\right)}{8a^{3/2}}+\frac{(2ax+b)\sqrt{ax^2+bx+c}}{4a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + b*x + a*x^2],x]
```

```
[Out] ((b + 2*a*x)*Sqrt[c + b*x + a*x^2])/(4*a) + ((b^2 - 4*a*c)*Log[a*b + 2*a^2*
x - 2*a^(3/2)*Sqrt[c + b*x + a*x^2]])/(8*a^(3/2))
```

fricas [A] time = 0.66, size = 177, normalized size = 2.42

$$\left[-\frac{\left(b^2-4\,ac\right)\sqrt{a}\,\log\left(-8\,a^2x^2-8\,abx-4\,\sqrt{ax^2+bx+c}\,(2\,ax+b)\sqrt{a}-b^2-4\,ac\right)-4\left(2\,a^2x+ab\right)\sqrt{ax^2+bx+c}}{16\,a^2},\ \frac{\left(b^2-4\,ac\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{ax^2+bx+c}\,(2\,ax+b)\sqrt{-a}}{2\left(a^2x^2+abx+ac\right)}\right)+2\left(2\,a^2x+ab\right)\sqrt{ax^2+bx+c}}{8\,a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((b^2 - 4*a*c)*sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x - 4*sqrt(a*x^2 + b*x
 + c)*(2*a*x + b)*sqrt(a) - b^2 - 4*a*c) - 4*(2*a^2*x + a*b)*sqrt(a*x^2 + b
*x + c))/a^2, 1/8*((b^2 - 4*a*c)*sqrt(-a)*arctan(1/2*sqrt(a*x^2 + b*x + c)*
(2*a*x + b)*sqrt(-a)/(a^2*x^2 + a*b*x + a*c)) + 2*(2*a^2*x + a*b)*sqrt(a*x^
2 + b*x + c))/a^2]
```

giac [A] time = 0.38, size = 68, normalized size = 0.93

$$\frac{1}{4}\sqrt{ax^2+bx+c}\left(2\,x+\frac{b}{a}\right)+\frac{\left(b^2-4\,ac\right)\log\left(\left|-2\left(\sqrt{a}\,x-\sqrt{ax^2+bx+c}\right)\sqrt{a}-b\right|\right)}{8\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(a*x^2 + b*x + c)*(2*x + b/a) + 1/8*(b^2 - 4*a*c)*log(abs(-2*(sqrt(
a)*x - sqrt(a*x^2 + b*x + c))*sqrt(a) - b))/a^(3/2)
```

maple [A] time = 0.00, size = 89, normalized size = 1.22

$$\frac{(2ax+b)\sqrt{a\,x^2+bx+c}}{4a}+\frac{\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx+c}\right)c}{2\sqrt{a}}-\frac{\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx+c}\right)b^2}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x+c)^(1/2),x)
```

```
[Out] 1/4*(2*a*x+b)*(a*x^2+b*x+c)^(1/2)/a+1/2/a^(1/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x
^2+b*x+c)^(1/2))*c-1/8/a^(3/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x+c)^(1/2))*
b^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [**B**] time = 0.12, size = 63, normalized size = 0.86

$$\left(\frac{x}{2}+\frac{b}{4\,a}\right)\,\sqrt{a\,x^2+b\,x+c}+\frac{\ln\left(\frac{\frac{b}{2}+a\,x}{\sqrt{a}}+\sqrt{a\,x^2+b\,x+c}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + b*x + a*x^2)^(1/2),x)
```

```
[Out] (x/2 + b/(4*a))*(c + b*x + a*x^2)^(1/2) + (log((b/2 + a*x)/a^(1/2) + (c + b
*x + a*x^2)^(1/2))*(a*c - b^2/4))/(2*a^(3/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^2+bx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**2 + b*x + c), x)
```

3.882 $\int \frac{-1+x}{\left(-1-2x+x^2\right)\sqrt{-x+x^3}}\,dx$

Optimal. Leaf size=73

$$\frac{1}{4}\left(\sqrt{2}-2\right)\tanh^{-1}\left(\frac{x-1}{\left(\sqrt{2}-1\right)\sqrt{x^3-x}}\right)+\frac{1}{4}\left(2+\sqrt{2}\right)\tanh^{-1}\left(\frac{x-1}{\left(1+\sqrt{2}\right)\sqrt{x^3-x}}\right)$$

Rubi [C] time = 0.51, antiderivative size = 103, normalized size of antiderivative = 1.41, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2056, 6728, 933, 168, 537}

$$\frac{\sqrt{x}\sqrt{1-x^2}\,\Pi\left(-\frac{1}{\sqrt{2}};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{2\sqrt{x^3-x}}-\frac{\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{\sqrt{2}};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{2\sqrt{x^3-x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/((-1 - 2*x + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] (Sqrt[x]*Sqrt[1 - x^2]*EllipticPi[-(1/Sqrt[2]), ArcSin[Sqrt[1 - x]], 1/2])/
(2*Sqrt[-x + x^3]) - (Sqrt[x]*Sqrt[1 - x^2]*EllipticPi[1/Sqrt[2], ArcSin[Sq
rt[1 - x]], 1/2])/(2*Sqrt[-x + x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-1+x}{\left(-1-2x+x^2\right)\sqrt{-x+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{-1+x}{\sqrt{x}\sqrt{-1+x^2}\left(-1-2x+x^2\right)}\,dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \left(\frac{1}{\sqrt{x}\left(-2-2\sqrt{2}+2x\right)\sqrt{-1+x^2}}+\frac{1}{\sqrt{x}\left(-2+2\sqrt{2}+2x\right)\sqrt{-1+x^2}}\right)dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{x}\left(-2-2\sqrt{2}+2x\right)\sqrt{-1+x^2}}\,dx}{\sqrt{-x+x^3}} + \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{x}\left(-2+2\sqrt{2}\cdots\right.}}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-x}\sqrt{x}\sqrt{1+x}\left(-2-2\sqrt{2}+2x\right)}\,dx}{\sqrt{-x+x^3}} + \frac{\left(\sqrt{x}\sqrt{1-x^2}\right)\int \frac{\cdots}{\sqrt{1-x}\sqrt{x}\sqrt{1+\cdots}}}{\sqrt{-x+x^3}}$$

$$= -\frac{\left(2\sqrt{x}\sqrt{1-x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(-2\sqrt{2}-2x^2\right)\sqrt{1-x^2}\sqrt{2-x^2}}\,dx, x, \sqrt{1-x}\right)}{\sqrt{-x+x^3}} - \frac{\left(2\sqrt{x}\sqrt{1\cdots\right.}}{\cdots}$$

$$= \frac{\sqrt{x}\sqrt{1-x^2}\,\Pi\left(-\frac{1}{\sqrt{2}};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{2\sqrt{-x+x^3}} - \frac{\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{\sqrt{2}};\sin^{-1}\left(\sqrt{1-\cdots}\right.\right.}{2\sqrt{-x+x^3}}$$

Mathematica [C] time = 0.80, size = 89, normalized size = 1.22

$$-\frac{\sqrt{1-\frac{1}{x^2}}\,x^{3/2}\left(2F\left(\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)\middle|-1\right)-\left(1+\sqrt{2}\right)\Pi\left(1-\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)\middle|-1\right)+\left(\sqrt{2}-1\right)\Pi\left(1+\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)\middle|-1\right)\right)}{\sqrt{x\left(x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/((-1 - 2*x + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] -((Sqrt[1 - x^(-2)]*x^(3/2)*(2*EllipticF[ArcSin[1/Sqrt[x]], -1] - (1 + Sqrt
[2])*EllipticPi[1 - Sqrt[2], ArcSin[1/Sqrt[x]], -1] + (-1 + Sqrt[2])*Ellipt
icPi[1 + Sqrt[2], ArcSin[1/Sqrt[x]], -1]))/Sqrt[x*(-1 + x^2)])
```

IntegrateAlgebraic [A] time = 0.61, size = 85, normalized size = 1.16

$$\frac{1}{4}\left(2+\sqrt{2}\right)\tanh^{-1}\left(\frac{\left(\sqrt{2}-1\right)x-\sqrt{2}+1}{\sqrt{x^3-x}}\right)+\frac{1}{4}\left(\sqrt{2}-2\right)\tanh^{-1}\left(\frac{\left(1+\sqrt{2}\right)x-\sqrt{2}-1}{\sqrt{x^3-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/((-1 - 2*x + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] ((2 + Sqrt[2])*ArcTanh[(1 - Sqrt[2] + (-1 + Sqrt[2])*x)/Sqrt[-x + x^3]])/4
 + ((-2 + Sqrt[2])*ArcTanh[(-1 - Sqrt[2] + (1 + Sqrt[2])*x)/Sqrt[-x + x^3]])
/4
```

fricas [B] time = 0.69, size = 126, normalized size = 1.73

$$\frac{1}{8}\sqrt{2}\log\left(\frac{x^4+12x^3+4\sqrt{2}\sqrt{x^3-x}\left(x^2+2x-1\right)+2x^2-12x+1}{x^4-4x^3+2x^2+4x+1}\right)+\frac{1}{4}\log\left(\frac{x^4+4x^3+2x^2-4\sqrt{x^3-x}\left(x^2+1\right)-4x+1}{x^4-4x^3+2x^2+4x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-2*x-1)/(x^3-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log((x^4 + 12*x^3 + 4*sqrt(2)*sqrt(x^3 - x)*(x^2 + 2*x - 1) + 2
*x^2 - 12*x + 1)/(x^4 - 4*x^3 + 2*x^2 + 4*x + 1)) + 1/4*log((x^4 + 4*x^3 +
2*x^2 - 4*sqrt(x^3 - x)*(x^2 + 1) - 4*x + 1)/(x^4 - 4*x^3 + 2*x^2 + 4*x + 1
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x^3-x}\left(x^2-2x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-2*x-1)/(x^3-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/(sqrt(x^3 - x)*(x^2 - 2*x - 1)), x)
```

maple [C] time = 0.03, size = 116, normalized size = 1.59

$$\frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\;\mathrm{EllipticPi}\left(\sqrt{1+x},-\frac{1}{-2-\sqrt{2}},\frac{\sqrt{2}}{2}\right)}{2\sqrt{x^3-x}\,\left(-2-\sqrt{2}\right)}+\frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\;\mathrm{EllipticPi}\left(\sqrt{1+x},-\frac{1}{-2+\sqrt{2}},\frac{\sqrt{2}}{2}\right)}{2\sqrt{x^3-x}\,\left(-2+\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(x^2-2*x-1)/(x^3-x)^(1/2),x)
```

```
[Out] 1/2*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)/(-2-2^(1/2))*Ellipti
cPi((1+x)^(1/2),-1/(-2-2^(1/2)),1/2*2^(1/2))+1/2*(1+x)^(1/2)*(2-2*x)^(1/2)*
(-x)^(1/2)/(x^3-x)^(1/2)/(-2+2^(1/2))*EllipticPi((1+x)^(1/2),-1/(-2+2^(1/2)
),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x^3-x}\left(x^2-2x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-2*x-1)/(x^3-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/(sqrt(x^3 - x)*(x^2 - 2*x - 1)), x)
```

mupad [B] time = 0.11, size = 102, normalized size = 1.40

$$\frac{\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\;\Pi\left(-\frac{1}{\sqrt{2}+1};\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{\sqrt{x^3-x}\,\left(\sqrt{2}+1\right)}-\frac{\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\;\Pi\left(\frac{1}{\sqrt{2}-1};\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{\sqrt{x^3-x}\,\left(\sqrt{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((x^3 - x)^(1/2)*(2*x - x^2 + 1)),x)
```

```
[Out] ((-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticPi(-1/(2^(1/2) + 1), asin((
-x)^(1/2)), -1))/((x^3 - x)^(1/2)*(2^(1/2) + 1)) - ((-x)^(1/2)*(1 - x)^(1/2
)*(x + 1)^(1/2)*ellipticPi(1/(2^(1/2) - 1), asin((-x)^(1/2)), -1))/((x^3 -
x)^(1/2)*(2^(1/2) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x\left(x-1\right)\left(x+1\right)}\,\left(x^2-2x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**2-2*x-1)/(x**3-x)**(1/2),x)
```

```
[Out] Integral((x - 1)/(sqrt(x*(x - 1)*(x + 1))*(x**2 - 2*x - 1)), x)
```

3.883 $\int \frac{-2+3x+x^2}{(-1-2x+x^2)\sqrt{-x+x^3}}\,dx$

Optimal. Leaf size=73

$$\frac{1}{4}\left(\sqrt{2}-6\right)\tanh^{-1}\left(\frac{x-1}{\left(\sqrt{2}-1\right)\sqrt{x^3-x}}\right)+\frac{1}{4}\left(6+\sqrt{2}\right)\tanh^{-1}\left(\frac{x-1}{\left(1+\sqrt{2}\right)\sqrt{x^3-x}}\right)$$

Rubi [C] time = 0.82, antiderivative size = 180, normalized size of antiderivative = 2.47, number of steps used = 13, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2056, 6728, 329, 222, 933, 168, 537}

$$\frac{\sqrt{2}\sqrt{x-1}\sqrt{x}\sqrt{x+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{x-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^3-x}}+\frac{\left(5+2\sqrt{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(-\frac{1}{\sqrt{2}};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{2\sqrt{x^3-x}}-\frac{\left(5-2\sqrt{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{\sqrt{2}};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{2\sqrt{x^3-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-2 + 3*x + x^2)/((-1 - 2*x + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] (Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]*Sqrt[1 + x]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[x]
)/Sqrt[-1 + x]], 1/2])/Sqrt[-x + x^3] + ((5 + 2*Sqrt[2])*Sqrt[x]*Sqrt[1 - x
^2]*EllipticPi[-(1/Sqrt[2]), ArcSin[Sqrt[1 - x]], 1/2])/(2*Sqrt[-x + x^3])
- ((5 - 2*Sqrt[2])*Sqrt[x]*Sqrt[1 - x^2]*EllipticPi[1/Sqrt[2], ArcSin[Sqrt[
1 - x]], 1/2])/(2*Sqrt[-x + x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
```

```
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-2+3x+x^2}{\left(-1-2x+x^2\right)\sqrt{-x+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{-2+3x+x^2}{\sqrt{x}\sqrt{-1+x^2}\left(-1-2x+x^2\right)}\,dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{-1+x^2}}-\frac{1-5x}{\sqrt{x}\sqrt{-1+x^2}\left(-1-2x+x^2\right)}\right)dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-1+x^2}}\,dx}{\sqrt{-x+x^3}} - \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1-5x}{\sqrt{x}\sqrt{-1+x^2}\left(-1-2x+x^2\right)}\,dx}{\sqrt{-x+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \left(\frac{-5-2\sqrt{2}}{\sqrt{x}\left(-2-2\sqrt{2}+2x\right)\sqrt{-1+x^2}}+\frac{-5+2\sqrt{2}}{\sqrt{x}\left(-2+2\sqrt{2}+2x\right)\sqrt{-1+x^2}}\right)dx}{\sqrt{-x+x^3}} + \cdots$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} - \frac{\left(\left(-5-2\sqrt{2}\right)\sqrt{x}\sqrt{-1+x^2}\right.\cdots}{\sqrt{-x\cdots}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} - \frac{\left(\left(-5-2\sqrt{2}\right)\sqrt{x}\sqrt{1-x^2}\right)\cdots}{\sqrt{-x\cdots}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} + \frac{\left(2\left(-5-2\sqrt{2}\right)\sqrt{x}\sqrt{1-x^2}\right.\cdots}{\cdots}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} + \frac{\left(5+2\sqrt{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\cdots\right.}{2\sqrt{-x+\cdots}}$$

Mathematica [C] time = 0.66, size = 93, normalized size = 1.27

$$-\frac{\sqrt{1-\frac{1}{x^2}}\,x^{3/2}\left(4F\left(\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)\middle|-1\right)-\left(1+3\sqrt{2}\right)\Pi\left(1-\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)\middle|-1\right)+\left(3\sqrt{2}-1\right)\Pi\left(1+\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)\middle|-1\right)\right)}{\sqrt{x\left(x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x + x^2)/((-1 - 2*x + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] -((Sqrt[1 - x^(-2)]*x^(3/2)*(4*EllipticF[ArcSin[1/Sqrt[x]], -1] - (1 + 3*Sq
rt[2])*EllipticPi[1 - Sqrt[2], ArcSin[1/Sqrt[x]], -1] + (-1 + 3*Sqrt[2])*El
lipticPi[1 + Sqrt[2], ArcSin[1/Sqrt[x]], -1]))/Sqrt[x*(-1 + x^2)])
```

IntegrateAlgebraic [A] time = 0.63, size = 85, normalized size = 1.16

$$\frac{1}{4}\left(6+\sqrt{2}\right)\tanh^{-1}\left(\frac{\left(\sqrt{2}-1\right)x-\sqrt{2}+1}{\sqrt{x^3-x}}\right)+\frac{1}{4}\left(\sqrt{2}-6\right)\tanh^{-1}\left(\frac{\left(1+\sqrt{2}\right)x-\sqrt{2}-1}{\sqrt{x^3-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + 3*x + x^2)/((-1 - 2*x + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] ((6 + Sqrt[2])*ArcTanh[(1 - Sqrt[2] + (-1 + Sqrt[2])*x)/Sqrt[-x + x^3]])/4
+ ((-6 + Sqrt[2])*ArcTanh[(-1 - Sqrt[2] + (1 + Sqrt[2])*x)/Sqrt[-x + x^3]])
/4
```

fricas [B] time = 0.70, size = 126, normalized size = 1.73

$$\frac{1}{8}\sqrt{2}\log\left(\frac{x^4+12x^3+4\sqrt{2}\sqrt{x^3-x}\left(x^2+2x-1\right)+2x^2-12x+1}{x^4-4x^3+2x^2+4x+1}\right)+\frac{3}{4}\log\left(\frac{x^4+4x^3+2x^2-4\sqrt{x^3-x}\left(x^2+1\right)-4x+1}{x^4-4x^3+2x^2+4x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-2)/(x^2-2*x-1)/(x^3-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log((x^4 + 12*x^3 + 4*sqrt(2)*sqrt(x^3 - x)*(x^2 + 2*x - 1) + 2
*x^2 - 12*x + 1)/(x^4 - 4*x^3 + 2*x^2 + 4*x + 1)) + 3/4*log((x^4 + 4*x^3 +
2*x^2 - 4*sqrt(x^3 - x)*(x^2 + 1) - 4*x + 1)/(x^4 - 4*x^3 + 2*x^2 + 4*x + 1
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+3x-2}{\sqrt{x^3-x}\left(x^2-2x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-2)/(x^2-2*x-1)/(x^3-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 3*x - 2)/(sqrt(x^3 - x)*(x^2 - 2*x - 1)), x)
```

maple [C] time = 0.03, size = 273, normalized size = 3.74

$$\frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\sqrt{1+x},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-x}}+\frac{\sqrt{2}\,\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\operatorname{EllipticPi}\left(\sqrt{1+x},-\frac{1}{-2-\sqrt{2}},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-x}\left(-2-\sqrt{2}\right)}+\frac{5\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\operatorname{EllipticPi}\left(\sqrt{1+x},-\frac{1}{-2-\sqrt{2}},\frac{\sqrt{2}}{2}\right)}{2\sqrt{x^3-x}\left(-2-\sqrt{2}\right)}-\frac{\sqrt{2}\,\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\operatorname{EllipticPi}\left(\sqrt{1+x},-\frac{1}{-2+\sqrt{2}},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-x}\left(-2+\sqrt{2}\right)}+\frac{5\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\operatorname{EllipticPi}\left(\sqrt{1+x},-\frac{1}{-2+\sqrt{2}},\frac{\sqrt{2}}{2}\right)}{2\sqrt{x^3-x}\left(-2+\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x-2)/(x^2-2*x-1)/(x^3-x)^(1/2),x)
```

```
[Out] (1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticF((1+x)^(1/2),1/
2*2^(1/2))+2^(1/2)*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)/(-2-2
^(1/2))*EllipticPi((1+x)^(1/2),-1/(-2-2^(1/2)),1/2*2^(1/2))+5/2*(1+x)^(1/2)
*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)/(-2-2^(1/2))*EllipticPi((1+x)^(1/2)
,-1/(-2-2^(1/2)),1/2*2^(1/2))-2^(1/2)*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/
(x^3-x)^(1/2)/(-2+2^(1/2))*EllipticPi((1+x)^(1/2),-1/(-2+2^(1/2)),1/2*2^(1/
2))+5/2*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)/(-2+2^(1/2))*Ell
ipticPi((1+x)^(1/2),-1/(-2+2^(1/2)),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+3x-2}{\sqrt{x^3-x}\left(x^2-2x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-2)/(x^2-2*x-1)/(x^3-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 3*x - 2)/(sqrt(x^3 - x)*(x^2 - 2*x - 1)), x)
```

mupad [B] time = 0.07, size = 159, normalized size = 2.18

$$\frac{\sqrt{2}\,\sqrt{-x}\,\left(5\,\sqrt{2}+4\right)\,\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(-\frac{1}{\sqrt{2}+1};\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{2\,\sqrt{x^3-x}\,\left(\sqrt{2}+1\right)}-\frac{\sqrt{2}\,\sqrt{-x}\,\left(5\,\sqrt{2}-4\right)\,\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(\frac{1}{\sqrt{2}-1};\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{2\,\sqrt{x^3-x}\,\left(\sqrt{2}-1\right)}-\frac{2\,\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{\sqrt{x^3-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x + x^2 - 2)/((x^3 - x)^(1/2)*(2*x - x^2 + 1)),x)
```

```
[Out] (2^(1/2)*(-x)^(1/2)*(5*2^(1/2) + 4)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticPi(
-1/(2^(1/2) + 1), asin((-x)^(1/2)), -1))/(2*(x^3 - x)^(1/2)*(2^(1/2) + 1))
- (2^(1/2)*(-x)^(1/2)*(5*2^(1/2) - 4)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticP
i(1/(2^(1/2) - 1), asin((-x)^(1/2)), -1))/(2*(x^3 - x)^(1/2)*(2^(1/2) - 1))
 - (2*(-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticF(asin((-x)^(1/2)), -1
))/(x^3 - x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+3x-2}{\sqrt{x\,(x-1)\,(x+1)}\,\left(x^2-2x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x-2)/(x**2-2*x-1)/(x**3-x)**(1/2),x)
```

```
[Out] Integral((x**2 + 3*x - 2)/(sqrt(x*(x - 1)*(x + 1))*(x**2 - 2*x - 1)), x)
```

3.884 $\int \frac{x(-3+x^2)}{(-1+x^2)^{2/3}(1-x^2+x^3)}\,dx$

Optimal. Leaf size=73

$$\log\left(\sqrt[3]{x^2-1}-x\right)-\frac{1}{2}\log\left(x^2+\sqrt[3]{x^2-1}\,x+\left(x^2-1\right)^{2/3}\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^2-1}+x}\right)$$

Rubi [F] time = 0.91, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\left(-3+x^2\right)}{\left(-1+x^2\right)^{2/3}\left(1-x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-3 + x^2))/((-1 + x^2)^(2/3)*(1 - x^2 + x^3)),x]
```

```
[Out] (3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + (-1 + x^2)^(1/3))*Sqrt[(1 - (-1 + x^2)^(1/3
) + (-1 + x^2)^(2/3))/(1 + Sqrt[3] + (-1 + x^2)^(1/3))^2]*EllipticF[ArcSin[
(1 - Sqrt[3] + (-1 + x^2)^(1/3))/(1 + Sqrt[3] + (-1 + x^2)^(1/3))], -7 - 4*
Sqrt[3]])/(x*Sqrt[(1 + (-1 + x^2)^(1/3))/(1 + Sqrt[3] + (-1 + x^2)^(1/3))^2
]) - Defer[Int][1/((-1 + x^2)^(2/3)*(1 - x^2 + x^3)), x] - 3*Defer[Int][x/(
(-1 + x^2)^(2/3)*(1 - x^2 + x^3)), x] + Defer[Int][x^2/((-1 + x^2)^(2/3)*(1
 - x^2 + x^3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(-3+x^2\right)}{\left(-1+x^2\right)^{2/3}\left(1-x^2+x^3\right)}\,dx &= \int \left(\frac{1}{\left(-1+x^2\right)^{2/3}}-\frac{1+3x-x^2}{\left(-1+x^2\right)^{2/3}\left(1-x^2+x^3\right)}\right)dx\\
&= \int \frac{1}{\left(-1+x^2\right)^{2/3}}\,dx-\int \frac{1+3x-x^2}{\left(-1+x^2\right)^{2/3}\left(1-x^2+x^3\right)}\,dx\\
&= \frac{\left(3\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^3}}\,dx,x,\sqrt[3]{-1+x^2}\right)}{2x}-\int \left(\frac{1}{\left(-1+x^2\right)^{2/3}\left(1-x^2+x^3\right)}+\right.\\
&= \frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(1+\sqrt[3]{-1+x^2}\right)\sqrt{\frac{1-\sqrt[3]{-1+x^2}+\left(-1+x^2\right)^{2/3}}{\left(1+\sqrt{3}+\sqrt[3]{-1+x^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+\sqrt[3]{-1+x^2}}{1+\sqrt{3}+\sqrt[3]{-1+x^2}}\right)\right.}{x\sqrt{\frac{1+\sqrt[3]{-1+x^2}}{\left(1+\sqrt{3}+\sqrt[3]{-1+x^2}\right)^2}}}
\end{aligned}$$

Mathematica [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x\left(-3+x^2\right)}{\left(-1+x^2\right)^{2/3}\left(1-x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(-3 + x^2))/((-1 + x^2)^(2/3)*(1 - x^2 + x^3)),x]
```

```
[Out] Integrate[(x*(-3 + x^2))/((-1 + x^2)^(2/3)*(1 - x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 1.76, size = 73, normalized size = 1.00

$$\log\left(\sqrt[3]{x^2-1}-x\right)-\frac{1}{2}\log\left(x^2+\sqrt[3]{x^2-1}\,x+\left(x^2-1\right)^{2/3}\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^2-1}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-3 + x^2))/((-1 + x^2)^(2/3)*(1 - x^2 + x^3)),x]
```

```
[Out] Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x^2)^(1/3))] + Log[-x + (-1 + x^2)^
(1/3)] - Log[x^2 + x*(-1 + x^2)^(1/3) + (-1 + x^2)^(2/3)]/2
```

fricas [A] time = 0.53, size = 74, normalized size = 1.01

$$-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,x+2\sqrt{3}\left(x^2-1\right)^{\frac{1}{3}}}{3x}\right)+\log\left(-\frac{x-\left(x^2-1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{2}\log\left(\frac{x^2+\left(x^2-1\right)^{\frac{1}{3}}x+\left(x^2-1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-3)/(x^2-1)^(2/3)/(x^3-x^2+1),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^2 - 1)^(1/3))/x) + log(-(x -
(x^2 - 1)^(1/3))/x) - 1/2*log((x^2 + (x^2 - 1)^(1/3)*x + (x^2 - 1)^(2/3))/x
^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2-3\right)x}{\left(x^3-x^2+1\right)\left(x^2-1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-3)/(x^2-1)^(2/3)/(x^3-x^2+1),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 3)*x/((x^3 - x^2 + 1)*(x^2 - 1)^(2/3)), x)
```

maple [C] time = 1.17, size = 227, normalized size = 3.11

$$\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3-3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2-1\right)^{\frac{1}{3}}x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3+3x\left(x^2-1\right)^{\frac{2}{3}}-3\left(x^2-1\right)^{\frac{1}{3}}x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2-x^2-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)+1}{x^3-x^2+1}\right)+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3-3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2-1\right)^{\frac{1}{3}}x^2+3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3+3x\left(x^2-1\right)^{\frac{2}{3}}-3\left(x^2-1\right)^{\frac{1}{3}}x^2+2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+x^3+x^2-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-1}{x^3-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2-3)/(x^2-1)^(2/3)/(x^3-x^2+1),x)
```

```
[Out] ln((-RootOf(_Z^2+_Z+1)^2*x^3-3*RootOf(_Z^2+_Z+1)*(x^2-1)^(1/3)*x^2+RootOf(_
Z^2+_Z+1)*x^3+3*x*(x^2-1)^(2/3)-3*(x^2-1)^(1/3)*x^2+RootOf(_Z^2+_Z+1)*x^2-x
^2-RootOf(_Z^2+_Z+1)+1)/(x^3-x^2+1))+RootOf(_Z^2+_Z+1)*ln((2*RootOf(_Z^2+_Z
+1)^2*x^3-3*RootOf(_Z^2+_Z+1)*(x^2-1)^(1/3)*x^2+3*RootOf(_Z^2+_Z+1)*x^3+3*x
*(x^2-1)^(2/3)-3*(x^2-1)^(1/3)*x^2+2*RootOf(_Z^2+_Z+1)*x^2+x^3+x^2-2*RootOf
(_Z^2+_Z+1)-1)/(x^3-x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2-3\right)x}{\left(x^3-x^2+1\right)\left(x^2-1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-3)/(x^2-1)^(2/3)/(x^3-x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 3)*x/((x^3 - x^2 + 1)*(x^2 - 1)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(x^2-3\right)}{\left(x^2-1\right)^{2/3}\left(x^3-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^2 - 3))/((x^2 - 1)^(2/3)*(x^3 - x^2 + 1)),x)
[Out] int((x*(x^2 - 3))/((x^2 - 1)^(2/3)*(x^3 - x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(x^2-3\right)}{\left(\left(x-1\right)\left(x+1\right)\right)^{\frac{2}{3}}\left(x^3-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2-3)/(x**2-1)**(2/3)/(x**3-x**2+1),x)
[Out] Integral(x*(x**2 - 3)/(((x - 1)*(x + 1))**(2/3)*(x**3 - x**2 + 1)), x)
```

3.885 $\int x^2 \sqrt[4]{-x^3+x^4}\, dx$

Optimal. Leaf size=73

$$\frac{77\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)}{1024}-\frac{77\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)}{1024}+\frac{\sqrt[4]{x^4-x^3}\,(384x^3-32x^2-44x-77)}{1536}$$

Rubi [B] time = 0.19, antiderivative size = 162, normalized size of antiderivative = 2.22, number of steps used = 10, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {2021, 2024, 2032, 63, 240, 212, 206, 203}

$$\frac{1}{4}\sqrt[4]{x^4-x^3}\,x^3-\frac{11}{384}\sqrt[4]{x^4-x^3}\,x-\frac{77\sqrt[4]{x^4-x^3}}{1536}-\frac{77(x-1)^{3/4}x^{9/4}\tan^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{1024\left(x^4-x^3\right)^{3/4}}-\frac{77(x-1)^{3/4}x^{9/4}\tanh^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{1024\left(x^4-x^3\right)^{3/4}}-\frac{1}{48}\sqrt[4]{x^4-x^3}\,x^2$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(-x^3 + x^4)^(1/4),x]
```

```
[Out] (-77*(-x^3 + x^4)^(1/4))/1536 - (11*x*(-x^3 + x^4)^(1/4))/384 - (x^2*(-x^3
+ x^4)^(1/4))/48 + (x^3*(-x^3 + x^4)^(1/4))/4 - (77*(-1 + x)^(3/4)*x^(9/4)*
ArcTan[(-1 + x)^(1/4)/x^(1/4)])/(1024*(-x^3 + x^4)^(3/4)) - (77*(-1 + x)^(3
/4)*x^(9/4)*ArcTanh[(-1 + x)^(1/4)/x^(1/4)])/(1024*(-x^3 + x^4)^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^2\sqrt[4]{-x^3+x^4}\,dx = \frac{1}{4}x^3\sqrt[4]{-x^3+x^4} - \frac{1}{16}\int \frac{x^5}{\left(-x^3+x^4\right)^{3/4}}\,dx$$

$$= -\frac{1}{48}x^2\sqrt[4]{-x^3+x^4} + \frac{1}{4}x^3\sqrt[4]{-x^3+x^4} - \frac{11}{192}\int \frac{x^4}{\left(-x^3+x^4\right)^{3/4}}\,dx$$

$$= -\frac{11}{384}x\sqrt[4]{-x^3+x^4} - \frac{1}{48}x^2\sqrt[4]{-x^3+x^4} + \frac{1}{4}x^3\sqrt[4]{-x^3+x^4} - \frac{77\int \frac{x^3}{\left(-x^3+x^4\right)^{3/4}}\,dx}{1536}$$

$$= -\frac{77\sqrt[4]{-x^3+x^4}}{1536} - \frac{11}{384}x\sqrt[4]{-x^3+x^4} - \frac{1}{48}x^2\sqrt[4]{-x^3+x^4} + \frac{1}{4}x^3\sqrt[4]{-x^3+x^4} - \frac{77\int \frac{x^2}{\left(-x^3+x^4\right)^{3/4}}\,d}{2048}$$

$$= -\frac{77\sqrt[4]{-x^3+x^4}}{1536} - \frac{11}{384}x\sqrt[4]{-x^3+x^4} - \frac{1}{48}x^2\sqrt[4]{-x^3+x^4} + \frac{1}{4}x^3\sqrt[4]{-x^3+x^4} - \frac{\left(77(-1+x)^{3/4}x\right.}{2048\left(-\right.}$$

$$= -\frac{77\sqrt[4]{-x^3+x^4}}{1536} - \frac{11}{384}x\sqrt[4]{-x^3+x^4} - \frac{1}{48}x^2\sqrt[4]{-x^3+x^4} + \frac{1}{4}x^3\sqrt[4]{-x^3+x^4} - \frac{\left(77(-1+x)^{3/4}x\right.}{}$$

$$= -\frac{77\sqrt[4]{-x^3+x^4}}{1536} - \frac{11}{384}x\sqrt[4]{-x^3+x^4} - \frac{1}{48}x^2\sqrt[4]{-x^3+x^4} + \frac{1}{4}x^3\sqrt[4]{-x^3+x^4} - \frac{\left(77(-1+x)^{3/4}x\right.}{5}$$

$$= -\frac{77\sqrt[4]{-x^3+x^4}}{1536} - \frac{11}{384}x\sqrt[4]{-x^3+x^4} - \frac{1}{48}x^2\sqrt[4]{-x^3+x^4} + \frac{1}{4}x^3\sqrt[4]{-x^3+x^4} - \frac{\left(77(-1+x)^{3/4}x\right.}{1}$$

$$= -\frac{77\sqrt[4]{-x^3+x^4}}{1536} - \frac{11}{384}x\sqrt[4]{-x^3+x^4} - \frac{1}{48}x^2\sqrt[4]{-x^3+x^4} + \frac{1}{4}x^3\sqrt[4]{-x^3+x^4} - \frac{77(-1+x)^{3/4}x^{9/}}{1024\left(-x\right.}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.48

$$\frac{4\left((x-1)x^3\right)^{5/4}\,{}_2F_1\left(-\frac{11}{4},\frac{5}{4};\frac{9}{4};1-x\right)}{5x^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(-x^3 + x^4)^(1/4),x]
```

```
[Out] (4*((-1 + x)*x^3)^(5/4)*Hypergeometric2F1[-11/4, 5/4, 9/4, 1 - x])/(5*x^(15
/4))
```

IntegrateAlgebraic [A] time = 0.31, size = 73, normalized size = 1.00

$$\frac{77\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)}{1024}-\frac{77\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)}{1024}+\frac{\sqrt[4]{x^4-x^3}\left(384x^3-32x^2-44x-77\right)}{1536}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(-x^3 + x^4)^(1/4),x]
```

```
[Out] ((-77 - 44*x - 32*x^2 + 384*x^3)*(-x^3 + x^4)^(1/4))/1536 + (77*ArcTan[x/(-
x^3 + x^4)^(1/4)])/1024 - (77*ArcTanh[x/(-x^3 + x^4)^(1/4)])/1024
```

fricas [A] time = 0.40, size = 90, normalized size = 1.23

$$\frac{1}{1536}\left(x^4-x^3\right)^{\frac{1}{4}}\left(384x^3-32x^2-44x-77\right)-\frac{77}{1024}\arctan\left(\frac{\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)-\frac{77}{2048}\log\left(\frac{x+\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{77}{2048}\log\left(-\frac{x-\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4-x^3)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/1536*(x^4 - x^3)^(1/4)*(384*x^3 - 32*x^2 - 44*x - 77) - 77/1024*arctan((x
^4 - x^3)^(1/4)/x) - 77/2048*log((x + (x^4 - x^3)^(1/4))/x) + 77/2048*log(-
(x - (x^4 - x^3)^(1/4))/x)
```

giac [A] time = 0.59, size = 106, normalized size = 1.45

$$\frac{1}{1536}\left(77\left(\frac{1}{x}-1\right)^3\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+275\left(\frac{1}{x}-1\right)^2\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-351\left(-\frac{1}{x}+1\right)^{\frac{5}{4}}-231\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)x^4+\frac{77}{1024}\arctan\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)+\frac{77}{2048}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right)-\frac{77}{2048}\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4-x^3)^(1/4),x, algorithm="giac")
```

```
[Out] 1/1536*(77*(1/x - 1)^3*(-1/x + 1)^(1/4) + 275*(1/x - 1)^2*(-1/x + 1)^(1/4)
- 351*(-1/x + 1)^(5/4) - 231*(-1/x + 1)^(1/4))*x^4 + 77/1024*arctan((-1/x +
 1)^(1/4)) + 77/2048*log((-1/x + 1)^(1/4) + 1) - 77/2048*log(abs((-1/x + 1)
^(1/4) - 1))
```

maple [C] time = 0.38, size = 407, normalized size = 5.58

$$\frac{\left(384x^3-32x^2-44x-77\right)\left(x^3(-1+x)\right)^{\frac{1}{4}}}{1536}+\frac{\left(-\frac{77\ln\left(\frac{2\left(x^4-3x^3+3x^2-x\right)^{\frac{3}{4}}+2\sqrt{x^4-3x^3+3x^2-x}\,x+2\left(x^4-3x^3+3x^2-x\right)^{\frac{1}{4}}x^2+2x^3-2\sqrt{x^4-3x^3+3x^2-x}-4\left(x^4-3x^3+3x^2-x\right)^{\frac{1}{4}}x-5x^2+2\left(x^4-3x^3+3x^2-x\right)^{\frac{1}{4}}+4x-1}{(-1+x)^2}\right)}{2048}+\frac{77\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\sqrt{x^4-3x^3+3x^2-x}\,\mathrm{RootOf}\left(\_Z^2+1\right)x-2\,\mathrm{RootOf}\left(\_Z^2+1\right)x^3+2\left(x^4-3x^3+3x^2-x\right)^{\frac{3}{4}}-2\sqrt{x^4-3x^3+3x^2-x}\,\mathrm{RootOf}\left(\_Z^2+1\right)-2\left(x^4-3x^3+3x^2-x\right)^{\frac{1}{4}}x^2+5\,\mathrm{RootOf}\left(\_Z^2+1\right)x^2+4\left(x^4-3x^3+3x^2-x\right)^{\frac{1}{4}}x-4\,\mathrm{RootOf}\left(\_Z^2+1\right)x-2\left(x^4-3x^3+3x^2-x\right)^{\frac{1}{4}}+\mathrm{RootOf}\left(\_Z^2+1\right)}{(-1+x)^2}\right)}{2048}\right)\left(x^3(-1+x)\right)^{\frac{1}{4}}\left(x(-1+x)^3\right)^{\frac{1}{4}}}{(-1+x)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4-x^3)^(1/4),x)
```

```
[Out] 1/1536*(384*x^3-32*x^2-44*x-77)*(x^3*(-1+x))^(1/4)+(-77/2048*ln((2*(x^4-3*x
^3+3*x^2-x)^(3/4)+2*(x^4-3*x^3+3*x^2-x)^(1/2)*x+2*(x^4-3*x^3+3*x^2-x)^(1/4)
```

```
*x^2+2*x^3-2*(x^4-3*x^3+3*x^2-x)^(1/2)-4*(x^4-3*x^3+3*x^2-x)^(1/4)*x-5*x^2+
2*(x^4-3*x^3+3*x^2-x)^(1/4)+4*x-1)/(-1+x)^2)+77/2048*RootOf(_Z^2+1)*ln((2*(
x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+1)*x-2*RootOf(_Z^2+1)*x^3+2*(x^4-3*x^3
+3*x^2-x)^(3/4)-2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+1)-2*(x^4-3*x^3+3*x
^2-x)^(1/4)*x^2+5*RootOf(_Z^2+1)*x^2+4*(x^4-3*x^3+3*x^2-x)^(1/4)*x-4*RootOf
(_Z^2+1)*x-2*(x^4-3*x^3+3*x^2-x)^(1/4)+RootOf(_Z^2+1))/(-1+x)^2))*(x^3*(-1+
x))^(1/4)/(-1+x)/x*(x*(-1+x)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4 - x^3\right)^{\frac{1}{4}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4-x^3)^(1/4),x, algorithm="maxima")
[Out] integrate((x^4 - x^3)^(1/4)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(x^4 - x^3\right)^{1/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4 - x^3)^(1/4),x)
[Out] int(x^2*(x^4 - x^3)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt[4]{x^3 (x-1)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**4-x**3)**(1/4),x)
[Out] Integral(x**2*(x**3*(x - 1))**(1/4), x)
```

3.886 $\int \frac{(3+x^4)\sqrt{x+x^4-x^5}}{(-1+x^4)(-1+x^3+x^4)}\,dx$

Optimal. Leaf size=73

$$2\tanh^{-1}\left(\frac{x\sqrt{-x^5+x^4+x}}{x^4-x^3-1}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt{-x^5+x^4+x}}{x^4-x^3-1}\right)$$

Rubi [F] time = 5.68, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(3+x^4)\sqrt{x+x^4-x^5}}{(-1+x^4)(-1+x^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((3 + x^4)*Sqrt[x + x^4 - x^5])/((-1 + x^4)*(-1 + x^3 + x^4)),x]
```

```
[Out] ((-I)*Sqrt[x + x^4 - x^5]*Defer[Subst][Defer[Int][Sqrt[1 + x^6 - x^8]/(I -
x), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + x^3 - x^4]) + ((-1)^(1/4)*Sqrt[x + x
^4 - x^5]*Defer[Subst][Defer[Int][Sqrt[1 + x^6 - x^8]/((-1)^(1/4) - x), x],
 x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + x^3 - x^4]) - ((-1)^(3/4)*Sqrt[x + x^4 - x^
5]*Defer[Subst][Defer[Int][Sqrt[1 + x^6 - x^8]/(-(-1)^(3/4) - x), x], x, Sq
rt[x]])/(Sqrt[x]*Sqrt[1 + x^3 - x^4]) + (Sqrt[x + x^4 - x^5]*Defer[Subst][D
efer[Int][Sqrt[1 + x^6 - x^8]/(-1 + x), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 +
x^3 - x^4]) - (I*Sqrt[x + x^4 - x^5]*Defer[Subst][Defer[Int][Sqrt[1 + x^6 -
 x^8]/(I + x), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + x^3 - x^4]) - (Sqrt[x + x
^4 - x^5]*Defer[Subst][Defer[Int][Sqrt[1 + x^6 - x^8]/(1 + x), x], x, Sqrt[
x]])/(Sqrt[x]*Sqrt[1 + x^3 - x^4]) + ((-1)^(1/4)*Sqrt[x + x^4 - x^5]*Defer[
Subst][Defer[Int][Sqrt[1 + x^6 - x^8]/((-1)^(1/4) + x), x], x, Sqrt[x]])/(S
qrt[x]*Sqrt[1 + x^3 - x^4]) - ((-1)^(3/4)*Sqrt[x + x^4 - x^5]*Defer[Subst][
Defer[Int][Sqrt[1 + x^6 - x^8]/(-(-1)^(3/4) + x), x], x, Sqrt[x]])/(Sqrt[x]
*Sqrt[1 + x^3 - x^4]) - (6*Sqrt[x + x^4 - x^5]*Defer[Subst][Defer[Int][(x^2
*Sqrt[1 + x^6 - x^8])/(-1 + x^6 + x^8), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 +
x^3 - x^4]) - (8*Sqrt[x + x^4 - x^5]*Defer[Subst][Defer[Int][(x^4*Sqrt[1 +
x^6 - x^8])/(-1 + x^6 + x^8), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + x^3 - x^4]
)
```

Rubi steps

$$\int \frac{(3+x^4)\sqrt{x+x^4-x^5}}{(-1+x^4)(-1+x^3+x^4)}\,dx = \frac{\sqrt{x+x^4-x^5}\int \frac{\sqrt{x}\sqrt{1+x^3-x^4}(3+x^4)}{(-1+x^4)(-1+x^3+x^4)}\,dx}{\sqrt{x}\sqrt{1+x^3-x^4}}$$

$$= \frac{\sqrt{x+x^4-x^5}\int \left(\frac{\sqrt{x}\sqrt{1+x^3-x^4}}{-1+x} + \frac{\sqrt{x}\sqrt{1+x^3-x^4}}{1+x} - \frac{2x^{3/2}\sqrt{1+x^3-x^4}}{1+x^2} + \frac{(-3-4x)\sqrt{x}\sqrt{1+x^3-x^4}}{-1+x^3+x^4}\right.}{\sqrt{x}\sqrt{1+x^3-x^4}}$$

$$= \frac{\sqrt{x+x^4-x^5}\int \frac{\sqrt{x}\sqrt{1+x^3-x^4}}{-1+x}\,dx}{\sqrt{x}\sqrt{1+x^3-x^4}} + \frac{\sqrt{x+x^4-x^5}\int \frac{\sqrt{x}\sqrt{1+x^3-x^4}}{1+x}\,dx}{\sqrt{x}\sqrt{1+x^3-x^4}} + \frac{\sqrt{x+x^4-}}{}$$

$$= -\frac{\left(2\sqrt{x+x^4-x^5}\right)\int \left(\frac{ix^{3/2}\sqrt{1+x^3-x^4}}{2(i-x)} + \frac{ix^{3/2}\sqrt{1+x^3-x^4}}{2(i+x)}\right)dx}{\sqrt{x}\sqrt{1+x^3-x^4}} + \frac{\left(2\sqrt{x+x^4-x^5}\right)\text{Sub}}{\sqrt{x}\sqrt{}}$$

$$= -\frac{\left(i\sqrt{x+x^4-x^5}\right)\int \frac{x^{3/2}\sqrt{1+x^3-x^4}}{i-x}\,dx}{\sqrt{x}\sqrt{1+x^3-x^4}} - \frac{\left(i\sqrt{x+x^4-x^5}\right)\int \frac{x^{3/2}\sqrt{1+x^3-x^4}}{i+x}\,dx}{\sqrt{x}\sqrt{1+x^3-x^4}} + \frac{(2}{}$$

$$= -\frac{\left(2i\sqrt{x+x^4-x^5}\right)\text{Subst}\left(\int \frac{x^4\sqrt{1+x^6-x^8}}{i-x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{1+x^3-x^4}} - \frac{\left(2i\sqrt{x+x^4-x^5}\right)\text{Subst}}{\sqrt{x}\sqrt{1+}}$$

$$= -\frac{\left(2i\sqrt{x+x^4-x^5}\right)\text{Subst}\left(\int \left(-i\sqrt{1+x^6-x^8} - x^2\sqrt{1+x^6-x^8} - \frac{\sqrt{1+x^6-x^8}}{i-x^2}\right)d\right.}{\sqrt{x}\sqrt{1+x^3-x^4}}$$

$$= -\frac{\left(i\sqrt{x+x^4-x^5}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^6-x^8}}{i-x}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{1+x^3-x^4}} - \frac{\left(i\sqrt{x+x^4-x^5}\right)\text{Subst}\left(\int\right.}{\sqrt{x}\sqrt{1+x^3}}$$

$$= -\frac{\left(i\sqrt{x+x^4-x^5}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^6-x^8}}{i-x}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{1+x^3-x^4}} - \frac{\left(i\sqrt{x+x^4-x^5}\right)\text{Subst}\left(\int\right.}{\sqrt{x}\sqrt{1+x^3}}$$

$$= -\frac{\left(i\sqrt{x+x^4-x^5}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^6-x^8}}{i-x}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{1+x^3-x^4}} - \frac{\left(i\sqrt{x+x^4-x^5}\right)\text{Subst}\left(\int\right.}{\sqrt{x}\sqrt{1+x^3}}$$

Mathematica [F] time = 1.15, size = 0, normalized size = 0.00

$$\int \frac{(3+x^4)\sqrt{x+x^4-x^5}}{(-1+x^4)(-1+x^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((3 + x^4)*Sqrt[x + x^4 - x^5])/((-1 + x^4)*(-1 + x^3 + x^4)),x]
[Out] Integrate[((3 + x^4)*Sqrt[x + x^4 - x^5])/((-1 + x^4)*(-1 + x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.16, size = 73, normalized size = 1.00

$$2\tanh^{-1}\left(\frac{x\sqrt{-x^5+x^4+x}}{x^4-x^3-1}\right) - 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{-x^5+x^4+x}}{x^4-x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3 + x^4)*Sqrt[x + x^4 - x^5])/((-1 + x^4)*(-1 + x^3 +
x^4)),x]
```

```
[Out] 2*ArcTanh[(x*Sqrt[x + x^4 - x^5])/(-1 - x^3 + x^4)] - 2*Sqrt[2]*ArcTanh[(Sq
rt[2]*x*Sqrt[x + x^4 - x^5])/(-1 - x^3 + x^4)]
```

fricas [A] time = 0.45, size = 122, normalized size = 1.67

$$\frac{1}{2}\sqrt{2}\log\left(\frac{x^8-14x^7+17x^6-2x^4+14x^3-4\sqrt{2}\left(x^5-3x^4-x\right)\sqrt{-x^5+x^4+x}+1}{x^8+2x^7+x^6-2x^4-2x^3+1}\right)+\log\left(-\frac{x^4-2x^3+2\sqrt{-x^5+x^4+x}\,x-1}{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)*(-x^5+x^4+x)^(1/2)/(x^4-1)/(x^4+x^3-1),x, algorithm="fric
as")
```

```
[Out] 1/2*sqrt(2)*log((x^8 - 14*x^7 + 17*x^6 - 2*x^4 + 14*x^3 - 4*sqrt(2)*(x^5 -
3*x^4 - x)*sqrt(-x^5 + x^4 + x) + 1)/(x^8 + 2*x^7 + x^6 - 2*x^4 - 2*x^3 + 1
)) + log(-(x^4 - 2*x^3 + 2*sqrt(-x^5 + x^4 + x)*x - 1)/(x^4 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^5+x^4+x}\left(x^4+3\right)}{\left(x^4+x^3-1\right)\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)*(-x^5+x^4+x)^(1/2)/(x^4-1)/(x^4+x^3-1),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-x^5 + x^4 + x)*(x^4 + 3)/((x^4 + x^3 - 1)*(x^4 - 1)), x)
```

maple [C] time = 0.78, size = 113, normalized size = 1.55

$$-\ln\left(\frac{-x^4+2x^3+2\sqrt{-x^5+x^4+x}\,x+1}{(-1+x)(1+x)\left(x^2+1\right)}\right)+\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2-2\right)x^4+3\,\mathrm{RootOf}\left(\_Z^2-2\right)x^3+4\sqrt{-x^5+x^4+x}\,x+\mathrm{RootOf}\left(\_Z^2-2\right)}{x^4+x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3)*(-x^5+x^4+x)^(1/2)/(x^4-1)/(x^4+x^3-1),x)
```

```
[Out] -ln((-x^4+2*x^3+2*(-x^5+x^4+x)^(1/2)*x+1)/(-1+x)/(1+x)/(x^2+1))+RootOf(_Z^2
-2)*ln((-RootOf(_Z^2-2)*x^4+3*RootOf(_Z^2-2)*x^3+4*(-x^5+x^4+x)^(1/2)*x+Roo
tOf(_Z^2-2))/(x^4+x^3-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^5+x^4+x}\left(x^4+3\right)}{\left(x^4+x^3-1\right)\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)*(-x^5+x^4+x)^(1/2)/(x^4-1)/(x^4+x^3-1),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-x^5 + x^4 + x)*(x^4 + 3)/((x^4 + x^3 - 1)*(x^4 - 1)), x)
```

mupad [B] time = 3.47, size = 81, normalized size = 1.11

$$\ln\left(\frac{2x\sqrt{-x^5+x^4+x}-2x^3+x^4-1}{x^4-1}\right)+\sqrt{2}\,\ln\left(\frac{3x^3-x^4+2\sqrt{2}\,x\sqrt{-x^5+x^4+x}+1}{x^4+x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 3)*(x + x^4 - x^5)^(1/2))/((x^4 - 1)*(x^3 + x^4 - 1)),x)
```

```
[Out] log((2*x*(x + x^4 - x^5)^(1/2) - 2*x^3 + x^4 - 1)/(x^4 - 1)) + 2^(1/2)*log(
(3*x^3 - x^4 + 2*2^(1/2)*x*(x + x^4 - x^5)^(1/2) + 1)/(x^3 + x^4 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3)*(-x**5+x**4+x)**(1/2)/(x**4-1)/(x**4+x**3-1),x)
```

```
[Out] Timed out
```

3.887 $\int \frac{(-b+ax^2)\sqrt{bx+ax^3}}{b^2x+2(-1+ab)x^3+a^2x^5}\,dx$

Optimal. Leaf size=73

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{ax^3+bx}}{ax^2+b}\right)}{\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{ax^3+bx}}{ax^2+b}\right)}{\sqrt[4]{2}}$$

Rubi [C] time = 15.30, antiderivative size = 2513, normalized size of antiderivative = 34.42, number of steps used = 23, number of rules used = 9, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1594, 2056, 6715, 6728, 406, 220, 409, 1217, 1707}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[((-b + a*x^2)*Sqrt[b*x + a*x^3])/(b^2*x + 2*(-1 + a*b)*x^3 + a^2*x^5),x
]

[Out] -1/4*((1 - Sqrt[1 - 2*a*b])^(3/2)*(2 + 2*a^2*b^2 - 2*Sqrt[1 - 2*a*b] - a*b*
(5 - 3*Sqrt[1 - 2*a*b]))*Sqrt[b*x + a*x^3]*ArcTan[(Sqrt[1 - Sqrt[1 - 2*a*b]
]*Sqrt[x])/((1 - a*b - Sqrt[1 - 2*a*b])^(1/4)*Sqrt[b + a*x^2])])/((1 - 2*a*
b - Sqrt[1 - 2*a*b])*(1 - a*b - Sqrt[1 - 2*a*b])^(7/4)*Sqrt[x]*Sqrt[b + a*x
^2]) - ((-1 + Sqrt[1 - 2*a*b])^(3/2)*(2 + 2*a^2*b^2 - 2*Sqrt[1 - 2*a*b] - a
*b*(5 - 3*Sqrt[1 - 2*a*b]))*Sqrt[b*x + a*x^3]*ArcTan[(Sqrt[-1 + Sqrt[1 - 2*
a*b]]*Sqrt[x])/((1 - a*b - Sqrt[1 - 2*a*b])^(1/4)*Sqrt[b + a*x^2])])/(4*(1
- 2*a*b - Sqrt[1 - 2*a*b])*(1 - a*b - Sqrt[1 - 2*a*b])^(7/4)*Sqrt[x]*Sqrt[b
 + a*x^2]) - ((-1 - Sqrt[1 - 2*a*b])^(3/2)*Sqrt[b*x + a*x^3]*ArcTan[(Sqrt[-
1 - Sqrt[1 - 2*a*b]]*Sqrt[x])/((1 - a*b + Sqrt[1 - 2*a*b])^(1/4)*Sqrt[b + a
*x^2])])/(4*(1 - a*b + Sqrt[1 - 2*a*b])^(3/4)*Sqrt[x]*Sqrt[b + a*x^2]) - ((
1 + Sqrt[1 - 2*a*b])^(3/2)*Sqrt[b*x + a*x^3]*ArcTan[(Sqrt[1 + Sqrt[1 - 2*a*
b]]*Sqrt[x])/((1 - a*b + Sqrt[1 - 2*a*b])^(1/4)*Sqrt[b + a*x^2])])/(4*(1 -
a*b + Sqrt[1 - 2*a*b])^(3/4)*Sqrt[x]*Sqrt[b + a*x^2]) + ((1 - Sqrt[1 - 2*a*
b])*(Sqrt[b] + Sqrt[a]*x)*Sqrt[(b + a*x^2)/(Sqrt[b] + Sqrt[a]*x)^2]*Sqrt[b*
x + a*x^3]*EllipticF[2*ArcTan[(a^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*a^(1/4)*
b^(1/4)*Sqrt[x]*(b + a*x^2)) + ((1 + Sqrt[1 - 2*a*b])*(Sqrt[b] + Sqrt[a]*x)
*Sqrt[(b + a*x^2)/(Sqrt[b] + Sqrt[a]*x)^2]*Sqrt[b*x + a*x^3]*EllipticF[2*Ar
cTan[(a^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[x]*(b + a*x^
2)) - ((1 - Sqrt[1 - 2*a*b])^2*(1 - (Sqrt[a]*Sqrt[b])/Sqrt[1 - a*b - Sqrt[1
 - 2*a*b]])*(Sqrt[b] + Sqrt[a]*x)*Sqrt[(b + a*x^2)/(Sqrt[b] + Sqrt[a]*x)^2]
*Sqrt[b*x + a*x^3]*EllipticF[2*ArcTan[(a^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(4*
a^(1/4)*b^(1/4)*(1 - 2*a*b - Sqrt[1 - 2*a*b])*Sqrt[x]*(b + a*x^2)) - ((1 -
Sqrt[1 - 2*a*b])^2*(1 + (Sqrt[a]*Sqrt[b])/Sqrt[1 - a*b - Sqrt[1 - 2*a*b]])*
(Sqrt[b] + Sqrt[a]*x)*Sqrt[(b + a*x^2)/(Sqrt[b] + Sqrt[a]*x)^2]*Sqrt[b*x +
a*x^3]*EllipticF[2*ArcTan[(a^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(4*a^(1/4)*b^(1
/4)*(1 - 2*a*b - Sqrt[1 - 2*a*b])*Sqrt[x]*(b + a*x^2)) - ((1 + Sqrt[1 - 2*a
*b])^2*(1 - (Sqrt[a]*Sqrt[b])/Sqrt[1 - a*b + Sqrt[1 - 2*a*b]])*(Sqrt[b] + S
qrt[a]*x)*Sqrt[(b + a*x^2)/(Sqrt[b] + Sqrt[a]*x)^2]*Sqrt[b*x + a*x^3]*Ellip
ticF[2*ArcTan[(a^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)*(1 - 2*a
*b + Sqrt[1 - 2*a*b])*Sqrt[x]*(b + a*x^2)) - ((1 + Sqrt[1 - 2*a*b])^2*(1 +
(Sqrt[a]*Sqrt[b])/Sqrt[1 - a*b + Sqrt[1 - 2*a*b]])*(Sqrt[b] + Sqrt[a]*x)*Sq
rt[(b + a*x^2)/(Sqrt[b] + Sqrt[a]*x)^2]*Sqrt[b*x + a*x^3]*EllipticF[2*ArcTa
n[(a^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)*(1 - 2*a*b + Sqrt[1
- 2*a*b])*Sqrt[x]*(b + a*x^2)) + ((1 - Sqrt[1 - 2*a*b] + 2*Sqrt[a]*Sqrt[b]*
Sqrt[1 - a*b - Sqrt[1 - 2*a*b]])*(Sqrt[b] + Sqrt[a]*x)*Sqrt[(b + a*x^2)/(Sq
rt[b] + Sqrt[a]*x)^2]*Sqrt[b*x + a*x^3]*EllipticPi[-1/4*(Sqrt[a]*Sqrt[b] -
Sqrt[1 - a*b - Sqrt[1 - 2*a*b]])^2/(Sqrt[a]*Sqrt[b]*Sqrt[1 - a*b - Sqrt[1 -
 2*a*b]]), 2*ArcTan[(a^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)*(1
 - 2*a*b - Sqrt[1 - 2*a*b])*Sqrt[x]*(b + a*x^2)) + ((1 - Sqrt[1 - 2*a*b] -
```

```
2*Sqrt[a]*Sqrt[b]*Sqrt[1 - a*b - Sqrt[1 - 2*a*b]])*(Sqrt[b] + Sqrt[a]*x)*Sq
rt[(b + a*x^2)/(Sqrt[b] + Sqrt[a]*x)^2]*Sqrt[b*x + a*x^3]*EllipticPi[(Sqrt[
a]*Sqrt[b] + Sqrt[1 - a*b - Sqrt[1 - 2*a*b]])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[1 -
 a*b - Sqrt[1 - 2*a*b]]), 2*ArcTan[(a^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(4*a^(
1/4)*b^(1/4)*(1 - 2*a*b - Sqrt[1 - 2*a*b])*Sqrt[x]*(b + a*x^2)) + ((1 + Sqr
t[1 - 2*a*b] + 2*Sqrt[a]*Sqrt[b]*Sqrt[1 - a*b + Sqrt[1 - 2*a*b]])*(Sqrt[b]
+ Sqrt[a]*x)*Sqrt[(b + a*x^2)/(Sqrt[b] + Sqrt[a]*x)^2]*Sqrt[b*x + a*x^3]*El
lipticPi[-1/4*(Sqrt[a]*Sqrt[b] - Sqrt[1 - a*b + Sqrt[1 - 2*a*b]])^2/(Sqrt[a
]*Sqrt[b]*Sqrt[1 - a*b + Sqrt[1 - 2*a*b]]), 2*ArcTan[(a^(1/4)*Sqrt[x])/b^(1
/4)], 1/2])/(4*a^(1/4)*b^(1/4)*(1 - 2*a*b + Sqrt[1 - 2*a*b])*Sqrt[x]*(b + a
*x^2)) + ((1 + Sqrt[1 - 2*a*b] - 2*Sqrt[a]*Sqrt[b]*Sqrt[1 - a*b + Sqrt[1 -
2*a*b]])*(Sqrt[b] + Sqrt[a]*x)*Sqrt[(b + a*x^2)/(Sqrt[b] + Sqrt[a]*x)^2]*Sq
rt[b*x + a*x^3]*EllipticPi[(Sqrt[a]*Sqrt[b] + Sqrt[1 - a*b + Sqrt[1 - 2*a*b
]])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[1 - a*b + Sqrt[1 - 2*a*b]]), 2*ArcTan[(a^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)*(1 - 2*a*b + Sqrt[1 - 2*a*b])
*Sqrt[x]*(b + a*x^2))
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(-b+ax^2\right)\sqrt{bx+ax^3}}{b^2x+2(-1+ab)x^3+a^2x^5}\,dx = \int \frac{\left(-b+ax^2\right)\sqrt{bx+ax^3}}{x\left(b^2+2(-1+ab)x^2+a^2x^4\right)}\,dx$$

$$= \frac{\sqrt{bx+ax^3}\int \frac{\left(-b+ax^2\right)\sqrt{b+ax^2}}{\sqrt{x}\left(b^2+2(-1+ab)x^2+a^2x^4\right)}\,dx}{\sqrt{x}\sqrt{b+ax^2}}$$

$$= \frac{\left(2\sqrt{bx+ax^3}\right)\text{Subst}\left(\int \frac{\left(-b+ax^4\right)\sqrt{b+ax^4}}{b^2+2(-1+ab)x^4+a^2x^8}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{b+ax^2}}$$

$$= \frac{\left(2\sqrt{bx+ax^3}\right)\text{Subst}\left(\int \left(\frac{\left(a+a\sqrt{1-2ab}\right)\sqrt{b+ax^4}}{-2\sqrt{1-2ab}+2(-1+ab)+2a^2x^4} + \frac{\left(a-a\sqrt{1-2ab}\right)\sqrt{b+ax^4}}{2\sqrt{1-2ab}+2(-1+ab)+2a^2x^4}\right) dx\right.}{\sqrt{x}\sqrt{b+ax^2}}$$

$$= \frac{\left(2a\left(1-\sqrt{1-2ab}\right)\sqrt{bx+ax^3}\right)\text{Subst}\left(\int \frac{\sqrt{b+ax^4}}{2\sqrt{1-2ab}+2(-1+ab)+2a^2x^4}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{b+ax^2}} +$$

$$= \frac{\left(\left(1-\sqrt{1-2ab}\right)\sqrt{bx+ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{b+ax^2}} + \frac{\left(2\left(1-\sqrt{1-2ab}\right.\right.}{}$$

$$= \frac{\left(1-\sqrt{1-2ab}\right)\left(\sqrt{b}+\sqrt{a}\,x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}\,x\right)^2}}\sqrt{bx+ax^3}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\left(b+ax^2\right)}$$

$$= \frac{\left(1-\sqrt{1-2ab}\right)\left(\sqrt{b}+\sqrt{a}\,x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}\,x\right)^2}}\sqrt{bx+ax^3}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\left(b+ax^2\right)}$$

$$= \frac{\left(1-\sqrt{1-2ab}\right)^{3/2}\left(\sqrt{a}\sqrt{b}-\sqrt{1-ab-\sqrt{1-2ab}}\right)\left(\sqrt{a}\sqrt{b}+\sqrt{1-ab-\sqrt{1-}}\right.}{4\left(1-2ab-\sqrt{1-2ab}\right)\left(1-ab-\sqrt{1-2ab}\right)^{3}}$$

Mathematica [C] time = 1.42, size = 372, normalized size = 5.10

$$-\frac{ix^{3/2}\sqrt{\frac{b}{ax^2}+1}\left(-\Pi\left(-\frac{i\sqrt{a}}{\sqrt{b}\sqrt{\frac{-ab+\sqrt{1-2ab}+1}{b^2}}};i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(\frac{i\sqrt{a}}{\sqrt{b}\sqrt{\frac{-ab+\sqrt{1-2ab}+1}{b^2}}};i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(-\frac{i\sqrt{a}}{\sqrt{b}\sqrt{-\frac{ab+\sqrt{1-2ab}-1}{b^2}}};i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(\frac{i\sqrt{a}}{\sqrt{b}\sqrt{-\frac{ab+\sqrt{1-2ab}-1}{b^2}}};i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{x}}\right)\middle|-1\right)+2F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{x\left(ax^2+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + a*x^2)*Sqrt[b*x + a*x^3])/(b^2*x + 2*(-1 + a*b)*x^3 + a^2*
x^5),x]
```

```
[Out] ((-I)*Sqrt[1 + b/(a*x^2)]*x^(3/2)*(2*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/S
qrt[a]]/Sqrt[x]], -1] - EllipticPi[((-I)*Sqrt[a])/(Sqrt[b]*Sqrt[(1 - a*b +
Sqrt[1 - 2*a*b])/b^2]), I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]/Sqrt[x]], -1] -
 EllipticPi[(I*Sqrt[a])/(Sqrt[b]*Sqrt[(1 - a*b + Sqrt[1 - 2*a*b])/b^2]), I*
ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]/Sqrt[x]], -1] - EllipticPi[((-I)*Sqrt[a])
/(Sqrt[b]*Sqrt[-((-1 + a*b + Sqrt[1 - 2*a*b])/b^2)]), I*ArcSinh[Sqrt[(I*Sqr
```

```
t[b])/Sqrt[a]]/Sqrt[x]], -1] - EllipticPi[(I*Sqrt[a])/(Sqrt[b]*Sqrt[-((-1 +
 a*b + Sqrt[1 - 2*a*b])/b^2)]), I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]/Sqrt[x]
], -1]))/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[x*(b + a*x^2)])
```

IntegrateAlgebraic [A] time = 0.41, size = 73, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{ax^3+bx}}{ax^2+b}\right)}{\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{ax^3+bx}}{ax^2+b}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^2)*Sqrt[b*x + a*x^3])/(b^2*x + 2*(-1 + a*b)*x
^3 + a^2*x^5),x]
```

```
[Out] -(ArcTan[(2^(1/4)*Sqrt[b*x + a*x^3])/(b + a*x^2)]/2^(1/4)) - ArcTanh[(2^(1/
4)*Sqrt[b*x + a*x^3])/(b + a*x^2)]/2^(1/4)
```

fricas [B] time = 0.54, size = 221, normalized size = 3.03

$$-\frac{1}{2}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{1}{4}}\sqrt{ax^3+bx}}{ax^2+b}\right)-\frac{1}{8}\cdot 2^{\frac{3}{4}}\log\left(\frac{a^2x^4+2(ab+1)x^2+b^2+2\sqrt{2}\left(ax^3+bx\right)+2\sqrt{ax^3+bx}\left(2^{\frac{3}{4}}x+2^{\frac{1}{4}}\left(ax^2+b\right)\right)}{a^2x^4+2(ab-1)x^2+b^2}\right)+\frac{1}{8}\cdot 2^{\frac{3}{4}}\log\left(\frac{a^2x^4+2(ab+1)x^2+b^2+2\sqrt{2}\left(ax^3+bx\right)-2\sqrt{ax^3+bx}\left(2^{\frac{3}{4}}x+2^{\frac{1}{4}}\left(ax^2+b\right)\right)}{a^2x^4+2(ab-1)x^2+b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(a*x^3+b*x)^(1/2)/(b^2*x+2*(a*b-1)*x^3+a^2*x^5),x, algo
rithm="fricas")
```

```
[Out] -1/2*2^(3/4)*arctan(2^(1/4)*sqrt(a*x^3 + b*x)/(a*x^2 + b)) - 1/8*2^(3/4)*lo
g((a^2*x^4 + 2*(a*b + 1)*x^2 + b^2 + 2*sqrt(2)*(a*x^3 + b*x) + 2*sqrt(a*x^3
 + b*x)*(2^(3/4)*x + 2^(1/4)*(a*x^2 + b)))/(a^2*x^4 + 2*(a*b - 1)*x^2 + b^2
)) + 1/8*2^(3/4)*log((a^2*x^4 + 2*(a*b + 1)*x^2 + b^2 + 2*sqrt(2)*(a*x^3 +
b*x) - 2*sqrt(a*x^3 + b*x)*(2^(3/4)*x + 2^(1/4)*(a*x^2 + b)))/(a^2*x^4 + 2*
(a*b - 1)*x^2 + b^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3+bx}\left(ax^2-b\right)}{a^2x^5+2(ab-1)x^3+b^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(a*x^3+b*x)^(1/2)/(b^2*x+2*(a*b-1)*x^3+a^2*x^5),x, algo
rithm="giac")
```

```
[Out] integrate(sqrt(a*x^3 + b*x)*(a*x^2 - b)/(a^2*x^5 + 2*(a*b - 1)*x^3 + b^2*x)
, x)
```

maple [C] time = 0.11, size = 530, normalized size = 7.26

$$-\frac{\frac{2\sqrt{ax^3+bx}}{3}+\frac{2b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3a\sqrt{ax^3+bx}}}{b}+\frac{\frac{2\sqrt{ax^3+bx}}{3}+\frac{5b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3a\sqrt{ax^3+bx}}-\frac{\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(a^2\_Z^4+(2ab-2)\_Z^2+b^2\right)}\frac{\left(\_\alpha^2ab-\_\alpha^2+b^2\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{\left(x-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{xa}{\sqrt{-ab}}}\left(a\left(a^2\_\alpha^3+\_\alpha ab-2\_\alpha\right)-a^2\sqrt{-ab}\,\_\alpha^2-\sqrt{-ab}\,ab+2\sqrt{-ab}\right)\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}\,\_\alpha^3a^2+\_\alpha^2a^2b+\sqrt{-ab}\,\_\alpha ab+ab^2-2\sqrt{-ab}\,\_\alpha-2b}{2b},\frac{\sqrt{2}}{2}\right)}{\_\alpha\left(\_\alpha^2a^2+ab-1\right)\sqrt{x\left(ax^2+b\right)}}\right)}{4a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)*(a*x^3+b*x)^(1/2)/(b^2*x+2*(a*b-1)*x^3+a^2*x^5),x)
```

```
[Out] -1/b*(2/3*(a*x^3+b*x)^(1/2)+2/3*b/a*(-a*b)^(1/2)*((x+1/a*(-a*b)^(1/2))*a/(-
a*b)^(1/2))^(1/2)*(-2*(x-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x*a/(-a*
b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)*EllipticF(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^
(1/2))^(1/2),1/2*2^(1/2)))+1/b*(2/3*(a*x^3+b*x)^(1/2)+5/3*b/a*(-a*b)^(1/2)*
((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x-1/a*(-a*b)^(1/2))*a/(-a*
```

```
b)^(1/2))^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)*EllipticF(((x+1
/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-1/4/a*2^(1/2)*sum((_alp
ha^2*a*b-_alpha^2+b^2)/_alpha/(_alpha^2*a^2+a*b-1)*(-a*b)^(1/2)*((x+1/a*(-a
*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-(x-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/
2)*(-x*a/(-a*b)^(1/2))^(1/2)/(x*(a*x^2+b))^(1/2)*(a*(_alpha^3*a^2+_alpha*a*
b-2*_alpha)-a^2*(-a*b)^(1/2)*_alpha^2-(-a*b)^(1/2)*a*b+2*(-a*b)^(1/2))*Elli
pticPi(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),-1/2*((-a*b)^(1/2)*_alph
a^3*a^2+_alpha^2*a^2*b+(-a*b)^(1/2)*_alpha*a*b+a*b^2-2*(-a*b)^(1/2)*_alpha-
2*b)/b,1/2*2^(1/2)),_alpha=RootOf(a^2*_Z^4+(2*a*b-2)*_Z^2+b^2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3+bx}\left(ax^2-b\right)}{a^2x^5+2\,(ab-1)x^3+b^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(a*x^3+b*x)^(1/2)/(b^2*x+2*(a*b-1)*x^3+a^2*x^5),x, algo
rithm="maxima")
```

```
[Out] integrate(sqrt(a*x^3 + b*x)*(a*x^2 - b)/(a^2*x^5 + 2*(a*b - 1)*x^3 + b^2*x)
, x)
```

mupad [B] time = 4.36, size = 119, normalized size = 1.63

$$\frac{2^{3/4}\,\ln\left(\frac{2^{3/4}\,b+2\,2^{1/4}\,x-4\,\sqrt{x\,\left(a\,x^2+b\right)}+2^{3/4}\,a\,x^2}{4\,a\,x^2-4\,\sqrt{2}\,x+4\,b}\right)}{4}+\frac{2^{3/4}\,\ln\left(\frac{2^{3/4}\,b\,1{}\mathrm{i}-2^{1/4}\,x\,2{}\mathrm{i}-4\,\sqrt{x\,\left(a\,x^2+b\right)}+2^{3/4}\,a\,x^2\,1{}\mathrm{i}}{a\,x^2+\sqrt{2}\,x+b}\right)\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b*x + a*x^3)^(1/2)*(b - a*x^2))/(b^2*x + 2*x^3*(a*b - 1) + a^2*x^5),
x)
```

```
[Out] (2^(3/4)*log((2^(3/4)*b*1i - 2^(1/4)*x*2i - 4*(x*(b + a*x^2))^(1/2) + 2^(3/
4)*a*x^2*1i)/(b + 2^(1/2)*x + a*x^2))*1i)/4 + (2^(3/4)*log((2^(3/4)*b + 2*2
^(1/4)*x - 4*(x*(b + a*x^2))^(1/2) + 2^(3/4)*a*x^2)/(4*b - 4*2^(1/2)*x + 4*
a*x^2)))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(ax^2+b\right)}\left(ax^2-b\right)}{x\left(a^2x^4+2abx^2+b^2-2x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)*(a*x**3+b*x)**(1/2)/(b**2*x+2*(a*b-1)*x**3+a**2*x**5),
x)
```

```
[Out] Integral(sqrt(x*(a*x**2 + b))*(a*x**2 - b)/(x*(a**2*x**4 + 2*a*b*x**2 + b**
2 - 2*x**2)), x)
```

3.888 $\int \frac{(-3+2x^5)\sqrt{x+2x^4+x^6}}{(1+x^5)(1+x^3+x^5)}\,dx$

Optimal. Leaf size=73

$$2\tanh^{-1}\left(\frac{x\sqrt{x^6+2x^4+x}}{x^5+2x^3+1}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{x^6+2x^4+x}}{x^5+2x^3+1}\right)$$

Rubi [F] time = 5.60, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-3+2x^5)\sqrt{x+2x^4+x^6}}{(1+x^5)(1+x^3+x^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + 2*x^5)*Sqrt[x + 2*x^4 + x^6])/((1 + x^5)*(1 + x^3 + x^5)),x]
```

```
[Out] ((-I)*Sqrt[x + 2*x^4 + x^6]*Defer[Subst][Defer[Int][Sqrt[1 + 2*x^6 + x^10]/
(I - x), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + 2*x^3 + x^5]) - (I*Sqrt[x + 2*x
^4 + x^6]*Defer[Subst][Defer[Int][Sqrt[1 + 2*x^6 + x^10]/(I + x), x], x, Sq
rt[x]])/(Sqrt[x]*Sqrt[1 + 2*x^3 + x^5]) + (2*Sqrt[x + 2*x^4 + x^6]*Defer[Su
bst][Defer[Int][Sqrt[1 + 2*x^6 + x^10]/(1 - x^2 + x^4 - x^6 + x^8), x], x,
Sqrt[x]])/(Sqrt[x]*Sqrt[1 + 2*x^3 + x^5]) - (4*Sqrt[x + 2*x^4 + x^6]*Defer[
Subst][Defer[Int][(x^2*Sqrt[1 + 2*x^6 + x^10])/(1 - x^2 + x^4 - x^6 + x^8),
 x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + 2*x^3 + x^5]) + (6*Sqrt[x + 2*x^4 + x^6
]*Defer[Subst][Defer[Int][(x^4*Sqrt[1 + 2*x^6 + x^10])/(1 - x^2 + x^4 - x^6
 + x^8), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + 2*x^3 + x^5]) + (2*Sqrt[x + 2*x
^4 + x^6]*Defer[Subst][Defer[Int][(x^6*Sqrt[1 + 2*x^6 + x^10])/(1 - x^2 + x
^4 - x^6 + x^8), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + 2*x^3 + x^5]) - (6*Sqrt
[x + 2*x^4 + x^6]*Defer[Subst][Defer[Int][(x^2*Sqrt[1 + 2*x^6 + x^10])/(1 +
 x^6 + x^10), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + 2*x^3 + x^5]) - (10*Sqrt[x
 + 2*x^4 + x^6]*Defer[Subst][Defer[Int][(x^6*Sqrt[1 + 2*x^6 + x^10])/(1 + x
^6 + x^10), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + 2*x^3 + x^5])
```

Rubi steps

$$\int \frac{(-3+2x^5)\sqrt{x+2x^4+x^6}}{(1+x^5)(1+x^3+x^5)}\,dx = \frac{\sqrt{x+2x^4+x^6}\int \frac{\sqrt{x}\sqrt{1+2x^3+x^5}(-3+2x^5)}{(1+x^5)(1+x^3+x^5)}\,dx}{\sqrt{x}\sqrt{1+2x^3+x^5}}$$

$$= \frac{\sqrt{x+2x^4+x^6}\int \left(\frac{\sqrt{x}\sqrt{1+2x^3+x^5}}{1+x} + \frac{\sqrt{x}(-1+2x+2x^2-x^3)\sqrt{1+2x^3+x^5}}{1-x+x^2-x^3+x^4} + \frac{\sqrt{x}(-3-5x^2)\sqrt{1+}}{1+x^3+x^5}\right.}{\sqrt{x}\sqrt{1+2x^3+x^5}}$$

$$= \frac{\sqrt{x+2x^4+x^6}\int \frac{\sqrt{x}\sqrt{1+2x^3+x^5}}{1+x}\,dx}{\sqrt{x}\sqrt{1+2x^3+x^5}} + \frac{\sqrt{x+2x^4+x^6}\int \frac{\sqrt{x}(-1+2x+2x^2-x^3)\sqrt{1+2x^3+}}{1-x+x^2-x^3+x^4}}{\sqrt{x}\sqrt{1+2x^3+x^5}}$$

$$= \frac{\left(2\sqrt{x+2x^4+x^6}\right)\text{Subst}\left(\int \frac{x^2\sqrt{1+2x^6+x^{10}}}{1+x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{1+2x^3+x^5}} + \frac{\left(2\sqrt{x+2x^4+x^6}\right)\text{Sul}}{}$$

$$= \frac{\left(2\sqrt{x+2x^4+x^6}\right)\text{Subst}\left(\int \left(\sqrt{1+2x^6+x^{10}} - \frac{\sqrt{1+2x^6+x^{10}}}{1+x^2}\right)dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{1+2x^3+x^5}} + \frac{(2}{}$$

$$= -\frac{\left(2\sqrt{x+2x^4+x^6}\right)\text{Subst}\left(\int \frac{\sqrt{1+2x^6+x^{10}}}{1+x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{1+2x^3+x^5}} + \frac{\left(2\sqrt{x+2x^4+x^6}\right)\text{Sul}}{}$$

$$= -\frac{\left(2\sqrt{x+2x^4+x^6}\right)\text{Subst}\left(\int \left(\frac{i\sqrt{1+2x^6+x^{10}}}{2(i-x)} + \frac{i\sqrt{1+2x^6+x^{10}}}{2(i+x)}\right)dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{1+2x^3+x^5}} + \frac{(2\sqrt{}}{}$$

$$= -\frac{\left(i\sqrt{x+2x^4+x^6}\right)\text{Subst}\left(\int \frac{\sqrt{1+2x^6+x^{10}}}{i-x}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{1+2x^3+x^5}} - \frac{\left(i\sqrt{x+2x^4+x^6}\right)\text{Subs}}{\sqrt{x}\sqrt{1}}$$

Mathematica [F] time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{(-3+2x^5)\sqrt{x+2x^4+x^6}}{(1+x^5)(1+x^3+x^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + 2*x^5)*Sqrt[x + 2*x^4 + x^6])/((1 + x^5)*(1 + x^3 + x^5)),x]
```

```
[Out] Integrate[((-3 + 2*x^5)*Sqrt[x + 2*x^4 + x^6])/((1 + x^5)*(1 + x^3 + x^5)), x]
```

IntegrateAlgebraic [A] time = 0.32, size = 73, normalized size = 1.00

$$2\tanh^{-1}\left(\frac{x\sqrt{x^6+2x^4+x}}{x^5+2x^3+1}\right) - 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{x^6+2x^4+x}}{x^5+2x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + 2*x^5)*Sqrt[x + 2*x^4 + x^6])/((1 + x^5)*(1 + x^3 + x^5)),x]
```

```
[Out] 2*ArcTanh[(x*Sqrt[x + 2*x^4 + x^6])/(1 + 2*x^3 + x^5)] - 2*Sqrt[2]*ArcTanh[(Sqrt[2]*x*Sqrt[x + 2*x^4 + x^6])/(1 + 2*x^3 + x^5)]
```

fricas [A] time = 0.48, size = 111, normalized size = 1.52

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{x^{10}+16x^8+32x^6+2x^5+16x^3-4\sqrt{2}\left(x^6+4x^4+x\right)\sqrt{x^6+2x^4+x}+1}{x^{10}+2x^5+1}\right)+\log\left(-\frac{x^5+3x^3+2\sqrt{x^6+2x^4+x}\,x+1}{x^5+x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^5-3)*(x^6+2*x^4+x)^(1/2)/(x^5+1)/(x^5+x^3+1),x, algorithm="f
ricas")
```

```
[Out] 1/2*sqrt(2)*log(-(x^10 + 16*x^8 + 32*x^6 + 2*x^5 + 16*x^3 - 4*sqrt(2)*(x^6
+ 4*x^4 + x)*sqrt(x^6 + 2*x^4 + x) + 1)/(x^10 + 2*x^5 + 1)) + log(-(x^5 + 3
*x^3 + 2*sqrt(x^6 + 2*x^4 + x)*x + 1)/(x^5 + x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^6+2x^4+x}\left(2x^5-3\right)}{\left(x^5+x^3+1\right)\left(x^5+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^5-3)*(x^6+2*x^4+x)^(1/2)/(x^5+1)/(x^5+x^3+1),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(x^6 + 2*x^4 + x)*(2*x^5 - 3)/((x^5 + x^3 + 1)*(x^5 + 1)), x)
```

maple [C] time = 0.86, size = 121, normalized size = 1.66

$$\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2-2\right)x^5-4\,\mathrm{RootOf}\left(\_Z^2-2\right)x^3+4\sqrt{x^6+2x^4+x}\,x-\mathrm{RootOf}\left(\_Z^2-2\right)}{(1+x)\left(x^4-x^3+x^2-x+1\right)}\right)-\ln\left(\frac{-x^5-3x^3+2\sqrt{x^6+2x^4+x}\,x-1}{x^5+x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^5-3)*(x^6+2*x^4+x)^(1/2)/(x^5+1)/(x^5+x^3+1),x)
```

```
[Out] RootOf(_Z^2-2)*ln((-RootOf(_Z^2-2)*x^5-4*RootOf(_Z^2-2)*x^3+4*(x^6+2*x^4+x)
^(1/2)*x-RootOf(_Z^2-2))/(1+x)/(x^4-x^3+x^2-x+1))-ln((-x^5-3*x^3+2*(x^6+2*x
^4+x)^(1/2)*x-1)/(x^5+x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^6+2x^4+x}\left(2x^5-3\right)}{\left(x^5+x^3+1\right)\left(x^5+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^5-3)*(x^6+2*x^4+x)^(1/2)/(x^5+1)/(x^5+x^3+1),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(x^6 + 2*x^4 + x)*(2*x^5 - 3)/((x^5 + x^3 + 1)*(x^5 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2x^5-3\right)\sqrt{x^6+2x^4+x}}{\left(x^5+1\right)\left(x^5+x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^5 - 3)*(x + 2*x^4 + x^6)^(1/2))/((x^5 + 1)*(x^3 + x^5 + 1)),x)
```

```
[Out] int(((2*x^5 - 3)*(x + 2*x^4 + x^6)^(1/2))/((x^5 + 1)*(x^3 + x^5 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**5-3)*(x**6+2*x**4+x)**(1/2)/(x**5+1)/(x**5+x**3+1),x)
```

```
[Out] Timed out
```

3.889 $\int \frac{\sqrt{-1+x^2+x^4+x^6}\left(1+x^4+2x^6\right)}{1-x^4-2x^6+x^8+2x^{10}+x^{12}}\,dx$

Optimal. Leaf size=73

$$-\frac{1}{2}\sqrt{1+i}\tan^{-1}\left(\frac{\sqrt{-1-i}\,x}{\sqrt{x^6+x^4+x^2-1}}\right)-\frac{1}{2}\sqrt{1-i}\tan^{-1}\left(\frac{\sqrt{-1+i}\,x}{\sqrt{x^6+x^4+x^2-1}}\right)$$

Rubi [F] time = 0.97, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-1+x^2+x^4+x^6}\left(1+x^4+2x^6\right)}{1-x^4-2x^6+x^8+2x^{10}+x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-1 + x^2 + x^4 + x^6]*(1 + x^4 + 2*x^6))/(1 - x^4 - 2*x^6 + x^8 +
 2*x^10 + x^12),x]
```

```
[Out] Defer[Int][Sqrt[-1 + x^2 + x^4 + x^6]/(1 - x^4 - 2*x^6 + x^8 + 2*x^10 + x^1
2), x] + Defer[Int][(x^4*Sqrt[-1 + x^2 + x^4 + x^6])/(1 - x^4 - 2*x^6 + x^8
 + 2*x^10 + x^12), x] + 2*Defer[Int][(x^6*Sqrt[-1 + x^2 + x^4 + x^6])/(1 -
x^4 - 2*x^6 + x^8 + 2*x^10 + x^12), x]
```

Rubi steps

$$\int \frac{\sqrt{-1+x^2+x^4+x^6}\left(1+x^4+2x^6\right)}{1-x^4-2x^6+x^8+2x^{10}+x^{12}}\,dx=\int\left(\frac{\sqrt{-1+x^2+x^4+x^6}}{1-x^4-2x^6+x^8+2x^{10}+x^{12}}+\frac{x^4\sqrt{-1+x^2+x^4+x^6}}{1-x^4-2x^6+x^8+2x^{10}+x\right.$$

$$=2\int\frac{x^6\sqrt{-1+x^2+x^4+x^6}}{1-x^4-2x^6+x^8+2x^{10}+x^{12}}\,dx+\int\frac{\sqrt{-1+x^2+x^4+}}{1-x^4-2x^6+x^8+2.}$$

Mathematica [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-1+x^2+x^4+x^6}\left(1+x^4+2x^6\right)}{1-x^4-2x^6+x^8+2x^{10}+x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[-1 + x^2 + x^4 + x^6]*(1 + x^4 + 2*x^6))/(1 - x^4 - 2*x^6 +
 x^8 + 2*x^10 + x^12),x]
```

```
[Out] Integrate[(Sqrt[-1 + x^2 + x^4 + x^6]*(1 + x^4 + 2*x^6))/(1 - x^4 - 2*x^6 +
 x^8 + 2*x^10 + x^12), x]
```

IntegrateAlgebraic [A] time = 0.47, size = 73, normalized size = 1.00

$$-\frac{1}{2}\sqrt{1+i}\tan^{-1}\left(\frac{\sqrt{-1-i}\,x}{\sqrt{x^6+x^4+x^2-1}}\right)-\frac{1}{2}\sqrt{1-i}\tan^{-1}\left(\frac{\sqrt{-1+i}\,x}{\sqrt{x^6+x^4+x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^2 + x^4 + x^6]*(1 + x^4 + 2*x^6))/(1 - x^4
- 2*x^6 + x^8 + 2*x^10 + x^12),x]
```

```
[Out] -1/2*(Sqrt[1 + I]*ArcTan[(Sqrt[-1 - I]*x)/Sqrt[-1 + x^2 + x^4 + x^6]]) - (S
qrt[1 - I]*ArcTan[(Sqrt[-1 + I]*x)/Sqrt[-1 + x^2 + x^4 + x^6]])/2
```

fricas [B] time = 8.82, size = 6596, normalized size = 90.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^4+x^2-1)^(1/2)*(2*x^6+x^4+1)/(x^12+2*x^10+x^8-2*x^6-x^4+1)
,x, algorithm="fricas")
```

```
[Out] 1/32*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(2*(8*x^8 + 8*x^6 + 8*x^4
 + 2*2^(1/4)*(x^7 + x^5 + sqrt(2)*x^3 + x^3 - x)*sqrt(x^6 + x^4 + x^2 - 1)*
sqrt(2*sqrt(2) + 4) - 8*x^2 + sqrt(2)*(x^12 + 2*x^10 + 5*x^8 + 2*x^6 + 3*x^
4 - 4*x^2 + 1))/(x^12 + 2*x^10 + x^8 - 2*x^6 - x^4 + 1)) - 1/32*2^(1/4)*sqr
t(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(2*(8*x^8 + 8*x^6 + 8*x^4 - 2*2^(1/4)*(x^
7 + x^5 + sqrt(2)*x^3 + x^3 - x)*sqrt(x^6 + x^4 + x^2 - 1)*sqrt(2*sqrt(2) +
 4) - 8*x^2 + sqrt(2)*(x^12 + 2*x^10 + 5*x^8 + 2*x^6 + 3*x^4 - 4*x^2 + 1))/
(x^12 + 2*x^10 + x^8 - 2*x^6 - x^4 + 1)) + 1/8*2^(3/4)*sqrt(2*sqrt(2) + 4)*
arctan(1/4*(4*x^72 + 48*x^70 + 296*x^68 + 1184*x^66 + 4100*x^64 + 14336*x^6
2 + 48576*x^60 + 136416*x^58 + 243584*x^56 + 55200*x^54 - 1076416*x^52 - 31
84640*x^50 - 3948592*x^48 + 641728*x^46 + 10292800*x^44 + 15128864*x^42 + 3
787784*x^40 - 17971776*x^38 - 25671216*x^36 - 5595392*x^34 + 21212056*x^32
+ 22744448*x^30 - 89792*x^28 - 16094688*x^26 - 9256416*x^24 + 4023008*x^22
+ 6035840*x^20 + 544896*x^18 - 1868656*x^16 - 554560*x^14 + 332224*x^12 + 1
16192*x^10 - 36524*x^8 - 5232*x^6 + 840*x^4 - 32*x^2 + 4*sqrt(x^6 + x^4 + x
^2 - 1)*(2^(3/4)*(4*x^67 + 44*x^65 + 254*x^63 + 956*x^61 + 2382*x^59 + 3356
*x^57 + 350*x^55 - 7792*x^53 - 5766*x^51 + 40296*x^49 + 137174*x^47 + 18675
6*x^45 + 20238*x^43 - 366284*x^41 - 611162*x^39 - 279864*x^37 + 486370*x^35
 + 879240*x^33 + 368218*x^31 - 485916*x^29 - 681958*x^27 - 123804*x^25 + 35
1834*x^23 + 245984*x^21 - 61090*x^19 - 118680*x^17 - 9422*x^15 + 30876*x^13
 + 4842*x^11 - 5236*x^9 + 194*x^7 + 72*x^5 + 34*x^3 - sqrt(2)*(3*x^67 + 33*
x^65 + 188*x^63 + 692*x^61 + 1414*x^59 - 98*x^57 - 11242*x^55 - 39830*x^53
- 72700*x^51 - 52668*x^49 + 87784*x^47 + 304536*x^45 + 361362*x^43 + 19402*
x^41 - 562182*x^39 - 769994*x^37 - 196922*x^35 + 665266*x^33 + 842764*x^31
+ 131812*x^29 - 569670*x^27 - 479358*x^25 + 68762*x^23 + 294150*x^21 + 9260
4*x^19 - 86324*x^17 - 54576*x^15 + 14160*x^13 + 14126*x^11 - 2090*x^9 - 167
4*x^7 + 314*x^5 + 23*x^3 - 3*x) - 4*x) + 32*2^(1/4)*(7*x^63 + 70*x^61 + 419
*x^59 + 1706*x^57 + 4942*x^55 + 9908*x^53 + 11825*x^51 + 580*x^49 - 28977*x
^47 - 58322*x^45 - 45399*x^43 + 29706*x^41 + 111540*x^39 + 100724*x^37 - 20
967*x^35 - 131220*x^33 - 100923*x^31 + 30198*x^29 + 100465*x^27 + 45314*x^2
5 - 34938*x^23 - 41936*x^21 - 1689*x^19 + 16480*x^17 + 5389*x^15 - 3690*x^1
3 - 1861*x^11 + 586*x^9 + 288*x^7 - 104*x^5 + 7*x^3 - sqrt(2)*(5*x^63 + 50*
x^61 + 298*x^59 + 1207*x^57 + 3407*x^55 + 6367*x^53 + 6040*x^51 - 4429*x^49
 - 25315*x^47 - 38326*x^45 - 15109*x^43 + 43760*x^41 + 83353*x^39 + 43063*x
^37 - 53901*x^35 - 101512*x^33 - 39859*x^31 + 55764*x^29 + 71846*x^27 + 849
3*x^25 - 38913*x^23 - 24125*x^21 + 7816*x^19 + 12629*x^17 + 1009*x^15 - 346
4*x^13 - 795*x^11 + 596*x^9 + 129*x^7 - 73*x^5 + 5*x^3)))*sqrt(2*sqrt(2) +
4) - sqrt(2)*(8*(96*x^63 + 960*x^61 + 5440*x^59 + 20640*x^57 + 49056*x^55 +
 51360*x^53 - 86656*x^51 - 440800*x^49 - 748192*x^47 - 344896*x^45 + 105052
8*x^43 + 2298624*x^41 + 1470048*x^39 - 1480800*x^37 - 3533152*x^35 - 190950
4*x^33 + 1775712*x^31 + 3135616*x^29 + 903360*x^27 - 1581856*x^25 - 1461344
*x^23 + 115232*x^21 + 733312*x^19 + 220384*x^17 - 187936*x^15 - 105984*x^13
 + 26720*x^11 + 22144*x^9 - 3744*x^7 - 1120*x^5 + 96*x^3 + sqrt(2)*(2*x^67
+ 22*x^65 + 118*x^63 + 388*x^61 + 242*x^59 - 4208*x^57 - 24094*x^55 - 72564
*x^53 - 137454*x^51 - 143968*x^49 + 17718*x^47 + 348676*x^45 + 593306*x^43
+ 379088*x^41 - 322846*x^39 - 921236*x^37 - 716514*x^35 + 200860*x^33 + 869
330*x^31 + 583276*x^29 - 209930*x^27 - 538448*x^25 - 193674*x^23 + 190852*x
^21 + 173798*x^19 - 18704*x^17 - 68382*x^15 - 6708*x^13 + 16990*x^11 + 2096
*x^9 - 3418*x^7 + 580*x^5 + 8*x^3 - sqrt(2)*(x^67 + 11*x^65 + 63*x^63 + 234
*x^61 + 817*x^59 + 2980*x^57 + 10977*x^55 + 33730*x^53 + 77885*x^51 + 12023
2*x^49 + 83383*x^47 - 117086*x^45 - 415651*x^43 - 508756*x^41 - 102871*x^39
 + 599162*x^37 + 876907*x^35 + 288542*x^33 - 600475*x^31 - 787586*x^29 - 13
```

6717*x^27 + 458364*x^25 + 357051*x^23 - 66490*x^21 - 195937*x^19 - 40496*x^17 + 57109*x^15 + 21718*x^13 - 11153*x^11 - 4332*x^9 + 2219*x^7 - 226*x^5 + 8*x^3 - x) - 2*x) - 64*sqrt(2)*(x^63 + 10*x^61 + 58*x^59 + 227*x^57 + 647*x^55 + 1339*x^53 + 1888*x^51 + 1247*x^49 - 1539*x^47 - 5654*x^45 - 7517*x^43 - 3320*x^41 + 5833*x^39 + 12123*x^37 + 8079*x^35 - 3712*x^33 - 11255*x^31 - 6900*x^29 + 3078*x^27 + 6825*x^25 + 2263*x^23 - 2521*x^21 - 2152*x^19 + 321*x^17 + 873*x^15 + 40*x^13 - 227*x^11 - 12*x^9 + 49*x^7 - 13*x^5 + x^3)) *sqrt(x^6 + x^4 + x^2 - 1) + (2^(3/4)*(2*x^72 + 24*x^70 + 136*x^68 + 460*x^66 + 346*x^64 - 5120*x^62 - 32464*x^60 - 110424*x^58 - 252208*x^56 - 389560*x^54 - 341144*x^52 + 79120*x^50 + 799728*x^48 + 1350768*x^46 + 1111936*x^44 - 74504*x^42 - 1522780*x^40 - 2032504*x^38 - 965272*x^36 + 913656*x^34 + 1874548*x^32 + 1026656*x^30 - 569936*x^28 - 1124264*x^26 - 356512*x^24 + 426616*x^22 + 348824*x^20 - 67920*x^18 - 144192*x^16 + 6448*x^14 + 39840*x^12 - 3832*x^10 - 9558*x^8 + 4384*x^6 - 624*x^4 - 4*x^2 - sqrt(2)*(x^72 + 12*x^70 + 70*x^68 + 252*x^66 + 797*x^64 + 2888*x^62 + 12884*x^60 + 51176*x^58 + 154948*x^56 + 333264*x^54 + 457924*x^52 + 205280*x^50 - 675088*x^48 - 1778088*x^46 - 1864860*x^44 + 4696*x^42 + 2766982*x^40 + 3558712*x^38 + 879248*x^36 - 2842552*x^34 - 3525414*x^32 - 580552*x^30 + 2245884*x^28 + 1903448*x^26 - 238828*x^24 - 1144976*x^22 - 379236*x^20 + 339296*x^18 + 231128*x^16 - 58328*x^14 - 66100*x^12 + 8808*x^10 + 11425*x^8 - 3332*x^6 + 202*x^4 - 4*x^2 + 1) + 2) + 32*2^(1/4)*(3*x^68 + 33*x^66 + 192*x^64 + 732*x^62 + 1667*x^60 + 999*x^58 - 7745*x^56 - 31675*x^54 - 60846*x^52 - 50794*x^50 + 47578*x^48 + 198202*x^46 + 236845*x^44 + 22241*x^42 - 317451*x^40 - 411513*x^38 - 79666*x^36 + 350286*x^34 + 387324*x^32 + 18696*x^30 - 277635*x^28 - 192343*x^26 + 56169*x^24 + 131091*x^22 + 30626*x^20 - 42978*x^18 - 24118*x^16 + 7618*x^14 + 7491*x^12 - 833*x^10 - 1325*x^8 + 241*x^6 + 27*x^4 - 3*x^2 - 2*sqrt(2)*(x^68 + 11*x^66 + 65*x^64 + 254*x^62 + 688*x^60 + 1229*x^58 + 1177*x^56 - 159*x^54 - 1409*x^52 + 1268*x^50 + 8391*x^48 + 9600*x^46 - 8210*x^44 - 35643*x^42 - 35291*x^40 + 12263*x^38 + 63145*x^36 + 50646*x^34 - 19181*x^32 - 62618*x^30 - 30972*x^28 + 23959*x^26 + 33055*x^24 + 3647*x^22 - 13803*x^20 - 6372*x^18 + 2797*x^16 + 2316*x^14 - 250*x^12 - 393*x^10 + 43*x^8 - 7*x^6 + 10*x^4 - x^2)))*sqrt(2*sqrt(2) + 4))*sqrt((8*x^8 + 8*x^6 + 8*x^4 - 2*2^(1/4)*(x^7 + x^5 + sqrt(2)*x^3 + x^3 - x)*sqrt(x^6 + x^4 + x^2 - 1)*sqrt(2*sqrt(2) + 4) - 8*x^2 + sqrt(2)*(x^12 + 2*x^10 + 5*x^8 + 2*x^6 + 3*x^4 - 4*x^2 + 1))/(x^12 + 2*x^10 + x^8 - 2*x^6 - x^4 + 1)) - 4*sqrt(2)*(x^72 + 12*x^70 + 70*x^68 + 252*x^66 + 289*x^64 - 2192*x^62 - 15608*x^60 - 55392*x^58 - 128904*x^56 - 195392*x^54 - 141296*x^52 + 145968*x^50 + 580196*x^48 + 769168*x^46 + 293368*x^44 - 697024*x^42 - 1316726*x^40 - 756072*x^38 + 603004*x^36 + 1358312*x^34 + 692582*x^32 - 542064*x^30 - 916232*x^28 - 242784*x^26 + 407920*x^24 + 337184*x^22 - 47200*x^20 - 157840*x^18 - 29164*x^16 + 44016*x^14 + 14280*x^12 - 8448*x^10 - 3635*x^8 + 2300*x^6 - 306*x^4 - 4*x^2 + 1) + 32*sqrt(2)*(4*x^68 + 44*x^66 + 270*x^64 + 1116*x^62 + 3296*x^60 + 6862*x^58 + 8948*x^56 + 2582*x^54 - 17690*x^52 - 42620*x^50 - 42854*x^48 + 6248*x^46 + 80340*x^44 + 102634*x^42 + 22328*x^40 - 99350*x^38 - 129878*x^36 - 25440*x^34 + 96146*x^32 + 97356*x^30 - 1816*x^28 - 66446*x^26 - 36828*x^24 + 16858*x^22 + 24794*x^20 + 2228*x^18 - 8218*x^16 - 2768*x^14 + 1700*x^12 + 822*x^10 - 272*x^8 - 122*x^6 + 50*x^4 - 4*x^2 - sqrt(2)*(3*x^68 + 33*x^66 + 199*x^64 + 802*x^62 + 2049*x^60 + 2372*x^58 - 4629*x^56 - 28494*x^54 - 65691*x^52 - 75144*x^50 + 8627*x^48 + 186586*x^46 + 305253*x^44 + 151484*x^42 - 254305*x^40 - 526582*x^38 - 293969*x^36 + 265274*x^34 + 530181*x^32 + 214406*x^30 - 247757*x^28 - 311140*x^26 - 30847*x^24 + 151478*x^22 + 79831*x^20 - 33520*x^18 - 37775*x^16 + 2014*x^14 + 9623*x^12 + 260*x^10 - 1531*x^8 + 174*x^6 + 34*x^4 - 3*x^2)) + 4)/(x^72 + 12*x^70 + 66*x^68 + 208*x^66 - 1231*x^64 - 15808*x^62 - 92240*x^60 - 353528*x^58 - 932160*x^56 - 1578120*x^54 - 1110816*x^52 + 2069088*x^50 + 7253380*x^48 + 8954960*x^46 + 771888*x^44 - 14239944*x^42 - 20370430*x^40 - 5559520*x^38 + 18409812*x^36 + 24321040*x^34 + 4205494*x^32 - 17469984*x^30 - 15945008*x^28 + 1564600*x^26 + 10561384*x^24 + 4478760*x^22 - 3021680*x^20 - 2938176*x^18 + 184980*x^16 + 919568*x^14 + 106256*x^12 - 172664*x^10 - 20075*x^8 + 19508*x^6 - 1606*x^4 + 1)) +

1/8*2^(3/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/4*(4*x^72 + 48*x^70 + 296*x^68 + 1184*x^66 + 4100*x^64 + 14336*x^62 + 48576*x^60 + 136416*x^58 + 243584*x^56 + 55200*x^54 - 1076416*x^52 - 3184640*x^50 - 3948592*x^48 + 641728*x^46 + 10292800*x^44 + 15128864*x^42 + 3787784*x^40 - 17971776*x^38 - 25671216*x^36 - 5595392*x^34 + 21212056*x^32 + 22744448*x^30 - 89792*x^28 - 16094688*x^26 - 9256416*x^24 + 4023008*x^22 + 6035840*x^20 + 544896*x^18 - 1868656*x^16 - 554560*x^14 + 332224*x^12 + 116192*x^10 - 36524*x^8 - 5232*x^6 + 840*x^4 - 32*x^2 - 4*sqrt(x^6 + x^4 + x^2 - 1)*(2^(3/4)*(4*x^67 + 44*x^65 + 254*x^63 + 956*x^61 + 2382*x^59 + 3356*x^57 + 350*x^55 - 7792*x^53 - 5766*x^51 + 40296*x^49 + 137174*x^47 + 186756*x^45 + 20238*x^43 - 366284*x^41 - 611162*x^39 - 279864*x^37 + 486370*x^35 + 879240*x^33 + 368218*x^31 - 485916*x^29 - 681958*x^27 - 123804*x^25 + 351834*x^23 + 245984*x^21 - 61090*x^19 - 118680*x^17 - 9422*x^15 + 30876*x^13 + 4842*x^11 - 5236*x^9 + 194*x^7 + 72*x^5 + 34*x^3 - sqrt(2)*(3*x^67 + 33*x^65 + 188*x^63 + 692*x^61 + 1414*x^59 - 98*x^57 - 11242*x^55 - 39830*x^53 - 72700*x^51 - 52668*x^49 + 87784*x^47 + 304536*x^45 + 361362*x^43 + 19402*x^41 - 562182*x^39 - 769994*x^37 - 196922*x^35 + 665266*x^33 + 842764*x^31 + 131812*x^29 - 569670*x^27 - 479358*x^25 + 68762*x^23 + 294150*x^21 + 92604*x^19 - 86324*x^17 - 54576*x^15 + 14160*x^13 + 14126*x^11 - 2090*x^9 - 1674*x^7 + 314*x^5 + 23*x^3 - 3*x) - 4*x) + 32*2^(1/4)*(7*x^63 + 70*x^61 + 419*x^59 + 1706*x^57 + 4942*x^55 + 9908*x^53 + 11825*x^51 + 580*x^49 - 28977*x^47 - 58322*x^45 - 45399*x^43 + 29706*x^41 + 111540*x^39 + 100724*x^37 - 20967*x^35 - 131220*x^33 - 100923*x^31 + 30198*x^29 + 100465*x^27 + 45314*x^25 - 34938*x^23 - 41936*x^21 - 1689*x^19 + 16480*x^17 + 5389*x^15 - 3690*x^13 - 1861*x^11 + 586*x^9 + 288*x^7 - 104*x^5 + 7*x^3 - sqrt(2)*(5*x^63 + 50*x^61 + 298*x^59 + 1207*x^57 + 3407*x^55 + 6367*x^53 + 6040*x^51 - 4429*x^49 - 25315*x^47 - 38326*x^45 - 15109*x^43 + 43760*x^41 + 83353*x^39 + 43063*x^37 - 53901*x^35 - 101512*x^33 - 39859*x^31 + 55764*x^29 + 71846*x^27 + 8493*x^25 - 38913*x^23 - 24125*x^21 + 7816*x^19 + 12629*x^17 + 1009*x^15 - 3464*x^13 - 795*x^11 + 596*x^9 + 129*x^7 - 73*x^5 + 5*x^3)))*sqrt(2*sqrt(2) + 4) - sqrt(2)*(8*(96*x^63 + 960*x^61 + 5440*x^59 + 20640*x^57 + 49056*x^55 + 51360*x^53 - 86656*x^51 - 440800*x^49 - 748192*x^47 - 344896*x^45 + 1050528*x^43 + 2298624*x^41 + 1470048*x^39 - 1480800*x^37 - 3533152*x^35 - 1909504*x^33 + 1775712*x^31 + 3135616*x^29 + 903360*x^27 - 1581856*x^25 - 1461344*x^23 + 115232*x^21 + 733312*x^19 + 220384*x^17 - 187936*x^15 - 105984*x^13 + 26720*x^11 + 22144*x^9 - 3744*x^7 - 1120*x^5 + 96*x^3 + sqrt(2)*(2*x^67 + 22*x^65 + 118*x^63 + 388*x^61 + 242*x^59 - 4208*x^57 - 24094*x^55 - 72564*x^53 - 137454*x^51 - 143968*x^49 + 17718*x^47 + 348676*x^45 + 593306*x^43 + 379088*x^41 - 322846*x^39 - 921236*x^37 - 716514*x^35 + 200860*x^33 + 869330*x^31 + 583276*x^29 - 209930*x^27 - 538448*x^25 - 193674*x^23 + 190852*x^21 + 173798*x^19 - 18704*x^17 - 68382*x^15 - 6708*x^13 + 16990*x^11 + 2096*x^9 - 3418*x^7 + 580*x^5 + 8*x^3 - sqrt(2)*(x^67 + 11*x^65 + 63*x^63 + 234*x^61 + 817*x^59 + 2980*x^57 + 10977*x^55 + 33730*x^53 + 77885*x^51 + 120232*x^49 + 83383*x^47 - 117086*x^45 - 415651*x^43 - 508756*x^41 - 102871*x^39 + 599162*x^37 + 876907*x^35 + 288542*x^33 - 600475*x^31 - 787586*x^29 - 136717*x^27 + 458364*x^25 + 357051*x^23 - 66490*x^21 - 195937*x^19 - 40496*x^17 + 57109*x^15 + 21718*x^13 - 11153*x^11 - 4332*x^9 + 2219*x^7 - 226*x^5 + 8*x^3 - x) - 2*x) - 64*sqrt(2)*(x^63 + 10*x^61 + 58*x^59 + 227*x^57 + 647*x^55 + 1339*x^53 + 1888*x^51 + 1247*x^49 - 1539*x^47 - 5654*x^45 - 7517*x^43 - 3320*x^41 + 5833*x^39 + 12123*x^37 + 8079*x^35 - 3712*x^33 - 11255*x^31 - 6900*x^29 + 3078*x^27 + 6825*x^25 + 2263*x^23 - 2521*x^21 - 2152*x^19 + 321*x^17 + 873*x^15 + 40*x^13 - 227*x^11 - 12*x^9 + 49*x^7 - 13*x^5 + x^3))*sqrt(x^6 + x^4 + x^2 - 1) - (2^(3/4)*(2*x^72 + 24*x^70 + 136*x^68 + 460*x^66 + 346*x^64 - 5120*x^62 - 32464*x^60 - 110424*x^58 - 252208*x^56 - 389560*x^54 - 341144*x^52 + 79120*x^50 + 799728*x^48 + 1350768*x^46 + 1111936*x^44 - 74504*x^42 - 1522780*x^40 - 2032504*x^38 - 965272*x^36 + 913656*x^34 + 1874548*x^32 + 1026656*x^30 - 569936*x^28 - 1124264*x^26 - 356512*x^24 + 426616*x^22 + 348824*x^20 - 67920*x^18 - 144192*x^16 + 6448*x^14 + 39840*x^12 - 3832*x^10 - 9558*x^8 + 4384*x^6 - 624*x^4 - 4*x^2 - sqrt(2)*(x^72 + 12*x^70 + 70*x^68 + 252*x^66 + 797*x^64 + 288

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8*x^62 + 12884*x^60 + 51176*x^58 + 154948*x^56 + 333264*x^54 + 457924*x^52
+ 205280*x^50 - 675088*x^48 - 1778088*x^46 - 1864860*x^44 + 4696*x^42 + 276
6982*x^40 + 3558712*x^38 + 879248*x^36 - 2842552*x^34 - 3525414*x^32 - 5805
52*x^30 + 2245884*x^28 + 1903448*x^26 - 238828*x^24 - 1144976*x^22 - 379236
*x^20 + 339296*x^18 + 231128*x^16 - 58328*x^14 - 66100*x^12 + 8808*x^10 + 1
1425*x^8 - 3332*x^6 + 202*x^4 - 4*x^2 + 1) + 2) + 32*2^(1/4)*(3*x^68 + 33*x
^66 + 192*x^64 + 732*x^62 + 1667*x^60 + 999*x^58 - 7745*x^56 - 31675*x^54 -
 60846*x^52 - 50794*x^50 + 47578*x^48 + 198202*x^46 + 236845*x^44 + 22241*x
^42 - 317451*x^40 - 411513*x^38 - 79666*x^36 + 350286*x^34 + 387324*x^32 +
18696*x^30 - 277635*x^28 - 192343*x^26 + 56169*x^24 + 131091*x^22 + 30626*x
^20 - 42978*x^18 - 24118*x^16 + 7618*x^14 + 7491*x^12 - 833*x^10 - 1325*x^8
 + 241*x^6 + 27*x^4 - 3*x^2 - 2*sqrt(2)*(x^68 + 11*x^66 + 65*x^64 + 254*x^6
2 + 688*x^60 + 1229*x^58 + 1177*x^56 - 159*x^54 - 1409*x^52 + 1268*x^50 + 8
391*x^48 + 9600*x^46 - 8210*x^44 - 35643*x^42 - 35291*x^40 + 12263*x^38 + 6
3145*x^36 + 50646*x^34 - 19181*x^32 - 62618*x^30 - 30972*x^28 + 23959*x^26
+ 33055*x^24 + 3647*x^22 - 13803*x^20 - 6372*x^18 + 2797*x^16 + 2316*x^14 -
 250*x^12 - 393*x^10 + 43*x^8 - 7*x^6 + 10*x^4 - x^2)))*sqrt(2*sqrt(2) + 4)
)*sqrt((8*x^8 + 8*x^6 + 8*x^4 + 2*2^(1/4)*(x^7 + x^5 + sqrt(2)*x^3 + x^3 -
x)*sqrt(x^6 + x^4 + x^2 - 1)*sqrt(2*sqrt(2) + 4) - 8*x^2 + sqrt(2)*(x^12 +
2*x^10 + 5*x^8 + 2*x^6 + 3*x^4 - 4*x^2 + 1))/(x^12 + 2*x^10 + x^8 - 2*x^6 -
 x^4 + 1)) - 4*sqrt(2)*(x^72 + 12*x^70 + 70*x^68 + 252*x^66 + 289*x^64 - 21
92*x^62 - 15608*x^60 - 55392*x^58 - 128904*x^56 - 195392*x^54 - 141296*x^52
 + 145968*x^50 + 580196*x^48 + 769168*x^46 + 293368*x^44 - 697024*x^42 - 13
16726*x^40 - 756072*x^38 + 603004*x^36 + 1358312*x^34 + 692582*x^32 - 54206
4*x^30 - 916232*x^28 - 242784*x^26 + 407920*x^24 + 337184*x^22 - 47200*x^20
 - 157840*x^18 - 29164*x^16 + 44016*x^14 + 14280*x^12 - 8448*x^10 - 3635*x^
8 + 2300*x^6 - 306*x^4 - 4*x^2 + 1) + 32*sqrt(2)*(4*x^68 + 44*x^66 + 270*x^
64 + 1116*x^62 + 3296*x^60 + 6862*x^58 + 8948*x^56 + 2582*x^54 - 17690*x^52
 - 42620*x^50 - 42854*x^48 + 6248*x^46 + 80340*x^44 + 102634*x^42 + 22328*x
^40 - 99350*x^38 - 129878*x^36 - 25440*x^34 + 96146*x^32 + 97356*x^30 - 181
6*x^28 - 66446*x^26 - 36828*x^24 + 16858*x^22 + 24794*x^20 + 2228*x^18 - 82
18*x^16 - 2768*x^14 + 1700*x^12 + 822*x^10 - 272*x^8 - 122*x^6 + 50*x^4 - 4
*x^2 - sqrt(2)*(3*x^68 + 33*x^66 + 199*x^64 + 802*x^62 + 2049*x^60 + 2372*x
^58 - 4629*x^56 - 28494*x^54 - 65691*x^52 - 75144*x^50 + 8627*x^48 + 186586
*x^46 + 305253*x^44 + 151484*x^42 - 254305*x^40 - 526582*x^38 - 293969*x^36
 + 265274*x^34 + 530181*x^32 + 214406*x^30 - 247757*x^28 - 311140*x^26 - 30
847*x^24 + 151478*x^22 + 79831*x^20 - 33520*x^18 - 37775*x^16 + 2014*x^14 +
 9623*x^12 + 260*x^10 - 1531*x^8 + 174*x^6 + 34*x^4 - 3*x^2)) + 4)/(x^72 +
12*x^70 + 66*x^68 + 208*x^66 - 1231*x^64 - 15808*x^62 - 92240*x^60 - 353528
*x^58 - 932160*x^56 - 1578120*x^54 - 1110816*x^52 + 2069088*x^50 + 7253380*
x^48 + 8954960*x^46 + 771888*x^44 - 14239944*x^42 - 20370430*x^40 - 5559520
*x^38 + 18409812*x^36 + 24321040*x^34 + 4205494*x^32 - 17469984*x^30 - 1594
5008*x^28 + 1564600*x^26 + 10561384*x^24 + 4478760*x^22 - 3021680*x^20 - 29
38176*x^18 + 184980*x^16 + 919568*x^14 + 106256*x^12 - 172664*x^10 - 20075*
x^8 + 19508*x^6 - 1606*x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{6}+x^{4}+1\right)\sqrt{x^{6}+x^{4}+x^{2}-1}}{x^{12}+2\,x^{10}+x^{8}-2\,x^{6}-x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^4+x^2-1)^(1/2)*(2*x^6+x^4+1)/(x^12+2*x^10+x^8-2*x^6-x^4+1)
,x, algorithm="giac")
```

```
[Out] integrate((2*x^6 + x^4 + 1)*sqrt(x^6 + x^4 + x^2 - 1)/(x^12 + 2*x^10 + x^8
 - 2*x^6 - x^4 + 1), x)
```

maple [C] time = 3.66, size = 589, normalized size = 8.07

$$-\frac{\mathrm{RootOf}\left(\_Z^2+16\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^2+2\right)\ln\left(\frac{4\,\mathrm{RootOf}\left(\_Z^2+16\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^2+2\right)\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^2x^6+64\,\mathrm{RootOf}\left(\_Z^2+16\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^2+2\right)\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^4x^2+4\,\mathrm{RootOf}\left(\_Z^2+16\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^2+2\right)\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^2x^4+12\,\mathrm{RootOf}\left(\_Z^2+16\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^2+2\right)\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^2x^2+16\sqrt{x^6+x^4+x^2-1}\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^2x-4\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^2\mathrm{RootOf}\left(\_Z^2+16\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^2+2\right)+\sqrt{x^6+x^4+x^2-1}\,x}{-x^6+16x^2\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^2-x^4+x^2+1}\right)}{4}-\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)\ln\left(-\frac{16\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^3x^6-256\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^5x^2+16\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^3x^4+2\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)x^6-16\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^3x^2+2\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)x^4+16\sqrt{x^6+x^4+x^2-1}\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^2x-16\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^3+2x^2\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)+\sqrt{x^6+x^4+x^2-1}\,x-2\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)}{x^6+16x^2\,\mathrm{RootOf}\left(128\_Z^4+16\_Z^2+1\right)^2+x^4+x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+x^4+x^2-1)^(1/2)*(2*x^6+x^4+1)/(x^12+2*x^10+x^8-2*x^6-x^4+1),x)
```

```
[Out] -1/4*RootOf(_Z^2+16*RootOf(128*_Z^4+16*_Z^2+1)^2+2)*ln((4*RootOf(_Z^2+16*Ro
otOf(128*_Z^4+16*_Z^2+1)^2+2)*RootOf(128*_Z^4+16*_Z^2+1)^2*x^6+64*RootOf(_Z
^2+16*RootOf(128*_Z^4+16*_Z^2+1)^2+2)*RootOf(128*_Z^4+16*_Z^2+1)^4*x^2+4*Ro
otOf(_Z^2+16*RootOf(128*_Z^4+16*_Z^2+1)^2+2)*RootOf(128*_Z^4+16*_Z^2+1)^2*x
^4+12*RootOf(_Z^2+16*RootOf(128*_Z^4+16*_Z^2+1)^2+2)*RootOf(128*_Z^4+16*_Z^
2+1)^2*x^2+16*(x^6+x^4+x^2-1)^(1/2)*RootOf(128*_Z^4+16*_Z^2+1)^2*x-4*RootOf
(128*_Z^4+16*_Z^2+1)^2*RootOf(_Z^2+16*RootOf(128*_Z^4+16*_Z^2+1)^2+2)+(x^6+
x^4+x^2-1)^(1/2)*x)/(-x^6+16*x^2*RootOf(128*_Z^4+16*_Z^2+1)^2-x^4+x^2+1))-R
ootOf(128*_Z^4+16*_Z^2+1)*ln(-(16*RootOf(128*_Z^4+16*_Z^2+1)^3*x^6-256*Root
Of(128*_Z^4+16*_Z^2+1)^5*x^2+16*RootOf(128*_Z^4+16*_Z^2+1)^3*x^4+2*RootOf(1
28*_Z^4+16*_Z^2+1)*x^6-16*RootOf(128*_Z^4+16*_Z^2+1)^3*x^2+2*RootOf(128*_Z^
4+16*_Z^2+1)*x^4+16*(x^6+x^4+x^2-1)^(1/2)*RootOf(128*_Z^4+16*_Z^2+1)^2*x-16
*RootOf(128*_Z^4+16*_Z^2+1)^3+2*x^2*RootOf(128*_Z^4+16*_Z^2+1)+(x^6+x^4+x^2
-1)^(1/2)*x-2*RootOf(128*_Z^4+16*_Z^2+1))/(x^6+16*x^2*RootOf(128*_Z^4+16*_Z
^2+1)^2+x^4+x^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{6}+x^{4}+1\right)\sqrt{x^{6}+x^{4}+x^{2}-1}}{x^{12}+2\,x^{10}+x^{8}-2\,x^{6}-x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^4+x^2-1)^(1/2)*(2*x^6+x^4+1)/(x^12+2*x^10+x^8-2*x^6-x^4+1)
,x, algorithm="maxima")
```

```
[Out] integrate((2*x^6 + x^4 + 1)*sqrt(x^6 + x^4 + x^2 - 1)/(x^12 + 2*x^10 + x^8
 - 2*x^6 - x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^6+x^4+1\right)\,\sqrt{x^6+x^4+x^2-1}}{x^{12}+2\,x^{10}+x^8-2\,x^6-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 2*x^6 + 1)*(x^2 + x^4 + x^6 - 1)^(1/2))/(x^8 - 2*x^6 - x^4 + 2*
x^10 + x^12 + 1),x)
```

```
[Out] int(((x^4 + 2*x^6 + 1)*(x^2 + x^4 + x^6 - 1)^(1/2))/(x^8 - 2*x^6 - x^4 + 2*
x^10 + x^12 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}+1\right)\left(2x^{4}-x^{2}+1\right)\sqrt{x^{6}+x^{4}+x^{2}-1}}{x^{12}+2x^{10}+x^{8}-2x^{6}-x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+x**4+x**2-1)**(1/2)*(2*x**6+x**4+1)/(x**12+2*x**10+x**8-2*x
**6-x**4+1),x)
```

```
[Out] Integral((x**2 + 1)*(2*x**4 - x**2 + 1)*sqrt(x**6 + x**4 + x**2 - 1)/(x**12
 + 2*x**10 + x**8 - 2*x**6 - x**4 + 1), x)
```

3.890 $\int \frac{\sqrt{x^2-x\sqrt{-x+x^2}}}{x^3}\,dx$

Optimal. Leaf size=73

$$\frac{4(x-3)\sqrt{-x\left(\sqrt{x^2-x}-x\right)}}{15x^2}-\frac{4\sqrt{x^2-x}\,\sqrt{-x\left(\sqrt{x^2-x}-x\right)}}{15x^2}$$

Rubi [F] time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{x^2-x\sqrt{-x+x^2}}}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[x^2 - x*Sqrt[-x + x^2]]/x^3,x]
[Out] Defer[Int][Sqrt[x^2 - x*Sqrt[-x + x^2]]/x^3, x]
```

Rubi steps

$$\int \frac{\sqrt{x^2-x\sqrt{-x+x^2}}}{x^3}\,dx=\int \frac{\sqrt{x^2-x\sqrt{-x+x^2}}}{x^3}\,dx$$

Mathematica [A] time = 0.21, size = 58, normalized size = 0.79

$$\frac{4x\left(4x^2-\left(4\sqrt{(x-1)x}+9\right)x+7\sqrt{(x-1)x}+3\right)}{15\left(x\left(x-\sqrt{(x-1)x}\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 - x*Sqrt[-x + x^2]]/x^3,x]
[Out] (4*x*(3 + 4*x^2 + 7*Sqrt[(-1 + x)*x] - x*(9 + 4*Sqrt[(-1 + x)*x])))/(15*(x*
(x - Sqrt[(-1 + x)*x]))^(3/2))
```

IntegrateAlgebraic [A] time = 3.42, size = 73, normalized size = 1.00

$$\frac{4(x-3)\sqrt{-x\left(\sqrt{x^2-x}-x\right)}}{15x^2}-\frac{4\sqrt{x^2-x}\,\sqrt{-x\left(\sqrt{x^2-x}-x\right)}}{15x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2 - x*Sqrt[-x + x^2]]/x^3,x]
[Out] (4*(-3 + x)*Sqrt[-(x*(-x + Sqrt[-x + x^2]))])/(15*x^2) - (4*Sqrt[-x + x^2]*
Sqrt[-(x*(-x + Sqrt[-x + x^2]))])/(15*x^2)
```

fricas [A] time = 0.41, size = 37, normalized size = 0.51

$$\frac{4\sqrt{x^2-\sqrt{x^2-x}\,x}\left(x-\sqrt{x^2-x}-3\right)}{15\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x*(x^2-x)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
[Out] 4/15*sqrt(x^2 - sqrt(x^2 - x)*x)*(x - sqrt(x^2 - x) - 3)/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 - \sqrt{x^2 - x}\,x}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x*(x^2-x)^(1/2))^(1/2)/x^3,x, algorithm="giac")
[Out] integrate(sqrt(x^2 - sqrt(x^2 - x)*x)/x^3, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 - x\sqrt{x^2 - x}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x*(x^2-x)^(1/2))^(1/2)/x^3,x)
[Out] int((x^2-x*(x^2-x)^(1/2))^(1/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 - \sqrt{x^2 - x}\,x}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x*(x^2-x)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(x^2 - sqrt(x^2 - x)*x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^2 - x\,\sqrt{x^2 - x}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x*(x^2 - x)^(1/2))^(1/2)/x^3,x)
[Out] int((x^2 - x*(x^2 - x)^(1/2))^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(x - \sqrt{x^2 - x}\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x*(x**2-x)**(1/2))**(1/2)/x**3,x)
[Out] Integral(sqrt(x*(x - sqrt(x**2 - x)))/x**3, x)
```

3.891 $\int \frac{1}{x\left(1+x^2\right)^{2/3}}\,dx$

Optimal. Leaf size=74

$$\frac{1}{2}\log\left(\sqrt[3]{x^2+1}-1\right)-\frac{1}{4}\log\left(\left(x^2+1\right)^{2/3}+\sqrt[3]{x^2+1}+1\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 0.73, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {266, 57, 618, 204, 31}

$$\frac{3}{4}\log\left(1-\sqrt[3]{x^2+1}\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^2)^(2/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + 2*(1 + x^2)^(1/3))/Sqrt[3]]) - Log[x]/2 + (3*Log[
1 - (1 + x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(1+x^2\right)^{2/3}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{2/3}}\,dx, x, x^2\right)$$
$$= -\frac{\log(x)}{2} - \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+x^2}\right) - \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1+x^2}\right)$$
$$= -\frac{\log(x)}{2} + \frac{3}{4}\log\left(1-\sqrt[3]{1+x^2}\right) + \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+x^2}\right)$$
$$= -\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^2}}{\sqrt{3}}\right) - \frac{\log(x)}{2} + \frac{3}{4}\log\left(1-\sqrt[3]{1+x^2}\right)$$

Mathematica [A] time = 0.02, size = 73, normalized size = 0.99

$$\frac{1}{2}\log\left(1-\sqrt[3]{x^2+1}\right) - \frac{1}{4}\log\left(\left(x^2+1\right)^{2/3} + \sqrt[3]{x^2+1} + 1\right) - \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^2)^(2/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + 2*(1 + x^2)^(1/3))/Sqrt[3]]) + Log[1 - (1 + x^2)^
(1/3)]/2 - Log[1 + (1 + x^2)^(1/3) + (1 + x^2)^(2/3)]/4
```

IntegrateAlgebraic [A] time = 0.05, size = 74, normalized size = 1.00

$$\frac{1}{2}\log\left(\sqrt[3]{x^2+1}-1\right) - \frac{1}{4}\log\left(\left(x^2+1\right)^{2/3} + \sqrt[3]{x^2+1} + 1\right) - \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(1 + x^2)^(2/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 + x^2)^(1/3))/Sqrt[3]]) + Log[-1 + (
1 + x^2)^(1/3)]/2 - Log[1 + (1 + x^2)^(1/3) + (1 + x^2)^(2/3)]/4
```

fricas [A] time = 0.43, size = 56, normalized size = 0.76

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^2+1\right)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right) - \frac{1}{4}\log\left(\left(x^2+1\right)^{\frac{2}{3}} + \left(x^2+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan(2/3*sqrt(3)*(x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/4*log((x
^2 + 1)^(2/3) + (x^2 + 1)^(1/3) + 1) + 1/2*log((x^2 + 1)^(1/3) - 1)
```

giac [A] time = 0.34, size = 54, normalized size = 0.73

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^2+1\right)^{\frac{1}{3}} + 1\right)\right) - \frac{1}{4}\log\left(\left(x^2+1\right)^{\frac{2}{3}} + \left(x^2+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^(2/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 + 1)^(1/3) + 1)) - 1/4*log((x^2 + 1
)^(2/3) + (x^2 + 1)^(1/3) + 1) + 1/2*log((x^2 + 1)^(1/3) - 1)
```

maple [C] time = 0.26, size = 46, normalized size = 0.62

$$\frac{-\frac{2\Gamma\left(\frac{2}{3}\right)x^2\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],-x^2\right)}{3}+\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)\right)\Gamma\left(\frac{2}{3}\right)}{2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+1)^(2/3),x)
```

```
[Out] 1/2/GAMMA(2/3)*(-2/3*GAMMA(2/3)*x^2*hypergeom([1,1,5/3],[2,2],-x^2)+(1/6*Pi
*3^(1/2)-3/2*ln(3)+2*ln(x))*GAMMA(2/3))
```

maxima [A] time = 0.44, size = 54, normalized size = 0.73

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(x^2+1\right)^{\frac{2}{3}}+\left(x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 + 1)^(1/3) + 1)) - 1/4*log((x^2 + 1
)^(2/3) + (x^2 + 1)^(1/3) + 1) + 1/2*log((x^2 + 1)^(1/3) - 1)
```

mupad [B] time = 0.87, size = 70, normalized size = 0.95

$$\frac{\ln\left(\frac{9\left(x^2+1\right)^{1/3}}{4}-\frac{9}{4}\right)}{2}+\ln\left(\frac{9\left(x^2+1\right)^{1/3}}{2}+\frac{9}{4}-\frac{\sqrt{3}\,9\mathrm{i}}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-\ln\left(\frac{9\left(x^2+1\right)^{1/3}}{2}+\frac{9}{4}+\frac{\sqrt{3}\,9\mathrm{i}}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 + 1)^(2/3)),x)
```

```
[Out] log((9*(x^2 + 1)^(1/3))/4 - 9/4)/2 + log((9*(x^2 + 1)^(1/3))/2 - (3^(1/2)*9
i)/4 + 9/4)*((3^(1/2)*1i)/4 - 1/4) - log((3^(1/2)*9i)/4 + (9*(x^2 + 1)^(1/3
))/2 + 9/4)*((3^(1/2)*1i)/4 + 1/4)
```

sympy [C] time = 0.82, size = 32, normalized size = 0.43

$$-\frac{\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^2}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+1)**(2/3),x)
```

```
[Out] -gamma(2/3)*hyper((2/3, 2/3), (5/3,), exp_polar(I*pi)/x**2)/(2*x**(4/3)*gam
ma(5/3))
```

3.892 $\int \frac{1}{x\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=74

$$\frac{1}{2}\log\left(\sqrt[3]{x^2+1}-1\right)-\frac{1}{4}\log\left(\left(x^2+1\right)^{2/3}+\sqrt[3]{x^2+1}+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 0.73, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {266, 55, 618, 204, 31}

$$\frac{3}{4}\log\left(1-\sqrt[3]{x^2+1}\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 + x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3*Log[1 - (1 + x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt[3]{1+x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx,x,x^2\right)\\
&= -\frac{\log(x)}{2}-\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1+x^2}\right)+\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,\sqrt[3]{1+x^2}\right)\\
&= -\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1+x^2}\right)-\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{1+x^2}\right)\\
&= \frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^2}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1+x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.72

$$
\frac{1}{2}\left(\frac{3}{2}\log\left(1-\sqrt[3]{x^2+1}\right)+\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}+1}{\sqrt{3}}\right)-\log(x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 + x^2)^(1/3))/Sqrt[3]] - Log[x] + (3*Log[1 - (1 +
 x^2)^(1/3)])/2)/2
```

IntegrateAlgebraic [A] time = 0.05, size = 74, normalized size = 1.00

$$
\frac{1}{2}\log\left(\sqrt[3]{x^2+1}-1\right)-\frac{1}{4}\log\left(\left(x^2+1\right)^{2/3}+\sqrt[3]{x^2+1}+1\right)+\frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(1 + x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 + x^2)^(1/3))/Sqrt[3]])/2 + Log[-1 + (1 +
 x^2)^(1/3)]/2 - Log[1 + (1 + x^2)^(1/3) + (1 + x^2)^(2/3)]/4
```

fricas [A] time = 0.43, size = 56, normalized size = 0.76

$$
\frac{1}{2}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(x^2+1\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right)-\frac{1}{4}\,\log\left(\left(x^2+1\right)^{\frac{2}{3}}+\left(x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\,\log\left(\left(x^2+1\right)^{\frac{1}{3}}-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan(2/3*sqrt(3)*(x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/4*log((x^
2 + 1)^(2/3) + (x^2 + 1)^(1/3) + 1) + 1/2*log((x^2 + 1)^(1/3) - 1)
```

giac [A] time = 0.43, size = 54, normalized size = 0.73

$$
\frac{1}{2}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\,\log\left(\left(x^2+1\right)^{\frac{2}{3}}+\left(x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\,\log\left(\left(x^2+1\right)^{\frac{1}{3}}-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 + 1)^(1/3) + 1)) - 1/4*log((x^2 + 1)
^(2/3) + (x^2 + 1)^(1/3) + 1) + 1/2*log((x^2 + 1)^(1/3) - 1)
```

maple [C] time = 0.24, size = 63, normalized size = 0.85

$$\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^2\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],-x^2\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{4\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+1)^(1/3),x)
```

```
[Out] 1/4/Pi*3^(1/2)*GAMMA(2/3)*(-2/9*Pi*3^(1/2)/GAMMA(2/3)*x^2*hypergeom([1,1,4/
3],[2,2],-x^2)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x))*Pi*3^(1/2)/GAMMA(2/3
))
```

maxima [A] time = 0.47, size = 54, normalized size = 0.73

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(x^2+1\right)^{\frac{2}{3}}+\left(x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 + 1)^(1/3) + 1)) - 1/4*log((x^2 + 1)
^(2/3) + (x^2 + 1)^(1/3) + 1) + 1/2*log((x^2 + 1)^(1/3) - 1)
```

mupad [B] time = 0.87, size = 80, normalized size = 1.08

$$\frac{\ln\left(\frac{9\left(x^2+1\right)^{1/3}}{4}-\frac{9}{4}\right)}{2}+\ln\left(\frac{9\left(x^2+1\right)^{1/3}}{4}-9\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)^2\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-\ln\left(\frac{9\left(x^2+1\right)^{1/3}}{4}-9\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)^2\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 + 1)^(1/3)),x)
```

```
[Out] log((9*(x^2 + 1)^(1/3))/4 - 9/4)/2 + log((9*(x^2 + 1)^(1/3))/4 - 9*((3^(1/2
)*1i)/4 - 1/4)^2)*((3^(1/2)*1i)/4 - 1/4) - log((9*(x^2 + 1)^(1/3))/4 - 9*((
3^(1/2)*1i)/4 + 1/4)^2)*((3^(1/2)*1i)/4 + 1/4)
```

sympy [C] time = 0.81, size = 32, normalized size = 0.43

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{e^{i\pi}}{x^2}\right)}{2x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+1)**(1/3),x)
```

```
[Out] -gamma(1/3)*hyper((1/3, 1/3), (4/3,), exp_polar(I*pi)/x**2)/(2*x**(2/3)*gam
ma(4/3))
```

3.893 $\int \frac{\sqrt{-1+x^3}\left(2+x^3\right)\left(-1-x^2+x^3\right)^2}{x^6\left(-2-3x^2+2x^3\right)}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{x^3-1}\left(12x^6-10x^5+15x^4-24x^3+10x^2+12\right)}{60x^5}-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt{x^3-1}}\right)$$

Rubi [F] time = 1.35, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-1+x^3}\left(2+x^3\right)\left(-1-x^2+x^3\right)^2}{x^6\left(-2-3x^2+2x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-1 + x^3]*(2 + x^3)*(-1 - x^2 + x^3)^2)/(x^6*(-2 - 3*x^2 + 2*x^3)
),x]
```

```
[Out] -1/6*Sqrt[-1 + x^3] + (3*Sqrt[-1 + x^3])/(4*(1 - Sqrt[3] - x)) + Sqrt[-1 +
x^3]/(5*x^5) + Sqrt[-1 + x^3]/(6*x^3) - (2*Sqrt[-1 + x^3])/(5*x^2) + Sqrt[-
1 + x^3]/(4*x) + (x*Sqrt[-1 + x^3])/5 - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x
)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x
)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(8*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^
2)]*Sqrt[-1 + x^3]) + (3^(3/4)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)
^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])
/(2*Sqrt[2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (3*Defer
[Int][Sqrt[-1 + x^3]/(2 + 3*x^2 - 2*x^3), x])/4 + (3*Defer[Int][(x*Sqrt[-1
+ x^3])/(-2 - 3*x^2 + 2*x^3), x])/4
```

Rubi steps

$$\int \frac{\sqrt{-1+x^3}\left(2+x^3\right)\left(-1-x^2+x^3\right)^2}{x^6\left(-2-3x^2+2x^3\right)}\,dx = \int \left(\frac{1}{2}\sqrt{-1+x^3}-\frac{\sqrt{-1+x^3}}{x^6}-\frac{\sqrt{-1+x^3}}{2x^4}+\frac{\sqrt{-1+x^3}}{2x^3}-\frac{\sqrt{-1+x^3}}{4x^2}\right.$$

$$=-\left(\frac{1}{4}\int \frac{\sqrt{-1+x^3}}{x^2}\,dx\right)-\frac{1}{4}\int \frac{\sqrt{-1+x^3}}{x}\,dx+\frac{1}{2}\int \sqrt{-1+x^3}\,dx-$$

$$=\frac{\sqrt{-1+x^3}}{5x^5}-\frac{\sqrt{-1+x^3}}{4x^2}+\frac{\sqrt{-1+x^3}}{4x}+\frac{1}{5}x\sqrt{-1+x^3}-\frac{1}{12}\text{Subst}\Big($$

$$=-\frac{1}{6}\sqrt{-1+x^3}+\frac{\sqrt{-1+x^3}}{5x^5}+\frac{\sqrt{-1+x^3}}{6x^3}-\frac{2\sqrt{-1+x^3}}{5x^2}+\frac{\sqrt{-1+x^3}}{4x}$$

$$=-\frac{1}{6}\sqrt{-1+x^3}+\frac{3\sqrt{-1+x^3}}{4\left(1-\sqrt{3}-x\right)}+\frac{\sqrt{-1+x^3}}{5x^5}+\frac{\sqrt{-1+x^3}}{6x^3}-\frac{2\sqrt{-1}}{5}$$

Mathematica [C] time = 6.22, size = 1814, normalized size = 24.51

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[-1 + x^3]*(2 + x^3)*(-1 - x^2 + x^3)^2)/(x^6*(-2 - 3*x^2 +
2*x^3)),x]
```

```
[Out] (-1/6 + 1/(5*x^5) + 1/(6*x^3) - 2/(5*x^2) + 1/(4*x) + x/5)*Sqrt[-1 + x^3] +
 (3*((Sqrt[(-1 + x)/(-1 - (-1)^(1/3))]*((-1)^(1/3) + x)*Sqrt[(-(-1)^(2/3) +
 x)/(-(-1)^(1/3) - (-1)^(2/3))]*EllipticF[ArcSin[Sqrt[-(((-1)^(2/3)*((-1)^(
1/3) + x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)])/(Sqrt[((-1)^(1/3) + x)/((-1)^(
1/3) + (-1)^(2/3))]*Sqrt[-1 + x^3]) + (2*(-(-1)^(1/3) - (-1)^(2/3))*Sqrt[(-
1 + x)/(-1 - (-1)^(1/3))]*Sqrt[(((-1)^(2/3) - x)*((-1)^(1/3) + x))/(-(-1)^(
1/3) - (-1)^(2/3))^2]*EllipticPi[((-1)^(1/3) + (-1)^(2/3))/((-1)^(1/3) + Ro
ot[-2 - 3*#1^2 + 2*#1^3 & , 1, 0]), ArcSin[Sqrt[-(((-1)^(2/3)*((-1)^(1/3) +
 x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)])/(Sqrt[-1 + x^3]*((-1)^(1/3) + Root[-
2 - 3*#1^2 + 2*#1^3 & , 1, 0])*(Root[-2 - 3*#1^2 + 2*#1^3 & , 1, 0] - Root[
-2 - 3*#1^2 + 2*#1^3 & , 2, 0])*(Root[-2 - 3*#1^2 + 2*#1^3 & , 1, 0] - Root
[-2 - 3*#1^2 + 2*#1^3 & , 3, 0])) + ((-(-1)^(1/3) - (-1)^(2/3))*Sqrt[(-1 +
x)/(-1 - (-1)^(1/3))]*Sqrt[(((-1)^(2/3) - x)*((-1)^(1/3) + x))/(-(-1)^(1/3)
 - (-1)^(2/3))^2]*EllipticPi[((-1)^(1/3) + (-1)^(2/3))/((-1)^(1/3) + Root[-
2 - 3*#1^2 + 2*#1^3 & , 1, 0]), ArcSin[Sqrt[-(((-1)^(2/3)*((-1)^(1/3) + x))
/(1 + (-1)^(1/3)))]], (-1)^(1/3)]*Root[-2 - 3*#1^2 + 2*#1^3 & , 1, 0]^3)/(S
qrt[-1 + x^3]*((-1)^(1/3) + Root[-2 - 3*#1^2 + 2*#1^3 & , 1, 0])*(Root[-2 -
 3*#1^2 + 2*#1^3 & , 1, 0] - Root[-2 - 3*#1^2 + 2*#1^3 & , 2, 0])*(Root[-2
- 3*#1^2 + 2*#1^3 & , 1, 0] - Root[-2 - 3*#1^2 + 2*#1^3 & , 3, 0])) + (2*(-
(-1)^(1/3) - (-1)^(2/3))*Sqrt[(-1 + x)/(-1 - (-1)^(1/3))]*Sqrt[(((-1)^(2/3)
 - x)*((-1)^(1/3) + x))/(-(-1)^(1/3) - (-1)^(2/3))^2]*EllipticPi[((-1)^(1/3
) + (-1)^(2/3))/((-1)^(1/3) + Root[-2 - 3*#1^2 + 2*#1^3 & , 2, 0]), ArcSin[
Sqrt[-(((-1)^(2/3)*((-1)^(1/3) + x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)])/(Sqr
t[-1 + x^3]*((-1)^(1/3) + Root[-2 - 3*#1^2 + 2*#1^3 & , 2, 0])*(-Root[-2 -
3*#1^2 + 2*#1^3 & , 1, 0] + Root[-2 - 3*#1^2 + 2*#1^3 & , 2, 0])*(Root[-2 -
 3*#1^2 + 2*#1^3 & , 2, 0] - Root[-2 - 3*#1^2 + 2*#1^3 & , 3, 0])) + ((-(-1
)^(1/3) - (-1)^(2/3))*Sqrt[(-1 + x)/(-1 - (-1)^(1/3))]*Sqrt[(((-1)^(2/3) -
x)*((-1)^(1/3) + x))/(-(-1)^(1/3) - (-1)^(2/3))^2]*EllipticPi[((-1)^(1/3) +
 (-1)^(2/3))/((-1)^(1/3) + Root[-2 - 3*#1^2 + 2*#1^3 & , 2, 0]), ArcSin[Sqr
t[-(((-1)^(2/3)*((-1)^(1/3) + x))/(1 + (-1)^(1/3)))]], (-1)^(1/3)]*Root[-2
- 3*#1^2 + 2*#1^3 & , 2, 0]^3)/(Sqrt[-1 + x^3]*((-1)^(1/3) + Root[-2 - 3*#1
^2 + 2*#1^3 & , 2, 0])*(-Root[-2 - 3*#1^2 + 2*#1^3 & , 1, 0] + Root[-2 - 3*
#1^2 + 2*#1^3 & , 2, 0])*(Root[-2 - 3*#1^2 + 2*#1^3 & , 2, 0] - Root[-2 - 3
*#1^2 + 2*#1^3 & , 3, 0])) + (2*(-(-1)^(1/3) - (-1)^(2/3))*Sqrt[(-1 + x)/(-
1 - (-1)^(1/3))]*Sqrt[(((-1)^(2/3) - x)*((-1)^(1/3) + x))/(-(-1)^(1/3) - (-
1)^(2/3))^2]*EllipticPi[((-1)^(1/3) + (-1)^(2/3))/((-1)^(1/3) + Root[-2 - 3
*#1^2 + 2*#1^3 & , 3, 0]), ArcSin[Sqrt[-(((-1)^(2/3)*((-1)^(1/3) + x))/(1 +
 (-1)^(1/3)))]], (-1)^(1/3)])/(Sqrt[-1 + x^3]*((-1)^(1/3) + Root[-2 - 3*#1^
2 + 2*#1^3 & , 3, 0])*(-Root[-2 - 3*#1^2 + 2*#1^3 & , 1, 0] + Root[-2 - 3*#
1^2 + 2*#1^3 & , 3, 0])*(-Root[-2 - 3*#1^2 + 2*#1^3 & , 2, 0] + Root[-2 - 3
*#1^2 + 2*#1^3 & , 3, 0])) + ((-(-1)^(1/3) - (-1)^(2/3))*Sqrt[(-1 + x)/(-1
- (-1)^(1/3))]*Sqrt[(((-1)^(2/3) - x)*((-1)^(1/3) + x))/(-(-1)^(1/3) - (-1)
^(2/3))^2]*EllipticPi[((-1)^(1/3) + (-1)^(2/3))/((-1)^(1/3) + Root[-2 - 3*#
1^2 + 2*#1^3 & , 3, 0]), ArcSin[Sqrt[-(((-1)^(2/3)*((-1)^(1/3) + x))/(1 + (
-1)^(1/3)))]], (-1)^(1/3)]*Root[-2 - 3*#1^2 + 2*#1^3 & , 3, 0]^3)/(Sqrt[-1
+ x^3]*((-1)^(1/3) + Root[-2 - 3*#1^2 + 2*#1^3 & , 3, 0])*(-Root[-2 - 3*#1^
2 + 2*#1^3 & , 1, 0] + Root[-2 - 3*#1^2 + 2*#1^3 & , 3, 0])*(-Root[-2 - 3*#
1^2 + 2*#1^3 & , 2, 0] + Root[-2 - 3*#1^2 + 2*#1^3 & , 3, 0]))))/8
```

IntegrateAlgebraic [A] time = 1.26, size = 74, normalized size = 1.00

$$\frac{\sqrt{x^3-1}\left(12x^6-10x^5+15x^4-24x^3+10x^2+12\right)}{60x^5}-\frac{1}{4}\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^3]*(2 + x^3)*(-1 - x^2 + x^3)^2)/(x^6*(-2 -
 3*x^2 + 2*x^3)),x]
```

```
[Out] (Sqrt[-1 + x^3]*(12 + 10*x^2 - 24*x^3 + 15*x^4 - 10*x^5 + 12*x^6))/(60*x^5)
 - (Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)/Sqrt[-1 + x^3]])/4
```

fricas [B] time = 0.53, size = 141, normalized size = 1.91

$$\frac{15 \sqrt{3} \sqrt{2} x^5 \log\left(-\frac{4 x^6+36 x^5+9 x^4-8 x^3-4 \sqrt{3} \sqrt{2}\left(2 x^4+3 x^3-2 x\right)\sqrt{x^3-1}-36 x^2+4}{4 x^6-12 x^5+9 x^4-8 x^3+12 x^2+4}\right)+8\left(12 x^6-10 x^5+15 x^4-24 x^3+10 x^2+12\right)\sqrt{x^3-1}}{480 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)*(x^3+2)*(x^3-x^2-1)^2/x^6/(2*x^3-3*x^2-2),x, algori
thm="fricas")
```

```
[Out] 1/480*(15*sqrt(3)*sqrt(2)*x^5*log(-(4*x^6 + 36*x^5 + 9*x^4 - 8*x^3 - 4*sqrt
(3)*sqrt(2)*(2*x^4 + 3*x^3 - 2*x)*sqrt(x^3 - 1) - 36*x^2 + 4)/(4*x^6 - 12*x
^5 + 9*x^4 - 8*x^3 + 12*x^2 + 4)) + 8*(12*x^6 - 10*x^5 + 15*x^4 - 24*x^3 +
10*x^2 + 12)*sqrt(x^3 - 1))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-x^2-1\right)^2\left(x^3+2\right)\sqrt{x^3-1}}{\left(2 x^3-3 x^2-2\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)*(x^3+2)*(x^3-x^2-1)^2/x^6/(2*x^3-3*x^2-2),x, algori
thm="giac")
```

```
[Out] integrate((x^3 - x^2 - 1)^2*(x^3 + 2)*sqrt(x^3 - 1)/((2*x^3 - 3*x^2 - 2)*x^
6), x)
```

maple [C] time = 0.40, size = 364, normalized size = 4.92

$$\frac{x\sqrt{x^3-1}}{5}+\frac{3\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{8\sqrt{x^3-1}}+\frac{\sqrt{x^3-1}}{6x^3}-\frac{2\sqrt{x^3-1}}{5x^2}+\frac{\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(2\_Z^3-3\_Z^2-2\right)}\frac{\_\alpha\left(-2\_\alpha^2+\_\alpha+1\right)\left(-3-i\sqrt{3}\right)\sqrt{\frac{-1+x}{-3-i\sqrt{3}}}\sqrt{\frac{1+2x-i\sqrt{3}}{3-i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}+2x+1}{3+i\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\_\alpha^2+\frac{\_\alpha}{2}+\frac{1}{2}-\frac{i\_\alpha^2\sqrt{3}}{3}+\frac{i\sqrt{3}\,\_\alpha}{6}+\frac{i\sqrt{3}}{6},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}\right)}{16}+\frac{\sqrt{x^3-1}}{4x}+\frac{\sqrt{x^3-1}}{5x^5}-\frac{\sqrt{x^3-1}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/2)*(x^3+2)*(x^3-x^2-1)^2/x^6/(2*x^3-3*x^2-2),x)
```

```
[Out] 1/5*x*(x^3-1)^(1/2)+3/8*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^
(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/
2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*
3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+1/6*(x^3-1
)^(1/2)/x^3-2/5/x^2*(x^3-1)^(1/2)+1/16*2^(1/2)*sum(_alpha*(-2*_alpha^2+_alp
ha+1)*(-3-I*3^(1/2))*((-1+x)/(-3-I*3^(1/2)))^(1/2)*((1+2*x-I*3^(1/2))/(3-I*
3^(1/2)))^(1/2)*((I*3^(1/2)+2*x+1)/(I*3^(1/2)+3))^(1/2)/(x^3-1)^(1/2)*Ellip
ticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-_alpha^2+1/2*_alpha+1/2-1/3*I*3^
(1/2)*_alpha^2+1/6*I*_alpha*3^(1/2)+1/6*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2
-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(2*_Z^3-3*_Z^2-2))+1/4*(x^3-1)^(1/2)/x
+1/5*(x^3-1)^(1/2)/x^5-1/6*(x^3-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-x^2-1\right)^2\left(x^3+2\right)\sqrt{x^3-1}}{\left(2 x^3-3 x^2-2\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/2)*(x^3+2)*(x^3-x^2-1)^2/x^6/(2*x^3-3*x^2-2),x, algori
thm="maxima")
```

```
[Out] integrate((x^3 - x^2 - 1)^2*(x^3 + 2)*sqrt(x^3 - 1)/((2*x^3 - 3*x^2 - 2)*x^
6), x)
```

mupad [B] time = 1.88, size = 117, normalized size = 1.58

$$\frac{x\sqrt{x^3-1}}{5}-\frac{\sqrt{x^3-1}}{6}+\frac{\sqrt{x^3-1}}{4x}-\frac{2\sqrt{x^3-1}}{5x^2}+\frac{\sqrt{x^3-1}}{6x^3}+\frac{\sqrt{x^3-1}}{5x^5}+\frac{\sqrt{2}\sqrt{3}\ln\left(\frac{3x^2+2x^3-2\sqrt{6}\,x\sqrt{x^3-1}-2}{-12x^3+18x^2+12}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3 - 1)^(1/2)*(x^3 + 2)*(x^2 - x^3 + 1)^2)/(x^6*(3*x^2 - 2*x^3 + 2)
),x)
```

```
[Out] (x*(x^3 - 1)^(1/2))/5 - (x^3 - 1)^(1/2)/6 + (x^3 - 1)^(1/2)/(4*x) - (2*(x^3
 - 1)^(1/2))/(5*x^2) + (x^3 - 1)^(1/2)/(6*x^3) + (x^3 - 1)^(1/2)/(5*x^5) +
(2^(1/2)*3^(1/2)*log((3*x^2 + 2*x^3 - 2*6^(1/2)*x*(x^3 - 1)^(1/2) - 2)/(18*
x^2 - 12*x^3 + 12)))/16
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/2)*(x**3+2)*(x**3-x**2-1)**2/x**6/(2*x**3-3*x**2-2),
x)
```

```
[Out] Timed out
```

3.894 $\int \frac{(1+x+x^2)(2x+x^2)\sqrt{1+2x+x^2-x^4}}{(1+x)^4}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{-x^4+x^2+2x+1}\,\left(2x^4+3x^3+x^2-4x-2\right)}{6(x+1)^3}-\tan^{-1}\left(\frac{\sqrt{-x^4+x^2+2x+1}}{x^2+x+1}\right)$$

Rubi [F] time = 1.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+x+x^2\right)\left(2x+x^2\right)\sqrt{1+2x+x^2-x^4}}{(1+x)^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x + x^2)*(2*x + x^2)*Sqrt[1 + 2*x + x^2 - x^4])/(1 + x)^4,x]
```

```
[Out] Defer[Int][Sqrt[1 + 2*x + x^2 - x^4]/(-1 - x), x] - Defer[Int][Sqrt[1 + 2*x
 + x^2 - x^4]/(1 + x)^4, x] + Defer[Int][Sqrt[1 + 2*x + x^2 - x^4]/(1 + x)^
3, x] - (64*Sqrt[1 + 2*x + x^2 - x^4]*Defer[Subst][Defer[Int][Sqrt[-240 - 1
28*x^2 + 256*x^4]/(2 - 4*x)^4, x], x, 1/2 + x^(-1)])/(Sqrt[-15 - 2*(1 + 2/x
)^2 + (1 + 2/x)^4]*x^2)
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+x+x^2\right)\left(2x+x^2\right)\sqrt{1+2x+x^2-x^4}}{(1+x)^4}\,dx &= \int \frac{x(2+x)\left(1+x+x^2\right)\sqrt{1+2x+x^2-x^4}}{(1+x)^4}\,dx\\
&= \int \left(\sqrt{1+2x+x^2-x^4}+\frac{\sqrt{1+2x+x^2-x^4}}{-1-x}-\frac{\sqrt{1+2x+x}}{(1+x)}\right.\\
&= \int \sqrt{1+2x+x^2-x^4}\,dx+\int \frac{\sqrt{1+2x+x^2-x^4}}{-1-x}\,dx-\int \\
&= -\left(16\,\text{Subst}\left(\int \frac{\sqrt{\frac{-240-128x^2+256x^4}{(2-4x)^4}}}{(2-4x)^2}\,dx,x,\frac{1}{2}+\frac{1}{x}\right)\right)+\int \frac{\sqrt{1}}{}\\
&= -\frac{\left(256\sqrt{1+2x+x^2-x^4}\right)\text{Subst}\left(\int \frac{\sqrt{-240-128x^2+256x^4}}{(2-4x)^4}\,dx,x,\right.}{\sqrt{-240-128\left(\frac{1}{2}+\frac{1}{x}\right)^2+256\left(\frac{1}{2}+\frac{1}{x}\right)^4}\,x^2}
\end{aligned}$$

Mathematica [C] time = 6.17, size = 1735, normalized size = 23.45

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + x + x^2)*(2*x + x^2)*Sqrt[1 + 2*x + x^2 - x^4])/(1 + x)^4,x
]
```

```
[Out] Sqrt[1 + 2*x + x^2 - x^4]*(-1/2 + x/3 + 1/(3*(1 + x)^3) - 5/(6*(1 + x)^2) +
 2/(3*(1 + x))) + ((1 + x)*Sqrt[1 + 2*x + x^2 - x^4]*(1/(2*Sqrt[1 + 2*x + x
^2 - x^4]) + x/(2*Sqrt[1 + 2*x + x^2 - x^4]) - 1/(2*(1 + x)*Sqrt[1 + 2*x +
x^2 - x^4]))*((2*(-(-1)^(1/3) + (-1 - Sqrt[5])/2)*Sqrt[((-(-1)^(2/3) + (1 +
```

```
 Sqrt[5])/2)*((-1)^(1/3) + x))/(((-1)^(1/3) + (1 + Sqrt[5])/2)*(-(-1)^(2/3)
 + x))]*(-(-1)^(2/3) + x)^2*Sqrt[(((-1)^(1/3) + (-1)^(2/3))*((-1 - Sqrt[5])
/2 + x))/(((-1)^(1/3) + (1 + Sqrt[5])/2)*(-(-1)^(2/3) + x))]*Sqrt[(((-1)^(1
/3) + (-1)^(2/3))*((-1 + Sqrt[5])/2 + x))/(((-1)^(1/3) + (1 - Sqrt[5])/2)*(
-(-1)^(2/3) + x))]*EllipticF[ArcSin[Sqrt[((-(-1)^(2/3) + (1 + Sqrt[5])/2)*(
(-1)^(1/3) + x))/(((-1)^(1/3) + (1 + Sqrt[5])/2)*(-(-1)^(2/3) + x))]], ((-(
-1)^(1/3) + (-1 - Sqrt[5])/2)*((-1)^(2/3) + (-1 + Sqrt[5])/2))/(((-1)^(2/3)
 + (-1 - Sqrt[5])/2)*(-(-1)^(1/3) + (-1 + Sqrt[5])/2))])/(((-1)^(1/3) + (-1
)^(2/3))*(-(-1)^(2/3) + (1 + Sqrt[5])/2)*Sqrt[1 + 2*x + x^2 - x^4]) + (2*(-
(-1)^(1/3) + (-1 - Sqrt[5])/2)*Sqrt[((-1 + 2*(-1)^(2/3) - Sqrt[5])*((-1)^(1
/3) + x))/((1 + 2*(-1)^(1/3) + Sqrt[5])*((-1)^(2/3) - x))]*(-(-1)^(2/3) + x
)^2*Sqrt[(((-1)^(1/3) + (-1)^(2/3))*((-1 - Sqrt[5])/2 + x))/(((-1)^(1/3) +
(1 + Sqrt[5])/2)*(-(-1)^(2/3) + x))]*Sqrt[(((-1)^(1/3) + (-1)^(2/3))*((-1 +
 Sqrt[5])/2 + x))/(((-1)^(1/3) + (1 - Sqrt[5])/2)*(-(-1)^(2/3) + x))]*(-((-
1)^(2/3)*EllipticF[ArcSin[Sqrt[((-1 + 2*(-1)^(2/3) - Sqrt[5])*((-1)^(1/3) +
 x))/((1 + 2*(-1)^(1/3) + Sqrt[5])*((-1)^(2/3) - x))]], ((1 + 2*(-1)^(1/3)
+ Sqrt[5])*(-1 + 2*(-1)^(2/3) + Sqrt[5]))/((1 + 2*(-1)^(1/3) - Sqrt[5])*(-1
 + 2*(-1)^(2/3) - Sqrt[5]))]) + ((-1)^(1/3) + (-1)^(2/3))*EllipticPi[((-1)^
(1/3) + (1 + Sqrt[5])/2)/(-(-1)^(2/3) + (1 + Sqrt[5])/2), ArcSin[Sqrt[((-1
+ 2*(-1)^(2/3) - Sqrt[5])*((-1)^(1/3) + x))/((1 + 2*(-1)^(1/3) + Sqrt[5])*(
(-1)^(2/3) - x))]], ((1 + 2*(-1)^(1/3) + Sqrt[5])*(-1 + 2*(-1)^(2/3) + Sqrt
[5]))/((1 + 2*(-1)^(1/3) - Sqrt[5])*(-1 + 2*(-1)^(2/3) - Sqrt[5]))]))/(((-1
)^(1/3) + (-1)^(2/3))*((-1)^(2/3) + (-1 - Sqrt[5])/2)*Sqrt[1 + 2*x + x^2 -
x^4]) - (2*((-1)^(1/3) + (1 + Sqrt[5])/2)*Sqrt[((-1 + 2*(-1)^(2/3) - Sqrt[5
])*((-1)^(1/3) + x))/((1 + 2*(-1)^(1/3) + Sqrt[5])*((-1)^(2/3) - x))]*(-(-1
)^(2/3) + x)^2*Sqrt[(((-1)^(1/3) + (-1)^(2/3))*((-1 - Sqrt[5])/2 + x))/(((-
1)^(1/3) + (1 + Sqrt[5])/2)*(-(-1)^(2/3) + x))]*Sqrt[(((-1)^(1/3) + (-1)^(2
/3))*((-1 + Sqrt[5])/2 + x))/(((-1)^(1/3) + (1 - Sqrt[5])/2)*(-(-1)^(2/3) +
 x))]*((-1 + (-1)^(1/3))*EllipticF[ArcSin[Sqrt[((-1 + 2*(-1)^(2/3) - Sqrt[5
])*((-1)^(1/3) + x))/((1 + 2*(-1)^(1/3) + Sqrt[5])*((-1)^(2/3) - x))]], ((1
 + 2*(-1)^(1/3) + Sqrt[5])*(-1 + 2*(-1)^(2/3) + Sqrt[5]))/((1 + 2*(-1)^(1/3
) - Sqrt[5])*(-1 + 2*(-1)^(2/3) - Sqrt[5]))] - ((-1)^(1/3) + (-1)^(2/3))*El
lipticPi[((1 + (-1)^(2/3))*((-1)^(1/3) + (1 + Sqrt[5])/2))/((1 - (-1)^(1/3)
)*(-(-1)^(2/3) + (1 + Sqrt[5])/2)), ArcSin[Sqrt[((-1 + 2*(-1)^(2/3) - Sqrt[
5])*((-1)^(1/3) + x))/((1 + 2*(-1)^(1/3) + Sqrt[5])*((-1)^(2/3) - x))]], ((
1 + 2*(-1)^(1/3) + Sqrt[5])*(-1 + 2*(-1)^(2/3) + Sqrt[5]))/((1 + 2*(-1)^(1/
3) - Sqrt[5])*(-1 + 2*(-1)^(2/3) - Sqrt[5]))]))/((1 - (-1)^(1/3))*(-1 - (-1
)^(2/3))*((-1)^(1/3) + (-1)^(2/3))*((-1)^(2/3) + (-1 - Sqrt[5])/2)*Sqrt[1 +
 2*x + x^2 - x^4])))/(x*(2 + x))
```

IntegrateAlgebraic [A] time = 1.18, size = 74, normalized size = 1.00

$$\frac{\sqrt{-x^4+x^2+2x+1}\left(2x^4+3x^3+x^2-4x-2\right)}{6(x+1)^3}-\tan^{-1}\left(\frac{\sqrt{-x^4+x^2+2x+1}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x + x^2)*(2*x + x^2)*Sqrt[1 + 2*x + x^2 - x^4])/(1
 + x)^4,x]
```

```
[Out] (Sqrt[1 + 2*x + x^2 - x^4]*(-2 - 4*x + x^2 + 3*x^3 + 2*x^4))/(6*(1 + x)^3)
- ArcTan[Sqrt[1 + 2*x + x^2 - x^4]/(1 + x + x^2)]
```

fricas [A] time = 0.48, size = 103, normalized size = 1.39

$$-\frac{3\left(x^3+3x^2+3x+1\right)\arctan\left(\frac{\sqrt{-x^4+x^2+2x+1}\,x^2}{x^4-x^2-2x-1}\right)-\left(2x^4+3x^3+x^2-4x-2\right)\sqrt{-x^4+x^2+2x+1}}{6\left(x^3+3x^2+3x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)*(x^2+2*x)*(-x^4+x^2+2*x+1)^(1/2)/(1+x)^4,x, algorithm="
fricas")
```

```
[Out] -1/6*(3*(x^3 + 3*x^2 + 3*x + 1)*arctan(sqrt(-x^4 + x^2 + 2*x + 1)*x^2/(x^4
- x^2 - 2*x - 1)) - (2*x^4 + 3*x^3 + x^2 - 4*x - 2)*sqrt(-x^4 + x^2 + 2*x +
 1))/(x^3 + 3*x^2 + 3*x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+x^2+2\,x+1}\,\left(x^2+2\,x\right)\left(x^2+x+1\right)}{\left(x+1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)*(x^2+2*x)*(-x^4+x^2+2*x+1)^(1/2)/(1+x)^4,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2*x + 1)*(x^2 + 2*x)*(x^2 + x + 1)/(x + 1)^4, x
)
```

maple [C] time = 0.54, size = 1431, normalized size = 19.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)*(x^2+2*x)*(-x^4+x^2+2*x+1)^(1/2)/(1+x)^4,x)
```

```
[Out] 1/3*x*(-x^4+x^2+2*x+1)^(1/2)-1/3*I*(-1-1/2*I*3^(1/2)-1/2*5^(1/2))*((1+1/2*5
^(1/2)-1/2*I*3^(1/2))*(x+1/2+1/2*I*3^(1/2))/(1+1/2*5^(1/2)+1/2*I*3^(1/2))/(
x+1/2-1/2*I*3^(1/2)))^(1/2)*(x+1/2-1/2*I*3^(1/2))^2*(I*3^(1/2)*(x-1/2+1/2*5
^(1/2))/(1-1/2*5^(1/2)+1/2*I*3^(1/2))/(x+1/2-1/2*I*3^(1/2)))^(1/2)*(I*3^(1/
2)*(x-1/2-1/2*5^(1/2))/(1+1/2*5^(1/2)+1/2*I*3^(1/2))/(x+1/2-1/2*I*3^(1/2)))
^(1/2)/(1+1/2*5^(1/2)-1/2*I*3^(1/2))*3^(1/2)/(-(x+1/2+1/2*I*3^(1/2))*(x+1/2
-1/2*I*3^(1/2))*(x-1/2+1/2*5^(1/2))*(x-1/2-1/2*5^(1/2)))^(1/2)*EllipticF(((
1+1/2*5^(1/2)-1/2*I*3^(1/2))*(x+1/2+1/2*I*3^(1/2))/(1+1/2*5^(1/2)+1/2*I*3^(
1/2))/(x+1/2-1/2*I*3^(1/2)))^(1/2),((-1+1/2*I*3^(1/2)+1/2*5^(1/2))*(-1-1/2*
I*3^(1/2)-1/2*5^(1/2))/(-1-1/2*I*3^(1/2)+1/2*5^(1/2))/(-1+1/2*I*3^(1/2)-1/2
*5^(1/2)))^(1/2))+1/3*I*(-1-1/2*I*3^(1/2)-1/2*5^(1/2))*((1+1/2*5^(1/2)-1/2*
I*3^(1/2))*(x+1/2+1/2*I*3^(1/2))/(1+1/2*5^(1/2)+1/2*I*3^(1/2))/(x+1/2-1/2*I
*3^(1/2)))^(1/2)*(x+1/2-1/2*I*3^(1/2))^2*(I*3^(1/2)*(x-1/2+1/2*5^(1/2))/(1-
1/2*5^(1/2)+1/2*I*3^(1/2))/(x+1/2-1/2*I*3^(1/2)))^(1/2)*(I*3^(1/2)*(x-1/2-1
/2*5^(1/2))/(1+1/2*5^(1/2)+1/2*I*3^(1/2))/(x+1/2-1/2*I*3^(1/2)))^(1/2)/(1+1
/2*5^(1/2)-1/2*I*3^(1/2))*3^(1/2)/(-(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1
/2))*(x-1/2+1/2*5^(1/2))*(x-1/2-1/2*5^(1/2)))^(1/2)/(1/2+1/2*I*3^(1/2))*(El
lipticF(((1+1/2*5^(1/2)-1/2*I*3^(1/2))*(x+1/2+1/2*I*3^(1/2))/(1+1/2*5^(1/2)
+1/2*I*3^(1/2))/(x+1/2-1/2*I*3^(1/2)))^(1/2),((-1+1/2*I*3^(1/2)+1/2*5^(1/2)
)*(-1-1/2*I*3^(1/2)-1/2*5^(1/2))/(-1-1/2*I*3^(1/2)+1/2*5^(1/2))/(-1+1/2*I*3
^(1/2)-1/2*5^(1/2)))^(1/2))+I*3^(1/2)/(1/2-1/2*I*3^(1/2))*EllipticPi(((1+1/
2*5^(1/2)-1/2*I*3^(1/2))*(x+1/2+1/2*I*3^(1/2))/(1+1/2*5^(1/2)+1/2*I*3^(1/2)
)/(x+1/2-1/2*I*3^(1/2)))^(1/2),(1/2+1/2*I*3^(1/2))*(1+1/2*5^(1/2)+1/2*I*3^(
1/2))/(1/2-1/2*I*3^(1/2))/(1+1/2*5^(1/2)-1/2*I*3^(1/2)),((-1+1/2*I*3^(1/2)+
1/2*5^(1/2))*(-1-1/2*I*3^(1/2)-1/2*5^(1/2))/(-1-1/2*I*3^(1/2)+1/2*5^(1/2))/
(-1+1/2*I*3^(1/2)-1/2*5^(1/2)))^(1/2)))-5/6*(-x^4+x^2+2*x+1)^(1/2)/(1+x)^2+
2/3/(1+x)*(-x^4+x^2+2*x+1)^(1/2)-1/3*I*(-1-1/2*I*3^(1/2)-1/2*5^(1/2))*((1+1
/2*5^(1/2)-1/2*I*3^(1/2))*(x+1/2+1/2*I*3^(1/2))/(1+1/2*5^(1/2)+1/2*I*3^(1/2
))/(x+1/2-1/2*I*3^(1/2)))^(1/2)*(x+1/2-1/2*I*3^(1/2))^2*(I*3^(1/2)*(x-1/2+1
/2*5^(1/2))/(1-1/2*5^(1/2)+1/2*I*3^(1/2))/(x+1/2-1/2*I*3^(1/2)))^(1/2)*(I*3
^(1/2)*(x-1/2-1/2*5^(1/2))/(1+1/2*5^(1/2)+1/2*I*3^(1/2))/(x+1/2-1/2*I*3^(1/
2)))^(1/2)/(1+1/2*5^(1/2)-1/2*I*3^(1/2))*3^(1/2)/(-(x+1/2+1/2*I*3^(1/2))*(x
+1/2-1/2*I*3^(1/2))*(x-1/2+1/2*5^(1/2))*(x-1/2-1/2*5^(1/2)))^(1/2)*((-1/2+1
/2*I*3^(1/2))*EllipticF(((1+1/2*5^(1/2)-1/2*I*3^(1/2))*(x+1/2+1/2*I*3^(1/2)
```

```
)/(1+1/2*5^(1/2)+1/2*I*3^(1/2))/(x+1/2-1/2*I*3^(1/2)))^(1/2),((-1+1/2*I*3^(
1/2)+1/2*5^(1/2))*(-1-1/2*I*3^(1/2)-1/2*5^(1/2))/(-1-1/2*I*3^(1/2)+1/2*5^(1
/2))/(-1+1/2*I*3^(1/2)-1/2*5^(1/2)))^(1/2))-I*3^(1/2)*EllipticPi(((1+1/2*5^
(1/2)-1/2*I*3^(1/2))*(x+1/2+1/2*I*3^(1/2))/(1+1/2*5^(1/2)+1/2*I*3^(1/2))/(x
+1/2-1/2*I*3^(1/2)))^(1/2),(1+1/2*5^(1/2)+1/2*I*3^(1/2))/(1+1/2*5^(1/2)-1/2
*I*3^(1/2)),((-1+1/2*I*3^(1/2)+1/2*5^(1/2))*(-1-1/2*I*3^(1/2)-1/2*5^(1/2))/
(-1-1/2*I*3^(1/2)+1/2*5^(1/2))/(-1+1/2*I*3^(1/2)-1/2*5^(1/2)))^(1/2)))-1/2*
(-x^4+x^2+2*x+1)^(1/2)+1/3/(1+x)^3*(-x^4+x^2+2*x+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4 + x^2 + 2\,x + 1}\left(x^2 + 2\,x\right)\left(x^2 + x + 1\right)}{(x + 1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)*(x^2+2*x)*(-x^4+x^2+2*x+1)^(1/2)/(1+x)^4,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2*x + 1)*(x^2 + 2*x)*(x^2 + x + 1)/(x + 1)^4, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2 + 2\,x\right)\,\left(x^2 + x + 1\right)\,\sqrt{-x^4 + x^2 + 2\,x + 1}}{(x + 1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + x^2)*(x + x^2 + 1)*(2*x + x^2 - x^4 + 1)^(1/2))/(x + 1)^4,x)
```

```
[Out] int(((2*x + x^2)*(x + x^2 + 1)*(2*x + x^2 - x^4 + 1)^(1/2))/(x + 1)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-\left(x^2 - x - 1\right)\left(x^2 + x + 1\right)}\,(x + 2)\left(x^2 + x + 1\right)}{(x + 1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)*(x**2+2*x)*(-x**4+x**2+2*x+1)**(1/2)/(1+x)**4,x)
```

```
[Out] Integral(x*sqrt(-(x**2 - x - 1)*(x**2 + x + 1))*(x + 2)*(x**2 + x + 1)/(x +
 1)**4, x)
```

3.895 $\int \frac{-1-x+x^4}{x\sqrt[4]{1+x^4}}\,dx$

Optimal. Leaf size=74

$$\frac{1}{3}\left(x^4+1\right)^{3/4}-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\tan^{-1}\left(\sqrt[4]{x^4+1}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)$$

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1833, 240, 212, 206, 203, 446, 80, 63, 298}

$$\frac{1}{3}\left(x^4+1\right)^{3/4}-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\tan^{-1}\left(\sqrt[4]{x^4+1}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - x + x^4)/(x*(1 + x^4)^(1/4)),x]
```

```
[Out] (1 + x^4)^(3/4)/3 - ArcTan[x/(1 + x^4)^(1/4)]/2 - ArcTan[(1 + x^4)^(1/4)]/2
 - ArcTanh[x/(1 + x^4)^(1/4)]/2 + ArcTanh[(1 + x^4)^(1/4)]/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
```

```
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1-x+x^4}{x\sqrt[4]{1+x^4}}\,dx &= \int \left(-\frac{1}{\sqrt[4]{1+x^4}}+\frac{-1+x^4}{x\sqrt[4]{1+x^4}}\right)dx \\
&= -\int \frac{1}{\sqrt[4]{1+x^4}}\,dx+\int \frac{-1+x^4}{x\sqrt[4]{1+x^4}}\,dx \\
&= \frac{1}{4}\operatorname{Subst}\left(\int \frac{-1+x}{x\sqrt[4]{1+x}}\,dx,x,x^4\right)-\operatorname{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right) \\
&= \frac{1}{3}\left(1+x^4\right)^{3/4}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{1+x}}\,dx,x,x^4\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{1}{2}\operatorname{Su} \\
&= \frac{1}{3}\left(1+x^4\right)^{3/4}-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\operatorname{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\sqrt[4]{1+} \right.\\
&= \frac{1}{3}\left(1+x^4\right)^{3/4}-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[4]{1+} \right.\\
&= \frac{1}{3}\left(1+x^4\right)^{3/4}-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{1}{2}\tan^{-1}\left(\sqrt[4]{1+x^4}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{2}\tanh^{-1}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 0.92

$$\frac{1}{6}\left(2\left(x^4+1\right)^{3/4}-3\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-3\tan^{-1}\left(\sqrt[4]{x^4+1}\right)-3\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+3\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - x + x^4)/(x*(1 + x^4)^(1/4)),x]
```

```
[Out] (2*(1 + x^4)^(3/4) - 3*ArcTan[x/(1 + x^4)^(1/4)] - 3*ArcTan[(1 + x^4)^(1/4)
] - 3*ArcTanh[x/(1 + x^4)^(1/4)] + 3*ArcTanh[(1 + x^4)^(1/4)])/6
```

IntegrateAlgebraic [A] time = 4.48, size = 74, normalized size = 1.00

$$\frac{1}{3}\left(x^4+1\right)^{3/4}-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\tan^{-1}\left(\sqrt[4]{x^4+1}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - x + x^4)/(x*(1 + x^4)^(1/4)),x]
```

```
[Out] (1 + x^4)^(3/4)/3 - ArcTan[x/(1 + x^4)^(1/4)]/2 - ArcTan[(1 + x^4)^(1/4)]/2
 - ArcTanh[x/(1 + x^4)^(1/4)]/2 + ArcTanh[(1 + x^4)^(1/4)]/2
```

fricas [B] time = 9.12, size = 123, normalized size = 1.66

$$\frac{1}{3}\left(x^4+1\right)^{\frac{3}{4}}-\frac{1}{2}\arctan\left(\frac{\left(x^4+1\right)^{\frac{3}{4}}(x-1)-\left(x^4+1\right)^{\frac{1}{4}}\left(x^2-x\right)}{x^4-x^2+1}\right)+\frac{1}{2}\log\left(-\frac{x^4-x^3-\left(x^4+1\right)^{\frac{3}{4}}(x-1)+\sqrt{x^4+1}\left(x^2-x+1\right)-\left(x^4+1\right)^{\frac{1}{4}}\left(x^3-x^2+x-1\right)-x+1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x-1)/x/(x^4+1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3*(x^4 + 1)^(3/4) - 1/2*arctan(((x^4 + 1)^(3/4)*(x - 1) - (x^4 + 1)^(1/4)
*(x^2 - x))/(x^4 - x^2 + 1)) + 1/2*log(-(x^4 - x^3 - (x^4 + 1)^(3/4)*(x - 1
) + sqrt(x^4 + 1)*(x^2 - x + 1) - (x^4 + 1)^(1/4)*(x^3 - x^2 + x - 1) - x +
 1)/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-x-1}{\left(x^4+1\right)^{\frac{1}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x-1)/x/(x^4+1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^4 - x - 1)/((x^4 + 1)^(1/4)*x), x)
```

maple [C] time = 0.24, size = 90, normalized size = 1.22

$$-\frac{\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)\left(-\frac{\pi\sqrt{2}\,x^4\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{4}\right],[2,2],-x^4\right)}{4\Gamma\left(\frac{3}{4}\right)}+\frac{\left(-3\ln(2)-\frac{\pi}{2}+4\ln(x)\right)\pi\sqrt{2}}{\Gamma\left(\frac{3}{4}\right)}\right)}{8\pi}+\frac{x^4\,\mathrm{hypergeom}\left(\left[\frac{1}{4},1\right],[2],-x^4\right)}{4}-x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{4}\right],\left[\frac{5}{4}\right],-x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x-1)/x/(x^4+1)^(1/4),x)
```

```
[Out] -1/8/Pi*2^(1/2)*GAMMA(3/4)*(-1/4*Pi*2^(1/2)/GAMMA(3/4)*x^4*hypergeom([1,1,5
/4],[2,2],-x^4)+(-3*ln(2)-1/2*Pi+4*ln(x))*Pi*2^(1/2)/GAMMA(3/4))+1/4*x^4*hy
pergeom([1/4,1],[2],-x^4)-x*hypergeom([1/4,1/4],[5/4],-x^4)
```

maxima [A] time = 0.42, size = 90, normalized size = 1.22

$$\frac{1}{3}\left(x^4+1\right)^{\frac{3}{4}}-\frac{1}{2}\arctan\left(\left(x^4+1\right)^{\frac{1}{4}}\right)+\frac{1}{2}\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{4}\log\left(\left(x^4+1\right)^{\frac{1}{4}}+1\right)-\frac{1}{4}\log\left(\left(x^4+1\right)^{\frac{1}{4}}-1\right)-\frac{1}{4}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}+1\right)+\frac{1}{4}\log\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x-1)/x/(x^4+1)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 1)^(3/4) - 1/2*arctan((x^4 + 1)^(1/4)) + 1/2*arctan((x^4 + 1)^(1
/4)/x) + 1/4*log((x^4 + 1)^(1/4) + 1) - 1/4*log((x^4 + 1)^(1/4) - 1) - 1/4*
log((x^4 + 1)^(1/4)/x + 1) + 1/4*log((x^4 + 1)^(1/4)/x - 1)
```

mupad [B] time = 1.24, size = 43, normalized size = 0.58

$$\frac{\operatorname{atanh}\left(\left(x^4+1\right)^{1/4}\right)}{2}-\frac{\operatorname{atan}\left(\left(x^4+1\right)^{1/4}\right)}{2}-x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -x^4\right)+\frac{\left(x^4+1\right)^{3/4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - x^4 + 1)/(x*(x^4 + 1)^(1/4)),x)
```

```
[Out] atanh((x^4 + 1)^(1/4))/2 - atan((x^4 + 1)^(1/4))/2 - x*hypergeom([1/4, 1/4]
, 5/4, -x^4) + (x^4 + 1)^(3/4)/3
```

sympy [C] time = 4.51, size = 66, normalized size = 0.89

$$-\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\left(x^4+1\right)^{\frac{3}{4}}}{3}+\frac{\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{4x\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x-1)/x/(x**4+1)**(1/4),x)
```

```
[Out] -x*gamma(1/4)*hyper((1/4, 1/4), (5/4,), x**4*exp_polar(I*pi))/(4*gamma(5/4)
) + (x**4 + 1)**(3/4)/3 + gamma(1/4)*hyper((1/4, 1/4), (5/4,), exp_polar(I*
pi)/x**4)/(4*x*gamma(5/4))
```

3.896 $\int \frac{\sqrt[4]{-x^3+x^4}}{x^2(-1+x^2)}\,dx$

Optimal. Leaf size=74

$$\frac{4\sqrt[4]{x^4-x^3}}{x}+\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-x^3}}\right)-\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-x^3}}\right)$$

Rubi [A] time = 0.15, antiderivative size = 120, normalized size of antiderivative = 1.62, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2056, 848, 96, 93, 298, 203, 206}

$$\frac{4\sqrt[4]{x^4-x^3}}{x}+\frac{\sqrt[4]{2}\sqrt[4]{x^4-x^3}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{\sqrt[4]{x-1}\,x^{3/4}}-\frac{\sqrt[4]{2}\sqrt[4]{x^4-x^3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{\sqrt[4]{x-1}\,x^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + x^4)^(1/4)/(x^2*(-1 + x^2)),x]
```

```
[Out] (4*(-x^3 + x^4)^(1/4))/x + (2^(1/4)*(-x^3 + x^4)^(1/4)*ArcTan[(2^(1/4)*x^(1
/4))/(-1 + x)^(1/4)])/((-1 + x)^(1/4)*x^(3/4)) - (2^(1/4)*(-x^3 + x^4)^(1/4
)*ArcTanh[(2^(1/4)*x^(1/4))/(-1 + x)^(1/4)])/((-1 + x)^(1/4)*x^(3/4))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{-x^3+x^4}}{x^2\left(-1+x^2\right)}\,dx &= \frac{\sqrt[4]{-x^3+x^4}\int \frac{\sqrt[4]{-1+x}}{x^{5/4}\left(-1+x^2\right)}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}}\\
&= \frac{\sqrt[4]{-x^3+x^4}\int \frac{1}{(-1+x)^{3/4}x^{5/4}(1+x)}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}}\\
&= \frac{4\sqrt[4]{-x^3+x^4}}{x} - \frac{\sqrt[4]{-x^3+x^4}\int \frac{1}{(-1+x)^{3/4}\sqrt[4]{x}\,(1+x)}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}}\\
&= \frac{4\sqrt[4]{-x^3+x^4}}{x} - \frac{\left(4\sqrt[4]{-x^3+x^4}\right)\mathrm{Subst}\left(\int \frac{x^2}{1-2x^4}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt[4]{-1+x}}\right)}{\sqrt[4]{-1+x}\,x^{3/4}}\\
&= \frac{4\sqrt[4]{-x^3+x^4}}{x} - \frac{\left(\sqrt{2}\sqrt[4]{-x^3+x^4}\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x^2}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt[4]{-1+x}}\right)}{\sqrt[4]{-1+x}\,x^{3/4}} + \frac{\left(\sqrt{2}\sqrt[4]{-x^3+x^4}\right)\mathrm{Subst}\left(\right.}{\sqrt[4]{-1+}}\\
&= \frac{4\sqrt[4]{-x^3+x^4}}{x} + \frac{\sqrt[4]{2}\sqrt[4]{-x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{-1+x}}\right)}{\sqrt[4]{-1+x}\,x^{3/4}} - \frac{\sqrt[4]{2}\sqrt[4]{-x^3+x^4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{-1+x}}\right)}{\sqrt[4]{-1+x}\,x^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.49

$$
-\frac{2\sqrt[4]{(x-1)x^3}\left({}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{x-1}{2x}\right)-2\right)}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + x^4)^(1/4)/(x^2*(-1 + x^2)),x]
```

```
[Out] (-2*((-1 + x)*x^3)^(1/4)*(-2 + Hypergeometric2F1[1/4, 1, 5/4, (-1 + x)/(2*x
)]))/x
```

IntegrateAlgebraic [A] time = 0.35, size = 74, normalized size = 1.00

$$
\frac{4\sqrt[4]{x^4-x^3}}{x}+\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-x^3}}\right)-\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-x^3}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^3 + x^4)^(1/4)/(x^2*(-1 + x^2)),x]
```

```
[Out] (4*(-x^3 + x^4)^(1/4))/x + 2^(1/4)*ArcTan[(2^(1/4)*x)/(-x^3 + x^4)^(1/4)] -
 2^(1/4)*ArcTanh[(2^(1/4)*x)/(-x^3 + x^4)^(1/4)]
```

fricas [B] time = 0.46, size = 138, normalized size = 1.86

$$\frac{4\cdot 8^{\frac{3}{4}} x \arctan\left(\frac{8^{\frac{1}{4}} x\sqrt{\frac{\sqrt{2}x^2+\sqrt{x^4-x^3}}{x^2}} - 8^{\frac{1}{4}}\left(x^4-x^3\right)^{\frac{1}{4}}}{2x}\right) - 8^{\frac{3}{4}}x\log\left(\frac{8^{\frac{3}{4}}x+4\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right) + 8^{\frac{3}{4}}x\log\left(-\frac{8^{\frac{3}{4}}x-4\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+32\left(x^4-x^3\right)^{\frac{1}{4}}}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x^2/(x^2-1),x, algorithm="fricas")
```

```
[Out] 1/8*(4*8^(3/4)*x*arctan(1/2*(8^(1/4)*x*sqrt((sqrt(2)*x^2 + sqrt(x^4 - x^3))
/x^2) - 8^(1/4)*(x^4 - x^3)^(1/4))/x) - 8^(3/4)*x*log((8^(3/4)*x + 4*(x^4 -
 x^3)^(1/4))/x) + 8^(3/4)*x*log(-(8^(3/4)*x - 4*(x^4 - x^3)^(1/4))/x) + 32*
(x^4 - x^3)^(1/4))/x
```

giac [A] time = 0.43, size = 72, normalized size = 0.97

$$2^{\frac{1}{4}} \arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right) + \frac{1}{2}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}} + \left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right) - \frac{1}{2}\cdot 2^{\frac{1}{4}}\log\left(\left|-2^{\frac{1}{4}} + \left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right|\right) - 4\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x^2/(x^2-1),x, algorithm="giac")
```

```
[Out] 2^(1/4)*arctan(1/2*2^(3/4)*(-1/x + 1)^(1/4)) + 1/2*2^(1/4)*log(2^(1/4) + (-
1/x + 1)^(1/4)) - 1/2*2^(1/4)*log(abs(-2^(1/4) + (-1/x + 1)^(1/4))) - 4*(-1
/x + 1)^(1/4)
```

maple [C] time = 1.38, size = 566, normalized size = 7.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3)^(1/4)/x^2/(x^2-1),x)
```

```
[Out] 4*(x^3*(-1+x))^(1/4)/x+(-1/2*RootOf(_Z^2+RootOf(_Z^4-2)^2)*ln(-(2*(x^4-3*x^
3+3*x^2-x)^(1/2)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2*x-2*(x^4-3*
x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2+2*(x^4-3*
x^3+3*x^2-x)^(1/4)*RootOf(_Z^4-2)^2*x^2-3*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^3
-4*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4-2)^2*x+7*RootOf(_Z^2+RootOf(_Z^4-2
)^2)*x^2-4*(x^4-3*x^3+3*x^2-x)^(3/4)+2*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^
4-2)^2-5*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x+RootOf(_Z^2+RootOf(_Z^4-2)^2))/(-1
+x)^2/(1+x))-1/2*RootOf(_Z^4-2)*ln((2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4
-2)^3*x-2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4-2)^3+2*(x^4-3*x^3+3*x^2-x)^
(1/4)*RootOf(_Z^4-2)^2*x^2-4*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4-2)^2*x+3
*RootOf(_Z^4-2)*x^3+4*(x^4-3*x^3+3*x^2-x)^(3/4)+2*(x^4-3*x^3+3*x^2-x)^(1/4)
*RootOf(_Z^4-2)^2-7*RootOf(_Z^4-2)*x^2+5*RootOf(_Z^4-2)*x-RootOf(_Z^4-2))/(
-1+x)^2/(1+x)))*(x^3*(-1+x))^(1/4)/(-1+x)/x*(x*(-1+x)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^3\right)^{\frac{1}{4}}}{\left(x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x^2/(x^2-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^3)^(1/4)/((x^2 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(x^4-x^3\right)^{1/4}}{x^2-x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x^3)^(1/4)/(x^2*(x^2 - 1)),x)
```

```
[Out] -int((x^4 - x^3)^(1/4)/(x^2 - x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\,(x-1)}}{x^2\,(x-1)\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3)**(1/4)/x**2/(x**2-1),x)
```

```
[Out] Integral((x**3*(x - 1))**(1/4)/(x**2*(x - 1)*(x + 1)), x)
```

3.897 $\int \frac{-3+x^4}{\sqrt[3]{1+x^4}\left(1-x^3+x^4\right)} \, dx$

Optimal. Leaf size=74

$$\log\left(\sqrt[3]{x^4+1}-x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+1}+x}\right)-\frac{1}{2}\log\left(\sqrt[3]{x^4+1}x+\left(x^4+1\right)^{2/3}+x^2\right)$$

Rubi [F] time = 0.59, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3+x^4}{\sqrt[3]{1+x^4}\left(1-x^3+x^4\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(-3 + x^4)/((1 + x^4)^(1/3)*(1 - x^3 + x^4)),x]
[Out] x*Hypergeometric2F1[1/4, 1/3, 5/4, -x^4] - 4*Defer[Int][1/((1 + x^4)^(1/3)*
(1 - x^3 + x^4)), x] + Defer[Int][x^3/((1 + x^4)^(1/3)*(1 - x^3 + x^4)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+x^4}{\sqrt[3]{1+x^4}\left(1-x^3+x^4\right)} \, dx &= \int \left(\frac{1}{\sqrt[3]{1+x^4}}-\frac{4-x^3}{\sqrt[3]{1+x^4}\left(1-x^3+x^4\right)}\right) dx\\ &= \int \frac{1}{\sqrt[3]{1+x^4}} \, dx-\int \frac{4-x^3}{\sqrt[3]{1+x^4}\left(1-x^3+x^4\right)} \, dx\\ &= x\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{5}{4};-x^4\right)-\int \left(\frac{4}{\sqrt[3]{1+x^4}\left(1-x^3+x^4\right)}-\frac{x^3}{\sqrt[3]{1+x^4}\left(1-x^3+x^4\right)}\right) dx\\ &= x\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{5}{4};-x^4\right)-4\int \frac{1}{\sqrt[3]{1+x^4}\left(1-x^3+x^4\right)} \, dx+\int \frac{x^3}{\sqrt[3]{1+x^4}\left(1-x^3+x^4\right)} \, dx\end{aligned}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{-3+x^4}{\sqrt[3]{1+x^4}\left(1-x^3+x^4\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3 + x^4)/((1 + x^4)^(1/3)*(1 - x^3 + x^4)),x]
[Out] Integrate[(-3 + x^4)/((1 + x^4)^(1/3)*(1 - x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 1.14, size = 74, normalized size = 1.00

$$\log\left(\sqrt[3]{x^4+1}-x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+1}+x}\right)-\frac{1}{2}\log\left(\sqrt[3]{x^4+1}x+\left(x^4+1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 + x^4)/((1 + x^4)^(1/3)*(1 - x^3 + x^4)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(1 + x^4)^(1/3))]) + Log[-x + (1 + x^4)
^(1/3)] - Log[x^2 + x*(1 + x^4)^(1/3) + (1 + x^4)^(2/3)]/2
```

fricas [A] time = 1.75, size = 110, normalized size = 1.49

$$-\sqrt{3}\arctan\left(-\frac{2\sqrt{3}\left(x^4+1\right)^{\frac{1}{3}}x^2-2\sqrt{3}\left(x^4+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(x^4-x^3+1\right)}{3\left(x^4+x^3+1\right)}\right)+\frac{1}{2}\log\left(\frac{x^4-x^3+3\left(x^4+1\right)^{\frac{1}{3}}x^2-3\left(x^4+1\right)^{\frac{2}{3}}x+1}{x^4-x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)/(x^4+1)^(1/3)/(x^4-x^3+1),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(-1/3*(2*sqrt(3)*(x^4 + 1)^(1/3)*x^2 - 2*sqrt(3)*(x^4 + 1)^(
2/3)*x + sqrt(3)*(x^4 - x^3 + 1))/(x^4 + x^3 + 1)) + 1/2*log((x^4 - x^3 + 3
*(x^4 + 1)^(1/3)*x^2 - 3*(x^4 + 1)^(2/3)*x + 1)/(x^4 - x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-3}{\left(x^4-x^3+1\right)\left(x^4+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)/(x^4+1)^(1/3)/(x^4-x^3+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 3)/((x^4 - x^3 + 1)*(x^4 + 1)^(1/3)), x)
```

maple [C] time = 2.63, size = 287, normalized size = 3.88

$$\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(-\frac{\left(x^4+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x+\left(x^4+1\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3+x^4+2\left(x^4+1\right)^{\frac{2}{3}}x+2x^2\left(x^4+1\right)^{\frac{1}{3}}+x^3+1}{x^4-x^3+1}\right)-\ln\left(\frac{\left(x^4+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x+\left(x^4+1\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3-x^4-\left(x^4+1\right)^{\frac{2}{3}}x-x^2\left(x^4+1\right)^{\frac{1}{3}}-1}{x^4-x^3+1}\right)\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-\ln\left(\frac{\left(x^4+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x+\left(x^4+1\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3-x^4-\left(x^4+1\right)^{\frac{2}{3}}x-x^2\left(x^4+1\right)^{\frac{1}{3}}-1}{x^4-x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)/(x^4+1)^(1/3)/(x^4-x^3+1),x)
```

```
[Out] RootOf(_Z^2+_Z+1)*ln(-((x^4+1)^(2/3)*RootOf(_Z^2+_Z+1)*x+(x^4+1)^(1/3)*Root
Of(_Z^2+_Z+1)*x^2+RootOf(_Z^2+_Z+1)*x^3+x^4+2*(x^4+1)^(2/3)*x+2*x^2*(x^4+1)
^(1/3)+x^3+1)/(x^4-x^3+1))-ln(((x^4+1)^(2/3)*RootOf(_Z^2+_Z+1)*x+(x^4+1)^(1
/3)*RootOf(_Z^2+_Z+1)*x^2+RootOf(_Z^2+_Z+1)*x^3-x^4-(x^4+1)^(2/3)*x-x^2*(x^
4+1)^(1/3)-1)/(x^4-x^3+1))*RootOf(_Z^2+_Z+1)-ln(((x^4+1)^(2/3)*RootOf(_Z^2+
_Z+1)*x+(x^4+1)^(1/3)*RootOf(_Z^2+_Z+1)*x^2+RootOf(_Z^2+_Z+1)*x^3-x^4-(x^4+
1)^(2/3)*x-x^2*(x^4+1)^(1/3)-1)/(x^4-x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-3}{\left(x^4-x^3+1\right)\left(x^4+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)/(x^4+1)^(1/3)/(x^4-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 3)/((x^4 - x^3 + 1)*(x^4 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4-3}{\left(x^4+1\right)^{1/3}\,\left(x^4-x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 3)/((x^4 + 1)^(1/3)*(x^4 - x^3 + 1)),x)
[Out] int((x^4 - 3)/((x^4 + 1)^(1/3)*(x^4 - x^3 + 1)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)/(x**4+1)**(1/3)/(x**4-x**3+1),x)
[Out] Timed out
```

3.898 $\int \frac{-b+2ax^4}{x^2\left(-b+ax^4\right)^{3/4}}\,dx$

Optimal. Leaf size=74

$$-\frac{\sqrt[4]{ax^4-b}}{x}-\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)$$

Rubi [A] time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {451, 331, 298, 203, 206}

$$-\frac{\sqrt[4]{ax^4-b}}{x}-\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-b + 2*a*x^4)/(x^2*(-b + a*x^4)^(3/4)),x]
```

```
[Out] -((-b + a*x^4)^(1/4)/x) - a^(1/4)*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)] +
a^(1/4)*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\int \frac{-b+2ax^4}{x^2\left(-b+ax^4\right)^{3/4}}\,dx = -\frac{\sqrt[4]{-b+ax^4}}{x}+(2a)\int \frac{x^2}{\left(-b+ax^4\right)^{3/4}}\,dx$$
$$= -\frac{\sqrt[4]{-b+ax^4}}{x}+(2a)\,\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)$$
$$= -\frac{\sqrt[4]{-b+ax^4}}{x}+\sqrt{a}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)-\sqrt{a}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{a}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)$$
$$= -\frac{\sqrt[4]{-b+ax^4}}{x}-\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)+\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)$$

Mathematica [A] time = 0.04, size = 74, normalized size = 1.00

$$-\frac{\sqrt[4]{ax^4-b}}{x}-\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + 2*a*x^4)/(x^2*(-b + a*x^4)^(3/4)),x]
[Out] -((-b + a*x^4)^(1/4)/x) - a^(1/4)*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)] + a^(1/4)*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)]
```

IntegrateAlgebraic [A] time = 0.36, size = 74, normalized size = 1.00

$$-\frac{\sqrt[4]{ax^4-b}}{x}-\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + 2*a*x^4)/(x^2*(-b + a*x^4)^(3/4)),x]
[Out] -((-b + a*x^4)^(1/4)/x) - a^(1/4)*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)] + a^(1/4)*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^4-b)/x^2/(a*x^4-b)^(3/4),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax^4-b}{\left(ax^4-b\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^4-b)/x^2/(a*x^4-b)^(3/4),x, algorithm="giac")
[Out] integrate((2*a*x^4 - b)/((a*x^4 - b)^(3/4)*x^2), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{2a\,x^{4}-b}{x^{2} \left(a\,x^{4}-b\right)^{\frac{3}{4}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x^4-b)/x^2/(a*x^4-b)^(3/4),x)
[Out] int((2*a*x^4-b)/x^2/(a*x^4-b)^(3/4),x)
```

maxima [A] time = 0.50, size = 94, normalized size = 1.27

$$\frac{1}{2}\,a\left(\frac{2\,\arctan\left(\frac{\left(ax^{4}-b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{3}{4}}}-\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^{4}-b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^{4}-b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{3}{4}}}\right)-\frac{\left(ax^{4}-b\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^4-b)/x^2/(a*x^4-b)^(3/4),x, algorithm="maxima")
[Out] 1/2*a*(2*arctan((a*x^4 - b)^(1/4)/(a^(1/4)*x))/a^(3/4) - log(-(a^(1/4) - (a
*x^4 - b)^(1/4)/x)/(a^(1/4) + (a*x^4 - b)^(1/4)/x))/a^(3/4)) - (a*x^4 - b)^
(1/4)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{b-2\,a\,x^{4}}{x^{2}\,\left(a\,x^{4}-b\right)^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - 2*a*x^4)/(x^2*(a*x^4 - b)^(3/4)),x)
[Out] -int((b - 2*a*x^4)/(x^2*(a*x^4 - b)^(3/4)), x)
```

sympy [C] time = 2.22, size = 126, normalized size = 1.70

$$\frac{ax^{3}e^{-\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{ax^{4}}{b}\right)}{2b^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}-b\left(\begin{cases}-\frac{\sqrt[4]{a}\sqrt[4]{-1+\frac{b}{ax^{4}}}e^{\frac{i\pi}{4}}\Gamma\left(-\frac{1}{4}\right)}{4b\Gamma\left(\frac{3}{4}\right)} & \text{for}\ \left|\frac{b}{ax^{4}}\right|>1\\ -\frac{\sqrt[4]{a}\sqrt[4]{1-\frac{b}{ax^{4}}}\Gamma\left(-\frac{1}{4}\right)}{4b\Gamma\left(\frac{3}{4}\right)} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x**4-b)/x**2/(a*x**4-b)**(3/4),x)
[Out] a*x**3*exp(-3*I*pi/4)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), a*x**4/b)/(2*b**
(3/4)*gamma(7/4)) - b*Piecewise((-a**(1/4)*(-1 + b/(a*x**4))**(1/4)*exp(I*p
i/4)*gamma(-1/4)/(4*b*gamma(3/4)), Abs(b/(a*x**4)) > 1), (-a**(1/4)*(1 - b/
(a*x**4))**(1/4)*gamma(-1/4)/(4*b*gamma(3/4)), True))
```

3.899 $\int \frac{1}{x \sqrt[3]{1+x^6}} \, dx$

Optimal. Leaf size=74

$$\frac{1}{6}\log\left(\sqrt[3]{x^6+1}-1\right)-\frac{1}{12}\log\left(\left(x^6+1\right)^{2/3}+\sqrt[3]{x^6+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^6+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 0.73, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {266, 55, 618, 204, 31}

$$\frac{1}{4}\log\left(1-\sqrt[3]{x^6+1}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^6+1}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^6)^(1/3)),x]
```

```
[Out] ArcTan[(1 + 2*(1 + x^6)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - Log[x]/2 + Log[1 - (1
 + x^6)^(1/3)]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt[3]{1+x^6}}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx, x, x^6\right)\\ &= -\frac{\log(x)}{2} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+x^6}\right) + \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1+x^6}\right)\\ &= -\frac{\log(x)}{2} + \frac{1}{4}\log\left(1-\sqrt[3]{1+x^6}\right) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+x^6}\right)\\ &= \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^6}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{4}\log\left(1-\sqrt[3]{1+x^6}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.72

$$\frac{1}{6}\left(\frac{3}{2}\log\left(1-\sqrt[3]{x^6+1}\right) + \sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^6+1}+1}{\sqrt{3}}\right) - 3\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^6)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 + x^6)^(1/3))/Sqrt[3]] - 3*Log[x] + (3*Log[1 - (1
 + x^6)^(1/3)])/2)/6
```

IntegrateAlgebraic [A] time = 0.04, size = 74, normalized size = 1.00

$$\frac{1}{6}\log\left(\sqrt[3]{x^6+1}-1\right) - \frac{1}{12}\log\left(\left(x^6+1\right)^{2/3} + \sqrt[3]{x^6+1}+1\right) + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^6+1}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(1 + x^6)^(1/3)),x]
```

```
[Out] ArcTan[1/Sqrt[3] + (2*(1 + x^6)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) + Log[-1 + (1 +
 x^6)^(1/3)]/6 - Log[1 + (1 + x^6)^(1/3) + (1 + x^6)^(2/3)]/12
```

fricas [A] time = 0.78, size = 56, normalized size = 0.76

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^6+1\right)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right) - \frac{1}{12}\log\left(\left(x^6+1\right)^{\frac{2}{3}} + \left(x^6+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{6}\log\left(\left(x^6+1\right)^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(2/3*sqrt(3)*(x^6 + 1)^(1/3) + 1/3*sqrt(3)) - 1/12*log((x
^6 + 1)^(2/3) + (x^6 + 1)^(1/3) + 1) + 1/6*log((x^6 + 1)^(1/3) - 1)
```

giac [A] time = 0.50, size = 54, normalized size = 0.73

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^6+1\right)^{\frac{1}{3}} + 1\right)\right) - \frac{1}{12}\log\left(\left(x^6+1\right)^{\frac{2}{3}} + \left(x^6+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{6}\log\left(\left(x^6+1\right)^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3) + 1)) - 1/12*log((x^6 + 1
)^(2/3) + (x^6 + 1)^(1/3) + 1) + 1/6*log((x^6 + 1)^(1/3) - 1)
```

maple [C] time = 0.24, size = 63, normalized size = 0.85

$$\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^{6}\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],-x^{6}\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+6\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{12\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6+1)^(1/3),x)
```

```
[Out] 1/12/Pi*3^(1/2)*GAMMA(2/3)*(-2/9*Pi*3^(1/2)/GAMMA(2/3)*x^6*hypergeom([1,1,4
/3],[2,2],-x^6)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+6*ln(x))*Pi*3^(1/2)/GAMMA(2/
3))
```

maxima [A] time = 0.53, size = 54, normalized size = 0.73

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^{6}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{12}\log\left(\left(x^{6}+1\right)^{\frac{2}{3}}+\left(x^{6}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(\left(x^{6}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3) + 1)) - 1/12*log((x^6 + 1
)^(2/3) + (x^6 + 1)^(1/3) + 1) + 1/6*log((x^6 + 1)^(1/3) - 1)
```

mupad [B] time = 0.86, size = 80, normalized size = 1.08

$$\frac{\ln\left(\frac{\left(x^{6}+1\right)^{1/3}}{4}-\frac{1}{4}\right)}{6}+\ln\left(\frac{\left(x^{6}+1\right)^{1/3}}{4}-9\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)^{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(\frac{\left(x^{6}+1\right)^{1/3}}{4}-9\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)^{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^6 + 1)^(1/3)),x)
```

```
[Out] log((x^6 + 1)^(1/3)/4 - 1/4)/6 + log((x^6 + 1)^(1/3)/4 - 9*((3^(1/2)*1i)/12
 - 1/12)^2)*((3^(1/2)*1i)/12 - 1/12) - log((x^6 + 1)^(1/3)/4 - 9*((3^(1/2)*
1i)/12 + 1/12)^2)*((3^(1/2)*1i)/12 + 1/12)
```

sympy [C] time = 0.79, size = 31, normalized size = 0.42

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^{6}}\right)}{6x^{2}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6+1)**(1/3),x)
```

```
[Out] -gamma(1/3)*hyper((1/3, 1/3), (4/3,), exp_polar(I*pi)/x**6)/(6*x**2*gamma(4
/3))
```

3.900 $\int \frac{3-9x^4+2x^6}{x(1+x^2)^2(-1+2x^2)\sqrt{\frac{1-2x^2}{1+2x^2}}(1+2x^2)}\,dx$

Optimal. Leaf size=74

$$3\tanh^{-1}\left(\sqrt{\frac{1-2x^2}{2x^2+1}}\right)-\frac{2\sqrt{\frac{1-2x^2}{2x^2+1}}\left(2x^4-x^2-1\right)}{3\left(2x^4+x^2-1\right)}$$

Rubi [A] time = 3.70, antiderivative size = 129, normalized size of antiderivative = 1.74, number of steps used = 17, number of rules used = 12, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {6688, 6719, 1586, 1606, 37, 96, 92, 206, 103, 21, 93, 203}

$$-\frac{4}{3\left(x^2+1\right)\sqrt{\frac{1-2x^2}{2x^2+1}}}+\frac{2}{3\sqrt{\frac{1-2x^2}{2x^2+1}}}+\frac{3\sqrt{1-2x^2}\tanh^{-1}\left(\sqrt{1-2x^2}\sqrt{2x^2+1}\right)}{2\sqrt{\frac{1-2x^2}{2x^2+1}}\sqrt{2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 9*x^4 + 2*x^6)/(x*(1 + x^2)^2*(-1 + 2*x^2)*Sqrt[(1 - 2*x^2)/(1 + 2
*x^2)]*(1 + 2*x^2)),x]
```

```
[Out] 2/(3*Sqrt[(1 - 2*x^2)/(1 + 2*x^2)]) - 4/(3*(1 + x^2)*Sqrt[(1 - 2*x^2)/(1 +
2*x^2)]) + (3*Sqrt[1 - 2*x^2]*ArcTanh[Sqrt[1 - 2*x^2]*Sqrt[1 + 2*x^2]])/(2*
Sqrt[(1 - 2*x^2)/(1 + 2*x^2)]*Sqrt[1 + 2*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1606

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegran
d[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b
, c, d, e, f, g, h, m, n, p, q}, x] && PolyQ[Px, x] && IntegersQ[m, n]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{3-9x^4+2x^6}{x\left(1+x^2\right)^2\left(-1+2x^2\right)\sqrt{\frac{1-2x^2}{1+2x^2}}\left(1+2x^2\right)}\,dx = \int \frac{-3+9x^4-2x^6}{x\left(1+x^2\right)^2\sqrt{\frac{1-2x^2}{1+2x^2}}\left(1-4x^4\right)}\,dx$$

$$= \frac{\sqrt{1-2x^2}\int \frac{\sqrt{1+2x^2}\left(-3+9x^4-2x^6\right)}{x\sqrt{1-2x^2}\left(1+x^2\right)^2\left(1-4x^4\right)}\,dx}{\sqrt{\frac{1-2x^2}{1+2x^2}}\sqrt{1+2x^2}}$$

$$= \frac{\sqrt{1-2x^2}\int \frac{-3+9x^4-2x^6}{x\left(1-2x^2\right)^{3/2}\left(1+x^2\right)^2\sqrt{1+2x^2}}\,dx}{\sqrt{\frac{1-2x^2}{1+2x^2}}\sqrt{1+2x^2}}$$

$$= \frac{\sqrt{1-2x^2}\,\text{Subst}\left(\int \frac{-3+9x^2-2x^3}{(1-2x)^{3/2}x(1+x)^2\sqrt{1+2x}}\,dx, x, x^2\right)}{2\sqrt{\frac{1-2x^2}{1+2x^2}}\sqrt{1+2x^2}}$$

$$= \frac{\sqrt{1-2x^2}\,\text{Subst}\left(\int\left(-\frac{2}{(1-2x)^{3/2}\sqrt{1+2x}} - \frac{3}{(1-2x)^{3/2}x\sqrt{1+2x}} - \frac{}{(1-2x)^{3/2}(}\right.\right.}{2\sqrt{\frac{1-2x^2}{1+2x^2}}\sqrt{1+2x^2}}$$

$$= -\frac{\sqrt{1-2x^2}\,\text{Subst}\left(\int \frac{1}{(1-2x)^{3/2}\sqrt{1+2x}}\,dx, x, x^2\right)}{\sqrt{\frac{1-2x^2}{1+2x^2}}\sqrt{1+2x^2}} - \frac{\left(3\sqrt{1-2x^2}\right)\text{Su}}{2}$$

$$= \frac{2}{3\sqrt{\frac{1-2x^2}{1+2x^2}}} - \frac{4}{3\left(1+x^2\right)\sqrt{\frac{1-2x^2}{1+2x^2}}} - \frac{\left(4\sqrt{1-2x^2}\right)\text{Subst}\left(\int \frac{}{(1-2x)^{3/}}\right.}{3\sqrt{\frac{1-2x^2}{1+2x^2}}\sqrt{1+}}$$

$$= \frac{2}{3\sqrt{\frac{1-2x^2}{1+2x^2}}} - \frac{4}{3\left(1+x^2\right)\sqrt{\frac{1-2x^2}{1+2x^2}}} - \frac{\left(8\sqrt{1-2x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-2x}(}\right.}{3\sqrt{\frac{1-2x^2}{1+2x^2}}\sqrt{1+}}$$

$$= \frac{2}{3\sqrt{\frac{1-2x^2}{1+2x^2}}} - \frac{4}{3\left(1+x^2\right)\sqrt{\frac{1-2x^2}{1+2x^2}}} + \frac{16\sqrt{1-2x^2}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{1+2x^2}}{\sqrt{1-2x^2}}\right.}{3\sqrt{3}\sqrt{\frac{1-2x^2}{1+2x^2}}\sqrt{1+2x^2}}$$

$$= \frac{2}{3\sqrt{\frac{1-2x^2}{1+2x^2}}} - \frac{4}{3\left(1+x^2\right)\sqrt{\frac{1-2x^2}{1+2x^2}}} + \frac{3\sqrt{1-2x^2}\tanh^{-1}\left(\sqrt{1-2x^2}\right.}{2\sqrt{\frac{1-2x^2}{1+2x^2}}\sqrt{1+2x^2}}$$

Mathematica [A] time = 0.25, size = 80, normalized size = 1.08

$$\frac{8x^4-4x^2+9\sqrt{4-\frac{1}{x^4}}\left(x^2+1\right)x^2\sin^{-1}\left(\frac{1}{2x^2}\right)-4}{6\sqrt{\frac{1-2x^2}{2x^2+1}}\left(2x^4+3x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 9*x^4 + 2*x^6)/(x*(1 + x^2)^2*(-1 + 2*x^2)*Sqrt[(1 - 2*x^2)/
(1 + 2*x^2)]*(1 + 2*x^2)),x]
```

```
[Out] (-4 - 4*x^2 + 8*x^4 + 9*Sqrt[4 - x^(-4)]*x^2*(1 + x^2)*ArcSin[1/(2*x^2)])/(
6*Sqrt[(1 - 2*x^2)/(1 + 2*x^2)]*(1 + 3*x^2 + 2*x^4))
```

IntegrateAlgebraic [A] time = 0.12, size = 74, normalized size = 1.00

$$3\tanh^{-1}\left(\sqrt{\frac{1-2x^2}{2x^2+1}}\right)-\frac{2\sqrt{\frac{1-2x^2}{2x^2+1}}\left(2x^4-x^2-1\right)}{3\left(2x^4+x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - 9*x^4 + 2*x^6)/(x*(1 + x^2)^2*(-1 + 2*x^2)*Sqrt[(1
- 2*x^2)/(1 + 2*x^2)]*(1 + 2*x^2)),x]
```

```
[Out] (-2*Sqrt[(1 - 2*x^2)/(1 + 2*x^2)]*(-1 - x^2 + 2*x^4))/(3*(-1 + x^2 + 2*x^4)
) + 3*ArcTanh[Sqrt[(1 - 2*x^2)/(1 + 2*x^2)]]
```

fricas [A] time = 0.54, size = 107, normalized size = 1.45

$$-\frac{8\,x^4+4\,x^2+9\left(2\,x^4+x^2-1\right)\log\left(\frac{\left(2\,x^2+1\right)\sqrt{-\frac{2\,x^2-1}{2\,x^2+1}}-1}{x^2}\right)+4\left(2\,x^4-x^2-1\right)\sqrt{-\frac{2\,x^2-1}{2\,x^2+1}}-4}{6\left(2\,x^4+x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6-9*x^4+3)/x/(x^2+1)^2/(2*x^2-1)/((-2*x^2+1)/(2*x^2+1))^(1/2
)/(2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/6*(8*x^4 + 4*x^2 + 9*(2*x^4 + x^2 - 1)*log(((2*x^2 + 1)*sqrt(-(2*x^2 - 1
)/(2*x^2 + 1)) - 1)/x^2) + 4*(2*x^4 - x^2 - 1)*sqrt(-(2*x^2 - 1)/(2*x^2 + 1
)) - 4)/(2*x^4 + x^2 - 1)
```

giac [A] time = 0.55, size = 44, normalized size = 0.59

$$-\frac{32}{3\left(\frac{\left(\sqrt{-4\,x^4+1}-1\right)^3}{x^6}+8\right)}-\frac{3}{2}\log\left(-\frac{\sqrt{-4\,x^4+1}-1}{2\,x^2}\right)+\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6-9*x^4+3)/x/(x^2+1)^2/(2*x^2-1)/((-2*x^2+1)/(2*x^2+1))^(1/2
)/(2*x^2+1),x, algorithm="giac")
```

```
[Out] -32/3/((sqrt(-4*x^4 + 1) - 1)^3/x^6 + 8) - 3/2*log(-1/2*(sqrt(-4*x^4 + 1) -
 1)/x^2) + 2/3
```

maple [B] time = 0.10, size = 169, normalized size = 2.28

$$-\frac{64\sqrt{-4x^4+1}\,x^6+16\left(-4x^4+1\right)^{\frac{3}{2}}x^2-32\sqrt{-4x^4+1}\,x^4+162\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-4x^4+1}}\right)x^4-8\left(-4x^4+1\right)^{\frac{3}{2}}-52x^2\sqrt{-4x^4+1}+81\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-4x^4+1}}\right)x^2+44\sqrt{-4x^4+1}-81\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-4x^4+1}}\right)}{54\left(x^2+1\right)\sqrt{-\left(2x^2+1\right)\left(2x^2-1\right)}\,\sqrt{-\frac{2x^2-1}{2x^2+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6-9*x^4+3)/x/(x^2+1)^2/(2*x^2-1)/((-2*x^2+1)/(2*x^2+1))^(1/2)/(2*x
^2+1),x)
```

```
[Out] -1/54*(64*(-4*x^4+1)^(1/2)*x^6+16*(-4*x^4+1)^(3/2)*x^2-32*(-4*x^4+1)^(1/2)*
x^4+162*arctanh(1/(-4*x^4+1)^(1/2))*x^4-8*(-4*x^4+1)^(3/2)-52*x^2*(-4*x^4+1
)^(1/2)+81*arctanh(1/(-4*x^4+1)^(1/2))*x^2+44*(-4*x^4+1)^(1/2)-81*arctanh(1
/(-4*x^4+1)^(1/2)))/(x^2+1)/(-(2*x^2+1)*(2*x^2-1))^(1/2)/(-(2*x^2-1)/(2*x^2
+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^{6} - 9\,x^{4} + 3}{\left(2\,x^{2} + 1\right)\left(2\,x^{2} - 1\right)\left(x^{2} + 1\right)^{2} x \sqrt{-\frac{2\,x^{2} - 1}{2\,x^{2} + 1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6-9*x^4+3)/x/(x^2+1)^2/(2*x^2-1)/((-2*x^2+1)/(2*x^2+1))^(1/2
)/(2*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^6 - 9*x^4 + 3)/((2*x^2 + 1)*(2*x^2 - 1)*(x^2 + 1)^2*x*sqrt(-
(2*x^2 - 1)/(2*x^2 + 1))), x)
```

mupad [B] time = 1.42, size = 270, normalized size = 3.65

$$3\,\mathrm{atanh}\left(\sqrt{-\frac{2\,x^{2} - 1}{2\,x^{2} + 1}}\right) - \sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\sqrt{-\frac{2\,x^{2}-1}{2\,x^{2}+1}}}{3}\right) + \frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\sqrt{1-2\,x^{2}}\,\sqrt{\frac{1}{2\,x^{2}+1}}}{3}\right)}{2\,x^{2} + 2} - \frac{\left(x^{2} + \frac{1}{2}\right)\left(\frac{x^{2}}{3} - \frac{1}{3}\right)\sqrt{-\frac{2\,x^{2}-1}{2\,x^{2}+1}}}{2\,x^{4} + x^{2} - 1} + \frac{3\,x^{2}}{\sqrt{1 - 2\,x^{2}}\,\left(2\,x^{2} + 2\right)\sqrt{\frac{1}{2\,x^{2}+1}}} + \frac{2\,\sqrt{3}\,x^{2}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\sqrt{1-2\,x^{2}}\,\sqrt{\frac{1}{2\,x^{2}+1}}}{3}\right)}{2\,x^{2} + 2} - \frac{2\mathrm{i}}{\sqrt{-\frac{2\,x^{2}-1}{2\,x^{2}+1}}\,3\mathrm{i} + \left(-\frac{2\,x^{2}-1}{2\,x^{2}+1}\right)^{3/2}\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6 - 9*x^4 + 3)/(x*(x^2 + 1)^2*(2*x^2 - 1)*(2*x^2 + 1)*(-(2*x^2 - 1
)/(2*x^2 + 1))^(1/2)),x)
```

```
[Out] 3*atanh((-(2*x^2 - 1)/(2*x^2 + 1))^(1/2)) - 2i/((-(2*x^2 - 1)/(2*x^2 + 1))^
(1/2)*3i + (-(2*x^2 - 1)/(2*x^2 + 1))^(3/2)*1i) - 3^(1/2)*atan((3^(1/2)*(-(
2*x^2 - 1)/(2*x^2 + 1))^(1/2))/3) + (2*3^(1/2)*atan((3^(1/2)*(1 - 2*x^2)^(1
/2)*(1/(2*x^2 + 1))^(1/2))/3))/(2*x^2 + 2) - ((x^2 + 1/2)*(x^2/3 - 1/3)*(-(
2*x^2 - 1)/(2*x^2 + 1))^(1/2))/(x^2 + 2*x^4 - 1) + (3*x^2)/((1 - 2*x^2)^(1/
2)*(2*x^2 + 2)*(1/(2*x^2 + 1))^(1/2)) + (2*3^(1/2)*x^2*atan((3^(1/2)*(1 - 2
*x^2)^(1/2)*(1/(2*x^2 + 1))^(1/2))/3))/(2*x^2 + 2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**6-9*x**4+3)/x/(x**2+1)**2/(2*x**2-1)/((-2*x**2+1)/(2*x**2+1
))**(1/2)/(2*x**2+1),x)
```

```
[Out] Timed out
```

3.901 $\int \frac{\sqrt[4]{-1+2x^4}\left(-2+x^8\right)}{x^6\left(-1+x^4\right)^2}\,dx$

Optimal. Leaf size=74

$$\frac{15}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{2x^4-1}}\right)-\frac{15}{8}\tanh^{-1}\left(\frac{x}{\sqrt[4]{2x^4-1}}\right)+\frac{\sqrt[4]{2x^4-1}\left(69x^8-56x^4-8\right)}{20x^5\left(x^4-1\right)}$$

Rubi [C] time = 2.88, antiderivative size = 198, normalized size of antiderivative = 2.68, number of steps used = 176, number of rules used = 33, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.222, Rules used = {6742, 2153, 1240, 412, 529, 237, 335, 275, 232, 407, 409, 1213, 537, 511, 510, 1248, 733, 844, 234, 220, 747, 400, 442, 444, 63, 212, 206, 203, 1336, 264, 277, 331, 298}

$$-\frac{4\sqrt[4]{2x^4-1}\,x^3F_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};2x^4,x^4\right)}{3\sqrt[4]{1-2x^4}}-\frac{\sqrt[4]{2x^4-1}\,x^3\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{x^4}{1-x^4}\right)}{3\sqrt[4]{1-2x^4}\left(1-x^4\right)^{3/4}}+\frac{4\sqrt[4]{2x^4-1}}{x}+2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{2x^4-1}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{2x^4-1}}\right)-\frac{2\left(2x^4-1\right)^{5/4}}{5x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + 2*x^4)^(1/4)*(-2 + x^8))/(x^6*(-1 + x^4)^2),x]
```

```
[Out] (4*(-1 + 2*x^4)^(1/4))/x - (2*(-1 + 2*x^4)^(5/4))/(5*x^5) - (4*x^3*(-1 + 2*
x^4)^(1/4)*AppellF1[3/4, -1/4, 1, 7/4, 2*x^4, x^4])/(3*(1 - 2*x^4)^(1/4)) +
 2*2^(1/4)*ArcTan[(2^(1/4)*x)/(-1 + 2*x^4)^(1/4)] - 2*2^(1/4)*ArcTanh[(2^(1
/4)*x)/(-1 + 2*x^4)^(1/4)] - (x^3*(-1 + 2*x^4)^(1/4)*Hypergeometric2F1[-1/4
, 3/4, 7/4, x^4/(1 - x^4)])/(3*(1 - 2*x^4)^(1/4)*(1 - x^4)^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[1/c, Int[1/(a + b*x^2)^(3/4), x], x] - Dist[d/c, Int[x^2/((a + b*x^2)^(3/
4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 529

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
```

```
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 2153

```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^nn)^p, (c/(c^2 - d^2*x^(2*n)) - (d*x^n)/
(c^2 - d^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !
IntegerQ[p] && ILtQ[q, 0] && IGtQ[Log[2, nn/n], 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt[4]{-1+2x^4}\left(-2+x^8\right)}{x^6\left(-1+x^4\right)^2}\,dx = \int\left(-\frac{\sqrt[4]{-1+2x^4}}{16(-1+x)^2} - \frac{2\sqrt[4]{-1+2x^4}}{x^6} - \frac{4\sqrt[4]{-1+2x^4}}{x^2} - \frac{\sqrt[4]{-1+2x^4}}{16(1+x)^2} + \frac{17\sqrt[4]{-1+2x^4}}{8\left(-1+x^2\right)}\right.$$

$$= -\left(\frac{1}{16}\int\frac{\sqrt[4]{-1+2x^4}}{(-1+x)^2}\,dx\right) - \frac{1}{16}\int\frac{\sqrt[4]{-1+2x^4}}{(1+x)^2}\,dx + \frac{1}{4}\int\frac{\sqrt[4]{-1+2x^4}}{\left(1+x^2\right)^2}\,dx - 2\int$$

$$= \frac{4\sqrt[4]{-1+2x^4}}{x} - \frac{2\left(-1+2x^4\right)^{5/4}}{5x^5} - \frac{1}{16}\int\left(\frac{\sqrt[4]{-1+2x^4}}{\left(-1+x^2\right)^2} - \frac{2x\sqrt[4]{-1+2x^4}}{\left(-1+x^2\right)^2} + \frac{x^2\sqrt[4]{-1}}{(-1}\right.$$

= rest of steps removed due to Latex formating problem

Mathematica [C] time = 0.23, size = 92, normalized size = 1.24

$$\frac{25\left(1-2x^4\right)^{3/4}\sqrt[4]{1-x^4}\,x^8\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{x^4}{1-x^4}\right)+138x^{12}-181x^8+40x^4+8}{20x^5\left(x^4-1\right)\left(2x^4-1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + 2*x^4)^(1/4)*(-2 + x^8))/(x^6*(-1 + x^4)^2),x]
```

```
[Out] (8 + 40*x^4 - 181*x^8 + 138*x^12 + 25*x^8*(1 - 2*x^4)^(3/4)*(1 - x^4)^(1/4)
*Hypergeometric2F1[3/4, 3/4, 7/4, x^4/(1 - x^4)])/(20*x^5*(-1 + x^4)*(-1 +
2*x^4)^(3/4))
```

IntegrateAlgebraic [A] time = 0.28, size = 74, normalized size = 1.00

$$\frac{15}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{2x^4-1}}\right)-\frac{15}{8}\tanh^{-1}\left(\frac{x}{\sqrt[4]{2x^4-1}}\right)+\frac{\sqrt[4]{2x^4-1}\left(69x^8-56x^4-8\right)}{20x^5\left(x^4-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + 2*x^4)^(1/4)*(-2 + x^8))/(x^6*(-1 + x^4)^2),x]
```

```
[Out] ((-1 + 2*x^4)^(1/4)*(-8 - 56*x^4 + 69*x^8))/(20*x^5*(-1 + x^4)) + (15*ArcTa
n[x/(-1 + 2*x^4)^(1/4)])/8 - (15*ArcTanh[x/(-1 + 2*x^4)^(1/4)])/8
```

fricas [B] time = 4.11, size = 151, normalized size = 2.04

$$\frac{75\left(x^9-x^5\right)\arctan\left(\frac{2\left(\left(2x^4-1\right)^{\frac{1}{4}}x^3+\left(2x^4-1\right)^{\frac{3}{4}}x\right)}{x^4-1}\right)+75\left(x^9-x^5\right)\log\left(-\frac{3x^4-2\left(2x^4-1\right)^{\frac{1}{4}}x^3+2\sqrt{2x^4-1}\,x^2-2\left(2x^4-1\right)^{\frac{3}{4}}x-1}{x^4-1}\right)+4\left(69x^8-56x^4-8\right)\left(2x^4-1\right)^{\frac{1}{4}}}{80\left(x^9-x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)^(1/4)*(x^8-2)/x^6/(x^4-1)^2,x, algorithm="fricas")
```

```
[Out] 1/80*(75*(x^9 - x^5)*arctan(2*((2*x^4 - 1)^(1/4)*x^3 + (2*x^4 - 1)^(3/4)*x)
/(x^4 - 1)) + 75*(x^9 - x^5)*log(-(3*x^4 - 2*(2*x^4 - 1)^(1/4)*x^3 + 2*sqrt
(2*x^4 - 1)*x^2 - 2*(2*x^4 - 1)^(3/4)*x - 1)/(x^4 - 1)) + 4*(69*x^8 - 56*x^
4 - 8)*(2*x^4 - 1)^(1/4))/(x^9 - x^5)
```

giac [A] time = 0.44, size = 108, normalized size = 1.46

$$-\frac{2\left(2x^4-1\right)^{\frac{1}{4}}\left(\frac{1}{x^4}-2\right)}{5x}-\frac{4\left(2x^4-1\right)^{\frac{1}{4}}}{x}+\frac{\left(2x^4-1\right)^{\frac{1}{4}}}{4x\left(\frac{1}{x^4}-1\right)}+\frac{15}{8}\arctan\left(\frac{\left(2x^4-1\right)^{\frac{1}{4}}}{x}\right)+\frac{15}{16}\log\left(\frac{\left(2x^4-1\right)^{\frac{1}{4}}}{x}+1\right)-\frac{15}{16}\log\left(\left|\frac{\left(2x^4-1\right)^{\frac{1}{4}}}{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)^(1/4)*(x^8-2)/x^6/(x^4-1)^2,x, algorithm="giac")
```

```
[Out] -2/5*(2*x^4 - 1)^(1/4)*(1/x^4 - 2)/x - 4*(2*x^4 - 1)^(1/4)/x + 1/4*(2*x^4 -
 1)^(1/4)/(x*(1/x^4 - 1)) + 15/8*arctan((2*x^4 - 1)^(1/4)/x) + 15/16*log((2
*x^4 - 1)^(1/4)/x + 1) - 15/16*log(abs((2*x^4 - 1)^(1/4)/x - 1))
```

maple [C] time = 1.07, size = 471, normalized size = 6.36

$$\frac{138x^{12}-181x^8+40x^4+8}{20\left(x^4-1\right)x^5\left(2x^4-1\right)^{\frac{3}{4}}}+\frac{\left(\frac{15\ln\left(-\frac{-12x^{12}+8\left(8x^{12}-12x^8+6x^4-1\right)^{\frac{1}{4}}x^9-4\sqrt{8x^{12}-12x^8+6x^4-1}\,x^6+16x^8+2\left(8x^{12}-12x^8+6x^4-1\right)^{\frac{3}{4}}x^3-8\left(8x^{12}-12x^8+6x^4-1\right)^{\frac{1}{4}}x^5+2\sqrt{8x^{12}-12x^8+6x^4-1}\,x^2-7x^4+2\left(8x^{12}-12x^8+6x^4-1\right)^{\frac{1}{4}}x+1}{\left(2x^4-1\right)^2\left(-1+x\right)\left(1+x\right)\left(x^2+1\right)}\right)}{16}-\frac{15\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-12x^{12}-8\,\mathrm{RootOf}\left(\_Z^2+1\right)\left(8x^{12}-12x^8+6x^4-1\right)^{\frac{1}{4}}x^9+4\sqrt{8x^{12}-12x^8+6x^4-1}\,x^6+16x^8+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\left(8x^{12}-12x^8+6x^4-1\right)^{\frac{3}{4}}x^3+8\,\mathrm{RootOf}\left(\_Z^2+1\right)\left(8x^{12}-12x^8+6x^4-1\right)^{\frac{1}{4}}x^5-2\sqrt{8x^{12}-12x^8+6x^4-1}\,x^2-7x^4-2\,\mathrm{RootOf}\left(\_Z^2+1\right)\left(8x^{12}-12x^8+6x^4-1\right)^{\frac{1}{4}}x+1}{\left(2x^4-1\right)^2\left(-1+x\right)\left(1+x\right)\left(x^2+1\right)}\right)}{16}\right)\left(\left(2x^4-1\right)^3\right)^{\frac{1}{4}}}{\left(2x^4-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-1)^(1/4)*(x^8-2)/x^6/(x^4-1)^2,x)
```

```
[Out] 1/20*(138*x^12-181*x^8+40*x^4+8)/(x^4-1)/x^5/(2*x^4-1)^(3/4)+(15/16*ln(-(-1
2*x^12+8*(8*x^12-12*x^8+6*x^4-1)^(1/4)*x^9-4*(8*x^12-12*x^8+6*x^4-1)^(1/2)*
x^6+16*x^8+2*(8*x^12-12*x^8+6*x^4-1)^(3/4)*x^3-8*(8*x^12-12*x^8+6*x^4-1)^(1
/4)*x^5+2*(8*x^12-12*x^8+6*x^4-1)^(1/2)*x^2-7*x^4+2*(8*x^12-12*x^8+6*x^4-1)
^(1/4)*x+1)/(2*x^4-1)^2/(-1+x)/(1+x)/(x^2+1))-15/16*RootOf(_Z^2+1)*ln((-12*
x^12-8*RootOf(_Z^2+1)*(8*x^12-12*x^8+6*x^4-1)^(1/4)*x^9+4*(8*x^12-12*x^8+6*
x^4-1)^(1/2)*x^6+16*x^8+2*RootOf(_Z^2+1)*(8*x^12-12*x^8+6*x^4-1)^(3/4)*x^3+
8*RootOf(_Z^2+1)*(8*x^12-12*x^8+6*x^4-1)^(1/4)*x^5-2*(8*x^12-12*x^8+6*x^4-1
)^(1/2)*x^2-7*x^4-2*RootOf(_Z^2+1)*(8*x^12-12*x^8+6*x^4-1)^(1/4)*x+1)/(2*x^
4-1)^2/(-1+x)/(1+x)/(x^2+1)))/(2*x^4-1)^(3/4)*((2*x^4-1)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8-2\right)\left(2x^4-1\right)^{\frac{1}{4}}}{\left(x^4-1\right)^2 x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)^(1/4)*(x^8-2)/x^6/(x^4-1)^2,x, algorithm="maxima")
```

```
[Out] integrate((x^8 - 2)*(2*x^4 - 1)^(1/4)/((x^4 - 1)^2*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^8-2\right)\,\left(2\,x^4-1\right)^{1/4}}{x^6\,\left(x^4-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^8 - 2)*(2*x^4 - 1)^(1/4))/(x^6*(x^4 - 1)^2),x)
```

```
[Out] int(((x^8 - 2)*(2*x^4 - 1)^(1/4))/(x^6*(x^4 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{2x^4-1}\left(x^8-2\right)}{x^6\left(x-1\right)^2\left(x+1\right)^2\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-1)**(1/4)*(x**8-2)/x**6/(x**4-1)**2,x)
```

```
[Out] Integral((2*x**4 - 1)**(1/4)*(x**8 - 2)/(x**6*(x - 1)**2*(x + 1)**2*(x**2 +
 1)**2), x)
```

3.902 $\int \frac{(b+ax^2)^{3/4}}{x^3} dx$

Optimal. Leaf size=75

$$-\frac{(ax^2+b)^{3/4}}{2x^2}+\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{4\sqrt[4]{b}}-\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{4\sqrt[4]{b}}$$

Rubi [A] time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 47, 63, 298, 203, 206}

$$-\frac{(ax^2+b)^{3/4}}{2x^2}+\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{4\sqrt[4]{b}}-\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{4\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^2)^(3/4)/x^3,x]
```

```
[Out] -1/2*(b + a*x^2)^(3/4)/x^2 + (3*a*ArcTan[(b + a*x^2)^(1/4)/b^(1/4)])/(4*b^(
1/4)) - (3*a*ArcTanh[(b + a*x^2)^(1/4)/b^(1/4)])/(4*b^(1/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(b+ax^2\right)^{3/4}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(b+ax)^{3/4}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\left(b+ax^2\right)^{3/4}}{2x^2} + \frac{1}{8}(3a)\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{b+ax}}\,dx, x, x^2\right)\\
&= -\frac{\left(b+ax^2\right)^{3/4}}{2x^2} + \frac{3}{2}\operatorname{Subst}\left(\int \frac{x^2}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^2}\right)\\
&= -\frac{\left(b+ax^2\right)^{3/4}}{2x^2} - \frac{1}{4}(3a)\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^2}\right) + \frac{1}{4}(3a)\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,d\right.\\
&= -\frac{\left(b+ax^2\right)^{3/4}}{2x^2} + \frac{3a\tan^{-1}\left(\frac{\sqrt[4]{b+ax^2}}{\sqrt[4]{b}}\right)}{4\sqrt[4]{b}} - \frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^2}}{\sqrt[4]{b}}\right)}{4\sqrt[4]{b}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.49

$$\frac{2a\left(ax^2+b\right)^{7/4}\,{}_2F_1\left(\frac{7}{4},2;\frac{11}{4};\frac{ax^2}{b}+1\right)}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^2)^(3/4)/x^3,x]
```

```
[Out] (2*a*(b + a*x^2)^(7/4)*Hypergeometric2F1[7/4, 2, 11/4, 1 + (a*x^2)/b])/(7*b
^2)
```

IntegrateAlgebraic [A] time = 0.13, size = 75, normalized size = 1.00

$$-\frac{\left(ax^2+b\right)^{3/4}}{2x^2} + \frac{3a\tan^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{4\sqrt[4]{b}} - \frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{4\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^2)^(3/4)/x^3,x]
```

```
[Out] -1/2*(b + a*x^2)^(3/4)/x^2 + (3*a*ArcTan[(b + a*x^2)^(1/4)/b^(1/4)])/(4*b^(
1/4)) - (3*a*ArcTanh[(b + a*x^2)^(1/4)/b^(1/4)])/(4*b^(1/4))
```

fricas [B] time = 0.42, size = 185, normalized size = 2.47

$$-\frac{12\left(\frac{a^4}{b}\right)^{\frac{1}{4}}x^2\arctan\left(-\frac{\left(\frac{a^4}{b}\right)^{\frac{1}{4}}\left(ax^2+b\right)^{\frac{1}{4}}a^3-\sqrt{\sqrt{ax^2+b}\,a^6+\sqrt{\frac{a^4}{b}}\,a^4b}\left(\frac{a^4}{b}\right)^{\frac{1}{4}}}{a^4}\right)+3\left(\frac{a^4}{b}\right)^{\frac{1}{4}}x^2\log\left(27\left(ax^2+b\right)^{\frac{1}{4}}a^3+27\left(\frac{a^4}{b}\right)^{\frac{3}{4}}b\right)-3\left(\frac{a^4}{b}\right)^{\frac{1}{4}}x^2\log\left(27\left(ax^2+b\right)^{\frac{1}{4}}a^3-27\left(\frac{a^4}{b}\right)^{\frac{3}{4}}b\right)+4\left(ax^2+b\right)^{\frac{3}{4}}}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)^(3/4)/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(12*(a^4/b)^(1/4)*x^2*arctan(-((a^4/b)^(1/4)*(a*x^2 + b)^(1/4)*a^3 - s
qrt(sqrt(a*x^2 + b)*a^6 + sqrt(a^4/b)*a^4*b)*(a^4/b)^(1/4))/a^4) + 3*(a^4/b
)^(1/4)*x^2*log(27*(a*x^2 + b)^(1/4)*a^3 + 27*(a^4/b)^(3/4)*b) - 3*(a^4/b)^
(1/4)*x^2*log(27*(a*x^2 + b)^(1/4)*a^3 - 27*(a^4/b)^(3/4)*b) + 4*(a*x^2 + b
)^(3/4))/x^2
```

giac [B] time = 0.31, size = 209, normalized size = 2.79

$$\frac{\frac{6\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}+2\left(ax^2+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{(-b)^{\frac{1}{4}}}+\frac{6\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}-2\left(ax^2+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{(-b)^{\frac{1}{4}}}+\frac{3\sqrt{2}a^2(-b)^{\frac{3}{4}}\log\left(\sqrt{2}\left(ax^2+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^2+b}+\sqrt{-b}\right)}{b}+\frac{3\sqrt{2}a^2\log\left(-\sqrt{2}\left(ax^2+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^2+b}+\sqrt{-b}\right)}{(-b)^{\frac{1}{4}}}-\frac{8\left(ax^2+b\right)^{\frac{3}{4}}a}{x^2}}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)^(3/4)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(6*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^2 + b)^
(1/4))/(-b)^(1/4))/(-b)^(1/4) + 6*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*
(-b)^(1/4) - 2*(a*x^2 + b)^(1/4))/(-b)^(1/4))/(-b)^(1/4) + 3*sqrt(2)*a^2*(-
b)^(3/4)*log(sqrt(2)*(a*x^2 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^2 + b) + sqrt(
-b))/b + 3*sqrt(2)*a^2*log(-sqrt(2)*(a*x^2 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x
^2 + b) + sqrt(-b))/(-b)^(1/4) - 8*(a*x^2 + b)^(3/4)*a/x^2)/a
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^2+b\right)^{\frac{3}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)^(3/4)/x^3,x)
```

```
[Out] int((a*x^2+b)^(3/4)/x^3,x)
```

maxima [A] time = 0.74, size = 74, normalized size = 0.99

$$\frac{3}{8}a\left(\frac{2\arctan\left(\frac{\left(ax^2+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(ax^2+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^2+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}\right)-\frac{\left(ax^2+b\right)^{\frac{3}{4}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)^(3/4)/x^3,x, algorithm="maxima")
```

```
[Out] 3/8*a*(2*arctan((a*x^2 + b)^(1/4)/b^(1/4))/b^(1/4) + log(((a*x^2 + b)^(1/4)
 - b^(1/4))/((a*x^2 + b)^(1/4) + b^(1/4)))/b^(1/4)) - 1/2*(a*x^2 + b)^(3/4)
/x^2
```

mupad [B] time = 0.96, size = 55, normalized size = 0.73

$$\frac{3\,a\,\mathrm{atan}\left(\frac{{\left(a\,x^2+b\right)}^{1/4}}{b^{1/4}}\right)}{4\,b^{1/4}}-\frac{{\left(a\,x^2+b\right)}^{3/4}}{2\,x^2}-\frac{3\,a\,\mathrm{atanh}\left(\frac{{\left(a\,x^2+b\right)}^{1/4}}{b^{1/4}}\right)}{4\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^2)^(3/4)/x^3,x)
```

[Out] (3*a*atan((b + a*x^2)^(1/4)/b^(1/4)))/(4*b^(1/4)) - (b + a*x^2)^(3/4)/(2*x^2) - (3*a*atanh((b + a*x^2)^(1/4)/b^(1/4)))/(4*b^(1/4))

sympy [C] time = 1.18, size = 42, normalized size = 0.56

$$-\frac{a^{\frac{3}{4}}\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^{2}}\right)}{2\sqrt{x}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**2+b)**(3/4)/x**3,x)

[Out] -a**(3/4)*gamma(1/4)*hyper((-3/4, 1/4), (5/4,), b*exp_polar(I*pi)/(a*x**2))/(2*sqrt(x)*gamma(5/4))

3.903 $\int \frac{3-2(1+k)x+kx^2}{\sqrt[4]{(1-x)x(1-kx)}\left(-d+d(1+k)x-dkx^2+x^3\right)}\,dx$

Optimal. Leaf size=75

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{x}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{x}\right)}{d^{3/4}}$$

Rubi [F] time = 19.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3-2(1+k)x+kx^2}{\sqrt[4]{(1-x)x(1-kx)}\left(-d+d(1+k)x-dkx^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3 - 2*(1 + k)*x + k*x^2)/(((1 - x)*x*(1 - k*x))^(1/4)*(-d + d*(1 + k)*
x - d*k*x^2 + x^3)),x]
```

```
[Out] (8*(1 + k)*(1 - x)^(1/4)*x^(1/4)*(1 - k*x)^(1/4)*Defer[Subst][Defer[Int][x^
6/((1 - x^4)^(1/4)*(1 - k*x^4)^(1/4)*(d - d*(1 + k)*x^4 + d*k*x^8 - x^12)),
 x], x, x^(1/4)])/((1 - x)*x*(1 - k*x))^(1/4) + (12*(1 - x)^(1/4)*x^(1/4)*(
1 - k*x)^(1/4)*Defer[Subst][Defer[Int][x^2/((1 - x^4)^(1/4)*(1 - k*x^4)^(1/
4)*(-d + d*(1 + k)*x^4 - d*k*x^8 + x^12)), x], x, x^(1/4)])/((1 - x)*x*(1 -
 k*x))^(1/4) + (4*k*(1 - x)^(1/4)*x^(1/4)*(1 - k*x)^(1/4)*Defer[Subst][Defe
r[Int][x^10/((1 - x^4)^(1/4)*(1 - k*x^4)^(1/4)*(-d + d*(1 + k)*x^4 - d*k*x^
8 + x^12)), x], x, x^(1/4)])/((1 - x)*x*(1 - k*x))^(1/4)
```

Rubi steps

$$\int \frac{3-2(1+k)x+kx^2}{\sqrt[4]{(1-x)x(1-kx)}\left(-d+d(1+k)x-dkx^2+x^3\right)}\,dx = \frac{\left(\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\int \frac{3-2(1+k)x+kx^2}{\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\left(-d+d(1+k)x-dkx^2+x^3\right)}}{\sqrt[4]{(1-x)x(1-kx)}}$$

$$=\frac{\left(4\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\text{Subst}\left(\int \frac{x^2\left(3-2(1+k)x^4+kx\right.}{\sqrt[4]{1-x^4}\sqrt[4]{1-kx^4}\left(-d+d(1+k)x\right.}\right.}{\sqrt[4]{(1-x)x(1-kx)}}$$

$$=\frac{\left(4\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\text{Subst}\left(\int\left(\frac{2(1+k)x^6}{\sqrt[4]{1-x^4}\sqrt[4]{1-kx^4}\left(d-d(1+k)x^4\right.}\right.\right.}{}$$

$$=\frac{\left(12\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{1-x^4}\sqrt[4]{1-kx^4}\left(-d+d(1+k)\right.}\right.}{\sqrt[4]{(1-x)x(1-kx)}}$$

Mathematica [F] time = 3.46, size = 0, normalized size = 0.00

$$\int \frac{3-2(1+k)x+kx^2}{\sqrt[4]{(1-x)x(1-kx)}\left(-d+d(1+k)x-dkx^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3 - 2*(1 + k)*x + k*x^2)/(((1 - x)*x*(1 - k*x))^(1/4)*(-d + d*(1
 + k)*x - d*k*x^2 + x^3)),x]
```

```
[Out] Integrate[(3 - 2*(1 + k)*x + k*x^2)/(((1 - x)*x*(1 - k*x))^(1/4)*(-d + d*(1
 + k)*x - d*k*x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.24, size = 75, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{kx^3+(-k-1)x^2+x}}{x}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{kx^3+(-k-1)x^2+x}}{x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - 2*(1 + k)*x + k*x^2)/(((1 - x)*x*(1 - k*x))^(1/4)*(
-d + d*(1 + k)*x - d*k*x^2 + x^3)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(x + (-1 - k)*x^2 + k*x^3)^(1/4))/x])/d^(3/4) - (2*ArcTa
nh[(d^(1/4)*(x + (-1 - k)*x^2 + k*x^3)^(1/4))/x])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*(1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(1/4)/(-d+d*(1+k)*x-d*k*x^
2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.68, size = 288, normalized size = 3.84

$$-\frac{\sqrt{2}\left(-d^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{1}{d}\right)^{\frac{1}{4}}+2\left(\frac{k}{x}-\frac{k}{x^2}-\frac{1}{x^2}+\frac{1}{x^3}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{1}{d}\right)^{\frac{1}{4}}}\right)}{d^3}-\frac{\sqrt{2}\left(-d^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{1}{d}\right)^{\frac{1}{4}}-2\left(\frac{k}{x}-\frac{k}{x^2}-\frac{1}{x^2}+\frac{1}{x^3}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{1}{d}\right)^{\frac{1}{4}}}\right)}{d^3}+\frac{\sqrt{2}\left(-d^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\left(\frac{k}{x}-\frac{k}{x^2}-\frac{1}{x^2}+\frac{1}{x^3}\right)^{\frac{1}{4}}\left(-\frac{1}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{k}{x}-\frac{k}{x^2}-\frac{1}{x^2}+\frac{1}{x^3}}+\sqrt{-\frac{1}{d}}\right)}{2\,d^3}-\frac{\sqrt{2}\left(-d^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\left(\frac{k}{x}-\frac{k}{x^2}-\frac{1}{x^2}+\frac{1}{x^3}\right)^{\frac{1}{4}}\left(-\frac{1}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{k}{x}-\frac{k}{x^2}-\frac{1}{x^2}+\frac{1}{x^3}}+\sqrt{-\frac{1}{d}}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*(1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(1/4)/(-d+d*(1+k)*x-d*k*x^
2+x^3),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(-d^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-1/d)^(1/4) + 2*(k/x - k
/x^2 - 1/x^2 + 1/x^3)^(1/4))/(-1/d)^(1/4))/d^3 - sqrt(2)*(-d^3)^(3/4)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(-1/d)^(1/4) - 2*(k/x - k/x^2 - 1/x^2 + 1/x^3)^(1/4
))/(-1/d)^(1/4))/d^3 + 1/2*sqrt(2)*(-d^3)^(3/4)*log(sqrt(2)*(k/x - k/x^2 -
1/x^2 + 1/x^3)^(1/4)*(-1/d)^(1/4) + sqrt(k/x - k/x^2 - 1/x^2 + 1/x^3) + sqr
t(-1/d))/d^3 - 1/2*sqrt(2)*(-d^3)^(3/4)*log(-sqrt(2)*(k/x - k/x^2 - 1/x^2 +
 1/x^3)^(1/4)*(-1/d)^(1/4) + sqrt(k/x - k/x^2 - 1/x^2 + 1/x^3) + sqrt(-1/d)
)/d^3
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{3-2\left(1+k\right)x+k\,x^{2}}{\left(\left(1-x\right)x\left(-kx+1\right)\right)^{\frac{1}{4}}\left(-d+d\left(1+k\right)x-dk\,x^{2}+x^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*(1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(1/4)/(-d+d*(1+k)*x-d*k*x^2+x^3)
,x)
```

```
[Out] int((3-2*(1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(1/4)/(-d+d*(1+k)*x-d*k*x^2+x^3)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{kx^2-2\,(k+1)x+3}{\left(dkx^2-d(k+1)x-x^3+d\right)\left((kx-1)(x-1)x\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*(1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(1/4)/(-d+d*(1+k)*x-d*k*x^
2+x^3),x, algorithm="maxima")
```

```
[Out] -integrate((k*x^2 - 2*(k + 1)*x + 3)/((d*k*x^2 - d*(k + 1)*x - x^3 + d)*((k
*x - 1)*(x - 1)*x)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{k\,x^2-2\,x\,\left(k+1\right)+3}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/4}\,\left(-x^3+d\,k\,x^2-d\,\left(k+1\right)\,x+d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(k*x^2 - 2*x*(k + 1) + 3)/((x*(k*x - 1)*(x - 1))^(1/4)*(d - x^3 - d*x*
(k + 1) + d*k*x^2)),x)
```

```
[Out] -int((k*x^2 - 2*x*(k + 1) + 3)/((x*(k*x - 1)*(x - 1))^(1/4)*(d - x^3 - d*x*
(k + 1) + d*k*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*(1+k)*x+k*x**2)/((1-x)*x*(-k*x+1))**(1/4)/(-d+d*(1+k)*x-d*k*
x**2+x**3),x)
```

```
[Out] Timed out
```

3.904 $\int \frac{(b+ax^3)^{3/4}}{x^4}\,dx$

Optimal. Leaf size=75

$$-\frac{(ax^3+b)^{3/4}}{3x^3}+\frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}-\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}$$

Rubi [A] time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 47, 63, 298, 203, 206}

$$-\frac{(ax^3+b)^{3/4}}{3x^3}+\frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}-\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^3)^(3/4)/x^4,x]
```

```
[Out] -1/3*(b + a*x^3)^(3/4)/x^3 + (a*ArcTan[(b + a*x^3)^(1/4)/b^(1/4)])/(2*b^(1/
4)) - (a*ArcTanh[(b + a*x^3)^(1/4)/b^(1/4)])/(2*b^(1/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(b+ax^3\right)^{3/4}}{x^4}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(b+ax)^{3/4}}{x^2}\,dx,x,x^3\right)\\
&= -\frac{\left(b+ax^3\right)^{3/4}}{3x^3}+\frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{b+ax}}\,dx,x,x^3\right)\\
&= -\frac{\left(b+ax^3\right)^{3/4}}{3x^3}+\text{Subst}\left(\int \frac{x^2}{-\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{b+ax^3}\right)\\
&= -\frac{\left(b+ax^3\right)^{3/4}}{3x^3}-\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx,x,\sqrt[4]{b+ax^3}\right)+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx,x,\sqrt[4]{b}\right.\\
&= -\frac{\left(b+ax^3\right)^{3/4}}{3x^3}+\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b+ax^3}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}-\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^3}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.49

$$\frac{4a\left(ax^3+b\right)^{7/4}\,{}_2F_1\left(\frac{7}{4},2;\frac{11}{4};\frac{ax^3}{b}+1\right)}{21b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^3)^(3/4)/x^4,x]
```

```
[Out] (4*a*(b + a*x^3)^(7/4)*Hypergeometric2F1[7/4, 2, 11/4, 1 + (a*x^3)/b])/(21*
b^2)
```

IntegrateAlgebraic [A] time = 0.08, size = 75, normalized size = 1.00

$$-\frac{\left(ax^3+b\right)^{3/4}}{3x^3}+\frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}-\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^3)^(3/4)/x^4,x]
```

```
[Out] -1/3*(b + a*x^3)^(3/4)/x^3 + (a*ArcTan[(b + a*x^3)^(1/4)/b^(1/4)])/(2*b^(1/
4)) - (a*ArcTanh[(b + a*x^3)^(1/4)/b^(1/4)])/(2*b^(1/4))
```

fricas [B] time = 0.44, size = 182, normalized size = 2.43

$$-\frac{12\left(\frac{a^4}{b}\right)^{\frac{1}{4}}x^3\arctan\left(-\frac{\left(ax^3+b\right)^{\frac{1}{4}}\left(\frac{a^4}{b}\right)^{\frac{1}{4}}a^3-\sqrt{\sqrt{ax^3+b}\,a^6+\sqrt{\frac{a^4}{b}}\,a^4b}\left(\frac{a^4}{b}\right)^{\frac{1}{4}}}{a^4}\right)+3\left(\frac{a^4}{b}\right)^{\frac{1}{4}}x^3\log\left(\left(ax^3+b\right)^{\frac{1}{4}}a^3+\left(\frac{a^4}{b}\right)^{\frac{3}{4}}b\right)-3\left(\frac{a^4}{b}\right)^{\frac{1}{4}}x^3\log\left(\left(ax^3+b\right)^{\frac{1}{4}}a^3-\left(\frac{a^4}{b}\right)^{\frac{3}{4}}b\right)+4\left(ax^3+b\right)^{\frac{3}{4}}}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)^(3/4)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(12*(a^4/b)^(1/4)*x^3*arctan(-((a*x^3 + b)^(1/4)*(a^4/b)^(1/4)*a^3 -
sqrt(sqrt(a*x^3 + b)*a^6 + sqrt(a^4/b)*a^4*b)*(a^4/b)^(1/4))/a^4) + 3*(a^4/
b)^(1/4)*x^3*log((a*x^3 + b)^(1/4)*a^3 + (a^4/b)^(3/4)*b) - 3*(a^4/b)^(1/4)
*x^3*log((a*x^3 + b)^(1/4)*a^3 - (a^4/b)^(3/4)*b) + 4*(a*x^3 + b)^(3/4))/x^
3
```

giac [B] time = 0.61, size = 209, normalized size = 2.79

$$\frac{\frac{6\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}+2\left(ax^3+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{(-b)^{\frac{1}{4}}}+\frac{6\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}-2\left(ax^3+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{(-b)^{\frac{1}{4}}}+\frac{3\sqrt{2}a^2(-b)^{\frac{3}{4}}\log\left(\sqrt{2}\left(ax^3+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^3+b}+\sqrt{-b}\right)}{b}+\frac{3\sqrt{2}a^2\log\left(-\sqrt{2}\left(ax^3+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^3+b}+\sqrt{-b}\right)}{(-b)^{\frac{1}{4}}}-\frac{8\left(ax^3+b\right)^{\frac{3}{4}}a}{x^3}}{24a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(6*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^3 + b)^
(1/4))/(-b)^(1/4))/(-b)^(1/4) + 6*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*
(-b)^(1/4) - 2*(a*x^3 + b)^(1/4))/(-b)^(1/4))/(-b)^(1/4) + 3*sqrt(2)*a^2*(-
b)^(3/4)*log(sqrt(2)*(a*x^3 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^3 + b) + sqrt(
-b))/b + 3*sqrt(2)*a^2*log(-sqrt(2)*(a*x^3 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x
^3 + b) + sqrt(-b))/(-b)^(1/4) - 8*(a*x^3 + b)^(3/4)*a/x^3)/a
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^3+b\right)^{\frac{3}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b)^(3/4)/x^4,x)
```

```
[Out] int((a*x^3+b)^(3/4)/x^4,x)
```

maxima [A] time = 0.44, size = 74, normalized size = 0.99

$$\frac{1}{4}a\left(\frac{2\arctan\left(\frac{\left(ax^3+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(ax^3+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^3+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}\right)-\frac{\left(ax^3+b\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)^(3/4)/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*a*(2*arctan((a*x^3 + b)^(1/4)/b^(1/4))/b^(1/4) + log(((a*x^3 + b)^(1/4)
 - b^(1/4))/((a*x^3 + b)^(1/4) + b^(1/4)))/b^(1/4)) - 1/3*(a*x^3 + b)^(3/4)
/x^3
```

mupad [B] time = 0.97, size = 55, normalized size = 0.73

$$\frac{a\,\mathrm{atan}\left(\frac{{\left(a\,x^3+b\right)}^{1/4}}{b^{1/4}}\right)}{2\,b^{1/4}}-\frac{{\left(a\,x^3+b\right)}^{3/4}}{3\,x^3}-\frac{a\,\mathrm{atanh}\left(\frac{{\left(a\,x^3+b\right)}^{1/4}}{b^{1/4}}\right)}{2\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^3)^(3/4)/x^4,x)
```

```
[Out] (a*atan((b + a*x^3)^(1/4)/b^(1/4)))/(2*b^(1/4)) - (b + a*x^3)^(3/4)/(3*x^3)
 - (a*atanh((b + a*x^3)^(1/4)/b^(1/4)))/(2*b^(1/4))
```

sympy [C] time = 1.23, size = 42, normalized size = 0.56

$$-\frac{a^{\frac{3}{4}}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3x^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b)**(3/4)/x**4,x)
```

```
[Out] -a**(3/4)*gamma(1/4)*hyper((-3/4, 1/4), (5/4,), b*exp_polar(I*pi)/(a*x**3))
/(3*x**(3/4)*gamma(5/4))
```

3.905 $\int \frac{\sqrt[4]{x^2+x^4}}{x^4\left(-1+x^4\right)}\,dx$

Optimal. Leaf size=75

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^2}}\right)}{2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^2}}\right)}{2^{3/4}}+\frac{2\sqrt[4]{x^4+x^2}\,\left(x^2+1\right)}{5x^3}$$

Rubi [A] time = 0.24, antiderivative size = 129, normalized size of antiderivative = 1.72, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2056, 1254, 466, 494, 461, 298, 203, 206}

$$\frac{\sqrt[4]{x^4+x^2}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt{x}}{\sqrt[4]{x^2+1}}\right)}{2^{3/4}\sqrt{x}\,\sqrt[4]{x^2+1}}-\frac{\sqrt[4]{x^4+x^2}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt{x}}{\sqrt[4]{x^2+1}}\right)}{2^{3/4}\sqrt{x}\,\sqrt[4]{x^2+1}}+\frac{2\sqrt[4]{x^4+x^2}\,\left(x^2+1\right)}{5x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2 + x^4)^(1/4)/(x^4*(-1 + x^4)),x]
```

```
[Out] (2*(1 + x^2)*(x^2 + x^4)^(1/4))/(5*x^3) + ((x^2 + x^4)^(1/4)*ArcTan[(2^(1/4
)*Sqrt[x])/(1 + x^2)^(1/4)])/(2^(3/4)*Sqrt[x]*(1 + x^2)^(1/4)) - ((x^2 + x^
4)^(1/4)*ArcTanh[(2^(1/4)*Sqrt[x])/(1 + x^2)^(1/4)])/(2^(3/4)*Sqrt[x]*(1 +
x^2)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 1254

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e)^p, x] /
; FreeQ[{a, c, d, e, f, q, m, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p
]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\sqrt[4]{x^2+x^4}}{x^4\left(-1+x^4\right)}\,dx = \frac{\sqrt[4]{x^2+x^4}\int \frac{\sqrt[4]{1+x^2}}{x^{7/2}\left(-1+x^4\right)}\,dx}{\sqrt{x}\sqrt[4]{1+x^2}}$$

$$= \frac{\sqrt[4]{x^2+x^4}\int \frac{1}{x^{7/2}\left(-1+x^2\right)\left(1+x^2\right)^{3/4}}\,dx}{\sqrt{x}\sqrt[4]{1+x^2}}$$

$$= \frac{\left(2\sqrt[4]{x^2+x^4}\right)\text{Subst}\left(\int \frac{1}{x^6\left(-1+x^4\right)\left(1+x^4\right)^{3/4}}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt[4]{1+x^2}}$$

$$= \frac{\left(2\sqrt[4]{x^2+x^4}\right)\text{Subst}\left(\int \frac{\left(1-x^4\right)^2}{x^6\left(-1+2x^4\right)}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{\sqrt{x}\sqrt[4]{1+x^2}}$$

$$= \frac{\left(2\sqrt[4]{x^2+x^4}\right)\text{Subst}\left(\int \left(-\frac{1}{x^6}+\frac{x^2}{-1+2x^4}\right)dx, x, \frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{\sqrt{x}\sqrt[4]{1+x^2}}$$

$$= \frac{2\left(1+x^2\right)\sqrt[4]{x^2+x^4}}{5x^3} + \frac{\left(2\sqrt[4]{x^2+x^4}\right)\text{Subst}\left(\int \frac{x^2}{-1+2x^4}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{\sqrt{x}\sqrt[4]{1+x^2}}$$

$$= \frac{2\left(1+x^2\right)\sqrt[4]{x^2+x^4}}{5x^3} - \frac{\sqrt[4]{x^2+x^4}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{\sqrt{2}\sqrt{x}\sqrt[4]{1+x^2}} + \frac{\sqrt[4]{x^2+x^4}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\cdots}\right.}{\sqrt{2}\sqrt{x}\sqrt[4]{1+\cdots}}$$

$$= \frac{2\left(1+x^2\right)\sqrt[4]{x^2+x^4}}{5x^3} + \frac{\sqrt[4]{x^2+x^4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{2^{3/4}\sqrt{x}\sqrt[4]{1+x^2}} - \frac{\sqrt[4]{x^2+x^4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{2^{3/4}\sqrt{x}\sqrt[4]{1+x^2}}$$

Mathematica [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2+x^4}}{x^4\left(-1+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2 + x^4)^(1/4)/(x^4*(-1 + x^4)),x]
```

```
[Out] Integrate[(x^2 + x^4)^(1/4)/(x^4*(-1 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.25, size = 75, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^2}}\right)}{2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^2}}\right)}{2^{3/4}}+\frac{2\sqrt[4]{x^4+x^2}\,\left(x^2+1\right)}{5x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2 + x^4)^(1/4)/(x^4*(-1 + x^4)),x]
```

```
[Out] (2*(1 + x^2)*(x^2 + x^4)^(1/4))/(5*x^3) + ArcTan[(2^(1/4)*x)/(x^2 + x^4)^(1
/4)]/2^(3/4) - ArcTanh[(2^(1/4)*x)/(x^2 + x^4)^(1/4)]/2^(3/4)
```

fricas [B] time = 1.99, size = 253, normalized size = 3.37

$$-\frac{20\cdot 8^{\frac{3}{4}}x^3\arctan\left(\frac{16\cdot 8^{\frac{1}{4}}\left(x^4+x^2\right)^{\frac{1}{4}}x^2+2^{\frac{3}{4}}\left(8^{\frac{3}{4}}\left(3x^3+x\right)+8\cdot 8^{\frac{1}{4}}\sqrt{x^4+x^2}\,x\right)+4\cdot 8^{\frac{3}{4}}\left(x^4+x^2\right)^{\frac{3}{4}}}{8\left(x^3-x\right)}\right)+5\cdot 8^{\frac{3}{4}}x^3\log\left(\frac{4\sqrt{2}\left(x^4+x^2\right)^{\frac{1}{4}}x^2+8^{\frac{3}{4}}\sqrt{x^4+x^2}\,x+8^{\frac{1}{4}}\left(3x^3+x\right)+4\left(x^4+x^2\right)^{\frac{3}{4}}}{x^3-x}\right)-5\cdot 8^{\frac{3}{4}}x^3\log\left(\frac{4\sqrt{2}\left(x^4+x^2\right)^{\frac{1}{4}}x^2-8^{\frac{3}{4}}\sqrt{x^4+x^2}\,x-8^{\frac{1}{4}}\left(3x^3+x\right)+4\left(x^4+x^2\right)^{\frac{3}{4}}}{x^3-x}\right)-64\left(x^4+x^2\right)^{\frac{1}{4}}\left(x^2+1\right)}{160\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2)^(1/4)/x^4/(x^4-1),x, algorithm="fricas")
```

```
[Out] -1/160*(20*8^(3/4)*x^3*arctan(1/8*(16*8^(1/4)*(x^4 + x^2)^(1/4)*x^2 + 2^(3/
4)*(8^(3/4)*(3*x^3 + x) + 8*8^(1/4)*sqrt(x^4 + x^2)*x) + 4*8^(3/4)*(x^4 + x
^2)^(3/4))/(x^3 - x)) + 5*8^(3/4)*x^3*log((4*sqrt(2)*(x^4 + x^2)^(1/4)*x^2
+ 8^(3/4)*sqrt(x^4 + x^2)*x + 8^(1/4)*(3*x^3 + x) + 4*(x^4 + x^2)^(3/4))/(x
^3 - x)) - 5*8^(3/4)*x^3*log((4*sqrt(2)*(x^4 + x^2)^(1/4)*x^2 - 8^(3/4)*sqr
t(x^4 + x^2)*x - 8^(1/4)*(3*x^3 + x) + 4*(x^4 + x^2)^(3/4))/(x^3 - x)) - 64
*(x^4 + x^2)^(1/4)*(x^2 + 1))/x^3
```

giac [A] time = 0.34, size = 65, normalized size = 0.87

$$\frac{2}{5}\left(\frac{1}{x^2}+1\right)^{\frac{5}{4}}-\frac{1}{2}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)-\frac{1}{4}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)+\frac{1}{4}\cdot 2^{\frac{1}{4}}\log\left(\left|-2^{\frac{1}{4}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2)^(1/4)/x^4/(x^4-1),x, algorithm="giac")
```

```
[Out] 2/5*(1/x^2 + 1)^(5/4) - 1/2*2^(1/4)*arctan(1/2*2^(3/4)*(1/x^2 + 1)^(1/4)) -
 1/4*2^(1/4)*log(2^(1/4) + (1/x^2 + 1)^(1/4)) + 1/4*2^(1/4)*log(abs(-2^(1/4
) + (1/x^2 + 1)^(1/4)))
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^2\right)^{\frac{1}{4}}}{x^4\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2)^(1/4)/x^4/(x^4-1),x)
```

```
[Out] int((x^4+x^2)^(1/4)/x^4/(x^4-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(32\,x^{7}-8\,x^{5}+5\,x^{3}+45\,x\right)\left(x^{2}+1\right)^{\frac{1}{4}}}{585\left(x^{\frac{15}{2}}-x^{\frac{7}{2}}\right)}-\int\frac{8\left(32\,x^{6}-8\,x^{4}+5\,x^{2}+45\right)\left(x^{2}+1\right)^{\frac{1}{4}}}{585\left(x^{\frac{23}{2}}-2\,x^{\frac{15}{2}}+x^{\frac{7}{2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2)^(1/4)/x^4/(x^4-1),x, algorithm="maxima")
```

```
[Out] -2/585*(32*x^7 - 8*x^5 + 5*x^3 + 45*x)*(x^2 + 1)^(1/4)/(x^(15/2) - x^(7/2))
 - integrate(8/585*(32*x^6 - 8*x^4 + 5*x^2 + 45)*(x^2 + 1)^(1/4)/(x^(23/2)
- 2*x^(15/2) + x^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int\frac{\left(x^4+x^2\right)^{1/4}}{x^4-x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4)^(1/4)/(x^4*(x^4 - 1)),x)
```

```
[Out] -int((x^2 + x^4)^(1/4)/(x^4 - x^8), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[4]{x^{2}\left(x^{2}+1\right)}}{x^{4}\left(x-1\right)\left(x+1\right)\left(x^{2}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2)**(1/4)/x**4/(x**4-1),x)
```

```
[Out] Integral((x**2*(x**2 + 1))**(1/4)/(x**4*(x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.906 $\int \frac{x(-3+x^4)}{(1+x^4)^{2/3}(1+x^3+x^4)}\,dx$

Optimal. Leaf size=75

$$\log\left(\sqrt[3]{x^4+1}+x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^4+1}-x}\right)-\frac{1}{2}\log\left(-\sqrt[3]{x^4+1}\,x+\left(x^4+1\right)^{2/3}+x^2\right)$$

Rubi [F] time = 0.98, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\left(-3+x^4\right)}{\left(1+x^4\right)^{2/3}\left(1+x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-3 + x^4))/((1 + x^4)^(2/3)*(1 + x^3 + x^4)),x]
```

```
[Out] -1/2*(3^(3/4)*Sqrt[2 - Sqrt[3]]*(1 - (1 + x^4)^(1/3))*Sqrt[(1 + (1 + x^4)^(
1/3) + (1 + x^4)^(2/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2]*EllipticF[ArcSin
[(1 + Sqrt[3] - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))], -7 + 4*S
qrt[3]])/(x^2*Sqrt[-((1 - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^
2)]) - x*Hypergeometric2F1[1/4, 2/3, 5/4, -x^4] + Defer[Int][1/((1 + x^4)^(
2/3)*(1 + x^3 + x^4)), x] - 4*Defer[Int][x/((1 + x^4)^(2/3)*(1 + x^3 + x^4)
), x] + Defer[Int][x^3/((1 + x^4)^(2/3)*(1 + x^3 + x^4)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(-3+x^4\right)}{\left(1+x^4\right)^{2/3}\left(1+x^3+x^4\right)}\,dx &= \int \left(-\frac{1}{\left(1+x^4\right)^{2/3}}+\frac{x}{\left(1+x^4\right)^{2/3}}+\frac{1-4x+x^3}{\left(1+x^4\right)^{2/3}\left(1+x^3+x^4\right)}\right)dx\\
&= -\int \frac{1}{\left(1+x^4\right)^{2/3}}\,dx+\int \frac{x}{\left(1+x^4\right)^{2/3}}\,dx+\int \frac{1-4x+x^3}{\left(1+x^4\right)^{2/3}\left(1+x^3+x^4\right)}\,dx\\
&= -x\,{}_2F_1\left(\frac{1}{4},\frac{2}{3};\frac{5}{4};-x^4\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{2/3}}\,dx,x,x^2\right)+\int \left(\frac{1}{\left(1+x^4\right)^{2/3}(\ldots}\right.\\
&= -x\,{}_2F_1\left(\frac{1}{4},\frac{2}{3};\frac{5}{4};-x^4\right)-4\int \frac{x}{\left(1+x^4\right)^{2/3}\left(1+x^3+x^4\right)}\,dx+\frac{\left(3\sqrt{x^4}\right)\text{Subst}\left(\int\ldots\right.}{\ldots}\\
&= -\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(1-\sqrt[3]{1+x^4}\right)\sqrt{\frac{1+\sqrt[3]{1+x^4}+\left(1+x^4\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1+x^4}\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-\sqrt[3]{1+x^4}}{1-\sqrt{3}-\sqrt[3]{1+x^4}}\right)\middle|-\ldots\right)}{2x^2\sqrt{-\frac{1-\sqrt[3]{1+x^4}}{\left(1-\sqrt{3}-\sqrt[3]{1+x^4}\right)^2}}}
\end{aligned}$$

Mathematica [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{x\left(-3+x^4\right)}{\left(1+x^4\right)^{2/3}\left(1+x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(-3 + x^4))/((1 + x^4)^(2/3)*(1 + x^3 + x^4)),x]
```

```
[Out] Integrate[(x*(-3 + x^4))/((1 + x^4)^(2/3)*(1 + x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 1.47, size = 75, normalized size = 1.00

$$\log\left(\sqrt[3]{x^4+1}+x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+1}-x}\right)-\frac{1}{2}\log\left(-\sqrt[3]{x^4+1}x+\left(x^4+1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-3 + x^4))/((1 + x^4)^(2/3)*(1 + x^3 + x^4)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(1 + x^4)^(1/3))]) + Log[x + (1 + x^4)
^(1/3)] - Log[x^2 - x*(1 + x^4)^(1/3) + (1 + x^4)^(2/3)]/2
```

fricas [A] time = 1.48, size = 102, normalized size = 1.36

$$-\sqrt{3}\arctan\left(\frac{4\sqrt{3}\left(x^4+1\right)^{\frac{1}{3}}x^2+2\sqrt{3}\left(x^4+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(x^4+1\right)}{x^4-8x^3+1}\right)+\frac{1}{2}\log\left(\frac{x^4+x^3+3\left(x^4+1\right)^{\frac{1}{3}}x^2+3\left(x^4+1\right)^{\frac{2}{3}}x+1}{x^4+x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4-3)/(x^4+1)^(2/3)/(x^4+x^3+1),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan((4*sqrt(3)*(x^4 + 1)^(1/3)*x^2 + 2*sqrt(3)*(x^4 + 1)^(2/3)*
x + sqrt(3)*(x^4 + 1))/(x^4 - 8*x^3 + 1)) + 1/2*log((x^4 + x^3 + 3*(x^4 + 1
)^(1/3)*x^2 + 3*(x^4 + 1)^(2/3)*x + 1)/(x^4 + x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-3\right)x}{\left(x^4+x^3+1\right)\left(x^4+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4-3)/(x^4+1)^(2/3)/(x^4+x^3+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 3)*x/((x^4 + x^3 + 1)*(x^4 + 1)^(2/3)), x)
```

maple [C] time = 2.68, size = 226, normalized size = 3.01

$$\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^4+\left(x^4+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x-\left(x^4+1\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2-\left(x^4+1\right)^{\frac{2}{3}}x-2x^2\left(x^4+1\right)^{\frac{1}{3}}+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)}{x^4+x^3+1}\right)+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^4+\left(x^4+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x+2\left(x^4+1\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3+2\left(x^4+1\right)^{\frac{2}{3}}x+x^2\left(x^4+1\right)^{\frac{1}{3}}-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)}{x^4+x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^4-3)/(x^4+1)^(2/3)/(x^4+x^3+1),x)
```

```
[Out] ln((RootOf(_Z^2+_Z+1)^2*x^3+RootOf(_Z^2+_Z+1)*x^4+(x^4+1)^(2/3)*RootOf(_Z^2
+_Z+1)*x-(x^4+1)^(1/3)*RootOf(_Z^2+_Z+1)*x^2-(x^4+1)^(2/3)*x-2*x^2*(x^4+1)^
(1/3)+RootOf(_Z^2+_Z+1))/(x^4+x^3+1))+RootOf(_Z^2+_Z+1)*ln(-(RootOf(_Z^2+_Z
+1)^2*x^3-RootOf(_Z^2+_Z+1)*x^4+(x^4+1)^(2/3)*RootOf(_Z^2+_Z+1)*x+2*(x^4+1)
^(1/3)*RootOf(_Z^2+_Z+1)*x^2+RootOf(_Z^2+_Z+1)*x^3+2*(x^4+1)^(2/3)*x+x^2*(x
^4+1)^(1/3)-RootOf(_Z^2+_Z+1))/(x^4+x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-3\right)x}{\left(x^4+x^3+1\right)\left(x^4+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4-3)/(x^4+1)^(2/3)/(x^4+x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 3)*x/((x^4 + x^3 + 1)*(x^4 + 1)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(x^4-3\right)}{\left(x^4+1\right)^{2/3}\left(x^4+x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^4 - 3))/((x^4 + 1)^(2/3)*(x^3 + x^4 + 1)),x)
```

```
[Out] int((x*(x^4 - 3))/((x^4 + 1)^(2/3)*(x^3 + x^4 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**4-3)/(x**4+1)**(2/3)/(x**4+x**3+1),x)
```

```
[Out] Timed out
```

3.907 $\displaystyle\int \frac{\sqrt{2-x^2-4x^4}\left(1+2x^4\right)}{\left(-1+2x^4\right)\left(-1-x^2+2x^4\right)}\,dx$

Optimal. Leaf size=75

$$\tan^{-1}\left(\frac{x\sqrt{-4x^4-x^2+2}}{4x^4+x^2-2}\right)-\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,x\sqrt{-4x^4-x^2+2}}{4x^4+x^2-2}\right)$$

Rubi [C] time = 1.79, antiderivative size = 475, normalized size of antiderivative = 6.33, number of steps used = 32, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.170, Rules used = {6725, 1208, 1180, 524, 424, 419, 1212, 537}

$$-\frac{1}{4}\sqrt{3\left(11\sqrt{33}-59\right)}F\left(\sin^{-1}\left(2\sqrt{\frac{2}{-1+\sqrt{33}}}x\right)|\frac{1}{16}\left(-17+\sqrt{33}\right)\right)+\frac{1}{4}\sqrt{3\left(13+3\sqrt{33}\right)}F\left(\sin^{-1}\left(2\sqrt{\frac{2}{-1+\sqrt{33}}}x\right)|\frac{1}{16}\left(-17+\sqrt{33}\right)\right)+\frac{\left(1+4\sqrt{2}-\sqrt{33}\right)F\left(\sin^{-1}\left(2\sqrt{\frac{2}{-1+\sqrt{33}}}x\right)|\frac{1}{16}\left(-17+\sqrt{33}\right)\right)}{\sqrt{2\left(1+\sqrt{33}\right)}}+\frac{\left(1-4\sqrt{2}-\sqrt{33}\right)F\left(\sin^{-1}\left(2\sqrt{\frac{2}{-1+\sqrt{33}}}x\right)|\frac{1}{16}\left(-17+\sqrt{33}\right)\right)}{\sqrt{2\left(1+\sqrt{33}\right)}}+3\sqrt{\frac{2}{1+\sqrt{33}}}\Pi\left(\frac{1}{4}\left(1-\sqrt{33}\right);\sin^{-1}\left(2\sqrt{\frac{2}{-1+\sqrt{33}}}x\right)|\frac{1}{16}\left(-17+\sqrt{33}\right)\right)-\sqrt{\frac{2}{1+\sqrt{33}}}\Pi\left(-\frac{1-\sqrt{33}}{4\sqrt{2}};\sin^{-1}\left(2\sqrt{\frac{2}{-1+\sqrt{33}}}x\right)|\frac{1}{16}\left(-17+\sqrt{33}\right)\right)-\sqrt{\frac{2}{1+\sqrt{33}}}\Pi\left(\frac{1-\sqrt{33}}{4\sqrt{2}};\sin^{-1}\left(2\sqrt{\frac{2}{-1+\sqrt{33}}}x\right)|\frac{1}{16}\left(-17+\sqrt{33}\right)\right)+3\sqrt{\frac{2}{1+\sqrt{33}}}\Pi\left(\frac{1}{8}\left(-1+\sqrt{33}\right);\sin^{-1}\left(2\sqrt{\frac{2}{-1+\sqrt{33}}}x\right)|\frac{1}{16}\left(-17+\sqrt{33}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[2 - x^2 - 4*x^4]*(1 + 2*x^4))/((-1 + 2*x^4)*(-1 - x^2 + 2*x^4)),x
]
```

```
[Out] ((1 - 4*Sqrt[2] - Sqrt[33])*EllipticF[ArcSin[2*Sqrt[2/(-1 + Sqrt[33])]*x],
(-17 + Sqrt[33])/16])/Sqrt[2*(1 + Sqrt[33])] + ((1 + 4*Sqrt[2] - Sqrt[33])*
EllipticF[ArcSin[2*Sqrt[2/(-1 + Sqrt[33])]*x], (-17 + Sqrt[33])/16])/Sqrt[2
*(1 + Sqrt[33])] + (Sqrt[3*(13 + 3*Sqrt[33])]*EllipticF[ArcSin[2*Sqrt[2/(-1
 + Sqrt[33])]*x], (-17 + Sqrt[33])/16])/4 - (Sqrt[3*(-59 + 11*Sqrt[33])]*El
lipticF[ArcSin[2*Sqrt[2/(-1 + Sqrt[33])]*x], (-17 + Sqrt[33])/16])/4 + 3*Sq
rt[2/(1 + Sqrt[33])]*EllipticPi[(1 - Sqrt[33])/4, ArcSin[2*Sqrt[2/(-1 + Sqr
t[33])]*x], (-17 + Sqrt[33])/16] - Sqrt[2/(1 + Sqrt[33])]*EllipticPi[-1/4*(
1 - Sqrt[33])/Sqrt[2], ArcSin[2*Sqrt[2/(-1 + Sqrt[33])]*x], (-17 + Sqrt[33]
)/16] - Sqrt[2/(1 + Sqrt[33])]*EllipticPi[(1 - Sqrt[33])/(4*Sqrt[2]), ArcSi
n[2*Sqrt[2/(-1 + Sqrt[33])]*x], (-17 + Sqrt[33])/16] + 3*Sqrt[2/(1 + Sqrt[3
3])]*EllipticPi[(-1 + Sqrt[33])/8, ArcSin[2*Sqrt[2/(-1 + Sqrt[33])]*x], (-1
7 + Sqrt[33])/16]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-x^2-4x^4}\left(1+2x^4\right)}{\left(-1+2x^4\right)\left(-1-x^2+2x^4\right)}\,dx = \int \left(\frac{\sqrt{2-x^2-4x^4}}{-1+x^2}+\frac{2\sqrt{2-x^2-4x^4}}{1+2x^2}-\frac{4x^2\sqrt{2-x^2-4x^4}}{-1+2x^4}\right)dx$$

$$= 2\int \frac{\sqrt{2-x^2-4x^4}}{1+2x^2}\,dx - 4\int \frac{x^2\sqrt{2-x^2-4x^4}}{-1+2x^4}\,dx + \int \frac{\sqrt{2-x^2-4x^4}}{-1+x^2}\,dx$$

$$= -\left(\frac{1}{2}\int \frac{-2+8x^2}{\sqrt{2-x^2-4x^4}}\,dx\right) - 3\int \frac{1}{\left(-1+x^2\right)\sqrt{2-x^2-4x^4}}\,dx + 3\int \frac{}{\left(1+2\right.}$$

$$= -\left(2\int \frac{-2+8x^2}{\sqrt{-1+\sqrt{33}-8x^2}\sqrt{1+\sqrt{33}+8x^2}}\,dx\right) - 4\int \frac{5+4x}{\sqrt{-1+\sqrt{33}-8x^2}}$$

$$= 3\sqrt{\frac{2}{1+\sqrt{33}}}\,\Pi\left(\frac{1}{4}\left(1-\sqrt{33}\right); \sin^{-1}\left(2\sqrt{\frac{2}{-1+\sqrt{33}}}\,x\right)\middle|\frac{1}{16}\left(-17+\sqrt{33}\right)\right) +$$

$$= -\sqrt{2\left(1+\sqrt{33}\right)}\,E\left(\sin^{-1}\left(2\sqrt{\frac{2}{-1+\sqrt{33}}}\,x\right)\middle|\frac{1}{16}\left(-17+\sqrt{33}\right)\right) + \frac{1}{4}\sqrt{3\left(13+\right.}$$

$$= -\sqrt{2\left(1+\sqrt{33}\right)}\,E\left(\sin^{-1}\left(2\sqrt{\frac{2}{-1+\sqrt{33}}}\,x\right)\middle|\frac{1}{16}\left(-17+\sqrt{33}\right)\right) + \frac{1}{4}\sqrt{3\left(13+\right.}$$

$$= \frac{\left(1-4\sqrt{2}-\sqrt{33}\right)F\left(\sin^{-1}\left(2\sqrt{\frac{2}{-1+\sqrt{33}}}\,x\right)\middle|\frac{1}{16}\left(-17+\sqrt{33}\right)\right)}{\sqrt{2\left(1+\sqrt{33}\right)}} + \frac{\left(1+4\sqrt{2}-\right.}{}$$

Mathematica [C] time = 7.04, size = 2667, normalized size = 35.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - x^2 - 4*x^4]*(1 + 2*x^4))/((-1 + 2*x^4)*(-1 - x^2 + 2*x
^4)),x]
```

```
[Out] -(((-1 + x)*(1 + x)*(1 + 2*x^2)*Sqrt[2 - x^2 - 4*x^4]*(-1 + 2*x^4)*(Sqrt[2
- x^2 - 4*x^4]/(2*(-1 + x)) - Sqrt[2 - x^2 - 4*x^4]/(2*(1 + x)) + (2*Sqrt[2
 - x^2 - 4*x^4])/(1 + 2*x^2) - (4*x^2*Sqrt[2 - x^2 - 4*x^4])/(-1 + 2*x^4))*
((I*Sqrt[(1 + Sqrt[33])/2]*Sqrt[1 - (8*x^2)/(-1 - Sqrt[33])]*Sqrt[1 - (8*x^
2)/(-1 + Sqrt[33])]*EllipticF[I*ArcSinh[2*Sqrt[2/(1 + Sqrt[33])]*x], (-1 -
Sqrt[33])/(-1 + Sqrt[33])])/Sqrt[2 - x^2 - 4*x^4] + ((I/8)*Sqrt[1 + Sqrt[33
]]*Sqrt[1 - (8*x^2)/(-1 - Sqrt[33])]*Sqrt[1 - (8*x^2)/(-1 + Sqrt[33])]*Elli
pticPi[-1/4*(-1 - Sqrt[33])/Sqrt[2], I*ArcSinh[2*Sqrt[2/(1 + Sqrt[33])]*x],
 (-1 - Sqrt[33])/(-1 + Sqrt[33])])/((-1 - 1/Sqrt[2])*(1/2 - 1/Sqrt[2])*Sqrt
[2 - x^2 - 4*x^4]) - ((I/8)*Sqrt[1 + Sqrt[33]]*Sqrt[1 - (8*x^2)/(-1 - Sqrt[
33])]*Sqrt[1 - (8*x^2)/(-1 + Sqrt[33])]*EllipticPi[(-1 - Sqrt[33])/(4*Sqrt[
2]), I*ArcSinh[2*Sqrt[2/(1 + Sqrt[33])]*x], (-1 - Sqrt[33])/(-1 + Sqrt[33])
])/((-1 + 1/Sqrt[2])*(1/2 + 1/Sqrt[2])*Sqrt[2 - x^2 - 4*x^4]) + (((3*I)/8)*
Sqrt[(1 + Sqrt[33])/2]*Sqrt[1 - (8*x^2)/(-1 - Sqrt[33])]*Sqrt[1 - (8*x^2)/(
-1 + Sqrt[33])]*EllipticPi[(1 + Sqrt[33])/4, I*ArcSinh[2*Sqrt[2/(1 + Sqrt[3
3])]*x], (-1 - Sqrt[33])/(-1 + Sqrt[33])])/((-1/2 - 1/Sqrt[2])*(-1/2 + 1/Sq
rt[2])*Sqrt[2 - x^2 - 4*x^4]) - (3*(((Sqrt[(-1 + Sqrt[33])/2]/2 + (I/2)*Sqr
t[(1 + Sqrt[33])/2])*(-1/2*Sqrt[(-1 + Sqrt[33])/2] + x)^2*Sqrt[((-1/2*I)*Sq
rt[(1 + Sqrt[33])/2] + x)/((Sqrt[(-1 + Sqrt[33])/2]/2 + (I/2)*Sqrt[(1 + Sqr
t[33])/2])*(-1/2*Sqrt[(-1 + Sqrt[33])/2] + x))]*Sqrt[((I/2)*Sqrt[(1 + Sqrt[
```

```
33])/2] + x)/((Sqrt[(-1 + Sqrt[33])/2]/2 - (I/2)*Sqrt[(1 + Sqrt[33])/2])*(-
1/2*Sqrt[(-1 + Sqrt[33])/2] + x))]*Sqrt[((Sqrt[-1 + Sqrt[33]] - I*Sqrt[1 +
Sqrt[33]])*(Sqrt[2*(-1 + Sqrt[33])] + 4*x))/((Sqrt[-1 + Sqrt[33]] + I*Sqrt[
1 + Sqrt[33]])*(Sqrt[2*(-1 + Sqrt[33])] - 4*x))]*((1 + Sqrt[(-1 + Sqrt[33])
/2]/2)*EllipticF[ArcSin[Sqrt[((Sqrt[-1 + Sqrt[33]] - I*Sqrt[1 + Sqrt[33]])*
(Sqrt[2*(-1 + Sqrt[33])] + 4*x))/((Sqrt[-1 + Sqrt[33]] + I*Sqrt[1 + Sqrt[33
]])*(Sqrt[2*(-1 + Sqrt[33])] - 4*x))]], (Sqrt[-1 + Sqrt[33]] + I*Sqrt[1 + S
qrt[33]])^2/(Sqrt[-1 + Sqrt[33]] - I*Sqrt[1 + Sqrt[33]])^2] - Sqrt[(-1 + Sq
rt[33])/2]*EllipticPi[((-1 + Sqrt[(-1 + Sqrt[33])/2]/2)*(Sqrt[(-1 + Sqrt[33
])/2]/2 + (I/2)*Sqrt[(1 + Sqrt[33])/2]))/((-1 - Sqrt[(-1 + Sqrt[33])/2]/2)*
(-1/2*Sqrt[(-1 + Sqrt[33])/2] + (I/2)*Sqrt[(1 + Sqrt[33])/2])), ArcSin[Sqrt
[((Sqrt[-1 + Sqrt[33]] - I*Sqrt[1 + Sqrt[33]])*(Sqrt[2*(-1 + Sqrt[33])] + 4
*x))/((Sqrt[-1 + Sqrt[33]] + I*Sqrt[1 + Sqrt[33]])*(Sqrt[2*(-1 + Sqrt[33])]
 - 4*x))]], (Sqrt[-1 + Sqrt[33]] + I*Sqrt[1 + Sqrt[33]])^2/(Sqrt[-1 + Sqrt[
33]] - I*Sqrt[1 + Sqrt[33]])^2]))/((-1 - Sqrt[(-1 + Sqrt[33])/2]/2)*(1 - Sq
rt[(-1 + Sqrt[33])/2]/2)*(Sqrt[(-1 + Sqrt[33])/2]/2 - (I/2)*Sqrt[(1 + Sqrt[
33])/2])*Sqrt[2 - x^2 - 4*x^4]) - ((Sqrt[(-1 + Sqrt[33])/2]/2 + (I/2)*Sqrt[
(1 + Sqrt[33])/2])*(-1/2*Sqrt[(-1 + Sqrt[33])/2] + x)^2*Sqrt[((-1/2*I)*Sqrt
[(1 + Sqrt[33])/2] + x)/((Sqrt[(-1 + Sqrt[33])/2]/2 + (I/2)*Sqrt[(1 + Sqrt[
33])/2])*(-1/2*Sqrt[(-1 + Sqrt[33])/2] + x))]*Sqrt[((I/2)*Sqrt[(1 + Sqrt[33
])/2] + x)/((Sqrt[(-1 + Sqrt[33])/2]/2 - (I/2)*Sqrt[(1 + Sqrt[33])/2])*(-1/
2*Sqrt[(-1 + Sqrt[33])/2] + x))]*Sqrt[((Sqrt[-1 + Sqrt[33]] - I*Sqrt[1 + Sq
rt[33]])*(Sqrt[2*(-1 + Sqrt[33])] + 4*x))/((Sqrt[-1 + Sqrt[33]] + I*Sqrt[1
+ Sqrt[33]])*(Sqrt[2*(-1 + Sqrt[33])] - 4*x))]*((-1 + Sqrt[(-1 + Sqrt[33])/
2]/2)*EllipticF[ArcSin[Sqrt[((Sqrt[-1 + Sqrt[33]] - I*Sqrt[1 + Sqrt[33]])*(
Sqrt[2*(-1 + Sqrt[33])] + 4*x))/((Sqrt[-1 + Sqrt[33]] + I*Sqrt[1 + Sqrt[33]
])*(Sqrt[2*(-1 + Sqrt[33])] - 4*x))]], (Sqrt[-1 + Sqrt[33]] + I*Sqrt[1 + Sq
rt[33]])^2/(Sqrt[-1 + Sqrt[33]] - I*Sqrt[1 + Sqrt[33]])^2] - Sqrt[(-1 + Sqr
t[33])/2]*EllipticPi[((1 + Sqrt[(-1 + Sqrt[33])/2]/2)*(Sqrt[(-1 + Sqrt[33])
/2]/2 + (I/2)*Sqrt[(1 + Sqrt[33])/2]))/((1 - Sqrt[(-1 + Sqrt[33])/2]/2)*(-1
/2*Sqrt[(-1 + Sqrt[33])/2] + (I/2)*Sqrt[(1 + Sqrt[33])/2])), ArcSin[Sqrt[((
Sqrt[-1 + Sqrt[33]] - I*Sqrt[1 + Sqrt[33]])*(Sqrt[2*(-1 + Sqrt[33])] + 4*x)
)/((Sqrt[-1 + Sqrt[33]] + I*Sqrt[1 + Sqrt[33]])*(Sqrt[2*(-1 + Sqrt[33])] -
4*x))]], (Sqrt[-1 + Sqrt[33]] + I*Sqrt[1 + Sqrt[33]])^2/(Sqrt[-1 + Sqrt[33]
] - I*Sqrt[1 + Sqrt[33]])^2]))/((-1 - Sqrt[(-1 + Sqrt[33])/2]/2)*(1 - Sqrt[
(-1 + Sqrt[33])/2]/2)*(Sqrt[(-1 + Sqrt[33])/2]/2 - (I/2)*Sqrt[(1 + Sqrt[33]
)/2])*Sqrt[2 - x^2 - 4*x^4])))/(2*(1 - 1/Sqrt[2])*(1 + 1/Sqrt[2]))))/((1 +
2*x^4)*(-2 + x^2 + 4*x^4)))
```

IntegrateAlgebraic [A] time = 0.43, size = 75, normalized size = 1.00

$$\tan^{-1}\left(\frac{x\sqrt{-4x^4-x^2+2}}{4x^4+x^2-2}\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x\sqrt{-4x^4-x^2+2}}{4x^4+x^2-2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[2 - x^2 - 4*x^4]*(1 + 2*x^4))/((-1 + 2*x^4)*(-1 -
x^2 + 2*x^4)),x]
```

```
[Out] ArcTan[(x*Sqrt[2 - x^2 - 4*x^4])/(-2 + x^2 + 4*x^4)] - Sqrt[3]*ArcTan[(Sqrt
[3]*x*Sqrt[2 - x^2 - 4*x^4])/(-2 + x^2 + 4*x^4)]
```

fricas [A] time = 0.51, size = 71, normalized size = 0.95

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-4x^4-x^2+2}\,x}{2x^4+2x^2-1}\right)+\frac{1}{2}\arctan\left(\frac{\sqrt{-4x^4-x^2+2}\,x}{2x^4+x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^4-x^2+2)^(1/2)*(2*x^4+1)/(2*x^4-1)/(2*x^4-x^2-1),x, algorit
hm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan(sqrt(3)*sqrt(-4*x^4 - x^2 + 2)*x/(2*x^4 + 2*x^2 - 1)) +
 1/2*arctan(sqrt(-4*x^4 - x^2 + 2)*x/(2*x^4 + x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^4+1\right)\sqrt{-4\,x^4-x^2+2}}{\left(2\,x^4-x^2-1\right)\left(2\,x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^4-x^2+2)^(1/2)*(2*x^4+1)/(2*x^4-1)/(2*x^4-x^2-1),x, algorit
hm="giac")
```

```
[Out] integrate((2*x^4 + 1)*sqrt(-4*x^4 - x^2 + 2)/((2*x^4 - x^2 - 1)*(2*x^4 - 1)
), x)
```

maple [C] time = 0.25, size = 823, normalized size = 10.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^4-x^2+2)^(1/2)*(2*x^4+1)/(2*x^4-1)/(2*x^4-x^2-1),x)
```

```
[Out] -6/(1+33^(1/2))^(1/2)*(1-(1/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(1/4-1/4*33^(1/2)
)*x^2)^(1/2)/(-4*x^4-x^2+2)^(1/2)*EllipticF(1/2*x*(1+33^(1/2))^(1/2),1/8*I*
66^(1/2)-1/8*I*2^(1/2))-32/(1+33^(1/2))^(1/2)*(1-(1/4+1/4*33^(1/2))*x^2)^(1
/2)*(1-(1/4-1/4*33^(1/2))*x^2)^(1/2)/(-4*x^4-x^2+2)^(1/2)/(-1+33^(1/2))*(El
lipticF(1/2*x*(1+33^(1/2))^(1/2),1/8*I*66^(1/2)-1/8*I*2^(1/2))-EllipticE(1/
2*x*(1+33^(1/2))^(1/2),1/8*I*66^(1/2)-1/8*I*2^(1/2)))+3/(1/4+1/4*33^(1/2))^
(1/2)*(1-(1/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(1/4-1/4*33^(1/2))*x^2)^(1/2)/(-4
*x^4-x^2+2)^(1/2)*EllipticPi((1/4+1/4*33^(1/2))^(1/2)*x,1/(1/4+1/4*33^(1/2)
),(1/4-1/4*33^(1/2))^(1/2)/(1/4+1/4*33^(1/2))^(1/2))+2/(1+33^(1/2))^(1/2)*(
1-1/4*x^2-1/4*x^2*33^(1/2))^(1/2)*(1-1/4*x^2+1/4*x^2*33^(1/2))^(1/2)/(-4*x^
4-x^2+2)^(1/2)*EllipticF(1/2*x*(1+33^(1/2))^(1/2),1/8*I*66^(1/2)-1/8*I*2^(1
/2))+32/(1+33^(1/2))^(1/2)*(1-1/4*x^2-1/4*x^2*33^(1/2))^(1/2)*(1-1/4*x^2+1/
4*x^2*33^(1/2))^(1/2)/(-4*x^4-x^2+2)^(1/2)/(-1+33^(1/2))*EllipticF(1/2*x*(1
+33^(1/2))^(1/2),1/8*I*66^(1/2)-1/8*I*2^(1/2))-32/(1+33^(1/2))^(1/2)*(1-1/4
*x^2-1/4*x^2*33^(1/2))^(1/2)*(1-1/4*x^2+1/4*x^2*33^(1/2))^(1/2)/(-4*x^4-x^2
+2)^(1/2)/(-1+33^(1/2))*EllipticE(1/2*x*(1+33^(1/2))^(1/2),1/8*I*66^(1/2)-1
/8*I*2^(1/2))+3/(1/4+1/4*33^(1/2))^(1/2)*(1-1/4*x^2-1/4*x^2*33^(1/2))^(1/2)
*(1-1/4*x^2+1/4*x^2*33^(1/2))^(1/2)/(-4*x^4-x^2+2)^(1/2)*EllipticPi((1/4+1/
4*33^(1/2))^(1/2)*x,-2/(1/4+1/4*33^(1/2)),(1/4-1/4*33^(1/2))^(1/2)/(1/4+1/4
*33^(1/2))^(1/2))+1/4*sum(_alpha*(1/(-_alpha^2)^(1/2)*arctanh(1/62*(8*_alph
a^2+1)*(-33*_alpha^2+31*x^2+8)/(-_alpha^2)^(1/2)/(-4*x^4-x^2+2)^(1/2))-4^(1
/2)*_alpha^3/(1+33^(1/2))^(1/2)*(-x^2+4-x^2*33^(1/2))^(1/2)*(-x^2+4+x^2*33^
(1/2))^(1/2)/(-4*x^4-x^2+2)^(1/2)*EllipticPi((1/4+1/4*33^(1/2))^(1/2)*x,1/4
*_alpha^2*33^(1/2)-1/4*_alpha^2,(1/4-1/4*33^(1/2))^(1/2)/(1/4+1/4*33^(1/2))
^(1/2))),_alpha=RootOf(2*_Z^4-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^4+1\right)\sqrt{-4\,x^4-x^2+2}}{\left(2\,x^4-x^2-1\right)\left(2\,x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^4-x^2+2)^(1/2)*(2*x^4+1)/(2*x^4-1)/(2*x^4-x^2-1),x, algorit
hm="maxima")
```

```
[Out] integrate((2*x^4 + 1)*sqrt(-4*x^4 - x^2 + 2)/((2*x^4 - x^2 - 1)*(2*x^4 - 1)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(2\,x^4+1\right)\,\sqrt{-4\,x^4-x^2+2}}{\left(2\,x^4-1\right)\,\left(-2\,x^4+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*x^4 + 1)*(2 - 4*x^4 - x^2)^(1/2))/((2*x^4 - 1)*(x^2 - 2*x^4 + 1)),
x)
```

```
[Out] -int(((2*x^4 + 1)*(2 - 4*x^4 - x^2)^(1/2))/((2*x^4 - 1)*(x^2 - 2*x^4 + 1)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^4+1\right)\sqrt{-4x^4-x^2+2}}{(x-1)\,(x+1)\left(2x^2+1\right)\left(2x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**4-x**2+2)**(1/2)*(2*x**4+1)/(2*x**4-1)/(2*x**4-x**2-1),x)
```

```
[Out] Integral((2*x**4 + 1)*sqrt(-4*x**4 - x**2 + 2)/((x - 1)*(x + 1)*(2*x**2 + 1
)*(2*x**4 - 1)), x)
```

3.908 $\int \left(b+ax^4\right)^{3/4} dx$

Optimal. Leaf size=75

$$\frac{1}{4}x\left(ax^4+b\right)^{3/4}+\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{8\sqrt[4]{a}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{8\sqrt[4]{a}}$$

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {195, 240, 212, 206, 203}

$$\frac{1}{4}x\left(ax^4+b\right)^{3/4}+\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{8\sqrt[4]{a}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{8\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^4)^(3/4),x]
```

```
[Out] (x*(b + a*x^4)^(3/4))/4 + (3*b*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(8*a^
(1/4)) + (3*b*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(8*a^(1/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$
\begin{aligned}
\int \left(b+ax^4\right)^{3/4} dx &= \frac{1}{4}x\left(b+ax^4\right)^{3/4}+\frac{1}{4}(3b)\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx \\
&= \frac{1}{4}x\left(b+ax^4\right)^{3/4}+\frac{1}{4}(3b)\,\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right) \\
&= \frac{1}{4}x\left(b+ax^4\right)^{3/4}+\frac{1}{8}(3b)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)+\frac{1}{8}(3b)\,\text{Subst}\left(\int \frac{1}{1+\sqrt{a}\right. \\
&= \frac{1}{4}x\left(b+ax^4\right)^{3/4}+\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{8\sqrt[4]{a}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{8\sqrt[4]{a}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.61

$$
\frac{x\left(ax^4+b\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};-\frac{ax^4}{b}\right)}{\left(\frac{ax^4}{b}+1\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^4)^(3/4),x]
```

```
[Out] (x*(b + a*x^4)^(3/4)*Hypergeometric2F1[-3/4, 1/4, 5/4, -((a*x^4)/b)])/(1 +
(a*x^4)/b)^(3/4)
```

IntegrateAlgebraic [A] time = 0.30, size = 75, normalized size = 1.00

$$
\frac{1}{4}x\left(ax^4+b\right)^{3/4}+\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8\sqrt[4]{a}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8\sqrt[4]{a}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^4)^(3/4),x]
```

```
[Out] (x*(b + a*x^4)^(3/4))/4 + (3*b*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(8*a^
(1/4)) + (3*b*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(8*a^(1/4))
```

fricas [B] time = 0.44, size = 192, normalized size = 2.56

$$
\frac{1}{4}\left(ax^4+b\right)^{\frac{3}{4}}x+\frac{3}{4}\left(\frac{b^4}{a}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^4+b\right)^{\frac{1}{4}}\left(\frac{b^4}{a}\right)^{\frac{1}{4}}b^3-\left(\frac{b^4}{a}\right)^{\frac{1}{4}}x\sqrt{\frac{\sqrt{\frac{b^4}{a}}\,ab^4x^2+\sqrt{ax^4+b}\,b^6}{x^2}}}{b^4x}\right)+\frac{3}{16}\left(\frac{b^4}{a}\right)^{\frac{1}{4}}\log\left(\frac{27\left(\left(ax^4+b\right)^{\frac{1}{4}}b^3+\left(\frac{b^4}{a}\right)^{\frac{3}{4}}ax\right)}{x}\right)-\frac{3}{16}\left(\frac{b^4}{a}\right)^{\frac{1}{4}}\log\left(\frac{27\left(\left(ax^4+b\right)^{\frac{1}{4}}b^3-\left(\frac{b^4}{a}\right)^{\frac{3}{4}}ax\right)}{x}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/4*(a*x^4 + b)^(3/4)*x + 3/4*(b^4/a)^(1/4)*arctan(-((a*x^4 + b)^(1/4)*(b^4
/a)^(1/4)*b^3 - (b^4/a)^(1/4)*x*sqrt((sqrt(b^4/a)*a*b^4*x^2 + sqrt(a*x^4 +
b)*b^6)/x^2))/(b^4*x)) + 3/16*(b^4/a)^(1/4)*log(27*((a*x^4 + b)^(1/4)*b^3 +
 (b^4/a)^(3/4)*a*x)/x) - 3/16*(b^4/a)^(1/4)*log(27*((a*x^4 + b)^(1/4)*b^3 -
 (b^4/a)^(3/4)*a*x)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \left(ax^4+b\right)^{\frac{3}{4}} dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^4 + b)^(3/4), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(a\,x^{4}+b\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b)^(3/4),x)
```

```
[Out] int((a*x^4+b)^(3/4),x)
```

maxima [A] time = 0.44, size = 102, normalized size = 1.36

$$-\frac{3}{16}\,b\left(\frac{2\,\arctan\left(\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right)-\frac{\left(ax^{4}+b\right)^{\frac{3}{4}}b}{4\left(a-\frac{ax^{4}+b}{x^{4}}\right)x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4),x, algorithm="maxima")
```

```
[Out] -3/16*b*(2*arctan((a*x^4 + b)^(1/4)/(a^(1/4)*x))/a^(1/4) + log(-(a^(1/4) -
(a*x^4 + b)^(1/4)/x)/(a^(1/4) + (a*x^4 + b)^(1/4)/x))/a^(1/4)) - 1/4*(a*x^4
 + b)^(3/4)*b/((a - (a*x^4 + b)/x^4)*x^3)
```

mupad [B] time = 0.78, size = 37, normalized size = 0.49

$$\frac{x\left(a\,x^{4}+b\right)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{a\,x^{4}}{b}\right)}{\left(\frac{a\,x^{4}}{b}+1\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^4)^(3/4),x)
```

```
[Out] (x*(b + a*x^4)^(3/4)*hypergeom([-3/4, 1/4], 5/4, -(a*x^4)/b))/((a*x^4)/b +
1)^(3/4)
```

sympy [C] time = 1.18, size = 37, normalized size = 0.49

$$\frac{b^{\frac{3}{4}}x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{ax^{4}e^{i\pi}}{b}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b)**(3/4),x)
```

```
[Out] b**(3/4)*x*gamma(1/4)*hyper((-3/4, 1/4), (5/4,), a*x**4*exp_polar(I*pi)/b)/
(4*gamma(5/4))
```

3.909 $\int \frac{(b+ax^4)^{3/4}}{x^4}\,dx$

Optimal. Leaf size=75

$$\frac{1}{2}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\frac{1}{2}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)-\frac{(ax^4+b)^{3/4}}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {277, 240, 212, 206, 203}

$$\frac{1}{2}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\frac{1}{2}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)-\frac{(ax^4+b)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^4)^(3/4)/x^4,x]
```

```
[Out] -1/3*(b + a*x^4)^(3/4)/x^3 + (a^(3/4)*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)]
)/2 + (a^(3/4)*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\left(b+ax^4\right)^{3/4}}{x^4}\,dx = -\frac{\left(b+ax^4\right)^{3/4}}{3x^3} + a\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx$$
$$= -\frac{\left(b+ax^4\right)^{3/4}}{3x^3} + a\,\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)$$
$$= -\frac{\left(b+ax^4\right)^{3/4}}{3x^3} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right) + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{1+\sqrt{a}\,x^2}\,dx, x, \right.$$
$$= -\frac{\left(b+ax^4\right)^{3/4}}{3x^3} + \frac{1}{2}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right) + \frac{1}{2}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.68

$$-\frac{\left(ax^4+b\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};-\frac{ax^4}{b}\right)}{3x^3\left(\frac{ax^4}{b}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^4)^(3/4)/x^4,x]
```

```
[Out] -1/3*((b + a*x^4)^(3/4)*Hypergeometric2F1[-3/4, -3/4, 1/4, -((a*x^4)/b)])/(
x^3*(1 + (a*x^4)/b)^(3/4))
```

IntegrateAlgebraic [A] time = 0.23, size = 75, normalized size = 1.00

$$\frac{1}{2}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right) + \frac{1}{2}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right) - \frac{\left(ax^4+b\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^4)^(3/4)/x^4,x]
```

```
[Out] -1/3*(b + a*x^4)^(3/4)/x^3 + (a^(3/4)*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)]
)/2 + (a^(3/4)*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4)/x^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4+b\right)^{\frac{3}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] integrate((a*x^4 + b)^(3/4)/x^4, x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{4}+b\right)^{\frac{3}{4}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b)^(3/4)/x^4,x)
```

```
[Out] int((a*x^4+b)^(3/4)/x^4,x)
```

maxima [A] time = 0.44, size = 85, normalized size = 1.13

$$-\frac{1}{4}\,a\left(\frac{2\,\arctan\left(\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right)-\frac{\left(ax^{4}+b\right)^{\frac{3}{4}}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4)/x^4,x, algorithm="maxima")
```

```
[Out] -1/4*a*(2*arctan((a*x^4 + b)^(1/4)/(a^(1/4)*x))/a^(1/4) + log(-(a^(1/4) - (
a*x^4 + b)^(1/4)/x)/(a^(1/4) + (a*x^4 + b)^(1/4)/x))/a^(1/4)) - 1/3*(a*x^4
+ b)^(3/4)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^4+b\right)^{3/4}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^4)^(3/4)/x^4,x)
```

```
[Out] int((b + a*x^4)^(3/4)/x^4, x)
```

sympy [C] time = 1.24, size = 42, normalized size = 0.56

$$\frac{b^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{ax^{4}e^{i\pi}}{b}\right)}{4x^{3}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b)**(3/4)/x**4,x)
```

```
[Out] b**(3/4)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), a*x**4*exp_polar(I*pi)/b)/
(4*x**3*gamma(1/4))
```

3.910 $\int \frac{\left(b+ax^5\right)^{3/4}}{x^6}\,dx$

Optimal. Leaf size=75

$$-\frac{\left(ax^5+b\right)^{3/4}}{5x^5}+\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{10\sqrt[4]{b}}-\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{10\sqrt[4]{b}}$$

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 47, 63, 298, 203, 206}

$$-\frac{\left(ax^5+b\right)^{3/4}}{5x^5}+\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{10\sqrt[4]{b}}-\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{10\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^5)^(3/4)/x^6,x]
```

```
[Out] -1/5*(b + a*x^5)^(3/4)/x^5 + (3*a*ArcTan[(b + a*x^5)^(1/4)/b^(1/4)])/(10*b^
(1/4)) - (3*a*ArcTanh[(b + a*x^5)^(1/4)/b^(1/4)])/(10*b^(1/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(b+ax^5\right)^{3/4}}{x^6}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{(b+ax)^{3/4}}{x^2}\,dx, x, x^5\right)\\
&= -\frac{\left(b+ax^5\right)^{3/4}}{5x^5}+\frac{1}{20}(3a)\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{b+ax}}\,dx, x, x^5\right)\\
&= -\frac{\left(b+ax^5\right)^{3/4}}{5x^5}+\frac{3}{5}\,\text{Subst}\left(\int \frac{x^2}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^5}\right)\\
&= -\frac{\left(b+ax^5\right)^{3/4}}{5x^5}-\frac{1}{10}(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^5}\right)+\frac{1}{10}(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+x\right.\\
&= -\frac{\left(b+ax^5\right)^{3/4}}{5x^5}+\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{b+ax^5}}{\sqrt[4]{b}}\right)}{10\sqrt[4]{b}}-\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^5}}{\sqrt[4]{b}}\right)}{10\sqrt[4]{b}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.49

$$\frac{4a\left(ax^5+b\right)^{7/4}\,{}_2F_1\left(\frac{7}{4},2;\frac{11}{4};\frac{ax^5}{b}+1\right)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^5)^(3/4)/x^6,x]
```

```
[Out] (4*a*(b + a*x^5)^(7/4)*Hypergeometric2F1[7/4, 2, 11/4, 1 + (a*x^5)/b])/(35*
b^2)
```

IntegrateAlgebraic [A] time = 0.12, size = 75, normalized size = 1.00

$$-\frac{\left(ax^5+b\right)^{3/4}}{5x^5}+\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{10\sqrt[4]{b}}-\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{10\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^5)^(3/4)/x^6,x]
```

```
[Out] -1/5*(b + a*x^5)^(3/4)/x^5 + (3*a*ArcTan[(b + a*x^5)^(1/4)/b^(1/4)])/(10*b^
(1/4)) - (3*a*ArcTanh[(b + a*x^5)^(1/4)/b^(1/4)])/(10*b^(1/4))
```

fricas [B] time = 0.49, size = 185, normalized size = 2.47

$$-\frac{12\left(\frac{a^4}{b}\right)^{\frac{1}{4}}x^5\arctan\left(-\frac{\left(ax^5+b\right)^{\frac{1}{4}}\left(\frac{a^4}{b}\right)^{\frac{1}{4}}a^3-\sqrt{\sqrt{ax^5+b}\,a^6+\sqrt{\frac{a^4}{b}}\,a^4b}\left(\frac{a^4}{b}\right)^{\frac{1}{4}}}{a^4}\right)+3\left(\frac{a^4}{b}\right)^{\frac{1}{4}}x^5\log\left(27\left(ax^5+b\right)^{\frac{1}{4}}a^3+27\left(\frac{a^4}{b}\right)^{\frac{3}{4}}b\right)-3\left(\frac{a^4}{b}\right)^{\frac{1}{4}}x^5\log\left(27\left(ax^5+b\right)^{\frac{1}{4}}a^3-27\left(\frac{a^4}{b}\right)^{\frac{3}{4}}b\right)+4\left(ax^5+b\right)^{\frac{3}{4}}}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5+b)^(3/4)/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(12*(a^4/b)^(1/4)*x^5*arctan(-((a*x^5 + b)^(1/4)*(a^4/b)^(1/4)*a^3 -
sqrt(sqrt(a*x^5 + b)*a^6 + sqrt(a^4/b)*a^4*b)*(a^4/b)^(1/4))/a^4) + 3*(a^4/
b)^(1/4)*x^5*log(27*(a*x^5 + b)^(1/4)*a^3 + 27*(a^4/b)^(3/4)*b) - 3*(a^4/b)
^(1/4)*x^5*log(27*(a*x^5 + b)^(1/4)*a^3 - 27*(a^4/b)^(3/4)*b) + 4*(a*x^5 +
b)^(3/4))/x^5
```

giac [B] time = 0.31, size = 209, normalized size = 2.79

$$\frac{\frac{6\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}+2\left(ax^5+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{(-b)^{\frac{1}{4}}}+\frac{6\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}-2\left(ax^5+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{(-b)^{\frac{1}{4}}}+\frac{3\sqrt{2}a^2(-b)^{\frac{3}{4}}\log\left(\sqrt{2}\left(ax^5+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^5+b}+\sqrt{-b}\right)}{b}+\frac{3\sqrt{2}a^2\log\left(-\sqrt{2}\left(ax^5+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^5+b}+\sqrt{-b}\right)}{(-b)^{\frac{1}{4}}}-\frac{8\left(ax^5+b\right)^{\frac{3}{4}}a}{x^5}}{40a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5+b)^(3/4)/x^6,x, algorithm="giac")
```

```
[Out] 1/40*(6*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^5 + b)^
(1/4))/(-b)^(1/4))/(-b)^(1/4) + 6*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*
(-b)^(1/4) - 2*(a*x^5 + b)^(1/4))/(-b)^(1/4))/(-b)^(1/4) + 3*sqrt(2)*a^2*(-
b)^(3/4)*log(sqrt(2)*(a*x^5 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^5 + b) + sqrt(
-b))/b + 3*sqrt(2)*a^2*log(-sqrt(2)*(a*x^5 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x
^5 + b) + sqrt(-b))/(-b)^(1/4) - 8*(a*x^5 + b)^(3/4)*a/x^5)/a
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^5+b\right)^{\frac{3}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^5+b)^(3/4)/x^6,x)
```

```
[Out] int((a*x^5+b)^(3/4)/x^6,x)
```

maxima [A] time = 0.44, size = 74, normalized size = 0.99

$$\frac{3}{20}a\left(\frac{2\arctan\left(\frac{\left(ax^5+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(ax^5+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^5+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}\right)-\frac{\left(ax^5+b\right)^{\frac{3}{4}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5+b)^(3/4)/x^6,x, algorithm="maxima")
```

```
[Out] 3/20*a*(2*arctan((a*x^5 + b)^(1/4)/b^(1/4))/b^(1/4) + log(((a*x^5 + b)^(1/4
) - b^(1/4))/((a*x^5 + b)^(1/4) + b^(1/4)))/b^(1/4)) - 1/5*(a*x^5 + b)^(3/4
)/x^5
```

mupad [B] time = 1.03, size = 55, normalized size = 0.73

$$\frac{3\,a\,\mathrm{atan}\left(\frac{{\left(a\,x^5+b\right)}^{1/4}}{b^{1/4}}\right)}{10\,b^{1/4}}-\frac{{\left(a\,x^5+b\right)}^{3/4}}{5\,x^5}-\frac{3\,a\,\mathrm{atanh}\left(\frac{{\left(a\,x^5+b\right)}^{1/4}}{b^{1/4}}\right)}{10\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^5)^(3/4)/x^6,x)
```

```
[Out] (3*a*atan((b + a*x^5)^(1/4)/b^(1/4)))/(10*b^(1/4)) - (b + a*x^5)^(3/4)/(5*x
^5) - (3*a*atanh((b + a*x^5)^(1/4)/b^(1/4)))/(10*b^(1/4))
```

sympy [C] time = 1.44, size = 42, normalized size = 0.56

$$-\frac{a^{\frac{3}{4}}\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^{5}}\right)}{5x^{\frac{5}{4}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**5+b)**(3/4)/x**6,x)
```

```
[Out] -a**(3/4)*gamma(1/4)*hyper((-3/4, 1/4), (5/4,), b*exp_polar(I*pi)/(a*x**5))
/(5*x**(5/4)*gamma(5/4))
```

3.911 $\int \frac{2-3x^5}{(1-x^2+x^5)\sqrt[3]{x+x^6}}\,dx$

Optimal. Leaf size=75

$$-\log\left(\sqrt[3]{x^6+x}-x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^6+x}+x}\right)+\frac{1}{2}\log\left(\sqrt[3]{x^6+x}\,x+\left(x^6+x\right)^{2/3}+x^2\right)$$

Rubi [F] time = 1.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2-3x^5}{\left(1-x^2+x^5\right)\sqrt[3]{x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2 - 3*x^5)/((1 - x^2 + x^5)*(x + x^6)^(1/3)),x]
```

```
[Out] (-9*x*(1 + x^5)^(1/3)*Hypergeometric2F1[2/15, 1/3, 17/15, -x^5])/(2*(x + x^
6)^(1/3)) + (15*x^(1/3)*(1 + x^5)^(1/3)*Defer[Subst][Defer[Int][x/((1 + x^1
5)^(1/3)*(1 - x^6 + x^15)), x], x, x^(1/3)])/(x + x^6)^(1/3) - (9*x^(1/3)*(
1 + x^5)^(1/3)*Defer[Subst][Defer[Int][x^7/((1 + x^15)^(1/3)*(1 - x^6 + x^1
5)), x], x, x^(1/3)])/(x + x^6)^(1/3)
```

Rubi steps

$$\begin{aligned}
\int \frac{2-3x^5}{\left(1-x^2+x^5\right)\sqrt[3]{x+x^6}}\,dx &= \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^5}\right)\int \frac{2-3x^5}{\sqrt[3]{x}\sqrt[3]{1+x^5}\left(1-x^2+x^5\right)}\,dx}{\sqrt[3]{x+x^6}}\\
&= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^5}\right)\text{Subst}\left(\int \frac{x\left(2-3x^{15}\right)}{\sqrt[3]{1+x^{15}}\left(1-x^6+x^{15}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x+x^6}}\\
&= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^5}\right)\text{Subst}\left(\int \left(-\frac{3x}{\sqrt[3]{1+x^{15}}}+\frac{x\left(5-3x^6\right)}{\sqrt[3]{1+x^{15}}\left(1-x^6+x^{15}\right)}\right)dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x+x^6}}\\
&= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^5}\right)\text{Subst}\left(\int \frac{x\left(5-3x^6\right)}{\sqrt[3]{1+x^{15}}\left(1-x^6+x^{15}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x+x^6}}-\frac{\left(9\sqrt[3]{x}\sqrt[3]{1+x^5}\right)\text{Subst}\left(\int\right.}{\sqrt[3]{x+x^6}}\\
&= -\frac{9x\sqrt[3]{1+x^5}\,{}_2F_1\left(\frac{2}{15},\frac{1}{3};\frac{17}{15};-x^5\right)}{2\sqrt[3]{x+x^6}}+\frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^5}\right)\text{Subst}\left(\int\left(\frac{5x}{\sqrt[3]{1+x^{15}}\left(1-x^6+x^{15}\right)}-\right.\right.}{\sqrt[3]{x+x^6}}\\
&= -\frac{9x\sqrt[3]{1+x^5}\,{}_2F_1\left(\frac{2}{15},\frac{1}{3};\frac{17}{15};-x^5\right)}{2\sqrt[3]{x+x^6}}-\frac{\left(9\sqrt[3]{x}\sqrt[3]{1+x^5}\right)\text{Subst}\left(\int \frac{x^7}{\sqrt[3]{1+x^{15}}\left(1-x^6+x^{15}\right)}\,dx,x\right.}{\sqrt[3]{x+x^6}}
\end{aligned}$$

Mathematica [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{2-3x^5}{\left(1-x^2+x^5\right)\sqrt[3]{x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2 - 3*x^5)/((1 - x^2 + x^5)*(x + x^6)^(1/3)),x]
```

```
[Out] Integrate[(2 - 3*x^5)/((1 - x^2 + x^5)*(x + x^6)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 2.69, size = 75, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^6+x}-x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^6+x}+x}\right)+\frac{1}{2}\log\left(\sqrt[3]{x^6+x}\,x+\left(x^6+x\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - 3*x^5)/((1 - x^2 + x^5)*(x + x^6)^(1/3)),x]
```

```
[Out] Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(x + x^6)^(1/3))] - Log[-x + (x + x^6)^(1
/3)] + Log[x^2 + x*(x + x^6)^(1/3) + (x + x^6)^(2/3)]/2
```

fricas [A] time = 2.35, size = 100, normalized size = 1.33

$$\sqrt{3}\arctan\left(-\frac{4\sqrt{3}\left(x^6+x\right)^{\frac{1}{3}}x+\sqrt{3}\left(x^5+1\right)-2\sqrt{3}\left(x^6+x\right)^{\frac{2}{3}}}{x^5+8x^2+1}\right)-\frac{1}{2}\log\left(\frac{x^5-x^2+3\left(x^6+x\right)^{\frac{1}{3}}x-3\left(x^6+x\right)^{\frac{2}{3}}+1}{x^5-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^5+2)/(x^5-x^2+1)/(x^6+x)^(1/3),x, algorithm="fricas")
```

```
[Out] sqrt(3)*arctan(-(4*sqrt(3)*(x^6 + x)^(1/3)*x + sqrt(3)*(x^5 + 1) - 2*sqrt(3
)*(x^6 + x)^(2/3))/(x^5 + 8*x^2 + 1)) - 1/2*log((x^5 - x^2 + 3*(x^6 + x)^(1
/3)*x - 3*(x^6 + x)^(2/3) + 1)/(x^5 - x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{3x^5-2}{\left(x^6+x\right)^{\frac{1}{3}}\left(x^5-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^5+2)/(x^5-x^2+1)/(x^6+x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(3*x^5 - 2)/((x^6 + x)^(1/3)*(x^5 - x^2 + 1)), x)
```

maple [C] time = 5.94, size = 347, normalized size = 4.63

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^5+2)/(x^5-x^2+1)/(x^6+x)^(1/3),x)
```

```
[Out] -ln((-34406523*RootOf(_Z^2-_Z+1)^2*x^5-43368040*RootOf(_Z^2-_Z+1)*x^5+50890
012*x^5+68813046*RootOf(_Z^2-_Z+1)^2*x^2+70252591*RootOf(_Z^2-_Z+1)*(x^6+x)
^(2/3)+92818507*RootOf(_Z^2-_Z+1)*(x^6+x)^(1/3)*x-154109581*RootOf(_Z^2-_Z+
1)*x^2-163071098*(x^6+x)^(2/3)+70252591*x*(x^6+x)^(1/3)-34406523*RootOf(_Z^
2-_Z+1)^2+76335018*x^2-43368040*RootOf(_Z^2-_Z+1)+50890012)/(x^5-x^2+1))+Ro
otOf(_Z^2-_Z+1)*ln(-(-25445006*RootOf(_Z^2-_Z+1)^2*x^5+110741541*RootOf(_Z^
2-_Z+1)*x^5-103219569*x^5+50890012*RootOf(_Z^2-_Z+1)^2*x^2+70252591*RootOf(
_Z^2-_Z+1)*(x^6+x)^(2/3)-163071098*RootOf(_Z^2-_Z+1)*(x^6+x)^(1/3)*x-433680
40*RootOf(_Z^2-_Z+1)*x^2+92818507*(x^6+x)^(2/3)+70252591*x*(x^6+x)^(1/3)-25
445006*RootOf(_Z^2-_Z+1)^2-34406523*x^2+110741541*RootOf(_Z^2-_Z+1)-1032195
69)/(x^5-x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3\,x^5-2}{\left(x^6+x\right)^{\frac{1}{3}}\left(x^5-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^5+2)/(x^5-x^2+1)/(x^6+x)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate((3*x^5 - 2)/((x^6 + x)^(1/3)*(x^5 - x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{3\,x^5-2}{{\left(x^6+x\right)}^{1/3}\,\left(x^5-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^5 - 2)/((x + x^6)^(1/3)*(x^5 - x^2 + 1)),x)
```

```
[Out] -int((3*x^5 - 2)/((x + x^6)^(1/3)*(x^5 - x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3x^5}{x^5\sqrt[3]{x^6+x}-x^2\sqrt[3]{x^6+x}+\sqrt[3]{x^6+x}}\,dx-\int \left(-\frac{2}{x^5\sqrt[3]{x^6+x}-x^2\sqrt[3]{x^6+x}+\sqrt[3]{x^6+x}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**5+2)/(x**5-x**2+1)/(x**6+x)**(1/3),x)
```

```
[Out] -Integral(3*x**5/(x**5*(x**6 + x)**(1/3) - x**2*(x**6 + x)**(1/3) + (x**6 +
 x)**(1/3)), x) - Integral(-2/(x**5*(x**6 + x)**(1/3) - x**2*(x**6 + x)**(1
/3) + (x**6 + x)**(1/3)), x)
```

3.912 $\int \frac{(b+ax^6)^{3/4}}{x^7}\,dx$

Optimal. Leaf size=75

$$-\frac{(ax^6+b)^{3/4}}{6x^6}+\frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{4\sqrt[4]{b}}-\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{4\sqrt[4]{b}}$$

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 47, 63, 298, 203, 206}

$$-\frac{(ax^6+b)^{3/4}}{6x^6}+\frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{4\sqrt[4]{b}}-\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{4\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^6)^(3/4)/x^7,x]
```

```
[Out] -1/6*(b + a*x^6)^(3/4)/x^6 + (a*ArcTan[(b + a*x^6)^(1/4)/b^(1/4)])/(4*b^(1/
4)) - (a*ArcTanh[(b + a*x^6)^(1/4)/b^(1/4)])/(4*b^(1/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(b+ax^6\right)^{3/4}}{x^7}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{(b+ax)^{3/4}}{x^2}\,dx, x, x^6\right)\\
&= -\frac{\left(b+ax^6\right)^{3/4}}{6x^6}+\frac{1}{8}a\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{b+ax}}\,dx, x, x^6\right)\\
&= -\frac{\left(b+ax^6\right)^{3/4}}{6x^6}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^6}\right)\\
&= -\frac{\left(b+ax^6\right)^{3/4}}{6x^6}-\frac{1}{4}a\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^6}\right)+\frac{1}{4}a\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx, x, \sqrt[4]{b}\right.\\
&= -\frac{\left(b+ax^6\right)^{3/4}}{6x^6}+\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b+ax^6}}{\sqrt[4]{b}}\right)}{4\sqrt[4]{b}}-\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^6}}{\sqrt[4]{b}}\right)}{4\sqrt[4]{b}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.49

$$\frac{2a\left(ax^6+b\right)^{7/4}\,{}_2F_1\left(\frac{7}{4},2;\frac{11}{4};\frac{ax^6}{b}+1\right)}{21b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^6)^(3/4)/x^7,x]
```

```
[Out] (2*a*(b + a*x^6)^(7/4)*Hypergeometric2F1[7/4, 2, 11/4, 1 + (a*x^6)/b])/(21*
b^2)
```

IntegrateAlgebraic [A] time = 0.07, size = 75, normalized size = 1.00

$$-\frac{\left(ax^6+b\right)^{3/4}}{6x^6}+\frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{4\sqrt[4]{b}}-\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{4\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^6)^(3/4)/x^7,x]
```

```
[Out] -1/6*(b + a*x^6)^(3/4)/x^6 + (a*ArcTan[(b + a*x^6)^(1/4)/b^(1/4)])/(4*b^(1/
4)) - (a*ArcTanh[(b + a*x^6)^(1/4)/b^(1/4)])/(4*b^(1/4))
```

fricas [B] time = 0.48, size = 182, normalized size = 2.43

$$-\frac{12\left(\frac{a^4}{b}\right)^{\frac{1}{4}}x^6\arctan\left(-\frac{\left(ax^6+b\right)^{\frac{1}{4}}\left(\frac{a^4}{b}\right)^{\frac{1}{4}}a^3-\sqrt{\sqrt{ax^6+b}\,a^6+\sqrt{\frac{a^4}{b}}\,a^4b}\left(\frac{a^4}{b}\right)^{\frac{1}{4}}}{a^4}\right)+3\left(\frac{a^4}{b}\right)^{\frac{1}{4}}x^6\log\left(\left(ax^6+b\right)^{\frac{1}{4}}a^3+\left(\frac{a^4}{b}\right)^{\frac{3}{4}}b\right)-3\left(\frac{a^4}{b}\right)^{\frac{1}{4}}x^6\log\left(\left(ax^6+b\right)^{\frac{1}{4}}a^3-\left(\frac{a^4}{b}\right)^{\frac{3}{4}}b\right)+4\left(ax^6+b\right)^{\frac{3}{4}}}{24\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)^(3/4)/x^7,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(a^4/b)^(1/4)*x^6*arctan(-((a*x^6 + b)^(1/4)*(a^4/b)^(1/4)*a^3 -
sqrt(sqrt(a*x^6 + b)*a^6 + sqrt(a^4/b)*a^4*b)*(a^4/b)^(1/4))/a^4) + 3*(a^4/
b)^(1/4)*x^6*log((a*x^6 + b)^(1/4)*a^3 + (a^4/b)^(3/4)*b) - 3*(a^4/b)^(1/4)
*x^6*log((a*x^6 + b)^(1/4)*a^3 - (a^4/b)^(3/4)*b) + 4*(a*x^6 + b)^(3/4))/x^
6
```

giac [B] time = 0.43, size = 209, normalized size = 2.79

$$\frac{\frac{6\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}+2\left(ax^6+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{(-b)^{\frac{1}{4}}}+\frac{6\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}-2\left(ax^6+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{(-b)^{\frac{1}{4}}}+\frac{3\sqrt{2}a^2(-b)^{\frac{3}{4}}\log\left(\sqrt{2}\left(ax^6+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^6+b}+\sqrt{-b}\right)}{b}+\frac{3\sqrt{2}a^2\log\left(-\sqrt{2}\left(ax^6+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^6+b}+\sqrt{-b}\right)}{(-b)^{\frac{1}{4}}}-\frac{8\left(ax^6+b\right)^{\frac{3}{4}}a}{x^6}}{48a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)^(3/4)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(6*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^6 + b)^
(1/4))/(-b)^(1/4))/(-b)^(1/4) + 6*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*
(-b)^(1/4) - 2*(a*x^6 + b)^(1/4))/(-b)^(1/4))/(-b)^(1/4) + 3*sqrt(2)*a^2*(-
b)^(3/4)*log(sqrt(2)*(a*x^6 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^6 + b) + sqrt(
-b))/b + 3*sqrt(2)*a^2*log(-sqrt(2)*(a*x^6 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x
^6 + b) + sqrt(-b))/(-b)^(1/4) - 8*(a*x^6 + b)^(3/4)*a/x^6)/a
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{6}+b\right)^{\frac{3}{4}}}{x^{7}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6+b)^(3/4)/x^7,x)
```

```
[Out] int((a*x^6+b)^(3/4)/x^7,x)
```

maxima [A] time = 0.41, size = 74, normalized size = 0.99

$$\frac{1}{8}\,a\left(\frac{2\,\arctan\left(\frac{\left(ax^6+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(ax^6+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^6+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}\right)-\frac{\left(ax^6+b\right)^{\frac{3}{4}}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)^(3/4)/x^7,x, algorithm="maxima")
```

```
[Out] 1/8*a*(2*arctan((a*x^6 + b)^(1/4)/b^(1/4))/b^(1/4) + log(((a*x^6 + b)^(1/4)
 - b^(1/4))/((a*x^6 + b)^(1/4) + b^(1/4)))/b^(1/4)) - 1/6*(a*x^6 + b)^(3/4)
/x^6
```

mupad [B] time = 1.04, size = 55, normalized size = 0.73

$$\frac{a\,\mathrm{atan}\left(\frac{{\left(a\,x^6+b\right)}^{1/4}}{b^{1/4}}\right)}{4\,b^{1/4}}-\frac{{\left(a\,x^6+b\right)}^{3/4}}{6\,x^6}-\frac{a\,\mathrm{atanh}\left(\frac{{\left(a\,x^6+b\right)}^{1/4}}{b^{1/4}}\right)}{4\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^6)^(3/4)/x^7,x)
```

```
[Out] (a*atan((b + a*x^6)^(1/4)/b^(1/4)))/(4*b^(1/4)) - (b + a*x^6)^(3/4)/(6*x^6)
 - (a*atanh((b + a*x^6)^(1/4)/b^(1/4)))/(4*b^(1/4))
```

sympy [C] time = 1.53, size = 42, normalized size = 0.56

$$-\frac{a^{\frac{3}{4}}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^6}\right)}{6x^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6+b)**(3/4)/x**7,x)
```

```
[Out] -a**(3/4)*gamma(1/4)*hyper((-3/4, 1/4), (5/4,), b*exp_polar(I*pi)/(a*x**6))
/(6*x**(3/2)*gamma(5/4))
```

3.913 $\int \frac{2b+ax^6}{\sqrt[4]{-b+ax^6}\left(-b-2x^4+ax^6\right)}\,dx$

Optimal. Leaf size=75

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x\left(ax^6-b\right)^{3/4}}{b-ax^6}\right)}{\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x\left(ax^6-b\right)^{3/4}}{b-ax^6}\right)}{\sqrt[4]{2}}$$

Rubi [F] time = 1.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2b+ax^6}{\sqrt[4]{-b+ax^6}\left(-b-2x^4+ax^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2*b + a*x^6)/((-b + a*x^6)^(1/4)*(-b - 2*x^4 + a*x^6)),x]
```

```
[Out] (x*(1 - (a*x^6)/b)^(1/4)*Hypergeometric2F1[1/6, 1/4, 7/6, (a*x^6)/b])/(-b +
 a*x^6)^(1/4) - 3*b*Defer[Int][1/((b + 2*x^4 - a*x^6)*(-b + a*x^6)^(1/4)),
x] + 2*Defer[Int][x^4/((-b + a*x^6)^(1/4)*(-b - 2*x^4 + a*x^6)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{2b+ax^6}{\sqrt[4]{-b+ax^6}\left(-b-2x^4+ax^6\right)}\,dx &= \int \left(\frac{1}{\sqrt[4]{-b+ax^6}}+\frac{3b+2x^4}{\sqrt[4]{-b+ax^6}\left(-b-2x^4+ax^6\right)}\right)dx\\
&= \int \frac{1}{\sqrt[4]{-b+ax^6}}\,dx+\int \frac{3b+2x^4}{\sqrt[4]{-b+ax^6}\left(-b-2x^4+ax^6\right)}\,dx\\
&= \frac{\sqrt[4]{1-\frac{ax^6}{b}}\int \frac{1}{\sqrt[4]{1-\frac{ax^6}{b}}}\,dx}{\sqrt[4]{-b+ax^6}}+\int \left(-\frac{3b}{\left(b+2x^4-ax^6\right)\sqrt[4]{-b+ax^6}}+\frac{}{\sqrt[4]{-b+ax}}\right.\\
&= \frac{x\sqrt[4]{1-\frac{ax^6}{b}}\,{}_2F_1\left(\frac{1}{6},\frac{1}{4};\frac{7}{6};\frac{ax^6}{b}\right)}{\sqrt[4]{-b+ax^6}}+2\int \frac{x^4}{\sqrt[4]{-b+ax^6}\left(-b-2x^4+ax^6\right)}\,dx-(3
\end{aligned}$$

Mathematica [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{2b+ax^6}{\sqrt[4]{-b+ax^6}\left(-b-2x^4+ax^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2*b + a*x^6)/((-b + a*x^6)^(1/4)*(-b - 2*x^4 + a*x^6)),x]
```

```
[Out] Integrate[(2*b + a*x^6)/((-b + a*x^6)^(1/4)*(-b - 2*x^4 + a*x^6)), x]
```

IntegrateAlgebraic [A] time = 4.65, size = 75, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x\left(ax^6-b\right)^{3/4}}{b-ax^6}\right)}{\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x\left(ax^6-b\right)^{3/4}}{b-ax^6}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*b + a*x^6)/((-b + a*x^6)^(1/4)*(-b - 2*x^4 + a*x^6)),
x]
```

```
[Out] ArcTan[(2^(1/4)*x*(-b + a*x^6)^(3/4))/(b - a*x^6)]/2^(1/4) + ArcTanh[(2^(1/
4)*x*(-b + a*x^6)^(3/4))/(b - a*x^6)]/2^(1/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+2*b)/(a*x^6-b)^(1/4)/(a*x^6-2*x^4-b),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^6 + 2\,b}{\left(ax^6 - 2\,x^4 - b\right)\left(ax^6 - b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+2*b)/(a*x^6-b)^(1/4)/(a*x^6-2*x^4-b),x, algorithm="giac")
```

```
[Out] integrate((a*x^6 + 2*b)/((a*x^6 - 2*x^4 - b)*(a*x^6 - b)^(1/4)), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{a\,x^6 + 2b}{\left(a\,x^6 - b\right)^{\frac{1}{4}}\left(a\,x^6 - 2x^4 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6+2*b)/(a*x^6-b)^(1/4)/(a*x^6-2*x^4-b),x)
```

```
[Out] int((a*x^6+2*b)/(a*x^6-b)^(1/4)/(a*x^6-2*x^4-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^6 + 2\,b}{\left(ax^6 - 2\,x^4 - b\right)\left(ax^6 - b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+2*b)/(a*x^6-b)^(1/4)/(a*x^6-2*x^4-b),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^6 + 2*b)/((a*x^6 - 2*x^4 - b)*(a*x^6 - b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a\,x^6 + 2\,b}{\left(a\,x^6 - b\right)^{1/4}\,\left(-a\,x^6 + 2\,x^4 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*b + a*x^6)/((a*x^6 - b)^(1/4)*(b - a*x^6 + 2*x^4)),x)
```

```
[Out] int(-(2*b + a*x^6)/((a*x^6 - b)^(1/4)*(b - a*x^6 + 2*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6+2*b)/(a*x**6-b)**(1/4)/(a*x**6-2*x**4-b),x)
```

```
[Out] Timed out
```

3.914 $\int \frac{(-1+x^4)\sqrt{1+x^4}}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=75

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x\sqrt{x^4+1}}{x^4-x^2+1}\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt{x^4+1}}{x^4+x^2+1}\right)}{2\sqrt{2}}$$

Rubi [C] time = 1.92, antiderivative size = 1128, normalized size of antiderivative = 15.04, number of steps used = 20, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6728, 406, 220, 409, 1217, 1707}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^4)*Sqrt[1 + x^4])/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((-1/4 + I/4)*ArcTan[((-1)^(1/4)*x)/Sqrt[1 + x^4]])/Sqrt[2] - ((-1)^(1/4)*(
2*I - Sqrt[6 - 2*Sqrt[5]])*(2*I - Sqrt[2*(3 + Sqrt[5])])*ArcTan[((-1)^(1/4)
*x)/Sqrt[1 + x^4]])/(16*Sqrt[5]) + ((1/4 + I/4)*ArcTan[((-1)^(3/4)*x)/Sqrt[
1 + x^4]])/Sqrt[2] + ((1/16 + I/16)*(2*I + Sqrt[6 - 2*Sqrt[5]])*(2 - I*Sqrt
[2*(3 + Sqrt[5])])*ArcTan[((-1)^(3/4)*x)/Sqrt[1 + x^4]])/Sqrt[10] + ((1 - S
qrt[5])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/
(4*Sqrt[1 + x^4]) + ((1 + Sqrt[5])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*El
lipticF[2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4]) - ((3 + Sqrt[5])*(1 - I*Sqrt[2
/(3 + Sqrt[5])])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x
], 1/2])/(4*(5 + Sqrt[5])*Sqrt[1 + x^4]) - ((3 + Sqrt[5])*(1 + I*Sqrt[2/(3
+ Sqrt[5])])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1
/2])/(4*(5 + Sqrt[5])*Sqrt[1 + x^4]) - ((3 - Sqrt[5])*(2 - I*Sqrt[2*(3 + Sq
rt[5])])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])
/(8*(5 - Sqrt[5])*Sqrt[1 + x^4]) - ((3 - Sqrt[5])*(2 + I*Sqrt[2*(3 + Sqrt[5
])])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(8*
(5 - Sqrt[5])*Sqrt[1 + x^4]) + ((5 - 2*Sqrt[5])*(3 + Sqrt[5])*(1 + Sqrt[5]
- (2*I)*Sqrt[2*(3 + Sqrt[5])])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*Ellipt
icPi[(I/4)*Sqrt[(3 + Sqrt[5])/2]*(I - Sqrt[2/(3 + Sqrt[5])])^2, 2*ArcTan[x]
, 1/2])/(160*Sqrt[1 + x^4]) + ((5 - 2*Sqrt[5])*(3 + Sqrt[5])*(1 + Sqrt[5] +
 (2*I)*Sqrt[2*(3 + Sqrt[5])])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*Ellipti
cPi[(-1/4*I)*Sqrt[(3 + Sqrt[5])/2]*(I + Sqrt[2/(3 + Sqrt[5])])^2, 2*ArcTan[
x], 1/2])/(160*Sqrt[1 + x^4]) - ((2 + I*Sqrt[6 - 2*Sqrt[5]])*(2*I + Sqrt[2*
(3 + Sqrt[5])])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticPi[(I/16)*Sqr
t[(3 - Sqrt[5])/2]*(2*I - Sqrt[2*(3 + Sqrt[5])])^2, 2*ArcTan[x], 1/2])/(32*
Sqrt[5]*Sqrt[1 + x^4]) + ((2*I + Sqrt[6 - 2*Sqrt[5]])*(2 + I*Sqrt[2*(3 + Sq
rt[5])])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticPi[(-1/16*I)*Sqrt[(3
 - Sqrt[5])/2]*(2*I + Sqrt[2*(3 + Sqrt[5])])^2, 2*ArcTan[x], 1/2])/(32*Sqrt
[5]*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(-1+x^4)\sqrt{1+x^4}}{1+3x^4+x^8}\,dx = \int \left(\frac{(1-\sqrt{5})\sqrt{1+x^4}}{3-\sqrt{5}+2x^4}+\frac{(1+\sqrt{5})\sqrt{1+x^4}}{3+\sqrt{5}+2x^4}\right)dx$$

$$= (1-\sqrt{5})\int \frac{\sqrt{1+x^4}}{3-\sqrt{5}+2x^4}\,dx + (1+\sqrt{5})\int \frac{\sqrt{1+x^4}}{3+\sqrt{5}+2x^4}\,dx$$

$$= (-3-\sqrt{5})\int \frac{1}{\sqrt{1+x^4}\,(3+\sqrt{5}+2x^4)}\,dx + \frac{1}{2}(1-\sqrt{5})\int \frac{1}{\sqrt{1+x^4}}\,dx + (-3+$$

$$= \frac{(1-\sqrt{5})(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{1+x^4}} + \frac{(1+\sqrt{5})(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan\right.}{4\sqrt{1+x^4}}$$

$$= \frac{(1-\sqrt{5})(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{1+x^4}} + \frac{(1+\sqrt{5})(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan\right.}{4\sqrt{1+x^4}}$$

$$= \frac{1}{4}(-1)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{-1}\,x}{\sqrt{1+x^4}}\right) - \frac{\sqrt[4]{-1}\left(2i-\sqrt{6-2\sqrt{5}}\right)\left(2i-\sqrt{2(3+\sqrt{5})}\right)\tan^{-1}\left(\right.}{16\sqrt{5}}$$

Mathematica [C] time = 0.55, size = 146, normalized size = 1.95

$$\frac{1}{2}\sqrt[4]{-1}\left(-2F\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)+\Pi\left(-\sqrt{\frac{2}{3+\sqrt{5}}};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)+\Pi\left(\sqrt{\frac{2}{3+\sqrt{5}}};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)+\Pi\left(-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)+\Pi\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)*Sqrt[1 + x^4])/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((-1)^(1/4)*(-2*EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] + EllipticPi[-Sqrt[2
/(3 + Sqrt[5])], I*ArcSinh[(-1)^(1/4)*x], -1] + EllipticPi[Sqrt[2/(3 + Sqrt
[5])], I*ArcSinh[(-1)^(1/4)*x], -1] + EllipticPi[-Sqrt[(3 + Sqrt[5])/2], I*
ArcSinh[(-1)^(1/4)*x], -1] + EllipticPi[Sqrt[(3 + Sqrt[5])/2], I*ArcSinh[(-
1)^(1/4)*x], -1]))/2
```

IntegrateAlgebraic [A] time = 0.31, size = 75, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x\sqrt{x^4+1}}{x^4-x^2+1}\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt{x^4+1}}{x^4+x^2+1}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*Sqrt[1 + x^4])/(1 + 3*x^4 + x^8),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2]*x*Sqrt[1 + x^4])/(1 - x^2 + x^4)]/Sqrt[2] - ArcTanh[(S
qrt[2]*x*Sqrt[1 + x^4])/(1 + x^2 + x^4)]/(2*Sqrt[2])
```

fricas [B] time = 0.59, size = 430, normalized size = 5.73

$$\frac{1}{4}\sqrt{2}\arctan\left(-\frac{x^8+3x^4+2\sqrt{2}\left(x^5-x^3+x\right)\sqrt{x^4+1}-\left(4\sqrt{x^4+1}x^3+\sqrt{2}\left(x^8-2x^6+x^4-2x^2+1\right)\right)\sqrt{\frac{x^8+4x^6+3x^4+2\sqrt{2}\left(x^5+x^3+x\right)\sqrt{x^4+1}+4x^2+1}{x^8+3x^4+1}}+1}{x^8-4x^6+3x^4-4x^2+1}\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{x^8+3x^4-2\sqrt{2}\left(x^5-x^3+x\right)\sqrt{x^4+1}-\left(4\sqrt{x^4+1}x^3-\sqrt{2}\left(x^8-2x^6+x^4-2x^2+1\right)\right)\sqrt{\frac{x^8+4x^6+3x^4-2\sqrt{2}\left(x^5+x^3+x\right)\sqrt{x^4+1}+4x^2+1}{x^8+3x^4+1}}+1}{x^8-4x^6+3x^4-4x^2+1}\right)-\frac{1}{16}\sqrt{2}\log\left(\frac{4\left(x^8+4x^6+3x^4+2\sqrt{2}\left(x^5+x^3+x\right)\sqrt{x^4+1}+4x^2+1\right)}{x^8+3x^4+1}\right)+\frac{1}{16}\sqrt{2}\log\left(\frac{4\left(x^8+4x^6+3x^4-2\sqrt{2}\left(x^5+x^3+x\right)\sqrt{x^4+1}+4x^2+1\right)}{x^8+3x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/2)/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(-(x^8 + 3*x^4 + 2*sqrt(2)*(x^5 - x^3 + x)*sqrt(x^4 + 1)
- (4*sqrt(x^4 + 1)*x^3 + sqrt(2)*(x^8 - 2*x^6 + x^4 - 2*x^2 + 1))*sqrt((x^8
 + 4*x^6 + 3*x^4 + 2*sqrt(2)*(x^5 + x^3 + x)*sqrt(x^4 + 1) + 4*x^2 + 1)/(x^
8 + 3*x^4 + 1)) + 1)/(x^8 - 4*x^6 + 3*x^4 - 4*x^2 + 1)) - 1/4*sqrt(2)*arcta
n(-(x^8 + 3*x^4 - 2*sqrt(2)*(x^5 - x^3 + x)*sqrt(x^4 + 1) - (4*sqrt(x^4 + 1
)*x^3 - sqrt(2)*(x^8 - 2*x^6 + x^4 - 2*x^2 + 1))*sqrt((x^8 + 4*x^6 + 3*x^4
- 2*sqrt(2)*(x^5 + x^3 + x)*sqrt(x^4 + 1) + 4*x^2 + 1)/(x^8 + 3*x^4 + 1)) +
 1)/(x^8 - 4*x^6 + 3*x^4 - 4*x^2 + 1)) - 1/16*sqrt(2)*log(4*(x^8 + 4*x^6 +
3*x^4 + 2*sqrt(2)*(x^5 + x^3 + x)*sqrt(x^4 + 1) + 4*x^2 + 1)/(x^8 + 3*x^4 +
 1)) + 1/16*sqrt(2)*log(4*(x^8 + 4*x^6 + 3*x^4 - 2*sqrt(2)*(x^5 + x^3 + x)*
sqrt(x^4 + 1) + 4*x^2 + 1)/(x^8 + 3*x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}\left(x^4-1\right)}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/2)/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 1)*(x^4 - 1)/(x^8 + 3*x^4 + 1), x)
```

maple [A] time = 0.03, size = 110, normalized size = 1.47

$$-\frac{\sqrt{2}\,\ln\left(\frac{x^4+1}{x^2}+\frac{\sqrt{2}\,\sqrt{x^4+1}}{x}+1\right)}{8}+\frac{\sqrt{2}\,\arctan\left(1+\frac{\sqrt{2}\,\sqrt{x^4+1}}{x}\right)}{4}+\frac{\sqrt{2}\,\ln\left(\frac{x^4+1}{x^2}-\frac{\sqrt{2}\,\sqrt{x^4+1}}{x}+1\right)}{8}-\frac{\sqrt{2}\,\arctan\left(1-\frac{\sqrt{2}\,\sqrt{x^4+1}}{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^4+1)^(1/2)/(x^8+3*x^4+1),x)
```

```
[Out] -1/8*2^(1/2)*ln((x^4+1)/x^2+2^(1/2)/x*(x^4+1)^(1/2)+1)+1/4*2^(1/2)*arctan(1
+2^(1/2)/x*(x^4+1)^(1/2))+1/8*2^(1/2)*ln((x^4+1)/x^2-2^(1/2)/x*(x^4+1)^(1/2
)+1)-1/4*2^(1/2)*arctan(1-2^(1/2)/x*(x^4+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}\,\left(x^4-1\right)}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/2)/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 1)*(x^4 - 1)/(x^8 + 3*x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-1\right)\,\sqrt{x^4+1}}{x^8+3\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(x^4 + 1)^(1/2))/(3*x^4 + x^8 + 1),x)
```

```
[Out] int(((x^4 - 1)*(x^4 + 1)^(1/2))/(3*x^4 + x^8 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)\left(x^2+1\right)\sqrt{x^4+1}}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**4+1)**(1/2)/(x**8+3*x**4+1),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)*sqrt(x**4 + 1)/(x**8 + 3*x**4 + 1), x)
```

3.915 $\int \frac{-1+k^2x^2}{\sqrt{(1-x)x(1-k^2x)}\left(a+bx+ak^2x^2\right)}\,dx$

Optimal. Leaf size=76

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{k^2x^3+(-k^2-1)x^2+x}\,\sqrt{ak^2+a+b}}{\sqrt{a}\,(x-1)(k^2x-1)}\right)}{\sqrt{a}\,\sqrt{ak^2+a+b}}$$

Rubi [C] time = 3.22, antiderivative size = 299, normalized size of antiderivative = 3.93, number of steps used = 16, number of rules used = 9, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.204, Rules used = {6718, 6688, 6728, 714, 115, 934, 12, 168, 537}

$$\frac{2(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,\Pi\left(-\frac{2a}{b-\sqrt{b^2-4a^2k^2}};\sin^{-1}\left(\sqrt{-k^2}\,\sqrt{-x}\right)|\frac{1}{k^2}\right)}{a\sqrt{-k^2}\,\sqrt{x-x^2}\,\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{2(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,\Pi\left(-\frac{2a}{b+\sqrt{b^2-4a^2k^2}};\sin^{-1}\left(\sqrt{-k^2}\,\sqrt{-x}\right)|\frac{1}{k^2}\right)}{a\sqrt{-k^2}\,\sqrt{x-x^2}\,\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{a\sqrt{(1-x)x\left(1-k^2x\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(a + b*x + a*k^2*x^2)),x]
```

```
[Out] (2*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticF[ArcSin[Sqrt[x]], k^2])/(a*
Sqrt[(1 - x)*x*(1 - k^2*x)]) + (2*(1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^2*x]*
EllipticPi[(-2*a)/(b - Sqrt[b^2 - 4*a^2*k^2]), ArcSin[Sqrt[-k^2]*Sqrt[-x]],
 k^(-2)])/(a*Sqrt[-k^2]*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2]) + (2*(1
- x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticPi[(-2*a)/(b + Sqrt[b^2 - 4*a
^2*k^2]), ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/(a*Sqrt[-k^2]*Sqrt[(1 - x)*
x*(1 - k^2*x)]*Sqrt[x - x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 714

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c*x)/b]), x] /; FreeQ[{b, c, d, e}, x
```

```
] && NeQ[c*d - b*e, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0]
 && RationalQ[b]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-1+k^2x^2}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(a+bx+ak^2x^2\right)}\,dx = \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{-1+k^2x^2}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,\left(a+bx+ak^2x^2\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{-1+k^2x^2}{\sqrt{1-k^2x}\,\sqrt{x-x^2}\,\left(a+bx+ak^2x^2\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \left(\frac{1}{a\sqrt{1-k^2x}\,\sqrt{x-x^2}} - \frac{2a+bx}{a\sqrt{1-k^2x}\,\sqrt{x-x^2}\,\left(a+bx+ak^2x^2\right)}\right.}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-k^2x}\,\sqrt{x-x^2}}\,dx}{a\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)}{a\sqrt{(1-}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}}\,dx}{a\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right.}{}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{a\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\left(b-\sqrt{b^2-4a^2k^2}\right)\sqrt{1}\right.}{}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{a\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{2}\left(b-\sqrt{b^2-4a^2k^2}\right)\right.}{}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{a\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\left(b-\sqrt{b^2-4a^2k^2}\right)\sqrt{2}\right.}{}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{a\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\left(b-\sqrt{b^2-4a^2k^2}\right)\right.}{}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{a\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{2(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-k}}{a\sqrt{-k^2}}$$

Mathematica [C] time = 6.04, size = 345, normalized size = 4.54

$$\frac{i\sqrt{\frac{1}{x-1}+1}\,(x-1)^{3/2}\sqrt{\frac{1-\frac{1}{k^2}}{x-1}+1}\left(2a\left(k^2-1\right)\sqrt{b^2-4a^2k^2}\,F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+\left(b\sqrt{b^2-4a^2k^2}+2a\left(\sqrt{b^2-4a^2k^2}-2ak^2\right)+b^2\right)\Pi\left(\frac{2\left(ak^2+a+b\right)}{2ak^2+b-\sqrt{b^2-4a^2k^2}};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+\left(b\sqrt{b^2-4a^2k^2}+2a\left(\sqrt{b^2-4a^2k^2}+2ak^2\right)-b^2\right)\Pi\left(\frac{2\left(ak^2+a+b\right)}{2ak^2+b+\sqrt{b^2-4a^2k^2}};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)}{a\sqrt{(x-1)x\left(k^2x-1\right)}\left(ak^2+a+b\right)\sqrt{b^2-4a^2k^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(a + b*x + a*k^2*x^2)
),x]
```

```
[Out] (I*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (1 - k^(-2))/(-1 + x)]*(-1 + x)^(3/2)*(
2*a*(-1 + k^2)*Sqrt[b^2 - 4*a^2*k^2]*EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], 1
 - k^(-2)] + (b^2 + b*Sqrt[b^2 - 4*a^2*k^2] + 2*a*(-2*a*k^2 + Sqrt[b^2 - 4*
a^2*k^2]))*EllipticPi[(2*(a + b + a*k^2))/(b + 2*a*k^2 - Sqrt[b^2 - 4*a^2*k
```

```
^2]), I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] + (-b^2 + b*Sqrt[b^2 - 4*a^2*k
^2] + 2*a*(2*a*k^2 + Sqrt[b^2 - 4*a^2*k^2]))*EllipticPi[(2*(a + b + a*k^2))
/(b + 2*a*k^2 + Sqrt[b^2 - 4*a^2*k^2]), I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-
2)]))/(a*(a + b + a*k^2)*Sqrt[b^2 - 4*a^2*k^2]*Sqrt[(-1 + x)*x*(-1 + k^2*x)
])
```

IntegrateAlgebraic [A] time = 0.20, size = 76, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}\,\sqrt{ak^2+a+b}}{\sqrt{a}\,(x-1)\left(k^2x-1\right)}\right)}{\sqrt{a}\,\sqrt{ak^2+a+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(a + b*x + a
*k^2*x^2)),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a + b + a*k^2]*Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3])/(Sqrt[a
]*(-1 + x)*(-1 + k^2*x))])/(Sqrt[a]*Sqrt[a + b + a*k^2])
```

fricas [B] time = 0.75, size = 413, normalized size = 5.43

$$\left[-\frac{\sqrt{-a^2k^2-a^2-ab}\,\log\left(\frac{a^2k^4x^4-2\left(4a^2k^4+\left(4a^2+3ab\right)k^2\right)x^3+\left(8a^2k^4+2\left(9a^2+4ab\right)k^2+8a^2+8ab+b^2\right)x^2-4\left(ak^2x^2-\left(2ak^2+2a+b\right)x+a\right)\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\,\sqrt{-a^2k^2-a^2-ab}+a^2-2\left(4a^2k^2+4a^2+3ab\right)x}{a^2k^4x^4+2abk^2x^3+2abx+\left(2a^2k^2+b^2\right)x^2+a^2}\right)}{2\left(a^2k^2+a^2+ab\right)}, \frac{\arctan\left(\frac{\left(ak^2x^2-\left(2ak^2+2a+b\right)x+a\right)\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\,\sqrt{a^2k^2+a^2+ab}}{2\left(\left(a^2k^4+\left(a^2+ab\right)k^2\right)x^3-\left(a^2k^4+\left(2a^2+ab\right)k^2+a^2+ab\right)x^2+\left(a^2k^2+a^2+ab\right)x\right)}\right)}{\sqrt{a^2k^2+a^2+ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(a*k^2*x^2+b*x+a),x, algor
ithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2*k^2 - a^2 - a*b)*log((a^2*k^4*x^4 - 2*(4*a^2*k^4 + (4*a^2 +
 3*a*b)*k^2)*x^3 + (8*a^2*k^4 + 2*(9*a^2 + 4*a*b)*k^2 + 8*a^2 + 8*a*b + b^2
)*x^2 - 4*(a*k^2*x^2 - (2*a*k^2 + 2*a + b)*x + a)*sqrt(k^2*x^3 - (k^2 + 1)*
x^2 + x)*sqrt(-a^2*k^2 - a^2 - a*b) + a^2 - 2*(4*a^2*k^2 + 4*a^2 + 3*a*b)*x
)/(a^2*k^4*x^4 + 2*a*b*k^2*x^3 + 2*a*b*x + (2*a^2*k^2 + b^2)*x^2 + a^2))/(a
^2*k^2 + a^2 + a*b), arctan(1/2*(a*k^2*x^2 - (2*a*k^2 + 2*a + b)*x + a)*sqr
t(k^2*x^3 - (k^2 + 1)*x^2 + x)*sqrt(a^2*k^2 + a^2 + a*b)/((a^2*k^4 + (a^2 +
 a*b)*k^2)*x^3 - (a^2*k^4 + (2*a^2 + a*b)*k^2 + a^2 + a*b)*x^2 + (a^2*k^2 +
 a^2 + a*b)*x))/sqrt(a^2*k^2 + a^2 + a*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^2-1}{\left(ak^2x^2+bx+a\right)\sqrt{\left(k^2x-1\right)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(a*k^2*x^2+b*x+a),x, algor
ithm="giac")
```

```
[Out] integrate((k^2*x^2 - 1)/((a*k^2*x^2 + b*x + a)*sqrt((k^2*x - 1)*(x - 1)*x))
, x)
```

maple [C] time = 0.07, size = 1178, normalized size = 15.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^2-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(a*k^2*x^2+b*x+a),x)
```

```
[Out] -2/a/k^2*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2
*x^3-k^2*x^2-x^2+x)^(1/2)*EllipticF((-(x-1/k^2)*k^2)^(1/2),(1/k^2/(1/k^2-1)
```

```
)^(1/2))+1/a*(-1/(-4*a^2*k^2+b^2)^(1/2)/k^4*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x
-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/2*
b/a/k^2-1/2/a/k^2*(-4*a^2*k^2+b^2)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2)
,1/k^2/(1/k^2-1/2/a/k^2*(-b+(-4*a^2*k^2+b^2)^(1/2))),(1/k^2/(1/k^2-1))^(1/2
))*b^2/a+1/k^4*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(
1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/2*b/a/k^2-1/2/a/k^2*(-4*a^2*k^2
+b^2)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-1/2/a/k^2*(-b+(
-4*a^2*k^2+b^2)^(1/2))),(1/k^2/(1/k^2-1))^(1/2))*b/a+4/(-4*a^2*k^2+b^2)^(1/
2)/k^2*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^
2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/2*b/a/k^2-1/2/a/k^2*(-4*a^2*k^2+b^2)^(1
/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-1/2/a/k^2*(-b+(-4*a^2*k
^2+b^2)^(1/2))),(1/k^2/(1/k^2-1))^(1/2))*a+1/(-4*a^2*k^2+b^2)^(1/2)/k^4*(-k
^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*
x^2-x^2+x)^(1/2)/(1/k^2+1/2*b/a/k^2+1/2/a/k^2*(-4*a^2*k^2+b^2)^(1/2))*Ellip
ticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+1/2*(b+(-4*a^2*k^2+b^2)^(1/2))/a/
k^2),(1/k^2/(1/k^2-1))^(1/2))*b^2/a+1/k^4*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1
/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/2*b/
a/k^2+1/2/a/k^2*(-4*a^2*k^2+b^2)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1
/k^2/(1/k^2+1/2*(b+(-4*a^2*k^2+b^2)^(1/2))/a/k^2),(1/k^2/(1/k^2-1))^(1/2))*
b/a-4/(-4*a^2*k^2+b^2)^(1/2)/k^2*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1
))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/2*b/a/k^2+1/2
/a/k^2*(-4*a^2*k^2+b^2)^(1/2))*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k
^2+1/2*(b+(-4*a^2*k^2+b^2)^(1/2))/a/k^2),(1/k^2/(1/k^2-1))^(1/2))*a)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(a*k^2*x^2+b*x+a),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*a*k-b>0)', see `assume?` for more
 details)Is 2*a*k-b positive, negative or zero?
```

mupad [**B**] time = 3.76, size = 90, normalized size = 1.18

$$\frac{\ln\left(\frac{a-2\,a\,x-b\,x-2\,a\,k^2\,x+a\,k^2\,x^2+\sqrt{a\,\left(a\,k^2+a+b\right)}\,\sqrt{x\,\left(k^2\,x-1\right)\,\left(x-1\right)}\,2{}\mathrm{i}}{a\,k^2\,x^2+b\,x+a}\right)\,1{}\mathrm{i}}{\sqrt{a^2\,k^2+a^2+b\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^2 - 1)/((a + b*x + a*k^2*x^2)*(x*(k^2*x - 1)*(x - 1))^(1/2)),x)
```

```
[Out] (log((a - 2*a*x - b*x + (a*(a + b + a*k^2))^(1/2)*(x*(k^2*x - 1)*(x - 1))^(
1/2)*2i - 2*a*k^2*x + a*k^2*x^2)/(a + b*x + a*k^2*x^2))*1i)/(a*b + a^2 + a^
2*k^2)^(1/2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k**2*x**2-1)/((1-x)*x*(-k**2*x+1))**(1/2)/(a*k**2*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.916 $\int \frac{1}{x\sqrt[3]{-1+x^4}}\,dx$

Optimal. Leaf size=76

$$-\frac{1}{4}\log\left(\sqrt[3]{x^4-1}+1\right)+\frac{1}{8}\log\left(\left(x^4-1\right)^{2/3}-\sqrt[3]{x^4-1}+1\right)-\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^4-1}}{\sqrt{3}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 52, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {266, 56, 618, 204, 31}

$$-\frac{3}{8}\log\left(\sqrt[3]{x^4-1}+1\right)-\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{x^4-1}}{\sqrt{3}}\right)+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-1 + x^4)^(1/3)),x]
```

```
[Out] -1/4*(Sqrt[3]*ArcTan[(1 - 2*(-1 + x^4)^(1/3))/Sqrt[3]]) + Log[x]/2 - (3*Log
[1 + (-1 + x^4)^(1/3)])/8
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-1+x^4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x}\,dx, x, x^4\right)$$
$$= \frac{\log(x)}{2} - \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[3]{-1+x^4}\right) + \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[3]{-1+x^4}\right)$$
$$= \frac{\log(x)}{2} - \frac{3}{8}\log\left(1+\sqrt[3]{-1+x^4}\right) - \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[3]{-1+x^4}\right)$$
$$= -\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^4}}{\sqrt{3}}\right) + \frac{\log(x)}{2} - \frac{3}{8}\log\left(1+\sqrt[3]{-1+x^4}\right)$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.37

$$\frac{3}{8}\left(x^4-1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-1 + x^4)^(1/3)),x]
```

```
[Out] (3*(-1 + x^4)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, 1 - x^4])/8
```

IntegrateAlgebraic [A] time = 0.05, size = 76, normalized size = 1.00

$$-\frac{1}{4}\log\left(\sqrt[3]{x^4-1}+1\right) + \frac{1}{8}\log\left(\left(x^4-1\right)^{2/3} - \sqrt[3]{x^4-1}+1\right) - \frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{x^4-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-1 + x^4)^(1/3)),x]
```

```
[Out] -1/4*(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(-1 + x^4)^(1/3))/Sqrt[3]]) - Log[1 + (
-1 + x^4)^(1/3)]/4 + Log[1 - (-1 + x^4)^(1/3) + (-1 + x^4)^(2/3)]/8
```

fricas [A] time = 0.44, size = 58, normalized size = 0.76

$$\frac{1}{4}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^4-1\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) + \frac{1}{8}\log\left(\left(x^4-1\right)^{\frac{2}{3}} - \left(x^4-1\right)^{\frac{1}{3}}+1\right) - \frac{1}{4}\log\left(\left(x^4-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*arctan(2/3*sqrt(3)*(x^4 - 1)^(1/3) - 1/3*sqrt(3)) + 1/8*log((x^
4 - 1)^(2/3) - (x^4 - 1)^(1/3) + 1) - 1/4*log((x^4 - 1)^(1/3) + 1)
```

giac [A] time = 0.45, size = 57, normalized size = 0.75

$$\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4-1\right)^{\frac{1}{3}}-1\right)\right) + \frac{1}{8}\log\left(\left(x^4-1\right)^{\frac{2}{3}} - \left(x^4-1\right)^{\frac{1}{3}}+1\right) - \frac{1}{4}\log\left(\left|\left(x^4-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 - 1)^(1/3) - 1)) + 1/8*log((x^4 - 1)
^(2/3) - (x^4 - 1)^(1/3) + 1) - 1/4*log(abs((x^4 - 1)^(1/3) + 1))
```

maple [C] time = 0.26, size = 83, normalized size = 1.09

$$\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\mathrm{signum}\left(x^{4}-1\right)\right)^{\frac{1}{3}}\left(\frac{2\pi\sqrt{3}\,x^{4}\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^{4}\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+4\ln(x)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{8\pi\mathrm{signum}\left(x^{4}-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4-1)^(1/3),x)
```

```
[Out] 1/8/Pi*3^(1/2)*GAMMA(2/3)/signum(x^4-1)^(1/3)*(-signum(x^4-1))^(1/3)*(2/9*P
i*3^(1/2)/GAMMA(2/3)*x^4*hypergeom([1,1,4/3],[2,2],x^4)+2/3*(-1/6*Pi*3^(1/2
)-3/2*ln(3)+4*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.42, size = 56, normalized size = 0.74

$$\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^{4}-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{1}{8}\log\left(\left(x^{4}-1\right)^{\frac{2}{3}}-\left(x^{4}-1\right)^{\frac{1}{3}}+1\right)-\frac{1}{4}\log\left(\left(x^{4}-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 - 1)^(1/3) - 1)) + 1/8*log((x^4 - 1)
^(2/3) - (x^4 - 1)^(1/3) + 1) - 1/4*log((x^4 - 1)^(1/3) + 1)
```

mupad [B] time = 0.98, size = 80, normalized size = 1.05

$$-\frac{\ln\left(\frac{9\left(x^{4}-1\right)^{1/3}}{16}+\frac{9}{16}\right)}{4}-\ln\left(9\left(-\frac{1}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8}\right)^{2}+\frac{9\left(x^{4}-1\right)^{1/3}}{16}\right)\left(-\frac{1}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8}\right)+\ln\left(9\left(\frac{1}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8}\right)^{2}+\frac{9\left(x^{4}-1\right)^{1/3}}{16}\right)\left(\frac{1}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^4 - 1)^(1/3)),x)
```

```
[Out] log(9*((3^(1/2)*1i)/8 + 1/8)^2 + (9*(x^4 - 1)^(1/3))/16)*((3^(1/2)*1i)/8 +
1/8) - log(9*((3^(1/2)*1i)/8 - 1/8)^2 + (9*(x^4 - 1)^(1/3))/16)*((3^(1/2)*1
i)/8 - 1/8) - log((9*(x^4 - 1)^(1/3))/16 + 9/16)/4
```

sympy [C] time = 0.81, size = 34, normalized size = 0.45

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^{4}}\right)}{4x^{\frac{4}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4-1)**(1/3),x)
```

```
[Out] -gamma(1/3)*hyper((1/3, 1/3), (4/3,), exp_polar(2*I*pi)/x**4)/(4*x**(4/3)*g
amma(4/3))
```

3.917 $\int \frac{x^2(-2b+ax^2)}{(-b+ax^2)^{3/4}(4b-4ax^2+x^4)}\,dx$

Optimal. Leaf size=76

$$\frac{\tanh^{-1}\left(\frac{x(ax^2-b)^{3/4}}{\sqrt{2}(b-ax^2)}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{x(ax^2-b)^{3/4}}{\sqrt{2}(b-ax^2)}\right)}{\sqrt{2}}$$

Rubi [C] time = 26.56, antiderivative size = 2421, normalized size of antiderivative = 31.86, number of steps used = 24, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {1692, 234, 220, 401, 108, 409, 1217, 1707}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(x^2*(-2*b + a*x^2))/((-b + a*x^2)^(3/4)*(4*b - 4*a*x^2 + x^4)),x]
```

```
[Out] -1/2*(Sqrt[b]*(4*a^4 - a*(4*a^2 - 3*b)*Sqrt[a^2 - b] - 5*a^2*b + b^2)*Sqrt[
(a*x^2)/b]*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[a - Sqrt[a^2 - b]]*(-b + a*x^2)^(1/
4))/((2*a^2 - 2*a*Sqrt[a^2 - b] - b)^(1/4)*Sqrt[b]*Sqrt[(a*x^2)/b])])/(Sqrt
[2]*Sqrt[a]*Sqrt[a - Sqrt[a^2 - b]]*(2*a^2 - 2*a*Sqrt[a^2 - b] - b)^(3/4)*(
a^2 - a*Sqrt[a^2 - b] - b)*x) - (Sqrt[b]*(4*a^4 - a*(4*a^2 - 3*b)*Sqrt[a^2
- b] - 5*a^2*b + b^2)*Sqrt[(a*x^2)/b]*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[-a + Sqr
t[a^2 - b]]*(-b + a*x^2)^(1/4))/((2*a^2 - 2*a*Sqrt[a^2 - b] - b)^(1/4)*Sqrt
[b]*Sqrt[(a*x^2)/b])])/(2*Sqrt[2]*Sqrt[a]*Sqrt[-a + Sqrt[a^2 - b]]*(2*a^2 -
 2*a*Sqrt[a^2 - b] - b)^(3/4)*(a^2 - a*Sqrt[a^2 - b] - b)*x) - ((2*a^2 + 2*
a*Sqrt[a^2 - b] - b)^(1/4)*Sqrt[b]*Sqrt[(a*x^2)/b]*ArcTan[(Sqrt[2]*Sqrt[a]*
Sqrt[-a - Sqrt[a^2 - b]]*(-b + a*x^2)^(1/4))/((2*a^2 + 2*a*Sqrt[a^2 - b] -
b)^(1/4)*Sqrt[b]*Sqrt[(a*x^2)/b])])/(2*Sqrt[2]*Sqrt[a]*Sqrt[-a - Sqrt[a^2 -
 b]]*x) - ((2*a^2 + 2*a*Sqrt[a^2 - b] - b)^(1/4)*Sqrt[b]*Sqrt[(a*x^2)/b]*Ar
cTan[(Sqrt[2]*Sqrt[a]*Sqrt[a + Sqrt[a^2 - b]]*(-b + a*x^2)^(1/4))/((2*a^2 +
 2*a*Sqrt[a^2 - b] - b)^(1/4)*Sqrt[b]*Sqrt[(a*x^2)/b])])/(2*Sqrt[2]*Sqrt[a]
*Sqrt[a + Sqrt[a^2 - b]]*x) + (Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[-b + a*x^2])^2]
*(Sqrt[b] + Sqrt[-b + a*x^2])*EllipticF[2*ArcTan[(-b + a*x^2)^(1/4)/b^(1/4)
], 1/2])/(b^(1/4)*x) - ((1 - Sqrt[b]/Sqrt[2*a^2 - 2*a*Sqrt[a^2 - b] - b])*(
2*a^2 - 2*a*Sqrt[a^2 - b] - b)*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[-b + a*x^2])^2]
*(Sqrt[b] + Sqrt[-b + a*x^2])*EllipticF[2*ArcTan[(-b + a*x^2)^(1/4)/b^(1/4)
], 1/2])/(4*(a^2 - a*Sqrt[a^2 - b] - b)*b^(1/4)*x) - ((1 + Sqrt[b]/Sqrt[2*a
^2 - 2*a*Sqrt[a^2 - b] - b])*(2*a^2 - 2*a*Sqrt[a^2 - b] - b)*Sqrt[(a*x^2)/(
Sqrt[b] + Sqrt[-b + a*x^2])^2]*(Sqrt[b] + Sqrt[-b + a*x^2])*EllipticF[2*Arc
Tan[(-b + a*x^2)^(1/4)/b^(1/4)], 1/2])/(4*(a^2 - a*Sqrt[a^2 - b] - b)*b^(1/
4)*x) - ((1 - Sqrt[b]/Sqrt[2*a^2 + 2*a*Sqrt[a^2 - b] - b])*(2*a^2 + 2*a*Sqr
t[a^2 - b] - b)*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[-b + a*x^2])^2]*(Sqrt[b] + Sqr
t[-b + a*x^2])*EllipticF[2*ArcTan[(-b + a*x^2)^(1/4)/b^(1/4)], 1/2])/(4*(a^
2 + a*Sqrt[a^2 - b] - b)*b^(1/4)*x) - ((1 + Sqrt[b]/Sqrt[2*a^2 + 2*a*Sqrt[a
^2 - b] - b])*(2*a^2 + 2*a*Sqrt[a^2 - b] - b)*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[
-b + a*x^2])^2]*(Sqrt[b] + Sqrt[-b + a*x^2])*EllipticF[2*ArcTan[(-b + a*x^2
)^(1/4)/b^(1/4)], 1/2])/(4*(a^2 + a*Sqrt[a^2 - b] - b)*b^(1/4)*x) + ((Sqrt[
2*a^2 - 2*a*Sqrt[a^2 - b] - b] + Sqrt[b])^2*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[-b
 + a*x^2])^2]*(Sqrt[b] + Sqrt[-b + a*x^2])*EllipticPi[-1/4*(Sqrt[2*a^2 - 2*
a*Sqrt[a^2 - b] - b] - Sqrt[b])^2/(Sqrt[2*a^2 - 2*a*Sqrt[a^2 - b] - b]*Sqrt
[b]), 2*ArcTan[(-b + a*x^2)^(1/4)/b^(1/4)], 1/2])/(8*(a^2 - a*Sqrt[a^2 - b]
 - b)*b^(1/4)*x) + ((Sqrt[2*a^2 - 2*a*Sqrt[a^2 - b] - b] - Sqrt[b])^2*Sqrt[
(a*x^2)/(Sqrt[b] + Sqrt[-b + a*x^2])^2]*(Sqrt[b] + Sqrt[-b + a*x^2])*Ellipt
icPi[(Sqrt[2*a^2 - 2*a*Sqrt[a^2 - b] - b] + Sqrt[b])^2/(4*Sqrt[2*a^2 - 2*a*
Sqrt[a^2 - b] - b]*Sqrt[b]), 2*ArcTan[(-b + a*x^2)^(1/4)/b^(1/4)], 1/2])/(8
*(a^2 - a*Sqrt[a^2 - b] - b)*b^(1/4)*x) + ((Sqrt[2*a^2 + 2*a*Sqrt[a^2 - b]
- b] + Sqrt[b])^2*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[-b + a*x^2])^2]*(Sqrt[b] + S
```

```
qrt[-b + a*x^2])*EllipticPi[-1/4*(Sqrt[2*a^2 + 2*a*Sqrt[a^2 - b] - b] - Sqr
t[b])^2/(Sqrt[2*a^2 + 2*a*Sqrt[a^2 - b] - b]*Sqrt[b]), 2*ArcTan[(-b + a*x^2
)^(1/4)/b^(1/4)], 1/2])/(8*(a^2 + a*Sqrt[a^2 - b] - b)*b^(1/4)*x) + ((Sqrt[
2*a^2 + 2*a*Sqrt[a^2 - b] - b] - Sqrt[b])^2*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[-b
 + a*x^2])^2]*(Sqrt[b] + Sqrt[-b + a*x^2])*EllipticPi[(Sqrt[2*a^2 + 2*a*Sqr
t[a^2 - b] - b] + Sqrt[b])^2/(4*Sqrt[2*a^2 + 2*a*Sqrt[a^2 - b] - b]*Sqrt[b]
), 2*ArcTan[(-b + a*x^2)^(1/4)/b^(1/4)], 1/2])/(8*(a^2 + a*Sqrt[a^2 - b] -
b)*b^(1/4)*x)
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(2*Sqrt[-((b*x^2)/a)])/
(b*x), Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[
{a, b}, x] && NegQ[a]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
```

```
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2\left(-2b+ax^2\right)}{\left(-b+ax^2\right)^{3/4}\left(4b-4ax^2+x^4\right)}\,dx = \int \left(\frac{a}{\left(-b+ax^2\right)^{3/4}} - \frac{2\left(2ab-\left(2a^2-b\right)x^2\right)}{\left(-b+ax^2\right)^{3/4}\left(4b-4ax^2+x^4\right)}\right)dx$$

$$= -\left(2\int \frac{2ab-\left(2a^2-b\right)x^2}{\left(-b+ax^2\right)^{3/4}\left(4b-4ax^2+x^4\right)}\,dx\right) + a\int \frac{1}{\left(-b+ax^2\right)^{3/4}}\,dx$$

$$= -\left(2\int\left(\frac{-2a^2-2a\sqrt{a^2-b}+b}{\left(-4a-4\sqrt{a^2-b}+2x^2\right)\left(-b+ax^2\right)^{3/4}} + \frac{-2a^2+2a\sqrt{a^2}\cdots}{\left(-4a+4\sqrt{a^2-b}+2x\cdots\right.}\right.\right.$$

$$= \frac{\sqrt{\frac{ax^2}{\left(\sqrt{b}+\sqrt{-b+ax^2}\right)^2}}\left(\sqrt{b}+\sqrt{-b+ax^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\,x} + \left(2\left(2a^2\cdots\right.\right.$$

$$= \frac{\sqrt{\frac{ax^2}{\left(\sqrt{b}+\sqrt{-b+ax^2}\right)^2}}\left(\sqrt{b}+\sqrt{-b+ax^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\,x} + \frac{\left(\left(2a^2-\cdots\right.\right.}{\cdots}$$

$$= \frac{\sqrt{\frac{ax^2}{\left(\sqrt{b}+\sqrt{-b+ax^2}\right)^2}}\left(\sqrt{b}+\sqrt{-b+ax^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\,x} - \frac{\left(4\left(2a^2\cdots\right.\right.}{\cdots}$$

$$= \frac{\sqrt{\frac{ax^2}{\left(\sqrt{b}+\sqrt{-b+ax^2}\right)^2}}\left(\sqrt{b}+\sqrt{-b+ax^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\,x} - \frac{\sqrt{\frac{ax^2}{b}}\ \mathrm{S}\cdots}{\cdots}$$

$$= \frac{\sqrt{\frac{ax^2}{\left(\sqrt{b}+\sqrt{-b+ax^2}\right)^2}}\left(\sqrt{b}+\sqrt{-b+ax^2}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\,x} - \frac{\left(\left(\frac{\cdots}{\sqrt{2a^2+\cdots}}\right.\right.}{\cdots}$$

$$= -\frac{\left(\sqrt{2a^2-2a\sqrt{a^2-b}-b}-\sqrt{b}\right)\left(\sqrt{2a^2-2a\sqrt{a^2-b}-b}+\sqrt{b}\right)\sqrt[4]{2a^2-\cdots}}{4\sqrt{2}\sqrt{a}\sqrt{a-\sqrt{a^2-b}}\left(a^2-\cdots\right.}$$

Mathematica [C] time = 0.60, size = 360, normalized size = 4.74

$$-\frac{\sqrt[4]{-b}\sqrt{\frac{ax^2}{b}}\,\Pi\left(-\frac{\sqrt{b}}{\sqrt{-2a^2-2\sqrt{a^2-b}\,a+b}};\sin^{-1}\left(\frac{\sqrt[4]{ax^2-b}}{\sqrt[4]{-b}}\right)\middle|-1\right)+\sqrt[4]{-b}\sqrt{\frac{ax^2}{b}}\,\Pi\left(\frac{\sqrt{b}}{\sqrt{-2a^2-2\sqrt{a^2-b}\,a+b}};\sin^{-1}\left(\frac{\sqrt[4]{ax^2-b}}{\sqrt[4]{-b}}\right)\middle|-1\right)+\sqrt[4]{-b}\sqrt{\frac{ax^2}{b}}\,\Pi\left(-\frac{\sqrt{b}}{\sqrt{-2a^2+2\sqrt{a^2-b}\,a+b}};\sin^{-1}\left(\frac{\sqrt[4]{ax^2-b}}{\sqrt[4]{-b}}\right)\middle|-1\right)+\sqrt[4]{-b}\sqrt{\frac{ax^2}{b}}\,\Pi\left(\frac{\sqrt{b}}{\sqrt{-2a^2+2\sqrt{a^2-b}\,a+b}};\sin^{-1}\left(\frac{\sqrt[4]{ax^2-b}}{\sqrt[4]{-b}}\right)\middle|-1\right)-\frac{ax^2\left(1-\frac{ax^2}{b}\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{ax^2}{b}\right)}{\left(ax^2-b\right)^{3/4}}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(-2*b + a*x^2))/((-b + a*x^2)^(3/4)*(4*b - 4*a*x^2 + x^4)),x
]
```

```
[Out] -(((-b)^(1/4)*Sqrt[(a*x^2)/b]*EllipticPi[-(Sqrt[b]/Sqrt[-2*a^2 - 2*a*Sqrt[a
^2 - b] + b]), ArcSin[(-b + a*x^2)^(1/4)/(-b)^(1/4)], -1] + (-b)^(1/4)*Sqrt
[(a*x^2)/b]*EllipticPi[Sqrt[b]/Sqrt[-2*a^2 - 2*a*Sqrt[a^2 - b] + b], ArcSin
[(-b + a*x^2)^(1/4)/(-b)^(1/4)], -1] + (-b)^(1/4)*Sqrt[(a*x^2)/b]*EllipticP
i[-(Sqrt[b]/Sqrt[-2*a^2 + 2*a*Sqrt[a^2 - b] + b]), ArcSin[(-b + a*x^2)^(1/4
)/(-b)^(1/4)], -1] + (-b)^(1/4)*Sqrt[(a*x^2)/b]*EllipticPi[Sqrt[b]/Sqrt[-2*
a^2 + 2*a*Sqrt[a^2 - b] + b], ArcSin[(-b + a*x^2)^(1/4)/(-b)^(1/4)], -1] -
(a*x^2*(1 - (a*x^2)/b)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (a*x^2)/b])/(
-b + a*x^2)^(3/4))/x)
```

IntegrateAlgebraic [A] time = 2.78, size = 76, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{x\left(ax^2-b\right)^{3/4}}{\sqrt{2}\left(b-ax^2\right)}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{x\left(ax^2-b\right)^{3/4}}{\sqrt{2}\left(b-ax^2\right)}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(-2*b + a*x^2))/((-b + a*x^2)^(3/4)*(4*b - 4*a*x^2
+ x^4)),x]
```

```
[Out] -(ArcTan[(x*(-b + a*x^2)^(3/4))/(Sqrt[2]*(b - a*x^2))]/Sqrt[2]) + ArcTanh[(
x*(-b + a*x^2)^(3/4))/(Sqrt[2]*(b - a*x^2))]/Sqrt[2]
```

fricas [B] time = 0.47, size = 113, normalized size = 1.49

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{4}\sqrt{2}\log\left(-\frac{x^4-2\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}x^3+4\,ax^2+4\sqrt{ax^2-b}\,x^2-4\sqrt{2}\left(ax^2-b\right)^{\frac{3}{4}}x-4\,b}{x^4-4\,ax^2+4\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^2-2*b)/(a*x^2-b)^(3/4)/(x^4-4*a*x^2+4*b),x, algorithm="f
ricas")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*(a*x^2 - b)^(1/4)/x) + 1/4*sqrt(2)*log(-(x^4 -
2*sqrt(2)*(a*x^2 - b)^(1/4)*x^3 + 4*a*x^2 + 4*sqrt(a*x^2 - b)*x^2 - 4*sqrt(
2)*(a*x^2 - b)^(3/4)*x - 4*b)/(x^4 - 4*a*x^2 + 4*b))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2-2\,b\right)x^2}{\left(x^4-4\,ax^2+4\,b\right)\left(ax^2-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^2-2*b)/(a*x^2-b)^(3/4)/(x^4-4*a*x^2+4*b),x, algorithm="g
iac")
```

```
[Out] integrate((a*x^2 - 2*b)*x^2/((x^4 - 4*a*x^2 + 4*b)*(a*x^2 - b)^(3/4)), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a\,x^2-2b\right)}{\left(a\,x^2-b\right)^{\frac{3}{4}}\left(x^4-4a\,x^2+4b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2-2*b)/(a*x^2-b)^(3/4)/(x^4-4*a*x^2+4*b),x)
```

```
[Out] int(x^2*(a*x^2-2*b)/(a*x^2-b)^(3/4)/(x^4-4*a*x^2+4*b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^2 - 2\,b)x^2}{(x^4 - 4\,ax^2 + 4\,b)(ax^2 - b)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^2-2*b)/(a*x^2-b)^(3/4)/(x^4-4*a*x^2+4*b),x, algorithm="m
axima")
```

```
[Out] integrate((a*x^2 - 2*b)*x^2/((x^4 - 4*a*x^2 + 4*b)*(a*x^2 - b)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2\,\left(2\,b-a\,x^2\right)}{{\left(a\,x^2-b\right)}^{3/4}\,\left(x^4-4\,a\,x^2+4\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(2*b - a*x^2))/((a*x^2 - b)^(3/4)*(4*b - 4*a*x^2 + x^4)),x)
```

```
[Out] -int((x^2*(2*b - a*x^2))/((a*x^2 - b)^(3/4)*(4*b - 4*a*x^2 + x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(ax^2 - 2b\right)}{\left(ax^2 - b\right)^{\frac{3}{4}}\left(-4ax^2 + 4b + x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x**2-2*b)/(a*x**2-b)**(3/4)/(x**4-4*a*x**2+4*b),x)
```

```
[Out] Integral(x**2*(a*x**2 - 2*b)/((a*x**2 - b)**(3/4)*(-4*a*x**2 + 4*b + x**4))
, x)
```

3.918

$$\int \frac{-abx+x^3}{\sqrt{x(-a+x)(-b+x)}\left(a^2b^2-2ab(a+b)x+\left(a^2+4ab+b^2-d\right)x^2-2(a+b)x^3+x^4\right)}\,dx$$

Optimal. Leaf size=76

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt{x^2(-a-b)+abx+x^3}}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt{x^2(-a-b)+abx+x^3}}\right)}{d^{3/4}}$$

Rubi [F] time = 17.66, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-abx+x^3}{\sqrt{x(-a+x)(-b+x)}\left(a^2b^2-2ab(a+b)x+\left(a^2+4ab+b^2-d\right)x^2-2(a+b)x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a*b*x) + x^3)/(Sqrt[x*(-a + x)*(-b + x)]*(a^2*b^2 - 2*a*b*(a + b)*x
+ (a^2 + 4*a*b + b^2 - d)*x^2 - 2*(a + b)*x^3 + x^4)),x]
```

```
[Out] (2*a*b*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][x^2/(Sqrt[
-a + x^2]*Sqrt[-b + x^2]*(-(a^2*(b - x^2)^2) + 2*a*(-(b*x) + x^3)^2 - x^4*(
b^2 - d - 2*b*x^2 + x^4))), x], x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x] + (2*S
qrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][x^6/(Sqrt[-a + x^2
]*Sqrt[-b + x^2]*(a^2*(b - x^2)^2 - 2*a*(-(b*x) + x^3)^2 + x^4*(b^2 - d - 2
*b*x^2 + x^4))), x], x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x]
```

Rubi steps

$$\int \frac{-abx+x^3}{\sqrt{x(-a+x)(-b+x)}\left(a^2b^2-2ab(a+b)x+\left(a^2+4ab+b^2-d\right)x^2-2(a+b)x^3+x^4\right)}\,dx = \int \frac{}{\sqrt{x(-a+x)(-}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-}\right.}{}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\right.}{}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\right.}{}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\right.}{}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\right.}{}$$

Mathematica [C] time = 15.24, size = 31019, normalized size = 408.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-(a*b*x) + x^3)/(Sqrt[x*(-a + x)*(-b + x)]*(a^2*b^2 - 2*a*b*(a +
 b)*x + (a^2 + 4*a*b + b^2 - d)*x^2 - 2*(a + b)*x^3 + x^4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.28, size = 76, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt{x^2(-a-b)+abx+x^3}}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt{x^2(-a-b)+abx+x^3}}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*b*x) + x^3)/(Sqrt[x*(-a + x)*(-b + x)]*(a^2*b^2 - 2
*a*b*(a + b)*x + (a^2 + 4*a*b + b^2 - d)*x^2 - 2*(a + b)*x^3 + x^4)),x]
```

```
[Out] ArcTan[(d^(1/4)*x)/Sqrt[a*b*x + (-a - b)*x^2 + x^3]]/d^(3/4) - ArcTanh[(d^(
1/4)*x)/Sqrt[a*b*x + (-a - b)*x^2 + x^3]]/d^(3/4)
```

fricas [B] time = 1.19, size = 444, normalized size = 5.84

$$\left(\frac{1}{d^3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{abx-(a+b)x^2+x^3}\,d\left(\frac{1}{d^3}\right)^{\frac{1}{4}}}{ab-(a+b)x+x^2}\right)-\frac{1}{4}\left(\frac{1}{d^3}\right)^{\frac{1}{4}}\log\left(\frac{a^2b^2-2\,(a+b)x^3+x^4+\left(a^2+4\,ab+b^2+d\right)x^2-2\left(a^2b+ab^2\right)x+2\left(d^3\left(\frac{1}{d^3}\right)^{\frac{3}{4}}x+\left(abd-(a+b)dx+dx^2\right)\left(\frac{1}{d^3}\right)^{\frac{1}{4}}\right)\sqrt{abx-(a+b)x^2+x^3}+2\left(abd^2x-(a+b)d^2x^2+d^2x^3\right)\sqrt{\frac{1}{d^3}}}{a^2b^2-2\,(a+b)x^3+x^4+\left(a^2+4\,ab+b^2-d\right)x^2-2\left(a^2b+ab^2\right)x}\right)+\frac{1}{4}\left(\frac{1}{d^3}\right)^{\frac{1}{4}}\log\left(\frac{a^2b^2-2\,(a+b)x^3+x^4+\left(a^2+4\,ab+b^2+d\right)x^2-2\left(a^2b+ab^2\right)x-2\left(d^3\left(\frac{1}{d^3}\right)^{\frac{3}{4}}x+\left(abd-(a+b)dx+dx^2\right)\left(\frac{1}{d^3}\right)^{\frac{1}{4}}\right)\sqrt{abx-(a+b)x^2+x^3}+2\left(abd^2x-(a+b)d^2x^2+d^2x^3\right)\sqrt{\frac{1}{d^3}}}{a^2b^2-2\,(a+b)x^3+x^4+\left(a^2+4\,ab+b^2-d\right)x^2-2\left(a^2b+ab^2\right)x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*x+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2-2*a*b*(a+b)*x+(a^2+
4*a*b+b^2-d)*x^2-2*(a+b)*x^3+x^4),x, algorithm="fricas")
```

```
[Out] (d^(-3))^(1/4)*arctan(sqrt(a*b*x - (a + b)*x^2 + x^3)*d*(d^(-3))^(1/4)/(a*b
 - (a + b)*x + x^2)) - 1/4*(d^(-3))^(1/4)*log((a^2*b^2 - 2*(a + b)*x^3 + x^
4 + (a^2 + 4*a*b + b^2 + d)*x^2 - 2*(a^2*b + a*b^2)*x + 2*(d^3*(d^(-3))^(3/
4)*x + (a*b*d - (a + b)*d*x + d*x^2)*(d^(-3))^(1/4))*sqrt(a*b*x - (a + b)*x
^2 + x^3) + 2*(a*b*d^2*x - (a + b)*d^2*x^2 + d^2*x^3)*sqrt(d^(-3)))/(a^2*b^
2 - 2*(a + b)*x^3 + x^4 + (a^2 + 4*a*b + b^2 - d)*x^2 - 2*(a^2*b + a*b^2)*x
)) + 1/4*(d^(-3))^(1/4)*log((a^2*b^2 - 2*(a + b)*x^3 + x^4 + (a^2 + 4*a*b +
 b^2 + d)*x^2 - 2*(a^2*b + a*b^2)*x - 2*(d^3*(d^(-3))^(3/4)*x + (a*b*d - (a
 + b)*d*x + d*x^2)*(d^(-3))^(1/4))*sqrt(a*b*x - (a + b)*x^2 + x^3) + 2*(a*b
*d^2*x - (a + b)*d^2*x^2 + d^2*x^3)*sqrt(d^(-3)))/(a^2*b^2 - 2*(a + b)*x^3
+ x^4 + (a^2 + 4*a*b + b^2 - d)*x^2 - 2*(a^2*b + a*b^2)*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{abx-x^3}{\left(a^2b^2-2\,(a+b)abx-2\,(a+b)x^3+x^4+\left(a^2+4\,ab+b^2-d\right)x^2\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*x+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2-2*a*b*(a+b)*x+(a^2+
4*a*b+b^2-d)*x^2-2*(a+b)*x^3+x^4),x, algorithm="giac")
```

```
[Out] integrate(-(a*b*x - x^3)/((a^2*b^2 - 2*(a + b)*a*b*x - 2*(a + b)*x^3 + x^4
+ (a^2 + 4*a*b + b^2 - d)*x^2)*sqrt((a - x)*(b - x)*x)), x)
```

maple [C] time = 0.06, size = 289, normalized size = 3.80

$$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4+(-2a-2b)\_Z^3+\left(a^2+4ab+b^2-d\right)\_Z^2+\left(-2a^2b-2a\,b^2\right)\_Z+a^2b^2\right)}\frac{\_\alpha\left(\_\alpha^2-ab\right)\left(-\_\alpha^3+\_\alpha^2a+2\_\alpha^2b-2\_\alpha ab-\_\alpha\,b^2+a\,b^2+\_\alpha d+ad\right)\sqrt{-\frac{-a+x}{a}}\,\sqrt{\frac{-b+x}{a-b}}\,\sqrt{\frac{x}{a}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-a+x}{a}},\frac{-\_\alpha^3+\_\alpha^2a+2\_\alpha^2b-2\_\alpha ab-\_\alpha\,b^2+a\,b^2+\_\alpha d+ad}{ad},\sqrt{\frac{a}{a-b}}\right)}{\left(-2\_\alpha^3+3\_\alpha^2a+3\_\alpha^2b-\_\alpha\,a^2-4\_\alpha ab-\_\alpha\,b^2+a^2b+a\,b^2+\_\alpha d\right)\sqrt{x\left(ab-ax-bx+x^2\right)}}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*b*x+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2-2*a*b*(a+b)*x+(a^2+4*a*b+
b^2-d)*x^2-2*(a+b)*x^3+x^4),x)
```

```
[Out] 1/a/d*sum(_alpha*(_alpha^2-a*b)/(-2*_alpha^3+3*_alpha^2*a+3*_alpha^2*b-_alp
ha*a^2-4*_alpha*a*b-_alpha*b^2+a^2*b+a*b^2+_alpha*d)*(-_alpha^3+_alpha^2*a+
2*_alpha^2*b-2*_alpha*a*b-_alpha*b^2+a*b^2+_alpha*d+a*d)*(-(-a+x)/a)^(1/2)*
((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(x*(a*b-a*x-b*x+x^2))^(1/2)*EllipticPi((
-(-a+x)/a)^(1/2),(-_alpha^3+_alpha^2*a+2*_alpha^2*b-2*_alpha*a*b-_alpha*b^2
+a*b^2+_alpha*d+a*d)/a/d,(a/(a-b))^(1/2)),_alpha=RootOf(_Z^4+(-2*a-2*b)*_Z^
3+(a^2+4*a*b+b^2-d)*_Z^2+(-2*a^2*b-2*a*b^2)*_Z+a^2*b^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{abx - x^3}{\left(a^2b^2 - 2\,(a+b)abx - 2\,(a+b)x^3 + x^4 + \left(a^2 + 4\,ab + b^2 - d\right)x^2\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*x+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2-2*a*b*(a+b)*x+(a^2+
4*a*b+b^2-d)*x^2-2*(a+b)*x^3+x^4),x, algorithm="maxima")
```

```
[Out] -integrate((a*b*x - x^3)/((a^2*b^2 - 2*(a + b)*a*b*x - 2*(a + b)*x^3 + x^4
+ (a^2 + 4*a*b + b^2 - d)*x^2)*sqrt((a - x)*(b - x)*x)), x)
```

mupad [B] time = 1.53, size = 714, normalized size = 9.39

$$\sum_{k=1}^{4}\left(-\frac{b\left(\mathrm{root}\left(z^4-z^3\,(2a+2b)+z^2\left(-d+4ab+a^2+b^2\right)-2abz\,(a+b)+a^2b^2,z,k\right)^3-ab\,\mathrm{root}\left(z^4-z^3\,(2a+2b)+z^2\left(-d+4ab+a^2+b^2\right)-2abz\,(a+b)+a^2b^2,z,k\right)\right)\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(-\frac{b}{\mathrm{root}\left(z^4-z^3\,(2a+2b)+z^2\left(-d+4ab+a^2+b^2\right)-2abz\,(a+b)+a^2b^2,z,k\right)-b};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)}{\left(\mathrm{root}\left(z^4-z^3\,(2a+2b)+z^2\left(-d+4ab+a^2+b^2\right)-2abz\,(a+b)+a^2b^2,z,k\right)-b\right)\sqrt{x\,(a-x)\,(b-x)}\left(a^2b-a^2\,\mathrm{root}\left(z^4-z^3\,(2a+2b)+z^2\left(-d+4ab+a^2+b^2\right)-2abz\,(a+b)+a^2b^2,z,k\right)+ab^2-4ab\,\mathrm{root}\left(z^4-z^3\,(2a+2b)+z^2\left(-d+4ab+a^2+b^2\right)-2abz\,(a+b)+a^2b^2,z,k\right)+3a\,\mathrm{root}\left(z^4-z^3\,(2a+2b)+z^2\left(-d+4ab+a^2+b^2\right)-2abz\,(a+b)+a^2b^2,z,k\right)^2-b^2\,\mathrm{root}\left(z^4-z^3\,(2a+2b)+z^2\left(-d+4ab+a^2+b^2\right)-2abz\,(a+b)+a^2b^2,z,k\right)+3b\,\mathrm{root}\left(z^4-z^3\,(2a+2b)+z^2\left(-d+4ab+a^2+b^2\right)-2abz\,(a+b)+a^2b^2,z,k\right)^2-2\,\mathrm{root}\left(z^4-z^3\,(2a+2b)+z^2\left(-d+4ab+a^2+b^2\right)-2abz\,(a+b)+a^2b^2,z,k\right)^3+d\,\mathrm{root}\left(z^4-z^3\,(2a+2b)+z^2\left(-d+4ab+a^2+b^2\right)-2abz\,(a+b)+a^2b^2,z,k\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - a*b*x)/((x*(a - x)*(b - x))^(1/2)*(x^4 - 2*x^3*(a + b) + a^2*b^2
 + x^2*(4*a*b - d + a^2 + b^2) - 2*a*b*x*(a + b))),x)
```

```
[Out] symsum(-(b*(root(z^4 - z^3*(2*a + 2*b) + z^2*(- d + 4*a*b + a^2 + b^2) - 2*
a*b*z*(a + b) + a^2*b^2, z, k)^3 - a*b*root(z^4 - z^3*(2*a + 2*b) + z^2*(-
d + 4*a*b + a^2 + b^2) - 2*a*b*z*(a + b) + a^2*b^2, z, k))*(x/b)^(1/2)*((b
- x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*ellipticPi(-b/(root(z^4 - z^3*(2*a +
2*b) + z^2*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(a + b) + a^2*b^2, z, k) - b
), asin(((b - x)/b)^(1/2)), -b/(a - b)))/((root(z^4 - z^3*(2*a + 2*b) + z^2
*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(a + b) + a^2*b^2, z, k) - b)*(x*(a -
x)*(b - x))^(1/2)*(3*a*root(z^4 - z^3*(2*a + 2*b) + z^2*(- d + 4*a*b + a^2
+ b^2) - 2*a*b*z*(a + b) + a^2*b^2, z, k)^2 - a^2*root(z^4 - z^3*(2*a + 2*b
) + z^2*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(a + b) + a^2*b^2, z, k) + 3*b*
root(z^4 - z^3*(2*a + 2*b) + z^2*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(a + b
) + a^2*b^2, z, k)^2 - b^2*root(z^4 - z^3*(2*a + 2*b) + z^2*(- d + 4*a*b +
a^2 + b^2) - 2*a*b*z*(a + b) + a^2*b^2, z, k) + a*b^2 + a^2*b - 2*root(z^4
- z^3*(2*a + 2*b) + z^2*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(a + b) + a^2*b
^2, z, k)^3 + d*root(z^4 - z^3*(2*a + 2*b) + z^2*(- d + 4*a*b + a^2 + b^2)
- 2*a*b*z*(a + b) + a^2*b^2, z, k) - 4*a*b*root(z^4 - z^3*(2*a + 2*b) + z^2
*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(a + b) + a^2*b^2, z, k))), k, 1, 4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*x+x**3)/(x*(-a+x)*(-b+x))**(1/2)/(a**2*b**2-2*a*b*(a+b)*x+(
a**2+4*a*b+b**2-d)*x**2-2*(a+b)*x**3+x**4),x)
```

```
[Out] Timed out
```

3.919 $\int \frac{\sqrt[4]{-1-x^4}\left(-1+x^4\right)}{x^6\left(1+2x^4\right)}\,dx$

Optimal. Leaf size=76

$$\frac{3}{2}\tan^{-1}\left(\frac{x\left(-x^4-1\right)^{3/4}}{x^4+1}\right)-\frac{3}{2}\tanh^{-1}\left(\frac{x\left(-x^4-1\right)^{3/4}}{x^4+1}\right)+\frac{\sqrt[4]{-x^4-1}\left(1-14x^4\right)}{5x^5}$$

Rubi [A] time = 0.10, antiderivative size = 73, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {580, 583, 12, 494, 298, 203, 206}

$$-\frac{14\sqrt[4]{-x^4-1}}{5x}-\frac{3}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{-x^4-1}}\right)+\frac{3}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{-x^4-1}}\right)+\frac{\sqrt[4]{-x^4-1}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 - x^4)^(1/4)*(-1 + x^4))/(x^6*(1 + 2*x^4)),x]
```

```
[Out] (-1 - x^4)^(1/4)/(5*x^5) - (14*(-1 - x^4)^(1/4))/(5*x) - (3*ArcTan[x/(-1 -
x^4)^(1/4)])/2 + (3*ArcTanh[x/(-1 - x^4)^(1/4)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{-1-x^4}\left(-1+x^4\right)}{x^6\left(1+2x^4\right)}\,dx &= \frac{\sqrt[4]{-1-x^4}}{5x^5}+\frac{1}{5}\int \frac{-14-13x^4}{x^2\left(-1-x^4\right)^{3/4}\left(1+2x^4\right)}\,dx\\
&= \frac{\sqrt[4]{-1-x^4}}{5x^5}-\frac{14\sqrt[4]{-1-x^4}}{5x}+\frac{1}{5}\int \frac{15x^2}{\left(-1-x^4\right)^{3/4}\left(1+2x^4\right)}\,dx\\
&= \frac{\sqrt[4]{-1-x^4}}{5x^5}-\frac{14\sqrt[4]{-1-x^4}}{5x}+3\int \frac{x^2}{\left(-1-x^4\right)^{3/4}\left(1+2x^4\right)}\,dx\\
&= \frac{\sqrt[4]{-1-x^4}}{5x^5}-\frac{14\sqrt[4]{-1-x^4}}{5x}+3\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\frac{x}{\sqrt[4]{-1-x^4}}\right)\\
&= \frac{\sqrt[4]{-1-x^4}}{5x^5}-\frac{14\sqrt[4]{-1-x^4}}{5x}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt[4]{-1-x^4}}\right)-\frac{3}{2}\text{Subst}\left(\int\right.\\
&= \frac{\sqrt[4]{-1-x^4}}{5x^5}-\frac{14\sqrt[4]{-1-x^4}}{5x}-\frac{3}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{-1-x^4}}\right)+\frac{3}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{-1-x^4}}\right)
\end{aligned}$$

Mathematica [C] time = 0.08, size = 78, normalized size = 1.03

$$\frac{\frac{5\left(x^4+1\right)^{3/4}x^8\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{x^4}{2x^4+1}\right)}{\left(2x^4+1\right)^{3/4}}+14x^8+13x^4-1}{5x^5\left(-x^4-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 - x^4)^(1/4)*(-1 + x^4))/(x^6*(1 + 2*x^4)),x]
```

```
[Out] (-1 + 13*x^4 + 14*x^8 + (5*x^8*(1 + x^4)^(3/4)*Hypergeometric2F1[3/4, 3/4,
7/4, x^4/(1 + 2*x^4)])/(1 + 2*x^4)^(3/4))/(5*x^5*(-1 - x^4)^(3/4))
```

IntegrateAlgebraic [A] time = 0.21, size = 76, normalized size = 1.00

$$\frac{3}{2}\tan^{-1}\left(\frac{x\left(-x^4-1\right)^{3/4}}{x^4+1}\right)-\frac{3}{2}\tanh^{-1}\left(\frac{x\left(-x^4-1\right)^{3/4}}{x^4+1}\right)+\frac{\sqrt[4]{-x^4-1}\left(1-14x^4\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 - x^4)^(1/4)*(-1 + x^4))/(x^6*(1 + 2*x^4)),x]
```

```
[Out] ((1 - 14*x^4)*(-1 - x^4)^(1/4))/(5*x^5) + (3*ArcTan[(x*(-1 - x^4)^(3/4))/(1
 + x^4)])/2 - (3*ArcTanh[(x*(-1 - x^4)^(3/4))/(1 + x^4)])/2
```

fricas [C] time = 3.55, size = 199, normalized size = 2.62

$$\frac{30\,x^5\log\left(-\frac{2\left(2\left(-x^4-1\right)^{\frac{1}{4}}x^3+2\sqrt{-x^4-1}\,x^2+2\left(-x^4-1\right)^{\frac{3}{4}}x-1\right)}{2x^4+1}\right)+15i\,x^5\log\left(\frac{4i\left(-x^4-1\right)^{\frac{1}{4}}x^3+4\sqrt{-x^4-1}\,x^2-4i\left(-x^4-1\right)^{\frac{3}{4}}x+2}{2x^4+1}\right)-15i\,x^5\log\left(\frac{-4i\left(-x^4-1\right)^{\frac{1}{4}}x^3+4\sqrt{-x^4-1}\,x^2+4i\left(-x^4-1\right)^{\frac{3}{4}}x+2}{2x^4+1}\right)-8\left(14\,x^4-1\right)\left(-x^4-1\right)^{\frac{1}{4}}}{40\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-1)^(1/4)*(x^4-1)/x^6/(2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*(30*x^5*log(-2*(2*(-x^4 - 1)^(1/4)*x^3 + 2*sqrt(-x^4 - 1)*x^2 + 2*(-x^
4 - 1)^(3/4)*x - 1)/(2*x^4 + 1)) + 15*I*x^5*log((4*I*(-x^4 - 1)^(1/4)*x^3 +
 4*sqrt(-x^4 - 1)*x^2 - 4*I*(-x^4 - 1)^(3/4)*x + 2)/(2*x^4 + 1)) - 15*I*x^5
*log((-4*I*(-x^4 - 1)^(1/4)*x^3 + 4*sqrt(-x^4 - 1)*x^2 + 4*I*(-x^4 - 1)^(3/
4)*x + 2)/(2*x^4 + 1)) - 8*(14*x^4 - 1)*(-x^4 - 1)^(1/4))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)\left(-x^4-1\right)^{\frac{1}{4}}}{\left(2\,x^4+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-1)^(1/4)*(x^4-1)/x^6/(2*x^4+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 1)*(-x^4 - 1)^(1/4)/((2*x^4 + 1)*x^6), x)
```

maple [C] time = 0.98, size = 429, normalized size = 5.64

$$\frac{14x^8+13x^4-1}{5x^5\left(-x^4-1\right)^{\frac{3}{4}}}+\frac{\left(-\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-2\,\mathrm{RootOf}\left(\_Z^2+1\right)\left(-x^{12}-3x^8-3x^4-1\right)^{\frac{1}{4}}x^9+2\sqrt{-x^{12}-3x^8-3x^4-1}\,x^6-x^8+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\left(-x^{12}-3x^8-3x^4-1\right)^{\frac{3}{4}}x^3-4\,\mathrm{RootOf}\left(\_Z^2+1\right)\left(-x^{12}-3x^8-3x^4-1\right)^{\frac{1}{4}}x^5+2\sqrt{-x^{12}-3x^8-3x^4-1}\,x^2-2x^4-2\,\mathrm{RootOf}\left(\_Z^2+1\right)\left(-x^{12}-3x^8-3x^4-1\right)^{\frac{1}{4}}x-1}{\left(x^4+1\right)^2\left(2x^4+1\right)}\right)}{4}+\frac{3\ln\left(-\frac{2\left(-x^{12}-3x^8-3x^4-1\right)^{\frac{1}{4}}x^9-2\sqrt{-x^{12}-3x^8-3x^4-1}\,x^6-x^8+2\left(-x^{12}-3x^8-3x^4-1\right)^{\frac{3}{4}}x^3+4\left(-x^{12}-3x^8-3x^4-1\right)^{\frac{1}{4}}x^5-2\sqrt{-x^{12}-3x^8-3x^4-1}\,x^2-2x^4+2\left(-x^{12}-3x^8-3x^4-1\right)^{\frac{1}{4}}x-1}{\left(x^4+1\right)^2\left(2x^4+1\right)}\right)}{4}\right)\left(-\left(x^4+1\right)^3\right)^{\frac{1}{4}}}{\left(-x^4-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4-1)^(1/4)*(x^4-1)/x^6/(2*x^4+1),x)
```

```
[Out] 1/5*(14*x^8+13*x^4-1)/x^5/(-x^4-1)^(3/4)+(-3/4*RootOf(_Z^2+1)*ln((-2*RootOf
(_Z^2+1)*(-x^12-3*x^8-3*x^4-1)^(1/4)*x^9+2*(-x^12-3*x^8-3*x^4-1)^(1/2)*x^6-
x^8+2*RootOf(_Z^2+1)*(-x^12-3*x^8-3*x^4-1)^(3/4)*x^3-4*RootOf(_Z^2+1)*(-x^1
2-3*x^8-3*x^4-1)^(1/4)*x^5+2*(-x^12-3*x^8-3*x^4-1)^(1/2)*x^2-2*x^4-2*RootOf
(_Z^2+1)*(-x^12-3*x^8-3*x^4-1)^(1/4)*x-1)/(x^4+1)^2/(2*x^4+1))+3/4*ln(-(2*(
-x^12-3*x^8-3*x^4-1)^(1/4)*x^9-2*(-x^12-3*x^8-3*x^4-1)^(1/2)*x^6-x^8+2*(-x^
12-3*x^8-3*x^4-1)^(3/4)*x^3+4*(-x^12-3*x^8-3*x^4-1)^(1/4)*x^5-2*(-x^12-3*x^
8-3*x^4-1)^(1/2)*x^2-2*x^4+2*(-x^12-3*x^8-3*x^4-1)^(1/4)*x-1)/(x^4+1)^2/(2*
x^4+1)))/(-x^4-1)^(3/4)*(-(x^4+1)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)\left(-x^4-1\right)^{\frac{1}{4}}}{\left(2\,x^4+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-1)^(1/4)*(x^4-1)/x^6/(2*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 1)*(-x^4 - 1)^(1/4)/((2*x^4 + 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-1\right)\left(-x^4-1\right)^{1/4}}{x^6\left(2\,x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(- x^4 - 1)^(1/4))/(x^6*(2*x^4 + 1)),x)
```

```
[Out] int(((x^4 - 1)*(- x^4 - 1)^(1/4))/(x^6*(2*x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2+1\right)\sqrt[4]{-x^4-1}}{x^6\left(2x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4-1)**(1/4)*(x**4-1)/x**6/(2*x**4+1),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)*(-x**4 - 1)**(1/4)/(x**6*(2*x**4 + 1)),
 x)
```

3.920

$$\int \frac{abx-x^3}{\sqrt{x(-a+x)(-b+x)}\left(a^2b^2d-2ab(a+b)dx+\left(-1+a^2d+4abd+b^2d\right)x^2-2(a+b)dx^3+dx^4\right)}\,dx$$

Optimal. Leaf size=76

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}\right)}{\sqrt[4]{d}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}\right)}{\sqrt[4]{d}}$$

Rubi [F] time = 21.99, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{abx-x^3}{\sqrt{x(-a+x)(-b+x)}\left(a^2b^2d-2ab(a+b)dx+\left(-1+a^2d+4abd+b^2d\right)x^2-2(a+b)dx^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*b*x - x^3)/(Sqrt[x*(-a + x)*(-b + x)]*(a^2*b^2*d - 2*a*b*(a + b)*d*x
 + (-1 + a^2*d + 4*a*b*d + b^2*d)*x^2 - 2*(a + b)*d*x^3 + d*x^4)),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][x^6/(Sqrt[-a +
 x^2]*Sqrt[-b + x^2]*(-(a^2*b^2*d) + 2*a^2*b*(1 + b/a)*d*x^2 + (1 - (a^2 +
4*a*b + b^2)*d)*x^4 + 2*a*(1 + b/a)*d*x^6 - d*x^8)), x], x, Sqrt[x]])/Sqrt[
(a - x)*(b - x)*x] + (2*a*b*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][
Defer[Int][x^2/(Sqrt[-a + x^2]*Sqrt[-b + x^2]*(a^2*b^2*d - 2*a^2*b*(1 + b/a
)*d*x^2 - (1 - (a^2 + 4*a*b + b^2)*d)*x^4 - 2*a*(1 + b/a)*d*x^6 + d*x^8)),
x], x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x]
```

Rubi steps

$$\int \frac{abx-x^3}{\sqrt{x(-a+x)(-b+x)}\left(a^2b^2d-2ab(a+b)dx+\left(-1+a^2d+4abd+b^2d\right)x^2-2(a+b)dx^3+dx^4\right)}\,dx = \int \ldots$$

Mathematica [C] time = 14.42, size = 32987, normalized size = 434.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*b*x - x^3)/(Sqrt[x*(-a + x)*(-b + x)]*(a^2*b^2*d - 2*a*b*(a + b)*d*x + (-1 + a^2*d + 4*a*b*d + b^2*d)*x^2 - 2*(a + b)*d*x^3 + d*x^4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.34, size = 76, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}\right)}{\sqrt[4]{d}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}\right)}{\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b*x - x^3)/(Sqrt[x*(-a + x)*(-b + x)]*(a^2*b^2*d - 2*a*b*(a + b)*d*x + (-1 + a^2*d + 4*a*b*d + b^2*d)*x^2 - 2*(a + b)*d*x^3 + d*x^4)),x]
```

```
[Out] -(ArcTan[x/(d^(1/4)*Sqrt[a*b*x + (-a - b)*x^2 + x^3])]/d^(1/4)) + ArcTanh[x/(d^(1/4)*Sqrt[a*b*x + (-a - b)*x^2 + x^3])]/d^(1/4)
```

fricas [B] time = 2.04, size = 434, normalized size = 5.71

$$-\frac{\arctan\left(\frac{\sqrt{abx-(a+b)x^2+x^3}}{\left(ab-(a+b)x+x^2\right)d^{\frac{1}{4}}}\right)}{d^{\frac{1}{4}}}+\frac{\log\left(\frac{a^2b^2d-2\,(a+b)dx^3+dx^4-2\left(a^2b+ab^2\right)dx+\left(\left(a^2+4\,ab+b^2\right)d+1\right)x^2+2\sqrt{abx-(a+b)x^2+x^3}\left(d^{\frac{1}{4}}x+\frac{abd-(a+b)dx+dx^2}{d^{\frac{1}{4}}}\right)+\frac{2\left(abdx-(a+b)dx^2+dx^3\right)}{\sqrt{d}}}{a^2b^2d-2\,(a+b)dx^3+dx^4-2\left(a^2b+ab^2\right)dx+\left(\left(a^2+4\,ab+b^2\right)d-1\right)x^2}\right)}{4\,d^{\frac{1}{4}}}-\frac{\log\left(\frac{a^2b^2d-2\,(a+b)dx^3+dx^4-2\left(a^2b+ab^2\right)dx+\left(\left(a^2+4\,ab+b^2\right)d+1\right)x^2-2\sqrt{abx-(a+b)x^2+x^3}\left(d^{\frac{1}{4}}x+\frac{abd-(a+b)dx+dx^2}{d^{\frac{1}{4}}}\right)+\frac{2\left(abdx-(a+b)dx^2+dx^3\right)}{\sqrt{d}}}{a^2b^2d-2\,(a+b)dx^3+dx^4-2\left(a^2b+ab^2\right)dx+\left(\left(a^2+4\,ab+b^2\right)d-1\right)x^2}\right)}{4\,d^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*x-x^3)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2*d-2*a*b*(a+b)*d*x+(a^2*d+4*a*b*d+b^2*d-1)*x^2-2*(a+b)*d*x^3+d*x^4),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(a*b*x - (a + b)*x^2 + x^3)/((a*b - (a + b)*x + x^2)*d^(1/4)))/d^(1/4) + 1/4*log((a^2*b^2*d - 2*(a + b)*d*x^3 + d*x^4 - 2*(a^2*b + a*b^2)*d*x + ((a^2 + 4*a*b + b^2)*d + 1)*x^2 + 2*sqrt(a*b*x - (a + b)*x^2 + x^3)*(d^(1/4)*x + (a*b*d - (a + b)*d*x + d*x^2)/d^(1/4)) + 2*(a*b*d*x - (a + b)*d*x^2 + d*x^3)/sqrt(d))/(a^2*b^2*d - 2*(a + b)*d*x^3 + d*x^4 - 2*(a^2*b + a*b^2)*d*x + ((a^2 + 4*a*b + b^2)*d - 1)*x^2))/d^(1/4) - 1/4*log((a^2*b^2*d - 2*(a + b)*d*x^3 + d*x^4 - 2*(a^2*b + a*b^2)*d*x + ((a^2 + 4*a*b + b^2)*d + 1)*x^2 - 2*sqrt(a*b*x - (a + b)*x^2 + x^3)*(d^(1/4)*x + (a*b*d - (a + b)*d*x + d*x^2)/d^(1/4)) + 2*(a*b*d*x - (a + b)*d*x^2 + d*x^3)/sqrt(d))/(a^2*b^2*d - 2*(a + b)*d*x^3 + d*x^4 - 2*(a^2*b + a*b^2)*d*x + ((a^2 + 4*a*b + b^2)*d - 1)*x^2))/d^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{abx-x^3}{\left(a^2b^2d-2\,(a+b)abdx-2\,(a+b)dx^3+dx^4+\left(a^2d+4\,abd+b^2d-1\right)x^2\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*x-x^3)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2*d-2*a*b*(a+b)*d*x+(a^2*d+4*a*b*d+b^2*d-1)*x^2-2*(a+b)*d*x^3+d*x^4),x, algorithm="giac")
```

```
[Out] integrate((a*b*x - x^3)/((a^2*b^2*d - 2*(a + b)*a*b*d*x - 2*(a + b)*d*x^3 + d*x^4 + (a^2*d + 4*a*b*d + b^2*d - 1)*x^2)*sqrt((a - x)*(b - x)*x)), x)
```

maple [C] time = 0.06, size = 304, normalized size = 4.00

$$-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+(-2ad-2bd)\_Z^3+\left(a^2d+4abd+b^2d-1\right)\_Z^2+\left(-2a^2bd-2a\,b^2d\right)\_Z+a^2b^2d\right)}\frac{\_\alpha\left(\_\alpha^2-ab\right)\left(-d\_\alpha^3+d\_\alpha^2a+2d\_\alpha^2b-2\_\alpha abd-\_\alpha\,b^2d+a\,b^2d+\_\alpha+a\right)\sqrt{-\frac{-a+x}{a}}\,\sqrt{\frac{-b+x}{a-b}}\,\sqrt{\frac{x}{a}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-a+x}{a}},\frac{-d\_\alpha^3+d\_\alpha^2a+2d\_\alpha^2b-2\_\alpha abd-\_\alpha\,b^2d+a\,b^2d+\_\alpha+a}{a},\sqrt{\frac{a}{a-b}}\right)}{\left(-2d\_\alpha^3+3d\_\alpha^2a+3d\_\alpha^2b-\_\alpha\,a^2d-4\_\alpha abd-\_\alpha\,b^2d+a^2bd+a\,b^2d+\_\alpha\right)\sqrt{x\left(ab-ax-bx+x^2\right)}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b*x-x^3)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2*d-2*a*b*(a+b)*d*x+(a^2*d+4
*a*b*d+b^2*d-1)*x^2-2*(a+b)*d*x^3+d*x^4),x)
```

```
[Out] -1/a*sum(_alpha*(_alpha^2-a*b)/(-2*_alpha^3*d+3*_alpha^2*a*d+3*_alpha^2*b*d
-_alpha*a^2*d-4*_alpha*a*b*d-_alpha*b^2*d+a^2*b*d+a*b^2*d+_alpha)*(-_alpha^
3*d+_alpha^2*a*d+2*_alpha^2*b*d-2*_alpha*a*b*d-_alpha*b^2*d+a*b^2*d+_alpha+
a)*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(x*(a*b-a*x-b*x+x^2
))^(1/2)*EllipticPi((-(-a+x)/a)^(1/2),(-_alpha^3*d+_alpha^2*a*d+2*_alpha^2*
b*d-2*_alpha*a*b*d-_alpha*b^2*d+a*b^2*d+_alpha+a)/a,(a/(a-b))^(1/2)),_alpha
=RootOf(d*_Z^4+(-2*a*d-2*b*d)*_Z^3+(a^2*d+4*a*b*d+b^2*d-1)*_Z^2+(-2*a^2*b*d
-2*a*b^2*d)*_Z+a^2*b^2*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{abx - x^3}{\left(a^2b^2d - 2\,(a+b)abdx - 2\,(a+b)dx^3 + dx^4 + \left(a^2d + 4\,abd + b^2d - 1\right)x^2\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*x-x^3)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2*d-2*a*b*(a+b)*d*x+(a
^2*d+4*a*b*d+b^2*d-1)*x^2-2*(a+b)*d*x^3+d*x^4),x, algorithm="maxima")
```

```
[Out] integrate((a*b*x - x^3)/((a^2*b^2*d - 2*(a + b)*a*b*d*x - 2*(a + b)*d*x^3 +
 d*x^4 + (a^2*d + 4*a*b*d + b^2*d - 1)*x^2)*sqrt((a - x)*(b - x)*x)), x)
```

mupad [B] time = 7.78, size = 175, normalized size = 2.30

$$\frac{\ln\left(\frac{x+2\,d^{1/4}\,\sqrt{x\,(a-x)\,(b-x)}+\sqrt{d}\,x^2+a\,b\,\sqrt{d}-a\,\sqrt{d}\,x-b\,\sqrt{d}\,x}{x-\sqrt{d}\,x^2-a\,b\,\sqrt{d}+a\,\sqrt{d}\,x+b\,\sqrt{d}\,x}\right)}{2\,d^{1/4}}+\frac{\ln\left(\frac{x-\sqrt{d}\,x^2-a\,b\,\sqrt{d}+a\,\sqrt{d}\,x+b\,\sqrt{d}\,x-d^{1/4}\,\sqrt{x\,(a-x)\,(b-x)}\,2\mathrm{i}}{x+\sqrt{d}\,x^2+a\,b\,\sqrt{d}-a\,\sqrt{d}\,x-b\,\sqrt{d}\,x}\right)\,1\mathrm{i}}{2\,d^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - a*b*x)/((x*(a - x)*(b - x))^(1/2)*(x^2*(a^2*d + b^2*d + 4*a*b*d
 - 1) + d*x^4 + a^2*b^2*d - 2*d*x^3*(a + b) - 2*a*b*d*x*(a + b))),x)
```

```
[Out] log((x + 2*d^(1/4)*(x*(a - x)*(b - x))^(1/2) + d^(1/2)*x^2 + a*b*d^(1/2) -
a*d^(1/2)*x - b*d^(1/2)*x)/(x - d^(1/2)*x^2 - a*b*d^(1/2) + a*d^(1/2)*x + b
*d^(1/2)*x))/(2*d^(1/4)) + (log((x - d^(1/4)*(x*(a - x)*(b - x))^(1/2)*2i -
 d^(1/2)*x^2 - a*b*d^(1/2) + a*d^(1/2)*x + b*d^(1/2)*x)/(x + d^(1/2)*x^2 +
a*b*d^(1/2) - a*d^(1/2)*x - b*d^(1/2)*x))*1i)/(2*d^(1/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b*x-x**3)/(x*(-a+x)*(-b+x))**(1/2)/(a**2*b**2*d-2*a*b*(a+b)*d*
x+(a**2*d+4*a*b*d+b**2*d-1)*x**2-2*(a+b)*d*x**3+d*x**4),x)
```

```
[Out] Timed out
```

3.921

$$\int \frac{-abx+x^3}{\sqrt{x(-a+x)(-b+x)}\left(a^2b^2d-2ab(a+b)dx+\left(-1+a^2d+4abd+b^2d\right)x^2-2(a+b)dx^3+dx^4\right)}\,dx$$

Optimal. Leaf size=76

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}\right)}{\sqrt[4]{d}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}\right)}{\sqrt[4]{d}}$$

Rubi [F] time = 23.92, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-abx+x^3}{\sqrt{x(-a+x)(-b+x)}\left(a^2b^2d-2ab(a+b)dx+\left(-1+a^2d+4abd+b^2d\right)x^2-2(a+b)dx^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a*b*x) + x^3)/(Sqrt[x*(-a + x)*(-b + x)]*(a^2*b^2*d - 2*a*b*(a + b)*
d*x + (-1 + a^2*d + 4*a*b*d + b^2*d)*x^2 - 2*(a + b)*d*x^3 + d*x^4)),x]
```

```
[Out] (2*a*b*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][x^2/(Sqrt[
-a + x^2]*Sqrt[-b + x^2]*(-(a^2*b^2*d) + 2*a^2*b*(1 + b/a)*d*x^2 + (1 - (a^
2 + 4*a*b + b^2)*d)*x^4 + 2*a*(1 + b/a)*d*x^6 - d*x^8)), x], x, Sqrt[x]])/S
qrt[(a - x)*(b - x)*x] + (2*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][
Defer[Int][x^6/(Sqrt[-a + x^2]*Sqrt[-b + x^2]*(a^2*b^2*d - 2*a^2*b*(1 + b/a
)*d*x^2 - (1 - (a^2 + 4*a*b + b^2)*d)*x^4 - 2*a*(1 + b/a)*d*x^6 + d*x^8)),
x], x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x]
```

Rubi steps

$$\int \frac{-abx+x^3}{\sqrt{x(-a+x)(-b+x)}\left(a^2b^2d-2ab(a+b)dx+\left(-1+a^2d+4abd+b^2d\right)x^2-2(a+b)dx^3+dx^4\right)}\,dx = \int \frac{\ldots}{\sqrt{x}\ldots}$$

$$= \frac{\left(\sqrt{x}\sqrt{\ldots}\right.}{\ldots}$$

$$= \frac{\left(2\sqrt{x}\ldots\right.}{\ldots}$$

$$= \frac{\left(2\sqrt{x}\ldots\right.}{\ldots}$$

$$= \frac{\left(2\sqrt{x}\ldots\right.}{\ldots}$$

Mathematica [C] time = 6.42, size = 32986, normalized size = 434.03

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-(a*b*x) + x^3)/(Sqrt[x*(-a + x)*(-b + x)]*(a^2*b^2*d - 2*a*b*(a
 + b)*d*x + (-1 + a^2*d + 4*a*b*d + b^2*d)*x^2 - 2*(a + b)*d*x^3 + d*x^4)),
x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.31, size = 76, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}\right)}{\sqrt[4]{d}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}\right)}{\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*b*x) + x^3)/(Sqrt[x*(-a + x)*(-b + x)]*(a^2*b^2*d -
 2*a*b*(a + b)*d*x + (-1 + a^2*d + 4*a*b*d + b^2*d)*x^2 - 2*(a + b)*d*x^3 +
 d*x^4)),x]
```

```
[Out] ArcTan[x/(d^(1/4)*Sqrt[a*b*x + (-a - b)*x^2 + x^3])]/d^(1/4) - ArcTanh[x/(d
^(1/4)*Sqrt[a*b*x + (-a - b)*x^2 + x^3])]/d^(1/4)
```

fricas [B] time = 1.93, size = 433, normalized size = 5.70

$$\frac{\arctan\left(\frac{\sqrt{abx-(a+b)x^2+x^3}}{(ab-(a+b)x+x^2)d^{\frac{1}{4}}}\right)}{d^{\frac{1}{4}}}-\frac{\log\left(\frac{a^2b^2d-2(a+b)dx^3+dx^4-2(a^2b+ab^2)dx+((a^2+4ab+b^2)d+1)x^2+2\sqrt{abx-(a+b)x^2+x^3}\left(d^{\frac{1}{4}}x+\frac{abd-(a+b)dx+dx^2}{d^{\frac{1}{4}}}\right)+\frac{2(abdx-(a+b)dx^2+dx^3)}{\sqrt{d}}}{a^2b^2d-2(a+b)dx^3+dx^4-2(a^2b+ab^2)dx+((a^2+4ab+b^2)d-1)x^2}\right)}{4d^{\frac{1}{4}}}+\frac{\log\left(\frac{a^2b^2d-2(a+b)dx^3+dx^4-2(a^2b+ab^2)dx+((a^2+4ab+b^2)d+1)x^2-2\sqrt{abx-(a+b)x^2+x^3}\left(d^{\frac{1}{4}}x+\frac{abd-(a+b)dx+dx^2}{d^{\frac{1}{4}}}\right)+\frac{2(abdx-(a+b)dx^2+dx^3)}{\sqrt{d}}}{a^2b^2d-2(a+b)dx^3+dx^4-2(a^2b+ab^2)dx+((a^2+4ab+b^2)d-1)x^2}\right)}{4d^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*x+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2*d-2*a*b*(a+b)*d*x+(
a^2*d+4*a*b*d+b^2*d-1)*x^2-2*(a+b)*d*x^3+d*x^4),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(a*b*x - (a + b)*x^2 + x^3)/((a*b - (a + b)*x + x^2)*d^(1/4)))/d
^(1/4) - 1/4*log((a^2*b^2*d - 2*(a + b)*d*x^3 + d*x^4 - 2*(a^2*b + a*b^2)*d
*x + ((a^2 + 4*a*b + b^2)*d + 1)*x^2 + 2*sqrt(a*b*x - (a + b)*x^2 + x^3)*(d
^(1/4)*x + (a*b*d - (a + b)*d*x + d*x^2)/d^(1/4)) + 2*(a*b*d*x - (a + b)*d*
x^2 + d*x^3)/sqrt(d))/(a^2*b^2*d - 2*(a + b)*d*x^3 + d*x^4 - 2*(a^2*b + a*b
^2)*d*x + ((a^2 + 4*a*b + b^2)*d - 1)*x^2))/d^(1/4) + 1/4*log((a^2*b^2*d -
2*(a + b)*d*x^3 + d*x^4 - 2*(a^2*b + a*b^2)*d*x + ((a^2 + 4*a*b + b^2)*d +
1)*x^2 - 2*sqrt(a*b*x - (a + b)*x^2 + x^3)*(d^(1/4)*x + (a*b*d - (a + b)*d*
x + d*x^2)/d^(1/4)) + 2*(a*b*d*x - (a + b)*d*x^2 + d*x^3)/sqrt(d))/(a^2*b^2
*d - 2*(a + b)*d*x^3 + d*x^4 - 2*(a^2*b + a*b^2)*d*x + ((a^2 + 4*a*b + b^2)
*d - 1)*x^2))/d^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{abx-x^3}{\left(a^2b^2d-2(a+b)abdx-2(a+b)dx^3+dx^4+\left(a^2d+4abd+b^2d-1\right)x^2\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*x+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2*d-2*a*b*(a+b)*d*x+(
a^2*d+4*a*b*d+b^2*d-1)*x^2-2*(a+b)*d*x^3+d*x^4),x, algorithm="giac")
```

```
[Out] integrate(-(a*b*x - x^3)/((a^2*b^2*d - 2*(a + b)*a*b*d*x - 2*(a + b)*d*x^3
+ d*x^4 + (a^2*d + 4*a*b*d + b^2*d - 1)*x^2)*sqrt((a - x)*(b - x)*x)), x)
```

maple [C] time = 0.02, size = 303, normalized size = 3.99

$$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4+(-2ad-2bd)\_Z^3+\left(a^2d+4abd+b^2d-1\right)\_Z^2+\left(-2a^2bd-2ab^2d\right)\_Z+a^2b^2d\right)}\frac{\_\alpha\left(\_\alpha^2-ab\right)\left(-d\_\alpha^3+d\_\alpha^2a+2d\_\alpha^2b-2\_\alpha abd-\_\alpha b^2d+ab^2d+\_\alpha+a\right)\sqrt{-\frac{-a+x}{a}}\sqrt{\frac{-b+x}{a-b}}\sqrt{\frac{x}{a}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-a+x}{a}},\frac{-d\_\alpha^3+d\_\alpha^2a+2d\_\alpha^2b-2\_\alpha abd-\_\alpha b^2d+ab^2d+\_\alpha+a}{a},\sqrt{\frac{a}{a-b}}\right)}{\left(-2d\_\alpha^3+3d\_\alpha^2a+3d\_\alpha^2b-\_\alpha a^2d-4\_\alpha abd-\_\alpha b^2d+a^2bd+ab^2d+\_\alpha\right)\sqrt{x\left(ab-ax-bx+x^2\right)}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*b*x+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2*d-2*a*b*(a+b)*d*x+(a^2*d+
4*a*b*d+b^2*d-1)*x^2-2*(a+b)*d*x^3+d*x^4),x)
```

```
[Out] 1/a*sum(_alpha*(_alpha^2-a*b)/(-2*_alpha^3*d+3*_alpha^2*a*d+3*_alpha^2*b*d-
_alpha*a^2*d-4*_alpha*a*b*d-_alpha*b^2*d+a^2*b*d+a*b^2*d+_alpha)*(-_alpha^3
*d+_alpha^2*a*d+2*_alpha^2*b*d-2*_alpha*a*b*d-_alpha*b^2*d+a*b^2*d+_alpha+a
)*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(x*(a*b-a*x-b*x+x^2)
)^(1/2)*EllipticPi((-(-a+x)/a)^(1/2),(-_alpha^3*d+_alpha^2*a*d+2*_alpha^2*b
*d-2*_alpha*a*b*d-_alpha*b^2*d+a*b^2*d+_alpha+a)/a,(a/(a-b))^(1/2)),_alpha=
RootOf(d*_Z^4+(-2*a*d-2*b*d)*_Z^3+(a^2*d+4*a*b*d+b^2*d-1)*_Z^2+(-2*a^2*b*d-
2*a*b^2*d)*_Z+a^2*b^2*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{abx - x^3}{\left(a^2b^2d - 2(a+b)abdx - 2(a+b)dx^3 + dx^4 + \left(a^2d + 4abd + b^2d - 1\right)x^2\right)\sqrt{(a-x)(b-x)x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*x+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2*d-2*a*b*(a+b)*d*x+(
a^2*d+4*a*b*d+b^2*d-1)*x^2-2*(a+b)*d*x^3+d*x^4),x, algorithm="maxima")
```

```
[Out] -integrate((a*b*x - x^3)/((a^2*b^2*d - 2*(a + b)*a*b*d*x - 2*(a + b)*d*x^3
+ d*x^4 + (a^2*d + 4*a*b*d + b^2*d - 1)*x^2)*sqrt((a - x)*(b - x)*x)), x)
```

mupad [B] time = 7.32, size = 175, normalized size = 2.30

$$\frac{\ln\left(\frac{x-2\,d^{1/4}\,\sqrt{x\,(a-x)\,(b-x)}+\sqrt{d}\,x^2+a\,b\,\sqrt{d}-a\,\sqrt{d}\,x-b\,\sqrt{d}\,x}{x-\sqrt{d}\,x^2-a\,b\,\sqrt{d}+a\,\sqrt{d}\,x+b\,\sqrt{d}\,x}\right)}{2\,d^{1/4}}+\frac{\ln\left(\frac{x-\sqrt{d}\,x^2-a\,b\,\sqrt{d}+a\,\sqrt{d}\,x+b\,\sqrt{d}\,x+d^{1/4}\,\sqrt{x\,(a-x)\,(b-x)}\,2\mathrm{i}}{x+\sqrt{d}\,x^2+a\,b\,\sqrt{d}-a\,\sqrt{d}\,x-b\,\sqrt{d}\,x}\right)\,1\mathrm{i}}{2\,d^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - a*b*x)/((x*(a - x)*(b - x))^(1/2)*(x^2*(a^2*d + b^2*d + 4*a*b*d
- 1) + d*x^4 + a^2*b^2*d - 2*d*x^3*(a + b) - 2*a*b*d*x*(a + b))),x)
```

```
[Out] log((x - 2*d^(1/4)*(x*(a - x)*(b - x))^(1/2) + d^(1/2)*x^2 + a*b*d^(1/2) -
a*d^(1/2)*x - b*d^(1/2)*x)/(x - d^(1/2)*x^2 - a*b*d^(1/2) + a*d^(1/2)*x + b
*d^(1/2)*x))/(2*d^(1/4)) + (log((x + d^(1/4)*(x*(a - x)*(b - x))^(1/2)*2i -
 d^(1/2)*x^2 - a*b*d^(1/2) + a*d^(1/2)*x + b*d^(1/2)*x)/(x + d^(1/2)*x^2 +
a*b*d^(1/2) - a*d^(1/2)*x - b*d^(1/2)*x))*1i)/(2*d^(1/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*x+x**3)/(x*(-a+x)*(-b+x))**(1/2)/(a**2*b**2*d-2*a*b*(a+b)*d
*x+(a**2*d+4*a*b*d+b**2*d-1)*x**2-2*(a+b)*d*x**3+d*x**4),x)
```

```
[Out] Timed out
```

3.922 $\int \frac{1}{x\sqrt[3]{-1+x^6}}\,dx$

Optimal. Leaf size=76

$$-\frac{1}{6}\log\left(\sqrt[3]{x^6-1}+1\right)+\frac{1}{12}\log\left(\left(x^6-1\right)^{2/3}-\sqrt[3]{x^6-1}+1\right)-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {266, 56, 618, 204, 31}

$$-\frac{1}{4}\log\left(\sqrt[3]{x^6-1}+1\right)-\frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-1 + x^6)^(1/3)),x]
```

```
[Out] -1/2*ArcTan[(1 - 2*(-1 + x^6)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[x]/2 - Log[1 +
(-1 + x^6)^(1/3)]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-1+x^6}}\,dx = \frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x}\,dx, x, x^6\right)$$
$$= \frac{\log(x)}{2} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[3]{-1+x^6}\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[3]{-1+x^6}\right)$$
$$= \frac{\log(x)}{2} - \frac{1}{4}\log\left(1+\sqrt[3]{-1+x^6}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[3]{-1+x^6}\right)$$
$$= -\frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^6}}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\log(x)}{2} - \frac{1}{4}\log\left(1+\sqrt[3]{-1+x^6}\right)$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.37

$$\frac{1}{4}\left(x^6-1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-1 + x^6)^(1/3)),x]
```

```
[Out] ((-1 + x^6)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, 1 - x^6])/4
```

IntegrateAlgebraic [A] time = 0.05, size = 76, normalized size = 1.00

$$-\frac{1}{6}\log\left(\sqrt[3]{x^6-1}+1\right) + \frac{1}{12}\log\left(\left(x^6-1\right)^{2/3} - \sqrt[3]{x^6-1}+1\right) - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-1 + x^6)^(1/3)),x]
```

```
[Out] -1/2*ArcTan[1/Sqrt[3] - (2*(-1 + x^6)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 + (-1
 + x^6)^(1/3)]/6 + Log[1 - (-1 + x^6)^(1/3) + (-1 + x^6)^(2/3)]/12
```

fricas [A] time = 0.42, size = 58, normalized size = 0.76

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) + \frac{1}{12}\log\left(\left(x^6-1\right)^{\frac{2}{3}} - \left(x^6-1\right)^{\frac{1}{3}}+1\right) - \frac{1}{6}\log\left(\left(x^6-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(2/3*sqrt(3)*(x^6 - 1)^(1/3) - 1/3*sqrt(3)) + 1/12*log((x
^6 - 1)^(2/3) - (x^6 - 1)^(1/3) + 1) - 1/6*log((x^6 - 1)^(1/3) + 1)
```

giac [A] time = 0.26, size = 57, normalized size = 0.75

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^6-1\right)^{\frac{1}{3}}-1\right)\right) + \frac{1}{12}\log\left(\left(x^6-1\right)^{\frac{2}{3}} - \left(x^6-1\right)^{\frac{1}{3}}+1\right) - \frac{1}{6}\log\left(\left|\left(x^6-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3) - 1)) + 1/12*log((x^6 - 1
)^(2/3) - (x^6 - 1)^(1/3) + 1) - 1/6*log(abs((x^6 - 1)^(1/3) + 1))
```

maple [C] time = 0.27, size = 83, normalized size = 1.09

$$\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\mathrm{signum}\left(x^6-1\right)\right)^{\frac{1}{3}}\left(\frac{2\pi\sqrt{3}\,x^6\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^6\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+6\ln(x)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{12\pi\mathrm{signum}\left(x^6-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6-1)^(1/3),x)
```

```
[Out] 1/12/Pi*3^(1/2)*GAMMA(2/3)/signum(x^6-1)^(1/3)*(-signum(x^6-1))^(1/3)*(2/9*
Pi*3^(1/2)/GAMMA(2/3)*x^6*hypergeom([1,1,4/3],[2,2],x^6)+2/3*(-1/6*Pi*3^(1/
2)-3/2*ln(3)+6*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.43, size = 56, normalized size = 0.74

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^6-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{1}{12}\log\left(\left(x^6-1\right)^{\frac{2}{3}}-\left(x^6-1\right)^{\frac{1}{3}}+1\right)-\frac{1}{6}\log\left(\left(x^6-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3) - 1)) + 1/12*log((x^6 - 1
)^(2/3) - (x^6 - 1)^(1/3) + 1) - 1/6*log((x^6 - 1)^(1/3) + 1)
```

mupad [B] time = 1.10, size = 80, normalized size = 1.05

$$-\frac{\ln\left(\frac{\left(x^6-1\right)^{1/3}}{4}+\frac{1}{4}\right)}{6}-\ln\left(9\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)^2+\frac{\left(x^6-1\right)^{1/3}}{4}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(9\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)^2+\frac{\left(x^6-1\right)^{1/3}}{4}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^6 - 1)^(1/3)),x)
```

```
[Out] log(9*((3^(1/2)*1i)/12 + 1/12)^2 + (x^6 - 1)^(1/3)/4)*((3^(1/2)*1i)/12 + 1/
12) - log(9*((3^(1/2)*1i)/12 - 1/12)^2 + (x^6 - 1)^(1/3)/4)*((3^(1/2)*1i)/1
2 - 1/12) - log((x^6 - 1)^(1/3)/4 + 1/4)/6
```

sympy [C] time = 0.80, size = 32, normalized size = 0.42

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^6}\right)}{6x^2\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6-1)**(1/3),x)
```

```
[Out] -gamma(1/3)*hyper((1/3, 1/3), (4/3,), exp_polar(2*I*pi)/x**6)/(6*x**2*gamma
(4/3))
```

3.923 $\int \frac{(-1+x^4)(1-x^2+x^4)\sqrt{4-x^2+4x^4}}{(1+x^4)(4+7x^4+4x^8)}\,dx$

Optimal. Leaf size=76

$$\tan^{-1}\left(\frac{x}{\sqrt{4x^4-x^2+4}}\right)-\frac{3}{4}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt{4x^4-x^2+4}}\right)+\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt{4x^4-x^2+4}}\right)$$

Rubi [C] time = 3.67, antiderivative size = 1134, normalized size of antiderivative = 14.92, number of steps used = 50, number of rules used = 8, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.151, Rules used = {6725, 1208, 1197, 1103, 1195, 1216, 1706, 6728}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4)*(1 - x^2 + x^4)*Sqrt[4 - x^2 + 4*x^4])/((1 + x^4)*(4 + 7*x^
4 + 4*x^8)),x]
```

```
[Out] ArcTan[x/Sqrt[4 - x^2 + 4*x^4]] - (9*Sqrt[5]*(7*I + Sqrt[15])*ArcTan[(Sqrt[
3]*x)/Sqrt[4 - x^2 + 4*x^4]])/(8*(15 + (7*I)*Sqrt[15])) - (9*Sqrt[5]*(I + S
qrt[15])^2*ArcTan[(Sqrt[3]*x)/Sqrt[4 - x^2 + 4*x^4]])/(16*(15*I + 7*Sqrt[15
])) + ArcTanh[x/Sqrt[4 - x^2 + 4*x^4]]/4 + (7*(1 + x^2)*Sqrt[(4 - x^2 + 4*x
^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 9/16])/(4*Sqrt[4 - x^2 + 4*x^4]) -
(9*(5*I - 3*Sqrt[15])*(1 + x^2)*Sqrt[(4 - x^2 + 4*x^4)/(1 + x^2)^2]*Ellipti
cF[2*ArcTan[x], 9/16])/(16*(15*I - Sqrt[15])*Sqrt[4 - x^2 + 4*x^4]) - (3*(2
 - I*Sqrt[15])*(1 + x^2)*Sqrt[(4 - x^2 + 4*x^4)/(1 + x^2)^2]*EllipticF[2*Ar
cTan[x], 9/16])/(16*Sqrt[4 - x^2 + 4*x^4]) - ((4 - I*Sqrt[15])*(1 + x^2)*Sq
rt[(4 - x^2 + 4*x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 9/16])/(16*Sqrt[4
- x^2 + 4*x^4]) - (3*(2 + I*Sqrt[15])*(1 + x^2)*Sqrt[(4 - x^2 + 4*x^4)/(1 +
 x^2)^2]*EllipticF[2*ArcTan[x], 9/16])/(16*Sqrt[4 - x^2 + 4*x^4]) - ((4 + I
*Sqrt[15])*(1 + x^2)*Sqrt[(4 - x^2 + 4*x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan
[x], 9/16])/(16*Sqrt[4 - x^2 + 4*x^4]) + ((9*I + Sqrt[15])*(1 + x^2)*Sqrt[(
4 - x^2 + 4*x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 9/16])/(16*(15*I - Sqr
t[15])*Sqrt[4 - x^2 + 4*x^4]) + ((9*I - Sqrt[15])*(1 + x^2)*Sqrt[(4 - x^2 +
 4*x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 9/16])/(16*(15*I + Sqrt[15])*Sq
rt[4 - x^2 + 4*x^4]) - (9*(5*I + 3*Sqrt[15])*(1 + x^2)*Sqrt[(4 - x^2 + 4*x^
4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 9/16])/(16*(15*I + Sqrt[15])*Sqrt[4
- x^2 + 4*x^4]) + (45*(7*I + Sqrt[15])*(1 + x^2)*Sqrt[(4 - x^2 + 4*x^4)/(1
+ x^2)^2]*EllipticPi[3/8, 2*ArcTan[x], 9/16])/(32*(15*I - 7*Sqrt[15])*Sqrt[
4 - x^2 + 4*x^4]) + (45*(7*I - Sqrt[15])*(1 + x^2)*Sqrt[(4 - x^2 + 4*x^4)/(
1 + x^2)^2]*EllipticPi[3/8, 2*ArcTan[x], 9/16])/(32*(15*I + 7*Sqrt[15])*Sqr
t[4 - x^2 + 4*x^4]) + (3*(7*I + Sqrt[15])*(1 + x^2)*Sqrt[(4 - x^2 + 4*x^4)/
(1 + x^2)^2]*EllipticPi[5/8, 2*ArcTan[x], 9/16])/(32*(15*I - 7*Sqrt[15])*Sq
rt[4 - x^2 + 4*x^4]) + (3*(7*I - Sqrt[15])*(1 + x^2)*Sqrt[(4 - x^2 + 4*x^4)
/(1 + x^2)^2]*EllipticPi[5/8, 2*ArcTan[x], 9/16])/(32*(15*I + 7*Sqrt[15])*S
qrt[4 - x^2 + 4*x^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
```

```
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(-1+x^4\right)\left(1-x^2+x^4\right)\sqrt{4-x^2+4x^4}}{\left(1+x^4\right)\left(4+7x^4+4x^8\right)}\,dx = \int\left(\frac{2x^2\sqrt{4-x^2+4x^4}}{1+x^4}+\frac{\left(1-4x^2\right)\sqrt{4-x^2+4x^4}}{4\left(2-x^2+2x^4\right)}-\frac{3\left(1+4x\right.}{4\left(2\right.}\right.$$

$$=\frac{1}{4}\int\frac{\left(1-4x^2\right)\sqrt{4-x^2+4x^4}}{2-x^2+2x^4}\,dx-\frac{3}{4}\int\frac{\left(1+4x^2\right)\sqrt{4-x^2+4}}{2+x^2+2x^4}$$

$$=\frac{1}{4}\int\left(-\frac{4\sqrt{4-x^2+4x^4}}{-1-i\sqrt{15}+4x^2}-\frac{4\sqrt{4-x^2+4x^4}}{-1+i\sqrt{15}+4x^2}\right)dx-\frac{3}{4}\int\left(\frac{4\sqrt{}}{1-}\right.$$

$$=-\left(3\int\frac{\sqrt{4-x^2+4x^4}}{1-i\sqrt{15}+4x^2}\,dx\right)-3\int\frac{\sqrt{4-x^2+4x^4}}{1+i\sqrt{15}+4x^2}\,dx-\int\frac{\sqrt{4}}{}$$

$$=i\int\frac{1}{\left(i-x^2\right)\sqrt{4-x^2+4x^4}}\,dx+i\int\frac{1}{\left(i+x^2\right)\sqrt{4-x^2+4x^4}}\,d$$

$$=-\left((-3+4i)\int\frac{1}{\sqrt{4-x^2+4x^4}}\,dx\right)+\left(-\frac{1}{2}+\frac{i}{2}\right)\int\frac{1+}{\left(i+x^2\right)\sqrt{4}}$$

$$=\tan^{-1}\left(\frac{x}{\sqrt{4-x^2+4x^4}}\right)-\frac{9\left(5\sqrt{3}+7i\sqrt{5}\right)\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt{4-x^2+4x^4}}\right)}{8\left(15+7i\sqrt{15}\right)}$$

Mathematica [C] time = 2.54, size = 605, normalized size = 7.96

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)*(1 - x^2 + x^4)*Sqrt[4 - x^2 + 4*x^4])/((1 + x^4)*(4
+ 7*x^4 + 4*x^8)),x]
```

```
[Out] ((-1/16*I)*Sqrt[2 - (16*x^2)/(1 - (3*I)*Sqrt[7])]*Sqrt[1 - (8*x^2)/(1 + (3*
I)*Sqrt[7])]*(4*EllipticF[I*ArcSinh[2*Sqrt[-2/(1 - (3*I)*Sqrt[7])]*x], (I +
 3*Sqrt[7])/(I - 3*Sqrt[7])] + 4*EllipticPi[(-I - 3*Sqrt[7])/8, I*ArcSinh[2
*Sqrt[-2/(1 - (3*I)*Sqrt[7])]*x], (I + 3*Sqrt[7])/(I - 3*Sqrt[7])] + 4*Elli
pticPi[(I + 3*Sqrt[7])/8, I*ArcSinh[2*Sqrt[-2/(1 - (3*I)*Sqrt[7])]*x], (I +
 3*Sqrt[7])/(I - 3*Sqrt[7])] + EllipticPi[(I + 3*Sqrt[7])/(2*I - 2*Sqrt[15]
), I*ArcSinh[2*Sqrt[-2/(1 - (3*I)*Sqrt[7])]*x], (I + 3*Sqrt[7])/(I - 3*Sqrt
[7])] - 9*EllipticPi[(I + 3*Sqrt[7])/(2*(-I + Sqrt[15])), I*ArcSinh[2*Sqrt[
-2/(1 - (3*I)*Sqrt[7])]*x], (I + 3*Sqrt[7])/(I - 3*Sqrt[7])] - 9*EllipticPi
[-((I + 3*Sqrt[7])/(2*I + 2*Sqrt[15])), I*ArcSinh[2*Sqrt[-2/(1 - (3*I)*Sqrt
[7])]*x], (I + 3*Sqrt[7])/(I - 3*Sqrt[7])] + EllipticPi[(I + 3*Sqrt[7])/(2*
I + 2*Sqrt[15]), I*ArcSinh[2*Sqrt[-2/(1 - (3*I)*Sqrt[7])]*x], (I + 3*Sqrt[7
])/(I - 3*Sqrt[7])]))/(Sqrt[(-I)/(I + 3*Sqrt[7])]*Sqrt[4 - x^2 + 4*x^4])
```

IntegrateAlgebraic [A] time = 0.78, size = 76, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt{4x^4-x^2+4}}\right)-\frac{3}{4}\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt{4x^4-x^2+4}}\right)+\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt{4x^4-x^2+4}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(1 - x^2 + x^4)*Sqrt[4 - x^2 + 4*x^4])/((1 +
 x^4)*(4 + 7*x^4 + 4*x^8)),x]
```

[Out] ArcTan[x/Sqrt[4 - x^2 + 4*x^4]] - (3*Sqrt[3]*ArcTan[(Sqrt[3]*x)/Sqrt[4 - x^2 + 4*x^4]])/4 + ArcTanh[x/Sqrt[4 - x^2 + 4*x^4]]/4

fricas [A] time = 0.59, size = 114, normalized size = 1.50

$$-\frac{3}{8}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{4x^4-x^2+4}\,x}{2\left(x^4-x^2+1\right)}\right)+\frac{1}{2}\arctan\left(\frac{\sqrt{4x^4-x^2+4}\,x}{2x^4-x^2+2}\right)+\frac{1}{8}\log\left(-\frac{2x^4+\sqrt{4x^4-x^2+4}\,x+2}{2x^4-x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-1)*(x^4-x^2+1)*(4*x^4-x^2+4)^(1/2)/(x^4+1)/(4*x^8+7*x^4+4),x, algorithm="fricas")

[Out] -3/8*sqrt(3)*arctan(1/2*sqrt(3)*sqrt(4*x^4 - x^2 + 4)*x/(x^4 - x^2 + 1)) + 1/2*arctan(sqrt(4*x^4 - x^2 + 4)*x/(2*x^4 - x^2 + 2)) + 1/8*log(-(2*x^4 + sqrt(4*x^4 - x^2 + 4)*x + 2)/(2*x^4 - x^2 + 2))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4x^4-x^2+4}\left(x^4-x^2+1\right)\left(x^4-1\right)}{\left(4x^8+7x^4+4\right)\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-1)*(x^4-x^2+1)*(4*x^4-x^2+4)^(1/2)/(x^4+1)/(4*x^8+7*x^4+4),x, algorithm="giac")

[Out] integrate(sqrt(4*x^4 - x^2 + 4)*(x^4 - x^2 + 1)*(x^4 - 1)/((4*x^8 + 7*x^4 + 4)*(x^4 + 1)), x)

maple [C] time = 0.60, size = 671, normalized size = 8.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-1)*(x^4-x^2+1)*(4*x^4-x^2+4)^(1/2)/(x^4+1)/(4*x^8+7*x^4+4),x)

[Out] 4/(2+6*I*7^(1/2))^(1/2)*(1-(1/8+3/8*I*7^(1/2))*x^2)^(1/2)*(1-(1/8-3/8*I*7^(1/2))*x^2)^(1/2)/(4*x^4-x^2+4)^(1/2)*EllipticF(1/4*x*(2+6*I*7^(1/2))^(1/2),1/8*(-62-6*I*7^(1/2))^(1/2))-1/16*sum(_alpha*(-4/(-_alpha^2)^(1/2)*arctanh(1/130*(8*_alpha^2-1)*(-63*_alpha^2+65*x^2-16)/(-_alpha^2)^(1/2)/(4*x^4-x^2+4)^(1/2))+8^(1/2)*_alpha^3/(1+3*I*7^(1/2))^(1/2)*(-x^2+8-3*I*x^2*7^(1/2))^(1/2)*(-x^2+8+3*I*x^2*7^(1/2))^(1/2)/(4*x^4-x^2+4)^(1/2)*EllipticPi((1/8+3/8*I*7^(1/2))^(1/2)*x,3/8*I*_alpha^2*7^(1/2)-1/8*_alpha^2,(1/8-3/8*I*7^(1/2))^(1/2)/(1/8+3/8*I*7^(1/2))^(1/2))),_alpha=RootOf(_Z^4+1))+3/128*sum(_alpha*(-8*3^(1/2)/(-_alpha^2)^(1/2)*arctanh(1/46*(8*_alpha^2-1)*(-21*_alpha^2+23*x^2-16)/(-3*_alpha^2)^(1/2)/(4*x^4-x^2+4)^(1/2))-3*8^(1/2)*(-2*_alpha^3-_alpha)/(1+3*I*7^(1/2))^(1/2)*(-x^2+8-3*I*x^2*7^(1/2))^(1/2)*(-x^2+8+3*I*x^2*7^(1/2))^(1/2)/(4*x^4-x^2+4)^(1/2)*EllipticPi((1/8+3/8*I*7^(1/2))^(1/2)*x,3/8*I*_alpha^2*7^(1/2)-1/8*_alpha^2+3/16*I*7^(1/2)-1/16,(1/8-3/8*I*7^(1/2))^(1/2)/(1/8+3/8*I*7^(1/2))^(1/2))),_alpha=RootOf(2*_Z^4+_Z^2+2))-1/128*sum(_alpha*(-8/(_alpha^2)^(1/2)*arctanh(1/122*(8*_alpha^2-1)*(-63*_alpha^2+61*x^2+16)/(_alpha^2)^(1/2)/(4*x^4-x^2+4)^(1/2))-8^(1/2)*(-2*_alpha^3+_alpha)/(1+3*I*7^(1/2))^(1/2)*(-x^2+8-3*I*x^2*7^(1/2))^(1/2)*(-x^2+8+3*I*x^2*7^(1/2))^(1/2)/(4*x^4-x^2+4)^(1/2)*EllipticPi((1/8+3/8*I*7^(1/2))^(1/2)*x,3/8*I*_alpha^2*7^(1/2)-1/8*_alpha^2-3/16*I*7^(1/2)+1/16,(1/8-3/8*I*7^(1/2))^(1/2)/(1/8+3/8*I*7^(1/2))^(1/2))),_alpha=RootOf(2*_Z^4-_Z^2+2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4x^4-x^2+4}\left(x^4-x^2+1\right)\left(x^4-1\right)}{\left(4x^8+7x^4+4\right)\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4-x^2+1)*(4*x^4-x^2+4)^(1/2)/(x^4+1)/(4*x^8+7*x^4+4),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*x^4 - x^2 + 4)*(x^4 - x^2 + 1)*(x^4 - 1)/((4*x^8 + 7*x^4 +
 4)*(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-1\right)\,\left(x^4-x^2+1\right)\,\sqrt{4\,x^4-x^2+4}}{\left(x^4+1\right)\,\left(4\,x^8+7\,x^4+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(x^4 - x^2 + 1)*(4*x^4 - x^2 + 4)^(1/2))/((x^4 + 1)*(7*x^4 +
 4*x^8 + 4)),x)
```

```
[Out] int(((x^4 - 1)*(x^4 - x^2 + 1)*(4*x^4 - x^2 + 4)^(1/2))/((x^4 + 1)*(7*x^4 +
 4*x^8 + 4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**4-x**2+1)*(4*x**4-x**2+4)**(1/2)/(x**4+1)/(4*x**8+7*
x**4+4),x)
```

```
[Out] Timed out
```

3.924 $\int \sqrt[3]{1-3x+3x^3-9x^4+3x^6-9x^7+x^9-3x^{10}}\,dx$

Optimal. Leaf size=76

$$\frac{\left(210x^4-7x^3-3x^2+681x-229\right)\sqrt[3]{-3x^{10}+x^9-9x^7+3x^6-9x^4+3x^3-3x+1}}{910(x+1)\left(x^2-x+1\right)}$$

Rubi [A] time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.83, number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {6688, 6719, 1850}

$$\frac{\sqrt[3]{(1-3x)\left(x^3+1\right)^3}(1-3x)^4}{351\left(x^3+1\right)}-\frac{\sqrt[3]{(1-3x)\left(x^3+1\right)^3}(1-3x)^3}{90\left(x^3+1\right)}+\frac{\sqrt[3]{(1-3x)\left(x^3+1\right)^3}(1-3x)^2}{63\left(x^3+1\right)}-\frac{7\sqrt[3]{(1-3x)\left(x^3+1\right)^3}(1-3x)}{27\left(x^3+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 3*x^3 - 9*x^4 + 3*x^6 - 9*x^7 + x^9 - 3*x^10)^(1/3),x]
```

```
[Out] (-7*(1 - 3*x)*((1 - 3*x)*(1 + x^3)^3)^(1/3))/(27*(1 + x^3)) + ((1 - 3*x)^2*
((1 - 3*x)*(1 + x^3)^3)^(1/3))/(63*(1 + x^3)) - ((1 - 3*x)^3*((1 - 3*x)*(1
+ x^3)^3)^(1/3))/(90*(1 + x^3)) + ((1 - 3*x)^4*((1 - 3*x)*(1 + x^3)^3)^(1/3
))/(351*(1 + x^3))
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{1-3x+3x^3-9x^4+3x^6-9x^7+x^9-3x^{10}}\,dx &= \int \sqrt[3]{-\left((-1+3x)\left(1+x^3\right)^3\right)}\,dx\\ &= \frac{\sqrt[3]{-\left((-1+3x)\left(1+x^3\right)^3\right)}\int \sqrt[3]{-1+3x}\left(1+x^3\right)dx}{\sqrt[3]{-1+3x}\left(1+x^3\right)}\\ &= \frac{\sqrt[3]{-\left((-1+3x)\left(1+x^3\right)^3\right)}\int \left(\frac{28}{27}\sqrt[3]{-1+3x}+\frac{1}{9}(-1+3x)^{4/3}\right.}{\sqrt[3]{-1+3x}\left(1+x^3\right)}\\ &= -\frac{7(1-3x)\sqrt[3]{(1-3x)\left(1+x^3\right)^3}}{27\left(1+x^3\right)}+\frac{(1-3x)^2\sqrt[3]{(1-3x)\left(1+\right.}}{63\left(1+x^3\right)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.58

$$-\frac{\left(-\left((3x-1)\left(x^3+1\right)^3\right)\right)^{4/3}\left(70x^3+21x^2+6x+229\right)}{910\left(x^3+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 3*x^3 - 9*x^4 + 3*x^6 - 9*x^7 + x^9 - 3*x^10)^(1/3),x]
```

```
[Out] -1/910*((-((-1 + 3*x)*(1 + x^3)^3))^(4/3)*(229 + 6*x + 21*x^2 + 70*x^3))/(1
 + x^3)^4
```

IntegrateAlgebraic [A] time = 0.07, size = 71, normalized size = 0.93

$$\frac{\left(-70x^3-21x^2-6x-229\right)\left(-3x^{10}+x^9-9x^7+3x^6-9x^4+3x^3-3x+1\right)^{4/3}}{910(x+1)^4\left(x^2-x+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - 3*x + 3*x^3 - 9*x^4 + 3*x^6 - 9*x^7 + x^9 - 3*x^10)
^(1/3),x]
```

```
[Out] ((-229 - 6*x - 21*x^2 - 70*x^3)*(1 - 3*x + 3*x^3 - 9*x^4 + 3*x^6 - 9*x^7 +
x^9 - 3*x^10)^(4/3))/(910*(1 + x)^4*(1 - x + x^2)^4)
```

fricas [A] time = 0.40, size = 64, normalized size = 0.84

$$\frac{\left(-3\,x^{10}+x^9-9\,x^7+3\,x^6-9\,x^4+3\,x^3-3\,x+1\right)^{\frac{1}{3}}\left(210\,x^4-7\,x^3-3\,x^2+681\,x-229\right)}{910\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^10+x^9-9*x^7+3*x^6-9*x^4+3*x^3-3*x+1)^(1/3),x, algorithm="f
ricas")
```

```
[Out] 1/910*(-3*x^10 + x^9 - 9*x^7 + 3*x^6 - 9*x^4 + 3*x^3 - 3*x + 1)^(1/3)*(210*
x^4 - 7*x^3 - 3*x^2 + 681*x - 229)/(x^3 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-3\,x^{10}+x^9-9\,x^7+3\,x^6-9\,x^4+3\,x^3-3\,x+1\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^10+x^9-9*x^7+3*x^6-9*x^4+3*x^3-3*x+1)^(1/3),x, algorithm="g
iac")
```

```
[Out] integrate((-3*x^10 + x^9 - 9*x^7 + 3*x^6 - 9*x^4 + 3*x^3 - 3*x + 1)^(1/3),
x)
```

maple [A] time = 0.00, size = 73, normalized size = 0.96

$$\frac{(-1+3x)\left(70x^3+21x^2+6x+229\right)\left(-3x^{10}+x^9-9x^7+3x^6-9x^4+3x^3-3x+1\right)^{\frac{1}{3}}}{910\,(1+x)\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^10+x^9-9*x^7+3*x^6-9*x^4+3*x^3-3*x+1)^(1/3),x)
```

```
[Out] 1/910*(-1+3*x)*(70*x^3+21*x^2+6*x+229)*(-3*x^10+x^9-9*x^7+3*x^6-9*x^4+3*x^3
-3*x+1)^(1/3)/(1+x)/(x^2-x+1)
```

maxima [A] time = 0.43, size = 29, normalized size = 0.38

$$-\frac{1}{910}\left(210\,x^4-7\,x^3-3\,x^2+681\,x-229\right)(3\,x-1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^10+x^9-9*x^7+3*x^6-9*x^4+3*x^3-3*x+1)^(1/3),x, algorithm="m
axima")
```

```
[Out] -1/910*(210*x^4 - 7*x^3 - 3*x^2 + 681*x - 229)*(3*x - 1)^(1/3)
```

mupad [B] time = 0.78, size = 64, normalized size = 0.84

$$-\frac{\left(-\frac{3\,x^4}{13}+\frac{x^3}{130}+\frac{3\,x^2}{910}-\frac{681\,x}{910}+\frac{229}{910}\right)\left(-3\,x^{10}+x^9-9\,x^7+3\,x^6-9\,x^4+3\,x^3-3\,x+1\right)^{1/3}}{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3 - 3*x - 9*x^4 + 3*x^6 - 9*x^7 + x^9 - 3*x^10 + 1)^(1/3),x)
```

```
[Out] -(((3*x^2)/910 - (681*x)/910 + x^3/130 - (3*x^4)/13 + 229/910)*(3*x^3 - 3*x
 - 9*x^4 + 3*x^6 - 9*x^7 + x^9 - 3*x^10 + 1)^(1/3))/(x^3 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt[3]{-3x^{10}+x^9-9x^7+3x^6-9x^4+3x^3-3x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**10+x**9-9*x**7+3*x**6-9*x**4+3*x**3-3*x+1)**(1/3),x)
```

```
[Out] Integral((-3*x**10 + x**9 - 9*x**7 + 3*x**6 - 9*x**4 + 3*x**3 - 3*x + 1)**(
1/3), x)
```

3.925 $\int \frac{x(-b+x)\left(ab-2ax+x^2\right)}{(-a+x)\sqrt{x(-a+x)(-b+x)}\left(ad+(-b-d)x+x^2\right)}\,dx$

Optimal. Leaf size=77

$$2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{x^2(-a-b)+abx+x^3}}{\sqrt{d}\,(a-x)}\right)-\frac{2\sqrt{x^2(-a-b)+abx+x^3}}{a-x}$$

Rubi [F] time = 9.68, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(-b+x)\left(ab-2ax+x^2\right)}{(-a+x)\sqrt{x(-a+x)(-b+x)}\left(ad+(-b-d)x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-b + x)*(a*b - 2*a*x + x^2))/((-a + x)*Sqrt[x*(-a + x)*(-b + x)]*(a
*d + (-b - d)*x + x^2)),x]
```

```
[Out] (2*(b - x)*x)/Sqrt[(a - x)*(b - x)*x] - (4*Sqrt[a]*(b - x)*Sqrt[x]*Sqrt[1 -
 x/a]*EllipticE[ArcSin[Sqrt[x]/Sqrt[a]], a/b])/(Sqrt[(a - x)*(b - x)*x]*Sqr
t[1 - x/b]) + (2*Sqrt[a]*b*Sqrt[x]*Sqrt[1 - x/a]*Sqrt[1 - x/b]*EllipticF[Ar
cSin[Sqrt[x]/Sqrt[a]], a/b])/Sqrt[(a - x)*(b - x)*x] - ((2*a - b - d - Sqrt
[b^2 - 4*a*d + 2*b*d + d^2])*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Int][(
Sqrt[x]*Sqrt[-b + x])/((-a + x)^(3/2)*(-b - d - Sqrt[b^2 - 4*a*d + 2*b*d +
d^2] + 2*x)), x])/Sqrt[(a - x)*(b - x)*x] - ((2*a - b - d + Sqrt[b^2 - 4*a*
d + 2*b*d + d^2])*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Int][(Sqrt[x]*Sqr
t[-b + x])/((-a + x)^(3/2)*(-b - d + Sqrt[b^2 - 4*a*d + 2*b*d + d^2] + 2*x)
), x])/Sqrt[(a - x)*(b - x)*x]
```

Rubi steps

$$\int \frac{x(-b+x)\left(ab-2ax+x^2\right)}{(-a+x)\sqrt{x(-a+x)(-b+x)}\left(ad+(-b-d)x+x^2\right)}dx = \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{\sqrt{x}\sqrt{-b+x}\left(ab-2ax+x^2\right)}{(-a+x)^{3/2}\left(ad+(-b-d)x+x^2\right)}dx}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int\left(\frac{\sqrt{x}\sqrt{-b+x}}{(-a+x)^{3/2}}+\frac{\sqrt{x}\sqrt{-b+x}\,(a(b-\ldots}{(-a+x)^{3/2}\left(ad+\ldots\right.}\right.}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{\sqrt{x}\sqrt{-b+x}}{(-a+x)^{3/2}}dx}{\sqrt{x(-a+x)(-b+x)}} + \frac{\left(\sqrt{x}\sqrt{-a+\ldots\right.}{\ldots}$$

$$= \frac{2(b-x)x}{\sqrt{(a-x)(b-x)x}} + \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int\left(\frac{(-2a+\ldots}{(-a+\ldots}\right.}{\ldots}$$

$$= \frac{2(b-x)x}{\sqrt{(a-x)(b-x)x}} + \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{\sqrt{-b\ldots}}{\sqrt{x}\sqrt{\ldots}}}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{2(b-x)x}{\sqrt{(a-x)(b-x)x}} + \frac{\left(\left(-2a+b+d-\sqrt{b^2-4ad+2b\ldots}\right.\right.}{\ldots}$$

$$= \frac{2(b-x)x}{\sqrt{(a-x)(b-x)x}} - \frac{4\sqrt{a}\,(b-x)\sqrt{x}\sqrt{1-\frac{x}{a}}\,E\left(\sin^{-1}\left(\ldots\right.\right.}{\sqrt{(a-x)(b-x)x}\sqrt{1-\frac{x}{b}}\ldots}$$

Mathematica [C] time = 7.60, size = 226, normalized size = 2.94

$$\frac{2\left(x(a-x)^2(x-b)-\frac{id(x-a)^3\sqrt{\frac{x-b}{a-b}}\left(-\Pi\left(-\frac{2a}{-2a+b+d-\sqrt{b^2+2db+d^2-4ad}};i\sinh^{-1}\left(\sqrt{\frac{x}{a}-1}\right)|\frac{a}{a-b}\right)-\Pi\left(-\frac{2a}{-2a+b+d+\sqrt{b^2+2db+d^2-4ad}};i\sinh^{-1}\left(\sqrt{\frac{x}{a}-1}\right)|\frac{a}{a-b}\right)+F\left(i\sinh^{-1}\left(\sqrt{\frac{x}{a}-1}\right)|\frac{a}{a-b}\right)\right)}{\sqrt{1-\frac{a}{x}}}\right)}{(a-x)^2\sqrt{x(x-a)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-b + x)*(a*b - 2*a*x + x^2))/((-a + x)*Sqrt[x*(-a + x)*(-b +
x)]*(a*d + (-b - d)*x + x^2)),x]
```

```
[Out] (2*((a - x)^2*x*(-b + x) - (I*d*(-a + x)^3*Sqrt[(-b + x)/(a - b)]*(Elliptic
F[I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] - EllipticPi[(-2*a)/(-2*a + b + d -
 Sqrt[b^2 - 4*a*d + 2*b*d + d^2]), I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] -
EllipticPi[(-2*a)/(-2*a + b + d + Sqrt[b^2 - 4*a*d + 2*b*d + d^2]), I*ArcSi
nh[Sqrt[-1 + x/a]], a/(a - b)]))/Sqrt[1 - a/x]))/((a - x)^2*Sqrt[x*(-a + x)
*(-b + x)])
```

IntegrateAlgebraic [A] time = 0.76, size = 77, normalized size = 1.00

$$2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{x^2(-a-b)+abx+x^3}}{\sqrt{d}\,(a-x)}\right)-\frac{2\sqrt{x^2(-a-b)+abx+x^3}}{a-x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-b + x)*(a*b - 2*a*x + x^2))/((-a + x)*Sqrt[x*(-a +
x)*(-b + x)]*(a*d + (-b - d)*x + x^2)),x]
```

```
[Out] (-2*Sqrt[a*b*x + (-a - b)*x^2 + x^3])/(a - x) + 2*Sqrt[d]*ArcTanh[Sqrt[a*b*
x + (-a - b)*x^2 + x^3]/(Sqrt[d]*(a - x))]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(a*b-2*a*x+x^2)/(-a+x)/(x*(-a+x)*(-b+x))^(1/2)/(a*d+(-b-
d)*x+x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ab - 2\,ax + x^2)(b - x)x}{\sqrt{(a - x)(b - x)x}\,(ad - (b + d)x + x^2)(a - x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(a*b-2*a*x+x^2)/(-a+x)/(x*(-a+x)*(-b+x))^(1/2)/(a*d+(-b-
d)*x+x^2),x, algorithm="giac")
```

```
[Out] integrate((a*b - 2*a*x + x^2)*(b - x)*x/(sqrt((a - x)*(b - x)*x)*(a*d - (b
+ d)*x + x^2)*(a - x)), x)
```

maple [C] time = 0.05, size = 2792, normalized size = 36.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b+x)*(a*b-2*a*x+x^2)/(-a+x)/(x*(-a+x)*(-b+x))^(1/2)/(a*d+(-b-d)*x+x
^2),x)
```

```
[Out] -2*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^
2+x^3)^(1/2)*((a-b)*EllipticE((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))+b*Elliptic
F((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2)))+2*a^2*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b)
)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(
1/2),(a/(a-b))^(1/2))-2*d*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^
(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(
1/2))-d*(-4/(-4*a*d+b^2+2*b*d+d^2)^(1/2)*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/
(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2*d-1
/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2*b-1/
2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*d-2*a^2*(1-1/a*x)^(1
/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2
)/(a-1/2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(
1/2),a/(a-1/2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))+1/
(-4*a*d+b^2+2*b*d+d^2)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)
*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2*d-1/2*(-4*a*d+b^2
+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2*b-1/2*d-1/2*(-4*a*
d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*b^2+2/(-4*a*d+b^2+2*b*d+d^2)^(1/2)
*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-
b*x^2+x^3)^(1/2)/(a-1/2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticP
i((-(-a+x)/a)^(1/2),a/(a-1/2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(
a-b))^(1/2))*b*d+a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/
2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^
(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+
d^2)^(1/2)),(a/(a-b))^(1/2))*b+1/(-4*a*d+b^2+2*b*d+d^2)^(1/2)*a*(1-1/a*x)^(
1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/
2)/(a-1/2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^
(1/2),a/(a-1/2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*d
```

```
^2+a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^
2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*Ellipti
cPi((-(-a+x)/a)^(1/2),a/(a-1/2*b-1/2*d-1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a
/(a-b))^(1/2))*d+4/(-4*a*d+b^2+2*b*d+d^2)^(1/2)*a^2*(1-1/a*x)^(1/2)*(-1/(a-
b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-
1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1
/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*d-2*a^2*(1-1/
a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^
3)^(1/2)/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+
x)/a)^(1/2),a/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1
/2))-1/(-4*a*d+b^2+2*b*d+d^2)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x
)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2*d+1/2*(-4*
a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2*b-1/2*d+1/2
*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*b^2-2/(-4*a*d+b^2+2*b*d+d^2
)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x
-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*El
lipticPi((-(-a+x)/a)^(1/2),a/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2
)),(a/(a-b))^(1/2))*b*d+a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a
*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*
d+d^2)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2
+2*b*d+d^2)^(1/2)),(a/(a-b))^(1/2))*b-1/(-4*a*d+b^2+2*b*d+d^2)^(1/2)*a*(1-1
/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x
^3)^(1/2)/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*EllipticPi((-(-a
+x)/a)^(1/2),a/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2)),(a/(a-b))^(
1/2))*d^2+a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b
*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1/2))*
EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2*b-1/2*d+1/2*(-4*a*d+b^2+2*b*d+d^2)^(1
/2)),(a/(a-b))^(1/2))*d)-a*(a-b)*(-2*(-b*x+x^2)/a/(a-b)/((-a+x)*(-b*x+x^2))
^(1/2)-2*(-1/a-1/a/(a-b)*b)*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x
)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))
^(1/2))-2/(a-b)*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x
-a*x^2-b*x^2+x^3)^(1/2)*((a-b)*EllipticE((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))
+b*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ab - 2\,ax + x^2\right)(b - x)x}{\sqrt{(a - x)(b - x)x}\left(ad - (b + d)x + x^2\right)(a - x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(a*b-2*a*x+x^2)/(-a+x)/(x*(-a+x)*(-b+x))^(1/2)/(a*d+(-b-
d)*x+x^2),x, algorithm="maxima")
```

```
[Out] integrate((a*b - 2*a*x + x^2)*(b - x)*x/(sqrt((a - x)*(b - x)*x)*(a*d - (b
+ d)*x + x^2)*(a - x)), x)
```

mupad [B]   time = 0.87, size = 754, normalized size = 9.79

$$\frac{2\,a\,b\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}}{\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}-\frac{2\,b\left(a\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)-(a-b)\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\right)\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}}{\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}-\frac{2\,b\,d\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}}{\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}+\frac{2\,b\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(\frac{b}{\frac{b}{2}-\frac{d}{2}+\frac{\sqrt{b^2+2\,b\,d+d^2-4\,a\,d}}{2}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\left(a\,d-\frac{b\,d}{2}+\frac{d\,\sqrt{b^2+2\,b\,d+d^2-4\,a\,d}}{2}-\frac{d^2}{2}\right)}{\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}\,\left(\frac{b}{2}-\frac{d}{2}+\frac{\sqrt{b^2+2\,b\,d+d^2-4\,a\,d}}{2}\right)}+\frac{2\,b\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(-\frac{b}{\frac{d}{2}-\frac{b}{2}+\frac{\sqrt{b^2+2\,b\,d+d^2-4\,a\,d}}{2}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\left(\frac{b\,d}{2}-a\,d+\frac{d\,\sqrt{b^2+2\,b\,d+d^2-4\,a\,d}}{2}+\frac{d^2}{2}\right)}{\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}\,\left(\frac{d}{2}-\frac{b}{2}+\frac{\sqrt{b^2+2\,b\,d+d^2-4\,a\,d}}{2}\right)}+\frac{2\,b\left(\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)+\frac{b\,\sin\left(2\,\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\right)}{2\,\sqrt{\frac{b-x}{a-b}+1}\,(a-b)}\right)\sqrt{\frac{x}{b}}\,\left(a\,b-a^2\right)\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}}{\left(\frac{b}{a-b}+1\right)(a-b)\,\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b - x)*(a*b - 2*a*x + x^2))/((a - x)*(x*(a - x)*(b - x))^(1/2)*(a*d
 + x^2 - x*(b + d))),x)
```

```
[Out] (2*a*b*ellipticF(asin(((b - x)/b)^(1/2)), -b/(a - b))*(x/b)^(1/2)*((b - x)/
b)^(1/2)*((a - x)/(a - b))^(1/2))/(x^3 - x^2*(a + b) + a*b*x)^(1/2) - (2*b*
(a*ellipticF(asin(((b - x)/b)^(1/2)), -b/(a - b)) - (a - b)*ellipticE(asin(
((b - x)/b)^(1/2)), -b/(a - b)))*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a
- b))^(1/2))/(x^3 - x^2*(a + b) + a*b*x)^(1/2) - (2*b*d*ellipticF(asin(((b
```

```
- x)/b)^(1/2)), -b/(a - b))*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))
^(1/2))/(x^3 - x^2*(a + b) + a*b*x)^(1/2) + (2*b*(x/b)^(1/2)*((b - x)/b)^(1
/2)*((a - x)/(a - b))^(1/2)*ellipticPi(b/(b/2 - d/2 + (2*b*d - 4*a*d + b^2
+ d^2)^(1/2)/2), asin(((b - x)/b)^(1/2)), -b/(a - b))*(a*d - (b*d)/2 + (d*(
2*b*d - 4*a*d + b^2 + d^2)^(1/2))/2 - d^2/2))/((x^3 - x^2*(a + b) + a*b*x)^
(1/2)*(b/2 - d/2 + (2*b*d - 4*a*d + b^2 + d^2)^(1/2)/2)) + (2*b*(x/b)^(1/2)
*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*ellipticPi(-b/(d/2 - b/2 + (2*b*
d - 4*a*d + b^2 + d^2)^(1/2)/2), asin(((b - x)/b)^(1/2)), -b/(a - b))*((b*d
)/2 - a*d + (d*(2*b*d - 4*a*d + b^2 + d^2)^(1/2))/2 + d^2/2))/((x^3 - x^2*(
a + b) + a*b*x)^(1/2)*(d/2 - b/2 + (2*b*d - 4*a*d + b^2 + d^2)^(1/2)/2)) +
(2*b*(ellipticE(asin(((b - x)/b)^(1/2)), -b/(a - b)) + (b*sin(2*asin(((b -
x)/b)^(1/2))))/(2*((b - x)/(a - b) + 1)^(1/2)*(a - b)))*(x/b)^(1/2)*(a*b -
a^2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2))/((b/(a - b) + 1)*(a - b)*(x
^3 - x^2*(a + b) + a*b*x)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(a*b-2*a*x+x**2)/(-a+x)/(x*(-a+x)*(-b+x))**(1/2)/(a*d+(-
b-d)*x+x**2),x)
```

```
[Out] Timed out
```

3.926 $\int \frac{1+k^2x^2}{\sqrt{(1-x)x(1-k^2x)}\,(-1+k^2x^2)}\,dx$

Optimal. Leaf size=77

$$\frac{\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^3+(-k^2-1)x^2+x}}\right)}{1-k}+\frac{\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^3+(-k^2-1)x^2+x}}\right)}{-k-1}$$

Rubi [C] time = 3.68, antiderivative size = 250, normalized size of antiderivative = 3.25, number of steps used = 15, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6718, 6725, 115, 6688, 934, 12, 168, 537}

$$\frac{2(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\,\Pi\left(-\frac{1}{k};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{\sqrt{-k^2}\sqrt{x-x^2}\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{2(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\,\Pi\left(\frac{1}{k};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{\sqrt{-k^2}\sqrt{x-x^2}\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^2*x^2)),x]
```

```
[Out] (2*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticF[ArcSin[Sqrt[x]], k^2])/Sqr
t[(1 - x)*x*(1 - k^2*x)] + (2*(1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^2*x]*Elli
pticPi[-k^(-1), ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/(Sqrt[-k^2]*Sqrt[(1 -
 x)*x*(1 - k^2*x)]*Sqrt[x - x^2]) + (2*(1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^
2*x]*EllipticPi[k^(-1), ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/(Sqrt[-k^2]*S
qrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
```

```
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+k^2x^2}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(-1+k^2x^2\right)}\,dx = \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{1+k^2x^2}{\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\left(-1+k^2x^2\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \left(\frac{1}{\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}} + \frac{2}{\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\left(-1+k^2x^2\right)}\right)dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int}{\sqrt{(1-x)x}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int}{\sqrt{(1-x)x\left(1\right.}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int\left(\right.}{\sqrt{}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int}{\sqrt{(1-x)x\left(1-\right.}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{2}\sqrt{2-2x}\sqrt{1-x}\sqrt{-x}\right.}{\sqrt{(1-x}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{2-2x}\sqrt{1-x}\sqrt{-x}\sqrt{x}\right.}{\sqrt{(1-x)x}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\sqrt{2-2x}\sqrt{1-x}\sqrt{-x}\sqrt{x}\right.}{\sqrt{}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{2(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\,\Pi}{\sqrt{-k^2}\sqrt{(1-x)x}}$$

Mathematica [C] time = 1.80, size = 152, normalized size = 1.97

$$\frac{2i\sqrt{\frac{1}{x-1}+1}\,(x-1)^{3/2}\sqrt{\frac{1-\frac{1}{k^2}}{x-1}+1}\left(\left(k^2+1\right)F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+(k-1)\Pi\left(1+\frac{1}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)-(k+1)\Pi\left(\frac{k-1}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)}{\left(k^2-1\right)\sqrt{(x-1)x\left(k^2x-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^2*x^2)),x]
```

```
[Out] ((2*I)*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (1 - k^(-2))/(-1 + x)]*(-1 + x)^(3/
2)*((1 + k^2)*EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] + (-1 + k)*E
llipticPi[1 + k^(-1), I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] - (1 + k)*Elli
pticPi[(-1 + k)/k, I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)]))/((-1 + k^2)*Sqr
t[(-1 + x)*x*(-1 + k^2*x)])
```

IntegrateAlgebraic [A] time = 0.18, size = 77, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^3+(-k^2-1)x^2+x}}\right)}{1-k}+\frac{\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^3+(-k^2-1)x^2+x}}\right)}{-k-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^2*x^2
)),x]
```

```
[Out] ArcTan[((-1 + k)*x)/Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3]]/(1 - k) + ArcTan[((
1 + k)*x)/Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3]]/(-1 - k)
```

fricas [B] time = 0.51, size = 174, normalized size = 2.26

$$\frac{(k-1)\arctan\left(\frac{\sqrt{k^2x^3-(k^2+1)x^2+x}\,(k^2x^2-2\,(k^2+k+1)x+1)}{2\,((k^3+k^2)x^3-(k^3+k^2+k+1)x^2+(k+1)x)}\right)+(k+1)\arctan\left(\frac{\sqrt{k^2x^3-(k^2+1)x^2+x}\,(k^2x^2-2\,(k^2-k+1)x+1)}{2\,((k^3-k^2)x^3-(k^3-k^2+k-1)x^2+(k-1)x)}\right)}{2\,(k^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-1),x, algorithm="
fricas")
```

```
[Out] 1/2*((k - 1)*arctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2 - 2*(k^2
 + k + 1)*x + 1)/((k^3 + k^2)*x^3 - (k^3 + k^2 + k + 1)*x^2 + (k + 1)*x)) +
 (k + 1)*arctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2 - 2*(k^2 - k
 + 1)*x + 1)/((k^3 - k^2)*x^3 - (k^3 - k^2 + k - 1)*x^2 + (k - 1)*x)))/(k^2
 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^2+1}{(k^2x^2-1)\sqrt{(k^2x-1)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-1),x, algorithm="
giac")
```

```
[Out] integrate((k^2*x^2 + 1)/((k^2*x^2 - 1)*sqrt((k^2*x - 1)*(x - 1)*x)), x)
```

maple [C] time = 0.03, size = 323, normalized size = 4.19

$$-\frac{2\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\operatorname{EllipticF}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^2\sqrt{k^2x^3-k^2x^2-x^2+x}}-\frac{2\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\operatorname{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}-\frac{1}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^3\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}-\frac{1}{k}\right)}+\frac{2\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\operatorname{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}+\frac{1}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^3\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}+\frac{1}{k}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^2+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-1),x)
```

```
[Out] -2/k^2*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x
^3-k^2*x^2-x^2+x)^(1/2)*EllipticF((-(x-1/k^2)*k^2)^(1/2),(1/k^2/(1/k^2-1))^
(1/2))-2/k^3*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/
(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2-1/k)*EllipticPi((-(x-1/k^2)*k^2)^(1/2)
,1/k^2/(1/k^2-1/k),(1/k^2/(1/k^2-1))^(1/2))+2/k^3*(-(x-1/k^2)*k^2)^(1/2)*((
-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1
/k)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+1/k),(1/k^2/(1/k^2-1))^(
1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^2+1}{\left(k^2x^2-1\right)\sqrt{\left(k^2x-1\right)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-1),x, algorithm="
maxima")
```

```
[Out] integrate((k^2*x^2 + 1)/((k^2*x^2 - 1)*sqrt((k^2*x - 1)*(x - 1)*x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^2 + 1)/((k^2*x^2 - 1)*(x*(k^2*x - 1)*(x - 1))^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^2+1}{\sqrt{x\,(x-1)\left(k^2x-1\right)}\,(kx-1)\,(kx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k**2*x**2+1)/((1-x)*x*(-k**2*x+1))**(1/2)/(k**2*x**2-1),x)
```

```
[Out] Integral((k**2*x**2 + 1)/(sqrt(x*(x - 1)*(k**2*x - 1))*(k*x - 1)*(k*x + 1))
, x)
```

3.927 $\int \frac{1}{\sqrt[3]{1+x^3}}\,dx$

Optimal. Leaf size=77

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.00, antiderivative size = 46, normalized size of antiderivative = 0.60, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {239}

$$\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(\sqrt[3]{x^3+1}-x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)^(-1/3),x]
```

```
[Out] ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[-x + (1 + x^3)^(1
/3)]/2
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1+x^3}}\,dx=\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(-x+\sqrt[3]{1+x^3}\right)$$

Mathematica [A] time = 0.04, size = 78, normalized size = 1.01

$$-\frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{x^3+1}}\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(\frac{x}{\sqrt[3]{x^3+1}}+\frac{x^2}{\left(x^3+1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)^(-1/3),x]
```

```
[Out] ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 - x/(1 + x^3)^(
1/3)]/3 + Log[1 + x^2/(1 + x^3)^(2/3) + x/(1 + x^3)^(1/3)]/6
```

IntegrateAlgebraic [A] time = 0.15, size = 77, normalized size = 1.00

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)^(-1/3),x]
```

```
[Out] ArcTan[(Sqrt[3]*x)/(x + 2*(1 + x^3)^(1/3))]/Sqrt[3] - Log[-x + (1 + x^3)^(1
/3)]/3 + Log[x^2 + x*(1 + x^3)^(1/3) + (1 + x^3)^(2/3)]/6
```

fricas [A] time = 0.42, size = 76, normalized size = 0.99

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\,x+2\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}}{3\,x}\right)-\frac{1}{3}\log\left(-\frac{x-\left(x^3+1\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{6}\log\left(\frac{x^2+\left(x^3+1\right)^{\frac{1}{3}}x+\left(x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 + 1)^(1/3))/x) - 1/3*lo
g(-(x - (x^3 + 1)^(1/3))/x) + 1/6*log((x^2 + (x^3 + 1)^(1/3)*x + (x^3 + 1)^
(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)^(-1/3), x)
```

maple [C] time = 0.26, size = 14, normalized size = 0.18

$$x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+1)^(1/3),x)
```

```
[Out] x*hypergeom([1/3,1/3],[4/3],-x^3)
```

maxima [A] time = 0.43, size = 69, normalized size = 0.90

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+1\right)^{\frac{1}{3}}}{x}+1\right)\right)+\frac{1}{6}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)-\frac{1}{3}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3)/x + 1)) + 1/6*log((x^3 +
 1)^(1/3)/x + (x^3 + 1)^(2/3)/x^2 + 1) - 1/3*log((x^3 + 1)^(1/3)/x - 1)
```

mupad [B] time = 0.77, size = 12, normalized size = 0.16

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ -x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 + 1)^(1/3),x)
```

```
[Out] x*hypergeom([1/3, 1/3], 4/3, -x^3)
```

sympy [C] time = 0.74, size = 27, normalized size = 0.35

$$\frac{x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+1)**(1/3),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
```

3.928 $\int \frac{x+x^2}{(-1-2x+x^2)\sqrt{-x+x^3}}\,dx$

Optimal. Leaf size=77

$$\frac{1}{4}\left(-2-\sqrt{2}\right)\tanh^{-1}\left(\frac{x-1}{\left(\sqrt{2}-1\right)\sqrt{x^3-x}}\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\tanh^{-1}\left(\frac{x-1}{\left(1+\sqrt{2}\right)\sqrt{x^3-x}}\right)$$

Rubi [C] time = 0.69, antiderivative size = 251, normalized size of antiderivative = 3.26, number of steps used = 16, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 2056, 6728, 944, 329, 222, 933, 168, 537}

$$\frac{\left(2+\sqrt{2}\right)\sqrt{x-1}\sqrt{x}\sqrt{x+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{x-1}}\right)|\frac{1}{2}\right)}{2\sqrt{x^3-x}}-\frac{\left(2-\sqrt{2}\right)\sqrt{x-1}\sqrt{x}\sqrt{x+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{x-1}}\right)|\frac{1}{2}\right)}{2\sqrt{x^3-x}}+\frac{\left(3+2\sqrt{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(-\frac{1}{\sqrt{2}};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{2\sqrt{x^3-x}}-\frac{\left(3-2\sqrt{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{\sqrt{2}};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{2\sqrt{x^3-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x + x^2)/((-1 - 2*x + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] -1/2*((2 - Sqrt[2])*Sqrt[-1 + x]*Sqrt[x]*Sqrt[1 + x]*EllipticF[ArcSin[(Sqrt
[2]*Sqrt[x])/Sqrt[-1 + x]], 1/2])/Sqrt[-x + x^3] + ((2 + Sqrt[2])*Sqrt[-1 +
 x]*Sqrt[x]*Sqrt[1 + x]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[x])/Sqrt[-1 + x]], 1
/2])/(2*Sqrt[-x + x^3]) + ((3 + 2*Sqrt[2])*Sqrt[x]*Sqrt[1 - x^2]*EllipticPi
[-(1/Sqrt[2]), ArcSin[Sqrt[1 - x]], 1/2])/(2*Sqrt[-x + x^3]) - ((3 - 2*Sqrt
[2])*Sqrt[x]*Sqrt[1 - x^2]*EllipticPi[1/Sqrt[2], ArcSin[Sqrt[1 - x]], 1/2])
/(2*Sqrt[-x + x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
```

```
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 944

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x+x^2}{\left(-1-2x+x^2\right)\sqrt{-x+x^3}}\,dx = \int \frac{x(1+x)}{\left(-1-2x+x^2\right)\sqrt{-x+x^3}}\,dx$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{\sqrt{x}(1+x)}{\sqrt{-1+x^2}\left(-1-2x+x^2\right)}\,dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \left(\frac{\left(1+\sqrt{2}\right)\sqrt{x}}{\left(-2-2\sqrt{2}+2x\right)\sqrt{-1+x^2}}+\frac{\left(1-\sqrt{2}\right)\sqrt{x}}{\left(-2+2\sqrt{2}+2x\right)\sqrt{-1+x^2}}\right)dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\left(1-\sqrt{2}\right)\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{\sqrt{x}}{\left(-2+2\sqrt{2}+2x\right)\sqrt{-1+x^2}}\,dx}{\sqrt{-x+x^3}} + \frac{\left(\left(1+\sqrt{2}\right)\sqrt{x}\sqrt{-1+x^2}\right.}{\sqrt{-x}}$$

$$= \frac{\left(\left(1-\sqrt{2}\right)\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-1+x^2}}\,dx}{2\sqrt{-x+x^3}} + \frac{\left(\left(1-\sqrt{2}\right)^2\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{}{\sqrt{x}(-}}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\left(1-\sqrt{2}\right)^2\sqrt{x}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-x}\sqrt{x}\sqrt{1+x}\left(-2+2\sqrt{2}+2x\right)}\,dx}{\sqrt{-x+x^3}} + \frac{\left(\left(1+\sqrt{2}\right)^2\sqrt{x}\sqrt{1-\right.}{}$$

$$= -\frac{\left(2-\sqrt{2}\right)\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)\middle|\frac{1}{2}\right)}{2\sqrt{-x+x^3}} + \frac{\left(2+\sqrt{2}\right)\sqrt{-1+x}\sqrt{x}}{2\sqrt{}}$$

$$= -\frac{\left(2-\sqrt{2}\right)\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)\middle|\frac{1}{2}\right)}{2\sqrt{-x+x^3}} + \frac{\left(2+\sqrt{2}\right)\sqrt{-1+x}\sqrt{x}}{2\sqrt{}}$$

Mathematica [C] time = 0.81, size = 88, normalized size = 1.14

$$\frac{\sqrt{x\left(x^2-1\right)}\left(-2F\left(\sin^{-1}\left(\sqrt{x}\right)\middle|-1\right)-\left(\sqrt{2}-1\right)\Pi\left(-1-\sqrt{2};\sin^{-1}\left(\sqrt{x}\right)\middle|-1\right)+\left(1+\sqrt{2}\right)\Pi\left(-1+\sqrt{2};\sin^{-1}\left(\sqrt{x}\right)\middle|-1\right)\right)}{\sqrt{x}\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^2)/((-1 - 2*x + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] (Sqrt[x*(-1 + x^2)]*(-2*EllipticF[ArcSin[Sqrt[x]], -1] - (-1 + Sqrt[2])*Ell
ipticPi[-1 - Sqrt[2], ArcSin[Sqrt[x]], -1] + (1 + Sqrt[2])*EllipticPi[-1 +
Sqrt[2], ArcSin[Sqrt[x]], -1]))/(Sqrt[x]*Sqrt[1 - x^2])
```

IntegrateAlgebraic [A] time = 0.42, size = 89, normalized size = 1.16

$$\frac{1}{4}\left(2-\sqrt{2}\right)\tanh^{-1}\left(\frac{\left(\sqrt{2}-1\right)x-\sqrt{2}+1}{\sqrt{x^3-x}}\right)+\frac{1}{4}\left(-2-\sqrt{2}\right)\tanh^{-1}\left(\frac{\left(1+\sqrt{2}\right)x-\sqrt{2}-1}{\sqrt{x^3-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + x^2)/((-1 - 2*x + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] ((2 - Sqrt[2])*ArcTanh[(1 - Sqrt[2] + (-1 + Sqrt[2])*x)/Sqrt[-x + x^3]])/4
+ ((-2 - Sqrt[2])*ArcTanh[(-1 - Sqrt[2] + (1 + Sqrt[2])*x)/Sqrt[-x + x^3]])
/4
```

fricas [B] time = 0.49, size = 126, normalized size = 1.64

$$\frac{1}{8}\sqrt{2}\log\left(\frac{x^4+12x^3-4\sqrt{2}\sqrt{x^3-x}\left(x^2+2x-1\right)+2x^2-12x+1}{x^4-4x^3+2x^2+4x+1}\right)+\frac{1}{4}\log\left(\frac{x^4+4x^3+2x^2-4\sqrt{x^3-x}\left(x^2+1\right)-4x+1}{x^4-4x^3+2x^2+4x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(x^2-2*x-1)/(x^3-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log((x^4 + 12*x^3 - 4*sqrt(2)*sqrt(x^3 - x)*(x^2 + 2*x - 1) + 2
*x^2 - 12*x + 1)/(x^4 - 4*x^3 + 2*x^2 + 4*x + 1)) + 1/4*log((x^4 + 4*x^3 +
2*x^2 - 4*sqrt(x^3 - x)*(x^2 + 1) - 4*x + 1)/(x^4 - 4*x^3 + 2*x^2 + 4*x + 1
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + x}{\sqrt{x^3 - x}\left(x^2 - 2x - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(x^2-2*x-1)/(x^3-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + x)/(sqrt(x^3 - x)*(x^2 - 2*x - 1)), x)
```

maple [C] time = 0.03, size = 273, normalized size = 3.55

$$\frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticF}\left(\sqrt{1+x}, \frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-x}} + \frac{\sqrt{2}\,\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{1+x}, -\frac{1}{-2-\sqrt{2}}, \frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-x}\left(-2-\sqrt{2}\right)} + \frac{3\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{1+x}, -\frac{1}{-2-\sqrt{2}}, \frac{\sqrt{2}}{2}\right)}{2\sqrt{x^3-x}\left(-2-\sqrt{2}\right)} - \frac{\sqrt{2}\,\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{1+x}, -\frac{1}{-2+\sqrt{2}}, \frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-x}\left(-2+\sqrt{2}\right)} + \frac{3\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{1+x}, -\frac{1}{-2+\sqrt{2}}, \frac{\sqrt{2}}{2}\right)}{2\sqrt{x^3-x}\left(-2+\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)/(x^2-2*x-1)/(x^3-x)^(1/2),x)
```

```
[Out] (1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticF((1+x)^(1/2),1/
2*2^(1/2))+2^(1/2)*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)/(-2-2
^(1/2))*EllipticPi((1+x)^(1/2),-1/(-2-2^(1/2)),1/2*2^(1/2))+3/2*(1+x)^(1/2)
*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)/(-2-2^(1/2))*EllipticPi((1+x)^(1/2)
,-1/(-2-2^(1/2)),1/2*2^(1/2))-2^(1/2)*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/
(x^3-x)^(1/2)/(-2+2^(1/2))*EllipticPi((1+x)^(1/2),-1/(-2+2^(1/2)),1/2*2^(1/
2))+3/2*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)/(-2+2^(1/2))*Ell
ipticPi((1+x)^(1/2),-1/(-2+2^(1/2)),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + x}{\sqrt{x^3 - x}\left(x^2 - 2x - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(x^2-2*x-1)/(x^3-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + x)/(sqrt(x^3 - x)*(x^2 - 2*x - 1)), x)
```

mupad [B] time = 0.82, size = 159, normalized size = 2.06

$$\frac{\sqrt{2}\,\sqrt{-x}\,\left(3\sqrt{2}+4\right)\sqrt{1-x}\,\sqrt{x+1}\;\Pi\left(-\frac{1}{\sqrt{2}+1};\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{2\sqrt{x^3-x}\,\left(\sqrt{2}+1\right)} - \frac{\sqrt{2}\,\sqrt{-x}\,\left(3\sqrt{2}-4\right)\sqrt{1-x}\,\sqrt{x+1}\;\Pi\left(\frac{1}{\sqrt{2}-1};\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{2\sqrt{x^3-x}\,\left(\sqrt{2}-1\right)} - \frac{2\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\;\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{\sqrt{x^3-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + x^2)/((x^3 - x)^(1/2)*(2*x - x^2 + 1)),x)
```

```
[Out] (2^(1/2)*(-x)^(1/2)*(3*2^(1/2) + 4)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticPi(
-1/(2^(1/2) + 1), asin((-x)^(1/2)), -1))/(2*(x^3 - x)^(1/2)*(2^(1/2) + 1))
 - (2^(1/2)*(-x)^(1/2)*(3*2^(1/2) - 4)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticP
i(1/(2^(1/2) - 1), asin((-x)^(1/2)), -1))/(2*(x^3 - x)^(1/2)*(2^(1/2) - 1))
 - (2*(-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticF(asin((-x)^(1/2)), -1
))/(x^3 - x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(x+1)}{\sqrt{x(x-1)(x+1)}\,\left(x^2-2x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)/(x**2-2*x-1)/(x**3-x)**(1/2),x)
```

```
[Out] Integral(x*(x + 1)/(sqrt(x*(x - 1)*(x + 1))*(x**2 - 2*x - 1)), x)
```

3.929 $\int \frac{x^2(3ab-2(a+b)x+x^2)}{(x(-a+x)(-b+x))^{3/4}(-ab+(a+b)x-x^2+dx^3)}\,dx$

Optimal. Leaf size=77

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{x^2(-a-b)+abx+x^3}}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{x^2(-a-b)+abx+x^3}}\right)}{d^{3/4}}$$

Rubi [F] time = 13.65, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(3ab-2(a+b)x+x^2\right)}{(x(-a+x)(-b+x))^{3/4}\left(-ab+(a+b)x-x^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(3*a*b - 2*(a + b)*x + x^2))/((x*(-a + x)*(-b + x))^(3/4)*(-(a*b)
+ (a + b)*x - x^2 + d*x^3)),x]
```

```
[Out] (4*(1 - 2*a*d - 2*b*d)*x*(1 - x/a)^(3/4)*(1 - x/b)^(3/4)*AppellF1[1/4, 3/4,
 3/4, 5/4, x/a, x/b])/(d^2*((a - x)*(b - x)*x)^(3/4)) + (4*x^2*(1 - x/a)^(3
/4)*(1 - x/b)^(3/4)*AppellF1[5/4, 3/4, 3/4, 9/4, x/a, x/b])/(5*d*((a - x)*(
b - x)*x)^(3/4)) - (4*a*b*(1 - 2*a*d - 2*b*d)*x^(3/4)*(-a + x)^(3/4)*(-b +
x)^(3/4)*Defer[Subst][Defer[Int][1/((-a + x^4)^(3/4)*(-b + x^4)^(3/4)*(a*b
- a*(1 + b/a)*x^4 + x^8 - d*x^12)), x], x, x^(1/4)])/(d^2*((a - x)*(b - x)*
x)^(3/4)) + (4*(a + b - 2*a^2*d - 5*a*b*d - 2*b^2*d)*x^(3/4)*(-a + x)^(3/4)
*(-b + x)^(3/4)*Defer[Subst][Defer[Int][x^4/((-a + x^4)^(3/4)*(-b + x^4)^(3
/4)*(a*b - a*(1 + b/a)*x^4 + x^8 - d*x^12)), x], x, x^(1/4)])/(d^2*((a - x)
*(b - x)*x)^(3/4)) - (4*(1 - 3*b*d - 3*a*d*(1 - b*d))*x^(3/4)*(-a + x)^(3/4
)*(-b + x)^(3/4)*Defer[Subst][Defer[Int][x^8/((-a + x^4)^(3/4)*(-b + x^4)^(
3/4)*(a*b - a*(1 + b/a)*x^4 + x^8 - d*x^12)), x], x, x^(1/4)])/(d^2*((a - x
)*(b - x)*x)^(3/4))
```

Rubi steps

$$\int \frac{x^2\left(3ab-2(a+b)x+x^2\right)}{(x(-a+x)(-b+x))^{3/4}\left(-ab+(a+b)x-x^2+dx^3\right)}dx = \frac{\left(x^{3/4}(-a+x)^{3/4}(-b+x)^{3/4}\right)\int \frac{x^{5/4}\left(3ab-2(a+b)x+x\ldots\right.}{(-a+x)^{3/4}(-b+x)^{3/4}\left(-ab+(a+b\ldots\right.}}{(x(-a+x)(-b+x))^{3/4}}$$

$$= \frac{\left(4x^{3/4}(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\left(\int \frac{x^8\left(3ab\ldots\right.}{\left(-a+x^4\right)^{3/4}\left(-b+x^4\ldots\right.}\ldots\right.}{(x(-a+x)(-b+x))^{3/\ldots}}$$

$$= \frac{\left(4x^{3/4}(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\left(\int\left(\frac{1-2ad-2b\ldots}{d^2\left(-a+x^4\right)^{3/4}\left(-b\ldots\right.}\ldots\right.\right.}{\ldots}$$

$$= \frac{\left(4x^{3/4}(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\left(\int \frac{ab(1-2ad-2bd)+(2\ldots}{\left(-a+x^4\ldots\right.}\ldots\right.}{d^2(x(-a+x)(\ldots}$$

$$= \frac{\left(4x^{3/4}(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\left(\int\left(\frac{ab\ldots}{\left(-a+x^4\right)^{3/4}\left(-b+x\ldots\right.}\ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(4ab(1-2ad-2bd)x^{3/4}(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subs}\ldots}{d^2(x(-a+x\ldots}$$

$$= \frac{4(1-2ad-2bd)x\left(1-\frac{x}{a}\right)^{3/4}\left(1-\frac{x}{b}\right)^{3/4}F_1\left(\frac{1}{4};\frac{3}{4},\frac{3}{4};\frac{5}{4};\frac{x}{a}\ldots\right.}{d^2((a-x)(b-x)x)^{3/4}}$$

Mathematica [F] time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(3ab-2(a+b)x+x^2\right)}{(x(-a+x)(-b+x))^{3/4}\left(-ab+(a+b)x-x^2+dx^3\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(3*a*b - 2*(a + b)*x + x^2))/((x*(-a + x)*(-b + x))^(3/4)*(-
(a*b) + (a + b)*x - x^2 + d*x^3)),x]
```

```
[Out] Integrate[(x^2*(3*a*b - 2*(a + b)*x + x^2))/((x*(-a + x)*(-b + x))^(3/4)*(-
(a*b) + (a + b)*x - x^2 + d*x^3)), x]
```

IntegrateAlgebraic [A] time = 3.20, size = 77, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{x^2(-a-b)+abx+x^3}}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{x^2(-a-b)+abx+x^3}}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(3*a*b - 2*(a + b)*x + x^2))/((x*(-a + x)*(-b + x))
^(3/4)*(-(a*b) + (a + b)*x - x^2 + d*x^3)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*x)/(a*b*x + (-a - b)*x^2 + x^3)^(1/4)])/d^(3/4) - (2*Arc
Tanh[(d^(1/4)*x)/(a*b*x + (-a - b)*x^2 + x^3)^(1/4)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*a*b-2*(a+b)*x+x^2)/(x*(-a+x)*(-b+x))^(3/4)/(-a*b+(a+b)*x-x
^2+d*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,ab - 2\,(a+b)x + x^2\right)x^2}{\left(dx^3 - ab + (a+b)x - x^2\right)\left((a-x)(b-x)x\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*a*b-2*(a+b)*x+x^2)/(x*(-a+x)*(-b+x))^(3/4)/(-a*b+(a+b)*x-x
^2+d*x^3),x, algorithm="giac")
```

```
[Out] integrate((3*a*b - 2*(a + b)*x + x^2)*x^2/((d*x^3 - a*b + (a + b)*x - x^2)*
((a - x)*(b - x)*x)^(3/4)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(3ab - 2\,(a+b)\,x + x^2\right)}{\left(x\,(-a+x)\,(-b+x)\right)^{\frac{3}{4}}\left(-ab + (a+b)\,x - x^2 + d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*a*b-2*(a+b)*x+x^2)/(x*(-a+x)*(-b+x))^(3/4)/(-a*b+(a+b)*x-x^2+d*x
^3),x)
```

```
[Out] int(x^2*(3*a*b-2*(a+b)*x+x^2)/(x*(-a+x)*(-b+x))^(3/4)/(-a*b+(a+b)*x-x^2+d*x
^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,ab - 2\,(a+b)x + x^2\right)x^2}{\left(dx^3 - ab + (a+b)x - x^2\right)\left((a-x)(b-x)x\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*a*b-2*(a+b)*x+x^2)/(x*(-a+x)*(-b+x))^(3/4)/(-a*b+(a+b)*x-x
^2+d*x^3),x, algorithm="maxima")
```

```
[Out] integrate((3*a*b - 2*(a + b)*x + x^2)*x^2/((d*x^3 - a*b + (a + b)*x - x^2)*
((a - x)*(b - x)*x)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2\,\left(3\,a\,b + x^2 - 2\,x\,(a+b)\right)}{{\left(x\,(a-x)\,(b-x)\right)}^{3/4}\,\left(-d\,x^3 + x^2 + (-a-b)\,x + a\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(3*a*b + x^2 - 2*x*(a + b)))/((x*(a - x)*(b - x))^(3/4)*(a*b - d*
x^3 + x^2 - x*(a + b))),x)
```

[Out] -int((x^2*(3*a*b + x^2 - 2*x*(a + b)))/((x*(a - x)*(b - x))^(3/4)*(a*b - d*
x^3 + x^2 - x*(a + b))), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(3*a*b-2*(a+b)*x+x**2)/(x*(-a+x)*(-b+x))**(3/4)/(-a*b+(a+b)*
x-x**2+d*x**3),x)

[Out] Timed out

3.930 $\int \frac{1}{x\sqrt[4]{-1+x^4}}\,dx$

Optimal. Leaf size=77

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^4-1}}{\sqrt{x^4-1}-1}\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+1}\right)}{2\sqrt{2}}$$

Rubi [A] time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.68, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {266, 63, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(\sqrt{x^4-1}-\sqrt{2}\sqrt[4]{x^4-1}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{x^4-1}+\sqrt{2}\sqrt[4]{x^4-1}+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{x^4-1}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt[4]{x^4-1}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-1 + x^4)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[1 - Sqrt[2]*(-1 + x^4)^(1/4)]/Sqrt[2] + ArcTan[1 + Sqrt[2]*(-1
+ x^4)^(1/4)]/(2*Sqrt[2]) + Log[1 - Sqrt[2]*(-1 + x^4)^(1/4) + Sqrt[-1 + x^
4]]/(4*Sqrt[2]) - Log[1 + Sqrt[2]*(-1 + x^4)^(1/4) + Sqrt[-1 + x^4]]/(4*Sqr
t[2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt[4]{-1+x^4}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{-1+x}\,x}\,dx, x, x^4\right)\\
&= \operatorname{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt[4]{-1+x^4}\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt[4]{-1+x^4}\right)\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt[4]{-1+x^4}\right)\\
&= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt[4]{-1+x^4}\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt[4]{-1+x^4}\right)\cdots\\
&= \frac{\log\left(1-\sqrt{2}\sqrt[4]{-1+x^4}+\sqrt{-1+x^4}\right)}{4\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt[4]{-1+x^4}+\sqrt{-1+x^4}\right)}{4\sqrt{2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{-1-\cdots}\right.}{\cdots}\\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+x^4}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+x^4}\right)}{2\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt[4]{-1+x^4}+\sqrt{-1+\cdots}\right.}{4\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.36

$$\frac{1}{3}\left(x^4-1\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};1-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-1 + x^4)^(1/4)),x]
[Out] ((-1 + x^4)^(3/4)*Hypergeometric2F1[3/4, 1, 7/4, 1 - x^4])/3
```

IntegrateAlgebraic [A] time = 0.09, size = 82, normalized size = 1.06

$$\frac{\tan^{-1}\left(\frac{\frac{\sqrt{x^4-1}}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+1}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-1 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[(-(1/Sqrt[2]) + Sqrt[-1 + x^4]/Sqrt[2])/(-1 + x^4)^(1/4)]/(2*Sqrt[2]
) - ArcTanh[(Sqrt[2]*(-1 + x^4)^(1/4))/(1 + Sqrt[-1 + x^4])]/(2*Sqrt[2])
```

fricas [B] time = 0.47, size = 155, normalized size = 2.01

$$-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+\sqrt{x^4-1}+1}-\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}-1\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+4\sqrt{x^4-1}+4}-\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+1\right)-\frac{1}{8}\sqrt{2}\log\left(4\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+4\sqrt{x^4-1}+4\right)+\frac{1}{8}\sqrt{2}\log\left(-4\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+4\sqrt{x^4-1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-1)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*(x^4 - 1)^(1/4) + sqrt(x^4 - 1) +
1) - sqrt(2)*(x^4 - 1)^(1/4) - 1) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*
sqrt(2)*(x^4 - 1)^(1/4) + 4*sqrt(x^4 - 1) + 4) - sqrt(2)*(x^4 - 1)^(1/4) +
1) - 1/8*sqrt(2)*log(4*sqrt(2)*(x^4 - 1)^(1/4) + 4*sqrt(x^4 - 1) + 4) + 1/8
*sqrt(2)*log(-4*sqrt(2)*(x^4 - 1)^(1/4) + 4*sqrt(x^4 - 1) + 4)
```

giac [A] time = 0.21, size = 102, normalized size = 1.32

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(x^4-1\right)^{\frac{1}{4}}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(x^4-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+\sqrt{x^4-1}+1\right)+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+\sqrt{x^4-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-1)^(1/4),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^4 - 1)^(1/4))) + 1/4*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^4 - 1)^(1/4))) - 1/8*sqrt(2)*log(sqrt(
2)*(x^4 - 1)^(1/4) + sqrt(x^4 - 1) + 1) + 1/8*sqrt(2)*log(-sqrt(2)*(x^4 - 1
)^(1/4) + sqrt(x^4 - 1) + 1)
```

maple [C] time = 0.31, size = 79, normalized size = 1.03

$$\frac{\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)\left(-\mathrm{signum}\left(x^4-1\right)\right)^{\frac{1}{4}}\left(\frac{\pi\sqrt{2}\,x^4\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{4}\right],[2,2],x^4\right)}{4\Gamma\left(\frac{3}{4}\right)}+\frac{\left(-3\ln(2)-\frac{\pi}{2}+4\ln(x)+i\pi\right)\pi\sqrt{2}}{\Gamma\left(\frac{3}{4}\right)}\right)}{8\pi\,\mathrm{signum}\left(x^4-1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4-1)^(1/4),x)
```

```
[Out] 1/8/Pi*2^(1/2)*GAMMA(3/4)/signum(x^4-1)^(1/4)*(-signum(x^4-1))^(1/4)*(1/4*P
i*2^(1/2)/GAMMA(3/4)*x^4*hypergeom([1,1,5/4],[2,2],x^4)+(-3*ln(2)-1/2*Pi+4*
ln(x)+I*Pi)*Pi*2^(1/2)/GAMMA(3/4))
```

maxima [A] time = 0.44, size = 102, normalized size = 1.32

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(x^4-1\right)^{\frac{1}{4}}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(x^4-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+\sqrt{x^4-1}+1\right)+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+\sqrt{x^4-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-1)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^4 - 1)^(1/4))) + 1/4*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^4 - 1)^(1/4))) - 1/8*sqrt(2)*log(sqrt(
2)*(x^4 - 1)^(1/4) + sqrt(x^4 - 1) + 1) + 1/8*sqrt(2)*log(-sqrt(2)*(x^4 - 1
)^(1/4) + sqrt(x^4 - 1) + 1)
```

mupad [B] time = 0.89, size = 45, normalized size = 0.58

$$\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(x^4-1\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(x^4-1\right)^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^4 - 1)^(1/4)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*(x^4 - 1)^(1/4)*(1/2 - 1i/2))*(1/4 - 1i/4) + 2^(1/2)*a
tan(2^(1/2)*(x^4 - 1)^(1/4)*(1/2 + 1i/2))*(1/4 + 1i/4)
```

sympy [C] time = 0.78, size = 31, normalized size = 0.40

$$-\frac{\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{e^{2i\pi}}{x^4}\right)}{4x\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4-1)**(1/4),x)
```

```
[Out] -gamma(1/4)*hyper((1/4, 1/4), (5/4,), exp_polar(2*I*pi)/x**4)/(4*x*gamma(5/
4))
```

3.931

$$\int \frac{(-a+x)(-b+x)\left(-ab+x^2\right)}{\sqrt{x(-a+x)(-b+x)}\left(a^2b^2-2ab(a+b)x+\left(a^2+4ab+b^2-d\right)x^2-2(a+b)x^3+x^4\right)}\,dx$$

Optimal. Leaf size=77

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt{x^2(-a-b)+abx+x^3}}\right)}{\sqrt[4]{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt{x^2(-a-b)+abx+x^3}}\right)}{\sqrt[4]{d}}$$

Rubi [F] time = 19.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-a+x)(-b+x)\left(-ab+x^2\right)}{\sqrt{x(-a+x)(-b+x)}\left(a^2b^2-2ab(a+b)x+\left(a^2+4ab+b^2-d\right)x^2-2(a+b)x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-a + x)*(-b + x)*(-(a*b) + x^2))/(Sqrt[x*(-a + x)*(-b + x)]*(a^2*b^2
- 2*a*b*(a + b)*x + (a^2 + 4*a*b + b^2 - d)*x^2 - 2*(a + b)*x^3 + x^4)),x]
```

```
[Out] (2*a*b*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][(Sqrt[-a +
 x^2]*Sqrt[-b + x^2])/(-(a^2*(b - x^2)^2) + 2*a*(-(b*x) + x^3)^2 - x^4*(b^2
 - d - 2*b*x^2 + x^4)), x], x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x] + (2*Sqrt[
x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][(x^4*Sqrt[-a + x^2]*Sq
rt[-b + x^2])/(a^2*(b - x^2)^2 - 2*a*(-(b*x) + x^3)^2 + x^4*(b^2 - d - 2*b*
x^2 + x^4)), x], x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x]
```

Rubi steps

$$\int \frac{(-a+x)(-b+x)\left(-ab+x^2\right)}{\sqrt{x(-a+x)(-b+x)}\left(a^2b^2-2ab(a+b)x+\left(a^2+4ab+b^2-d\right)x^2-2(a+b)x^3+x^4\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{\ldots}\right.}{\ldots}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\ldots\right.}{\ldots}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\ldots\right.}{\ldots}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\ldots\right.}{\ldots}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\ldots\right.}{\ldots}$$

Mathematica [C] time = 13.75, size = 22729, normalized size = 295.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-a + x)*(-b + x)*(-(a*b) + x^2))/(Sqrt[x*(-a + x)*(-b + x)]*(a^
2*b^2 - 2*a*b*(a + b)*x + (a^2 + 4*a*b + b^2 - d)*x^2 - 2*(a + b)*x^3 + x^4
)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.28, size = 77, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt{x^2(-a-b)+abx+x^3}}\right)}{\sqrt[4]{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt{x^2(-a-b)+abx+x^3}}\right)}{\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-a + x)*(-b + x)*(-(a*b) + x^2))/(Sqrt[x*(-a + x)*(-b
+ x)]*(a^2*b^2 - 2*a*b*(a + b)*x + (a^2 + 4*a*b + b^2 - d)*x^2 - 2*(a + b)*
x^3 + x^4)),x]
```

```
[Out] -(ArcTan[(d^(1/4)*x)/Sqrt[a*b*x + (-a - b)*x^2 + x^3]]/d^(1/4)) - ArcTanh[(
d^(1/4)*x)/Sqrt[a*b*x + (-a - b)*x^2 + x^3]]/d^(1/4)
```

fricas [B] time = 0.93, size = 406, normalized size = 5.27

$$-\frac{\arctan\left(\frac{\sqrt{abx-(a+b)x^2+x^3}\,d^{\frac{1}{4}}}{ab-(a+b)x+x^2}\right)}{d^{\frac{1}{4}}}-\frac{\log\left(\frac{a^2b^2-2\,(a+b)x^3+x^4+\left(a^2+4\,ab+b^2+d\right)x^2-2\,\left(a^2b+ab^2\right)x+2\,\sqrt{abx-(a+b)x^2+x^3}\left(d^{\frac{3}{4}}x+\frac{abd-(a+b)dx+dx^2}{d^{\frac{3}{4}}}\right)+\frac{2\left(abdx-(a+b)dx^2+dx^3\right)}{\sqrt{d}}}{a^2b^2-2\,(a+b)x^3+x^4+\left(a^2+4\,ab+b^2-d\right)x^2-2\,\left(a^2b+ab^2\right)x}\right)}{4\,d^{\frac{1}{4}}}+\frac{\log\left(\frac{a^2b^2-2\,(a+b)x^3+x^4+\left(a^2+4\,ab+b^2+d\right)x^2-2\,\left(a^2b+ab^2\right)x-2\,\sqrt{abx-(a+b)x^2+x^3}\left(d^{\frac{3}{4}}x+\frac{abd-(a+b)dx+dx^2}{d^{\frac{3}{4}}}\right)+\frac{2\left(abdx-(a+b)dx^2+dx^3\right)}{\sqrt{d}}}{a^2b^2-2\,(a+b)x^3+x^4+\left(a^2+4\,ab+b^2-d\right)x^2-2\,\left(a^2b+ab^2\right)x}\right)}{4\,d^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-a*b+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2-2*a*b*(
a+b)*x+(a^2+4*a*b+b^2-d)*x^2-2*(a+b)*x^3+x^4),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(a*b*x - (a + b)*x^2 + x^3)*d^(1/4)/(a*b - (a + b)*x + x^2))/d^
(1/4) - 1/4*log((a^2*b^2 - 2*(a + b)*x^3 + x^4 + (a^2 + 4*a*b + b^2 + d)*x^
2 - 2*(a^2*b + a*b^2)*x + 2*sqrt(a*b*x - (a + b)*x^2 + x^3)*(d^(3/4)*x + (a
*b*d - (a + b)*d*x + d*x^2)/d^(3/4)) + 2*(a*b*d*x - (a + b)*d*x^2 + d*x^3)/
sqrt(d))/(a^2*b^2 - 2*(a + b)*x^3 + x^4 + (a^2 + 4*a*b + b^2 - d)*x^2 - 2*(
a^2*b + a*b^2)*x))/d^(1/4) + 1/4*log((a^2*b^2 - 2*(a + b)*x^3 + x^4 + (a^2
+ 4*a*b + b^2 + d)*x^2 - 2*(a^2*b + a*b^2)*x - 2*sqrt(a*b*x - (a + b)*x^2 +
 x^3)*(d^(3/4)*x + (a*b*d - (a + b)*d*x + d*x^2)/d^(3/4)) + 2*(a*b*d*x - (a
 + b)*d*x^2 + d*x^3)/sqrt(d))/(a^2*b^2 - 2*(a + b)*x^3 + x^4 + (a^2 + 4*a*b
 + b^2 - d)*x^2 - 2*(a^2*b + a*b^2)*x))/d^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(ab-x^2\right)(a-x)(b-x)}{\left(a^2b^2-2\,(a+b)abx-2\,(a+b)x^3+x^4+\left(a^2+4\,ab+b^2-d\right)x^2\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-a*b+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2-2*a*b*(
a+b)*x+(a^2+4*a*b+b^2-d)*x^2-2*(a+b)*x^3+x^4),x, algorithm="giac")
```

```
[Out] integrate(-(a*b - x^2)*(a - x)*(b - x)/((a^2*b^2 - 2*(a + b)*a*b*x - 2*(a +
 b)*x^3 + x^4 + (a^2 + 4*a*b + b^2 - d)*x^2)*sqrt((a - x)*(b - x)*x)), x)
```

maple [C] time = 0.06, size = 427, normalized size = 5.55

$$-\frac{2a\sqrt{-\frac{-a+x}{a}}\,\sqrt{\frac{-b+x}{a-b}}\,\sqrt{\frac{x}{a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{-a+x}{a}},\sqrt{\frac{a}{a-b}}\right)}{\sqrt{abx-a\,x^2-b\,x^2+x^3}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4+(-2a-2b)\_Z^3+\left(a^2+4ab+b^2-d\right)\_Z^2+\left(-2a^2b-2a\,b^2\right)\_Z+a^2b^2\right)}\frac{\left(-\_\alpha^3a-\_\alpha^3b+\_\alpha^2a^2+4\_\alpha^2ab+\_\alpha^2b^2-3\_\alpha\,a^2b-3\_\alpha a\,b^2+2a^2b^2-\_\alpha^2d\right)\left(-\_\alpha^3+\_\alpha^2a+2\_\alpha^2b-2\_\alpha ab-\_\alpha\,b^2+a\,b^2+\_\alpha d+ad\right)\sqrt{-\frac{-a+x}{a}}\,\sqrt{\frac{-b+x}{a-b}}\,\sqrt{\frac{x}{a}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-a+x}{a}},\frac{-\_\alpha^3+\_\alpha^2a+2\_\alpha^2b-2\_\alpha ab-\_\alpha\,b^2+a\,b^2+\_\alpha d+ad}{ad},\sqrt{\frac{a}{a-b}}\right)}{\left(-2\_\alpha^3+3\_\alpha^2a+3\_\alpha^2b-\_\alpha\,a^2-4\_\alpha ab-\_\alpha\,b^2+a^2b+a\,b^2+\_\alpha d\right)\sqrt{x\left(ab-ax-bx+x^2\right)}}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x)*(-b+x)*(-a*b+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2-2*a*b*(a+b)*x
+(a^2+4*a*b+b^2-d)*x^2-2*(a+b)*x^3+x^4),x)
```

```
[Out] -2*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^
2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))-1/a/d*sum((-_alph
a^3*a-_alpha^3*b+_alpha^2*a^2+4*_alpha^2*a*b+_alpha^2*b^2-3*_alpha*a^2*b-3*
_alpha*a*b^2+2*a^2*b^2-_alpha^2*d)/(-2*_alpha^3+3*_alpha^2*a+3*_alpha^2*b-_
alpha*a^2-4*_alpha*a*b-_alpha*b^2+a^2*b+a*b^2+_alpha*d)*(-_alpha^3+_alpha^2
*a+2*_alpha^2*b-2*_alpha*a*b-_alpha*b^2+a*b^2+_alpha*d+a*d)*(-(-a+x)/a)^(1/
2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(x*(a*b-a*x-b*x+x^2))^(1/2)*EllipticP
i((-(-a+x)/a)^(1/2),(-_alpha^3+_alpha^2*a+2*_alpha^2*b-2*_alpha*a*b-_alpha*
b^2+a*b^2+_alpha*d+a*d)/a/d,(a/(a-b))^(1/2)),_alpha=RootOf(_Z^4+(-2*a-2*b)*
_Z^3+(a^2+4*a*b+b^2-d)*_Z^2+(-2*a^2*b-2*a*b^2)*_Z+a^2*b^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(ab-x^2\right)(a-x)(b-x)}{\left(a^2b^2-2\,(a+b)abx-2\,(a+b)x^3+x^4+\left(a^2+4\,ab+b^2-d\right)x^2\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-a*b+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2-2*a*b*(
a+b)*x+(a^2+4*a*b+b^2-d)*x^2-2*(a+b)*x^3+x^4),x, algorithm="maxima")
```

```
[Out] -integrate((a*b - x^2)*(a - x)*(b - x)/((a^2*b^2 - 2*(a + b)*a*b*x - 2*(a +
 b)*x^3 + x^4 + (a^2 + 4*a*b + b^2 - d)*x^2)*sqrt((a - x)*(b - x)*x)), x)
```

mupad [B] time = 1.78, size = 1150, normalized size = 14.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a - x)*(b - x)*(a*b - x^2))/((x*(a - x)*(b - x))^(1/2)*(x^4 - 2*x^3*
(a + b) + a^2*b^2 + x^2*(4*a*b - d + a^2 + b^2) - 2*a*b*x*(a + b))),x)
```

```
[Out] symsum(-(b*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*ellipticPi
(-b/(root(z^4 - z^3*(2*a + 2*b) + z^2*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(
a + b) + a^2*b^2, z, k) - b), asin(((b - x)/b)^(1/2)), -b/(a - b))*(a*root(
z^4 - z^3*(2*a + 2*b) + z^2*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(a + b) + a
^2*b^2, z, k)^3 + b*root(z^4 - z^3*(2*a + 2*b) + z^2*(- d + 4*a*b + a^2 + b
^2) - 2*a*b*z*(a + b) + a^2*b^2, z, k)^3 + d*root(z^4 - z^3*(2*a + 2*b) + z
^2*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(a + b) + a^2*b^2, z, k)^2 - a^2*roo
t(z^4 - z^3*(2*a + 2*b) + z^2*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(a + b) +
 a^2*b^2, z, k)^2 - b^2*root(z^4 - z^3*(2*a + 2*b) + z^2*(- d + 4*a*b + a^2
 + b^2) - 2*a*b*z*(a + b) + a^2*b^2, z, k)^2 - 2*a^2*b^2 - 4*a*b*root(z^4 -
 z^3*(2*a + 2*b) + z^2*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(a + b) + a^2*b^
2, z, k)^2 + 3*a*b^2*root(z^4 - z^3*(2*a + 2*b) + z^2*(- d + 4*a*b + a^2 +
b^2) - 2*a*b*z*(a + b) + a^2*b^2, z, k) + 3*b*a^2*root(z^4 - z^3*(2*a + 2*b
) + z^2*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(a + b) + a^2*b^2, z, k)))/((ro
ot(z^4 - z^3*(2*a + 2*b) + z^2*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(a + b)
+ a^2*b^2, z, k) - b)*(x*(a - x)*(b - x))^(1/2)*(3*a*root(z^4 - z^3*(2*a +
2*b) + z^2*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(a + b) + a^2*b^2, z, k)^2 -
 a^2*root(z^4 - z^3*(2*a + 2*b) + z^2*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(
a + b) + a^2*b^2, z, k) + 3*b*root(z^4 - z^3*(2*a + 2*b) + z^2*(- d + 4*a*b
 + a^2 + b^2) - 2*a*b*z*(a + b) + a^2*b^2, z, k)^2 - b^2*root(z^4 - z^3*(2*
a + 2*b) + z^2*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(a + b) + a^2*b^2, z, k)
 + a*b^2 + a^2*b - 2*root(z^4 - z^3*(2*a + 2*b) + z^2*(- d + 4*a*b + a^2 +
b^2) - 2*a*b*z*(a + b) + a^2*b^2, z, k)^3 + d*root(z^4 - z^3*(2*a + 2*b) +
```

```
z^2*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(a + b) + a^2*b^2, z, k) - 4*a*b*ro
ot(z^4 - z^3*(2*a + 2*b) + z^2*(- d + 4*a*b + a^2 + b^2) - 2*a*b*z*(a + b)
+ a^2*b^2, z, k))), k, 1, 4) - (2*b*ellipticF(asin(((b - x)/b)^(1/2)), -b/(
a - b))*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2))/(x^3 - x^2*(
a + b) + a*b*x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-a*b+x**2)/(x*(-a+x)*(-b+x))**(1/2)/(a**2*b**2-2*a
*b*(a+b)*x+(a**2+4*a*b+b**2-d)*x**2-2*(a+b)*x**3+x**4),x)
```

```
[Out] Timed out
```

3.932 $\int \frac{x^2(-b+ax^4)}{(b+ax^4)^{3/4}}\,dx$

Optimal. Leaf size=77

$$\frac{7b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{3/4}}-\frac{7b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{3/4}}+\frac{1}{4}x^3\sqrt[4]{ax^4+b}$$

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {459, 331, 298, 203, 206}

$$\frac{7b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{3/4}}-\frac{7b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{3/4}}+\frac{1}{4}x^3\sqrt[4]{ax^4+b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(-b + a*x^4))/(b + a*x^4)^(3/4),x]
```

```
[Out] (x^3*(b + a*x^4)^(1/4))/4 + (7*b*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(8*
a^(3/4)) - (7*b*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(8*a^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^2 \left(-b+ax^4\right)}{\left(b+ax^4\right)^{3/4}}\,dx = \frac{1}{4}x^3\sqrt[4]{b+ax^4} - \frac{1}{4}(7b)\int \frac{x^2}{\left(b+ax^4\right)^{3/4}}\,dx$$
$$= \frac{1}{4}x^3\sqrt[4]{b+ax^4} - \frac{1}{4}(7b)\,\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)$$
$$= \frac{1}{4}x^3\sqrt[4]{b+ax^4} - \frac{(7b)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{8\sqrt{a}} + \frac{(7b)\,\text{Subst}\left(\int \frac{1}{1+\sqrt{a}x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{8\sqrt{a}}$$
$$= \frac{1}{4}x^3\sqrt[4]{b+ax^4} + \frac{7b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{8a^{3/4}} - \frac{7b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{8a^{3/4}}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 1.00

$$\frac{7b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{3/4}} - \frac{7b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{3/4}} + \frac{1}{4}x^3\sqrt[4]{ax^4+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(-b + a*x^4))/(b + a*x^4)^(3/4),x]
```

```
[Out] (x^3*(b + a*x^4)^(1/4))/4 + (7*b*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(8*
a^(3/4)) - (7*b*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(8*a^(3/4))
```

IntegrateAlgebraic [A] time = 0.43, size = 77, normalized size = 1.00

$$\frac{7b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{3/4}} - \frac{7b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{3/4}} + \frac{1}{4}x^3\sqrt[4]{ax^4+b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(-b + a*x^4))/(b + a*x^4)^(3/4),x]
```

```
[Out] (x^3*(b + a*x^4)^(1/4))/4 + (7*b*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(8*
a^(3/4)) - (7*b*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(8*a^(3/4))
```

fricas [B] time = 0.51, size = 192, normalized size = 2.49

$$\frac{1}{4}\left(ax^4+b\right)^{\frac{1}{4}}x^3 + \frac{7}{4}\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2\left(\frac{b^4}{a^3}\right)^{\frac{3}{4}}x\sqrt{\frac{a^2\sqrt{\frac{b^4}{a^3}}x^2+\sqrt{ax^4+b}\,b^2}{x^2}} - \left(ax^4+b\right)^{\frac{1}{4}}a^2b\left(\frac{b^4}{a^3}\right)^{\frac{3}{4}}}{b^4x}\right) - \frac{7}{16}\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{7\left(a\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}x+\left(ax^4+b\right)^{\frac{1}{4}}b\right)}{x}\right) + \frac{7}{16}\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}\log\left(-\frac{7\left(a\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}x-\left(ax^4+b\right)^{\frac{1}{4}}b\right)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^4-b)/(a*x^4+b)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/4*(a*x^4 + b)^(1/4)*x^3 + 7/4*(b^4/a^3)^(1/4)*arctan((a^2*(b^4/a^3)^(3/4)
*x*sqrt((a^2*sqrt(b^4/a^3)*x^2 + sqrt(a*x^4 + b)*b^2)/x^2) - (a*x^4 + b)^(1
/4)*a^2*b*(b^4/a^3)^(3/4))/(b^4*x)) - 7/16*(b^4/a^3)^(1/4)*log(7*(a*(b^4/a^
3)^(1/4)*x + (a*x^4 + b)^(1/4)*b)/x) + 7/16*(b^4/a^3)^(1/4)*log(-7*(a*(b^4/
a^3)^(1/4)*x - (a*x^4 + b)^(1/4)*b)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4-b\right)x^2}{\left(ax^4+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^4-b)/(a*x^4+b)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^4 - b)*x^2/(a*x^4 + b)^(3/4), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a \,x^{4}-b \right)}{\left(a \,x^{4}+b \right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^4-b)/(a*x^4+b)^(3/4),x)
```

```
[Out] int(x^2*(a*x^4-b)/(a*x^4+b)^(3/4),x)
```

maxima [B] time = 0.44, size = 185, normalized size = 2.40

$$-\frac{1}{4}\,b\left(\frac{2\,\arctan\left(\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{3}{4}}}-\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{3}{4}}}\right)-\frac{1}{16}\,a\left(\frac{3\left(\frac{2\,b\arctan\left(\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{3}{4}}}-\frac{b\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{3}{4}}}\right)}{a}+\frac{4\left(ax^{4}+b\right)^{\frac{1}{4}}b}{\left(a^{2}-\frac{\left(ax^{4}+b\right)a}{x^{4}}\right)x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^4-b)/(a*x^4+b)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*arctan((a*x^4 + b)^(1/4)/(a^(1/4)*x))/a^(3/4) - log(-(a^(1/4) - (
a*x^4 + b)^(1/4)/x)/(a^(1/4) + (a*x^4 + b)^(1/4)/x))/a^(3/4)) - 1/16*a*(3*(
2*b*arctan((a*x^4 + b)^(1/4)/(a^(1/4)*x))/a^(3/4) - b*log(-(a^(1/4) - (a*x^
4 + b)^(1/4)/x)/(a^(1/4) + (a*x^4 + b)^(1/4)/x))/a^(3/4))/a + 4*(a*x^4 + b)
^(1/4)*b/((a^2 - (a*x^4 + b)*a/x^4)*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2\,\left(b-a\,x^4\right)}{{\left(a\,x^4+b\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(b - a*x^4))/(b + a*x^4)^(3/4),x)
```

```
[Out] -int((x^2*(b - a*x^4))/(b + a*x^4)^(3/4), x)
```

sympy [C] time = 2.74, size = 78, normalized size = 1.01

$$\frac{a x^{7} \Gamma\left(\frac{7}{4}\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| {\frac{a x^{4} e^{i \pi}}{b}} \right)}}{4 b^{\frac{3}{4}} \Gamma\left(\frac{11}{4}\right)} - \frac{\sqrt[4]{b} x^{3} \Gamma\left(\frac{3}{4}\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| {\frac{a x^{4} e^{i \pi}}{b}} \right)}}{4 \Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x**4-b)/(a*x**4+b)**(3/4),x)
```

```
[Out] a*x**7*gamma(7/4)*hyper((3/4, 7/4), (11/4,), a*x**4*exp_polar(I*pi)/b)/(4*b
**(3/4)*gamma(11/4)) - b**(1/4)*x**3*gamma(3/4)*hyper((3/4, 3/4), (7/4,), a
*x**4*exp_polar(I*pi)/b)/(4*gamma(7/4))
```

3.933 $\int \frac{1}{\sqrt[4]{bx+ax^4}}\,dx$

Optimal. Leaf size=77

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}$$

Rubi [A] time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.60, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2011, 329, 275, 240, 212, 206, 203}

$$\frac{2\sqrt[4]{x}\sqrt[4]{ax^3+b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{a}\sqrt[4]{ax^4+bx}}+\frac{2\sqrt[4]{x}\sqrt[4]{ax^3+b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{a}\sqrt[4]{ax^4+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + a*x^4)^(-1/4),x]
```

```
[Out] (2*x^(1/4)*(b + a*x^3)^(1/4)*ArcTan[(a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)])/(
3*a^(1/4)*(b*x + a*x^4)^(1/4)) + (2*x^(1/4)*(b + a*x^3)^(1/4)*ArcTanh[(a^(1
/4)*x^(3/4))/(b + a*x^3)^(1/4)])/(3*a^(1/4)*(b*x + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{bx+ax^4}}\,dx = \frac{\left(\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\int \frac{1}{\sqrt[4]{x}\,\sqrt[4]{b+ax^3}}\,dx}{\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{b+ax^{12}}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx, x, x^{3/4}\right)}{3\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx, x, \frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(2\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}} + \frac{\left(2\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{a}\,x^2}\,dx, x,\right.}{3\sqrt[4]{bx+ax^4}}$$

$$= \frac{2\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{a}\,\sqrt[4]{bx+ax^4}} + \frac{2\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{a}\,\sqrt[4]{bx+ax^4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.66

$$\frac{4x\sqrt[4]{\frac{ax^3}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{ax^3}{b}\right)}{3\sqrt[4]{x\left(ax^3+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + a*x^4)^(-1/4),x]
```

```
[Out] (4*x*(1 + (a*x^3)/b)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, -((a*x^3)/b)])/
(3*(x*(b + a*x^3))^(1/4))
```

IntegrateAlgebraic [A] time = 0.28, size = 77, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x + a*x^4)^(-1/4),x]
```

```
[Out] (2*ArcTan[(a^(1/4)*(b*x + a*x^4)^(3/4))/(b + a*x^3)])/(3*a^(1/4)) + (2*ArcT
anh[(a^(1/4)*(b*x + a*x^4)^(3/4))/(b + a*x^3)])/(3*a^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.29, size = 186, normalized size = 2.42

$$\frac{\sqrt{2}\left(-a\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}+2\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)}{3\,a}+\frac{\sqrt{2}\left(-a\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}-2\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)}{3\,a}-\frac{\sqrt{2}\left(-a\right)^{\frac{3}{4}}\log\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^{3}}}\right)}{6\,a}+\frac{\sqrt{2}\left(-a\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\left(-a\right)^{\frac{1}{4}}\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^{3}}}\right)}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^
3)^(1/4))/(-a)^(1/4))/a + 1/3*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(
2)*(-a)^(1/4) - 2*(a + b/x^3)^(1/4))/(-a)^(1/4))/a - 1/6*sqrt(2)*(-a)^(3/4)
*log(sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a) + sqrt(a + b/x^3))/a +
 1/6*sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a
) + sqrt(a + b/x^3))/a
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,x^{4}+b x\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^4+b*x)^(1/4),x)
```

```
[Out] int(1/(a*x^4+b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^{4}+bx\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + b*x)^(-1/4), x)
```

mupad [B] time = 0.83, size = 40, normalized size = 0.52

$$\frac{4\,x\,{\left(\frac{a\,x^3}{b}+1\right)}^{1/4}\,{{}}_2{\mathrm{F}}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{a\,x^3}{b}\right)}{3\,{\left(a\,x^4+b\,x\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + a*x^4)^(1/4),x)
```

```
[Out] (4*x*((a*x^3)/b + 1)^(1/4)*hypergeom([1/4, 1/4], 5/4, -(a*x^3)/b))/(3*(b*x
+ a*x^4)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ax^4 + bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**4+b*x)**(1/4),x)
```

```
[Out] Integral((a*x**4 + b*x)**(-1/4), x)
```

3.934

$$\int \frac{(-a+x)(-b+x)\left(-ab+x^2\right)}{\sqrt{x(-a+x)(-b+x)}\left(a^2b^2d-2ab(a+b)dx+\left(-1+a^2d+4abd+b^2d\right)x^2-2(a+b)dx^3+dx^4\right)}\,dx$$

Optimal. Leaf size=77

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}\right)}{d^{3/4}}$$

Rubi [F] time = 31.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-a+x)(-b+x)\left(-ab+x^2\right)}{\sqrt{x(-a+x)(-b+x)}\left(a^2b^2d-2ab(a+b)dx+\left(-1+a^2d+4abd+b^2d\right)x^2-2(a+b)dx^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-a + x)*(-b + x)*(-(a*b) + x^2))/(Sqrt[x*(-a + x)*(-b + x)]*(a^2*b^2*
d - 2*a*b*(a + b)*d*x + (-1 + a^2*d + 4*a*b*d + b^2*d)*x^2 - 2*(a + b)*d*x^
3 + d*x^4)),x]
```

```
[Out] (2*a*b*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][(Sqrt[-a +
 x^2]*Sqrt[-b + x^2])/(-(a^2*b^2*d) + 2*a^2*b*(1 + b/a)*d*x^2 + (1 - (a^2 +
 4*a*b + b^2)*d)*x^4 + 2*a*(1 + b/a)*d*x^6 - d*x^8), x], x, Sqrt[x]])/Sqrt[
(a - x)*(b - x)*x] + (2*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defe
r[Int][(x^4*Sqrt[-a + x^2]*Sqrt[-b + x^2])/(a^2*b^2*d - 2*a^2*b*(1 + b/a)*d
*x^2 - (1 - (a^2 + 4*a*b + b^2)*d)*x^4 - 2*a*(1 + b/a)*d*x^6 + d*x^8), x],
x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x]
```

Rubi steps

$$\int \frac{(-a+x)(-b+x)\left(-ab+x^2\right)}{\sqrt{x(-a+x)(-b+x)}\left(a^2b^2d-2ab(a+b)dx+\left(-1+a^2d+4abd+b^2d\right)x^2-2(a+b)dx^3+dx^4\right)}\,dx = \frac{\left(\sqrt{x}\right.\ldots}{\ldots}$$

$$= \frac{\left(2\sqrt{x}\right.\ldots}{\ldots}$$

$$= \frac{\left(2\sqrt{x}\right.\ldots}{\ldots}$$

$$= \frac{\left(2\sqrt{x}\right.\ldots}{\ldots}$$

Mathematica [C] time = 13.85, size = 24546, normalized size = 318.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-a + x)*(-b + x)*(-(a*b) + x^2))/(Sqrt[x*(-a + x)*(-b + x)]*(a^
2*b^2*d - 2*a*b*(a + b)*d*x + (-1 + a^2*d + 4*a*b*d + b^2*d)*x^2 - 2*(a + b
)*d*x^3 + d*x^4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.29, size = 77, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-a + x)*(-b + x)*(-(a*b) + x^2))/(Sqrt[x*(-a + x)*(-b
+ x)]*(a^2*b^2*d - 2*a*b*(a + b)*d*x + (-1 + a^2*d + 4*a*b*d + b^2*d)*x^2 -
 2*(a + b)*d*x^3 + d*x^4)),x]
```

```
[Out] -(ArcTan[x/(d^(1/4)*Sqrt[a*b*x + (-a - b)*x^2 + x^3])]/d^(3/4)) - ArcTanh[x
/(d^(1/4)*Sqrt[a*b*x + (-a - b)*x^2 + x^3])]/d^(3/4)
```

fricas [B] time = 1.26, size = 483, normalized size = 6.27

$$-\left(\frac{1}{d^3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{abx-(a+b)x^2+x^3}\,d^2\left(\frac{1}{d^3}\right)^{\frac{3}{4}}}{ab-(a+b)x+x^2}\right)-\frac{1}{4}\left(\frac{1}{d^3}\right)^{\frac{1}{4}}\log\left(\frac{a^2b^2d-2(a+b)dx^3+dx^4-2\left(a^2b+ab^2\right)dx+\left(\left(a^2+4ab+b^2\right)d+1\right)x^2+2\sqrt{abx-(a+b)x^2+x^3}\left(d\left(\frac{1}{d^3}\right)^{\frac{1}{4}}x+\left(abd^3-(a+b)d^3x+d^3x^2\right)\left(\frac{1}{d^3}\right)^{\frac{3}{4}}\right)+2\left(abd^2x-(a+b)d^2x^2+d^2x^3\right)\sqrt{\frac{1}{d^3}}}{a^2b^2d-2(a+b)dx^3+dx^4-2\left(a^2b+ab^2\right)dx+\left(\left(a^2+4ab+b^2\right)d-1\right)x^2}\right)+\frac{1}{4}\left(\frac{1}{d^3}\right)^{\frac{1}{4}}\log\left(\frac{a^2b^2d-2(a+b)dx^3+dx^4-2\left(a^2b+ab^2\right)dx+\left(\left(a^2+4ab+b^2\right)d+1\right)x^2-2\sqrt{abx-(a+b)x^2+x^3}\left(d\left(\frac{1}{d^3}\right)^{\frac{1}{4}}x+\left(abd^3-(a+b)d^3x+d^3x^2\right)\left(\frac{1}{d^3}\right)^{\frac{3}{4}}\right)+2\left(abd^2x-(a+b)d^2x^2+d^2x^3\right)\sqrt{\frac{1}{d^3}}}{a^2b^2d-2(a+b)dx^3+dx^4-2\left(a^2b+ab^2\right)dx+\left(\left(a^2+4ab+b^2\right)d-1\right)x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-a*b+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2*d-2*a*b
*(a+b)*d*x+(a^2*d+4*a*b*d+b^2*d-1)*x^2-2*(a+b)*d*x^3+d*x^4),x, algorithm="f
ricas")
```

```
[Out] -(d^(-3))^(1/4)*arctan(sqrt(a*b*x - (a + b)*x^2 + x^3)*d^2*(d^(-3))^(3/4)/(
a*b - (a + b)*x + x^2)) - 1/4*(d^(-3))^(1/4)*log((a^2*b^2*d - 2*(a + b)*d*x
^3 + d*x^4 - 2*(a^2*b + a*b^2)*d*x + ((a^2 + 4*a*b + b^2)*d + 1)*x^2 + 2*sq
rt(a*b*x - (a + b)*x^2 + x^3)*(d*(d^(-3))^(1/4)*x + (a*b*d^3 - (a + b)*d^3*
x + d^3*x^2)*(d^(-3))^(3/4)) + 2*(a*b*d^2*x - (a + b)*d^2*x^2 + d^2*x^3)*sq
rt(d^(-3)))/(a^2*b^2*d - 2*(a + b)*d*x^3 + d*x^4 - 2*(a^2*b + a*b^2)*d*x +
((a^2 + 4*a*b + b^2)*d - 1)*x^2)) + 1/4*(d^(-3))^(1/4)*log((a^2*b^2*d - 2*(
a + b)*d*x^3 + d*x^4 - 2*(a^2*b + a*b^2)*d*x + ((a^2 + 4*a*b + b^2)*d + 1)*
x^2 - 2*sqrt(a*b*x - (a + b)*x^2 + x^3)*(d*(d^(-3))^(1/4)*x + (a*b*d^3 - (a
 + b)*d^3*x + d^3*x^2)*(d^(-3))^(3/4)) + 2*(a*b*d^2*x - (a + b)*d^2*x^2 + d
^2*x^3)*sqrt(d^(-3)))/(a^2*b^2*d - 2*(a + b)*d*x^3 + d*x^4 - 2*(a^2*b + a*b
^2)*d*x + ((a^2 + 4*a*b + b^2)*d - 1)*x^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(ab-x^2\right)(a-x)(b-x)}{\left(a^2b^2d-2\,(a+b)abdx-2\,(a+b)dx^3+dx^4+\left(a^2d+4\,abd+b^2d-1\right)x^2\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-a*b+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2*d-2*a*b
*(a+b)*d*x+(a^2*d+4*a*b*d+b^2*d-1)*x^2-2*(a+b)*d*x^3+d*x^4),x, algorithm="g
iac")
```

```
[Out] integrate(-(a*b - x^2)*(a - x)*(b - x)/((a^2*b^2*d - 2*(a + b)*a*b*d*x - 2*
(a + b)*d*x^3 + d*x^4 + (a^2*d + 4*a*b*d + b^2*d - 1)*x^2)*sqrt((a - x)*(b
- x)*x)), x)
```

maple [C] time = 0.06, size = 454, normalized size = 5.90

$$-\frac{2a\sqrt{-\frac{-a+x}{a}}\,\sqrt{\frac{-b+x}{a-b}}\,\sqrt{\frac{x}{a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{-a+x}{a}},\sqrt{\frac{a}{a-b}}\right)}{d\sqrt{abx-a\,x^2-b\,x^2+x^3}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\,\_Z^4+(-2ad-2bd)\_Z^3+\left(a^2d+4abd+b^2d-1\right)\_Z^2+\left(-2a^2bd-2a\,b^2d\right)\_Z+a^2b^2d\right)}\frac{\left(-\_\alpha^3ad-\_\alpha^3bd+\_\alpha^2a^2d+4\_\alpha^2abd+\_\alpha^2b^2d-3\_\alpha\,a^2bd-3\_\alpha a\,b^2d+2a^2b^2d-\_\alpha^2\right)\left(-d\,\_\alpha^3+d\,\_\alpha^2a+2d\,\_\alpha^2b-2\_\alpha bda-\_\alpha\,b^2d+a\,b^2d+\_\alpha+a\right)\sqrt{-\frac{-a+x}{a}}\,\sqrt{\frac{-b+x}{a-b}}\,\sqrt{\frac{x}{a}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-a+x}{a}},\frac{-d\,\_\alpha^3+d\,\_\alpha^2a+2d\,\_\alpha^2b-2\_\alpha bda-\_\alpha\,b^2d+a\,b^2d+\_\alpha+a}{a},\sqrt{\frac{a}{a-b}}\right)}{\left(-2d\,\_\alpha^3+3d\,\_\alpha^2a+3d\,\_\alpha^2b-\_\alpha\,a^2d-4\_\alpha bda-\_\alpha\,b^2d+a^2bd+a\,b^2d+\_\alpha\right)\sqrt{x\left(ab-ax-bx+x^2\right)}}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x)*(-b+x)*(-a*b+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2*d-2*a*b*(a+b)
*d*x+(a^2*d+4*a*b*d+b^2*d-1)*x^2-2*(a+b)*d*x^3+d*x^4),x)
```

```
[Out] -2/d*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*
x^2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))-1/d/a*sum((-_al
pha^3*a*d-_alpha^3*b*d+_alpha^2*a^2*d+4*_alpha^2*a*b*d+_alpha^2*b^2*d-3*_al
pha*a^2*b*d-3*_alpha*a*b^2*d+2*a^2*b^2*d-_alpha^2)/(-2*_alpha^3*d+3*_alpha^
2*a*d+3*_alpha^2*b*d-_alpha*a^2*d-4*_alpha*a*b*d-_alpha*b^2*d+a^2*b*d+a*b^2
*d+_alpha)*(-_alpha^3*d+_alpha^2*a*d+2*_alpha^2*b*d-2*_alpha*a*b*d-_alpha*b
^2*d+a*b^2*d+_alpha+a)*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)
/(x*(a*b-a*x-b*x+x^2))^(1/2)*EllipticPi((-(-a+x)/a)^(1/2),(-_alpha^3*d+_alp
ha^2*a*d+2*_alpha^2*b*d-2*_alpha*a*b*d-_alpha*b^2*d+a*b^2*d+_alpha+a)/a,(a/
(a-b))^(1/2)),_alpha=RootOf(d*_Z^4+(-2*a*d-2*b*d)*_Z^3+(a^2*d+4*a*b*d+b^2*d
-1)*_Z^2+(-2*a^2*b*d-2*a*b^2*d)*_Z+a^2*b^2*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(ab-x^2\right)(a-x)(b-x)}{\left(a^2b^2d-2\,(a+b)abdx-2\,(a+b)dx^3+dx^4+\left(a^2d+4\,abd+b^2d-1\right)x^2\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-a*b+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(a^2*b^2*d-2*a*b
*(a+b)*d*x+(a^2*d+4*a*b*d+b^2*d-1)*x^2-2*(a+b)*d*x^3+d*x^4),x, algorithm="m
axima")
```

```
[Out] -integrate((a*b - x^2)*(a - x)*(b - x)/((a^2*b^2*d - 2*(a + b)*a*b*d*x - 2*
(a + b)*d*x^3 + d*x^4 + (a^2*d + 4*a*b*d + b^2*d - 1)*x^2)*sqrt((a - x)*(b
- x)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{(a-x)\ (b-x)\ \left(a\,b-x^2\right)}{\sqrt{x\ (a-x)\ (b-x)}\ \left(x^2\ \left(d\,a^2+4\,d\,a\,b+d\,b^2-1\right)+d\,x^4+a^2\,b^2\,d-2\,d\,x^3\ (a+b)-2\,a\,b\,d\,x\ (a+b)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a - x)*(b - x)*(a*b - x^2))/((x*(a - x)*(b - x))^(1/2)*(x^2*(a^2*d +
 b^2*d + 4*a*b*d - 1) + d*x^4 + a^2*b^2*d - 2*d*x^3*(a + b) - 2*a*b*d*x*(a
+ b))),x)
```

```
[Out] -int(((a - x)*(b - x)*(a*b - x^2))/((x*(a - x)*(b - x))^(1/2)*(x^2*(a^2*d +
 b^2*d + 4*a*b*d - 1) + d*x^4 + a^2*b^2*d - 2*d*x^3*(a + b) - 2*a*b*d*x*(a
+ b))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-a*b+x**2)/(x*(-a+x)*(-b+x))**(1/2)/(a**2*b**2*d-2
*a*b*(a+b)*d*x+(a**2*d+4*a*b*d+b**2*d-1)*x**2-2*(a+b)*d*x**3+d*x**4),x)
```

```
[Out] Timed out
```

3.935 $\int \frac{1}{x\sqrt[4]{-1+x^6}}\,dx$

Optimal. Leaf size=77

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^6-1}}{\sqrt{x^6-1}-1}\right)}{3\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^6-1}}{\sqrt{x^6-1}+1}\right)}{3\sqrt{2}}$$

Rubi [A] time = 0.11, antiderivative size = 129, normalized size of antiderivative = 1.68, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {266, 63, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(\sqrt{x^6-1}-\sqrt{2}\sqrt[4]{x^6-1}+1\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{x^6-1}+\sqrt{2}\sqrt[4]{x^6-1}+1\right)}{6\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{x^6-1}\right)}{3\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt[4]{x^6-1}+1\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-1 + x^6)^(1/4)),x]
```

```
[Out] -1/3*ArcTan[1 - Sqrt[2]*(-1 + x^6)^(1/4)]/Sqrt[2] + ArcTan[1 + Sqrt[2]*(-1
+ x^6)^(1/4)]/(3*Sqrt[2]) + Log[1 - Sqrt[2]*(-1 + x^6)^(1/4) + Sqrt[-1 + x^
6]]/(6*Sqrt[2]) - Log[1 + Sqrt[2]*(-1 + x^6)^(1/4) + Sqrt[-1 + x^6]]/(6*Sqr
t[2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt[4]{-1+x^6}}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{-1+x}\,x}\,dx,x,x^6\right)\\
&= \frac{2}{3}\operatorname{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt[4]{-1+x^6}\right)\\
&= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt[4]{-1+x^6}\right)\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt[4]{-1+x^6}\right)\\
&= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt[4]{-1+x^6}\right)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\sqrt[4]{-1+x^6}\right.\\
&= \frac{\log\left(1-\sqrt{2}\sqrt[4]{-1+x^6}+\sqrt{-1+x^6}\right)}{6\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt[4]{-1+x^6}+\sqrt{-1+x^6}\right)}{6\sqrt{2}}+\frac{\operatorname{Subst}\left(\int\right.}{}\\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+x^6}\right)}{3\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+x^6}\right)}{3\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt[4]{-1+x^6}+\sqrt{-1}\right.}{6\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.36

$$\frac{2}{9}\left(x^6-1\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-1 + x^6)^(1/4)),x]
[Out] (2*(-1 + x^6)^(3/4)*Hypergeometric2F1[3/4, 1, 7/4, 1 - x^6])/9
```

IntegrateAlgebraic [A] time = 0.09, size = 82, normalized size = 1.06

$$\frac{\tan^{-1}\left(\frac{\frac{\sqrt{x^6-1}}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt[4]{x^6-1}}\right)}{3\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^6-1}}{\sqrt{x^6-1}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-1 + x^6)^(1/4)),x]
```

```
[Out] ArcTan[(-(1/Sqrt[2]) + Sqrt[-1 + x^6]/Sqrt[2])/(-1 + x^6)^(1/4)]/(3*Sqrt[2]
) - ArcTanh[(Sqrt[2]*(-1 + x^6)^(1/4))/(1 + Sqrt[-1 + x^6])]/(3*Sqrt[2])
```

fricas [B] time = 0.47, size = 155, normalized size = 2.01

$$-\frac{1}{3}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(x^{6}-1\right)^{\frac{1}{4}}+\sqrt{x^{6}-1}+1}-\sqrt{2}\left(x^{6}-1\right)^{\frac{1}{4}}-1\right)-\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(x^{6}-1\right)^{\frac{1}{4}}+4\sqrt{x^{6}-1}+4}-\sqrt{2}\left(x^{6}-1\right)^{\frac{1}{4}}+1\right)-\frac{1}{12}\sqrt{2}\log\left(4\sqrt{2}\left(x^{6}-1\right)^{\frac{1}{4}}+4\sqrt{x^{6}-1}+4\right)+\frac{1}{12}\sqrt{2}\log\left(-4\sqrt{2}\left(x^{6}-1\right)^{\frac{1}{4}}+4\sqrt{x^{6}-1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-1)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*(x^6 - 1)^(1/4) + sqrt(x^6 - 1) +
1) - sqrt(2)*(x^6 - 1)^(1/4) - 1) - 1/3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*
sqrt(2)*(x^6 - 1)^(1/4) + 4*sqrt(x^6 - 1) + 4) - sqrt(2)*(x^6 - 1)^(1/4) +
1) - 1/12*sqrt(2)*log(4*sqrt(2)*(x^6 - 1)^(1/4) + 4*sqrt(x^6 - 1) + 4) + 1/
12*sqrt(2)*log(-4*sqrt(2)*(x^6 - 1)^(1/4) + 4*sqrt(x^6 - 1) + 4)
```

giac [A] time = 0.25, size = 102, normalized size = 1.32

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(x^{6}-1\right)^{\frac{1}{4}}\right)\right)+\frac{1}{6}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(x^{6}-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{12}\sqrt{2}\log\left(\sqrt{2}\left(x^{6}-1\right)^{\frac{1}{4}}+\sqrt{x^{6}-1}+1\right)+\frac{1}{12}\sqrt{2}\log\left(-\sqrt{2}\left(x^{6}-1\right)^{\frac{1}{4}}+\sqrt{x^{6}-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-1)^(1/4),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^6 - 1)^(1/4))) + 1/6*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^6 - 1)^(1/4))) - 1/12*sqrt(2)*log(sqrt
(2)*(x^6 - 1)^(1/4) + sqrt(x^6 - 1) + 1) + 1/12*sqrt(2)*log(-sqrt(2)*(x^6 -
 1)^(1/4) + sqrt(x^6 - 1) + 1)
```

maple [C] time = 0.28, size = 79, normalized size = 1.03

$$\frac{\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)\left(-\mathrm{signum}\left(x^{6}-1\right)\right)^{\frac{1}{4}}\left(\frac{\pi\sqrt{2}\,x^{6}\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{4}\right],[2,2],x^{6}\right)}{4\Gamma\left(\frac{3}{4}\right)}+\frac{\left(-3\ln(2)-\frac{\pi}{2}+6\ln(x)+i\pi\right)\pi\sqrt{2}}{\Gamma\left(\frac{3}{4}\right)}\right)}{12\pi\,\mathrm{signum}\left(x^{6}-1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6-1)^(1/4),x)
```

```
[Out] 1/12/Pi*2^(1/2)*GAMMA(3/4)/signum(x^6-1)^(1/4)*(-signum(x^6-1))^(1/4)*(1/4*
Pi*2^(1/2)/GAMMA(3/4)*x^6*hypergeom([1,1,5/4],[2,2],x^6)+(-3*ln(2)-1/2*Pi+6
*ln(x)+I*Pi)*Pi*2^(1/2)/GAMMA(3/4))
```

maxima [A] time = 0.43, size = 102, normalized size = 1.32

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(x^{6}-1\right)^{\frac{1}{4}}\right)\right)+\frac{1}{6}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(x^{6}-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{12}\sqrt{2}\log\left(\sqrt{2}\left(x^{6}-1\right)^{\frac{1}{4}}+\sqrt{x^{6}-1}+1\right)+\frac{1}{12}\sqrt{2}\log\left(-\sqrt{2}\left(x^{6}-1\right)^{\frac{1}{4}}+\sqrt{x^{6}-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-1)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^6 - 1)^(1/4))) + 1/6*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^6 - 1)^(1/4))) - 1/12*sqrt(2)*log(sqrt
(2)*(x^6 - 1)^(1/4) + sqrt(x^6 - 1) + 1) + 1/12*sqrt(2)*log(-sqrt(2)*(x^6 -
 1)^(1/4) + sqrt(x^6 - 1) + 1)
```

mupad [B] time = 0.90, size = 45, normalized size = 0.58

$$\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(x^6-1\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{6}-\frac{1}{6}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(x^6-1\right)^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{6}+\frac{1}{6}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^6 - 1)^(1/4)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*(x^6 - 1)^(1/4)*(1/2 - 1i/2))*(1/6 - 1i/6) + 2^(1/2)*a
tan(2^(1/2)*(x^6 - 1)^(1/4)*(1/2 + 1i/2))*(1/6 + 1i/6)
```

sympy [C] time = 0.79, size = 34, normalized size = 0.44

$$-\frac{\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{4}\\\frac{5}{4}\end{matrix}\right|\frac{e^{2i\pi}}{x^6}\right)}{6x^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6-1)**(1/4),x)
```

```
[Out] -gamma(1/4)*hyper((1/4, 1/4), (5/4,), exp_polar(2*I*pi)/x**6)/(6*x**(3/2)*g
amma(5/4))
```

3.936 $\int \left(-1+x^2\right)\left(-1+x\sqrt{-1+3x^2-x^4}\right)\,dx$

Optimal. Leaf size=77

$$-\frac{x^3}{3}+\frac{1}{48}\sqrt{-x^4+3x^2-1}\,\left(8x^4-18x^2-1\right)+\frac{5}{32}i\log\left(-2ix^2+2\sqrt{-x^4+3x^2-1}\,+3i\right)+x$$

Rubi [A] time = 0.19, antiderivative size = 74, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6742, 1107, 612, 619, 216, 14, 1114, 640}

$$-\frac{x^3}{3}-\frac{5}{32}\sin^{-1}\left(\frac{3-2x^2}{\sqrt{5}}\right)-\frac{1}{6}\left(-x^4+3x^2-1\right)^{3/2}-\frac{1}{16}\left(3-2x^2\right)\sqrt{-x^4+3x^2-1}\,+x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)*(-1 + x*Sqrt[-1 + 3*x^2 - x^4]),x]
```

```
[Out] x - x^3/3 - ((3 - 2*x^2)*Sqrt[-1 + 3*x^2 - x^4])/16 - (-1 + 3*x^2 - x^4)^(3
/2)/6 - (5*ArcSin[(3 - 2*x^2)/Sqrt[5]])/32
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
```

```
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \left(-1+x^2\right)\left(-1+x\sqrt{-1+3x^2-x^4}\right)dx = \int \left(1-x\sqrt{-1+3x^2-x^4}+x^2\left(-1+x\sqrt{-1+3x^2-x^4}\right)\right)dx$$

$$= x-\int x\sqrt{-1+3x^2-x^4}\,dx+\int x^2\left(-1+x\sqrt{-1+3x^2-x^4}\right)dx$$

$$= x-\frac{1}{2}\text{Subst}\left(\int \sqrt{-1+3x-x^2}\,dx,x,x^2\right)+\int \left(-x^2+x^3\sqrt{-1+3\right.$$

$$= x-\frac{x^3}{3}+\frac{1}{8}\left(3-2x^2\right)\sqrt{-1+3x^2-x^4}-\frac{5}{16}\text{Subst}\left(\int \frac{1}{\sqrt{-1+3x-}}\right.$$

$$= x-\frac{x^3}{3}+\frac{1}{8}\left(3-2x^2\right)\sqrt{-1+3x^2-x^4}+\frac{1}{2}\text{Subst}\left(\int x\sqrt{-1+3x-}\right.$$

$$= x-\frac{x^3}{3}+\frac{1}{8}\left(3-2x^2\right)\sqrt{-1+3x^2-x^4}-\frac{1}{6}\left(-1+3x^2-x^4\right)^{3/2}+\frac{5}{16}$$

$$= x-\frac{x^3}{3}-\frac{1}{16}\left(3-2x^2\right)\sqrt{-1+3x^2-x^4}-\frac{1}{6}\left(-1+3x^2-x^4\right)^{3/2}+\frac{5}{1}$$

$$= x-\frac{x^3}{3}-\frac{1}{16}\left(3-2x^2\right)\sqrt{-1+3x^2-x^4}-\frac{1}{6}\left(-1+3x^2-x^4\right)^{3/2}+\frac{5}{1}$$

$$= x-\frac{x^3}{3}-\frac{1}{16}\left(3-2x^2\right)\sqrt{-1+3x^2-x^4}-\frac{1}{6}\left(-1+3x^2-x^4\right)^{3/2}-\frac{5}{3}$$

Mathematica [A] time = 0.06, size = 89, normalized size = 1.16

$$\frac{1}{96}\left(-32x^3-15\sin^{-1}\left(\frac{3-2x^2}{\sqrt{5}}\right)+16\sqrt{-x^4+3x^2-1}\,x^4-36\sqrt{-x^4+3x^2-1}\,x^2-2\sqrt{-x^4+3x^2-1}+96x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)*(-1 + x*Sqrt[-1 + 3*x^2 - x^4]),x]
```

```
[Out] (96*x - 32*x^3 - 2*Sqrt[-1 + 3*x^2 - x^4] - 36*x^2*Sqrt[-1 + 3*x^2 - x^4] +
 16*x^4*Sqrt[-1 + 3*x^2 - x^4] - 15*ArcSin[(3 - 2*x^2)/Sqrt[5]])/96
```

IntegrateAlgebraic [A] time = 0.35, size = 77, normalized size = 1.00

$$-\frac{x^3}{3}+\frac{1}{48}\sqrt{-x^4+3x^2-1}\left(8x^4-18x^2-1\right)+\frac{5}{32}i\log\left(-2ix^2+2\sqrt{-x^4+3x^2-1}+3i\right)+x$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)*(-1 + x*Sqrt[-1 + 3*x^2 - x^4]),x]
```

```
[Out] x - x^3/3 + (Sqrt[-1 + 3*x^2 - x^4]*(-1 - 18*x^2 + 8*x^4))/48 + ((5*I)/32)*
Log[3*I - (2*I)*x^2 + 2*Sqrt[-1 + 3*x^2 - x^4]]
```

fricas [A] time = 0.42, size = 73, normalized size = 0.95

$$-\frac{1}{3}x^3 + \frac{1}{48}\left(8x^4 - 18x^2 - 1\right)\sqrt{-x^4 + 3x^2 - 1} + x - \frac{5}{32}\arctan\left(\frac{\sqrt{-x^4 + 3x^2 - 1}\left(2x^2 - 3\right)}{2\left(x^4 - 3x^2 + 1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(-1+x*(-x^4+3*x^2-1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/3*x^3 + 1/48*(8*x^4 - 18*x^2 - 1)*sqrt(-x^4 + 3*x^2 - 1) + x - 5/32*arct
an(1/2*sqrt(-x^4 + 3*x^2 - 1)*(2*x^2 - 3)/(x^4 - 3*x^2 + 1))
```

giac [A] time = 0.45, size = 52, normalized size = 0.68

$$-\frac{1}{3}x^3 + \frac{1}{48}\sqrt{-x^4 + 3x^2 - 1}\left(2\left(4x^2 - 9\right)x^2 - 1\right) + x + \frac{5}{32}\arcsin\left(\frac{1}{5}\sqrt{5}\left(2x^2 - 3\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(-1+x*(-x^4+3*x^2-1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/3*x^3 + 1/48*sqrt(-x^4 + 3*x^2 - 1)*(2*(4*x^2 - 9)*x^2 - 1) + x + 5/32*a
rcsin(1/5*sqrt(5)*(2*x^2 - 3))
```

maple [A] time = 0.02, size = 98, normalized size = 1.27

$$\frac{x^4\sqrt{-x^4+3x^2-1}}{6} - \frac{x^2\sqrt{-x^4+3x^2-1}}{8} - \frac{19\sqrt{-x^4+3x^2-1}}{48} + \frac{5\arcsin\left(\frac{2\sqrt{5}\left(x^2-\frac{3}{2}\right)}{5}\right)}{32} + \frac{\left(-2x^2+3\right)\sqrt{-x^4+3x^2-1}}{8} - \frac{x^3}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(-1+x*(-x^4+3*x^2-1)^(1/2)),x)
```

```
[Out] 1/6*x^4*(-x^4+3*x^2-1)^(1/2)-1/8*x^2*(-x^4+3*x^2-1)^(1/2)-19/48*(-x^4+3*x^2
-1)^(1/2)+5/32*arcsin(2/5*5^(1/2)*(x^2-3/2))+1/8*(-2*x^2+3)*(-x^4+3*x^2-1)^
(1/2)-1/3*x^3+x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}x^3 + x + \int \left(x^3 - x\right)\sqrt{x^2 + x - 1}\sqrt{-x^2 + x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(-1+x*(-x^4+3*x^2-1)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/3*x^3 + x + integrate((x^3 - x)*sqrt(x^2 + x - 1)*sqrt(-x^2 + x + 1), x)
```

mupad [B] time = 0.40, size = 111, normalized size = 1.44

$$x - \frac{\left(\frac{x^2}{2} - \frac{3}{4}\right)\sqrt{-x^4+3x^2-1}}{2} - \frac{\sqrt{-x^4+3x^2-1}\,\left(-8x^4+6x^2+19\right)}{48} - \frac{x^3}{3} - \frac{\ln\left(x^2 - \frac{3}{2} - \sqrt{-x^4+3x^2-1}\,1\mathrm{i}\right)15\mathrm{i}}{32} + \frac{\ln\left(\sqrt{-x^4+3x^2-1} + x^2\,1\mathrm{i} - \frac{3}{2}\mathrm{i}\right)5\mathrm{i}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)*(x*(3*x^2 - x^4 - 1)^(1/2) - 1),x)
```

```
[Out] x - (log(x^2 - (3*x^2 - x^4 - 1)^(1/2)*1i - 3/2)*15i)/32 + (log((3*x^2 - x^
4 - 1)^(1/2) + x^2*1i - 3i/2)*5i)/16 - ((x^2/2 - 3/4)*(3*x^2 - x^4 - 1)^(1/
2))/2 - ((3*x^2 - x^4 - 1)^(1/2)*(6*x^2 - 8*x^4 + 19))/48 - x^3/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x-1)(x+1)\left(x\sqrt{-x^4+3x^2-1}-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(-1+x*(-x**4+3*x**2-1)**(1/2)),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x*sqrt(-x**4 + 3*x**2 - 1) - 1), x)
```

3.937 $\int \frac{\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{\sqrt{b+a^2x^2}}\,dx$

Optimal. Leaf size=77

$$\frac{4\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{a}-\frac{4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{\sqrt{c}}\right)}{a}$$

Rubi [F] time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{\sqrt{b+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]]/Sqrt[b + a^2*x^2],x]
[Out] Defer[Int][Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]]/Sqrt[b + a^2*x^2], x]
```

Rubi steps

$$\int \frac{\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{\sqrt{b+a^2x^2}}\,dx=\int \frac{\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}}{\sqrt{b+a^2x^2}}\,dx$$

Mathematica [A] time = 0.13, size = 74, normalized size = 0.96

$$\frac{4\left(\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}-\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{\sqrt{c}}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]]/Sqrt[b + a^2*x^2],x]
[Out] (4*(Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]] - Sqrt[c]*ArcTanh[Sqrt[c + Sqrt
[a*x + Sqrt[b + a^2*x^2]]]/Sqrt[c]]))/a
```

IntegrateAlgebraic [A] time = 0.21, size = 77, normalized size = 1.00

$$\frac{4\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{a}-\frac{4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]]/Sqrt[b + a^2*x^2
],x]
```

[Out] (4*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]])/a - (4*Sqrt[c]*ArcTanh[Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]]/Sqrt[c]])/a

fricas [A] time = 0.51, size = 202, normalized size = 2.62

$$\left[\frac{2\left(\sqrt{c}\log\left(2\left(a\sqrt{c}x-\sqrt{a^2x^2+b}\sqrt{c}\right)\sqrt{ax+\sqrt{a^2x^2+b}}\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}-2\left(acx-\sqrt{a^2x^2+b}\,c\right)\sqrt{ax+\sqrt{a^2x^2+b}}+b\right)+2\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}\right)}{a}, \frac{4\left(\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{c}\right)+\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2)/(a^2*x^2+b)^(1/2),x, algorithm="fricas")

[Out] [2*(sqrt(c)*log(2*(a*sqrt(c)*x - sqrt(a^2*x^2 + b)*sqrt(c))*sqrt(a*x + sqrt(a^2*x^2 + b))*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b))) - 2*(a*c*x - sqrt(a^2*x^2 + b)*c)*sqrt(a*x + sqrt(a^2*x^2 + b)) + b) + 2*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b))))/a, 4*(sqrt(-c)*arctan(sqrt(-c)*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b)))/c) + sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b))))/a]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2)/(a^2*x^2+b)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{\sqrt{a^2x^2+b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2)/(a^2*x^2+b)^(1/2),x)

[Out] int((c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2)/(a^2*x^2+b)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{\sqrt{a^2x^2+b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2)/(a^2*x^2+b)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b)))/sqrt(a^2*x^2 + b), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+\sqrt{\sqrt{a^2\,x^2+b}+a\,x}}}{\sqrt{a^2\,x^2+b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + ((b + a^2*x^2)^(1/2) + a*x)^(1/2))^(1/2)/(b + a^2*x^2)^(1/2),x)
```

```
[Out] int((c + ((b + a^2*x^2)^(1/2) + a*x)^(1/2))^(1/2)/(b + a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{\sqrt{a^2x^2+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a**2*x**2+b)**(1/2))**(1/2))**(1/2)/(a**2*x**2+b)**(1/2)
,x)
```

```
[Out] Integral(sqrt(c + sqrt(a*x + sqrt(a**2*x**2 + b)))/sqrt(a**2*x**2 + b), x)
```

3.938 $\int \frac{\sqrt{-1+x^2}}{(-i+x)^2} \, dx$

Optimal. Leaf size=78

$$\frac{\sqrt{x^2-1}}{-x+i} - \log\left(\sqrt{x^2-1}-x\right) - \sqrt{2}\tanh^{-1}\left(-\frac{i\sqrt{x^2-1}}{\sqrt{2}} + \frac{ix}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {733, 844, 217, 206, 725, 204}

$$\frac{\sqrt{x^2-1}}{-x+i} - \frac{i\tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^2]/(-I + x)^2,x]
```

```
[Out] Sqrt[-1 + x^2]/(I - x) - (I*ArcTan[(1 - I*x)/(Sqrt[2]*Sqrt[-1 + x^2])])/Sqr
t[2] + ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx &= \frac{\sqrt{-1+x^2}}{i-x}+\int \frac{x}{(-i+x)\sqrt{-1+x^2}}\,dx\\ &= \frac{\sqrt{-1+x^2}}{i-x}+i\int \frac{1}{(-i+x)\sqrt{-1+x^2}}\,dx+\int \frac{1}{\sqrt{-1+x^2}}\,dx\\ &= \frac{\sqrt{-1+x^2}}{i-x}-i\,\text{Subst}\left(\int \frac{1}{-2-x^2}\,dx,x,\frac{-1+ix}{\sqrt{-1+x^2}}\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right)\\ &= \frac{\sqrt{-1+x^2}}{i-x}-\frac{i\tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{-1+x^2}}\right)}{\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.06, size = 59, normalized size = 0.76

$$-\frac{\sqrt{x^2-1}}{x-i}+\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)-\frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^2]/(-I + x)^2,x]
[Out] -(Sqrt[-1 + x^2]/(-I + x)) + ArcTanh[x/Sqrt[-1 + x^2]] - ArcTanh[(I + x)/(S
qrt[2]*Sqrt[-1 + x^2])]/Sqrt[2]
```

IntegrateAlgebraic [A] time = 0.32, size = 78, normalized size = 1.00

$$\frac{\sqrt{x^2-1}}{-x+i}-\log\left(\sqrt{x^2-1}-x\right)-\sqrt{2}\tanh^{-1}\left(-\frac{i\sqrt{x^2-1}}{\sqrt{2}}+\frac{ix}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x^2]/(-I + x)^2,x]
[Out] Sqrt[-1 + x^2]/(I - x) - Sqrt[2]*ArcTanh[1/Sqrt[2] + (I*x)/Sqrt[2] - (I*Sqr
t[-1 + x^2])/Sqrt[2]] - Log[-x + Sqrt[-1 + x^2]]
```

fricas [A] time = 0.45, size = 92, normalized size = 1.18

$$-\frac{\sqrt{2}(x-i)\log\left(-x+i\sqrt{2}+\sqrt{x^2-1}+i\right)-\sqrt{2}(x-i)\log\left(-x-i\sqrt{2}+\sqrt{x^2-1}+i\right)+(2x-2i)\log\left(-x+\sqrt{x^2-1}\right)+2x+2\sqrt{x^2-1}-2i}{2x-2i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="fricas")
[Out] -(sqrt(2)*(x - I)*log(-x + I*sqrt(2) + sqrt(x^2 - 1) + I) - sqrt(2)*(x - I)
*log(-x - I*sqrt(2) + sqrt(x^2 - 1) + I) + (2*x - 2*I)*log(-x + sqrt(x^2 -
1)) + 2*x + 2*sqrt(x^2 - 1) - 2*I)/(2*x - 2*I)
```

giac [A] time = 0.38, size = 84, normalized size = 1.08

$$i\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2-1}-i\right)\right)+\frac{2\left(ix-i\sqrt{x^2-1}-1\right)}{\left(x-\sqrt{x^2-1}\right)^2-2ix+2i\sqrt{x^2-1}+1}-\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="giac")
```

```
[Out] I*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 - 1) - I)) + 2*(I*x - I*sqrt(x^
2 - 1) - 1)/((x - sqrt(x^2 - 1))^2 - 2*I*x + 2*I*sqrt(x^2 - 1) + 1) - log(a
bs(-x + sqrt(x^2 - 1)))
```

maple [A] time = 0.04, size = 125, normalized size = 1.60

$$\frac{\left((-i+x)^{2}+2i\,(-i+x)-2\right)^{\frac{3}{2}}}{-2i+2x}+\frac{i\sqrt{2}\,\arctan\left(\frac{(-4+2i(-i+x))\sqrt{2}}{4\sqrt{(-i+x)^{2}+2i(-i+x)-2}}\right)}{2}+\ln\left(x+\sqrt{(-i+x)^{2}+2i\,(-i+x)-2}\right)-\frac{i\sqrt{(-i+x)^{2}+2i\,(-i+x)-2}}{2}-\frac{x\sqrt{(-i+x)^{2}+2i\,(-i+x)-2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(1/2)/(-I+x)^2,x)
```

```
[Out] 1/2/(-I+x)*((-I+x)^2+2*I*(-I+x)-2)^(3/2)+1/2*I*2^(1/2)*arctan(1/4*(-4+2*I*(
-I+x))*2^(1/2)/((-I+x)^2+2*I*(-I+x)-2)^(1/2))+ln(x+((-I+x)^2+2*I*(-I+x)-2)^
(1/2))-1/2*I*((-I+x)^2+2*I*(-I+x)-2)^(1/2)-1/2*x*((-I+x)^2+2*I*(-I+x)-2)^(1
/2)
```

maxima [A] time = 0.44, size = 53, normalized size = 0.68

$$\frac{1}{2}i\sqrt{2}\,\arcsin\left(\frac{i\,x}{|x-i|}-\frac{1}{|x-i|}\right)-\frac{\sqrt{x^{2}-1}}{x-i}+\log\left(2\,x+2\,\sqrt{x^{2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*I*sqrt(2)*arcsin(I*x/abs(x - I) - 1/abs(x - I)) - sqrt(x^2 - 1)/(x - I)
 + log(2*x + 2*sqrt(x^2 - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^{2}-1}}{(x-\mathrm{i})^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^(1/2)/(x - 1i)^2,x)
```

```
[Out] int((x^2 - 1)^(1/2)/(x - 1i)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)\,(x+1)}}{(x-i)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(1/2)/(-I+x)**2,x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1))/(x - I)**2, x)
```

3.939 $\int \frac{1}{x^3\left(b+ax^2\right)^{3/4}}\,dx$

Optimal. Leaf size=78

$$\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{4b^{7/4}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{4b^{7/4}}-\frac{\sqrt[4]{ax^2+b}}{2bx^2}$$

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 63, 212, 206, 203}

$$\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{4b^{7/4}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{4b^{7/4}}-\frac{\sqrt[4]{ax^2+b}}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b + a*x^2)^(3/4)),x]
```

```
[Out] -1/2*(b + a*x^2)^(1/4)/(b*x^2) + (3*a*ArcTan[(b + a*x^2)^(1/4)/b^(1/4)])/(4
*b^(7/4)) + (3*a*ArcTanh[(b + a*x^2)^(1/4)/b^(1/4)])/(4*b^(7/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(b+ax^2\right)^{3/4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(b+ax)^{3/4}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt[4]{b+ax^2}}{2bx^2} - \frac{(3a)\operatorname{Subst}\left(\int \frac{1}{x(b+ax)^{3/4}}\,dx, x, x^2\right)}{8b}\\
&= -\frac{\sqrt[4]{b+ax^2}}{2bx^2} - \frac{3\operatorname{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^2}\right)}{2b}\\
&= -\frac{\sqrt[4]{b+ax^2}}{2bx^2} + \frac{(3a)\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^2}\right)}{4b^{3/2}} + \frac{(3a)\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx, x, \sqrt[4]{b\cdots}\right.}{4b^{3/2}}\\
&= -\frac{\sqrt[4]{b+ax^2}}{2bx^2} + \frac{3a\tan^{-1}\left(\frac{\sqrt[4]{b+ax^2}}{\sqrt[4]{b}}\right)}{4b^{7/4}} + \frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^2}}{\sqrt[4]{b}}\right)}{4b^{7/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.45

$$\frac{2a\sqrt[4]{ax^2+b}\;{}_2F_1\left(\frac{1}{4},2;\frac{5}{4};\frac{ax^2}{b}+1\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b + a*x^2)^(3/4)),x]
```

```
[Out] (2*a*(b + a*x^2)^(1/4)*Hypergeometric2F1[1/4, 2, 5/4, 1 + (a*x^2)/b])/b^2
```

IntegrateAlgebraic [A] time = 0.13, size = 78, normalized size = 1.00

$$\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{4b^{7/4}} + \frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)}{4b^{7/4}} - \frac{\sqrt[4]{ax^2+b}}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(b + a*x^2)^(3/4)),x]
```

```
[Out] -1/2*(b + a*x^2)^(1/4)/(b*x^2) + (3*a*ArcTan[(b + a*x^2)^(1/4)/b^(1/4)])/(4
*b^(7/4)) + (3*a*ArcTanh[(b + a*x^2)^(1/4)/b^(1/4)])/(4*b^(7/4))
```

fricas [B] time = 0.46, size = 194, normalized size = 2.49

$$-\frac{12\,bx^2\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^2+b\right)^{\frac{1}{4}}ab^5\left(\frac{a^4}{b^7}\right)^{\frac{3}{4}} - \sqrt{b^4\sqrt{\frac{a^4}{b^7}}+\sqrt{ax^2+b}\,a^2}\,b^5\left(\frac{a^4}{b^7}\right)^{\frac{3}{4}}}{a^4}\right) - 3\,bx^2\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(3\,b^2\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}} + 3\left(ax^2+b\right)^{\frac{1}{4}}a\right) + 3\,bx^2\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(-3\,b^2\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}} + 3\left(ax^2+b\right)^{\frac{1}{4}}a\right) + 4\left(ax^2+b\right)^{\frac{1}{4}}}{8\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^2+b)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/8*(12*b*x^2*(a^4/b^7)^(1/4)*arctan(-((a*x^2 + b)^(1/4)*a*b^5*(a^4/b^7)^(
3/4) - sqrt(b^4*sqrt(a^4/b^7) + sqrt(a*x^2 + b)*a^2)*b^5*(a^4/b^7)^(3/4))/a
^4) - 3*b*x^2*(a^4/b^7)^(1/4)*log(3*b^2*(a^4/b^7)^(1/4) + 3*(a*x^2 + b)^(1/
4)*a) + 3*b*x^2*(a^4/b^7)^(1/4)*log(-3*b^2*(a^4/b^7)^(1/4) + 3*(a*x^2 + b)^
(1/4)*a) + 4*(a*x^2 + b)^(1/4))/(b*x^2)
```

giac [B] time = 0.36, size = 221, normalized size = 2.83

$$\frac{\frac{6\sqrt{2}a^2(-b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}+2\left(ax^2+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{b^2}+\frac{6\sqrt{2}a^2(-b)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}-2\left(ax^2+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{b^2}+\frac{3\sqrt{2}a^2(-b)^{\frac{1}{4}}\log\left(\sqrt{2}\left(ax^2+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^2+b}+\sqrt{-b}\right)}{b^2}+\frac{3\sqrt{2}a^2\log\left(-\sqrt{2}\left(ax^2+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^2+b}+\sqrt{-b}\right)}{(-b)^{\frac{3}{4}}b}-\frac{8\left(ax^2+b\right)^{\frac{1}{4}}a}{bx^2}}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^2+b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/16*(6*sqrt(2)*a^2*(-b)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(
a*x^2 + b)^(1/4))/(-b)^(1/4))/b^2 + 6*sqrt(2)*a^2*(-b)^(1/4)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(-b)^(1/4) - 2*(a*x^2 + b)^(1/4))/(-b)^(1/4))/b^2 + 3*sqrt(2
)*a^2*(-b)^(1/4)*log(sqrt(2)*(a*x^2 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^2 + b)
 + sqrt(-b))/b^2 + 3*sqrt(2)*a^2*log(-sqrt(2)*(a*x^2 + b)^(1/4)*(-b)^(1/4)
+ sqrt(a*x^2 + b) + sqrt(-b))/((-b)^(3/4)*b) - 8*(a*x^2 + b)^(1/4)*a/(b*x^2
))/a
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a\,x^2+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x^2+b)^(3/4),x)
```

```
[Out] int(1/x^3/(a*x^2+b)^(3/4),x)
```

maxima [A] time = 0.42, size = 94, normalized size = 1.21

$$-\frac{\left(ax^2+b\right)^{\frac{1}{4}}a}{2\left(\left(ax^2+b\right)b-b^2\right)}+\frac{3\left(\frac{2\,a\arctan\left(\frac{\left(ax^2+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}-\frac{a\log\left(\frac{\left(ax^2+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^2+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^2+b)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/2*(a*x^2 + b)^(1/4)*a/((a*x^2 + b)*b - b^2) + 3/8*(2*a*arctan((a*x^2 + b
)^(1/4)/b^(1/4))/b^(3/4) - a*log(((a*x^2 + b)^(1/4) - b^(1/4))/((a*x^2 + b)
^(1/4) + b^(1/4)))/b^(3/4))/b
```

mupad [B] time = 0.99, size = 58, normalized size = 0.74

$$\frac{3\,a\,\mathrm{atan}\left(\frac{{\left(a\,x^2+b\right)}^{1/4}}{b^{1/4}}\right)}{4\,b^{7/4}}-\frac{{\left(a\,x^2+b\right)}^{1/4}}{2\,b\,x^2}+\frac{3\,a\,\mathrm{atanh}\left(\frac{{\left(a\,x^2+b\right)}^{1/4}}{b^{1/4}}\right)}{4\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b + a*x^2)^(3/4)),x)
```

```
[Out] (3*a*atan((b + a*x^2)^(1/4)/b^(1/4)))/(4*b^(7/4)) - (b + a*x^2)^(1/4)/(2*b*
x^2) + (3*a*atanh((b + a*x^2)^(1/4)/b^(1/4)))/(4*b^(7/4))
```

sympy [C] time = 1.44, size = 41, normalized size = 0.53

$$-\frac{\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^2}\right)}{2a^{\frac{3}{4}}x^{\frac{7}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*x**2+b)**(3/4),x)
```

```
[Out] -gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*exp_polar(I*pi)/(a*x**2))/(2*a**(3
/4)*x**(7/2)*gamma(11/4))
```

3.940 $$\int \frac{(-1-2x+x^2+3x^3)^4}{\sqrt[4]{-1+3x-3x^2+x^3}}\,dx$$

Optimal. Leaf size=78

$$4\left((x-1)^3\right)^{3/4}\left(1949108765175x^{12}+4908866519700x^{11}+609206533650x^{10}-9283999210200x^9-8805988591\right.$$

Rubi [B] time = 1.00, antiderivative size = 248, normalized size of antiderivative = 3.18, number of steps used = 53, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6742, 2067, 15, 30, 2081, 43}

$$-\frac{324(1-x)^{13}}{49\sqrt[4]{(x-1)^3}}+\frac{96(1-x)^{12}}{\sqrt[4]{(x-1)^3}}-\frac{25488(1-x)^{11}}{41\sqrt[4]{(x-1)^3}}+\frac{87312(1-x)^{10}}{37\sqrt[4]{(x-1)^3}}-\frac{191416(1-x)^9}{33\sqrt[4]{(x-1)^3}}+\frac{278928(1-x)^8}{29\sqrt[4]{(x-1)^3}}-\frac{271528(1-x)^7}{25\sqrt[4]{(x-1)^3}}+\frac{57648(1-x)^6}{7\sqrt[4]{(x-1)^3}}-\frac{68820(1-x)^5}{17\sqrt[4]{(x-1)^3}}+\frac{16032(1-x)^4}{13\sqrt[4]{(x-1)^3}}+\frac{144(1-x)^2}{5\sqrt[4]{(x-1)^3}}-\frac{4(1-x)}{\sqrt[4]{(x-1)^3}}+\frac{2104}{9}\left((x-1)^3\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 2*x + x^2 + 3*x^3)^4/(-1 + 3*x - 3*x^2 + x^3)^(1/4),x]
```

```
[Out] (-4*(1 - x))/((-1 + x)^3)^(1/4) + (144*(1 - x)^2)/(5*((-1 + x)^3)^(1/4)) +
(16032*(1 - x)^4)/(13*((-1 + x)^3)^(1/4)) - (68820*(1 - x)^5)/(17*((-1 + x)
^3)^(1/4)) + (57648*(1 - x)^6)/(7*((-1 + x)^3)^(1/4)) - (271528*(1 - x)^7)/
(25*((-1 + x)^3)^(1/4)) + (278928*(1 - x)^8)/(29*((-1 + x)^3)^(1/4)) - (191
416*(1 - x)^9)/(33*((-1 + x)^3)^(1/4)) + (87312*(1 - x)^10)/(37*((-1 + x)^3
)^(1/4)) - (25488*(1 - x)^11)/(41*((-1 + x)^3)^(1/4)) + (96*(1 - x)^12)/((-
1 + x)^3)^(1/4) - (324*(1 - x)^13)/(49*((-1 + x)^3)^(1/4)) + (2104*((-1 + x
)^3)^(3/4))/9
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 2081

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(-1-2x+x^2+3x^3\right)^4}{\sqrt[4]{-1+3x-3x^2+x^3}}\,dx = \int \left(\frac{1}{\sqrt[4]{-1+3x-3x^2+x^3}} + \frac{8x}{\sqrt[4]{-1+3x-3x^2+x^3}} + \frac{20x^2}{\sqrt[4]{-1+3x-3x^2+x^3}} - \right.$$

$$= -\left(4\int \frac{x^3}{\sqrt[4]{-1+3x-3x^2+x^3}}\,dx\right) + 8\int \frac{x}{\sqrt[4]{-1+3x-3x^2+x^3}}\,dx + 20\int \frac{}{\sqrt[4]{-}}$$

$$= -\left(4\,\text{Subst}\left(\int \frac{(1+x)^3}{\sqrt[4]{x^3}}\,dx, x, -1+x\right)\right) + 8\,\text{Subst}\left(\int \frac{1+x}{\sqrt[4]{x^3}}\,dx, x, -1+x\right) + 20$$

$$= \frac{(-1+x)^{3/4}\,\text{Subst}\left(\int \frac{1}{x^{3/4}}\,dx, x, -1+x\right)}{\sqrt[4]{(-1+x)^3}} - \frac{\left(4(-1+x)^{3/4}\right)\text{Subst}\left(\int \frac{(1+x)^3}{x^{3/4}}\,dx, x, -\right.}{\sqrt[4]{(-1+x)^3}}$$

$$= -\frac{4(1-x)}{\sqrt[4]{(-1+x)^3}} - \frac{\left(4(-1+x)^{3/4}\right)\text{Subst}\left(\int \left(\frac{1}{x^{3/4}} + 3\sqrt[4]{x} + 3x^{5/4} + x^{9/4}\right)dx, x, -1\right.}{\sqrt[4]{(-1+x)^3}}$$

$$= -\frac{4(1-x)}{\sqrt[4]{(-1+x)^3}} + \frac{144(1-x)^2}{5\sqrt[4]{(-1+x)^3}} + \frac{16032(1-x)^4}{13\sqrt[4]{(-1+x)^3}} - \frac{68820(1-x)^5}{17\sqrt[4]{(-1+x)^3}} + \frac{57648(1-}{7\sqrt[4]{(-1+x}}$$

Mathematica [A] time = 0.10, size = 156, normalized size = 2.00

$$\frac{4\left(\frac{81}{49}(x-1)^{49/4}+24(x-1)^{45/4}+\frac{6372}{41}(x-1)^{41/4}+\frac{21828}{37}(x-1)^{37/4}+\frac{47854}{33}(x-1)^{33/4}+\frac{69732}{29}(x-1)^{29/4}+\frac{67882}{25}(x-1)^{25/4}+\frac{14412}{7}(x-1)^{21/4}+\frac{17205}{17}(x-1)^{17/4}+\frac{4008}{13}(x-1)^{13/4}+\frac{526}{9}(x-1)^{9/4}+\frac{36}{5}(x-1)^{5/4}+\sqrt[4]{x-1}\right)(x-1)^{3/4}}{\sqrt[4]{(x-1)^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x + x^2 + 3*x^3)^4/(-1 + 3*x - 3*x^2 + x^3)^(1/4),x]
```

```
[Out] (4*((-1 + x)^(1/4) + (36*(-1 + x)^(5/4))/5 + (526*(-1 + x)^(9/4))/9 + (4008
*(-1 + x)^(13/4))/13 + (17205*(-1 + x)^(17/4))/17 + (14412*(-1 + x)^(21/4))
/7 + (67882*(-1 + x)^(25/4))/25 + (69732*(-1 + x)^(29/4))/29 + (47854*(-1 +
 x)^(33/4))/33 + (21828*(-1 + x)^(37/4))/37 + (6372*(-1 + x)^(41/4))/41 + 2
4*(-1 + x)^(45/4) + (81*(-1 + x)^(49/4))/49)*(-1 + x)^(3/4))/((-1 + x)^3)^(
1/4)
```

IntegrateAlgebraic [A] time = 5.06, size = 138, normalized size = 1.77

$$\frac{4\left(1949108765175(x-1)^{49/4}+28298171701800(x-1)^{45/4}+183247916751900(x-1)^{41/4}+695599653048300(x-1)^{37/4}+1709824127042850(x-1)^{33/4}+2835184064813100(x-1)^{29/4}+3201560819102646(x-1)^{25/4}+2427578872418700(x-1)^{21/4}+1193308931689875(x-1)^{17/4}+363522667246200(x-1)^{13/4}+68911288496050(x-1)^{9/4}+8489451510540(x-1)^{5/4}+1179090487575\sqrt[4]{x-1}\right)\left((x-1)^3\right)^{3/4}}{1179090487575(x-1)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 2*x + x^2 + 3*x^3)^4/(-1 + 3*x - 3*x^2 + x^3)^(1/4
),x]
```

```
[Out] (4*(1179090487575*(-1 + x)^(1/4) + 8489451510540*(-1 + x)^(5/4) + 689112884
96050*(-1 + x)^(9/4) + 363522667246200*(-1 + x)^(13/4) + 1193308931689875*(
-1 + x)^(17/4) + 2427578872418700*(-1 + x)^(21/4) + 3201560819102646*(-1 +
x)^(25/4) + 2835184064813100*(-1 + x)^(29/4) + 1709824127042850*(-1 + x)^(3
3/4) + 695599653048300*(-1 + x)^(37/4) + 183247916751900*(-1 + x)^(41/4) +
28298171701800*(-1 + x)^(45/4) + 1949108765175*(-1 + x)^(49/4))*((-1 + x)^3
)^(3/4))/(1179090487575*(-1 + x)^(9/4))
```

fricas [A] time = 0.43, size = 87, normalized size = 1.12

$$\frac{4\left(1949108765175\,x^{12}+4908866519700\,x^{11}+609206533650\,x^{10}-9283999210200\,x^{9}-8805988591725\,x^{8}+3131067556500\,x^{7}+9260757242646\,x^{6}+4070651298324\,x^{5}-2008108342110\,x^{4}-2834315032620\,x^{3}-1158885626660\,x^{2}+32327777464\,x+1308401597431\right)\left(x^{3}-3\,x^{2}+3\,x-1\right)^{\frac{3}{4}}}{1179090487575\,\left(x^{2}-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3+x^2-2*x-1)^4/(x^3-3*x^2+3*x-1)^(1/4),x, algorithm="fricas"
)
```

```
[Out] 4/1179090487575*(1949108765175*x^12 + 4908866519700*x^11 + 609206533650*x^1
0 - 9283999210200*x^9 - 8805988591725*x^8 + 3131067556500*x^7 + 92607572426
46*x^6 + 4070651298324*x^5 - 2008108342110*x^4 - 2834315032620*x^3 - 115888
5626660*x^2 + 32327777464*x + 1308401597431)*(x^3 - 3*x^2 + 3*x - 1)^(3/4)/
(x^2 - 2*x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^3+x^2-2\,x-1\right)^4}{\left(x^3-3\,x^2+3\,x-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3+x^2-2*x-1)^4/(x^3-3*x^2+3*x-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((3*x^3 + x^2 - 2*x - 1)^4/(x^3 - 3*x^2 + 3*x - 1)^(1/4), x)
```

maple [A] time = 0.01, size = 81, normalized size = 1.04

$$\frac{4\left(-1+x\right)\left(1949108765175x^{12}+4908866519700x^{11}+609206533650x^{10}-9283999210200x^{9}-8805988591725x^{8}+3131067556500x^{7}+9260757242646x^{6}+4070651298324x^{5}-2008108342110x^{4}-2834315032620x^{3}-1158885626660x^{2}+32327777464x+1308401597431\right)}{1179090487575\left(x^{3}-3x^{2}+3x-1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3+x^2-2*x-1)^4/(x^3-3*x^2+3*x-1)^(1/4),x)
```

```
[Out] 4/1179090487575*(-1+x)*(1949108765175*x^12+4908866519700*x^11+609206533650*
x^10-9283999210200*x^9-8805988591725*x^8+3131067556500*x^7+9260757242646*x^
6+4070651298324*x^5-2008108342110*x^4-2834315032620*x^3-1158885626660*x^2+3
2327777464*x+1308401597431)/(x^3-3*x^2+3*x-1)^(1/4)
```

maxima [B] time = 0.36, size = 540, normalized size = 6.92

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3+x^2-2*x-1)^4/(x^3-3*x^2+3*x-1)^(1/4),x, algorithm="maxima"
)
```

```
[Out] 108/11910004925*(729183975*x^13 + 48612265*x^12 + 56911920*x^11 + 67679040*
x^10 + 82035200*x^9 + 101836800*x^8 + 130351104*x^7 + 173801472*x^6 + 24536
6784*x^5 + 377487360*x^4 + 671088640*x^3 + 1610612736*x^2 + 12884901888*x -
 17179869184)/(x - 1)^(3/4) + 48/243061325*(48612265*x^12 + 3556995*x^11 +
4229940*x^10 + 5127200*x^9 + 6364800*x^8 + 8146944*x^7 + 10862592*x^6 + 153
35424*x^5 + 23592960*x^4 + 41943040*x^3 + 100663296*x^2 + 805306368*x - 107
3741824)/(x - 1)^(3/4) - 648/534734915*(13042315*x^11 + 1057485*x^10 + 1281
800*x^9 + 1591200*x^8 + 2036736*x^7 + 2715648*x^6 + 3833856*x^5 + 5898240*x
^4 + 10485760*x^3 + 25165824*x^2 + 201326592*x - 268435456)/(x - 1)^(3/4) -
 96/5016275*(1762475*x^10 + 160225*x^9 + 198900*x^8 + 254592*x^7 + 339456*x
^6 + 479232*x^5 + 737280*x^4 + 1310720*x^3 + 3145728*x^2 + 25165824*x - 335
54432)/(x - 1)^(3/4) + 148/15862275*(480675*x^9 + 49725*x^8 + 63648*x^7 + 8
4864*x^6 + 119808*x^5 + 184320*x^4 + 327680*x^3 + 786432*x^2 + 6291456*x -
8388608)/(x - 1)^(3/4) + 1264/480675*(16575*x^8 + 1989*x^7 + 2652*x^6 + 374
4*x^5 + 5760*x^4 + 10240*x^3 + 24576*x^2 + 196608*x - 262144)/(x - 1)^(3/4)
 + 488/116025*(4641*x^7 + 663*x^6 + 936*x^5 + 1440*x^4 + 2560*x^3 + 6144*x^
2 + 49152*x - 65536)/(x - 1)^(3/4) - 464/13923*(663*x^6 + 117*x^5 + 180*x^4
```

```
 + 320*x^3 + 768*x^2 + 6144*x - 8192)/(x - 1)^(3/4) - 392/3315*(195*x^5 + 4
5*x^4 + 80*x^3 + 192*x^2 + 1536*x - 2048)/(x - 1)^(3/4) - 16/195*(15*x^4 +
5*x^3 + 12*x^2 + 96*x - 128)/(x - 1)^(3/4) + 16/9*(5*x^3 + 3*x^2 + 24*x - 3
2)/(x - 1)^(3/4) + 32/5*(x^2 + 3*x - 4)/(x - 1)^(3/4) + 4*(x - 1)^(1/4)
```

mupad [B] time = 0.90, size = 86, normalized size = 1.10

$$\frac{\left(x^3-3x^2+3x-1\right)^{3/4}\left(\frac{324x^{12}}{49}+\frac{816x^{11}}{49}+\frac{4152x^{10}}{2009}-\frac{2341152x^9}{74333}-\frac{73280188x^8}{2452989}+\frac{755612080x^7}{71136681}+\frac{7981691224x^6}{254059575}+\frac{24558982192x^5}{1778417025}-\frac{41191965992x^4}{6046617885}-\frac{755817342032x^3}{78606032505}-\frac{927108501328x^2}{235818097515}+\frac{129311109856x}{1179090487575}+\frac{5233606389724}{1179090487575}\right)}{x^2-2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 - 3*x^3 + 1)^4/(3*x - 3*x^2 + x^3 - 1)^(1/4),x)
```

```
[Out] ((3*x - 3*x^2 + x^3 - 1)^(3/4)*((129311109856*x)/1179090487575 - (927108501
328*x^2)/235818097515 - (755817342032*x^3)/78606032505 - (41191965992*x^4)/
6046617885 + (24558982192*x^5)/1778417025 + (7981691224*x^6)/254059575 + (7
55612080*x^7)/71136681 - (73280188*x^8)/2452989 - (2341152*x^9)/74333 + (41
52*x^10)/2009 + (816*x^11)/49 + (324*x^12)/49 + 5233606389724/1179090487575
))/(x^2 - 2*x + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^3+x^2-2x-1\right)^4}{\sqrt[4]{(x-1)^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3+x**2-2*x-1)**4/(x**3-3*x**2+3*x-1)**(1/4),x)
```

```
[Out] Integral((3*x**3 + x**2 - 2*x - 1)**4/((x - 1)**3)**(1/4), x)
```

3.941 $\int \frac{1}{x^4\left(b+ax^3\right)^{3/4}}\,dx$

Optimal. Leaf size=78

$$\frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{2b^{7/4}}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{2b^{7/4}}-\frac{\sqrt[4]{ax^3+b}}{3bx^3}$$

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 63, 212, 206, 203}

$$\frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{2b^{7/4}}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{2b^{7/4}}-\frac{\sqrt[4]{ax^3+b}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(b + a*x^3)^(3/4)),x]
```

```
[Out] -1/3*(b + a*x^3)^(1/4)/(b*x^3) + (a*ArcTan[(b + a*x^3)^(1/4)/b^(1/4)])/(2*b
^(7/4)) + (a*ArcTanh[(b + a*x^3)^(1/4)/b^(1/4)])/(2*b^(7/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(b+ax^3\right)^{3/4}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2(b+ax)^{3/4}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt[4]{b+ax^3}}{3bx^3} - \frac{a\operatorname{Subst}\left(\int \frac{1}{x(b+ax)^{3/4}}\,dx, x, x^3\right)}{4b}\\
&= -\frac{\sqrt[4]{b+ax^3}}{3bx^3} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^3}\right)}{b}\\
&= -\frac{\sqrt[4]{b+ax^3}}{3bx^3} + \frac{a\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^3}\right)}{2b^{3/2}} + \frac{a\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx, x, \sqrt[4]{b+ax^3}\right)}{2b^{3/2}}\\
&= -\frac{\sqrt[4]{b+ax^3}}{3bx^3} + \frac{a\tan^{-1}\left(\frac{\sqrt[4]{b+ax^3}}{\sqrt[4]{b}}\right)}{2b^{7/4}} + \frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^3}}{\sqrt[4]{b}}\right)}{2b^{7/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.47

$$\frac{4a\sqrt[4]{ax^3+b}\,{}_2F_1\left(\frac{1}{4},2;\frac{5}{4};\frac{ax^3}{b}+1\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(b + a*x^3)^(3/4)),x]
```

```
[Out] (4*a*(b + a*x^3)^(1/4)*Hypergeometric2F1[1/4, 2, 5/4, 1 + (a*x^3)/b])/(3*b^
2)
```

IntegrateAlgebraic [A] time = 0.08, size = 78, normalized size = 1.00

$$\frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{2b^{7/4}} + \frac{a\tanh^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{2b^{7/4}} - \frac{\sqrt[4]{ax^3+b}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(b + a*x^3)^(3/4)),x]
```

```
[Out] -1/3*(b + a*x^3)^(1/4)/(b*x^3) + (a*ArcTan[(b + a*x^3)^(1/4)/b^(1/4)])/(2*b
^(7/4)) + (a*ArcTanh[(b + a*x^3)^(1/4)/b^(1/4)])/(2*b^(7/4))
```

fricas [B] time = 0.44, size = 191, normalized size = 2.45

$$-\frac{12\,bx^3\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{(ax^3+b)^{\frac{1}{4}}ab^5\left(\frac{a^4}{b^7}\right)^{\frac{3}{4}}-\sqrt{b^4\sqrt{\frac{a^4}{b^7}}+\sqrt{ax^3+b}\,a^2}\,b^5\left(\frac{a^4}{b^7}\right)^{\frac{3}{4}}}{a^4}\right)-3\,bx^3\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(b^2\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}+\left(ax^3+b\right)^{\frac{1}{4}}a\right)+3\,bx^3\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(-b^2\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}+\left(ax^3+b\right)^{\frac{1}{4}}a\right)+4\left(ax^3+b\right)^{\frac{1}{4}}}{12\,bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*x^3+b)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/12*(12*b*x^3*(a^4/b^7)^(1/4)*arctan(-((a*x^3 + b)^(1/4)*a*b^5*(a^4/b^7)^
(3/4) - sqrt(b^4*sqrt(a^4/b^7) + sqrt(a*x^3 + b)*a^2)*b^5*(a^4/b^7)^(3/4))/
a^4) - 3*b*x^3*(a^4/b^7)^(1/4)*log(b^2*(a^4/b^7)^(1/4) + (a*x^3 + b)^(1/4)*
a) + 3*b*x^3*(a^4/b^7)^(1/4)*log(-b^2*(a^4/b^7)^(1/4) + (a*x^3 + b)^(1/4)*a
) + 4*(a*x^3 + b)^(1/4))/(b*x^3)
```

giac [B] time = 0.97, size = 221, normalized size = 2.83

$$\frac{\frac{6\sqrt{2}a^2(-b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}+2\left(ax^3+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{b^2}+\frac{6\sqrt{2}a^2(-b)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}-2\left(ax^3+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{b^2}+\frac{3\sqrt{2}a^2(-b)^{\frac{1}{4}}\log\left(\sqrt{2}\left(ax^3+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^3+b}+\sqrt{-b}\right)}{b^2}+\frac{3\sqrt{2}a^2\log\left(-\sqrt{2}\left(ax^3+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^3+b}+\sqrt{-b}\right)}{(-b)^{\frac{3}{4}}b}-\frac{8\left(ax^3+b\right)^{\frac{1}{4}}a}{bx^3}}{24a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*x^3+b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/24*(6*sqrt(2)*a^2*(-b)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(
a*x^3 + b)^(1/4))/(-b)^(1/4))/b^2 + 6*sqrt(2)*a^2*(-b)^(1/4)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(-b)^(1/4) - 2*(a*x^3 + b)^(1/4))/(-b)^(1/4))/b^2 + 3*sqrt(2
)*a^2*(-b)^(1/4)*log(sqrt(2)*(a*x^3 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^3 + b)
 + sqrt(-b))/b^2 + 3*sqrt(2)*a^2*log(-sqrt(2)*(a*x^3 + b)^(1/4)*(-b)^(1/4)
+ sqrt(a*x^3 + b) + sqrt(-b))/((-b)^(3/4)*b) - 8*(a*x^3 + b)^(1/4)*a/(b*x^3
))/a
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a\,x^3+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a*x^3+b)^(3/4),x)
```

```
[Out] int(1/x^4/(a*x^3+b)^(3/4),x)
```

maxima [A] time = 0.43, size = 94, normalized size = 1.21

$$-\frac{\left(ax^3+b\right)^{\frac{1}{4}}a}{3\left(\left(ax^3+b\right)b-b^2\right)}+\frac{\frac{2\,a\arctan\left(\frac{\left(ax^3+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}-\frac{a\log\left(\frac{\left(ax^3+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^3+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*x^3+b)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/3*(a*x^3 + b)^(1/4)*a/((a*x^3 + b)*b - b^2) + 1/4*(2*a*arctan((a*x^3 + b
)^(1/4)/b^(1/4))/b^(3/4) - a*log(((a*x^3 + b)^(1/4) - b^(1/4))/((a*x^3 + b)
^(1/4) + b^(1/4)))/b^(3/4))/b
```

mupad [B] time = 1.01, size = 58, normalized size = 0.74

$$\frac{a\,\mathrm{atan}\left(\frac{{\left(a\,x^3+b\right)}^{1/4}}{b^{1/4}}\right)}{2\,b^{7/4}}-\frac{{\left(a\,x^3+b\right)}^{1/4}}{3\,b\,x^3}+\frac{a\,\mathrm{atanh}\left(\frac{{\left(a\,x^3+b\right)}^{1/4}}{b^{1/4}}\right)}{2\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(b + a*x^3)^(3/4)),x)
```

```
[Out] (a*atan((b + a*x^3)^(1/4)/b^(1/4)))/(2*b^(7/4)) - (b + a*x^3)^(1/4)/(3*b*x^
3) + (a*atanh((b + a*x^3)^(1/4)/b^(1/4)))/(2*b^(7/4))
```

sympy [C] time = 1.20, size = 41, normalized size = 0.53

$$-\frac{\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{4}}x^{\frac{21}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a*x**3+b)**(3/4),x)
```

```
[Out] -gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*exp_polar(I*pi)/(a*x**3))/(3*a**(3
/4)*x**(21/4)*gamma(11/4))
```

3.942 $\int \frac{x}{\sqrt{-71-96x+10x^2+x^4}}\,dx$

Optimal. Leaf size=78

$$-\frac{1}{8}\log\left(-x^8-20x^6+128x^5-54x^4+1408x^3-3124x^2+\sqrt{x^4+10x^2-96x-71}\left(x^6+15x^4-80x^3+27x^2-52\right.\right.$$

Rubi [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 0.97, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2082}

$$\frac{1}{8}\log\left(x^8+20x^6-128x^5+54x^4-1408x^3+3124x^2+\sqrt{x^4+10x^2-96x-71}\left(x^6+15x^4-80x^3+27x^2-528x+781\right)+10001\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-71 - 96*x + 10*x^2 + x^4],x]
```

```
[Out] Log[10001 + 3124*x^2 - 1408*x^3 + 54*x^4 - 128*x^5 + 20*x^6 + x^8 + Sqrt[-7
1 - 96*x + 10*x^2 + x^4]*(781 - 528*x + 27*x^2 - 80*x^3 + 15*x^4 + x^6)]/8
```

Rule 2082

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (e_.)*(x_)^4], x_Symbol] :
> With[{Px = (1*(33*b^2*c + 6*a*c^2 + 40*a^2*e))/320 - (22*a*c*e*x^2)/5 + (
22*b*c*e*x^3)/15 + (1*e*(5*c^2 + 4*a*e)*x^4)/4 + (4*b*e^2*x^5)/3 + 2*c*e^2*
x^6 + e^3*x^8}, Simp[(1*Log[Px + Dist[1/(8*Rt[e, 2]*x), D[Px, x], x]*Sqrt[a
 + b*x + c*x^2 + e*x^4]])/(8*Rt[e, 2]), x]] /; FreeQ[{a, b, c, e}, x] && Eq
Q[71*c^2 + 100*a*e, 0] && EqQ[1152*c^3 - 125*b^2*e, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{-71-96x+10x^2+x^4}}\,dx = \frac{1}{8}\log\left(10001+3124x^2-1408x^3+54x^4-128x^5+20x^6+x^8+\sqrt{-71-96x+\right.$$

Mathematica [C] time = 2.68, size = 1226, normalized size = 15.72

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[-71 - 96*x + 10*x^2 + x^4],x]
```

```
[Out] (-2*(Sqrt[3] + 2*Sqrt[2*(-1 + Sqrt[3])] - x)*((Sqrt[3] + 2*Sqrt[2*(-1 + Sqr
t[3])])*EllipticF[ArcSin[Sqrt[((Sqrt[3] - 2*Sqrt[2*(-1 + Sqrt[3])] - x)*(Sq
rt[3] + 2*Sqrt[2*(-1 + Sqrt[3])] - Root[-71 - 96*#1 + 10*#1^2 + #1^4 & , 4,
 0]))/((Sqrt[3] + 2*Sqrt[2*(-1 + Sqrt[3])] - x)*(Sqrt[3] - 2*Sqrt[2*(-1 + S
qrt[3])] - Root[-71 - 96*#1 + 10*#1^2 + #1^4 & , 4, 0]))]], ((Sqrt[3] + 2*S
qrt[2*(-1 + Sqrt[3])] - Root[-71 - 96*#1 + 10*#1^2 + #1^4 & , 3, 0])*(Sqrt[
3] - 2*Sqrt[2*(-1 + Sqrt[3])] - Root[-71 - 96*#1 + 10*#1^2 + #1^4 & , 4, 0]
))/((Sqrt[3] - 2*Sqrt[2*(-1 + Sqrt[3])] - Root[-71 - 96*#1 + 10*#1^2 + #1^4
 & , 3, 0])*(Sqrt[3] + 2*Sqrt[2*(-1 + Sqrt[3])] - Root[-71 - 96*#1 + 10*#1^
2 + #1^4 & , 4, 0]))] - 4*Sqrt[2*(-1 + Sqrt[3])]*EllipticPi[(Sqrt[3] - 2*Sq
rt[2*(-1 + Sqrt[3])] - Root[-71 - 96*#1 + 10*#1^2 + #1^4 & , 4, 0])/(Sqrt[3
] + 2*Sqrt[2*(-1 + Sqrt[3])] - Root[-71 - 96*#1 + 10*#1^2 + #1^4 & , 4, 0])
, ArcSin[Sqrt[((Sqrt[3] - 2*Sqrt[2*(-1 + Sqrt[3])] - x)*(Sqrt[3] + 2*Sqrt[2
*(-1 + Sqrt[3])] - Root[-71 - 96*#1 + 10*#1^2 + #1^4 & , 4, 0]))/((Sqrt[3]
+ 2*Sqrt[2*(-1 + Sqrt[3])] - x)*(Sqrt[3] - 2*Sqrt[2*(-1 + Sqrt[3])] - Root[
-71 - 96*#1 + 10*#1^2 + #1^4 & , 4, 0]))]], ((Sqrt[3] + 2*Sqrt[2*(-1 + Sqrt
```

```
[3])] - Root[-71 - 96*#1 + 10*#1^2 + #1^4 & , 3, 0])*(Sqrt[3] - 2*Sqrt[2*(-
1 + Sqrt[3])] - Root[-71 - 96*#1 + 10*#1^2 + #1^4 & , 4, 0]))/((Sqrt[3] - 2
*Sqrt[2*(-1 + Sqrt[3])] - Root[-71 - 96*#1 + 10*#1^2 + #1^4 & , 3, 0])*(Sqr
t[3] + 2*Sqrt[2*(-1 + Sqrt[3])] - Root[-71 - 96*#1 + 10*#1^2 + #1^4 & , 4,
0]))])*Sqrt[(x - Root[-71 - 96*#1 + 10*#1^2 + #1^4 & , 3, 0])/((Sqrt[3] + 2
*Sqrt[2*(-1 + Sqrt[3])] - x)*(Sqrt[3] - 2*Sqrt[2*(-1 + Sqrt[3])] - Root[-71
 - 96*#1 + 10*#1^2 + #1^4 & , 3, 0]))]*Sqrt[((Sqrt[3] - 2*Sqrt[2*(-1 + Sqrt
[3])] - x)*(Sqrt[3] + 2*Sqrt[2*(-1 + Sqrt[3])] - Root[-71 - 96*#1 + 10*#1^2
 + #1^4 & , 4, 0]))/((Sqrt[3] + 2*Sqrt[2*(-1 + Sqrt[3])] - x)*(Sqrt[3] - 2*
Sqrt[2*(-1 + Sqrt[3])] - Root[-71 - 96*#1 + 10*#1^2 + #1^4 & , 4, 0]))]*(x
- Root[-71 - 96*#1 + 10*#1^2 + #1^4 & , 4, 0]))/(Sqrt[-71 - 96*x + 10*x^2 +
 x^4]*(Sqrt[3] + 2*Sqrt[2*(-1 + Sqrt[3])] - Root[-71 - 96*#1 + 10*#1^2 + #1
^4 & , 4, 0])*Sqrt[(x - Root[-71 - 96*#1 + 10*#1^2 + #1^4 & , 4, 0])/((Sqrt
[3] + 2*Sqrt[2*(-1 + Sqrt[3])] - x)*(Sqrt[3] - 2*Sqrt[2*(-1 + Sqrt[3])] - R
oot[-71 - 96*#1 + 10*#1^2 + #1^4 & , 4, 0]))])
```

IntegrateAlgebraic [A] time = 5.04, size = 78, normalized size = 1.00

$$-\frac{1}{8}\log\left(-x^8-20x^6+128x^5-54x^4+1408x^3-3124x^2+\sqrt{x^4+10x^2-96x-71}\,\left(x^6+15x^4-80x^3+27x^2-528x+781\right)-10001\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[-71 - 96*x + 10*x^2 + x^4],x]
```

```
[Out] -1/8*Log[-10001 - 3124*x^2 + 1408*x^3 - 54*x^4 + 128*x^5 - 20*x^6 - x^8 + S
qrt[-71 - 96*x + 10*x^2 + x^4]*(781 - 528*x + 27*x^2 - 80*x^3 + 15*x^4 + x^
6)]
```

fricas [A] time = 0.50, size = 72, normalized size = 0.92

$$\frac{1}{8}\log\left(x^8+20\,x^6-128\,x^5+54\,x^4-1408\,x^3+3124\,x^2+\left(x^6+15\,x^4-80\,x^3+27\,x^2-528\,x+781\right)\sqrt{x^4+10\,x^2-96\,x-71}+10001\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+10*x^2-96*x-71)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*log(x^8 + 20*x^6 - 128*x^5 + 54*x^4 - 1408*x^3 + 3124*x^2 + (x^6 + 15*x
^4 - 80*x^3 + 27*x^2 - 528*x + 781)*sqrt(x^4 + 10*x^2 - 96*x - 71) + 10001)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^4+10\,x^2-96\,x-71}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+10*x^2-96*x-71)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(x^4 + 10*x^2 - 96*x - 71), x)
```

maple [C] time = 0.90, size = 1290, normalized size = 16.54

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+10*x^2-96*x-71)^(1/2),x)
```

```
[Out] 2*(-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=4)+RootOf(_Z^4+10*_Z^2-96*_Z-71,inde
x=1))*((RootOf(_Z^4+10*_Z^2-96*_Z-71,index=4)-RootOf(_Z^4+10*_Z^2-96*_Z-71,
index=2))*(x-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=1))/(RootOf(_Z^4+10*_Z^2-96
*_Z-71,index=4)-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=1))/(x-RootOf(_Z^4+10*_Z
```

```
^2-96*_Z-71,index=2)))^(1/2)*(x-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=2))^2*(-
(RootOf(_Z^4+10*_Z^2-96*_Z-71,index=2)-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=1
))*(x-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=3))/(-RootOf(_Z^4+10*_Z^2-96*_Z-71
,index=3)+RootOf(_Z^4+10*_Z^2-96*_Z-71,index=1))/(x-RootOf(_Z^4+10*_Z^2-96*
_Z-71,index=2)))^(1/2)*((RootOf(_Z^4+10*_Z^2-96*_Z-71,index=2)-RootOf(_Z^4+
10*_Z^2-96*_Z-71,index=1))*(x-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=4))/(RootO
f(_Z^4+10*_Z^2-96*_Z-71,index=4)-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=1))/(x-
RootOf(_Z^4+10*_Z^2-96*_Z-71,index=2)))^(1/2)/(RootOf(_Z^4+10*_Z^2-96*_Z-71
,index=4)-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=2))/(RootOf(_Z^4+10*_Z^2-96*_Z
-71,index=2)-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=1))/((x-RootOf(_Z^4+10*_Z^2
-96*_Z-71,index=1))*(x-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=2))*(x-RootOf(_Z^
4+10*_Z^2-96*_Z-71,index=3))*(x-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=4)))^(1/
2)*(RootOf(_Z^4+10*_Z^2-96*_Z-71,index=2)*EllipticF(((RootOf(_Z^4+10*_Z^2-9
6*_Z-71,index=4)-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=2))*(x-RootOf(_Z^4+10*_
Z^2-96*_Z-71,index=1))/(RootOf(_Z^4+10*_Z^2-96*_Z-71,index=4)-RootOf(_Z^4+1
0*_Z^2-96*_Z-71,index=1))/(x-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=2)))^(1/2),
((RootOf(_Z^4+10*_Z^2-96*_Z-71,index=2)-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=
3))*(-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=4)+RootOf(_Z^4+10*_Z^2-96*_Z-71,in
dex=1))/(-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=3)+RootOf(_Z^4+10*_Z^2-96*_Z-7
1,index=1))/(RootOf(_Z^4+10*_Z^2-96*_Z-71,index=2)-RootOf(_Z^4+10*_Z^2-96*_
Z-71,index=4)))^(1/2))+(-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=2)+RootOf(_Z^4+
10*_Z^2-96*_Z-71,index=1))*EllipticPi(((RootOf(_Z^4+10*_Z^2-96*_Z-71,index=
4)-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=2))*(x-RootOf(_Z^4+10*_Z^2-96*_Z-71,i
ndex=1))/(RootOf(_Z^4+10*_Z^2-96*_Z-71,index=4)-RootOf(_Z^4+10*_Z^2-96*_Z-7
1,index=1))/(x-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=2)))^(1/2),(RootOf(_Z^4+1
0*_Z^2-96*_Z-71,index=4)-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=1))/(RootOf(_Z^
4+10*_Z^2-96*_Z-71,index=4)-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=2)),((RootOf
(_Z^4+10*_Z^2-96*_Z-71,index=2)-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=3))*(-Ro
otOf(_Z^4+10*_Z^2-96*_Z-71,index=4)+RootOf(_Z^4+10*_Z^2-96*_Z-71,index=1))/
(-RootOf(_Z^4+10*_Z^2-96*_Z-71,index=3)+RootOf(_Z^4+10*_Z^2-96*_Z-71,index=
1))/(RootOf(_Z^4+10*_Z^2-96*_Z-71,index=2)-RootOf(_Z^4+10*_Z^2-96*_Z-71,ind
ex=4)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^4 + 10\,x^2 - 96\,x - 71}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+10*x^2-96*x-71)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(x^4 + 10*x^2 - 96*x - 71), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{x^4 + 10\,x^2 - 96\,x - 71}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(10*x^2 - 96*x + x^4 - 71)^(1/2),x)
```

```
[Out] int(x/(10*x^2 - 96*x + x^4 - 71)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^4 + 10x^2 - 96x - 71}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+10*x**2-96*x-71)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**4 + 10*x**2 - 96*x - 71), x)
```

3.943 $\int \frac{(8+3x)\sqrt[4]{-2-x+2x^4}}{x^2(2+x+x^4)}\,dx$

Optimal. Leaf size=78

$$-\frac{4\sqrt[4]{2x^4-x-2}}{x}-2\sqrt[4]{3}\tan^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{2x^4-x-2}}\right)+2\sqrt[4]{3}\tanh^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{2x^4-x-2}}\right)$$

Rubi [F] time = 1.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(8+3x)\sqrt[4]{-2-x+2x^4}}{x^2(2+x+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((8 + 3*x)*(-2 - x + 2*x^4)^(1/4))/(x^2*(2 + x + x^4)),x]
```

```
[Out] 4*Defer[Int][(-2 - x + 2*x^4)^(1/4)/x^2, x] - Defer[Int][(-2 - x + 2*x^4)^(
1/4)/x, x]/2 + Defer[Int][(-2 - x + 2*x^4)^(1/4)/(2 + x + x^4), x]/2 - 4*De
fer[Int][(x^2*(-2 - x + 2*x^4)^(1/4))/(2 + x + x^4), x] + Defer[Int][(x^3*(
-2 - x + 2*x^4)^(1/4))/(2 + x + x^4), x]/2
```

Rubi steps

$$\begin{aligned}\int \frac{(8+3x)\sqrt[4]{-2-x+2x^4}}{x^2(2+x+x^4)}\,dx &= \int \left(\frac{4\sqrt[4]{-2-x+2x^4}}{x^2}-\frac{\sqrt[4]{-2-x+2x^4}}{2x}+\frac{(1-8x^2+x^3)\sqrt[4]{-2-x+2x^4}}{2(2+x+x^4)}\right) dx\\ &= -\left(\frac{1}{2}\int \frac{\sqrt[4]{-2-x+2x^4}}{x}\,dx\right)+\frac{1}{2}\int \frac{(1-8x^2+x^3)\sqrt[4]{-2-x+2x^4}}{2+x+x^4}\,dx+4\int\\ &= -\left(\frac{1}{2}\int \frac{\sqrt[4]{-2-x+2x^4}}{x}\,dx\right)+\frac{1}{2}\int \left(\frac{\sqrt[4]{-2-x+2x^4}}{2+x+x^4}-\frac{8x^2\sqrt[4]{-2-x+2x^4}}{2+x+x^4}+\right.\\ &= -\left(\frac{1}{2}\int \frac{\sqrt[4]{-2-x+2x^4}}{x}\,dx\right)+\frac{1}{2}\int \frac{\sqrt[4]{-2-x+2x^4}}{2+x+x^4}\,dx+\frac{1}{2}\int \frac{x^3\sqrt[4]{-2-x+2}}{2+x+x^4}\end{aligned}$$

Mathematica [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(8+3x)\sqrt[4]{-2-x+2x^4}}{x^2(2+x+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((8 + 3*x)*(-2 - x + 2*x^4)^(1/4))/(x^2*(2 + x + x^4)),x]
```

```
[Out] Integrate[((8 + 3*x)*(-2 - x + 2*x^4)^(1/4))/(x^2*(2 + x + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.53, size = 78, normalized size = 1.00

$$-\frac{4\sqrt[4]{2x^4-x-2}}{x}-2\sqrt[4]{3}\tan^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{2x^4-x-2}}\right)+2\sqrt[4]{3}\tanh^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{2x^4-x-2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((8 + 3*x)*(-2 - x + 2*x^4)^(1/4))/(x^2*(2 + x + x^4)),x
]
```

```
[Out] (-4*(-2 - x + 2*x^4)^(1/4))/x - 2*3^(1/4)*ArcTan[(3^(1/4)*x)/(-2 - x + 2*x^
4)^(1/4)] + 2*3^(1/4)*ArcTanh[(3^(1/4)*x)/(-2 - x + 2*x^4)^(1/4)]
```

fricas [B] time = 14.72, size = 287, normalized size = 3.68

$$\frac{4\cdot 3^{\frac{1}{4}}x\arctan\left(\frac{6\cdot 3^{\frac{3}{4}}\left(2x^4-x-2\right)^{\frac{1}{4}}x^3+6\cdot 3^{\frac{1}{4}}\left(2x^4-x-2\right)^{\frac{3}{4}}x+3^{\frac{3}{4}}\left(2\cdot 3^{\frac{3}{4}}\sqrt{2x^4-x-2}\,x^2+3^{\frac{1}{4}}\left(5x^4-x-2\right)\right)}{3\left(x^4+x+2\right)}\right)+3^{\frac{1}{4}}x\log\left(\frac{6\sqrt{3}\left(2x^4-x-2\right)^{\frac{1}{4}}x^3+6\cdot 3^{\frac{1}{4}}\sqrt{2x^4-x-2}\,x^2+3^{\frac{3}{4}}\left(5x^4-x-2\right)+6\left(2x^4-x-2\right)^{\frac{3}{4}}x}{x^4+x+2}\right)-3^{\frac{1}{4}}x\log\left(\frac{6\sqrt{3}\left(2x^4-x-2\right)^{\frac{1}{4}}x^3-6\cdot 3^{\frac{1}{4}}\sqrt{2x^4-x-2}\,x^2-3^{\frac{3}{4}}\left(5x^4-x-2\right)+6\left(2x^4-x-2\right)^{\frac{3}{4}}x}{x^4+x+2}\right)-8\left(2x^4-x-2\right)^{\frac{1}{4}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8+3*x)*(2*x^4-x-2)^(1/4)/x^2/(x^4+x+2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*3^(1/4)*x*arctan(1/3*(6*3^(3/4)*(2*x^4 - x - 2)^(1/4)*x^3 + 6*3^(1/4
)*(2*x^4 - x - 2)^(3/4)*x + 3^(3/4)*(2*3^(3/4)*sqrt(2*x^4 - x - 2)*x^2 + 3^
(1/4)*(5*x^4 - x - 2)))/(x^4 + x + 2)) + 3^(1/4)*x*log((6*sqrt(3)*(2*x^4 -
x - 2)^(1/4)*x^3 + 6*3^(1/4)*sqrt(2*x^4 - x - 2)*x^2 + 3^(3/4)*(5*x^4 - x -
 2) + 6*(2*x^4 - x - 2)^(3/4)*x)/(x^4 + x + 2)) - 3^(1/4)*x*log((6*sqrt(3)*
(2*x^4 - x - 2)^(1/4)*x^3 - 6*3^(1/4)*sqrt(2*x^4 - x - 2)*x^2 - 3^(3/4)*(5*
x^4 - x - 2) + 6*(2*x^4 - x - 2)^(3/4)*x)/(x^4 + x + 2)) - 8*(2*x^4 - x - 2
)^(1/4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^4-x-2\right)^{\frac{1}{4}}(3\,x+8)}{\left(x^4+x+2\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8+3*x)*(2*x^4-x-2)^(1/4)/x^2/(x^4+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^4 - x - 2)^(1/4)*(3*x + 8)/((x^4 + x + 2)*x^2), x)
```

maple [C] time = 3.05, size = 1594, normalized size = 20.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8+3*x)*(2*x^4-x-2)^(1/4)/x^2/(x^4+x+2),x)
```

```
[Out] -4*(2*x^4-x-2)^(1/4)/x+(RootOf(_Z^4-3)*ln(-(20*RootOf(_Z^4-3)^2*x^12+8*(8*x
^12-12*x^9-24*x^8+6*x^6+24*x^5+24*x^4-x^3-6*x^2-12*x-8)^(1/4)*RootOf(_Z^4-3
)^3*x^9-24*RootOf(_Z^4-3)^2*x^9-8*(8*x^12-12*x^9-24*x^8+6*x^6+24*x^5+24*x^4
-x^3-6*x^2-12*x-8)^(1/4)*RootOf(_Z^4-3)^3*x^6-48*x^8*RootOf(_Z^4-3)^2-16*(8
*x^12-12*x^9-24*x^8+6*x^6+24*x^5+24*x^4-x^3-6*x^2-12*x-8)^(1/4)*RootOf(_Z^4
-3)^3*x^5+12*(8*x^12-12*x^9-24*x^8+6*x^6+24*x^5+24*x^4-x^3-6*x^2-12*x-8)^(1
/2)*x^6+9*RootOf(_Z^4-3)^2*x^6+6*(8*x^12-12*x^9-24*x^8+6*x^6+24*x^5+24*x^4-
x^3-6*x^2-12*x-8)^(3/4)*RootOf(_Z^4-3)*x^3+2*(8*x^12-12*x^9-24*x^8+6*x^6+24
*x^5+24*x^4-x^3-6*x^2-12*x-8)^(1/4)*RootOf(_Z^4-3)^3*x^3+36*RootOf(_Z^4-3)^
2*x^5+8*(8*x^12-12*x^9-24*x^8+6*x^6+24*x^5+24*x^4-x^3-6*x^2-12*x-8)^(1/4)*R
ootOf(_Z^4-3)^3*x^2+36*RootOf(_Z^4-3)^2*x^4-6*(8*x^12-12*x^9-24*x^8+6*x^6+2
4*x^5+24*x^4-x^3-6*x^2-12*x-8)^(1/2)*x^3+8*(8*x^12-12*x^9-24*x^8+6*x^6+24*x
^5+24*x^4-x^3-6*x^2-12*x-8)^(1/4)*RootOf(_Z^4-3)^3*x-x^3*RootOf(_Z^4-3)^2-1
2*(8*x^12-12*x^9-24*x^8+6*x^6+24*x^5+24*x^4-x^3-6*x^2-12*x-8)^(1/2)*x^2-6*x
^2*RootOf(_Z^4-3)^2-12*RootOf(_Z^4-3)^2*x-8*RootOf(_Z^4-3)^2)/(x^4+x+2)/(2*
x^4-x-2)^2)+RootOf(_Z^2+RootOf(_Z^4-3)^2)*ln(-(-20*RootOf(_Z^4-3)^2*x^12-8*
(8*x^12-12*x^9-24*x^8+6*x^6+24*x^5+24*x^4-x^3-6*x^2-12*x-8)^(1/4)*RootOf(_Z
^2+RootOf(_Z^4-3)^2)*RootOf(_Z^4-3)^2*x^9+24*RootOf(_Z^4-3)^2*x^9+8*(8*x^12
-12*x^9-24*x^8+6*x^6+24*x^5+24*x^4-x^3-6*x^2-12*x-8)^(1/4)*RootOf(_Z^2+Root
Of(_Z^4-3)^2)*RootOf(_Z^4-3)^2*x^6+48*x^8*RootOf(_Z^4-3)^2+16*(8*x^12-12*x^
```

```
9-24*x^8+6*x^6+24*x^5+24*x^4-x^3-6*x^2-12*x-8)^(1/4)*RootOf(_Z^2+RootOf(_Z^
4-3)^2)*RootOf(_Z^4-3)^2*x^5+12*(8*x^12-12*x^9-24*x^8+6*x^6+24*x^5+24*x^4-x
^3-6*x^2-12*x-8)^(1/2)*x^6-9*RootOf(_Z^4-3)^2*x^6+6*(8*x^12-12*x^9-24*x^8+6
*x^6+24*x^5+24*x^4-x^3-6*x^2-12*x-8)^(3/4)*RootOf(_Z^2+RootOf(_Z^4-3)^2)*x^
3-2*(8*x^12-12*x^9-24*x^8+6*x^6+24*x^5+24*x^4-x^3-6*x^2-12*x-8)^(1/4)*RootO
f(_Z^2+RootOf(_Z^4-3)^2)*RootOf(_Z^4-3)^2*x^3-36*RootOf(_Z^4-3)^2*x^5-8*(8*
x^12-12*x^9-24*x^8+6*x^6+24*x^5+24*x^4-x^3-6*x^2-12*x-8)^(1/4)*RootOf(_Z^2+
RootOf(_Z^4-3)^2)*RootOf(_Z^4-3)^2*x^2-36*RootOf(_Z^4-3)^2*x^4-6*(8*x^12-12
*x^9-24*x^8+6*x^6+24*x^5+24*x^4-x^3-6*x^2-12*x-8)^(1/2)*x^3-8*(8*x^12-12*x^
9-24*x^8+6*x^6+24*x^5+24*x^4-x^3-6*x^2-12*x-8)^(1/4)*RootOf(_Z^2+RootOf(_Z^
4-3)^2)*RootOf(_Z^4-3)^2*x+x^3*RootOf(_Z^4-3)^2-12*(8*x^12-12*x^9-24*x^8+6*
x^6+24*x^5+24*x^4-x^3-6*x^2-12*x-8)^(1/2)*x^2+6*x^2*RootOf(_Z^4-3)^2+12*Roo
tOf(_Z^4-3)^2*x+8*RootOf(_Z^4-3)^2)/(x^4+x+2)/(2*x^4-x-2)^2))/(2*x^4-x-2)^(
3/4)*((2*x^4-x-2)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^4 - x - 2\right)^{\frac{1}{4}}(3\,x + 8)}{\left(x^4 + x + 2\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8+3*x)*(2*x^4-x-2)^(1/4)/x^2/(x^4+x+2),x, algorithm="maxima")
[Out] integrate((2*x^4 - x - 2)^(1/4)*(3*x + 8)/((x^4 + x + 2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3\,x+8)\,\left(2\,x^4-x-2\right)^{1/4}}{x^2\,\left(x^4+x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 8)*(2*x^4 - x - 2)^(1/4))/(x^2*(x + x^4 + 2)),x)
[Out] int(((3*x + 8)*(2*x^4 - x - 2)^(1/4))/(x^2*(x + x^4 + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8+3*x)*(2*x**4-x-2)**(1/4)/x**2/(x**4+x+2),x)
[Out] Timed out
```

3.944 $\int \frac{1}{x^5\left(b+ax^4\right)^{3/4}}\,dx$

Optimal. Leaf size=78

$$\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{8b^{7/4}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{8b^{7/4}}-\frac{\sqrt[4]{ax^4+b}}{4bx^4}$$

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 63, 212, 206, 203}

$$\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{8b^{7/4}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{8b^{7/4}}-\frac{\sqrt[4]{ax^4+b}}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(b + a*x^4)^(3/4)),x]
```

```
[Out] -1/4*(b + a*x^4)^(1/4)/(b*x^4) + (3*a*ArcTan[(b + a*x^4)^(1/4)/b^(1/4)])/(8
*b^(7/4)) + (3*a*ArcTanh[(b + a*x^4)^(1/4)/b^(1/4)])/(8*b^(7/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(b+ax^4\right)^{3/4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2(b+ax)^{3/4}}\,dx, x, x^4\right)\\
&= -\frac{\sqrt[4]{b+ax^4}}{4bx^4} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{x(b+ax)^{3/4}}\,dx, x, x^4\right)}{16b}\\
&= -\frac{\sqrt[4]{b+ax^4}}{4bx^4} - \frac{3\,\text{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^4}\right)}{4b}\\
&= -\frac{\sqrt[4]{b+ax^4}}{4bx^4} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^4}\right)}{8b^{3/2}} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx, x, \sqrt[4]{b-\right.}{8b^{3/2}}\\
&= -\frac{\sqrt[4]{b+ax^4}}{4bx^4} + \frac{3a\tan^{-1}\left(\frac{\sqrt[4]{b+ax^4}}{\sqrt[4]{b}}\right)}{8b^{7/4}} + \frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^4}}{\sqrt[4]{b}}\right)}{8b^{7/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.44

$$\frac{a\sqrt[4]{ax^4+b}\;{}_2F_1\left(\frac{1}{4},2;\frac{5}{4};\frac{ax^4}{b}+1\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(b + a*x^4)^(3/4)),x]
```

```
[Out] (a*(b + a*x^4)^(1/4)*Hypergeometric2F1[1/4, 2, 5/4, 1 + (a*x^4)/b])/b^2
```

IntegrateAlgebraic [A] time = 0.07, size = 78, normalized size = 1.00

$$\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{8b^{7/4}} + \frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{8b^{7/4}} - \frac{\sqrt[4]{ax^4+b}}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(b + a*x^4)^(3/4)),x]
```

```
[Out] -1/4*(b + a*x^4)^(1/4)/(b*x^4) + (3*a*ArcTan[(b + a*x^4)^(1/4)/b^(1/4)])/(8
*b^(7/4)) + (3*a*ArcTanh[(b + a*x^4)^(1/4)/b^(1/4)])/(8*b^(7/4))
```

fricas [B] time = 0.59, size = 194, normalized size = 2.49

$$-\frac{12\,bx^4\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^4+b\right)^{\frac{1}{4}}ab^5\left(\frac{a^4}{b^7}\right)^{\frac{3}{4}}-\sqrt{b^4\sqrt{\frac{a^4}{b^7}}+\sqrt{ax^4+b}\,a^2}\,b^5\left(\frac{a^4}{b^7}\right)^{\frac{3}{4}}}{a^4}\right)-3\,bx^4\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(3\,b^2\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}+3\left(ax^4+b\right)^{\frac{1}{4}}a\right)+3\,bx^4\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(-3\,b^2\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}+3\left(ax^4+b\right)^{\frac{1}{4}}a\right)+4\left(ax^4+b\right)^{\frac{1}{4}}}{16\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a*x^4+b)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/16*(12*b*x^4*(a^4/b^7)^(1/4)*arctan(-((a*x^4 + b)^(1/4)*a*b^5*(a^4/b^7)^
(3/4) - sqrt(b^4*sqrt(a^4/b^7) + sqrt(a*x^4 + b)*a^2)*b^5*(a^4/b^7)^(3/4))/
a^4) - 3*b*x^4*(a^4/b^7)^(1/4)*log(3*b^2*(a^4/b^7)^(1/4) + 3*(a*x^4 + b)^(1
/4)*a) + 3*b*x^4*(a^4/b^7)^(1/4)*log(-3*b^2*(a^4/b^7)^(1/4) + 3*(a*x^4 + b)
^(1/4)*a) + 4*(a*x^4 + b)^(1/4))/(b*x^4)
```

giac [B] time = 0.46, size = 221, normalized size = 2.83

$$\frac{\frac{6\,\sqrt{2}a^{2}(-b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}+2\left(ax^{4}+b\right)^{\frac{1}{4}}\right)}{2\,(-b)^{\frac{1}{4}}}\right)}{b^{2}}+\frac{6\,\sqrt{2}a^{2}(-b)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}-2\left(ax^{4}+b\right)^{\frac{1}{4}}\right)}{2\,(-b)^{\frac{1}{4}}}\right)}{b^{2}}+\frac{3\,\sqrt{2}a^{2}(-b)^{\frac{1}{4}}\log\left(\sqrt{2}\left(ax^{4}+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^{4}+b}+\sqrt{-b}\right)}{b^{2}}+\frac{3\,\sqrt{2}a^{2}\log\left(-\sqrt{2}\left(ax^{4}+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^{4}+b}+\sqrt{-b}\right)}{(-b)^{\frac{3}{4}}b}-\frac{8\left(ax^{4}+b\right)^{\frac{1}{4}}a}{bx^{4}}}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a*x^4+b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/32*(6*sqrt(2)*a^2*(-b)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(
a*x^4 + b)^(1/4))/(-b)^(1/4))/b^2 + 6*sqrt(2)*a^2*(-b)^(1/4)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(-b)^(1/4) - 2*(a*x^4 + b)^(1/4))/(-b)^(1/4))/b^2 + 3*sqrt(2
)*a^2*(-b)^(1/4)*log(sqrt(2)*(a*x^4 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^4 + b)
 + sqrt(-b))/b^2 + 3*sqrt(2)*a^2*log(-sqrt(2)*(a*x^4 + b)^(1/4)*(-b)^(1/4)
+ sqrt(a*x^4 + b) + sqrt(-b))/((-b)^(3/4)*b) - 8*(a*x^4 + b)^(1/4)*a/(b*x^4
))/a
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5}\left(a\,x^{4}+b\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(a*x^4+b)^(3/4),x)
```

```
[Out] int(1/x^5/(a*x^4+b)^(3/4),x)
```

maxima [A] time = 0.43, size = 94, normalized size = 1.21

$$-\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}a}{4\left(\left(ax^{4}+b\right)b-b^{2}\right)}+\frac{3\left(\frac{2\,a\arctan\left(\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}-\frac{a\log\left(\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^{4}+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a*x^4+b)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/4*(a*x^4 + b)^(1/4)*a/((a*x^4 + b)*b - b^2) + 3/16*(2*a*arctan((a*x^4 +
b)^(1/4)/b^(1/4))/b^(3/4) - a*log(((a*x^4 + b)^(1/4) - b^(1/4))/((a*x^4 + b
)^(1/4) + b^(1/4)))/b^(3/4))/b
```

mupad [B] time = 1.17, size = 58, normalized size = 0.74

$$\frac{3\,a\,\mathrm{atan}\left(\frac{{\left(a\,x^4+b\right)}^{1/4}}{b^{1/4}}\right)}{8\,b^{7/4}}-\frac{{\left(a\,x^4+b\right)}^{1/4}}{4\,b\,x^4}+\frac{3\,a\,\mathrm{atanh}\left(\frac{{\left(a\,x^4+b\right)}^{1/4}}{b^{1/4}}\right)}{8\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(b + a*x^4)^(3/4)),x)
```

```
[Out] (3*a*atan((b + a*x^4)^(1/4)/b^(1/4)))/(8*b^(7/4)) - (b + a*x^4)^(1/4)/(4*b*
x^4) + (3*a*atanh((b + a*x^4)^(1/4)/b^(1/4)))/(8*b^(7/4))
```

sympy [C] time = 1.40, size = 39, normalized size = 0.50

$$-\frac{\Gamma\left(\frac{7}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| {\frac{b e^{i\pi}}{a x^{4}}} \right)}{4 a^{\frac{3}{4}} x^{7} \Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(a*x**4+b)**(3/4),x)
```

```
[Out] -gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*exp_polar(I*pi)/(a*x**4))/(4*a**(3
/4)*x**7*gamma(11/4))
```

3.945 $\int \frac{1}{x^6 \left(b+ax^5\right)^{3/4}} \, dx$

Optimal. Leaf size=78

$$\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{10b^{7/4}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{10b^{7/4}}-\frac{\sqrt[4]{ax^5+b}}{5bx^5}$$

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 63, 212, 206, 203}

$$\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{10b^{7/4}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{10b^{7/4}}-\frac{\sqrt[4]{ax^5+b}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(b + a*x^5)^(3/4)),x]
```

```
[Out] -1/5*(b + a*x^5)^(1/4)/(b*x^5) + (3*a*ArcTan[(b + a*x^5)^(1/4)/b^(1/4)])/(1
0*b^(7/4)) + (3*a*ArcTanh[(b + a*x^5)^(1/4)/b^(1/4)])/(10*b^(7/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^6 \left(b+ax^5\right)^{3/4}}\,dx = \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x^2(b+ax)^{3/4}}\,dx, x, x^5\right)$$

$$= -\frac{\sqrt[4]{b+ax^5}}{5bx^5} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{x(b+ax)^{3/4}}\,dx, x, x^5\right)}{20b}$$

$$= -\frac{\sqrt[4]{b+ax^5}}{5bx^5} - \frac{3\,\text{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^5}\right)}{5b}$$

$$= -\frac{\sqrt[4]{b+ax^5}}{5bx^5} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^5}\right)}{10b^{3/2}} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx, x, \sqrt[4]{b-\right.}{10b^{3/2}}$$

$$= -\frac{\sqrt[4]{b+ax^5}}{5bx^5} + \frac{3a\tan^{-1}\left(\frac{\sqrt[4]{b+ax^5}}{\sqrt[4]{b}}\right)}{10b^{7/4}} + \frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^5}}{\sqrt[4]{b}}\right)}{10b^{7/4}}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.47

$$\frac{4a\sqrt[4]{ax^5+b}\;{}_2F_1\left(\frac{1}{4},2;\frac{5}{4};\frac{ax^5}{b}+1\right)}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(b + a*x^5)^(3/4)),x]
```

```
[Out] (4*a*(b + a*x^5)^(1/4)*Hypergeometric2F1[1/4, 2, 5/4, 1 + (a*x^5)/b])/(5*b^
2)
```

IntegrateAlgebraic [A] time = 0.06, size = 78, normalized size = 1.00

$$\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{10b^{7/4}} + \frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{10b^{7/4}} - \frac{\sqrt[4]{ax^5+b}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^6*(b + a*x^5)^(3/4)),x]
```

```
[Out] -1/5*(b + a*x^5)^(1/4)/(b*x^5) + (3*a*ArcTan[(b + a*x^5)^(1/4)/b^(1/4)])/(1
0*b^(7/4)) + (3*a*ArcTanh[(b + a*x^5)^(1/4)/b^(1/4)])/(10*b^(7/4))
```

fricas [B] time = 0.61, size = 194, normalized size = 2.49

$$-\frac{12\,bx^5\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^5+b\right)^{\frac{1}{4}}ab^5\left(\frac{a^4}{b^7}\right)^{\frac{3}{4}}-\sqrt{b^4\sqrt{\frac{a^4}{b^7}}+\sqrt{ax^5+b}\,a^2}\,b^5\left(\frac{a^4}{b^7}\right)^{\frac{3}{4}}}{a^4}\right)-3\,bx^5\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(3\,b^2\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}+3\left(ax^5+b\right)^{\frac{1}{4}}a\right)+3\,bx^5\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(-3\,b^2\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}+3\left(ax^5+b\right)^{\frac{1}{4}}a\right)+4\left(ax^5+b\right)^{\frac{1}{4}}}{20\,bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(a*x^5+b)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/20*(12*b*x^5*(a^4/b^7)^(1/4)*arctan(-((a*x^5 + b)^(1/4)*a*b^5*(a^4/b^7)^
(3/4) - sqrt(b^4*sqrt(a^4/b^7) + sqrt(a*x^5 + b)*a^2)*b^5*(a^4/b^7)^(3/4))/
a^4) - 3*b*x^5*(a^4/b^7)^(1/4)*log(3*b^2*(a^4/b^7)^(1/4) + 3*(a*x^5 + b)^(1
/4)*a) + 3*b*x^5*(a^4/b^7)^(1/4)*log(-3*b^2*(a^4/b^7)^(1/4) + 3*(a*x^5 + b)
^(1/4)*a) + 4*(a*x^5 + b)^(1/4))/(b*x^5)
```

giac [B] time = 0.98, size = 221, normalized size = 2.83

$$\frac{\frac{6\,\sqrt{2}a^{2}(-b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}+2\left(ax^{5}+b\right)^{\frac{1}{4}}\right)}{2\,(-b)^{\frac{1}{4}}}\right)}{b^{2}}+\frac{6\,\sqrt{2}a^{2}(-b)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}-2\left(ax^{5}+b\right)^{\frac{1}{4}}\right)}{2\,(-b)^{\frac{1}{4}}}\right)}{b^{2}}+\frac{3\,\sqrt{2}a^{2}(-b)^{\frac{1}{4}}\log\left(\sqrt{2}\left(ax^{5}+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^{5}+b}+\sqrt{-b}\right)}{b^{2}}+\frac{3\,\sqrt{2}a^{2}\log\left(-\sqrt{2}\left(ax^{5}+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^{5}+b}+\sqrt{-b}\right)}{(-b)^{\frac{3}{4}}b}-\frac{8\left(ax^{5}+b\right)^{\frac{1}{4}}a}{bx^{5}}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(a*x^5+b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/40*(6*sqrt(2)*a^2*(-b)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(
a*x^5 + b)^(1/4))/(-b)^(1/4))/b^2 + 6*sqrt(2)*a^2*(-b)^(1/4)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(-b)^(1/4) - 2*(a*x^5 + b)^(1/4))/(-b)^(1/4))/b^2 + 3*sqrt(2
)*a^2*(-b)^(1/4)*log(sqrt(2)*(a*x^5 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^5 + b)
 + sqrt(-b))/b^2 + 3*sqrt(2)*a^2*log(-sqrt(2)*(a*x^5 + b)^(1/4)*(-b)^(1/4)
+ sqrt(a*x^5 + b) + sqrt(-b))/((-b)^(3/4)*b) - 8*(a*x^5 + b)^(1/4)*a/(b*x^5
))/a
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{6}\left(a\,x^{5}+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(a*x^5+b)^(3/4),x)
```

```
[Out] int(1/x^6/(a*x^5+b)^(3/4),x)
```

maxima [A] time = 0.43, size = 94, normalized size = 1.21

$$-\frac{\left(ax^{5}+b\right)^{\frac{1}{4}}a}{5\left(\left(ax^{5}+b\right)b-b^{2}\right)}+\frac{3\left(\frac{2\,a\arctan\left(\frac{\left(ax^{5}+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}-\frac{a\log\left(\frac{\left(ax^{5}+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^{5}+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}\right)}{20\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(a*x^5+b)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/5*(a*x^5 + b)^(1/4)*a/((a*x^5 + b)*b - b^2) + 3/20*(2*a*arctan((a*x^5 +
b)^(1/4)/b^(1/4))/b^(3/4) - a*log(((a*x^5 + b)^(1/4) - b^(1/4))/((a*x^5 + b
)^(1/4) + b^(1/4)))/b^(3/4))/b
```

mupad [B] time = 1.04, size = 58, normalized size = 0.74

$$\frac{3\,a\,\mathrm{atan}\left(\frac{{\left(a\,x^5+b\right)}^{1/4}}{b^{1/4}}\right)}{10\,b^{7/4}}-\frac{{\left(a\,x^5+b\right)}^{1/4}}{5\,b\,x^5}+\frac{3\,a\,\mathrm{atanh}\left(\frac{{\left(a\,x^5+b\right)}^{1/4}}{b^{1/4}}\right)}{10\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(b + a*x^5)^(3/4)),x)
```

```
[Out] (3*a*atan((b + a*x^5)^(1/4)/b^(1/4)))/(10*b^(7/4)) - (b + a*x^5)^(1/4)/(5*b
*x^5) + (3*a*atanh((b + a*x^5)^(1/4)/b^(1/4)))/(10*b^(7/4))
```

sympy [C] time = 1.33, size = 41, normalized size = 0.53

$$-\frac{\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^{5}}\right)}{5a^{\frac{3}{4}}x^{\frac{35}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(a*x**5+b)**(3/4),x)
```

```
[Out] -gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*exp_polar(I*pi)/(a*x**5))/(5*a**(3
/4)*x**(35/4)*gamma(11/4))
```

3.946 $\int \frac{1}{x^7\left(b+ax^6\right)^{3/4}}\,dx$

Optimal. Leaf size=78

$$\frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{4b^{7/4}}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{4b^{7/4}}-\frac{\sqrt[4]{ax^6+b}}{6bx^6}$$

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 63, 212, 206, 203}

$$\frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{4b^{7/4}}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{4b^{7/4}}-\frac{\sqrt[4]{ax^6+b}}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(b + a*x^6)^(3/4)),x]
```

```
[Out] -1/6*(b + a*x^6)^(1/4)/(b*x^6) + (a*ArcTan[(b + a*x^6)^(1/4)/b^(1/4)])/(4*b
^(7/4)) + (a*ArcTanh[(b + a*x^6)^(1/4)/b^(1/4)])/(4*b^(7/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^7\left(b+ax^6\right)^{3/4}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x^2(b+ax)^{3/4}}\,dx, x, x^6\right)\\
&= -\frac{\sqrt[4]{b+ax^6}}{6bx^6} - \frac{a\,\text{Subst}\left(\int \frac{1}{x(b+ax)^{3/4}}\,dx, x, x^6\right)}{8b}\\
&= -\frac{\sqrt[4]{b+ax^6}}{6bx^6} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^6}\right)}{2b}\\
&= -\frac{\sqrt[4]{b+ax^6}}{6bx^6} + \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^6}\right)}{4b^{3/2}} + \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+x^2}\,dx, x, \sqrt[4]{b+ax^6}\right)}{4b^{3/2}}\\
&= -\frac{\sqrt[4]{b+ax^6}}{6bx^6} + \frac{a\tan^{-1}\left(\frac{\sqrt[4]{b+ax^6}}{\sqrt[4]{b}}\right)}{4b^{7/4}} + \frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^6}}{\sqrt[4]{b}}\right)}{4b^{7/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.47

$$
\frac{2a\sqrt[4]{ax^6+b}\,{}_2F_1\left(\frac{1}{4},2;\frac{5}{4};\frac{ax^6}{b}+1\right)}{3b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(b + a*x^6)^(3/4)),x]
```

```
[Out] (2*a*(b + a*x^6)^(1/4)*Hypergeometric2F1[1/4, 2, 5/4, 1 + (a*x^6)/b])/(3*b^
2)
```

IntegrateAlgebraic [A] time = 0.07, size = 78, normalized size = 1.00

$$
\frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{4b^{7/4}} + \frac{a\tanh^{-1}\left(\frac{\sqrt[4]{ax^6+b}}{\sqrt[4]{b}}\right)}{4b^{7/4}} - \frac{\sqrt[4]{ax^6+b}}{6bx^6}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(b + a*x^6)^(3/4)),x]
```

```
[Out] -1/6*(b + a*x^6)^(1/4)/(b*x^6) + (a*ArcTan[(b + a*x^6)^(1/4)/b^(1/4)])/(4*b
^(7/4)) + (a*ArcTanh[(b + a*x^6)^(1/4)/b^(1/4)])/(4*b^(7/4))
```

fricas [B] time = 0.46, size = 191, normalized size = 2.45

$$
-\frac{12\,bx^6\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^6+b\right)^{\frac{1}{4}}ab^5\left(\frac{a^4}{b^7}\right)^{\frac{3}{4}} - \sqrt{b^4\sqrt{\frac{a^4}{b^7}}+\sqrt{ax^6+b}\,a^2}\,b^5\left(\frac{a^4}{b^7}\right)^{\frac{3}{4}}}{a^4}\right) - 3\,bx^6\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(b^2\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}} + \left(ax^6+b\right)^{\frac{1}{4}}a\right) + 3\,bx^6\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(-b^2\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}} + \left(ax^6+b\right)^{\frac{1}{4}}a\right) + 4\left(ax^6+b\right)^{\frac{1}{4}}}{24\,bx^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a*x^6+b)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/24*(12*b*x^6*(a^4/b^7)^(1/4)*arctan(-((a*x^6 + b)^(1/4)*a*b^5*(a^4/b^7)^
(3/4) - sqrt(b^4*sqrt(a^4/b^7) + sqrt(a*x^6 + b)*a^2)*b^5*(a^4/b^7)^(3/4))/
a^4) - 3*b*x^6*(a^4/b^7)^(1/4)*log(b^2*(a^4/b^7)^(1/4) + (a*x^6 + b)^(1/4)*
a) + 3*b*x^6*(a^4/b^7)^(1/4)*log(-b^2*(a^4/b^7)^(1/4) + (a*x^6 + b)^(1/4)*a
) + 4*(a*x^6 + b)^(1/4))/(b*x^6)
```

giac [B] time = 0.30, size = 221, normalized size = 2.83

$$\frac{\frac{6\,\sqrt{2}a^{2}(-b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}+2\left(ax^{6}+b\right)^{\frac{1}{4}}\right)}{2\,(-b)^{\frac{1}{4}}}\right)}{b^{2}}+\frac{6\,\sqrt{2}a^{2}(-b)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}-2\left(ax^{6}+b\right)^{\frac{1}{4}}\right)}{2\,(-b)^{\frac{1}{4}}}\right)}{b^{2}}+\frac{3\,\sqrt{2}a^{2}(-b)^{\frac{1}{4}}\log\left(\sqrt{2}\left(ax^{6}+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^{6}+b}+\sqrt{-b}\right)}{b^{2}}+\frac{3\,\sqrt{2}a^{2}\log\left(-\sqrt{2}\left(ax^{6}+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^{6}+b}+\sqrt{-b}\right)}{(-b)^{\frac{3}{4}}b}-\frac{8\left(ax^{6}+b\right)^{\frac{1}{4}}a}{bx^{6}}}{48\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a*x^6+b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/48*(6*sqrt(2)*a^2*(-b)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(
a*x^6 + b)^(1/4))/(-b)^(1/4))/b^2 + 6*sqrt(2)*a^2*(-b)^(1/4)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(-b)^(1/4) - 2*(a*x^6 + b)^(1/4))/(-b)^(1/4))/b^2 + 3*sqrt(2
)*a^2*(-b)^(1/4)*log(sqrt(2)*(a*x^6 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^6 + b)
 + sqrt(-b))/b^2 + 3*sqrt(2)*a^2*log(-sqrt(2)*(a*x^6 + b)^(1/4)*(-b)^(1/4)
+ sqrt(a*x^6 + b) + sqrt(-b))/((-b)^(3/4)*b) - 8*(a*x^6 + b)^(1/4)*a/(b*x^6
))/a
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{7} \left(a\,x^{6}+b\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(a*x^6+b)^(3/4),x)
```

```
[Out] int(1/x^7/(a*x^6+b)^(3/4),x)
```

maxima [A] time = 0.43, size = 94, normalized size = 1.21

$$-\frac{\left(ax^{6}+b\right)^{\frac{1}{4}}a}{6\left(\left(ax^{6}+b\right)b-b^{2}\right)}+\frac{\frac{2\,a\arctan\left(\frac{\left(ax^{6}+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}-\frac{a\log\left(\frac{\left(ax^{6}+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^{6}+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a*x^6+b)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/6*(a*x^6 + b)^(1/4)*a/((a*x^6 + b)*b - b^2) + 1/8*(2*a*arctan((a*x^6 + b
)^(1/4)/b^(1/4))/b^(3/4) - a*log(((a*x^6 + b)^(1/4) - b^(1/4))/((a*x^6 + b)
^(1/4) + b^(1/4)))/b^(3/4))/b
```

mupad [B] time = 1.07, size = 58, normalized size = 0.74

$$\frac{a\,\mathrm{atan}\left(\frac{{\left(a\,x^6+b\right)}^{1/4}}{b^{1/4}}\right)}{4\,b^{7/4}}-\frac{{\left(a\,x^6+b\right)}^{1/4}}{6\,b\,x^6}+\frac{a\,\mathrm{atanh}\left(\frac{{\left(a\,x^6+b\right)}^{1/4}}{b^{1/4}}\right)}{4\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(b + a*x^6)^(3/4)),x)
```

```
[Out] (a*atan((b + a*x^6)^(1/4)/b^(1/4)))/(4*b^(7/4)) - (b + a*x^6)^(1/4)/(6*b*x^
6) + (a*atanh((b + a*x^6)^(1/4)/b^(1/4)))/(4*b^(7/4))
```

sympy [C] time = 1.41, size = 41, normalized size = 0.53

$$-\frac{\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^6}\right)}{6a^{\frac{3}{4}}x^{\frac{21}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(a*x**6+b)**(3/4),x)
```

```
[Out] -gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*exp_polar(I*pi)/(a*x**6))/(6*a**(3
/4)*x**(21/2)*gamma(11/4))
```

3.947 $\int \frac{(1+2x^4)\sqrt{1+2x^8}}{x}\,dx$

Optimal. Leaf size=78

$$\frac{1}{4}\sqrt{2x^8+1}\,(x^4+1)+\frac{\log\left(\sqrt{2x^8+1}+\sqrt{2}\,x^4\right)}{4\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\sqrt{2x^8+1}+\sqrt{2}\,x^4\right)$$

Rubi [A] time = 0.06, antiderivative size = 56, normalized size of antiderivative = 0.72, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1475, 815, 844, 215, 266, 63, 207}

$$-\frac{1}{4}\tanh^{-1}\left(\sqrt{2x^8+1}\right)+\frac{\sinh^{-1}\left(\sqrt{2}\,x^4\right)}{4\sqrt{2}}+\frac{1}{4}\sqrt{2x^8+1}\,(x^4+1)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x^4)*Sqrt[1 + 2*x^8])/x,x]
```

```
[Out] ((1 + x^4)*Sqrt[1 + 2*x^8])/4 + ArcSinh[Sqrt[2]*x^4]/(4*Sqrt[2]) - ArcTanh[
Sqrt[1 + 2*x^8]]/4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+2x^4\right)\sqrt{1+2x^8}}{x}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{(1+2x)\sqrt{1+2x^2}}{x}\,dx, x, x^4\right)\\
&= \frac{1}{4}\left(1+x^4\right)\sqrt{1+2x^8} + \frac{1}{16}\,\text{Subst}\left(\int \frac{4+4x}{x\sqrt{1+2x^2}}\,dx, x, x^4\right)\\
&= \frac{1}{4}\left(1+x^4\right)\sqrt{1+2x^8} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+2x^2}}\,dx, x, x^4\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+2x^2}}\right.\\
&= \frac{1}{4}\left(1+x^4\right)\sqrt{1+2x^8} + \frac{\sinh^{-1}\left(\sqrt{2}\,x^4\right)}{4\sqrt{2}} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+2x}}\,dx, x, x^8\right)\\
&= \frac{1}{4}\left(1+x^4\right)\sqrt{1+2x^8} + \frac{\sinh^{-1}\left(\sqrt{2}\,x^4\right)}{4\sqrt{2}} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{x^2}{2}}\,dx, x, \sqrt{1+2x^8}\right)\\
&= \frac{1}{4}\left(1+x^4\right)\sqrt{1+2x^8} + \frac{\sinh^{-1}\left(\sqrt{2}\,x^4\right)}{4\sqrt{2}} - \frac{1}{4}\tanh^{-1}\left(\sqrt{1+2x^8}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.68

$$\frac{1}{8}\left(-2\tanh^{-1}\left(\sqrt{2x^8+1}\right)+\sqrt{2}\,\sinh^{-1}\left(\sqrt{2}\,x^4\right)+2\sqrt{2x^8+1}\,\left(x^4+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x^4)*Sqrt[1 + 2*x^8])/x,x]
```

```
[Out] (2*(1 + x^4)*Sqrt[1 + 2*x^8] + Sqrt[2]*ArcSinh[Sqrt[2]*x^4] - 2*ArcTanh[Sqr
t[1 + 2*x^8]])/8
```

IntegrateAlgebraic [A] time = 0.15, size = 81, normalized size = 1.04

$$\frac{1}{4}\sqrt{2x^8+1}\,\left(x^4+1\right)-\frac{\log\left(\sqrt{2x^8+1}-\sqrt{2}\,x^4\right)}{4\sqrt{2}}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2}\,x^4-\sqrt{2x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + 2*x^4)*Sqrt[1 + 2*x^8])/x,x]
```

```
[Out] ((1 + x^4)*Sqrt[1 + 2*x^8])/4 + ArcTanh[Sqrt[2]*x^4 - Sqrt[1 + 2*x^8]]/2 -
Log[-(Sqrt[2]*x^4) + Sqrt[1 + 2*x^8]]/(4*Sqrt[2])
```

fricas [A] time = 0.42, size = 61, normalized size = 0.78

$$\frac{1}{4}\sqrt{2x^8+1}\left(x^4+1\right)+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}x^4-\sqrt{2x^8+1}\right)+\frac{1}{4}\log\left(\frac{\sqrt{2x^8+1}-1}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+1)*(2*x^8+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2*x^8 + 1)*(x^4 + 1) + 1/8*sqrt(2)*log(-sqrt(2)*x^4 - sqrt(2*x^8 +
 1)) + 1/4*log((sqrt(2*x^8 + 1) - 1)/x^4)
```

giac [A] time = 0.32, size = 86, normalized size = 1.10

$$\frac{1}{4}\sqrt{2x^8+1}\left(x^4+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}x^4+\sqrt{2x^8+1}\right)+\frac{1}{4}\log\left(\sqrt{2}x^4-\sqrt{2x^8+1}+1\right)-\frac{1}{4}\log\left(-\sqrt{2}x^4+\sqrt{2x^8+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+1)*(2*x^8+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2*x^8 + 1)*(x^4 + 1) - 1/8*sqrt(2)*log(-sqrt(2)*x^4 + sqrt(2*x^8 +
 1)) + 1/4*log(sqrt(2)*x^4 - sqrt(2*x^8 + 1) + 1) - 1/4*log(-sqrt(2)*x^4 +
sqrt(2*x^8 + 1) + 1)
```

maple [A] time = 0.28, size = 103, normalized size = 1.32

$$-\frac{4\sqrt{\pi}-4\sqrt{\pi}\sqrt{2x^8+1}+4\sqrt{\pi}\ \ln\left(\frac{1}{2}+\frac{\sqrt{2x^8+1}}{2}\right)-2\left(2-\ln(2)+8\ln(x)\right)\sqrt{\pi}}{16\sqrt{\pi}}-\frac{\sqrt{2}\left(-2\sqrt{\pi}\sqrt{2}\,x^4\sqrt{2x^8+1}-2\sqrt{\pi}\ \operatorname{arcsinh}\left(\sqrt{2}\,x^4\right)\right)}{16\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+1)*(2*x^8+1)^(1/2)/x,x)
```

```
[Out] -1/16/Pi^(1/2)*(4*Pi^(1/2)-4*Pi^(1/2)*(2*x^8+1)^(1/2)+4*Pi^(1/2)*ln(1/2+1/2
*(2*x^8+1)^(1/2))-2*(2-ln(2)+8*ln(x))*Pi^(1/2))-1/16*2^(1/2)/Pi^(1/2)*(-2*P
i^(1/2)*2^(1/2)*x^4*(2*x^8+1)^(1/2)-2*Pi^(1/2)*arcsinh(2^(1/2)*x^4))
```

maxima [A] time = 0.43, size = 114, normalized size = 1.46

$$-\frac{1}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2x^8+1}}{x^4}}{\sqrt{2}+\frac{\sqrt{2x^8+1}}{x^4}}\right)+\frac{1}{4}\sqrt{2x^8+1}+\frac{\sqrt{2x^8+1}}{4x^4\left(\frac{2x^8+1}{x^8}-2\right)}-\frac{1}{8}\log\left(\sqrt{2x^8+1}+1\right)+\frac{1}{8}\log\left(\sqrt{2x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+1)*(2*x^8+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*log(-(sqrt(2) - sqrt(2*x^8 + 1)/x^4)/(sqrt(2) + sqrt(2*x^8 +
1)/x^4)) + 1/4*sqrt(2*x^8 + 1) + 1/4*sqrt(2*x^8 + 1)/(x^4*((2*x^8 + 1)/x^8
- 2)) - 1/8*log(sqrt(2*x^8 + 1) + 1) + 1/8*log(sqrt(2*x^8 + 1) - 1)
```

mupad [B] time = 1.24, size = 47, normalized size = 0.60

$$\frac{\sqrt{2}\ \operatorname{asinh}\left(\sqrt{2}\ x^4\right)}{8}-\frac{\operatorname{atanh}\left(\sqrt{2}\ \sqrt{x^8+\frac{1}{2}}\right)}{4}+\frac{\sqrt{2}\ \sqrt{x^8+\frac{1}{2}}\ \left(\frac{x^4}{2}+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^4 + 1)*(2*x^8 + 1)^(1/2))/x,x)
```

```
[Out] (2^(1/2)*asinh(2^(1/2)*x^4))/8 - atanh(2^(1/2)*(x^8 + 1/2)^(1/2))/4 + (2^(1
/2)*(x^8 + 1/2)^(1/2)*(x^4/2 + 1/2))/2
```

sympy [A] time = 31.28, size = 65, normalized size = 0.83

$$\frac{x^4\sqrt{2x^8+1}}{4}+\frac{\sqrt{2x^8+1}}{4}+\frac{\log\left(x^8\right)}{8}-\frac{\log\left(\sqrt{2x^8+1}\ +1\right)}{4}+\frac{\sqrt{2}\ \operatorname{asinh}\left(\sqrt{2}\,x^4\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+1)*(2*x**8+1)**(1/2)/x,x)
```

```
[Out] x**4*sqrt(2*x**8 + 1)/4 + sqrt(2*x**8 + 1)/4 + log(x**8)/8 - log(sqrt(2*x**
8 + 1) + 1)/4 + sqrt(2)*asinh(sqrt(2)*x**4)/8
```

3.948 $\int \sqrt{x+\sqrt{1+x}}\,dx$

Optimal. Leaf size=78

$$\frac{1}{12}\sqrt{x+\sqrt{x+1}}\,(8x-3)+\frac{1}{6}\sqrt{x+1}\sqrt{x+\sqrt{x+1}}-\frac{5}{8}\log\left(2\sqrt{x+1}-2\sqrt{x+\sqrt{x+1}}+1\right)$$

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {640, 612, 621, 206}

$$\frac{2}{3}\left(x+\sqrt{x+1}\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}}+\frac{5}{8}\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[1 + x]],x]
```

```
[Out] (2*(x + Sqrt[1 + x])^(3/2))/3 - (Sqrt[x + Sqrt[1 + x]]*(1 + 2*Sqrt[1 + x]))
/4 + (5*ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/8
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x+\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int x\sqrt{-1+x+x^2}\,dx, x, \sqrt{1+x}\right)\\
&= \frac{2}{3}\left(x+\sqrt{1+x}\right)^{3/2} - \text{Subst}\left(\int \sqrt{-1+x+x^2}\,dx, x, \sqrt{1+x}\right)\\
&= \frac{2}{3}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{1}{4}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) + \frac{5}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x+x^2}}\,dx, x\right.\\
&= \frac{2}{3}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{1}{4}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) + \frac{5}{4}\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{1+2\sqrt{}}{\sqrt{x+}}\right.\\
&= \frac{2}{3}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{1}{4}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) + \frac{5}{8}\tanh^{-1}\left(\frac{1+2\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 0.83

$$\frac{1}{12}\sqrt{x+\sqrt{x+1}}\left(8x+2\sqrt{x+1}-3\right)+\frac{5}{8}\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[1 + x]],x]
[Out] (Sqrt[x + Sqrt[1 + x]]*(-3 + 8*x + 2*Sqrt[1 + x]))/12 + (5*ArcTanh[(1 + 2*S
qrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/8
```

IntegrateAlgebraic [A] time = 0.11, size = 65, normalized size = 0.83

$$\frac{1}{12}\sqrt{x+\sqrt{x+1}}\left(8(x+1)+2\sqrt{x+1}-11\right)-\frac{5}{8}\log\left(-2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x + Sqrt[1 + x]],x]
[Out] (Sqrt[x + Sqrt[1 + x]]*(-11 + 2*Sqrt[1 + x] + 8*(1 + x)))/12 - (5*Log[-1 -
2*Sqrt[1 + x] + 2*Sqrt[x + Sqrt[1 + x]]])/8
```

fricas [A] time = 0.88, size = 59, normalized size = 0.76

$$\frac{1}{12}\left(8x+2\sqrt{x+1}-3\right)\sqrt{x+\sqrt{x+1}}+\frac{5}{16}\log\left(4\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)+8x+8\sqrt{x+1}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 1/12*(8*x + 2*sqrt(x + 1) - 3)*sqrt(x + sqrt(x + 1)) + 5/16*log(4*sqrt(x +
sqrt(x + 1))*(2*sqrt(x + 1) + 1) + 8*x + 8*sqrt(x + 1) + 5)
```

giac [A] time = 0.29, size = 53, normalized size = 0.68

$$\frac{1}{12}\left(2\sqrt{x+1}\left(4\sqrt{x+1}+1\right)-11\right)\sqrt{x+\sqrt{x+1}}-\frac{5}{8}\log\left(-2\sqrt{x+\sqrt{x+1}}+2\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x + 1)*(4*sqrt(x + 1) + 1) - 11)*sqrt(x + sqrt(x + 1)) - 5/8*l
og(-2*sqrt(x + sqrt(x + 1)) + 2*sqrt(x + 1) + 1)
```

maple [A] time = 0.01, size = 52, normalized size = 0.67

$$\frac{2\left(x+\sqrt{1+x}\right)^{\frac{3}{2}}}{3}-\frac{\left(2\sqrt{1+x}\ +1\right)\sqrt{x+\sqrt{1+x}}}{4}+\frac{5\ln\left(\frac{1}{2}+\sqrt{1+x}\ +\sqrt{x+\sqrt{1+x}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(1+x)^(1/2))^(1/2),x)
```

```
[Out] 2/3*(x+(1+x)^(1/2))^(3/2)-1/4*(2*(1+x)^(1/2)+1)*(x+(1+x)^(1/2))^(1/2)+5/8*l
n(1/2+(1+x)^(1/2)+(x+(1+x)^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x+\sqrt{x+1}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x+\sqrt{x+1}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x + 1)^(1/2))^(1/2),x)
```

```
[Out] int((x + (x + 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x+\sqrt{x+1}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x + sqrt(x + 1)), x)
```

3.949 $\int \frac{2b+ax^2}{x\left(b^2+a^2x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt[4]{a^2x^2+b^2}}{a}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a^2x^2+b^2}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a^2x^2+b^2}}{\sqrt{b}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {446, 80, 63, 212, 206, 203}

$$\frac{2\sqrt[4]{a^2x^2+b^2}}{a}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a^2x^2+b^2}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a^2x^2+b^2}}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*b + a*x^2)/(x*(b^2 + a^2*x^2)^(3/4)),x]
```

```
[Out] (2*(b^2 + a^2*x^2)^(1/4))/a - (2*ArcTan[(b^2 + a^2*x^2)^(1/4)/Sqrt[b]])/Sqr
t[b] - (2*ArcTanh[(b^2 + a^2*x^2)^(1/4)/Sqrt[b]])/Sqrt[b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{2b+ax^2}{x\left(b^2+a^2x^2\right)^{3/4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2b+ax}{x\left(b^2+a^2x\right)^{3/4}}\,dx,x,x^2\right)\\
&= \frac{2\sqrt[4]{b^2+a^2x^2}}{a}+b\operatorname{Subst}\left(\int \frac{1}{x\left(b^2+a^2x\right)^{3/4}}\,dx,x,x^2\right)\\
&= \frac{2\sqrt[4]{b^2+a^2x^2}}{a}+\frac{(4b)\operatorname{Subst}\left(\int \frac{1}{-\frac{b^2}{a^2}+\frac{x^4}{a^2}}\,dx,x,\sqrt[4]{b^2+a^2x^2}\right)}{a^2}\\
&= \frac{2\sqrt[4]{b^2+a^2x^2}}{a}-2\operatorname{Subst}\left(\int \frac{1}{b-x^2}\,dx,x,\sqrt[4]{b^2+a^2x^2}\right)-2\operatorname{Subst}\left(\int \frac{1}{b+x^2}\,dx,x,\sqrt[4]{b^2+\right.\\
&= \frac{2\sqrt[4]{b^2+a^2x^2}}{a}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{b^2+a^2x^2}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{b^2+a^2x^2}}{\sqrt{b}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 1.00

$$\frac{2\sqrt[4]{a^2x^2+b^2}}{a}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a^2x^2+b^2}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a^2x^2+b^2}}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*b + a*x^2)/(x*(b^2 + a^2*x^2)^(3/4)),x]
```

```
[Out] (2*(b^2 + a^2*x^2)^(1/4))/a - (2*ArcTan[(b^2 + a^2*x^2)^(1/4)/Sqrt[b]])/Sqr
t[b] - (2*ArcTanh[(b^2 + a^2*x^2)^(1/4)/Sqrt[b]])/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.07, size = 79, normalized size = 1.00

$$\frac{2\sqrt[4]{a^2x^2+b^2}}{a}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a^2x^2+b^2}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a^2x^2+b^2}}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*b + a*x^2)/(x*(b^2 + a^2*x^2)^(3/4)),x]
```

```
[Out] (2*(b^2 + a^2*x^2)^(1/4))/a - (2*ArcTan[(b^2 + a^2*x^2)^(1/4)/Sqrt[b]])/Sqr
t[b] - (2*ArcTanh[(b^2 + a^2*x^2)^(1/4)/Sqrt[b]])/Sqrt[b]
```

fricas [A] time = 0.43, size = 264, normalized size = 3.34

$$\left[-\frac{2\,a\sqrt{b}\,\arctan\left(\frac{\left(a^2x^2+b^2\right)^{\frac{1}{4}}}{\sqrt{b}}\right)-a\sqrt{b}\,\log\left(\frac{a^2x^2+2\,b^2-2\left(a^2x^2+b^2\right)^{\frac{1}{4}}b^{\frac{3}{2}}+2\,\sqrt{a^2x^2+b^2}\,b-2\left(a^2x^2+b^2\right)^{\frac{3}{4}}\sqrt{b}}{x^2}\right)-2\left(a^2x^2+b^2\right)^{\frac{1}{4}}b}{ab},\ \frac{2\,a\sqrt{-b}\,\arctan\left(\frac{\left(a^2x^2+b^2\right)^{\frac{1}{4}}\sqrt{-b}}{b}\right)-a\sqrt{-b}\,\log\left(\frac{a^2x^2+2\,b^2-2\left(a^2x^2+b^2\right)^{\frac{1}{4}}\sqrt{-b}\,b-2\,\sqrt{a^2x^2+b^2}\,b+2\left(a^2x^2+b^2\right)^{\frac{3}{4}}\sqrt{-b}}{x^2}\right)+2\left(a^2x^2+b^2\right)^{\frac{1}{4}}b}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b)/x/(a^2*x^2+b^2)^(3/4),x, algorithm="fricas")
```

```
[Out] [-(2*a*sqrt(b)*arctan((a^2*x^2 + b^2)^(1/4)/sqrt(b)) - a*sqrt(b)*log((a^2*x
^2 + 2*b^2 - 2*(a^2*x^2 + b^2)^(1/4)*b^(3/2) + 2*sqrt(a^2*x^2 + b^2)*b - 2*
(a^2*x^2 + b^2)^(3/4)*sqrt(b))/x^2) - 2*(a^2*x^2 + b^2)^(1/4)*b)/(a*b), (2*
a*sqrt(-b)*arctan((a^2*x^2 + b^2)^(1/4)*sqrt(-b)/b) - a*sqrt(-b)*log((a^2*x
^2 + 2*b^2 - 2*(a^2*x^2 + b^2)^(1/4)*sqrt(-b)*b - 2*sqrt(a^2*x^2 + b^2)*b +
 2*(a^2*x^2 + b^2)^(3/4)*sqrt(-b))/x^2) + 2*(a^2*x^2 + b^2)^(1/4)*b)/(a*b)]
```

giac [A] time = 0.35, size = 69, normalized size = 0.87

$$\frac{2\arctan\left(\frac{\left(a^2x^2+b^2\right)^{\frac{1}{4}}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{2\arctan\left(\frac{\left(a^2x^2+b^2\right)^{\frac{1}{4}}}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{2\left(a^2x^2+b^2\right)^{\frac{1}{4}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b)/x/(a^2*x^2+b^2)^(3/4),x, algorithm="giac")
```

```
[Out] 2*arctan((a^2*x^2 + b^2)^(1/4)/sqrt(-b))/sqrt(-b) - 2*arctan((a^2*x^2 + b^2
)^(1/4)/sqrt(b))/sqrt(b) + 2*(a^2*x^2 + b^2)^(1/4)/a
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2+2b}{x\left(a^2x^2+b^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+2*b)/x/(a^2*x^2+b^2)^(3/4),x)
```

```
[Out] int((a*x^2+2*b)/x/(a^2*x^2+b^2)^(3/4),x)
```

maxima [A] time = 0.42, size = 92, normalized size = 1.16

$$-b\left(\frac{2\arctan\left(\frac{\left(a^2x^2+b^2\right)^{\frac{1}{4}}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}}-\frac{\log\left(-\frac{\sqrt{b}-\left(a^2x^2+b^2\right)^{\frac{1}{4}}}{\sqrt{b}+\left(a^2x^2+b^2\right)^{\frac{1}{4}}}\right)}{b^{\frac{3}{2}}}\right)+\frac{2\left(a^2x^2+b^2\right)^{\frac{1}{4}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b)/x/(a^2*x^2+b^2)^(3/4),x, algorithm="maxima")
```

```
[Out] -b*(2*arctan((a^2*x^2 + b^2)^(1/4)/sqrt(b))/b^(3/2) - log(-(sqrt(b) - (a^2*
x^2 + b^2)^(1/4))/(sqrt(b) + (a^2*x^2 + b^2)^(1/4)))/b^(3/2)) + 2*(a^2*x^2
+ b^2)^(1/4)/a
```

mupad [B] time = 1.14, size = 65, normalized size = 0.82

$$\frac{2\left(a^2\,x^2+b^2\right)^{1/4}}{a}-\frac{2\,\mathrm{atanh}\left(\frac{\left(a^2\,x^2+b^2\right)^{1/4}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{2\,\mathrm{atan}\left(\frac{\left(a^2\,x^2+b^2\right)^{1/4}}{\sqrt{b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b + a*x^2)/(x*(b^2 + a^2*x^2)^(3/4)),x)
```

```
[Out] (2*(b^2 + a^2*x^2)^(1/4))/a - (2*atanh((b^2 + a^2*x^2)^(1/4)/b^(1/2)))/b^(1
/2) - (2*atan((b^2 + a^2*x^2)^(1/4)/b^(1/2)))/b^(1/2)
```

sympy [A] time = 7.86, size = 76, normalized size = 0.96

$$a\left(\begin{cases}\frac{x^2}{2(b^2)^{\frac{3}{4}}} & \text{for } a^2=0\\ \frac{2\sqrt[4]{a^2x^2+b^2}}{a^2} & \text{otherwise}\end{cases}\right)-\frac{b\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{b^2e^{i\pi}}{a^2x^2}\right)}{a^{\frac{3}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+2*b)/x/(a**2*x**2+b**2)**(3/4),x)
```

```
[Out] a*Piecewise((x**2/(2*(b**2)**(3/4)), Eq(a**2, 0)), (2*(a**2*x**2 + b**2)**(
1/4)/a**2, True)) - b*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b**2*exp_polar(I
*pi)/(a**2*x**2))/(a**(3/2)*x**(3/2)*gamma(7/4))
```

3.950 $\int \frac{-abx+x^3}{(-a+x)(-b+x)\sqrt{x(-a+x)(-b+x)}\left(abd-(1+ad+bd)x+dx^2\right)}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt{abx-ax^2-bx^2+x^3}}{(x-a)(x-b)}-2\sqrt{d}\,\tanh^{-1}\left(\frac{x}{\sqrt{d}\sqrt{x^2(-a-b)+abx+x^3}}\right)$$

Rubi [F] time = 13.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-abx+x^3}{(-a+x)(-b+x)\sqrt{x(-a+x)(-b+x)}\left(abd-(1+ad+bd)x+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a*b*x) + x^3)/((-a + x)*(-b + x)*Sqrt[x*(-a + x)*(-b + x)]*(a*b*d -
(1 + a*d + b*d)*x + d*x^2)),x]
```

```
[Out] (-2*x)/((a - b)*d*Sqrt[(a - x)*(b - x)*x]) + (4*(a - x)*x)/((a - b)^2*d*Sqr
t[(a - x)*(b - x)*x]) - (4*Sqrt[a]*(b - x)*Sqrt[x]*Sqrt[1 - x/a]*EllipticE[
ArcSin[Sqrt[x]/Sqrt[a]], a/b])/((a - b)^2*d*Sqrt[(a - x)*(b - x)*x]*Sqrt[1
- x/b]) - (2*Sqrt[a]*Sqrt[x]*Sqrt[1 - x/a]*Sqrt[1 - x/b]*EllipticF[ArcSin[S
qrt[x]/Sqrt[a]], a/b])/((a - b)*d*Sqrt[(a - x)*(b - x)*x]) + ((1 + a*d + b*
d + Sqrt[a^2*d^2 + 2*a*d*(1 - b*d) + (1 + b*d)^2])*Sqrt[x]*Sqrt[-a + x]*Sqr
t[-b + x]*Defer[Int][Sqrt[x]/((-a + x)^(3/2)*(-b + x)^(3/2)*(-1 - a*d - b*d
 - Sqrt[1 + 2*a*d + 2*b*d + a^2*d^2 - 2*a*b*d^2 + b^2*d^2] + 2*d*x)), x])/(
d*Sqrt[(a - x)*(b - x)*x]) + ((1 + a*d + b*d - Sqrt[a^2*d^2 + 2*a*d*(1 - b*
d) + (1 + b*d)^2])*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Int][Sqrt[x]/((-
a + x)^(3/2)*(-b + x)^(3/2)*(-1 - a*d - b*d + Sqrt[1 + 2*a*d + 2*b*d + a^2*
d^2 - 2*a*b*d^2 + b^2*d^2] + 2*d*x)), x])/(d*Sqrt[(a - x)*(b - x)*x])
```

Rubi steps

$$\int \frac{-abx+x^3}{(-a+x)(-b+x)\sqrt{x(-a+x)(-b+x)}\left(abd-(1+ad+bd)x+dx^2\right)}\,dx = \int \frac{x\left(-ab\right.}{(-a+x)(-b+x)\sqrt{x(-a+x)(-b+}}$$

$$= \int \frac{x^2\left(-ab+x^2\right)}{(x(-a+x)(-b+x))^{3/2}\left(abd-(1+\right.}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{}{(-a+x)^{3/2}(-b+}}{\sqrt{x(-a+x)(-b}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int\left(\frac{\sqrt{x}}{d(-a+x)^{3/2}(-}\right.}{\sqrt{x(-}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{\sqrt{x}}{(-a+x)^{3/2}(-b+}}{d\sqrt{x(-a+x)(-b+x)}}$$

$$= -\frac{2x}{(a-b)d\sqrt{(a-x)(b-x)x}} - \frac{\left(\sqrt{x}\sqrt{-\right.}{}$$

$$= -\frac{2x}{(a-b)d\sqrt{(a-x)(b-x)x}} + \frac{}{(a-b)^2}$$

$$= -\frac{2x}{(a-b)d\sqrt{(a-x)(b-x)x}} + \frac{}{(a-b)^2}$$

$$= -\frac{2x}{(a-b)d\sqrt{(a-x)(b-x)x}} + \frac{}{(a-b)^2}$$

$$= -\frac{2x}{(a-b)d\sqrt{(a-x)(b-x)x}} + \frac{}{(a-b)^2}$$

Mathematica [C] time = 6.96, size = 280, normalized size = 3.54

$$\frac{2ix^{3/2}\sqrt{1-\frac{a}{x}}\sqrt{1-\frac{b}{x}}\,\Pi\left(\frac{2bd}{ad+bd-\sqrt{(ad+bd+1)^2-4abd^2}+1};i\sinh^{-1}\left(\frac{\sqrt{-a}}{\sqrt{x}}\right)|\frac{b}{a}\right)+2ix^{3/2}\sqrt{1-\frac{a}{x}}\sqrt{1-\frac{b}{x}}\,\Pi\left(\frac{2bd}{ad+bd+\sqrt{(ad+bd+1)^2-4abd^2}+1};i\sinh^{-1}\left(\frac{\sqrt{-a}}{\sqrt{x}}\right)|\frac{b}{a}\right)-2ix^{3/2}\sqrt{1-\frac{a}{x}}\sqrt{1-\frac{b}{x}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-a}}{\sqrt{x}}\right)|\frac{b}{a}\right)+2\sqrt{-a}\,x}{\sqrt{-a}\sqrt{x(x-a)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-(a*b*x) + x^3)/((-a + x)*(-b + x)*Sqrt[x*(-a + x)*(-b + x)]*(a*
b*d - (1 + a*d + b*d)*x + d*x^2)),x]
```

```
[Out] (2*Sqrt[-a]*x - (2*I)*Sqrt[1 - a/x]*Sqrt[1 - b/x]*x^(3/2)*EllipticF[I*ArcSi
nh[Sqrt[-a]/Sqrt[x]], b/a] + (2*I)*Sqrt[1 - a/x]*Sqrt[1 - b/x]*x^(3/2)*Elli
pticPi[(2*b*d)/(1 + a*d + b*d - Sqrt[-4*a*b*d^2 + (1 + a*d + b*d)^2]), I*Ar
cSinh[Sqrt[-a]/Sqrt[x]], b/a] + (2*I)*Sqrt[1 - a/x]*Sqrt[1 - b/x]*x^(3/2)*E
llipticPi[(2*b*d)/(1 + a*d + b*d + Sqrt[-4*a*b*d^2 + (1 + a*d + b*d)^2]), I
*ArcSinh[Sqrt[-a]/Sqrt[x]], b/a])/(Sqrt[-a]*Sqrt[x*(-a + x)*(-b + x)])
```

IntegrateAlgebraic [A] time = 0.30, size = 79, normalized size = 1.00

$$\frac{2\sqrt{abx-ax^2-bx^2+x^3}}{(x-a)(x-b)} - 2\sqrt{d}\,\tanh^{-1}\left(\frac{x}{\sqrt{d}\sqrt{x^2(-a-b)+abx+x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*b*x) + x^3)/((-a + x)*(-b + x)*Sqrt[x*(-a + x)*(-b
+ x)]*(a*b*d - (1 + a*d + b*d)*x + d*x^2)),x]
```

```
[Out] (2*Sqrt[a*b*x - a*x^2 - b*x^2 + x^3])/((-a + x)*(-b + x)) - 2*Sqrt[d]*ArcTa
nh[x/(Sqrt[d]*Sqrt[a*b*x + (-a - b)*x^2 + x^3])]
```

fricas [A] time = 0.77, size = 395, normalized size = 5.00

$$\left[\frac{\left(ab-(a+b)x+x^2\right)\sqrt{d}\,\log\left(\frac{a^2b^2d^2+d^2x^4-2\left((a+b)d^2-3\,d\right)x^3+\left(\left(a^2+4\,ab+b^2\right)d^2-6\,(a+b)d+1\right)x^2-4\left(abd+dx^2-((a+b)d-1)x\right)\sqrt{abx-(a+b)x^2+x^3}\,\sqrt{d}+2\left(3\,abd-\left(a^2b+ab^2\right)d^2\right)x}{a^2b^2d^2+d^2x^4-2\left((a+b)d^2+d\right)x^3+\left(\left(a^2+4\,ab+b^2\right)d^2+2\,(a+b)d+1\right)x^2-2\left(abd+\left(a^2b+ab^2\right)d^2\right)x}\right)+4\,\sqrt{abx-(a+b)x^2+x^3}}{2\left(ab-(a+b)x+x^2\right)},\ \frac{\left(ab-(a+b)x+x^2\right)\sqrt{-d}\,\arctan\left(\frac{\left(abd+dx^2-((a+b)d-1)x\right)\sqrt{abx-(a+b)x^2+x^3}\,\sqrt{-d}}{2\left(abdx-(a+b)dx^2+dx^3\right)}\right)+2\,\sqrt{abx-(a+b)x^2+x^3}}{ab-(a+b)x+x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*x+x^3)/(-a+x)/(-b+x)/(x*(-a+x)*(-b+x))^(1/2)/(a*b*d-(a*d+b*
d+1)*x+d*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b - (a + b)*x + x^2)*sqrt(d)*log((a^2*b^2*d^2 + d^2*x^4 - 2*((a +
b)*d^2 - 3*d)*x^3 + ((a^2 + 4*a*b + b^2)*d^2 - 6*(a + b)*d + 1)*x^2 - 4*(a*
b*d + d*x^2 - ((a + b)*d - 1)*x)*sqrt(a*b*x - (a + b)*x^2 + x^3)*sqrt(d) +
2*(3*a*b*d - (a^2*b + a*b^2)*d^2)*x)/(a^2*b^2*d^2 + d^2*x^4 - 2*((a + b)*d^
2 + d)*x^3 + ((a^2 + 4*a*b + b^2)*d^2 + 2*(a + b)*d + 1)*x^2 - 2*(a*b*d + (
a^2*b + a*b^2)*d^2)*x)) + 4*sqrt(a*b*x - (a + b)*x^2 + x^3))/(a*b - (a + b)
*x + x^2), ((a*b - (a + b)*x + x^2)*sqrt(-d)*arctan(1/2*(a*b*d + d*x^2 - ((
a + b)*d - 1)*x)*sqrt(a*b*x - (a + b)*x^2 + x^3)*sqrt(-d)/(a*b*d*x - (a + b
)*d*x^2 + d*x^3)) + 2*sqrt(a*b*x - (a + b)*x^2 + x^3))/(a*b - (a + b)*x + x
^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{abx - x^3}{\left(abd + dx^2 - (ad + bd + 1)x\right)\sqrt{(a-x)(b-x)x}\,(a-x)(b-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*x+x^3)/(-a+x)/(-b+x)/(x*(-a+x)*(-b+x))^(1/2)/(a*b*d-(a*d+b*
d+1)*x+d*x^2),x, algorithm="giac")
```

```
[Out] integrate(-(a*b*x - x^3)/((a*b*d + d*x^2 - (a*d + b*d + 1)*x)*sqrt((a - x)*
(b - x)*x)*(a - x)*(b - x)), x)
```

maple [C] time = 0.04, size = 4446, normalized size = 56.28

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*b*x+x^3)/(-a+x)/(-b+x)/(x*(-a+x)*(-b+x))^(1/2)/(a*b*d-(a*d+b*d+1)*x
+d*x^2),x)
```

```
[Out] -a*(-2*(-b*x+x^2)/a/(a-b)/((-a+x)*(-b*x+x^2))^(1/2)-2*(-1/a-1/a/(a-b)*b)*a*
(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3
)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))-2/(a-b)*(-(-a+x)/a)^(1
/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)*((a-b)
*EllipticE((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))+b*EllipticF((-(-a+x)/a)^(1/2)
,(a/(a-b))^(1/2))))-b*(2*(-a*x+x^2)/(a-b)/b/((-b+x)*(-a*x+x^2))^(1/2)-2*(-1
/b+a/(a-b)/b)*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x
-a*x^2-b*x^2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))+2/(a-b
)/b*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x
^2+x^3)^(1/2)*((a-b)*EllipticE((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))+b*Ellipti
cF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))))-1/(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+
2*b*d+1)^(1/2)*a^3*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/
```

```
2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2/d-1/2/d*(a^2*d^2-2*a*b*d^
2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(a*
d+b*d+1+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))*
d+2/(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)*a^2*(1-1/a*x)^(1/2)*(-1
/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*
a-1/2*b-1/2/d-1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))*Ellipt
icPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(a*d+b*d+1+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a
*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))*b*d-2/(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d
+2*b*d+1)^(1/2)*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1
/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2/d-1/2/d*(a^2*d^2-2*a*b*d
^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(a
*d+b*d+1+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))
-1/(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a
-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1
/2*b-1/2/d-1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))*EllipticP
i((-(-a+x)/a)^(1/2),a/(a-1/2/d*(a*d+b*d+1+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+
2*b*d+1)^(1/2))),(a/(a-b))^(1/2))*b^2*d-2/(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+
2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)
/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2/d-1/2/d*(a^2*d^2-2*a*b*d^2+
b^2*d^2+2*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(a*d+
b*d+1+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))*b-
1/(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-
b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/
2*b-1/2/d-1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))*EllipticPi
((-(-a+x)/a)^(1/2),a/(a-1/2/d*(a*d+b*d+1+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2
*b*d+1)^(1/2))),(a/(a-b))^(1/2))/d-a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*
x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2/d-1/2
/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^
(1/2),a/(a-1/2/d*(a*d+b*d+1+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)
)),(a/(a-b))^(1/2))-a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^
(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2/d-1/2/d*(a^2*d^2-2*a*b
*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*
(a*d+b*d+1+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2
))*b-a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*
x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b-1/2/d-1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*
a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(a*d+b*d+1+(a^2
*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))/d+1/(a^2*d^2
-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)*a^3*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(
a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b+1/2/
d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x
)/a)^(1/2),a/(a+1/2*(-a*d-b*d+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/
2)-1)/d),(a/(a-b))^(1/2))*d-2/(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/
2)*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*
x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b+1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*
b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-a*d-b*d+(a^2*d^
2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))*b*d+2/(a^2*
d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)*a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+
1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b+1
/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-
a+x)/a)^(1/2),a/(a+1/2*(-a*d-b*d+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^
(1/2)-1)/d),(a/(a-b))^(1/2))+1/(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1
/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x
^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b+1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b
*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-a*d-b*d+(a^2*d^2
-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))*b^2*d+2/(a^2
*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1
/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b+1/
2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a
+x)/a)^(1/2),a/(a+1/2*(-a*d-b*d+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(
```

```
1/2)-1)/d),(a/(a-b))^(1/2))*b-a^2*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1
/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b+1/2/d*(a^2*d^2
-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2)
,a/(a+1/2*(-a*d-b*d+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1)/d),(
a/(a-b))^(1/2))-a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2
)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b+1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d
^2+2*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-a*d-
b*d+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))*
b-a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2
-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b+1/2/d*(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d
+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-a*d-b*d+(a^2*d^2-2
*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))/d+1/(a^2*d^2-2
*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)
*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(1/2*a-1/2*b+1/2/d*(a
^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)
^(1/2),a/(a+1/2*(-a*d-b*d+(a^2*d^2-2*a*b*d^2+b^2*d^2+2*a*d+2*b*d+1)^(1/2)-1
)/d),(a/(a-b))^(1/2))/d
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{abx - x^3}{\left(abd + dx^2 - (ad + bd + 1)x\right)\sqrt{(a-x)(b-x)x}\,(a-x)(b-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*x+x^3)/(-a+x)/(-b+x)/(x*(-a+x)*(-b+x))^(1/2)/(a*b*d-(a*d+b*
d+1)*x+d*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((a*b*x - x^3)/((a*b*d + d*x^2 - (a*d + b*d + 1)*x)*sqrt((a - x)*
(b - x)*x)*(a - x)*(b - x)), x)
```

mupad [B]   time = 0.87, size = 695, normalized size = 8.80

$$-\frac{2a\sqrt{\frac{x}{a}}\left(\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x}{a}}\right)\middle|\frac{a}{b}\right)-\frac{a\sin\left(2\,\mathrm{asin}\left(\sqrt{\frac{x}{a}}\right)\right)}{2b\sqrt{1-\frac{x}{b}}}\right)\sqrt{\frac{a-x}{a}}\sqrt{\frac{b-x}{b}}}{\left(\frac{a}{b}-1\right)\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}-\frac{b\sqrt{\frac{x}{b}}\sqrt{\frac{b-x}{b}}\sqrt{\frac{a-x}{a-b}}\,\Pi\left(\frac{b}{b-\frac{a\,d+b\,d+\sqrt{a^2d^2-2abd^2+2ad+b^2d^2+2bd+1}+1}{2d}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\left(a\,d+b\,d+\sqrt{a^2d^2-2\,a\,b\,d^2+2\,a\,d+b^2d^2+2\,b\,d+1}+1\right)}{d\left(b-\frac{a\,d+b\,d+\sqrt{a^2d^2-2abd^2+2ad+b^2d^2+2bd+1}+1}{2d}\right)\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}-\frac{b\sqrt{\frac{x}{b}}\sqrt{\frac{b-x}{b}}\sqrt{\frac{a-x}{a-b}}\,\Pi\left(\frac{b}{b-\frac{a\,d+b\,d-\sqrt{a^2d^2-2abd^2+2ad+b^2d^2+2bd+1}+1}{2d}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\left(a\,d+b\,d-\sqrt{a^2d^2-2\,a\,b\,d^2+2\,a\,d+b^2d^2+2\,b\,d+1}+1\right)}{d\left(b-\frac{a\,d+b\,d-\sqrt{a^2d^2-2abd^2+2ad+b^2d^2+2bd+1}+1}{2d}\right)\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}-\frac{2\,a\,b\left(\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)+\frac{b\sin\left(2\,\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\right)}{2\sqrt{\frac{b-x}{a-b}+1}\,(a-b)}\right)\sqrt{\frac{x}{b}}\sqrt{\frac{b-x}{b}}\sqrt{\frac{a-x}{a-b}}}{\left(\frac{b}{a-b}+1\right)(a-b)\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - a*b*x)/((a - x)*(b - x)*(d*x^2 - x*(a*d + b*d + 1) + a*b*d)*(x*(
a - x)*(b - x))^(1/2)),x)
```

```
[Out] - (2*a*(x/a)^(1/2)*(ellipticE(asin((x/a)^(1/2)), a/b) - (a*sin(2*asin((x/a)
^(1/2))))/(2*b*(1 - x/b)^(1/2)))*((a - x)/a)^(1/2)*((b - x)/b)^(1/2))/((a/b
 - 1)*(x^3 - x^2*(a + b) + a*b*x)^(1/2)) - (b*(x/b)^(1/2)*((b - x)/b)^(1/2)
*((a - x)/(a - b))^(1/2)*ellipticPi(b/(b - (a*d + b*d + (2*a*d + 2*b*d + a^
2*d^2 + b^2*d^2 - 2*a*b*d^2 + 1)^(1/2) + 1)/(2*d)), asin(((b - x)/b)^(1/2))
, -b/(a - b))*(a*d + b*d + (2*a*d + 2*b*d + a^2*d^2 + b^2*d^2 - 2*a*b*d^2 +
 1)^(1/2) + 1))/(d*(b - (a*d + b*d + (2*a*d + 2*b*d + a^2*d^2 + b^2*d^2 - 2
*a*b*d^2 + 1)^(1/2) + 1)/(2*d))*(x^3 - x^2*(a + b) + a*b*x)^(1/2)) - (b*(x/
b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*ellipticPi(b/(b - (a*d +
 b*d - (2*a*d + 2*b*d + a^2*d^2 + b^2*d^2 - 2*a*b*d^2 + 1)^(1/2) + 1)/(2*d)
), asin(((b - x)/b)^(1/2)), -b/(a - b))*(a*d + b*d - (2*a*d + 2*b*d + a^2*d
^2 + b^2*d^2 - 2*a*b*d^2 + 1)^(1/2) + 1))/(d*(b - (a*d + b*d - (2*a*d + 2*b
*d + a^2*d^2 + b^2*d^2 - 2*a*b*d^2 + 1)^(1/2) + 1)/(2*d))*(x^3 - x^2*(a + b
) + a*b*x)^(1/2)) - (2*a*b*(ellipticE(asin(((b - x)/b)^(1/2)), -b/(a - b))
+ (b*sin(2*asin(((b - x)/b)^(1/2))))/(2*((b - x)/(a - b) + 1)^(1/2)*(a - b)
))*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2))/((b/(a - b) + 1)*
(a - b)*(x^3 - x^2*(a + b) + a*b*x)^(1/2))
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*x+x**3)/(-a+x)/(-b+x)/(x*(-a+x)*(-b+x))**(1/2)/(a*b*d-(a*d+
b*d+1)*x+d*x**2),x)
```

```
[Out] Timed out
```

3.951 $\int \frac{-1+2(-1+k)x+kx^2}{\sqrt[4]{(1-x)x(1-kx)}\left(-1+(3+d)x-(3+dk)x^2+x^3\right)}\,dx$

Optimal. Leaf size=79

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{x-1}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{x-1}\right)}{d^{3/4}}$$

Rubi [F] time = 12.58, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+2(-1+k)x+kx^2}{\sqrt[4]{(1-x)x(1-kx)}\left(-1+(3+d)x-(3+dk)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 2*(-1 + k)*x + k*x^2)/(((1 - x)*x*(1 - k*x))^(1/4)*(-1 + (3 + d)*
x - (3 + d*k)*x^2 + x^3)),x]
```

```
[Out] (4*(1 - x)^(1/4)*x^(1/4)*(1 - k*x)^(1/4)*Defer[Subst][Defer[Int][x^2/((1 -
x^4)^(1/4)*(1 - k*x^4)^(1/4)*(1 - 3*(1 + d/3)*x^4 + 3*(1 + (d*k)/3)*x^8 - x
^12)), x], x, x^(1/4)])/((1 - x)*x*(1 - k*x))^(1/4) + (8*(1 - k)*(1 - x)^(1
/4)*x^(1/4)*(1 - k*x)^(1/4)*Defer[Subst][Defer[Int][x^6/((1 - x^4)^(1/4)*(1
 - k*x^4)^(1/4)*(1 - 3*(1 + d/3)*x^4 + 3*(1 + (d*k)/3)*x^8 - x^12)), x], x,
 x^(1/4)])/((1 - x)*x*(1 - k*x))^(1/4) + (4*k*(1 - x)^(1/4)*x^(1/4)*(1 - k*
x)^(1/4)*Defer[Subst][Defer[Int][x^10/((1 - x^4)^(1/4)*(1 - k*x^4)^(1/4)*(-
1 + 3*(1 + d/3)*x^4 - 3*(1 + (d*k)/3)*x^8 + x^12)), x], x, x^(1/4)])/((1 -
x)*x*(1 - k*x))^(1/4)
```

Rubi steps

$$\int \frac{-1+2(-1+k)x+kx^2}{\sqrt[4]{(1-x)x(1-kx)}\left(-1+(3+d)x-(3+dk)x^2+x^3\right)}\,dx=\frac{\left(\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\int\frac{-1+2(-1+k)x+kx^2}{\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\left(-1+(3+d)x-\cdots\right.}}{\sqrt[4]{(1-x)x(1-kx)}}$$

$$=\frac{\left(4\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\text{Subst}\left(\int\frac{x^2\left(-1+2\cdots\right.}{\sqrt[4]{1-x^4}\sqrt[4]{1-kx^4}\left(-1\cdots\right.}\right.}{\sqrt[4]{(1-x)x(1-kx)}}$$

$$=\frac{\left(4\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\text{Subst}\left(\int\left(\frac{\cdots}{\sqrt[4]{1-x^4}\sqrt[4]{1-kx^4}\left(1\cdots\right.}\right.\right.}{\cdots}$$

$$=\frac{\left(4\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\text{Subst}\left(\int\frac{\cdots}{\sqrt[4]{1-x^4}\sqrt[4]{1-kx^4}\left(1-\cdots\right.}\right.}{\sqrt[4]{(1-x)x(1-kx}\cdots}$$

Mathematica [F] time = 3.32, size = 0, normalized size = 0.00

$$\int \frac{-1+2(-1+k)x+kx^2}{\sqrt[4]{(1-x)x(1-kx)}\left(-1+(3+d)x-(3+dk)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + 2*(-1 + k)*x + k*x^2)/(((1 - x)*x*(1 - k*x))^(1/4)*(-1 + (3
 + d)*x - (3 + d*k)*x^2 + x^3)),x]
```

```
[Out] Integrate[(-1 + 2*(-1 + k)*x + k*x^2)/(((1 - x)*x*(1 - k*x))^(1/4)*(-1 + (3
 + d)*x - (3 + d*k)*x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.25, size = 79, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{x-1}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{x-1}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*(-1 + k)*x + k*x^2)/(((1 - x)*x*(1 - k*x))^(1/4)
*(-1 + (3 + d)*x - (3 + d*k)*x^2 + x^3)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(x + (-1 - k)*x^2 + k*x^3)^(1/4))/(-1 + x)])/d^(3/4) - (
2*ArcTanh[(d^(1/4)*(x + (-1 - k)*x^2 + k*x^3)^(1/4))/(-1 + x)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(1/4)/(-1+(3+d)*x-(d*k+3
)*x^2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{kx^2+2(k-1)x-1}{((kx-1)(x-1)x)^{\frac{1}{4}}\left((dk+3)x^2-x^3-(d+3)x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(1/4)/(-1+(3+d)*x-(d*k+3
)*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate(-(k*x^2 + 2*(k - 1)*x - 1)/(((k*x - 1)*(x - 1)*x)^(1/4)*((d*k + 3
)*x^2 - x^3 - (d + 3)*x + 1)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{-1+2(-1+k)x+kx^2}{((1-x)x(-kx+1))^{\frac{1}{4}}\left(-1+(3+d)x-(dk+3)x^2+x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(1/4)/(-1+(3+d)*x-(d*k+3)*x^2+
x^3),x)
```

```
[Out] int((-1+2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(1/4)/(-1+(3+d)*x-(d*k+3)*x^2+
x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{kx^2+2(k-1)x-1}{((kx-1)(x-1)x)^{\frac{1}{4}}\left((dk+3)x^2-x^3-(d+3)x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(1/4)/(-1+(3+d)*x-(d*k+3
)*x^2+x^3),x, algorithm="maxima")
```

```
[Out] -integrate((k*x^2 + 2*(k - 1)*x - 1)/(((k*x - 1)*(x - 1)*x)^(1/4)*((d*k + 3
)*x^2 - x^3 - (d + 3)*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x\,\left(k-1\right)+k\,x^2-1}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/4}\,\left(x^3+\left(-d\,k-3\right)\,x^2+\left(d+3\right)\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x*(k - 1) + k*x^2 - 1)/((x*(k*x - 1)*(x - 1))^(1/4)*(x*(d + 3) - x^2
*(d*k + 3) + x^3 - 1)),x)
```

```
[Out] int((2*x*(k - 1) + k*x^2 - 1)/((x*(k*x - 1)*(x - 1))^(1/4)*(x*(d + 3) - x^2
*(d*k + 3) + x^3 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*(-1+k)*x+k*x**2)/((1-x)*x*(-k*x+1))**(1/4)/(-1+(3+d)*x-(d*k
+3)*x**2+x**3),x)
```

```
[Out] Timed out
```

3.952 $\int \frac{1+x^4}{\sqrt{-x+x^3}\left(-1+x^4\right)}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{x^3-x}}{1-x^2}-\frac{1}{4}\tan^{-1}\left(\frac{2\sqrt{x^3-x}}{x^2-2x-1}\right)-\frac{1}{4}\tanh^{-1}\left(\frac{\frac{x^2}{2}+x-\frac{1}{2}}{\sqrt{x^3-x}}\right)$$

Rubi [C] time = 0.68, antiderivative size = 119, normalized size of antiderivative = 1.51, number of steps used = 18, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2056, 6715, 6725, 222, 1404, 414, 523, 409, 1211, 1699, 206, 203}

$$-\frac{x}{\sqrt{x^3-x}}-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\sqrt{x^2-1}\sqrt{x}\tan^{-1}\left(\frac{(1+i)\sqrt{x}}{\sqrt{x^2-1}}\right)}{\sqrt{x^3-x}}-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\sqrt{x^2-1}\sqrt{x}\tanh^{-1}\left(\frac{(1+i)\sqrt{x}}{\sqrt{x^2-1}}\right)}{\sqrt{x^3-x}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(Sqrt[-x + x^3]*(-1 + x^4)),x]
```

```
[Out] -(x/Sqrt[-x + x^3]) - ((1/4 - I/4)*Sqrt[x]*Sqrt[-1 + x^2]*ArcTan[((1 + I)*S
qrt[x])/Sqrt[-1 + x^2]])/Sqrt[-x + x^3] - ((1/4 - I/4)*Sqrt[x]*Sqrt[-1 + x^
2]*ArcTanh[((1 + I)*Sqrt[x])/Sqrt[-1 + x^2]])/Sqrt[-x + x^3]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
```

```
d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1404

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^4}{\sqrt{-x+x^3}\left(-1+x^4\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1+x^4}{\sqrt{x}\sqrt{-1+x^2}\left(-1+x^4\right)}\,dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1+x^8}{\sqrt{-1+x^4}\left(-1+x^8\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{-x+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt{-1+x^4}} + \frac{2}{\sqrt{-1+x^4}\left(-1+x^8\right)}\right) dx, x, \sqrt{x}\right)}{\sqrt{-x+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{-x+x^3}} + \frac{\left(4\sqrt{x}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^4}\,(}\right.}{\sqrt{-x+x^3}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} + \frac{\left(4\sqrt{x}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\left(-1+x^4\right)^{3/2}}\right.}{\sqrt{-x+x^3}}$$

$$= -\frac{x}{\sqrt{-x+x^3}} + \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} + \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\text{Subs}}{\sqrt{}}$$

$$= -\frac{x}{\sqrt{-x+x^3}} + \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} - \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\text{Subs}}{\sqrt{-x}}$$

$$= -\frac{x}{\sqrt{-x+x^3}} - \frac{\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x+x^3}} + \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F}{\sqrt{-x+}}$$

$$= -\frac{x}{\sqrt{-x+x^3}} - \frac{\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x+x^3}} + \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F}{\sqrt{-x+}}$$

$$= -\frac{x}{\sqrt{-x+x^3}} - \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-2ix^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{-1+x^2}}\right)}{2\sqrt{-x+x^3}} - \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\text{Sul}}{2}$$

$$= -\frac{x}{\sqrt{-x+x^3}} - \frac{\left(\frac{1}{4}-\frac{i}{4}\right)\sqrt{x}\sqrt{-1+x^2}\,\tan^{-1}\left(\frac{(1+i)\sqrt{x}}{\sqrt{-1+x^2}}\right)}{\sqrt{-x+x^3}} - \frac{\left(\frac{1}{4}-\frac{i}{4}\right)\sqrt{x}\sqrt{-1+x^2}\,\tanh^{-}}{\sqrt{-x+x^3}}$$

Mathematica [C] time = 0.25, size = 63, normalized size = 0.80

$$\frac{x\left(x^2-1\right)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};x^2,-x^2\right)}{\sqrt{1-x^2}\sqrt{x\left(x^2-1\right)}} - \frac{x}{\sqrt{x\left(x^2-1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^4)/(Sqrt[-x + x^3]*(-1 + x^4)),x]
```

```
[Out] -(x/Sqrt[x*(-1 + x^2)]) + (x*(-1 + x^2)*AppellF1[1/4, -1/2, 1, 5/4, x^2, -x
^2])/(Sqrt[1 - x^2]*Sqrt[x*(-1 + x^2)])
```

IntegrateAlgebraic [A] time = 0.28, size = 79, normalized size = 1.00

$$\frac{\sqrt{x^3-x}}{1-x^2} - \frac{1}{4}\tan^{-1}\left(\frac{2\sqrt{x^3-x}}{x^2-2x-1}\right) - \frac{1}{4}\tanh^{-1}\left(\frac{\frac{x^2}{2}+x-\frac{1}{2}}{\sqrt{x^3-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)/(Sqrt[-x + x^3]*(-1 + x^4)),x]
```

```
[Out] Sqrt[-x + x^3]/(1 - x^2) - ArcTan[(2*Sqrt[-x + x^3])/(-1 - 2*x + x^2)]/4 -
ArcTanh[(-1/2 + x + x^2/2)/Sqrt[-x + x^3]]/4
```

fricas [A] time = 0.49, size = 105, normalized size = 1.33

$$\frac{2\left(x^{2}-1\right)\arctan\left(\frac{x^{2}-2\,x-1}{2\,\sqrt{x^{3}-x}}\right)+\left(x^{2}-1\right)\log\left(\frac{x^{4}+8\,x^{3}+2\,x^{2}-4\,\sqrt{x^{3}-x}\left(x^{2}+2\,x-1\right)-8\,x+1}{x^{4}+2\,x^{2}+1}\right)-8\,\sqrt{x^{3}-x}}{8\left(x^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^3-x)^(1/2)/(x^4-1),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(x^2 - 1)*arctan(1/2*(x^2 - 2*x - 1)/sqrt(x^3 - x)) + (x^2 - 1)*log(
(x^4 + 8*x^3 + 2*x^2 - 4*sqrt(x^3 - x)*(x^2 + 2*x - 1) - 8*x + 1)/(x^4 + 2*
x^2 + 1)) - 8*sqrt(x^3 - x))/(x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}+1}{\left(x^{4}-1\right)\sqrt{x^{3}-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^3-x)^(1/2)/(x^4-1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/((x^4 - 1)*sqrt(x^3 - x)), x)
```

maple [C] time = 0.05, size = 251, normalized size = 3.18

$$\frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\sqrt{1+x},\frac{\sqrt{2}}{2}\right)}{2\sqrt{x^{3}-x}}-\frac{x^{2}+x}{2\sqrt{(-1+x)\left(x^{2}+x\right)}}+\frac{x^{2}-x}{2\sqrt{(1+x)\left(x^{2}-x\right)}}-\frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\operatorname{EllipticPi}\left(\sqrt{1+x},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^{3}-x}}-\frac{i\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\operatorname{EllipticPi}\left(\sqrt{1+x},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^{3}-x}}-\frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\operatorname{EllipticPi}\left(\sqrt{1+x},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^{3}-x}}+\frac{i\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\operatorname{EllipticPi}\left(\sqrt{1+x},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^{3}-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^3-x)^(1/2)/(x^4-1),x)
```

```
[Out] 1/2*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticF((1+x)^(1/2
),1/2*2^(1/2))-1/2*(x^2+x)/((-1+x)*(x^2+x))^(1/2)+1/2*(x^2-x)/((1+x)*(x^2-x
))^(1/2)-1/4*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticPi(
(1+x)^(1/2),1/2-1/2*I,1/2*2^(1/2))-1/4*I*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/
2)/(x^3-x)^(1/2)*EllipticPi((1+x)^(1/2),1/2-1/2*I,1/2*2^(1/2))-1/4*(1+x)^(1
/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticPi((1+x)^(1/2),1/2+1/2*I
,1/2*2^(1/2))+1/4*I*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*Elli
pticPi((1+x)^(1/2),1/2+1/2*I,1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}+1}{\left(x^{4}-1\right)\sqrt{x^{3}-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^3-x)^(1/2)/(x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/((x^4 - 1)*sqrt(x^3 - x)), x)
```

mupad [B] time = 0.76, size = 232, normalized size = 2.94

$$\frac{\sqrt{-x}\,\left(\frac{\sin\left(2\,\mathrm{asin}\left(\sqrt{-x}\right)\right)}{4\,\sqrt{1-x}}+\frac{\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{2}\right)\,\sqrt{1-x}\,\sqrt{x+1}}{\sqrt{x^3-x}}+\frac{\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(-\mathrm{i};\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{\sqrt{x^3-x}}+\frac{\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(1\mathrm{i};\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{\sqrt{x^3-x}}-\frac{2\,\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{\sqrt{x^3-x}}+\frac{\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)-\frac{\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{2}+\frac{\sqrt{-x}\,\sqrt{1-x}}{2\,\sqrt{x+1}}\right)}{\sqrt{x^3-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/((x^3 - x)^(1/2)*(x^4 - 1)),x)
```

```
[Out] ((-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticPi(-1i, asin((-x)^(1/2)), -
1))/(x^3 - x)^(1/2) - (2*(-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticF(a
sin((-x)^(1/2)), -1))/(x^3 - x)^(1/2) + ((-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(
1/2)*ellipticPi(1i, asin((-x)^(1/2)), -1))/(x^3 - x)^(1/2) + ((-x)^(1/2)*(s
in(2*asin((-x)^(1/2)))/(4*(1 - x)^(1/2)) + ellipticE(asin((-x)^(1/2)), -1)/
2)*(1 - x)^(1/2)*(x + 1)^(1/2))/(x^3 - x)^(1/2) + ((-x)^(1/2)*(1 - x)^(1/2)
*(x + 1)^(1/2)*(ellipticF(asin((-x)^(1/2)), -1) - ellipticE(asin((-x)^(1/2)
), -1)/2 + ((-x)^(1/2)*(1 - x)^(1/2))/(2*(x + 1)^(1/2))))/(x^3 - x)^(1/2)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\sqrt{x(x-1)(x+1)}\,(x-1)(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**3-x)**(1/2)/(x**4-1),x)
```

```
[Out] Integral((x**4 + 1)/(sqrt(x*(x - 1)*(x + 1))*(x - 1)*(x + 1)*(x**2 + 1)), x
)
```

3.953 $\int \frac{(2+x^2)\sqrt{4-5x^2+x^4}}{x^2(-2+2x+x^2)}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{x^4-5x^2+4}}{x}-4\tanh^{-1}\left(\frac{x^2+x-2}{\sqrt{x^4-5x^2+4}}\right)+2\sqrt{3}\,\tanh^{-1}\left(\frac{\sqrt{x^4-5x^2+4}}{\sqrt{3}\,(x^2+x-2)}\right)$$

Rubi [C] time = 2.14, antiderivative size = 938, normalized size of antiderivative = 11.87, number of steps used = 57, number of rules used = 21, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {6728, 1117, 1183, 1096, 1182, 1114, 734, 843, 621, 206, 724, 1728, 1208, 1214, 1456, 540, 421, 420, 538, 537, 1247}

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Int[((2 + x^2)*Sqrt[4 - 5*x^2 + x^4])/(x^2*(-2 + 2*x + x^2)),x]
```

```
[Out] -1/2*Sqrt[4 - 5*x^2 + x^4] + ((1 - Sqrt[3])*Sqrt[4 - 5*x^2 + x^4])/4 + ((1
+ Sqrt[3])*Sqrt[4 - 5*x^2 + x^4])/4 + Sqrt[4 - 5*x^2 + x^4]/x + ArcTanh[(8
- 5*x^2)/(4*Sqrt[4 - 5*x^2 + x^4])] - (5*ArcTanh[(5 - 2*x^2)/(2*Sqrt[4 - 5*
x^2 + x^4])])/4 + ((9 - Sqrt[3])*ArcTanh[(5 - 2*x^2)/(2*Sqrt[4 - 5*x^2 + x^
4])])/8 + ((9 + Sqrt[3])*ArcTanh[(5 - 2*x^2)/(2*Sqrt[4 - 5*x^2 + x^4])])/8
+ (Sqrt[3]*ArcTanh[(2*(6 - 5*Sqrt[3]) - (3 - 4*Sqrt[3])*x^2)/(2*Sqrt[6*(2 -
 Sqrt[3])]*Sqrt[4 - 5*x^2 + x^4])])/2 - (Sqrt[3]*ArcTanh[(2*(6 + 5*Sqrt[3])
 - (3 + 4*Sqrt[3])*x^2)/(2*Sqrt[6*(2 + Sqrt[3])]*Sqrt[4 - 5*x^2 + x^4])])/2
 + ((2 - Sqrt[3])*Sqrt[4 - x^2]*Sqrt[-1 + x^2]*EllipticF[ArcCos[x/2], 4/3])
/Sqrt[4 - 5*x^2 + x^4] - ((2 + Sqrt[3])*Sqrt[4 - x^2]*Sqrt[-1 + x^2]*Ellipt
icF[ArcCos[x/2], 4/3])/Sqrt[4 - 5*x^2 + x^4] + ((2 + x^2)*Sqrt[(4 - 5*x^2 +
 x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 9/8])/(2*Sqrt[2]*Sqrt[4 -
 5*x^2 + x^4]) + ((1 - 2*Sqrt[3])*(2 + x^2)*Sqrt[(4 - 5*x^2 + x^4)/(2 + x^2
)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 9/8])/(2*Sqrt[2]*Sqrt[4 - 5*x^2 + x^4])
 + (Sqrt[3/2]*(2 - Sqrt[3])*(2 + x^2)*Sqrt[(4 - 5*x^2 + x^4)/(2 + x^2)^2]*E
llipticF[2*ArcTan[x/Sqrt[2]], 9/8])/(2*Sqrt[4 - 5*x^2 + x^4]) - (Sqrt[3/2]*
(2 + Sqrt[3])*(2 + x^2)*Sqrt[(4 - 5*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*Arc
Tan[x/Sqrt[2]], 9/8])/(2*Sqrt[4 - 5*x^2 + x^4]) + ((1 + 2*Sqrt[3])*(2 + x^2
)*Sqrt[(4 - 5*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 9/8])/
(2*Sqrt[2]*Sqrt[4 - 5*x^2 + x^4]) - (3*Sqrt[1 - x^2]*Sqrt[4 - x^2]*Elliptic
Pi[(2 - Sqrt[3])/2, ArcSin[x], 1/4])/(2*Sqrt[4 - 5*x^2 + x^4]) - (3*Sqrt[1
- x^2]*Sqrt[4 - x^2]*EllipticPi[(2 + Sqrt[3])/2, ArcSin[x], 1/4])/(2*Sqrt[4
 - 5*x^2 + x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 420

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> -
Simp[EllipticF[ArcCos[Rt[-(d/c), 2]*x], (b*c)/(b*c - a*d)]/(Sqrt[c]*Rt[-(d/
c), 2]*Sqrt[a - (b*c)/d]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ
[c, 0] && GtQ[a - (b*c)/d, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 540

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x
] + Dist[(b*c - a*d)/b, Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1096

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
```

```
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1117

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^p)/(d*(m + 1)), x] - Dist[(2*p)
/(d^2*(m + 1)), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 -
4*a*c, 0] && GtQ[c/a, 0] && LtQ[b/a, 0]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c
/a, 0] && LtQ[b/a, 0]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1728

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbo
l] :> Dist[d, Int[(a + b*x^2 + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, I
nt[(x*(a + b*x^2 + c*x^4)^p)/(d^2 - e^2*x^2), x], x] /; FreeQ[{a, b, c, d,
e}, x] && IntegerQ[p + 1/2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(2+x^2\right)\sqrt{4-5x^2+x^4}}{x^2\left(-2+2x+x^2\right)}\,dx = \int\left(-\frac{\sqrt{4-5x^2+x^4}}{x^2}-\frac{\sqrt{4-5x^2+x^4}}{x}+\frac{(4+x)\sqrt{4-5x^2+x^4}}{-2+2x+x^2}\right)dx$$

$$= -\int\frac{\sqrt{4-5x^2+x^4}}{x^2}\,dx-\int\frac{\sqrt{4-5x^2+x^4}}{x}\,dx+\int\frac{(4+x)\sqrt{4-5x^2+x^4}}{-2+2x+x^2}\,dx$$

$$= \frac{\sqrt{4-5x^2+x^4}}{x}-\frac{1}{2}\text{Subst}\left(\int\frac{\sqrt{4-5x+x^2}}{x}\,dx,x,x^2\right)-\int\frac{-5+2x^2}{\sqrt{4-5x^2+x^4}}\,dx$$

$$= -\frac{1}{2}\sqrt{4-5x^2+x^4}+\frac{\sqrt{4-5x^2+x^4}}{x}+\frac{1}{4}\text{Subst}\left(\int\frac{-8+5x}{x\sqrt{4-5x+x^2}}\,dx,x,x^2\right)$$

$$= -\frac{1}{2}\sqrt{4-5x^2+x^4}+\frac{\sqrt{4-5x^2+x^4}}{x}-\frac{2x\sqrt{4-5x^2+x^4}}{2+x^2}+\frac{2\sqrt{2}\left(2+x^2\right)\sqrt{\cdots}}{\sqrt{4\cdots}}$$

$$= -\frac{1}{2}\sqrt{4-5x^2+x^4}+\frac{\sqrt{4-5x^2+x^4}}{x}-\frac{2x\sqrt{4-5x^2+x^4}}{2+x^2}+\frac{2\sqrt{2}\left(2+x^2\right)\sqrt{\cdots}}{\sqrt{4\cdots}}$$

$$= -\frac{1}{2}\sqrt{4-5x^2+x^4}+\frac{1}{4}\left(1-\sqrt{3}\right)\sqrt{4-5x^2+x^4}+\frac{1}{4}\left(1+\sqrt{3}\right)\sqrt{4-5x^2+x^4}$$

$$= -\frac{1}{2}\sqrt{4-5x^2+x^4}+\frac{1}{4}\left(1-\sqrt{3}\right)\sqrt{4-5x^2+x^4}+\frac{1}{4}\left(1+\sqrt{3}\right)\sqrt{4-5x^2+x^4}$$

$$= -\frac{1}{2}\sqrt{4-5x^2+x^4}+\frac{1}{4}\left(1-\sqrt{3}\right)\sqrt{4-5x^2+x^4}+\frac{1}{4}\left(1+\sqrt{3}\right)\sqrt{4-5x^2+x^4}$$

$$= -\frac{1}{2}\sqrt{4-5x^2+x^4}+\frac{1}{4}\left(1-\sqrt{3}\right)\sqrt{4-5x^2+x^4}+\frac{1}{4}\left(1+\sqrt{3}\right)\sqrt{4-5x^2+x^4}$$

$$= -\frac{1}{2}\sqrt{4-5x^2+x^4}+\frac{1}{4}\left(1-\sqrt{3}\right)\sqrt{4-5x^2+x^4}+\frac{1}{4}\left(1+\sqrt{3}\right)\sqrt{4-5x^2+x^4}$$

$$= -\frac{1}{2}\sqrt{4-5x^2+x^4}+\frac{1}{4}\left(1-\sqrt{3}\right)\sqrt{4-5x^2+x^4}+\frac{1}{4}\left(1+\sqrt{3}\right)\sqrt{4-5x^2+x^4}$$

Mathematica [C] time = 0.91, size = 390, normalized size = 4.94

$$\frac{2x^4-10x^2+3\sqrt{1-x^2}\sqrt{4-x^2}\,xF\left(\sin^{-1}(x)|\frac{1}{4}\right)-3\sqrt{1-x^2}\sqrt{4-x^2}\,x\Pi\left(1-\frac{\sqrt{3}}{2};\sin^{-1}(x)|\frac{1}{4}\right)-3\sqrt{1-x^2}\sqrt{4-x^2}\,x\Pi\left(1+\frac{\sqrt{3}}{2};\sin^{-1}(x)|\frac{1}{4}\right)+2\sqrt{x^4-5x^2+4}\,x\tanh^{-1}\left(\frac{8-5x^2}{4\sqrt{x^4-5x^2+4}}\right)-2\sqrt{x^4-5x^2+4}\,x\tanh^{-1}\left(\frac{2x^2-5}{2\sqrt{x^4-5x^2+4}}\right)-\sqrt{3}\sqrt{x^4-5x^2+4}\,x\tanh^{-1}\left(\frac{2\left(6+5\sqrt{3}\right)-\left(3+4\sqrt{3}\right)x^2}{2\sqrt{6\left(2+\sqrt{3}\right)}\sqrt{x^4-5x^2+4}}\right)+\sqrt{3}\sqrt{x^4-5x^2+4}\,x\tanh^{-1}\left(\frac{\left(4\sqrt{3}-3\right)x^2-10\sqrt{3}+12}{2\sqrt{6}\sqrt{-\left(\left(\sqrt{3}-2\right)\left(x^4-5x^2+4\right)\right)}}\right)+8}{2x\sqrt{x^4-5x^2+4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + x^2)*Sqrt[4 - 5*x^2 + x^4])/(x^2*(-2 + 2*x + x^2)),x]
```

```
[Out] (8 - 10*x^2 + 2*x^4 + 2*x*Sqrt[4 - 5*x^2 + x^4]*ArcTanh[(8 - 5*x^2)/(4*Sqrt
[4 - 5*x^2 + x^4])] - 2*x*Sqrt[4 - 5*x^2 + x^4]*ArcTanh[(-5 + 2*x^2)/(2*Sqr
t[4 - 5*x^2 + x^4])] - Sqrt[3]*x*Sqrt[4 - 5*x^2 + x^4]*ArcTanh[(2*(6 + 5*Sq
rt[3]) - (3 + 4*Sqrt[3])*x^2)/(2*Sqrt[6*(2 + Sqrt[3])]*Sqrt[4 - 5*x^2 + x^4
])] + Sqrt[3]*x*Sqrt[4 - 5*x^2 + x^4]*ArcTanh[(12 - 10*Sqrt[3] + (-3 + 4*Sq
rt[3])*x^2)/(2*Sqrt[6]*Sqrt[-((-2 + Sqrt[3])*(4 - 5*x^2 + x^4))])] + 3*x*Sq
rt[1 - x^2]*Sqrt[4 - x^2]*EllipticF[ArcSin[x], 1/4] - 3*x*Sqrt[1 - x^2]*Sqr
t[4 - x^2]*EllipticPi[1 - Sqrt[3]/2, ArcSin[x], 1/4] - 3*x*Sqrt[1 - x^2]*Sq
rt[4 - x^2]*EllipticPi[1 + Sqrt[3]/2, ArcSin[x], 1/4])/(2*x*Sqrt[4 - 5*x^2
+ x^4])
```

IntegrateAlgebraic [A] time = 0.59, size = 79, normalized size = 1.00

$$\frac{\sqrt{x^4-5x^2+4}}{x}-4\tanh^{-1}\left(\frac{x^2+x-2}{\sqrt{x^4-5x^2+4}}\right)+2\sqrt{3}\,\tanh^{-1}\left(\frac{\sqrt{x^4-5x^2+4}}{\sqrt{3}\left(x^2+x-2\right)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^2)*Sqrt[4 - 5*x^2 + x^4])/(x^2*(-2 + 2*x + x^2))
,x]
```

```
[Out] Sqrt[4 - 5*x^2 + x^4]/x - 4*ArcTanh[(-2 + x + x^2)/Sqrt[4 - 5*x^2 + x^4]] +
 2*Sqrt[3]*ArcTanh[Sqrt[4 - 5*x^2 + x^4]/(Sqrt[3]*(-2 + x + x^2))]
```

fricas [A] time = 0.58, size = 115, normalized size = 1.46

$$\frac{\sqrt{3}\,x\log\left(-\frac{7\,x^4+4\,x^3+2\,\sqrt{3}\,\sqrt{x^4-5\,x^2+4}\left(2\,x^2+x-4\right)-30\,x^2-8\,x+28}{x^4+4\,x^3-8\,x+4}\right)+4\,x\log\left(\frac{x^2-\sqrt{x^4-5\,x^2+4}-2}{x}\right)+2\,\sqrt{x^4-5\,x^2+4}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)*(x^4-5*x^2+4)^(1/2)/x^2/(x^2+2*x-2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(sqrt(3)*x*log(-(7*x^4 + 4*x^3 + 2*sqrt(3)*sqrt(x^4 - 5*x^2 + 4)*(2*x^2
 + x - 4) - 30*x^2 - 8*x + 28)/(x^4 + 4*x^3 - 8*x + 4)) + 4*x*log((x^2 - sq
rt(x^4 - 5*x^2 + 4) - 2)/x) + 2*sqrt(x^4 - 5*x^2 + 4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4-5\,x^2+4}\left(x^2+2\right)}{\left(x^2+2\,x-2\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)*(x^4-5*x^2+4)^(1/2)/x^2/(x^2+2*x-2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 - 5*x^2 + 4)*(x^2 + 2)/((x^2 + 2*x - 2)*x^2), x)
```

maple [C] time = 0.07, size = 915, normalized size = 11.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)*(x^4-5*x^2+4)^(1/2)/x^2/(x^2+2*x-2),x)
```

```
[Out] 11/2*(-x^2+1)^(1/2)*(-x^2+4)^(1/2)/(x^4-5*x^2+4)^(1/2)*EllipticF(x,1/2)+9*l
n(2)-9/2*ln(2*x^2-5+2*(x^4-5*x^2+4)^(1/2))-4*(-x^2+1)^(1/2)*(-x^2+4)^(1/2)/
(x^4-5*x^2+4)^(1/2)*EllipticE(x,1/2)-3/2/(-3/2*3^(1/2)+3)^(1/2)*arctanh(3/2
/(-3/2*3^(1/2)+3)^(1/2)/(1/4*x^4-5/4*x^2+1)^(1/2)-5/4/(-3/2*3^(1/2)+3)^(1/2
)/(1/4*x^4-5/4*x^2+1)^(1/2)*3^(1/2)-3/8/(-3/2*3^(1/2)+3)^(1/2)/(1/4*x^4-5/4
*x^2+1)^(1/2)*x^2+1/2/(-3/2*3^(1/2)+3)^(1/2)/(1/4*x^4-5/4*x^2+1)^(1/2)*x^2*
3^(1/2))+3/2*(-x^2+1)^(1/2)*(-x^2+4)^(1/2)/(x^4-5*x^2+4)^(1/2)/(3^(1/2)-1)*
EllipticPi(x,1/(3^(1/2)-1)^2,1/2)+3/2*3^(1/2)/(-3/2*3^(1/2)+3)^(1/2)*arctan
h(3/2/(-3/2*3^(1/2)+3)^(1/2)/(1/4*x^4-5/4*x^2+1)^(1/2)-5/4/(-3/2*3^(1/2)+3)
^(1/2)/(1/4*x^4-5/4*x^2+1)^(1/2)*3^(1/2)-3/8/(-3/2*3^(1/2)+3)^(1/2)/(1/4*x^
4-5/4*x^2+1)^(1/2)*x^2+1/2/(-3/2*3^(1/2)+3)^(1/2)/(1/4*x^4-5/4*x^2+1)^(1/2)
*x^2*3^(1/2))-3/2*3^(1/2)*(-x^2+1)^(1/2)*(-x^2+4)^(1/2)/(x^4-5*x^2+4)^(1/2)
/(3^(1/2)-1)*EllipticPi(x,1/(3^(1/2)-1)^2,1/2)+3/2*3^(1/2)/(3/2*3^(1/2)+3)^
(1/2)*arctanh(-3/2/(3/2*3^(1/2)+3)^(1/2)/(1/4*x^4-5/4*x^2+1)^(1/2)-5/4/(3/2
*3^(1/2)+3)^(1/2)/(1/4*x^4-5/4*x^2+1)^(1/2)*3^(1/2)+3/8/(3/2*3^(1/2)+3)^(1/
2)/(1/4*x^4-5/4*x^2+1)^(1/2)*x^2+1/2/(3/2*3^(1/2)+3)^(1/2)/(1/4*x^4-5/4*x^2
+1)^(1/2)*x^2*3^(1/2))+3/2*3^(1/2)*(-x^2+1)^(1/2)*(-x^2+4)^(1/2)/(x^4-5*x^2
+4)^(1/2)/(-1-3^(1/2))*EllipticPi(x,1/(-1-3^(1/2))^2,1/2)+3/2/(3/2*3^(1/2)+
3)^(1/2)*arctanh(-3/2/(3/2*3^(1/2)+3)^(1/2)/(1/4*x^4-5/4*x^2+1)^(1/2)-5/4/(
3/2*3^(1/2)+3)^(1/2)/(1/4*x^4-5/4*x^2+1)^(1/2)*3^(1/2)+3/8/(3/2*3^(1/2)+3)^
(1/2)/(1/4*x^4-5/4*x^2+1)^(1/2)*x^2+1/2/(3/2*3^(1/2)+3)^(1/2)/(1/4*x^4-5/4*
x^2+1)^(1/2)*x^2*3^(1/2))+3/2*(-x^2+1)^(1/2)*(-x^2+4)^(1/2)/(x^4-5*x^2+4)^(
1/2)/(-1-3^(1/2))*EllipticPi(x,1/(-1-3^(1/2))^2,1/2)+(x^4-5*x^2+4)^(1/2)/x-
4*(-x^2+1)^(1/2)*(-x^2+4)^(1/2)/(x^4-5*x^2+4)^(1/2)*(EllipticF(x,1/2)-Ellip
ticE(x,1/2))+5/4*ln(-5/2+x^2+(x^4-5*x^2+4)^(1/2))+arctanh(1/4*(-5*x^2+8)/(x
^4-5*x^2+4)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4-5\,x^2+4}\,\left(x^2+2\right)}{\left(x^2+2\,x-2\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)*(x^4-5*x^2+4)^(1/2)/x^2/(x^2+2*x-2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(x^4 - 5*x^2 + 4)*(x^2 + 2)/((x^2 + 2*x - 2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+2\right)\,\sqrt{x^4-5\,x^2+4}}{x^2\,\left(x^2+2\,x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 2)*(x^4 - 5*x^2 + 4)^(1/2))/(x^2*(2*x + x^2 - 2)),x)
```

```
[Out] int(((x^2 + 2)*(x^4 - 5*x^2 + 4)^(1/2))/(x^2*(2*x + x^2 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-2)(x-1)(x+1)(x+2)}\,\left(x^2+2\right)}{x^2\left(x^2+2x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)*(x**4-5*x**2+4)**(1/2)/x**2/(x**2+2*x-2),x)
```

```
[Out] Integral(sqrt((x - 2)*(x - 1)*(x + 1)*(x + 2))*(x**2 + 2)/(x**2*(x**2 + 2*x
 - 2)), x)
```

3.954 $\int \frac{x^2}{(-1+x^4)\sqrt[4]{x^2+x^4}}\,dx$

Optimal. Leaf size=79

$$\frac{(x^4+x^2)^{3/4}}{x(x^2+1)}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^2}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^2}}\right)}{2\sqrt[4]{2}}$$

Rubi [C] time = 0.17, antiderivative size = 45, normalized size of antiderivative = 0.57, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2056, 1254, 466, 510}

$$-\frac{2x^3\,{}_2F_1\left(1,\frac{5}{4};\frac{9}{4};\frac{2x^2}{x^2+1}\right)}{5(x^2+1)\sqrt[4]{x^4+x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-1 + x^4)*(x^2 + x^4)^(1/4)),x]
```

```
[Out] (-2*x^3*Hypergeometric2F1[1, 5/4, 9/4, (2*x^2)/(1 + x^2)])/(5*(1 + x^2)*(x^
2 + x^4)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1254

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e)^p, x] /
; FreeQ[{a, c, d, e, f, q, m, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p
]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{x^2}{\left(-1+x^4\right)\sqrt[4]{x^2+x^4}}\,dx = \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^2}\right)\int \frac{x^{3/2}}{\sqrt[4]{1+x^2}\left(-1+x^4\right)}\,dx}{\sqrt[4]{x^2+x^4}}$$
$$= \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^2}\right)\int \frac{x^{3/2}}{\left(-1+x^2\right)\left(1+x^2\right)^{5/4}}\,dx}{\sqrt[4]{x^2+x^4}}$$
$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^2}\right)\mathrm{Subst}\left(\int \frac{x^4}{\left(-1+x^4\right)\left(1+x^4\right)^{5/4}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^4}}$$
$$= -\frac{2x^3\,{}_2F_1\left(1,\frac{5}{4};\frac{9}{4};\frac{2x^2}{1+x^2}\right)}{5\left(1+x^2\right)\sqrt[4]{x^2+x^4}}$$

Mathematica [A] time = 0.32, size = 68, normalized size = 0.86

$$\frac{x\left(2^{3/4}\sqrt[4]{\frac{1}{x^2}+1}\left(\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{x^2}+1}}{\sqrt[4]{2}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{x^2}+1}}{\sqrt[4]{2}}\right)\right)+4\right)}{4\sqrt[4]{x^4+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-1 + x^4)*(x^2 + x^4)^(1/4)),x]
```

```
[Out] (x*(4 + 2^(3/4)*(1 + x^(-2))^(1/4)*(ArcTan[(1 + x^(-2))^(1/4)/2^(1/4)] - Ar
cTanh[(1 + x^(-2))^(1/4)/2^(1/4)])))/(4*(x^2 + x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.25, size = 79, normalized size = 1.00

$$\frac{\left(x^4+x^2\right)^{3/4}}{x\left(x^2+1\right)}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^2}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^2}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-1 + x^4)*(x^2 + x^4)^(1/4)),x]
```

```
[Out] (x^2 + x^4)^(3/4)/(x*(1 + x^2)) - ArcTan[(2^(1/4)*x)/(x^2 + x^4)^(1/4)]/(2*
2^(1/4)) - ArcTanh[(2^(1/4)*x)/(x^2 + x^4)^(1/4)]/(2*2^(1/4))
```

fricas [B] time = 1.74, size = 258, normalized size = 3.27

$$\frac{4\cdot 2^{\frac{3}{4}}\left(x^3+x\right)\arctan\left(\frac{4\cdot 2^{\frac{3}{4}}\left(x^4+x^2\right)^{\frac{1}{4}}x^2+2^{\frac{3}{4}}\left(2\cdot 2^{\frac{3}{4}}\sqrt{x^4+x^2}\,x+2^{\frac{1}{4}}\left(3x^3+x\right)\right)+4\cdot 2^{\frac{1}{4}}\left(x^4+x^2\right)^{\frac{3}{4}}}{2\left(x^3-x\right)}\right)-2^{\frac{3}{4}}\left(x^3+x\right)\log\left(\frac{4\sqrt{2}\left(x^4+x^2\right)^{\frac{1}{4}}x^2+2^{\frac{3}{4}}\left(3x^3+x\right)+4\cdot 2^{\frac{1}{4}}\sqrt{x^4+x^2}\,x+4\left(x^4+x^2\right)^{\frac{3}{4}}}{x^3-x}\right)+2^{\frac{3}{4}}\left(x^3+x\right)\log\left(\frac{4\sqrt{2}\left(x^4+x^2\right)^{\frac{1}{4}}x^2-2^{\frac{3}{4}}\left(3x^3+x\right)-4\cdot 2^{\frac{1}{4}}\sqrt{x^4+x^2}\,x+4\left(x^4+x^2\right)^{\frac{3}{4}}}{x^3-x}\right)+16\left(x^4+x^2\right)^{\frac{3}{4}}}{16\left(x^3+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-1)/(x^4+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/16*(4*2^(3/4)*(x^3 + x)*arctan(1/2*(4*2^(3/4)*(x^4 + x^2)^(1/4)*x^2 + 2^(
3/4)*(2*2^(3/4)*sqrt(x^4 + x^2)*x + 2^(1/4)*(3*x^3 + x)) + 4*2^(1/4)*(x^4 +
 x^2)^(3/4))/(x^3 - x)) - 2^(3/4)*(x^3 + x)*log((4*sqrt(2)*(x^4 + x^2)^(1/4
)*x^2 + 2^(3/4)*(3*x^3 + x) + 4*2^(1/4)*sqrt(x^4 + x^2)*x + 4*(x^4 + x^2)^(
3/4))/(x^3 - x)) + 2^(3/4)*(x^3 + x)*log((4*sqrt(2)*(x^4 + x^2)^(1/4)*x^2 -
```

```
2^(3/4)*(3*x^3 + x) - 4*2^(1/4)*sqrt(x^4 + x^2)*x + 4*(x^4 + x^2)^(3/4))/(
x^3 - x)) + 16*(x^4 + x^2)^(3/4))/(x^3 + x)
```

giac [A] time = 0.51, size = 63, normalized size = 0.80

$$\frac{1}{4}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)-\frac{1}{8}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)+\frac{1}{8}\cdot 2^{\frac{3}{4}}\log\left(\left|-2^{\frac{1}{4}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right|\right)+\frac{1}{\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-1)/(x^4+x^2)^(1/4),x, algorithm="giac")
```

```
[Out] 1/4*2^(3/4)*arctan(1/2*2^(3/4)*(1/x^2 + 1)^(1/4)) - 1/8*2^(3/4)*log(2^(1/4)
 + (1/x^2 + 1)^(1/4)) + 1/8*2^(3/4)*log(abs(-2^(1/4) + (1/x^2 + 1)^(1/4)))
+ 1/(1/x^2 + 1)^(1/4)
```

maple [C] time = 3.95, size = 241, normalized size = 3.05

$$\frac{x}{\left(x^2\left(x^2+1\right)\right)^{\frac{1}{4}}}-\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\ln\left(-\frac{\sqrt{x^4+x^2}\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\mathrm{RootOf}\left(\_Z^4-8\right)^2 x+2\,\mathrm{RootOf}\left(\_Z^4-8\right)^2\left(x^4+x^2\right)^{\frac{1}{4}}x^2-3\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)x^3-4\left(x^4+x^2\right)^{\frac{3}{4}}-\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)x}{(1+x)x(-1+x)}\right)}{8}-\frac{\mathrm{RootOf}\left(\_Z^4-8\right)\ln\left(\frac{\sqrt{x^4+x^2}\,\mathrm{RootOf}\left(\_Z^4-8\right)^3 x+2\,\mathrm{RootOf}\left(\_Z^4-8\right)^2\left(x^4+x^2\right)^{\frac{1}{4}}x^2+3\,\mathrm{RootOf}\left(\_Z^4-8\right)x^3+4\left(x^4+x^2\right)^{\frac{3}{4}}+\mathrm{RootOf}\left(\_Z^4-8\right)x}{(1+x)x(-1+x)}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4-1)/(x^4+x^2)^(1/4),x)
```

```
[Out] x/(x^2*(x^2+1))^(1/4)-1/8*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln(-((x^4+x^2)^(1/2
)*RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x+2*RootOf(_Z^4-8)^2*(x^4+
x^2)^(1/4)*x^2-3*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^3-4*(x^4+x^2)^(3/4)-RootOf
(_Z^2+RootOf(_Z^4-8)^2)*x)/(1+x)/x/(-1+x))-1/8*RootOf(_Z^4-8)*ln(((x^4+x^2)
^(1/2)*RootOf(_Z^4-8)^3*x+2*RootOf(_Z^4-8)^2*(x^4+x^2)^(1/4)*x^2+3*RootOf(_
Z^4-8)*x^3+4*(x^4+x^2)^(3/4)+RootOf(_Z^4-8)*x)/(1+x)/x/(-1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(x^3+x\right)x^{\frac{3}{2}}}{3\left(x^4-1\right)\left(x^2+1\right)^{\frac{1}{4}}}-\int\frac{8\left(x^2+1\right)^{\frac{3}{4}}x^{\frac{3}{2}}}{3\left(x^8-2x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-1)/(x^4+x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] -2/3*(x^3 + x)*x^(3/2)/((x^4 - 1)*(x^2 + 1)^(1/4)) - integrate(8/3*(x^2 + 1
)^(3/4)*x^(3/2)/(x^8 - 2*x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^2}{{\left(x^4+x^2\right)}^{1/4}\,\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + x^4)^(1/4)*(x^4 - 1)),x)
```

```
[Out] int(x^2/((x^2 + x^4)^(1/4)*(x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt[4]{x^2\left(x^2+1\right)}\,(x-1)\,(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4-1)/(x**4+x**2)**(1/4),x)
[Out] Integral(x**2/((x**2*(x**2 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.955 $\int \frac{(-2+x^6)\sqrt{-1+x^6}}{x^4(2+x^6)} dx$

Optimal. Leaf size=79

$$\frac{\sqrt{x^6-1}}{3x^3}+\frac{1}{3}\log\left(\sqrt{x^6-1}+x^3\right)+\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{x^6}{\sqrt{6}}+\frac{\sqrt{x^6-1}\,x^3}{\sqrt{6}}+\sqrt{\frac{2}{3}}\right)$$

Rubi [A] time = 0.09, antiderivative size = 65, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {575, 580, 523, 217, 206, 377}

$$\frac{\sqrt{x^6-1}}{3x^3}+\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)-\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-2 + x^6)*Sqrt[-1 + x^6])/(x^4*(2 + x^6)),x]
```

```
[Out] Sqrt[-1 + x^6]/(3*x^3) + ArcTanh[x^3/Sqrt[-1 + x^6]]/3 - Sqrt[2/3]*ArcTanh[
(Sqrt[3/2]*x^3)/Sqrt[-1 + x^6]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 575

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Di
st[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q*(e
+ f*x^(n/k))^r, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, f, p, q
, r}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rubi steps

$$\begin{aligned}\int \frac{(-2+x^6)\sqrt{-1+x^6}}{x^4(2+x^6)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(-2+x^2)\sqrt{-1+x^2}}{x^2(2+x^2)}\,dx, x, x^3\right)\\ &= \frac{\sqrt{-1+x^6}}{3x^3}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{-8+2x^2}{\sqrt{-1+x^2}\,(2+x^2)}\,dx, x, x^3\right)\\ &= \frac{\sqrt{-1+x^6}}{3x^3}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, x^3\right)-2\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}\,(2+x^2)}\right.\\ &= \frac{\sqrt{-1+x^6}}{3x^3}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^3}{\sqrt{-1+x^6}}\right)-2\operatorname{Subst}\left(\int \frac{1}{2-3x^2}\,dx, x, -\right.\\ &= \frac{\sqrt{-1+x^6}}{3x^3}+\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)-\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x^3}{\sqrt{-1+x^6}}\right)\end{aligned}$$

Mathematica [C] time = 0.10, size = 94, normalized size = 1.19

$$\frac{\sqrt{1-x^6}\,x^{12}F_1\left(\frac{3}{2};\frac{1}{2},1;\frac{5}{2};x^6,-\frac{x^6}{2}\right)+6x^6-4\sqrt{6-6x^6}\,x^3\sin^{-1}\left(\frac{\sqrt{3}x^3}{\sqrt{x^6+2}}\right)-6}{18x^3\sqrt{x^6-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-2 + x^6)*Sqrt[-1 + x^6])/(x^4*(2 + x^6)),x]
```

```
[Out] (-6 + 6*x^6 + x^12*Sqrt[1 - x^6]*AppellF1[3/2, 1/2, 1, 5/2, x^6, -1/2*x^6]
- 4*x^3*Sqrt[6 - 6*x^6]*ArcSin[(Sqrt[3]*x^3)/Sqrt[2 + x^6]])/(18*x^3*Sqrt[-
1 + x^6])
```

IntegrateAlgebraic [A] time = 0.36, size = 79, normalized size = 1.00

$$\frac{\sqrt{x^6-1}}{3x^3}+\frac{1}{3}\log\left(\sqrt{x^6-1}+x^3\right)+\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{x^6}{\sqrt{6}}+\frac{\sqrt{x^6-1}\,x^3}{\sqrt{6}}+\sqrt{\frac{2}{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^6)*Sqrt[-1 + x^6])/(x^4*(2 + x^6)),x]
```

```
[Out] Sqrt[-1 + x^6]/(3*x^3) + Sqrt[2/3]*ArcTanh[Sqrt[2/3] + x^6/Sqrt[6] + (x^3*S
qrt[-1 + x^6])/Sqrt[6]] + Log[x^3 + Sqrt[-1 + x^6]]/3
```

fricas [A] time = 0.43, size = 106, normalized size = 1.34

$$\frac{\sqrt{3}\sqrt{2}\,x^3\log\left(\frac{25\,x^6-2\,\sqrt{3}\,\sqrt{2}\,(5\,x^6-2)-2\,\sqrt{x^6-1}\,(5\,\sqrt{3}\,\sqrt{2}\,x^3-12\,x^3)-10}{x^6+2}\right)-2\,x^3\log\left(-x^3+\sqrt{x^6-1}\right)+2\,x^3+2\,\sqrt{x^6-1}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-1)^(1/2)/x^4/(x^6+2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(3)*sqrt(2)*x^3*log((25*x^6 - 2*sqrt(3)*sqrt(2)*(5*x^6 - 2) - 2*sq
rt(x^6 - 1)*(5*sqrt(3)*sqrt(2)*x^3 - 12*x^3) - 10)/(x^6 + 2)) - 2*x^3*log(-
x^3 + sqrt(x^6 - 1)) + 2*x^3 + 2*sqrt(x^6 - 1))/x^3
```

giac [A] time = 0.31, size = 97, normalized size = 1.23

$$\frac{\sqrt{6}\log\left(\frac{\sqrt{6}-2\sqrt{-\frac{1}{x^6}+1}}{\sqrt{6}+2\sqrt{-\frac{1}{x^6}+1}}\right)}{6\operatorname{sgn}(x)}+\frac{\log\left(\sqrt{-\frac{1}{x^6}+1}+1\right)}{6\operatorname{sgn}(x)}-\frac{\log\left(-\sqrt{-\frac{1}{x^6}+1}+1\right)}{6\operatorname{sgn}(x)}+\frac{\sqrt{-\frac{1}{x^6}+1}}{3\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-1)^(1/2)/x^4/(x^6+2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(6)*log((sqrt(6) - 2*sqrt(-1/x^6 + 1))/(sqrt(6) + 2*sqrt(-1/x^6 + 1
)))/sgn(x) + 1/6*log(sqrt(-1/x^6 + 1) + 1)/sgn(x) - 1/6*log(-sqrt(-1/x^6 +
1) + 1)/sgn(x) + 1/3*sqrt(-1/x^6 + 1)/sgn(x)
```

maple [C] time = 0.54, size = 80, normalized size = 1.01

$$\frac{\sqrt{x^6-1}}{3x^3}-\frac{\ln\left(x^3-\sqrt{x^6-1}\right)}{3}+\frac{\operatorname{RootOf}\left(\_Z^2-6\right)\ln\left(-\frac{5\operatorname{RootOf}\left(\_Z^2-6\right)x^6-12x^3\sqrt{x^6-1}-2\operatorname{RootOf}\left(\_Z^2-6\right)}{x^6+2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-2)*(x^6-1)^(1/2)/x^4/(x^6+2),x)
```

```
[Out] 1/3*(x^6-1)^(1/2)/x^3-1/3*ln(x^3-(x^6-1)^(1/2))+1/6*RootOf(_Z^2-6)*ln(-(5*R
ootOf(_Z^2-6)*x^6-12*x^3*(x^6-1)^(1/2)-2*RootOf(_Z^2-6))/(x^6+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^6-1}\left(x^6-2\right)}{\left(x^6+2\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-1)^(1/2)/x^4/(x^6+2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^6 - 1)*(x^6 - 2)/((x^6 + 2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{x^6-1}\,\left(x^6-2\right)}{x^4\,\left(x^6+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/2)*(x^6 - 2))/(x^4*(x^6 + 2)),x)
```

```
[Out] int(((x^6 - 1)^(1/2)*(x^6 - 2))/(x^4*(x^6 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{(x-1)(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,\left(x^6-2\right)}{x^4\left(x^6+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-2)*(x**6-1)**(1/2)/x**4/(x**6+2),x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1))*(x**6 - 2)/(x*
*4*(x**6 + 2)), x)
```

3.956 $\int \frac{(-4+x^6)(2-x^4+x^6)^{5/2}}{x^7(2+x^6)^2}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{x^6-x^4+2}\,(2x^{12}-14x^{10}-3x^8+8x^6-28x^4+8)}{6x^6(x^6+2)}-\frac{5}{2}\tan^{-1}\left(\frac{x^2}{\sqrt{x^6-x^4+2}}\right)$$

Rubi [F] time = 4.39, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-4+x^6)(2-x^4+x^6)^{5/2}}{x^7(2+x^6)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-4 + x^6)*(2 - x^4 + x^6)^(5/2))/(x^7*(2 + x^6)^2),x]
```

```
[Out] (5*Defer[Int][(2 - x^4 + x^6)^(5/2)/((-2)^(1/6) - x), x])/24 + (5*Defer[Int
][(2 - x^4 + x^6)^(5/2)/(-((-2)^(1/6)*(-1)^(1/3)) - x), x])/24 + (5*Defer[I
nt][(2 - x^4 + x^6)^(5/2)/((-2)^(1/6)*(-1)^(2/3) - x), x])/24 - Defer[Int][
(2 - x^4 + x^6)^(5/2)/x^7, x] - (5*Defer[Int][(2 - x^4 + x^6)^(5/2)/((-2)^(
1/6) + x), x])/24 - (5*Defer[Int][(2 - x^4 + x^6)^(5/2)/(-((-2)^(1/6)*(-1)^
(1/3)) + x), x])/24 - (5*Defer[Int][(2 - x^4 + x^6)^(5/2)/((-2)^(1/6)*(-1)^
(2/3) + x), x])/24 + (10935*Sqrt[3]*(2 - x^4 + x^6)^(5/2)*Defer[Subst][Defe
r[Int][(((1 + (26 - 15*Sqrt[3])^(2/3))/(3*(26 - 15*Sqrt[3])^(1/3)) + x)^(5/
2)*((-1 + (26 - 15*Sqrt[3])^(-2/3) + (26 - 15*Sqrt[3])^(2/3))/9 - ((1 + (26
 - 15*Sqrt[3])^(2/3))*x)/(3*(26 - 15*Sqrt[3])^(1/3)) + x^2)^(5/2))/(1/3 + x
), x], x, (-1 + 3*x^2)/3])/(8*(-1 + (26 - 15*Sqrt[3])^(-1/3) + (26 - 15*Sqr
t[3])^(1/3) + 3*x^2)^(5/2)*(-1 + (26 - 15*Sqrt[3])^(-2/3) + (26 - 15*Sqrt[3
])^(2/3) + ((1 + (26 - 15*Sqrt[3])^(2/3))*(1 - 3*x^2))/(26 - 15*Sqrt[3])^(1
/3) + (-1 + 3*x^2)^2)^(5/2)) - (3*Defer[Subst][Defer[Int][(x^2*(2 - x^2 + x
^3)^(5/2))/(2 + x^3)^2, x], x, x^2])/4
```

Rubi steps

$$\int \frac{\left(-4+x^6\right)\left(2-x^4+x^6\right)^{5/2}}{x^7\left(2+x^6\right)^2}\,dx = \int\left(-\frac{\left(2-x^4+x^6\right)^{5/2}}{x^7}+\frac{5\left(2-x^4+x^6\right)^{5/2}}{4x}-\frac{3x^5\left(2-x^4+x^6\right)^{5/2}}{2\left(2+x^6\right)^2}-\frac{5x^5\left(2\right.}{4\left(\right.}\right.$$

$$= \frac{5}{4}\int \frac{\left(2-x^4+x^6\right)^{5/2}}{x}\,dx - \frac{5}{4}\int \frac{x^5\left(2-x^4+x^6\right)^{5/2}}{2+x^6}\,dx - \frac{3}{2}\int \frac{x^5\left(2-x^4+x\right.}{\left(2+x^6\right)^2}$$

$$= \frac{5}{8}\,\mathrm{Subst}\left(\int \frac{\left(2-x^2+x^3\right)^{5/2}}{x}\,dx, x, x^2\right) - \frac{3}{4}\,\mathrm{Subst}\left(\int \frac{x^2\left(2-x^2+x^3\right)^{5/2}}{\left(2+x^3\right)^2}\,dx\right.$$

$$= -\left(\frac{5}{8}\int \frac{x^2\left(2-x^4+x^6\right)^{5/2}}{-i\sqrt{2}+x^3}\,dx\right) - \frac{5}{8}\int \frac{x^2\left(2-x^4+x^6\right)^{5/2}}{i\sqrt{2}+x^3}\,dx + \frac{5}{8}\,\mathrm{Subst}\left(\int\right.$$

$$= -\left(\frac{5}{8}\int\left(-\frac{\left(2-x^4+x^6\right)^{5/2}}{3\left(\sqrt[6]{-2}-x\right)} - \frac{\left(2-x^4+x^6\right)^{5/2}}{3\left(-\sqrt[6]{-2}\,\sqrt[3]{-1}-x\right)} - \frac{\left(2-x^4+x^6\right)^{5/2}}{3\left(\sqrt[6]{-2}\,(-1)^{2/3}-x\right)}\right)d\right.$$

$$= \frac{5}{24}\int \frac{\left(2-x^4+x^6\right)^{5/2}}{\sqrt[6]{-2}-x}\,dx + \frac{5}{24}\int \frac{\left(2-x^4+x^6\right)^{5/2}}{-\sqrt[6]{-2}\,\sqrt[3]{-1}-x}\,dx + \frac{5}{24}\int \frac{\left(2-x^4+x^6\right.}{\sqrt[6]{-2}\,(-1)^{2/3}}$$

Mathematica [C] time = 1.70, size = 354, normalized size = 4.48

$$\frac{1}{6}\sqrt{x^6-x^4+2}\left(\frac{4}{x^6}-\frac{14}{x^2}-\frac{3x^2}{x^6+2}+2\right)+\frac{\left(\frac{1}{2}+i\right)\sqrt{ix^2+(1-i)}\sqrt{(-3-4i)\left(x^4+ix^2-(1-i)\right)}\left(iF\left(\sin^{-1}\left(\frac{\sqrt{(-2-i)\left(x^2-(1-i)\right)}}{\sqrt{5}}\right)\middle|\frac{1}{2}+i\right)+\sqrt[3]{2}\left(\frac{3\sqrt[6]{-1}\,\Pi\left(\frac{2-i}{(-1+i)+\sqrt[3]{-2}};\sin^{-1}\left(\frac{\sqrt{(-2-i)\left(x^2-(1-i)\right)}}{\sqrt{5}}\right)\middle|\frac{1}{2}+i\right)}{\left(\sqrt[3]{-2}+(-1+i)\right)\left(1+\sqrt[3]{-1}\right)^2}-\frac{i\Pi\left(\frac{2-i}{(1-i)+\sqrt[3]{2}};\sin^{-1}\left(\frac{\sqrt{(-2-i)\left(x^2-(1-i)\right)}}{\sqrt{5}}\right)\middle|\frac{1}{2}+i\right)}{\sqrt[3]{2}+(1-i)}-\frac{2\sqrt[6]{-1}\,\Pi\left(\frac{2-i}{(1-i)+(-1)^{2/3}\sqrt[3]{2}};\sin^{-1}\left(\frac{\sqrt{(-2-i)\left(x^2-(1-i)\right)}}{\sqrt{5}}\right)\middle|\frac{1}{2}+i\right)}{(-2+2i)+\sqrt[3]{2}-i\sqrt[3]{2}\sqrt{3}}\right)\right)}{\sqrt{2}\sqrt{x^6-x^4+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-4 + x^6)*(2 - x^4 + x^6)^(5/2))/(x^7*(2 + x^6)^2),x]
```

```
[Out] (Sqrt[2 - x^4 + x^6]*(2 + 4/x^6 - 14/x^2 - (3*x^2)/(2 + x^6)))/6 + ((1/2 +
I)*Sqrt[(1 - I) + I*x^2]*Sqrt[(-3 - 4*I)*((-1 + I) + I*x^2 + x^4)]*(I*Ellip
ticF[ArcSin[Sqrt[(-2 - I)*((-1 + I) + x^2)]/Sqrt[5]], 1/2 + I] + 2^(1/3)*((
3*(-1)^(1/6)*EllipticPi[(-2 + I)/((-1 + I) + (-2)^(1/3)), ArcSin[Sqrt[(-2 -
 I)*((-1 + I) + x^2)]/Sqrt[5]], 1/2 + I])/(((-1 + I) + (-2)^(1/3))*(1 + (-1
)^(1/3))^2) - (I*EllipticPi[(2 - I)/((1 - I) + 2^(1/3)), ArcSin[Sqrt[(-2 -
I)*((-1 + I) + x^2)]/Sqrt[5]], 1/2 + I])/((1 - I) + 2^(1/3)) - (2*(-1)^(1/6
)*EllipticPi[(2 - I)/((1 - I) + (-1)^(2/3)*2^(1/3)), ArcSin[Sqrt[(-2 - I)*(
(-1 + I) + x^2)]/Sqrt[5]], 1/2 + I])/((-2 + 2*I) + 2^(1/3) - I*2^(1/3)*Sqrt
[3]))))/(Sqrt[2]*Sqrt[2 - x^4 + x^6])
```

IntegrateAlgebraic [A] time = 3.09, size = 79, normalized size = 1.00

$$\frac{\sqrt{x^6-x^4+2}\left(2x^{12}-14x^{10}-3x^8+8x^6-28x^4+8\right)}{6x^6\left(x^6+2\right)}-\frac{5}{2}\tan^{-1}\left(\frac{x^2}{\sqrt{x^6-x^4+2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + x^6)*(2 - x^4 + x^6)^(5/2))/(x^7*(2 + x^6)^2),x]
```

```
[Out] (Sqrt[2 - x^4 + x^6]*(8 - 28*x^4 + 8*x^6 - 3*x^8 - 14*x^10 + 2*x^12))/(6*x^
6*(2 + x^6)) - (5*ArcTan[x^2/Sqrt[2 - x^4 + x^6]])/2
```

fricas [A] time = 0.56, size = 96, normalized size = 1.22

$$-\frac{15\left(x^{12}+2\,x^{6}\right)\arctan\left(\frac{2\,\sqrt{x^{6}-x^{4}+2}\,x^{2}}{x^{6}-2\,x^{4}+2}\right)-2\left(2\,x^{12}-14\,x^{10}-3\,x^{8}+8\,x^{6}-28\,x^{4}+8\right)\sqrt{x^{6}-x^{4}+2}}{12\left(x^{12}+2\,x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-4)*(x^6-x^4+2)^(5/2)/x^7/(x^6+2)^2,x, algorithm="fricas")
```

```
[Out] -1/12*(15*(x^12 + 2*x^6)*arctan(2*sqrt(x^6 - x^4 + 2)*x^2/(x^6 - 2*x^4 + 2)
) - 2*(2*x^12 - 14*x^10 - 3*x^8 + 8*x^6 - 28*x^4 + 8)*sqrt(x^6 - x^4 + 2))/
(x^12 + 2*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}-x^{4}+2\right)^{\frac{5}{2}}\left(x^{6}-4\right)}{\left(x^{6}+2\right)^{2} x^{7}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-4)*(x^6-x^4+2)^(5/2)/x^7/(x^6+2)^2,x, algorithm="giac")
```

```
[Out] integrate((x^6 - x^4 + 2)^(5/2)*(x^6 - 4)/((x^6 + 2)^2*x^7), x)
```

maple [C] time = 0.51, size = 133, normalized size = 1.68

$$\frac{2x^{18}-16x^{16}+11x^{14}+15x^{12}-64x^{10}+22x^{8}+24x^{6}-64x^{4}+16}{6x^{6}\sqrt{x^{6}-x^{4}+2}\,\left(x^{6}+2\right)}-\frac{5\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right)x^{6}-2\,\mathrm{RootOf}\left(\_Z^{2}+1\right)x^{4}+2\sqrt{x^{6}-x^{4}+2}\,x^{2}+2\,\mathrm{RootOf}\left(\_Z^{2}+1\right)}{x^{6}+2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-4)*(x^6-x^4+2)^(5/2)/x^7/(x^6+2)^2,x)
```

```
[Out] 1/6*(2*x^18-16*x^16+11*x^14+15*x^12-64*x^10+22*x^8+24*x^6-64*x^4+16)/x^6/(x
^6-x^4+2)^(1/2)/(x^6+2)-5/4*RootOf(_Z^2+1)*ln(-(RootOf(_Z^2+1)*x^6-2*RootOf
(_Z^2+1)*x^4+2*(x^6-x^4+2)^(1/2)*x^2+2*RootOf(_Z^2+1))/(x^6+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}-x^{4}+2\right)^{\frac{5}{2}}\left(x^{6}-4\right)}{\left(x^{6}+2\right)^{2} x^{7}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-4)*(x^6-x^4+2)^(5/2)/x^7/(x^6+2)^2,x, algorithm="maxima")
```

```
[Out] integrate((x^6 - x^4 + 2)^(5/2)*(x^6 - 4)/((x^6 + 2)^2*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^{6}-4\right)\left(x^{6}-x^{4}+2\right)^{5/2}}{x^{7}\left(x^{6}+2\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 4)*(x^6 - x^4 + 2)^(5/2))/(x^7*(x^6 + 2)^2),x)
[Out] int(((x^6 - 4)*(x^6 - x^4 + 2)^(5/2))/(x^7*(x^6 + 2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-4)*(x**6-x**4+2)**(5/2)/x**7/(x**6+2)**2,x)
[Out] Timed out
```

3.957 $\int \frac{x+x^7}{\left(-1+x^6\right)^{2/3}\left(-1+x^3+x^6\right)}\,dx$

Optimal. Leaf size=79

$$\frac{1}{3}\log\left(\sqrt[3]{x^6-1}+x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^6-1}-x}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(-\sqrt[3]{x^6-1}\,x+\left(x^6-1\right)^{2/3}+x^2\right)$$

Rubi [C] time = 1.74, antiderivative size = 601, normalized size of antiderivative = 7.61, number of steps used = 40, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1593, 6728, 275, 246, 245, 1562, 465, 430, 429, 511, 510}

$$\frac{\left(5-\sqrt{5}\right)\left(1-x^6\right)^{2/3}x^5F_1\left(\frac{5}{6};\frac{2}{3},1;\frac{11}{6};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{25\left(3-\sqrt{5}\right)\left(x^6-1\right)^{2/3}}-\frac{4\left(1-x^6\right)^{2/3}x^5F_1\left(\frac{5}{6};\frac{2}{3},1;\frac{11}{6};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{5\sqrt{5}\left(3-\sqrt{5}\right)\left(x^6-1\right)^{2/3}}+\frac{\left(5+\sqrt{5}\right)\left(1-x^6\right)^{2/3}x^5F_1\left(\frac{5}{6};\frac{2}{3},1;\frac{11}{6};x^6,\frac{2x^6}{3+\sqrt{5}}\right)}{25\left(3+\sqrt{5}\right)\left(x^6-1\right)^{2/3}}+\frac{4\left(1-x^6\right)^{2/3}x^5F_1\left(\frac{5}{6};\frac{2}{3},1;\frac{11}{6};x^6,\frac{2x^6}{3+\sqrt{5}}\right)}{5\sqrt{5}\left(3+\sqrt{5}\right)\left(x^6-1\right)^{2/3}}-\frac{\left(5-\sqrt{5}\right)\left(1-x^6\right)^{2/3}x^2F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{5\left(3-\sqrt{5}\right)\left(x^6-1\right)^{2/3}}+\frac{\left(1-x^6\right)^{2/3}x^2F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{2\sqrt{5}\left(x^6-1\right)^{2/3}}-\frac{\left(5+\sqrt{5}\right)\left(1-x^6\right)^{2/3}x^2F_1\left(\frac{1}{3};1,\frac{2}{3};\frac{4}{3};\frac{2x^6}{3+\sqrt{5}},x^6\right)}{5\left(3+\sqrt{5}\right)\left(x^6-1\right)^{2/3}}-\frac{\left(1-x^6\right)^{2/3}x^2F_1\left(\frac{1}{3};1,\frac{2}{3};\frac{4}{3};\frac{2x^6}{3+\sqrt{5}},x^6\right)}{2\sqrt{5}\left(x^6-1\right)^{2/3}}+\frac{\left(1-x^6\right)^{2/3}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^6\right)}{2\left(x^6-1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x + x^7)/((-1 + x^6)^(2/3)*(-1 + x^3 + x^6)),x]
```

```
[Out] (x^2*(1 - x^6)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, x^6, (2*x^6)/(3 - Sqrt[5])]
)/(2*Sqrt[5]*(-1 + x^6)^(2/3)) - ((5 - Sqrt[5])*x^2*(1 - x^6)^(2/3)*AppellF
1[1/3, 2/3, 1, 4/3, x^6, (2*x^6)/(3 - Sqrt[5])])/(5*(3 - Sqrt[5])*(-1 + x^6
)^(2/3)) - (x^2*(1 - x^6)^(2/3)*AppellF1[1/3, 1, 2/3, 4/3, (2*x^6)/(3 + Sqr
t[5]), x^6])/(2*Sqrt[5]*(-1 + x^6)^(2/3)) - ((5 + Sqrt[5])*x^2*(1 - x^6)^(2
/3)*AppellF1[1/3, 1, 2/3, 4/3, (2*x^6)/(3 + Sqrt[5]), x^6])/(5*(3 + Sqrt[5]
)*(-1 + x^6)^(2/3)) - (4*x^5*(1 - x^6)^(2/3)*AppellF1[5/6, 2/3, 1, 11/6, x^
6, (2*x^6)/(3 - Sqrt[5])])/(5*Sqrt[5]*(3 - Sqrt[5])*(-1 + x^6)^(2/3)) + ((5
 - Sqrt[5])*x^5*(1 - x^6)^(2/3)*AppellF1[5/6, 2/3, 1, 11/6, x^6, (2*x^6)/(3
 - Sqrt[5])])/(25*(3 - Sqrt[5])*(-1 + x^6)^(2/3)) + (4*x^5*(1 - x^6)^(2/3)*
AppellF1[5/6, 2/3, 1, 11/6, x^6, (2*x^6)/(3 + Sqrt[5])])/(5*Sqrt[5]*(3 + Sq
rt[5])*(-1 + x^6)^(2/3)) + ((5 + Sqrt[5])*x^5*(1 - x^6)^(2/3)*AppellF1[5/6,
 2/3, 1, 11/6, x^6, (2*x^6)/(3 + Sqrt[5])])/(25*(3 + Sqrt[5])*(-1 + x^6)^(2
/3)) + (x^2*(1 - x^6)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, x^6])/(2*(-1 +
 x^6)^(2/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1562

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(
2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)/(d^2 - e^2*x^(2*n)))^(-q), x], x]
, x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !IntegerQ[p]
 && ILtQ[q, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x+x^7}{\left(-1+x^6\right)^{2/3}\left(-1+x^3+x^6\right)}\,dx = \int \frac{x\left(1+x^6\right)}{\left(-1+x^6\right)^{2/3}\left(-1+x^3+x^6\right)}\,dx$$

$$= \int \left(\frac{x}{\left(-1+x^6\right)^{2/3}} + \frac{x\left(2-x^3\right)}{\left(-1+x^6\right)^{2/3}\left(-1+x^3+x^6\right)}\right) dx$$

$$= \int \frac{x}{\left(-1+x^6\right)^{2/3}}\,dx + \int \frac{x\left(2-x^3\right)}{\left(-1+x^6\right)^{2/3}\left(-1+x^3+x^6\right)}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(-1+x^3\right)^{2/3}}\,dx, x, x^2\right) + \int \left(\frac{2x}{\left(-1+x^6\right)^{2/3}\left(-1+x^3+x^6\right)} - \frac{}{(-1}\right.$$

$$= 2\int \frac{x}{\left(-1+x^6\right)^{2/3}\left(-1+x^3+x^6\right)}\,dx + \frac{\left(1-x^6\right)^{2/3}\text{Subst}\left(\int \frac{1}{\left(1-x^3\right)^{2/3}}\,dx, x, x^2\right)}{2\left(-1+x^6\right)^{2/3}}$$

$$= \frac{x^2\left(1-x^6\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^6\right)}{2\left(-1+x^6\right)^{2/3}} + 2\int \left(-\frac{2x}{\sqrt{5}\left(-1+\sqrt{5}-2x^3\right)\left(-1+x^6\right)^{2/3}} - \right.$$

$$= \frac{x^2\left(1-x^6\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^6\right)}{2\left(-1+x^6\right)^{2/3}} - \frac{4\int \frac{x}{\left(-1+\sqrt{5}-2x^3\right)\left(-1+x^6\right)^{2/3}}\,dx}{\sqrt{5}} - \frac{4\int \frac{x}{\left(1+\sqrt{5}+2x^3\right.}}{\sqrt{5}}$$

$$= \frac{x^2\left(1-x^6\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^6\right)}{2\left(-1+x^6\right)^{2/3}} - \frac{4\int \left(\frac{\left(1+\sqrt{5}\right)x}{2\left(3+\sqrt{5}-2x^6\right)\left(-1+x^6\right)^{2/3}} + \frac{x^4}{\left(-1+x^6\right)^{2/3}\left(-3-\sqrt{5}+\right.}\right.}{\sqrt{5}}$$

$$= \frac{x^2\left(1-x^6\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^6\right)}{2\left(-1+x^6\right)^{2/3}} - \frac{4\int \frac{x^4}{\left(-1+x^6\right)^{2/3}\left(-3-\sqrt{5}+2x^6\right)}\,dx}{\sqrt{5}} + \frac{4\int \frac{}{\left(-1+x^6\right)^{2/3}(}}{\sqrt{}}$$

$$= \frac{x^2\left(1-x^6\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^6\right)}{2\left(-1+x^6\right)^{2/3}} + \frac{1}{10}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\left(-1+x^3\right)^{2/3}\left(-3+\right.}\right.$$

$$= -\frac{4x^5\left(1-x^6\right)^{2/3} F_1\left(\frac{5}{6};\frac{2}{3},1;\frac{11}{6};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{5\sqrt{5}\left(3-\sqrt{5}\right)\left(-1+x^6\right)^{2/3}} + \frac{\left(5-\sqrt{5}\right)x^5\left(1-x^6\right)^{2/3} F_1\left(\frac{5}{6};\frac{2}{3},1\right.}{25\left(3-\sqrt{5}\right)\left(-1+x^6\right.}$$

$$= \frac{x^2\left(1-x^6\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{2\sqrt{5}\left(-1+x^6\right)^{2/3}} - \frac{\left(5-\sqrt{5}\right)x^2\left(1-x^6\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};\right.}{5\left(3-\sqrt{5}\right)\left(-1+x^6\right)^{2/3}}$$

Mathematica [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x+x^7}{\left(-1+x^6\right)^{2/3}\left(-1+x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x + x^7)/((-1 + x^6)^(2/3)*(-1 + x^3 + x^6)),x]
[Out] Integrate[(x + x^7)/((-1 + x^6)^(2/3)*(-1 + x^3 + x^6)), x]
```

IntegrateAlgebraic [A] time = 1.51, size = 79, normalized size = 1.00

$$\frac{1}{3}\log\left(\sqrt[3]{x^6-1}+x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^6-1}-x}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(-\sqrt[3]{x^6-1}\,x+\left(x^6-1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + x^7)/((-1 + x^6)^(2/3)*(-1 + x^3 + x^6)),x]
```

```
[Out] -(ArcTan[(Sqrt[3]*x)/(-x + 2*(-1 + x^6)^(1/3))]/Sqrt[3]) + Log[x + (-1 + x^
6)^(1/3)]/3 - Log[x^2 - x*(-1 + x^6)^(1/3) + (-1 + x^6)^(2/3)]/6
```

fricas [A] time = 2.44, size = 102, normalized size = 1.29

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}x^2+2\sqrt{3}\left(x^6-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(x^6-1\right)}{x^6-8x^3-1}\right)+\frac{1}{6}\log\left(\frac{x^6+x^3+3\left(x^6-1\right)^{\frac{1}{3}}x^2+3\left(x^6-1\right)^{\frac{2}{3}}x-1}{x^6+x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^7+x)/(x^6-1)^(2/3)/(x^6+x^3-1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan((4*sqrt(3)*(x^6 - 1)^(1/3)*x^2 + 2*sqrt(3)*(x^6 - 1)^(2
/3)*x + sqrt(3)*(x^6 - 1))/(x^6 - 8*x^3 - 1)) + 1/6*log((x^6 + x^3 + 3*(x^6
 - 1)^(1/3)*x^2 + 3*(x^6 - 1)^(2/3)*x - 1)/(x^6 + x^3 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7+x}{\left(x^6+x^3-1\right)\left(x^6-1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^7+x)/(x^6-1)^(2/3)/(x^6+x^3-1),x, algorithm="giac")
```

```
[Out] integrate((x^7 + x)/((x^6 + x^3 - 1)*(x^6 - 1)^(2/3)), x)
```

maple [C] time = 1.67, size = 396, normalized size = 5.01

$$\frac{\ln\left(\frac{3\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)x^6-x^6-9\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)^2x^3+9\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)(x^6-1)^{\frac{2}{3}}x+9\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)(x^6-1)^{\frac{1}{3}}x^2+3x^2(x^6-1)^{\frac{1}{3}}+x^3-3\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)+1}{x^6+x^3-1}\right)}{3}-\frac{\ln\left(-\frac{3\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)x^6-x^6+9\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)^2x^3+9\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)(x^6-1)^{\frac{2}{3}}x-3\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)x^3+3x(x^6-1)^{\frac{2}{3}}+3x^2(x^6-1)^{\frac{1}{3}}-3\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)+1}{x^6+x^3-1}\right)}{3}-\ln\left(-\frac{3\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)x^6-x^6+9\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)^2x^3+9\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)(x^6-1)^{\frac{2}{3}}x-3\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)x^3+3x(x^6-1)^{\frac{2}{3}}+3x^2(x^6-1)^{\frac{1}{3}}-3\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)+1}{x^6+x^3-1}\right)\mathrm{RootOf}(9\_Z^2+3\_Z+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7+x)/(x^6-1)^(2/3)/(x^6+x^3-1),x)
```

```
[Out] 1/3*ln((3*RootOf(9*_Z^2+3*_Z+1)*x^6-x^6-9*RootOf(9*_Z^2+3*_Z+1)^2*x^3+9*Roo
tOf(9*_Z^2+3*_Z+1)*(x^6-1)^(2/3)*x+9*RootOf(9*_Z^2+3*_Z+1)*(x^6-1)^(1/3)*x^
2+3*x^2*(x^6-1)^(1/3)+x^3-3*RootOf(9*_Z^2+3*_Z+1)+1)/(x^6+x^3-1))-1/3*ln(-(
3*RootOf(9*_Z^2+3*_Z+1)*x^6-x^6+9*RootOf(9*_Z^2+3*_Z+1)^2*x^3+9*RootOf(9*_Z
^2+3*_Z+1)*(x^6-1)^(2/3)*x-3*RootOf(9*_Z^2+3*_Z+1)*x^3+3*x*(x^6-1)^(2/3)+3*
x^2*(x^6-1)^(1/3)-3*RootOf(9*_Z^2+3*_Z+1)+1)/(x^6+x^3-1))-ln(-(3*RootOf(9*_
Z^2+3*_Z+1)*x^6-x^6+9*RootOf(9*_Z^2+3*_Z+1)^2*x^3+9*RootOf(9*_Z^2+3*_Z+1)*(
x^6-1)^(2/3)*x-3*RootOf(9*_Z^2+3*_Z+1)*x^3+3*x*(x^6-1)^(2/3)+3*x^2*(x^6-1)^
(1/3)-3*RootOf(9*_Z^2+3*_Z+1)+1)/(x^6+x^3-1))*RootOf(9*_Z^2+3*_Z+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7+x}{\left(x^6+x^3-1\right)\left(x^6-1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^7+x)/(x^6-1)^(2/3)/(x^6+x^3-1),x, algorithm="maxima")
```

```
[Out] integrate((x^7 + x)/((x^6 + x^3 - 1)*(x^6 - 1)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7+x}{\left(x^6-1\right)^{2/3}\,\left(x^6+x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^7)/((x^6 - 1)^(2/3)*(x^3 + x^6 - 1)),x)
```

```
[Out] int((x + x^7)/((x^6 - 1)^(2/3)*(x^3 + x^6 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**7+x)/(x**6-1)**(2/3)/(x**6+x**3-1),x)
```

```
[Out] Timed out
```

3.958 $\int \frac{\sqrt{1-x^6}\left(1+2x^6\right)}{1+x^4-2x^6+x^{12}}\,dx$

Optimal. Leaf size=79

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x\sqrt{1-x^6}}{x^6+x^2-1}\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt{1-x^6}}{x^6-x^2-1}\right)}{2\sqrt{2}}$$

Rubi [F] time = 0.51, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1-x^6}\left(1+2x^6\right)}{1+x^4-2x^6+x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[1 - x^6]*(1 + 2*x^6))/(1 + x^4 - 2*x^6 + x^12),x]
```

```
[Out] Defer[Int][Sqrt[1 - x^6]/(1 + x^4 - 2*x^6 + x^12), x] + 2*Defer[Int][(x^6*S
qrt[1 - x^6])/(1 + x^4 - 2*x^6 + x^12), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^6}\left(1+2x^6\right)}{1+x^4-2x^6+x^{12}}\,dx &= \int \left(\frac{\sqrt{1-x^6}}{1+x^4-2x^6+x^{12}}+\frac{2x^6\sqrt{1-x^6}}{1+x^4-2x^6+x^{12}}\right)dx\\ &= 2\int \frac{x^6\sqrt{1-x^6}}{1+x^4-2x^6+x^{12}}\,dx+\int \frac{\sqrt{1-x^6}}{1+x^4-2x^6+x^{12}}\,dx\end{aligned}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x^6}\left(1+2x^6\right)}{1+x^4-2x^6+x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[1 - x^6]*(1 + 2*x^6))/(1 + x^4 - 2*x^6 + x^12),x]
```

```
[Out] Integrate[(Sqrt[1 - x^6]*(1 + 2*x^6))/(1 + x^4 - 2*x^6 + x^12), x]
```

IntegrateAlgebraic [A] time = 4.74, size = 91, normalized size = 1.15

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x\sqrt{1-x^6}}{x^6+x^2-1}\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^6}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{1-x^6}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 - x^6]*(1 + 2*x^6))/(1 + x^4 - 2*x^6 + x^12),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2]*x*Sqrt[1 - x^6])/(-1 + x^2 + x^6)]/Sqrt[2] - ArcTanh[(
-(1/Sqrt[2]) - x^2/Sqrt[2] + x^6/Sqrt[2])/(x*Sqrt[1 - x^6])]/(2*Sqrt[2])
```

fricas [B] time = 0.73, size = 512, normalized size = 6.48

$$\frac{1}{4}\sqrt{2}\arctan\left(-\frac{x^{12}-2x^6+x^4+2\sqrt{2}\left(x^7+x^3-x\right)\sqrt{-x^6+1}-\left(4\sqrt{-x^6+1}x^3-\sqrt{2}\left(x^{12}+2x^8-2x^6-x^4-2x^2+1\right)\right)\sqrt{\frac{x^{12}-4x^8-2x^6+x^4+2\sqrt{2}\left(x^7-x^3-x\right)\sqrt{-x^6+1}+4x^2+1}{x^{12}-2x^6+x^4+1}}+1}{x^{12}+4x^8-2x^6+x^4-4x^2+1}\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{x^{12}-2x^6+x^4-2\sqrt{2}\left(x^7+x^3-x\right)\sqrt{-x^6+1}-\left(4\sqrt{-x^6+1}x^3+\sqrt{2}\left(x^{12}+2x^8-2x^6-x^4-2x^2+1\right)\right)\sqrt{\frac{x^{12}-4x^8-2x^6+x^4-2\sqrt{2}\left(x^7-x^3-x\right)\sqrt{-x^6+1}+4x^2+1}{x^{12}-2x^6+x^4+1}}+1}{x^{12}+4x^8-2x^6+x^4-4x^2+1}\right)-\frac{1}{16}\sqrt{2}\log\left(\frac{4\left(x^{12}-4x^8-2x^6+x^4+2\sqrt{2}\left(x^7-x^3-x\right)\sqrt{-x^6+1}+4x^2+1\right)}{x^{12}-2x^6+x^4+1}\right)+\frac{1}{16}\sqrt{2}\log\left(\frac{4\left(x^{12}-4x^8-2x^6+x^4-2\sqrt{2}\left(x^7-x^3-x\right)\sqrt{-x^6+1}+4x^2+1\right)}{x^{12}-2x^6+x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(1/2)*(2*x^6+1)/(x^12-2*x^6+x^4+1),x, algorithm="fricas"
)
```

```
[Out] 1/4*sqrt(2)*arctan(-(x^12 - 2*x^6 + x^4 + 2*sqrt(2)*(x^7 + x^3 - x)*sqrt(-x
^6 + 1) - (4*sqrt(-x^6 + 1)*x^3 - sqrt(2)*(x^12 + 2*x^8 - 2*x^6 - x^4 - 2*x
^2 + 1))*sqrt((x^12 - 4*x^8 - 2*x^6 + x^4 + 2*sqrt(2)*(x^7 - x^3 - x)*sqrt(
-x^6 + 1) + 4*x^2 + 1)/(x^12 - 2*x^6 + x^4 + 1)) + 1)/(x^12 + 4*x^8 - 2*x^6
 + x^4 - 4*x^2 + 1)) - 1/4*sqrt(2)*arctan(-(x^12 - 2*x^6 + x^4 - 2*sqrt(2)*
(x^7 + x^3 - x)*sqrt(-x^6 + 1) - (4*sqrt(-x^6 + 1)*x^3 + sqrt(2)*(x^12 + 2*
x^8 - 2*x^6 - x^4 - 2*x^2 + 1))*sqrt((x^12 - 4*x^8 - 2*x^6 + x^4 - 2*sqrt(2
)*(x^7 - x^3 - x)*sqrt(-x^6 + 1) + 4*x^2 + 1)/(x^12 - 2*x^6 + x^4 + 1)) + 1
)/(x^12 + 4*x^8 - 2*x^6 + x^4 - 4*x^2 + 1)) - 1/16*sqrt(2)*log(4*(x^12 - 4*
x^8 - 2*x^6 + x^4 + 2*sqrt(2)*(x^7 - x^3 - x)*sqrt(-x^6 + 1) + 4*x^2 + 1)/(
x^12 - 2*x^6 + x^4 + 1)) + 1/16*sqrt(2)*log(4*(x^12 - 4*x^8 - 2*x^6 + x^4 -
 2*sqrt(2)*(x^7 - x^3 - x)*sqrt(-x^6 + 1) + 4*x^2 + 1)/(x^12 - 2*x^6 + x^4
+ 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^6+1\right)\sqrt{-x^6+1}}{x^{12}-2\,x^6+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(1/2)*(2*x^6+1)/(x^12-2*x^6+x^4+1),x, algorithm="giac")
```

```
[Out] integrate((2*x^6 + 1)*sqrt(-x^6 + 1)/(x^12 - 2*x^6 + x^4 + 1), x)
```

maple [C] time = 1.68, size = 157, normalized size = 1.99

$$-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)x^6+\mathrm{RootOf}\left(\_Z^4+1\right)^3x^2+2\sqrt{-x^6+1}\,x-\mathrm{RootOf}\left(\_Z^4+1\right)}{-x^6+\mathrm{RootOf}\left(\_Z^4+1\right)^2x^2+1}\right)}{4}-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^3x^6-\mathrm{RootOf}\left(\_Z^4+1\right)^5x^2-\mathrm{RootOf}\left(\_Z^4+1\right)^3+2\sqrt{-x^6+1}\,x}{x^6+\mathrm{RootOf}\left(\_Z^4+1\right)^2x^2-1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^6+1)^(1/2)*(2*x^6+1)/(x^12-2*x^6+x^4+1),x)
```

```
[Out] -1/4*RootOf(_Z^4+1)*ln(-(RootOf(_Z^4+1)*x^6+RootOf(_Z^4+1)^3*x^2+2*(-x^6+1)
^(1/2)*x-RootOf(_Z^4+1))/(-x^6+RootOf(_Z^4+1)^2*x^2+1))-1/4*RootOf(_Z^4+1)^
3*ln(-(RootOf(_Z^4+1)^3*x^6-RootOf(_Z^4+1)^5*x^2-RootOf(_Z^4+1)^3+2*(-x^6+1
)^(1/2)*x)/(x^6+RootOf(_Z^4+1)^2*x^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^6+1\right)\sqrt{-x^6+1}}{x^{12}-2\,x^6+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(1/2)*(2*x^6+1)/(x^12-2*x^6+x^4+1),x, algorithm="maxima"
)
```

```
[Out] integrate((2*x^6 + 1)*sqrt(-x^6 + 1)/(x^12 - 2*x^6 + x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-x^6}\,\left(2\,x^6+1\right)}{x^{12}-2\,x^6+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - x^6)^(1/2)*(2*x^6 + 1))/(x^4 - 2*x^6 + x^12 + 1),x)
```

```
[Out] int(((1 - x^6)^(1/2)*(2*x^6 + 1))/(x^4 - 2*x^6 + x^12 + 1), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\left(2x^6+1\right)}{x^{12}-2x^6+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**6+1)**(1/2)*(2*x**6+1)/(x**12-2*x**6+x**4+1),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1))*(2*x**6 + 1)/
(x**12 - 2*x**6 + x**4 + 1), x)
```

3.959 $\int \frac{\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt{x^2+1}\,x}{3\sqrt{\sqrt{x^2+1}+1}}-\frac{2x}{3\sqrt{\sqrt{x^2+1}+1}}+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}\right)$$

Rubi [F] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + x^2]/Sqrt[1 + Sqrt[1 + x^2]],x]
```

```
[Out] Defer[Int][Sqrt[1 + x^2]/Sqrt[1 + Sqrt[1 + x^2]], x]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\,dx=\int \frac{\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\,dx$$

Mathematica [C] time = 0.18, size = 111, normalized size = 1.41

$$\frac{\sqrt{\sqrt{x^2+1}+1}\left(6\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}-\frac{\sqrt{x^2+1}}{2}\right)+4x^2-8\sqrt{x^2+1}+3\sqrt{2}\sqrt{\sqrt{x^2+1}-1}\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}-1}}{\sqrt{2}}\right)+2\right)}{6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[1 + Sqrt[1 + x^2]],x]
```

```
[Out] (Sqrt[1 + Sqrt[1 + x^2]]*(2 + 4*x^2 - 8*Sqrt[1 + x^2] + 3*Sqrt[2]*Sqrt[-1 +
 Sqrt[1 + x^2]]*ArcTan[Sqrt[-1 + Sqrt[1 + x^2]]/Sqrt[2]] + 6*Hypergeometric
2F1[-1/2, 1, 1/2, 1/2 - Sqrt[1 + x^2]/2]))/(6*x)
```

IntegrateAlgebraic [A] time = 0.14, size = 79, normalized size = 1.00

$$\frac{2\sqrt{x^2+1}\,x}{3\sqrt{\sqrt{x^2+1}+1}}-\frac{2x}{3\sqrt{\sqrt{x^2+1}+1}}+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^2]/Sqrt[1 + Sqrt[1 + x^2]],x]
```

```
[Out] (-2*x)/(3*Sqrt[1 + Sqrt[1 + x^2]]) + (2*x*Sqrt[1 + x^2])/(3*Sqrt[1 + Sqrt[1
 + x^2]]) + Sqrt[2]*ArcTan[x/(Sqrt[2]*Sqrt[1 + Sqrt[1 + x^2]])]
```

fricas [A] time = 1.17, size = 58, normalized size = 0.73

$$-\frac{3\sqrt{2}x\arctan\left(\frac{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}{x}\right)-2\left(x^2-2\sqrt{x^2+1}+2\right)\sqrt{\sqrt{x^2+1}+1}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
[Out] -1/3*(3*sqrt(2)*x*arctan(sqrt(2)*sqrt(sqrt(x^2 + 1) + 1)/x) - 2*(x^2 - 2*sq
rt(x^2 + 1) + 2)*sqrt(sqrt(x^2 + 1) + 1))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2+1}}{\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 1)/sqrt(sqrt(x^2 + 1) + 1), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2+1}}{\sqrt{1+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(1+(x^2+1)^(1/2))^(1/2),x)
[Out] int((x^2+1)^(1/2)/(1+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2+1}}{\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 1)/sqrt(sqrt(x^2 + 1) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{x^2+1}}{\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/2)/((x^2 + 1)^(1/2) + 1)^(1/2),x)
[Out] int((x^2 + 1)^(1/2)/((x^2 + 1)^(1/2) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(1+(x**2+1)**(1/2))**(1/2),x)
[Out] Integral(sqrt(x**2 + 1)/sqrt(sqrt(x**2 + 1) + 1), x)
```

3.960 $\int \frac{1+x^2}{\sqrt{x+\sqrt{1+x^2}}} dx$

Optimal. Leaf size=79

$$\frac{2(28x^6+147x^4+112x^2+9)}{35\left(\sqrt{x^2+1}+x\right)^{7/2}}+\frac{2\sqrt{x^2+1}\,(4x^5+19x^3+7x)}{5\left(\sqrt{x^2+1}+x\right)^{7/2}}$$

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2122, 270}

$$\frac{1}{20}\left(\sqrt{x^2+1}+x\right)^{5/2}+\frac{3}{4}\sqrt{\sqrt{x^2+1}+x}-\frac{1}{4\left(\sqrt{x^2+1}+x\right)^{3/2}}-\frac{1}{28\left(\sqrt{x^2+1}+x\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/Sqrt[x + Sqrt[1 + x^2]],x]
```

```
[Out] -1/28*1/(x + Sqrt[1 + x^2])^(7/2) - 1/(4*(x + Sqrt[1 + x^2])^(3/2)) + (3*Sq
rt[x + Sqrt[1 + x^2]])/4 + (x + Sqrt[1 + x^2])^(5/2)/20
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{\sqrt{x+\sqrt{1+x^2}}}\,dx &= \frac{1}{8}\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^{9/2}}\,dx, x, x+\sqrt{1+x^2}\right)\\ &= \frac{1}{8}\text{Subst}\left(\int \left(\frac{1}{x^{9/2}}+\frac{3}{x^{5/2}}+\frac{3}{\sqrt{x}}+x^{3/2}\right)dx, x, x+\sqrt{1+x^2}\right)\\ &= -\frac{1}{28\left(x+\sqrt{1+x^2}\right)^{7/2}}-\frac{1}{4\left(x+\sqrt{1+x^2}\right)^{3/2}}+\frac{3}{4}\sqrt{x+\sqrt{1+x^2}}+\frac{1}{20}\left(x+\sqrt{1+x^2}\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 0.84

$$\frac{7\left(\sqrt{x^2+1}+x\right)^6+105\left(\sqrt{x^2+1}+x\right)^4-35\left(\sqrt{x^2+1}+x\right)^2-5}{140\left(\sqrt{x^2+1}+x\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/Sqrt[x + Sqrt[1 + x^2]],x]
```

```
[Out] (-5 - 35*(x + Sqrt[1 + x^2])^2 + 105*(x + Sqrt[1 + x^2])^4 + 7*(x + Sqrt[1
+ x^2])^6)/(140*(x + Sqrt[1 + x^2])^(7/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 79, normalized size = 1.00

$$\frac{2\left(28x^6+147x^4+112x^2+9\right)}{35\left(\sqrt{x^2+1}+x\right)^{7/2}}+\frac{2\sqrt{x^2+1}\left(4x^5+19x^3+7x\right)}{5\left(\sqrt{x^2+1}+x\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/Sqrt[x + Sqrt[1 + x^2]],x]
```

```
[Out] (2*Sqrt[1 + x^2]*(7*x + 19*x^3 + 4*x^5))/(5*(x + Sqrt[1 + x^2])^(7/2)) + (2
*(9 + 112*x^2 + 147*x^4 + 28*x^6))/(35*(x + Sqrt[1 + x^2])^(7/2))
```

fricas [A] time = 0.44, size = 43, normalized size = 0.54

$$-\frac{2}{35}\left(5\,x^4+12\,x^2-\left(5\,x^3+13\,x\right)\sqrt{x^2+1}-9\right)\sqrt{x+\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/35*(5*x^4 + 12*x^2 - (5*x^3 + 13*x)*sqrt(x^2 + 1) - 9)*sqrt(x + sqrt(x^2
 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/sqrt(x + sqrt(x^2 + 1)), x)
```

maple [C] time = 0.05, size = 84, normalized size = 1.06

$$-\frac{-\frac{32\sqrt{\pi}\,\sqrt{2}\,\cosh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{1}{x}\right)}{2}\right)}{3x^{\frac{3}{2}}}-\frac{8\sqrt{\pi}\,\sqrt{2}\,x^{\frac{3}{2}}\left(-\frac{4}{3x^4}-\frac{2}{3x^2}+\frac{2}{3}\right)\sinh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{1}{x}\right)}{2}\right)}{\sqrt{1+\frac{1}{x^2}}}}{8\sqrt{\pi}}+\frac{\sqrt{2}\,x^{\frac{5}{2}}\,\mathrm{hypergeom}\left(\left[-\frac{5}{4},\frac{1}{4},\frac{3}{4}\right],\left[-\frac{1}{4},\frac{3}{2}\right],-\frac{1}{x^2}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] -1/8/Pi^(1/2)*(-32/3*Pi^(1/2)*2^(1/2)/x^(3/2)*cosh(3/2*arcsinh(1/x))-8*Pi^(
1/2)*2^(1/2)*x^(3/2)*(-4/3/x^4-2/3/x^2+2/3)*sinh(3/2*arcsinh(1/x))/(1+1/x^2
)^(1/2))+1/5*2^(1/2)*x^(5/2)*hypergeom([-5/4,1/4,3/4],[-1/4,3/2],-1/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/sqrt(x + sqrt(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+1}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x + (x^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int((x^2 + 1)/(x + (x^2 + 1)^(1/2))^(1/2), x)
```

sympy [A] time = 0.62, size = 92, normalized size = 1.16

$$\frac{12x^3}{35\sqrt{x+\sqrt{x^2+1}}}+\frac{2x^2\sqrt{x^2+1}}{35\sqrt{x+\sqrt{x^2+1}}}+\frac{44x}{35\sqrt{x+\sqrt{x^2+1}}}+\frac{18\sqrt{x^2+1}}{35\sqrt{x+\sqrt{x^2+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] 12*x**3/(35*sqrt(x + sqrt(x**2 + 1))) + 2*x**2*sqrt(x**2 + 1)/(35*sqrt(x +
sqrt(x**2 + 1))) + 44*x/(35*sqrt(x + sqrt(x**2 + 1))) + 18*sqrt(x**2 + 1)/(
35*sqrt(x + sqrt(x**2 + 1)))
```

3.961 $\int \frac{x^2}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal. Leaf size=79

$$\frac{1}{4}\sqrt{\sqrt{x^4+1}+x^2}\,x-\frac{x}{8\left(\sqrt{x^4+1}+x^2\right)^{3/2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}\right)}{8\sqrt{2}}$$

Rubi [F] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

```
[Out] Defer[Int][x^2/Sqrt[x^2 + Sqrt[1 + x^4]], x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{x^2}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Mathematica [B] time = 3.30, size = 194, normalized size = 2.46

$$\frac{\sqrt{2}x^2\left(\sqrt{x^4+1}+x^2\right)\left(8x^8+9x^4+8\sqrt{x^4+1}x^6+5\sqrt{x^4+1}x^2+1\right)-\sqrt{x^2\left(\sqrt{x^4+1}+x^2\right)}\left(4x^8+5x^4+4\sqrt{x^4+1}x^6+3\sqrt{x^4+1}x^2+1\right)\tan^{-1}\left(\sqrt{\left(\sqrt{x^4+1}+x^2\right)^2-1}\right)}{8\sqrt{2}\left(\sqrt{x^4+1}+x^2\right)^{5/2}\left(x^5+\sqrt{x^4+1}x^3+x\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

```
[Out] (Sqrt[2]*x^2*(x^2 + Sqrt[1 + x^4])*(1 + 9*x^4 + 8*x^8 + 5*x^2*Sqrt[1 + x^4]
 + 8*x^6*Sqrt[1 + x^4]) - Sqrt[x^2*(x^2 + Sqrt[1 + x^4])]*(1 + 5*x^4 + 4*x^
8 + 3*x^2*Sqrt[1 + x^4] + 4*x^6*Sqrt[1 + x^4])*ArcTan[Sqrt[-1 + (x^2 + Sqrt
[1 + x^4])^2]])/(8*Sqrt[2]*(x^2 + Sqrt[1 + x^4])^(5/2)*(x + x^5 + x^3*Sqrt[
1 + x^4]))
```

IntegrateAlgebraic [A] time = 0.17, size = 79, normalized size = 1.00

$$\frac{1}{4}\sqrt{\sqrt{x^4+1}+x^2}\,x-\frac{x}{8\left(\sqrt{x^4+1}+x^2\right)^{3/2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

```
[Out] -1/8*x/(x^2 + Sqrt[1 + x^4])^(3/2) + (x*Sqrt[x^2 + Sqrt[1 + x^4]])/4 - ArcT
an[Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]]]/(8*Sqrt[2])
```

fricas [A] time = 0.81, size = 81, normalized size = 1.03

$$-\frac{1}{8}\left(2x^5-2\sqrt{x^4+1}\,x^3-x\right)\sqrt{x^2+\sqrt{x^4+1}}+\frac{1}{16}\sqrt{2}\arctan\left(-\frac{\left(\sqrt{2}x^2-\sqrt{2}\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(2*x^5 - 2*sqrt(x^4 + 1)*x^3 - x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1/16*sqr
t(2)*arctan(-1/2*(sqrt(2)*x^2 - sqrt(2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4
+ 1))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

maple [C] time = 0.03, size = 22, normalized size = 0.28

$$\frac{\sqrt{2}\,x^2\,\mathrm{hypergeom}\left(\left[-\frac{1}{2},\frac{1}{4},\frac{3}{4}\right],\left[\frac{1}{2},\frac{3}{2}\right],-\frac{1}{x^4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] 1/4*2^(1/2)*x^2*hypergeom([-1/2,1/4,3/4],[1/2,3/2],-1/x^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

```
[Out] int(x^2/((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

sympy [A] time = 0.73, size = 15, normalized size = 0.19

$$\frac{G_{3,3}^{2,2}\left(\begin{matrix}\frac{3}{2},1 & 2\\ \frac{3}{4},\frac{5}{4} & 0\end{matrix}\middle| x^4\right)}{16\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] meijerg(((3/2, 1), (2,)), ((3/4, 5/4), (0,)), x**4)/(16*sqrt(pi))
```

3.962 $\int \frac{\sqrt{1+x^4}}{\sqrt{x^2+\sqrt{1+x^4}}} \, dx$

Optimal. Leaf size=79

$$\frac{1}{4}\sqrt{\sqrt{x^4+1}+x^2}\,x+\frac{x}{8\left(\sqrt{x^4+1}+x^2\right)^{3/2}}+\frac{5\tan^{-1}\left(\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}\right)}{8\sqrt{2}}$$

Rubi [F] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+x^4}}{\sqrt{x^2+\sqrt{1+x^4}}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + x^4]/Sqrt[x^2 + Sqrt[1 + x^4]],x]
[Out] Defer[Int][Sqrt[1 + x^4]/Sqrt[x^2 + Sqrt[1 + x^4]], x]
```

Rubi steps

$$\int \frac{\sqrt{1+x^4}}{\sqrt{x^2+\sqrt{1+x^4}}} \, dx=\int \frac{\sqrt{1+x^4}}{\sqrt{x^2+\sqrt{1+x^4}}} \, dx$$

Mathematica [B] time = 0.96, size = 200, normalized size = 2.53

$$\frac{\sqrt{2}\left(\sqrt{x^4+1}+x^2\right)\left(16x^8+18x^4+16\sqrt{x^4+1}\,x^6+10\sqrt{x^4+1}\,x^2+3\right)x^2+5\sqrt{x^2\left(\sqrt{x^4+1}+x^2\right)}\left(8x^8+8x^4+8\sqrt{x^4+1}\,x^6+4\sqrt{x^4+1}\,x^2+1\right)\tan^{-1}\left(\sqrt{\left(\sqrt{x^4+1}+x^2\right)^2-1}\right)}{8\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}\left(2x^4+2\sqrt{x^4+1}\,x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^4]/Sqrt[x^2 + Sqrt[1 + x^4]],x]
[Out] (Sqrt[2]*x^2*(x^2 + Sqrt[1 + x^4])*(3 + 18*x^4 + 16*x^8 + 10*x^2*Sqrt[1 + x
^4] + 16*x^6*Sqrt[1 + x^4]) + 5*Sqrt[x^2*(x^2 + Sqrt[1 + x^4])]*(1 + 8*x^4
+ 8*x^8 + 4*x^2*Sqrt[1 + x^4] + 8*x^6*Sqrt[1 + x^4])*ArcTan[Sqrt[-1 + (x^2
+ Sqrt[1 + x^4])^2]])/(8*Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]]*(1 + 2*x^4 + 2
*x^2*Sqrt[1 + x^4])^2)
```

IntegrateAlgebraic [A] time = 0.18, size = 79, normalized size = 1.00

$$\frac{1}{4}\sqrt{\sqrt{x^4+1}+x^2}\,x+\frac{x}{8\left(\sqrt{x^4+1}+x^2\right)^{3/2}}+\frac{5\tan^{-1}\left(\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^4]/Sqrt[x^2 + Sqrt[1 + x^4]],x]
[Out] x/(8*(x^2 + Sqrt[1 + x^4])^(3/2)) + (x*Sqrt[x^2 + Sqrt[1 + x^4]])/4 + (5*Ar
cTan[Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]]])/(8*Sqrt[2])
```

fricas [A] time = 0.91, size = 81, normalized size = 1.03

$$\frac{1}{8}\left(2x^5-2\sqrt{x^4+1}\,x^3+3x\right)\sqrt{x^2+\sqrt{x^4+1}}-\frac{5}{16}\sqrt{2}\arctan\left(-\frac{\left(\sqrt{2}\,x^2-\sqrt{2}\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^5 - 2*sqrt(x^4 + 1)*x^3 + 3*x)*sqrt(x^2 + sqrt(x^4 + 1)) - 5/16*sq
rt(2)*arctan(-1/2*(sqrt(2)*x^2 - sqrt(2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4
 + 1))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^4+1}}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 1)/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^4+1}}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int((x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^4+1}}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 1)/sqrt(x^2 + sqrt(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{x^4+1}}{\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)^(1/2)/((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

```
[Out] int((x^4 + 1)^(1/2)/((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}}{\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x**4 + 1)/sqrt(x**2 + sqrt(x**4 + 1)), x)
```

3.963 $\int \frac{-3+x^2}{\sqrt[3]{-1+x^2}\left(-1+x^2+x^3\right)}\,dx$

Optimal. Leaf size=80

$$\log\left(\sqrt[3]{x^2-1}+x\right)-\frac{1}{2}\log\left(x^2-\sqrt[3]{x^2-1}\,x+\left(x^2-1\right)^{2/3}\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^2-1}}{\sqrt[3]{x^2-1}-2x}\right)$$

Rubi [F] time = 0.41, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3+x^2}{\sqrt[3]{-1+x^2}\left(-1+x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3 + x^2)/((-1 + x^2)^(1/3)*(-1 + x^2 + x^3)),x]
```

```
[Out] -3*Defer[Int][1/((-1 + x^2)^(1/3)*(-1 + x^2 + x^3)), x] + Defer[Int][x^2/((
-1 + x^2)^(1/3)*(-1 + x^2 + x^3)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+x^2}{\sqrt[3]{-1+x^2}\left(-1+x^2+x^3\right)}\,dx &= \int\left(-\frac{3}{\sqrt[3]{-1+x^2}\left(-1+x^2+x^3\right)}+\frac{x^2}{\sqrt[3]{-1+x^2}\left(-1+x^2+x^3\right)}\right)dx\\ &= -\left(3\int\frac{1}{\sqrt[3]{-1+x^2}\left(-1+x^2+x^3\right)}\,dx\right)+\int\frac{x^2}{\sqrt[3]{-1+x^2}\left(-1+x^2+x^3\right)}\,dx\end{aligned}$$

Mathematica [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{-3+x^2}{\sqrt[3]{-1+x^2}\left(-1+x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3 + x^2)/((-1 + x^2)^(1/3)*(-1 + x^2 + x^3)),x]
```

```
[Out] Integrate[(-3 + x^2)/((-1 + x^2)^(1/3)*(-1 + x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.14, size = 80, normalized size = 1.00

$$\log\left(\sqrt[3]{x^2-1}+x\right)-\frac{1}{2}\log\left(x^2-\sqrt[3]{x^2-1}\,x+\left(x^2-1\right)^{2/3}\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^2-1}}{\sqrt[3]{x^2-1}-2x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 + x^2)/((-1 + x^2)^(1/3)*(-1 + x^2 + x^3)),x]
```

```
[Out] Sqrt[3]*ArcTan[(Sqrt[3]*(-1 + x^2)^(1/3))/(-2*x + (-1 + x^2)^(1/3))] + Log[
x + (-1 + x^2)^(1/3)] - Log[x^2 - x*(-1 + x^2)^(1/3) + (-1 + x^2)^(2/3)]/2
```

fricas [A] time = 1.30, size = 100, normalized size = 1.25

$$-\sqrt{3}\arctan\left(\frac{\sqrt{3}x^3+2\sqrt{3}\left(x^2-1\right)^{\frac{1}{3}}x^2+4\sqrt{3}\left(x^2-1\right)^{\frac{2}{3}}x}{x^3-8x^2+8}\right)+\frac{1}{2}\log\left(\frac{x^3+3\left(x^2-1\right)^{\frac{1}{3}}x^2+x^2+3\left(x^2-1\right)^{\frac{2}{3}}x-1}{x^3+x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)/(x^2-1)^(1/3)/(x^3+x^2-1),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan((sqrt(3)*x^3 + 2*sqrt(3)*(x^2 - 1)^(1/3)*x^2 + 4*sqrt(3)*(x
^2 - 1)^(2/3)*x)/(x^3 - 8*x^2 + 8)) + 1/2*log((x^3 + 3*(x^2 - 1)^(1/3)*x^2
+ x^2 + 3*(x^2 - 1)^(2/3)*x - 1)/(x^3 + x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-3}{\left(x^3+x^2-1\right)\left(x^2-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)/(x^2-1)^(1/3)/(x^3+x^2-1),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 3)/((x^3 + x^2 - 1)*(x^2 - 1)^(1/3)), x)
```

maple [C] time = 1.50, size = 255, normalized size = 3.19

$$\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x^3+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2-1\right)^{\frac{2}{3}} x-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2-1\right)^{\frac{1}{3}} x^2-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right) x^3+2x\left(x^2-1\right)^{\frac{2}{3}}+\left(x^2-1\right)^{\frac{1}{3}} x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right) x^2-x^3+x^2-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-1}{x^3+x^2-1}\right)+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x^3+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2-1\right)^{\frac{2}{3}} x+2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2-1\right)^{\frac{1}{3}} x^2-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right) x^3-x\left(x^2-1\right)^{\frac{2}{3}}+\left(x^2-1\right)^{\frac{1}{3}} x^2-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right) x^2-x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)+1}{x^3+x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3)/(x^2-1)^(1/3)/(x^3+x^2-1),x)
```

```
[Out] ln(-(-RootOf(_Z^2+_Z+1)^2*x^3+RootOf(_Z^2+_Z+1)*(x^2-1)^(2/3)*x-RootOf(_Z^2
+_Z+1)*(x^2-1)^(1/3)*x^2-2*RootOf(_Z^2+_Z+1)*x^3+2*x*(x^2-1)^(2/3)+(x^2-1)^
(1/3)*x^2+RootOf(_Z^2+_Z+1)*x^2-x^3+x^2-RootOf(_Z^2+_Z+1)-1)/(x^3+x^2-1))+R
ootOf(_Z^2+_Z+1)*ln((-RootOf(_Z^2+_Z+1)^2*x^3+RootOf(_Z^2+_Z+1)*(x^2-1)^(2/
3)*x+2*RootOf(_Z^2+_Z+1)*(x^2-1)^(1/3)*x^2-RootOf(_Z^2+_Z+1)*x^3-x*(x^2-1)^
(2/3)+(x^2-1)^(1/3)*x^2-RootOf(_Z^2+_Z+1)*x^2-x^2+RootOf(_Z^2+_Z+1)+1)/(x^3
+x^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-3}{\left(x^3+x^2-1\right)\left(x^2-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)/(x^2-1)^(1/3)/(x^3+x^2-1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 3)/((x^3 + x^2 - 1)*(x^2 - 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2-3}{{\left(x^2-1\right)}^{1/3}\,\left(x^3+x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 3)/((x^2 - 1)^(1/3)*(x^2 + x^3 - 1)),x)
```

```
[Out] int((x^2 - 3)/((x^2 - 1)^(1/3)*(x^2 + x^3 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-3}{\sqrt[3]{\left(x-1\right)\left(x+1\right)}\,\left(x^3+x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3)/(x**2-1)**(1/3)/(x**3+x**2-1),x)
```

```
[Out] Integral((x**2 - 3)/(((x - 1)*(x + 1))**(1/3)*(x**3 + x**2 - 1)), x)
```

3.964 $\int \frac{-a^2b+a(2a+b)x-3ax^2+x^3}{x(-b+x)\sqrt{x(-a+x)(-b+x)}\left(a-(1+bd)x+dx^2\right)}\,dx$

Optimal. Leaf size=80

$$2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{x^2(-a-b)+abx+x^3}}{a-x}\right)+\frac{2\sqrt{x^2(-a-b)+abx+x^3}}{x(x-b)}$$

Rubi [F] time = 12.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-a^2b+a(2a+b)x-3ax^2+x^3}{x(-b+x)\sqrt{x(-a+x)(-b+x)}\left(a-(1+bd)x+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a^2*b) + a*(2*a + b)*x - 3*a*x^2 + x^3)/(x*(-b + x)*Sqrt[x*(-a + x)*
(-b + x)]*(a - (1 + b*d)*x + d*x^2)),x]
```

```
[Out] (-4*Sqrt[(a - x)*(b - x)*x])/(b^2*d*x) + (2*Sqrt[(a - x)*(b - x)*x])/(b*d*(
b - x)*x) - (4*Sqrt[a]*Sqrt[(a - x)*(b - x)*x]*Sqrt[1 - x/a]*EllipticE[ArcS
in[Sqrt[x]/Sqrt[a]], a/b])/(b^2*d*(a - x)*Sqrt[x]*Sqrt[1 - x/b]) + (2*Sqrt[
a]*Sqrt[(a - x)*(b - x)*x]*Sqrt[1 - x/a]*Sqrt[1 - x/b]*EllipticF[ArcSin[Sqr
t[x]/Sqrt[a]], a/b])/(b*d*(a - x)*(b - x)*Sqrt[x]) + ((1 - 2*a*d + b*d + Sq
rt[-4*a*d + (1 + b*d)^2])*Sqrt[(a - x)*(b - x)*x]*Defer[Int][Sqrt[a - x]/((
b - x)^(3/2)*x^(3/2)*(-1 - b*d - Sqrt[1 - 4*a*d + 2*b*d + b^2*d^2] + 2*d*x)
), x])/(d*Sqrt[a - x]*Sqrt[b - x]*Sqrt[x]) + ((1 - 2*a*d + b*d - Sqrt[-4*a*
d + (1 + b*d)^2])*Sqrt[(a - x)*(b - x)*x]*Defer[Int][Sqrt[a - x]/((b - x)^(
3/2)*x^(3/2)*(-1 - b*d + Sqrt[1 - 4*a*d + 2*b*d + b^2*d^2] + 2*d*x)), x])/(
d*Sqrt[a - x]*Sqrt[b - x]*Sqrt[x])
```

Rubi steps

$$\int \frac{-a^2b+a(2a+b)x-3ax^2+x^3}{x(-b+x)\sqrt{x(-a+x)(-b+x)}\left(a-(1+bd)x+dx^2\right)}\,dx = \int \frac{\sqrt{(a-x)(b-x)x}\left(ab-2ax+x^2\right)}{(b-x)^2x^2\left(a-(1+bd)x+dx^2\right)}\,dx$$

$$= \frac{\sqrt{(a-x)(b-x)x}\int \frac{\sqrt{a-x}\left(ab-2ax+x^2\right)}{(b-x)^{3/2}x^{3/2}\left(a-(1+bd)x+dx^2\right)}\,dx}{\sqrt{a-x}\sqrt{b-x}\sqrt{x}}$$

$$= \frac{\sqrt{(a-x)(b-x)x}\int \left(\frac{\sqrt{a-x}}{d(b-x)^{3/2}x^{3/2}} - \frac{\sqrt{a-x}\,(a-abd-(1-2ad}{d(b-x)^{3/2}x^{3/2}\left(a+(-1-bc\right.}\right.}{\sqrt{a-x}\sqrt{b-x}\sqrt{x}}$$

$$= \frac{\sqrt{(a-x)(b-x)x}\int \frac{\sqrt{a-x}}{(b-x)^{3/2}x^{3/2}}\,dx}{d\sqrt{a-x}\sqrt{b-x}\sqrt{x}} - \frac{\sqrt{(a-x)(b-x)x}}{d\sqrt{}}$$

$$= \frac{2\sqrt{(a-x)(b-x)x}}{bd(b-x)x} - \frac{\sqrt{(a-x)(b-x)x}\int\left(\frac{(-1+2ac}{(b-x)^{3/2}x^{3/2}(}\right.}{}$$

$$= -\frac{4\sqrt{(a-x)(b-x)x}}{b^2dx} + \frac{2\sqrt{(a-x)(b-x)x}}{bd(b-x)x} + \frac{\left(4\sqrt{(a\right.}{}$$

$$= -\frac{4\sqrt{(a-x)(b-x)x}}{b^2dx} + \frac{2\sqrt{(a-x)(b-x)x}}{bd(b-x)x} - \frac{\left(2\sqrt{(a\right.}{b}$$

$$= -\frac{4\sqrt{(a-x)(b-x)x}}{b^2dx} + \frac{2\sqrt{(a-x)(b-x)x}}{bd(b-x)x} - \frac{\left(\left(-1+\right.\right.}{}$$

$$= -\frac{4\sqrt{(a-x)(b-x)x}}{b^2dx} + \frac{2\sqrt{(a-x)(b-x)x}}{bd(b-x)x} - \frac{4\sqrt{a}\sqrt{(}}{}$$

Mathematica [C] time = 4.76, size = 277, normalized size = 3.46

$$\frac{2(x-a)\left(i\sqrt{\frac{x}{a}}\sqrt{\frac{x-b}{a-b}}\Pi\left(\frac{2ad}{2ad-bd+\sqrt{(bd+1)^2-4ad}-1};i\sinh^{-1}\left(\sqrt{\frac{x}{a}-1}\right)|\frac{a}{a-b}\right)+i\sqrt{\frac{x}{a}}\sqrt{\frac{x-b}{a-b}}\Pi\left(-\frac{2ad}{-2ad+bd+\sqrt{(bd+1)^2-4ad}+1};i\sinh^{-1}\left(\sqrt{\frac{x}{a}-1}\right)|\frac{a}{a-b}\right)-i\sqrt{\frac{x}{a}}\sqrt{\frac{x-b}{a-b}}F\left(i\sinh^{-1}\left(\sqrt{\frac{x}{a}-1}\right)|\frac{a}{a-b}\right)+\sqrt{\frac{x}{a}-1}\right)}{\sqrt{\frac{x}{a}-1}\sqrt{x(x-a)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-(a^2*b) + a*(2*a + b)*x - 3*a*x^2 + x^3)/(x*(-b + x)*Sqrt[x*(-a
 + x)*(-b + x)]*(a - (1 + b*d)*x + d*x^2)),x]
```

```
[Out] (2*(-a + x)*(Sqrt[-1 + x/a] - I*Sqrt[x/a]*Sqrt[(-b + x)/(a - b)]*EllipticF[
I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] + I*Sqrt[x/a]*Sqrt[(-b + x)/(a - b)]*
EllipticPi[(2*a*d)/(-1 + 2*a*d - b*d + Sqrt[-4*a*d + (1 + b*d)^2]), I*ArcSi
nh[Sqrt[-1 + x/a]], a/(a - b)] + I*Sqrt[x/a]*Sqrt[(-b + x)/(a - b)]*Ellipti
cPi[(-2*a*d)/(1 - 2*a*d + b*d + Sqrt[-4*a*d + (1 + b*d)^2]), I*ArcSinh[Sqrt
[-1 + x/a]], a/(a - b)]))/(Sqrt[x*(-a + x)*(-b + x)]*Sqrt[-1 + x/a])
```

IntegrateAlgebraic [A] time = 1.18, size = 80, normalized size = 1.00

$$2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{x^2(-a-b)+abx+x^3}}{a-x}\right)+\frac{2\sqrt{x^2(-a-b)+abx+x^3}}{x(x-b)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a^2*b) + a*(2*a + b)*x - 3*a*x^2 + x^3)/(x*(-b + x)*S
qrt[x*(-a + x)*(-b + x)]*(a - (1 + b*d)*x + d*x^2)),x]
```

```
[Out] (2*Sqrt[a*b*x + (-a - b)*x^2 + x^3])/(x*(-b + x)) + 2*Sqrt[d]*ArcTanh[(Sqrt
[d]*Sqrt[a*b*x + (-a - b)*x^2 + x^3])/(a - x)]
```

fricas [A]   time = 0.85, size = 312, normalized size = 3.90

$$\left[\frac{\left(bx-x^2\right)\sqrt{d}\log\left(\frac{d^2x^4-2\left(bd^2-3d\right)x^3+\left(b^2d^2-6(a+b)d+1\right)x^2+a^2-4\sqrt{abx-(a+b)x^2+x^3}\left(dx^2-(bd-1)x-a\right)\sqrt{d}+2(3abd-a)x}{d^2x^4-2\left(bd^2+d\right)x^3+\left(b^2d^2+2(a+b)d+1\right)x^2+a^2-2(abd+a)x}\right)-4\sqrt{abx-(a+b)x^2+x^3}}{2\left(bx-x^2\right)}, \frac{\left(bx-x^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{abx-(a+b)x^2+x^3}\left(dx^2-(bd-1)x-a\right)\sqrt{-d}}{2\left(abdx-(a+b)dx^2+dx^3\right)}\right)-2\sqrt{abx-(a+b)x^2+x^3}}{bx-x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*b+a*(2*a+b)*x-3*a*x^2+x^3)/x/(-b+x)/(x*(-a+x)*(-b+x))^(1/2)
/(a-(b*d+1)*x+d*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*x - x^2)*sqrt(d)*log((d^2*x^4 - 2*(b*d^2 - 3*d)*x^3 + (b^2*d^2 - 6
*(a + b)*d + 1)*x^2 + a^2 - 4*sqrt(a*b*x - (a + b)*x^2 + x^3)*(d*x^2 - (b*d
 - 1)*x - a)*sqrt(d) + 2*(3*a*b*d - a)*x)/(d^2*x^4 - 2*(b*d^2 + d)*x^3 + (b
^2*d^2 + 2*(a + b)*d + 1)*x^2 + a^2 - 2*(a*b*d + a)*x)) - 4*sqrt(a*b*x - (a
 + b)*x^2 + x^3))/(b*x - x^2), ((b*x - x^2)*sqrt(-d)*arctan(1/2*sqrt(a*b*x
- (a + b)*x^2 + x^3)*(d*x^2 - (b*d - 1)*x - a)*sqrt(-d)/(a*b*d*x - (a + b)*
d*x^2 + d*x^3)) - 2*sqrt(a*b*x - (a + b)*x^2 + x^3))/(b*x - x^2)]
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2b-(2a+b)ax+3ax^2-x^3}{\sqrt{(a-x)(b-x)x}\left(dx^2-(bd+1)x+a\right)(b-x)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*b+a*(2*a+b)*x-3*a*x^2+x^3)/x/(-b+x)/(x*(-a+x)*(-b+x))^(1/2)
/(a-(b*d+1)*x+d*x^2),x, algorithm="giac")
```

```
[Out] integrate((a^2*b - (2*a + b)*a*x + 3*a*x^2 - x^3)/(sqrt((a - x)*(b - x)*x)*
(d*x^2 - (b*d + 1)*x + a)*(b - x)*x), x)
```

maple [C]   time = 0.03, size = 2958, normalized size = 36.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*b+a*(2*a+b)*x-3*a*x^2+x^3)/x/(-b+x)/(x*(-a+x)*(-b+x))^(1/2)/(a-(b
*d+1)*x+d*x^2),x)
```

```
[Out] -1/(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(
1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2/d-1/2/d*(b^2*
d^2-4*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(b*d+1+(b
^2*d^2-4*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))*b^2*d-2/(b^2*d^2-4*a*d+2*b*d
+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b
*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2/d-1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2
))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(b*d+1+(b^2*d^2-4*a*d+2*b*d+1)^(
1/2))),(a/(a-b))^(1/2))*b+4/(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*a^2*(1-1/a*x)^(1/
2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)
/(a-1/2*b-1/2/d-1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)
^(1/2),a/(a-1/2/d*(b*d+1+(b^2*d^2-4*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))-1
/(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/
2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2/d-1/2/d*(b^2*d^
2-4*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(b*d+1+(b^2
```

```
*d^2-4*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))/d+2*a^2*(1-1/a*x)^(1/2)*(-1/(a
-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b
-1/2/d-1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/
(a-1/2/d*(b*d+1+(b^2*d^2-4*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))-a*(1-1/a*x
)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^
(1/2)/(a-1/2*b-1/2/d-1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+
x)/a)^(1/2),a/(a-1/2/d*(b*d+1+(b^2*d^2-4*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/
2))*b-a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a
*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2/d-1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2))*E
llipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(b*d+1+(b^2*d^2-4*a*d+2*b*d+1)^(1/2)
)),(a/(a-b))^(1/2))/d+1/(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1
/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/
2*b+1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2)
,a/(a+1/2*(-b*d+(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))*b^2*d+
2/(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1
/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b+1/2/d*(b^2*d^2-4*a
*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-b*d+(b^2*d
^2-4*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))*b+2*a^2*(1-1/a*x)^(1/2)*(-1/
(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2
*b+1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),
a/(a+1/2*(-b*d+(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))-a*(1-1/
a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^
3)^(1/2)/(a-1/2*b+1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(
-a+x)/a)^(1/2),a/(a+1/2*(-b*d+(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))
^(1/2))*b-a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b
*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b+1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1/2/
d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-b*d+(b^2*d^2-4*a*d+2*b*d+1)^(1/2
)-1)/d),(a/(a-b))^(1/2))/d-4/(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*a^2*(1-1/a*x)^(1
/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2
)/(a-1/2*b+1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a
)^(1/2),a/(a+1/2*(-b*d+(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))
+1/(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(
1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b+1/2/d*(b^2*d^2-4*
a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-b*d+(b^2*
d^2-4*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))/d+(a-b)*(2*(-a*x+x^2)/(a-b)
/b/((-b+x)*(-a*x+x^2))^(1/2)-2*(-1/b+a/(a-b)/b)*a*(-(-a+x)/a)^(1/2)*((-b+x)
/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)*EllipticF((-(-a+x
)/a)^(1/2),(a/(a-b))^(1/2))+2/(a-b)/b*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1
/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)*((a-b)*EllipticE((-(-a+x)/a
)^(1/2),(a/(a-b))^(1/2))+b*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))))+a
*(-2*(a*b-a*x-b*x+x^2)/a/b/(x*(a*b-a*x-b*x+x^2))^(1/2)-2*((a+b)/a/b+(-a-b)/
a/b)*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*
x^2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))-2/b*(-(-a+x)/a)
^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)*((a
-b)*EllipticE((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))+b*EllipticF((-(-a+x)/a)^(1
/2),(a/(a-b))^(1/2))))
```

<u>**maxima**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2b - (2\,a + b)ax + 3\,ax^2 - x^3}{\sqrt{(a-x)(b-x)x}\left(dx^2 - (bd+1)x + a\right)(b-x)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*b+a*(2*a+b)*x-3*a*x^2+x^3)/x/(-b+x)/(x*(-a+x)*(-b+x))^(1/2)
/(a-(b*d+1)*x+d*x^2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*b - (2*a + b)*a*x + 3*a*x^2 - x^3)/(sqrt((a - x)*(b - x)*x)*
(d*x^2 - (b*d + 1)*x + a)*(b - x)*x), x)
```

mupad [B] time = 0.87, size = 628, normalized size = 7.85

$$\frac{2\,a\,\left(\frac{\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)-\frac{\sqrt{\frac{b-x}{a-b}+1}\,\sqrt{\frac{b-x}{b}}}{\sqrt{1-\frac{b-x}{b}}}}{\frac{b}{a-b}+1}-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\right)\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}}{\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}+\frac{b\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(\frac{b}{b-\frac{b\,d-\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}+1}{2\,d}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\,\left(2\,a\,d-b\,d+\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}-1\right)}{d\,\left(b-\frac{b\,d-\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}+1}{2\,d}\right)\,\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}+\frac{2\,a\,(a-b)\,\sqrt{\frac{x}{a}}\,\left(\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x}{a}}\right)\middle|\frac{a}{b}\right)-\frac{a\,\sin\left(2\,\mathrm{asin}\left(\sqrt{\frac{x}{a}}\right)\right)}{2\,b\,\sqrt{1-\frac{x}{b}}}\right)\,\sqrt{\frac{a-x}{a}}\,\sqrt{\frac{b-x}{b}}}{b\,\left(\frac{a}{b}-1\right)\,\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}-\frac{b\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(\frac{b}{b-\frac{b\,d+\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}+1}{2\,d}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\,\left(b\,d-2\,a\,d+\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}+1\right)}{d\,\left(b-\frac{b\,d+\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}+1}{2\,d}\right)\,\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*b + 3*a*x^2 - x^3 - a*x*(2*a + b))/(x*(b - x)*(x*(a - x)*(b - x))^
(1/2)*(a - x*(b*d + 1) + d*x^2)),x)
```

```
[Out] (2*a*((ellipticE(asin(((b - x)/b)^(1/2)), -b/(a - b)) - (((b - x)/(a - b) +
 1)^(1/2)*((b - x)/b)^(1/2))/(1 - (b - x)/b)^(1/2))/(b/(a - b) + 1) - ellip
ticF(asin(((b - x)/b)^(1/2)), -b/(a - b)))*(x/b)^(1/2)*((b - x)/b)^(1/2)*((
a - x)/(a - b))^(1/2))/(x^3 - x^2*(a + b) + a*b*x)^(1/2) + (b*(x/b)^(1/2)*(
(b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*ellipticPi(b/(b - (b*d - (2*b*d -
4*a*d + b^2*d^2 + 1)^(1/2) + 1)/(2*d)), asin(((b - x)/b)^(1/2)), -b/(a - b)
)*(2*a*d - b*d + (2*b*d - 4*a*d + b^2*d^2 + 1)^(1/2) - 1))/(d*(b - (b*d - (
2*b*d - 4*a*d + b^2*d^2 + 1)^(1/2) + 1)/(2*d))*(x^3 - x^2*(a + b) + a*b*x)^
(1/2)) + (2*a*(a - b)*(x/a)^(1/2)*(ellipticE(asin((x/a)^(1/2)), a/b) - (a*s
in(2*asin((x/a)^(1/2))))/(2*b*(1 - x/b)^(1/2)))*((a - x)/a)^(1/2)*((b - x)/
b)^(1/2))/(b*(a/b - 1)*(x^3 - x^2*(a + b) + a*b*x)^(1/2)) - (b*(x/b)^(1/2)*
((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*ellipticPi(b/(b - (b*d + (2*b*d -
 4*a*d + b^2*d^2 + 1)^(1/2) + 1)/(2*d)), asin(((b - x)/b)^(1/2)), -b/(a - b
))*(b*d - 2*a*d + (2*b*d - 4*a*d + b^2*d^2 + 1)^(1/2) + 1))/(d*(b - (b*d +
(2*b*d - 4*a*d + b^2*d^2 + 1)^(1/2) + 1)/(2*d))*(x^3 - x^2*(a + b) + a*b*x)
^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*b+a*(2*a+b)*x-3*a*x**2+x**3)/x/(-b+x)/(x*(-a+x)*(-b+x))**(
1/2)/(a-(b*d+1)*x+d*x**2),x)
```

```
[Out] Timed out
```

3.965 $\int \frac{x^2(-4+x^3)}{(-1+x^3)^{3/4}(-1+x^3+x^4)}\,dx$

Optimal. Leaf size=80

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^3-1}}{\sqrt{x^3-1}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^3-1}}{\sqrt{x^3-1}+x^2}\right)$$

Rubi [F] time = 1.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2(-4+x^3)}{(-1+x^3)^{3/4}(-1+x^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(-4 + x^3))/((-1 + x^3)^(3/4)*(-1 + x^3 + x^4)),x]
```

```
[Out] -((x*(1 - x^3)^(3/4)*Hypergeometric2F1[1/3, 3/4, 4/3, x^3])/(-1 + x^3)^(3/4
)) + (x^2*(1 - x^3)^(3/4)*Hypergeometric2F1[2/3, 3/4, 5/3, x^3])/(2*(-1 + x
^3)^(3/4)) - Defer[Int][1/((-1 + x^3)^(3/4)*(-1 + x^3 + x^4)), x] + Defer[I
nt][x/((-1 + x^3)^(3/4)*(-1 + x^3 + x^4)), x] - 4*Defer[Int][x^2/((-1 + x^3
)^(3/4)*(-1 + x^3 + x^4)), x] + Defer[Int][x^3/((-1 + x^3)^(3/4)*(-1 + x^3
+ x^4)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(-4+x^3)}{(-1+x^3)^{3/4}(-1+x^3+x^4)}\,dx &= \int\left(-\frac{1}{(-1+x^3)^{3/4}}+\frac{x}{(-1+x^3)^{3/4}}-\frac{1-x+4x^2-x^3}{(-1+x^3)^{3/4}(-1+x^3+x^4)}\right)dx\\ &= -\int\frac{1}{(-1+x^3)^{3/4}}\,dx+\int\frac{x}{(-1+x^3)^{3/4}}\,dx-\int\frac{1-x+4x^2-x^3}{(-1+x^3)^{3/4}(-1+x^3+x^4)}\,d\\ &= -\frac{(1-x^3)^{3/4}\int\frac{1}{(1-x^3)^{3/4}}\,dx}{(-1+x^3)^{3/4}}+\frac{(1-x^3)^{3/4}\int\frac{x}{(1-x^3)^{3/4}}\,dx}{(-1+x^3)^{3/4}}-\int\left(\frac{1}{(-1+x^3)^{3/4}(-1\right.\\ &= -\frac{x(1-x^3)^{3/4}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};x^3\right)}{(-1+x^3)^{3/4}}+\frac{x^2(1-x^3)^{3/4}\,{}_2F_1\left(\frac{2}{3},\frac{3}{4};\frac{5}{3};x^3\right)}{2(-1+x^3)^{3/4}}-4\int\frac{}{(-1+}\end{aligned}$$

Mathematica [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x^2(-4+x^3)}{(-1+x^3)^{3/4}(-1+x^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(-4 + x^3))/((-1 + x^3)^(3/4)*(-1 + x^3 + x^4)),x]
```

```
[Out] Integrate[(x^2*(-4 + x^3))/((-1 + x^3)^(3/4)*(-1 + x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 2.62, size = 80, normalized size = 1.00

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x\sqrt[4]{x^3-1}}{\sqrt{x^3-1}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt[4]{x^3-1}}{\sqrt{x^3-1}+x^2}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(-4 + x^3))/((-1 + x^3)^(3/4)*(-1 + x^3 + x^4)),x]

[Out] Sqrt[2]*ArcTan[(Sqrt[2]*x*(-1 + x^3)^(1/4))/(-x^2 + Sqrt[-1 + x^3])] - Sqrt[2]*ArcTanh[(Sqrt[2]*x*(-1 + x^3)^(1/4))/(x^2 + Sqrt[-1 + x^3])]

fricas [B] time = 0.44, size = 189, normalized size = 2.36

$$2\sqrt{2}\arctan\left(\frac{\sqrt{2}x\sqrt{\frac{x^2+\sqrt{2}(x^3-1)^{\frac{1}{4}}x+\sqrt{x^3-1}}{x^2}}-x-\sqrt{2}(x^3-1)^{\frac{1}{4}}}{x}\right)+2\sqrt{2}\arctan\left(\frac{\sqrt{2}x\sqrt{\frac{x^2-\sqrt{2}(x^3-1)^{\frac{1}{4}}x+\sqrt{x^3-1}}{x^2}}+x-\sqrt{2}(x^3-1)^{\frac{1}{4}}}{x}\right)-\frac{1}{2}\sqrt{2}\log\left(\frac{4\left(x^2+\sqrt{2}(x^3-1)^{\frac{1}{4}}x+\sqrt{x^3-1}\right)}{x^2}\right)+\frac{1}{2}\sqrt{2}\log\left(\frac{4\left(x^2-\sqrt{2}(x^3-1)^{\frac{1}{4}}x+\sqrt{x^3-1}\right)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(x^3-4)/(x^3-1)^(3/4)/(x^4+x^3-1),x, algorithm="fricas")

[Out] 2*sqrt(2)*arctan((sqrt(2)*x*sqrt((x^2 + sqrt(2)*(x^3 - 1)^(1/4)*x + sqrt(x^3 - 1))/x^2) - x - sqrt(2)*(x^3 - 1)^(1/4))/x) + 2*sqrt(2)*arctan((sqrt(2)*x*sqrt((x^2 - sqrt(2)*(x^3 - 1)^(1/4)*x + sqrt(x^3 - 1))/x^2) + x - sqrt(2)*(x^3 - 1)^(1/4))/x) - 1/2*sqrt(2)*log(4*(x^2 + sqrt(2)*(x^3 - 1)^(1/4)*x + sqrt(x^3 - 1))/x^2) + 1/2*sqrt(2)*log(4*(x^2 - sqrt(2)*(x^3 - 1)^(1/4)*x + sqrt(x^3 - 1))/x^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x^3-4)x^2}{(x^4+x^3-1)(x^3-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(x^3-4)/(x^3-1)^(3/4)/(x^4+x^3-1),x, algorithm="giac")

[Out] integrate((x^3 - 4)*x^2/((x^4 + x^3 - 1)*(x^3 - 1)^(3/4)), x)

maple [C] time = 1.60, size = 207, normalized size = 2.59

$$-\mathrm{RootOf}(\_Z^4+1)\ln\left(\frac{\mathrm{RootOf}(\_Z^4+1)^3x^4+2(x^3-1)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^4+1)^2x^3-\mathrm{RootOf}(\_Z^4+1)^3x^3+2\sqrt{x^3-1}\,\mathrm{RootOf}(\_Z^4+1)x^2+2(x^3-1)^{\frac{3}{4}}x+\mathrm{RootOf}(\_Z^4+1)^3}{x^4+x^3-1}\right)+\mathrm{RootOf}(\_Z^4+1)^3\ln\left(\frac{-2\sqrt{x^3-1}\,\mathrm{RootOf}(\_Z^4+1)^3x^2-2(x^3-1)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^4+1)^2x^3-\mathrm{RootOf}(\_Z^4+1)x^4+2(x^3-1)^{\frac{3}{4}}x+\mathrm{RootOf}(\_Z^4+1)x^3-\mathrm{RootOf}(\_Z^4+1)}{x^4+x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(x^3-4)/(x^3-1)^(3/4)/(x^4+x^3-1),x)

[Out] -RootOf(_Z^4+1)*ln(-(RootOf(_Z^4+1)^3*x^4+2*(x^3-1)^(1/4)*RootOf(_Z^4+1)^2*x^3-RootOf(_Z^4+1)^3*x^3+2*(x^3-1)^(1/2)*RootOf(_Z^4+1)*x^2+2*(x^3-1)^(3/4)*x+RootOf(_Z^4+1)^3)/(x^4+x^3-1))+RootOf(_Z^4+1)^3*ln(-(-2*(x^3-1)^(1/2)*RootOf(_Z^4+1)^3*x^2-2*(x^3-1)^(1/4)*RootOf(_Z^4+1)^2*x^3-RootOf(_Z^4+1)*x^4+2*(x^3-1)^(3/4)*x+RootOf(_Z^4+1)*x^3-RootOf(_Z^4+1))/(x^4+x^3-1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x^3-4)x^2}{(x^4+x^3-1)(x^3-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-4)/(x^3-1)^(3/4)/(x^4+x^3-1),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 4)*x^2/((x^4 + x^3 - 1)*(x^3 - 1)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\left(x^3-4\right)}{\left(x^3-1\right)^{3/4}\left(x^4+x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^3 - 4))/((x^3 - 1)^(3/4)*(x^3 + x^4 - 1)),x)
```

```
[Out] int((x^2*(x^3 - 4))/((x^3 - 1)^(3/4)*(x^3 + x^4 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3-4)/(x**3-1)**(3/4)/(x**4+x**3-1),x)
```

```
[Out] Timed out
```

3.966 $\int \frac{x^6}{\left(b+ax^4\right)^{3/4}}\,dx$

Optimal. Leaf size=80

$$\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{7/4}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{7/4}}+\frac{x^3\sqrt[4]{ax^4+b}}{4a}$$

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {321, 331, 298, 203, 206}

$$\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{7/4}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{7/4}}+\frac{x^3\sqrt[4]{ax^4+b}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b + a*x^4)^(3/4),x]
```

```
[Out] (x^3*(b + a*x^4)^(1/4))/(4*a) + (3*b*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)])
/(8*a^(7/4)) - (3*b*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(8*a^(7/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{x^6}{\left(b+ax^4\right)^{3/4}}\,dx = \frac{x^3\sqrt[4]{b+ax^4}}{4a} - \frac{(3b)\int \frac{x^2}{\left(b+ax^4\right)^{3/4}}\,dx}{4a}$$

$$= \frac{x^3\sqrt[4]{b+ax^4}}{4a} - \frac{(3b)\,\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{4a}$$

$$= \frac{x^3\sqrt[4]{b+ax^4}}{4a} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{8a^{3/2}} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{1+\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{8a^{3/2}}$$

$$= \frac{x^3\sqrt[4]{b+ax^4}}{4a} + \frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{8a^{7/4}} - \frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{8a^{7/4}}$$

Mathematica [A] time = 0.02, size = 75, normalized size = 0.94

$$\frac{2a^{3/4}x^3\sqrt[4]{ax^4+b} + 3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right) - 3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b + a*x^4)^(3/4),x]
```

```
[Out] (2*a^(3/4)*x^3*(b + a*x^4)^(1/4) + 3*b*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)
] - 3*b*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(8*a^(7/4))
```

IntegrateAlgebraic [A] time = 0.40, size = 80, normalized size = 1.00

$$\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{7/4}} - \frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{7/4}} + \frac{x^3\sqrt[4]{ax^4+b}}{4a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/(b + a*x^4)^(3/4),x]
```

```
[Out] (x^3*(b + a*x^4)^(1/4))/(4*a) + (3*b*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)])
/(8*a^(7/4)) - (3*b*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(8*a^(7/4))
```

fricas [B] time = 0.47, size = 204, normalized size = 2.55

$$\frac{4\left(ax^4+b\right)^{\frac{1}{4}}x^3 + 12\,a\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}\arctan\left(\frac{a^5x\sqrt{\frac{a^4x^2\sqrt{\frac{b^4}{a^7}}+\sqrt{ax^4+b}\,b^2}{x^2}}\left(\frac{b^4}{a^7}\right)^{\frac{3}{4}} - \left(ax^4+b\right)^{\frac{1}{4}}a^5b\left(\frac{b^4}{a^7}\right)^{\frac{3}{4}}}{b^4x}\right) - 3\,a\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}\log\left(\frac{3\left(a^2x\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}+\left(ax^4+b\right)^{\frac{1}{4}}b\right)}{x}\right) + 3\,a\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}\log\left(-\frac{3\left(a^2x\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}-\left(ax^4+b\right)^{\frac{1}{4}}b\right)}{x}\right)}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a*x^4+b)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/16*(4*(a*x^4 + b)^(1/4)*x^3 + 12*a*(b^4/a^7)^(1/4)*arctan((a^5*x*sqrt((a^
4*x^2*sqrt(b^4/a^7) + sqrt(a*x^4 + b)*b^2)/x^2)*(b^4/a^7)^(3/4) - (a*x^4 +
b)^(1/4)*a^5*b*(b^4/a^7)^(3/4))/(b^4*x)) - 3*a*(b^4/a^7)^(1/4)*log(3*(a^2*x
*(b^4/a^7)^(1/4) + (a*x^4 + b)^(1/4)*b)/x) + 3*a*(b^4/a^7)^(1/4)*log(-3*(a^
2*x*(b^4/a^7)^(1/4) - (a*x^4 + b)^(1/4)*b)/x))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(ax^4+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a*x^4+b)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(a*x^4 + b)^(3/4), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a\,x^4+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^4+b)^(3/4),x)
```

```
[Out] int(x^6/(a*x^4+b)^(3/4),x)
```

maxima [A] time = 0.42, size = 110, normalized size = 1.38

$$-\frac{3\left(\frac{2\,b\arctan\left(\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{3}{4}}}-\frac{b\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{3}{4}}}\right)}{16\,a}-\frac{\left(ax^4+b\right)^{\frac{1}{4}}b}{4\left(a^2-\frac{\left(ax^4+b\right)a}{x^4}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a*x^4+b)^(3/4),x, algorithm="maxima")
```

```
[Out] -3/16*(2*b*arctan((a*x^4 + b)^(1/4)/(a^(1/4)*x))/a^(3/4) - b*log(-(a^(1/4)
- (a*x^4 + b)^(1/4)/x)/(a^(1/4) + (a*x^4 + b)^(1/4)/x))/a^(3/4))/a - 1/4*(a
*x^4 + b)^(1/4)*b/((a^2 - (a*x^4 + b)*a/x^4)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(a\,x^4+b\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b + a*x^4)^(3/4),x)
```

```
[Out] int(x^6/(b + a*x^4)^(3/4), x)
```

sympy [C] time = 1.08, size = 37, normalized size = 0.46

$$\frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{ax^4e^{i\pi}}{b}\right)}{4b^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(a*x**4+b)**(3/4),x)
```

```
[Out] x**7*gamma(7/4)*hyper((3/4, 7/4), (11/4,), a*x**4*exp_polar(I*pi)/b)/(4*b**
(3/4)*gamma(11/4))
```

3.967 $\int \frac{\sqrt[4]{bx^2+ax^4}}{x^2}\,dx$

Optimal. Leaf size=80

$$-\frac{2\sqrt[4]{ax^4+bx^2}}{x}-\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)+\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)$$

Rubi [A] time = 0.15, antiderivative size = 142, normalized size of antiderivative = 1.78, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2020, 2032, 329, 331, 298, 203, 206}

$$-\frac{2\sqrt[4]{ax^4+bx^2}}{x}-\frac{\sqrt[4]{a}\,x^{3/2}\left(ax^2+b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{\left(ax^4+bx^2\right)^{3/4}}+\frac{\sqrt[4]{a}\,x^{3/2}\left(ax^2+b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{\left(ax^4+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + a*x^4)^(1/4)/x^2,x]
```

```
[Out] (-2*(b*x^2 + a*x^4)^(1/4))/x - (a^(1/4)*x^(3/2)*(b + a*x^2)^(3/4)*ArcTan[(a
^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(b*x^2 + a*x^4)^(3/4) + (a^(1/4)*x^(3/2
)*(b + a*x^2)^(3/4)*ArcTanh[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(b*x^2 +
a*x^4)^(3/4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{bx^2+ax^4}}{x^2}\,dx &= -\frac{2\sqrt[4]{bx^2+ax^4}}{x} + a\int \frac{x^2}{\left(bx^2+ax^4\right)^{3/4}}\,dx \\
&= -\frac{2\sqrt[4]{bx^2+ax^4}}{x} + \frac{\left(ax^{3/2}\left(b+ax^2\right)^{3/4}\right)\int \frac{\sqrt{x}}{\left(b+ax^2\right)^{3/4}}\,dx}{\left(bx^2+ax^4\right)^{3/4}} \\
&= -\frac{2\sqrt[4]{bx^2+ax^4}}{x} + \frac{\left(2ax^{3/2}\left(b+ax^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{\left(b+ax^4\right)^{3/4}}\,dx, x, \sqrt{x}\right)}{\left(bx^2+ax^4\right)^{3/4}} \\
&= -\frac{2\sqrt[4]{bx^2+ax^4}}{x} + \frac{\left(2ax^{3/2}\left(b+ax^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{\left(bx^2+ax^4\right)^{3/4}} \\
&= -\frac{2\sqrt[4]{bx^2+ax^4}}{x} + \frac{\left(\sqrt{a}\,x^{3/2}\left(b+ax^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{\left(bx^2+ax^4\right)^{3/4}} - \frac{\left(\sqrt{a}\,x^{3/2}\left(b+a\right.\right.}{} \\
&= -\frac{2\sqrt[4]{bx^2+ax^4}}{x} - \frac{\sqrt[4]{a}\,x^{3/2}\left(b+ax^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{\left(bx^2+ax^4\right)^{3/4}} + \frac{\sqrt[4]{a}\,x^{3/2}\left(b+ax^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}}{\sqrt[4]{b+}}\right.}{\left(bx^2+ax^4\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 53, normalized size = 0.66

$$
-\frac{2\sqrt[4]{x^2\left(ax^2+b\right)}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};-\frac{ax^2}{b}\right)}{x\sqrt[4]{\frac{ax^2}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + a*x^4)^(1/4)/x^2,x]
```

```
[Out] (-2*(x^2*(b + a*x^2))^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, -((a*x^2)/b)
])/(x*(1 + (a*x^2)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.23, size = 80, normalized size = 1.00

$$
-\frac{2\sqrt[4]{ax^4+bx^2}}{x} - \sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right) + \sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^2 + a*x^4)^(1/4)/x^2,x]
```

```
[Out] (-2*(b*x^2 + a*x^4)^(1/4))/x - a^(1/4)*ArcTan[(a^(1/4)*x)/(b*x^2 + a*x^4)^(
1/4)] + a^(1/4)*ArcTanh[(a^(1/4)*x)/(b*x^2 + a*x^4)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^2)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.37, size = 185, normalized size = 2.31

$$\frac{1}{2}\sqrt{2}(-a)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)+\frac{1}{2}\sqrt{2}(-a)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)+\frac{1}{4}\sqrt{2}(-a)^{\frac{1}{4}}\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)-\frac{1}{4}\sqrt{2}(-a)^{\frac{1}{4}}\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)-2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^2)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(-a)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^
2)^(1/4))/(-a)^(1/4)) + 1/2*sqrt(2)*(-a)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(-a)^(1/4) - 2*(a + b/x^2)^(1/4))/(-a)^(1/4)) + 1/4*sqrt(2)*(-a)^(1/4)*log
(sqrt(2)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sqrt(a + b/x^2)) - 1/4*s
qrt(2)*(-a)^(1/4)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sq
rt(a + b/x^2)) - 2*(a + b/x^2)^(1/4)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{4}+b\,x^{2}\right)^{\frac{1}{4}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b*x^2)^(1/4)/x^2,x)
```

```
[Out] int((a*x^4+b*x^2)^(1/4)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{4}+bx^{2}\right)^{\frac{1}{4}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^2)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + b*x^2)^(1/4)/x^2, x)
```

mupad [B] time = 0.98, size = 44, normalized size = 0.55

$$-\frac{2\left(a\,x^4+b\,x^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ -\frac{a\,x^2}{b}\right)}{x\,{\left(\frac{a\,x^2}{b}+1\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4 + b*x^2)^(1/4)/x^2,x)
```

```
[Out] -(2*(a*x^4 + b*x^2)^(1/4)*hypergeom([-1/4, -1/4], 3/4, -(a*x^2)/b))/(x*((a*
x^2)/b + 1)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\left(ax^2+b\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b*x**2)**(1/4)/x**2,x)
```

```
[Out] Integral((x**2*(a*x**2 + b))**(1/4)/x**2, x)
```

3.968 $\int \frac{-2ab+(a+b)x}{\sqrt[4]{x^2(-a+x)(-b+x)}\left(abd-(a+b)dx+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=81

$$-\frac{2\tan^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt[4]{x^3(-a-b)+abx^2+x^4}}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt[4]{x^3(-a-b)+abx^2+x^4}}\right)}{d^{3/4}}$$

Rubi [F] time = 5.91, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2ab+(a+b)x}{\sqrt[4]{x^2(-a+x)(-b+x)}\left(abd-(a+b)dx+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*a*b + (a + b)*x)/((x^2*(-a + x)*(-b + x))^(1/4)*(a*b*d - (a + b)*d*
x + (-1 + d)*x^2)),x]
```

```
[Out] ((a + b + Sqrt[2*a*b*(2 - d) + a^2*d + b^2*d]/Sqrt[d])*Sqrt[x]*(-a + x)^(1/
4)*(-b + x)^(1/4)*Defer[Int][1/(Sqrt[x]*(-a + x)^(1/4)*(-b + x)^(1/4)*(-((a
 + b)*d) - Sqrt[d]*Sqrt[4*a*b + a^2*d - 2*a*b*d + b^2*d] + 2*(-1 + d)*x)),
x])/((a - x)*(b - x)*x^2)^(1/4) + ((a + b - Sqrt[2*a*b*(2 - d) + a^2*d + b^
2*d]/Sqrt[d])*Sqrt[x]*(-a + x)^(1/4)*(-b + x)^(1/4)*Defer[Int][1/(Sqrt[x]*(
-a + x)^(1/4)*(-b + x)^(1/4)*(-((a + b)*d) + Sqrt[d]*Sqrt[4*a*b + a^2*d - 2
*a*b*d + b^2*d] + 2*(-1 + d)*x)), x])/((a - x)*(b - x)*x^2)^(1/4)
```

Rubi steps

$$\int \frac{-2ab+(a+b)x}{\sqrt[4]{x^2(-a+x)(-b+x)}\left(abd-(a+b)dx+(-1+d)x^2\right)}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\int \frac{-2ab+(a+b)x}{\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\left(abd-(a+b)d\ldots\right.}}{\sqrt[4]{x^2(-a+x)(-b+x)}}$$

$$= \frac{\left(\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\int \left(\frac{a+b+\ldots}{\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\left(-((a+b)d)\ldots\right.}\right.}{\ldots}$$

$$= \frac{\left(\left(a+b-\frac{\sqrt{2ab(2-d)+a^2d+b^2d}}{\sqrt{d}}\right)\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right.\ldots}{\sqrt[4]{x^2(-\ldots}}$$

Mathematica [F] time = 8.40, size = 0, normalized size = 0.00

$$\int \frac{-2ab+(a+b)x}{\sqrt[4]{x^2(-a+x)(-b+x)}\left(abd-(a+b)dx+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*a*b + (a + b)*x)/((x^2*(-a + x)*(-b + x))^(1/4)*(a*b*d - (a +
 b)*d*x + (-1 + d)*x^2)),x]
```

```
[Out] Integrate[(-2*a*b + (a + b)*x)/((x^2*(-a + x)*(-b + x))^(1/4)*(a*b*d - (a +
 b)*d*x + (-1 + d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.37, size = 81, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt[4]{x^3(-a-b)+abx^2+x^4}}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt[4]{x^3(-a-b)+abx^2+x^4}}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*a*b + (a + b)*x)/((x^2*(-a + x)*(-b + x))^(1/4)*(a*b
*d - (a + b)*d*x + (-1 + d)*x^2)),x]
```

```
[Out] (-2*ArcTan[x/(d^(1/4)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/4))])/d^(3/4) - (2*
ArcTanh[x/(d^(1/4)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/4))])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b+(a+b)*x)/(x^2*(-a+x)*(-b+x))^(1/4)/(a*b*d-(a+b)*d*x+(-1+d
)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,ab-(a+b)x}{\left((a-x)(b-x)x^2\right)^{\frac{1}{4}}\left(abd-(a+b)dx+(d-1)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b+(a+b)*x)/(x^2*(-a+x)*(-b+x))^(1/4)/(a*b*d-(a+b)*d*x+(-1+d
)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-(2*a*b - (a + b)*x)/(((a - x)*(b - x)*x^2)^(1/4)*(a*b*d - (a + b
)*d*x + (d - 1)*x^2)), x)
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{-2ab+(a+b)\,x}{\left(x^2\,(-a+x)\,(-b+x)\right)^{\frac{1}{4}}\left(abd-(a+b)\,dx+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a*b+(a+b)*x)/(x^2*(-a+x)*(-b+x))^(1/4)/(a*b*d-(a+b)*d*x+(-1+d)*x^2)
,x)
```

```
[Out] int((-2*a*b+(a+b)*x)/(x^2*(-a+x)*(-b+x))^(1/4)/(a*b*d-(a+b)*d*x+(-1+d)*x^2)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,ab-(a+b)x}{\left((a-x)(b-x)x^2\right)^{\frac{1}{4}}\left(abd-(a+b)dx+(d-1)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b+(a+b)*x)/(x^2*(-a+x)*(-b+x))^(1/4)/(a*b*d-(a+b)*d*x+(-1+d
)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((2*a*b - (a + b)*x)/((((a - x)*(b - x)*x^2)^(1/4)*(a*b*d - (a + b
)*d*x + (d - 1)*x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{2\,a\,b-x\,\left(a+b\right)}{\left(\left(d-1\right)\,x^2-d\,\left(a+b\right)\,x+a\,b\,d\right)\,{\left(x^2\,\left(a-x\right)\,\left(b-x\right)\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*a*b - x*(a + b))/((x^2*(d - 1) - d*x*(a + b) + a*b*d)*(x^2*(a - x)*
(b - x))^(1/4)),x)
```

```
[Out] int(-(2*a*b - x*(a + b))/((x^2*(d - 1) - d*x*(a + b) + a*b*d)*(x^2*(a - x)*
(b - x))^(1/4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b+(a+b)*x)/(x**2*(-a+x)*(-b+x))**(1/4)/(a*b*d-(a+b)*d*x+(-1
+d)*x**2),x)
```

```
[Out] Timed out
```

3.969 $\int \frac{\sqrt[3]{1+x^3}}{x} \, dx$

Optimal. Leaf size=81

$$\sqrt[3]{x^3+1}+\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-1\right)-\frac{1}{6}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 50, 57, 618, 204, 31}

$$\sqrt[3]{x^3+1}+\frac{1}{2}\log\left(1-\sqrt[3]{x^3+1}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)^(1/3)/x,x]
```

```
[Out] (1 + x^3)^(1/3) - ArcTan[(1 + 2*(1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/
2 + Log[1 - (1 + x^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{1+x^3}}{x}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt[3]{1+x}}{x}\,dx, x, x^3\right)\\
&= \sqrt[3]{1+x^3} + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x(1+x)^{2/3}}\,dx, x, x^3\right)\\
&= \sqrt[3]{1+x^3} - \frac{\log(x)}{2} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+x^3}\right) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1+x^3}\right)\\
&= \sqrt[3]{1+x^3} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1+x^3}\right) + \operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+x^3}\right)\\
&= \sqrt[3]{1+x^3} - \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1+x^3}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 80, normalized size = 0.99

$$
\sqrt[3]{x^3+1} + \frac{1}{3}\log\left(1-\sqrt[3]{x^3+1}\right) - \frac{1}{6}\log\left(\left(x^3+1\right)^{2/3} + \sqrt[3]{x^3+1} + 1\right) - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)^(1/3)/x,x]
```

```
[Out] (1 + x^3)^(1/3) - ArcTan[(1 + 2*(1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[1 -
 (1 + x^3)^(1/3)]/3 - Log[1 + (1 + x^3)^(1/3) + (1 + x^3)^(2/3)]/6
```

IntegrateAlgebraic [A] time = 0.05, size = 81, normalized size = 1.00

$$
\sqrt[3]{x^3+1} + \frac{1}{3}\log\left(\sqrt[3]{x^3+1}-1\right) - \frac{1}{6}\log\left(\left(x^3+1\right)^{2/3} + \sqrt[3]{x^3+1} + 1\right) - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)}{\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)^(1/3)/x,x]
```

```
[Out] (1 + x^3)^(1/3) - ArcTan[1/Sqrt[3] + (2*(1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] +
 Log[-1 + (1 + x^3)^(1/3)]/3 - Log[1 + (1 + x^3)^(1/3) + (1 + x^3)^(2/3)]/6
```

fricas [A] time = 0.45, size = 63, normalized size = 0.78

$$
-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right) + \left(x^3+1\right)^{\frac{1}{3}} - \frac{1}{6}\log\left(\left(x^3+1\right)^{\frac{2}{3}} + \left(x^3+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{3}\log\left(\left(x^3+1\right)^{\frac{1}{3}} - 1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*(x^3 + 1)^(1/3) + 1/3*sqrt(3)) + (x^3 + 1)^
(1/3) - 1/6*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) + 1/3*log((x^3 + 1)^
(1/3) - 1)
```

giac [A] time = 0.30, size = 62, normalized size = 0.77

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\left(x^3+1\right)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) + (x^3 + 1)^(1/3)
- 1/6*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) + 1/3*log(abs((x^3 + 1)^(1
/3) - 1))
```

maple [C] time = 0.27, size = 48, normalized size = 0.59

$$-\frac{-\Gamma\left(\frac{2}{3}\right)x^3\,\mathrm{hypergeom}\left(\left[\frac{2}{3},1,1\right],[2,2],-x^3\right)-3\left(3+\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)\right)\Gamma\left(\frac{2}{3}\right)}{9\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(1/3)/x,x)
```

```
[Out] -1/9/GAMMA(2/3)*(-GAMMA(2/3)*x^3*hypergeom([2/3,1,1],[2,2],-x^3)-3*(3+1/6*P
i*3^(1/2)-3/2*ln(3)+3*ln(x))*GAMMA(2/3))
```

maxima [A] time = 0.43, size = 61, normalized size = 0.75

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\left(x^3+1\right)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) + (x^3 + 1)^(1/3)
- 1/6*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) + 1/3*log((x^3 + 1)^(1/3)
- 1)
```

mupad [B] time = 0.83, size = 75, normalized size = 0.93

$$\frac{\ln\left(\left(x^3+1\right)^{1/3}-1\right)}{3}+\left(x^3+1\right)^{1/3}+\ln\left(3\left(x^3+1\right)^{1/3}+\frac{3}{2}-\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(3\left(x^3+1\right)^{1/3}+\frac{3}{2}+\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)^(1/3)/x,x)
```

```
[Out] log((x^3 + 1)^(1/3) - 1)/3 + (x^3 + 1)^(1/3) + log(3*(x^3 + 1)^(1/3) - (3^(
1/2)*3i)/2 + 3/2)*((3^(1/2)*1i)/6 - 1/6) - log((3^(1/2)*3i)/2 + 3*(x^3 + 1)
^(1/3) + 3/2)*((3^(1/2)*1i)/6 + 1/6)
```

sympy [C] time = 0.94, size = 34, normalized size = 0.42

$$-\frac{x\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^3}\right)}{3\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(1/3)/x,x)
```

```
[Out] -x*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), exp_polar(I*pi)/x**3)/(3*gamma(2
/3))
```

3.970 $\int \frac{3ab-2(a+b)x+x^2}{\sqrt[4]{x(-a+x)(-b+x)}\left(-abd+(a+b)dx-dx^2+x^3\right)}\,dx$

Optimal. Leaf size=81

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2(-a-b)+abx+x^3}}{x}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2(-a-b)+abx+x^3}}{x}\right)}{d^{3/4}}$$

Rubi [F] time = 16.81, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3ab-2(a+b)x+x^2}{\sqrt[4]{x(-a+x)(-b+x)}\left(-abd+(a+b)dx-dx^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3*a*b - 2*(a + b)*x + x^2)/((x*(-a + x)*(-b + x))^(1/4)*(-(a*b*d) + (a
 + b)*d*x - d*x^2 + x^3)),x]
```

```
[Out] (8*(a + b)*x^(1/4)*(-a + x)^(1/4)*(-b + x)^(1/4)*Defer[Subst][Defer[Int][x^
6/((-a + x^4)^(1/4)*(-b + x^4)^(1/4)*(a*b*d - a*(1 + b/a)*d*x^4 + d*x^8 - x
^12)), x], x, x^(1/4)])/((a - x)*(b - x)*x)^(1/4) + (12*a*b*x^(1/4)*(-a + x
)^(1/4)*(-b + x)^(1/4)*Defer[Subst][Defer[Int][x^2/((-a + x^4)^(1/4)*(-b +
x^4)^(1/4)*(-(a*b*d) + a*(1 + b/a)*d*x^4 - d*x^8 + x^12)), x], x, x^(1/4)])
/((a - x)*(b - x)*x)^(1/4) + (4*x^(1/4)*(-a + x)^(1/4)*(-b + x)^(1/4)*Defer
[Subst][Defer[Int][x^10/((-a + x^4)^(1/4)*(-b + x^4)^(1/4)*(-(a*b*d) + a*(1
 + b/a)*d*x^4 - d*x^8 + x^12)), x], x, x^(1/4)])/((a - x)*(b - x)*x)^(1/4)
```

Rubi steps

$$\begin{aligned}\int \frac{3ab-2(a+b)x+x^2}{\sqrt[4]{x(-a+x)(-b+x)}\left(-abd+(a+b)dx-dx^2+x^3\right)}\,dx &= \frac{\left(\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\int \frac{3ab-2(a+b)x+x^2}{\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\left(-abd+(a+b)d\right.}}{\sqrt[4]{x(-a+x)(-b+x)}}\\ &= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\text{Subst}\left(\int \frac{x^2\left(3ab-2(\right.}{\sqrt[4]{-a+x^4}\sqrt[4]{-b+x^4}\left(-a\right.}\right.}{\sqrt[4]{x(-a+x)(-b+x)}}\\ &= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\text{Subst}\left(\int\left(\frac{2(}{\sqrt[4]{-a+x^4}\sqrt[4]{-b+x^4}\left(a\right.}\right.\right.}{}\\ &= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{-a+x^4}\sqrt[4]{-b+x^4}\left(-a\right.}\right.}{\sqrt[4]{x(-a+x)(-b+x)}}\end{aligned}$$

Mathematica [F] time = 2.93, size = 0, normalized size = 0.00

$$\int \frac{3ab-2(a+b)x+x^2}{\sqrt[4]{x(-a+x)(-b+x)}\left(-abd+(a+b)dx-dx^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3*a*b - 2*(a + b)*x + x^2)/((x*(-a + x)*(-b + x))^(1/4)*(-(a*b*d
) + (a + b)*d*x - d*x^2 + x^3)),x]
```

```
[Out] Integrate[(3*a*b - 2*(a + b)*x + x^2)/((x*(-a + x)*(-b + x))^(1/4)*(-(a*b*d
) + (a + b)*d*x - d*x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.28, size = 81, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2(-a-b)+abx+x^3}}{x}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2(-a-b)+abx+x^3}}{x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*a*b - 2*(a + b)*x + x^2)/((x*(-a + x)*(-b + x))^(1/4)
*(-(a*b*d) + (a + b)*d*x - d*x^2 + x^3)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(a*b*x + (-a - b)*x^2 + x^3)^(1/4))/x])/d^(3/4) - (2*Arc
Tanh[(d^(1/4)*(a*b*x + (-a - b)*x^2 + x^3)^(1/4))/x])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*b-2*(a+b)*x+x^2)/(x*(-a+x)*(-b+x))^(1/4)/(-a*b*d+(a+b)*d*x-d
*x^2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{3\,ab-2\,(a+b)x+x^2}{\left(abd-(a+b)dx+dx^2-x^3\right)((a-x)(b-x)x)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*b-2*(a+b)*x+x^2)/(x*(-a+x)*(-b+x))^(1/4)/(-a*b*d+(a+b)*d*x-d
*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate(-(3*a*b - 2*(a + b)*x + x^2)/((a*b*d - (a + b)*d*x + d*x^2 - x^3)
*((a - x)*(b - x)*x)^(1/4)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{3ab-2\,(a+b)\,x+x^2}{\left(x\,(-a+x)\,(-b+x)\right)^{\frac{1}{4}}\left(-abd+(a+b)\,dx-d\,x^2+x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a*b-2*(a+b)*x+x^2)/(x*(-a+x)*(-b+x))^(1/4)/(-a*b*d+(a+b)*d*x-d*x^2+x
^3),x)
```

```
[Out] int((3*a*b-2*(a+b)*x+x^2)/(x*(-a+x)*(-b+x))^(1/4)/(-a*b*d+(a+b)*d*x-d*x^2+x
^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3\,ab-2\,(a+b)x+x^2}{\left(abd-(a+b)dx+dx^2-x^3\right)((a-x)(b-x)x)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*b-2*(a+b)*x+x^2)/(x*(-a+x)*(-b+x))^(1/4)/(-a*b*d+(a+b)*d*x-d
*x^2+x^3),x, algorithm="maxima")
```

```
[Out] -integrate((3*a*b - 2*(a + b)*x + x^2)/((a*b*d - (a + b)*d*x + d*x^2 - x^3)
*((a - x)*(b - x)*x)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{3\,a\,b+x^2-2\,x\,\left(a+b\right)}{{\left(x\,\left(a-x\right)\,\left(b-x\right)\right)}^{1/4}\,\left(-x^3+d\,x^2-d\,\left(a+b\right)\,x+a\,b\,d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*a*b + x^2 - 2*x*(a + b))/((x*(a - x)*(b - x))^(1/4)*(d*x^2 - x^3 -
d*x*(a + b) + a*b*d)),x)
```

```
[Out] -int((3*a*b + x^2 - 2*x*(a + b))/((x*(a - x)*(b - x))^(1/4)*(d*x^2 - x^3 -
d*x*(a + b) + a*b*d)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*b-2*(a+b)*x+x**2)/(x*(-a+x)*(-b+x))**(1/4)/(-a*b*d+(a+b)*d*x
-d*x**2+x**3),x)
```

```
[Out] Timed out
```

3.971 $\int \frac{-2abx^2+(a+b)x^3}{\left(x^2(-a+x)(-b+x)\right)^{3/4}\left(-ab+(a+b)x+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=81

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{x^3(-a-b)+abx^2+x^4}}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{x^3(-a-b)+abx^2+x^4}}\right)}{d^{3/4}}$$

Rubi [F] time = 5.35, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2abx^2+(a+b)x^3}{\left(x^2(-a+x)(-b+x)\right)^{3/4}\left(-ab+(a+b)x+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*a*b*x^2 + (a + b)*x^3)/((x^2*(-a + x)*(-b + x))^(3/4)*(-(a*b) + (a
+ b)*x + (-1 + d)*x^2)),x]
```

```
[Out] ((a + b - Sqrt[a^2 - 2*a*b + b^2 + 4*a*b*d])*x^(3/2)*(-a + x)^(3/4)*(-b + x
)^(3/4)*Defer[Int][Sqrt[x]/((-a + x)^(3/4)*(-b + x)^(3/4)*(a + b - Sqrt[a^2
 - 2*a*b + b^2 + 4*a*b*d] + 2*(-1 + d)*x)), x])/((a - x)*(b - x)*x^2)^(3/4)
 + ((a + b + Sqrt[a^2 - 2*a*b + b^2 + 4*a*b*d])*x^(3/2)*(-a + x)^(3/4)*(-b
+ x)^(3/4)*Defer[Int][Sqrt[x]/((-a + x)^(3/4)*(-b + x)^(3/4)*(a + b + Sqrt[
a^2 - 2*a*b + b^2 + 4*a*b*d] + 2*(-1 + d)*x)), x])/((a - x)*(b - x)*x^2)^(3
/4)
```

Rubi steps

$$\int \frac{-2abx^2+(a+b)x^3}{\left(x^2(-a+x)(-b+x)\right)^{3/4}\left(-ab+(a+b)x+(-1+d)x^2\right)}\,dx=\int \frac{x^2(-2ab+(a+b)x)}{\left(x^2(-a+x)(-b+x)\right)^{3/4}\left(-ab+(a+b)x+(\cdots\right.}$$

$$=\frac{\left(x^{3/2}(-a+x)^{3/4}(-b+x)^{3/4}\right)\int \frac{\sqrt{x}\,(-2a\cdots}{(-a+x)^{3/4}(-b+x)^{3/4}(-\cdots}}{\left(x^2(-a+x)(-b+x)\right)^{3/4}}$$

$$=\frac{\left(x^{3/2}(-a+x)^{3/4}(-b+x)^{3/4}\right)\int\left(\frac{\left(a+b-\cdots\right.}{(-a+x)^{3/4}(-b+x)^{3/4}(\cdots}\right.}{\cdots}$$

$$=\frac{\left(\left(a+b-\sqrt{a^2-2ab+b^2+4abd}\right)x^{3/2}(-a+x\cdots\right.}{\left(x^2\cdots\right.}$$

Mathematica [F] time = 11.07, size = 0, normalized size = 0.00

$$\int \frac{-2abx^2+(a+b)x^3}{\left(x^2(-a+x)(-b+x)\right)^{3/4}\left(-ab+(a+b)x+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*a*b*x^2 + (a + b)*x^3)/((x^2*(-a + x)*(-b + x))^(3/4)*(-(a*b)
 + (a + b)*x + (-1 + d)*x^2)),x]
```

```
[Out] Integrate[(-2*a*b*x^2 + (a + b)*x^3)/((x^2*(-a + x)*(-b + x))^(3/4)*(-(a*b)
 + (a + b)*x + (-1 + d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 2.52, size = 81, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{x^3(-a-b)+abx^2+x^4}}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{x^3(-a-b)+abx^2+x^4}}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*a*b*x^2 + (a + b)*x^3)/((x^2*(-a + x)*(-b + x))^(3/4
)*(-(a*b) + (a + b)*x + (-1 + d)*x^2)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*x)/(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/4)])/d^(3/4) + (2*
ArcTanh[(d^(1/4)*x)/(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/4)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b*x^2+(a+b)*x^3)/(x^2*(-a+x)*(-b+x))^(3/4)/(-a*b+(a+b)*x+(-
1+d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,abx^2-(a+b)x^3}{\left((a-x)(b-x)x^2\right)^{\frac{3}{4}}\left((d-1)x^2-ab+(a+b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b*x^2+(a+b)*x^3)/(x^2*(-a+x)*(-b+x))^(3/4)/(-a*b+(a+b)*x+(-
1+d)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-(2*a*b*x^2 - (a + b)*x^3)/(((a - x)*(b - x)*x^2)^(3/4)*((d - 1)*
x^2 - a*b + (a + b)*x)), x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{-2ab\,x^2+(a+b)\,x^3}{\left(x^2\,(-a+x)\,(-b+x)\right)^{\frac{3}{4}}\left(-ab+(a+b)\,x+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a*b*x^2+(a+b)*x^3)/(x^2*(-a+x)*(-b+x))^(3/4)/(-a*b+(a+b)*x+(-1+d)*x
^2),x)
```

```
[Out] int((-2*a*b*x^2+(a+b)*x^3)/(x^2*(-a+x)*(-b+x))^(3/4)/(-a*b+(a+b)*x+(-1+d)*x
^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,abx^2-(a+b)x^3}{\left((a-x)(b-x)x^2\right)^{\frac{3}{4}}\left((d-1)x^2-ab+(a+b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b*x^2+(a+b)*x^3)/(x^2*(-a+x)*(-b+x))^(3/4)/(-a*b+(a+b)*x+(-
1+d)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((2*a*b*x^2 - (a + b)*x^3)/(((a - x)*(b - x)*x^2)^(3/4)*((d - 1)*
x^2 - a*b + (a + b)*x)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,(a+b)-2\,a\,b\,x^2}{{\left(x^2\,(a-x)\,(b-x)\right)}^{3/4}\,\left((d-1)\,x^2+(a+b)\,x-a\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b) - 2*a*b*x^2)/((x^2*(a - x)*(b - x))^(3/4)*(x*(a + b) - a*b
 + x^2*(d - 1))),x)
```

```
[Out] int((x^3*(a + b) - 2*a*b*x^2)/((x^2*(a - x)*(b - x))^(3/4)*(x*(a + b) - a*b
 + x^2*(d - 1))), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b*x**2+(a+b)*x**3)/(x**2*(-a+x)*(-b+x))**(3/4)/(-a*b+(a+b)*
x+(-1+d)*x**2),x)
```

```
[Out] Timed out
```

3.972 $\int \frac{(-1+x^4)^{3/4}}{1+x^4}\,dx$

Optimal. Leaf size=81

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-1}}\right)}{\sqrt[4]{2}}+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-1}}\right)}{\sqrt[4]{2}}$$

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {408, 240, 212, 206, 203, 377}

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-1}}\right)}{\sqrt[4]{2}}+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-1}}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)^(3/4)/(1 + x^4),x]
```

```
[Out] ArcTan[x/(-1 + x^4)^(1/4)]/2 - ArcTan[(2^(1/4)*x)/(-1 + x^4)^(1/4)]/2^(1/4)
 + ArcTanh[x/(-1 + x^4)^(1/4)]/2 - ArcTanh[(2^(1/4)*x)/(-1 + x^4)^(1/4)]/2^
(1/4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 408

```
Int[((a_) + (b_.)*(x_)^4)^(p_)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d
, Int[(a + b*x^4)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(p
- 1)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[p, 3/4] || EqQ[p, 5/4])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-1+x^4\right)^{3/4}}{1+x^4}\,dx &= -\left(2\int \frac{1}{\sqrt[4]{-1+x^4}\left(1+x^4\right)}\,dx\right)+\int \frac{1}{\sqrt[4]{-1+x^4}}\,dx\\ &= -\left(2\,\mathrm{Subst}\left(\int \frac{1}{1-2x^4}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)\right)+\mathrm{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)-\mathrm{Subst}\left(\int\right.\\ &= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt[4]{2}}+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt[4]{2}}\end{aligned}$$

Mathematica [C] time = 0.11, size = 109, normalized size = 1.35

$$-\frac{5x\left(x^4-1\right)^{3/4}F_1\left(\frac{1}{4};-\frac{3}{4},1;\frac{5}{4};x^4,-x^4\right)}{\left(x^4+1\right)\left(x^4\left(4F_1\left(\frac{5}{4};-\frac{3}{4},2;\frac{9}{4};x^4,-x^4\right)+3F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};x^4,-x^4\right)\right)-5F_1\left(\frac{1}{4};-\frac{3}{4},1;\frac{5}{4};x^4,-x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + x^4)^(3/4)/(1 + x^4),x]
```

```
[Out] (-5*x*(-1 + x^4)^(3/4)*AppellF1[1/4, -3/4, 1, 5/4, x^4, -x^4])/((1 + x^4)*(
-5*AppellF1[1/4, -3/4, 1, 5/4, x^4, -x^4] + x^4*(4*AppellF1[5/4, -3/4, 2, 9
/4, x^4, -x^4] + 3*AppellF1[5/4, 1/4, 1, 9/4, x^4, -x^4])))
```

IntegrateAlgebraic [A] time = 0.28, size = 81, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-1}}\right)}{\sqrt[4]{2}}+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-1}}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)^(3/4)/(1 + x^4),x]
```

```
[Out] ArcTan[x/(-1 + x^4)^(1/4)]/2 - ArcTan[(2^(1/4)*x)/(-1 + x^4)^(1/4)]/2^(1/4)
 + ArcTanh[x/(-1 + x^4)^(1/4)]/2 - ArcTanh[(2^(1/4)*x)/(-1 + x^4)^(1/4)]/2^
(1/4)
```

fricas [B] time = 0.43, size = 149, normalized size = 1.84

$$-2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}x\sqrt{\frac{\sqrt{2}x^2+\sqrt{x^4-1}}{x^2}}-2^{\frac{3}{4}}\left(x^4-1\right)^{\frac{1}{4}}}{2x}\right)-\frac{1}{4}\cdot 2^{\frac{3}{4}}\log\left(\frac{2^{\frac{1}{4}}x+\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{4}\cdot 2^{\frac{3}{4}}\log\left(-\frac{2^{\frac{1}{4}}x-\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{2}\arctan\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{4}\log\left(\frac{x+\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{4}\log\left(-\frac{x-\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(3/4)/(x^4+1),x, algorithm="fricas")
```

```
[Out] -2^(3/4)*arctan(1/2*(2^(3/4)*x*sqrt((sqrt(2)*x^2 + sqrt(x^4 - 1))/x^2) - 2^
(3/4)*(x^4 - 1)^(1/4))/x) - 1/4*2^(3/4)*log((2^(1/4)*x + (x^4 - 1)^(1/4))/x
```

```
) + 1/4*2^(3/4)*log(-(2^(1/4)*x - (x^4 - 1)^(1/4))/x) - 1/2*arctan((x^4 - 1
)^(1/4)/x) + 1/4*log((x + (x^4 - 1)^(1/4))/x) - 1/4*log(-(x - (x^4 - 1)^(1/
4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)^{\frac{3}{4}}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(3/4)/(x^4+1),x, algorithm="giac")
[Out] integrate((x^4 - 1)^(3/4)/(x^4 + 1), x)
```

maple [F] time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)^{\frac{3}{4}}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(3/4)/(x^4+1),x)
[Out] int((x^4-1)^(3/4)/(x^4+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)^{\frac{3}{4}}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(3/4)/(x^4+1),x, algorithm="maxima")
[Out] integrate((x^4 - 1)^(3/4)/(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-1\right)^{3/4}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)^(3/4)/(x^4 + 1),x)
[Out] int((x^4 - 1)^(3/4)/(x^4 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x-1\right)\left(x+1\right)\left(x^2+1\right)\right)^{\frac{3}{4}}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(3/4)/(x**4+1),x)
[Out] Integral(((x - 1)*(x + 1)*(x**2 + 1))**(3/4)/(x**4 + 1), x)
```

3.973 $\int \frac{\sqrt[4]{-x^2+x^4}}{x^4(-1+x^4)}\,dx$

Optimal. Leaf size=81

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^2}}\right)}{2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^2}}\right)}{2^{3/4}}-\frac{2\sqrt[4]{x^4-x^2}\,(x^2-1)}{5x^3}$$

Rubi [A] time = 0.22, antiderivative size = 137, normalized size of antiderivative = 1.69, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2056, 1254, 466, 494, 461, 298, 203, 206}

$$-\frac{\sqrt[4]{x^4-x^2}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{2^{3/4}\sqrt{x}\sqrt[4]{x^2-1}}+\frac{\sqrt[4]{x^4-x^2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{2^{3/4}\sqrt{x}\sqrt[4]{x^2-1}}+\frac{2\sqrt[4]{x^4-x^2}\,(1-x^2)}{5x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + x^4)^(1/4)/(x^4*(-1 + x^4)),x]
```

```
[Out] (2*(1 - x^2)*(-x^2 + x^4)^(1/4))/(5*x^3) - ((-x^2 + x^4)^(1/4)*ArcTan[(2^(1
/4)*Sqrt[x])/(-1 + x^2)^(1/4)])/(2^(3/4)*Sqrt[x]*(-1 + x^2)^(1/4)) + ((-x^2
 + x^4)^(1/4)*ArcTanh[(2^(1/4)*Sqrt[x])/(-1 + x^2)^(1/4)])/(2^(3/4)*Sqrt[x]
*(-1 + x^2)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 1254

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e)^p, x] /
; FreeQ[{a, c, d, e, f, q, m, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p
]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{-x^2+x^4}}{x^4\left(-1+x^4\right)}\,dx &= \frac{\sqrt[4]{-x^2+x^4}\int \frac{\sqrt[4]{-1+x^2}}{x^{7/2}\left(-1+x^4\right)}\,dx}{\sqrt{x}\,\sqrt[4]{-1+x^2}}\\
&= \frac{\sqrt[4]{-x^2+x^4}\int \frac{1}{x^{7/2}\left(-1+x^2\right)^{3/4}\left(1+x^2\right)}\,dx}{\sqrt{x}\,\sqrt[4]{-1+x^2}}\\
&= \frac{\left(2\sqrt[4]{-x^2+x^4}\right)\text{Subst}\left(\int \frac{1}{x^6\left(-1+x^4\right)^{3/4}\left(1+x^4\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\,\sqrt[4]{-1+x^2}}\\
&= \frac{\left(2\sqrt[4]{-x^2+x^4}\right)\text{Subst}\left(\int \frac{\left(1-x^4\right)^2}{x^6\left(1-2x^4\right)}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt{x}\,\sqrt[4]{-1+x^2}}\\
&= \frac{\left(2\sqrt[4]{-x^2+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{x^6}-\frac{x^2}{-1+2x^4}\right)dx,x,\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt{x}\,\sqrt[4]{-1+x^2}}\\
&= \frac{2\left(1-x^2\right)\sqrt[4]{-x^2+x^4}}{5x^3}-\frac{\left(2\sqrt[4]{-x^2+x^4}\right)\text{Subst}\left(\int \frac{x^2}{-1+2x^4}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt{x}\,\sqrt[4]{-1+x^2}}\\
&= \frac{2\left(1-x^2\right)\sqrt[4]{-x^2+x^4}}{5x^3}+\frac{\sqrt[4]{-x^2+x^4}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt{2}\sqrt{x}\,\sqrt[4]{-1+x^2}}-\frac{\sqrt[4]{-x^2+x^4}\,\text{Subst}\left(\int\right.}{\sqrt{2}\sqrt{x}}\\
&= \frac{2\left(1-x^2\right)\sqrt[4]{-x^2+x^4}}{5x^3}-\frac{\sqrt[4]{-x^2+x^4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{2^{3/4}\sqrt{x}\,\sqrt[4]{-1+x^2}}+\frac{\sqrt[4]{-x^2+x^4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{2^{3/4}\sqrt{x}\,\sqrt[4]{-1+x^2}}
\end{aligned}
$$

Mathematica [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{-x^2+x^4}}{x^4\left(-1+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-x^2 + x^4)^(1/4)/(x^4*(-1 + x^4)),x]
```

```
[Out] Integrate[(-x^2 + x^4)^(1/4)/(x^4*(-1 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.24, size = 81, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-x^2}}\right)}{2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-x^2}}\right)}{2^{3/4}}-\frac{2\sqrt[4]{x^4-x^2}\,\left(x^2-1\right)}{5x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^2 + x^4)^(1/4)/(x^4*(-1 + x^4)),x]
```

```
[Out] (-2*(-1 + x^2)*(-x^2 + x^4)^(1/4))/(5*x^3) - ArcTan[(2^(1/4)*x)/(-x^2 + x^4
)^(1/4)]/2^(3/4) + ArcTanh[(2^(1/4)*x)/(-x^2 + x^4)^(1/4)]/2^(3/4)
```

fricas [B] time = 1.93, size = 273, normalized size = 3.37

$$\frac{20\cdot 8^{\frac{3}{4}} x^3 \arctan\left(\frac{16\cdot 8^{\frac{1}{4}}\left(x^4-x^2\right)^{\frac{1}{4}}x^2+2^{\frac{3}{4}}\left(8^{\frac{3}{4}}\left(3x^3-x\right)+8\cdot 8^{\frac{1}{4}}\sqrt{x^4-x^2}\,x\right)+4\cdot 8^{\frac{3}{4}}\left(x^4-x^2\right)^{\frac{3}{4}}}{8\left(x^3+x\right)}\right)+5\cdot 8^{\frac{3}{4}}x^3\log\left(\frac{4\sqrt{2}\left(x^4-x^2\right)^{\frac{1}{4}}x^2+8^{\frac{3}{4}}\sqrt{x^4-x^2}\,x+8^{\frac{1}{4}}\left(3x^3-x\right)+4\left(x^4-x^2\right)^{\frac{3}{4}}}{x^3+x}\right)-5\cdot 8^{\frac{3}{4}}x^3\log\left(\frac{4\sqrt{2}\left(x^4-x^2\right)^{\frac{1}{4}}x^2-8^{\frac{3}{4}}\sqrt{x^4-x^2}\,x-8^{\frac{1}{4}}\left(3x^3-x\right)+4\left(x^4-x^2\right)^{\frac{3}{4}}}{x^3+x}\right)-64\left(x^4-x^2\right)^{\frac{1}{4}}\left(x^2-1\right)}{160\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2)^(1/4)/x^4/(x^4-1),x, algorithm="fricas")
```

```
[Out] 1/160*(20*8^(3/4)*x^3*arctan(1/8*(16*8^(1/4)*(x^4 - x^2)^(1/4)*x^2 + 2^(3/4
)*(8^(3/4)*(3*x^3 - x) + 8*8^(1/4)*sqrt(x^4 - x^2)*x) + 4*8^(3/4)*(x^4 - x^
2)^(3/4))/(x^3 + x)) + 5*8^(3/4)*x^3*log((4*sqrt(2)*(x^4 - x^2)^(1/4)*x^2 +
 8^(3/4)*sqrt(x^4 - x^2)*x + 8^(1/4)*(3*x^3 - x) + 4*(x^4 - x^2)^(3/4))/(x^
3 + x)) - 5*8^(3/4)*x^3*log((4*sqrt(2)*(x^4 - x^2)^(1/4)*x^2 - 8^(3/4)*sqrt
(x^4 - x^2)*x - 8^(1/4)*(3*x^3 - x) + 4*(x^4 - x^2)^(3/4))/(x^3 + x)) - 64*
(x^4 - x^2)^(1/4)*(x^2 - 1))/x^3
```

giac [A] time = 1.14, size = 72, normalized size = 0.89

$$\frac{2}{5}\left(-\frac{1}{x^2}+1\right)^{\frac{5}{4}}-\frac{1}{2}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)-\frac{1}{4}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)+\frac{1}{4}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}-\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2)^(1/4)/x^4/(x^4-1),x, algorithm="giac")
```

```
[Out] 2/5*(-1/x^2 + 1)^(5/4) - 1/2*2^(1/4)*arctan(1/2*2^(3/4)*(-1/x^2 + 1)^(1/4))
 - 1/4*2^(1/4)*log(2^(1/4) + (-1/x^2 + 1)^(1/4)) + 1/4*2^(1/4)*log(2^(1/4)
- (-1/x^2 + 1)^(1/4))
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^2\right)^{\frac{1}{4}}}{x^4\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^2)^(1/4)/x^4/(x^4-1),x)
```

```
[Out] int((x^4-x^2)^(1/4)/x^4/(x^4-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^2\right)^{\frac{1}{4}}}{\left(x^4-1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2)^(1/4)/x^4/(x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^2)^(1/4)/((x^4 - 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(x^4-x^2\right)^{1/4}}{x^4-x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x^2)^(1/4)/(x^4*(x^4 - 1)),x)
```

```
[Out] -int((x^4 - x^2)^(1/4)/(x^4 - x^8), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\left(x-1\right)\left(x+1\right)}}{x^4\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**2)**(1/4)/x**4/(x**4-1),x)
```

```
[Out] Integral((x**2*(x - 1)*(x + 1))**(1/4)/(x**4*(x - 1)*(x + 1)*(x**2 + 1)), x
)
```

3.974 $\int \frac{(1-x^2)^2}{(1+x^2)(1+6x^2+x^4)^{3/4}}\,dx$

Optimal. Leaf size=81

$$-\tan^{-1}\left(\frac{x-1}{\sqrt[4]{x^4+6x^2+1}}\right)-\tan^{-1}\left(\frac{x+1}{\sqrt[4]{x^4+6x^2+1}}\right)+\tanh^{-1}\left(\frac{x-1}{\sqrt[4]{x^4+6x^2+1}}\right)+\tanh^{-1}\left(\frac{x+1}{\sqrt[4]{x^4+6x^2+1}}\right)$$

Rubi [F] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-x^2)^2}{(1+x^2)(1+6x^2+x^4)^{3/4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - x^2)^2/((1 + x^2)*(1 + 6*x^2 + x^4)^(3/4)),x]
[Out] Defer[Int][(1 - x^2)^2/((1 + x^2)*(1 + 6*x^2 + x^4)^(3/4)), x]
```

Rubi steps

$$\int \frac{(1-x^2)^2}{(1+x^2)(1+6x^2+x^4)^{3/4}}\,dx = \int \frac{(1-x^2)^2}{(1+x^2)(1+6x^2+x^4)^{3/4}}\,dx$$

Mathematica [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(1-x^2)^2}{(1+x^2)(1+6x^2+x^4)^{3/4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^2)^2/((1 + x^2)*(1 + 6*x^2 + x^4)^(3/4)),x]
[Out] Integrate[(1 - x^2)^2/((1 + x^2)*(1 + 6*x^2 + x^4)^(3/4)), x]
```

IntegrateAlgebraic [A] time = 5.35, size = 81, normalized size = 1.00

$$-\tan^{-1}\left(\frac{x-1}{\sqrt[4]{x^4+6x^2+1}}\right)-\tan^{-1}\left(\frac{x+1}{\sqrt[4]{x^4+6x^2+1}}\right)+\tanh^{-1}\left(\frac{x-1}{\sqrt[4]{x^4+6x^2+1}}\right)+\tanh^{-1}\left(\frac{x+1}{\sqrt[4]{x^4+6x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^2)^2/((1 + x^2)*(1 + 6*x^2 + x^4)^(3/4)),x]
[Out] -ArcTan[(-1 + x)/(1 + 6*x^2 + x^4)^(1/4)] - ArcTan[(1 + x)/(1 + 6*x^2 + x^4
)^(1/4)] + ArcTanh[(-1 + x)/(1 + 6*x^2 + x^4)^(1/4)] + ArcTanh[(1 + x)/(1 +
 6*x^2 + x^4)^(1/4)]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2/(x^2+1)/(x^4+6*x^2+1)^(3/4),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-1\right)^2}{\left(x^4+6\,x^2+1\right)^{\frac{3}{4}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2/(x^2+1)/(x^4+6*x^2+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)^2/((x^4 + 6*x^2 + 1)^(3/4)*(x^2 + 1)), x)
```

maple [C]   time = 3.41, size = 383, normalized size = 4.73

$$\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\mathrm{RootOf}(\_Z^2+1)^3\sqrt{x^4+6x^2+1}\,x^4-\mathrm{RootOf}(\_Z^2+1)^3x^6-\mathrm{RootOf}(\_Z^2+1)^2(x^4+6x^2+1)^{\frac{1}{4}}x^5+\mathrm{RootOf}(\_Z^2+1)^3\sqrt{x^4+6x^2+1}\,x^2-5\,\mathrm{RootOf}(\_Z^2+1)^3x^4-3\,\mathrm{RootOf}(\_Z^2+1)^2(x^4+6x^2+1)^{\frac{1}{4}}x^3-(x^4+6x^2+1)^{\frac{3}{4}}x^3-\mathrm{RootOf}(\_Z^2+1)\sqrt{x^4+6x^2+1}\,x^2+\mathrm{RootOf}(\_Z^2+1)x^4-(x^4+6x^2+1)^{\frac{3}{4}}x+(x^4+6x^2+1)^{\frac{1}{4}}x^3-\mathrm{RootOf}(\_Z^2+1)\sqrt{x^4+6x^2+1}+5\,\mathrm{RootOf}(\_Z^2+1)x^2+3(x^4+6x^2+1)^{\frac{1}{4}}x}{(\mathrm{RootOf}(\_Z^2+1)x-1)^2(\mathrm{RootOf}(\_Z^2+1)x+1)^2}\right)}{2}+\frac{\ln\left(-\frac{(x^4+6x^2+1)^{\frac{3}{4}}x+x^2\sqrt{x^4+6x^2+1}+(x^4+6x^2+1)^{\frac{1}{4}}x^3+x^4+\sqrt{x^4+6x^2+1}+3(x^4+6x^2+1)^{\frac{1}{4}}x+5x^2}{x^2+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^2/(x^2+1)/(x^4+6*x^2+1)^(3/4),x)
```

```
[Out] 1/2*RootOf(_Z^2+1)*ln(-(RootOf(_Z^2+1)^3*(x^4+6*x^2+1)^(1/2)*x^4-RootOf(_Z^
2+1)^3*x^6-RootOf(_Z^2+1)^2*(x^4+6*x^2+1)^(1/4)*x^5+RootOf(_Z^2+1)^3*(x^4+6
*x^2+1)^(1/2)*x^2-5*RootOf(_Z^2+1)^3*x^4-3*RootOf(_Z^2+1)^2*(x^4+6*x^2+1)^(
1/4)*x^3-(x^4+6*x^2+1)^(3/4)*x^3-RootOf(_Z^2+1)*(x^4+6*x^2+1)^(1/2)*x^2+Roo
tOf(_Z^2+1)*x^4-(x^4+6*x^2+1)^(3/4)*x+(x^4+6*x^2+1)^(1/4)*x^3-RootOf(_Z^2+1
)*(x^4+6*x^2+1)^(1/2)+5*RootOf(_Z^2+1)*x^2+3*(x^4+6*x^2+1)^(1/4)*x)/(RootOf
(_Z^2+1)*x-1)^2/(RootOf(_Z^2+1)*x+1)^2)+1/2*ln(-((x^4+6*x^2+1)^(3/4)*x+x^2*
(x^4+6*x^2+1)^(1/2)+(x^4+6*x^2+1)^(1/4)*x^3+x^4+(x^4+6*x^2+1)^(1/2)+3*(x^4+
6*x^2+1)^(1/4)*x+5*x^2)/(x^2+1))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-1\right)^2}{\left(x^4+6\,x^2+1\right)^{\frac{3}{4}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2/(x^2+1)/(x^4+6*x^2+1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)^2/((x^4 + 6*x^2 + 1)^(3/4)*(x^2 + 1)), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2-1\right)^2}{\left(x^2+1\right)\,\left(x^4+6\,x^2+1\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^2/((x^2 + 1)*(6*x^2 + x^4 + 1)^(3/4)),x)
```

```
[Out] int((x^2 - 1)^2/((x^2 + 1)*(6*x^2 + x^4 + 1)^(3/4)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-1\right)^2\left(x+1\right)^2}{\left(x^2+1\right)\left(x^4+6x^2+1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**2/(x**2+1)/(x**4+6*x**2+1)**(3/4),x)
```

```
[Out] Integral((x - 1)**2*(x + 1)**2/((x**2 + 1)*(x**4 + 6*x**2 + 1)**(3/4)), x)
```

3.975 $\int \frac{1}{(1+x)\sqrt[4]{1+6x^2+x^4}}\,dx$

Optimal. Leaf size=81

$$\frac{\tan^{-1}\left(\frac{\frac{x}{\sqrt[4]{2}}-\frac{1}{\sqrt[4]{2}}}{\sqrt[4]{x^4+6x^2+1}}\right)}{2\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\frac{x}{\sqrt[4]{2}}-\frac{1}{\sqrt[4]{2}}}{\sqrt[4]{x^4+6x^2+1}}\right)}{2\,2^{3/4}}$$

Rubi [F] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(1+x)\sqrt[4]{1+6x^2+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 + x)*(1 + 6*x^2 + x^4)^(1/4)),x]
```

```
[Out] Defer[Int][1/((1 + x)*(1 + 6*x^2 + x^4)^(1/4)), x]
```

Rubi steps

$$\int \frac{1}{(1+x)\sqrt[4]{1+6x^2+x^4}}\,dx=\int \frac{1}{(1+x)\sqrt[4]{1+6x^2+x^4}}\,dx$$

Mathematica [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x)\sqrt[4]{1+6x^2+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 + x)*(1 + 6*x^2 + x^4)^(1/4)),x]
```

```
[Out] Integrate[1/((1 + x)*(1 + 6*x^2 + x^4)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 1.02, size = 81, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\frac{x}{\sqrt[4]{2}}-\frac{1}{\sqrt[4]{2}}}{\sqrt[4]{x^4+6x^2+1}}\right)}{2\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\frac{x}{\sqrt[4]{2}}-\frac{1}{\sqrt[4]{2}}}{\sqrt[4]{x^4+6x^2+1}}\right)}{2\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x)*(1 + 6*x^2 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[(-2^(-1/4) + x/2^(1/4))/(1 + 6*x^2 + x^4)^(1/4)]/(2*2^(3/4)) + ArcTa
nh[(-2^(-1/4) + x/2^(1/4))/(1 + 6*x^2 + x^4)^(1/4)]/(2*2^(3/4))
```

fricas [B] time = 8.09, size = 382, normalized size = 4.72

$$-\frac{1}{16}\cdot 8^{\frac{3}{4}}\arctan\left(-\frac{8^{\frac{3}{4}}\left(x^4+6x^2+1\right)^{\frac{1}{4}}\left(x^3-3x^2+3x-1\right)+4\cdot 8^{\frac{1}{4}}\left(x^4+6x^2+1\right)^{\frac{3}{4}}(x-1)-2^{\frac{1}{4}}\left(8^{\frac{3}{4}}\sqrt{x^4+6x^2+1}\left(x^2-2x+1\right)+8^{\frac{1}{4}}\left(3x^4-4x^3+18x^2-4x+3\right)\right)}{2\left(x^4+4x^3+6x^2+4x+1\right)}\right)+\frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(\frac{8^{\frac{3}{4}}\left(3x^4-4x^3+18x^2-4x+3\right)+8\sqrt{2}\left(x^4+6x^2+1\right)^{\frac{1}{4}}\left(x^3-3x^2+3x-1\right)+8\cdot 8^{\frac{1}{4}}\sqrt{x^4+6x^2+1}\left(x^2-2x+1\right)+16\left(x^4+6x^2+1\right)^{\frac{3}{4}}(x-1)}{x^4+4x^3+6x^2+4x+1}\right)-\frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(-\frac{8^{\frac{3}{4}}\left(3x^4-4x^3+18x^2-4x+3\right)-8\sqrt{2}\left(x^4+6x^2+1\right)^{\frac{1}{4}}\left(x^3-3x^2+3x-1\right)+8\cdot 8^{\frac{1}{4}}\sqrt{x^4+6x^2+1}\left(x^2-2x+1\right)-16\left(x^4+6x^2+1\right)^{\frac{3}{4}}(x-1)}{x^4+4x^3+6x^2+4x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^4+6*x^2+1)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/16*8^(3/4)*arctan(-1/2*(8^(3/4)*(x^4 + 6*x^2 + 1)^(1/4)*(x^3 - 3*x^2 + 3
*x - 1) + 4*8^(1/4)*(x^4 + 6*x^2 + 1)^(3/4)*(x - 1) - 2^(1/4)*(8^(3/4)*sqrt
(x^4 + 6*x^2 + 1)*(x^2 - 2*x + 1) + 8^(1/4)*(3*x^4 - 4*x^3 + 18*x^2 - 4*x +
 3)))/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1)) + 1/64*8^(3/4)*log((8^(3/4)*(3*x^4 -
 4*x^3 + 18*x^2 - 4*x + 3) + 8*sqrt(2)*(x^4 + 6*x^2 + 1)^(1/4)*(x^3 - 3*x^2
 + 3*x - 1) + 8*8^(1/4)*sqrt(x^4 + 6*x^2 + 1)*(x^2 - 2*x + 1) + 16*(x^4 + 6
*x^2 + 1)^(3/4)*(x - 1))/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1)) - 1/64*8^(3/4)*lo
g(-(8^(3/4)*(3*x^4 - 4*x^3 + 18*x^2 - 4*x + 3) - 8*sqrt(2)*(x^4 + 6*x^2 + 1
)^(1/4)*(x^3 - 3*x^2 + 3*x - 1) + 8*8^(1/4)*sqrt(x^4 + 6*x^2 + 1)*(x^2 - 2*
x + 1) - 16*(x^4 + 6*x^2 + 1)^(3/4)*(x - 1))/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4 + 6\,x^2 + 1\right)^{\frac{1}{4}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^4+6*x^2+1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 6*x^2 + 1)^(1/4)*(x + 1)), x)
```

maple [F] time = 2.40, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x)\left(x^4 + 6x^2 + 1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^4+6*x^2+1)^(1/4),x)
```

```
[Out] int(1/(1+x)/(x^4+6*x^2+1)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4 + 6\,x^2 + 1\right)^{\frac{1}{4}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^4+6*x^2+1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 6*x^2 + 1)^(1/4)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x+1)\,\left(x^4 + 6\,x^2 + 1\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)*(6*x^2 + x^4 + 1)^(1/4)),x)
```

```
[Out] int(1/((x + 1)*(6*x^2 + x^4 + 1)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)\sqrt[4]{x^4 + 6x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**4+6*x**2+1)**(1/4),x)
```

```
[Out] Integral(1/((x + 1)*(x**4 + 6*x**2 + 1)**(1/4)), x)
```

3.976 $\int \frac{1+x}{\sqrt{-7+4x+14x^2-12x^3+x^4}}\,dx$

Optimal. Leaf size=81

$$-\log\left(-x^2+\sqrt{x^4-12x^3+14x^2+4x-7}+6x-5\right)+\tan^{-1}\left(\frac{4x-4}{x^2-\sqrt{x^4-12x^3+14x^2+4x-7}-2x+1}\right)+\log$$

Rubi [F] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x}{\sqrt{-7+4x+14x^2-12x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x)/Sqrt[-7 + 4*x + 14*x^2 - 12*x^3 + x^4],x]
```

```
[Out] Defer[Int][1/Sqrt[-7 + 4*x + 14*x^2 - 12*x^3 + x^4], x] + Defer[Int][x/Sqrt
[-7 + 4*x + 14*x^2 - 12*x^3 + x^4], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{\sqrt{-7+4x+14x^2-12x^3+x^4}}\,dx &= \int \left(\frac{1}{\sqrt{-7+4x+14x^2-12x^3+x^4}}+\frac{x}{\sqrt{-7+4x+14x^2-12x^3+x^4}}\right)d\\ &= \int \frac{1}{\sqrt{-7+4x+14x^2-12x^3+x^4}}\,dx+\int \frac{x}{\sqrt{-7+4x+14x^2-12x^3+x}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 76, normalized size = 0.94

$$\frac{(x-1)\sqrt{x^2-10x-7}\left(\tan^{-1}\left(\frac{-x-3}{\sqrt{x^2-10x-7}}\right)+2\tanh^{-1}\left(\frac{x-5}{\sqrt{x^2-10x-7}}\right)\right)}{2\sqrt{(x-1)^2\left(x^2-10x-7\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/Sqrt[-7 + 4*x + 14*x^2 - 12*x^3 + x^4],x]
```

```
[Out] ((-1 + x)*Sqrt[-7 - 10*x + x^2]*(ArcTan[(-3 - x)/Sqrt[-7 - 10*x + x^2]] + 2
*ArcTanh[(-5 + x)/Sqrt[-7 - 10*x + x^2]]))/(2*Sqrt[(-1 + x)^2*(-7 - 10*x +
x^2)])
```

IntegrateAlgebraic [A] time = 0.30, size = 81, normalized size = 1.00

$$-\log\left(-x^2+\sqrt{x^4-12x^3+14x^2+4x-7}+6x-5\right)+\tan^{-1}\left(\frac{4x-4}{x^2-\sqrt{x^4-12x^3+14x^2+4x-7}-2x+1}\right)+\log(x-1)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/Sqrt[-7 + 4*x + 14*x^2 - 12*x^3 + x^4],x]
```

```
[Out] ArcTan[(-4 + 4*x)/(1 - 2*x + x^2 - Sqrt[-7 + 4*x + 14*x^2 - 12*x^3 + x^4])]
 + Log[-1 + x] - Log[-5 + 6*x - x^2 + Sqrt[-7 + 4*x + 14*x^2 - 12*x^3 + x^4
]]
```

fricas [A] time = 0.41, size = 79, normalized size = 0.98

$$\arctan\left(-\frac{x^2-2\,x-\sqrt{x^4-12\,x^3+14\,x^2+4\,x-7}+1}{4\,(x-1)}\right)-\log\left(-\frac{x^2-6\,x-\sqrt{x^4-12\,x^3+14\,x^2+4\,x-7}+5}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^4-12*x^3+14*x^2+4*x-7)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(-1/4*(x^2 - 2*x - sqrt(x^4 - 12*x^3 + 14*x^2 + 4*x - 7) + 1)/(x - 1)
) - log(-(x^2 - 6*x - sqrt(x^4 - 12*x^3 + 14*x^2 + 4*x - 7) + 5)/(x - 1))
```

giac [B] time = 0.46, size = 183, normalized size = 2.26

$$-\frac{\arctan\left(\frac{1}{7}\sqrt{7}\left(\sqrt{2}+\frac{3\left(\sqrt{7}\sqrt{-\frac{10}{x}-\frac{7}{x^2}+1}-4\sqrt{2}\right)}{\frac{7}{x}+5}\right)\right)}{\operatorname{sgn}\left(-\frac{1}{x^2}+\frac{1}{x^3}\right)}-\frac{\log\left(\left|10\sqrt{7}+40\sqrt{2}+\frac{50\left(\sqrt{7}\sqrt{-\frac{10}{x}-\frac{7}{x^2}+1}-4\sqrt{2}\right)}{\frac{7}{x}+5}\right|\right)}{\operatorname{sgn}\left(-\frac{1}{x^2}+\frac{1}{x^3}\right)}+\frac{\log\left(\left|-2\sqrt{7}+8\sqrt{2}+\frac{10\left(\sqrt{7}\sqrt{-\frac{10}{x}-\frac{7}{x^2}+1}-4\sqrt{2}\right)}{\frac{7}{x}+5}\right|\right)}{\operatorname{sgn}\left(-\frac{1}{x^2}+\frac{1}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^4-12*x^3+14*x^2+4*x-7)^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(1/7*sqrt(7)*(sqrt(2) + 3*(sqrt(7)*sqrt(-10/x - 7/x^2 + 1) - 4*sqrt(
2))/(7/x + 5)))/sgn(-1/x^2 + 1/x^3) - log(abs(10*sqrt(7) + 40*sqrt(2) + 50*
(sqrt(7)*sqrt(-10/x - 7/x^2 + 1) - 4*sqrt(2))/(7/x + 5)))/sgn(-1/x^2 + 1/x^
3) + log(abs(-2*sqrt(7) + 8*sqrt(2) + 10*(sqrt(7)*sqrt(-10/x - 7/x^2 + 1) -
 4*sqrt(2))/(7/x + 5)))/sgn(-1/x^2 + 1/x^3)
```

maple [A] time = 0.01, size = 70, normalized size = 0.86

$$\frac{(-1+x)\sqrt{x^2-10x-7}\,\left(2\ln\left(x-5+\sqrt{x^2-10x-7}\right)-\arctan\left(\frac{3+x}{\sqrt{x^2-10x-7}}\right)\right)}{2\sqrt{x^4-12x^3+14x^2+4x-7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^4-12*x^3+14*x^2+4*x-7)^(1/2),x)
```

```
[Out] 1/2*(-1+x)*(x^2-10*x-7)^(1/2)*(2*ln(x-5+(x^2-10*x-7)^(1/2))-arctan((3+x)/(x
^2-10*x-7)^(1/2)))/(x^4-12*x^3+14*x^2+4*x-7)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^4-12\,x^3+14\,x^2+4\,x-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^4-12*x^3+14*x^2+4*x-7)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/sqrt(x^4 - 12*x^3 + 14*x^2 + 4*x - 7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+1}{\sqrt{x^4-12\,x^3+14\,x^2+4\,x-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(4*x + 14*x^2 - 12*x^3 + x^4 - 7)^(1/2),x)
```

```
[Out] int((x + 1)/(4*x + 14*x^2 - 12*x^3 + x^4 - 7)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{(x-1)^2\left(x^2-10x-7\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**4-12*x**3+14*x**2+4*x-7)**(1/2),x)
```

```
[Out] Integral((x + 1)/sqrt((x - 1)**2*(x**2 - 10*x - 7)), x)
```

3.977 $\int \frac{3+x^4}{\sqrt[3]{-1+x^4}\left(-1-8x^3+x^4\right)}\,dx$

Optimal. Leaf size=81

$$\frac{1}{2}\log\left(\sqrt[3]{x^4-1}-2x\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt[3]{x^4-1}+x}\right)-\frac{1}{4}\log\left(2\sqrt[3]{x^4-1}x+\left(x^4-1\right)^{2/3}+4x^2\right)$$

Rubi [F] time = 0.64, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3+x^4}{\sqrt[3]{-1+x^4}\left(-1-8x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3 + x^4)/((-1 + x^4)^(1/3)*(-1 - 8*x^3 + x^4)),x]
```

```
[Out] (x*(1 - x^4)^(1/3)*Hypergeometric2F1[1/4, 1/3, 5/4, x^4])/(-1 + x^4)^(1/3)
+ 4*Defer[Int][1/((-1 + x^4)^(1/3)*(-1 - 8*x^3 + x^4)), x] + 8*Defer[Int][x
^3/((-1 + x^4)^(1/3)*(-1 - 8*x^3 + x^4)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{3+x^4}{\sqrt[3]{-1+x^4}\left(-1-8x^3+x^4\right)}\,dx &= \int \left(\frac{1}{\sqrt[3]{-1+x^4}}+\frac{4\left(1+2x^3\right)}{\sqrt[3]{-1+x^4}\left(-1-8x^3+x^4\right)}\right)dx\\
&= 4\int \frac{1+2x^3}{\sqrt[3]{-1+x^4}\left(-1-8x^3+x^4\right)}\,dx+\int \frac{1}{\sqrt[3]{-1+x^4}}\,dx\\
&= 4\int \left(\frac{1}{\sqrt[3]{-1+x^4}\left(-1-8x^3+x^4\right)}+\frac{2x^3}{\sqrt[3]{-1+x^4}\left(-1-8x^3+x^4\right)}\right)dx+\frac{\sqrt[3]{1-x^4}}{\sqrt[3]{-}}\\
&= \frac{x\sqrt[3]{1-x^4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{5}{4};x^4\right)}{\sqrt[3]{-1+x^4}}+4\int \frac{1}{\sqrt[3]{-1+x^4}\left(-1-8x^3+x^4\right)}\,dx+8\int \frac{}{\sqrt[3]{-1+}}
\end{aligned}$$

Mathematica [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{3+x^4}{\sqrt[3]{-1+x^4}\left(-1-8x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3 + x^4)/((-1 + x^4)^(1/3)*(-1 - 8*x^3 + x^4)),x]
```

```
[Out] Integrate[(3 + x^4)/((-1 + x^4)^(1/3)*(-1 - 8*x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 1.17, size = 81, normalized size = 1.00

$$\frac{1}{2}\log\left(\sqrt[3]{x^4-1}-2x\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt[3]{x^4-1}+x}\right)-\frac{1}{4}\log\left(2\sqrt[3]{x^4-1}x+\left(x^4-1\right)^{2/3}+4x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + x^4)/((-1 + x^4)^(1/3)*(-1 - 8*x^3 + x^4)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + (-1 + x^4)^(1/3))]) + Log[-2*x + (-1
+ x^4)^(1/3)]/2 - Log[4*x^2 + 2*x*(-1 + x^4)^(1/3) + (-1 + x^4)^(2/3)]/4
```

fricas [A] time = 2.30, size = 112, normalized size = 1.38

$$-\frac{1}{2}\sqrt{3}\arctan\left(-\frac{8\sqrt{3}\left(x^{4}-1\right)^{\frac{1}{3}}x^{2}-4\sqrt{3}\left(x^{4}-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(x^{4}-8x^{3}-1\right)}{3\left(x^{4}+8x^{3}-1\right)}\right)+\frac{1}{4}\log\left(\frac{x^{4}-8x^{3}+12\left(x^{4}-1\right)^{\frac{1}{3}}x^{2}-6\left(x^{4}-1\right)^{\frac{2}{3}}x-1}{x^{4}-8x^{3}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)/(x^4-1)^(1/3)/(x^4-8*x^3-1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan(-1/3*(8*sqrt(3)*(x^4 - 1)^(1/3)*x^2 - 4*sqrt(3)*(x^4 -
1)^(2/3)*x + sqrt(3)*(x^4 - 8*x^3 - 1))/(x^4 + 8*x^3 - 1)) + 1/4*log((x^4 -
 8*x^3 + 12*(x^4 - 1)^(1/3)*x^2 - 6*(x^4 - 1)^(2/3)*x - 1)/(x^4 - 8*x^3 - 1
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}+3}{\left(x^{4}-8x^{3}-1\right)\left(x^{4}-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)/(x^4-1)^(1/3)/(x^4-8*x^3-1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3)/((x^4 - 8*x^3 - 1)*(x^4 - 1)^(1/3)), x)
```

maple [C] time = 3.11, size = 320, normalized size = 3.95

$$\frac{\ln\left(-\frac{32\,\mathrm{RootOf}\left(4\_Z^{2}+2\_Z+1\right)^{2}x^{3}+2\,\mathrm{RootOf}\left(4\_Z^{2}+2\_Z+1\right)x^{4}-4\,\mathrm{RootOf}\left(4\_Z^{2}+2\_Z+1\right)\left(x^{4}-1\right)^{\frac{2}{3}}x-8\,\mathrm{RootOf}\left(4\_Z^{2}+2\_Z+1\right)\left(x^{4}-1\right)^{\frac{1}{3}}x^{2}+32x^{3}\,\mathrm{RootOf}\left(4\_Z^{2}+2\_Z+1\right)+x^{4}-4\left(x^{4}-1\right)^{\frac{2}{3}}x+4\left(x^{4}-1\right)^{\frac{1}{3}}x^{2}+8x^{3}-2\,\mathrm{RootOf}\left(4\_Z^{2}+2\_Z+1\right)-1}{x^{4}-8x^{3}-1}\right)}{2}+\mathrm{RootOf}\left(4\_Z^{2}+2\_Z+1\right)\ln\left(\frac{32\,\mathrm{RootOf}\left(4\_Z^{2}+2\_Z+1\right)^{2}x^{3}-2\,\mathrm{RootOf}\left(4\_Z^{2}+2\_Z+1\right)x^{4}-4\,\mathrm{RootOf}\left(4\_Z^{2}+2\_Z+1\right)\left(x^{4}-1\right)^{\frac{2}{3}}x+16\,\mathrm{RootOf}\left(4\_Z^{2}+2\_Z+1\right)\left(x^{4}-1\right)^{\frac{1}{3}}x^{2}+16x^{3}\,\mathrm{RootOf}\left(4\_Z^{2}+2\_Z+1\right)-x^{4}+2\left(x^{4}-1\right)^{\frac{2}{3}}x+4\left(x^{4}-1\right)^{\frac{1}{3}}x^{2}+2\,\mathrm{RootOf}\left(4\_Z^{2}+2\_Z+1\right)+1}{x^{4}-8x^{3}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3)/(x^4-1)^(1/3)/(x^4-8*x^3-1),x)
```

```
[Out] 1/2*ln(-(32*RootOf(4*_Z^2+2*_Z+1)^2*x^3+2*RootOf(4*_Z^2+2*_Z+1)*x^4-4*RootO
f(4*_Z^2+2*_Z+1)*(x^4-1)^(2/3)*x-8*RootOf(4*_Z^2+2*_Z+1)*(x^4-1)^(1/3)*x^2+
32*x^3*RootOf(4*_Z^2+2*_Z+1)+x^4-4*(x^4-1)^(2/3)*x+4*(x^4-1)^(1/3)*x^2+8*x^
3-2*RootOf(4*_Z^2+2*_Z+1)-1)/(x^4-8*x^3-1))+RootOf(4*_Z^2+2*_Z+1)*ln((32*Ro
otOf(4*_Z^2+2*_Z+1)^2*x^3-2*RootOf(4*_Z^2+2*_Z+1)*x^4-4*RootOf(4*_Z^2+2*_Z+
1)*(x^4-1)^(2/3)*x+16*RootOf(4*_Z^2+2*_Z+1)*(x^4-1)^(1/3)*x^2+16*x^3*RootOf
(4*_Z^2+2*_Z+1)-x^4+2*(x^4-1)^(2/3)*x+4*(x^4-1)^(1/3)*x^2+2*RootOf(4*_Z^2+2
*_Z+1)+1)/(x^4-8*x^3-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}+3}{\left(x^{4}-8x^{3}-1\right)\left(x^{4}-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)/(x^4-1)^(1/3)/(x^4-8*x^3-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3)/((x^4 - 8*x^3 - 1)*(x^4 - 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^{4}+3}{\left(x^{4}-1\right)^{1/3}\,\left(-x^{4}+8x^{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 + 3)/((x^4 - 1)^(1/3)*(8*x^3 - x^4 + 1)),x)
```

```
[Out] int(-(x^4 + 3)/((x^4 - 1)^(1/3)*(8*x^3 - x^4 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3)/(x**4-1)**(1/3)/(x**4-8*x**3-1),x)
```

```
[Out] Timed out
```

3.978 $\int \frac{(-3+2x)\sqrt{-2x+2x^2+3x^4}}{\left(-2+2x+x^3\right)^2}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{3x^4+2x^2-2x}\,x}{2\left(x^3+2x-2\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{3x^4+2x^2-2x}}{3x^3+2x-2}\right)}{2\sqrt{2}}$$

Rubi [F] time = 1.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-3+2x)\sqrt{-2x+2x^2+3x^4}}{\left(-2+2x+x^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + 2*x)*Sqrt[-2*x + 2*x^2 + 3*x^4])/(-2 + 2*x + x^3)^2,x]
```

```
[Out] (-6*Sqrt[-2*x + 2*x^2 + 3*x^4]*Defer[Subst][Defer[Int][(x^2*Sqrt[-2 + 2*x^2
 + 3*x^6])/(-2 + 2*x^2 + x^6)^2, x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[-2 + 2*x +
3*x^3]) + (4*Sqrt[-2*x + 2*x^2 + 3*x^4]*Defer[Subst][Defer[Int][(x^4*Sqrt[-
2 + 2*x^2 + 3*x^6])/(-2 + 2*x^2 + x^6)^2, x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[-2
 + 2*x + 3*x^3])
```

Rubi steps

$$\begin{aligned}
\int \frac{(-3+2x)\sqrt{-2x+2x^2+3x^4}}{\left(-2+2x+x^3\right)^2}\,dx &= \frac{\sqrt{-2x+2x^2+3x^4}\int \frac{\sqrt{x}(-3+2x)\sqrt{-2+2x+3x^3}}{\left(-2+2x+x^3\right)^2}\,dx}{\sqrt{x}\sqrt{-2+2x+3x^3}}\\
&= \frac{\left(2\sqrt{-2x+2x^2+3x^4}\right)\text{Subst}\left(\int \frac{x^2\left(-3+2x^2\right)\sqrt{-2+2x^2+3x^6}}{\left(-2+2x^2+x^6\right)^2}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{-2+2x+3x^3}}\\
&= \frac{\left(2\sqrt{-2x+2x^2+3x^4}\right)\text{Subst}\left(\int \left(-\frac{3x^2\sqrt{-2+2x^2+3x^6}}{\left(-2+2x^2+x^6\right)^2}+\frac{2x^4\sqrt{-2+2x^2+3x^6}}{\left(-2+2x^2+x^6\right)^2}\right)dx,\right.}{\sqrt{x}\sqrt{-2+2x+3x^3}}\\
&= \frac{\left(4\sqrt{-2x+2x^2+3x^4}\right)\text{Subst}\left(\int \frac{x^4\sqrt{-2+2x^2+3x^6}}{\left(-2+2x^2+x^6\right)^2}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{-2+2x+3x^3}}-\frac{\left(6\sqrt{-2x+2\right.}}{}
\end{aligned}$$

Mathematica [A] time = 0.47, size = 85, normalized size = 1.05

$$-\frac{x^2\left(-6x^3+\sqrt{\frac{4}{x^3}-\frac{4}{x^2}-6}\left(x^3+2x-2\right)\tan^{-1}\left(\sqrt{\frac{1}{x^3}-\frac{1}{x^2}-\frac{3}{2}}\right)-4x+4\right)}{4\left(x^3+2x-2\right)\sqrt{x\left(3x^3+2x-2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + 2*x)*Sqrt[-2*x + 2*x^2 + 3*x^4])/(-2 + 2*x + x^3)^2,x]
```

```
[Out] -1/4*(x^2*(4 - 4*x - 6*x^3 + Sqrt[-6 + 4/x^3 - 4/x^2]*(-2 + 2*x + x^3)*ArcT
an[Sqrt[-3/2 + x^(-3) - x^(-2)]]))/((-2 + 2*x + x^3)*Sqrt[x*(-2 + 2*x + 3*x
^3)])
```

IntegrateAlgebraic [A] time = 0.45, size = 81, normalized size = 1.00

$$\frac{\sqrt{3x^4+2x^2-2x}\,x}{2\left(x^3+2x-2\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt{3x^4+2x^2-2x}}{3x^3+2x-2}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + 2*x)*Sqrt[-2*x + 2*x^2 + 3*x^4])/(-2 + 2*x + x^3)
^2,x]
```

```
[Out] (x*Sqrt[-2*x + 2*x^2 + 3*x^4])/(2*(-2 + 2*x + x^3)) + ArcTanh[(Sqrt[2]*x*Sq
rt[-2*x + 2*x^2 + 3*x^4])/(-2 + 2*x + 3*x^3)]/(2*Sqrt[2])
```

fricas [A] time = 0.46, size = 132, normalized size = 1.63

$$\frac{\sqrt{2}\left(x^3+2x-2\right)\log\left(-\frac{49x^6+36x^4-36x^3+4\sqrt{2}\left(5x^4+2x^2-2x\right)\sqrt{3x^4+2x^2-2x}+4x^2-8x+4}{x^6+4x^4-4x^3+4x^2-8x+4}\right)+8\sqrt{3x^4+2x^2-2x}\,x}{16\left(x^3+2x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(3*x^4+2*x^2-2*x)^(1/2)/(x^3+2*x-2)^2,x, algorithm="fric
as")
```

```
[Out] 1/16*(sqrt(2)*(x^3 + 2*x - 2)*log(-(49*x^6 + 36*x^4 - 36*x^3 + 4*sqrt(2)*(5
*x^4 + 2*x^2 - 2*x)*sqrt(3*x^4 + 2*x^2 - 2*x) + 4*x^2 - 8*x + 4)/(x^6 + 4*x
^4 - 4*x^3 + 4*x^2 - 8*x + 4)) + 8*sqrt(3*x^4 + 2*x^2 - 2*x)*x)/(x^3 + 2*x
- 2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{3x^4+2x^2-2x}\,(2x-3)}{\left(x^3+2x-2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(3*x^4+2*x^2-2*x)^(1/2)/(x^3+2*x-2)^2,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(3*x^4 + 2*x^2 - 2*x)*(2*x - 3)/(x^3 + 2*x - 2)^2, x)
```

maple [C] time = 0.43, size = 1689, normalized size = 20.85

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*x)*(3*x^4+2*x^2-2*x)^(1/2)/(x^3+2*x-2)^2,x)
```

```
[Out] 1/2*x*(3*x^4+2*x^2-2*x)^(1/2)/(x^3+2*x-2)-1/1728*108^(1/2)*sum(_alpha^2*((9
+89^(1/2))^(1/3)-2/(9+89^(1/2))^(1/3)+I*3^(1/2)*((9+89^(1/2))^(1/3)+2/(9+89
^(1/2))^(1/3)))*(x*(-3*(9+89^(1/2))^(1/3)+6/(9+89^(1/2))^(1/3)-I*3^(1/2)*((
9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)))/(-(9+89^(1/2))^(1/3)+2/(9+89^(1/2)
)^(1/3)-I*3^(1/2)*((9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)))/(3*x-(9+89^(1/
2))^(1/3)+2/(9+89^(1/2))^(1/3)))^(1/2)*(3*x-(9+89^(1/2))^(1/3)+2/(9+89^(1/2
))^(1/3))^2*(((9+89^(1/2))^(1/3)-2/(9+89^(1/2))^(1/3))*(6*x+(9+89^(1/2))^(1
```

```
/3)-2/(9+89^(1/2))^(1/3)-I*3^(1/2)*((9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)
))/(-(9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)+I*3^(1/2)*((9+89^(1/2))^(1/3)+
2/(9+89^(1/2))^(1/3)))/(3*x-(9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)))^(1/2)
*(((9+89^(1/2))^(1/3)-2/(9+89^(1/2))^(1/3))*(6*x+(9+89^(1/2))^(1/3)-2/(9+89
^(1/2))^(1/3)+I*3^(1/2)*((9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)))/(-(9+89^
(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)-I*3^(1/2)*((9+89^(1/2))^(1/3)+2/(9+89^(1/
2))^(1/3)))/(3*x-(9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)))^(1/2)/(-3*(9+89^
(1/2))^(1/3)+6/(9+89^(1/2))^(1/3)-I*3^(1/2)*((9+89^(1/2))^(1/3)+2/(9+89^(1/
2))^(1/3)))/((9+89^(1/2))^(1/3)-2/(9+89^(1/2))^(1/3))/(x*(3*x-(9+89^(1/2))^
(1/3)+2/(9+89^(1/2))^(1/3))*(6*x+(9+89^(1/2))^(1/3)-2/(9+89^(1/2))^(1/3)-I*
3^(1/2)*((9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3)))*(6*x+(9+89^(1/2))^(1/3)-
2/(9+89^(1/2))^(1/3)+I*3^(1/2)*((9+89^(1/2))^(1/3)+2/(9+89^(1/2))^(1/3))))^
(1/2)*(-44*_alpha^2-8*_alpha-96-31*_alpha^2*(9+89^(1/2))^(2/3)+3*_alpha^2*(
9+89^(1/2))^(2/3)*89^(1/2)+6*_alpha^2*(9+89^(1/2))^(1/3)-2*_alpha^2*(9+89^(
1/2))^(1/3)*89^(1/2)-2*_alpha*(9+89^(1/2))^(1/3)*89^(1/2)+6*_alpha*(9+89^(1
/2))^(1/3)-31*_alpha*(9+89^(1/2))^(2/3)+3*_alpha*(9+89^(1/2))^(2/3)*89^(1/2
)+30*(9+89^(1/2))^(1/3)-6*(9+89^(1/2))^(1/3)*89^(1/2)-66*(9+89^(1/2))^(2/3)
+6*(9+89^(1/2))^(2/3)*89^(1/2))*(6*EllipticF(((-1/2*(9+89^(1/2))^(1/3)+1/(9
+89^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(9+89^(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/
3)))*x/(-1/6*(9+89^(1/2))^(1/3)+1/3/(9+89^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(
9+89^(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/3)))/(x-1/3*(9+89^(1/2))^(1/3)+2/3/(9
+89^(1/2))^(1/3)))^(1/2),((1/2*(9+89^(1/2))^(1/3)-1/(9+89^(1/2))^(1/3)-1/2*
I*3^(1/2)*(1/3*(9+89^(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/3)))*(1/6*(9+89^(1/2)
)^(1/3)-1/3/(9+89^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/3*(9+89^(1/2))^(1/3)+2/3/(9
+89^(1/2))^(1/3)))/(1/6*(9+89^(1/2))^(1/3)-1/3/(9+89^(1/2))^(1/3)-1/2*I*3^(
1/2)*(1/3*(9+89^(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/3)))/(1/2*(9+89^(1/2))^(1/
3)-1/(9+89^(1/2))^(1/3)+1/2*I*3^(1/2)*(1/3*(9+89^(1/2))^(1/3)+2/3/(9+89^(1/
2))^(1/3))))^(1/2))-(_alpha^2+2)*((9+89^(1/2))^(1/3)-2/(9+89^(1/2))^(1/3))*
EllipticPi(((-1/2*(9+89^(1/2))^(1/3)+1/(9+89^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/
3*(9+89^(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/3)))*x/(-1/6*(9+89^(1/2))^(1/3)+1/
3/(9+89^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(9+89^(1/2))^(1/3)+2/3/(9+89^(1/2))
^(1/3)))/(x-1/3*(9+89^(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/3)))^(1/2),1/2-1/356
*(9+89^(1/2))^(1/3)*89^(1/2)+29/1068*(9+89^(1/2))^(2/3)*89^(1/2)-1/712*I*3^
(1/2)*(9+89^(1/2))^(1/3)*89^(1/2)*_alpha^2+1/16*I*3^(1/2)*(9+89^(1/2))^(2/3
)*_alpha^2+31/1602*I*3^(1/2)*89^(1/2)-1/72*I*(9+89^(1/2))^(1/3)*3^(1/2)*_al
pha^2-1/4*(9+89^(1/2))^(2/3)-1/36*I*(9+89^(1/2))^(1/3)*3^(1/2)-3/16*_alpha^
2*(9+89^(1/2))^(2/3)-1/12*(9+89^(1/2))^(1/3)+1/801*I*3^(1/2)*89^(1/2)*_alph
a^2+1/12*I*(9+89^(1/2))^(2/3)*3^(1/2)-29/3204*I*(9+89^(1/2))^(2/3)*89^(1/2)
*3^(1/2)-3/712*_alpha^2*(9+89^(1/2))^(1/3)*89^(1/2)-1/24*_alpha^2*(9+89^(1/
2))^(1/3)+85/4272*_alpha^2*(9+89^(1/2))^(2/3)*89^(1/2)-1/1068*I*3^(1/2)*(9+
89^(1/2))^(1/3)*89^(1/2)-85/12816*I*3^(1/2)*(9+89^(1/2))^(2/3)*89^(1/2)*_al
pha^2,((1/2*(9+89^(1/2))^(1/3)-1/(9+89^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(9+8
9^(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/3)))*(1/6*(9+89^(1/2))^(1/3)-1/3/(9+89^(
1/2))^(1/3)+1/2*I*3^(1/2)*(1/3*(9+89^(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/3)))/
(1/6*(9+89^(1/2))^(1/3)-1/3/(9+89^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/3*(9+89^(1/
2))^(1/3)+2/3/(9+89^(1/2))^(1/3)))/(1/2*(9+89^(1/2))^(1/3)-1/(9+89^(1/2))^(
1/3)+1/2*I*3^(1/2)*(1/3*(9+89^(1/2))^(1/3)+2/3/(9+89^(1/2))^(1/3))))^(1/2))
),_alpha=RootOf(_Z^3+2*_Z-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3x^4+2x^2-2x}\,(2x-3)}{\left(x^3+2x-2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(3*x^4+2*x^2-2*x)^(1/2)/(x^3+2*x-2)^2,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(3*x^4 + 2*x^2 - 2*x)*(2*x - 3)/(x^3 + 2*x - 2)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x-3)\sqrt{3x^4+2x^2-2x}}{\left(x^3+2x-2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x - 3)*(2*x^2 - 2*x + 3*x^4)^(1/2))/(2*x + x^3 - 2)^2,x)
[Out] int(((2*x - 3)*(2*x^2 - 2*x + 3*x^4)^(1/2))/(2*x + x^3 - 2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(3*x**4+2*x**2-2*x)**(1/2)/(x**3+2*x-2)**2,x)
[Out] Timed out
```

3.979 $\int \frac{-2b+ax^4}{x^4\sqrt[4]{-b+ax^4}}\,dx$

Optimal. Leaf size=81

$$\frac{1}{2}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+\frac{1}{2}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)-\frac{2\left(ax^4-b\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {451, 240, 212, 206, 203}

$$\frac{1}{2}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+\frac{1}{2}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)-\frac{2\left(ax^4-b\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*b + a*x^4)/(x^4*(-b + a*x^4)^(1/4)),x]
```

```
[Out] (-2*(-b + a*x^4)^(3/4))/(3*x^3) + (a^(3/4)*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^
(1/4)])/2 + (a^(3/4)*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{-2b+ax^4}{x^4\sqrt[4]{-b+ax^4}}\,dx &= -\frac{2\left(-b+ax^4\right)^{3/4}}{3x^3}+a\int \frac{1}{\sqrt[4]{-b+ax^4}}\,dx\\ &= -\frac{2\left(-b+ax^4\right)^{3/4}}{3x^3}+a\,\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)\\ &= -\frac{2\left(-b+ax^4\right)^{3/4}}{3x^3}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{1+\sqrt{a}\,x^2}\right.\\ &= -\frac{2\left(-b+ax^4\right)^{3/4}}{3x^3}+\frac{1}{2}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)+\frac{1}{2}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 81, normalized size = 1.00

$$\frac{1}{2}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+\frac{1}{2}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)-\frac{2\left(ax^4-b\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*b + a*x^4)/(x^4*(-b + a*x^4)^(1/4)),x]
[Out] (-2*(-b + a*x^4)^(3/4))/(3*x^3) + (a^(3/4)*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^
(1/4)])/2 + (a^(3/4)*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/2
```

IntegrateAlgebraic [A] time = 0.30, size = 81, normalized size = 1.00

$$\frac{1}{2}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+\frac{1}{2}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)-\frac{2\left(ax^4-b\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*b + a*x^4)/(x^4*(-b + a*x^4)^(1/4)),x]
[Out] (-2*(-b + a*x^4)^(3/4))/(3*x^3) + (a^(3/4)*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^
(1/4)])/2 + (a^(3/4)*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-2*b)/x^4/(a*x^4-b)^(1/4),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4-2\,b}{\left(ax^4-b\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-2*b)/x^4/(a*x^4-b)^(1/4),x, algorithm="giac")
[Out] integrate((a*x^4 - 2*b)/((a*x^4 - b)^(1/4)*x^4), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{a\,x^{4}-2b}{x^{4}\left(a\,x^{4}-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-2*b)/x^4/(a*x^4-b)^(1/4),x)
```

```
[Out] int((a*x^4-2*b)/x^4/(a*x^4-b)^(1/4),x)
```

maxima [A] time = 0.43, size = 93, normalized size = 1.15

$$-\frac{1}{4}\,a\left(\frac{2\arctan\left(\frac{\left(ax^{4}-b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^{4}-b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^{4}-b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right)-\frac{2\left(ax^{4}-b\right)^{\frac{3}{4}}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-2*b)/x^4/(a*x^4-b)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/4*a*(2*arctan((a*x^4 - b)^(1/4)/(a^(1/4)*x))/a^(1/4) + log(-(a^(1/4) - (
a*x^4 - b)^(1/4)/x)/(a^(1/4) + (a*x^4 - b)^(1/4)/x))/a^(1/4)) - 2/3*(a*x^4
- b)^(3/4)/x^3
```

mupad [B] time = 1.15, size = 57, normalized size = 0.70

$$\frac{a\,x\left(1-\frac{a\,x^{4}}{b}\right)^{1/4}{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \frac{a\,x^{4}}{b}\right)}{\left(a\,x^{4}-b\right)^{1/4}}-\frac{2\left(a\,x^{4}-b\right)^{3/4}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*b - a*x^4)/(x^4*(a*x^4 - b)^(1/4)),x)
```

```
[Out] (a*x*(1 - (a*x^4)/b)^(1/4)*hypergeom([1/4, 1/4], 5/4, (a*x^4)/b))/(a*x^4 -
b)^(1/4) - (2*(a*x^4 - b)^(3/4))/(3*x^3)
```

sympy [C] time = 2.60, size = 126, normalized size = 1.56

$$\frac{axe^{-\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{ax^{4}}{b}\right)}{4\sqrt[4]{b}\,\Gamma\left(\frac{5}{4}\right)}-2b\left(\begin{cases}-\frac{a^{\frac{3}{4}}\left(-1+\frac{b}{ax^{4}}\right)^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\Gamma\left(-\frac{3}{4}\right)}{4b\Gamma\left(\frac{1}{4}\right)} & \text{for}\ \left|\frac{b}{ax^{4}}\right|>1\\-\frac{a^{\frac{3}{4}}\left(1-\frac{b}{ax^{4}}\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{4b\Gamma\left(\frac{1}{4}\right)} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-2*b)/x**4/(a*x**4-b)**(1/4),x)
```

```
[Out] a*x*exp(-I*pi/4)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), a*x**4/b)/(4*b**(1/4)
*gamma(5/4)) - 2*b*Piecewise((-a**(3/4)*(-1 + b/(a*x**4))**(3/4)*exp(3*I*pi
/4)*gamma(-3/4)/(4*b*gamma(1/4)), Abs(b/(a*x**4)) > 1), (-a**(3/4)*(1 - b/(
a*x**4))**(3/4)*gamma(-3/4)/(4*b*gamma(1/4)), True))
```

3.980 $\int \left(-b+ax^4\right)^{3/4}\,dx$

Optimal. Leaf size=81

$$\frac{1}{4}x\left(ax^4-b\right)^{3/4}-\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{8\sqrt[4]{a}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{8\sqrt[4]{a}}$$

Rubi [A] time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {195, 240, 212, 206, 203}

$$\frac{1}{4}x\left(ax^4-b\right)^{3/4}-\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{8\sqrt[4]{a}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{8\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^4)^(3/4),x]
```

```
[Out] (x*(-b + a*x^4)^(3/4))/4 - (3*b*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(8*
a^(1/4)) - (3*b*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(8*a^(1/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\begin{aligned}\int \left(-b+ax^4\right)^{3/4}\,dx &= \frac{1}{4}x\left(-b+ax^4\right)^{3/4}-\frac{1}{4}(3b)\int \frac{1}{\sqrt[4]{-b+ax^4}}\,dx\\ &= \frac{1}{4}x\left(-b+ax^4\right)^{3/4}-\frac{1}{4}(3b)\,\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)\\ &= \frac{1}{4}x\left(-b+ax^4\right)^{3/4}-\frac{1}{8}(3b)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)-\frac{1}{8}(3b)\,\text{Subst}\left(\int \frac{}{1+}\right.\\ &= \frac{1}{4}x\left(-b+ax^4\right)^{3/4}-\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{8\sqrt[4]{a}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{8\sqrt[4]{a}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.59

$$\frac{x\left(ax^4-b\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};\frac{ax^4}{b}\right)}{\left(1-\frac{ax^4}{b}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^4)^(3/4),x]
```

```
[Out] (x*(-b + a*x^4)^(3/4)*Hypergeometric2F1[-3/4, 1/4, 5/4, (a*x^4)/b])/(1 - (a
*x^4)/b)^(3/4)
```

IntegrateAlgebraic [A] time = 0.29, size = 81, normalized size = 1.00

$$\frac{1}{4}x\left(ax^4-b\right)^{3/4}-\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{8\sqrt[4]{a}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{8\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^4)^(3/4),x]
```

```
[Out] (x*(-b + a*x^4)^(3/4))/4 - (3*b*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(8*
a^(1/4)) - (3*b*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(8*a^(1/4))
```

fricas [B] time = 0.42, size = 202, normalized size = 2.49

$$\frac{1}{4}\left(ax^4-b\right)^{\frac{3}{4}}x-\frac{3}{4}\left(\frac{b^4}{a}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^4-b\right)^{\frac{1}{4}}\left(\frac{b^4}{a}\right)^{\frac{1}{4}}b^3-\left(\frac{b^4}{a}\right)^{\frac{1}{4}}x\sqrt{\frac{\sqrt{\frac{b^4}{a}}ab^4x^2+\sqrt{ax^4-b}\,b^6}{x^2}}}{b^4x}\right)-\frac{3}{16}\left(\frac{b^4}{a}\right)^{\frac{1}{4}}\log\left(\frac{27\left(\left(ax^4-b\right)^{\frac{1}{4}}b^3+\left(\frac{b^4}{a}\right)^{\frac{3}{4}}ax\right)}{x}\right)+\frac{3}{16}\left(\frac{b^4}{a}\right)^{\frac{1}{4}}\log\left(\frac{27\left(\left(ax^4-b\right)^{\frac{1}{4}}b^3-\left(\frac{b^4}{a}\right)^{\frac{3}{4}}ax\right)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/4*(a*x^4 - b)^(3/4)*x - 3/4*(b^4/a)^(1/4)*arctan(-((a*x^4 - b)^(1/4)*(b^4
/a)^(1/4)*b^3 - (b^4/a)^(1/4)*x*sqrt((sqrt(b^4/a)*a*b^4*x^2 + sqrt(a*x^4 -
b)*b^6)/x^2))/(b^4*x)) - 3/16*(b^4/a)^(1/4)*log(27*((a*x^4 - b)^(1/4)*b^3 +
 (b^4/a)^(3/4)*a*x)/x) + 3/16*(b^4/a)^(1/4)*log(27*((a*x^4 - b)^(1/4)*b^3 -
 (b^4/a)^(3/4)*a*x)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^4-b\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4),x, algorithm="giac")
[Out] integrate((a*x^4 - b)^(3/4), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \left(a\,x^{4}-b\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b)^(3/4),x)
[Out] int((a*x^4-b)^(3/4),x)
```

maxima [A] time = 0.43, size = 112, normalized size = 1.38

$$\frac{3}{16}\,b\left(\frac{2\,\arctan\left(\frac{\left(ax^{4}-b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^{4}-b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^{4}-b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right)+\frac{\left(ax^{4}-b\right)^{\frac{3}{4}}b}{4\left(a-\frac{ax^{4}-b}{x^{4}}\right)x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4),x, algorithm="maxima")
[Out] 3/16*b*(2*arctan((a*x^4 - b)^(1/4)/(a^(1/4)*x))/a^(1/4) + log(-(a^(1/4) - (
a*x^4 - b)^(1/4)/x)/(a^(1/4) + (a*x^4 - b)^(1/4)/x))/a^(1/4)) + 1/4*(a*x^4
- b)^(3/4)*b/((a - (a*x^4 - b)/x^4)*x^3)
```

mupad [B] time = 0.78, size = 39, normalized size = 0.48

$$\frac{x\left(a\,x^{4}-b\right)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{4};\ \frac{5}{4};\ \frac{a\,x^{4}}{b}\right)}{\left(1-\frac{a\,x^{4}}{b}\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4 - b)^(3/4),x)
[Out] (x*(a*x^4 - b)^(3/4)*hypergeom([-3/4, 1/4], 5/4, (a*x^4)/b))/(1 - (a*x^4)/b
)^(3/4)
```

sympy [C] time = 1.42, size = 41, normalized size = 0.51

$$-\frac{b^{\frac{3}{4}}xe^{-\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{ax^{4}}{b}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b)**(3/4),x)
[Out] -b**(3/4)*x*exp(-I*pi/4)*gamma(1/4)*hyper((-3/4, 1/4), (5/4,), a*x**4/b)/(4
*gamma(5/4))
```

3.981 $\int \frac{(-b+ax^4)^{3/4}}{x^4}\,dx$

Optimal. Leaf size=81

$$\frac{1}{2}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+\frac{1}{2}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)-\frac{(ax^4-b)^{3/4}}{3x^3}$$

Rubi [A] time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {277, 240, 212, 206, 203}

$$\frac{1}{2}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+\frac{1}{2}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)-\frac{(ax^4-b)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^4)^(3/4)/x^4,x]
```

```
[Out] -1/3*(-b + a*x^4)^(3/4)/x^3 + (a^(3/4)*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4
)])/2 + (a^(3/4)*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(-b+ax^4)^{3/4}}{x^4}\,dx = -\frac{(-b+ax^4)^{3/4}}{3x^3} + a\int \frac{1}{\sqrt[4]{-b+ax^4}}\,dx$$
$$= -\frac{(-b+ax^4)^{3/4}}{3x^3} + a\,\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)$$
$$= -\frac{(-b+ax^4)^{3/4}}{3x^3} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right) + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{1+\sqrt{a}\,x^2}\,dx\right.$$
$$= -\frac{(-b+ax^4)^{3/4}}{3x^3} + \frac{1}{2}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right) + \frac{1}{2}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)$$

Mathematica [C] time = 0.01, size = 53, normalized size = 0.65

$$-\frac{(ax^4-b)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};\frac{ax^4}{b}\right)}{3x^3\left(1-\frac{ax^4}{b}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^4)^(3/4)/x^4,x]
```

```
[Out] -1/3*((-b + a*x^4)^(3/4)*Hypergeometric2F1[-3/4, -3/4, 1/4, (a*x^4)/b])/(x^
3*(1 - (a*x^4)/b)^(3/4))
```

IntegrateAlgebraic [A] time = 0.23, size = 81, normalized size = 1.00

$$\frac{1}{2}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right) + \frac{1}{2}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right) - \frac{(ax^4-b)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^4)^(3/4)/x^4,x]
```

```
[Out] -1/3*(-b + a*x^4)^(3/4)/x^3 + (a^(3/4)*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4
)])/2 + (a^(3/4)*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4)/x^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^4-b)^{\frac{3}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] integrate((a*x^4 - b)^(3/4)/x^4, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{4}-b\right)^{\frac{3}{4}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b)^(3/4)/x^4,x)
```

```
[Out] int((a*x^4-b)^(3/4)/x^4,x)
```

maxima [A] time = 0.44, size = 93, normalized size = 1.15

$$-\frac{1}{4}\,a\left(\frac{2\,\arctan\left(\frac{\left(ax^{4}-b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^{4}-b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^{4}-b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right)-\frac{\left(ax^{4}-b\right)^{\frac{3}{4}}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4)/x^4,x, algorithm="maxima")
```

```
[Out] -1/4*a*(2*arctan((a*x^4 - b)^(1/4)/(a^(1/4)*x))/a^(1/4) + log(-(a^(1/4) - (
a*x^4 - b)^(1/4)/x)/(a^(1/4) + (a*x^4 - b)^(1/4)/x))/a^(1/4)) - 1/3*(a*x^4
- b)^(3/4)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^4-b\right)^{3/4}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4 - b)^(3/4)/x^4,x)
```

```
[Out] int((a*x^4 - b)^(3/4)/x^4, x)
```

sympy [C] time = 1.27, size = 46, normalized size = 0.57

$$\frac{b^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\right|\frac{ax^{4}}{b}\right)}{4x^{3}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b)**(3/4)/x**4,x)
```

```
[Out] b**(3/4)*exp(3*I*pi/4)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), a*x**4/b)/(4
*x**3*gamma(1/4))
```

3.982 $\int \frac{\sqrt[4]{bx^3+ax^4}}{x^2} dx$

Optimal. Leaf size=81

$$-\frac{4\sqrt[4]{ax^4+bx^3}}{x}-2\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)+2\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)$$

Rubi [A] time = 0.15, antiderivative size = 135, normalized size of antiderivative = 1.67, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2020, 2032, 63, 331, 298, 203, 206}

$$-\frac{4\sqrt[4]{ax^4+bx^3}}{x}-\frac{2\sqrt[4]{a}\,x^{9/4}(ax+b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{\left(ax^4+bx^3\right)^{3/4}}+\frac{2\sqrt[4]{a}\,x^{9/4}(ax+b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{\left(ax^4+bx^3\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^3 + a*x^4)^(1/4)/x^2,x]
```

```
[Out] (-4*(b*x^3 + a*x^4)^(1/4))/x - (2*a^(1/4)*x^(9/4)*(b + a*x)^(3/4)*ArcTan[(a
^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(b*x^3 + a*x^4)^(3/4) + (2*a^(1/4)*x^(9/4
)*(b + a*x)^(3/4)*ArcTanh[(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(b*x^3 + a*x^
4)^(3/4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt[4]{bx^3+ax^4}}{x^2}\,dx = -\frac{4\sqrt[4]{bx^3+ax^4}}{x} + a\int \frac{x^2}{\left(bx^3+ax^4\right)^{3/4}}\,dx$$

$$= -\frac{4\sqrt[4]{bx^3+ax^4}}{x} + \frac{\left(ax^{9/4}(b+ax)^{3/4}\right)\int \frac{1}{\sqrt[4]{x}\,(b+ax)^{3/4}}\,dx}{\left(bx^3+ax^4\right)^{3/4}}$$

$$= -\frac{4\sqrt[4]{bx^3+ax^4}}{x} + \frac{\left(4ax^{9/4}(b+ax)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{\left(b+ax^4\right)^{3/4}}\,dx, x, \sqrt[4]{x}\right)}{\left(bx^3+ax^4\right)^{3/4}}$$

$$= -\frac{4\sqrt[4]{bx^3+ax^4}}{x} + \frac{\left(4ax^{9/4}(b+ax)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)}{\left(bx^3+ax^4\right)^{3/4}}$$

$$= -\frac{4\sqrt[4]{bx^3+ax^4}}{x} + \frac{\left(2\sqrt{a}\,x^{9/4}(b+ax)^{3/4}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)}{\left(bx^3+ax^4\right)^{3/4}} - \frac{\left(2\sqrt{a}\,x^{9/4}(b+a\cdots\right.}{}$$

$$= -\frac{4\sqrt[4]{bx^3+ax^4}}{x} - \frac{2\sqrt[4]{a}\,x^{9/4}(b+ax)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)}{\left(bx^3+ax^4\right)^{3/4}} + \frac{2\sqrt[4]{a}\,x^{9/4}(b+ax)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)}{\left(bx^3+ax^4\right)^{3/4}}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.58

$$-\frac{4\sqrt[4]{x^3(ax+b)}\;{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};-\frac{ax}{b}\right)}{x\sqrt[4]{\frac{ax}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^3 + a*x^4)^(1/4)/x^2,x]
```

```
[Out] (-4*(x^3*(b + a*x))^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, -((a*x)/b)])/(
x*(1 + (a*x)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.31, size = 81, normalized size = 1.00

$$-\frac{4\sqrt[4]{ax^4+bx^3}}{x} - 2\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right) + 2\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^3 + a*x^4)^(1/4)/x^2,x]
```

```
[Out] (-4*(b*x^3 + a*x^4)^(1/4))/x - 2*a^(1/4)*ArcTan[(a^(1/4)*x)/(b*x^3 + a*x^4)
^(1/4)] + 2*a^(1/4)*ArcTanh[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)]
```

fricas [B] time = 0.43, size = 148, normalized size = 1.83

$$-\frac{4\,a^{\frac{1}{4}}x\arctan\left(\frac{a^{\frac{3}{4}}x\sqrt{\frac{\sqrt{a}\,x^2+\sqrt{ax^4+bx^3}}{x^2}}-\left(ax^4+bx^3\right)^{\frac{1}{4}}a^{\frac{3}{4}}}{ax}\right)-a^{\frac{1}{4}}x\log\left(\frac{a^{\frac{1}{4}}x+\left(ax^4+bx^3\right)^{\frac{1}{4}}}{x}\right)+a^{\frac{1}{4}}x\log\left(-\frac{a^{\frac{1}{4}}x-\left(ax^4+bx^3\right)^{\frac{1}{4}}}{x}\right)+4\left(ax^4+bx^3\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^3)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] -(4*a^(1/4)*x*arctan((a^(3/4)*x*sqrt((sqrt(a)*x^2 + sqrt(a*x^4 + b*x^3))/x^
2) - (a*x^4 + b*x^3)^(1/4)*a^(3/4))/(a*x)) - a^(1/4)*x*log((a^(1/4)*x + (a*
x^4 + b*x^3)^(1/4))/x) + a^(1/4)*x*log(-(a^(1/4)*x - (a*x^4 + b*x^3)^(1/4))
/x) + 4*(a*x^4 + b*x^3)^(1/4))/x
```

giac [B] time = 0.42, size = 183, normalized size = 2.26

$$\sqrt{2}\,(-a)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)+\sqrt{2}\,(-a)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)+\frac{1}{2}\sqrt{2}\,(-a)^{\frac{1}{4}}\log\left(\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)-\frac{1}{2}\sqrt{2}\,(-a)^{\frac{1}{4}}\log\left(-\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)-4\left(a+\frac{b}{x}\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^3)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] sqrt(2)*(-a)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x)^(1/
4))/(-a)^(1/4)) + sqrt(2)*(-a)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4
) - 2*(a + b/x)^(1/4))/(-a)^(1/4)) + 1/2*sqrt(2)*(-a)^(1/4)*log(sqrt(2)*(-a
)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a + b/x)) - 1/2*sqrt(2)*(-a)^(1/4
)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a + b/x)) - 4*(
a + b/x)^(1/4)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{4}+b\,x^{3}\right)^{\frac{1}{4}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b*x^3)^(1/4)/x^2,x)
```

```
[Out] int((a*x^4+b*x^3)^(1/4)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{4}+bx^{3}\right)^{\frac{1}{4}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^3)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + b*x^3)^(1/4)/x^2, x)
```

mupad [B] time = 0.99, size = 40, normalized size = 0.49

$$-\frac{4\left(a\,x^4+b\,x^3\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ -\frac{a\,x}{b}\right)}{x\,\left(\frac{a\,x}{b}+1\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4 + b*x^3)^(1/4)/x^2,x)
```

```
[Out] -(4*(a*x^4 + b*x^3)^(1/4)*hypergeom([-1/4, -1/4], 3/4, -(a*x)/b))/(x*((a*x)
/b + 1)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[4]{x^3\left(ax+b\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b*x**3)**(1/4)/x**2,x)
```

```
[Out] Integral((x**3*(a*x + b))**(1/4)/x**2, x)
```

3.983 $$\int \frac{\left(1-x^3+x^4+x^6\right)^{3/4}\left(-4+x^3+2x^6\right)}{\left(1-x^3+x^6\right)^2}\,dx$$

Optimal. Leaf size=81

$$-\frac{\left(x^6+x^4-x^3+1\right)^{3/4}x}{x^6-x^3+1}-\frac{3}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^6+x^4-x^3+1}}\right)-\frac{3}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^6+x^4-x^3+1}}\right)$$

Rubi [F] time = 18.76, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-x^3+x^4+x^6\right)^{3/4}\left(-4+x^3+2x^6\right)}{\left(1-x^3+x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 - x^3 + x^4 + x^6)^(3/4)*(-4 + x^3 + 2*x^6))/(1 - x^3 + x^6)^2,x]

[Out] (-4*Defer[Int][(1 - x^3 + x^4 + x^6)^(3/4)/(2^(2/3)*(1 - I*Sqrt[3])^(1/3) -
 2*x)^2, x])/(9*((1 - I*Sqrt[3])/2)^(1/3)) + (16*2^(1/3)*Defer[Int][(1 - x^
3 + x^4 + x^6)^(3/4)/(2^(2/3)*(1 - I*Sqrt[3])^(1/3) - 2*x)^2, x])/(9*(1 - I
*Sqrt[3])^(4/3)) - (4*Defer[Int][(1 - x^3 + x^4 + x^6)^(3/4)/(2^(2/3)*(1 +
I*Sqrt[3])^(1/3) - 2*x)^2, x])/(9*((1 + I*Sqrt[3])/2)^(1/3)) + (16*2^(1/3)*
Defer[Int][(1 - x^3 + x^4 + x^6)^(3/4)/(2^(2/3)*(1 + I*Sqrt[3])^(1/3) - 2*x
)^2, x])/(9*(1 + I*Sqrt[3])^(4/3)) - (4*Defer[Int][(1 - x^3 + x^4 + x^6)^(3
/4)/(2^(2/3)*(1 + I*Sqrt[3])^(1/3) - 2*x), x])/(9*((1 + I*Sqrt[3])/2)^(2/3)
) + (16*2^(2/3)*Defer[Int][(1 - x^3 + x^4 + x^6)^(3/4)/(2^(2/3)*(1 + I*Sqrt
[3])^(1/3) - 2*x), x])/(9*(1 + I*Sqrt[3])^(5/3)) + (((2*I)/3)*Defer[Int][(1
 - x^3 + x^4 + x^6)^(3/4)/((1 - I*Sqrt[3])^(1/3) + (-2)^(1/3)*x), x])/(Sqrt
[3]*(1 - I*Sqrt[3])^(2/3)) - (((2*I)/3)*Defer[Int][(1 - x^3 + x^4 + x^6)^(3
/4)/((1 + I*Sqrt[3])^(1/3) + (-2)^(1/3)*x), x])/(Sqrt[3]*(1 + I*Sqrt[3])^(2
/3)) + (8*(-2)^(1/3)*Defer[Int][(1 - x^3 + x^4 + x^6)^(3/4)/(2^(2/3)*(1 - I
*Sqrt[3])^(1/3) + 2*(-1)^(1/3)*x)^2, x])/(9*(1 - I*Sqrt[3])^(1/3)) - (2*(-2
)^(1/3)*(1 - I*Sqrt[3])^(2/3)*Defer[Int][(1 - x^3 + x^4 + x^6)^(3/4)/(2^(2/
3)*(1 - I*Sqrt[3])^(1/3) + 2*(-1)^(1/3)*x)^2, x])/9 - (4*2^(1/3)*Defer[Int]
[(1 - x^3 + x^4 + x^6)^(3/4)/(2^(2/3)*(1 + I*Sqrt[3])^(1/3) + 2*(-1)^(1/3)*
x)^2, x])/(9*((-I)/(I - Sqrt[3]))^(2/3)*(1 - I*Sqrt[3])) - (16*((-2*I)/(I -
 Sqrt[3]))^(1/3)*Defer[Int][(1 - x^3 + x^4 + x^6)^(3/4)/(2^(2/3)*(1 + I*Sqr
t[3])^(1/3) + 2*(-1)^(1/3)*x)^2, x])/(9*(1 - I*Sqrt[3])) - (16*(-1)^(1/6)*2
^(1/3)*Defer[Int][(1 - x^3 + x^4 + x^6)^(3/4)/(-(2^(2/3)*(1 - I*Sqrt[3])^(1
/3)) + 2*(-1)^(2/3)*x)^2, x])/(9*(I - Sqrt[3])*(1 - I*Sqrt[3])^(1/3)) + (4*
(-1)^(1/6)*2^(1/3)*(1 - I*Sqrt[3])^(2/3)*Defer[Int][(1 - x^3 + x^4 + x^6)^(
3/4)/(-(2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + 2*(-1)^(2/3)*x)^2, x])/(9*(I - Sqr
t[3])) - (8*(-1)^(2/3)*Defer[Int][(1 - x^3 + x^4 + x^6)^(3/4)/(-(2^(2/3)*(1
 + I*Sqrt[3])^(1/3)) + 2*(-1)^(2/3)*x)^2, x])/(9*((1 + I*Sqrt[3])/2)^(1/3))
 + (2*(-1)^(2/3)*2^(1/3)*(1 + I*Sqrt[3])^(2/3)*Defer[Int][(1 - x^3 + x^4 +
x^6)^(3/4)/(-(2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + 2*(-1)^(2/3)*x)^2, x])/9 + (
((2*I)/3)*Defer[Int][(1 - x^3 + x^4 + x^6)^(3/4)/((1 - I*Sqrt[3])^(1/3) - 2
^(1/3)*x), x])/(Sqrt[3]*(1 - I*Sqrt[3])^(2/3)) - (((2*I)/3)*Defer[Int][(1 -
 x^3 + x^4 + x^6)^(3/4)/((1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x), x])/(Sqrt[3]*(
1 + I*Sqrt[3])^(2/3)) + (((2*I)/3)*Defer[Int][(1 - x^3 + x^4 + x^6)^(3/4)/(
(1 - I*Sqrt[3])^(1/3) - (-1)^(2/3)*2^(1/3)*x), x])/(Sqrt[3]*(1 - I*Sqrt[3])
^(2/3)) - (((2*I)/3)*Defer[Int][(1 - x^3 + x^4 + x^6)^(3/4)/((1 + I*Sqrt[3]
)^(1/3) - (-1)^(2/3)*2^(1/3)*x), x])/(Sqrt[3]*(1 + I*Sqrt[3])^(2/3)) + (4*(
-2)^(2/3)*Defer[Int][(1 - x^3 + x^4 + x^6)^(3/4)/(2^(2/3) + (1 - I*Sqrt[3])
^(2/3)*x), x])/9 - (2^(2/3)*(1 + I*Sqrt[3])*Defer[Int][(1 - x^3 + x^4 + x^6
```

```
)^(3/4)/(2^(2/3) + (1 - I*Sqrt[3])^(2/3)*x), x])/9 + (4*Defer[Int][(1 - x^3
 + x^4 + x^6)^(3/4)/(3*I - Sqrt[3] + 2^(1/3)*Sqrt[3]*(1 - I*Sqrt[3])^(2/3)*
x), x])/(3*Sqrt[3]) - (4*(3*I + Sqrt[3])*Defer[Int][(1 - x^3 + x^4 + x^6)^(
3/4)/(3*I - Sqrt[3] + 2^(1/3)*Sqrt[3]*(1 - I*Sqrt[3])^(2/3)*x), x])/9 - (4*
(1 + I*Sqrt[3])*Defer[Int][(1 - x^3 + x^4 + x^6)^(3/4)/(2 + 2^(1/3)*(1 + I*
Sqrt[3])^(2/3)*x), x])/9 + ((1 + I*Sqrt[3])^2*Defer[Int][(1 - x^3 + x^4 + x
^6)^(3/4)/(2 + 2^(1/3)*(1 + I*Sqrt[3])^(2/3)*x), x])/9 - (4*(-1)^(1/9)*(I -
 Sqrt[3])*Defer[Int][(1 - x^3 + x^4 + x^6)^(3/4)/(I*2^(2/3)*(1 + I*Sqrt[3])
^(1/3) + (I + Sqrt[3])*x), x])/9 - (2*(-1)^(1/9)*(I + Sqrt[3])*Defer[Int][(
1 - x^3 + x^4 + x^6)^(3/4)/(I*2^(2/3)*(1 + I*Sqrt[3])^(1/3) + (I + Sqrt[3])
*x), x])/9 + (8*Defer[Int][(1 - x^3 + x^4 + x^6)^(3/4)/(2*(3*I - Sqrt[3]) -
 2^(1/3)*(1 - I*Sqrt[3])^(2/3)*(3*I + Sqrt[3])*x), x])/(3*Sqrt[3]) - (8*(3*
I + Sqrt[3])*Defer[Int][(1 - x^3 + x^4 + x^6)^(3/4)/(2*(3*I - Sqrt[3]) - 2^
(1/3)*(1 - I*Sqrt[3])^(2/3)*(3*I + Sqrt[3])*x), x])/9
```

Rubi steps

$$\int \frac{\left(1-x^3+x^4+x^6\right)^{3/4}\left(-4+x^3+2x^6\right)}{\left(1-x^3+x^6\right)^2}\,dx = \int \left(\frac{3\left(-2+x^3\right)\left(1-x^3+x^4+x^6\right)^{3/4}}{\left(1-x^3+x^6\right)^2}+\frac{2\left(1-x^3+x^4+x^6\right)^{3/4}}{1-x^3+x^6}\right)$$

$$= 2\int \frac{\left(1-x^3+x^4+x^6\right)^{3/4}}{1-x^3+x^6}\,dx + 3\int \frac{\left(-2+x^3\right)\left(1-x^3+x^4+x^6\right.}{\left(1-x^3+x^6\right)^2}$$

$$= 2\int \left(\frac{2i\left(1-x^3+x^4+x^6\right)^{3/4}}{\sqrt{3}\left(1+i\sqrt{3}-2x^3\right)}+\frac{2i\left(1-x^3+x^4+x^6\right)^{3/4}}{\sqrt{3}\left(-1+i\sqrt{3}+2x^3\right)}\right)dx + 3$$

$$= 3\int \frac{x^3\left(1-x^3+x^4+x^6\right)^{3/4}}{\left(1-x^3+x^6\right)^2}\,dx - 6\int \frac{\left(1-x^3+x^4+x^6\right)^{3/4}}{\left(1-x^3+x^6\right)^2}\,dx$$

$$= 3\int \left(-\frac{2\left(1+i\sqrt{3}\right)\left(1-x^3+x^4+x^6\right)^{3/4}}{3\left(1+i\sqrt{3}-2x^3\right)^2}+\frac{2i\left(1-x^3+x^4+x^6\right)}{3\sqrt{3}\left(1+i\sqrt{3}-2x\right.}\right.$$

$$= 8\int \frac{\left(1-x^3+x^4+x^6\right)^{3/4}}{\left(1+i\sqrt{3}-2x^3\right)^2}\,dx + 8\int \frac{\left(1-x^3+x^4+x^6\right)^{3/4}}{\left(-1+i\sqrt{3}+2x^3\right)^2}\,dx + $$

$$= 8\int \left(-\frac{2i\left(1-x^3+x^4+x^6\right)^{3/4}}{9\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}\left(-i+\sqrt{3}\right)\left(2^{2/3}\sqrt[3]{1+i\sqrt{3}}-2x\right)^2} - \frac{}{9(}\right.$$

$$= \frac{1}{9}\left(4(-2)^{2/3}\right)\int \frac{\left(1-x^3+x^4+x^6\right)^{3/4}}{2^{2/3}+\left(1-i\sqrt{3}\right)^{2/3}x}\,dx + \frac{4\int \frac{\left(1-x^3+x^4+x^6\right)^{3/4}}{3i-\sqrt{3}+\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{}\right.}}{3\sqrt{3}}$$

Mathematica [F] time = 1.95, size = 0, normalized size = 0.00

$$\int \frac{\left(1-x^3+x^4+x^6\right)^{3/4}\left(-4+x^3+2x^6\right)}{\left(1-x^3+x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 - x^3 + x^4 + x^6)^(3/4)*(-4 + x^3 + 2*x^6))/(1 - x^3 + x^6)^
2,x]
```

```
[Out] Integrate[((1 - x^3 + x^4 + x^6)^(3/4)*(-4 + x^3 + 2*x^6))/(1 - x^3 + x^6)^
2, x]
```

IntegrateAlgebraic [A] time = 0.19, size = 81, normalized size = 1.00

$$-\frac{\left(x^6+x^4-x^3+1\right)^{3/4}x}{x^6-x^3+1}-\frac{3}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^6+x^4-x^3+1}}\right)-\frac{3}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^6+x^4-x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 - x^3 + x^4 + x^6)^(3/4)*(-4 + x^3 + 2*x^6))/(1 - x^
3 + x^6)^2,x]
```

```
[Out] -((x*(1 - x^3 + x^4 + x^6)^(3/4))/(1 - x^3 + x^6)) - (3*ArcTan[x/(1 - x^3 +
 x^4 + x^6)^(1/4)])/2 - (3*ArcTanh[x/(1 - x^3 + x^4 + x^6)^(1/4)])/2
```

fricas [B] time = 52.04, size = 196, normalized size = 2.42

$$-\frac{3\left(x^6-x^3+1\right)\arctan\left(\frac{2\left(\left(x^6+x^4-x^3+1\right)^{\frac{1}{4}}x^3+\left(x^6+x^4-x^3+1\right)^{\frac{3}{4}}x\right)}{x^6-x^3+1}\right)-3\left(x^6-x^3+1\right)\log\left(\frac{x^6+2x^4-2\left(x^6+x^4-x^3+1\right)^{\frac{1}{4}}x^3-x^3+2\sqrt{x^6+x^4-x^3+1}\,x^2-2\left(x^6+x^4-x^3+1\right)^{\frac{3}{4}}x+1}{x^6-x^3+1}\right)+4\left(x^6+x^4-x^3+1\right)^{\frac{3}{4}}x}{4\left(x^6-x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^4-x^3+1)^(3/4)*(2*x^6+x^3-4)/(x^6-x^3+1)^2,x, algorithm="f
ricas")
```

```
[Out] -1/4*(3*(x^6 - x^3 + 1)*arctan(2*((x^6 + x^4 - x^3 + 1)^(1/4)*x^3 + (x^6 +
x^4 - x^3 + 1)^(3/4)*x)/(x^6 - x^3 + 1)) - 3*(x^6 - x^3 + 1)*log((x^6 + 2*x
^4 - 2*(x^6 + x^4 - x^3 + 1)^(1/4)*x^3 - x^3 + 2*sqrt(x^6 + x^4 - x^3 + 1)*
x^2 - 2*(x^6 + x^4 - x^3 + 1)^(3/4)*x + 1)/(x^6 - x^3 + 1)) + 4*(x^6 + x^4
- x^3 + 1)^(3/4)*x)/(x^6 - x^3 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^6+x^3-4\right)\left(x^6+x^4-x^3+1\right)^{\frac{3}{4}}}{\left(x^6-x^3+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^4-x^3+1)^(3/4)*(2*x^6+x^3-4)/(x^6-x^3+1)^2,x, algorithm="g
iac")
```

```
[Out] integrate((2*x^6 + x^3 - 4)*(x^6 + x^4 - x^3 + 1)^(3/4)/(x^6 - x^3 + 1)^2,
x)
```

maple [C] time = 5.37, size = 250, normalized size = 3.09

$$-\frac{x\left(x^6+x^4-x^3+1\right)^{\frac{3}{4}}}{x^6-x^3+1}+\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{-\mathrm{RootOf}(\_Z^2+1)x^6+2\,\mathrm{RootOf}(\_Z^2+1)\sqrt{x^6+x^4-x^3+1}\,x^2-2\,\mathrm{RootOf}(\_Z^2+1)x^4+\mathrm{RootOf}(\_Z^2+1)x^3+2\left(x^6+x^4-x^3+1\right)^{\frac{3}{4}}x-2\left(x^6+x^4-x^3+1\right)^{\frac{1}{4}}x^3-\mathrm{RootOf}(\_Z^2+1)}{x^6-x^3+1}\right)}{4}-\frac{3\ln\left(-\frac{x^6+2\left(x^6+x^4-x^3+1\right)^{\frac{3}{4}}x+2\sqrt{x^6+x^4-x^3+1}\,x^2+2\left(x^6+x^4-x^3+1\right)^{\frac{1}{4}}x^3+2x^4-x^3+1}{x^6-x^3+1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+x^4-x^3+1)^(3/4)*(2*x^6+x^3-4)/(x^6-x^3+1)^2,x)
```

```
[Out] -x*(x^6+x^4-x^3+1)^(3/4)/(x^6-x^3+1)+3/4*RootOf(_Z^2+1)*ln(-(-RootOf(_Z^2+1
)*x^6+2*RootOf(_Z^2+1)*(x^6+x^4-x^3+1)^(1/2)*x^2-2*RootOf(_Z^2+1)*x^4+RootO
f(_Z^2+1)*x^3+2*(x^6+x^4-x^3+1)^(3/4)*x-2*(x^6+x^4-x^3+1)^(1/4)*x^3-RootOf(
_Z^2+1))/(x^6-x^3+1))-3/4*ln(-(x^6+2*(x^6+x^4-x^3+1)^(3/4)*x+2*(x^6+x^4-x^3
+1)^(1/2)*x^2+2*(x^6+x^4-x^3+1)^(1/4)*x^3+2*x^4-x^3+1)/(x^6-x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{6}+x^{3}-4\right)\left(x^{6}+x^{4}-x^{3}+1\right)^{\frac{3}{4}}}{\left(x^{6}-x^{3}+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^4-x^3+1)^(3/4)*(2*x^6+x^3-4)/(x^6-x^3+1)^2,x, algorithm="m
axima")
```

```
[Out] integrate((2*x^6 + x^3 - 4)*(x^6 + x^4 - x^3 + 1)^(3/4)/(x^6 - x^3 + 1)^2,
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^6+x^3-4\right)\,{\left(x^6+x^4-x^3+1\right)}^{3/4}}{{\left(x^6-x^3+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 2*x^6 - 4)*(x^4 - x^3 + x^6 + 1)^(3/4))/(x^6 - x^3 + 1)^2,x)
```

```
[Out] int(((x^3 + 2*x^6 - 4)*(x^4 - x^3 + x^6 + 1)^(3/4))/(x^6 - x^3 + 1)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+x**4-x**3+1)**(3/4)*(2*x**6+x**3-4)/(x**6-x**3+1)**2,x)
```

```
[Out] Timed out
```

3.984 $\int \frac{1+2x^4}{\sqrt[4]{1+x^4}\left(-2-x^4+x^8\right)}\,dx$

Optimal. Leaf size=81

$$\frac{x}{3\sqrt[4]{x^4+1}}-\frac{5\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}x}{\sqrt[4]{x^4+1}}\right)}{6\,2^{3/4}\sqrt[4]{3}}-\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}x}{\sqrt[4]{x^4+1}}\right)}{6\,2^{3/4}\sqrt[4]{3}}$$

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1454, 527, 12, 377, 212, 206, 203}

$$\frac{x}{3\sqrt[4]{x^4+1}}-\frac{5\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}x}{\sqrt[4]{x^4+1}}\right)}{6\,2^{3/4}\sqrt[4]{3}}-\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}x}{\sqrt[4]{x^4+1}}\right)}{6\,2^{3/4}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^4)/((1 + x^4)^(1/4)*(-2 - x^4 + x^8)),x]
```

```
[Out] x/(3*(1 + x^4)^(1/4)) - (5*ArcTan[((3/2)^(1/4)*x)/(1 + x^4)^(1/4)])/(6*2^(3
/4)*3^(1/4)) - (5*ArcTanh[((3/2)^(1/4)*x)/(1 + x^4)^(1/4)])/(6*2^(3/4)*3^(1
/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1454

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(d + e*x^n)^(p +
 q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, g, n,
 q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*
e^2, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+2x^4}{\sqrt[4]{1+x^4}\left(-2-x^4+x^8\right)}\,dx &= \int \frac{1+2x^4}{\left(-2+x^4\right)\left(1+x^4\right)^{5/4}}\,dx \\
&= \frac{x}{3\sqrt[4]{1+x^4}}+\frac{1}{3}\int \frac{5}{\left(-2+x^4\right)\sqrt[4]{1+x^4}}\,dx \\
&= \frac{x}{3\sqrt[4]{1+x^4}}+\frac{5}{3}\int \frac{1}{\left(-2+x^4\right)\sqrt[4]{1+x^4}}\,dx \\
&= \frac{x}{3\sqrt[4]{1+x^4}}+\frac{5}{3}\operatorname{Subst}\left(\int \frac{1}{-2+3x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right) \\
&= \frac{x}{3\sqrt[4]{1+x^4}}-\frac{5\operatorname{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{3}x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)}{6\sqrt{2}}-\frac{5\operatorname{Subst}\left(\int \frac{1}{\sqrt{2}+\sqrt{3}x^2}\,dx,x,\ldots\right)}{6\sqrt{2}} \\
&= \frac{x}{3\sqrt[4]{1+x^4}}-\frac{5\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}x}{\sqrt[4]{1+x^4}}\right)}{6\,2^{3/4}\sqrt[4]{3}}-\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}x}{\sqrt[4]{1+x^4}}\right)}{6\,2^{3/4}\sqrt[4]{3}}
\end{aligned}
$$

Mathematica [A] time = 0.28, size = 102, normalized size = 1.26

$$
\frac{x}{3\sqrt[4]{x^4+1}}-\frac{5\left(-\log\left(2-\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt[4]{x^4+1}}\right)+\log\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt[4]{x^4+1}}+2\right)+2\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}x}{\sqrt[4]{x^4+1}}\right)\right)}{12\,2^{3/4}\sqrt[4]{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^4)/((1 + x^4)^(1/4)*(-2 - x^4 + x^8)),x]
```

```
[Out] x/(3*(1 + x^4)^(1/4)) - (5*(2*ArcTan[((3/2)^(1/4)*x)/(1 + x^4)^(1/4)] - Log
[2 - (2^(3/4)*3^(1/4)*x)/(1 + x^4)^(1/4)] + Log[2 + (2^(3/4)*3^(1/4)*x)/(1
+ x^4)^(1/4)]))/(12*2^(3/4)*3^(1/4))
```

IntegrateAlgebraic [A] time = 0.41, size = 81, normalized size = 1.00

$$
\frac{x}{3\sqrt[4]{x^4+1}}-\frac{5\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}x}{\sqrt[4]{x^4+1}}\right)}{6\,2^{3/4}\sqrt[4]{3}}-\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}x}{\sqrt[4]{x^4+1}}\right)}{6\,2^{3/4}\sqrt[4]{3}}
$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 2*x^4)/((1 + x^4)^(1/4)*(-2 - x^4 + x^8)),x]

[Out] x/(3*(1 + x^4)^(1/4)) - (5*ArcTan[((3/2)^(1/4)*x)/(1 + x^4)^(1/4)])/(6*2^(3/4)*3^(1/4)) - (5*ArcTanh[((3/2)^(1/4)*x)/(1 + x^4)^(1/4)])/(6*2^(3/4)*3^(1/4))

fricas [B] time = 6.36, size = 245, normalized size = 3.02

$$\frac{20\cdot 24^{\frac{3}{4}}\left(x^4+1\right)\arctan\left(\frac{3\cdot 24^{\frac{3}{4}}\left(x^4+1\right)^{\frac{1}{4}}x^3+12\cdot 24^{\frac{1}{4}}\left(x^4+1\right)^{\frac{3}{4}}x+6^{\frac{1}{4}}\sqrt{3}\left(24^{\frac{3}{4}}\sqrt{x^4+1}\,x^2+24^{\frac{1}{4}}\left(5x^4+2\right)\right)}{6\left(x^4-2\right)}\right)-5\cdot 24^{\frac{3}{4}}\left(x^4+1\right)\log\left(\frac{24\sqrt{6}\left(x^4+1\right)^{\frac{1}{4}}x^3+24\cdot 24^{\frac{1}{4}}\sqrt{x^4+1}\,x^2+24^{\frac{3}{4}}\left(5x^4+2\right)+48\left(x^4+1\right)^{\frac{3}{4}}x}{x^4-2}\right)+5\cdot 24^{\frac{3}{4}}\left(x^4+1\right)\log\left(\frac{24\sqrt{6}\left(x^4+1\right)^{\frac{1}{4}}x^3-24\cdot 24^{\frac{1}{4}}\sqrt{x^4+1}\,x^2-24^{\frac{3}{4}}\left(5x^4+2\right)+48\left(x^4+1\right)^{\frac{3}{4}}x}{x^4-2}\right)+192\left(x^4+1\right)^{\frac{3}{4}}x}{576\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^4+1)/(x^4+1)^(1/4)/(x^8-x^4-2),x, algorithm="fricas")

[Out] 1/576*(20*24^(3/4)*(x^4 + 1)*arctan(1/6*(3*24^(3/4)*(x^4 + 1)^(1/4)*x^3 + 12*24^(1/4)*(x^4 + 1)^(3/4)*x + 6^(1/4)*sqrt(3)*(24^(3/4)*sqrt(x^4 + 1)*x^2 + 24^(1/4)*(5*x^4 + 2)))/(x^4 - 2)) - 5*24^(3/4)*(x^4 + 1)*log((24*sqrt(6)*(x^4 + 1)^(1/4)*x^3 + 24*24^(1/4)*sqrt(x^4 + 1)*x^2 + 24^(3/4)*(5*x^4 + 2) + 48*(x^4 + 1)^(3/4)*x)/(x^4 - 2)) + 5*24^(3/4)*(x^4 + 1)*log((24*sqrt(6)*(x^4 + 1)^(1/4)*x^3 - 24*24^(1/4)*sqrt(x^4 + 1)*x^2 - 24^(3/4)*(5*x^4 + 2) + 48*(x^4 + 1)^(3/4)*x)/(x^4 - 2)) + 192*(x^4 + 1)^(3/4)*x)/(x^4 + 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^4+1}{\left(x^8-x^4-2\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^4+1)/(x^4+1)^(1/4)/(x^8-x^4-2),x, algorithm="giac")

[Out] integrate((2*x^4 + 1)/((x^8 - x^4 - 2)*(x^4 + 1)^(1/4)), x)

maple [C] time = 1.70, size = 220, normalized size = 2.72

$$\frac{x}{3\left(x^4+1\right)^{\frac{1}{4}}}+\frac{5\,\mathrm{RootOf}\left(\_Z^4-54\right)\ln\left(\frac{2\sqrt{x^4+1}\,\mathrm{RootOf}\left(\_Z^4-54\right)^3x^2-6\left(x^4+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-54\right)^2x^3+15\,\mathrm{RootOf}\left(\_Z^4-54\right)x^4-36\left(x^4+1\right)^{\frac{3}{4}}x+6\,\mathrm{RootOf}\left(\_Z^4-54\right)}{x^4-2}\right)}{72}-\frac{5\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-54\right)^2\right)\ln\left(\frac{2\sqrt{x^4+1}\,\mathrm{RootOf}\left(\_Z^4-54\right)^2\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-54\right)^2\right)x^2+6\left(x^4+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-54\right)^2x^3-15\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-54\right)^2\right)x^4-36\left(x^4+1\right)^{\frac{3}{4}}x-6\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-54\right)^2\right)}{x^4-2}\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^4+1)/(x^4+1)^(1/4)/(x^8-x^4-2),x)

[Out] 1/3*x/(x^4+1)^(1/4)+5/72*RootOf(_Z^4-54)*ln((2*(x^4+1)^(1/2)*RootOf(_Z^4-54)^3*x^2-6*(x^4+1)^(1/4)*RootOf(_Z^4-54)^2*x^3+15*RootOf(_Z^4-54)*x^4-36*(x^4+1)^(3/4)*x+6*RootOf(_Z^4-54))/(x^4-2))-5/72*RootOf(_Z^2+RootOf(_Z^4-54)^2)*ln((2*(x^4+1)^(1/2)*RootOf(_Z^4-54)^2*RootOf(_Z^2+RootOf(_Z^4-54)^2)*x^2+6*(x^4+1)^(1/4)*RootOf(_Z^4-54)^2*x^3-15*RootOf(_Z^2+RootOf(_Z^4-54)^2)*x^4-36*(x^4+1)^(3/4)*x-6*RootOf(_Z^2+RootOf(_Z^4-54)^2))/(x^4-2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^4+1}{\left(x^8-x^4-2\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^4+1)/(x^4+1)^(1/4)/(x^8-x^4-2),x, algorithm="maxima")

[Out] integrate((2*x^4 + 1)/((x^8 - x^4 - 2)*(x^4 + 1)^(1/4)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{2\,x^4+1}{\left(x^4+1\right)^{1/4}\,\left(-x^8+x^4+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^4 + 1)/((x^4 + 1)^(1/4)*(x^4 - x^8 + 2)),x)
[Out] int(-(2*x^4 + 1)/((x^4 + 1)^(1/4)*(x^4 - x^8 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^4+1}{\left(x^4-2\right)\left(x^4+1\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+1)/(x**4+1)**(1/4)/(x**8-x**4-2),x)
[Out] Integral((2*x**4 + 1)/((x**4 - 2)*(x**4 + 1)**(5/4)), x)
```

3.985 $\int \frac{(-b^4+a^4x^4)\sqrt{b^4+a^4x^4}}{b^8+a^8x^8}\,dx$

Optimal. Leaf size=81

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,abx}{\sqrt{a^4x^4+b^4}}\right)}{2\sqrt[4]{2}\,ab}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,abx}{\sqrt{a^4x^4+b^4}}\right)}{2\sqrt[4]{2}\,ab}$$

Rubi [C] time = 3.53, antiderivative size = 1639, normalized size of antiderivative = 20.23, number of steps used = 20, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6725, 406, 220, 409, 1217, 1707}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[((-b^4 + a^4*x^4)*Sqrt[b^4 + a^4*x^4])/(b^8 + a^8*x^8),x]
```

```
[Out] ((-Sqrt[-a^8])^(5/4)*(a^4 - Sqrt[-a^8])^(3/2)*ArcTan[(Sqrt[a^4 - Sqrt[-a^8]
]*b*x)/((-Sqrt[-a^8])^(1/4)*Sqrt[b^4 + a^4*x^4])])/(8*a^12*b) - (a^6*(-a^4
+ Sqrt[-a^8])^(3/2)*ArcTan[((-a^8)^(1/8)*Sqrt[-a^4 + Sqrt[-a^8]]*b*x)/(a^2*
Sqrt[b^4 + a^4*x^4])])/(8*(-a^8)^(13/8)*b) - ((-a^4 + Sqrt[-a^8])^(3/2)*Arc
Tan[(Sqrt[-a^4 + Sqrt[-a^8]]*b*x)/((-Sqrt[-a^8])^(1/4)*Sqrt[b^4 + a^4*x^4])
])/(8*a^4*(-Sqrt[-a^8])^(3/4)*b) - ((a^4 + Sqrt[-a^8])^(3/2)*ArcTan[(Sqrt[a
^4 + Sqrt[-a^8]]*b*x)/((-a^8)^(1/8)*Sqrt[b^4 + a^4*x^4])])/(8*a^4*(-a^8)^(3
/8)*b) + ((a^4 - Sqrt[-a^8])*(b^2 + a^2*x^2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^
2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2])/(4*a^5*b*Sqrt[b^4 + a^4*x^4])
- (Sqrt[-a^8]*(a^2 + (-a^8)^(1/4))*(a^4 - Sqrt[-a^8])*(b^2 + a^2*x^2)*Sqrt[
(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2])/(8*a^
11*b*Sqrt[b^4 + a^4*x^4]) + ((a^4 + Sqrt[-a^8])*(b^2 + a^2*x^2)*Sqrt[(b^4 +
 a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2])/(4*a^5*b*Sq
rt[b^4 + a^4*x^4]) - ((a^2 - (-a^8)^(1/4))*(a^4 + Sqrt[-a^8])*(b^2 + a^2*x^
2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2
])/(8*a^7*b*Sqrt[b^4 + a^4*x^4]) + (Sqrt[-a^8]*(a^4 + Sqrt[-a^8])*(a^2 - Sq
rt[-Sqrt[-a^8]])*(b^2 + a^2*x^2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*El
lipticF[2*ArcTan[(a*x)/b], 1/2])/(8*a^11*b*Sqrt[b^4 + a^4*x^4]) + (Sqrt[-a^
8]*(a^4 + Sqrt[-a^8])*(a^2 + Sqrt[-Sqrt[-a^8]])*(b^2 + a^2*x^2)*Sqrt[(b^4 +
 a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2])/(8*a^11*b*S
qrt[b^4 + a^4*x^4]) + (Sqrt[-a^8]*(a^2 + (-a^8)^(1/4))^2*(a^4 - Sqrt[-a^8])
*(b^2 + a^2*x^2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticPi[(a^6*(a
^2 - (-a^8)^(1/4))^2)/(4*(-a^8)^(5/4)), 2*ArcTan[(a*x)/b], 1/2])/(16*a^13*b
*Sqrt[b^4 + a^4*x^4]) - ((a^2 - (-a^8)^(1/4))^3*(a^2 + (-a^8)^(1/4))*(b^2 +
 a^2*x^2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticPi[(a^2 + (-a^8)^
(1/4))^2/(4*a^2*(-a^8)^(1/4)), 2*ArcTan[(a*x)/b], 1/2])/(16*a^5*Sqrt[-a^8]*
b*Sqrt[b^4 + a^4*x^4]) - (Sqrt[-a^8]*(a^4 + Sqrt[-a^8])*(a^2 + Sqrt[-Sqrt[-
a^8]])^2*(b^2 + a^2*x^2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticPi
[-1/4*(a^2 - Sqrt[-Sqrt[-a^8]])^2/(a^2*Sqrt[-Sqrt[-a^8]]), 2*ArcTan[(a*x)/b
], 1/2])/(16*a^13*b*Sqrt[b^4 + a^4*x^4]) - (Sqrt[-a^8]*(a^4 + Sqrt[-a^8])*(
a^2 - Sqrt[-Sqrt[-a^8]])^2*(b^2 + a^2*x^2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*
x^2)^2]*EllipticPi[(a^2 + Sqrt[-Sqrt[-a^8]])^2/(4*a^2*Sqrt[-Sqrt[-a^8]]), 2
*ArcTan[(a*x)/b], 1/2])/(16*a^13*b*Sqrt[b^4 + a^4*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(-b^4+a^4x^4\right)\sqrt{b^4+a^4x^4}}{b^8+a^8x^8}\,dx = \int \left(-\frac{\sqrt{-a^8}\left(a^4b^4-\sqrt{-a^8}\,b^4\right)\sqrt{b^4+a^4x^4}}{2a^8b^4\left(b^4-\sqrt{-a^8}\,x^4\right)}+\frac{\sqrt{-a^8}\left(a^4b^4+\sqrt{-a^8}\,b^4\right)\sqrt{b^4+a^4}}{2a^8b^4\left(b^4+\sqrt{-a^8}\,x^4\right)}\right.$$

$$= -\frac{\left(a^4+\sqrt{-a^8}\right)\int \frac{\sqrt{b^4+a^4x^4}}{b^4-\sqrt{-a^8}\,x^4}\,dx}{2a^4}+\frac{\left(\sqrt{-a^8}\left(a^4b^4+\sqrt{-a^8}\,b^4\right)\right)\int \frac{\sqrt{b^4+a^4x^4}}{b^4+\sqrt{-a^8}\,x^4}\,dx}{2a^8b^4}$$

$$= \frac{1}{2}\left(1+\frac{a^4}{\sqrt{-a^8}}\right)\int \frac{1}{\sqrt{b^4+a^4x^4}}\,dx+\frac{\left(a^4+\sqrt{-a^8}\right)\int \frac{1}{\sqrt{b^4+a^4x^4}}\,dx}{2a^4}-b^4\int \frac{}{\sqrt{b^4+}}$$

$$= \frac{\left(1+\frac{a^4}{\sqrt{-a^8}}\right)\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{4ab\sqrt{b^4+a^4x^4}}+\frac{\left(a^4+\sqrt{-a^8}\right)\left(b^2+a^2\right.}{4a}$$

$$= \frac{\left(1+\frac{a^4}{\sqrt{-a^8}}\right)\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{4ab\sqrt{b^4+a^4x^4}}+\frac{\left(a^4+\sqrt{-a^8}\right)\left(b^2+a^2\right.}{4a}$$

$$= -\frac{\sqrt[4]{-\sqrt{-a^8}}\,\tan^{-1}\left(\frac{\sqrt{a^4-\sqrt{-a^8}}\,bx}{\sqrt[4]{-\sqrt{-a^8}}\,\sqrt{b^4+a^4x^4}}\right)}{4\sqrt{a^4-\sqrt{-a^8}}\,b}-\frac{a^2\tan^{-1}\left(\frac{\sqrt[8]{-a^8}\,\sqrt{-a^4+\sqrt{-a^8}}\,bx}{a^2\sqrt{b^4+a^4x^4}}\right)}{4\sqrt[8]{-a^8}\,\sqrt{-a^4+\sqrt{-a^8}}\,b}-\frac{\sqrt[4]{-\sqrt{-}}}{}$$

Mathematica [C] time = 0.85, size = 201, normalized size = 2.48

$$-\frac{i\sqrt{\frac{a^4x^4}{b^4}+1}\left(2F\left(i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}x\right)\middle|-1\right)-\Pi\left(-\sqrt[4]{-1};i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}x\right)\middle|-1\right)-\Pi\left(\sqrt[4]{-1};i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}x\right)\middle|-1\right)-\Pi\left(-(-1)^{3/4};i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}x\right)\middle|-1\right)-\Pi\left((-1)^{3/4};i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}x\right)\middle|-1\right)\right)}{2\sqrt{\frac{ia^2}{b^2}}\sqrt{a^4x^4+b^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b^4 + a^4*x^4)*Sqrt[b^4 + a^4*x^4])/(b^8 + a^8*x^8),x]
```

```
[Out] ((-1/2*I)*Sqrt[1 + (a^4*x^4)/b^4]*(2*EllipticF[I*ArcSinh[Sqrt[(I*a^2)/b^2]*
x], -1] - EllipticPi[-(-1)^(1/4), I*ArcSinh[Sqrt[(I*a^2)/b^2]*x], -1] - Ell
ipticPi[(-1)^(1/4), I*ArcSinh[Sqrt[(I*a^2)/b^2]*x], -1] - EllipticPi[-(-1)^
(3/4), I*ArcSinh[Sqrt[(I*a^2)/b^2]*x], -1] - EllipticPi[(-1)^(3/4), I*ArcSi
nh[Sqrt[(I*a^2)/b^2]*x], -1]))/(Sqrt[(I*a^2)/b^2]*Sqrt[b^4 + a^4*x^4])
```

IntegrateAlgebraic [A] time = 0.44, size = 81, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,abx}{\sqrt{a^4x^4+b^4}}\right)}{2\sqrt[4]{2}\,ab}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,abx}{\sqrt{a^4x^4+b^4}}\right)}{2\sqrt[4]{2}\,ab}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b^4 + a^4*x^4)*Sqrt[b^4 + a^4*x^4])/(b^8 + a^8*x^8),x
]
```

```
[Out] -1/2*ArcTan[(2^(1/4)*a*b*x)/Sqrt[b^4 + a^4*x^4]]/(2^(1/4)*a*b) - ArcTanh[(2
^(1/4)*a*b*x)/Sqrt[b^4 + a^4*x^4]]/(2*2^(1/4)*a*b)
```

fricas [B] time = 5.10, size = 500, normalized size = 6.17

$$-\frac{1}{2}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{a^4b^4}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(2\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}a^4b^4x^3\left(\frac{1}{a^4b^4}\right)^{\frac{1}{4}}+\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(a^8b^4x^5+a^4b^8x\right)\left(\frac{1}{a^4b^4}\right)^{\frac{3}{4}}\right)\sqrt{a^4x^4+b^4}+\left(\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(a^{12}b^4x^8+4a^8b^8x^4+a^4b^{12}\right)\left(\frac{1}{a^4b^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(a^8b^4x^6+a^4b^8x^2\right)\left(\frac{1}{a^4b^4}\right)^{\frac{1}{4}}\right)\right)\sqrt{\sqrt{\frac{1}{2}}\sqrt{\frac{1}{a^4b^4}}}}{a^8x^8+b^8}\right)-\frac{1}{8}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{a^4b^4}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(a^8b^4x^6+a^4b^8x^2\right)\left(\frac{1}{a^4b^4}\right)^{\frac{3}{4}}+2\left(2\sqrt{\frac{1}{2}}a^4b^4x^3\sqrt{\frac{1}{a^4b^4}}+a^4x^5+b^4x\right)\sqrt{a^4x^4+b^4}+\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(a^8x^8+4a^4b^4x^4+b^8\right)\left(\frac{1}{a^4b^4}\right)^{\frac{1}{4}}}{2\left(a^8x^8+b^8\right)}\right)+\frac{1}{8}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{a^4b^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(a^8b^4x^6+a^4b^8x^2\right)\left(\frac{1}{a^4b^4}\right)^{\frac{3}{4}}-2\left(2\sqrt{\frac{1}{2}}a^4b^4x^3\sqrt{\frac{1}{a^4b^4}}+a^4x^5+b^4x\right)\sqrt{a^4x^4+b^4}+\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(a^8x^8+4a^4b^4x^4+b^8\right)\left(\frac{1}{a^4b^4}\right)^{\frac{1}{4}}}{2\left(a^8x^8+b^8\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^4)*(a^4*x^4+b^4)^(1/2)/(a^8*x^8+b^8),x, algorithm="fri
cas")
```

```
[Out] -1/2*(1/2)^(1/4)*(1/(a^4*b^4))^(1/4)*arctan(2*(2*((1/2)^(1/4)*a^4*b^4*x^3*(
1/(a^4*b^4))^(1/4) + (1/2)^(3/4)*(a^8*b^4*x^5 + a^4*b^8*x)*(1/(a^4*b^4))^(3
/4))*sqrt(a^4*x^4 + b^4) + ((1/2)^(3/4)*(a^12*b^4*x^8 + 4*a^8*b^8*x^4 + a^4
*b^12)*(1/(a^4*b^4))^(3/4) + 2*(1/2)^(1/4)*(a^8*b^4*x^6 + a^4*b^8*x^2)*(1/(
a^4*b^4))^(1/4))*sqrt(sqrt(1/2)*sqrt(1/(a^4*b^4))))/(a^8*x^8 + b^8)) - 1/8*
(1/2)^(1/4)*(1/(a^4*b^4))^(1/4)*log(-1/2*(4*(1/2)^(3/4)*(a^8*b^4*x^6 + a^4*
b^8*x^2)*(1/(a^4*b^4))^(3/4) + 2*(2*sqrt(1/2)*a^4*b^4*x^3*sqrt(1/(a^4*b^4))
 + a^4*x^5 + b^4*x)*sqrt(a^4*x^4 + b^4) + (1/2)^(1/4)*(a^8*x^8 + 4*a^4*b^4*
x^4 + b^8)*(1/(a^4*b^4))^(1/4))/(a^8*x^8 + b^8)) + 1/8*(1/2)^(1/4)*(1/(a^4*
b^4))^(1/4)*log(1/2*(4*(1/2)^(3/4)*(a^8*b^4*x^6 + a^4*b^8*x^2)*(1/(a^4*b^4)
)^(3/4) - 2*(2*sqrt(1/2)*a^4*b^4*x^3*sqrt(1/(a^4*b^4)) + a^4*x^5 + b^4*x)*s
qrt(a^4*x^4 + b^4) + (1/2)^(1/4)*(a^8*x^8 + 4*a^4*b^4*x^4 + b^8)*(1/(a^4*b^
4))^(1/4))/(a^8*x^8 + b^8))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^4x^4+b^4}\left(a^4x^4-b^4\right)}{a^8x^8+b^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^4)*(a^4*x^4+b^4)^(1/2)/(a^8*x^8+b^8),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(a^4*x^4 + b^4)*(a^4*x^4 - b^4)/(a^8*x^8 + b^8), x)
```

maple [B] time = 0.05, size = 157, normalized size = 1.94

$$\frac{\arctan\left(\frac{\sqrt{a^4x^4+b^4}}{x\sqrt{\sqrt{2}\,\sqrt{a^4b^4}}}\right)}{2\sqrt{\sqrt{2}\,\sqrt{a^4b^4}}}-\frac{\ln\left(\frac{\frac{\sqrt{a^4x^4+b^4}\,\sqrt{2}}{2x}+\frac{\sqrt{2}\,\sqrt{\sqrt{2}\,\sqrt{a^4b^4}}}{2}}{\frac{\sqrt{a^4x^4+b^4}\,\sqrt{2}}{2x}-\frac{\sqrt{2}\,\sqrt{\sqrt{2}\,\sqrt{a^4b^4}}}{2}}\right)}{4\sqrt{\sqrt{2}\,\sqrt{a^4b^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^4*x^4-b^4)*(a^4*x^4+b^4)^(1/2)/(a^8*x^8+b^8),x)
```

```
[Out] 1/2/(2^(1/2)*(a^4*b^4)^(1/2))^(1/2)*arctan((a^4*x^4+b^4)^(1/2)/x/(2^(1/2)*(
a^4*b^4)^(1/2))^(1/2))-1/4/(2^(1/2)*(a^4*b^4)^(1/2))^(1/2)*ln((1/2*(a^4*x^4
+b^4)^(1/2)*2^(1/2)/x+1/2*2^(1/2)*(2^(1/2)*(a^4*b^4)^(1/2))^(1/2))/(1/2*(a^
4*x^4+b^4)^(1/2)*2^(1/2)/x-1/2*2^(1/2)*(2^(1/2)*(a^4*b^4)^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^4x^4+b^4}\left(a^4x^4-b^4\right)}{a^8x^8+b^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^4)*(a^4*x^4+b^4)^(1/2)/(a^8*x^8+b^8),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(a^4*x^4 + b^4)*(a^4*x^4 - b^4)/(a^8*x^8 + b^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{a^4 x^4 + b^4}\, \left(b^4 - a^4 x^4\right)}{a^8 x^8 + b^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b^4 + a^4*x^4)^(1/2)*(b^4 - a^4*x^4))/(b^8 + a^8*x^8),x)
```

```
[Out] int(-((b^4 + a^4*x^4)^(1/2)*(b^4 - a^4*x^4))/(b^8 + a^8*x^8), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax - b)\,(ax + b)\left(a^2x^2 + b^2\right)\sqrt{a^4x^4 + b^4}}{a^8x^8 + b^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**4*x**4-b**4)*(a**4*x**4+b**4)**(1/2)/(a**8*x**8+b**8),x)
```

```
[Out] Integral((a*x - b)*(a*x + b)*(a**2*x**2 + b**2)*sqrt(a**4*x**4 + b**4)/(a**
8*x**8 + b**8), x)
```

3.986 $\int \frac{-b^8+a^8x^8}{\sqrt{b^4+a^4x^4}\left(b^8+a^8x^8\right)}\,dx$

Optimal. Leaf size=81

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}abx}{\sqrt{a^4x^4+b^4}}\right)}{2\sqrt[4]{2}ab}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}abx}{\sqrt{a^4x^4+b^4}}\right)}{2\sqrt[4]{2}ab}$$

Rubi [C] time = 2.74, antiderivative size = 1639, normalized size of antiderivative = 20.23, number of steps used = 21, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1586, 6725, 406, 220, 409, 1217, 1707}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(-b^8 + a^8*x^8)/(Sqrt[b^4 + a^4*x^4]*(b^8 + a^8*x^8)),x]
```

```
[Out] ((-Sqrt[-a^8])^(5/4)*(a^4 - Sqrt[-a^8])^(3/2)*ArcTan[(Sqrt[a^4 - Sqrt[-a^8]
]*b*x)/((-Sqrt[-a^8])^(1/4)*Sqrt[b^4 + a^4*x^4])])/(8*a^12*b) - (a^6*(-a^4
+ Sqrt[-a^8])^(3/2)*ArcTan[((-a^8)^(1/8)*Sqrt[-a^4 + Sqrt[-a^8]]*b*x)/(a^2*
Sqrt[b^4 + a^4*x^4])])/(8*(-a^8)^(13/8)*b) - ((-a^4 + Sqrt[-a^8])^(3/2)*Arc
Tan[(Sqrt[-a^4 + Sqrt[-a^8]]*b*x)/((-Sqrt[-a^8])^(1/4)*Sqrt[b^4 + a^4*x^4])
])/(8*a^4*(-Sqrt[-a^8])^(3/4)*b) - ((a^4 + Sqrt[-a^8])^(3/2)*ArcTan[(Sqrt[a
^4 + Sqrt[-a^8]]*b*x)/((-a^8)^(1/8)*Sqrt[b^4 + a^4*x^4])])/(8*a^4*(-a^8)^(3
/8)*b) + ((a^4 - Sqrt[-a^8])*(b^2 + a^2*x^2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^
2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2])/(4*a^5*b*Sqrt[b^4 + a^4*x^4])
- (Sqrt[-a^8]*(a^2 + (-a^8)^(1/4))*(a^4 - Sqrt[-a^8])*(b^2 + a^2*x^2)*Sqrt[
(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2])/(8*a^
11*b*Sqrt[b^4 + a^4*x^4]) + ((a^4 + Sqrt[-a^8])*(b^2 + a^2*x^2)*Sqrt[(b^4 +
 a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2])/(4*a^5*b*Sq
rt[b^4 + a^4*x^4]) - ((a^2 - (-a^8)^(1/4))*(a^4 + Sqrt[-a^8])*(b^2 + a^2*x^
2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2
])/(8*a^7*b*Sqrt[b^4 + a^4*x^4]) + (Sqrt[-a^8]*(a^4 + Sqrt[-a^8])*(a^2 - Sq
rt[-Sqrt[-a^8]])*(b^2 + a^2*x^2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*El
lipticF[2*ArcTan[(a*x)/b], 1/2])/(8*a^11*b*Sqrt[b^4 + a^4*x^4]) + (Sqrt[-a^
8]*(a^4 + Sqrt[-a^8])*(a^2 + Sqrt[-Sqrt[-a^8]])*(b^2 + a^2*x^2)*Sqrt[(b^4 +
 a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2])/(8*a^11*b*S
qrt[b^4 + a^4*x^4]) + (Sqrt[-a^8]*(a^2 + (-a^8)^(1/4))^2*(a^4 - Sqrt[-a^8])
*(b^2 + a^2*x^2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticPi[(a^6*(a
^2 - (-a^8)^(1/4))^2)/(4*(-a^8)^(5/4)), 2*ArcTan[(a*x)/b], 1/2])/(16*a^13*b
*Sqrt[b^4 + a^4*x^4]) - ((a^2 - (-a^8)^(1/4))^3*(a^2 + (-a^8)^(1/4))*(b^2 +
 a^2*x^2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticPi[(a^2 + (-a^8)^
(1/4))^2/(4*a^2*(-a^8)^(1/4)), 2*ArcTan[(a*x)/b], 1/2])/(16*a^5*Sqrt[-a^8]*
b*Sqrt[b^4 + a^4*x^4]) - (Sqrt[-a^8]*(a^4 + Sqrt[-a^8])*(a^2 + Sqrt[-Sqrt[-
a^8]])^2*(b^2 + a^2*x^2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticPi
[-1/4*(a^2 - Sqrt[-Sqrt[-a^8]])^2/(a^2*Sqrt[-Sqrt[-a^8]]), 2*ArcTan[(a*x)/b
], 1/2])/(16*a^13*b*Sqrt[b^4 + a^4*x^4]) - (Sqrt[-a^8]*(a^4 + Sqrt[-a^8])*(
a^2 - Sqrt[-Sqrt[-a^8]])^2*(b^2 + a^2*x^2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*
x^2)^2]*EllipticPi[(a^2 + Sqrt[-Sqrt[-a^8]])^2/(4*a^2*Sqrt[-Sqrt[-a^8]]), 2
*ArcTan[(a*x)/b], 1/2])/(16*a^13*b*Sqrt[b^4 + a^4*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-b^8+a^8x^8}{\sqrt{b^4+a^4x^4}\left(b^8+a^8x^8\right)}\,dx=\int \frac{\left(-b^4+a^4x^4\right)\sqrt{b^4+a^4x^4}}{b^8+a^8x^8}\,dx$$

$$=\int \left(-\frac{\sqrt{-a^8}\left(a^4b^4-\sqrt{-a^8}\,b^4\right)\sqrt{b^4+a^4x^4}}{2a^8b^4\left(b^4-\sqrt{-a^8}\,x^4\right)}+\frac{\sqrt{-a^8}\left(a^4b^4+\sqrt{-a^8}\,b^4\right)\sqrt{b^4+a}}{2a^8b^4\left(b^4+\sqrt{-a^8}\,x^4\right)}\right.$$

$$=-\frac{\left(a^4+\sqrt{-a^8}\right)\int \frac{\sqrt{b^4+a^4x^4}}{b^4-\sqrt{-a^8}\,x^4}\,dx}{2a^4}+\frac{\left(\sqrt{-a^8}\left(a^4b^4+\sqrt{-a^8}\,b^4\right)\right)\int \frac{\sqrt{b^4+a^4x^4}}{b^4+\sqrt{-a^8}\,x^4}\,dx}{2a^8b^4}$$

$$=\frac{1}{2}\left(1+\frac{a^4}{\sqrt{-a^8}}\right)\int \frac{1}{\sqrt{b^4+a^4x^4}}\,dx+\frac{\left(a^4+\sqrt{-a^8}\right)\int \frac{1}{\sqrt{b^4+a^4x^4}}\,dx}{2a^4}-b^4\int \frac{}{\sqrt{b^4}}$$

$$=\frac{\left(1+\frac{a^4}{\sqrt{-a^8}}\right)\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{4ab\sqrt{b^4+a^4x^4}}+\frac{\left(a^4+\sqrt{-a^8}\right)\left(b^2+\right.}{}$$

$$=\frac{\left(1+\frac{a^4}{\sqrt{-a^8}}\right)\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{4ab\sqrt{b^4+a^4x^4}}+\frac{\left(a^4+\sqrt{-a^8}\right)\left(b^2+\right.}{}$$

$$=-\frac{\sqrt[4]{-\sqrt{-a^8}}\,\tan^{-1}\left(\frac{\sqrt{a^4-\sqrt{-a^8}}\,bx}{\sqrt[4]{-\sqrt{-a^8}}\,\sqrt{b^4+a^4x^4}}\right)}{4\sqrt{a^4-\sqrt{-a^8}}\,b}-\frac{a^2\tan^{-1}\left(\frac{\sqrt[8]{-a^8}\,\sqrt{-a^4+\sqrt{-a^8}}\,bx}{a^2\sqrt{b^4+a^4x^4}}\right)}{4\sqrt[8]{-a^8}\,\sqrt{-a^4+\sqrt{-a^8}}\,b}-\frac{\sqrt[4]{-\sqrt{}}}{}$$

Mathematica [C] time = 0.56, size = 201, normalized size = 2.48

$$-\frac{i\sqrt{\frac{a^4x^4}{b^4}+1}\left(2F\left(i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}x\right)|-1\right)-\Pi\left(-\sqrt[4]{-1};i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}x\right)|-1\right)-\Pi\left(\sqrt[4]{-1};i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}x\right)|-1\right)-\Pi\left(-(-1)^{3/4};i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}x\right)|-1\right)-\Pi\left((-1)^{3/4};i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}x\right)|-1\right)\right)}{2\sqrt{\frac{ia^2}{b^2}}\sqrt{a^4x^4+b^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b^8 + a^8*x^8)/(Sqrt[b^4 + a^4*x^4]*(b^8 + a^8*x^8)),x]
```

```
[Out] ((-1/2*I)*Sqrt[1 + (a^4*x^4)/b^4]*(2*EllipticF[I*ArcSinh[Sqrt[(I*a^2)/b^2]*
x], -1] - EllipticPi[-(-1)^(1/4), I*ArcSinh[Sqrt[(I*a^2)/b^2]*x], -1] - Ell
ipticPi[(-1)^(1/4), I*ArcSinh[Sqrt[(I*a^2)/b^2]*x], -1] - EllipticPi[-(-1)^
(3/4), I*ArcSinh[Sqrt[(I*a^2)/b^2]*x], -1] - EllipticPi[(-1)^(3/4), I*ArcSi
nh[Sqrt[(I*a^2)/b^2]*x], -1]))/(Sqrt[(I*a^2)/b^2]*Sqrt[b^4 + a^4*x^4])
```

IntegrateAlgebraic [A] time = 0.55, size = 81, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,abx}{\sqrt{a^4x^4+b^4}}\right)}{2\sqrt[4]{2}\,ab}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,abx}{\sqrt{a^4x^4+b^4}}\right)}{2\sqrt[4]{2}\,ab}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b^8 + a^8*x^8)/(Sqrt[b^4 + a^4*x^4]*(b^8 + a^8*x^8)),x
]
```

```
[Out] -1/2*ArcTan[(2^(1/4)*a*b*x)/Sqrt[b^4 + a^4*x^4]]/(2^(1/4)*a*b) - ArcTanh[(2
^(1/4)*a*b*x)/Sqrt[b^4 + a^4*x^4]]/(2*2^(1/4)*a*b)
```

fricas [B] time = 5.37, size = 500, normalized size = 6.17

$$-\frac{1}{2}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{a^4b^4}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(2\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}a^4b^4x^3\left(\frac{1}{a^4b^4}\right)^{\frac{1}{4}}+\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(a^8b^4x^5+a^4b^8x\right)\left(\frac{1}{a^4b^4}\right)^{\frac{3}{4}}\right)\sqrt{a^4x^4+b^4}+\left(\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(a^{12}b^4x^8+4a^8b^8x^4+a^4b^{12}\right)\left(\frac{1}{a^4b^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(a^8b^4x^6+a^4b^8x^2\right)\left(\frac{1}{a^4b^4}\right)^{\frac{1}{4}}\right)\sqrt{\sqrt{\frac{1}{2}}\sqrt{\frac{1}{a^4b^4}}}\right)}{a^8x^8+b^8}\right)-\frac{1}{8}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{a^4b^4}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(a^8b^4x^6+a^4b^8x^2\right)\left(\frac{1}{a^4b^4}\right)^{\frac{3}{4}}+2\left(2\sqrt{\frac{1}{2}}a^4b^4x^3\sqrt{\frac{1}{a^4b^4}}+a^4x^5+b^4x\right)\sqrt{a^4x^4+b^4}+\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(a^8x^8+4a^4b^4x^4+b^8\right)\left(\frac{1}{a^4b^4}\right)^{\frac{1}{4}}}{2\left(a^8x^8+b^8\right)}\right)+\frac{1}{8}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{a^4b^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(a^8b^4x^6+a^4b^8x^2\right)\left(\frac{1}{a^4b^4}\right)^{\frac{3}{4}}-2\left(2\sqrt{\frac{1}{2}}a^4b^4x^3\sqrt{\frac{1}{a^4b^4}}+a^4x^5+b^4x\right)\sqrt{a^4x^4+b^4}+\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(a^8x^8+4a^4b^4x^4+b^8\right)\left(\frac{1}{a^4b^4}\right)^{\frac{1}{4}}}{2\left(a^8x^8+b^8\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^8*x^8-b^8)/(a^4*x^4+b^4)^(1/2)/(a^8*x^8+b^8),x, algorithm="fri
cas")
```

```
[Out] -1/2*(1/2)^(1/4)*(1/(a^4*b^4))^(1/4)*arctan(2*(2*((1/2)^(1/4)*a^4*b^4*x^3*(
1/(a^4*b^4))^(1/4) + (1/2)^(3/4)*(a^8*b^4*x^5 + a^4*b^8*x)*(1/(a^4*b^4))^(3
/4))*sqrt(a^4*x^4 + b^4) + ((1/2)^(3/4)*(a^12*b^4*x^8 + 4*a^8*b^8*x^4 + a^4
*b^12)*(1/(a^4*b^4))^(3/4) + 2*(1/2)^(1/4)*(a^8*b^4*x^6 + a^4*b^8*x^2)*(1/(
a^4*b^4))^(1/4))*sqrt(sqrt(1/2)*sqrt(1/(a^4*b^4))))/(a^8*x^8 + b^8)) - 1/8*
(1/2)^(1/4)*(1/(a^4*b^4))^(1/4)*log(-1/2*(4*(1/2)^(3/4)*(a^8*b^4*x^6 + a^4*
b^8*x^2)*(1/(a^4*b^4))^(3/4) + 2*(2*sqrt(1/2)*a^4*b^4*x^3*sqrt(1/(a^4*b^4))
 + a^4*x^5 + b^4*x)*sqrt(a^4*x^4 + b^4) + (1/2)^(1/4)*(a^8*x^8 + 4*a^4*b^4*
x^4 + b^8)*(1/(a^4*b^4))^(1/4))/(a^8*x^8 + b^8)) + 1/8*(1/2)^(1/4)*(1/(a^4*
b^4))^(1/4)*log(1/2*(4*(1/2)^(3/4)*(a^8*b^4*x^6 + a^4*b^8*x^2)*(1/(a^4*b^4)
)^(3/4) - 2*(2*sqrt(1/2)*a^4*b^4*x^3*sqrt(1/(a^4*b^4)) + a^4*x^5 + b^4*x)*s
qrt(a^4*x^4 + b^4) + (1/2)^(1/4)*(a^8*x^8 + 4*a^4*b^4*x^4 + b^8)*(1/(a^4*b^
4))^(1/4))/(a^8*x^8 + b^8))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^8x^8-b^8}{\left(a^8x^8+b^8\right)\sqrt{a^4x^4+b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^8*x^8-b^8)/(a^4*x^4+b^4)^(1/2)/(a^8*x^8+b^8),x, algorithm="gia
c")
```

```
[Out] integrate((a^8*x^8 - b^8)/((a^8*x^8 + b^8)*sqrt(a^4*x^4 + b^4)), x)
```

maple [C] time = 0.11, size = 287, normalized size = 3.54

$$\frac{\sqrt{1-\frac{ia^2x^2}{b^2}}\,\sqrt{1+\frac{ia^2x^2}{b^2}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{ia^2}{b^2}},i\right)}{\sqrt{\frac{ia^2}{b^2}}\,\sqrt{a^4x^4+b^4}}-\frac{b^8\left(\sum_{\_\alpha=\mathrm{RootOf}\left(a^8\_Z^8+b^8\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{\_\alpha^2\left(-a^4\_\alpha^6+b^4x^2\right)a^4}{b^4\sqrt{a^4\_\alpha^4+b^4}\,\sqrt{a^4x^4+b^4}}\right)}{\sqrt{a^4\_\alpha^4+b^4}}+\frac{2\_\alpha^7a^8\sqrt{1-\frac{ia^2x^2}{b^2}}\,\sqrt{1+\frac{ia^2x^2}{b^2}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{ia^2}{b^2}},\frac{i\_\alpha^6a^6}{b^6},\frac{\sqrt{-\frac{ia^2}{b^2}}}{\sqrt{\frac{ia^2}{b^2}}}\right)}{\sqrt{\frac{ia^2}{b^2}}\,b^8\sqrt{a^4x^4+b^4}}}{\_\alpha^7}\right)}{8a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^8*x^8-b^8)/(a^4*x^4+b^4)^(1/2)/(a^8*x^8+b^8),x)
```

```
[Out] 1/(I*a^2/b^2)^(1/2)*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/b^2*x^2)^(1/2)/(a^4*x^
4+b^4)^(1/2)*EllipticF(x*(I*a^2/b^2)^(1/2),I)-1/8*b^8/a^8*sum(1/_alpha^7*(-
1/(_alpha^4*a^4+b^4)^(1/2)*arctanh(_alpha^2/b^4*(-_alpha^6*a^4+b^4*x^2)*a^4
/(_alpha^4*a^4+b^4)^(1/2)/(a^4*x^4+b^4)^(1/2))+2/(I*a^2/b^2)^(1/2)*_alpha^7
*a^8/b^8*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/b^2*x^2)^(1/2)/(a^4*x^4+b^4)^(1/2
)*EllipticPi(x*(I*a^2/b^2)^(1/2),I*_alpha^6*a^6/b^6,(-I*a^2/b^2)^(1/2)/(I*a
^2/b^2)^(1/2))),_alpha=RootOf(_Z^8*a^8+b^8))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^8x^8 - b^8}{\left(a^8x^8 + b^8\right)\sqrt{a^4x^4 + b^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^8*x^8-b^8)/(a^4*x^4+b^4)^(1/2)/(a^8*x^8+b^8),x, algorithm="max
ima")
```

```
[Out] integrate((a^8*x^8 - b^8)/((a^8*x^8 + b^8)*sqrt(a^4*x^4 + b^4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{b^8 - a^8\,x^8}{\sqrt{a^4\,x^4 + b^4}\,\left(a^8\,x^8 + b^8\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^8 - a^8*x^8)/((b^4 + a^4*x^4)^(1/2)*(b^8 + a^8*x^8)),x)
```

```
[Out] int(-(b^8 - a^8*x^8)/((b^4 + a^4*x^4)^(1/2)*(b^8 + a^8*x^8)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax - b)(ax + b)\left(a^2x^2 + b^2\right)\sqrt{a^4x^4 + b^4}}{a^8x^8 + b^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**8*x**8-b**8)/(a**4*x**4+b**4)**(1/2)/(a**8*x**8+b**8),x)
```

```
[Out] Integral((a*x - b)*(a*x + b)*(a**2*x**2 + b**2)*sqrt(a**4*x**4 + b**4)/(a**
8*x**8 + b**8), x)
```

3.987 $\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^{3/2}}\,dx$

Optimal. Leaf size=81

$$\frac{x}{b\sqrt{ax^2+b^2}\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}\,b^{3/2}}$$

Rubi [F] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^(3/2),x]
[Out] Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^{3/2}}\,dx=\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^{3/2}}\,dx$$

Mathematica [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^(3/2),x]
[Out] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^(3/2), x]
```

IntegrateAlgebraic [A] time = 0.22, size = 81, normalized size = 1.00

$$\frac{x}{b\sqrt{ax^2+b^2}\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^(3/2),x]
[Out] x/(b*Sqrt[b^2 + a*x^2]*Sqrt[b + Sqrt[b^2 + a*x^2]]) + ArcTan[(Sqrt[a]*x)/(S
qrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])]/(Sqrt[a]*b^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\left(ax^2+b^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 + b^2)^(3/2), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{\left(a\,x^2+b^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^(3/2),x)
```

```
[Out] int((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\left(ax^2+b^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 + b^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b+\sqrt{b^2+a\,x^2}}}{{\left(b^2+a\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(a*x^2 + b^2)^(3/2),x)
```

```
[Out] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(a*x^2 + b^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\left(ax^2+b^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x**2+b**2)**(1/2))**(1/2)/(a*x**2+b**2)**(3/2),x)
[Out] Integral(sqrt(b + sqrt(a*x**2 + b**2))/(a*x**2 + b**2)**(3/2), x)
```

3.988 $\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx$

Optimal. Leaf size=81

$$\frac{\log\left(ia^{3/2}x^2+i\sqrt{a}\sqrt{a^2x^4+b}+i\sqrt{2}\,ax\sqrt{\sqrt{a^2x^4+b}+ax^2}\right)}{\sqrt{2}\sqrt{a}}$$

Rubi [A] time = 0.12, antiderivative size = 47, normalized size of antiderivative = 0.58, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2132, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}x}{\sqrt{\sqrt{a^2x^4+b}+ax^2}}\right)}{\sqrt{2}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]/Sqrt[b + a^2*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[a]*x)/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]]/(Sqrt[2]*Sqrt[
a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1-2ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}x}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\right)}{\sqrt{2}\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.58

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}x}{\sqrt{\sqrt{a^2x^4+b}+ax^2}}\right)}{\sqrt{2}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]/Sqrt[b + a^2*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[a]*x)/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]]/(Sqrt[2]*Sqrt[
a])
```

IntegrateAlgebraic [A] time = 0.39, size = 81, normalized size = 1.00

$$\frac{\log\left(ia^{3/2}x^2 + i\sqrt{a}\sqrt{a^2x^4+b} + i\sqrt{2}\,ax\sqrt{\sqrt{a^2x^4+b}+ax^2}\right)}{\sqrt{2}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]/Sqrt[b + a^2*x^4],x]
```

```
[Out] Log[I*a^(3/2)*x^2 + I*Sqrt[a]*Sqrt[b + a^2*x^4] + I*Sqrt[2]*a*x*Sqrt[a*x^2
+ Sqrt[b + a^2*x^4]]]/(Sqrt[2]*Sqrt[a])
```

fricas [A] time = 1.89, size = 135, normalized size = 1.67

$$\left[\frac{\sqrt{2}\log\left(4a^2x^4 + 4\sqrt{a^2x^4+b}\,ax^2 + 2\left(\sqrt{2}\,a^{\frac{3}{2}}x^3 + \sqrt{2}\sqrt{a^2x^4+b}\sqrt{a}\,x\right)\sqrt{ax^2+\sqrt{a^2x^4+b}} + b\right)}{4\sqrt{a}}, -\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{ax^2+\sqrt{a^2x^4+b}}\sqrt{-\frac{1}{a}}}{2x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/(a^2*x^4+b)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/4*sqrt(2)*log(4*a^2*x^4 + 4*sqrt(a^2*x^4 + b)*a*x^2 + 2*(sqrt(2)*a^(3/2)
*x^3 + sqrt(2)*sqrt(a^2*x^4 + b)*sqrt(a)*x)*sqrt(a*x^2 + sqrt(a^2*x^4 + b))
 + b)/sqrt(a), -1/2*sqrt(2)*sqrt(-1/a)*arctan(1/2*sqrt(2)*sqrt(a*x^2 + sqrt
(a^2*x^4 + b))*sqrt(-1/a)/x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2+\sqrt{a^2x^4+b}}}{\sqrt{a^2x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/(a^2*x^4+b)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^4 + b))/sqrt(a^2*x^4 + b), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}{\sqrt{a^2x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/(a^2*x^4+b)^(1/2),x)
```

```
[Out] int((a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/(a^2*x^4+b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2+\sqrt{a^2x^4+b}}}{\sqrt{a^2x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/(a^2*x^4+b)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^4 + b))/sqrt(a^2*x^4 + b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{a^2\,x^4+b}+a\,x^2}}{\sqrt{a^2\,x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a^2*x^4)^(1/2) + a*x^2)^(1/2)/(b + a^2*x^4)^(1/2),x)
```

```
[Out] int(((b + a^2*x^4)^(1/2) + a*x^2)^(1/2)/(b + a^2*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2+\sqrt{a^2x^4+b}}}{\sqrt{a^2x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+(a**2*x**4+b)**(1/2))**(1/2)/(a**2*x**4+b)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**2 + sqrt(a**2*x**4 + b))/sqrt(a**2*x**4 + b), x)
```

3.989 $\int \frac{\sqrt[4]{bx^3+ax^4}}{x}\,dx$

Optimal. Leaf size=82

$$-\frac{b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{2a^{3/4}}+\frac{b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{2a^{3/4}}+\sqrt[4]{ax^4+bx^3}$$

Rubi [A] time = 0.15, antiderivative size = 136, normalized size of antiderivative = 1.66, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2021, 2032, 63, 331, 298, 203, 206}

$$-\frac{bx^{9/4}(ax+b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{2a^{3/4}\left(ax^4+bx^3\right)^{3/4}}+\frac{bx^{9/4}(ax+b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{2a^{3/4}\left(ax^4+bx^3\right)^{3/4}}+\sqrt[4]{ax^4+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^3 + a*x^4)^(1/4)/x,x]
```

```
[Out] (b*x^3 + a*x^4)^(1/4) - (b*x^(9/4)*(b + a*x)^(3/4)*ArcTan[(a^(1/4)*x^(1/4))
/(b + a*x)^(1/4)])/(2*a^(3/4)*(b*x^3 + a*x^4)^(3/4)) + (b*x^(9/4)*(b + a*x)
^(3/4)*ArcTanh[(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(2*a^(3/4)*(b*x^3 + a*x^
4)^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{bx^3+ax^4}}{x}\,dx &= \sqrt[4]{bx^3+ax^4}+\frac{1}{4}b\int \frac{x^2}{\left(bx^3+ax^4\right)^{3/4}}\,dx \\
&= \sqrt[4]{bx^3+ax^4}+\frac{\left(bx^{9/4}(b+ax)^{3/4}\right)\int \frac{1}{\sqrt[4]{x}\,(b+ax)^{3/4}}\,dx}{4\left(bx^3+ax^4\right)^{3/4}} \\
&= \sqrt[4]{bx^3+ax^4}+\frac{\left(bx^{9/4}(b+ax)^{3/4}\right)\mathrm{Subst}\left(\int \frac{x^2}{\left(b+ax^4\right)^{3/4}}\,dx,x,\sqrt[4]{x}\right)}{\left(bx^3+ax^4\right)^{3/4}} \\
&= \sqrt[4]{bx^3+ax^4}+\frac{\left(bx^{9/4}(b+ax)^{3/4}\right)\mathrm{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx,x,\frac{\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)}{\left(bx^3+ax^4\right)^{3/4}} \\
&= \sqrt[4]{bx^3+ax^4}+\frac{\left(bx^{9/4}(b+ax)^{3/4}\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)}{2\sqrt{a}\left(bx^3+ax^4\right)^{3/4}}-\frac{\left(bx^{9/4}(b+ax)^{3/4}\right)\mathrm{Subs}}{2\sqrt{a}\,(bx}\\
&= \sqrt[4]{bx^3+ax^4}-\frac{bx^{9/4}(b+ax)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)}{2a^{3/4}\left(bx^3+ax^4\right)^{3/4}}+\frac{bx^{9/4}(b+ax)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)}{2a^{3/4}\left(bx^3+ax^4\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 46, normalized size = 0.56

$$
\frac{4\sqrt[4]{x^3(ax+b)}\;{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{ax}{b}\right)}{3\sqrt[4]{\frac{ax}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^3 + a*x^4)^(1/4)/x,x]
```

```
[Out] (4*(x^3*(b + a*x))^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, -((a*x)/b)])/(3*
(1 + (a*x)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.36, size = 82, normalized size = 1.00

$$
-\frac{b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{2a^{3/4}}+\frac{b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{2a^{3/4}}+\sqrt[4]{ax^4+bx^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^3 + a*x^4)^(1/4)/x,x]
```

```
[Out] (b*x^3 + a*x^4)^(1/4) - (b*ArcTan[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(2*a^
(3/4)) + (b*ArcTanh[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(2*a^(3/4))
```

fricas [B] time = 0.43, size = 206, normalized size = 2.51

$$-\left(\frac{b^4}{a^3}\right)^{\frac14}\arctan\left(\frac{a^2\left(\frac{b^4}{a^3}\right)^{\frac34}x\sqrt{\frac{a^2\sqrt{\frac{b^4}{a^3}}x^2+\sqrt{ax^4+bx^3}b^2}{x^2}}-\left(ax^4+bx^3\right)^{\frac14}a^2b\left(\frac{b^4}{a^3}\right)^{\frac34}}{b^4x}\right)+\frac14\left(\frac{b^4}{a^3}\right)^{\frac14}\log\left(\frac{a\left(\frac{b^4}{a^3}\right)^{\frac14}x+\left(ax^4+bx^3\right)^{\frac14}b}{x}\right)-\frac14\left(\frac{b^4}{a^3}\right)^{\frac14}\log\left(-\frac{a\left(\frac{b^4}{a^3}\right)^{\frac14}x-\left(ax^4+bx^3\right)^{\frac14}b}{x}\right)+\left(ax^4+bx^3\right)^{\frac14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^3)^(1/4)/x,x, algorithm="fricas")
```

```
[Out] -(b^4/a^3)^(1/4)*arctan((a^2*(b^4/a^3)^(3/4)*x*sqrt((a^2*sqrt(b^4/a^3)*x^2
+ sqrt(a*x^4 + b*x^3)*b^2)/x^2) - (a*x^4 + b*x^3)^(1/4)*a^2*b*(b^4/a^3)^(3/
4))/(b^4*x)) + 1/4*(b^4/a^3)^(1/4)*log((a*(b^4/a^3)^(1/4)*x + (a*x^4 + b*x^
3)^(1/4)*b)/x) - 1/4*(b^4/a^3)^(1/4)*log(-(a*(b^4/a^3)^(1/4)*x - (a*x^4 + b
*x^3)^(1/4)*b)/x) + (a*x^4 + b*x^3)^(1/4)
```

giac [B] time = 0.21, size = 211, normalized size = 2.57

$$\frac{\frac{2\sqrt2(-a)^{\frac14}b^2\arctan\left(\frac{\sqrt2\left(\sqrt2(-a)^{\frac14}+2\left(a+\frac bx\right)^{\frac14}\right)}{2(-a)^{\frac14}}\right)}{a}+\frac{2\sqrt2(-a)^{\frac14}b^2\arctan\left(-\frac{\sqrt2\left(\sqrt2(-a)^{\frac14}-2\left(a+\frac bx\right)^{\frac14}\right)}{2(-a)^{\frac14}}\right)}{a}+\frac{\sqrt2(-a)^{\frac14}b^2\log\left(\sqrt2(-a)^{\frac14}\left(a+\frac bx\right)^{\frac14}+\sqrt{-a}+\sqrt{a+\frac bx}\right)}{a}+\frac{\sqrt2b^2\log\left(-\sqrt2(-a)^{\frac14}\left(a+\frac bx\right)^{\frac14}+\sqrt{-a}+\sqrt{a+\frac bx}\right)}{(-a)^{\frac34}}+8\left(a+\frac bx\right)^{\frac14}bx}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^3)^(1/4)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(2)*(-a)^(1/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a
 + b/x)^(1/4))/(-a)^(1/4))/a + 2*sqrt(2)*(-a)^(1/4)*b^2*arctan(-1/2*sqrt(2)
*(sqrt(2)*(-a)^(1/4) - 2*(a + b/x)^(1/4))/(-a)^(1/4))/a + sqrt(2)*(-a)^(1/4
)*b^2*log(sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a + b/x))/a
+ sqrt(2)*b^2*log(-sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a +
 b/x))/(-a)^(3/4) + 8*(a + b/x)^(1/4)*b*x)/b
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{4}+b\,x^{3}\right)^{\frac{1}{4}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b*x^3)^(1/4)/x,x)
```

```
[Out] int((a*x^4+b*x^3)^(1/4)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{4}+bx^{3}\right)^{\frac{1}{4}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^3)^(1/4)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + b*x^3)^(1/4)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^4+b\,x^3\right)^{1/4}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4 + b*x^3)^(1/4)/x,x)
```

```
[Out] int((a*x^4 + b*x^3)^(1/4)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\,(ax+b)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b*x**3)**(1/4)/x,x)
```

```
[Out] Integral((x**3*(a*x + b))**(1/4)/x, x)
```

3.990 $\int \frac{(-b+ax)\sqrt[4]{bx^3+ax^4}}{x(b+ax)}\,dx$

Optimal. Leaf size=82

$$\frac{7b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{2a^{3/4}}-\frac{7b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{2a^{3/4}}+\sqrt[4]{ax^4+bx^3}$$

Rubi [A] time = 0.36, antiderivative size = 136, normalized size of antiderivative = 1.66, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2056, 80, 63, 331, 298, 203, 206}

$$\frac{7b\sqrt[4]{ax^4+bx^3}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{2a^{3/4}x^{3/4}\sqrt[4]{ax+b}}-\frac{7b\sqrt[4]{ax^4+bx^3}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{2a^{3/4}x^{3/4}\sqrt[4]{ax+b}}+\sqrt[4]{ax^4+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a*x)*(b*x^3 + a*x^4)^(1/4))/(x*(b + a*x)),x]
```

```
[Out] (b*x^3 + a*x^4)^(1/4) + (7*b*(b*x^3 + a*x^4)^(1/4)*ArcTan[(a^(1/4)*x^(1/4))
/(b + a*x)^(1/4)])/(2*a^(3/4)*x^(3/4)*(b + a*x)^(1/4)) - (7*b*(b*x^3 + a*x^
4)^(1/4)*ArcTanh[(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(2*a^(3/4)*x^(3/4)*(b
+ a*x)^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-b+ax)\sqrt[4]{bx^3+ax^4}}{x(b+ax)}\,dx &= \frac{\sqrt[4]{bx^3+ax^4}\int \frac{-b+ax}{\sqrt[4]{x}(b+ax)^{3/4}}\,dx}{x^{3/4}\sqrt[4]{b+ax}}\\
&= \sqrt[4]{bx^3+ax^4} - \frac{\left(7b\sqrt[4]{bx^3+ax^4}\right)\int \frac{1}{\sqrt[4]{x}(b+ax)^{3/4}}\,dx}{4x^{3/4}\sqrt[4]{b+ax}}\\
&= \sqrt[4]{bx^3+ax^4} - \frac{\left(7b\sqrt[4]{bx^3+ax^4}\right)\mathrm{Subst}\left(\int \frac{x^2}{\left(b+ax^4\right)^{3/4}}\,dx, x, \sqrt[4]{x}\right)}{x^{3/4}\sqrt[4]{b+ax}}\\
&= \sqrt[4]{bx^3+ax^4} - \frac{\left(7b\sqrt[4]{bx^3+ax^4}\right)\mathrm{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)}{x^{3/4}\sqrt[4]{b+ax}}\\
&= \sqrt[4]{bx^3+ax^4} - \frac{\left(7b\sqrt[4]{bx^3+ax^4}\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)}{2\sqrt{a}\,x^{3/4}\sqrt[4]{b+ax}} + \frac{\left(7b\sqrt[4]{bx^3+ax\right.}}{}\\
&= \sqrt[4]{bx^3+ax^4} + \frac{7b\sqrt[4]{bx^3+ax^4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)}{2a^{3/4}x^{3/4}\sqrt[4]{b+ax}} - \frac{7b\sqrt[4]{bx^3+ax^4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)}{2a^{3/4}x^{3/4}\sqrt[4]{b+ax}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 60, normalized size = 0.73

$$\frac{x^3\left(3(ax+b)-7b\left(\frac{ax}{b}+1\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{ax}{b}\right)\right)}{3\left(x^3(ax+b)\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + a*x)*(b*x^3 + a*x^4)^(1/4))/(x*(b + a*x)),x]
```

```
[Out] (x^3*(3*(b + a*x) - 7*b*(1 + (a*x)/b)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4
, -((a*x)/b)]))/(3*(x^3*(b + a*x))^(3/4))
```

IntegrateAlgebraic [A] time = 0.39, size = 82, normalized size = 1.00

$$\frac{7b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{2a^{3/4}} - \frac{7b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{2a^{3/4}} + \sqrt[4]{ax^4+bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x)*(b*x^3 + a*x^4)^(1/4))/(x*(b + a*x)),x]
```

```
[Out] (b*x^3 + a*x^4)^(1/4) + (7*b*ArcTan[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(2*
a^(3/4)) - (7*b*ArcTanh[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(2*a^(3/4))
```

fricas [B] time = 0.44, size = 207, normalized size = 2.52

$$7\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2\left(\frac{b^4}{a^3}\right)^{\frac{3}{4}} x\sqrt{\frac{a^2\sqrt{\frac{b^4}{a^3}}x^2+\sqrt{ax^4+bx^3}\,b^2}{x^2}} - \left(ax^4+bx^3\right)^{\frac{1}{4}}a^2b\left(\frac{b^4}{a^3}\right)^{\frac{3}{4}}}{b^4x}\right) - \frac{7}{4}\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{7\left(a\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}x+\left(ax^4+bx^3\right)^{\frac{1}{4}}b\right)}{x}\right)+\frac{7}{4}\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}\log\left(-\frac{7\left(a\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}x-\left(ax^4+bx^3\right)^{\frac{1}{4}}b\right)}{x}\right)+\left(ax^4+bx^3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)*(a*x^4+b*x^3)^(1/4)/x/(a*x+b),x, algorithm="fricas")
```

```
[Out] 7*(b^4/a^3)^(1/4)*arctan((a^2*(b^4/a^3)^(3/4)*x*sqrt((a^2*sqrt(b^4/a^3)*x^2
 + sqrt(a*x^4 + b*x^3)*b^2)/x^2) - (a*x^4 + b*x^3)^(1/4)*a^2*b*(b^4/a^3)^(3
/4))/(b^4*x)) - 7/4*(b^4/a^3)^(1/4)*log(7*(a*(b^4/a^3)^(1/4)*x + (a*x^4 + b
*x^3)^(1/4)*b)/x) + 7/4*(b^4/a^3)^(1/4)*log(-7*(a*(b^4/a^3)^(1/4)*x - (a*x^
4 + b*x^3)^(1/4)*b)/x) + (a*x^4 + b*x^3)^(1/4)
```

giac [B] time = 0.38, size = 207, normalized size = 2.52

$$\frac{\frac{14\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}} + \frac{14\sqrt{2}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}} + \frac{7\sqrt{2}b^2\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{(-a)^{\frac{3}{4}}} + \frac{7\sqrt{2}(-a)^{\frac{1}{4}}b^2\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{a} + 8\left(a+\frac{b}{x}\right)^{\frac{1}{4}}bx}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)*(a*x^4+b*x^3)^(1/4)/x/(a*x+b),x, algorithm="giac")
```

```
[Out] 1/8*(14*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x)^(1
/4))/(-a)^(1/4))/(-a)^(3/4) + 14*sqrt(2)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(
-a)^(1/4) - 2*(a + b/x)^(1/4))/(-a)^(1/4))/(-a)^(3/4) + 7*sqrt(2)*b^2*log(s
qrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a + b/x))/(-a)^(3/4) +
7*sqrt(2)*(-a)^(1/4)*b^2*log(-sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a)
 + sqrt(a + b/x))/a + 8*(a + b/x)^(1/4)*b*x)/b
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(ax-b)\left(a\,x^4+b\,x^3\right)^{\frac{1}{4}}}{x\,(ax+b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-b)*(a*x^4+b*x^3)^(1/4)/x/(a*x+b),x)
```

```
[Out] int((a*x-b)*(a*x^4+b*x^3)^(1/4)/x/(a*x+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4+bx^3\right)^{\frac{1}{4}}(ax-b)}{(ax+b)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)*(a*x^4+b*x^3)^(1/4)/x/(a*x+b),x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + b*x^3)^(1/4)*(a*x - b)/((a*x + b)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(a\,x^4+b\,x^3\right)^{1/4}\,\left(b-a\,x\right)}{x\,\left(b+a\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*x^4 + b*x^3)^(1/4)*(b - a*x))/(x*(b + a*x)),x)
[Out] int(-((a*x^4 + b*x^3)^(1/4)*(b - a*x))/(x*(b + a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^{3}\left(ax+b\right)}\,\left(ax-b\right)}{x\left(ax+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)*(a*x**4+b*x**3)**(1/4)/x/(a*x+b),x)
[Out] Integral((x**3*(a*x + b))**(1/4)*(a*x - b)/(x*(a*x + b)), x)
```

3.991 $\int \frac{-1+3x^4}{\left(1-x+x^4\right)\sqrt[3]{x^2+x^6}}\,dx$

Optimal. Leaf size=82

$$\log\left(\sqrt[3]{x^6+x^2}-x\right)-\frac{1}{2}\log\left(x^2+\sqrt[3]{x^6+x^2}\,x+\left(x^6+x^2\right)^{2/3}\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^6+x^2}+x}\right)$$

Rubi [F] time = 1.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+3x^4}{\left(1-x+x^4\right)\sqrt[3]{x^2+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 3*x^4)/((1 - x + x^4)*(x^2 + x^6)^(1/3)),x]
```

```
[Out] (9*x*(1 + x^4)^(1/3)*Hypergeometric2F1[1/12, 1/3, 13/12, -x^4])/(x^2 + x^6)
^(1/3) - (12*x^(2/3)*(1 + x^4)^(1/3)*Defer[Subst][Defer[Int][1/((1 + x^12)^
(1/3)*(1 - x^3 + x^12)), x], x, x^(1/3)])/(x^2 + x^6)^(1/3) + (9*x^(2/3)*(1
 + x^4)^(1/3)*Defer[Subst][Defer[Int][x^3/((1 + x^12)^(1/3)*(1 - x^3 + x^12
)), x], x, x^(1/3)])/(x^2 + x^6)^(1/3)
```

Rubi steps

$$\int \frac{-1+3x^4}{\left(1-x+x^4\right)\sqrt[3]{x^2+x^6}}\,dx=\frac{\left(x^{2/3}\sqrt[3]{1+x^4}\right)\int \frac{-1+3x^4}{x^{2/3}\sqrt[3]{1+x^4}\left(1-x+x^4\right)}\,dx}{\sqrt[3]{x^2+x^6}}$$

$$=\frac{\left(3x^{2/3}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{-1+3x^{12}}{\sqrt[3]{1+x^{12}}\left(1-x^3+x^{12}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^6}}$$

$$=\frac{\left(3x^{2/3}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{3}{\sqrt[3]{1+x^{12}}}-\frac{4-3x^3}{\sqrt[3]{1+x^{12}}\left(1-x^3+x^{12}\right)}\right)dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^6}}$$

$$=-\frac{\left(3x^{2/3}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{4-3x^3}{\sqrt[3]{1+x^{12}}\left(1-x^3+x^{12}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^6}}+\frac{\left(9x^{2/3}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\ldots\right.}{\sqrt[3]{x^2+\ldots}}$$

$$=\frac{9x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{12},\frac{1}{3};\frac{13}{12};-x^4\right)}{\sqrt[3]{x^2+x^6}}-\frac{\left(3x^{2/3}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int\left(\frac{4}{\sqrt[3]{1+x^{12}}\left(1-x^3+x^{12}\right)}-\ldots\right.\right.}{\sqrt[3]{x^2+x^6}}$$

$$=\frac{9x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{12},\frac{1}{3};\frac{13}{12};-x^4\right)}{\sqrt[3]{x^2+x^6}}+\frac{\left(9x^{2/3}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{1+x^{12}}\left(1-x^3+x^{12}\right)}\,dx,x,\ldots\right.}{\sqrt[3]{x^2+x^6}}$$

Mathematica [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{-1+3x^4}{\left(1-x+x^4\right)\sqrt[3]{x^2+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + 3*x^4)/((1 - x + x^4)*(x^2 + x^6)^(1/3)),x]
```

```
[Out] Integrate[(-1 + 3*x^4)/((1 - x + x^4)*(x^2 + x^6)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 2.67, size = 82, normalized size = 1.00

$$\log\left(\sqrt[3]{x^6+x^2}-x\right)-\frac{1}{2}\log\left(x^2+\sqrt[3]{x^6+x^2}\,x+\left(x^6+x^2\right)^{2/3}\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^6+x^2}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 3*x^4)/((1 - x + x^4)*(x^2 + x^6)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(x^2 + x^6)^(1/3))]) + Log[-x + (x^2 +
x^6)^(1/3)] - Log[x^2 + x*(x^2 + x^6)^(1/3) + (x^2 + x^6)^(2/3)]/2
```

fricas [A] time = 1.71, size = 112, normalized size = 1.37

$$-\sqrt{3}\arctan\left(-\frac{2\sqrt{3}\left(x^6+x^2\right)^{\frac{1}{3}}x+\sqrt{3}\left(x^5-x^2+x\right)-2\sqrt{3}\left(x^6+x^2\right)^{\frac{2}{3}}}{3\left(x^5+x^2+x\right)}\right)+\frac{1}{2}\log\left(\frac{x^5-x^2+3\left(x^6+x^2\right)^{\frac{1}{3}}x+x-3\left(x^6+x^2\right)^{\frac{2}{3}}}{x^5-x^2+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)/(x^4-x+1)/(x^6+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(-1/3*(2*sqrt(3)*(x^6 + x^2)^(1/3)*x + sqrt(3)*(x^5 - x^2 +
x) - 2*sqrt(3)*(x^6 + x^2)^(2/3))/(x^5 + x^2 + x)) + 1/2*log((x^5 - x^2 + 3
*(x^6 + x^2)^(1/3)*x + x - 3*(x^6 + x^2)^(2/3))/(x^5 - x^2 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^4-1}{\left(x^6+x^2\right)^{\frac{1}{3}}\left(x^4-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)/(x^4-x+1)/(x^6+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((3*x^4 - 1)/((x^6 + x^2)^(1/3)*(x^4 - x + 1)), x)
```

maple [C] time = 4.43, size = 337, normalized size = 4.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4-1)/(x^4-x+1)/(x^6+x^2)^(1/3),x)
```

```
[Out] ln(-(4747*RootOf(_Z^2+_Z+1)^2*x^5-6570*RootOf(_Z^2+_Z+1)*x^5-5848*x^5-9494*
RootOf(_Z^2+_Z+1)^2*x^2+12039*RootOf(_Z^2+_Z+1)*(x^6+x^2)^(2/3)+9873*RootOf
(_Z^2+_Z+1)*(x^6+x^2)^(1/3)*x+4747*RootOf(_Z^2+_Z+1)^2*x-20089*RootOf(_Z^2+
_Z+1)*x^2+21912*(x^6+x^2)^(2/3)-12039*x*(x^6+x^2)^(1/3)-6570*RootOf(_Z^2+_Z
+1)*x-8772*x^2-5848*x)/x/(x^4-x+1))+RootOf(_Z^2+_Z+1)*ln((2924*RootOf(_Z^2+
_Z+1)^2*x^5+13519*RootOf(_Z^2+_Z+1)*x^5+14241*x^5-5848*RootOf(_Z^2+_Z+1)^2*
x^2+12039*RootOf(_Z^2+_Z+1)*(x^6+x^2)^(2/3)-21912*RootOf(_Z^2+_Z+1)*(x^6+x^
2)^(1/3)*x+2924*RootOf(_Z^2+_Z+1)^2*x-6570*RootOf(_Z^2+_Z+1)*x^2-9873*(x^6+
x^2)^(2/3)-12039*x*(x^6+x^2)^(1/3)+13519*RootOf(_Z^2+_Z+1)*x+4747*x^2+14241
*x)/x/(x^4-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^4-1}{\left(x^6+x^2\right)^{\frac{1}{3}}\left(x^4-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)/(x^4-x+1)/(x^6+x^2)^(1/3),x, algorithm="maxima")
[Out] integrate((3*x^4 - 1)/((x^6 + x^2)^(1/3)*(x^4 - x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x^4-1}{{\left(x^6+x^2\right)}^{1/3}\,\left(x^4-x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4 - 1)/((x^2 + x^6)^(1/3)*(x^4 - x + 1)),x)
[Out] int((3*x^4 - 1)/((x^2 + x^6)^(1/3)*(x^4 - x + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4-1)/(x**4-x+1)/(x**6+x**2)**(1/3),x)
[Out] Timed out
```

3.992 $\int \frac{-b+ax^8}{x^2(b+ax^4)^{3/4}}\,dx$

Optimal. Leaf size=82

$$\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{3/4}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{3/4}}+\frac{(x^4+4)\sqrt[4]{ax^4+b}}{4x}$$

Rubi [A] time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1487, 451, 331, 298, 203, 206}

$$\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{3/4}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{3/4}}+\frac{\sqrt[4]{ax^4+b}}{x}+\frac{1}{4}x^3\sqrt[4]{ax^4+b}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^8)/(x^2*(b + a*x^4)^(3/4)),x]
```

```
[Out] (b + a*x^4)^(1/4)/x + (x^3*(b + a*x^4)^(1/4))/4 + (3*b*ArcTan[(a^(1/4)*x)/(
b + a*x^4)^(1/4)])/(8*a^(3/4)) - (3*b*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)
])/(8*a^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1487

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 2*n*p - n + 1)*(d + e*x^n)^(
q + 1))/(e*f^(2*n*p - n + 1)*(m + 2*n*p + n*q + 1)), x] + Dist[1/(e*(m + 2*
n*p + n*q + 1)), Int[(f*x)^m*(d + e*x^n)^q*ExpandToSum[e*(m + 2*n*p + n*q +
 1)*((a + c*x^(2*n))^p - c^p*x^(2*n*p)) - d*c^p*(m + 2*n*p - n + 1)*x^(2*n*
p - n), x], x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IG
tQ[n, 0] && IGtQ[p, 0] && GtQ[2*n*p, n - 1] &&  !IntegerQ[q] && NeQ[m + 2*n
*p + n*q + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-b+ax^8}{x^2\left(b+ax^4\right)^{3/4}}\,dx &= \frac{1}{4}x^3\sqrt[4]{b+ax^4}+\frac{\int \frac{-4ab-3abx^4}{x^2\left(b+ax^4\right)^{3/4}}\,dx}{4a}\\
&= \frac{\sqrt[4]{b+ax^4}}{x}+\frac{1}{4}x^3\sqrt[4]{b+ax^4}-\frac{1}{4}(3b)\int \frac{x^2}{\left(b+ax^4\right)^{3/4}}\,dx\\
&= \frac{\sqrt[4]{b+ax^4}}{x}+\frac{1}{4}x^3\sqrt[4]{b+ax^4}-\frac{1}{4}(3b)\,\mathrm{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)\\
&= \frac{\sqrt[4]{b+ax^4}}{x}+\frac{1}{4}x^3\sqrt[4]{b+ax^4}-\frac{(3b)\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)}{8\sqrt{a}}+\frac{(3b)\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{a}x^2}\right.}{8\sqrt{a}}\\
&= \frac{\sqrt[4]{b+ax^4}}{x}+\frac{1}{4}x^3\sqrt[4]{b+ax^4}+\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b+ax^4}}\right)}{8a^{3/4}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b+ax^4}}\right)}{8a^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 82, normalized size = 1.00

$$\frac{2a^{3/4}\left(x^4+4\right)\sqrt[4]{ax^4+b}+3bx\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)-3bx\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{8a^{3/4}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^8)/(x^2*(b + a*x^4)^(3/4)),x]
```

```
[Out] (2*a^(3/4)*(4 + x^4)*(b + a*x^4)^(1/4) + 3*b*x*ArcTan[(a^(1/4)*x)/(b + a*x^
4)^(1/4)] - 3*b*x*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(8*a^(3/4)*x)
```

IntegrateAlgebraic [A] time = 0.56, size = 82, normalized size = 1.00

$$\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{8a^{3/4}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{8a^{3/4}}+\frac{\left(x^4+4\right)\sqrt[4]{ax^4+b}}{4x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^8)/(x^2*(b + a*x^4)^(3/4)),x]
```

```
[Out] ((4 + x^4)*(b + a*x^4)^(1/4))/(4*x) + (3*b*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(
1/4)])/(8*a^(3/4)) - (3*b*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(8*a^(3/4
))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-b)/x^2/(a*x^4+b)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^8 - b}{\left(ax^4 + b\right)^{\frac{3}{4}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-b)/x^2/(a*x^4+b)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^8 - b)/((a*x^4 + b)^(3/4)*x^2), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{a\,x^8 - b}{x^2 \left(a\,x^4 + b\right)^{\frac{3}{4}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^8-b)/x^2/(a*x^4+b)^(3/4),x)
```

```
[Out] int((a*x^8-b)/x^2/(a*x^4+b)^(3/4),x)
```

maxima [B] time = 0.42, size = 127, normalized size = 1.55

$$-\frac{1}{16} a \left(\frac{3 \left(\frac{2\,b \arctan\left(\frac{(ax^4+b)^{\frac{1}{4}}}{a^{\frac{1}{4}} x}\right)}{a^{\frac{3}{4}}} - \frac{b \log\left(-\frac{a^{\frac{1}{4}} - \frac{(ax^4+b)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}} + \frac{(ax^4+b)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{3}{4}}} \right)}{a} + \frac{4 \left(ax^4 + b\right)^{\frac{1}{4}} b}{\left(a^2 - \frac{(ax^4+b)a}{x^4}\right) x} + \frac{\left(ax^4 + b\right)^{\frac{1}{4}}}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-b)/x^2/(a*x^4+b)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/16*a*(3*(2*b*arctan((a*x^4 + b)^(1/4)/(a^(1/4)*x))/a^(3/4) - b*log(-(a^(
1/4) - (a*x^4 + b)^(1/4)/x)/(a^(1/4) + (a*x^4 + b)^(1/4)/x))/a^(3/4))/a + 4
*(a*x^4 + b)^(1/4)*b/((a^2 - (a*x^4 + b)*a/x^4)*x)) + (a*x^4 + b)^(1/4)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{b - a\,x^8}{x^2 \left(a\,x^4 + b\right)^{3/4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^8)/(x^2*(b + a*x^4)^(3/4)),x)
```

```
[Out] -int((b - a*x^8)/(x^2*(b + a*x^4)^(3/4)), x)
```

sympy [C] time = 2.37, size = 70, normalized size = 0.85

$$-\frac{\sqrt[4]{a}\,\sqrt[4]{1+\frac{b}{ax^4}}\,\Gamma\left(-\frac{1}{4}\right)}{4\Gamma\left(\frac{3}{4}\right)}+\frac{ax^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{ax^4e^{i\pi}}{b}\right)}{4b^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**8-b)/x**2/(a*x**4+b)**(3/4),x)
```

```
[Out] -a**(1/4)*(1 + b/(a*x**4))**(1/4)*gamma(-1/4)/(4*gamma(3/4)) + a*x**7*gamma
(7/4)*hyper((3/4, 7/4), (11/4,), a*x**4*exp_polar(I*pi)/b)/(4*b**(3/4)*gamm
a(11/4))
```

3.993 $\int \frac{1}{\sqrt[3]{x^2+x^3}} \, dx$

Optimal. Leaf size=83

$$-\log\left(\sqrt[3]{x^3+x^2}-x\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^3+x^2}\,x+\left(x^3+x^2\right)^{2/3}\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3+x^2}+x}\right)$$

Rubi [A] time = 0.02, antiderivative size = 129, normalized size of antiderivative = 1.55, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2011, 59}

$$-\frac{x^{2/3}\sqrt[3]{x+1}\log(x)}{2\sqrt[3]{x^3+x^2}}-\frac{3x^{2/3}\sqrt[3]{x+1}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x}}-1\right)}{2\sqrt[3]{x^3+x^2}}-\frac{\sqrt{3}\,x^{2/3}\sqrt[3]{x+1}\tan^{-1}\left(\frac{2\sqrt[3]{x+1}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{x^3+x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2 + x^3)^(-1/3),x]
```

```
[Out] -((Sqrt[3]*x^(2/3)*(1 + x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(1 + x)^(1/3))/(Sqrt
[3]*x^(1/3))])/(x^2 + x^3)^(1/3)) - (x^(2/3)*(1 + x)^(1/3)*Log[x])/(2*(x^2
+ x^3)^(1/3)) - (3*x^(2/3)*(1 + x)^(1/3)*Log[-1 + (1 + x)^(1/3)/x^(1/3)])/(
2*(x^2 + x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x^2+x^3}} \, dx = \frac{\left(x^{2/3}\sqrt[3]{1+x}\right)\int \frac{1}{x^{2/3}\sqrt[3]{1+x}} \, dx}{\sqrt[3]{x^2+x^3}}$$

$$= -\frac{\sqrt{3}\,x^{2/3}\sqrt[3]{1+x}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1+x}}{\sqrt{3}\sqrt[3]{x}}\right)}{\sqrt[3]{x^2+x^3}}-\frac{x^{2/3}\sqrt[3]{1+x}\log(x)}{2\sqrt[3]{x^2+x^3}}-\frac{3x^{2/3}\sqrt[3]{1+x}\log\left(-1+\frac{\sqrt[3]{1+x}}{\sqrt[3]{x}}\right)}{2\sqrt[3]{x^2+x^3}}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.41

$$\frac{3x\sqrt[3]{x+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-x\right)}{\sqrt[3]{x^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2 + x^3)^(-1/3),x]
```

```
[Out] (3*x*(1 + x)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, -x])/(x^2*(1 + x))^(1/3
)
```

IntegrateAlgebraic [A] time = 0.17, size = 83, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^3+x^2}-x\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^3+x^2}\,x+\left(x^3+x^2\right)^{2/3}\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3+x^2}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2 + x^3)^(-1/3),x]
```

```
[Out] Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(x^2 + x^3)^(1/3))] - Log[-x + (x^2 + x^3
)^(1/3)] + Log[x^2 + x*(x^2 + x^3)^(1/3) + (x^2 + x^3)^(2/3)]/2
```

fricas [A] time = 0.61, size = 84, normalized size = 1.01

$$-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,x+2\sqrt{3}\left(x^3+x^2\right)^{\frac{1}{3}}}{3\,x}\right)-\log\left(-\frac{x-\left(x^3+x^2\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{2}\log\left(\frac{x^2+\left(x^3+x^2\right)^{\frac{1}{3}}x+\left(x^3+x^2\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 + x^2)^(1/3))/x) - log(-(x
- (x^3 + x^2)^(1/3))/x) + 1/2*log((x^2 + (x^3 + x^2)^(1/3)*x + (x^3 + x^2)^
(2/3))/x^2)
```

giac [A] time = 3.03, size = 55, normalized size = 0.66

$$-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2}\log\left(\left(\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)-\log\left(\left|\left(\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*(1/x + 1)^(1/3) + 1)) + 1/2*log((1/x + 1)^(2
/3) + (1/x + 1)^(1/3) + 1) - log(abs((1/x + 1)^(1/3) - 1))
```

maple [C] time = 0.31, size = 15, normalized size = 0.18

$$3x^{\frac{1}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+x^2)^(1/3),x)
```

```
[Out] 3*x^(1/3)*hypergeom([1/3,1/3],[4/3],-x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(x^3+x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + x^2)^(-1/3), x)
```

mupad [B] time = 0.95, size = 25, normalized size = 0.30

$$\frac{3\,x\,{\left(x+1\right)}^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ -x\right)}{{\left(x^3+x^2\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + x^3)^(1/3),x)
[Out] (3*x*(x + 1)^(1/3)*hypergeom([1/3, 1/3], 4/3, -x))/(x^2 + x^3)^(1/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x^3+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+x**2)**(1/3),x)
[Out] Integral((x**3 + x**2)**(-1/3), x)
```

3.994 $\int \frac{a-3b+2x}{\sqrt[4]{(-a+x)(-b+x)}\left(-a^3+bd-\left(-3a^2+d\right)x-3ax^2+x^3\right)}\,dx$

Optimal. Leaf size=83

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{x(-a-b)+ab+x^2}}{a-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{x(-a-b)+ab+x^2}}{a-x}\right)}{d^{3/4}}$$

Rubi [F] time = 6.76, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a-3b+2x}{\sqrt[4]{(-a+x)(-b+x)}\left(-a^3+bd-\left(-3a^2+d\right)x-3ax^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a - 3*b + 2*x)/(((-a + x)*(-b + x))^(1/4)*(-a^3 + b*d - (-3*a^2 + d)*x
 - 3*a*x^2 + x^3)),x]
```

```
[Out] (8*a*(-a + x)^(1/4)*(-b + x)^(1/4)*Defer[Subst][Defer[Int][x^2/((a - b + x^
4)^(1/4)*(-(a*(1 - b/a)*d) - d*x^4 + x^12)), x], x, (-a + x)^(1/4)])/((a -
x)*(b - x))^(1/4) + (8*(-a + x)^(1/4)*(-b + x)^(1/4)*Defer[Subst][Defer[Int
][x^6/((a - b + x^4)^(1/4)*(-(a*(1 - b/a)*d) - d*x^4 + x^12)), x], x, (-a +
 x)^(1/4)])/((a - x)*(b - x))^(1/4) - (4*(a - 3*b)*(-a + x)^(1/4)*(-b + x)^
(1/4)*Defer[Subst][Defer[Int][x^2/((a - b + x^4)^(1/4)*(a*(1 - b/a)*d + x^4
*(d - x^8))), x], x, (-a + x)^(1/4)])/((a - x)*(b - x))^(1/4)
```

Rubi steps

$$\int \frac{a-3b+2x}{\sqrt[4]{(-a+x)(-b+x)}\left(-a^3+bd-\left(-3a^2+d\right)x-3ax^2+x^3\right)}\,dx = \frac{\left(\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\int \frac{a-3b+\cdots}{\sqrt[4]{-a+x}\sqrt[4]{-b+x}\left(-a^3+bd-\cdots\right)}}{\sqrt[4]{(-a+x)(-b+x)}}$$

$$= \frac{\left(\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\int \frac{-a+3b-\cdots}{\sqrt[4]{-a+x}\sqrt[4]{-b+x}\left(a^3-bd-\cdots\right)}}{\sqrt[4]{(-a+x)(-b+x)}}$$

$$= \frac{\left(\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\int \left(\frac{3\left(1-\cdots\right)}{\sqrt[4]{-a+x}\sqrt[4]{-b+x}\left(a^3-bd-\cdots\right)}\cdots\right.}{\cdots}$$

$$= \frac{\left(2\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\int \frac{x\cdots}{\sqrt[4]{-a+x}\sqrt[4]{-b+x}\left(-a^3+b\cdots\right)}}{\sqrt[4]{(-a+x)(-b+x)}}$$

$$= \frac{\left(8\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\mathrm{Subst}\left(\int \frac{x^2(\cdots}{\sqrt[4]{a-b+x^4}\left(-a\cdots\right.}\cdots\right.}{\sqrt[4]{(-a+x)(-b+\cdots}}$$

$$= \frac{\left(8\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\mathrm{Subst}\left(\int \frac{x^2(\cdots}{\sqrt[4]{a-b+x^4}\left(a(\cdots\right.}\cdots\right.}{\sqrt[4]{(-a+x)(-b+\cdots}}$$

$$= \frac{\left(8\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\mathrm{Subst}\left(\int\left(\frac{\cdots}{\sqrt[4]{a-b+x^4}\left(-\cdots\right.}\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(8\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\mathrm{Subst}\left(\int \frac{\cdots}{\sqrt[4]{a-b+x^4}\left(-a\cdots\right.}\cdots\right.}{\sqrt[4]{(-a+x)(-b+\cdots}}$$

Mathematica [F] time = 1.84, size = 0, normalized size = 0.00

$$\int \frac{a-3b+2x}{\sqrt[4]{(-a+x)(-b+x)}\left(-a^3+bd-\left(-3a^2+d\right)x-3ax^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a - 3*b + 2*x)/(((-a + x)*(-b + x))^(1/4)*(-a^3 + b*d - (-3*a^2
 + d)*x - 3*a*x^2 + x^3)),x]
```

```
[Out] Integrate[(a - 3*b + 2*x)/(((-a + x)*(-b + x))^(1/4)*(-a^3 + b*d - (-3*a^2
 + d)*x - 3*a*x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.39, size = 83, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x(-a-b)+ab+x^2}}{a-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x(-a-b)+ab+x^2}}{a-x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - 3*b + 2*x)/(((-a + x)*(-b + x))^(1/4)*(-a^3 + b*d -
 (-3*a^2 + d)*x - 3*a*x^2 + x^3)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(a*b + (-a - b)*x + x^2)^(1/4))/(a - x)])/d^(3/4) + (2*
ArcTanh[(d^(1/4)*(a*b + (-a - b)*x + x^2)^(1/4))/(a - x)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-3*b+2*x)/((-a+x)*(-b+x))^(1/4)/(-a^3+b*d-(-3*a^2+d)*x-3*a*x^2+
x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{a-3\,b+2\,x}{\left(a^3+3\,ax^2-x^3-bd-\left(3\,a^2-d\right)x\right)\left((a-x)(b-x)\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-3*b+2*x)/((-a+x)*(-b+x))^(1/4)/(-a^3+b*d-(-3*a^2+d)*x-3*a*x^2+
x^3),x, algorithm="giac")
```

```
[Out] integrate(-(a - 3*b + 2*x)/((a^3 + 3*a*x^2 - x^3 - b*d - (3*a^2 - d)*x)*((a
 - x)*(b - x))^(1/4)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{a-3b+2x}{\left(\left(-a+x\right)\left(-b+x\right)\right)^{\frac{1}{4}}\left(-a^{3}+bd-\left(-3a^{2}+d\right)x-3a\,x^{2}+x^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-3*b+2*x)/((-a+x)*(-b+x))^(1/4)/(-a^3+b*d-(-3*a^2+d)*x-3*a*x^2+x^3),x
)
```

```
[Out] int((a-3*b+2*x)/((-a+x)*(-b+x))^(1/4)/(-a^3+b*d-(-3*a^2+d)*x-3*a*x^2+x^3),x
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a-3\,b+2\,x}{\left(a^3+3\,ax^2-x^3-bd-\left(3\,a^2-d\right)x\right)\left((a-x)(b-x)\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-3*b+2*x)/((-a+x)*(-b+x))^(1/4)/(-a^3+b*d-(-3*a^2+d)*x-3*a*x^2+
x^3),x, algorithm="maxima")
```

```
[Out] -integrate((a - 3*b + 2*x)/((a^3 + 3*a*x^2 - x^3 - b*d - (3*a^2 - d)*x)*((a
 - x)*(b - x))^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a-3\,b+2\,x}{{\left(\left(a-x\right)\,\left(b-x\right)\right)}^{1/4}\,\left(3\,a\,x^2-b\,d+x\,\left(d-3\,a^2\right)+a^3-x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a - 3*b + 2*x)/(((a - x)*(b - x))^(1/4)*(3*a*x^2 - b*d + x*(d - 3*a^2
) + a^3 - x^3)),x)
```

```
[Out] int(-(a - 3*b + 2*x)/(((a - x)*(b - x))^(1/4)*(3*a*x^2 - b*d + x*(d - 3*a^2
) + a^3 - x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-3*b+2*x)/((-a+x)*(-b+x))**(1/4)/(-a**3+b*d-(-3*a**2+d)*x-3*a*x
**2+x**3),x)
```

```
[Out] Timed out
```

3.995 $$\int \frac{(a-3b+2x)\left(a^2-2ax+x^2\right)}{((-a+x)(-b+x))^{3/4}\left(-b+a^3d+\left(1-3a^2d\right)x+3adx^2-dx^3\right)}\,dx$$

Optimal. Leaf size=83

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x(-a-b)+ab+x^2\right)^{3/4}}{b-x}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x(-a-b)+ab+x^2\right)^{3/4}}{b-x}\right)}{d^{3/4}}$$

Rubi [F] time = 7.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a-3b+2x)\left(a^2-2ax+x^2\right)}{((-a+x)(-b+x))^{3/4}\left(-b+a^3d+\left(1-3a^2d\right)x+3adx^2-dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a - 3*b + 2*x)*(a^2 - 2*a*x + x^2))/(((-a + x)*(-b + x))^(3/4)*(-b +
a^3*d + (1 - 3*a^2*d)*x + 3*a*d*x^2 - d*x^3)),x]
```

```
[Out] (8*(1 - (a - b)/(a - x))^(3/4)*(-a + x)^(3/2)*EllipticF[ArcCot[Sqrt[-a + x]
/Sqrt[a - b]]/2, 2])/(Sqrt[a - b]*d*((a - x)*(b - x))^(3/4)) + (8*(a - b)*(
-a + x)^(3/4)*(-b + x)^(3/4)*Defer[Subst][Defer[Int][1/((a - b + x^4)^(3/4)
*(a*(1 - b/a) + x^4 - d*x^12)), x], x, (-a + x)^(1/4)])/(d*((a - x)*(b - x)
)^(3/4)) + (8*(-a + x)^(3/4)*(-b + x)^(3/4)*Defer[Subst][Defer[Int][x^4/((a
 - b + x^4)^(3/4)*(a*(1 - b/a) + x^4 - d*x^12)), x], x, (-a + x)^(1/4)])/(d
*((a - x)*(b - x))^(3/4)) + (8*a*(-a + x)^(3/4)*(-b + x)^(3/4)*Defer[Subst]
[Defer[Int][x^8/((a - b + x^4)^(3/4)*(a*(1 - b/a) + x^4 - d*x^12)), x], x,
(-a + x)^(1/4)])/((a - x)*(b - x))^(3/4) + (4*(a - 3*b)*(-a + x)^(3/4)*(-b
+ x)^(3/4)*Defer[Subst][Defer[Int][x^8/((a - b + x^4)^(3/4)*(a*(1 - b/a) +
x^4 - d*x^12)), x], x, (-a + x)^(1/4)])/((a - x)*(b - x))^(3/4)
```

Rubi steps

$$\int \frac{(a-3b+2x)\left(a^2-2ax+x^2\right)}{((-a+x)(-b+x))^{3/4}\left(-b+a^3d+\left(1-3a^2d\right)x+3adx^2-dx^3\right)}dx = \int \frac{(-a+x)^2(a-3b \cdots}{((-a+x)(-b+x))^{3/4}\left(-b+a^3d+\left(1 \cdots\right.\right.}$$

$$= \frac{\left((-a+x)^{3/4}(-b+x)^{3/4}\right)\int \frac{(-a \cdots}{(-b+x)^{3/4}\left(-b+a \cdots\right.}}{((-a+x)(-b+x) \cdots}$$

$$= \frac{\left((-a+x)^{3/4}(-b+x)^{3/4}\right)\int \left(\frac{a(\cdots}{(-b+x)^{3/4}\left(-b+ \cdots\right.} \cdots\right.}{\cdots}$$

$$= \frac{\left(2(-a+x)^{3/4}(-b+x)^{3/4}\right)\int \frac{\cdots}{(-b+x)^{3/4}\left(-b+ \cdots\right.}}{((-a+x)(-b+x \cdots}$$

$$= -\frac{\left(8(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\left(\int \frac{\cdots}{(a \cdots} \cdots\right.}{((-a+x) \cdots}$$

$$= -\frac{\left(8(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\left(\int \frac{\cdots}{(a \cdots} \cdots\right.}{((-a+x \cdots}$$

$$= -\frac{\left(8(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\left(\int \left(\frac{\cdots}{a \cdots} \cdots\right.\right.}{\cdots}$$

$$= \frac{\left(4(a-3b)(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subs} \cdots}{((-a+ \cdots}$$

$$= \frac{\left(4(a-3b)(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subs} \cdots}{((-a+ \cdots}$$

$$= \frac{\left(8a(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\left(\int \frac{\cdots}{(a- \cdots} \cdots\right.}{((-a+x)(\cdots}$$

$$= \frac{\left(8a(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\left(\int \frac{\cdots}{(a- \cdots} \cdots\right.}{((-a+x)(\cdots}$$

$$= \frac{8\left(1-\frac{a-b}{a-x}\right)^{3/4}(-a+x)^{3/2}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\cdots}{\sqrt{\cdots}} \cdots\right.\right.}{\sqrt{a-b}\,d((a-x)(b-x))^{3/4}}$$

Mathematica [F] time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{(a-3b+2x)\left(a^2-2ax+x^2\right)}{((-a+x)(-b+x))^{3/4}\left(-b+a^3d+\left(1-3a^2d\right)x+3adx^2-dx^3\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a - 3*b + 2*x)*(a^2 - 2*a*x + x^2))/(((-a + x)*(-b + x))^(3/4)*
(-b + a^3*d + (1 - 3*a^2*d)*x + 3*a*d*x^2 - d*x^3)),x]
```

```
[Out] Integrate[((a - 3*b + 2*x)*(a^2 - 2*a*x + x^2))/((((-a + x)*(-b + x))^(3/4)*
(-b + a^3*d + (1 - 3*a^2*d)*x + 3*a*d*x^2 - d*x^3)), x]
```

IntegrateAlgebraic [A] time = 0.63, size = 83, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x(-a-b)+ab+x^2\right)^{3/4}}{b-x}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x(-a-b)+ab+x^2\right)^{3/4}}{b-x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a - 3*b + 2*x)*(a^2 - 2*a*x + x^2))/((((-a + x)*(-b + x
))^(3/4)*(-b + a^3*d + (1 - 3*a^2*d)*x + 3*a*d*x^2 - d*x^3)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(a*b + (-a - b)*x + x^2)^(3/4))/(b - x)])/d^(3/4) - (2*A
rcTanh[(d^(1/4)*(a*b + (-a - b)*x + x^2)^(3/4))/(b - x)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-3*b+2*x)*(a^2-2*a*x+x^2)/((-a+x)*(-b+x))^(3/4)/(-b+a^3*d+(-3*a
^2*d+1)*x+3*a*d*x^2-d*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2-2\,ax+x^2\right)(a-3\,b+2\,x)}{\left(a^3d+3\,adx^2-dx^3-\left(3\,a^2d-1\right)x-b\right)\left((a-x)(b-x)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-3*b+2*x)*(a^2-2*a*x+x^2)/((-a+x)*(-b+x))^(3/4)/(-b+a^3*d+(-3*a
^2*d+1)*x+3*a*d*x^2-d*x^3),x, algorithm="giac")
```

```
[Out] integrate((a^2 - 2*a*x + x^2)*(a - 3*b + 2*x)/((a^3*d + 3*a*d*x^2 - d*x^3 -
 (3*a^2*d - 1)*x - b)*((a - x)*(b - x))^(3/4)), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(a-3b+2x)\left(a^2-2ax+x^2\right)}{\left((-a+x)\,(-b+x)\right)^{\frac{3}{4}}\left(-b+a^3d+\left(-3a^2d+1\right)x+3ad\,x^2-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-3*b+2*x)*(a^2-2*a*x+x^2)/((-a+x)*(-b+x))^(3/4)/(-b+a^3*d+(-3*a^2*d+1
)*x+3*a*d*x^2-d*x^3),x)
```

```
[Out] int((a-3*b+2*x)*(a^2-2*a*x+x^2)/((-a+x)*(-b+x))^(3/4)/(-b+a^3*d+(-3*a^2*d+1
)*x+3*a*d*x^2-d*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2-2\,ax+x^2\right)(a-3\,b+2\,x)}{\left(a^3d+3\,adx^2-dx^3-\left(3\,a^2d-1\right)x-b\right)\left((a-x)(b-x)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-3*b+2*x)*(a^2-2*a*x+x^2)/((-a+x)*(-b+x))^(3/4)/(-b+a^3*d+(-3*a
^2*d+1)*x+3*a*d*x^2-d*x^3),x, algorithm="maxima")
```

```
[Out] integrate((a^2 - 2*a*x + x^2)*(a - 3*b + 2*x)/((a^3*d + 3*a*d*x^2 - d*x^3 -
 (3*a^2*d - 1)*x - b)*((a - x)*(b - x))^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(a^2-2\,a\,x+x^2\right)\,\left(a-3\,b+2\,x\right)}{{\left(\left(a-x\right)\,\left(b-x\right)\right)}^{3/4}\,\left(b-a^3\,d+d\,x^3+x\,\left(3\,a^2\,d-1\right)-3\,a\,d\,x^2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a^2 - 2*a*x + x^2)*(a - 3*b + 2*x))/(((a - x)*(b - x))^(3/4)*(b - a^
3*d + d*x^3 + x*(3*a^2*d - 1) - 3*a*d*x^2)),x)
```

```
[Out] -int(((a^2 - 2*a*x + x^2)*(a - 3*b + 2*x))/(((a - x)*(b - x))^(3/4)*(b - a^
3*d + d*x^3 + x*(3*a^2*d - 1) - 3*a*d*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-3*b+2*x)*(a**2-2*a*x+x**2)/((-a+x)*(-b+x))**(3/4)/(-b+a**3*d+(
-3*a**2*d+1)*x+3*a*d*x**2-d*x**3),x)
```

```
[Out] Timed out
```

3.996 $\int \frac{-1-2(-1+k)x+kx^2}{\sqrt[4]{(1-x)x(1-kx)}\left(-1+(d+3k)x-\left(d+3k^2\right)x^2+k^3x^3\right)}\,dx$

Optimal. Leaf size=83

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{kx-1}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{kx-1}\right)}{d^{3/4}}$$

Rubi [F] time = 15.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1-2(-1+k)x+kx^2}{\sqrt[4]{(1-x)x(1-kx)}\left(-1+(d+3k)x-\left(d+3k^2\right)x^2+k^3x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 - 2*(-1 + k)*x + k*x^2)/(((1 - x)*x*(1 - k*x))^(1/4)*(-1 + (d + 3*k
)*x - (d + 3*k^2)*x^2 + k^3*x^3)),x]
```

```
[Out] (4*(1 - x)^(1/4)*x^(1/4)*(1 - k*x)^(1/4)*Defer[Subst][Defer[Int][x^2/((1 -
x^4)^(1/4)*(1 - k*x^4)^(1/4)*(-(-1 + k*x^4)^3 - d*(x^4 - x^8))), x], x, x^(
1/4)])/((1 - x)*x*(1 - k*x))^(1/4) - (8*(1 - k)*(1 - x)^(1/4)*x^(1/4)*(1 -
k*x)^(1/4)*Defer[Subst][Defer[Int][x^6/((1 - x^4)^(1/4)*(1 - k*x^4)^(1/4)*(
-(-1 + k*x^4)^3 - d*(x^4 - x^8))), x], x, x^(1/4)])/((1 - x)*x*(1 - k*x))^(
1/4) + (4*k*(1 - x)^(1/4)*x^(1/4)*(1 - k*x)^(1/4)*Defer[Subst][Defer[Int][x
^10/((1 - x^4)^(1/4)*(1 - k*x^4)^(1/4)*((-1 + k*x^4)^3 + d*(x^4 - x^8))), x
], x, x^(1/4)])/((1 - x)*x*(1 - k*x))^(1/4)
```

Rubi steps

$$\int \frac{-1-2(-1+k)x+kx^2}{\sqrt[4]{(1-x)x(1-kx)}\left(-1+(d+3k)x-\left(d+3k^2\right)x^2+k^3x^3\right)}\,dx = \frac{\left(\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\int \frac{-1-2(-1+k)x}{\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\left(-1+(d+3k\right.}}{\sqrt[4]{(1-x)x(1-kx)}}$$

$$= \frac{\left(4\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\text{Subst}\left(\int \frac{x^2(}{\sqrt[4]{1-x^4}\sqrt[4]{1-kx^4}(}\right.}{\sqrt[4]{(1-x)x(1-}}$$

$$= \frac{\left(4\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\text{Subst}\left(\int \frac{x^2(-1-2(}{\sqrt[4]{1-x^4}\sqrt[4]{1-kx^4}(}\right.}{\sqrt[4]{(1-x)x(1-kx)}}$$

$$= \frac{\left(4\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\text{Subst}\left(\int\left(\frac{}{\sqrt[4]{1-x^4}\sqrt[4]{1-kx^4}}\right.\right.}{}$$

$$= \frac{\left(4\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{1-x^4}\sqrt[4]{1-kx^4}(}\right.}{\sqrt[4]{(1-x)x(1-}}$$

$$= \frac{\left(4\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{1-x^4}\sqrt[4]{1-kx^4}(}\right.}{\sqrt[4]{(1-x)x(1-kx)}}$$

Mathematica [F] time = 3.75, size = 0, normalized size = 0.00

$$\int \frac{-1-2(-1+k)x+kx^2}{\sqrt[4]{(1-x)x(1-kx)}\left(-1+(d+3k)x-\left(d+3k^2\right)x^2+k^3x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 - 2*(-1 + k)*x + k*x^2)/(((1 - x)*x*(1 - k*x))^(1/4)*(-1 + (d
 + 3*k)*x - (d + 3*k^2)*x^2 + k^3*x^3)),x]
```

```
[Out] Integrate[(-1 - 2*(-1 + k)*x + k*x^2)/(((1 - x)*x*(1 - k*x))^(1/4)*(-1 + (d
 + 3*k)*x - (d + 3*k^2)*x^2 + k^3*x^3)), x]
```

IntegrateAlgebraic [A] time = 0.26, size = 83, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{kx-1}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{kx-1}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 2*(-1 + k)*x + k*x^2)/(((1 - x)*x*(1 - k*x))^(1/4)
*(-1 + (d + 3*k)*x - (d + 3*k^2)*x^2 + k^3*x^3)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(x + (-1 - k)*x^2 + k*x^3)^(1/4))/(-1 + k*x)])/d^(3/4) -
 (2*ArcTanh[(d^(1/4)*(x + (-1 - k)*x^2 + k*x^3)^(1/4))/(-1 + k*x)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(1/4)/(-1+(d+3*k)*x-(3*k
^2+d)*x^2+k^3*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx^2-2(k-1)x-1}{\left(k^3x^3-\left(3k^2+d\right)x^2+(d+3k)x-1\right)((kx-1)(x-1)x)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(1/4)/(-1+(d+3*k)*x-(3*k
^2+d)*x^2+k^3*x^3),x, algorithm="giac")
```

```
[Out] integrate((k*x^2 - 2*(k - 1)*x - 1)/((k^3*x^3 - (3*k^2 + d)*x^2 + (d + 3*k)
*x - 1)*((k*x - 1)*(x - 1)*x)^(1/4)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{-1-2(-1+k)x+kx^2}{((1-x)x(-kx+1))^{\frac{1}{4}}\left(-1+(d+3k)x-\left(3k^2+d\right)x^2+k^3x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(1/4)/(-1+(d+3*k)*x-(3*k^2+d)*
x^2+k^3*x^3),x)
```

```
[Out] int((-1-2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(1/4)/(-1+(d+3*k)*x-(3*k^2+d)*
x^2+k^3*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx^2 - 2\,(k-1)x - 1}{\left(k^3x^3 - \left(3\,k^2 + d\right)x^2 + (d + 3\,k)x - 1\right)\left((kx-1)(x-1)x\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(1/4)/(-1+(d+3*k)*x-(3*k
^2+d)*x^2+k^3*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k*x^2 - 2*(k - 1)*x - 1)/((k^3*x^3 - (3*k^2 + d)*x^2 + (d + 3*k)
*x - 1)*((k*x - 1)*(x - 1)*x)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{2\,x\,\left(k-1\right)-k\,x^2+1}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/4}\,\left(k^3\,x^3-x^2\,\left(3\,k^2+d\right)+x\,\left(d+3\,k\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x*(k - 1) - k*x^2 + 1)/((x*(k*x - 1)*(x - 1))^(1/4)*(k^3*x^3 - x^2*
(d + 3*k^2) + x*(d + 3*k) - 1)),x)
```

```
[Out] int(-(2*x*(k - 1) - k*x^2 + 1)/((x*(k*x - 1)*(x - 1))^(1/4)*(k^3*x^3 - x^2*
(d + 3*k^2) + x*(d + 3*k) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-2*(-1+k)*x+k*x**2)/((1-x)*x*(-k*x+1))**(1/4)/(-1+(d+3*k)*x-(3
*k**2+d)*x**2+k**3*x**3),x)
```

```
[Out] Timed out
```

3.997 $\int \frac{\sqrt{1+6x^2+x^4}}{x(1+x^2)}\,dx$

Optimal. Leaf size=83

$$-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+6x^2+1}+1\right)-2\tan^{-1}\left(\frac{x^2}{2}-\frac{1}{2}\sqrt{x^4+6x^2+1}+\frac{1}{2}\right)-\tanh^{-1}\left(x^2-\sqrt{x^4+6x^2+1}+2\right)$$

Rubi [A] time = 0.11, antiderivative size = 78, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1251, 895, 724, 206, 843, 621, 204}

$$-\tan^{-1}\left(\frac{1-x^2}{\sqrt{x^4+6x^2+1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x^2+3}{\sqrt{x^4+6x^2+1}}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{3x^2+1}{\sqrt{x^4+6x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 6*x^2 + x^4]/(x*(1 + x^2)),x]
```

```
[Out] -ArcTan[(1 - x^2)/Sqrt[1 + 6*x^2 + x^4]] + ArcTanh[(3 + x^2)/Sqrt[1 + 6*x^2
 + x^4]]/2 - ArcTanh[(1 + 3*x^2)/Sqrt[1 + 6*x^2 + x^4]]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
```

```
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+6x^2+x^4}}{x\left(1+x^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{1+6x+x^2}}{x(1+x)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+6x+x^2}}\,dx,x,x^2\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{-5-x}{(1+x)\sqrt{1+6x+x^2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+6x+x^2}}\,dx,x,x^2\right)+2\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{1+6x+x^2}}\,dx,x,x^2\right)-\text{Sub}\\
&= -\frac{1}{2}\tanh^{-1}\left(\frac{1+3x^2}{\sqrt{1+6x^2+x^4}}\right)-4\,\text{Subst}\left(\int \frac{1}{-16-x^2}\,dx,x,\frac{4\left(-1+x^2\right)}{\sqrt{1+6x^2+x^4}}\right)+\text{Subst}\left(\int\right.\\
&= -\tan^{-1}\left(\frac{1-x^2}{\sqrt{1+6x^2+x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{3+x^2}{\sqrt{1+6x^2+x^4}}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{1+3x^2}{\sqrt{1+6x^2+x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 89, normalized size = 1.07

$$-\tan^{-1}\left(\frac{4-4x^2}{4\sqrt{x^4+6x^2+1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{2x^2+6}{2\sqrt{x^4+6x^2+1}}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{6x^2+2}{2\sqrt{x^4+6x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 6*x^2 + x^4]/(x*(1 + x^2)),x]
```

```
[Out] -ArcTan[(4 - 4*x^2)/(4*Sqrt[1 + 6*x^2 + x^4])] + ArcTanh[(6 + 2*x^2)/(2*Sqr
t[1 + 6*x^2 + x^4])]/2 - ArcTanh[(2 + 6*x^2)/(2*Sqrt[1 + 6*x^2 + x^4])]/2
```

IntegrateAlgebraic [A] time = 0.18, size = 83, normalized size = 1.00

$$-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+6x^2+1}+1\right)-2\tan^{-1}\left(\frac{x^2}{2}-\frac{1}{2}\sqrt{x^4+6x^2+1}+\frac{1}{2}\right)-\tanh^{-1}\left(x^2-\sqrt{x^4+6x^2+1}+2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + 6*x^2 + x^4]/(x*(1 + x^2)),x]
```

```
[Out] -2*ArcTan[1/2 + x^2/2 - Sqrt[1 + 6*x^2 + x^4]/2] - ArcTanh[2 + x^2 - Sqrt[1
 + 6*x^2 + x^4]] - Log[1 - x^2 + Sqrt[1 + 6*x^2 + x^4]]/2
```

fricas [A] time = 0.40, size = 79, normalized size = 0.95

$$2\arctan\left(-\frac{1}{2}x^2+\frac{1}{2}\sqrt{x^4+6x^2+1}-\frac{1}{2}\right)-\frac{1}{2}\log\left(x^4+4x^2-\sqrt{x^4+6x^2+1}\left(x^2+1\right)-1\right)+\frac{1}{2}\log\left(-x^2+\sqrt{x^4+6x^2+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^2+1)^(1/2)/x/(x^2+1),x, algorithm="fricas")
```

```
[Out] 2*arctan(-1/2*x^2 + 1/2*sqrt(x^4 + 6*x^2 + 1) - 1/2) - 1/2*log(x^4 + 4*x^2
- sqrt(x^4 + 6*x^2 + 1)*(x^2 + 1) - 1) + 1/2*log(-x^2 + sqrt(x^4 + 6*x^2 +
1) - 1)
```

giac [A] time = 0.18, size = 91, normalized size = 1.10

$$2 \arctan\left(-\frac{1}{2} x^2 + \frac{1}{2} \sqrt{x^4 + 6 x^2 + 1} - \frac{1}{2}\right) - \frac{1}{2} \log\left(x^2 - \sqrt{x^4 + 6 x^2 + 1} + 3\right) - \frac{1}{2} \log\left(-x^2 + \sqrt{x^4 + 6 x^2 + 1} + 1\right) + \frac{1}{2} \log\left(-x^2 + \sqrt{x^4 + 6 x^2 + 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^2+1)^(1/2)/x/(x^2+1),x, algorithm="giac")
```

```
[Out] 2*arctan(-1/2*x^2 + 1/2*sqrt(x^4 + 6*x^2 + 1) - 1/2) - 1/2*log(x^2 - sqrt(x
^4 + 6*x^2 + 1) + 3) - 1/2*log(-x^2 + sqrt(x^4 + 6*x^2 + 1) + 1) + 1/2*log(
-x^2 + sqrt(x^4 + 6*x^2 + 1) - 1)
```

maple [A] time = 0.03, size = 125, normalized size = 1.51

$$-\frac{\sqrt{\left(x^2+1\right)^2+4x^2}}{2} - \ln\left(3+x^2+\sqrt{\left(x^2+1\right)^2+4x^2}\right) + \arctan\left(\frac{4x^2-4}{4\sqrt{\left(x^2+1\right)^2+4x^2}}\right) + \frac{\sqrt{x^4+6x^2+1}}{2} + \frac{3\ln\left(x^2+3+\sqrt{x^4+6x^2+1}\right)}{2} - \frac{\operatorname{arctanh}\left(\frac{6x^2+2}{2\sqrt{x^4+6x^2+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+6*x^2+1)^(1/2)/x/(x^2+1),x)
```

```
[Out] -1/2*((x^2+1)^2+4*x^2)^(1/2)-ln(3+x^2+((x^2+1)^2+4*x^2)^(1/2))+arctan(1/4*(
4*x^2-4)/((x^2+1)^2+4*x^2)^(1/2))+1/2*(x^4+6*x^2+1)^(1/2)+3/2*ln(x^2+3+(x^4
+6*x^2+1)^(1/2))-1/2*arctanh(1/2*(6*x^2+2)/(x^4+6*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4 + 6 x^2 + 1}}{\left(x^2 + 1\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^2+1)^(1/2)/x/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 6*x^2 + 1)/((x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^4 + 6 x^2 + 1}}{x \left(x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x^2 + x^4 + 1)^(1/2)/(x*(x^2 + 1)),x)
```

```
[Out] int((6*x^2 + x^4 + 1)^(1/2)/(x*(x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4 + 6x^2 + 1}}{x \left(x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+6*x**2+1)**(1/2)/x/(x**2+1),x)
```

```
[Out] Integral(sqrt(x**4 + 6*x**2 + 1)/(x*(x**2 + 1)), x)
```

3.998 $\int \frac{(-1+x^2)\sqrt[4]{-1+2x^2+2x^4}}{x^2(-1+2x^2)}\,dx$

Optimal. Leaf size=83

$$-\frac{\sqrt[4]{2x^4+2x^2-1}}{x}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{2x^4+2x^2-1}}\right)}{2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{2x^4+2x^2-1}}\right)}{2^{3/4}}$$

Rubi [F] time = 0.51, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^2)\sqrt[4]{-1+2x^2+2x^4}}{x^2(-1+2x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^2)*(-1 + 2*x^2 + 2*x^4)^(1/4))/(x^2*(-1 + 2*x^2)),x]
```

```
[Out] -(((-1 + 2*x^2 + 2*x^4)^(1/4)*AppellF1[-1/2, -1/4, -1/4, 1/2, (-2*x^2)/(1 -
 Sqrt[3]), (-2*x^2)/(1 + Sqrt[3])])/(x*(1 + (2*x^2)/(1 - Sqrt[3]))^(1/4)*(1
 + (2*x^2)/(1 + Sqrt[3]))^(1/4))) + Defer[Int][(-1 + 2*x^2 + 2*x^4)^(1/4)/(
1 - 2*x^2), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^2)\sqrt[4]{-1+2x^2+2x^4}}{x^2(-1+2x^2)}\,dx &= \int \left(\frac{\sqrt[4]{-1+2x^2+2x^4}}{x^2}+\frac{\sqrt[4]{-1+2x^2+2x^4}}{1-2x^2}\right) dx\\
&= \int \frac{\sqrt[4]{-1+2x^2+2x^4}}{x^2}\,dx+\int \frac{\sqrt[4]{-1+2x^2+2x^4}}{1-2x^2}\,dx\\
&= \frac{\sqrt[4]{-1+2x^2+2x^4}\int \frac{\sqrt[4]{1+\frac{4x^2}{2-2\sqrt{3}}}\sqrt[4]{1+\frac{4x^2}{2+2\sqrt{3}}}}{x^2}\,dx}{\sqrt[4]{1+\frac{4x^2}{2-2\sqrt{3}}}\sqrt[4]{1+\frac{4x^2}{2+2\sqrt{3}}}}+\int \frac{\sqrt[4]{-1+2x^2+2x^4}}{1-2x^2}\,dx\\
&= -\frac{\sqrt[4]{-1+2x^2+2x^4}\,F_1\left(-\frac{1}{2};-\frac{1}{4},-\frac{1}{4};\frac{1}{2};-\frac{2x^2}{1-\sqrt{3}},-\frac{2x^2}{1+\sqrt{3}}\right)}{x\sqrt[4]{1+\frac{2x^2}{1-\sqrt{3}}}\sqrt[4]{1+\frac{2x^2}{1+\sqrt{3}}}}+\int \frac{\sqrt[4]{-1+2x^2+2x^4}}{1-2x^2}\,dx
\end{aligned}$$

Mathematica [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^2)\sqrt[4]{-1+2x^2+2x^4}}{x^2(-1+2x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^2)*(-1 + 2*x^2 + 2*x^4)^(1/4))/(x^2*(-1 + 2*x^2)),x]
```

```
[Out] Integrate[((-1 + x^2)*(-1 + 2*x^2 + 2*x^4)^(1/4))/(x^2*(-1 + 2*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.25, size = 83, normalized size = 1.00

$$-\frac{\sqrt[4]{2x^4+2x^2-1}}{x}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{2x^4+2x^2-1}}\right)}{2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{2x^4+2x^2-1}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*(-1 + 2*x^2 + 2*x^4)^(1/4))/(x^2*(-1 + 2*x^2
)),x]
```

```
[Out] -((-1 + 2*x^2 + 2*x^4)^(1/4)/x) - ArcTan[(2^(1/4)*x)/(-1 + 2*x^2 + 2*x^4)^(
1/4)]/2^(3/4) + ArcTanh[(2^(1/4)*x)/(-1 + 2*x^2 + 2*x^4)^(1/4)]/2^(3/4)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(2*x^4+2*x^2-1)^(1/4)/x^2/(2*x^2-1),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^4+2\,x^2-1\right)^{\frac{1}{4}}\left(x^2-1\right)}{\left(2\,x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(2*x^4+2*x^2-1)^(1/4)/x^2/(2*x^2-1),x, algorithm="giac")
```

```
[Out] integrate((2*x^4 + 2*x^2 - 1)^(1/4)*(x^2 - 1)/((2*x^2 - 1)*x^2), x)
```

maple [C] time = 111.57, size = 1059, normalized size = 12.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(2*x^4+2*x^2-1)^(1/4)/x^2/(2*x^2-1),x)
```

```
[Out] -(2*x^4+2*x^2-1)^(1/4)/x+(1/4*RootOf(_Z^2+RootOf(_Z^4-2)^2)*ln(-(16*x^12*Ro
otOf(_Z^4-2)^2+8*(8*x^12+24*x^10+12*x^8-16*x^6-6*x^4+6*x^2-1)^(1/4)*RootOf(
_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2*x^9+40*x^10*RootOf(_Z^4-2)^2+16*(8*
x^12+24*x^10+12*x^8-16*x^6-6*x^4+6*x^2-1)^(1/4)*RootOf(_Z^2+RootOf(_Z^4-2)^
2)*RootOf(_Z^4-2)^2*x^7+12*x^8*RootOf(_Z^4-2)^2-24*RootOf(_Z^4-2)^2*x^6-8*(
8*x^12+24*x^10+12*x^8-16*x^6-6*x^4+6*x^2-1)^(1/2)*x^6-8*(8*x^12+24*x^10+12*
x^8-16*x^6-6*x^4+6*x^2-1)^(1/4)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2
)^2*x^3-4*RootOf(_Z^2+RootOf(_Z^4-2)^2)*(8*x^12+24*x^10+12*x^8-16*x^6-6*x^4
+6*x^2-1)^(3/4)*x^3-4*RootOf(_Z^4-2)^2*x^4-8*(8*x^12+24*x^10+12*x^8-16*x^6-
6*x^4+6*x^2-1)^(1/2)*x^4+2*(8*x^12+24*x^10+12*x^8-16*x^6-6*x^4+6*x^2-1)^(1/
4)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2*x+6*RootOf(_Z^4-2)^2*x^2+
4*(8*x^12+24*x^10+12*x^8-16*x^6-6*x^4+6*x^2-1)^(1/2)*x^2-RootOf(_Z^4-2)^2)/
(2*x^2-1)/(2*x^4+2*x^2-1)^2)-1/4*RootOf(_Z^4-2)*ln(-(-16*x^12*RootOf(_Z^4-2
)^2+8*RootOf(_Z^4-2)^3*(8*x^12+24*x^10+12*x^8-16*x^6-6*x^4+6*x^2-1)^(1/4)*x
^9-40*x^10*RootOf(_Z^4-2)^2+16*RootOf(_Z^4-2)^3*(8*x^12+24*x^10+12*x^8-16*x
^6-6*x^4+6*x^2-1)^(1/4)*x^7-12*x^8*RootOf(_Z^4-2)^2+24*RootOf(_Z^4-2)^2*x^6
-8*(8*x^12+24*x^10+12*x^8-16*x^6-6*x^4+6*x^2-1)^(1/2)*x^6-8*RootOf(_Z^4-2)^
3*(8*x^12+24*x^10+12*x^8-16*x^6-6*x^4+6*x^2-1)^(1/4)*x^3+4*RootOf(_Z^4-2)*(
8*x^12+24*x^10+12*x^8-16*x^6-6*x^4+6*x^2-1)^(3/4)*x^3+4*RootOf(_Z^4-2)^2*x^
4-8*(8*x^12+24*x^10+12*x^8-16*x^6-6*x^4+6*x^2-1)^(1/2)*x^4+2*RootOf(_Z^4-2)
^3*(8*x^12+24*x^10+12*x^8-16*x^6-6*x^4+6*x^2-1)^(1/4)*x-6*RootOf(_Z^4-2)^2*
x^2+4*(8*x^12+24*x^10+12*x^8-16*x^6-6*x^4+6*x^2-1)^(1/2)*x^2+RootOf(_Z^4-2)
```

```
^2)/(2*x^2-1)/(2*x^4+2*x^2-1)^2))/(2*x^4+2*x^2-1)^(3/4)*((2*x^4+2*x^2-1)^3)
^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{4}+2\,x^{2}-1\right)^{\frac{1}{4}}\left(x^{2}-1\right)}{\left(2\,x^{2}-1\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(2*x^4+2*x^2-1)^(1/4)/x^2/(2*x^2-1),x, algorithm="maxima"
)
```

```
[Out] integrate((2*x^4 + 2*x^2 - 1)^(1/4)*(x^2 - 1)/((2*x^2 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^{2}-1\right)\,\left(2\,x^{4}+2\,x^{2}-1\right)^{1/4}}{x^{2}\,\left(2\,x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*(2*x^2 + 2*x^4 - 1)^(1/4))/(x^2*(2*x^2 - 1)),x)
```

```
[Out] int(((x^2 - 1)*(2*x^2 + 2*x^4 - 1)^(1/4))/(x^2*(2*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)\,\sqrt[4]{2x^{4}+2x^{2}-1}}{x^{2}\left(2x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(2*x**4+2*x**2-1)**(1/4)/x**2/(2*x**2-1),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(2*x**4 + 2*x**2 - 1)**(1/4)/(x**2*(2*x**2 - 1)),
x)
```

3.999 $\int \frac{1}{\sqrt[4]{b+ax^4}\left(2b+ax^4\right)}\,dx$

Optimal. Leaf size=83

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{2}\,\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}\sqrt[4]{a}\,b}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{2}\,\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}\sqrt[4]{a}\,b}$$

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {377, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{2}\,\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}\sqrt[4]{a}\,b}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{2}\,\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}\sqrt[4]{a}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b + a*x^4)^(1/4)*(2*b + a*x^4)),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/(2^(1/4)*(b + a*x^4)^(1/4))]/(2*2^(3/4)*a^(1/4)*b) + Arc
Tanh[(a^(1/4)*x)/(2^(1/4)*(b + a*x^4)^(1/4))]/(2*2^(3/4)*a^(1/4)*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{b+ax^4}\left(2b+ax^4\right)}\,dx = \text{Subst}\left(\int \frac{1}{2b-abx^4}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{2\sqrt{2}\,b} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}+\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{2\sqrt{2}\,b}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{2}\,\sqrt[4]{b+ax^4}}\right)}{2\,2^{3/4}\sqrt[4]{a}\,b} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{2}\,\sqrt[4]{b+ax^4}}\right)}{2\,2^{3/4}\sqrt[4]{a}\,b}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.80

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{2}\,\sqrt[4]{ax^4+b}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{2}\,\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}\sqrt[4]{a}\,b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b + a*x^4)^(1/4)*(2*b + a*x^4)),x]
[Out] (ArcTan[(a^(1/4)*x)/(2^(1/4)*(b + a*x^4)^(1/4))] + ArcTanh[(a^(1/4)*x)/(2^(
1/4)*(b + a*x^4)^(1/4))])/(2*2^(3/4)*a^(1/4)*b)
```

IntegrateAlgebraic [A] time = 0.35, size = 83, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{2}\,\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}\sqrt[4]{a}\,b}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{2}\,\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}\sqrt[4]{a}\,b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((b + a*x^4)^(1/4)*(2*b + a*x^4)),x]
[Out] ArcTan[(a^(1/4)*x)/(2^(1/4)*(b + a*x^4)^(1/4))]/(2*2^(3/4)*a^(1/4)*b) + Arc
Tanh[(a^(1/4)*x)/(2^(1/4)*(b + a*x^4)^(1/4))]/(2*2^(3/4)*a^(1/4)*b)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+b)^(1/4)/(a*x^4+2*b),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^4+2\,b\right)\left(ax^4+b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+b)^(1/4)/(a*x^4+2*b),x, algorithm="giac")
[Out] integrate(1/((a*x^4 + 2*b)*(a*x^4 + b)^(1/4)), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,x^{4}+b\right)^{\frac{1}{4}} \left(a\,x^{4}+2b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^4+b)^(1/4)/(a*x^4+2*b),x)
```

```
[Out] int(1/(a*x^4+b)^(1/4)/(a*x^4+2*b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^{4}+2\,b\right)\left(ax^{4}+b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+b)^(1/4)/(a*x^4+2*b),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x^4 + 2*b)*(a*x^4 + b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\,x^4+b\right)^{1/4}\,\left(a\,x^4+2\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b + a*x^4)^(1/4)*(2*b + a*x^4)),x)
```

```
[Out] int(1/((b + a*x^4)^(1/4)*(2*b + a*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ax^{4}+b}\,\left(ax^{4}+2b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**4+b)**(1/4)/(a*x**4+2*b),x)
```

```
[Out] Integral(1/((a*x**4 + b)**(1/4)*(a*x**4 + 2*b)), x)
```

3.1000 $\int \frac{\sqrt[4]{-bx^2+ax^4}}{x^2}\,dx$

Optimal. Leaf size=83

$$-\frac{2\sqrt[4]{ax^4-bx^2}}{x}-\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)+\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)$$

Rubi [A] time = 0.17, antiderivative size = 153, normalized size of antiderivative = 1.84, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2020, 2032, 329, 331, 298, 203, 206}

$$-\frac{2\sqrt[4]{ax^4-bx^2}}{x}-\frac{\sqrt[4]{a}\,x^{3/2}\left(ax^2-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2-b}}\right)}{\left(ax^4-bx^2\right)^{3/4}}+\frac{\sqrt[4]{a}\,x^{3/2}\left(ax^2-b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2-b}}\right)}{\left(ax^4-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-(b*x^2) + a*x^4)^(1/4)/x^2,x]
```

```
[Out] (-2*(-(b*x^2) + a*x^4)^(1/4))/x - (a^(1/4)*x^(3/2)*(-b + a*x^2)^(3/4)*ArcTa
n[(a^(1/4)*Sqrt[x])/(-b + a*x^2)^(1/4)])/(-(b*x^2) + a*x^4)^(3/4) + (a^(1/4
)*x^(3/2)*(-b + a*x^2)^(3/4)*ArcTanh[(a^(1/4)*Sqrt[x])/(-b + a*x^2)^(1/4)])
/(-(b*x^2) + a*x^4)^(3/4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt[4]{-bx^2+ax^4}}{x^2}\,dx = -\frac{2\sqrt[4]{-bx^2+ax^4}}{x} + a\int \frac{x^2}{\left(-bx^2+ax^4\right)^{3/4}}\,dx$$
$$= -\frac{2\sqrt[4]{-bx^2+ax^4}}{x} + \frac{\left(ax^{3/2}\left(-b+ax^2\right)^{3/4}\right)\int \frac{\sqrt{x}}{\left(-b+ax^2\right)^{3/4}}\,dx}{\left(-bx^2+ax^4\right)^{3/4}}$$
$$= -\frac{2\sqrt[4]{-bx^2+ax^4}}{x} + \frac{\left(2ax^{3/2}\left(-b+ax^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{\left(-b+ax^4\right)^{3/4}}\,dx, x, \sqrt{x}\right)}{\left(-bx^2+ax^4\right)^{3/4}}$$
$$= -\frac{2\sqrt[4]{-bx^2+ax^4}}{x} + \frac{\left(2ax^{3/2}\left(-b+ax^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{-b+ax^2}}\right)}{\left(-bx^2+ax^4\right)^{3/4}}$$
$$= -\frac{2\sqrt[4]{-bx^2+ax^4}}{x} + \frac{\left(\sqrt{a}\,x^{3/2}\left(-b+ax^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{-b+ax^2}}\right)}{\left(-bx^2+ax^4\right)^{3/4}} - \frac{\left(\sqrt{a}\,x^{3/2}\right.}{}$$
$$= -\frac{2\sqrt[4]{-bx^2+ax^4}}{x} - \frac{\sqrt[4]{a}\,x^{3/2}\left(-b+ax^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{-b+ax^2}}\right)}{\left(-bx^2+ax^4\right)^{3/4}} + \frac{\sqrt[4]{a}\,x^{3/2}\left(-b+ax^2\right)^{3/4}\tanh}{\left(-bx^2+ax^4\right)^{3/}}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.65

$$-\frac{2\sqrt[4]{ax^4-bx^2}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};\frac{ax^2}{b}\right)}{x\sqrt[4]{1-\frac{ax^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-(b*x^2) + a*x^4)^(1/4)/x^2,x]
```

```
[Out] (-2*(-(b*x^2) + a*x^4)^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, (a*x^2)/b])
/(x*(1 - (a*x^2)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.21, size = 83, normalized size = 1.00

$$-\frac{2\sqrt[4]{ax^4-bx^2}}{x} - \sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right) + \sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(b*x^2) + a*x^4)^(1/4)/x^2,x]
```

```
[Out] (-2*(-(b*x^2) + a*x^4)^(1/4))/x - a^(1/4)*ArcTan[(a^(1/4)*x)/(-(b*x^2) + a*
x^4)^(1/4)] + a^(1/4)*ArcTanh[(a^(1/4)*x)/(-(b*x^2) + a*x^4)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x^2)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.19, size = 192, normalized size = 2.31

$$\frac{1}{2}\sqrt{2}\,(-a)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)+\frac{1}{2}\sqrt{2}\,(-a)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)+\frac{1}{4}\sqrt{2}\,(-a)^{\frac{1}{4}}\log\left(\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^2}}\right)-\frac{1}{4}\sqrt{2}\,(-a)^{\frac{1}{4}}\log\left(-\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^2}}\right)-2\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x^2)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(-a)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a - b/x^
2)^(1/4))/(-a)^(1/4)) + 1/2*sqrt(2)*(-a)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(-a)^(1/4) - 2*(a - b/x^2)^(1/4))/(-a)^(1/4)) + 1/4*sqrt(2)*(-a)^(1/4)*log
(sqrt(2)*(-a)^(1/4)*(a - b/x^2)^(1/4) + sqrt(-a) + sqrt(a - b/x^2)) - 1/4*s
qrt(2)*(-a)^(1/4)*log(-sqrt(2)*(-a)^(1/4)*(a - b/x^2)^(1/4) + sqrt(-a) + sq
rt(a - b/x^2)) - 2*(a - b/x^2)^(1/4)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{4}-b\,x^{2}\right)^{\frac{1}{4}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b*x^2)^(1/4)/x^2,x)
```

```
[Out] int((a*x^4-b*x^2)^(1/4)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{4}-bx^{2}\right)^{\frac{1}{4}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x^2)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 - b*x^2)^(1/4)/x^2, x)
```

mupad [B] time = 1.03, size = 45, normalized size = 0.54

$$-\frac{2\left(a\,x^4-b\,x^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ \frac{a\,x^2}{b}\right)}{x\left(1-\frac{a\,x^2}{b}\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4 - b*x^2)^(1/4)/x^2,x)
```

```
[Out] -(2*(a*x^4 - b*x^2)^(1/4)*hypergeom([-1/4, -1/4], 3/4, (a*x^2)/b))/(x*(1 -
(a*x^2)/b)^(1/4))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\left(ax^2-b\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b*x**2)**(1/4)/x**2,x)
```

```
[Out] Integral((x**2*(a*x**2 - b))**(1/4)/x**2, x)
```

3.1001 $\int \frac{(-1+x)\sqrt{x+\sqrt{1+x}}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=83

$$\frac{1}{96}\sqrt{x+\sqrt{x+1}}\,(8x-21)+\frac{1}{48}\sqrt{x+1}\,(24x-89)\sqrt{x+\sqrt{x+1}}-\frac{115}{64}\log\left(2\sqrt{x+1}-2\sqrt{x+\sqrt{x+1}}+1\right)$$

Rubi [A] time = 0.23, antiderivative size = 103, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1661, 640, 612, 621, 206}

$$\frac{1}{2}\sqrt{x+1}\left(x+\sqrt{x+1}\right)^{3/2}-\frac{5}{12}\left(x+\sqrt{x+1}\right)^{3/2}-\frac{23}{32}\left(2\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}}+\frac{115}{64}\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)*Sqrt[x + Sqrt[1 + x]])/Sqrt[1 + x],x]
```

```
[Out] (-5*(x + Sqrt[1 + x])^(3/2))/12 + (Sqrt[1 + x]*(x + Sqrt[1 + x])^(3/2))/2 -
 (23*Sqrt[x + Sqrt[1 + x]]*(1 + 2*Sqrt[1 + x]))/32 + (115*ArcTanh[(1 + 2*Sq
rt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/64
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(-1+x)\sqrt{x+\sqrt{1+x}}}{\sqrt{1+x}}\,dx = 2\,\text{Subst}\left(\int \left(-2+x^2\right)\sqrt{-1+x+x^2}\,dx, x, \sqrt{1+x}\right)$$
$$= \frac{1}{2}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{2}\,\text{Subst}\left(\int \left(-7-\frac{5x}{2}\right)\sqrt{-1+x+x^2}\,dx, x, \sqrt{1+\right.$$
$$= -\frac{5}{12}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{2}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{23}{8}\,\text{Subst}\left(\int \sqrt{-1+x+x\right.$$
$$= -\frac{5}{12}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{2}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{23}{32}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1\right.$$
$$= -\frac{5}{12}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{2}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{23}{32}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1\right.$$
$$= -\frac{5}{12}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{2}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{23}{32}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1\right.$$

Mathematica [A] time = 0.08, size = 76, normalized size = 0.92

$$\frac{1}{192}\left(2\sqrt{x+\sqrt{x+1}}\left(8x\left(6\sqrt{x+1}+1\right)-178\sqrt{x+1}-21\right)+345\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)*Sqrt[x + Sqrt[1 + x]])/Sqrt[1 + x],x]
```

```
[Out] (2*Sqrt[x + Sqrt[1 + x]]*(-21 - 178*Sqrt[1 + x] + 8*x*(1 + 6*Sqrt[1 + x]))
+ 345*ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/192
```

IntegrateAlgebraic [A] time = 0.18, size = 74, normalized size = 0.89

$$\frac{1}{96}\sqrt{x+\sqrt{x+1}}\left(48(x+1)^{3/2}+8(x+1)-226\sqrt{x+1}-29\right)-\frac{115}{64}\log\left(-2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)*Sqrt[x + Sqrt[1 + x]])/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[x + Sqrt[1 + x]]*(-29 - 226*Sqrt[1 + x] + 8*(1 + x) + 48*(1 + x)^(3/2
)))/96 - (115*Log[-1 - 2*Sqrt[1 + x] + 2*Sqrt[x + Sqrt[1 + x]]])/64
```

fricas [A] time = 1.63, size = 64, normalized size = 0.77

$$\frac{1}{96}\left(2\,(24\,x-89)\sqrt{x+1}+8\,x-21\right)\sqrt{x+\sqrt{x+1}}+\frac{115}{128}\log\left(4\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)+8\,x+8\sqrt{x+1}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(x+(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(2*(24*x - 89)*sqrt(x + 1) + 8*x - 21)*sqrt(x + sqrt(x + 1)) + 115/128
*log(4*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) + 1) + 8*x + 8*sqrt(x + 1) + 5)
```

giac [A] time = 0.35, size = 82, normalized size = 0.99

$$\frac{1}{96}\left(2\left(4\sqrt{x+1}\left(6\sqrt{x+1}+1\right)-65\right)\sqrt{x+1}+19\right)\sqrt{x+\sqrt{x+1}}-\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)-\frac{115}{64}\log\left(-2\sqrt{x+\sqrt{x+1}}+2\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(x+(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*(4*sqrt(x + 1)*(6*sqrt(x + 1) + 1) - 65)*sqrt(x + 1) + 19)*sqrt(x +
 sqrt(x + 1)) - 1/2*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) + 1) - 115/64*log(
-2*sqrt(x + sqrt(x + 1)) + 2*sqrt(x + 1) + 1)
```

maple [A] time = 0.01, size = 68, normalized size = 0.82

$$\frac{\sqrt{1+x}\,\left(x+\sqrt{1+x}\right)^{\frac{3}{2}}}{2}-\frac{5\left(x+\sqrt{1+x}\right)^{\frac{3}{2}}}{12}-\frac{23\left(1+2\sqrt{1+x}\right)\sqrt{x+\sqrt{1+x}}}{32}+\frac{115\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)*(x+(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x)
```

```
[Out] 1/2*(1+x)^(1/2)*(x+(1+x)^(1/2))^(3/2)-5/12*(x+(1+x)^(1/2))^(3/2)-23/32*(1+2
*(1+x)^(1/2))*(x+(1+x)^(1/2))^(1/2)+115/64*ln(1/2+(1+x)^(1/2)+(x+(1+x)^(1/2
))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x+1}}\,(x-1)}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(x+(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1))*(x - 1)/sqrt(x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x+\sqrt{x+1}}\,\,(x-1)}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + (x + 1)^(1/2))^(1/2)*(x - 1))/(x + 1)^(1/2),x)
```

```
[Out] int(((x + (x + 1)^(1/2))^(1/2)*(x - 1))/(x + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(x+(1+x)**(1/2))**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Integral((x - 1)*sqrt(x + sqrt(x + 1))/sqrt(x + 1), x)
```

3.1002 $\int \frac{x\sqrt{x+\sqrt{1+x}}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=83

$$\frac{1}{96}\sqrt{x+\sqrt{x+1}}\,(8x+27)+\frac{1}{48}\sqrt{x+1}\,(24x-41)\sqrt{x+\sqrt{x+1}}-\frac{35}{64}\log\left(2\sqrt{x+1}-2\sqrt{x+\sqrt{x+1}}+1\right)$$

Rubi [A] time = 0.21, antiderivative size = 103, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1661, 640, 612, 621, 206}

$$\frac{1}{2}\sqrt{x+1}\left(x+\sqrt{x+1}\right)^{3/2}-\frac{5}{12}\left(x+\sqrt{x+1}\right)^{3/2}-\frac{7}{32}\left(2\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}}+\frac{35}{64}\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[x + Sqrt[1 + x]])/Sqrt[1 + x],x]
```

```
[Out] (-5*(x + Sqrt[1 + x])^(3/2))/12 + (Sqrt[1 + x]*(x + Sqrt[1 + x])^(3/2))/2 -
 (7*Sqrt[x + Sqrt[1 + x]]*(1 + 2*Sqrt[1 + x]))/32 + (35*ArcTanh[(1 + 2*Sqrt
[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/64
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x\sqrt{x+\sqrt{1+x}}}{\sqrt{1+x}}\,dx = 2\,\text{Subst}\left(\int \left(-1+x^2\right)\sqrt{-1+x+x^2}\,dx, x, \sqrt{1+x}\right)$$

$$= \frac{1}{2}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{2}\text{Subst}\left(\int \left(-3-\frac{5x}{2}\right)\sqrt{-1+x+x^2}\,dx, x, \sqrt{1+x}\right)$$

$$= -\frac{5}{12}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{2}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{7}{8}\text{Subst}\left(\int \sqrt{-1+x+x^2}\,dx, x, \sqrt{1\right.$$

$$= -\frac{5}{12}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{2}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{7}{32}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) + \frac{3}{6}$$

$$= -\frac{5}{12}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{2}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{7}{32}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) + \frac{3}{3}$$

$$= -\frac{5}{12}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{2}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{7}{32}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) + \frac{3}{6}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 0.92

$$\frac{1}{96}\sqrt{x+\sqrt{x+1}}\left(8x\left(6\sqrt{x+1}+1\right)-82\sqrt{x+1}+27\right)+\frac{35}{64}\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[x + Sqrt[1 + x]])/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[x + Sqrt[1 + x]]*(27 - 82*Sqrt[1 + x] + 8*x*(1 + 6*Sqrt[1 + x])))/96
+ (35*ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/64
```

IntegrateAlgebraic [A] time = 0.21, size = 74, normalized size = 0.89

$$\frac{1}{96}\sqrt{x+\sqrt{x+1}}\left(48(x+1)^{3/2}+8(x+1)-130\sqrt{x+1}+19\right)-\frac{35}{64}\log\left(-2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[x + Sqrt[1 + x]])/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[x + Sqrt[1 + x]]*(19 - 130*Sqrt[1 + x] + 8*(1 + x) + 48*(1 + x)^(3/2)
))/96 - (35*Log[-1 - 2*Sqrt[1 + x] + 2*Sqrt[x + Sqrt[1 + x]]])/64
```

fricas [A] time = 1.72, size = 64, normalized size = 0.77

$$\frac{1}{96}\left(2\,(24\,x-41)\sqrt{x+1}+8\,x+27\right)\sqrt{x+\sqrt{x+1}}+\frac{35}{128}\log\left(4\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)+8\,x+8\sqrt{x+1}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(2*(24*x - 41)*sqrt(x + 1) + 8*x + 27)*sqrt(x + sqrt(x + 1)) + 35/128*
log(4*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) + 1) + 8*x + 8*sqrt(x + 1) + 5)
```

giac [A] time = 0.15, size = 62, normalized size = 0.75

$$\frac{1}{96}\left(2\left(4\sqrt{x+1}\left(6\sqrt{x+1}+1\right)-65\right)\sqrt{x+1}+19\right)\sqrt{x+\sqrt{x+1}}-\frac{35}{64}\log\left(-2\sqrt{x+\sqrt{x+1}}+2\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*(4*sqrt(x + 1)*(6*sqrt(x + 1) + 1) - 65)*sqrt(x + 1) + 19)*sqrt(x +
 sqrt(x + 1)) - 35/64*log(-2*sqrt(x + sqrt(x + 1)) + 2*sqrt(x + 1) + 1)
```

maple [A] time = 0.01, size = 68, normalized size = 0.82

$$\frac{\sqrt{1+x}\,\left(x+\sqrt{1+x}\right)^{\frac{3}{2}}}{2}-\frac{5\left(x+\sqrt{1+x}\right)^{\frac{3}{2}}}{12}-\frac{7\left(1+2\sqrt{1+x}\right)\sqrt{x+\sqrt{1+x}}}{32}+\frac{35\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x+(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x)
```

```
[Out] 1/2*(1+x)^(1/2)*(x+(1+x)^(1/2))^(3/2)-5/12*(x+(1+x)^(1/2))^(3/2)-7/32*(1+2*
(1+x)^(1/2))*(x+(1+x)^(1/2))^(1/2)+35/64*ln(1/2+(1+x)^(1/2)+(x+(1+x)^(1/2))
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x+1}}\,x}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1))*x/sqrt(x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + (x + 1)^(1/2))^(1/2))/(x + 1)^(1/2),x)
```

```
[Out] int((x*(x + (x + 1)^(1/2))^(1/2))/(x + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+(1+x)**(1/2))**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x + sqrt(x + 1))/sqrt(x + 1), x)
```

3.1003 $\int \sqrt{1+x}\sqrt{x+\sqrt{1+x}}\,dx$

Optimal. Leaf size=83

$$\frac{1}{96}\sqrt{x+\sqrt{x+1}}\,(8x+75)+\frac{1}{48}\sqrt{x+1}\,(24x+7)\sqrt{x+\sqrt{x+1}}+\frac{45}{64}\log\left(2\sqrt{x+1}-2\sqrt{x+\sqrt{x+1}}+1\right)$$

Rubi [A] time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {742, 640, 612, 621, 206}

$$\frac{1}{2}\sqrt{x+1}\left(x+\sqrt{x+1}\right)^{3/2}-\frac{5}{12}\left(x+\sqrt{x+1}\right)^{3/2}+\frac{9}{32}\left(2\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}}-\frac{45}{64}\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]],x]
```

```
[Out] (-5*(x + Sqrt[1 + x])^(3/2))/12 + (Sqrt[1 + x]*(x + Sqrt[1 + x])^(3/2))/2 +
 (9*Sqrt[x + Sqrt[1 + x]]*(1 + 2*Sqrt[1 + x]))/32 - (45*ArcTanh[(1 + 2*Sqrt
[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/64
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rubi steps

$$\int \sqrt{1+x}\sqrt{x+\sqrt{1+x}}\,dx = 2\,\text{Subst}\left(\int x^2\sqrt{-1+x+x^2}\,dx, x, \sqrt{1+x}\right)$$
$$= \frac{1}{2}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{2}\text{Subst}\left(\int\left(1-\frac{5x}{2}\right)\sqrt{-1+x+x^2}\,dx, x, \sqrt{1+x}\right)$$
$$= -\frac{5}{12}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{2}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{9}{8}\text{Subst}\left(\int \sqrt{-1+x+x^2}\right.$$
$$= -\frac{5}{12}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{2}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{9}{32}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1}\right.$$
$$= -\frac{5}{12}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{2}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{9}{32}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1}\right.$$
$$= -\frac{5}{12}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{2}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{9}{32}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1}\right.$$

Mathematica [A] time = 0.04, size = 76, normalized size = 0.92

$$\frac{1}{96}\sqrt{x+\sqrt{x+1}}\left(8x\left(6\sqrt{x+1}+1\right)+14\sqrt{x+1}+75\right)-\frac{45}{64}\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]],x]
```

```
[Out] (Sqrt[x + Sqrt[1 + x]]*(75 + 14*Sqrt[1 + x] + 8*x*(1 + 6*Sqrt[1 + x])))/96
- (45*ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/64
```

IntegrateAlgebraic [A] time = 0.13, size = 74, normalized size = 0.89

$$\frac{1}{96}\sqrt{x+\sqrt{x+1}}\left(48(x+1)^{3/2}+8(x+1)-34\sqrt{x+1}+67\right)+\frac{45}{64}\log\left(-2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]],x]
```

```
[Out] (Sqrt[x + Sqrt[1 + x]]*(67 - 34*Sqrt[1 + x] + 8*(1 + x) + 48*(1 + x)^(3/2))
)/96 + (45*Log[-1 - 2*Sqrt[1 + x] + 2*Sqrt[x + Sqrt[1 + x]]])/64
```

fricas [A] time = 1.57, size = 64, normalized size = 0.77

$$\frac{1}{96}\left(2\,(24\,x+7)\sqrt{x+1}+8\,x+75\right)\sqrt{x+\sqrt{x+1}}+\frac{45}{128}\log\left(4\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)-8\,x-8\sqrt{x+1}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(2*(24*x + 7)*sqrt(x + 1) + 8*x + 75)*sqrt(x + sqrt(x + 1)) + 45/128*l
og(4*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) + 1) - 8*x - 8*sqrt(x + 1) - 5)
```

giac [A] time = 0.16, size = 82, normalized size = 0.99

$$\frac{1}{96}\left(2\left(4\sqrt{x+1}\left(6\sqrt{x+1}+1\right)-65\right)\sqrt{x+1}+19\right)\sqrt{x+\sqrt{x+1}}+\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)+\frac{45}{64}\log\left(-2\sqrt{x+\sqrt{x+1}}+2\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x+(1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*(4*sqrt(x + 1)*(6*sqrt(x + 1) + 1) - 65)*sqrt(x + 1) + 19)*sqrt(x +
 sqrt(x + 1)) + 1/2*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) + 1) + 45/64*log(-
2*sqrt(x + sqrt(x + 1)) + 2*sqrt(x + 1) + 1)
```

maple [A] time = 0.00, size = 68, normalized size = 0.82

$$\frac{\sqrt{1+x}\,\left(x+\sqrt{1+x}\right)^{\frac{3}{2}}}{2}-\frac{5\left(x+\sqrt{1+x}\right)^{\frac{3}{2}}}{12}+\frac{9\left(1+2\sqrt{1+x}\right)\sqrt{x+\sqrt{1+x}}}{32}-\frac{45\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)*(x+(1+x)^(1/2))^(1/2),x)
```

```
[Out] 1/2*(1+x)^(1/2)*(x+(1+x)^(1/2))^(3/2)-5/12*(x+(1+x)^(1/2))^(3/2)+9/32*(1+2*
(1+x)^(1/2))*(x+(1+x)^(1/2))^(1/2)-45/64*ln(1/2+(1+x)^(1/2)+(x+(1+x)^(1/2))
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x+\sqrt{x+1}}\,\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1))*sqrt(x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x+\sqrt{x+1}}\,\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x + 1)^(1/2))^(1/2)*(x + 1)^(1/2),x)
```

```
[Out] int((x + (x + 1)^(1/2))^(1/2)*(x + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x+1}\,\sqrt{x+\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x + 1)*sqrt(x + sqrt(x + 1)), x)
```

3.1004 $\int \frac{1+\sqrt{1+x^2}}{\sqrt{x+\sqrt{1+x^2}}} \, dx$

Optimal. Leaf size=83

$$\frac{2\sqrt{x^2+1}\left(2x^3+6x^2-2x+1\right)}{3\left(\sqrt{x^2+1}+x\right)^{5/2}}+\frac{2\left(10x^4+30x^3-5x^2+20x-7\right)}{15\left(\sqrt{x^2+1}+x\right)^{5/2}}$$

Rubi [A] time = 0.17, antiderivative size = 90, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6742, 2117, 14, 2122, 270}

$$\frac{1}{6}\left(\sqrt{x^2+1}+x\right)^{3/2}+\sqrt{\sqrt{x^2+1}+x}-\frac{1}{\sqrt{\sqrt{x^2+1}+x}}-\frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}}-\frac{1}{10\left(\sqrt{x^2+1}+x\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[1 + x^2])/Sqrt[x + Sqrt[1 + x^2]],x]
```

```
[Out] -1/10*1/(x + Sqrt[1 + x^2])^(5/2) - 1/(3*(x + Sqrt[1 + x^2])^(3/2)) - 1/Sqr
t[x + Sqrt[1 + x^2]] + Sqrt[x + Sqrt[1 + x^2]] + (x + Sqrt[1 + x^2])^(3/2)/
6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1+\sqrt{1+x^2}}{\sqrt{x+\sqrt{1+x^2}}}\,dx = \int \left(\frac{1}{\sqrt{x+\sqrt{1+x^2}}}+\frac{\sqrt{1+x^2}}{\sqrt{x+\sqrt{1+x^2}}}\right)dx$$
$$= \int \frac{1}{\sqrt{x+\sqrt{1+x^2}}}\,dx + \int \frac{\sqrt{1+x^2}}{\sqrt{x+\sqrt{1+x^2}}}\,dx$$
$$= \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^{7/2}}\,dx, x, x+\sqrt{1+x^2}\right) + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1+x^2}{x^{5/2}}\,dx, x, x+\sqrt{1+x^2}\right)$$
$$= \frac{1}{4}\,\mathrm{Subst}\left(\int \left(\frac{1}{x^{7/2}}+\frac{2}{x^{3/2}}+\sqrt{x}\right)dx, x, x+\sqrt{1+x^2}\right) + \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{1}{x^{5/2}}+\frac{1}{\sqrt{x}}\right)dx, x, x\right.$$
$$= -\frac{1}{10\left(x+\sqrt{1+x^2}\right)^{5/2}} - \frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} - \frac{1}{\sqrt{x+\sqrt{1+x^2}}} + \sqrt{x+\sqrt{1+x^2}} + \frac{1}{6}\left(x+\right.$$

Mathematica [A] time = 0.06, size = 85, normalized size = 1.02

$$\frac{2\left(10x^4+5\left(6\sqrt{x^2+1}-1\right)x^2-10\left(\sqrt{x^2+1}-2\right)x+5\sqrt{x^2+1}+10\left(\sqrt{x^2+1}+3\right)x^3-7\right)}{15\left(\sqrt{x^2+1}+x\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[1 + x^2])/Sqrt[x + Sqrt[1 + x^2]],x]
```

```
[Out] (2*(-7 + 10*x^4 + 5*Sqrt[1 + x^2] - 10*x*(-2 + Sqrt[1 + x^2]) + 10*x^3*(3 +
 Sqrt[1 + x^2]) + 5*x^2*(-1 + 6*Sqrt[1 + x^2])))/(15*(x + Sqrt[1 + x^2])^(5
/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 83, normalized size = 1.00

$$\frac{2\sqrt{x^2+1}\left(2x^3+6x^2-2x+1\right)}{3\left(\sqrt{x^2+1}+x\right)^{5/2}}+\frac{2\left(10x^4+30x^3-5x^2+20x-7\right)}{15\left(\sqrt{x^2+1}+x\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[1 + x^2])/Sqrt[x + Sqrt[1 + x^2]],x]
```

```
[Out] (2*Sqrt[1 + x^2]*(1 - 2*x + 6*x^2 + 2*x^3))/(3*(x + Sqrt[1 + x^2])^(5/2)) +
 (2*(-7 + 20*x - 5*x^2 + 30*x^3 + 10*x^4))/(15*(x + Sqrt[1 + x^2])^(5/2))
```

fricas [A] time = 0.39, size = 47, normalized size = 0.57

$$\frac{2}{15}\left(3\,x^3-5\,x^2-\left(3\,x^2-5\,x+7\right)\sqrt{x^2+1}+11\,x+5\right)\sqrt{x+\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^3 - 5*x^2 - (3*x^2 - 5*x + 7)*sqrt(x^2 + 1) + 11*x + 5)*sqrt(x +
sqrt(x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}+1}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate((sqrt(x^2 + 1) + 1)/sqrt(x + sqrt(x^2 + 1)), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1+\sqrt{x^2+1}}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x^2+1)^(1/2))/(x+(x^2+1)^(1/2))^(1/2),x)
[Out] int((1+(x^2+1)^(1/2))/(x+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}+1}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate((sqrt(x^2 + 1) + 1)/sqrt(x + sqrt(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^2+1}+1}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)^(1/2) + 1)/(x + (x^2 + 1)^(1/2))^(1/2),x)
[Out] int(((x^2 + 1)^(1/2) + 1)/(x + (x^2 + 1)^(1/2))^(1/2), x)
```

sympy [A] time = 0.55, size = 107, normalized size = 1.29

$$\frac{2x^2}{15\sqrt{x+\sqrt{x^2+1}}}+\frac{8x\sqrt{x^2+1}}{15\sqrt{x+\sqrt{x^2+1}}}+\frac{4x}{3\sqrt{x+\sqrt{x^2+1}}}+\frac{2\sqrt{x^2+1}}{3\sqrt{x+\sqrt{x^2+1}}}-\frac{14}{15\sqrt{x+\sqrt{x^2+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x**2+1)**(1/2))/(x+(x**2+1)**(1/2))**(1/2),x)
[Out] 2*x**2/(15*sqrt(x + sqrt(x**2 + 1))) + 8*x*sqrt(x**2 + 1)/(15*sqrt(x + sqrt
(x**2 + 1))) + 4*x/(3*sqrt(x + sqrt(x**2 + 1))) + 2*sqrt(x**2 + 1)/(3*sqrt(
x + sqrt(x**2 + 1))) - 14/(15*sqrt(x + sqrt(x**2 + 1)))
```

3.1005 $\int \frac{x^2}{\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=83

$$\frac{4\sqrt{a^2x^2-bx}\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{b^2x}-\frac{4a\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{b^2}$$

Rubi [F] time = 5.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/(Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2)),x
]
```

```
[Out] (2*Sqrt[x]*Sqrt[-b + a^2*x]*Defer[Subst][Defer[Int][x^4/(Sqrt[-b + a^2*x^2]
*(a*x^4 + x^2*Sqrt[-(b*x^2) + a^2*x^4])^(3/2)), x], x, Sqrt[x]])/Sqrt[-(b*x
) + a^2*x^2]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx=\frac{\left(\sqrt{x}\sqrt{-b+a^2x}\right)\int \frac{x^{3/2}}{\sqrt{-b+a^2x}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx}{\sqrt{-bx+a^2x^2}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{-b+a^2x}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{-b+a^2x^2}\left(ax^4+x^2\sqrt{-bx^2+a^2x^4}\right)^{3/2}}\,dx,x,\sqrt{\right.}{\sqrt{-bx+a^2x^2}}$$

Mathematica [F] time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/(Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3
/2)),x]
```

```
[Out] Integrate[x^2/(Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3
/2)), x]
```

IntegrateAlgebraic [A] time = 4.04, size = 83, normalized size = 1.00

$$\frac{4\sqrt{a^2x^2-bx}\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{b^2x}-\frac{4a\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2
*x^2])^(3/2)),x]
```

```
[Out] (-4*a*Sqrt[x*(a*x + Sqrt[-(b*x) + a^2*x^2])])/b^2 + (4*Sqrt[-(b*x) + a^2*x^
2]*Sqrt[x*(a*x + Sqrt[-(b*x) + a^2*x^2])])/(b^2*x)
```

fricas [A] time = 0.42, size = 52, normalized size = 0.63

$$-\frac{4\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}\left(ax-\sqrt{a^2x^2-bx}\right)}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, al
gorithm="fricas")
```

```
[Out] -4*sqrt(a*x^2 + sqrt(a^2*x^2 - b*x)*x)*(a*x - sqrt(a^2*x^2 - b*x))/(b^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a^2x^2-bx}\left(ax^2+\sqrt{a^2x^2-bx}\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, al
gorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(a^2*x^2 - b*x)*(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2)),
x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a^2x^2-bx}\,\left(a\,x^2+x\sqrt{a^2x^2-bx}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

```
[Out] int(x^2/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a^2x^2-bx}\left(ax^2+\sqrt{a^2x^2-bx}\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, al
gorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(a^2*x^2 - b*x)*(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{a^2\,x^2-b\,x}\,{\left(a\,x^2+x\,\sqrt{a^2\,x^2-b\,x}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a^2*x^2 - b*x)^(1/2)*(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2)),x)
```

```
[Out] int(x^2/((a^2*x^2 - b*x)^(1/2)*(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(x \left(ax + \sqrt{a^{2}x^{2} - bx}\right)\right)^{\frac{3}{2}} \sqrt{x \left(a^{2}x - b\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*x**2-b*x)**(1/2)/(a*x**2+x*(a**2*x**2-b*x)**(1/2))**(3
/2),x)
```

```
[Out] Integral(x**2/((x*(a*x + sqrt(a**2*x**2 - b*x)))**(3/2)*sqrt(x*(a**2*x - b)
)), x)
```

3.1006 $\int \frac{a^2b-a(2a+b)x+3ax^2-x^3}{x(-b+x)\sqrt{x(-a+x)(-b+x)}\left(a+(-1-bd)x+dx^2\right)}\,dx$

Optimal. Leaf size=84

$$2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{x^2(-a-b)+abx+x^3}}{\sqrt{d}\,x(x-b)}\right)+\frac{2\sqrt{abx-ax^2-bx^2+x^3}}{x(b-x)}$$

Rubi [F] time = 12.66, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a^2b-a(2a+b)x+3ax^2-x^3}{x(-b+x)\sqrt{x(-a+x)(-b+x)}\left(a+(-1-bd)x+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a^2*b - a*(2*a + b)*x + 3*a*x^2 - x^3)/(x*(-b + x)*Sqrt[x*(-a + x)*(-b
 + x)]*(a + (-1 - b*d)*x + d*x^2)),x]
```

```
[Out] (2*(a - x))/(b*d*Sqrt[(a - x)*(b - x)*x]) - (4*(a - x)*x)/(b^2*d*Sqrt[(a -
x)*(b - x)*x]) + (4*Sqrt[a]*(b - x)*Sqrt[x]*Sqrt[1 - x/a]*EllipticE[ArcSin[
Sqrt[x]/Sqrt[a]], a/b])/(b^2*d*Sqrt[(a - x)*(b - x)*x]*Sqrt[1 - x/b]) - (2*
Sqrt[a]*Sqrt[x]*Sqrt[1 - x/a]*Sqrt[1 - x/b]*EllipticF[ArcSin[Sqrt[x]/Sqrt[a
]], a/b])/(b*d*Sqrt[(a - x)*(b - x)*x]) - ((1 - 2*a*d + b*d + Sqrt[-4*a*d +
 (1 + b*d)^2])*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Int][Sqrt[-a + x]/(x
^(3/2)*(-b + x)^(3/2)*(-1 - b*d - Sqrt[1 - 4*a*d + 2*b*d + b^2*d^2] + 2*d*x
)), x])/(d*Sqrt[(a - x)*(b - x)*x]) - ((1 - 2*a*d + b*d - Sqrt[-4*a*d + (1
+ b*d)^2])*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Int][Sqrt[-a + x]/(x^(3/
2)*(-b + x)^(3/2)*(-1 - b*d + Sqrt[1 - 4*a*d + 2*b*d + b^2*d^2] + 2*d*x)),
x])/(d*Sqrt[(a - x)*(b - x)*x])
```

Rubi steps

$$\int \frac{a^2b-a(2a+b)x+3ax^2-x^3}{x(-b+x)\sqrt{x(-a+x)(-b+x)}\left(a+(-1-bd)x+dx^2\right)}dx = \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{a^2b-a(2a+b)x+3ax^2-x^3}{x^{3/2}\sqrt{-a+x}(-b+x)^{3/2}\left(a+(-1-bd)x+\cdots\right)}}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{\sqrt{-a+x}\left(-ab+2ax-x^2\right)}{x^{3/2}(-b+x)^{3/2}\left(a+(-1-bd)x+dx^2\right)}\cdots}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int\left(-\frac{\sqrt{-a+x}}{dx^{3/2}(-b+x)^{3/2}}+\frac{\sqrt{-a+x}}{dx^{3/2}(-b\cdots}\cdots\right)}{\sqrt{x(-a+x)(-b+x)}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int\frac{\sqrt{-a+x}}{x^{3/2}(-b+x)^{3/2}}dx}{d\sqrt{x(-a+x)(-b+x)}}+\frac{\left(\sqrt{x}\cdots\right.}{\cdots}$$

$$= \frac{2(a-x)}{bd\sqrt{(a-x)(b-x)x}}+\frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int\left(\cdots\right.}{\cdots}$$

$$= \frac{2(a-x)}{bd\sqrt{(a-x)(b-x)x}}-\frac{4(a-x)x}{b^2d\sqrt{(a-x)(b-x)x}}+\frac{\left(4\sqrt{\cdots}\right.}{\cdots}$$

$$= \frac{2(a-x)}{bd\sqrt{(a-x)(b-x)x}}-\frac{4(a-x)x}{b^2d\sqrt{(a-x)(b-x)x}}-\frac{\left(\sqrt{x}\cdots\right.}{\cdots}$$

$$= \frac{2(a-x)}{bd\sqrt{(a-x)(b-x)x}}-\frac{4(a-x)x}{b^2d\sqrt{(a-x)(b-x)x}}+\frac{\left(\left(-1\cdots\right.\right.}{\cdots}$$

$$= \frac{2(a-x)}{bd\sqrt{(a-x)(b-x)x}}-\frac{4(a-x)x}{b^2d\sqrt{(a-x)(b-x)x}}+\frac{4\sqrt{a\cdots}}{\cdots}$$

Mathematica [C] time = 2.57, size = 277, normalized size = 3.30

$$-\frac{2(x-a)\left(i\sqrt{\frac{x}{a}}\sqrt{\frac{x-b}{a-b}}\Pi\left(\frac{2ad}{2ad-bd+\sqrt{(bd+1)^2-4ad}-1};i\sinh^{-1}\left(\sqrt{\frac{x}{a}-1}\right)|\frac{a}{a-b}\right)+i\sqrt{\frac{x}{a}}\sqrt{\frac{x-b}{a-b}}\Pi\left(-\frac{2ad}{-2ad+bd+\sqrt{(bd+1)^2-4ad}+1};i\sinh^{-1}\left(\sqrt{\frac{x}{a}-1}\right)|\frac{a}{a-b}\right)-i\sqrt{\frac{x}{a}}\sqrt{\frac{x-b}{a-b}}F\left(i\sinh^{-1}\left(\sqrt{\frac{x}{a}-1}\right)|\frac{a}{a-b}\right)+\sqrt{\frac{x}{a}-1}\right)}{\sqrt{\frac{x}{a}-1}\sqrt{x(x-a)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*b - a*(2*a + b)*x + 3*a*x^2 - x^3)/(x*(-b + x)*Sqrt[x*(-a +
x)*(-b + x)]*(a + (-1 - b*d)*x + d*x^2)),x]
```

```
[Out] (-2*(-a + x)*(Sqrt[-1 + x/a] - I*Sqrt[x/a]*Sqrt[(-b + x)/(a - b)]*EllipticF
[I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)] + I*Sqrt[x/a]*Sqrt[(-b + x)/(a - b)]
*EllipticPi[(2*a*d)/(-1 + 2*a*d - b*d + Sqrt[-4*a*d + (1 + b*d)^2]), I*ArcS
inh[Sqrt[-1 + x/a]], a/(a - b)] + I*Sqrt[x/a]*Sqrt[(-b + x)/(a - b)]*Ellipt
icPi[(-2*a*d)/(1 - 2*a*d + b*d + Sqrt[-4*a*d + (1 + b*d)^2]), I*ArcSinh[Sqr
t[-1 + x/a]], a/(a - b)]))/(Sqrt[x*(-a + x)*(-b + x)]*Sqrt[-1 + x/a])
```

IntegrateAlgebraic [A] time = 1.00, size = 84, normalized size = 1.00

$$2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{x^2(-a-b)+abx+x^3}}{\sqrt{d}\,x(x-b)}\right)+\frac{2\sqrt{abx-ax^2-bx^2+x^3}}{x(b-x)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2*b - a*(2*a + b)*x + 3*a*x^2 - x^3)/(x*(-b + x)*Sqrt
[x*(-a + x)*(-b + x)]*(a + (-1 - b*d)*x + d*x^2)),x]
```

```
[Out] (2*Sqrt[a*b*x - a*x^2 - b*x^2 + x^3])/((b - x)*x) + 2*Sqrt[d]*ArcTanh[Sqrt[
a*b*x + (-a - b)*x^2 + x^3]/(Sqrt[d]*x*(-b + x))]
```

fricas [A] time = 0.82, size = 313, normalized size = 3.73

$$\left[\frac{\left(bx-x^2\right)\sqrt{d}\log\left(\frac{d^2x^4-2\left(bd^2-3\,d\right)x^3+\left(b^2d^2-6\,(a+b)d+1\right)x^2+a^2+4\,\sqrt{abx-(a+b)x^2+x^3}\,\left(dx^2-(bd-1)x-a\right)\sqrt{d}+2\,(3\,abd-a)x}{d^2x^4-2\left(bd^2+d\right)x^3+\left(b^2d^2+2\,(a+b)d+1\right)x^2+a^2-2\,(abd+a)x}\right)+4\,\sqrt{abx-(a+b)x^2+x^3}}{2\left(bx-x^2\right)}, -\frac{\left(bx-x^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{abx-(a+b)x^2+x^3}\,\left(dx^2-(bd-1)x-a\right)\sqrt{-d}}{2\left(abdx-(a+b)dx^2+dx^3\right)}\right)-2\,\sqrt{abx-(a+b)x^2+x^3}}{bx-x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*b-a*(2*a+b)*x+3*a*x^2-x^3)/x/(-b+x)/(x*(-a+x)*(-b+x))^(1/2)/
(a+(-b*d-1)*x+d*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*x - x^2)*sqrt(d)*log((d^2*x^4 - 2*(b*d^2 - 3*d)*x^3 + (b^2*d^2 - 6
*(a + b)*d + 1)*x^2 + a^2 + 4*sqrt(a*b*x - (a + b)*x^2 + x^3)*(d*x^2 - (b*d
 - 1)*x - a)*sqrt(d) + 2*(3*a*b*d - a)*x)/(d^2*x^4 - 2*(b*d^2 + d)*x^3 + (b
^2*d^2 + 2*(a + b)*d + 1)*x^2 + a^2 - 2*(a*b*d + a)*x)) + 4*sqrt(a*b*x - (a
 + b)*x^2 + x^3))/(b*x - x^2), -((b*x - x^2)*sqrt(-d)*arctan(1/2*sqrt(a*b*x
 - (a + b)*x^2 + x^3)*(d*x^2 - (b*d - 1)*x - a)*sqrt(-d)/(a*b*d*x - (a + b)
*d*x^2 + d*x^3)) - 2*sqrt(a*b*x - (a + b)*x^2 + x^3))/(b*x - x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{a^2b-(2\,a+b)ax+3\,ax^2-x^3}{\sqrt{(a-x)(b-x)x}\,\left(dx^2-(bd+1)x+a\right)(b-x)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*b-a*(2*a+b)*x+3*a*x^2-x^3)/x/(-b+x)/(x*(-a+x)*(-b+x))^(1/2)/
(a+(-b*d-1)*x+d*x^2),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*b - (2*a + b)*a*x + 3*a*x^2 - x^3)/(sqrt((a - x)*(b - x)*x)
*(d*x^2 - (b*d + 1)*x + a)*(b - x)*x), x)
```

maple [C] time = 0.03, size = 2956, normalized size = 35.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*b-a*(2*a+b)*x+3*a*x^2-x^3)/x/(-b+x)/(x*(-a+x)*(-b+x))^(1/2)/(a+(-b
*d-1)*x+d*x^2),x)
```

```
[Out] 1/(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1
/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2/d-1/2/d*(b^2*d
^2-4*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(b*d+1+(b^
2*d^2-4*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))*b^2*d+2/(b^2*d^2-4*a*d+2*b*d+
1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*
x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2/d-1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2)
)*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(b*d+1+(b^2*d^2-4*a*d+2*b*d+1)^(1
/2))),(a/(a-b))^(1/2))*b-4/(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*a^2*(1-1/a*x)^(1/2
)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/
(a-1/2*b-1/2/d-1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^
(1/2),a/(a-1/2/d*(b*d+1+(b^2*d^2-4*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))+1/
(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2
)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2/d-1/2/d*(b^2*d^2
-4*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(b*d+1+(b^2*
```

```
d^2-4*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))/d-2*a^2*(1-1/a*x)^(1/2)*(-1/(a-
b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-
1/2/d-1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x)/a)^(1/2),a/(
a-1/2/d*(b*d+1+(b^2*d^2-4*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2))+a*(1-1/a*x)
^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(
1/2)/(a-1/2*b-1/2/d-1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2))*EllipticPi((-(-a+x
)/a)^(1/2),a/(a-1/2/d*(b*d+1+(b^2*d^2-4*a*d+2*b*d+1)^(1/2))),(a/(a-b))^(1/2
))*b+a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*
x^2-b*x^2+x^3)^(1/2)/(a-1/2*b-1/2/d-1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2))*El
lipticPi((-(-a+x)/a)^(1/2),a/(a-1/2/d*(b*d+1+(b^2*d^2-4*a*d+2*b*d+1)^(1/2))
),(a/(a-b))^(1/2))/d-1/(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/
(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2
*b+1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),
a/(a+1/2*(-b*d+(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))*b^2*d-2
/(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/
2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b+1/2/d*(b^2*d^2-4*a*
d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-b*d+(b^2*d^
2-4*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))*b-2*a^2*(1-1/a*x)^(1/2)*(-1/(
a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*
b+1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a
/(a+1/2*(-b*d+(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))+a*(1-1/a
*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3
)^(1/2)/(a-1/2*b+1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-
a+x)/a)^(1/2),a/(a+1/2*(-b*d+(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^
(1/2))*b+a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*
x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b+1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1/2/d
)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-b*d+(b^2*d^2-4*a*d+2*b*d+1)^(1/2)
-1)/d),(a/(a-b))^(1/2))/d+4/(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*a^2*(1-1/a*x)^(1/
2)*(-1/(a-b)*b+1/(a-b)*x)^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)
/(a-1/2*b+1/2/d*(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)
^(1/2),a/(a+1/2*(-b*d+(b^2*d^2-4*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))-
1/(b^2*d^2-4*a*d+2*b*d+1)^(1/2)*a*(1-1/a*x)^(1/2)*(-1/(a-b)*b+1/(a-b)*x)^(1
/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)/(a-1/2*b+1/2/d*(b^2*d^2-4*a
*d+2*b*d+1)^(1/2)-1/2/d)*EllipticPi((-(-a+x)/a)^(1/2),a/(a+1/2*(-b*d+(b^2*d
^2-4*a*d+2*b*d+1)^(1/2)-1)/d),(a/(a-b))^(1/2))/d-(a-b)*(2*(-a*x+x^2)/(a-b)/
b/((-b+x)*(-a*x+x^2))^(1/2)-2*(-1/b+a/(a-b)/b)*a*(-(-a+x)/a)^(1/2)*((-b+x)/
(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)*EllipticF((-(-a+x)
/a)^(1/2),(a/(a-b))^(1/2))+2/(a-b)/b*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/
2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)*((a-b)*EllipticE((-(-a+x)/a)
^(1/2),(a/(a-b))^(1/2))+b*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))))-a*
(-2*(a*b-a*x-b*x+x^2)/a/b/(x*(a*b-a*x-b*x+x^2))^(1/2)-2*((a+b)/a/b+(-a-b)/a
/b)*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x
^2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))-2/b*(-(-a+x)/a)^
(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^2+x^3)^(1/2)*((a-
b)*EllipticE((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))+b*EllipticF((-(-a+x)/a)^(1/
2),(a/(a-b))^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2b - (2\,a + b)ax + 3\,ax^2 - x^3}{\sqrt{(a-x)(b-x)x}\left(dx^2 - (bd+1)x + a\right)(b-x)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*b-a*(2*a+b)*x+3*a*x^2-x^3)/x/(-b+x)/(x*(-a+x)*(-b+x))^(1/2)/
(a+(-b*d-1)*x+d*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*b - (2*a + b)*a*x + 3*a*x^2 - x^3)/(sqrt((a - x)*(b - x)*x)
*(d*x^2 - (b*d + 1)*x + a)*(b - x)*x), x)
```

mupad [B] time = 0.18, size = 628, normalized size = 7.48

$$\frac{b\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(\frac{b}{b-\frac{b\,d+\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}+1}{2\,d}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\left(b\,d-2\,a\,d+\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}+1\right)}{d\left(b-\frac{b\,d+\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}+1}{2\,d}\right)\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}-\frac{b\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(\frac{b}{b-\frac{b\,d-\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}+1}{2\,d}};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\left(2\,a\,d-b\,d+\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}-1\right)}{d\left(b-\frac{b\,d-\sqrt{b^2\,d^2+2\,b\,d-4\,a\,d+1}+1}{2\,d}\right)\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}-\frac{2\,a\,(a-b)\,\sqrt{\frac{x}{a}}\left(\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x}{a}}\right)\middle|\frac{a}{b}\right)-\frac{a\,\sin\left(2\,\mathrm{asin}\left(\sqrt{\frac{x}{a}}\right)\right)}{2\,b\,\sqrt{1-\frac{x}{b}}}\right)\sqrt{\frac{a-x}{a}}\,\sqrt{\frac{b-x}{b}}}{b\left(\frac{a}{b}-1\right)\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}-\frac{2\,a\left(\frac{\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)-\frac{\sqrt{\frac{b-x}{a-b}+1}\,\sqrt{\frac{b-x}{b}}}{\sqrt{1-\frac{b-x}{b}}}}{\frac{b}{a-b}+1}-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\right)\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}}{\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*b + 3*a*x^2 - x^3 - a*x*(2*a + b))/(x*(b - x)*(x*(a - x)*(b - x))
^(1/2)*(a - x*(b*d + 1) + d*x^2)),x)
```

```
[Out] (b*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*ellipticPi(b/(b -
(b*d + (2*b*d - 4*a*d + b^2*d^2 + 1)^(1/2) + 1)/(2*d)), asin(((b - x)/b)^(1
/2)), -b/(a - b))*(b*d - 2*a*d + (2*b*d - 4*a*d + b^2*d^2 + 1)^(1/2) + 1))/
(d*(b - (b*d + (2*b*d - 4*a*d + b^2*d^2 + 1)^(1/2) + 1)/(2*d))*(x^3 - x^2*(
a + b) + a*b*x)^(1/2)) - (b*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))
^(1/2)*ellipticPi(b/(b - (b*d - (2*b*d - 4*a*d + b^2*d^2 + 1)^(1/2) + 1)/(2
*d)), asin(((b - x)/b)^(1/2)), -b/(a - b))*(2*a*d - b*d + (2*b*d - 4*a*d +
b^2*d^2 + 1)^(1/2) - 1))/(d*(b - (b*d - (2*b*d - 4*a*d + b^2*d^2 + 1)^(1/2)
 + 1)/(2*d))*(x^3 - x^2*(a + b) + a*b*x)^(1/2)) - (2*a*(a - b)*(x/a)^(1/2)*
(ellipticE(asin((x/a)^(1/2)), a/b) - (a*sin(2*asin((x/a)^(1/2))))/(2*b*(1 -
 x/b)^(1/2)))*((a - x)/a)^(1/2)*((b - x)/b)^(1/2))/(b*(a/b - 1)*(x^3 - x^2*
(a + b) + a*b*x)^(1/2)) - (2*a*((ellipticE(asin(((b - x)/b)^(1/2)), -b/(a -
 b)) - (((b - x)/(a - b) + 1)^(1/2)*((b - x)/b)^(1/2))/(1 - (b - x)/b)^(1/2
))/(b/(a - b) + 1) - ellipticF(asin(((b - x)/b)^(1/2)), -b/(a - b)))*(x/b)^
(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2))/(x^3 - x^2*(a + b) + a*b*x
)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*b-a*(2*a+b)*x+3*a*x**2-x**3)/x/(-b+x)/(x*(-a+x)*(-b+x))**(1
/2)/(a+(-b*d-1)*x+d*x**2),x)
```

```
[Out] Timed out
```

3.1007 $\int \frac{(1+x^3)^{2/3}}{x}\,dx$

Optimal. Leaf size=84

$$\frac{1}{2}\left(x^3+1\right)^{2/3}+\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-1\right)-\frac{1}{6}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 50, 55, 618, 204, 31}

$$\frac{1}{2}\left(x^3+1\right)^{2/3}+\frac{1}{2}\log\left(1-\sqrt[3]{x^3+1}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)^(2/3)/x,x]
```

```
[Out] (1 + x^3)^(2/3)/2 + ArcTan[(1 + 2*(1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x
]/2 + Log[1 - (1 + x^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+x^3\right)^{2/3}}{x} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(1+x)^{2/3}}{x} dx, x, x^3\right)\\
&= \frac{1}{2}\left(1+x^3\right)^{2/3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}} dx, x, x^3\right)\\
&= \frac{1}{2}\left(1+x^3\right)^{2/3} - \frac{\log(x)}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x} dx, x, \sqrt[3]{1+x^3}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2} dx, x\right.\\
&= \frac{1}{2}\left(1+x^3\right)^{2/3} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1+x^3}\right) - \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2\sqrt[3]{1+x^3}\right)\\
&= \frac{1}{2}\left(1+x^3\right)^{2/3} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1+x^3}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 59, normalized size = 0.70

$$\frac{1}{2}\left(\left(x^3+1\right)^{2/3}+\log\left(1-\sqrt[3]{x^3+1}\right)-\log(x)\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)^(2/3)/x,x]
```

```
[Out] ArcTan[(1 + 2*(1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + ((1 + x^3)^(2/3) - Log[x]
 + Log[1 - (1 + x^3)^(1/3)])/2
```

IntegrateAlgebraic [A] time = 0.06, size = 84, normalized size = 1.00

$$\frac{1}{2}\left(x^3+1\right)^{2/3}+\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-1\right)-\frac{1}{6}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)^(2/3)/x,x]
```

```
[Out] (1 + x^3)^(2/3)/2 + ArcTan[1/Sqrt[3] + (2*(1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3]
 + Log[-1 + (1 + x^3)^(1/3)]/3 - Log[1 + (1 + x^3)^(1/3) + (1 + x^3)^(2/3)]
/6
```

fricas [A] time = 0.40, size = 65, normalized size = 0.77

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{1}{2}\left(x^3+1\right)^{\frac{2}{3}}-\frac{1}{6}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 1/2*(x^3 +
1)^(2/3) - 1/6*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) + 1/3*log((x^3 +
1)^(1/3) - 1)
```

giac [A] time = 0.21, size = 64, normalized size = 0.76

$$\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2}\left(x^3+1\right)^{\frac{2}{3}}-\frac{1}{6}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)/x,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) + 1/2*(x^3 + 1)^(2/
3) - 1/6*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) + 1/3*log(abs((x^3 + 1)
^(1/3) - 1))
```

maple [C] time = 0.30, size = 64, normalized size = 0.76

$$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{3},1,1\right],[2,2],-x^3\right)}{3\Gamma\left(\frac{2}{3}\right)}-\frac{\left(\frac{3}{2}-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)\right)\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}\right)}{9\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)/x,x)
```

```
[Out] -1/9/Pi*3^(1/2)*GAMMA(2/3)*(-2/3*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1/3,1
,1],[2,2],-x^3)-(3/2-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x))*Pi*3^(1/2)/GAMMA(2/3
))
```

maxima [A] time = 0.49, size = 63, normalized size = 0.75

$$\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2}\left(x^3+1\right)^{\frac{2}{3}}-\frac{1}{6}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) + 1/2*(x^3 + 1)^(2/
3) - 1/6*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) + 1/3*log((x^3 + 1)^(1/
3) - 1)
```

mupad [B] time = 0.82, size = 83, normalized size = 0.99

$$\frac{\ln\left(\left(x^3+1\right)^{1/3}-1\right)}{3}+\frac{\left(x^3+1\right)^{2/3}}{2}+\ln\left(\left(x^3+1\right)^{1/3}-9\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(\left(x^3+1\right)^{1/3}-9\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)^(2/3)/x,x)
```

```
[Out] log((x^3 + 1)^(1/3) - 1)/3 + (x^3 + 1)^(2/3)/2 + log((x^3 + 1)^(1/3) - 9*((
3^(1/2)*1i)/6 - 1/6)^2)*((3^(1/2)*1i)/6 - 1/6) - log((x^3 + 1)^(1/3) - 9*((
3^(1/2)*1i)/6 + 1/6)^2)*((3^(1/2)*1i)/6 + 1/6)
```

sympy [C] time = 0.84, size = 36, normalized size = 0.43

$$-\frac{x^2\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^3}\right)}{3\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)/x,x)
```

```
[Out] -x**2*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), exp_polar(I*pi)/x**3)/(3*gamm
a(1/3))
```

3.1008 $\int \frac{\left(-b+2ax^2\right)\sqrt[4]{bx^2+ax^4}}{b+ax^2}\,dx$

Optimal. Leaf size=84

$$\frac{5b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{2a^{3/4}}-\frac{5b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{2a^{3/4}}+x\sqrt[4]{ax^4+bx^2}$$

Rubi [A] time = 0.22, antiderivative size = 146, normalized size of antiderivative = 1.74, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2056, 459, 329, 331, 298, 203, 206}

$$\frac{5b\sqrt[4]{ax^4+bx^2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{2a^{3/4}\sqrt{x}\sqrt[4]{ax^2+b}}-\frac{5b\sqrt[4]{ax^4+bx^2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{2a^{3/4}\sqrt{x}\sqrt[4]{ax^2+b}}+x\sqrt[4]{ax^4+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + 2*a*x^2)*(b*x^2 + a*x^4)^(1/4))/(b + a*x^2),x]
```

```
[Out] x*(b*x^2 + a*x^4)^(1/4) + (5*b*(b*x^2 + a*x^4)^(1/4)*ArcTan[(a^(1/4)*Sqrt[x
])/(b + a*x^2)^(1/4)])/(2*a^(3/4)*Sqrt[x]*(b + a*x^2)^(1/4)) - (5*b*(b*x^2
+ a*x^4)^(1/4)*ArcTanh[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(2*a^(3/4)*Sqr
t[x]*(b + a*x^2)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\left(-b+2ax^2\right)\sqrt[4]{bx^2+ax^4}}{b+ax^2}\,dx = \frac{\sqrt[4]{bx^2+ax^4}\int \frac{\sqrt{x}\left(-b+2ax^2\right)}{\left(b+ax^2\right)^{3/4}}\,dx}{\sqrt{x}\sqrt[4]{b+ax^2}}$$

$$= x\sqrt[4]{bx^2+ax^4} - \frac{\left(5b\sqrt[4]{bx^2+ax^4}\right)\int \frac{\sqrt{x}}{\left(b+ax^2\right)^{3/4}}\,dx}{2\sqrt{x}\sqrt[4]{b+ax^2}}$$

$$= x\sqrt[4]{bx^2+ax^4} - \frac{\left(5b\sqrt[4]{bx^2+ax^4}\right)\operatorname{Subst}\left(\int \frac{x^2}{\left(b+ax^4\right)^{3/4}}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt[4]{b+ax^2}}$$

$$= x\sqrt[4]{bx^2+ax^4} - \frac{\left(5b\sqrt[4]{bx^2+ax^4}\right)\operatorname{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{\sqrt{x}\sqrt[4]{b+ax^2}}$$

$$= x\sqrt[4]{bx^2+ax^4} - \frac{\left(5b\sqrt[4]{bx^2+ax^4}\right)\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{2\sqrt{a}\sqrt{x}\sqrt[4]{b+ax^2}} + \frac{\left(5b\sqrt[4]{bx^2\right.}}{}$$

$$= x\sqrt[4]{bx^2+ax^4} + \frac{5b\sqrt[4]{bx^2+ax^4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{2a^{3/4}\sqrt{x}\sqrt[4]{b+ax^2}} - \frac{5b\sqrt[4]{bx^2+ax^4}\tanh^{-1}\left(\frac{\sqrt[4]{a}}{\sqrt[4]{b}}\right.}{2a^{3/4}\sqrt{x}\sqrt[4]{b+ax^2}}$$

Mathematica [A] time = 0.13, size = 123, normalized size = 1.46

$$\frac{x^{3/2}\left(2a^{3/4}x^{3/2}\left(ax^2+b\right)+5b\left(ax^2+b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)-5b\left(ax^2+b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)\right)}{2a^{3/4}\left(x^2\left(ax^2+b\right)\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + 2*a*x^2)*(b*x^2 + a*x^4)^(1/4))/(b + a*x^2),x]
```

```
[Out] (x^(3/2)*(2*a^(3/4)*x^(3/2)*(b + a*x^2) + 5*b*(b + a*x^2)^(3/4)*ArcTan[(a^(
1/4)*Sqrt[x])/(b + a*x^2)^(1/4)] - 5*b*(b + a*x^2)^(3/4)*ArcTanh[(a^(1/4)*S
qrt[x])/(b + a*x^2)^(1/4)]))/(2*a^(3/4)*(x^2*(b + a*x^2))^(3/4))
```

IntegrateAlgebraic [A] time = 0.31, size = 84, normalized size = 1.00

$$\frac{5b\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+bx^2}}\right)}{2a^{3/4}} - \frac{5b\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+bx^2}}\right)}{2a^{3/4}} + x\sqrt[4]{ax^4+bx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + 2*a*x^2)*(b*x^2 + a*x^4)^(1/4))/(b + a*x^2),x]
```

```
[Out] x*(b*x^2 + a*x^4)^(1/4) + (5*b*ArcTan[(a^(1/4)*x)/(b*x^2 + a*x^4)^(1/4)])/(
2*a^(3/4)) - (5*b*ArcTanh[(a^(1/4)*x)/(b*x^2 + a*x^4)^(1/4)])/(2*a^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2-b)*(a*x^4+b*x^2)^(1/4)/(a*x^2+b),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.19, size = 209, normalized size = 2.49

$$\frac{8\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}} b x^2 + \frac{10\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}} + \frac{10\sqrt{2}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}} + \frac{5\sqrt{2}b^2\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)}{(-a)^{\frac{3}{4}}} + \frac{5\sqrt{2}(-a)^{\frac{1}{4}}b^2\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)}{a}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2-b)*(a*x^4+b*x^2)^(1/4)/(a*x^2+b),x, algorithm="giac")
```

```
[Out] 1/8*(8*(a + b/x^2)^(1/4)*b*x^2 + 10*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)
*(-a)^(1/4) + 2*(a + b/x^2)^(1/4))/(-a)^(1/4))/(-a)^(3/4) + 10*sqrt(2)*b^2*
arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(a + b/x^2)^(1/4))/(-a)^(1/4))/
(-a)^(3/4) + 5*sqrt(2)*b^2*log(sqrt(2)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(
-a) + sqrt(a + b/x^2))/(-a)^(3/4) + 5*sqrt(2)*(-a)^(1/4)*b^2*log(-sqrt(2)*(
-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sqrt(a + b/x^2))/a)/b
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(2a\,x^2-b\right)\left(a\,x^4+b\,x^2\right)^{\frac{1}{4}}}{a\,x^2+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x^2-b)*(a*x^4+b*x^2)^(1/4)/(a*x^2+b),x)
```

```
[Out] int((2*a*x^2-b)*(a*x^4+b*x^2)^(1/4)/(a*x^2+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4+bx^2\right)^{\frac{1}{4}}\left(2\,ax^2-b\right)}{ax^2+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2-b)*(a*x^4+b*x^2)^(1/4)/(a*x^2+b),x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + b*x^2)^(1/4)*(2*a*x^2 - b)/(a*x^2 + b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(b-2\,a\,x^2\right)\,{\left(a\,x^4+b\,x^2\right)}^{1/4}}{a\,x^2+b} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b - 2*a*x^2)*(a*x^4 + b*x^2)^(1/4))/(b + a*x^2),x)
[Out] int(-((b - 2*a*x^2)*(a*x^4 + b*x^2)^(1/4))/(b + a*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\left(ax^2+b\right)}\left(2ax^2-b\right)}{ax^2+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x**2-b)*(a*x**4+b*x**2)**(1/4)/(a*x**2+b),x)
[Out] Integral((x**2*(a*x**2 + b))**(1/4)*(2*a*x**2 - b)/(a*x**2 + b), x)
```

3.1009 $\int \frac{(-1+x^4)\sqrt{1+x^2+x^4}(1+x^2+3x^4+x^6+x^8)}{(1+x^4)^3(1-x^2+x^4)}\,dx$

Optimal. Leaf size=84

$$\frac{31}{8}\tanh^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+x^2+1}}\right)+\frac{\sqrt{x^4+x^2+1}\,(9x^5+2x^3+9x)}{8(x^4+1)^2}$$

Rubi [F] time = 3.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^4)\sqrt{1+x^2+x^4}\,(1+x^2+3x^4+x^6+x^8)}{(1+x^4)^3(1-x^2+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^4)*Sqrt[1 + x^2 + x^4]*(1 + x^2 + 3*x^4 + x^6 + x^8))/((1 + x^
4)^3*(1 - x^2 + x^4)),x]
```

```
[Out] ((I/2)*x*Sqrt[1 + x^2 + x^4])/(I - x^2) + ((I/2)*x*Sqrt[1 + x^2 + x^4])/(I
+ x^2) + 4*ArcTanh[x/Sqrt[1 + x^2 + x^4]] - 3*Sqrt[2]*ArcTanh[(Sqrt[2]*x)/S
qrt[1 + x^2 + x^4]] - (9*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Ellipt
icF[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4]) - (3*(I - Sqrt[3])*(1 + x^2)
*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/((3*I - Sqr
t[3])*Sqrt[1 + x^2 + x^4]) + (3*(5 - I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^2 + x
^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4]) + (3*
(5 + I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*Arc
Tan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4]) - (3*(I + Sqrt[3])*(1 + x^2)*Sqrt[(1
+ x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/((3*I + Sqrt[3])*Sqr
t[1 + x^2 + x^4]) + 2*Defer[Int][(x^2*Sqrt[1 + x^2 + x^4])/(1 + x^4)^3, x]
- Defer[Int][(x^2*Sqrt[1 + x^2 + x^4])/(1 + x^4)^2, x]
```

Rubi steps

$$\int \frac{(-1+x^4)\sqrt{1+x^2+x^4}\,(1+x^2+3x^4+x^6+x^8)}{(1+x^4)^3(1-x^2+x^4)}\,dx = \int \left(\frac{2x^2\sqrt{1+x^2+x^4}}{(1+x^4)^3} + \frac{(4-x^2)\sqrt{1+x^2+x^4}}{(1+x^4)^2} - \frac{2(\cdots}{\cdots}\right.$$

$$= 2\int \frac{x^2\sqrt{1+x^2+x^4}}{(1+x^4)^3}\,dx - 2\int \frac{(1+3x^2)\sqrt{1+x^2+\cdots}}{1+x^4}$$

$$= 2\int \frac{x^2\sqrt{1+x^2+x^4}}{(1+x^4)^3}\,dx - 2\int \left(-\frac{\left(\frac{3}{2}-\frac{i}{2}\right)\sqrt{1+x^2+\cdots}}{i-x^2}\right.$$

$$= -\left((-3+i)\int \frac{\sqrt{1+x^2+x^4}}{i-x^2}\,dx\right) + 2\int \frac{x^2\sqrt{1+x^2+\cdots}}{(1+x^4)^3}$$

$$= -\left((-3-i)\int \frac{(-1+i)-x^2}{\sqrt{1+x^2+x^4}}\,dx\right) - (-1-3i)\int \frac{\cdots}{(i-\cdots}$$

$$= -\left((-3-i)\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx\right) - (-3+i)\int \frac{1\cdots}{\sqrt{1+\cdots}}$$

$$= \frac{ix\sqrt{1+x^2+x^4}}{2(i-x^2)} + \frac{ix\sqrt{1+x^2+x^4}}{2(i+x^2)} - \frac{6x\sqrt{1+x^2+\cdots}}{1+x^2}$$

$$= \frac{ix\sqrt{1+x^2+x^4}}{2(i-x^2)} + \frac{ix\sqrt{1+x^2+x^4}}{2(i+x^2)} - \frac{6x\sqrt{1+x^2+\cdots}}{1+x^2}$$

$$= \frac{ix\sqrt{1+x^2+x^4}}{2(i-x^2)} + \frac{ix\sqrt{1+x^2+x^4}}{2(i+x^2)} - \frac{6x\sqrt{1+x^2+\cdots}}{1+x^2}$$

$$= \frac{ix\sqrt{1+x^2+x^4}}{2(i-x^2)} + \frac{ix\sqrt{1+x^2+x^4}}{2(i+x^2)} - \frac{6x\sqrt{1+x^2+\cdots}}{1+x^2}$$

$$= \frac{ix\sqrt{1+x^2+x^4}}{2(i-x^2)} + \frac{ix\sqrt{1+x^2+x^4}}{2(i+x^2)} - \frac{6x\sqrt{1+x^2+\cdots}}{1+x^2}$$

Mathematica [C] time = 1.46, size = 345, normalized size = 4.11

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^4)*Sqrt[1 + x^2 + x^4]*(1 + x^2 + 3*x^4 + x^6 + x^8))/((
1 + x^4)^3*(1 - x^2 + x^4)),x]
```

```
[Out] ((x*(1 + x^2 + x^4)*(9 + 2*x^2 + 9*x^4))/(1 + x^4)^2 + Sqrt[1 + (-1)^(1/3)*
x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(((17*I)/2)*(I + Sqrt[3])*EllipticF[I*ArcSinh
[(-1)^(5/6)*x], (-1)^(2/3)] - 48*(-1 + (-1)^(1/3))*EllipticPi[-1, I*ArcSinh
[(-1)^(5/6)*x], (-1)^(2/3)] - 48*EllipticPi[-(-1)^(2/3), I*ArcSinh[(-1)^(5/
6)*x], (-1)^(2/3)] + 48*(-1)^(1/3)*EllipticPi[-(-1)^(2/3), I*ArcSinh[(-1)^(
5/6)*x], (-1)^(2/3)] - 96*(-1)^(2/3)*EllipticPi[-(-1)^(2/3), I*ArcSinh[(-1)
^(5/6)*x], (-1)^(2/3)] - 31*EllipticPi[-(-1)^(5/6), I*ArcSinh[(-1)^(5/6)*x]
, (-1)^(2/3)] + 31*(-1)^(1/3)*EllipticPi[-(-1)^(5/6), I*ArcSinh[(-1)^(5/6)*
x], (-1)^(2/3)] - 31*EllipticPi[(-1)^(5/6), I*ArcSinh[(-1)^(5/6)*x], (-1)^(
2/3)] + 31*(-1)^(1/3)*EllipticPi[(-1)^(5/6), I*ArcSinh[(-1)^(5/6)*x], (-1)^
(2/3)]))/(8*Sqrt[1 + x^2 + x^4])
```

IntegrateAlgebraic [A] time = 1.55, size = 84, normalized size = 1.00

$$\frac{31}{8}\tanh^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+x^2+1}}\right)+\frac{\sqrt{x^4+x^2+1}\,\left(9x^5+2x^3+9x\right)}{8\left(x^4+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*Sqrt[1 + x^2 + x^4]*(1 + x^2 + 3*x^4 + x^6 +
 x^8))/((1 + x^4)^3*(1 - x^2 + x^4)),x]
```

```
[Out] (Sqrt[1 + x^2 + x^4]*(9*x + 2*x^3 + 9*x^5))/(8*(1 + x^4)^2) + (31*ArcTanh[x
/Sqrt[1 + x^2 + x^4]])/8 - 3*Sqrt[2]*ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^2 + x^4
]]
```

fricas [B] time = 0.53, size = 171, normalized size = 2.04

$$\frac{12\sqrt{2}\left(x^8+2x^4+1\right)\log\left(-\frac{x^8+14x^6+19x^4-4\sqrt{2}\left(x^5+3x^3+x\right)\sqrt{x^4+x^2+1}+14x^2+1}{x^8-2x^6+3x^4-2x^2+1}\right)+31\left(x^8+2x^4+1\right)\log\left(-\frac{x^4+2x^2+2\sqrt{x^4+x^2+1}\,x+1}{x^4+1}\right)+2\left(9x^5+2x^3+9x\right)\sqrt{x^4+x^2+1}}{16\left(x^8+2x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+x^2+1)^(1/2)*(x^8+x^6+3*x^4+x^2+1)/(x^4+1)^3/(x^4-x^
2+1),x, algorithm="fricas")
```

```
[Out] 1/16*(12*sqrt(2)*(x^8 + 2*x^4 + 1)*log(-(x^8 + 14*x^6 + 19*x^4 - 4*sqrt(2)*
(x^5 + 3*x^3 + x)*sqrt(x^4 + x^2 + 1) + 14*x^2 + 1)/(x^8 - 2*x^6 + 3*x^4 -
2*x^2 + 1)) + 31*(x^8 + 2*x^4 + 1)*log(-(x^4 + 2*x^2 + 2*sqrt(x^4 + x^2 + 1
)*x + 1)/(x^4 + 1)) + 2*(9*x^5 + 2*x^3 + 9*x)*sqrt(x^4 + x^2 + 1))/(x^8 + 2
*x^4 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^8+x^6+3x^4+x^2+1\right)\sqrt{x^4+x^2+1}\,\left(x^4-1\right)}{\left(x^4-x^2+1\right)\left(x^4+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+x^2+1)^(1/2)*(x^8+x^6+3*x^4+x^2+1)/(x^4+1)^3/(x^4-x^
2+1),x, algorithm="giac")
```

```
[Out] integrate((x^8 + x^6 + 3*x^4 + x^2 + 1)*sqrt(x^4 + x^2 + 1)*(x^4 - 1)/((x^4
 - x^2 + 1)*(x^4 + 1)^3), x)
```

maple [C] time = 0.18, size = 862, normalized size = 10.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^4+x^2+1)^(1/2)*(x^8+x^6+3*x^4+x^2+1)/(x^4+1)^3/(x^4-x^2+1),x
)
```

```
[Out] 17/4/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2
*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(
1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+1/4*sum(_alpha*(_alpha^2-3)*(-1/(_alpha^2)
^(1/2)*arctanh(1/10*(2*_alpha^2+1)*(-3*_alpha^2+5*x^2+4)/(_alpha^2)^(1/2)/(
x^4+x^2+1)^(1/2))+2^(1/2)*_alpha^3/(I*3^(1/2)-1)^(1/2)*(x^2+2-I*3^(1/2)*x^2
)^(1/2)*(x^2+2+I*3^(1/2)*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*
I*3^(1/2))^(1/2)*x,1/2*_alpha^2+1/2*I*_alpha^2*3^(1/2),(-1/2-1/2*I*3^(1/2))
^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))),_alpha=RootOf(_Z^4+1))+1/4*x^3*(x^4+x^2
+1)^(1/2)/(x^4+1)^2-2*(-1/8*x^3-1/16*x)*(x^4+x^2+1)^(1/2)/(x^4+1)-1/32*sum(
_alpha*(2*_alpha^2-1)*(-1/(_alpha^2)^(1/2)*arctanh(1/10*(2*_alpha^2+1)*(-3*
_alpha^2+5*x^2+4)/(_alpha^2)^(1/2)/(x^4+x^2+1)^(1/2))+2^(1/2)*_alpha^3/(I*3
^(1/2)-1)^(1/2)*(x^2+2-I*3^(1/2)*x^2)^(1/2)*(x^2+2+I*3^(1/2)*x^2)^(1/2)/(x^
4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,1/2*_alpha^2+1/2*I*_
alpha^2*3^(1/2),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))),_al
pha=RootOf(_Z^4+1))+3/4*2^(1/2)*sum(_alpha*(-1/(_alpha^2)^(1/2)*arctanh(1/2
8*(2*_alpha^2+1)*(-3*_alpha^2+7*x^2+8)*2^(1/2)/(_alpha^2)^(1/2)/(x^4+x^2+1)
^(1/2))-2*(-_alpha^3+_alpha)/(I*3^(1/2)-1)^(1/2)*(x^2+2-I*3^(1/2)*x^2)^(1/2
)*(x^2+2+I*3^(1/2)*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1
/2))^(1/2)*x,1/2*I*_alpha^2*3^(1/2)+1/2*_alpha^2-1/2-1/2*I*3^(1/2),(-1/2-1/
2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))),_alpha=RootOf(_Z^4-_Z^2+1))
+(-1/4*x^3+x)*(x^4+x^2+1)^(1/2)/(x^4+1)-1/16*sum(_alpha*(3*_alpha^2+4)*(-1/
(_alpha^2)^(1/2)*arctanh(1/10*(2*_alpha^2+1)*(-3*_alpha^2+5*x^2+4)/(_alpha^
2)^(1/2)/(x^4+x^2+1)^(1/2))+2^(1/2)*_alpha^3/(I*3^(1/2)-1)^(1/2)*(x^2+2-I*3
^(1/2)*x^2)^(1/2)*(x^2+2+I*3^(1/2)*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi(
(-1/2+1/2*I*3^(1/2))^(1/2)*x,1/2*_alpha^2+1/2*I*_alpha^2*3^(1/2),(-1/2-1/2*
I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))),_alpha=RootOf(_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+x^6+3\,x^4+x^2+1\right)\sqrt{x^4+x^2+1}\,\left(x^4-1\right)}{\left(x^4-x^2+1\right)\left(x^4+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+x^2+1)^(1/2)*(x^8+x^6+3*x^4+x^2+1)/(x^4+1)^3/(x^4-x^
2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^8 + x^6 + 3*x^4 + x^2 + 1)*sqrt(x^4 + x^2 + 1)*(x^4 - 1)/((x^4
 - x^2 + 1)*(x^4 + 1)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-1\right)\,\sqrt{x^4+x^2+1}\,\left(x^8+x^6+3\,x^4+x^2+1\right)}{{\left(x^4+1\right)}^3\,\left(x^4-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(x^2 + x^4 + 1)^(1/2)*(x^2 + 3*x^4 + x^6 + x^8 + 1))/((x^4 +
 1)^3*(x^4 - x^2 + 1)),x)
```

```
[Out] int(((x^4 - 1)*(x^2 + x^4 + 1)^(1/2)*(x^2 + 3*x^4 + x^6 + x^8 + 1))/((x^4 +
 1)^3*(x^4 - x^2 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**4+x**2+1)**(1/2)*(x**8+x**6+3*x**4+x**2+1)/(x**4+1)*
*3/(x**4-x**2+1),x)
```

```
[Out] Timed out
```

3.1010 $\int \frac{(1+x^6)(-1+x^3+x^6)\sqrt{1+x^{12}}}{x^7(-1-x^3+x^6)}\,dx$

Optimal. Leaf size=84

$$\log\left(\sqrt{x^{12}+1}+x^6-1\right)+\frac{\sqrt{x^{12}+1}\left(x^6+4x^3-1\right)}{6x^6}-\frac{4\tanh^{-1}\left(\frac{\sqrt{3}x^3}{\sqrt{x^{12}+1}+x^6-x^3-1}\right)}{\sqrt{3}}-3\log(x)$$

Rubi [C] time = 2.34, antiderivative size = 666, normalized size of antiderivative = 7.93, number of steps used = 45, number of rules used = 22, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {6728, 275, 277, 215, 305, 220, 1196, 266, 50, 63, 207, 6715, 1729, 1209, 1198, 1217, 1707, 1248, 735, 844, 725, 206}

$$-\frac{1}{6}\left(1+\sqrt{5}\right)\sqrt{x^{12}+1}-\frac{1}{6}\left(1-\sqrt{5}\right)\sqrt{x^{12}+1}+\frac{\sqrt{x^{12}+1}}{2}-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^{12}+1}\right)+\frac{1}{6}\left(1+\sqrt{5}\right)\sinh^{-1}\left(x^6\right)+\frac{1}{6}\left(1-\sqrt{5}\right)\sinh^{-1}\left(x^6\right)+\frac{1}{6}\sinh^{-1}\left(x^6\right)-\frac{\sqrt{x^{12}+1}}{6x^6}+\frac{\tanh^{-1}\left(\frac{\left(3-\sqrt{5}\right)x^6+2}{\sqrt{6\left(3-\sqrt{5}\right)}\sqrt{x^{12}+1}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\left(3+\sqrt{5}\right)x^6+2}{\sqrt{6\left(3+\sqrt{5}\right)}\sqrt{x^{12}+1}}\right)}{\sqrt{3}}+\frac{2\sqrt{x^{12}+1}}{3x^3}-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}x^3}{\sqrt{x^{12}+1}}\right)}{\sqrt{3}}+\frac{\left(5+\sqrt{5}\right)\left(x^6+1\right)\sqrt{\frac{x^{12}+1}{\left(x^6+1\right)^2}}F\left(2\tan^{-1}\left(x^3\right)|\frac{1}{2}\right)}{6\sqrt{x^{12}+1}}-\frac{\left(3+\sqrt{5}\right)\left(x^6+1\right)\sqrt{\frac{x^{12}+1}{\left(x^6+1\right)^2}}F\left(2\tan^{-1}\left(x^3\right)|\frac{1}{2}\right)}{\left(5+\sqrt{5}\right)\sqrt{x^{12}+1}}+\frac{\left(5-\sqrt{5}\right)\left(x^6+1\right)\sqrt{\frac{x^{12}+1}{\left(x^6+1\right)^2}}F\left(2\tan^{-1}\left(x^3\right)|\frac{1}{2}\right)}{6\sqrt{x^{12}+1}}-\frac{\left(3-\sqrt{5}\right)\left(x^6+1\right)\sqrt{\frac{x^{12}+1}{\left(x^6+1\right)^2}}F\left(2\tan^{-1}\left(x^3\right)|\frac{1}{2}\right)}{\left(5-\sqrt{5}\right)\sqrt{x^{12}+1}}-\frac{2\left(x^6+1\right)\sqrt{\frac{x^{12}+1}{\left(x^6+1\right)^2}}F\left(2\tan^{-1}\left(x^3\right)|\frac{1}{2}\right)}{3\sqrt{x^{12}+1}}+\frac{\left(3+\sqrt{5}\right)\left(x^6+1\right)\sqrt{\frac{x^{12}+1}{\left(x^6+1\right)^2}}\Pi\left(\frac{5}{4};2\tan^{-1}\left(x^3\right)|\frac{1}{2}\right)}{2\left(5+3\sqrt{5}\right)\sqrt{x^{12}+1}}+\frac{\left(3-\sqrt{5}\right)\left(x^6+1\right)\sqrt{\frac{x^{12}+1}{\left(x^6+1\right)^2}}\Pi\left(\frac{5}{4};2\tan^{-1}\left(x^3\right)|\frac{1}{2}\right)}{2\left(5-3\sqrt{5}\right)\sqrt{x^{12}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((1 + x^6)*(-1 + x^3 + x^6)*Sqrt[1 + x^12])/(x^7*(-1 - x^3 + x^6)),x]
```

```
[Out] Sqrt[1 + x^12]/2 - ((1 - Sqrt[5])*Sqrt[1 + x^12])/6 - ((1 + Sqrt[5])*Sqrt[1
 + x^12])/6 - Sqrt[1 + x^12]/(6*x^6) + (2*Sqrt[1 + x^12])/(3*x^3) + ArcSinh
[x^6]/6 + ((1 - Sqrt[5])*ArcSinh[x^6])/6 + ((1 + Sqrt[5])*ArcSinh[x^6])/6 -
 (2*ArcTanh[(Sqrt[3]*x^3)/Sqrt[1 + x^12]])/Sqrt[3] + ArcTanh[(2 + (3 - Sqrt
[5])*x^6)/(Sqrt[6*(3 - Sqrt[5])]*Sqrt[1 + x^12])]/Sqrt[3] - ArcTanh[(2 + (3
 + Sqrt[5])*x^6)/(Sqrt[6*(3 + Sqrt[5])]*Sqrt[1 + x^12])]/Sqrt[3] - ArcTanh[
Sqrt[1 + x^12]]/2 - (2*(1 + x^6)*Sqrt[(1 + x^12)/(1 + x^6)^2]*EllipticF[2*A
rcTan[x^3], 1/2])/(3*Sqrt[1 + x^12]) - ((3 - Sqrt[5])*(1 + x^6)*Sqrt[(1 + x
^12)/(1 + x^6)^2]*EllipticF[2*ArcTan[x^3], 1/2])/((5 - Sqrt[5])*Sqrt[1 + x^
12]) + ((5 - Sqrt[5])*(1 + x^6)*Sqrt[(1 + x^12)/(1 + x^6)^2]*EllipticF[2*Ar
cTan[x^3], 1/2])/(6*Sqrt[1 + x^12]) - ((3 + Sqrt[5])*(1 + x^6)*Sqrt[(1 + x^
12)/(1 + x^6)^2]*EllipticF[2*ArcTan[x^3], 1/2])/((5 + Sqrt[5])*Sqrt[1 + x^1
2]) + ((5 + Sqrt[5])*(1 + x^6)*Sqrt[(1 + x^12)/(1 + x^6)^2]*EllipticF[2*Arc
Tan[x^3], 1/2])/(6*Sqrt[1 + x^12]) + ((3 - Sqrt[5])*(1 + x^6)*Sqrt[(1 + x^1
2)/(1 + x^6)^2]*EllipticPi[5/4, 2*ArcTan[x^3], 1/2])/(2*(5 - 3*Sqrt[5])*Sqr
t[1 + x^12]) + ((3 + Sqrt[5])*(1 + x^6)*Sqrt[(1 + x^12)/(1 + x^6)^2]*Ellipt
icPi[5/4, 2*ArcTan[x^3], 1/2])/(2*(5 + 3*Sqrt[5])*Sqrt[1 + x^12])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Dist[d, I
nt[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, Int[(x*(a + c*x^4)^p)/(d
^2 - e^2*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(1+x^6\right)\left(-1+x^3+x^6\right)\sqrt{1+x^{12}}}{x^7\left(-1-x^3+x^6\right)}\,dx = \int\left(\frac{\sqrt{1+x^{12}}}{x^7}-\frac{2\sqrt{1+x^{12}}}{x^4}+\frac{3\sqrt{1+x^{12}}}{x}-\frac{2x^2\left(-3+x^3\right)\sqrt{1+x^{12}}}{-1-x^3+x^6}\right.$$

$$= -\left(2\int\frac{\sqrt{1+x^{12}}}{x^4}\,dx\right)-2\int\frac{x^2\left(-3+x^3\right)\sqrt{1+x^{12}}}{-1-x^3+x^6}\,dx+3\int\frac{\sqrt{1+}}{x}$$

$$= \frac{1}{6}\,\text{Subst}\left(\int\frac{\sqrt{1+x^2}}{x^2}\,dx,x,x^6\right)+\frac{1}{4}\,\text{Subst}\left(\int\frac{\sqrt{1+x}}{x}\,dx,x,x^{12}\right)-$$

$$= \frac{\sqrt{1+x^{12}}}{2}-\frac{\sqrt{1+x^{12}}}{6x^6}+\frac{2\sqrt{1+x^{12}}}{3x^3}+\frac{1}{6}\,\text{Subst}\left(\int\frac{1}{\sqrt{1+x^2}}\,dx,x,\right.$$

$$= \frac{\sqrt{1+x^{12}}}{2}-\frac{\sqrt{1+x^{12}}}{6x^6}+\frac{2\sqrt{1+x^{12}}}{3x^3}+\frac{1}{6}\sinh^{-1}\left(x^6\right)+\frac{1}{2}\,\text{Subst}\left(\int\right.$$

$$= \frac{\sqrt{1+x^{12}}}{2}-\frac{\sqrt{1+x^{12}}}{6x^6}+\frac{2\sqrt{1+x^{12}}}{3x^3}-\frac{4x^3\sqrt{1+x^{12}}}{3\left(1+x^6\right)}+\frac{1}{6}\sinh^{-1}\left(x^6\right.$$

$$= \frac{\sqrt{1+x^{12}}}{2}-\frac{\sqrt{1+x^{12}}}{6x^6}+\frac{2\sqrt{1+x^{12}}}{3x^3}-\frac{4x^3\sqrt{1+x^{12}}}{3\left(1+x^6\right)}+\frac{1}{6}\sinh^{-1}\left(x^6\right.$$

$$= \frac{\sqrt{1+x^{12}}}{2}-\frac{1}{6}\left(1-\sqrt{5}\right)\sqrt{1+x^{12}}-\frac{1}{6}\left(1+\sqrt{5}\right)\sqrt{1+x^{12}}-\frac{\sqrt{1+x}}{6x^6}$$

$$= \frac{\sqrt{1+x^{12}}}{2}-\frac{1}{6}\left(1-\sqrt{5}\right)\sqrt{1+x^{12}}-\frac{1}{6}\left(1+\sqrt{5}\right)\sqrt{1+x^{12}}-\frac{\sqrt{1+x}}{6x^6}$$

$$= \frac{\sqrt{1+x^{12}}}{2}-\frac{1}{6}\left(1-\sqrt{5}\right)\sqrt{1+x^{12}}-\frac{1}{6}\left(1+\sqrt{5}\right)\sqrt{1+x^{12}}-\frac{\sqrt{1+x}}{6x^6}$$

$$= \frac{\sqrt{1+x^{12}}}{2}-\frac{1}{6}\left(1-\sqrt{5}\right)\sqrt{1+x^{12}}-\frac{1}{6}\left(1+\sqrt{5}\right)\sqrt{1+x^{12}}-\frac{\sqrt{1+x}}{6x^6}$$

Mathematica [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(1+x^6\right)\left(-1+x^3+x^6\right)\sqrt{1+x^{12}}}{x^7\left(-1-x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^6)*(-1 + x^3 + x^6)*Sqrt[1 + x^12])/(x^7*(-1 - x^3 + x^6)
),x]
```

```
[Out] Integrate[((1 + x^6)*(-1 + x^3 + x^6)*Sqrt[1 + x^12])/(x^7*(-1 - x^3 + x^6)
), x]
```

IntegrateAlgebraic [A] time = 37.15, size = 84, normalized size = 1.00

$$\log\left(\sqrt{x^{12}+1}+x^6-1\right)+\frac{\sqrt{x^{12}+1}\left(x^6+4x^3-1\right)}{6x^6}-\frac{4\tanh^{-1}\left(\frac{\sqrt{3}x^3}{\sqrt{x^{12}+1}+x^6-x^3-1}\right)}{\sqrt{3}}-3\log(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^6)*(-1 + x^3 + x^6)*Sqrt[1 + x^12])/(x^7*(-1 - x
^3 + x^6)),x]
```

```
[Out] ((-1 + 4*x^3 + x^6)*Sqrt[1 + x^12])/(6*x^6) - (4*ArcTanh[(Sqrt[3]*x^3)/(-1
- x^3 + x^6 + Sqrt[1 + x^12])])/Sqrt[3] - 3*Log[x] + Log[-1 + x^6 + Sqrt[1
+ x^12]]
```

fricas [A] time = 0.52, size = 120, normalized size = 1.43

$$\frac{2\sqrt{3}\,x^6\log\left(\frac{2\,x^{12}+2\,x^9+x^6-2\,x^3-\sqrt{3}\,\sqrt{x^{12}+1}\left(x^6+2\,x^3-1\right)+2}{x^{12}-2\,x^9-x^6+2\,x^3+1}\right)+6\,x^6\log\left(\frac{x^6+\sqrt{x^{12}+1}-1}{x^3}\right)+\sqrt{x^{12}+1}\left(x^6+4\,x^3-1\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)*(x^6+x^3-1)*(x^12+1)^(1/2)/x^7/(x^6-x^3-1),x, algorithm="
fricas")
```

```
[Out] 1/6*(2*sqrt(3)*x^6*log((2*x^12 + 2*x^9 + x^6 - 2*x^3 - sqrt(3)*sqrt(x^12 +
1)*(x^6 + 2*x^3 - 1) + 2)/(x^12 - 2*x^9 - x^6 + 2*x^3 + 1)) + 6*x^6*log((x^
6 + sqrt(x^12 + 1) - 1)/x^3) + sqrt(x^12 + 1)*(x^6 + 4*x^3 - 1))/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{12}+1}\left(x^6+x^3-1\right)\left(x^6+1\right)}{\left(x^6-x^3-1\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)*(x^6+x^3-1)*(x^12+1)^(1/2)/x^7/(x^6-x^3-1),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(x^12 + 1)*(x^6 + x^3 - 1)*(x^6 + 1)/((x^6 - x^3 - 1)*x^7), x
)
```

maple [C] time = 1.14, size = 122, normalized size = 1.45

$$\frac{4x^{15}-x^{12}+4x^3-1}{6x^6\sqrt{x^{12}+1}}+\frac{\sqrt{x^{12}+1}}{6}-\ln\left(\frac{-x^6+\sqrt{x^{12}+1}+1}{x^3}\right)-\frac{2\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2-3\right)x^6+2\,\mathrm{RootOf}\left(\_Z^2-3\right)x^3+3\sqrt{x^{12}+1}-\mathrm{RootOf}\left(\_Z^2-3\right)}{x^6-x^3-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)*(x^6+x^3-1)*(x^12+1)^(1/2)/x^7/(x^6-x^3-1),x)
```

```
[Out] 1/6*(4*x^15-x^12+4*x^3-1)/x^6/(x^12+1)^(1/2)+1/6*(x^12+1)^(1/2)-ln((-x^6+(x
^12+1)^(1/2)+1)/x^3)-2/3*RootOf(_Z^2-3)*ln((RootOf(_Z^2-3)*x^6+2*RootOf(_Z^
2-3)*x^3+3*(x^12+1)^(1/2)-RootOf(_Z^2-3))/(x^6-x^3-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{12}+1}\left(x^6+x^3-1\right)\left(x^6+1\right)}{\left(x^6-x^3-1\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)*(x^6+x^3-1)*(x^12+1)^(1/2)/x^7/(x^6-x^3-1),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(x^12 + 1)*(x^6 + x^3 - 1)*(x^6 + 1)/((x^6 - x^3 - 1)*x^7), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^6+1\right)\,\sqrt{x^{12}+1}\,\left(x^6+x^3-1\right)}{x^7\,\left(-x^6+x^3+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^6 + 1)*(x^12 + 1)^(1/2)*(x^3 + x^6 - 1))/(x^7*(x^3 - x^6 + 1)),x)
```

```
[Out] int(-((x^6 + 1)*(x^12 + 1)^(1/2)*(x^3 + x^6 - 1))/(x^7*(x^3 - x^6 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)*(x**6+x**3-1)*(x**12+1)**(1/2)/x**7/(x**6-x**3-1),x)
```

```
[Out] Timed out
```

3.1011 $\int \sqrt{d+c\sqrt{b+ax}}\,dx$

Optimal. Leaf size=84

$$\frac{4\left(3ac^2x+3bc^2-2d^2\right)\sqrt{c\sqrt{ax+b}+d}}{15ac^2}+\frac{4d\sqrt{ax+b}\sqrt{c\sqrt{ax+b}+d}}{15ac}$$

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 0.67, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {247, 190, 43}

$$\frac{4\left(c\sqrt{ax+b}+d\right)^{5/2}}{5ac^2}-\frac{4d\left(c\sqrt{ax+b}+d\right)^{3/2}}{3ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + c*Sqrt[b + a*x]],x]
```

```
[Out] (-4*d*(d + c*Sqrt[b + a*x])^(3/2))/(3*a*c^2) + (4*(d + c*Sqrt[b + a*x])^(5/
2))/(5*a*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{d+c\sqrt{b+ax}}\,dx &= \frac{\text{Subst}\left(\int \sqrt{d+c\sqrt{x}}\,dx,x,b+ax\right)}{a}\\
&= \frac{2\,\text{Subst}\left(\int x\sqrt{d+cx}\,dx,x,\sqrt{b+ax}\right)}{a}\\
&= \frac{2\,\text{Subst}\left(\int \left(-\frac{d\sqrt{d+cx}}{c}+\frac{(d+cx)^{3/2}}{c}\right)dx,x,\sqrt{b+ax}\right)}{a}\\
&= -\frac{4d\left(d+c\sqrt{b+ax}\right)^{3/2}}{3ac^2}+\frac{4\left(d+c\sqrt{b+ax}\right)^{5/2}}{5ac^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.51

$$\frac{4\left(c\sqrt{ax+b}+d\right)^{3/2}\left(3c\sqrt{ax+b}-2d\right)}{15ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + c*Sqrt[b + a*x]],x]
```

```
[Out] (4*(d + c*Sqrt[b + a*x])^(3/2)*(-2*d + 3*c*Sqrt[b + a*x]))/(15*a*c^2)
```

IntegrateAlgebraic [A] time = 0.03, size = 55, normalized size = 0.65

$$\frac{4\sqrt{c\sqrt{ax+b}+d}\left(3c^2(ax+b)+cd\sqrt{ax+b}-2d^2\right)}{15ac^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + c*Sqrt[b + a*x]],x]
```

```
[Out] (4*Sqrt[d + c*Sqrt[b + a*x]]*(-2*d^2 + c*d*Sqrt[b + a*x] + 3*c^2*(b + a*x))
)/(15*a*c^2)
```

fricas [A] time = 0.53, size = 50, normalized size = 0.60

$$\frac{4\left(3\,ac^2x+3\,bc^2+\sqrt{ax+b}\,cd-2\,d^2\right)\sqrt{\sqrt{ax+b}\,c+d}}{15\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+c*(a*x+b)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/15*(3*a*c^2*x + 3*b*c^2 + sqrt(a*x + b)*c*d - 2*d^2)*sqrt(sqrt(a*x + b)*c
 + d)/(a*c^2)
```

giac [A] time = 0.22, size = 99, normalized size = 1.18

$$\frac{4\left(\frac{5\left(\left(\sqrt{ax+b}\,c+d\right)^{\frac{3}{2}}-3\,\sqrt{\sqrt{ax+b}\,c+d}\,d\right)d}{c}+\frac{3\left(\sqrt{ax+b}\,c+d\right)^{\frac{5}{2}}-10\left(\sqrt{ax+b}\,c+d\right)^{\frac{3}{2}}d+15\,\sqrt{\sqrt{ax+b}\,c+d}\,d^2}{c}\right)}{15\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+c*(a*x+b)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(5*((sqrt(a*x + b)*c + d)^(3/2) - 3*sqrt(sqrt(a*x + b)*c + d)*d)*d/c +
 (3*(sqrt(a*x + b)*c + d)^(5/2) - 10*(sqrt(a*x + b)*c + d)^(3/2)*d + 15*sqr
t(sqrt(a*x + b)*c + d)*d^2)/c)/(a*c)
```

maple [A] time = 0.01, size = 41, normalized size = 0.49

$$\frac{\frac{4\left(d+c\sqrt{ax+b}\right)^{\frac{5}{2}}}{5}-\frac{4d\left(d+c\sqrt{ax+b}\right)^{\frac{3}{2}}}{3}}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+c*(a*x+b)^(1/2))^(1/2),x)
```

```
[Out] 4/a/c^2*(1/5*(d+c*(a*x+b)^(1/2))^(5/2)-1/3*d*(d+c*(a*x+b)^(1/2))^(3/2))
```

maxima [A] time = 0.32, size = 43, normalized size = 0.51

$$\frac{4\left(\frac{3\left(\sqrt{ax+b}\,c+d\right)^{\frac{5}{2}}}{c^2}-\frac{5\left(\sqrt{ax+b}\,c+d\right)^{\frac{3}{2}}d}{c^2}\right)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+c*(a*x+b)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/15*(3*(sqrt(a*x + b)*c + d)^(5/2)/c^2 - 5*(sqrt(a*x + b)*c + d)^(3/2)*d/c
^2)/a
```

mupad [B] time = 0.86, size = 44, normalized size = 0.52

$$\frac{4\left(d+c\sqrt{b+a\,x}\right)^{5/2}}{5\,a\,c^2}-\frac{4\,d\left(d+c\sqrt{b+a\,x}\right)^{3/2}}{3\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + c*(b + a*x)^(1/2))^(1/2),x)
```

```
[Out] (4*(d + c*(b + a*x)^(1/2))^(5/2))/(5*a*c^2) - (4*d*(d + c*(b + a*x)^(1/2))^
(3/2))/(3*a*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c\sqrt{ax+b}+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+c*(a*x+b)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(c*sqrt(a*x + b) + d), x)
```

3.1012 $\int \frac{1+2x^2}{x\left(1+x^2\right)^{2/3}}\,dx$

Optimal. Leaf size=85

$$3\sqrt[3]{x^2+1}+\frac{1}{2}\log\left(\sqrt[3]{x^2+1}-1\right)-\frac{1}{4}\log\left(\left(x^2+1\right)^{2/3}+\sqrt[3]{x^2+1}+1\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {446, 80, 57, 618, 204, 31}

$$3\sqrt[3]{x^2+1}+\frac{3}{4}\log\left(1-\sqrt[3]{x^2+1}\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(x*(1 + x^2)^(2/3)),x]
```

```
[Out] 3*(1 + x^2)^(1/3) - (Sqrt[3]*ArcTan[(1 + 2*(1 + x^2)^(1/3))/Sqrt[3]])/2 - L
og[x]/2 + (3*Log[1 - (1 + x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1+2x^2}{x\left(1+x^2\right)^{2/3}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1+2x}{x(1+x)^{2/3}}\,dx,x,x^2\right)$$
$$= 3\sqrt[3]{1+x^2}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{2/3}}\,dx,x,x^2\right)$$
$$= 3\sqrt[3]{1+x^2}-\frac{\log(x)}{2}-\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1+x^2}\right)-\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,\sqrt[3]{1\right.$$
$$= 3\sqrt[3]{1+x^2}-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1+x^2}\right)+\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{1+x^2}\right)$$
$$= 3\sqrt[3]{1+x^2}-\frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^2}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1+x^2}\right)$$

Mathematica [A] time = 0.02, size = 84, normalized size = 0.99

$$3\sqrt[3]{x^2+1}+\frac{1}{2}\log\left(1-\sqrt[3]{x^2+1}\right)-\frac{1}{4}\log\left(\left(x^2+1\right)^{2/3}+\sqrt[3]{x^2+1}+1\right)-\frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(x*(1 + x^2)^(2/3)),x]
```

```
[Out] 3*(1 + x^2)^(1/3) - (Sqrt[3]*ArcTan[(1 + 2*(1 + x^2)^(1/3))/Sqrt[3]])/2 + L
og[1 - (1 + x^2)^(1/3)]/2 - Log[1 + (1 + x^2)^(1/3) + (1 + x^2)^(2/3)]/4
```

IntegrateAlgebraic [A] time = 0.05, size = 85, normalized size = 1.00

$$3\sqrt[3]{x^2+1}+\frac{1}{2}\log\left(\sqrt[3]{x^2+1}-1\right)-\frac{1}{4}\log\left(\left(x^2+1\right)^{2/3}+\sqrt[3]{x^2+1}+1\right)-\frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x^2)/(x*(1 + x^2)^(2/3)),x]
```

```
[Out] 3*(1 + x^2)^(1/3) - (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 + x^2)^(1/3))/Sqrt[3]
])/2 + Log[-1 + (1 + x^2)^(1/3)]/2 - Log[1 + (1 + x^2)^(1/3) + (1 + x^2)^(2
/3)]/4
```

fricas [A] time = 0.41, size = 65, normalized size = 0.76

$$-\frac{1}{2}\sqrt{3}\,\arctan\left(\frac{2}{3}\sqrt{3}\left(x^2+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+3\left(x^2+1\right)^{\frac{1}{3}}-\frac{1}{4}\log\left(\left(x^2+1\right)^{\frac{2}{3}}+\left(x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/x/(x^2+1)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan(2/3*sqrt(3)*(x^2 + 1)^(1/3) + 1/3*sqrt(3)) + 3*(x^2 + 1
)^(1/3) - 1/4*log((x^2 + 1)^(2/3) + (x^2 + 1)^(1/3) + 1) + 1/2*log((x^2 + 1
)^(1/3) - 1)
```

giac [A] time = 0.24, size = 63, normalized size = 0.74

$$-\frac{1}{2}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^2+1\right)^{\frac{1}{3}}+1\right)\right)+3\left(x^2+1\right)^{\frac{1}{3}}-\frac{1}{4}\log\left(\left(x^2+1\right)^{\frac{2}{3}}+\left(x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/x/(x^2+1)^(2/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 + 1)^(1/3) + 1)) + 3*(x^2 + 1)^(1/3
) - 1/4*log((x^2 + 1)^(2/3) + (x^2 + 1)^(1/3) + 1) + 1/2*log((x^2 + 1)^(1/3
) - 1)
```

maple [C] time = 0.28, size = 62, normalized size = 0.73

$$x^2\,\mathrm{hypergeom}\left(\left[\frac{2}{3},1\right],[2],-x^2\right)+\frac{-\frac{2\Gamma\left(\frac{2}{3}\right)x^2\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],-x^2\right)}{3}+\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)\right)\Gamma\left(\frac{2}{3}\right)}{2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/x/(x^2+1)^(2/3),x)
```

```
[Out] x^2*hypergeom([2/3,1],[2],-x^2)+1/2/GAMMA(2/3)*(-2/3*GAMMA(2/3)*x^2*hyperge
om([1,1,5/3],[2,2],-x^2)+(1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x))*GAMMA(2/3))
```

maxima [A] time = 0.43, size = 63, normalized size = 0.74

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^2+1\right)^{\frac{1}{3}}+1\right)\right)+3\left(x^2+1\right)^{\frac{1}{3}}-\frac{1}{4}\log\left(\left(x^2+1\right)^{\frac{2}{3}}+\left(x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/x/(x^2+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 + 1)^(1/3) + 1)) + 3*(x^2 + 1)^(1/3
) - 1/4*log((x^2 + 1)^(2/3) + (x^2 + 1)^(1/3) + 1) + 1/2*log((x^2 + 1)^(1/3
) - 1)
```

mupad [B] time = 1.00, size = 79, normalized size = 0.93

$$\frac{\ln\left(\frac{9\left(x^2+1\right)^{1/3}}{4}-\frac{9}{4}\right)}{2}+3\left(x^2+1\right)^{1/3}+\ln\left(\frac{9\left(x^2+1\right)^{1/3}}{2}+\frac{9}{4}-\frac{\sqrt{3}\,9\mathrm{i}}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-\ln\left(\frac{9\left(x^2+1\right)^{1/3}}{2}+\frac{9}{4}+\frac{\sqrt{3}\,9\mathrm{i}}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(x*(x^2 + 1)^(2/3)),x)
```

```
[Out] log((9*(x^2 + 1)^(1/3))/4 - 9/4)/2 + 3*(x^2 + 1)^(1/3) + log((9*(x^2 + 1)^(
1/3))/2 - (3^(1/2)*9i)/4 + 9/4)*((3^(1/2)*1i)/4 - 1/4) - log((3^(1/2)*9i)/4
 + (9*(x^2 + 1)^(1/3))/2 + 9/4)*((3^(1/2)*1i)/4 + 1/4)
```

sympy [C] time = 7.39, size = 41, normalized size = 0.48

$$3\sqrt[3]{x^2+1}-\frac{\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^2}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/x/(x**2+1)**(2/3),x)
```

```
[Out] 3*(x**2 + 1)**(1/3) - gamma(2/3)*hyper((2/3, 2/3), (5/3,), exp_polar(I*pi)/
x**2)/(2*x**(4/3)*gamma(5/3))
```

3.1013 $\int \frac{x}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(-1+k^2x^2\right)}\,dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}\right)}{2k(k+1)}-\frac{\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}\right)}{2(k-1)k}$$

Rubi [C] time = 1.69, antiderivative size = 203, normalized size of antiderivative = 2.39, number of steps used = 13, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {6718, 6725, 175, 115, 168, 538, 537}

$$\frac{\sqrt{1-x}\sqrt{x}\sqrt{\frac{k^2(1-x)}{1-k^2}+1}\,\Pi\left(-\frac{k}{1-k};\sin^{-1}\left(\sqrt{1-x}\right)|-\frac{k^2}{1-k^2}\right)}{(1-k)k\sqrt{(1-x)x\left(1-k^2x\right)}}-\frac{\sqrt{1-x}\sqrt{x}\sqrt{\frac{k^2(1-x)}{1-k^2}+1}\,\Pi\left(\frac{k}{k+1};\sin^{-1}\left(\sqrt{1-x}\right)|-\frac{k^2}{1-k^2}\right)}{k(k+1)\sqrt{(1-x)x\left(1-k^2x\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^2*x^2)),x]
```

```
[Out] (Sqrt[1 + (k^2*(1 - x))/(1 - k^2)]*Sqrt[1 - x]*Sqrt[x]*EllipticPi[-(k/(1 -
k)), ArcSin[Sqrt[1 - x]], -(k^2/(1 - k^2))])/((1 - k)*k*Sqrt[(1 - x)*x*(1 -
 k^2*x)]) - (Sqrt[1 + (k^2*(1 - x))/(1 - k^2)]*Sqrt[1 - x]*Sqrt[x]*Elliptic
Pi[k/(1 + k), ArcSin[Sqrt[1 - x]], -(k^2/(1 - k^2))])/(k*(1 + k)*Sqrt[(1 -
x)*x*(1 - k^2*x)])
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 175

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))*Sqrt[(e_.) + (f_.)*(x_)]
*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x]*Sqr
t[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(b*c - a*d)/b, Int[1/((a + b*x)*Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(-1+k^2x^2\right)}\,dx = \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{\sqrt{x}}{\sqrt{1-x}\sqrt{1-k^2x}\left(-1+k^2x^2\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \left(-\frac{\sqrt{x}}{2\sqrt{1-x}(1-kx)\sqrt{1-k^2x}}-\frac{\sqrt{x}}{2\sqrt{1-x}(1+kx)\sqrt{1-k^2x}}\right) d}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= -\frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{\sqrt{x}}{\sqrt{1-x}(1-kx)\sqrt{1-k^2x}}\,dx}{2\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k\right.}{2\sqrt{(1-}$$

$$= -\frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-x}\sqrt{x}(1-kx)\sqrt{1-k^2x}}\,dx}{2k\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-\right.}{2k\sqrt{}}$$

$$= -\frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\left(1+k-kx^2\right)\sqrt{1-k^2+k^2x^2}}\,dx, x, \sqrt{1}\right.}{k\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= -\frac{\left(\sqrt{1+\frac{k^2(-1+x)}{-1+k^2}}\sqrt{1-x}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\left(1+k-kx^2\right)\sqrt{1+\frac{k^2x^2}{1-k^2}}}\,dx, x, \right.}{k\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\sqrt{1+\frac{k^2(1-x)}{1-k^2}}\sqrt{1-x}\sqrt{x}\,\Pi\left(-\frac{k}{1-k};\sin^{-1}\left(\sqrt{1-x}\right)|-\frac{k^2}{1-k^2}\right)}{(1-k)k\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\sqrt{1+\frac{k^2}{1}}}{}$$

Mathematica [C] time = 1.39, size = 154, normalized size = 1.81

$$\frac{i\sqrt{x-1}\,x\sqrt{\frac{1-\frac{1}{k^2}}{x-1}+1}\left(2kF\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)-(k-1)\Pi\left(1+\frac{1}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)-(k+1)\Pi\left(\frac{k-1}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)}{k\left(k^2-1\right)\sqrt{\frac{1}{x-1}+1}\sqrt{(x-1)x\left(k^2x-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^2*x^2)),x]
```

```
[Out] (I*Sqrt[1 + (1 - k^(-2))/(-1 + x)]*Sqrt[-1 + x]*x*(2*k*EllipticF[I*ArcSinh[
1/Sqrt[-1 + x]], 1 - k^(-2)] - (-1 + k)*EllipticPi[1 + k^(-1), I*ArcSinh[1/
Sqrt[-1 + x]], 1 - k^(-2)] - (1 + k)*EllipticPi[(-1 + k)/k, I*ArcSinh[1/Sqr
t[-1 + x]], 1 - k^(-2)]))/(k*(-1 + k^2)*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[(-1 +
x)*x*(-1 + k^2*x)])
```

IntegrateAlgebraic [A] time = 0.19, size = 85, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}\right)}{2k(k+1)}-\frac{\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}\right)}{2(k-1)k}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^2*x^2)),x]
```

```
[Out] -1/2*ArcTan[((-1 + k)*x)/Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3]]/((-1 + k)*k) +
 ArcTan[((1 + k)*x)/Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3]]/(2*k*(1 + k))
```

fricas [B] time = 0.47, size = 177, normalized size = 2.08

$$-\frac{(k-1)\arctan\left(\frac{\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\left(k^2x^2-2\left(k^2+k+1\right)x+1\right)}{2\left(\left(k^3+k^2\right)x^3-\left(k^3+k^2+k+1\right)x^2+(k+1)x\right)}\right)-(k+1)\arctan\left(\frac{\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\left(k^2x^2-2\left(k^2-k+1\right)x+1\right)}{2\left(\left(k^3-k^2\right)x^3-\left(k^3-k^2+k-1\right)x^2+(k-1)x\right)}\right)}{4\left(k^3-k\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-1),x, algorithm="fricas")
```

```
[Out] -1/4*((k - 1)*arctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2 - 2*(k^
2 + k + 1)*x + 1)/((k^3 + k^2)*x^3 - (k^3 + k^2 + k + 1)*x^2 + (k + 1)*x))
- (k + 1)*arctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2 - 2*(k^2 -
k + 1)*x + 1)/((k^3 - k^2)*x^3 - (k^3 - k^2 + k - 1)*x^2 + (k - 1)*x)))/(k^
3 - k)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(k^2x^2-1\right)\sqrt{\left(k^2x-1\right)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-1),x, algorithm="giac")
```

```
[Out] integrate(x/((k^2*x^2 - 1)*sqrt((k^2*x - 1)*(x - 1)*x)), x)
```

maple [C] time = 0.03, size = 232, normalized size = 2.73

$$-\frac{\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\;\mathrm{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}-\frac{1}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^4\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}-\frac{1}{k}\right)}-\frac{\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\;\mathrm{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}+\frac{1}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^4\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}+\frac{1}{k}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-1),x)
```

```
[Out] -1/k^4*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x
^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2-1/k)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2
```

```
/(1/k^2-1/k),(1/k^2/(1/k^2-1))^(1/2))-1/k^4*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/
(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/k)*El
lipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+1/k),(1/k^2/(1/k^2-1))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(k^2x^2-1)\sqrt{(k^2x-1)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-1),x, algorithm="maxima")
[Out] integrate(x/((k^2*x^2 - 1)*sqrt((k^2*x - 1)*(x - 1)*x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((k^2*x^2 - 1)*(x*(k^2*x - 1)*(x - 1))^(1/2)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x(x-1)(k^2x-1)}\,(kx-1)(kx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((1-x)*x*(-k**2*x+1))**(1/2)/(k**2*x**2-1),x)
[Out] Integral(x/(sqrt(x*(x - 1)*(k**2*x - 1))*(k*x - 1)*(k*x + 1)), x)
```

3.1014 $\int \frac{1+x}{(-1+2x+x^2)\sqrt{-x+x^3}}\,dx$

Optimal. Leaf size=85

$$\frac{1}{4}\left(\sqrt{2}-2\right)\tan^{-1}\left(\frac{\sqrt{3-2\sqrt{2}}\sqrt{x^3-x}}{x+1}\right)+\frac{1}{4}\left(2+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{2}}\sqrt{x^3-x}}{x+1}\right)$$

Rubi [C] time = 0.56, antiderivative size = 137, normalized size of antiderivative = 1.61, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2056, 6728, 933, 168, 537}

$$-\frac{\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{2+\sqrt{2}};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{\sqrt{2}\left(2+\sqrt{2}\right)\sqrt{x^3-x}}-\frac{\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{2}\left(2+\sqrt{2}\right);\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{\sqrt{2}\left(2-\sqrt{2}\right)\sqrt{x^3-x}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-1 + 2*x + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] -((Sqrt[x]*Sqrt[1 - x^2]*EllipticPi[(2 + Sqrt[2])^(-1), ArcSin[Sqrt[1 - x]]
, 1/2])/(Sqrt[2]*(2 + Sqrt[2])*Sqrt[-x + x^3])) - (Sqrt[x]*Sqrt[1 - x^2]*El
lipticPi[(2 + Sqrt[2])/2, ArcSin[Sqrt[1 - x]], 1/2])/(Sqrt[2]*(2 - Sqrt[2])
*Sqrt[-x + x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1+x}{\left(-1+2x+x^2\right)\sqrt{-x+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1+x}{\sqrt{x}\sqrt{-1+x^2}\left(-1+2x+x^2\right)}\,dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \left(\frac{1}{\sqrt{x}\left(2-2\sqrt{2}+2x\right)\sqrt{-1+x^2}}+\frac{1}{\sqrt{x}\left(2+2\sqrt{2}+2x\right)\sqrt{-1+x^2}}\right)dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{x}\left(2-2\sqrt{2}+2x\right)\sqrt{-1+x^2}}\,dx}{\sqrt{-x+x^3}}+\frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{x}\left(2+2\sqrt{2}+2x\right)\ldots}}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-x}\sqrt{x}\sqrt{1+x}\left(2-2\sqrt{2}+2x\right)}\,dx}{\sqrt{-x+x^3}}+\frac{\left(\sqrt{x}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-x}\sqrt{x}\sqrt{1+x}\ldots}}{\sqrt{-x+x^3}}$$

$$= -\frac{\left(2\sqrt{x}\sqrt{1-x^2}\right)\text{Subst}\left(\int \frac{1}{\left(2\left(2-\sqrt{2}\right)-2x^2\right)\sqrt{1-x^2}\sqrt{2-x^2}}\,dx,x,\sqrt{1-x}\right)}{\sqrt{-x+x^3}}-\frac{\left(2\sqrt{x}\ldots\right.}{\ldots}$$

$$= -\frac{\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{2+\sqrt{2}};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{\sqrt{2}\left(2+\sqrt{2}\right)\sqrt{-x+x^3}}-\frac{\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{2}\left(2+\sqrt{2}\right);\sin\ldots\right.}{\sqrt{2}\left(2-\sqrt{2}\right)\sqrt{-x\ldots}}$$

Mathematica [C] time = 0.78, size = 89, normalized size = 1.05

$$-\frac{\sqrt{1-\frac{1}{x^2}}\,x^{3/2}\left(-2F\left(\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)\middle|-1\right)-\left(\sqrt{2}-1\right)\Pi\left(-1-\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)\middle|-1\right)+\left(1+\sqrt{2}\right)\Pi\left(-1+\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)\middle|-1\right)\right)}{\sqrt{x\left(x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-1 + 2*x + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] -((Sqrt[1 - x^(-2)]*x^(3/2)*(-2*EllipticF[ArcSin[1/Sqrt[x]], -1] - (-1 + Sq
rt[2])*EllipticPi[-1 - Sqrt[2], ArcSin[1/Sqrt[x]], -1] + (1 + Sqrt[2])*Elli
pticPi[-1 + Sqrt[2], ArcSin[1/Sqrt[x]], -1]))/Sqrt[x*(-1 + x^2)])
```

IntegrateAlgebraic [A] time = 0.59, size = 73, normalized size = 0.86

$$\frac{1}{4}\left(\sqrt{2}-2\right)\tan^{-1}\left(\frac{\left(\sqrt{2}-1\right)\sqrt{x^3-x}}{x+1}\right)+\frac{1}{4}\left(2+\sqrt{2}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{2}\right)\sqrt{x^3-x}}{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((-1 + 2*x + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] ((-2 + Sqrt[2])*ArcTan[((-1 + Sqrt[2])*Sqrt[-x + x^3])/(1 + x)])/4 + ((2 +
Sqrt[2])*ArcTan[((1 + Sqrt[2])*Sqrt[-x + x^3])/(1 + x)])/4
```

fricas [A] time = 0.49, size = 101, normalized size = 1.19

$$\frac{1}{4}\sqrt{2}\sqrt{2\sqrt{2}+3}\,\arctan\left(\frac{\sqrt{x^3-x}\sqrt{2\sqrt{2}+3}\left(2\sqrt{2}-3\right)}{x^2-x}\right)+\frac{1}{4}\sqrt{2}\sqrt{-2\sqrt{2}+3}\,\arctan\left(\frac{\sqrt{x^3-x}\left(2\sqrt{2}+3\right)\sqrt{-2\sqrt{2}+3}}{x^2-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x-1)/(x^3-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt(2*sqrt(2) + 3)*arctan(sqrt(x^3 - x)*sqrt(2*sqrt(2) + 3)*(2
*sqrt(2) - 3)/(x^2 - x)) + 1/4*sqrt(2)*sqrt(-2*sqrt(2) + 3)*arctan(sqrt(x^3
 - x)*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 3)/(x^2 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^3-x}\left(x^2+2x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x-1)/(x^3-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 - x)*(x^2 + 2*x - 1)), x)
```

maple [C] time = 0.03, size = 96, normalized size = 1.13

$$-\frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\sqrt{2}\,\operatorname{EllipticPi}\left(\sqrt{1+x},\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^3-x}}+\frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\sqrt{2}\,\operatorname{EllipticPi}\left(\sqrt{1+x},-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^3-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2+2*x-1)/(x^3-x)^(1/2),x)
```

```
[Out] -1/4*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*2^(1/2)*EllipticPi(
(1+x)^(1/2),1/2*2^(1/2),1/2*2^(1/2))+1/4*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/
2)/(x^3-x)^(1/2)*2^(1/2)*EllipticPi((1+x)^(1/2),-1/2*2^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^3-x}\left(x^2+2x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x-1)/(x^3-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 - x)*(x^2 + 2*x - 1)), x)
```

mupad [B] time = 0.11, size = 102, normalized size = 1.20

$$\frac{\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(-\frac{1}{\sqrt{2}-1};\operatorname{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{\sqrt{x^3-x}\,\left(\sqrt{2}-1\right)}-\frac{\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(\frac{1}{\sqrt{2}+1};\operatorname{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{\sqrt{x^3-x}\,\left(\sqrt{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^3 - x)^(1/2)*(2*x + x^2 - 1)),x)
```

```
[Out] ((-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticPi(-1/(2^(1/2) - 1), asin((
-x)^(1/2)), -1))/((x^3 - x)^(1/2)*(2^(1/2) - 1)) - ((-x)^(1/2)*(1 - x)^(1/2
)*(x + 1)^(1/2)*ellipticPi(1/(2^(1/2) + 1), asin((-x)^(1/2)), -1))/((x^3 -
x)^(1/2)*(2^(1/2) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x\left(x-1\right)\left(x+1\right)}\,\left(x^2+2x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+2*x-1)/(x**3-x)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt(x*(x - 1)*(x + 1))*(x**2 + 2*x - 1)), x)
```

3.1015 $\int \frac{-x+x^2}{(-1+2x+x^2)\sqrt{-x+x^3}}\,dx$

Optimal. Leaf size=85

$$\frac{1}{4}\left(2+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{3-2\sqrt{2}}\,\sqrt{x^3-x}}{x+1}\right)+\frac{1}{4}\left(\sqrt{2}-2\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{2}}\,\sqrt{x^3-x}}{x+1}\right)$$

Rubi [C] time = 0.73, antiderivative size = 257, normalized size of antiderivative = 3.02, number of steps used = 16, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1593, 2056, 6728, 944, 329, 222, 933, 168, 537}

$$\frac{\left(2+\sqrt{2}\right)\sqrt{x-1}\sqrt{x}\sqrt{x+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{x-1}}\right)|\frac{1}{2}\right)}{2\sqrt{x^3-x}}-\frac{\left(2-\sqrt{2}\right)\sqrt{x-1}\sqrt{x}\sqrt{x+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{x-1}}\right)|\frac{1}{2}\right)}{2\sqrt{x^3-x}}+\frac{\left(1+\sqrt{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{2+\sqrt{2}};\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{2\sqrt{x^3-x}}-\frac{\left(1-\sqrt{2}\right)\sqrt{x}\sqrt{1-x^2}\,\Pi\left(\frac{1}{2}\left(2+\sqrt{2}\right);\sin^{-1}\left(\sqrt{1-x}\right)|\frac{1}{2}\right)}{2\sqrt{x^3-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-x + x^2)/((-1 + 2*x + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] -1/2*((2 - Sqrt[2])*Sqrt[-1 + x]*Sqrt[x]*Sqrt[1 + x]*EllipticF[ArcSin[(Sqrt
[2]*Sqrt[x])/Sqrt[-1 + x]], 1/2])/Sqrt[-x + x^3] + ((2 + Sqrt[2])*Sqrt[-1 +
 x]*Sqrt[x]*Sqrt[1 + x]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[x])/Sqrt[-1 + x]], 1
/2])/(2*Sqrt[-x + x^3]) + ((1 + Sqrt[2])*Sqrt[x]*Sqrt[1 - x^2]*EllipticPi[(
2 + Sqrt[2])^(-1), ArcSin[Sqrt[1 - x]], 1/2])/(2*Sqrt[-x + x^3]) - ((1 - Sq
rt[2])*Sqrt[x]*Sqrt[1 - x^2]*EllipticPi[(2 + Sqrt[2])/2, ArcSin[Sqrt[1 - x]
], 1/2])/(2*Sqrt[-x + x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 944

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-x+x^2}{\left(-1+2x+x^2\right)\sqrt{-x+x^3}}\,dx &= \int \frac{(-1+x)x}{\left(-1+2x+x^2\right)\sqrt{-x+x^3}}\,dx \\
&= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{(-1+x)\sqrt{x}}{\sqrt{-1+x^2}\left(-1+2x+x^2\right)}\,dx}{\sqrt{-x+x^3}} \\
&= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \left(\frac{\left(1-\sqrt{2}\right)\sqrt{x}}{\left(2-2\sqrt{2}+2x\right)\sqrt{-1+x^2}}+\frac{\left(1+\sqrt{2}\right)\sqrt{x}}{\left(2+2\sqrt{2}+2x\right)\sqrt{-1+x^2}}\right)dx}{\sqrt{-x+x^3}} \\
&= \frac{\left(\left(1-\sqrt{2}\right)\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{\sqrt{x}}{\left(2-2\sqrt{2}+2x\right)\sqrt{-1+x^2}}\,dx}{\sqrt{-x+x^3}} + \frac{\left(\left(1+\sqrt{2}\right)\sqrt{x}\sqrt{-1+x^2}\right)\cdots}{\sqrt{-x+\cdots}} \\
&= \frac{\left(\left(1-\sqrt{2}\right)\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-1+x^2}}\,dx}{2\sqrt{-x+x^3}} + \frac{\left(\left(1-\sqrt{2}\right)\left(-1+\sqrt{2}\right)\sqrt{x}\sqrt{-1+\cdots}\right.}{\sqrt{-x+\cdots}} \\
&= \frac{\left(\left(1-\sqrt{2}\right)\left(-1+\sqrt{2}\right)\sqrt{x}\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-x}\sqrt{x}\sqrt{1+x}\left(2-2\sqrt{2}+2x\right)}\,dx}{\sqrt{-x+x^3}} - \frac{\left(\left(1+\sqrt{2}\right)^2\cdots\right.}{\cdots} \\
&= -\frac{\left(2-\sqrt{2}\right)\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{2\sqrt{-x+x^3}} + \frac{\left(2+\sqrt{2}\right)\sqrt{-1+x}\sqrt{x}\cdots}{2\sqrt{\cdots}} \\
&= -\frac{\left(2-\sqrt{2}\right)\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{2\sqrt{-x+x^3}} + \frac{\left(2+\sqrt{2}\right)\sqrt{-1+x}\sqrt{x}\cdots}{2\sqrt{\cdots}}
\end{aligned}
$$

Mathematica [C] time = 0.73, size = 88, normalized size = 1.04

$$
\frac{\sqrt{x}\sqrt{1-x^2}\left(2F\left(\sin^{-1}\left(\sqrt{x}\right)\middle|-1\right)-\left(1+\sqrt{2}\right)\Pi\left(1-\sqrt{2};\sin^{-1}\left(\sqrt{x}\right)\middle|-1\right)+\left(\sqrt{2}-1\right)\Pi\left(1+\sqrt{2};\sin^{-1}\left(\sqrt{x}\right)\middle|-1\right)\right)}{\sqrt{x\left(x^2-1\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^2)/((-1 + 2*x + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] (Sqrt[x]*Sqrt[1 - x^2]*(2*EllipticF[ArcSin[Sqrt[x]], -1] - (1 + Sqrt[2])*El
lipticPi[1 - Sqrt[2], ArcSin[Sqrt[x]], -1] + (-1 + Sqrt[2])*EllipticPi[1 +
Sqrt[2], ArcSin[Sqrt[x]], -1]))/Sqrt[x*(-1 + x^2)]
```

IntegrateAlgebraic [A] time = 0.39, size = 73, normalized size = 0.86

$$
\frac{1}{4}\left(2+\sqrt{2}\right)\tan^{-1}\left(\frac{\left(\sqrt{2}-1\right)\sqrt{x^3-x}}{x+1}\right)+\frac{1}{4}\left(\sqrt{2}-2\right)\tan^{-1}\left(\frac{\left(1+\sqrt{2}\right)\sqrt{x^3-x}}{x+1}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x + x^2)/((-1 + 2*x + x^2)*Sqrt[-x + x^3]),x]
```

```
[Out] ((2 + Sqrt[2])*ArcTan[((-1 + Sqrt[2])*Sqrt[-x + x^3])/(1 + x)])/4 + ((-2 +
Sqrt[2])*ArcTan[((1 + Sqrt[2])*Sqrt[-x + x^3])/(1 + x)])/4
```

fricas [A] time = 0.49, size = 87, normalized size = 1.02

$$
-\frac{1}{4}\sqrt{2}\sqrt{2\sqrt{2}+3}\arctan\left(\frac{\sqrt{x^3-x}\sqrt{2\sqrt{2}+3}}{x^2-x}\right)+\frac{1}{4}\sqrt{2}\sqrt{-2\sqrt{2}+3}\arctan\left(\frac{\sqrt{x^3-x}\sqrt{-2\sqrt{2}+3}}{x^2-x}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)/(x^2+2*x-1)/(x^3-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt(2*sqrt(2) + 3)*arctan(sqrt(x^3 - x)*sqrt(2*sqrt(2) + 3)/(
x^2 - x)) + 1/4*sqrt(2)*sqrt(-2*sqrt(2) + 3)*arctan(sqrt(x^3 - x)*sqrt(-2*s
qrt(2) + 3)/(x^2 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - x}{\sqrt{x^3 - x}\left(x^2 + 2x - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)/(x^2+2*x-1)/(x^3-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - x)/(sqrt(x^3 - x)*(x^2 + 2*x - 1)), x)
```

maple [C] time = 0.03, size = 222, normalized size = 2.61

$$\frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\sqrt{1+x},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-x}} - \frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\operatorname{EllipticPi}\left(\sqrt{1+x},\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-x}} + \frac{3\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\sqrt{2}\,\operatorname{EllipticPi}\left(\sqrt{1+x},\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^3-x}} - \frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\operatorname{EllipticPi}\left(\sqrt{1+x},-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-x}} - \frac{3\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\sqrt{2}\,\operatorname{EllipticPi}\left(\sqrt{1+x},-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)}{4\sqrt{x^3-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x)/(x^2+2*x-1)/(x^3-x)^(1/2),x)
```

```
[Out] (1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticF((1+x)^(1/2),1/
2*2^(1/2))-(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticPi((1
+x)^(1/2),1/2*2^(1/2),1/2*2^(1/2))+3/4*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)
/(x^3-x)^(1/2)*2^(1/2)*EllipticPi((1+x)^(1/2),1/2*2^(1/2),1/2*2^(1/2))-(1+x
)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticPi((1+x)^(1/2),-1/2*
2^(1/2),1/2*2^(1/2))-3/4*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)
*2^(1/2)*EllipticPi((1+x)^(1/2),-1/2*2^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - x}{\sqrt{x^3 - x}\left(x^2 + 2x - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)/(x^2+2*x-1)/(x^3-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x)/(sqrt(x^3 - x)*(x^2 + 2*x - 1)), x)
```

mupad [B] time = 0.80, size = 159, normalized size = 1.87

$$\frac{\sqrt{2}\,\sqrt{-x}\,\left(3\sqrt{2}+4\right)\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(\frac{1}{\sqrt{2}+1};\operatorname{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{2\sqrt{x^3-x}\,\left(\sqrt{2}+1\right)} - \frac{2\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{\sqrt{x^3-x}} - \frac{\sqrt{2}\,\sqrt{-x}\,\left(3\sqrt{2}-4\right)\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(-\frac{1}{\sqrt{2}-1};\operatorname{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{2\sqrt{x^3-x}\,\left(\sqrt{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - x^2)/((x^3 - x)^(1/2)*(2*x + x^2 - 1)),x)
```

```
[Out] (2^(1/2)*(-x)^(1/2)*(3*2^(1/2) + 4)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticPi(
1/(2^(1/2) + 1), asin((-x)^(1/2)), -1))/(2*(x^3 - x)^(1/2)*(2^(1/2) + 1)) -
 (2*(-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticF(asin((-x)^(1/2)), -1))
/(x^3 - x)^(1/2) - (2^(1/2)*(-x)^(1/2)*(3*2^(1/2) - 4)*(1 - x)^(1/2)*(x + 1
)^(1/2)*ellipticPi(-1/(2^(1/2) - 1), asin((-x)^(1/2)), -1))/(2*(x^3 - x)^(1
/2)*(2^(1/2) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(x-1)}{\sqrt{x(x-1)(x+1)}\,(x^2+2x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x)/(x**2+2*x-1)/(x**3-x)**(1/2),x)
[Out] Integral(x*(x - 1)/(sqrt(x*(x - 1)*(x + 1))*(x**2 + 2*x - 1)), x)
```

3.1016 $\int \frac{3+x^2}{\sqrt[3]{1+x^2}\left(1+x^2+x^3\right)} \, dx$

Optimal. Leaf size=85

$$\log \left(\sqrt[3]{x^2+1}+x\right)-\frac{1}{2} \log \left(x^2-\sqrt[3]{x^2+1} x+\left(x^2+1\right)^{2/3}\right)-\sqrt{3} \tan ^{-1}\left(\frac{\frac{\sqrt[3]{x^2+1}}{\sqrt{3}}-\frac{2 x}{\sqrt{3}}}{\sqrt[3]{x^2+1}}\right)$$

Rubi [F] time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3+x^2}{\sqrt[3]{1+x^2}\left(1+x^2+x^3\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(3 + x^2)/((1 + x^2)^(1/3)*(1 + x^2 + x^3)),x]
```

```
[Out] 3*Defer[Int][1/((1 + x^2)^(1/3)*(1 + x^2 + x^3)), x] + Defer[Int][x^2/((1 +
 x^2)^(1/3)*(1 + x^2 + x^3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{3+x^2}{\sqrt[3]{1+x^2}\left(1+x^2+x^3\right)} \, dx &= \int \left(\frac{3}{\sqrt[3]{1+x^2}\left(1+x^2+x^3\right)}+\frac{x^2}{\sqrt[3]{1+x^2}\left(1+x^2+x^3\right)}\right) dx \\
&= 3 \int \frac{1}{\sqrt[3]{1+x^2}\left(1+x^2+x^3\right)} \, dx+\int \frac{x^2}{\sqrt[3]{1+x^2}\left(1+x^2+x^3\right)} \, dx
\end{aligned}$$

Mathematica [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{3+x^2}{\sqrt[3]{1+x^2}\left(1+x^2+x^3\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3 + x^2)/((1 + x^2)^(1/3)*(1 + x^2 + x^3)),x]
```

```
[Out] Integrate[(3 + x^2)/((1 + x^2)^(1/3)*(1 + x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.12, size = 85, normalized size = 1.00

$$\log \left(\sqrt[3]{x^2+1}+x\right)-\frac{1}{2} \log \left(x^2-\sqrt[3]{x^2+1} x+\left(x^2+1\right)^{2/3}\right)-\sqrt{3} \tan ^{-1}\left(\frac{\frac{\sqrt[3]{x^2+1}}{\sqrt{3}}-\frac{2 x}{\sqrt{3}}}{\sqrt[3]{x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + x^2)/((1 + x^2)^(1/3)*(1 + x^2 + x^3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[((-2*x)/Sqrt[3] + (1 + x^2)^(1/3)/Sqrt[3])/(1 + x^2)^(1/3)
]) + Log[x + (1 + x^2)^(1/3)] - Log[x^2 - x*(1 + x^2)^(1/3) + (1 + x^2)^(2/
3)]/2
```

fricas [A] time = 1.16, size = 100, normalized size = 1.18

$$-\sqrt{3}\arctan\left(\frac{\sqrt{3}x^3+2\sqrt{3}\left(x^2+1\right)^{\frac{1}{3}}x^2+4\sqrt{3}\left(x^2+1\right)^{\frac{2}{3}}x}{x^3-8x^2-8}\right)+\frac{1}{2}\log\left(\frac{x^3+3\left(x^2+1\right)^{\frac{1}{3}}x^2+x^2+3\left(x^2+1\right)^{\frac{2}{3}}x+1}{x^3+x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2+1)^(1/3)/(x^3+x^2+1),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan((sqrt(3)*x^3 + 2*sqrt(3)*(x^2 + 1)^(1/3)*x^2 + 4*sqrt(3)*(x
^2 + 1)^(2/3)*x)/(x^3 - 8*x^2 - 8)) + 1/2*log((x^3 + 3*(x^2 + 1)^(1/3)*x^2
+ x^2 + 3*(x^2 + 1)^(2/3)*x + 1)/(x^3 + x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+3}{\left(x^3+x^2+1\right)\left(x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2+1)^(1/3)/(x^3+x^2+1),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 3)/((x^3 + x^2 + 1)*(x^2 + 1)^(1/3)), x)
```

maple [C] time = 1.40, size = 267, normalized size = 3.14

$$\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3+3\left(x^2+1\right)^{\frac{2}{3}}x-3\left(x^2+1\right)^{\frac{1}{3}}x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+x^3-x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-1}{x^3+x^2+1}\right)-\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2+1\right)^{\frac{2}{3}}x-\left(x^2+1\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3-\left(x^2+1\right)^{\frac{2}{3}}x+\left(x^2+1\right)^{\frac{1}{3}}x^2+x^2+1}{x^3+x^2+1}\right)\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2+1\right)^{\frac{2}{3}}x-\left(x^2+1\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3-\left(x^2+1\right)^{\frac{2}{3}}x+\left(x^2+1\right)^{\frac{1}{3}}x^2+x^2+1}{x^3+x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3)/(x^2+1)^(1/3)/(x^3+x^2+1),x)
```

```
[Out] RootOf(_Z^2+_Z+1)*ln((-RootOf(_Z^2+_Z+1)^2*x^3+3*(x^2+1)^(2/3)*x-3*(x^2+1)^
(1/3)*x^2+RootOf(_Z^2+_Z+1)*x^2+x^3-x^2+RootOf(_Z^2+_Z+1)-1)/(x^3+x^2+1))-l
n(-(RootOf(_Z^2+_Z+1)*(x^2+1)^(2/3)*x-(x^2+1)^(1/3)*RootOf(_Z^2+_Z+1)*x^2+R
ootOf(_Z^2+_Z+1)*x^3-(x^2+1)^(2/3)*x+(x^2+1)^(1/3)*x^2+x^2+1)/(x^3+x^2+1))*
RootOf(_Z^2+_Z+1)-ln(-(RootOf(_Z^2+_Z+1)*(x^2+1)^(2/3)*x-(x^2+1)^(1/3)*Root
Of(_Z^2+_Z+1)*x^2+RootOf(_Z^2+_Z+1)*x^3-(x^2+1)^(2/3)*x+(x^2+1)^(1/3)*x^2+x
^2+1)/(x^3+x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+3}{\left(x^3+x^2+1\right)\left(x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2+1)^(1/3)/(x^3+x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 3)/((x^3 + x^2 + 1)*(x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^2+3}{{\left(x^2+1\right)}^{1/3}\,\left(x^3+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3)/((x^2 + 1)^(1/3)*(x^2 + x^3 + 1)),x)
```

```
[Out] int((x^2 + 3)/((x^2 + 1)^(1/3)*(x^2 + x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+3}{\sqrt[3]{x^2+1}\,\left(x^3+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3)/(x**2+1)**(1/3)/(x**3+x**2+1),x)
```

```
[Out] Integral((x**2 + 3)/((x**2 + 1)**(1/3)*(x**3 + x**2 + 1)), x)
```

3.1017 $\int \frac{x}{(-1+x^3)(-1+2x^3)^{2/3}}\,dx$

Optimal. Leaf size=85

$$\frac{1}{3}\log\left(\sqrt[3]{2x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{2x^3-1}+x}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(\sqrt[3]{2x^3-1}\,x+\left(2x^3-1\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {494, 292, 31, 634, 618, 204, 628}

$$\frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{2x^3-1}}\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{2x^3-1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(\frac{x}{\sqrt[3]{2x^3-1}}+\frac{x^2}{\left(2x^3-1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((-1 + x^3)*(-1 + 2*x^3)^(2/3)),x]
```

```
[Out] ArcTan[(1 + (2*x)/(-1 + 2*x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[1 - x/(-1 + 2*
x^3)^(1/3)]/3 - Log[1 + x^2/(-1 + 2*x^3)^(2/3) + x/(-1 + 2*x^3)^(1/3)]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\left(-1+x^3\right)\left(-1+2x^3\right)^{2/3}}\,dx = \text{Subst}\left(\int \frac{x}{-1+x^3}\,dx, x, \frac{x}{\sqrt[3]{-1+2x^3}}\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \frac{x}{\sqrt[3]{-1+2x^3}}\right) - \frac{1}{3}\text{Subst}\left(\int \frac{-1+x}{1+x+x^2}\,dx, x, \frac{x}{\sqrt[3]{-1 \cdots}}\right.$$

$$= \frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{-1+2x^3}}\right) - \frac{1}{6}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{x}{\sqrt[3]{-1+2x^3}}\right) + \frac{1}{2}\text{Sub}\cdots$$

$$= \frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{-1+2x^3}}\right) - \frac{1}{6}\log\left(1+\frac{x^2}{\left(-1+2x^3\right)^{2/3}}+\frac{x}{\sqrt[3]{-1+2x^3}}\right) - \text{Subst}\Big(\cdots$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+2x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{-1+2x^3}}\right) - \frac{1}{6}\log\left(1+\frac{x^2}{\left(-1+2x^3\right)^{2/3}}+\cdots\right.$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.47

$$-\frac{x^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{x^3}{1-2x^3}\right)}{2\left(2x^3-1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-1 + x^3)*(-1 + 2*x^3)^(2/3)),x]
```

```
[Out] -1/2*(x^2*Hypergeometric2F1[2/3, 1, 5/3, -(x^3/(1 - 2*x^3))])/(-1 + 2*x^3)^
(2/3)
```

IntegrateAlgebraic [A] time = 0.19, size = 85, normalized size = 1.00

$$\frac{1}{3}\log\left(\sqrt[3]{2x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2x^3-1}+x}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(\sqrt[3]{2x^3-1}\,x+\left(2x^3-1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((-1 + x^3)*(-1 + 2*x^3)^(2/3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + 2*x^3)^(1/3))]/Sqrt[3] + Log[-x + (-1 + 2*x
^3)^(1/3)]/3 - Log[x^2 + x*(-1 + 2*x^3)^(1/3) + (-1 + 2*x^3)^(2/3)]/6
```

fricas [A] time = 0.85, size = 104, normalized size = 1.22

$$\frac{1}{3}\sqrt{3}\arctan\left(-\frac{4\sqrt{3}\left(2x^3-1\right)^{\frac{1}{3}}x^2-2\sqrt{3}\left(2x^3-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(2x^3-1\right)}{10x^3-1}\right)+\frac{1}{6}\log\left(\frac{x^3+3\left(2x^3-1\right)^{\frac{1}{3}}x^2-3\left(2x^3-1\right)^{\frac{2}{3}}x-1}{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-1)/(2*x^3-1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(-(4*sqrt(3)*(2*x^3 - 1)^(1/3)*x^2 - 2*sqrt(3)*(2*x^3 - 1
)^(2/3)*x + sqrt(3)*(2*x^3 - 1))/(10*x^3 - 1)) + 1/6*log((x^3 + 3*(2*x^3 -
1)^(1/3)*x^2 - 3*(2*x^3 - 1)^(2/3)*x - 1)/(x^3 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2\,x^3-1\right)^{\frac{2}{3}}\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-1)/(2*x^3-1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x/((2*x^3 - 1)^(2/3)*(x^3 - 1)), x)
```

maple [C] time = 0.91, size = 298, normalized size = 3.51

$$\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\ln\left(\frac{9\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)^2 x^3-3\left(2x^3-1\right)^{\frac{2}{3}} x-3\left(2x^3-1\right)^{\frac{1}{3}} x^2-4x^3+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)+2}{\left(-1+x\right)\left(x^2+x+1\right)}\right)-\frac{\ln\left(\frac{9\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)^2 x^3+6\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3-3\left(2x^3-1\right)^{\frac{2}{3}} x-3\left(2x^3-1\right)^{\frac{1}{3}} x^2-3x^3-3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)+1}{\left(-1+x\right)\left(x^2+x+1\right)}\right)}{3}-\ln\left(\frac{9\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)^2 x^3+6\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3-3\left(2x^3-1\right)^{\frac{2}{3}} x-3\left(2x^3-1\right)^{\frac{1}{3}} x^2-3x^3-3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)+1}{\left(-1+x\right)\left(x^2+x+1\right)}\right)\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3-1)/(2*x^3-1)^(2/3),x)
```

```
[Out] RootOf(9*_Z^2+3*_Z+1)*ln((9*RootOf(9*_Z^2+3*_Z+1)^2*x^3-3*(2*x^3-1)^(2/3)*x
-3*(2*x^3-1)^(1/3)*x^2-4*x^3+3*RootOf(9*_Z^2+3*_Z+1)+2)/(-1+x)/(x^2+x+1))-1
/3*ln((9*RootOf(9*_Z^2+3*_Z+1)^2*x^3+6*RootOf(9*_Z^2+3*_Z+1)*x^3-3*(2*x^3-1
)^(2/3)*x-3*(2*x^3-1)^(1/3)*x^2-3*x^3-3*RootOf(9*_Z^2+3*_Z+1)+1)/(-1+x)/(x^
2+x+1))-ln((9*RootOf(9*_Z^2+3*_Z+1)^2*x^3+6*RootOf(9*_Z^2+3*_Z+1)*x^3-3*(2*
x^3-1)^(2/3)*x-3*(2*x^3-1)^(1/3)*x^2-3*x^3-3*RootOf(9*_Z^2+3*_Z+1)+1)/(-1+x
)/(x^2+x+1))*RootOf(9*_Z^2+3*_Z+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2\,x^3-1\right)^{\frac{2}{3}}\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-1)/(2*x^3-1)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(x/((2*x^3 - 1)^(2/3)*(x^3 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(x^3-1\right)\,{\left(2\,x^3-1\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 - 1)*(2*x^3 - 1)^(2/3)),x)
```

```
[Out] int(x/((x^3 - 1)*(2*x^3 - 1)^(2/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x - 1\right) \left(2 x^{3} - 1\right)^{\frac{2}{3}} \left(x^{2} + x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3-1)/(2*x**3-1)**(2/3),x)
```

```
[Out] Integral(x/((x - 1)*(2*x**3 - 1)**(2/3)*(x**2 + x + 1)), x)
```

3.1018 $\int \frac{\left(-1-2(-1+k)x+kx^2\right)\left(1-2kx+k^2x^2\right)}{((1-x)x(1-kx))^{3/4}\left(-d+(1+3dk)x-\left(1+3dk^2\right)x^2+dk^3x^3\right)}\,dx$

Optimal. Leaf size=85

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(kx^3+(-k-1)x^2+x\right)^{3/4}}{(x-1)x}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(kx^3+(-k-1)x^2+x\right)^{3/4}}{(x-1)x}\right)}{d^{3/4}}$$

Rubi [F] time = 15.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1-2(-1+k)x+kx^2\right)\left(1-2kx+k^2x^2\right)}{((1-x)x(1-kx))^{3/4}\left(-d+(1+3dk)x-\left(1+3dk^2\right)x^2+dk^3x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 - 2*(-1 + k)*x + k*x^2)*(1 - 2*k*x + k^2*x^2))/(((1 - x)*x*(1 - k*
x))^(3/4)*(-d + (1 + 3*d*k)*x - (1 + 3*d*k^2)*x^2 + d*k^3*x^3)),x]
```

```
[Out] (4*(1 - x)^(3/4)*x^(3/4)*(1 - k*x)^(3/4)*Defer[Subst][Defer[Int][(1 - k*x^4
)^(5/4)/((1 - x^4)^(3/4)*(-x^4 + x^8 - d*(-1 + k*x^4)^3)), x], x, x^(1/4)])
/((1 - x)*x*(1 - k*x))^(3/4) - (8*(1 - k)*(1 - x)^(3/4)*x^(3/4)*(1 - k*x)^(
3/4)*Defer[Subst][Defer[Int][(x^4*(1 - k*x^4)^(5/4))/((1 - x^4)^(3/4)*(-x^4
 + x^8 - d*(-1 + k*x^4)^3)), x], x, x^(1/4)])/((1 - x)*x*(1 - k*x))^(3/4) +
 (4*k*(1 - x)^(3/4)*x^(3/4)*(1 - k*x)^(3/4)*Defer[Subst][Defer[Int][(x^8*(1
 - k*x^4)^(5/4))/((1 - x^4)^(3/4)*(x^4 - x^8 + d*(-1 + k*x^4)^3)), x], x, x
^(1/4)])/((1 - x)*x*(1 - k*x))^(3/4)
```

Rubi steps

$$\int \frac{\left(-1-2(-1+k)x+kx^2\right)\left(1-2kx+k^2x^2\right)}{((1-x)x(1-kx))^{3/4}\left(-d+(1+3dk)x-\left(1+3dk^2\right)x^2+dk^3x^3\right)}\,dx = \int \frac{(-1+kx)^2\left(-1-2(-\right.}{((1-x)x(1-kx))^{3/4}\left(-d+(1+3dk\right.}$$

$$= \frac{\left((1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\int \frac{}{(1-x)^{3/4}x^{3/4}(}}{((1-x)x(1}$$

$$= \frac{\left((1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\int \frac{(1-}{(1-x)^{3/4}x^{3/4}(}}{((1-x)x(1-k}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\mathrm{Subst}\left(\int \right.}{((1-}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\mathrm{Subst}\left(\int \right.}{((1-x)x(1-}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\mathrm{Subst}\left(\int\left(\right.\right.}{}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\mathrm{Subst}\left(\int \right.}{((1-x}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\mathrm{Subst}\left(\int \right.}{((1-x)x(1-}$$

Mathematica [F] time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{\left(-1-2(-1+k)x+kx^2\right)\left(1-2kx+k^2x^2\right)}{((1-x)x(1-kx))^{3/4}\left(-d+(1+3dk)x-\left(1+3dk^2\right)x^2+dk^3x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 - 2*(-1 + k)*x + k*x^2)*(1 - 2*k*x + k^2*x^2))/(((1 - x)*x*(
1 - k*x))^(3/4)*(-d + (1 + 3*d*k)*x - (1 + 3*d*k^2)*x^2 + d*k^3*x^3)),x]
```

```
[Out] Integrate[((-1 - 2*(-1 + k)*x + k*x^2)*(1 - 2*k*x + k^2*x^2))/(((1 - x)*x*(
1 - k*x))^(3/4)*(-d + (1 + 3*d*k)*x - (1 + 3*d*k^2)*x^2 + d*k^3*x^3)), x]
```

IntegrateAlgebraic [A] time = 3.17, size = 85, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(kx^3+(-k-1)x^2+x\right)^{3/4}}{(x-1)x}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(kx^3+(-k-1)x^2+x\right)^{3/4}}{(x-1)x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 - 2*(-1 + k)*x + k*x^2)*(1 - 2*k*x + k^2*x^2))/(((1
 - x)*x*(1 - k*x))^(3/4)*(-d + (1 + 3*d*k)*x - (1 + 3*d*k^2)*x^2 + d*k^3*x^
3)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(x + (-1 - k)*x^2 + k*x^3)^(3/4))/((-1 + x)*x)])/d^(3/4)
 - (2*ArcTanh[(d^(1/4)*(x + (-1 - k)*x^2 + k*x^3)^(3/4))/((-1 + x)*x)])/d^(
3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-2*(-1+k)*x+k*x^2)*(k^2*x^2-2*k*x+1)/((1-x)*x*(-k*x+1))^(3/4)/
(-d+(3*d*k+1)*x-(3*d*k^2+1)*x^2+d*k^3*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(k^2x^2-2\,kx+1\right)\left(kx^2-2\,(k-1)x-1\right)}{\left(dk^3x^3-\left(3\,dk^2+1\right)x^2+(3\,dk+1)x-d\right)((kx-1)(x-1)x)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-2*(-1+k)*x+k*x^2)*(k^2*x^2-2*k*x+1)/((1-x)*x*(-k*x+1))^(3/4)/
(-d+(3*d*k+1)*x-(3*d*k^2+1)*x^2+d*k^3*x^3),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^2 - 2*k*x + 1)*(k*x^2 - 2*(k - 1)*x - 1)/((d*k^3*x^3 - (3*
d*k^2 + 1)*x^2 + (3*d*k + 1)*x - d)*((k*x - 1)*(x - 1)*x)^(3/4)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(-1-2\,(-1+k)\,x+k\,x^2\right)\left(k^2x^2-2kx+1\right)}{((1-x)\,x\,(-kx+1))^{\frac{3}{4}}\left(-d+(3dk+1)\,x-\left(3d\,k^2+1\right)x^2+d\,k^3x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-2*(-1+k)*x+k*x^2)*(k^2*x^2-2*k*x+1)/((1-x)*x*(-k*x+1))^(3/4)/(-d+(3
*d*k+1)*x-(3*d*k^2+1)*x^2+d*k^3*x^3),x)
```

```
[Out] int((-1-2*(-1+k)*x+k*x^2)*(k^2*x^2-2*k*x+1)/((1-x)*x*(-k*x+1))^(3/4)/(-d+(3
*d*k+1)*x-(3*d*k^2+1)*x^2+d*k^3*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(k^2x^2-2\,kx+1\right)\left(kx^2-2\,(k-1)x-1\right)}{\left(dk^3x^3-\left(3\,dk^2+1\right)x^2+(3\,dk+1)x-d\right)((kx-1)(x-1)x)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-2*(-1+k)*x+k*x^2)*(k^2*x^2-2*k*x+1)/((1-x)*x*(-k*x+1))^(3/4)/
(-d+(3*d*k+1)*x-(3*d*k^2+1)*x^2+d*k^3*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^2 - 2*k*x + 1)*(k*x^2 - 2*(k - 1)*x - 1)/((d*k^3*x^3 - (3*
d*k^2 + 1)*x^2 + (3*d*k + 1)*x - d)*((k*x - 1)*(x - 1)*x)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x\,\left(k-1\right)-k\,x^2+1\right)\,\left(k^2\,x^2-2\,k\,x+1\right)}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{3/4}\,\left(d+x^2\,\left(3\,d\,k^2+1\right)-x\,\left(3\,d\,k+1\right)-d\,k^3\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x*(k - 1) - k*x^2 + 1)*(k^2*x^2 - 2*k*x + 1))/((x*(k*x - 1)*(x - 1)
)^(3/4)*(d + x^2*(3*d*k^2 + 1) - x*(3*d*k + 1) - d*k^3*x^3)),x)
```

```
[Out] int(((2*x*(k - 1) - k*x^2 + 1)*(k^2*x^2 - 2*k*x + 1))/((x*(k*x - 1)*(x - 1)
)^(3/4)*(d + x^2*(3*d*k^2 + 1) - x*(3*d*k + 1) - d*k^3*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-2*(-1+k)*x+k*x**2)*(k**2*x**2-2*k*x+1)/((1-x)*x*(-k*x+1))**(3
/4)/(-d+(3*d*k+1)*x-(3*d*k**2+1)*x**2+d*k**3*x**3),x)
```

```
[Out] Timed out
```

3.1019 $\int \frac{-1+x^4}{\sqrt{-x+x^3}\left(1+x^4\right)}\,dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{2^{3/4}\sqrt{x^3-x}}{-x^2+\sqrt{2}\,x+1}\right)}{2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{2^{3/4}}+\frac{x}{\sqrt[4]{2}}-\frac{1}{2^{3/4}}}{\sqrt{x^3-x}}\right)}{2^{3/4}}$$

Rubi [C] time = 1.66, antiderivative size = 654, normalized size of antiderivative = 7.69, number of steps used = 43, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {2056, 1586, 6715, 6725, 406, 222, 409, 1215, 1457, 540, 253, 538, 537}

$$\frac{2\sqrt{x-1}\sqrt{x}\sqrt{x+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{x-1}}\right)|\frac{1}{2}\right)}{\left(2\sqrt{2}+(2+2i)\right)\sqrt{x^3-x}}+\frac{2\sqrt{x-1}\sqrt{x}\sqrt{x+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{x-1}}\right)|\frac{1}{2}\right)}{\left(2\sqrt{2}+(2-2i)\right)\sqrt{x^3-x}}-\frac{\sqrt{x-1}\sqrt{x}\sqrt{x+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{x-1}}\right)|\frac{1}{2}\right)}{\left(\sqrt{2}+(1+i)\right)\sqrt{x^3-x}}-\frac{\sqrt{x-1}\sqrt{x}\sqrt{x+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{x-1}}\right)|\frac{1}{2}\right)}{\left(\sqrt{2}+(1-i)\right)\sqrt{x^3-x}}-\frac{\sqrt{x-1}\sqrt{x}\sqrt{x+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{x-1}}\right)|\frac{1}{2}\right)}{\left(\sqrt{2}+(-1-i)\right)\sqrt{x^3-x}}-\frac{2\sqrt{x-1}\sqrt{x}\sqrt{x+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{x-1}}\right)|\frac{1}{2}\right)}{\left(-2\sqrt{2}+(2+2i)\right)\sqrt{x^3-x}}+\frac{\sqrt{2}\sqrt{x-1}\sqrt{x}\sqrt{x+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{x-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^3-x}}-\frac{\sqrt{1-x}\sqrt{x}\sqrt{x+1}\Pi\left(-\sqrt[4]{-1};\sin^{-1}\left(\sqrt{x}\right)|-1\right)}{\sqrt{x^3-x}}-\frac{\sqrt{1-x}\sqrt{x}\sqrt{x+1}\Pi\left(\sqrt[4]{-1};\sin^{-1}\left(\sqrt{x}\right)|-1\right)}{\sqrt{x^3-x}}-\frac{\sqrt{1-x}\sqrt{x}\sqrt{x+1}\Pi\left(-(-1)^{3/4};\sin^{-1}\left(\sqrt{x}\right)|-1\right)}{\sqrt{x^3-x}}-\frac{\sqrt{1-x}\sqrt{x}\sqrt{x+1}\Pi\left((-1)^{3/4};\sin^{-1}\left(\sqrt{x}\right)|-1\right)}{\sqrt{x^3-x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)/(Sqrt[-x + x^3]*(1 + x^4)),x]
```

```
[Out] (Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]*Sqrt[1 + x]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[x]
)/Sqrt[-1 + x]], 1/2])/Sqrt[-x + x^3] - (2*Sqrt[-1 + x]*Sqrt[x]*Sqrt[1 + x]
*EllipticF[ArcSin[(Sqrt[2]*Sqrt[x])/Sqrt[-1 + x]], 1/2])/(((2 + 2*I) - 2*Sq
rt[2])*Sqrt[-x + x^3]) - (Sqrt[-1 + x]*Sqrt[x]*Sqrt[1 + x]*EllipticF[ArcSin
[(Sqrt[2]*Sqrt[x])/Sqrt[-1 + x]], 1/2])/(((-1 - I) + Sqrt[2])*Sqrt[-x + x^3
]) - (Sqrt[-1 + x]*Sqrt[x]*Sqrt[1 + x]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[x])/S
qrt[-1 + x]], 1/2])/(((1 - I) + Sqrt[2])*Sqrt[-x + x^3]) - (Sqrt[-1 + x]*Sq
rt[x]*Sqrt[1 + x]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[x])/Sqrt[-1 + x]], 1/2])/(
((1 + I) + Sqrt[2])*Sqrt[-x + x^3]) + (2*Sqrt[-1 + x]*Sqrt[x]*Sqrt[1 + x]*E
llipticF[ArcSin[(Sqrt[2]*Sqrt[x])/Sqrt[-1 + x]], 1/2])/(((2 - 2*I) + 2*Sqrt
[2])*Sqrt[-x + x^3]) + (2*Sqrt[-1 + x]*Sqrt[x]*Sqrt[1 + x]*EllipticF[ArcSin
[(Sqrt[2]*Sqrt[x])/Sqrt[-1 + x]], 1/2])/(((2 + 2*I) + 2*Sqrt[2])*Sqrt[-x +
x^3]) - (Sqrt[1 - x]*Sqrt[x]*Sqrt[1 + x]*EllipticPi[-(-1)^(1/4), ArcSin[Sqr
t[x]], -1])/Sqrt[-x + x^3] - (Sqrt[1 - x]*Sqrt[x]*Sqrt[1 + x]*EllipticPi[(-
1)^(1/4), ArcSin[Sqrt[x]], -1])/Sqrt[-x + x^3] - (Sqrt[1 - x]*Sqrt[x]*Sqrt[
1 + x]*EllipticPi[-(-1)^(3/4), ArcSin[Sqrt[x]], -1])/Sqrt[-x + x^3] - (Sqrt
[1 - x]*Sqrt[x]*Sqrt[1 + x]*EllipticPi[(-1)^(3/4), ArcSin[Sqrt[x]], -1])/Sq
rt[-x + x^3]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && !IntegerQ[p]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 540

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x
] + Dist[(b*c - a*d)/b, Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

Rule 1215

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[c/(c*d + e*q), Int[1/Sqrt[a + c*x^4], x], x] + Di
st[e/(c*d + e*q), Int[(q - c*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
 FreeQ[{a, c, d, e}, x] && GtQ[-(a*c), 0] &&  !LtQ[c, 0]
```

Rule 1457

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + c*x^(2*n))^FracPart[p]/((d +
e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p]), Int[(d + e*x^n)^(p + q)*
(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, n, p,
 q, r}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+x^4}{\sqrt{-x+x^3}\left(1+x^4\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{-1+x^4}{\sqrt{x}\sqrt{-1+x^2}\left(1+x^4\right)}\,dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\int \frac{\sqrt{-1+x^2}\left(1+x^2\right)}{\sqrt{x}\left(1+x^4\right)}\,dx}{\sqrt{-x+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{\sqrt{-1+x^4}\left(1+x^4\right)}{1+x^8}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \left(-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{-1+x^4}}{i-x^4}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-1+x^4}}{i+x^4}\right)dx,x,\sqrt{x}\right)}{\sqrt{-x+x^3}}$$

$$= -\frac{\left((1-i)\sqrt{x}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{\sqrt{-1+x^4}}{i-x^4}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^3}} + \frac{\left((1+i)\sqrt{x}\sqrt{-1+x^2}\right)\text{Subs}\cdots}{\sqrt{-x+\cdots}}$$

$$= -\frac{\left(2i\sqrt{x}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\left(i-x^4\right)\sqrt{-1+x^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{-x+x^3}} - \frac{\left(2i\sqrt{x}\sqrt{-1+x^2}\right)\text{Subst}\left(\cdots\right.}{\sqrt{-x\cdots}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} - \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\left(1-\sqrt[4]{-1}\,x^2\cdots\right.}\cdots\right.}{\sqrt{-x+x^3}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} - \frac{\left(\sqrt{x}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^4}}\,d\cdots\right.}{\left(1-\sqrt[4]{-1}\right)\sqrt{-x+x^3}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} - \frac{\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{-1+\cdots}}\right.\right.}{\sqrt{2}\left(1-\sqrt[4]{-1}\right)\sqrt{-x+x^3}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} - \frac{\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{-1+\cdots}}\right.\right.}{\sqrt{2}\left(1-\sqrt[4]{-1}\right)\sqrt{-x+x^3}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} - \frac{\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{-1+\cdots}}\right.\right.}{\sqrt{2}\left(1-\sqrt[4]{-1}\right)\sqrt{-x+x^3}}$$

$$= \frac{\sqrt{2}\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{-1+x}}\right)|\frac{1}{2}\right)}{\sqrt{-x+x^3}} - \frac{\sqrt{-1+x}\sqrt{x}\sqrt{1+x}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{-1+\cdots}}\right.\right.}{\sqrt{2}\left(1-(-1)^{3/4}\right)\sqrt{-x+x^3}}$$

Mathematica [C] time = 0.72, size = 96, normalized size = 1.13

$$-\frac{\sqrt{1-\frac{1}{x^2}}\,x^{3/2}\left(-2F\left(\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)\middle|-1\right)+\Pi\left(-\sqrt[4]{-1};\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)\middle|-1\right)+\Pi\left(\sqrt[4]{-1};\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)\middle|-1\right)+\Pi\left(-(-1)^{3/4};\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)\middle|-1\right)+\Pi\left((-1)^{3/4};\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)\middle|-1\right)\right)}{\sqrt{x\left(x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)/(Sqrt[-x + x^3]*(1 + x^4)),x]
```

```
[Out] -((Sqrt[1 - x^(-2)]*x^(3/2)*(-2*EllipticF[ArcSin[1/Sqrt[x]], -1] + Elliptic
Pi[-(-1)^(1/4), ArcSin[1/Sqrt[x]], -1] + EllipticPi[(-1)^(1/4), ArcSin[1/Sq
rt[x]], -1] + EllipticPi[-(-1)^(3/4), ArcSin[1/Sqrt[x]], -1] + EllipticPi[(
-1)^(3/4), ArcSin[1/Sqrt[x]], -1]))/Sqrt[x*(-1 + x^2)])
```

IntegrateAlgebraic [A] time = 0.36, size = 85, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2^{3/4}\sqrt{x^3-x}}{-x^2+\sqrt{2}\,x+1}\right)}{2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{2^{3/4}}+\frac{x}{\sqrt[4]{2}}-\frac{1}{2^{3/4}}}{\sqrt{x^3-x}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/(Sqrt[-x + x^3]*(1 + x^4)),x]
```

```
[Out] ArcTan[(2^(3/4)*Sqrt[-x + x^3])/(1 + Sqrt[2]*x - x^2)]/2^(3/4) - ArcTanh[(-
2^(-3/4) + x/2^(1/4) + x^2/2^(3/4))/Sqrt[-x + x^3]]/2^(3/4)
```

fricas [B] time = 0.48, size = 385, normalized size = 4.53

$$-\frac{1}{2}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{\sqrt{x^3-x}\left(2^{\frac{3}{4}}x-2^{\frac{1}{4}}\left(x^2-1\right)\right)-\left(2\,x^3-\sqrt{x^3-x}\left(2^{\frac{3}{4}}x+2^{\frac{1}{4}}\left(x^2-1\right)\right)-2\,x\right)\sqrt{\frac{x^4+4\,\sqrt{2}\left(x^3-x\right)+2\,\sqrt{x^3-x}\left(2^{\frac{3}{4}}\left(x^2-1\right)+2\cdot 2^{\frac{1}{4}}x\right)+1}{x^4+1}}}{2\left(x^3-x\right)}\right)-\frac{1}{2}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{\sqrt{x^3-x}\left(2^{\frac{3}{4}}x-2^{\frac{1}{4}}\left(x^2-1\right)\right)+\left(2\,x^3+\sqrt{x^3-x}\left(2^{\frac{3}{4}}x+2^{\frac{1}{4}}\left(x^2-1\right)\right)-2\,x\right)\sqrt{\frac{x^4+4\,\sqrt{2}\left(x^3-x\right)-2\,\sqrt{x^3-x}\left(2^{\frac{3}{4}}\left(x^2-1\right)+2\cdot 2^{\frac{1}{4}}x\right)+1}{x^4+1}}}{2\left(x^3-x\right)}\right)-\frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\frac{x^4+4\,\sqrt{2}\left(x^3-x\right)+2\,\sqrt{x^3-x}\left(2^{\frac{3}{4}}\left(x^2-1\right)+2\cdot 2^{\frac{1}{4}}x\right)+1}{x^4+1}\right)+\frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\frac{x^4+4\,\sqrt{2}\left(x^3-x\right)-2\,\sqrt{x^3-x}\left(2^{\frac{3}{4}}\left(x^2-1\right)+2\cdot 2^{\frac{1}{4}}x\right)+1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^3-x)^(1/2)/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*2^(1/4)*arctan(1/2*(sqrt(x^3 - x)*(2^(3/4)*x - 2^(1/4)*(x^2 - 1)) - (2
*x^3 - sqrt(x^3 - x)*(2^(3/4)*x + 2^(1/4)*(x^2 - 1)) - 2*x)*sqrt((x^4 + 4*s
qrt(2)*(x^3 - x) + 2*sqrt(x^3 - x)*(2^(3/4)*(x^2 - 1) + 2*2^(1/4)*x) + 1)/(
x^4 + 1)))/(x^3 - x)) - 1/2*2^(1/4)*arctan(1/2*(sqrt(x^3 - x)*(2^(3/4)*x -
2^(1/4)*(x^2 - 1)) + (2*x^3 + sqrt(x^3 - x)*(2^(3/4)*x + 2^(1/4)*(x^2 - 1))
 - 2*x)*sqrt((x^4 + 4*sqrt(2)*(x^3 - x) - 2*sqrt(x^3 - x)*(2^(3/4)*(x^2 - 1
) + 2*2^(1/4)*x) + 1)/(x^4 + 1)))/(x^3 - x)) - 1/8*2^(1/4)*log((x^4 + 4*sqr
t(2)*(x^3 - x) + 2*sqrt(x^3 - x)*(2^(3/4)*(x^2 - 1) + 2*2^(1/4)*x) + 1)/(x^
4 + 1)) + 1/8*2^(1/4)*log((x^4 + 4*sqrt(2)*(x^3 - x) - 2*sqrt(x^3 - x)*(2^(
3/4)*(x^2 - 1) + 2*2^(1/4)*x) + 1)/(x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\left(x^4+1\right)\sqrt{x^3-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^3-x)^(1/2)/(x^4+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 1)/((x^4 + 1)*sqrt(x^3 - x)), x)
```

maple [C] time = 0.03, size = 119, normalized size = 1.40

$$\frac{\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\sqrt{1+x},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-x}}+\frac{\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4+1\right)}\frac{\_\alpha\left(\_\alpha^3-\_\alpha^2+\_\alpha-1\right)\sqrt{1+x}\,\sqrt{1-x}\,\sqrt{-x}\,\operatorname{EllipticPi}\left(\sqrt{1+x},-\frac{1}{2}\_\alpha^3+\frac{1}{2}\_\alpha^2-\frac{1}{2}\_\alpha+\frac{1}{2},\frac{\sqrt{2}}{2}\right)}{\sqrt{x\left(x^2-1\right)}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/(x^3-x)^(1/2)/(x^4+1),x)
```

```
[Out] (1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1/2)/(x^3-x)^(1/2)*EllipticF((1+x)^(1/2),1/
2*2^(1/2))+1/4*2^(1/2)*sum(_alpha*(_alpha^3-_alpha^2+_alpha-1)*(1+x)^(1/2)*
```

```
(1-x)^(1/2)*(-x)^(1/2)/(x*(x^2-1))^(1/2)*EllipticPi((1+x)^(1/2),-1/2*_alpha
^3+1/2*_alpha^2-1/2*_alpha+1/2,1/2*2^(1/2)),_alpha=RootOf(_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\left(x^4+1\right)\sqrt{x^3-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^3-x)^(1/2)/(x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 1)/((x^4 + 1)*sqrt(x^3 - x)), x)
```

mupad [B] time = 0.03, size = 205, normalized size = 2.41

$$\frac{\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(\sqrt{2}\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right);\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{\sqrt{x^3-x}}+\frac{\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(\sqrt{2}\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right);\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{\sqrt{x^3-x}}+\frac{\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(\sqrt{2}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right);\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{\sqrt{x^3-x}}+\frac{\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\Pi\left(\sqrt{2}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right);\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{\sqrt{x^3-x}}-\frac{2\,\sqrt{-x}\,\sqrt{1-x}\,\sqrt{x+1}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-x}\right)\middle|-1\right)}{\sqrt{x^3-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/((x^3 - x)^(1/2)*(x^4 + 1)),x)
```

```
[Out] ((-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticPi(2^(1/2)*(- 1/2 - 1i/2),
asin((-x)^(1/2)), -1))/(x^3 - x)^(1/2) + ((-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^
(1/2)*ellipticPi(2^(1/2)*(- 1/2 + 1i/2), asin((-x)^(1/2)), -1))/(x^3 - x)^(
1/2) + ((-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticPi(2^(1/2)*(1/2 - 1i
/2), asin((-x)^(1/2)), -1))/(x^3 - x)^(1/2) + ((-x)^(1/2)*(1 - x)^(1/2)*(x
+ 1)^(1/2)*ellipticPi(2^(1/2)*(1/2 + 1i/2), asin((-x)^(1/2)), -1))/(x^3 - x
)^(1/2) - (2*(-x)^(1/2)*(1 - x)^(1/2)*(x + 1)^(1/2)*ellipticF(asin((-x)^(1/
2)), -1))/(x^3 - x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)\,\left(x^2+1\right)}{\sqrt{x\,(x-1)\,(x+1)}\,\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/(x**3-x)**(1/2)/(x**4+1),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)/(sqrt(x*(x - 1)*(x + 1))*(x**4 + 1)), x
)
```

3.1020 $\int \frac{-1+x^2}{(1+x+x^2)\sqrt[3]{x^2+x^4}}\,dx$

Optimal. Leaf size=85

$$-\log\left(\sqrt[3]{x^4+x^2}+x\right)+\frac{1}{2}\log\left(x^2-\sqrt[3]{x^4+x^2}\,x+\left(x^4+x^2\right)^{2/3}\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^4+x^2}-x}\right)$$

Rubi [C] time = 1.15, antiderivative size = 297, normalized size of antiderivative = 3.49, number of steps used = 18, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2056, 6728, 364, 959, 466, 429, 465, 510}

$$\frac{3\sqrt[3]{x^2+1}\,x^2F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};-\frac{4x^2}{\left(i-\sqrt{3}\right)^2},-x^2\right)}{2\left(1+i\sqrt{3}\right)\sqrt[3]{x^4+x^2}}+\frac{3\sqrt[3]{x^2+1}\,x^2F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};-\frac{4x^2}{\left(i+\sqrt{3}\right)^2},-x^2\right)}{2\left(1-i\sqrt{3}\right)\sqrt[3]{x^4+x^2}}-\frac{3\sqrt[3]{x^2+1}\,xF_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{4x^2}{\left(i-\sqrt{3}\right)^2},-x^2\right)}{\sqrt[3]{x^4+x^2}}-\frac{3\sqrt[3]{x^2+1}\,xF_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{4x^2}{\left(i+\sqrt{3}\right)^2},-x^2\right)}{\sqrt[3]{x^4+x^2}}+\frac{3\sqrt[3]{x^2+1}\,x\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^4+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^2)/((1 + x + x^2)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] (-3*x*(1 + x^2)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6, (-4*x^2)/(I - Sqrt[3])^2,
-x^2])/(x^2 + x^4)^(1/3) - (3*x*(1 + x^2)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6,
(-4*x^2)/(I + Sqrt[3])^2, -x^2])/(x^2 + x^4)^(1/3) + (3*x^2*(1 + x^2)^(1/3)
*AppellF1[2/3, 1, 1/3, 5/3, (-4*x^2)/(I - Sqrt[3])^2, -x^2])/(2*(1 + I*Sqrt
[3])*(x^2 + x^4)^(1/3)) + (3*x^2*(1 + x^2)^(1/3)*AppellF1[2/3, 1, 1/3, 5/3,
 (-4*x^2)/(I + Sqrt[3])^2, -x^2])/(2*(1 - I*Sqrt[3])*(x^2 + x^4)^(1/3)) + (
3*x*(1 + x^2)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -x^2])/(x^2 + x^4)^(1/
3)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-1+x^2}{\left(1+x+x^2\right)\sqrt[3]{x^2+x^4}}\,dx = \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{-1+x^2}{x^{2/3}\sqrt[3]{1+x^2}\left(1+x+x^2\right)}\,dx}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \left(\frac{1}{x^{2/3}\sqrt[3]{1+x^2}}-\frac{2+x}{x^{2/3}\sqrt[3]{1+x^2}\left(1+x+x^2\right)}\right)dx}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{1}{x^{2/3}\sqrt[3]{1+x^2}}\,dx}{\sqrt[3]{x^2+x^4}} - \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{2+x}{x^{2/3}\sqrt[3]{1+x^2}\left(1+x+x^2\right)}\,dx}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \left(\frac{1-i\sqrt{3}}{x^{2/3}\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1+x^2}}+\frac{1+i}{x^{2/3}\left(1+i\sqrt{3}\right.}\right.}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(\left(1-i\sqrt{3}\right)x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{1}{x^{2/3}\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1+x^2}}\,dx}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(2\left(1-i\sqrt{3}\right)x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{\sqrt[3]{x}}{\left(\left(1-i\sqrt{3}\right)^2-4x^2\right)\sqrt[3]{1+x^2}}\,d}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(6\left(1-i\sqrt{3}\right)x^{2/3}\sqrt[3]{1+x^2}\right)\mathrm{Subst}\left(\int \frac{x^3}{\left(\left(1-i\sqrt{3}\right)^2-4x^6\right)}\right.}{\sqrt[3]{x^2+x^4}}$$

$$= -\frac{3x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{4x^2}{\left(i-\sqrt{3}\right)^2},-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{3x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{4x^2}{\left(i+\sqrt{3}\right)^2},-x^2\right.}{\sqrt[3]{x^2+x^4}}$$

$$= -\frac{3x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{4x^2}{\left(i-\sqrt{3}\right)^2},-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{3x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{4x^2}{\left(i+\sqrt{3}\right)^2},-x^2\right.}{\sqrt[3]{x^2+x^4}}$$

Mathematica [F] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{-1+x^2}{\left(1+x+x^2\right)\sqrt[3]{x^2+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^2)/((1 + x + x^2)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] Integrate[(-1 + x^2)/((1 + x + x^2)*(x^2 + x^4)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.33, size = 85, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^4+x^2}+x\right)+\frac{1}{2}\log\left(x^2-\sqrt[3]{x^4+x^2}\,x+\left(x^4+x^2\right)^{2/3}\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^4+x^2}-x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x + x^2)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(x^2 + x^4)^(1/3))]) - Log[x + (x^2 + 
x^4)^(1/3)] + Log[x^2 - x*(x^2 + x^4)^(1/3) + (x^2 + x^4)^(2/3)]/2
```

fricas [A] time = 1.20, size = 106, normalized size = 1.25

$$-\sqrt{3} \arctan\left(\frac{\sqrt{3}\,x^2+2\,\sqrt{3}\left(x^4+x^2\right)^{\frac{1}{3}}x+4\,\sqrt{3}\left(x^4+x^2\right)^{\frac{2}{3}}}{8\,x^3-x^2+8\,x}\right)-\frac{1}{2}\,\log\left(\frac{x^3+x^2+3\left(x^4+x^2\right)^{\frac{1}{3}}x+x+3\left(x^4+x^2\right)^{\frac{2}{3}}}{x^3+x^2+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+x+1)/(x^4+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan((sqrt(3)*x^2 + 2*sqrt(3)*(x^4 + x^2)^(1/3)*x + 4*sqrt(3)*(x
^4 + x^2)^(2/3))/(8*x^3 - x^2 + 8*x)) - 1/2*log((x^3 + x^2 + 3*(x^4 + x^2)^
(1/3)*x + x + 3*(x^4 + x^2)^(2/3))/(x^3 + x^2 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^4+x^2\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+x+1)/(x^4+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/((x^4 + x^2)^(1/3)*(x^2 + x + 1)), x)
```

maple [C] time = 1.79, size = 307, normalized size = 3.61

$$-\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)^2 x^3-2\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)^2 x^2-3\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)x^3+\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)^2 x-3\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^4+x^2\right)^{\frac{2}{3}}+3\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)x^2+2x^3-3\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)x+3\left(x^4+x^2\right)^{\frac{2}{3}}+3x\left(x^4+x^2\right)^{\frac{1}{3}}-x^2+2x}{x\left(x^2+x+1\right)}\right)+\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)^2 x^3-2\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)^2 x^2+\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)^2 x-3\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^4+x^2\right)^{\frac{2}{3}}-3\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^4+x^2\right)^{\frac{1}{3}}x+3\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)x^2-x^3+3x\left(x^4+x^2\right)^{\frac{1}{3}}-x^2-x}{x\left(x^2+x+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+x+1)/(x^4+x^2)^(1/3),x)
```

```
[Out] -ln((RootOf(_Z^2-_Z+1)^2*x^3-2*RootOf(_Z^2-_Z+1)^2*x^2-3*RootOf(_Z^2-_Z+1)*
x^3+RootOf(_Z^2-_Z+1)^2*x-3*RootOf(_Z^2-_Z+1)*(x^4+x^2)^(2/3)+3*RootOf(_Z^2
-_Z+1)*x^2+2*x^3-3*RootOf(_Z^2-_Z+1)*x+3*(x^4+x^2)^(2/3)+3*x*(x^4+x^2)^(1/3
)-x^2+2*x)/x/(x^2+x+1))+RootOf(_Z^2-_Z+1)*ln(-(RootOf(_Z^2-_Z+1)^2*x^3-2*Ro
otOf(_Z^2-_Z+1)^2*x^2+RootOf(_Z^2-_Z+1)^2*x-3*RootOf(_Z^2-_Z+1)*(x^4+x^2)^(
2/3)-3*RootOf(_Z^2-_Z+1)*(x^4+x^2)^(1/3)*x+3*RootOf(_Z^2-_Z+1)*x^2-x^3+3*x*
(x^4+x^2)^(1/3)-x^2-x)/x/(x^2+x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^4+x^2\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+x+1)/(x^4+x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)/((x^4 + x^2)^(1/3)*(x^2 + x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2-1}{{\left(x^4+x^2\right)}^{1/3}\,\left(x^2+x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + x^4)^(1/3)*(x + x^2 + 1)),x)
```

```
[Out] int((x^2 - 1)/((x^2 + x^4)^(1/3)*(x + x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{\sqrt[3]{x^2(x^2+1)}\,(x^2+x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+x+1)/(x**4+x**2)**(1/3),x)
```

```
[Out] Integral((x - 1)*(x + 1)/((x**2*(x**2 + 1))**(1/3)*(x**2 + x + 1)), x)
```

3.1021 $\int \frac{(-2b+ax^4)(b+ax^4)^{3/4}}{x^8}\,dx$

Optimal. Leaf size=85

$$\frac{1}{2}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\frac{1}{2}a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\frac{(6b-ax^4)(ax^4+b)^{3/4}}{21x^7}$$

Rubi [A] time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {451, 277, 240, 212, 206, 203}

$$\frac{1}{2}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\frac{1}{2}a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\frac{2(ax^4+b)^{7/4}}{7x^7}-\frac{a(ax^4+b)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-2*b + a*x^4)*(b + a*x^4)^(3/4))/x^8,x]
```

```
[Out] -1/3*(a*(b + a*x^4)^(3/4))/x^3 + (2*(b + a*x^4)^(7/4))/(7*x^7) + (a^(7/4)*A
rcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/2 + (a^(7/4)*ArcTanh[(a^(1/4)*x)/(b +
 a*x^4)^(1/4)])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-2b+ax^4\right)\left(b+ax^4\right)^{3/4}}{x^8}\,dx &= \frac{2\left(b+ax^4\right)^{7/4}}{7x^7}+a\int \frac{\left(b+ax^4\right)^{3/4}}{x^4}\,dx \\
&= -\frac{a\left(b+ax^4\right)^{3/4}}{3x^3}+\frac{2\left(b+ax^4\right)^{7/4}}{7x^7}+a^2\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx \\
&= -\frac{a\left(b+ax^4\right)^{3/4}}{3x^3}+\frac{2\left(b+ax^4\right)^{7/4}}{7x^7}+a^2\operatorname{Subst}\left(\int \frac{1}{1-ax^4}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right) \\
&= -\frac{a\left(b+ax^4\right)^{3/4}}{3x^3}+\frac{2\left(b+ax^4\right)^{7/4}}{7x^7}+\frac{1}{2}a^2\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)+ \\
&= -\frac{a\left(b+ax^4\right)^{3/4}}{3x^3}+\frac{2\left(b+ax^4\right)^{7/4}}{7x^7}+\frac{1}{2}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)+\frac{1}{2}a^{7/4}\tanh^{-1}\left(\frac{}{\sqrt[4]{b}}\right.
\end{aligned}
$$

Mathematica [C] time = 0.06, size = 67, normalized size = 0.79

$$
\frac{\left(ax^4+b\right)^{3/4}\left(6\left(ax^4+b\right)-\frac{7ax^4\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};-\frac{ax^4}{b}\right)}{\left(\frac{ax^4}{b}+1\right)^{3/4}}\right)}{21x^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((-2*b + a*x^4)*(b + a*x^4)^(3/4))/x^8,x]
```

```
[Out] ((b + a*x^4)^(3/4)*(6*(b + a*x^4) - (7*a*x^4*Hypergeometric2F1[-3/4, -3/4,
1/4, -((a*x^4)/b)])/(1 + (a*x^4)/b)^(3/4)))/(21*x^7)
```

IntegrateAlgebraic [A] time = 0.26, size = 85, normalized size = 1.00

$$
\frac{1}{2}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\frac{1}{2}a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\frac{\left(6b-ax^4\right)\left(ax^4+b\right)^{3/4}}{21x^7}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*b + a*x^4)*(b + a*x^4)^(3/4))/x^8,x]
```

```
[Out] ((6*b - a*x^4)*(b + a*x^4)^(3/4))/(21*x^7) + (a^(7/4)*ArcTan[(a^(1/4)*x)/(b
 + a*x^4)^(1/4)])/2 + (a^(7/4)*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-2*b)*(a*x^4+b)^(3/4)/x^8,x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4+b\right)^{\frac{3}{4}}\left(ax^4-2\,b\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-2*b)*(a*x^4+b)^(3/4)/x^8,x, algorithm="giac")
[Out] integrate((a*x^4 + b)^(3/4)*(a*x^4 - 2*b)/x^8, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^4-2b\right)\left(a\,x^4+b\right)^{\frac{3}{4}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-2*b)*(a*x^4+b)^(3/4)/x^8,x)
[Out] int((a*x^4-2*b)*(a*x^4+b)^(3/4)/x^8,x)
```

maxima [A] time = 0.41, size = 103, normalized size = 1.21

$$-\frac{1}{12}\left(3\,a\left(\frac{2\,\arctan\left(\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right)+\frac{4\left(ax^4+b\right)^{\frac{3}{4}}}{x^3}\right)a+\frac{2\left(ax^4+b\right)^{\frac{7}{4}}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-2*b)*(a*x^4+b)^(3/4)/x^8,x, algorithm="maxima")
[Out] -1/12*(3*a*(2*arctan((a*x^4 + b)^(1/4)/(a^(1/4)*x))/a^(1/4) + log(-(a^(1/4)
 - (a*x^4 + b)^(1/4)/x)/(a^(1/4) + (a*x^4 + b)^(1/4)/x))/a^(1/4)) + 4*(a*x^
4 + b)^(3/4)/x^3)*a + 2/7*(a*x^4 + b)^(7/4)/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(a\,x^4+b\right)^{3/4}\,\left(2\,b-a\,x^4\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b + a*x^4)^(3/4)*(2*b - a*x^4))/x^8,x)
[Out] -int(((b + a*x^4)^(3/4)*(2*b - a*x^4))/x^8, x)
```

sympy [C] time = 3.26, size = 114, normalized size = 1.34

$$-\frac{a^{\frac{7}{4}}\left(1+\frac{b}{ax^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{2\Gamma\left(-\frac{3}{4}\right)}-\frac{a^{\frac{3}{4}}b\left(1+\frac{b}{ax^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{2x^4\Gamma\left(-\frac{3}{4}\right)}+\frac{ab^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{ax^4e^{i\pi}}{b}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-2*b)*(a*x**4+b)**(3/4)/x**8,x)
```

```
[Out] -a**(7/4)*(1 + b/(a*x**4))**(3/4)*gamma(-7/4)/(2*gamma(-3/4)) - a**(3/4)*b*
(1 + b/(a*x**4))**(3/4)*gamma(-7/4)/(2*x**4*gamma(-3/4)) + a*b**(3/4)*gamma
(-3/4)*hyper((-3/4, -3/4), (1/4,), a*x**4*exp_polar(I*pi)/b)/(4*x**3*gamma(
1/4))
```

3.1022 $\int \frac{\left(-b+ax^4\right)^{3/4}\left(-b+2ax^4\right)}{x^8}\,dx$

Optimal. Leaf size=85

$$a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+\frac{\left(3b-17ax^4\right)\left(ax^4-b\right)^{3/4}}{21x^7}$$

Rubi [A] time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {451, 277, 240, 212, 206, 203}

$$a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)-\frac{\left(ax^4-b\right)^{7/4}}{7x^7}-\frac{2a\left(ax^4-b\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a*x^4)^(3/4)*(-b + 2*a*x^4))/x^8,x]
```

```
[Out] (-2*a*(-b + a*x^4)^(3/4))/(3*x^3) - (-b + a*x^4)^(7/4)/(7*x^7) + a^(7/4)*Ar
cTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)] + a^(7/4)*ArcTanh[(a^(1/4)*x)/(-b + a*
x^4)^(1/4)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\int \frac{\left(-b+ax^4\right)^{3/4}\left(-b+2ax^4\right)}{x^8}\,dx = -\frac{\left(-b+ax^4\right)^{7/4}}{7x^7}+(2a)\int \frac{\left(-b+ax^4\right)^{3/4}}{x^4}\,dx$$

$$= -\frac{2a\left(-b+ax^4\right)^{3/4}}{3x^3}-\frac{\left(-b+ax^4\right)^{7/4}}{7x^7}+\left(2a^2\right)\int \frac{1}{\sqrt[4]{-b+ax^4}}\,dx$$

$$= -\frac{2a\left(-b+ax^4\right)^{3/4}}{3x^3}-\frac{\left(-b+ax^4\right)^{7/4}}{7x^7}+\left(2a^2\right)\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx, x, \frac{x}{\sqrt[4]{-b+ax}}\right.$$

$$= -\frac{2a\left(-b+ax^4\right)^{3/4}}{3x^3}-\frac{\left(-b+ax^4\right)^{7/4}}{7x^7}+a^2\,\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right.$$

$$= -\frac{2a\left(-b+ax^4\right)^{3/4}}{3x^3}-\frac{\left(-b+ax^4\right)^{7/4}}{7x^7}+a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)+a^{7/4}\tanh^{-1}\left(\right.$$

Mathematica [C] time = 0.09, size = 69, normalized size = 0.81

$$-\frac{\left(ax^4-b\right)^{3/4}\left(\frac{14ax^4\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};\frac{ax^4}{b}\right)}{\left(1-\frac{ax^4}{b}\right)^{3/4}}+3ax^4-3b\right)}{21x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + a*x^4)^(3/4)*(-b + 2*a*x^4))/x^8,x]
```

```
[Out] -1/21*((-b + a*x^4)^(3/4)*(-3*b + 3*a*x^4 + (14*a*x^4*Hypergeometric2F1[-3/
4, -3/4, 1/4, (a*x^4)/b])/(1 - (a*x^4)/b)^(3/4)))/x^7
```

IntegrateAlgebraic [A] time = 0.26, size = 85, normalized size = 1.00

$$a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+\frac{\left(3b-17ax^4\right)\left(ax^4-b\right)^{3/4}}{21x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^4)^(3/4)*(-b + 2*a*x^4))/x^8,x]
```

```
[Out] ((3*b - 17*a*x^4)*(-b + a*x^4)^(3/4))/(21*x^7) + a^(7/4)*ArcTan[(a^(1/4)*x)
/(-b + a*x^4)^(1/4)] + a^(7/4)*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4)*(2*a*x^4-b)/x^8,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,ax^4 - b\right)\left(ax^4 - b\right)^{\frac{3}{4}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4)*(2*a*x^4-b)/x^8,x, algorithm="giac")
[Out] integrate((2*a*x^4 - b)*(a*x^4 - b)^(3/4)/x^8, x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^4 - b\right)^{\frac{3}{4}}\left(2a\,x^4 - b\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b)^(3/4)*(2*a*x^4-b)/x^8,x)
[Out] int((a*x^4-b)^(3/4)*(2*a*x^4-b)/x^8,x)
```

maxima [A] time = 0.42, size = 113, normalized size = 1.33

$$-\frac{1}{6}\left(3\,a\left(\frac{2\,\arctan\left(\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right)+\frac{4\left(ax^4-b\right)^{\frac{3}{4}}}{x^3}\right)a-\frac{\left(ax^4-b\right)^{\frac{7}{4}}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4)*(2*a*x^4-b)/x^8,x, algorithm="maxima")
[Out] -1/6*(3*a*(2*arctan((a*x^4 - b)^(1/4)/(a^(1/4)*x))/a^(1/4) + log(-(a^(1/4)
- (a*x^4 - b)^(1/4)/x)/(a^(1/4) + (a*x^4 - b)^(1/4)/x))/a^(1/4)) + 4*(a*x^4
 - b)^(3/4)/x^3)*a - 1/7*(a*x^4 - b)^(7/4)/x^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(a\,x^4-b\right)^{3/4}\,\left(b-2\,a\,x^4\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*x^4 - b)^(3/4)*(b - 2*a*x^4))/x^8,x)
[Out] -int(((a*x^4 - b)^(3/4)*(b - 2*a*x^4))/x^8, x)
```

sympy [C] time = 3.44, size = 209, normalized size = 2.46

$$\frac{ab^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{ax^4}{b}\right)}{2x^3\Gamma\left(\frac{1}{4}\right)}-b\left(\begin{cases}\frac{a^{\frac{7}{4}}\left(-1+\frac{b}{ax^4}\right)^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{7}{4}\right)}{4b\Gamma\left(-\frac{3}{4}\right)}-\frac{a^{\frac{3}{4}}\left(-1+\frac{b}{ax^4}\right)^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{7}{4}\right)}{4x^4\Gamma\left(-\frac{3}{4}\right)} & \text{for}\ \left|\frac{b}{ax^4}\right|>1\\ -\frac{a^{\frac{7}{4}}\left(1-\frac{b}{ax^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4b\Gamma\left(-\frac{3}{4}\right)}+\frac{a^{\frac{3}{4}}\left(1-\frac{b}{ax^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4x^4\Gamma\left(-\frac{3}{4}\right)} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b)**(3/4)*(2*a*x**4-b)/x**8,x)
```

```
[Out] a*b**(3/4)*exp(3*I*pi/4)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), a*x**4/b)/
(2*x**3*gamma(1/4)) - b*Piecewise((a**(7/4)*(-1 + b/(a*x**4))**(3/4)*exp(-I
*pi/4)*gamma(-7/4)/(4*b*gamma(-3/4)) - a**(3/4)*(-1 + b/(a*x**4))**(3/4)*ex
p(-I*pi/4)*gamma(-7/4)/(4*x**4*gamma(-3/4)), Abs(b/(a*x**4)) > 1), (-a**(7/
4)*(1 - b/(a*x**4))**(3/4)*gamma(-7/4)/(4*b*gamma(-3/4)) + a**(3/4)*(1 - b/
(a*x**4))**(3/4)*gamma(-7/4)/(4*x**4*gamma(-3/4)), True))
```

3.1023 $\int \frac{(-q+px^2)(aq+bx+apx^2)\sqrt{q^2+p^2x^4}}{x^4}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{p^2x^4+q^2}\,(2ap^2x^4+2aq^2+3bpx^3+3bqx)}{6x^3}-bpq\log\left(\sqrt{p^2x^4+q^2}+px^2+q\right)+bpq\log(x)$$

Rubi [A] time = 0.24, antiderivative size = 109, normalized size of antiderivative = 1.28, number of steps used = 11, number of rules used = 10, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.244, Rules used = {1833, 1252, 813, 844, 217, 206, 266, 63, 208, 449}

$$\frac{a(p^2x^4+q^2)^{3/2}}{3x^3}-\frac{1}{2}bpq\tanh^{-1}\left(\frac{\sqrt{p^2x^4+q^2}}{q}\right)+\frac{b(px^2+q)\sqrt{p^2x^4+q^2}}{2x^2}-\frac{1}{2}bpq\tanh^{-1}\left(\frac{px^2}{\sqrt{p^2x^4+q^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-q + p*x^2)*(a*q + b*x + a*p*x^2)*Sqrt[q^2 + p^2*x^4])/x^4,x]
```

```
[Out] (b*(q + p*x^2)*Sqrt[q^2 + p^2*x^4])/(2*x^2) + (a*(q^2 + p^2*x^4)^(3/2))/(3*
x^3) - (b*p*q*ArcTanh[(p*x^2)/Sqrt[q^2 + p^2*x^4]])/2 - (b*p*q*ArcTanh[Sqrt
[q^2 + p^2*x^4]/q])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{\left(-q+px^2\right)\left(aq+bx+apx^2\right)\sqrt{q^2+p^2x^4}}{x^4}\,dx = \int \left(\frac{\left(-bq+bpx^2\right)\sqrt{q^2+p^2x^4}}{x^3} + \frac{\sqrt{q^2+p^2x^4}\,\left(-aq^2+ap^2x^4\right.}{x^4}\right.$$

$$= \int \frac{\left(-bq+bpx^2\right)\sqrt{q^2+p^2x^4}}{x^3}\,dx + \int \frac{\sqrt{q^2+p^2x^4}\,\left(-aq^2+a\right.}{x^4}$$

$$= \frac{a\left(q^2+p^2x^4\right)^{3/2}}{3x^3} + \frac{1}{2}\,\text{Subst}\left(\int \frac{(-bq+bpx)\sqrt{q^2+p^2x^2}}{x^2}\,dx,\right.$$

$$= \frac{b\left(q+px^2\right)\sqrt{q^2+p^2x^4}}{2x^2} + \frac{a\left(q^2+p^2x^4\right)^{3/2}}{3x^3} - \frac{1}{4}\,\text{Subst}\left(\int \right.$$

$$= \frac{b\left(q+px^2\right)\sqrt{q^2+p^2x^4}}{2x^2} + \frac{a\left(q^2+p^2x^4\right)^{3/2}}{3x^3} - \frac{1}{2}\left(bp^2q\right)\text{Subst}$$

$$= \frac{b\left(q+px^2\right)\sqrt{q^2+p^2x^4}}{2x^2} + \frac{a\left(q^2+p^2x^4\right)^{3/2}}{3x^3} - \frac{1}{2}\left(bp^2q\right)\text{Subst}$$

$$= \frac{b\left(q+px^2\right)\sqrt{q^2+p^2x^4}}{2x^2} + \frac{a\left(q^2+p^2x^4\right)^{3/2}}{3x^3} - \frac{1}{2}bpq\tanh^{-1}\left(\right.$$

$$= \frac{b\left(q+px^2\right)\sqrt{q^2+p^2x^4}}{2x^2} + \frac{a\left(q^2+p^2x^4\right)^{3/2}}{3x^3} - \frac{1}{2}bpq\tanh^{-1}\left(\right.$$

Mathematica [C] time = 0.28, size = 226, normalized size = 2.66

$$\frac{6ap^2x^4\sqrt{p^2x^4+q^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{p^2x^4}{q^2}\right)+2aq^2\sqrt{p^2x^4+q^2}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{p^2x^4}{q^2}\right)+3bpx^3\sqrt{\frac{p^2x^4}{q^2}+1}\left(\sqrt{p^2x^4+q^2}-q\tanh^{-1}\left(\frac{\sqrt{p^2x^4+q^2}}{q}\right)\right)+3bx\sqrt{p^2x^4+q^2}\left(q\sqrt{\frac{p^2x^4}{q^2}+1}-px^2\sinh^{-1}\left(\frac{px^2}{q}\right)\right)}{6x^3\sqrt{\frac{p^2x^4}{q^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-q + p*x^2)*(a*q + b*x + a*p*x^2)*Sqrt[q^2 + p^2*x^4])/x^4,x]
```

```
[Out] (3*b*x*Sqrt[q^2 + p^2*x^4]*(q*Sqrt[1 + (p^2*x^4)/q^2] - p*x^2*ArcSinh[(p*x^
2)/q]) + 3*b*p*x^3*Sqrt[1 + (p^2*x^4)/q^2]*(Sqrt[q^2 + p^2*x^4] - q*ArcTanh
[Sqrt[q^2 + p^2*x^4]/q]) + 2*a*q^2*Sqrt[q^2 + p^2*x^4]*Hypergeometric2F1[-3
/4, -1/2, 1/4, -((p^2*x^4)/q^2)] + 6*a*p^2*x^4*Sqrt[q^2 + p^2*x^4]*Hypergeo
metric2F1[-1/2, 1/4, 5/4, -((p^2*x^4)/q^2)])/(6*x^3*Sqrt[1 + (p^2*x^4)/q^2]
)
```

IntegrateAlgebraic [A] time = 1.18, size = 85, normalized size = 1.00

$$\frac{\sqrt{p^2x^4+q^2}\,\left(2ap^2x^4+2aq^2+3bpx^3+3bqx\right)}{6x^3} - bpq\log\left(\sqrt{p^2x^4+q^2}+px^2+q\right) + bpq\log(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-q + p*x^2)*(a*q + b*x + a*p*x^2)*Sqrt[q^2 + p^2*x^4])
/x^4,x]
```

```
[Out] (Sqrt[q^2 + p^2*x^4]*(2*a*q^2 + 3*b*q*x + 3*b*p*x^3 + 2*a*p^2*x^4))/(6*x^3)
 + b*p*q*Log[x] - b*p*q*Log[q + p*x^2 + Sqrt[q^2 + p^2*x^4]]
```

fricas [A] time = 0.71, size = 83, normalized size = 0.98

$$\frac{6\,bpqx^3\log\left(\frac{px^2+q-\sqrt{p^2x^4+q^2}}{x}\right)+\left(2\,ap^2x^4+3\,bpx^3+2\,aq^2+3\,bqx\right)\sqrt{p^2x^4+q^2}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^2-q)*(a*p*x^2+a*q+b*x)*(p^2*x^4+q^2)^(1/2)/x^4,x, algorithm=
"fricas")
```

```
[Out] 1/6*(6*b*p*q*x^3*log((p*x^2 + q - sqrt(p^2*x^4 + q^2))/x) + (2*a*p^2*x^4 +
3*b*p*x^3 + 2*a*q^2 + 3*b*q*x)*sqrt(p^2*x^4 + q^2))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^4+q^2}\left(apx^2+aq+bx\right)\left(px^2-q\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^2-q)*(a*p*x^2+a*q+b*x)*(p^2*x^4+q^2)^(1/2)/x^4,x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(p^2*x^4 + q^2)*(a*p*x^2 + a*q + b*x)*(p*x^2 - q)/x^4, x)
```

maple [B] time = 0.08, size = 190, normalized size = 2.24

$$\frac{a\,p^2x\sqrt{p^2x^4+q^2}}{3}+\frac{a\,q^2\sqrt{p^2x^4+q^2}}{3x^3}+\frac{b\left(p^2x^4+q^2\right)^{\frac{3}{2}}}{2q\,x^2}-\frac{b\,p^2x^2\sqrt{p^2x^4+q^2}}{2q}-\frac{qb\,p^2\ln\left(\frac{p^2x^2}{\sqrt{p^2}}+\sqrt{p^2x^4+q^2}\right)}{2\sqrt{p^2}}+\frac{pb\sqrt{p^2x^4+q^2}}{2}-\frac{pb\,q^2\ln\left(\frac{2q^2+2\sqrt{q^2}\,\sqrt{p^2x^4+q^2}}{x^2}\right)}{2\sqrt{q^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^2-q)*(a*p*x^2+a*q+b*x)*(p^2*x^4+q^2)^(1/2)/x^4,x)
```

```
[Out] 1/3*a*p^2*x*(p^2*x^4+q^2)^(1/2)+1/3*a*q^2*(p^2*x^4+q^2)^(1/2)/x^3+1/2/q*b/x
^2*(p^2*x^4+q^2)^(3/2)-1/2/q*b*p^2*x^2*(p^2*x^4+q^2)^(1/2)-1/2*q*b*p^2*ln(p
^2*x^2/(p^2)^(1/2)+(p^2*x^4+q^2)^(1/2))/(p^2)^(1/2)+1/2*p*b*(p^2*x^4+q^2)^(
1/2)-1/2*p*b*q^2/(q^2)^(1/2)*ln((2*q^2+2*(q^2)^(1/2)*(p^2*x^4+q^2)^(1/2))/x
^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^4+q^2}\left(apx^2+aq+bx\right)\left(px^2-q\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^2-q)*(a*p*x^2+a*q+b*x)*(p^2*x^4+q^2)^(1/2)/x^4,x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(p^2*x^4 + q^2)*(a*p*x^2 + a*q + b*x)*(p*x^2 - q)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{p^2\,x^4+q^2}\,\left(q-p\,x^2\right)\,\left(a\,p\,x^2+b\,x+a\,q\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((p^2*x^4 + q^2)^(1/2)*(q - p*x^2)*(a*q + b*x + a*p*x^2))/x^4,x)
```

```
[Out] -int(((p^2*x^4 + q^2)^(1/2)*(q - p*x^2)*(a*q + b*x + a*p*x^2))/x^4, x)
```

sympy [C] time = 5.45, size = 223, normalized size = 2.62

$$\frac{ap^2qx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{p^2x^4e^{i\pi}}{q^2}\right)}{4\Gamma\left(\frac{5}{4}\right)}-\frac{aq^3\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{p^2x^4e^{i\pi}}{q^2}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}+\frac{bp^2x^2}{2\sqrt{\frac{p^2x^4}{q^2}+1}}+\frac{bp^2x^2}{2\sqrt{1+\frac{q^2}{p^2x^4}}}-\frac{bpq\operatorname{asinh}\left(\frac{q}{px^2}\right)}{2}-\frac{bpq\operatorname{asinh}\left(\frac{px^2}{q}\right)}{2}+\frac{bq^2}{2x^2\sqrt{\frac{p^2x^4}{q^2}+1}}+\frac{bq^2}{2x^2\sqrt{1+\frac{q^2}{p^2x^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**2-q)*(a*p*x**2+a*q+b*x)*(p**2*x**4+q**2)**(1/2)/x**4,x)
```

```
[Out] a*p**2*q*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), p**2*x**4*exp_polar(I*pi)/
q**2)/(4*gamma(5/4)) - a*q**3*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), p**2*
x**4*exp_polar(I*pi)/q**2)/(4*x**3*gamma(1/4)) + b*p**2*x**2/(2*sqrt(p**2*x
**4/q**2 + 1)) + b*p**2*x**2/(2*sqrt(1 + q**2/(p**2*x**4))) - b*p*q*asinh(q
/(p*x**2))/2 - b*p*q*asinh(p*x**2/q)/2 + b*q**2/(2*x**2*sqrt(p**2*x**4/q**2
 + 1)) + b*q**2/(2*x**2*sqrt(1 + q**2/(p**2*x**4)))
```

3.1024 $\int \frac{-1+2x^3}{(1+x+x^3)\sqrt[3]{x^2+x^5}}\,dx$

Optimal. Leaf size=85

$$-\log\left(\sqrt[3]{x^5+x^2}+x\right)+\frac{1}{2}\log\left(x^2-\sqrt[3]{x^5+x^2}\,x+\left(x^5+x^2\right)^{2/3}\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^5+x^2}-x}\right)$$

Rubi [F] time = 1.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+2x^3}{(1+x+x^3)\sqrt[3]{x^2+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 2*x^3)/((1 + x + x^3)*(x^2 + x^5)^(1/3)),x]
```

```
[Out] (6*x*(1 + x^3)^(1/3)*Hypergeometric2F1[1/9, 1/3, 10/9, -x^3])/(x^2 + x^5)^(
1/3) - (9*x^(2/3)*(1 + x^3)^(1/3)*Defer[Subst][Defer[Int][1/((1 + x^9)^(1/3
)*(1 + x^3 + x^9)), x], x, x^(1/3)])/(x^2 + x^5)^(1/3) - (6*x^(2/3)*(1 + x^
3)^(1/3)*Defer[Subst][Defer[Int][x^3/((1 + x^9)^(1/3)*(1 + x^3 + x^9)), x],
 x, x^(1/3)])/(x^2 + x^5)^(1/3)
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+2x^3}{(1+x+x^3)\sqrt[3]{x^2+x^5}}\,dx &= \frac{\left(x^{2/3}\sqrt[3]{1+x^3}\right)\int \frac{-1+2x^3}{x^{2/3}\sqrt[3]{1+x^3}\,(1+x+x^3)}\,dx}{\sqrt[3]{x^2+x^5}}\\
&= \frac{\left(3x^{2/3}\sqrt[3]{1+x^3}\right)\operatorname{Subst}\left(\int \frac{-1+2x^9}{\sqrt[3]{1+x^9}\,(1+x^3+x^9)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^5}}\\
&= \frac{\left(3x^{2/3}\sqrt[3]{1+x^3}\right)\operatorname{Subst}\left(\int \left(\frac{2}{\sqrt[3]{1+x^9}}-\frac{3+2x^3}{\sqrt[3]{1+x^9}\,(1+x^3+x^9)}\right)dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^5}}\\
&= -\frac{\left(3x^{2/3}\sqrt[3]{1+x^3}\right)\operatorname{Subst}\left(\int \frac{3+2x^3}{\sqrt[3]{1+x^9}\,(1+x^3+x^9)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^5}}+\frac{\left(6x^{2/3}\sqrt[3]{1+x^3}\right)\operatorname{Subst}\left(\int \right.}{\sqrt[3]{x^2+x}}\\
&= \frac{6x\sqrt[3]{1+x^3}\;{}_2F_1\left(\frac{1}{9},\frac{1}{3};\frac{10}{9};-x^3\right)}{\sqrt[3]{x^2+x^5}}-\frac{\left(3x^{2/3}\sqrt[3]{1+x^3}\right)\operatorname{Subst}\left(\int \left(\frac{3}{\sqrt[3]{1+x^9}\,(1+x^3+x^9)}+\frac{}{\sqrt[3]{1+}}\right.\right.}{\sqrt[3]{x^2+x^5}}\\
&= \frac{6x\sqrt[3]{1+x^3}\;{}_2F_1\left(\frac{1}{9},\frac{1}{3};\frac{10}{9};-x^3\right)}{\sqrt[3]{x^2+x^5}}-\frac{\left(6x^{2/3}\sqrt[3]{1+x^3}\right)\operatorname{Subst}\left(\int \frac{x^3}{\sqrt[3]{1+x^9}\,(1+x^3+x^9)}\,dx,x,\sqrt[3]{\right.}{\sqrt[3]{x^2+x^5}}
\end{aligned}$$

Mathematica [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{-1+2x^3}{(1+x+x^3)\sqrt[3]{x^2+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + 2*x^3)/((1 + x + x^3)*(x^2 + x^5)^(1/3)),x]
```

```
[Out] Integrate[(-1 + 2*x^3)/((1 + x + x^3)*(x^2 + x^5)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.91, size = 85, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^5+x^2}+x\right)+\frac{1}{2}\log\left(x^2-\sqrt[3]{x^5+x^2}\,x+\left(x^5+x^2\right)^{2/3}\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^5+x^2}-x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x^3)/((1 + x + x^3)*(x^2 + x^5)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(x^2 + x^5)^(1/3))]) - Log[x + (x^2 +
x^5)^(1/3)] + Log[x^2 - x*(x^2 + x^5)^(1/3) + (x^2 + x^5)^(2/3)]/2
```

fricas [A] time = 1.22, size = 108, normalized size = 1.27

$$-\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(x^5+x^2\right)^{\frac{1}{3}}x+\sqrt{3}\left(x^4+x^2+x\right)+2\sqrt{3}\left(x^5+x^2\right)^{\frac{2}{3}}}{3\left(x^4-x^2+x\right)}\right)-\frac{1}{2}\log\left(\frac{x^4+x^2+3\left(x^5+x^2\right)^{\frac{1}{3}}x+x+3\left(x^5+x^2\right)^{\frac{2}{3}}}{x^4+x^2+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-1)/(x^3+x+1)/(x^5+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(1/3*(2*sqrt(3)*(x^5 + x^2)^(1/3)*x + sqrt(3)*(x^4 + x^2 + x
) + 2*sqrt(3)*(x^5 + x^2)^(2/3))/(x^4 - x^2 + x)) - 1/2*log((x^4 + x^2 + 3*
(x^5 + x^2)^(1/3)*x + x + 3*(x^5 + x^2)^(2/3))/(x^4 + x^2 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3-1}{\left(x^5+x^2\right)^{\frac{1}{3}}\left(x^3+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-1)/(x^3+x+1)/(x^5+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 - 1)/((x^5 + x^2)^(1/3)*(x^3 + x + 1)), x)
```

maple [C] time = 3.05, size = 555, normalized size = 6.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-1)/(x^3+x+1)/(x^5+x^2)^(1/3),x)
```

```
[Out] RootOf(_Z^2-_Z+1)*ln(-(257*RootOf(_Z^2-_Z+1)^2*x^4-672*RootOf(_Z^2-_Z+1)*x^
4-514*RootOf(_Z^2-_Z+1)^2*x^2+316*x^4+474*RootOf(_Z^2-_Z+1)*(x^5+x^2)^(2/3)
-474*RootOf(_Z^2-_Z+1)*(x^5+x^2)^(1/3)*x+257*RootOf(_Z^2-_Z+1)^2*x+573*Root
Of(_Z^2-_Z+1)*x^2+297*(x^5+x^2)^(2/3)-297*x*(x^5+x^2)^(1/3)-672*RootOf(_Z^2
-_Z+1)*x-158*x^2+316*x)/(x^3+x+1)/x)-ln((-257*RootOf(_Z^2-_Z+1)^2*x^4-158*R
ootOf(_Z^2-_Z+1)*x^4+514*RootOf(_Z^2-_Z+1)^2*x^2+99*x^4+474*RootOf(_Z^2-_Z+
1)*(x^5+x^2)^(2/3)-474*RootOf(_Z^2-_Z+1)*(x^5+x^2)^(1/3)*x-257*RootOf(_Z^2-
_Z+1)^2*x-455*RootOf(_Z^2-_Z+1)*x^2-771*(x^5+x^2)^(2/3)+771*x*(x^5+x^2)^(1/
3)-158*RootOf(_Z^2-_Z+1)*x+99*x^2+99*x)/(x^3+x+1)/x)*RootOf(_Z^2-_Z+1)+ln((
-257*RootOf(_Z^2-_Z+1)^2*x^4-158*RootOf(_Z^2-_Z+1)*x^4+514*RootOf(_Z^2-_Z+1
)^2*x^2+99*x^4+474*RootOf(_Z^2-_Z+1)*(x^5+x^2)^(2/3)-474*RootOf(_Z^2-_Z+1)*
(x^5+x^2)^(1/3)*x-257*RootOf(_Z^2-_Z+1)^2*x-455*RootOf(_Z^2-_Z+1)*x^2-771*(
x^5+x^2)^(2/3)+771*x*(x^5+x^2)^(1/3)-158*RootOf(_Z^2-_Z+1)*x+99*x^2+99*x)/(
x^3+x+1)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^{3}-1}{\left(x^{5}+x^{2}\right)^{\frac{1}{3}}\left(x^{3}+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-1)/(x^3+x+1)/(x^5+x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((2*x^3 - 1)/((x^5 + x^2)^(1/3)*(x^3 + x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x^3-1}{{\left(x^5+x^2\right)}^{1/3}\,\left(x^3+x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3 - 1)/((x^2 + x^5)^(1/3)*(x + x^3 + 1)),x)
```

```
[Out] int((2*x^3 - 1)/((x^2 + x^5)^(1/3)*(x + x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^{3}-1}{\sqrt[3]{x^{2}\left(x+1\right)\left(x^{2}-x+1\right)}\left(x^{3}+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-1)/(x**3+x+1)/(x**5+x**2)**(1/3),x)
```

```
[Out] Integral((2*x**3 - 1)/((x**2*(x + 1)*(x**2 - x + 1))**(1/3)*(x**3 + x + 1))
, x)
```

3.1025 $\int \frac{2+3x^5}{\left(-1+x^2+x^5\right)\sqrt[3]{-x+x^6}}\,dx$

Optimal. Leaf size=85

$$-\log\left(\sqrt[3]{x^6-x}+x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^6-x}-x}\right)+\frac{1}{2}\log\left(-\sqrt[3]{x^6-x}\,x+\left(x^6-x\right)^{2/3}+x^2\right)$$

Rubi [F] time = 1.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2+3x^5}{\left(-1+x^2+x^5\right)\sqrt[3]{-x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2 + 3*x^5)/((-1 + x^2 + x^5)*(-x + x^6)^(1/3)),x]
```

```
[Out] (9*x*(1 - x^5)^(1/3)*Hypergeometric2F1[2/15, 1/3, 17/15, x^5])/(2*(-x + x^6
)^(1/3)) + (15*x^(1/3)*(-1 + x^5)^(1/3)*Defer[Subst][Defer[Int][x/((-1 + x^
15)^(1/3)*(-1 + x^6 + x^15)), x], x, x^(1/3)])/(-x + x^6)^(1/3) - (9*x^(1/3
)*(-1 + x^5)^(1/3)*Defer[Subst][Defer[Int][x^7/((-1 + x^15)^(1/3)*(-1 + x^6
 + x^15)), x], x, x^(1/3)])/(-x + x^6)^(1/3)
```

Rubi steps

$$\int \frac{2+3x^5}{\left(-1+x^2+x^5\right)\sqrt[3]{-x+x^6}}\,dx=\frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^5}\right)\int \frac{2+3x^5}{\sqrt[3]{x}\sqrt[3]{-1+x^5}\left(-1+x^2+x^5\right)}\,dx}{\sqrt[3]{-x+x^6}}$$

$$=\frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^5}\right)\text{Subst}\left(\int \frac{x\left(2+3x^{15}\right)}{\sqrt[3]{-1+x^{15}}\left(-1+x^6+x^{15}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x+x^6}}$$

$$=\frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^5}\right)\text{Subst}\left(\int \left(\frac{3x}{\sqrt[3]{-1+x^{15}}}+\frac{x\left(5-3x^6\right)}{\sqrt[3]{-1+x^{15}}\left(-1+x^6+x^{15}\right)}\right)dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x+x^6}}$$

$$=\frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^5}\right)\text{Subst}\left(\int \frac{x\left(5-3x^6\right)}{\sqrt[3]{-1+x^{15}}\left(-1+x^6+x^{15}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x+x^6}}+\frac{\left(9\sqrt[3]{x}\sqrt[3]{-1+x^5}\right.}{}$$

$$=\frac{\left(9\sqrt[3]{x}\sqrt[3]{1-x^5}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x^{15}}}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x+x^6}}+\frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^5}\right)\text{Subst}\left(\int\left(\right.\right.}{}$$

$$=\frac{9x\sqrt[3]{1-x^5}\,{}_2F_1\left(\frac{2}{15},\frac{1}{3};\frac{17}{15};x^5\right)}{2\sqrt[3]{-x+x^6}}-\frac{\left(9\sqrt[3]{x}\sqrt[3]{-1+x^5}\right)\text{Subst}\left(\int \frac{x^7}{\sqrt[3]{-1+x^{15}}\left(-1+x^6+x^{\cdot}\right.}\right.}{\sqrt[3]{-x+x^6}}$$

Mathematica [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{2+3x^5}{\left(-1+x^2+x^5\right)\sqrt[3]{-x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2 + 3*x^5)/((-1 + x^2 + x^5)*(-x + x^6)^(1/3)),x]
```

```
[Out] Integrate[(2 + 3*x^5)/((-1 + x^2 + x^5)*(-x + x^6)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 2.69, size = 85, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^6-x}+x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^6-x}-x}\right)+\frac{1}{2}\log\left(-\sqrt[3]{x^6-x}\,x+\left(x^6-x\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x^5)/((-1 + x^2 + x^5)*(-x + x^6)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(-x + x^6)^(1/3))]) - Log[x + (-x + x^
6)^(1/3)] + Log[x^2 - x*(-x + x^6)^(1/3) + (-x + x^6)^(2/3)]/2
```

fricas [A] time = 2.07, size = 104, normalized size = 1.22

$$-\sqrt{3}\arctan\left(\frac{4\sqrt{3}\left(x^6-x\right)^{\frac{1}{3}}x+\sqrt{3}\left(x^5-1\right)+2\sqrt{3}\left(x^6-x\right)^{\frac{2}{3}}}{x^5-8\,x^2-1}\right)-\frac{1}{2}\log\left(\frac{x^5+x^2+3\left(x^6-x\right)^{\frac{1}{3}}x+3\left(x^6-x\right)^{\frac{2}{3}}-1}{x^5+x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+2)/(x^5+x^2-1)/(x^6-x)^(1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan((4*sqrt(3)*(x^6 - x)^(1/3)*x + sqrt(3)*(x^5 - 1) + 2*sqrt(3
)*(x^6 - x)^(2/3))/(x^5 - 8*x^2 - 1)) - 1/2*log((x^5 + x^2 + 3*(x^6 - x)^(1
/3)*x + 3*(x^6 - x)^(2/3) - 1)/(x^5 + x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{3\,x^5+2}{\left(x^6-x\right)^{\frac{1}{3}}\left(x^5+x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+2)/(x^5+x^2-1)/(x^6-x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((3*x^5 + 2)/((x^6 - x)^(1/3)*(x^5 + x^2 - 1)), x)
```

maple [C] time = 7.78, size = 360, normalized size = 4.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^5+2)/(x^5+x^2-1)/(x^6-x)^(1/3),x)
```

```
[Out] -ln(-(8327084306326444968*RootOf(_Z^2-_Z+1)^2*x^5-10176976877377096586*Root
Of(_Z^2-_Z+1)*x^5-4212739303490596602349*x^5-6453490337402994850 2*RootOf(_Z^
2-_Z+1)^2*x^2-4333007997974873505 53*RootOf(_Z^2-_Z+1)*(x^6-x)^(2/3)-8610519
21881822746252*RootOf(_Z^2-_Z+1)*(x^6-x)^(1/3)*x-36506611128135609881 5*Root
Of(_Z^2-_Z+1)*x^2-8327084306326444968*RootOf(_Z^2-_Z+1)^2-42775112208433539
5699*(x^6-x)^(2/3)+433300799797487350553*x*(x^6-x)^(1/3)+366916003852406750
433*x^2+10176976877377096586*RootOf(_Z^2-_Z+1)+421273930349059602349)/(x^5+
x^2-1))+RootOf(_Z^2-_Z+1)*ln(-(543579264966528519 16*RootOf(_Z^2-_Z+1)^2*x^5
+375243088158733195401*RootOf(_Z^2-_Z+1)*x^5+562078190677035035 34*x^5-42127
3930349059602349*RootOf(_Z^2-_Z+1)^2*x^2+433300799797487350553*RootOf(_Z^2-
_Z+1)*(x^6-x)^(2/3)-427751122084335395699*RootOf(_Z^2-_Z+1)*(x^6-x)^(1/3)*x
-10176976877377096586*RootOf(_Z^2-_Z+1)*x^2-54357926496652851916*RootOf(_Z^
2-_Z+1)^2-861051921881822746252*(x^6-x)^(2/3)-433300799797487350553*x*(x^6-
```

```
x)^(1/3)+83270843063264449 68*x^2-37524308815873319 5401*RootOf(_Z^2-_Z+1)-56
207819067703503534)/(x^5+x^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^5+2}{\left(x^6-x\right)^{\frac{1}{3}}\left(x^5+x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5+2)/(x^5+x^2-1)/(x^6-x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((3*x^5 + 2)/((x^6 - x)^(1/3)*(x^5 + x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x^5+2}{\left(x^6-x\right)^{1/3}\,\left(x^5+x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^5 + 2)/((x^6 - x)^(1/3)*(x^2 + x^5 - 1)),x)
```

```
[Out] int((3*x^5 + 2)/((x^6 - x)^(1/3)*(x^2 + x^5 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^5+2}{\sqrt[3]{x\,(x-1)\left(x^4+x^3+x^2+x+1\right)}\,\left(x^5+x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**5+2)/(x**5+x**2-1)/(x**6-x)**(1/3),x)
```

```
[Out] Integral((3*x**5 + 2)/((x*(x - 1)*(x**4 + x**3 + x**2 + x + 1))**(1/3)*(x**
5 + x**2 - 1)), x)
```

3.1026 $\int \frac{\sqrt{-1+x^6}\left(-1+2x^6\right)^2}{x^4\left(-1+4x^6\right)}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{x^6-1}\,\left(x^6+2\right)}{6x^3}-\frac{5}{12}\log\left(\sqrt{x^6-1}+x^3\right)+\frac{\tan^{-1}\left(-\frac{4x^6}{\sqrt{3}}-\frac{4\sqrt{x^6-1}\,x^3}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Rubi [A] time = 0.23, antiderivative size = 79, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {575, 586, 580, 523, 217, 206, 377, 204, 528}

$$\frac{1}{6}\sqrt{x^6-1}\,x^3+\frac{\sqrt{x^6-1}}{3x^3}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x^3}{\sqrt{x^6-1}}\right)}{4\sqrt{3}}-\frac{5}{12}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^6]*(-1 + 2*x^6)^2)/(x^4*(-1 + 4*x^6)),x]
```

```
[Out] Sqrt[-1 + x^6]/(3*x^3) + (x^3*Sqrt[-1 + x^6])/6 + ArcTan[(Sqrt[3]*x^3)/Sqrt
[-1 + x^6]]/(4*Sqrt[3]) - (5*ArcTanh[x^3/Sqrt[-1 + x^6]])/12
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
```

```
a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 575

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Di
st[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q*(e
+ f*x^(n/k))^r, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, f, p, q
, r}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 586

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[e, Int[(g*x)^m*
(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^(r - 1), x], x] + Dist[f/e^n, Int[(
g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^(r - 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g, m, p, q}, x] && IGtQ[n, 0] && IGtQ[r, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-1+x^6}\left(-1+2x^6\right)^2}{x^4\left(-1+4x^6\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{-1+x^2}\left(-1+2x^2\right)^2}{x^2\left(-1+4x^2\right)}\,dx, x, x^3\right)\\
&= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{-1+x^2}\left(-1+2x^2\right)}{x^2\left(-1+4x^2\right)}\,dx, x, x^3\right)\right) + \frac{2}{3}\operatorname{Subst}\left(\int \frac{\sqrt{-1+x^2}\,(-1}{-1+4x^2}\right.\\
&= \frac{\sqrt{-1+x^6}}{3x^3} + \frac{1}{6}x^3\sqrt{-1+x^6} + \frac{1}{12}\operatorname{Subst}\left(\int \frac{6-12x^2}{\sqrt{-1+x^2}\left(-1+4x^2\right)}\,dx, x, x^3\right) + \\
&= \frac{\sqrt{-1+x^6}}{3x^3} + \frac{1}{6}x^3\sqrt{-1+x^6} - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, x^3\right) - \frac{1}{4}\operatorname{Subst}\left(\int \right.\\
&= \frac{\sqrt{-1+x^6}}{3x^3} + \frac{1}{6}x^3\sqrt{-1+x^6} - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^3}{\sqrt{-1+x^6}}\right) + \frac{1}{4}\operatorname{Subst}\\
&= \frac{\sqrt{-1+x^6}}{3x^3} + \frac{1}{6}x^3\sqrt{-1+x^6} + \frac{\tan^{-1}\left(\frac{\sqrt{3}x^3}{\sqrt{-1+x^6}}\right)}{4\sqrt{3}} - \frac{5}{12}\tanh^{-1}\left(\frac{x^3}{\sqrt{-1+x^6}}\right)
\end{aligned}
$$

Mathematica [B] time = 0.18, size = 201, normalized size = 2.36

$$
\frac{4\sqrt{1-x^6}\,x^6-8\sqrt{1-x^6}+4\sqrt{1-x^6}\,x^{12}+12\left(x^6-1\right)x^3\sin^{-1}\left(x^3\right)+\sqrt{3}\sqrt{-\left(x^6-1\right)^2}\,x^3\tan^{-1}\left(\frac{2-x^3}{\sqrt{3}\sqrt{x^6-1}}\right)-\sqrt{3}\sqrt{-\left(x^6-1\right)^2}\,x^3\tan^{-1}\left(\frac{x^3+2}{\sqrt{3}\sqrt{x^6-1}}\right)+2\sqrt{-\left(x^6-1\right)^2}\,x^3\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-1}}\right)}{24x^3\sqrt{-\left(x^6-1\right)^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^6]*(-1 + 2*x^6)^2)/(x^4*(-1 + 4*x^6)),x]
```

```
[Out] (-8*Sqrt[1 - x^6] + 4*x^6*Sqrt[1 - x^6] + 4*x^12*Sqrt[1 - x^6] + 12*x^3*(-1
 + x^6)*ArcSin[x^3] + Sqrt[3]*x^3*Sqrt[-(-1 + x^6)^2]*ArcTan[(2 - x^3)/(Sqr
t[3]*Sqrt[-1 + x^6])] - Sqrt[3]*x^3*Sqrt[-(-1 + x^6)^2]*ArcTan[(2 + x^3)/(S
qrt[3]*Sqrt[-1 + x^6])] + 2*x^3*Sqrt[-(-1 + x^6)^2]*ArcTanh[x^3/Sqrt[-1 + x
^6]])/(24*x^3*Sqrt[-(-1 + x^6)^2])
```

IntegrateAlgebraic [A] time = 0.20, size = 87, normalized size = 1.02

$$\frac{\sqrt{x^6-1}\left(x^6+2\right)}{6x^3}+\frac{5}{12}\log\left(\sqrt{x^6-1}-x^3\right)-\frac{\tan^{-1}\left(-\frac{4x^6}{\sqrt{3}}+\frac{4\sqrt{x^6-1}\,x^3}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^6]*(-1 + 2*x^6)^2)/(x^4*(-1 + 4*x^6)),x]
```

```
[Out] (Sqrt[-1 + x^6]*(2 + x^6))/(6*x^3) - ArcTan[1/Sqrt[3] - (4*x^6)/Sqrt[3] + (
4*x^3*Sqrt[-1 + x^6])/Sqrt[3]]/(4*Sqrt[3]) + (5*Log[-x^3 + Sqrt[-1 + x^6]])
/12
```

fricas [A] time = 0.41, size = 80, normalized size = 0.94

$$-\frac{\sqrt{3}\,x^3\arctan\left(\frac{4}{3}\sqrt{3}\sqrt{x^6-1}\,x^3-\frac{1}{3}\sqrt{3}\left(4\,x^6-1\right)\right)-5\,x^3\log\left(-x^3+\sqrt{x^6-1}\right)-4\,x^3-2\left(x^6+2\right)\sqrt{x^6-1}}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)*(2*x^6-1)^2/x^4/(4*x^6-1),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(3)*x^3*arctan(4/3*sqrt(3)*sqrt(x^6 - 1)*x^3 - 1/3*sqrt(3)*(4*x^
6 - 1)) - 5*x^3*log(-x^3 + sqrt(x^6 - 1)) - 4*x^3 - 2*(x^6 + 2)*sqrt(x^6 -
1))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)*(2*x^6-1)^2/x^4/(4*x^6-1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:rootof minimal polynomial must be unitary Er
ror: Bad Argument Valuerootof minimal polynomial must be unitary Error: Bad
 Argument Valuerootof minimal polynomial must be unitary Error: Bad Argumen
t Valuerootof minimal polynomial must be unitary Error: Bad Argument Valuer
ootof minimal polynomial must be unitary Error: Bad Argument Valuerootof mi
nimal polynomial must be unitary Error: Bad Argument Valuerootof minimal po
lynomial must be unitary Error: Bad Argument Valuerootof minimal polynomial
 must be unitary Error: Bad Argument Valuerootof minimal polynomial must be
 unitary Error: Bad Argument Valuerootof minimal polynomial must be unitary
 Error: Bad Argument Valuerootof minimal polynomial must be unitary Error:
Bad Argument Valuerootof minimal polynomial must be unitary Error: Bad Argu
ment ValueDone
```

maple [C] time = 0.54, size = 97, normalized size = 1.14

$$\frac{x^{12}+x^6-2}{6x^3\sqrt{x^6-1}}+\frac{5\ln\left(x^3-\sqrt{x^6-1}\right)}{12}-\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+3\right)x^6+6x^3\sqrt{x^6-1}+\mathrm{RootOf}\left(\_Z^2+3\right)}{\left(2x^3-1\right)\left(2x^3+1\right)}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/2)*(2*x^6-1)^2/x^4/(4*x^6-1),x)
```

```
[Out] 1/6*(x^12+x^6-2)/x^3/(x^6-1)^(1/2)+5/12*ln(x^3-(x^6-1)^(1/2))-1/24*RootOf(_
Z^2+3)*ln(-(2*RootOf(_Z^2+3)*x^6+6*x^3*(x^6-1)^(1/2)+RootOf(_Z^2+3))/(2*x^3
-1)/(2*x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^6-1\right)^2\sqrt{x^6-1}}{\left(4\,x^6-1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/2)*(2*x^6-1)^2/x^4/(4*x^6-1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^6 - 1)^2*sqrt(x^6 - 1)/((4*x^6 - 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^6-1}\,\left(2\,x^6-1\right)^2}{x^4\,\left(4\,x^6-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/2)*(2*x^6 - 1)^2)/(x^4*(4*x^6 - 1)),x)
```

```
[Out] int(((x^6 - 1)^(1/2)*(2*x^6 - 1)^2)/(x^4*(4*x^6 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)\,(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,\left(2x^6-1\right)^2}{x^4\left(2x^3-1\right)\left(2x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/2)*(2*x**6-1)**2/x**4/(4*x**6-1),x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1))*(2*x**6 - 1)**
2/(x**4*(2*x**3 - 1)*(2*x**3 + 1)), x)
```

3.1027 $\int \frac{(4+x^5)\sqrt[4]{-2+x^4+2x^5}\,(2-4x^5+x^8+2x^{10})}{x^{10}(-1+x^5)}\,dx$

Optimal. Leaf size=85

$$2\tan^{-1}\left(\frac{x}{\sqrt[4]{2x^5+x^4-2}}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt[4]{2x^5+x^4-2}}\right)+\frac{4\sqrt[4]{2x^5+x^4-2}\,(10x^{10}+x^9+43x^8-20x^5-x^4+10)}{45x^9}$$

Rubi [F] time = 1.96, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(4+x^5)\sqrt[4]{-2+x^4+2x^5}\,(2-4x^5+x^8+2x^{10})}{x^{10}(-1+x^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((4 + x^5)*(-2 + x^4 + 2*x^5)^(1/4)*(2 - 4*x^5 + x^8 + 2*x^10))/(x^10*(
-1 + x^5)),x]
```

```
[Out] 2*Defer[Int][(-2 + x^4 + 2*x^5)^(1/4), x] + Defer[Int][(-2 + x^4 + 2*x^5)^(
1/4)/(-1 + x), x] - 8*Defer[Int][(-2 + x^4 + 2*x^5)^(1/4)/x^10, x] + 6*Defe
r[Int][(-2 + x^4 + 2*x^5)^(1/4)/x^5, x] - 4*Defer[Int][(-2 + x^4 + 2*x^5)^(
1/4)/x^2, x] + Defer[Int][(-2 + x^4 + 2*x^5)^(1/4)/(1 + x + x^2 + x^3 + x^4
), x] + 2*Defer[Int][(x*(-2 + x^4 + 2*x^5)^(1/4))/(1 + x + x^2 + x^3 + x^4)
, x] + 3*Defer[Int][(x^2*(-2 + x^4 + 2*x^5)^(1/4))/(1 + x + x^2 + x^3 + x^4
), x] - Defer[Int][(x^3*(-2 + x^4 + 2*x^5)^(1/4))/(1 + x + x^2 + x^3 + x^4)
, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(4+x^5)\sqrt[4]{-2+x^4+2x^5}\,(2-4x^5+x^8+2x^{10})}{x^{10}(-1+x^5)}\,dx &= \int \left(2\sqrt[4]{-2+x^4+2x^5}+\frac{\sqrt[4]{-2+x^4+2x^5}}{-1+x}-\frac{8\sqrt[4]{-2+x^4}}{x^{10}}\right.\\ &= 2\int \sqrt[4]{-2+x^4+2x^5}\,dx-4\int \frac{\sqrt[4]{-2+x^4+2x^5}}{x^2}\,dx+6\\ &= 2\int \sqrt[4]{-2+x^4+2x^5}\,dx-4\int \frac{\sqrt[4]{-2+x^4+2x^5}}{x^2}\,dx+6\\ &= 2\int \sqrt[4]{-2+x^4+2x^5}\,dx+2\int \frac{x\sqrt[4]{-2+x^4+2x^5}}{1+x+x^2+x^3+x^4}\,dx+\end{aligned}$$

Mathematica [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{(4+x^5)\sqrt[4]{-2+x^4+2x^5}\,(2-4x^5+x^8+2x^{10})}{x^{10}(-1+x^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((4 + x^5)*(-2 + x^4 + 2*x^5)^(1/4)*(2 - 4*x^5 + x^8 + 2*x^10))/(
x^10*(-1 + x^5)),x]
```

```
[Out] Integrate[((4 + x^5)*(-2 + x^4 + 2*x^5)^(1/4)*(2 - 4*x^5 + x^8 + 2*x^10))/(
x^10*(-1 + x^5)), x]
```

IntegrateAlgebraic [A] time = 2.67, size = 85, normalized size = 1.00

$$2\tan^{-1}\left(\frac{x}{\sqrt[4]{2x^5+x^4-2}}\right)-2\tanh^{-1}\left(\frac{x}{\sqrt[4]{2x^5+x^4-2}}\right)+\frac{4\sqrt[4]{2x^5+x^4-2}\,\left(10x^{10}+x^9+43x^8-20x^5-x^4+10\right)}{45x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((4 + x^5)*(-2 + x^4 + 2*x^5)^(1/4)*(2 - 4*x^5 + x^8 + 2
*x^10))/(x^10*(-1 + x^5)),x]
```

```
[Out] (4*(-2 + x^4 + 2*x^5)^(1/4)*(10 - x^4 - 20*x^5 + 43*x^8 + x^9 + 10*x^10))/(
45*x^9) + 2*ArcTan[x/(-2 + x^4 + 2*x^5)^(1/4)] - 2*ArcTanh[x/(-2 + x^4 + 2*
x^5)^(1/4)]
```

fricas [B] time = 47.99, size = 161, normalized size = 1.89

$$\frac{45\,x^9\arctan\left(\frac{\left(2\,x^5+x^4-2\right)^{\frac{1}{4}}x^3+\left(2\,x^5+x^4-2\right)^{\frac{3}{4}}x}{x^5-1}\right)+45\,x^9\log\left(-\frac{x^5+x^4-\left(2\,x^5+x^4-2\right)^{\frac{1}{4}}x^3+\sqrt{2\,x^5+x^4-2}\,x^2-\left(2\,x^5+x^4-2\right)^{\frac{3}{4}}x-1}{x^5-1}\right)+4\left(10\,x^{10}+x^9+43\,x^8-20\,x^5-x^4+10\right)\left(2\,x^5+x^4-2\right)^{\frac{1}{4}}}{45\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+4)*(2*x^5+x^4-2)^(1/4)*(2*x^10+x^8-4*x^5+2)/x^10/(x^5-1),x,
algorithm="fricas")
```

```
[Out] 1/45*(45*x^9*arctan(((2*x^5 + x^4 - 2)^(1/4)*x^3 + (2*x^5 + x^4 - 2)^(3/4)*
x)/(x^5 - 1)) + 45*x^9*log(-(x^5 + x^4 - (2*x^5 + x^4 - 2)^(1/4)*x^3 + sqrt
(2*x^5 + x^4 - 2)*x^2 - (2*x^5 + x^4 - 2)^(3/4)*x - 1)/(x^5 - 1)) + 4*(10*x
^10 + x^9 + 43*x^8 - 20*x^5 - x^4 + 10)*(2*x^5 + x^4 - 2)^(1/4))/x^9
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{10}+x^8-4\,x^5+2\right)\left(2\,x^5+x^4-2\right)^{\frac{1}{4}}\left(x^5+4\right)}{\left(x^5-1\right)x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+4)*(2*x^5+x^4-2)^(1/4)*(2*x^10+x^8-4*x^5+2)/x^10/(x^5-1),x,
algorithm="giac")
```

```
[Out] integrate((2*x^10 + x^8 - 4*x^5 + 2)*(2*x^5 + x^4 - 2)^(1/4)*(x^5 + 4)/((x^
5 - 1)*x^10), x)
```

maple [C] time = 4.35, size = 1334, normalized size = 15.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+4)*(2*x^5+x^4-2)^(1/4)*(2*x^10+x^8-4*x^5+2)/x^10/(x^5-1),x)
```

```
[Out] 4/45*(20*x^15+12*x^14+87*x^13+43*x^12-60*x^10-24*x^9-87*x^8+60*x^5+12*x^4-2
0)/x^9/(2*x^5+x^4-2)^(3/4)+(ln((-4*x^15-8*x^14-5*x^13+4*(8*x^15+12*x^14+6*x
^13+x^12-24*x^10-24*x^9-6*x^8+24*x^5+12*x^4-8)^(1/4)*x^11-x^12+4*(8*x^15+12
*x^14+6*x^13+x^12-24*x^10-24*x^9-6*x^8+24*x^5+12*x^4-8)^(1/4)*x^10+(8*x^15+
12*x^14+6*x^13+x^12-24*x^10-24*x^9-6*x^8+24*x^5+12*x^4-8)^(1/4)*x^9+12*x^10
-2*(8*x^15+12*x^14+6*x^13+x^12-24*x^10-24*x^9-6*x^8+24*x^5+12*x^4-8)^(1/2)*
x^7+16*x^9-(8*x^15+12*x^14+6*x^13+x^12-24*x^10-24*x^9-6*x^8+24*x^5+12*x^4-8
)^(1/2)*x^6+5*x^8-8*(8*x^15+12*x^14+6*x^13+x^12-24*x^10-24*x^9-6*x^8+24*x^5
+12*x^4-8)^(1/4)*x^6+(8*x^15+12*x^14+6*x^13+x^12-24*x^10-24*x^9-6*x^8+24*x^
5+12*x^4-8)^(3/4)*x^3-4*(8*x^15+12*x^14+6*x^13+x^12-24*x^10-24*x^9-6*x^8+24
*x^5+12*x^4-8)^(1/4)*x^5-12*x^5+2*(8*x^15+12*x^14+6*x^13+x^12-24*x^10-24*x^
9-6*x^8+24*x^5+12*x^4-8)^(1/2)*x^2-8*x^4+4*(8*x^15+12*x^14+6*x^13+x^12-24*x
```

```
^10-24*x^9-6*x^8+24*x^5+12*x^4-8)^(1/4)*x+4)/(-1+x)/(x^4+x^3+x^2+x+1)/(2*x^
5+x^4-2)^2)+RootOf(_Z^2+1)*ln((4*x^15+8*x^14-4*RootOf(_Z^2+1)*(8*x^15+12*x^
14+6*x^13+x^12-24*x^10-24*x^9-6*x^8+24*x^5+12*x^4-8)^(1/4)*x^11+5*x^13-4*Ro
otOf(_Z^2+1)*(8*x^15+12*x^14+6*x^13+x^12-24*x^10-24*x^9-6*x^8+24*x^5+12*x^4
-8)^(1/4)*x^10+x^12-RootOf(_Z^2+1)*(8*x^15+12*x^14+6*x^13+x^12-24*x^10-24*x
^9-6*x^8+24*x^5+12*x^4-8)^(1/4)*x^9-12*x^10-2*(8*x^15+12*x^14+6*x^13+x^12-2
4*x^10-24*x^9-6*x^8+24*x^5+12*x^4-8)^(1/2)*x^7-16*x^9-(8*x^15+12*x^14+6*x^1
3+x^12-24*x^10-24*x^9-6*x^8+24*x^5+12*x^4-8)^(1/2)*x^6+8*RootOf(_Z^2+1)*(8*
x^15+12*x^14+6*x^13+x^12-24*x^10-24*x^9-6*x^8+24*x^5+12*x^4-8)^(1/4)*x^6-5*
x^8+(8*x^15+12*x^14+6*x^13+x^12-24*x^10-24*x^9-6*x^8+24*x^5+12*x^4-8)^(3/4)
*RootOf(_Z^2+1)*x^3+4*RootOf(_Z^2+1)*(8*x^15+12*x^14+6*x^13+x^12-24*x^10-24
*x^9-6*x^8+24*x^5+12*x^4-8)^(1/4)*x^5+12*x^5+2*(8*x^15+12*x^14+6*x^13+x^12-
24*x^10-24*x^9-6*x^8+24*x^5+12*x^4-8)^(1/2)*x^2+8*x^4-4*RootOf(_Z^2+1)*(8*x
^15+12*x^14+6*x^13+x^12-24*x^10-24*x^9-6*x^8+24*x^5+12*x^4-8)^(1/4)*x-4)/(-
1+x)/(x^4+x^3+x^2+x+1)/(2*x^5+x^4-2)^2))/(2*x^5+x^4-2)^(3/4)*((2*x^5+x^4-2)
^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{10}+x^{8}-4\,x^{5}+2\right)\left(2\,x^{5}+x^{4}-2\right)^{\frac{1}{4}}\left(x^{5}+4\right)}{\left(x^{5}-1\right)x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+4)*(2*x^5+x^4-2)^(1/4)*(2*x^10+x^8-4*x^5+2)/x^10/(x^5-1),x,
algorithm="maxima")
```

```
[Out] integrate((2*x^10 + x^8 - 4*x^5 + 2)*(2*x^5 + x^4 - 2)^(1/4)*(x^5 + 4)/((x^
5 - 1)*x^10), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^5+4\right)\,{\left(2\,x^5+x^4-2\right)}^{1/4}\,\left(2\,x^{10}+x^8-4\,x^5+2\right)}{x^{10}\,\left(x^5-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 + 4)*(x^4 + 2*x^5 - 2)^(1/4)*(x^8 - 4*x^5 + 2*x^10 + 2))/(x^10*(x
^5 - 1)),x)
```

```
[Out] int(((x^5 + 4)*(x^4 + 2*x^5 - 2)^(1/4)*(x^8 - 4*x^5 + 2*x^10 + 2))/(x^10*(x
^5 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+4)*(2*x**5+x**4-2)**(1/4)*(2*x**10+x**8-4*x**5+2)/x**10/(x*
*5-1),x)
```

```
[Out] Timed out
```

3.1028 $\int \frac{1+x^{12}}{\sqrt{1+x^4}\left(-1+x^{12}\right)}\,dx$

Optimal. Leaf size=85

$$-\frac{1}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{6\sqrt{2}}-\frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{6\sqrt{2}}$$

Rubi [C] time = 3.07, antiderivative size = 420, normalized size of antiderivative = 4.94, number of steps used = 171, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {1586, 6725, 1729, 1209, 1198, 220, 1196, 1211, 1699, 206, 1248, 735, 844, 215, 725, 203, 1217, 1707}

$$-\frac{1}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{6\sqrt{2}}-\frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{6\sqrt{2}}+\frac{\left(3+i\sqrt{3}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{12\sqrt{x^4+1}}-\frac{\left(1+i\sqrt{3}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\sqrt{x^4+1}}+\frac{\left(3-i\sqrt{3}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{12\sqrt{x^4+1}}-\frac{\left(1-i\sqrt{3}\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\sqrt{x^4+1}}-\frac{\left(-\sqrt{3}+i\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\left(-\sqrt{3}+3i\right)\sqrt{x^4+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\left(1+\sqrt[3]{-1}\right)\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^12)/(Sqrt[1 + x^4]*(-1 + x^12)),x]
```

```
[Out] -1/3*ArcTan[x/Sqrt[1 + x^4]] - ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(6*Sqrt[2]
) - ArcTanh[x/Sqrt[1 + x^4]]/3 - ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(6*Sqrt
[2]) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/
(6*(1 + (-1)^(1/3))*Sqrt[1 + x^4]) - ((I - Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4
)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(6*(3*I - Sqrt[3])*Sqrt[1 + x^4
]) - ((1 - I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*Arc
Tan[x], 1/2])/(6*Sqrt[1 + x^4]) + ((3 - I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)
/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(12*Sqrt[1 + x^4]) - ((1 + I*Sqr
t[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(6
*Sqrt[1 + x^4]) + ((3 + I*Sqrt[3])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*El
lipticF[2*ArcTan[x], 1/2])/(12*Sqrt[1 + x^4])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Dist[d, I
nt[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, Int[(x*(a + c*x^4)^p)/(d
^2 - e^2*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^{12}}{\sqrt{1+x^4}\left(-1+x^{12}\right)}\,dx = \int \frac{\sqrt{1+x^4}\left(1-x^4+x^8\right)}{-1+x^{12}}\,dx$$

$$= \int \left(-\frac{\sqrt{1+x^4}}{12(1-x)} - \frac{\sqrt{1+x^4}}{12(1-ix)} - \frac{\sqrt{1+x^4}}{12(1+ix)} - \frac{\sqrt{1+x^4}}{12(1+x)} - \frac{\left(1+\sqrt[3]{-1}+(-1)^{2/3}\right)\sqrt{1+x^4}}{12\left(1-\sqrt[6]{-1}\,x\right)}\right.$$

$$= -\left(\frac{1}{12}\int \frac{\sqrt{1+x^4}}{1-x}\,dx\right) - \frac{1}{12}\int \frac{\sqrt{1+x^4}}{1-ix}\,dx - \frac{1}{12}\int \frac{\sqrt{1+x^4}}{1+ix}\,dx - \frac{1}{12}\int \frac{\sqrt{1+x^4}}{1+x}\,dx$$

$$= -2\left(\frac{1}{12}\int \frac{\sqrt{1+x^4}}{1-x^2}\,dx\right) - 2\left(\frac{1}{12}\int \frac{\sqrt{1+x^4}}{1+x^2}\,dx\right) + 2\left(\frac{1}{12}\left(-1-i\sqrt{3}\right)\int \frac{\sqrt{1+x^4}}{1-\sqrt[3]{-1}\,x^2}\right.$$

$$= -2\left(-\left(\frac{1}{12}\int \frac{1+x^2}{\sqrt{1+x^4}}\,dx\right) + \frac{1}{6}\int \frac{1}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx\right) - 2\left(-\left(\frac{1}{12}\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx\right)\right.$$

$$= -2\left(\frac{x\sqrt{1+x^4}}{12\left(1+x^2\right)} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{12\sqrt{1+x^4}} + \frac{1}{12}\int \frac{1}{\sqrt{1+x^4}}\,dx + \frac{1}{12}\int\right.$$

$$= 2\left(-\frac{x\sqrt{1+x^4}}{6\left(1+x^2\right)} - \frac{1}{12}\tan^{-1}\left(\frac{x}{\sqrt{1+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\sqrt{1+x^4}} - \right.$$

$$= -2\left(-\frac{x\sqrt{1+x^4}}{12\left(1+x^2\right)} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{12\sqrt{2}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{12\sqrt{1+x^4}} - \right.$$

Mathematica [C] time = 0.92, size = 204, normalized size = 2.40

$$\frac{(-1)^{5/12}\left(-3F\left(i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)|-1\right)+\Pi\left(-i;i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)|-1\right)+\Pi\left(i;i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)|-1\right)+\Pi\left(-\frac{i}{2}-\frac{\sqrt{3}}{2};i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)|-1\right)+\Pi\left(\frac{i}{2}-\frac{\sqrt{3}}{2};i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)|-1\right)+\Pi\left(\frac{1}{2}\left(-i+\sqrt{3}\right);i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)|-1\right)+\Pi\left(\frac{1}{2}\left(i+\sqrt{3}\right);i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)|-1\right)\right)}{\sqrt{3}\left(1+\sqrt[3]{-1}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^12)/(Sqrt[1 + x^4]*(-1 + x^12)),x]
```

```
[Out] ((-1)^(5/12)*(-3*EllipticF[I*ArcSinh[((1 + I)*x)/Sqrt[2]], -1] + EllipticPi
[-I, I*ArcSinh[((1 + I)*x)/Sqrt[2]], -1] + EllipticPi[I, I*ArcSinh[((1 + I)
*x)/Sqrt[2]], -1] + EllipticPi[-1/2*I - Sqrt[3]/2, I*ArcSinh[((1 + I)*x)/Sq
rt[2]], -1] + EllipticPi[I/2 - Sqrt[3]/2, I*ArcSinh[((1 + I)*x)/Sqrt[2]], -
1] + EllipticPi[(-I + Sqrt[3])/2, I*ArcSinh[((1 + I)*x)/Sqrt[2]], -1] + Ell
ipticPi[(I + Sqrt[3])/2, I*ArcSinh[((1 + I)*x)/Sqrt[2]], -1]))/(Sqrt[3]*(1
+ (-1)^(1/3)))
```

IntegrateAlgebraic [A] time = 0.42, size = 85, normalized size = 1.00

$$-\frac{1}{3}\tan^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right) - \frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{6\sqrt{2}} - \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{x^4+1}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^12)/(Sqrt[1 + x^4]*(-1 + x^12)),x]
```

```
[Out] -1/3*ArcTan[x/Sqrt[1 + x^4]] - ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(6*Sqrt[2]
) - ArcTanh[x/Sqrt[1 + x^4]]/3 - ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(6*Sqrt
[2])
```

fricas [A] time = 0.50, size = 120, normalized size = 1.41

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)+\frac{1}{24}\sqrt{2}\log\left(\frac{x^4-2\sqrt{2}\sqrt{x^4+1}x+2x^2+1}{x^4-2x^2+1}\right)-\frac{1}{6}\arctan\left(\frac{2\sqrt{x^4+1}x}{x^4-x^2+1}\right)+\frac{1}{6}\log\left(\frac{x^4+x^2-2\sqrt{x^4+1}x+1}{x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^12+1)/(x^4+1)^(1/2)/(x^12-1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) + 1/24*sqrt(2)*log((x^4 - 2*s
qrt(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1)) - 1/6*arctan(2*sqrt(
x^4 + 1)*x/(x^4 - x^2 + 1)) + 1/6*log((x^4 + x^2 - 2*sqrt(x^4 + 1)*x + 1)/(
x^4 - x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{12}+1}{(x^{12}-1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^12+1)/(x^4+1)^(1/2)/(x^12-1),x, algorithm="giac")
```

```
[Out] integrate((x^12 + 1)/((x^12 - 1)*sqrt(x^4 + 1)), x)
```

maple [C] time = 0.04, size = 730, normalized size = 8.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^12+1)/(x^4+1)^(1/2)/(x^12-1),x)
```

```
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+1/3*(-1)^(3/4)*(1-I*x^2)^(1/2)*
(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1
/4))+1/6*(-1/2+1/2*I*3^(1/2))*(1/2/(1/2+1/2*I*3^(1/2)))^(1/2)*arctanh((1/2+1
/2*I*3^(1/2))^(1/2)*(x^2-1/2+1/2*I*3^(1/2))/(x^4+1)^(1/2))+(-1)^(3/4)*(-1/2
-1/2*I*3^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-
1)^(1/4)*x,-I*(-1/2+1/2*I*3^(1/2)),I))+1/6*(-1/2-1/2*I*3^(1/2))*(1/2/(1/2-1
/2*I*3^(1/2))^(1/2)*arctanh((1/2-1/2*I*3^(1/2))^(1/2)*(x^2-1/2-1/2*I*3^(1/2
))/(x^4+1)^(1/2))+(-1)^(3/4)*(-1/2+1/2*I*3^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)
^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I*(-1/2-1/2*I*3^(1/2)),I))+1/
12*sum(_alpha*(-1/(_alpha^2)^(1/2)*arctanh(_alpha^2*(-_alpha^2+x^2+1)/(_alp
ha^2)^(1/2)/(x^4+1)^(1/2))+2*(-1)^(3/4)*(-_alpha^3+_alpha)*(1-I*x^2)^(1/2)*
(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I*_alpha^2-I,I)),_alp
ha=RootOf(_Z^4-_Z^2+1))+1/3*(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4
+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4))+1/6*(1/2+1/2*I*3
^(1/2))*(-1/2/(1/2-1/2*I*3^(1/2))^(1/2)*arctanh((-1/2+1/2*I*3^(1/2))*(x^2-1
/2-1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))^(1/2)/(x^4+1)^(1/2))+(-1)^(3/4)*(1/2-
1/2*I*3^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1
)^(1/4)*x,I*(1/2+1/2*I*3^(1/2)),I))+1/6*(1/2-1/2*I*3^(1/2))*(-1/2/(1/2+1/2*
I*3^(1/2))^(1/2)*arctanh((-1/2-1/2*I*3^(1/2))*(x^2-1/2+1/2*I*3^(1/2))/(1/2+
1/2*I*3^(1/2))^(1/2)/(x^4+1)^(1/2))+(-1)^(3/4)*(1/2+1/2*I*3^(1/2))*(1-I*x^2
)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I*(1/2-1/2*I*
3^(1/2)),I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{12}+1}{(x^{12}-1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^12+1)/(x^4+1)^(1/2)/(x^12-1),x, algorithm="maxima")
```

```
[Out] integrate((x^12 + 1)/((x^12 - 1)*sqrt(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{12}+1}{\sqrt{x^4+1}\,\left(x^{12}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^12 + 1)/((x^4 + 1)^(1/2)*(x^12 - 1)),x)
```

```
[Out] int((x^12 + 1)/((x^4 + 1)^(1/2)*(x^12 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}\,\left(x^8-x^4+1\right)}{(x-1)(x+1)\left(x^2+1\right)\left(x^2-x+1\right)\left(x^2+x+1\right)\left(x^4-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**12+1)/(x**4+1)**(1/2)/(x**12-1),x)
```

```
[Out] Integral(sqrt(x**4 + 1)*(x**8 - x**4 + 1)/((x - 1)*(x + 1)*(x**2 + 1)*(x**2
 - x + 1)*(x**2 + x + 1)*(x**4 - x**2 + 1)), x)
```

3.1029 $$\int \frac{\sqrt{1-x^6}\left(1+2x^6\right)\left(1+x^2-x^4-2x^6-x^8+x^{12}\right)}{\left(-1+x^6\right)\left(-1+2x^6-3x^{12}+x^{18}\right)}\,dx$$

Optimal. Leaf size=85

$$-\frac{1}{3}\tan^{-1}\left(\frac{x}{\sqrt{1-x^6}}\right)-\frac{1}{3}\tan^{-1}\left(\frac{x\sqrt{1-x^6}}{x^6+x^2-1}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}x\sqrt{1-x^6}}{x^6-x^2-1}\right)}{\sqrt{3}}$$

Rubi [F] time = 2.68, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1-x^6}\left(1+2x^6\right)\left(1+x^2-x^4-2x^6-x^8+x^{12}\right)}{\left(-1+x^6\right)\left(-1+2x^6-3x^{12}+x^{18}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[1 - x^6]*(1 + 2*x^6)*(1 + x^2 - x^4 - 2*x^6 - x^8 + x^12))/((-1 +
 x^6)*(-1 + 2*x^6 - 3*x^12 + x^18)),x]
```

```
[Out] -((x*(1 - x^2)*Sqrt[(1 + x^2 + x^4)/(1 - (1 + Sqrt[3])*x^2)^2]*EllipticF[Ar
cCos[(1 - (1 - Sqrt[3])*x^2)/(1 - (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(3
^(1/4)*Sqrt[-((x^2*(1 - x^2))/(1 - (1 + Sqrt[3])*x^2)^2)]*Sqrt[1 - x^6])) +
 Defer[Int][1/(Sqrt[1 - x^6]*(-1 - x^2 + x^6)), x] + (2*Defer[Int][x^2/(Sqr
t[1 - x^6]*(-1 - x^2 + x^6)), x])/3 + 4*Defer[Int][1/(Sqrt[1 - x^6]*(1 - x^
2 + x^4 - 2*x^6 + x^8 + x^12)), x] - (10*Defer[Int][x^2/(Sqrt[1 - x^6]*(1 -
 x^2 + x^4 - 2*x^6 + x^8 + x^12)), x])/3 + (8*Defer[Int][x^4/(Sqrt[1 - x^6]
*(1 - x^2 + x^4 - 2*x^6 + x^8 + x^12)), x])/3 - 4*Defer[Int][x^6/(Sqrt[1 -
x^6]*(1 - x^2 + x^4 - 2*x^6 + x^8 + x^12)), x] + (4*Defer[Int][x^8/(Sqrt[1
- x^6]*(1 - x^2 + x^4 - 2*x^6 + x^8 + x^12)), x])/3
```

Rubi steps

$$\int \frac{\sqrt{1-x^6}\left(1+2x^6\right)\left(1+x^2-x^4-2x^6-x^8+x^{12}\right)}{\left(-1+x^6\right)\left(-1+2x^6-3x^{12}+x^{18}\right)}\,dx = -\int \frac{\left(1+2x^6\right)\left(1+x^2-x^4-2x^6-x^8+x^{12}\right)}{\sqrt{1-x^6}\left(-1+2x^6-3x^{12}+x^{18}\right)}\,dx$$

$$= -\int \left(\frac{2}{\sqrt{1-x^6}} + \frac{3+x^2-x^4-4x^6+x^8-2x^{10}+3x^{12}}{\sqrt{1-x^6}\left(-1+2x^6-3x^{12}+x^{18}\right.}\right.$$

$$= -\left(2\int \frac{1}{\sqrt{1-x^6}}\,dx\right) - \int \frac{3+x^2-x^4-4x^6+x^8-2x^{1(}}{\sqrt{1-x^6}\left(-1+2x^6-3x\right.}$$

$$= -\frac{x\left(1-x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x^2}{1-\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2\right.\right.}{\sqrt[4]{3}\sqrt{-\frac{x^2\left(1-x^2\right)}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1-x^6}}$$

$$= -\frac{x\left(1-x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x^2}{1-\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2\right.\right.}{\sqrt[4]{3}\sqrt{-\frac{x^2\left(1-x^2\right)}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1-x^6}}$$

$$= -\frac{x\left(1-x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x^2}{1-\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2\right.\right.}{\sqrt[4]{3}\sqrt{-\frac{x^2\left(1-x^2\right)}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1-x^6}}$$

$$= -\frac{x\left(1-x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{1-\left(1-\sqrt{3}\right)x^2}{1-\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2\right.\right.}{\sqrt[4]{3}\sqrt{-\frac{x^2\left(1-x^2\right)}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1-x^6}}$$

Mathematica [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x^6}\left(1+2x^6\right)\left(1+x^2-x^4-2x^6-x^8+x^{12}\right)}{\left(-1+x^6\right)\left(-1+2x^6-3x^{12}+x^{18}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[1 - x^6]*(1 + 2*x^6)*(1 + x^2 - x^4 - 2*x^6 - x^8 + x^12))/
((-1 + x^6)*(-1 + 2*x^6 - 3*x^12 + x^18)),x]
```

```
[Out] Integrate[(Sqrt[1 - x^6]*(1 + 2*x^6)*(1 + x^2 - x^4 - 2*x^6 - x^8 + x^12))/
((-1 + x^6)*(-1 + 2*x^6 - 3*x^12 + x^18)), x]
```

IntegrateAlgebraic [C] time = 18.21, size = 105, normalized size = 1.24

$$-\frac{1}{3}\tan^{-1}\left(\frac{x}{\sqrt{1-x^6}}\right)+\frac{1}{3}\left(1+i\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-i\sqrt{3}\right)x}{2\sqrt{1-x^6}}\right)+\frac{1}{3}\left(1-i\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+i\sqrt{3}\right)x}{2\sqrt{1-x^6}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 - x^6]*(1 + 2*x^6)*(1 + x^2 - x^4 - 2*x^6 - x^8
+ x^12))/((-1 + x^6)*(-1 + 2*x^6 - 3*x^12 + x^18)),x]
```

```
[Out] -1/3*ArcTan[x/Sqrt[1 - x^6]] + ((1 + I*Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x)/
(2*Sqrt[1 - x^6])])/3 + ((1 - I*Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x)/(2*Sqrt
[1 - x^6])])/3
```

fricas [B] time = 1.16, size = 440, normalized size = 5.18

$$-\frac{1}{12}\sqrt{3}\log\left(\frac{16\left(x^{12}-5x^{8}-2x^{6}+x^{4}+2\sqrt{3}\left(x^{7}-x^{3}-x\right)\sqrt{-x^{6}+1}+5x^{2}+1\right)}{x^{12}+x^{8}-2x^{6}+x^{4}-x^{2}+1}\right)+\frac{1}{12}\sqrt{3}\log\left(\frac{16\left(x^{12}-5x^{8}-2x^{6}+x^{4}-2\sqrt{3}\left(x^{7}-x^{3}-x\right)\sqrt{-x^{6}+1}+5x^{2}+1\right)}{x^{12}+x^{8}-2x^{6}+x^{4}-x^{2}+1}\right)+\frac{1}{6}\arctan\left(\frac{2\sqrt{-x^{6}+1}\,x}{x^{6}+x^{2}-1}\right)+\frac{1}{3}\arctan\left(-\frac{\sqrt{-x^{6}+1}\,x+\left(x^{6}-\sqrt{3}\sqrt{-x^{6}+1}\,x-x^{2}-1\right)\sqrt{\frac{x^{12}-5x^{8}-2x^{6}+x^{4}+2\sqrt{3}\left(x^{7}-x^{3}-x\right)\sqrt{-x^{6}+1}+5x^{2}+1}{x^{12}+x^{8}-2x^{6}+x^{4}-x^{2}+1}}}{x^{6}+x^{2}-1}\right)-\frac{1}{3}\arctan\left(\frac{\sqrt{-x^{6}+1}\,x+\left(x^{6}+\sqrt{3}\sqrt{-x^{6}+1}\,x-x^{2}-1\right)\sqrt{\frac{x^{12}-5x^{8}-2x^{6}+x^{4}-2\sqrt{3}\left(x^{7}-x^{3}-x\right)\sqrt{-x^{6}+1}+5x^{2}+1}{x^{12}+x^{8}-2x^{6}+x^{4}-x^{2}+1}}}{x^{6}+x^{2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(1/2)*(2*x^6+1)*(x^12-x^8-2*x^6-x^4+x^2+1)/(x^6-1)/(x^18
-3*x^12+2*x^6-1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*log(16*(x^12 - 5*x^8 - 2*x^6 + x^4 + 2*sqrt(3)*(x^7 - x^3 - x
)*sqrt(-x^6 + 1) + 5*x^2 + 1)/(x^12 + x^8 - 2*x^6 + x^4 - x^2 + 1)) + 1/12*
sqrt(3)*log(16*(x^12 - 5*x^8 - 2*x^6 + x^4 - 2*sqrt(3)*(x^7 - x^3 - x)*sqrt
(-x^6 + 1) + 5*x^2 + 1)/(x^12 + x^8 - 2*x^6 + x^4 - x^2 + 1)) + 1/6*arctan(
2*sqrt(-x^6 + 1)*x/(x^6 + x^2 - 1)) + 1/3*arctan(-(sqrt(-x^6 + 1)*x + (x^6
- sqrt(3)*sqrt(-x^6 + 1)*x - x^2 - 1)*sqrt((x^12 - 5*x^8 - 2*x^6 + x^4 + 2*
sqrt(3)*(x^7 - x^3 - x)*sqrt(-x^6 + 1) + 5*x^2 + 1)/(x^12 + x^8 - 2*x^6 + x
^4 - x^2 + 1)))/(x^6 + x^2 - 1)) - 1/3*arctan((sqrt(-x^6 + 1)*x + (x^6 + sq
rt(3)*sqrt(-x^6 + 1)*x - x^2 - 1)*sqrt((x^12 - 5*x^8 - 2*x^6 + x^4 - 2*sqrt
(3)*(x^7 - x^3 - x)*sqrt(-x^6 + 1) + 5*x^2 + 1)/(x^12 + x^8 - 2*x^6 + x^4 -
 x^2 + 1)))/(x^6 + x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{12}-x^{8}-2\,x^{6}-x^{4}+x^{2}+1\right)\left(2\,x^{6}+1\right)\sqrt{-x^{6}+1}}{\left(x^{18}-3\,x^{12}+2\,x^{6}-1\right)\left(x^{6}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(1/2)*(2*x^6+1)*(x^12-x^8-2*x^6-x^4+x^2+1)/(x^6-1)/(x^18
-3*x^12+2*x^6-1),x, algorithm="giac")
```

```
[Out] integrate((x^12 - x^8 - 2*x^6 - x^4 + x^2 + 1)*(2*x^6 + 1)*sqrt(-x^6 + 1)/(
(x^18 - 3*x^12 + 2*x^6 - 1)*(x^6 - 1)), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^{6}+1}\,\left(2x^{6}+1\right)\left(x^{12}-x^{8}-2x^{6}-x^{4}+x^{2}+1\right)}{\left(x^{6}-1\right)\left(x^{18}-3x^{12}+2x^{6}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^6+1)^(1/2)*(2*x^6+1)*(x^12-x^8-2*x^6-x^4+x^2+1)/(x^6-1)/(x^18-3*x^1
2+2*x^6-1),x)
```

```
[Out] int((-x^6+1)^(1/2)*(2*x^6+1)*(x^12-x^8-2*x^6-x^4+x^2+1)/(x^6-1)/(x^18-3*x^1
2+2*x^6-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{12}-x^{8}-2\,x^{6}-x^{4}+x^{2}+1\right)\left(2\,x^{6}+1\right)\sqrt{-x^{6}+1}}{\left(x^{18}-3\,x^{12}+2\,x^{6}-1\right)\left(x^{6}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(1/2)*(2*x^6+1)*(x^12-x^8-2*x^6-x^4+x^2+1)/(x^6-1)/(x^18
-3*x^12+2*x^6-1),x, algorithm="maxima")
```

```
[Out] integrate((x^12 - x^8 - 2*x^6 - x^4 + x^2 + 1)*(2*x^6 + 1)*sqrt(-x^6 + 1)/(
(x^18 - 3*x^12 + 2*x^6 - 1)*(x^6 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(2\,x^6+1\right)\,\left(x^{12}-x^8-2\,x^6-x^4+x^2+1\right)}{\sqrt{1-x^6}\,\left(x^{18}-3\,x^{12}+2\,x^6-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*x^6 + 1)*(x^2 - x^4 - 2*x^6 - x^8 + x^12 + 1))/((1 - x^6)^(1/2)*(2
*x^6 - 3*x^12 + x^18 - 1)),x)
```

```
[Out] int(-((2*x^6 + 1)*(x^2 - x^4 - 2*x^6 - x^8 + x^12 + 1))/((1 - x^6)^(1/2)*(2
*x^6 - 3*x^12 + x^18 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**6+1)**(1/2)*(2*x**6+1)*(x**12-x**8-2*x**6-x**4+x**2+1)/(x**6
-1)/(x**18-3*x**12+2*x**6-1),x)
```

```
[Out] Timed out
```

3.1030 $\int \frac{x^2-\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=85

$$\frac{2\left(x^2+2\right)x}{5\sqrt{\sqrt{x^2+1}+1}}-\frac{4\sqrt{x^2+1}\,x}{5\sqrt{\sqrt{x^2+1}+1}}-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}\right)$$

Rubi [F] time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2-\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2 - Sqrt[1 + x^2])/Sqrt[1 + Sqrt[1 + x^2]],x]
```

```
[Out] Defer[Int][x^2/Sqrt[1 + Sqrt[1 + x^2]], x] - Defer[Int][Sqrt[1 + x^2]/Sqrt[
1 + Sqrt[1 + x^2]], x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2-\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\,dx &= \int \left(\frac{x^2}{\sqrt{1+\sqrt{1+x^2}}}-\frac{\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\right)dx\\ &= \int \frac{x^2}{\sqrt{1+\sqrt{1+x^2}}}\,dx-\int \frac{\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\,dx\end{aligned}$$

Mathematica [C] time = 0.21, size = 125, normalized size = 1.47

$$\frac{\sqrt{\sqrt{x^2+1}+1}\left(-10\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}-\frac{\sqrt{x^2+1}}{2}\right)+4\sqrt{x^2+1}\,x^2-12x^2+16\sqrt{x^2+1}-5\sqrt{2}\sqrt{\sqrt{x^2+1}-1}\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}-1}}{\sqrt{2}}\right)-6\right)}{10x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2 - Sqrt[1 + x^2])/Sqrt[1 + Sqrt[1 + x^2]],x]
```

```
[Out] (Sqrt[1 + Sqrt[1 + x^2]]*(-6 - 12*x^2 + 16*Sqrt[1 + x^2] + 4*x^2*Sqrt[1 + x
^2] - 5*Sqrt[2]*Sqrt[-1 + Sqrt[1 + x^2]]*ArcTan[Sqrt[-1 + Sqrt[1 + x^2]]/Sq
rt[2]] - 10*Hypergeometric2F1[-1/2, 1, 1/2, 1/2 - Sqrt[1 + x^2]/2]))/(10*x)
```

IntegrateAlgebraic [A] time = 0.40, size = 85, normalized size = 1.00

$$\frac{2\left(x^2+2\right)x}{5\sqrt{\sqrt{x^2+1}+1}}-\frac{4\sqrt{x^2+1}\,x}{5\sqrt{\sqrt{x^2+1}+1}}-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2 - Sqrt[1 + x^2])/Sqrt[1 + Sqrt[1 + x^2]],x]
```

```
[Out] (-4*x*Sqrt[1 + x^2])/(5*Sqrt[1 + Sqrt[1 + x^2]]) + (2*x*(2 + x^2))/(5*Sqrt[
1 + Sqrt[1 + x^2]]) - Sqrt[2]*ArcTan[x/(Sqrt[2]*Sqrt[1 + Sqrt[1 + x^2]])]
```

fricas [A] time = 1.06, size = 65, normalized size = 0.76

$$\frac{5\sqrt{2}\,x\arctan\left(\frac{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}{x}\right)-2\left(3\,x^2-\left(x^2+4\right)\sqrt{x^2+1}+4\right)\sqrt{\sqrt{x^2+1}+1}}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-(x^2+1)^(1/2))/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/5*(5*sqrt(2)*x*arctan(sqrt(2)*sqrt(sqrt(x^2 + 1) + 1)/x) - 2*(3*x^2 - (x^
2 + 4)*sqrt(x^2 + 1) + 4)*sqrt(sqrt(x^2 + 1) + 1))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-\sqrt{x^2+1}}{\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-(x^2+1)^(1/2))/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - sqrt(x^2 + 1))/sqrt(sqrt(x^2 + 1) + 1), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x^2-\sqrt{x^2+1}}{\sqrt{1+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-(x^2+1)^(1/2))/(1+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int((x^2-(x^2+1)^(1/2))/(1+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-\sqrt{x^2+1}}{\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-(x^2+1)^(1/2))/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((x^2 - sqrt(x^2 + 1))/sqrt(sqrt(x^2 + 1) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{x^2+1}-x^2}{\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^2 + 1)^(1/2) - x^2)/((x^2 + 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int(-((x^2 + 1)^(1/2) - x^2)/((x^2 + 1)^(1/2) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - \sqrt{x^2 + 1}}{\sqrt{\sqrt{x^2 + 1} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-(x**2+1)**(1/2))/(1+(x**2+1)**(1/2))**(1/2),x)
[Out] Integral((x**2 - sqrt(x**2 + 1))/sqrt(sqrt(x**2 + 1) + 1), x)
```

3.1031 $\int \frac{1}{(-1+x)\sqrt[3]{2-2x+x^2}}\,dx$

Optimal. Leaf size=86

$$\frac{1}{2}\log\left(\sqrt[3]{x^2-2x+2}-1\right)-\frac{1}{4}\log\left(\left(x^2-2x+2\right)^{2/3}+\sqrt[3]{x^2-2x+2}+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^2-2x+2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {694, 266, 55, 618, 204, 31}

$$\frac{3}{4}\log\left(1-\sqrt[3]{(x-1)^2+1}\right)-\frac{1}{2}\log(1-x)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{(x-1)^2+1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)*(2 - 2*x + x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 + (-1 + x)^2)^(1/3))/Sqrt[3]])/2 + (3*Log[1 - (1
+ (-1 + x)^2)^(1/3)])/4 - Log[1 - x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(-1+x)\sqrt[3]{2-2x+x^2}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x^2}}\,dx, x, -1+x\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx, x, (-1+x)^2\right)$$
$$= -\frac{1}{2}\log(1-x) - \frac{3}{4}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+(-1+x)^2}\right) + \frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x}\right.$$
$$= \frac{3}{4}\log\left(1-\sqrt[3]{1+(-1+x)^2}\right) - \frac{1}{2}\log(1-x) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{}\right.$$
$$= \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1+(-1+x)^2}}{\sqrt{3}}\right) + \frac{3}{4}\log\left(1-\sqrt[3]{1+(-1+x)^2}\right) - \frac{1}{2}\log(1-$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.71

$$\frac{1}{2}\left(\frac{3}{2}\log\left(1-\sqrt[3]{(x-1)^2+1}\right) - \log(1-x) + \sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{(x-1)^2+1}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)*(2 - 2*x + x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 + (-1 + x)^2)^(1/3))/Sqrt[3]] + (3*Log[1 - (1 + (
-1 + x)^2)^(1/3)])/2 - Log[1 - x])/2
```

IntegrateAlgebraic [A] time = 0.08, size = 86, normalized size = 1.00

$$\frac{1}{2}\log\left(\sqrt[3]{x^2-2x+2}-1\right) - \frac{1}{4}\log\left(\left(x^2-2x+2\right)^{2/3} + \sqrt[3]{x^2-2x+2}+1\right) + \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^2-2x+2}}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x)*(2 - 2*x + x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(2 - 2*x + x^2)^(1/3))/Sqrt[3]])/2 + Log[-1
+ (2 - 2*x + x^2)^(1/3)]/2 - Log[1 + (2 - 2*x + x^2)^(1/3) + (2 - 2*x + x^2
)^(2/3)]/4
```

fricas [A] time = 0.41, size = 68, normalized size = 0.79

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^2-2x+2\right)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right) - \frac{1}{4}\log\left(\left(x^2-2x+2\right)^{\frac{2}{3}} + \left(x^2-2x+2\right)^{\frac{1}{3}}+1\right) + \frac{1}{2}\log\left(\left(x^2-2x+2\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^2-2*x+2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan(2/3*sqrt(3)*(x^2 - 2*x + 2)^(1/3) + 1/3*sqrt(3)) - 1/4*l
og((x^2 - 2*x + 2)^(2/3) + (x^2 - 2*x + 2)^(1/3) + 1) + 1/2*log((x^2 - 2*x
+ 2)^(1/3) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-2x+2\right)^{\frac{1}{3}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^2-2*x+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - 2*x + 2)^(1/3)*(x - 1)), x)
```

maple [C] time = 1.90, size = 285, normalized size = 3.31

$$\frac{\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x^2-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x-15\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2-2x+2\right)^{\frac{2}{3}}+3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+24\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2-2x+2\right)^{\frac{1}{3}}-6\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x+9\left(x^2-2x+2\right)^{\frac{2}{3}}-10x^2-7\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)+15\left(x^2-2x+2\right)^{\frac{1}{3}}+20x-35}{(-1+x)^2}\right)}{2}+\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(\frac{5\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x^2-10\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x-15\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2-2x+2\right)^{\frac{2}{3}}+16\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2-9\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2-2x+2\right)^{\frac{1}{3}}-32\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x-24\left(x^2-2x+2\right)^{\frac{2}{3}}+3x^2+35\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)+15\left(x^2-2x+2\right)^{\frac{1}{3}}-6x+7}{(-1+x)^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)/(x^2-2*x+2)^(1/3),x)
```

```
[Out] 1/2*ln(-(RootOf(_Z^2+_Z+1)^2*x^2-2*RootOf(_Z^2+_Z+1)^2*x-15*RootOf(_Z^2+_Z+
1)*(x^2-2*x+2)^(2/3)+3*RootOf(_Z^2+_Z+1)*x^2+24*RootOf(_Z^2+_Z+1)*(x^2-2*x+
2)^(1/3)-6*RootOf(_Z^2+_Z+1)*x+9*(x^2-2*x+2)^(2/3)-10*x^2-7*RootOf(_Z^2+_Z+
1)+15*(x^2-2*x+2)^(1/3)+20*x-35)/(-1+x)^2)+1/2*RootOf(_Z^2+_Z+1)*ln((5*Root
Of(_Z^2+_Z+1)^2*x^2-10*RootOf(_Z^2+_Z+1)^2*x-15*RootOf(_Z^2+_Z+1)*(x^2-2*x+
2)^(2/3)+16*RootOf(_Z^2+_Z+1)*x^2-9*RootOf(_Z^2+_Z+1)*(x^2-2*x+2)^(1/3)-32*
RootOf(_Z^2+_Z+1)*x-24*(x^2-2*x+2)^(2/3)+3*x^2+35*RootOf(_Z^2+_Z+1)+15*(x^2
-2*x+2)^(1/3)-6*x+7)/(-1+x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-2\,x+2\right)^{\frac{1}{3}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^2-2*x+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - 2*x + 2)^(1/3)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x-1)\,\left(x^2-2\,x+2\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)*(x^2 - 2*x + 2)^(1/3)),x)
```

```
[Out] int(1/((x - 1)*(x^2 - 2*x + 2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x-1)\sqrt[3]{x^2-2x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x**2-2*x+2)**(1/3),x)
```

```
[Out] Integral(1/((x - 1)*(x**2 - 2*x + 2)**(1/3)), x)
```

3.1032 $\int \frac{1}{(1+x)\sqrt[3]{2+2x+x^2}}\,dx$

Optimal. Leaf size=86

$$\frac{1}{2}\log\left(\sqrt[3]{x^2+2x+2}-1\right)-\frac{1}{4}\log\left(\left(x^2+2x+2\right)^{2/3}+\sqrt[3]{x^2+2x+2}+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^2+2x+2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {694, 266, 55, 618, 204, 31}

$$-\frac{1}{2}\log(x+1)+\frac{3}{4}\log\left(1-\sqrt[3]{(x+1)^2+1}\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{(x+1)^2+1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + 2*x + x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 + (1 + x)^2)^(1/3))/Sqrt[3]])/2 - Log[1 + x]/2 +
(3*Log[1 - (1 + (1 + x)^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)\sqrt[3]{2+2x+x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x^2}}\,dx,x,1+x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx,x,(1+x)^2\right)\\ &= -\frac{1}{2}\log(1+x)-\frac{3}{4}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1+(1+x)^2}\right)+\frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\right.\\ &= -\frac{1}{2}\log(1+x)+\frac{3}{4}\log\left(1-\sqrt[3]{1+(1+x)^2}\right)-\frac{3}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{1+}\right.\\ &= \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1+(1+x)^2}}{\sqrt{3}}\right)-\frac{1}{2}\log(1+x)+\frac{3}{4}\log\left(1-\sqrt[3]{1+(1+x)^2}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 0.69

$$\frac{1}{2}\left(-\log(x+1)+\frac{3}{2}\log\left(1-\sqrt[3]{(x+1)^2+1}\right)+\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{(x+1)^2+1}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(2 + 2*x + x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 + (1 + x)^2)^(1/3))/Sqrt[3]] - Log[1 + x] + (3*Lo
g[1 - (1 + (1 + x)^2)^(1/3)])/2)/2
```

IntegrateAlgebraic [A] time = 0.08, size = 86, normalized size = 1.00

$$\frac{1}{2}\log\left(\sqrt[3]{x^2+2x+2}-1\right)-\frac{1}{4}\log\left(\left(x^2+2x+2\right)^{2/3}+\sqrt[3]{x^2+2x+2}+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^2+2x+2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x)*(2 + 2*x + x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(2 + 2*x + x^2)^(1/3))/Sqrt[3]])/2 + Log[-1
+ (2 + 2*x + x^2)^(1/3)]/2 - Log[1 + (2 + 2*x + x^2)^(1/3) + (2 + 2*x + x^2
)^(2/3)]/4
```

fricas [A] time = 0.41, size = 68, normalized size = 0.79

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^2+2x+2\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{4}\log\left(\left(x^2+2x+2\right)^{\frac{2}{3}}+\left(x^2+2x+2\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(x^2+2x+2\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x+2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan(2/3*sqrt(3)*(x^2 + 2*x + 2)^(1/3) + 1/3*sqrt(3)) - 1/4*l
og((x^2 + 2*x + 2)^(2/3) + (x^2 + 2*x + 2)^(1/3) + 1) + 1/2*log((x^2 + 2*x
+ 2)^(1/3) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+2x+2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x^2+2*x+2)^(1/3),x, algorithm="giac")

[Out] integrate(1/((x^2 + 2*x + 2)^(1/3)*(x + 1)), x)

maple [C] time = 1.90, size = 285, normalized size = 3.31

$$\frac{\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x^2+2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x-15\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2+2x+2\right)^{\frac{2}{3}}+3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+24\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2+2x+2\right)^{\frac{1}{3}}+6\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x+9\left(x^2+2x+2\right)^{\frac{2}{3}}-10x^2-7\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)+15\left(x^2+2x+2\right)^{\frac{1}{3}}-20x-35}{(1+x)^2}\right)}{2}+\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(\frac{5\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x^2+10\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x-15\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2+2x+2\right)^{\frac{2}{3}}+16\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2-9\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2+2x+2\right)^{\frac{1}{3}}+32\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x-24\left(x^2+2x+2\right)^{\frac{2}{3}}+3x^2+35\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)+15\left(x^2+2x+2\right)^{\frac{1}{3}}+6x+7}{(1+x)^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+x)/(x^2+2*x+2)^(1/3),x)

[Out] 1/2*ln(-(RootOf(_Z^2+_Z+1)^2*x^2+2*RootOf(_Z^2+_Z+1)^2*x-15*RootOf(_Z^2+_Z+1)*(x^2+2*x+2)^(2/3)+3*RootOf(_Z^2+_Z+1)*x^2+24*RootOf(_Z^2+_Z+1)*(x^2+2*x+2)^(1/3)+6*RootOf(_Z^2+_Z+1)*x+9*(x^2+2*x+2)^(2/3)-10*x^2-7*RootOf(_Z^2+_Z+1)+15*(x^2+2*x+2)^(1/3)-20*x-35)/(1+x)^2)+1/2*RootOf(_Z^2+_Z+1)*ln((5*RootOf(_Z^2+_Z+1)^2*x^2+10*RootOf(_Z^2+_Z+1)^2*x-15*RootOf(_Z^2+_Z+1)*(x^2+2*x+2)^(2/3)+16*RootOf(_Z^2+_Z+1)*x^2-9*RootOf(_Z^2+_Z+1)*(x^2+2*x+2)^(1/3)+32*RootOf(_Z^2+_Z+1)*x-24*(x^2+2*x+2)^(2/3)+3*x^2+35*RootOf(_Z^2+_Z+1)+15*(x^2+2*x+2)^(1/3)+6*x+7)/(1+x)^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+2\,x+2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x^2+2*x+2)^(1/3),x, algorithm="maxima")

[Out] integrate(1/((x^2 + 2*x + 2)^(1/3)*(x + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x+1)\,\left(x^2+2\,x+2\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x + 1)*(2*x + x^2 + 2)^(1/3)),x)

[Out] int(1/((x + 1)*(2*x + x^2 + 2)^(1/3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)\sqrt[3]{x^2+2x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x**2+2*x+2)**(1/3),x)

[Out] Integral(1/((x + 1)*(x**2 + 2*x + 2)**(1/3)), x)

3.1033 $\int \frac{(-1+x^3)^{2/3}}{x}\,dx$

Optimal. Leaf size=86

$$\frac{1}{2}\left(x^3-1\right)^{2/3}+\frac{1}{3}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{1}{6}\log\left(\left(x^3-1\right)^{2/3}-\sqrt[3]{x^3-1}+1\right)+\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 50, 56, 618, 204, 31}

$$\frac{1}{2}\left(x^3-1\right)^{2/3}+\frac{1}{2}\log\left(\sqrt[3]{x^3-1}+1\right)+\frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)^(2/3)/x,x]
```

```
[Out] (-1 + x^3)^(2/3)/2 + ArcTan[(1 - 2*(-1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log
[x]/2 + Log[1 + (-1 + x^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^3\right)^{2/3}}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(-1+x)^{2/3}}{x}\,dx,x,x^3\right)\\
&= \frac{1}{2}\left(-1+x^3\right)^{2/3}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x}\,dx,x,x^3\right)\\
&= \frac{1}{2}\left(-1+x^3\right)^{2/3}-\frac{\log(x)}{2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt[3]{-1+x^3}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\right.\\
&= \frac{1}{2}\left(-1+x^3\right)^{2/3}-\frac{\log(x)}{2}+\frac{1}{2}\log\left(1+\sqrt[3]{-1+x^3}\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2\sqrt[3]{-1}\right.\\
&= \frac{1}{2}\left(-1+x^3\right)^{2/3}-\frac{\tan^{-1}\left(\frac{-1+2\sqrt[3]{-1+x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}+\frac{1}{2}\log\left(1+\sqrt[3]{-1+x^3}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.35

$$-\frac{1}{2}\left(x^3-1\right)^{2/3}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-x^3\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)^(2/3)/x,x]
```

```
[Out] -1/2*((-1 + x^3)^(2/3)*(-1 + Hypergeometric2F1[2/3, 1, 5/3, 1 - x^3]))
```

IntegrateAlgebraic [A] time = 0.06, size = 86, normalized size = 1.00

$$\frac{1}{2}\left(x^3-1\right)^{2/3}+\frac{1}{3}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{1}{6}\log\left(\left(x^3-1\right)^{2/3}-\sqrt[3]{x^3-1}+1\right)+\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)^(2/3)/x,x]
```

```
[Out] (-1 + x^3)^(2/3)/2 + ArcTan[1/Sqrt[3] - (2*(-1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[
3] + Log[1 + (-1 + x^3)^(1/3)]/3 - Log[1 - (-1 + x^3)^(1/3) + (-1 + x^3)^(2
/3)]/6
```

fricas [A] time = 0.41, size = 67, normalized size = 0.78

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+\frac{1}{2}\left(x^3-1\right)^{\frac{2}{3}}-\frac{1}{6}\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(x^3-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*(x^3 - 1)^(1/3) - 1/3*sqrt(3)) + 1/2*(x^3 -
 1)^(2/3) - 1/6*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 1/3*log((x^3 -
 1)^(1/3) + 1)
```

giac [A] time = 0.41, size = 66, normalized size = 0.77

$$-\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,\left(x^{3}-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{1}{2}\,\left(x^{3}-1\right)^{\frac{2}{3}}-\frac{1}{6}\,\log\left(\left(x^{3}-1\right)^{\frac{2}{3}}-\left(x^{3}-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\,\log\left(\left|\left(x^{3}-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)/x,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) + 1/2*(x^3 - 1)^(2
/3) - 1/6*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 1/3*log(abs((x^3 - 1
)^(1/3) + 1))
```

maple [C] time = 0.28, size = 84, normalized size = 0.98

$$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\operatorname{signum}\left(x^{3}-1\right)^{\frac{2}{3}}\left(\frac{2\pi\sqrt{3}\,x^{3}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},1,1\right],[2,2],x^{3}\right)}{3\Gamma\left(\frac{2}{3}\right)}-\frac{\left(\frac{3}{2}-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}\right)}{9\pi\left(-\operatorname{signum}\left(x^{3}-1\right)\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)/x,x)
```

```
[Out] -1/9/Pi*3^(1/2)*GAMMA(2/3)*signum(x^3-1)^(2/3)/(-signum(x^3-1))^(2/3)*(2/3*
Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1/3,1,1],[2,2],x^3)-(3/2-1/6*Pi*3^(1/2
)-3/2*ln(3)+3*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.72, size = 65, normalized size = 0.76

$$-\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,\left(x^{3}-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{1}{2}\,\left(x^{3}-1\right)^{\frac{2}{3}}-\frac{1}{6}\,\log\left(\left(x^{3}-1\right)^{\frac{2}{3}}-\left(x^{3}-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\,\log\left(\left(x^{3}-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) + 1/2*(x^3 - 1)^(2
/3) - 1/6*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 1/3*log((x^3 - 1)^(1
/3) + 1)
```

mupad [B] time = 0.85, size = 83, normalized size = 0.97

$$\frac{\ln\left(\left(x^{3}-1\right)^{1/3}+1\right)}{3}+\frac{\left(x^{3}-1\right)^{2/3}}{2}+\ln\left(9\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)^{2}+\left(x^{3}-1\right)^{1/3}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)-\ln\left(9\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)^{2}+\left(x^{3}-1\right)^{1/3}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)^(2/3)/x,x)
```

```
[Out] log((x^3 - 1)^(1/3) + 1)/3 + (x^3 - 1)^(2/3)/2 + log(9*((3^(1/2)*1i)/6 - 1/
6)^2 + (x^3 - 1)^(1/3))*((3^(1/2)*1i)/6 - 1/6) - log(9*((3^(1/2)*1i)/6 + 1/
6)^2 + (x^3 - 1)^(1/3))*((3^(1/2)*1i)/6 + 1/6)
```

sympy [C] time = 0.87, size = 37, normalized size = 0.43

$$-\frac{x^{2}\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^{3}}\right)}{3\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)/x,x)
```

```
[Out] -x**2*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), exp_polar(2*I*pi)/x**3)/(3*ga
mma(1/3))
```

3.1034 $\int \frac{3+x^2}{\sqrt[3]{1+x^2}\left(-1-x^2+x^3\right)}\,dx$

Optimal. Leaf size=86

$$\log\left(\sqrt[3]{x^2+1}-x\right)-\frac{1}{2}\log\left(x^2+\sqrt[3]{x^2+1}\,x+\left(x^2+1\right)^{2/3}\right)-\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2+1}}{\sqrt{3}}+\frac{2x}{\sqrt{3}}}{\sqrt[3]{x^2+1}}\right)$$

Rubi [F] time = 0.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3+x^2}{\sqrt[3]{1+x^2}\left(-1-x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3 + x^2)/((1 + x^2)^(1/3)*(-1 - x^2 + x^3)),x]
```

```
[Out] 3*Defer[Int][1/((1 + x^2)^(1/3)*(-1 - x^2 + x^3)), x] + Defer[Int][x^2/((1
+ x^2)^(1/3)*(-1 - x^2 + x^3)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{3+x^2}{\sqrt[3]{1+x^2}\left(-1-x^2+x^3\right)}\,dx &= \int \left(\frac{3}{\sqrt[3]{1+x^2}\left(-1-x^2+x^3\right)}+\frac{x^2}{\sqrt[3]{1+x^2}\left(-1-x^2+x^3\right)}\right)dx\\ &= 3\int \frac{1}{\sqrt[3]{1+x^2}\left(-1-x^2+x^3\right)}\,dx+\int \frac{x^2}{\sqrt[3]{1+x^2}\left(-1-x^2+x^3\right)}\,dx\end{aligned}$$

Mathematica [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{3+x^2}{\sqrt[3]{1+x^2}\left(-1-x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3 + x^2)/((1 + x^2)^(1/3)*(-1 - x^2 + x^3)),x]
```

```
[Out] Integrate[(3 + x^2)/((1 + x^2)^(1/3)*(-1 - x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.12, size = 86, normalized size = 1.00

$$\log\left(\sqrt[3]{x^2+1}-x\right)-\frac{1}{2}\log\left(x^2+\sqrt[3]{x^2+1}\,x+\left(x^2+1\right)^{2/3}\right)-\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2+1}}{\sqrt{3}}+\frac{2x}{\sqrt{3}}}{\sqrt[3]{x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + x^2)/((1 + x^2)^(1/3)*(-1 - x^2 + x^3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[((2*x)/Sqrt[3] + (1 + x^2)^(1/3)/Sqrt[3])/(1 + x^2)^(1/3)]
) + Log[-x + (1 + x^2)^(1/3)] - Log[x^2 + x*(1 + x^2)^(1/3) + (1 + x^2)^(2/
3)]/2
```

fricas [A] time = 1.14, size = 104, normalized size = 1.21

$$-\sqrt{3}\arctan\left(\frac{\sqrt{3}x^3-2\sqrt{3}\left(x^2+1\right)^{\frac{1}{3}}x^2+4\sqrt{3}\left(x^2+1\right)^{\frac{2}{3}}x}{x^3+8x^2+8}\right)+\frac{1}{2}\log\left(\frac{x^3-3\left(x^2+1\right)^{\frac{1}{3}}x^2-x^2+3\left(x^2+1\right)^{\frac{2}{3}}x-1}{x^3-x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2+1)^(1/3)/(x^3-x^2-1),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan((sqrt(3)*x^3 - 2*sqrt(3)*(x^2 + 1)^(1/3)*x^2 + 4*sqrt(3)*(x
^2 + 1)^(2/3)*x)/(x^3 + 8*x^2 + 8)) + 1/2*log((x^3 - 3*(x^2 + 1)^(1/3)*x^2
- x^2 + 3*(x^2 + 1)^(2/3)*x - 1)/(x^3 - x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+3}{\left(x^3-x^2-1\right)\left(x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2+1)^(1/3)/(x^3-x^2-1),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 3)/((x^3 - x^2 - 1)*(x^2 + 1)^(1/3)), x)
```

maple [C] time = 1.56, size = 260, normalized size = 3.02

$$\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2+1\right)^{\frac{2}{3}}x+\left(x^2+1\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3+2\left(x^2+1\right)^{\frac{2}{3}}x-\left(x^2+1\right)^{\frac{1}{3}}x^2-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2-x^3-x^2-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-1}{x^3-x^2-1}\right)+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^2+1\right)^{\frac{2}{3}}x-2\left(x^2+1\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3-\left(x^2+1\right)^{\frac{2}{3}}x-\left(x^2+1\right)^{\frac{1}{3}}x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)+1}{x^3-x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3)/(x^2+1)^(1/3)/(x^3-x^2-1),x)
```

```
[Out] ln(-(-RootOf(_Z^2+_Z+1)^2*x^3+RootOf(_Z^2+_Z+1)*(x^2+1)^(2/3)*x+(x^2+1)^(1/
3)*RootOf(_Z^2+_Z+1)*x^2-2*RootOf(_Z^2+_Z+1)*x^3+2*(x^2+1)^(2/3)*x-(x^2+1)^
(1/3)*x^2-RootOf(_Z^2+_Z+1)*x^2-x^3-x^2-RootOf(_Z^2+_Z+1)-1)/(x^3-x^2-1))+R
ootOf(_Z^2+_Z+1)*ln((-RootOf(_Z^2+_Z+1)^2*x^3+RootOf(_Z^2+_Z+1)*(x^2+1)^(2/
3)*x-2*(x^2+1)^(1/3)*RootOf(_Z^2+_Z+1)*x^2-RootOf(_Z^2+_Z+1)*x^3-(x^2+1)^(2
/3)*x-(x^2+1)^(1/3)*x^2+RootOf(_Z^2+_Z+1)*x^2+x^2+RootOf(_Z^2+_Z+1)+1)/(x^3
-x^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+3}{\left(x^3-x^2-1\right)\left(x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2+1)^(1/3)/(x^3-x^2-1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 3)/((x^3 - x^2 - 1)*(x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^2+3}{\left(x^2+1\right)^{1/3}\left(-x^3+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 + 3)/((x^2 + 1)^(1/3)*(x^2 - x^3 + 1)),x)
```

```
[Out] int(-(x^2 + 3)/((x^2 + 1)^(1/3)*(x^2 - x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+3}{\sqrt[3]{x^2+1}\,\left(x^3-x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3)/(x**2+1)**(1/3)/(x**3-x**2-1),x)
[Out] Integral((x**2 + 3)/((x**2 + 1)**(1/3)*(x**3 - x**2 - 1)), x)
```

3.1035 $\int \frac{-2-(-1+k)(1+k)x+2k^2x^2}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\left(-1+d-(3+d)x-(3+dk^2)x^2+(-1+dk^2)x^3\right)}\,dx$

Optimal. Leaf size=86

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{x+1}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{x+1}\right)}{d^{3/4}}$$

Rubi [F] time = 5.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2-(-1+k)(1+k)x+2k^2x^2}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\left(-1+d-(3+d)x-(3+dk^2)x^2+(-1+dk^2)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2 - (-1 + k)*(1 + k)*x + 2*k^2*x^2)/(((1 - x^2)*(1 - k^2*x^2))^(1/4)*
(-1 + d - (3 + d)*x - (3 + d*k^2)*x^2 + (-1 + d*k^2)*x^3)),x]
```

```
[Out] (2*k^2*(1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*Defer[Int][x^2/((1 - x^2)^(1/4)*
(1 - k^2*x^2)^(1/4)*(-1 + d - (3 + d)*x - (3 + d*k^2)*x^2 - (1 - d*k^2)*x^3
)), x])/((1 - x^2)*(1 - k^2*x^2))^(1/4) + (2*(1 - x^2)^(1/4)*(1 - k^2*x^2)^
(1/4)*Defer[Int][1/((1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*(1 - d + (3 + d)*x
+ (3 + d*k^2)*x^2 + (1 - d*k^2)*x^3)), x])/((1 - x^2)*(1 - k^2*x^2))^(1/4)
- ((1 - k^2)*(1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*Defer[Int][x/((1 - x^2)^(1
/4)*(1 - k^2*x^2)^(1/4)*(1 - d + (3 + d)*x + (3 + d*k^2)*x^2 + (1 - d*k^2)*
x^3)), x])/((1 - x^2)*(1 - k^2*x^2))^(1/4)
```

Rubi steps

$$\int \frac{-2-(-1+k)(1+k)x+2k^2x^2}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\left(-1+d-(3+d)x-(3+dk^2)x^2+(-1+dk^2)x^3\right)}\,dx = \frac{\left(\sqrt[4]{1-x^2}\sqrt[4]{1-k^2x^2}\right)\int \cdots}{\sqrt[4]{(\cdots}}$$

$$= \frac{\left(\sqrt[4]{1-x^2}\sqrt[4]{1-k^2x^2}\right)\int \cdots}{\sqrt[4]{(1\cdots}}$$

$$= \frac{\left(\sqrt[4]{1-x^2}\sqrt[4]{1-k^2x^2}\right)\int \left(\cdots\right.}{\cdots}$$

$$= \frac{\left(2\sqrt[4]{1-x^2}\sqrt[4]{1-k^2x^2}\right)\int \cdots}{\sqrt[4]{(\cdots}}$$

Mathematica [F] time = 5.19, size = 0, normalized size = 0.00

$$\int \frac{-2-(-1+k)(1+k)x+2k^2x^2}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\left(-1+d-(3+d)x-(3+dk^2)x^2+(-1+dk^2)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2 - (-1 + k)*(1 + k)*x + 2*k^2*x^2)/(((1 - x^2)*(1 - k^2*x^2))^
(1/4)*(-1 + d - (3 + d)*x - (3 + d*k^2)*x^2 + (-1 + d*k^2)*x^3)),x]
```

```
[Out] Integrate[(-2 - (-1 + k)*(1 + k)*x + 2*k^2*x^2)/(((1 - x^2)*(1 - k^2*x^2))^
(1/4)*(-1 + d - (3 + d)*x - (3 + d*k^2)*x^2 + (-1 + d*k^2)*x^3)), x]
```

IntegrateAlgebraic [A] time = 12.91, size = 86, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{x+1}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{x+1}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 - (-1 + k)*(1 + k)*x + 2*k^2*x^2)/(((1 - x^2)*(1 - k
^2*x^2))^(1/4)*(-1 + d - (3 + d)*x - (3 + d*k^2)*x^2 + (-1 + d*k^2)*x^3)),x
]
```

```
[Out] ArcTan[(d^(1/4)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4))/(1 + x)]/d^(3/4) - Ar
cTanh[(d^(1/4)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4))/(1 + x)]/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2-(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d
-(3+d)*x-(d*k^2+3)*x^2+(d*k^2-1)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,k^2x^2-(k+1)(k-1)x-2}{\left((dk^2-1)x^3-(dk^2+3)x^2-(d+3)x+d-1\right)\left((k^2x^2-1)(x^2-1)\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2-(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d
-(3+d)*x-(d*k^2+3)*x^2+(d*k^2-1)*x^3),x, algorithm="giac")
```

```
[Out] integrate((2*k^2*x^2 - (k + 1)*(k - 1)*x - 2)/(((d*k^2 - 1)*x^3 - (d*k^2 +
3)*x^2 - (d + 3)*x + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/4)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{-2-(-1+k)\,(1+k)\,x+2k^2x^2}{\left((-x^2+1)\,(-k^2x^2+1)\right)^{\frac{1}{4}}\left(-1+d-(3+d)\,x-\left(d\,k^2+3\right)x^2+\left(d\,k^2-1\right)x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2-(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d-(3+d)
*x-(d*k^2+3)*x^2+(d*k^2-1)*x^3),x)
```

```
[Out] int((-2-(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d-(3+d)
*x-(d*k^2+3)*x^2+(d*k^2-1)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,k^2x^2-(k+1)(k-1)x-2}{\left((dk^2-1)x^3-(dk^2+3)x^2-(d+3)x+d-1\right)\left((k^2x^2-1)(x^2-1)\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2-(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d
-(3+d)*x-(d*k^2+3)*x^2+(d*k^2-1)*x^3),x, algorithm="maxima")
```

```
[Out] integrate((2*k^2*x^2 - (k + 1)*(k - 1)*x - 2)/(((d*k^2 - 1)*x^3 - (d*k^2 +
3)*x^2 - (d + 3)*x + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(k-1)\,(k+1)-2\,k^2\,x^2+2}{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)^{1/4}\,\left(\left(1-d\,k^2\right)\,x^3+\left(d\,k^2+3\right)\,x^2+(d+3)\,x-d+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(k - 1)*(k + 1) - 2*k^2*x^2 + 2)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/4)*(x
^2*(d*k^2 + 3) - x^3*(d*k^2 - 1) - d + x*(d + 3) + 1)),x)
```

```
[Out] int((x*(k - 1)*(k + 1) - 2*k^2*x^2 + 2)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/4)*(x
^2*(d*k^2 + 3) - x^3*(d*k^2 - 1) - d + x*(d + 3) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2-(-1+k)*(1+k)*x+2*k**2*x**2)/((-x**2+1)*(-k**2*x**2+1))**(1/4)
/(-1+d-(3+d)*x-(d*k**2+3)*x**2+(d*k**2-1)*x**3),x)
```

```
[Out] Timed out
```

3.1036 $$\int \frac{-2+(-1+k)(1+k)x+2k^2x^2}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\left(1-d-(3+d)x+(3+dk^2)x^2+(-1+dk^2)x^3\right)}\,dx$$

Optimal. Leaf size=86

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{x-1}\right)}{d^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{x-1}\right)}{d^{3/4}}$$

Rubi [F] time = 5.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2+(-1+k)(1+k)x+2k^2x^2}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\left(1-d-(3+d)x+(3+dk^2)x^2+(-1+dk^2)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2 + (-1 + k)*(1 + k)*x + 2*k^2*x^2)/(((1 - x^2)*(1 - k^2*x^2))^(1/4)*
(1 - d - (3 + d)*x + (3 + d*k^2)*x^2 + (-1 + d*k^2)*x^3)),x]
```

```
[Out] -(((1 - k)*(1 + k)*(1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*Defer[Int][x/((1 - x
^2)^(1/4)*(1 - k^2*x^2)^(1/4)*(1 - d - (3 + d)*x + (3 + d*k^2)*x^2 - (1 - d
*k^2)*x^3)), x])/((1 - x^2)*(1 - k^2*x^2))^(1/4)) + (2*k^2*(1 - x^2)^(1/4)*
(1 - k^2*x^2)^(1/4)*Defer[Int][x^2/((1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*(1
 - d - (3 + d)*x + (3 + d*k^2)*x^2 - (1 - d*k^2)*x^3)), x])/((1 - x^2)*(1 -
k^2*x^2))^(1/4) + (2*(1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*Defer[Int][1/((1 -
 x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*(-1 + d + (3 + d)*x - (3 + d*k^2)*x^2 + (1
 - d*k^2)*x^3)), x])/((1 - x^2)*(1 - k^2*x^2))^(1/4)
```

Rubi steps

$$\int \frac{-2+(-1+k)(1+k)x+2k^2x^2}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\left(1-d-(3+d)x+(3+dk^2)x^2+(-1+dk^2)x^3\right)}\,dx = \frac{\left(\sqrt[4]{1-x^2}\sqrt[4]{1-k^2x^2}\right)\int \frac{\ldots}{\sqrt[4]{1-x^2}\sqrt[4]{1-\ldots}}}{\sqrt[4]{(1-x\ldots}}$$

$$= \frac{\left(\sqrt[4]{1-x^2}\sqrt[4]{1-k^2x^2}\right)\int \frac{\ldots}{\sqrt[4]{1-x^2}\sqrt[4]{1-\ldots}}}{\sqrt[4]{(1-x^2\ldots}}$$

$$= \frac{\left(\sqrt[4]{1-x^2}\sqrt[4]{1-k^2x^2}\right)\int \left(\frac{\ldots}{\sqrt[4]{1-x^2}\ldots}\right.}{\ldots}$$

$$= \frac{\left(2\sqrt[4]{1-x^2}\sqrt[4]{1-k^2x^2}\right)\int \frac{\ldots}{\sqrt[4]{1-x^2}\ldots}}{\sqrt[4]{(1-x\ldots}}$$

Mathematica [F] time = 4.97, size = 0, normalized size = 0.00

$$\int \frac{-2+(-1+k)(1+k)x+2k^2x^2}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\left(1-d-(3+d)x+(3+dk^2)x^2+(-1+dk^2)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2 + (-1 + k)*(1 + k)*x + 2*k^2*x^2)/(((1 - x^2)*(1 - k^2*x^2))^
(1/4)*(1 - d - (3 + d)*x + (3 + d*k^2)*x^2 + (-1 + d*k^2)*x^3)),x]
```

```
[Out] Integrate[(-2 + (-1 + k)*(1 + k)*x + 2*k^2*x^2)/(((1 - x^2)*(1 - k^2*x^2))^
(1/4)*(1 - d - (3 + d)*x + (3 + d*k^2)*x^2 + (-1 + d*k^2)*x^3)), x]
```

IntegrateAlgebraic [A] time = 12.93, size = 86, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{x-1}\right)}{d^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{x-1}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + (-1 + k)*(1 + k)*x + 2*k^2*x^2)/(((1 - x^2)*(1 - k
^2*x^2))^(1/4)*(1 - d - (3 + d)*x + (3 + d*k^2)*x^2 + (-1 + d*k^2)*x^3)),x]
```

```
[Out] -(ArcTan[(d^(1/4)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4))/(-1 + x)]/d^(3/4))
+ ArcTanh[(d^(1/4)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4))/(-1 + x)]/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(1-d-
(3+d)*x+(d*k^2+3)*x^2+(d*k^2-1)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,k^2x^2+(k+1)(k-1)x-2}{\left((dk^2-1)x^3+(dk^2+3)x^2-(d+3)x-d+1\right)\left((k^2x^2-1)(x^2-1)\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(1-d-
(3+d)*x+(d*k^2+3)*x^2+(d*k^2-1)*x^3),x, algorithm="giac")
```

```
[Out] integrate((2*k^2*x^2 + (k + 1)*(k - 1)*x - 2)/(((d*k^2 - 1)*x^3 + (d*k^2 +
3)*x^2 - (d + 3)*x - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/4)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{-2+(-1+k)\,(1+k)\,x+2k^2x^2}{\left((-x^2+1)\left(-k^2x^2+1\right)\right)^{\frac{1}{4}}\left(1-d-(3+d)\,x+\left(d\,k^2+3\right)x^2+\left(d\,k^2-1\right)x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(1-d-(3+d)*
x+(d*k^2+3)*x^2+(d*k^2-1)*x^3),x)
```

```
[Out] int((-2+(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(1-d-(3+d)*
x+(d*k^2+3)*x^2+(d*k^2-1)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,k^2x^2+(k+1)(k-1)x-2}{\left((dk^2-1)x^3+(dk^2+3)x^2-(d+3)x-d+1\right)\left((k^2x^2-1)(x^2-1)\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(1-d-
(3+d)*x+(d*k^2+3)*x^2+(d*k^2-1)*x^3),x, algorithm="maxima")
```

```
[Out] integrate((2*k^2*x^2 + (k + 1)*(k - 1)*x - 2)/(((d*k^2 - 1)*x^3 + (d*k^2 +
3)*x^2 - (d + 3)*x - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2k^2x^2+x\,(k-1)\,(k+1)-2}{\left(\left(x^2-1\right)\left(k^2x^2-1\right)\right)^{1/4}\left(\left(d\,k^2-1\right)x^3+\left(d\,k^2+3\right)x^2+(-d-3)\,x-d+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*k^2*x^2 + x*(k - 1)*(k + 1) - 2)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/4)*(x
^3*(d*k^2 - 1) - d + x^2*(d*k^2 + 3) - x*(d + 3) + 1)),x)
```

```
[Out] int((2*k^2*x^2 + x*(k - 1)*(k + 1) - 2)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/4)*(x
^3*(d*k^2 - 1) - d + x^2*(d*k^2 + 3) - x*(d + 3) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(-1+k)*(1+k)*x+2*k**2*x**2)/((-x**2+1)*(-k**2*x**2+1))**(1/4)
/(1-d-(3+d)*x+(d*k**2+3)*x**2+(d*k**2-1)*x**3),x)
```

```
[Out] Timed out
```

3.1037 $\int \frac{\sqrt[4]{-1+x^4}}{x}\,dx$

Optimal. Leaf size=86

$$\sqrt[4]{x^4-1}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^4-1}}{\sqrt{x^4-1}-1}\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+1}\right)}{2\sqrt{2}}$$

Rubi [A] time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.60, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {266, 50, 63, 211, 1165, 628, 1162, 617, 204}

$$\sqrt[4]{x^4-1}+\frac{\log\left(\sqrt{x^4-1}-\sqrt{2}\sqrt[4]{x^4-1}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{x^4-1}+\sqrt{2}\sqrt[4]{x^4-1}+1\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{x^4-1}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt[4]{x^4-1}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)^(1/4)/x,x]
```

```
[Out] (-1 + x^4)^(1/4) + ArcTan[1 - Sqrt[2]*(-1 + x^4)^(1/4)]/(2*Sqrt[2]) - ArcTa
n[1 + Sqrt[2]*(-1 + x^4)^(1/4)]/(2*Sqrt[2]) + Log[1 - Sqrt[2]*(-1 + x^4)^(1
/4) + Sqrt[-1 + x^4]]/(4*Sqrt[2]) - Log[1 + Sqrt[2]*(-1 + x^4)^(1/4) + Sqrt
[-1 + x^4]]/(4*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{-1+x^4}}{x}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{\sqrt[4]{-1+x}}{x}\,dx,x,x^4\right)\\
&= \sqrt[4]{-1+x^4}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{(-1+x)^{3/4}x}\,dx,x,x^4\right)\\
&= \sqrt[4]{-1+x^4}-\operatorname{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt[4]{-1+x^4}\right)\\
&= \sqrt[4]{-1+x^4}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt[4]{-1+x^4}\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt[4]{-1+x^4}\right)\\
&= \sqrt[4]{-1+x^4}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt[4]{-1+x^4}\right)-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,x,\right.\\
&= \sqrt[4]{-1+x^4}+\frac{\log\left(1-\sqrt{2}\sqrt[4]{-1+x^4}+\sqrt{-1+x^4}\right)}{4\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt[4]{-1+x^4}+\sqrt{-1+x^4}\right)}{4\sqrt{2}}-\frac{\text{Su}}{}\\
&= \sqrt[4]{-1+x^4}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+x^4}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+x^4}\right)}{2\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt[4]{-1+x^4}\right.}{4\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 135, normalized size = 1.57

$$\frac{1}{8}\left(8\sqrt[4]{x^4-1}+\sqrt{2}\log\left(\sqrt{x^4-1}-\sqrt{2}\sqrt[4]{x^4-1}+1\right)-\sqrt{2}\log\left(\sqrt{x^4-1}+\sqrt{2}\sqrt[4]{x^4-1}+1\right)+2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{x^4-1}\right)-2\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt[4]{x^4-1}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)^(1/4)/x,x]
```

```
[Out] (8*(-1 + x^4)^(1/4) + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*(-1 + x^4)^(1/4)] - 2*Sq
rt[2]*ArcTan[1 + Sqrt[2]*(-1 + x^4)^(1/4)] + Sqrt[2]*Log[1 - Sqrt[2]*(-1 +
x^4)^(1/4) + Sqrt[-1 + x^4]] - Sqrt[2]*Log[1 + Sqrt[2]*(-1 + x^4)^(1/4) + S
qrt[-1 + x^4]])/8
```

IntegrateAlgebraic [A] time = 0.11, size = 91, normalized size = 1.06

$$\sqrt[4]{x^4-1}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{x^4-1}}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+1}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)^(1/4)/x,x]
```

```
[Out] (-1 + x^4)^(1/4) - ArcTan[(-(1/Sqrt[2]) + Sqrt[-1 + x^4]/Sqrt[2])/(-1 + x^4
)^(1/4)]/(2*Sqrt[2]) - ArcTanh[(Sqrt[2]*(-1 + x^4)^(1/4))/(1 + Sqrt[-1 + x^
4])]/(2*Sqrt[2])
```

fricas [B] time = 0.43, size = 162, normalized size = 1.88

$$\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+\sqrt{x^4-1}+1}-\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}-1\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+4\sqrt{x^4-1}+4}-\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+1\right)-\frac{1}{8}\sqrt{2}\log\left(4\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+4\sqrt{x^4-1}+4\right)+\frac{1}{8}\sqrt{2}\log\left(-4\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+4\sqrt{x^4-1}+4\right)+\left(x^4-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/4)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*(x^4 - 1)^(1/4) + sqrt(x^4 - 1) + 1
) - sqrt(2)*(x^4 - 1)^(1/4) - 1) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*s
qrt(2)*(x^4 - 1)^(1/4) + 4*sqrt(x^4 - 1) + 4) - sqrt(2)*(x^4 - 1)^(1/4) + 1
) - 1/8*sqrt(2)*log(4*sqrt(2)*(x^4 - 1)^(1/4) + 4*sqrt(x^4 - 1) + 4) + 1/8*
sqrt(2)*log(-4*sqrt(2)*(x^4 - 1)^(1/4) + 4*sqrt(x^4 - 1) + 4) + (x^4 - 1)^(
1/4)
```

giac [A] time = 0.16, size = 109, normalized size = 1.27

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(x^4-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(x^4-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+\sqrt{x^4-1}+1\right)+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+\sqrt{x^4-1}+1\right)+\left(x^4-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/4)/x,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^4 - 1)^(1/4))) - 1/4*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^4 - 1)^(1/4))) - 1/8*sqrt(2)*log(sqrt
(2)*(x^4 - 1)^(1/4) + sqrt(x^4 - 1) + 1) + 1/8*sqrt(2)*log(-sqrt(2)*(x^4 -
1)^(1/4) + sqrt(x^4 - 1) + 1) + (x^4 - 1)^(1/4)
```

maple [C] time = 0.30, size = 64, normalized size = 0.74

$$-\frac{\operatorname{signum}\left(x^4-1\right)^{\frac{1}{4}}\left(\Gamma\left(\frac{3}{4}\right)x^4\operatorname{hypergeom}\left(\left[\frac{3}{4},1,1\right],[2,2],x^4\right)-4\left(4-3\ln(2)+\frac{\pi}{2}+4\ln(x)+i\pi\right)\Gamma\left(\frac{3}{4}\right)\right)}{16\Gamma\left(\frac{3}{4}\right)\left(-\operatorname{signum}\left(x^4-1\right)\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(1/4)/x,x)
```

```
[Out] -1/16/GAMMA(3/4)*signum(x^4-1)^(1/4)/(-signum(x^4-1))^(1/4)*(GAMMA(3/4)*x^4
*hypergeom([3/4,1,1],[2,2],x^4)-4*(4-3*ln(2)+1/2*Pi+4*ln(x)+I*Pi)*GAMMA(3/4
))
```

maxima [A] time = 0.49, size = 109, normalized size = 1.27

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(x^4-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(x^4-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+\sqrt{x^4-1}+1\right)+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+\sqrt{x^4-1}+1\right)+\left(x^4-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/4)/x,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^4 - 1)^(1/4))) - 1/4*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^4 - 1)^(1/4))) - 1/8*sqrt(2)*log(sqrt
(2)*(x^4 - 1)^(1/4) + sqrt(x^4 - 1) + 1) + 1/8*sqrt(2)*log(-sqrt(2)*(x^4 -
1)^(1/4) + sqrt(x^4 - 1) + 1) + (x^4 - 1)^(1/4)
```

mupad [B] time = 0.85, size = 52, normalized size = 0.60

$$\left(x^4-1\right)^{1/4}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(x^4-1\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(x^4-1\right)^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)^(1/4)/x,x)
```

```
[Out] (x^4 - 1)^(1/4) - 2^(1/2)*atan(2^(1/2)*(x^4 - 1)^(1/4)*(1/2 + 1i/2))*(1/4 -
 1i/4) - 2^(1/2)*atan(2^(1/2)*(x^4 - 1)^(1/4)*(1/2 - 1i/2))*(1/4 + 1i/4)
```

sympy [C] time = 0.83, size = 36, normalized size = 0.42

$$-\frac{x\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\right|\frac{e^{2i\pi}}{x^4}\right)}{4\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(1/4)/x,x)
```

```
[Out] -x*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), exp_polar(2*I*pi)/x**4)/(4*gamma
(3/4))
```

3.1038 $\int \frac{\sqrt{-x+x^4}}{-b+ax^3} \, dx$

Optimal. Leaf size=86

$$\frac{2\sqrt{a-b}\tan^{-1}\left(\frac{x\sqrt{x^4-x}\sqrt{a-b}}{\sqrt{b}(x-1)(x^2+x+1)}\right)}{3a\sqrt{b}}+\frac{2\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)}{3a}$$

Rubi [A] time = 0.15, antiderivative size = 125, normalized size of antiderivative = 1.45, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2056, 466, 465, 402, 217, 206, 377, 205}

$$\frac{2\sqrt{x^4-x}\sqrt{a-b}\tan^{-1}\left(\frac{x^{3/2}\sqrt{a-b}}{\sqrt{b}\sqrt{x^3-1}}\right)}{3a\sqrt{b}\sqrt{x}\sqrt{x^3-1}}+\frac{2\sqrt{x^4-x}\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{x^3-1}}\right)}{3a\sqrt{x}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-x + x^4]/(-b + a*x^3),x]
```

```
[Out] (2*Sqrt[a - b]*Sqrt[-x + x^4]*ArcTan[(Sqrt[a - b]*x^(3/2))/(Sqrt[b]*Sqrt[-1
 + x^3])])/(3*a*Sqrt[b]*Sqrt[x]*Sqrt[-1 + x^3]) + (2*Sqrt[-x + x^4]*ArcTanh
[x^(3/2)/Sqrt[-1 + x^3]])/(3*a*Sqrt[x]*Sqrt[-1 + x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\sqrt{-x+x^4}}{-b+ax^3}\,dx = \frac{\sqrt{-x+x^4}\int \frac{\sqrt{x}\sqrt{-1+x^3}}{-b+ax^3}\,dx}{\sqrt{x}\sqrt{-1+x^3}}$$

$$= \frac{\left(2\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{x^2\sqrt{-1+x^6}}{-b+ax^6}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{-1+x^3}}$$

$$= \frac{\left(2\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{\sqrt{-1+x^2}}{-b+ax^2}\,dx, x, x^{3/2}\right)}{3\sqrt{x}\sqrt{-1+x^3}}$$

$$= \frac{\left(2\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, x^{3/2}\right)}{3a\sqrt{x}\sqrt{-1+x^3}} - \frac{\left(2(a-b)\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}\left(-b+ax^2\right)}\,dx,\right.}{3a\sqrt{x}\sqrt{-1+x^3}}$$

$$= \frac{\left(2\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^{3/2}}{\sqrt{-1+x^3}}\right)}{3a\sqrt{x}\sqrt{-1+x^3}} - \frac{\left(2(a-b)\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{1}{-b-(a-b)x^2}\,dx, x, \right.}{3a\sqrt{x}\sqrt{-1+x^3}}$$

$$= \frac{2\sqrt{a-b}\sqrt{-x+x^4}\tan^{-1}\left(\frac{\sqrt{a-b}\,x^{3/2}}{\sqrt{b}\sqrt{-1+x^3}}\right)}{3a\sqrt{b}\sqrt{x}\sqrt{-1+x^3}} + \frac{2\sqrt{-x+x^4}\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{-1+x^3}}\right)}{3a\sqrt{x}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.60

$$-\frac{2x\sqrt{x\left(x^3-1\right)}F_1\left(\frac{1}{2};-\frac{1}{2},1;\frac{3}{2};x^3,\frac{ax^3}{b}\right)}{3b\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-x + x^4]/(-b + a*x^3),x]
```

```
[Out] (-2*x*Sqrt[x*(-1 + x^3)]*AppellF1[1/2, -1/2, 1, 3/2, x^3, (a*x^3)/b])/(3*b*
Sqrt[1 - x^3])
```

IntegrateAlgebraic [A] time = 0.52, size = 86, normalized size = 1.00

$$\frac{2\sqrt{a-b}\tan^{-1}\left(\frac{x\sqrt{x^4-x}\sqrt{a-b}}{\sqrt{b}(x-1)\left(x^2+x+1\right)}\right)}{3a\sqrt{b}} + \frac{2\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-x + x^4]/(-b + a*x^3),x]
```

```
[Out] (2*Sqrt[a - b]*ArcTan[(Sqrt[a - b]*x*Sqrt[-x + x^4])/(Sqrt[b]*(-1 + x)*(1 +
 x + x^2))])/(3*a*Sqrt[b]) + (2*ArcTanh[x^2/Sqrt[-x + x^4]])/(3*a)
```

fricas [A] time = 0.73, size = 219, normalized size = 2.55

$$\left[\frac{\sqrt{-\frac{a-b}{b}}\log\left(-\frac{\left(a^2-8\,ab+8\,b^2\right)x^6+2\left(3\,ab-4\,b^2\right)x^3+b^2+4\left(\left(ab-2\,b^2\right)x^4+b^2x\right)\sqrt{x^4-x}\,\sqrt{-\frac{a-b}{b}}}{a^2x^6-2\,abx^3+b^2}\right)+2\,\log\left(-2\,x^3-2\,\sqrt{x^4-x}\,x+1\right)}{6\,a},\ \frac{\sqrt{\frac{a-b}{b}}\,\arctan\left(-\frac{2\,\sqrt{x^4-x}\,bx\sqrt{\frac{a-b}{b}}}{(a-2\,b)x^3+b}\right)+\log\left(-2\,x^3-2\,\sqrt{x^4-x}\,x+1\right)}{3\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)/(a*x^3-b),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(-(a - b)/b)*log(-((a^2 - 8*a*b + 8*b^2)*x^6 + 2*(3*a*b - 4*b^2)*
x^3 + b^2 + 4*((a*b - 2*b^2)*x^4 + b^2*x)*sqrt(x^4 - x)*sqrt(-(a - b)/b))/(
a^2*x^6 - 2*a*b*x^3 + b^2)) + 2*log(-2*x^3 - 2*sqrt(x^4 - x)*x + 1))/a, 1/3
*(sqrt((a - b)/b)*arctan(-2*sqrt(x^4 - x)*b*x*sqrt((a - b)/b)/((a - 2*b)*x^
3 + b)) + log(-2*x^3 - 2*sqrt(x^4 - x)*x + 1))/a]
```

giac [A] time = 0.30, size = 80, normalized size = 0.93

$$-\frac{2\,(a-b)\arctan\left(\frac{b\sqrt{-\frac{1}{x^3}+1}}{\sqrt{ab-b^2}}\right)}{3\,\sqrt{ab-b^2}\,a}+\frac{\log\left(\sqrt{-\frac{1}{x^3}+1}+1\right)}{3\,a}-\frac{\log\left(\left|\sqrt{-\frac{1}{x^3}+1}-1\right|\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)/(a*x^3-b),x, algorithm="giac")
```

```
[Out] -2/3*(a - b)*arctan(b*sqrt(-1/x^3 + 1)/sqrt(a*b - b^2))/(sqrt(a*b - b^2)*a)
 + 1/3*log(sqrt(-1/x^3 + 1) + 1)/a - 1/3*log(abs(sqrt(-1/x^3 + 1) - 1))/a
```

maple [C] time = 0.40, size = 636, normalized size = 7.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x)^(1/2)/(a*x^3-b),x)
```

```
[Out] 2/a*(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x)
)^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2))/(-1+x))^(1/2)*
((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2)/(-3/2+1/2*I*3^(1/
2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/2)))^(1/2)*(EllipticF
(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3^(
1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-E
llipticPi(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),(-1/2+
1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2)
)/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)))-2/3/a*4^(1/2)*sum(1/_alp
ha*(-1+x)^2*(_alpha^2+_alpha+1)*(1-I*3^(1/2))*(x/(-1+x)*(-3+I*3^(1/2))/(I*3
^(1/2)-1))^(1/2)*(1/(-1+x)*(I*3^(1/2)+2*x+1)/(-1-I*3^(1/2)))^(1/2)*(1/(-1+x
)*(1+2*x-I*3^(1/2))/(I*3^(1/2)-1))^(1/2)/(-3+I*3^(1/2))/(x*(-1+x)*(I*3^(1/2
)+2*x+1)*(1+2*x-I*3^(1/2)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+
1/2*I*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-_alpha^2*a/b*EllipticPi(((-3/2+1
/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),1/6*(I*_alpha^2*3^(1/2)*
a-I*3^(1/2)*b-3*_alpha^2*a+3*b)/b,((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/
(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))),_alpha=RootOf(_Z^3*a-b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4 - x}}{ax^3 - b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)/(a*x^3-b),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 - x)/(a*x^3 - b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{x^4 - x}}{b - a\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - x)^(1/2)/(b - a*x^3),x)
```

```
[Out] -int((x^4 - x)^(1/2)/(b - a*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\,(x-1)\left(x^2+x+1\right)}}{ax^3 - b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x)**(1/2)/(a*x**3-b),x)
```

```
[Out] Integral(sqrt(x*(x - 1)*(x**2 + x + 1))/(a*x**3 - b), x)
```

3.1039 $\int \frac{1}{\left(-1+x^4\right)\sqrt[4]{-x^2+x^4}}\,dx$

Optimal. Leaf size=86

$$-\frac{\left(x^4-x^2\right)^{3/4}}{x\left(x^2-1\right)}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^2}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^2}}\right)}{2\sqrt[4]{2}}$$

Rubi [A] time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.52, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2056, 1254, 466, 382, 377, 212, 206, 203}

$$-\frac{x}{\sqrt[4]{x^4-x^2}}-\frac{\sqrt[4]{x^2-1}\sqrt{x}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{2\sqrt[4]{2}\sqrt[4]{x^4-x^2}}-\frac{\sqrt[4]{x^2-1}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{2\sqrt[4]{2}\sqrt[4]{x^4-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x^4)*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] -(x/(-x^2 + x^4)^(1/4)) - (Sqrt[x]*(-1 + x^2)^(1/4)*ArcTan[(2^(1/4)*Sqrt[x]
)/(-1 + x^2)^(1/4)])/(2*2^(1/4)*(-x^2 + x^4)^(1/4)) - (Sqrt[x]*(-1 + x^2)^(
1/4)*ArcTanh[(2^(1/4)*Sqrt[x])/(-1 + x^2)^(1/4)])/(2*2^(1/4)*(-x^2 + x^4)^(
1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1254

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e)^p, x] /
; FreeQ[{a, c, d, e, f, q, m, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p
]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(-1+x^4\right)\sqrt[4]{-x^2+x^4}}\,dx &= \frac{\left(\sqrt{x}\,\sqrt[4]{-1+x^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt[4]{-1+x^2}\left(-1+x^4\right)}\,dx}{\sqrt[4]{-x^2+x^4}}\\
&= \frac{\left(\sqrt{x}\,\sqrt[4]{-1+x^2}\right)\int \frac{1}{\sqrt{x}\left(-1+x^2\right)^{5/4}\left(1+x^2\right)}\,dx}{\sqrt[4]{-x^2+x^4}}\\
&= \frac{\left(2\sqrt{x}\,\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\left(-1+x^4\right)^{5/4}\left(1+x^4\right)}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{-x^2+x^4}}\\
&= -\frac{x}{\sqrt[4]{-x^2+x^4}}-\frac{\left(\sqrt{x}\,\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-1+x^4}\left(1+x^4\right)}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{-x^2+x^4}}\\
&= -\frac{x}{\sqrt[4]{-x^2+x^4}}-\frac{\left(\sqrt{x}\,\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-2x^4}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}}\\
&= -\frac{x}{\sqrt[4]{-x^2+x^4}}-\frac{\left(\sqrt{x}\,\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{2\sqrt[4]{-x^2+x^4}}-\left(\sqrt{x}\,\sqrt[4]{-1+x^2}\right)\\
&= -\frac{x}{\sqrt[4]{-x^2+x^4}}-\frac{\sqrt{x}\,\sqrt[4]{-1+x^2}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{2\sqrt[4]{2}\,\sqrt[4]{-x^2+x^4}}-\frac{\sqrt{x}\,\sqrt[4]{-1+x^2}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{2\sqrt[4]{2}\,\sqrt[4]{-x^2+x^4}}
\end{aligned}$$

Mathematica [C] time = 0.23, size = 69, normalized size = 0.80

$$-\frac{x\left(\sqrt[4]{1-x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{2x^2}{x^2+1}\right)+\sqrt[4]{x^2+1}\right)}{\sqrt[4]{x^2\left(x^2-1\right)}\,\sqrt[4]{x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-1 + x^4)*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] -((x*((1 + x^2)^(1/4) + (1 - x^2)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (2
*x^2)/(1 + x^2)]))/((x^2*(-1 + x^2))^(1/4)*(1 + x^2)^(1/4)))
```

IntegrateAlgebraic [A] time = 0.26, size = 86, normalized size = 1.00

$$-\frac{\left(x^4-x^2\right)^{3/4}}{x\left(x^2-1\right)}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^2}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^2}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x^4)*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] -((-x^2 + x^4)^(3/4)/(x*(-1 + x^2))) - ArcTan[(2^(1/4)*x)/(-x^2 + x^4)^(1/4
)]/(2*2^(1/4)) - ArcTanh[(2^(1/4)*x)/(-x^2 + x^4)^(1/4)]/(2*2^(1/4))
```

fricas [B] time = 1.65, size = 286, normalized size = 3.33

$$\frac{4\cdot 2^{\frac{3}{4}}\left(x^3-x\right)\arctan\left(\frac{4\cdot 2^{\frac{3}{4}}\left(x^4-x^2\right)^{\frac{1}{4}}x^2+2^{\frac{3}{4}}\left(2\cdot 2^{\frac{3}{4}}\sqrt{x^4-x^2}\,x+2^{\frac{1}{4}}\left(3x^3-x\right)\right)+4\cdot 2^{\frac{1}{4}}\left(x^4-x^2\right)^{\frac{3}{4}}}{2\left(x^3+x\right)}\right)-2^{\frac{3}{4}}\left(x^3-x\right)\log\left(\frac{4\sqrt{2}\left(x^4-x^2\right)^{\frac{1}{4}}x^2+2^{\frac{3}{4}}\left(3x^3-x\right)+4\cdot 2^{\frac{1}{4}}\sqrt{x^4-x^2}\,x+4\left(x^4-x^2\right)^{\frac{3}{4}}}{x^3+x}\right)+2^{\frac{3}{4}}\left(x^3-x\right)\log\left(\frac{4\sqrt{2}\left(x^4-x^2\right)^{\frac{1}{4}}x^2-2^{\frac{3}{4}}\left(3x^3-x\right)-4\cdot 2^{\frac{1}{4}}\sqrt{x^4-x^2}\,x+4\left(x^4-x^2\right)^{\frac{3}{4}}}{x^3+x}\right)-16\left(x^4-x^2\right)^{\frac{3}{4}}}{16\left(x^3-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)/(x^4-x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/16*(4*2^(3/4)*(x^3 - x)*arctan(1/2*(4*2^(3/4)*(x^4 - x^2)^(1/4)*x^2 + 2^(
3/4)*(2*2^(3/4)*sqrt(x^4 - x^2)*x + 2^(1/4)*(3*x^3 - x)) + 4*2^(1/4)*(x^4 -
 x^2)^(3/4))/(x^3 + x)) - 2^(3/4)*(x^3 - x)*log((4*sqrt(2)*(x^4 - x^2)^(1/4
)*x^2 + 2^(3/4)*(3*x^3 - x) + 4*2^(1/4)*sqrt(x^4 - x^2)*x + 4*(x^4 - x^2)^(
3/4))/(x^3 + x)) + 2^(3/4)*(x^3 - x)*log((4*sqrt(2)*(x^4 - x^2)^(1/4)*x^2 -
 2^(3/4)*(3*x^3 - x) - 4*2^(1/4)*sqrt(x^4 - x^2)*x + 4*(x^4 - x^2)^(3/4))/(
x^3 + x)) - 16*(x^4 - x^2)^(3/4))/(x^3 - x)
```

giac [A] time = 0.23, size = 70, normalized size = 0.81

$$-\frac{1}{4}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)+\frac{1}{8}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)-\frac{1}{8}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}-\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)+\frac{1}{\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)/(x^4-x^2)^(1/4),x, algorithm="giac")
```

```
[Out] -1/4*2^(3/4)*arctan(1/2*2^(3/4)*(-1/x^2 + 1)^(1/4)) + 1/8*2^(3/4)*log(2^(1/
4) + (-1/x^2 + 1)^(1/4)) - 1/8*2^(3/4)*log(2^(1/4) - (-1/x^2 + 1)^(1/4)) +
1/(-1/x^2 + 1)^(1/4)
```

maple [C] time = 4.02, size = 247, normalized size = 2.87

$$-\frac{x}{\left(x^2\left(x^2-1\right)\right)^{\frac{1}{4}}}-\frac{\mathrm{RootOf}\left(\_Z^4-8\right)\ln\left(\frac{\sqrt{x^4-x^2}\,\mathrm{RootOf}\left(\_Z^4-8\right)^3x+2\,\mathrm{RootOf}\left(\_Z^4-8\right)^2\left(x^4-x^2\right)^{\frac{1}{4}}x^2+3\,\mathrm{RootOf}\left(\_Z^4-8\right)x^3+4\left(x^4-x^2\right)^{\frac{3}{4}}-\mathrm{RootOf}\left(\_Z^4-8\right)x}{x\left(x^2+1\right)}\right)}{8}+\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\ln\left(\frac{\sqrt{x^4-x^2}\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\mathrm{RootOf}\left(\_Z^4-8\right)^2x-2\,\mathrm{RootOf}\left(\_Z^4-8\right)^2\left(x^4-x^2\right)^{\frac{1}{4}}x^2-3\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)x^3+4\left(x^4-x^2\right)^{\frac{3}{4}}+\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)x}{x\left(x^2+1\right)}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-1)/(x^4-x^2)^(1/4),x)
```

```
[Out] -x/(x^2*(x^2-1))^(1/4)-1/8*RootOf(_Z^4-8)*ln(((x^4-x^2)^(1/2)*RootOf(_Z^4-8
)^3*x+2*RootOf(_Z^4-8)^2*(x^4-x^2)^(1/4)*x^2+3*RootOf(_Z^4-8)*x^3+4*(x^4-x^
2)^(3/4)-RootOf(_Z^4-8)*x)/x/(x^2+1))+1/8*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln(
((x^4-x^2)^(1/2)*RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x-2*RootOf(
```

```
_Z^4-8)^2*(x^4-x^2)^(1/4)*x^2-3*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^3+4*(x^4-x^
2)^(3/4)+RootOf(_Z^2+RootOf(_Z^4-8)^2)*x)/x/(x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4-x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)/(x^4-x^2)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((x^4 - x^2)^(1/4)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^4-1\right)\,\left(x^4-x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^4 - 1)*(x^4 - x^2)^(1/4)),x)
[Out] int(1/((x^4 - 1)*(x^4 - x^2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^2\,(x-1)\,(x+1)}\;(x-1)\,(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-1)/(x**4-x**2)**(1/4),x)
[Out] Integral(1/((x**2*(x - 1)*(x + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.1040 $\int \frac{x^6}{\left(-b+ax^4\right)^{3/4}}\,dx$

Optimal. Leaf size=86

$$-\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{8a^{7/4}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{8a^{7/4}}+\frac{x^3\sqrt[4]{ax^4-b}}{4a}$$

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {321, 331, 298, 203, 206}

$$-\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{8a^{7/4}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{8a^{7/4}}+\frac{x^3\sqrt[4]{ax^4-b}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(-b + a*x^4)^(3/4),x]
```

```
[Out] (x^3*(-b + a*x^4)^(1/4))/(4*a) - (3*b*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)
])/(8*a^(7/4)) + (3*b*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(8*a^(7/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{x^6}{\left(-b+ax^4\right)^{3/4}}\,dx = \frac{x^3\sqrt[4]{-b+ax^4}}{4a} + \frac{(3b)\int \frac{x^2}{\left(-b+ax^4\right)^{3/4}}\,dx}{4a}$$

$$= \frac{x^3\sqrt[4]{-b+ax^4}}{4a} + \frac{(3b)\,\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{4a}$$

$$= \frac{x^3\sqrt[4]{-b+ax^4}}{4a} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{8a^{3/2}} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{1+\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-b+a}}\right.}{8a^{3/2}}$$

$$= \frac{x^3\sqrt[4]{-b+ax^4}}{4a} - \frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{8a^{7/4}} + \frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{8a^{7/4}}$$

Mathematica [A] time = 0.03, size = 81, normalized size = 0.94

$$\frac{2a^{3/4}x^3\sqrt[4]{ax^4-b} - 3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right) + 3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{8a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(-b + a*x^4)^(3/4),x]
```

```
[Out] (2*a^(3/4)*x^3*(-b + a*x^4)^(1/4) - 3*b*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/
4)] + 3*b*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(8*a^(7/4))
```

IntegrateAlgebraic [A] time = 0.39, size = 86, normalized size = 1.00

$$-\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{8a^{7/4}} + \frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{8a^{7/4}} + \frac{x^3\sqrt[4]{ax^4-b}}{4a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/(-b + a*x^4)^(3/4),x]
```

```
[Out] (x^3*(-b + a*x^4)^(1/4))/(4*a) - (3*b*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)
])/(8*a^(7/4)) + (3*b*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(8*a^(7/4))
```

fricas [B] time = 0.46, size = 214, normalized size = 2.49

$$\frac{4\left(ax^4-b\right)^{\frac{1}{4}}x^3 - 12\,a\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}\arctan\left(\frac{a^5x\sqrt{\frac{a^4x^2\sqrt{\frac{b^4}{a^7}}+\sqrt{ax^4-b}\,b^2}{x^2}}\left(\frac{b^4}{a^7}\right)^{\frac{3}{4}} - \left(ax^4-b\right)^{\frac{1}{4}}a^5b\left(\frac{b^4}{a^7}\right)^{\frac{3}{4}}}{b^4x}\right) + 3\,a\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}\log\left(\frac{3\left(a^2x\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}+\left(ax^4-b\right)^{\frac{1}{4}}b\right)}{x}\right) - 3\,a\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}\log\left(-\frac{3\left(a^2x\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}-\left(ax^4-b\right)^{\frac{1}{4}}b\right)}{x}\right)}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a*x^4-b)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/16*(4*(a*x^4 - b)^(1/4)*x^3 - 12*a*(b^4/a^7)^(1/4)*arctan((a^5*x*sqrt((a^
4*x^2*sqrt(b^4/a^7) + sqrt(a*x^4 - b)*b^2)/x^2)*(b^4/a^7)^(3/4) - (a*x^4 -
b)^(1/4)*a^5*b*(b^4/a^7)^(3/4))/(b^4*x)) + 3*a*(b^4/a^7)^(1/4)*log(3*(a^2*x
*(b^4/a^7)^(1/4) + (a*x^4 - b)^(1/4)*b)/x) - 3*a*(b^4/a^7)^(1/4)*log(-3*(a^
2*x*(b^4/a^7)^(1/4) - (a*x^4 - b)^(1/4)*b)/x))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(ax^4-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a*x^4-b)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(a*x^4 - b)^(3/4), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(a\,x^{4}-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^4-b)^(3/4),x)
```

```
[Out] int(x^6/(a*x^4-b)^(3/4),x)
```

maxima [A] time = 0.41, size = 120, normalized size = 1.40

$$\frac{3\left(\frac{2\,b\arctan\left(\frac{\left(ax^{4}-b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{3}{4}}}-\frac{b\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^{4}-b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^{4}-b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{3}{4}}}\right)}{16\,a}+\frac{\left(ax^{4}-b\right)^{\frac{1}{4}}b}{4\left(a^{2}-\frac{\left(ax^{4}-b\right)a}{x^{4}}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a*x^4-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 3/16*(2*b*arctan((a*x^4 - b)^(1/4)/(a^(1/4)*x))/a^(3/4) - b*log(-(a^(1/4) -
 (a*x^4 - b)^(1/4)/x)/(a^(1/4) + (a*x^4 - b)^(1/4)/x))/a^(3/4))/a + 1/4*(a*
x^4 - b)^(1/4)*b/((a^2 - (a*x^4 - b)*a/x^4)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{6}}{\left(a\,x^{4}-b\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^4 - b)^(3/4),x)
```

```
[Out] int(x^6/(a*x^4 - b)^(3/4), x)
```

sympy [C] time = 1.09, size = 41, normalized size = 0.48

$$-\frac{x^{7}e^{\frac{i\pi}{4}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{ax^{4}}{b}\right)}{4b^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(a*x**4-b)**(3/4),x)
```

```
[Out] -x**7*exp(I*pi/4)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), a*x**4/b)/(4*b**(3/
4)*gamma(11/4))
```

3.1041 $\int \frac{(-b+ax^4)\sqrt[4]{b+ax^4}}{x^2}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt[4]{ax^4+b}\,(ax^4+4b)}{4x}+\frac{3}{8}\sqrt[4]{a}\,b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)-\frac{3}{8}\sqrt[4]{a}\,b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {453, 279, 331, 298, 203, 206}

$$\frac{(ax^4+b)^{5/4}}{x}+\frac{3}{8}\sqrt[4]{a}\,b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)-\frac{3}{8}\sqrt[4]{a}\,b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)-\frac{3}{4}ax^3\sqrt[4]{ax^4+b}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a*x^4)*(b + a*x^4)^(1/4))/x^2,x]
```

```
[Out] (-3*a*x^3*(b + a*x^4)^(1/4))/4 + (b + a*x^4)^(5/4)/x + (3*a^(1/4)*b*ArcTan[
(a^(1/4)*x)/(b + a*x^4)^(1/4)])/8 - (3*a^(1/4)*b*ArcTanh[(a^(1/4)*x)/(b + a
*x^4)^(1/4)])/8
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-b+ax^4\right)\sqrt[4]{b+ax^4}}{x^2}\,dx &= \frac{\left(b+ax^4\right)^{5/4}}{x}-(3a)\int x^2\sqrt[4]{b+ax^4}\,dx\\ &= -\frac{3}{4}ax^3\sqrt[4]{b+ax^4}+\frac{\left(b+ax^4\right)^{5/4}}{x}-\frac{1}{4}(3ab)\int \frac{x^2}{\left(b+ax^4\right)^{3/4}}\,dx\\ &= -\frac{3}{4}ax^3\sqrt[4]{b+ax^4}+\frac{\left(b+ax^4\right)^{5/4}}{x}-\frac{1}{4}(3ab)\,\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)\\ &= -\frac{3}{4}ax^3\sqrt[4]{b+ax^4}+\frac{\left(b+ax^4\right)^{5/4}}{x}-\frac{1}{8}\left(3\sqrt{a}\,b\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{b+ax}}\right.\\ &= -\frac{3}{4}ax^3\sqrt[4]{b+ax^4}+\frac{\left(b+ax^4\right)^{5/4}}{x}+\frac{3}{8}\sqrt[4]{a}\,b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)-\frac{3}{8}\sqrt[4]{a}\,b\tanh^{-1}\left(\right.\end{aligned}$$

Mathematica [C] time = 0.06, size = 61, normalized size = 0.71

$$\frac{\sqrt[4]{ax^4+b}\left(-\frac{ax^4\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{ax^4}{b}\right)}{\sqrt[4]{\frac{ax^4}{b}+1}}+ax^4+b\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + a*x^4)*(b + a*x^4)^(1/4))/x^2,x]
```

```
[Out] ((b + a*x^4)^(1/4)*(b + a*x^4 - (a*x^4*Hypergeometric2F1[-1/4, 3/4, 7/4, -(
(a*x^4)/b)])/(1 + (a*x^4)/b)^(1/4)))/x
```

IntegrateAlgebraic [A] time = 0.33, size = 86, normalized size = 1.00

$$\frac{\sqrt[4]{ax^4+b}\left(ax^4+4b\right)}{4x}+\frac{3}{8}\sqrt[4]{a}\,b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)-\frac{3}{8}\sqrt[4]{a}\,b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^4)*(b + a*x^4)^(1/4))/x^2,x]
```

```
[Out] ((b + a*x^4)^(1/4)*(4*b + a*x^4))/(4*x) + (3*a^(1/4)*b*ArcTan[(a^(1/4)*x)/(
b + a*x^4)^(1/4)])/8 - (3*a^(1/4)*b*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)])
/8
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)*(a*x^4+b)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4+b\right)^{\frac{1}{4}}\left(ax^4-b\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)*(a*x^4+b)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a*x^4 + b)^(1/4)*(a*x^4 - b)/x^2, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^4-b\right)\left(a\,x^4+b\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b)*(a*x^4+b)^(1/4)/x^2,x)
```

```
[Out] int((a*x^4-b)*(a*x^4+b)^(1/4)/x^2,x)
```

maxima [B] time = 0.42, size = 190, normalized size = 2.21

$$\frac{1}{16}\left(\frac{2\,b\arctan\left(\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{3}{4}}}-\frac{b\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{3}{4}}}-\frac{4\left(ax^4+b\right)^{\frac{1}{4}}b}{\left(a-\frac{ax^4+b}{x^4}\right)x}\right)a-\frac{1}{4}\left(2\,a^{\frac{1}{4}}\arctan\left(\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)-a^{\frac{1}{4}}\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}\right)-\frac{4\left(ax^4+b\right)^{\frac{1}{4}}}{x}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)*(a*x^4+b)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*b*arctan((a*x^4 + b)^(1/4)/(a^(1/4)*x))/a^(3/4) - b*log(-(a^(1/4) -
 (a*x^4 + b)^(1/4)/x)/(a^(1/4) + (a*x^4 + b)^(1/4)/x))/a^(3/4) - 4*(a*x^4 +
 b)^(1/4)*b/((a - (a*x^4 + b)/x^4)*x))*a - 1/4*(2*a^(1/4)*arctan((a*x^4 + b
)^(1/4)/(a^(1/4)*x)) - a^(1/4)*log(-(a^(1/4) - (a*x^4 + b)^(1/4)/x)/(a^(1/4
) + (a*x^4 + b)^(1/4)/x)) - 4*(a*x^4 + b)^(1/4)/x)*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(a\,x^4+b\right)^{1/4}\,\left(b-a\,x^4\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b + a*x^4)^(1/4)*(b - a*x^4))/x^2,x)
```

```
[Out] -int(((b + a*x^4)^(1/4)*(b - a*x^4))/x^2, x)
```

sympy [C] time = 3.16, size = 83, normalized size = 0.97

$$\frac{a\sqrt[4]{b}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{ax^4e^{i\pi}}{b}\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{b^{\frac{5}{4}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{ax^4e^{i\pi}}{b}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b)*(a*x**4+b)**(1/4)/x**2,x)
```

```
[Out] a*b**(1/4)*x**3*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), a*x**4*exp_polar(I*pi
)/b)/(4*gamma(7/4)) - b**(5/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), a*x*
*4*exp_polar(I*pi)/b)/(4*x*gamma(3/4))
```

3.1042 $\int \frac{-b+ax^8}{x^6\left(b+ax^4\right)^{3/4}}\,dx$

Optimal. Leaf size=86

$$-\frac{1}{2}\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\frac{1}{2}\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\frac{\sqrt[4]{ax^4+b}\,\left(b-4ax^4\right)}{5bx^5}$$

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1489, 271, 264, 331, 298, 203, 206}

$$-\frac{4a\sqrt[4]{ax^4+b}}{5bx}-\frac{1}{2}\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\frac{1}{2}\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\frac{\sqrt[4]{ax^4+b}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^8)/(x^6*(b + a*x^4)^(3/4)),x]
```

```
[Out] (b + a*x^4)^(1/4)/(5*x^5) - (4*a*(b + a*x^4)^(1/4))/(5*b*x) - (a^(1/4)*ArcT
an[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/2 + (a^(1/4)*ArcTanh[(a^(1/4)*x)/(b + a*
x^4)^(1/4)])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 1489

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IG
tQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-b+ax^8}{x^6\left(b+ax^4\right)^{3/4}}\,dx &= \int \left(-\frac{b}{x^6\left(b+ax^4\right)^{3/4}}+\frac{ax^2}{\left(b+ax^4\right)^{3/4}}\right)dx\\
&= a\int \frac{x^2}{\left(b+ax^4\right)^{3/4}}\,dx-b\int \frac{1}{x^6\left(b+ax^4\right)^{3/4}}\,dx\\
&= \frac{\sqrt[4]{b+ax^4}}{5x^5}+\frac{1}{5}(4a)\int \frac{1}{x^2\left(b+ax^4\right)^{3/4}}\,dx+a\,\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)\\
&= \frac{\sqrt[4]{b+ax^4}}{5x^5}-\frac{4a\sqrt[4]{b+ax^4}}{5bx}+\frac{1}{2}\sqrt{a}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)-\frac{1}{2}\sqrt{a}\,\text{Subst}\\
&= \frac{\sqrt[4]{b+ax^4}}{5x^5}-\frac{4a\sqrt[4]{b+ax^4}}{5bx}-\frac{1}{2}\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)+\frac{1}{2}\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.10, size = 84, normalized size = 0.98

$$\frac{1}{10}\left(-5\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+5\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\frac{2\sqrt[4]{ax^4+b}\left(b-4ax^4\right)}{bx^5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^8)/(x^6*(b + a*x^4)^(3/4)),x]
```

```
[Out] ((2*(b - 4*a*x^4)*(b + a*x^4)^(1/4))/(b*x^5) - 5*a^(1/4)*ArcTan[(a^(1/4)*x)
/(b + a*x^4)^(1/4)] + 5*a^(1/4)*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/10
```

IntegrateAlgebraic [A] time = 0.52, size = 86, normalized size = 1.00

$$-\frac{1}{2}\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\frac{1}{2}\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\frac{\sqrt[4]{ax^4+b}\left(b-4ax^4\right)}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^8)/(x^6*(b + a*x^4)^(3/4)),x]
```

```
[Out] ((b - 4*a*x^4)*(b + a*x^4)^(1/4))/(5*b*x^5) - (a^(1/4)*ArcTan[(a^(1/4)*x)/(
b + a*x^4)^(1/4)])/2 + (a^(1/4)*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-b)/x^6/(a*x^4+b)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^8 - b}{\left(ax^4 + b\right)^{\frac{3}{4}} x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-b)/x^6/(a*x^4+b)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^8 - b)/((a*x^4 + b)^(3/4)*x^6), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{a\,x^8 - b}{x^6 \left(a\,x^4 + b\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^8-b)/x^6/(a*x^4+b)^(3/4),x)
```

```
[Out] int((a*x^8-b)/x^6/(a*x^4+b)^(3/4),x)
```

maxima [A] time = 0.49, size = 107, normalized size = 1.24

$$\frac{1}{4}\,a\left(\frac{2\,\arctan\left(\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}\,x}\right)}{a^{\frac{3}{4}}} - \frac{\log\left(-\frac{a^{\frac{1}{4}} - \frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}} + \frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{3}{4}}}\right) - \frac{\frac{5\left(ax^4+b\right)^{\frac{1}{4}}a}{x} - \frac{\left(ax^4+b\right)^{\frac{5}{4}}}{x^5}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-b)/x^6/(a*x^4+b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/4*a*(2*arctan((a*x^4 + b)^(1/4)/(a^(1/4)*x))/a^(3/4) - log(-(a^(1/4) - (a
*x^4 + b)^(1/4)/x)/(a^(1/4) + (a*x^4 + b)^(1/4)/x))/a^(3/4)) - 1/5*(5*(a*x^
4 + b)^(1/4)*a/x - (a*x^4 + b)^(5/4)/x^5)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{b - a\,x^8}{x^6\left(a\,x^4 + b\right)^{3/4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^8)/(x^6*(b + a*x^4)^(3/4)),x)
```

```
[Out] -int((b - a*x^8)/(x^6*(b + a*x^4)^(3/4)), x)
```

sympy [C] time = 2.32, size = 105, normalized size = 1.22

$$-\frac{a^{\frac{5}{4}} \sqrt[4]{1 + \frac{b}{ax^4}}\,\Gamma\left(-\frac{5}{4}\right)}{4b\Gamma\left(\frac{3}{4}\right)} + \frac{\sqrt[4]{a}\,\sqrt[4]{1 + \frac{b}{ax^4}}\,\Gamma\left(-\frac{5}{4}\right)}{16x^4\Gamma\left(\frac{3}{4}\right)} + \frac{ax^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{ax^4e^{i\pi}}{b}\right)}{4b^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**8-b)/x**6/(a*x**4+b)**(3/4),x)
```

```
[Out] -a**(5/4)*(1 + b/(a*x**4))**(1/4)*gamma(-5/4)/(4*b*gamma(3/4)) + a**(1/4)*(
1 + b/(a*x**4))**(1/4)*gamma(-5/4)/(16*x**4*gamma(3/4)) + a*x**3*gamma(3/4)
*hyper((3/4, 3/4), (7/4,), a*x**4*exp_polar(I*pi)/b)/(4*b**(3/4)*gamma(7/4)
)
```

3.1043 $\int \frac{\sqrt{x}}{(-1+x)\sqrt{-\sqrt{x}+x}}\,dx$

Optimal. Leaf size=87

$$-\frac{2\sqrt{x-\sqrt{x}}}{\sqrt{x}-1}+4\tanh^{-1}\left(\frac{\sqrt{x-\sqrt{x}}}{\sqrt{x}-1}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x-\sqrt{x}}}{\sqrt{x}-1}\right)$$

Rubi [A] time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.41, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2056, 1549, 848, 98, 157, 54, 215, 93, 206}

$$-\frac{2\sqrt{x}}{\sqrt{x-\sqrt{x}}}+\frac{4\sqrt{\sqrt{x}-1}\,\sqrt[4]{x}\sinh^{-1}\left(\sqrt{\sqrt{x}-1}\right)}{\sqrt{x-\sqrt{x}}}-\frac{\sqrt{2}\sqrt{\sqrt{x}-1}\,\sqrt[4]{x}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x}}{\sqrt{\sqrt{x}-1}}\right)}{\sqrt{x-\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/((-1 + x)*Sqrt[-Sqrt[x] + x]),x]
```

```
[Out] (-2*Sqrt[x])/Sqrt[-Sqrt[x] + x] + (4*Sqrt[-1 + Sqrt[x]]*x^(1/4)*ArcSinh[Sqr
t[-1 + Sqrt[x]]])/Sqrt[-Sqrt[x] + x] - (Sqrt[2]*Sqrt[-1 + Sqrt[x]]*x^(1/4)*
ArcTanh[(Sqrt[2]*x^(1/4))/Sqrt[-1 + Sqrt[x]]])/Sqrt[-Sqrt[x] + x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 1549

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g*(m + 1)
- 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{
a, c, d, e, m, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(-1+x)\sqrt{-\sqrt{x}+x}}\,dx = \frac{\left(\sqrt{-1+\sqrt{x}}\,\sqrt[4]{x}\right)\int \frac{\sqrt[4]{x}}{\sqrt{-1+\sqrt{x}}\,(-1+x)}\,dx}{\sqrt{-\sqrt{x}+x}}$$

$$= \frac{\left(2\sqrt{-1+\sqrt{x}}\,\sqrt[4]{x}\right)\text{Subst}\left(\int \frac{x^{3/2}}{\sqrt{-1+x}\,(-1+x^2)}\,dx, x, \sqrt{x}\right)}{\sqrt{-\sqrt{x}+x}}$$

$$= \frac{\left(2\sqrt{-1+\sqrt{x}}\,\sqrt[4]{x}\right)\text{Subst}\left(\int \frac{x^{3/2}}{(-1+x)^{3/2}(1+x)}\,dx, x, \sqrt{x}\right)}{\sqrt{-\sqrt{x}+x}}$$

$$= -\frac{2\sqrt{x}}{\sqrt{-\sqrt{x}+x}} - \frac{\left(2\sqrt{-1+\sqrt{x}}\,\sqrt[4]{x}\right)\text{Subst}\left(\int \frac{-\frac{1}{2}-x}{\sqrt{-1+x}\,\sqrt{x}\,(1+x)}\,dx, x, \sqrt{x}\right)}{\sqrt{-\sqrt{x}+x}}$$

$$= -\frac{2\sqrt{x}}{\sqrt{-\sqrt{x}+x}} - \frac{\left(\sqrt{-1+\sqrt{x}}\,\sqrt[4]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{x}\,(1+x)}\,dx, x, \sqrt{x}\right)}{\sqrt{-\sqrt{x}+x}} + \frac{\left(2\sqrt{-1+\sqrt{\cdots}}\right.}{\cdots}$$

$$= -\frac{2\sqrt{x}}{\sqrt{-\sqrt{x}+x}} - \frac{\left(2\sqrt{-1+\sqrt{x}}\,\sqrt[4]{x}\right)\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt{-1+\sqrt{x}}}\right)}{\sqrt{-\sqrt{x}+x}} + \frac{\left(4\sqrt{-1+\sqrt{x}}\cdots\right.}{\cdots}$$

$$= -\frac{2\sqrt{x}}{\sqrt{-\sqrt{x}+x}} + \frac{4\sqrt{-1+\sqrt{x}}\,\sqrt[4]{x}\,\sinh^{-1}\left(\sqrt{-1+\sqrt{x}}\right)}{\sqrt{-\sqrt{x}+x}} - \frac{\sqrt{2}\,\sqrt{-1+\sqrt{x}}\,\sqrt[4]{x}\,\tanh^{-1}\cdots}{\sqrt{-\sqrt{x}+x}}$$

Mathematica [A] time = 0.09, size = 97, normalized size = 1.11

$$-\frac{\sqrt[4]{x}\left(2\sqrt[4]{x}+4\sqrt{1-\sqrt{x}}\,\sin^{-1}\left(\sqrt{1-\sqrt{x}}\right)+\sqrt{2}\,\sqrt{\sqrt{x}-1}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}}{\sqrt{2}\,\sqrt[4]{x}}\right)\right)}{\sqrt{x-\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/((-1 + x)*Sqrt[-Sqrt[x] + x]),x]
```

```
[Out] -((x^(1/4)*(2*x^(1/4) + 4*Sqrt[1 - Sqrt[x]]*ArcSin[Sqrt[1 - Sqrt[x]]] + Sqr
t[2]*Sqrt[-1 + Sqrt[x]]*ArcTanh[Sqrt[-1 + Sqrt[x]]/(Sqrt[2]*x^(1/4))]))/Sqr
t[-Sqrt[x] + x])
```

IntegrateAlgebraic [A] time = 0.27, size = 87, normalized size = 1.00

$$-\frac{2\sqrt{x-\sqrt{x}}}{\sqrt{x}-1}+4\tanh^{-1}\left(\frac{\sqrt{x-\sqrt{x}}}{\sqrt{x}-1}\right)-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{x-\sqrt{x}}}{\sqrt{x}-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/((-1 + x)*Sqrt[-Sqrt[x] + x]),x]
```

```
[Out] (-2*Sqrt[-Sqrt[x] + x])/(-1 + Sqrt[x]) + 4*ArcTanh[Sqrt[-Sqrt[x] + x]/(-1 +
 Sqrt[x])] - Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[-Sqrt[x] + x])/(-1 + Sqrt[x])]
```

fricas [B] time = 1.97, size = 132, normalized size = 1.52

$$\frac{\sqrt{2}\,(x-1)\log\left(-\frac{17\,x^{2}-4\left(\sqrt{2}\,(3\,x+5)\,\sqrt{x}-\sqrt{2}\,(7\,x+1)\right)\sqrt{x-\sqrt{x}}-16\,(3\,x+1)\,\sqrt{x}+46\,x+1}{x^{2}-2\,x+1}\right)+4\,(x-1)\log\left(-4\,\sqrt{x-\sqrt{x}}\,\left(2\,\sqrt{x}-1\right)-8\,x+8\,\sqrt{x}-1\right)-8\,\sqrt{x-\sqrt{x}}\,\left(\sqrt{x}+1\right)}{4\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1+x)/(-x^(1/2)+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(x - 1)*log(-(17*x^2 - 4*(sqrt(2)*(3*x + 5)*sqrt(x) - sqrt(2)*
(7*x + 1))*sqrt(x - sqrt(x)) - 16*(3*x + 1)*sqrt(x) + 46*x + 1)/(x^2 - 2*x
+ 1)) + 4*(x - 1)*log(-4*sqrt(x - sqrt(x))*(2*sqrt(x) - 1) - 8*x + 8*sqrt(x
) - 1) - 8*sqrt(x - sqrt(x))*(sqrt(x) + 1))/(x - 1)
```

giac [A] time = 0.86, size = 96, normalized size = 1.10

$$-\frac{1}{2}\,\sqrt{2}\,\log\left(\frac{2\left(\sqrt{2}-\sqrt{x-\sqrt{x}}+\sqrt{x}+1\right)}{\left|2\,\sqrt{2}+2\,\sqrt{x-\sqrt{x}}-2\,\sqrt{x}-2\right|}\right)-\frac{2}{\sqrt{x-\sqrt{x}}-\sqrt{x}+1}-2\,\log\left(\left|2\,\sqrt{x-\sqrt{x}}-2\,\sqrt{x}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1+x)/(-x^(1/2)+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(2*(sqrt(2) - sqrt(x - sqrt(x)) + sqrt(x) + 1)/abs(2*sqrt(2
) + 2*sqrt(x - sqrt(x)) - 2*sqrt(x) - 2)) - 2/(sqrt(x - sqrt(x)) - sqrt(x)
+ 1) - 2*log(abs(2*sqrt(x - sqrt(x)) - 2*sqrt(x) + 1))
```

maple [B] time = 0.02, size = 219, normalized size = 2.52

$$\frac{\sqrt{-\sqrt{x}+x}\,\left(2\sqrt{2}\,\sqrt{x}\,\operatorname{arctanh}\left(\frac{\left(-1+3\sqrt{x}\right)\sqrt{2}}{4\sqrt{-\sqrt{x}+x}}\right)-\sqrt{2}\,x\operatorname{arctanh}\left(\frac{\left(-1+3\sqrt{x}\right)\sqrt{2}}{4\sqrt{-\sqrt{x}+x}}\right)-4\left(-\sqrt{x}+x\right)^{\frac{3}{2}}-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-1+3\sqrt{x}\right)\sqrt{2}}{4\sqrt{-\sqrt{x}+x}}\right)-8\sqrt{x}\,\ln\left(-\frac{1}{2}+\sqrt{x}+\sqrt{-\sqrt{x}+x}\right)-8\sqrt{x}\,\sqrt{-\sqrt{x}+x}+4x\ln\left(-\frac{1}{2}+\sqrt{x}+\sqrt{-\sqrt{x}+x}\right)+4x\sqrt{-\sqrt{x}+x}+4\ln\left(-\frac{1}{2}+\sqrt{x}+\sqrt{-\sqrt{x}+x}\right)+4\sqrt{-\sqrt{x}+x}\right)}{2\sqrt{\sqrt{x}\,\left(-1+\sqrt{x}\right)}\,\left(-1+\sqrt{x}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-1+x)/(-x^(1/2)+x)^(1/2),x)
```

```
[Out] 1/2*(-x^(1/2)+x)^(1/2)*(2*2^(1/2)*x^(1/2)*arctanh(1/4*(-1+3*x^(1/2))*2^(1/2
)/(-x^(1/2)+x)^(1/2))-2^(1/2)*x*arctanh(1/4*(-1+3*x^(1/2))*2^(1/2)/(-x^(1/2
)+x)^(1/2))-4*(-x^(1/2)+x)^(3/2)-2^(1/2)*arctanh(1/4*(-1+3*x^(1/2))*2^(1/2)
/(-x^(1/2)+x)^(1/2))-8*x^(1/2)*ln(-1/2+x^(1/2)+(-x^(1/2)+x)^(1/2))-8*x^(1/2
)*(-x^(1/2)+x)^(1/2)+4*x*ln(-1/2+x^(1/2)+(-x^(1/2)+x)^(1/2))+4*x*(-x^(1/2)+
x)^(1/2)+4*ln(-1/2+x^(1/2)+(-x^(1/2)+x)^(1/2))+4*(-x^(1/2)+x)^(1/2))/(x^(1/
2)*(-1+x^(1/2)))^(1/2)/(-1+x^(1/2))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{x-\sqrt{x}}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1+x)/(-x^(1/2)+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(sqrt(x - sqrt(x))*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}}{\sqrt{x-\sqrt{x}}\,\left(x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((x - x^(1/2))^(1/2)*(x - 1)),x)
```

```
[Out] int(x^(1/2)/((x - x^(1/2))^(1/2)*(x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{-\sqrt{x}+x}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-1+x)/(-x**(1/2)+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/(sqrt(-sqrt(x) + x)*(x - 1)), x)
```

3.1044 $\int \frac{1}{x^3(1+x^2)^{2/3}}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt[3]{x^2+1}}{2x^2}-\frac{1}{3}\log\left(\sqrt[3]{x^2+1}-1\right)+\frac{1}{6}\log\left(\left(x^2+1\right)^{2/3}+\sqrt[3]{x^2+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 51, 57, 618, 204, 31}

$$-\frac{\sqrt[3]{x^2+1}}{2x^2}-\frac{1}{2}\log\left(1-\sqrt[3]{x^2+1}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x^2)^(2/3)),x]
```

```
[Out] -1/2*(1 + x^2)^(1/3)/x^2 + ArcTan[(1 + 2*(1 + x^2)^(1/3))/Sqrt[3]]/Sqrt[3]
+ Log[x]/3 - Log[1 - (1 + x^2)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(1+x^2\right)^{2/3}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(1+x)^{2/3}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt[3]{1+x^2}}{2x^2}-\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{2/3}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt[3]{1+x^2}}{2x^2}+\frac{\log(x)}{3}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1+x^2}\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,\sqrt[3]{\right.\\
&= -\frac{\sqrt[3]{1+x^2}}{2x^2}+\frac{\log(x)}{3}-\frac{1}{2}\log\left(1-\sqrt[3]{1+x^2}\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{1+x^2}\right)\\
&= -\frac{\sqrt[3]{1+x^2}}{2x^2}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^2}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log(x)}{3}-\frac{1}{2}\log\left(1-\sqrt[3]{1+x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.30

$$\frac{3}{2}\sqrt[3]{x^2+1}\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x^2)^(2/3)),x]
[Out] (3*(1 + x^2)^(1/3)*Hypergeometric2F1[1/3, 2, 4/3, 1 + x^2])/2
```

IntegrateAlgebraic [A] time = 0.09, size = 87, normalized size = 1.00

$$-\frac{\sqrt[3]{x^2+1}}{2x^2}-\frac{1}{3}\log\left(\sqrt[3]{x^2+1}-1\right)+\frac{1}{6}\log\left(\left(x^2+1\right)^{2/3}+\sqrt[3]{x^2+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(1 + x^2)^(2/3)),x]
[Out] -1/2*(1 + x^2)^(1/3)/x^2 + ArcTan[1/Sqrt[3] + (2*(1 + x^2)^(1/3))/Sqrt[3]]/
Sqrt[3] - Log[-1 + (1 + x^2)^(1/3)]/3 + Log[1 + (1 + x^2)^(1/3) + (1 + x^2)
^(2/3)]/6
```

fricas [A] time = 0.41, size = 78, normalized size = 0.90

$$\frac{2\sqrt{3}x^2\arctan\left(\frac{2}{3}\sqrt{3}\left(x^2+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+x^2\log\left(\left(x^2+1\right)^{\frac{2}{3}}+\left(x^2+1\right)^{\frac{1}{3}}+1\right)-2x^2\log\left(\left(x^2+1\right)^{\frac{1}{3}}-1\right)-3\left(x^2+1\right)^{\frac{1}{3}}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+1)^(2/3),x, algorithm="fricas")
[Out] 1/6*(2*sqrt(3)*x^2*arctan(2/3*sqrt(3)*(x^2 + 1)^(1/3) + 1/3*sqrt(3)) + x^2*
log((x^2 + 1)^(2/3) + (x^2 + 1)^(1/3) + 1) - 2*x^2*log((x^2 + 1)^(1/3) - 1)
 - 3*(x^2 + 1)^(1/3))/x^2
```

giac [A] time = 0.29, size = 66, normalized size = 0.76

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^2+1\right)^{\frac{1}{3}}}{2x^2}+\frac{1}{6}\log\left(\left(x^2+1\right)^{\frac{2}{3}}+\left(x^2+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{3}\log\left(\left(x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+1)^(2/3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 + 1)^(1/3) + 1)) - 1/2*(x^2 + 1)^(1/
3)/x^2 + 1/6*log((x^2 + 1)^(2/3) + (x^2 + 1)^(1/3) + 1) - 1/3*log((x^2 + 1)
^(1/3) - 1)
```

maple [C] time = 0.28, size = 59, normalized size = 0.68

$$-\frac{\left(x^2+1\right)^{\frac{1}{3}}}{2x^2}-\frac{-\frac{2\Gamma\left(\frac{2}{3}\right)x^2\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],-x^2\right)}{3}+\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)\right)\Gamma\left(\frac{2}{3}\right)}{3\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^2+1)^(2/3),x)
```

```
[Out] -1/2*(x^2+1)^(1/3)/x^2-1/3/GAMMA(2/3)*(-2/3*GAMMA(2/3)*x^2*hypergeom([1,1,5
/3],[2,2],-x^2)+(1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x))*GAMMA(2/3))
```

maxima [A] time = 0.57, size = 66, normalized size = 0.76

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^2+1\right)^{\frac{1}{3}}}{2x^2}+\frac{1}{6}\log\left(\left(x^2+1\right)^{\frac{2}{3}}+\left(x^2+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{3}\log\left(\left(x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+1)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 + 1)^(1/3) + 1)) - 1/2*(x^2 + 1)^(1/
3)/x^2 + 1/6*log((x^2 + 1)^(2/3) + (x^2 + 1)^(1/3) + 1) - 1/3*log((x^2 + 1)
^(1/3) - 1)
```

mupad [B] time = 0.89, size = 80, normalized size = 0.92

$$-\frac{\ln\left(\left(x^2+1\right)^{1/3}-1\right)}{3}-\frac{\left(x^2+1\right)^{1/3}}{2x^2}-\ln\left(3\left(x^2+1\right)^{1/3}+\frac{3}{2}-\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\ln\left(3\left(x^2+1\right)^{1/3}+\frac{3}{2}+\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^2 + 1)^(2/3)),x)
```

```
[Out] log((3^(1/2)*3i)/2 + 3*(x^2 + 1)^(1/3) + 3/2)*((3^(1/2)*1i)/6 + 1/6) - (x^2
 + 1)^(1/3)/(2*x^2) - log(3*(x^2 + 1)^(1/3) - (3^(1/2)*3i)/2 + 3/2)*((3^(1/
2)*1i)/6 - 1/6) - log((x^2 + 1)^(1/3) - 1)/3
```

sympy [C] time = 0.92, size = 32, normalized size = 0.37

$$-\frac{\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^2}\right)}{2x^{\frac{10}{3}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**2+1)**(2/3),x)
```

```
[Out] -gamma(5/3)*hyper((2/3, 5/3), (8/3,), exp_polar(I*pi)/x**2)/(2*x**(10/3)*ga
mma(8/3))
```

3.1045 $\int \frac{(1+x^2)^{2/3}}{x^3} dx$

Optimal. Leaf size=87

$$-\frac{(x^2+1)^{2/3}}{2x^2}+\frac{1}{3}\log\left(\sqrt[3]{x^2+1}-1\right)-\frac{1}{6}\log\left(\left(x^2+1\right)^{2/3}+\sqrt[3]{x^2+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 47, 55, 618, 204, 31}

$$-\frac{(x^2+1)^{2/3}}{2x^2}+\frac{1}{2}\log\left(1-\sqrt[3]{x^2+1}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^(2/3)/x^3,x]
```

```
[Out] -1/2*(1 + x^2)^(2/3)/x^2 + ArcTan[(1 + 2*(1 + x^2)^(1/3))/Sqrt[3]]/Sqrt[3]
- Log[x]/3 + Log[1 - (1 + x^2)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+x^2\right)^{2/3}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(1+x)^{2/3}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\left(1+x^2\right)^{2/3}}{2x^2} + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx, x, x^2\right)\\
&= -\frac{\left(1+x^2\right)^{2/3}}{2x^2} - \frac{\log(x)}{3} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+x^2}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{\right.\\
&= -\frac{\left(1+x^2\right)^{2/3}}{2x^2} - \frac{\log(x)}{3} + \frac{1}{2}\log\left(1-\sqrt[3]{1+x^2}\right) - \operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+x^2}\right)\\
&= -\frac{\left(1+x^2\right)^{2/3}}{2x^2} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^2}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{3} + \frac{1}{2}\log\left(1-\sqrt[3]{1+x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.30

$$\frac{3}{10}\left(x^2+1\right)^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^(2/3)/x^3,x]
```

```
[Out] (3*(1 + x^2)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 + x^2])/10
```

IntegrateAlgebraic [A] time = 0.08, size = 87, normalized size = 1.00

$$-\frac{\left(x^2+1\right)^{2/3}}{2x^2} + \frac{1}{3}\log\left(\sqrt[3]{x^2+1}-1\right) - \frac{1}{6}\log\left(\left(x^2+1\right)^{2/3}+\sqrt[3]{x^2+1}+1\right) + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)^(2/3)/x^3,x]
```

```
[Out] -1/2*(1 + x^2)^(2/3)/x^2 + ArcTan[1/Sqrt[3] + (2*(1 + x^2)^(1/3))/Sqrt[3]]/
Sqrt[3] + Log[-1 + (1 + x^2)^(1/3)]/3 - Log[1 + (1 + x^2)^(1/3) + (1 + x^2)
^(2/3)]/6
```

fricas [A] time = 0.41, size = 79, normalized size = 0.91

$$\frac{2\sqrt{3}x^2\arctan\left(\frac{2}{3}\sqrt{3}\left(x^2+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right) - x^2\log\left(\left(x^2+1\right)^{\frac{2}{3}}+\left(x^2+1\right)^{\frac{1}{3}}+1\right) + 2x^2\log\left(\left(x^2+1\right)^{\frac{1}{3}}-1\right) - 3\left(x^2+1\right)^{\frac{2}{3}}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*x^2*arctan(2/3*sqrt(3)*(x^2 + 1)^(1/3) + 1/3*sqrt(3)) - x^2*
log((x^2 + 1)^(2/3) + (x^2 + 1)^(1/3) + 1) + 2*x^2*log((x^2 + 1)^(1/3) - 1)
 - 3*(x^2 + 1)^(2/3))/x^2
```

giac [A] time = 0.35, size = 66, normalized size = 0.76

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^2+1\right)^{\frac{2}{3}}}{2x^2}-\frac{1}{6}\log\left(\left(x^2+1\right)^{\frac{2}{3}}+\left(x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 + 1)^(1/3) + 1)) - 1/2*(x^2 + 1)^(2/
3)/x^2 - 1/6*log((x^2 + 1)^(2/3) + (x^2 + 1)^(1/3) + 1) + 1/3*log((x^2 + 1)
^(1/3) - 1)
```

maple [C] time = 0.31, size = 76, normalized size = 0.87

$$-\frac{\left(x^2+1\right)^{\frac{2}{3}}}{2x^2}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^2\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],-x^2\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{6\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(2/3)/x^3,x)
```

```
[Out] -1/2*(x^2+1)^(2/3)/x^2+1/6/Pi*3^(1/2)*GAMMA(2/3)*(-2/9*Pi*3^(1/2)/GAMMA(2/3
)*x^2*hypergeom([1,1,4/3],[2,2],-x^2)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x
))*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.41, size = 66, normalized size = 0.76

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^2+1\right)^{\frac{2}{3}}}{2x^2}-\frac{1}{6}\log\left(\left(x^2+1\right)^{\frac{2}{3}}+\left(x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 + 1)^(1/3) + 1)) - 1/2*(x^2 + 1)^(2/
3)/x^2 - 1/6*log((x^2 + 1)^(2/3) + (x^2 + 1)^(1/3) + 1) + 1/3*log((x^2 + 1)
^(1/3) - 1)
```

mupad [B] time = 0.95, size = 86, normalized size = 0.99

$$\frac{\ln\left(\left(x^2+1\right)^{1/3}-1\right)}{3}+\ln\left(\left(x^2+1\right)^{1/3}-9\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^2\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)-\ln\left(\left(x^2+1\right)^{1/3}-9\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^2\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)-\frac{\left(x^2+1\right)^{2/3}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(2/3)/x^3,x)
```

```
[Out] log((x^2 + 1)^(1/3) - 1)/3 + log((x^2 + 1)^(1/3) - 9*((3^(1/2)*1i)/6 - 1/6)
^2)*((3^(1/2)*1i)/6 - 1/6) - log((x^2 + 1)^(1/3) - 9*((3^(1/2)*1i)/6 + 1/6)
^2)*((3^(1/2)*1i)/6 + 1/6) - (x^2 + 1)^(2/3)/(2*x^2)
```

sympy [C] time = 0.90, size = 34, normalized size = 0.39

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^2}\right)}{2x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(2/3)/x**3,x)
```

```
[Out] -gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_polar(I*pi)/x**2)/(2*x**(2/3)*ga
mma(4/3))
```

3.1046 $\int \frac{(1+x^2)^{2/3}}{x} dx$

Optimal. Leaf size=87

$$\frac{3}{4}\left(x^2+1\right)^{2/3}+\frac{1}{2}\log\left(\sqrt[3]{x^2+1}-1\right)-\frac{1}{4}\log\left(\left(x^2+1\right)^{2/3}+\sqrt[3]{x^2+1}+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 50, 55, 618, 204, 31}

$$\frac{3}{4}\left(x^2+1\right)^{2/3}+\frac{3}{4}\log\left(1-\sqrt[3]{x^2+1}\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^(2/3)/x,x]
```

```
[Out] (3*(1 + x^2)^(2/3))/4 + (Sqrt[3]*ArcTan[(1 + 2*(1 + x^2)^(1/3))/Sqrt[3]])/2
 - Log[x]/2 + (3*Log[1 - (1 + x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(1+x^2\right)^{2/3}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(1+x)^{2/3}}{x}\,dx, x, x^2\right)\\
&= \frac{3}{4}\left(1+x^2\right)^{2/3}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx, x, x^2\right)\\
&= \frac{3}{4}\left(1+x^2\right)^{2/3}-\frac{\log(x)}{2}-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+x^2}\right)+\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1+x^2}\right)\\
&= \frac{3}{4}\left(1+x^2\right)^{2/3}-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1+x^2}\right)-\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+x^2}\right)\\
&= \frac{3}{4}\left(1+x^2\right)^{2/3}+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^2}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1+x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.71

$$\frac{1}{4}\left(3\left(\left(x^2+1\right)^{2/3}+\log\left(1-\sqrt[3]{x^2+1}\right)\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}+1}{\sqrt{3}}\right)-2\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^(2/3)/x,x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2*(1 + x^2)^(1/3))/Sqrt[3]] - 2*Log[x] + 3*((1 + x^2
)^(2/3) + Log[1 - (1 + x^2)^(1/3)]))/4
```

IntegrateAlgebraic [A] time = 0.05, size = 87, normalized size = 1.00

$$\frac{3}{4}\left(x^2+1\right)^{2/3}+\frac{1}{2}\log\left(\sqrt[3]{x^2+1}-1\right)-\frac{1}{4}\log\left(\left(x^2+1\right)^{2/3}+\sqrt[3]{x^2+1}+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)^(2/3)/x,x]
```

```
[Out] (3*(1 + x^2)^(2/3))/4 + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 + x^2)^(1/3))/Sqr
t[3]])/2 + Log[-1 + (1 + x^2)^(1/3)]/2 - Log[1 + (1 + x^2)^(1/3) + (1 + x^2
)^(2/3)]/4
```

fricas [A] time = 0.40, size = 65, normalized size = 0.75

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^2+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{3}{4}\left(x^2+1\right)^{\frac{2}{3}}-\frac{1}{4}\log\left(\left(x^2+1\right)^{\frac{2}{3}}+\left(x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan(2/3*sqrt(3)*(x^2 + 1)^(1/3) + 1/3*sqrt(3)) + 3/4*(x^2 +
1)^(2/3) - 1/4*log((x^2 + 1)^(2/3) + (x^2 + 1)^(1/3) + 1) + 1/2*log((x^2 +
1)^(1/3) - 1)
```

giac [A] time = 0.22, size = 63, normalized size = 0.72

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^2+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{3}{4}\left(x^2+1\right)^{\frac{2}{3}}-\frac{1}{4}\log\left(\left(x^2+1\right)^{\frac{2}{3}}+\left(x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(2/3)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 + 1)^(1/3) + 1)) + 3/4*(x^2 + 1)^(2/
3) - 1/4*log((x^2 + 1)^(2/3) + (x^2 + 1)^(1/3) + 1) + 1/2*log((x^2 + 1)^(1/
3) - 1)
```

maple [C] time = 0.28, size = 64, normalized size = 0.74

$$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^{2}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},1,1\right],[2,2],-x^{2}\right)}{3\Gamma\left(\frac{2}{3}\right)}-\frac{\left(\frac{3}{2}-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)\right)\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}\right)}{6\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(2/3)/x,x)
```

```
[Out] -1/6/Pi*3^(1/2)*GAMMA(2/3)*(-2/3*Pi*3^(1/2)/GAMMA(2/3)*x^2*hypergeom([1/3,1
,1],[2,2],-x^2)-(3/2-1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x))*Pi*3^(1/2)/GAMMA(2/3
))
```

maxima [A] time = 0.41, size = 63, normalized size = 0.72

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^{2}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{3}{4}\left(x^{2}+1\right)^{\frac{2}{3}}-\frac{1}{4}\log\left(\left(x^{2}+1\right)^{\frac{2}{3}}+\left(x^{2}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(x^{2}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 + 1)^(1/3) + 1)) + 3/4*(x^2 + 1)^(2/
3) - 1/4*log((x^2 + 1)^(2/3) + (x^2 + 1)^(1/3) + 1) + 1/2*log((x^2 + 1)^(1/
3) - 1)
```

mupad [B] time = 0.87, size = 89, normalized size = 1.02

$$\frac{\ln\left(\frac{9\left(x^{2}+1\right)^{1/3}}{4}-\frac{9}{4}\right)}{2}+\ln\left(\frac{9\left(x^{2}+1\right)^{1/3}}{4}-9\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)^{2}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-\ln\left(\frac{9\left(x^{2}+1\right)^{1/3}}{4}-9\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)^{2}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)+\frac{3\left(x^{2}+1\right)^{2/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(2/3)/x,x)
```

```
[Out] log((9*(x^2 + 1)^(1/3))/4 - 9/4)/2 + log((9*(x^2 + 1)^(1/3))/4 - 9*((3^(1/2
)*1i)/4 - 1/4)^2)*((3^(1/2)*1i)/4 - 1/4) - log((9*(x^2 + 1)^(1/3))/4 - 9*((
3^(1/2)*1i)/4 + 1/4)^2)*((3^(1/2)*1i)/4 + 1/4) + (3*(x^2 + 1)^(2/3))/4
```

sympy [C] time = 0.81, size = 37, normalized size = 0.43

$$-\frac{x^{\frac{4}{3}}\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\right|\frac{e^{i\pi}}{x^{2}}\right)}{2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(2/3)/x,x)
```

```
[Out] -x**(4/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), exp_polar(I*pi)/x**2)/(2*
gamma(1/3))
```

3.1047 $\int \frac{(3+4x)\sqrt{x+2x^2-2x^4}}{(1+2x)(1+2x+x^3)}\,dx$

Optimal. Leaf size=87

$$2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt{-2x^4+2x^2+x}}{2x^3-2x-1}\right)-2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x\sqrt{-2x^4+2x^2+x}}{2x^3-2x-1}\right)$$

Rubi [F] time = 3.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(3+4x)\sqrt{x+2x^2-2x^4}}{(1+2x)(1+2x+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((3 + 4*x)*Sqrt[x + 2*x^2 - 2*x^4])/((1 + 2*x)*(1 + 2*x + x^3)),x]
```

```
[Out] ((4*I)*Sqrt[x + 2*x^2 - 2*x^4]*Defer[Subst][Defer[Int][Sqrt[1 + 2*x^2 - 2*x
^6]/(I - Sqrt[2]*x), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + 2*x - 2*x^3]) + ((4
*I)*Sqrt[x + 2*x^2 - 2*x^4]*Defer[Subst][Defer[Int][Sqrt[1 + 2*x^2 - 2*x^6]
/(I + Sqrt[2]*x), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + 2*x - 2*x^3]) - (8*Sqr
t[x + 2*x^2 - 2*x^4]*Defer[Subst][Defer[Int][Sqrt[1 + 2*x^2 - 2*x^6]/(1 + 2
*x^2 + x^6), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + 2*x - 2*x^3]) + (6*Sqrt[x +
 2*x^2 - 2*x^4]*Defer[Subst][Defer[Int][(x^2*Sqrt[1 + 2*x^2 - 2*x^6])/(1 +
2*x^2 + x^6), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + 2*x - 2*x^3]) - (4*Sqrt[x
+ 2*x^2 - 2*x^4]*Defer[Subst][Defer[Int][(x^4*Sqrt[1 + 2*x^2 - 2*x^6])/(1 +
 2*x^2 + x^6), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + 2*x - 2*x^3])
```

Rubi steps

$$\int \frac{(3+4x)\sqrt{x+2x^2-2x^4}}{(1+2x)(1+2x+x^3)}\,dx=\frac{\sqrt{x+2x^2-2x^4}\int \frac{\sqrt{x}(3+4x)\sqrt{1+2x-2x^3}}{(1+2x)(1+2x+x^3)}\,dx}{\sqrt{x}\sqrt{1+2x-2x^3}}$$
$$=\frac{\left(2\sqrt{x+2x^2-2x^4}\right)\text{Subst}\left(\int \frac{x^2(3+4x^2)\sqrt{1+2x^2-2x^6}}{(1+2x^2)(1+2x^2+x^6)}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1+2x-2x^3}}$$
$$=\frac{\left(2\sqrt{x+2x^2-2x^4}\right)\text{Subst}\left(\int \left(\frac{4\sqrt{1+2x^2-2x^6}}{1+2x^2}+\frac{(-4+3x^2-2x^4)\sqrt{1+2x^2-2x^6}}{1+2x^2+x^6}\right)dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1+2x-2x^3}}$$
$$=\frac{\left(2\sqrt{x+2x^2-2x^4}\right)\text{Subst}\left(\int \frac{(-4+3x^2-2x^4)\sqrt{1+2x^2-2x^6}}{1+2x^2+x^6}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1+2x-2x^3}}+\frac{\left(8\sqrt{x+2x^2-\right.}{}$$
$$=\frac{\left(2\sqrt{x+2x^2-2x^4}\right)\text{Subst}\left(\int \left(-\frac{4\sqrt{1+2x^2-2x^6}}{1+2x^2+x^6}+\frac{3x^2\sqrt{1+2x^2-2x^6}}{1+2x^2+x^6}-\frac{2x^4\sqrt{1+2x^2-2x^6}}{1+2x^2+x^6}\right)d\right.}{\sqrt{x}\sqrt{1+2x-2x^3}}$$
$$=\frac{\left(4i\sqrt{x+2x^2-2x^4}\right)\text{Subst}\left(\int \frac{\sqrt{1+2x^2-2x^6}}{i-\sqrt{2}\,x}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1+2x-2x^3}}+\frac{\left(4i\sqrt{x+2x^2-2x^4}\right)\text{Sub}}{\sqrt{x}\sqrt{1}}$$

Mathematica [C] time = 6.70, size = 18077, normalized size = 207.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((3 + 4*x)*Sqrt[x + 2*x^2 - 2*x^4])/((1 + 2*x)*(1 + 2*x + x^3)),x
]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.42, size = 87, normalized size = 1.00

$$2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x\sqrt{-2x^4+2x^2+x}}{2x^3-2x-1}\right)-2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x\sqrt{-2x^4+2x^2+x}}{2x^3-2x-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3 + 4*x)*Sqrt[x + 2*x^2 - 2*x^4])/((1 + 2*x)*(1 + 2*x
+ x^3)),x]
```

```
[Out] 2*Sqrt[2]*ArcTan[(Sqrt[2]*x*Sqrt[x + 2*x^2 - 2*x^4])/(-1 - 2*x + 2*x^3)] -
2*Sqrt[3]*ArcTan[(Sqrt[3]*x*Sqrt[x + 2*x^2 - 2*x^4])/(-1 - 2*x + 2*x^3)]
```

fricas [B] time = 0.58, size = 168, normalized size = 1.93

$$\frac{2}{5}\sqrt{2}\arctan\left(\frac{2\sqrt{2}\sqrt{-2x^4+2x^2+x}\left(4x^3-4x^2-x+1\right)}{16x^5-16x^4-12x^3+8x^2+4x-1}\right)-\frac{1}{5}\sqrt{2}\arctan\left(\frac{2\sqrt{2}\sqrt{-2x^4+2x^2+x}\left(4x^2+5x+2\right)}{32x^5+80x^4+84x^3+40x^2+6x-1}\right)-\sqrt{3}\arctan\left(\frac{2\sqrt{3}\sqrt{-2x^4+2x^2+x}\,x}{5x^3-2x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)*(-2*x^4+2*x^2+x)^(1/2)/(1+2*x)/(x^3+2*x+1),x, algorithm="
fricas")
```

```
[Out] 2/5*sqrt(2)*arctan(2*sqrt(2)*sqrt(-2*x^4 + 2*x^2 + x)*(4*x^3 - 4*x^2 - x +
1)/(16*x^5 - 16*x^4 - 12*x^3 + 8*x^2 + 4*x - 1)) - 1/5*sqrt(2)*arctan(2*sqr
t(2)*sqrt(-2*x^4 + 2*x^2 + x)*(4*x^2 + 5*x + 2)/(32*x^5 + 80*x^4 + 84*x^3 +
 40*x^2 + 6*x - 1)) - sqrt(3)*arctan(2*sqrt(3)*sqrt(-2*x^4 + 2*x^2 + x)*x/(
5*x^3 - 2*x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-2x^4+2x^2+x}\,(4x+3)}{\left(x^3+2x+1\right)(2x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)*(-2*x^4+2*x^2+x)^(1/2)/(1+2*x)/(x^3+2*x+1),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-2*x^4 + 2*x^2 + x)*(4*x + 3)/((x^3 + 2*x + 1)*(2*x + 1)), x
)
```

maple [C] time = 4.61, size = 6278, normalized size = 72.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+4*x)*(-2*x^4+2*x^2+x)^(1/2)/(1+2*x)/(x^3+2*x+1),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-2x^4+2x^2+x}\,(4x+3)}{\left(x^3+2x+1\right)(2x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)*(-2*x^4+2*x^2+x)^(1/2)/(1+2*x)/(x^3+2*x+1),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-2*x^4 + 2*x^2 + x)*(4*x + 3)/((x^3 + 2*x + 1)*(2*x + 1)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(4x+3)\sqrt{-2x^4+2x^2+x}}{(2x+1)\left(x^3+2x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((4*x + 3)*(x + 2*x^2 - 2*x^4)^(1/2))/((2*x + 1)*(2*x + x^3 + 1)),x)
```

```
[Out] int(((4*x + 3)*(x + 2*x^2 - 2*x^4)^(1/2))/((2*x + 1)*(2*x + x^3 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)*(-2*x**4+2*x**2+x)**(1/2)/(1+2*x)/(x**3+2*x+1),x)
```

```
[Out] Timed out
```

3.1048 $\int \frac{(1+x^4)^{2/3}}{x} dx$

Optimal. Leaf size=87

$$\frac{3}{8}\left(x^4+1\right)^{2/3}+\frac{1}{4}\log\left(\sqrt[3]{x^4+1}-1\right)-\frac{1}{8}\log\left(\left(x^4+1\right)^{2/3}+\sqrt[3]{x^4+1}+1\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 50, 55, 618, 204, 31}

$$\frac{3}{8}\left(x^4+1\right)^{2/3}+\frac{3}{8}\log\left(1-\sqrt[3]{x^4+1}\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)^(2/3)/x,x]
```

```
[Out] (3*(1 + x^4)^(2/3))/8 + (Sqrt[3]*ArcTan[(1 + 2*(1 + x^4)^(1/3))/Sqrt[3]])/4
 - Log[x]/2 + (3*Log[1 - (1 + x^4)^(1/3)])/8
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(1+x^4\right)^{2/3}}{x}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{(1+x)^{2/3}}{x}\,dx,x,x^4\right)\\
&= \frac{3}{8}\left(1+x^4\right)^{2/3}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx,x,x^4\right)\\
&= \frac{3}{8}\left(1+x^4\right)^{2/3}-\frac{\log(x)}{2}-\frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1+x^4}\right)+\frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,\sqrt[3]{\right.\\
&= \frac{3}{8}\left(1+x^4\right)^{2/3}-\frac{\log(x)}{2}+\frac{3}{8}\log\left(1-\sqrt[3]{1+x^4}\right)-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{1+x^4}\right)\\
&= \frac{3}{8}\left(1+x^4\right)^{2/3}+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^4}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{8}\log\left(1-\sqrt[3]{1+x^4}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.71

$$\frac{1}{8}\left(3\left(\left(x^4+1\right)^{2/3}+\log\left(1-\sqrt[3]{x^4+1}\right)\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}+1}{\sqrt{3}}\right)-4\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)^(2/3)/x,x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2*(1 + x^4)^(1/3))/Sqrt[3]] - 4*Log[x] + 3*((1 + x^4
)^(2/3) + Log[1 - (1 + x^4)^(1/3)]))/8
```

IntegrateAlgebraic [A] time = 0.04, size = 87, normalized size = 1.00

$$\frac{3}{8}\left(x^4+1\right)^{2/3}+\frac{1}{4}\log\left(\sqrt[3]{x^4+1}-1\right)-\frac{1}{8}\log\left(\left(x^4+1\right)^{2/3}+\sqrt[3]{x^4+1}+1\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)^(2/3)/x,x]
```

```
[Out] (3*(1 + x^4)^(2/3))/8 + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 + x^4)^(1/3))/Sqr
t[3]])/4 + Log[-1 + (1 + x^4)^(1/3)]/4 - Log[1 + (1 + x^4)^(1/3) + (1 + x^4
)^(2/3)]/8
```

fricas [A] time = 0.41, size = 65, normalized size = 0.75

$$\frac{1}{4}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^4+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{3}{8}\left(x^4+1\right)^{\frac{2}{3}}-\frac{1}{8}\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{4}\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*arctan(2/3*sqrt(3)*(x^4 + 1)^(1/3) + 1/3*sqrt(3)) + 3/8*(x^4 +
1)^(2/3) - 1/8*log((x^4 + 1)^(2/3) + (x^4 + 1)^(1/3) + 1) + 1/4*log((x^4 +
1)^(1/3) - 1)
```

giac [A] time = 0.26, size = 63, normalized size = 0.72

$$\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{3}{8}\left(x^4+1\right)^{\frac{2}{3}}-\frac{1}{8}\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{4}\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(2/3)/x,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 + 1)^(1/3) + 1)) + 3/8*(x^4 + 1)^(2/
3) - 1/8*log((x^4 + 1)^(2/3) + (x^4 + 1)^(1/3) + 1) + 1/4*log((x^4 + 1)^(1/
3) - 1)
```

maple [C] time = 0.27, size = 64, normalized size = 0.74

$$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^4\,\mathrm{hypergeom}\left(\left[\frac{1}{3},1,1\right],[2,2],-x^4\right)}{3\Gamma\left(\frac{2}{3}\right)}-\frac{\left(\frac{3}{2}-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+4\ln(x)\right)\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}\right)}{12\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(2/3)/x,x)
```

```
[Out] -1/12/Pi*3^(1/2)*GAMMA(2/3)*(-2/3*Pi*3^(1/2)/GAMMA(2/3)*x^4*hypergeom([1/3,
1,1],[2,2],-x^4)-(3/2-1/6*Pi*3^(1/2)-3/2*ln(3)+4*ln(x))*Pi*3^(1/2)/GAMMA(2/
3))
```

maxima [A] time = 0.44, size = 63, normalized size = 0.72

$$\frac{1}{4}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{3}{8}\left(x^4+1\right)^{\frac{2}{3}}-\frac{1}{8}\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{4}\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 + 1)^(1/3) + 1)) + 3/8*(x^4 + 1)^(2/
3) - 1/8*log((x^4 + 1)^(2/3) + (x^4 + 1)^(1/3) + 1) + 1/4*log((x^4 + 1)^(1/
3) - 1)
```

mupad [B] time = 0.82, size = 89, normalized size = 1.02

$$\frac{\ln\left(\frac{9\left(x^4+1\right)^{1/3}}{16}-\frac{9}{16}\right)}{4}+\ln\left(\frac{9\left(x^4+1\right)^{1/3}}{16}-9\left(-\frac{1}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8}\right)^2\right)\left(-\frac{1}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8}\right)-\ln\left(\frac{9\left(x^4+1\right)^{1/3}}{16}-9\left(\frac{1}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8}\right)^2\right)\left(\frac{1}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8}\right)+\frac{3\left(x^4+1\right)^{2/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)^(2/3)/x,x)
```

```
[Out] log((9*(x^4 + 1)^(1/3))/16 - 9/16)/4 + log((9*(x^4 + 1)^(1/3))/16 - 9*((3^(
1/2)*1i)/8 - 1/8)^2)*((3^(1/2)*1i)/8 - 1/8) - log((9*(x^4 + 1)^(1/3))/16 -
9*((3^(1/2)*1i)/8 + 1/8)^2)*((3^(1/2)*1i)/8 + 1/8) + (3*(x^4 + 1)^(2/3))/8
```

sympy [C] time = 0.83, size = 37, normalized size = 0.43

$$-\frac{x^{\frac{8}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\\frac{1}{3}\end{matrix}\right|\frac{e^{i\pi}}{x^4}\right)}{4\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(2/3)/x,x)
```

```
[Out] -x**(8/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), exp_polar(I*pi)/x**4)/(4*
gamma(1/3))
```

3.1049 $\int \frac{\sqrt[3]{-1+x^4}\left(3+x^4\right)}{x^2\left(-1-x^3+x^4\right)}\,dx$

Optimal. Leaf size=87

$$\frac{3\sqrt[3]{x^4-1}}{x}+\log\left(\sqrt[3]{x^4-1}-x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^4-1}+x}\right)-\frac{1}{2}\log\left(\sqrt[3]{x^4-1}\,x+\left(x^4-1\right)^{2/3}+x^2\right)$$

Rubi [F] time = 0.75, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{-1+x^4}\left(3+x^4\right)}{x^2\left(-1-x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^4)^(1/3)*(3 + x^4))/(x^2*(-1 - x^3 + x^4)),x]
```

```
[Out] (3*(-1 + x^4)^(1/3)*Hypergeometric2F1[-1/3, -1/4, 3/4, x^4])/(x*(1 - x^4)^(
1/3)) - 3*Defer[Int][(x*(-1 + x^4)^(1/3))/(-1 - x^3 + x^4), x] + 4*Defer[In
t][(x^2*(-1 + x^4)^(1/3))/(-1 - x^3 + x^4), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{-1+x^4}\left(3+x^4\right)}{x^2\left(-1-x^3+x^4\right)}\,dx &= \int \left(-\frac{3\sqrt[3]{-1+x^4}}{x^2}+\frac{x(-3+4x)\sqrt[3]{-1+x^4}}{-1-x^3+x^4}\right)dx\\
&= -\left(3\int \frac{\sqrt[3]{-1+x^4}}{x^2}\,dx\right)+\int \frac{x(-3+4x)\sqrt[3]{-1+x^4}}{-1-x^3+x^4}\,dx\\
&= -\frac{\left(3\sqrt[3]{-1+x^4}\right)\int \frac{\sqrt[3]{1-x^4}}{x^2}\,dx}{\sqrt[3]{1-x^4}}+\int \left(-\frac{3x\sqrt[3]{-1+x^4}}{-1-x^3+x^4}+\frac{4x^2\sqrt[3]{-1+x^4}}{-1-x^3+x^4}\right)dx\\
&= \frac{3\sqrt[3]{-1+x^4}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{4};\frac{3}{4};x^4\right)}{x\sqrt[3]{1-x^4}}-3\int \frac{x\sqrt[3]{-1+x^4}}{-1-x^3+x^4}\,dx+4\int \frac{x^2\sqrt[3]{-1+x^4}}{-1-x^3+x^4}\,dx
\end{aligned}$$

Mathematica [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-1+x^4}\left(3+x^4\right)}{x^2\left(-1-x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^4)^(1/3)*(3 + x^4))/(x^2*(-1 - x^3 + x^4)),x]
```

```
[Out] Integrate[((-1 + x^4)^(1/3)*(3 + x^4))/(x^2*(-1 - x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.83, size = 87, normalized size = 1.00

$$\frac{3\sqrt[3]{x^4-1}}{x}+\log\left(\sqrt[3]{x^4-1}-x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^4-1}+x}\right)-\frac{1}{2}\log\left(\sqrt[3]{x^4-1}\,x+\left(x^4-1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)^(1/3)*(3 + x^4))/(x^2*(-1 - x^3 + x^4)),x]
```

```
[Out] (3*(-1 + x^4)^(1/3))/x + Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x^4)^(1/3)
)] + Log[-x + (-1 + x^4)^(1/3)] - Log[x^2 + x*(-1 + x^4)^(1/3) + (-1 + x^4)
^(2/3)]/2
```

fricas [A] time = 3.82, size = 132, normalized size = 1.52

$$\frac{2\sqrt{3}\,x\arctan\left(-\frac{14106128635054532\,\sqrt{3}\left(x^4-1\right)^{\frac{1}{3}}x^2-89654043956484782\,\sqrt{3}\left(x^4-1\right)^{\frac{2}{3}}x-\sqrt{3}\left(35416555940707109\,x^4+2357401720008016\,x^3-35416555940707109\right)}{3\left(51678794422160641\,x^4+201291873609016\,x^3-51678794422160641\right)}\right)+x\log\left(\frac{x^4-x^3+3\left(x^4-1\right)^{\frac{1}{3}}x^2-3\left(x^4-1\right)^{\frac{2}{3}}x-1}{x^4-x^3-1}\right)+6\left(x^4-1\right)^{\frac{1}{3}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/3)*(x^4+3)/x^2/(x^4-x^3-1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*x*arctan(-1/3*(14106128635054532*sqrt(3)*(x^4 - 1)^(1/3)*x^2
 - 89654043956484782*sqrt(3)*(x^4 - 1)^(2/3)*x - sqrt(3)*(35416555940707109
*x^4 + 2357401720008016*x^3 - 35416555940707109))/(51678794422160641*x^4 +
201291873609016*x^3 - 51678794422160641)) + x*log((x^4 - x^3 + 3*(x^4 - 1)^
(1/3)*x^2 - 3*(x^4 - 1)^(2/3)*x - 1)/(x^4 - x^3 - 1)) + 6*(x^4 - 1)^(1/3))/
x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\right)\left(x^4-1\right)^{\frac{1}{3}}}{\left(x^4-x^3-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/3)*(x^4+3)/x^2/(x^4-x^3-1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3)*(x^4 - 1)^(1/3)/((x^4 - x^3 - 1)*x^2), x)
```

maple [C] time = 3.03, size = 754, normalized size = 8.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(1/3)*(x^4+3)/x^2/(x^4-x^3-1),x)
```

```
[Out] 3*(x^4-1)^(1/3)/x+(RootOf(_Z^2+_Z+1)*ln(-(RootOf(_Z^2+_Z+1)^2*x^7-RootOf(_Z
^2+_Z+1)*x^8+x^7*RootOf(_Z^2+_Z+1)-x^8-RootOf(_Z^2+_Z+1)*(x^8-2*x^4+1)^(1/3
)*x^5-2*(x^8-2*x^4+1)^(1/3)*x^5-RootOf(_Z^2+_Z+1)^2*x^3-RootOf(_Z^2+_Z+1)*(
x^8-2*x^4+1)^(2/3)*x^2+2*RootOf(_Z^2+_Z+1)*x^4-RootOf(_Z^2+_Z+1)*x^3-2*(x^8
-2*x^4+1)^(2/3)*x^2+2*x^4+RootOf(_Z^2+_Z+1)*(x^8-2*x^4+1)^(1/3)*x+2*(x^8-2*
x^4+1)^(1/3)*x-RootOf(_Z^2+_Z+1)-1)/(x^4-x^3-1)/(-1+x)/(1+x)/(x^2+1))-ln(-(
RootOf(_Z^2+_Z+1)^2*x^7+RootOf(_Z^2+_Z+1)*x^8+x^7*RootOf(_Z^2+_Z+1)+RootOf(
_Z^2+_Z+1)*(x^8-2*x^4+1)^(1/3)*x^5-(x^8-2*x^4+1)^(1/3)*x^5-RootOf(_Z^2+_Z+1
)^2*x^3+RootOf(_Z^2+_Z+1)*(x^8-2*x^4+1)^(2/3)*x^2-2*RootOf(_Z^2+_Z+1)*x^4-R
ootOf(_Z^2+_Z+1)*x^3-(x^8-2*x^4+1)^(2/3)*x^2-RootOf(_Z^2+_Z+1)*(x^8-2*x^4+1
)^(1/3)*x+(x^8-2*x^4+1)^(1/3)*x+RootOf(_Z^2+_Z+1))/(x^4-x^3-1)/(-1+x)/(1+x)
/(x^2+1))*RootOf(_Z^2+_Z+1)-ln(-(RootOf(_Z^2+_Z+1)^2*x^7+RootOf(_Z^2+_Z+1)*
x^8+x^7*RootOf(_Z^2+_Z+1)+RootOf(_Z^2+_Z+1)*(x^8-2*x^4+1)^(1/3)*x^5-(x^8-2*
x^4+1)^(1/3)*x^5-RootOf(_Z^2+_Z+1)^2*x^3+RootOf(_Z^2+_Z+1)*(x^8-2*x^4+1)^(2
/3)*x^2-2*RootOf(_Z^2+_Z+1)*x^4-RootOf(_Z^2+_Z+1)*x^3-(x^8-2*x^4+1)^(2/3)*x
^2-RootOf(_Z^2+_Z+1)*(x^8-2*x^4+1)^(1/3)*x+(x^8-2*x^4+1)^(1/3)*x+RootOf(_Z^
2+_Z+1))/(x^4-x^3-1)/(-1+x)/(1+x)/(x^2+1)))/(x^4-1)^(2/3)*((x^4-1)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\right)\left(x^4-1\right)^{\frac{1}{3}}}{\left(x^4-x^3-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/3)*(x^4+3)/x^2/(x^4-x^3-1),x, algorithm="maxima")
[Out] integrate((x^4 + 3)*(x^4 - 1)^(1/3)/((x^4 - x^3 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^4-1\right)^{1/3}\left(x^4+3\right)}{x^2\left(-x^4+x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^4 - 1)^(1/3)*(x^4 + 3))/(x^2*(x^3 - x^4 + 1)),x)
[Out] int(-((x^4 - 1)^(1/3)*(x^4 + 3))/(x^2*(x^3 - x^4 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(1/3)*(x**4+3)/x**2/(x**4-x**3-1),x)
[Out] Timed out
```

3.1050 $\int \frac{x^2(4+x^3)}{(1+x^3)^{3/4}(1+x^3+x^4)} dx$

Optimal. Leaf size=87

$$-\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{x^3+1}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{x^3+1}}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^3+1}}{\sqrt{x^3+1}+x^2}\right)$$

Rubi [F] time = 1.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2(4+x^3)}{(1+x^3)^{3/4}(1+x^3+x^4)} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(4 + x^3))/((1 + x^3)^(3/4)*(1 + x^3 + x^4)),x]
```

```
[Out] -(x*Hypergeometric2F1[1/3, 3/4, 4/3, -x^3]) + (x^2*Hypergeometric2F1[2/3, 3
/4, 5/3, -x^3])/2 + Defer[Int][1/((1 + x^3)^(3/4)*(1 + x^3 + x^4)), x] - De
fer[Int][x/((1 + x^3)^(3/4)*(1 + x^3 + x^4)), x] + 4*Defer[Int][x^2/((1 + x
^3)^(3/4)*(1 + x^3 + x^4)), x] + Defer[Int][x^3/((1 + x^3)^(3/4)*(1 + x^3 +
 x^4)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(4+x^3)}{(1+x^3)^{3/4}(1+x^3+x^4)} dx &= \int \left(-\frac{1}{(1+x^3)^{3/4}}+\frac{x}{(1+x^3)^{3/4}}+\frac{1-x+4x^2+x^3}{(1+x^3)^{3/4}(1+x^3+x^4)}\right) dx \\
&= -\int \frac{1}{(1+x^3)^{3/4}} dx+\int \frac{x}{(1+x^3)^{3/4}} dx+\int \frac{1-x+4x^2+x^3}{(1+x^3)^{3/4}(1+x^3+x^4)} dx \\
&= -x\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-x^3\right)+\frac{1}{2}x^2\,{}_2F_1\left(\frac{2}{3},\frac{3}{4};\frac{5}{3};-x^3\right)+\int \left(\frac{1}{(1+x^3)^{3/4}(1+x^3+x^4)}\right. \\
&= -x\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-x^3\right)+\frac{1}{2}x^2\,{}_2F_1\left(\frac{2}{3},\frac{3}{4};\frac{5}{3};-x^3\right)+4\int \frac{x^2}{(1+x^3)^{3/4}(1+x^3+x}
\end{aligned}$$

Mathematica [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^2(4+x^3)}{(1+x^3)^{3/4}(1+x^3+x^4)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(4 + x^3))/((1 + x^3)^(3/4)*(1 + x^3 + x^4)),x]
```

```
[Out] Integrate[(x^2*(4 + x^3))/((1 + x^3)^(3/4)*(1 + x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 2.62, size = 87, normalized size = 1.00

$$-\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{x^3+1}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{x^3+1}}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^3+1}}{\sqrt{x^3+1}+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(4 + x^3))/((1 + x^3)^(3/4)*(1 + x^3 + x^4)),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(-(x^2/Sqrt[2]) + Sqrt[1 + x^3]/Sqrt[2])/(x*(1 + x^3)^(1/4
))]) - Sqrt[2]*ArcTanh[(Sqrt[2]*x*(1 + x^3)^(1/4))/(x^2 + Sqrt[1 + x^3])]
```

fricas [B] time = 0.42, size = 189, normalized size = 2.17

$$2\sqrt{2}\arctan\left(\frac{\sqrt{2}x\sqrt{\frac{x^2+\sqrt{2}(x^3+1)^{\frac{1}{4}}x+\sqrt{x^3+1}}{x^2}}-x-\sqrt{2}(x^3+1)^{\frac{1}{4}}}{x}\right)+2\sqrt{2}\arctan\left(\frac{\sqrt{2}x\sqrt{\frac{x^2-\sqrt{2}(x^3+1)^{\frac{1}{4}}x+\sqrt{x^3+1}}{x^2}}+x-\sqrt{2}(x^3+1)^{\frac{1}{4}}}{x}\right)-\frac{1}{2}\sqrt{2}\log\left(\frac{4\left(x^2+\sqrt{2}(x^3+1)^{\frac{1}{4}}x+\sqrt{x^3+1}\right)}{x^2}\right)+\frac{1}{2}\sqrt{2}\log\left(\frac{4\left(x^2-\sqrt{2}(x^3+1)^{\frac{1}{4}}x+\sqrt{x^3+1}\right)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+4)/(x^3+1)^(3/4)/(x^4+x^3+1),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan((sqrt(2)*x*sqrt((x^2 + sqrt(2)*(x^3 + 1)^(1/4)*x + sqrt(x^
3 + 1))/x^2) - x - sqrt(2)*(x^3 + 1)^(1/4))/x) + 2*sqrt(2)*arctan((sqrt(2)*
x*sqrt((x^2 - sqrt(2)*(x^3 + 1)^(1/4)*x + sqrt(x^3 + 1))/x^2) + x - sqrt(2)
*(x^3 + 1)^(1/4))/x) - 1/2*sqrt(2)*log(4*(x^2 + sqrt(2)*(x^3 + 1)^(1/4)*x +
 sqrt(x^3 + 1))/x^2) + 1/2*sqrt(2)*log(4*(x^2 - sqrt(2)*(x^3 + 1)^(1/4)*x +
 sqrt(x^3 + 1))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+4\right)x^2}{\left(x^4+x^3+1\right)\left(x^3+1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+4)/(x^3+1)^(3/4)/(x^4+x^3+1),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 4)*x^2/((x^4 + x^3 + 1)*(x^3 + 1)^(3/4)), x)
```

maple [C] time = 1.62, size = 202, normalized size = 2.32

$$\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(\frac{-2\sqrt{x^3+1}\,\mathrm{RootOf}\left(\_Z^4+1\right)^3x^2-2\left(x^3+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+1\right)^2x^3-\mathrm{RootOf}\left(\_Z^4+1\right)x^4+2\left(x^3+1\right)^{\frac{3}{4}}x+\mathrm{RootOf}\left(\_Z^4+1\right)x^3+\mathrm{RootOf}\left(\_Z^4+1\right)}{x^4+x^3+1}\right)+\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^4+1\right)^3x^4+2\left(x^3+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+1\right)^2x^3+\mathrm{RootOf}\left(\_Z^4+1\right)^3x^3-2\sqrt{x^3+1}\,\mathrm{RootOf}\left(\_Z^4+1\right)x^2+2\left(x^3+1\right)^{\frac{3}{4}}x+\mathrm{RootOf}\left(\_Z^4+1\right)^3}{x^4+x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3+4)/(x^3+1)^(3/4)/(x^4+x^3+1),x)
```

```
[Out] RootOf(_Z^4+1)^3*ln((-2*(x^3+1)^(1/2)*RootOf(_Z^4+1)^3*x^2-2*(x^3+1)^(1/4)*
RootOf(_Z^4+1)^2*x^3-RootOf(_Z^4+1)*x^4+2*(x^3+1)^(3/4)*x+RootOf(_Z^4+1)*x^
3+RootOf(_Z^4+1))/(x^4+x^3+1))+RootOf(_Z^4+1)*ln((-RootOf(_Z^4+1)^3*x^4+2*(
x^3+1)^(1/4)*RootOf(_Z^4+1)^2*x^3+RootOf(_Z^4+1)^3*x^3-2*(x^3+1)^(1/2)*Root
Of(_Z^4+1)*x^2+2*(x^3+1)^(3/4)*x+RootOf(_Z^4+1)^3)/(x^4+x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+4\right)x^2}{\left(x^4+x^3+1\right)\left(x^3+1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+4)/(x^3+1)^(3/4)/(x^4+x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 4)*x^2/((x^4 + x^3 + 1)*(x^3 + 1)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\left(x^3+4\right)}{\left(x^3+1\right)^{3/4}\left(x^4+x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^3 + 4))/((x^3 + 1)^(3/4)*(x^3 + x^4 + 1)),x)
[Out] int((x^2*(x^3 + 4))/((x^3 + 1)^(3/4)*(x^3 + x^4 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3+4)/(x**3+1)**(3/4)/(x**4+x**3+1),x)
[Out] Timed out
```

3.1051 $\int \frac{x^2}{\left(-b+ax^4\right)^{3/4}\left(b+ax^4\right)}\,dx$

Optimal. Leaf size=87

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2\,2^{3/4}a^{3/4}b}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2\,2^{3/4}a^{3/4}b}$$

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {494, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2\,2^{3/4}a^{3/4}b}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2\,2^{3/4}a^{3/4}b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-b + a*x^4)^(3/4)*(b + a*x^4)),x]
```

```
[Out] -1/2*ArcTan[(2^(1/4)*a^(1/4)*x)/(-b + a*x^4)^(1/4)]/(2^(3/4)*a^(3/4)*b) + A
rcTanh[(2^(1/4)*a^(1/4)*x)/(-b + a*x^4)^(1/4)]/(2*2^(3/4)*a^(3/4)*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(-b+ax^4\right)^{3/4}\left(b+ax^4\right)}\,dx = \text{Subst}\left(\int \frac{x^2}{b-2abx^4}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{2\sqrt{2}\sqrt{a}\,b} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{2\sqrt{2}\sqrt{a}\,b}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{2\,2^{3/4}a^{3/4}b} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{2\,2^{3/4}a^{3/4}b}$$

Mathematica [C] time = 0.04, size = 77, normalized size = 0.89

$$\frac{x^3\left(1-\frac{ax^4}{b}\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{2ax^4}{ax^4+b}\right)}{3b\left(ax^4-b\right)^{3/4}\left(\frac{ax^4}{b}+1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-b + a*x^4)^(3/4)*(b + a*x^4)),x]
```

```
[Out] (x^3*(1 - (a*x^4)/b)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (2*a*x^4)/(b +
a*x^4)])/(3*b*(-b + a*x^4)^(3/4)*(1 + (a*x^4)/b)^(3/4))
```

IntegrateAlgebraic [A] time = 0.53, size = 87, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2\,2^{3/4}a^{3/4}b} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2\,2^{3/4}a^{3/4}b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-b + a*x^4)^(3/4)*(b + a*x^4)),x]
```

```
[Out] -1/2*ArcTan[(2^(1/4)*a^(1/4)*x)/(-b + a*x^4)^(1/4)]/(2^(3/4)*a^(3/4)*b) + A
rcTanh[(2^(1/4)*a^(1/4)*x)/(-b + a*x^4)^(1/4)]/(2*2^(3/4)*a^(3/4)*b)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4-b)^(3/4)/(a*x^4+b),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax^4+b\right)\left(ax^4-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4-b)^(3/4)/(a*x^4+b),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a*x^4 + b)*(a*x^4 - b)^(3/4)), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(a\,x^{4}-b\right)^{\frac{3}{4}} \left(a\,x^{4}+b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^4-b)^(3/4)/(a*x^4+b),x)
[Out] int(x^2/(a*x^4-b)^(3/4)/(a*x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(ax^{4}+b\right)\left(ax^{4}-b\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4-b)^(3/4)/(a*x^4+b),x, algorithm="maxima")
[Out] integrate(x^2/((a*x^4 + b)*(a*x^4 - b)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2}}{\left(a\,x^{4}+b\right)\,\left(a\,x^{4}-b\right)^{3/4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b + a*x^4)*(a*x^4 - b)^(3/4)),x)
[Out] int(x^2/((b + a*x^4)*(a*x^4 - b)^(3/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(ax^{4}-b\right)^{\frac{3}{4}} \left(ax^{4}+b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**4-b)**(3/4)/(a*x**4+b),x)
[Out] Integral(x**2/((a*x**4 - b)**(3/4)*(a*x**4 + b)), x)
```

3.1052 $\int \frac{1}{\sqrt[4]{-b+ax^4}\left(b+ax^4\right)}\,dx$

Optimal. Leaf size=87

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{a}\,b}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{a}\,b}$$

Rubi [A] time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {377, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{a}\,b}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{a}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-b + a*x^4)^(1/4)*(b + a*x^4)),x]
```

```
[Out] ArcTan[(2^(1/4)*a^(1/4)*x)/(-b + a*x^4)^(1/4)]/(2*2^(1/4)*a^(1/4)*b) + ArcT
anh[(2^(1/4)*a^(1/4)*x)/(-b + a*x^4)^(1/4)]/(2*2^(1/4)*a^(1/4)*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{-b+ax^4}\left(b+ax^4\right)}\,dx = \text{Subst}\left(\int \frac{1}{b-2abx^4}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\sqrt{a}x^2}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\sqrt{a}x^2}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{2b}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{-b+ax^4}}\right)}{2\sqrt[4]{2}\sqrt[4]{a}b} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{-b+ax^4}}\right)}{2\sqrt[4]{2}\sqrt[4]{a}b}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.80

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{a}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-b + a*x^4)^(1/4)*(b + a*x^4)),x]
```

```
[Out] (ArcTan[(2^(1/4)*a^(1/4)*x)/(-b + a*x^4)^(1/4)] + ArcTanh[(2^(1/4)*a^(1/4)*
x)/(-b + a*x^4)^(1/4)])/(2*2^(1/4)*a^(1/4)*b)
```

IntegrateAlgebraic [A] time = 0.33, size = 87, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{a}b}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{a}b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-b + a*x^4)^(1/4)*(b + a*x^4)),x]
```

```
[Out] ArcTan[(2^(1/4)*a^(1/4)*x)/(-b + a*x^4)^(1/4)]/(2*2^(1/4)*a^(1/4)*b) + ArcT
anh[(2^(1/4)*a^(1/4)*x)/(-b + a*x^4)^(1/4)]/(2*2^(1/4)*a^(1/4)*b)
```

fricas [B] time = 172.99, size = 431, normalized size = 4.95

$$-\frac{1}{2}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(ax^4-b\right)^{\frac{3}{4}}ab^3x\left(\frac{1}{ab^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(ax^4-b\right)^{\frac{1}{4}}abx^3\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}+\left(2\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{ax^4-b}\,abx^2\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}+\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(3a^2b^3x^4-ab^4\right)\left(\frac{1}{ab^4}\right)^{\frac{3}{4}}\right)\sqrt{\sqrt{\frac{1}{2}}b^2\sqrt{\frac{1}{ab^4}}}\right)}{ax^4+b}\right)+\frac{1}{8}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{ax^4-b}\,ab^3x^2\left(\frac{1}{ab^4}\right)^{\frac{3}{4}}+4\sqrt{\frac{1}{2}}\left(ax^4-b\right)^{\frac{1}{4}}ab^2x^3\sqrt{\frac{1}{ab^4}}+2\left(ax^4-b\right)^{\frac{3}{4}}x+\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(3abx^4-b^2\right)\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}}{2\left(ax^4+b\right)}\right)-\frac{1}{8}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{ax^4-b}\,ab^3x^2\left(\frac{1}{ab^4}\right)^{\frac{3}{4}}-4\sqrt{\frac{1}{2}}\left(ax^4-b\right)^{\frac{1}{4}}ab^2x^3\sqrt{\frac{1}{ab^4}}-2\left(ax^4-b\right)^{\frac{3}{4}}x+\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(3abx^4-b^2\right)\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}}{2\left(ax^4+b\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4-b)^(1/4)/(a*x^4+b),x, algorithm="fricas")
```

```
[Out] -1/2*(1/2)^(1/4)*(1/(a*b^4))^(1/4)*arctan(2*(2*(1/2)^(3/4)*(a*x^4 - b)^(3/4
)*a*b^3*x*(1/(a*b^4))^(3/4) + 2*(1/2)^(1/4)*(a*x^4 - b)^(1/4)*a*b*x^3*(1/(a
*b^4))^(1/4) + (2*(1/2)^(1/4)*sqrt(a*x^4 - b)*a*b*x^2*(1/(a*b^4))^(1/4) + (
1/2)^(3/4)*(3*a^2*b^3*x^4 - a*b^4)*(1/(a*b^4))^(3/4))*sqrt(sqrt(1/2)*b^2*sq
rt(1/(a*b^4))))/(a*x^4 + b)) + 1/8*(1/2)^(1/4)*(1/(a*b^4))^(1/4)*log(1/2*(4
*(1/2)^(3/4)*sqrt(a*x^4 - b)*a*b^3*x^2*(1/(a*b^4))^(3/4) + 4*sqrt(1/2)*(a*x
^4 - b)^(1/4)*a*b^2*x^3*sqrt(1/(a*b^4)) + 2*(a*x^4 - b)^(3/4)*x + (1/2)^(1/
4)*(3*a*b*x^4 - b^2)*(1/(a*b^4))^(1/4))/(a*x^4 + b)) - 1/8*(1/2)^(1/4)*(1/(
a*b^4))^(1/4)*log(-1/2*(4*(1/2)^(3/4)*sqrt(a*x^4 - b)*a*b^3*x^2*(1/(a*b^4))
^(3/4) - 4*sqrt(1/2)*(a*x^4 - b)^(1/4)*a*b^2*x^3*sqrt(1/(a*b^4)) - 2*(a*x^4
 - b)^(3/4)*x + (1/2)^(1/4)*(3*a*b*x^4 - b^2)*(1/(a*b^4))^(1/4))/(a*x^4 + b
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^4+b)(ax^4-b)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4-b)^(1/4)/(a*x^4+b),x, algorithm="giac")
[Out] integrate(1/((a*x^4 + b)*(a*x^4 - b)^(1/4)), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\,x^4-b)^{\frac{1}{4}}\,(a\,x^4+b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^4-b)^(1/4)/(a*x^4+b),x)
[Out] int(1/(a*x^4-b)^(1/4)/(a*x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^4+b)(ax^4-b)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4-b)^(1/4)/(a*x^4+b),x, algorithm="maxima")
[Out] integrate(1/((a*x^4 + b)*(a*x^4 - b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a\,x^4+b)\,(a\,x^4-b)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b + a*x^4)*(a*x^4 - b)^(1/4)),x)
[Out] int(1/((b + a*x^4)*(a*x^4 - b)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ax^4-b}\,(ax^4+b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**4-b)**(1/4)/(a*x**4+b),x)
[Out] Integral(1/((a*x**4 - b)**(1/4)*(a*x**4 + b)), x)
```

3.1053 $$\int \frac{-a^2b+a(2a+b)x-3ax^2+x^3}{\sqrt{x(-a+x)(-b+x)}\left(-a^2+2ax+\left(-1+b^2d\right)x^2-2bdx^3+dx^4\right)}\,dx$$

Optimal. Leaf size=87

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}{a-x}\right)}{\sqrt[4]{d}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}{a-x}\right)}{\sqrt[4]{d}}$$

Rubi [F] time = 9.95, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-a^2b+a(2a+b)x-3ax^2+x^3}{\sqrt{x(-a+x)(-b+x)}\left(-a^2+2ax+\left(-1+b^2d\right)x^2-2bdx^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a^2*b) + a*(2*a + b)*x - 3*a*x^2 + x^3)/(Sqrt[x*(-a + x)*(-b + x)]*(
-a^2 + 2*a*x + (-1 + b^2*d)*x^2 - 2*b*d*x^3 + d*x^4)),x]
```

```
[Out] (4*a*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][(x^2*Sqrt[-a
 + x^2])/(Sqrt[-b + x^2]*(a^2 - 2*a*x^2 + (1 - b^2*d)*x^4 + 2*b*d*x^6 - d*x
^8)), x], x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x] + (2*a*b*Sqrt[x]*Sqrt[-a + x
]*Sqrt[-b + x]*Defer[Subst][Defer[Int][Sqrt[-a + x^2]/(Sqrt[-b + x^2]*(-a^2
 + 2*a*x^2 - (1 - b^2*d)*x^4 - 2*b*d*x^6 + d*x^8)), x], x, Sqrt[x]])/Sqrt[(
a - x)*(b - x)*x] + (2*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer
[Int][(x^4*Sqrt[-a + x^2])/(Sqrt[-b + x^2]*(-a^2 + 2*a*x^2 - (1 - b^2*d)*x^
4 - 2*b*d*x^6 + d*x^8)), x], x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x]
```

Rubi steps

$$\int \frac{-a^2b+a(2a+b)x-3ax^2+x^3}{\sqrt{x(-a+x)(-b+x)}\left(-a^2+2ax+\left(-1+b^2d\right)x^2-2bdx^3+dx^4\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{-}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+}}}{\sqrt{x(-a+x)(-}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{\sqrt{-}}{\sqrt{x}\sqrt{-b+x}\left(-a^2+\right.}}{\sqrt{x(-a+x)(-b+}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt{-b-}}\right.}{\sqrt{x(-a+}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}\left(\int \left(\frac{}{\sqrt{-}}\right.\right.}{}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt{-b-}}\right.}{\sqrt{x(-a+}}$$

Mathematica [C] time = 8.08, size = 5341, normalized size = 61.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-(a^2*b) + a*(2*a + b)*x - 3*a*x^2 + x^3)/(Sqrt[x*(-a + x)*(-b +
 x)]*(-a^2 + 2*a*x + (-1 + b^2*d)*x^2 - 2*b*d*x^3 + d*x^4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 2.06, size = 87, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}{a-x}\right)}{\sqrt[4]{d}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}{a-x}\right)}{\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a^2*b) + a*(2*a + b)*x - 3*a*x^2 + x^3)/(Sqrt[x*(-a +
 x)*(-b + x)]*(-a^2 + 2*a*x + (-1 + b^2*d)*x^2 - 2*b*d*x^3 + d*x^4)),x]
```

```
[Out] ArcTan[(d^(1/4)*Sqrt[a*b*x + (-a - b)*x^2 + x^3])/(a - x)]/d^(1/4) + ArcTan
h[(d^(1/4)*Sqrt[a*b*x + (-a - b)*x^2 + x^3])/(a - x)]/d^(1/4)
```

fricas [B] time = 1.59, size = 340, normalized size = 3.91

$$\frac{\arctan\left(-\frac{\sqrt{abx-(a+b)x^2+x^3}}{(bx-x^2)d^{\frac{1}{4}}}\right)}{d^{\frac{1}{4}}}-\frac{\log\left(\frac{2\,bdx^3-dx^4-(b^2d+1)x^2-a^2+2\,ax+2\,\sqrt{abx-(a+b)x^2+x^3}\left(\frac{bdx-dx^2}{d^{\frac{1}{4}}}+\frac{ad-dx}{d^{\frac{3}{4}}}\right)-\frac{2\left(abdx-(a+b)dx^2+dx^3\right)}{\sqrt{d}}}{2\,bdx^3-dx^4-(b^2d-1)x^2+a^2-2\,ax}\right)}{4\,d^{\frac{1}{4}}}+\frac{\log\left(\frac{2\,bdx^3-dx^4-(b^2d+1)x^2-a^2+2\,ax-2\,\sqrt{abx-(a+b)x^2+x^3}\left(\frac{bdx-dx^2}{d^{\frac{1}{4}}}+\frac{ad-dx}{d^{\frac{3}{4}}}\right)-\frac{2\left(abdx-(a+b)dx^2+dx^3\right)}{\sqrt{d}}}{2\,bdx^3-dx^4-(b^2d-1)x^2+a^2-2\,ax}\right)}{4\,d^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*b+a*(2*a+b)*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a^2+2*
a*x+(b^2*d-1)*x^2-2*b*d*x^3+d*x^4),x, algorithm="fricas")
```

```
[Out] arctan(-sqrt(a*b*x - (a + b)*x^2 + x^3)/((b*x - x^2)*d^(1/4)))/d^(1/4) - 1/
4*log((2*b*d*x^3 - d*x^4 - (b^2*d + 1)*x^2 - a^2 + 2*a*x + 2*sqrt(a*b*x - (
a + b)*x^2 + x^3)*((b*d*x - d*x^2)/d^(1/4) + (a*d - d*x)/d^(3/4)) - 2*(a*b*
d*x - (a + b)*d*x^2 + d*x^3)/sqrt(d))/(2*b*d*x^3 - d*x^4 - (b^2*d - 1)*x^2
+ a^2 - 2*a*x))/d^(1/4) + 1/4*log((2*b*d*x^3 - d*x^4 - (b^2*d + 1)*x^2 - a^
2 + 2*a*x - 2*sqrt(a*b*x - (a + b)*x^2 + x^3)*((b*d*x - d*x^2)/d^(1/4) + (a
*d - d*x)/d^(3/4)) - 2*(a*b*d*x - (a + b)*d*x^2 + d*x^3)/sqrt(d))/(2*b*d*x^
3 - d*x^4 - (b^2*d - 1)*x^2 + a^2 - 2*a*x))/d^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2b-(2\,a+b)ax+3\,ax^2-x^3}{\left(2\,bdx^3-dx^4-\left(b^2d-1\right)x^2+a^2-2\,ax\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*b+a*(2*a+b)*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a^2+2*
a*x+(b^2*d-1)*x^2-2*b*d*x^3+d*x^4),x, algorithm="giac")
```

```
[Out] integrate((a^2*b - (2*a + b)*a*x + 3*a*x^2 - x^3)/((2*b*d*x^3 - d*x^4 - (b^
2*d - 1)*x^2 + a^2 - 2*a*x)*sqrt((a - x)*(b - x)*x)), x)
```

maple [C] time = 0.06, size = 330, normalized size = 3.79

$$-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^4-2bd\_Z^3+\left(b^2d-1\right)\_Z^2+2a\_Z-a^2\right)}\frac{\left(-\_\alpha^3+3\_\alpha^2a-2\_\alpha a^2-\_\alpha ab+a^2b\right)\left(\_\alpha^3d+\_\alpha^2ad-2\_\alpha^2bd+\_\alpha a^2d-2\_\alpha bad+\_\alpha b^2d+a^3d-2a^2bd+ab^2d-\_\alpha+a\right)\sqrt{-\frac{-a+x}{a}}\sqrt{\frac{-b+x}{a-b}}\sqrt{\frac{x}{a}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-a+x}{a}},\frac{\_\alpha^3d+\_\alpha^2ad-2\_\alpha^2bd+\_\alpha a^2d-2\_\alpha bad+\_\alpha b^2d+a^3d-2a^2bd+ab^2d-\_\alpha+a}{ad\left(a^2-2ab+b^2\right)},\sqrt{\frac{a}{a-b}}\right)}{\left(-2\_\alpha^3d+3\_\alpha^2bd-\_\alpha b^2d+\_\alpha-a\right)\left(a^2-2ab+b^2\right)\sqrt{x\left(ab-ax-bx+x^2\right)}}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*b+a*(2*a+b)*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a^2+2*a*x+(b
^2*d-1)*x^2-2*b*d*x^3+d*x^4),x)
```

```
[Out] -1/a/d*sum((-_alpha^3+3*_alpha^2*a-2*_alpha*a^2-_alpha*a*b+a^2*b)/(-2*_alph
a^3*d+3*_alpha^2*b*d-_alpha*b^2*d+_alpha-a)*(_alpha^3*d+_alpha^2*a*d-2*_alp
ha^2*b*d+_alpha*a^2*d-2*_alpha*a*b*d+_alpha*b^2*d+a^3*d-2*a^2*b*d+a*b^2*d-_
alpha+a)/(a^2-2*a*b+b^2)*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/
2)/(x*(a*b-a*x-b*x+x^2))^(1/2)*EllipticPi((-(-a+x)/a)^(1/2),(_alpha^3*d+_al
pha^2*a*d-2*_alpha^2*b*d+_alpha*a^2*d-2*_alpha*a*b*d+_alpha*b^2*d+a^3*d-2*a
^2*b*d+a*b^2*d-_alpha+a)/a/d/(a^2-2*a*b+b^2),(a/(a-b))^(1/2)),_alpha=RootOf
(d*_Z^4-2*b*d*_Z^3+(b^2*d-1)*_Z^2+2*a*_Z-a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2b - (2\,a + b)ax + 3\,ax^2 - x^3}{\left(2\,bdx^3 - dx^4 - \left(b^2d - 1\right)x^2 + a^2 - 2\,ax\right)\sqrt{(a - x)(b - x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*b+a*(2*a+b)*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a^2+2*
a*x+(b^2*d-1)*x^2-2*b*d*x^3+d*x^4),x, algorithm="maxima")
```

```
[Out] integrate((a^2*b - (2*a + b)*a*x + 3*a*x^2 - x^3)/((2*b*d*x^3 - d*x^4 - (b^
2*d - 1)*x^2 + a^2 - 2*a*x)*sqrt((a - x)*(b - x)*x)), x)
```

mupad [B] time = 7.32, size = 135, normalized size = 1.55

$$\frac{\ln\left(\frac{a-x+2\,d^{1/4}\,\sqrt{x\,(a-x)\,(b-x)}-\sqrt{d}\,x^2+b\,\sqrt{d}\,x}{a-x+\sqrt{d}\,x^2-b\,\sqrt{d}\,x}\right)}{2\,d^{1/4}}+\frac{\ln\left(\frac{x-a-\sqrt{d}\,x^2+b\,\sqrt{d}\,x+d^{1/4}\,\sqrt{x\,(a-x)\,(b-x)}\,2{}\mathrm{i}}{a-x-\sqrt{d}\,x^2+b\,\sqrt{d}\,x}\right)\,1{}\mathrm{i}}{2\,d^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^2*b + 3*a*x^2 - x^3 - a*x*(2*a + b))/((x*(a - x)*(b - x))^(1/2)*(x^
2*(b^2*d - 1) + 2*a*x + d*x^4 - a^2 - 2*b*d*x^3)),x)
```

```
[Out] log((a - x + 2*d^(1/4)*(x*(a - x)*(b - x))^(1/2) - d^(1/2)*x^2 + b*d^(1/2)*
x)/(a - x + d^(1/2)*x^2 - b*d^(1/2)*x))/(2*d^(1/4)) + (log((x - a + d^(1/4)
*(x*(a - x)*(b - x))^(1/2)*2i - d^(1/2)*x^2 + b*d^(1/2)*x)/(a - x - d^(1/2)
*x^2 + b*d^(1/2)*x))*1i)/(2*d^(1/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*b+a*(2*a+b)*x-3*a*x**2+x**3)/(x*(-a+x)*(-b+x))**(1/2)/(-a*
*2+2*a*x+(b**2*d-1)*x**2-2*b*d*x**3+d*x**4),x)
```

```
[Out] Timed out
```

3.1054 $\int \frac{1+k^2x^4}{\sqrt{(1-x^2)(1-k^2x^2)}\,(-1+k^2x^4)}\,dx$

Optimal. Leaf size=87

$$\frac{\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}+kx^2+1}\right)}{-k-1}-\frac{\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}}\right)}{2(k-1)}$$

Rubi [C] time = 1.46, antiderivative size = 169, normalized size of antiderivative = 1.94, number of steps used = 8, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {6719, 6725, 419, 537}

$$\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}-\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi\left(-k;\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}-\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi\left(k;\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + k^2*x^4)/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(-1 + k^2*x^4)),x]
```

```
[Out] (Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticF[ArcSin[x], k^2])/Sqrt[(1 - x^2)*
(1 - k^2*x^2)] - (Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[-k, ArcSin[x],
 k^2])/Sqrt[(1 - x^2)*(1 - k^2*x^2)] - (Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*Ell
ipticPi[k, ArcSin[x], k^2])/Sqrt[(1 - x^2)*(1 - k^2*x^2)]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+k^2x^4}{\sqrt{(1-x^2)(1-k^2x^2)}\,(-1+k^2x^4)}\,dx = \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1+k^2x^4}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,(-1+k^2x^4)}\,dx}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

$$= \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \left(\frac{1}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}} + \frac{2}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,(-1+k^2x^4)}\right)dx}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

$$= \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}}\,dx}{\sqrt{(1-x^2)(1-k^2x^2)}} + \frac{\left(2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \ldots}{\sqrt{(1-x^2)(1\ldots}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}} + \frac{\left(2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \left(-\frac{\ldots}{2\sqrt{1-x\ldots}}\ldots\right.}{\sqrt{(1-\ldots}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}} - \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1}{\sqrt{1-x^2}\,(1-k\ldots}}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}} - \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(-k;\sin^{-1}(x\ldots\right.}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

Mathematica [C] time = 0.52, size = 72, normalized size = 0.83

$$\frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\left(F\left(\sin^{-1}(x)|k^2\right) - \Pi\left(-k;\sin^{-1}(x)|k^2\right) - \Pi\left(k;\sin^{-1}(x)|k^2\right)\right)}{\sqrt{(x^2-1)(k^2x^2-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + k^2*x^4)/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(-1 + k^2*x^4)),x]
```

```
[Out] (Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*(EllipticF[ArcSin[x], k^2] - EllipticPi[-k
, ArcSin[x], k^2] - EllipticPi[k, ArcSin[x], k^2]))/Sqrt[(-1 + x^2)*(-1 + k
^2*x^2)]
```

IntegrateAlgebraic [A] time = 1.84, size = 87, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}+kx^2+1}\right)}{-k-1} - \frac{\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}}\right)}{2(k-1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + k^2*x^4)/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(-1 + k^2*x
^4)),x]
```

```
[Out] -1/2*ArcTan[((-1 + k)*x)/Sqrt[1 + (-1 - k^2)*x^2 + k^2*x^4]]/(-1 + k) + Arc
Tan[((1 + k)*x)/(1 + k*x^2 + Sqrt[1 + (-1 - k^2)*x^2 + k^2*x^4])]/(-1 - k)
```

fricas [A] time = 0.52, size = 80, normalized size = 0.92

$$\frac{(k-1)\arctan\left(\frac{\sqrt{k^2x^4-(k^2+1)x^2+1}}{(k+1)x}\right) + (k+1)\arctan\left(\frac{\sqrt{k^2x^4-(k^2+1)x^2+1}}{(k-1)x}\right)}{2\,(k^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^4+1)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^4-1),x, algorith
m="fricas")
```

```
[Out] 1/2*((k - 1)*arctan(sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)/((k + 1)*x)) + (k + 1
)*arctan(sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)/((k - 1)*x)))/(k^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^4+1}{\left(k^2x^4-1\right)\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^4+1)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^4-1),x, algorith
m="giac")
```

```
[Out] integrate((k^2*x^4 + 1)/((k^2*x^4 - 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))), x)
```

maple [C] time = 0.04, size = 155, normalized size = 1.78

$$\frac{\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\operatorname{EllipticF}(x,k)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}-\frac{\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\operatorname{EllipticPi}(x,k,k)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}-\frac{\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\operatorname{EllipticPi}(x,-k,k)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^4+1)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^4-1),x)
```

```
[Out] (-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*EllipticF(x
,k)-(-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*Ellipti
cPi(x,k,k)-(-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*
EllipticPi(x,-k,k)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^4+1}{\left(k^2x^4-1\right)\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^4+1)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^4-1),x, algorith
m="maxima")
```

```
[Out] integrate((k^2*x^4 + 1)/((k^2*x^4 - 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{k^2\,x^4+1}{\left(k^2\,x^4-1\right)\,\sqrt{\left(x^2-1\right)\,\left(k^2\,x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^4 + 1)/((k^2*x^4 - 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)),x)
```

```
[Out] int((k^2*x^4 + 1)/((k^2*x^4 - 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^4+1}{\sqrt{(x-1)(x+1)(kx-1)(kx+1)}\,\left(kx^2-1\right)\left(kx^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k**2*x**4+1)/((-x**2+1)*(-k**2*x**2+1))**(1/2)/(k**2*x**4-1),x)
```

```
[Out] Integral((k**2*x**4 + 1)/(sqrt((x - 1)*(x + 1)*(k*x - 1)*(k*x + 1))*(k*x**2
 - 1)*(k*x**2 + 1)), x)
```

3.1055 $\int \frac{(1+x^5)^{2/3}}{x}\,dx$

Optimal. Leaf size=87

$$\frac{3}{10}\left(x^5+1\right)^{2/3}+\frac{1}{5}\log\left(\sqrt[3]{x^5+1}-1\right)-\frac{1}{10}\log\left(\left(x^5+1\right)^{2/3}+\sqrt[3]{x^5+1}+1\right)+\frac{1}{5}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^5+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 50, 55, 618, 204, 31}

$$\frac{3}{10}\left(x^5+1\right)^{2/3}+\frac{3}{10}\log\left(1-\sqrt[3]{x^5+1}\right)+\frac{1}{5}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^5+1}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^5)^(2/3)/x,x]
```

```
[Out] (3*(1 + x^5)^(2/3))/10 + (Sqrt[3]*ArcTan[(1 + 2*(1 + x^5)^(1/3))/Sqrt[3]])/
5 - Log[x]/2 + (3*Log[1 - (1 + x^5)^(1/3)])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+x^5\right)^{2/3}}{x}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{(1+x)^{2/3}}{x}\,dx,x,x^5\right)\\
&= \frac{3}{10}\left(1+x^5\right)^{2/3}+\frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx,x,x^5\right)\\
&= \frac{3}{10}\left(1+x^5\right)^{2/3}-\frac{\log(x)}{2}-\frac{3}{10}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1+x^5}\right)+\frac{3}{10}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x\right.\\
&= \frac{3}{10}\left(1+x^5\right)^{2/3}-\frac{\log(x)}{2}+\frac{3}{10}\log\left(1-\sqrt[3]{1+x^5}\right)-\frac{3}{5}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{1+x^5}\right)\\
&= \frac{3}{10}\left(1+x^5\right)^{2/3}+\frac{1}{5}\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^5}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{10}\log\left(1-\sqrt[3]{1+x^5}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.71

$$\frac{1}{10}\left(3\left(\left(x^5+1\right)^{2/3}+\log\left(1-\sqrt[3]{x^5+1}\right)\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^5+1}+1}{\sqrt{3}}\right)-5\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^5)^(2/3)/x,x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2*(1 + x^5)^(1/3))/Sqrt[3]] - 5*Log[x] + 3*((1 + x^5
)^(2/3) + Log[1 - (1 + x^5)^(1/3)]))/10
```

IntegrateAlgebraic [A] time = 0.05, size = 87, normalized size = 1.00

$$\frac{3}{10}\left(x^5+1\right)^{2/3}+\frac{1}{5}\log\left(\sqrt[3]{x^5+1}-1\right)-\frac{1}{10}\log\left(\left(x^5+1\right)^{2/3}+\sqrt[3]{x^5+1}+1\right)+\frac{1}{5}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^5+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^5)^(2/3)/x,x]
```

```
[Out] (3*(1 + x^5)^(2/3))/10 + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 + x^5)^(1/3))/Sq
rt[3]])/5 + Log[-1 + (1 + x^5)^(1/3)]/5 - Log[1 + (1 + x^5)^(1/3) + (1 + x^
5)^(2/3)]/10
```

fricas [A] time = 0.41, size = 65, normalized size = 0.75

$$\frac{1}{5}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^5+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{3}{10}\left(x^5+1\right)^{\frac{2}{3}}-\frac{1}{10}\log\left(\left(x^5+1\right)^{\frac{2}{3}}+\left(x^5+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{5}\log\left(\left(x^5+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(3)*arctan(2/3*sqrt(3)*(x^5 + 1)^(1/3) + 1/3*sqrt(3)) + 3/10*(x^5 +
 1)^(2/3) - 1/10*log((x^5 + 1)^(2/3) + (x^5 + 1)^(1/3) + 1) + 1/5*log((x^5
+ 1)^(1/3) - 1)
```

giac [A] time = 0.17, size = 64, normalized size = 0.74

$$\frac{1}{5}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^5+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{3}{10}\left(x^5+1\right)^{\frac{2}{3}}-\frac{1}{10}\log\left(\left(x^5+1\right)^{\frac{2}{3}}+\left(x^5+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{5}\log\left(\left|\left(x^5+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)/x,x, algorithm="giac")
```

```
[Out] 1/5*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^5 + 1)^(1/3) + 1)) + 3/10*(x^5 + 1)^(2
/3) - 1/10*log((x^5 + 1)^(2/3) + (x^5 + 1)^(1/3) + 1) + 1/5*log(abs((x^5 +
1)^(1/3) - 1))
```

maple [C] time = 0.26, size = 64, normalized size = 0.74

$$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^5\,\mathrm{hypergeom}\left(\left[\frac{1}{3},1,1\right],[2,2],-x^5\right)}{3\Gamma\left(\frac{2}{3}\right)}-\frac{\left(\frac{3}{2}-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+5\ln(x)\right)\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}\right)}{15\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)^(2/3)/x,x)
```

```
[Out] -1/15/Pi*3^(1/2)*GAMMA(2/3)*(-2/3*Pi*3^(1/2)/GAMMA(2/3)*x^5*hypergeom([1/3,
1,1],[2,2],-x^5)-(3/2-1/6*Pi*3^(1/2)-3/2*ln(3)+5*ln(x))*Pi*3^(1/2)/GAMMA(2/
3))
```

maxima [A] time = 0.45, size = 63, normalized size = 0.72

$$\frac{1}{5}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^5+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{3}{10}\left(x^5+1\right)^{\frac{2}{3}}-\frac{1}{10}\log\left(\left(x^5+1\right)^{\frac{2}{3}}+\left(x^5+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{5}\log\left(\left(x^5+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^5 + 1)^(1/3) + 1)) + 3/10*(x^5 + 1)^(2
/3) - 1/10*log((x^5 + 1)^(2/3) + (x^5 + 1)^(1/3) + 1) + 1/5*log((x^5 + 1)^(
1/3) - 1)
```

mupad [B] time = 1.00, size = 89, normalized size = 1.02

$$\frac{\ln\left(\frac{9\left(x^5+1\right)^{1/3}}{25}-\frac{9}{25}\right)}{5}+\ln\left(\frac{9\left(x^5+1\right)^{1/3}}{25}-9\left(-\frac{1}{10}+\frac{\sqrt{3}\,1i}{10}\right)^2\right)\left(-\frac{1}{10}+\frac{\sqrt{3}\,1i}{10}\right)-\ln\left(\frac{9\left(x^5+1\right)^{1/3}}{25}-9\left(\frac{1}{10}+\frac{\sqrt{3}\,1i}{10}\right)^2\right)\left(\frac{1}{10}+\frac{\sqrt{3}\,1i}{10}\right)+\frac{3\left(x^5+1\right)^{2/3}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5 + 1)^(2/3)/x,x)
```

```
[Out] log((9*(x^5 + 1)^(1/3))/25 - 9/25)/5 + log((9*(x^5 + 1)^(1/3))/25 - 9*((3^(
1/2)*1i)/10 - 1/10)^2)*((3^(1/2)*1i)/10 - 1/10) - log((9*(x^5 + 1)^(1/3))/2
5 - 9*((3^(1/2)*1i)/10 + 1/10)^2)*((3^(1/2)*1i)/10 + 1/10) + (3*(x^5 + 1)^(
2/3))/10
```

sympy [C] time = 0.83, size = 37, normalized size = 0.43

$$-\frac{x^{\frac{10}{3}}\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^5}\right)}{5\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+1)**(2/3)/x,x)
```

```
[Out] -x**(10/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), exp_polar(I*pi)/x**5)/(5
*gamma(1/3))
```

3.1056 $\int \frac{(1+x^6)^{2/3}}{x} dx$

Optimal. Leaf size=87

$$\frac{1}{4}\left(x^6+1\right)^{2/3}+\frac{1}{6}\log\left(\sqrt[3]{x^6+1}-1\right)-\frac{1}{12}\log\left(\left(x^6+1\right)^{2/3}+\sqrt[3]{x^6+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^6+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 50, 55, 618, 204, 31}

$$\frac{1}{4}\left(x^6+1\right)^{2/3}+\frac{1}{4}\log\left(1-\sqrt[3]{x^6+1}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^6+1}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)^(2/3)/x,x]
```

```
[Out] (1 + x^6)^(2/3)/4 + ArcTan[(1 + 2*(1 + x^6)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - L
og[x]/2 + Log[1 - (1 + x^6)^(1/3)]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1+x^6\right)^{2/3}}{x}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{(1+x)^{2/3}}{x}\,dx,x,x^6\right)\\ &= \frac{1}{4}\left(1+x^6\right)^{2/3}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx,x,x^6\right)\\ &= \frac{1}{4}\left(1+x^6\right)^{2/3}-\frac{\log(x)}{2}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1+x^6}\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x\right.\\ &= \frac{1}{4}\left(1+x^6\right)^{2/3}-\frac{\log(x)}{2}+\frac{1}{4}\log\left(1-\sqrt[3]{1+x^6}\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{1+x^6}\right)\\ &= \frac{1}{4}\left(1+x^6\right)^{2/3}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^6}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log(x)}{2}+\frac{1}{4}\log\left(1-\sqrt[3]{1+x^6}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.71

$$\frac{1}{12}\left(3\left(\left(x^6+1\right)^{2/3}+\log\left(1-\sqrt[3]{x^6+1}\right)-2\log(x)\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^6+1}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)^(2/3)/x,x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2*(1 + x^6)^(1/3))/Sqrt[3]] + 3*((1 + x^6)^(2/3) - 2
*Log[x] + Log[1 - (1 + x^6)^(1/3)]))/12
```

IntegrateAlgebraic [A] time = 0.05, size = 87, normalized size = 1.00

$$\frac{1}{4}\left(x^6+1\right)^{2/3}+\frac{1}{6}\log\left(\sqrt[3]{x^6+1}-1\right)-\frac{1}{12}\log\left(\left(x^6+1\right)^{2/3}+\sqrt[3]{x^6+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^6+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^6)^(2/3)/x,x]
```

```
[Out] (1 + x^6)^(2/3)/4 + ArcTan[1/Sqrt[3] + (2*(1 + x^6)^(1/3))/Sqrt[3]]/(2*Sqrt
[3]) + Log[-1 + (1 + x^6)^(1/3)]/6 - Log[1 + (1 + x^6)^(1/3) + (1 + x^6)^(2
/3)]/12
```

fricas [A] time = 0.43, size = 65, normalized size = 0.75

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^6+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{1}{4}\left(x^6+1\right)^{\frac{2}{3}}-\frac{1}{12}\log\left(\left(x^6+1\right)^{\frac{2}{3}}+\left(x^6+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(\left(x^6+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(2/3*sqrt(3)*(x^6 + 1)^(1/3) + 1/3*sqrt(3)) + 1/4*(x^6 +
1)^(2/3) - 1/12*log((x^6 + 1)^(2/3) + (x^6 + 1)^(1/3) + 1) + 1/6*log((x^6 +
 1)^(1/3) - 1)
```

giac [A] time = 0.28, size = 63, normalized size = 0.72

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^6+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{4}\left(x^6+1\right)^{\frac{2}{3}}-\frac{1}{12}\log\left(\left(x^6+1\right)^{\frac{2}{3}}+\left(x^6+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(\left(x^6+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(2/3)/x,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3) + 1)) + 1/4*(x^6 + 1)^(2/
3) - 1/12*log((x^6 + 1)^(2/3) + (x^6 + 1)^(1/3) + 1) + 1/6*log((x^6 + 1)^(1
/3) - 1)
```

maple [C] time = 0.29, size = 64, normalized size = 0.74

$$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^6\,\mathrm{hypergeom}\left(\left[\frac{1}{3},1,1\right],[2,2],-x^6\right)}{3\Gamma\left(\frac{2}{3}\right)}-\frac{\left(\frac{3}{2}-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+6\ln(x)\right)\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}\right)}{18\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)^(2/3)/x,x)
```

```
[Out] -1/18/Pi*3^(1/2)*GAMMA(2/3)*(-2/3*Pi*3^(1/2)/GAMMA(2/3)*x^6*hypergeom([1/3,
1,1],[2,2],-x^6)-(3/2-1/6*Pi*3^(1/2)-3/2*ln(3)+6*ln(x))*Pi*3^(1/2)/GAMMA(2/
3))
```

maxima [A] time = 0.42, size = 63, normalized size = 0.72

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^6+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{4}\left(x^6+1\right)^{\frac{2}{3}}-\frac{1}{12}\log\left(\left(x^6+1\right)^{\frac{2}{3}}+\left(x^6+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(\left(x^6+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3) + 1)) + 1/4*(x^6 + 1)^(2/
3) - 1/12*log((x^6 + 1)^(2/3) + (x^6 + 1)^(1/3) + 1) + 1/6*log((x^6 + 1)^(1
/3) - 1)
```

mupad [B] time = 0.84, size = 89, normalized size = 1.02

$$\frac{\ln\left(\frac{\left(x^6+1\right)^{1/3}}{4}-\frac{1}{4}\right)}{6}+\ln\left(\frac{\left(x^6+1\right)^{1/3}}{4}-9\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)^2\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(\frac{\left(x^6+1\right)^{1/3}}{4}-9\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)^2\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\frac{\left(x^6+1\right)^{2/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)^(2/3)/x,x)
```

```
[Out] log((x^6 + 1)^(1/3)/4 - 1/4)/6 + log((x^6 + 1)^(1/3)/4 - 9*((3^(1/2)*1i)/12
 - 1/12)^2)*((3^(1/2)*1i)/12 - 1/12) - log((x^6 + 1)^(1/3)/4 - 9*((3^(1/2)*
1i)/12 + 1/12)^2)*((3^(1/2)*1i)/12 + 1/12) + (x^6 + 1)^(2/3)/4
```

sympy [C] time = 0.81, size = 36, normalized size = 0.41

$$-\frac{x^4\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^6}\right)}{6\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)**(2/3)/x,x)
```

```
[Out] -x**4*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), exp_polar(I*pi)/x**6)/(6*gamm
a(1/3))
```

3.1057 $\int \frac{(1-x^2+2x^4)\sqrt{1-x^2-x^4-x^6}}{(-1+x^2)(1+x^2)(-1+x^4+x^6)}\,dx$

Optimal. Leaf size=87

$$\tan^{-1}\left(\frac{x\sqrt{-x^6-x^4-x^2+1}}{x^6+x^4+x^2-1}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt{-x^6-x^4-x^2+1}}{x^6+x^4+x^2-1}\right)$$

Rubi [F] time = 2.59, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-x^2+2x^4)\sqrt{1-x^2-x^4-x^6}}{(-1+x^2)(1+x^2)(-1+x^4+x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 - x^2 + 2*x^4)*Sqrt[1 - x^2 - x^4 - x^6])/((-1 + x^2)*(1 + x^2)*(-1
 + x^4 + x^6)),x]
```

```
[Out] I*Defer[Int][Sqrt[1 - x^2 - x^4 - x^6]/(I - x), x] + Defer[Int][Sqrt[1 - x^
2 - x^4 - x^6]/(-1 + x), x]/2 + I*Defer[Int][Sqrt[1 - x^2 - x^4 - x^6]/(I +
 x), x] - Defer[Int][Sqrt[1 - x^2 - x^4 - x^6]/(1 + x), x]/2 - 2*Defer[Int]
[(x^2*Sqrt[1 - x^2 - x^4 - x^6])/(-1 + x^4 + x^6), x] - 3*Defer[Int][(x^4*S
qrt[1 - x^2 - x^4 - x^6])/(-1 + x^4 + x^6), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x^2+2x^4)\sqrt{1-x^2-x^4-x^6}}{(-1+x^2)(1+x^2)(-1+x^4+x^6)}\,dx &= \int \left(\frac{\sqrt{1-x^2-x^4-x^6}}{-1+x^2}+\frac{2\sqrt{1-x^2-x^4-x^6}}{1+x^2}-\frac{x^2(2+3x^2)\sqrt{1-x^2-}}{-1+x^4+x^6}\right.\\
&= 2\int \frac{\sqrt{1-x^2-x^4-x^6}}{1+x^2}\,dx+\int \frac{\sqrt{1-x^2-x^4-x^6}}{-1+x^2}\,dx-\int \frac{x^2(2+3x^2}{-1}\\
&= 2\int \left(\frac{i\sqrt{1-x^2-x^4-x^6}}{2(i-x)}+\frac{i\sqrt{1-x^2-x^4-x^6}}{2(i+x)}\right)dx+\int \left(\frac{\sqrt{1-x^2-x}}{2(-1+}\right.\\
&= i\int \frac{\sqrt{1-x^2-x^4-x^6}}{i-x}\,dx+i\int \frac{\sqrt{1-x^2-x^4-x^6}}{i+x}\,dx+\frac{1}{2}\int \frac{\sqrt{1-x^2}}{-1}
\end{aligned}$$

Mathematica [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{(1-x^2+2x^4)\sqrt{1-x^2-x^4-x^6}}{(-1+x^2)(1+x^2)(-1+x^4+x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 - x^2 + 2*x^4)*Sqrt[1 - x^2 - x^4 - x^6])/((-1 + x^2)*(1 + x^
2)*(-1 + x^4 + x^6)),x]
```

```
[Out] Integrate[((1 - x^2 + 2*x^4)*Sqrt[1 - x^2 - x^4 - x^6])/((-1 + x^2)*(1 + x^
2)*(-1 + x^4 + x^6)), x]
```

IntegrateAlgebraic [A] time = 0.20, size = 87, normalized size = 1.00

$$\tan^{-1}\left(\frac{x\sqrt{-x^6-x^4-x^2+1}}{x^6+x^4+x^2-1}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt{-x^6-x^4-x^2+1}}{x^6+x^4+x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 - x^2 + 2*x^4)*Sqrt[1 - x^2 - x^4 - x^6])/((-1 + x^2
)*(1 + x^2)*(-1 + x^4 + x^6)),x]
```

```
[Out] ArcTan[(x*Sqrt[1 - x^2 - x^4 - x^6])/(-1 + x^2 + x^4 + x^6)] - Sqrt[2]*ArcT
an[(Sqrt[2]*x*Sqrt[1 - x^2 - x^4 - x^6])/(-1 + x^2 + x^4 + x^6)]
```

fricas [B] time = 0.57, size = 184, normalized size = 2.11

$$-\frac{1}{10}\,\sqrt{2}\arctan\left(\frac{2\,\sqrt{2}\left(6\,x^{7}+x^{5}-4\,x^{3}+x\right)\sqrt{-x^{6}-x^{4}-x^{2}+1}}{17\,x^{10}+11\,x^{8}-2\,x^{6}-18\,x^{4}+9\,x^{2}-1}\right)+\frac{1}{5}\,\sqrt{2}\arctan\left(\frac{2\,\sqrt{2}\left(x^{7}+x^{3}-2\,x\right)\sqrt{-x^{6}-x^{4}-x^{2}+1}}{3\,x^{10}+3\,x^{8}+10\,x^{6}+6\,x^{4}+11\,x^{2}-1}\right)+\frac{1}{2}\arctan\left(\frac{2\,\sqrt{-x^{6}-x^{4}-x^{2}+1}\,x}{x^{6}+x^{4}+2\,x^{2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x^2+1)*(-x^6-x^4-x^2+1)^(1/2)/(x^2-1)/(x^2+1)/(x^6+x^4-1),
x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(2)*arctan(2*sqrt(2)*(6*x^7 + x^5 - 4*x^3 + x)*sqrt(-x^6 - x^4 -
x^2 + 1)/(17*x^10 + 11*x^8 - 2*x^6 - 18*x^4 + 9*x^2 - 1)) + 1/5*sqrt(2)*arc
tan(2*sqrt(2)*(x^7 + x^3 - 2*x)*sqrt(-x^6 - x^4 - x^2 + 1)/(3*x^10 + 3*x^8
+ 10*x^6 + 6*x^4 + 11*x^2 - 1)) + 1/2*arctan(2*sqrt(-x^6 - x^4 - x^2 + 1)*x
/(x^6 + x^4 + 2*x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^{6}-x^{4}-x^{2}+1}\left(2\,x^{4}-x^{2}+1\right)}{\left(x^{6}+x^{4}-1\right)\left(x^{2}+1\right)\left(x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x^2+1)*(-x^6-x^4-x^2+1)^(1/2)/(x^2-1)/(x^2+1)/(x^6+x^4-1),
x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^6 - x^4 - x^2 + 1)*(2*x^4 - x^2 + 1)/((x^6 + x^4 - 1)*(x^
2 + 1)*(x^2 - 1)), x)
```

maple [C] time = 1.12, size = 174, normalized size = 2.00

$$\frac{\mathrm{RootOf}\left(\_Z^{2}+2\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^{2}+2\right)x^{6}+\mathrm{RootOf}\left(\_Z^{2}+2\right)x^{4}+3\,\mathrm{RootOf}\left(\_Z^{2}+2\right)x^{2}-4\sqrt{-x^{6}-x^{4}-x^{2}+1}\,x-\mathrm{RootOf}\left(\_Z^{2}+2\right)}{\left(-1+x\right)\left(1+x\right)\left(x^{2}+1\right)^{2}}\right)}{2}+\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right)x^{6}+\mathrm{RootOf}\left(\_Z^{2}+1\right)x^{4}+2\,\mathrm{RootOf}\left(\_Z^{2}+1\right)x^{2}+2\sqrt{-x^{6}-x^{4}-x^{2}+1}\,x-\mathrm{RootOf}\left(\_Z^{2}+1\right)}{x^{6}+x^{4}-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-x^2+1)*(-x^6-x^4-x^2+1)^(1/2)/(x^2-1)/(x^2+1)/(x^6+x^4-1),x)
```

```
[Out] 1/2*RootOf(_Z^2+2)*ln((RootOf(_Z^2+2)*x^6+RootOf(_Z^2+2)*x^4+3*RootOf(_Z^2+
2)*x^2-4*(-x^6-x^4-x^2+1)^(1/2)*x-RootOf(_Z^2+2))/(-1+x)/(1+x)/(x^2+1)^2)+1
/2*RootOf(_Z^2+1)*ln(-(RootOf(_Z^2+1)*x^6+RootOf(_Z^2+1)*x^4+2*RootOf(_Z^2+
1)*x^2+2*(-x^6-x^4-x^2+1)^(1/2)*x-RootOf(_Z^2+1))/(x^6+x^4-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^{6}-x^{4}-x^{2}+1}\left(2\,x^{4}-x^{2}+1\right)}{\left(x^{6}+x^{4}-1\right)\left(x^{2}+1\right)\left(x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x^2+1)*(-x^6-x^4-x^2+1)^(1/2)/(x^2-1)/(x^2+1)/(x^6+x^4-1),
x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^6 - x^4 - x^2 + 1)*(2*x^4 - x^2 + 1)/((x^6 + x^4 - 1)*(x^
2 + 1)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^4-x^2+1\right)\,\sqrt{-x^6-x^4-x^2+1}}{\left(x^2-1\right)\,\left(x^2+1\right)\,\left(x^6+x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^4 - x^2 + 1)*(1 - x^4 - x^6 - x^2)^(1/2))/((x^2 - 1)*(x^2 + 1)*(x
^4 + x^6 - 1)),x)
```

```
[Out] int(((2*x^4 - x^2 + 1)*(1 - x^4 - x^6 - x^2)^(1/2))/((x^2 - 1)*(x^2 + 1)*(x
^4 + x^6 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-x**2+1)*(-x**6-x**4-x**2+1)**(1/2)/(x**2-1)/(x**2+1)/(x**
6+x**4-1),x)
```

```
[Out] Timed out
```

3.1058 $\int \frac{\sqrt[3]{-1+x^8}\,(3+5x^8)}{x^2(-1-x^3+x^8)}\,dx$

Optimal. Leaf size=87

$$\frac{3\sqrt[3]{x^8-1}}{x}+\log\left(\sqrt[3]{x^8-1}-x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^8-1}+x}\right)-\frac{1}{2}\log\left(\sqrt[3]{x^8-1}\,x+\left(x^8-1\right)^{2/3}+x^2\right)$$

Rubi [F] time = 0.79, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{-1+x^8}\,\left(3+5x^8\right)}{x^2\left(-1-x^3+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^8)^(1/3)*(3 + 5*x^8))/(x^2*(-1 - x^3 + x^8)),x]
```

```
[Out] (3*(-1 + x^8)^(1/3)*Hypergeometric2F1[-1/3, -1/8, 7/8, x^8])/(x*(1 - x^8)^(
1/3)) - 3*Defer[Int][(x*(-1 + x^8)^(1/3))/(-1 - x^3 + x^8), x] + 8*Defer[In
t][(x^6*(-1 + x^8)^(1/3))/(-1 - x^3 + x^8), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{-1+x^8}\,\left(3+5x^8\right)}{x^2\left(-1-x^3+x^8\right)}\,dx &= \int \left(-\frac{3\sqrt[3]{-1+x^8}}{x^2}+\frac{x\left(3-8x^5\right)\sqrt[3]{-1+x^8}}{1+x^3-x^8}\right)dx\\
&= -\left(3\int \frac{\sqrt[3]{-1+x^8}}{x^2}\,dx\right)+\int \frac{x\left(3-8x^5\right)\sqrt[3]{-1+x^8}}{1+x^3-x^8}\,dx\\
&= -\frac{\left(3\sqrt[3]{-1+x^8}\right)\int \frac{\sqrt[3]{1-x^8}}{x^2}\,dx}{\sqrt[3]{1-x^8}}+\int \left(-\frac{3x\sqrt[3]{-1+x^8}}{-1-x^3+x^8}+\frac{8x^6\sqrt[3]{-1+x^8}}{-1-x^3+x^8}\right)dx\\
&= \frac{3\sqrt[3]{-1+x^8}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{8};\frac{7}{8};x^8\right)}{x\sqrt[3]{1-x^8}}-3\int \frac{x\sqrt[3]{-1+x^8}}{-1-x^3+x^8}\,dx+8\int \frac{x^6\sqrt[3]{-1+x^8}}{-1-x^3+x^8}\,dx
\end{aligned}$$

Mathematica [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-1+x^8}\,\left(3+5x^8\right)}{x^2\left(-1-x^3+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^8)^(1/3)*(3 + 5*x^8))/(x^2*(-1 - x^3 + x^8)),x]
```

```
[Out] Integrate[((-1 + x^8)^(1/3)*(3 + 5*x^8))/(x^2*(-1 - x^3 + x^8)), x]
```

IntegrateAlgebraic [A] time = 20.31, size = 87, normalized size = 1.00

$$\frac{3\sqrt[3]{x^8-1}}{x}+\log\left(\sqrt[3]{x^8-1}-x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^8-1}+x}\right)-\frac{1}{2}\log\left(\sqrt[3]{x^8-1}\,x+\left(x^8-1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^8)^(1/3)*(3 + 5*x^8))/(x^2*(-1 - x^3 + x^8)),x]
```

```
[Out] (3*(-1 + x^8)^(1/3))/x + Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x^8)^(1/3)
)] + Log[-x + (-1 + x^8)^(1/3)] - Log[x^2 + x*(-1 + x^8)^(1/3) + (-1 + x^8)
^(2/3)]/2
```

fricas [A] time = 38.50, size = 131, normalized size = 1.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)^(1/3)*(5*x^8+3)/x^2/(x^8-x^3-1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*x*arctan(-1/3*(3106938903853179838301239309474736261657506409
14347519620206018375075582395161384253253772397893174953288579030579571412
06059288722620160721093489516063746612973182*sqrt(3)*(x^8 - 1)^(1/3)*x^2 -
246201421639630874524477268583691780300309670232508566228491053906496528172
6856794736217850308008582186678460057234561120056845593902299988319207916477
97236311980480*sqrt(3)*(x^8 - 1)^(2/3)*x + sqrt(3)*(140987309082699875979177
444503559024317602059990008201354952906276698907411739058023966360620238764
18322337000958016148565005886294703209808664629857632230121011200*x^8 - 10
874107470985632132635411332166810138488157464908872465909542404240938030050
120563415036693669260581591300349715210383562260469902904629389713924681998
974970514849*x^3 - 140987309082699875979177444503559024317602059990008201354
952906276698907411739058023966360620238764183223370009580161485650058862947
03209808664629857632230121011200))/(925174252329000529539497147880028099971
575379940528322350174780642887015458970839351473228174375453657494234708017
77464311573812087758030109633333654700796272640000*x^8 + 185930230779574376
223350884977579893235872617579375210689331058076497353738026447928290455896
909471220228789047349736297721564911220457772911794509749604118352128311*x^3
 - 92517425232900052953949714788002809997157537994052832235017478064288701545
897083935147322817437545365749423470801777464311573812087758030109633333654
70079627264000)) + x*log((x^8 - x^3 + 3*(x^8 - 1)^(1/3)*x^2 - 3*(x^8 - 1)
^(2/3)*x - 1)/(x^8 - x^3 - 1)) + 6*(x^8 - 1)^(1/3))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^{8}+3\right)\left(x^{8}-1\right)^{\frac{1}{3}}}{\left(x^{8}-x^{3}-1\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)^(1/3)*(5*x^8+3)/x^2/(x^8-x^3-1),x, algorithm="giac")
```

```
[Out] integrate((5*x^8 + 3)*(x^8 - 1)^(1/3)/((x^8 - x^3 - 1)*x^2), x)
```

maple [C] time = 6.78, size = 725, normalized size = 8.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-1)^(1/3)*(5*x^8+3)/x^2/(x^8-x^3-1),x)
```

```
[Out] 3*(x^8-1)^(1/3)/x+(RootOf(_Z^2+_Z+1)*ln(-(2*x^16*RootOf(_Z^2+_Z+1)+x^16-2*R
ootOf(_Z^2+_Z+1)^2*x^11-x^11*RootOf(_Z^2+_Z+1)+3*(x^16-2*x^8+1)^(1/3)*RootO
f(_Z^2+_Z+1)*x^9+3*(x^16-2*x^8+1)^(1/3)*x^9-4*RootOf(_Z^2+_Z+1)*x^8-2*x^8+3
*(x^16-2*x^8+1)^(2/3)*RootOf(_Z^2+_Z+1)*x^2+2*RootOf(_Z^2+_Z+1)^2*x^3+3*(x^
16-2*x^8+1)^(2/3)*x^2+RootOf(_Z^2+_Z+1)*x^3-3*(x^16-2*x^8+1)^(1/3)*RootOf(_
Z^2+_Z+1)*x-3*(x^16-2*x^8+1)^(1/3)*x+2*RootOf(_Z^2+_Z+1)+1)/(-1+x)/(1+x)/(x
^2+1)/(x^4+1)/(x^8-x^3-1))-ln((2*x^16*RootOf(_Z^2+_Z+1)+x^16+2*RootOf(_Z^2+
_Z+1)^2*x^11+3*x^11*RootOf(_Z^2+_Z+1)+3*(x^16-2*x^8+1)^(1/3)*RootOf(_Z^2+_Z
```

```
+1)*x^9+x^11-4*RootOf(_Z^2+_Z+1)*x^8-2*x^8+3*(x^16-2*x^8+1)^(2/3)*RootOf(_Z
^2+_Z+1)*x^2-2*RootOf(_Z^2+_Z+1)^2*x^3-3*RootOf(_Z^2+_Z+1)*x^3-3*(x^16-2*x^
8+1)^(1/3)*RootOf(_Z^2+_Z+1)*x-x^3+2*RootOf(_Z^2+_Z+1)+1)/(-1+x)/(1+x)/(x^2
+1)/(x^4+1)/(x^8-x^3-1))*RootOf(_Z^2+_Z+1)-ln((2*x^16*RootOf(_Z^2+_Z+1)+x^1
6+2*RootOf(_Z^2+_Z+1)^2*x^11+3*x^11*RootOf(_Z^2+_Z+1)+3*(x^16-2*x^8+1)^(1/3
)*RootOf(_Z^2+_Z+1)*x^9+x^11-4*RootOf(_Z^2+_Z+1)*x^8-2*x^8+3*(x^16-2*x^8+1)
^(2/3)*RootOf(_Z^2+_Z+1)*x^2-2*RootOf(_Z^2+_Z+1)^2*x^3-3*RootOf(_Z^2+_Z+1)*
x^3-3*(x^16-2*x^8+1)^(1/3)*RootOf(_Z^2+_Z+1)*x-x^3+2*RootOf(_Z^2+_Z+1)+1)/(
-1+x)/(1+x)/(x^2+1)/(x^4+1)/(x^8-x^3-1)))/(x^8-1)^(2/3)*((x^8-1)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^8+3\right)\left(x^8-1\right)^{\frac{1}{3}}}{\left(x^8-x^3-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)^(1/3)*(5*x^8+3)/x^2/(x^8-x^3-1),x, algorithm="maxima")
[Out] integrate((5*x^8 + 3)*(x^8 - 1)^(1/3)/((x^8 - x^3 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^8-1\right)^{1/3}\,\left(5\,x^8+3\right)}{x^2\,\left(-x^8+x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^8 - 1)^(1/3)*(5*x^8 + 3))/(x^2*(x^3 - x^8 + 1)),x)
[Out] int(-((x^8 - 1)^(1/3)*(5*x^8 + 3))/(x^2*(x^3 - x^8 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{(x-1)\,(x+1)\left(x^2+1\right)\left(x^4+1\right)}\,\left(5x^8+3\right)}{x^2\left(x^8-x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-1)**(1/3)*(5*x**8+3)/x**2/(x**8-x**3-1),x)
[Out] Integral(((x - 1)*(x + 1)*(x**2 + 1)*(x**4 + 1))**(1/3)*(5*x**8 + 3)/(x**2*
(x**8 - x**3 - 1)), x)
```

3.1059 $\int \frac{(2+x^8)\sqrt{4-2x^8+x^{16}}}{x^9}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{x^{16}-2x^8+4}\,(x^8-2)}{8x^8}-\frac{1}{8}\log\left(-x^8+\sqrt{x^{16}-2x^8+4}+2\right)+\frac{1}{4}\tanh^{-1}\left(\frac{2x^8}{3}-\frac{2}{3}\sqrt{x^{16}-2x^8+4}+\frac{1}{3}\right)$$

Rubi [A] time = 0.08, antiderivative size = 77, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1474, 812, 843, 619, 215, 724, 206}

$$-\frac{1}{8}\sinh^{-1}\left(\frac{1-x^8}{\sqrt{3}}\right)-\frac{\sqrt{x^{16}-2x^8+4}\,(2-x^8)}{8x^8}-\frac{1}{8}\tanh^{-1}\left(\frac{4-x^8}{2\sqrt{x^{16}-2x^8+4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x^8)*Sqrt[4 - 2*x^8 + x^16])/x^9,x]
```

```
[Out] -1/8*((2 - x^8)*Sqrt[4 - 2*x^8 + x^16])/x^8 - ArcSinh[(1 - x^8)/Sqrt[3]]/8
- ArcTanh[(4 - x^8)/(2*Sqrt[4 - 2*x^8 + x^16])]/8
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(2+x^8\right)\sqrt{4-2x^8+x^{16}}}{x^9}\,dx = \frac{1}{8}\text{Subst}\left(\int \frac{(2+x)\sqrt{4-2x+x^2}}{x^2}\,dx, x, x^8\right)$$

$$= -\frac{\left(2-x^8\right)\sqrt{4-2x^8+x^{16}}}{8x^8} - \frac{1}{16}\text{Subst}\left(\int \frac{-4-2x}{x\sqrt{4-2x+x^2}}\,dx, x, x^8\right)$$

$$= -\frac{\left(2-x^8\right)\sqrt{4-2x^8+x^{16}}}{8x^8} + \frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{4-2x+x^2}}\,dx, x, x^8\right) + \frac{1}{4}\text{Subst}$$

$$= -\frac{\left(2-x^8\right)\sqrt{4-2x^8+x^{16}}}{8x^8} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{16-x^2}\,dx, x, \frac{2\left(4-x^8\right)}{\sqrt{4-2x^8+x^{16}}}\right) + \cdots$$

$$= -\frac{\left(2-x^8\right)\sqrt{4-2x^8+x^{16}}}{8x^8} - \frac{1}{8}\sinh^{-1}\left(\frac{1-x^8}{\sqrt{3}}\right) - \frac{1}{8}\tanh^{-1}\left(\frac{4-x^8}{2\sqrt{4-2x^8+x^{16}}}\right)$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.78

$$\frac{1}{8}\left(\sinh^{-1}\left(\frac{x^8-1}{\sqrt{3}}\right) + \frac{\sqrt{x^{16}-2x^8+4}\left(x^8-2\right)}{x^8} - \tanh^{-1}\left(\frac{4-x^8}{2\sqrt{x^{16}-2x^8+4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x^8)*Sqrt[4 - 2*x^8 + x^16])/x^9,x]
```

```
[Out] (((-2 + x^8)*Sqrt[4 - 2*x^8 + x^16])/x^8 + ArcSinh[(-1 + x^8)/Sqrt[3]] - Ar
cTanh[(4 - x^8)/(2*Sqrt[4 - 2*x^8 + x^16])])/8
```

IntegrateAlgebraic [A] time = 0.15, size = 87, normalized size = 1.00

$$\frac{\sqrt{x^{16}-2x^8+4}\left(x^8-2\right)}{8x^8} - \frac{1}{8}\log\left(-x^8+\sqrt{x^{16}-2x^8+4}+2\right) + \frac{1}{4}\tanh^{-1}\left(\frac{2x^8}{3} - \frac{2}{3}\sqrt{x^{16}-2x^8+4} + \frac{1}{3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^8)*Sqrt[4 - 2*x^8 + x^16])/x^9,x]
```

```
[Out] ((-2 + x^8)*Sqrt[4 - 2*x^8 + x^16])/(8*x^8) + ArcTanh[1/3 + (2*x^8)/3 - (2*
Sqrt[4 - 2*x^8 + x^16])/3]/4 - Log[2 - x^8 + Sqrt[4 - 2*x^8 + x^16]]/8
```

fricas [A] time = 0.42, size = 94, normalized size = 1.08

$$-\frac{2\,x^{8}\log\left(2\,x^{16}-5\,x^{8}-\sqrt{x^{16}-2\,x^{8}+4}\left(2\,x^{8}-3\right)+6\right)-2\,x^{8}\log\left(-x^{8}+\sqrt{x^{16}-2\,x^{8}+4}-2\right)+5\,x^{8}-2\,\sqrt{x^{16}-2\,x^{8}+4}\left(x^{8}-2\right)}{16\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+2)*(x^16-2*x^8+4)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/16*(2*x^8*log(2*x^16 - 5*x^8 - sqrt(x^16 - 2*x^8 + 4)*(2*x^8 - 3) + 6) -
 2*x^8*log(-x^8 + sqrt(x^16 - 2*x^8 + 4) - 2) + 5*x^8 - 2*sqrt(x^16 - 2*x^8
 + 4)*(x^8 - 2))/x^8
```

giac [A] time = 0.18, size = 126, normalized size = 1.45

$$\frac{1}{8}\sqrt{x^{16}-2\,x^{8}+4}-\frac{x^{8}-\sqrt{x^{16}-2\,x^{8}+4}-4}{2\left(\left(x^{8}-\sqrt{x^{16}-2\,x^{8}+4}\right)^{2}-4\right)}+\frac{1}{8}\log\left(x^{8}-\sqrt{x^{16}-2\,x^{8}+4}+2\right)-\frac{1}{8}\log\left(-x^{8}+\sqrt{x^{16}-2\,x^{8}+4}+2\right)-\frac{1}{8}\log\left(-x^{8}+\sqrt{x^{16}-2\,x^{8}+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+2)*(x^16-2*x^8+4)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^16 - 2*x^8 + 4) - 1/2*(x^8 - sqrt(x^16 - 2*x^8 + 4) - 4)/((x^8 -
 sqrt(x^16 - 2*x^8 + 4))^2 - 4) + 1/8*log(x^8 - sqrt(x^16 - 2*x^8 + 4) + 2)
 - 1/8*log(-x^8 + sqrt(x^16 - 2*x^8 + 4) + 2) - 1/8*log(-x^8 + sqrt(x^16 -
2*x^8 + 4) + 1)
```

maple [A] time = 0.54, size = 60, normalized size = 0.69

$$\frac{x^{24}-4x^{16}+8x^{8}-8}{8x^{8}\sqrt{x^{16}-2x^{8}+4}}-\frac{\ln\left(\frac{2-x^{8}+\sqrt{x^{16}-2x^{8}+4}}{x^{4}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8+2)*(x^16-2*x^8+4)^(1/2)/x^9,x)
```

```
[Out] 1/8*(x^24-4*x^16+8*x^8-8)/x^8/(x^16-2*x^8+4)^(1/2)-1/4*ln((2-x^8+(x^16-2*x^
8+4)^(1/2))/x^4)
```

maxima [A] time = 0.45, size = 62, normalized size = 0.71

$$\frac{1}{8}\sqrt{x^{16}-2\,x^{8}+4}-\frac{\sqrt{x^{16}-2\,x^{8}+4}}{4\,x^{8}}+\frac{1}{8}\,\mathrm{arsinh}\left(\frac{1}{3}\,\sqrt{3}\left(x^{8}-1\right)\right)-\frac{1}{8}\,\mathrm{arsinh}\left(-\frac{1}{3}\,\sqrt{3}+\frac{4\,\sqrt{3}}{3\,x^{8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+2)*(x^16-2*x^8+4)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(x^16 - 2*x^8 + 4) - 1/4*sqrt(x^16 - 2*x^8 + 4)/x^8 + 1/8*arcsinh(1
/3*sqrt(3)*(x^8 - 1)) - 1/8*arcsinh(-1/3*sqrt(3) + 4/3*sqrt(3)/x^8)
```

mupad [B] time = 1.97, size = 80, normalized size = 0.92

$$\frac{\ln\left(\sqrt{x^{16}-2\,x^{8}+4}+x^{8}-1\right)}{8}-\frac{\ln\left(\frac{2\,\sqrt{x^{16}-2\,x^{8}+4}-x^{8}+4}{x^{8}}\right)}{8}-\frac{\sqrt{x^{16}-2\,x^{8}+4}}{4\,x^{8}}+\frac{\sqrt{x^{16}-2\,x^{8}+4}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^8 + 2)*(x^16 - 2*x^8 + 4)^(1/2))/x^9,x)
```

```
[Out] log((x^16 - 2*x^8 + 4)^(1/2) + x^8 - 1)/8 - log((2*(x^16 - 2*x^8 + 4)^(1/2)
 - x^8 + 4)/x^8)/8 - (x^16 - 2*x^8 + 4)^(1/2)/(4*x^8) + (x^16 - 2*x^8 + 4)^
(1/2)/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+2\right)\sqrt{x^{16}-2x^8+4}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8+2)*(x**16-2*x**8+4)**(1/2)/x**9,x)
```

```
[Out] Integral((x**8 + 2)*sqrt(x**16 - 2*x**8 + 4)/x**9, x)
```

3.1060 $\int \frac{\sqrt{x+\sqrt{1+x}}}{1-\sqrt{1+x}}\,dx$

Optimal. Leaf size=87

$$\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(-2\sqrt{x+1}-5\right)+\frac{7}{4}\log\left(-2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}}-1\right)+4\tanh^{-1}\left(-\sqrt{x+1}+\sqrt{x+\sqrt{x+1}}\right.$$

Rubi [A] time = 0.15, antiderivative size = 93, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {814, 843, 621, 206, 724}

$$-\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+5\right)-2\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)-\frac{7}{4}\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[1 + x]]/(1 - Sqrt[1 + x]),x]
```

```
[Out] -1/2*(Sqrt[x + Sqrt[1 + x]]*(5 + 2*Sqrt[1 + x])) - 2*ArcTanh[(1 - 3*Sqrt[1
+ x])/(2*Sqrt[x + Sqrt[1 + x]])] - (7*ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x
 + Sqrt[1 + x]])])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{x+\sqrt{1+x}}}{1-\sqrt{1+x}}\,dx = 2\,\text{Subst}\left(\int \frac{x\sqrt{-1+x+x^2}}{1-x}\,dx, x, \sqrt{1+x}\right)$$

$$= -\frac{1}{2}\sqrt{x+\sqrt{1+x}}\left(5+2\sqrt{1+x}\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{7x}{2}}{(1-x)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)$$

$$= -\frac{1}{2}\sqrt{x+\sqrt{1+x}}\left(5+2\sqrt{1+x}\right)-\frac{7}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)+2\,\text{Subs}$$

$$= -\frac{1}{2}\sqrt{x+\sqrt{1+x}}\left(5+2\sqrt{1+x}\right)-\frac{7}{2}\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{1+2\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right)-4\,\text{Subst}$$

$$= -\frac{1}{2}\sqrt{x+\sqrt{1+x}}\left(5+2\sqrt{1+x}\right)-2\tanh^{-1}\left(\frac{1-3\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)-\frac{7}{4}\tanh^{-1}\left(\frac{1+2\sqrt{1-}}{2\sqrt{x+\sqrt{1}}}\right.$$

Mathematica [A] time = 0.07, size = 93, normalized size = 1.07

$$\frac{1}{4}\left(-2\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+5\right)-8\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)-7\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[1 + x]]/(1 - Sqrt[1 + x]),x]
```

```
[Out] (-2*Sqrt[x + Sqrt[1 + x]]*(5 + 2*Sqrt[1 + x]) - 8*ArcTanh[(1 - 3*Sqrt[1 + x
])/(2*Sqrt[x + Sqrt[1 + x]])] - 7*ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + S
qrt[1 + x]])])/4
```

IntegrateAlgebraic [A] time = 0.23, size = 87, normalized size = 1.00

$$\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(-2\sqrt{x+1}-5\right)+\frac{7}{4}\log\left(-2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}}-1\right)+4\tanh^{-1}\left(-\sqrt{x+1}+\sqrt{x+\sqrt{x+1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x + Sqrt[1 + x]]/(1 - Sqrt[1 + x]),x]
```

```
[Out] ((-5 - 2*Sqrt[1 + x])*Sqrt[x + Sqrt[1 + x]])/2 + 4*ArcTanh[1 - Sqrt[1 + x]
+ Sqrt[x + Sqrt[1 + x]]] + (7*Log[-1 - 2*Sqrt[1 + x] + 2*Sqrt[x + Sqrt[1 +
x]]])/4
```

fricas [A] time = 1.16, size = 93, normalized size = 1.07

$$-\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+5\right)+\frac{7}{8}\log\left(4\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)-8x-8\sqrt{x+1}-5\right)+2\log\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}+1\right)+3x+2\sqrt{x+1}+2}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(1-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) + 5) + 7/8*log(4*sqrt(x + sqrt(x
+ 1))*(2*sqrt(x + 1) + 1) - 8*x - 8*sqrt(x + 1) - 5) + 2*log((2*sqrt(x + sq
rt(x + 1))*(sqrt(x + 1) + 1) + 3*x + 2*sqrt(x + 1) + 2)/x)
```

giac [A] time = 0.61, size = 87, normalized size = 1.00

$$-\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+5\right)+\frac{7}{4}\log\left(-2\sqrt{x+\sqrt{x+1}}+2\sqrt{x+1}+1\right)+2\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}+2\right|\right)-2\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(1-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) + 5) + 7/4*log(-2*sqrt(x + sqrt(x
 + 1)) + 2*sqrt(x + 1) + 1) + 2*log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1)
 + 2)) - 2*log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1)))
```

maple [A] time = 0.01, size = 127, normalized size = 1.46

$$-\frac{\left(1+2\sqrt{1+x}\right)\sqrt{x+\sqrt{1+x}}}{2}+\frac{5\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}\right)}{4}-2\sqrt{\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2}-3\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2}\right)+2\,\mathrm{arctanh}\left(\frac{-1+3\sqrt{1+x}}{2\sqrt{\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(1+x)^(1/2))^(1/2)/(1-(1+x)^(1/2)),x)
```

```
[Out] -1/2*(1+2*(1+x)^(1/2))*(x+(1+x)^(1/2))^(1/2)+5/4*ln(1/2+(1+x)^(1/2)+(x+(1+x
)^(1/2))^(1/2))-2*((-1+(1+x)^(1/2))^2+3*(1+x)^(1/2)-2)^(1/2)-3*ln(1/2+(1+x)
^(1/2)+((-1+(1+x)^(1/2))^2+3*(1+x)^(1/2)-2)^(1/2))+2*arctanh(1/2*(-1+3*(1+x
)^(1/2))/((-1+(1+x)^(1/2))^2+3*(1+x)^(1/2)-2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(1-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x + sqrt(x + 1))/(sqrt(x + 1) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int\frac{\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + (x + 1)^(1/2))^(1/2)/((x + 1)^(1/2) - 1),x)
```

```
[Out] -int((x + (x + 1)^(1/2))^(1/2)/((x + 1)^(1/2) - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)**(1/2))**(1/2)/(1-(1+x)**(1/2)),x)
```

```
[Out] -Integral(sqrt(x + sqrt(x + 1))/(sqrt(x + 1) - 1), x)
```

3.1061 $\int \frac{\sqrt{c+\sqrt{b+ax}}}{d-\sqrt{b+ax}}\,dx$

Optimal. Leaf size=87

$$-\frac{4\sqrt{\sqrt{ax+b}+c}\left(\sqrt{ax+b}+c+3d\right)}{3a}-\frac{4d\sqrt{-c-d}\tan^{-1}\left(\frac{\sqrt{-c-d}\sqrt{\sqrt{ax+b}+c}}{c+d}\right)}{a}$$

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {431, 376, 80, 50, 63, 206}

$$-\frac{4d\sqrt{\sqrt{ax+b}+c}}{a}+\frac{4d\sqrt{c+d}\tanh^{-1}\left(\frac{\sqrt{\sqrt{ax+b}+c}}{\sqrt{c+d}}\right)}{a}-\frac{4\left(\sqrt{ax+b}+c\right)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + Sqrt[b + a*x]]/(d - Sqrt[b + a*x]),x]
```

```
[Out] (-4*d*Sqrt[c + Sqrt[b + a*x]])/a - (4*(c + Sqrt[b + a*x])^(3/2))/(3*a) + (4
*d*Sqrt[c + d]*ArcTanh[Sqrt[c + Sqrt[b + a*x]]/Sqrt[c + d]])/a
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
```

```
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+\sqrt{b+ax}}}{d-\sqrt{b+ax}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{c+\sqrt{x}}}{d-\sqrt{x}}\,dx, x, b+ax\right)}{a} \\
&= \frac{2\,\text{Subst}\left(\int \frac{x\sqrt{c+x}}{d-x}\,dx, x, \sqrt{b+ax}\right)}{a} \\
&= -\frac{4\left(c+\sqrt{b+ax}\right)^{3/2}}{3a} + \frac{(2d)\,\text{Subst}\left(\int \frac{\sqrt{c+x}}{d-x}\,dx, x, \sqrt{b+ax}\right)}{a} \\
&= -\frac{4d\sqrt{c+\sqrt{b+ax}}}{a} - \frac{4\left(c+\sqrt{b+ax}\right)^{3/2}}{3a} + \frac{(2d(c+d))\,\text{Subst}\left(\int \frac{1}{(d-x)\sqrt{c+x}}\,dx, x, \sqrt{b+ax}\right)}{a} \\
&= -\frac{4d\sqrt{c+\sqrt{b+ax}}}{a} - \frac{4\left(c+\sqrt{b+ax}\right)^{3/2}}{3a} + \frac{(4d(c+d))\,\text{Subst}\left(\int \frac{1}{c+d-x^2}\,dx, x, \sqrt{c+\sqrt{b+ax}}\right)}{a} \\
&= -\frac{4d\sqrt{c+\sqrt{b+ax}}}{a} - \frac{4\left(c+\sqrt{b+ax}\right)^{3/2}}{3a} + \frac{4d\sqrt{c+d}\,\tanh^{-1}\left(\frac{\sqrt{c+\sqrt{b+ax}}}{\sqrt{c+d}}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 73, normalized size = 0.84

$$\frac{12d\sqrt{c+d}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{ax+b}+c}}{\sqrt{c+d}}\right) - 4\sqrt{\sqrt{ax+b}+c}\left(\sqrt{ax+b}+c+3d\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + Sqrt[b + a*x]]/(d - Sqrt[b + a*x]),x]
```

```
[Out] (-4*Sqrt[c + Sqrt[b + a*x]]*(c + 3*d + Sqrt[b + a*x]) + 12*d*Sqrt[c + d]*Ar
cTanh[Sqrt[c + Sqrt[b + a*x]]/Sqrt[c + d]])/(3*a)
```

IntegrateAlgebraic [A] time = 0.11, size = 87, normalized size = 1.00

$$-\frac{4\sqrt{\sqrt{ax+b}+c}\left(\sqrt{ax+b}+c+3d\right)}{3a} - \frac{4d\sqrt{-c-d}\,\tan^{-1}\left(\frac{\sqrt{-c-d}\sqrt{\sqrt{ax+b}+c}}{c+d}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + Sqrt[b + a*x]]/(d - Sqrt[b + a*x]),x]
```

```
[Out] (-4*Sqrt[c + Sqrt[b + a*x]]*(c + 3*d + Sqrt[b + a*x]))/(3*a) - (4*Sqrt[-c -
 d]*d*ArcTan[(Sqrt[-c - d]*Sqrt[c + Sqrt[b + a*x]])/(c + d)])/a
```

fricas [A] time = 0.43, size = 183, normalized size = 2.10

$$\left[\frac{2\left(3\sqrt{c+d}\,d\log\left(-\frac{2cd+d^2+ax+2\sqrt{ax+b}(c+d)+2\left(\sqrt{c+d}\,d+\sqrt{ax+b}\sqrt{c+d}\right)\sqrt{c+\sqrt{ax+b}}+b}{d^2-ax-b}\right)-2\left(c+3d+\sqrt{ax+b}\right)\sqrt{c+\sqrt{ax+b}}\right)}{3a},-\frac{4\left(3\sqrt{-c-d}\,d\arctan\left(\frac{\sqrt{c+\sqrt{ax+b}}\sqrt{-c-d}}{c+d}\right)+\left(c+3d+\sqrt{ax+b}\right)\sqrt{c+\sqrt{ax+b}}\right)}{3a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+b)^(1/2))^(1/2)/(d-(a*x+b)^(1/2)),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(c + d)*d*log(-(2*c*d + d^2 + a*x + 2*sqrt(a*x + b)*(c + d) + 2
*(sqrt(c + d)*d + sqrt(a*x + b)*sqrt(c + d))*sqrt(c + sqrt(a*x + b)) + b)/(
d^2 - a*x - b)) - 2*(c + 3*d + sqrt(a*x + b))*sqrt(c + sqrt(a*x + b)))/a, -
4/3*(3*sqrt(-c - d)*d*arctan(sqrt(c + sqrt(a*x + b))*sqrt(-c - d)/(c + d))
+ (c + 3*d + sqrt(a*x + b))*sqrt(c + sqrt(a*x + b)))/a]
```

giac [A] time = 0.16, size = 82, normalized size = 0.94

$$-\frac{4\left(cd+d^2\right)\arctan\left(\frac{\sqrt{c+\sqrt{ax+b}}}{\sqrt{-c-d}}\right)}{a\sqrt{-c-d}}-\frac{4\left(a^2\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}+3\,a^2\sqrt{c+\sqrt{ax+b}}\,d\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+b)^(1/2))^(1/2)/(d-(a*x+b)^(1/2)),x, algorithm="giac")
```

```
[Out] -4*(c*d + d^2)*arctan(sqrt(c + sqrt(a*x + b))/sqrt(-c - d))/(a*sqrt(-c - d)
) - 4/3*(a^2*(c + sqrt(a*x + b))^(3/2) + 3*a^2*sqrt(c + sqrt(a*x + b))*d)/a
^3
```

maple [A] time = 0.01, size = 60, normalized size = 0.69

$$-\frac{2\left(\frac{2\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}}{3}+2\sqrt{c+\sqrt{ax+b}}\,d-2d\sqrt{c+d}\,\operatorname{arctanh}\left(\frac{\sqrt{c+\sqrt{ax+b}}}{\sqrt{c+d}}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+(a*x+b)^(1/2))^(1/2)/(d-(a*x+b)^(1/2)),x)
```

```
[Out] -2/a*(2/3*(c+(a*x+b)^(1/2))^(3/2)+2*(c+(a*x+b)^(1/2))^(1/2)*d-2*d*(c+d)^(1/
2)*arctanh((c+(a*x+b)^(1/2))^(1/2)/(c+d)^(1/2)))
```

maxima [A] time = 0.41, size = 88, normalized size = 1.01

$$-\frac{2\left(2\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}+6\sqrt{c+\sqrt{ax+b}}\,d+\frac{3\left(cd+d^2\right)\log\left(-\frac{\sqrt{c+d}-\sqrt{c+\sqrt{ax+b}}}{\sqrt{c+d}+\sqrt{c+\sqrt{ax+b}}}\right)}{\sqrt{c+d}}\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+b)^(1/2))^(1/2)/(d-(a*x+b)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2/3*(2*(c + sqrt(a*x + b))^(3/2) + 6*sqrt(c + sqrt(a*x + b))*d + 3*(c*d +
d^2)*log(-(sqrt(c + d) - sqrt(c + sqrt(a*x + b)))/(sqrt(c + d) + sqrt(c + s
qrt(a*x + b))))/sqrt(c + d))/a
```

mupad [B] time = 0.96, size = 63, normalized size = 0.72

$$\frac{4\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\sqrt{b+a\,x}}}{\sqrt{c+d}}\right)\,\sqrt{c+d}}{a}-\frac{4\,d\,\sqrt{c+\sqrt{b+a\,x}}}{a}-\frac{4\,{\left(c+\sqrt{b+a\,x}\right)}^{3/2}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + (b + a*x)^(1/2))^(1/2)/(d - (b + a*x)^(1/2)),x)
```

```
[Out] (4*d*atanh((c + (b + a*x)^(1/2))^(1/2)/(c + d)^(1/2))*(c + d)^(1/2))/a - (4
*d*(c + (b + a*x)^(1/2))^(1/2))/a - (4*(c + (b + a*x)^(1/2))^(3/2))/(3*a)
```

sympy [A] time = 3.54, size = 78, normalized size = 0.90

$$-\frac{4d\sqrt{c+\sqrt{ax+b}}}{a}-\frac{4d\,(c+d)\,\mathrm{atan}\left(\frac{\sqrt{c+\sqrt{ax+b}}}{\sqrt{-c-d}}\right)}{a\sqrt{-c-d}}-\frac{4\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+b)**(1/2))**(1/2)/(d-(a*x+b)**(1/2)),x)
```

```
[Out] -4*d*sqrt(c + sqrt(a*x + b))/a - 4*d*(c + d)*atan(sqrt(c + sqrt(a*x + b))/s
qrt(-c - d))/(a*sqrt(-c - d)) - 4*(c + sqrt(a*x + b))**(3/2)/(3*a)
```

3.1062 $\int \left(b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}\,dx$

Optimal. Leaf size=87

$$\frac{2x\sqrt{ax^2+b^2}\left(5ax^2+13b^2\right)}{35\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{4x\left(3abx^2+11b^3\right)}{35\sqrt{\sqrt{ax^2+b^2}+b}}$$

Rubi [F] time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] (2*a*b^2*x^3)/(3*(b + Sqrt[b^2 + a*x^2])^(3/2)) + (2*b^3*x)/Sqrt[b + Sqrt[b
^2 + a*x^2]] + a*Defer[Int][x^2*Sqrt[b + Sqrt[b^2 + a*x^2]], x]
```

Rubi steps

$$\begin{aligned}
\int \left(b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}\,dx &= \int \left(b^2\sqrt{b+\sqrt{b^2+ax^2}}+ax^2\sqrt{b+\sqrt{b^2+ax^2}}\right)dx\\
&= a\int x^2\sqrt{b+\sqrt{b^2+ax^2}}\,dx+b^2\int \sqrt{b+\sqrt{b^2+ax^2}}\,dx\\
&= \frac{2ab^2x^3}{3\left(b+\sqrt{b^2+ax^2}\right)^{3/2}}+\frac{2b^3x}{\sqrt{b+\sqrt{b^2+ax^2}}}+a\int x^2\sqrt{b+\sqrt{b^2+ax^2}}\,dx
\end{aligned}$$

Mathematica [A] time = 0.21, size = 85, normalized size = 0.98

$$\frac{2x\left(5a^2x^4+24ab^2x^2+11abx^2\sqrt{ax^2+b^2}+35b^3\sqrt{ax^2+b^2}+35b^4\right)}{35\left(\sqrt{ax^2+b^2}+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] (2*x*(35*b^4 + 24*a*b^2*x^2 + 5*a^2*x^4 + 35*b^3*Sqrt[b^2 + a*x^2] + 11*a*b
*x^2*Sqrt[b^2 + a*x^2]))/(35*(b + Sqrt[b^2 + a*x^2])^(3/2))
```

IntegrateAlgebraic [A] time = 0.16, size = 87, normalized size = 1.00

$$\frac{2x\sqrt{ax^2+b^2}\left(5ax^2+13b^2\right)}{35\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{4x\left(3abx^2+11b^3\right)}{35\sqrt{\sqrt{ax^2+b^2}+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] (2*x*Sqrt[b^2 + a*x^2]*(13*b^2 + 5*a*x^2))/(35*Sqrt[b + Sqrt[b^2 + a*x^2]])
 + (4*x*(11*b^3 + 3*a*b*x^2))/(35*Sqrt[b + Sqrt[b^2 + a*x^2]])
```

fricas [A] time = 0.47, size = 70, normalized size = 0.80

$$\frac{2\left(5\,a^2x^4+12\,ab^2x^2-9\,b^4+\left(abx^2+9\,b^3\right)\sqrt{ax^2+b^2}\right)\sqrt{b+\sqrt{ax^2+b^2}}}{35\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)*(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^2*x^4 + 12*a*b^2*x^2 - 9*b^4 + (a*b*x^2 + 9*b^3)*sqrt(a*x^2 + b^2
))*sqrt(b + sqrt(a*x^2 + b^2))/(a*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax^2+b^2\right)\sqrt{b+\sqrt{ax^2+b^2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)*(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^2 + b^2)*sqrt(b + sqrt(a*x^2 + b^2)), x)
```

maple [C] time = 0.04, size = 153, normalized size = 1.76

$$\frac{\left(b^2\right)^{\frac{1}{4}}a\sqrt{2}\,x^3\,\mathrm{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4},\frac{3}{2}\right],\left[\frac{1}{2},\frac{5}{2}\right],-\frac{x^2a}{b^2}\right)}{3}-\frac{b^2\left(b^2\right)^{\frac{1}{4}}\left(-\frac{32\sqrt{\pi}\,\sqrt{2}\,x^3\sqrt{\frac{a}{b^2}}\,a\cosh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{x\sqrt{a}}{b}\right)}{2}\right)}{3b^2}-\frac{8\sqrt{\pi}\,\sqrt{2}\,\sqrt{\frac{a}{b^2}}\left(-\frac{4x^4a^2}{3b^4}-\frac{2x^2a}{3b^2}+\frac{2}{3}\right)\sinh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{x\sqrt{a}}{b}\right)}{2}\right)b}{\sqrt{a}\,\sqrt{\frac{x^2a}{b^2}+1}}\right)}{8\sqrt{\pi}\,\sqrt{\frac{a}{b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b^2)*(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] 1/3*(b^2)^(1/4)*a*2^(1/2)*x^3*hypergeom([-1/4,1/4,3/2],[1/2,5/2],-x^2*a/b^2
)-1/8*b^2*(b^2)^(1/4)/Pi^(1/2)/(a/b^2)^(1/2)*(-32/3*Pi^(1/2)*2^(1/2)*x^3*(a
/b^2)^(1/2)*a/b^2*cosh(3/2*arcsinh(x*a^(1/2)/b))-8*Pi^(1/2)*2^(1/2)*(a/b^2)
^(1/2)*(-4/3*x^4*a^2/b^4-2/3*x^2*a/b^2+2/3)*sinh(3/2*arcsinh(x*a^(1/2)/b))/
a^(1/2)*b/(x^2*a/b^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax^2+b^2\right)\sqrt{b+\sqrt{ax^2+b^2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)*(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 + b^2)*sqrt(b + sqrt(a*x^2 + b^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(b^2+a\,x^2\right)\,\sqrt{b+\sqrt{b^2+a\,x^2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b^2)*(b + (a*x^2 + b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x^2 + b^2)*(b + (a*x^2 + b^2)^(1/2))^(1/2), x)
```

sympy [B] time = 3.71, size = 581, normalized size = 6.68

$$-\frac{15\sqrt{2}a^{2}\sqrt{b}x^{5}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{420\pi b^{2}\sqrt{\frac{ax^{2}}{b^{2}}+1}\sqrt{\sqrt{\frac{ax^{2}}{b^{2}}+1}+1}+420\pi b^{2}\sqrt{\sqrt{\frac{ax^{2}}{b^{2}}+1}+1}}-\frac{33\sqrt{2}ab^{\frac{5}{2}}x^{3}\sqrt{\frac{ax^{2}}{b^{2}}+1}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{420\pi b^{2}\sqrt{\frac{ax^{2}}{b^{2}}+1}\sqrt{\sqrt{\frac{ax^{2}}{b^{2}}+1}+1}+420\pi b^{2}\sqrt{\sqrt{\frac{ax^{2}}{b^{2}}+1}+1}}-\frac{37\sqrt{2}ab^{\frac{5}{2}}x^{3}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{420\pi b^{2}\sqrt{\frac{ax^{2}}{b^{2}}+1}\sqrt{\sqrt{\frac{ax^{2}}{b^{2}}+1}+1}+420\pi b^{2}\sqrt{\sqrt{\frac{ax^{2}}{b^{2}}+1}+1}}-\frac{\sqrt{2}ab^{\frac{5}{2}}x^{3}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi b^{2}\sqrt{\frac{ax^{2}}{b^{2}}+1}\sqrt{\sqrt{\frac{ax^{2}}{b^{2}}+1}+1}+12\pi b^{2}\sqrt{\sqrt{\frac{ax^{2}}{b^{2}}+1}+1}}-\frac{3\sqrt{2}b^{\frac{9}{2}}x\sqrt{\frac{ax^{2}}{b^{2}}+1}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi b^{2}\sqrt{\frac{ax^{2}}{b^{2}}+1}\sqrt{\sqrt{\frac{ax^{2}}{b^{2}}+1}+1}+12\pi b^{2}\sqrt{\sqrt{\frac{ax^{2}}{b^{2}}+1}+1}}-\frac{3\sqrt{2}b^{\frac{9}{2}}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi b^{2}\sqrt{\frac{ax^{2}}{b^{2}}+1}\sqrt{\sqrt{\frac{ax^{2}}{b^{2}}+1}+1}+12\pi b^{2}\sqrt{\sqrt{\frac{ax^{2}}{b^{2}}+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b**2)*(b+(a*x**2+b**2)**(1/2))**(1/2),x)
```

```
[Out] -15*sqrt(2)*a**2*sqrt(b)*x**5*gamma(-1/4)*gamma(1/4)/(420*pi*b**2*sqrt(a*x*
*2/b**2 + 1)*sqrt(sqrt(a*x**2/b**2 + 1) + 1) + 420*pi*b**2*sqrt(sqrt(a*x**2
/b**2 + 1) + 1)) - 33*sqrt(2)*a*b**(5/2)*x**3*sqrt(a*x**2/b**2 + 1)*gamma(-
1/4)*gamma(1/4)/(420*pi*b**2*sqrt(a*x**2/b**2 + 1)*sqrt(sqrt(a*x**2/b**2 +
1) + 1) + 420*pi*b**2*sqrt(sqrt(a*x**2/b**2 + 1) + 1)) - 37*sqrt(2)*a*b**(5
/2)*x**3*gamma(-1/4)*gamma(1/4)/(420*pi*b**2*sqrt(a*x**2/b**2 + 1)*sqrt(sqr
t(a*x**2/b**2 + 1) + 1) + 420*pi*b**2*sqrt(sqrt(a*x**2/b**2 + 1) + 1)) - sq
rt(2)*a*b**(5/2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*b**2*sqrt(a*x**2/b**2 +
 1)*sqrt(sqrt(a*x**2/b**2 + 1) + 1) + 12*pi*b**2*sqrt(sqrt(a*x**2/b**2 + 1)
 + 1)) - 3*sqrt(2)*b**(9/2)*x*sqrt(a*x**2/b**2 + 1)*gamma(-1/4)*gamma(1/4)/
(12*pi*b**2*sqrt(a*x**2/b**2 + 1)*sqrt(sqrt(a*x**2/b**2 + 1) + 1) + 12*pi*b
**2*sqrt(sqrt(a*x**2/b**2 + 1) + 1)) - 3*sqrt(2)*b**(9/2)*x*gamma(-1/4)*gam
ma(1/4)/(12*pi*b**2*sqrt(a*x**2/b**2 + 1)*sqrt(sqrt(a*x**2/b**2 + 1) + 1) +
 12*pi*b**2*sqrt(sqrt(a*x**2/b**2 + 1) + 1))
```

3.1063 $\int \frac{1}{(-2+x)\sqrt[3]{-4-4x+x^2}}\,dx$

Optimal. Leaf size=88

$$-\frac{1}{4}\log\left(\sqrt[3]{x^2-4x-4}+2\right)+\frac{1}{8}\log\left(\left(x^2-4x-4\right)^{2/3}-2\sqrt[3]{x^2-4x-4}+4\right)-\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{x^2-4x-4}}{\sqrt{3}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 0.68, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {694, 266, 56, 618, 204, 31}

$$-\frac{3}{8}\log\left(\sqrt[3]{(x-2)^2-8}+2\right)+\frac{1}{4}\log(2-x)-\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1-\sqrt[3]{(x-2)^2-8}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + x)*(-4 - 4*x + x^2)^(1/3)),x]
```

```
[Out] -1/4*(Sqrt[3]*ArcTan[(1 - (-8 + (-2 + x)^2)^(1/3))/Sqrt[3]]) - (3*Log[2 + (
-8 + (-2 + x)^2)^(1/3)])/8 + Log[2 - x]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(-2+x)\sqrt[3]{-4-4x+x^2}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt[3]{-8+x^2}}\,dx, x, -2+x\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{-8+x}\,x}\,dx, x, (-2+x)^2\right)$$
$$= \frac{1}{4}\log(2-x) - \frac{3}{8}\text{Subst}\left(\int \frac{1}{2+x}\,dx, x, \sqrt[3]{-8+(-2+x)^2}\right) + \frac{3}{4}\text{Subst}\left(\int \frac{}{4-}\right.$$
$$= -\frac{3}{8}\log\left(2+\sqrt[3]{-8+(-2+x)^2}\right) + \frac{1}{4}\log(2-x) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x,\right.$$
$$= -\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1-\sqrt[3]{-8+(-2+x)^2}}{\sqrt{3}}\right) - \frac{3}{8}\log\left(2+\sqrt[3]{-8+(-2+x)^2}\right) + \frac{1}{4}\text{lo}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.41

$$\frac{3}{32}\left((x-2)^2-8\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{1}{8}\left(8-(x-2)^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + x)*(-4 - 4*x + x^2)^(1/3)),x]
```

```
[Out] (3*(-8 + (-2 + x)^2)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (8 - (-2 + x)^2)/
8])/32
```

IntegrateAlgebraic [A] time = 0.09, size = 88, normalized size = 1.00

$$-\frac{1}{4}\log\left(\sqrt[3]{x^2-4x-4}+2\right)+\frac{1}{8}\log\left(\left(x^2-4x-4\right)^{2/3}-2\sqrt[3]{x^2-4x-4}+4\right)-\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{x^2-4x-4}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2 + x)*(-4 - 4*x + x^2)^(1/3)),x]
```

```
[Out] -1/4*(Sqrt[3]*ArcTan[1/Sqrt[3] - (-4 - 4*x + x^2)^(1/3)/Sqrt[3]]) - Log[2 +
 (-4 - 4*x + x^2)^(1/3)]/4 + Log[4 - 2*(-4 - 4*x + x^2)^(1/3) + (-4 - 4*x +
 x^2)^(2/3)]/8
```

fricas [A] time = 0.42, size = 70, normalized size = 0.80

$$\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(x^2-4x-4\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+\frac{1}{8}\log\left(\left(x^2-4x-4\right)^{\frac{2}{3}}-2\left(x^2-4x-4\right)^{\frac{1}{3}}+4\right)-\frac{1}{4}\log\left(\left(x^2-4x-4\right)^{\frac{1}{3}}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(x^2-4*x-4)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(x^2 - 4*x - 4)^(1/3) - 1/3*sqrt(3)) + 1/8*l
og((x^2 - 4*x - 4)^(2/3) - 2*(x^2 - 4*x - 4)^(1/3) + 4) - 1/4*log((x^2 - 4*
x - 4)^(1/3) + 2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-4x-4\right)^{\frac{1}{3}}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(x^2-4*x-4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - 4*x - 4)^(1/3)*(x - 2)), x)
```

maple [C] time = 1.98, size = 522, normalized size = 5.93

$$\frac{\ln\left(\frac{20\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)^2 x^2-80\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)^2 x+48\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)\left(x^2-4x-4\right)^{\frac{2}{3}}-23\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)x^2-96\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)\left(x^2-4x-4\right)^{\frac{1}{3}}+92\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)x-15\left(x^2-4x-4\right)^{\frac{2}{3}}+6x^2+140\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)+30\left(x^2-4x-4\right)^{\frac{1}{3}}-24x-56}{\left(-2+x\right)^2}\right)}{4}-\frac{\ln\left(\frac{20\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)^2 x^2-80\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)^2 x+48\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)\left(x^2-4x-4\right)^{\frac{2}{3}}-23\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)x^2-96\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)\left(x^2-4x-4\right)^{\frac{1}{3}}+92\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)x-15\left(x^2-4x-4\right)^{\frac{2}{3}}+6x^2+140\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)+30\left(x^2-4x-4\right)^{\frac{1}{3}}-24x-56}{\left(-2+x\right)^2}\right)\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)}{2}+\frac{\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)\ln\left(\frac{40\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)^2 x^2-160\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)^2 x-96\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)\left(x^2-4x-4\right)^{\frac{2}{3}}+6\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)x^2+192\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)\left(x^2-4x-4\right)^{\frac{1}{3}}-24\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)x+18\left(x^2-4x-4\right)^{\frac{2}{3}}-x^2-280\,\mathrm{RootOf}\left(4\_Z^2-2\_Z+1\right)-36\left(x^2-4x-4\right)^{\frac{1}{3}}+4x+28}{\left(-2+x\right)^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2+x)/(x^2-4*x-4)^(1/3),x)
```

```
[Out] 1/4*ln((20*RootOf(4*_Z^2-2*_Z+1)^2*x^2-80*RootOf(4*_Z^2-2*_Z+1)^2*x+48*Root
Of(4*_Z^2-2*_Z+1)*(x^2-4*x-4)^(2/3)-23*RootOf(4*_Z^2-2*_Z+1)*x^2-96*RootOf(
4*_Z^2-2*_Z+1)*(x^2-4*x-4)^(1/3)+92*RootOf(4*_Z^2-2*_Z+1)*x-15*(x^2-4*x-4)^
(2/3)+6*x^2+140*RootOf(4*_Z^2-2*_Z+1)+30*(x^2-4*x-4)^(1/3)-24*x-56)/(-2+x)^
2)-1/2*ln((20*RootOf(4*_Z^2-2*_Z+1)^2*x^2-80*RootOf(4*_Z^2-2*_Z+1)^2*x+48*R
ootOf(4*_Z^2-2*_Z+1)*(x^2-4*x-4)^(2/3)-23*RootOf(4*_Z^2-2*_Z+1)*x^2-96*Root
Of(4*_Z^2-2*_Z+1)*(x^2-4*x-4)^(1/3)+92*RootOf(4*_Z^2-2*_Z+1)*x-15*(x^2-4*x-
4)^(2/3)+6*x^2+140*RootOf(4*_Z^2-2*_Z+1)+30*(x^2-4*x-4)^(1/3)-24*x-56)/(-2+
x)^2)*RootOf(4*_Z^2-2*_Z+1)+1/2*RootOf(4*_Z^2-2*_Z+1)*ln((40*RootOf(4*_Z^2-
2*_Z+1)^2*x^2-160*RootOf(4*_Z^2-2*_Z+1)^2*x-96*RootOf(4*_Z^2-2*_Z+1)*(x^2-4
*x-4)^(2/3)+6*RootOf(4*_Z^2-2*_Z+1)*x^2+192*RootOf(4*_Z^2-2*_Z+1)*(x^2-4*x-
4)^(1/3)-24*RootOf(4*_Z^2-2*_Z+1)*x+18*(x^2-4*x-4)^(2/3)-x^2-280*RootOf(4*_
Z^2-2*_Z+1)-36*(x^2-4*x-4)^(1/3)+4*x+28)/(-2+x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-4\,x-4\right)^{\frac{1}{3}}\left(x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(x^2-4*x-4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - 4*x - 4)^(1/3)*(x - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x-2\right)\,\left(x^2-4\,x-4\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)*(x^2 - 4*x - 4)^(1/3)),x)
```

```
[Out] int(1/((x - 2)*(x^2 - 4*x - 4)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x-2\right)\sqrt[3]{x^2-4x-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(x**2-4*x-4)**(1/3),x)
```

```
[Out] Integral(1/((x - 2)*(x**2 - 4*x - 4)**(1/3)), x)
```

3.1064 $\int \frac{-1+kx^2}{(a+bx)\sqrt{(1-x)x(1-kx)}\,(b+akx)}\,dx$

Optimal. Leaf size=88

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{kx^3+(-k-1)x^2+x}\,\sqrt{ak+b}}{\sqrt{a}\,\sqrt{b}\,(x-1)(kx-1)}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{a+b}\,\sqrt{ak+b}}$$

Rubi [C] time = 6.83, antiderivative size = 253, normalized size of antiderivative = 2.88, number of steps used = 14, number of rules used = 9, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {6718, 6688, 6742, 714, 115, 934, 12, 168, 537}

$$\frac{2(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-kx}\,\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{-k}\,\sqrt{-x}\right)|\frac{1}{k}\right)}{ab\sqrt{-k}\,\sqrt{x-x^2}\,\sqrt{(1-x)x(1-kx)}}+\frac{2(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-kx}\,\Pi\left(-\frac{b}{ak};\sin^{-1}\left(\sqrt{-k}\,\sqrt{-x}\right)|\frac{1}{k}\right)}{ab\sqrt{-k}\,\sqrt{x-x^2}\,\sqrt{(1-x)x(1-kx)}}+\frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-kx}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k\right)}{ab\sqrt{(1-x)x(1-kx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + k*x^2)/((a + b*x)*Sqrt[(1 - x)*x*(1 - k*x)]*(b + a*k*x)),x]
```

```
[Out] (2*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k*x]*EllipticF[ArcSin[Sqrt[x]], k])/(a*b*Sq
rt[(1 - x)*x*(1 - k*x)]) + (2*(1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k*x]*Ellipt
icPi[-(a/b), ArcSin[Sqrt[-k]*Sqrt[-x]], k^(-1)])/(a*b*Sqrt[-k]*Sqrt[(1 - x)
*x*(1 - k*x)]*Sqrt[x - x^2]) + (2*(1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k*x]*El
lipticPi[-(b/(a*k)), ArcSin[Sqrt[-k]*Sqrt[-x]], k^(-1)])/(a*b*Sqrt[-k]*Sqrt
[(1 - x)*x*(1 - k*x)]*Sqrt[x - x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 714

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c*x)/b]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0]
```

```
&& RationalQ[b]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-1+kx^2}{(a+bx)\sqrt{(1-x)x(1-kx)}\,(b+akx)}\,dx = \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-kx}\right)\int \frac{-1+kx^2}{\sqrt{1-x}\,\sqrt{x}\,(a+bx)\sqrt{1-kx}\,(b+akx)}\,dx}{\sqrt{(1-x)x(1-kx)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-kx}\right)\int \frac{-1+kx^2}{(a+bx)\sqrt{1-kx}\,(b+akx)\sqrt{x-x^2}}\,dx}{\sqrt{(1-x)x(1-kx)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-kx}\right)\int \left(\frac{1}{ab\sqrt{1-kx}\,\sqrt{x-x^2}} - \frac{1}{b(a+bx)\sqrt{1-kx}\,\sqrt{x-x^2}} - \frac{}{a\sqrt{1-kx}\,(}\right.}{\sqrt{(1-x)x(1-kx)}}$$

$$= -\frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-kx}\right)\int \frac{1}{\sqrt{1-kx}\,(b+akx)\sqrt{x-x^2}}\,dx}{a\sqrt{(1-x)x(1-kx)}} - \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k\right.}{b\sqrt{(1-}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-kx}\right)\int \frac{1}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-kx}}\,dx}{ab\sqrt{(1-x)x(1-kx)}} - \frac{\left(\sqrt{2}\,\sqrt{2-2x}\,\sqrt{1-x}\,\sqrt{-\right.}{a\sqrt{(}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-kx}\,F\left(\sin^{-1}\left(\sqrt{x}\right)\middle|k\right)}{ab\sqrt{(1-x)x(1-kx)}} - \frac{\left(\sqrt{2-2x}\,\sqrt{1-x}\,\sqrt{-x}\,\sqrt{x}\right.}{a\sqrt{(1-x)}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-kx}\,F\left(\sin^{-1}\left(\sqrt{x}\right)\middle|k\right)}{ab\sqrt{(1-x)x(1-kx)}} + \frac{\left(2\sqrt{2-2x}\,\sqrt{1-x}\,\sqrt{-x}\,\sqrt{x}\right.}{a}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-kx}\,F\left(\sin^{-1}\left(\sqrt{x}\right)\middle|k\right)}{ab\sqrt{(1-x)x(1-kx)}} + \frac{2(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-kx}\,\Pi}{ab\sqrt{-k}\,\sqrt{(1-x)x}}$$

Mathematica [C] time = 1.88, size = 169, normalized size = 1.92

$$\frac{2i(x-1)^{3/2}\sqrt{\frac{x}{x-1}}\sqrt{\frac{1-kx}{k-kx}}\left(ab(k-1)F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|\frac{k-1}{k}\right)+a(ak+b)\Pi\left(\frac{a+b}{b};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|\frac{k-1}{k}\right)+b(a+b)\Pi\left(\frac{b}{ak}+1;i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|\frac{k-1}{k}\right)\right)}{ab(a+b)\sqrt{(x-1)x(kx-1)}\,(ak+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + k*x^2)/((a + b*x)*Sqrt[(1 - x)*x*(1 - k*x)]*(b + a*k*x)),x]
```

```
[Out] ((2*I)*(-1 + x)^(3/2)*Sqrt[x/(-1 + x)]*Sqrt[(1 - k*x)/(k - k*x)]*(a*b*(-1 +
 k)*EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], (-1 + k)/k] + a*(b + a*k)*Elliptic
Pi[(a + b)/b, I*ArcSinh[1/Sqrt[-1 + x]], (-1 + k)/k] + b*(a + b)*EllipticPi
[1 + b/(a*k), I*ArcSinh[1/Sqrt[-1 + x]], (-1 + k)/k]))/(a*b*(a + b)*(b + a*
k)*Sqrt[(-1 + x)*x*(-1 + k*x)])
```

IntegrateAlgebraic [A] time = 0.29, size = 88, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{kx^3+(-k-1)x^2+x}\,\sqrt{ak+b}}{\sqrt{a}\,\sqrt{b}\,(x-1)(kx-1)}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{a+b}\,\sqrt{ak+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + k*x^2)/((a + b*x)*Sqrt[(1 - x)*x*(1 - k*x)]*(b + a
*k*x)),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a + b]*Sqrt[b + a*k]*Sqrt[x + (-1 - k)*x^2 + k*x^3])/(Sqrt
[a]*Sqrt[b]*(-1 + x)*(-1 + k*x))])/(Sqrt[a]*Sqrt[b]*Sqrt[a + b]*Sqrt[b + a*
k])
```

fricas [B] time = 1.54, size = 639, normalized size = 7.26

$$\left[-\frac{\sqrt{-a^2b^2-ab^3-(a^3b+a^2b^2)k}\log\left(\frac{a^2b^2k^2x^4+a^2b^2-2\left((3a^3b+4a^2b^2)k^2+(4a^2b^2+3ab^3)k\right)x^3+\left(8a^2b^2+8ab^3+b^4+(a^4+8a^3b+8a^2b^2)k^2+4(2a^3b+5a^2b^2+2ab^3)k\right)x^2-4\left(abkx^2+ab-(2ab+b^2+(a^2+2ab)k)x\right)\sqrt{-a^2b^2-ab^3-(a^3b+a^2b^2)k}\sqrt{kx^3-(k+1)x^2+x}-2\left(4a^2b^2+3ab^3+(3a^3b+4a^2b^2)k\right)x}{a^2b^2k^2x^4+a^2b^2+2(a^3bk^2+ab^3k)x^3+(a^4k^2+4a^2b^2k+b^4)x^2+2(a^3bk+ab^3)x}\right)}{2\left(a^2b^2+ab^3+(a^3b+a^2b^2)k\right)},\ \frac{\arctan\left(\frac{\left(abkx^2+ab-(2ab+b^2+(a^2+2ab)k)x\right)\sqrt{a^2b^2+ab^3+(a^3b+a^2b^2)k}\sqrt{kx^3-(k+1)x^2+x}}{2\left(((a^3b+a^2b^2)k^2+(a^2b^2+ab^3)k)x^3-(a^2b^2+ab^3+(a^3b+a^2b^2)k^2+(a^3b+2a^2b^2+ab^3)k)x^2+(a^2b^2+ab^3+(a^3b+a^2b^2)k)x\right)}\right)}{\sqrt{a^2b^2+ab^3+(a^3b+a^2b^2)k}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^2-1)/(b*x+a)/((1-x)*x*(-k*x+1))^(1/2)/(a*k*x+b),x, algorithm
="fricas")
```

```
[Out] [-1/2*sqrt(-a^2*b^2 - a*b^3 - (a^3*b + a^2*b^2)*k)*log((a^2*b^2*k^2*x^4 + a
^2*b^2 - 2*((3*a^3*b + 4*a^2*b^2)*k^2 + (4*a^2*b^2 + 3*a*b^3)*k)*x^3 + (8*a
^2*b^2 + 8*a*b^3 + b^4 + (a^4 + 8*a^3*b + 8*a^2*b^2)*k^2 + 4*(2*a^3*b + 5*a
^2*b^2 + 2*a*b^3)*k)*x^2 - 4*(a*b*k*x^2 + a*b - (2*a*b + b^2 + (a^2 + 2*a*b
)*k)*x)*sqrt(-a^2*b^2 - a*b^3 - (a^3*b + a^2*b^2)*k)*sqrt(k*x^3 - (k + 1)*x
^2 + x) - 2*(4*a^2*b^2 + 3*a*b^3 + (3*a^3*b + 4*a^2*b^2)*k)*x)/(a^2*b^2*k^2
*x^4 + a^2*b^2 + 2*(a^3*b*k^2 + a*b^3*k)*x^3 + (a^4*k^2 + 4*a^2*b^2*k + b^4
)*x^2 + 2*(a^3*b*k + a*b^3)*x))/(a^2*b^2 + a*b^3 + (a^3*b + a^2*b^2)*k), ar
ctan(1/2*(a*b*k*x^2 + a*b - (2*a*b + b^2 + (a^2 + 2*a*b)*k)*x)*sqrt(a^2*b^2
 + a*b^3 + (a^3*b + a^2*b^2)*k)*sqrt(k*x^3 - (k + 1)*x^2 + x)/(((a^3*b + a^
2*b^2)*k^2 + (a^2*b^2 + a*b^3)*k)*x^3 - (a^2*b^2 + a*b^3 + (a^3*b + a^2*b^2
)*k^2 + (a^3*b + 2*a^2*b^2 + a*b^3)*k)*x^2 + (a^2*b^2 + a*b^3 + (a^3*b + a^
2*b^2)*k)*x))/sqrt(a^2*b^2 + a*b^3 + (a^3*b + a^2*b^2)*k)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx^2-1}{(akx+b)\sqrt{(kx-1)(x-1)x}\,(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^2-1)/(b*x+a)/((1-x)*x*(-k*x+1))^(1/2)/(a*k*x+b),x, algorithm
="giac")
```

```
[Out] integrate((k*x^2 - 1)/((a*k*x + b)*sqrt((k*x - 1)*(x - 1)*x)*(b*x + a)), x)
```

maple [C] time = 0.07, size = 315, normalized size = 3.58

$$-\frac{2\sqrt{-\left(x-\frac{1}{k}\right)k}\,\sqrt{\frac{-1+x}{\frac{1}{k}-1}}\,\sqrt{kx}\,\operatorname{EllipticF}\left(\sqrt{-\left(x-\frac{1}{k}\right)k},\sqrt{\frac{1}{k\left(\frac{1}{k}-1\right)}}\right)}{abk\sqrt{k\,x^3-k\,x^2-x^2+x}}+\frac{2\sqrt{-\left(x-\frac{1}{k}\right)k}\,\sqrt{\frac{-1+x}{\frac{1}{k}-1}}\,\sqrt{kx}\,\operatorname{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k}\right)k},\frac{1}{k\left(\frac{1}{k}+\frac{b}{ak}\right)},\sqrt{\frac{1}{k\left(\frac{1}{k}-1\right)}}\right)}{a^2k^2\sqrt{k\,x^3-k\,x^2-x^2+x}\,\left(\frac{1}{k}+\frac{b}{ak}\right)}+\frac{2\sqrt{-\left(x-\frac{1}{k}\right)k}\,\sqrt{\frac{-1+x}{\frac{1}{k}-1}}\,\sqrt{kx}\,\operatorname{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k}\right)k},\frac{1}{k\left(\frac{1}{k}+\frac{a}{b}\right)},\sqrt{\frac{1}{k\left(\frac{1}{k}-1\right)}}\right)}{b^2k\sqrt{k\,x^3-k\,x^2-x^2+x}\,\left(\frac{1}{k}+\frac{a}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^2-1)/(b*x+a)/((1-x)*x*(-k*x+1))^(1/2)/(a*k*x+b),x)
```

```
[Out] -2/a/b/k*(-(x-1/k)*k)^(1/2)*((-1+x)/(1/k-1))^(1/2)*(k*x)^(1/2)/(k*x^3-k*x^2
-x^2+x)^(1/2)*EllipticF((-(x-1/k)*k)^(1/2),(1/k/(1/k-1))^(1/2))+2/a^2/k^2*(
-(x-1/k)*k)^(1/2)*((-1+x)/(1/k-1))^(1/2)*(k*x)^(1/2)/(k*x^3-k*x^2-x^2+x)^(1
/2)/(1/k+b/a/k)*EllipticPi((-(x-1/k)*k)^(1/2),1/k/(1/k+b/a/k),(1/k/(1/k-1))
^(1/2))+2/b^2/k*(-(x-1/k)*k)^(1/2)*((-1+x)/(1/k-1))^(1/2)*(k*x)^(1/2)/(k*x^
3-k*x^2-x^2+x)^(1/2)/(1/k+a/b)*EllipticPi((-(x-1/k)*k)^(1/2),1/k/(1/k+a/b),
(1/k/(1/k-1))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx^2-1}{(akx+b)\sqrt{(kx-1)(x-1)x}\,(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^2-1)/(b*x+a)/((1-x)*x*(-k*x+1))^(1/2)/(a*k*x+b),x, algorithm
="maxima")
```

```
[Out] integrate((k*x^2 - 1)/((a*k*x + b)*sqrt((k*x - 1)*(x - 1)*x)*(b*x + a)), x)
```

mupad [B] time = 4.31, size = 103, normalized size = 1.17

$$\frac{\ln\left(\frac{2\,\sqrt{x\,(k\,x-1)\,(x-1)}\,\sqrt{a\,b\,(a+b)\,(b+a\,k)}+b^2\,x\,1\mathrm{i}-a\,b\,1\mathrm{i}+a^2\,k\,x\,1\mathrm{i}+a\,b\,x\,2\mathrm{i}+a\,b\,k\,x\,2\mathrm{i}-a\,b\,k\,x^2\,1\mathrm{i}}{(b+a\,k\,x)\,(a+b\,x)}\right)\,1\mathrm{i}}{\sqrt{a\,b\,\left(a+b\right)\,\left(b+a\,k\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^2 - 1)/((b + a*k*x)*(a + b*x)*(x*(k*x - 1)*(x - 1))^(1/2)),x)
```

```
[Out] (log((b^2*x*1i - a*b*1i + 2*(x*(k*x - 1)*(x - 1))^(1/2)*(a*b*(a + b)*(b + a
*k))^(1/2) + a^2*k*x*1i + a*b*x*2i + a*b*k*x*2i - a*b*k*x^2*1i)/((b + a*k*x
)*(a + b*x)))*1i)/(a*b*(a + b)*(b + a*k))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx^2-1}{\sqrt{x\,(x-1)\,(kx-1)}\,\left(a+bx\right)\left(akx+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x**2-1)/(b*x+a)/((1-x)*x*(-k*x+1))**(1/2)/(a*k*x+b),x)
```

```
[Out] Integral((k*x**2 - 1)/(sqrt(x*(x - 1)*(k*x - 1))*(a + b*x)*(a*k*x + b)), x)
```

3.1065 $\int \frac{x(-b+x)\left(ab-2ax+x^2\right)}{\sqrt{x(-a+x)(-b+x)}\left(-a^2+2ax+\left(-1+b^2d\right)x^2-2bdx^3+dx^4\right)}\,dx$

Optimal. Leaf size=88

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{x^2(-a-b)+abx+x^3}}{a-x}\right)}{d^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{x^2(-a-b)+abx+x^3}}{a-x}\right)}{d^{3/4}}$$

Rubi [F] time = 16.93, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(-b+x)\left(ab-2ax+x^2\right)}{\sqrt{x(-a+x)(-b+x)}\left(-a^2+2ax+\left(-1+b^2d\right)x^2-2bdx^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-b + x)*(a*b - 2*a*x + x^2))/(Sqrt[x*(-a + x)*(-b + x)]*(-a^2 + 2*a
*x + (-1 + b^2*d)*x^2 - 2*b*d*x^3 + d*x^4)),x]
```

```
[Out] (4*a*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][(x^4*Sqrt[-b
 + x^2])/(Sqrt[-a + x^2]*(a^2 - 2*a*x^2 + (1 - b^2*d)*x^4 + 2*b*d*x^6 - d*x
^8)), x], x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x] + (2*a*b*Sqrt[x]*Sqrt[-a + x
]*Sqrt[-b + x]*Defer[Subst][Defer[Int][(x^2*Sqrt[-b + x^2])/(Sqrt[-a + x^2]
*(-a^2 + 2*a*x^2 - (1 - b^2*d)*x^4 - 2*b*d*x^6 + d*x^8)), x], x, Sqrt[x]])/
Sqrt[(a - x)*(b - x)*x] + (2*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst]
[Defer[Int][(x^6*Sqrt[-b + x^2])/(Sqrt[-a + x^2]*(-a^2 + 2*a*x^2 - (1 - b^2
*d)*x^4 - 2*b*d*x^6 + d*x^8)), x], x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x]
```

Rubi steps

$$\int \frac{x(-b+x)\left(ab-2ax+x^2\right)}{\sqrt{x(-a+x)(-b+x)}\left(-a^2+2ax+\left(-1+b^2d\right)x^2-2bdx^3+dx^4\right)}\,dx=\frac{\left(\sqrt{x}\,\sqrt{-a+x}\,\sqrt{-b+x}\right)\int\ldots}{\sqrt{x(-a+x)(-b\ldots}}$$

$$=\frac{\left(2\sqrt{x}\,\sqrt{-a+x}\,\sqrt{-b+x}\right)\text{Subst}\left(\int\ldots\right.}{\sqrt{x(-\ldots}}$$

$$=\frac{\left(2\sqrt{x}\,\sqrt{-a+x}\,\sqrt{-b+x}\right)\text{Subst}\left(\int\left(\ldots\right.\right.}{}$$

$$=\frac{\left(2\sqrt{x}\,\sqrt{-a+x}\,\sqrt{-b+x}\right)\text{Subst}\left(\int\ldots\right.}{\sqrt{x(-a\ldots}}$$

Mathematica [C] time = 15.22, size = 31196, normalized size = 354.50

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(-b + x)*(a*b - 2*a*x + x^2))/(Sqrt[x*(-a + x)*(-b + x)]*(-a^2
 + 2*a*x + (-1 + b^2*d)*x^2 - 2*b*d*x^3 + d*x^4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.93, size = 88, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}{a-x}\right)}{d^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}{a-x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-b + x)*(a*b - 2*a*x + x^2))/(Sqrt[x*(-a + x)*(-b +
x)]*(-a^2 + 2*a*x + (-1 + b^2*d)*x^2 - 2*b*d*x^3 + d*x^4)),x]
```

```
[Out] -(ArcTan[(d^(1/4)*Sqrt[a*b*x + (-a - b)*x^2 + x^3])/(a - x)]/d^(3/4)) + Arc
Tanh[(d^(1/4)*Sqrt[a*b*x + (-a - b)*x^2 + x^3])/(a - x)]/d^(3/4)
```

fricas [B] time = 1.20, size = 384, normalized size = 4.36

$$-\left(\frac{1}{d^3}\right)^{\frac14}\arctan\left(-\frac{\sqrt{abx-(a+b)x^2+x^3}\,d^2\left(\frac{1}{d^3}\right)^{\frac34}}{bx-x^2}\right)-\frac14\left(\frac{1}{d^3}\right)^{\frac14}\log\left(\frac{2bdx^3-dx^4-(b^2d+1)x^2-a^2+2ax+2\sqrt{abx-(a+b)x^2+x^3}\left((bd^3x-d^3x^2)\left(\frac{1}{d^3}\right)^{\frac34}+(ad-dx)\left(\frac{1}{d^3}\right)^{\frac14}\right)-2(abd^2x-(a+b)d^2x^2+d^2x^3)\sqrt{\frac{1}{d^3}}}{2bdx^3-dx^4-(b^2d-1)x^2+a^2-2ax}\right)+\frac14\left(\frac{1}{d^3}\right)^{\frac14}\log\left(\frac{2bdx^3-dx^4-(b^2d+1)x^2-a^2+2ax-2\sqrt{abx-(a+b)x^2+x^3}\left((bd^3x-d^3x^2)\left(\frac{1}{d^3}\right)^{\frac34}+(ad-dx)\left(\frac{1}{d^3}\right)^{\frac14}\right)-2(abd^2x-(a+b)d^2x^2+d^2x^3)\sqrt{\frac{1}{d^3}}}{2bdx^3-dx^4-(b^2d-1)x^2+a^2-2ax}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(a*b-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(-a^2+2*a*x+(b^2
*d-1)*x^2-2*b*d*x^3+d*x^4),x, algorithm="fricas")
```

```
[Out] -(d^(-3))^(1/4)*arctan(-sqrt(a*b*x - (a + b)*x^2 + x^3)*d^2*(d^(-3))^(3/4)/
(b*x - x^2)) - 1/4*(d^(-3))^(1/4)*log((2*b*d*x^3 - d*x^4 - (b^2*d + 1)*x^2
- a^2 + 2*a*x + 2*sqrt(a*b*x - (a + b)*x^2 + x^3)*((b*d^3*x - d^3*x^2)*(d^(
-3))^(3/4) + (a*d - d*x)*(d^(-3))^(1/4)) - 2*(a*b*d^2*x - (a + b)*d^2*x^2 +
 d^2*x^3)*sqrt(d^(-3)))/(2*b*d*x^3 - d*x^4 - (b^2*d - 1)*x^2 + a^2 - 2*a*x)
) + 1/4*(d^(-3))^(1/4)*log((2*b*d*x^3 - d*x^4 - (b^2*d + 1)*x^2 - a^2 + 2*a
*x - 2*sqrt(a*b*x - (a + b)*x^2 + x^3)*((b*d^3*x - d^3*x^2)*(d^(-3))^(3/4)
+ (a*d - d*x)*(d^(-3))^(1/4)) - 2*(a*b*d^2*x - (a + b)*d^2*x^2 + d^2*x^3)*s
qrt(d^(-3)))/(2*b*d*x^3 - d*x^4 - (b^2*d - 1)*x^2 + a^2 - 2*a*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ab-2\,ax+x^2)(b-x)x}{(2\,bdx^3-dx^4-(b^2d-1)x^2+a^2-2\,ax)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(a*b-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(-a^2+2*a*x+(b^2
*d-1)*x^2-2*b*d*x^3+d*x^4),x, algorithm="giac")
```

```
[Out] integrate((a*b - 2*a*x + x^2)*(b - x)*x/((2*b*d*x^3 - d*x^4 - (b^2*d - 1)*x
^2 + a^2 - 2*a*x)*sqrt((a - x)*(b - x)*x)), x)
```

maple [C] time = 0.06, size = 437, normalized size = 4.97

$$-\frac{2a\sqrt{-\frac{-a+x}{a}}\sqrt{\frac{-b+x}{a-b}}\sqrt{\frac{x}{a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{-a+x}{a}},\sqrt{\frac{a}{a-b}}\right)}{d\sqrt{abx-a\,x^2-b\,x^2+x^3}}+\frac{\sum_{\_\alpha=\mathrm{RootOf}(d\_Z^4-2bd\_Z^3+(b^2d-1)\_Z^2+2a\_Z-a^2)}\frac{(-2\_\alpha^3ad+\_\alpha^3bd+3\_\alpha^2abd-\_\alpha^2b^2d-\_\alpha ab^2d+\_\alpha^2-2\_\alpha a+a^2)(\_\alpha^3d+\_\alpha^2ad-2\_\alpha^2bd+\_\alpha a^2d-2\_\alpha abd+\_\alpha b^2d+a^3d-2a^2bd+ab^2d-\_\alpha+a)\sqrt{-\frac{-a+x}{a}}\sqrt{\frac{-b+x}{a-b}}\sqrt{\frac{x}{a}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-a+x}{a}},\ldots,\sqrt{\frac{a}{a-b}}\right)}{(-2\_\alpha^3d+3\_\alpha^2bd-\_\alpha b^2d+\_\alpha-a)(a^2-2ab+b^2)\sqrt{x(ab-ax-bx+x^2)}}}{d^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b+x)*(a*b-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(-a^2+2*a*x+(b^2*d-1)*
x^2-2*b*d*x^3+d*x^4),x)
```

```
[Out] -2/d*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*
x^2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))+1/d^2/a*sum((-2
*_alpha^3*a*d+_alpha^3*b*d+3*_alpha^2*a*b*d-_alpha^2*b^2*d-_alpha*a*b^2*d+_
alpha^2-2*_alpha*a+a^2)/(-2*_alpha^3*d+3*_alpha^2*b*d-_alpha*b^2*d+_alpha-a
)*(_alpha^3*d+_alpha^2*a*d-2*_alpha^2*b*d+_alpha*a^2*d-2*_alpha*a*b*d+_alph
a*b^2*d+a^3*d-2*a^2*b*d+a*b^2*d-_alpha+a)/(a^2-2*a*b+b^2)*(-(-a+x)/a)^(1/2)
```

```
*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(x*(a*b-a*x-b*x+x^2))^(1/2)*EllipticPi(
(-(-a+x)/a)^(1/2),(_alpha^3*d+_alpha^2*a*d-2*_alpha^2*b*d+_alpha*a^2*d-2*_a
lpha*a*b*d+_alpha*b^2*d+a^3*d-2*a^2*b*d+a*b^2*d-_alpha+a)/a/d/(a^2-2*a*b+b^
2),(a/(a-b))^(1/2)),_alpha=RootOf(d*_Z^4-2*b*d*_Z^3+(b^2*d-1)*_Z^2+2*a*_Z-a
^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ab - 2\,ax + x^2\right)(b - x)x}{\left(2\,bdx^3 - dx^4 - \left(b^2d - 1\right)x^2 + a^2 - 2\,ax\right)\sqrt{(a - x)(b - x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(a*b-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(-a^2+2*a*x+(b^2
*d-1)*x^2-2*b*d*x^3+d*x^4),x, algorithm="maxima")
```

```
[Out] integrate((a*b - 2*a*x + x^2)*(b - x)*x/((2*b*d*x^3 - d*x^4 - (b^2*d - 1)*x
^2 + a^2 - 2*a*x)*sqrt((a - x)*(b - x)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x\,\left(b-x\right)\,\left(x^2-2\,a\,x+a\,b\right)}{\sqrt{x\,\left(a-x\right)\,\left(b-x\right)}\,\left(-a^2+2\,a\,x+d\,x^4-2\,b\,d\,x^3+\left(b^2\,d-1\right)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(b - x)*(a*b - 2*a*x + x^2))/((x*(a - x)*(b - x))^(1/2)*(x^2*(b^2*d
 - 1) + 2*a*x + d*x^4 - a^2 - 2*b*d*x^3)),x)
```

```
[Out] int(-(x*(b - x)*(a*b - 2*a*x + x^2))/((x*(a - x)*(b - x))^(1/2)*(x^2*(b^2*d
 - 1) + 2*a*x + d*x^4 - a^2 - 2*b*d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(a*b-2*a*x+x**2)/(x*(-a+x)*(-b+x))**(1/2)/(-a**2+2*a*x+(
b**2*d-1)*x**2-2*b*d*x**3+d*x**4),x)
```

```
[Out] Timed out
```

3.1066 $\int \frac{1+x^4}{\left(-1-x^2+x^4\right)\sqrt[3]{-x+x^5}}\,dx$

Optimal. Leaf size=88

$$\frac{1}{2}\log\left(\sqrt[3]{x^5-x}-x\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^5-x}+x}\right)-\frac{1}{4}\log\left(\sqrt[3]{x^5-x}\,x+\left(x^5-x\right)^{2/3}+x^2\right)$$

Rubi [C] time = 1.16, antiderivative size = 305, normalized size of antiderivative = 3.47, number of steps used = 22, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2056, 6715, 6728, 246, 245, 1438, 430, 429, 465, 511, 510}

$$-\frac{3\sqrt[3]{1-x^4}\,xF_1\left(\frac{1}{6};\frac{1}{3},1;\frac{7}{6};x^4,\frac{2x^4}{3-\sqrt{5}}\right)}{2\sqrt[3]{x^5-x}}-\frac{3\sqrt[3]{1-x^4}\,xF_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};\frac{2x^4}{3+\sqrt{5}},x^4\right)}{2\sqrt[3]{x^5-x}}-\frac{3\left(1-\sqrt{5}\right)\sqrt[3]{1-x^4}\,x^3F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^4,\frac{2x^4}{3-\sqrt{5}}\right)}{8\left(3-\sqrt{5}\right)\sqrt[3]{x^5-x}}-\frac{3\left(1+\sqrt{5}\right)\sqrt[3]{1-x^4}\,x^3F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^4,\frac{2x^4}{3+\sqrt{5}}\right)}{8\left(3+\sqrt{5}\right)\sqrt[3]{x^5-x}}+\frac{3\sqrt[3]{1-x^4}\,x\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^4\right)}{2\sqrt[3]{x^5-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^4)/((-1 - x^2 + x^4)*(-x + x^5)^(1/3)),x]
```

```
[Out] (-3*x*(1 - x^4)^(1/3)*AppellF1[1/6, 1/3, 1, 7/6, x^4, (2*x^4)/(3 - Sqrt[5])
])/(2*(-x + x^5)^(1/3)) - (3*x*(1 - x^4)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6, (
2*x^4)/(3 + Sqrt[5]), x^4])/(2*(-x + x^5)^(1/3)) - (3*(1 - Sqrt[5])*x^3*(1
- x^4)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, x^4, (2*x^4)/(3 - Sqrt[5])])/(8*(3
- Sqrt[5])*(-x + x^5)^(1/3)) - (3*(1 + Sqrt[5])*x^3*(1 - x^4)^(1/3)*AppellF
1[2/3, 1/3, 1, 5/3, x^4, (2*x^4)/(3 + Sqrt[5])])/(8*(3 + Sqrt[5])*(-x + x^5
)^(1/3)) + (3*x*(1 - x^4)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, x^4])/(2*(
-x + x^5)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
```

```
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1+x^4}{\left(-1-x^2+x^4\right)\sqrt[3]{-x+x^5}}\,dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\int \frac{1+x^4}{\sqrt[3]{x}\sqrt[3]{-1+x^4}\left(-1-x^2+x^4\right)}\,dx}{\sqrt[3]{-x+x^5}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \frac{1+x^6}{\sqrt[3]{-1+x^6}\left(-1-x^3+x^6\right)}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{-x+x^5}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{-1+x^6}}+\frac{2+x^3}{\sqrt[3]{-1+x^6}\left(-1-x^3+x^6\right)}\right)dx, x, x^{2/3}\right)}{2\sqrt[3]{-x+x^5}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x^6}}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{-x+x^5}} + \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \ldots\right.}{2\sqrt[3]{-x+\ldots}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1-x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x^6}}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{-x+x^5}} + \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \left(\frac{\ldots}{(-1-\ldots}\right.\right.}{\ldots}$$

$$= \frac{3x\sqrt[3]{1-x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^4\right)}{2\sqrt[3]{-x+x^5}} + \frac{\left(3\left(1-\sqrt{5}\right)\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(-1+\sqrt{5}+2x^3\ldots\right.}\right.}{2\sqrt[3]{-x+x^5}}$$

$$= \frac{3x\sqrt[3]{1-x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^4\right)}{2\sqrt[3]{-x+x^5}} + \frac{\left(3\left(1-\sqrt{5}\right)\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \left(\frac{1-\ldots}{2\sqrt[3]{-1+x^6}\left(-\ldots\right.}\right.\right.}{2\sqrt[3]{-x+\ldots}}$$

$$= \frac{3x\sqrt[3]{1-x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^4\right)}{2\sqrt[3]{-x+x^5}} + \frac{\left(3\left(1-\sqrt{5}\right)\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{-1+x^6}\left(-3+\ldots\right.}\right.}{2\sqrt[3]{-x+x^5}}$$

$$= \frac{3x\sqrt[3]{1-x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^4\right)}{2\sqrt[3]{-x+x^5}} + \frac{\left(3\left(1-\sqrt{5}\right)^2\sqrt[3]{x}\sqrt[3]{1-x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x^6}\left(-3+\sqrt{\ldots}\right.}\right.}{4\sqrt[3]{-x+x^5}}$$

$$= -\frac{3x\sqrt[3]{1-x^4}\,F_1\left(\frac{1}{6};\frac{1}{3},1;\frac{7}{6};x^4,\frac{2x^4}{3-\sqrt{5}}\right)}{2\sqrt[3]{-x+x^5}} - \frac{3x\sqrt[3]{1-x^4}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};\frac{2x^4}{3+\sqrt{5}},x^4\right)}{2\sqrt[3]{-x+x^5}} + \frac{3x\ldots}{\ldots}$$

$$= -\frac{3x\sqrt[3]{1-x^4}\,F_1\left(\frac{1}{6};\frac{1}{3},1;\frac{7}{6};x^4,\frac{2x^4}{3-\sqrt{5}}\right)}{2\sqrt[3]{-x+x^5}} - \frac{3x\sqrt[3]{1-x^4}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};\frac{2x^4}{3+\sqrt{5}},x^4\right)}{2\sqrt[3]{-x+x^5}} - \frac{3\left(\ldots\right.}{\ldots}$$

Mathematica [F] time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{\left(-1-x^2+x^4\right)\sqrt[3]{-x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^4)/((-1 - x^2 + x^4)*(-x + x^5)^(1/3)),x]
[Out] Integrate[(1 + x^4)/((-1 - x^2 + x^4)*(-x + x^5)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.45, size = 88, normalized size = 1.00

$$\frac{1}{2}\log\left(\sqrt[3]{x^5-x}-x\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^5-x}+x}\right)-\frac{1}{4}\log\left(\sqrt[3]{x^5-x}\,x+\left(x^5-x\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)/((-1 - x^2 + x^4)*(-x + x^5)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-x + x^5)^(1/3))]) + Log[-x + (-x
+ x^5)^(1/3)]/2 - Log[x^2 + x*(-x + x^5)^(1/3) + (-x + x^5)^(2/3)]/4
```

fricas [A] time = 1.20, size = 109, normalized size = 1.24

$$-\frac{1}{2}\sqrt{3}\arctan\left(-\frac{4\sqrt{3}\left(x^5-x\right)^{\frac{1}{3}}x+\sqrt{3}\left(x^4-1\right)-2\sqrt{3}\left(x^5-x\right)^{\frac{2}{3}}}{x^4+8x^2-1}\right)+\frac{1}{4}\log\left(\frac{x^4-x^2+3\left(x^5-x\right)^{\frac{1}{3}}x-3\left(x^5-x\right)^{\frac{2}{3}}-1}{x^4-x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-x^2-1)/(x^5-x)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan(-(4*sqrt(3)*(x^5 - x)^(1/3)*x + sqrt(3)*(x^4 - 1) - 2*s
qrt(3)*(x^5 - x)^(2/3))/(x^4 + 8*x^2 - 1)) + 1/4*log((x^4 - x^2 + 3*(x^5 -
x)^(1/3)*x - 3*(x^5 - x)^(2/3) - 1)/(x^4 - x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^5-x\right)^{\frac{1}{3}}\left(x^4-x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-x^2-1)/(x^5-x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/((x^5 - x)^(1/3)*(x^4 - x^2 - 1)), x)
```

maple [C] time = 5.48, size = 591, normalized size = 6.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^4-x^2-1)/(x^5-x)^(1/3),x)
```

```
[Out] 1/2*ln((11545253597330080*RootOf(4*_Z^2+2*_Z+1)^2*x^4+31816769631648996*Roo
tOf(4*_Z^2+2*_Z+1)*x^4-43294700989987800*RootOf(4*_Z^2+2*_Z+1)^2*x^2+104423
81802726383*x^4-41156279639771682*RootOf(4*_Z^2+2*_Z+1)*(x^5-x)^(2/3)+41801
30780591496*RootOf(4*_Z^2+2*_Z+1)*(x^5-x)^(1/3)*x-10715344468797670*RootOf(
4*_Z^2+2*_Z+1)*x^2-11545253597330080*RootOf(4*_Z^2+2*_Z+1)^2-20900653902957
48*(x^5-x)^(2/3)-18488074429590093*x*(x^5-x)^(1/3)+2198396168995028*x^2-318
16769631648996*RootOf(4*_Z^2+2*_Z+1)-10442381802726383)/(x^4-x^2-1))-1/2*ln
(-(2751668921349968*RootOf(4*_Z^2+2*_Z+1)^2*x^4-9556171565521246*RootOf(4*_
Z^2+2*_Z+1)*x^4-10318758455062380*RootOf(4*_Z^2+2*_Z+1)^2*x^2+8243985633731
355*x^4-41156279639771682*RootOf(4*_Z^2+2*_Z+1)*(x^5-x)^(2/3)+3697614885918
0186*RootOf(4*_Z^2+2*_Z+1)*(x^5-x)^(1/3)*x+9952757579256536*RootOf(4*_Z^2+2
*_Z+1)*x^2-2751668921349968*RootOf(4*_Z^2+2*_Z+1)^2-18488074429590093*(x^5-
x)^(2/3)-2090065390295748*x*(x^5-x)^(1/3)+10442381802726383*x^2+95561715655
21246*RootOf(4*_Z^2+2*_Z+1)-8243985633731355)/(x^4-x^2-1))-ln(-(27516689213
49968*RootOf(4*_Z^2+2*_Z+1)^2*x^4-9556171565521246*RootOf(4*_Z^2+2*_Z+1)*x^
4-10318758455062380*RootOf(4*_Z^2+2*_Z+1)^2*x^2+8243985633731355*x^4-411562
79639771682*RootOf(4*_Z^2+2*_Z+1)*(x^5-x)^(2/3)+36976148859180186*RootOf(4*
_Z^2+2*_Z+1)*(x^5-x)^(1/3)*x+9952757579256536*RootOf(4*_Z^2+2*_Z+1)*x^2-275
1668921349968*RootOf(4*_Z^2+2*_Z+1)^2-18488074429590093*(x^5-x)^(2/3)-20900
65390295748*x*(x^5-x)^(1/3)+10442381802726383*x^2+9556171565521246*RootOf(4
*_Z^2+2*_Z+1)-8243985633731355)/(x^4-x^2-1))*RootOf(4*_Z^2+2*_Z+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^5-x\right)^{\frac{1}{3}}\left(x^4-x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-x^2-1)/(x^5-x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/((x^5 - x)^(1/3)*(x^4 - x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^4+1}{\left(x^5-x\right)^{1/3}\,\left(-x^4+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 + 1)/((x^5 - x)^(1/3)*(x^2 - x^4 + 1)),x)
```

```
[Out] int(-(x^4 + 1)/((x^5 - x)^(1/3)*(x^2 - x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\sqrt[3]{x\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}\,\left(x^4-x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**4-x**2-1)/(x**5-x)**(1/3),x)
```

```
[Out] Integral((x**4 + 1)/((x*(x - 1)*(x + 1)*(x**2 + 1))**(1/3)*(x**4 - x**2 - 1
)), x)
```

3.1067 $\int \frac{x^3}{\left(-1+x^6\right)^{2/3}}\,dx$

Optimal. Leaf size=88

$$-\frac{1}{6}\log\left(\sqrt[3]{x^6-1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)$$

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {275, 331, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(1-\frac{x^2}{\sqrt[3]{x^6-1}}\right)-\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6-1}}+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(\frac{x^4}{\left(x^6-1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6-1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(-1 + x^6)^(2/3),x]
```

```
[Out] -1/2*ArcTan[(1 + (2*x^2)/(-1 + x^6)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 - x^2/(
-1 + x^6)^(1/3)]/6 + Log[1 + x^4/(-1 + x^6)^(2/3) + x^2/(-1 + x^6)^(1/3)]/1
2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(-1+x^6\right)^{2/3}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\left(-1+x^3\right)^{2/3}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{x^2}{\sqrt[3]{-1+x^6}}\right)\\
&= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{x^2}{\sqrt[3]{-1+x^6}}\right) - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1-x}{1+x+x^2}\,dx, x, \frac{x^2}{\sqrt[3]{-1+x^6}}\right)\\
&= -\frac{1}{6}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right) + \frac{1}{12}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{x^2}{\sqrt[3]{-1+x^6}}\right) - \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x+\right.\\
&= -\frac{1}{6}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right) + \frac{1}{12}\log\left(1+\frac{x^4}{\left(-1+x^6\right)^{2/3}}+\frac{x^2}{\sqrt[3]{-1+x^6}}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.\\
&= -\frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{-1+x^6}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{6}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right) + \frac{1}{12}\log\left(1+\frac{x^4}{\left(-1+x^6\right)^{2/3}}+\frac{x^2}{\sqrt[3]{-1+x^6}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.40

$$\frac{x^4\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x^6}{x^6-1}\right)}{4\left(x^6-1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(-1 + x^6)^(2/3),x]
[Out] (x^4*Hypergeometric2F1[2/3, 1, 5/3, x^6/(-1 + x^6)])/(4*(-1 + x^6)^(2/3))
```

IntegrateAlgebraic [A] time = 0.67, size = 88, normalized size = 1.00

$$-\frac{1}{6}\log\left(\sqrt[3]{x^6-1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(-1 + x^6)^(2/3),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(-1 + x^6)^(1/3))]/Sqrt[3] - Log[-x^2 + (-1 + x^6)^(1/3)]/6 + Log[x^4 + x^2*(-1 + x^6)^(1/3) + (-1 + x^6)^(2/3)]/12
```

fricas [A] time = 0.41, size = 82, normalized size = 0.93

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}}{3x^2}\right)-\frac{1}{6}\log\left(-\frac{x^2-\left(x^6-1\right)^{\frac{1}{3}}}{x^2}\right)+\frac{1}{12}\log\left(\frac{x^4+\left(x^6-1\right)^{\frac{1}{3}}x^2+\left(x^6-1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6-1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*(sqrt(3)*x^2 + 2*sqrt(3)*(x^6 - 1)^(1/3))/x^2) - 1/6*log(-(x^2 - (x^6 - 1)^(1/3))/x^2) + 1/12*log((x^4 + (x^6 - 1)^(1/3)*x^2 + (x^6 - 1)^(2/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\left(x^6-1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6-1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x^3/(x^6 - 1)^(2/3), x)
```

maple [C] time = 0.32, size = 33, normalized size = 0.38

$$\frac{\left(-\mathrm{signum}\left(x^6-1\right)\right)^{\frac{2}{3}}x^4\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^6\right)}{4\mathrm{signum}\left(x^6-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6-1)^(2/3),x)
```

```
[Out] 1/4/signum(x^6-1)^(2/3)*(-signum(x^6-1))^(2/3)*x^4*hypergeom([2/3,2/3],[5/3],x^6)
```

maxima [A] time = 0.49, size = 69, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)+\frac{1}{12}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6-1\right)^{\frac{2}{3}}}{x^4}+1\right)-\frac{1}{6}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6-1)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3)/x^2 + 1)) + 1/12*log((x^6 - 1)^(1/3)/x^2 + (x^6 - 1)^(2/3)/x^4 + 1) - 1/6*log((x^6 - 1)^(1/3)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^3}{{\left(x^6-1\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6 - 1)^(2/3),x)
```

```
[Out] int(x^3/(x^6 - 1)^(2/3), x)
```

sympy [C] time = 0.79, size = 32, normalized size = 0.36

$$\frac{x^4 e^{-\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| x^6\right)}{6\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6-1)**(2/3),x)
```

```
[Out] x**4*exp(-2*I*pi/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), x**6)/(6*gamma(5/3
))
```

3.1068 $\int \frac{x}{\sqrt[3]{-1+x^6}}\,dx$

Optimal. Leaf size=88

$$-\frac{1}{6}\log\left(\sqrt[3]{x^6-1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}x^2\right)$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 0.60, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {275, 239}

$$\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6-1}}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}\log\left(x^2-\sqrt[3]{x^6-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x^6)^(1/3),x]
```

```
[Out] ArcTan[(1 + (2*x^2)/(-1 + x^6)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - Log[x^2 - (-1
+ x^6)^(1/3)]/4
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt[3]{-1+x^6}}\,dx&=\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x^3}}\,dx,x,x^2\right)\\&=\frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{-1+x^6}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}\log\left(x^2-\sqrt[3]{-1+x^6}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.94

$$\frac{1}{12}\left(-2\log\left(1-\frac{x^2}{\sqrt[3]{x^6-1}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6-1}}+1}{\sqrt{3}}\right)+\log\left(\frac{x^4}{\left(x^6-1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6-1}}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x^6)^(1/3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*x^2)/(-1 + x^6)^(1/3))/Sqrt[3]] - 2*Log[1 - x^2/(
-1 + x^6)^(1/3)] + Log[1 + x^4/(-1 + x^6)^(2/3) + x^2/(-1 + x^6)^(1/3)])/12
```

IntegrateAlgebraic [A] time = 0.55, size = 88, normalized size = 1.00

$$-\frac{1}{6}\log\left(\sqrt[3]{x^6-1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(-1 + x^6)^(1/3),x]
```

```
[Out] ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(-1 + x^6)^(1/3))]/(2*Sqrt[3]) - Log[-x^2 + (
-1 + x^6)^(1/3)]/6 + Log[x^4 + x^2*(-1 + x^6)^(1/3) + (-1 + x^6)^(2/3)]/12
```

fricas [A] time = 0.41, size = 82, normalized size = 0.93

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}}{3x^2}\right)-\frac{1}{6}\log\left(-\frac{x^2-\left(x^6-1\right)^{\frac{1}{3}}}{x^2}\right)+\frac{1}{12}\log\left(\frac{x^4+\left(x^6-1\right)^{\frac{1}{3}}x^2+\left(x^6-1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*(sqrt(3)*x^2 + 2*sqrt(3)*(x^6 - 1)^(1/3))/x^2) - 1/
6*log(-(x^2 - (x^6 - 1)^(1/3))/x^2) + 1/12*log((x^4 + (x^6 - 1)^(1/3)*x^2 +
 (x^6 - 1)^(2/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^6-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x/(x^6 - 1)^(1/3), x)
```

maple [C] time = 0.31, size = 33, normalized size = 0.38

$$\frac{\left(-\mathrm{signum}\left(x^6-1\right)\right)^{\frac{1}{3}}x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^6\right)}{2\mathrm{signum}\left(x^6-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6-1)^(1/3),x)
```

```
[Out] 1/2/signum(x^6-1)^(1/3)*(-signum(x^6-1))^(1/3)*x^2*hypergeom([1/3,1/3],[4/3
],x^6)
```

maxima [A] time = 0.40, size = 69, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)+\frac{1}{12}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6-1\right)^{\frac{2}{3}}}{x^4}+1\right)-\frac{1}{6}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3)/x^2 + 1)) + 1/12*log((x^
6 - 1)^(1/3)/x^2 + (x^6 - 1)^(2/3)/x^4 + 1) - 1/6*log((x^6 - 1)^(1/3)/x^2 -
 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(x^6-1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6 - 1)^(1/3),x)
```

```
[Out] int(x/(x^6 - 1)^(1/3), x)
```

sympy [C] time = 0.81, size = 31, normalized size = 0.35

$$\frac{x^2 e^{-\frac{i\pi}{3}} \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{3}, \frac{1}{3}\\ \frac{4}{3}\end{matrix}\right| x^6\right)}{6\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6-1)**(1/3),x)
```

```
[Out] x**2*exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**6)/(6*gamma(4/3))
```

3.1069 $\int \frac{x^3}{\left(1+x^6\right)^{2/3}}\,dx$

Optimal. Leaf size=88

$$-\frac{1}{6}\log\left(\sqrt[3]{x^6+1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right)$$

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {275, 331, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(1-\frac{x^2}{\sqrt[3]{x^6+1}}\right)-\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6+1}}+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(\frac{x^4}{\left(x^6+1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6+1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^6)^(2/3),x]
```

```
[Out] -1/2*ArcTan[(1 + (2*x^2)/(1 + x^6)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 - x^2/(1
 + x^6)^(1/3)]/6 + Log[1 + x^4/(1 + x^6)^(2/3) + x^2/(1 + x^6)^(1/3)]/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(1+x^6\right)^{2/3}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{\left(1+x^3\right)^{2/3}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{x^2}{\sqrt[3]{1+x^6}}\right)\\
&= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{x^2}{\sqrt[3]{1+x^6}}\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1-x}{1+x+x^2}\,dx, x, \frac{x^2}{\sqrt[3]{1+x^6}}\right)\\
&= -\frac{1}{6}\log\left(1-\frac{x^2}{\sqrt[3]{1+x^6}}\right)+\frac{1}{12}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{x^2}{\sqrt[3]{1+x^6}}\right)-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x+}\right.\\
&= -\frac{1}{6}\log\left(1-\frac{x^2}{\sqrt[3]{1+x^6}}\right)+\frac{1}{12}\log\left(1+\frac{x^4}{\left(1+x^6\right)^{2/3}}+\frac{x^2}{\sqrt[3]{1+x^6}}\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x\right.\\
&= -\frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{1+x^6}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{6}\log\left(1-\frac{x^2}{\sqrt[3]{1+x^6}}\right)+\frac{1}{12}\log\left(1+\frac{x^4}{\left(1+x^6\right)^{2/3}}+\frac{x^2}{\sqrt[3]{1+x^6}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.40

$$
\frac{x^4\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x^6}{x^6+1}\right)}{4\left(x^6+1\right)^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^6)^(2/3),x]
[Out] (x^4*Hypergeometric2F1[2/3, 1, 5/3, x^6/(1 + x^6)])/(4*(1 + x^6)^(2/3))
```

IntegrateAlgebraic [A] time = 0.71, size = 88, normalized size = 1.00

$$
-\frac{1}{6}\log\left(\sqrt[3]{x^6+1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(1 + x^6)^(2/3),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(1 + x^6)^(1/3))]/Sqrt[3] - Log[-x^2 + (
1 + x^6)^(1/3)]/6 + Log[x^4 + x^2*(1 + x^6)^(1/3) + (1 + x^6)^(2/3)]/12
```

fricas [A] time = 0.41, size = 82, normalized size = 0.93

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6+1\right)^{\frac{1}{3}}}{3x^2}\right)-\frac{1}{6}\log\left(-\frac{x^2-\left(x^6+1\right)^{\frac{1}{3}}}{x^2}\right)+\frac{1}{12}\log\left(\frac{x^4+\left(x^6+1\right)^{\frac{1}{3}}x^2+\left(x^6+1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*(sqrt(3)*x^2 + 2*sqrt(3)*(x^6 + 1)^(1/3))/x^2) - 1/6
*log(-(x^2 - (x^6 + 1)^(1/3))/x^2) + 1/12*log((x^4 + (x^6 + 1)^(1/3)*x^2 +
(x^6 + 1)^(2/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^6+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x^3/(x^6 + 1)^(2/3), x)
```

maple [C] time = 0.28, size = 17, normalized size = 0.19

$$\frac{x^4\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^6\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+1)^(2/3),x)
```

```
[Out] 1/4*x^4*hypergeom([2/3,2/3],[5/3],-x^6)
```

maxima [A] time = 0.45, size = 69, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)+\frac{1}{12}\log\left(\frac{\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6+1\right)^{\frac{2}{3}}}{x^4}+1\right)-\frac{1}{6}\log\left(\frac{\left(x^6+1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+1)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3)/x^2 + 1)) + 1/12*log((x^6
 + 1)^(1/3)/x^2 + (x^6 + 1)^(2/3)/x^4 + 1) - 1/6*log((x^6 + 1)^(1/3)/x^2 -
1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\left(x^6+1\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6 + 1)^(2/3),x)
```

```
[Out] int(x^3/(x^6 + 1)^(2/3), x)
```

sympy [C] time = 0.78, size = 29, normalized size = 0.33

$$\frac{x^4\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^6e^{i\pi}\right)}{6\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+1)**(2/3),x)
```

```
[Out] x**4*gamma(2/3)*hyper((2/3, 2/3), (5/3,), x**6*exp_polar(I*pi))/(6*gamma(5/
3))
```

3.1070 $\int \frac{x}{\sqrt[3]{1+x^6}} \, dx$

Optimal. Leaf size=88

$$-\frac{1}{6}\log\left(\sqrt[3]{x^6+1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}x^2\right)$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 0.60, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {275, 239}

$$\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6+1}}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}\log\left(x^2-\sqrt[3]{x^6+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^6)^(1/3),x]
```

```
[Out] ArcTan[(1 + (2*x^2)/(1 + x^6)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - Log[x^2 - (1 +
x^6)^(1/3)]/4
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt[3]{1+x^6}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^3}} \, dx, x, x^2\right)\\ &= \frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{1+x^6}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}\log\left(x^2-\sqrt[3]{1+x^6}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.94

$$\frac{1}{12}\left(-2\log\left(1-\frac{x^2}{\sqrt[3]{x^6+1}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6+1}}+1}{\sqrt{3}}\right)+\log\left(\frac{x^4}{\left(x^6+1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6+1}}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^6)^(1/3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*x^2)/(1 + x^6)^(1/3))/Sqrt[3]] - 2*Log[1 - x^2/(1
 + x^6)^(1/3)] + Log[1 + x^4/(1 + x^6)^(2/3) + x^2/(1 + x^6)^(1/3)])/12
```

IntegrateAlgebraic [A] time = 0.56, size = 88, normalized size = 1.00

$$-\frac{1}{6}\log\left(\sqrt[3]{x^6+1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(1 + x^6)^(1/3),x]
```

```
[Out] ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(1 + x^6)^(1/3))]/(2*Sqrt[3]) - Log[-x^2 + (1
 + x^6)^(1/3)]/6 + Log[x^4 + x^2*(1 + x^6)^(1/3) + (1 + x^6)^(2/3)]/12
```

fricas [A] time = 0.41, size = 82, normalized size = 0.93

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6+1\right)^{\frac{1}{3}}}{3x^2}\right)-\frac{1}{6}\log\left(-\frac{x^2-\left(x^6+1\right)^{\frac{1}{3}}}{x^2}\right)+\frac{1}{12}\log\left(\frac{x^4+\left(x^6+1\right)^{\frac{1}{3}}x^2+\left(x^6+1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*(sqrt(3)*x^2 + 2*sqrt(3)*(x^6 + 1)^(1/3))/x^2) - 1/
6*log(-(x^2 - (x^6 + 1)^(1/3))/x^2) + 1/12*log((x^4 + (x^6 + 1)^(1/3)*x^2 +
 (x^6 + 1)^(2/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(x^6+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x/(x^6 + 1)^(1/3), x)
```

maple [C] time = 0.28, size = 17, normalized size = 0.19

$$\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x^6\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6+1)^(1/3),x)
```

```
[Out] 1/2*x^2*hypergeom([1/3,1/3],[4/3],-x^6)
```

maxima [A] time = 0.44, size = 69, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)+\frac{1}{12}\log\left(\frac{\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6+1\right)^{\frac{2}{3}}}{x^4}+1\right)-\frac{1}{6}\log\left(\frac{\left(x^6+1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3)/x^2 + 1)) + 1/12*log((x^
6 + 1)^(1/3)/x^2 + (x^6 + 1)^(2/3)/x^4 + 1) - 1/6*log((x^6 + 1)^(1/3)/x^2 -
 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(x^6+1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6 + 1)^(1/3),x)
```

```
[Out] int(x/(x^6 + 1)^(1/3), x)
```

sympy [C] time = 0.74, size = 29, normalized size = 0.33

$$\frac{x^2\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|x^6e^{i\pi}\right)}{6\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6+1)**(1/3),x)
```

```
[Out] x**2*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**6*exp_polar(I*pi))/(6*gamma(4/
3))
```

3.1071 $\int \frac{-b+ax^8}{x^2\left(-b+ax^4\right)^{3/4}}\,dx$

Optimal. Leaf size=88

$$-\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{8a^{3/4}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{8a^{3/4}}+\frac{\left(x^4-4\right)\sqrt[4]{ax^4-b}}{4x}$$

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1487, 451, 331, 298, 203, 206}

$$-\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{8a^{3/4}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{8a^{3/4}}-\frac{\sqrt[4]{ax^4-b}}{x}+\frac{1}{4}x^3\sqrt[4]{ax^4-b}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^8)/(x^2*(-b + a*x^4)^(3/4)),x]
```

```
[Out] -((-b + a*x^4)^(1/4)/x) + (x^3*(-b + a*x^4)^(1/4))/4 - (3*b*ArcTan[(a^(1/4)
*x)/(-b + a*x^4)^(1/4)])/(8*a^(3/4)) + (3*b*ArcTanh[(a^(1/4)*x)/(-b + a*x^4
)^(1/4)])/(8*a^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1487

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 2*n*p - n + 1)*(d + e*x^n)^(
q + 1))/(e*f^(2*n*p - n + 1)*(m + 2*n*p + n*q + 1)), x] + Dist[1/(e*(m + 2*
n*p + n*q + 1)), Int[(f*x)^m*(d + e*x^n)^q*ExpandToSum[e*(m + 2*n*p + n*q +
 1)*((a + c*x^(2*n))^p - c^p*x^(2*n*p)) - d*c^p*(m + 2*n*p - n + 1)*x^(2*n*
p - n), x], x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IG
tQ[n, 0] && IGtQ[p, 0] && GtQ[2*n*p, n - 1] &&  !IntegerQ[q] && NeQ[m + 2*n
*p + n*q + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-b+ax^8}{x^2\left(-b+ax^4\right)^{3/4}}\,dx &= \frac{1}{4}x^3\sqrt[4]{-b+ax^4}+\frac{\int \frac{-4ab+3abx^4}{x^2\left(-b+ax^4\right)^{3/4}}\,dx}{4a}\\
&= -\frac{\sqrt[4]{-b+ax^4}}{x}+\frac{1}{4}x^3\sqrt[4]{-b+ax^4}+\frac{1}{4}(3b)\int \frac{x^2}{\left(-b+ax^4\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{-b+ax^4}}{x}+\frac{1}{4}x^3\sqrt[4]{-b+ax^4}+\frac{1}{4}(3b)\,\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)\\
&= -\frac{\sqrt[4]{-b+ax^4}}{x}+\frac{1}{4}x^3\sqrt[4]{-b+ax^4}+\frac{(3b)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)}{8\sqrt{a}}-\frac{(3b)\,\text{Subst}\left(\int\right.}{}\\
&= -\frac{\sqrt[4]{-b+ax^4}}{x}+\frac{1}{4}x^3\sqrt[4]{-b+ax^4}-\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{-b+ax^4}}\right)}{8a^{3/4}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{-b+ax^4}}\right)}{8a^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 88, normalized size = 1.00

$$\frac{2a^{3/4}\left(x^4-4\right)\sqrt[4]{ax^4-b}-3bx\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)+3bx\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{8a^{3/4}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^8)/(x^2*(-b + a*x^4)^(3/4)),x]
```

```
[Out] (2*a^(3/4)*(-4 + x^4)*(-b + a*x^4)^(1/4) - 3*b*x*ArcTan[(a^(1/4)*x)/(-b + a
*x^4)^(1/4)] + 3*b*x*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(8*a^(3/4)*x)
```

IntegrateAlgebraic [A] time = 0.56, size = 88, normalized size = 1.00

$$-\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{8a^{3/4}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{8a^{3/4}}+\frac{\left(x^4-4\right)\sqrt[4]{ax^4-b}}{4x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^8)/(x^2*(-b + a*x^4)^(3/4)),x]
```

```
[Out] ((-4 + x^4)*(-b + a*x^4)^(1/4))/(4*x) - (3*b*ArcTan[(a^(1/4)*x)/(-b + a*x^4
)^(1/4)])/(8*a^(3/4)) + (3*b*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(8*a^
(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-b)/x^2/(a*x^4-b)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^8 - b}{\left(ax^4 - b\right)^{\frac{3}{4}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-b)/x^2/(a*x^4-b)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^8 - b)/((a*x^4 - b)^(3/4)*x^2), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{a\,x^8 - b}{x^2 \left(a\,x^4 - b\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^8-b)/x^2/(a*x^4-b)^(3/4),x)
```

```
[Out] int((a*x^8-b)/x^2/(a*x^4-b)^(3/4),x)
```

maxima [B] time = 0.45, size = 140, normalized size = 1.59

$$\frac{1}{16}\,a\left(\frac{3\left(\frac{2\,b\arctan\left(\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{3}{4}}} - \frac{b\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{3}{4}}}\right)}{a} + \frac{4\left(ax^4-b\right)^{\frac{1}{4}}b}{\left(a^2 - \frac{\left(ax^4-b\right)a}{x^4}\right)x}\right) - \frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-b)/x^2/(a*x^4-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/16*a*(3*(2*b*arctan((a*x^4 - b)^(1/4)/(a^(1/4)*x))/a^(3/4) - b*log(-(a^(1
/4) - (a*x^4 - b)^(1/4)/x)/(a^(1/4) + (a*x^4 - b)^(1/4)/x))/a^(3/4))/a + 4*
(a*x^4 - b)^(1/4)*b/((a^2 - (a*x^4 - b)*a/x^4)*x)) - (a*x^4 - b)^(1/4)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{b - a\,x^8}{x^2\left(a\,x^4 - b\right)^{3/4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^8)/(x^2*(a*x^4 - b)^(3/4)),x)
[Out] -int((b - a*x^8)/(x^2*(a*x^4 - b)^(3/4)), x)
```

sympy [C] time = 2.41, size = 126, normalized size = 1.43

$$-\frac{ax^7e^{\frac{i\pi}{4}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{ax^4}{b}\right)}{4b^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}-b\left(\begin{cases}-\frac{\sqrt[4]{a}\sqrt[4]{-1+\frac{b}{ax^4}}e^{\frac{i\pi}{4}}\Gamma\left(-\frac{1}{4}\right)}{4b\Gamma\left(\frac{3}{4}\right)} & \text{for } \left|\frac{b}{ax^4}\right|>1\\ -\frac{\sqrt[4]{a}\sqrt[4]{1-\frac{b}{ax^4}}\Gamma\left(-\frac{1}{4}\right)}{4b\Gamma\left(\frac{3}{4}\right)} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**8-b)/x**2/(a*x**4-b)**(3/4),x)
[Out] -a*x**7*exp(I*pi/4)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), a*x**4/b)/(4*b**(
3/4)*gamma(11/4)) - b*Piecewise((-a**(1/4)*(-1 + b/(a*x**4))**(1/4)*exp(I*p
i/4)*gamma(-1/4)/(4*b*gamma(3/4)), Abs(b/(a*x**4)) > 1), (-a**(1/4)*(1 - b/
(a*x**4))**(1/4)*gamma(-1/4)/(4*b*gamma(3/4)), True))
```

3.1072 $\int \frac{-3b+2ax^8}{x^8 \sqrt[4]{-b+ax^4}}\,dx$

Optimal. Leaf size=88

$$a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+\frac{\left(ax^4-b\right)^{3/4}\left(-4ax^4-3b\right)}{7bx^7}$$

Rubi [A] time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1489, 240, 212, 206, 203, 271, 264}

$$a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)-\frac{3\left(ax^4-b\right)^{3/4}}{7x^7}-\frac{4a\left(ax^4-b\right)^{3/4}}{7bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-3*b + 2*a*x^8)/(x^8*(-b + a*x^4)^(1/4)),x]
```

```
[Out] (-3*(-b + a*x^4)^(3/4))/(7*x^7) - (4*a*(-b + a*x^4)^(3/4))/(7*b*x^3) + a^(3
/4)*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)] + a^(3/4)*ArcTanh[(a^(1/4)*x)/(-
b + a*x^4)^(1/4)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
```

```
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1489

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IG
tQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-3b+2ax^8}{x^8\sqrt[4]{-b+ax^4}}\,dx &= \int \left(\frac{2a}{\sqrt[4]{-b+ax^4}}-\frac{3b}{x^8\sqrt[4]{-b+ax^4}}\right)dx\\ &= (2a)\int \frac{1}{\sqrt[4]{-b+ax^4}}\,dx-(3b)\int \frac{1}{x^8\sqrt[4]{-b+ax^4}}\,dx\\ &= -\frac{3\left(-b+ax^4\right)^{3/4}}{7x^7}-\frac{1}{7}(12a)\int \frac{1}{x^4\sqrt[4]{-b+ax^4}}\,dx+(2a)\,\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right.\\ &= -\frac{3\left(-b+ax^4\right)^{3/4}}{7x^7}-\frac{4a\left(-b+ax^4\right)^{3/4}}{7bx^3}+a\,\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)+a\,\text{Subs}\\ &= -\frac{3\left(-b+ax^4\right)^{3/4}}{7x^7}-\frac{4a\left(-b+ax^4\right)^{3/4}}{7bx^3}+a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)+a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right.\end{aligned}$$

Mathematica [A] time = 0.08, size = 88, normalized size = 1.00

$$a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)-\frac{\left(ax^4-b\right)^{3/4}\left(4ax^4+3b\right)}{7bx^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3*b + 2*a*x^8)/(x^8*(-b + a*x^4)^(1/4)),x]
```

```
[Out] -1/7*((-b + a*x^4)^(3/4)*(3*b + 4*a*x^4))/(b*x^7) + a^(3/4)*ArcTan[(a^(1/4)
*x)/(-b + a*x^4)^(1/4)] + a^(3/4)*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)]
```

IntegrateAlgebraic [A] time = 0.36, size = 88, normalized size = 1.00

$$a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+\frac{\left(ax^4-b\right)^{3/4}\left(-4ax^4-3b\right)}{7bx^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*b + 2*a*x^8)/(x^8*(-b + a*x^4)^(1/4)),x]
```

```
[Out] ((-3*b - 4*a*x^4)*(-b + a*x^4)^(3/4))/(7*b*x^7) + a^(3/4)*ArcTan[(a^(1/4)*x
)/(-b + a*x^4)^(1/4)] + a^(3/4)*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^8-3*b)/x^8/(a*x^4-b)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax^8 - 3\,b}{\left(ax^4 - b\right)^{\frac{1}{4}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^8-3*b)/x^8/(a*x^4-b)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((2*a*x^8 - 3*b)/((a*x^4 - b)^(1/4)*x^8), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{2a\,x^8 - 3b}{x^8\left(a\,x^4 - b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x^8-3*b)/x^8/(a*x^4-b)^(1/4),x)
```

```
[Out] int((2*a*x^8-3*b)/x^8/(a*x^4-b)^(1/4),x)
```

maxima [A] time = 0.43, size = 116, normalized size = 1.32

$$-\frac{1}{2}\,a\left(\frac{2\,\arctan\left(\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}} + \frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right) - \frac{\frac{7\left(ax^4-b\right)^{\frac{3}{4}}a}{x^3} - \frac{3\left(ax^4-b\right)^{\frac{7}{4}}}{x^7}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^8-3*b)/x^8/(a*x^4-b)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*arctan((a*x^4 - b)^(1/4)/(a^(1/4)*x))/a^(1/4) + log(-(a^(1/4) - (
a*x^4 - b)^(1/4)/x)/(a^(1/4) + (a*x^4 - b)^(1/4)/x))/a^(1/4)) - 1/7*(7*(a*x
^4 - b)^(3/4)*a/x^3 - 3*(a*x^4 - b)^(7/4)/x^7)/b
```

mupad [B] time = 1.30, size = 71, normalized size = 0.81

$$\frac{2\,a\,x\left(1-\frac{a\,x^4}{b}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \frac{a\,x^4}{b}\right)}{\left(a\,x^4-b\right)^{1/4}} - \frac{\left(a\,x^4-b\right)^{3/4}\,\left(4\,a\,x^4+3\,b\right)}{7\,b\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*b - 2*a*x^8)/(x^8*(a*x^4 - b)^(1/4)),x)
```

```
[Out] (2*a*x*(1 - (a*x^4)/b)^(1/4)*hypergeom([1/4, 1/4], 5/4, (a*x^4)/b))/(a*x^4
- b)^(1/4) - ((a*x^4 - b)^(3/4)*(3*b + 4*a*x^4))/(7*b*x^7)
```

sympy [C] time = 2.52, size = 320, normalized size = 3.64

$$\frac{axe^{-\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{ax^4}{b}\right)}{2\sqrt[4]{b}\,\Gamma\left(\frac{5}{4}\right)} - 3b\left(\begin{cases} -\frac{a^{\frac{7}{4}}\left(-1+\frac{b}{ax^4}\right)^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{7}{4}\right)}{4b^2\Gamma\left(\frac{1}{4}\right)} - \frac{3a^{\frac{3}{4}}\left(-1+\frac{b}{ax^4}\right)^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{7}{4}\right)}{16bx^4\Gamma\left(\frac{1}{4}\right)} & \text{for}\ \left|\frac{b}{ax^4}\right| > 1 \\ \frac{4a^{\frac{15}{4}}x^8\left(1-\frac{b}{ax^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{16a^2b^2x^8\Gamma\left(\frac{1}{4}\right)-16ab^3x^4\Gamma\left(\frac{1}{4}\right)} - \frac{a^{\frac{11}{4}}bx^4\left(1-\frac{b}{ax^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{16a^2b^2x^8\Gamma\left(\frac{1}{4}\right)-16ab^3x^4\Gamma\left(\frac{1}{4}\right)} - \frac{3a^{\frac{7}{4}}b^2\left(1-\frac{b}{ax^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{16a^2b^2x^8\Gamma\left(\frac{1}{4}\right)-16ab^3x^4\Gamma\left(\frac{1}{4}\right)} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x**8-3*b)/x**8/(a*x**4-b)**(1/4),x)
```

```
[Out] a*x*exp(-I*pi/4)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), a*x**4/b)/(2*b**(1/4)
*gamma(5/4)) - 3*b*Piecewise((-a**(7/4)*(-1 + b/(a*x**4))**(3/4)*exp(-I*pi/
4)*gamma(-7/4)/(4*b**2*gamma(1/4)) - 3*a**(3/4)*(-1 + b/(a*x**4))**(3/4)*ex
p(-I*pi/4)*gamma(-7/4)/(16*b*x**4*gamma(1/4)), Abs(b/(a*x**4)) > 1), (4*a**
(15/4)*x**8*(1 - b/(a*x**4))**(3/4)*gamma(-7/4)/(16*a**2*b**2*x**8*gamma(1/
4) - 16*a*b**3*x**4*gamma(1/4)) - a**(11/4)*b*x**4*(1 - b/(a*x**4))**(3/4)*
gamma(-7/4)/(16*a**2*b**2*x**8*gamma(1/4) - 16*a*b**3*x**4*gamma(1/4)) - 3*
a**(7/4)*b**2*(1 - b/(a*x**4))**(3/4)*gamma(-7/4)/(16*a**2*b**2*x**8*gamma(
1/4) - 16*a*b**3*x**4*gamma(1/4)), True))
```

3.1073 $$\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Optimal. Leaf size=88

$$-\frac{\sqrt{2}\,b\log\left(b\left(-\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)+\sqrt{2}\,\sqrt{a}\,\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}-ax\right)}{\sqrt{a}}$$

Rubi [A] time = 0.64, antiderivative size = 46, normalized size of antiderivative = 0.52, number of steps used = 2, number of rules used = 2, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2130, 215}

$$\frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-(a/b^2) + (a^
2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[
a]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2130

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx &= \frac{\left(\sqrt{2}\,b\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}\\ &= \frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 1.08, size = 148, normalized size = 1.68

$$\frac{\sqrt{2}x\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(bx\sqrt{\frac{a(ax^2-1)}{b^2}}+ax^2-1\right)\tanh^{-1}\left(\frac{\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}}{\sqrt{2}ax}\right)}{\sqrt{\frac{a(ax^2-1)}{b^2}}\left(x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-(a/b^2)
 + (a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*x*Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*(-1 + a*x^2 + b*x
*Sqrt[(a*(-1 + a*x^2))/b^2])*ArcTanh[Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2)
)/b^2])]/(Sqrt[2]*a*x)])/(Sqrt[(a*(-1 + a*x^2))/b^2]*(x*(a*x + b*Sqrt[(a*(-
1 + a*x^2))/b^2]))^(3/2))
```

IntegrateAlgebraic [A] time = 0.00, size = 88, normalized size = 1.00

$$-\frac{\sqrt{2}b\log\left(b\left(-\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)+\sqrt{2}\sqrt{a}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}-ax\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt
[-(a/b^2) + (a^2*x^2)/b^2]),x]
```

```
[Out] -((Sqrt[2]*b*Log[-(a*x) - b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2] + Sqrt[2]*Sqrt[a
]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]])/Sqrt[a])
```

fricas [A] time = 12.92, size = 161, normalized size = 1.83

$$\left[\frac{\sqrt{2}b\log\left(-4ax^2-4bx\sqrt{\frac{a^2x^2-a}{b^2}}-2\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\left(\sqrt{2}\sqrt{a}x+\frac{\sqrt{2}b\sqrt{\frac{a^2x^2-a}{b^2}}}{\sqrt{a}}\right)+1\right)}{2\sqrt{a}},-\sqrt{2}b\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\sqrt{-\frac{1}{a}}}{2x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*log(-4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b^2) - 2*sqrt(a*x^2
+ b*x*sqrt((a^2*x^2 - a)/b^2))*(sqrt(2)*sqrt(a)*x + sqrt(2)*b*sqrt((a^2*x^2
 - a)/b^2)/sqrt(a)) + 1)/sqrt(a), -sqrt(2)*b*sqrt(-1/a)*arctan(1/2*sqrt(2)*
sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*sqrt(-1/a)/x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,bx}}{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\,x^{2}+bx\sqrt{-\frac{a}{b^{2}}+\frac{a^{2}x^{2}}{b^{2}}}}}{x\sqrt{-\frac{a}{b^{2}}+\frac{a^{2}x^{2}}{b^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^2)^(1/
2),x)
```

```
[Out] int((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^2)^(1/
2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{2}+\sqrt{\frac{a^{2}x^{2}}{b^{2}}-\frac{a}{b^{2}}}\,bx}}{\sqrt{\frac{a^{2}x^{2}}{b^{2}}-\frac{a}{b^{2}}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x^2+b\,x\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}}}{x\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)),x)
```

```
[Out] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(ax+b\sqrt{\frac{a^{2}x^{2}}{b^{2}}-\frac{a}{b^{2}}}\right)}}{x\sqrt{\frac{a\left(ax^{2}-1\right)}{b^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x*(-a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(-a/b**2+a*
*2*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))/(x*sqrt(a*(a*x**2
- 1)/b**2)), x)
```

3.1074 $\int \frac{\left(-1+x^2\right)^{2/3}}{x}\,dx$

Optimal. Leaf size=89

$$\frac{3}{4}\left(x^2-1\right)^{2/3}+\frac{1}{2}\log\left(\sqrt[3]{x^2-1}+1\right)-\frac{1}{4}\log\left(\left(x^2-1\right)^{2/3}-\sqrt[3]{x^2-1}+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^2-1}}{\sqrt{3}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 50, 56, 618, 204, 31}

$$\frac{3}{4}\left(x^2-1\right)^{2/3}+\frac{3}{4}\log\left(\sqrt[3]{x^2-1}+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{x^2-1}}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)^(2/3)/x,x]
```

```
[Out] (3*(-1 + x^2)^(2/3))/4 + (Sqrt[3]*ArcTan[(1 - 2*(-1 + x^2)^(1/3))/Sqrt[3]])
/2 - Log[x]/2 + (3*Log[1 + (-1 + x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-1+x^2\right)^{2/3}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(-1+x)^{2/3}}{x}\,dx, x, x^2\right)\\
&= \frac{3}{4}\left(-1+x^2\right)^{2/3}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x}\,dx, x, x^2\right)\\
&= \frac{3}{4}\left(-1+x^2\right)^{2/3}-\frac{\log(x)}{2}+\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[3]{-1+x^2}\right)-\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\right.\\
&= \frac{3}{4}\left(-1+x^2\right)^{2/3}-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1+\sqrt[3]{-1+x^2}\right)+\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[3]{-}\right.\\
&= \frac{3}{4}\left(-1+x^2\right)^{2/3}+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^2}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1+\sqrt[3]{-1+x^2}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.34

$$-\frac{3}{4}\left(x^2-1\right)^{2/3}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-x^2\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)^(2/3)/x,x]
```

```
[Out] (-3*(-1 + x^2)^(2/3)*(-1 + Hypergeometric2F1[2/3, 1, 5/3, 1 - x^2]))/4
```

IntegrateAlgebraic [A] time = 0.06, size = 89, normalized size = 1.00

$$\frac{3}{4}\left(x^2-1\right)^{2/3}+\frac{1}{2}\log\left(\sqrt[3]{x^2-1}+1\right)-\frac{1}{4}\log\left(\left(x^2-1\right)^{2/3}-\sqrt[3]{x^2-1}+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^2-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)^(2/3)/x,x]
```

```
[Out] (3*(-1 + x^2)^(2/3))/4 + (Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(-1 + x^2)^(1/3))/S
qrt[3]])/2 + Log[1 + (-1 + x^2)^(1/3)]/2 - Log[1 - (-1 + x^2)^(1/3) + (-1 +
 x^2)^(2/3)]/4
```

fricas [A] time = 0.41, size = 67, normalized size = 0.75

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^2-1\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+\frac{3}{4}\left(x^2-1\right)^{\frac{2}{3}}-\frac{1}{4}\log\left(\left(x^2-1\right)^{\frac{2}{3}}-\left(x^2-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(x^2-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan(2/3*sqrt(3)*(x^2 - 1)^(1/3) - 1/3*sqrt(3)) + 3/4*(x^2 -
 1)^(2/3) - 1/4*log((x^2 - 1)^(2/3) - (x^2 - 1)^(1/3) + 1) + 1/2*log((x^2 -
 1)^(1/3) + 1)
```

giac [A] time = 0.35, size = 66, normalized size = 0.74

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^2-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{3}{4}\left(x^2-1\right)^{\frac{2}{3}}-\frac{1}{4}\log\left(\left(x^2-1\right)^{\frac{2}{3}}-\left(x^2-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left|\left(x^2-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(2/3)/x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 - 1)^(1/3) - 1)) + 3/4*(x^2 - 1)^(2
/3) - 1/4*log((x^2 - 1)^(2/3) - (x^2 - 1)^(1/3) + 1) + 1/2*log(abs((x^2 - 1
)^(1/3) + 1))
```

maple [C] time = 0.30, size = 84, normalized size = 0.94

$$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\operatorname{signum}\left(x^2-1\right)^{\frac{2}{3}}\left(\frac{2\pi\sqrt{3}\,x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{3},1,1\right],[2,2],x^2\right)}{3\Gamma\left(\frac{2}{3}\right)}-\frac{\left(\frac{3}{2}-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)+i\pi\right)\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}\right)}{6\pi\left(-\operatorname{signum}\left(x^2-1\right)\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(2/3)/x,x)
```

```
[Out] -1/6/Pi*3^(1/2)*GAMMA(2/3)*signum(x^2-1)^(2/3)/(-signum(x^2-1))^(2/3)*(2/3*
Pi*3^(1/2)/GAMMA(2/3)*x^2*hypergeom([1/3,1,1],[2,2],x^2)-(3/2-1/6*Pi*3^(1/2
)-3/2*ln(3)+2*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.44, size = 65, normalized size = 0.73

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^2-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{3}{4}\left(x^2-1\right)^{\frac{2}{3}}-\frac{1}{4}\log\left(\left(x^2-1\right)^{\frac{2}{3}}-\left(x^2-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(x^2-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 - 1)^(1/3) - 1)) + 3/4*(x^2 - 1)^(2
/3) - 1/4*log((x^2 - 1)^(2/3) - (x^2 - 1)^(1/3) + 1) + 1/2*log((x^2 - 1)^(1
/3) + 1)
```

mupad [B] time = 0.83, size = 89, normalized size = 1.00

$$\frac{\ln\left(\frac{9\left(x^2-1\right)^{1/3}}{4}+\frac{9}{4}\right)}{2}+\ln\left(9\left(-\frac{1}{4}+\frac{\sqrt{3}\,1i}{4}\right)^2+\frac{9\left(x^2-1\right)^{1/3}}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1i}{4}\right)-\ln\left(9\left(\frac{1}{4}+\frac{\sqrt{3}\,1i}{4}\right)^2+\frac{9\left(x^2-1\right)^{1/3}}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1i}{4}\right)+\frac{3\left(x^2-1\right)^{2/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^(2/3)/x,x)
```

```
[Out] log((9*(x^2 - 1)^(1/3))/4 + 9/4)/2 + log(9*((3^(1/2)*1i)/4 - 1/4)^2 + (9*(x
^2 - 1)^(1/3))/4)*((3^(1/2)*1i)/4 - 1/4) - log(9*((3^(1/2)*1i)/4 + 1/4)^2 +
 (9*(x^2 - 1)^(1/3))/4)*((3^(1/2)*1i)/4 + 1/4) + (3*(x^2 - 1)^(2/3))/4
```

sympy [C] time = 0.87, size = 39, normalized size = 0.44

$$-\frac{x^{\frac{4}{3}}\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\right|\frac{e^{2i\pi}}{x^2}\right)}{2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(2/3)/x,x)
```

```
[Out] -x**(4/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), exp_polar(2*I*pi)/x**2)/(
2*gamma(1/3))
```

3.1075 $\int \frac{-3+x}{\sqrt[3]{-1+x^2}\left(2+x+x^2\right)} dx$

Optimal. Leaf size=89

$$\log \left(\sqrt[3]{x^2-1}-x-1\right)-\frac{1}{2} \log \left(x^2+\left(x^2-1\right)^{2/3}+(x+1) \sqrt[3]{x^2-1}+2 x+1\right)+\sqrt{3} \tan ^{-1}\left(\frac{\sqrt{3} \sqrt[3]{x^2-1}}{\sqrt[3]{x^2-1}+2 x+2}\right)$$

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3+x}{\sqrt[3]{-1+x^2}\left(2+x+x^2\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(-3 + x)/((-1 + x^2)^(1/3)*(2 + x + x^2)),x]
[Out] Defer[Int][(-3 + x)/((-1 + x^2)^(1/3)*(2 + x + x^2)), x]
```

Rubi steps

$$\int \frac{-3+x}{\sqrt[3]{-1+x^2}\left(2+x+x^2\right)} dx=\int \frac{-3+x}{\sqrt[3]{-1+x^2}\left(2+x+x^2\right)} dx$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{-3+x}{\sqrt[3]{-1+x^2}\left(2+x+x^2\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3 + x)/((-1 + x^2)^(1/3)*(2 + x + x^2)),x]
[Out] Integrate[(-3 + x)/((-1 + x^2)^(1/3)*(2 + x + x^2)), x]
```

IntegrateAlgebraic [A] time = 0.12, size = 89, normalized size = 1.00

$$\log \left(\sqrt[3]{x^2-1}-x-1\right)-\frac{1}{2} \log \left(x^2+\left(x^2-1\right)^{2/3}+(x+1) \sqrt[3]{x^2-1}+2 x+1\right)+\sqrt{3} \tan ^{-1}\left(\frac{\sqrt{3} \sqrt[3]{x^2-1}}{\sqrt[3]{x^2-1}+2 x+2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 + x)/((-1 + x^2)^(1/3)*(2 + x + x^2)),x]
[Out] Sqrt[3]*ArcTan[(Sqrt[3]*(-1 + x^2)^(1/3))/(2 + 2*x + (-1 + x^2)^(1/3))] + L
og[-1 - x + (-1 + x^2)^(1/3)] - Log[1 + 2*x + x^2 + (1 + x)*(-1 + x^2)^(1/3
) + (-1 + x^2)^(2/3)]/2
```

fricas [A] time = 0.85, size = 95, normalized size = 1.07

$$-\sqrt{3} \arctan \left(-\frac{4 \sqrt{3} \left(x^2-1\right)^{\frac{1}{3}}(x+1)+\sqrt{3}(x-1)-2 \sqrt{3} \left(x^2-1\right)^{\frac{2}{3}}}{8 x^2+17 x+7}\right)+\frac{1}{2} \log \left(\frac{x^2-3 \left(x^2-1\right)^{\frac{1}{3}}(x+1)+x+3 \left(x^2-1\right)^{\frac{2}{3}}+2}{x^2+x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^2-1)^(1/3)/(x^2+x+2),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(-(4*sqrt(3)*(x^2 - 1)^(1/3)*(x + 1) + sqrt(3)*(x - 1) - 2*s
qrt(3)*(x^2 - 1)^(2/3))/(8*x^2 + 17*x + 7)) + 1/2*log((x^2 - 3*(x^2 - 1)^(1
/3)*(x + 1) + x + 3*(x^2 - 1)^(2/3) + 2)/(x^2 + x + 2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-3}{\left(x^2+x+2\right)\left(x^2-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^2-1)^(1/3)/(x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((x - 3)/((x^2 + x + 2)*(x^2 - 1)^(1/3)), x)
```

maple [C] time = 2.08, size = 334, normalized size = 3.75

$$\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(-\frac{-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^2-6\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+3\left(x^2-1\right)^{\frac{2}{3}}+3x\left(x^2-1\right)^{\frac{1}{3}}-5\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x+3\left(x^2-1\right)^{\frac{1}{3}}+2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-x+1}{x^2+x+2}\right)-\ln\left(-\frac{-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^2-6\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x-3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+3\left(x^2-1\right)^{\frac{2}{3}}+3x\left(x^2-1\right)^{\frac{1}{3}}-7\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x-x^2+3\left(x^2-1\right)^{\frac{1}{3}}-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-2x-1}{x^2+x+2}\right)\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-\ln\left(-\frac{-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^2-6\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x-3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+3\left(x^2-1\right)^{\frac{2}{3}}+3x\left(x^2-1\right)^{\frac{1}{3}}-7\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x-x^2+3\left(x^2-1\right)^{\frac{1}{3}}-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-2x-1}{x^2+x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+x)/(x^2-1)^(1/3)/(x^2+x+2),x)
```

```
[Out] RootOf(_Z^2+_Z+1)*ln(-(-2*RootOf(_Z^2+_Z+1)^2*x^2-6*RootOf(_Z^2+_Z+1)^2*x-R
ootOf(_Z^2+_Z+1)*x^2+3*(x^2-1)^(2/3)+3*x*(x^2-1)^(1/3)-5*RootOf(_Z^2+_Z+1)*
x+3*(x^2-1)^(1/3)+2*RootOf(_Z^2+_Z+1)-x+1)/(x^2+x+2))-ln(-(-2*RootOf(_Z^2+_
Z+1)^2*x^2-6*RootOf(_Z^2+_Z+1)^2*x-3*RootOf(_Z^2+_Z+1)*x^2+3*(x^2-1)^(2/3)+
3*x*(x^2-1)^(1/3)-7*RootOf(_Z^2+_Z+1)*x-x^2+3*(x^2-1)^(1/3)-2*RootOf(_Z^2+_
Z+1)-2*x-1)/(x^2+x+2))*RootOf(_Z^2+_Z+1)-ln(-(-2*RootOf(_Z^2+_Z+1)^2*x^2-6*
RootOf(_Z^2+_Z+1)^2*x-3*RootOf(_Z^2+_Z+1)*x^2+3*(x^2-1)^(2/3)+3*x*(x^2-1)^(
1/3)-7*RootOf(_Z^2+_Z+1)*x-x^2+3*(x^2-1)^(1/3)-2*RootOf(_Z^2+_Z+1)-2*x-1)/(
x^2+x+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-3}{\left(x^2+x+2\right)\left(x^2-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^2-1)^(1/3)/(x^2+x+2),x, algorithm="maxima")
```

```
[Out] integrate((x - 3)/((x^2 + x + 2)*(x^2 - 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x-3}{{\left(x^2-1\right)}^{1/3}\,\left(x^2+x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3)/((x^2 - 1)^(1/3)*(x + x^2 + 2)),x)
```

```
[Out] int((x - 3)/((x^2 - 1)^(1/3)*(x + x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-3}{\sqrt[3]{\left(x-1\right)\left(x+1\right)}\,\left(x^2+x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x**2-1)**(1/3)/(x**2+x+2),x)
```

```
[Out] Integral((x - 3)/(((x - 1)*(x + 1))**(1/3)*(x**2 + x + 2)), x)
```

3.1076 $\int \frac{2+x^2}{(-2-2x+x^2)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=89

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{x^3-1}}{x^2+x+1}\right)}{3^{3/4}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{x^3-1}}{x^2+x+1}\right)}{3^{3/4}}$$

Rubi [C] time = 0.94, antiderivative size = 433, normalized size of antiderivative = 4.87, number of steps used = 13, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6728, 219, 2135, 2140, 206, 203}

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{x^3-1}}\right)}{3^{3/4}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{x^3-1}}\right)}{3^{3/4}}-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{\sqrt{2\left(7-4\sqrt{3}\right)}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{\sqrt{2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(2 + x^2)/((-2 - 2*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/3^(3/4) + (
Sqrt[2]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/3^(3/4) + (S
qrt[2]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1
+ Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 - x)
/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (Sqrt[2*(7 - 4*Sqrt[3])]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] -
x)^2)]*Sqrt[-1 + x^3]) - (2*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -
7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3
])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2135

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-
6*a*d^3)/(c*(b*c^3 - 28*a*d^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
```

```
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{2+x^2}{\left(-2-2x+x^2\right)\sqrt{-1+x^3}}\,dx = \int \left(\frac{1}{\sqrt{-1+x^3}}+\frac{2(2+x)}{\left(-2-2x+x^2\right)\sqrt{-1+x^3}}\right)dx$$

$$= 2\int \frac{2+x}{\left(-2-2x+x^2\right)\sqrt{-1+x^3}}\,dx + \int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + 2\int \Bigg(\frac{\quad}{(-\cdots}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \left(2\left(1-\cdots\right.\right.$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{1}{288}\left(-3\cdots\right.$$

$$= \frac{\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{\sqrt{14-8\sqrt{3}}\,(1-\cdots}{\cdots}$$

$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{3^{3/4}} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{3^{3/4}} + \frac{\sqrt{2}\,(1-x)\sqrt{\frac{1+\cdots}{(1-\cdots}}}{3^{3/4}\cdots}$$

Mathematica [C] time = 0.80, size = 279, normalized size = 3.13

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left((1+i)\left(3-i\sqrt{3}\right)\sqrt{x^2+x+1}\left(\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)-i\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)\right)+\frac{3\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\left(\left(\sqrt{3}+i\right)x+2i\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}\right)}{3\left(\sqrt{3}+i\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + x^2)/((-2 - 2*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((3*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/(1 +
 (-1)^(1/3))]*(2*I + (I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)
*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3
))] + (1 + I)*(3 - I*Sqrt[3])*Sqrt[1 + x + x^2]*(EllipticPi[(2*Sqrt[3])/(-3
*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]],
 (-1)^(1/3)] - I*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[S
qrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])))/(3*(I + Sqrt[3])*
Sqrt[-1 + x^3])
```

IntegrateAlgebraic [A] time = 1.54, size = 89, normalized size = 1.00

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{x^3-1}}{x^2+x+1}\right)}{3^{3/4}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{x^3-1}}{x^2+x+1}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x^2)/((-2 - 2*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[-1 + x^3])/(1 + x + x^2)])/3^(
3/4)) - (Sqrt[2]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[-1 + x^3])/(1 + x + x^2)
])/3^(3/4)
```

fricas [B] time = 0.48, size = 230, normalized size = 2.58

$$\frac{1}{27}\cdot 27^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{x^3-1}\left(27^{\frac{3}{4}}\sqrt{2}-9\cdot 27^{\frac{1}{4}}\sqrt{2}\right)}{18\left(x^2+x+1\right)}\right)-\frac{1}{108}\cdot 27^{\frac{3}{4}}\sqrt{2}\log\left(\frac{2\left(9x^4+18x^3+54x^2+36\sqrt{3}\left(x^3-1\right)+\sqrt{x^3-1}\left(27^{\frac{3}{4}}\sqrt{2}\left(x^2+4x-2\right)+9\cdot 27^{\frac{1}{4}}\sqrt{2}\left(x^2+2\right)\right)-36x+36\right)}{x^4-4x^3+8x+4}\right)+\frac{1}{108}\cdot 27^{\frac{3}{4}}\sqrt{2}\log\left(\frac{2\left(9x^4+18x^3+54x^2+36\sqrt{3}\left(x^3-1\right)-\sqrt{x^3-1}\left(27^{\frac{3}{4}}\sqrt{2}\left(x^2+4x-2\right)+9\cdot 27^{\frac{1}{4}}\sqrt{2}\left(x^2+2\right)\right)-36x+36\right)}{x^4-4x^3+8x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(x^2-2*x-2)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*27^(3/4)*sqrt(2)*arctan(1/18*sqrt(x^3 - 1)*(27^(3/4)*sqrt(2) - 9*27^(1
/4)*sqrt(2))/(x^2 + x + 1)) - 1/108*27^(3/4)*sqrt(2)*log(2*(9*x^4 + 18*x^3
+ 54*x^2 + 36*sqrt(3)*(x^3 - 1) + sqrt(x^3 - 1)*(27^(3/4)*sqrt(2)*(x^2 + 4*
x - 2) + 9*27^(1/4)*sqrt(2)*(x^2 + 2)) - 36*x + 36)/(x^4 - 4*x^3 + 8*x + 4)
) + 1/108*27^(3/4)*sqrt(2)*log(2*(9*x^4 + 18*x^3 + 54*x^2 + 36*sqrt(3)*(x^3
 - 1) - sqrt(x^3 - 1)*(27^(3/4)*sqrt(2)*(x^2 + 4*x - 2) + 9*27^(1/4)*sqrt(2
)*(x^2 + 2)) - 36*x + 36)/(x^4 - 4*x^3 + 8*x + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2}{\sqrt{x^3-1}\left(x^2-2x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(x^2-2*x-2)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 2)/(sqrt(x^3 - 1)*(x^2 - 2*x - 2)), x)
```

maple [C] time = 0.32, size = 1516, normalized size = 17.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)/(x^2-2*x-2)/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
```

```
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/
2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/
2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/
2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*Ellipti
cPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((
3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+3*(1/(-3/2-1/2*I*3^(
1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2
*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))
*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)
^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/
2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+I*(1/(-3/2-1/2
*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3
/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^
(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(
x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I
*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+I*(1/(-3
/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+
1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/
2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2
+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^
(1/2)+3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2
*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1
/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/
2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(
1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2
)))^(1/2))-I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/
2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*
3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2
)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3
^(1/2)))^(1/2))+I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1
/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3
^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/
2*I*3^(1/2)))^(1/2))*3^(1/2)-(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2
)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1
/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-
3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/
2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2}{\sqrt{x^3-1}\left(x^2-2\,x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(x^2-2*x-2)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 2)/(sqrt(x^3 - 1)*(x^2 - 2*x - 2)), x)
```

mupad [B] time = 0.20, size = 509, normalized size = 5.72

$$-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\frac{\left(2\sqrt{3}-6\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{3};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}+\frac{\left(2\sqrt{3}+6\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(-\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{3};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 + 2)/((x^3 - 1)^(1/2)*(2*x - x^2 + 2)),x)
```

```
[Out] ((2*3^(1/2) + 6)*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1
/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-(3^(1/2)*((3^(1/
2)*1i)/2 + 3/2))/3, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/
2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x
^3)^(1/2)) - ((2*3^(1/2) - 6)*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2
+ 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*
1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(
1/2)*((3^(1/2)*1i)/2 + 3/2))/3, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(((3^(1/2)*1i)/2 -
1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) + 1) + x^3)^(1/2)) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 +
1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-
(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1
i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)
*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x^2-2x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)/(x**2-2*x-2)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x**2 + 2)/(sqrt((x - 1)*(x**2 + x + 1))*(x**2 - 2*x - 2)), x)
```

3.1077 $\int \frac{\left(-1+x^3\right)\sqrt[3]{1+x^3}}{x}\,dx$

Optimal. Leaf size=89

$$\frac{1}{4}\sqrt[3]{x^3+1}\left(x^3-3\right)-\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-1\right)+\frac{1}{6}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {446, 80, 50, 57, 618, 204, 31}

$$\frac{1}{4}\left(x^3+1\right)^{4/3}-\sqrt[3]{x^3+1}-\frac{1}{2}\log\left(1-\sqrt[3]{x^3+1}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)*(1 + x^3)^(1/3))/x,x]
```

```
[Out] -(1 + x^3)^(1/3) + (1 + x^3)^(4/3)/4 + ArcTan[(1 + 2*(1 + x^3)^(1/3))/Sqrt[
3]]/Sqrt[3] + Log[x]/2 - Log[1 - (1 + x^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-1+x^3\right)\sqrt[3]{1+x^3}}{x}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(-1+x)\sqrt[3]{1+x}}{x}\,dx,x,x^3\right)\\
&= \frac{1}{4}\left(1+x^3\right)^{4/3}-\frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt[3]{1+x}}{x}\,dx,x,x^3\right)\\
&= -\sqrt[3]{1+x^3}+\frac{1}{4}\left(1+x^3\right)^{4/3}-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x(1+x)^{2/3}}\,dx,x,x^3\right)\\
&= -\sqrt[3]{1+x^3}+\frac{1}{4}\left(1+x^3\right)^{4/3}+\frac{\log(x)}{2}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1+x^3}\right)+\frac{1}{2}\operatorname{Subst}\Big(\\
&= -\sqrt[3]{1+x^3}+\frac{1}{4}\left(1+x^3\right)^{4/3}+\frac{\log(x)}{2}-\frac{1}{2}\log\left(1-\sqrt[3]{1+x^3}\right)-\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x\right.\\
&= -\sqrt[3]{1+x^3}+\frac{1}{4}\left(1+x^3\right)^{4/3}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^3}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log(x)}{2}-\frac{1}{2}\log\left(1-\sqrt[3]{1+x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 96, normalized size = 1.08

$$\frac{1}{12}\left(3\sqrt[3]{x^3+1}\,x^3-9\sqrt[3]{x^3+1}-4\log\left(1-\sqrt[3]{x^3+1}\right)+2\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)+4\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)*(1 + x^3)^(1/3))/x,x]
```

```
[Out] (-9*(1 + x^3)^(1/3) + 3*x^3*(1 + x^3)^(1/3) + 4*Sqrt[3]*ArcTan[(1 + 2*(1 +
x^3)^(1/3))/Sqrt[3]] - 4*Log[1 - (1 + x^3)^(1/3)] + 2*Log[1 + (1 + x^3)^(1/
3) + (1 + x^3)^(2/3)])/12
```

IntegrateAlgebraic [A] time = 0.06, size = 89, normalized size = 1.00

$$\frac{1}{4}\sqrt[3]{x^3+1}\left(x^3-3\right)-\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-1\right)+\frac{1}{6}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)*(1 + x^3)^(1/3))/x,x]
```

```
[Out] ((-3 + x^3)*(1 + x^3)^(1/3))/4 + ArcTan[1/Sqrt[3] + (2*(1 + x^3)^(1/3))/Sqr
t[3]]/Sqrt[3] - Log[-1 + (1 + x^3)^(1/3)]/3 + Log[1 + (1 + x^3)^(1/3) + (1
+ x^3)^(2/3)]/6
```

fricas [A] time = 0.44, size = 70, normalized size = 0.79

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{1}{4}\left(x^3+1\right)^{\frac{1}{3}}\left(x^3-3\right)+\frac{1}{6}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{3}\log\left(\left(x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 1/4*(x^3 +
1)^(1/3)*(x^3 - 3) + 1/6*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) - 1/3*l
og((x^3 + 1)^(1/3) - 1)
```

giac [A] time = 0.29, size = 73, normalized size = 0.82

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{4}\left(x^3+1\right)^{\frac{4}{3}}-\left(x^3+1\right)^{\frac{1}{3}}+\frac{1}{6}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{3}\log\left(\left|\left(x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) + 1/4*(x^3 + 1)^(4/
3) - (x^3 + 1)^(1/3) + 1/6*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) - 1/3
*log(abs((x^3 + 1)^(1/3) - 1))
```

maple [C] time = 0.32, size = 65, normalized size = 0.73

$$\frac{-\Gamma\left(\frac{2}{3}\right)x^3\,\mathrm{hypergeom}\left(\left[\frac{2}{3},1,1\right],[2,2],-x^3\right)-3\left(3+\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)\right)\Gamma\left(\frac{2}{3}\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{x^3\,\mathrm{hypergeom}\left(\left[-\frac{1}{3},1\right],[2],-x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)*(x^3+1)^(1/3)/x,x)
```

```
[Out] 1/9/GAMMA(2/3)*(-GAMMA(2/3)*x^3*hypergeom([2/3,1,1],[2,2],-x^3)-3*(3+1/6*Pi
*3^(1/2)-3/2*ln(3)+3*ln(x))*GAMMA(2/3))+1/3*x^3*hypergeom([-1/3,1],[2],-x^3
)
```

maxima [A] time = 0.44, size = 72, normalized size = 0.81

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{4}\left(x^3+1\right)^{\frac{4}{3}}-\left(x^3+1\right)^{\frac{1}{3}}+\frac{1}{6}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{3}\log\left(\left(x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) + 1/4*(x^3 + 1)^(4/
3) - (x^3 + 1)^(1/3) + 1/6*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) - 1/3
*log((x^3 + 1)^(1/3) - 1)
```

mupad [B] time = 0.95, size = 86, normalized size = 0.97

$$\frac{\left(x^3+1\right)^{4/3}}{4}-\left(x^3+1\right)^{1/3}-\frac{\ln\left(\left(x^3+1\right)^{1/3}-1\right)}{3}-\ln\left(3\left(x^3+1\right)^{1/3}+\frac{3}{2}-\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\ln\left(3\left(x^3+1\right)^{1/3}+\frac{3}{2}+\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)*(x^3 + 1)^(1/3))/x,x)
```

```
[Out] (x^3 + 1)^(4/3)/4 - (x^3 + 1)^(1/3) - log((x^3 + 1)^(1/3) - 1)/3 - log(3*(x
^3 + 1)^(1/3) - (3^(1/2)*3i)/2 + 3/2)*((3^(1/2)*1i)/6 - 1/6) + log((3^(1/2)
*3i)/2 + 3*(x^3 + 1)^(1/3) + 3/2)*((3^(1/2)*1i)/6 + 1/6)
```

sympy [A] time = 26.65, size = 82, normalized size = 0.92

$$\frac{\left(x^{3}+1\right)^{\frac{4}{3}}}{4}-\sqrt[3]{x^{3}+1}-\frac{\log\left(\sqrt[3]{x^{3}+1}-1\right)}{3}+\frac{\log\left(\left(x^{3}+1\right)^{\frac{2}{3}}+\sqrt[3]{x^{3}+1}+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\left(\sqrt[3]{x^{3}+1}+\frac{1}{2}\right)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)*(x**3+1)**(1/3)/x,x)
```

```
[Out] (x**3 + 1)**(4/3)/4 - (x**3 + 1)**(1/3) - log((x**3 + 1)**(1/3) - 1)/3 + lo
g((x**3 + 1)**(2/3) + (x**3 + 1)**(1/3) + 1)/6 + sqrt(3)*atan(2*sqrt(3)*((x
**3 + 1)**(1/3) + 1/2)/3)/3
```

3.1078 $\int \frac{-ab+2(a-b)x+x^2}{\sqrt[4]{x(-a+x)(-b+x)}\left(-a^3+\left(3a^2+bd\right)x-(3a+d)x^2+x^3\right)}\,dx$

Optimal. Leaf size=89

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2(-a-b)+abx+x^3}}{a-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2(-a-b)+abx+x^3}}{a-x}\right)}{d^{3/4}}$$

Rubi [F] time = 13.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-ab+2(a-b)x+x^2}{\sqrt[4]{x(-a+x)(-b+x)}\left(-a^3+\left(3a^2+bd\right)x-(3a+d)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a*b) + 2*(a - b)*x + x^2)/((x*(-a + x)*(-b + x))^(1/4)*(-a^3 + (3*a^
2 + b*d)*x - (3*a + d)*x^2 + x^3)),x]
```

```
[Out] (4*a*b*x^(1/4)*(-a + x)^(1/4)*(-b + x)^(1/4)*Defer[Subst][Defer[Int][x^2/((
-a + x^4)^(1/4)*(-b + x^4)^(1/4)*(a^3 - 3*a^2*(1 + (b*d)/(3*a^2))*x^4 + 3*a
*(1 + d/(3*a))*x^8 - x^12)), x], x, x^(1/4)])/((a - x)*(b - x)*x)^(1/4) - (
8*(a - b)*x^(1/4)*(-a + x)^(1/4)*(-b + x)^(1/4)*Defer[Subst][Defer[Int][x^6
/((-a + x^4)^(1/4)*(-b + x^4)^(1/4)*(a^3 - 3*a^2*(1 + (b*d)/(3*a^2))*x^4 +
3*a*(1 + d/(3*a))*x^8 - x^12)), x], x, x^(1/4)])/((a - x)*(b - x)*x)^(1/4)
+ (4*x^(1/4)*(-a + x)^(1/4)*(-b + x)^(1/4)*Defer[Subst][Defer[Int][x^10/((-
a + x^4)^(1/4)*(-b + x^4)^(1/4)*(-a^3 + 3*a^2*(1 + (b*d)/(3*a^2))*x^4 - 3*a
*(1 + d/(3*a))*x^8 + x^12)), x], x, x^(1/4)])/((a - x)*(b - x)*x)^(1/4)
```

Rubi steps

$$\int \frac{-ab+2(a-b)x+x^2}{\sqrt[4]{x(-a+x)(-b+x)}\left(-a^3+\left(3a^2+bd\right)x-(3a+d)x^2+x^3\right)}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\int \frac{-}{\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}}}{\sqrt[4]{x(-a+x)(-b}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{-a+x}}\right.}{\sqrt[4]{x(-a}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\text{Subst}\left(\int\left(\frac{}{\sqrt[4]{-a+}}\right.\right.}{}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{-a+x}}\right.}{\sqrt[4]{x(-}}$$

Mathematica [F] time = 3.35, size = 0, normalized size = 0.00

$$\int \frac{-ab+2(a-b)x+x^2}{\sqrt[4]{x(-a+x)(-b+x)}\left(-a^3+\left(3a^2+bd\right)x-(3a+d)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-(a*b) + 2*(a - b)*x + x^2)/((x*(-a + x)*(-b + x))^(1/4)*(-a^3 +
 (3*a^2 + b*d)*x - (3*a + d)*x^2 + x^3)),x]
```

```
[Out] Integrate[(-(a*b) + 2*(a - b)*x + x^2)/((x*(-a + x)*(-b + x))^(1/4)*(-a^3 +
 (3*a^2 + b*d)*x - (3*a + d)*x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.28, size = 89, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2(-a-b)+abx+x^3}}{a-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2(-a-b)+abx+x^3}}{a-x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*b) + 2*(a - b)*x + x^2)/((x*(-a + x)*(-b + x))^(1/4
)*(-a^3 + (3*a^2 + b*d)*x - (3*a + d)*x^2 + x^3)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(a*b*x + (-a - b)*x^2 + x^3)^(1/4))/(a - x)])/d^(3/4) +
 (2*ArcTanh[(d^(1/4)*(a*b*x + (-a - b)*x^2 + x^3)^(1/4))/(a - x)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+2*(a-b)*x+x^2)/(x*(-a+x)*(-b+x))^(1/4)/(-a^3+(3*a^2+b*d)*x-
(3*a+d)*x^2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab-2(a-b)x-x^2}{\left(a^3+(3a+d)x^2-x^3-\left(3a^2+bd\right)x\right)((a-x)(b-x)x)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+2*(a-b)*x+x^2)/(x*(-a+x)*(-b+x))^(1/4)/(-a^3+(3*a^2+b*d)*x-
(3*a+d)*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate((a*b - 2*(a - b)*x - x^2)/((a^3 + (3*a + d)*x^2 - x^3 - (3*a^2 +
b*d)*x)*((a - x)*(b - x)*x)^(1/4)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{-ab+2(a-b)x+x^2}{(x(-a+x)(-b+x))^{\frac{1}{4}}\left(-a^3+\left(3a^2+bd\right)x-(3a+d)x^2+x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*b+2*(a-b)*x+x^2)/(x*(-a+x)*(-b+x))^(1/4)/(-a^3+(3*a^2+b*d)*x-(3*a+d
)*x^2+x^3),x)
```

```
[Out] int((-a*b+2*(a-b)*x+x^2)/(x*(-a+x)*(-b+x))^(1/4)/(-a^3+(3*a^2+b*d)*x-(3*a+d
)*x^2+x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab-2(a-b)x-x^2}{\left(a^3+(3a+d)x^2-x^3-\left(3a^2+bd\right)x\right)((a-x)(b-x)x)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+2*(a-b)*x+x^2)/(x*(-a+x)*(-b+x))^(1/4)/(-a^3+(3*a^2+b*d)*x-
(3*a+d)*x^2+x^3),x, algorithm="maxima")
```

```
[Out] integrate((a*b - 2*(a - b)*x - x^2)/((a^3 + (3*a + d)*x^2 - x^3 - (3*a^2 +
b*d)*x)*((a - x)*(b - x)*x)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x\,\left(a-b\right)-a\,b+x^2}{{\left(x\,\left(a-x\right)\,\left(b-x\right)\right)}^{1/4}\,\left(x\,\left(3\,a^2+b\,d\right)-x^2\,\left(3\,a+d\right)-a^3+x^3\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x*(a - b) - a*b + x^2)/((x*(a - x)*(b - x))^(1/4)*(x*(b*d + 3*a^2) -
 x^2*(3*a + d) - a^3 + x^3)),x)
```

```
[Out] int((2*x*(a - b) - a*b + x^2)/((x*(a - x)*(b - x))^(1/4)*(x*(b*d + 3*a^2) -
 x^2*(3*a + d) - a^3 + x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+2*(a-b)*x+x**2)/(x*(-a+x)*(-b+x))**(1/4)/(-a**3+(3*a**2+b*d
)*x-(3*a+d)*x**2+x**3),x)
```

```
[Out] Timed out
```

3.1079 $\int \frac{(1-2x+x^2)(-1+2(-1+k)x+kx^2)}{((1-x)x(1-kx))^{3/4}(-d+(1+3d)x-(3d+k)x^2+dx^3)}\,dx$

Optimal. Leaf size=89

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x-\sqrt[4]{d}}{\sqrt[4]{kx^3+(-k-1)x^2+x}}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x-\sqrt[4]{d}}{\sqrt[4]{kx^3+(-k-1)x^2+x}}\right)}{d^{3/4}}$$

Rubi [F] time = 13.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-2x+x^2)(-1+2(-1+k)x+kx^2)}{((1-x)x(1-kx))^{3/4}(-d+(1+3d)x-(3d+k)x^2+dx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 - 2*x + x^2)*(-1 + 2*(-1 + k)*x + k*x^2))/(((1 - x)*x*(1 - k*x))^(3
/4)*(-d + (1 + 3*d)*x - (3*d + k)*x^2 + d*x^3)),x]
```

```
[Out] (4*(1 - x)^(3/4)*x^(3/4)*(1 - k*x)^(3/4)*Defer[Subst][Defer[Int][(1 - x^4)^
(5/4)/((1 - k*x^4)^(3/4)*(-x^4 + k*x^8 - d*(-1 + x^4)^3)), x], x, x^(1/4)])
/((1 - x)*x*(1 - k*x))^(3/4) + (8*(1 - k)*(1 - x)^(3/4)*x^(3/4)*(1 - k*x)^(
3/4)*Defer[Subst][Defer[Int][(x^4*(1 - x^4)^(5/4))/((1 - k*x^4)^(3/4)*(-x^4
 + k*x^8 - d*(-1 + x^4)^3)), x], x, x^(1/4)])/((1 - x)*x*(1 - k*x))^(3/4) +
 (4*k*(1 - x)^(3/4)*x^(3/4)*(1 - k*x)^(3/4)*Defer[Subst][Defer[Int][(x^8*(1
 - x^4)^(5/4))/((1 - k*x^4)^(3/4)*(x^4 - k*x^8 + d*(-1 + x^4)^3)), x], x, x
^(1/4)])/((1 - x)*x*(1 - k*x))^(3/4)
```

Rubi steps

$$\int \frac{(1-2x+x^2)(-1+2(-1+k)x+kx^2)}{((1-x)x(1-kx))^{3/4}(-d+(1+3d)x-(3d+k)x^2+dx^3)}dx = \int \frac{(-1+x)^2(-1+2(-1+k)x+ \ldots}{((1-x)x(1-kx))^{3/4}(-d+(1+3d)x-(3 \ldots}$$

$$= \frac{\left((1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\int \frac{(-1+x)^2(\ldots}{(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\ldots}}{((1-x)x(1-kx))^{3/4}}$$

$$= \frac{\left((1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\int \frac{(1-x)^{5/4}(-1+2\ldots}{x^{3/4}(1-kx)^{3/4}(-d+(1+\ldots}}{((1-x)x(1-kx))^{3/4}}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subst}\left(\int \frac{(1-\ldots}{(1-kx^4)^{3/4}\ldots}\right.}{((1-x)x(1-\ldots}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subst}\left(\int \frac{(1-x^4)^{5/4}\ldots}{(1-kx^4)^{3/4}\ldots}\right.}{((1-x)x(1-kx))^{3\ldots}}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subst}\left(\int \left(\frac{\ldots}{(1-kx^4)\ldots}\right.\right.}{\ldots}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subst}\left(\int \frac{\ldots}{(1-kx^4)^{3/4}\ldots}\right.}{((1-x)x(1-\ldots}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subst}\left(\int \frac{\ldots}{(1-kx^4)^{3/4}\ldots}\right.}{((1-x)x(1-kx))\ldots}$$

Mathematica [F] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{(1-2x+x^2)(-1+2(-1+k)x+kx^2)}{((1-x)x(1-kx))^{3/4}(-d+(1+3d)x-(3d+k)x^2+dx^3)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 - 2*x + x^2)*(-1 + 2*(-1 + k)*x + k*x^2))/(((1 - x)*x*(1 - k*
x))^(3/4)*(-d + (1 + 3*d)*x - (3*d + k)*x^2 + d*x^3)),x]
```

```
[Out] Integrate[((1 - 2*x + x^2)*(-1 + 2*(-1 + k)*x + k*x^2))/(((1 - x)*x*(1 - k*
x))^(3/4)*(-d + (1 + 3*d)*x - (3*d + k)*x^2 + d*x^3)), x]
```

IntegrateAlgebraic [A] time = 3.14, size = 89, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}x-\sqrt[4]{d}}{\sqrt[4]{kx^3+(-k-1)x^2+x}}\right)}{d^{3/4}} - \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}x-\sqrt[4]{d}}{\sqrt[4]{kx^3+(-k-1)x^2+x}}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 - 2*x + x^2)*(-1 + 2*(-1 + k)*x + k*x^2))/(((1 - x)*
x*(1 - k*x))^(3/4)*(-d + (1 + 3*d)*x - (3*d + k)*x^2 + d*x^3)),x]
```

```
[Out] (2*ArcTan[(-d^(1/4) + d^(1/4)*x)/(x + (-1 - k)*x^2 + k*x^3)^(1/4)])/d^(3/4)
 - (2*ArcTanh[(-d^(1/4) + d^(1/4)*x)/(x + (-1 - k)*x^2 + k*x^3)^(1/4)])/d^(
3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x+1)*(-1+2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(3/4)/(-d+(1
+3*d)*x-(3*d+k)*x^2+d*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(kx^2 + 2\,(k-1)x - 1\right)\left(x^2 - 2\,x + 1\right)}{\left(dx^3 - (3\,d + k)x^2 + (3\,d + 1)x - d\right)\left((kx - 1)(x - 1)x\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x+1)*(-1+2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(3/4)/(-d+(1
+3*d)*x-(3*d+k)*x^2+d*x^3),x, algorithm="giac")
```

```
[Out] integrate((k*x^2 + 2*(k - 1)*x - 1)*(x^2 - 2*x + 1)/((d*x^3 - (3*d + k)*x^2
 + (3*d + 1)*x - d)*((k*x - 1)*(x - 1)*x)^(3/4)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2 - 2x + 1\right)\left(-1 + 2\,(-1 + k)\,x + k\,x^2\right)}{\left((1 - x)\,x\,(-kx + 1)\right)^{\frac{3}{4}}\left(-d + (1 + 3d)\,x - (3d + k)\,x^2 + d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x+1)*(-1+2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(3/4)/(-d+(1+3*d)*
x-(3*d+k)*x^2+d*x^3),x)
```

```
[Out] int((x^2-2*x+1)*(-1+2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(3/4)/(-d+(1+3*d)*
x-(3*d+k)*x^2+d*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(kx^2 + 2\,(k-1)x - 1\right)\left(x^2 - 2\,x + 1\right)}{\left(dx^3 - (3\,d + k)x^2 + (3\,d + 1)x - d\right)\left((kx - 1)(x - 1)x\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x+1)*(-1+2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(3/4)/(-d+(1
+3*d)*x-(3*d+k)*x^2+d*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k*x^2 + 2*(k - 1)*x - 1)*(x^2 - 2*x + 1)/((d*x^3 - (3*d + k)*x^2
 + (3*d + 1)*x - d)*((k*x - 1)*(x - 1)*x)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^2 - 2\,x + 1\right)\,\left(2\,x\,\left(k - 1\right) + k\,x^2 - 1\right)}{{\left(x\,\left(k\,x - 1\right)\,\left(x - 1\right)\right)}^{3/4}\,\left(-d\,x^3 + \left(3\,d + k\right)\,x^2 + \left(-3\,d - 1\right)\,x + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^2 - 2*x + 1)*(2*x*(k - 1) + k*x^2 - 1))/((x*(k*x - 1)*(x - 1))^(3/
4)*(d - d*x^3 + x^2*(3*d + k) - x*(3*d + 1))),x)
```

```
[Out] int(-((x^2 - 2*x + 1)*(2*x*(k - 1) + k*x^2 - 1))/((x*(k*x - 1)*(x - 1))^(3/
4)*(d - d*x^3 + x^2*(3*d + k) - x*(3*d + 1))), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x+1)*(-1+2*(-1+k)*x+k*x**2)/((1-x)*x*(-k*x+1))**(3/4)/(-d
+(1+3*d)*x-(3*d+k)*x**2+d*x**3),x)
```

```
[Out] Timed out
```

3.1080 $\int \frac{x^2}{\sqrt{x+x^2+x^3}\left(-1+x^4\right)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{4}\tan^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x^3+x^2+x}}{x^2+x+1}\right)}{4\sqrt{3}}$$

Rubi [C] time = 2.39, antiderivative size = 541, normalized size of antiderivative = 6.08, number of steps used = 59, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2056, 6725, 957, 716, 1103, 934, 169, 538, 537, 1139, 1195}

$$\frac{\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\,\Pi\left(\frac{1}{2}\left(-i-\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}-\frac{\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\,\Pi\left(\frac{1}{2}\left(1-i\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}-\frac{\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\,\Pi\left(\frac{1}{2}\left(-1+i\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}+\frac{\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\,\Pi\left(\frac{1}{2}\left(i+\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2/(Sqrt[x + x^2 + x^3]*(-1 + x^4)),x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + (2*x)/(1 - I*Sqrt[3])]*Sqrt[1 + (2*x)/(1 + I*Sqrt[3])]*El
lipticPi[(-I - Sqrt[3])/2, ArcSin[((1 - I*Sqrt[3])*Sqrt[x])/2], (I + Sqrt[3
])/(I - Sqrt[3])])/((1 - I*Sqrt[3])*Sqrt[x + x^2 + x^3]) - (Sqrt[x]*Sqrt[1
+ (2*x)/(1 - I*Sqrt[3])]*Sqrt[1 + (2*x)/(1 + I*Sqrt[3])]*EllipticPi[(1 - I*
Sqrt[3])/2, ArcSin[((1 - I*Sqrt[3])*Sqrt[x])/2], (I + Sqrt[3])/(I - Sqrt[3]
)])/((1 - I*Sqrt[3])*Sqrt[x + x^2 + x^3]) - (Sqrt[x]*Sqrt[1 + (2*x)/(1 - I*
Sqrt[3])]*Sqrt[1 + (2*x)/(1 + I*Sqrt[3])]*EllipticPi[(-1 + I*Sqrt[3])/2, Ar
cSin[((1 - I*Sqrt[3])*Sqrt[x])/2], (I + Sqrt[3])/(I - Sqrt[3])])/((1 - I*Sq
rt[3])*Sqrt[x + x^2 + x^3]) + (Sqrt[x]*Sqrt[1 + (2*x)/(1 - I*Sqrt[3])]*Sqrt
[1 + (2*x)/(1 + I*Sqrt[3])]*EllipticPi[(I + Sqrt[3])/2, ArcSin[((1 - I*Sqrt
[3])*Sqrt[x])/2], (I + Sqrt[3])/(I - Sqrt[3])])/((1 - I*Sqrt[3])*Sqrt[x + x
^2 + x^3])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 716

```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2,
Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; FreeQ
```

```
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 957

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{x+x^2+x^3}\left(-1+x^4\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{x^{3/2}}{\sqrt{1+x+x^2}\left(-1+x^4\right)}\,dx}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \left(-\frac{x^{3/2}}{2\left(1-x^2\right)\sqrt{1+x+x^2}}-\frac{x^{3/2}}{2\left(1+x^2\right)\sqrt{1+x+x^2}}\right)dx}{\sqrt{x+x^2+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{x^{3/2}}{\left(1-x^2\right)\sqrt{1+x+x^2}}\,dx}{2\sqrt{x+x^2+x^3}} - \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{x^{3/2}}{\left(1+x^2\right)\sqrt{1+x+x^2}}\,dx}{2\sqrt{x+x^2+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \left(\frac{ix^{3/2}}{2(i-x)\sqrt{1+x+x^2}}+\frac{ix^{3/2}}{2(i+x)\sqrt{1+x+x^2}}\right)dx}{2\sqrt{x+x^2+x^3}} - \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int}{}$$

$$= -\frac{\left(i\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{x^{3/2}}{(i-x)\sqrt{1+x+x^2}}\,dx}{4\sqrt{x+x^2+x^3}} - \frac{\left(i\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{x^{3/2}}{(i+x)\sqrt{1+x+x^2}}\,dx}{4\sqrt{x+x^2+x^3}} -$$

$$= -\frac{\left(i\sqrt{x}\sqrt{1+x+x^2}\right)\int \left(-\frac{i}{\sqrt{x}\sqrt{1+x+x^2}}+\frac{1}{(-i-x)\sqrt{x}\sqrt{1+x+x^2}}+\frac{\sqrt{x}}{\sqrt{1+x+x^2}}\right)dx}{4\sqrt{x+x^2+x^3}} - \frac{\left(i\sqrt{x}\right.}{}$$

$$= -\frac{\left(i\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1}{(-i-x)\sqrt{x}\sqrt{1+x+x^2}}\,dx}{4\sqrt{x+x^2+x^3}} - \frac{\left(i\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1}{\sqrt{x}(-i+x)\sqrt{1+x+}}}{4\sqrt{x+x^2+x^3}}$$

$$= -\frac{\left(i\sqrt{x}\sqrt{1-i\sqrt{3}+2x}\sqrt{1+i\sqrt{3}+2x}\right)\int \frac{1}{(-i-x)\sqrt{x}\sqrt{1-i\sqrt{3}+2x}\sqrt{1+i\sqrt{3}+2x}}\,dx}{4\sqrt{x+x^2+x^3}} - \frac{\left(i\right.}{}$$

$$= \frac{\left(i\sqrt{x}\sqrt{1-i\sqrt{3}+2x}\sqrt{1+i\sqrt{3}+2x}\right)\text{Subst}\left(\int \frac{1}{\left(i-x^2\right)\sqrt{1-i\sqrt{3}+2x^2}\sqrt{1+i\sqrt{3}+2x^2}}\,dx,\right.}{2\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(i\sqrt{x}\sqrt{1+i\sqrt{3}+2x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\right)\text{Subst}\left(\int \frac{1}{\left(i-x^2\right)\sqrt{1+i\sqrt{3}+2x^2}\sqrt{1+\frac{2x^2}{1-i\sqrt{3}}}}\,dx,x,\sqrt{\right.}{2\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(i\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\right)\text{Subst}\left(\int \frac{1}{\left(i-x^2\right)\sqrt{1+\frac{2x^2}{1-i\sqrt{3}}}\sqrt{1+\frac{2x^2}{1+i\sqrt{3}}}}\,dx,x,\sqrt{x}\right)}{2\sqrt{x+x^2+x^3}} +$$

$$= \frac{\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\,\Pi\left(\frac{1}{2}\left(-i-\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{\left(1-i\sqrt{3}\right)\sqrt{x+x^2+x^3}} -$$

Mathematica [C] time = 0.62, size = 153, normalized size = 1.72

$$\frac{\sqrt[3]{-1}\sqrt{\sqrt[3]{-1}x\left(x+\sqrt[3]{-1}\right)}\sqrt{\sqrt[3]{-1}x+1}\left(\Pi\left(-\sqrt[3]{-1};\sin^{-1}\left(\sqrt{(-1)^{2/3}x}\right)|(-1)^{2/3}\right)+\Pi\left(\sqrt[3]{-1};\sin^{-1}\left(\sqrt{(-1)^{2/3}x}\right)|(-1)^{2/3}\right)-\Pi\left(-(-1)^{5/6};\sin^{-1}\left(\sqrt{(-1)^{2/3}x}\right)|(-1)^{2/3}\right)-\Pi\left((-1)^{5/6};\sin^{-1}\left(\sqrt{(-1)^{2/3}x}\right)|(-1)^{2/3}\right)\right)}{2\sqrt{x\left(x^2+x+1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[x + x^2 + x^3]*(-1 + x^4)),x]
```

```
[Out] ((-1)^(1/3)*Sqrt[(-1)^(1/3)*x*((-1)^(1/3) + x)]*Sqrt[1 + (-1)^(1/3)*x]*(Ell
ipticPi[-(-1)^(1/3), ArcSin[Sqrt[(-1)^(2/3)*x]], (-1)^(2/3)] + EllipticPi[(
```

```
-1)^(1/3), ArcSin[Sqrt[(-1)^(2/3)*x]], (-1)^(2/3)] - EllipticPi[-(-1)^(5/6)
, ArcSin[Sqrt[(-1)^(2/3)*x]], (-1)^(2/3)] - EllipticPi[(-1)^(5/6), ArcSin[S
qrt[(-1)^(2/3)*x]], (-1)^(2/3)]))/(2*Sqrt[x*(1 + x + x^2)])
```

IntegrateAlgebraic [A] time = 0.14, size = 89, normalized size = 1.00

$$-\frac{1}{4}\tan^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x^3+x^2+x}}{x^2+x+1}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[x + x^2 + x^3]*(-1 + x^4)),x]
```

```
[Out] -1/4*ArcTan[Sqrt[x + x^2 + x^3]/(1 + x + x^2)] + ArcTanh[Sqrt[x + x^2 + x^3
]/(1 + x + x^2)]/2 - ArcTanh[(Sqrt[3]*Sqrt[x + x^2 + x^3])/(1 + x + x^2)]/(
4*Sqrt[3])
```

fricas [A] time = 0.57, size = 120, normalized size = 1.35

$$\frac{1}{48}\sqrt{3}\log\left(\frac{x^4+20x^3-4\sqrt{3}\sqrt{x^3+x^2+x}\left(x^2+4x+1\right)+30x^2+20x+1}{x^4-4x^3+6x^2-4x+1}\right)+\frac{1}{8}\arctan\left(\frac{x^2+1}{2\sqrt{x^3+x^2+x}}\right)+\frac{1}{4}\log\left(\frac{x^2+2x+2\sqrt{x^3+x^2+x}+1}{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+x^2+x)^(1/2)/(x^4-1),x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(3)*log((x^4 + 20*x^3 - 4*sqrt(3)*sqrt(x^3 + x^2 + x)*(x^2 + 4*x +
 1) + 30*x^2 + 20*x + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)) + 1/8*arctan(1/2*
(x^2 + 1)/sqrt(x^3 + x^2 + x)) + 1/4*log((x^2 + 2*x + 2*sqrt(x^3 + x^2 + x)
 + 1)/(x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^4-1\right)\sqrt{x^3+x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+x^2+x)^(1/2)/(x^4-1),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^4 - 1)*sqrt(x^3 + x^2 + x)), x)
```

maple [C] time = 0.03, size = 960, normalized size = 10.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3+x^2+x)^(1/2)/(x^4-1),x)
```

```
[Out] 1/6*(1/2+1/2*I*3^(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3
^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/
2)/(x^3+x^2+x)^(1/2)/(-3/2-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))
/(1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)),1/3*3
^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))-1/6*(1/2+1/2*I*3^(1/2))*((x+
1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2)/(1/2-1/
2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),(
-1/2-1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2))-1/12*I*(1/(1/2+1/2*I*3^(1/2))*x+1/2/(1/2+1/2*I*3^(1/2))+1/
2*I/(1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3
/2)^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2)/(-1/2-I-1/2*I*3^
```

```
(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1
/2*I*3^(1/2))/(-1/2-I-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2))+1/4*(1/(1/2+1/2*I*3^(1/2))*x+1/2/(1/2+1/2*I*3^(1/2))+1/2*I/(1
/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(x/(
-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2)/(-1/2-I-1/2*I*3^(1/2))*Ellipti
cPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1/2*I*3^(1/2))/
(-1/2-I-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))+
1/12*I*(1/(1/2+1/2*I*3^(1/2))*x+1/2/(1/2+1/2*I*3^(1/2))+1/2*I/(1/2+1/2*I*3^
(1/2))*3^(1/2))^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(x/(-1/
2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2)/(-1/2+I-1/2*I*3^(1/2))*EllipticPi
(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-1
/2+I-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))-1/4
*(1/(1/2+1/2*I*3^(1/2))*x+1/2/(1/2+1/2*I*3^(1/2))+1/2*I/(1/2+1/2*I*3^(1/2))
*3^(1/2))^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(x/(-1/2-1/2*I*3^(1/2
)))^(1/2)/(x^3+x^2+x)^(1/2)/(-1/2+I-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I
*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-1/2+I-1/2*I*3^(
1/2)),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^4-1\right)\sqrt{x^3+x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+x^2+x)^(1/2)/(x^4-1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^4 - 1)*sqrt(x^3 + x^2 + x)), x)
```

mupad [B] time = 0.07, size = 565, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^4 - 1)*(x + x^2 + x^3)^(1/2)),x)
```

```
[Out] (((3^(1/2)*1i)/2 - 1/2)*(x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i
)/2 + 1/2)/((3^(1/2)*1i)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1
/2)*1i)/2 + 1/2))^(1/2)*ellipticPi(- 3^(1/2)/2 - 1i/2, asin((x/((3^(1/2)*1i
)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))/(2*(x^
2 + x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) + (((3^(1
/2)*1i)/2 - 1/2)*(x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 + 1/2))^(1/2)*ellipticPi(3^(1/2)/2 + 1i/2, asin((x/((3^(1/2)*1i)/2 - 1/2
))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))/(2*(x^2 + x^3 -
 x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (((3^(1/2)*1i)/2
 - 1/2)*(x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(
1/2)*1i)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2)
)^(1/2)*ellipticPi(1/2 - (3^(1/2)*1i)/2, asin((x/((3^(1/2)*1i)/2 - 1/2))^(1
/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))/(2*(x^2 + x^3 - x*((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (((3^(1/2)*1i)/2 - 1/
2)*(x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*
1i)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/
2)*ellipticPi((3^(1/2)*1i)/2 - 1/2, asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)),
 -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))/(2*(x^2 + x^3 - x*((3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x\left(x^2+x+1\right)}\,(x-1)\,(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**3+x**2+x)**(1/2)/(x**4-1),x)
[Out] Integral(x**2/(sqrt(x*(x**2 + x + 1))*(x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.1081 $\int \frac{\sqrt{x+x^2+x^3}}{-1+x^4}\,dx$

Optimal. Leaf size=89

$$\frac{1}{4}\tan^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)-\frac{1}{4}\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x^3+x^2+x}}{x^2+x+1}\right)$$

Rubi [A] time = 3.13, antiderivative size = 155, normalized size of antiderivative = 1.74, number of steps used = 41, number of rules used = 18, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {2056, 6725, 918, 27, 6733, 1712, 1197, 1103, 1195, 1700, 1698, 206, 12, 1210, 203, 1714, 1708, 1706}

$$\frac{\sqrt{x^3+x^2+x}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x^2+x+1}}\right)}{4\sqrt{x}\sqrt{x^2+x+1}}+\frac{\sqrt{x^3+x^2+x}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x^2+x+1}}\right)}{2\sqrt{x}\sqrt{x^2+x+1}}-\frac{\sqrt{3}\sqrt{x^3+x^2+x}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x}}{\sqrt{x^2+x+1}}\right)}{4\sqrt{x}\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + x^2 + x^3]/(-1 + x^4),x]
```

```
[Out] (Sqrt[x + x^2 + x^3]*ArcTan[Sqrt[x]/Sqrt[1 + x + x^2]])/(4*Sqrt[x]*Sqrt[1 +
 x + x^2]) + (Sqrt[x + x^2 + x^3]*ArcTanh[Sqrt[x]/Sqrt[1 + x + x^2]])/(2*Sq
rt[x]*Sqrt[1 + x + x^2]) - (Sqrt[3]*Sqrt[x + x^2 + x^3]*ArcTanh[(Sqrt[3]*Sq
rt[x])/Sqrt[1 + x + x^2]])/(4*Sqrt[x]*Sqrt[1 + x + x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 918

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/(e*(2*m + 5)), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m*Simp[b*d*f - 3*a*e*f + a*d*g + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x -
 (c*e*f - 3*c*d*g + b*e*g)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1210

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1700

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
```

```
[c*A^2 - a*B^2, 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1712

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{\sqrt{x+x^2+x^3}}{-1+x^4}\,dx = \frac{\sqrt{x+x^2+x^3}\int \frac{\sqrt{x}\sqrt{1+x+x^2}}{-1+x^4}\,dx}{\sqrt{x}\sqrt{1+x+x^2}}$$

$$= \frac{\sqrt{x+x^2+x^3}\int \left(-\frac{\sqrt{x}\sqrt{1+x+x^2}}{2(1-x^2)} - \frac{\sqrt{x}\sqrt{1+x+x^2}}{2(1+x^2)}\right)dx}{\sqrt{x}\sqrt{1+x+x^2}}$$

$$= -\frac{\sqrt{x+x^2+x^3}\int \frac{\sqrt{x}\sqrt{1+x+x^2}}{1-x^2}\,dx}{2\sqrt{x}\sqrt{1+x+x^2}} - \frac{\sqrt{x+x^2+x^3}\int \frac{\sqrt{x}\sqrt{1+x+x^2}}{1+x^2}\,dx}{2\sqrt{x}\sqrt{1+x+x^2}}$$

$$= -\frac{\sqrt{x+x^2+x^3}\int \left(\frac{i\sqrt{x}\sqrt{1+x+x^2}}{2(i-x)} + \frac{i\sqrt{x}\sqrt{1+x+x^2}}{2(i+x)}\right)dx}{2\sqrt{x}\sqrt{1+x+x^2}} - \frac{\sqrt{x+x^2+x^3}\int \left(\frac{\sqrt{x}\sqrt{1+x+x^2}}{2(1-x)} + \frac{\sqrt{x}\ \cdots}{\cdots}\right.}{2\sqrt{x}\sqrt{1+x+x^2}}$$

$$= -\frac{\left(i\sqrt{x+x^2+x^3}\right)\int \frac{\sqrt{x}\sqrt{1+x+x^2}}{i-x}\,dx}{4\sqrt{x}\sqrt{1+x+x^2}} - \frac{\left(i\sqrt{x+x^2+x^3}\right)\int \frac{\sqrt{x}\sqrt{1+x+x^2}}{i+x}\,dx}{4\sqrt{x}\sqrt{1+x+x^2}} - \frac{\sqrt{x+x^2+x^3}\ \cdots}{4\sqrt{x}\sqrt{\cdots}}$$

$$= \frac{\left(i\sqrt{x+x^2+x^3}\right)\int \frac{i-(2-2i)x-(1-3i)x^2}{\sqrt{x}(i+x)\sqrt{1+x+x^2}}\,dx}{12\sqrt{x}\sqrt{1+x+x^2}} - \frac{\left(i\sqrt{x+x^2+x^3}\right)\int \frac{i+(2+2i)x+(1+3i)x^2}{(i-x)\sqrt{x}\sqrt{1+x+x^2}}\,dx}{12\sqrt{x}\sqrt{1+x+x^2}} + \frac{\sqrt{x+\cdots}}{\cdots}$$

$$= \frac{\left(i\sqrt{x+x^2+x^3}\right)\mathrm{Subst}\left(\int \frac{i-(2-2i)x^2-(1-3i)x^4}{(i+x^2)\sqrt{1+x^2+x^4}}\,dx, x, \sqrt{x}\right)}{6\sqrt{x}\sqrt{1+x+x^2}} - \frac{\left(i\sqrt{x+x^2+x^3}\right)\mathrm{Subst}\left(\int \frac{i+(2\cdots}{(i-\cdots}\right.}{6\sqrt{x}\sqrt{1+x\cdots}}$$

$$= -\left(-\frac{\left(\left(\frac{1}{2}-\frac{i}{6}\right)\sqrt{x+x^2+x^3}\right)\mathrm{Subst}\left(\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{1+x+x^2}}\right) + \frac{\left(i\sqrt{x+x^2+x^3}\right)\mathrm{Subst}\left(\cdots\right.}{6\sqrt{x}\sqrt{\cdots}}$$

$$= -\frac{2\sqrt{x+x^2+x^3}}{3(1+x)} + \frac{2(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\sqrt{x+x^2+x^3}\,E\left(2\tan^{-1}\left(\sqrt{x}\right)\middle|\frac{1}{4}\right)}{3\sqrt{x}\left(1+x+x^2\right)} + -\frac{\left(\left(\frac{3}{4}+\frac{i}{4}\right)\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{2\sqrt{x+x^2+x^3}}{3(1+x)} + \frac{\sqrt{x+x^2+x^3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x+x^2}}\right)}{2\sqrt{x}\sqrt{1+x+x^2}} + \frac{2(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\sqrt{x+x^2+x^3}\ \cdots}{3\sqrt{x}\left(1+x+\cdots\right.}$$

$$= \frac{\sqrt{x+x^2+x^3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x+x^2}}\right)}{2\sqrt{x}\sqrt{1+x+x^2}} + \frac{\sqrt{x+x^2+x^3}\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{1+x+x^2}}\right)}{4\sqrt{x}\sqrt{1+x+x^2}} - \frac{\left(3\sqrt{\cdots}\right.}{\cdots}$$

$$= \frac{\sqrt{x+x^2+x^3}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x+x^2}}\right)}{4\sqrt{x}\sqrt{1+x+x^2}} + \frac{\sqrt{x+x^2+x^3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x+x^2}}\right)}{2\sqrt{x}\sqrt{1+x+x^2}} - \frac{\sqrt{3}\sqrt{x+x^2+x^3}\ \cdots}{4\sqrt{x}\sqrt{1\cdots}}$$

Mathematica [C] time = 3.22, size = 743, normalized size = 8.35

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[x + x^2 + x^3]/(-1 + x^4),x]
```

```
[Out] ((-1)^(2/3)*(-1 + Sqrt[x])*(1 + Sqrt[x])*Sqrt[x]*(1 + x)*(1 + x^2)*((3*((-1
)^(1/3) - Sqrt[x])^2*Sqrt[((-1)^(2/3) - Sqrt[x])/((1 + (-1)^(1/3))*((-1)^(1
/3) - Sqrt[x]))]*Sqrt[(1 - (-1 + (-1)^(1/3))*Sqrt[x])/((1 + (-1)^(1/3))*((-
1)^(1/3) - Sqrt[x]))]*Sqrt[-(((-1)^(2/3) + Sqrt[x])/(1 + (-1 + (-1)^(1/3))*
```

```
Sqrt[x]))]*((1 + (-1)^(1/3))*EllipticF[ArcSin[Sqrt[(1 - (-1 + (-1)^(1/3))*S
qrt[x])/((1 + (-1)^(1/3))*((-1)^(1/3) - Sqrt[x]))]], -3] - 2*(-1)^(1/3)*Ell
ipticPi[-1, ArcSin[Sqrt[(1 - (-1 + (-1)^(1/3))*Sqrt[x])/((1 + (-1)^(1/3))*(
(-1)^(1/3) - Sqrt[x]))]], -3]))/(-1 + (-1)^(1/3))^2 - (3*((-1)^(1/3) - Sqrt
[x])^2*Sqrt[((-1)^(2/3) - Sqrt[x])/((1 + (-1)^(1/3))*((-1)^(1/3) - Sqrt[x])
)]*Sqrt[(1 - (-1 + (-1)^(1/3))*Sqrt[x])/((1 + (-1)^(1/3))*((-1)^(1/3) - Sqr
t[x]))]*Sqrt[-(((-1)^(2/3) + Sqrt[x])/(1 + (-1 + (-1)^(1/3))*Sqrt[x]))]*((-
1 + (-1)^(1/3))*EllipticF[ArcSin[Sqrt[(1 - (-1 + (-1)^(1/3))*Sqrt[x])/((1 +
 (-1)^(1/3))*((-1)^(1/3) - Sqrt[x]))]], -3] - 2*(-1)^(1/3)*EllipticPi[3, Ar
cSin[Sqrt[(1 - (-1 + (-1)^(1/3))*Sqrt[x])/((1 + (-1)^(1/3))*((-1)^(1/3) - S
qrt[x]))]], -3]))/(-1 + (-1)^(1/3))^2 + Sqrt[1 + (-1)^(1/3)*x]*Sqrt[1 - (-1
)^(2/3)*x]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*Sqrt[x]], (-1)^(2/3)
] + Sqrt[1 + (-1)^(1/3)*x]*Sqrt[1 - (-1)^(2/3)*x]*EllipticPi[-(-1)^(5/6), I
*ArcSinh[(-1)^(5/6)*Sqrt[x]], (-1)^(2/3)] + Sqrt[1 + (-1)^(1/3)*x]*Sqrt[1 -
 (-1)^(2/3)*x]*EllipticPi[(-1)^(5/6), I*ArcSinh[(-1)^(5/6)*Sqrt[x]], (-1)^(
2/3)]))/(2*Sqrt[x*(1 + x + x^2)]*(-1 + x^4))
```

IntegrateAlgebraic [A] time = 0.13, size = 89, normalized size = 1.00

$$\frac{1}{4}\tan^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)-\frac{1}{4}\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x^3+x^2+x}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x + x^2 + x^3]/(-1 + x^4),x]
```

```
[Out] ArcTan[Sqrt[x + x^2 + x^3]/(1 + x + x^2)]/4 + ArcTanh[Sqrt[x + x^2 + x^3]/(
1 + x + x^2)]/2 - (Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[x + x^2 + x^3])/(1 + x + x
^2)])/4
```

fricas [A] time = 0.72, size = 120, normalized size = 1.35

$$\frac{1}{16}\sqrt{3}\log\left(\frac{x^4+20x^3-4\sqrt{3}\sqrt{x^3+x^2+x}\left(x^2+4x+1\right)+30x^2+20x+1}{x^4-4x^3+6x^2-4x+1}\right)-\frac{1}{8}\arctan\left(\frac{x^2+1}{2\sqrt{x^3+x^2+x}}\right)+\frac{1}{4}\log\left(\frac{x^2+2x+2\sqrt{x^3+x^2+x}+1}{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x)^(1/2)/(x^4-1),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(3)*log((x^4 + 20*x^3 - 4*sqrt(3)*sqrt(x^3 + x^2 + x)*(x^2 + 4*x +
 1) + 30*x^2 + 20*x + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)) - 1/8*arctan(1/2*
(x^2 + 1)/sqrt(x^3 + x^2 + x)) + 1/4*log((x^2 + 2*x + 2*sqrt(x^3 + x^2 + x)
 + 1)/(x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^3+x^2+x}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x)^(1/2)/(x^4-1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^3 + x^2 + x)/(x^4 - 1), x)
```

maple [C] time = 0.34, size = 1788, normalized size = 20.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x)^(1/2)/(x^4-1),x)
```

```
[Out] 1/3*(1/2+1/2*I*3^(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3
^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/
2)/(x^3+x^2+x)^(1/2)*EllipticF(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^
(1/2),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))+1/3*(1/2+1/2*I*3^
(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3^(1/2)*(I*(x+1/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1
/2)*(-I*3^(1/2)*EllipticE(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)
,1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))+(-1/2+1/2*I*3^(1/2))*E
llipticF(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),1/3*3^(1/2)*(I*(
-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)))+1/2*(1/2+1/2*I*3^(1/2))*((x+1/2+1/2*I*
3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2)/(-3/2-1/2*I*3^(1/
2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1/2*
I*3^(1/2))/(-3/2-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2)
)^(1/2))+1/6*(1/2+1/2*I*3^(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))
)^(1/2)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1
/2)))^(1/2)/(x^3+x^2+x)^(1/2)/(1/2-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*
3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2)
),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))+1/6*(1/(1/2+1/2*I*3^(
1/2))*x+1/2/(1/2+1/2*I*3^(1/2))+1/2*I/(1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)*3^
(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/
(x^3+x^2+x)^(1/2)*EllipticF(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/
2),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))-1/2*I*(1/(1/2+1/2*I*
3^(1/2))*x+1/2/(1/2+1/2*I*3^(1/2))+1/2*I/(1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)
*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+
x^2+x)^(1/2)*EllipticF(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),1/
3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))+1/12*I*(1/(1/2+1/2*I*3^(1
/2))*x+1/2/(1/2+1/2*I*3^(1/2))+1/2*I/(1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)*3^(
1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(
x^3+x^2+x)^(1/2)/(-1/2+I-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(
1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-1/2+I-1/2*I*3^(1/2)),1/3*3
^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))-1/2*(1/(1/2+1/2*I*3^(1/2))*x
+1/2/(1/2+1/2*I*3^(1/2))+1/2*I/(1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)*3^(1/2)*(
I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^
2+x)^(1/2)*EllipticE(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),1/3*
3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))-1/12*I*(1/(1/2+1/2*I*3^(1/2
))*x+1/2/(1/2+1/2*I*3^(1/2))+1/2*I/(1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)*3^(1/
2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^
3+x^2+x)^(1/2)/(-1/2-I-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/
2+1/2*I*3^(1/2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-1/2-I-1/2*I*3^(1/2)),1/3*3^(
1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))+1/4*(1/(1/2+1/2*I*3^(1/2))*x+1
/2/(1/2+1/2*I*3^(1/2))+1/2*I/(1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)*(I*3^(1/2)*
x+1/2*I*3^(1/2)+3/2)^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2)
/(-1/2-I-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2
)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-1/2-I-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2))+1/2*I*(1/(1/2+1/2*I*3^(1/2))*x+1/2/(1/2+1/2*
I*3^(1/2))+1/2*I/(1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1
/2)+3/2)^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2)*EllipticE((
(x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),1/3*3^(1/2)*(I*(-1/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2))-1/4*(1/(1/2+1/2*I*3^(1/2))*x+1/2/(1/2+1/2*I*3^(1/
2))+1/2*I/(1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2
)^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2)/(-1/2+I-1/2*I*3^(1
/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1/2
*I*3^(1/2))/(-1/2+I-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^3 + x^2 + x}}{x^4 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x)^(1/2)/(x^4-1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^3 + x^2 + x)/(x^4 - 1), x)
```

mupad [B] time = 0.06, size = 565, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3)^(1/2)/(x^4 - 1),x)
```

```
[Out] (((3^(1/2)*1i)/2 - 1/2)*(x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i
)/2 + 1/2)/((3^(1/2)*1i)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1
/2)*1i)/2 + 1/2))^(1/2)*ellipticPi(- 3^(1/2)/2 - 1i/2, asin((x/((3^(1/2)*1i
)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))/(2*(x^
2 + x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) + (((3^(1
/2)*1i)/2 - 1/2)*(x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 + 1/2))^(1/2)*ellipticPi(3^(1/2)/2 + 1i/2, asin((x/((3^(1/2)*1i)/2 - 1/2
))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))/(2*(x^2 + x^3 -
 x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) + (((3^(1/2)*1i)/2
 - 1/2)*(x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(
1/2)*1i)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2)
)^(1/2)*ellipticPi(1/2 - (3^(1/2)*1i)/2, asin((x/((3^(1/2)*1i)/2 - 1/2))^(1
/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))/(2*(x^2 + x^3 - x*((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (3*((3^(1/2)*1i)/2 -
1/2)*(x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2
)*1i)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(
1/2)*ellipticPi((3^(1/2)*1i)/2 - 1/2, asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)
), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))/(2*(x^2 + x^3 - x*((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(x^2+x+1\right)}}{(x-1)(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x)**(1/2)/(x**4-1),x)
```

```
[Out] Integral(sqrt(x*(x**2 + x + 1))/((x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.1082 $\int \frac{(-1+x^4)^{2/3}}{x} \, dx$

Optimal. Leaf size=89

$$\frac{3}{8}\left(x^4-1\right)^{2/3}+\frac{1}{4}\log\left(\sqrt[3]{x^4-1}+1\right)-\frac{1}{8}\log\left(\left(x^4-1\right)^{2/3}-\sqrt[3]{x^4-1}+1\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^4-1}}{\sqrt{3}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 50, 56, 618, 204, 31}

$$\frac{3}{8}\left(x^4-1\right)^{2/3}+\frac{3}{8}\log\left(\sqrt[3]{x^4-1}+1\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{x^4-1}}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)^(2/3)/x,x]
```

```
[Out] (3*(-1 + x^4)^(2/3))/8 + (Sqrt[3]*ArcTan[(1 - 2*(-1 + x^4)^(1/3))/Sqrt[3]])
/4 - Log[x]/2 + (3*Log[1 + (-1 + x^4)^(1/3)])/8
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-1+x^4\right)^{2/3}}{x}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{(-1+x)^{2/3}}{x}\,dx,x,x^4\right)\\ &= \frac{3}{8}\left(-1+x^4\right)^{2/3}-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x}\,dx,x,x^4\right)\\ &= \frac{3}{8}\left(-1+x^4\right)^{2/3}-\frac{\log(x)}{2}+\frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt[3]{-1+x^4}\right)-\frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,\right.\\ &= \frac{3}{8}\left(-1+x^4\right)^{2/3}-\frac{\log(x)}{2}+\frac{3}{8}\log\left(1+\sqrt[3]{-1+x^4}\right)+\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2\sqrt[3]{-1+}\right.\\ &= \frac{3}{8}\left(-1+x^4\right)^{2/3}+\frac{1}{4}\sqrt{3}\,\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^4}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{8}\log\left(1+\sqrt[3]{-1+x^4}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.34

$$-\frac{3}{8}\left(x^4-1\right)^{2/3}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-x^4\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)^(2/3)/x,x]
[Out] (-3*(-1 + x^4)^(2/3)*(-1 + Hypergeometric2F1[2/3, 1, 5/3, 1 - x^4]))/8
```

IntegrateAlgebraic [A] time = 0.05, size = 89, normalized size = 1.00

$$\frac{3}{8}\left(x^4-1\right)^{2/3}+\frac{1}{4}\log\left(\sqrt[3]{x^4-1}+1\right)-\frac{1}{8}\log\left(\left(x^4-1\right)^{2/3}-\sqrt[3]{x^4-1}+1\right)+\frac{1}{4}\sqrt{3}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^4-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)^(2/3)/x,x]
[Out] (3*(-1 + x^4)^(2/3))/8 + (Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(-1 + x^4)^(1/3))/S
qrt[3]])/4 + Log[1 + (-1 + x^4)^(1/3)]/4 - Log[1 - (-1 + x^4)^(1/3) + (-1 +
 x^4)^(2/3)]/8
```

fricas [A] time = 0.76, size = 67, normalized size = 0.75

$$-\frac{1}{4}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^4-1\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+\frac{3}{8}\left(x^4-1\right)^{\frac{2}{3}}-\frac{1}{8}\log\left(\left(x^4-1\right)^{\frac{2}{3}}-\left(x^4-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{4}\log\left(\left(x^4-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)/x,x, algorithm="fricas")
[Out] -1/4*sqrt(3)*arctan(2/3*sqrt(3)*(x^4 - 1)^(1/3) - 1/3*sqrt(3)) + 3/8*(x^4 -
 1)^(2/3) - 1/8*log((x^4 - 1)^(2/3) - (x^4 - 1)^(1/3) + 1) + 1/4*log((x^4 -
 1)^(1/3) + 1)
```

giac [A] time = 0.13, size = 66, normalized size = 0.74

$$-\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{3}{8}\left(x^4-1\right)^{\frac{2}{3}}-\frac{1}{8}\log\left(\left(x^4-1\right)^{\frac{2}{3}}-\left(x^4-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{4}\log\left(\left|\left(x^4-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)/x,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 - 1)^(1/3) - 1)) + 3/8*(x^4 - 1)^(2
/3) - 1/8*log((x^4 - 1)^(2/3) - (x^4 - 1)^(1/3) + 1) + 1/4*log(abs((x^4 - 1
)^(1/3) + 1))
```

maple [C] time = 0.29, size = 84, normalized size = 0.94

$$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\operatorname{signum}\left(x^4-1\right)^{\frac{2}{3}}\left(\frac{2\pi\sqrt{3}\,x^4\operatorname{hypergeom}\left(\left[\frac{1}{3},1,1\right],[2,2],x^4\right)}{3\Gamma\left(\frac{2}{3}\right)}-\frac{\left(\frac{3}{2}-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+4\ln(x)+i\pi\right)\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}\right)}{12\pi\left(-\operatorname{signum}\left(x^4-1\right)\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(2/3)/x,x)
```

```
[Out] -1/12/Pi*3^(1/2)*GAMMA(2/3)*signum(x^4-1)^(2/3)/(-signum(x^4-1))^(2/3)*(2/3
*Pi*3^(1/2)/GAMMA(2/3)*x^4*hypergeom([1/3,1,1],[2,2],x^4)-(3/2-1/6*Pi*3^(1/
2)-3/2*ln(3)+4*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.56, size = 65, normalized size = 0.73

$$-\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{3}{8}\left(x^4-1\right)^{\frac{2}{3}}-\frac{1}{8}\log\left(\left(x^4-1\right)^{\frac{2}{3}}-\left(x^4-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{4}\log\left(\left(x^4-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 - 1)^(1/3) - 1)) + 3/8*(x^4 - 1)^(2
/3) - 1/8*log((x^4 - 1)^(2/3) - (x^4 - 1)^(1/3) + 1) + 1/4*log((x^4 - 1)^(1
/3) + 1)
```

mupad [B] time = 0.83, size = 89, normalized size = 1.00

$$\frac{\ln\left(\frac{9\left(x^4-1\right)^{1/3}}{16}+\frac{9}{16}\right)}{4}+\ln\left(9\left(-\frac{1}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8}\right)^2+\frac{9\left(x^4-1\right)^{1/3}}{16}\right)\left(-\frac{1}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8}\right)-\ln\left(9\left(\frac{1}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8}\right)^2+\frac{9\left(x^4-1\right)^{1/3}}{16}\right)\left(\frac{1}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8}\right)+\frac{3\left(x^4-1\right)^{2/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)^(2/3)/x,x)
```

```
[Out] log((9*(x^4 - 1)^(1/3))/16 + 9/16)/4 + log(9*((3^(1/2)*1i)/8 - 1/8)^2 + (9*
(x^4 - 1)^(1/3))/16)*((3^(1/2)*1i)/8 - 1/8) - log(9*((3^(1/2)*1i)/8 + 1/8)^
2 + (9*(x^4 - 1)^(1/3))/16)*((3^(1/2)*1i)/8 + 1/8) + (3*(x^4 - 1)^(2/3))/8
```

sympy [C] time = 0.86, size = 39, normalized size = 0.44

$$-\frac{x^{\frac{8}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^4}\right)}{4\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(2/3)/x,x)
```

```
[Out] -x**(8/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), exp_polar(2*I*pi)/x**4)/(
4*gamma(1/3))
```

3.1083 $\int \frac{1-x^2+x^4}{\sqrt{x+x^2+x^3}\left(-1+x^4\right)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{4}\tan^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)-\frac{3}{2}\tanh^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x^3+x^2+x}}{x^2+x+1}\right)}{4\sqrt{3}}$$

Rubi [C] time = 2.23, antiderivative size = 592, normalized size of antiderivative = 6.65, number of steps used = 31, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2056, 6725, 716, 1103, 934, 169, 538, 537}

$$\frac{\sqrt{x}(x+1)\sqrt{\frac{x^2+x+1}{(x+1)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x^3+x^2+x}}-\frac{3\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\Pi\left(\frac{1}{2}\left(-i-\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}-\frac{\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\Pi\left(\frac{1}{2}\left(1-i\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}-\frac{\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\Pi\left(\frac{1}{2}\left(-1+i\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}-\frac{3\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\Pi\left(\frac{1}{2}\left(i+\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x^2 + x^4)/(Sqrt[x + x^2 + x^3]*(-1 + x^4)),x]
```

```
[Out] (Sqrt[x]*(1 + x)*Sqrt[(1 + x + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]],
 1/4])/Sqrt[x + x^2 + x^3] - (3*Sqrt[x]*Sqrt[1 + (2*x)/(1 - I*Sqrt[3])]*Sqr
t[1 + (2*x)/(1 + I*Sqrt[3])]*EllipticPi[(-I - Sqrt[3])/2, ArcSin[((1 - I*Sq
rt[3])*Sqrt[x])/2], (I + Sqrt[3])/(I - Sqrt[3])])/((1 - I*Sqrt[3])*Sqrt[x +
 x^2 + x^3]) - (Sqrt[x]*Sqrt[1 + (2*x)/(1 - I*Sqrt[3])]*Sqrt[1 + (2*x)/(1 +
 I*Sqrt[3])]*EllipticPi[(1 - I*Sqrt[3])/2, ArcSin[((1 - I*Sqrt[3])*Sqrt[x])
/2], (I + Sqrt[3])/(I - Sqrt[3])])/((1 - I*Sqrt[3])*Sqrt[x + x^2 + x^3]) -
(Sqrt[x]*Sqrt[1 + (2*x)/(1 - I*Sqrt[3])]*Sqrt[1 + (2*x)/(1 + I*Sqrt[3])]*El
lipticPi[(-1 + I*Sqrt[3])/2, ArcSin[((1 - I*Sqrt[3])*Sqrt[x])/2], (I + Sqrt
[3])/(I - Sqrt[3])])/((1 - I*Sqrt[3])*Sqrt[x + x^2 + x^3]) - (3*Sqrt[x]*Sqr
t[1 + (2*x)/(1 - I*Sqrt[3])]*Sqrt[1 + (2*x)/(1 + I*Sqrt[3])]*EllipticPi[(I
+ Sqrt[3])/2, ArcSin[((1 - I*Sqrt[3])*Sqrt[x])/2], (I + Sqrt[3])/(I - Sqrt[
3])])/((1 - I*Sqrt[3])*Sqrt[x + x^2 + x^3])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 716

```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2,
Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; FreeQ
```

```
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1-x^2+x^4}{\sqrt{x+x^2+x^3}\left(-1+x^4\right)}dx = \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1-x^2+x^4}{\sqrt{x}\sqrt{1+x+x^2}\left(-1+x^4\right)}dx}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{1+x+x^2}}+\frac{2-x^2}{\sqrt{x}\sqrt{1+x+x^2}\left(-1+x^4\right)}\right)dx}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+x+x^2}}dx}{\sqrt{x+x^2+x^3}} + \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{2-x^2}{\sqrt{x}\sqrt{1+x+x^2}\left(-1+x^4\right)}dx}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \left(-\frac{1}{2\sqrt{x}\left(1-x^2\right)\sqrt{1+x+x^2}}-\frac{3}{2\sqrt{x}\left(1+x^2\right)\sqrt{1+x+x^2}}\right)dx}{\sqrt{x+x^2+x^3}} + \frac{\left(2\sqrt{x}\sqrt{1+ \right.}{}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} - \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1}{\sqrt{x}\left(1-x^2\right)\sqrt{1+x+x^2}}}{2\sqrt{x+x^2+x^3}}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} - \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \left(\frac{1}{2(1-x)\sqrt{x}\sqrt{1+x+x^2}}\right.}{2\sqrt{x+x^2+x^3}}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} - \frac{\left(3i\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1}{(i-x)\sqrt{x}\sqrt{1+x+x^2}}}{4\sqrt{x+x^2+x^3}}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} - \frac{\left(3i\sqrt{x}\sqrt{1-i\sqrt{3}+2x}\sqrt{1+i\sqrt{3}+2\right.}{4\sqrt{x+}}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} + \frac{\left(3i\sqrt{x}\sqrt{1-i\sqrt{3}+2x}\sqrt{1+i\sqrt{3}+2\right.}{}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} + \frac{\left(3i\sqrt{x}\sqrt{1+i\sqrt{3}+2x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\right)}{2\sqrt{}}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} + \frac{\left(3i\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\right)\mathrm{Sub}}{2\sqrt{x}}$$

$$= \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} - \frac{3\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\Pi\left(\frac{1}{2}\left(\right.\right.}{\left(1-i\sqrt{3}\right)}$$

Mathematica [C] time = 3.07, size = 774, normalized size = 8.70

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^2 + x^4)/(Sqrt[x + x^2 + x^3]*(-1 + x^4)),x]
```

```
[Out] -1/2*((-1)^(2/3)*x^(3/2)*(-4*Sqrt[1 - (-1)^(2/3)/x]*Sqrt[((-1)^(1/3) + x)/x
]*EllipticF[I*ArcSinh[(-1)^(5/6)/Sqrt[x]], (-1)^(2/3)] - (Sqrt[(1 - (-1)^(1
```

```
/3) + Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x]))]*(-1 + (-1)^(1/
3)*Sqrt[x])^2*Sqrt[(-1 + (-1)^(2/3)*Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-1)^(
1/3)*Sqrt[x]))]*Sqrt[-((1 + (-1)^(2/3)*Sqrt[x])/(-1 + (-1)^(1/3) + Sqrt[x])
)]*((1 + (-1)^(1/3))*EllipticF[ArcSin[Sqrt[(1 - (-1)^(1/3) + Sqrt[x])/((1 +
 (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x]))]], -3] - 2*(-1)^(1/3)*EllipticPi[-1
, ArcSin[Sqrt[(1 - (-1)^(1/3) + Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-1)^(1/3)
*Sqrt[x]))]], -3]))/((-1 + (-1)^(1/3))^2*x) + (Sqrt[(1 - (-1)^(1/3) + Sqrt[
x])/((1 + (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x]))]*(-1 + (-1)^(1/3)*Sqrt[x])
^2*Sqrt[(-1 + (-1)^(2/3)*Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x
]))]*Sqrt[-((1 + (-1)^(2/3)*Sqrt[x])/(-1 + (-1)^(1/3) + Sqrt[x]))]*((-1 + (
-1)^(1/3))*EllipticF[ArcSin[Sqrt[(1 - (-1)^(1/3) + Sqrt[x])/((1 + (-1)^(1/3
))*(-1 + (-1)^(1/3)*Sqrt[x]))]], -3] - 2*(-1)^(1/3)*EllipticPi[3, ArcSin[Sq
rt[(1 - (-1)^(1/3) + Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x]))]
], -3]))/((-1 + (-1)^(1/3))^2*x) + Sqrt[1 - (-1)^(2/3)/x]*Sqrt[((-1)^(1/3)
+ x)/x]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)/Sqrt[x]], (-1)^(2/3)] +
 3*Sqrt[1 - (-1)^(2/3)/x]*Sqrt[((-1)^(1/3) + x)/x]*EllipticPi[-(-1)^(5/6),
I*ArcSinh[(-1)^(5/6)/Sqrt[x]], (-1)^(2/3)] + 3*Sqrt[1 - (-1)^(2/3)/x]*Sqrt[
((-1)^(1/3) + x)/x]*EllipticPi[(-1)^(5/6), I*ArcSinh[(-1)^(5/6)/Sqrt[x]], (
-1)^(2/3)]))/Sqrt[x*(1 + x + x^2)]
```

IntegrateAlgebraic [A] time = 0.15, size = 89, normalized size = 1.00

$$-\frac{1}{4}\tan^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)-\frac{3}{2}\tanh^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x^3+x^2+x}}{x^2+x+1}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^2 + x^4)/(Sqrt[x + x^2 + x^3]*(-1 + x^4)),x]
```

```
[Out] -1/4*ArcTan[Sqrt[x + x^2 + x^3]/(1 + x + x^2)] - (3*ArcTanh[Sqrt[x + x^2 +
x^3]/(1 + x + x^2)])/2 - ArcTanh[(Sqrt[3]*Sqrt[x + x^2 + x^3])/(1 + x + x^2
)]/(4*Sqrt[3])
```

fricas [A] time = 0.69, size = 120, normalized size = 1.35

$$\frac{1}{48}\sqrt{3}\log\left(\frac{x^4+20x^3-4\sqrt{3}\sqrt{x^3+x^2+x}\left(x^2+4x+1\right)+30x^2+20x+1}{x^4-4x^3+6x^2-4x+1}\right)+\frac{1}{8}\arctan\left(\frac{x^2+1}{2\sqrt{x^3+x^2+x}}\right)+\frac{3}{4}\log\left(\frac{x^2+2x-2\sqrt{x^3+x^2+x}+1}{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2+1)/(x^3+x^2+x)^(1/2)/(x^4-1),x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(3)*log((x^4 + 20*x^3 - 4*sqrt(3)*sqrt(x^3 + x^2 + x)*(x^2 + 4*x +
 1) + 30*x^2 + 20*x + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)) + 1/8*arctan(1/2*
(x^2 + 1)/sqrt(x^3 + x^2 + x)) + 3/4*log((x^2 + 2*x - 2*sqrt(x^3 + x^2 + x)
 + 1)/(x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4-x^2+1}{\left(x^4-1\right)\sqrt{x^3+x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2+1)/(x^3+x^2+x)^(1/2)/(x^4-1),x, algorithm="giac")
```

```
[Out] integrate((x^4 - x^2 + 1)/((x^4 - 1)*sqrt(x^3 + x^2 + x)), x)
```

maple [C] time = 0.03, size = 1080, normalized size = 12.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^2+1)/(x^3+x^2+x)^(1/2)/(x^4-1),x)
```

```
[Out] 2/3*(1/2+1/2*I*3^(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3
^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/
2)/(x^3+x^2+x)^(1/2)*EllipticF(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^
(1/2),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))+1/6*(1/2+1/2*I*3^
(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3^(1/2)*(I*(x+1/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1
/2)/(-3/2-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/
2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2))-1/6*(1/2+1/2*I*3^(1/2))*((x+1/2+1/2*I*3^(1/2)
)/(1/2+1/2*I*3^(1/2)))^(1/2)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2)/(1/2-1/2*I*3^(1/2))*Elli
pticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1/2*I*3^(1/2
))/(1/2-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))+
1/4*I*(1/(1/2+1/2*I*3^(1/2))*x+1/2/(1/2+1/2*I*3^(1/2))+1/2*I/(1/2+1/2*I*3^(
1/2))*3^(1/2))^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(x/(-1/2
-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2)/(-1/2-I-1/2*I*3^(1/2))*EllipticPi(
((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-1/
2-I-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))-3/4*
(1/(1/2+1/2*I*3^(1/2))*x+1/2/(1/2+1/2*I*3^(1/2))+1/2*I/(1/2+1/2*I*3^(1/2))*
3^(1/2))^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(x/(-1/2-1/2*I*3^(1/2)
))^(1/2)/(x^3+x^2+x)^(1/2)/(-1/2-I-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*
3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-1/2-I-1/2*I*3^(1
/2)),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))-1/4*I*(1/(1/2+1/2*
I*3^(1/2))*x+1/2/(1/2+1/2*I*3^(1/2))+1/2*I/(1/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(
1/2)/(x^3+x^2+x)^(1/2)/(-1/2+I-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1
/2))/(1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-1/2+I-1/2*I*3^(1/2))
,1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))+3/4*(1/(1/2+1/2*I*3^(1
/2))*x+1/2/(1/2+1/2*I*3^(1/2))+1/2*I/(1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)*(I*
3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+
x)^(1/2)/(-1/2+I-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*
I*3^(1/2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-1/2+I-1/2*I*3^(1/2)),1/3*3^(1/2)*(
I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 - x^2 + 1}{\left(x^4 - 1\right)\sqrt{x^3 + x^2 + x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2+1)/(x^3+x^2+x)^(1/2)/(x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^2 + 1)/((x^4 - 1)*sqrt(x^3 + x^2 + x)), x)
```

mupad [B] time = 0.03, size = 698, normalized size = 7.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x^2 + 1)/((x^4 - 1)*(x + x^2 + x^3)^(1/2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 - 1/2)*(x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*
1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 + 1/2))^(1/2)*ellipticF(asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)),
 -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))/(x^2 + x^3 - x*((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (3*((3^(1/2)*1i)/2 - 1/2)*(x/(
(3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
```

```
 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/2)*elli
pticPi(- 3^(1/2)/2 - 1i/2, asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/
2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))/(2*(x^2 + x^3 - x*((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (3*((3^(1/2)*1i)/2 - 1/2)*(x/((3^(1
/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 1/2)
)^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/2)*ellipticP
i(3^(1/2)/2 + 1i/2, asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/
2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))/(2*(x^2 + x^3 - x*((3^(1/2)*1i)/2 - 1/2)*
((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (((3^(1/2)*1i)/2 - 1/2)*(x/((3^(1/2)*1i)/2
 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(
(x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/2)*ellipticPi(1/2 - (
3^(1/2)*1i)/2, asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1
/2)/((3^(1/2)*1i)/2 + 1/2)))/(2*(x^2 + x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2))^(1/2)) - (((3^(1/2)*1i)/2 - 1/2)*(x/((3^(1/2)*1i)/2 - 1/
2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 1/2))^(1/2)*((x +
(3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/2)*ellipticPi((3^(1/2)*1i)
/2 - 1/2, asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/(
(3^(1/2)*1i)/2 + 1/2)))/(2*(x^2 + x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 - x^2 + 1}{\sqrt{x\left(x^2 + x + 1\right)}\,(x-1)\,(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**2+1)/(x**3+x**2+x)**(1/2)/(x**4-1),x)
```

```
[Out] Integral((x**4 - x**2 + 1)/(sqrt(x*(x**2 + x + 1))*(x - 1)*(x + 1)*(x**2 +
1)), x)
```

3.1084 $\int \frac{1-x+x^2}{\left(-1+x^2\right)\sqrt{1-x+x^2-x^3+x^4}}\,dx$

Optimal. Leaf size=89

$$\frac{1}{2}\tanh^{-1}\left(\frac{x}{x^2-\sqrt{x^4-x^3+x^2-x+1}-2x+1}\right)+\frac{3\tanh^{-1}\left(\frac{\sqrt{5}x}{x^2-\sqrt{x^4-x^3+x^2-x+1}+2x+1}\right)}{2\sqrt{5}}$$

Rubi [F] time = 0.62, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1-x+x^2}{\left(-1+x^2\right)\sqrt{1-x+x^2-x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - x + x^2)/((-1 + x^2)*Sqrt[1 - x + x^2 - x^3 + x^4]),x]
```

```
[Out] Defer[Int][1/Sqrt[1 - x + x^2 - x^3 + x^4], x] - Defer[Int][1/((1 - x)*Sqrt
[1 - x + x^2 - x^3 + x^4]), x]/2 - (3*Defer[Int][1/((1 + x)*Sqrt[1 - x + x^
2 - x^3 + x^4]), x])/2
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x+x^2}{\left(-1+x^2\right)\sqrt{1-x+x^2-x^3+x^4}}\,dx &= \int \left(\frac{1}{\sqrt{1-x+x^2-x^3+x^4}}+\frac{2-x}{\left(-1+x^2\right)\sqrt{1-x+x^2-x^3+x^4}}\right)dx\\
&= \int \frac{1}{\sqrt{1-x+x^2-x^3+x^4}}\,dx+\int \frac{2-x}{\left(-1+x^2\right)\sqrt{1-x+x^2-x^3+x^4}}\,dx\\
&= \int \frac{1}{\sqrt{1-x+x^2-x^3+x^4}}\,dx+\int \left(-\frac{1}{2(1-x)\sqrt{1-x+x^2-x^3+x^4}}-\ldots\right.\\
&= -\left(\frac{1}{2}\int \frac{1}{(1-x)\sqrt{1-x+x^2-x^3+x^4}}\,dx\right)-\frac{3}{2}\int \frac{1}{(1+x)\sqrt{1-x+x^2-\ldots}}
\end{aligned}$$

Mathematica [C] time = 1.59, size = 743, normalized size = 8.35

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x + x^2)/((-1 + x^2)*Sqrt[1 - x + x^2 - x^3 + x^4]),x]
```

```
[Out] -(((-1)^(3/5)*Sqrt[(1 - (-1)^(1/5) + (-1)^(2/5) + (-1)^(4/5) - x)/((-1 + (-
1)^(1/5))*((-1)^(2/5) + x))]*((-1)^(2/5) + x)*((-1)^(4/5) + x)*(2*(2*Sqrt[-
(((-1)^(1/5)*(-1 + (-1)^(1/5))*((-1)^(1/5) - x))/((1 - (-1)^(1/5) + (-1)^(2
/5))*((-1)^(2/5) + x)))] + (-1)^(2/5)*Sqrt[-(((-1)^(1/5)*(-1 + (-1)^(1/5))*
((-1)^(1/5) - x))/((1 - (-1)^(1/5) + (-1)^(2/5))*((-1)^(2/5) + x)))] - Sqrt
[(-1 + (-1)^(1/5) - (-1)^(4/5) + (-1)^(1/5)*(-1 + (-1)^(1/5))*x)/((1 - (-1)
^(1/5) + (-1)^(2/5))*((-1)^(2/5) + x))] + (-1)^(4/5)*Sqrt[(-1 + (-1)^(1/5)
- (-1)^(4/5) + (-1)^(1/5)*(-1 + (-1)^(1/5))*x)/((1 - (-1)^(1/5) + (-1)^(2/5
))*((-1)^(2/5) + x))])*EllipticF[ArcSin[Sqrt[-(((-1)^(1/5)*(-1 + (-1)^(1/5)
)*((-1)^(1/5) - x))/((1 - (-1)^(1/5) + (-1)^(2/5))*((-1)^(2/5) + x)))]], (1
 - (-1)^(1/5) + (-1)^(2/5))/(-1 + (-1)^(1/5))^2] + (-1)^(1/5)*Sqrt[-(((-1)^
(1/5)*(-1 + (-1)^(1/5))*((-1)^(1/5) - x))/((1 - (-1)^(1/5) + (-1)^(2/5))*((
```

```
-1)^(2/5) + x)))]*((1 + (-1)^(1/5))^2*EllipticPi[((-1)^(4/5)*(1 + (-1)^(2/5
))*(1 - (-1)^(1/5) + (-1)^(2/5)))/(-1 + (-1)^(1/5))^2, ArcSin[Sqrt[-(((-1)^
(1/5)*(-1 + (-1)^(1/5))*((-1)^(1/5) - x))/((1 - (-1)^(1/5) + (-1)^(2/5))*((
-1)^(2/5) + x)))]], (1 - (-1)^(1/5) + (-1)^(2/5))/(-1 + (-1)^(1/5))^2] - 3*
(1 + (-1)^(2/5))*EllipticPi[1 - (-1)^(1/5) + (-1)^(4/5), ArcSin[Sqrt[-(((-1
)^(1/5)*(-1 + (-1)^(1/5))*((-1)^(1/5) - x))/((1 - (-1)^(1/5) + (-1)^(2/5))*
((-1)^(2/5) + x)))]], (1 - (-1)^(1/5) + (-1)^(2/5))/(-1 + (-1)^(1/5))^2]))
/((-1 + (-1)^(2/5))^2*(1 + (-1)^(2/5))*Sqrt[((-1)^(4/5) + x)/((1 - (-1)^(1/
5) + (-1)^(2/5))*((-1)^(2/5) + x))]*Sqrt[1 - x + x^2 - x^3 + x^4]))
```

IntegrateAlgebraic [A] time = 0.44, size = 89, normalized size = 1.00

$$\frac{1}{2}\tanh^{-1}\left(\frac{x}{x^2-\sqrt{x^4-x^3+x^2-x+1}-2x+1}\right)+\frac{3\tanh^{-1}\left(\frac{\sqrt{5}x}{x^2-\sqrt{x^4-x^3+x^2-x+1}+2x+1}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x + x^2)/((-1 + x^2)*Sqrt[1 - x + x^2 - x^3 + x^4])
,x]
```

```
[Out] ArcTanh[x/(1 - 2*x + x^2 - Sqrt[1 - x + x^2 - x^3 + x^4])]/2 + (3*ArcTanh[(
Sqrt[5]*x)/(1 + 2*x + x^2 - Sqrt[1 - x + x^2 - x^3 + x^4])])/(2*Sqrt[5])
```

fricas [A] time = 0.51, size = 121, normalized size = 1.36

$$\frac{3}{40}\sqrt{5}\log\left(-\frac{9x^4-4x^3+4\sqrt{5}\sqrt{x^4-x^3+x^2-x+1}\left(x^2+1\right)+14x^2-4x+9}{x^4+4x^3+6x^2+4x+1}\right)+\frac{1}{4}\log\left(\frac{3x^2-4x-2\sqrt{x^4-x^3+x^2-x+1}+3}{x^2-2x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2-1)/(x^4-x^3+x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 3/40*sqrt(5)*log(-(9*x^4 - 4*x^3 + 4*sqrt(5)*sqrt(x^4 - x^3 + x^2 - x + 1)*
(x^2 + 1) + 14*x^2 - 4*x + 9)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1)) + 1/4*log((3
*x^2 - 4*x - 2*sqrt(x^4 - x^3 + x^2 - x + 1) + 3)/(x^2 - 2*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-x+1}{\sqrt{x^4-x^3+x^2-x+1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2-1)/(x^4-x^3+x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)/(sqrt(x^4 - x^3 + x^2 - x + 1)*(x^2 - 1)), x)
```

maple [C] time = 0.73, size = 2852, normalized size = 32.04

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+1)/(x^2-1)/(x^4-x^3+x^2-x+1)^(1/2),x)
```

```
[Out] I*2^(1/2)*(5-5^(1/2))^(1/2)*(I*(1/2*5^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2)
-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))*(x-1/4*5^(1/2)-1/4-1/4*I*2^(1/2)*(5-5^(1/
2))^(1/2))*2^(1/2)/(5-5^(1/2))^(1/2)/(x+1/4*5^(1/2)-1/4-1/4*I*2^(1/2)*(5+5^
(1/2))^(1/2)))^(1/2)*(x+1/4*5^(1/2)-1/4-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))^2*
((-1/2*5^(1/2)+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1
/2))*(x+1/4*5^(1/2)-1/4+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))/(-1/2*5^(1/2)-1/4*
```

```
I*2^(1/2)*(5+5^(1/2))^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))/(x+1/4*5^(1/2)
-1/4-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)))^(1/2)*(I*(-1/2*5^(1/2)+1/4*I*2^(1/2)
*(5+5^(1/2))^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))*(x-1/4*5^(1/2)-1/4+1/4*
I*2^(1/2)*(5-5^(1/2))^(1/2))*2^(1/2)/(5-5^(1/2))^(1/2)/(x+1/4*5^(1/2)-1/4-1
/4*I*2^(1/2)*(5+5^(1/2))^(1/2)))^(1/2)/(1/2*5^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2
))^(1/2)-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))/(-1/2*5^(1/2)+1/4*I*2^(1/2)*(5+5^
(1/2))^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))/((x-1/4*5^(1/2)-1/4-1/4*I*2^(
1/2)*(5-5^(1/2))^(1/2))*(x+1/4*5^(1/2)-1/4-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))
*(x+1/4*5^(1/2)-1/4+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))*(x-1/4*5^(1/2)-1/4+1/4
*I*2^(1/2)*(5-5^(1/2))^(1/2)))^(1/2)*EllipticF((I*(1/2*5^(1/2)-1/4*I*2^(1/2
)*(5-5^(1/2))^(1/2)-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))*(x-1/4*5^(1/2)-1/4-1/4
*I*2^(1/2)*(5-5^(1/2))^(1/2))*2^(1/2)/(5-5^(1/2))^(1/2)/(x+1/4*5^(1/2)-1/4-
1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)))^(1/2),1/2*(-2*(5+5^(1/2))^(1/2)*(5-5^(1/2
))^(1/2)/(1/2*5^(1/2)+1/4*I*2^(1/2)*(5-5^(1/2))^(1/2)+1/4*I*2^(1/2)*(5+5^(1
/2))^(1/2))/(-1/2*5^(1/2)+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)+1/4*I*2^(1/2)*(5-
5^(1/2))^(1/2)))^(1/2))+1/2*I*2^(1/2)*(5-5^(1/2))^(1/2)*(I*(1/2*5^(1/2)-1/4
*I*2^(1/2)*(5-5^(1/2))^(1/2)-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))*(x-1/4*5^(1/2
)-1/4-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))*2^(1/2)/(5-5^(1/2))^(1/2)/(x+1/4*5^(
1/2)-1/4-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)))^(1/2)*(x+1/4*5^(1/2)-1/4-1/4*I*2
^(1/2)*(5+5^(1/2))^(1/2))^2*((-1/2*5^(1/2)+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)-
1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))*(x+1/4*5^(1/2)-1/4+1/4*I*2^(1/2)*(5+5^(1/2
))^(1/2))/(-1/2*5^(1/2)-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)-1/4*I*2^(1/2)*(5-5^
(1/2))^(1/2))/(x+1/4*5^(1/2)-1/4-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)))^(1/2)*(I
*(-1/2*5^(1/2)+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1
/2))*(x-1/4*5^(1/2)-1/4+1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))*2^(1/2)/(5-5^(1/2)
)^(1/2)/(x+1/4*5^(1/2)-1/4-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)))^(1/2)/(1/2*5^(
1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2)-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))/(-1/2
*5^(1/2)+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))/(
(x-1/4*5^(1/2)-1/4-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))*(x+1/4*5^(1/2)-1/4-1/4*
I*2^(1/2)*(5+5^(1/2))^(1/2))*(x+1/4*5^(1/2)-1/4+1/4*I*2^(1/2)*(5+5^(1/2))^(
1/2))*(x-1/4*5^(1/2)-1/4+1/4*I*2^(1/2)*(5-5^(1/2))^(1/2)))^(1/2)/(-1/4*5^(1
/2)-3/4+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))*(EllipticF((I*(1/2*5^(1/2)-1/4*I*2
^(1/2)*(5-5^(1/2))^(1/2)-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))*(x-1/4*5^(1/2)-1/
4-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))*2^(1/2)/(5-5^(1/2))^(1/2)/(x+1/4*5^(1/2)
-1/4-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)))^(1/2),1/2*(-2*(5+5^(1/2))^(1/2)*(5-5
^(1/2))^(1/2)/(1/2*5^(1/2)+1/4*I*2^(1/2)*(5-5^(1/2))^(1/2)+1/4*I*2^(1/2)*(5
+5^(1/2))^(1/2))/(-1/2*5^(1/2)+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)+1/4*I*2^(1/2
)*(5-5^(1/2))^(1/2)))^(1/2))+(-1/2*5^(1/2)+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)-
1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))/(1/4*5^(1/2)-3/4+1/4*I*2^(1/2)*(5-5^(1/2))
^(1/2))*EllipticPi((I*(1/2*5^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2)-1/4*I*2^
(1/2)*(5+5^(1/2))^(1/2))*(x-1/4*5^(1/2)-1/4-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2)
)*2^(1/2)/(5-5^(1/2))^(1/2)/(x+1/4*5^(1/2)-1/4-1/4*I*2^(1/2)*(5+5^(1/2))^(1
/2)))^(1/2),-1/2*I*(-1/4*5^(1/2)-3/4+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))*2^(1/
2)*(5-5^(1/2))^(1/2)/(1/4*5^(1/2)-3/4+1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))/(1/2
*5^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2)-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)),1
/2*(-2*(5+5^(1/2))^(1/2)*(5-5^(1/2))^(1/2)/(1/2*5^(1/2)+1/4*I*2^(1/2)*(5-5^
(1/2))^(1/2)+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))/(-1/2*5^(1/2)+1/4*I*2^(1/2)*(
5+5^(1/2))^(1/2)+1/4*I*2^(1/2)*(5-5^(1/2))^(1/2)))^(1/2)))-3/2*I*2^(1/2)*(5
-5^(1/2))^(1/2)*(I*(1/2*5^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2)-1/4*I*2^(1/
2)*(5+5^(1/2))^(1/2))*(x-1/4*5^(1/2)-1/4-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))*2
^(1/2)/(5-5^(1/2))^(1/2)/(x+1/4*5^(1/2)-1/4-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)
))^(1/2)*(x+1/4*5^(1/2)-1/4-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))^2*((-1/2*5^(1/
2)+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))*(x+1/4*
5^(1/2)-1/4+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))/(-1/2*5^(1/2)-1/4*I*2^(1/2)*(5
+5^(1/2))^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))/(x+1/4*5^(1/2)-1/4-1/4*I*2
^(1/2)*(5+5^(1/2))^(1/2)))^(1/2)*(I*(-1/2*5^(1/2)+1/4*I*2^(1/2)*(5+5^(1/2))
^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))*(x-1/4*5^(1/2)-1/4+1/4*I*2^(1/2)*(5
-5^(1/2))^(1/2))*2^(1/2)/(5-5^(1/2))^(1/2)/(x+1/4*5^(1/2)-1/4-1/4*I*2^(1/2)
*(5+5^(1/2))^(1/2)))^(1/2)/(1/2*5^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2)-1/4
```

```
*I*2^(1/2)*(5+5^(1/2))^(1/2))/(-1/2*5^(1/2)+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)
-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))/((x-1/4*5^(1/2)-1/4-1/4*I*2^(1/2)*(5-5^(1
/2))^(1/2))*(x+1/4*5^(1/2)-1/4-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))*(x+1/4*5^(1
/2)-1/4+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))*(x-1/4*5^(1/2)-1/4+1/4*I*2^(1/2)*(
5-5^(1/2))^(1/2)))^(1/2)/(-1/4*5^(1/2)+5/4+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))
*(EllipticF((I*(1/2*5^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2)-1/4*I*2^(1/2)*(
5+5^(1/2))^(1/2))*(x-1/4*5^(1/2)-1/4-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))*2^(1/
2)/(5-5^(1/2))^(1/2)/(x+1/4*5^(1/2)-1/4-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)))^(
1/2),1/2*(-2*(5+5^(1/2))^(1/2)*(5-5^(1/2))^(1/2)/(1/2*5^(1/2)+1/4*I*2^(1/2)
*(5-5^(1/2))^(1/2)+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))/(-1/2*5^(1/2)+1/4*I*2^(
1/2)*(5+5^(1/2))^(1/2)+1/4*I*2^(1/2)*(5-5^(1/2))^(1/2)))^(1/2))+(-1/2*5^(1/
2)+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))/(1/4*5^
(1/2)+5/4+1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))*EllipticPi((I*(1/2*5^(1/2)-1/4*I
*2^(1/2)*(5-5^(1/2))^(1/2)-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2))*(x-1/4*5^(1/2)-
1/4-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2))*2^(1/2)/(5-5^(1/2))^(1/2)/(x+1/4*5^(1/
2)-1/4-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)))^(1/2),-1/2*I*(-1/4*5^(1/2)+5/4+1/4
*I*2^(1/2)*(5+5^(1/2))^(1/2))*2^(1/2)*(5-5^(1/2))^(1/2)/(1/4*5^(1/2)+5/4+1/
4*I*2^(1/2)*(5-5^(1/2))^(1/2))/(1/2*5^(1/2)-1/4*I*2^(1/2)*(5-5^(1/2))^(1/2)
-1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)),1/2*(-2*(5+5^(1/2))^(1/2)*(5-5^(1/2))^(1/
2)/(1/2*5^(1/2)+1/4*I*2^(1/2)*(5-5^(1/2))^(1/2)+1/4*I*2^(1/2)*(5+5^(1/2))^(
1/2))/(-1/2*5^(1/2)+1/4*I*2^(1/2)*(5+5^(1/2))^(1/2)+1/4*I*2^(1/2)*(5-5^(1/2
))^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - x + 1}{\sqrt{x^4 - x^3 + x^2 - x + 1}\,(x^2 - 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2-1)/(x^4-x^3+x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 - x + 1)/(sqrt(x^4 - x^3 + x^2 - x + 1)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 - x + 1}{(x^2 - 1)\,\sqrt{x^4 - x^3 + x^2 - x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x + 1)/((x^2 - 1)*(x^2 - x - x^3 + x^4 + 1)^(1/2)),x)
[Out] int((x^2 - x + 1)/((x^2 - 1)*(x^2 - x - x^3 + x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - x + 1}{(x - 1)(x + 1)\sqrt{x^4 - x^3 + x^2 - x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+1)/(x**2-1)/(x**4-x**3+x**2-x+1)**(1/2),x)
[Out] Integral((x**2 - x + 1)/((x - 1)*(x + 1)*sqrt(x**4 - x**3 + x**2 - x + 1)),
 x)
```

3.1085 $\int \frac{(-1+x)^2\left(x-2x^2+2x^3\right)}{(-1+2x)\sqrt{\frac{1-2x}{1+2x^2}}\left(-2+4x+3x^2-4x^3+2x^4\right)}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{\frac{1-2x}{2x^2+1}}\left(-2x^3+2x^2-x+1\right)}{3(2x-1)}-\frac{\tanh^{-1}\left(\frac{\frac{x}{\sqrt{3}}-\frac{1}{\sqrt{3}}}{\sqrt{\frac{1-2x}{2x^2+1}}}\right)}{\sqrt{3}}$$

Rubi [F] time = 11.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x)^2\left(x-2x^2+2x^3\right)}{(-1+2x)\sqrt{\frac{1-2x}{1+2x^2}}\left(-2+4x+3x^2-4x^3+2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x)^2*(x - 2*x^2 + 2*x^3))/((-1 + 2*x)*Sqrt[(1 - 2*x)/(1 + 2*x^2)
]*(-2 + 4*x + 3*x^2 - 4*x^3 + 2*x^4)),x]
```

```
[Out] 1/(3*Sqrt[(1 - 2*x)/(1 + 2*x^2)]) + (4*(1 - 2*x))/(3*(1 + Sqrt[3] - 2*x)*Sq
rt[(1 - 2*x)/(1 + 2*x^2)]) - (2 - x)/(3*Sqrt[(1 - 2*x)/(1 + 2*x^2)]) + ((I/
3)*Sqrt[2]*(1 - 2*x)*EllipticE[ArcSin[Sqrt[1 - I*Sqrt[2]*x]/Sqrt[2]], (-2*S
qrt[2])/(I - Sqrt[2])])/(Sqrt[(1 - 2*x)/(1 + I*Sqrt[2])]*Sqrt[(1 - 2*x)/(1
+ 2*x^2)]*Sqrt[1 + 2*x^2]) - (2*Sqrt[2]*Sqrt[1 - 2*x]*(1 + Sqrt[3] - 2*x)*S
qrt[(1 + 2*x^2)/(1 + Sqrt[3] - 2*x)^2]*EllipticE[2*ArcTan[Sqrt[1 - 2*x]/3^(
1/4)], (3 + Sqrt[3])/6])/(3^(3/4)*Sqrt[(1 - 2*x)/(1 + 2*x^2)]*(1 + 2*x^2))
 - (Sqrt[2]*(1 - Sqrt[3])*Sqrt[1 - 2*x]*(1 + Sqrt[3] - 2*x)*Sqrt[(1 + 2*x^2)
/(1 + Sqrt[3] - 2*x)^2]*EllipticF[2*ArcTan[Sqrt[1 - 2*x]/3^(1/4)], (3 + Sqr
t[3])/6])/(3*3^(1/4)*Sqrt[(1 - 2*x)/(1 + 2*x^2)]*(1 + 2*x^2)) + (8*Sqrt[2]*
Sqrt[1 - 2*x]*Defer[Subst][Defer[Int][Sqrt[3 - 2*x^2 + x^4]/(3 - 20*x^2 + x
^8), x], x, Sqrt[1 - 2*x]])/(3*Sqrt[(1 - 2*x)/(1 + 2*x^2)]*Sqrt[1 + 2*x^2])
 - (5*Sqrt[2]*Sqrt[1 - 2*x]*Defer[Subst][Defer[Int][(x^2*Sqrt[3 - 2*x^2 + x
^4])/(3 - 20*x^2 + x^8), x], x, Sqrt[1 - 2*x]])/(Sqrt[(1 - 2*x)/(1 + 2*x^2)
]*Sqrt[1 + 2*x^2]) - (Sqrt[2]*Sqrt[1 - 2*x]*Defer[Subst][Defer[Int][(x^6*Sq
rt[3 - 2*x^2 + x^4])/(3 - 20*x^2 + x^8), x], x, Sqrt[1 - 2*x]])/(3*Sqrt[(1
- 2*x)/(1 + 2*x^2)]*Sqrt[1 + 2*x^2])
```

Rubi steps

$$\int \frac{(-1+x)^2\left(x-2x^2+2x^3\right)}{(-1+2x)\sqrt{\frac{1-2x}{1+2x^2}}\left(-2+4x+3x^2-4x^3+2x^4\right)}\,dx = \int \frac{(-1+x)^2x\left(1-2x+2x^2\right)}{(-1+2x)\sqrt{\frac{1-2x}{1+2x^2}}\left(-2+4x+3x^2-4x^3+2x^4\right)}\,dx$$

$$= \frac{\sqrt{1-2x}\int \frac{(-1+x)^2x\sqrt{1+2x^2}\left(1-2x+2x^2\right)}{\sqrt{1-2x}(-1+2x)\left(-2+4x+3x^2-4x^3+2x^4\right)}\,dx}{\sqrt{\frac{1-2x}{1+2x^2}}\sqrt{1+2x^2}}$$

$$= -\frac{\sqrt{1-2x}\int \frac{(-1+x)^2x\sqrt{1+2x^2}\left(1-2x+2x^2\right)}{(1-2x)^{3/2}\left(-2+4x+3x^2-4x^3+2x^4\right)}\,dx}{\sqrt{\frac{1-2x}{1+2x^2}}\sqrt{1+2x^2}}$$

$$= -\frac{\sqrt{1-2x}\int\left(-\frac{\sqrt{1+2x^2}}{(1-2x)^{3/2}}+\frac{x\sqrt{1+2x^2}}{(1-2x)^{3/2}}-\frac{\sqrt{1+2x^2}\left(2-7x+\ldots\right.}{(1-2x)^{3/2}\left(-2+4x+3x^2-\ldots\right.}\right.}{\sqrt{\frac{1-2x}{1+2x^2}}\sqrt{1+2x^2}}$$

$$= \frac{\sqrt{1-2x}\int \frac{\sqrt{1+2x^2}}{(1-2x)^{3/2}}\,dx}{\sqrt{\frac{1-2x}{1+2x^2}}\sqrt{1+2x^2}} - \frac{\sqrt{1-2x}\int \frac{x\sqrt{1+2x^2}}{(1-2x)^{3/2}}\,dx}{\sqrt{\frac{1-2x}{1+2x^2}}\sqrt{1+2x^2}} + \ldots$$

$$= \frac{1}{\sqrt{\frac{1-2x}{1+2x^2}}} - \frac{2-x}{3\sqrt{\frac{1-2x}{1+2x^2}}} + \frac{\sqrt{1-2x}\int \frac{2+8x}{\sqrt{1-2x}\sqrt{1+2x^2}}\,dx}{6\sqrt{\frac{1-2x}{1+2x^2}}\sqrt{1+2x^2}} + \ldots$$

$$= \frac{1}{\sqrt{\frac{1-2x}{1+2x^2}}} - \frac{2-x}{3\sqrt{\frac{1-2x}{1+2x^2}}} - \frac{\left(2\sqrt{1-2x}\right)\int \frac{\sqrt{1-2x}}{\sqrt{1+2x^2}}\,dx}{3\sqrt{\frac{1-2x}{1+2x^2}}\sqrt{1+2x^2}} + \ldots$$

$$= \frac{1}{\sqrt{\frac{1-2x}{1+2x^2}}} - \frac{2-x}{3\sqrt{\frac{1-2x}{1+2x^2}}} - \frac{\left(5\sqrt{2}\sqrt{1-2x}\right)\text{Subst}\left(\int \frac{(-1+x\ldots}{x^2(\ldots}\right.}{\sqrt{\frac{1-2x}{1+2x^2}}\ldots}$$

$$= \frac{1}{\sqrt{\frac{1-2x}{1+2x^2}}} - \frac{2-x}{3\sqrt{\frac{1-2x}{1+2x^2}}} + \frac{i\sqrt{2}(1-2x)E\left(\sin^{-1}\left(\frac{\sqrt{1-i\sqrt{2}x}}{\sqrt{2}}\ldots\right.\right.}{3\sqrt{\frac{1-2x}{1+i\sqrt{2}}}\sqrt{\frac{1-2x}{1+2x^2}}\sqrt{1+\ldots}}$$

$$= \frac{1}{\sqrt{\frac{1-2x}{1+2x^2}}} - \frac{2-x}{3\sqrt{\frac{1-2x}{1+2x^2}}} + \frac{i\sqrt{2}(1-2x)E\left(\sin^{-1}\left(\frac{\sqrt{1-i\sqrt{2}x}}{\sqrt{2}}\ldots\right.\right.}{3\sqrt{\frac{1-2x}{1+i\sqrt{2}}}\sqrt{\frac{1-2x}{1+2x^2}}\sqrt{1+\ldots}}$$

$$= \frac{1}{3\sqrt{\frac{1-2x}{1+2x^2}}} - \frac{2-x}{3\sqrt{\frac{1-2x}{1+2x^2}}} + \frac{i\sqrt{2}(1-2x)E\left(\sin^{-1}\left(\frac{\sqrt{1-i\sqrt{2}\ldots}}{\sqrt{2}}\right.\right.}{3\sqrt{\frac{1-2x}{1+i\sqrt{2}}}\sqrt{\frac{1-2x}{1+2x^2}}\sqrt{1+\ldots}}$$

$$= \frac{1}{3\sqrt{\frac{1-2x}{1+2x^2}}} - \frac{2-x}{3\sqrt{\frac{1-2x}{1+2x^2}}} + \frac{i\sqrt{2}(1-2x)E\left(\sin^{-1}\left(\frac{\sqrt{1-i\sqrt{2}\ldots}}{\sqrt{2}}\right.\right.}{3\sqrt{\frac{1-2x}{1+i\sqrt{2}}}\sqrt{\frac{1-2x}{1+2x^2}}\sqrt{1+\ldots}}$$

$$= \frac{1}{3\sqrt{\frac{1-2x}{1+2x^2}}} + \frac{4(1-2x)}{3\left(1+\sqrt{3}-2x\right)\sqrt{\frac{1-2x}{1+2x^2}}} - \frac{2-x}{3\sqrt{\frac{1-2x}{1+2x^2}}} + \ldots$$

Mathematica [C] time = 10.47, size = 2703, normalized size = 30.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x)^2*(x - 2*x^2 + 2*x^3))/((-1 + 2*x)*Sqrt[(1 - 2*x)/(1 +
2*x^2)]*(-2 + 4*x + 3*x^2 - 4*x^3 + 2*x^4)),x]
```

```
[Out] ((1 - 2*x)^2*(1/6 - 1/(6*(-1 + 2*x))))/((-1 + 2*x)*Sqrt[(1 - 2*x)/(1 + 2*x^
2)]) + ((6*I)*Sqrt[2]*Sqrt[1 + (I*(1 - 2*x))/(-I + Sqrt[2])]*Sqrt[1 - (I*(1
 - 2*x))/(I + Sqrt[2])]*(1 - 2*x)^(3/2)*((Sqrt[(-I)/(I + Sqrt[2])]*Sqrt[1 -
 2*x]*EllipticPi[(1 - I*Sqrt[2])/Root[3 - 20*#1 + #1^4 & , 1, 0], I*ArcSinh
[Sqrt[((-I)*(1 - 2*x))/(I + Sqrt[2])]], (I + Sqrt[2])/(I - Sqrt[2])]*(-1 +
Root[3 - 20*#1 + #1^4 & , 1, 0] - Root[3 - 20*#1 + #1^4 & , 1, 0]^2 + Root[
3 - 20*#1 + #1^4 & , 1, 0]^3))/(Sqrt[((-I)*(1 - 2*x))/(I + Sqrt[2])]*Root[3
 - 20*#1 + #1^4 & , 1, 0]*(Root[3 - 20*#1 + #1^4 & , 1, 0] - Root[3 - 20*#1
 + #1^4 & , 2, 0])*(Root[3 - 20*#1 + #1^4 & , 1, 0] - Root[3 - 20*#1 + #1^4
 & , 3, 0])*(Root[3 - 20*#1 + #1^4 & , 1, 0] - Root[3 - 20*#1 + #1^4 & , 4,
 0])) + (-((Sqrt[(-I)/(I + Sqrt[2])]*Sqrt[1 - 2*x]*EllipticPi[(1 - I*Sqrt[2
])/Root[3 - 20*#1 + #1^4 & , 2, 0], I*ArcSinh[Sqrt[((-I)*(1 - 2*x))/(I + Sq
rt[2])]], (I + Sqrt[2])/(I - Sqrt[2])]*(-1 + Root[3 - 20*#1 + #1^4 & , 2, 0
] - Root[3 - 20*#1 + #1^4 & , 2, 0]^2 + Root[3 - 20*#1 + #1^4 & , 2, 0]^3))
/(Sqrt[((-I)*(1 - 2*x))/(I + Sqrt[2])]*(Root[3 - 20*#1 + #1^4 & , 1, 0] - R
oot[3 - 20*#1 + #1^4 & , 2, 0])*Root[3 - 20*#1 + #1^4 & , 2, 0])) + ((Sqrt[
(-I)/(I + Sqrt[2])]*Sqrt[1 - 2*x]*EllipticPi[(1 - I*Sqrt[2])/Root[3 - 20*#1
 + #1^4 & , 4, 0], I*ArcSinh[Sqrt[((-I)*(1 - 2*x))/(I + Sqrt[2])]], (I + Sq
rt[2])/(I - Sqrt[2])]*(Root[3 - 20*#1 + #1^4 & , 1, 0] - Root[3 - 20*#1 + #
1^4 & , 3, 0])*(Root[3 - 20*#1 + #1^4 & , 2, 0] - Root[3 - 20*#1 + #1^4 & ,
 3, 0])*Root[3 - 20*#1 + #1^4 & , 3, 0])/Sqrt[((-I)*(1 - 2*x))/(I + Sqrt[2]
)] + (Sqrt[(-I)/(I + Sqrt[2])]*Sqrt[1 - 2*x]*EllipticPi[(1 - I*Sqrt[2])/Roo
t[3 - 20*#1 + #1^4 & , 4, 0], I*ArcSinh[Sqrt[((-I)*(1 - 2*x))/(I + Sqrt[2])
]], (I + Sqrt[2])/(I - Sqrt[2])]*(Root[3 - 20*#1 + #1^4 & , 1, 0] - Root[3
- 20*#1 + #1^4 & , 3, 0])*Root[3 - 20*#1 + #1^4 & , 3, 0]*(-Root[3 - 20*#1
+ #1^4 & , 2, 0] + Root[3 - 20*#1 + #1^4 & , 3, 0])*Root[3 - 20*#1 + #1^4 &
 , 4, 0])/Sqrt[((-I)*(1 - 2*x))/(I + Sqrt[2])] + (Sqrt[(-I)/(I + Sqrt[2])]*
Sqrt[1 - 2*x]*EllipticPi[(1 - I*Sqrt[2])/Root[3 - 20*#1 + #1^4 & , 3, 0], I
*ArcSinh[Sqrt[((-I)*(1 - 2*x))/(I + Sqrt[2])]], (I + Sqrt[2])/(I - Sqrt[2])
]*Root[3 - 20*#1 + #1^4 & , 3, 0]*(Root[3 - 20*#1 + #1^4 & , 1, 0] - Root[3
 - 20*#1 + #1^4 & , 4, 0])*(Root[3 - 20*#1 + #1^4 & , 2, 0] - Root[3 - 20*#
1 + #1^4 & , 4, 0])*Root[3 - 20*#1 + #1^4 & , 4, 0])/Sqrt[((-I)*(1 - 2*x))/
(I + Sqrt[2])] + (Sqrt[(-I)/(I + Sqrt[2])]*Sqrt[1 - 2*x]*EllipticPi[(1 - I*
Sqrt[2])/Root[3 - 20*#1 + #1^4 & , 3, 0], I*ArcSinh[Sqrt[((-I)*(1 - 2*x))/(
I + Sqrt[2])]], (I + Sqrt[2])/(I - Sqrt[2])]*Root[3 - 20*#1 + #1^4 & , 3, 0
]^3*(Root[3 - 20*#1 + #1^4 & , 1, 0] - Root[3 - 20*#1 + #1^4 & , 4, 0])*(Ro
ot[3 - 20*#1 + #1^4 & , 2, 0] - Root[3 - 20*#1 + #1^4 & , 4, 0])*Root[3 - 2
0*#1 + #1^4 & , 4, 0])/Sqrt[((-I)*(1 - 2*x))/(I + Sqrt[2])] + (Sqrt[(-I)/(I
 + Sqrt[2])]*Sqrt[1 - 2*x]*EllipticPi[(1 - I*Sqrt[2])/Root[3 - 20*#1 + #1^4
 & , 4, 0], I*ArcSinh[Sqrt[((-I)*(1 - 2*x))/(I + Sqrt[2])]], (I + Sqrt[2])/
(I - Sqrt[2])]*(Root[3 - 20*#1 + #1^4 & , 1, 0] - Root[3 - 20*#1 + #1^4 & ,
 3, 0])*(Root[3 - 20*#1 + #1^4 & , 2, 0] - Root[3 - 20*#1 + #1^4 & , 3, 0])
*Root[3 - 20*#1 + #1^4 & , 3, 0]*Root[3 - 20*#1 + #1^4 & , 4, 0]^2)/Sqrt[((
-I)*(1 - 2*x))/(I + Sqrt[2])] + (Sqrt[(-I)/(I + Sqrt[2])]*Sqrt[1 - 2*x]*Ell
ipticPi[(1 - I*Sqrt[2])/Root[3 - 20*#1 + #1^4 & , 4, 0], I*ArcSinh[Sqrt[((-
I)*(1 - 2*x))/(I + Sqrt[2])]], (I + Sqrt[2])/(I - Sqrt[2])]*(Root[3 - 20*#1
 + #1^4 & , 1, 0] - Root[3 - 20*#1 + #1^4 & , 3, 0])*Root[3 - 20*#1 + #1^4
& , 3, 0]*(-Root[3 - 20*#1 + #1^4 & , 2, 0] + Root[3 - 20*#1 + #1^4 & , 3,
0])*Root[3 - 20*#1 + #1^4 & , 4, 0]^3)/Sqrt[((-I)*(1 - 2*x))/(I + Sqrt[2])]
 + (Sqrt[(-I)/(I + Sqrt[2])]*Sqrt[1 - 2*x]*EllipticPi[(1 - I*Sqrt[2])/Root[
3 - 20*#1 + #1^4 & , 3, 0], I*ArcSinh[Sqrt[((-I)*(1 - 2*x))/(I + Sqrt[2])]]
```

```
, (I + Sqrt[2])/(I - Sqrt[2])]*(Root[3 - 20*#1 + #1^4 & , 1, 0] - Root[3 -
20*#1 + #1^4 & , 4, 0])*Root[3 - 20*#1 + #1^4 & , 4, 0]*(-Root[3 - 20*#1 +
#1^4 & , 2, 0] + Root[3 - 20*#1 + #1^4 & , 4, 0]))/Sqrt[((-I)*(1 - 2*x))/(I
 + Sqrt[2])] + (Sqrt[(-I)/(I + Sqrt[2])]*Sqrt[1 - 2*x]*EllipticPi[(1 - I*Sq
rt[2])/Root[3 - 20*#1 + #1^4 & , 3, 0], I*ArcSinh[Sqrt[((-I)*(1 - 2*x))/(I
+ Sqrt[2])]], (I + Sqrt[2])/(I - Sqrt[2])]*Root[3 - 20*#1 + #1^4 & , 3, 0]^
2*(Root[3 - 20*#1 + #1^4 & , 1, 0] - Root[3 - 20*#1 + #1^4 & , 4, 0])*Root[
3 - 20*#1 + #1^4 & , 4, 0]*(-Root[3 - 20*#1 + #1^4 & , 2, 0] + Root[3 - 20*
#1 + #1^4 & , 4, 0]))/Sqrt[((-I)*(1 - 2*x))/(I + Sqrt[2])]))/((Root[3 - 20*#
1 + #1^4 & , 1, 0] - Root[3 - 20*#1 + #1^4 & , 3, 0])*Root[3 - 20*#1 + #1^4
 & , 3, 0]*(Root[3 - 20*#1 + #1^4 & , 1, 0] - Root[3 - 20*#1 + #1^4 & , 4,
0])*(Root[3 - 20*#1 + #1^4 & , 3, 0] - Root[3 - 20*#1 + #1^4 & , 4, 0])*Roo
t[3 - 20*#1 + #1^4 & , 4, 0]))/((Root[3 - 20*#1 + #1^4 & , 2, 0] - Root[3 -
 20*#1 + #1^4 & , 3, 0])*(Root[3 - 20*#1 + #1^4 & , 2, 0] - Root[3 - 20*#1
+ #1^4 & , 4, 0]))))/(Sqrt[(-I)/(I + Sqrt[2])]*Sqrt[3 - 2*(1 - 2*x) + (1 -
2*x)^2]*(-1 + 2*x)*Sqrt[(1 - 2*x)/(1 + 2*x^2)]*Sqrt[1 + 2*x^2])
```

IntegrateAlgebraic [A] time = 0.41, size = 89, normalized size = 1.00

$$\frac{\sqrt{\frac{1-2x}{2x^2+1}}\left(-2x^3+2x^2-x+1\right)}{3(2x-1)}-\frac{\tanh^{-1}\left(\frac{\frac{x}{\sqrt{3}}-\frac{1}{\sqrt{3}}}{\sqrt{\frac{1-2x}{2x^2+1}}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)^2*(x - 2*x^2 + 2*x^3))/((-1 + 2*x)*Sqrt[(1 - 2
*x)/(1 + 2*x^2)]*(-2 + 4*x + 3*x^2 - 4*x^3 + 2*x^4)),x]
```

```
[Out] (Sqrt[(1 - 2*x)/(1 + 2*x^2)]*(1 - x + 2*x^2 - 2*x^3))/(3*(-1 + 2*x)) - ArcT
anh[(-(1/Sqrt[3]) + x/Sqrt[3])/Sqrt[(1 - 2*x)/(1 + 2*x^2)]]/Sqrt[3]
```

fricas [B] time = 0.51, size = 195, normalized size = 2.19

$$\frac{\sqrt{3}\,(2\,x-1)\log\left(-\frac{4\,x^8-16\,x^7+28\,x^6-104\,x^5+209\,x^4-200\,x^3+172\,x^2-4\,\sqrt{3}\left(4\,x^7-12\,x^6+16\,x^5-28\,x^4+31\,x^3-19\,x^2+12\,x-4\right)\sqrt{-\frac{2\,x-1}{2\,x^2+1}}-112\,x+28}{4\,x^8-16\,x^7+28\,x^6-8\,x^5-31\,x^4+40\,x^3+4\,x^2-16\,x+4}\right)-4\left(2\,x^3-2\,x^2+x-1\right)\sqrt{-\frac{2\,x-1}{2\,x^2+1}}}{12\,(2\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^2*(2*x^3-2*x^2+x)/(-1+2*x)/((1-2*x)/(2*x^2+1))^(1/2)/(2*x^
4-4*x^3+3*x^2+4*x-2),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(3)*(2*x - 1)*log(-(4*x^8 - 16*x^7 + 28*x^6 - 104*x^5 + 209*x^4 -
 200*x^3 + 172*x^2 - 4*sqrt(3)*(4*x^7 - 12*x^6 + 16*x^5 - 28*x^4 + 31*x^3 -
 19*x^2 + 12*x - 4)*sqrt(-(2*x - 1)/(2*x^2 + 1)) - 112*x + 28)/(4*x^8 - 16*
x^7 + 28*x^6 - 8*x^5 - 31*x^4 + 40*x^3 + 4*x^2 - 16*x + 4)) - 4*(2*x^3 - 2*
x^2 + x - 1)*sqrt(-(2*x - 1)/(2*x^2 + 1)))/(2*x - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^3-2\,x^2+x\right)(x-1)^2}{\left(2\,x^4-4\,x^3+3\,x^2+4\,x-2\right)(2\,x-1)\sqrt{-\frac{2\,x-1}{2\,x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^2*(2*x^3-2*x^2+x)/(-1+2*x)/((1-2*x)/(2*x^2+1))^(1/2)/(2*x^
4-4*x^3+3*x^2+4*x-2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 - 2*x^2 + x)*(x - 1)^2/((2*x^4 - 4*x^3 + 3*x^2 + 4*x - 2)*
(2*x - 1)*sqrt(-(2*x - 1)/(2*x^2 + 1))), x)
```

maple [C] time = 0.51, size = 314, normalized size = 3.53

$$\frac{i \left(\sum_{\_\alpha=\mathrm{RootOf}\left(2\_Z^{4}-4\_Z^{3}+3\_Z^{2}+4\_Z-2\right)}\frac{\left(2\_\alpha^{3}-2\_\alpha^{2}+\_\alpha-1\right)\sqrt{-i\left(2x+i\sqrt{2}\right)}\,\sqrt{-\frac{-1+2x}{1+i\sqrt{2}}}\,\sqrt{-i\left(i\sqrt{2}-2x\right)}\,\left(-2\_\alpha^{3}+6\_\alpha^{2}-6\_\alpha-4+i\sqrt{2}\,\left(2\_\alpha^{3}-3\_\alpha^{2}+7\right)\right)\mathrm{EllipticPi}\left(\frac{\sqrt{-i\left(2x+i\sqrt{2}\right)}\,\sqrt{2}}{2},\frac{4\_\alpha^{3}}{9}-\frac{2\_\alpha^{2}}{3}+\frac{2i\sqrt{2}\,\_\alpha^{3}}{9}-\frac{2i\sqrt{2}\,\_\alpha^{2}}{3}+\frac{14}{9}+\frac{2i\_\alpha\sqrt{2}}{3}+\frac{4i\sqrt{2}}{9},\sqrt{2}\,\sqrt{\frac{i\sqrt{2}}{1+i\sqrt{2}}}\right)}{\sqrt{-4x^{3}+2x^{2}-2x+1}}\right)\sqrt{-\left(-1+2x\right)\left(2x^{2}+1\right)}-9\sqrt{-4x^{3}+2x^{2}-2x+1}\,\sqrt{-\left(-1+2x\right)\left(2x^{2}+1\right)}-18x^{2}-9}{54\sqrt{-\frac{-1+2x}{2x^{2}+1}}\,\left(2x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^2*(2*x^3-2*x^2+x)/(-1+2*x)/((1-2*x)/(2*x^2+1))^(1/2)/(2*x^4-4*x^
3+3*x^2+4*x-2),x)
```

```
[Out] 1/54*(I*sum((2*_alpha^3-2*_alpha^2+_alpha-1)*(-I*(2*x+I*2^(1/2)))^(1/2)*(-(
-1+2*x)/(1+I*2^(1/2)))^(1/2)*(-I*(I*2^(1/2)-2*x))^(1/2)*(-2*_alpha^3+6*_alp
ha^2-6*_alpha-4+I*2^(1/2)*(2*_alpha^3-3*_alpha^2+7))*EllipticPi(1/2*(-I*(2*
x+I*2^(1/2))*2^(1/2))^(1/2),4/9*_alpha^3-2/3*_alpha^2+2/9*I*2^(1/2)*_alpha^
3-2/3*I*2^(1/2)*_alpha^2+14/9+2/3*I*_alpha*2^(1/2)+4/9*I*2^(1/2),2^(1/2)*(I
*2^(1/2)/(1+I*2^(1/2)))^(1/2))/(-4*x^3+2*x^2-2*x+1)^(1/2),_alpha=RootOf(2*_
Z^4-4*_Z^3+3*_Z^2+4*_Z-2))*(-(-1+2*x)*(2*x^2+1))^(1/2)-9*(-4*x^3+2*x^2-2*x+
1)^(1/2)*(-(-1+2*x)*(2*x^2+1))^(1/2)-18*x^2-9)/(-(-1+2*x)/(2*x^2+1))^(1/2)/
(2*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{3}-2\,x^{2}+x\right)\left(x-1\right)^{2}}{\left(2\,x^{4}-4\,x^{3}+3\,x^{2}+4\,x-2\right)\left(2\,x-1\right)\sqrt{-\frac{2\,x-1}{2\,x^{2}+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^2*(2*x^3-2*x^2+x)/(-1+2*x)/((1-2*x)/(2*x^2+1))^(1/2)/(2*x^
4-4*x^3+3*x^2+4*x-2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^3 - 2*x^2 + x)*(x - 1)^2/((2*x^4 - 4*x^3 + 3*x^2 + 4*x - 2)*
(2*x - 1)*sqrt(-(2*x - 1)/(2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(x-1\right)}^2\,\left(2\,x^3-2\,x^2+x\right)}{\left(2\,x-1\right)\,\sqrt{-\frac{2\,x-1}{2\,x^2+1}}\,\left(2\,x^4-4\,x^3+3\,x^2+4\,x-2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)^2*(x - 2*x^2 + 2*x^3))/((2*x - 1)*(-(2*x - 1)/(2*x^2 + 1))^(1/
2)*(4*x + 3*x^2 - 4*x^3 + 2*x^4 - 2)),x)
```

```
[Out] int(((x - 1)^2*(x - 2*x^2 + 2*x^3))/((2*x - 1)*(-(2*x - 1)/(2*x^2 + 1))^(1/
2)*(4*x + 3*x^2 - 4*x^3 + 2*x^4 - 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**2*(2*x**3-2*x**2+x)/(-1+2*x)/((1-2*x)/(2*x**2+1))**(1/2)/
(2*x**4-4*x**3+3*x**2+4*x-2),x)
```

```
[Out] Timed out
```

3.1086 $\int \frac{(-1+x^5)^{2/3}}{x} \, dx$

Optimal. Leaf size=89

$$\frac{3}{10}\left(x^5-1\right)^{2/3}+\frac{1}{5}\log\left(\sqrt[3]{x^5-1}+1\right)-\frac{1}{10}\log\left(\left(x^5-1\right)^{2/3}-\sqrt[3]{x^5-1}+1\right)+\frac{1}{5}\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^5-1}}{\sqrt{3}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 50, 56, 618, 204, 31}

$$\frac{3}{10}\left(x^5-1\right)^{2/3}+\frac{3}{10}\log\left(\sqrt[3]{x^5-1}+1\right)+\frac{1}{5}\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{x^5-1}}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^5)^(2/3)/x,x]
```

```
[Out] (3*(-1 + x^5)^(2/3))/10 + (Sqrt[3]*ArcTan[(1 - 2*(-1 + x^5)^(1/3))/Sqrt[3]]
)/5 - Log[x]/2 + (3*Log[1 + (-1 + x^5)^(1/3)])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^5\right)^{2/3}}{x}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{(-1+x)^{2/3}}{x}\,dx,x,x^5\right)\\
&= \frac{3}{10}\left(-1+x^5\right)^{2/3}-\frac{1}{5}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x}\,dx,x,x^5\right)\\
&= \frac{3}{10}\left(-1+x^5\right)^{2/3}-\frac{\log(x)}{2}+\frac{3}{10}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt[3]{-1+x^5}\right)-\frac{3}{10}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\right.\\
&= \frac{3}{10}\left(-1+x^5\right)^{2/3}-\frac{\log(x)}{2}+\frac{3}{10}\log\left(1+\sqrt[3]{-1+x^5}\right)+\frac{3}{5}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2\sqrt[3]{-1}\right.\\
&= \frac{3}{10}\left(-1+x^5\right)^{2/3}+\frac{1}{5}\sqrt{3}\,\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^5}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{10}\log\left(1+\sqrt[3]{-1+x^5}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.34

$$-\frac{3}{10}\left(x^5-1\right)^{2/3}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-x^5\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^5)^(2/3)/x,x]
```

```
[Out] (-3*(-1 + x^5)^(2/3)*(-1 + Hypergeometric2F1[2/3, 1, 5/3, 1 - x^5]))/10
```

IntegrateAlgebraic [A] time = 0.05, size = 89, normalized size = 1.00

$$\frac{3}{10}\left(x^5-1\right)^{2/3}+\frac{1}{5}\log\left(\sqrt[3]{x^5-1}+1\right)-\frac{1}{10}\log\left(\left(x^5-1\right)^{2/3}-\sqrt[3]{x^5-1}+1\right)+\frac{1}{5}\sqrt{3}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^5-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^5)^(2/3)/x,x]
```

```
[Out] (3*(-1 + x^5)^(2/3))/10 + (Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(-1 + x^5)^(1/3))/
Sqrt[3]])/5 + Log[1 + (-1 + x^5)^(1/3)]/5 - Log[1 - (-1 + x^5)^(1/3) + (-1
+ x^5)^(2/3)]/10
```

fricas [A] time = 0.42, size = 67, normalized size = 0.75

$$-\frac{1}{5}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^5-1\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+\frac{3}{10}\left(x^5-1\right)^{\frac{2}{3}}-\frac{1}{10}\log\left(\left(x^5-1\right)^{\frac{2}{3}}-\left(x^5-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{5}\log\left(\left(x^5-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(3)*arctan(2/3*sqrt(3)*(x^5 - 1)^(1/3) - 1/3*sqrt(3)) + 3/10*(x^5
- 1)^(2/3) - 1/10*log((x^5 - 1)^(2/3) - (x^5 - 1)^(1/3) + 1) + 1/5*log((x^5
 - 1)^(1/3) + 1)
```

giac [A] time = 0.30, size = 66, normalized size = 0.74

$$-\frac{1}{5}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^5-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{3}{10}\left(x^5-1\right)^{\frac{2}{3}}-\frac{1}{10}\log\left(\left(x^5-1\right)^{\frac{2}{3}}-\left(x^5-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{5}\log\left(\left|\left(x^5-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(2/3)/x,x, algorithm="giac")
```

```
[Out] -1/5*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^5 - 1)^(1/3) - 1)) + 3/10*(x^5 - 1)^(
2/3) - 1/10*log((x^5 - 1)^(2/3) - (x^5 - 1)^(1/3) + 1) + 1/5*log(abs((x^5 -
 1)^(1/3) + 1))
```

maple [C] time = 0.31, size = 84, normalized size = 0.94

$$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\mathrm{signum}\left(x^5-1\right)^{\frac{2}{3}}\left(\frac{2\pi\sqrt{3}\,x^5\,\mathrm{hypergeom}\left(\left[\frac{1}{3},1,1\right],[2,2],x^5\right)}{3\Gamma\left(\frac{2}{3}\right)}-\frac{\left(\frac{3}{2}-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+5\ln(x)+i\pi\right)\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}\right)}{15\pi\left(-\mathrm{signum}\left(x^5-1\right)\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)^(2/3)/x,x)
```

```
[Out] -1/15/Pi*3^(1/2)*GAMMA(2/3)*signum(x^5-1)^(2/3)/(-signum(x^5-1))^(2/3)*(2/3
*Pi*3^(1/2)/GAMMA(2/3)*x^5*hypergeom([1/3,1,1],[2,2],x^5)-(3/2-1/6*Pi*3^(1/
2)-3/2*ln(3)+5*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.41, size = 65, normalized size = 0.73

$$-\frac{1}{5}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^5-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{3}{10}\left(x^5-1\right)^{\frac{2}{3}}-\frac{1}{10}\log\left(\left(x^5-1\right)^{\frac{2}{3}}-\left(x^5-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{5}\log\left(\left(x^5-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^5 - 1)^(1/3) - 1)) + 3/10*(x^5 - 1)^(
2/3) - 1/10*log((x^5 - 1)^(2/3) - (x^5 - 1)^(1/3) + 1) + 1/5*log((x^5 - 1)^
(1/3) + 1)
```

mupad [B] time = 0.85, size = 89, normalized size = 1.00

$$\frac{\ln\left(\frac{9\left(x^5-1\right)^{1/3}}{25}+\frac{9}{25}\right)}{5}+\ln\left(9\left(-\frac{1}{10}+\frac{\sqrt{3}\,1\mathrm{i}}{10}\right)^2+\frac{9\left(x^5-1\right)^{1/3}}{25}\right)\left(-\frac{1}{10}+\frac{\sqrt{3}\,1\mathrm{i}}{10}\right)-\ln\left(9\left(\frac{1}{10}+\frac{\sqrt{3}\,1\mathrm{i}}{10}\right)^2+\frac{9\left(x^5-1\right)^{1/3}}{25}\right)\left(\frac{1}{10}+\frac{\sqrt{3}\,1\mathrm{i}}{10}\right)+\frac{3\left(x^5-1\right)^{2/3}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5 - 1)^(2/3)/x,x)
```

```
[Out] log((9*(x^5 - 1)^(1/3))/25 + 9/25)/5 + log(9*((3^(1/2)*1i)/10 - 1/10)^2 + (
9*(x^5 - 1)^(1/3))/25)*((3^(1/2)*1i)/10 - 1/10) - log(9*((3^(1/2)*1i)/10 +
1/10)^2 + (9*(x^5 - 1)^(1/3))/25)*((3^(1/2)*1i)/10 + 1/10) + (3*(x^5 - 1)^(
2/3))/10
```

sympy [C] time = 0.91, size = 39, normalized size = 0.44

$$-\frac{x^{\frac{10}{3}}\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^5}\right)}{5\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)**(2/3)/x,x)
```

```
[Out] -x**(10/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), exp_polar(2*I*pi)/x**5)/
(5*gamma(1/3))
```

3.1087 $\int \frac{(-1+x^6)^{2/3}}{x}\,dx$

Optimal. Leaf size=89

$$\frac{1}{4}\left(x^6-1\right)^{2/3}+\frac{1}{6}\log\left(\sqrt[3]{x^6-1}+1\right)-\frac{1}{12}\log\left(\left(x^6-1\right)^{2/3}-\sqrt[3]{x^6-1}+1\right)+\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 50, 56, 618, 204, 31}

$$\frac{1}{4}\left(x^6-1\right)^{2/3}+\frac{1}{4}\log\left(\sqrt[3]{x^6-1}+1\right)+\frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)^(2/3)/x,x]
```

```
[Out] (-1 + x^6)^(2/3)/4 + ArcTan[(1 - 2*(-1 + x^6)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) -
 Log[x]/2 + Log[1 + (-1 + x^6)^(1/3)]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^6\right)^{2/3}}{x} dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{(-1+x)^{2/3}}{x} dx, x, x^6\right) \\
&= \frac{1}{4}\left(-1+x^6\right)^{2/3} - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x} dx, x, x^6\right) \\
&= \frac{1}{4}\left(-1+x^6\right)^{2/3} - \frac{\log(x)}{2} + \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x} dx, x, \sqrt[3]{-1+x^6}\right) - \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\right. \\
&= \frac{1}{4}\left(-1+x^6\right)^{2/3} - \frac{\log(x)}{2} + \frac{1}{4}\log\left(1+\sqrt[3]{-1+x^6}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2\sqrt[3]{-}\right. \\
&= \frac{1}{4}\left(-1+x^6\right)^{2/3} + \frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^6}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{4}\log\left(1+\sqrt[3]{-1+x^6}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.34

$$-\frac{1}{4}\left(x^6-1\right)^{2/3}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-x^6\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)^(2/3)/x,x]
```

```
[Out] -1/4*((-1 + x^6)^(2/3)*(-1 + Hypergeometric2F1[2/3, 1, 5/3, 1 - x^6]))
```

IntegrateAlgebraic [A] time = 0.05, size = 89, normalized size = 1.00

$$\frac{1}{4}\left(x^6-1\right)^{2/3}+\frac{1}{6}\log\left(\sqrt[3]{x^6-1}+1\right)-\frac{1}{12}\log\left(\left(x^6-1\right)^{2/3}-\sqrt[3]{x^6-1}+1\right)+\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^6)^(2/3)/x,x]
```

```
[Out] (-1 + x^6)^(2/3)/4 + ArcTan[1/Sqrt[3] - (2*(-1 + x^6)^(1/3))/Sqrt[3]]/(2*Sq
rt[3]) + Log[1 + (-1 + x^6)^(1/3)]/6 - Log[1 - (-1 + x^6)^(1/3) + (-1 + x^6
)^(2/3)]/12
```

fricas [A] time = 0.41, size = 67, normalized size = 0.75

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+\frac{1}{4}\left(x^6-1\right)^{\frac{2}{3}}-\frac{1}{12}\log\left(\left(x^6-1\right)^{\frac{2}{3}}-\left(x^6-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(\left(x^6-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan(2/3*sqrt(3)*(x^6 - 1)^(1/3) - 1/3*sqrt(3)) + 1/4*(x^6 -
 1)^(2/3) - 1/12*log((x^6 - 1)^(2/3) - (x^6 - 1)^(1/3) + 1) + 1/6*log((x^6
- 1)^(1/3) + 1)
```

giac [A] time = 0.34, size = 66, normalized size = 0.74

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^6-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{1}{4}\left(x^6-1\right)^{\frac{2}{3}}-\frac{1}{12}\log\left(\left(x^6-1\right)^{\frac{2}{3}}-\left(x^6-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(\left|\left(x^6-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(2/3)/x,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3) - 1)) + 1/4*(x^6 - 1)^(2
/3) - 1/12*log((x^6 - 1)^(2/3) - (x^6 - 1)^(1/3) + 1) + 1/6*log(abs((x^6 -
1)^(1/3) + 1))
```

maple [C] time = 0.34, size = 84, normalized size = 0.94

$$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\mathrm{signum}\left(x^6-1\right)^{\frac{2}{3}}\left(\frac{2\pi\sqrt{3}\,x^6\,\mathrm{hypergeom}\left(\left[\frac{1}{3},1,1\right],[2,2],x^6\right)}{3\Gamma\left(\frac{2}{3}\right)}-\frac{\left(\frac{3}{2}-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+6\ln(x)+i\pi\right)\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}\right)}{18\pi\left(-\mathrm{signum}\left(x^6-1\right)\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(2/3)/x,x)
```

```
[Out] -1/18/Pi*3^(1/2)*GAMMA(2/3)*signum(x^6-1)^(2/3)/(-signum(x^6-1))^(2/3)*(2/3
*Pi*3^(1/2)/GAMMA(2/3)*x^6*hypergeom([1/3,1,1],[2,2],x^6)-(3/2-1/6*Pi*3^(1/
2)-3/2*ln(3)+6*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.41, size = 65, normalized size = 0.73

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^6-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{1}{4}\left(x^6-1\right)^{\frac{2}{3}}-\frac{1}{12}\log\left(\left(x^6-1\right)^{\frac{2}{3}}-\left(x^6-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(\left(x^6-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3) - 1)) + 1/4*(x^6 - 1)^(2
/3) - 1/12*log((x^6 - 1)^(2/3) - (x^6 - 1)^(1/3) + 1) + 1/6*log((x^6 - 1)^(
1/3) + 1)
```

mupad [B] time = 0.84, size = 89, normalized size = 1.00

$$\frac{\ln\left(\frac{\left(x^6-1\right)^{1/3}}{4}+\frac{1}{4}\right)}{6}+\ln\left(9\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)^2+\frac{\left(x^6-1\right)^{1/3}}{4}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(9\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)^2+\frac{\left(x^6-1\right)^{1/3}}{4}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\frac{\left(x^6-1\right)^{2/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)^(2/3)/x,x)
```

```
[Out] log((x^6 - 1)^(1/3)/4 + 1/4)/6 + log(9*((3^(1/2)*1i)/12 - 1/12)^2 + (x^6 -
1)^(1/3)/4)*((3^(1/2)*1i)/12 - 1/12) - log(9*((3^(1/2)*1i)/12 + 1/12)^2 + (
x^6 - 1)^(1/3)/4)*((3^(1/2)*1i)/12 + 1/12) + (x^6 - 1)^(2/3)/4
```

sympy [C] time = 0.84, size = 37, normalized size = 0.42

$$-\frac{x^4\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^6}\right)}{6\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(2/3)/x,x)
```

```
[Out] -x**4*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), exp_polar(2*I*pi)/x**6)/(6*ga
mma(1/3))
```

3.1088 $\int \sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$

Optimal. Leaf size=89

$$\frac{1}{2}x\sqrt{\sqrt{a^2x^4+b}+ax^2}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}}{\sqrt{b}}\right)}{2\sqrt{2}\sqrt{a}}$$

Rubi [F] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] Defer[Int][Sqrt[a*x^2 + Sqrt[b + a^2*x^4]], x]
```

Rubi steps

$$\int \sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx=\int \sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Mathematica [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] Integrate[Sqrt[a*x^2 + Sqrt[b + a^2*x^4]], x]
```

IntegrateAlgebraic [A] time = 0.17, size = 89, normalized size = 1.00

$$\frac{1}{2}x\sqrt{\sqrt{a^2x^4+b}+ax^2}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}}{\sqrt{b}}\right)}{2\sqrt{2}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] (x*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]])/2 + (Sqrt[b]*ArcTan[(Sqrt[2]*Sqrt[a]*x*
Sqrt[a*x^2 + Sqrt[b + a^2*x^4]])/Sqrt[b]])/(2*Sqrt[2]*Sqrt[a])
```

fricas [A] time = 3.76, size = 249, normalized size = 2.80

$$\left[\frac{1}{8}\sqrt{2}\sqrt{-\frac{b}{a}}\log\left(4\,a^2bx^4-4\,\sqrt{a^2x^4+b}\,abx^2+b^2+2\left(2\,\sqrt{2}\,\sqrt{a^2x^4+b}\,a^2x^3\sqrt{-\frac{b}{a}}-\sqrt{2}\left(2\,a^3x^5+abx\right)\sqrt{-\frac{b}{a}}\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}\right)+\frac{1}{2}\sqrt{ax^2+\sqrt{a^2x^4+b}}\,x,-\frac{1}{4}\sqrt{2}\sqrt{\frac{b}{a}}\arctan\left(-\frac{\left(\sqrt{2}\,ax^2\sqrt{\frac{b}{a}}-\sqrt{2}\,\sqrt{a^2x^4+b}\,\sqrt{\frac{b}{a}}\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{2\,bx}\right)+\frac{1}{2}\sqrt{ax^2+\sqrt{a^2x^4+b}}\,x\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(2)*sqrt(-b/a)*log(4*a^2*b*x^4 - 4*sqrt(a^2*x^4 + b)*a*b*x^2 + b^2
 + 2*(2*sqrt(2)*sqrt(a^2*x^4 + b)*a^2*x^3*sqrt(-b/a) - sqrt(2)*(2*a^3*x^5 +
 a*b*x)*sqrt(-b/a))*sqrt(a*x^2 + sqrt(a^2*x^4 + b))) + 1/2*sqrt(a*x^2 + sqr
t(a^2*x^4 + b))*x, -1/4*sqrt(2)*sqrt(b/a)*arctan(-1/2*(sqrt(2)*a*x^2*sqrt(b
/a) - sqrt(2)*sqrt(a^2*x^4 + b)*sqrt(b/a))*sqrt(a*x^2 + sqrt(a^2*x^4 + b))/
(b*x)) + 1/2*sqrt(a*x^2 + sqrt(a^2*x^4 + b))*x]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^2 + \sqrt{a^2x^4 + b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^4 + b)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,x^2 + \sqrt{a^2x^4 + b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x)
```

```
[Out] int((a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^2 + \sqrt{a^2x^4 + b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^4 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sqrt{a^2\,x^4 + b} + a\,x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a^2*x^4)^(1/2) + a*x^2)^(1/2),x)
```

```
[Out] int(((b + a^2*x^4)^(1/2) + a*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^2 + \sqrt{a^2x^4 + b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+(a**2*x**4+b)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**2 + sqrt(a**2*x**4 + b)), x)
```

3.1089 $\int \frac{(-1+x^2)^{2/3}}{x^3} \, dx$

Optimal. Leaf size=90

$$-\frac{(x^2-1)^{2/3}}{2x^2}-\frac{1}{3}\log\left(\sqrt[3]{x^2-1}+1\right)+\frac{1}{6}\log\left((x^2-1)^{2/3}-\sqrt[3]{x^2-1}+1\right)-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^2-1}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 47, 56, 618, 204, 31}

$$-\frac{(x^2-1)^{2/3}}{2x^2}-\frac{1}{2}\log\left(\sqrt[3]{x^2-1}+1\right)-\frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{x^2-1}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)^(2/3)/x^3,x]
```

```
[Out] -1/2*(-1 + x^2)^(2/3)/x^2 - ArcTan[(1 - 2*(-1 + x^2)^(1/3))/Sqrt[3]]/Sqrt[3
] + Log[x]/3 - Log[1 + (-1 + x^2)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-1+x^2\right)^{2/3}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(-1+x)^{2/3}}{x^2}\,dx,x,x^2\right)\\ &= -\frac{\left(-1+x^2\right)^{2/3}}{2x^2}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x}\,dx,x,x^2\right)\\ &= -\frac{\left(-1+x^2\right)^{2/3}}{2x^2}+\frac{\log(x)}{3}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt[3]{-1+x^2}\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\right.\\ &= -\frac{\left(-1+x^2\right)^{2/3}}{2x^2}+\frac{\log(x)}{3}-\frac{1}{2}\log\left(1+\sqrt[3]{-1+x^2}\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2\sqrt[3]{-1\right.\\ &= -\frac{\left(-1+x^2\right)^{2/3}}{2x^2}+\frac{\tan^{-1}\left(\frac{-1+2\sqrt[3]{-1+x^2}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log(x)}{3}-\frac{1}{2}\log\left(1+\sqrt[3]{-1+x^2}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.31

$$\frac{3}{10}\left(x^2-1\right)^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)^(2/3)/x^3,x]
```

```
[Out] (3*(-1 + x^2)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 - x^2])/10
```

IntegrateAlgebraic [A] time = 0.11, size = 90, normalized size = 1.00

$$-\frac{\left(x^2-1\right)^{2/3}}{2x^2}-\frac{1}{3}\log\left(\sqrt[3]{x^2-1}+1\right)+\frac{1}{6}\log\left(\left(x^2-1\right)^{2/3}-\sqrt[3]{x^2-1}+1\right)-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^2-1}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)^(2/3)/x^3,x]
```

```
[Out] -1/2*(-1 + x^2)^(2/3)/x^2 - ArcTan[1/Sqrt[3] - (2*(-1 + x^2)^(1/3))/Sqrt[3]
]/Sqrt[3] - Log[1 + (-1 + x^2)^(1/3)]/3 + Log[1 - (-1 + x^2)^(1/3) + (-1 +
x^2)^(2/3)]/6
```

fricas [A] time = 0.42, size = 80, normalized size = 0.89

$$\frac{2\sqrt{3}\,x^2\arctan\left(\frac{2}{3}\sqrt{3}\left(x^2-1\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x^2\log\left(\left(x^2-1\right)^{\frac{2}{3}}-\left(x^2-1\right)^{\frac{1}{3}}+1\right)-2\,x^2\log\left(\left(x^2-1\right)^{\frac{1}{3}}+1\right)-3\left(x^2-1\right)^{\frac{2}{3}}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*x^2*arctan(2/3*sqrt(3)*(x^2 - 1)^(1/3) - 1/3*sqrt(3)) + x^2*
log((x^2 - 1)^(2/3) - (x^2 - 1)^(1/3) + 1) - 2*x^2*log((x^2 - 1)^(1/3) + 1)
 - 3*(x^2 - 1)^(2/3))/x^2
```

giac [A] time = 0.45, size = 69, normalized size = 0.77

$$\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,{\left(x^{2}-1\right)}^{\frac{1}{3}}-1\right)\right)-\frac{{\left(x^{2}-1\right)}^{\frac{2}{3}}}{2\,x^{2}}+\frac{1}{6}\,\log\left({\left(x^{2}-1\right)}^{\frac{2}{3}}-{\left(x^{2}-1\right)}^{\frac{1}{3}}+1\right)-\frac{1}{3}\,\log\left(\left|{\left(x^{2}-1\right)}^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 - 1)^(1/3) - 1)) - 1/2*(x^2 - 1)^(2/
3)/x^2 + 1/6*log((x^2 - 1)^(2/3) - (x^2 - 1)^(1/3) + 1) - 1/3*log(abs((x^2
- 1)^(1/3) + 1))
```

maple [C] time = 0.34, size = 96, normalized size = 1.07

$$-\frac{\left(x^{2}-1\right)^{\frac{2}{3}}}{2x^{2}}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\mathrm{signum}\left(x^{2}-1\right)\right)^{\frac{1}{3}}\left(\frac{2\pi\sqrt{3}\,x^{2}\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^{2}\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{6\pi\mathrm{signum}\left(x^{2}-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(2/3)/x^3,x)
```

```
[Out] -1/2*(x^2-1)^(2/3)/x^2+1/6/Pi*3^(1/2)*GAMMA(2/3)/signum(x^2-1)^(1/3)*(-sign
um(x^2-1))^(1/3)*(2/9*Pi*3^(1/2)/GAMMA(2/3)*x^2*hypergeom([1,1,4/3],[2,2],x
^2)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.41, size = 68, normalized size = 0.76

$$\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,{\left(x^{2}-1\right)}^{\frac{1}{3}}-1\right)\right)-\frac{{\left(x^{2}-1\right)}^{\frac{2}{3}}}{2\,x^{2}}+\frac{1}{6}\,\log\left({\left(x^{2}-1\right)}^{\frac{2}{3}}-{\left(x^{2}-1\right)}^{\frac{1}{3}}+1\right)-\frac{1}{3}\,\log\left({\left(x^{2}-1\right)}^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^2 - 1)^(1/3) - 1)) - 1/2*(x^2 - 1)^(2/
3)/x^2 + 1/6*log((x^2 - 1)^(2/3) - (x^2 - 1)^(1/3) + 1) - 1/3*log((x^2 - 1)
^(1/3) + 1)
```

mupad [B] time = 0.88, size = 86, normalized size = 0.96

$$-\frac{\ln\left({\left(x^2-1\right)}^{1/3}+1\right)}{3}-\ln\left(9\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2+{\left(x^2-1\right)}^{1/3}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)+\ln\left(9\,{\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2+{\left(x^2-1\right)}^{1/3}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)-\frac{{\left(x^2-1\right)}^{2/3}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^(2/3)/x^3,x)
```

```
[Out] log(9*((3^(1/2)*1i)/6 + 1/6)^2 + (x^2 - 1)^(1/3))*((3^(1/2)*1i)/6 + 1/6) -
log(9*((3^(1/2)*1i)/6 - 1/6)^2 + (x^2 - 1)^(1/3))*((3^(1/2)*1i)/6 - 1/6) -
log((x^2 - 1)^(1/3) + 1)/3 - (x^2 - 1)^(2/3)/(2*x^2)
```

sympy [C] time = 0.93, size = 36, normalized size = 0.40

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^{2}}\right)}{2x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(2/3)/x**3,x)
```

```
[Out] -gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_polar(2*I*pi)/x**2)/(2*x**(2/3)*
gamma(4/3))
```

3.1090 $\int \frac{-2+x+x^2}{x^2\left(-1+x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=90

$$-\frac{2\sqrt[4]{x^2-1}}{x}+\frac{\tan^{-1}\left(\frac{\frac{\sqrt{x^2-1}}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt[4]{x^2-1}}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^2-1}}{\sqrt{x^2-1}+1}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.13, antiderivative size = 138, normalized size of antiderivative = 1.53, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1807, 266, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2\sqrt[4]{x^2-1}}{x}-\frac{\log\left(\sqrt{x^2-1}-\sqrt{2}\sqrt[4]{x^2-1}+1\right)}{2\sqrt{2}}+\frac{\log\left(\sqrt{x^2-1}+\sqrt{2}\sqrt[4]{x^2-1}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{x^2-1}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt[4]{x^2-1}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x + x^2)/(x^2*(-1 + x^2)^(3/4)),x]
```

```
[Out] (-2*(-1 + x^2)^(1/4))/x - ArcTan[1 - Sqrt[2]*(-1 + x^2)^(1/4)]/Sqrt[2] + Ar
cTan[1 + Sqrt[2]*(-1 + x^2)^(1/4)]/Sqrt[2] - Log[1 - Sqrt[2]*(-1 + x^2)^(1/
4) + Sqrt[-1 + x^2]]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*(-1 + x^2)^(1/4) + Sqrt[
-1 + x^2]]/(2*Sqrt[2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{-2+x+x^2}{x^2\left(-1+x^2\right)^{3/4}}\,dx &= -\frac{2\sqrt[4]{-1+x^2}}{x}+\int \frac{1}{x\left(-1+x^2\right)^{3/4}}\,dx\\
&= -\frac{2\sqrt[4]{-1+x^2}}{x}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(-1+x)^{3/4}x}\,dx,x,x^2\right)\\
&= -\frac{2\sqrt[4]{-1+x^2}}{x}+2\operatorname{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt[4]{-1+x^2}\right)\\
&= -\frac{2\sqrt[4]{-1+x^2}}{x}+\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt[4]{-1+x^2}\right)+\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt[4]{-1+x^2}\right)\\
&= -\frac{2\sqrt[4]{-1+x^2}}{x}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt[4]{-1+x^2}\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x\cdots}\right.\\
&= -\frac{2\sqrt[4]{-1+x^2}}{x}-\frac{\log\left(1-\sqrt{2}\sqrt[4]{-1+x^2}+\sqrt{-1+x^2}\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt[4]{-1+x^2}+\sqrt{-1\cdots}\right.}{2\sqrt{2}}\\
&= -\frac{2\sqrt[4]{-1+x^2}}{x}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+x^2}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+x^2}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\cdots\right.}{}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 138, normalized size = 1.53

$$-\frac{2\sqrt[4]{x^2-1}}{x}-\frac{\log\left(\sqrt{x^2-1}-\sqrt{2}\sqrt[4]{x^2-1}+1\right)}{2\sqrt{2}}+\frac{\log\left(\sqrt{x^2-1}+\sqrt{2}\sqrt[4]{x^2-1}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{x^2-1}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt[4]{x^2-1}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x + x^2)/(x^2*(-1 + x^2)^(3/4)),x]
```

```
[Out] (-2*(-1 + x^2)^(1/4))/x - ArcTan[1 - Sqrt[2]*(-1 + x^2)^(1/4)]/Sqrt[2] + Ar
cTan[1 + Sqrt[2]*(-1 + x^2)^(1/4)]/Sqrt[2] - Log[1 - Sqrt[2]*(-1 + x^2)^(1/
4) + Sqrt[-1 + x^2]]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*(-1 + x^2)^(1/4) + Sqrt[
-1 + x^2]]/(2*Sqrt[2])
```

IntegrateAlgebraic [A] time = 18.60, size = 86, normalized size = 0.96

$$-\frac{2\sqrt[4]{x^2-1}}{x}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^2-1}}{\sqrt{x^2-1}-1}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^2-1}}{\sqrt{x^2-1}+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x + x^2)/(x^2*(-1 + x^2)^(3/4)),x]
```

```
[Out] (-2*(-1 + x^2)^(1/4))/x - ArcTan[(Sqrt[2]*(-1 + x^2)^(1/4))/(-1 + Sqrt[-1 +
 x^2])]/Sqrt[2] + ArcTanh[(Sqrt[2]*(-1 + x^2)^(1/4))/(1 + Sqrt[-1 + x^2])]/
Sqrt[2]
```

fricas [B] time = 1.34, size = 451, normalized size = 5.01

$$-\frac{4\sqrt{2}\,x\arctan\left(\frac{x^4+4\sqrt{x^2-1}\,x^2+2\sqrt{2}(x^2-1)^{\frac{3}{4}}(x^2-4)+2\sqrt{2}(3x^2-4)(x^2-1)^{\frac{1}{4}}+\left(4(x^2-1)^{\frac{1}{4}}x^2+2\sqrt{2}\sqrt{x^2-1}(x^2-4)+\sqrt{2}(x^4-10x^2+8)+16(x^2-1)^{\frac{3}{4}}\right)\sqrt{\frac{x^2+2\sqrt{2}(x^2-1)^{\frac{3}{4}}+2\sqrt{2}(x^2-1)^{\frac{1}{4}}+4\sqrt{x^2-1}}{x^2}}}{x^4-16x^2+16}\right)-4\sqrt{2}\,x\arctan\left(\frac{x^4+4\sqrt{x^2-1}\,x^2-2\sqrt{2}(x^2-1)^{\frac{3}{4}}(x^2-4)-2\sqrt{2}(3x^2-4)(x^2-1)^{\frac{1}{4}}+\left(4(x^2-1)^{\frac{1}{4}}x^2-2\sqrt{2}\sqrt{x^2-1}(x^2-4)-\sqrt{2}(x^4-10x^2+8)+16(x^2-1)^{\frac{3}{4}}\right)\sqrt{\frac{x^2-2\sqrt{2}(x^2-1)^{\frac{3}{4}}-2\sqrt{2}(x^2-1)^{\frac{1}{4}}+4\sqrt{x^2-1}}{x^2}}}{x^4-16x^2+16}\right)-\sqrt{2}\,x\log\left(\frac{4\left(x^2+2\sqrt{2}(x^2-1)^{\frac{3}{4}}+2\sqrt{2}(x^2-1)^{\frac{1}{4}}+4\sqrt{x^2-1}\right)}{x^2}\right)+\sqrt{2}\,x\log\left(\frac{4\left(x^2-2\sqrt{2}(x^2-1)^{\frac{3}{4}}-2\sqrt{2}(x^2-1)^{\frac{1}{4}}+4\sqrt{x^2-1}\right)}{x^2}\right)+16(x^2-1)^{\frac{1}{4}}}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-2)/x^2/(x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/8*(4*sqrt(2)*x*arctan((x^4 + 4*sqrt(x^2 - 1)*x^2 + 2*sqrt(2)*(x^2 - 1)^(
3/4)*(x^2 - 4) + 2*sqrt(2)*(3*x^2 - 4)*(x^2 - 1)^(1/4) + (4*(x^2 - 1)^(1/4)
*x^2 + 2*sqrt(2)*sqrt(x^2 - 1)*(x^2 - 4) + sqrt(2)*(x^4 - 10*x^2 + 8) + 16*
(x^2 - 1)^(3/4))*sqrt((x^2 + 2*sqrt(2)*(x^2 - 1)^(3/4) + 2*sqrt(2)*(x^2 - 1
)^(1/4) + 4*sqrt(x^2 - 1))/x^2))/(x^4 - 16*x^2 + 16)) - 4*sqrt(2)*x*arctan(
(x^4 + 4*sqrt(x^2 - 1)*x^2 - 2*sqrt(2)*(x^2 - 1)^(3/4)*(x^2 - 4) - 2*sqrt(2
)*(3*x^2 - 4)*(x^2 - 1)^(1/4) + (4*(x^2 - 1)^(1/4)*x^2 - 2*sqrt(2)*sqrt(x^2
 - 1)*(x^2 - 4) - sqrt(2)*(x^4 - 10*x^2 + 8) + 16*(x^2 - 1)^(3/4))*sqrt((x^
2 - 2*sqrt(2)*(x^2 - 1)^(3/4) - 2*sqrt(2)*(x^2 - 1)^(1/4) + 4*sqrt(x^2 - 1)
)/x^2))/(x^4 - 16*x^2 + 16)) - sqrt(2)*x*log(4*(x^2 + 2*sqrt(2)*(x^2 - 1)^(
3/4) + 2*sqrt(2)*(x^2 - 1)^(1/4) + 4*sqrt(x^2 - 1))/x^2) + sqrt(2)*x*log(4*
(x^2 - 2*sqrt(2)*(x^2 - 1)^(3/4) - 2*sqrt(2)*(x^2 - 1)^(1/4) + 4*sqrt(x^2 -
 1))/x^2) + 16*(x^2 - 1)^(1/4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+x-2}{\left(x^2-1\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-2)/x^2/(x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((x^2 + x - 2)/((x^2 - 1)^(3/4)*x^2), x)
```

maple [C] time = 0.36, size = 76, normalized size = 0.84

$$-\frac{2\left(x^{2}-1\right)^{\frac{1}{4}}}{x}+\frac{\left(-\mathrm{signum}\left(x^{2}-1\right)\right)^{\frac{3}{4}}\left(\frac{3\Gamma\left(\frac{3}{4}\right)x^{2}\,\mathrm{hypergeom}\left(\left[1,1,\frac{7}{4}\right],[2,2],x^{2}\right)}{4}+\left(-3\ln(2)+\frac{\pi}{2}+2\ln(x)+i\pi\right)\Gamma\left(\frac{3}{4}\right)\right)}{2\Gamma\left(\frac{3}{4}\right)\mathrm{signum}\left(x^{2}-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-2)/x^2/(x^2-1)^(3/4),x)
```

```
[Out] -2*(x^2-1)^(1/4)/x+1/2/GAMMA(3/4)/signum(x^2-1)^(3/4)*(-signum(x^2-1))^(3/4
)*(3/4*GAMMA(3/4)*x^2*hypergeom([1,1,7/4],[2,2],x^2)+(-3*ln(2)+1/2*Pi+2*ln(
x)+I*Pi)*GAMMA(3/4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}+x-2}{\left(x^{2}-1\right)^{\frac{3}{4}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-2)/x^2/(x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + x - 2)/((x^2 - 1)^(3/4)*x^2), x)
```

mupad [B] time = 1.33, size = 89, normalized size = 0.99

$$\frac{4\left(\frac{1}{x^{2}}\right)^{3/4}{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ \frac{1}{x^{2}}\right)}{5\,x}+\frac{x\left(1-x^{2}\right)^{3/4}{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ x^{2}\right)}{\left(x^{2}-1\right)^{3/4}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(x^{2}-1\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(x^{2}-1\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 - 2)/(x^2*(x^2 - 1)^(3/4)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*(x^2 - 1)^(1/4)*(1/2 - 1i/2))*(1/2 + 1i/2) + 2^(1/2)*a
tan(2^(1/2)*(x^2 - 1)^(1/4)*(1/2 + 1i/2))*(1/2 - 1i/2) + (4*(1/x^2)^(3/4)*h
ypergeom([3/4, 5/4], 9/4, 1/x^2))/(5*x) + (x*(1 - x^2)^(3/4)*hypergeom([1/2
, 3/4], 3/2, x^2))/(x^2 - 1)^(3/4)
```

sympy [C] time = 2.90, size = 75, normalized size = 0.83

$$xe^{-\frac{3i\pi}{4}}{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\right|x^{2}\right)-\frac{2e^{\frac{i\pi}{4}}{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{1}{2}\end{matrix}\right|x^{2}\right)}{x}-\frac{\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{e^{2i\pi}}{x^{2}}\right)}{2x^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x-2)/x**2/(x**2-1)**(3/4),x)
```

```
[Out] x*exp(-3*I*pi/4)*hyper((1/2, 3/4), (3/2,), x**2) - 2*exp(I*pi/4)*hyper((-1/
2, 3/4), (1/2,), x**2)/x - gamma(3/4)*hyper((3/4, 3/4), (7/4,), exp_polar(2
*I*pi)/x**2)/(2*x**(3/2)*gamma(7/4))
```

3.1091 $\int \frac{\sqrt[3]{1+x^3}}{x^4} dx$

Optimal. Leaf size=90

$$-\frac{\sqrt[3]{x^3+1}}{3x^3}+\frac{1}{9}\log\left(\sqrt[3]{x^3+1}-1\right)-\frac{1}{18}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 47, 57, 618, 204, 31}

$$-\frac{\sqrt[3]{x^3+1}}{3x^3}+\frac{1}{6}\log\left(1-\sqrt[3]{x^3+1}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\log(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)^(1/3)/x^4,x]
```

```
[Out] -1/3*(1 + x^3)^(1/3)/x^3 - ArcTan[(1 + 2*(1 + x^3)^(1/3))/Sqrt[3]]/(3*Sqrt[
3]) - Log[x]/6 + Log[1 - (1 + x^3)^(1/3)]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{1+x^3}}{x^4}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{x^2}\,dx, x, x^3\right)\\ &= -\frac{\sqrt[3]{1+x^3}}{3x^3}+\frac{1}{9}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{2/3}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt[3]{1+x^3}}{3x^3}-\frac{\log(x)}{6}-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+x^3}\right)-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1\right.\\ &= -\frac{\sqrt[3]{1+x^3}}{3x^3}-\frac{\log(x)}{6}+\frac{1}{6}\log\left(1-\sqrt[3]{1+x^3}\right)+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+x^3}\right)\\ &= -\frac{\sqrt[3]{1+x^3}}{3x^3}-\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^3}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\log(x)}{6}+\frac{1}{6}\log\left(1-\sqrt[3]{1+x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.29

$$\frac{1}{4}\left(x^3+1\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)^(1/3)/x^4,x]
```

```
[Out] ((1 + x^3)^(4/3)*Hypergeometric2F1[4/3, 2, 7/3, 1 + x^3])/4
```

IntegrateAlgebraic [A] time = 0.08, size = 90, normalized size = 1.00

$$-\frac{\sqrt[3]{x^3+1}}{3x^3}+\frac{1}{9}\log\left(\sqrt[3]{x^3+1}-1\right)-\frac{1}{18}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)^(1/3)/x^4,x]
```

```
[Out] -1/3*(1 + x^3)^(1/3)/x^3 - ArcTan[1/Sqrt[3] + (2*(1 + x^3)^(1/3))/Sqrt[3]]/
(3*Sqrt[3]) + Log[-1 + (1 + x^3)^(1/3)]/9 - Log[1 + (1 + x^3)^(1/3) + (1 +
x^3)^(2/3)]/18
```

fricas [A] time = 0.41, size = 78, normalized size = 0.87

$$-\frac{2\sqrt{3}\,x^3\arctan\left(\frac{2}{3}\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+x^3\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)-2\,x^3\log\left(\left(x^3+1\right)^{\frac{1}{3}}-1\right)+6\left(x^3+1\right)^{\frac{1}{3}}}{18\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/3)/x^4,x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*x^3*arctan(2/3*sqrt(3)*(x^3 + 1)^(1/3) + 1/3*sqrt(3)) + x^
3*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) - 2*x^3*log((x^3 + 1)^(1/3) -
1) + 6*(x^3 + 1)^(1/3))/x^3
```

giac [A] time = 0.16, size = 67, normalized size = 0.74

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^3+1\right)^{\frac{1}{3}}}{3x^3}-\frac{1}{18}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{9}\log\left(\left|\left(x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/3)/x^4,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) - 1/3*(x^3 + 1)^(1
/3)/x^3 - 1/18*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) + 1/9*log(abs((x^
3 + 1)^(1/3) - 1))
```

maple [C] time = 0.29, size = 59, normalized size = 0.66

$$-\frac{\left(x^3+1\right)^{\frac{1}{3}}}{3x^3}+\frac{-\frac{2\Gamma\left(\frac{2}{3}\right)x^3\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],-x^3\right)}{3}+\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)\right)\Gamma\left(\frac{2}{3}\right)}{9\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(1/3)/x^4,x)
```

```
[Out] -1/3*(x^3+1)^(1/3)/x^3+1/9/GAMMA(2/3)*(-2/3*GAMMA(2/3)*x^3*hypergeom([1,1,5
/3],[2,2],-x^3)+(1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x))*GAMMA(2/3))
```

maxima [A] time = 0.44, size = 66, normalized size = 0.73

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^3+1\right)^{\frac{1}{3}}}{3x^3}-\frac{1}{18}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{9}\log\left(\left(x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/3)/x^4,x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) - 1/3*(x^3 + 1)^(1
/3)/x^3 - 1/18*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) + 1/9*log((x^3 +
1)^(1/3) - 1)
```

mupad [B] time = 0.90, size = 78, normalized size = 0.87

$$\frac{\ln\left(\frac{\left(x^3+1\right)^{1/3}}{9}-\frac{1}{9}\right)}{9}+\ln\left(\left(x^3+1\right)^{1/3}+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)-\ln\left(\left(x^3+1\right)^{1/3}+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)-\frac{\left(x^3+1\right)^{1/3}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)^(1/3)/x^4,x)
```

```
[Out] log((x^3 + 1)^(1/3)/9 - 1/9)/9 + log((x^3 + 1)^(1/3) - (3^(1/2)*1i)/2 + 1/2
)*((3^(1/2)*1i)/18 - 1/18) - log((3^(1/2)*1i)/2 + (x^3 + 1)^(1/3) + 1/2)*((
3^(1/2)*1i)/18 + 1/18) - (x^3 + 1)^(1/3)/(3*x^3)
```

sympy [C] time = 0.88, size = 32, normalized size = 0.36

$$-\frac{\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^3}\right)}{3x^2\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(1/3)/x**4,x)
```

```
[Out] -gamma(2/3)*hyper((-1/3, 2/3), (5/3,), exp_polar(I*pi)/x**3)/(3*x**2*gamma(
5/3))
```

3.1092 $\int \frac{(1+x^3)^{2/3}}{x^4} \, dx$

Optimal. Leaf size=90

$$-\frac{(x^3+1)^{2/3}}{3x^3}+\frac{2}{9}\log\left(\sqrt[3]{x^3+1}-1\right)-\frac{1}{9}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 47, 55, 618, 204, 31}

$$-\frac{(x^3+1)^{2/3}}{3x^3}+\frac{1}{3}\log\left(1-\sqrt[3]{x^3+1}\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)^(2/3)/x^4,x]
```

```
[Out] -1/3*(1 + x^3)^(2/3)/x^3 + (2*ArcTan[(1 + 2*(1 + x^3)^(1/3))/Sqrt[3]])/(3*S
qrt[3]) - Log[x]/3 + Log[1 - (1 + x^3)^(1/3)]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+x^3\right)^{2/3}}{x^4} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(1+x)^{2/3}}{x^2} dx, x, x^3\right) \\
&= -\frac{\left(1+x^3\right)^{2/3}}{3x^3} + \frac{2}{9}\text{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}} dx, x, x^3\right) \\
&= -\frac{\left(1+x^3\right)^{2/3}}{3x^3} - \frac{\log(x)}{3} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x} dx, x, \sqrt[3]{1+x^3}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x+x^2} dx, x\right. \\
&= -\frac{\left(1+x^3\right)^{2/3}}{3x^3} - \frac{\log(x)}{3} + \frac{1}{3}\log\left(1-\sqrt[3]{1+x^3}\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2\sqrt[3]{1+x^3}\right) \\
&= -\frac{\left(1+x^3\right)^{2/3}}{3x^3} + \frac{2\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\log(x)}{3} + \frac{1}{3}\log\left(1-\sqrt[3]{1+x^3}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.29

$$\frac{1}{5}\left(x^3+1\right)^{5/3} {}_2F_1\left(\frac{5}{3},2;\frac{8}{3};x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)^(2/3)/x^4,x]
```

```
[Out] ((1 + x^3)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 + x^3])/5
```

IntegrateAlgebraic [A] time = 0.09, size = 90, normalized size = 1.00

$$-\frac{\left(x^3+1\right)^{2/3}}{3x^3} + \frac{2}{9}\log\left(\sqrt[3]{x^3+1}-1\right) - \frac{1}{9}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right) + \frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)^(2/3)/x^4,x]
```

```
[Out] -1/3*(1 + x^3)^(2/3)/x^3 + (2*ArcTan[1/Sqrt[3] + (2*(1 + x^3)^(1/3))/Sqrt[3
]])/(3*Sqrt[3]) + (2*Log[-1 + (1 + x^3)^(1/3)])/9 - Log[1 + (1 + x^3)^(1/3)
 + (1 + x^3)^(2/3)]/9
```

fricas [A] time = 0.42, size = 79, normalized size = 0.88

$$\frac{2\sqrt{3}x^3\arctan\left(\frac{2}{3}\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right) - x^3\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right) + 2x^3\log\left(\left(x^3+1\right)^{\frac{1}{3}}-1\right) - 3\left(x^3+1\right)^{\frac{2}{3}}}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)/x^4,x, algorithm="fricas")
```

```
[Out] 1/9*(2*sqrt(3)*x^3*arctan(2/3*sqrt(3)*(x^3 + 1)^(1/3) + 1/3*sqrt(3)) - x^3*
log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) + 2*x^3*log((x^3 + 1)^(1/3) - 1)
 - 3*(x^3 + 1)^(2/3))/x^3
```

giac [A] time = 0.16, size = 67, normalized size = 0.74

$$\frac{2}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,{\left(x^{3}+1\right)}^{\frac{1}{3}}+1\right)\right)-\frac{{\left(x^{3}+1\right)}^{\frac{2}{3}}}{3\,x^{3}}-\frac{1}{9}\,\log\left({\left(x^{3}+1\right)}^{\frac{2}{3}}+{\left(x^{3}+1\right)}^{\frac{1}{3}}+1\right)+\frac{2}{9}\,\log\left(\left|{\left(x^{3}+1\right)}^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)/x^4,x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) - 1/3*(x^3 + 1)^(2/
3)/x^3 - 1/9*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) + 2/9*log(abs((x^3
+ 1)^(1/3) - 1))
```

maple [C] time = 0.28, size = 76, normalized size = 0.84

$$-\frac{\left(x^{3}+1\right)^{\frac{2}{3}}}{3x^{3}}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^{3}\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],\left[2,2\right],-x^{3}\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln\left(3\right)}{2}+3\ln\left(x\right)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{9\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)/x^4,x)
```

```
[Out] -1/3*(x^3+1)^(2/3)/x^3+1/9/Pi*3^(1/2)*GAMMA(2/3)*(-2/9*Pi*3^(1/2)/GAMMA(2/3
)*x^3*hypergeom([1,1,4/3],[2,2],-x^3)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x
))*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.49, size = 66, normalized size = 0.73

$$\frac{2}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,{\left(x^{3}+1\right)}^{\frac{1}{3}}+1\right)\right)-\frac{{\left(x^{3}+1\right)}^{\frac{2}{3}}}{3\,x^{3}}-\frac{1}{9}\,\log\left({\left(x^{3}+1\right)}^{\frac{2}{3}}+{\left(x^{3}+1\right)}^{\frac{1}{3}}+1\right)+\frac{2}{9}\,\log\left({\left(x^{3}+1\right)}^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)/x^4,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) - 1/3*(x^3 + 1)^(2/
3)/x^3 - 1/9*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) + 2/9*log((x^3 + 1)
^(1/3) - 1)
```

mupad [B] time = 0.90, size = 92, normalized size = 1.02

$$\frac{2\,\ln\left(\frac{4\,{\left(x^3+1\right)}^{1/3}}{9}-\frac{4}{9}\right)}{9}+\ln\left(\frac{4\,{\left(x^3+1\right)}^{1/3}}{9}-9\,{\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}^2\right)\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)-\ln\left(\frac{4\,{\left(x^3+1\right)}^{1/3}}{9}-9\,{\left(\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}^2\right)\,\left(\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)-\frac{{\left(x^3+1\right)}^{2/3}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)^(2/3)/x^4,x)
```

```
[Out] (2*log((4*(x^3 + 1)^(1/3))/9 - 4/9))/9 + log((4*(x^3 + 1)^(1/3))/9 - 9*((3^
(1/2)*1i)/9 - 1/9)^2)*((3^(1/2)*1i)/9 - 1/9) - log((4*(x^3 + 1)^(1/3))/9 -
9*((3^(1/2)*1i)/9 + 1/9)^2)*((3^(1/2)*1i)/9 + 1/9) - (x^3 + 1)^(2/3)/(3*x^3
)
```

sympy [C] time = 0.93, size = 31, normalized size = 0.34

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^{3}}\right)}{3x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)/x**4,x)
```

```
[Out] -gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_polar(I*pi)/x**3)/(3*x*gamma(4/3
))
```

3.1093 $$\int \frac{-2k-(-1+k)(1+k)x+2kx^2}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\left(-1+d+(3+d)kx-(d+3k^2)x^2+k(-d+k^2)x^3\right)}\,dx$$

Optimal. Leaf size=90

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{kx-1}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{kx-1}\right)}{d^{3/4}}$$

Rubi [F] time = 5.85, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2k-(-1+k)(1+k)x+2kx^2}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\left(-1+d+(3+d)kx-(d+3k^2)x^2+k(-d+k^2)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*k - (-1 + k)*(1 + k)*x + 2*k*x^2)/(((1 - x^2)*(1 - k^2*x^2))^(1/4)*
(-1 + d + (3 + d)*k*x - (d + 3*k^2)*x^2 + k*(-d + k^2)*x^3)),x]
```

```
[Out] (2*k*(1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*Defer[Int][x^2/((1 - x^2)^(1/4)*(1
 - k^2*x^2)^(1/4)*(-1 + d + (3 + d)*k*x - (d + 3*k^2)*x^2 - k*(d - k^2)*x^3
)), x])/((1 - x^2)*(1 - k^2*x^2))^(1/4) + (2*k*(1 - x^2)^(1/4)*(1 - k^2*x^2
)^(1/4)*Defer[Int][1/((1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*(1 - d - (3 + d)*
k*x + (d + 3*k^2)*x^2 + k*(d - k^2)*x^3)), x])/((1 - x^2)*(1 - k^2*x^2))^(1
/4) - ((1 - k^2)*(1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*Defer[Int][x/((1 - x^2
)^(1/4)*(1 - k^2*x^2)^(1/4)*(1 - d - (3 + d)*k*x + (d + 3*k^2)*x^2 + k*(d -
 k^2)*x^3)), x])/((1 - x^2)*(1 - k^2*x^2))^(1/4)
```

Rubi steps

$$\int \frac{-2k-(-1+k)(1+k)x+2kx^2}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\left(-1+d+(3+d)kx-(d+3k^2)x^2+k(-d+k^2)x^3\right)}\,dx=\frac{\left(\sqrt[4]{1-x^2}\sqrt[4]{1-k^2x^2}\right)\int \cdots}{\cdots}$$
$$=\frac{\left(\sqrt[4]{1-x^2}\sqrt[4]{1-k^2x^2}\right)\int \cdots}{\cdots}$$
$$=\frac{\left(\sqrt[4]{1-x^2}\sqrt[4]{1-k^2x^2}\right)\int \left(\cdots\right)}{\cdots}$$
$$=\frac{\left(2k\sqrt[4]{1-x^2}\sqrt[4]{1-k^2x^2}\right)\int \cdots}{\cdots}$$

Mathematica [F] time = 1.51, size = 0, normalized size = 0.00

$$\int \frac{-2k-(-1+k)(1+k)x+2kx^2}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\left(-1+d+(3+d)kx-(d+3k^2)x^2+k(-d+k^2)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*k - (-1 + k)*(1 + k)*x + 2*k*x^2)/(((1 - x^2)*(1 - k^2*x^2))^
(1/4)*(-1 + d + (3 + d)*k*x - (d + 3*k^2)*x^2 + k*(-d + k^2)*x^3)),x]
```

```
[Out] Integrate[(-2*k - (-1 + k)*(1 + k)*x + 2*k*x^2)/(((1 - x^2)*(1 - k^2*x^2))^
(1/4)*(-1 + d + (3 + d)*k*x - (d + 3*k^2)*x^2 + k*(-d + k^2)*x^3)), x]
```

IntegrateAlgebraic [A] time = 12.97, size = 90, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{kx-1}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{kx-1}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*k - (-1 + k)*(1 + k)*x + 2*k*x^2)/(((1 - x^2)*(1 - k
^2*x^2))^(1/4)*(-1 + d + (3 + d)*k*x - (d + 3*k^2)*x^2 + k*(-d + k^2)*x^3))
,x]
```

```
[Out] ArcTan[(d^(1/4)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4))/(-1 + k*x)]/d^(3/4) -
 ArcTanh[(d^(1/4)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4))/(-1 + k*x)]/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*k-(-1+k)*(1+k)*x+2*k*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d
+(3+d)*k*x-(3*k^2+d)*x^2+k*(k^2-d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(k+1)(k-1)x-2\,kx^2+2\,k}{\left(\left(k^2-d\right)kx^3+(d+3)kx-\left(3\,k^2+d\right)x^2+d-1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*k-(-1+k)*(1+k)*x+2*k*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d
+(3+d)*k*x-(3*k^2+d)*x^2+k*(k^2-d)*x^3),x, algorithm="giac")
```

```
[Out] integrate(-((k + 1)*(k - 1)*x - 2*k*x^2 + 2*k)/(((k^2 - d)*k*x^3 + (d + 3)*
k*x - (3*k^2 + d)*x^2 + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/4)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{-2k-(-1+k)\,(1+k)\,x+2k\,x^2}{\left(\left(-x^2+1\right)\left(-k^2x^2+1\right)\right)^{\frac{1}{4}}\left(-1+d+(3+d)\,kx-\left(3k^2+d\right)x^2+k\left(k^2-d\right)x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*k-(-1+k)*(1+k)*x+2*k*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d+(3+d)
*k*x-(3*k^2+d)*x^2+k*(k^2-d)*x^3),x)
```

```
[Out] int((-2*k-(-1+k)*(1+k)*x+2*k*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d+(3+d)
*k*x-(3*k^2+d)*x^2+k*(k^2-d)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(k+1)(k-1)x-2\,kx^2+2\,k}{\left(\left(k^2-d\right)kx^3+(d+3)kx-\left(3\,k^2+d\right)x^2+d-1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*k-(-1+k)*(1+k)*x+2*k*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d
+(3+d)*k*x-(3*k^2+d)*x^2+k*(k^2-d)*x^3),x, algorithm="maxima")
```

```
[Out] -integrate(((k + 1)*(k - 1)*x - 2*k*x^2 + 2*k)/(((k^2 - d)*k*x^3 + (d + 3)*
k*x - (3*k^2 + d)*x^2 + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/4)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{-2\,k\,x^2+(k-1)\,(k+1)\,x+2\,k}{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)^{1/4}\,\left(k\,\left(d-k^2\right)\,x^3+\left(3\,k^2+d\right)\,x^2-k\,(d+3)\,x-d+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*k - 2*k*x^2 + x*(k - 1)*(k + 1))/(((x^2 - 1)*(k^2*x^2 - 1))^(1/4)*(x
^2*(d + 3*k^2) - d - k*x*(d + 3) + k*x^3*(d - k^2) + 1)),x)
```

```
[Out] int((2*k - 2*k*x^2 + x*(k - 1)*(k + 1))/(((x^2 - 1)*(k^2*x^2 - 1))^(1/4)*(x
^2*(d + 3*k^2) - d - k*x*(d + 3) + k*x^3*(d - k^2) + 1)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*k-(-1+k)*(1+k)*x+2*k*x**2)/((-x**2+1)*(-k**2*x**2+1))**(1/4)/
(-1+d+(3+d)*k*x-(3*k**2+d)*x**2+k*(k**2-d)*x**3),x)
```

```
[Out] Timed out
```

3.1094 $\int \frac{-2k+(-1+k)(1+k)x+2kx^2}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\,(1-d+(3+d)kx+(d+3k^2)x^2+k(-d+k^2)x^3)}\,dx$

Optimal. Leaf size=90

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{kx+1}\right)}{d^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{kx+1}\right)}{d^{3/4}}$$

Rubi [F] time = 5.81, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2k+(-1+k)(1+k)x+2kx^2}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\,\left(1-d+(3+d)kx+\left(d+3k^2\right)x^2+k\left(-d+k^2\right)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*k + (-1 + k)*(1 + k)*x + 2*k*x^2)/(((1 - x^2)*(1 - k^2*x^2))^(1/4)*
(1 - d + (3 + d)*k*x + (d + 3*k^2)*x^2 + k*(-d + k^2)*x^3)),x]
```

```
[Out] -(((1 - k)*(1 + k)*(1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*Defer[Int][x/((1 - x
^2)^(1/4)*(1 - k^2*x^2)^(1/4)*(1 - d + (3 + d)*k*x + (d + 3*k^2)*x^2 - k*(d
 - k^2)*x^3)), x])/((1 - x^2)*(1 - k^2*x^2))^(1/4)) + (2*k*(1 - x^2)^(1/4)*
(1 - k^2*x^2)^(1/4)*Defer[Int][x^2/((1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*(1
- d + (3 + d)*k*x + (d + 3*k^2)*x^2 - k*(d - k^2)*x^3)), x])/((1 - x^2)*(1
- k^2*x^2))^(1/4) + (2*k*(1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*Defer[Int][1/(
(1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*(-1 + d - (3 + d)*k*x - (d + 3*k^2)*x^2
 + k*(d - k^2)*x^3)), x])/((1 - x^2)*(1 - k^2*x^2))^(1/4)
```

Rubi steps

$$\int \frac{-2k+(-1+k)(1+k)x+2kx^2}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\,\left(1-d+(3+d)kx+\left(d+3k^2\right)x^2+k\left(-d+k^2\right)x^3\right)}\,dx = \frac{\left(\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\int \cdots}{\sqrt[4]{(\cdots}}$$

$$= \frac{\left(\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\int \cdots}{\sqrt[4]{(\cdots}}$$

$$= \frac{\left(\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\int \left(\cdots\right.}{\cdots}$$

$$= \frac{\left((-1-k)(1-k)\sqrt[4]{1-x^2}\,\sqrt[4]{1-\cdots}\right.}{\cdots}$$

Mathematica [F] time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{-2k+(-1+k)(1+k)x+2kx^2}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\,\left(1-d+(3+d)kx+\left(d+3k^2\right)x^2+k\left(-d+k^2\right)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*k + (-1 + k)*(1 + k)*x + 2*k*x^2)/(((1 - x^2)*(1 - k^2*x^2))^
(1/4)*(1 - d + (3 + d)*k*x + (d + 3*k^2)*x^2 + k*(-d + k^2)*x^3)),x]
```

```
[Out] Integrate[(-2*k + (-1 + k)*(1 + k)*x + 2*k*x^2)/(((1 - x^2)*(1 - k^2*x^2))^
(1/4)*(1 - d + (3 + d)*k*x + (d + 3*k^2)*x^2 + k*(-d + k^2)*x^3)), x]
```

IntegrateAlgebraic [A] time = 12.97, size = 90, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{kx+1}\right)}{d^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{kx+1}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*k + (-1 + k)*(1 + k)*x + 2*k*x^2)/(((1 - x^2)*(1 - k
^2*x^2))^(1/4)*(1 - d + (3 + d)*k*x + (d + 3*k^2)*x^2 + k*(-d + k^2)*x^3)),
x]
```

```
[Out] -(ArcTan[(d^(1/4)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4))/(1 + k*x)]/d^(3/4))
 + ArcTanh[(d^(1/4)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4))/(1 + k*x)]/d^(3/4
)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*k+(-1+k)*(1+k)*x+2*k*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(1-d+
(3+d)*k*x+(3*k^2+d)*x^2+k*(k^2-d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(k+1)(k-1)x+2\,kx^2-2\,k}{\left(\left(k^2-d\right)kx^3+(d+3)kx+\left(3\,k^2+d\right)x^2-d+1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*k+(-1+k)*(1+k)*x+2*k*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(1-d+
(3+d)*k*x+(3*k^2+d)*x^2+k*(k^2-d)*x^3),x, algorithm="giac")
```

```
[Out] integrate(((k + 1)*(k - 1)*x + 2*k*x^2 - 2*k)/(((k^2 - d)*k*x^3 + (d + 3)*k
*x + (3*k^2 + d)*x^2 - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/4)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{-2k+(-1+k)\,(1+k)\,x+2k\,x^2}{\left(\left(-x^2+1\right)\left(-k^2x^2+1\right)\right)^{\frac{1}{4}}\left(1-d+(3+d)\,kx+\left(3k^2+d\right)x^2+k\left(k^2-d\right)x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*k+(-1+k)*(1+k)*x+2*k*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(1-d+(3+d)*
k*x+(3*k^2+d)*x^2+k*(k^2-d)*x^3),x)
```

```
[Out] int((-2*k+(-1+k)*(1+k)*x+2*k*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(1-d+(3+d)*
k*x+(3*k^2+d)*x^2+k*(k^2-d)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(k+1)(k-1)x+2\,kx^2-2\,k}{\left(\left(k^2-d\right)kx^3+(d+3)kx+\left(3\,k^2+d\right)x^2-d+1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*k+(-1+k)*(1+k)*x+2*k*x^2)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(1-d+
(3+d)*k*x+(3*k^2+d)*x^2+k*(k^2-d)*x^3),x, algorithm="maxima")
```

```
[Out] integrate(((k + 1)*(k - 1)*x + 2*k*x^2 - 2*k)/(((k^2 - d)*k*x^3 + (d + 3)*k
*x + (3*k^2 + d)*x^2 - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,k\,x^2+(k-1)\,(k+1)\,x-2\,k}{{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)}^{1/4}\,\left(-k\,\left(d-k^2\right)\,x^3+\left(3\,k^2+d\right)\,x^2+k\,(d+3)\,x-d+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*k*x^2 - 2*k + x*(k - 1)*(k + 1))/(((x^2 - 1)*(k^2*x^2 - 1))^(1/4)*(x
^2*(d + 3*k^2) - d + k*x*(d + 3) - k*x^3*(d - k^2) + 1)),x)
```

```
[Out] int((2*k*x^2 - 2*k + x*(k - 1)*(k + 1))/(((x^2 - 1)*(k^2*x^2 - 1))^(1/4)*(x
^2*(d + 3*k^2) - d + k*x*(d + 3) - k*x^3*(d - k^2) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*k+(-1+k)*(1+k)*x+2*k*x**2)/((-x**2+1)*(-k**2*x**2+1))**(1/4)/
(1-d+(3+d)*k*x+(3*k**2+d)*x**2+k*(k**2-d)*x**3),x)
```

```
[Out] Timed out
```

3.1095 $\int \frac{(1+x^2)\sqrt{1-2x^4}}{x^5}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{1-2x^4}\left(-2x^2-1\right)}{4x^4}+\frac{i\log\left(\sqrt{1-2x^4}+i\sqrt{2}\,x^2\right)}{\sqrt{2}}+i\tan^{-1}\left(\sqrt{2}\,x^2-i\sqrt{1-2x^4}\right)$$

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 0.66, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1252, 811, 844, 216, 266, 63, 206}

$$\frac{1}{2}\tanh^{-1}\left(\sqrt{1-2x^4}\right)-\frac{\sin^{-1}\left(\sqrt{2}\,x^2\right)}{\sqrt{2}}-\frac{\sqrt{1-2x^4}\left(2x^2+1\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*Sqrt[1 - 2*x^4])/x^5,x]
```

```
[Out] -1/4*((1 + 2*x^2)*Sqrt[1 - 2*x^4])/x^4 - ArcSin[Sqrt[2]*x^2]/Sqrt[2] + ArcT
anh[Sqrt[1 - 2*x^4]]/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 && !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(1+x^2\right)\sqrt{1-2x^4}}{x^5}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(1+x)\sqrt{1-2x^2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{\left(1+2x^2\right)\sqrt{1-2x^4}}{4x^4}-\frac{1}{8}\,\text{Subst}\left(\int \frac{4+8x}{x\sqrt{1-2x^2}}\,dx,x,x^2\right)\\
&= -\frac{\left(1+2x^2\right)\sqrt{1-2x^4}}{4x^4}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-2x^2}}\,dx,x,x^2\right)-\text{Subst}\left(\int \frac{1}{\sqrt{1-2x^2}}\,d\right.\\
&= -\frac{\left(1+2x^2\right)\sqrt{1-2x^4}}{4x^4}-\frac{\sin^{-1}\left(\sqrt{2}\,x^2\right)}{\sqrt{2}}-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-2x}\,x}\,dx,x,x^4\right)\\
&= -\frac{\left(1+2x^2\right)\sqrt{1-2x^4}}{4x^4}-\frac{\sin^{-1}\left(\sqrt{2}\,x^2\right)}{\sqrt{2}}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\frac{1}{2}-\frac{x^2}{2}}\,dx,x,\sqrt{1-2x^4}\right)\\
&= -\frac{\left(1+2x^2\right)\sqrt{1-2x^4}}{4x^4}-\frac{\sin^{-1}\left(\sqrt{2}\,x^2\right)}{\sqrt{2}}+\frac{1}{2}\tanh^{-1}\left(\sqrt{1-2x^4}\right)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.66

$$\frac{1}{4}\left(2\tanh^{-1}\left(\sqrt{1-2x^4}\right)-2\sqrt{2}\,\sin^{-1}\left(\sqrt{2}\,x^2\right)-\frac{\sqrt{1-2x^4}\,\left(2x^2+1\right)}{x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*Sqrt[1 - 2*x^4])/x^5,x]
```

```
[Out] (-(((1 + 2*x^2)*Sqrt[1 - 2*x^4])/x^4) - 2*Sqrt[2]*ArcSin[Sqrt[2]*x^2] + 2*A
rcTanh[Sqrt[1 - 2*x^4]])/4
```

IntegrateAlgebraic [A] time = 0.17, size = 84, normalized size = 0.93

$$-\frac{1}{2}\log\left(\sqrt{1-2x^4}-1\right)+\frac{\log\left(x^2\right)}{2}+\frac{\sqrt{1-2x^4}\,\left(-2x^2-1\right)}{4x^4}-\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,x^2}{\sqrt{1-2x^4}-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^2)*Sqrt[1 - 2*x^4])/x^5,x]
```

```
[Out] ((-1 - 2*x^2)*Sqrt[1 - 2*x^4])/(4*x^4) - Sqrt[2]*ArcTan[(Sqrt[2]*x^2)/(-1 +
 Sqrt[1 - 2*x^4])] + Log[x^2]/2 - Log[-1 + Sqrt[1 - 2*x^4]]/2
```

fricas [A] time = 0.58, size = 78, normalized size = 0.87

$$\frac{4\sqrt{2}\,x^4\arctan\left(\frac{\sqrt{2}\sqrt{-2\,x^4+1}-\sqrt{2}}{2\,x^2}\right)-2\,x^4\log\left(\frac{\sqrt{-2\,x^4+1}-1}{x^2}\right)-\sqrt{-2\,x^4+1}\left(2\,x^2+1\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(-2*x^4+1)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*x^4*arctan(1/2*(sqrt(2)*sqrt(-2*x^4 + 1) - sqrt(2))/x^2) - 2
*x^4*log((sqrt(-2*x^4 + 1) - 1)/x^2) - sqrt(-2*x^4 + 1)*(2*x^2 + 1))/x^4
```

giac [B] time = 0.25, size = 148, normalized size = 1.64

$$\frac{x^4\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-2\,x^4+1}-\sqrt{2}\right)}{x^2}-1\right)}{2\left(\sqrt{2}\sqrt{-2\,x^4+1}-\sqrt{2}\right)^2}-\frac{1}{2}\sqrt{2}\arcsin\left(\sqrt{2}\,x^2\right)-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-2\,x^4+1}-\sqrt{2}\right)}{8\,x^2}+\frac{\left(\sqrt{2}\sqrt{-2\,x^4+1}-\sqrt{2}\right)^2}{32\,x^4}-\frac{1}{2}\log\left(-\frac{\sqrt{2}\sqrt{-2\,x^4+1}-\sqrt{2}}{2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(-2*x^4+1)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*x^4*(sqrt(2)*(sqrt(2)*sqrt(-2*x^4 + 1) - sqrt(2))/x^2 - 1)/(sqrt(2)*sqr
t(-2*x^4 + 1) - sqrt(2))^2 - 1/2*sqrt(2)*arcsin(sqrt(2)*x^2) - 1/8*sqrt(2)*
(sqrt(2)*sqrt(-2*x^4 + 1) - sqrt(2))/x^2 + 1/32*(sqrt(2)*sqrt(-2*x^4 + 1) -
 sqrt(2))^2/x^4 - 1/2*log(-1/2*(sqrt(2)*sqrt(-2*x^4 + 1) - sqrt(2))/x^2)
```

maple [A] time = 0.03, size = 80, normalized size = 0.89

$$-\frac{\left(-2x^4+1\right)^{\frac{3}{2}}}{2x^2}-x^2\sqrt{-2x^4+1}-\frac{\sqrt{2}\ \arcsin\left(\sqrt{2}\ x^2\right)}{2}-\frac{\left(-2x^4+1\right)^{\frac{3}{2}}}{4x^4}-\frac{\sqrt{-2x^4+1}}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-2x^4+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(-2*x^4+1)^(1/2)/x^5,x)
```

```
[Out] -1/2/x^2*(-2*x^4+1)^(3/2)-x^2*(-2*x^4+1)^(1/2)-1/2*2^(1/2)*arcsin(2^(1/2)*x
^2)-1/4/x^4*(-2*x^4+1)^(3/2)-1/2*(-2*x^4+1)^(1/2)+1/2*arctanh(1/(-2*x^4+1)^
(1/2))
```

maxima [A] time = 0.45, size = 80, normalized size = 0.89

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-2\,x^4+1}}{2\,x^2}\right)-\frac{\sqrt{-2\,x^4+1}}{2\,x^2}-\frac{\sqrt{-2\,x^4+1}}{4\,x^4}+\frac{1}{4}\log\left(\sqrt{-2\,x^4+1}+1\right)-\frac{1}{4}\log\left(\sqrt{-2\,x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(-2*x^4+1)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-2*x^4 + 1)/x^2) - 1/2*sqrt(-2*x^4 + 1)
/x^2 - 1/4*sqrt(-2*x^4 + 1)/x^4 + 1/4*log(sqrt(-2*x^4 + 1) + 1) - 1/4*log(s
qrt(-2*x^4 + 1) - 1)
```

mupad [B] time = 1.11, size = 71, normalized size = 0.79

$$-\frac{\ln\left(\sqrt{\frac{1}{2\,x^4}-1}-\sqrt{\frac{1}{2\,x^4}}\right)}{2}-\frac{\sqrt{2}\,\mathrm{asin}\left(\sqrt{2}\,x^2\right)}{2}-\frac{\sqrt{2}\,\sqrt{\frac{1}{2}-x^4}}{2\,x^2}-\frac{\sqrt{2}\,\sqrt{\frac{1}{2}-x^4}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*(1 - 2*x^4)^(1/2))/x^5,x)
```

```
[Out] - log((1/(2*x^4) - 1)^(1/2) - (1/(2*x^4))^(1/2))/2 - (2^(1/2)*asin(2^(1/2)*
x^2))/2 - (2^(1/2)*(1/2 - x^4)^(1/2))/(2*x^2) - (2^(1/2)*(1/2 - x^4)^(1/2))
/(4*x^4)
```

sympy [A] time = 5.12, size = 230, normalized size = 2.56

$$\begin{cases} -\frac{ix^2}{\sqrt{2x^4-1}} + \frac{\sqrt{2}\,i\operatorname{acosh}\left(\sqrt{2}\,x^2\right)}{2} + \frac{i}{2x^2\sqrt{2x^4-1}} & \text{for } 2\left|x^4\right| > 1 \\ \frac{x^2}{\sqrt{1-2x^4}} - \frac{\sqrt{2}\operatorname{asin}\left(\sqrt{2}\,x^2\right)}{2} - \frac{1}{2x^2\sqrt{1-2x^4}} & \text{otherwise} \end{cases} + \begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{2}}{2x^2}\right)}{2} + \frac{\sqrt{2}}{4x^2\sqrt{-1+\frac{1}{2x^4}}} - \frac{\sqrt{2}}{8x^6\sqrt{-1+\frac{1}{2x^4}}} & \text{for } \frac{1}{2|x^4|} > 1 \\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{2}}{2x^2}\right)}{2} - \frac{\sqrt{2}\,i}{4x^2\sqrt{1-\frac{1}{2x^4}}} + \frac{\sqrt{2}\,i}{8x^6\sqrt{1-\frac{1}{2x^4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(-2*x**4+1)**(1/2)/x**5,x)
```

```
[Out] Piecewise((-I*x**2/sqrt(2*x**4 - 1) + sqrt(2)*I*acosh(sqrt(2)*x**2)/2 + I/(
2*x**2*sqrt(2*x**4 - 1)), 2*Abs(x**4) > 1), (x**2/sqrt(1 - 2*x**4) - sqrt(2
)*asin(sqrt(2)*x**2)/2 - 1/(2*x**2*sqrt(1 - 2*x**4)), True)) + Piecewise((a
cosh(sqrt(2)/(2*x**2))/2 + sqrt(2)/(4*x**2*sqrt(-1 + 1/(2*x**4))) - sqrt(2)
/(8*x**6*sqrt(-1 + 1/(2*x**4))), 1/(2*Abs(x**4)) > 1), (-I*asin(sqrt(2)/(2*
x**2))/2 - sqrt(2)*I/(4*x**2*sqrt(1 - 1/(2*x**4))) + sqrt(2)*I/(8*x**6*sqrt
(1 - 1/(2*x**4))), True))
```

3.1096 $\int \frac{(1+x^4)^{2/3}}{x^5}\,dx$

Optimal. Leaf size=90

$$-\frac{(x^4+1)^{2/3}}{4x^4}+\frac{1}{6}\log\left(\sqrt[3]{x^4+1}-1\right)-\frac{1}{12}\log\left(\left(x^4+1\right)^{2/3}+\sqrt[3]{x^4+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 47, 55, 618, 204, 31}

$$-\frac{(x^4+1)^{2/3}}{4x^4}+\frac{1}{4}\log\left(1-\sqrt[3]{x^4+1}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)^(2/3)/x^5,x]
```

```
[Out] -1/4*(1 + x^4)^(2/3)/x^4 + ArcTan[(1 + 2*(1 + x^4)^(1/3))/Sqrt[3]]/(2*Sqrt[
3]) - Log[x]/3 + Log[1 - (1 + x^4)^(1/3)]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(1+x^4\right)^{2/3}}{x^5}\,dx &= \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{(1+x)^{2/3}}{x^2}\,dx, x, x^4\right)\\
&= -\frac{\left(1+x^4\right)^{2/3}}{4x^4}+\frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx, x, x^4\right)\\
&= -\frac{\left(1+x^4\right)^{2/3}}{4x^4}-\frac{\log(x)}{3}-\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+x^4}\right)+\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x\right.\\
&= -\frac{\left(1+x^4\right)^{2/3}}{4x^4}-\frac{\log(x)}{3}+\frac{1}{4}\log\left(1-\sqrt[3]{1+x^4}\right)-\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+x^4}\right)\\
&= -\frac{\left(1+x^4\right)^{2/3}}{4x^4}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^4}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log(x)}{3}+\frac{1}{4}\log\left(1-\sqrt[3]{1+x^4}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.29

$$\frac{3}{20}\left(x^4+1\right)^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)^(2/3)/x^5,x]
```

```
[Out] (3*(1 + x^4)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 + x^4])/20
```

IntegrateAlgebraic [A] time = 0.06, size = 90, normalized size = 1.00

$$-\frac{\left(x^4+1\right)^{2/3}}{4x^4}+\frac{1}{6}\log\left(\sqrt[3]{x^4+1}-1\right)-\frac{1}{12}\log\left(\left(x^4+1\right)^{2/3}+\sqrt[3]{x^4+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)^(2/3)/x^5,x]
```

```
[Out] -1/4*(1 + x^4)^(2/3)/x^4 + ArcTan[1/Sqrt[3] + (2*(1 + x^4)^(1/3))/Sqrt[3]]/
(2*Sqrt[3]) + Log[-1 + (1 + x^4)^(1/3)]/6 - Log[1 + (1 + x^4)^(1/3) + (1 +
x^4)^(2/3)]/12
```

fricas [A] time = 0.50, size = 79, normalized size = 0.88

$$\frac{2\sqrt{3}\,x^4\arctan\left(\frac{2}{3}\sqrt{3}\left(x^4+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-x^4\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)+2\,x^4\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)-3\left(x^4+1\right)^{\frac{2}{3}}}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(2/3)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*sqrt(3)*x^4*arctan(2/3*sqrt(3)*(x^4 + 1)^(1/3) + 1/3*sqrt(3)) - x^4
*log((x^4 + 1)^(2/3) + (x^4 + 1)^(1/3) + 1) + 2*x^4*log((x^4 + 1)^(1/3) - 1
) - 3*(x^4 + 1)^(2/3))/x^4
```

giac [A] time = 0.33, size = 66, normalized size = 0.73

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^4+1\right)^{\frac{2}{3}}}{4x^4}-\frac{1}{12}\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(2/3)/x^5,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 + 1)^(1/3) + 1)) - 1/4*(x^4 + 1)^(2/
3)/x^4 - 1/12*log((x^4 + 1)^(2/3) + (x^4 + 1)^(1/3) + 1) + 1/6*log((x^4 + 1
)^(1/3) - 1)
```

maple [C] time = 0.29, size = 76, normalized size = 0.84

$$-\frac{\left(x^4+1\right)^{\frac{2}{3}}}{4x^4}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^4\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],-x^4\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+4\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{12\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(2/3)/x^5,x)
```

```
[Out] -1/4*(x^4+1)^(2/3)/x^4+1/12/Pi*3^(1/2)*GAMMA(2/3)*(-2/9*Pi*3^(1/2)/GAMMA(2/
3)*x^4*hypergeom([1,1,4/3],[2,2],-x^4)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+4*ln(
x))*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.41, size = 66, normalized size = 0.73

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^4+1\right)^{\frac{2}{3}}}{4x^4}-\frac{1}{12}\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(2/3)/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 + 1)^(1/3) + 1)) - 1/4*(x^4 + 1)^(2/
3)/x^4 - 1/12*log((x^4 + 1)^(2/3) + (x^4 + 1)^(1/3) + 1) + 1/6*log((x^4 + 1
)^(1/3) - 1)
```

mupad [B] time = 0.89, size = 92, normalized size = 1.02

$$\frac{\ln\left(\frac{\left(x^4+1\right)^{1/3}}{4}-\frac{1}{4}\right)}{6}+\ln\left(\frac{\left(x^4+1\right)^{1/3}}{4}-9\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)^2\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(\frac{\left(x^4+1\right)^{1/3}}{4}-9\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)^2\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\frac{\left(x^4+1\right)^{2/3}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)^(2/3)/x^5,x)
```

```
[Out] log((x^4 + 1)^(1/3)/4 - 1/4)/6 + log((x^4 + 1)^(1/3)/4 - 9*((3^(1/2)*1i)/12
 - 1/12)^2)*((3^(1/2)*1i)/12 - 1/12) - log((x^4 + 1)^(1/3)/4 - 9*((3^(1/2)*
1i)/12 + 1/12)^2)*((3^(1/2)*1i)/12 + 1/12) - (x^4 + 1)^(2/3)/(4*x^4)
```

sympy [C] time = 1.01, size = 34, normalized size = 0.38

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^4}\right)}{4x^{\frac{4}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(2/3)/x**5,x)
```

```
[Out] -gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_polar(I*pi)/x**4)/(4*x**(4/3)*ga
mma(4/3))
```

3.1097 $\int \frac{(-3+x^4)(1+x^4)^{2/3}}{x^3(1-x^3+x^4)}\,dx$

Optimal. Leaf size=90

$$\log\left(\sqrt[3]{x^4+1}-x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^4+1}+x}\right)+\frac{3\left(x^4+1\right)^{2/3}}{2x^2}-\frac{1}{2}\log\left(\sqrt[3]{x^4+1}\,x+\left(x^4+1\right)^{2/3}+x^2\right)$$

Rubi [F] time = 1.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}}{x^3\left(1-x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + x^4)*(1 + x^4)^(2/3))/(x^3*(1 - x^3 + x^4)),x]
```

```
[Out] (3*(1 + x^4)^(2/3))/(2*x^2) + (6*x^2)/(1 - Sqrt[3] - (1 + x^4)^(1/3)) - (3*
3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 + x^4)^(1/3))*Sqrt[(1 + (1 + x^4)^(1/3) +
 (1 + x^4)^(2/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2]*EllipticE[ArcSin[(1 +
Sqrt[3] - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))], -7 + 4*Sqrt[3]
])/(x^2*Sqrt[-((1 - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2)]) +
 (2*Sqrt[2]*3^(3/4)*(1 - (1 + x^4)^(1/3))*Sqrt[(1 + (1 + x^4)^(1/3) + (1 +
x^4)^(2/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3
] - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))], -7 + 4*Sqrt[3]])/(x^
2*Sqrt[-((1 - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2)]) - 3*Def
er[Int][(1 + x^4)^(2/3)/(1 - x^3 + x^4), x] + 4*Defer[Int][(x*(1 + x^4)^(2/
3))/(1 - x^3 + x^4), x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}}{x^3\left(1-x^3+x^4\right)}\,dx &= \int \left(-\frac{3\left(1+x^4\right)^{2/3}}{x^3}+\frac{(-3+4x)\left(1+x^4\right)^{2/3}}{1-x^3+x^4}\right)dx\\
&= -\left(3\int \frac{\left(1+x^4\right)^{2/3}}{x^3}\,dx\right)+\int \frac{(-3+4x)\left(1+x^4\right)^{2/3}}{1-x^3+x^4}\,dx\\
&= -\left(\frac{3}{2}\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^{2/3}}{x^2}\,dx,x,x^2\right)\right)+\int \left(-\frac{3\left(1+x^4\right)^{2/3}}{1-x^3+x^4}+\frac{4x\left(1+x^4\right)^{2/3}}{1-x^3+x^4}\right)dx\\
&= \frac{3\left(1+x^4\right)^{2/3}}{2x^2}-2\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^2}}\,dx,x,x^2\right)-3\int \frac{\left(1+x^4\right)^{2/3}}{1-x^3+x^4}\,dx+4\int \frac{x\left(1+x\right.}{1-x^3}\\
&= \frac{3\left(1+x^4\right)^{2/3}}{2x^2}-3\int \frac{\left(1+x^4\right)^{2/3}}{1-x^3+x^4}\,dx+4\int \frac{x\left(1+x^4\right)^{2/3}}{1-x^3+x^4}\,dx-\frac{\left(3\sqrt{x^4}\right)\operatorname{Subst}\left(\int \frac{x}{\sqrt{-1+}}\right.}{x^2}\\
&= \frac{3\left(1+x^4\right)^{2/3}}{2x^2}-3\int \frac{\left(1+x^4\right)^{2/3}}{1-x^3+x^4}\,dx+4\int \frac{x\left(1+x^4\right)^{2/3}}{1-x^3+x^4}\,dx+\frac{\left(3\sqrt{x^4}\right)\operatorname{Subst}\left(\int \frac{1+\sqrt{3}}{\sqrt{-1-}}\right.}{x^2}\\
&= \frac{3\left(1+x^4\right)^{2/3}}{2x^2}+\frac{6x^2}{1-\sqrt{3}-\sqrt[3]{1+x^4}}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1+x^4}\right)\sqrt{\frac{1+\sqrt[3]{1+x^4}+(1+x}{\left(1-\sqrt{3}-\sqrt[3]{1+x\right.}}}}{x^2\sqrt{-\frac{1-}{\left(1-\sqrt{3}-\right.}}}
\end{aligned}
$$

Mathematica [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}}{x^3\left(1-x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + x^4)*(1 + x^4)^(2/3))/(x^3*(1 - x^3 + x^4)),x]
```

```
[Out] Integrate[((-3 + x^4)*(1 + x^4)^(2/3))/(x^3*(1 - x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 1.00, size = 90, normalized size = 1.00

$$\log\left(\sqrt[3]{x^4+1}-x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+1}+x}\right)+\frac{3\left(x^4+1\right)^{2/3}}{2x^2}-\frac{1}{2}\log\left(\sqrt[3]{x^4+1}\,x+\left(x^4+1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 + x^4)^(2/3))/(x^3*(1 - x^3 + x^4)),x]
```

```
[Out] (3*(1 + x^4)^(2/3))/(2*x^2) - Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(1 + x^4)^(
1/3))] + Log[-x + (1 + x^4)^(1/3)] - Log[x^2 + x*(1 + x^4)^(1/3) + (1 + x^4
)^(2/3)]/2
```

fricas [A] time = 3.03, size = 134, normalized size = 1.49

$$-\frac{2\sqrt{3}x^2\arctan\left(-\frac{13034\sqrt{3}\left(x^4+1\right)^{\frac{1}{3}}x^2-686\sqrt{3}\left(x^4+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(37x^4+6137x^3+37\right)}{3\left(x^4+6859x^3+1\right)}\right)-x^2\log\left(\frac{x^4-x^3+3\left(x^4+1\right)^{\frac{1}{3}}x^2-3\left(x^4+1\right)^{\frac{2}{3}}x+1}{x^4-x^3+1}\right)-3\left(x^4+1\right)^{\frac{2}{3}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)/x^3/(x^4-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*x^2*arctan(-1/3*(13034*sqrt(3)*(x^4 + 1)^(1/3)*x^2 - 686*sq
rt(3)*(x^4 + 1)^(2/3)*x + sqrt(3)*(37*x^4 + 6137*x^3 + 37))/(x^4 + 6859*x^3
 + 1)) - x^2*log((x^4 - x^3 + 3*(x^4 + 1)^(1/3)*x^2 - 3*(x^4 + 1)^(2/3)*x +
 1)/(x^4 - x^3 + 1)) - 3*(x^4 + 1)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)^{\frac{2}{3}}\left(x^4-3\right)}{\left(x^4-x^3+1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)/x^3/(x^4-x^3+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)^(2/3)*(x^4 - 3)/((x^4 - x^3 + 1)*x^3), x)
```

maple [C] time = 3.08, size = 272, normalized size = 3.02

$$\frac{3\left(x^4+1\right)^{\frac{2}{3}}}{2x^2}+\ln\left(-\frac{-\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3-\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)x^4+\left(x^4+1\right)^{\frac{2}{3}}\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)x+\left(x^4+1\right)^{\frac{1}{3}}\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)x^2-2\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)x^3-x^4+2\left(x^4+1\right)^{\frac{2}{3}}x-x^2\left(x^4+1\right)^{\frac{1}{3}}-x^3-\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)-1}{x^4-x^3+1}\right)+\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3+\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)x^4+\left(x^4+1\right)^{\frac{2}{3}}\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)x-2\left(x^4+1\right)^{\frac{1}{3}}\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)x^2-\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)x^3+x^4-\left(x^4+1\right)^{\frac{2}{3}}x-x^2\left(x^4+1\right)^{\frac{1}{3}}+\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)+1}{x^4-x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4+1)^(2/3)/x^3/(x^4-x^3+1),x)
```

```
[Out] 3/2*(x^4+1)^(2/3)/x^2+ln(-(-RootOf(_Z^2+_Z+1)^2*x^3-RootOf(_Z^2+_Z+1)*x^4+(
x^4+1)^(2/3)*RootOf(_Z^2+_Z+1)*x+(x^4+1)^(1/3)*RootOf(_Z^2+_Z+1)*x^2-2*Root
Of(_Z^2+_Z+1)*x^3-x^4+2*(x^4+1)^(2/3)*x-x^2*(x^4+1)^(1/3)-x^3-RootOf(_Z^2+_
```

```
Z+1)-1)/(x^4-x^3+1))+RootOf(_Z^2+_Z+1)*ln((-RootOf(_Z^2+_Z+1)^2*x^3+RootOf(
_Z^2+_Z+1)*x^4+(x^4+1)^(2/3)*RootOf(_Z^2+_Z+1)*x-2*(x^4+1)^(1/3)*RootOf(_Z^
2+_Z+1)*x^2-RootOf(_Z^2+_Z+1)*x^3+x^4-(x^4+1)^(2/3)*x-x^2*(x^4+1)^(1/3)+Roo
tOf(_Z^2+_Z+1)+1)/(x^4-x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)^{\frac{2}{3}}\left(x^4-3\right)}{\left(x^4-x^3+1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)/x^3/(x^4-x^3+1),x, algorithm="maxima")
[Out] integrate((x^4 + 1)^(2/3)*(x^4 - 3)/((x^4 - x^3 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4+1\right)^{2/3}\,\left(x^4-3\right)}{x^3\,\left(x^4-x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(2/3)*(x^4 - 3))/(x^3*(x^4 - x^3 + 1)),x)
[Out] int(((x^4 + 1)^(2/3)*(x^4 - 3))/(x^3*(x^4 - x^3 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4+1)**(2/3)/x**3/(x**4-x**3+1),x)
[Out] Timed out
```

3.1098 $\int \frac{\sqrt[3]{-1+x^4}\left(3+x^4\right)}{x^2\left(-1+x^3+x^4\right)}\,dx$

Optimal. Leaf size=90

$$\frac{3\sqrt[3]{x^4-1}}{x}-\log\left(\sqrt[3]{x^4-1}+x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^4-1}-x}\right)+\frac{1}{2}\log\left(-\sqrt[3]{x^4-1}\,x+\left(x^4-1\right)^{2/3}+x^2\right)$$

Rubi [F] time = 0.78, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{-1+x^4}\left(3+x^4\right)}{x^2\left(-1+x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^4)^(1/3)*(3 + x^4))/(x^2*(-1 + x^3 + x^4)),x]
```

```
[Out] (3*(-1 + x^4)^(1/3)*Hypergeometric2F1[-1/3, -1/4, 3/4, x^4])/(x*(1 - x^4)^(
1/3)) + 3*Defer[Int][(x*(-1 + x^4)^(1/3))/(-1 + x^3 + x^4), x] + 4*Defer[In
t][(x^2*(-1 + x^4)^(1/3))/(-1 + x^3 + x^4), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{-1+x^4}\left(3+x^4\right)}{x^2\left(-1+x^3+x^4\right)}\,dx &= \int \left(-\frac{3\sqrt[3]{-1+x^4}}{x^2}+\frac{x(3+4x)\sqrt[3]{-1+x^4}}{-1+x^3+x^4}\right)dx\\
&= -\left(3\int \frac{\sqrt[3]{-1+x^4}}{x^2}\,dx\right)+\int \frac{x(3+4x)\sqrt[3]{-1+x^4}}{-1+x^3+x^4}\,dx\\
&= -\frac{\left(3\sqrt[3]{-1+x^4}\right)\int \frac{\sqrt[3]{1-x^4}}{x^2}\,dx}{\sqrt[3]{1-x^4}}+\int \left(\frac{3x\sqrt[3]{-1+x^4}}{-1+x^3+x^4}+\frac{4x^2\sqrt[3]{-1+x^4}}{-1+x^3+x^4}\right)dx\\
&= \frac{3\sqrt[3]{-1+x^4}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{4};\frac{3}{4};x^4\right)}{x\sqrt[3]{1-x^4}}+3\int \frac{x\sqrt[3]{-1+x^4}}{-1+x^3+x^4}\,dx+4\int \frac{x^2\sqrt[3]{-1+x^4}}{-1+x^3+x^4}\,dx
\end{aligned}$$

Mathematica [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-1+x^4}\left(3+x^4\right)}{x^2\left(-1+x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^4)^(1/3)*(3 + x^4))/(x^2*(-1 + x^3 + x^4)),x]
```

```
[Out] Integrate[((-1 + x^4)^(1/3)*(3 + x^4))/(x^2*(-1 + x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.85, size = 90, normalized size = 1.00

$$\frac{3\sqrt[3]{x^4-1}}{x}-\log\left(\sqrt[3]{x^4-1}+x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^4-1}-x}\right)+\frac{1}{2}\log\left(-\sqrt[3]{x^4-1}\,x+\left(x^4-1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)^(1/3)*(3 + x^4))/(x^2*(-1 + x^3 + x^4)),x]
```

```
[Out] (3*(-1 + x^4)^(1/3))/x + Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(-1 + x^4)^(1/3
))] - Log[x + (-1 + x^4)^(1/3)] + Log[x^2 - x*(-1 + x^4)^(1/3) + (-1 + x^4)
^(2/3)]/2
```

fricas [A] time = 4.35, size = 128, normalized size = 1.42

$$\frac{2\sqrt{3}\,x\arctan\left(-\frac{33798185694614068\sqrt{3}\left(x^4-1\right)^{\frac{1}{3}}x^2-35774000716806898\sqrt{3}\left(x^4-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(18215948833549379\,x^4-16570144372161104\,x^3-18215948833549379\right)}{18912305915671589\,x^4+15948583382382344\,x^3-18912305915671589}\right)-x\log\left(\frac{x^4+x^3+3\left(x^4-1\right)^{\frac{1}{3}}x^2+3\left(x^4-1\right)^{\frac{2}{3}}x-1}{x^4+x^3-1}\right)+6\left(x^4-1\right)^{\frac{1}{3}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/3)*(x^4+3)/x^2/(x^4+x^3-1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*x*arctan(-(33798185694614068*sqrt(3)*(x^4 - 1)^(1/3)*x^2 - 3
5774000716806898*sqrt(3)*(x^4 - 1)^(2/3)*x + sqrt(3)*(18215948833549379*x^4
 - 16570144372161104*x^3 - 18215948833549379))/(18912305915671589*x^4 + 159
48583382382344*x^3 - 18912305915671589)) - x*log((x^4 + x^3 + 3*(x^4 - 1)^(
1/3)*x^2 + 3*(x^4 - 1)^(2/3)*x - 1)/(x^4 + x^3 - 1)) + 6*(x^4 - 1)^(1/3))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\right)\left(x^4-1\right)^{\frac{1}{3}}}{\left(x^4+x^3-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/3)*(x^4+3)/x^2/(x^4+x^3-1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3)*(x^4 - 1)^(1/3)/((x^4 + x^3 - 1)*x^2), x)
```

maple [C] time = 3.63, size = 806, normalized size = 8.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(1/3)*(x^4+3)/x^2/(x^4+x^3-1),x)
```

```
[Out] 3*(x^4-1)^(1/3)/x+(RootOf(_Z^2-_Z+1)*ln(-(RootOf(_Z^2-_Z+1)^2*x^7-RootOf(_Z
^2-_Z+1)*x^8-x^7*RootOf(_Z^2-_Z+1)+x^8+RootOf(_Z^2-_Z+1)*(x^8-2*x^4+1)^(1/3
)*x^5-2*(x^8-2*x^4+1)^(1/3)*x^5-RootOf(_Z^2-_Z+1)^2*x^3-RootOf(_Z^2-_Z+1)*(
x^8-2*x^4+1)^(2/3)*x^2+2*RootOf(_Z^2-_Z+1)*x^4+RootOf(_Z^2-_Z+1)*x^3+2*(x^8
-2*x^4+1)^(2/3)*x^2-2*x^4-RootOf(_Z^2-_Z+1)*(x^8-2*x^4+1)^(1/3)*x+2*(x^8-2*
x^4+1)^(1/3)*x-RootOf(_Z^2-_Z+1)+1)/(x^4+x^3-1)/(-1+x)/(1+x)/(x^2+1))-ln(-(
RootOf(_Z^2-_Z+1)^2*x^7+RootOf(_Z^2-_Z+1)*x^8-x^7*RootOf(_Z^2-_Z+1)-RootOf(
_Z^2-_Z+1)*(x^8-2*x^4+1)^(1/3)*x^5-(x^8-2*x^4+1)^(1/3)*x^5-RootOf(_Z^2-_Z+1
)^2*x^3+RootOf(_Z^2-_Z+1)*(x^8-2*x^4+1)^(2/3)*x^2-2*RootOf(_Z^2-_Z+1)*x^4+R
ootOf(_Z^2-_Z+1)*x^3+(x^8-2*x^4+1)^(2/3)*x^2+RootOf(_Z^2-_Z+1)*(x^8-2*x^4+1
)^(1/3)*x+(x^8-2*x^4+1)^(1/3)*x+RootOf(_Z^2-_Z+1))/(x^4+x^3-1)/(-1+x)/(1+x)
/(x^2+1))*RootOf(_Z^2-_Z+1)+ln(-(RootOf(_Z^2-_Z+1)^2*x^7+RootOf(_Z^2-_Z+1)*
x^8-x^7*RootOf(_Z^2-_Z+1)-RootOf(_Z^2-_Z+1)*(x^8-2*x^4+1)^(1/3)*x^5-(x^8-2*
x^4+1)^(1/3)*x^5-RootOf(_Z^2-_Z+1)^2*x^3+RootOf(_Z^2-_Z+1)*(x^8-2*x^4+1)^(2
/3)*x^2-2*RootOf(_Z^2-_Z+1)*x^4+RootOf(_Z^2-_Z+1)*x^3+(x^8-2*x^4+1)^(2/3)*x
^2+RootOf(_Z^2-_Z+1)*(x^8-2*x^4+1)^(1/3)*x+(x^8-2*x^4+1)^(1/3)*x+RootOf(_Z^
2-_Z+1))/(x^4+x^3-1)/(-1+x)/(1+x)/(x^2+1)))/(x^4-1)^(2/3)*((x^4-1)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\right)\left(x^4-1\right)^{\frac{1}{3}}}{\left(x^4+x^3-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/3)*(x^4+3)/x^2/(x^4+x^3-1),x, algorithm="maxima")
[Out] integrate((x^4 + 3)*(x^4 - 1)^(1/3)/((x^4 + x^3 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-1\right)^{1/3}\left(x^4+3\right)}{x^2\left(x^4+x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)^(1/3)*(x^4 + 3))/(x^2*(x^3 + x^4 - 1)),x)
[Out] int(((x^4 - 1)^(1/3)*(x^4 + 3))/(x^2*(x^3 + x^4 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(1/3)*(x**4+3)/x**2/(x**4+x**3-1),x)
[Out] Timed out
```

3.1099 $\int \frac{(-1+x^4)^{2/3}(3+x^4)}{x^3(-1+x^3+x^4)}\,dx$

Optimal. Leaf size=90

$$\log\left(\sqrt[3]{x^4-1}+x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^4-1}-x}\right)+\frac{3\left(x^4-1\right)^{2/3}}{2x^2}-\frac{1}{2}\log\left(-\sqrt[3]{x^4-1}\,x+\left(x^4-1\right)^{2/3}+x^2\right)$$

Rubi [F] time = 1.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)}{x^3\left(-1+x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^4)^(2/3)*(3 + x^4))/(x^3*(-1 + x^3 + x^4)),x]
```

```
[Out] (3*(-1 + x^4)^(2/3))/(2*x^2) - (6*x^2)/(1 + Sqrt[3] + (-1 + x^4)^(1/3)) + (
3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + (-1 + x^4)^(1/3))*Sqrt[(1 - (-1 + x^4)^(1/
3) + (-1 + x^4)^(2/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))^2]*EllipticE[ArcSin
[(1 - Sqrt[3] + (-1 + x^4)^(1/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))], -7 - 4
*Sqrt[3]])/(x^2*Sqrt[(1 + (-1 + x^4)^(1/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3)
)^2]) - (2*Sqrt[2]*3^(3/4)*(1 + (-1 + x^4)^(1/3))*Sqrt[(1 - (-1 + x^4)^(1/3
) + (-1 + x^4)^(2/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))^2]*EllipticF[ArcSin[
(1 - Sqrt[3] + (-1 + x^4)^(1/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))], -7 - 4*
Sqrt[3]])/(x^2*Sqrt[(1 + (-1 + x^4)^(1/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))
^2]) + 3*Defer[Int][(-1 + x^4)^(2/3)/(-1 + x^3 + x^4), x] + 4*Defer[Int][(x
*(-1 + x^4)^(2/3))/(-1 + x^3 + x^4), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)}{x^3\left(-1+x^3+x^4\right)}\,dx &= \int \left(-\frac{3\left(-1+x^4\right)^{2/3}}{x^3}+\frac{(3+4x)\left(-1+x^4\right)^{2/3}}{-1+x^3+x^4}\right)dx\\
&= -\left(3\int \frac{\left(-1+x^4\right)^{2/3}}{x^3}\,dx\right)+\int \frac{(3+4x)\left(-1+x^4\right)^{2/3}}{-1+x^3+x^4}\,dx\\
&= -\left(\frac{3}{2}\operatorname{Subst}\left(\int \frac{\left(-1+x^2\right)^{2/3}}{x^2}\,dx,x,x^2\right)\right)+\int \left(\frac{3\left(-1+x^4\right)^{2/3}}{-1+x^3+x^4}+\frac{4x\left(-1+x^4\right)^{2/3}}{-1+x^3+x^4}\right)dx\\
&= \frac{3\left(-1+x^4\right)^{2/3}}{2x^2}-2\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x^2}}\,dx,x,x^2\right)+3\int \frac{\left(-1+x^4\right)^{2/3}}{-1+x^3+x^4}\,dx+4\int \frac{x(\ldots}{-1\ldots}\\
&= \frac{3\left(-1+x^4\right)^{2/3}}{2x^2}+3\int \frac{\left(-1+x^4\right)^{2/3}}{-1+x^3+x^4}\,dx+4\int \frac{x\left(-1+x^4\right)^{2/3}}{-1+x^3+x^4}\,dx-\frac{\left(3\sqrt{x^4}\right)\operatorname{Subst}\left(\int\ldots\right.}{\ldots}\\
&= \frac{3\left(-1+x^4\right)^{2/3}}{2x^2}+3\int \frac{\left(-1+x^4\right)^{2/3}}{-1+x^3+x^4}\,dx+4\int \frac{x\left(-1+x^4\right)^{2/3}}{-1+x^3+x^4}\,dx-\frac{\left(3\sqrt{x^4}\right)\operatorname{Subst}\left(\int\ldots\right.}{\ldots}\\
&= \frac{3\left(-1+x^4\right)^{2/3}}{2x^2}-\frac{6x^2}{1+\sqrt{3}+\sqrt[3]{-1+x^4}}+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1+\sqrt[3]{-1+x^4}\right)\sqrt{\frac{1-\sqrt[3]{-1+x\ldots}}{\left(1+\sqrt{3}\ldots\right.}}}{x^2\sqrt{\frac{\ldots}{(1\ldots}}}
\end{aligned}$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^4\right)^{2/3} \left(3+x^4\right)}{x^3 \left(-1+x^3+x^4\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^4)^(2/3)*(3 + x^4))/(x^3*(-1 + x^3 + x^4)),x]
```

```
[Out] Integrate[((-1 + x^4)^(2/3)*(3 + x^4))/(x^3*(-1 + x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 1.00, size = 90, normalized size = 1.00

$$\log \left(\sqrt[3]{x^4-1}+x\right)+\sqrt{3} \tan ^{-1}\left(\frac{\sqrt{3} x}{2 \sqrt[3]{x^4-1}-x}\right)+\frac{3 \left(x^4-1\right)^{2/3}}{2 x^2}-\frac{1}{2} \log \left(-\sqrt[3]{x^4-1} x+\left(x^4-1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)^(2/3)*(3 + x^4))/(x^3*(-1 + x^3 + x^4)),x]
```

```
[Out] (3*(-1 + x^4)^(2/3))/(2*x^2) + Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(-1 + x^4
)^(1/3))] + Log[x + (-1 + x^4)^(1/3)] - Log[x^2 - x*(-1 + x^4)^(1/3) + (-1
+ x^4)^(2/3)]/2
```

fricas [A] time = 5.51, size = 131, normalized size = 1.46

$$\frac{2 \sqrt{3} x^2 \arctan\left(-\frac{33798185694614068 \sqrt{3} \left(x^4-1\right)^{\frac{1}{3}} x^2-35774000716806898 \sqrt{3} \left(x^4-1\right)^{\frac{2}{3}} x+\sqrt{3} \left(18215948833549379 x^4-16570144372161104 x^3-18215948833549379\right)}{18912305915671589 x^4+15948583382382344 x^3-18912305915671589}\right)+x^2 \log\left(\frac{x^4+x^3+3 \left(x^4-1\right)^{\frac{1}{3}} x^2+3 \left(x^4-1\right)^{\frac{2}{3}} x-1}{x^4+x^3-1}\right)+3 \left(x^4-1\right)^{\frac{2}{3}}}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)/x^3/(x^4+x^3-1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*x^2*arctan(-(33798185694614068*sqrt(3)*(x^4 - 1)^(1/3)*x^2 -
 35774000716806898*sqrt(3)*(x^4 - 1)^(2/3)*x + sqrt(3)*(18215948833549379*x
^4 - 16570144372161104*x^3 - 18215948833549379))/(18912305915671589*x^4 + 1
5948583382382344*x^3 - 18912305915671589)) + x^2*log((x^4 + x^3 + 3*(x^4 -
1)^(1/3)*x^2 + 3*(x^4 - 1)^(2/3)*x - 1)/(x^4 + x^3 - 1)) + 3*(x^4 - 1)^(2/3
))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\right) \left(x^4-1\right)^{\frac{2}{3}}}{\left(x^4+x^3-1\right) x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)/x^3/(x^4+x^3-1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3)*(x^4 - 1)^(2/3)/((x^4 + x^3 - 1)*x^3), x)
```

maple [C] time = 2.88, size = 293, normalized size = 3.26

$$\frac{3 \left(x^4-1\right)^{\frac{2}{3}}}{2 x^2}+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right) \ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right) \left(x^4-1\right)^{\frac{2}{3}} x-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right) \left(x^4-1\right)^{\frac{1}{3}} x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right) x^3-x^4+2 \left(x^4-1\right)^{\frac{2}{3}} x-2 \left(x^4-1\right)^{\frac{1}{3}} x^2+x^3+1}{x^4+x^3-1}\right)-\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right) \left(x^4-1\right)^{\frac{2}{3}} x-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right) \left(x^4-1\right)^{\frac{1}{3}} x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right) x^3+x^4-\left(x^4-1\right)^{\frac{2}{3}} x+\left(x^4-1\right)^{\frac{1}{3}} x^2-1}{x^4+x^3-1}\right) \mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right) \left(x^4-1\right)^{\frac{2}{3}} x-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right) \left(x^4-1\right)^{\frac{1}{3}} x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right) x^3+x^4-\left(x^4-1\right)^{\frac{2}{3}} x+\left(x^4-1\right)^{\frac{1}{3}} x^2-1}{x^4+x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(2/3)*(x^4+3)/x^3/(x^4+x^3-1),x)
```

```
[Out] 3/2*(x^4-1)^(2/3)/x^2+RootOf(_Z^2+_Z+1)*ln((RootOf(_Z^2+_Z+1)*(x^4-1)^(2/3)
*x-RootOf(_Z^2+_Z+1)*(x^4-1)^(1/3)*x^2+RootOf(_Z^2+_Z+1)*x^3-x^4+2*(x^4-1)^
```

```
(2/3)*x-2*(x^4-1)^(1/3)*x^2+x^3+1)/(x^4+x^3-1))-ln(-(RootOf(_Z^2+_Z+1)*(x^4
-1)^(2/3)*x-RootOf(_Z^2+_Z+1)*(x^4-1)^(1/3)*x^2+RootOf(_Z^2+_Z+1)*x^3+x^4-(
x^4-1)^(2/3)*x+(x^4-1)^(1/3)*x^2-1)/(x^4+x^3-1))*RootOf(_Z^2+_Z+1)-ln(-(Roo
tOf(_Z^2+_Z+1)*(x^4-1)^(2/3)*x-RootOf(_Z^2+_Z+1)*(x^4-1)^(1/3)*x^2+RootOf(_
Z^2+_Z+1)*x^3+x^4-(x^4-1)^(2/3)*x+(x^4-1)^(1/3)*x^2-1)/(x^4+x^3-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\right)\left(x^4-1\right)^{\frac{2}{3}}}{\left(x^4+x^3-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)/x^3/(x^4+x^3-1),x, algorithm="maxima")
[Out] integrate((x^4 + 3)*(x^4 - 1)^(2/3)/((x^4 + x^3 - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-1\right)^{2/3}\,\left(x^4+3\right)}{x^3\,\left(x^4+x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)^(2/3)*(x^4 + 3))/(x^3*(x^3 + x^4 - 1)),x)
[Out] int(((x^4 - 1)^(2/3)*(x^4 + 3))/(x^3*(x^3 + x^4 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(2/3)*(x**4+3)/x**3/(x**4+x**3-1),x)
[Out] Timed out
```

3.1100 $\int \frac{(-3+x^4)\sqrt[3]{1+x^4}}{x^2(1+x^3+x^4)}\,dx$

Optimal. Leaf size=90

$$\frac{3\sqrt[3]{x^4+1}}{x}-\log\left(\sqrt[3]{x^4+1}+x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^4+1}-x}\right)+\frac{1}{2}\log\left(-\sqrt[3]{x^4+1}\,x+\left(x^4+1\right)^{2/3}+x^2\right)$$

Rubi [F] time = 0.74, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-3+x^4)\sqrt[3]{1+x^4}}{x^2(1+x^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + x^4)*(1 + x^4)^(1/3))/(x^2*(1 + x^3 + x^4)),x]
```

```
[Out] (3*Hypergeometric2F1[-1/3, -1/4, 3/4, -x^4])/x + 3*Defer[Int][(x*(1 + x^4)^
(1/3))/(1 + x^3 + x^4), x] + 4*Defer[Int][(x^2*(1 + x^4)^(1/3))/(1 + x^3 +
x^4), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-3+x^4)\sqrt[3]{1+x^4}}{x^2(1+x^3+x^4)}\,dx &= \int \left(-\frac{3\sqrt[3]{1+x^4}}{x^2}+\frac{x(3+4x)\sqrt[3]{1+x^4}}{1+x^3+x^4}\right)dx\\
&= -\left(3\int \frac{\sqrt[3]{1+x^4}}{x^2}\,dx\right)+\int \frac{x(3+4x)\sqrt[3]{1+x^4}}{1+x^3+x^4}\,dx\\
&= \frac{3\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{4};\frac{3}{4};-x^4\right)}{x}+\int \left(\frac{3x\sqrt[3]{1+x^4}}{1+x^3+x^4}+\frac{4x^2\sqrt[3]{1+x^4}}{1+x^3+x^4}\right)dx\\
&= \frac{3\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{4};\frac{3}{4};-x^4\right)}{x}+3\int \frac{x\sqrt[3]{1+x^4}}{1+x^3+x^4}\,dx+4\int \frac{x^2\sqrt[3]{1+x^4}}{1+x^3+x^4}\,dx
\end{aligned}$$

Mathematica [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(-3+x^4)\sqrt[3]{1+x^4}}{x^2(1+x^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + x^4)*(1 + x^4)^(1/3))/(x^2*(1 + x^3 + x^4)),x]
```

```
[Out] Integrate[((-3 + x^4)*(1 + x^4)^(1/3))/(x^2*(1 + x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.85, size = 90, normalized size = 1.00

$$\frac{3\sqrt[3]{x^4+1}}{x}-\log\left(\sqrt[3]{x^4+1}+x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^4+1}-x}\right)+\frac{1}{2}\log\left(-\sqrt[3]{x^4+1}\,x+\left(x^4+1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 + x^4)^(1/3))/(x^2*(1 + x^3 + x^4)),x]
```

```
[Out] (3*(1 + x^4)^(1/3))/x + Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(1 + x^4)^(1/3))
] - Log[x + (1 + x^4)^(1/3)] + Log[x^2 - x*(1 + x^4)^(1/3) + (1 + x^4)^(2/3
)]/2
```

fricas [A] time = 2.79, size = 122, normalized size = 1.36

$$\frac{2\sqrt{3}\,x\arctan\left(\frac{2\sqrt{3}\left(x^4+1\right)^{\frac{1}{3}}x^2+2\sqrt{3}\left(x^4+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(x^4+x^3+1\right)}{3\left(x^4-x^3+1\right)}\right)-x\log\left(\frac{x^4+x^3+3\left(x^4+1\right)^{\frac{1}{3}}x^2+3\left(x^4+1\right)^{\frac{2}{3}}x+1}{x^4+x^3+1}\right)+6\left(x^4+1\right)^{\frac{1}{3}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(1/3)/x^2/(x^4+x^3+1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*x*arctan(1/3*(2*sqrt(3)*(x^4 + 1)^(1/3)*x^2 + 2*sqrt(3)*(x^4
 + 1)^(2/3)*x + sqrt(3)*(x^4 + x^3 + 1))/(x^4 - x^3 + 1)) - x*log((x^4 + x^
3 + 3*(x^4 + 1)^(1/3)*x^2 + 3*(x^4 + 1)^(2/3)*x + 1)/(x^4 + x^3 + 1)) + 6*(
x^4 + 1)^(1/3))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)^{\frac{1}{3}}\left(x^4-3\right)}{\left(x^4+x^3+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(1/3)/x^2/(x^4+x^3+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)^(1/3)*(x^4 - 3)/((x^4 + x^3 + 1)*x^2), x)
```

maple [C] time = 3.50, size = 477, normalized size = 5.30

$$\frac{3\left(x^4+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-\ln\left(-\frac{x^7\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)-x^8+\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^8+2x^4+1\right)^{\frac{1}{3}}x^5-2\left(x^8+2x^4+1\right)^{\frac{1}{3}}x^5+2\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^8+2x^4+1\right)^{\frac{2}{3}}x^2+\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)x^3-\left(x^8+2x^4+1\right)^{\frac{2}{3}}x^2-2x^4+\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^8+2x^4+1\right)^{\frac{1}{3}}x-2\left(x^8+2x^4+1\right)^{\frac{1}{3}}x-1}{\left(x^4+x^3+1\right)\left(x^4+1\right)}\right)+\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)^2x^7+\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)x^8-2x^7\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)-x^8+\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^8+2x^4+1\right)^{\frac{1}{3}}x^5+x^7+\left(x^8+2x^4+1\right)^{\frac{1}{3}}x^5+\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)^2x^3-\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^8+2x^4+1\right)^{\frac{2}{3}}x^2+2\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)x^4-2\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)x^3+2\left(x^8+2x^4+1\right)^{\frac{2}{3}}x^2-2x^4+\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^8+2x^4+1\right)^{\frac{1}{3}}x+x^3+\left(x^8+2x^4+1\right)^{\frac{1}{3}}x+\operatorname{RootOf}\left(\_Z^2-\_Z+1\right)-1}{\left(x^4+x^3+1\right)\left(x^4+1\right)}\right)\right)\left(\left(x^4+1\right)^2\right)^{\frac{1}{3}}}{\left(x^4+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4+1)^(1/3)/x^2/(x^4+x^3+1),x)
```

```
[Out] 3*(x^4+1)^(1/3)/x+(-ln(-(x^7*RootOf(_Z^2-_Z+1)-x^8+RootOf(_Z^2-_Z+1)*(x^8+2
*x^4+1)^(1/3)*x^5-2*(x^8+2*x^4+1)^(1/3)*x^5+2*RootOf(_Z^2-_Z+1)*(x^8+2*x^4+
1)^(2/3)*x^2+RootOf(_Z^2-_Z+1)*x^3-(x^8+2*x^4+1)^(2/3)*x^2-2*x^4+RootOf(_Z^
2-_Z+1)*(x^8+2*x^4+1)^(1/3)*x-2*(x^8+2*x^4+1)^(1/3)*x-1)/(x^4+x^3+1)/(x^4+1
))+RootOf(_Z^2-_Z+1)*ln((RootOf(_Z^2-_Z+1)^2*x^7+RootOf(_Z^2-_Z+1)*x^8-2*x^
7*RootOf(_Z^2-_Z+1)-x^8+RootOf(_Z^2-_Z+1)*(x^8+2*x^4+1)^(1/3)*x^5+x^7+(x^8+
2*x^4+1)^(1/3)*x^5+RootOf(_Z^2-_Z+1)^2*x^3-RootOf(_Z^2-_Z+1)*(x^8+2*x^4+1)^
(2/3)*x^2+2*RootOf(_Z^2-_Z+1)*x^4-2*RootOf(_Z^2-_Z+1)*x^3+2*(x^8+2*x^4+1)^(
2/3)*x^2-2*x^4+RootOf(_Z^2-_Z+1)*(x^8+2*x^4+1)^(1/3)*x+x^3+(x^8+2*x^4+1)^(1
/3)*x+RootOf(_Z^2-_Z+1)-1)/(x^4+x^3+1)/(x^4+1)))/(x^4+1)^(2/3)*((x^4+1)^2)^
(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)^{\frac{1}{3}}\left(x^4-3\right)}{\left(x^4+x^3+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(1/3)/x^2/(x^4+x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)^(1/3)*(x^4 - 3)/((x^4 + x^3 + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4+1\right)^{1/3}\,\left(x^4-3\right)}{x^2\,\left(x^4+x^3+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/3)*(x^4 - 3))/(x^2*(x^3 + x^4 + 1)),x)
[Out] int(((x^4 + 1)^(1/3)*(x^4 - 3))/(x^2*(x^3 + x^4 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4+1)**(1/3)/x**2/(x**4+x**3+1),x)
[Out] Timed out
```

3.1101 $\int \frac{(4+3x)\left(-1-x+x^4\right)\sqrt[4]{-1-x+2x^4}}{x^6\left(1+x+x^4\right)}\,dx$

Optimal. Leaf size=90

$$-4\sqrt[4]{3}\tan^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{2x^4-x-1}}\right)+4\sqrt[4]{3}\tanh^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{2x^4-x-1}}\right)-\frac{4\sqrt[4]{2x^4-x-1}\left(12x^4-x-1\right)}{5x^5}$$

Rubi [F] time = 1.71, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(4+3x)\left(-1-x+x^4\right)\sqrt[4]{-1-x+2x^4}}{x^6\left(1+x+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((4 + 3*x)*(-1 - x + x^4)*(-1 - x + 2*x^4)^(1/4))/(x^6*(1 + x + x^4)),x
]
```

```
[Out] -4*Defer[Int][(-1 - x + 2*x^4)^(1/4)/x^6, x] - 3*Defer[Int][(-1 - x + 2*x^4
)^(1/4)/x^5, x] + 8*Defer[Int][(-1 - x + 2*x^4)^(1/4)/x^2, x] - 2*Defer[Int
][(-1 - x + 2*x^4)^(1/4)/x, x] + 2*Defer[Int][(-1 - x + 2*x^4)^(1/4)/(1 + x
 + x^4), x] - 8*Defer[Int][(x^2*(-1 - x + 2*x^4)^(1/4))/(1 + x + x^4), x] +
 2*Defer[Int][(x^3*(-1 - x + 2*x^4)^(1/4))/(1 + x + x^4), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(4+3x)\left(-1-x+x^4\right)\sqrt[4]{-1-x+2x^4}}{x^6\left(1+x+x^4\right)}\,dx &= \int \left(-\frac{4\sqrt[4]{-1-x+2x^4}}{x^6}-\frac{3\sqrt[4]{-1-x+2x^4}}{x^5}+\frac{8\sqrt[4]{-1-x+2x^4}}{x^2}-\frac{2\right.\\
&= -\left(2\int \frac{\sqrt[4]{-1-x+2x^4}}{x}\,dx\right)+2\int \frac{\left(1-4x^2+x^3\right)\sqrt[4]{-1-x+2x^4}}{1+x+x^4}\,d\\
&= -\left(2\int \frac{\sqrt[4]{-1-x+2x^4}}{x}\,dx\right)+2\int \left(\frac{\sqrt[4]{-1-x+2x^4}}{1+x+x^4}-\frac{4x^2\sqrt[4]{-1-x}}{1+x+}\right.\\
&= -\left(2\int \frac{\sqrt[4]{-1-x+2x^4}}{x}\,dx\right)+2\int \frac{\sqrt[4]{-1-x+2x^4}}{1+x+x^4}\,dx+2\int \frac{x^3\sqrt[4]{-}}{1}
\end{aligned}$$

Mathematica [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{(4+3x)\left(-1-x+x^4\right)\sqrt[4]{-1-x+2x^4}}{x^6\left(1+x+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((4 + 3*x)*(-1 - x + x^4)*(-1 - x + 2*x^4)^(1/4))/(x^6*(1 + x + x
^4)),x]
```

```
[Out] Integrate[((4 + 3*x)*(-1 - x + x^4)*(-1 - x + 2*x^4)^(1/4))/(x^6*(1 + x + x
^4)), x]
```

IntegrateAlgebraic [A] time = 2.42, size = 90, normalized size = 1.00

$$-4\sqrt[4]{3}\tan^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{2x^4-x-1}}\right)+4\sqrt[4]{3}\tanh^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{2x^4-x-1}}\right)-\frac{4\sqrt[4]{2x^4-x-1}\left(12x^4-x-1\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((4 + 3*x)*(-1 - x + x^4)*(-1 - x + 2*x^4)^(1/4))/(x^6*(
1 + x + x^4)),x]
```

```
[Out] (-4*(-1 - x + 2*x^4)^(1/4)*(-1 - x + 12*x^4))/(5*x^5) - 4*3^(1/4)*ArcTan[(3
^(1/4)*x)/(-1 - x + 2*x^4)^(1/4)] + 4*3^(1/4)*ArcTanh[(3^(1/4)*x)/(-1 - x +
 2*x^4)^(1/4)]
```

fricas [B] time = 13.54, size = 304, normalized size = 3.38

$$\frac{20 \cdot 3^{\frac{1}{4}} x^{5} \arctan\left(\frac{6 \cdot 3^{\frac{3}{4}} \left(2 x^{4}-x-1\right)^{\frac{1}{4}} x^{3}+6 \cdot 3^{\frac{1}{4}} \left(2 x^{4}-x-1\right)^{\frac{3}{4}} x+3^{\frac{3}{4}} \left(2 \cdot 3^{\frac{3}{4}} \sqrt{2 x^{4}-x-1}\, x^{2}+3^{\frac{1}{4}} \left(5 x^{4}-x-1\right)\right)}{3 \left(x^{4}+x+1\right)}\right)+5 \cdot 3^{\frac{1}{4}} x^{5} \log\left(\frac{6 \sqrt{3} \left(2 x^{4}-x-1\right)^{\frac{1}{4}} x^{3}+6 \cdot 3^{\frac{1}{4}} \sqrt{2 x^{4}-x-1}\, x^{2}+3^{\frac{3}{4}} \left(5 x^{4}-x-1\right)+6 \left(2 x^{4}-x-1\right)^{\frac{3}{4}} x}{x^{4}+x+1}\right)-5 \cdot 3^{\frac{1}{4}} x^{5} \log\left(\frac{6 \sqrt{3} \left(2 x^{4}-x-1\right)^{\frac{1}{4}} x^{3}-6 \cdot 3^{\frac{1}{4}} \sqrt{2 x^{4}-x-1}\, x^{2}-3^{\frac{3}{4}} \left(5 x^{4}-x-1\right)+6 \left(2 x^{4}-x-1\right)^{\frac{3}{4}} x}{x^{4}+x+1}\right)-4 \left(12 x^{4}-x-1\right) \left(2 x^{4}-x-1\right)^{\frac{1}{4}}}{5 x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+3*x)*(x^4-x-1)*(2*x^4-x-1)^(1/4)/x^6/(x^4+x+1),x, algorithm="f
ricas")
```

```
[Out] 1/5*(20*3^(1/4)*x^5*arctan(1/3*(6*3^(3/4)*(2*x^4 - x - 1)^(1/4)*x^3 + 6*3^(
1/4)*(2*x^4 - x - 1)^(3/4)*x + 3^(3/4)*(2*3^(3/4)*sqrt(2*x^4 - x - 1)*x^2 +
 3^(1/4)*(5*x^4 - x - 1)))/(x^4 + x + 1)) + 5*3^(1/4)*x^5*log((6*sqrt(3)*(2
*x^4 - x - 1)^(1/4)*x^3 + 6*3^(1/4)*sqrt(2*x^4 - x - 1)*x^2 + 3^(3/4)*(5*x^
4 - x - 1) + 6*(2*x^4 - x - 1)^(3/4)*x)/(x^4 + x + 1)) - 5*3^(1/4)*x^5*log(
(6*sqrt(3)*(2*x^4 - x - 1)^(1/4)*x^3 - 6*3^(1/4)*sqrt(2*x^4 - x - 1)*x^2 -
3^(3/4)*(5*x^4 - x - 1) + 6*(2*x^4 - x - 1)^(3/4)*x)/(x^4 + x + 1)) - 4*(12
*x^4 - x - 1)*(2*x^4 - x - 1)^(1/4))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 x^{4}-x-1\right)^{\frac{1}{4}} \left(x^{4}-x-1\right) (3 x+4)}{\left(x^{4}+x+1\right) x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+3*x)*(x^4-x-1)*(2*x^4-x-1)^(1/4)/x^6/(x^4+x+1),x, algorithm="g
iac")
```

```
[Out] integrate((2*x^4 - x - 1)^(1/4)*(x^4 - x - 1)*(3*x + 4)/((x^4 + x + 1)*x^6)
, x)
```

maple [C] time = 3.14, size = 1633, normalized size = 18.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4+3*x)*(x^4-x-1)*(2*x^4-x-1)^(1/4)/x^6/(x^4+x+1),x)
```

```
[Out] -4/5*(24*x^8-14*x^5-14*x^4+x^2+2*x+1)/x^5/(2*x^4-x-1)^(3/4)+(-2*RootOf(_Z^4
-3)*ln((-20*RootOf(_Z^4-3)^2*x^12+8*(8*x^12-12*x^9-12*x^8+6*x^6+12*x^5+6*x^
4-x^3-3*x^2-3*x-1)^(1/4)*RootOf(_Z^4-3)^3*x^9+24*RootOf(_Z^4-3)^2*x^9-8*(8*
x^12-12*x^9-12*x^8+6*x^6+12*x^5+6*x^4-x^3-3*x^2-3*x-1)^(1/4)*RootOf(_Z^4-3)
^3*x^6+24*x^8*RootOf(_Z^4-3)^2-8*(8*x^12-12*x^9-12*x^8+6*x^6+12*x^5+6*x^4-x
^3-3*x^2-3*x-1)^(1/4)*RootOf(_Z^4-3)^3*x^5-9*RootOf(_Z^4-3)^2*x^6-12*(8*x^1
2-12*x^9-12*x^8+6*x^6+12*x^5+6*x^4-x^3-3*x^2-3*x-1)^(1/2)*x^6+2*(8*x^12-12*
x^9-12*x^8+6*x^6+12*x^5+6*x^4-x^3-3*x^2-3*x-1)^(1/4)*RootOf(_Z^4-3)^3*x^3-1
8*RootOf(_Z^4-3)^2*x^5+6*(8*x^12-12*x^9-12*x^8+6*x^6+12*x^5+6*x^4-x^3-3*x^2
-3*x-1)^(3/4)*RootOf(_Z^4-3)*x^3+4*(8*x^12-12*x^9-12*x^8+6*x^6+12*x^5+6*x^4
-x^3-3*x^2-3*x-1)^(1/4)*RootOf(_Z^4-3)^3*x^2-9*RootOf(_Z^4-3)^2*x^4+2*(8*x^
12-12*x^9-12*x^8+6*x^6+12*x^5+6*x^4-x^3-3*x^2-3*x-1)^(1/4)*RootOf(_Z^4-3)^3
*x+x^3*RootOf(_Z^4-3)^2+6*(8*x^12-12*x^9-12*x^8+6*x^6+12*x^5+6*x^4-x^3-3*x^
2-3*x-1)^(1/2)*x^3+3*x^2*RootOf(_Z^4-3)^2+6*(8*x^12-12*x^9-12*x^8+6*x^6+12*
```

```
x^5+6*x^4-x^3-3*x^2-3*x-1)^(1/2)*x^2+3*RootOf(_Z^4-3)^2*x+RootOf(_Z^4-3)^2)
/(2*x^3+2*x^2+2*x+1)^2/(x^4+x+1)/(-1+x)^2)-2*RootOf(_Z^2+RootOf(_Z^4-3)^2)*
ln(-(-20*RootOf(_Z^4-3)^2*x^12+8*(8*x^12-12*x^9-12*x^8+6*x^6+12*x^5+6*x^4-x
^3-3*x^2-3*x-1)^(1/4)*RootOf(_Z^2+RootOf(_Z^4-3)^2)*RootOf(_Z^4-3)^2*x^9+24
*RootOf(_Z^4-3)^2*x^9-8*(8*x^12-12*x^9-12*x^8+6*x^6+12*x^5+6*x^4-x^3-3*x^2-
3*x-1)^(1/4)*RootOf(_Z^2+RootOf(_Z^4-3)^2)*RootOf(_Z^4-3)^2*x^6+24*x^8*Root
Of(_Z^4-3)^2-8*(8*x^12-12*x^9-12*x^8+6*x^6+12*x^5+6*x^4-x^3-3*x^2-3*x-1)^(1
/4)*RootOf(_Z^2+RootOf(_Z^4-3)^2)*RootOf(_Z^4-3)^2*x^5-9*RootOf(_Z^4-3)^2*x
^6+12*(8*x^12-12*x^9-12*x^8+6*x^6+12*x^5+6*x^4-x^3-3*x^2-3*x-1)^(1/2)*x^6+2
*(8*x^12-12*x^9-12*x^8+6*x^6+12*x^5+6*x^4-x^3-3*x^2-3*x-1)^(1/4)*RootOf(_Z^
2+RootOf(_Z^4-3)^2)*RootOf(_Z^4-3)^2*x^3-6*(8*x^12-12*x^9-12*x^8+6*x^6+12*x
^5+6*x^4-x^3-3*x^2-3*x-1)^(3/4)*RootOf(_Z^2+RootOf(_Z^4-3)^2)*x^3-18*RootOf
(_Z^4-3)^2*x^5+4*(8*x^12-12*x^9-12*x^8+6*x^6+12*x^5+6*x^4-x^3-3*x^2-3*x-1)^
(1/4)*RootOf(_Z^2+RootOf(_Z^4-3)^2)*RootOf(_Z^4-3)^2*x^2-9*RootOf(_Z^4-3)^2
*x^4+2*(8*x^12-12*x^9-12*x^8+6*x^6+12*x^5+6*x^4-x^3-3*x^2-3*x-1)^(1/4)*Root
Of(_Z^2+RootOf(_Z^4-3)^2)*RootOf(_Z^4-3)^2*x+x^3*RootOf(_Z^4-3)^2-6*(8*x^12
-12*x^9-12*x^8+6*x^6+12*x^5+6*x^4-x^3-3*x^2-3*x-1)^(1/2)*x^3+3*x^2*RootOf(_
Z^4-3)^2-6*(8*x^12-12*x^9-12*x^8+6*x^6+12*x^5+6*x^4-x^3-3*x^2-3*x-1)^(1/2)*
x^2+3*RootOf(_Z^4-3)^2*x+RootOf(_Z^4-3)^2)/(2*x^3+2*x^2+2*x+1)^2/(x^4+x+1)/
(-1+x)^2))/(2*x^4-x-1)^(3/4)*((2*x^4-x-1)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^4-x-1\right)^{\frac{1}{4}}\left(x^4-x-1\right)(3\,x+4)}{\left(x^4+x+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+3*x)*(x^4-x-1)*(2*x^4-x-1)^(1/4)/x^6/(x^4+x+1),x, algorithm="m
axima")
```

```
[Out] integrate((2*x^4 - x - 1)^(1/4)*(x^4 - x - 1)*(3*x + 4)/((x^4 + x + 1)*x^6)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(3\,x+4)\,\left(-x^4+x+1\right)\,{\left(2\,x^4-x-1\right)}^{1/4}}{x^6\,\left(x^4+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((3*x + 4)*(x - x^4 + 1)*(2*x^4 - x - 1)^(1/4))/(x^6*(x + x^4 + 1)),x)
```

```
[Out] int(-((3*x + 4)*(x - x^4 + 1)*(2*x^4 - x - 1)^(1/4))/(x^6*(x + x^4 + 1)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+3*x)*(x**4-x-1)*(2*x**4-x-1)**(1/4)/x**6/(x**4+x+1),x)
```

```
[Out] Timed out
```

3.1102 $\int \frac{\sqrt{1+3x^4}}{-1+3x^4}\,dx$

Optimal. Leaf size=90

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}x}{\sqrt{3x^4+1}}\right)}{2\sqrt{2}\sqrt[4]{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3x^4+1}}{\sqrt{3}x^2-\sqrt{2}\sqrt[4]{3}x+1}\right)}{\sqrt{2}\sqrt[4]{3}}$$

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {404, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}x}{\sqrt{3x^4+1}}\right)}{2\sqrt{2}\sqrt[4]{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}x}{\sqrt{3x^4+1}}\right)}{2\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 3*x^4]/(-1 + 3*x^4),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2]*3^(1/4)*x)/Sqrt[1 + 3*x^4]]/(Sqrt[2]*3^(1/4)) - ArcTan
h[(Sqrt[2]*3^(1/4)*x)/Sqrt[1 + 3*x^4]]/(2*Sqrt[2]*3^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 404

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[a/c,
 Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+3x^4}}{-1+3x^4}\,dx &= -\text{Subst}\left(\int \frac{1}{1-12x^4}\,dx, x, \frac{x}{\sqrt{1+3x^4}}\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2\sqrt{3}x^2}\,dx, x, \frac{x}{\sqrt{1+3x^4}}\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2\sqrt{3}x^2}\,dx, x, \frac{x}{\sqrt{1+3x^4}}\right)\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}x}{\sqrt{1+3x^4}}\right)}{2\sqrt{2}\sqrt[4]{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}x}{\sqrt{1+3x^4}}\right)}{2\sqrt{2}\sqrt[4]{3}}\end{aligned}$$

Mathematica [C] time = 0.10, size = 120, normalized size = 1.33

$$\frac{5x\sqrt{3x^4+1}\,F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-3x^4,3x^4\right)}{\left(3x^4-1\right)\left(6x^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-3x^4,3x^4\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-3x^4,3x^4\right)\right)+5F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-3x^4,3x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 + 3*x^4]/(-1 + 3*x^4),x]
```

```
[Out] (5*x*Sqrt[1 + 3*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -3*x^4, 3*x^4])/((-1 + 3*x
^4)*(5*AppellF1[1/4, -1/2, 1, 5/4, -3*x^4, 3*x^4] + 6*x^4*(2*AppellF1[5/4,
-1/2, 2, 9/4, -3*x^4, 3*x^4] + AppellF1[5/4, 1/2, 1, 9/4, -3*x^4, 3*x^4])))
```

IntegrateAlgebraic [A] time = 0.41, size = 77, normalized size = 0.86

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}x}{\sqrt{3x^4+1}}\right)}{2\sqrt{2}\sqrt[4]{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}x}{\sqrt{3x^4+1}}\right)}{2\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + 3*x^4]/(-1 + 3*x^4),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2]*3^(1/4)*x)/Sqrt[1 + 3*x^4]]/(Sqrt[2]*3^(1/4)) - ArcTan
h[(Sqrt[2]*3^(1/4)*x)/Sqrt[1 + 3*x^4]]/(2*Sqrt[2]*3^(1/4))
```

fricas [B] time = 0.48, size = 146, normalized size = 1.62

$$\frac{1}{12}\cdot 12^{\frac{3}{4}}\arctan\left(-\frac{12^{\frac{3}{4}}\sqrt{3}x^2-12^{\frac{3}{4}}\sqrt{3x^4+1}+2\cdot 12^{\frac{1}{4}}\sqrt{3}}{12x}\right)-\frac{1}{48}\cdot 12^{\frac{3}{4}}\log\left(\frac{6\cdot 12^{\frac{1}{4}}x^3+12^{\frac{3}{4}}x+2\sqrt{3x^4+1}\left(3x^2+\sqrt{3}\right)}{3x^4-1}\right)+\frac{1}{48}\cdot 12^{\frac{3}{4}}\log\left(-\frac{6\cdot 12^{\frac{1}{4}}x^3+12^{\frac{3}{4}}x-2\sqrt{3x^4+1}\left(3x^2+\sqrt{3}\right)}{3x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+1)^(1/2)/(3*x^4-1),x, algorithm="fricas")
```

```
[Out] 1/12*12^(3/4)*arctan(-1/12*(12^(3/4)*sqrt(3)*x^2 - 12^(3/4)*sqrt(3*x^4 + 1)
 + 2*12^(1/4)*sqrt(3))/x) - 1/48*12^(3/4)*log((6*12^(1/4)*x^3 + 12^(3/4)*x
+ 2*sqrt(3*x^4 + 1)*(3*x^2 + sqrt(3)))/(3*x^4 - 1)) + 1/48*12^(3/4)*log(-(6
*12^(1/4)*x^3 + 12^(3/4)*x - 2*sqrt(3*x^4 + 1)*(3*x^2 + sqrt(3)))/(3*x^4 -
1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3x^4+1}}{3x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+1)^(1/2)/(3*x^4-1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*x^4 + 1)/(3*x^4 - 1), x)
```

maple [A] time = 0.02, size = 87, normalized size = 0.97

$$\frac{\sqrt{2}\,3^{\frac{3}{4}}\arctan\left(\frac{\sqrt{3x^4+1}\,\sqrt{2}\,3^{\frac{3}{4}}}{6x}\right)}{12}-\frac{\sqrt{2}\,3^{\frac{3}{4}}\ln\left(\frac{\frac{\sqrt{3x^4+1}\,\sqrt{2}}{2x}+3^{\frac{1}{4}}}{\frac{\sqrt{3x^4+1}\,\sqrt{2}}{2x}-3^{\frac{1}{4}}}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4+1)^(1/2)/(3*x^4-1),x)
```

```
[Out] 1/12*2^(1/2)*3^(3/4)*arctan(1/6*(3*x^4+1)^(1/2)*2^(1/2)/x*3^(3/4))-1/24*2^(
1/2)*3^(3/4)*ln((1/2*(3*x^4+1)^(1/2)*2^(1/2)/x+3^(1/4))/(1/2*(3*x^4+1)^(1/2
)*2^(1/2)/x-3^(1/4)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3\,x^4+1}}{3\,x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+1)^(1/2)/(3*x^4-1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x^4 + 1)/(3*x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3\,x^4+1}}{3\,x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4 + 1)^(1/2)/(3*x^4 - 1),x)
```

```
[Out] int((3*x^4 + 1)^(1/2)/(3*x^4 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3x^4+1}}{3x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4+1)**(1/2)/(3*x**4-1),x)
```

```
[Out] Integral(sqrt(3*x**4 + 1)/(3*x**4 - 1), x)
```

3.1103 $\int \frac{(b+ax^2)\sqrt[4]{-bx^2+ax^4}}{-b+ax^2}\,dx$

Optimal. Leaf size=90

$$-\frac{7b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{4a^{3/4}}+\frac{7b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{4a^{3/4}}+\frac{1}{2}x\sqrt[4]{ax^4-bx^2}$$

Rubi [A] time = 0.24, antiderivative size = 160, normalized size of antiderivative = 1.78, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2056, 459, 329, 331, 298, 203, 206}

$$-\frac{7b\sqrt[4]{ax^4-bx^2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2-b}}\right)}{4a^{3/4}\sqrt{x}\sqrt[4]{ax^2-b}}+\frac{7b\sqrt[4]{ax^4-bx^2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2-b}}\right)}{4a^{3/4}\sqrt{x}\sqrt[4]{ax^2-b}}+\frac{1}{2}x\sqrt[4]{ax^4-bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4))/(-b + a*x^2),x]
```

```
[Out] (x*(-(b*x^2) + a*x^4)^(1/4))/2 - (7*b*(-(b*x^2) + a*x^4)^(1/4)*ArcTan[(a^(1
/4)*Sqrt[x])/(-b + a*x^2)^(1/4)])/(4*a^(3/4)*Sqrt[x]*(-b + a*x^2)^(1/4)) +
(7*b*(-(b*x^2) + a*x^4)^(1/4)*ArcTanh[(a^(1/4)*Sqrt[x])/(-b + a*x^2)^(1/4)]
)/(4*a^(3/4)*Sqrt[x]*(-b + a*x^2)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\left(b+ax^2\right)\sqrt[4]{-bx^2+ax^4}}{-b+ax^2}\,dx = \frac{\sqrt[4]{-bx^2+ax^4}\int \frac{\sqrt{x}\left(b+ax^2\right)}{\left(-b+ax^2\right)^{3/4}}\,dx}{\sqrt{x}\sqrt[4]{-b+ax^2}}$$

$$= \frac{1}{2}x\sqrt[4]{-bx^2+ax^4} + \frac{\left(7b\sqrt[4]{-bx^2+ax^4}\right)\int \frac{\sqrt{x}}{\left(-b+ax^2\right)^{3/4}}\,dx}{4\sqrt{x}\sqrt[4]{-b+ax^2}}$$

$$= \frac{1}{2}x\sqrt[4]{-bx^2+ax^4} + \frac{\left(7b\sqrt[4]{-bx^2+ax^4}\right)\text{Subst}\left(\int \frac{x^2}{\left(-b+ax^4\right)^{3/4}}\,dx, x, \sqrt{x}\right)}{2\sqrt{x}\sqrt[4]{-b+ax^2}}$$

$$= \frac{1}{2}x\sqrt[4]{-bx^2+ax^4} + \frac{\left(7b\sqrt[4]{-bx^2+ax^4}\right)\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{-b+ax^2}}\right)}{2\sqrt{x}\sqrt[4]{-b+ax^2}}$$

$$= \frac{1}{2}x\sqrt[4]{-bx^2+ax^4} + \frac{\left(7b\sqrt[4]{-bx^2+ax^4}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{-b+ax^2}}\right)}{4\sqrt{a}\sqrt{x}\sqrt[4]{-b+ax^2}} - \frac{(7b}{}$$

$$= \frac{1}{2}x\sqrt[4]{-bx^2+ax^4} - \frac{7b\sqrt[4]{-bx^2+ax^4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{-b+ax^2}}\right)}{4a^{3/4}\sqrt{x}\sqrt[4]{-b+ax^2}} + \frac{7b\sqrt[4]{-bx^2+ax^4}\tanh}{4a^{3/4}\sqrt{x}\sqrt[4]{-b+}}$$

Mathematica [A] time = 0.11, size = 125, normalized size = 1.39

$$\frac{\sqrt[4]{ax^4-bx^2}\left(2a^{3/4}x^{3/2}\sqrt[4]{ax^2-b}-7b\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2-b}}\right)+7b\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2-b}}\right)\right)}{4a^{3/4}\sqrt{x}\sqrt[4]{ax^2-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4))/(-b + a*x^2),x]
```

```
[Out] ((-(b*x^2) + a*x^4)^(1/4)*(2*a^(3/4)*x^(3/2)*(-b + a*x^2)^(1/4) - 7*b*ArcTa
n[(a^(1/4)*Sqrt[x])/(-b + a*x^2)^(1/4)] + 7*b*ArcTanh[(a^(1/4)*Sqrt[x])/(-b
 + a*x^2)^(1/4)]))/(4*a^(3/4)*Sqrt[x]*(-b + a*x^2)^(1/4))
```

IntegrateAlgebraic [A] time = 0.33, size = 90, normalized size = 1.00

$$-\frac{7b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{4a^{3/4}}+\frac{7b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{4a^{3/4}}+\frac{1}{2}x\sqrt[4]{ax^4-bx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4))/(-b + a*x^2),x]
```

```
[Out] (x*(-(b*x^2) + a*x^4)^(1/4))/2 - (7*b*ArcTan[(a^(1/4)*x)/(-(b*x^2) + a*x^4)
^(1/4)])/(4*a^(3/4)) + (7*b*ArcTanh[(a^(1/4)*x)/(-(b*x^2) + a*x^4)^(1/4)])/
(4*a^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)*(a*x^4-b*x^2)^(1/4)/(a*x^2-b),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.35, size = 222, normalized size = 2.47

$$\frac{8\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}} b x^2 + \frac{14\sqrt{2}(-a)^{\frac{1}{4}} b^2 \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a} + \frac{14\sqrt{2}(-a)^{\frac{1}{4}} b^2 \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a} + \frac{7\sqrt{2}(-a)^{\frac{1}{4}} b^2 \log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^2}}\right)}{a} + \frac{7\sqrt{2} b^2 \log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^2}}\right)}{(-a)^{\frac{3}{4}}}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)*(a*x^4-b*x^2)^(1/4)/(a*x^2-b),x, algorithm="giac")
```

```
[Out] 1/16*(8*(a - b/x^2)^(1/4)*b*x^2 + 14*sqrt(2)*(-a)^(1/4)*b^2*arctan(1/2*sqrt
(2)*(sqrt(2)*(-a)^(1/4) + 2*(a - b/x^2)^(1/4))/(-a)^(1/4))/a + 14*sqrt(2)*(
-a)^(1/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(a - b/x^2)^(1/4)
)/(-a)^(1/4))/a + 7*sqrt(2)*(-a)^(1/4)*b^2*log(sqrt(2)*(-a)^(1/4)*(a - b/x^
2)^(1/4) + sqrt(-a) + sqrt(a - b/x^2))/a + 7*sqrt(2)*b^2*log(-sqrt(2)*(-a)^
(1/4)*(a - b/x^2)^(1/4) + sqrt(-a) + sqrt(a - b/x^2))/(-a)^(3/4))/b
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{2}+b\right)\left(a\,x^{4}-b\,x^{2}\right)^{\frac{1}{4}}}{a\,x^{2}-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)*(a*x^4-b*x^2)^(1/4)/(a*x^2-b),x)
```

```
[Out] int((a*x^2+b)*(a*x^4-b*x^2)^(1/4)/(a*x^2-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{4}-bx^{2}\right)^{\frac{1}{4}}\left(ax^{2}+b\right)}{ax^{2}-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)*(a*x^4-b*x^2)^(1/4)/(a*x^2-b),x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 - b*x^2)^(1/4)*(a*x^2 + b)/(a*x^2 - b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(a\,x^2+b\right)\,{\left(a\,x^4-b\,x^2\right)}^{1/4}}{b-a\,x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b + a*x^2)*(a*x^4 - b*x^2)^(1/4))/(b - a*x^2),x)
```

```
[Out] int(-((b + a*x^2)*(a*x^4 - b*x^2)^(1/4))/(b - a*x^2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\left(ax^2-b\right)}\left(ax^2+b\right)}{ax^2-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b)*(a*x**4-b*x**2)**(1/4)/(a*x**2-b),x)
```

```
[Out] Integral((x**2*(a*x**2 - b))**(1/4)*(a*x**2 + b)/(a*x**2 - b), x)
```

3.1104 $\int \frac{-b+2ax^2}{(b+ax^2)\sqrt[4]{bx^2+ax^4}}\,dx$

Optimal. Leaf size=90

$$-\frac{6\left(ax^4+bx^2\right)^{3/4}}{x\left(ax^2+b\right)}+\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{\sqrt[4]{a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{\sqrt[4]{a}}$$

Rubi [A] time = 0.21, antiderivative size = 141, normalized size of antiderivative = 1.57, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2056, 452, 329, 240, 212, 206, 203}

$$-\frac{6x}{\sqrt[4]{ax^4+bx^2}}+\frac{2\sqrt{x}\,\sqrt[4]{ax^2+b}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{\sqrt[4]{a}\,\sqrt[4]{ax^4+bx^2}}+\frac{2\sqrt{x}\,\sqrt[4]{ax^2+b}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{\sqrt[4]{a}\,\sqrt[4]{ax^4+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + 2*a*x^2)/((b + a*x^2)*(b*x^2 + a*x^4)^(1/4)),x]
```

```
[Out] (-6*x)/(b*x^2 + a*x^4)^(1/4) + (2*Sqrt[x]*(b + a*x^2)^(1/4)*ArcTan[(a^(1/4)
*Sqrt[x])/(b + a*x^2)^(1/4)])/(a^(1/4)*(b*x^2 + a*x^4)^(1/4)) + (2*Sqrt[x]*
(b + a*x^2)^(1/4)*ArcTanh[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(a^(1/4)*(b
*x^2 + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{-b+2ax^2}{\left(b+ax^2\right)\sqrt[4]{bx^2+ax^4}}\,dx = \frac{\left(\sqrt{x}\,\sqrt[4]{b+ax^2}\right)\int \frac{-b+2ax^2}{\sqrt{x}\left(b+ax^2\right)^{5/4}}\,dx}{\sqrt[4]{bx^2+ax^4}}$$

$$= -\frac{6x}{\sqrt[4]{bx^2+ax^4}} + \frac{\left(2\sqrt{x}\,\sqrt[4]{b+ax^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt[4]{b+ax^2}}\,dx}{\sqrt[4]{bx^2+ax^4}}$$

$$= -\frac{6x}{\sqrt[4]{bx^2+ax^4}} + \frac{\left(4\sqrt{x}\,\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{bx^2+ax^4}}$$

$$= -\frac{6x}{\sqrt[4]{bx^2+ax^4}} + \frac{\left(4\sqrt{x}\,\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{\sqrt[4]{bx^2+ax^4}}$$

$$= -\frac{6x}{\sqrt[4]{bx^2+ax^4}} + \frac{\left(2\sqrt{x}\,\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{\sqrt[4]{bx^2+ax^4}} + \frac{\left(2\sqrt{x}\,\sqrt[4]{b-\right.}{}$$

$$= -\frac{6x}{\sqrt[4]{bx^2+ax^4}} + \frac{2\sqrt{x}\,\sqrt[4]{b+ax^2}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{\sqrt[4]{a}\,\sqrt[4]{bx^2+ax^4}} + \frac{2\sqrt{x}\,\sqrt[4]{b+ax^2}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{}}{\sqrt[4]{b+a}}\right.}{\sqrt[4]{a}\,\sqrt[4]{bx^2+ax^4}}$$

Mathematica [C] time = 0.08, size = 66, normalized size = 0.73

$$\frac{2x\left(2ax^2\sqrt[4]{\frac{ax^2}{b}+1}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};-\frac{ax^2}{b}\right)-5b\right)}{5b\sqrt[4]{x^2\left(ax^2+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + 2*a*x^2)/((b + a*x^2)*(b*x^2 + a*x^4)^(1/4)),x]
```

```
[Out] (2*x*(-5*b + 2*a*x^2*(1 + (a*x^2)/b)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4,
 -((a*x^2)/b)]))/(5*b*(x^2*(b + a*x^2))^(1/4))
```

IntegrateAlgebraic [A] time = 0.28, size = 90, normalized size = 1.00

$$-\frac{6\left(ax^4+bx^2\right)^{3/4}}{x\left(ax^2+b\right)} + \frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{\sqrt[4]{a}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + 2*a*x^2)/((b + a*x^2)*(b*x^2 + a*x^4)^(1/4)),x]
```

```
[Out] (-6*(b*x^2 + a*x^4)^(3/4))/(x*(b + a*x^2)) + (2*ArcTan[(a^(1/4)*x)/(b*x^2 +
 a*x^4)^(1/4)])/a^(1/4) + (2*ArcTanh[(a^(1/4)*x)/(b*x^2 + a*x^4)^(1/4)])/a^
(1/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2-b)/(a*x^2+b)/(a*x^4+b*x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.22, size = 195, normalized size = 2.17

$$\frac{\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a}+\frac{\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a}-\frac{\sqrt{2}\,(-a)^{\frac{3}{4}}\log\left(\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)}{2\,a}+\frac{\sqrt{2}\,(-a)^{\frac{3}{4}}\log\left(-\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)}{2\,a}-\frac{6}{\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2-b)/(a*x^2+b)/(a*x^4+b*x^2)^(1/4),x, algorithm="giac")
```

```
[Out] sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^2)^(
1/4))/(-a)^(1/4))/a + sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^
(1/4) - 2*(a + b/x^2)^(1/4))/(-a)^(1/4))/a - 1/2*sqrt(2)*(-a)^(3/4)*log(sqr
t(2)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sqrt(a + b/x^2))/a + 1/2*sqr
t(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sqrt
(a + b/x^2))/a - 6/(a + b/x^2)^(1/4)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{2 a \,x^{2}-b}{\left(a \,x^{2}+b \right) \left(a \,x^{4}+b \,x^{2}\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x^2-b)/(a*x^2+b)/(a*x^4+b*x^2)^(1/4),x)
```

```
[Out] int((2*a*x^2-b)/(a*x^2+b)/(a*x^4+b*x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 \, a x^{2} - b}{{\left(a x^{4} + b x^{2}\right)}^{\frac{1}{4}} {\left(a x^{2} + b\right)}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2-b)/(a*x^2+b)/(a*x^4+b*x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((2*a*x^2 - b)/((a*x^4 + b*x^2)^(1/4)*(a*x^2 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{b-2\,a\,x^2}{\left(a\,x^2+b\right)\,{\left(a\,x^4+b\,x^2\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - 2*a*x^2)/((b + a*x^2)*(a*x^4 + b*x^2)^(1/4)),x)
[Out] int(-(b - 2*a*x^2)/((b + a*x^2)*(a*x^4 + b*x^2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2ax^2 - b}{\sqrt[4]{x^2 \left(ax^2 + b\right)} \left(ax^2 + b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x**2-b)/(a*x**2+b)/(a*x**4+b*x**2)**(1/4),x)
[Out] Integral((2*a*x**2 - b)/((x**2*(a*x**2 + b))**(1/4)*(a*x**2 + b)), x)
```

3.1105 $\int \frac{(1+x^5)^{2/3}}{x^6}\,dx$

Optimal. Leaf size=90

$$-\frac{(x^5+1)^{2/3}}{5x^5}+\frac{2}{15}\log\left(\sqrt[3]{x^5+1}-1\right)-\frac{1}{15}\log\left((x^5+1)^{2/3}+\sqrt[3]{x^5+1}+1\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^5+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{5\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 47, 55, 618, 204, 31}

$$-\frac{(x^5+1)^{2/3}}{5x^5}+\frac{1}{5}\log\left(1-\sqrt[3]{x^5+1}\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^5+1}+1}{\sqrt{3}}\right)}{5\sqrt{3}}-\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^5)^(2/3)/x^6,x]
```

```
[Out] -1/5*(1 + x^5)^(2/3)/x^5 + (2*ArcTan[(1 + 2*(1 + x^5)^(1/3))/Sqrt[3]])/(5*S
qrt[3]) - Log[x]/3 + Log[1 - (1 + x^5)^(1/3)]/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(1+x^5\right)^{2/3}}{x^6}\,dx = \frac{1}{5}\,\text{Subst}\left(\int \frac{(1+x)^{2/3}}{x^2}\,dx, x, x^5\right)$$
$$= -\frac{\left(1+x^5\right)^{2/3}}{5x^5} + \frac{2}{15}\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx, x, x^5\right)$$
$$= -\frac{\left(1+x^5\right)^{2/3}}{5x^5} - \frac{\log(x)}{3} - \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+x^5}\right) + \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x\right.$$
$$= -\frac{\left(1+x^5\right)^{2/3}}{5x^5} - \frac{\log(x)}{3} + \frac{1}{5}\log\left(1-\sqrt[3]{1+x^5}\right) - \frac{2}{5}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+x^5}\right)$$
$$= -\frac{\left(1+x^5\right)^{2/3}}{5x^5} + \frac{2\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^5}}{\sqrt{3}}\right)}{5\sqrt{3}} - \frac{\log(x)}{3} + \frac{1}{5}\log\left(1-\sqrt[3]{1+x^5}\right)$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.29

$$\frac{3}{25}\left(x^5+1\right)^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};x^5+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^5)^(2/3)/x^6,x]
```

```
[Out] (3*(1 + x^5)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 + x^5])/25
```

IntegrateAlgebraic [A] time = 0.09, size = 90, normalized size = 1.00

$$-\frac{\left(x^5+1\right)^{2/3}}{5x^5} + \frac{2}{15}\log\left(\sqrt[3]{x^5+1}-1\right) - \frac{1}{15}\log\left(\left(x^5+1\right)^{2/3}+\sqrt[3]{x^5+1}+1\right) + \frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^5+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{5\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^5)^(2/3)/x^6,x]
```

```
[Out] -1/5*(1 + x^5)^(2/3)/x^5 + (2*ArcTan[1/Sqrt[3] + (2*(1 + x^5)^(1/3))/Sqrt[3
]])/(5*Sqrt[3]) + (2*Log[-1 + (1 + x^5)^(1/3)])/15 - Log[1 + (1 + x^5)^(1/3
) + (1 + x^5)^(2/3)]/15
```

fricas [A] time = 0.41, size = 79, normalized size = 0.88

$$\frac{2\sqrt{3}\,x^5\arctan\left(\frac{2}{3}\sqrt{3}\left(x^5+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right) - x^5\log\left(\left(x^5+1\right)^{\frac{2}{3}}+\left(x^5+1\right)^{\frac{1}{3}}+1\right) + 2x^5\log\left(\left(x^5+1\right)^{\frac{1}{3}}-1\right) - 3\left(x^5+1\right)^{\frac{2}{3}}}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(2*sqrt(3)*x^5*arctan(2/3*sqrt(3)*(x^5 + 1)^(1/3) + 1/3*sqrt(3)) - x^5
*log((x^5 + 1)^(2/3) + (x^5 + 1)^(1/3) + 1) + 2*x^5*log((x^5 + 1)^(1/3) - 1
) - 3*(x^5 + 1)^(2/3))/x^5
```

giac [A] time = 0.16, size = 67, normalized size = 0.74

$$\frac{2}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^5+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^5+1\right)^{\frac{2}{3}}}{5x^5}-\frac{1}{15}\log\left(\left(x^5+1\right)^{\frac{2}{3}}+\left(x^5+1\right)^{\frac{1}{3}}+1\right)+\frac{2}{15}\log\left(\left|\left(x^5+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)/x^6,x, algorithm="giac")
```

```
[Out] 2/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^5 + 1)^(1/3) + 1)) - 1/5*(x^5 + 1)^(2
/3)/x^5 - 1/15*log((x^5 + 1)^(2/3) + (x^5 + 1)^(1/3) + 1) + 2/15*log(abs((x
^5 + 1)^(1/3) - 1))
```

maple [C] time = 0.30, size = 76, normalized size = 0.84

$$-\frac{\left(x^5+1\right)^{\frac{2}{3}}}{5x^5}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^5\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],-x^5\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+5\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{15\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)^(2/3)/x^6,x)
```

```
[Out] -1/5*(x^5+1)^(2/3)/x^5+1/15/Pi*3^(1/2)*GAMMA(2/3)*(-2/9*Pi*3^(1/2)/GAMMA(2/
3)*x^5*hypergeom([1,1,4/3],[2,2],-x^5)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+5*ln(
x))*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.42, size = 66, normalized size = 0.73

$$\frac{2}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^5+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^5+1\right)^{\frac{2}{3}}}{5x^5}-\frac{1}{15}\log\left(\left(x^5+1\right)^{\frac{2}{3}}+\left(x^5+1\right)^{\frac{1}{3}}+1\right)+\frac{2}{15}\log\left(\left(x^5+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)/x^6,x, algorithm="maxima")
```

```
[Out] 2/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^5 + 1)^(1/3) + 1)) - 1/5*(x^5 + 1)^(2
/3)/x^5 - 1/15*log((x^5 + 1)^(2/3) + (x^5 + 1)^(1/3) + 1) + 2/15*log((x^5 +
 1)^(1/3) - 1)
```

mupad [B] time = 0.94, size = 92, normalized size = 1.02

$$\frac{2\ln\left(\frac{4\left(x^5+1\right)^{1/3}}{25}-\frac{4}{25}\right)}{15}+\ln\left(\frac{4\left(x^5+1\right)^{1/3}}{25}-9\left(-\frac{1}{15}+\frac{\sqrt{3}\,1\mathrm{i}}{15}\right)^2\right)\left(-\frac{1}{15}+\frac{\sqrt{3}\,1\mathrm{i}}{15}\right)-\ln\left(\frac{4\left(x^5+1\right)^{1/3}}{25}-9\left(\frac{1}{15}+\frac{\sqrt{3}\,1\mathrm{i}}{15}\right)^2\right)\left(\frac{1}{15}+\frac{\sqrt{3}\,1\mathrm{i}}{15}\right)-\frac{\left(x^5+1\right)^{2/3}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5 + 1)^(2/3)/x^6,x)
```

```
[Out] (2*log((4*(x^5 + 1)^(1/3))/25 - 4/25))/15 + log((4*(x^5 + 1)^(1/3))/25 - 9*
((3^(1/2)*1i)/15 - 1/15)^2)*((3^(1/2)*1i)/15 - 1/15) - log((4*(x^5 + 1)^(1/
3))/25 - 9*((3^(1/2)*1i)/15 + 1/15)^2)*((3^(1/2)*1i)/15 + 1/15) - (x^5 + 1)
^(2/3)/(5*x^5)
```

sympy [C] time = 1.03, size = 34, normalized size = 0.38

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^5}\right)}{5x^{\frac{5}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+1)**(2/3)/x**6,x)
```

```
[Out] -gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_polar(I*pi)/x**5)/(5*x**(5/3)*ga
mma(4/3))
```

3.1106 $\int \frac{(1+x^5)^{2/3}(-3+2x^5)}{x^3(1-x^3+x^5)}\,dx$

Optimal. Leaf size=90

$$\log\left(\sqrt[3]{x^5+1}-x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^5+1}+x}\right)+\frac{3\left(x^5+1\right)^{2/3}}{2x^2}-\frac{1}{2}\log\left(\sqrt[3]{x^5+1}\,x+\left(x^5+1\right)^{2/3}+x^2\right)$$

Rubi [F] time = 0.76, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+x^5\right)^{2/3}\left(-3+2x^5\right)}{x^3\left(1-x^3+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^5)^(2/3)*(-3 + 2*x^5))/(x^3*(1 - x^3 + x^5)),x]
```

```
[Out] (3*Hypergeometric2F1[-2/3, -2/5, 3/5, -x^5])/(2*x^2) - 3*Defer[Int][(1 + x^
5)^(2/3)/(1 - x^3 + x^5), x] + 5*Defer[Int][(x^2*(1 + x^5)^(2/3))/(1 - x^3
+ x^5), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+x^5\right)^{2/3}\left(-3+2x^5\right)}{x^3\left(1-x^3+x^5\right)}\,dx &= \int \left(-\frac{3\left(1+x^5\right)^{2/3}}{x^3}+\frac{\left(-3+5x^2\right)\left(1+x^5\right)^{2/3}}{1-x^3+x^5}\right)dx\\
&= -\left(3\int \frac{\left(1+x^5\right)^{2/3}}{x^3}\,dx\right)+\int \frac{\left(-3+5x^2\right)\left(1+x^5\right)^{2/3}}{1-x^3+x^5}\,dx\\
&= \frac{3\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{5};\frac{3}{5};-x^5\right)}{2x^2}+\int \left(-\frac{3\left(1+x^5\right)^{2/3}}{1-x^3+x^5}+\frac{5x^2\left(1+x^5\right)^{2/3}}{1-x^3+x^5}\right)dx\\
&= \frac{3\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{5};\frac{3}{5};-x^5\right)}{2x^2}-3\int \frac{\left(1+x^5\right)^{2/3}}{1-x^3+x^5}\,dx+5\int \frac{x^2\left(1+x^5\right)^{2/3}}{1-x^3+x^5}\,dx
\end{aligned}$$

Mathematica [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(1+x^5\right)^{2/3}\left(-3+2x^5\right)}{x^3\left(1-x^3+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^5)^(2/3)*(-3 + 2*x^5))/(x^3*(1 - x^3 + x^5)),x]
```

```
[Out] Integrate[((1 + x^5)^(2/3)*(-3 + 2*x^5))/(x^3*(1 - x^3 + x^5)), x]
```

IntegrateAlgebraic [A] time = 1.50, size = 90, normalized size = 1.00

$$\log\left(\sqrt[3]{x^5+1}-x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^5+1}+x}\right)+\frac{3\left(x^5+1\right)^{2/3}}{2x^2}-\frac{1}{2}\log\left(\sqrt[3]{x^5+1}\,x+\left(x^5+1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^5)^(2/3)*(-3 + 2*x^5))/(x^3*(1 - x^3 + x^5)),x]
```

```
[Out] (3*(1 + x^5)^(2/3))/(2*x^2) - Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(1 + x^5)^(
1/3))] + Log[-x + (1 + x^5)^(1/3)] - Log[x^2 + x*(1 + x^5)^(1/3) + (1 + x^5
)^(2/3)]/2
```

fricas [A] time = 5.37, size = 135, normalized size = 1.50

$$-\frac{2\sqrt{3}x^2\arctan\left(\frac{67616276\sqrt{3}\left(x^5+1\right)^{\frac{1}{3}}x^2+10249526\sqrt{3}\left(x^5+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(1423013x^5+37509888x^3+1423013\right)}{300763x^5-86350888x^3+300763}\right)-x^2\log\left(\frac{x^5-x^3+3\left(x^5+1\right)^{\frac{1}{3}}x^2-3\left(x^5+1\right)^{\frac{2}{3}}x+1}{x^5-x^3+1}\right)-3\left(x^5+1\right)^{\frac{2}{3}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)*(2*x^5-3)/x^3/(x^5-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*x^2*arctan((67616276*sqrt(3)*(x^5 + 1)^(1/3)*x^2 + 10249526
*sqrt(3)*(x^5 + 1)^(2/3)*x + sqrt(3)*(1423013*x^5 + 37509888*x^3 + 1423013)
)/(300763*x^5 - 86350888*x^3 + 300763)) - x^2*log((x^5 - x^3 + 3*(x^5 + 1)^
(1/3)*x^2 - 3*(x^5 + 1)^(2/3)*x + 1)/(x^5 - x^3 + 1)) - 3*(x^5 + 1)^(2/3))/
x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^5-3\right)\left(x^5+1\right)^{\frac{2}{3}}}{\left(x^5-x^3+1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)*(2*x^5-3)/x^3/(x^5-x^3+1),x, algorithm="giac")
```

```
[Out] integrate((2*x^5 - 3)*(x^5 + 1)^(2/3)/((x^5 - x^3 + 1)*x^3), x)
```

maple [C] time = 3.82, size = 289, normalized size = 3.21

$$\frac{3\left(x^5+1\right)^{\frac{2}{3}}}{2x^2}+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^5-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3+x^5+3\left(x^5+1\right)^{\frac{2}{3}}x+3\left(x^5+1\right)^{\frac{1}{3}}x^2+x^3-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)+1}{x^5-x^3+1}\right)-\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^5-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3+2x^5-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3+3\left(x^5+1\right)^{\frac{2}{3}}x+3\left(x^5+1\right)^{\frac{1}{3}}x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)+2}{x^5-x^3+1}\right)\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^5-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3+2x^5-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3+3\left(x^5+1\right)^{\frac{2}{3}}x+3\left(x^5+1\right)^{\frac{1}{3}}x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)+2}{x^5-x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)^(2/3)*(2*x^5-3)/x^3/(x^5-x^3+1),x)
```

```
[Out] 3/2*(x^5+1)^(2/3)/x^2+RootOf(_Z^2+_Z+1)*ln(-(-RootOf(_Z^2+_Z+1)*x^5-RootOf(
_Z^2+_Z+1)^2*x^3+x^5+3*(x^5+1)^(2/3)*x+3*(x^5+1)^(1/3)*x^2+x^3-RootOf(_Z^2+
_Z+1)+1)/(x^5-x^3+1))-ln(-(RootOf(_Z^2+_Z+1)*x^5-RootOf(_Z^2+_Z+1)^2*x^3+2*
x^5-2*RootOf(_Z^2+_Z+1)*x^3+3*(x^5+1)^(2/3)*x+3*(x^5+1)^(1/3)*x^2+RootOf(_Z
^2+_Z+1)+2)/(x^5-x^3+1))*RootOf(_Z^2+_Z+1)-ln(-(RootOf(_Z^2+_Z+1)*x^5-RootO
f(_Z^2+_Z+1)^2*x^3+2*x^5-2*RootOf(_Z^2+_Z+1)*x^3+3*(x^5+1)^(2/3)*x+3*(x^5+1
)^(1/3)*x^2+RootOf(_Z^2+_Z+1)+2)/(x^5-x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^5-3\right)\left(x^5+1\right)^{\frac{2}{3}}}{\left(x^5-x^3+1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)*(2*x^5-3)/x^3/(x^5-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^5 - 3)*(x^5 + 1)^(2/3)/((x^5 - x^3 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^5+1\right)^{2/3}\,\left(2\,x^5-3\right)}{x^3\,\left(x^5-x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 + 1)^(2/3)*(2*x^5 - 3))/(x^3*(x^5 - x^3 + 1)),x)
```

```
[Out] int(((x^5 + 1)^(2/3)*(2*x^5 - 3))/(x^3*(x^5 - x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x+1\right)\left(x^4-x^3+x^2-x+1\right)\right)^{\frac{2}{3}}\left(2x^5-3\right)}{x^3\left(x^5-x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+1)**(2/3)*(2*x**5-3)/x**3/(x**5-x**3+1),x)
```

```
[Out] Integral(((x + 1)*(x**4 - x**3 + x**2 - x + 1))**(2/3)*(2*x**5 - 3)/(x**3*(
x**5 - x**3 + 1)), x)
```

3.1107 $\int \frac{(-1+x^5)^{2/3}(3+2x^5)}{x^3(-1-x^3+x^5)}\,dx$

Optimal. Leaf size=90

$$\log\left(\sqrt[3]{x^5-1}-x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^5-1}+x}\right)+\frac{3\left(x^5-1\right)^{2/3}}{2x^2}-\frac{1}{2}\log\left(\sqrt[3]{x^5-1}\,x+\left(x^5-1\right)^{2/3}+x^2\right)$$

Rubi [F] time = 0.79, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1+x^5\right)^{2/3}\left(3+2x^5\right)}{x^3\left(-1-x^3+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^5)^(2/3)*(3 + 2*x^5))/(x^3*(-1 - x^3 + x^5)),x]
```

```
[Out] (3*(-1 + x^5)^(2/3)*Hypergeometric2F1[-2/3, -2/5, 3/5, x^5])/(2*x^2*(1 - x^
5)^(2/3)) - 3*Defer[Int][(-1 + x^5)^(2/3)/(-1 - x^3 + x^5), x] + 5*Defer[In
t][(x^2*(-1 + x^5)^(2/3))/(-1 - x^3 + x^5), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^5\right)^{2/3}\left(3+2x^5\right)}{x^3\left(-1-x^3+x^5\right)}\,dx &= \int \left(-\frac{3\left(-1+x^5\right)^{2/3}}{x^3}+\frac{\left(-3+5x^2\right)\left(-1+x^5\right)^{2/3}}{-1-x^3+x^5}\right)dx\\
&= -\left(3\int \frac{\left(-1+x^5\right)^{2/3}}{x^3}\,dx\right)+\int \frac{\left(-3+5x^2\right)\left(-1+x^5\right)^{2/3}}{-1-x^3+x^5}\,dx\\
&= -\frac{\left(3\left(-1+x^5\right)^{2/3}\right)\int \frac{\left(1-x^5\right)^{2/3}}{x^3}\,dx}{\left(1-x^5\right)^{2/3}}+\int \left(-\frac{3\left(-1+x^5\right)^{2/3}}{-1-x^3+x^5}+\frac{5x^2\left(-1+x^5\right)^{2/3}}{-1-x^3+x^5}\right)dx\\
&= \frac{3\left(-1+x^5\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{5};\frac{3}{5};x^5\right)}{2x^2\left(1-x^5\right)^{2/3}}-3\int \frac{\left(-1+x^5\right)^{2/3}}{-1-x^3+x^5}\,dx+5\int \frac{x^2\left(-1+x^5\right)^{2/3}}{-1-x^3+x^5}
\end{aligned}$$

Mathematica [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^5\right)^{2/3}\left(3+2x^5\right)}{x^3\left(-1-x^3+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^5)^(2/3)*(3 + 2*x^5))/(x^3*(-1 - x^3 + x^5)),x]
```

```
[Out] Integrate[((-1 + x^5)^(2/3)*(3 + 2*x^5))/(x^3*(-1 - x^3 + x^5)), x]
```

IntegrateAlgebraic [A] time = 1.38, size = 90, normalized size = 1.00

$$\log\left(\sqrt[3]{x^5-1}-x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^5-1}+x}\right)+\frac{3\left(x^5-1\right)^{2/3}}{2x^2}-\frac{1}{2}\log\left(\sqrt[3]{x^5-1}\,x+\left(x^5-1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^5)^(2/3)*(3 + 2*x^5))/(x^3*(-1 - x^3 + x^5)),x]
```

```
[Out] (3*(-1 + x^5)^(2/3))/(2*x^2) - Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x^5)
^(1/3))] + Log[-x + (-1 + x^5)^(1/3)] - Log[x^2 + x*(-1 + x^5)^(1/3) + (-1
+ x^5)^(2/3)]/2
```

fricas [A] time = 5.37, size = 135, normalized size = 1.50

$$-\frac{2 \sqrt{3} x^{2} \arctan\left(\frac{67616276 \sqrt{3} \left(x^{5}-1\right)^{\frac{1}{3}} x^{2}+10249526 \sqrt{3} \left(x^{5}-1\right)^{\frac{2}{3}} x+\sqrt{3} \left(1423013 x^{5}+37509888 x^{3}-1423013\right)}{300763 x^{5}-86350888 x^{3}-300763}\right)-x^{2} \log\left(\frac{x^{5}-x^{3}+3 \left(x^{5}-1\right)^{\frac{1}{3}} x^{2}-3 \left(x^{5}-1\right)^{\frac{2}{3}} x-1}{x^{5}-x^{3}-1}\right)-3 \left(x^{5}-1\right)^{\frac{2}{3}}}{2 x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(2/3)*(2*x^5+3)/x^3/(x^5-x^3-1),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*x^2*arctan((67616276*sqrt(3)*(x^5 - 1)^(1/3)*x^2 + 10249526
*sqrt(3)*(x^5 - 1)^(2/3)*x + sqrt(3)*(1423013*x^5 + 37509888*x^3 - 1423013)
)/(300763*x^5 - 86350888*x^3 - 300763)) - x^2*log((x^5 - x^3 + 3*(x^5 - 1)^
(1/3)*x^2 - 3*(x^5 - 1)^(2/3)*x - 1)/(x^5 - x^3 - 1)) - 3*(x^5 - 1)^(2/3))/
x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 x^{5}+3\right) \left(x^{5}-1\right)^{\frac{2}{3}}}{\left(x^{5}-x^{3}-1\right) x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(2/3)*(2*x^5+3)/x^3/(x^5-x^3-1),x, algorithm="giac")
```

```
[Out] integrate((2*x^5 + 3)*(x^5 - 1)^(2/3)/((x^5 - x^3 - 1)*x^3), x)
```

maple [C] time = 3.96, size = 288, normalized size = 3.20

$$\frac{3 \left(x^{5}-1\right)^{\frac{2}{3}}}{2 x^{2}}+\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right) \ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{5}+\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right)^{2} x^{3}-x^{5}-3 \left(x^{5}-1\right)^{\frac{2}{3}} x-3 \left(x^{5}-1\right)^{\frac{1}{3}} x^{2}-x^{3}-\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right)+1}{x^{5}-x^{3}-1}\right)-\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{5}+\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right)^{2} x^{3}-2 x^{5}+2 \operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{3}-3 \left(x^{5}-1\right)^{\frac{2}{3}} x-3 \left(x^{5}-1\right)^{\frac{1}{3}} x^{2}+\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right)+2}{x^{5}-x^{3}-1}\right) \operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right)-\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{5}+\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right)^{2} x^{3}-2 x^{5}+2 \operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{3}-3 \left(x^{5}-1\right)^{\frac{2}{3}} x-3 \left(x^{5}-1\right)^{\frac{1}{3}} x^{2}+\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right)+2}{x^{5}-x^{3}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)^(2/3)*(2*x^5+3)/x^3/(x^5-x^3-1),x)
```

```
[Out] 3/2*(x^5-1)^(2/3)/x^2+RootOf(_Z^2+_Z+1)*ln((RootOf(_Z^2+_Z+1)*x^5+RootOf(_Z
^2+_Z+1)^2*x^3-x^5-3*(x^5-1)^(2/3)*x-3*(x^5-1)^(1/3)*x^2-x^3-RootOf(_Z^2+_Z
+1)+1)/(x^5-x^3-1))-ln((-RootOf(_Z^2+_Z+1)*x^5+RootOf(_Z^2+_Z+1)^2*x^3-2*x^
5+2*RootOf(_Z^2+_Z+1)*x^3-3*(x^5-1)^(2/3)*x-3*(x^5-1)^(1/3)*x^2+RootOf(_Z^2
+_Z+1)+2)/(x^5-x^3-1))*RootOf(_Z^2+_Z+1)-ln((-RootOf(_Z^2+_Z+1)*x^5+RootOf(
_Z^2+_Z+1)^2*x^3-2*x^5+2*RootOf(_Z^2+_Z+1)*x^3-3*(x^5-1)^(2/3)*x-3*(x^5-1)^
(1/3)*x^2+RootOf(_Z^2+_Z+1)+2)/(x^5-x^3-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 x^{5}+3\right) \left(x^{5}-1\right)^{\frac{2}{3}}}{\left(x^{5}-x^{3}-1\right) x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(2/3)*(2*x^5+3)/x^3/(x^5-x^3-1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^5 + 3)*(x^5 - 1)^(2/3)/((x^5 - x^3 - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^5-1\right)^{2/3}\,\left(2\,x^5+3\right)}{x^3\,\left(-x^5+x^3+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^5 - 1)^(2/3)*(2*x^5 + 3))/(x^3*(x^3 - x^5 + 1)),x)
```

```
[Out] int(-((x^5 - 1)^(2/3)*(2*x^5 + 3))/(x^3*(x^3 - x^5 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((x-1)\left(x^4+x^3+x^2+x+1\right)\right)^{\frac{2}{3}}\left(2x^5+3\right)}{x^3\left(x^5-x^3-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)**(2/3)*(2*x**5+3)/x**3/(x**5-x**3-1),x)
```

```
[Out] Integral(((x - 1)*(x**4 + x**3 + x**2 + x + 1))**(2/3)*(2*x**5 + 3)/(x**3*(
x**5 - x**3 - 1)), x)
```

3.1108 $\int \frac{\sqrt[4]{-1+x^6}}{x}\,dx$

Optimal. Leaf size=90

$$\frac{2}{3}\sqrt[4]{x^6-1}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^6-1}}{\sqrt{x^6-1}-1}\right)}{3\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^6-1}}{\sqrt{x^6-1}+1}\right)}{3\sqrt{2}}$$

Rubi [A] time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.58, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {266, 50, 63, 211, 1165, 628, 1162, 617, 204}

$$\frac{2}{3}\sqrt[4]{x^6-1}+\frac{\log\left(\sqrt{x^6-1}-\sqrt{2}\sqrt[4]{x^6-1}+1\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{x^6-1}+\sqrt{2}\sqrt[4]{x^6-1}+1\right)}{6\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{x^6-1}\right)}{3\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt[4]{x^6-1}+1\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)^(1/4)/x,x]
```

```
[Out] (2*(-1 + x^6)^(1/4))/3 + ArcTan[1 - Sqrt[2]*(-1 + x^6)^(1/4)]/(3*Sqrt[2]) -
 ArcTan[1 + Sqrt[2]*(-1 + x^6)^(1/4)]/(3*Sqrt[2]) + Log[1 - Sqrt[2]*(-1 + x
^6)^(1/4) + Sqrt[-1 + x^6]]/(6*Sqrt[2]) - Log[1 + Sqrt[2]*(-1 + x^6)^(1/4)
+ Sqrt[-1 + x^6]]/(6*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{-1+x^6}}{x}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{\sqrt[4]{-1+x}}{x}\,dx, x, x^6\right)\\
&= \frac{2}{3}\sqrt[4]{-1+x^6} - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{(-1+x)^{3/4}x}\,dx, x, x^6\right)\\
&= \frac{2}{3}\sqrt[4]{-1+x^6} - \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt[4]{-1+x^6}\right)\\
&= \frac{2}{3}\sqrt[4]{-1+x^6} - \frac{1}{3}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt[4]{-1+x^6}\right) - \frac{1}{3}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt[4]{-1+x^6}\right)\\
&= \frac{2}{3}\sqrt[4]{-1+x^6} - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt[4]{-1+x^6}\right) - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx\right.\\
&= \frac{2}{3}\sqrt[4]{-1+x^6} + \frac{\log\left(1-\sqrt{2}\sqrt[4]{-1+x^6}+\sqrt{-1+x^6}\right)}{6\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt[4]{-1+x^6}+\sqrt{-1+x^6}\right)}{6\sqrt{2}}\\
&= \frac{2}{3}\sqrt[4]{-1+x^6} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+x^6}\right)}{3\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+x^6}\right)}{3\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt[4]{-1\right.}}{6\sqrt{}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 135, normalized size = 1.50

$$\frac{1}{12}\left(8\sqrt[4]{x^6-1}+\sqrt{2}\log\left(\sqrt{x^6-1}-\sqrt{2}\sqrt[4]{x^6-1}+1\right)-\sqrt{2}\log\left(\sqrt{x^6-1}+\sqrt{2}\sqrt[4]{x^6-1}+1\right)+2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{x^6-1}\right)-2\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt[4]{x^6-1}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)^(1/4)/x,x]
```

```
[Out] (8*(-1 + x^6)^(1/4) + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*(-1 + x^6)^(1/4)] - 2*Sq
rt[2]*ArcTan[1 + Sqrt[2]*(-1 + x^6)^(1/4)] + Sqrt[2]*Log[1 - Sqrt[2]*(-1 +
x^6)^(1/4) + Sqrt[-1 + x^6]] - Sqrt[2]*Log[1 + Sqrt[2]*(-1 + x^6)^(1/4) + S
qrt[-1 + x^6]])/12
```

IntegrateAlgebraic [A] time = 0.09, size = 95, normalized size = 1.06

$$\frac{2}{3}\sqrt[4]{x^6-1}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{x^6-1}}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt[4]{x^6-1}}\right)}{3\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^6-1}}{\sqrt{x^6-1}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^6)^(1/4)/x,x]
```

```
[Out] (2*(-1 + x^6)^(1/4))/3 - ArcTan[(-(1/Sqrt[2]) + Sqrt[-1 + x^6]/Sqrt[2])/(-1
 + x^6)^(1/4)]/(3*Sqrt[2]) - ArcTanh[(Sqrt[2]*(-1 + x^6)^(1/4))/(1 + Sqrt[-
1 + x^6])]/(3*Sqrt[2])
```

fricas [B] time = 0.41, size = 164, normalized size = 1.82

$$\frac{1}{3}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+\sqrt{x^6-1}+1}-\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}-1\right)+\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+4\sqrt{x^6-1}+4}-\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+1\right)-\frac{1}{12}\sqrt{2}\log\left(4\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+4\sqrt{x^6-1}+4\right)+\frac{1}{12}\sqrt{2}\log\left(-4\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+4\sqrt{x^6-1}+4\right)+\frac{2}{3}\left(x^6-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/4)/x,x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*(x^6 - 1)^(1/4) + sqrt(x^6 - 1) + 1
) - sqrt(2)*(x^6 - 1)^(1/4) - 1) + 1/3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*s
qrt(2)*(x^6 - 1)^(1/4) + 4*sqrt(x^6 - 1) + 4) - sqrt(2)*(x^6 - 1)^(1/4) + 1
) - 1/12*sqrt(2)*log(4*sqrt(2)*(x^6 - 1)^(1/4) + 4*sqrt(x^6 - 1) + 4) + 1/1
2*sqrt(2)*log(-4*sqrt(2)*(x^6 - 1)^(1/4) + 4*sqrt(x^6 - 1) + 4) + 2/3*(x^6
- 1)^(1/4)
```

giac [A] time = 0.16, size = 111, normalized size = 1.23

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(x^6-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{6}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(x^6-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{12}\sqrt{2}\log\left(\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+\sqrt{x^6-1}+1\right)+\frac{1}{12}\sqrt{2}\log\left(-\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+\sqrt{x^6-1}+1\right)+\frac{2}{3}\left(x^6-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/4)/x,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^6 - 1)^(1/4))) - 1/6*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^6 - 1)^(1/4))) - 1/12*sqrt(2)*log(sqr
t(2)*(x^6 - 1)^(1/4) + sqrt(x^6 - 1) + 1) + 1/12*sqrt(2)*log(-sqrt(2)*(x^6
- 1)^(1/4) + sqrt(x^6 - 1) + 1) + 2/3*(x^6 - 1)^(1/4)
```

maple [C] time = 0.31, size = 64, normalized size = 0.71

$$-\frac{\operatorname{signum}\left(x^6-1\right)^{\frac{1}{4}}\left(\Gamma\left(\frac{3}{4}\right)x^6\,\mathrm{hypergeom}\left(\left[\frac{3}{4},1,1\right],[2,2],x^6\right)-4\left(4-3\ln(2)+\frac{\pi}{2}+6\ln(x)+i\pi\right)\Gamma\left(\frac{3}{4}\right)\right)}{24\Gamma\left(\frac{3}{4}\right)\left(-\operatorname{signum}\left(x^6-1\right)\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/4)/x,x)
```

```
[Out] -1/24/GAMMA(3/4)*signum(x^6-1)^(1/4)/(-signum(x^6-1))^(1/4)*(GAMMA(3/4)*x^6
*hypergeom([3/4,1,1],[2,2],x^6)-4*(4-3*ln(2)+1/2*Pi+6*ln(x)+I*Pi)*GAMMA(3/4
))
```

maxima [A] time = 0.41, size = 111, normalized size = 1.23

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(x^6-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{6}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(x^6-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{12}\sqrt{2}\log\left(\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+\sqrt{x^6-1}+1\right)+\frac{1}{12}\sqrt{2}\log\left(-\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+\sqrt{x^6-1}+1\right)+\frac{2}{3}\left(x^6-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/4)/x,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^6 - 1)^(1/4))) - 1/6*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^6 - 1)^(1/4))) - 1/12*sqrt(2)*log(sqr
t(2)*(x^6 - 1)^(1/4) + sqrt(x^6 - 1) + 1) + 1/12*sqrt(2)*log(-sqrt(2)*(x^6
- 1)^(1/4) + sqrt(x^6 - 1) + 1) + 2/3*(x^6 - 1)^(1/4)
```

mupad [B] time = 0.91, size = 54, normalized size = 0.60

$$\frac{2\left(x^6-1\right)^{1/4}}{3}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(x^6-1\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{6}-\frac{1}{6}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(x^6-1\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{6}+\frac{1}{6}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)^(1/4)/x,x)
```

```
[Out] (2*(x^6 - 1)^(1/4))/3 - 2^(1/2)*atan(2^(1/2)*(x^6 - 1)^(1/4)*(1/2 + 1i/2))*
(1/6 - 1i/6) - 2^(1/2)*atan(2^(1/2)*(x^6 - 1)^(1/4)*(1/2 - 1i/2))*(1/6 + 1i
/6)
```

sympy [C] time = 0.86, size = 39, normalized size = 0.43

$$-\frac{x^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{e^{2i\pi}}{x^6}\right)}{6\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/4)/x,x)
```

```
[Out] -x**(3/2)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), exp_polar(2*I*pi)/x**6)/(
6*gamma(3/4))
```

3.1109 $\int \frac{1}{x^7 \sqrt[3]{1+x^6}} \, dx$

Optimal. Leaf size=90

$$-\frac{\left(x^6+1\right)^{2/3}}{6x^6}-\frac{1}{18}\log\left(\sqrt[3]{x^6+1}-1\right)+\frac{1}{36}\log\left(\left(x^6+1\right)^{2/3}+\sqrt[3]{x^6+1}+1\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^6+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{6\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 51, 55, 618, 204, 31}

$$-\frac{\left(x^6+1\right)^{2/3}}{6x^6}-\frac{1}{12}\log\left(1-\sqrt[3]{x^6+1}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^6+1}+1}{\sqrt{3}}\right)}{6\sqrt{3}}+\frac{\log(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + x^6)^(1/3)),x]
```

```
[Out] -1/6*(1 + x^6)^(2/3)/x^6 - ArcTan[(1 + 2*(1 + x^6)^(1/3))/Sqrt[3]]/(6*Sqrt[
3]) + Log[x]/6 - Log[1 - (1 + x^6)^(1/3)]/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^7\sqrt[3]{1+x^6}}\,dx = \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt[3]{1+x}}\,dx, x, x^6\right)$$
$$= -\frac{\left(1+x^6\right)^{2/3}}{6x^6} - \frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx, x, x^6\right)$$
$$= -\frac{\left(1+x^6\right)^{2/3}}{6x^6} + \frac{\log(x)}{6} + \frac{1}{12}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+x^6}\right) - \frac{1}{12}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx\right.$$
$$= -\frac{\left(1+x^6\right)^{2/3}}{6x^6} + \frac{\log(x)}{6} - \frac{1}{12}\log\left(1-\sqrt[3]{1+x^6}\right) + \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+x^6}\right.$$
$$= -\frac{\left(1+x^6\right)^{2/3}}{6x^6} - \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^6}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{\log(x)}{6} - \frac{1}{12}\log\left(1-\sqrt[3]{1+x^6}\right)$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.29

$$\frac{1}{4}\left(x^6+1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};x^6+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + x^6)^(1/3)),x]
```

```
[Out] ((1 + x^6)^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, 1 + x^6])/4
```

IntegrateAlgebraic [A] time = 0.08, size = 90, normalized size = 1.00

$$-\frac{\left(x^6+1\right)^{2/3}}{6x^6} - \frac{1}{18}\log\left(\sqrt[3]{x^6+1}-1\right) + \frac{1}{36}\log\left(\left(x^6+1\right)^{2/3}+\sqrt[3]{x^6+1}+1\right) - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^6+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(1 + x^6)^(1/3)),x]
```

```
[Out] -1/6*(1 + x^6)^(2/3)/x^6 - ArcTan[1/Sqrt[3] + (2*(1 + x^6)^(1/3))/Sqrt[3]]/
(6*Sqrt[3]) - Log[-1 + (1 + x^6)^(1/3)]/18 + Log[1 + (1 + x^6)^(1/3) + (1 +
 x^6)^(2/3)]/36
```

fricas [A] time = 0.41, size = 79, normalized size = 0.88

$$-\frac{2\sqrt{3}\,x^6\arctan\left(\frac{2}{3}\sqrt{3}\left(x^6+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right) - x^6\log\left(\left(x^6+1\right)^{\frac{2}{3}}+\left(x^6+1\right)^{\frac{1}{3}}+1\right) + 2x^6\log\left(\left(x^6+1\right)^{\frac{1}{3}}-1\right) + 6\left(x^6+1\right)^{\frac{2}{3}}}{36\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/36*(2*sqrt(3)*x^6*arctan(2/3*sqrt(3)*(x^6 + 1)^(1/3) + 1/3*sqrt(3)) - x^
6*log((x^6 + 1)^(2/3) + (x^6 + 1)^(1/3) + 1) + 2*x^6*log((x^6 + 1)^(1/3) -
1) + 6*(x^6 + 1)^(2/3))/x^6
```

giac [A] time = 0.31, size = 66, normalized size = 0.73

$$-\frac{1}{18}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(x^{6}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^{6}+1\right)^{\frac{2}{3}}}{6\,x^{6}}+\frac{1}{36}\,\log\left(\left(x^{6}+1\right)^{\frac{2}{3}}+\left(x^{6}+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{18}\,\log\left(\left(x^{6}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+1)^(1/3),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3) + 1)) - 1/6*(x^6 + 1)^(
2/3)/x^6 + 1/36*log((x^6 + 1)^(2/3) + (x^6 + 1)^(1/3) + 1) - 1/18*log((x^6
+ 1)^(1/3) - 1)
```

maple [C] time = 0.28, size = 76, normalized size = 0.84

$$-\frac{\left(x^{6}+1\right)^{\frac{2}{3}}}{6x^{6}}-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^{6}\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],-x^{6}\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+6\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{36\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^6+1)^(1/3),x)
```

```
[Out] -1/6*(x^6+1)^(2/3)/x^6-1/36/Pi*3^(1/2)*GAMMA(2/3)*(-2/9*Pi*3^(1/2)/GAMMA(2/
3)*x^6*hypergeom([1,1,4/3],[2,2],-x^6)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+6*ln(
x))*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.42, size = 66, normalized size = 0.73

$$-\frac{1}{18}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(x^{6}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^{6}+1\right)^{\frac{2}{3}}}{6\,x^{6}}+\frac{1}{36}\,\log\left(\left(x^{6}+1\right)^{\frac{2}{3}}+\left(x^{6}+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{18}\,\log\left(\left(x^{6}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3) + 1)) - 1/6*(x^6 + 1)^(
2/3)/x^6 + 1/36*log((x^6 + 1)^(2/3) + (x^6 + 1)^(1/3) + 1) - 1/18*log((x^6
+ 1)^(1/3) - 1)
```

mupad [B] time = 0.94, size = 92, normalized size = 1.02

$$-\frac{\ln\left(\frac{\left(x^6+1\right)^{1/3}}{36}-\frac{1}{36}\right)}{18}-\ln\left(\frac{\left(x^6+1\right)^{1/3}}{36}-9\left(-\frac{1}{36}+\frac{\sqrt{3}\,1{}\mathrm{i}}{36}\right)^2\right)\left(-\frac{1}{36}+\frac{\sqrt{3}\,1{}\mathrm{i}}{36}\right)+\ln\left(\frac{\left(x^6+1\right)^{1/3}}{36}-9\left(\frac{1}{36}+\frac{\sqrt{3}\,1{}\mathrm{i}}{36}\right)^2\right)\left(\frac{1}{36}+\frac{\sqrt{3}\,1{}\mathrm{i}}{36}\right)-\frac{\left(x^6+1\right)^{2/3}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^6 + 1)^(1/3)),x)
```

```
[Out] log((x^6 + 1)^(1/3)/36 - 9*((3^(1/2)*1i)/36 + 1/36)^2)*((3^(1/2)*1i)/36 + 1
/36) - log((x^6 + 1)^(1/3)/36 - 9*((3^(1/2)*1i)/36 - 1/36)^2)*((3^(1/2)*1i)
/36 - 1/36) - log((x^6 + 1)^(1/3)/36 - 1/36)/18 - (x^6 + 1)^(2/3)/(6*x^6)
```

sympy [C] time = 1.01, size = 31, normalized size = 0.34

$$-\frac{\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{e^{i\pi}}{x^{6}}\right)}{6x^{8}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**6+1)**(1/3),x)
```

```
[Out] -gamma(4/3)*hyper((1/3, 4/3), (7/3,), exp_polar(I*pi)/x**6)/(6*x**8*gamma(7
/3))
```

3.1110 $\int \frac{(1+x^6)^{2/3}}{x^7}\,dx$

Optimal. Leaf size=90

$$-\frac{(x^6+1)^{2/3}}{6x^6}+\frac{1}{9}\log\left(\sqrt[3]{x^6+1}-1\right)-\frac{1}{18}\log\left(\left(x^6+1\right)^{2/3}+\sqrt[3]{x^6+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^6+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 47, 55, 618, 204, 31}

$$-\frac{(x^6+1)^{2/3}}{6x^6}+\frac{1}{6}\log\left(1-\sqrt[3]{x^6+1}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^6+1}+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)^(2/3)/x^7,x]
```

```
[Out] -1/6*(1 + x^6)^(2/3)/x^6 + ArcTan[(1 + 2*(1 + x^6)^(1/3))/Sqrt[3]]/(3*Sqrt[
3]) - Log[x]/3 + Log[1 - (1 + x^6)^(1/3)]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1+x^6\right)^{2/3}}{x^7}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{(1+x)^{2/3}}{x^2}\,dx,x,x^6\right)\\ &= -\frac{\left(1+x^6\right)^{2/3}}{6x^6}+\frac{1}{9}\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx,x,x^6\right)\\ &= -\frac{\left(1+x^6\right)^{2/3}}{6x^6}-\frac{\log(x)}{3}-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1+x^6}\right)+\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x\right.\\ &= -\frac{\left(1+x^6\right)^{2/3}}{6x^6}-\frac{\log(x)}{3}+\frac{1}{6}\log\left(1-\sqrt[3]{1+x^6}\right)-\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{1+x^6}\right)\\ &= -\frac{\left(1+x^6\right)^{2/3}}{6x^6}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^6}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\log(x)}{3}+\frac{1}{6}\log\left(1-\sqrt[3]{1+x^6}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.29

$$\frac{1}{10}\left(x^6+1\right)^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};x^6+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)^(2/3)/x^7,x]
```

```
[Out] ((1 + x^6)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 + x^6])/10
```

IntegrateAlgebraic [A] time = 0.06, size = 90, normalized size = 1.00

$$-\frac{\left(x^6+1\right)^{2/3}}{6x^6}+\frac{1}{9}\log\left(\sqrt[3]{x^6+1}-1\right)-\frac{1}{18}\log\left(\left(x^6+1\right)^{2/3}+\sqrt[3]{x^6+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^6+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^6)^(2/3)/x^7,x]
```

```
[Out] -1/6*(1 + x^6)^(2/3)/x^6 + ArcTan[1/Sqrt[3] + (2*(1 + x^6)^(1/3))/Sqrt[3]]/
(3*Sqrt[3]) + Log[-1 + (1 + x^6)^(1/3)]/9 - Log[1 + (1 + x^6)^(1/3) + (1 +
x^6)^(2/3)]/18
```

fricas [A] time = 0.41, size = 79, normalized size = 0.88

$$\frac{2\sqrt{3}x^6\arctan\left(\frac{2}{3}\sqrt{3}\left(x^6+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-x^6\log\left(\left(x^6+1\right)^{\frac{2}{3}}+\left(x^6+1\right)^{\frac{1}{3}}+1\right)+2x^6\log\left(\left(x^6+1\right)^{\frac{1}{3}}-1\right)-3\left(x^6+1\right)^{\frac{2}{3}}}{18x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(2/3)/x^7,x, algorithm="fricas")
```

```
[Out] 1/18*(2*sqrt(3)*x^6*arctan(2/3*sqrt(3)*(x^6 + 1)^(1/3) + 1/3*sqrt(3)) - x^6
*log((x^6 + 1)^(2/3) + (x^6 + 1)^(1/3) + 1) + 2*x^6*log((x^6 + 1)^(1/3) - 1
) - 3*(x^6 + 1)^(2/3))/x^6
```

giac [A] time = 0.16, size = 66, normalized size = 0.73

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^6+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^6+1\right)^{\frac{2}{3}}}{6x^6}-\frac{1}{18}\log\left(\left(x^6+1\right)^{\frac{2}{3}}+\left(x^6+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{9}\log\left(\left(x^6+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(2/3)/x^7,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3) + 1)) - 1/6*(x^6 + 1)^(2/
3)/x^6 - 1/18*log((x^6 + 1)^(2/3) + (x^6 + 1)^(1/3) + 1) + 1/9*log((x^6 + 1
)^(1/3) - 1)
```

maple [C] time = 0.34, size = 76, normalized size = 0.84

$$-\frac{\left(x^6+1\right)^{\frac{2}{3}}}{6x^6}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^6\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],-x^6\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+6\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{18\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)^(2/3)/x^7,x)
```

```
[Out] -1/6*(x^6+1)^(2/3)/x^6+1/18/Pi*3^(1/2)*GAMMA(2/3)*(-2/9*Pi*3^(1/2)/GAMMA(2/
3)*x^6*hypergeom([1,1,4/3],[2,2],-x^6)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+6*ln(
x))*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.41, size = 66, normalized size = 0.73

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^6+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^6+1\right)^{\frac{2}{3}}}{6x^6}-\frac{1}{18}\log\left(\left(x^6+1\right)^{\frac{2}{3}}+\left(x^6+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{9}\log\left(\left(x^6+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^(2/3)/x^7,x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3) + 1)) - 1/6*(x^6 + 1)^(2/
3)/x^6 - 1/18*log((x^6 + 1)^(2/3) + (x^6 + 1)^(1/3) + 1) + 1/9*log((x^6 + 1
)^(1/3) - 1)
```

mupad [B] time = 0.90, size = 92, normalized size = 1.02

$$\frac{\ln\left(\frac{\left(x^6+1\right)^{1/3}}{9}-\frac{1}{9}\right)}{9}+\ln\left(\frac{\left(x^6+1\right)^{1/3}}{9}-9\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2\right)\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)-\ln\left(\frac{\left(x^6+1\right)^{1/3}}{9}-9\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2\right)\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)-\frac{\left(x^6+1\right)^{2/3}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)^(2/3)/x^7,x)
```

```
[Out] log((x^6 + 1)^(1/3)/9 - 1/9)/9 + log((x^6 + 1)^(1/3)/9 - 9*((3^(1/2)*1i)/18
 - 1/18)^2)*((3^(1/2)*1i)/18 - 1/18) - log((x^6 + 1)^(1/3)/9 - 9*((3^(1/2)*
1i)/18 + 1/18)^2)*((3^(1/2)*1i)/18 + 1/18) - (x^6 + 1)^(2/3)/(6*x^6)
```

sympy [C] time = 1.09, size = 32, normalized size = 0.36

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^6}\right)}{6x^2\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)**(2/3)/x**7,x)
```

```
[Out] -gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_polar(I*pi)/x**6)/(6*x**2*gamma(
4/3))
```

3.1111 $\int \frac{(-3+k^2)x+2k^2x^3}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\,(-1+d+(3-dk^2)x^2-3x^4+x^6)}\,dx$

Optimal. Leaf size=90

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{x^2-1}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{x^2-1}\right)}{d^{3/4}}$$

Rubi [F] time = 6.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-3+k^2)\,x+2k^2x^3}{\sqrt[4]{(1-x^2)\,(1-k^2x^2)}\,\left(-1+d+(3-dk^2)\,x^2-3x^4+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + k^2)*x + 2*k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/4)*(-1 + d + (
3 - d*k^2)*x^2 - 3*x^4 + x^6)),x]
```

```
[Out] (-4*k^2*(1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*Defer[Subst][Defer[Int][x^2/((1
 - k^2 + k^2*x^4)^(1/4)*(d*(1 - k^2) + d*k^2*x^4 - x^12)), x], x, (1 - x^2)
^(1/4)])/((1 - x^2)*(1 - k^2*x^2))^(1/4) - (4*k^2*(1 - x^2)^(1/4)*(1 - k^2*
x^2)^(1/4)*Defer[Subst][Defer[Int][x^6/((1 - k^2 + k^2*x^4)^(1/4)*(-(d*(1 -
 k^2)) - d*k^2*x^4 + x^12)), x], x, (1 - x^2)^(1/4)])/((1 - x^2)*(1 - k^2*x
^2))^(1/4) + (2*(3 - k^2)*(1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*Defer[Subst][
Defer[Int][x^2/((1 - k^2 + k^2*x^4)^(1/4)*(d - x^12 + d*k^2*(-1 + x^4))), x
], x, (1 - x^2)^(1/4)])/((1 - x^2)*(1 - k^2*x^2))^(1/4)
```

Rubi steps

$$\int \frac{\left(-3+k^2\right)x+2k^2x^3}{\sqrt[4]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d+\left(3-dk^2\right)x^2-3x^4+x^6\right)}\,dx = \int \frac{x\left(-3+k^2+2k^2x^2\right)}{\sqrt[4]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d+\left(3-dk^2\right)\right.}$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{-3+k^2+2k}{\sqrt[4]{(1-x)\left(1-k^2x\right)}\left(-1+d+\left(3\right.\right.}\right.$$

$$= \frac{\left(\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\text{Subst}\left(\int \frac{-3}{\sqrt[4]{1-x}\,\sqrt[4]{1-k^2x}\,(-}\right.}{2\sqrt[4]{\left(1-x^2\right)\left(1-k^2\right.}}$$

$$= \frac{\left(\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\text{Subst}\left(\int\left(\frac{}{\sqrt[4]{1-x}\,\sqrt[4]{1-k^2x}\,(}\right.\right.}{2}$$

$$= \frac{\left(k^2\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{1-x}\,\sqrt[4]{1-k^2}}\right.}{\sqrt[4]{\left(1-x^2\right)\left(1-k^2\right.}}$$

$$= -\frac{\left(4k^2\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{1+k^2(-}}\right.}{\sqrt[4]{\left(1-x^2\right)\left(1\right.}}$$

$$= -\frac{\left(4k^2\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{1+k^2(-}}\right.}{\sqrt[4]{\left(1-x^2\right)\left(1\right.}}$$

$$= -\frac{\left(4k^2\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{1-k^2+k^2}}\right.}{\sqrt[4]{\left(1-x^2\right)\left(1-\right.}}$$

$$= -\frac{\left(4k^2\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\text{Subst}\left(\int\left(\frac{}{\sqrt[4]{1-k^2+}}\right.\right.}{}$$

$$= -\frac{\left(4k^2\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\text{Subst}\left(\int \frac{-}{\sqrt[4]{1-k^2+k^2}}\right.}{\sqrt[4]{\left(1-x^2\right)\left(1-\right.}}$$

Mathematica [F] time = 2.91, size = 0, normalized size = 0.00

$$\int \frac{\left(-3+k^2\right)x+2k^2x^3}{\sqrt[4]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d+\left(3-dk^2\right)x^2-3x^4+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + k^2)*x + 2*k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/4)*(-1 +
 d + (3 - d*k^2)*x^2 - 3*x^4 + x^6)),x]
```

```
[Out] Integrate[((-3 + k^2)*x + 2*k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/4)*(-1 +
 d + (3 - d*k^2)*x^2 - 3*x^4 + x^6)), x]
```

IntegrateAlgebraic [A] time = 12.93, size = 90, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{x^2-1}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{x^2-1}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + k^2)*x + 2*k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1
/4)*(-1 + d + (3 - d*k^2)*x^2 - 3*x^4 + x^6)),x]
```

```
[Out] ArcTan[(d^(1/4)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4))/(-1 + x^2)]/d^(3/4) -
 ArcTanh[(d^(1/4)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4))/(-1 + x^2)]/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((k^2-3)*x+2*k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d+(-d*k^2
+3)*x^2-3*x^4+x^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,k^2x^3+\left(k^2-3\right)x}{\left(x^6-3\,x^4-\left(dk^2-3\right)x^2+d-1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((k^2-3)*x+2*k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d+(-d*k^2
+3)*x^2-3*x^4+x^6),x, algorithm="giac")
```

```
[Out] integrate((2*k^2*x^3 + (k^2 - 3)*x)/((x^6 - 3*x^4 - (d*k^2 - 3)*x^2 + d - 1
)*((k^2*x^2 - 1)*(x^2 - 1))^(1/4)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(k^2-3\right)x+2k^2x^3}{\left(\left(-x^2+1\right)\left(-k^2x^2+1\right)\right)^{\frac{1}{4}}\left(-1+d+\left(-d\,k^2+3\right)x^2-3x^4+x^6\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((k^2-3)*x+2*k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d+(-d*k^2+3)*x^
2-3*x^4+x^6),x)
```

```
[Out] int(((k^2-3)*x+2*k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d+(-d*k^2+3)*x^
2-3*x^4+x^6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,k^2x^3+\left(k^2-3\right)x}{\left(x^6-3\,x^4-\left(dk^2-3\right)x^2+d-1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((k^2-3)*x+2*k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d+(-d*k^2
+3)*x^2-3*x^4+x^6),x, algorithm="maxima")
```

```
[Out] integrate((2*k^2*x^3 + (k^2 - 3)*x)/((x^6 - 3*x^4 - (d*k^2 - 3)*x^2 + d - 1
)*((k^2*x^2 - 1)*(x^2 - 1))^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x\left(k^2-3\right)+2k^2x^3}{\left(\left(x^2-1\right)\left(k^2x^2-1\right)\right)^{1/4}\left(-x^6+3x^4+\left(d\,k^2-3\right)x^2-d+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(k^2 - 3) + 2*k^2*x^3)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/4)*(x^2*(d*k^2
 - 3) - d + 3*x^4 - x^6 + 1)),x)
```

```
[Out] -int((x*(k^2 - 3) + 2*k^2*x^3)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/4)*(x^2*(d*k^2
 - 3) - d + 3*x^4 - x^6 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((k**2-3)*x+2*k**2*x**3)/((-x**2+1)*(-k**2*x**2+1))**(1/4)/(-1+d+
(-d*k**2+3)*x**2-3*x**4+x**6),x)
```

```
[Out] Timed out
```

3.1112 $\int \frac{(-1+x^7)^{2/3}(3+4x^7)}{x^3(-1+x^3+x^7)}\,dx$

Optimal. Leaf size=90

$$\log\left(\sqrt[3]{x^7-1}+x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^7-1}-x}\right)+\frac{3\left(x^7-1\right)^{2/3}}{2x^2}-\frac{1}{2}\log\left(-\sqrt[3]{x^7-1}\,x+\left(x^7-1\right)^{2/3}+x^2\right)$$

Rubi [F] time = 0.72, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1+x^7\right)^{2/3}\left(3+4x^7\right)}{x^3\left(-1+x^3+x^7\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^7)^(2/3)*(3 + 4*x^7))/(x^3*(-1 + x^3 + x^7)),x]
```

```
[Out] (3*(-1 + x^7)^(2/3)*Hypergeometric2F1[-2/3, -2/7, 5/7, x^7])/(2*x^2*(1 - x^
7)^(2/3)) + 3*Defer[Int][(-1 + x^7)^(2/3)/(-1 + x^3 + x^7), x] + 7*Defer[In
t][(x^4*(-1 + x^7)^(2/3))/(-1 + x^3 + x^7), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^7\right)^{2/3}\left(3+4x^7\right)}{x^3\left(-1+x^3+x^7\right)}\,dx &= \int \left(-\frac{3\left(-1+x^7\right)^{2/3}}{x^3}+\frac{\left(3+7x^4\right)\left(-1+x^7\right)^{2/3}}{-1+x^3+x^7}\right)dx\\
&= -\left(3\int \frac{\left(-1+x^7\right)^{2/3}}{x^3}\,dx\right)+\int \frac{\left(3+7x^4\right)\left(-1+x^7\right)^{2/3}}{-1+x^3+x^7}\,dx\\
&= -\frac{\left(3\left(-1+x^7\right)^{2/3}\right)\int \frac{\left(1-x^7\right)^{2/3}}{x^3}\,dx}{\left(1-x^7\right)^{2/3}}+\int \left(\frac{3\left(-1+x^7\right)^{2/3}}{-1+x^3+x^7}+\frac{7x^4\left(-1+x^7\right)^{2/3}}{-1+x^3+x^7}\right)dx\\
&= \frac{3\left(-1+x^7\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{7};\frac{5}{7};x^7\right)}{2x^2\left(1-x^7\right)^{2/3}}+3\int \frac{\left(-1+x^7\right)^{2/3}}{-1+x^3+x^7}\,dx+7\int \frac{x^4\left(-1+x^7\right)^{2/3}}{-1+x^3+x^7}\,d
\end{aligned}$$

Mathematica [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^7\right)^{2/3}\left(3+4x^7\right)}{x^3\left(-1+x^3+x^7\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^7)^(2/3)*(3 + 4*x^7))/(x^3*(-1 + x^3 + x^7)),x]
```

```
[Out] Integrate[((-1 + x^7)^(2/3)*(3 + 4*x^7))/(x^3*(-1 + x^3 + x^7)), x]
```

IntegrateAlgebraic [A] time = 17.79, size = 90, normalized size = 1.00

$$\log\left(\sqrt[3]{x^7-1}+x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^7-1}-x}\right)+\frac{3\left(x^7-1\right)^{2/3}}{2x^2}-\frac{1}{2}\log\left(-\sqrt[3]{x^7-1}\,x+\left(x^7-1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^7)^(2/3)*(3 + 4*x^7))/(x^3*(-1 + x^3 + x^7)),x]
```

```
[Out] (3*(-1 + x^7)^(2/3))/(2*x^2) + Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(-1 + x^7
)^(1/3))] + Log[x + (-1 + x^7)^(1/3)] - Log[x^2 - x*(-1 + x^7)^(1/3) + (-1
+ x^7)^(2/3)]/2
```

fricas [A] time = 16.09, size = 131, normalized size = 1.46

$$\frac{2\sqrt{3}x^2\arctan\left(-\frac{26962\sqrt{3}\left(x^7-1\right)^{\frac{1}{3}}x^2-60268\sqrt{3}\left(x^7-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(34656x^7-8959x^3-34656\right)}{54872x^7+4913x^3-54872}\right)+x^2\log\left(\frac{x^7+x^3+3\left(x^7-1\right)^{\frac{1}{3}}x^2+3\left(x^7-1\right)^{\frac{2}{3}}x-1}{x^7+x^3-1}\right)+3\left(x^7-1\right)^{\frac{2}{3}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^7-1)^(2/3)*(4*x^7+3)/x^3/(x^7+x^3-1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*x^2*arctan(-(26962*sqrt(3)*(x^7 - 1)^(1/3)*x^2 - 60268*sqrt(
3)*(x^7 - 1)^(2/3)*x + sqrt(3)*(34656*x^7 - 8959*x^3 - 34656))/(54872*x^7 +
 4913*x^3 - 54872)) + x^2*log((x^7 + x^3 + 3*(x^7 - 1)^(1/3)*x^2 + 3*(x^7 -
 1)^(2/3)*x - 1)/(x^7 + x^3 - 1)) + 3*(x^7 - 1)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4x^7+3\right)\left(x^7-1\right)^{\frac{2}{3}}}{\left(x^7+x^3-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^7-1)^(2/3)*(4*x^7+3)/x^3/(x^7+x^3-1),x, algorithm="giac")
```

```
[Out] integrate((4*x^7 + 3)*(x^7 - 1)^(2/3)/((x^7 + x^3 - 1)*x^3), x)
```

maple [C] time = 5.13, size = 292, normalized size = 3.24

$$\frac{3\left(x^7-1\right)^{\frac{2}{3}}}{2x^2}+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(-\frac{-x^7+\left(x^7-1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^7-1\right)^{\frac{1}{3}}x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3+2\left(x^7-1\right)^{\frac{2}{3}}x-2\left(x^7-1\right)^{\frac{1}{3}}x^2+x^3+1}{x^7+x^3-1}\right)-\ln\left(\frac{x^7+\left(x^7-1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^7-1\right)^{\frac{1}{3}}x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3-\left(x^7-1\right)^{\frac{2}{3}}x+\left(x^7-1\right)^{\frac{1}{3}}x^2-1}{x^7+x^3-1}\right)\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-\ln\left(\frac{x^7+\left(x^7-1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^7-1\right)^{\frac{1}{3}}x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3-\left(x^7-1\right)^{\frac{2}{3}}x+\left(x^7-1\right)^{\frac{1}{3}}x^2-1}{x^7+x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7-1)^(2/3)*(4*x^7+3)/x^3/(x^7+x^3-1),x)
```

```
[Out] 3/2*(x^7-1)^(2/3)/x^2+RootOf(_Z^2+_Z+1)*ln(-(-x^7+(x^7-1)^(2/3)*RootOf(_Z^2
+_Z+1)*x-RootOf(_Z^2+_Z+1)*(x^7-1)^(1/3)*x^2+RootOf(_Z^2+_Z+1)*x^3+2*(x^7-1
)^(2/3)*x-2*(x^7-1)^(1/3)*x^2+x^3+1)/(x^7+x^3-1))-ln((x^7+(x^7-1)^(2/3)*Roo
tOf(_Z^2+_Z+1)*x-RootOf(_Z^2+_Z+1)*(x^7-1)^(1/3)*x^2+RootOf(_Z^2+_Z+1)*x^3-
(x^7-1)^(2/3)*x+(x^7-1)^(1/3)*x^2-1)/(x^7+x^3-1))*RootOf(_Z^2+_Z+1)-ln((x^7
+(x^7-1)^(2/3)*RootOf(_Z^2+_Z+1)*x-RootOf(_Z^2+_Z+1)*(x^7-1)^(1/3)*x^2+Root
Of(_Z^2+_Z+1)*x^3-(x^7-1)^(2/3)*x+(x^7-1)^(1/3)*x^2-1)/(x^7+x^3-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4x^7+3\right)\left(x^7-1\right)^{\frac{2}{3}}}{\left(x^7+x^3-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^7-1)^(2/3)*(4*x^7+3)/x^3/(x^7+x^3-1),x, algorithm="maxima")
```

```
[Out] integrate((4*x^7 + 3)*(x^7 - 1)^(2/3)/((x^7 + x^3 - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^7-1\right)^{2/3}\,\left(4\,x^7+3\right)}{x^3\,\left(x^7+x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^7 - 1)^(2/3)*(4*x^7 + 3))/(x^3*(x^3 + x^7 - 1)),x)
```

```
[Out] int(((x^7 - 1)^(2/3)*(4*x^7 + 3))/(x^3*(x^3 + x^7 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((x-1)\left(x^6+x^5+x^4+x^3+x^2+x+1\right)\right)^{\frac{2}{3}}\left(4x^7+3\right)}{x^3\left(x^7+x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**7-1)**(2/3)*(4*x**7+3)/x**3/(x**7+x**3-1),x)
```

```
[Out] Integral(((x - 1)*(x**6 + x**5 + x**4 + x**3 + x**2 + x + 1))**(2/3)*(4*x**
7 + 3)/(x**3*(x**7 + x**3 - 1)), x)
```

3.1113 $\int \frac{x}{\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{20a\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{3b^2x}-\frac{4\sqrt{a^2x^2-bx}\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{3b^2x^2}$$

Rubi [F] time = 4.39, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/(Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2)),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[-b + a^2*x]*Defer[Subst][Defer[Int][x^2/(Sqrt[-b + a^2*x^2]
*(a*x^4 + x^2*Sqrt[-(b*x^2) + a^2*x^4])^(3/2)), x], x, Sqrt[x]])/Sqrt[-(b*x
) + a^2*x^2]
```

Rubi steps

$$\int \frac{x}{\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx=\frac{\left(\sqrt{x}\sqrt{-b+a^2x}\right)\int \frac{\sqrt{x}}{\sqrt{-b+a^2x}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx}{\sqrt{-bx+a^2x^2}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{-b+a^2x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-b+a^2x^2}\left(ax^4+x^2\sqrt{-bx^2+a^2x^4}\right)^{3/2}}\,dx,\right.}{\sqrt{-bx+a^2x^2}}$$

Mathematica [F] time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/(Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2
)),x]
```

```
[Out] Integrate[x/(Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2
)), x]
```

IntegrateAlgebraic [A] time = 5.02, size = 90, normalized size = 1.00

$$\frac{20a\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{3b^2x}-\frac{4\sqrt{a^2x^2-bx}\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{3b^2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x
^2])^(3/2)),x]
```

```
[Out] (20*a*Sqrt[x*(a*x + Sqrt[-(b*x) + a^2*x^2])])/(3*b^2*x) - (4*Sqrt[-(b*x) +
a^2*x^2]*Sqrt[x*(a*x + Sqrt[-(b*x) + a^2*x^2])])/(3*b^2*x^2)
```

fricas [A] time = 0.41, size = 53, normalized size = 0.59

$$\frac{4\sqrt{ax^2 + \sqrt{a^2x^2 - bx}\,x}\left(5\,ax - \sqrt{a^2x^2 - bx}\right)}{3\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algo
rithm="fricas")
```

```
[Out] 4/3*sqrt(a*x^2 + sqrt(a^2*x^2 - b*x)*x)*(5*a*x - sqrt(a^2*x^2 - b*x))/(b^2*
x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a^2x^2 - bx}\left(ax^2 + \sqrt{a^2x^2 - bx}\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algo
rithm="giac")
```

```
[Out] integrate(x/(sqrt(a^2*x^2 - b*x)*(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2)), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a^2x^2 - bx}\,\left(a\,x^2 + x\sqrt{a^2x^2 - bx}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

```
[Out] int(x/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a^2x^2 - bx}\left(ax^2 + \sqrt{a^2x^2 - bx}\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algo
rithm="maxima")
```

```
[Out] integrate(x/(sqrt(a^2*x^2 - b*x)*(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a^2\,x^2 - b\,x}\,\left(a\,x^2 + x\,\sqrt{a^2\,x^2 - b\,x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a^2*x^2 - b*x)^(1/2)*(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2)),x)
```

```
[Out] int(x/((a^2*x^2 - b*x)^(1/2)*(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x\left(ax+\sqrt{a^2x^2-bx}\right)\right)^{\frac{3}{2}}\sqrt{x\left(a^2x-b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2*x**2-b*x)**(1/2)/(a*x**2+x*(a**2*x**2-b*x)**(1/2))**(3/2)
,x)
```

```
[Out] Integral(x/((x*(a*x + sqrt(a**2*x**2 - b*x)))**(3/2)*sqrt(x*(a**2*x - b))),
 x)
```

3.1114 $\int \frac{-\sqrt{ab}+x}{\sqrt{x(a+x)(b+x)}\left(\sqrt{ab}+x\right)}\,dx$

Optimal. Leaf size=91

$$\frac{2\tan^{-1}\left(\frac{(b+x)\left(\frac{x}{\sqrt{2\sqrt{ab}-a-b}}+\frac{a}{\sqrt{2\sqrt{ab}-a-b}}\right)}{\sqrt{x^2(a+b)+abx+x^3}}\right)}{\sqrt{2\sqrt{ab}-a-b}}$$

Rubi [C] time = 0.87, antiderivative size = 146, normalized size of antiderivative = 1.60, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6718, 1607, 169, 538, 537, 117, 116}

$$\frac{2\sqrt{-a}\sqrt{x}\sqrt{\frac{x}{a}+1}\sqrt{\frac{x}{b}+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{-a}}\right)|\frac{a}{b}\right)}{\sqrt{x(a+x)(b+x)}}-\frac{4\sqrt{-a}\sqrt{x}\sqrt{\frac{x}{a}+1}\sqrt{\frac{x}{b}+1}\,\Pi\left(\frac{a}{\sqrt{ab}};\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{-a}}\right)|\frac{a}{b}\right)}{\sqrt{x(a+x)(b+x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[a*b] + x)/(Sqrt[x*(a + x)*(b + x)]*(Sqrt[a*b] + x)),x]
```

```
[Out] (2*Sqrt[-a]*Sqrt[x]*Sqrt[1 + x/a]*Sqrt[1 + x/b]*EllipticF[ArcSin[Sqrt[x]/Sq
rt[-a]], a/b])/Sqrt[x*(a + x)*(b + x)] - (4*Sqrt[-a]*Sqrt[x]*Sqrt[1 + x/a]*
Sqrt[1 + x/b]*EllipticPi[a/Sqrt[a*b], ArcSin[Sqrt[x]/Sqrt[-a]], a/b])/Sqrt[
x*(a + x)*(b + x)]
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] && !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !FreeQ[w, x] &&  !Fr
eeQ[z, x]
```

Rubi steps

$$\int \frac{-\sqrt{ab}+x}{\sqrt{x(a+x)(b+x)}\left(\sqrt{ab}+x\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{a+x}\sqrt{b+x}\right)\int \frac{-\sqrt{ab}+x}{\sqrt{x}\sqrt{a+x}\sqrt{b+x}\left(\sqrt{ab}+x\right)}\,dx}{\sqrt{x(a+x)(b+x)}}$$

$$= \frac{\left(\sqrt{x}\sqrt{a+x}\sqrt{b+x}\right)\int \frac{1}{\sqrt{x}\sqrt{a+x}\sqrt{b+x}}\,dx}{\sqrt{x(a+x)(b+x)}} - \frac{\left(2\sqrt{ab}\sqrt{x}\sqrt{a+x}\sqrt{b+x}\right)\int \ldots}{\sqrt{x(a+x)(b\ldots}}$$

$$= \frac{\left(4\sqrt{ab}\sqrt{x}\sqrt{a+x}\sqrt{b+x}\right)\mathrm{Subst}\left(\int \frac{1}{\left(-\sqrt{ab}-x^2\right)\sqrt{a+x^2}\sqrt{b+x^2}}\,dx, x, \sqrt{x}\right)}{\sqrt{x(a+x)(b+x)}} + \ldots$$

$$= \frac{2\sqrt{-a}\sqrt{x}\sqrt{1+\frac{x}{a}}\sqrt{1+\frac{x}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{-a}}\right)|\frac{a}{b}\right)}{\sqrt{x(a+x)(b+x)}} + \frac{\left(4\sqrt{ab}\sqrt{x}\sqrt{b+x}\sqrt{1+\ldots}\right.}{\ldots}$$

$$= \frac{2\sqrt{-a}\sqrt{x}\sqrt{1+\frac{x}{a}}\sqrt{1+\frac{x}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{-a}}\right)|\frac{a}{b}\right)}{\sqrt{x(a+x)(b+x)}} + \frac{\left(4\sqrt{ab}\sqrt{x}\sqrt{1+\frac{x}{a}}\sqrt{1+\ldots}\right.}{\ldots}$$

$$= \frac{2\sqrt{-a}\sqrt{x}\sqrt{1+\frac{x}{a}}\sqrt{1+\frac{x}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{-a}}\right)|\frac{a}{b}\right)}{\sqrt{x(a+x)(b+x)}} - \frac{4\sqrt{-a}\sqrt{x}\sqrt{1+\frac{x}{a}}\sqrt{1+\frac{x}{b}}\ldots}{\sqrt{x(a+\ldots}}$$

Mathematica [C] time = 0.58, size = 101, normalized size = 1.11

$$-\frac{2ax^{3/2}\sqrt{\frac{a}{x}+1}\sqrt{\frac{b}{x}+1}\left(F\left(\sin^{-1}\left(\frac{\sqrt{-a}}{\sqrt{x}}\right)|\frac{b}{a}\right)-2\Pi\left(\frac{b}{\sqrt{ab}};\sin^{-1}\left(\frac{\sqrt{-a}}{\sqrt{x}}\right)|\frac{b}{a}\right)\right)}{(-a)^{3/2}\sqrt{x(a+x)(b+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[a*b] + x)/(Sqrt[x*(a + x)*(b + x)]*(Sqrt[a*b] + x)),x]
```

```
[Out] (-2*a*Sqrt[1 + a/x]*Sqrt[1 + b/x]*x^(3/2)*(EllipticF[ArcSin[Sqrt[-a]/Sqrt[x
]], b/a] - 2*EllipticPi[b/Sqrt[a*b], ArcSin[Sqrt[-a]/Sqrt[x]], b/a]))/((-a)
^(3/2)*Sqrt[x*(a + x)*(b + x)])
```

IntegrateAlgebraic [A] time = 0.76, size = 91, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{(b+x)\left(\frac{x}{\sqrt{2\sqrt{ab}-a-b}}+\frac{a}{\sqrt{2\sqrt{ab}-a-b}}\right)}{\sqrt{x^2(a+b)+abx+x^3}}\right)}{\sqrt{2\sqrt{ab}-a-b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-Sqrt[a*b] + x)/(Sqrt[x*(a + x)*(b + x)]*(Sqrt[a*b] + x
)),x]
```

```
[Out] (2*ArcTan[((b + x)*(a/Sqrt[-a - b + 2*Sqrt[a*b]] + x/Sqrt[-a - b + 2*Sqrt[a
*b]]))/Sqrt[a*b*x + (a + b)*x^2 + x^3]])/Sqrt[-a - b + 2*Sqrt[a*b]]
```

fricas [A] time = 0.57, size = 791, normalized size = 8.69

$$\left[-\frac{\sqrt{a+b+2\sqrt{ab}}\log\left(-\frac{a^5b^4-a^4b^5+(a-b)x^8+8(a^2-b^2)x^7+4(2a^3+17a^2b-17ab^2-2b^3)x^6+120(a^3b-ab^3)x^5+2(24a^4b+91a^3b^2-91a^2b^3-24ab^4)x^4+120(a^4b^2-a^2b^4)x^3+4(2a^5b^2+17a^4b^3-17a^3b^4-2a^2b^5)x^2+4\left(a^4b^3+a^3b^4+(a+b)x^6+2(a^2+8ab+b^2)x^5+31(a^2b+ab^2)x^4+4(3a^3b+16a^2b^2+3ab^3)x^3+31(a^3b^2+a^2b^3)x^2+2(a^4b^2+8a^3b^3+a^2b^4)x-2\left(a^3b^3+5(a+b)x^5+x^6+(4a^2+23ab+4b^2)x^4+22(a^2b+ab^2)x^3+(4a^3b+23a^2b^2+4ab^3)x^2+5(a^3b^2+a^2b^3)x\right)\sqrt{ab}\right)\sqrt{abx+(a+b)x^2+x^3}\sqrt{a+b+2\sqrt{ab}}+8(a^5b^3-a^3b^5)x-16\left((a-b)x^7+3(a^2-b^2)x^6+(2a^3+9a^2b-9ab^2-2b^3)x^5+10(a^3b-ab^3)x^4+(2a^4b+9a^3b^2-9a^2b^3-2ab^4)x^3+3(a^4b^2-a^2b^4)x^2+(a^4b^3-a^3b^4)x\right)\sqrt{ab}}{a^4b^4-4a^3b^3x^2+6a^2b^2x^4-4abx^6+x^8}\right)}{2(a-b)}, \frac{\sqrt{-a-b-2\sqrt{ab}}\arctan\left(\frac{\sqrt{abx+(a+b)x^2+x^3}\left(ab+2(a+b)x+x^2-2\sqrt{ab}x\right)\sqrt{-a-b-2\sqrt{ab}}}{2\left((a-b)x^3+(a^2-b^2)x^2+(a^2b-ab^2)x\right)}\right)}{a-b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(a*b)^(1/2)+x)/(x*(a+x)*(b+x))^(1/2)/((a*b)^(1/2)+x),x, algorit
hm="fricas")
```

```
[Out] [-1/2*sqrt(a + b + 2*sqrt(a*b))*log(-(a^5*b^4 - a^4*b^5 + (a - b)*x^8 + 8*(
a^2 - b^2)*x^7 + 4*(2*a^3 + 17*a^2*b - 17*a*b^2 - 2*b^3)*x^6 + 120*(a^3*b -
 a*b^3)*x^5 + 2*(24*a^4*b + 91*a^3*b^2 - 91*a^2*b^3 - 24*a*b^4)*x^4 + 120*(
a^4*b^2 - a^2*b^4)*x^3 + 4*(2*a^5*b^2 + 17*a^4*b^3 - 17*a^3*b^4 - 2*a^2*b^5
)*x^2 + 4*(a^4*b^3 + a^3*b^4 + (a + b)*x^6 + 2*(a^2 + 8*a*b + b^2)*x^5 + 31
*(a^2*b + a*b^2)*x^4 + 4*(3*a^3*b + 16*a^2*b^2 + 3*a*b^3)*x^3 + 31*(a^3*b^2
 + a^2*b^3)*x^2 + 2*(a^4*b^2 + 8*a^3*b^3 + a^2*b^4)*x - 2*(a^3*b^3 + 5*(a +
 b)*x^5 + x^6 + (4*a^2 + 23*a*b + 4*b^2)*x^4 + 22*(a^2*b + a*b^2)*x^3 + (4*
a^3*b + 23*a^2*b^2 + 4*a*b^3)*x^2 + 5*(a^3*b^2 + a^2*b^3)*x)*sqrt(a*b))*sqr
t(a*b*x + (a + b)*x^2 + x^3)*sqrt(a + b + 2*sqrt(a*b)) + 8*(a^5*b^3 - a^3*b
^5)*x - 16*((a - b)*x^7 + 3*(a^2 - b^2)*x^6 + (2*a^3 + 9*a^2*b - 9*a*b^2 -
2*b^3)*x^5 + 10*(a^3*b - a*b^3)*x^4 + (2*a^4*b + 9*a^3*b^2 - 9*a^2*b^3 - 2*
a*b^4)*x^3 + 3*(a^4*b^2 - a^2*b^4)*x^2 + (a^4*b^3 - a^3*b^4)*x)*sqrt(a*b))/
(a^4*b^4 - 4*a^3*b^3*x^2 + 6*a^2*b^2*x^4 - 4*a*b*x^6 + x^8))/(a - b), sqrt(
-a - b - 2*sqrt(a*b))*arctan(1/2*sqrt(a*b*x + (a + b)*x^2 + x^3)*(a*b + 2*(
a + b)*x + x^2 - 2*sqrt(a*b)*x)*sqrt(-a - b - 2*sqrt(a*b))/((a - b)*x^3 + (
a^2 - b^2)*x^2 + (a^2*b - a*b^2)*x))/(a - b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{ab}}{\sqrt{(a+x)(b+x)x}\left(x+\sqrt{ab}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(a*b)^(1/2)+x)/(x*(a+x)*(b+x))^(1/2)/((a*b)^(1/2)+x),x, algorit
hm="giac")
```

```
[Out] integrate((x - sqrt(a*b))/(sqrt((a + x)*(b + x)*x)*(x + sqrt(a*b))), x)
```

maple [C] time = 0.11, size = 564, normalized size = 6.20

$$\frac{2a\sqrt{\frac{a+x}{a}}\,\sqrt{\frac{b+x}{-a+b}}\,\sqrt{-\frac{x}{a}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a+x}{a}},\sqrt{-\frac{a}{-a+b}}\right)}{\sqrt{abx+a\,x^2+b\,x^2+x^3}}-2\sqrt{ab}\left(\frac{a\sqrt{1+\frac{x}{a}}\,\sqrt{\frac{b}{-a+b}+\frac{x}{-a+b}}\,\sqrt{-\frac{x}{a}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{a+x}{a}},-\frac{a}{-a-\sqrt{ab}},\sqrt{-\frac{a}{-a+b}}\right)}{\sqrt{abx+a\,x^2+b\,x^2+x^3}\,\left(-a-\sqrt{ab}\right)}+\frac{a\sqrt{1+\frac{x}{a}}\,\sqrt{\frac{b}{-a+b}+\frac{x}{-a+b}}\,\sqrt{-\frac{x}{a}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{a+x}{a}},-\frac{a}{-a+\sqrt{ab}},\sqrt{-\frac{a}{-a+b}}\right)}{\sqrt{abx+a\,x^2+b\,x^2+x^3}\,\left(-a+\sqrt{ab}\right)}-\frac{a^2b\sqrt{1+\frac{x}{a}}\,\sqrt{\frac{b}{-a+b}+\frac{x}{-a+b}}\,\sqrt{-\frac{x}{a}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{a+x}{a}},-\frac{a}{-a-\sqrt{ab}},\sqrt{-\frac{a}{-a+b}}\right)}{\sqrt{ab}\,\sqrt{a^2b^2x+a^2b\,x^2+a\,b^2x^2+x^3ab}\,\left(-a-\sqrt{ab}\right)}+\frac{a^2b\sqrt{1+\frac{x}{a}}\,\sqrt{\frac{b}{-a+b}+\frac{x}{-a+b}}\,\sqrt{-\frac{x}{a}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{a+x}{a}},-\frac{a}{-a+\sqrt{ab}},\sqrt{-\frac{a}{-a+b}}\right)}{\sqrt{ab}\,\sqrt{a^2b^2x+a^2b\,x^2+a\,b^2x^2+x^3ab}\,\left(-a+\sqrt{ab}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a*b)^(1/2)+x)/(x*(a+x)*(b+x))^(1/2)/((a*b)^(1/2)+x),x)
```

```
[Out] 2*a*((a+x)/a)^(1/2)*((b+x)/(-a+b))^(1/2)*(-1/a*x)^(1/2)/(a*b*x+a*x^2+b*x^2+
x^3)^(1/2)*EllipticF(((a+x)/a)^(1/2),(-1/(-a+b)*a)^(1/2))-2*(a*b)^(1/2)*(a*
(1+1/a*x)^(1/2)*(1/(-a+b)*b+1/(-a+b)*x)^(1/2)*(-1/a*x)^(1/2)/(a*b*x+a*x^2+b
*x^2+x^3)^(1/2)/(-a-(a*b)^(1/2))*EllipticPi(((a+x)/a)^(1/2),-a/(-a-(a*b)^(1
/2)),(-1/(-a+b)*a)^(1/2))+a*(1+1/a*x)^(1/2)*(1/(-a+b)*b+1/(-a+b)*x)^(1/2)*(
-1/a*x)^(1/2)/(a*b*x+a*x^2+b*x^2+x^3)^(1/2)/(-a+(a*b)^(1/2))*EllipticPi(((a
+x)/a)^(1/2),-a/(-a+(a*b)^(1/2)),(-1/(-a+b)*a)^(1/2))-a^2*b/(a*b)^(1/2)*(1+
1/a*x)^(1/2)*(1/(-a+b)*b+1/(-a+b)*x)^(1/2)*(-1/a*x)^(1/2)/(a^2*b^2*x+a^2*b*
x^2+a*b^2*x^2+a*b*x^3)^(1/2)/(-a-(a*b)^(1/2))*EllipticPi(((a+x)/a)^(1/2),-a
/(-a-(a*b)^(1/2)),(-1/(-a+b)*a)^(1/2))+a^2*b/(a*b)^(1/2)*(1+1/a*x)^(1/2)*(1
/(-a+b)*b+1/(-a+b)*x)^(1/2)*(-1/a*x)^(1/2)/(a^2*b^2*x+a^2*b*x^2+a*b^2*x^2+a
*b*x^3)^(1/2)/(-a+(a*b)^(1/2))*EllipticPi(((a+x)/a)^(1/2),-a/(-a+(a*b)^(1/2
)),(-1/(-a+b)*a)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{ab}}{\sqrt{(a+x)(b+x)x}\left(x+\sqrt{ab}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(a*b)^(1/2)+x)/(x*(a+x)*(b+x))^(1/2)/((a*b)^(1/2)+x),x, algorit
hm="maxima")
```

```
[Out] integrate((x - sqrt(a*b))/(sqrt((a + x)*(b + x)*x)*(x + sqrt(a*b))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x-\sqrt{a\,b}}{\left(x+\sqrt{a\,b}\right)\,\sqrt{x\,\left(a+x\right)\,\left(b+x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a*b)^(1/2))/((x + (a*b)^(1/2))*(x*(a + x)*(b + x))^(1/2)),x)
```

```
[Out] int((x - (a*b)^(1/2))/((x + (a*b)^(1/2))*(x*(a + x)*(b + x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(a*b)**(1/2)+x)/(x*(a+x)*(b+x))**(1/2)/((a*b)**(1/2)+x),x)
```

```
[Out] Timed out
```

3.1115 $\int \frac{1}{\sqrt[3]{-x^2+x^3}}\,dx$

Optimal. Leaf size=91

$$-\log\left(\sqrt[3]{x^3-x^2}-x\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-x^2}+x}\right)$$

Rubi [A] time = 0.02, antiderivative size = 135, normalized size of antiderivative = 1.48, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2011, 59}

$$-\frac{3\sqrt[3]{x-1}\,x^{2/3}\log\left(\frac{\sqrt[3]{x-1}}{\sqrt[3]{x}}-1\right)}{2\sqrt[3]{x^3-x^2}}-\frac{\sqrt[3]{x-1}\,x^{2/3}\log(x)}{2\sqrt[3]{x^3-x^2}}-\frac{\sqrt{3}\sqrt[3]{x-1}\,x^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{x-1}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{x^3-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + x^3)^(-1/3),x]
```

```
[Out] -((Sqrt[3]*(-1 + x)^(1/3)*x^(2/3)*ArcTan[1/Sqrt[3] + (2*(-1 + x)^(1/3))/(Sq
rt[3]*x^(1/3))])/(-x^2 + x^3)^(1/3)) - (3*(-1 + x)^(1/3)*x^(2/3)*Log[-1 + (
-1 + x)^(1/3)/x^(1/3)])/(2*(-x^2 + x^3)^(1/3)) - ((-1 + x)^(1/3)*x^(2/3)*Lo
g[x])/(2*(-x^2 + x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{-x^2+x^3}}\,dx &= \frac{\left(\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{1}{\sqrt[3]{-1+x}\,x^{2/3}}\,dx}{\sqrt[3]{-x^2+x^3}}\\ &= -\frac{\sqrt{3}\sqrt[3]{-1+x}\,x^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-1+x}}{\sqrt{3}\sqrt[3]{x}}\right)}{\sqrt[3]{-x^2+x^3}}-\frac{3\sqrt[3]{-1+x}\,x^{2/3}\log\left(-1+\frac{\sqrt[3]{-1+x}}{\sqrt[3]{x}}\right)}{2\sqrt[3]{-x^2+x^3}}-\frac{\sqrt[3]{-1+x}\,x^{2/3}\,\mathrm{l}}{2\sqrt[3]{-x^2+}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.38

$$\frac{3\left((x-1)x^2\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};1-x\right)}{2x^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + x^3)^(-1/3),x]
```

```
[Out] (3*((-1 + x)*x^2)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, 1 - x])/(2*x^(4/3)
)
```

IntegrateAlgebraic [A] time = 0.15, size = 91, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^3-x^2}-x\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x^2}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^2 + x^3)^(-1/3),x]
```

```
[Out] Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-x^2 + x^3)^(1/3))] - Log[-x + (-x^2 + x
^3)^(1/3)] + Log[x^2 + x*(-x^2 + x^3)^(1/3) + (-x^2 + x^3)^(2/3)]/2
```

fricas [A] time = 0.40, size = 92, normalized size = 1.01

$$-\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-x^2\right)^{\frac{1}{3}}}{3x}\right)-\log\left(-\frac{x-\left(x^3-x^2\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{2}\log\left(\frac{x^2+\left(x^3-x^2\right)^{\frac{1}{3}}x+\left(x^3-x^2\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 - x^2)^(1/3))/x) - log(-(x
- (x^3 - x^2)^(1/3))/x) + 1/2*log((x^2 + (x^3 - x^2)^(1/3)*x + (x^3 - x^2)^
(2/3))/x^2)
```

giac [A] time = 0.18, size = 63, normalized size = 0.69

$$-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)-\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x^2)^(1/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*(-1/x + 1)^(1/3) + 1)) + 1/2*log((-1/x + 1)^
(2/3) + (-1/x + 1)^(1/3) + 1) - log(abs((-1/x + 1)^(1/3) - 1))
```

maple [C] time = 0.30, size = 27, normalized size = 0.30

$$\frac{3\left(-\mathrm{signum}\left(-1+x\right)\right)^{\frac{1}{3}}x^{\frac{1}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x\right)}{\mathrm{signum}\left(-1+x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-x^2)^(1/3),x)
```

```
[Out] 3/signum(-1+x)^(1/3)*(-signum(-1+x))^(1/3)*x^(1/3)*hypergeom([1/3,1/3],[4/3
],x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(x^3-x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x^2)^(1/3),x, algorithm="maxima")
[Out] integrate((x^3 - x^2)^(-1/3), x)
```

mupad [B] time = 0.98, size = 27, normalized size = 0.30

$$\frac{3\,x\,{\left(1-x\right)}^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ x\right)}{{\left(x^3-x^2\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 - x^2)^(1/3),x)
[Out] (3*x*(1 - x)^(1/3)*hypergeom([1/3, 1/3], 4/3, x))/(x^3 - x^2)^(1/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x^3-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-x**2)**(1/3),x)
[Out] Integral((x**3 - x**2)**(-1/3), x)
```

3.1116 $\int \frac{3-x^2}{(1-x^2)\sqrt[4]{1-6x^2+x^4}}\,dx$

Optimal. Leaf size=91

$$\tan^{-1}\left(\frac{x+i}{\sqrt[4]{x^4-6x^2+1}}\right)-\tan^{-1}\left(\frac{\sqrt[4]{x^4-6x^2+1}}{x-i}\right)+\tanh^{-1}\left(\frac{x+i}{\sqrt[4]{x^4-6x^2+1}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{x^4-6x^2+1}}{x-i}\right)$$

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3-x^2}{(1-x^2)\sqrt[4]{1-6x^2+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3 - x^2)/((1 - x^2)*(1 - 6*x^2 + x^4)^(1/4)),x]
[Out] Defer[Int][(3 - x^2)/((1 - x^2)*(1 - 6*x^2 + x^4)^(1/4)), x]
```

Rubi steps

$$\int \frac{3-x^2}{(1-x^2)\sqrt[4]{1-6x^2+x^4}}\,dx=\int \frac{3-x^2}{(1-x^2)\sqrt[4]{1-6x^2+x^4}}\,dx$$

Mathematica [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{3-x^2}{(1-x^2)\sqrt[4]{1-6x^2+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3 - x^2)/((1 - x^2)*(1 - 6*x^2 + x^4)^(1/4)),x]
[Out] Integrate[(3 - x^2)/((1 - x^2)*(1 - 6*x^2 + x^4)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 4.95, size = 91, normalized size = 1.00

$$\tan^{-1}\left(\frac{x+i}{\sqrt[4]{x^4-6x^2+1}}\right)-\tan^{-1}\left(\frac{\sqrt[4]{x^4-6x^2+1}}{x-i}\right)+\tanh^{-1}\left(\frac{x+i}{\sqrt[4]{x^4-6x^2+1}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{x^4-6x^2+1}}{x-i}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x^2)/((1 - x^2)*(1 - 6*x^2 + x^4)^(1/4)),x]
[Out] ArcTan[(I + x)/(1 - 6*x^2 + x^4)^(1/4)] - ArcTan[(1 - 6*x^2 + x^4)^(1/4)/(-
I + x)] + ArcTanh[(I + x)/(1 - 6*x^2 + x^4)^(1/4)] + ArcTanh[(1 - 6*x^2 + x
^4)^(1/4)/(-I + x)]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^2+1)/(x^4-6*x^2+1)^(1/4),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-3}{\left(x^4-6\,x^2+1\right)^{\frac{1}{4}}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^2+1)/(x^4-6*x^2+1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 3)/((x^4 - 6*x^2 + 1)^(1/4)*(x^2 - 1)), x)
```

maple [C] time = 4.39, size = 232, normalized size = 2.55

$$\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\mathrm{RootOf}(\_Z^2+1)\sqrt{x^4-6x^2+1}\,x^2-\mathrm{RootOf}(\_Z^2+1)x^4-(x^4-6x^2+1)^{\frac{3}{4}}x+(x^4-6x^2+1)^{\frac{1}{4}}x^3-\mathrm{RootOf}(\_Z^2+1)\sqrt{x^4-6x^2+1}+5\,\mathrm{RootOf}(\_Z^2+1)x^2-3(x^4-6x^2+1)^{\frac{1}{4}}x}{(1+x)(-1+x)}\right)}{2}+\frac{\ln\left(-\frac{(x^4-6x^2+1)^{\frac{3}{4}}x+\sqrt{x^4-6x^2+1}\,x^2+(x^4-6x^2+1)^{\frac{1}{4}}x^3+x^4-\sqrt{x^4-6x^2+1}-3(x^4-6x^2+1)^{\frac{1}{4}}x-5x^2}{(1+x)(-1+x)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^2+1)/(x^4-6*x^2+1)^(1/4),x)
```

```
[Out] 1/2*RootOf(_Z^2+1)*ln((RootOf(_Z^2+1)*(x^4-6*x^2+1)^(1/2)*x^2-RootOf(_Z^2+1
)*x^4-(x^4-6*x^2+1)^(3/4)*x+(x^4-6*x^2+1)^(1/4)*x^3-RootOf(_Z^2+1)*(x^4-6*x
^2+1)^(1/2)+5*RootOf(_Z^2+1)*x^2-3*(x^4-6*x^2+1)^(1/4)*x)/(1+x)/(-1+x))+1/2
*ln(-((x^4-6*x^2+1)^(3/4)*x+(x^4-6*x^2+1)^(1/2)*x^2+(x^4-6*x^2+1)^(1/4)*x^3
+x^4-(x^4-6*x^2+1)^(1/2)-3*(x^4-6*x^2+1)^(1/4)*x-5*x^2)/(1+x)/(-1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-3}{\left(x^4-6\,x^2+1\right)^{\frac{1}{4}}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^2+1)/(x^4-6*x^2+1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 3)/((x^4 - 6*x^2 + 1)^(1/4)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2-3}{\left(x^2-1\right)\,{\left(x^4-6\,x^2+1\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 3)/((x^2 - 1)*(x^4 - 6*x^2 + 1)^(1/4)),x)
```

```
[Out] int((x^2 - 3)/((x^2 - 1)*(x^4 - 6*x^2 + 1)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-3}{\sqrt[4]{\left(x^2-2x-1\right)\left(x^2+2x-1\right)}\,\left(x-1\right)\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**2+1)/(x**4-6*x**2+1)**(1/4),x)
```

```
[Out] Integral((x**2 - 3)/(((x**2 - 2*x - 1)*(x**2 + 2*x - 1))**(1/4)*(x - 1)*(x
+ 1)), x)
```

3.1117 $\int \frac{-3+x^4}{\left(1+x^4\right)\sqrt[4]{-3x+4x^4-3x^5}}\,dx$

Optimal. Leaf size=91

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\left(-3x^5+4x^4-3x\right)^{3/4}}{3x^4-4x^3+3}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\left(-3x^5+4x^4-3x\right)^{3/4}}{3x^4-4x^3+3}\right)$$

Rubi [F] time = 8.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3+x^4}{\left(1+x^4\right)\sqrt[4]{-3x+4x^4-3x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3 + x^4)/((1 + x^4)*(-3*x + 4*x^4 - 3*x^5)^(1/4)),x]
```

```
[Out] -(((-1)^(3/16)*x^(1/4)*(-3 + 4*x^3 - 3*x^4)^(1/4)*Defer[Subst][Defer[Int][1
/(((-1)^(1/16) - x)*(-3 + 4*x^12 - 3*x^16)^(1/4)), x], x, x^(1/4)])/(-3*x +
 4*x^4 - 3*x^5)^(1/4)) - ((-1)^(9/16)*x^(1/4)*(-3 + 4*x^3 - 3*x^4)^(1/4)*De
fer[Subst][Defer[Int][1/(((-1)^(3/16) - x)*(-3 + 4*x^12 - 3*x^16)^(1/4)), x
], x, x^(1/4)])/(-3*x + 4*x^4 - 3*x^5)^(1/4) - ((-1)^(15/16)*x^(1/4)*(-3 +
4*x^3 - 3*x^4)^(1/4)*Defer[Subst][Defer[Int][1/(((-1)^(5/16) - x)*(-3 + 4*x
^12 - 3*x^16)^(1/4)), x], x, x^(1/4)])/(-3*x + 4*x^4 - 3*x^5)^(1/4) + ((-1)
^(5/16)*x^(1/4)*(-3 + 4*x^3 - 3*x^4)^(1/4)*Defer[Subst][Defer[Int][1/(((-1)
^(7/16) - x)*(-3 + 4*x^12 - 3*x^16)^(1/4)), x], x, x^(1/4)])/(-3*x + 4*x^4
- 3*x^5)^(1/4) - ((-1)^(11/16)*x^(1/4)*(-3 + 4*x^3 - 3*x^4)^(1/4)*Defer[Sub
st][Defer[Int][1/((-(-1)^(9/16) - x)*(-3 + 4*x^12 - 3*x^16)^(1/4)), x], x,
x^(1/4)])/(-3*x + 4*x^4 - 3*x^5)^(1/4) + ((-1)^(1/16)*x^(1/4)*(-3 + 4*x^3 -
 3*x^4)^(1/4)*Defer[Subst][Defer[Int][1/((-(-1)^(11/16) - x)*(-3 + 4*x^12 -
 3*x^16)^(1/4)), x], x, x^(1/4)])/(-3*x + 4*x^4 - 3*x^5)^(1/4) + ((-1)^(7/1
6)*x^(1/4)*(-3 + 4*x^3 - 3*x^4)^(1/4)*Defer[Subst][Defer[Int][1/((-(-1)^(13
/16) - x)*(-3 + 4*x^12 - 3*x^16)^(1/4)), x], x, x^(1/4)])/(-3*x + 4*x^4 - 3
*x^5)^(1/4) + ((-1)^(13/16)*x^(1/4)*(-3 + 4*x^3 - 3*x^4)^(1/4)*Defer[Subst]
[Defer[Int][1/((-(-1)^(15/16) - x)*(-3 + 4*x^12 - 3*x^16)^(1/4)), x], x, x^
(1/4)])/(-3*x + 4*x^4 - 3*x^5)^(1/4) + (4*x^(1/4)*(-3 + 4*x^3 - 3*x^4)^(1/4
)*Defer[Subst][Defer[Int][x^2/(-3 + 4*x^12 - 3*x^16)^(1/4), x], x, x^(1/4)]
)/(-3*x + 4*x^4 - 3*x^5)^(1/4) - ((-1)^(3/16)*x^(1/4)*(-3 + 4*x^3 - 3*x^4)^
(1/4)*Defer[Subst][Defer[Int][1/(((-1)^(1/16) + x)*(-3 + 4*x^12 - 3*x^16)^(
1/4)), x], x, x^(1/4)])/(-3*x + 4*x^4 - 3*x^5)^(1/4) - ((-1)^(9/16)*x^(1/4)
*(-3 + 4*x^3 - 3*x^4)^(1/4)*Defer[Subst][Defer[Int][1/(((-1)^(3/16) + x)*(-
3 + 4*x^12 - 3*x^16)^(1/4)), x], x, x^(1/4)])/(-3*x + 4*x^4 - 3*x^5)^(1/4)
- ((-1)^(15/16)*x^(1/4)*(-3 + 4*x^3 - 3*x^4)^(1/4)*Defer[Subst][Defer[Int][
1/(((-1)^(5/16) + x)*(-3 + 4*x^12 - 3*x^16)^(1/4)), x], x, x^(1/4)])/(-3*x
+ 4*x^4 - 3*x^5)^(1/4) + ((-1)^(5/16)*x^(1/4)*(-3 + 4*x^3 - 3*x^4)^(1/4)*De
fer[Subst][Defer[Int][1/(((-1)^(7/16) + x)*(-3 + 4*x^12 - 3*x^16)^(1/4)), x
], x, x^(1/4)])/(-3*x + 4*x^4 - 3*x^5)^(1/4) - ((-1)^(11/16)*x^(1/4)*(-3 +
4*x^3 - 3*x^4)^(1/4)*Defer[Subst][Defer[Int][1/((-(-1)^(9/16) + x)*(-3 + 4*
x^12 - 3*x^16)^(1/4)), x], x, x^(1/4)])/(-3*x + 4*x^4 - 3*x^5)^(1/4) + ((-1
)^(1/16)*x^(1/4)*(-3 + 4*x^3 - 3*x^4)^(1/4)*Defer[Subst][Defer[Int][1/((-(-
1)^(11/16) + x)*(-3 + 4*x^12 - 3*x^16)^(1/4)), x], x, x^(1/4)])/(-3*x + 4*x
^4 - 3*x^5)^(1/4) + ((-1)^(7/16)*x^(1/4)*(-3 + 4*x^3 - 3*x^4)^(1/4)*Defer[S
ubst][Defer[Int][1/((-(-1)^(13/16) + x)*(-3 + 4*x^12 - 3*x^16)^(1/4)), x],
x, x^(1/4)])/(-3*x + 4*x^4 - 3*x^5)^(1/4) + ((-1)^(13/16)*x^(1/4)*(-3 + 4*x
^3 - 3*x^4)^(1/4)*Defer[Subst][Defer[Int][1/((-(-1)^(15/16) + x)*(-3 + 4*x^
12 - 3*x^16)^(1/4)), x], x, x^(1/4)])/(-3*x + 4*x^4 - 3*x^5)^(1/4)
```

Rubi steps

$$\int \frac{-3+x^4}{\left(1+x^4\right)\sqrt[4]{-3x+4x^4-3x^5}}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}\right)\int \frac{-3+x^4}{\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}\left(1+x^4\right)}\,dx}{\sqrt[4]{-3x+4x^4-3x^5}}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}\right)\int \left(\frac{1}{\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}}-\frac{4}{\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}\left(1+x^4\right)}\right)dx}{\sqrt[4]{-3x+4x^4-3x^5}}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}\right)\int \frac{1}{\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}}\,dx}{\sqrt[4]{-3x+4x^4-3x^5}} - \frac{\left(4\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}\right)\int \frac{}{\sqrt[4]{}}}{\sqrt[4]{-3x+4x^4-}}$$

$$= -\frac{\left(4\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}\right)\int \left(\frac{i}{2\sqrt[4]{x}\left(i-x^2\right)\sqrt[4]{-3+4x^3-3x^4}}+\frac{i}{2\sqrt[4]{x}\left(i+x^2\right)\sqrt[4]{-3+4x^3-3x^4}}\right)}{\sqrt[4]{-3x+4x^4-3x^5}}$$

$$= -\frac{\left(2i\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}\right)\int \frac{1}{\sqrt[4]{x}\left(i-x^2\right)\sqrt[4]{-3+4x^3-3x^4}}\,dx}{\sqrt[4]{-3x+4x^4-3x^5}} - \frac{\left(2i\sqrt[4]{x}\sqrt[4]{-3+4x^3-3}\right.}{\sqrt[4]{-3}}$$

$$= -\frac{\left(2i\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}\right)\int \left(-\frac{(-1)^{3/4}}{2\left(\sqrt[4]{-1}-x\right)\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}}-\frac{(-1)^{3/4}}{2\sqrt[4]{x}\left(\sqrt[4]{-1}+x\right)\sqrt[4]{-3+4x}}\right.}{\sqrt[4]{-3x+4x^4-3x^5}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{-3+4x^{12}-3x^{16}}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-3x+4x^4-3x^5}} - \frac{\left(\sqrt[4]{-1}\sqrt[4]{x}\sqrt[4]{-3}\right.}{}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{-3+4x^{12}-3x^{16}}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-3x+4x^4-3x^5}} - \frac{\left(4\sqrt[4]{-1}\sqrt[4]{x}\sqrt[4]{-}\right.}{}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{-3+4x^{12}-3x^{16}}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-3x+4x^4-3x^5}} - \frac{\left(4\sqrt[4]{-1}\sqrt[4]{x}\sqrt[4]{-}\right.}{}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{-3+4x^{12}-3x^{16}}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-3x+4x^4-3x^5}} - \frac{\left(2\sqrt[4]{-1}\sqrt[4]{x}\sqrt[4]{-}\right.}{}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{-3+4x^{12}-3x^{16}}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-3x+4x^4-3x^5}} - \frac{\left(2\sqrt[4]{-1}\sqrt[4]{x}\sqrt[4]{-}\right.}{}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-3+4x^3-3x^4}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{-3+4x^{12}-3x^{16}}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-3x+4x^4-3x^5}} + \frac{\left(\sqrt[16]{-1}\sqrt[4]{x}\sqrt[4]{-}\right.}{}$$

Mathematica [F] time = 1.94, size = 0, normalized size = 0.00

$$\int \frac{-3+x^4}{\left(1+x^4\right)\sqrt[4]{-3x+4x^4-3x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3 + x^4)/((1 + x^4)*(-3*x + 4*x^4 - 3*x^5)^(1/4)),x]
```

```
[Out] Integrate[(-3 + x^4)/((1 + x^4)*(-3*x + 4*x^4 - 3*x^5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.15, size = 91, normalized size = 1.00

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\left(-3x^5+4x^4-3x\right)^{3/4}}{3x^4-4x^3+3}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\left(-3x^5+4x^4-3x\right)^{3/4}}{3x^4-4x^3+3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 + x^4)/((1 + x^4)*(-3*x + 4*x^4 - 3*x^5)^(1/4)),x]
```

```
[Out] Sqrt[2]*ArcTan[(Sqrt[2]*(-3*x + 4*x^4 - 3*x^5)^(3/4))/(3 - 4*x^3 + 3*x^4)]
+ Sqrt[2]*ArcTanh[(Sqrt[2]*(-3*x + 4*x^4 - 3*x^5)^(3/4))/(3 - 4*x^3 + 3*x^4
)]
```

fricas [B] time = 84.16, size = 228, normalized size = 2.51

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{2\sqrt{2}\left(-3x^5+4x^4-3x\right)^{\frac{3}{4}}x+\sqrt{2}\left(-3x^5+4x^4-3x\right)^{\frac{1}{4}}\left(3x^4-4x^3+3\right)}{4\left(3x^5-4x^4+3x\right)}\right)+\frac{1}{4}\sqrt{2}\log\left(\frac{9x^8-192x^7+256x^6+18x^4-192x^3+4\sqrt{2}\left(-3x^5+4x^4-3x\right)^{\frac{3}{4}}\left(3x^4-16x^3+3\right)+8\sqrt{2}\left(9x^6-16x^5+9x^2\right)\left(-3x^5+4x^4-3x\right)^{\frac{1}{4}}-16\left(3x^5-8x^4+3x\right)\sqrt{-3x^5+4x^4-3x}+9}{x^8+2x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)/(x^4+1)/(-3*x^5+4*x^4-3*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/4*(2*sqrt(2)*(-3*x^5 + 4*x^4 - 3*x)^(3/4)*x + sqrt(2)*
(-3*x^5 + 4*x^4 - 3*x)^(1/4)*(3*x^4 - 4*x^3 + 3))/(3*x^5 - 4*x^4 + 3*x)) +
1/4*sqrt(2)*log((9*x^8 - 192*x^7 + 256*x^6 + 18*x^4 - 192*x^3 + 4*sqrt(2)*(
-3*x^5 + 4*x^4 - 3*x)^(3/4)*(3*x^4 - 16*x^3 + 3) + 8*sqrt(2)*(9*x^6 - 16*x^
5 + 9*x^2)*(-3*x^5 + 4*x^4 - 3*x)^(1/4) - 16*(3*x^5 - 8*x^4 + 3*x)*sqrt(-3*
x^5 + 4*x^4 - 3*x) + 9)/(x^8 + 2*x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-3}{\left(-3\,x^5+4\,x^4-3\,x\right)^{\frac{1}{4}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)/(x^4+1)/(-3*x^5+4*x^4-3*x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 3)/((-3*x^5 + 4*x^4 - 3*x)^(1/4)*(x^4 + 1)), x)
```

maple [C] time = 4.55, size = 226, normalized size = 2.48

$$\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^2-2\right)x^4-4\operatorname{RootOf}\left(\_Z^2-2\right)\sqrt{-3x^5+4x^4-3x}\,x-8\operatorname{RootOf}\left(\_Z^2-2\right)x^3+4\left(-3x^5+4x^4-3x\right)^{\frac{3}{4}}+8\left(-3x^5+4x^4-3x\right)^{\frac{1}{4}}x^2+3\operatorname{RootOf}\left(\_Z^2-2\right)}{x^4+1}\right)}{2}+\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^2+2\right)x^4+4\operatorname{RootOf}\left(\_Z^2+2\right)\sqrt{-3x^5+4x^4-3x}\,x-8\operatorname{RootOf}\left(\_Z^2+2\right)x^3+4\left(-3x^5+4x^4-3x\right)^{\frac{3}{4}}-8\left(-3x^5+4x^4-3x\right)^{\frac{1}{4}}x^2+3\operatorname{RootOf}\left(\_Z^2+2\right)}{x^4+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)/(x^4+1)/(-3*x^5+4*x^4-3*x)^(1/4),x)
```

```
[Out] 1/2*RootOf(_Z^2-2)*ln((3*RootOf(_Z^2-2)*x^4-4*RootOf(_Z^2-2)*(-3*x^5+4*x^4-
3*x)^(1/2)*x-8*RootOf(_Z^2-2)*x^3+4*(-3*x^5+4*x^4-3*x)^(3/4)+8*(-3*x^5+4*x^
4-3*x)^(1/4)*x^2+3*RootOf(_Z^2-2))/(x^4+1))+1/2*RootOf(_Z^2+2)*ln((3*RootOf
(_Z^2+2)*x^4+4*RootOf(_Z^2+2)*(-3*x^5+4*x^4-3*x)^(1/2)*x-8*RootOf(_Z^2+2)*x
^3+4*(-3*x^5+4*x^4-3*x)^(3/4)-8*(-3*x^5+4*x^4-3*x)^(1/4)*x^2+3*RootOf(_Z^2+
2))/(x^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-3}{\left(-3\,x^5+4\,x^4-3\,x\right)^{\frac{1}{4}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)/(x^4+1)/(-3*x^5+4*x^4-3*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 3)/((-3*x^5 + 4*x^4 - 3*x)^(1/4)*(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4-3}{\left(x^4+1\right)\left(-3\,x^5+4\,x^4-3\,x\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 3)/((x^4 + 1)*(4*x^4 - 3*x - 3*x^5)^(1/4)),x)
```

```
[Out] int((x^4 - 3)/((x^4 + 1)*(4*x^4 - 3*x - 3*x^5)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-3}{\sqrt[4]{-x\left(3x^4-4x^3+3\right)}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)/(x**4+1)/(-3*x**5+4*x**4-3*x)**(1/4),x)
```

```
[Out] Integral((x**4 - 3)/((-x*(3*x**4 - 4*x**3 + 3))**(1/4)*(x**4 + 1)), x)
```

3.1118 $\int \frac{\sqrt[3]{-1+x^6}\left(1+x^6\right)}{x^2\left(-1+x^3+x^6\right)}\,dx$

Optimal. Leaf size=91

$$\frac{\sqrt[3]{x^6-1}}{x}-\frac{1}{3}\log\left(\sqrt[3]{x^6-1}+x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^6-1}-x}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(-\sqrt[3]{x^6-1}\,x+\left(x^6-1\right)^{2/3}+x^2\right)$$

Rubi [C] time = 1.70, antiderivative size = 593, normalized size of antiderivative = 6.52, number of steps used = 38, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {6728, 365, 364, 1562, 465, 430, 429, 511, 510}

$$-\frac{2\left(5-\sqrt{5}\right)\sqrt[3]{x^6-1}\,x^5F_1\left(\frac{5}{6};-\frac{1}{3},1;\frac{11}{6};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{25\left(3-\sqrt{5}\right)\sqrt[3]{1-x^6}}-\frac{2\sqrt[3]{x^6-1}\,x^5F_1\left(\frac{5}{6};-\frac{1}{3},1;\frac{11}{6};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{5\sqrt{5}\left(3-\sqrt{5}\right)\sqrt[3]{1-x^6}}-\frac{2\left(5+\sqrt{5}\right)\sqrt[3]{x^6-1}\,x^5F_1\left(\frac{5}{6};-\frac{1}{3},1;\frac{11}{6};x^6,\frac{2x^6}{3+\sqrt{5}}\right)}{25\left(3+\sqrt{5}\right)\sqrt[3]{1-x^6}}+\frac{2\sqrt[3]{x^6-1}\,x^5F_1\left(\frac{5}{6};-\frac{1}{3},1;\frac{11}{6};x^6,\frac{2x^6}{3+\sqrt{5}}\right)}{5\sqrt{5}\left(3+\sqrt{5}\right)\sqrt[3]{1-x^6}}-\frac{\left(5-\sqrt{5}\right)\sqrt[3]{x^6-1}\,x^2F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{10\left(3-\sqrt{5}\right)\sqrt[3]{1-x^6}}-\frac{\sqrt[3]{x^6-1}\,x^2F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{\sqrt{5}\sqrt[3]{1-x^6}}-\frac{\left(5+\sqrt{5}\right)\sqrt[3]{x^6-1}\,x^2F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};\frac{2x^6}{3+\sqrt{5}},x^6\right)}{10\left(3+\sqrt{5}\right)\sqrt[3]{1-x^6}}+\frac{\sqrt[3]{x^6-1}\,x^2F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};\frac{2x^6}{3+\sqrt{5}},x^6\right)}{\sqrt{5}\sqrt[3]{1-x^6}}+\frac{\sqrt[3]{x^6-1}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};x^6\right)}{\sqrt[3]{1-x^6}\,x}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^6)^(1/3)*(1 + x^6))/(x^2*(-1 + x^3 + x^6)),x]
```

```
[Out] -((x^2*(-1 + x^6)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, x^6, (2*x^6)/(3 - Sqrt[
5])])/(Sqrt[5]*(1 - x^6)^(1/3))) - ((5 - Sqrt[5])*x^2*(-1 + x^6)^(1/3)*Appe
llF1[1/3, -1/3, 1, 4/3, x^6, (2*x^6)/(3 - Sqrt[5])])/(10*(3 - Sqrt[5])*(1 -
 x^6)^(1/3)) + (x^2*(-1 + x^6)^(1/3)*AppellF1[1/3, 1, -1/3, 4/3, (2*x^6)/(3
 + Sqrt[5]), x^6])/(Sqrt[5]*(1 - x^6)^(1/3)) - ((5 + Sqrt[5])*x^2*(-1 + x^6
)^(1/3)*AppellF1[1/3, 1, -1/3, 4/3, (2*x^6)/(3 + Sqrt[5]), x^6])/(10*(3 + S
qrt[5])*(1 - x^6)^(1/3)) - (2*x^5*(-1 + x^6)^(1/3)*AppellF1[5/6, -1/3, 1, 1
1/6, x^6, (2*x^6)/(3 - Sqrt[5])])/(5*Sqrt[5]*(3 - Sqrt[5])*(1 - x^6)^(1/3))
 - (2*(5 - Sqrt[5])*x^5*(-1 + x^6)^(1/3)*AppellF1[5/6, -1/3, 1, 11/6, x^6,
(2*x^6)/(3 - Sqrt[5])])/(25*(3 - Sqrt[5])*(1 - x^6)^(1/3)) + (2*x^5*(-1 + x
^6)^(1/3)*AppellF1[5/6, -1/3, 1, 11/6, x^6, (2*x^6)/(3 + Sqrt[5])])/(5*Sqrt
[5]*(3 + Sqrt[5])*(1 - x^6)^(1/3)) - (2*(5 + Sqrt[5])*x^5*(-1 + x^6)^(1/3)*
AppellF1[5/6, -1/3, 1, 11/6, x^6, (2*x^6)/(3 + Sqrt[5])])/(25*(3 + Sqrt[5])
*(1 - x^6)^(1/3)) + ((-1 + x^6)^(1/3)*Hypergeometric2F1[-1/3, -1/6, 5/6, x^
6])/(x*(1 - x^6)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
```

```
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1562

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(
2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)/(d^2 - e^2*x^(2*n)))^(-q), x], x]
, x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !IntegerQ[p]
 && ILtQ[q, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{-1+x^6}\left(1+x^6\right)}{x^2\left(-1+x^3+x^6\right)}dx = \int \left(-\frac{\sqrt[3]{-1+x^6}}{x^2}+\frac{x\left(-1-2x^3\right)\sqrt[3]{-1+x^6}}{1-x^3-x^6}\right)dx$$
$$= -\int \frac{\sqrt[3]{-1+x^6}}{x^2}dx+\int \frac{x\left(-1-2x^3\right)\sqrt[3]{-1+x^6}}{1-x^3-x^6}dx$$
$$= -\frac{\sqrt[3]{-1+x^6}\int \frac{\sqrt[3]{1-x^6}}{x^2}dx}{\sqrt[3]{1-x^6}}+\int \left(\frac{x\sqrt[3]{-1+x^6}}{-1+x^3+x^6}+\frac{2x^4\sqrt[3]{-1+x^6}}{-1+x^3+x^6}\right)dx$$
$$= \frac{\sqrt[3]{-1+x^6}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};x^6\right)}{x\sqrt[3]{1-x^6}}+2\int \frac{x^4\sqrt[3]{-1+x^6}}{-1+x^3+x^6}dx+\int \frac{x\sqrt[3]{-1+x^6}}{-1+x^3+x^6}dx$$
$$= \frac{\sqrt[3]{-1+x^6}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};x^6\right)}{x\sqrt[3]{1-x^6}}+2\int \left(-\frac{\left(-1+\sqrt{5}\right)x\sqrt[3]{-1+x^6}}{\sqrt{5}\left(-1+\sqrt{5}-2x^3\right)}+\frac{\left(1+\sqrt{5}\right)x\sqrt[3]{-1}}{\sqrt{5}\left(1+\sqrt{5}+2\right.}\right.$$
$$= \frac{\sqrt[3]{-1+x^6}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};x^6\right)}{x\sqrt[3]{1-x^6}}-\frac{2\int \frac{x\sqrt[3]{-1+x^6}}{-1+\sqrt{5}-2x^3}dx}{\sqrt{5}}-\frac{2\int \frac{x\sqrt[3]{-1+x^6}}{1+\sqrt{5}+2x^3}dx}{\sqrt{5}}-\frac{1}{5}\left(2\left(5-\right.\right.$$
$$= \frac{\sqrt[3]{-1+x^6}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};x^6\right)}{x\sqrt[3]{1-x^6}}-\frac{2\int \left(\frac{\left(1+\sqrt{5}\right)x\sqrt[3]{-1+x^6}}{2\left(3+\sqrt{5}-2x^6\right)}+\frac{x^4\sqrt[3]{-1+x^6}}{-3-\sqrt{5}+2x^6}\right)dx}{\sqrt{5}}-\frac{2\int \left(\frac{(1-\sqrt{}}{2(-3}\right.}{}$$
$$= \frac{\sqrt[3]{-1+x^6}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};x^6\right)}{x\sqrt[3]{1-x^6}}-\frac{2\int \frac{x^4\sqrt[3]{-1+x^6}}{-3-\sqrt{5}+2x^6}dx}{\sqrt{5}}+\frac{2\int \frac{x^4\sqrt[3]{-1+x^6}}{-3+\sqrt{5}+2x^6}dx}{\sqrt{5}}-\frac{1}{5}\left(2\left(5-\right.\right.$$
$$= \frac{\sqrt[3]{-1+x^6}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};x^6\right)}{x\sqrt[3]{1-x^6}}-\frac{1}{5}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{\sqrt[3]{-1+x^3}}{-3+\sqrt{5}+2x^3}dx,x,x^2\right)$$
$$= -\frac{2x^5\sqrt[3]{-1+x^6}\,F_1\left(\frac{5}{6};-\frac{1}{3},1;\frac{11}{6};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{5\sqrt{5}\left(3-\sqrt{5}\right)\sqrt[3]{1-x^6}}-\frac{2\left(5-\sqrt{5}\right)x^5\sqrt[3]{-1+x^6}\,F_1\left(\frac{5}{6};-\frac{1}{3},1;\frac{11}{6}\right.}{25\left(3-\sqrt{5}\right)\sqrt[3]{1-x^6}}$$
$$= -\frac{x^2\sqrt[3]{-1+x^6}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{\sqrt{5}\sqrt[3]{1-x^6}}-\frac{\left(5-\sqrt{5}\right)x^2\sqrt[3]{-1+x^6}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^6,\right.}{10\left(3-\sqrt{5}\right)\sqrt[3]{1-x^6}}$$

Mathematica [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-1+x^6}\left(1+x^6\right)}{x^2\left(-1+x^3+x^6\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^6)^(1/3)*(1 + x^6))/(x^2*(-1 + x^3 + x^6)),x]
[Out] Integrate[((-1 + x^6)^(1/3)*(1 + x^6))/(x^2*(-1 + x^3 + x^6)), x]
```

IntegrateAlgebraic [A] time = 1.01, size = 91, normalized size = 1.00

$$\frac{\sqrt[3]{x^6-1}}{x}-\frac{1}{3}\log\left(\sqrt[3]{x^6-1}+x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^6-1}-x}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(-\sqrt[3]{x^6-1}\,x+\left(x^6-1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((-1 + x^6)^(1/3)*(1 + x^6))/(x^2*(-1 + x^3 + x^6)),x]

[Out] (-1 + x^6)^(1/3)/x + ArcTan[(Sqrt[3]*x)/(-x + 2*(-1 + x^6)^(1/3))]/Sqrt[3] - Log[x + (-1 + x^6)^(1/3)]/3 + Log[x^2 - x*(-1 + x^6)^(1/3) + (-1 + x^6)^(2/3)]/6

fricas [A] time = 14.35, size = 128, normalized size = 1.41

$$\frac{2\sqrt{3}\,x\arctan\left(\frac{17707979315346691547103487078601066282657059082726673278841963389300888497059669011634\,\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}x^2+18779074824464902023518972945875034013564452605964125877184864112405550428883609929964\,\sqrt{3}\left(x^6-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(8791266734992875261237504664599259772605087326251698970792557525513888268399719816592\,x^6+9326814489551980499445247598236243638058784087870425269964007887066219234311690275757\,x^3-8791266734992875261237504664599259772605087326251698970792557525513888268399719816592\right)}{3\left(9923243904393545413458713816471868889492119646716071835561526356358143878603884871272\,x^6-8320283165512251371852516195766181258618636197629327742451887394495075584367754599527\,x^3-9923243904393545413458713816471868889492119646716071835561526356358143878603884871272\right)}\right)-x\log\left(\frac{x^6+x^3+3\left(x^6-1\right)^{\frac{1}{3}}x^2+3\left(x^6-1\right)^{\frac{2}{3}}x-1}{x^6+x^3-1}\right)+6\left(x^6-1\right)^{\frac{1}{3}}}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6-1)^(1/3)*(x^6+1)/x^2/(x^6+x^3-1),x, algorithm="fricas")

[Out] 1/6*(2*sqrt(3)*x*arctan(1/3*(17707979315346691547103487078601066282657059082726673278841963389300888497059669011634*sqrt(3)*(x^6 - 1)^(1/3)*x^2 + 18779074824464902023518972945875034013564452605964125877184864112405550428883609929964*sqrt(3)*(x^6 - 1)^(2/3)*x + sqrt(3)*(8791266734992875261237504664599259772605087326251698970792557525513888268399719816592*x^6 + 9326814489551980499445247598236243638058784087870425269964007887066219234311690275757*x^3 - 8791266734992875261237504664599259772605087326251698970792557525513888268399719816592))/(9923243904393545413458713816471868889492119646716071835561526356358143878603884871272*x^6 - 8320283165512251371852516195766181258618636197629327742451887394495075584367754599527*x^3 - 9923243904393545413458713816471868889492119646716071835561526356358143878603884871272)) - x*log((x^6 + x^3 + 3*(x^6 - 1)^(1/3)*x^2 + 3*(x^6 - 1)^(2/3)*x - 1)/(x^6 + x^3 - 1)) + 6*(x^6 - 1)^(1/3))/x

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+1\right)\left(x^6-1\right)^{\frac{1}{3}}}{\left(x^6+x^3-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6-1)^(1/3)*(x^6+1)/x^2/(x^6+x^3-1),x, algorithm="giac")

[Out] integrate((x^6 + 1)*(x^6 - 1)^(1/3)/((x^6 + x^3 - 1)*x^2), x)

maple [C] time = 1.71, size = 865, normalized size = 9.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6-1)^(1/3)*(x^6+1)/x^2/(x^6+x^3-1),x)

[Out] (x^6-1)^(1/3)/x+(RootOf(9*_Z^2-3*_Z+1)*ln(-(-6*RootOf(9*_Z^2-3*_Z+1)*x^12+x^12+18*RootOf(9*_Z^2-3*_Z+1)^2*x^9-3*RootOf(9*_Z^2-3*_Z+1)*x^9+9*RootOf(9*_Z^2-3*_Z+1)*(x^12-2*x^6+1)^(1/3)*x^7-3*(x^12-2*x^6+1)^(1/3)*x^7+12*RootOf(9*_Z^2-3*_Z+1)*x^6-2*x^6-18*RootOf(9*_Z^2-3*_Z+1)^2*x^3-9*RootOf(9*_Z^2-3*_Z+1)*(x^12-2*x^6+1)^(2/3)*x^2+3*RootOf(9*_Z^2-3*_Z+1)*x^3+3*(x^12-2*x^6+1)^(2/3)*x^2-9*RootOf(9*_Z^2-3*_Z+1)*(x^12-2*x^6+1)^(1/3)*x+3*(x^12-2*x^6+1)^(1/3)*x-6*RootOf(9*_Z^2-3*_Z+1)+1)/(-1+x)/(1+x)/(x^2+x+1)/(x^2-x+1)/(x^6+x^3-1))+1/3*ln(-(6*RootOf(9*_Z^2-3*_Z+1)*x^12-x^12+18*RootOf(9*_Z^2-3*_Z+1)^2*x^9-9*RootOf(9*_Z^2-3*_Z+1)*x^9-9*RootOf(9*_Z^2-3*_Z+1)*(x^12-2*x^6+1)^(1/3)*x^7+x^9-12*RootOf(9*_Z^2-3*_Z+1)*x^6+2*x^6-18*RootOf(9*_Z^2-3*_Z+1)^2*x^3+9*RootOf(9*_Z^2-3*_Z+1)*(x^12-2*x^6+1)^(2/3)*x^2+9*RootOf(9*_Z^2-3*_Z+1)*x^3+9*RootOf(9*_Z^2-3*_Z+1)*(x^12-2*x^6+1)^(1/3)*x-x^3+6*RootOf(9*_Z^2-3*_Z+1)-1)/(-1+x)/(1+x)/(x^2+x+1)/(x^2-x+1)/(x^6+x^3-1))-ln(-(6*RootOf(9*_Z^2-3*_Z+1)*x^12-x^12+18*RootOf(9*_Z^2-3*_Z+1)^2*x^9-9*RootOf(9*_Z^2-3*_Z+1)*x^9-9*RootOf(9*_Z^2-3*_Z+1)*(x^12-2*x^6+1)^(1/3)*x^7+x^9-12*RootOf(9*_Z^2-3*_Z+1)*x^6+2*x^6-18*RootOf(9*_Z^2-3*_Z+1)^2*x^3+9*RootOf(9*_Z^2-3*_Z+1)*(x^12-2*

```
x^6+1)^(2/3)*x^2+9*RootOf(9*_Z^2-3*_Z+1)*x^3+9*RootOf(9*_Z^2-3*_Z+1)*(x^12-
2*x^6+1)^(1/3)*x-x^3+6*RootOf(9*_Z^2-3*_Z+1)-1)/(-1+x)/(1+x)/(x^2+x+1)/(x^2
-x+1)/(x^6+x^3-1))*RootOf(9*_Z^2-3*_Z+1))/(x^6-1)^(2/3)*((x^6-1)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+1\right)\left(x^6-1\right)^{\frac{1}{3}}}{\left(x^6+x^3-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(x^6+1)/x^2/(x^6+x^3-1),x, algorithm="maxima")
[Out] integrate((x^6 + 1)*(x^6 - 1)^(1/3)/((x^6 + x^3 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6-1\right)^{1/3}\,\left(x^6+1\right)}{x^2\,\left(x^6+x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/3)*(x^6 + 1))/(x^2*(x^3 + x^6 - 1)),x)
[Out] int(((x^6 - 1)^(1/3)*(x^6 + 1))/(x^2*(x^3 + x^6 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/3)*(x**6+1)/x**2/(x**6+x**3-1),x)
[Out] Timed out
```

3.1119 $\int \frac{(-1+x^4)\sqrt{1+x^4}}{1+x^2+3x^4+x^6+x^8}\,dx$

Optimal. Leaf size=91

$$-\sqrt{\frac{1}{6}\left(-1-i\sqrt{3}\right)}\tan^{-1}\left(\frac{2x}{\left(\sqrt{3}-i\right)\sqrt{x^4+1}}\right)-\sqrt{\frac{1}{6}\left(-1+i\sqrt{3}\right)}\tan^{-1}\left(\frac{2x}{\left(\sqrt{3}+i\right)\sqrt{x^4+1}}\right)$$

Rubi [F] time = 0.49, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^4)\sqrt{1+x^4}}{1+x^2+3x^4+x^6+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^4)*Sqrt[1 + x^4])/(1 + x^2 + 3*x^4 + x^6 + x^8),x]
```

```
[Out] Defer[Int][Sqrt[1 + x^4]/(-1 - x^2 - 3*x^4 - x^6 - x^8), x] + Defer[Int][(x
^4*Sqrt[1 + x^4])/(1 + x^2 + 3*x^4 + x^6 + x^8), x]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+x^4)\sqrt{1+x^4}}{1+x^2+3x^4+x^6+x^8}\,dx &= \int \left(\frac{\sqrt{1+x^4}}{-1-x^2-3x^4-x^6-x^8}+\frac{x^4\sqrt{1+x^4}}{1+x^2+3x^4+x^6+x^8}\right)dx\\ &= \int \frac{\sqrt{1+x^4}}{-1-x^2-3x^4-x^6-x^8}\,dx+\int \frac{x^4\sqrt{1+x^4}}{1+x^2+3x^4+x^6+x^8}\,dx\end{aligned}$$

Mathematica [C] time = 6.24, size = 2093, normalized size = 23.00

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^4)*Sqrt[1 + x^4])/(1 + x^2 + 3*x^4 + x^6 + x^8),x]
```

```
[Out] -(((-1)^(1/4)*Sqrt[1 - I*x^2]*Sqrt[1 + I*x^2]*EllipticF[I*ArcSinh[(-1)^(1/4
)*x], -1])/Sqrt[1 + x^4]) - ((-1)^(1/4)*Sqrt[1 - I*x^2]*Sqrt[1 + I*x^2]*Ell
ipticPi[I/Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 1, 0], I*ArcSinh[(-1)^(1/4
)*x], -1])/(Sqrt[1 + x^4]*Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 1, 0]*(Roo
t[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 1, 0] - Root[1 + #1 + 3*#1^2 + #1^3 + #
1^4 & , 2, 0])*(Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 1, 0] - Root[1 + #1
+ 3*#1^2 + #1^3 + #1^4 & , 3, 0])*(Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 1
, 0] - Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 4, 0])) + ((-1)^(1/4)*Sqrt[1
- I*x^2]*Sqrt[1 + I*x^2]*EllipticPi[I/Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 &
, 1, 0], I*ArcSinh[(-1)^(1/4)*x], -1]*Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 &
, 1, 0]^3)/(Sqrt[1 + x^4]*(Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 1, 0] - R
oot[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 2, 0])*(Root[1 + #1 + 3*#1^2 + #1^3 +
 #1^4 & , 1, 0] - Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 3, 0])*(Root[1 + #
1 + 3*#1^2 + #1^3 + #1^4 & , 1, 0] - Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & ,
 4, 0])) - ((-1)^(1/4)*Sqrt[1 - I*x^2]*Sqrt[1 + I*x^2]*EllipticPi[I/Root[1
+ #1 + 3*#1^2 + #1^3 + #1^4 & , 2, 0], I*ArcSinh[(-1)^(1/4)*x], -1])/(Sqrt[
1 + x^4]*Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 2, 0]*(-Root[1 + #1 + 3*#1^
2 + #1^3 + #1^4 & , 1, 0] + Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 2, 0])*(
Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 2, 0] - Root[1 + #1 + 3*#1^2 + #1^3
```

```
+ #1^4 & , 3, 0])*(Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 2, 0] - Root[1 +
#1 + 3*#1^2 + #1^3 + #1^4 & , 4, 0])) + ((-1)^(1/4)*Sqrt[1 - I*x^2]*Sqrt[1
+ I*x^2]*EllipticPi[I/Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 2, 0], I*ArcSi
nh[(-1)^(1/4)*x], -1]*Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 2, 0]^3)/(Sqrt
[1 + x^4]*(-Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 1, 0] + Root[1 + #1 + 3*
#1^2 + #1^3 + #1^4 & , 2, 0])*(Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 2, 0]
 - Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 3, 0])*(Root[1 + #1 + 3*#1^2 + #1
^3 + #1^4 & , 2, 0] - Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 4, 0])) - ((-1
)^(1/4)*Sqrt[1 - I*x^2]*Sqrt[1 + I*x^2]*EllipticPi[I/Root[1 + #1 + 3*#1^2 +
 #1^3 + #1^4 & , 3, 0], I*ArcSinh[(-1)^(1/4)*x], -1])/(Sqrt[1 + x^4]*Root[1
 + #1 + 3*#1^2 + #1^3 + #1^4 & , 3, 0]*(-Root[1 + #1 + 3*#1^2 + #1^3 + #1^4
 & , 1, 0] + Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 3, 0])*(-Root[1 + #1 +
3*#1^2 + #1^3 + #1^4 & , 2, 0] + Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 3,
0])*(Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 3, 0] - Root[1 + #1 + 3*#1^2 +
#1^3 + #1^4 & , 4, 0])) + ((-1)^(1/4)*Sqrt[1 - I*x^2]*Sqrt[1 + I*x^2]*Ellip
ticPi[I/Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 3, 0], I*ArcSinh[(-1)^(1/4)*
x], -1]*Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 3, 0]^3)/(Sqrt[1 + x^4]*(-Ro
ot[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 1, 0] + Root[1 + #1 + 3*#1^2 + #1^3 +
#1^4 & , 3, 0])*(-Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 2, 0] + Root[1 + #
1 + 3*#1^2 + #1^3 + #1^4 & , 3, 0])*(Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & ,
 3, 0] - Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 4, 0])) - ((-1)^(1/4)*Sqrt[
1 - I*x^2]*Sqrt[1 + I*x^2]*EllipticPi[I/Root[1 + #1 + 3*#1^2 + #1^3 + #1^4
& , 4, 0], I*ArcSinh[(-1)^(1/4)*x], -1])/(Sqrt[1 + x^4]*Root[1 + #1 + 3*#1^
2 + #1^3 + #1^4 & , 4, 0]*(-Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 1, 0] +
Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 4, 0])*(-Root[1 + #1 + 3*#1^2 + #1^3
 + #1^4 & , 2, 0] + Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 4, 0])*(-Root[1
+ #1 + 3*#1^2 + #1^3 + #1^4 & , 3, 0] + Root[1 + #1 + 3*#1^2 + #1^3 + #1^4
& , 4, 0])) + ((-1)^(1/4)*Sqrt[1 - I*x^2]*Sqrt[1 + I*x^2]*EllipticPi[I/Root
[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 4, 0], I*ArcSinh[(-1)^(1/4)*x], -1]*Root
[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 4, 0]^3)/(Sqrt[1 + x^4]*(-Root[1 + #1 +
3*#1^2 + #1^3 + #1^4 & , 1, 0] + Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 4,
0])*(-Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 2, 0] + Root[1 + #1 + 3*#1^2 +
 #1^3 + #1^4 & , 4, 0])*(-Root[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 3, 0] + Ro
ot[1 + #1 + 3*#1^2 + #1^3 + #1^4 & , 4, 0]))
```

IntegrateAlgebraic [A] time = 0.59, size = 65, normalized size = 0.71

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}x\sqrt{x^4+1}}{x^4-x^2+1}\right)}{2\sqrt{3}}-\frac{1}{2}\tanh^{-1}\left(\frac{x\sqrt{x^4+1}}{x^4+x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*Sqrt[1 + x^4])/(1 + x^2 + 3*x^4 + x^6 + x^8)
,x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3]*x*Sqrt[1 + x^4])/(1 - x^2 + x^4)]/Sqrt[3] - ArcTanh[(x
*Sqrt[1 + x^4])/(1 + x^2 + x^4)]/2
```

fricas [A] time = 0.55, size = 95, normalized size = 1.04

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x^4-x^2+1\right)\sqrt{x^4+1}}{3\left(x^5+x\right)}\right)+\frac{1}{4}\log\left(\frac{x^8+3x^6+3x^4+3x^2-2\left(x^5+x^3+x\right)\sqrt{x^4+1}+1}{x^8+x^6+3x^4+x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/2)/(x^8+x^6+3*x^4+x^2+1),x, algorithm="fricas"
)
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^4 - x^2 + 1)*sqrt(x^4 + 1)/(x^5 + x)) + 1
/4*log((x^8 + 3*x^6 + 3*x^4 + 3*x^2 - 2*(x^5 + x^3 + x)*sqrt(x^4 + 1) + 1)/
(x^8 + x^6 + 3*x^4 + x^2 + 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/2)/(x^8+x^6+3*x^4+x^2+1),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

maple [A] time = 0.07, size = 121, normalized size = 1.33

$$\frac{\ln\left(\frac{x^4+1}{x^2}-\frac{\sqrt{x^4+1}}{x}+1\right)}{4}+\frac{\sqrt{2}\,\sqrt{6}\,\arctan\left(\frac{\left(\frac{2\sqrt{2}\,\sqrt{x^4+1}}{x}-\sqrt{2}\right)\sqrt{6}}{6}\right)}{12}-\frac{\ln\left(\frac{x^4+1}{x^2}+\frac{\sqrt{x^4+1}}{x}+1\right)}{4}+\frac{\sqrt{2}\,\sqrt{6}\,\arctan\left(\frac{\left(\frac{2\sqrt{2}\,\sqrt{x^4+1}}{x}+\sqrt{2}\right)\sqrt{6}}{6}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^4+1)^(1/2)/(x^8+x^6+3*x^4+x^2+1),x)
[Out] 1/4*ln((x^4+1)/x^2-(x^4+1)^(1/2)/x+1)+1/12*2^(1/2)*6^(1/2)*arctan(1/6*(2*2^
(1/2)/x*(x^4+1)^(1/2)-2^(1/2))*6^(1/2))-1/4*ln((x^4+1)/x^2+(x^4+1)^(1/2)/x+
1)+1/12*2^(1/2)*6^(1/2)*arctan(1/6*(2*2^(1/2)/x*(x^4+1)^(1/2)+2^(1/2))*6^(1
/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}\left(x^4-1\right)}{x^8+x^6+3\,x^4+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+1)^(1/2)/(x^8+x^6+3*x^4+x^2+1),x, algorithm="maxima"
)
[Out] integrate(sqrt(x^4 + 1)*(x^4 - 1)/(x^8 + x^6 + 3*x^4 + x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-1\right)\sqrt{x^4+1}}{x^8+x^6+3\,x^4+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(x^4 + 1)^(1/2))/(x^2 + 3*x^4 + x^6 + x^8 + 1),x)
[Out] int(((x^4 - 1)*(x^4 + 1)^(1/2))/(x^2 + 3*x^4 + x^6 + x^8 + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**4+1)**(1/2)/(x**8+x**6+3*x**4+x**2+1),x)
[Out] Timed out
```

3.1120 $\int \frac{-1+2x^8}{\sqrt[4]{1+x^4}\left(-1+x^8\right)}\,dx$

Optimal. Leaf size=91

$$-\frac{x}{2\sqrt[4]{x^4+1}}+\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[4]{2}}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[4]{2}}$$

Rubi [A] time = 0.20, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6725, 240, 212, 206, 203, 1404, 382, 377}

$$-\frac{x}{2\sqrt[4]{x^4+1}}+\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[4]{2}}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x^8)/((1 + x^4)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -1/2*x/(1 + x^4)^(1/4) + ArcTan[x/(1 + x^4)^(1/4)] - ArcTan[(2^(1/4)*x)/(1
+ x^4)^(1/4)]/(4*2^(1/4)) + ArcTanh[x/(1 + x^4)^(1/4)] - ArcTanh[(2^(1/4)*x
)/(1 + x^4)^(1/4)]/(4*2^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 1404

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+2x^8}{\sqrt[4]{1+x^4}\left(-1+x^8\right)}\,dx &= \int \left(\frac{2}{\sqrt[4]{1+x^4}}+\frac{1}{\sqrt[4]{1+x^4}\left(-1+x^8\right)}\right) dx\\
&= 2\int \frac{1}{\sqrt[4]{1+x^4}}\,dx+\int \frac{1}{\sqrt[4]{1+x^4}\left(-1+x^8\right)}\,dx\\
&= 2\,\mathrm{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)+\int \frac{1}{\left(-1+x^4\right)\left(1+x^4\right)^{5/4}}\,dx\\
&= -\frac{x}{2\sqrt[4]{1+x^4}}+\frac{1}{2}\int \frac{1}{\left(-1+x^4\right)\sqrt[4]{1+x^4}}\,dx+\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)+\mathrm{Subs}\\
&= -\frac{x}{2\sqrt[4]{1+x^4}}+\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{-1+2x^4}\,dx,x,\right.\\
&= -\frac{x}{2\sqrt[4]{1+x^4}}+\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x^2}\,dx,x,\right.\\
&= -\frac{x}{2\sqrt[4]{1+x^4}}+\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{4\sqrt[4]{2}}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{\tanh^{-1}\left(\right.}{4\sqrt[4]{2}}
\end{aligned}
$$

Mathematica [C] time = 0.29, size = 111, normalized size = 1.22

$$
-\frac{2}{5}x^5F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-x^4,x^4\right)-\frac{x}{2\sqrt[4]{x^4+1}}+\frac{3\left(-\log\left(1-\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+1}}\right)+\log\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+1}}+1\right)+2\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+1}}\right)\right)}{8\sqrt[4]{2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 2*x^8)/((1 + x^4)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -1/2*x/(1 + x^4)^(1/4) - (2*x^5*AppellF1[5/4, 1/4, 1, 9/4, -x^4, x^4])/5 +
(3*(2*ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)] - Log[1 - (2^(1/4)*x)/(1 + x^4)^(
1/4)] + Log[1 + (2^(1/4)*x)/(1 + x^4)^(1/4)]))/(8*2^(1/4))
```

IntegrateAlgebraic [A] time = 0.38, size = 91, normalized size = 1.00

$$-\frac{x}{2\sqrt[4]{x^4+1}}+\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[4]{2}}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x^8)/((1 + x^4)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -1/2*x/(1 + x^4)^(1/4) + ArcTan[x/(1 + x^4)^(1/4)] - ArcTan[(2^(1/4)*x)/(1
+ x^4)^(1/4)]/(4*2^(1/4)) + ArcTanh[x/(1 + x^4)^(1/4)] - ArcTanh[(2^(1/4)*x
)/(1 + x^4)^(1/4)]/(4*2^(1/4))
```

fricas [B] time = 0.43, size = 197, normalized size = 2.16

$$-\frac{4\cdot 2^{\frac{3}{4}}\left(x^4+1\right)\arctan\left(\frac{2^{\frac{3}{4}}x\sqrt{\frac{\sqrt{2}x^2+\sqrt{x^4+1}}{x^2}}-2^{\frac{3}{4}}\left(x^4+1\right)^{\frac{1}{4}}}{2x}\right)+2^{\frac{3}{4}}\left(x^4+1\right)\log\left(\frac{2^{\frac{1}{4}}x+\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-2^{\frac{3}{4}}\left(x^4+1\right)\log\left(-\frac{2^{\frac{1}{4}}x-\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+16\left(x^4+1\right)\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-8\left(x^4+1\right)\log\left(\frac{x+\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+8\left(x^4+1\right)\log\left(-\frac{x-\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+8\left(x^4+1\right)^{\frac{3}{4}}x}{16\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-1)/(x^4+1)^(1/4)/(x^8-1),x, algorithm="fricas")
```

```
[Out] -1/16*(4*2^(3/4)*(x^4 + 1)*arctan(1/2*(2^(3/4)*x*sqrt((sqrt(2)*x^2 + sqrt(x
^4 + 1))/x^2) - 2^(3/4)*(x^4 + 1)^(1/4))/x) + 2^(3/4)*(x^4 + 1)*log((2^(1/4
)*x + (x^4 + 1)^(1/4))/x) - 2^(3/4)*(x^4 + 1)*log(-(2^(1/4)*x - (x^4 + 1)^(
1/4))/x) + 16*(x^4 + 1)*arctan((x^4 + 1)^(1/4)/x) - 8*(x^4 + 1)*log((x + (x
^4 + 1)^(1/4))/x) + 8*(x^4 + 1)*log(-(x - (x^4 + 1)^(1/4))/x) + 8*(x^4 + 1)
^(3/4)*x)/(x^4 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^8-1}{\left(x^8-1\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-1)/(x^4+1)^(1/4)/(x^8-1),x, algorithm="giac")
```

```
[Out] integrate((2*x^8 - 1)/((x^8 - 1)*(x^4 + 1)^(1/4)), x)
```

maple [F] time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{2x^8-1}{\left(x^4+1\right)^{\frac{1}{4}}\left(x^8-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8-1)/(x^4+1)^(1/4)/(x^8-1),x)
```

```
[Out] int((2*x^8-1)/(x^4+1)^(1/4)/(x^8-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^8-1}{\left(x^8-1\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-1)/(x^4+1)^(1/4)/(x^8-1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^8 - 1)/((x^8 - 1)*(x^4 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x^8-1}{{\left(x^4+1\right)}^{1/4}\,\left(x^8-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8 - 1)/((x^4 + 1)^(1/4)*(x^8 - 1)),x)
[Out] int((2*x^8 - 1)/((x^4 + 1)^(1/4)*(x^8 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^{8} - 1}{\left(x - 1\right) \left(x + 1\right) \left(x^{2} + 1\right) \left(x^{4} + 1\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8-1)/(x**4+1)**(1/4)/(x**8-1),x)
[Out] Integral((2*x**8 - 1)/((x - 1)*(x + 1)*(x**2 + 1)*(x**4 + 1)**(5/4)), x)
```

3.1121 $\int \sqrt[4]{bx^5 + ax^8}\, dx$

Optimal. Leaf size=91

$$\frac{b\tan^{-1}\left(\frac{\sqrt[4]{ax^8+bx^5}}{\sqrt[4]{a}\,x^2}\right)}{6a^{3/4}}+\frac{b\tanh^{-1}\left(\frac{\sqrt[4]{ax^8+bx^5}}{\sqrt[4]{a}\,x^2}\right)}{6a^{3/4}}+\frac{1}{3}x\sqrt[4]{ax^8+bx^5}$$

Rubi [A] time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.64, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2004, 2032, 329, 275, 331, 298, 203, 206}

$$-\frac{bx^{15/4}\left(ax^3+b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{6a^{3/4}\left(ax^8+bx^5\right)^{3/4}}+\frac{bx^{15/4}\left(ax^3+b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{6a^{3/4}\left(ax^8+bx^5\right)^{3/4}}+\frac{1}{3}x\sqrt[4]{ax^8+bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^5 + a*x^8)^(1/4),x]
```

```
[Out] (x*(b*x^5 + a*x^8)^(1/4))/3 - (b*x^(15/4)*(b + a*x^3)^(3/4)*ArcTan[(a^(1/4)
*x^(3/4))/(b + a*x^3)^(1/4)])/(6*a^(3/4)*(b*x^5 + a*x^8)^(3/4)) + (b*x^(15/
4)*(b + a*x^3)^(3/4)*ArcTanh[(a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)])/(6*a^(3/
4)*(b*x^5 + a*x^8)^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt[4]{bx^5+ax^8}\,dx &= \frac{1}{3}x\sqrt[4]{bx^5+ax^8}+\frac{1}{4}b\int \frac{x^5}{\left(bx^5+ax^8\right)^{3/4}}\,dx\\
&= \frac{1}{3}x\sqrt[4]{bx^5+ax^8}+\frac{\left(bx^{15/4}\left(b+ax^3\right)^{3/4}\right)\int \frac{x^{5/4}}{\left(b+ax^3\right)^{3/4}}\,dx}{4\left(bx^5+ax^8\right)^{3/4}}\\
&= \frac{1}{3}x\sqrt[4]{bx^5+ax^8}+\frac{\left(bx^{15/4}\left(b+ax^3\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^8}{\left(b+ax^{12}\right)^{3/4}}\,dx,x,\sqrt[4]{x}\right)}{\left(bx^5+ax^8\right)^{3/4}}\\
&= \frac{1}{3}x\sqrt[4]{bx^5+ax^8}+\frac{\left(bx^{15/4}\left(b+ax^3\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{\left(b+ax^4\right)^{3/4}}\,dx,x,x^{3/4}\right)}{3\left(bx^5+ax^8\right)^{3/4}}\\
&= \frac{1}{3}x\sqrt[4]{bx^5+ax^8}+\frac{\left(bx^{15/4}\left(b+ax^3\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\left(bx^5+ax^8\right)^{3/4}}\\
&= \frac{1}{3}x\sqrt[4]{bx^5+ax^8}+\frac{\left(bx^{15/4}\left(b+ax^3\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{6\sqrt{a}\left(bx^5+ax^8\right)^{3/4}}-\frac{\left(bx^{15/4}\left(b+ax^3\right)^{3/4}\right.}{6\sqrt{}}\\
&= \frac{1}{3}x\sqrt[4]{bx^5+ax^8}-\frac{bx^{15/4}\left(b+ax^3\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{6a^{3/4}\left(bx^5+ax^8\right)^{3/4}}+\frac{bx^{15/4}\left(b+ax^3\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{6a^{3/4}\left(bx^5+ax^8\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 53, normalized size = 0.58

$$
\frac{4x\sqrt[4]{x^5\left(ax^3+b\right)}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{ax^3}{b}\right)}{9\sqrt[4]{\frac{ax^3}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^5 + a*x^8)^(1/4),x]
```

```
[Out] (4*x*(x^5*(b + a*x^3))^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, -((a*x^3)/b)
])/(9*(1 + (a*x^3)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.45, size = 91, normalized size = 1.00

$$\frac{b\tan^{-1}\left(\frac{\sqrt[4]{ax^8+bx^5}}{\sqrt[4]{a}\,x^2}\right)}{6a^{3/4}}+\frac{b\tanh^{-1}\left(\frac{\sqrt[4]{ax^8+bx^5}}{\sqrt[4]{a}\,x^2}\right)}{6a^{3/4}}+\frac{1}{3}x\sqrt[4]{ax^8+bx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^5 + a*x^8)^(1/4),x]
```

```
[Out] (x*(b*x^5 + a*x^8)^(1/4))/3 + (b*ArcTan[(b*x^5 + a*x^8)^(1/4)/(a^(1/4)*x^2)
])/(6*a^(3/4)) + (b*ArcTanh[(b*x^5 + a*x^8)^(1/4)/(a^(1/4)*x^2)])/(6*a^(3/4
))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+b*x^5)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.29, size = 213, normalized size = 2.34

$$\frac{8\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}bx^3+\frac{2\sqrt{2}(-a)^{\frac{1}{4}}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a}+\frac{2\sqrt{2}(-a)^{\frac{1}{4}}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a}+\frac{\sqrt{2}(-a)^{\frac{1}{4}}b^2\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)}{a}+\frac{\sqrt{2}b^2\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)}{(-a)^{\frac{3}{4}}}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+b*x^5)^(1/4),x, algorithm="giac")
```

```
[Out] 1/24*(8*(a + b/x^3)^(1/4)*b*x^3 + 2*sqrt(2)*(-a)^(1/4)*b^2*arctan(1/2*sqrt(
2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^3)^(1/4))/(-a)^(1/4))/a + 2*sqrt(2)*(-a
)^(1/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(a + b/x^3)^(1/4))/
(-a)^(1/4))/a + sqrt(2)*(-a)^(1/4)*b^2*log(sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(
1/4) + sqrt(-a) + sqrt(a + b/x^3))/a + sqrt(2)*b^2*log(-sqrt(2)*(-a)^(1/4)*
(a + b/x^3)^(1/4) + sqrt(-a) + sqrt(a + b/x^3))/(-a)^(3/4))/b
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \left(a\,x^{8}+b\,x^{5}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^8+b*x^5)^(1/4),x)
```

```
[Out] int((a*x^8+b*x^5)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^{8}+bx^{5}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+b*x^5)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^8 + b*x^5)^(1/4), x)
```

mupad [B] time = 0.93, size = 42, normalized size = 0.46

$$\frac{4\,x\,{\left(a\,x^8+b\,x^5\right)}^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{a\,x^3}{b}\right)}{9\,{\left(\frac{a\,x^3}{b}+1\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^8 + b*x^5)^(1/4),x)
```

```
[Out] (4*x*(a*x^8 + b*x^5)^(1/4)*hypergeom([-1/4, 3/4], 7/4, -(a*x^3)/b))/(9*((a*
x^3)/b + 1)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{ax^{8} + bx^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**8+b*x**5)**(1/4),x)
```

```
[Out] Integral((a*x**8 + b*x**5)**(1/4), x)
```

3.1122 $\int \frac{\sqrt{x-\sqrt{1+x^2}}}{1-\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=91

$$\frac{2\sqrt{x-\sqrt{x^2+1}}\,(-x-2)+2\sqrt{x^2+1}\,\sqrt{x-\sqrt{x^2+1}}}{\sqrt{x^2+1}-x-1}-2\tan^{-1}\left(\sqrt{x-\sqrt{x^2+1}}\right)$$

Rubi [A] time = 0.44, antiderivative size = 83, normalized size of antiderivative = 0.91, number of steps used = 16, number of rules used = 12, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6742, 2119, 457, 329, 298, 203, 206, 2120, 463, 12, 321, 212}

$$\frac{\sqrt{x-\sqrt{x^2+1}}}{x}+2\sqrt{x-\sqrt{x^2+1}}-\frac{1}{x\sqrt{x-\sqrt{x^2+1}}}-2\tan^{-1}\left(\sqrt{x-\sqrt{x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x - Sqrt[1 + x^2]]/(1 - Sqrt[1 + x^2]),x]
```

```
[Out] -(1/(x*Sqrt[x - Sqrt[1 + x^2]])) + 2*Sqrt[x - Sqrt[1 + x^2]] + Sqrt[x - Sqr
t[1 + x^2]]/x - 2*ArcTan[Sqrt[x - Sqrt[1 + x^2]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{x-\sqrt{1+x^2}}}{1-\sqrt{1+x^2}}\,dx = \int \left(-\frac{\sqrt{x-\sqrt{1+x^2}}}{x^2}-\frac{\sqrt{1+x^2}\sqrt{x-\sqrt{1+x^2}}}{x^2}\right)dx$$

$$= -\int \frac{\sqrt{x-\sqrt{1+x^2}}}{x^2}\,dx - \int \frac{\sqrt{1+x^2}\sqrt{x-\sqrt{1+x^2}}}{x^2}\,dx$$

$$= -\left(2\,\text{Subst}\left(\int \frac{\sqrt{x}\left(1+x^2\right)}{\left(-1+x^2\right)^2}\,dx, x, x-\sqrt{1+x^2}\right)\right) + \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\sqrt{x}\left(-1+x^2\right)^2}\,dx, x, x -$$

$$= \frac{\sqrt{x-\sqrt{1+x^2}}}{x} + \frac{\sqrt{x-\sqrt{1+x^2}}}{x\left(-x+\sqrt{1+x^2}\right)} + \frac{1}{2}\text{Subst}\left(\int \frac{2x^{3/2}}{-1+x^2}\,dx, x, x-\sqrt{1+x^2}\right) - \text{Sul}$$

$$= \frac{\sqrt{x-\sqrt{1+x^2}}}{x} + \frac{\sqrt{x-\sqrt{1+x^2}}}{x\left(-x+\sqrt{1+x^2}\right)} - 2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{x-\sqrt{1+x^2}}\right) + \text{S}$$

$$= 2\sqrt{x-\sqrt{1+x^2}} + \frac{\sqrt{x-\sqrt{1+x^2}}}{x} + \frac{\sqrt{x-\sqrt{1+x^2}}}{x\left(-x+\sqrt{1+x^2}\right)} + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x\cdot}\right.$$

$$= 2\sqrt{x-\sqrt{1+x^2}} + \frac{\sqrt{x-\sqrt{1+x^2}}}{x} + \frac{\sqrt{x-\sqrt{1+x^2}}}{x\left(-x+\sqrt{1+x^2}\right)} - \tan^{-1}\left(\sqrt{x-\sqrt{1+x^2}}\right) + \text{ta}$$

$$= 2\sqrt{x-\sqrt{1+x^2}} + \frac{\sqrt{x-\sqrt{1+x^2}}}{x} + \frac{\sqrt{x-\sqrt{1+x^2}}}{x\left(-x+\sqrt{1+x^2}\right)} - \tan^{-1}\left(\sqrt{x-\sqrt{1+x^2}}\right) + \text{ta}$$

$$= 2\sqrt{x-\sqrt{1+x^2}} + \frac{\sqrt{x-\sqrt{1+x^2}}}{x} + \frac{\sqrt{x-\sqrt{1+x^2}}}{x\left(-x+\sqrt{1+x^2}\right)} - 2\tan^{-1}\left(\sqrt{x-\sqrt{1+x^2}}\right)$$

Mathematica [C] time = 14.17, size = 1129, normalized size = 12.41

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[x - Sqrt[1 + x^2]]/(1 - Sqrt[1 + x^2]),x]
```

```
[Out] -1/4*((-1 + (x - Sqrt[1 + x^2])^2)^2*((-2*(x - Sqrt[1 + x^2])^(3/2))/(-1 +
(x - Sqrt[1 + x^2])^2) + ArcTan[Sqrt[x - Sqrt[1 + x^2]]] - ArcTanh[Sqrt[x -
 Sqrt[1 + x^2]]]))/(x^2*(1 - x/Sqrt[1 + x^2])*(x - Sqrt[1 + x^2])^2*(1 - (-
1 + (x - Sqrt[1 + x^2])^2)/(2*(x - Sqrt[1 + x^2])^2))) - (159120*(x - Sqrt[
1 + x^2])^(23/2)*(1 + x^2 - x*Sqrt[1 + x^2])*((-3645 - 6769*(x - Sqrt[1 + x
^2])^2 - 1483*(x - Sqrt[1 + x^2])^4 + 681*(x - Sqrt[1 + x^2])^6 + 5*(729 +
1208*(x - Sqrt[1 + x^2])^2 + 102*(x - Sqrt[1 + x^2])^4 - 248*(x - Sqrt[1 +
x^2])^6 + (x - Sqrt[1 + x^2])^8)*Hypergeometric2F1[1/4, 1, 5/4, (x - Sqrt[1
 + x^2])^2])/(640*(x - Sqrt[1 + x^2])^4) + (16*(x - Sqrt[1 + x^2] + (x - Sq
rt[1 + x^2])^3)^2*HypergeometricPFQ[{5/4, 2, 2, 2}, {1, 1, 17/4}, (x - Sqrt
[1 + x^2])^2])/585))/(x*(1 - x/Sqrt[1 + x^2])*(1 + (x - Sqrt[1 + x^2])^2)*(
1989*(140 - 8563*x^2 - 99050*x^4 - 331584*x^6 - 565792*x^8 - 540032*x^10 -
217600*x^12 + 687*x*Sqrt[1 + x^2] + 36170*x^3*Sqrt[1 + x^2] + 183200*x^5*Sq
rt[1 + x^2] + 377376*x^7*Sqrt[1 + x^2] + 431232*x^9*Sqrt[1 + x^2] + 217600*
x^11*Sqrt[1 + x^2] + 10*x*(1335*x + 13240*x^3 + 44684*x^5 + 79168*x^7 + 758
```

```
40*x^9 + 28672*x^11 - 1024*x^13 - 182*Sqrt[1 + x^2] - 4978*x^2*Sqrt[1 + x^2
] - 24300*x^4*Sqrt[1 + x^2] - 52320*x^6*Sqrt[1 + x^2] - 61120*x^8*Sqrt[1 +
x^2] - 29184*x^10*Sqrt[1 + x^2] + 1024*x^12*Sqrt[1 + x^2])*Hypergeometric2F
1[1/4, 1, 5/4, (x - Sqrt[1 + x^2])^2]) + 4352*x*(-130*x - 4550*x^3 - 46592*
x^5 - 212160*x^7 - 499200*x^9 - 632320*x^11 - 409600*x^13 - 106496*x^15 + 9
*Sqrt[1 + x^2] + 938*x^2*Sqrt[1 + x^2] + 15904*x^4*Sqrt[1 + x^2] + 100800*x
^6*Sqrt[1 + x^2] + 303360*x^8*Sqrt[1 + x^2] + 467456*x^10*Sqrt[1 + x^2] + 3
56352*x^12*Sqrt[1 + x^2] + 106496*x^14*Sqrt[1 + x^2])*HypergeometricPFQ[{5/
4, 2, 2, 2}, {1, 1, 17/4}, (x - Sqrt[1 + x^2])^2] + 40960*x*(-16*x - 688*x^
3 - 8736*x^5 - 50304*x^7 - 154880*x^9 - 272384*x^11 - 274432*x^13 - 147456*
x^15 - 32768*x^17 + Sqrt[1 + x^2] + 128*x^2*Sqrt[1 + x^2] + 2688*x^4*Sqrt[1
 + x^2] + 21504*x^6*Sqrt[1 + x^2] + 84480*x^8*Sqrt[1 + x^2] + 180224*x^10*S
qrt[1 + x^2] + 212992*x^12*Sqrt[1 + x^2] + 131072*x^14*Sqrt[1 + x^2] + 3276
8*x^16*Sqrt[1 + x^2])*HypergeometricPFQ[{9/4, 3, 3, 3}, {2, 2, 21/4}, (x -
Sqrt[1 + x^2])^2]))
```

IntegrateAlgebraic [A] time = 0.10, size = 91, normalized size = 1.00

$$\frac{2\sqrt{x-\sqrt{x^2+1}}\,(-x-2)+2\sqrt{x^2+1}\,\sqrt{x-\sqrt{x^2+1}}}{\sqrt{x^2+1}-x-1}-2\tan^{-1}\left(\sqrt{x-\sqrt{x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x - Sqrt[1 + x^2]]/(1 - Sqrt[1 + x^2]),x]
```

```
[Out] (2*(-2 - x)*Sqrt[x - Sqrt[1 + x^2]] + 2*Sqrt[1 + x^2]*Sqrt[x - Sqrt[1 + x^2
]])/(-1 - x + Sqrt[1 + x^2]) - 2*ArcTan[Sqrt[x - Sqrt[1 + x^2]]]
```

fricas [A] time = 0.45, size = 50, normalized size = 0.55

$$-\frac{2\,x\arctan\left(\sqrt{x-\sqrt{x^2+1}}\right)-\left(3\,x+\sqrt{x^2+1}+1\right)\sqrt{x-\sqrt{x^2+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+1)^(1/2))^(1/2)/(1-(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -(2*x*arctan(sqrt(x - sqrt(x^2 + 1))) - (3*x + sqrt(x^2 + 1) + 1)*sqrt(x -
sqrt(x^2 + 1)))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{x-\sqrt{x^2+1}}}{\sqrt{x^2+1}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+1)^(1/2))^(1/2)/(1-(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(x - sqrt(x^2 + 1))/(sqrt(x^2 + 1) - 1), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x-\sqrt{x^2+1}}}{1-\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+1)^(1/2))^(1/2)/(1-(x^2+1)^(1/2)),x)
```

```
[Out] int((x-(x^2+1)^(1/2))^(1/2)/(1-(x^2+1)^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x-\sqrt{x^2+1}}}{\sqrt{x^2+1}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+1)^(1/2))^(1/2)/(1-(x^2+1)^(1/2)),x, algorithm="maxima")
[Out] -integrate(sqrt(x - sqrt(x^2 + 1))/(sqrt(x^2 + 1) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{x-\sqrt{x^2+1}}}{\sqrt{x^2+1}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - (x^2 + 1)^(1/2))^(1/2)/((x^2 + 1)^(1/2) - 1),x)
[Out] -int((x - (x^2 + 1)^(1/2))^(1/2)/((x^2 + 1)^(1/2) - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x-\sqrt{x^2+1}}}{\sqrt{x^2+1}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+1)**(1/2))**(1/2)/(1-(x**2+1)**(1/2)),x)
[Out] -Integral(sqrt(x - sqrt(x**2 + 1))/(sqrt(x**2 + 1) - 1), x)
```

3.1123 $\int \frac{(-1+x)(3+x)}{\left(-1+x^2\right)^{2/3}\left(2-x+x^2\right)}\,dx$

Optimal. Leaf size=92

$$-\log\left(\left(x^2-1\right)^{2/3}+x+1\right)+\frac{1}{2}\log\left(x^2+\left(x^2-1\right)^{4/3}+(-x-1)\left(x^2-1\right)^{2/3}+2x+1\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(x^2-1\right)^{2/3}}{\left(x^2-1\right)^{2/3}-2x-\right.}\right.$$

Rubi [F] time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x)(3+x)}{\left(-1+x^2\right)^{2/3}\left(2-x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x)*(3 + x))/((-1 + x^2)^(2/3)*(2 - x + x^2)),x]
```

```
[Out] (3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + (-1 + x^2)^(1/3))*Sqrt[(1 - (-1 + x^2)^(1/3
) + (-1 + x^2)^(2/3))/(1 + Sqrt[3] + (-1 + x^2)^(1/3))^2]*EllipticF[ArcSin[
(1 - Sqrt[3] + (-1 + x^2)^(1/3))/(1 + Sqrt[3] + (-1 + x^2)^(1/3))], -7 - 4*
Sqrt[3]])/(x*Sqrt[(1 + (-1 + x^2)^(1/3))/(1 + Sqrt[3] + (-1 + x^2)^(1/3))^2
]) - Defer[Int][(5 - 3*x)/((-1 + x^2)^(2/3)*(2 - x + x^2)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+x)(3+x)}{\left(-1+x^2\right)^{2/3}\left(2-x+x^2\right)}\,dx &= \int \left(\frac{1}{\left(-1+x^2\right)^{2/3}}-\frac{5-3x}{\left(-1+x^2\right)^{2/3}\left(2-x+x^2\right)}\right)dx\\ &= \int \frac{1}{\left(-1+x^2\right)^{2/3}}\,dx-\int \frac{5-3x}{\left(-1+x^2\right)^{2/3}\left(2-x+x^2\right)}\,dx\\ &= \frac{\left(3\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^3}}\,dx,x,\sqrt[3]{-1+x^2}\right)}{2x}-\int \frac{5-3x}{\left(-1+x^2\right)^{2/3}\left(2-x+x^2\right)}\,dx\\ &= \frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(1+\sqrt[3]{-1+x^2}\right)\sqrt{\frac{1-\sqrt[3]{-1+x^2}+\left(-1+x^2\right)^{2/3}}{\left(1+\sqrt{3}+\sqrt[3]{-1+x^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+\sqrt[3]{-1+x^2}}{1+\sqrt{3}+\sqrt[3]{-1+x^2}}\right)\right|}{x\sqrt{\frac{1+\sqrt[3]{-1+x^2}}{\left(1+\sqrt{3}+\sqrt[3]{-1+x^2}\right)^2}}}\end{aligned}$$

Mathematica [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{(-1+x)(3+x)}{\left(-1+x^2\right)^{2/3}\left(2-x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x)*(3 + x))/((-1 + x^2)^(2/3)*(2 - x + x^2)),x]
```

```
[Out] Integrate[((-1 + x)*(3 + x))/((-1 + x^2)^(2/3)*(2 - x + x^2)), x]
```

IntegrateAlgebraic [A] time = 0.14, size = 92, normalized size = 1.00

$$-\log\left(\left(x^2-1\right)^{2/3}+x+1\right)+\frac{1}{2}\log\left(x^2+\left(x^2-1\right)^{4/3}+(-x-1)\left(x^2-1\right)^{2/3}+2x+1\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(x^2-1\right)^{2/3}}{\left(x^2-1\right)^{2/3}-2x-2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)*(3 + x))/((-1 + x^2)^(2/3)*(2 - x + x^2)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*(-1 + x^2)^(2/3))/(-2 - 2*x + (-1 + x^2)^(2/3))])
 - Log[1 + x + (-1 + x^2)^(2/3)] + Log[1 + 2*x + x^2 + (-1 - x)*(-1 + x^2)^
(2/3) + (-1 + x^2)^(4/3)]/2
```

fricas [A] time = 0.88, size = 98, normalized size = 1.07

$$-\sqrt{3}\arctan\left(\frac{4\sqrt{3}\left(x^2-1\right)^{\frac{1}{3}}(x-1)+\sqrt{3}(x+1)+2\sqrt{3}\left(x^2-1\right)^{\frac{2}{3}}}{8x^2-17x+7}\right)-\frac{1}{2}\log\left(\frac{x^2+3\left(x^2-1\right)^{\frac{1}{3}}(x-1)-x+3\left(x^2-1\right)^{\frac{2}{3}}+2}{x^2-x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(3+x)/(x^2-1)^(2/3)/(x^2-x+2),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan((4*sqrt(3)*(x^2 - 1)^(1/3)*(x - 1) + sqrt(3)*(x + 1) + 2*sq
rt(3)*(x^2 - 1)^(2/3))/(8*x^2 - 17*x + 7)) - 1/2*log((x^2 + 3*(x^2 - 1)^(1/
3)*(x - 1) - x + 3*(x^2 - 1)^(2/3) + 2)/(x^2 - x + 2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x+3)(x-1)}{\left(x^2-x+2\right)\left(x^2-1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(3+x)/(x^2-1)^(2/3)/(x^2-x+2),x, algorithm="giac")
```

```
[Out] integrate((x + 3)*(x - 1)/((x^2 - x + 2)*(x^2 - 1)^(2/3)), x)
```

maple [C] time = 2.39, size = 315, normalized size = 3.42

$$-\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x^2-3\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x-3\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^2-1\right)^{\frac{1}{3}}x-3\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x^2+3\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^2-1\right)^{\frac{1}{3}}+8\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x+3\left(x^2-1\right)^{\frac{2}{3}}+3x\left(x^2-1\right)^{\frac{1}{3}}+2x^2-\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)-3\left(x^2-1\right)^{\frac{1}{3}}-4x+2}{x^2-x+2}\right)+\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x^2-3\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x-3\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^2-1\right)^{\frac{1}{3}}x-2\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x^2+3\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^2-1\right)^{\frac{1}{3}}+7\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x+3\left(x^2-1\right)^{\frac{2}{3}}+3x\left(x^2-1\right)^{\frac{1}{3}}+\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)-3\left(x^2-1\right)^{\frac{1}{3}}-2x-2}{x^2-x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)*(3+x)/(x^2-1)^(2/3)/(x^2-x+2),x)
```

```
[Out] -ln(-(RootOf(_Z^2-_Z+1)^2*x^2-3*RootOf(_Z^2-_Z+1)^2*x-3*RootOf(_Z^2-_Z+1)*(
x^2-1)^(1/3)*x-3*RootOf(_Z^2-_Z+1)*x^2+3*RootOf(_Z^2-_Z+1)*(x^2-1)^(1/3)+8*
RootOf(_Z^2-_Z+1)*x+3*(x^2-1)^(2/3)+3*x*(x^2-1)^(1/3)+2*x^2-RootOf(_Z^2-_Z+
1)-3*(x^2-1)^(1/3)-4*x+2)/(x^2-x+2))+RootOf(_Z^2-_Z+1)*ln(-(RootOf(_Z^2-_Z+
1)^2*x^2-3*RootOf(_Z^2-_Z+1)^2*x-3*RootOf(_Z^2-_Z+1)*(x^2-1)^(1/3)*x-2*Root
Of(_Z^2-_Z+1)*x^2+3*RootOf(_Z^2-_Z+1)*(x^2-1)^(1/3)+7*RootOf(_Z^2-_Z+1)*x+3
*(x^2-1)^(2/3)+3*x*(x^2-1)^(1/3)+RootOf(_Z^2-_Z+1)-3*(x^2-1)^(1/3)-2*x-2)/(
x^2-x+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x+3)(x-1)}{\left(x^2-x+2\right)\left(x^2-1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(3+x)/(x^2-1)^(2/3)/(x^2-x+2),x, algorithm="maxima")
```

```
[Out] integrate((x + 3)*(x - 1)/((x^2 - x + 2)*(x^2 - 1)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x-1)\ (x+3)}{\left(x^2-1\right)^{2/3}\ \left(x^2-x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)*(x + 3))/((x^2 - 1)^(2/3)*(x^2 - x + 2)),x)
```

```
[Out] int(((x - 1)*(x + 3))/((x^2 - 1)^(2/3)*(x^2 - x + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+3)}{((x-1)\,(x+1))^{\frac{2}{3}}\,\left(x^2-x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(3+x)/(x**2-1)**(2/3)/(x**2-x+2),x)
```

```
[Out] Integral((x - 1)*(x + 3)/(((x - 1)*(x + 1))**(2/3)*(x**2 - x + 2)), x)
```

3.1124 $\int \frac{\sqrt[3]{-1+x^3}}{x^4}\,dx$

Optimal. Leaf size=92

$$-\frac{\sqrt[3]{x^3-1}}{3x^3}+\frac{1}{9}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{1}{18}\log\left(\left(x^3-1\right)^{2/3}-\sqrt[3]{x^3-1}+1\right)-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 47, 58, 618, 204, 31}

$$-\frac{\sqrt[3]{x^3-1}}{3x^3}+\frac{1}{6}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\log(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)^(1/3)/x^4,x]
```

```
[Out] -1/3*(-1 + x^3)^(1/3)/x^3 - ArcTan[(1 - 2*(-1 + x^3)^(1/3))/Sqrt[3]]/(3*Sqr
t[3]) - Log[x]/6 + Log[1 + (-1 + x^3)^(1/3)]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{-1+x^3}}{x^4}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt[3]{-1+x}}{x^2}\,dx,x,x^3\right)\\
&= -\frac{\sqrt[3]{-1+x^3}}{3x^3}+\frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{(-1+x)^{2/3}x}\,dx,x,x^3\right)\\
&= -\frac{\sqrt[3]{-1+x^3}}{3x^3}-\frac{\log(x)}{6}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt[3]{-1+x^3}\right)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,\sqrt[3]{}\right.\\
&= -\frac{\sqrt[3]{-1+x^3}}{3x^3}-\frac{\log(x)}{6}+\frac{1}{6}\log\left(1+\sqrt[3]{-1+x^3}\right)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2\sqrt[3]{-1+x^3}\right)\\
&= -\frac{\sqrt[3]{-1+x^3}}{3x^3}-\frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^3}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\log(x)}{6}+\frac{1}{6}\log\left(1+\sqrt[3]{-1+x^3}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.30

$$\frac{1}{4}\left(x^3-1\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)^(1/3)/x^4,x]
```

```
[Out] ((-1 + x^3)^(4/3)*Hypergeometric2F1[4/3, 2, 7/3, 1 - x^3])/4
```

IntegrateAlgebraic [A] time = 0.11, size = 92, normalized size = 1.00

$$-\frac{\sqrt[3]{x^3-1}}{3x^3}+\frac{1}{9}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{1}{18}\log\left(\left(x^3-1\right)^{2/3}-\sqrt[3]{x^3-1}+1\right)-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)^(1/3)/x^4,x]
```

```
[Out] -1/3*(-1 + x^3)^(1/3)/x^3 - ArcTan[1/Sqrt[3] - (2*(-1 + x^3)^(1/3))/Sqrt[3]
]/(3*Sqrt[3]) + Log[1 + (-1 + x^3)^(1/3)]/9 - Log[1 - (-1 + x^3)^(1/3) + (-
1 + x^3)^(2/3)]/18
```

fricas [A] time = 0.41, size = 81, normalized size = 0.88

$$\frac{2\sqrt{3}\,x^3\arctan\left(\frac{2}{3}\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-x^3\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+2\,x^3\log\left(\left(x^3-1\right)^{\frac{1}{3}}+1\right)-6\left(x^3-1\right)^{\frac{1}{3}}}{18\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)/x^4,x, algorithm="fricas")
```

```
[Out] 1/18*(2*sqrt(3)*x^3*arctan(2/3*sqrt(3)*(x^3 - 1)^(1/3) - 1/3*sqrt(3)) - x^3
*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 2*x^3*log((x^3 - 1)^(1/3) + 1
) - 6*(x^3 - 1)^(1/3))/x^3
```

giac [A] time = 0.35, size = 69, normalized size = 0.75

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3-1\right)^{\frac{1}{3}}-1\right)\right)-\frac{\left(x^3-1\right)^{\frac{1}{3}}}{3x^3}-\frac{1}{18}\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{9}\log\left(\left|\left(x^3-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)/x^4,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) - 1/3*(x^3 - 1)^(1/
3)/x^3 - 1/18*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 1/9*log(abs((x^3
 - 1)^(1/3) + 1))
```

maple [C] time = 0.31, size = 79, normalized size = 0.86

$$-\frac{\left(x^3-1\right)^{\frac{1}{3}}}{3x^3}+\frac{\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{2}{3}}\left(\frac{2\Gamma\left(\frac{2}{3}\right)x^3\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],x^3\right)}{3}+\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)\right)}{9\Gamma\left(\frac{2}{3}\right)\mathrm{signum}\left(x^3-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/3)/x^4,x)
```

```
[Out] -1/3*(x^3-1)^(1/3)/x^3+1/9/GAMMA(2/3)/signum(x^3-1)^(2/3)*(-signum(x^3-1))^
(2/3)*(2/3*GAMMA(2/3)*x^3*hypergeom([1,1,5/3],[2,2],x^3)+(1/6*Pi*3^(1/2)-3/
2*ln(3)+3*ln(x)+I*Pi)*GAMMA(2/3))
```

maxima [A] time = 0.62, size = 68, normalized size = 0.74

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3-1\right)^{\frac{1}{3}}-1\right)\right)-\frac{\left(x^3-1\right)^{\frac{1}{3}}}{3x^3}-\frac{1}{18}\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{9}\log\left(\left(x^3-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) - 1/3*(x^3 - 1)^(1/
3)/x^3 - 1/18*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 1/9*log((x^3 - 1
)^(1/3) + 1)
```

mupad [B] time = 0.89, size = 80, normalized size = 0.87

$$\frac{\ln\left(\frac{\left(x^3-1\right)^{1/3}}{9}+\frac{1}{9}\right)}{9}+\ln\left(\left(x^3-1\right)^{1/3}-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)-\frac{\left(x^3-1\right)^{1/3}}{3x^3}-\ln\left(\frac{1}{2}-\left(x^3-1\right)^{1/3}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)^(1/3)/x^4,x)
```

```
[Out] log((x^3 - 1)^(1/3)/9 + 1/9)/9 + log((3^(1/2)*1i)/2 + (x^3 - 1)^(1/3) - 1/2
)*((3^(1/2)*1i)/18 - 1/18) - (x^3 - 1)^(1/3)/(3*x^3) - log((3^(1/2)*1i)/2 -
 (x^3 - 1)^(1/3) + 1/2)*((3^(1/2)*1i)/18 + 1/18)
```

sympy [C] time = 0.95, size = 34, normalized size = 0.37

$$-\frac{\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^3}\right)}{3x^2\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/3)/x**4,x)
```

```
[Out] -gamma(2/3)*hyper((-1/3, 2/3), (5/3,), exp_polar(2*I*pi)/x**3)/(3*x**2*gamm
a(5/3))
```

3.1125 $\int \frac{(-1+x^3)^{2/3}}{x^4}\,dx$

Optimal. Leaf size=92

$$-\frac{(x^3-1)^{2/3}}{3x^3}-\frac{2}{9}\log\left(\sqrt[3]{x^3-1}+1\right)+\frac{1}{9}\log\left((x^3-1)^{2/3}-\sqrt[3]{x^3-1}+1\right)-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 47, 56, 618, 204, 31}

$$-\frac{(x^3-1)^{2/3}}{3x^3}-\frac{1}{3}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{2\tan^{-1}\left(\frac{1-2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)^(2/3)/x^4,x]
```

```
[Out] -1/3*(-1 + x^3)^(2/3)/x^3 - (2*ArcTan[(1 - 2*(-1 + x^3)^(1/3))/Sqrt[3]])/(3
*Sqrt[3]) + Log[x]/3 - Log[1 + (-1 + x^3)^(1/3)]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(-1+x^3\right)^{2/3}}{x^4}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(-1+x)^{2/3}}{x^2}\,dx, x, x^3\right)$$
$$= -\frac{\left(-1+x^3\right)^{2/3}}{3x^3} + \frac{2}{9}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x}\,dx, x, x^3\right)$$
$$= -\frac{\left(-1+x^3\right)^{2/3}}{3x^3} + \frac{\log(x)}{3} - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[3]{-1+x^3}\right) + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,\right.$$
$$= -\frac{\left(-1+x^3\right)^{2/3}}{3x^3} + \frac{\log(x)}{3} - \frac{1}{3}\log\left(1+\sqrt[3]{-1+x^3}\right) - \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[3]{-1+\right.$$
$$= -\frac{\left(-1+x^3\right)^{2/3}}{3x^3} - \frac{2\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\log(x)}{3} - \frac{1}{3}\log\left(1+\sqrt[3]{-1+x^3}\right)$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.30

$$\frac{1}{5}\left(x^3-1\right)^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)^(2/3)/x^4,x]
```

```
[Out] ((-1 + x^3)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 - x^3])/5
```

IntegrateAlgebraic [A] time = 0.08, size = 92, normalized size = 1.00

$$-\frac{\left(x^3-1\right)^{2/3}}{3x^3} - \frac{2}{9}\log\left(\sqrt[3]{x^3-1}+1\right) + \frac{1}{9}\log\left(\left(x^3-1\right)^{2/3} - \sqrt[3]{x^3-1}+1\right) - \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)^(2/3)/x^4,x]
```

```
[Out] -1/3*(-1 + x^3)^(2/3)/x^3 - (2*ArcTan[1/Sqrt[3] - (2*(-1 + x^3)^(1/3))/Sqrt
[3]])/(3*Sqrt[3]) - (2*Log[1 + (-1 + x^3)^(1/3)])/9 + Log[1 - (-1 + x^3)^(1
/3) + (-1 + x^3)^(2/3)]/9
```

fricas [A] time = 0.41, size = 80, normalized size = 0.87

$$\frac{2\sqrt{3}\,x^3\arctan\left(\frac{2}{3}\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) + x^3\log\left(\left(x^3-1\right)^{\frac{2}{3}} - \left(x^3-1\right)^{\frac{1}{3}}+1\right) - 2x^3\log\left(\left(x^3-1\right)^{\frac{1}{3}}+1\right) - 3\left(x^3-1\right)^{\frac{2}{3}}}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)/x^4,x, algorithm="fricas")
```

```
[Out] 1/9*(2*sqrt(3)*x^3*arctan(2/3*sqrt(3)*(x^3 - 1)^(1/3) - 1/3*sqrt(3)) + x^3*
log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) - 2*x^3*log((x^3 - 1)^(1/3) + 1)
 - 3*(x^3 - 1)^(2/3))/x^3
```

giac [A] time = 0.28, size = 69, normalized size = 0.75

$$\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(x^3-1\right)^{\frac{1}{3}}-1\right)\right)-\frac{\left(x^3-1\right)^{\frac{2}{3}}}{3\,x^3}+\frac{1}{9}\,\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\,\log\left(\left|\left(x^3-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)/x^4,x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) - 1/3*(x^3 - 1)^(2/
3)/x^3 + 1/9*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) - 2/9*log(abs((x^3
- 1)^(1/3) + 1))
```

maple [C] time = 0.37, size = 96, normalized size = 1.04

$$-\frac{\left(x^3-1\right)^{\frac{2}{3}}}{3x^3}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{1}{3}}\left(\frac{2\pi\sqrt{3}\,x^3\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^3\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{9\pi\,\mathrm{signum}\left(x^3-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)/x^4,x)
```

```
[Out] -1/3*(x^3-1)^(2/3)/x^3+1/9/Pi*3^(1/2)*GAMMA(2/3)/signum(x^3-1)^(1/3)*(-sign
um(x^3-1))^(1/3)*(2/9*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1,1,4/3],[2,2],x
^3)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.94, size = 68, normalized size = 0.74

$$\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(x^3-1\right)^{\frac{1}{3}}-1\right)\right)-\frac{\left(x^3-1\right)^{\frac{2}{3}}}{3\,x^3}+\frac{1}{9}\,\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\,\log\left(\left(x^3-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)/x^4,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) - 1/3*(x^3 - 1)^(2/
3)/x^3 + 1/9*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) - 2/9*log((x^3 - 1)
^(1/3) + 1)
```

mupad [B] time = 0.89, size = 92, normalized size = 1.00

$$-\frac{2\,\ln\left(\frac{4\,{\left(x^3-1\right)}^{1/3}}{9}+\frac{4}{9}\right)}{9}-\ln\left(9\,{\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}^2+\frac{4\,{\left(x^3-1\right)}^{1/3}}{9}\right)\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)+\ln\left(9\,{\left(\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}^2+\frac{4\,{\left(x^3-1\right)}^{1/3}}{9}\right)\,\left(\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)-\frac{{\left(x^3-1\right)}^{2/3}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)^(2/3)/x^4,x)
```

```
[Out] log(9*((3^(1/2)*1i)/9 + 1/9)^2 + (4*(x^3 - 1)^(1/3))/9)*((3^(1/2)*1i)/9 + 1
/9) - log(9*((3^(1/2)*1i)/9 - 1/9)^2 + (4*(x^3 - 1)^(1/3))/9)*((3^(1/2)*1i)
/9 - 1/9) - (2*log((4*(x^3 - 1)^(1/3))/9 + 4/9))/9 - (x^3 - 1)^(2/3)/(3*x^3
)
```

sympy [C] time = 0.96, size = 32, normalized size = 0.35

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^3}\right)}{3x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)/x**4,x)
```

```
[Out] -gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_polar(2*I*pi)/x**3)/(3*x*gamma(4
/3))
```

3.1126 $\int \frac{\sqrt[3]{1+x^3}}{x^2}\,dx$

Optimal. Leaf size=92

$$-\frac{\sqrt[3]{x^3+1}}{x}-\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {277, 331, 292, 31, 634, 618, 204, 628}

$$-\frac{\sqrt[3]{x^3+1}}{x}-\frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{x^3+1}}\right)-\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(\frac{x}{\sqrt[3]{x^3+1}}+\frac{x^2}{\left(x^3+1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)^(1/3)/x^2,x]
```

```
[Out] -((1 + x^3)^(1/3)/x) - ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3]
- Log[1 - x/(1 + x^3)^(1/3)]/3 + Log[1 + x^2/(1 + x^3)^(2/3) + x/(1 + x^3)^
(1/3)]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{1+x^3}}{x^2}\,dx &= -\frac{\sqrt[3]{1+x^3}}{x}+\int \frac{x}{\left(1+x^3\right)^{2/3}}\,dx\\
&= -\frac{\sqrt[3]{1+x^3}}{x}+\text{Subst}\left(\int \frac{x}{1-x^3}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right)\\
&= -\frac{\sqrt[3]{1+x^3}}{x}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1-x}{1+x+x^2}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right)\\
&= -\frac{\sqrt[3]{1+x^3}}{x}-\frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\right)+\frac{1}{6}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{}{1+}\right.\\
&= -\frac{\sqrt[3]{1+x^3}}{x}-\frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\right)+\frac{1}{6}\log\left(1+\frac{x^2}{\left(1+x^3\right)^{2/3}}+\frac{x}{\sqrt[3]{1+x^3}}\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.\\
&= -\frac{\sqrt[3]{1+x^3}}{x}-\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\right)+\frac{1}{6}\log\left(1+\frac{x^2}{\left(1+x^3\right)^{2/3}}+\frac{x}{\sqrt[3]{1+x^3}}\right)
\end{aligned}$$

Mathematica [C] time = 0.00, size = 20, normalized size = 0.22

$$-\frac{{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-x^3\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)^(1/3)/x^2,x]
[Out] -(Hypergeometric2F1[-1/3, -1/3, 2/3, -x^3]/x)
```

IntegrateAlgebraic [A] time = 0.13, size = 92, normalized size = 1.00

$$-\frac{\sqrt[3]{x^3+1}}{x}-\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)^(1/3)/x^2,x]
```

```
[Out] -((1 + x^3)^(1/3)/x) - ArcTan[(Sqrt[3]*x)/(x + 2*(1 + x^3)^(1/3))]/Sqrt[3]
- Log[-x + (1 + x^3)^(1/3)]/3 + Log[x^2 + x*(1 + x^3)^(1/3) + (1 + x^3)^(2/
3)]/6
```

fricas [A] time = 0.63, size = 100, normalized size = 1.09

$$-\frac{2\sqrt{3}\,x\arctan\left(-\frac{25382\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}x^2-13720\sqrt{3}\left(x^3+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(5831\,x^3+7200\right)}{58653\,x^3+8000}\right)+x\log\left(3\left(x^3+1\right)^{\frac{1}{3}}x^2-3\left(x^3+1\right)^{\frac{2}{3}}x+1\right)+6\left(x^3+1\right)^{\frac{1}{3}}}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x*arctan(-(25382*sqrt(3)*(x^3 + 1)^(1/3)*x^2 - 13720*sqrt(3
)*(x^3 + 1)^(2/3)*x + sqrt(3)*(5831*x^3 + 7200))/(58653*x^3 + 8000)) + x*lo
g(3*(x^3 + 1)^(1/3)*x^2 - 3*(x^3 + 1)^(2/3)*x + 1) + 6*(x^3 + 1)^(1/3))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+1\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)^(1/3)/x^2, x)
```

maple [C] time = 0.31, size = 30, normalized size = 0.33

$$-\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(1/3)/x^2,x)
```

```
[Out] -(x^3+1)^(1/3)/x+1/2*x^2*hypergeom([2/3,2/3],[5/3],-x^3)
```

maxima [A] time = 1.02, size = 81, normalized size = 0.88

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{1}{6}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)-\frac{1}{3}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3)/x + 1)) - (x^3 + 1)^(1/3)
/x + 1/6*log((x^3 + 1)^(1/3)/x + (x^3 + 1)^(2/3)/x^2 + 1) - 1/3*log((x^3 +
1)^(1/3)/x - 1)
```

mupad [B] time = 0.90, size = 15, normalized size = 0.16

$$-\frac{{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\ \frac{2}{3};\ -x^3\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)^(1/3)/x^2,x)
```

```
[Out] -hypergeom([-1/3, -1/3], 2/3, -x^3)/x
```

sympy **[C]** time = 0.82, size = 32, normalized size = 0.35

$$\frac{\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(1/3)/x**2,x)
```

```
[Out] gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), x**3*exp_polar(I*pi))/(3*x*gamma(2/
3))
```

3.1127 $\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=92

$$-\log\left(\sqrt[3]{x^3+2}-x-2\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^3+2}}{\sqrt[3]{x^3+2}+2x+4}\right)+\frac{1}{2}\log\left(\left(x^3+2\right)^{2/3}+(x+2)\sqrt[3]{x^3+2}+x^2+4x+4\right)$$

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 0.58, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2151}

$$-\frac{3}{2}\log\left(-\sqrt[3]{x^3+2}+x+2\right)+\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]] + Log[1 + x] - (3
*Log[2 + x - (2 + x^3)^(1/3)])/2
```

Rule 2151

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[(Sqrt[3]*f*ArcTan[(1 + (2*Rt[b, 3]*(2*c + d*x))/(d*(a + b
*x^3)^(1/3)))/Sqrt[3]])/(Rt[b, 3]*d), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx=\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)+\log(1+x)-\frac{3}{2}\log\left(2+x-\sqrt[3]{2+x^3}\right)$$

Mathematica [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.84, size = 92, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^3+2}-x-2\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^3+2}}{\sqrt[3]{x^3+2}+2x+4}\right)+\frac{1}{2}\log\left(\left(x^3+2\right)^{2/3}+(x+2)\sqrt[3]{x^3+2}+x^2+4x+4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*(2 + x^3)^(1/3))/(4 + 2*x + (2 + x^3)^(1/3))]) -
Log[-2 - x + (2 + x^3)^(1/3)] + Log[4 + 4*x + x^2 + (2 + x)*(2 + x^3)^(1/3)
 + (2 + x^3)^(2/3)]/2
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^3+2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

maple [C] time = 3.69, size = 543, normalized size = 5.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(1+x)/(x^3+2)^(1/3),x)
```

```
[Out] -ln(-(787*RootOf(_Z^2-_Z+1)^2*x^3+4504*RootOf(_Z^2-_Z+1)*(x^3+2)^(2/3)*x-48
39*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x^2-1574*RootOf(_Z^2-_Z+1)^2*x^2-452*Roo
tOf(_Z^2-_Z+1)*x^3+9008*RootOf(_Z^2-_Z+1)*(x^3+2)^(2/3)+335*x*(x^3+2)^(2/3)
-19356*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x+4504*(x^3+2)^(1/3)*x^2-3148*RootOf
(_Z^2-_Z+1)^2*x+11922*RootOf(_Z^2-_Z+1)*x^2-4052*x^3+670*(x^3+2)^(2/3)-1935
6*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)+18016*x*(x^3+2)^(1/3)+23844*RootOf(_Z^2-_
Z+1)*x-20260*x^2+18016*(x^3+2)^(1/3)+11018*RootOf(_Z^2-_Z+1)-40520*x-28364)
/(1+x)^2)+RootOf(_Z^2-_Z+1)*ln((2026*RootOf(_Z^2-_Z+1)^2*x^3+4504*RootOf(_Z
^2-_Z+1)*(x^3+2)^(2/3)*x+335*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x^2-4052*RootO
f(_Z^2-_Z+1)^2*x^2-6865*RootOf(_Z^2-_Z+1)*x^3+9008*RootOf(_Z^2-_Z+1)*(x^3+2
)^(2/3)-4839*x*(x^3+2)^(2/3)+1340*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x+4504*(x
^3+2)^(1/3)*x^2-8104*RootOf(_Z^2-_Z+1)^2*x-14634*RootOf(_Z^2-_Z+1)*x^2+2361
*x^3-9678*(x^3+2)^(2/3)+1340*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)+18016*x*(x^3+2
)^(1/3)-29268*RootOf(_Z^2-_Z+1)*x+6296*x^2+18016*(x^3+2)^(1/3)-28364*RootOf
(_Z^2-_Z+1)+12592*x+11018)/(1+x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^3+2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x-1}{\left(x^3+2\right)^{1/3}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x^3 + 2)^(1/3)*(x + 1)),x)
[Out] int((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{(x+1)\,\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x**3+2)**(1/3),x)
[Out] Integral((x - 1)/((x + 1)*(x**3 + 2)**(1/3)), x)
```

3.1128 $\int \frac{1}{x^7\left(b+ax^3\right)^{3/4}}\,dx$

Optimal. Leaf size=92

$$-\frac{7a^2\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{16b^{11/4}}-\frac{7a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{16b^{11/4}}+\frac{\sqrt[4]{ax^3+b}\,\left(7ax^3-4b\right)}{24b^2x^6}$$

Rubi [A] time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 63, 212, 206, 203}

$$-\frac{7a^2\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{16b^{11/4}}-\frac{7a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{16b^{11/4}}+\frac{7a\sqrt[4]{ax^3+b}}{24b^2x^3}-\frac{\sqrt[4]{ax^3+b}}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(b + a*x^3)^(3/4)),x]
```

```
[Out] -1/6*(b + a*x^3)^(1/4)/(b*x^6) + (7*a*(b + a*x^3)^(1/4))/(24*b^2*x^3) - (7*
a^2*ArcTan[(b + a*x^3)^(1/4)/b^(1/4)])/(16*b^(11/4)) - (7*a^2*ArcTanh[(b +
a*x^3)^(1/4)/b^(1/4)])/(16*b^(11/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^7\left(b+ax^3\right)^{3/4}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^3(b+ax)^{3/4}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt[4]{b+ax^3}}{6bx^6} - \frac{(7a)\operatorname{Subst}\left(\int \frac{1}{x^2(b+ax)^{3/4}}\,dx, x, x^3\right)}{24b}\\
&= -\frac{\sqrt[4]{b+ax^3}}{6bx^6} + \frac{7a\sqrt[4]{b+ax^3}}{24b^2x^3} + \frac{\left(7a^2\right)\operatorname{Subst}\left(\int \frac{1}{x(b+ax)^{3/4}}\,dx, x, x^3\right)}{32b^2}\\
&= -\frac{\sqrt[4]{b+ax^3}}{6bx^6} + \frac{7a\sqrt[4]{b+ax^3}}{24b^2x^3} + \frac{(7a)\operatorname{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^3}\right)}{8b^2}\\
&= -\frac{\sqrt[4]{b+ax^3}}{6bx^6} + \frac{7a\sqrt[4]{b+ax^3}}{24b^2x^3} - \frac{\left(7a^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^3}\right)}{16b^{5/2}} - \frac{\left(7a^2\right)\operatorname{Subst}\left(\int\right.}{}\\
&= -\frac{\sqrt[4]{b+ax^3}}{6bx^6} + \frac{7a\sqrt[4]{b+ax^3}}{24b^2x^3} - \frac{7a^2\tan^{-1}\left(\frac{\sqrt[4]{b+ax^3}}{\sqrt[4]{b}}\right)}{16b^{11/4}} - \frac{7a^2\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^3}}{\sqrt[4]{b}}\right)}{16b^{11/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.42

$$
-\frac{4a^2\sqrt[4]{ax^3+b}\;{}_2F_1\left(\frac{1}{4},3;\frac{5}{4};\frac{ax^3}{b}+1\right)}{3b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(b + a*x^3)^(3/4)),x]
```

```
[Out] (-4*a^2*(b + a*x^3)^(1/4)*Hypergeometric2F1[1/4, 3, 5/4, 1 + (a*x^3)/b])/(3
*b^3)
```

IntegrateAlgebraic [A] time = 0.15, size = 92, normalized size = 1.00

$$
-\frac{7a^2\tan^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{16b^{11/4}} - \frac{7a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax^3+b}}{\sqrt[4]{b}}\right)}{16b^{11/4}} + \frac{\sqrt[4]{ax^3+b}\left(7ax^3-4b\right)}{24b^2x^6}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(b + a*x^3)^(3/4)),x]
```

```
[Out] ((b + a*x^3)^(1/4)*(-4*b + 7*a*x^3))/(24*b^2*x^6) - (7*a^2*ArcTan[(b + a*x^
3)^(1/4)/b^(1/4)])/(16*b^(11/4)) - (7*a^2*ArcTanh[(b + a*x^3)^(1/4)/b^(1/4)
])/(16*b^(11/4))
```

fricas [B] time = 0.43, size = 216, normalized size = 2.35

$$
\frac{84\,b^2x^6\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^3+b\right)^{\frac{1}{4}}a^2b^8\left(\frac{a^8}{b^{11}}\right)^{\frac{3}{4}}-\sqrt{b^6\sqrt{\frac{a^8}{b^{11}}}+\sqrt{ax^3+b}\,a^4}\,b^8\left(\frac{a^8}{b^{11}}\right)^{\frac{3}{4}}}{a^8}\right)-21\,b^2x^6\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\log\left(7\,b^3\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}+7\left(ax^3+b\right)^{\frac{1}{4}}a^2\right)+21\,b^2x^6\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\log\left(-7\,b^3\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}+7\left(ax^3+b\right)^{\frac{1}{4}}a^2\right)+4\left(7\,ax^3-4\,b\right)\left(ax^3+b\right)^{\frac{1}{4}}}{96\,b^2x^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a*x^3+b)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/96*(84*b^2*x^6*(a^8/b^11)^(1/4)*arctan(-((a*x^3 + b)^(1/4)*a^2*b^8*(a^8/b
^11)^(3/4) - sqrt(b^6*sqrt(a^8/b^11) + sqrt(a*x^3 + b)*a^4)*b^8*(a^8/b^11)^
(3/4))/a^8) - 21*b^2*x^6*(a^8/b^11)^(1/4)*log(7*b^3*(a^8/b^11)^(1/4) + 7*(a
*x^3 + b)^(1/4)*a^2) + 21*b^2*x^6*(a^8/b^11)^(1/4)*log(-7*b^3*(a^8/b^11)^(1
/4) + 7*(a*x^3 + b)^(1/4)*a^2) + 4*(7*a*x^3 - 4*b)*(a*x^3 + b)^(1/4))/(b^2*
x^6)
```

giac [B] time = 0.24, size = 244, normalized size = 2.65

$$\frac{\frac{42\sqrt{2}a^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}+2\left(ax^3+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{(-b)^{\frac{3}{4}}b^2}+\frac{42\sqrt{2}a^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}-2\left(ax^3+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{(-b)^{\frac{3}{4}}b^2}+\frac{21\sqrt{2}a^3\log\left(\sqrt{2}\left(ax^3+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^3+b}+\sqrt{-b}\right)}{(-b)^{\frac{3}{4}}b^2}+\frac{21\sqrt{2}a^3(-b)^{\frac{1}{4}}\log\left(-\sqrt{2}\left(ax^3+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^3+b}+\sqrt{-b}\right)}{b^3}+\frac{8\left(7\left(ax^3+b\right)^{\frac{5}{4}}a^3-11\left(ax^3+b\right)^{\frac{1}{4}}a^3b\right)}{a^2b^2x^6}}{192a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a*x^3+b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/192*(42*sqrt(2)*a^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^3 + b
)^(1/4))/(-b)^(1/4))/((-b)^(3/4)*b^2) + 42*sqrt(2)*a^3*arctan(-1/2*sqrt(2)*
(sqrt(2)*(-b)^(1/4) - 2*(a*x^3 + b)^(1/4))/(-b)^(1/4))/((-b)^(3/4)*b^2) + 2
1*sqrt(2)*a^3*log(sqrt(2)*(a*x^3 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^3 + b) +
sqrt(-b))/((-b)^(3/4)*b^2) + 21*sqrt(2)*a^3*(-b)^(1/4)*log(-sqrt(2)*(a*x^3
+ b)^(1/4)*(-b)^(1/4) + sqrt(a*x^3 + b) + sqrt(-b))/b^3 + 8*(7*(a*x^3 + b)^
(5/4)*a^3 - 11*(a*x^3 + b)^(1/4)*a^3*b)/(a^2*b^2*x^6))/a
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(a\,x^3+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(a*x^3+b)^(3/4),x)
```

```
[Out] int(1/x^7/(a*x^3+b)^(3/4),x)
```

maxima [A] time = 0.50, size = 132, normalized size = 1.43

$$\frac{7\left(ax^3+b\right)^{\frac{5}{4}}a^2-11\left(ax^3+b\right)^{\frac{1}{4}}a^2b}{24\left(\left(ax^3+b\right)^2b^2-2\left(ax^3+b\right)b^3+b^4\right)}-\frac{7\left(\frac{2\,a^2\arctan\left(\frac{\left(ax^3+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}-\frac{a^2\log\left(\frac{\left(ax^3+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^3+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}\right)}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a*x^3+b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/24*(7*(a*x^3 + b)^(5/4)*a^2 - 11*(a*x^3 + b)^(1/4)*a^2*b)/((a*x^3 + b)^2*
b^2 - 2*(a*x^3 + b)*b^3 + b^4) - 7/32*(2*a^2*arctan((a*x^3 + b)^(1/4)/b^(1/
4))/b^(3/4) - a^2*log(((a*x^3 + b)^(1/4) - b^(1/4))/((a*x^3 + b)^(1/4) + b^
(1/4)))/b^(3/4))/b^2
```

mupad [B] time = 1.09, size = 82, normalized size = 0.89

$$\frac{7\left(a\,x^3+b\right)^{5/4}}{24\,b^2\,x^6}-\frac{11\left(a\,x^3+b\right)^{1/4}}{24\,b\,x^6}-\frac{7\,a^2\,\mathrm{atan}\left(\frac{\left(a\,x^3+b\right)^{1/4}}{b^{1/4}}\right)}{16\,b^{11/4}}+\frac{a^2\,\mathrm{atan}\left(\frac{\left(a\,x^3+b\right)^{1/4}\,1{}\mathrm{i}}{b^{1/4}}\right)\,7{}\mathrm{i}}{16\,b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(b + a*x^3)^(3/4)),x)
```

```
[Out] (a^2*atan(((b + a*x^3)^(1/4)*1i)/b^(1/4))*7i)/(16*b^(11/4)) - (7*a^2*atan((
b + a*x^3)^(1/4)/b^(1/4)))/(16*b^(11/4)) - (11*(b + a*x^3)^(1/4))/(24*b*x^6
) + (7*(b + a*x^3)^(5/4))/(24*b^2*x^6)
```

sympy [C] time = 1.47, size = 41, normalized size = 0.45

$$-\frac{\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{4}}x^{\frac{33}{4}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(a*x**3+b)**(3/4),x)
```

```
[Out] -gamma(11/4)*hyper((3/4, 11/4), (15/4,), b*exp_polar(I*pi)/(a*x**3))/(3*a**
(3/4)*x**(33/4)*gamma(15/4))
```

3.1129 $$\int \frac{(1+2x+x^2)(-2-(-1+k)(1+k)x+2k^2x^2)}{((1-x^2)(1-k^2x^2))^{3/4}(1-d-(1+3d)x-(3d+k^2)x^2+(-d+k^2)x^3)}\,dx$$

Optimal. Leaf size=92

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,x+\sqrt[4]{d}}{\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x+\sqrt[4]{d}}{\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}\right)}{d^{3/4}}$$

Rubi [F] time = 28.76, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+2x+x^2)(-2-(-1+k)(1+k)x+2k^2x^2)}{((1-x^2)(1-k^2x^2))^{3/4}(1-d-(1+3d)x-(3d+k^2)x^2+(-d+k^2)x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + 2*x + x^2)*(-2 - (-1 + k)*(1 + k)*x + 2*k^2*x^2))/(((1 - x^2)*(1
 - k^2*x^2))^(3/4)*(1 - d - (1 + 3*d)*x - (3*d + k^2)*x^2 + (-d + k^2)*x^3))
,x]
```

```
[Out] (Sqrt[2]*k^(3/2)*Sqrt[-1 + k^2]*Sqrt[(-1 - k^2 + 2*k^2*x^2)^2]*Sqrt[(1 + k^
2*(1 - 2*x^2))^2/((1 - k^2)^2*(1 - (2*k*Sqrt[(1 - x^2)*(1 - k^2*x^2)])/(1 -
 k^2))^2)]*(1 - (2*k*Sqrt[(1 - x^2)*(1 - k^2*x^2)])/(1 - k^2))*EllipticF[2*
ArcTan[(Sqrt[2]*Sqrt[k]*((1 - x^2)*(1 - k^2*x^2))^(1/4))/Sqrt[-1 + k^2]], 1
/2])/((d - k^2)*(1 + k^2 - 2*k^2*x^2)*Sqrt[(-1 - k^2*(1 - 2*x^2))^2]) + ((k
^2 + 5*k^4 - d*(1 - 3*k^2))*x*((1 - x^2)/(1 - k^2*x^2))^(3/4)*(1 - k^2*x^2)
*Hypergeometric2F1[1/2, 3/4, 3/2, ((1 - k^2)*x^2)/(1 - k^2*x^2)])/((d - k^2
)^2*((1 - x^2)*(1 - k^2*x^2))^(3/4)) - ((k^2 + 7*k^4 + 3*d^2*(1 - k^2) - d*
(1 + 2*k^2 + 5*k^4))*(1 - x^2)^(3/4)*(1 - k^2*x^2)^(3/4)*Defer[Int][1/((1 -
 x^2)^(3/4)*(1 - k^2*x^2)^(3/4)*(1 - d - (1 + 3*d)*x - (3*d + k^2)*x^2 - (d
 - k^2)*x^3)), x])/((d - k^2)^2*((1 - x^2)*(1 - k^2*x^2))^(3/4)) + ((k^2 -
k^6 - 6*d^2*(1 - k^2) - d*(1 - 14*k^2 - 19*k^4))*(1 - x^2)^(3/4)*(1 - k^2*x
^2)^(3/4)*Defer[Int][x/((1 - x^2)^(3/4)*(1 - k^2*x^2)^(3/4)*(1 - d - (1 + 3
*d)*x - (3*d + k^2)*x^2 - (d - k^2)*x^3)), x])/((d - k^2)^2*((1 - x^2)*(1 -
 k^2*x^2))^(3/4)) + ((24*d*k^4 - 3*d^2*(1 - k^2) + k^4*(3 + 5*k^2))*(1 - x^
2)^(3/4)*(1 - k^2*x^2)^(3/4)*Defer[Int][x^2/((1 - x^2)^(3/4)*(1 - k^2*x^2)^
(3/4)*(1 - d - (1 + 3*d)*x - (3*d + k^2)*x^2 - (d - k^2)*x^3)), x])/((d - k
^2)^2*((1 - x^2)*(1 - k^2*x^2))^(3/4))
```

Rubi steps

$$\int \frac{\left(1+2x+x^2\right)\left(-2-(-1+k)(1+k)x+2k^2x^2\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{3/4}\left(1-d-(1+3d)x-\left(3d+k^2\right)x^2+\left(-d+k^2\right)x^3\right)}\,dx = \int \frac{(1+x)^2 \left(\cdots\right.}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{3/4}\left(1\cdots\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right)^{3/4}\right)\int \cdots}{\left(\cdots\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right)^{3/4}\right)\int \cdots}{\left(\left(\cdots\right.\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right)^{3/4}\right)\int \cdots}{\cdots}$$

$$= \frac{\left(\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right)^{3/4}\right)\int \cdots}{\cdots}$$

$$= \frac{\left(k^2+5k^4-d\left(1-3k^2\right)\right)x\left(\cdots\right.}{\left(d-k^2\right)^2\left(\cdots\right.}$$

$$= \frac{\left(k^2+5k^4-d\left(1-3k^2\right)\right)x\left(\cdots\right.}{\left(d-k^2\right)^2\left(\cdots\right.}$$

$$= \frac{\left(k^2+5k^4-d\left(1-3k^2\right)\right)x\left(\cdots\right.}{\left(d-k^2\right)^2\left(\cdots\right.}$$

$$= \frac{\sqrt{2}\,k^{3/2}\sqrt{-1+k^2}\sqrt{\left(-1-k^2\cdots\right.}}{\cdots}$$

Mathematica [F] time = 2.07, size = 0, normalized size = 0.00

$$\int \frac{\left(1+2x+x^2\right)\left(-2-(-1+k)(1+k)x+2k^2x^2\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{3/4}\left(1-d-(1+3d)x-\left(3d+k^2\right)x^2+\left(-d+k^2\right)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + 2*x + x^2)*(-2 - (-1 + k)*(1 + k)*x + 2*k^2*x^2))/(((1 - x^
2)*(1 - k^2*x^2))^(3/4)*(1 - d - (1 + 3*d)*x - (3*d + k^2)*x^2 + (-d + k^2)
*x^3)),x]
```

```
[Out] Integrate[((1 + 2*x + x^2)*(-2 - (-1 + k)*(1 + k)*x + 2*k^2*x^2))/(((1 - x^
2)*(1 - k^2*x^2))^(3/4)*(1 - d - (1 + 3*d)*x - (3*d + k^2)*x^2 + (-d + k^2)
*x^3)), x]
```

IntegrateAlgebraic [A] time = 15.68, size = 92, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,x+\sqrt[4]{d}}{\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x+\sqrt[4]{d}}{\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + 2*x + x^2)*(-2 - (-1 + k)*(1 + k)*x + 2*k^2*x^2))/
(((1 - x^2)*(1 - k^2*x^2))^(3/4)*(1 - d - (1 + 3*d)*x - (3*d + k^2)*x^2 + (
-d + k^2)*x^3)),x]
```

```
[Out] ArcTan[(d^(1/4) + d^(1/4)*x)/(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4)]/d^(3/4)
- ArcTanh[(d^(1/4) + d^(1/4)*x)/(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4)]/d^(3/
4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)*(-2-(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))
^(3/4)/(1-d-(1+3*d)*x-(k^2+3*d)*x^2+(k^2-d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,k^2x^2-(k+1)(k-1)x-2\right)\left(x^2+2\,x+1\right)}{\left(\left(k^2-d\right)x^3-\left(k^2+3\,d\right)x^2-(3\,d+1)x-d+1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)*(-2-(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))
^(3/4)/(1-d-(1+3*d)*x-(k^2+3*d)*x^2+(k^2-d)*x^3),x, algorithm="giac")
```

```
[Out] integrate((2*k^2*x^2 - (k + 1)*(k - 1)*x - 2)*(x^2 + 2*x + 1)/(((k^2 - d)*x
^3 - (k^2 + 3*d)*x^2 - (3*d + 1)*x - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(3/4)
), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+2x+1\right)\left(-2-(-1+k)\,(1+k)\,x+2k^2x^2\right)}{\left(\left(-x^2+1\right)\left(-k^2x^2+1\right)\right)^{\frac{3}{4}}\left(1-d-(1+3d)\,x-\left(k^2+3d\right)x^2+\left(k^2-d\right)x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x+1)*(-2-(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))^(3/4)
/(1-d-(1+3*d)*x-(k^2+3*d)*x^2+(k^2-d)*x^3),x)
```

```
[Out] int((x^2+2*x+1)*(-2-(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))^(3/4)
/(1-d-(1+3*d)*x-(k^2+3*d)*x^2+(k^2-d)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,k^2x^2-(k+1)(k-1)x-2\right)\left(x^2+2\,x+1\right)}{\left(\left(k^2-d\right)x^3-\left(k^2+3\,d\right)x^2-(3\,d+1)x-d+1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)*(-2-(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))
^(3/4)/(1-d-(1+3*d)*x-(k^2+3*d)*x^2+(k^2-d)*x^3),x, algorithm="maxima")
```

```
[Out] integrate((2*k^2*x^2 - (k + 1)*(k - 1)*x - 2)*(x^2 + 2*x + 1)/((((k^2 - d)*x
^3 - (k^2 + 3*d)*x^2 - (3*d + 1)*x - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(3/4)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+2\,x+1\right)\,\left(x\,\left(k-1\right)\,\left(k+1\right)-2\,k^2\,x^2+2\right)}{{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)}^{3/4}\,\left(\left(d-k^2\right)\,x^3+\left(k^2+3\,d\right)\,x^2+\left(3\,d+1\right)\,x+d-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + x^2 + 1)*(x*(k - 1)*(k + 1) - 2*k^2*x^2 + 2))/(((x^2 - 1)*(k^2*
x^2 - 1))^(3/4)*(d + x^3*(d - k^2) + x^2*(3*d + k^2) + x*(3*d + 1) - 1)),x)
```

```
[Out] int(((2*x + x^2 + 1)*(x*(k - 1)*(k + 1) - 2*k^2*x^2 + 2))/(((x^2 - 1)*(k^2*
x^2 - 1))^(3/4)*(d + x^3*(d - k^2) + x^2*(3*d + k^2) + x*(3*d + 1) - 1)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x+1)*(-2-(-1+k)*(1+k)*x+2*k**2*x**2)/((-x**2+1)*(-k**2*x*
*2+1))**(3/4)/(1-d-(1+3*d)*x-(k**2+3*d)*x**2+(k**2-d)*x**3),x)
```

```
[Out] Timed out
```

3.1130 $\int \frac{1}{x^5 \sqrt[3]{-1+x^4}} \, dx$

Optimal. Leaf size=92

$$\frac{(x^4-1)^{2/3}}{4x^4} - \frac{1}{12}\log\left(\sqrt[3]{x^4-1}+1\right) + \frac{1}{24}\log\left((x^4-1)^{2/3} - \sqrt[3]{x^4-1}+1\right) - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{x^4-1}}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 51, 56, 618, 204, 31}

$$\frac{(x^4-1)^{2/3}}{4x^4} - \frac{1}{8}\log\left(\sqrt[3]{x^4-1}+1\right) - \frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{x^4-1}}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\log(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(-1 + x^4)^(1/3)),x]
```

```
[Out] (-1 + x^4)^(2/3)/(4*x^4) - ArcTan[(1 - 2*(-1 + x^4)^(1/3))/Sqrt[3]]/(4*Sqrt
[3]) + Log[x]/6 - Log[1 + (-1 + x^4)^(1/3)]/8
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt[3]{-1+x^4}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x^2}\,dx, x, x^4\right)\\ &= \frac{\left(-1+x^4\right)^{2/3}}{4x^4} + \frac{1}{12}\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x}\,dx, x, x^4\right)\\ &= \frac{\left(-1+x^4\right)^{2/3}}{4x^4} + \frac{\log(x)}{6} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[3]{-1+x^4}\right) + \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,d\right.\\ &= \frac{\left(-1+x^4\right)^{2/3}}{4x^4} + \frac{\log(x)}{6} - \frac{1}{8}\log\left(1+\sqrt[3]{-1+x^4}\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[3]{-1}\right.\\ &= \frac{\left(-1+x^4\right)^{2/3}}{4x^4} - \frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^4}}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{\log(x)}{6} - \frac{1}{8}\log\left(1+\sqrt[3]{-1+x^4}\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 28, normalized size = 0.30

$$\frac{3}{8}\left(x^4-1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};1-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(-1 + x^4)^(1/3)),x]
```

```
[Out] (3*(-1 + x^4)^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, 1 - x^4])/8
```

IntegrateAlgebraic [A] time = 0.07, size = 92, normalized size = 1.00

$$\frac{\left(x^4-1\right)^{2/3}}{4x^4} - \frac{1}{12}\log\left(\sqrt[3]{x^4-1}+1\right) + \frac{1}{24}\log\left(\left(x^4-1\right)^{2/3} - \sqrt[3]{x^4-1}+1\right) - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{x^4-1}}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(-1 + x^4)^(1/3)),x]
```

```
[Out] (-1 + x^4)^(2/3)/(4*x^4) - ArcTan[1/Sqrt[3] - (2*(-1 + x^4)^(1/3))/Sqrt[3]]
/(4*Sqrt[3]) - Log[1 + (-1 + x^4)^(1/3)]/12 + Log[1 - (-1 + x^4)^(1/3) + (-
1 + x^4)^(2/3)]/24
```

fricas [A] time = 0.42, size = 80, normalized size = 0.87

$$\frac{2\sqrt{3}x^4\arctan\left(\frac{2}{3}\sqrt{3}\left(x^4-1\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) + x^4\log\left(\left(x^4-1\right)^{\frac{2}{3}} - \left(x^4-1\right)^{\frac{1}{3}}+1\right) - 2x^4\log\left(\left(x^4-1\right)^{\frac{1}{3}}+1\right) + 6\left(x^4-1\right)^{\frac{2}{3}}}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(3)*x^4*arctan(2/3*sqrt(3)*(x^4 - 1)^(1/3) - 1/3*sqrt(3)) + x^4
*log((x^4 - 1)^(2/3) - (x^4 - 1)^(1/3) + 1) - 2*x^4*log((x^4 - 1)^(1/3) + 1
) + 6*(x^4 - 1)^(2/3))/x^4
```

giac [A] time = 0.43, size = 69, normalized size = 0.75

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{\left(x^4-1\right)^{\frac{2}{3}}}{4x^4}+\frac{1}{24}\log\left(\left(x^4-1\right)^{\frac{2}{3}}-\left(x^4-1\right)^{\frac{1}{3}}+1\right)-\frac{1}{12}\log\left(\left|\left(x^4-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4-1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 - 1)^(1/3) - 1)) + 1/4*(x^4 - 1)^(2
/3)/x^4 + 1/24*log((x^4 - 1)^(2/3) - (x^4 - 1)^(1/3) + 1) - 1/12*log(abs((x
^4 - 1)^(1/3) + 1))
```

maple [C] time = 0.31, size = 96, normalized size = 1.04

$$\frac{\left(x^4-1\right)^{\frac{2}{3}}}{4x^4}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\mathrm{signum}\left(x^4-1\right)\right)^{\frac{1}{3}}\left(\frac{2\pi\sqrt{3}\,x^4\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^4\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+4\ln(x)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{24\pi\,\mathrm{signum}\left(x^4-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^4-1)^(1/3),x)
```

```
[Out] 1/4*(x^4-1)^(2/3)/x^4+1/24/Pi*3^(1/2)*GAMMA(2/3)/signum(x^4-1)^(1/3)*(-sign
um(x^4-1))^(1/3)*(2/9*Pi*3^(1/2)/GAMMA(2/3)*x^4*hypergeom([1,1,4/3],[2,2],x
^4)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+4*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.50, size = 68, normalized size = 0.74

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{\left(x^4-1\right)^{\frac{2}{3}}}{4x^4}+\frac{1}{24}\log\left(\left(x^4-1\right)^{\frac{2}{3}}-\left(x^4-1\right)^{\frac{1}{3}}+1\right)-\frac{1}{12}\log\left(\left(x^4-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4-1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 - 1)^(1/3) - 1)) + 1/4*(x^4 - 1)^(2
/3)/x^4 + 1/24*log((x^4 - 1)^(2/3) - (x^4 - 1)^(1/3) + 1) - 1/12*log((x^4 -
 1)^(1/3) + 1)
```

mupad [B] time = 0.94, size = 92, normalized size = 1.00

$$\frac{\left(x^4-1\right)^{2/3}}{4x^4}-\ln\left(9\left(-\frac{1}{24}+\frac{\sqrt{3}\,1i}{24}\right)^2+\frac{\left(x^4-1\right)^{1/3}}{16}\right)\left(-\frac{1}{24}+\frac{\sqrt{3}\,1i}{24}\right)+\ln\left(9\left(\frac{1}{24}+\frac{\sqrt{3}\,1i}{24}\right)^2+\frac{\left(x^4-1\right)^{1/3}}{16}\right)\left(\frac{1}{24}+\frac{\sqrt{3}\,1i}{24}\right)-\frac{\ln\left(\frac{\left(x^4-1\right)^{1/3}}{16}+\frac{1}{16}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^4 - 1)^(1/3)),x)
```

```
[Out] log(9*((3^(1/2)*1i)/24 + 1/24)^2 + (x^4 - 1)^(1/3)/16)*((3^(1/2)*1i)/24 + 1
/24) - log(9*((3^(1/2)*1i)/24 - 1/24)^2 + (x^4 - 1)^(1/3)/16)*((3^(1/2)*1i)
/24 - 1/24) - log((x^4 - 1)^(1/3)/16 + 1/16)/12 + (x^4 - 1)^(2/3)/(4*x^4)
```

sympy [C] time = 0.99, size = 34, normalized size = 0.37

$$-\frac{\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{e^{2i\pi}}{x^4}\right)}{4x^{\frac{16}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**4-1)**(1/3),x)
```

```
[Out] -gamma(4/3)*hyper((1/3, 4/3), (7/3,), exp_polar(2*I*pi)/x**4)/(4*x**(16/3)*
gamma(7/3))
```

3.1131 $\int \frac{x(-b+x)\left(ab-2ax+x^2\right)}{\sqrt{x(-a+x)(-b+x)}\left(-a^2d+2adx+\left(b^2-d\right)x^2-2bx^3+x^4\right)}\,dx$

Optimal. Leaf size=92

$$-\frac{\tan^{-1}\left(\frac{\sqrt{x^2(-a-b)+abx+x^3}}{\sqrt[4]{d}\,(a-x)}\right)}{\sqrt[4]{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{x^2(-a-b)+abx+x^3}}{x(x-b)}\right)}{\sqrt[4]{d}}$$

Rubi [F] time = 19.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(-b+x)\left(ab-2ax+x^2\right)}{\sqrt{x(-a+x)(-b+x)}\left(-a^2d+2adx+\left(b^2-d\right)x^2-2bx^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-b + x)*(a*b - 2*a*x + x^2))/(Sqrt[x*(-a + x)*(-b + x)]*(-(a^2*d) +
 2*a*d*x + (b^2 - d)*x^2 - 2*b*x^3 + x^4)),x]
```

```
[Out] (4*a*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][(x^4*Sqrt[-b
 + x^2])/(Sqrt[-a + x^2]*(a^2*d - 2*a*d*x^2 - b^2*(1 - d/b^2)*x^4 + 2*b*x^6
 - x^8)), x], x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x] + (2*a*b*Sqrt[x]*Sqrt[-a
 + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][(x^2*Sqrt[-b + x^2])/(Sqrt[-a +
x^2]*(-(a^2*d) + 2*a*d*x^2 + b^2*(1 - d/b^2)*x^4 - 2*b*x^6 + x^8)), x], x,
Sqrt[x]])/Sqrt[(a - x)*(b - x)*x] + (2*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*De
fer[Subst][Defer[Int][(x^6*Sqrt[-b + x^2])/(Sqrt[-a + x^2]*(-(a^2*d) + 2*a*
d*x^2 + b^2*(1 - d/b^2)*x^4 - 2*b*x^6 + x^8)), x], x, Sqrt[x]])/Sqrt[(a - x
)*(b - x)*x]
```

Rubi steps

$$\int \frac{x(-b+x)\left(ab-2ax+x^2\right)}{\sqrt{x(-a+x)(-b+x)}\left(-a^2d+2adx+\left(b^2-d\right)x^2-2bx^3+x^4\right)}\,dx = \frac{\left(\sqrt{x}\,\sqrt{-a+x}\,\sqrt{-b+x}\right)\int \frac{\sqrt{x}\,\sqrt{-b+x}\,(}{\sqrt{-a+x}\left(-a^2d+2adx\right.}}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt{-a+x}\,\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt{-a+x^2}}\right.}{\sqrt{x(-a+x)(}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt{-a+x}\,\sqrt{-b+x}\right)\text{Subst}\left(\int\left(\frac{}{\sqrt{-a+x}}\right.\right.}{}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt{-a+x}\,\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt{-a+x^2}}\right.}{\sqrt{x(-a+x)}}$$

Mathematica [C] time = 14.41, size = 28005, normalized size = 304.40

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(-b + x)*(a*b - 2*a*x + x^2))/(Sqrt[x*(-a + x)*(-b + x)]*(-(a^
2*d) + 2*a*d*x + (b^2 - d)*x^2 - 2*b*x^3 + x^4)),x]
```

[Out] Result too large to show

IntegrateAlgebraic [A] time = 1.03, size = 92, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{x^2(-a-b)+abx+x^3}}{\sqrt[4]{d}\,(a-x)}\right)}{\sqrt[4]{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{x^2(-a-b)+abx+x^3}}{x(x-b)}\right)}{\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-b + x)*(a*b - 2*a*x + x^2))/(Sqrt[x*(-a + x)*(-b +
x)]*(-(a^2*d) + 2*a*d*x + (b^2 - d)*x^2 - 2*b*x^3 + x^4)),x]
```

```
[Out] -(ArcTan[Sqrt[a*b*x + (-a - b)*x^2 + x^3]/(d^(1/4)*(a - x))]/d^(1/4)) - Arc
Tanh[(d^(1/4)*Sqrt[a*b*x + (-a - b)*x^2 + x^3])/(x*(-b + x))]/d^(1/4)
```

fricas [B] time = 0.93, size = 339, normalized size = 3.68

$$-\frac{\arctan\left(-\frac{\sqrt{abx-(a+b)x^2+x^3}\,d^{\frac{1}{4}}}{bx-x^2}\right)}{d^{\frac{1}{4}}}-\frac{\log\left(\frac{2\,bx^3-x^4-a^2d+2\,adx-(b^2+d)x^2+2\,\sqrt{abx-(a+b)x^2+x^3}\left(\frac{ad-dx}{d^{\frac{1}{4}}}+\frac{bdx-dx^2}{d^{\frac{3}{4}}}\right)-\frac{2\,(abdx-(a+b)dx^2+dx^3)}{\sqrt{d}}}{2\,bx^3-x^4+a^2d-2\,adx-(b^2-d)x^2}\right)}{4\,d^{\frac{1}{4}}}+\frac{\log\left(\frac{2\,bx^3-x^4-a^2d+2\,adx-(b^2+d)x^2-2\,\sqrt{abx-(a+b)x^2+x^3}\left(\frac{ad-dx}{d^{\frac{1}{4}}}+\frac{bdx-dx^2}{d^{\frac{3}{4}}}\right)-\frac{2\,(abdx-(a+b)dx^2+dx^3)}{\sqrt{d}}}{2\,bx^3-x^4+a^2d-2\,adx-(b^2-d)x^2}\right)}{4\,d^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(a*b-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(-a^2*d+2*a*d*x+
(b^2-d)*x^2-2*b*x^3+x^4),x, algorithm="fricas")
```

```
[Out] -arctan(-sqrt(a*b*x - (a + b)*x^2 + x^3)*d^(1/4)/(b*x - x^2))/d^(1/4) - 1/4
*log((2*b*x^3 - x^4 - a^2*d + 2*a*d*x - (b^2 + d)*x^2 + 2*sqrt(a*b*x - (a +
 b)*x^2 + x^3)*((a*d - d*x)/d^(1/4) + (b*d*x - d*x^2)/d^(3/4)) - 2*(a*b*d*x
 - (a + b)*d*x^2 + d*x^3)/sqrt(d))/(2*b*x^3 - x^4 + a^2*d - 2*a*d*x - (b^2
- d)*x^2))/d^(1/4) + 1/4*log((2*b*x^3 - x^4 - a^2*d + 2*a*d*x - (b^2 + d)*x
^2 - 2*sqrt(a*b*x - (a + b)*x^2 + x^3)*((a*d - d*x)/d^(1/4) + (b*d*x - d*x^
2)/d^(3/4)) - 2*(a*b*d*x - (a + b)*d*x^2 + d*x^3)/sqrt(d))/(2*b*x^3 - x^4 +
 a^2*d - 2*a*d*x - (b^2 - d)*x^2))/d^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ab-2\,ax+x^2\right)(b-x)x}{\left(2\,bx^3-x^4+a^2d-2\,adx-\left(b^2-d\right)x^2\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(a*b-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(-a^2*d+2*a*d*x+
(b^2-d)*x^2-2*b*x^3+x^4),x, algorithm="giac")
```

```
[Out] integrate((a*b - 2*a*x + x^2)*(b - x)*x/((2*b*x^3 - x^4 + a^2*d - 2*a*d*x -
 (b^2 - d)*x^2)*sqrt((a - x)*(b - x)*x)), x)
```

maple [C] time = 0.07, size = 411, normalized size = 4.47

$$-\frac{2a\sqrt{-\frac{-a+x}{a}}\sqrt{\frac{-b+x}{a-b}}\sqrt{\frac{x}{a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{-a+x}{a}},\sqrt{\frac{a}{a-b}}\right)}{\sqrt{abx-a\,x^2-b\,x^2+x^3}}+\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4-2b\,\_Z^3+\left(b^2-d\right)\_Z^2+2ad\,\_Z-a^2d\right)}\frac{\left(-2\_\alpha^3a+\_\alpha^3b+3\_\alpha^2ab-\_\alpha^2b^2-\_\alpha a\,b^2+\_\alpha^2d-2\_\alpha ad+a^2d\right)\left(\_\alpha^3+\_\alpha^2a-2\_\alpha^2b+\_\alpha\,a^2-2\_\alpha ab+\_\alpha\,b^2+a^3-2a^2b+a\,b^2-\_\alpha d+ad\right)\sqrt{-\frac{-a+x}{a}}\sqrt{\frac{-b+x}{a-b}}\sqrt{\frac{x}{a}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-a+x}{a}},\frac{\_\alpha^3+\_\alpha^2a-2\_\alpha^2b+\_\alpha\,a^2-2\_\alpha ab+\_\alpha\,b^2+a^3-2a^2b+a\,b^2-\_\alpha d+ad}{a\left(a^2-2ab+b^2\right)},\sqrt{\frac{a}{a-b}}\right)}{\left(-2\_\alpha^3+3\_\alpha^2b-\_\alpha\,b^2+\_\alpha d-ad\right)\left(a^2-2ab+b^2\right)\sqrt{x\left(ab-ax-bx+x^2\right)}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b+x)*(a*b-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(-a^2*d+2*a*d*x+(b^2-d
)*x^2-2*b*x^3+x^4),x)
```

```
[Out] -2*a*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(a*b*x-a*x^2-b*x^
2+x^3)^(1/2)*EllipticF((-(-a+x)/a)^(1/2),(a/(a-b))^(1/2))+1/a*sum((-2*_alph
a^3*a+_alpha^3*b+3*_alpha^2*a*b-_alpha^2*b^2-_alpha*a*b^2+_alpha^2*d-2*_alp
```

```
ha*a*d+a^2*d)/(-2*_alpha^3+3*_alpha^2*b-_alpha*b^2+_alpha*d-a*d)*(_alpha^3+
_alpha^2*a-2*_alpha^2*b+_alpha*a^2-2*_alpha*a*b+_alpha*b^2+a^3-2*a^2*b+a*b^
2-_alpha*d+a*d)/(a^2-2*a*b+b^2)*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a
*x)^(1/2)/(x*(a*b-a*x-b*x+x^2))^(1/2)*EllipticPi((-(-a+x)/a)^(1/2),(_alpha^
3+_alpha^2*a-2*_alpha^2*b+_alpha*a^2-2*_alpha*a*b+_alpha*b^2+a^3-2*a^2*b+a*
b^2-_alpha*d+a*d)/a/(a^2-2*a*b+b^2),(a/(a-b))^(1/2)),_alpha=RootOf(_Z^4-2*b
*_Z^3+(b^2-d)*_Z^2+2*a*d*_Z-a^2*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ab - 2\,ax + x^2\right)(b - x)x}{\left(2\,bx^3 - x^4 + a^2d - 2\,adx - \left(b^2 - d\right)x^2\right)\sqrt{(a - x)(b - x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(a*b-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/2)/(-a^2*d+2*a*d*x+
(b^2-d)*x^2-2*b*x^3+x^4),x, algorithm="maxima")
```

```
[Out] integrate((a*b - 2*a*x + x^2)*(b - x)*x/((2*b*x^3 - x^4 + a^2*d - 2*a*d*x -
 (b^2 - d)*x^2)*sqrt((a - x)*(b - x)*x)), x)
```

mupad [B] time = 1.38, size = 705, normalized size = 7.66

$$\left(\sum_{k=1}^{4} -\frac{2\,b\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(-\frac{b}{\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,(d-b^2)+2\,a\,d\,z-a^2\,d,z,k\right)-b};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\left(-d\,a^2+a\,b^2\,\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,(d-b^2)+2\,a\,d\,z-a^2\,d,z,k\right)-3\,a\,b\,\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,(d-b^2)+2\,a\,d\,z-a^2\,d,z,k\right)^2+2\,a\,\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,(d-b^2)+2\,a\,d\,z-a^2\,d,z,k\right)^3+2\,d\,a\,\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,(d-b^2)+2\,a\,d\,z-a^2\,d,z,k\right)+b^2\,\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,(d-b^2)+2\,a\,d\,z-a^2\,d,z,k\right)^2-b\,\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,(d-b^2)+2\,a\,d\,z-a^2\,d,z,k\right)^3-d\,\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,(d-b^2)+2\,a\,d\,z-a^2\,d,z,k\right)^2\right)}{\left(\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,(d-b^2)+2\,a\,d\,z-a^2\,d,z,k\right)-b\right)\sqrt{x\,(a-x)\,(b-x)}\,\left(2\,b^2\,\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,(d-b^2)+2\,a\,d\,z-a^2\,d,z,k\right)-6\,b\,\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,(d-b^2)+2\,a\,d\,z-a^2\,d,z,k\right)^2+4\,\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,(d-b^2)+2\,a\,d\,z-a^2\,d,z,k\right)^3-2\,d\,\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,(d-b^2)+2\,a\,d\,z-a^2\,d,z,k\right)+2\,a\,d\right)}\right)-\frac{2\,b\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}}{\sqrt{x^3+(-a-b)\,x^2+a\,b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b - x)*(a*b - 2*a*x + x^2))/((x*(a - x)*(b - x))^(1/2)*(x^2*(d - b^
2) + a^2*d + 2*b*x^3 - x^4 - 2*a*d*x)),x)
```

```
[Out] symsum(-(2*b*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*elliptic
Pi(-b/(root(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z - a^2*d, z, k) - b), as
in(((b - x)/b)^(1/2)), -b/(a - b))*(2*a*root(z^4 - 2*b*z^3 - z^2*(d - b^2)
+ 2*a*d*z - a^2*d, z, k)^3 - b*root(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z
 - a^2*d, z, k)^3 - d*root(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z - a^2*d,
 z, k)^2 - a^2*d + b^2*root(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z - a^2*d
, z, k)^2 + 2*a*d*root(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z - a^2*d, z,
k) - 3*a*b*root(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z - a^2*d, z, k)^2 +
a*b^2*root(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z - a^2*d, z, k)))/((root(
z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z - a^2*d, z, k) - b)*(x*(a - x)*(b -
 x))^(1/2)*(2*a*d - 2*d*root(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z - a^2*
d, z, k) - 6*b*root(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z - a^2*d, z, k)^
2 + 2*b^2*root(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z - a^2*d, z, k) + 4*r
oot(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z - a^2*d, z, k)^3)), k, 1, 4) -
(2*b*ellipticF(asin(((b - x)/b)^(1/2)), -b/(a - b))*(x/b)^(1/2)*((b - x)/b)
^(1/2)*((a - x)/(a - b))^(1/2))/(x^3 - x^2*(a + b) + a*b*x)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(a*b-2*a*x+x**2)/(x*(-a+x)*(-b+x))**(1/2)/(-a**2*d+2*a*d
*x+(b**2-d)*x**2-2*b*x**3+x**4),x)
```

```
[Out] Timed out
```

3.1132 $\int \frac{1}{x^9\left(b+ax^4\right)^{3/4}}\,dx$

Optimal. Leaf size=92

$$-\frac{21a^2\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{64b^{11/4}}-\frac{21a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{64b^{11/4}}+\frac{\sqrt[4]{ax^4+b}\,\left(7ax^4-4b\right)}{32b^2x^8}$$

Rubi [A] time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 63, 212, 206, 203}

$$-\frac{21a^2\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{64b^{11/4}}-\frac{21a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{64b^{11/4}}+\frac{7a\sqrt[4]{ax^4+b}}{32b^2x^4}-\frac{\sqrt[4]{ax^4+b}}{8bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(b + a*x^4)^(3/4)),x]
```

```
[Out] -1/8*(b + a*x^4)^(1/4)/(b*x^8) + (7*a*(b + a*x^4)^(1/4))/(32*b^2*x^4) - (21
*a^2*ArcTan[(b + a*x^4)^(1/4)/b^(1/4)])/(64*b^(11/4)) - (21*a^2*ArcTanh[(b
+ a*x^4)^(1/4)/b^(1/4)])/(64*b^(11/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^9\left(b+ax^4\right)^{3/4}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x^3(b+ax)^{3/4}}\,dx, x, x^4\right)\\
&= -\frac{\sqrt[4]{b+ax^4}}{8bx^8} - \frac{(7a)\operatorname{Subst}\left(\int \frac{1}{x^2(b+ax)^{3/4}}\,dx, x, x^4\right)}{32b}\\
&= -\frac{\sqrt[4]{b+ax^4}}{8bx^8} + \frac{7a\sqrt[4]{b+ax^4}}{32b^2x^4} + \frac{\left(21a^2\right)\operatorname{Subst}\left(\int \frac{1}{x(b+ax)^{3/4}}\,dx, x, x^4\right)}{128b^2}\\
&= -\frac{\sqrt[4]{b+ax^4}}{8bx^8} + \frac{7a\sqrt[4]{b+ax^4}}{32b^2x^4} + \frac{(21a)\operatorname{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^4}\right)}{32b^2}\\
&= -\frac{\sqrt[4]{b+ax^4}}{8bx^8} + \frac{7a\sqrt[4]{b+ax^4}}{32b^2x^4} - \frac{\left(21a^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^4}\right)}{64b^{5/2}} - \frac{\left(21a^2\right)\operatorname{Subst}\left(\int\right.}{}\\
&= -\frac{\sqrt[4]{b+ax^4}}{8bx^8} + \frac{7a\sqrt[4]{b+ax^4}}{32b^2x^4} - \frac{21a^2\tan^{-1}\left(\frac{\sqrt[4]{b+ax^4}}{\sqrt[4]{b}}\right)}{64b^{11/4}} - \frac{21a^2\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^4}}{\sqrt[4]{b}}\right)}{64b^{11/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.40

$$-\frac{a^2\sqrt[4]{ax^4+b}\;{}_2F_1\left(\frac{1}{4},3;\frac{5}{4};\frac{ax^4}{b}+1\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(b + a*x^4)^(3/4)),x]
```

```
[Out] -((a^2*(b + a*x^4)^(1/4)*Hypergeometric2F1[1/4, 3, 5/4, 1 + (a*x^4)/b])/b^3
)
```

IntegrateAlgebraic [A] time = 0.08, size = 92, normalized size = 1.00

$$-\frac{21a^2\tan^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{64b^{11/4}} - \frac{21a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax^4+b}}{\sqrt[4]{b}}\right)}{64b^{11/4}} + \frac{\sqrt[4]{ax^4+b}\left(7ax^4-4b\right)}{32b^2x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^9*(b + a*x^4)^(3/4)),x]
```

```
[Out] ((b + a*x^4)^(1/4)*(-4*b + 7*a*x^4))/(32*b^2*x^8) - (21*a^2*ArcTan[(b + a*x
^4)^(1/4)/b^(1/4)])/(64*b^(11/4)) - (21*a^2*ArcTanh[(b + a*x^4)^(1/4)/b^(1/
4)])/(64*b^(11/4))
```

fricas [B] time = 0.43, size = 216, normalized size = 2.35

$$\frac{84\,b^2x^8\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^4+b\right)^{\frac{1}{4}}a^2b^8\left(\frac{a^8}{b^{11}}\right)^{\frac{3}{4}}-\sqrt{b^6\sqrt{\frac{a^8}{b^{11}}}+\sqrt{ax^4+b}\,a^4}\,b^8\left(\frac{a^8}{b^{11}}\right)^{\frac{3}{4}}}{a^8}\right)-21\,b^2x^8\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\log\left(21\,b^3\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}+21\left(ax^4+b\right)^{\frac{1}{4}}a^2\right)+21\,b^2x^8\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\log\left(-21\,b^3\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}+21\left(ax^4+b\right)^{\frac{1}{4}}a^2\right)+4\left(7\,ax^4-4\,b\right)\left(ax^4+b\right)^{\frac{1}{4}}}{128\,b^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(a*x^4+b)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/128*(84*b^2*x^8*(a^8/b^11)^(1/4)*arctan(-((a*x^4 + b)^(1/4)*a^2*b^8*(a^8/
b^11)^(3/4) - sqrt(b^6*sqrt(a^8/b^11) + sqrt(a*x^4 + b)*a^4)*b^8*(a^8/b^11)
^(3/4))/a^8) - 21*b^2*x^8*(a^8/b^11)^(1/4)*log(21*b^3*(a^8/b^11)^(1/4) + 21
*(a*x^4 + b)^(1/4)*a^2) + 21*b^2*x^8*(a^8/b^11)^(1/4)*log(-21*b^3*(a^8/b^11
)^(1/4) + 21*(a*x^4 + b)^(1/4)*a^2) + 4*(7*a*x^4 - 4*b)*(a*x^4 + b)^(1/4))/
(b^2*x^8)
```

giac [B] time = 0.18, size = 244, normalized size = 2.65

$$\frac{\frac{42 \sqrt{2} a^{3} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}+2\left(ax^{4}+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{(-b)^{\frac{3}{4}} b^{2}}+\frac{42 \sqrt{2} a^{3} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}-2\left(ax^{4}+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{(-b)^{\frac{3}{4}} b^{2}}+\frac{21 \sqrt{2} a^{3} \log\left(\sqrt{2}\left(ax^{4}+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^{4}+b}+\sqrt{-b}\right)}{(-b)^{\frac{3}{4}} b^{2}}+\frac{21 \sqrt{2} a^{3}(-b)^{\frac{1}{4}} \log\left(-\sqrt{2}\left(ax^{4}+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^{4}+b}+\sqrt{-b}\right)}{b^{3}}+\frac{8\left(7\left(ax^{4}+b\right)^{\frac{5}{4}} a^{3}-11\left(ax^{4}+b\right)^{\frac{1}{4}} a^{3} b\right)}{a^{2} b^{2} x^{8}}}{256 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(a*x^4+b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/256*(42*sqrt(2)*a^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^4 + b
)^(1/4))/(-b)^(1/4))/((-b)^(3/4)*b^2) + 42*sqrt(2)*a^3*arctan(-1/2*sqrt(2)*
(sqrt(2)*(-b)^(1/4) - 2*(a*x^4 + b)^(1/4))/(-b)^(1/4))/((-b)^(3/4)*b^2) + 2
1*sqrt(2)*a^3*log(sqrt(2)*(a*x^4 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^4 + b) +
sqrt(-b))/((-b)^(3/4)*b^2) + 21*sqrt(2)*a^3*(-b)^(1/4)*log(-sqrt(2)*(a*x^4
+ b)^(1/4)*(-b)^(1/4) + sqrt(a*x^4 + b) + sqrt(-b))/b^3 + 8*(7*(a*x^4 + b)^
(5/4)*a^3 - 11*(a*x^4 + b)^(1/4)*a^3*b)/(a^2*b^2*x^8))/a
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{9} \left(a\,x^{4}+b\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(a*x^4+b)^(3/4),x)
```

```
[Out] int(1/x^9/(a*x^4+b)^(3/4),x)
```

maxima [A] time = 0.55, size = 132, normalized size = 1.43

$$\frac{7\left(ax^{4}+b\right)^{\frac{5}{4}} a^{2}-11\left(ax^{4}+b\right)^{\frac{1}{4}} a^{2} b}{32\left(\left(ax^{4}+b\right)^{2} b^{2}-2\left(ax^{4}+b\right) b^{3}+b^{4}\right)}-\frac{21\left(\frac{2 a^{2} \arctan\left(\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}-\frac{a^{2} \log\left(\frac{\left(ax^{4}+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^{4}+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}\right)}{128 b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(a*x^4+b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/32*(7*(a*x^4 + b)^(5/4)*a^2 - 11*(a*x^4 + b)^(1/4)*a^2*b)/((a*x^4 + b)^2*
b^2 - 2*(a*x^4 + b)*b^3 + b^4) - 21/128*(2*a^2*arctan((a*x^4 + b)^(1/4)/b^(
1/4))/b^(3/4) - a^2*log(((a*x^4 + b)^(1/4) - b^(1/4))/((a*x^4 + b)^(1/4) +
b^(1/4)))/b^(3/4))/b^2
```

mupad [B] time = 1.19, size = 82, normalized size = 0.89

$$\frac{7\left(a\,x^{4}+b\right)^{5/4}}{32\,b^{2}\,x^{8}}-\frac{11\left(a\,x^{4}+b\right)^{1/4}}{32\,b\,x^{8}}-\frac{21\,a^{2}\,\mathrm{atan}\left(\frac{\left(a\,x^{4}+b\right)^{1/4}}{b^{1/4}}\right)}{64\,b^{11/4}}+\frac{a^{2}\,\mathrm{atan}\left(\frac{\left(a\,x^{4}+b\right)^{1/4}\,1{}\mathrm{i}}{b^{1/4}}\right)\,21{}\mathrm{i}}{64\,b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(b + a*x^4)^(3/4)),x)
```

```
[Out] (a^2*atan(((b + a*x^4)^(1/4)*1i)/b^(1/4))*21i)/(64*b^(11/4)) - (21*a^2*atan
((b + a*x^4)^(1/4)/b^(1/4)))/(64*b^(11/4)) - (11*(b + a*x^4)^(1/4))/(32*b*x
^8) + (7*(b + a*x^4)^(5/4))/(32*b^2*x^8)
```

sympy [C] time = 1.60, size = 39, normalized size = 0.42

$$-\frac{\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^4}\right)}{4a^{\frac{3}{4}}x^{11}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(a*x**4+b)**(3/4),x)
```

```
[Out] -gamma(11/4)*hyper((3/4, 11/4), (15/4,), b*exp_polar(I*pi)/(a*x**4))/(4*a**
(3/4)*x**11*gamma(15/4))
```

3.1133 $\int x^4 \left(b+ax^4\right)^{3/4} \, dx$

Optimal. Leaf size=92

$$-\frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{64a^{5/4}}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{64a^{5/4}}+\frac{\left(ax^4+b\right)^{3/4}\left(4ax^5+3bx\right)}{32a}$$

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {279, 321, 240, 212, 206, 203}

$$-\frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{64a^{5/4}}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{64a^{5/4}}+\frac{3bx\left(ax^4+b\right)^{3/4}}{32a}+\frac{1}{8}x^5\left(ax^4+b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(b + a*x^4)^(3/4),x]
```

```
[Out] (3*b*x*(b + a*x^4)^(3/4))/(32*a) + (x^5*(b + a*x^4)^(3/4))/8 - (3*b^2*ArcTa
n[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(64*a^(5/4)) - (3*b^2*ArcTanh[(a^(1/4)*x)
/(b + a*x^4)^(1/4)])/(64*a^(5/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^4\left(b+ax^4\right)^{3/4}\,dx &= \frac{1}{8}x^5\left(b+ax^4\right)^{3/4}+\frac{1}{8}(3b)\int\frac{x^4}{\sqrt[4]{b+ax^4}}\,dx\\
&= \frac{3bx\left(b+ax^4\right)^{3/4}}{32a}+\frac{1}{8}x^5\left(b+ax^4\right)^{3/4}-\frac{\left(3b^2\right)\int\frac{1}{\sqrt[4]{b+ax^4}}\,dx}{32a}\\
&= \frac{3bx\left(b+ax^4\right)^{3/4}}{32a}+\frac{1}{8}x^5\left(b+ax^4\right)^{3/4}-\frac{\left(3b^2\right)\mathrm{Subst}\left(\int\frac{1}{1-ax^4}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)}{32a}\\
&= \frac{3bx\left(b+ax^4\right)^{3/4}}{32a}+\frac{1}{8}x^5\left(b+ax^4\right)^{3/4}-\frac{\left(3b^2\right)\mathrm{Subst}\left(\int\frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)}{64a}-\frac{\left(3b^2\right)\mathrm{Subs}}{}\\
&= \frac{3bx\left(b+ax^4\right)^{3/4}}{32a}+\frac{1}{8}x^5\left(b+ax^4\right)^{3/4}-\frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{64a^{5/4}}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{64a^{5/4}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 62, normalized size = 0.67

$$\frac{x\left(ax^4+b\right)^{3/4}\left(-\frac{b\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};-\frac{ax^4}{b}\right)}{\left(\frac{ax^4}{b}+1\right)^{3/4}}+ax^4+b\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(b + a*x^4)^(3/4),x]
```

```
[Out] (x*(b + a*x^4)^(3/4)*(b + a*x^4 - (b*Hypergeometric2F1[-3/4, 1/4, 5/4, -((a
*x^4)/b)])/(1 + (a*x^4)/b)^(3/4)))/(8*a)
```

IntegrateAlgebraic [A] time = 0.36, size = 92, normalized size = 1.00

$$-\frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{64a^{5/4}}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{64a^{5/4}}+\frac{\left(ax^4+b\right)^{3/4}\left(4ax^5+3bx\right)}{32a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(b + a*x^4)^(3/4),x]
```

```
[Out] ((b + a*x^4)^(3/4)*(3*b*x + 4*a*x^5))/(32*a) - (3*b^2*ArcTan[(a^(1/4)*x)/(b
 + a*x^4)^(1/4)])/(64*a^(5/4)) - (3*b^2*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/
4)])/(64*a^(5/4))
```

fricas [B] time = 0.45, size = 218, normalized size = 2.37

$$-\frac{12\left(\frac{b^8}{a^5}\right)^{\frac{1}{4}}a\arctan\left(-\frac{\left(ax^4+b\right)^{\frac{1}{4}}\left(\frac{b^8}{a^5}\right)^{\frac{1}{4}}ab^6-\left(\frac{b^8}{a^5}\right)^{\frac{1}{4}}ax\sqrt{\frac{\sqrt{\frac{b^8}{a^5}}a^3b^8x^2+\sqrt{ax^4+b}\,b^{12}}{x^2}}}{b^8x}\right)+3\left(\frac{b^8}{a^5}\right)^{\frac{1}{4}}a\log\left(\frac{27\left(\left(ax^4+b\right)^{\frac{1}{4}}b^6+\left(\frac{b^8}{a^5}\right)^{\frac{3}{4}}a^4x\right)}{x}\right)-3\left(\frac{b^8}{a^5}\right)^{\frac{1}{4}}a\log\left(\frac{27\left(\left(ax^4+b\right)^{\frac{1}{4}}b^6-\left(\frac{b^8}{a^5}\right)^{\frac{3}{4}}a^4x\right)}{x}\right)-4\left(4ax^5+3bx\right)\left(ax^4+b\right)^{\frac{3}{4}}}{128a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^4+b)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/128*(12*(b^8/a^5)^(1/4)*a*arctan(-((a*x^4 + b)^(1/4)*(b^8/a^5)^(1/4)*a*b
^6 - (b^8/a^5)^(1/4)*a*x*sqrt((sqrt(b^8/a^5)*a^3*b^8*x^2 + sqrt(a*x^4 + b)*
b^12)/x^2))/(b^8*x)) + 3*(b^8/a^5)^(1/4)*a*log(27*((a*x^4 + b)^(1/4)*b^6 +
(b^8/a^5)^(3/4)*a^4*x)/x) - 3*(b^8/a^5)^(1/4)*a*log(27*((a*x^4 + b)^(1/4)*b
^6 - (b^8/a^5)^(3/4)*a^4*x)/x) - 4*(4*a*x^5 + 3*b*x)*(a*x^4 + b)^(3/4))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^4+b\right)^{\frac{3}{4}} x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^4+b)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^4 + b)^(3/4)*x^4, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int x^4 \left(a\,x^4+b\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a*x^4+b)^(3/4),x)
```

```
[Out] int(x^4*(a*x^4+b)^(3/4),x)
```

maxima [B] time = 0.70, size = 148, normalized size = 1.61

$$\frac{3\,b^2\left(\frac{2\arctan\left(\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right)}{128\,a}+\frac{\frac{\left(ax^4+b\right)^{\frac{3}{4}}ab^2}{x^3}+\frac{3\left(ax^4+b\right)^{\frac{7}{4}}b^2}{x^7}}{32\left(a^3-\frac{2\left(ax^4+b\right)a^2}{x^4}+\frac{\left(ax^4+b\right)^2a}{x^8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^4+b)^(3/4),x, algorithm="maxima")
```

```
[Out] 3/128*b^2*(2*arctan((a*x^4 + b)^(1/4)/(a^(1/4)*x))/a^(1/4) + log(-(a^(1/4)
- (a*x^4 + b)^(1/4)/x)/(a^(1/4) + (a*x^4 + b)^(1/4)/x))/a^(1/4))/a + 1/32*(
(a*x^4 + b)^(3/4)*a*b^2/x^3 + 3*(a*x^4 + b)^(7/4)*b^2/x^7)/(a^3 - 2*(a*x^4
+ b)*a^2/x^4 + (a*x^4 + b)^2*a/x^8)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,{\left(a\,x^4+b\right)}^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b + a*x^4)^(3/4),x)
```

```
[Out] int(x^4*(b + a*x^4)^(3/4), x)
```

sympy [C] time = 1.35, size = 39, normalized size = 0.42

$$\frac{b^{\frac{3}{4}}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{ax^4e^{i\pi}}{b}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a*x**4+b)**(3/4),x)
```

```
[Out] b**(3/4)*x**5*gamma(5/4)*hyper((-3/4, 5/4), (9/4,), a*x**4*exp_polar(I*pi)/
b)/(4*gamma(9/4))
```

3.1134 $\int \frac{\left(-1+x^5\right)^{2/3}}{x^6}\,dx$

Optimal. Leaf size=92

$$-\frac{\left(x^5-1\right)^{2/3}}{5x^5}-\frac{2}{15}\log\left(\sqrt[3]{x^5-1}+1\right)+\frac{1}{15}\log\left(\left(x^5-1\right)^{2/3}-\sqrt[3]{x^5-1}+1\right)-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^5-1}}{\sqrt{3}}\right)}{5\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 47, 56, 618, 204, 31}

$$-\frac{\left(x^5-1\right)^{2/3}}{5x^5}-\frac{1}{5}\log\left(\sqrt[3]{x^5-1}+1\right)-\frac{2\tan^{-1}\left(\frac{1-2\sqrt[3]{x^5-1}}{\sqrt{3}}\right)}{5\sqrt{3}}+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^5)^(2/3)/x^6,x]
```

```
[Out] -1/5*(-1 + x^5)^(2/3)/x^5 - (2*ArcTan[(1 - 2*(-1 + x^5)^(1/3))/Sqrt[3]])/(5
*Sqrt[3]) + Log[x]/3 - Log[1 + (-1 + x^5)^(1/3)]/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^5\right)^{2/3}}{x^6}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{(-1+x)^{2/3}}{x^2}\,dx,x,x^5\right)\\
&= -\frac{\left(-1+x^5\right)^{2/3}}{5x^5}+\frac{2}{15}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x}\,dx,x,x^5\right)\\
&= -\frac{\left(-1+x^5\right)^{2/3}}{5x^5}+\frac{\log(x)}{3}-\frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt[3]{-1+x^5}\right)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,\right.\\
&= -\frac{\left(-1+x^5\right)^{2/3}}{5x^5}+\frac{\log(x)}{3}-\frac{1}{5}\log\left(1+\sqrt[3]{-1+x^5}\right)-\frac{2}{5}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2\sqrt[3]{-1+}\right.\\
&= -\frac{\left(-1+x^5\right)^{2/3}}{5x^5}-\frac{2\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^5}}{\sqrt{3}}\right)}{5\sqrt{3}}+\frac{\log(x)}{3}-\frac{1}{5}\log\left(1+\sqrt[3]{-1+x^5}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.30

$$\frac{3}{25}\left(x^5-1\right)^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};1-x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^5)^(2/3)/x^6,x]
```

```
[Out] (3*(-1 + x^5)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 - x^5])/25
```

IntegrateAlgebraic [A] time = 0.07, size = 92, normalized size = 1.00

$$-\frac{\left(x^5-1\right)^{2/3}}{5x^5}-\frac{2}{15}\log\left(\sqrt[3]{x^5-1}+1\right)+\frac{1}{15}\log\left(\left(x^5-1\right)^{2/3}-\sqrt[3]{x^5-1}+1\right)-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^5-1}}{\sqrt{3}}\right)}{5\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^5)^(2/3)/x^6,x]
```

```
[Out] -1/5*(-1 + x^5)^(2/3)/x^5 - (2*ArcTan[1/Sqrt[3] - (2*(-1 + x^5)^(1/3))/Sqrt
[3]])/(5*Sqrt[3]) - (2*Log[1 + (-1 + x^5)^(1/3)])/15 + Log[1 - (-1 + x^5)^(
1/3) + (-1 + x^5)^(2/3)]/15
```

fricas [A] time = 0.49, size = 80, normalized size = 0.87

$$\frac{2\sqrt{3}\,x^5\arctan\left(\frac{2}{3}\sqrt{3}\left(x^5-1\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x^5\log\left(\left(x^5-1\right)^{\frac{2}{3}}-\left(x^5-1\right)^{\frac{1}{3}}+1\right)-2\,x^5\log\left(\left(x^5-1\right)^{\frac{1}{3}}+1\right)-3\left(x^5-1\right)^{\frac{2}{3}}}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(2/3)/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(2*sqrt(3)*x^5*arctan(2/3*sqrt(3)*(x^5 - 1)^(1/3) - 1/3*sqrt(3)) + x^5
*log((x^5 - 1)^(2/3) - (x^5 - 1)^(1/3) + 1) - 2*x^5*log((x^5 - 1)^(1/3) + 1
) - 3*(x^5 - 1)^(2/3))/x^5
```

giac [A] time = 0.30, size = 69, normalized size = 0.75

$$\frac{2}{15}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(x^5-1\right)^{\frac{1}{3}}-1\right)\right)-\frac{\left(x^5-1\right)^{\frac{2}{3}}}{5\,x^5}+\frac{1}{15}\,\log\left(\left(x^5-1\right)^{\frac{2}{3}}-\left(x^5-1\right)^{\frac{1}{3}}+1\right)-\frac{2}{15}\,\log\left(\left|\left(x^5-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(2/3)/x^6,x, algorithm="giac")
```

```
[Out] 2/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^5 - 1)^(1/3) - 1)) - 1/5*(x^5 - 1)^(2
/3)/x^5 + 1/15*log((x^5 - 1)^(2/3) - (x^5 - 1)^(1/3) + 1) - 2/15*log(abs((x
^5 - 1)^(1/3) + 1))
```

maple [C] time = 0.31, size = 96, normalized size = 1.04

$$-\frac{\left(x^5-1\right)^{\frac{2}{3}}}{5x^5}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\mathrm{signum}\left(x^5-1\right)\right)^{\frac{1}{3}}\left(\frac{2\pi\sqrt{3}\,x^5\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^5\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+5\ln(x)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{15\pi\,\mathrm{signum}\left(x^5-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)^(2/3)/x^6,x)
```

```
[Out] -1/5*(x^5-1)^(2/3)/x^5+1/15/Pi*3^(1/2)*GAMMA(2/3)/signum(x^5-1)^(1/3)*(-sig
num(x^5-1))^(1/3)*(2/9*Pi*3^(1/2)/GAMMA(2/3)*x^5*hypergeom([1,1,4/3],[2,2],
x^5)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+5*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.47, size = 68, normalized size = 0.74

$$\frac{2}{15}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(x^5-1\right)^{\frac{1}{3}}-1\right)\right)-\frac{\left(x^5-1\right)^{\frac{2}{3}}}{5\,x^5}+\frac{1}{15}\,\log\left(\left(x^5-1\right)^{\frac{2}{3}}-\left(x^5-1\right)^{\frac{1}{3}}+1\right)-\frac{2}{15}\,\log\left(\left(x^5-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(2/3)/x^6,x, algorithm="maxima")
```

```
[Out] 2/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^5 - 1)^(1/3) - 1)) - 1/5*(x^5 - 1)^(2
/3)/x^5 + 1/15*log((x^5 - 1)^(2/3) - (x^5 - 1)^(1/3) + 1) - 2/15*log((x^5 -
 1)^(1/3) + 1)
```

mupad [B] time = 0.94, size = 92, normalized size = 1.00

$$-\frac{2\ln\left(\frac{4\left(x^5-1\right)^{1/3}}{25}+\frac{4}{25}\right)}{15}-\ln\left(9\left(-\frac{1}{15}+\frac{\sqrt{3}\,1\mathrm{i}}{15}\right)^2+\frac{4\left(x^5-1\right)^{1/3}}{25}\right)\left(-\frac{1}{15}+\frac{\sqrt{3}\,1\mathrm{i}}{15}\right)+\ln\left(9\left(\frac{1}{15}+\frac{\sqrt{3}\,1\mathrm{i}}{15}\right)^2+\frac{4\left(x^5-1\right)^{1/3}}{25}\right)\left(\frac{1}{15}+\frac{\sqrt{3}\,1\mathrm{i}}{15}\right)-\frac{\left(x^5-1\right)^{2/3}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5 - 1)^(2/3)/x^6,x)
```

```
[Out] log(9*((3^(1/2)*1i)/15 + 1/15)^2 + (4*(x^5 - 1)^(1/3))/25)*((3^(1/2)*1i)/15
 + 1/15) - log(9*((3^(1/2)*1i)/15 - 1/15)^2 + (4*(x^5 - 1)^(1/3))/25)*((3^(
1/2)*1i)/15 - 1/15) - (2*log((4*(x^5 - 1)^(1/3))/25 + 4/25))/15 - (x^5 - 1)
^(2/3)/(5*x^5)
```

sympy [C] time = 1.09, size = 36, normalized size = 0.39

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^5}\right)}{5x^{\frac{5}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)**(2/3)/x**6,x)
```

```
[Out] -gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_polar(2*I*pi)/x**5)/(5*x**(5/3)*
gamma(4/3))
```

3.1135 $\int \frac{1}{x^{11}\left(b+ax^5\right)^{3/4}}\,dx$

Optimal. Leaf size=92

$$-\frac{21a^2\tan^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{80b^{11/4}}-\frac{21a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{80b^{11/4}}+\frac{\sqrt[4]{ax^5+b}\left(7ax^5-4b\right)}{40b^2x^{10}}$$

Rubi [A] time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 51, 63, 212, 206, 203}

$$-\frac{21a^2\tan^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{80b^{11/4}}-\frac{21a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{80b^{11/4}}+\frac{7a\sqrt[4]{ax^5+b}}{40b^2x^5}-\frac{\sqrt[4]{ax^5+b}}{10bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^11*(b + a*x^5)^(3/4)),x]
```

```
[Out] -1/10*(b + a*x^5)^(1/4)/(b*x^10) + (7*a*(b + a*x^5)^(1/4))/(40*b^2*x^5) - (
21*a^2*ArcTan[(b + a*x^5)^(1/4)/b^(1/4)])/(80*b^(11/4)) - (21*a^2*ArcTanh[(
b + a*x^5)^(1/4)/b^(1/4)])/(80*b^(11/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^{11}\left(b+ax^5\right)^{3/4}}\,dx = \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x^3(b+ax)^{3/4}}\,dx, x, x^5\right)$$

$$= -\frac{\sqrt[4]{b+ax^5}}{10bx^{10}} - \frac{(7a)\,\text{Subst}\left(\int \frac{1}{x^2(b+ax)^{3/4}}\,dx, x, x^5\right)}{40b}$$

$$= -\frac{\sqrt[4]{b+ax^5}}{10bx^{10}} + \frac{7a\sqrt[4]{b+ax^5}}{40b^2x^5} + \frac{\left(21a^2\right)\text{Subst}\left(\int \frac{1}{x(b+ax)^{3/4}}\,dx, x, x^5\right)}{160b^2}$$

$$= -\frac{\sqrt[4]{b+ax^5}}{10bx^{10}} + \frac{7a\sqrt[4]{b+ax^5}}{40b^2x^5} + \frac{(21a)\,\text{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{b+ax^5}\right)}{40b^2}$$

$$= -\frac{\sqrt[4]{b+ax^5}}{10bx^{10}} + \frac{7a\sqrt[4]{b+ax^5}}{40b^2x^5} - \frac{\left(21a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-x^2}\,dx, x, \sqrt[4]{b+ax^5}\right)}{80b^{5/2}} - \frac{\left(21a^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt[4]{b+ax^5}}{10bx^{10}} + \frac{7a\sqrt[4]{b+ax^5}}{40b^2x^5} - \frac{21a^2\tan^{-1}\left(\frac{\sqrt[4]{b+ax^5}}{\sqrt[4]{b}}\right)}{80b^{11/4}} - \frac{21a^2\tanh^{-1}\left(\frac{\sqrt[4]{b+ax^5}}{\sqrt[4]{b}}\right)}{80b^{11/4}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.42

$$-\frac{4a^2\sqrt[4]{ax^5+b}\;{}_2F_1\left(\frac{1}{4},3;\frac{5}{4};\frac{ax^5}{b}+1\right)}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^11*(b + a*x^5)^(3/4)),x]
```

```
[Out] (-4*a^2*(b + a*x^5)^(1/4)*Hypergeometric2F1[1/4, 3, 5/4, 1 + (a*x^5)/b])/(5
*b^3)
```

IntegrateAlgebraic [A] time = 0.11, size = 92, normalized size = 1.00

$$-\frac{21a^2\tan^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{80b^{11/4}} - \frac{21a^2\tanh^{-1}\left(\frac{\sqrt[4]{ax^5+b}}{\sqrt[4]{b}}\right)}{80b^{11/4}} + \frac{\sqrt[4]{ax^5+b}\left(7ax^5-4b\right)}{40b^2x^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^11*(b + a*x^5)^(3/4)),x]
```

```
[Out] ((b + a*x^5)^(1/4)*(-4*b + 7*a*x^5))/(40*b^2*x^10) - (21*a^2*ArcTan[(b + a*
x^5)^(1/4)/b^(1/4)])/(80*b^(11/4)) - (21*a^2*ArcTanh[(b + a*x^5)^(1/4)/b^(1
/4)])/(80*b^(11/4))
```

fricas [B] time = 0.49, size = 216, normalized size = 2.35

$$\frac{84\,b^2x^{10}\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^5+b\right)^{\frac{1}{4}}a^2b^8\left(\frac{a^8}{b^{11}}\right)^{\frac{3}{4}}-\sqrt{b^6\sqrt{\frac{a^8}{b^{11}}}+\sqrt{ax^5+b}\,a^4}\,b^8\left(\frac{a^8}{b^{11}}\right)^{\frac{3}{4}}}{a^8}\right) - 21\,b^2x^{10}\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\log\left(21\,b^3\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}+21\left(ax^5+b\right)^{\frac{1}{4}}a^2\right) + 21\,b^2x^{10}\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\log\left(-21\,b^3\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}+21\left(ax^5+b\right)^{\frac{1}{4}}a^2\right) + 4\left(7\,ax^5-4\,b\right)\left(ax^5+b\right)^{\frac{1}{4}}}{160\,b^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(a*x^5+b)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/160*(84*b^2*x^10*(a^8/b^11)^(1/4)*arctan(-((a*x^5 + b)^(1/4)*a^2*b^8*(a^8
/b^11)^(3/4) - sqrt(b^6*sqrt(a^8/b^11) + sqrt(a*x^5 + b)*a^4)*b^8*(a^8/b^11
)^(3/4))/a^8) - 21*b^2*x^10*(a^8/b^11)^(1/4)*log(21*b^3*(a^8/b^11)^(1/4) +
21*(a*x^5 + b)^(1/4)*a^2) + 21*b^2*x^10*(a^8/b^11)^(1/4)*log(-21*b^3*(a^8/b
^11)^(1/4) + 21*(a*x^5 + b)^(1/4)*a^2) + 4*(7*a*x^5 - 4*b)*(a*x^5 + b)^(1/4
))/(b^2*x^10)
```

giac [B] time = 0.25, size = 244, normalized size = 2.65

$$\frac{\frac{42\sqrt{2}a^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}+2\left(ax^5+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{(-b)^{\frac{3}{4}}b^2}+\frac{42\sqrt{2}a^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}-2\left(ax^5+b\right)^{\frac{1}{4}}\right)}{2(-b)^{\frac{1}{4}}}\right)}{(-b)^{\frac{3}{4}}b^2}+\frac{21\sqrt{2}a^3\log\left(\sqrt{2}\left(ax^5+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^5+b}+\sqrt{-b}\right)}{(-b)^{\frac{3}{4}}b^2}+\frac{21\sqrt{2}a^3(-b)^{\frac{1}{4}}\log\left(-\sqrt{2}\left(ax^5+b\right)^{\frac{1}{4}}(-b)^{\frac{1}{4}}+\sqrt{ax^5+b}+\sqrt{-b}\right)}{b^3}+\frac{8\left(7\left(ax^5+b\right)^{\frac{5}{4}}a^3-11\left(ax^5+b\right)^{\frac{1}{4}}a^3b\right)}{a^2b^2x^{10}}}{320\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(a*x^5+b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/320*(42*sqrt(2)*a^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4) + 2*(a*x^5 + b
)^(1/4))/(-b)^(1/4))/((-b)^(3/4)*b^2) + 42*sqrt(2)*a^3*arctan(-1/2*sqrt(2)*
(sqrt(2)*(-b)^(1/4) - 2*(a*x^5 + b)^(1/4))/(-b)^(1/4))/((-b)^(3/4)*b^2) + 2
1*sqrt(2)*a^3*log(sqrt(2)*(a*x^5 + b)^(1/4)*(-b)^(1/4) + sqrt(a*x^5 + b) +
sqrt(-b))/((-b)^(3/4)*b^2) + 21*sqrt(2)*a^3*(-b)^(1/4)*log(-sqrt(2)*(a*x^5
+ b)^(1/4)*(-b)^(1/4) + sqrt(a*x^5 + b) + sqrt(-b))/b^3 + 8*(7*(a*x^5 + b)^
(5/4)*a^3 - 11*(a*x^5 + b)^(1/4)*a^3*b)/(a^2*b^2*x^10))/a
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{11}\left(a\,x^5+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^11/(a*x^5+b)^(3/4),x)
```

```
[Out] int(1/x^11/(a*x^5+b)^(3/4),x)
```

maxima [A] time = 0.48, size = 132, normalized size = 1.43

$$\frac{7\left(ax^5+b\right)^{\frac{5}{4}}a^2-11\left(ax^5+b\right)^{\frac{1}{4}}a^2b}{40\left(\left(ax^5+b\right)^2b^2-2\left(ax^5+b\right)b^3+b^4\right)}-\frac{21\left(\frac{2\,a^2\arctan\left(\frac{\left(ax^5+b\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}-\frac{a^2\log\left(\frac{\left(ax^5+b\right)^{\frac{1}{4}}-b^{\frac{1}{4}}}{\left(ax^5+b\right)^{\frac{1}{4}}+b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}\right)}{160\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(a*x^5+b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/40*(7*(a*x^5 + b)^(5/4)*a^2 - 11*(a*x^5 + b)^(1/4)*a^2*b)/((a*x^5 + b)^2*
b^2 - 2*(a*x^5 + b)*b^3 + b^4) - 21/160*(2*a^2*arctan((a*x^5 + b)^(1/4)/b^(
1/4))/b^(3/4) - a^2*log(((a*x^5 + b)^(1/4) - b^(1/4))/((a*x^5 + b)^(1/4) +
b^(1/4)))/b^(3/4))/b^2
```

mupad [B] time = 1.18, size = 82, normalized size = 0.89

$$\frac{7\left(a\,x^5+b\right)^{5/4}}{40\,b^2\,x^{10}}-\frac{11\left(a\,x^5+b\right)^{1/4}}{40\,b\,x^{10}}-\frac{21\,a^2\,\mathrm{atan}\left(\frac{\left(a\,x^5+b\right)^{1/4}}{b^{1/4}}\right)}{80\,b^{11/4}}+\frac{a^2\,\mathrm{atan}\left(\frac{\left(a\,x^5+b\right)^{1/4}\,1{}\mathrm{i}}{b^{1/4}}\right)\,21{}\mathrm{i}}{80\,b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11*(b + a*x^5)^(3/4)),x)
```

```
[Out] (a^2*atan(((b + a*x^5)^(1/4)*1i)/b^(1/4))*21i)/(80*b^(11/4)) - (21*a^2*atan
((b + a*x^5)^(1/4)/b^(1/4)))/(80*b^(11/4)) - (11*(b + a*x^5)^(1/4))/(40*b*x
^10) + (7*(b + a*x^5)^(5/4))/(40*b^2*x^10)
```

sympy [C] time = 1.82, size = 41, normalized size = 0.45

$$-\frac{\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^5}\right)}{5a^{\frac{3}{4}}x^{\frac{55}{4}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**11/(a*x**5+b)**(3/4),x)
```

```
[Out] -gamma(11/4)*hyper((3/4, 11/4), (15/4,), b*exp_polar(I*pi)/(a*x**5))/(5*a**
(3/4)*x**(55/4)*gamma(15/4))
```

3.1136 $\int \frac{1}{x^7 \sqrt[3]{-1+x^6}}\, dx$

Optimal. Leaf size=92

$$\frac{(x^6-1)^{2/3}}{6x^6}-\frac{1}{18}\log\left(\sqrt[3]{x^6-1}+1\right)+\frac{1}{36}\log\left((x^6-1)^{2/3}-\sqrt[3]{x^6-1}+1\right)-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{6\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 51, 56, 618, 204, 31}

$$\frac{(x^6-1)^{2/3}}{6x^6}-\frac{1}{12}\log\left(\sqrt[3]{x^6-1}+1\right)-\frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{6\sqrt{3}}+\frac{\log(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(-1 + x^6)^(1/3)),x]
```

```
[Out] (-1 + x^6)^(2/3)/(6*x^6) - ArcTan[(1 - 2*(-1 + x^6)^(1/3))/Sqrt[3]]/(6*Sqrt
[3]) + Log[x]/6 - Log[1 + (-1 + x^6)^(1/3)]/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\sqrt[3]{-1+x^6}}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x^2}\,dx,x,x^6\right)\\ &= \frac{\left(-1+x^6\right)^{2/3}}{6x^6}+\frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x}\,dx,x,x^6\right)\\ &= \frac{\left(-1+x^6\right)^{2/3}}{6x^6}+\frac{\log(x)}{6}-\frac{1}{12}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt[3]{-1+x^6}\right)+\frac{1}{12}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx\right.\\ &= \frac{\left(-1+x^6\right)^{2/3}}{6x^6}+\frac{\log(x)}{6}-\frac{1}{12}\log\left(1+\sqrt[3]{-1+x^6}\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2\sqrt[3]{-1+\right.\\ &= \frac{\left(-1+x^6\right)^{2/3}}{6x^6}-\frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^6}}{\sqrt{3}}\right)}{6\sqrt{3}}+\frac{\log(x)}{6}-\frac{1}{12}\log\left(1+\sqrt[3]{-1+x^6}\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 28, normalized size = 0.30

$$\frac{1}{4}\left(x^6-1\right)^{2/3}{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(-1 + x^6)^(1/3)),x]
```

```
[Out] ((-1 + x^6)^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, 1 - x^6])/4
```

IntegrateAlgebraic [A] time = 0.06, size = 92, normalized size = 1.00

$$\frac{\left(x^6-1\right)^{2/3}}{6x^6}-\frac{1}{18}\log\left(\sqrt[3]{x^6-1}+1\right)+\frac{1}{36}\log\left(\left(x^6-1\right)^{2/3}-\sqrt[3]{x^6-1}+1\right)-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(-1 + x^6)^(1/3)),x]
```

```
[Out] (-1 + x^6)^(2/3)/(6*x^6) - ArcTan[1/Sqrt[3] - (2*(-1 + x^6)^(1/3))/Sqrt[3]]
/(6*Sqrt[3]) - Log[1 + (-1 + x^6)^(1/3)]/18 + Log[1 - (-1 + x^6)^(1/3) + (-
1 + x^6)^(2/3)]/36
```

fricas [A] time = 0.43, size = 80, normalized size = 0.87

$$\frac{2\sqrt{3}\,x^6\arctan\left(\frac{2}{3}\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x^6\log\left(\left(x^6-1\right)^{\frac{2}{3}}-\left(x^6-1\right)^{\frac{1}{3}}+1\right)-2\,x^6\log\left(\left(x^6-1\right)^{\frac{1}{3}}+1\right)+6\left(x^6-1\right)^{\frac{2}{3}}}{36\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/36*(2*sqrt(3)*x^6*arctan(2/3*sqrt(3)*(x^6 - 1)^(1/3) - 1/3*sqrt(3)) + x^6
*log((x^6 - 1)^(2/3) - (x^6 - 1)^(1/3) + 1) - 2*x^6*log((x^6 - 1)^(1/3) + 1
) + 6*(x^6 - 1)^(2/3))/x^6
```

giac [A] time = 0.15, size = 69, normalized size = 0.75

$$\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^6-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{\left(x^6-1\right)^{\frac{2}{3}}}{6x^6}+\frac{1}{36}\log\left(\left(x^6-1\right)^{\frac{2}{3}}-\left(x^6-1\right)^{\frac{1}{3}}+1\right)-\frac{1}{18}\log\left(\left|\left(x^6-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6-1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3) - 1)) + 1/6*(x^6 - 1)^(2
/3)/x^6 + 1/36*log((x^6 - 1)^(2/3) - (x^6 - 1)^(1/3) + 1) - 1/18*log(abs((x
^6 - 1)^(1/3) + 1))
```

maple [C] time = 0.43, size = 96, normalized size = 1.04

$$\frac{\left(x^6-1\right)^{\frac{2}{3}}}{6x^6}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\mathrm{signum}\left(x^6-1\right)\right)^{\frac{1}{3}}\left(\frac{2\pi\sqrt{3}\,x^6\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^6\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+6\ln(x)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{36\pi\,\mathrm{signum}\left(x^6-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^6-1)^(1/3),x)
```

```
[Out] 1/6*(x^6-1)^(2/3)/x^6+1/36/Pi*3^(1/2)*GAMMA(2/3)/signum(x^6-1)^(1/3)*(-sign
um(x^6-1))^(1/3)*(2/9*Pi*3^(1/2)/GAMMA(2/3)*x^6*hypergeom([1,1,4/3],[2,2],x
^6)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+6*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.63, size = 68, normalized size = 0.74

$$\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^6-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{\left(x^6-1\right)^{\frac{2}{3}}}{6x^6}+\frac{1}{36}\log\left(\left(x^6-1\right)^{\frac{2}{3}}-\left(x^6-1\right)^{\frac{1}{3}}+1\right)-\frac{1}{18}\log\left(\left(x^6-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6-1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3) - 1)) + 1/6*(x^6 - 1)^(2
/3)/x^6 + 1/36*log((x^6 - 1)^(2/3) - (x^6 - 1)^(1/3) + 1) - 1/18*log((x^6 -
 1)^(1/3) + 1)
```

mupad [B] time = 0.91, size = 92, normalized size = 1.00

$$\frac{\left(x^6-1\right)^{2/3}}{6x^6}-\ln\left(9\left(-\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{36}\right)^2+\frac{\left(x^6-1\right)^{1/3}}{36}\right)\left(-\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{36}\right)+\ln\left(9\left(\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{36}\right)^2+\frac{\left(x^6-1\right)^{1/3}}{36}\right)\left(\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{36}\right)-\frac{\ln\left(\frac{\left(x^6-1\right)^{1/3}}{36}+\frac{1}{36}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^6 - 1)^(1/3)),x)
```

```
[Out] log(9*((3^(1/2)*1i)/36 + 1/36)^2 + (x^6 - 1)^(1/3)/36)*((3^(1/2)*1i)/36 + 1
/36) - log(9*((3^(1/2)*1i)/36 - 1/36)^2 + (x^6 - 1)^(1/3)/36)*((3^(1/2)*1i)
/36 - 1/36) - log((x^6 - 1)^(1/3)/36 + 1/36)/18 + (x^6 - 1)^(2/3)/(6*x^6)
```

sympy [C] time = 1.05, size = 32, normalized size = 0.35

$$-\frac{\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^6}\right)}{6x^8\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**6-1)**(1/3),x)
```

```
[Out] -gamma(4/3)*hyper((1/3, 4/3), (7/3,), exp_polar(2*I*pi)/x**6)/(6*x**8*gamma
(7/3))
```

3.1137 $\int \frac{(-1+x^6)^{2/3}}{x^7} \, dx$

Optimal. Leaf size=92

$$-\frac{\left(x^6-1\right)^{2/3}}{6x^6}-\frac{1}{9}\log\left(\sqrt[3]{x^6-1}+1\right)+\frac{1}{18}\log\left(\left(x^6-1\right)^{2/3}-\sqrt[3]{x^6-1}+1\right)-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {266, 47, 56, 618, 204, 31}

$$-\frac{\left(x^6-1\right)^{2/3}}{6x^6}-\frac{1}{6}\log\left(\sqrt[3]{x^6-1}+1\right)-\frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)^(2/3)/x^7,x]
```

```
[Out] -1/6*(-1 + x^6)^(2/3)/x^6 - ArcTan[(1 - 2*(-1 + x^6)^(1/3))/Sqrt[3]]/(3*Sqr
t[3]) + Log[x]/3 - Log[1 + (-1 + x^6)^(1/3)]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^6\right)^{2/3}}{x^7}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{(-1+x)^{2/3}}{x^2}\,dx,x,x^6\right)\\
&= -\frac{\left(-1+x^6\right)^{2/3}}{6x^6}+\frac{1}{9}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x}\,dx,x,x^6\right)\\
&= -\frac{\left(-1+x^6\right)^{2/3}}{6x^6}+\frac{\log(x)}{3}-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt[3]{-1+x^6}\right)+\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,\right.\\
&= -\frac{\left(-1+x^6\right)^{2/3}}{6x^6}+\frac{\log(x)}{3}-\frac{1}{6}\log\left(1+\sqrt[3]{-1+x^6}\right)-\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2\sqrt[3]{-1+\right.\\
&= -\frac{\left(-1+x^6\right)^{2/3}}{6x^6}-\frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^6}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\log(x)}{3}-\frac{1}{6}\log\left(1+\sqrt[3]{-1+x^6}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.30

$$\frac{1}{10}\left(x^6-1\right)^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)^(2/3)/x^7,x]
```

```
[Out] ((-1 + x^6)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 - x^6])/10
```

IntegrateAlgebraic [A] time = 0.06, size = 92, normalized size = 1.00

$$-\frac{\left(x^6-1\right)^{2/3}}{6x^6}-\frac{1}{9}\log\left(\sqrt[3]{x^6-1}+1\right)+\frac{1}{18}\log\left(\left(x^6-1\right)^{2/3}-\sqrt[3]{x^6-1}+1\right)-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^6)^(2/3)/x^7,x]
```

```
[Out] -1/6*(-1 + x^6)^(2/3)/x^6 - ArcTan[1/Sqrt[3] - (2*(-1 + x^6)^(1/3))/Sqrt[3]
]/(3*Sqrt[3]) - Log[1 + (-1 + x^6)^(1/3)]/9 + Log[1 - (-1 + x^6)^(1/3) + (-
1 + x^6)^(2/3)]/18
```

fricas [A] time = 0.44, size = 80, normalized size = 0.87

$$\frac{2\sqrt{3}\,x^6\arctan\left(\frac{2}{3}\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x^6\log\left(\left(x^6-1\right)^{\frac{2}{3}}-\left(x^6-1\right)^{\frac{1}{3}}+1\right)-2\,x^6\log\left(\left(x^6-1\right)^{\frac{1}{3}}+1\right)-3\left(x^6-1\right)^{\frac{2}{3}}}{18\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(2/3)/x^7,x, algorithm="fricas")
```

```
[Out] 1/18*(2*sqrt(3)*x^6*arctan(2/3*sqrt(3)*(x^6 - 1)^(1/3) - 1/3*sqrt(3)) + x^6
*log((x^6 - 1)^(2/3) - (x^6 - 1)^(1/3) + 1) - 2*x^6*log((x^6 - 1)^(1/3) + 1
) - 3*(x^6 - 1)^(2/3))/x^6
```

giac [A] time = 0.17, size = 69, normalized size = 0.75

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^6-1\right)^{\frac{1}{3}}-1\right)\right)-\frac{\left(x^6-1\right)^{\frac{2}{3}}}{6x^6}+\frac{1}{18}\log\left(\left(x^6-1\right)^{\frac{2}{3}}-\left(x^6-1\right)^{\frac{1}{3}}+1\right)-\frac{1}{9}\log\left(\left|\left(x^6-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(2/3)/x^7,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3) - 1)) - 1/6*(x^6 - 1)^(2/
3)/x^6 + 1/18*log((x^6 - 1)^(2/3) - (x^6 - 1)^(1/3) + 1) - 1/9*log(abs((x^6
 - 1)^(1/3) + 1))
```

maple [C] time = 0.35, size = 96, normalized size = 1.04

$$-\frac{\left(x^6-1\right)^{\frac{2}{3}}}{6x^6}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\mathrm{signum}\left(x^6-1\right)\right)^{\frac{1}{3}}\left(\frac{2\pi\sqrt{3}\,x^6\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^6\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+6\ln(x)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{18\pi\,\mathrm{signum}\left(x^6-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(2/3)/x^7,x)
```

```
[Out] -1/6*(x^6-1)^(2/3)/x^6+1/18/Pi*3^(1/2)*GAMMA(2/3)/signum(x^6-1)^(1/3)*(-sig
num(x^6-1))^(1/3)*(2/9*Pi*3^(1/2)/GAMMA(2/3)*x^6*hypergeom([1,1,4/3],[2,2],
x^6)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+6*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.51, size = 68, normalized size = 0.74

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^6-1\right)^{\frac{1}{3}}-1\right)\right)-\frac{\left(x^6-1\right)^{\frac{2}{3}}}{6x^6}+\frac{1}{18}\log\left(\left(x^6-1\right)^{\frac{2}{3}}-\left(x^6-1\right)^{\frac{1}{3}}+1\right)-\frac{1}{9}\log\left(\left(x^6-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(2/3)/x^7,x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3) - 1)) - 1/6*(x^6 - 1)^(2/
3)/x^6 + 1/18*log((x^6 - 1)^(2/3) - (x^6 - 1)^(1/3) + 1) - 1/9*log((x^6 - 1
)^(1/3) + 1)
```

mupad [B] time = 0.90, size = 92, normalized size = 1.00

$$-\frac{\ln\left(\frac{\left(x^6-1\right)^{1/3}}{9}+\frac{1}{9}\right)}{9}-\ln\left(9\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2+\frac{\left(x^6-1\right)^{1/3}}{9}\right)\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)+\ln\left(9\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2+\frac{\left(x^6-1\right)^{1/3}}{9}\right)\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)-\frac{\left(x^6-1\right)^{2/3}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)^(2/3)/x^7,x)
```

```
[Out] log(9*((3^(1/2)*1i)/18 + 1/18)^2 + (x^6 - 1)^(1/3)/9)*((3^(1/2)*1i)/18 + 1/
18) - log(9*((3^(1/2)*1i)/18 - 1/18)^2 + (x^6 - 1)^(1/3)/9)*((3^(1/2)*1i)/1
8 - 1/18) - log((x^6 - 1)^(1/3)/9 + 1/9)/9 - (x^6 - 1)^(2/3)/(6*x^6)
```

sympy [C] time = 1.12, size = 34, normalized size = 0.37

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^6}\right)}{6x^2\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(2/3)/x**7,x)
```

```
[Out] -gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_polar(2*I*pi)/x**6)/(6*x**2*gamm
a(4/3))
```

3.1138 $\int \frac{\left(-1+x^4+2x^6\right)\sqrt[3]{x+x^5+x^7}}{\left(1+x^4+x^6\right)\left(1-x^2+x^4+x^6\right)}\,dx$

Optimal. Leaf size=92

$$\frac{1}{2}\log\left(\sqrt[3]{x^7+x^5+x}-x\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^7+x^5+x}+x}\right)-\frac{1}{4}\log\left(x^2+\sqrt[3]{x^7+x^5+x}\,x+\left(x^7+x^5+x\right)^{2/3}\right)$$

Rubi [F] time = 2.39, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1+x^4+2x^6\right)\sqrt[3]{x+x^5+x^7}}{\left(1+x^4+x^6\right)\left(1-x^2+x^4+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^4 + 2*x^6)*(x + x^5 + x^7)^(1/3))/((1 + x^4 + x^6)*(1 - x^2 +
x^4 + x^6)),x]
```

```
[Out] (3*(x + x^5 + x^7)^(1/3)*Defer[Subst][Defer[Int][x/(1 + x^6 + x^9)^(2/3), x
], x, x^(2/3)])/(x^(1/3)*(1 + x^4 + x^6)^(1/3)) - (9*(x + x^5 + x^7)^(1/3)*
Defer[Subst][Defer[Int][x/((1 + x^6 + x^9)^(2/3)*(1 - x^3 + x^6 + x^9)), x]
, x, x^(2/3)])/(2*x^(1/3)*(1 + x^4 + x^6)^(1/3)) + (3*(x + x^5 + x^7)^(1/3)
*Defer[Subst][Defer[Int][x^4/((1 + x^6 + x^9)^(2/3)*(1 - x^3 + x^6 + x^9)),
 x], x, x^(2/3)])/(x^(1/3)*(1 + x^4 + x^6)^(1/3)) - (3*(x + x^5 + x^7)^(1/3
)*Defer[Subst][Defer[Int][x^7/((1 + x^6 + x^9)^(2/3)*(1 - x^3 + x^6 + x^9))
, x], x, x^(2/3)])/(2*x^(1/3)*(1 + x^4 + x^6)^(1/3))
```

Rubi steps

$$\int \frac{\left(-1+x^4+2x^6\right)\sqrt[3]{x+x^5+x^7}}{\left(1+x^4+x^6\right)\left(1-x^2+x^4+x^6\right)}\,dx = \frac{\sqrt[3]{x+x^5+x^7}\int \frac{\sqrt[3]{x}\left(-1+x^4+2x^6\right)}{\left(1+x^4+x^6\right)^{2/3}\left(1-x^2+x^4+x^6\right)}\,dx}{\sqrt[3]{x}\sqrt[3]{1+x^4+x^6}}$$

$$= \frac{\left(3\sqrt[3]{x+x^5+x^7}\right)\text{Subst}\left(\int \frac{x^3\left(-1+x^{12}+2x^{18}\right)}{\left(1+x^{12}+x^{18}\right)^{2/3}\left(1-x^6+x^{12}+x^{18}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1+x^4+x^6}}$$

$$= \frac{\left(3\sqrt[3]{x+x^5+x^7}\right)\text{Subst}\left(\int \frac{x\left(-1+x^6+2x^9\right)}{\left(1+x^6+x^9\right)^{2/3}\left(1-x^3+x^6+x^9\right)}\,dx,x,x^{2/3}\right)}{2\sqrt[3]{x}\sqrt[3]{1+x^4+x^6}}$$

$$= \frac{\left(3\sqrt[3]{x+x^5+x^7}\right)\text{Subst}\left(\int\left(\frac{2x}{\left(1+x^6+x^9\right)^{2/3}}+\frac{x\left(-3+2x^3-x^6\right)}{\left(1+x^6+x^9\right)^{2/3}\left(1-x^3+x^6+x^9\right)}\right)dx,\right.}{2\sqrt[3]{x}\sqrt[3]{1+x^4+x^6}}$$

$$= \frac{\left(3\sqrt[3]{x+x^5+x^7}\right)\text{Subst}\left(\int \frac{x\left(-3+2x^3-x^6\right)}{\left(1+x^6+x^9\right)^{2/3}\left(1-x^3+x^6+x^9\right)}\,dx,x,x^{2/3}\right)}{2\sqrt[3]{x}\sqrt[3]{1+x^4+x^6}}+\frac{\left(3\sqrt[3]{x\right.}}{}$$

$$= \frac{\left(3\sqrt[3]{x+x^5+x^7}\right)\text{Subst}\left(\int\left(-\frac{3x}{\left(1+x^6+x^9\right)^{2/3}\left(1-x^3+x^6+x^9\right)}+\frac{2x^4}{\left(1+x^6+x^9\right)^{2/3}\left(1-x\right.}\right.\right.}{2\sqrt[3]{x}\sqrt[3]{1+x^4+x^6}}$$

$$= -\frac{\left(3\sqrt[3]{x+x^5+x^7}\right)\text{Subst}\left(\int \frac{x^7}{\left(1+x^6+x^9\right)^{2/3}\left(1-x^3+x^6+x^9\right)}\,dx,x,x^{2/3}\right)}{2\sqrt[3]{x}\sqrt[3]{1+x^4+x^6}}+\frac{\left(3\sqrt[3]{\right.}}{}$$

Mathematica [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^4+2x^6)\sqrt[3]{x+x^5+x^7}}{(1+x^4+x^6)(1-x^2+x^4+x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^4 + 2*x^6)*(x + x^5 + x^7)^(1/3))/((1 + x^4 + x^6)*(1 -
x^2 + x^4 + x^6)),x]
```

```
[Out] Integrate[((-1 + x^4 + 2*x^6)*(x + x^5 + x^7)^(1/3))/((1 + x^4 + x^6)*(1 -
x^2 + x^4 + x^6)), x]
```

IntegrateAlgebraic [A] time = 0.19, size = 92, normalized size = 1.00

$$\frac{1}{2}\log\left(\sqrt[3]{x^7+x^5+x}-x\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^7+x^5+x}+x}\right)-\frac{1}{4}\log\left(x^2+\sqrt[3]{x^7+x^5+x}\,x+\left(x^7+x^5+x\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4 + 2*x^6)*(x + x^5 + x^7)^(1/3))/((1 + x^4 + x
^6)*(1 - x^2 + x^4 + x^6)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(x + x^5 + x^7)^(1/3))])/2 + Log[-x + (x
 + x^5 + x^7)^(1/3)]/2 - Log[x^2 + x*(x + x^5 + x^7)^(1/3) + (x + x^5 + x^7
)^(2/3)]/4
```

fricas [A] time = 3.07, size = 127, normalized size = 1.38

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(x^7+x^5+x\right)^{\frac{1}{3}}x+\sqrt{3}\left(x^6+x^4+x^2+1\right)+2\sqrt{3}\left(x^7+x^5+x\right)^{\frac{2}{3}}}{x^6+x^4-x^2+1}\right)+\frac{1}{4}\log\left(\frac{x^6+x^4-x^2+3\left(x^7+x^5+x\right)^{\frac{1}{3}}x-3\left(x^7+x^5+x\right)^{\frac{2}{3}}+1}{x^6+x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+x^4-1)*(x^7+x^5+x)^(1/3)/(x^6+x^4+1)/(x^6+x^4-x^2+1),x, al
gorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan((2*sqrt(3)*(x^7 + x^5 + x)^(1/3)*x + sqrt(3)*(x^6 + x^4
+ x^2 + 1) + 2*sqrt(3)*(x^7 + x^5 + x)^(2/3))/(x^6 + x^4 - x^2 + 1)) + 1/4*
log((x^6 + x^4 - x^2 + 3*(x^7 + x^5 + x)^(1/3)*x - 3*(x^7 + x^5 + x)^(2/3)
+ 1)/(x^6 + x^4 - x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^7+x^5+x\right)^{\frac{1}{3}}\left(2x^6+x^4-1\right)}{\left(x^6+x^4-x^2+1\right)\left(x^6+x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+x^4-1)*(x^7+x^5+x)^(1/3)/(x^6+x^4+1)/(x^6+x^4-x^2+1),x, al
gorithm="giac")
```

```
[Out] integrate((x^7 + x^5 + x)^(1/3)*(2*x^6 + x^4 - 1)/((x^6 + x^4 - x^2 + 1)*(x
^6 + x^4 + 1)), x)
```

maple [C] time = 8.38, size = 489, normalized size = 5.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6+x^4-1)*(x^7+x^5+x)^(1/3)/(x^6+x^4+1)/(x^6+x^4-x^2+1),x)
```

```
[Out] 1/2*ln((20906848965368*RootOf(4*_Z^2+2*_Z+1)^2*x^6+44817676777674*RootOf(4*
_Z^2+2*_Z+1)*x^6+20906848965368*RootOf(4*_Z^2+2*_Z+1)^2*x^4+6007957693876*x
^6+44817676777674*RootOf(4*_Z^2+2*_Z+1)*x^4-62720546896104*RootOf(4*_Z^2+2*
_Z+1)^2*x^2+6007957693876*x^4-35926743200058*RootOf(4*_Z^2+2*_Z+1)*(x^7+x^5
+x)^(2/3)-31239270484854*RootOf(4*_Z^2+2*_Z+1)*(x^7+x^5+x)^(1/3)*x+14414879
41870*RootOf(4*_Z^2+2*_Z+1)*x^2+20906848965368*RootOf(4*_Z^2+2*_Z+1)^2+1561
9635242427*(x^7+x^5+x)^(2/3)-33583006842456*x*(x^7+x^5+x)^(1/3)+15019894234
69*x^2+44817676777674*RootOf(4*_Z^2+2*_Z+1)+6007957693876)/(x^6+x^4-x^2+1))
+RootOf(4*_Z^2+2*_Z+1)*ln(-(14898891271492*RootOf(4*_Z^2+2*_Z+1)^2*x^6-2535
2315754176*RootOf(4*_Z^2+2*_Z+1)*x^6+14898891271492*RootOf(4*_Z^2+2*_Z+1)^2
*x^4+4505968270407*x^6-25352315754176*RootOf(4*_Z^2+2*_Z+1)*x^4-44696673814
476*RootOf(4*_Z^2+2*_Z+1)^2*x^2+4505968270407*x^4-35926743200058*RootOf(4*_
Z^2+2*_Z+1)*(x^7+x^5+x)^(2/3)+67166013684912*RootOf(4*_Z^2+2*_Z+1)*(x^7+x^5
+x)^(1/3)*x-20785846002170*RootOf(4*_Z^2+2*_Z+1)*x^2+14898891271492*RootOf(
4*_Z^2+2*_Z+1)^2-33583006842456*(x^7+x^5+x)^(2/3)+15619635242427*x*(x^7+x^5
+x)^(1/3)+6007957693876*x^2-25352315754176*RootOf(4*_Z^2+2*_Z+1)+4505968270
407)/(x^6+x^4-x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^7+x^5+x\right)^{\frac{1}{3}}\left(2\,x^6+x^4-1\right)}{\left(x^6+x^4-x^2+1\right)\left(x^6+x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+x^4-1)*(x^7+x^5+x)^(1/3)/(x^6+x^4+1)/(x^6+x^4-x^2+1),x, al
gorithm="maxima")
```

```
[Out] integrate((x^7 + x^5 + x)^(1/3)*(2*x^6 + x^4 - 1)/((x^6 + x^4 - x^2 + 1)*(x
^6 + x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^6+x^4-1\right)\,{\left(x^7+x^5+x\right)}^{1/3}}{\left(x^6+x^4+1\right)\,\left(x^6+x^4-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 2*x^6 - 1)*(x + x^5 + x^7)^(1/3))/((x^4 + x^6 + 1)*(x^4 - x^2 +
 x^6 + 1)),x)
```

```
[Out] int(((x^4 + 2*x^6 - 1)*(x + x^5 + x^7)^(1/3))/((x^4 + x^6 + 1)*(x^4 - x^2 +
 x^6 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**6+x**4-1)*(x**7+x**5+x)**(1/3)/(x**6+x**4+1)/(x**6+x**4-x**
2+1),x)
```

```
[Out] Timed out
```

3.1139 $\int \frac{\sqrt{1+\sqrt{1+x}}}{x-\sqrt{1+x}}\,dx$

Optimal. Leaf size=92

$$4\sqrt{\sqrt{x+1}+1}-\frac{2}{5}\left(3\sqrt{5}-5\right)\tanh^{-1}\left(\frac{2\sqrt{\sqrt{x+1}+1}}{\sqrt{5}-1}\right)-\frac{2}{5}\left(5+3\sqrt{5}\right)\tanh^{-1}\left(\frac{2\sqrt{\sqrt{x+1}+1}}{1+\sqrt{5}}\right)$$

Rubi [A] time = 0.29, antiderivative size = 114, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {824, 826, 1166, 207}

$$4\sqrt{\sqrt{x+1}+1}-2\sqrt{\frac{2}{5}\left(7+3\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\sqrt{\sqrt{x+1}+1}\right)-2\sqrt{\frac{2}{5}\left(7-3\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\sqrt{\sqrt{x+1}+1}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[1 + Sqrt[1 + x]]/(x - Sqrt[1 + x]),x]
```

```
[Out] 4*Sqrt[1 + Sqrt[1 + x]] - 2*Sqrt[(2*(7 + 3*Sqrt[5]))/5]*ArcTanh[Sqrt[2/(3 +
 Sqrt[5])]*Sqrt[1 + Sqrt[1 + x]]] - 2*Sqrt[(2*(7 - 3*Sqrt[5]))/5]*ArcTanh[S
qrt[(3 + Sqrt[5])/2]*Sqrt[1 + Sqrt[1 + x]]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{1+\sqrt{1+x}}}{x-\sqrt{1+x}}\,dx = 2\,\text{Subst}\left(\int \frac{x\sqrt{1+x}}{-1-x+x^2}\,dx, x, \sqrt{1+x}\right)$$
$$= 4\sqrt{1+\sqrt{1+x}} + 2\,\text{Subst}\left(\int \frac{1+2x}{\sqrt{1+x}\,(-1-x+x^2)}\,dx, x, \sqrt{1+x}\right)$$
$$= 4\sqrt{1+\sqrt{1+x}} + 4\,\text{Subst}\left(\int \frac{-1+2x^2}{1-3x^2+x^4}\,dx, x, \sqrt{1+\sqrt{1+x}}\right)$$
$$= 4\sqrt{1+\sqrt{1+x}} + \frac{1}{5}\left(4\left(5-2\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, \sqrt{1+\sqrt{1+x}}\right) + \frac{1}{5}\Big($$
$$= 4\sqrt{1+\sqrt{1+x}} - 2\sqrt{\frac{2}{5}\left(7+3\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\sqrt{1+\sqrt{1+x}}\right) - \frac{2}{5}\sqrt{70-30\sqrt{5}}$$

Mathematica [A] time = 0.28, size = 125, normalized size = 1.36

$$\frac{1}{5}\left(20\sqrt{\sqrt{x+1}+1} + \sqrt{6-2\sqrt{5}}\left(\sqrt{5}-5\right)\tanh^{-1}\left(\sqrt{\frac{2}{3-\sqrt{5}}}\sqrt{\sqrt{x+1}+1}\right) - \sqrt{2\left(3+\sqrt{5}\right)}\left(5+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\sqrt{\sqrt{x+1}+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + x]]/(x - Sqrt[1 + x]),x]
```

```
[Out] (20*Sqrt[1 + Sqrt[1 + x]] + Sqrt[6 - 2*Sqrt[5]]*(-5 + Sqrt[5])*ArcTanh[Sqrt
[2/(3 - Sqrt[5])]*Sqrt[1 + Sqrt[1 + x]]] - Sqrt[2*(3 + Sqrt[5])]*(5 + Sqrt[
5])*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*Sqrt[1 + Sqrt[1 + x]]])/5
```

IntegrateAlgebraic [A] time = 0.15, size = 92, normalized size = 1.00

$$4\sqrt{\sqrt{x+1}+1} - \frac{2}{5}\left(5+3\sqrt{5}\right)\tanh^{-1}\left(\frac{1}{2}\left(\sqrt{5}-1\right)\sqrt{\sqrt{x+1}+1}\right) - \frac{2}{5}\left(3\sqrt{5}-5\right)\tanh^{-1}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt{\sqrt{x+1}+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 + x]]/(x - Sqrt[1 + x]),x]
```

```
[Out] 4*Sqrt[1 + Sqrt[1 + x]] - (2*(5 + 3*Sqrt[5])*ArcTanh[((-1 + Sqrt[5])*Sqrt[1
 + Sqrt[1 + x]])/2])/5 - (2*(-5 + 3*Sqrt[5])*ArcTanh[((1 + Sqrt[5])*Sqrt[1
+ Sqrt[1 + x]])/2])/5
```

fricas [B] time = 0.43, size = 234, normalized size = 2.54

$$\frac{3}{5}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(3x+1)+\left(\sqrt{5}(x+2)+5x\right)\sqrt{x+1}-\left(\sqrt{5}(x+2)+\left(\sqrt{5}(2x-1)+5\right)\sqrt{x+1}+5x\right)\sqrt{\sqrt{x+1}+1}+3x+3}{x^2-x-1}\right) + \frac{3}{5}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}(3x+1)-\left(\sqrt{5}(x+2)-5x\right)\sqrt{x+1}-\left(\sqrt{5}(x+2)+\left(\sqrt{5}(2x-1)-5\right)\sqrt{x+1}-5x\right)\sqrt{\sqrt{x+1}+1}+3x+3}{x^2-x-1}\right) + 4\sqrt{\sqrt{x+1}+1} - \log\left(\sqrt{x+1}+\sqrt{\sqrt{x+1}+1}\right) + \log\left(\sqrt{x+1}-\sqrt{\sqrt{x+1}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+x)^(1/2))^(1/2)/(x-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 3/5*sqrt(5)*log((2*x^2 + sqrt(5)*(3*x + 1) + (sqrt(5)*(x + 2) + 5*x)*sqrt(x
 + 1) - (sqrt(5)*(x + 2) + (sqrt(5)*(2*x - 1) + 5)*sqrt(x + 1) + 5*x)*sqrt(
sqrt(x + 1) + 1) + 3*x + 3)/(x^2 - x - 1)) + 3/5*sqrt(5)*log((2*x^2 - sqrt(
5)*(3*x + 1) - (sqrt(5)*(x + 2) - 5*x)*sqrt(x + 1) - (sqrt(5)*(x + 2) + (sq
rt(5)*(2*x - 1) - 5)*sqrt(x + 1) - 5*x)*sqrt(sqrt(x + 1) + 1) + 3*x + 3)/(x
^2 - x - 1)) + 4*sqrt(sqrt(x + 1) + 1) - log(sqrt(x + 1) + sqrt(sqrt(x + 1)
 + 1)) + log(sqrt(x + 1) - sqrt(sqrt(x + 1) + 1))
```

giac [A] time = 0.56, size = 136, normalized size = 1.48

$$\frac{3}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{\sqrt{x+1}+1}-1}{\sqrt{5}+2\sqrt{\sqrt{x+1}+1}+1}\right) + \frac{3}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{\sqrt{x+1}+1}-1\right|}{\left|\sqrt{5}+2\sqrt{\sqrt{x+1}+1}-1\right|}\right) + 4\sqrt{\sqrt{x+1}+1} - \log\left(\sqrt{x+1}+\sqrt{\sqrt{x+1}+1}\right) + \log\left(\left|\sqrt{x+1}-\sqrt{\sqrt{x+1}+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+x)^(1/2))^(1/2)/(x-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 3/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(sqrt(x + 1) + 1) - 1)/(sqrt(5) + 2*sqrt(
sqrt(x + 1) + 1) + 1)) + 3/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(sqrt(x + 1)
+ 1) - 1)/abs(sqrt(5) + 2*sqrt(sqrt(x + 1) + 1) - 1)) + 4*sqrt(sqrt(x + 1)
+ 1) - log(sqrt(x + 1) + sqrt(sqrt(x + 1) + 1)) + log(abs(sqrt(x + 1) - sqr
t(sqrt(x + 1) + 1)))
```

maple [A] time = 0.01, size = 97, normalized size = 1.05

$$4\sqrt{1+\sqrt{1+x}}-\ln\left(\sqrt{1+x}+\sqrt{1+\sqrt{1+x}}\right)-\frac{6\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(1+2\sqrt{1+\sqrt{1+x}}\right)\sqrt{5}}{5}\right)}{5}+\ln\left(\sqrt{1+x}-\sqrt{1+\sqrt{1+x}}\right)-\frac{6\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(-1+2\sqrt{1+\sqrt{1+x}}\right)\sqrt{5}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(1+x)^(1/2))^(1/2)/(x-(1+x)^(1/2)),x)
```

```
[Out] 4*(1+(1+x)^(1/2))^(1/2)-ln((1+x)^(1/2)+(1+(1+x)^(1/2))^(1/2))-6/5*5^(1/2)*a
rctanh(1/5*(1+2*(1+(1+x)^(1/2))^(1/2))*5^(1/2))+ln((1+x)^(1/2)-(1+(1+x)^(1/
2))^(1/2))-6/5*5^(1/2)*arctanh(1/5*(-1+2*(1+(1+x)^(1/2))^(1/2))*5^(1/2))
```

maxima [A] time = 1.06, size = 132, normalized size = 1.43

$$\frac{3}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{\sqrt{x+1}+1}+1}{\sqrt{5}+2\sqrt{\sqrt{x+1}+1}-1}\right)+\frac{3}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{\sqrt{x+1}+1}-1}{\sqrt{5}+2\sqrt{\sqrt{x+1}+1}+1}\right)+4\sqrt{\sqrt{x+1}+1}-\log\left(\sqrt{x+1}+\sqrt{\sqrt{x+1}+1}\right)+\log\left(\sqrt{x+1}-\sqrt{\sqrt{x+1}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+x)^(1/2))^(1/2)/(x-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 3/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(sqrt(x + 1) + 1) + 1)/(sqrt(5) + 2*sqrt(
sqrt(x + 1) + 1) - 1)) + 3/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(sqrt(x + 1) + 1
) - 1)/(sqrt(5) + 2*sqrt(sqrt(x + 1) + 1) + 1)) + 4*sqrt(sqrt(x + 1) + 1) -
 log(sqrt(x + 1) + sqrt(sqrt(x + 1) + 1)) + log(sqrt(x + 1) - sqrt(sqrt(x +
 1) + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{x+1}+1}}{x-\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + 1)^(1/2)/(x - (x + 1)^(1/2)),x)
```

```
[Out] int(((x + 1)^(1/2) + 1)^(1/2)/(x - (x + 1)^(1/2)), x)
```

sympy [A] time = 12.35, size = 260, normalized size = 2.83

$$4\sqrt{\sqrt{x+1}+1}+12\left(\begin{cases}-\frac{\sqrt{5}\operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{\sqrt{x+1}+1}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{\sqrt{x+1}+1}-\frac{1}{2}\right)^2>\frac{5}{4}\\ -\frac{\sqrt{5}\operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{\sqrt{x+1}+1}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{\sqrt{x+1}+1}-\frac{1}{2}\right)^2<\frac{5}{4}\end{cases}\right)+12\left(\begin{cases}-\frac{\sqrt{5}\operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)^2>\frac{5}{4}\\ -\frac{\sqrt{5}\operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)^2<\frac{5}{4}\end{cases}\right)+\log\left(\sqrt{x+1}-\sqrt{\sqrt{x+1}+1}\right)-\log\left(\sqrt{x+1}+\sqrt{\sqrt{x+1}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+x)**(1/2))**(1/2)/(x-(1+x)**(1/2)),x)
```

```
[Out] 4*sqrt(sqrt(x + 1) + 1) + 12*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(sqrt
(x + 1) + 1) - 1/2)/5)/10, (sqrt(sqrt(x + 1) + 1) - 1/2)**2 > 5/4), (-sqrt(
```

```
5)*atanh(2*sqrt(5)*(sqrt(sqrt(x + 1) + 1) - 1/2)/5)/10, (sqrt(sqrt(x + 1) +
 1) - 1/2)**2 < 5/4)) + 12*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(sqrt(x
 + 1) + 1) + 1/2)/5)/10, (sqrt(sqrt(x + 1) + 1) + 1/2)**2 > 5/4), (-sqrt(5)
*atanh(2*sqrt(5)*(sqrt(sqrt(x + 1) + 1) + 1/2)/5)/10, (sqrt(sqrt(x + 1) + 1
) + 1/2)**2 < 5/4)) + log(sqrt(x + 1) - sqrt(sqrt(x + 1) + 1)) - log(sqrt(x
 + 1) + sqrt(sqrt(x + 1) + 1))
```

3.1140 $\int \frac{1}{x^3\left(-1+x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=93

$$\frac{\sqrt[4]{x^2-1}}{2x^2}-\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^2-1}}{\sqrt{x^2-1}-1}\right)}{4\sqrt{2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^2-1}}{\sqrt{x^2-1}+1}\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.56, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {266, 51, 63, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{x^2-1}}{2x^2}-\frac{3\log\left(\sqrt{x^2-1}-\sqrt{2}\sqrt[4]{x^2-1}+1\right)}{8\sqrt{2}}+\frac{3\log\left(\sqrt{x^2-1}+\sqrt{2}\sqrt[4]{x^2-1}+1\right)}{8\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{x^2-1}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt[4]{x^2-1}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-1 + x^2)^(3/4)),x]
```

```
[Out] (-1 + x^2)^(1/4)/(2*x^2) - (3*ArcTan[1 - Sqrt[2]*(-1 + x^2)^(1/4)])/(4*Sqrt
[2]) + (3*ArcTan[1 + Sqrt[2]*(-1 + x^2)^(1/4)])/(4*Sqrt[2]) - (3*Log[1 - Sq
rt[2]*(-1 + x^2)^(1/4) + Sqrt[-1 + x^2]])/(8*Sqrt[2]) + (3*Log[1 + Sqrt[2]*
(-1 + x^2)^(1/4) + Sqrt[-1 + x^2]])/(8*Sqrt[2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(-1+x^2\right)^{3/4}}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{(-1+x)^{3/4}x^2}\,dx, x, x^2\right)\\
&= \frac{\sqrt[4]{-1+x^2}}{2x^2}+\frac{3}{8}\,\mathrm{Subst}\left(\int \frac{1}{(-1+x)^{3/4}x}\,dx, x, x^2\right)\\
&= \frac{\sqrt[4]{-1+x^2}}{2x^2}+\frac{3}{2}\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt[4]{-1+x^2}\right)\\
&= \frac{\sqrt[4]{-1+x^2}}{2x^2}+\frac{3}{4}\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt[4]{-1+x^2}\right)+\frac{3}{4}\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt[4]{-1+\right.\\
&= \frac{\sqrt[4]{-1+x^2}}{2x^2}+\frac{3}{8}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt[4]{-1+x^2}\right)+\frac{3}{8}\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x\right.\\
&= \frac{\sqrt[4]{-1+x^2}}{2x^2}-\frac{3\log\left(1-\sqrt{2}\sqrt[4]{-1+x^2}+\sqrt{-1+x^2}\right)}{8\sqrt{2}}+\frac{3\log\left(1+\sqrt{2}\sqrt[4]{-1+x^2}+\sqrt{-1\right.}{8\sqrt{2}}\\
&= \frac{\sqrt[4]{-1+x^2}}{2x^2}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+x^2}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+x^2}\right)}{4\sqrt{2}}-\frac{3\log\left(1-\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 26, normalized size = 0.28

$$2\sqrt[4]{x^2-1}\;{}_2F_1\left(\frac{1}{4},2;\frac{5}{4};1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-1 + x^2)^(3/4)),x]
```

```
[Out] 2*(-1 + x^2)^(1/4)*Hypergeometric2F1[1/4, 2, 5/4, 1 - x^2]
```

IntegrateAlgebraic [A] time = 0.18, size = 98, normalized size = 1.05

$$\frac{\sqrt[4]{x^2-1}}{2x^2}+\frac{3\tan^{-1}\left(\frac{\frac{\sqrt{x^2-1}}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt[4]{x^2-1}}\right)}{4\sqrt{2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^2-1}}{\sqrt{x^2-1}+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(-1 + x^2)^(3/4)),x]
```

```
[Out] (-1 + x^2)^(1/4)/(2*x^2) + (3*ArcTan[(-(1/Sqrt[2]) + Sqrt[-1 + x^2]/Sqrt[2]
)/(-1 + x^2)^(1/4)])/(4*Sqrt[2]) + (3*ArcTanh[(Sqrt[2]*(-1 + x^2)^(1/4))/(1
 + Sqrt[-1 + x^2])])/(4*Sqrt[2])
```

fricas [B] time = 0.44, size = 181, normalized size = 1.95

$$-\frac{12\sqrt{2}x^2\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}+\sqrt{x^2-1}+1}-\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}-1\right)+12\sqrt{2}x^2\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}+4\sqrt{x^2-1}+4}-\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}+1\right)-3\sqrt{2}x^2\log\left(4\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}+4\sqrt{x^2-1}+4\right)+3\sqrt{2}x^2\log\left(-4\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}+4\sqrt{x^2-1}+4\right)-8\left(x^2-1\right)^{\frac{1}{4}}}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/16*(12*sqrt(2)*x^2*arctan(sqrt(2)*sqrt(sqrt(2)*(x^2 - 1)^(1/4) + sqrt(x^
2 - 1) + 1) - sqrt(2)*(x^2 - 1)^(1/4) - 1) + 12*sqrt(2)*x^2*arctan(1/2*sqrt
(2)*sqrt(-4*sqrt(2)*(x^2 - 1)^(1/4) + 4*sqrt(x^2 - 1) + 4) - sqrt(2)*(x^2 -
 1)^(1/4) + 1) - 3*sqrt(2)*x^2*log(4*sqrt(2)*(x^2 - 1)^(1/4) + 4*sqrt(x^2 -
 1) + 4) + 3*sqrt(2)*x^2*log(-4*sqrt(2)*(x^2 - 1)^(1/4) + 4*sqrt(x^2 - 1) +
 4) - 8*(x^2 - 1)^(1/4))/x^2
```

giac [A] time = 0.28, size = 114, normalized size = 1.23

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(x^2-1\right)^{\frac{1}{4}}\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(x^2-1\right)^{\frac{1}{4}}\right)\right)+\frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}+\sqrt{x^2-1}+1\right)-\frac{3}{16}\sqrt{2}\log\left(-\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}+\sqrt{x^2-1}+1\right)+\frac{\left(x^2-1\right)^{\frac{1}{4}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^2 - 1)^(1/4))) + 3/8*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^2 - 1)^(1/4))) + 3/16*sqrt(2)*log(sqrt
(2)*(x^2 - 1)^(1/4) + sqrt(x^2 - 1) + 1) - 3/16*sqrt(2)*log(-sqrt(2)*(x^2 -
 1)^(1/4) + sqrt(x^2 - 1) + 1) + 1/2*(x^2 - 1)^(1/4)/x^2
```

maple [C] time = 0.35, size = 76, normalized size = 0.82

$$\frac{\left(x^2-1\right)^{\frac{1}{4}}}{2x^2}+\frac{3\left(-\mathrm{signum}\left(x^2-1\right)\right)^{\frac{3}{4}}\left(\frac{3\Gamma\left(\frac{3}{4}\right)x^2\,\mathrm{hypergeom}\left(\left[1,1,\frac{7}{4}\right],[2,2],x^2\right)}{4}+\left(-3\ln(2)+\frac{\pi}{2}+2\ln(x)+i\pi\right)\Gamma\left(\frac{3}{4}\right)\right)}{8\Gamma\left(\frac{3}{4}\right)\mathrm{signum}\left(x^2-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^2-1)^(3/4),x)
```

```
[Out] 1/2*(x^2-1)^(1/4)/x^2+3/8/GAMMA(3/4)/signum(x^2-1)^(3/4)*(-signum(x^2-1))^(
3/4)*(3/4*GAMMA(3/4)*x^2*hypergeom([1,1,7/4],[2,2],x^2)+(-3*ln(2)+1/2*Pi+2*
ln(x)+I*Pi)*GAMMA(3/4))
```

maxima [A] time = 0.76, size = 114, normalized size = 1.23

$$\frac{3}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\left(x^2-1\right)^{\frac{1}{4}}\right)\right)+\frac{3}{8}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\left(x^2-1\right)^{\frac{1}{4}}\right)\right)+\frac{3}{16}\,\sqrt{2}\log\left(\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}+\sqrt{x^2-1}+1\right)-\frac{3}{16}\,\sqrt{2}\log\left(-\sqrt{2}\left(x^2-1\right)^{\frac{1}{4}}+\sqrt{x^2-1}+1\right)+\frac{\left(x^2-1\right)^{\frac{1}{4}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^2 - 1)^(1/4))) + 3/8*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^2 - 1)^(1/4))) + 3/16*sqrt(2)*log(sqrt
(2)*(x^2 - 1)^(1/4) + sqrt(x^2 - 1) + 1) - 3/16*sqrt(2)*log(-sqrt(2)*(x^2 -
 1)^(1/4) + sqrt(x^2 - 1) + 1) + 1/2*(x^2 - 1)^(1/4)/x^2
```

mupad [B] time = 0.95, size = 57, normalized size = 0.61

$$\frac{\left(x^2-1\right)^{1/4}}{2\,x^2}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(x^2-1\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{8}+\frac{3}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(x^2-1\right)^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{8}-\frac{3}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^2 - 1)^(3/4)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*(x^2 - 1)^(1/4)*(1/2 - 1i/2))*(3/8 + 3i/8) + 2^(1/2)*a
tan(2^(1/2)*(x^2 - 1)^(1/4)*(1/2 + 1i/2))*(3/8 - 3i/8) + (x^2 - 1)^(1/4)/(2
*x^2)
```

sympy [C] time = 0.95, size = 34, normalized size = 0.37

$$-\frac{\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{e^{2i\pi}}{x^2}\right)}{2x^{\frac{7}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**2-1)**(3/4),x)
```

```
[Out] -gamma(7/4)*hyper((3/4, 7/4), (11/4,), exp_polar(2*I*pi)/x**2)/(2*x**(7/2)*
gamma(11/4))
```

3.1141 $\int \frac{(-1+x^3)^{2/3}}{x^3}\,dx$

Optimal. Leaf size=93

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}-\frac{\left(x^3-1\right)^{2/3}}{2x^2}+\frac{1}{6}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.01, antiderivative size = 62, normalized size of antiderivative = 0.67, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {277, 239}

$$-\frac{1}{2}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\left(x^3-1\right)^{2/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)^(2/3)/x^3,x]
```

```
[Out] -1/2*(-1 + x^3)^(2/3)/x^2 + ArcTan[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]]/Sq
rt[3] - Log[-x + (-1 + x^3)^(1/3)]/2
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-1+x^3\right)^{2/3}}{x^3}\,dx &= -\frac{\left(-1+x^3\right)^{2/3}}{2x^2}+\int \frac{1}{\sqrt[3]{-1+x^3}}\,dx\\ &= -\frac{\left(-1+x^3\right)^{2/3}}{2x^2}+\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(-x+\sqrt[3]{-1+x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.43

$$-\frac{\left(x^3-1\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};x^3\right)}{2x^2\left(1-x^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)^(2/3)/x^3,x]
```

```
[Out] -1/2*((-1 + x^3)^(2/3)*Hypergeometric2F1[-2/3, -2/3, 1/3, x^3])/(x^2*(1 - x
^3)^(2/3))
```

IntegrateAlgebraic [A] time = 0.15, size = 93, normalized size = 1.00

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}-\frac{\left(x^3-1\right)^{2/3}}{2x^2}+\frac{1}{6}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)^(2/3)/x^3,x]
```

```
[Out] -1/2*(-1 + x^3)^(2/3)/x^2 + ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x^3)^(1/3))]/Sq
rt[3] - Log[-x + (-1 + x^3)^(1/3)]/3 + Log[x^2 + x*(-1 + x^3)^(1/3) + (-1 +
 x^3)^(2/3)]/6
```

fricas [A] time = 0.75, size = 105, normalized size = 1.13

$$\frac{2\sqrt{3}x^2\arctan\left(-\frac{25382\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}x^2-13720\sqrt{3}\left(x^3-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(5831x^3-7200\right)}{58653x^3-8000}\right)-x^2\log\left(-3\left(x^3-1\right)^{\frac{1}{3}}x^2+3\left(x^3-1\right)^{\frac{2}{3}}x+1\right)-3\left(x^3-1\right)^{\frac{2}{3}}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*x^2*arctan(-(25382*sqrt(3)*(x^3 - 1)^(1/3)*x^2 - 13720*sqrt(
3)*(x^3 - 1)^(2/3)*x + sqrt(3)*(5831*x^3 - 7200))/(58653*x^3 - 8000)) - x^2
*log(-3*(x^3 - 1)^(1/3)*x^2 + 3*(x^3 - 1)^(2/3)*x + 1) - 3*(x^3 - 1)^(2/3))
/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^3-1\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate((x^3 - 1)^(2/3)/x^3, x)
```

maple [C] time = 0.35, size = 43, normalized size = 0.46

$$-\frac{\left(x^3-1\right)^{\frac{2}{3}}}{2x^2}+\frac{\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{1}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^3\right)}{\mathrm{signum}\left(x^3-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)/x^3,x)
```

```
[Out] -1/2*(x^3-1)^(2/3)/x^2+1/signum(x^3-1)^(1/3)*(-signum(x^3-1))^(1/3)*x*hyper
geom([1/3,1/3],[4/3],x^3)
```

maxima [A] time = 0.57, size = 81, normalized size = 0.87

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3-1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\left(x^3-1\right)^{\frac{2}{3}}}{2x^2}+\frac{1}{6}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3-1\right)^{\frac{2}{3}}}{x^2}+1\right)-\frac{1}{3}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3)/x + 1)) - 1/2*(x^3 - 1)^
(2/3)/x^2 + 1/6*log((x^3 - 1)^(1/3)/x + (x^3 - 1)^(2/3)/x^2 + 1) - 1/3*log(
(x^3 - 1)^(1/3)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3-1\right)^{2/3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)^(2/3)/x^3,x)
```

```
[Out] int((x^3 - 1)^(2/3)/x^3, x)
```

sympy [C] time = 0.91, size = 37, normalized size = 0.40

$$\frac{e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|x^3\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)/x**3,x)
```

```
[Out] exp(2*I*pi/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), x**3)/(3*x**2*gamma(1
/3))
```

3.1142 $\int \frac{2+x^2}{(-2+2x+x^2)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=93

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{x^3+1}}{x^2-x+1}\right)}{3^{3/4}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{x^3+1}}{x^2-x+1}\right)}{3^{3/4}}$$

Rubi [C] time = 0.94, antiderivative size = 386, normalized size of antiderivative = 4.15, number of steps used = 13, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6728, 218, 2135, 2140, 206, 203}

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}-\frac{\sqrt{2\left(7+4\sqrt{3}\right)}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(2 + x^2)/((-2 + 2*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/3^(3/4)) -
(Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/3^(3/4) - (
Sqrt[2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1
 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/
(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1
- x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqr
t[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqr
t[1 + x^3]) - (Sqrt[2*(7 + 4*Sqrt[3])]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt
[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*S
qrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2135

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-
6*a*d^3)/(c*(b*c^3 - 28*a*d^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x^2}{\left(-2+2x+x^2\right)\sqrt{1+x^3}}\,dx &= \int \left(\frac{1}{\sqrt{1+x^3}}+\frac{2(2-x)}{\left(-2+2x+x^2\right)\sqrt{1+x^3}}\right)dx \\
&= 2\int \frac{2-x}{\left(-2+2x+x^2\right)\sqrt{1+x^3}}\,dx+\int \frac{1}{\sqrt{1+x^3}}\,dx \\
&= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+2\int\Bigg(\frac{}{\left(2-2\sqrt{\right.}} \\
&= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}-\left(2\left(1-\sqrt{3}\right)\right) \\
&= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}-\frac{1}{288}\left(-3+\sqrt{3}\right. \\
&= -\frac{\sqrt{2}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+\frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{}}{\sqrt[4]{}} \\
&= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}}-\frac{\sqrt{2}\,(1+x)\sqrt{\frac{1-x+x}{\left(1+\sqrt{3}+\right.}}}{3^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.79, size = 274, normalized size = 2.95

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left((1+i)\left(\sqrt{3}+3i\right)\sqrt{x^2-x+1}\left(i\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)+\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)\right)+\frac{3\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(\left(\sqrt{3}+i\right)x-2i\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}\right)}{3\left(\sqrt{3}+i\right)\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + x^2)/((-2 + 2*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((3*Sqrt[((-1)^(1/3) - (-1)^(2/3)*x)/(1 +
 (-1)^(1/3))]*(-2*I + (I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3
)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/
3))] + (1 + I)*(3*I + Sqrt[3])*Sqrt[1 - x + x^2]*(I*EllipticPi[(2*Sqrt[3])/
(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))
]], (-1)^(1/3)] + EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[
Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])))/(3*(I + Sqrt[3])
*Sqrt[1 + x^3])
```

IntegrateAlgebraic [A] time = 1.51, size = 93, normalized size = 1.00

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{x^3+1}}{x^2-x+1}\right)}{3^{3/4}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{x^3+1}}{x^2-x+1}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x^2)/((-2 + 2*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[1 + x^3])/(1 - x + x^2)])/3^(3/
4)) - (Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[1 + x^3])/(1 - x + x^2)])
/3^(3/4)
```

fricas [B] time = 0.49, size = 232, normalized size = 2.49

$$-\frac{1}{27}\cdot 27^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{x^3+1}\left(27^{\frac{3}{4}}\sqrt{2}+9\cdot 27^{\frac{1}{4}}\sqrt{2}\right)}{18\left(x^2-x+1\right)}\right)+\frac{1}{108}\cdot 27^{\frac{3}{4}}\sqrt{2}\log\left(\frac{2\left(9x^4-18x^3+54x^2+36\sqrt{3}\left(x^3+1\right)+\sqrt{x^3+1}\left(27^{\frac{3}{4}}\sqrt{2}\left(x^2-4x-2\right)-9\cdot 27^{\frac{1}{4}}\sqrt{2}\left(x^2+2\right)\right)+36x+36\right)}{x^4+4x^3-8x+4}\right)-\frac{1}{108}\cdot 27^{\frac{3}{4}}\sqrt{2}\log\left(\frac{2\left(9x^4-18x^3+54x^2+36\sqrt{3}\left(x^3+1\right)-\sqrt{x^3+1}\left(27^{\frac{3}{4}}\sqrt{2}\left(x^2-4x-2\right)-9\cdot 27^{\frac{1}{4}}\sqrt{2}\left(x^2+2\right)\right)+36x+36\right)}{x^4+4x^3-8x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(x^2+2*x-2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/27*27^(3/4)*sqrt(2)*arctan(1/18*sqrt(x^3 + 1)*(27^(3/4)*sqrt(2) + 9*27^(
1/4)*sqrt(2))/(x^2 - x + 1)) + 1/108*27^(3/4)*sqrt(2)*log(2*(9*x^4 - 18*x^3
 + 54*x^2 + 36*sqrt(3)*(x^3 + 1) + sqrt(x^3 + 1)*(27^(3/4)*sqrt(2)*(x^2 - 4
*x - 2) - 9*27^(1/4)*sqrt(2)*(x^2 + 2)) + 36*x + 36)/(x^4 + 4*x^3 - 8*x + 4
)) - 1/108*27^(3/4)*sqrt(2)*log(2*(9*x^4 - 18*x^3 + 54*x^2 + 36*sqrt(3)*(x^
3 + 1) - sqrt(x^3 + 1)*(27^(3/4)*sqrt(2)*(x^2 - 4*x - 2) - 9*27^(1/4)*sqrt(
2)*(x^2 + 2)) + 36*x + 36)/(x^4 + 4*x^3 - 8*x + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2}{\sqrt{x^3+1}\left(x^2+2x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(x^2+2*x-2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 2)/(sqrt(x^3 + 1)*(x^2 + 2*x - 2)), x)
```

maple [C] time = 0.33, size = 1500, normalized size = 16.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)/(x^2+2*x-2)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
```

```
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1
/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2
*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+
1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*Elli
pticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),
((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(1/(3/2-1/2*I*
3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2
-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^
(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)
/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*
I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-I*(1/
(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*
x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3
/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(
1/2))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3
*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(
1/2))+I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*
I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3
^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))
^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*
3^(1/2)))^(1/2))*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^
(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*
3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+
1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(
3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*
3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-
3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I
*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*EllipticP
i(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+I*(1/(3/2-1/2*I*3^(1/2))*x+1/(
3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2)
)-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(
-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)
*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1
/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-(1/(3/2-1/2*
I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3
/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*
3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2
*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2}{\sqrt{x^3+1}\,\left(x^2+2\,x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(x^2+2*x-2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 2)/(sqrt(x^3 + 1)*(x^2 + 2*x - 2)), x)
```

mupad [B] time = 0.20, size = 509, normalized size = 5.47

$$\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}+\frac{\left(2\sqrt{3}-6\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{3};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\frac{\left(2\sqrt{3}+6\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(-\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{3};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 2)/((x^3 + 1)^(1/2)*(2*x + x^2 - 2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2))^(1/2) + ((2*3^(1/2) - 6)*((3^(1/2)*1i)/2 + 3/2)*((x +
(3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*e
llipticPi((3^(1/2)*((3^(1/2)*1i)/2 + 3/2))/3, asin(((x + 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(x^3 -
 x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 -
1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - ((2*3^(1/2) + 6)*((3^(1/2)*1i)/2 + 3/
2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*ellipticPi(-(3^(1/2)*((3^(1/2)*1i)/2 + 3/2))/3, asin(((x + 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))
/(3*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x^2+2x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)/(x**2+2*x-2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x**2 + 2)/(sqrt((x + 1)*(x**2 - x + 1))*(x**2 + 2*x - 2)), x)
```

3.1143 $\int \frac{(1+x^3)^{2/3}}{x^3} dx$

Optimal. Leaf size=93

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}-\frac{(x^3+1)^{2/3}}{2x^2}+\frac{1}{6}\log\left(\sqrt[3]{x^3+1}x+(x^3+1)^{2/3}+x^2\right)$$

Rubi [A] time = 0.01, antiderivative size = 62, normalized size of antiderivative = 0.67, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {277, 239}

$$-\frac{1}{2}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{(x^3+1)^{2/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)^(2/3)/x^3,x]
```

```
[Out] -1/2*(1 + x^3)^(2/3)/x^2 + ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]]/Sqrt
[3] - Log[-x + (1 + x^3)^(1/3)]/2
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x^3)^{2/3}}{x^3} dx &= -\frac{(1+x^3)^{2/3}}{2x^2}+\int \frac{1}{\sqrt[3]{1+x^3}} dx\\ &= -\frac{(1+x^3)^{2/3}}{2x^2}+\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(-x+\sqrt[3]{1+x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.24

$$-\frac{{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-x^3\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)^(2/3)/x^3,x]
```

`[Out] -1/2*Hypergeometric2F1[-2/3, -2/3, 1/3, -x^3]/x^2`

IntegrateAlgebraic [A] time = 0.13, size = 93, normalized size = 1.00

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}-\frac{\left(x^3+1\right)^{2/3}}{2x^2}+\frac{1}{6}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)^(2/3)/x^3,x]
```

```
[Out] -1/2*(1 + x^3)^(2/3)/x^2 + ArcTan[(Sqrt[3]*x)/(x + 2*(1 + x^3)^(1/3))]/Sqrt
[3] - Log[-x + (1 + x^3)^(1/3)]/3 + Log[x^2 + x*(1 + x^3)^(1/3) + (1 + x^3)
^(2/3)]/6
```

fricas [A] time = 0.73, size = 105, normalized size = 1.13

$$\frac{2\sqrt{3}x^2\arctan\left(-\frac{25382\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}x^2-13720\sqrt{3}\left(x^3+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(5831x^3+7200\right)}{58653x^3+8000}\right)-x^2\log\left(3\left(x^3+1\right)^{\frac{1}{3}}x^2-3\left(x^3+1\right)^{\frac{2}{3}}x+1\right)-3\left(x^3+1\right)^{\frac{2}{3}}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*x^2*arctan(-(25382*sqrt(3)*(x^3 + 1)^(1/3)*x^2 - 13720*sqrt(
3)*(x^3 + 1)^(2/3)*x + sqrt(3)*(5831*x^3 + 7200))/(58653*x^3 + 8000)) - x^2
*log(3*(x^3 + 1)^(1/3)*x^2 - 3*(x^3 + 1)^(2/3)*x + 1) - 3*(x^3 + 1)^(2/3))/
x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^3+1\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)^(2/3)/x^3, x)
```

maple [C] time = 0.33, size = 27, normalized size = 0.29

$$-\frac{\left(x^3+1\right)^{\frac{2}{3}}}{2x^2}+x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)/x^3,x)
```

```
[Out] -1/2*(x^3+1)^(2/3)/x^2+x*hypergeom([1/3,1/3],[4/3],-x^3)
```

maxima [A] time = 1.52, size = 81, normalized size = 0.87

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\left(x^3+1\right)^{\frac{2}{3}}}{2x^2}+\frac{1}{6}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)-\frac{1}{3}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)/x^3,x, algorithm="maxima")
```

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3)/x + 1)) - 1/2*(x^3 + 1)^(2/3)/x^2 + 1/6*log((x^3 + 1)^(1/3)/x + (x^3 + 1)^(2/3)/x^2 + 1) - 1/3*log((x^3 + 1)^(1/3)/x - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3+1\right)^{2/3}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 + 1)^(2/3)/x^3,x)

[Out] int((x^3 + 1)^(2/3)/x^3, x)

sympy [C] time = 0.86, size = 34, normalized size = 0.37

$$\frac{\Gamma\left(-\frac{2}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, -\frac{2}{3} \\ \frac{1}{3} \end{matrix}\middle| {x^{3} e^{i \pi}} \right)}{3 x^{2} \Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+1)**(2/3)/x**3,x)

[Out] gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), x**3*exp_polar(I*pi))/(3*x**2*gamma(1/3))

3.1144 $\int \frac{2abx+(-3a+b)x^2}{\sqrt[4]{x^2(-a+x)(-b+x)}\left(a^3-3a^2x+(3a-bd)x^2+(-1+d)x^3\right)}\,dx$

Optimal. Leaf size=93

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^3(-a-b)+abx^2+x^4}}{a-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^3(-a-b)+abx^2+x^4}}{a-x}\right)}{d^{3/4}}$$

Rubi [F] time = 10.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2abx+(-3a+b)x^2}{\sqrt[4]{x^2(-a+x)(-b+x)}\left(a^3-3a^2x+(3a-bd)x^2+(-1+d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2*a*b*x + (-3*a + b)*x^2)/((x^2*(-a + x)*(-b + x))^(1/4)*(a^3 - 3*a^2*
x + (3*a - b*d)*x^2 + (-1 + d)*x^3)),x]
```

```
[Out] (4*a*b*Sqrt[x]*(-a + x)^(1/4)*(-b + x)^(1/4)*Defer[Subst][Defer[Int][x^2/((
-a + x^2)^(1/4)*(-b + x^2)^(1/4)*(a^3 - 3*a^2*x^2 + 3*a*(1 - (b*d)/(3*a))*x
^4 - (1 - d)*x^6)), x], x, Sqrt[x]])/((a - x)*(b - x)*x^2)^(1/4) - (2*(3*a
- b)*Sqrt[x]*(-a + x)^(1/4)*(-b + x)^(1/4)*Defer[Subst][Defer[Int][x^4/((-a
 + x^2)^(1/4)*(-b + x^2)^(1/4)*(a^3 - 3*a^2*x^2 + 3*a*(1 - (b*d)/(3*a))*x^4
 - (1 - d)*x^6)), x], x, Sqrt[x]])/((a - x)*(b - x)*x^2)^(1/4)
```

Rubi steps

$$\int \frac{2abx+(-3a+b)x^2}{\sqrt[4]{x^2(-a+x)(-b+x)}\left(a^3-3a^2x+(3a-bd)x^2+(-1+d)x^3\right)}\,dx = \int \frac{x(2ab+(-3a+}{\sqrt[4]{x^2(-a+x)(-b+x)}\left(a^3-3a^2x+(3\right.}$$

$$= \frac{\left(\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\int \frac{\sqrt{}}{\sqrt[4]{-a+x}\sqrt[4]{-b+x}(}}{\sqrt[4]{x^2(-a+x)(-b}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{-a-}}\right.}{\sqrt[4]{x^2(-}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\text{Subst}\left(\int\left(\frac{}{\sqrt[4]{-}}\right.\right.}{}$$

$$= \frac{\left(4ab\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{}}\right.}{\sqrt[4]{x^2(-}}$$

Mathematica [F] time = 4.03, size = 0, normalized size = 0.00

$$\int \frac{2abx+(-3a+b)x^2}{\sqrt[4]{x^2(-a+x)(-b+x)}\left(a^3-3a^2x+(3a-bd)x^2+(-1+d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2*a*b*x + (-3*a + b)*x^2)/((x^2*(-a + x)*(-b + x))^(1/4)*(a^3 -
3*a^2*x + (3*a - b*d)*x^2 + (-1 + d)*x^3)),x]
```

```
[Out] Integrate[(2*a*b*x + (-3*a + b)*x^2)/((x^2*(-a + x)*(-b + x))^(1/4)*(a^3 -
3*a^2*x + (3*a - b*d)*x^2 + (-1 + d)*x^3)), x]
```

IntegrateAlgebraic [A] time = 2.88, size = 93, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^3(-a-b)+abx^2+x^4}}{a-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^3(-a-b)+abx^2+x^4}}{a-x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*a*b*x + (-3*a + b)*x^2)/((x^2*(-a + x)*(-b + x))^(1/4
)*(a^3 - 3*a^2*x + (3*a - b*d)*x^2 + (-1 + d)*x^3)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/4))/(a - x)])/d^(3/4)
 + (2*ArcTanh[(d^(1/4)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/4))/(a - x)])/d^(3
/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b*x+(-3*a+b)*x^2)/(x^2*(-a+x)*(-b+x))^(1/4)/(a^3-3*a^2*x+(-b
*d+3*a)*x^2+(-1+d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,abx-(3\,a-b)x^2}{\left((a-x)(b-x)x^2\right)^{\frac{1}{4}}\left((d-1)x^3+a^3-3\,a^2x-(bd-3\,a)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b*x+(-3*a+b)*x^2)/(x^2*(-a+x)*(-b+x))^(1/4)/(a^3-3*a^2*x+(-b
*d+3*a)*x^2+(-1+d)*x^3),x, algorithm="giac")
```

```
[Out] integrate((2*a*b*x - (3*a - b)*x^2)/(((a - x)*(b - x)*x^2)^(1/4)*((d - 1)*x
^3 + a^3 - 3*a^2*x - (b*d - 3*a)*x^2)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{2abx+(-3a+b)\,x^2}{\left(x^2\,(-a+x)\,(-b+x)\right)^{\frac{1}{4}}\left(a^3-3a^2x+(-bd+3a)\,x^2+(-1+d)\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*b*x+(-3*a+b)*x^2)/(x^2*(-a+x)*(-b+x))^(1/4)/(a^3-3*a^2*x+(-b*d+3*a
)*x^2+(-1+d)*x^3),x)
```

```
[Out] int((2*a*b*x+(-3*a+b)*x^2)/(x^2*(-a+x)*(-b+x))^(1/4)/(a^3-3*a^2*x+(-b*d+3*a
)*x^2+(-1+d)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,abx-(3\,a-b)x^2}{\left((a-x)(b-x)x^2\right)^{\frac{1}{4}}\left((d-1)x^3+a^3-3\,a^2x-(bd-3\,a)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b*x+(-3*a+b)*x^2)/(x^2*(-a+x)*(-b+x))^(1/4)/(a^3-3*a^2*x+(-b
*d+3*a)*x^2+(-1+d)*x^3),x, algorithm="maxima")
```

```
[Out] integrate((2*a*b*x - (3*a - b)*x^2)/(((a - x)*(b - x)*x^2)^(1/4)*((d - 1)*x
^3 + a^3 - 3*a^2*x - (b*d - 3*a)*x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^2\,\left(3\,a-b\right)-2\,a\,b\,x}{{\left(x^2\,\left(a-x\right)\,\left(b-x\right)\right)}^{1/4}\,\left(x^2\,\left(3\,a-b\,d\right)-3\,a^2\,x+a^3+x^3\,\left(d-1\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(3*a - b) - 2*a*b*x)/((x^2*(a - x)*(b - x))^(1/4)*(x^2*(3*a - b*d
) - 3*a^2*x + a^3 + x^3*(d - 1))),x)
```

```
[Out] int(-(x^2*(3*a - b) - 2*a*b*x)/((x^2*(a - x)*(b - x))^(1/4)*(x^2*(3*a - b*d
) - 3*a^2*x + a^3 + x^3*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b*x+(-3*a+b)*x**2)/(x**2*(-a+x)*(-b+x))**(1/4)/(a**3-3*a**2*
x+(-b*d+3*a)*x**2+(-1+d)*x**3),x)
```

```
[Out] Timed out
```

3.1145 $\int \frac{\sqrt[4]{-1+x^4}}{x^5}\,dx$

Optimal. Leaf size=93

$$-\frac{\sqrt[4]{x^4-1}}{4x^4}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^4-1}}{\sqrt{x^4-1}-1}\right)}{8\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+1}\right)}{8\sqrt{2}}$$

Rubi [A] time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.56, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {266, 47, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{x^4-1}}{4x^4}-\frac{\log\left(\sqrt{x^4-1}-\sqrt{2}\sqrt[4]{x^4-1}+1\right)}{16\sqrt{2}}+\frac{\log\left(\sqrt{x^4-1}+\sqrt{2}\sqrt[4]{x^4-1}+1\right)}{16\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{x^4-1}\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt[4]{x^4-1}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)^(1/4)/x^5,x]
```

```
[Out] -1/4*(-1 + x^4)^(1/4)/x^4 - ArcTan[1 - Sqrt[2]*(-1 + x^4)^(1/4)]/(8*Sqrt[2]
) + ArcTan[1 + Sqrt[2]*(-1 + x^4)^(1/4)]/(8*Sqrt[2]) - Log[1 - Sqrt[2]*(-1
+ x^4)^(1/4) + Sqrt[-1 + x^4]]/(16*Sqrt[2]) + Log[1 + Sqrt[2]*(-1 + x^4)^(1
/4) + Sqrt[-1 + x^4]]/(16*Sqrt[2])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt[4]{-1+x^4}}{x^5}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt[4]{-1+x}}{x^2}\,dx, x, x^4\right)$$
$$= -\frac{\sqrt[4]{-1+x^4}}{4x^4} + \frac{1}{16}\,\text{Subst}\left(\int \frac{1}{(-1+x)^{3/4}x}\,dx, x, x^4\right)$$
$$= -\frac{\sqrt[4]{-1+x^4}}{4x^4} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt[4]{-1+x^4}\right)$$
$$= -\frac{\sqrt[4]{-1+x^4}}{4x^4} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt[4]{-1+x^4}\right) + \frac{1}{8}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt[4]{-1+x^4}\right)$$
$$= -\frac{\sqrt[4]{-1+x^4}}{4x^4} + \frac{1}{16}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt[4]{-1+x^4}\right) + \frac{1}{16}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\right.$$
$$= -\frac{\sqrt[4]{-1+x^4}}{4x^4} - \frac{\log\left(1-\sqrt{2}\sqrt[4]{-1+x^4}+\sqrt{-1+x^4}\right)}{16\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt[4]{-1+x^4}+\sqrt{-1+x^4}\right)}{16\sqrt{2}}$$
$$= -\frac{\sqrt[4]{-1+x^4}}{4x^4} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+x^4}\right)}{8\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+x^4}\right)}{8\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\sqrt[4]{-1\right.}{16}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.30

$$\frac{1}{5}\left(x^4-1\right)^{5/4}\,{}_2F_1\left(\frac{5}{4},2;\frac{9}{4};1-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)^(1/4)/x^5,x]
```

```
[Out] ((-1 + x^4)^(5/4)*Hypergeometric2F1[5/4, 2, 9/4, 1 - x^4])/5
```

IntegrateAlgebraic [A] time = 0.17, size = 98, normalized size = 1.05

$$-\frac{\sqrt[4]{x^4-1}}{4x^4}+\frac{\tan^{-1}\left(\frac{\frac{\sqrt{x^4-1}}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt[4]{x^4-1}}\right)}{8\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+1}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)^(1/4)/x^5,x]
```

```
[Out] -1/4*(-1 + x^4)^(1/4)/x^4 + ArcTan[(-(1/Sqrt[2]) + Sqrt[-1 + x^4]/Sqrt[2])/
(-1 + x^4)^(1/4)]/(8*Sqrt[2]) + ArcTanh[(Sqrt[2]*(-1 + x^4)^(1/4))/(1 + Sqr
t[-1 + x^4])]/(8*Sqrt[2])
```

fricas [B] time = 0.43, size = 180, normalized size = 1.94

$$-\frac{4\sqrt{2}x^4\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+\sqrt{x^4-1}+1}-\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}-1\right)+4\sqrt{2}x^4\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+4\sqrt{x^4-1}+4}-\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+1\right)-\sqrt{2}x^4\log\left(4\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+4\sqrt{x^4-1}+4\right)+\sqrt{2}x^4\log\left(-4\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+4\sqrt{x^4-1}+4\right)+8\left(x^4-1\right)^{\frac{1}{4}}}{32x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/4)/x^5,x, algorithm="fricas")
```

```
[Out] -1/32*(4*sqrt(2)*x^4*arctan(sqrt(2)*sqrt(sqrt(2)*(x^4 - 1)^(1/4) + sqrt(x^4
 - 1) + 1) - sqrt(2)*(x^4 - 1)^(1/4) - 1) + 4*sqrt(2)*x^4*arctan(1/2*sqrt(2
)*sqrt(-4*sqrt(2)*(x^4 - 1)^(1/4) + 4*sqrt(x^4 - 1) + 4) - sqrt(2)*(x^4 - 1
)^(1/4) + 1) - sqrt(2)*x^4*log(4*sqrt(2)*(x^4 - 1)^(1/4) + 4*sqrt(x^4 - 1)
+ 4) + sqrt(2)*x^4*log(-4*sqrt(2)*(x^4 - 1)^(1/4) + 4*sqrt(x^4 - 1) + 4) +
8*(x^4 - 1)^(1/4))/x^4
```

giac [A] time = 0.17, size = 114, normalized size = 1.23

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(x^4-1\right)^{\frac{1}{4}}\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(x^4-1\right)^{\frac{1}{4}}\right)\right)+\frac{1}{32}\sqrt{2}\log\left(\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+\sqrt{x^4-1}+1\right)-\frac{1}{32}\sqrt{2}\log\left(-\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+\sqrt{x^4-1}+1\right)-\frac{\left(x^4-1\right)^{\frac{1}{4}}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/4)/x^5,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^4 - 1)^(1/4))) + 1/16*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^4 - 1)^(1/4))) + 1/32*sqrt(2)*log(sq
rt(2)*(x^4 - 1)^(1/4) + sqrt(x^4 - 1) + 1) - 1/32*sqrt(2)*log(-sqrt(2)*(x^4
 - 1)^(1/4) + sqrt(x^4 - 1) + 1) - 1/4*(x^4 - 1)^(1/4)/x^4
```

maple [C] time = 0.33, size = 76, normalized size = 0.82

$$-\frac{\left(x^4-1\right)^{\frac{1}{4}}}{4x^4}+\frac{\left(-\mathrm{signum}\left(x^4-1\right)\right)^{\frac{3}{4}}\left(\frac{3\Gamma\left(\frac{3}{4}\right)x^4\,\mathrm{hypergeom}\left(\left[1,1,\frac{7}{4}\right],[2,2],x^4\right)}{4}+\left(-3\ln(2)+\frac{\pi}{2}+4\ln(x)+i\pi\right)\Gamma\left(\frac{3}{4}\right)\right)}{16\Gamma\left(\frac{3}{4}\right)\mathrm{signum}\left(x^4-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(1/4)/x^5,x)
```

```
[Out] -1/4*(x^4-1)^(1/4)/x^4+1/16/GAMMA(3/4)/signum(x^4-1)^(3/4)*(-signum(x^4-1))
^(3/4)*(3/4*GAMMA(3/4)*x^4*hypergeom([1,1,7/4],[2,2],x^4)+(-3*ln(2)+1/2*Pi+
4*ln(x)+I*Pi)*GAMMA(3/4))
```

maxima [A] time = 0.66, size = 114, normalized size = 1.23

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(x^4-1\right)^{\frac{1}{4}}\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(x^4-1\right)^{\frac{1}{4}}\right)\right)+\frac{1}{32}\sqrt{2}\log\left(\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+\sqrt{x^4-1}+1\right)-\frac{1}{32}\sqrt{2}\log\left(-\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+\sqrt{x^4-1}+1\right)-\frac{\left(x^4-1\right)^{\frac{1}{4}}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/4)/x^5,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^4 - 1)^(1/4))) + 1/16*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^4 - 1)^(1/4))) + 1/32*sqrt(2)*log(sq
rt(2)*(x^4 - 1)^(1/4) + sqrt(x^4 - 1) + 1) - 1/32*sqrt(2)*log(-sqrt(2)*(x^4
 - 1)^(1/4) + sqrt(x^4 - 1) + 1) - 1/4*(x^4 - 1)^(1/4)/x^4
```

mupad [B] time = 0.97, size = 57, normalized size = 0.61

$$-\frac{\left(x^4-1\right)^{1/4}}{4\,x^4}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(x^4-1\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{16}+\frac{1}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(x^4-1\right)^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{16}-\frac{1}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)^(1/4)/x^5,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*(x^4 - 1)^(1/4)*(1/2 - 1i/2))*(1/16 + 1i/16) + 2^(1/2)
*atan(2^(1/2)*(x^4 - 1)^(1/4)*(1/2 + 1i/2))*(1/16 - 1i/16) - (x^4 - 1)^(1/4
)/(4*x^4)
```

sympy [C] time = 0.98, size = 34, normalized size = 0.37

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^{2i\pi}}{x^4}\right)}{4x^3\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(1/4)/x**5,x)
```

```
[Out] -gamma(3/4)*hyper((-1/4, 3/4), (7/4,), exp_polar(2*I*pi)/x**4)/(4*x**3*gamm
a(7/4))
```

3.1146 $\int \frac{1+x^2}{\left(-1+x+x^2\right)\sqrt[3]{-x^2+x^4}}\,dx$

Optimal. Leaf size=93

$$-\log\left(\sqrt[3]{x^4-x^2}+x\right)+\frac{1}{2}\log\left(x^2-\sqrt[3]{x^4-x^2}\,x+\left(x^4-x^2\right)^{2/3}\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^4-x^2}-x}\right)$$

Rubi [C] time = 1.25, antiderivative size = 293, normalized size of antiderivative = 3.15, number of steps used = 23, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2056, 6728, 365, 364, 959, 466, 430, 429, 465, 511, 510}

$$\frac{3\sqrt[3]{1-x^2}\,x^2F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};\frac{4x^2}{\left(1-\sqrt{5}\right)^2},x^2\right)}{2\left(1-\sqrt{5}\right)\sqrt[3]{x^4-x^2}}+\frac{3\sqrt[3]{1-x^2}\,x^2F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};\frac{4x^2}{\left(1+\sqrt{5}\right)^2},x^2\right)}{2\left(1+\sqrt{5}\right)\sqrt[3]{x^4-x^2}}-\frac{3\sqrt[3]{1-x^2}\,xF_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};\frac{4x^2}{\left(1-\sqrt{5}\right)^2},x^2\right)}{\sqrt[3]{x^4-x^2}}-\frac{3\sqrt[3]{1-x^2}\,xF_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};\frac{4x^2}{\left(1+\sqrt{5}\right)^2},x^2\right)}{\sqrt[3]{x^4-x^2}}+\frac{3\sqrt[3]{1-x^2}\,x\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^2\right)}{\sqrt[3]{x^4-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^2)/((-1 + x + x^2)*(-x^2 + x^4)^(1/3)),x]
```

```
[Out] (-3*x*(1 - x^2)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6, (4*x^2)/(1 - Sqrt[5])^2, x
^2])/(-x^2 + x^4)^(1/3) - (3*x*(1 - x^2)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6, (
4*x^2)/(1 + Sqrt[5])^2, x^2])/(-x^2 + x^4)^(1/3) + (3*x^2*(1 - x^2)^(1/3)*A
ppellF1[2/3, 1, 1/3, 5/3, (4*x^2)/(1 - Sqrt[5])^2, x^2])/(2*(1 - Sqrt[5])*(
-x^2 + x^4)^(1/3)) + (3*x^2*(1 - x^2)^(1/3)*AppellF1[2/3, 1, 1/3, 5/3, (4*x
^2)/(1 + Sqrt[5])^2, x^2])/(2*(1 + Sqrt[5])*(-x^2 + x^4)^(1/3)) + (3*x*(1 -
 x^2)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, x^2])/(-x^2 + x^4)^(1/3)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
```

```
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1+x^2}{\left(-1+x+x^2\right)\sqrt[3]{-x^2+x^4}}\,dx = \frac{\left(x^{2/3}\sqrt[3]{-1+x^2}\right)\int \frac{1+x^2}{x^{2/3}\sqrt[3]{-1+x^2}\left(-1+x+x^2\right)}\,dx}{\sqrt[3]{-x^2+x^4}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{-1+x^2}\right)\int \left(\frac{1}{x^{2/3}\sqrt[3]{-1+x^2}}+\frac{2-x}{x^{2/3}\sqrt[3]{-1+x^2}\left(-1+x+x^2\right)}\right)dx}{\sqrt[3]{-x^2+x^4}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{-1+x^2}\right)\int \frac{1}{x^{2/3}\sqrt[3]{-1+x^2}}\,dx}{\sqrt[3]{-x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{-1+x^2}\right)\int \frac{2-x}{x^{2/3}\sqrt[3]{-1+x^2}\left(-1+x+x^2\right)}\,dx}{\sqrt[3]{-x^2+x^4}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{1-x^2}\right)\int \frac{1}{x^{2/3}\sqrt[3]{1-x^2}}\,dx}{\sqrt[3]{-x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{-1+x^2}\right)\int \left(\frac{-1+\sqrt{5}}{x^{2/3}\left(1-\sqrt{5}+2x\right)\sqrt[3]{-1+x^2}} + \frac{}{x^{2/3}(\cdots}\right.}{\sqrt[3]{-x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1-x^2}\;{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^2\right)}{\sqrt[3]{-x^2+x^4}} + \frac{\left(\left(-1-\sqrt{5}\right)x^{2/3}\sqrt[3]{-1+x^2}\right)\int \frac{1}{x^{2/3}\left(1+\sqrt{5}+2x\right)\sqrt[3]{-1+x\cdots}}}{\sqrt[3]{-x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1-x^2}\;{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^2\right)}{\sqrt[3]{-x^2+x^4}} - \frac{\left(2\left(-1-\sqrt{5}\right)x^{2/3}\sqrt[3]{-1+x^2}\right)\int \frac{\sqrt[3]{x}}{\left(\left(1+\sqrt{5}\right)^2-4x^2\right)\sqrt[3]{-1\cdots}}}{\sqrt[3]{-x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1-x^2}\;{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^2\right)}{\sqrt[3]{-x^2+x^4}} - \frac{\left(6\left(-1-\sqrt{5}\right)x^{2/3}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{}{\left(\left(1+\sqrt{5}\right)^2-\cdots\right.}\right.}{\sqrt[3]{-x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1-x^2}\;{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^2\right)}{\sqrt[3]{-x^2+x^4}} + \frac{\left(3\left(1-\sqrt{5}\right)\left(-1+\sqrt{5}\right)x^{2/3}\sqrt[3]{1-x^2}\right)\text{Subst}\left(\int \cdots\right.}{\sqrt[3]{-x^2+x^4}}$$

$$= -\frac{3x\sqrt[3]{1-x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};\frac{4x^2}{\left(1-\sqrt{5}\right)^2},x^2\right)}{\sqrt[3]{-x^2+x^4}} - \frac{3x\sqrt[3]{1-x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};\frac{4x^2}{\left(1+\sqrt{5}\right)^2},x^2\right)}{\sqrt[3]{-x^2+x^4}} \cdots$$

$$= -\frac{3x\sqrt[3]{1-x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};\frac{4x^2}{\left(1-\sqrt{5}\right)^2},x^2\right)}{\sqrt[3]{-x^2+x^4}} - \frac{3x\sqrt[3]{1-x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};\frac{4x^2}{\left(1+\sqrt{5}\right)^2},x^2\right)}{\sqrt[3]{-x^2+x^4}} \cdots$$

Mathematica [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{\left(-1+x+x^2\right)\sqrt[3]{-x^2+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^2)/((-1 + x + x^2)*(-x^2 + x^4)^(1/3)),x]
[Out] Integrate[(1 + x^2)/((-1 + x + x^2)*(-x^2 + x^4)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.35, size = 93, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^4-x^2}+x\right)+\frac{1}{2}\log\left(x^2-\sqrt[3]{x^4-x^2}\,x+\left(x^4-x^2\right)^{2/3}\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^4-x^2}-x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 + x + x^2)*(-x^2 + x^4)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(-x^2 + x^4)^(1/3))]) - Log[x + (-x^2
+ x^4)^(1/3)] + Log[x^2 - x*(-x^2 + x^4)^(1/3) + (-x^2 + x^4)^(2/3)]/2
```

fricas [A] time = 1.23, size = 130, normalized size = 1.40

$$-\sqrt{3}\arctan\left(-\frac{128537192\sqrt{3}\left(x^4-x^2\right)^{\frac{1}{3}}x+\sqrt{3}\left(1454911\,x^3-69864736\,x^2-1454911\,x\right)-14102102\sqrt{3}\left(x^4-x^2\right)^{\frac{2}{3}}}{226981\,x^3+171879616\,x^2-226981\,x}\right)-\frac{1}{2}\log\left(\frac{x^3+x^2+3\left(x^4-x^2\right)^{\frac{1}{3}}x-x+3\left(x^4-x^2\right)^{\frac{2}{3}}}{x^3+x^2-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x-1)/(x^4-x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(-(128537192*sqrt(3)*(x^4 - x^2)^(1/3)*x + sqrt(3)*(1454911*
x^3 - 69864736*x^2 - 1454911*x) - 14102102*sqrt(3)*(x^4 - x^2)^(2/3))/(2269
81*x^3 + 171879616*x^2 - 226981*x)) - 1/2*log((x^3 + x^2 + 3*(x^4 - x^2)^(1
/3)*x - x + 3*(x^4 - x^2)^(2/3))/(x^3 + x^2 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^4-x^2\right)^{\frac{1}{3}}\left(x^2+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x-1)/(x^4-x^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/((x^4 - x^2)^(1/3)*(x^2 + x - 1)), x)
```

maple [C] time = 2.80, size = 386, normalized size = 4.15

$$-\ln\left(\frac{-370\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)^2x^3+555\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)^2x^2+862\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)x^3+1356\left(x^4-x^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)+2346\left(x^4-x^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)x+370\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)^2x+2\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)x^2+744x^3+1980\left(x^4-x^2\right)^{\frac{2}{3}}-2712x\left(x^4-x^2\right)^{\frac{1}{3}}-862\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)x-248x^2-744x}{x\left(x^2+x-1\right)}\right)+\frac{\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)\ln\left(-\frac{62\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)^2x^3-93\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)^2x^2+432\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)x^3+678\left(x^4-x^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)-495\left(x^4-x^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)x-62\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)^2x-431\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)x^2+370x^3-2346\left(x^4-x^2\right)^{\frac{2}{3}}-1356x\left(x^4-x^2\right)^{\frac{1}{3}}-432\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)x+740x^2-370x}{x\left(x^2+x-1\right)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2+x-1)/(x^4-x^2)^(1/3),x)
```

```
[Out] -ln((-370*RootOf(_Z^2-2*_Z+4)^2*x^3+555*RootOf(_Z^2-2*_Z+4)^2*x^2+862*RootO
f(_Z^2-2*_Z+4)*x^3+1356*(x^4-x^2)^(2/3)*RootOf(_Z^2-2*_Z+4)+2346*(x^4-x^2)^
(1/3)*RootOf(_Z^2-2*_Z+4)*x+370*RootOf(_Z^2-2*_Z+4)^2*x+2*RootOf(_Z^2-2*_Z+
4)*x^2+744*x^3+1980*(x^4-x^2)^(2/3)-2712*x*(x^4-x^2)^(1/3)-862*RootOf(_Z^2-
2*_Z+4)*x-248*x^2-744*x)/x/(x^2+x-1))+1/2*RootOf(_Z^2-2*_Z+4)*ln(-(62*RootO
f(_Z^2-2*_Z+4)^2*x^3-93*RootOf(_Z^2-2*_Z+4)^2*x^2+432*RootOf(_Z^2-2*_Z+4)*x
^3+678*(x^4-x^2)^(2/3)*RootOf(_Z^2-2*_Z+4)-495*(x^4-x^2)^(1/3)*RootOf(_Z^2-
2*_Z+4)*x-62*RootOf(_Z^2-2*_Z+4)^2*x-431*RootOf(_Z^2-2*_Z+4)*x^2+370*x^3-23
46*(x^4-x^2)^(2/3)-1356*x*(x^4-x^2)^(1/3)-432*RootOf(_Z^2-2*_Z+4)*x+740*x^2
-370*x)/x/(x^2+x-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^4-x^2\right)^{\frac{1}{3}}\left(x^2+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x-1)/(x^4-x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/((x^4 - x^2)^(1/3)*(x^2 + x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+1}{\left(x^4-x^2\right)^{1/3}\left(x^2+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((x^4 - x^2)^(1/3)*(x + x^2 - 1)),x)
```

```
[Out] int((x^2 + 1)/((x^4 - x^2)^(1/3)*(x + x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt[3]{x^2\left(x-1\right)\left(x+1\right)}\left(x^2+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2+x-1)/(x**4-x**2)**(1/3),x)
```

```
[Out] Integral((x**2 + 1)/((x**2*(x - 1)*(x + 1))**(1/3)*(x**2 + x - 1)), x)
```

3.1147 $\int \frac{(-1+2x)(2-x+x^2)\sqrt{-2+x^2-2x^3+x^4}}{3-2x+2x^2}\,dx$

Optimal. Leaf size=93

$$\frac{1}{4}\sqrt{x^4-2x^3+x^2-2}\left(x^2-x+1\right)-\frac{7}{8}\log\left(x^2+\sqrt{x^4-2x^3+x^2-2}-x\right)-\frac{1}{4}\tanh^{-1}\left(2x^2+2\sqrt{x^4-2x^3+x^2-\right.$$

Rubi [F] time = 1.83, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+2x)\left(2-x+x^2\right)\sqrt{-2+x^2-2x^3+x^4}}{3-2x+2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + 2*x)*(2 - x + x^2)*Sqrt[-2 + x^2 - 2*x^3 + x^4])/(3 - 2*x + 2*x^
2),x]
```

```
[Out] -1/4*((1 - x)*x*Sqrt[-2 + x^2 - 2*x^3 + x^4]) + ArcTanh[((1 - x)*x)/Sqrt[-2
 + x^2 - 2*x^3 + x^4]]/2 + Defer[Int][Sqrt[-2 + x^2 - 2*x^3 + x^4]/(-2 - (2
*I)*Sqrt[5] + 4*x), x] + Defer[Int][Sqrt[-2 + x^2 - 2*x^3 + x^4]/(-2 + (2*I
)*Sqrt[5] + 4*x), x]
```

Rubi steps

$$\int \frac{(-1+2x)\left(2-x+x^2\right)\sqrt{-2+x^2-2x^3+x^4}}{3-2x+2x^2}\,dx = \int \left(-\frac{1}{2}\sqrt{-2+x^2-2x^3+x^4}+x\sqrt{-2+x^2-2x^3+x^4}-\right.$$

$$= -\left(\frac{1}{2}\int \sqrt{-2+x^2-2x^3+x^4}\,dx\right)-\frac{1}{2}\int \frac{(1-2x)\sqrt{-2+}}{3-2x}$$

$$= -\left(\frac{1}{2}\int \left(-\frac{2\sqrt{-2+x^2-2x^3+x^4}}{-2-2i\sqrt{5}+4x}-\frac{2\sqrt{-2+x^2-2x^3+}}{-2+2i\sqrt{5}+4x}\right.\right.$$

$$= \frac{1}{12}(1-2x)\sqrt{-2+x^2-2x^3+x^4}-\frac{1}{6}\text{Subst}\left(\int \frac{-\frac{31}{8}}{\sqrt{-\frac{31}{16}-}}\right.$$

$$= \frac{1}{12}(1-2x)\sqrt{-2+x^2-2x^3+x^4}+\frac{1}{24}\text{Subst}\left(\int \frac{-\frac{1}{2}-2}{\sqrt{-\frac{31}{16}-}}\right.$$

$$= -\frac{(1-2x)\left(\sqrt{2}-x+x^2\right)}{24\sqrt{-2+x^2-2x^3+x^4}}-\frac{1}{4}(1-x)x\sqrt{-2+x^2-2x^3+}$$

$$= -\frac{(1-2x)\left(\sqrt{2}-x+x^2\right)}{24\sqrt{-2+x^2-2x^3+x^4}}-\frac{1}{4}(1-x)x\sqrt{-2+x^2-2x^3+}$$

$$= -\frac{1}{4}(1-x)x\sqrt{-2+x^2-2x^3+x^4}-\frac{1}{2}\tanh^{-1}\left(\frac{(-1}{\sqrt{-2+x^2}}\right.$$

Mathematica [A] time = 0.18, size = 88, normalized size = 0.95

$$\frac{1}{8}\left(2\sqrt{x^4-2x^3+x^2-2}\left(x^2-x+1\right)-7\tanh^{-1}\left(\frac{(x-1)x}{\sqrt{x^4-2x^3+x^2-2}}\right)-\tanh^{-1}\left(\frac{-3x^2+3x-4}{\sqrt{x^4-2x^3+x^2-2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + 2*x)*(2 - x + x^2)*Sqrt[-2 + x^2 - 2*x^3 + x^4])/(3 - 2*x
+ 2*x^2),x]
```

```
[Out] (2*(1 - x + x^2)*Sqrt[-2 + x^2 - 2*x^3 + x^4] - 7*ArcTanh[((-1 + x)*x)/Sqrt
[-2 + x^2 - 2*x^3 + x^4]] - ArcTanh[(-4 + 3*x - 3*x^2)/Sqrt[-2 + x^2 - 2*x^
3 + x^4]])/8
```

IntegrateAlgebraic [A] time = 0.27, size = 93, normalized size = 1.00

$$\frac{1}{4}\sqrt{x^4-2x^3+x^2-2}\left(x^2-x+1\right)-\frac{7}{8}\log\left(x^2+\sqrt{x^4-2x^3+x^2-2}-x\right)-\frac{1}{4}\tanh^{-1}\left(2x^2+2\sqrt{x^4-2x^3+x^2-2}-2x+3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + 2*x)*(2 - x + x^2)*Sqrt[-2 + x^2 - 2*x^3 + x^4])/
(3 - 2*x + 2*x^2),x]
```

```
[Out] ((1 - x + x^2)*Sqrt[-2 + x^2 - 2*x^3 + x^4])/4 - ArcTanh[3 - 2*x + 2*x^2 +
2*Sqrt[-2 + x^2 - 2*x^3 + x^4]]/4 - (7*Log[-x + x^2 + Sqrt[-2 + x^2 - 2*x^3
 + x^4]])/8
```

fricas [A] time = 0.49, size = 92, normalized size = 0.99

$$\frac{1}{4}\sqrt{x^4-2x^3+x^2-2}\left(x^2-x+1\right)+\frac{7}{8}\log\left(-x^2+x+\sqrt{x^4-2x^3+x^2-2}\right)+\frac{1}{8}\log\left(\frac{3x^2-3x+\sqrt{x^4-2x^3+x^2-2}+4}{2x^2-2x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*(x^2-x+2)*(x^4-2*x^3+x^2-2)^(1/2)/(2*x^2-2*x+3),x, algor
ithm="fricas")
```

```
[Out] 1/4*sqrt(x^4 - 2*x^3 + x^2 - 2)*(x^2 - x + 1) + 7/8*log(-x^2 + x + sqrt(x^4
 - 2*x^3 + x^2 - 2)) + 1/8*log((3*x^2 - 3*x + sqrt(x^4 - 2*x^3 + x^2 - 2) +
 4)/(2*x^2 - 2*x + 3))
```

giac [A] time = 0.35, size = 100, normalized size = 1.08

$$\frac{1}{4}\sqrt{\left(x^2-x\right)^2-2}\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2-x-\sqrt{\left(x^2-x\right)^2-2}+2\right)-\frac{1}{8}\log\left(x^2-x-\sqrt{\left(x^2-x\right)^2-2}+1\right)+\frac{7}{8}\log\left(\left|-x^2+x+\sqrt{\left(x^2-x\right)^2-2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*(x^2-x+2)*(x^4-2*x^3+x^2-2)^(1/2)/(2*x^2-2*x+3),x, algor
ithm="giac")
```

```
[Out] 1/4*sqrt((x^2 - x)^2 - 2)*(x^2 - x + 1) + 1/8*log(x^2 - x - sqrt((x^2 - x)^
2 - 2) + 2) - 1/8*log(x^2 - x - sqrt((x^2 - x)^2 - 2) + 1) + 7/8*log(abs(-x
^2 + x + sqrt((x^2 - x)^2 - 2)))
```

maple [C] time = 1.41, size = 3475, normalized size = 37.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x)*(x^2-x+2)*(x^4-2*x^3+x^2-2)^(1/2)/(2*x^2-2*x+3),x)
```

```
[Out] 1/4*x^2*(x^4-2*x^3+x^2-2)^(1/2)-1/4*x*(x^4-2*x^3+x^2-2)^(1/2)+1/4*(x^4-2*x^
3+x^2-2)^(1/2)+7/4*(-1/2*(1+4*2^(1/2))^(1/2)-1/2*I*(-1+4*2^(1/2))^(1/2))*((
1/2*I*(-1+4*2^(1/2))^(1/2)-1/2*(1+4*2^(1/2))^(1/2))*(x-1/2+1/2*(1+4*2^(1/2)
)^(1/2))/(1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))/(x-1/2-1/2*(1
+4*2^(1/2))^(1/2)))^(1/2)*(x-1/2-1/2*(1+4*2^(1/2))^(1/2))^2*((1+4*2^(1/2))^
(1/2)*(x-1/2+1/2*I*(-1+4*2^(1/2))^(1/2))/(-1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(
1+4*2^(1/2))^(1/2))/(x-1/2-1/2*(1+4*2^(1/2))^(1/2)))^(1/2)*((1+4*2^(1/2))^(
1/2)*(x-1/2-1/2*I*(-1+4*2^(1/2))^(1/2))/(1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+
4*2^(1/2))^(1/2))/(x-1/2-1/2*(1+4*2^(1/2))^(1/2)))^(1/2)/(1/2*I*(-1+4*2^(1/
2))^(1/2)-1/2*(1+4*2^(1/2))^(1/2))/(1+4*2^(1/2))^(1/2)/((x-1/2+1/2*(1+4*2^(
1/2))^(1/2))*(x-1/2-1/2*(1+4*2^(1/2))^(1/2))*(x-1/2+1/2*I*(-1+4*2^(1/2))^(1
/2))*(x-1/2-1/2*I*(-1+4*2^(1/2))^(1/2)))^(1/2)*EllipticF(((1/2*I*(-1+4*2^(1
/2))^(1/2)-1/2*(1+4*2^(1/2))^(1/2))*(x-1/2+1/2*(1+4*2^(1/2))^(1/2))/(1/2*I*
(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))/(x-1/2-1/2*(1+4*2^(1/2))^(1/2
)))^(1/2),((1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))*(-1/2*(1+4*
2^(1/2))^(1/2)-1/2*I*(-1+4*2^(1/2))^(1/2))/(1/2*I*(-1+4*2^(1/2))^(1/2)-1/2*
(1+4*2^(1/2))^(1/2))/(-1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2)))
^(1/2))-7/2*(-1/2*(1+4*2^(1/2))^(1/2)-1/2*I*(-1+4*2^(1/2))^(1/2))*((1/2*I*(
-1+4*2^(1/2))^(1/2)-1/2*(1+4*2^(1/2))^(1/2))*(x-1/2+1/2*(1+4*2^(1/2))^(1/2)
)/(1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))/(x-1/2-1/2*(1+4*2^(1
/2))^(1/2)))^(1/2)*(x-1/2-1/2*(1+4*2^(1/2))^(1/2))^2*((1+4*2^(1/2))^(1/2)*(
x-1/2+1/2*I*(-1+4*2^(1/2))^(1/2))/(-1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(
1/2))^(1/2))/(x-1/2-1/2*(1+4*2^(1/2))^(1/2)))^(1/2)*((1+4*2^(1/2))^(1/2)*(x
-1/2-1/2*I*(-1+4*2^(1/2))^(1/2))/(1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/
2))^(1/2))/(x-1/2-1/2*(1+4*2^(1/2))^(1/2)))^(1/2)/(1/2*I*(-1+4*2^(1/2))^(1/
2)-1/2*(1+4*2^(1/2))^(1/2))/(1+4*2^(1/2))^(1/2)/((x-1/2+1/2*(1+4*2^(1/2))^(
1/2))*(x-1/2-1/2*(1+4*2^(1/2))^(1/2))*(x-1/2+1/2*I*(-1+4*2^(1/2))^(1/2))*(x
-1/2-1/2*I*(-1+4*2^(1/2))^(1/2)))^(1/2)*((1/2+1/2*(1+4*2^(1/2))^(1/2))*Elli
pticF(((1/2*I*(-1+4*2^(1/2))^(1/2)-1/2*(1+4*2^(1/2))^(1/2))*(x-1/2+1/2*(1+4
*2^(1/2))^(1/2))/(1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))/(x-1/
2-1/2*(1+4*2^(1/2))^(1/2)))^(1/2),((1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(
1/2))^(1/2))*(-1/2*(1+4*2^(1/2))^(1/2)-1/2*I*(-1+4*2^(1/2))^(1/2))/(1/2*I*(
-1+4*2^(1/2))^(1/2)-1/2*(1+4*2^(1/2))^(1/2))/(-1/2*I*(-1+4*2^(1/2))^(1/2)+1
/2*(1+4*2^(1/2))^(1/2)))^(1/2))-(1+4*2^(1/2))^(1/2)*EllipticPi(((1/2*I*(-1+
4*2^(1/2))^(1/2)-1/2*(1+4*2^(1/2))^(1/2))*(x-1/2+1/2*(1+4*2^(1/2))^(1/2))/(
1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))/(x-1/2-1/2*(1+4*2^(1/2)
)^(1/2)))^(1/2),(1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))/(1/2*I
*(-1+4*2^(1/2))^(1/2)-1/2*(1+4*2^(1/2))^(1/2)),((1/2*I*(-1+4*2^(1/2))^(1/2)
+1/2*(1+4*2^(1/2))^(1/2))*(-1/2*(1+4*2^(1/2))^(1/2)-1/2*I*(-1+4*2^(1/2))^(1
/2))/(1/2*I*(-1+4*2^(1/2))^(1/2)-1/2*(1+4*2^(1/2))^(1/2))/(-1/2*I*(-1+4*2^(
1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2)))^(1/2)))+1/8*(-1/2*(1+4*2^(1/2))^(1/2)
-1/2*I*(-1+4*2^(1/2))^(1/2))*((1/2*I*(-1+4*2^(1/2))^(1/2)-1/2*(1+4*2^(1/2))
^(1/2))*(x-1/2+1/2*(1+4*2^(1/2))^(1/2))/(1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+
4*2^(1/2))^(1/2))/(x-1/2-1/2*(1+4*2^(1/2))^(1/2)))^(1/2)*(x-1/2-1/2*(1+4*2^
(1/2))^(1/2))^2*((1+4*2^(1/2))^(1/2)*(x-1/2+1/2*I*(-1+4*2^(1/2))^(1/2))/(-1
/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))/(x-1/2-1/2*(1+4*2^(1/2))
^(1/2)))^(1/2)*((1+4*2^(1/2))^(1/2)*(x-1/2-1/2*I*(-1+4*2^(1/2))^(1/2))/(1/2
*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))/(x-1/2-1/2*(1+4*2^(1/2))^(
1/2)))^(1/2)/(1/2*I*(-1+4*2^(1/2))^(1/2)-1/2*(1+4*2^(1/2))^(1/2))/(1+4*2^(1
/2))^(1/2)/((x-1/2+1/2*(1+4*2^(1/2))^(1/2))*(x-1/2-1/2*(1+4*2^(1/2))^(1/2))
*(x-1/2+1/2*I*(-1+4*2^(1/2))^(1/2))*(x-1/2-1/2*I*(-1+4*2^(1/2))^(1/2)))^(1/
2)*(-4*(1/2+1/2*I*5^(1/2))*2^(1/2)+3*I*5^(1/2)-2*2^(1/2)*(1+4*2^(1/2))^(1/2
)+2*2^(1/2)+3*(1+4*2^(1/2))^(1/2))*(EllipticF(((1/2*I*(-1+4*2^(1/2))^(1/2)-
1/2*(1+4*2^(1/2))^(1/2))*(x-1/2+1/2*(1+4*2^(1/2))^(1/2))/(1/2*I*(-1+4*2^(1/
2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))/(x-1/2-1/2*(1+4*2^(1/2))^(1/2)))^(1/2),(
(1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))*(-1/2*(1+4*2^(1/2))^(1
/2)-1/2*I*(-1+4*2^(1/2))^(1/2))/(1/2*I*(-1+4*2^(1/2))^(1/2)-1/2*(1+4*2^(1/2
))^(1/2))/(-1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2)))^(1/2))+(1+
4*2^(1/2))^(1/2)*(-4*(1/2+1/2*I*5^(1/2))*2^(1/2)+3*I*5^(1/2)+2*2^(1/2)*(1+4
```

```
*2^(1/2))^(1/2)+2*2^(1/2)-3*(1+4*2^(1/2))^(1/2))*EllipticPi(((1/2*I*(-1+4*2
^(1/2))^(1/2)-1/2*(1+4*2^(1/2))^(1/2))*(x-1/2+1/2*(1+4*2^(1/2))^(1/2))/(1/2
*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))/(x-1/2-1/2*(1+4*2^(1/2))^(
1/2)))^(1/2),5/2+5/2*I*(-1+4*2^(1/2))^(1/2)-7/4*2^(1/2)-13/8*I*(-1+4*2^(1/2
))^(1/2)*2^(1/2)-5*I*(1/2+1/2*I*5^(1/2))*(-1+4*2^(1/2))^(1/2)+13/4*I*(1/2+1
/2*I*5^(1/2))*(-1+4*2^(1/2))^(1/2)*2^(1/2)-1/2*(1+4*2^(1/2))^(1/2)-7/4*I*(1
+4*2^(1/2))^(1/2)*(-1+4*2^(1/2))^(1/2)*2^(1/2)+3/8*2^(1/2)*(1+4*2^(1/2))^(1
/2)+5/2*I*(1+4*2^(1/2))^(1/2)*(-1+4*2^(1/2))^(1/2)-3/4*(1/2+1/2*I*5^(1/2))*
2^(1/2)*(1+4*2^(1/2))^(1/2)+(1/2+1/2*I*5^(1/2))*(1+4*2^(1/2))^(1/2),((1/2*I
*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))*(-1/2*(1+4*2^(1/2))^(1/2)-1/
2*I*(-1+4*2^(1/2))^(1/2))/(1/2*I*(-1+4*2^(1/2))^(1/2)-1/2*(1+4*2^(1/2))^(1/
2))/(-1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2)))^(1/2)))+1/8*(-1/
2*(1+4*2^(1/2))^(1/2)-1/2*I*(-1+4*2^(1/2))^(1/2))*((1/2*I*(-1+4*2^(1/2))^(1
/2)-1/2*(1+4*2^(1/2))^(1/2))*(x-1/2+1/2*(1+4*2^(1/2))^(1/2))/(1/2*I*(-1+4*2
^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))/(x-1/2-1/2*(1+4*2^(1/2))^(1/2)))^(1/
2)*(x-1/2-1/2*(1+4*2^(1/2))^(1/2))^2*((1+4*2^(1/2))^(1/2)*(x-1/2+1/2*I*(-1+
4*2^(1/2))^(1/2))/(-1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))/(x-
1/2-1/2*(1+4*2^(1/2))^(1/2)))^(1/2)*((1+4*2^(1/2))^(1/2)*(x-1/2-1/2*I*(-1+4
*2^(1/2))^(1/2))/(1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))/(x-1/
2-1/2*(1+4*2^(1/2))^(1/2)))^(1/2)/(1/2*I*(-1+4*2^(1/2))^(1/2)-1/2*(1+4*2^(1
/2))^(1/2))/(1+4*2^(1/2))^(1/2)/((x-1/2+1/2*(1+4*2^(1/2))^(1/2))*(x-1/2-1/2
*(1+4*2^(1/2))^(1/2))*(x-1/2+1/2*I*(-1+4*2^(1/2))^(1/2))*(x-1/2-1/2*I*(-1+4
*2^(1/2))^(1/2)))^(1/2)*(-4*(1/2-1/2*I*5^(1/2))*2^(1/2)-3*I*5^(1/2)-2*2^(1/
2)*(1+4*2^(1/2))^(1/2)+2*2^(1/2)+3*(1+4*2^(1/2))^(1/2))*(EllipticF(((1/2*I*
(-1+4*2^(1/2))^(1/2)-1/2*(1+4*2^(1/2))^(1/2))*(x-1/2+1/2*(1+4*2^(1/2))^(1/2
))/(1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))/(x-1/2-1/2*(1+4*2^(
1/2))^(1/2)))^(1/2),((1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))*(
-1/2*(1+4*2^(1/2))^(1/2)-1/2*I*(-1+4*2^(1/2))^(1/2))/(1/2*I*(-1+4*2^(1/2))^
(1/2)-1/2*(1+4*2^(1/2))^(1/2))/(-1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2
))^(1/2)))^(1/2))+(1+4*2^(1/2))^(1/2)*(-4*(1/2-1/2*I*5^(1/2))*2^(1/2)-3*I*5
^(1/2)+2*2^(1/2)*(1+4*2^(1/2))^(1/2)+2*2^(1/2)-3*(1+4*2^(1/2))^(1/2))*Ellip
ticPi(((1/2*I*(-1+4*2^(1/2))^(1/2)-1/2*(1+4*2^(1/2))^(1/2))*(x-1/2+1/2*(1+4
*2^(1/2))^(1/2))/(1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))/(x-1/
2-1/2*(1+4*2^(1/2))^(1/2)))^(1/2),5/2+5/2*I*(-1+4*2^(1/2))^(1/2)-7/4*2^(1/2
)-13/8*I*(-1+4*2^(1/2))^(1/2)*2^(1/2)-5*I*(1/2-1/2*I*5^(1/2))*(-1+4*2^(1/2)
)^(1/2)+13/4*I*(1/2-1/2*I*5^(1/2))*(-1+4*2^(1/2))^(1/2)*2^(1/2)-1/2*(1+4*2^
(1/2))^(1/2)-7/4*I*(1+4*2^(1/2))^(1/2)*(-1+4*2^(1/2))^(1/2)*2^(1/2)+3/8*2^(
1/2)*(1+4*2^(1/2))^(1/2)+5/2*I*(1+4*2^(1/2))^(1/2)*(-1+4*2^(1/2))^(1/2)-3/4
*(1/2-1/2*I*5^(1/2))*2^(1/2)*(1+4*2^(1/2))^(1/2)+(1/2-1/2*I*5^(1/2))*(1+4*2
^(1/2))^(1/2),((1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/2))*(-1/2*(
1+4*2^(1/2))^(1/2)-1/2*I*(-1+4*2^(1/2))^(1/2))/(1/2*I*(-1+4*2^(1/2))^(1/2)-
1/2*(1+4*2^(1/2))^(1/2))/(-1/2*I*(-1+4*2^(1/2))^(1/2)+1/2*(1+4*2^(1/2))^(1/
2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4 - 2x^3 + x^2 - 2}\,(x^2 - x + 2)(2x - 1)}{2x^2 - 2x + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*(x^2-x+2)*(x^4-2*x^3+x^2-2)^(1/2)/(2*x^2-2*x+3),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(x^4 - 2*x^3 + x^2 - 2)*(x^2 - x + 2)*(2*x - 1)/(2*x^2 - 2*x
+ 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x-1)\,(x^2 - x + 2)\,\sqrt{x^4 - 2x^3 + x^2 - 2}}{2x^2 - 2x + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x - 1)*(x^2 - x + 2)*(x^2 - 2*x^3 + x^4 - 2)^(1/2))/(2*x^2 - 2*x +
3),x)
```

```
[Out] int(((2*x - 1)*(x^2 - x + 2)*(x^2 - 2*x^3 + x^4 - 2)^(1/2))/(2*x^2 - 2*x +
3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x-1)\left(x^2-x+2\right)\sqrt{x^4-2x^3+x^2-2}}{2x^2-2x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*(x**2-x+2)*(x**4-2*x**3+x**2-2)**(1/2)/(2*x**2-2*x+3),x)
```

```
[Out] Integral((2*x - 1)*(x**2 - x + 2)*sqrt(x**4 - 2*x**3 + x**2 - 2)/(2*x**2 -
2*x + 3), x)
```

3.1148 $\int \frac{\sqrt[4]{-x^3+x^4}}{x(1+x)} \, dx$

Optimal. Leaf size=93

$$-2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)+2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^3}}\right)+2\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^3}}\right)$$

Rubi [A] time = 0.15, antiderivative size = 185, normalized size of antiderivative = 1.99, number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {2042, 105, 63, 240, 212, 206, 203, 93, 298}

$$\frac{2\sqrt[4]{x^4-x^3}\tan^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{\sqrt[4]{x-1}\,x^{3/4}}+\frac{2\sqrt[4]{2}\sqrt[4]{x^4-x^3}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{\sqrt[4]{x-1}\,x^{3/4}}+\frac{2\sqrt[4]{x^4-x^3}\tanh^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{\sqrt[4]{x-1}\,x^{3/4}}-\frac{2\sqrt[4]{2}\sqrt[4]{x^4-x^3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{\sqrt[4]{x-1}\,x^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + x^4)^(1/4)/(x*(1 + x)),x]
```

```
[Out] (2*(-x^3 + x^4)^(1/4)*ArcTan[(-1 + x)^(1/4)/x^(1/4)])/((-1 + x)^(1/4)*x^(3/
4)) + (2*2^(1/4)*(-x^3 + x^4)^(1/4)*ArcTan[(2^(1/4)*x^(1/4))/(-1 + x)^(1/4)
])/((-1 + x)^(1/4)*x^(3/4)) + (2*(-x^3 + x^4)^(1/4)*ArcTanh[(-1 + x)^(1/4)/
x^(1/4)])/((-1 + x)^(1/4)*x^(3/4)) - (2*2^(1/4)*(-x^3 + x^4)^(1/4)*ArcTanh[
(2^(1/4)*x^(1/4))/(-1 + x)^(1/4)])/((-1 + x)^(1/4)*x^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\int \frac{\sqrt[4]{-x^3+x^4}}{x(1+x)}\,dx = \frac{\sqrt[4]{-x^3+x^4}\int \frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}\,(1+x)}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{\sqrt[4]{-x^3+x^4}\int \frac{1}{(-1+x)^{3/4}\sqrt[4]{x}}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}} - \frac{\left(2\sqrt[4]{-x^3+x^4}\right)\int \frac{1}{(-1+x)^{3/4}\sqrt[4]{x}\,(1+x)}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{\left(4\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^4}}\,dx, x, \sqrt[4]{-1+x}\right)}{\sqrt[4]{-1+x}\,x^{3/4}} - \frac{\left(8\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\int \frac{x^2}{1-2x^4}\,dx, x, \frac{\sqrt[4]{\ldots}}{\sqrt[4]{-\ldots}}\right)}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{\left(4\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{\sqrt[4]{-1+x}\,x^{3/4}} - \frac{\left(2\sqrt{2}\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x^2}\,dx, x, \frac{\ldots}{\sqrt[4]{\ldots}}\right)}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{2\sqrt[4]{2}\sqrt[4]{-x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{-1+x}}\right)}{\sqrt[4]{-1+x}\,x^{3/4}} - \frac{2\sqrt[4]{2}\sqrt[4]{-x^3+x^4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{-1+x}}\right)}{\sqrt[4]{-1+x}\,x^{3/4}} + \frac{\left(2\sqrt[4]{-x^3+x^4}\right)\text{Sub}\ldots}{\sqrt[4]{-\ldots}}$$

$$= \frac{2\sqrt[4]{-x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{\sqrt[4]{-1+x}\,x^{3/4}} + \frac{2\sqrt[4]{2}\sqrt[4]{-x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{-1+x}}\right)}{\sqrt[4]{-1+x}\,x^{3/4}} + \frac{2\sqrt[4]{-x^3+x^4}\tanh^{-1}\left(\ldots\right)}{\sqrt[4]{-1+x}\,x^{3/4}}$$

Mathematica [C] time = 0.01, size = 58, normalized size = 0.62

$$\frac{4\sqrt[4]{(x-1)x^3}\left(\sqrt[4]{x}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};1-x\right)-{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{x-1}{2x}\right)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + x^4)^(1/4)/(x*(1 + x)),x]
```

```
[Out] (4*((-1 + x)*x^3)^(1/4)*(x^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, 1 - x] -
Hypergeometric2F1[1/4, 1, 5/4, (-1 + x)/(2*x)]))/x
```

IntegrateAlgebraic [A] time = 0.32, size = 93, normalized size = 1.00

$$-2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)+2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^3}}\right)+2\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^3 + x^4)^(1/4)/(x*(1 + x)),x]
```

```
[Out] -2*ArcTan[x/(-x^3 + x^4)^(1/4)] + 2*2^(1/4)*ArcTan[(2^(1/4)*x)/(-x^3 + x^4)
^(1/4)] + 2*ArcTanh[x/(-x^3 + x^4)^(1/4)] - 2*2^(1/4)*ArcTanh[(2^(1/4)*x)/(
-x^3 + x^4)^(1/4)]
```

fricas [B] time = 0.44, size = 174, normalized size = 1.87

$$4\cdot 2^{\frac{1}{4}}\arctan\left(\frac{2^{\frac{3}{4}}x\sqrt{\frac{\sqrt{2}x^2+\sqrt{x^4-x^3}}{x^2}}-2^{\frac{3}{4}}\left(x^4-x^3\right)^{\frac{1}{4}}}{2x}\right)-2^{\frac{1}{4}}\log\left(\frac{2^{\frac{1}{4}}x+\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+2^{\frac{1}{4}}\log\left(-\frac{2^{\frac{1}{4}}x-\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+2\arctan\left(\frac{\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+\log\left(\frac{x+\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)-\log\left(-\frac{x-\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x/(1+x),x, algorithm="fricas")
```

```
[Out] 4*2^(1/4)*arctan(1/2*(2^(3/4)*x*sqrt((sqrt(2)*x^2 + sqrt(x^4 - x^3))/x^2) -
 2^(3/4)*(x^4 - x^3)^(1/4))/x) - 2^(1/4)*log((2^(1/4)*x + (x^4 - x^3)^(1/4)
)/x) + 2^(1/4)*log(-(2^(1/4)*x - (x^4 - x^3)^(1/4))/x) + 2*arctan((x^4 - x^
3)^(1/4)/x) + log((x + (x^4 - x^3)^(1/4))/x) - log(-(x - (x^4 - x^3)^(1/4))
/x)
```

giac [A] time = 0.23, size = 100, normalized size = 1.08

$$2\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)+2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)-2^{\frac{1}{4}}\log\left(\left|-2^{\frac{1}{4}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right|\right)-2\arctan\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)-\log\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right)+\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x/(1+x),x, algorithm="giac")
```

```
[Out] 2*2^(1/4)*arctan(1/2*2^(3/4)*(-1/x + 1)^(1/4)) + 2^(1/4)*log(2^(1/4) + (-1/
x + 1)^(1/4)) - 2^(1/4)*log(abs(-2^(1/4) + (-1/x + 1)^(1/4))) - 2*arctan((-
1/x + 1)^(1/4)) - log((-1/x + 1)^(1/4) + 1) + log(abs((-1/x + 1)^(1/4) - 1)
)
```

maple [C] time = 1.70, size = 387, normalized size = 4.16

$$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\sqrt{x^4-x^3}\,\mathrm{RootOf}\left(\_Z^2+1\right)x-2\,\mathrm{RootOf}\left(\_Z^2+1\right)x^3+\mathrm{RootOf}\left(\_Z^2+1\right)x^2+2\left(x^4-x^3\right)^{\frac{3}{4}}-2x^2\left(x^4-x^3\right)^{\frac{1}{4}}}{x^2}\right)+\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\ln\left(-\frac{-3\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\mathrm{RootOf}\left(\_Z^4-2\right)^2x^3+\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\mathrm{RootOf}\left(\_Z^4-2\right)^2x^2+4\left(x^4-x^3\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-2\right)^2x^2+4\sqrt{x^4-x^3}\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)x-4\left(x^4-x^3\right)^{\frac{3}{4}}}{x^2(1+x)}\right)-\mathrm{RootOf}\left(\_Z^4-2\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^4-2\right)^3x^3+4\left(x^4-x^3\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-2\right)^2x^2-\mathrm{RootOf}\left(\_Z^4-2\right)^3x^2+4\sqrt{x^4-x^3}\,\mathrm{RootOf}\left(\_Z^4-2\right)x+4\left(x^4-x^3\right)^{\frac{3}{4}}}{x^2(1+x)}\right)-\ln\left(\frac{2\left(x^4-x^3\right)^{\frac{3}{4}}-2\sqrt{x^4-x^3}\,x+2x^2\left(x^4-x^3\right)^{\frac{1}{4}}-2x^3+x^2}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3)^(1/4)/x/(1+x),x)
```

```
[Out] RootOf(_Z^2+1)*ln((2*(x^4-x^3)^(1/2)*RootOf(_Z^2+1)*x-2*RootOf(_Z^2+1)*x^3+
RootOf(_Z^2+1)*x^2+2*(x^4-x^3)^(3/4)-2*x^2*(x^4-x^3)^(1/4))/x^2)+RootOf(_Z^
2+RootOf(_Z^4-2)^2)*ln(-(-3*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2*
x^3+RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2*x^2+4*(x^4-x^3)^(1/4)*Ro
otOf(_Z^4-2)^2*x^2+4*(x^4-x^3)^(1/2)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x-4*(x^4
-x^3)^(3/4))/x^2/(1+x))-RootOf(_Z^4-2)*ln((3*RootOf(_Z^4-2)^3*x^3+4*(x^4-x^
3)^(1/4)*RootOf(_Z^4-2)^2*x^2-RootOf(_Z^4-2)^3*x^2+4*(x^4-x^3)^(1/2)*RootOf
(_Z^4-2)*x+4*(x^4-x^3)^(3/4))/x^2/(1+x))-ln((2*(x^4-x^3)^(3/4)-2*(x^4-x^3)^
(1/2)*x+2*x^2*(x^4-x^3)^(1/4)-2*x^3+x^2)/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^3\right)^{\frac{1}{4}}}{(x+1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x/(1+x),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^3)^(1/4)/((x + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-x^3\right)^{1/4}}{x\,\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x^3)^(1/4)/(x*(x + 1)),x)
```

```
[Out] int((x^4 - x^3)^(1/4)/(x*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3(x-1)}}{x(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3)**(1/4)/x/(1+x),x)
```

```
[Out] Integral((x**3*(x - 1))**(1/4)/(x*(x + 1)), x)
```

3.1149 $\int \frac{1+2x^3}{\left(-1+x+x^3\right)\sqrt[3]{-x^2+x^5}}\,dx$

Optimal. Leaf size=93

$$-\log\left(\sqrt[3]{x^5-x^2}+x\right)+\frac{1}{2}\log\left(x^2-\sqrt[3]{x^5-x^2}\,x+\left(x^5-x^2\right)^{2/3}\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^5-x^2}-x}\right)$$

Rubi [F] time = 1.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+2x^3}{\left(-1+x+x^3\right)\sqrt[3]{-x^2+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + 2*x^3)/((-1 + x + x^3)*(-x^2 + x^5)^(1/3)),x]
```

```
[Out] (6*x*(1 - x^3)^(1/3)*Hypergeometric2F1[1/9, 1/3, 10/9, x^3])/(-x^2 + x^5)^(
1/3) + (9*x^(2/3)*(-1 + x^3)^(1/3)*Defer[Subst][Defer[Int][1/((-1 + x^9)^(1
/3)*(-1 + x^3 + x^9)), x], x, x^(1/3)])/(-x^2 + x^5)^(1/3) - (6*x^(2/3)*(-1
 + x^3)^(1/3)*Defer[Subst][Defer[Int][x^3/((-1 + x^9)^(1/3)*(-1 + x^3 + x^9
)), x], x, x^(1/3)])/(-x^2 + x^5)^(1/3)
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^3}{\left(-1+x+x^3\right)\sqrt[3]{-x^2+x^5}}\,dx &= \frac{\left(x^{2/3}\sqrt[3]{-1+x^3}\right)\int \frac{1+2x^3}{x^{2/3}\sqrt[3]{-1+x^3}\left(-1+x+x^3\right)}\,dx}{\sqrt[3]{-x^2+x^5}}\\
&= \frac{\left(3x^{2/3}\sqrt[3]{-1+x^3}\right)\text{Subst}\left(\int \frac{1+2x^9}{\sqrt[3]{-1+x^9}\left(-1+x^3+x^9\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x^2+x^5}}\\
&= \frac{\left(3x^{2/3}\sqrt[3]{-1+x^3}\right)\text{Subst}\left(\int \left(\frac{2}{\sqrt[3]{-1+x^9}}+\frac{3-2x^3}{\sqrt[3]{-1+x^9}\left(-1+x^3+x^9\right)}\right)dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x^2+x^5}}\\
&= \frac{\left(3x^{2/3}\sqrt[3]{-1+x^3}\right)\text{Subst}\left(\int \frac{3-2x^3}{\sqrt[3]{-1+x^9}\left(-1+x^3+x^9\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x^2+x^5}}+\frac{\left(6x^{2/3}\sqrt[3]{-1+x^3}\right)\text{Su}}{\sqrt[3]{-}}\\
&= \frac{\left(6x^{2/3}\sqrt[3]{1-x^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x^9}}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x^2+x^5}}+\frac{\left(3x^{2/3}\sqrt[3]{-1+x^3}\right)\text{Subst}\left(\int\left(\frac{}{\sqrt[3]{-1}}\right.\right.}{}\\
&= \frac{6x\sqrt[3]{1-x^3}\,{}_2F_1\left(\frac{1}{9},\frac{1}{3};\frac{10}{9};x^3\right)}{\sqrt[3]{-x^2+x^5}}-\frac{\left(6x^{2/3}\sqrt[3]{-1+x^3}\right)\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{-1+x^9}\left(-1+x^3+x^9\right)}\,dx\right.}{\sqrt[3]{-x^2+x^5}}
\end{aligned}$$

Mathematica [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1+2x^3}{\left(-1+x+x^3\right)\sqrt[3]{-x^2+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + 2*x^3)/((-1 + x + x^3)*(-x^2 + x^5)^(1/3)),x]
```

```
[Out] Integrate[(1 + 2*x^3)/((-1 + x + x^3)*(-x^2 + x^5)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 1.05, size = 93, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^5-x^2}+x\right)+\frac{1}{2}\log\left(x^2-\sqrt[3]{x^5-x^2}\,x+\left(x^5-x^2\right)^{2/3}\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^5-x^2}-x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x^3)/((-1 + x + x^3)*(-x^2 + x^5)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(-x^2 + x^5)^(1/3))]) - Log[x + (-x^2
+ x^5)^(1/3)] + Log[x^2 - x*(-x^2 + x^5)^(1/3) + (-x^2 + x^5)^(2/3)]/2
```

fricas [A] time = 1.37, size = 124, normalized size = 1.33

$$-\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(x^5-x^2\right)^{\frac{1}{3}}x+\sqrt{3}\left(x^4+x^2-x\right)+2\sqrt{3}\left(x^5-x^2\right)^{\frac{2}{3}}}{3\left(x^4-x^2-x\right)}\right)-\frac{1}{2}\log\left(\frac{x^4+x^2+3\left(x^5-x^2\right)^{\frac{1}{3}}x-x+3\left(x^5-x^2\right)^{\frac{2}{3}}}{x^4+x^2-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)/(x^3+x-1)/(x^5-x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(1/3*(2*sqrt(3)*(x^5 - x^2)^(1/3)*x + sqrt(3)*(x^4 + x^2 - x
) + 2*sqrt(3)*(x^5 - x^2)^(2/3))/(x^4 - x^2 - x)) - 1/2*log((x^4 + x^2 + 3*
(x^5 - x^2)^(1/3)*x - x + 3*(x^5 - x^2)^(2/3))/(x^4 + x^2 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3+1}{\left(x^5-x^2\right)^{\frac{1}{3}}\left(x^3+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)/(x^3+x-1)/(x^5-x^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 1)/((x^5 - x^2)^(1/3)*(x^3 + x - 1)), x)
```

maple [C] time = 4.37, size = 385, normalized size = 4.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+1)/(x^3+x-1)/(x^5-x^2)^(1/3),x)
```

```
[Out] -ln((1462231700*RootOf(_Z^2-2*_Z+4)^2*x^4-9984360926*RootOf(_Z^2-2*_Z+4)*x^
4-5117810950*RootOf(_Z^2-2*_Z+4)^2*x^2-1758826818*x^4-1462231700*RootOf(_Z^
2-2*_Z+4)^2*x-17923671861*RootOf(_Z^2-2*_Z+4)*(x^5-x^2)^(2/3)-14667651144*R
ootOf(_Z^2-2*_Z+4)*(x^5-x^2)^(1/3)*x+6431745091*RootOf(_Z^2-2*_Z+4)*x^2+998
4360926*RootOf(_Z^2-2*_Z+4)*x+6512041434*(x^5-x^2)^(2/3)+35847343722*x*(x^5
-x^2)^(1/3)+1256304870*x^2+1758826818*x)/x/(x^3+x-1))+1/2*RootOf(_Z^2-2*_Z+
4)*ln((251260974*RootOf(_Z^2-2*_Z+4)^2*x^4+7105231670*RootOf(_Z^2-2*_Z+4)*x
^4-879413409*RootOf(_Z^2-2*_Z+4)^2*x^2+29244634000*x^4-251260974*RootOf(_Z^
2-2*_Z+4)^2*x+35847343722*RootOf(_Z^2-2*_Z+4)*(x^5-x^2)^(2/3)+6512041434*Ro
otOf(_Z^2-2*_Z+4)*(x^5-x^2)^(1/3)*x-19968721852*RootOf(_Z^2-2*_Z+4)*x^2-710
5231670*RootOf(_Z^2-2*_Z+4)*x-58670604576*(x^5-x^2)^(2/3)-71694687444*x*(x^
5-x^2)^(1/3)+11697853600*x^2-29244634000*x)/x/(x^3+x-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^3+1}{\left(x^5-x^2\right)^{\frac{1}{3}}\left(x^3+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)/(x^3+x-1)/(x^5-x^2)^(1/3),x, algorithm="maxima")
[Out] integrate((2*x^3 + 1)/((x^5 - x^2)^(1/3)*(x^3 + x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x^3+1}{\left(x^5-x^2\right)^{1/3}\,\left(x^3+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3 + 1)/((x^5 - x^2)^(1/3)*(x + x^3 - 1)),x)
[Out] int((2*x^3 + 1)/((x^5 - x^2)^(1/3)*(x + x^3 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+1)/(x**3+x-1)/(x**5-x**2)**(1/3),x)
[Out] Timed out
```

3.1150 $\int \frac{\sqrt[4]{-1+x^6}}{x^7}\,dx$

Optimal. Leaf size=93

$$-\frac{\sqrt[4]{x^6-1}}{6x^6}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^6-1}}{\sqrt{x^6-1}-1}\right)}{12\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^6-1}}{\sqrt{x^6-1}+1}\right)}{12\sqrt{2}}$$

Rubi [A] time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.56, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {266, 47, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{x^6-1}}{6x^6}-\frac{\log\left(\sqrt{x^6-1}-\sqrt{2}\sqrt[4]{x^6-1}+1\right)}{24\sqrt{2}}+\frac{\log\left(\sqrt{x^6-1}+\sqrt{2}\sqrt[4]{x^6-1}+1\right)}{24\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{x^6-1}\right)}{12\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt[4]{x^6-1}+1\right)}{12\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)^(1/4)/x^7,x]
```

```
[Out] -1/6*(-1 + x^6)^(1/4)/x^6 - ArcTan[1 - Sqrt[2]*(-1 + x^6)^(1/4)]/(12*Sqrt[2
]) + ArcTan[1 + Sqrt[2]*(-1 + x^6)^(1/4)]/(12*Sqrt[2]) - Log[1 - Sqrt[2]*(-
1 + x^6)^(1/4) + Sqrt[-1 + x^6]]/(24*Sqrt[2]) + Log[1 + Sqrt[2]*(-1 + x^6)^
(1/4) + Sqrt[-1 + x^6]]/(24*Sqrt[2])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{-1+x^6}}{x^7}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{\sqrt[4]{-1+x}}{x^2}\,dx, x, x^6\right)\\
&= -\frac{\sqrt[4]{-1+x^6}}{6x^6} + \frac{1}{24}\,\text{Subst}\left(\int \frac{1}{(-1+x)^{3/4}x}\,dx, x, x^6\right)\\
&= -\frac{\sqrt[4]{-1+x^6}}{6x^6} + \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt[4]{-1+x^6}\right)\\
&= -\frac{\sqrt[4]{-1+x^6}}{6x^6} + \frac{1}{12}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt[4]{-1+x^6}\right) + \frac{1}{12}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt[4]{-1+x^6}\right)\\
&= -\frac{\sqrt[4]{-1+x^6}}{6x^6} + \frac{1}{24}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt[4]{-1+x^6}\right) + \frac{1}{24}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx\right.\\
&= -\frac{\sqrt[4]{-1+x^6}}{6x^6} - \frac{\log\left(1-\sqrt{2}\,\sqrt[4]{-1+x^6}+\sqrt{-1+x^6}\right)}{24\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\,\sqrt[4]{-1+x^6}+\sqrt{-1+x^6}\right)}{24\sqrt{2}} +\\
&= -\frac{\sqrt[4]{-1+x^6}}{6x^6} - \frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt[4]{-1+x^6}\right)}{12\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt[4]{-1+x^6}\right)}{12\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\,\sqrt[4]{-1+x^6}\right.}{24\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.30

$$\frac{2}{15}\left(x^6-1\right)^{5/4}\,{}_2F_1\left(\frac{5}{4},2;\frac{9}{4};1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)^(1/4)/x^7,x]
```

```
[Out] (2*(-1 + x^6)^(5/4)*Hypergeometric2F1[5/4, 2, 9/4, 1 - x^6])/15
```

IntegrateAlgebraic [A] time = 0.19, size = 98, normalized size = 1.05

$$-\frac{\sqrt[4]{x^6-1}}{6x^6}+\frac{\tan^{-1}\left(\frac{\frac{\sqrt{x^6-1}}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt[4]{x^6-1}}\right)}{12\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^6-1}}{\sqrt{x^6-1}+1}\right)}{12\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^6)^(1/4)/x^7,x]
```

```
[Out] -1/6*(-1 + x^6)^(1/4)/x^6 + ArcTan[(-(1/Sqrt[2]) + Sqrt[-1 + x^6]/Sqrt[2])/
(-1 + x^6)^(1/4)]/(12*Sqrt[2]) + ArcTanh[(Sqrt[2]*(-1 + x^6)^(1/4))/(1 + Sq
rt[-1 + x^6])]/(12*Sqrt[2])
```

fricas [B] time = 0.43, size = 180, normalized size = 1.94

$$-\frac{4\sqrt{2}x^6\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+\sqrt{x^6-1}+1}-\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}-1\right)+4\sqrt{2}x^6\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+4\sqrt{x^6-1}+4}-\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+1\right)-\sqrt{2}x^6\log\left(4\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+4\sqrt{x^6-1}+4\right)+\sqrt{2}x^6\log\left(-4\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+4\sqrt{x^6-1}+4\right)+8\left(x^6-1\right)^{\frac{1}{4}}}{48x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/4)/x^7,x, algorithm="fricas")
```

```
[Out] -1/48*(4*sqrt(2)*x^6*arctan(sqrt(2)*sqrt(sqrt(2)*(x^6 - 1)^(1/4) + sqrt(x^6
 - 1) + 1) - sqrt(2)*(x^6 - 1)^(1/4) - 1) + 4*sqrt(2)*x^6*arctan(1/2*sqrt(2
)*sqrt(-4*sqrt(2)*(x^6 - 1)^(1/4) + 4*sqrt(x^6 - 1) + 4) - sqrt(2)*(x^6 - 1
)^(1/4) + 1) - sqrt(2)*x^6*log(4*sqrt(2)*(x^6 - 1)^(1/4) + 4*sqrt(x^6 - 1)
+ 4) + sqrt(2)*x^6*log(-4*sqrt(2)*(x^6 - 1)^(1/4) + 4*sqrt(x^6 - 1) + 4) +
8*(x^6 - 1)^(1/4))/x^6
```

giac [A] time = 0.20, size = 114, normalized size = 1.23

$$\frac{1}{24}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(x^6-1\right)^{\frac{1}{4}}\right)\right)+\frac{1}{24}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(x^6-1\right)^{\frac{1}{4}}\right)\right)+\frac{1}{48}\sqrt{2}\log\left(\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+\sqrt{x^6-1}+1\right)-\frac{1}{48}\sqrt{2}\log\left(-\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+\sqrt{x^6-1}+1\right)-\frac{\left(x^6-1\right)^{\frac{1}{4}}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/4)/x^7,x, algorithm="giac")
```

```
[Out] 1/24*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^6 - 1)^(1/4))) + 1/24*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^6 - 1)^(1/4))) + 1/48*sqrt(2)*log(sq
rt(2)*(x^6 - 1)^(1/4) + sqrt(x^6 - 1) + 1) - 1/48*sqrt(2)*log(-sqrt(2)*(x^6
 - 1)^(1/4) + sqrt(x^6 - 1) + 1) - 1/6*(x^6 - 1)^(1/4)/x^6
```

maple [C] time = 0.37, size = 76, normalized size = 0.82

$$-\frac{\left(x^6-1\right)^{\frac{1}{4}}}{6x^6}+\frac{\left(-\mathrm{signum}\left(x^6-1\right)\right)^{\frac{3}{4}}\left(\frac{3\Gamma\left(\frac{3}{4}\right)x^6\,\mathrm{hypergeom}\left(\left[1,1,\frac{7}{4}\right],[2,2],x^6\right)}{4}+\left(-3\ln(2)+\frac{\pi}{2}+6\ln(x)+i\pi\right)\Gamma\left(\frac{3}{4}\right)\right)}{24\Gamma\left(\frac{3}{4}\right)\mathrm{signum}\left(x^6-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/4)/x^7,x)
```

```
[Out] -1/6*(x^6-1)^(1/4)/x^6+1/24/GAMMA(3/4)/signum(x^6-1)^(3/4)*(-signum(x^6-1))
^(3/4)*(3/4*GAMMA(3/4)*x^6*hypergeom([1,1,7/4],[2,2],x^6)+(-3*ln(2)+1/2*Pi+
6*ln(x)+I*Pi)*GAMMA(3/4))
```

maxima [A] time = 0.58, size = 114, normalized size = 1.23

$$\frac{1}{24}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(x^6-1\right)^{\frac{1}{4}}\right)\right)+\frac{1}{24}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(x^6-1\right)^{\frac{1}{4}}\right)\right)+\frac{1}{48}\sqrt{2}\log\left(\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+\sqrt{x^6-1}+1\right)-\frac{1}{48}\sqrt{2}\log\left(-\sqrt{2}\left(x^6-1\right)^{\frac{1}{4}}+\sqrt{x^6-1}+1\right)-\frac{\left(x^6-1\right)^{\frac{1}{4}}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/4)/x^7,x, algorithm="maxima")
```

```
[Out] 1/24*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^6 - 1)^(1/4))) + 1/24*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^6 - 1)^(1/4))) + 1/48*sqrt(2)*log(sq
rt(2)*(x^6 - 1)^(1/4) + sqrt(x^6 - 1) + 1) - 1/48*sqrt(2)*log(-sqrt(2)*(x^6
 - 1)^(1/4) + sqrt(x^6 - 1) + 1) - 1/6*(x^6 - 1)^(1/4)/x^6
```

mupad [B] time = 0.98, size = 57, normalized size = 0.61

$$-\frac{\left(x^6-1\right)^{1/4}}{6x^6}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(x^6-1\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{24}+\frac{1}{24}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(x^6-1\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{24}-\frac{1}{24}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)^(1/4)/x^7,x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*(x^6 - 1)^(1/4)*(1/2 - 1i/2))*(1/24 + 1i/24) + 2^(1/2)
*atan(2^(1/2)*(x^6 - 1)^(1/4)*(1/2 + 1i/2))*(1/24 - 1i/24) - (x^6 - 1)^(1/4
)/(6*x^6)
```

sympy [C] time = 1.11, size = 36, normalized size = 0.39

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{e^{2i\pi}}{x^6}\right)}{6x^{\frac{9}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/4)/x**7,x)
```

```
[Out] -gamma(3/4)*hyper((-1/4, 3/4), (7/4,), exp_polar(2*I*pi)/x**6)/(6*x**(9/2)*
gamma(7/4))
```

3.1151 $\int \frac{1+3x^4}{(-1+x+x^4)\sqrt[3]{-x^2+x^6}}\,dx$

Optimal. Leaf size=93

$$-\log\left(\sqrt[3]{x^6-x^2}+x\right)+\frac{1}{2}\log\left(x^2-\sqrt[3]{x^6-x^2}\,x+\left(x^6-x^2\right)^{2/3}\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^6-x^2}-x}\right)$$

Rubi [F] time = 0.99, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+3x^4}{(-1+x+x^4)\sqrt[3]{-x^2+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + 3*x^4)/((-1 + x + x^4)*(-x^2 + x^6)^(1/3)),x]
```

```
[Out] (9*x*(1 - x^4)^(1/3)*Hypergeometric2F1[1/12, 1/3, 13/12, x^4])/(-x^2 + x^6)
^(1/3) + (12*x^(2/3)*(-1 + x^4)^(1/3)*Defer[Subst][Defer[Int][1/((-1 + x^12
)^(1/3)*(-1 + x^3 + x^12)), x], x, x^(1/3)])/(-x^2 + x^6)^(1/3) - (9*x^(2/3
)*(-1 + x^4)^(1/3)*Defer[Subst][Defer[Int][x^3/((-1 + x^12)^(1/3)*(-1 + x^3
 + x^12)), x], x, x^(1/3)])/(-x^2 + x^6)^(1/3)
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x^4}{(-1+x+x^4)\sqrt[3]{-x^2+x^6}}\,dx &= \frac{\left(x^{2/3}\sqrt[3]{-1+x^4}\right)\int \frac{1+3x^4}{x^{2/3}\sqrt[3]{-1+x^4}\,(-1+x+x^4)}\,dx}{\sqrt[3]{-x^2+x^6}}\\
&= \frac{\left(3x^{2/3}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \frac{1+3x^{12}}{\sqrt[3]{-1+x^{12}}\,(-1+x^3+x^{12})}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x^2+x^6}}\\
&= \frac{\left(3x^{2/3}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \left(\frac{3}{\sqrt[3]{-1+x^{12}}}+\frac{4-3x^3}{\sqrt[3]{-1+x^{12}}\,(-1+x^3+x^{12})}\right)dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x^2+x^6}}\\
&= \frac{\left(3x^{2/3}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \frac{4-3x^3}{\sqrt[3]{-1+x^{12}}\,(-1+x^3+x^{12})}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x^2+x^6}}+\frac{\left(9x^{2/3}\sqrt[3]{-1+x^4}\right.}{}\\
&= \frac{\left(9x^{2/3}\sqrt[3]{1-x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x^{12}}}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x^2+x^6}}+\frac{\left(3x^{2/3}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int\left(\right.\right.}{}\\
&= \frac{9x\sqrt[3]{1-x^4}\,{}_2F_1\left(\frac{1}{12},\frac{1}{3};\frac{13}{12};x^4\right)}{\sqrt[3]{-x^2+x^6}}-\frac{\left(9x^{2/3}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{-1+x^{12}}\,(-1+x^3+x^{\cdot}}\right.}{\sqrt[3]{-x^2+x^6}}
\end{aligned}$$

Mathematica [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1+3x^4}{(-1+x+x^4)\sqrt[3]{-x^2+x^6}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(1 + 3*x^4)/((-1 + x + x^4)*(-x^2 + x^6)^(1/3)),x]

[Out] Integrate[(1 + 3*x^4)/((-1 + x + x^4)*(-x^2 + x^6)^(1/3)), x]

IntegrateAlgebraic [A] time = 2.70, size = 93, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^6-x^2}+x\right)+\frac{1}{2}\log\left(x^2-\sqrt[3]{x^6-x^2}\,x+\left(x^6-x^2\right)^{2/3}\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^6-x^2}-x}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 3*x^4)/((-1 + x + x^4)*(-x^2 + x^6)^(1/3)),x]

[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(-x^2 + x^6)^(1/3))]) - Log[x + (-x^2 + x^6)^(1/3)] + Log[x^2 - x*(-x^2 + x^6)^(1/3) + (-x^2 + x^6)^(2/3)]/2

fricas [A] time = 1.70, size = 124, normalized size = 1.33

$$-\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(x^6-x^2\right)^{\frac{1}{3}}x+\sqrt{3}\left(x^5+x^2-x\right)+2\sqrt{3}\left(x^6-x^2\right)^{\frac{2}{3}}}{3\left(x^5-x^2-x\right)}\right)-\frac{1}{2}\log\left(\frac{x^5+x^2+3\left(x^6-x^2\right)^{\frac{1}{3}}x-x+3\left(x^6-x^2\right)^{\frac{2}{3}}}{x^5+x^2-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^4+1)/(x^4+x-1)/(x^6-x^2)^(1/3),x, algorithm="fricas")

[Out] -sqrt(3)*arctan(1/3*(2*sqrt(3)*(x^6 - x^2)^(1/3)*x + sqrt(3)*(x^5 + x^2 - x) + 2*sqrt(3)*(x^6 - x^2)^(2/3))/(x^5 - x^2 - x)) - 1/2*log((x^5 + x^2 + 3*(x^6 - x^2)^(1/3)*x - x + 3*(x^6 - x^2)^(2/3))/(x^5 + x^2 - x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^4+1}{\left(x^6-x^2\right)^{\frac{1}{3}}\left(x^4+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^4+1)/(x^4+x-1)/(x^6-x^2)^(1/3),x, algorithm="giac")

[Out] integrate((3*x^4 + 1)/((x^6 - x^2)^(1/3)*(x^4 + x - 1)), x)

maple [C] time = 5.48, size = 580, normalized size = 6.24

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^4+1)/(x^4+x-1)/(x^6-x^2)^(1/3),x)

[Out] 1/2*RootOf(_Z^2-2*_Z+4)*ln(-(-40235029434530*RootOf(_Z^2-2*_Z+4)^2*x^5-213844410247318*RootOf(_Z^2-2*_Z+4)*x^5+12260547776479020*x^5+301762720758975*RootOf(_Z^2-2*_Z+4)^2*x^2+6505058416224948*RootOf(_Z^2-2*_Z+4)*(x^6-x^2)^(2/3)-6505058416224948*RootOf(_Z^2-2*_Z+4)*(x^6-x^2)^(1/3)*x+40235029434530*RootOf(_Z^2-2*_Z+4)^2*x+5607218505590620*RootOf(_Z^2-2*_Z+4)*x^2-24254346850201524*(x^6-x^2)^(2/3)+24254346850201524*x*(x^6-x^2)^(1/3)+213844410247318*RootOf(_Z^2-2*_Z+4)*x-10625808072948484*x^2-12260547776479020*x)/(x^4+x-1)/x)-1/2*ln((103130*RootOf(_Z^2-2*_Z+4)^2*x^5-226806*RootOf(_Z^2-2*_Z+4)*x^5-20377448*x^5-773475*RootOf(_Z^2-2*_Z+4)^2*x^2+10436076*RootOf(_Z^2-2*_Z+4)*(x^6-x^2)^(2/3)-10436076*RootOf(_Z^2-2*_Z+4)*(x^6-x^2)^(1/3)*x-103130*RootOf(_Z^2-2*_Z+4)^2*x+11962480*RootOf(_Z^2-2*_Z+4)*x^2+20748876*(x^6-x^2)^(2/3)-20748876*x*(x^6-x^2)^(1/3)+226806*RootOf(_Z^2-2*_Z+4)*x-3134992*x^2+20377448*x)/(x^4+x-1)/x)*RootOf(_Z^2-2*_Z+4)+ln((103130*RootOf(_Z^2-2*_Z+4)^2*x^5

```
-226806*RootOf(_Z^2-2*_Z+4)*x^5-20377448*x^5-773475*RootOf(_Z^2-2*_Z+4)^2*x
^2+10436076*RootOf(_Z^2-2*_Z+4)*(x^6-x^2)^(2/3)-10436076*RootOf(_Z^2-2*_Z+4
)*(x^6-x^2)^(1/3)*x-103130*RootOf(_Z^2-2*_Z+4)^2*x+11962480*RootOf(_Z^2-2*_
Z+4)*x^2+20748876*(x^6-x^2)^(2/3)-20748876*x*(x^6-x^2)^(1/3)+226806*RootOf(
_Z^2-2*_Z+4)*x-3134992*x^2+20377448*x)/(x^4+x-1)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^4+1}{\left(x^6-x^2\right)^{\frac{1}{3}}\left(x^4+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+1)/(x^4+x-1)/(x^6-x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((3*x^4 + 1)/((x^6 - x^2)^(1/3)*(x^4 + x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x^4+1}{{\left(x^6-x^2\right)}^{1/3}\,\left(x^4+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4 + 1)/((x^6 - x^2)^(1/3)*(x + x^4 - 1)),x)
```

```
[Out] int((3*x^4 + 1)/((x^6 - x^2)^(1/3)*(x + x^4 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4+1)/(x**4+x-1)/(x**6-x**2)**(1/3),x)
```

```
[Out] Timed out
```

3.1152 $\int \frac{(2+x^3)\sqrt{-1-x^2+x^3}}{1-x^2-2x^3-x^4+x^5+x^6}\,dx$

Optimal. Leaf size=93

$$\frac{1}{5}\left(-5-\sqrt{5}\right)\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt{x^3-x^2-1}}\right)+\frac{1}{5}\left(5-\sqrt{5}\right)\tan^{-1}\left(\frac{\sqrt{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt{x^3-x^2-1}}\right)$$

Rubi [F] time = 0.67, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(2+x^3)\sqrt{-1-x^2+x^3}}{1-x^2-2x^3-x^4+x^5+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((2 + x^3)*Sqrt[-1 - x^2 + x^3])/(1 - x^2 - 2*x^3 - x^4 + x^5 + x^6),x]
```

```
[Out] 2*Defer[Int][Sqrt[-1 - x^2 + x^3]/(1 - x^2 - 2*x^3 - x^4 + x^5 + x^6), x] +
 Defer[Int][(x^3*Sqrt[-1 - x^2 + x^3])/(1 - x^2 - 2*x^3 - x^4 + x^5 + x^6),
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+x^3)\sqrt{-1-x^2+x^3}}{1-x^2-2x^3-x^4+x^5+x^6}\,dx &= \int \left(\frac{2\sqrt{-1-x^2+x^3}}{1-x^2-2x^3-x^4+x^5+x^6}+\frac{x^3\sqrt{-1-x^2+x^3}}{1-x^2-2x^3-x^4+x^5+x^6}\right)dx\\ &= 2\int \frac{\sqrt{-1-x^2+x^3}}{1-x^2-2x^3-x^4+x^5+x^6}\,dx+\int \frac{x^3\sqrt{-1-x^2+x^3}}{1-x^2-2x^3-x^4+x^5+x^6}\,dx\end{aligned}$$

Mathematica [C] time = 5.18, size = 25746, normalized size = 276.84

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + x^3)*Sqrt[-1 - x^2 + x^3])/(1 - x^2 - 2*x^3 - x^4 + x^5 + x
^6),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.54, size = 81, normalized size = 0.87

$$\frac{1}{5}\left(5-\sqrt{5}\right)\tan^{-1}\left(\frac{\left(\sqrt{5}-1\right)x}{2\sqrt{x^3-x^2-1}}\right)+\frac{1}{5}\left(-5-\sqrt{5}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{5}\right)x}{2\sqrt{x^3-x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^3)*Sqrt[-1 - x^2 + x^3])/(1 - x^2 - 2*x^3 - x^4
+ x^5 + x^6),x]
```

```
[Out] ((5 - Sqrt[5])*ArcTan[((-1 + Sqrt[5])*x)/(2*Sqrt[-1 - x^2 + x^3])])/5 + ((-
5 - Sqrt[5])*ArcTan[((1 + Sqrt[5])*x)/(2*Sqrt[-1 - x^2 + x^3])])/5
```

fricas [A] time = 0.52, size = 79, normalized size = 0.85

$$-\frac{1}{5}\,\sqrt{5}\,\sqrt{2\,\sqrt{5}+6}\,\arctan\left(\frac{x\sqrt{2\,\sqrt{5}+6}}{2\,\sqrt{x^3-x^2-1}}\right)+\frac{1}{5}\,\sqrt{5}\,\sqrt{-2\,\sqrt{5}+6}\,\arctan\left(\frac{x\sqrt{-2\,\sqrt{5}+6}}{2\,\sqrt{x^3-x^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^3-x^2-1)^(1/2)/(x^6+x^5-x^4-2*x^3-x^2+1),x, algorithm=
"fricas")
```

```
[Out] -1/5*sqrt(5)*sqrt(2*sqrt(5) + 6)*arctan(1/2*x*sqrt(2*sqrt(5) + 6)/sqrt(x^3
- x^2 - 1)) + 1/5*sqrt(5)*sqrt(-2*sqrt(5) + 6)*arctan(1/2*x*sqrt(-2*sqrt(5)
 + 6)/sqrt(x^3 - x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^3-x^2-1}\,\left(x^3+2\right)}{x^6+x^5-x^4-2\,x^3-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^3-x^2-1)^(1/2)/(x^6+x^5-x^4-2*x^3-x^2+1),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(x^3 - x^2 - 1)*(x^3 + 2)/(x^6 + x^5 - x^4 - 2*x^3 - x^2 + 1)
, x)
```

maple [C] time = 5.74, size = 2877, normalized size = 30.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2)*(x^3-x^2-1)^(1/2)/(x^6+x^5-x^4-2*x^3-x^2+1),x)
```

```
[Out] 2/3*I*3^(1/2)*(1/6*(116+12*93^(1/2))^(1/3)-2/3/(116+12*93^(1/2))^(1/3))*(-I
*(x+1/12*(116+12*93^(1/2))^(1/3)+1/3/(116+12*93^(1/2))^(1/3)-1/3+1/2*I*3^(1
/2)*(1/6*(116+12*93^(1/2))^(1/3)-2/3/(116+12*93^(1/2))^(1/3)))*3^(1/2)/(1/6
*(116+12*93^(1/2))^(1/3)-2/3/(116+12*93^(1/2))^(1/3)))^(1/2)*((x-1/6*(116+1
2*93^(1/2))^(1/3)-2/3/(116+12*93^(1/2))^(1/3)-1/3)/(-1/4*(116+12*93^(1/2))^
(1/3)-1/(116+12*93^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/6*(116+12*93^(1/2))^(1/3)-
2/3/(116+12*93^(1/2))^(1/3))))^(1/2)*(I*(x+1/12*(116+12*93^(1/2))^(1/3)+1/3
/(116+12*93^(1/2))^(1/3)-1/3-1/2*I*3^(1/2)*(1/6*(116+12*93^(1/2))^(1/3)-2/3
/(116+12*93^(1/2))^(1/3)))*3^(1/2)/(1/6*(116+12*93^(1/2))^(1/3)-2/3/(116+12
*93^(1/2))^(1/3)))^(1/2)/(x^3-x^2-1)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/1
2*(116+12*93^(1/2))^(1/3)+1/3/(116+12*93^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(1/
6*(116+12*93^(1/2))^(1/3)-2/3/(116+12*93^(1/2))^(1/3)))*3^(1/2)/(1/6*(116+1
2*93^(1/2))^(1/3)-2/3/(116+12*93^(1/2))^(1/3)))^(1/2),(-I*3^(1/2)*(1/6*(116
+12*93^(1/2))^(1/3)-2/3/(116+12*93^(1/2))^(1/3))/(-1/4*(116+12*93^(1/2))^(1
/3)-1/(116+12*93^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/6*(116+12*93^(1/2))^(1/3)-2/
3/(116+12*93^(1/2))^(1/3))))^(1/2))-1/360*I*2^(1/2)*4^(1/3)*sum(_alpha*(3*_
alpha^4+3*_alpha^3-3*_alpha^2-3*_alpha-4)*((29+3*93^(1/2))^(1/3)-4^(1/3)/(2
9+3*93^(1/2))^(1/3))*(-1/8*I*(-4+12*x+4^(1/3)*((29+3*93^(1/2))^(1/3)+4^(1/3
)/(29+3*93^(1/2))^(1/3)+I*3^(1/2)*((29+3*93^(1/2))^(1/3)-4^(1/3)/(29+3*93^(
1/2))^(1/3))))/((29+3*93^(1/2))^(1/3)-4^(1/3)/(29+3*93^(1/2))^(1/3)))^(1/2)
*((-2+6*x+4^(1/3)*(-(29+3*93^(1/2))^(1/3)-4^(1/3)/(29+3*93^(1/2))^(1/3)))/(
-3*(29+3*93^(1/2))^(1/3)-3*4^(1/3)/(29+3*93^(1/2))^(1/3)-I*3^(1/2)*((29+3*9
3^(1/2))^(1/3)-4^(1/3)/(29+3*93^(1/2))^(1/3))))^(1/2)*(1/8*I*(-4+12*x+4^(1/
3)*((29+3*93^(1/2))^(1/3)+4^(1/3)/(29+3*93^(1/2))^(1/3)-I*3^(1/2)*((29+3*93
^(1/2))^(1/3)-4^(1/3)/(29+3*93^(1/2))^(1/3))))/((29+3*93^(1/2))^(1/3)-4^(1/
```

3)/(29+3*93^(1/2))^(1/3)))^(1/2)/(x^3-x^2-1)^(1/2)*(32*_alpha^5+80*_alpha^4
-16*_alpha^3-144*_alpha^2-48*_alpha+80+4^(1/3)*(4*(3*3^(1/2)*31^(1/2)+29)^(
1/3)+4*I*3^(1/2)*(3*3^(1/2)*31^(1/2)+29)^(1/3)+18*4^(1/3)*_alpha*(3*3^(1/2)
*31^(1/2)+29)^(2/3)-76*4^(1/3)*_alpha^3*(3*3^(1/2)*31^(1/2)+29)^(2/3)-87*4^
(1/3)*_alpha^2*(3*3^(1/2)*31^(1/2)+29)^(2/3)+29*4^(1/3)*_alpha^4*(3*3^(1/2)
*31^(1/2)+29)^(2/3)+38*4^(1/3)*_alpha^5*(3*3^(1/2)*31^(1/2)+29)^(2/3)+4*_al
pha^4*(3*3^(1/2)*31^(1/2)+29)^(1/3)-14*_alpha^5*(3*3^(1/2)*31^(1/2)+29)^(1/
3)+29*4^(1/3)*(3*3^(1/2)*31^(1/2)+29)^(2/3)-36*_alpha*(3*3^(1/2)*31^(1/2)+2
9)^(1/3)+8*4^(1/3)*31^(1/2)*3^(1/2)*_alpha^3*(3*3^(1/2)*31^(1/2)+29)^(2/3)-
2*4^(1/3)*31^(1/2)*3^(1/2)*_alpha*(3*3^(1/2)*31^(1/2)+29)^(2/3)+9*4^(1/3)*3
1^(1/2)*3^(1/2)*_alpha^2*(3*3^(1/2)*31^(1/2)+29)^(2/3)-4*4^(1/3)*31^(1/2)*3
^(1/2)*_alpha^5*(3*3^(1/2)*31^(1/2)+29)^(2/3)-3*4^(1/3)*31^(1/2)*3^(1/2)*_a
lpha^4*(3*3^(1/2)*31^(1/2)+29)^(2/3)+9*I*4^(1/3)*31^(1/2)*_alpha^4*(3*3^(1/
2)*31^(1/2)+29)^(2/3)-29*I*3^(1/2)*4^(1/3)*_alpha^4*(3*3^(1/2)*31^(1/2)+29)
^(2/3)-24*I*4^(1/3)*31^(1/2)*_alpha^3*(3*3^(1/2)*31^(1/2)+29)^(2/3)+76*I*3^
(1/2)*4^(1/3)*_alpha^3*(3*3^(1/2)*31^(1/2)+29)^(2/3)-27*I*4^(1/3)*31^(1/2)*
_alpha^2*(3*3^(1/2)*31^(1/2)+29)^(2/3)+87*I*3^(1/2)*4^(1/3)*_alpha^2*(3*3^(
1/2)*31^(1/2)+29)^(2/3)-18*I*3^(1/2)*4^(1/3)*_alpha*(3*3^(1/2)*31^(1/2)+29)
^(2/3)+6*I*4^(1/3)*31^(1/2)*_alpha*(3*3^(1/2)*31^(1/2)+29)^(2/3)+12*I*4^(1/
3)*31^(1/2)*_alpha^5*(3*3^(1/2)*31^(1/2)+29)^(2/3)-38*I*3^(1/2)*4^(1/3)*_al
pha^5*(3*3^(1/2)*31^(1/2)+29)^(2/3)-12*_alpha^2*(3*3^(1/2)*31^(1/2)+29)^(1/
3)+28*_alpha^3*(3*3^(1/2)*31^(1/2)+29)^(1/3)-3*4^(1/3)*31^(1/2)*3^(1/2)*(3*
3^(1/2)*31^(1/2)+29)^(2/3)-4*3^(1/2)*31^(1/2)*_alpha^3*(3*3^(1/2)*31^(1/2)+
29)^(1/3)+4*3^(1/2)*31^(1/2)*_alpha*(3*3^(1/2)*31^(1/2)+29)^(1/3)-29*I*3^(1
/2)*4^(1/3)*(3*3^(1/2)*31^(1/2)+29)^(2/3)+9*I*4^(1/3)*31^(1/2)*(3*3^(1/2)*3
1^(1/2)+29)^(2/3)+2*3^(1/2)*31^(1/2)*_alpha^5*(3*3^(1/2)*31^(1/2)+29)^(1/3)
+4*I*3^(1/2)*_alpha^4*(3*3^(1/2)*31^(1/2)+29)^(1/3)-12*I*31^(1/2)*_alpha^3*
(3*3^(1/2)*31^(1/2)+29)^(1/3)+28*I*3^(1/2)*_alpha^3*(3*3^(1/2)*31^(1/2)+29)
^(1/3)-12*I*3^(1/2)*_alpha^2*(3*3^(1/2)*31^(1/2)+29)^(1/3)+12*I*31^(1/2)*_a
lpha*(3*3^(1/2)*31^(1/2)+29)^(1/3)-36*I*3^(1/2)*_alpha*(3*3^(1/2)*31^(1/2)+
29)^(1/3)+6*I*31^(1/2)*_alpha^5*(3*3^(1/2)*31^(1/2)+29)^(1/3)-14*I*3^(1/2)*
_alpha^5*(3*3^(1/2)*31^(1/2)+29)^(1/3)))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/12
*(116+12*93^(1/2))^(1/3)+1/3/(116+12*93^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(1/6
*(116+12*93^(1/2))^(1/3)-2/3/(116+12*93^(1/2))^(1/3)))*3^(1/2)/(1/6*(116+12
*93^(1/2))^(1/3)-2/3/(116+12*93^(1/2))^(1/3)))^(1/2),-1/3+3/4*_alpha*(116+1
2*3^(1/2)*31^(1/2))^(1/3)-3/8*_alpha*(116+12*3^(1/2)*31^(1/2))^(2/3)-29/8*_
alpha^2*(116+12*3^(1/2)*31^(1/2))^(1/3)-19/6*_alpha^3*(116+12*3^(1/2)*31^(1
/2))^(1/3)+7/24*_alpha^3*(116+12*3^(1/2)*31^(1/2))^(2/3)-1/8*_alpha^2*(116+
12*3^(1/2)*31^(1/2))^(2/3)+1/24*_alpha^4*(116+12*3^(1/2)*31^(1/2))^(2/3)+29
/24*_alpha^4*(116+12*3^(1/2)*31^(1/2))^(1/3)+19/12*_alpha^5*(116+12*3^(1/2)
*31^(1/2))^(1/3)-7/48*_alpha^5*(116+12*3^(1/2)*31^(1/2))^(2/3)-1/8*3^(1/2)*
31^(1/2)*(116+12*3^(1/2)*31^(1/2))^(1/3)+1/3*I*_alpha*31^(1/2)+7/6*_alpha^5
-1/3*_alpha^4-7/3*_alpha^3+_alpha^2+3*_alpha-1/8*I*(116+12*3^(1/2)*31^(1/2)
)^(1/3)*31^(1/2)+1/8*I*3^(1/2)*(116+12*3^(1/2)*31^(1/2))^(1/3)+2*I*3^(1/2)*
(116+12*3^(1/2)*31^(1/2))^(2/3)-5/8*I*(116+12*3^(1/2)*31^(1/2))^(2/3)*31^(1
/2)+29/24*(116+12*3^(1/2)*31^(1/2))^(1/3)+1/24*(116+12*3^(1/2)*31^(1/2))^(2
/3)+1/3*_alpha^3*(116+12*3^(1/2)*31^(1/2))^(1/3)*31^(1/2)*3^(1/2)-1/24*_alp
ha^3*(116+12*3^(1/2)*31^(1/2))^(2/3)*31^(1/2)*3^(1/2)+1/24*_alpha*(116+12*3
^(1/2)*31^(1/2))^(2/3)*31^(1/2)*3^(1/2)+3/8*_alpha^2*(116+12*3^(1/2)*31^(1/
2))^(1/3)*31^(1/2)*3^(1/2)-1/12*_alpha*(116+12*3^(1/2)*31^(1/2))^(1/3)*31^(
1/2)*3^(1/2)-1/3*I*_alpha^3*31^(1/2)+1/6*I*_alpha^5*31^(1/2)-1/6*_alpha^5*(
116+12*3^(1/2)*31^(1/2))^(1/3)*31^(1/2)*3^(1/2)+1/48*_alpha^5*(116+12*3^(1/
2)*31^(1/2))^(2/3)*31^(1/2)*3^(1/2)-1/8*_alpha^4*(116+12*3^(1/2)*31^(1/2))^
(1/3)*31^(1/2)*3^(1/2)+1/8*I*_alpha^4*3^(1/2)*(116+12*3^(1/2)*31^(1/2))^(1/
3)-5/8*I*_alpha^4*(116+12*3^(1/2)*31^(1/2))^(2/3)*31^(1/2)+2*I*_alpha^4*3^(
1/2)*(116+12*3^(1/2)*31^(1/2))^(2/3)+1/3*I*_alpha^3*(116+12*3^(1/2)*31^(1/2
))^(1/3)*31^(1/2)-I*_alpha^3*3^(1/2)*(116+12*3^(1/2)*31^(1/2))^(1/3)+1/6*I*
_alpha^3*(116+12*3^(1/2)*31^(1/2))^(2/3)*31^(1/2)-1/2*I*_alpha^3*3^(1/2)*(1
16+12*3^(1/2)*31^(1/2))^(2/3)-17/24*I*_alpha^2*3^(1/2)*(116+12*3^(1/2)*31^(

```
1/2))^(1/3)+9/8*I*_alpha^2*(116+12*3^(1/2)*31^(1/2))^(2/3)*31^(1/2)-43/12*I
*_alpha^2*3^(1/2)*(116+12*3^(1/2)*31^(1/2))^(2/3)-13/12*I*_alpha*3^(1/2)*(1
16+12*3^(1/2)*31^(1/2))^(2/3)+1/3*I*_alpha*(116+12*3^(1/2)*31^(1/2))^(2/3)*
31^(1/2)-1/12*I*_alpha*(116+12*3^(1/2)*31^(1/2))^(1/3)*31^(1/2)+5/12*I*_alp
ha*3^(1/2)*(116+12*3^(1/2)*31^(1/2))^(1/3)-1/6*I*_alpha^5*(116+12*3^(1/2)*3
1^(1/2))^(1/3)*31^(1/2)+5/12*I*_alpha^5*3^(1/2)*(116+12*3^(1/2)*31^(1/2))^(
1/3)-13/48*I*_alpha^5*(116+12*3^(1/2)*31^(1/2))^(2/3)*31^(1/2)+41/48*I*_alp
ha^5*3^(1/2)*(116+12*3^(1/2)*31^(1/2))^(2/3)+3/8*I*(116+12*3^(1/2)*31^(1/2)
)^(1/3)*_alpha^2*31^(1/2)-1/8*I*(116+12*3^(1/2)*31^(1/2))^(1/3)*_alpha^4*31
^(1/2),(-I*3^(1/2)*(1/6*(116+12*93^(1/2))^(1/3)-2/3/(116+12*93^(1/2))^(1/3)
)/(-1/4*(116+12*93^(1/2))^(1/3)-1/(116+12*93^(1/2))^(1/3)-1/2*I*3^(1/2)*(1/
6*(116+12*93^(1/2))^(1/3)-2/3/(116+12*93^(1/2))^(1/3))))^(1/2)),_alpha=Root
Of(_Z^6+_Z^5-_Z^4-2*_Z^3-_Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^3 - x^2 - 1}\,(x^3 + 2)}{x^6 + x^5 - x^4 - 2\,x^3 - x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(x^3-x^2-1)^(1/2)/(x^6+x^5-x^4-2*x^3-x^2+1),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(x^3 - x^2 - 1)*(x^3 + 2)/(x^6 + x^5 - x^4 - 2*x^3 - x^2 + 1)
, x)
```

mupad [B] time = 3.20, size = 2803, normalized size = 30.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3 + 2)*(x^3 - x^2 - 1)^(1/2))/(x^2 + 2*x^3 + x^4 - x^5 - x^6 - 1),
x)
```

```
[Out] symsum(-(2*((x - (3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) -
 ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*1i)/2 + 1/(18*((31^(1/2)*108^(1/
2))/108 + 29/54)^(1/3)) + ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)/2 - 1/3)
/(1/(6*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - (3^(1/2)*(1/(9*((31^(1/2
)*108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)
)*1i)/2 + (3*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))/2))^(1/2)*((1/(9*((3
1^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - x + ((31^(1/2)*108^(1/2))/108 + 29
/54)^(1/3) + 1/3)/(1/(6*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - (3^(1/2
)*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/1
08 + 29/54)^(1/3))*1i)/2 + (3*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))/2))
^(1/2)*ellipticPi((1/(6*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - (3^(1/2
)*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/1
08 + 29/54)^(1/3))*1i)/2 + (3*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))/2)/
(root(z^6 + z^5 - z^4 - 2*z^3 - z^2 + 1, z, k) - (3^(1/2)*(1/(9*((31^(1/2)*
108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*
1i)/2 + 1/(18*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) + ((31^(1/2)*108^(1
/2))/108 + 29/54)^(1/3)/2 - 1/3), asin(((x - (3^(1/2)*(1/(9*((31^(1/2)*108^
(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*1i)/
2 + 1/(18*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) + ((31^(1/2)*108^(1/2))
/108 + 29/54)^(1/3)/2 - 1/3)/(1/(6*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)
) - (3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*1
08^(1/2))/108 + 29/54)^(1/3))*1i)/2 + (3*((31^(1/2)*108^(1/2))/108 + 29/54)
^(1/3))/2))^(1/2)), (3^(1/2)*(1/(6*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)
) - (3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*1
08^(1/2))/108 + 29/54)^(1/3))*1i)/2 + (3*((31^(1/2)*108^(1/2))/108 + 29/54)
^(1/3))/2)*1i)/(3*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1
```

```
/2)*108^(1/2))/108 + 29/54)^(1/3))))*(1/(6*((31^(1/2)*108^(1/2))/108 + 29/5
4)^(1/3)) - (3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((31
^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*1i)/2 + (3*((31^(1/2)*108^(1/2))/108
+ 29/54)^(1/3))/2)*(-(3^(1/2)*(x + (3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/108
 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*1i)/2 + 1/(18*
((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) + ((31^(1/2)*108^(1/2))/108 + 29/
54)^(1/3)/2 - 1/3)*1i)/(3*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) -
 ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))))^(1/2)*(root(z^6 + z^5 - z^4 -
2*z^3 - z^2 + 1, z, k)^2 - 3*root(z^6 + z^5 - z^4 - 2*z^3 - z^2 + 1, z, k)^
3 - root(z^6 + z^5 - z^4 - 2*z^3 - z^2 + 1, z, k)^4 + 2*root(z^6 + z^5 - z^
4 - 2*z^3 - z^2 + 1, z, k)^5 + 3))/((x^3 - x^2 - x*(((3^(1/2)*(1/(9*((31^(1
/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/
3))*1i)/2 - 1/(18*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*10
8^(1/2))/108 + 29/54)^(1/3)/2 + 1/3)*((3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/
108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*1i)/2 + 1/(
18*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) + ((31^(1/2)*108^(1/2))/108 +
29/54)^(1/3)/2 - 1/3) - (1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) + (
(31^(1/2)*108^(1/2))/108 + 29/54)^(1/3) + 1/3)*((3^(1/2)*(1/(9*((31^(1/2)*1
08^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*1
i)/2 - 1/(18*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/
2))/108 + 29/54)^(1/3)/2 + 1/3) + (1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^
(1/3)) + ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3) + 1/3)*((3^(1/2)*(1/(9*((
31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54
)^(1/3))*1i)/2 + 1/(18*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) + ((31^(1/
2)*108^(1/2))/108 + 29/54)^(1/3)/2 - 1/3)) + (1/(9*((31^(1/2)*108^(1/2))/10
8 + 29/54)^(1/3)) + ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3) + 1/3)*((3^(1/
2)*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/
108 + 29/54)^(1/3))*1i)/2 - 1/(18*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))
 - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)/2 + 1/3)*((3^(1/2)*(1/(9*((31^(
1/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1
/3))*1i)/2 + 1/(18*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) + ((31^(1/2)*1
08^(1/2))/108 + 29/54)^(1/3)/2 - 1/3))^(1/2)*(root(z^6 + z^5 - z^4 - 2*z^3
- z^2 + 1, z, k) - (3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))
 - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*1i)/2 + 1/(18*((31^(1/2)*108^(
1/2))/108 + 29/54)^(1/3)) + ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)/2 - 1/
3)*(6*root(z^6 + z^5 - z^4 - 2*z^3 - z^2 + 1, z, k)^2 + 4*root(z^6 + z^5 -
z^4 - 2*z^3 - z^2 + 1, z, k)^3 - 5*root(z^6 + z^5 - z^4 - 2*z^3 - z^2 + 1,
z, k)^4 - 6*root(z^6 + z^5 - z^4 - 2*z^3 - z^2 + 1, z, k)^5 + 2*root(z^6 +
z^5 - z^4 - 2*z^3 - z^2 + 1, z, k))), k, 1, 6) + (2*((x - (3^(1/2)*(1/(9*((
31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54
)^(1/3))*1i)/2 + 1/(18*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) + ((31^(1/
2)*108^(1/2))/108 + 29/54)^(1/3)/2 - 1/3)/(1/(6*((31^(1/2)*108^(1/2))/108 +
 29/54)^(1/3)) - (3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) -
 ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*1i)/2 + (3*((31^(1/2)*108^(1/2))
/108 + 29/54)^(1/3))/2))^(1/2)*((1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1
/3)) - x + ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3) + 1/3)/(1/(6*((31^(1/2)
*108^(1/2))/108 + 29/54)^(1/3)) - (3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/108
+ 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*1i)/2 + (3*((31
^(1/2)*108^(1/2))/108 + 29/54)^(1/3))/2))^(1/2)*ellipticF(asin(((x - (3^(1/
2)*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/
108 + 29/54)^(1/3))*1i)/2 + 1/(18*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))
 + ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)/2 - 1/3)/(1/(6*((31^(1/2)*108^(
1/2))/108 + 29/54)^(1/3)) - (3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/5
4)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*1i)/2 + (3*((31^(1/2)
*108^(1/2))/108 + 29/54)^(1/3))/2))^(1/2)), (3^(1/2)*(1/(6*((31^(1/2)*108^(
1/2))/108 + 29/54)^(1/3)) - (3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/5
4)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*1i)/2 + (3*((31^(1/2)
*108^(1/2))/108 + 29/54)^(1/3))/2)*1i)/(3*(1/(9*((31^(1/2)*108^(1/2))/108 +
 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))))*(1/(6*((31^(1/
```

```
2)*108^(1/2))/108 + 29/54)^(1/3)) - (3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/10
8 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*1i)/2 + (3*((
31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))/2)*(-(3^(1/2)*(x + (3^(1/2)*(1/(9*(
(31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/5
4)^(1/3))*1i)/2 + 1/(18*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) + ((31^(1
/2)*108^(1/2))/108 + 29/54)^(1/3)/2 - 1/3)*1i)/(3*(1/(9*((31^(1/2)*108^(1/2
))/108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))))^(1/2))
/(x^3 - x^2 - x*(((3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))
- ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*1i)/2 - 1/(18*((31^(1/2)*108^(1
/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)/2 + 1/3
)*((3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*10
8^(1/2))/108 + 29/54)^(1/3))*1i)/2 + 1/(18*((31^(1/2)*108^(1/2))/108 + 29/5
4)^(1/3)) + ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)/2 - 1/3) - (1/(9*((31^
(1/2)*108^(1/2))/108 + 29/54)^(1/3)) + ((31^(1/2)*108^(1/2))/108 + 29/54)^(
1/3) + 1/3)*((3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((3
1^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*1i)/2 - 1/(18*((31^(1/2)*108^(1/2))/
108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)/2 + 1/3) + (
1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) + ((31^(1/2)*108^(1/2))/108
+ 29/54)^(1/3) + 1/3)*((3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1
/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*1i)/2 + 1/(18*((31^(1/2)*1
08^(1/2))/108 + 29/54)^(1/3)) + ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)/2
- 1/3)) + (1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) + ((31^(1/2)*108^
(1/2))/108 + 29/54)^(1/3) + 1/3)*((3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/108
+ 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*1i)/2 - 1/(18*(
(31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)) - ((31^(1/2)*108^(1/2))/108 + 29/5
4)^(1/3)/2 + 1/3)*((3^(1/2)*(1/(9*((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))
 - ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3))*1i)/2 + 1/(18*((31^(1/2)*108^(
1/2))/108 + 29/54)^(1/3)) + ((31^(1/2)*108^(1/2))/108 + 29/54)^(1/3)/2 - 1/
3))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+2\right)\sqrt{x^3-x^2-1}}{x^6+x^5-x^4-2x^3-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2)*(x**3-x**2-1)**(1/2)/(x**6+x**5-x**4-2*x**3-x**2+1),x)
```

```
[Out] Integral((x**3 + 2)*sqrt(x**3 - x**2 - 1)/(x**6 + x**5 - x**4 - 2*x**3 - x*
*2 + 1), x)
```

3.1153 $\int \frac{(1+x^4)\sqrt{-1-x^2+x^4}}{1-x^2-3x^4+x^6+x^8}\,dx$

Optimal. Leaf size=93

$$\frac{1}{10}\left(-5-\sqrt{5}\right)\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt{x^4-x^2-1}}\right)+\frac{1}{10}\left(5-\sqrt{5}\right)\tan^{-1}\left(\frac{\sqrt{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt{x^4-x^2-1}}\right)$$

Rubi [F] time = 0.59, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+x^4\right)\sqrt{-1-x^2+x^4}}{1-x^2-3x^4+x^6+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^4)*Sqrt[-1 - x^2 + x^4])/(1 - x^2 - 3*x^4 + x^6 + x^8),x]
```

```
[Out] Defer[Int][Sqrt[-1 - x^2 + x^4]/(1 - x^2 - 3*x^4 + x^6 + x^8), x] + Defer[I
nt][(x^4*Sqrt[-1 - x^2 + x^4])/(1 - x^2 - 3*x^4 + x^6 + x^8), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+x^4\right)\sqrt{-1-x^2+x^4}}{1-x^2-3x^4+x^6+x^8}\,dx &= \int \left(\frac{\sqrt{-1-x^2+x^4}}{1-x^2-3x^4+x^6+x^8}+\frac{x^4\sqrt{-1-x^2+x^4}}{1-x^2-3x^4+x^6+x^8}\right)dx\\
&= \int \frac{\sqrt{-1-x^2+x^4}}{1-x^2-3x^4+x^6+x^8}\,dx+\int \frac{x^4\sqrt{-1-x^2+x^4}}{1-x^2-3x^4+x^6+x^8}\,dx
\end{aligned}$$

Mathematica [C] time = 6.48, size = 5470, normalized size = 58.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + x^4)*Sqrt[-1 - x^2 + x^4])/(1 - x^2 - 3*x^4 + x^6 + x^8),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.42, size = 81, normalized size = 0.87

$$\frac{1}{10}\left(5-\sqrt{5}\right)\tan^{-1}\left(\frac{\left(\sqrt{5}-1\right)x}{2\sqrt{x^4-x^2-1}}\right)+\frac{1}{10}\left(-5-\sqrt{5}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{5}\right)x}{2\sqrt{x^4-x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^4)*Sqrt[-1 - x^2 + x^4])/(1 - x^2 - 3*x^4 + x^6
+ x^8),x]
```

```
[Out] ((5 - Sqrt[5])*ArcTan[((-1 + Sqrt[5])*x)/(2*Sqrt[-1 - x^2 + x^4])])/10 + ((
-5 - Sqrt[5])*ArcTan[((1 + Sqrt[5])*x)/(2*Sqrt[-1 - x^2 + x^4])])/10
```

fricas [B] time = 0.64, size = 313, normalized size = 3.37

$$\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+3}\arctan\left(-\frac{2\sqrt{10}\sqrt{x^4-x^2-1}\left(5x^3+\sqrt{5}\left(2x^5-5x^3-2x\right)\right)\sqrt{\sqrt{5}+3}+\sqrt{10}\left(15x^8-65x^6+5x^4+65x^2-\sqrt{5}\left(7x^8-29x^6+x^4+29x^2+7\right)+15\right)\sqrt{4\sqrt{5}+9}\sqrt{\sqrt{5}+3}}{20\left(x^8-5x^6+3x^4+5x^2+1\right)}\right)+\frac{1}{10}\sqrt{10}\sqrt{-\sqrt{5}+3}\arctan\left(-\frac{40\sqrt{10}\sqrt{x^4-x^2-1}\left(5x^3-\sqrt{5}\left(2x^5-5x^3-2x\right)\right)\sqrt{-\sqrt{5}+3}+\sqrt{10}\left(15x^8-65x^6+5x^4+65x^2+\sqrt{5}\left(7x^8-29x^6+x^4+29x^2+7\right)+15\right)\sqrt{-\sqrt{5}+3}\sqrt{-1600\sqrt{5}+3600}}{400\left(x^8-5x^6+3x^4+5x^2+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^4-x^2-1)^(1/2)/(x^8+x^6-3*x^4-x^2+1),x, algorithm="fri
cas")
```

```
[Out] 1/10*sqrt(10)*sqrt(sqrt(5) + 3)*arctan(-1/20*(2*sqrt(10)*sqrt(x^4 - x^2 - 1
)*(5*x^3 + sqrt(5)*(2*x^5 - 5*x^3 - 2*x))*sqrt(sqrt(5) + 3) + sqrt(10)*(15*
x^8 - 65*x^6 + 5*x^4 + 65*x^2 - sqrt(5)*(7*x^8 - 29*x^6 + x^4 + 29*x^2 + 7)
 + 15)*sqrt(4*sqrt(5) + 9)*sqrt(sqrt(5) + 3))/(x^8 - 5*x^6 + 3*x^4 + 5*x^2
+ 1)) + 1/10*sqrt(10)*sqrt(-sqrt(5) + 3)*arctan(-1/400*(40*sqrt(10)*sqrt(x^
4 - x^2 - 1)*(5*x^3 - sqrt(5)*(2*x^5 - 5*x^3 - 2*x))*sqrt(-sqrt(5) + 3) + s
qrt(10)*(15*x^8 - 65*x^6 + 5*x^4 + 65*x^2 + sqrt(5)*(7*x^8 - 29*x^6 + x^4 +
 29*x^2 + 7) + 15)*sqrt(-sqrt(5) + 3)*sqrt(-1600*sqrt(5) + 3600))/(x^8 - 5*
x^6 + 3*x^4 + 5*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4-x^2-1}\left(x^4+1\right)}{x^8+x^6-3\,x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^4-x^2-1)^(1/2)/(x^8+x^6-3*x^4-x^2+1),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(x^4 - x^2 - 1)*(x^4 + 1)/(x^8 + x^6 - 3*x^4 - x^2 + 1), x)
```

maple [B] time = 0.09, size = 216, normalized size = 2.32

$$-\frac{6\sqrt{2}\,\sqrt{5}\,\arctan\left(\frac{4\sqrt{x^4-x^2-1}\,\sqrt{2}}{x\left(2\sqrt{10}-2\sqrt{2}\right)}\right)}{5\left(2\sqrt{10}-2\sqrt{2}\right)}+\frac{2\sqrt{2}\,\arctan\left(\frac{4\sqrt{x^4-x^2-1}\,\sqrt{2}}{x\left(2\sqrt{10}-2\sqrt{2}\right)}\right)}{2\sqrt{10}-2\sqrt{2}}+\frac{2\sqrt{2}\,\arctan\left(\frac{4\sqrt{x^4-x^2-1}\,\sqrt{2}}{x\left(2\sqrt{10}+2\sqrt{2}\right)}\right)}{2\sqrt{10}+2\sqrt{2}}+\frac{6\sqrt{2}\,\sqrt{5}\,\arctan\left(\frac{4\sqrt{x^4-x^2-1}\,\sqrt{2}}{x\left(2\sqrt{10}+2\sqrt{2}\right)}\right)}{5\left(2\sqrt{10}+2\sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)*(x^4-x^2-1)^(1/2)/(x^8+x^6-3*x^4-x^2+1),x)
```

```
[Out] -6/5*2^(1/2)*5^(1/2)/(2*10^(1/2)-2*2^(1/2))*arctan(4*(x^4-x^2-1)^(1/2)*2^(1
/2)/x/(2*10^(1/2)-2*2^(1/2)))+2*2^(1/2)/(2*10^(1/2)-2*2^(1/2))*arctan(4*(x^
4-x^2-1)^(1/2)*2^(1/2)/x/(2*10^(1/2)-2*2^(1/2)))+2*2^(1/2)/(2*10^(1/2)+2*2^
(1/2))*arctan(4*(x^4-x^2-1)^(1/2)*2^(1/2)/x/(2*10^(1/2)+2*2^(1/2)))+6/5*2^(
1/2)*5^(1/2)/(2*10^(1/2)+2*2^(1/2))*arctan(4*(x^4-x^2-1)^(1/2)*2^(1/2)/x/(2
*10^(1/2)+2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4-x^2-1}\left(x^4+1\right)}{x^8+x^6-3\,x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^4-x^2-1)^(1/2)/(x^8+x^6-3*x^4-x^2+1),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x^4 - x^2 - 1)*(x^4 + 1)/(x^8 + x^6 - 3*x^4 - x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4+1\right)\,\sqrt{x^4-x^2-1}}{x^8+x^6-3\,x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)*(x^4 - x^2 - 1)^(1/2))/(x^6 - 3*x^4 - x^2 + x^8 + 1),x)
[Out] int(((x^4 + 1)*(x^4 - x^2 - 1)^(1/2))/(x^6 - 3*x^4 - x^2 + x^8 + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)*(x**4-x**2-1)**(1/2)/(x**8+x**6-3*x**4-x**2+1),x)
[Out] Timed out
```

3.1154 $\int \frac{x^2\sqrt{x+\sqrt{1+x}}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=93

$$\frac{\sqrt{x+\sqrt{x+1}}\left(128x^2+328x+563\right)}{3840}+\frac{\sqrt{x+1}\left(640x^2-872x+975\right)\sqrt{x+\sqrt{x+1}}}{1920}+\frac{385}{512}\log\left(2\sqrt{x+1}-2\sqrt{\right.$$

Rubi [A] time = 0.41, antiderivative size = 147, normalized size of antiderivative = 1.58, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1661, 640, 612, 621, 206}

$$\frac{1}{3}(x+1)^{3/2}\left(x+\sqrt{x+1}\right)^{3/2}-\frac{3}{10}(x+1)\left(x+\sqrt{x+1}\right)^{3/2}-\frac{39}{80}\sqrt{x+1}\left(x+\sqrt{x+1}\right)^{3/2}+\frac{33}{160}\left(x+\sqrt{x+1}\right)^{3/2}+\frac{77}{256}\left(2\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}}-\frac{385}{512}\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[x + Sqrt[1 + x]])/Sqrt[1 + x],x]
```

```
[Out] (33*(x + Sqrt[1 + x])^(3/2))/160 - (39*Sqrt[1 + x]*(x + Sqrt[1 + x])^(3/2))
/80 - (3*(1 + x)*(x + Sqrt[1 + x])^(3/2))/10 + ((1 + x)^(3/2)*(x + Sqrt[1 +
 x])^(3/2))/3 + (77*Sqrt[x + Sqrt[1 + x]]*(1 + 2*Sqrt[1 + x]))/256 - (385*A
rcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/512
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2\sqrt{x+\sqrt{1+x}}}{\sqrt{1+x}}\,dx = 2\,\text{Subst}\left(\int \left(-1+x^2\right)^2\sqrt{-1+x+x^2}\,dx, x, \sqrt{1+x}\right)$$

$$= \frac{1}{3}(1+x)^{3/2}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{3}\,\text{Subst}\left(\int \sqrt{-1+x+x^2}\left(6-9x^2-\frac{9x^3}{2}\right)dx, x, \sqrt{1+x}\right.$$

$$= -\frac{3}{10}(1+x)\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{3}(1+x)^{3/2}\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{15}\,\text{Subst}\left(\int \left(30-9x-\right.\right.$$

$$= -\frac{39}{80}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{3}{10}(1+x)\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{3}(1+x)^{3/2}\left(x+\sqrt{1+x}\right)^{3/2}$$

$$= \frac{33}{160}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{39}{80}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{3}{10}(1+x)\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{3}(1+$$

$$= \frac{33}{160}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{39}{80}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{3}{10}(1+x)\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{3}(1+$$

$$= \frac{33}{160}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{39}{80}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{3}{10}(1+x)\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{3}(1+$$

$$= \frac{33}{160}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{39}{80}\sqrt{1+x}\left(x+\sqrt{1+x}\right)^{3/2} - \frac{3}{10}(1+x)\left(x+\sqrt{1+x}\right)^{3/2} + \frac{1}{3}(1+$$

Mathematica [A] time = 0.11, size = 92, normalized size = 0.99

$$\frac{2\sqrt{x+\sqrt{x+1}}\left(128\left(10\sqrt{x+1}+1\right)x^2-8\left(218\sqrt{x+1}-41\right)x+1950\sqrt{x+1}+563\right)-5775\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)}{7680}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[x + Sqrt[1 + x]])/Sqrt[1 + x],x]
```

```
[Out] (2*Sqrt[x + Sqrt[1 + x]]*(563 + 1950*Sqrt[1 + x] + 128*x^2*(1 + 10*Sqrt[1 +
 x]) - 8*x*(-41 + 218*Sqrt[1 + x])) - 5775*ArcTanh[(1 + 2*Sqrt[1 + x])/(2*S
qrt[x + Sqrt[1 + x]])])/7680
```

IntegrateAlgebraic [A] time = 0.25, size = 90, normalized size = 0.97

$$\frac{\sqrt{x+\sqrt{x+1}}\left(1280(x+1)^{5/2}+128(x+1)^2-4304(x+1)^{3/2}+72(x+1)+4974\sqrt{x+1}+363\right)}{3840}+\frac{385}{512}\log\left(-2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[x + Sqrt[1 + x]])/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[x + Sqrt[1 + x]]*(363 + 4974*Sqrt[1 + x] + 72*(1 + x) - 4304*(1 + x)^
(3/2) + 128*(1 + x)^2 + 1280*(1 + x)^(5/2)))/3840 + (385*Log[-1 - 2*Sqrt[1
+ x] + 2*Sqrt[x + Sqrt[1 + x]]])/512
```

fricas [A] time = 1.15, size = 74, normalized size = 0.80

$$\frac{1}{3840}\left(128\,x^2+2\left(640\,x^2-872\,x+975\right)\sqrt{x+1}+328\,x+563\right)\sqrt{x+\sqrt{x+1}}+\frac{385}{1024}\log\left(4\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)-8\,x-8\sqrt{x+1}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x+(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3840*(128*x^2 + 2*(640*x^2 - 872*x + 975)*sqrt(x + 1) + 328*x + 563)*sqrt
(x + sqrt(x + 1)) + 385/1024*log(4*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) + 1
) - 8*x - 8*sqrt(x + 1) - 5)
```

giac [A] time = 0.16, size = 80, normalized size = 0.86

$$\frac{1}{3840}\left(2\left(4\left(2\left(8\sqrt{x+1}\left(10\sqrt{x+1}+1\right)-269\right)\sqrt{x+1}+9\right)\sqrt{x+1}+2487\right)\sqrt{x+1}+363\right)\sqrt{x+\sqrt{x+1}}+\frac{385}{512}\log\left(-2\sqrt{x+\sqrt{x+1}}+2\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x+(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(2*(8*sqrt(x + 1)*(10*sqrt(x + 1) + 1) - 269)*sqrt(x + 1) + 9)
*sqrt(x + 1) + 2487)*sqrt(x + 1) + 363)*sqrt(x + sqrt(x + 1)) + 385/512*log
(-2*sqrt(x + sqrt(x + 1)) + 2*sqrt(x + 1) + 1)
```

maple [A] time = 0.01, size = 98, normalized size = 1.05

$$\frac{(1+x)^{\frac{3}{2}}\left(x+\sqrt{1+x}\right)^{\frac{3}{2}}}{3}-\frac{3(1+x)\left(x+\sqrt{1+x}\right)^{\frac{3}{2}}}{10}-\frac{39\sqrt{1+x}\,\left(x+\sqrt{1+x}\right)^{\frac{3}{2}}}{80}+\frac{33\left(x+\sqrt{1+x}\right)^{\frac{3}{2}}}{160}+\frac{77\left(1+2\sqrt{1+x}\right)\sqrt{x+\sqrt{1+x}}}{256}-\frac{385\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x+(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x)
```

```
[Out] 1/3*(1+x)^(3/2)*(x+(1+x)^(1/2))^(3/2)-3/10*(1+x)*(x+(1+x)^(1/2))^(3/2)-39/8
0*(1+x)^(1/2)*(x+(1+x)^(1/2))^(3/2)+33/160*(x+(1+x)^(1/2))^(3/2)+77/256*(1+
2*(1+x)^(1/2))*(x+(1+x)^(1/2))^(1/2)-385/512*ln(1/2+(1+x)^(1/2)+(x+(1+x)^(1
/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x+1}}\,x^2}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x+(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1))*x^2/sqrt(x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x + (x + 1)^(1/2))^(1/2))/(x + 1)^(1/2),x)
```

```
[Out] int((x^2*(x + (x + 1)^(1/2))^(1/2))/(x + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x+(1+x)**(1/2))**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x + sqrt(x + 1))/sqrt(x + 1), x)
```

3.1155 $\int \left(-1+x^3\right)^{2/3} \, dx$

Optimal. Leaf size=94

$$\frac{1}{3} \left(x^3-1\right)^{2/3} x+\frac{2}{9} \log \left(\sqrt[3]{x^3-1}-x\right)-\frac{2 \tan ^{-1}\left(\frac{\sqrt{3} x}{2 \sqrt[3]{x^3-1}+x}\right)}{3 \sqrt{3}}-\frac{1}{9} \log \left(\sqrt[3]{x^3-1} x+\left(x^3-1\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 0.67, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {195, 239}

$$\frac{1}{3} \left(x^3-1\right)^{2/3} x+\frac{1}{3} \log \left(\sqrt[3]{x^3-1}-x\right)-\frac{2 \tan ^{-1}\left(\frac{\frac{2 x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{3 \sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)^(2/3),x]
```

```
[Out] (x*(-1 + x^3)^(2/3))/3 - (2*ArcTan[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]])/(
3*Sqrt[3]) + Log[-x + (-1 + x^3)^(1/3)]/3
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \left(-1+x^3\right)^{2/3} \, dx &= \frac{1}{3} x \left(-1+x^3\right)^{2/3}-\frac{2}{3} \int \frac{1}{\sqrt[3]{-1+x^3}} \, dx\\ &= \frac{1}{3} x \left(-1+x^3\right)^{2/3}-\frac{2 \tan ^{-1}\left(\frac{1+\frac{2 x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{3 \sqrt{3}}+\frac{1}{3} \log \left(-x+\sqrt[3]{-1+x^3}\right)\\ \end{aligned}$$

Mathematica [C] time = 0.10, size = 99, normalized size = 1.05

$$\frac{3 (x-1) \left(x^3-1\right)^{2/3} F_1\left(\frac{5}{3};-\frac{2}{3},-\frac{2}{3};\frac{8}{3};-\frac{x-1}{1-(-1)^{2/3}},-\frac{x-1}{1+\sqrt[3]{-1}}\right)}{5 \left(\frac{x-1}{1+\sqrt[3]{-1}}+1\right)^{2/3} \left(\frac{x-1}{1-(-1)^{2/3}}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + x^3)^(2/3),x]
```

```
[Out] (3*(-1 + x)*(-1 + x^3)^(2/3)*AppellF1[5/3, -2/3, -2/3, 8/3, -((-1 + x)/(1 -
 (-1)^(2/3))), -((-1 + x)/(1 + (-1)^(1/3)))])/(5*(1 + (-1 + x)/(1 + (-1)^(1
/3)))^(2/3)*(1 + (-1 + x)/(1 - (-1)^(2/3)))^(2/3))
```

IntegrateAlgebraic [A] time = 0.19, size = 94, normalized size = 1.00

$$\frac{1}{3}\left(x^3-1\right)^{2/3}x+\frac{2}{9}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{2\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{3\sqrt{3}}-\frac{1}{9}\log\left(\sqrt[3]{x^3-1}x+\left(x^3-1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)^(2/3),x]
```

```
[Out] (x*(-1 + x^3)^(2/3))/3 - (2*ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x^3)^(1/3))])/(
3*Sqrt[3]) + (2*Log[-x + (-1 + x^3)^(1/3)])/9 - Log[x^2 + x*(-1 + x^3)^(1/3
) + (-1 + x^3)^(2/3)]/9
```

fricas [A] time = 0.43, size = 86, normalized size = 0.91

$$\frac{1}{3}\left(x^3-1\right)^{\frac{2}{3}}x+\frac{2}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}}{3x}\right)+\frac{2}{9}\log\left(-\frac{x-\left(x^3-1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{9}\log\left(\frac{x^2+\left(x^3-1\right)^{\frac{1}{3}}x+\left(x^3-1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 - 1)^(2/3)*x + 2/9*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3
- 1)^(1/3))/x) + 2/9*log(-(x - (x^3 - 1)^(1/3))/x) - 1/9*log((x^2 + (x^3 -
1)^(1/3)*x + (x^3 - 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^3-1\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 1)^(2/3), x)
```

maple [C] time = 0.33, size = 30, normalized size = 0.32

$$\frac{\mathrm{signum}\left(x^3-1\right)^{\frac{2}{3}}x\,\mathrm{hypergeom}\left(\left[-\frac{2}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^3\right)}{\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3),x)
```

```
[Out] signum(x^3-1)^(2/3)/(-signum(x^3-1))^(2/3)*x*hypergeom([-2/3,1/3],[4/3],x^3
)
```

maxima [A] time = 0.78, size = 94, normalized size = 1.00

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3-1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\left(x^3-1\right)^{\frac{2}{3}}}{3x^2\left(\frac{x^3-1}{x^3}-1\right)}-\frac{1}{9}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3-1\right)^{\frac{2}{3}}}{x^2}+1\right)+\frac{2}{9}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3)/x + 1)) - 1/3*(x^3 - 1)^(
2/3)/(x^2*((x^3 - 1)/x^3 - 1)) - 1/9*log((x^3 - 1)^(1/3)/x + (x^3 - 1)^(2/3
)/x^2 + 1) + 2/9*log((x^3 - 1)^(1/3)/x - 1)
```

mupad [B] time = 0.85, size = 26, normalized size = 0.28

$$\frac{x\left(x^3-1\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\ \frac{4}{3};\ x^3\right)}{\left(1-x^3\right)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)^(2/3),x)
```

```
[Out] (x*(x^3 - 1)^(2/3)*hypergeom([-2/3, 1/3], 4/3, x^3))/(1 - x^3)^(2/3)
```

sympy [C] time = 0.86, size = 32, normalized size = 0.34

$$-\frac{xe^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3),x)
```

```
[Out] -x*exp(-I*pi/3)*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), x**3)/(3*gamma(4/3))
```

3.1156 $\int \left(1+x^3\right)^{2/3} dx$

Optimal. Leaf size=94

$$\frac{1}{3}\left(x^3+1\right)^{2/3}x-\frac{2}{9}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{2\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{3\sqrt{3}}+\frac{1}{9}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 0.67, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {195, 239}

$$\frac{1}{3}\left(x^3+1\right)^{2/3}x-\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{2\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)^(2/3),x]
```

```
[Out] (x*(1 + x^3)^(2/3))/3 + (2*ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]])/(3*
Sqrt[3]) - Log[-x + (1 + x^3)^(1/3)]/3
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \left(1+x^3\right)^{2/3} dx &= \frac{1}{3}x\left(1+x^3\right)^{2/3}+\frac{2}{3}\int \frac{1}{\sqrt[3]{1+x^3}}\,dx\\ &= \frac{1}{3}x\left(1+x^3\right)^{2/3}+\frac{2\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3}\log\left(-x+\sqrt[3]{1+x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.06, size = 99, normalized size = 1.05

$$\frac{3(x+1)\left(x^3+1\right)^{2/3}F_1\left(\frac{5}{3};-\frac{2}{3},-\frac{2}{3};\frac{8}{3};-\frac{x+1}{-1+(-1)^{2/3}},-\frac{x+1}{-1-\sqrt[3]{-1}}\right)}{5\left(\frac{x+1}{-1-\sqrt[3]{-1}}+1\right)^{2/3}\left(\frac{x+1}{(-1)^{2/3}-1}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^3)^(2/3),x]
```

```
[Out] (3*(1 + x)*(1 + x^3)^(2/3)*AppellF1[5/3, -2/3, -2/3, 8/3, -((1 + x)/(-1 + (
-1)^(2/3))), -((1 + x)/(-1 - (-1)^(1/3)))])/(5*(1 + (1 + x)/(-1 - (-1)^(1/3
)))^(2/3)*(1 + (1 + x)/(-1 + (-1)^(2/3)))^(2/3))
```

IntegrateAlgebraic [A] time = 0.19, size = 94, normalized size = 1.00

$$\frac{1}{3}\left(x^3+1\right)^{2/3}x-\frac{2}{9}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{2\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{3\sqrt{3}}+\frac{1}{9}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)^(2/3),x]
```

```
[Out] (x*(1 + x^3)^(2/3))/3 + (2*ArcTan[(Sqrt[3]*x)/(x + 2*(1 + x^3)^(1/3))])/(3*
Sqrt[3]) - (2*Log[-x + (1 + x^3)^(1/3)])/9 + Log[x^2 + x*(1 + x^3)^(1/3) +
(1 + x^3)^(2/3)]/9
```

fricas [A] time = 0.44, size = 86, normalized size = 0.91

$$\frac{1}{3}\left(x^3+1\right)^{\frac{2}{3}}x-\frac{2}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}}{3x}\right)-\frac{2}{9}\log\left(-\frac{x-\left(x^3+1\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{9}\log\left(\frac{x^2+\left(x^3+1\right)^{\frac{1}{3}}x+\left(x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 1)^(2/3)*x - 2/9*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3
+ 1)^(1/3))/x) - 2/9*log(-(x - (x^3 + 1)^(1/3))/x) + 1/9*log((x^2 + (x^3 +
1)^(1/3)*x + (x^3 + 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^3+1\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)^(2/3), x)
```

maple [C] time = 0.32, size = 14, normalized size = 0.15

$$x\,\mathrm{hypergeom}\left(\left[-\frac{2}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3),x)
```

```
[Out] x*hypergeom([-2/3,1/3],[4/3],-x^3)
```

maxima [A] time = 0.76, size = 94, normalized size = 1.00

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+1\right)^{\frac{1}{3}}}{x}+1\right)\right)+\frac{\left(x^3+1\right)^{\frac{2}{3}}}{3x^2\left(\frac{x^3+1}{x^3}-1\right)}+\frac{1}{9}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)-\frac{2}{9}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3)/x + 1)) + 1/3*(x^3 + 1)^
(2/3)/(x^2*((x^3 + 1)/x^3 - 1)) + 1/9*log((x^3 + 1)^(1/3)/x + (x^3 + 1)^(2/
3)/x^2 + 1) - 2/9*log((x^3 + 1)^(1/3)/x - 1)
```

mupad [B] time = 0.77, size = 12, normalized size = 0.13

$$x\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{1}{3};\ \frac{4}{3};\ -x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)^(2/3),x)
```

```
[Out] x*hypergeom([-2/3, 1/3], 4/3, -x^3)
```

sympy [C] time = 0.81, size = 29, normalized size = 0.31

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3),x)
```

```
[Out] x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3)
)
```

3.1157 $\int \frac{-1+x^3}{\sqrt[3]{2+x^3}} \, dx$

Optimal. Leaf size=94

$$\frac{1}{3}\left(x^3+2\right)^{2/3}x+\frac{5}{9}\log\left(\sqrt[3]{x^3+2}-x\right)-\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+2}+x}\right)}{3\sqrt{3}}-\frac{5}{18}\log\left(\sqrt[3]{x^3+2}\,x+\left(x^3+2\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 0.67, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {388, 239}

$$\frac{1}{3}\left(x^3+2\right)^{2/3}x+\frac{5}{6}\log\left(\sqrt[3]{x^3+2}-x\right)-\frac{5\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(2 + x^3)^(1/3),x]
```

```
[Out] (x*(2 + x^3)^(2/3))/3 - (5*ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]])/(3*
Sqrt[3]) + (5*Log[-x + (2 + x^3)^(1/3)])/6
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^3}{\sqrt[3]{2+x^3}}\,dx &= \frac{1}{3}x\left(2+x^3\right)^{2/3}-\frac{5}{3}\int \frac{1}{\sqrt[3]{2+x^3}}\,dx\\ &= \frac{1}{3}x\left(2+x^3\right)^{2/3}-\frac{5\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{5}{6}\log\left(-x+\sqrt[3]{2+x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 91, normalized size = 0.97

$$\frac{1}{18}\left(6\left(x^3+2\right)^{2/3}x+10\log\left(1-\frac{x}{\sqrt[3]{x^3+2}}\right)-10\sqrt{3}\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)-5\log\left(\frac{x}{\sqrt[3]{x^3+2}}+\frac{x^2}{\left(x^3+2\right)^{2/3}}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(2 + x^3)^(1/3),x]
```

```
[Out] (6*x*(2 + x^3)^(2/3) - 10*Sqrt[3]*ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3
]] + 10*Log[1 - x/(2 + x^3)^(1/3)] - 5*Log[1 + x^2/(2 + x^3)^(2/3) + x/(2 +
 x^3)^(1/3)])/18
```

IntegrateAlgebraic [A] time = 0.21, size = 94, normalized size = 1.00

$$\frac{1}{3}\left(x^3+2\right)^{2/3}x+\frac{5}{9}\log\left(\sqrt[3]{x^3+2}-x\right)-\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+2}+x}\right)}{3\sqrt{3}}-\frac{5}{18}\log\left(\sqrt[3]{x^3+2}\,x+\left(x^3+2\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)/(2 + x^3)^(1/3),x]
```

```
[Out] (x*(2 + x^3)^(2/3))/3 - (5*ArcTan[(Sqrt[3]*x)/(x + 2*(2 + x^3)^(1/3))])/(3*
Sqrt[3]) + (5*Log[-x + (2 + x^3)^(1/3)])/9 - (5*Log[x^2 + x*(2 + x^3)^(1/3)
 + (2 + x^3)^(2/3)])/18
```

fricas [A] time = 0.43, size = 86, normalized size = 0.91

$$\frac{1}{3}\left(x^3+2\right)^{\frac{2}{3}}x+\frac{5}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+2\right)^{\frac{1}{3}}}{3x}\right)+\frac{5}{9}\log\left(-\frac{x-\left(x^3+2\right)^{\frac{1}{3}}}{x}\right)-\frac{5}{18}\log\left(\frac{x^2+\left(x^3+2\right)^{\frac{1}{3}}x+\left(x^3+2\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 2)^(2/3)*x + 5/9*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3
+ 2)^(1/3))/x) + 5/9*log(-(x - (x^3 + 2)^(1/3))/x) - 5/18*log((x^2 + (x^3 +
 2)^(1/3)*x + (x^3 + 2)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3-1}{\left(x^3+2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 1)/(x^3 + 2)^(1/3), x)
```

maple [C] time = 0.31, size = 29, normalized size = 0.31

$$\frac{x\left(x^3+2\right)^{\frac{2}{3}}}{3}-\frac{5\,2^{\frac{2}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-\frac{x^3}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x^3+2)^(1/3),x)
```

```
[Out] 1/3*x*(x^3+2)^(2/3)-5/6*2^(2/3)*x*hypergeom([1/3,1/3],[4/3],-1/2*x^3)
```

maxima [A] time = 0.90, size = 94, normalized size = 1.00

$$\frac{5}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+2\right)^{\frac{1}{3}}}{x}+1\right)\right)+\frac{2\left(x^3+2\right)^{\frac{2}{3}}}{3x^2\left(\frac{x^3+2}{x^3}-1\right)}-\frac{5}{18}\log\left(\frac{\left(x^3+2\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+2\right)^{\frac{2}{3}}}{x^2}+1\right)+\frac{5}{9}\log\left(\frac{\left(x^3+2\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="maxima")
```

```
[Out] 5/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 2)^(1/3)/x + 1)) + 2/3*(x^3 + 2)^(
2/3)/(x^2*((x^3 + 2)/x^3 - 1)) - 5/18*log((x^3 + 2)^(1/3)/x + (x^3 + 2)^(2/
3)/x^2 + 1) + 5/9*log((x^3 + 2)^(1/3)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3-1}{\left(x^3+2\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x^3 + 2)^(1/3),x)
```

```
[Out] int((x^3 - 1)/(x^3 + 2)^(1/3), x)
```

sympy [C] time = 1.61, size = 71, normalized size = 0.76

$$\frac{2^{\frac{2}{3}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{x^3e^{i\pi}}{2}\right)}{6\Gamma\left(\frac{7}{3}\right)}-\frac{2^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{x^3e^{i\pi}}{2}\right)}{6\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(x**3+2)**(1/3),x)
```

```
[Out] 2**(2/3)*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), x**3*exp_polar(I*pi)/2)/
(6*gamma(7/3)) - 2**(2/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_p
olar(I*pi)/2)/(6*gamma(4/3))
```

3.1158 $\int \sqrt[3]{x+x^3}\,dx$

Optimal. Leaf size=94

$$\frac{1}{2}\sqrt[3]{x^3+x}\,x-\frac{1}{6}\log\left(\sqrt[3]{x^3+x}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x}+x}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(\sqrt[3]{x^3+x}\,x+\left(x^3+x\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.13, antiderivative size = 178, normalized size of antiderivative = 1.89, number of steps used = 11, number of rules used = 11, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.222, Rules used = {2004, 2032, 329, 275, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{2}\sqrt[3]{x^3+x}\,x-\frac{\left(x^2+1\right)^{2/3}x^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2+1}}\right)}{6\left(x^3+x\right)^{2/3}}+\frac{\left(x^2+1\right)^{2/3}x^{2/3}\log\left(\frac{x^{4/3}}{\left(x^2+1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2+1}}+1\right)}{12\left(x^3+x\right)^{2/3}}-\frac{\left(x^2+1\right)^{2/3}x^{2/3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)}{2\sqrt{3}\left(x^3+x\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^3)^(1/3),x]
```

```
[Out] (x*(x + x^3)^(1/3))/2 - (x^(2/3)*(1 + x^2)^(2/3)*ArcTan[(1 + (2*x^(2/3))/(1
 + x^2)^(1/3))/Sqrt[3]])/(2*Sqrt[3]*(x + x^3)^(2/3)) - (x^(2/3)*(1 + x^2)^(
2/3)*Log[1 - x^(2/3)/(1 + x^2)^(1/3)])/(6*(x + x^3)^(2/3)) + (x^(2/3)*(1 +
x^2)^(2/3)*Log[1 + x^(4/3)/(1 + x^2)^(2/3) + x^(2/3)/(1 + x^2)^(1/3)])/(12*
(x + x^3)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \sqrt[3]{x+x^3}\,dx = \frac{1}{2}x\sqrt[3]{x+x^3} + \frac{1}{3}\int \frac{x}{\left(x+x^3\right)^{2/3}}\,dx$$

$$= \frac{1}{2}x\sqrt[3]{x+x^3} + \frac{\left(x^{2/3}\left(1+x^2\right)^{2/3}\right)\int \frac{\sqrt[3]{x}}{\left(1+x^2\right)^{2/3}}\,dx}{3\left(x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}x\sqrt[3]{x+x^3} + \frac{\left(x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x^3}{\left(1+x^6\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{\left(x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}x\sqrt[3]{x+x^3} + \frac{\left(x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{\left(1+x^3\right)^{2/3}}\,dx, x, x^{2/3}\right)}{2\left(x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}x\sqrt[3]{x+x^3} + \frac{\left(x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2\left(x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}x\sqrt[3]{x+x^3} + \frac{\left(x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{6\left(x+x^3\right)^{2/3}} - \frac{\left(x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1-}{1+x}\right.}{6\left(x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}x\sqrt[3]{x+x^3} - \frac{x^{2/3}\left(1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{6\left(x+x^3\right)^{2/3}} + \frac{\left(x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{x^{2/}}{\sqrt[3]{1+}}\right.}{12\left(x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}x\sqrt[3]{x+x^3} - \frac{x^{2/3}\left(1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{6\left(x+x^3\right)^{2/3}} + \frac{x^{2/3}\left(1+x^2\right)^{2/3}\log\left(1+\frac{x^{4/3}}{\left(1+x^2\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{12\left(x+x^3\right)^{2/3}} -$$

$$= \frac{1}{2}x\sqrt[3]{x+x^3} - \frac{x^{2/3}\left(1+x^2\right)^{2/3}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\left(x+x^3\right)^{2/3}} - \frac{x^{2/3}\left(1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{6\left(x+x^3\right)^{2/3}} + \frac{x^{2/3}\left(1+\right.}{}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.40

$$\frac{3x\sqrt[3]{x^3+x}\;{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-x^2\right)}{4\sqrt[3]{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^3)^(1/3),x]
```

```
[Out] (3*x*(x + x^3)^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/3, -x^2])/(4*(1 + x^2)^
(1/3))
```

IntegrateAlgebraic [A] time = 0.18, size = 94, normalized size = 1.00

$$\frac{1}{2}\sqrt[3]{x^3+x}\,x - \frac{1}{6}\log\left(\sqrt[3]{x^3+x}-x\right) - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x}+x}\right)}{2\sqrt{3}} + \frac{1}{12}\log\left(\sqrt[3]{x^3+x}\,x+\left(x^3+x\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + x^3)^(1/3),x]
```

```
[Out] (x*(x + x^3)^(1/3))/2 - ArcTan[(Sqrt[3]*x)/(x + 2*(x + x^3)^(1/3))]/(2*Sqrt
[3]) - Log[-x + (x + x^3)^(1/3)]/6 + Log[x^2 + x*(x + x^3)^(1/3) + (x + x^3
)^(2/3)]/12
```

fricas [A] time = 0.62, size = 90, normalized size = 0.96

$$-\frac{1}{6}\sqrt{3}\arctan\left(-\frac{196\sqrt{3}\left(x^{3}+x\right)^{\frac{1}{3}}x-\sqrt{3}\left(539x^{2}+507\right)-1274\sqrt{3}\left(x^{3}+x\right)^{\frac{2}{3}}}{2205x^{2}+2197}\right)+\frac{1}{2}\left(x^{3}+x\right)^{\frac{1}{3}}x-\frac{1}{12}\log\left(3\left(x^{3}+x\right)^{\frac{1}{3}}x-3\left(x^{3}+x\right)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan(-(196*sqrt(3)*(x^3 + x)^(1/3)*x - sqrt(3)*(539*x^2 + 50
7) - 1274*sqrt(3)*(x^3 + x)^(2/3))/(2205*x^2 + 2197)) + 1/2*(x^3 + x)^(1/3)
*x - 1/12*log(3*(x^3 + x)^(1/3)*x - 3*(x^3 + x)^(2/3) + 1)
```

giac [A] time = 0.36, size = 67, normalized size = 0.71

$$\frac{1}{2}x^{2}\left(\frac{1}{x^{2}}+1\right)^{\frac{1}{3}}+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x^{2}}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{12}\log\left(\left(\frac{1}{x^{2}}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x^{2}}+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{6}\log\left(\left|\left(\frac{1}{x^{2}}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*x^2*(1/x^2 + 1)^(1/3) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(1/x^2 + 1)^(
1/3) + 1)) + 1/12*log((1/x^2 + 1)^(2/3) + (1/x^2 + 1)^(1/3) + 1) - 1/6*log(
abs((1/x^2 + 1)^(1/3) - 1))
```

maple [C] time = 0.30, size = 17, normalized size = 0.18

$$\frac{3x^{\frac{4}{3}}\,\mathrm{hypergeom}\left(\left[-\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x)^(1/3),x)
```

```
[Out] 3/4*x^(4/3)*hypergeom([-1/3,2/3],[5/3],-x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^{3}+x\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + x)^(1/3), x)
```

mupad [B] time = 0.80, size = 27, normalized size = 0.29

$$\frac{3\,x\,\left(x^{3}+x\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{2}{3};\ \frac{5}{3};\ -x^{2}\right)}{4\,\left(x^{2}+1\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3)^(1/3),x)
[Out] (3*x*(x + x^3)^(1/3)*hypergeom([-1/3, 2/3], 5/3, -x^2))/(4*(x^2 + 1)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{x^3 + x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x)**(1/3),x)
[Out] Integral((x**3 + x)**(1/3), x)
```

3.1159 $\int \frac{\left(-2k+(-1+k)(1+k)x+2kx^2\right)\left(1+2kx+k^2x^2\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{3/4}\left(-1+d+(1+3d)kx+\left(1+3dk^2\right)x^2+k\left(-1+dk^2\right)x^3\right)}\,dx$

Optimal. Leaf size=94

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,kx+\sqrt[4]{d}}{\sqrt[4]{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{d^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,kx+\sqrt[4]{d}}{\sqrt[4]{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{d^{3/4}}$$

Rubi [F] time = 35.35, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-2k+(-1+k)(1+k)x+2kx^2\right)\left(1+2kx+k^2x^2\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{3/4}\left(-1+d+(1+3d)kx+\left(1+3dk^2\right)x^2+k\left(-1+dk^2\right)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*k + (-1 + k)*(1 + k)*x + 2*k*x^2)*(1 + 2*k*x + k^2*x^2))/(((1 - x^
2)*(1 - k^2*x^2))^(3/4)*(-1 + d + (1 + 3*d)*k*x + (1 + 3*d*k^2)*x^2 + k*(-1
 + d*k^2)*x^3)),x]
```

```
[Out] (Sqrt[2]*k^(3/2)*Sqrt[-1 + k^2]*Sqrt[(-1 - k^2 + 2*k^2*x^2)^2]*Sqrt[(1 + k^
2*(1 - 2*x^2))^2/((1 - k^2)^2*(1 - (2*k*Sqrt[(1 - x^2)*(1 - k^2*x^2)])/(1 -
 k^2))^2)]*(1 - (2*k*Sqrt[(1 - x^2)*(1 - k^2*x^2)])/(1 - k^2))*EllipticF[2*
ArcTan[(Sqrt[2]*Sqrt[k]*((1 - x^2)*(1 - k^2*x^2))^(1/4))/Sqrt[-1 + k^2]], 1
/2])/((1 - d*k^2)*(1 + k^2 - 2*k^2*x^2)*Sqrt[(-1 - k^2*(1 - 2*x^2))^2]) - (
k*(5 + (1 + 3*d)*k^2 - d*k^4)*x*((1 - x^2)/(1 - k^2*x^2))^(3/4)*(1 - k^2*x^
2)*Hypergeometric2F1[1/2, 3/4, 3/2, ((1 - k^2)*x^2)/(1 - k^2*x^2)])/((1 - d
*k^2)^2*((1 - x^2)*(1 - k^2*x^2))^(3/4)) + (k*(7 + k^2 - 3*d^2*k^2*(1 - k^2
) - d*(5 + 2*k^2 + k^4))*(1 - x^2)^(3/4)*(1 - k^2*x^2)^(3/4)*Defer[Int][1/(
(1 - x^2)^(3/4)*(1 - k^2*x^2)^(3/4)*(1 - d - (1 + 3*d)*k*x - (1 + 3*d*k^2)*
x^2 + k*(1 - d*k^2)*x^3)), x])/((1 - d*k^2)^2*((1 - x^2)*(1 - k^2*x^2))^(3/
4)) + ((1 - k^4 - 6*d^2*k^4*(1 - k^2) - d*k^2*(19 + 14*k^2 - k^4))*(1 - x^2
)^(3/4)*(1 - k^2*x^2)^(3/4)*Defer[Int][x/((1 - x^2)^(3/4)*(1 - k^2*x^2)^(3/
4)*(1 - d - (1 + 3*d)*k*x - (1 + 3*d*k^2)*x^2 + k*(1 - d*k^2)*x^3)), x])/((
1 - d*k^2)^2*((1 - x^2)*(1 - k^2*x^2))^(3/4)) - (k*(5 + 3*(1 + 8*d)*k^2 + 3
*d^2*k^4 - 3*d^2*k^6)*(1 - x^2)^(3/4)*(1 - k^2*x^2)^(3/4)*Defer[Int][x^2/((
1 - x^2)^(3/4)*(1 - k^2*x^2)^(3/4)*(1 - d - (1 + 3*d)*k*x - (1 + 3*d*k^2)*x
^2 + k*(1 - d*k^2)*x^3)), x])/((1 - d*k^2)^2*((1 - x^2)*(1 - k^2*x^2))^(3/4
))
```

Rubi steps

$$\int \frac{\left(-2k+(-1+k)(1+k)x+2kx^2\right)\left(1+2kx+k^2x^2\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{3/4}\left(-1+d+(1+3d)kx+\left(1+3dk^2\right)x^2+k\left(-1+dk^2\right)x^3\right)}\,dx = \int \frac{}{\left(\left(1-x^2\right)\left(1-k^2x^2 \right.\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right.\right.}{}$$

$$= \frac{\left(\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right.\right.}{}$$

$$= \frac{\left(\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right.\right.}{}$$

$$= -\frac{\left(2k^2\left(1-x^2\right)^{3/4}\left(1-\right.\right.}{\left(1-dk^2\right)\left(\right.}$$

$$= -\frac{k\left(5+(1+3d)k^2-\right.}{\left(1\right.}$$

$$= -\frac{k\left(5+(1+3d)k^2-\right.}{\left(1\right.}$$

$$= -\frac{k\left(5+(1+3d)k^2-\right.}{\left(1\right.}$$

$$= \frac{\sqrt{2}\,k^{3/2}\sqrt{-1+k^2}\,\sqrt{\left(-\right.}}{}$$

<u>Mathematica</u> [F] time = 1.88, size = 0, normalized size = 0.00

$$\int \frac{\left(-2k+(-1+k)(1+k)x+2kx^2\right)\left(1+2kx+k^2x^2\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{3/4}\left(-1+d+(1+3d)kx+\left(1+3dk^2\right)x^2+k\left(-1+dk^2\right)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2*k + (-1 + k)*(1 + k)*x + 2*k*x^2)*(1 + 2*k*x + k^2*x^2))/(((
1 - x^2)*(1 - k^2*x^2))^(3/4)*(-1 + d + (1 + 3*d)*k*x + (1 + 3*d*k^2)*x^2 +
 k*(-1 + d*k^2)*x^3)),x]
```

```
[Out] Integrate[((-2*k + (-1 + k)*(1 + k)*x + 2*k*x^2)*(1 + 2*k*x + k^2*x^2))/(((
1 - x^2)*(1 - k^2*x^2))^(3/4)*(-1 + d + (1 + 3*d)*k*x + (1 + 3*d*k^2)*x^2 +
 k*(-1 + d*k^2)*x^3)), x]
```

IntegrateAlgebraic [A] time = 15.74, size = 94, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,kx+\sqrt[4]{d}}{\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}\right)}{d^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,kx+\sqrt[4]{d}}{\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*k + (-1 + k)*(1 + k)*x + 2*k*x^2)*(1 + 2*k*x + k^2*
x^2))/(((1 - x^2)*(1 - k^2*x^2))^(3/4)*(-1 + d + (1 + 3*d)*k*x + (1 + 3*d*k
^2)*x^2 + k*(-1 + d*k^2)*x^3)),x]
```

```
[Out] -(ArcTan[(d^(1/4) + d^(1/4)*k*x)/(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4)]/d^(3
/4)) + ArcTanh[(d^(1/4) + d^(1/4)*k*x)/(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4)
]/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*k+(-1+k)*(1+k)*x+2*k*x^2)*(k^2*x^2+2*k*x+1)/((-x^2+1)*(-k^2*x
^2+1))^(3/4)/(-1+d+(1+3*d)*k*x+(3*d*k^2+1)*x^2+k*(d*k^2-1)*x^3),x, algorith
m="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(k^2x^2+2\,kx+1\right)\left((k+1)(k-1)x+2\,kx^2-2\,k\right)}{\left(\left(dk^2-1\right)kx^3+(3\,d+1)kx+\left(3\,dk^2+1\right)x^2+d-1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*k+(-1+k)*(1+k)*x+2*k*x^2)*(k^2*x^2+2*k*x+1)/((-x^2+1)*(-k^2*x
^2+1))^(3/4)/(-1+d+(1+3*d)*k*x+(3*d*k^2+1)*x^2+k*(d*k^2-1)*x^3),x, algorith
m="giac")
```

```
[Out] integrate((k^2*x^2 + 2*k*x + 1)*((k + 1)*(k - 1)*x + 2*k*x^2 - 2*k)/(((d*k^
2 - 1)*k*x^3 + (3*d + 1)*k*x + (3*d*k^2 + 1)*x^2 + d - 1)*((k^2*x^2 - 1)*(x
^2 - 1))^(3/4)), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(-2k+(-1+k)\,(1+k)\,x+2k\,x^2\right)\left(k^2x^2+2kx+1\right)}{\left(\left(-x^2+1\right)\left(-k^2x^2+1\right)\right)^{\frac{3}{4}}\left(-1+d+(1+3d)\,kx+\left(3d\,k^2+1\right)x^2+k\left(d\,k^2-1\right)x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*k+(-1+k)*(1+k)*x+2*k*x^2)*(k^2*x^2+2*k*x+1)/((-x^2+1)*(-k^2*x^2+1))
^(3/4)/(-1+d+(1+3*d)*k*x+(3*d*k^2+1)*x^2+k*(d*k^2-1)*x^3),x)
```

```
[Out] int((-2*k+(-1+k)*(1+k)*x+2*k*x^2)*(k^2*x^2+2*k*x+1)/((-x^2+1)*(-k^2*x^2+1))
^(3/4)/(-1+d+(1+3*d)*k*x+(3*d*k^2+1)*x^2+k*(d*k^2-1)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(k^2x^2+2\,kx+1\right)\left((k+1)(k-1)x+2\,kx^2-2\,k\right)}{\left(\left(dk^2-1\right)kx^3+(3\,d+1)kx+\left(3\,dk^2+1\right)x^2+d-1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*k+(-1+k)*(1+k)*x+2*k*x^2)*(k^2*x^2+2*k*x+1)/((-x^2+1)*(-k^2*x
^2+1))^(3/4)/(-1+d+(1+3*d)*k*x+(3*d*k^2+1)*x^2+k*(d*k^2-1)*x^3),x, algorith
m="maxima")
```

```
[Out] integrate((k^2*x^2 + 2*k*x + 1)*((k + 1)*(k - 1)*x + 2*k*x^2 - 2*k)/(((d*k^
2 - 1)*k*x^3 + (3*d + 1)*k*x + (3*d*k^2 + 1)*x^2 + d - 1)*((k^2*x^2 - 1)*(x
^2 - 1))^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,k\,x^2+\left(k-1\right)\,\left(k+1\right)\,x-2\,k\right)\,\left(k^2\,x^2+2\,k\,x+1\right)}{{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)}^{3/4}\,\left(k\,\left(d\,k^2-1\right)\,x^3+\left(3\,d\,k^2+1\right)\,x^2+k\,\left(3\,d+1\right)\,x+d-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*k*x^2 - 2*k + x*(k - 1)*(k + 1))*(k^2*x^2 + 2*k*x + 1))/(((x^2 - 1)
*(k^2*x^2 - 1))^(3/4)*(d + x^2*(3*d*k^2 + 1) + k*x*(3*d + 1) + k*x^3*(d*k^2
 - 1) - 1)),x)
```

```
[Out] int(((2*k*x^2 - 2*k + x*(k - 1)*(k + 1))*(k^2*x^2 + 2*k*x + 1))/(((x^2 - 1)
*(k^2*x^2 - 1))^(3/4)*(d + x^2*(3*d*k^2 + 1) + k*x*(3*d + 1) + k*x^3*(d*k^2
 - 1) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*k+(-1+k)*(1+k)*x+2*k*x**2)*(k**2*x**2+2*k*x+1)/((-x**2+1)*(-k
**2*x**2+1))**(3/4)/(-1+d+(1+3*d)*k*x+(3*d*k**2+1)*x**2+k*(d*k**2-1)*x**3),
x)
```

```
[Out] Timed out
```

3.1160 $\int \frac{(-1+x^4)^{3/4}(4+x^4)}{x^8(-4+x^4)}\,dx$

Optimal. Leaf size=94

$$-\frac{3^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{3}x}{\sqrt{2}\sqrt[4]{x^4-1}}\right)}{8\sqrt{2}}-\frac{3^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{3}x}{\sqrt{2}\sqrt[4]{x^4-1}}\right)}{8\sqrt{2}}+\frac{(x^4-1)^{3/4}(x^4+6)}{42x^7}$$

Rubi [A] time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {580, 583, 12, 377, 212, 206, 203}

$$-\frac{3^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{3}x}{\sqrt{2}\sqrt[4]{x^4-1}}\right)}{8\sqrt{2}}-\frac{3^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{3}x}{\sqrt{2}\sqrt[4]{x^4-1}}\right)}{8\sqrt{2}}+\frac{(x^4-1)^{3/4}}{7x^7}+\frac{(x^4-1)^{3/4}}{42x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^4)^(3/4)*(4 + x^4))/(x^8*(-4 + x^4)),x]
```

```
[Out] (-1 + x^4)^(3/4)/(7*x^7) + (-1 + x^4)^(3/4)/(42*x^3) - (3^(3/4)*ArcTan[(3^(
1/4)*x)/(Sqrt[2]*(-1 + x^4)^(1/4))])/(8*Sqrt[2]) - (3^(3/4)*ArcTanh[(3^(1/4
)*x)/(Sqrt[2]*(-1 + x^4)^(1/4))])/(8*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^4\right)^{3/4}\left(4+x^4\right)}{x^8\left(-4+x^4\right)}\,dx &= \frac{\left(-1+x^4\right)^{3/4}}{7x^7}-\frac{1}{28}\int \frac{8-44x^4}{x^4\left(-4+x^4\right)\sqrt[4]{-1+x^4}}\,dx\\
&= \frac{\left(-1+x^4\right)^{3/4}}{7x^7}+\frac{\left(-1+x^4\right)^{3/4}}{42x^3}+\frac{1}{336}\int \frac{504}{\left(-4+x^4\right)\sqrt[4]{-1+x^4}}\,dx\\
&= \frac{\left(-1+x^4\right)^{3/4}}{7x^7}+\frac{\left(-1+x^4\right)^{3/4}}{42x^3}+\frac{3}{2}\int \frac{1}{\left(-4+x^4\right)\sqrt[4]{-1+x^4}}\,dx\\
&= \frac{\left(-1+x^4\right)^{3/4}}{7x^7}+\frac{\left(-1+x^4\right)^{3/4}}{42x^3}+\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{-4+3x^4}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)\\
&= \frac{\left(-1+x^4\right)^{3/4}}{7x^7}+\frac{\left(-1+x^4\right)^{3/4}}{42x^3}-\frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{2-\sqrt{3}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)-\frac{3}{8}\operatorname{Sub}\\
&= \frac{\left(-1+x^4\right)^{3/4}}{7x^7}+\frac{\left(-1+x^4\right)^{3/4}}{42x^3}-\frac{3^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt{2}\sqrt[4]{-1+x^4}}\right)}{8\sqrt{2}}-\frac{3^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt{2}\sqrt[4]{-1+x^4}}\right.}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 118, normalized size = 1.26

$$\frac{\left(x^4-1\right)^{3/4}\left(x^4+6\right)}{42x^7}-\frac{3^{3/4}\left(-\log\left(2-\frac{\sqrt{2}\sqrt[4]{3}\,x}{\sqrt[4]{1-x^4}}\right)+\log\left(\frac{\sqrt{2}\sqrt[4]{3}\,x}{\sqrt[4]{1-x^4}}+2\right)+2\tan^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt{2}\sqrt[4]{1-x^4}}\right)\right)}{16\sqrt{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^4)^(3/4)*(4 + x^4))/(x^8*(-4 + x^4)),x]
```

```
[Out] ((-1 + x^4)^(3/4)*(6 + x^4))/(42*x^7) - (3^(3/4)*(2*ArcTan[(3^(1/4)*x)/(Sqr
t[2]*(1 - x^4)^(1/4))] - Log[2 - (Sqrt[2]*3^(1/4)*x)/(1 - x^4)^(1/4)] + Log
[2 + (Sqrt[2]*3^(1/4)*x)/(1 - x^4)^(1/4)]))/(16*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.46, size = 94, normalized size = 1.00

$$-\frac{3^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt{2}\sqrt[4]{x^4-1}}\right)}{8\sqrt{2}}-\frac{3^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt{2}\sqrt[4]{x^4-1}}\right)}{8\sqrt{2}}+\frac{\left(x^4-1\right)^{3/4}\left(x^4+6\right)}{42x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)^(3/4)*(4 + x^4))/(x^8*(-4 + x^4)),x]
```

```
[Out] ((-1 + x^4)^(3/4)*(6 + x^4))/(42*x^7) - (3^(3/4)*ArcTan[(3^(1/4)*x)/(Sqrt[2
]*(-1 + x^4)^(1/4))])/(8*Sqrt[2]) - (3^(3/4)*ArcTanh[(3^(1/4)*x)/(Sqrt[2]*(
-1 + x^4)^(1/4))])/(8*Sqrt[2])
```

fricas [B] time = 6.28, size = 277, normalized size = 2.95

$$\frac{84 \cdot 27^{\frac{1}{4}} \sqrt{2} x^{7} \arctan\left(-\frac{108 \cdot 27^{\frac{1}{4}} \sqrt{2} \left(x^{4}-1\right)^{\frac{1}{4}} x^{3}+24 \cdot 27^{\frac{3}{4}} \sqrt{2} \left(x^{4}-1\right)^{\frac{3}{4}} x-\sqrt{6} 3^{\frac{1}{4}} \left(36 \cdot 27^{\frac{1}{4}} \sqrt{2} \sqrt{x^{4}-1} x^{2}+27^{\frac{3}{4}} \sqrt{2} \left(7 x^{4}-4\right)\right)}{54 \left(x^{4}-4\right)}\right)-21 \cdot 27^{\frac{1}{4}} \sqrt{2} x^{7} \log\left(\frac{2 \left(4 \cdot 27^{\frac{3}{4}} \sqrt{2} \sqrt{x^{4}-1} x^{2}+36 \sqrt{3} \left(x^{4}-1\right)^{\frac{1}{4}} x^{3}+3 \cdot 27^{\frac{1}{4}} \sqrt{2} \left(7 x^{4}-4\right)+72 \left(x^{4}-1\right)^{\frac{3}{4}} x\right)}{x^{4}-4}\right)+21 \cdot 27^{\frac{1}{4}} \sqrt{2} x^{7} \log\left(-\frac{2 \left(4 \cdot 27^{\frac{3}{4}} \sqrt{2} \sqrt{x^{4}-1} x^{2}-36 \sqrt{3} \left(x^{4}-1\right)^{\frac{1}{4}} x^{3}+3 \cdot 27^{\frac{1}{4}} \sqrt{2} \left(7 x^{4}-4\right)-72 \left(x^{4}-1\right)^{\frac{3}{4}} x\right)}{x^{4}-4}\right)+32 \left(x^{4}+6\right) \left(x^{4}-1\right)^{\frac{3}{4}}}{1344 x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(3/4)*(x^4+4)/x^8/(x^4-4),x, algorithm="fricas")
```

```
[Out] 1/1344*(84*27^(1/4)*sqrt(2)*x^7*arctan(-1/54*(108*27^(1/4)*sqrt(2)*(x^4 - 1
)^(1/4)*x^3 + 24*27^(3/4)*sqrt(2)*(x^4 - 1)^(3/4)*x - sqrt(6)*3^(1/4)*(36*2
7^(1/4)*sqrt(2)*sqrt(x^4 - 1)*x^2 + 27^(3/4)*sqrt(2)*(7*x^4 - 4)))/(x^4 - 4
)) - 21*27^(1/4)*sqrt(2)*x^7*log(2*(4*27^(3/4)*sqrt(2)*sqrt(x^4 - 1)*x^2 +
36*sqrt(3)*(x^4 - 1)^(1/4)*x^3 + 3*27^(1/4)*sqrt(2)*(7*x^4 - 4) + 72*(x^4 -
 1)^(3/4)*x)/(x^4 - 4)) + 21*27^(1/4)*sqrt(2)*x^7*log(-2*(4*27^(3/4)*sqrt(2
)*sqrt(x^4 - 1)*x^2 - 36*sqrt(3)*(x^4 - 1)^(1/4)*x^3 + 3*27^(1/4)*sqrt(2)*(
7*x^4 - 4) - 72*(x^4 - 1)^(3/4)*x)/(x^4 - 4)) + 32*(x^4 + 6)*(x^4 - 1)^(3/4
))/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}+4\right)\left(x^{4}-1\right)^{\frac{3}{4}}}{\left(x^{4}-4\right)x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(3/4)*(x^4+4)/x^8/(x^4-4),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 4)*(x^4 - 1)^(3/4)/((x^4 - 4)*x^8), x)
```

maple [C] time = 1.84, size = 247, normalized size = 2.63

$$\frac{x^{8}+5x^{4}-6}{42x^{7}\left(x^{4}-1\right)^{\frac{1}{4}}}+\frac{\operatorname{RootOf}\left(\_Z^{4}-108\right)\ln\left(-\frac{-2\sqrt{x^{4}-1}\,\operatorname{RootOf}\left(\_Z^{4}-108\right)^{3}x^{2}+6\left(x^{4}-1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^{4}-108\right)^{2}x^{3}-21\operatorname{RootOf}\left(\_Z^{4}-108\right)x^{4}+72\left(x^{4}-1\right)^{\frac{3}{4}}x+12\operatorname{RootOf}\left(\_Z^{4}-108\right)}{\left(x^{2}+2\right)\left(x^{2}-2\right)}\right)}{32}+\frac{\operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{4}-108\right)^{2}\right)\ln\left(\frac{2\sqrt{x^{4}-1}\,\operatorname{RootOf}\left(\_Z^{4}-108\right)^{2}\operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{4}-108\right)^{2}\right)x^{2}-6\left(x^{4}-1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^{4}-108\right)^{2}x^{3}-21\operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{4}-108\right)^{2}\right)x^{4}+72\left(x^{4}-1\right)^{\frac{3}{4}}x+12\operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{4}-108\right)^{2}\right)}{\left(x^{2}+2\right)\left(x^{2}-2\right)}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(3/4)*(x^4+4)/x^8/(x^4-4),x)
```

```
[Out] 1/42*(x^8+5*x^4-6)/x^7/(x^4-1)^(1/4)+1/32*RootOf(_Z^4-108)*ln(-(-2*(x^4-1)^
(1/2)*RootOf(_Z^4-108)^3*x^2+6*(x^4-1)^(1/4)*RootOf(_Z^4-108)^2*x^3-21*Root
Of(_Z^4-108)*x^4+72*(x^4-1)^(3/4)*x+12*RootOf(_Z^4-108))/(x^2+2)/(x^2-2))+1
/32*RootOf(_Z^2+RootOf(_Z^4-108)^2)*ln((2*(x^4-1)^(1/2)*RootOf(_Z^4-108)^2*
RootOf(_Z^2+RootOf(_Z^4-108)^2)*x^2-6*(x^4-1)^(1/4)*RootOf(_Z^4-108)^2*x^3-
21*RootOf(_Z^2+RootOf(_Z^4-108)^2)*x^4+72*(x^4-1)^(3/4)*x+12*RootOf(_Z^2+Ro
otOf(_Z^4-108)^2))/(x^2+2)/(x^2-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}+4\right)\left(x^{4}-1\right)^{\frac{3}{4}}}{\left(x^{4}-4\right)x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(3/4)*(x^4+4)/x^8/(x^4-4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 4)*(x^4 - 1)^(3/4)/((x^4 - 4)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-1\right)^{3/4}\,\left(x^4+4\right)}{x^8\,\left(x^4-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)^(3/4)*(x^4 + 4))/(x^8*(x^4 - 4)),x)
```

```
[Out] int(((x^4 - 1)^(3/4)*(x^4 + 4))/(x^8*(x^4 - 4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x-1\right)\left(x+1\right)\left(x^2+1\right)\right)^{\frac{3}{4}}\left(x^2-2x+2\right)\left(x^2+2x+2\right)}{x^8\left(x^2-2\right)\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(3/4)*(x**4+4)/x**8/(x**4-4),x)
```

```
[Out] Integral(((x - 1)*(x + 1)*(x**2 + 1))**(3/4)*(x**2 - 2*x + 2)*(x**2 + 2*x +
 2)/(x**8*(x**2 - 2)*(x**2 + 2)), x)
```

3.1161 $\int \frac{4b+ax^3}{\sqrt[4]{b+ax^3}\left(b+ax^3+x^4\right)}\,dx$

Optimal. Leaf size=94

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^3+b}}{\sqrt{ax^3+b}+x^2}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3+b}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3+b}}\right)$$

Rubi [F] time = 0.72, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{4b+ax^3}{\sqrt[4]{b+ax^3}\left(b+ax^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(4*b + a*x^3)/((b + a*x^3)^(1/4)*(b + a*x^3 + x^4)),x]
```

```
[Out] 4*b*Defer[Int][1/((b + a*x^3)^(1/4)*(b + a*x^3 + x^4)), x] + a*Defer[Int][x
^3/((b + a*x^3)^(1/4)*(b + a*x^3 + x^4)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{4b+ax^3}{\sqrt[4]{b+ax^3}\left(b+ax^3+x^4\right)}\,dx &= \int \left(\frac{4b}{\sqrt[4]{b+ax^3}\left(b+ax^3+x^4\right)}+\frac{ax^3}{\sqrt[4]{b+ax^3}\left(b+ax^3+x^4\right)}\right)dx\\ &= a\int \frac{x^3}{\sqrt[4]{b+ax^3}\left(b+ax^3+x^4\right)}\,dx+(4b)\int \frac{1}{\sqrt[4]{b+ax^3}\left(b+ax^3+x^4\right)}\,dx\end{aligned}$$

Mathematica [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{4b+ax^3}{\sqrt[4]{b+ax^3}\left(b+ax^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(4*b + a*x^3)/((b + a*x^3)^(1/4)*(b + a*x^3 + x^4)),x]
```

```
[Out] Integrate[(4*b + a*x^3)/((b + a*x^3)^(1/4)*(b + a*x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.75, size = 94, normalized size = 1.00

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^3+b}}{\sqrt{ax^3+b}+x^2}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3+b}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3+b}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(4*b + a*x^3)/((b + a*x^3)^(1/4)*(b + a*x^3 + x^4)),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(-(x^2/Sqrt[2]) + Sqrt[b + a*x^3]/Sqrt[2])/(x*(b + a*x^3)^
(1/4))]) + Sqrt[2]*ArcTanh[(Sqrt[2]*x*(b + a*x^3)^(1/4))/(x^2 + Sqrt[b + a*
x^3])]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+4*b)/(a*x^3+b)^(1/4)/(a*x^3+x^4+b),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 + 4b}{\left(ax^3 + x^4 + b\right)\left(ax^3 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+4*b)/(a*x^3+b)^(1/4)/(a*x^3+x^4+b),x, algorithm="giac")
[Out] integrate((a*x^3 + 4*b)/((a*x^3 + x^4 + b)*(a*x^3 + b)^(1/4)), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{a\,x^3 + 4b}{\left(a\,x^3 + b\right)^{\frac{1}{4}}\left(a\,x^3 + x^4 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+4*b)/(a*x^3+b)^(1/4)/(a*x^3+x^4+b),x)
[Out] int((a*x^3+4*b)/(a*x^3+b)^(1/4)/(a*x^3+x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 + 4b}{\left(ax^3 + x^4 + b\right)\left(ax^3 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+4*b)/(a*x^3+b)^(1/4)/(a*x^3+x^4+b),x, algorithm="maxima")
[Out] integrate((a*x^3 + 4*b)/((a*x^3 + x^4 + b)*(a*x^3 + b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x^3 + 4\,b}{\left(a\,x^3 + b\right)^{1/4}\,\left(x^4 + a\,x^3 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*b + a*x^3)/((b + a*x^3)^(1/4)*(b + a*x^3 + x^4)),x)
[Out] int((4*b + a*x^3)/((b + a*x^3)^(1/4)*(b + a*x^3 + x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+4*b)/(a*x**3+b)**(1/4)/(a*x**3+x**4+b),x)
[Out] Timed out
```

3.1162 $\int \frac{-a^2b+a(2a+b)x-3ax^2+x^3}{\sqrt{x(-a+x)(-b+x)}\left(-a^2d+2adx+\left(b^2-d\right)x^2-2bx^3+x^4\right)}\,dx$

Optimal. Leaf size=94

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}{x(x-b)}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt{x^2(-a-b)+abx+x^3}}{x(x-b)}\right)}{d^{3/4}}$$

Rubi [F] time = 10.49, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-a^2b+a(2a+b)x-3ax^2+x^3}{\sqrt{x(-a+x)(-b+x)}\left(-a^2d+2adx+\left(b^2-d\right)x^2-2bx^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a^2*b) + a*(2*a + b)*x - 3*a*x^2 + x^3)/(Sqrt[x*(-a + x)*(-b + x)]*(
-(a^2*d) + 2*a*d*x + (b^2 - d)*x^2 - 2*b*x^3 + x^4)),x]
```

```
[Out] (4*a*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][(x^2*Sqrt[-a
 + x^2])/(Sqrt[-b + x^2]*(a^2*d - 2*a*d*x^2 - b^2*(1 - d/b^2)*x^4 + 2*b*x^6
 - x^8)), x], x, Sqrt[x]])/Sqrt[(a - x)*(b - x)*x] + (2*a*b*Sqrt[x]*Sqrt[-a
 + x]*Sqrt[-b + x]*Defer[Subst][Defer[Int][Sqrt[-a + x^2]/(Sqrt[-b + x^2]*(
-(a^2*d) + 2*a*d*x^2 + b^2*(1 - d/b^2)*x^4 - 2*b*x^6 + x^8)), x], x, Sqrt[x
]])/Sqrt[(a - x)*(b - x)*x] + (2*Sqrt[x]*Sqrt[-a + x]*Sqrt[-b + x]*Defer[Su
bst][Defer[Int][(x^4*Sqrt[-a + x^2])/(Sqrt[-b + x^2]*(-(a^2*d) + 2*a*d*x^2
+ b^2*(1 - d/b^2)*x^4 - 2*b*x^6 + x^8)), x], x, Sqrt[x]])/Sqrt[(a - x)*(b -
 x)*x]
```

Rubi steps

$$\int \frac{-a^2b+a(2a+b)x-3ax^2+x^3}{\sqrt{x(-a+x)(-b+x)}\left(-a^2d+2adx+\left(b^2-d\right)x^2-2bx^3+x^4\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{-a^2b+}{\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}(}}{\sqrt{x(-a+x)(-b+}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\int \frac{\sqrt{-a+x}(}{\sqrt{x}\sqrt{-b+x}\left(-a^2d+2\right.}}{\sqrt{x(-a+x)(-b+x)}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt{-b+x^2}}\right.}{\sqrt{x(-a+x)(}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}\left(\int\left(\frac{}{\sqrt{-b+x^2}}\right.\right.}{}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-a+x}\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt{-b+x^2}}\right.}{\sqrt{x(-a+x)}}$$

Mathematica [C] time = 7.41, size = 4636, normalized size = 49.32

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-(a^2*b) + a*(2*a + b)*x - 3*a*x^2 + x^3)/(Sqrt[x*(-a + x)*(-b +
 x)]*(-(a^2*d) + 2*a*d*x + (b^2 - d)*x^2 - 2*b*x^3 + x^4)),x]
```

```
[Out] ((-2*I)*(a - x)*Sqrt[(-b + x)/(a - b)]*((EllipticPi[a/Root[a^4 - 2*a^3*b +
a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2
+ (-4*a + 2*b)*#1^3 + #1^4 & , 1], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)]*(a
*(a - b) - Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 +
 (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 1]^2))/((Roo
t[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*
b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 1] - Root[a^4 - 2*a^3*b +
a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2
+ (-4*a + 2*b)*#1^3 + #1^4 & , 2])*(Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3
+ 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^
3 + #1^4 & , 1] - Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^
2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 3])*(
Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6
*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 1] - Root[a^4 - 2*a^3*b
 + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1
^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 4])) + ((EllipticPi[a/Root[a^4 - 2*a^3*b
+ a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^
2 + (-4*a + 2*b)*#1^3 + #1^4 & , 2], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)]*
(a*(-a + b) + Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#
1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 2]^2))/(R
oot[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*
a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 1] - Root[a^4 - 2*a^3*b
+ a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^
2 + (-4*a + 2*b)*#1^3 + #1^4 & , 2]) + (a*b*EllipticPi[a/Root[a^4 - 2*a^3*b
 + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1
^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 4], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)]
*(Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 -
 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 1] - Root[a^4 - 2*a^3
*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*
#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 3])*(Root[a^4 - 2*a^3*b + a^2*b^2 + (-4
*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b
)*#1^3 + #1^4 & , 2] - Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2
*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & ,
3]) + a^2*EllipticPi[a/Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2
*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & ,
4], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)]*(Root[a^4 - 2*a^3*b + a^2*b^2 + (
-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2
*b)*#1^3 + #1^4 & , 1] - Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b -
 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 &
, 3])*(-Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6
*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 2] + Root[a^4 -
 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2
 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 3]) + a^2*EllipticPi[a/Root[a^4 -
 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2
 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 3], I*ArcSinh[Sqrt[-1 + x/a]], a/
(a - b)]*(Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 +
(6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 1] - Root[a^4
 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b
^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 4])*(Root[a^4 - 2*a^3*b + a^2*b
^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4
*a + 2*b)*#1^3 + #1^4 & , 2] - Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a
^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #
1^4 & , 4]) + EllipticPi[a/Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b
 - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4
& , 4], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)]*(Root[a^4 - 2*a^3*b + a^2*b^2
 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a
 + 2*b)*#1^3 + #1^4 & , 1] - Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2
```

```
*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^
4 & , 3])*(Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 +
 (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 2] - Root[a^
4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b +
b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 3])*Root[a^4 - 2*a^3*b + a^2*b
^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4
*a + 2*b)*#1^3 + #1^4 & , 4]^2 + a*b*EllipticPi[a/Root[a^4 - 2*a^3*b + a^2*
b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-
4*a + 2*b)*#1^3 + #1^4 & , 3], I*ArcSinh[Sqrt[-1 + x/a]], a/(a - b)]*(Root[
a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b
+ b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 1] - Root[a^4 - 2*a^3*b + a^
2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 +
(-4*a + 2*b)*#1^3 + #1^4 & , 4])*(-Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 +
 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3
 + #1^4 & , 2] + Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2
)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 4]) +
EllipticPi[a/Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1
 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 3], I*ArcS
inh[Sqrt[-1 + x/a]], a/(a - b)]*Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*
a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 +
#1^4 & , 3]^2*(Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*
#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 1] - Roo
t[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*
b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 4])*(-Root[a^4 - 2*a^3*b +
 a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2
 + (-4*a + 2*b)*#1^3 + #1^4 & , 2] + Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3
 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1
^3 + #1^4 & , 4]))/((Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a
*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 1]
 - Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2
- 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 3])*(Root[a^4 - 2*a^
3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)
*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 1] - Root[a^4 - 2*a^3*b + a^2*b^2 + (-
4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*
b)*#1^3 + #1^4 & , 4])*(Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b -
2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & ,
 3] - Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a
^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 4])))/((Root[a^4
- 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^
2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 2] - Root[a^4 - 2*a^3*b + a^2*b^
2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*
a + 2*b)*#1^3 + #1^4 & , 3])*(Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^
2*b - 2*a*b^2)*#1 + (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1
^4 & , 2] - Root[a^4 - 2*a^3*b + a^2*b^2 + (-4*a^3 + 6*a^2*b - 2*a*b^2)*#1
+ (6*a^2 - 6*a*b + b^2 - d)*#1^2 + (-4*a + 2*b)*#1^3 + #1^4 & , 4]))))/(Sqr
t[1 - a/x]*Sqrt[x*(-a + x)*(-b + x)])
```

IntegrateAlgebraic [A] time = 1.90, size = 91, normalized size = 0.97

$$\frac{\tan^{-1}\left(\frac{\sqrt{x^2(-a-b)+abx+x^3}}{\sqrt[4]{d}\,(a-x)}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{x^2(-a-b)+abx+x^3}}{x(x-b)}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a^2*b) + a*(2*a + b)*x - 3*a*x^2 + x^3)/(Sqrt[x*(-a +
 x)*(-b + x)]*(-(a^2*d) + 2*a*d*x + (b^2 - d)*x^2 - 2*b*x^3 + x^4)),x]
```

```
[Out] ArcTan[Sqrt[a*b*x + (-a - b)*x^2 + x^3]/(d^(1/4)*(a - x))]/d^(3/4) - ArcTan
h[(d^(1/4)*Sqrt[a*b*x + (-a - b)*x^2 + x^3])/(x*(-b + x))]/d^(3/4)
```

fricas [B] time = 1.10, size = 379, normalized size = 4.03

$$\left(\frac{1}{d^{3}}\right)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{abx-(a+b)x^{2}+x^{3}}\, d \left(\frac{1}{d^{3}}\right)^{\frac{1}{4}}}{bx-x^{2}}\right) - \frac{1}{4} \left(\frac{1}{d^{3}}\right)^{\frac{1}{4}} \log\left(\frac{2\,bx^{3}-x^{4}-a^{2}d+2\,adx-\left(b^{2}+d\right)x^{2}+2\,\sqrt{abx-(a+b)x^{2}+x^{3}}\left(\left(ad^{3}-d^{3}x\right)\left(\frac{1}{d^{3}}\right)^{\frac{3}{4}}+\left(bdx-dx^{2}\right)\left(\frac{1}{d^{3}}\right)^{\frac{1}{4}}\right)-2\left(abd^{2}x-(a+b)d^{2}x^{2}+d^{2}x^{3}\right)\sqrt{\frac{1}{d^{3}}}}{2\,bx^{3}-x^{4}+a^{2}d-2\,adx-\left(b^{2}-d\right)x^{2}}\right) + \frac{1}{4} \left(\frac{1}{d^{3}}\right)^{\frac{1}{4}} \log\left(\frac{2\,bx^{3}-x^{4}-a^{2}d+2\,adx-\left(b^{2}+d\right)x^{2}-2\,\sqrt{abx-(a+b)x^{2}+x^{3}}\left(\left(ad^{3}-d^{3}x\right)\left(\frac{1}{d^{3}}\right)^{\frac{3}{4}}+\left(bdx-dx^{2}\right)\left(\frac{1}{d^{3}}\right)^{\frac{1}{4}}\right)-2\left(abd^{2}x-(a+b)d^{2}x^{2}+d^{2}x^{3}\right)\sqrt{\frac{1}{d^{3}}}}{2\,bx^{3}-x^{4}+a^{2}d-2\,adx-\left(b^{2}-d\right)x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*b+a*(2*a+b)*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a^2*d+
2*a*d*x+(b^2-d)*x^2-2*b*x^3+x^4),x, algorithm="fricas")
```

```
[Out] (d^(-3))^(1/4)*arctan(-sqrt(a*b*x - (a + b)*x^2 + x^3)*d*(d^(-3))^(1/4)/(b*
x - x^2)) - 1/4*(d^(-3))^(1/4)*log((2*b*x^3 - x^4 - a^2*d + 2*a*d*x - (b^2
+ d)*x^2 + 2*sqrt(a*b*x - (a + b)*x^2 + x^3)*((a*d^3 - d^3*x)*(d^(-3))^(3/4
) + (b*d*x - d*x^2)*(d^(-3))^(1/4)) - 2*(a*b*d^2*x - (a + b)*d^2*x^2 + d^2*
x^3)*sqrt(d^(-3)))/(2*b*x^3 - x^4 + a^2*d - 2*a*d*x - (b^2 - d)*x^2)) + 1/4
*(d^(-3))^(1/4)*log((2*b*x^3 - x^4 - a^2*d + 2*a*d*x - (b^2 + d)*x^2 - 2*sq
rt(a*b*x - (a + b)*x^2 + x^3)*((a*d^3 - d^3*x)*(d^(-3))^(3/4) + (b*d*x - d*
x^2)*(d^(-3))^(1/4)) - 2*(a*b*d^2*x - (a + b)*d^2*x^2 + d^2*x^3)*sqrt(d^(-3
)))/(2*b*x^3 - x^4 + a^2*d - 2*a*d*x - (b^2 - d)*x^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^{2}b-(2\,a+b)ax+3\,ax^{2}-x^{3}}{\left(2\,bx^{3}-x^{4}+a^{2}d-2\,adx-\left(b^{2}-d\right)x^{2}\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*b+a*(2*a+b)*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a^2*d+
2*a*d*x+(b^2-d)*x^2-2*b*x^3+x^4),x, algorithm="giac")
```

```
[Out] integrate((a^2*b - (2*a + b)*a*x + 3*a*x^2 - x^3)/((2*b*x^3 - x^4 + a^2*d -
 2*a*d*x - (b^2 - d)*x^2)*sqrt((a - x)*(b - x)*x)), x)
```

maple [C] time = 0.06, size = 307, normalized size = 3.27

$$-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}-2b\,\_Z^{3}+\left(b^{2}-d\right)\_Z^{2}+2ad\,\_Z-a^{2}d\right)}\frac{\left(-\_\alpha^{3}+3\_\alpha^{2}a-2\_\alpha a^{2}-\_\alpha ab+a^{2}b\right)\left(\_\alpha^{3}+\_\alpha^{2}a-2\_\alpha^{2}b+\_\alpha a^{2}-2\_\alpha ab+\_\alpha b^{2}+a^{3}-2a^{2}b+a b^{2}-\_\alpha d+ad\right)\sqrt{-\frac{-a+x}{a}}\,\sqrt{\frac{-b+x}{a-b}}\,\sqrt{\frac{x}{a}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-a+x}{a}},\frac{\_\alpha^{3}+\_\alpha^{2}a-2\_\alpha^{2}b+\_\alpha a^{2}-2\_\alpha ab+\_\alpha b^{2}+a^{3}-2a^{2}b+a b^{2}-\_\alpha d+ad}{a\left(a^{2}-2ab+b^{2}\right)},\sqrt{\frac{a}{a-b}}\right)}{\left(-2\_\alpha^{3}+3\_\alpha^{2}b-\_\alpha b^{2}+\_\alpha d-ad\right)\left(a^{2}-2ab+b^{2}\right)\sqrt{x\left(ab-ax-bx+x^{2}\right)}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*b+a*(2*a+b)*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a^2*d+2*a*d*
x+(b^2-d)*x^2-2*b*x^3+x^4),x)
```

```
[Out] -1/a*sum((-_alpha^3+3*_alpha^2*a-2*_alpha*a^2-_alpha*a*b+a^2*b)/(-2*_alpha^
3+3*_alpha^2*b-_alpha*b^2+_alpha*d-a*d)*(_alpha^3+_alpha^2*a-2*_alpha^2*b+_
alpha*a^2-2*_alpha*a*b+_alpha*b^2+a^3-2*a^2*b+a*b^2-_alpha*d+a*d)/(a^2-2*a*
b+b^2)*(-(-a+x)/a)^(1/2)*((-b+x)/(a-b))^(1/2)*(1/a*x)^(1/2)/(x*(a*b-a*x-b*x
+x^2))^(1/2)*EllipticPi((-(-a+x)/a)^(1/2),(_alpha^3+_alpha^2*a-2*_alpha^2*b
+_alpha*a^2-2*_alpha*a*b+_alpha*b^2+a^3-2*a^2*b+a*b^2-_alpha*d+a*d)/a/(a^2-
2*a*b+b^2),(a/(a-b))^(1/2)),_alpha=RootOf(_Z^4-2*b*_Z^3+(b^2-d)*_Z^2+2*a*d*
_Z-a^2*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^{2}b-(2\,a+b)ax+3\,ax^{2}-x^{3}}{\left(2\,bx^{3}-x^{4}+a^{2}d-2\,adx-\left(b^{2}-d\right)x^{2}\right)\sqrt{(a-x)(b-x)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*b+a*(2*a+b)*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x))^(1/2)/(-a^2*d+
2*a*d*x+(b^2-d)*x^2-2*b*x^3+x^4),x, algorithm="maxima")
```

```
[Out] integrate((a^2*b - (2*a + b)*a*x + 3*a*x^2 - x^3)/((2*b*x^3 - x^4 + a^2*d -
 2*a*d*x - (b^2 - d)*x^2)*sqrt((a - x)*(b - x)*x)), x)
```

mupad [B] time = 1.44, size = 453, normalized size = 4.82

$$\sum_{k=1}^{4}\frac{2\,b\,\left(\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,\left(d-b^2\right)+2\,a\,d\,z-a^2\,d,z,k\right)-a\right)\,\sqrt{\frac{x}{b}}\,\sqrt{\frac{b-x}{b}}\,\sqrt{\frac{a-x}{a-b}}\,\Pi\left(-\frac{b}{\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,\left(d-b^2\right)+2\,a\,d\,z-a^2\,d,z,k\right)-b};\mathrm{asin}\left(\sqrt{\frac{b-x}{b}}\right)\middle|-\frac{b}{a-b}\right)\,\left(\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,\left(d-b^2\right)+2\,a\,d\,z-a^2\,d,z,k\right)^2-2\,a\,\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,\left(d-b^2\right)+2\,a\,d\,z-a^2\,d,z,k\right)+a\,b\right)}{\left(\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,\left(d-b^2\right)+2\,a\,d\,z-a^2\,d,z,k\right)-b\right)\,\sqrt{x\,\left(a-x\right)\,\left(b-x\right)}\,\left(2\,b^2\,\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,\left(d-b^2\right)+2\,a\,d\,z-a^2\,d,z,k\right)-6\,b\,\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,\left(d-b^2\right)+2\,a\,d\,z-a^2\,d,z,k\right)^2+4\,\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,\left(d-b^2\right)+2\,a\,d\,z-a^2\,d,z,k\right)^3-2\,d\,\mathrm{root}\left(z^4-2\,b\,z^3-z^2\,\left(d-b^2\right)+2\,a\,d\,z-a^2\,d,z,k\right)+2\,a\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*b + 3*a*x^2 - x^3 - a*x*(2*a + b))/((x*(a - x)*(b - x))^(1/2)*(x^2
*(d - b^2) + a^2*d + 2*b*x^3 - x^4 - 2*a*d*x)),x)
```

```
[Out] symsum((2*b*(root(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z - a^2*d, z, k) -
a)*(x/b)^(1/2)*((b - x)/b)^(1/2)*((a - x)/(a - b))^(1/2)*ellipticPi(-b/(roo
t(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z - a^2*d, z, k) - b), asin(((b - x
)/b)^(1/2)), -b/(a - b))*(a*b - 2*a*root(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*
a*d*z - a^2*d, z, k) + root(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z - a^2*d
, z, k)^2))/((root(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z - a^2*d, z, k) -
 b)*(x*(a - x)*(b - x))^(1/2)*(2*a*d - 2*d*root(z^4 - 2*b*z^3 - z^2*(d - b^
2) + 2*a*d*z - a^2*d, z, k) - 6*b*root(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*
d*z - a^2*d, z, k)^2 + 2*b^2*root(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z -
 a^2*d, z, k) + 4*root(z^4 - 2*b*z^3 - z^2*(d - b^2) + 2*a*d*z - a^2*d, z,
k)^3)), k, 1, 4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*b+a*(2*a+b)*x-3*a*x**2+x**3)/(x*(-a+x)*(-b+x))**(1/2)/(-a*
*2*d+2*a*d*x+(b**2-d)*x**2-2*b*x**3+x**4),x)
```

```
[Out] Timed out
```

3.1163 $\int \frac{-2b+ax^4}{x^4 \sqrt[4]{bx^2+ax^4}} \, dx$

Optimal. Leaf size=94

$$a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)+a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)-\frac{4\left(ax^4+bx^2\right)^{3/4}\left(4ax^2-3b\right)}{21bx^5}$$

Rubi [A] time = 0.29, antiderivative size = 169, normalized size of antiderivative = 1.80, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {2052, 2011, 329, 240, 212, 206, 203, 2016, 2014}

$$\frac{a^{3/4}\sqrt{x}\,\sqrt[4]{ax^2+b}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{\sqrt[4]{ax^4+bx^2}}+\frac{a^{3/4}\sqrt{x}\,\sqrt[4]{ax^2+b}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{\sqrt[4]{ax^4+bx^2}}+\frac{4\left(ax^4+bx^2\right)^{3/4}}{7x^5}-\frac{16a\left(ax^4+bx^2\right)^{3/4}}{21bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*b + a*x^4)/(x^4*(b*x^2 + a*x^4)^(1/4)),x]
```

```
[Out] (4*(b*x^2 + a*x^4)^(3/4))/(7*x^5) - (16*a*(b*x^2 + a*x^4)^(3/4))/(21*b*x^3)
 + (a^(3/4)*Sqrt[x]*(b + a*x^2)^(1/4)*ArcTan[(a^(1/4)*Sqrt[x])/(b + a*x^2)^
(1/4)])/(b*x^2 + a*x^4)^(1/4) + (a^(3/4)*Sqrt[x]*(b + a*x^2)^(1/4)*ArcTanh[
(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(b*x^2 + a*x^4)^(1/4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-2b+ax^4}{x^4\sqrt[4]{bx^2+ax^4}}\,dx &= \int \left(\frac{a}{\sqrt[4]{bx^2+ax^4}}-\frac{2b}{x^4\sqrt[4]{bx^2+ax^4}}\right)dx\\
&= a\int \frac{1}{\sqrt[4]{bx^2+ax^4}}\,dx-(2b)\int \frac{1}{x^4\sqrt[4]{bx^2+ax^4}}\,dx\\
&= \frac{4\left(bx^2+ax^4\right)^{3/4}}{7x^5}+\frac{1}{7}(8a)\int \frac{1}{x^2\sqrt[4]{bx^2+ax^4}}\,dx+\frac{\left(a\sqrt{x}\sqrt[4]{b+ax^2}\right)\int \frac{1}{\sqrt{x}\sqrt[4]{b+ax^2}}\,dx}{\sqrt[4]{bx^2+ax^4}}\\
&= \frac{4\left(bx^2+ax^4\right)^{3/4}}{7x^5}-\frac{16a\left(bx^2+ax^4\right)^{3/4}}{21bx^3}+\frac{\left(2a\sqrt{x}\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{bx^2+ax^4}}\\
&= \frac{4\left(bx^2+ax^4\right)^{3/4}}{7x^5}-\frac{16a\left(bx^2+ax^4\right)^{3/4}}{21bx^3}+\frac{\left(2a\sqrt{x}\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{\sqrt[4]{bx^2+ax^4}}\\
&= \frac{4\left(bx^2+ax^4\right)^{3/4}}{7x^5}-\frac{16a\left(bx^2+ax^4\right)^{3/4}}{21bx^3}+\frac{\left(a\sqrt{x}\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{\sqrt[4]{bx^2+ax^4}}\\
&= \frac{4\left(bx^2+ax^4\right)^{3/4}}{7x^5}-\frac{16a\left(bx^2+ax^4\right)^{3/4}}{21bx^3}+\frac{a^{3/4}\sqrt{x}\sqrt[4]{b+ax^2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{\sqrt[4]{bx^2+ax^4}}+\frac{a^{3/4}\sqrt{x}\sqrt[4]{b+}}{\sqrt[4]{}}
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 142, normalized size = 1.51

$$
\frac{21a^{3/4}bx^{7/2}\sqrt[4]{ax^2+b}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)+21a^{3/4}bx^{7/2}\sqrt[4]{ax^2+b}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)-4\left(4a^2x^4+abx^2-3b^2\right)}{21bx^3\sqrt[4]{x^2\left(ax^2+b\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*b + a*x^4)/(x^4*(b*x^2 + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(-3*b^2 + a*b*x^2 + 4*a^2*x^4) + 21*a^(3/4)*b*x^(7/2)*(b + a*x^2)^(1/4)
*ArcTan[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)] + 21*a^(3/4)*b*x^(7/2)*(b + a*
x^2)^(1/4)*ArcTanh[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(21*b*x^3*(x^2*(b
+ a*x^2))^(1/4))
```

IntegrateAlgebraic [A] time = 0.51, size = 94, normalized size = 1.00

$$a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)+a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)-\frac{4\left(ax^4+bx^2\right)^{3/4}\left(4ax^2-3b\right)}{21bx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*b + a*x^4)/(x^4*(b*x^2 + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(-3*b + 4*a*x^2)*(b*x^2 + a*x^4)^(3/4))/(21*b*x^5) + a^(3/4)*ArcTan[(a^
(1/4)*x)/(b*x^2 + a*x^4)^(1/4)] + a^(3/4)*ArcTanh[(a^(1/4)*x)/(b*x^2 + a*x^
4)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-2*b)/x^4/(a*x^4+b*x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.22, size = 209, normalized size = 2.22

$$\frac{1}{2}\,\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)+\frac{1}{2}\,\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)-\frac{1}{4}\,\sqrt{2}\,(-a)^{\frac{3}{4}}\log\left(\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)+\frac{1}{4}\,\sqrt{2}\,(-a)^{\frac{3}{4}}\log\left(-\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)+\frac{4\left(3\left(a+\frac{b}{x^2}\right)^{\frac{7}{4}}b^6-7\left(a+\frac{b}{x^2}\right)^{\frac{3}{4}}ab^6\right)}{21\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-2*b)/x^4/(a*x^4+b*x^2)^(1/4),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^
2)^(1/4))/(-a)^(1/4)) + 1/2*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(-a)^(1/4) - 2*(a + b/x^2)^(1/4))/(-a)^(1/4)) - 1/4*sqrt(2)*(-a)^(3/4)*log
(sqrt(2)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sqrt(a + b/x^2)) + 1/4*s
qrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sq
rt(a + b/x^2)) + 4/21*(3*(a + b/x^2)^(7/4)*b^6 - 7*(a + b/x^2)^(3/4)*a*b^6)
/b^7
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{a\,x^4-2b}{x^4\left(a\,x^4+b\,x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-2*b)/x^4/(a*x^4+b*x^2)^(1/4),x)
```

```
[Out] int((a*x^4-2*b)/x^4/(a*x^4+b*x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4 - 2b}{\left(ax^4 + bx^2\right)^{\frac{1}{4}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-2*b)/x^4/(a*x^4+b*x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 - 2*b)/((a*x^4 + b*x^2)^(1/4)*x^4), x)
```

mupad [B] time = 1.25, size = 75, normalized size = 0.80

$$\frac{4\left(a\,x^4+b\,x^2\right)^{3/4}\left(3\,b-4\,a\,x^2\right)}{21\,b\,x^5}+\frac{2\,a\,x\left(\frac{a\,x^2}{b}+1\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{a\,x^2}{b}\right)}{\left(a\,x^4+b\,x^2\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*b - a*x^4)/(x^4*(a*x^4 + b*x^2)^(1/4)),x)
```

```
[Out] (4*(a*x^4 + b*x^2)^(3/4)*(3*b - 4*a*x^2))/(21*b*x^5) + (2*a*x*((a*x^2)/b +
1)^(1/4)*hypergeom([1/4, 1/4], 5/4, -(a*x^2)/b))/(a*x^4 + b*x^2)^(1/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4 - 2b}{x^4 \sqrt[4]{x^2\left(ax^2 + b\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-2*b)/x**4/(a*x**4+b*x**2)**(1/4),x)
```

```
[Out] Integral((a*x**4 - 2*b)/(x**4*(x**2*(a*x**2 + b))**(1/4)), x)
```

3.1164 $\int \frac{2b+ax^4}{x^4 \sqrt[4]{bx^2+ax^4}} \, dx$

Optimal. Leaf size=94

$$a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)+a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)+\frac{4\left(ax^4+bx^2\right)^{3/4}\left(4ax^2-3b\right)}{21bx^5}$$

Rubi [A] time = 0.29, antiderivative size = 169, normalized size of antiderivative = 1.80, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {2052, 2011, 329, 240, 212, 206, 203, 2016, 2014}

$$\frac{a^{3/4}\sqrt{x}\sqrt[4]{ax^2+b}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{\sqrt[4]{ax^4+bx^2}}+\frac{a^{3/4}\sqrt{x}\sqrt[4]{ax^2+b}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{\sqrt[4]{ax^4+bx^2}}-\frac{4\left(ax^4+bx^2\right)^{3/4}}{7x^5}+\frac{16a\left(ax^4+bx^2\right)^{3/4}}{21bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2*b + a*x^4)/(x^4*(b*x^2 + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(b*x^2 + a*x^4)^(3/4))/(7*x^5) + (16*a*(b*x^2 + a*x^4)^(3/4))/(21*b*x^3
) + (a^(3/4)*Sqrt[x]*(b + a*x^2)^(1/4)*ArcTan[(a^(1/4)*Sqrt[x])/(b + a*x^2)
^(1/4)])/(b*x^2 + a*x^4)^(1/4) + (a^(3/4)*Sqrt[x]*(b + a*x^2)^(1/4)*ArcTanh
[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(b*x^2 + a*x^4)^(1/4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{2b+ax^4}{x^4\sqrt[4]{bx^2+ax^4}}\,dx &= \int \left(\frac{a}{\sqrt[4]{bx^2+ax^4}}+\frac{2b}{x^4\sqrt[4]{bx^2+ax^4}}\right)dx\\
&= a\int \frac{1}{\sqrt[4]{bx^2+ax^4}}\,dx+(2b)\int \frac{1}{x^4\sqrt[4]{bx^2+ax^4}}\,dx\\
&= -\frac{4\left(bx^2+ax^4\right)^{3/4}}{7x^5}-\frac{1}{7}(8a)\int \frac{1}{x^2\sqrt[4]{bx^2+ax^4}}\,dx+\frac{\left(a\sqrt{x}\,\sqrt[4]{b+ax^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt[4]{b+ax^2}}\,dx}{\sqrt[4]{bx^2+ax^4}}\\
&= -\frac{4\left(bx^2+ax^4\right)^{3/4}}{7x^5}+\frac{16a\left(bx^2+ax^4\right)^{3/4}}{21bx^3}+\frac{\left(2a\sqrt{x}\,\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{bx^2+ax^4}}\\
&= -\frac{4\left(bx^2+ax^4\right)^{3/4}}{7x^5}+\frac{16a\left(bx^2+ax^4\right)^{3/4}}{21bx^3}+\frac{\left(2a\sqrt{x}\,\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{\sqrt[4]{bx^2+ax^4}}\\
&= -\frac{4\left(bx^2+ax^4\right)^{3/4}}{7x^5}+\frac{16a\left(bx^2+ax^4\right)^{3/4}}{21bx^3}+\frac{\left(a\sqrt{x}\,\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt[4]{b+ax^2}}\right.}{\sqrt[4]{bx^2+ax^4}}\\
&= -\frac{4\left(bx^2+ax^4\right)^{3/4}}{7x^5}+\frac{16a\left(bx^2+ax^4\right)^{3/4}}{21bx^3}+\frac{a^{3/4}\sqrt{x}\,\sqrt[4]{b+ax^2}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{\sqrt[4]{bx^2+ax^4}}+\frac{a^{3/4}\sqrt{x}\,\sqrt[4]{b}}{}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 142, normalized size = 1.51

$$\frac{21a^{3/4}bx^{7/2}\sqrt[4]{ax^2+b}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)+21a^{3/4}bx^{7/2}\sqrt[4]{ax^2+b}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)+4\left(4a^2x^4+abx^2-3b^2\right)}{21bx^3\sqrt[4]{x^2\left(ax^2+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*b + a*x^4)/(x^4*(b*x^2 + a*x^4)^(1/4)),x]
```

```
[Out] (4*(-3*b^2 + a*b*x^2 + 4*a^2*x^4) + 21*a^(3/4)*b*x^(7/2)*(b + a*x^2)^(1/4)*
ArcTan[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)] + 21*a^(3/4)*b*x^(7/2)*(b + a*x
^2)^(1/4)*ArcTanh[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(21*b*x^3*(x^2*(b +
 a*x^2))^(1/4))
```

IntegrateAlgebraic [A] time = 0.47, size = 94, normalized size = 1.00

$$a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)+a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)+\frac{4\left(ax^4+bx^2\right)^{3/4}\left(4ax^2-3b\right)}{21bx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*b + a*x^4)/(x^4*(b*x^2 + a*x^4)^(1/4)),x]
```

```
[Out] (4*(-3*b + 4*a*x^2)*(b*x^2 + a*x^4)^(3/4))/(21*b*x^5) + a^(3/4)*ArcTan[(a^(
1/4)*x)/(b*x^2 + a*x^4)^(1/4)] + a^(3/4)*ArcTanh[(a^(1/4)*x)/(b*x^2 + a*x^4
)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+2*b)/x^4/(a*x^4+b*x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.23, size = 209, normalized size = 2.22

$$\frac{1}{2}\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)+\frac{1}{2}\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)-\frac{1}{4}\sqrt{2}\,(-a)^{\frac{3}{4}}\log\left(\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)+\frac{1}{4}\sqrt{2}\,(-a)^{\frac{3}{4}}\log\left(-\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)-\frac{4\left(3\left(a+\frac{b}{x^2}\right)^{\frac{7}{4}}b^6-7\left(a+\frac{b}{x^2}\right)^{\frac{3}{4}}ab^6\right)}{21\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+2*b)/x^4/(a*x^4+b*x^2)^(1/4),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^
2)^(1/4))/(-a)^(1/4)) + 1/2*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(-a)^(1/4) - 2*(a + b/x^2)^(1/4))/(-a)^(1/4)) - 1/4*sqrt(2)*(-a)^(3/4)*log
(sqrt(2)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sqrt(a + b/x^2)) + 1/4*s
qrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sq
rt(a + b/x^2)) - 4/21*(3*(a + b/x^2)^(7/4)*b^6 - 7*(a + b/x^2)^(3/4)*a*b^6)
/b^7
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{a\,x^4+2b}{x^4\left(a\,x^4+b\,x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+2*b)/x^4/(a*x^4+b*x^2)^(1/4),x)
```

```
[Out] int((a*x^4+2*b)/x^4/(a*x^4+b*x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4 + 2\,b}{\left(ax^4 + bx^2\right)^{\frac{1}{4}} x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+2*b)/x^4/(a*x^4+b*x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + 2*b)/((a*x^4 + b*x^2)^(1/4)*x^4), x)
```

mupad [B] time = 1.05, size = 75, normalized size = 0.80

$$\frac{2\,a\,x\,\left(\frac{a\,x^2}{b}+1\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{a\,x^2}{b}\right)}{\left(a\,x^4+b\,x^2\right)^{1/4}}-\frac{4\,\left(a\,x^4+b\,x^2\right)^{3/4}\,\left(3\,b-4\,a\,x^2\right)}{21\,b\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b + a*x^4)/(x^4*(a*x^4 + b*x^2)^(1/4)),x)
```

```
[Out] (2*a*x*((a*x^2)/b + 1)^(1/4)*hypergeom([1/4, 1/4], 5/4, -(a*x^2)/b))/(a*x^4
 + b*x^2)^(1/4) - (4*(a*x^4 + b*x^2)^(3/4)*(3*b - 4*a*x^2))/(21*b*x^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4 + 2b}{x^4 \sqrt[4]{x^2\left(ax^2 + b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+2*b)/x**4/(a*x**4+b*x**2)**(1/4),x)
```

```
[Out] Integral((a*x**4 + 2*b)/(x**4*(x**2*(a*x**2 + b))**(1/4)), x)
```

3.1165 $\int \frac{-1+x}{(1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

Optimal. Leaf size=94

$$-\frac{2\sqrt{-2a+2b-c}\tan^{-1}\left(\frac{x\sqrt{-2a+2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}x^2+2\sqrt{a}x+\sqrt{a}}\right)}{2a-2b+c}$$

Rubi [F] time = 0.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+x}{(1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x)/((1 + x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] Defer[Int][1/Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4], x] - 2*Defer[Int][1/((1
 + x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x}{(1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx &= \int \left(\frac{1}{\sqrt{a+bx+cx^2+bx^3+ax^4}} - \frac{2}{(1+x)\sqrt{a+bx+cx^2+bx^3+ax}}\right.\\ &= -\left(2\int \frac{1}{(1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx\right) + \int \frac{1}{\sqrt{a+bx+cx^2}}\end{aligned}$$

Mathematica [C] time = 6.49, size = 6023, normalized size = 64.07

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/((1 + x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.74, size = 94, normalized size = 1.00

$$-\frac{2\sqrt{-2a+2b-c}\tan^{-1}\left(\frac{x\sqrt{-2a+2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}x^2+2\sqrt{a}x+\sqrt{a}}\right)}{2a-2b+c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/((1 + x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])
,x]
```

```
[Out] (-2*Sqrt[-2*a + 2*b - c]*ArcTan[(Sqrt[-2*a + 2*b - c]*x)/(Sqrt[a] + 2*Sqrt[
a]*x + Sqrt[a]*x^2 - Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(2*a - 2*b +
c)
```

fricas [B] time = 0.93, size = 392, normalized size = 4.17

$$\left[\frac{\log\left(\frac{\left(24a^2-16ab+b^2+4ac\right)x^4+4\left(8a^2+4ab-3b^2-2(2a-b)c\right)x^3+2\left(24a^2+3b^2-4(a+2b)c+4c^2\right)x^2+4\sqrt{ax^4+bx^3+cx^2+bx+a}\left((4a-b)x^2+2(2a+b-c)x+4a-b\right)\sqrt{2a-2b+c}+24a^2-16ab+b^2+4ac+4\left(8a^2+4ab-3b^2-2(2a-b)c\right)x}{x^4+4x^3+6x^2+4x+1}\right)}{2\sqrt{2a-2b+c}}, \frac{\sqrt{-2a+2b-c}\arctan\left(-\frac{\sqrt{ax^4+bx^3+cx^2+bx+a}\left((4a-b)x^2+2(2a+b-c)x+4a-b\right)\sqrt{-2a+2b-c}}{2\left(\left(2a^2-2ab+ac\right)x^4+\left(2ab-2b^2+bc\right)x^3+\left(2(a-b)c+c^2\right)x^2+2a^2-2ab+ac+\left(2ab-2b^2+bc\right)x\right)}\right)}{2a-2b+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

```
[Out] [1/2*log(((24*a^2 - 16*a*b + b^2 + 4*a*c)*x^4 + 4*(8*a^2 + 4*a*b - 3*b^2 -
2*(2*a - b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a + 2*b)*c + 4*c^2)*x^2 + 4*sqr
t(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x^2 + 2*(2*a + b - c)*x + 4*a
 - b)*sqrt(2*a - 2*b + c) + 24*a^2 - 16*a*b + b^2 + 4*a*c + 4*(8*a^2 + 4*a*
b - 3*b^2 - 2*(2*a - b)*c)*x)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1))/sqrt(2*a - 2
*b + c), sqrt(-2*a + 2*b - c)*arctan(-1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x
+ a)*((4*a - b)*x^2 + 2*(2*a + b - c)*x + 4*a - b)*sqrt(-2*a + 2*b - c)/((2
*a^2 - 2*a*b + a*c)*x^4 + (2*a*b - 2*b^2 + b*c)*x^3 + (2*(a - b)*c + c^2)*x
^2 + 2*a^2 - 2*a*b + a*c + (2*a*b - 2*b^2 + b*c)*x))/(2*a - 2*b + c)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.10, size = 2813, normalized size = 29.93

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(1+x)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 2*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c
+_Z*b+a,index=1))*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4
*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,ind
ex=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z
^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2
)*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))^2*((RootOf(_Z^4*a+_Z^3*b+
_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-Root
Of(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+
a,index=3)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z
^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,in
dex=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-RootOf(_Z^4*a+_Z^3*b
+_Z^2*c+_Z*b+a,index=4))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootO
f(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b
+a,index=2)))^(1/2)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^
4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,inde
x=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(a*x^4+b*x^3+c*x^2+b*x+a)
^(1/2)*EllipticF(((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*
a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,inde
x=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^
2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)
,((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+
_Z*b+a,index=3))*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*
a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index
=3)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^
2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2))-4*
(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_
Z*b+a,index=1))*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a
+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index
=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2
```

```
*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)*
(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))^2*((RootOf(_Z^4*a+_Z^3*b+_Z
^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-RootOf
(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,
index=3)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3
*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,inde
x=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-RootOf(_Z^4*a+_Z^3*b+_
Z^2*c+_Z*b+a,index=4))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(
_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a
,index=2)))^(1/2)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*
a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=
2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(a*x^4+b*x^3+c*x^2+b*x+a)^(
1/2)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)+1)*(EllipticF(((RootOf(_Z
^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index
=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+
_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-Root
Of(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2),((RootOf(_Z^4*a+_Z^3*b+_Z^2
*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))*(-RootOf(_Z
^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index
=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3)+RootOf(_Z^4*a+_Z^3*b+_Z^
2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_
Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2))+(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_
Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+
_Z^3*b+_Z^2*c+_Z*b+a,index=1)+1)*EllipticPi(((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_
Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*
a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=
4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^
2*c+_Z*b+a,index=2)))^(1/2),(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)+1)
*(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_
Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1)+1)/(RootOf(_Z^
4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=
2)),((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2
*c+_Z*b+a,index=3))*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z
^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,in
dex=3)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+
_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2))
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{ax^4+bx^3+cx^2+bx+a}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((x - 1)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x-1}{(x+1)\,\sqrt{a\,x^4+b\,x^3+c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x + 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)),x)
```

```
[Out] int((x - 1)/((x + 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{(x+1)\sqrt{ax^4+a+bx^3+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(a*x**4+b*x**3+c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((x - 1)/((x + 1)*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2)), x)
```

3.1166 $\int \frac{1+x}{(-1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

Optimal. Leaf size=94

$$-\frac{2\sqrt{-2a-2b-c}\,\tan^{-1}\left(\frac{x\sqrt{-2a-2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2-2\sqrt{a}\,x+\sqrt{a}}\right)}{2a+2b+c}$$

Rubi [F] time = 0.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x}{(-1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x)/((-1 + x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] Defer[Int][1/Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4], x] + 2*Defer[Int][1/((-
1 + x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x]
```

Rubi steps

$$\int \frac{1+x}{(-1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx = \int \left(\frac{1}{\sqrt{a+bx+cx^2+bx^3+ax^4}} + \frac{2}{(-1+x)\sqrt{a+bx+cx^2+bx^3+}}\right.$$
$$= 2\int \frac{1}{(-1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx + \int \frac{1}{\sqrt{a+bx+cx^2-}}$$

Mathematica [C] time = 6.40, size = 6023, normalized size = 64.07

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-1 + x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.75, size = 94, normalized size = 1.00

$$-\frac{2\sqrt{-2a-2b-c}\,\tan^{-1}\left(\frac{x\sqrt{-2a-2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2-2\sqrt{a}\,x+\sqrt{a}}\right)}{2a+2b+c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((-1 + x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])
,x]
```

```
[Out] (-2*Sqrt[-2*a - 2*b - c]*ArcTan[(Sqrt[-2*a - 2*b - c]*x)/(Sqrt[a] - 2*Sqrt[
a]*x + Sqrt[a]*x^2 - Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(2*a + 2*b +
c)
```

fricas [B] time = 0.97, size = 382, normalized size = 4.06

$$\left[\frac{\log\left(\frac{\left(24a^2+16ab+b^2+4ac\right)x^4-4\left(8a^2-4ab-3b^2-2(2a+b)c\right)x^3+2\left(24a^2+3b^2-4(a-2b)c+4c^2\right)x^2-4\sqrt{ax^4+bx^3+cx^2+bx+a}\left((4a+b)x^2-2(2a-b-c)x+4a+b\right)\sqrt{2a+2b+c}+24a^2+16ab+b^2+4ac-4\left(8a^2-4ab-3b^2-2(2a+b)c\right)x}{x^4-4x^3+6x^2-4x+1}\right)}{2\sqrt{2a+2b+c}}, \frac{\sqrt{-2a-2b-c}\arctan\left(\frac{\sqrt{ax^4+bx^3+cx^2+bx+a}\left((4a+b)x^2-2(2a-b-c)x+4a+b\right)\sqrt{-2a-2b-c}}{2\left(\left(2a^2+2ab+ac\right)x^4+\left(2ab+2b^2+bc\right)x^3+\left(2(a+b)c+c^2\right)x^2+2a^2+2ab+ac+\left(2ab+2b^2+bc\right)x\right)}\right)}{2a+2b+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

```
[Out] [1/2*log(((24*a^2 + 16*a*b + b^2 + 4*a*c)*x^4 - 4*(8*a^2 - 4*a*b - 3*b^2 -
2*(2*a + b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a - 2*b)*c + 4*c^2)*x^2 - 4*sqr
t(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a
 + b)*sqrt(2*a + 2*b + c) + 24*a^2 + 16*a*b + b^2 + 4*a*c - 4*(8*a^2 - 4*a*
b - 3*b^2 - 2*(2*a + b)*c)*x)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1))/sqrt(2*a + 2
*b + c), sqrt(-2*a - 2*b - c)*arctan(1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x +
 a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(-2*a - 2*b - c)/((2*
a^2 + 2*a*b + a*c)*x^4 + (2*a*b + 2*b^2 + b*c)*x^3 + (2*(a + b)*c + c^2)*x^
2 + 2*a^2 + 2*a*b + a*c + (2*a*b + 2*b^2 + b*c)*x))/(2*a + 2*b + c)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.09, size = 2813, normalized size = 29.93

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-1+x)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 2*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c
+_Z*b+a,index=1))*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4
*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,ind
ex=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z
^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2
)*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))^2*((RootOf(_Z^4*a+_Z^3*b+
_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-Root
Of(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+
a,index=3)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z
^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,in
dex=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-RootOf(_Z^4*a+_Z^3*b
+_Z^2*c+_Z*b+a,index=4))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootO
f(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b
+a,index=2)))^(1/2)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^
4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,inde
x=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(a*x^4+b*x^3+c*x^2+b*x+a)
^(1/2)*EllipticF(((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*
a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,inde
x=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^
2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)
,((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+
_Z*b+a,index=3))*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*
a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index
=3)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^
2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2))+4*
(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_
Z*b+a,index=1))*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a
+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index
=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2
```

```
*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)*
(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))^2*((RootOf(_Z^4*a+_Z^3*b+_Z
^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-RootOf
(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,
index=3)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3
*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,inde
x=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-RootOf(_Z^4*a+_Z^3*b+_
Z^2*c+_Z*b+a,index=4))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(
_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a
,index=2)))^(1/2)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*
a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=
2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(a*x^4+b*x^3+c*x^2+b*x+a)^(
1/2)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-1)*(EllipticF(((RootOf(_Z
^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index
=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+
_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-Root
Of(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2),((RootOf(_Z^4*a+_Z^3*b+_Z^2
*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))*(-RootOf(_Z
^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index
=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3)+RootOf(_Z^4*a+_Z^3*b+_Z^
2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_
Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2))+(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_
Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+
_Z^3*b+_Z^2*c+_Z*b+a,index=1)-1)*EllipticPi(((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_
Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*
a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=
4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^
2*c+_Z*b+a,index=2)))^(1/2),(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-1)
*(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_
Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1)-1)/(RootOf(_Z^
4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=
2)),((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2
*c+_Z*b+a,index=3))*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z
^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,in
dex=3)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+
_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2))
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{ax^4+bx^3+cx^2+bx+a}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((x + 1)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+1}{(x-1)\,\sqrt{a\,x^4+b\,x^3+c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x - 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)),x)
```

```
[Out] int((x + 1)/((x - 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{(x-1)\sqrt{ax^4+a+bx^3+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(a*x**4+b*x**3+c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((x + 1)/((x - 1)*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2)), x)
```

3.1167 $\int \frac{x^2}{\sqrt{(1-x^2)(1-k^2x^2)}\,(-1+k^2x^4)}\,dx$

Optimal. Leaf size=94

$$\frac{\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}+kx^2+1}\right)}{2k(k+1)}-\frac{\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}}\right)}{4(k-1)k}$$

Rubi [A] time = 1.10, antiderivative size = 87, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {1992, 6725, 1210, 1103, 1698, 204, 203}

$$\frac{\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^4-(k^2+1)x^2+1}}\right)}{4k(k+1)}-\frac{\tan^{-1}\left(\frac{(1-k)x}{\sqrt{k^2x^4-(k^2+1)x^2+1}}\right)}{4(1-k)k}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(-1 + k^2*x^4)),x]
```

```
[Out] -1/4*ArcTan[((1 - k)*x)/Sqrt[1 - (1 + k^2)*x^2 + k^2*x^4]]/((1 - k)*k) + Ar
cTan[((1 + k)*x)/Sqrt[1 - (1 + k^2)*x^2 + k^2*x^4]]/(4*k*(1 + k))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1210

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1992

```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{(1-x^2)(1-k^2x^2)}\,(-1+k^2x^4)}\,dx = \int \frac{x^2}{(-1+k^2x^4)\sqrt{1-(1+k^2)x^2+k^2x^4}}\,dx$$

$$= \int \left(\frac{1}{2k(-1+kx^2)\sqrt{1-(1+k^2)x^2+k^2x^4}} + \frac{1}{2k(1+kx^2)\sqrt{1-(1+k}\right.$$

$$= \frac{\int \frac{1}{(-1+kx^2)\sqrt{1+(-1-k^2)x^2+k^2x^4}}\,dx}{2k} + \frac{\int \frac{1}{(1+kx^2)\sqrt{1+(-1-k^2)x^2+k^2x^4}}\,dx}{2k}$$

$$= -\frac{\int \frac{-1-kx^2}{(-1+kx^2)\sqrt{1+(-1-k^2)x^2+k^2x^4}}\,dx}{4k} + \frac{\int \frac{1-kx^2}{(1+kx^2)\sqrt{1+(-1-k^2)x^2+k^2x^4}}\,dx}{4k}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-(-1-2k-k^2)x^2}\,dx, x, \frac{x}{\sqrt{1+(-1-k^2)x^2+k^2x^4}}\right)}{4k} + \frac{\text{Subst}\left(\int \frac{1}{-1-(1-2k+}\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{(1-k)x}{\sqrt{1-(1+k^2)x^2+k^2x^4}}\right)}{4(1-k)k} + \frac{\tan^{-1}\left(\frac{(1+k)x}{\sqrt{1-(1+k^2)x^2+k^2x^4}}\right)}{4k(1+k)}$$

Mathematica [C] time = 0.39, size = 70, normalized size = 0.74

$$\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\left(\Pi\left(-k;\sin^{-1}(x)|k^2\right)-\Pi\left(k;\sin^{-1}(x)|k^2\right)\right)}{2k\sqrt{(x^2-1)(k^2x^2-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(-1 + k^2*x^4)),x]
```

```
[Out] (Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*(EllipticPi[-k, ArcSin[x], k^2] - Elliptic
Pi[k, ArcSin[x], k^2]))/(2*k*Sqrt[(-1 + x^2)*(-1 + k^2*x^2)])
```

IntegrateAlgebraic [A] time = 0.98, size = 94, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}+kx^2+1}\right)}{2k(k+1)} - \frac{\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}}\right)}{4(k-1)k}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(-1 + k^2*x^4)),x]
```

```
[Out] -1/4*ArcTan[((-1 + k)*x)/Sqrt[1 + (-1 - k^2)*x^2 + k^2*x^4]]/((-1 + k)*k) +
 ArcTan[((1 + k)*x)/(1 + k*x^2 + Sqrt[1 + (-1 - k^2)*x^2 + k^2*x^4])]/(2*k*
(1 + k))
```

fricas [A] time = 0.94, size = 83, normalized size = 0.88

$$-\frac{(k-1)\arctan\left(\frac{\sqrt{k^2x^4-(k^2+1)x^2+1}}{(k+1)x}\right)-(k+1)\arctan\left(\frac{\sqrt{k^2x^4-(k^2+1)x^2+1}}{(k-1)x}\right)}{4\left(k^3-k\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^4-1),x, algorithm="frica
s")
```

```
[Out] -1/4*((k - 1)*arctan(sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)/((k + 1)*x)) - (k +
1)*arctan(sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)/((k - 1)*x)))/(k^3 - k)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(k^2x^4-1\right)\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^4-1),x, algorithm="giac"
)
```

```
[Out] integrate(x^2/((k^2*x^4 - 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))), x)
```

maple [C] time = 0.03, size = 112, normalized size = 1.19

$$-\frac{\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\operatorname{EllipticPi}(x,k,k)}{2k\sqrt{k^2x^4-k^2x^2-x^2+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\operatorname{EllipticPi}(x,-k,k)}{2k\sqrt{k^2x^4-k^2x^2-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^4-1),x)
```

```
[Out] -1/2/k*(-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*Elli
pticPi(x,k,k)+1/2/k*(-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+
1)^(1/2)*EllipticPi(x,-k,k)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(k^2x^4-1\right)\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^4-1),x, algorithm="maxim
a")
```

```
[Out] integrate(x^2/((k^2*x^4 - 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^2}{\left(k^2\,x^4-1\right)\,\sqrt{\left(x^2-1\right)\,\left(k^2\,x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((k^2*x^4 - 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)),x)
```

```
[Out] int(x^2/((k^2*x^4 - 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{(x-1)(x+1)(kx-1)(kx+1)}\left(kx^2-1\right)\left(kx^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((-x**2+1)*(-k**2*x**2+1))**(1/2)/(k**2*x**4-1),x)
```

```
[Out] Integral(x**2/(sqrt((x - 1)*(x + 1)*(k*x - 1)*(k*x + 1))*(k*x**2 - 1)*(k*x*
*2 + 1)), x)
```

3.1168 $\int \frac{(-1+x^6)^{2/3}(1+x^6)}{x^3(-1-x^3+x^6)}\,dx$

Optimal. Leaf size=94

$$\frac{1}{3}\log\left(\sqrt[3]{x^6-1}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^6-1}+x}\right)}{\sqrt{3}}+\frac{\left(x^6-1\right)^{2/3}}{2x^2}-\frac{1}{6}\log\left(\sqrt[3]{x^6-1}\,x+\left(x^6-1\right)^{2/3}+x^2\right)$$

Rubi [C] time = 0.91, antiderivative size = 313, normalized size of antiderivative = 3.33, number of steps used = 21, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6728, 275, 365, 364, 1438, 430, 429, 465, 511, 510}

$$-\frac{\left(1-\sqrt{5}\right)\left(x^6-1\right)^{2/3}xF_1\left(\frac{1}{6};-\frac{2}{3},1;\frac{7}{6};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{\left(3-\sqrt{5}\right)\left(1-x^6\right)^{2/3}}-\frac{\left(1+\sqrt{5}\right)\left(x^6-1\right)^{2/3}xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};\frac{2x^6}{3+\sqrt{5}},x^6\right)}{\left(3+\sqrt{5}\right)\left(1-x^6\right)^{2/3}}-\frac{\left(x^6-1\right)^{2/3}x^4F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{2\left(3-\sqrt{5}\right)\left(1-x^6\right)^{2/3}}-\frac{\left(x^6-1\right)^{2/3}x^4F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^6,\frac{2x^6}{3+\sqrt{5}}\right)}{2\left(3+\sqrt{5}\right)\left(1-x^6\right)^{2/3}}+\frac{\left(x^6-1\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};x^6\right)}{2\left(1-x^6\right)^{2/3}x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^6)^(2/3)*(1 + x^6))/(x^3*(-1 - x^3 + x^6)),x]
```

```
[Out] -(((1 - Sqrt[5])*x*(-1 + x^6)^(2/3)*AppellF1[1/6, -2/3, 1, 7/6, x^6, (2*x^6
)/(3 - Sqrt[5])])/((3 - Sqrt[5])*(1 - x^6)^(2/3))) - ((1 + Sqrt[5])*x*(-1 +
 x^6)^(2/3)*AppellF1[1/6, 1, -2/3, 7/6, (2*x^6)/(3 + Sqrt[5]), x^6])/((3 +
Sqrt[5])*(1 - x^6)^(2/3)) - (x^4*(-1 + x^6)^(2/3)*AppellF1[2/3, -2/3, 1, 5/
3, x^6, (2*x^6)/(3 - Sqrt[5])])/(2*(3 - Sqrt[5])*(1 - x^6)^(2/3)) - (x^4*(-
1 + x^6)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, x^6, (2*x^6)/(3 + Sqrt[5])])/(2*
(3 + Sqrt[5])*(1 - x^6)^(2/3)) + ((-1 + x^6)^(2/3)*Hypergeometric2F1[-2/3,
-1/3, 2/3, x^6])/(2*x^2*(1 - x^6)^(2/3))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
```

```
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(-1+x^6)^{2/3}(1+x^6)}{x^3(-1-x^3+x^6)}\,dx = \int \left(-\frac{(-1+x^6)^{2/3}}{x^3}+\frac{(-1+2x^3)(-1+x^6)^{2/3}}{-1-x^3+x^6}\right)dx$$

$$= -\int \frac{(-1+x^6)^{2/3}}{x^3}\,dx + \int \frac{(-1+2x^3)(-1+x^6)^{2/3}}{-1-x^3+x^6}\,dx$$

$$= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{(-1+x^3)^{2/3}}{x^2}\,dx, x, x^2\right)\right) + \int \left(\frac{2(-1+x^6)^{2/3}}{-1-\sqrt{5}+2x^3} + \frac{2(-1+x^6)^{2/3}}{-1+\sqrt{5}+2x^3}\right.$$

$$= 2\int \frac{(-1+x^6)^{2/3}}{-1-\sqrt{5}+2x^3}\,dx + 2\int \frac{(-1+x^6)^{2/3}}{-1+\sqrt{5}+2x^3}\,dx - \frac{(-1+x^6)^{2/3}\,\text{Subst}\left(\int \frac{(1-x^3)}{x^2}\right.}{2(1-x^6)^{2/3}}$$

$$= \frac{(-1+x^6)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};x^6\right)}{2x^2(1-x^6)^{2/3}} + 2\int \left(\frac{(-1-\sqrt{5})(-1+x^6)^{2/3}}{2(3+\sqrt{5}-2x^6)} + \frac{x^3(-1+x^6)}{-3-\sqrt{5}+2}\right.$$

$$= \frac{(-1+x^6)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};x^6\right)}{2x^2(1-x^6)^{2/3}} + 2\int \frac{x^3(-1+x^6)^{2/3}}{-3-\sqrt{5}+2x^6}\,dx + 2\int \frac{x^3(-1+x^6)^{2/}}{-3+\sqrt{5}+2x}$$

$$= \frac{(-1+x^6)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};x^6\right)}{2x^2(1-x^6)^{2/3}} + \frac{\left((-1-\sqrt{5})(-1+x^6)^{2/3}\right)\int \frac{(1-x^6)^{2/3}}{3+\sqrt{5}-2x^6}\,dx}{(1-x^6)^{2/3}} + \frac{(}{}$$

$$= -\frac{(1-\sqrt{5})\,x\,(-1+x^6)^{2/3}\,F_1\left(\frac{1}{6};-\frac{2}{3},1;\frac{7}{6};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{(3-\sqrt{5})(1-x^6)^{2/3}} - \frac{(1+\sqrt{5})\,x\,(-1+x^6)^{2/3}\,F_1}{(3+\sqrt{5})(1}$$

$$= -\frac{(1-\sqrt{5})\,x\,(-1+x^6)^{2/3}\,F_1\left(\frac{1}{6};-\frac{2}{3},1;\frac{7}{6};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{(3-\sqrt{5})(1-x^6)^{2/3}} - \frac{(1+\sqrt{5})\,x\,(-1+x^6)^{2/3}\,F_1}{(3+\sqrt{5})(1}$$

Mathematica [F] time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^6)^{2/3}(1+x^6)}{x^3(-1-x^3+x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^6)^(2/3)*(1 + x^6))/(x^3*(-1 - x^3 + x^6)),x]
```

```
[Out] Integrate[((-1 + x^6)^(2/3)*(1 + x^6))/(x^3*(-1 - x^3 + x^6)), x]
```

IntegrateAlgebraic [A] time = 0.94, size = 94, normalized size = 1.00

$$\frac{1}{3}\log\left(\sqrt[3]{x^6-1}-x\right) - \frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^6-1}+x}\right)}{\sqrt{3}} + \frac{(x^6-1)^{2/3}}{2x^2} - \frac{1}{6}\log\left(\sqrt[3]{x^6-1}\,x + (x^6-1)^{2/3} + x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^6)^(2/3)*(1 + x^6))/(x^3*(-1 - x^3 + x^6)),x]
```

```
[Out] (-1 + x^6)^(2/3)/(2*x^2) - ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x^6)^(1/3))]/Sqr
t[3] + Log[-x + (-1 + x^6)^(1/3)]/3 - Log[x^2 + x*(-1 + x^6)^(1/3) + (-1 +
x^6)^(2/3)]/6
```

fricas [A] time = 17.21, size = 136, normalized size = 1.45

$$-\frac{2\,\sqrt{3}\,x^{2}\arctan\left(\frac{47399638863594863345242891761429898599688622451126011503668045351488814414825 0\,\sqrt{3}\,(x^{6}-1)^{\frac{1}{3}}x^{2}+19325031480489228255674265966448835967818926087643600184123099965366515892788\,\sqrt{3}\,(x^{6}-1)^{\frac{2}{3}}x+\sqrt{3}\,(771225779807741020855977802972631216428368740202755221603971931588718036144\,x^{6}+245889484278411189833195613987401279765924206559249102388797804808538611984375\,x^{3}-771225779807741020855977802972631216428368740202755221603971931588718036144)}{3\,(15407513785538665202033017569552164636906896740149986002803824712402669144\,x^{6}-227351086091515241263579358841494627179170556108548407412281480599473216796875\,x^{3}-15407513785538665202033017569552164636906896740149986002803824712402669144)}\right)-x^{2}\log\left(\frac{x^{6}-x^{3}+3\,(x^{6}-1)^{\frac{1}{3}}x^{2}-3\,(x^{6}-1)^{\frac{2}{3}}x-1}{x^{6}-x^{3}-1}\right)-3\,(x^{6}-1)^{\frac{2}{3}}}{6\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(2/3)*(x^6+1)/x^3/(x^6-x^3-1),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x^2*arctan(1/3*(4739963886359486334524289176142989859968862
245112601150366804535148881441482 50*sqrt(3)*(x^6 - 1)^(1/3)*x^2 + 193250314
804892282556742659664488359678189260876436001841230999653665158927 88*sqrt(3
)*(x^6 - 1)^(2/3)*x + sqrt(3)*(77122577980774102085597780297263121642836874
0202755221603971931588718036144*x^6 + 2458894842784111898331956139874012797
659242065592491023887978048085386119843 75*x^3 - 771225779807741020855977802
97263121642836874020275522160397193158871803614 4))/(1540751378553866520203 3
01756955216463690689674014998600280382471240266914 4*x^6 - 22735108609151524
1263579358841494627179170556108548407412281480599473216796875*x^3 - 1540751
378553866520203301756955216463690689674014998600280382471240266914 4)) - x^2
*log((x^6 - x^3 + 3*(x^6 - 1)^(1/3)*x^2 - 3*(x^6 - 1)^(2/3)*x - 1)/(x^6 - x
^3 - 1)) - 3*(x^6 - 1)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}+1\right)\left(x^{6}-1\right)^{\frac{2}{3}}}{\left(x^{6}-x^{3}-1\right)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(2/3)*(x^6+1)/x^3/(x^6-x^3-1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 1)*(x^6 - 1)^(2/3)/((x^6 - x^3 - 1)*x^3), x)
```

maple [C] time = 2.10, size = 413, normalized size = 4.39

$$\frac{\left(x^{6}-1\right)^{\frac{2}{3}}}{2x^{2}}+\frac{\ln\left(\frac{-3\operatorname{RootOf}(9\_Z^{2}+3\_Z+1)x^{6}-2x^{6}+9\operatorname{RootOf}(9\_Z^{2}+3\_Z+1)^{2}x^{3}+9\operatorname{RootOf}(9\_Z^{2}+3\_Z+1)(x^{6}-1)^{\frac{2}{3}}x-9\operatorname{RootOf}(9\_Z^{2}+3\_Z+1)(x^{6}-1)^{\frac{1}{3}}x^{2}+6\operatorname{RootOf}(9\_Z^{2}+3\_Z+1)x^{3}+3x(x^{6}-1)^{\frac{2}{3}}+3\operatorname{RootOf}(9\_Z^{2}+3\_Z+1)+2}{x^{6}-x^{3}-1}\right)}{3}-\frac{\ln\left(\frac{3\operatorname{RootOf}(9\_Z^{2}+3\_Z+1)x^{6}+2x^{6}+9\operatorname{RootOf}(9\_Z^{2}+3\_Z+1)^{2}x^{3}-9\operatorname{RootOf}(9\_Z^{2}+3\_Z+1)(x^{6}-1)^{\frac{2}{3}}x+9\operatorname{RootOf}(9\_Z^{2}+3\_Z+1)x^{3}-3x^{2}(x^{6}-1)^{\frac{1}{3}}+2x^{3}-3\operatorname{RootOf}(9\_Z^{2}+3\_Z+1)-2}{x^{6}-x^{3}-1}\right)}{3}-\ln\left(\frac{3\operatorname{RootOf}(9\_Z^{2}+3\_Z+1)x^{6}+2x^{6}+9\operatorname{RootOf}(9\_Z^{2}+3\_Z+1)^{2}x^{3}-9\operatorname{RootOf}(9\_Z^{2}+3\_Z+1)(x^{6}-1)^{\frac{2}{3}}x+9\operatorname{RootOf}(9\_Z^{2}+3\_Z+1)x^{3}-3x^{2}(x^{6}-1)^{\frac{1}{3}}+2x^{3}-3\operatorname{RootOf}(9\_Z^{2}+3\_Z+1)-2}{x^{6}-x^{3}-1}\right)\operatorname{RootOf}(9\_Z^{2}+3\_Z+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(2/3)*(x^6+1)/x^3/(x^6-x^3-1),x)
```

```
[Out] 1/2*(x^6-1)^(2/3)/x^2+1/3*ln((-3*RootOf(9*_Z^2+3*_Z+1)*x^6-2*x^6+9*RootOf(9
*_Z^2+3*_Z+1)^2*x^3+9*RootOf(9*_Z^2+3*_Z+1)*(x^6-1)^(2/3)*x-9*RootOf(9*_Z^2
+3*_Z+1)*(x^6-1)^(1/3)*x^2+6*RootOf(9*_Z^2+3*_Z+1)*x^3+3*x*(x^6-1)^(2/3)+3*
RootOf(9*_Z^2+3*_Z+1)+2)/(x^6-x^3-1))-1/3*ln((3*RootOf(9*_Z^2+3*_Z+1)*x^6+2
*x^6+9*RootOf(9*_Z^2+3*_Z+1)^2*x^3-9*RootOf(9*_Z^2+3*_Z+1)*(x^6-1)^(2/3)*x+
9*RootOf(9*_Z^2+3*_Z+1)*x^3-3*x^2*(x^6-1)^(1/3)+2*x^3-3*RootOf(9*_Z^2+3*_Z+
1)-2)/(x^6-x^3-1))-ln((3*RootOf(9*_Z^2+3*_Z+1)*x^6+2*x^6+9*RootOf(9*_Z^2+3*
_Z+1)^2*x^3-9*RootOf(9*_Z^2+3*_Z+1)*(x^6-1)^(2/3)*x+9*RootOf(9*_Z^2+3*_Z+1)
*x^3-3*x^2*(x^6-1)^(1/3)+2*x^3-3*RootOf(9*_Z^2+3*_Z+1)-2)/(x^6-x^3-1))*Root
Of(9*_Z^2+3*_Z+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}+1\right)\left(x^{6}-1\right)^{\frac{2}{3}}}{\left(x^{6}-x^{3}-1\right)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(2/3)*(x^6+1)/x^3/(x^6-x^3-1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + 1)*(x^6 - 1)^(2/3)/((x^6 - x^3 - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^6-1\right)^{2/3}\,\left(x^6+1\right)}{x^3\,\left(-x^6+x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^6 - 1)^(2/3)*(x^6 + 1))/(x^3*(x^3 - x^6 + 1)),x)
[Out] int(-((x^6 - 1)^(2/3)*(x^6 + 1))/(x^3*(x^3 - x^6 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(2/3)*(x**6+1)/x**3/(x**6-x**3-1),x)
[Out] Timed out
```

3.1169 $\int \frac{(-1+x^6)(1+x^6)^{2/3}}{x^3(1-x^3+x^6)}\,dx$

Optimal. Leaf size=94

$$\frac{1}{3}\log\left(\sqrt[3]{x^6+1}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^6+1}+x}\right)}{\sqrt{3}}+\frac{\left(x^6+1\right)^{2/3}}{2x^2}-\frac{1}{6}\log\left(\sqrt[3]{x^6+1}\,x+\left(x^6+1\right)^{2/3}+x^2\right)$$

Rubi [C] time = 0.79, antiderivative size = 247, normalized size of antiderivative = 2.63, number of steps used = 16, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {6728, 275, 364, 1438, 429, 465, 510}

$$\frac{\left(-\sqrt{3}+i\right)xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};-\frac{2x^6}{1-i\sqrt{3}},-x^6\right)}{\sqrt{3}+i}+\frac{\left(\sqrt{3}+i\right)xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};-\frac{2x^6}{1+i\sqrt{3}},-x^6\right)}{-\sqrt{3}+i}+\frac{x^4F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-x^6,-\frac{2x^6}{1-i\sqrt{3}}\right)}{2\left(1-i\sqrt{3}\right)}+\frac{x^4F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-x^6,-\frac{2x^6}{1+i\sqrt{3}}\right)}{2\left(1+i\sqrt{3}\right)}+\frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};-x^6\right)}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^6)*(1 + x^6)^(2/3))/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] ((I - Sqrt[3])*x*AppellF1[1/6, 1, -2/3, 7/6, (-2*x^6)/(1 - I*Sqrt[3]), -x^6
])/(I + Sqrt[3]) + ((I + Sqrt[3])*x*AppellF1[1/6, 1, -2/3, 7/6, (-2*x^6)/(1
 + I*Sqrt[3]), -x^6])/(I - Sqrt[3]) + (x^4*AppellF1[2/3, -2/3, 1, 5/3, -x^6
, (-2*x^6)/(1 - I*Sqrt[3])])/(2*(1 - I*Sqrt[3])) + (x^4*AppellF1[2/3, -2/3,
 1, 5/3, -x^6, (-2*x^6)/(1 + I*Sqrt[3])])/(2*(1 + I*Sqrt[3])) + Hypergeomet
ric2F1[-2/3, -1/3, 2/3, -x^6]/(2*x^2)
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(-1+x^6)(1+x^6)^{2/3}}{x^3(1-x^3+x^6)}\,dx = \int \left(-\frac{(1+x^6)^{2/3}}{x^3}+\frac{(-1+2x^3)(1+x^6)^{2/3}}{1-x^3+x^6}\right)dx$$

$$= -\int \frac{(1+x^6)^{2/3}}{x^3}\,dx + \int \frac{(-1+2x^3)(1+x^6)^{2/3}}{1-x^3+x^6}\,dx$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(1+x^3)^{2/3}}{x^2}\,dx, x, x^2\right)\right) + \int \left(\frac{2(1+x^6)^{2/3}}{-1-i\sqrt{3}+2x^3}+\frac{2(1+x^6)^{2/3}}{-1+i\sqrt{3}+2x^3}\right.$$

$$= \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};-x^6\right)}{2x^2} + 2\int \frac{(1+x^6)^{2/3}}{-1-i\sqrt{3}+2x^3}\,dx + 2\int \frac{(1+x^6)^{2/3}}{-1+i\sqrt{3}+2x^3}\,dx$$

$$= \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};-x^6\right)}{2x^2} + 2\int \left(\frac{(i-\sqrt{3})(1+x^6)^{2/3}}{2(i+\sqrt{3}+2ix^6)}+\frac{x^3(1+x^6)^{2/3}}{1-i\sqrt{3}+2x^6}\right)dx + 2\int\Bigg($$

$$= \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};-x^6\right)}{2x^2} + 2\int \frac{x^3(1+x^6)^{2/3}}{1-i\sqrt{3}+2x^6}\,dx + 2\int \frac{x^3(1+x^6)^{2/3}}{1+i\sqrt{3}+2x^6}\,dx + \left(-i-\right.$$

$$= \frac{(i-\sqrt{3})\,xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};-\frac{2x^6}{1-i\sqrt{3}},-x^6\right)}{i+\sqrt{3}} + \frac{(i+\sqrt{3})\,xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};-\frac{2x^6}{1+i\sqrt{3}},-x^6\right.}{i-\sqrt{3}}$$

$$= \frac{(i-\sqrt{3})\,xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};-\frac{2x^6}{1-i\sqrt{3}},-x^6\right)}{i+\sqrt{3}} + \frac{(i+\sqrt{3})\,xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};-\frac{2x^6}{1+i\sqrt{3}},-x^6\right.}{i-\sqrt{3}}$$

Mathematica [F] time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^6)(1+x^6)^{2/3}}{x^3(1-x^3+x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^6)*(1 + x^6)^(2/3))/(x^3*(1 - x^3 + x^6)),x]
[Out] Integrate[((-1 + x^6)*(1 + x^6)^(2/3))/(x^3*(1 - x^3 + x^6)), x]
```

IntegrateAlgebraic [A] time = 0.91, size = 94, normalized size = 1.00

$$\frac{1}{3}\log\left(\sqrt[3]{x^6+1}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^6+1}+x}\right)}{\sqrt{3}}+\frac{\left(x^6+1\right)^{2/3}}{2x^2}-\frac{1}{6}\log\left(\sqrt[3]{x^6+1}\,x+\left(x^6+1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^6)*(1 + x^6)^(2/3))/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] (1 + x^6)^(2/3)/(2*x^2) - ArcTan[(Sqrt[3]*x)/(x + 2*(1 + x^6)^(1/3))]/Sqrt[
3] + Log[-x + (1 + x^6)^(1/3)]/3 - Log[x^2 + x*(1 + x^6)^(1/3) + (1 + x^6)^
(2/3)]/6
```

fricas [A] time = 9.02, size = 135, normalized size = 1.44

$$-\frac{2\sqrt{3}x^2\arctan\left(\frac{1078\sqrt{3}\left(x^6+1\right)^{\frac{1}{3}}x^2+196\sqrt{3}\left(x^6+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(32x^6+605x^3+32\right)}{8x^6-1331x^3+8}\right)-x^2\log\left(\frac{x^6-x^3+3\left(x^6+1\right)^{\frac{1}{3}}x^2-3\left(x^6+1\right)^{\frac{2}{3}}x+1}{x^6-x^3+1}\right)-3\left(x^6+1\right)^{\frac{2}{3}}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)*(x^6+1)^(2/3)/x^3/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x^2*arctan((1078*sqrt(3)*(x^6 + 1)^(1/3)*x^2 + 196*sqrt(3)*
(x^6 + 1)^(2/3)*x + sqrt(3)*(32*x^6 + 605*x^3 + 32))/(8*x^6 - 1331*x^3 + 8)
) - x^2*log((x^6 - x^3 + 3*(x^6 + 1)^(1/3)*x^2 - 3*(x^6 + 1)^(2/3)*x + 1)/(
x^6 - x^3 + 1)) - 3*(x^6 + 1)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^6+1\right)^{\frac{2}{3}}\left(x^6-1\right)}{\left(x^6-x^3+1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)*(x^6+1)^(2/3)/x^3/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 1)^(2/3)*(x^6 - 1)/((x^6 - x^3 + 1)*x^3), x)
```

maple [C] time = 1.90, size = 282, normalized size = 3.00

$$\frac{\left(x^6+1\right)^{\frac{2}{3}}}{2x^2}+\frac{\ln\left(\frac{3\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)x^6-x^6+9\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)^2x^3-9\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)\left(x^6+1\right)^{\frac{2}{3}}x+9\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)\left(x^6+1\right)^{\frac{1}{3}}x^2+3x^2\left(x^6+1\right)^{\frac{1}{3}}-x^3+3\,\mathrm{RootOf}(9\_Z^2+3\_Z+1)-1}{x^6-x^3+1}\right)}{3}+\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\ln\left(\frac{-3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)x^6+x^6+9\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3+9\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^6+1\right)^{\frac{2}{3}}x-3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3+3x\left(x^6+1\right)^{\frac{2}{3}}-3x^2\left(x^6+1\right)^{\frac{1}{3}}-3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)+1}{x^6-x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)*(x^6+1)^(2/3)/x^3/(x^6-x^3+1),x)
```

```
[Out] 1/2*(x^6+1)^(2/3)/x^2+1/3*ln((3*RootOf(9*_Z^2+3*_Z+1)*x^6-x^6+9*RootOf(9*_Z
^2+3*_Z+1)^2*x^3-9*RootOf(9*_Z^2+3*_Z+1)*(x^6+1)^(2/3)*x+9*RootOf(9*_Z^2+3*
_Z+1)*(x^6+1)^(1/3)*x^2+3*x^2*(x^6+1)^(1/3)-x^3+3*RootOf(9*_Z^2+3*_Z+1)-1)/
(x^6-x^3+1))+RootOf(9*_Z^2+3*_Z+1)*ln((-3*RootOf(9*_Z^2+3*_Z+1)*x^6+x^6+9*R
ootOf(9*_Z^2+3*_Z+1)^2*x^3+9*RootOf(9*_Z^2+3*_Z+1)*(x^6+1)^(2/3)*x-3*RootOf
(9*_Z^2+3*_Z+1)*x^3+3*x*(x^6+1)^(2/3)-3*x^2*(x^6+1)^(1/3)-3*RootOf(9*_Z^2+3
*_Z+1)+1)/(x^6-x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^6+1\right)^{\frac{2}{3}}\left(x^6-1\right)}{\left(x^6-x^3+1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)*(x^6+1)^(2/3)/x^3/(x^6-x^3+1),x, algorithm="maxima")
[Out] integrate((x^6 + 1)^(2/3)*(x^6 - 1)/((x^6 - x^3 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6-1\right)\left(x^6+1\right)^{2/3}}{x^3\left(x^6-x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)*(x^6 + 1)^(2/3))/(x^3*(x^6 - x^3 + 1)),x)
[Out] int(((x^6 - 1)*(x^6 + 1)^(2/3))/(x^3*(x^6 - x^3 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)*(x**6+1)**(2/3)/x**3/(x**6-x**3+1),x)
[Out] Timed out
```

3.1170 $\int \frac{1+3x^4+x^8}{x^2(1+x^4)^{3/4}(1+3x^4+3x^8)}\,dx$

Optimal. Leaf size=94

$$-\frac{\sqrt[4]{x^4+1}}{x}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}-x^2}\right)}{\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{3}\,x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}+x^2}\right)}{\sqrt{3}}$$

Rubi [C] time = 0.79, antiderivative size = 347, normalized size of antiderivative = 3.69, number of steps used = 13, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {6728, 264, 1528, 494, 298, 205, 208}

$$-\frac{\sqrt[4]{x^4+1}}{x}-\frac{1}{6}\left(3+i\sqrt{3}\right)\sqrt[4]{-\frac{-\sqrt{3}+3i}{\sqrt{3}+3i}}\tan^{-1}\left(\frac{x}{\sqrt[4]{-\frac{-\sqrt{3}+3i}{\sqrt{3}+3i}}\sqrt[4]{x^4+1}}\right)-\frac{\left(3-i\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{-\frac{-\sqrt{3}+3i}{\sqrt{3}+3i}}x}{\sqrt[4]{x^4+1}}\right)}{6\sqrt[4]{-\frac{-\sqrt{3}+3i}{\sqrt{3}+3i}}}+\frac{1}{6}\left(3+i\sqrt{3}\right)\sqrt[4]{-\frac{-\sqrt{3}+3i}{\sqrt{3}+3i}}\tanh^{-1}\left(\frac{x}{\sqrt[4]{-\frac{-\sqrt{3}+3i}{\sqrt{3}+3i}}\sqrt[4]{x^4+1}}\right)+\frac{\left(3-i\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{-\frac{-\sqrt{3}+3i}{\sqrt{3}+3i}}x}{\sqrt[4]{x^4+1}}\right)}{6\sqrt[4]{-\frac{-\sqrt{3}+3i}{\sqrt{3}+3i}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + 3*x^4 + x^8)/(x^2*(1 + x^4)^(3/4)*(1 + 3*x^4 + 3*x^8)),x]
```

```
[Out] -((1 + x^4)^(1/4)/x) - ((3 + I*Sqrt[3])*(-((3*I - Sqrt[3])/(3*I + Sqrt[3])))^(1/4)*ArcTan[x/((-((3*I - Sqrt[3])/(3*I + Sqrt[3])))^(1/4)*(1 + x^4)^(1/4))])/6 - ((3 - I*Sqrt[3])*ArcTan[((-((3*I - Sqrt[3])/(3*I + Sqrt[3])))^(1/4)*x)/(1 + x^4)^(1/4)])/(6*(-((3*I - Sqrt[3])/(3*I + Sqrt[3])))^(1/4)) + ((3 + I*Sqrt[3])*(-((3*I - Sqrt[3])/(3*I + Sqrt[3])))^(1/4)*ArcTanh[x/((-((3*I - Sqrt[3])/(3*I + Sqrt[3])))^(1/4)*(1 + x^4)^(1/4))])/6 + ((3 - I*Sqrt[3])*ArcTanh[((-((3*I - Sqrt[3])/(3*I + Sqrt[3])))^(1/4)*x)/(1 + x^4)^(1/4)])/(6*(-((3*I - Sqrt[3])/(3*I + Sqrt[3])))^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Subst[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q + (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
```

```
tQ[-1, p, 0]
```

Rule 1528

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q, (
f*x)^m/(a + b*x^n + c*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, e, f, q, n}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&  !IntegerQ[q] &&
IntegerQ[m]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1+3x^4+x^8}{x^2\left(1+x^4\right)^{3/4}\left(1+3x^4+3x^8\right)}\,dx = \int \left(\frac{1}{x^2\left(1+x^4\right)^{3/4}} - \frac{2x^6}{\left(1+x^4\right)^{3/4}\left(1+3x^4+3x^8\right)}\right)dx$$

$$= -\left(2\int \frac{x^6}{\left(1+x^4\right)^{3/4}\left(1+3x^4+3x^8\right)}\,dx\right) + \int \frac{1}{x^2\left(1+x^4\right)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{1+x^4}}{x} - 2\int \left(\frac{i\left(-3+i\sqrt{3}\right)x^2}{\sqrt{3}\left(-3+i\sqrt{3}-6x^4\right)\left(1+x^4\right)^{3/4}} - \frac{i\left(3+i\cdots\right.}{\sqrt{3}\left(1+x^4\right)^{3/4}\left(\cdots\right.}\right.$$

$$= -\frac{\sqrt[4]{1+x^4}}{x} - \left(2\left(1-i\sqrt{3}\right)\right)\int \frac{x^2}{\left(1+x^4\right)^{3/4}\left(3+i\sqrt{3}+6x^4\right)}\,dx + \left(2\left(1+\cdots\right.\right.$$

$$= -\frac{\sqrt[4]{1+x^4}}{x} - \left(2\left(1-i\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{x^2}{3+i\sqrt{3}-\left(-3+i\sqrt{3}\right)x^4}\,dx, x, \cdots\right.$$

$$= -\frac{\sqrt[4]{1+x^4}}{x} - \frac{\left(i-\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{3i+\sqrt{3}}-\sqrt{-3i+\sqrt{3}}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{-3i+\sqrt{3}}} + \frac{\left(i-\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt[4]{1+x^4}}{x} - \frac{1}{2}\left(1+\frac{i}{\sqrt{3}}\right)\sqrt[4]{-\frac{3i-\sqrt{3}}{3i+\sqrt{3}}}\tan^{-1}\left(\frac{x}{\sqrt[4]{-\frac{3i-\sqrt{3}}{3i+\sqrt{3}}}\sqrt[4]{1+x^4}}\right) - \cdots$$

Mathematica [F] time = 7.11, size = 0, normalized size = 0.00

$$\int \frac{1+3x^4+x^8}{x^2\left(1+x^4\right)^{3/4}\left(1+3x^4+3x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + 3*x^4 + x^8)/(x^2*(1 + x^4)^(3/4)*(1 + 3*x^4 + 3*x^8)),x]
[Out] Integrate[(1 + 3*x^4 + x^8)/(x^2*(1 + x^4)^(3/4)*(1 + 3*x^4 + 3*x^8)), x]
```

IntegrateAlgebraic [A] time = 0.51, size = 115, normalized size = 1.22

$$-\frac{\sqrt[4]{x^4+1}}{x}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[4]{x^4+1}-x}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[4]{x^4+1}+x}\right)}{\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{3}\,x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}+x^2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 3*x^4 + x^8)/(x^2*(1 + x^4)^(3/4)*(1 + 3*x^4 + 3*x^
8)),x]
```

```
[Out] -((1 + x^4)^(1/4)/x) - ArcTan[(Sqrt[3]*x)/(-x + 2*(1 + x^4)^(1/4))]/Sqrt[3]
 - ArcTan[(Sqrt[3]*x)/(x + 2*(1 + x^4)^(1/4))]/Sqrt[3] + ArcTanh[(Sqrt[3]*x
*(1 + x^4)^(1/4))/(x^2 + Sqrt[1 + x^4])]/Sqrt[3]
```

fricas [B] time = 19.35, size = 227, normalized size = 2.41

$$-\frac{2\sqrt{3}\,x\arctan\left(\frac{2\left(\sqrt{3}\left(3x^5-x\right)\left(x^4+1\right)^{\frac{3}{4}}-\sqrt{3}\left(3x^7+4x^3\right)\left(x^4+1\right)^{\frac{1}{4}}\right)}{21x^8+21x^4-1}\right)-\sqrt{3}\,x\log\left(-\frac{441x^{16}+882x^{12}+543x^8+102x^4+4\sqrt{3}\left(63x^{13}+78x^9+24x^5+x\right)\left(x^4+1\right)^{\frac{3}{4}}+4\sqrt{3}\left(63x^{15}+111x^{11}+57x^7+8x^3\right)\left(x^4+1\right)^{\frac{1}{4}}+24\left(18x^{14}+27x^{10}+11x^6+x^2\right)\sqrt{x^4+1}+1}{9x^{16}+18x^{12}+15x^8+6x^4+1}\right)+12\left(x^4+1\right)^{\frac{1}{4}}}{12x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+3*x^4+1)/x^2/(x^4+1)^(3/4)/(3*x^8+3*x^4+1),x, algorithm="fri
cas")
```

```
[Out] -1/12*(2*sqrt(3)*x*arctan(2*(sqrt(3)*(3*x^5 - x)*(x^4 + 1)^(3/4) - sqrt(3)*
(3*x^7 + 4*x^3)*(x^4 + 1)^(1/4))/(21*x^8 + 21*x^4 - 1)) - sqrt(3)*x*log(-(4
41*x^16 + 882*x^12 + 543*x^8 + 102*x^4 + 4*sqrt(3)*(63*x^13 + 78*x^9 + 24*x
^5 + x)*(x^4 + 1)^(3/4) + 4*sqrt(3)*(63*x^15 + 111*x^11 + 57*x^7 + 8*x^3)*(
x^4 + 1)^(1/4) + 24*(18*x^14 + 27*x^10 + 11*x^6 + x^2)*sqrt(x^4 + 1) + 1)/(
9*x^16 + 18*x^12 + 15*x^8 + 6*x^4 + 1)) + 12*(x^4 + 1)^(1/4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8+3x^4+1}{\left(3x^8+3x^4+1\right)\left(x^4+1\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+3*x^4+1)/x^2/(x^4+1)^(3/4)/(3*x^8+3*x^4+1),x, algorithm="gia
c")
```

```
[Out] integrate((x^8 + 3*x^4 + 1)/((3*x^8 + 3*x^4 + 1)*(x^4 + 1)^(3/4)*x^2), x)
```

maple [C] time = 2.38, size = 624, normalized size = 6.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8+3*x^4+1)/x^2/(x^4+1)^(3/4)/(3*x^8+3*x^4+1),x)
```

```
[Out] -(x^4+1)^(1/4)/x+(1/6*RootOf(_Z^2+3)*ln((9*RootOf(_Z^2+3)*x^16-18*(x^12+3*x
^8+3*x^4+1)^(1/4)*x^13-12*(x^12+3*x^8+3*x^4+1)^(1/2)*RootOf(_Z^2+3)*x^10+27
*RootOf(_Z^2+3)*x^12+18*(x^12+3*x^8+3*x^4+1)^(3/4)*x^7-42*(x^12+3*x^8+3*x^4
+1)^(1/4)*x^9-18*(x^12+3*x^8+3*x^4+1)^(1/2)*RootOf(_Z^2+3)*x^6+28*RootOf(_Z
^2+3)*x^8+12*(x^12+3*x^8+3*x^4+1)^(3/4)*x^3-30*(x^12+3*x^8+3*x^4+1)^(1/4)*x
^5-6*(x^12+3*x^8+3*x^4+1)^(1/2)*RootOf(_Z^2+3)*x^2+11*RootOf(_Z^2+3)*x^4-6*
(x^12+3*x^8+3*x^4+1)^(1/4)*x+RootOf(_Z^2+3))/(x^4+1)^2/(3*x^8+3*x^4+1))+1/6
*RootOf(_Z^2-3)*ln((9*RootOf(_Z^2-3)*x^16+18*(x^12+3*x^8+3*x^4+1)^(1/4)*x^1
```

```
3+12*(x^12+3*x^8+3*x^4+1)^(1/2)*RootOf(_Z^2-3)*x^10+27*RootOf(_Z^2-3)*x^12+
18*(x^12+3*x^8+3*x^4+1)^(3/4)*x^7+42*(x^12+3*x^8+3*x^4+1)^(1/4)*x^9+18*(x^1
2+3*x^8+3*x^4+1)^(1/2)*RootOf(_Z^2-3)*x^6+28*RootOf(_Z^2-3)*x^8+12*(x^12+3*
x^8+3*x^4+1)^(3/4)*x^3+30*(x^12+3*x^8+3*x^4+1)^(1/4)*x^5+6*(x^12+3*x^8+3*x^
4+1)^(1/2)*RootOf(_Z^2-3)*x^2+11*RootOf(_Z^2-3)*x^4+6*(x^12+3*x^8+3*x^4+1)^
(1/4)*x+RootOf(_Z^2-3))/(x^4+1)^2/(3*x^8+3*x^4+1)))/(x^4+1)^(3/4)*((x^4+1)^
3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8+3x^4+1}{(3x^8+3x^4+1)(x^4+1)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+3*x^4+1)/x^2/(x^4+1)^(3/4)/(3*x^8+3*x^4+1),x, algorithm="max
ima")
```

```
[Out] integrate((x^8 + 3*x^4 + 1)/((3*x^8 + 3*x^4 + 1)*(x^4 + 1)^(3/4)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8+3x^4+1}{x^2(x^4+1)^{3/4}(3x^8+3x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4 + x^8 + 1)/(x^2*(x^4 + 1)^(3/4)*(3*x^4 + 3*x^8 + 1)),x)
```

```
[Out] int((3*x^4 + x^8 + 1)/(x^2*(x^4 + 1)^(3/4)*(3*x^4 + 3*x^8 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8+3*x**4+1)/x**2/(x**4+1)**(3/4)/(3*x**8+3*x**4+1),x)
```

```
[Out] Timed out
```

3.1171 $\int \frac{-1+x}{x^4 \sqrt[3]{1+x^3}} \, dx$

Optimal. Leaf size=95

$$\frac{(x^3+1)^{2/3}(2-3x)}{6x^3}+\frac{1}{9}\log\left(\sqrt[3]{x^3+1}-1\right)-\frac{1}{18}\log\left((x^3+1)^{2/3}+\sqrt[3]{x^3+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1844, 266, 51, 55, 618, 204, 31, 264}

$$\frac{(x^3+1)^{2/3}}{3x^3}+\frac{1}{6}\log\left(1-\sqrt[3]{x^3+1}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{(x^3+1)^{2/3}}{2x^2}-\frac{\log(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(x^4*(1 + x^3)^(1/3)),x]
```

```
[Out] (1 + x^3)^(2/3)/(3*x^3) - (1 + x^3)^(2/3)/(2*x^2) + ArcTan[(1 + 2*(1 + x^3)
^(1/3))/Sqrt[3]]/(3*Sqrt[3]) - Log[x]/6 + Log[1 - (1 + x^3)^(1/3)]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x}{x^4\sqrt[3]{1+x^3}}\,dx &= \int\left(-\frac{1}{x^4\sqrt[3]{1+x^3}}+\frac{1}{x^3\sqrt[3]{1+x^3}}\right)dx\\
&= -\int \frac{1}{x^4\sqrt[3]{1+x^3}}\,dx+\int \frac{1}{x^3\sqrt[3]{1+x^3}}\,dx\\
&= -\frac{\left(1+x^3\right)^{2/3}}{2x^2}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{1+x}}\,dx,x,x^3\right)\\
&= \frac{\left(1+x^3\right)^{2/3}}{3x^3}-\frac{\left(1+x^3\right)^{2/3}}{2x^2}+\frac{1}{9}\text{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx,x,x^3\right)\\
&= \frac{\left(1+x^3\right)^{2/3}}{3x^3}-\frac{\left(1+x^3\right)^{2/3}}{2x^2}-\frac{\log(x)}{6}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1+x^3}\right)+\frac{1}{6}\text{Subst}\left(\int \frac{}{1+\cdots}\right.\\
&= \frac{\left(1+x^3\right)^{2/3}}{3x^3}-\frac{\left(1+x^3\right)^{2/3}}{2x^2}-\frac{\log(x)}{6}+\frac{1}{6}\log\left(1-\sqrt[3]{1+x^3}\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\cdots\right.\\
&= \frac{\left(1+x^3\right)^{2/3}}{3x^3}-\frac{\left(1+x^3\right)^{2/3}}{2x^2}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^3}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\log(x)}{6}+\frac{1}{6}\log\left(1-\sqrt[3]{1+x^3}\right)
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 35, normalized size = 0.37

$$
-\frac{\left(x^3+1\right)^{2/3}\left(x^2\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};x^3+1\right)+1\right)}{2x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(x^4*(1 + x^3)^(1/3)),x]
```

```
[Out] -1/2*((1 + x^3)^(2/3)*(1 + x^2*Hypergeometric2F1[2/3, 2, 5/3, 1 + x^3]))/x^
2
```

IntegrateAlgebraic [A] time = 7.95, size = 95, normalized size = 1.00

$$
\frac{\left(x^3+1\right)^{2/3}(2-3x)}{6x^3}+\frac{1}{9}\log\left(\sqrt[3]{x^3+1}-1\right)-\frac{1}{18}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/(x^4*(1 + x^3)^(1/3)),x]
```

```
[Out] ((2 - 3*x)*(1 + x^3)^(2/3))/(6*x^3) + ArcTan[1/Sqrt[3] + (2*(1 + x^3)^(1/3)
)/Sqrt[3]]/(3*Sqrt[3]) + Log[-1 + (1 + x^3)^(1/3)]/9 - Log[1 + (1 + x^3)^(1
/3) + (1 + x^3)^(2/3)]/18
```

fricas [A] time = 0.70, size = 104, normalized size = 1.09

$$-\frac{2\sqrt{3}x^3\arctan\left(-\frac{\sqrt{3}\left(x^3+1\right)-2\sqrt{3}\left(x^3+1\right)^{\frac{2}{3}}+4\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}}{x^3+9}\right)-x^3\log\left(\frac{x^3-3\left(x^3+1\right)^{\frac{2}{3}}+3\left(x^3+1\right)^{\frac{1}{3}}}{x^3}\right)+3\left(x^3+1\right)^{\frac{2}{3}}(3x-2)}{18x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x^4/(x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*x^3*arctan(-(sqrt(3)*(x^3 + 1) - 2*sqrt(3)*(x^3 + 1)^(2/3)
 + 4*sqrt(3)*(x^3 + 1)^(1/3))/(x^3 + 9)) - x^3*log((x^3 - 3*(x^3 + 1)^(2/3)
 + 3*(x^3 + 1)^(1/3))/x^3) + 3*(x^3 + 1)^(2/3)*(3*x - 2))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^3+1\right)^{\frac{1}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x^4/(x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/((x^3 + 1)^(1/3)*x^4), x)
```

maple [C] time = 0.40, size = 91, normalized size = 0.96

$$-\frac{3x^4-2x^3+3x-2}{6x^3\left(x^3+1\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^3\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],-x^3\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{18\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/x^4/(x^3+1)^(1/3),x)
```

```
[Out] -1/6*(3*x^4-2*x^3+3*x-2)/x^3/(x^3+1)^(1/3)+1/18/Pi*3^(1/2)*GAMMA(2/3)*(-2/9
*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1,1,4/3],[2,2],-x^3)+2/3*(-1/6*Pi*3^(
1/2)-3/2*ln(3)+3*ln(x))*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.47, size = 78, normalized size = 0.82

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^3+1\right)^{\frac{2}{3}}}{2x^2}+\frac{\left(x^3+1\right)^{\frac{2}{3}}}{3x^3}-\frac{1}{18}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{9}\log\left(\left(x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x^4/(x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) - 1/2*(x^3 + 1)^(2/
3)/x^2 + 1/3*(x^3 + 1)^(2/3)/x^3 - 1/18*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/
3) + 1) + 1/9*log((x^3 + 1)^(1/3) - 1)
```

mupad [B] time = 1.34, size = 104, normalized size = 1.09

$$\frac{\ln\left(\frac{(x^3+1)^{1/3}}{9}-\frac{1}{9}\right)}{9}+\ln\left(\frac{(x^3+1)^{1/3}}{9}-9\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2\right)\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)-\ln\left(\frac{(x^3+1)^{1/3}}{9}-9\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2\right)\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)-\frac{(x^3+1)^{2/3}}{2x^2}+\frac{(x^3+1)^{2/3}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(x^4*(x^3 + 1)^(1/3)),x)
```

```
[Out] log((x^3 + 1)^(1/3)/9 - 1/9)/9 + log((x^3 + 1)^(1/3)/9 - 9*((3^(1/2)*1i)/18
 - 1/18)^2)*((3^(1/2)*1i)/18 - 1/18) - log((x^3 + 1)^(1/3)/9 - 9*((3^(1/2)*
1i)/18 + 1/18)^2)*((3^(1/2)*1i)/18 + 1/18) - (x^3 + 1)^(2/3)/(2*x^2) + (x^3
 + 1)^(2/3)/(3*x^3)
```

sympy [C] time = 1.81, size = 53, normalized size = 0.56

$$\frac{\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{2}{3}\right)}{3\Gamma\left(\frac{1}{3}\right)}+\frac{\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^3}\right)}{3x^4\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x**4/(x**3+1)**(1/3),x)
```

```
[Out] (1 + x**(-3))**(2/3)*gamma(-2/3)/(3*gamma(1/3)) + gamma(4/3)*hyper((1/3, 4/
3), (7/3,), exp_polar(I*pi)/x**3)/(3*x**4*gamma(7/3))
```

3.1172 $\int \frac{(3+2x)\sqrt[3]{1+x+x^3}}{x^2(1+x)}\,dx$

Optimal. Leaf size=95

$$-\frac{3\sqrt[3]{x^3+x+1}}{x}-\log\left(\sqrt[3]{x^3+x+1}-x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x+1}+x}\right)+\frac{1}{2}\log\left(\sqrt[3]{x^3+x+1}\,x+\left(x^3+x+1\right)^{2/3}\right.$$

Rubi [F] time = 1.47, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(3+2x)\sqrt[3]{1+x+x^3}}{x^2(1+x)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((3 + 2*x)*(1 + x + x^3)^(1/3))/(x^2*(1 + x)),x]
```

```
[Out] (9*2^(2/3)*(1 + x + x^3)^(1/3)*Defer[Int][(((2*(3/(-9 + Sqrt[93]))^(1/3) -
(2*(-9 + Sqrt[93]))^(1/3))/6^(2/3) + x)^(1/3)*((6 + 6*3^(1/3)*(2/(-9 + Sqrt
[93]))^(2/3) + 2^(1/3)*(3*(-9 + Sqrt[93]))^(2/3))/18 - (((6/(-9 + Sqrt[93])
)^(1/3) - ((-9 + Sqrt[93])/2)^(1/3))*x)/3^(2/3) + x^2)^(1/3))/x^2, x])/((6^
(1/3)*(2*(3/(-9 + Sqrt[93]))^(1/3) - (2*(-9 + Sqrt[93]))^(1/3)) + 6*x)^(1/3
)*(6 + 6*3^(1/3)*(2/(-9 + Sqrt[93]))^(2/3) + 2^(1/3)*(3*(-9 + Sqrt[93]))^(2
/3) - 6*3^(1/3)*((6/(-9 + Sqrt[93]))^(1/3) - ((-9 + Sqrt[93])/2)^(1/3))*x +
 18*x^2)^(1/3)) - (3*2^(2/3)*(1 + x + x^3)^(1/3)*Defer[Int][(((2*(3/(-9 + S
qrt[93]))^(1/3) - (2*(-9 + Sqrt[93]))^(1/3))/6^(2/3) + x)^(1/3)*((6 + 6*3^(
1/3)*(2/(-9 + Sqrt[93]))^(2/3) + 2^(1/3)*(3*(-9 + Sqrt[93]))^(2/3))/18 - ((
(6/(-9 + Sqrt[93]))^(1/3) - ((-9 + Sqrt[93])/2)^(1/3))*x)/3^(2/3) + x^2)^(1
/3))/x, x])/((6^(1/3)*(2*(3/(-9 + Sqrt[93]))^(1/3) - (2*(-9 + Sqrt[93]))^(1
/3)) + 6*x)^(1/3)*(6 + 6*3^(1/3)*(2/(-9 + Sqrt[93]))^(2/3) + 2^(1/3)*(3*(-9
 + Sqrt[93]))^(2/3) - 6*3^(1/3)*((6/(-9 + Sqrt[93]))^(1/3) - ((-9 + Sqrt[93
])/2)^(1/3))*x + 18*x^2)^(1/3)) + (3*2^(2/3)*(1 + x + x^3)^(1/3)*Defer[Int]
[(((2*(3/(-9 + Sqrt[93]))^(1/3) - (2*(-9 + Sqrt[93]))^(1/3))/6^(2/3) + x)^(
1/3)*((6 + 6*3^(1/3)*(2/(-9 + Sqrt[93]))^(2/3) + 2^(1/3)*(3*(-9 + Sqrt[93])
)^(2/3))/18 - (((6/(-9 + Sqrt[93]))^(1/3) - ((-9 + Sqrt[93])/2)^(1/3))*x)/3
^(2/3) + x^2)^(1/3))/(1 + x), x])/((6^(1/3)*(2*(3/(-9 + Sqrt[93]))^(1/3) -
(2*(-9 + Sqrt[93]))^(1/3)) + 6*x)^(1/3)*(6 + 6*3^(1/3)*(2/(-9 + Sqrt[93]))^
(2/3) + 2^(1/3)*(3*(-9 + Sqrt[93]))^(2/3) - 6*3^(1/3)*((6/(-9 + Sqrt[93]))^
(1/3) - ((-9 + Sqrt[93])/2)^(1/3))*x + 18*x^2)^(1/3))
```

Rubi steps

$$\begin{aligned}
\int \frac{(3+2x)\sqrt[3]{1+x+x^3}}{x^2(1+x)}\,dx &= \int \left(\frac{3\sqrt[3]{1+x+x^3}}{x^2}-\frac{\sqrt[3]{1+x+x^3}}{x}+\frac{\sqrt[3]{1+x+x^3}}{1+x}\right)dx\\
&= 3\int \frac{\sqrt[3]{1+x+x^3}}{x^2}\,dx-\int \frac{\sqrt[3]{1+x+x^3}}{x}\,dx+\int \frac{\sqrt[3]{1+x+x^3}}{1+x}\,dx\\
&= -\frac{\sqrt[3]{1+x+x^3}\int \frac{\sqrt[3]{\frac{2\sqrt[3]{\frac{3}{-9+\sqrt{93}}}-\sqrt[3]{2\left(-9+\sqrt{93}\right)}}{6^{2/3}}+x}\sqrt[3]{\frac{1}{18}\left(6+6\sqrt[3]{3}\left(\frac{2}{-9+\sqrt{93}}\right)^{2/3}+\sqrt[3]{2}\left(3\left(-9+\sqrt{93}\right)\right)^{2/3}\right.}}{x}}{\sqrt[3]{\frac{2\sqrt[3]{\frac{3}{-9+\sqrt{93}}}-\sqrt[3]{2\left(-9+\sqrt{93}\right)}}{6^{2/3}}+x}\sqrt[3]{\frac{1}{18}\left(6+6\sqrt[3]{3}\left(\frac{2}{-9+\sqrt{93}}\right)^{2/3}+\sqrt[3]{2}\left(3\left(-9+\sqrt{93}\right)\right)^{2/}\right.}}
\end{aligned}$$

Mathematica [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(3+2x)\sqrt[3]{1+x+x^3}}{x^2(1+x)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((3 + 2*x)*(1 + x + x^3)^(1/3))/(x^2*(1 + x)),x]
[Out] Integrate[((3 + 2*x)*(1 + x + x^3)^(1/3))/(x^2*(1 + x)), x]
```

IntegrateAlgebraic [A] time = 0.26, size = 95, normalized size = 1.00

$$-\frac{3\sqrt[3]{x^3+x+1}}{x}-\log\left(\sqrt[3]{x^3+x+1}-x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x+1}+x}\right)+\frac{1}{2}\log\left(\sqrt[3]{x^3+x+1}\,x+\left(x^3+x+1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3 + 2*x)*(1 + x + x^3)^(1/3))/(x^2*(1 + x)),x]
[Out] (-3*(1 + x + x^3)^(1/3))/x - Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(1 + x + x^3
)^(1/3))] - Log[-x + (1 + x + x^3)^(1/3)] + Log[x^2 + x*(1 + x + x^3)^(1/3)
 + (1 + x + x^3)^(2/3)]/2
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)*(x^3+x+1)^(1/3)/x^2/(1+x),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+x+1\right)^{\frac{1}{3}}(2x+3)}{(x+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)*(x^3+x+1)^(1/3)/x^2/(1+x),x, algorithm="giac")
[Out] integrate((x^3 + x + 1)^(1/3)*(2*x + 3)/((x + 1)*x^2), x)
```

maple [C] time = 3.11, size = 1096, normalized size = 11.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)*(x^3+x+1)^(1/3)/x^2/(1+x),x)
[Out] -3*(x^3+x+1)^(1/3)/x+(1/2*RootOf(_Z^2-2*_Z+4)*ln((RootOf(_Z^2-2*_Z+4)^2*x^6
+RootOf(_Z^2-2*_Z+4)*x^6+RootOf(_Z^2-2*_Z+4)^2*x^4+3*RootOf(_Z^2-2*_Z+4)*(x
^6+2*x^4+2*x^3+x^2+2*x+1)^(1/3)*x^4-2*x^6+RootOf(_Z^2-2*_Z+4)^2*x^3+3*RootO
f(_Z^2-2*_Z+4)*(x^6+2*x^4+2*x^3+x^2+2*x+1)^(2/3)*x^2+3*RootOf(_Z^2-2*_Z+4)*
x^4-6*(x^6+2*x^4+2*x^3+x^2+2*x+1)^(1/3)*x^4+3*RootOf(_Z^2-2*_Z+4)*(x^6+2*x^
4+2*x^3+x^2+2*x+1)^(1/3)*x^2+3*RootOf(_Z^2-2*_Z+4)*x^3-6*x^2*(x^6+2*x^4+2*x
^3+x^2+2*x+1)^(2/3)-4*x^4+3*RootOf(_Z^2-2*_Z+4)*(x^6+2*x^4+2*x^3+x^2+2*x+1)
^(1/3)*x+2*RootOf(_Z^2-2*_Z+4)*x^2-6*(x^6+2*x^4+2*x^3+x^2+2*x+1)^(1/3)*x^2-
```

```
4*x^3+4*RootOf(_Z^2-2*_Z+4)*x-6*(x^6+2*x^4+2*x^3+x^2+2*x+1)^(1/3)*x-2*x^2+2
*RootOf(_Z^2-2*_Z+4)-4*x-2)/(x^3+x+1)/(1+x))+ln((RootOf(_Z^2-2*_Z+4)^2*x^6-
5*RootOf(_Z^2-2*_Z+4)*x^6+RootOf(_Z^2-2*_Z+4)^2*x^4-3*RootOf(_Z^2-2*_Z+4)*(
x^6+2*x^4+2*x^3+x^2+2*x+1)^(1/3)*x^4+4*x^6+RootOf(_Z^2-2*_Z+4)^2*x^3-3*Root
Of(_Z^2-2*_Z+4)*(x^6+2*x^4+2*x^3+x^2+2*x+1)^(2/3)*x^2-7*RootOf(_Z^2-2*_Z+4)
*x^4-3*RootOf(_Z^2-2*_Z+4)*(x^6+2*x^4+2*x^3+x^2+2*x+1)^(1/3)*x^2-7*RootOf(_
Z^2-2*_Z+4)*x^3+6*x^4-3*RootOf(_Z^2-2*_Z+4)*(x^6+2*x^4+2*x^3+x^2+2*x+1)^(1/
3)*x-2*RootOf(_Z^2-2*_Z+4)*x^2+6*x^3-4*RootOf(_Z^2-2*_Z+4)*x+2*x^2-2*RootOf
(_Z^2-2*_Z+4)+4*x+2)/(x^3+x+1)/(1+x))-1/2*ln((RootOf(_Z^2-2*_Z+4)^2*x^6-5*R
ootOf(_Z^2-2*_Z+4)*x^6+RootOf(_Z^2-2*_Z+4)^2*x^4-3*RootOf(_Z^2-2*_Z+4)*(x^6
+2*x^4+2*x^3+x^2+2*x+1)^(1/3)*x^4+4*x^6+RootOf(_Z^2-2*_Z+4)^2*x^3-3*RootOf(
_Z^2-2*_Z+4)*(x^6+2*x^4+2*x^3+x^2+2*x+1)^(2/3)*x^2-7*RootOf(_Z^2-2*_Z+4)*x^
4-3*RootOf(_Z^2-2*_Z+4)*(x^6+2*x^4+2*x^3+x^2+2*x+1)^(1/3)*x^2-7*RootOf(_Z^2
-2*_Z+4)*x^3+6*x^4-3*RootOf(_Z^2-2*_Z+4)*(x^6+2*x^4+2*x^3+x^2+2*x+1)^(1/3)*
x-2*RootOf(_Z^2-2*_Z+4)*x^2+6*x^3-4*RootOf(_Z^2-2*_Z+4)*x+2*x^2-2*RootOf(_Z
^2-2*_Z+4)+4*x+2)/(x^3+x+1)/(1+x))*RootOf(_Z^2-2*_Z+4))/(x^3+x+1)^(2/3)*((x
^3+x+1)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+x+1\right)^{\frac{1}{3}}(2\,x+3)}{(x+1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)*(x^3+x+1)^(1/3)/x^2/(1+x),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + x + 1)^(1/3)*(2*x + 3)/((x + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x+3)\,\left(x^3+x+1\right)^{1/3}}{x^2\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 3)*(x + x^3 + 1)^(1/3))/(x^2*(x + 1)),x)
```

```
[Out] int(((2*x + 3)*(x + x^3 + 1)^(1/3))/(x^2*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+3)\,\sqrt[3]{x^3+x+1}}{x^2\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)*(x**3+x+1)**(1/3)/x**2/(1+x),x)
```

```
[Out] Integral((2*x + 3)*(x**3 + x + 1)**(1/3)/(x**2*(x + 1)), x)
```

3.1173 $\int \frac{(a^2-2ax+x^2)(-ab+2(a-b)x+x^2)}{(x(-a+x)(-b+x))^{3/4}(-a^3d+(b+3a^2d)x-(1+3ad)x^2+dx^3)}\,dx$

Optimal. Leaf size=95

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x^2(-a-b)+abx+x^3\right)^{3/4}}{x(x-b)}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x^2(-a-b)+abx+x^3\right)^{3/4}}{x(x-b)}\right)}{d^{3/4}}$$

Rubi [F] time = 11.60, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a^2-2ax+x^2)(-ab+2(a-b)x+x^2)}{(x(-a+x)(-b+x))^{3/4}(-a^3d+(b+3a^2d)x-(1+3ad)x^2+dx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a^2 - 2*a*x + x^2)*(-(a*b) + 2*(a - b)*x + x^2))/((x*(-a + x)*(-b + x
))^(3/4)*(-(a^3*d) + (b + 3*a^2*d)*x - (1 + 3*a*d)*x^2 + d*x^3)),x]
```

```
[Out] (4*a*b*x^(3/4)*(-a + x)^(3/4)*(-b + x)^(3/4)*Defer[Subst][Defer[Int][(-a +
x^4)^(5/4)/((-b + x^4)^(3/4)*(a^3*d - b*(1 + (3*a^2*d)/b)*x^4 + (1 + 3*a*d)
*x^8 - d*x^12)), x], x, x^(1/4)])/((a - x)*(b - x)*x)^(3/4) - (8*(a - b)*x^
(3/4)*(-a + x)^(3/4)*(-b + x)^(3/4)*Defer[Subst][Defer[Int][(x^4*(-a + x^4)
^(5/4))/((-b + x^4)^(3/4)*(a^3*d - b*(1 + (3*a^2*d)/b)*x^4 + (1 + 3*a*d)*x^
8 - d*x^12)), x], x, x^(1/4)])/((a - x)*(b - x)*x)^(3/4) + (4*x^(3/4)*(-a +
 x)^(3/4)*(-b + x)^(3/4)*Defer[Subst][Defer[Int][(x^8*(-a + x^4)^(5/4))/((-
b + x^4)^(3/4)*(-(a^3*d) + b*(1 + (3*a^2*d)/b)*x^4 - (1 + 3*a*d)*x^8 + d*x^
12)), x], x, x^(1/4)])/((a - x)*(b - x)*x)^(3/4)
```

Rubi steps

$$\int \frac{(a^2-2ax+x^2)(-ab+2(a-b)x+x^2)}{(x(-a+x)(-b+x))^{3/4}(-a^3d+(b+3a^2d)x-(1+3ad)x^2+dx^3)}\,dx = \int \frac{(-a+x)^2(-ab+\ldots}{(x(-a+x)(-b+x))^{3/4}(-a^3d+(\ldots}$$

$$= \frac{\left(x^{3/4}(-a+x)^{3/4}(-b+x)^{3/4}\right)\int \frac{\ldots}{x^{3/4}(-\ldots}}{(x(-a+x)(\ldots}$$

$$= \frac{\left(4x^{3/4}(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\ldots}{(x(\ldots}$$

$$= \frac{\left(4x^{3/4}(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\ldots}{\ldots}$$

$$= \frac{\left(4x^{3/4}(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\ldots}{(x\ldots}$$

Mathematica [F] time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{(a^2-2ax+x^2)(-ab+2(a-b)x+x^2)}{(x(-a+x)(-b+x))^{3/4}(-a^3d+(b+3a^2d)x-(1+3ad)x^2+dx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a^2 - 2*a*x + x^2)*(-(a*b) + 2*(a - b)*x + x^2))/((x*(-a + x)*(
-b + x))^(3/4)*(-(a^3*d) + (b + 3*a^2*d)*x - (1 + 3*a*d)*x^2 + d*x^3)),x]
```

```
[Out] Integrate[((a^2 - 2*a*x + x^2)*(-(a*b) + 2*(a - b)*x + x^2))/((x*(-a + x)*(
-b + x))^(3/4)*(-(a^3*d) + (b + 3*a^2*d)*x - (1 + 3*a*d)*x^2 + d*x^3)), x]
```

IntegrateAlgebraic [A] time = 3.38, size = 95, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x^2(-a-b)+abx+x^3\right)^{3/4}}{x(x-b)}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x^2(-a-b)+abx+x^3\right)^{3/4}}{x(x-b)}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a^2 - 2*a*x + x^2)*(-(a*b) + 2*(a - b)*x + x^2))/((x*(
-a + x)*(-b + x))^(3/4)*(-(a^3*d) + (b + 3*a^2*d)*x - (1 + 3*a*d)*x^2 + d*x
^3)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(a*b*x + (-a - b)*x^2 + x^3)^(3/4))/(x*(-b + x))])/d^(3/
4) - (2*ArcTanh[(d^(1/4)*(a*b*x + (-a - b)*x^2 + x^3)^(3/4))/(x*(-b + x))])
/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-2*a*x+x^2)*(-a*b+2*(a-b)*x+x^2)/(x*(-a+x)*(-b+x))^(3/4)/(-a^
3*d+(3*a^2*d+b)*x-(3*a*d+1)*x^2+d*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2-2\,ax+x^2\right)\left(ab-2\,(a-b)x-x^2\right)}{\left(a^3d-dx^3+(3\,ad+1)x^2-\left(3\,a^2d+b\right)x\right)((a-x)(b-x)x)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-2*a*x+x^2)*(-a*b+2*(a-b)*x+x^2)/(x*(-a+x)*(-b+x))^(3/4)/(-a^
3*d+(3*a^2*d+b)*x-(3*a*d+1)*x^2+d*x^3),x, algorithm="giac")
```

```
[Out] integrate((a^2 - 2*a*x + x^2)*(a*b - 2*(a - b)*x - x^2)/((a^3*d - d*x^3 + (
3*a*d + 1)*x^2 - (3*a^2*d + b)*x)*((a - x)*(b - x)*x)^(3/4)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2-2ax+x^2\right)\left(-ab+2\,(a-b)\,x+x^2\right)}{\left(x\,(-a+x)\,(-b+x)\right)^{\frac{3}{4}}\left(-a^3d+\left(3a^2d+b\right)x-(3ad+1)\,x^2+d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-2*a*x+x^2)*(-a*b+2*(a-b)*x+x^2)/(x*(-a+x)*(-b+x))^(3/4)/(-a^3*d+(3
*a^2*d+b)*x-(3*a*d+1)*x^2+d*x^3),x)
```

```
[Out] int((a^2-2*a*x+x^2)*(-a*b+2*(a-b)*x+x^2)/(x*(-a+x)*(-b+x))^(3/4)/(-a^3*d+(3
*a^2*d+b)*x-(3*a*d+1)*x^2+d*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2 - 2\,ax + x^2\right)\left(ab - 2\,(a-b)x - x^2\right)}{\left(a^3d - dx^3 + (3\,ad + 1)x^2 - \left(3\,a^2d + b\right)x\right)((a-x)(b-x)x)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-2*a*x+x^2)*(-a*b+2*(a-b)*x+x^2)/(x*(-a+x)*(-b+x))^(3/4)/(-a^
3*d+(3*a^2*d+b)*x-(3*a*d+1)*x^2+d*x^3),x, algorithm="maxima")
```

```
[Out] integrate((a^2 - 2*a*x + x^2)*(a*b - 2*(a - b)*x - x^2)/((a^3*d - d*x^3 + (
3*a*d + 1)*x^2 - (3*a^2*d + b)*x)*((a - x)*(b - x)*x)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2 - 2\,a\,x + x^2\right)\,\left(2\,x\,\left(a-b\right) - a\,b + x^2\right)}{{\left(x\,\left(a-x\right)\,\left(b-x\right)\right)}^{3/4}\,\left(x\,\left(3\,d\,a^2 + b\right) - a^3\,d + d\,x^3 - x^2\,\left(3\,a\,d + 1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a^2 - 2*a*x + x^2)*(2*x*(a - b) - a*b + x^2))/((x*(a - x)*(b - x))^(3
/4)*(x*(b + 3*a^2*d) - a^3*d + d*x^3 - x^2*(3*a*d + 1))),x)
```

```
[Out] int(((a^2 - 2*a*x + x^2)*(2*x*(a - b) - a*b + x^2))/((x*(a - x)*(b - x))^(3
/4)*(x*(b + 3*a^2*d) - a^3*d + d*x^3 - x^2*(3*a*d + 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-2*a*x+x**2)*(-a*b+2*(a-b)*x+x**2)/(x*(-a+x)*(-b+x))**(3/4)/
(-a**3*d+(3*a**2*d+b)*x-(3*a*d+1)*x**2+d*x**3),x)
```

```
[Out] Timed out
```

3.1174 $\int \frac{\left(1+x^2\right)^2}{\left(1-x^2\right)\left(1-6x^2+x^4\right)^{3/4}}\,dx$

Optimal. Leaf size=95

$$\tan^{-1}\left(\frac{x+i}{\sqrt[4]{x^4-6x^2+1}}\right)-\tan^{-1}\left(\frac{\sqrt[4]{x^4-6x^2+1}}{x-i}\right)-\tanh^{-1}\left(\frac{x+i}{\sqrt[4]{x^4-6x^2+1}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{x^4-6x^2+1}}{x-i}\right)$$

Rubi [F] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+x^2\right)^2}{\left(1-x^2\right)\left(1-6x^2+x^4\right)^{3/4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^2)^2/((1 - x^2)*(1 - 6*x^2 + x^4)^(3/4)),x]
```

```
[Out] Defer[Int][(1 + x^2)^2/((1 - x^2)*(1 - 6*x^2 + x^4)^(3/4)), x]
```

Rubi steps

$$\int \frac{\left(1+x^2\right)^2}{\left(1-x^2\right)\left(1-6x^2+x^4\right)^{3/4}}\,dx=\int \frac{\left(1+x^2\right)^2}{\left(1-x^2\right)\left(1-6x^2+x^4\right)^{3/4}}\,dx$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\left(1+x^2\right)^2}{\left(1-x^2\right)\left(1-6x^2+x^4\right)^{3/4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^2)^2/((1 - x^2)*(1 - 6*x^2 + x^4)^(3/4)),x]
```

```
[Out] Integrate[(1 + x^2)^2/((1 - x^2)*(1 - 6*x^2 + x^4)^(3/4)), x]
```

IntegrateAlgebraic [A] time = 5.32, size = 95, normalized size = 1.00

$$\tan^{-1}\left(\frac{x+i}{\sqrt[4]{x^4-6x^2+1}}\right)-\tan^{-1}\left(\frac{\sqrt[4]{x^4-6x^2+1}}{x-i}\right)-\tanh^{-1}\left(\frac{x+i}{\sqrt[4]{x^4-6x^2+1}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{x^4-6x^2+1}}{x-i}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)^2/((1 - x^2)*(1 - 6*x^2 + x^4)^(3/4)),x]
```

```
[Out] ArcTan[(I + x)/(1 - 6*x^2 + x^4)^(1/4)] - ArcTan[(1 - 6*x^2 + x^4)^(1/4)/(-
I + x)] - ArcTanh[(I + x)/(1 - 6*x^2 + x^4)^(1/4)] - ArcTanh[(1 - 6*x^2 + x
^4)^(1/4)/(-I + x)]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(-x^2+1)/(x^4-6*x^2+1)^(3/4),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(x^{2}+1\right)^{2}}{\left(x^{4}-6\,x^{2}+1\right)^{\frac{3}{4}}\left(x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(-x^2+1)/(x^4-6*x^2+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 1)^2/((x^4 - 6*x^2 + 1)^(3/4)*(x^2 - 1)), x)
```

maple [C] time = 3.74, size = 232, normalized size = 2.44

$$-\frac{\ln\left(-\frac{\left(x^{4}-6x^{2}+1\right)^{\frac{3}{4}}x+\sqrt{x^{4}-6x^{2}+1}\,x^{2}+\left(x^{4}-6x^{2}+1\right)^{\frac{1}{4}}x^{3}+x^{4}-\sqrt{x^{4}-6x^{2}+1}-3\left(x^{4}-6x^{2}+1\right)^{\frac{1}{4}}x-5x^{2}}{(1+x)(-1+x)}\right)}{2}+\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{x^{4}-6x^{2}+1}\,x^{2}-\mathrm{RootOf}\left(\_Z^{2}+1\right)x^{4}-\left(x^{4}-6x^{2}+1\right)^{\frac{3}{4}}x+\left(x^{4}-6x^{2}+1\right)^{\frac{1}{4}}x^{3}-\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{x^{4}-6x^{2}+1}+5\,\mathrm{RootOf}\left(\_Z^{2}+1\right)x^{2}-3\left(x^{4}-6x^{2}+1\right)^{\frac{1}{4}}x}{(1+x)(-1+x)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2/(-x^2+1)/(x^4-6*x^2+1)^(3/4),x)
```

```
[Out] -1/2*ln(-((x^4-6*x^2+1)^(3/4)*x+(x^4-6*x^2+1)^(1/2)*x^2+(x^4-6*x^2+1)^(1/4)
*x^3+x^4-(x^4-6*x^2+1)^(1/2)-3*(x^4-6*x^2+1)^(1/4)*x-5*x^2)/(1+x)/(-1+x))+1
/2*RootOf(_Z^2+1)*ln((RootOf(_Z^2+1)*(x^4-6*x^2+1)^(1/2)*x^2-RootOf(_Z^2+1)
*x^4-(x^4-6*x^2+1)^(3/4)*x+(x^4-6*x^2+1)^(1/4)*x^3-RootOf(_Z^2+1)*(x^4-6*x^
2+1)^(1/2)+5*RootOf(_Z^2+1)*x^2-3*(x^4-6*x^2+1)^(1/4)*x)/(1+x)/(-1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(x^{2}+1\right)^{2}}{\left(x^{4}-6\,x^{2}+1\right)^{\frac{3}{4}}\left(x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(-x^2+1)/(x^4-6*x^2+1)^(3/4),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 1)^2/((x^4 - 6*x^2 + 1)^(3/4)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{{\left(x^2+1\right)}^2}{\left(x^2-1\right)\,{\left(x^4-6\,x^2+1\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 + 1)^2/((x^2 - 1)*(x^4 - 6*x^2 + 1)^(3/4)),x)
```

```
[Out] int(-(x^2 + 1)^2/((x^2 - 1)*(x^4 - 6*x^2 + 1)^(3/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x^{2}}{x^{2}\left(x^{4}-6x^{2}+1\right)^{\frac{3}{4}}-\left(x^{4}-6x^{2}+1\right)^{\frac{3}{4}}}\,dx-\int \frac{x^{4}}{x^{2}\left(x^{4}-6x^{2}+1\right)^{\frac{3}{4}}-\left(x^{4}-6x^{2}+1\right)^{\frac{3}{4}}}\,dx-\int \frac{1}{x^{2}\left(x^{4}-6x^{2}+1\right)^{\frac{3}{4}}-\left(x^{4}-6x^{2}+1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2/(-x**2+1)/(x**4-6*x**2+1)**(3/4),x)
```

```
[Out] -Integral(2*x**2/(x**2*(x**4 - 6*x**2 + 1)**(3/4) - (x**4 - 6*x**2 + 1)**(3
/4)), x) - Integral(x**4/(x**2*(x**4 - 6*x**2 + 1)**(3/4) - (x**4 - 6*x**2
+ 1)**(3/4)), x) - Integral(1/(x**2*(x**4 - 6*x**2 + 1)**(3/4) - (x**4 - 6*
x**2 + 1)**(3/4)), x)
```

3.1175 $\int \frac{(-4+x^3)(1-x^3+x^4)}{x^2(-1+x^3)^{3/4}(-1+x^3+x^4)} \, dx$

Optimal. Leaf size=95

$$\frac{4\sqrt[4]{x^3-1}}{x}+2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^3-1}}{\sqrt{x^3-1}-x^2}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^3-1}}{\sqrt{x^3-1}+x^2}\right)$$

Rubi [F] time = 1.41, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-4+x^3)(1-x^3+x^4)}{x^2(-1+x^3)^{3/4}(-1+x^3+x^4)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[((-4 + x^3)*(1 - x^3 + x^4))/(x^2*(-1 + x^3)^(3/4)*(-1 + x^3 + x^4)),x]
```

```
[Out] (-4*(1 - x^3)^(3/4)*Hypergeometric2F1[-1/3, 3/4, 2/3, x^3])/(x*(-1 + x^3)^(
3/4)) - (2*x*(1 - x^3)^(3/4)*Hypergeometric2F1[1/3, 3/4, 4/3, x^3])/(-1 + x
^3)^(3/4) + (x^2*(1 - x^3)^(3/4)*Hypergeometric2F1[2/3, 3/4, 5/3, x^3])/(2*
(-1 + x^3)^(3/4)) + 2*Defer[Int][1/((-1 + x^3)^(3/4)*(1 - x^3 - x^4)), x] +
 2*Defer[Int][x/((-1 + x^3)^(3/4)*(-1 + x^3 + x^4)), x] - 8*Defer[Int][x^2/
((-1 + x^3)^(3/4)*(-1 + x^3 + x^4)), x] + 2*Defer[Int][x^3/((-1 + x^3)^(3/4
)*(-1 + x^3 + x^4)), x]
```

Rubi steps

$$\int \frac{(-4+x^3)(1-x^3+x^4)}{x^2(-1+x^3)^{3/4}(-1+x^3+x^4)} \, dx = \int \left(-\frac{2}{(-1+x^3)^{3/4}}+\frac{4}{x^2(-1+x^3)^{3/4}}+\frac{x}{(-1+x^3)^{3/4}}+\frac{2(-1+x-4}{(-1+x^3)^{3/4}(-1}\right.$$

$$= -\left(2\int \frac{1}{(-1+x^3)^{3/4}} \, dx\right)+2\int \frac{-1+x-4x^2+x^3}{(-1+x^3)^{3/4}(-1+x^3+x^4)} \, dx+4\int$$

$$= 2\int \left(\frac{1}{(-1+x^3)^{3/4}(1-x^3-x^4)}+\frac{x}{(-1+x^3)^{3/4}(-1+x^3+x^4)}-\frac{}{(-1+}\right.$$

$$= -\frac{4(1-x^3)^{3/4}\,{}_2F_1\left(-\frac{1}{3},\frac{3}{4};\frac{2}{3};x^3\right)}{x(-1+x^3)^{3/4}}-\frac{2x(1-x^3)^{3/4}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};x^3\right)}{(-1+x^3)^{3/4}}+\frac{x^2}{}$$

Mathematica [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{(-4+x^3)(1-x^3+x^4)}{x^2(-1+x^3)^{3/4}(-1+x^3+x^4)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-4 + x^3)*(1 - x^3 + x^4))/(x^2*(-1 + x^3)^(3/4)*(-1 + x^3 + x^
4)),x]
```

```
[Out] Integrate[((-4 + x^3)*(1 - x^3 + x^4))/(x^2*(-1 + x^3)^(3/4)*(-1 + x^3 + x^
4)), x]
```

IntegrateAlgebraic [A] time = 4.36, size = 95, normalized size = 1.00

$$\frac{4\sqrt[4]{x^3-1}}{x}+2\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^3-1}}{\sqrt{x^3-1}-x^2}\right)-2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^3-1}}{\sqrt{x^3-1}+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + x^3)*(1 - x^3 + x^4))/(x^2*(-1 + x^3)^(3/4)*(-1 +
 x^3 + x^4)),x]
```

```
[Out] (4*(-1 + x^3)^(1/4))/x + 2*Sqrt[2]*ArcTan[(Sqrt[2]*x*(-1 + x^3)^(1/4))/(-x^
2 + Sqrt[-1 + x^3])] - 2*Sqrt[2]*ArcTanh[(Sqrt[2]*x*(-1 + x^3)^(1/4))/(x^2
+ Sqrt[-1 + x^3])]
```

fricas [B] time = 24.61, size = 459, normalized size = 4.83

$$-\frac{4\sqrt{2}\,x\arctan\left(\frac{\sqrt{2}(x^3-1)^{\frac14}x^3+\sqrt{2}(x^3-1)^{\frac34}x-\left(x^4-\sqrt{2}(x^3-1)^{\frac14}x^3+x^3+2\sqrt{x^3-1}\,x^2-\sqrt{2}(x^3-1)^{\frac34}x-1\right)\sqrt{\frac{x^4+2\sqrt{2}(x^3-1)^{\frac14}x^3+x^3+4\sqrt{x^3-1}\,x^2+2\sqrt{2}(x^3-1)^{\frac34}x-1}{x^4+x^3-1}}}{x^4-x^3+1}\right)+4\sqrt{2}\,x\arctan\left(\frac{\sqrt{2}(x^3-1)^{\frac14}x^3+\sqrt{2}(x^3-1)^{\frac34}x+\left(x^4+\sqrt{2}(x^3-1)^{\frac14}x^3+x^3+2\sqrt{x^3-1}\,x^2+\sqrt{2}(x^3-1)^{\frac34}x-1\right)\sqrt{\frac{x^4-2\sqrt{2}(x^3-1)^{\frac14}x^3+x^3+4\sqrt{x^3-1}\,x^2-2\sqrt{2}(x^3-1)^{\frac34}x-1}{x^4+x^3-1}}}{x^4-x^3+1}\right)+\sqrt{2}\,x\log\left(\frac{x^4+2\sqrt{2}(x^3-1)^{\frac14}x^3+x^3+4\sqrt{x^3-1}\,x^2+2\sqrt{2}(x^3-1)^{\frac34}x-1}{x^4+x^3-1}\right)-\sqrt{2}\,x\log\left(\frac{x^4-2\sqrt{2}(x^3-1)^{\frac14}x^3+x^3+4\sqrt{x^3-1}\,x^2-2\sqrt{2}(x^3-1)^{\frac34}x-1}{x^4+x^3-1}\right)-8\left(x^3-1\right)^{\frac14}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^4-x^3+1)/x^2/(x^3-1)^(3/4)/(x^4+x^3-1),x, algorithm="f
ricas")
```

```
[Out] -1/2*(4*sqrt(2)*x*arctan((sqrt(2)*(x^3 - 1)^(1/4)*x^3 + sqrt(2)*(x^3 - 1)^(
3/4)*x - (x^4 - sqrt(2)*(x^3 - 1)^(1/4)*x^3 + x^3 + 2*sqrt(x^3 - 1)*x^2 - s
qrt(2)*(x^3 - 1)^(3/4)*x - 1)*sqrt((x^4 + 2*sqrt(2)*(x^3 - 1)^(1/4)*x^3 + x
^3 + 4*sqrt(x^3 - 1)*x^2 + 2*sqrt(2)*(x^3 - 1)^(3/4)*x - 1)/(x^4 + x^3 - 1)
))/(x^4 - x^3 + 1)) + 4*sqrt(2)*x*arctan((sqrt(2)*(x^3 - 1)^(1/4)*x^3 + sqr
t(2)*(x^3 - 1)^(3/4)*x + (x^4 + sqrt(2)*(x^3 - 1)^(1/4)*x^3 + x^3 + 2*sqrt(
x^3 - 1)*x^2 + sqrt(2)*(x^3 - 1)^(3/4)*x - 1)*sqrt((x^4 - 2*sqrt(2)*(x^3 -
1)^(1/4)*x^3 + x^3 + 4*sqrt(x^3 - 1)*x^2 - 2*sqrt(2)*(x^3 - 1)^(3/4)*x - 1)
/(x^4 + x^3 - 1)))/(x^4 - x^3 + 1)) + sqrt(2)*x*log((x^4 + 2*sqrt(2)*(x^3 -
 1)^(1/4)*x^3 + x^3 + 4*sqrt(x^3 - 1)*x^2 + 2*sqrt(2)*(x^3 - 1)^(3/4)*x - 1
)/(x^4 + x^3 - 1)) - sqrt(2)*x*log((x^4 - 2*sqrt(2)*(x^3 - 1)^(1/4)*x^3 + x
^3 + 4*sqrt(x^3 - 1)*x^2 - 2*sqrt(2)*(x^3 - 1)^(3/4)*x - 1)/(x^4 + x^3 - 1)
) - 8*(x^3 - 1)^(1/4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^3+1\right)\left(x^3-4\right)}{\left(x^4+x^3-1\right)\left(x^3-1\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^4-x^3+1)/x^2/(x^3-1)^(3/4)/(x^4+x^3-1),x, algorithm="g
iac")
```

```
[Out] integrate((x^4 - x^3 + 1)*(x^3 - 4)/((x^4 + x^3 - 1)*(x^3 - 1)^(3/4)*x^2),
x)
```

maple [C] time = 2.31, size = 593, normalized size = 6.24

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-4)*(x^4-x^3+1)/x^2/(x^3-1)^(3/4)/(x^4+x^3-1),x)
```

```
[Out] 4*(x^3-1)^(1/4)/x+(-2*RootOf(_Z^4+1)^3*ln(-(-x^10*RootOf(_Z^4+1)^2+RootOf(_
Z^4+1)^2*x^9+2*(x^9-3*x^6+3*x^3-1)^(3/4)*RootOf(_Z^4+1)^3*x^3+2*RootOf(_Z^4
```

```
+1)^2*x^7-2*(x^9-3*x^6+3*x^3-1)^(1/4)*RootOf(_Z^4+1)*x^7-3*RootOf(_Z^4+1)^2
*x^6+2*(x^9-3*x^6+3*x^3-1)^(1/2)*x^5-RootOf(_Z^4+1)^2*x^4+4*(x^9-3*x^6+3*x^
3-1)^(1/4)*RootOf(_Z^4+1)*x^4+3*RootOf(_Z^4+1)^2*x^3-2*(x^9-3*x^6+3*x^3-1)^
(1/2)*x^2-2*(x^9-3*x^6+3*x^3-1)^(1/4)*RootOf(_Z^4+1)*x-RootOf(_Z^4+1)^2)/(x
^4+x^3-1)/(-1+x)^2/(x^2+x+1)^2)-2*RootOf(_Z^4+1)*ln((-x^10*RootOf(_Z^4+1)^2
+2*RootOf(_Z^4+1)^3*(x^9-3*x^6+3*x^3-1)^(1/4)*x^7+RootOf(_Z^4+1)^2*x^9+2*Ro
otOf(_Z^4+1)^2*x^7-4*RootOf(_Z^4+1)^3*(x^9-3*x^6+3*x^3-1)^(1/4)*x^4-3*RootO
f(_Z^4+1)^2*x^6-2*RootOf(_Z^4+1)*(x^9-3*x^6+3*x^3-1)^(3/4)*x^3-2*(x^9-3*x^6
+3*x^3-1)^(1/2)*x^5-RootOf(_Z^4+1)^2*x^4+2*RootOf(_Z^4+1)^3*(x^9-3*x^6+3*x^
3-1)^(1/4)*x+3*RootOf(_Z^4+1)^2*x^3+2*(x^9-3*x^6+3*x^3-1)^(1/2)*x^2-RootOf(
_Z^4+1)^2)/(x^4+x^3-1)/(-1+x)^2/(x^2+x+1)^2))/(x^3-1)^(3/4)*((x^3-1)^3)^(1/
4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4 - x^3 + 1\right)\left(x^3 - 4\right)}{\left(x^4 + x^3 - 1\right)\left(x^3 - 1\right)^{\frac{3}{4}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^4-x^3+1)/x^2/(x^3-1)^(3/4)/(x^4+x^3-1),x, algorithm="m
axima")
```

```
[Out] integrate((x^4 - x^3 + 1)*(x^3 - 4)/((x^4 + x^3 - 1)*(x^3 - 1)^(3/4)*x^2),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3 - 4\right)\,\left(x^4 - x^3 + 1\right)}{x^2\,{\left(x^3 - 1\right)}^{3/4}\,\left(x^4 + x^3 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 4)*(x^4 - x^3 + 1))/(x^2*(x^3 - 1)^(3/4)*(x^3 + x^4 - 1)),x)
```

```
[Out] int(((x^3 - 4)*(x^4 - x^3 + 1))/(x^2*(x^3 - 1)^(3/4)*(x^3 + x^4 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-4)*(x**4-x**3+1)/x**2/(x**3-1)**(3/4)/(x**4+x**3-1),x)
```

```
[Out] Timed out
```

3.1176 $\int \frac{(-1+x^4)(3+x^4)(-1-x^3+x^4)}{x^6(-1-2x^3+x^4)\sqrt[4]{-x+x^5}}\,dx$

Optimal. Leaf size=95

$$-2\,2^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\left(x^5-x\right)^{3/4}}{x^4-1}\right)-2\,2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\left(x^5-x\right)^{3/4}}{x^4-1}\right)+\frac{4\left(x^5-x\right)^{3/4}\left(3x^4+7x^3-3\right)}{21x^6}$$

Rubi [F] time = 2.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^4)(3+x^4)(-1-x^3+x^4)}{x^6(-1-2x^3+x^4)\sqrt[4]{-x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^4)*(3 + x^4)*(-1 - x^3 + x^4))/(x^6*(-1 - 2*x^3 + x^4)*(-x + x
^5)^(1/4)),x]
```

```
[Out] (4*(1 - x^4)^(1/4)*Hypergeometric2F1[-21/16, -3/4, -5/16, x^4])/(7*x^5*(-x
+ x^5)^(1/4)) - (4*(1 - x^4)^(1/4)*Hypergeometric2F1[-3/4, -9/16, 7/16, x^4
])/(3*x^2*(-x + x^5)^(1/4)) + (4*(1 - x^4)^(1/4)*Hypergeometric2F1[-3/4, -5
/16, 11/16, x^4])/(5*x*(-x + x^5)^(1/4)) - (24*x^(1/4)*(-1 + x^4)^(1/4)*Def
er[Subst][Defer[Int][(x^2*(-1 + x^16)^(3/4))/(-1 - 2*x^12 + x^16), x], x, x
^(1/4)])/(-x + x^5)^(1/4) + (16*x^(1/4)*(-1 + x^4)^(1/4)*Defer[Subst][Defer
[Int][(x^6*(-1 + x^16)^(3/4))/(-1 - 2*x^12 + x^16), x], x, x^(1/4)])/(-x +
x^5)^(1/4)
```

Rubi steps

$$\int \frac{(-1+x^4)(3+x^4)(-1-x^3+x^4)}{x^6(-1-2x^3+x^4)\sqrt[4]{-x+x^5}}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{-1+x^4}\right)\int \frac{(-1+x^4)^{3/4}(3+x^4)(-1-x^3+x^4)}{x^{25/4}(-1-2x^3+x^4)}\,dx}{\sqrt[4]{-x+x^5}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-1+x^4}\right)\text{Subst}\left(\int \frac{(-1+x^{16})^{3/4}(3+x^{16})(-1-x^{12}+x^{16})}{x^{22}(-1-2x^{12}+x^{16})}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-x+x^5}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-1+x^4}\right)\text{Subst}\left(\int \left(\frac{3(-1+x^{16})^{3/4}}{x^{22}}-\frac{3(-1+x^{16})^{3/4}}{x^{10}}+\frac{(-1+x^{16})^{3/4}}{x^6}+\frac{2x^2}{\cdots}\right.\right.}{\sqrt[4]{-x+x^5}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-1+x^4}\right)\text{Subst}\left(\int \frac{(-1+x^{16})^{3/4}}{x^6}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-x+x^5}}+\frac{\left(8\sqrt[4]{x}\sqrt[4]{-1+x^4}\right)\text{S}\cdots}{\cdots}$$

$$= \frac{\left(8\sqrt[4]{x}\sqrt[4]{-1+x^4}\right)\text{Subst}\left(\int \left(-\frac{3x^2(-1+x^{16})^{3/4}}{-1-2x^{12}+x^{16}}+\frac{2x^6(-1+x^{16})^{3/4}}{-1-2x^{12}+x^{16}}\right)dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-x+x^5}}$$

$$= \frac{4\sqrt[4]{1-x^4}\,{}_2F_1\left(-\frac{21}{16},-\frac{3}{4};-\frac{5}{16};x^4\right)}{7x^5\sqrt[4]{-x+x^5}}-\frac{4\sqrt[4]{1-x^4}\,{}_2F_1\left(-\frac{3}{4},-\frac{9}{16};\frac{7}{16};x^4\right)}{3x^2\sqrt[4]{-x+x^5}}+\cdots$$

Mathematica [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^4\right)\left(3+x^4\right)\left(-1-x^3+x^4\right)}{x^6\left(-1-2x^3+x^4\right)\sqrt[4]{-x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^4)*(3 + x^4)*(-1 - x^3 + x^4))/(x^6*(-1 - 2*x^3 + x^4)*(
-x + x^5)^(1/4)),x]
```

```
[Out] Integrate[((-1 + x^4)*(3 + x^4)*(-1 - x^3 + x^4))/(x^6*(-1 - 2*x^3 + x^4)*(
-x + x^5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 2.76, size = 95, normalized size = 1.00

$$-2\,2^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\left(x^5-x\right)^{3/4}}{x^4-1}\right)-2\,2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\left(x^5-x\right)^{3/4}}{x^4-1}\right)+\frac{4\left(x^5-x\right)^{3/4}\left(3x^4+7x^3-3\right)}{21x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(3 + x^4)*(-1 - x^3 + x^4))/(x^6*(-1 - 2*x^3
 + x^4)*(-x + x^5)^(1/4)),x]
```

```
[Out] (4*(-3 + 7*x^3 + 3*x^4)*(-x + x^5)^(3/4))/(21*x^6) - 2*2^(3/4)*ArcTan[(2^(1
/4)*(-x + x^5)^(3/4))/(-1 + x^4)] - 2*2^(3/4)*ArcTanh[(2^(1/4)*(-x + x^5)^(
3/4))/(-1 + x^4)]
```

fricas [B] time = 102.44, size = 280, normalized size = 2.95

$$-\frac{84\cdot 8^{\frac{1}{4}}x^6\arctan\left(\frac{16\cdot 8^{\frac{1}{4}}\left(x^5-x\right)^{\frac{1}{4}}x^2+2^{\frac{3}{4}}\left(8^{\frac{3}{4}}\left(x^4+2x^3-1\right)+8\cdot 8^{\frac{1}{4}}\sqrt{x^5-x}\,x\right)+4\cdot 8^{\frac{3}{4}}\left(x^5-x\right)^{\frac{3}{4}}}{8\left(x^4-2x^3-1\right)}\right)+21\cdot 8^{\frac{1}{4}}x^6\log\left(-\frac{4\sqrt{2}\left(x^5-x\right)^{\frac{1}{4}}x^2+8^{\frac{3}{4}}\sqrt{x^5-x}\,x+8^{\frac{1}{4}}\left(x^4+2x^3-1\right)+4\left(x^5-x\right)^{\frac{3}{4}}}{x^4-2x^3-1}\right)-21\cdot 8^{\frac{1}{4}}x^6\log\left(-\frac{4\sqrt{2}\left(x^5-x\right)^{\frac{1}{4}}x^2-8^{\frac{3}{4}}\sqrt{x^5-x}\,x-8^{\frac{1}{4}}\left(x^4+2x^3-1\right)+4\left(x^5-x\right)^{\frac{3}{4}}}{x^4-2x^3-1}\right)-8\left(x^5-x\right)^{\frac{3}{4}}\left(3x^4+7x^3-3\right)}{42x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+3)*(x^4-x^3-1)/x^6/(x^4-2*x^3-1)/(x^5-x)^(1/4),x, al
gorithm="fricas")
```

```
[Out] -1/42*(84*8^(1/4)*x^6*arctan(1/8*(16*8^(1/4)*(x^5 - x)^(1/4)*x^2 + 2^(3/4)*
(8^(3/4)*(x^4 + 2*x^3 - 1) + 8*8^(1/4)*sqrt(x^5 - x)*x) + 4*8^(3/4)*(x^5 -
x)^(3/4))/(x^4 - 2*x^3 - 1)) + 21*8^(1/4)*x^6*log(-(4*sqrt(2)*(x^5 - x)^(1/
4)*x^2 + 8^(3/4)*sqrt(x^5 - x)*x + 8^(1/4)*(x^4 + 2*x^3 - 1) + 4*(x^5 - x)^
(3/4))/(x^4 - 2*x^3 - 1)) - 21*8^(1/4)*x^6*log(-(4*sqrt(2)*(x^5 - x)^(1/4)*
x^2 - 8^(3/4)*sqrt(x^5 - x)*x - 8^(1/4)*(x^4 + 2*x^3 - 1) + 4*(x^5 - x)^(3/
4))/(x^4 - 2*x^3 - 1)) - 8*(x^5 - x)^(3/4)*(3*x^4 + 7*x^3 - 3))/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^3-1\right)\left(x^4+3\right)\left(x^4-1\right)}{\left(x^5-x\right)^{\frac{1}{4}}\left(x^4-2\,x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+3)*(x^4-x^3-1)/x^6/(x^4-2*x^3-1)/(x^5-x)^(1/4),x, al
gorithm="giac")
```

```
[Out] integrate((x^4 - x^3 - 1)*(x^4 + 3)*(x^4 - 1)/((x^5 - x)^(1/4)*(x^4 - 2*x^3
 - 1)*x^6), x)
```

maple [C] time = 11.08, size = 285, normalized size = 3.00

$$\frac{\frac{4}{7}x^8-\frac{8}{7}x^4+\frac{4}{3}x^7-\frac{4}{3}x^3+\frac{4}{7}}{x^5\left(x\left(x^4-1\right)\right)^{\frac{1}{4}}}+\mathrm{RootOf}\left(\_Z^4-8\right)\ln\left(\frac{\sqrt{x^5-x}\;\mathrm{RootOf}\left(\_Z^4-8\right)^3x-2\,\mathrm{RootOf}\left(\_Z^4-8\right)^2\left(x^5-x\right)^{\frac{1}{4}}x^2+\mathrm{RootOf}\left(\_Z^4-8\right)x^4+2\,\mathrm{RootOf}\left(\_Z^4-8\right)x^3-4\left(x^5-x\right)^{\frac{3}{4}}-\mathrm{RootOf}\left(\_Z^4-8\right)}{x^4-2x^3-1}\right)-\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\ln\left(\frac{\sqrt{x^5-x}\;\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\mathrm{RootOf}\left(\_Z^4-8\right)^2x+2\,\mathrm{RootOf}\left(\_Z^4-8\right)^2\left(x^5-x\right)^{\frac{1}{4}}x^2-\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)x^4-2\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)x^3-4\left(x^5-x\right)^{\frac{3}{4}}+\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)}{x^4-2x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^4+3)*(x^4-x^3-1)/x^6/(x^4-2*x^3-1)/(x^5-x)^(1/4),x)
```

```
[Out] 4/21*(3*x^8+7*x^7-6*x^4-7*x^3+3)/x^5/(x*(x^4-1))^(1/4)+RootOf(_Z^4-8)*ln(((
x^5-x)^(1/2)*RootOf(_Z^4-8)^3*x-2*RootOf(_Z^4-8)^2*(x^5-x)^(1/4)*x^2+RootOf
(_Z^4-8)*x^4+2*RootOf(_Z^4-8)*x^3-4*(x^5-x)^(3/4)-RootOf(_Z^4-8))/(x^4-2*x^
3-1))-RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln(((x^5-x)^(1/2)*RootOf(_Z^2+RootOf(_Z
^4-8)^2)*RootOf(_Z^4-8)^2*x+2*RootOf(_Z^4-8)^2*(x^5-x)^(1/4)*x^2-RootOf(_Z^
2+RootOf(_Z^4-8)^2)*x^4-2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^3-4*(x^5-x)^(3/4)
+RootOf(_Z^2+RootOf(_Z^4-8)^2))/(x^4-2*x^3-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^3-1\right)\left(x^4+3\right)\left(x^4-1\right)}{\left(x^5-x\right)^{\frac{1}{4}}\left(x^4-2\,x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4+3)*(x^4-x^3-1)/x^6/(x^4-2*x^3-1)/(x^5-x)^(1/4),x, al
gorithm="maxima")
```

```
[Out] integrate((x^4 - x^3 - 1)*(x^4 + 3)*(x^4 - 1)/((x^5 - x)^(1/4)*(x^4 - 2*x^3
 - 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-1\right)\,\left(x^4+3\right)\,\left(-x^4+x^3+1\right)}{x^6\,{\left(x^5-x\right)}^{1/4}\,\left(-x^4+2\,x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(x^4 + 3)*(x^3 - x^4 + 1))/(x^6*(x^5 - x)^(1/4)*(2*x^3 - x^4
 + 1)),x)
```

```
[Out] int(((x^4 - 1)*(x^4 + 3)*(x^3 - x^4 + 1))/(x^6*(x^5 - x)^(1/4)*(2*x^3 - x^4
 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**4+3)*(x**4-x**3-1)/x**6/(x**4-2*x**3-1)/(x**5-x)**(1
/4),x)
```

```
[Out] Timed out
```

3.1177 $\int \frac{(4b+ax^5)(-b+cx^4+ax^5)}{x^2(-b+ax^5)^{3/4}(-b-cx^4+ax^5)}\,dx$

Optimal. Leaf size=95

$$-4\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{c}\,x\left(ax^5-b\right)^{3/4}}{b-ax^5}\right)+4\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x\left(ax^5-b\right)^{3/4}}{b-ax^5}\right)+\frac{4\sqrt[4]{ax^5-b}}{x}$$

Rubi [F] time = 4.96, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(4b+ax^5\right)\left(-b+cx^4+ax^5\right)}{x^2\left(-b+ax^5\right)^{3/4}\left(-b-cx^4+ax^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((4*b + a*x^5)*(-b + c*x^4 + a*x^5))/(x^2*(-b + a*x^5)^(3/4)*(-b - c*x^
4 + a*x^5)),x]
```

```
[Out] (-4*b*(1 - (a*x^5)/b)^(3/4)*Hypergeometric2F1[-1/5, 3/4, 4/5, (a*x^5)/b])/(
x*(-b + a*x^5)^(3/4)) + (2*c^3*x*(1 - (a*x^5)/b)^(3/4)*Hypergeometric2F1[1/
5, 3/4, 6/5, (a*x^5)/b])/(a^2*(-b + a*x^5)^(3/4)) + (c^2*x^2*(1 - (a*x^5)/b
)^(3/4)*Hypergeometric2F1[2/5, 3/4, 7/5, (a*x^5)/b])/(a*(-b + a*x^5)^(3/4))
 + (2*c*x^3*(1 - (a*x^5)/b)^(3/4)*Hypergeometric2F1[3/5, 3/4, 8/5, (a*x^5)/
b])/(3*(-b + a*x^5)^(3/4)) + (a*x^4*(1 - (a*x^5)/b)^(3/4)*Hypergeometric2F1
[3/4, 4/5, 9/5, (a*x^5)/b])/(4*(-b + a*x^5)^(3/4)) - (2*b*c^3*Defer[Int][1/
((b + c*x^4 - a*x^5)*(-b + a*x^5)^(3/4)), x])/a^2 - (2*c^4*Defer[Int][x^4/(
(b + c*x^4 - a*x^5)*(-b + a*x^5)^(3/4)), x])/a^2 + (2*b*c^2*Defer[Int][x/((
-b + a*x^5)^(3/4)*(-b - c*x^4 + a*x^5)), x])/a + 10*b*c*Defer[Int][x^2/((-b
 + a*x^5)^(3/4)*(-b - c*x^4 + a*x^5)), x]
```

Rubi steps

$$\int \frac{\left(4b+ax^5\right)\left(-b+cx^4+ax^5\right)}{x^2\left(-b+ax^5\right)^{3/4}\left(-b-cx^4+ax^5\right)}\,dx=\int\left(\frac{2c^3}{a^2\left(-b+ax^5\right)^{3/4}}+\frac{4b}{x^2\left(-b+ax^5\right)^{3/4}}+\frac{2c^2x}{a\left(-b+ax^5\right)^{3/4}}+\frac{2c}{\left(-b+\right.}\right.$$

$$=\frac{2\int\frac{bc^3+abc^2x+5a^2bcx^2+c^4x^4}{\left(-b+ax^5\right)^{3/4}\left(-b-cx^4+ax^5\right)}\,dx}{a^2}+a\int\frac{x^3}{\left(-b+ax^5\right)^{3/4}}\,dx+(4b)\int\frac{}{x^2\left(-\right.}$$

$$=\frac{2\int\left(-\frac{bc^3}{\left(b+cx^4-ax^5\right)\left(-b+ax^5\right)^{3/4}}-\frac{c^4x^4}{\left(b+cx^4-ax^5\right)\left(-b+ax^5\right)^{3/4}}+\frac{abc^2x}{\left(-b+ax^5\right)^{3/4}\left(-b-cx^4+\right.}\right.}{a^2}$$

$$=-\frac{4b\left(1-\frac{ax^5}{b}\right)^{3/4}{}_2F_1\left(-\frac{1}{5},\frac{3}{4};\frac{4}{5};\frac{ax^5}{b}\right)}{x\left(-b+ax^5\right)^{3/4}}+\frac{2c^3x\left(1-\frac{ax^5}{b}\right)^{3/4}{}_2F_1\left(\frac{1}{5},\frac{3}{4};\frac{6}{5};\right.}{a^2\left(-b+ax^5\right)^{3/4}}$$

Mathematica [F] time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{\left(4b+ax^5\right)\left(-b+cx^4+ax^5\right)}{x^2\left(-b+ax^5\right)^{3/4}\left(-b-cx^4+ax^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((4*b + a*x^5)*(-b + c*x^4 + a*x^5))/(x^2*(-b + a*x^5)^(3/4)*(-b
- c*x^4 + a*x^5)),x]
```

```
[Out] Integrate[((4*b + a*x^5)*(-b + c*x^4 + a*x^5))/(x^2*(-b + a*x^5)^(3/4)*(-b
- c*x^4 + a*x^5)), x]
```

IntegrateAlgebraic [A] time = 12.99, size = 95, normalized size = 1.00

$$-4\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{c}\,x\left(ax^5-b\right)^{3/4}}{b-ax^5}\right)+4\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x\left(ax^5-b\right)^{3/4}}{b-ax^5}\right)+\frac{4\sqrt[4]{ax^5-b}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((4*b + a*x^5)*(-b + c*x^4 + a*x^5))/(x^2*(-b + a*x^5)^(
3/4)*(-b - c*x^4 + a*x^5)),x]
```

```
[Out] (4*(-b + a*x^5)^(1/4))/x - 4*c^(1/4)*ArcTan[(c^(1/4)*x*(-b + a*x^5)^(3/4))/
(b - a*x^5)] + 4*c^(1/4)*ArcTanh[(c^(1/4)*x*(-b + a*x^5)^(3/4))/(b - a*x^5)
]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5+4*b)*(a*x^5+c*x^4-b)/x^2/(a*x^5-b)^(3/4)/(a*x^5-c*x^4-b),x
, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^5+cx^4-b\right)\left(ax^5+4\,b\right)}{\left(ax^5-cx^4-b\right)\left(ax^5-b\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5+4*b)*(a*x^5+c*x^4-b)/x^2/(a*x^5-b)^(3/4)/(a*x^5-c*x^4-b),x
, algorithm="giac")
```

```
[Out] integrate((a*x^5 + c*x^4 - b)*(a*x^5 + 4*b)/((a*x^5 - c*x^4 - b)*(a*x^5 - b
)^(3/4)*x^2), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^5+4b\right)\left(a\,x^5+c\,x^4-b\right)}{x^2\left(a\,x^5-b\right)^{\frac{3}{4}}\left(a\,x^5-c\,x^4-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^5+4*b)*(a*x^5+c*x^4-b)/x^2/(a*x^5-b)^(3/4)/(a*x^5-c*x^4-b),x)
```

```
[Out] int((a*x^5+4*b)*(a*x^5+c*x^4-b)/x^2/(a*x^5-b)^(3/4)/(a*x^5-c*x^4-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^5+cx^4-b\right)\left(ax^5+4\,b\right)}{\left(ax^5-cx^4-b\right)\left(ax^5-b\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5+4*b)*(a*x^5+c*x^4-b)/x^2/(a*x^5-b)^(3/4)/(a*x^5-c*x^4-b),x
, algorithm="maxima")
```

```
[Out] integrate((a*x^5 + c*x^4 - b)*(a*x^5 + 4*b)/((a*x^5 - c*x^4 - b)*(a*x^5 - b
)^(3/4)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(a\,x^5+4\,b\right)\,\left(a\,x^5+c\,x^4-b\right)}{x^2\,{\left(a\,x^5-b\right)}^{3/4}\,\left(-a\,x^5+c\,x^4+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((4*b + a*x^5)*(a*x^5 - b + c*x^4))/(x^2*(a*x^5 - b)^(3/4)*(b - a*x^5
+ c*x^4)),x)
```

```
[Out] int(-((4*b + a*x^5)*(a*x^5 - b + c*x^4))/(x^2*(a*x^5 - b)^(3/4)*(b - a*x^5
+ c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{5} + 4b\right) \left(ax^{5} - b + cx^{4}\right)}{x^{2} \left(ax^{5} - b\right)^{\frac{3}{4}} \left(ax^{5} - b - cx^{4}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**5+4*b)*(a*x**5+c*x**4-b)/x**2/(a*x**5-b)**(3/4)/(a*x**5-c*x
**4-b),x)
```

```
[Out] Integral((a*x**5 + 4*b)*(a*x**5 - b + c*x**4)/(x**2*(a*x**5 - b)**(3/4)*(a*
x**5 - b - c*x**4)), x)
```

3.1178 $\int \frac{(-2+x^6)(1-x^4+x^6)}{x^4\sqrt[4]{1+x^6}(1+x^4+x^6)}\,dx$

Optimal. Leaf size=95

$$\frac{2\left(x^6+1\right)^{3/4}}{3x^3}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6+1}}{\sqrt{x^6+1}-x^2}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6+1}}{\sqrt{x^6+1}+x^2}\right)$$

Rubi [F] time = 1.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-2+x^6\right)\left(1-x^4+x^6\right)}{x^4\sqrt[4]{1+x^6}\left(1+x^4+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + x^6)*(1 - x^4 + x^6))/(x^4*(1 + x^6)^(1/4)*(1 + x^4 + x^6)),x]
```

```
[Out] (2*(1 + x^6)^(3/4))/(3*x^3) - 2*x*Hypergeometric2F1[1/6, 1/4, 7/6, -x^6] +
6*Defer[Int][1/((1 + x^6)^(1/4)*(1 + x^4 + x^6)), x] + 2*Defer[Int][x^4/((1
 + x^6)^(1/4)*(1 + x^4 + x^6)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-2+x^6\right)\left(1-x^4+x^6\right)}{x^4\sqrt[4]{1+x^6}\left(1+x^4+x^6\right)}\,dx &= \int \left(-\frac{2}{\sqrt[4]{1+x^6}}-\frac{2}{x^4\sqrt[4]{1+x^6}}+\frac{x^2}{\sqrt[4]{1+x^6}}+\frac{2\left(3+x^4\right)}{\sqrt[4]{1+x^6}\left(1+x^4+x^6\right)}\right)dx\\
&= -\left(2\int \frac{1}{\sqrt[4]{1+x^6}}\,dx\right)-2\int \frac{1}{x^4\sqrt[4]{1+x^6}}\,dx+2\int \frac{3+x^4}{\sqrt[4]{1+x^6}\left(1+x^4+x^6\right)}\,dx+\int\\
&= -2x\,{}_2F_1\left(\frac{1}{6},\frac{1}{4};\frac{7}{6};-x^6\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^2}}\,dx,x,x^3\right)-\frac{2}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt[4]{1+}}\right.\\
&= \frac{2x^3}{3\sqrt[4]{1+x^6}}+\frac{2\left(1+x^6\right)^{3/4}}{3x^3}-2x\,{}_2F_1\left(\frac{1}{6},\frac{1}{4};\frac{7}{6};-x^6\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{5/4}}\,dx,\right.\\
&= \frac{2\left(1+x^6\right)^{3/4}}{3x^3}-\frac{2}{3}E\left(\frac{1}{2}\tan^{-1}\left(x^3\right)\middle|2\right)-2x\,{}_2F_1\left(\frac{1}{6},\frac{1}{4};\frac{7}{6};-x^6\right)+\frac{1}{3}\text{Subst}\left(\int \frac{}{(1+}\right.\\
&= \frac{2\left(1+x^6\right)^{3/4}}{3x^3}-2x\,{}_2F_1\left(\frac{1}{6},\frac{1}{4};\frac{7}{6};-x^6\right)+2\int \frac{x^4}{\sqrt[4]{1+x^6}\left(1+x^4+x^6\right)}\,dx+6\int \frac{}{\sqrt[4]{}}
\end{aligned}$$

Mathematica [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(-2+x^6\right)\left(1-x^4+x^6\right)}{x^4\sqrt[4]{1+x^6}\left(1+x^4+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + x^6)*(1 - x^4 + x^6))/(x^4*(1 + x^6)^(1/4)*(1 + x^4 + x^6)
),x]
```

[Out] Integrate[((-2 + x^6)*(1 - x^4 + x^6))/(x^4*(1 + x^6)^(1/4)*(1 + x^4 + x^6)
), x]

IntegrateAlgebraic [A] time = 6.67, size = 95, normalized size = 1.00

$$\frac{2\left(x^6+1\right)^{3/4}}{3x^3}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6+1}}{\sqrt{x^6+1}-x^2}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6+1}}{\sqrt{x^6+1}+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^6)*(1 - x^4 + x^6))/(x^4*(1 + x^6)^(1/4)*(1 + x
^4 + x^6)),x]
```

```
[Out] (2*(1 + x^6)^(3/4))/(3*x^3) + Sqrt[2]*ArcTan[(Sqrt[2]*x*(1 + x^6)^(1/4))/(-
x^2 + Sqrt[1 + x^6])] + Sqrt[2]*ArcTanh[(Sqrt[2]*x*(1 + x^6)^(1/4))/(x^2 +
Sqrt[1 + x^6])]
```

fricas [B] time = 173.49, size = 702, normalized size = 7.39

$$\frac{12\sqrt{2}x^3\arctan\left(\frac{x^{12}+2x^{10}+x^8+2x^6+2x^4+2\sqrt{2}(x^7-3x^5+x)(x^6+1)^{3/4}+2\sqrt{2}(3x^9-x^7+3x^3)(x^6+1)^{1/4}+4(x^8+x^6+x^2)\sqrt{x^6+1}+\left(16(x^6+1)^{3/4}x^5+2\sqrt{2}(x^8-3x^6+x^2)\sqrt{x^6+1}+\sqrt{2}(x^{12}-8x^{10}-x^8+2x^6-8x^4+1)+4(x^9+x^7+x^3)(x^6+1)^{1/4}\right)\sqrt{\frac{x^6+x^4+2\sqrt{2}(x^6+1)^{1/4}x^3+4\sqrt{x^6+1}x^2+2\sqrt{2}(x^6+1)^{3/4}x+1}{x^6+x^4+1}}+1}{x^{12}-14x^{10}+x^8+2x^6-14x^4+1}\right)-12\sqrt{2}x^3\arctan\left(\frac{x^{12}+2x^{10}+x^8+2x^6+2x^4-2\sqrt{2}(x^7-3x^5+x)(x^6+1)^{3/4}-2\sqrt{2}(3x^9-x^7+3x^3)(x^6+1)^{1/4}+4(x^8+x^6+x^2)\sqrt{x^6+1}+\left(16(x^6+1)^{3/4}x^5-2\sqrt{2}(x^8-3x^6+x^2)\sqrt{x^6+1}-\sqrt{2}(x^{12}-8x^{10}-x^8+2x^6-8x^4+1)+4(x^9+x^7+x^3)(x^6+1)^{1/4}\right)\sqrt{\frac{x^6+x^4-2\sqrt{2}(x^6+1)^{1/4}x^3+4\sqrt{x^6+1}x^2-2\sqrt{2}(x^6+1)^{3/4}x+1}{x^6+x^4+1}}+1}{x^{12}-14x^{10}+x^8+2x^6-14x^4+1}\right)+3\sqrt{2}x^3\log\left(\frac{4\left(x^6+x^4+2\sqrt{2}(x^6+1)^{1/4}x^3+4\sqrt{x^6+1}x^2+2\sqrt{2}(x^6+1)^{3/4}x+1\right)}{x^6+x^4+1}\right)-3\sqrt{2}x^3\log\left(\frac{4\left(x^6+x^4-2\sqrt{2}(x^6+1)^{1/4}x^3+4\sqrt{x^6+1}x^2-2\sqrt{2}(x^6+1)^{3/4}x+1\right)}{x^6+x^4+1}\right)+8(x^6+1)^{3/4}}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-x^4+1)/x^4/(x^6+1)^(1/4)/(x^6+x^4+1),x, algorithm="f
ricas")
```

```
[Out] 1/12*(12*sqrt(2)*x^3*arctan((x^12 + 2*x^10 + x^8 + 2*x^6 + 2*x^4 + 2*sqrt(2
)*(x^7 - 3*x^5 + x)*(x^6 + 1)^(3/4) + 2*sqrt(2)*(3*x^9 - x^7 + 3*x^3)*(x^6
+ 1)^(1/4) + 4*(x^8 + x^6 + x^2)*sqrt(x^6 + 1) + (16*(x^6 + 1)^(3/4)*x^5 +
2*sqrt(2)*(x^8 - 3*x^6 + x^2)*sqrt(x^6 + 1) + sqrt(2)*(x^12 - 8*x^10 - x^8
+ 2*x^6 - 8*x^4 + 1) + 4*(x^9 + x^7 + x^3)*(x^6 + 1)^(1/4))*sqrt((x^6 + x^4
 + 2*sqrt(2)*(x^6 + 1)^(1/4)*x^3 + 4*sqrt(x^6 + 1)*x^2 + 2*sqrt(2)*(x^6 + 1
)^(3/4)*x + 1)/(x^6 + x^4 + 1)) + 1)/(x^12 - 14*x^10 + x^8 + 2*x^6 - 14*x^4
 + 1)) - 12*sqrt(2)*x^3*arctan((x^12 + 2*x^10 + x^8 + 2*x^6 + 2*x^4 - 2*sqr
t(2)*(x^7 - 3*x^5 + x)*(x^6 + 1)^(3/4) - 2*sqrt(2)*(3*x^9 - x^7 + 3*x^3)*(x
^6 + 1)^(1/4) + 4*(x^8 + x^6 + x^2)*sqrt(x^6 + 1) + (16*(x^6 + 1)^(3/4)*x^5
 - 2*sqrt(2)*(x^8 - 3*x^6 + x^2)*sqrt(x^6 + 1) - sqrt(2)*(x^12 - 8*x^10 - x
^8 + 2*x^6 - 8*x^4 + 1) + 4*(x^9 + x^7 + x^3)*(x^6 + 1)^(1/4))*sqrt((x^6 +
x^4 - 2*sqrt(2)*(x^6 + 1)^(1/4)*x^3 + 4*sqrt(x^6 + 1)*x^2 - 2*sqrt(2)*(x^6
+ 1)^(3/4)*x + 1)/(x^6 + x^4 + 1)) + 1)/(x^12 - 14*x^10 + x^8 + 2*x^6 - 14*
x^4 + 1)) + 3*sqrt(2)*x^3*log(4*(x^6 + x^4 + 2*sqrt(2)*(x^6 + 1)^(1/4)*x^3
+ 4*sqrt(x^6 + 1)*x^2 + 2*sqrt(2)*(x^6 + 1)^(3/4)*x + 1)/(x^6 + x^4 + 1)) -
 3*sqrt(2)*x^3*log(4*(x^6 + x^4 - 2*sqrt(2)*(x^6 + 1)^(1/4)*x^3 + 4*sqrt(x^
6 + 1)*x^2 - 2*sqrt(2)*(x^6 + 1)^(3/4)*x + 1)/(x^6 + x^4 + 1)) + 8*(x^6 + 1
)^(3/4))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6-x^4+1\right)\left(x^6-2\right)}{\left(x^6+x^4+1\right)\left(x^6+1\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-x^4+1)/x^4/(x^6+1)^(1/4)/(x^6+x^4+1),x, algorithm="g
iac")
```

```
[Out] integrate((x^6 - x^4 + 1)*(x^6 - 2)/((x^6 + x^4 + 1)*(x^6 + 1)^(1/4)*x^4),
x)
```

maple [C] time = 14.88, size = 217, normalized size = 2.28

$$\frac{2\left(x^6+1\right)^{\frac{3}{4}}}{3x^3}+\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3\sqrt{x^6+1}\,x^2-\mathrm{RootOf}\left(\_Z^4+1\right)x^6+2\left(x^6+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+1\right)^2x^3+\mathrm{RootOf}\left(\_Z^4+1\right)x^4-2\left(x^6+1\right)^{\frac{3}{4}}x-\mathrm{RootOf}\left(\_Z^4+1\right)}{x^6+x^4+1}\right)+\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^3x^6-\mathrm{RootOf}\left(\_Z^4+1\right)^3x^4+2\left(x^6+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+1\right)^2x^3-2\,\mathrm{RootOf}\left(\_Z^4+1\right)\sqrt{x^6+1}\,x^2+2\left(x^6+1\right)^{\frac{3}{4}}x+\mathrm{RootOf}\left(\_Z^4+1\right)^3}{x^6+x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-2)*(x^6-x^4+1)/x^4/(x^6+1)^(1/4)/(x^6+x^4+1),x)
```

```
[Out] 2/3*(x^6+1)^(3/4)/x^3+RootOf(_Z^4+1)*ln(-(2*RootOf(_Z^4+1)^3*(x^6+1)^(1/2)*
x^2-RootOf(_Z^4+1)*x^6+2*(x^6+1)^(1/4)*RootOf(_Z^4+1)^2*x^3+RootOf(_Z^4+1)*
x^4-2*(x^6+1)^(3/4)*x-RootOf(_Z^4+1))/(x^6+x^4+1))+RootOf(_Z^4+1)^3*ln((Roo
tOf(_Z^4+1)^3*x^6-RootOf(_Z^4+1)^3*x^4+2*(x^6+1)^(1/4)*RootOf(_Z^4+1)^2*x^3
-2*RootOf(_Z^4+1)*(x^6+1)^(1/2)*x^2+2*(x^6+1)^(3/4)*x+RootOf(_Z^4+1)^3)/(x^
6+x^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6 - x^4 + 1\right)\left(x^6 - 2\right)}{\left(x^6 + x^4 + 1\right)\left(x^6 + 1\right)^{\frac{1}{4}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-x^4+1)/x^4/(x^6+1)^(1/4)/(x^6+x^4+1),x, algorithm="m
axima")
```

```
[Out] integrate((x^6 - x^4 + 1)*(x^6 - 2)/((x^6 + x^4 + 1)*(x^6 + 1)^(1/4)*x^4),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6-2\right)\,\left(x^6-x^4+1\right)}{x^4\,{\left(x^6+1\right)}^{1/4}\,\left(x^6+x^4+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 2)*(x^6 - x^4 + 1))/(x^4*(x^6 + 1)^(1/4)*(x^4 + x^6 + 1)),x)
```

```
[Out] int(((x^6 - 2)*(x^6 - x^4 + 1))/(x^4*(x^6 + 1)^(1/4)*(x^4 + x^6 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-2)*(x**6-x**4+1)/x**4/(x**6+1)**(1/4)/(x**6+x**4+1),x)
```

```
[Out] Timed out
```

3.1179 $\int \frac{x^4(2b+ax^6)}{\sqrt[4]{-b+ax^6}\left(-b-x^4+ax^6\right)^2}\,dx$

Optimal. Leaf size=95

$$-\frac{1}{4}\tan^{-1}\left(\frac{x\left(ax^6-b\right)^{3/4}}{b-ax^6}\right)-\frac{1}{4}\tanh^{-1}\left(\frac{x\left(ax^6-b\right)^{3/4}}{b-ax^6}\right)-\frac{x\left(ax^6-b\right)^{3/4}}{2\left(ax^6-b-x^4\right)}$$

Rubi [F] time = 2.94, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4\left(2b+ax^6\right)}{\sqrt[4]{-b+ax^6}\left(-b-x^4+ax^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(2*b + a*x^6))/((-b + a*x^6)^(1/4)*(-b - x^4 + a*x^6)^2),x]
```

```
[Out] (b*Defer[Int][1/((b + x^4 - a*x^6)^2*(-b + a*x^6)^(1/4)), x])/a^2 + (b*Defe
r[Int][x^2/((b + x^4 - a*x^6)^2*(-b + a*x^6)^(1/4)), x])/a + (a^(-2) + 3*b)
*Defer[Int][x^4/((b + x^4 - a*x^6)^2*(-b + a*x^6)^(1/4)), x] + Defer[Int][1
/((-b + a*x^6)^(1/4)*(-b - x^4 + a*x^6)), x]/a^2 + Defer[Int][x^2/((-b + a*
x^6)^(1/4)*(-b - x^4 + a*x^6)), x]/a + Defer[Int][x^4/((-b + a*x^6)^(1/4)*(
-b - x^4 + a*x^6)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(2b+ax^6\right)}{\sqrt[4]{-b+ax^6}\left(-b-x^4+ax^6\right)^2}\,dx &= \int \left(\frac{b+abx^2+\left(1+3a^2b\right)x^4}{a^2\left(b+x^4-ax^6\right)^2\sqrt[4]{-b+ax^6}}+\frac{1+ax^2+a^2x^4}{a^2\sqrt[4]{-b+ax^6}\left(-b-x^4+ax^6\right)}\right)dx\\
&= \frac{\int \frac{b+abx^2+\left(1+3a^2b\right)x^4}{\left(b+x^4-ax^6\right)^2\sqrt[4]{-b+ax^6}}\,dx}{a^2}+\frac{\int \frac{1+ax^2+a^2x^4}{\sqrt[4]{-b+ax^6}\left(-b-x^4+ax^6\right)}\,dx}{a^2}\\
&= \frac{\int \left(\frac{b}{\left(b+x^4-ax^6\right)^2\sqrt[4]{-b+ax^6}}+\frac{abx^2}{\left(b+x^4-ax^6\right)^2\sqrt[4]{-b+ax^6}}+\frac{\left(1+3a^2b\right)x^4}{\left(b+x^4-ax^6\right)^2\sqrt[4]{-b+ax^6}}\right)dx}{a^2}+\frac{\int}{}\\
&= \frac{\int \frac{1}{\sqrt[4]{-b+ax^6}\left(-b-x^4+ax^6\right)}\,dx}{a^2}+\frac{\int \frac{x^2}{\sqrt[4]{-b+ax^6}\left(-b-x^4+ax^6\right)}\,dx}{a}+\frac{b\int \frac{1}{\left(b+x^4-ax^6\right)^2\sqrt[4]{-b+}}}{a^2}\end{aligned}$$

Mathematica [F] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(2b+ax^6\right)}{\sqrt[4]{-b+ax^6}\left(-b-x^4+ax^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(2*b + a*x^6))/((-b + a*x^6)^(1/4)*(-b - x^4 + a*x^6)^2),x]
```

```
[Out] Integrate[(x^4*(2*b + a*x^6))/((-b + a*x^6)^(1/4)*(-b - x^4 + a*x^6)^2), x]
```

IntegrateAlgebraic [A] time = 15.54, size = 95, normalized size = 1.00

$$-\frac{1}{4}\tan^{-1}\left(\frac{x\left(ax^6-b\right)^{3/4}}{b-ax^6}\right)-\frac{1}{4}\tanh^{-1}\left(\frac{x\left(ax^6-b\right)^{3/4}}{b-ax^6}\right)-\frac{x\left(ax^6-b\right)^{3/4}}{2\left(ax^6-b-x^4\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(2*b + a*x^6))/((-b + a*x^6)^(1/4)*(-b - x^4 + a*x^
6)^2),x]
```

```
[Out] -1/2*(x*(-b + a*x^6)^(3/4))/(-b - x^4 + a*x^6) - ArcTan[(x*(-b + a*x^6)^(3/
4))/(b - a*x^6)]/4 - ArcTanh[(x*(-b + a*x^6)^(3/4))/(b - a*x^6)]/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^6+2*b)/(a*x^6-b)^(1/4)/(a*x^6-x^4-b)^2,x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^6+2\,b\right)x^4}{\left(ax^6-x^4-b\right)^2\left(ax^6-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^6+2*b)/(a*x^6-b)^(1/4)/(a*x^6-x^4-b)^2,x, algorithm="gia
c")
```

```
[Out] integrate((a*x^6 + 2*b)*x^4/((a*x^6 - x^4 - b)^2*(a*x^6 - b)^(1/4)), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a\,x^6+2b\right)}{\left(a\,x^6-b\right)^{\frac{1}{4}}\left(a\,x^6-x^4-b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a*x^6+2*b)/(a*x^6-b)^(1/4)/(a*x^6-x^4-b)^2,x)
```

```
[Out] int(x^4*(a*x^6+2*b)/(a*x^6-b)^(1/4)/(a*x^6-x^4-b)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^6+2\,b\right)x^4}{\left(ax^6-x^4-b\right)^2\left(ax^6-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^6+2*b)/(a*x^6-b)^(1/4)/(a*x^6-x^4-b)^2,x, algorithm="max
ima")
```

```
[Out] integrate((a*x^6 + 2*b)*x^4/((a*x^6 - x^4 - b)^2*(a*x^6 - b)^(1/4)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\left(a x^6+2 b\right)}{\left(a x^6-b\right)^{1/4}\left(-a x^6+x^4+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(2*b + a*x^6))/((a*x^6 - b)^(1/4)*(b - a*x^6 + x^4)^2),x)
[Out] int((x^4*(2*b + a*x^6))/((a*x^6 - b)^(1/4)*(b - a*x^6 + x^4)^2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a*x**6+2*b)/(a*x**6-b)**(1/4)/(a*x**6-x**4-b)**2,x)
[Out] Timed out
```

3.1180 $\int \frac{(2b+ax^6)(-b-x^4+ax^6)}{x^4\sqrt[4]{-b+ax^6}\,(-b-2x^4+ax^6)}\,dx$

Optimal. Leaf size=95

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x\left(ax^6-b\right)^{3/4}}{b-ax^6}\right)}{\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x\left(ax^6-b\right)^{3/4}}{b-ax^6}\right)}{\sqrt[4]{2}}+\frac{2\left(ax^6-b\right)^{3/4}}{3x^3}$$

Rubi [F] time = 3.78, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(2b+ax^6)(-b-x^4+ax^6)}{x^4\sqrt[4]{-b+ax^6}\,(-b-2x^4+ax^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((2*b + a*x^6)*(-b - x^4 + a*x^6))/(x^4*(-b + a*x^6)^(1/4)*(-b - 2*x^4
+ a*x^6)),x]
```

```
[Out] (2*(-b + a*x^6)^(3/4))/(3*x^3) + (x*(1 - (a*x^6)/b)^(1/4)*Hypergeometric2F1
[1/6, 1/4, 7/6, (a*x^6)/b])/(-b + a*x^6)^(1/4) - 3*b*Defer[Int][1/((b + 2*x
^4 - a*x^6)*(-b + a*x^6)^(1/4)), x] + 2*Defer[Int][x^4/((-b + a*x^6)^(1/4)*
(-b - 2*x^4 + a*x^6)), x]
```

Rubi steps

$$\int \frac{(2b+ax^6)(-b-x^4+ax^6)}{x^4\sqrt[4]{-b+ax^6}\,(-b-2x^4+ax^6)}\,dx=\int\left(\frac{1}{\sqrt[4]{-b+ax^6}}+\frac{2b}{x^4\sqrt[4]{-b+ax^6}}+\frac{ax^2}{\sqrt[4]{-b+ax^6}}+\frac{-3b-2x^4}{\left(b+2x^4-ax^6\right)\sqrt[4]{-b+}}\right.$$

$$=a\int\frac{x^2}{\sqrt[4]{-b+ax^6}}\,dx+(2b)\int\frac{1}{x^4\sqrt[4]{-b+ax^6}}\,dx+\int\frac{1}{\sqrt[4]{-b+ax^6}}\,dx+\int$$

$$=\frac{1}{3}a\,\text{Subst}\left(\int\frac{1}{\sqrt[4]{-b+ax^2}}\,dx,x,x^3\right)+\frac{1}{3}(2b)\,\text{Subst}\left(\int\frac{1}{x^2\sqrt[4]{-b+ax^2}}\,dx,\right.$$

$$=\frac{2\left(-b+ax^6\right)^{3/4}}{3x^3}+\frac{x\sqrt[4]{1-\frac{ax^6}{b}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{4};\frac{7}{6};\frac{ax^6}{b}\right)}{\sqrt[4]{-b+ax^6}}+2\int\frac{x^4}{\sqrt[4]{-b+ax^6}\,(-b-2}$$

$$=\frac{2\left(-b+ax^6\right)^{3/4}}{3x^3}+\frac{x\sqrt[4]{1-\frac{ax^6}{b}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{4};\frac{7}{6};\frac{ax^6}{b}\right)}{\sqrt[4]{-b+ax^6}}+2\int\frac{x^4}{\sqrt[4]{-b+ax^6}\,(-b-2}$$

$$=\frac{2\left(-b+ax^6\right)^{3/4}}{3x^3}+\frac{2ax^3\sqrt[4]{-b+ax^6}}{3\left(\sqrt{b}+\sqrt{-b+ax^6}\right)}-\frac{2\sqrt[4]{b}\sqrt{\frac{ax^6}{\left(\sqrt{b}+\sqrt{-b+ax^6}\right)^2}}\left(\sqrt{b}+\right.}{}$$

$$=\frac{2\left(-b+ax^6\right)^{3/4}}{3x^3}+\frac{x\sqrt[4]{1-\frac{ax^6}{b}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{4};\frac{7}{6};\frac{ax^6}{b}\right)}{\sqrt[4]{-b+ax^6}}+2\int\frac{x^4}{\sqrt[4]{-b+ax^6}\,(-b-2}$$

Mathematica [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(2b+ax^6)(-b-x^4+ax^6)}{x^4\sqrt[4]{-b+ax^6}\,(-b-2x^4+ax^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((2*b + a*x^6)*(-b - x^4 + a*x^6))/(x^4*(-b + a*x^6)^(1/4)*(-b -
2*x^4 + a*x^6)),x]
```

```
[Out] Integrate[((2*b + a*x^6)*(-b - x^4 + a*x^6))/(x^4*(-b + a*x^6)^(1/4)*(-b -
2*x^4 + a*x^6)), x]
```

IntegrateAlgebraic [A] time = 15.41, size = 95, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x(ax^6-b)^{3/4}}{b-ax^6}\right)}{\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x(ax^6-b)^{3/4}}{b-ax^6}\right)}{\sqrt[4]{2}}+\frac{2\left(ax^6-b\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2*b + a*x^6)*(-b - x^4 + a*x^6))/(x^4*(-b + a*x^6)^(1/
4)*(-b - 2*x^4 + a*x^6)),x]
```

```
[Out] (2*(-b + a*x^6)^(3/4))/(3*x^3) + ArcTan[(2^(1/4)*x*(-b + a*x^6)^(3/4))/(b -
 a*x^6)]/2^(1/4) + ArcTanh[(2^(1/4)*x*(-b + a*x^6)^(3/4))/(b - a*x^6)]/2^(1
/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+2*b)*(a*x^6-x^4-b)/x^4/(a*x^6-b)^(1/4)/(a*x^6-2*x^4-b),x,
algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^6-x^4-b)(ax^6+2\,b)}{(ax^6-2\,x^4-b)(ax^6-b)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+2*b)*(a*x^6-x^4-b)/x^4/(a*x^6-b)^(1/4)/(a*x^6-2*x^4-b),x,
algorithm="giac")
```

```
[Out] integrate((a*x^6 - x^4 - b)*(a*x^6 + 2*b)/((a*x^6 - 2*x^4 - b)*(a*x^6 - b)^
(1/4)*x^4), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(a\,x^6+2b)(a\,x^6-x^4-b)}{x^4(a\,x^6-b)^{\frac{1}{4}}(a\,x^6-2x^4-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6+2*b)*(a*x^6-x^4-b)/x^4/(a*x^6-b)^(1/4)/(a*x^6-2*x^4-b),x)
```

```
[Out] int((a*x^6+2*b)*(a*x^6-x^4-b)/x^4/(a*x^6-b)^(1/4)/(a*x^6-2*x^4-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^6 - x^4 - b\right)\left(ax^6 + 2\,b\right)}{\left(ax^6 - 2\,x^4 - b\right)\left(ax^6 - b\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+2*b)*(a*x^6-x^4-b)/x^4/(a*x^6-b)^(1/4)/(a*x^6-2*x^4-b),x,
algorithm="maxima")
```

```
[Out] integrate((a*x^6 - x^4 - b)*(a*x^6 + 2*b)/((a*x^6 - 2*x^4 - b)*(a*x^6 - b)^
(1/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^6 + 2\,b\right)\,\left(-a\,x^6 + x^4 + b\right)}{x^4\,{\left(a\,x^6 - b\right)}^{1/4}\,\left(-a\,x^6 + 2\,x^4 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*b + a*x^6)*(b - a*x^6 + x^4))/(x^4*(a*x^6 - b)^(1/4)*(b - a*x^6 + 2
*x^4)),x)
```

```
[Out] int(((2*b + a*x^6)*(b - a*x^6 + x^4))/(x^4*(a*x^6 - b)^(1/4)*(b - a*x^6 + 2
*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6+2*b)*(a*x**6-x**4-b)/x**4/(a*x**6-b)**(1/4)/(a*x**6-2*x**
4-b),x)
```

```
[Out] Timed out
```

3.1181 $\int \frac{\sqrt{-1-2x^2-2x^3-x^8}\left(-1+x^3+3x^8\right)}{\left(1+2x^3+x^8\right)\left(1+x^2+2x^3+x^8\right)}\,dx$

Optimal. Leaf size=95

$$\tan^{-1}\left(\frac{x\sqrt{-x^8-2x^3-2x^2-1}}{x^8+2x^3+2x^2+1}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt{-x^8-2x^3-2x^2-1}}{x^8+2x^3+2x^2+1}\right)$$

Rubi [F] time = 4.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-1-2x^2-2x^3-x^8}\left(-1+x^3+3x^8\right)}{\left(1+2x^3+x^8\right)\left(1+x^2+2x^3+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-1 - 2*x^2 - 2*x^3 - x^8]*(-1 + x^3 + 3*x^8))/((1 + 2*x^3 + x^8)*
(1 + x^2 + 2*x^3 + x^8)),x]
```

```
[Out] -1/2*Defer[Int][Sqrt[-1 - 2*x^2 - 2*x^3 - x^8]/(1 + x), x] + Defer[Int][Sqr
t[-1 - 2*x^2 - 2*x^3 - x^8]/(1 - x + x^2 + x^3 - x^4 + x^5 - x^6 + x^7), x]
/2 + 2*Defer[Int][(x*Sqrt[-1 - 2*x^2 - 2*x^3 - x^8])/(1 - x + x^2 + x^3 - x
^4 + x^5 - x^6 + x^7), x] - (3*Defer[Int][(x^2*Sqrt[-1 - 2*x^2 - 2*x^3 - x^
8])/(1 - x + x^2 + x^3 - x^4 + x^5 - x^6 + x^7), x])/2 + 2*Defer[Int][(x^3*
Sqrt[-1 - 2*x^2 - 2*x^3 - x^8])/(1 - x + x^2 + x^3 - x^4 + x^5 - x^6 + x^7)
, x] - (5*Defer[Int][(x^4*Sqrt[-1 - 2*x^2 - 2*x^3 - x^8])/(1 - x + x^2 + x^
3 - x^4 + x^5 - x^6 + x^7), x])/2 + 3*Defer[Int][(x^5*Sqrt[-1 - 2*x^2 - 2*x
^3 - x^8])/(1 - x + x^2 + x^3 - x^4 + x^5 - x^6 + x^7), x] + Defer[Int][(x^
6*Sqrt[-1 - 2*x^2 - 2*x^3 - x^8])/(1 - x + x^2 + x^3 - x^4 + x^5 - x^6 + x^
7), x]/2 + Defer[Int][Sqrt[-1 - 2*x^2 - 2*x^3 - x^8]/(-1 - x^2 - 2*x^3 - x^
8), x] - 3*Defer[Int][(x*Sqrt[-1 - 2*x^2 - 2*x^3 - x^8])/(1 + x^2 + 2*x^3 +
 x^8), x] - 4*Defer[Int][(x^6*Sqrt[-1 - 2*x^2 - 2*x^3 - x^8])/(1 + x^2 + 2*
x^3 + x^8), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1-2x^2-2x^3-x^8}\left(-1+x^3+3x^8\right)}{\left(1+2x^3+x^8\right)\left(1+x^2+2x^3+x^8\right)}\,dx &= \int \left(-\frac{\sqrt{-1-2x^2-2x^3-x^8}}{2(1+x)}+\frac{\left(1+4x-3x^2+4x^3-5x^4+6x\right.}{2\left(1-x+x^2+x^3-\right.}\right.\\ &= -\left(\frac{1}{2}\int \frac{\sqrt{-1-2x^2-2x^3-x^8}}{1+x}\,dx\right)+\frac{1}{2}\int \frac{\left(1+4x-3x^2+4x^3\right.}{1-x+}\\ &= -\left(\frac{1}{2}\int \frac{\sqrt{-1-2x^2-2x^3-x^8}}{1+x}\,dx\right)+\frac{1}{2}\int \left(\frac{\sqrt{-1-2x^2-}}{1-x+x^2+x^3-}\right.\\ &= -\left(\frac{1}{2}\int \frac{\sqrt{-1-2x^2-2x^3-x^8}}{1+x}\,dx\right)+\frac{1}{2}\int \frac{\sqrt{-1-2x^2-}}{1-x+x^2+x^3-x^4}\end{aligned}$$

Mathematica [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-1-2x^2-2x^3-x^8}\left(-1+x^3+3x^8\right)}{\left(1+2x^3+x^8\right)\left(1+x^2+2x^3+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[-1 - 2*x^2 - 2*x^3 - x^8]*(-1 + x^3 + 3*x^8))/((1 + 2*x^3 +
 x^8)*(1 + x^2 + 2*x^3 + x^8)),x]
```

```
[Out] Integrate[(Sqrt[-1 - 2*x^2 - 2*x^3 - x^8]*(-1 + x^3 + 3*x^8))/((1 + 2*x^3 +
 x^8)*(1 + x^2 + 2*x^3 + x^8)), x]
```

IntegrateAlgebraic [A] time = 0.52, size = 95, normalized size = 1.00

$$\tan^{-1}\left(\frac{x\sqrt{-x^8-2x^3-2x^2-1}}{x^8+2x^3+2x^2+1}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt{-x^8-2x^3-2x^2-1}}{x^8+2x^3+2x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 - 2*x^2 - 2*x^3 - x^8]*(-1 + x^3 + 3*x^8))/((1
+ 2*x^3 + x^8)*(1 + x^2 + 2*x^3 + x^8)),x]
```

```
[Out] ArcTan[(x*Sqrt[-1 - 2*x^2 - 2*x^3 - x^8])/(1 + 2*x^2 + 2*x^3 + x^8)] - Sqrt
[2]*ArcTan[(Sqrt[2]*x*Sqrt[-1 - 2*x^2 - 2*x^3 - x^8])/(1 + 2*x^2 + 2*x^3 +
x^8)]
```

fricas [C] time = 1.59, size = 241, normalized size = 2.54

$$-\frac{1}{4}\sqrt{-2}\log\left(-\frac{2\left(\sqrt{-2}\left(x^8+2x^3+4x^2+1\right)+4\sqrt{-x^8-2x^3-2x^2-1}\,x\right)}{x^8+2x^3+1}\right)+\frac{1}{4}\sqrt{-2}\log\left(\frac{2\left(\sqrt{-2}\left(x^8+2x^3+4x^2+1\right)-4\sqrt{-x^8-2x^3-2x^2-1}\,x\right)}{x^8+2x^3+1}\right)-\frac{1}{4}i\log\left(\frac{ix^8+2ix^3+3ix^2-2\sqrt{-x^8-2x^3-2x^2-1}\,x+i}{x^8+2x^3+x^2+1}\right)+\frac{1}{4}i\log\left(\frac{-ix^8-2ix^3-3ix^2-2\sqrt{-x^8-2x^3-2x^2-1}\,x-i}{x^8+2x^3+x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^8-2*x^3-2*x^2-1)^(1/2)*(3*x^8+x^3-1)/(x^8+2*x^3+1)/(x^8+2*x^3
+x^2+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-2)*log(-2*(sqrt(-2)*(x^8 + 2*x^3 + 4*x^2 + 1) + 4*sqrt(-x^8 - 2*
x^3 - 2*x^2 - 1)*x)/(x^8 + 2*x^3 + 1)) + 1/4*sqrt(-2)*log(2*(sqrt(-2)*(x^8
+ 2*x^3 + 4*x^2 + 1) - 4*sqrt(-x^8 - 2*x^3 - 2*x^2 - 1)*x)/(x^8 + 2*x^3 + 1
)) - 1/4*I*log((I*x^8 + 2*I*x^3 + 3*I*x^2 - 2*sqrt(-x^8 - 2*x^3 - 2*x^2 - 1
)*x + I)/(x^8 + 2*x^3 + x^2 + 1)) + 1/4*I*log((-I*x^8 - 2*I*x^3 - 3*I*x^2 -
 2*sqrt(-x^8 - 2*x^3 - 2*x^2 - 1)*x - I)/(x^8 + 2*x^3 + x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^8+x^3-1\right)\sqrt{-x^8-2\,x^3-2\,x^2-1}}{\left(x^8+2\,x^3+x^2+1\right)\left(x^8+2\,x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^8-2*x^3-2*x^2-1)^(1/2)*(3*x^8+x^3-1)/(x^8+2*x^3+1)/(x^8+2*x^3
+x^2+1),x, algorithm="giac")
```

```
[Out] integrate((3*x^8 + x^3 - 1)*sqrt(-x^8 - 2*x^3 - 2*x^2 - 1)/((x^8 + 2*x^3 +
x^2 + 1)*(x^8 + 2*x^3 + 1)), x)
```

maple [C] time = 1.17, size = 195, normalized size = 2.05

$$-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)x^8+2\,\mathrm{RootOf}\left(\_Z^2+2\right)x^3+4\,\mathrm{RootOf}\left(\_Z^2+2\right)x^2+4\sqrt{-x^8-2x^3-2x^2-1}\,x+\mathrm{RootOf}\left(\_Z^2+2\right)}{(1+x)\left(x^7-x^6+x^5-x^4+x^3+x^2-x+1\right)}\right)}{2}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)x^8+2\,\mathrm{RootOf}\left(\_Z^2+1\right)x^3+3\,\mathrm{RootOf}\left(\_Z^2+1\right)x^2+2\sqrt{-x^8-2x^3-2x^2-1}\,x+\mathrm{RootOf}\left(\_Z^2+1\right)}{x^8+2x^3+x^2+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^8-2*x^3-2*x^2-1)^(1/2)*(3*x^8+x^3-1)/(x^8+2*x^3+1)/(x^8+2*x^3+x^2+1
),x)
```

```
[Out] -1/2*RootOf(_Z^2+2)*ln(-(RootOf(_Z^2+2)*x^8+2*RootOf(_Z^2+2)*x^3+4*RootOf(_
Z^2+2)*x^2+4*(-x^8-2*x^3-2*x^2-1)^(1/2)*x+RootOf(_Z^2+2))/(1+x)/(x^7-x^6+x^
5-x^4+x^3+x^2-x+1))+1/2*RootOf(_Z^2+1)*ln(-(RootOf(_Z^2+1)*x^8+2*RootOf(_Z^
```

```
2+1)*x^3+3*RootOf(_Z^2+1)*x^2+2*(-x^8-2*x^3-2*x^2-1)^(1/2)*x+RootOf(_Z^2+1)
)/(x^8+2*x^3+x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^{8}+x^{3}-1\right)\sqrt{-x^{8}-2\,x^{3}-2\,x^{2}-1}}{\left(x^{8}+2\,x^{3}+x^{2}+1\right)\left(x^{8}+2\,x^{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^8-2*x^3-2*x^2-1)^(1/2)*(3*x^8+x^3-1)/(x^8+2*x^3+1)/(x^8+2*x^3
+x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((3*x^8 + x^3 - 1)*sqrt(-x^8 - 2*x^3 - 2*x^2 - 1)/((x^8 + 2*x^3 +
x^2 + 1)*(x^8 + 2*x^3 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^{8}+x^{3}-1\right)\,\sqrt{-x^{8}-2\,x^{3}-2\,x^{2}-1}}{\left(x^{8}+2\,x^{3}+1\right)\,\left(x^{8}+2\,x^{3}+x^{2}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 3*x^8 - 1)*(- 2*x^2 - 2*x^3 - x^8 - 1)^(1/2))/((2*x^3 + x^8 + 1
)*(x^2 + 2*x^3 + x^8 + 1)),x)
```

```
[Out] int(((x^3 + 3*x^8 - 1)*(- 2*x^2 - 2*x^3 - x^8 - 1)^(1/2))/((2*x^3 + x^8 + 1
)*(x^2 + 2*x^3 + x^8 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^{8}+x^{3}-1\right)\sqrt{-x^{8}-2x^{3}-2x^{2}-1}}{\left(x+1\right)\left(x^{8}+2x^{3}+x^{2}+1\right)\left(x^{7}-x^{6}+x^{5}-x^{4}+x^{3}+x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**8-2*x**3-2*x**2-1)**(1/2)*(3*x**8+x**3-1)/(x**8+2*x**3+1)/(x
**8+2*x**3+x**2+1),x)
```

```
[Out] Integral((3*x**8 + x**3 - 1)*sqrt(-x**8 - 2*x**3 - 2*x**2 - 1)/((x + 1)*(x*
*8 + 2*x**3 + x**2 + 1)*(x**7 - x**6 + x**5 - x**4 + x**3 + x**2 - x + 1)),
 x)
```

3.1182 $\int \frac{(1+x^6)\sqrt{-2-x^2+x^6}}{4-3x^4-4x^6+x^{12}}\,dx$

Optimal. Leaf size=95

$$-\frac{1}{4}\sqrt{\frac{1}{3}\left(1+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{1+\sqrt{3}}\,x}{\sqrt{x^6-x^2-2}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(\sqrt{3}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\sqrt{3}-1}\,x}{\sqrt{x^6-x^2-2}}\right)$$

Rubi [F] time = 0.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+x^6\right)\sqrt{-2-x^2+x^6}}{4-3x^4-4x^6+x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^6)*Sqrt[-2 - x^2 + x^6])/(4 - 3*x^4 - 4*x^6 + x^12),x]
```

```
[Out] Defer[Int][Sqrt[-2 - x^2 + x^6]/(4 - 3*x^4 - 4*x^6 + x^12), x] + Defer[Int]
[(x^6*Sqrt[-2 - x^2 + x^6])/(4 - 3*x^4 - 4*x^6 + x^12), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1+x^6\right)\sqrt{-2-x^2+x^6}}{4-3x^4-4x^6+x^{12}}\,dx &= \int \left(\frac{\sqrt{-2-x^2+x^6}}{4-3x^4-4x^6+x^{12}}+\frac{x^6\sqrt{-2-x^2+x^6}}{4-3x^4-4x^6+x^{12}}\right)dx\\ &= \int \frac{\sqrt{-2-x^2+x^6}}{4-3x^4-4x^6+x^{12}}\,dx+\int \frac{x^6\sqrt{-2-x^2+x^6}}{4-3x^4-4x^6+x^{12}}\,dx\end{aligned}$$

Mathematica [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\left(1+x^6\right)\sqrt{-2-x^2+x^6}}{4-3x^4-4x^6+x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^6)*Sqrt[-2 - x^2 + x^6])/(4 - 3*x^4 - 4*x^6 + x^12),x]
```

```
[Out] Integrate[((1 + x^6)*Sqrt[-2 - x^2 + x^6])/(4 - 3*x^4 - 4*x^6 + x^12), x]
```

IntegrateAlgebraic [A] time = 2.19, size = 95, normalized size = 1.00

$$-\frac{1}{4}\sqrt{\frac{1}{3}\left(1+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{1+\sqrt{3}}\,x}{\sqrt{x^6-x^2-2}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(\sqrt{3}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\sqrt{3}-1}\,x}{\sqrt{x^6-x^2-2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^6)*Sqrt[-2 - x^2 + x^6])/(4 - 3*x^4 - 4*x^6 + x^
12),x]
```

```
[Out] -1/4*(Sqrt[(1 + Sqrt[3])/3]*ArcTan[(Sqrt[1 + Sqrt[3]]*x)/Sqrt[-2 - x^2 + x^
6]]) - (Sqrt[(-1 + Sqrt[3])/3]*ArcTanh[(Sqrt[-1 + Sqrt[3]]*x)/Sqrt[-2 - x^2
 + x^6]])/4
```

fricas [B] time = 0.87, size = 438, normalized size = 4.61

$$-\frac{1}{12}\sqrt{3}\sqrt{\sqrt{3}+1}\arctan\left(\frac{4\left(x^{7}+\sqrt{3}x^{3}-2x^{3}-2x\right)\sqrt{x^{6}-x^{2}-2}\sqrt{\sqrt{3}+1}-\left(2x^{12}-10x^{8}-8x^{6}+12x^{4}+20x^{2}-\sqrt{3}\left(x^{12}-6x^{8}-4x^{6}+7x^{4}+12x^{2}+4\right)+8\right)\sqrt{6\sqrt{3}+10}\sqrt{\sqrt{3}+1}}{2\left(x^{12}-4x^{8}-4x^{6}+x^{4}+8x^{2}+4\right)}\right)+\frac{1}{48}\sqrt{3}\sqrt{\sqrt{3}-1}\log\left(\frac{4\left(2x^{7}-3x^{3}-\sqrt{3}\left(x^{7}-2x^{3}-2x\right)-4x\right)\sqrt{x^{6}-x^{2}-2}+\left(x^{12}-8x^{8}-4x^{6}+9x^{4}+16x^{2}-\sqrt{3}\left(x^{12}-4x^{8}-4x^{6}+5x^{4}+8x^{2}+4\right)+4\right)\sqrt{\sqrt{3}-1}}{x^{12}-4x^{6}-3x^{4}+4}\right)-\frac{1}{48}\sqrt{3}\sqrt{\sqrt{3}-1}\log\left(\frac{4\left(2x^{7}-3x^{3}-\sqrt{3}\left(x^{7}-2x^{3}-2x\right)-4x\right)\sqrt{x^{6}-x^{2}-2}-\left(x^{12}-8x^{8}-4x^{6}+9x^{4}+16x^{2}-\sqrt{3}\left(x^{12}-4x^{8}-4x^{6}+5x^{4}+8x^{2}+4\right)+4\right)\sqrt{\sqrt{3}-1}}{x^{12}-4x^{6}-3x^{4}+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)*(x^6-x^2-2)^(1/2)/(x^12-4*x^6-3*x^4+4),x, algorithm="fric
as")
```

```
[Out] -1/12*sqrt(3)*sqrt(sqrt(3) + 1)*arctan(1/2*(4*(x^7 + sqrt(3)*x^3 - 2*x^3 -
2*x)*sqrt(x^6 - x^2 - 2)*sqrt(sqrt(3) + 1) - (2*x^12 - 10*x^8 - 8*x^6 + 12*
x^4 + 20*x^2 - sqrt(3)*(x^12 - 6*x^8 - 4*x^6 + 7*x^4 + 12*x^2 + 4) + 8)*sqr
t(6*sqrt(3) + 10)*sqrt(sqrt(3) + 1))/(x^12 - 4*x^8 - 4*x^6 + x^4 + 8*x^2 +
4)) + 1/48*sqrt(3)*sqrt(sqrt(3) - 1)*log((4*(2*x^7 - 3*x^3 - sqrt(3)*(x^7 -
 2*x^3 - 2*x) - 4*x)*sqrt(x^6 - x^2 - 2) + (x^12 - 8*x^8 - 4*x^6 + 9*x^4 +
16*x^2 - sqrt(3)*(x^12 - 4*x^8 - 4*x^6 + 5*x^4 + 8*x^2 + 4) + 4)*sqrt(sqrt(
3) - 1))/(x^12 - 4*x^6 - 3*x^4 + 4)) - 1/48*sqrt(3)*sqrt(sqrt(3) - 1)*log((
4*(2*x^7 - 3*x^3 - sqrt(3)*(x^7 - 2*x^3 - 2*x) - 4*x)*sqrt(x^6 - x^2 - 2) -
 (x^12 - 8*x^8 - 4*x^6 + 9*x^4 + 16*x^2 - sqrt(3)*(x^12 - 4*x^8 - 4*x^6 + 5
*x^4 + 8*x^2 + 4) + 4)*sqrt(sqrt(3) - 1))/(x^12 - 4*x^6 - 3*x^4 + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{6}-x^{2}-2}\left(x^{6}+1\right)}{x^{12}-4\,x^{6}-3\,x^{4}+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)*(x^6-x^2-2)^(1/2)/(x^12-4*x^6-3*x^4+4),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(x^6 - x^2 - 2)*(x^6 + 1)/(x^12 - 4*x^6 - 3*x^4 + 4), x)
```

maple [C] time = 3.73, size = 578, normalized size = 6.08

$$-\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)\ln\left(-\frac{384\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{3}x^{6}+73728\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{5}x^{2}-4\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)x^{6}-1152\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{3}x^{2}+192\sqrt{x^{6}-x^{2}-2}\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{2}x-768\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{3}+4\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)x^{2}-\sqrt{x^{6}-x^{2}-2}\,x+8\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)}{-x^{6}+192x^{2}\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{2}+x^{2}+2}\right)-\frac{\mathrm{RootOf}\left(\_Z^{2}+576\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{2}+6\right)\ln\left(\frac{48\,\mathrm{RootOf}\left(\_Z^{2}+576\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{2}+6\right)\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{2}x^{6}-9216\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{4}\mathrm{RootOf}\left(\_Z^{2}+576\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{2}+6\right)x^{2}+\mathrm{RootOf}\left(\_Z^{2}+576\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{2}+6\right)x^{6}-336\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{2}\mathrm{RootOf}\left(\_Z^{2}+576\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{2}+6\right)x^{2}+576\sqrt{x^{6}-x^{2}-2}\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{2}x-96\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{2}\mathrm{RootOf}\left(\_Z^{2}+576\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{2}+6\right)-3\,\mathrm{RootOf}\left(\_Z^{2}+576\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{2}+6\right)x^{2}+9\sqrt{x^{6}-x^{2}-2}\,x-2\,\mathrm{RootOf}\left(\_Z^{2}+576\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{2}+6\right)}{x^{6}+192x^{2}\,\mathrm{RootOf}\left(18432\_Z^{4}+192\_Z^{2}-1\right)^{2}+x^{2}-2}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)*(x^6-x^2-2)^(1/2)/(x^12-4*x^6-3*x^4+4),x)
```

```
[Out] -RootOf(18432*_Z^4+192*_Z^2-1)*ln(-(384*RootOf(18432*_Z^4+192*_Z^2-1)^3*x^6
+73728*RootOf(18432*_Z^4+192*_Z^2-1)^5*x^2-4*RootOf(18432*_Z^4+192*_Z^2-1)*
x^6-1152*RootOf(18432*_Z^4+192*_Z^2-1)^3*x^2+192*(x^6-x^2-2)^(1/2)*RootOf(1
8432*_Z^4+192*_Z^2-1)^2*x-768*RootOf(18432*_Z^4+192*_Z^2-1)^3+4*RootOf(1843
2*_Z^4+192*_Z^2-1)*x^2-(x^6-x^2-2)^(1/2)*x+8*RootOf(18432*_Z^4+192*_Z^2-1))
/(-x^6+192*x^2*RootOf(18432*_Z^4+192*_Z^2-1)^2+x^2+2))-1/24*RootOf(_Z^2+576
*RootOf(18432*_Z^4+192*_Z^2-1)^2+6)*ln((48*RootOf(_Z^2+576*RootOf(18432*_Z^
4+192*_Z^2-1)^2+6)*RootOf(18432*_Z^4+192*_Z^2-1)^2*x^6-9216*RootOf(18432*_Z
^4+192*_Z^2-1)^4*RootOf(_Z^2+576*RootOf(18432*_Z^4+192*_Z^2-1)^2+6)*x^2+Roo
tOf(_Z^2+576*RootOf(18432*_Z^4+192*_Z^2-1)^2+6)*x^6-336*RootOf(18432*_Z^4+1
92*_Z^2-1)^2*RootOf(_Z^2+576*RootOf(18432*_Z^4+192*_Z^2-1)^2+6)*x^2+576*(x^
6-x^2-2)^(1/2)*RootOf(18432*_Z^4+192*_Z^2-1)^2*x-96*RootOf(18432*_Z^4+192*_
Z^2-1)^2*RootOf(_Z^2+576*RootOf(18432*_Z^4+192*_Z^2-1)^2+6)-3*RootOf(_Z^2+5
76*RootOf(18432*_Z^4+192*_Z^2-1)^2+6)*x^2+9*(x^6-x^2-2)^(1/2)*x-2*RootOf(_Z
^2+576*RootOf(18432*_Z^4+192*_Z^2-1)^2+6))/(x^6+192*x^2*RootOf(18432*_Z^4+1
92*_Z^2-1)^2+x^2-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{6}-x^{2}-2}\left(x^{6}+1\right)}{x^{12}-4\,x^{6}-3\,x^{4}+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)*(x^6-x^2-2)^(1/2)/(x^12-4*x^6-3*x^4+4),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(x^6 - x^2 - 2)*(x^6 + 1)/(x^12 - 4*x^6 - 3*x^4 + 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^6+1\right)\,\sqrt{x^6-x^2-2}}{-x^{12}+4\,x^6+3\,x^4-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^6 + 1)*(x^6 - x^2 - 2)^(1/2))/(3*x^4 + 4*x^6 - x^12 - 4),x)
```

```
[Out] int(-((x^6 + 1)*(x^6 - x^2 - 2)^(1/2))/(3*x^4 + 4*x^6 - x^12 - 4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)\left(x^4-x^2+1\right)\sqrt{x^6-x^2-2}}{x^{12}-4x^6-3x^4+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)*(x**6-x**2-2)**(1/2)/(x**12-4*x**6-3*x**4+4),x)
```

```
[Out] Integral((x**2 + 1)*(x**4 - x**2 + 1)*sqrt(x**6 - x**2 - 2)/(x**12 - 4*x**6
 - 3*x**4 + 4), x)
```

3.1183 $\int x\sqrt[3]{-1+x^3}\,dx$

Optimal. Leaf size=96

$$\frac{1}{9}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{3\sqrt{3}}+\frac{1}{3}\sqrt[3]{x^3-1}x^2-\frac{1}{18}\log\left(\sqrt[3]{x^3-1}x+\left(x^3-1\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {279, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{9}\log\left(1-\frac{x}{\sqrt[3]{x^3-1}}\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\sqrt[3]{x^3-1}x^2-\frac{1}{18}\log\left(\frac{x}{\sqrt[3]{x^3-1}}+\frac{x^2}{\left(x^3-1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(-1 + x^3)^(1/3),x]
```

```
[Out] (x^2*(-1 + x^3)^(1/3))/3 + ArcTan[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]]/(3*
Sqrt[3]) + Log[1 - x/(-1 + x^3)^(1/3)]/9 - Log[1 + x^2/(-1 + x^3)^(2/3) + x
/(-1 + x^3)^(1/3)]/18
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int x\sqrt[3]{-1+x^3}\,dx &= \frac{1}{3}x^2\sqrt[3]{-1+x^3}-\frac{1}{3}\int\frac{x}{\left(-1+x^3\right)^{2/3}}\,dx\\
&= \frac{1}{3}x^2\sqrt[3]{-1+x^3}-\frac{1}{3}\,\mathrm{Subst}\left(\int\frac{x}{1-x^3}\,dx,x,\frac{x}{\sqrt[3]{-1+x^3}}\right)\\
&= \frac{1}{3}x^2\sqrt[3]{-1+x^3}-\frac{1}{9}\,\mathrm{Subst}\left(\int\frac{1}{1-x}\,dx,x,\frac{x}{\sqrt[3]{-1+x^3}}\right)+\frac{1}{9}\,\mathrm{Subst}\left(\int\frac{1-x}{1+x+x^2}\,dx,x,\frac{x}{\sqrt[3]{-1+\ldots}}\right.\\
&= \frac{1}{3}x^2\sqrt[3]{-1+x^3}+\frac{1}{9}\log\left(1-\frac{x}{\sqrt[3]{-1+x^3}}\right)-\frac{1}{18}\,\mathrm{Subst}\left(\int\frac{1+2x}{1+x+x^2}\,dx,x,\frac{x}{\sqrt[3]{-1+x^3}}\right)+\frac{1}{6}\,\mathrm{Sub}\ldots\\
&= \frac{1}{3}x^2\sqrt[3]{-1+x^3}+\frac{1}{9}\log\left(1-\frac{x}{\sqrt[3]{-1+x^3}}\right)-\frac{1}{18}\log\left(1+\frac{x^2}{\left(-1+x^3\right)^{2/3}}+\frac{x}{\sqrt[3]{-1+x^3}}\right)-\frac{1}{3}\,\mathrm{Subst}\left(\ldots\right.\\
&= \frac{1}{3}x^2\sqrt[3]{-1+x^3}+\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{9}\log\left(1-\frac{x}{\sqrt[3]{-1+x^3}}\right)-\frac{1}{18}\log\left(1+\frac{x^2}{\left(-1+x^3\right)^{2/3}}+\ldots\right.
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.42

$$
\frac{x^2\sqrt[3]{x^3-1}\;{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)}{2\sqrt[3]{1-x^3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-1 + x^3)^(1/3),x]
```

```
[Out] (x^2*(-1 + x^3)^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/3, x^3])/(2*(1 - x^3)^
(1/3))
```

IntegrateAlgebraic [A] time = 0.16, size = 96, normalized size = 1.00

$$
\frac{1}{9}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-1}+x}\right)}{3\sqrt{3}}+\frac{1}{3}\sqrt[3]{x^3-1}\,x^2-\frac{1}{18}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(-1 + x^3)^(1/3),x]
```

```
[Out] (x^2*(-1 + x^3)^(1/3))/3 + ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x^3)^(1/3))]/(3*
Sqrt[3]) + Log[-x + (-1 + x^3)^(1/3)]/9 - Log[x^2 + x*(-1 + x^3)^(1/3) + (-
1 + x^3)^(2/3)]/18
```

fricas [A] time = 0.56, size = 88, normalized size = 0.92

$$\frac{1}{3}\left(x^3-1\right)^{\frac{1}{3}}x^2-\frac{1}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{9}\log\left(-\frac{x-\left(x^3-1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{18}\log\left(\frac{x^2+\left(x^3-1\right)^{\frac{1}{3}}x+\left(x^3-1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^3-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 - 1)^(1/3)*x^2 - 1/9*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^
3 - 1)^(1/3))/x) + 1/9*log(-(x - (x^3 - 1)^(1/3))/x) - 1/18*log((x^2 + (x^3
 - 1)^(1/3)*x + (x^3 - 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^3-1\right)^{\frac{1}{3}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^3-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 1)^(1/3)*x, x)
```

maple [C] time = 0.37, size = 46, normalized size = 0.48

$$\frac{x^2\left(x^3-1\right)^{\frac{1}{3}}}{3}-\frac{\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{2}{3}}x^2\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{6\mathrm{signum}\left(x^3-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^3-1)^(1/3),x)
```

```
[Out] 1/3*x^2*(x^3-1)^(1/3)-1/6/signum(x^3-1)^(2/3)*(-signum(x^3-1))^(2/3)*x^2*hy
pergeom([2/3,2/3],[5/3],x^3)
```

maxima [A] time = 0.42, size = 94, normalized size = 0.98

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3-1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\left(x^3-1\right)^{\frac{1}{3}}}{3x\left(\frac{x^3-1}{x^3}-1\right)}-\frac{1}{18}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3-1\right)^{\frac{2}{3}}}{x^2}+1\right)+\frac{1}{9}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^3-1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3)/x + 1)) - 1/3*(x^3 - 1)^
(1/3)/(x*((x^3 - 1)/x^3 - 1)) - 1/18*log((x^3 - 1)^(1/3)/x + (x^3 - 1)^(2/3
)/x^2 + 1) + 1/9*log((x^3 - 1)^(1/3)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(x^3-1\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^3 - 1)^(1/3),x)
```

```
[Out] int(x*(x^3 - 1)^(1/3), x)
```

sympy [C] time = 0.87, size = 36, normalized size = 0.38

$$-\frac{x^{2}e^{-\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^{3}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**3-1)**(1/3),x)
```

```
[Out] -x**2*exp(-2*I*pi/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), x**3)/(3*gamma(5
/3))
```

3.1184 $\int x\sqrt[3]{1+x^3}\,dx$

Optimal. Leaf size=96

$$-\frac{1}{9}\log\left(\sqrt[3]{x^3+1}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{3\sqrt{3}}+\frac{1}{3}\sqrt[3]{x^3+1}x^2+\frac{1}{18}\log\left(\sqrt[3]{x^3+1}x+\left(x^3+1\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {279, 331, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{9}\log\left(1-\frac{x}{\sqrt[3]{x^3+1}}\right)-\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\sqrt[3]{x^3+1}x^2+\frac{1}{18}\log\left(\frac{x}{\sqrt[3]{x^3+1}}+\frac{x^2}{\left(x^3+1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + x^3)^(1/3),x]
```

```
[Out] (x^2*(1 + x^3)^(1/3))/3 - ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]]/(3*Sq
rt[3]) - Log[1 - x/(1 + x^3)^(1/3)]/9 + Log[1 + x^2/(1 + x^3)^(2/3) + x/(1
+ x^3)^(1/3)]/18
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt[3]{1+x^3}\,dx &= \frac{1}{3}x^2\sqrt[3]{1+x^3}+\frac{1}{3}\int\frac{x}{\left(1+x^3\right)^{2/3}}\,dx\\
&= \frac{1}{3}x^2\sqrt[3]{1+x^3}+\frac{1}{3}\text{Subst}\left(\int\frac{x}{1-x^3}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right)\\
&= \frac{1}{3}x^2\sqrt[3]{1+x^3}+\frac{1}{9}\text{Subst}\left(\int\frac{1}{1-x}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right)-\frac{1}{9}\text{Subst}\left(\int\frac{1-x}{1+x+x^2}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right)\\
&= \frac{1}{3}x^2\sqrt[3]{1+x^3}-\frac{1}{9}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\right)+\frac{1}{18}\text{Subst}\left(\int\frac{1+2x}{1+x+x^2}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right)-\frac{1}{6}\text{Subst}\Bigg(\int\\
&= \frac{1}{3}x^2\sqrt[3]{1+x^3}-\frac{1}{9}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\right)+\frac{1}{18}\log\left(1+\frac{x^2}{\left(1+x^3\right)^{2/3}}+\frac{x}{\sqrt[3]{1+x^3}}\right)+\frac{1}{3}\text{Subst}\Bigg(\int\frac{1}{-3}\\
&= \frac{1}{3}x^2\sqrt[3]{1+x^3}-\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{9}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\right)+\frac{1}{18}\log\left(1+\frac{x^2}{\left(1+x^3\right)^{2/3}}+\frac{x}{\sqrt[3]{1+x^3}}\right)
\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.23

$$\frac{1}{2}x^2\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + x^3)^(1/3),x]
```

```
[Out] (x^2*Hypergeometric2F1[-1/3, 2/3, 5/3, -x^3])/2
```

IntegrateAlgebraic [A] time = 0.16, size = 96, normalized size = 1.00

$$-\frac{1}{9}\log\left(\sqrt[3]{x^3+1}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3+1}+x}\right)}{3\sqrt{3}}+\frac{1}{3}\sqrt[3]{x^3+1}\,x^2+\frac{1}{18}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(1 + x^3)^(1/3),x]
```

```
[Out] (x^2*(1 + x^3)^(1/3))/3 - ArcTan[(Sqrt[3]*x)/(x + 2*(1 + x^3)^(1/3))]/(3*Sq
rt[3]) - Log[-x + (1 + x^3)^(1/3)]/9 + Log[x^2 + x*(1 + x^3)^(1/3) + (1 + x
^3)^(2/3)]/18
```

fricas [A] time = 0.58, size = 88, normalized size = 0.92

$$\frac{1}{3}\left(x^3+1\right)^{\frac{1}{3}}x^2+\frac{1}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}}{3x}\right)-\frac{1}{9}\log\left(-\frac{x-\left(x^3+1\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{18}\log\left(\frac{x^2+\left(x^3+1\right)^{\frac{1}{3}}x+\left(x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 1)^(1/3)*x^2 + 1/9*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^
3 + 1)^(1/3))/x) - 1/9*log(-(x - (x^3 + 1)^(1/3))/x) + 1/18*log((x^2 + (x^3
 + 1)^(1/3)*x + (x^3 + 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^3+1\right)^{\frac{1}{3}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)^(1/3)*x, x)
```

maple [C] time = 0.34, size = 30, normalized size = 0.31

$$\frac{x^2\left(x^3+1\right)^{\frac{1}{3}}}{3}+\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^3+1)^(1/3),x)
```

```
[Out] 1/3*x^2*(x^3+1)^(1/3)+1/6*x^2*hypergeom([2/3,2/3],[5/3],-x^3)
```

maxima [A] time = 0.45, size = 94, normalized size = 0.98

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+1\right)^{\frac{1}{3}}}{x}+1\right)\right)+\frac{\left(x^3+1\right)^{\frac{1}{3}}}{3x\left(\frac{x^3+1}{x^3}-1\right)}+\frac{1}{18}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)-\frac{1}{9}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3)/x + 1)) + 1/3*(x^3 + 1)^(
1/3)/(x*((x^3 + 1)/x^3 - 1)) + 1/18*log((x^3 + 1)^(1/3)/x + (x^3 + 1)^(2/3)
/x^2 + 1) - 1/9*log((x^3 + 1)^(1/3)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(x^3+1\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^3 + 1)^(1/3),x)
```

```
[Out] int(x*(x^3 + 1)^(1/3), x)
```

sympy [C] time = 0.83, size = 31, normalized size = 0.32

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**3+1)**(1/3),x)
```

```
[Out] x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5
/3))
```

3.1185 $\int \frac{\sqrt[3]{x+x^3}}{x^2}\,dx$

Optimal. Leaf size=96

$$-\frac{3\sqrt[3]{x^3+x}}{2x}-\frac{1}{2}\log\left(\sqrt[3]{x^3+x}-x\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x}+x}\right)+\frac{1}{4}\log\left(\sqrt[3]{x^3+x}\,x+\left(x^3+x\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.13, antiderivative size = 180, normalized size of antiderivative = 1.88, number of steps used = 11, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {2020, 2032, 329, 275, 331, 292, 31, 634, 618, 204, 628}

$$-\frac{3\sqrt[3]{x^3+x}}{2x}-\frac{x^{2/3}\left(x^2+1\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2+1}}\right)}{2\left(x^3+x\right)^{2/3}}+\frac{x^{2/3}\left(x^2+1\right)^{2/3}\log\left(\frac{x^{4/3}}{\left(x^2+1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2+1}}+1\right)}{4\left(x^3+x\right)^{2/3}}-\frac{\sqrt{3}x^{2/3}\left(x^2+1\right)^{2/3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)}{2\left(x^3+x\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^3)^(1/3)/x^2,x]
```

```
[Out] (-3*(x + x^3)^(1/3))/(2*x) - (Sqrt[3]*x^(2/3)*(1 + x^2)^(2/3)*ArcTan[(1 + (
2*x^(2/3))/(1 + x^2)^(1/3))/Sqrt[3]])/(2*(x + x^3)^(2/3)) - (x^(2/3)*(1 + x
^2)^(2/3)*Log[1 - x^(2/3)/(1 + x^2)^(1/3)])/(2*(x + x^3)^(2/3)) + (x^(2/3)*
(1 + x^2)^(2/3)*Log[1 + x^(4/3)/(1 + x^2)^(2/3) + x^(2/3)/(1 + x^2)^(1/3)])
/(4*(x + x^3)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt[3]{x+x^3}}{x^2}\,dx = -\frac{3\sqrt[3]{x+x^3}}{2x} + \int \frac{x}{\left(x+x^3\right)^{2/3}}\,dx$$
$$= -\frac{3\sqrt[3]{x+x^3}}{2x} + \frac{\left(x^{2/3}\left(1+x^2\right)^{2/3}\right)\int \frac{\sqrt[3]{x}}{\left(1+x^2\right)^{2/3}}\,dx}{\left(x+x^3\right)^{2/3}}$$
$$= -\frac{3\sqrt[3]{x+x^3}}{2x} + \frac{\left(3x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x^3}{\left(1+x^6\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{\left(x+x^3\right)^{2/3}}$$
$$= -\frac{3\sqrt[3]{x+x^3}}{2x} + \frac{\left(3x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{\left(1+x^3\right)^{2/3}}\,dx, x, x^{2/3}\right)}{2\left(x+x^3\right)^{2/3}}$$
$$= -\frac{3\sqrt[3]{x+x^3}}{2x} + \frac{\left(3x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2\left(x+x^3\right)^{2/3}}$$
$$= -\frac{3\sqrt[3]{x+x^3}}{2x} + \frac{\left(x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2\left(x+x^3\right)^{2/3}} - \frac{\left(x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+x}\right.}{2\left(x+x^3\right)^{2/3}}$$
$$= -\frac{3\sqrt[3]{x+x^3}}{2x} - \frac{x^{2/3}\left(1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2\left(x+x^3\right)^{2/3}} + \frac{\left(x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \frac{x^2}{\sqrt[3]{1}}\right.}{4\left(x+x^3\right)^{2/3}}$$
$$= -\frac{3\sqrt[3]{x+x^3}}{2x} - \frac{x^{2/3}\left(1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2\left(x+x^3\right)^{2/3}} + \frac{x^{2/3}\left(1+x^2\right)^{2/3}\log\left(1+\frac{x^{4/3}}{\left(1+x^2\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{4\left(x+x^3\right)^{2/3}}$$
$$= -\frac{3\sqrt[3]{x+x^3}}{2x} - \frac{\sqrt{3}\,x^{2/3}\left(1+x^2\right)^{2/3}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2\left(x+x^3\right)^{2/3}} - \frac{x^{2/3}\left(1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2\left(x+x^3\right)^{2/3}} + \frac{x^{2/3}}{}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.42

$$-\frac{3\sqrt[3]{x^3+x}\;{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-x^2\right)}{2x\sqrt[3]{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^3)^(1/3)/x^2,x]
```

```
[Out] (-3*(x + x^3)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, -x^2])/(2*x*(1 + x^2
)^(1/3))
```

IntegrateAlgebraic [A] time = 0.21, size = 96, normalized size = 1.00

$$-\frac{3\sqrt[3]{x^3+x}}{2x} - \frac{1}{2}\log\left(\sqrt[3]{x^3+x}-x\right) - \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3+x}+x}\right) + \frac{1}{4}\log\left(\sqrt[3]{x^3+x}\,x+\left(x^3+x\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + x^3)^(1/3)/x^2,x]
```

```
[Out] (-3*(x + x^3)^(1/3))/(2*x) - (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(x + x^3)^(
1/3))])/2 - Log[-x + (x + x^3)^(1/3)]/2 + Log[x^2 + x*(x + x^3)^(1/3) + (x
+ x^3)^(2/3)]/4
```

fricas [A] time = 0.71, size = 95, normalized size = 0.99

$$-\frac{2\,\sqrt{3}\,x\arctan\left(-\frac{196\,\sqrt{3}\,\left(x^3+x\right)^{\frac{1}{3}}x-\sqrt{3}\,\left(539\,x^2+507\right)-1274\,\sqrt{3}\,\left(x^3+x\right)^{\frac{2}{3}}}{2205\,x^2+2197}\right)+x\log\left(3\left(x^3+x\right)^{\frac{1}{3}}x-3\left(x^3+x\right)^{\frac{2}{3}}+1\right)+6\left(x^3+x\right)^{\frac{1}{3}}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*x*arctan(-(196*sqrt(3)*(x^3 + x)^(1/3)*x - sqrt(3)*(539*x^2
 + 507) - 1274*sqrt(3)*(x^3 + x)^(2/3))/(2205*x^2 + 2197)) + x*log(3*(x^3 +
 x)^(1/3)*x - 3*(x^3 + x)^(2/3) + 1) + 6*(x^3 + x)^(1/3))/x
```

giac [A] time = 0.19, size = 64, normalized size = 0.67

$$\frac{1}{2}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{3}{2}\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+\frac{1}{4}\,\log\left(\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{2}\,\log\left(\left|\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(1/x^2 + 1)^(1/3) + 1)) - 3/2*(1/x^2 + 1)
^(1/3) + 1/4*log((1/x^2 + 1)^(2/3) + (1/x^2 + 1)^(1/3) + 1) - 1/2*log(abs((
1/x^2 + 1)^(1/3) - 1))
```

maple [C] time = 2.27, size = 729, normalized size = 7.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x)^(1/3)/x^2,x)
```

```
[Out] -3/2*(x*(x^2+1))^(1/3)/x+(1/4*RootOf(_Z^2-2*_Z+4)*ln(-(2*RootOf(_Z^2-2*_Z+4
)^2*x^4+11*RootOf(_Z^2-2*_Z+4)*x^4+15*(x^6+2*x^4+x^2)^(1/3)*RootOf(_Z^2-2*_
Z+4)*x^2-40*x^4+15*RootOf(_Z^2-2*_Z+4)*(x^6+2*x^4+x^2)^(2/3)-48*(x^6+2*x^4+
x^2)^(1/3)*x^2+28*RootOf(_Z^2-2*_Z+4)*x^2-48*(x^6+2*x^4+x^2)^(2/3)+15*RootO
f(_Z^2-2*_Z+4)*(x^6+2*x^4+x^2)^(1/3)-2*RootOf(_Z^2-2*_Z+4)^2-70*x^2-48*(x^6
+2*x^4+x^2)^(1/3)+17*RootOf(_Z^2-2*_Z+4)-30)/(x^2+1))-1/4*ln((-2*RootOf(_Z^
2-2*_Z+4)^2*x^4+19*RootOf(_Z^2-2*_Z+4)*x^4+15*(x^6+2*x^4+x^2)^(1/3)*RootOf(
_Z^2-2*_Z+4)*x^2+10*x^4+15*RootOf(_Z^2-2*_Z+4)*(x^6+2*x^4+x^2)^(2/3)+18*(x^
6+2*x^4+x^2)^(1/3)*x^2+28*RootOf(_Z^2-2*_Z+4)*x^2+18*(x^6+2*x^4+x^2)^(2/3)+
15*RootOf(_Z^2-2*_Z+4)*(x^6+2*x^4+x^2)^(1/3)+2*RootOf(_Z^2-2*_Z+4)^2+14*x^2
+18*(x^6+2*x^4+x^2)^(1/3)+9*RootOf(_Z^2-2*_Z+4)+4)/(x^2+1))*RootOf(_Z^2-2*_
Z+4)+1/2*ln((-2*RootOf(_Z^2-2*_Z+4)^2*x^4+19*RootOf(_Z^2-2*_Z+4)*x^4+15*(x^
6+2*x^4+x^2)^(1/3)*RootOf(_Z^2-2*_Z+4)*x^2+10*x^4+15*RootOf(_Z^2-2*_Z+4)*(x
^6+2*x^4+x^2)^(2/3)+18*(x^6+2*x^4+x^2)^(1/3)*x^2+28*RootOf(_Z^2-2*_Z+4)*x^2
+18*(x^6+2*x^4+x^2)^(2/3)+15*RootOf(_Z^2-2*_Z+4)*(x^6+2*x^4+x^2)^(1/3)+2*Ro
otOf(_Z^2-2*_Z+4)^2+14*x^2+18*(x^6+2*x^4+x^2)^(1/3)+9*RootOf(_Z^2-2*_Z+4)+4
)/(x^2+1)))*(x*(x^2+1))^(1/3)/x*(x^2*(x^2+1)^2)^(1/3)/(x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^3+x\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x^3 + x)^(1/3)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3 + x\right)^{1/3}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3)^(1/3)/x^2,x)
```

```
[Out] int((x + x^3)^(1/3)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x\left(x^2 + 1\right)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x)**(1/3)/x**2,x)
```

```
[Out] Integral((x*(x**2 + 1))**(1/3)/x**2, x)
```

3.1186 $\int \frac{(-1+x)(-1+kx)\left(3-2(1+k)x+kx^2\right)}{x((1-x)x(1-kx))^{3/4}\left(-1+(1+k)x-kx^2+dx^3\right)}\,dx$

Optimal. Leaf size=96

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{kx^3+(-k-1)x^2+x}}\right)-2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{kx^3+(-k-1)x^2+x}}\right)+\frac{4\sqrt[4]{kx^3+(-k-1)x^2+x}}{x}$$

Rubi [F] time = 16.65, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x)(-1+kx)\left(3-2(1+k)x+kx^2\right)}{x((1-x)x(1-kx))^{3/4}\left(-1+(1+k)x-kx^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x)*(-1 + k*x)*(3 - 2*(1 + k)*x + k*x^2))/(x*((1 - x)*x*(1 - k*x)
)^(3/4)*(-1 + (1 + k)*x - k*x^2 + d*x^3)),x]
```

```
[Out] (4*(1 - x)^(3/4)*(1 - k*x)^(3/4)*AppellF1[-3/4, -1/4, -1/4, 1/4, x, k*x])/(
(1 - x)*x*(1 - k*x))^(3/4) + (8*k*(1 - x)^(3/4)*x^(3/4)*(1 - k*x)^(3/4)*Def
er[Subst][Defer[Int][(x^4*(1 - x^4)^(1/4)*(1 - k*x^4)^(1/4))/(1 - (1 + k)*x
^4 + k*x^8 - d*x^12), x], x, x^(1/4)])/((1 - x)*x*(1 - k*x))^(3/4) + (4*(1
+ k)*(1 - x)^(3/4)*x^(3/4)*(1 - k*x)^(3/4)*Defer[Subst][Defer[Int][((1 - x^
4)^(1/4)*(1 - k*x^4)^(1/4))/(-1 + (1 + k)*x^4 - k*x^8 + d*x^12), x], x, x^(
1/4)])/((1 - x)*x*(1 - k*x))^(3/4) + (12*d*(1 - x)^(3/4)*x^(3/4)*(1 - k*x)^
(3/4)*Defer[Subst][Defer[Int][(x^8*(1 - x^4)^(1/4)*(1 - k*x^4)^(1/4))/(-1 +
 (1 + k)*x^4 - k*x^8 + d*x^12), x], x, x^(1/4)])/((1 - x)*x*(1 - k*x))^(3/4
)
```

Rubi steps

$$\int \frac{(-1+x)(-1+kx)\left(3-2(1+k)x+kx^2\right)}{x((1-x)x(1-kx))^{3/4}\left(-1+(1+k)x-kx^2+dx^3\right)}\,dx = \frac{\left((1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\int \frac{(-1+x)(-1+kx)\left(3-2(1+k\right.}{(1-x)^{3/4}x^{7/4}(1-kx)^{3/4}\left(-1+(1+k\right.}}{((1-x)x(1-kx))^{3/4}}$$

$$= -\frac{\left((1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\int \frac{\sqrt[4]{1-x}\,(-1+kx)\left(3-2(1+k)x\right.}{x^{7/4}(1-kx)^{3/4}\left(-1+(1+k)x-kx\right.}}{((1-x)x(1-kx))^{3/4}}$$

$$= \frac{\left((1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\int \frac{\sqrt[4]{1-x}\,\sqrt[4]{1-kx}\left(3-2(1+k)x+kx^2\right.}{x^{7/4}\left(-1+(1+k)x-kx^2+dx^3\right)}}{((1-x)x(1-kx))^{3/4}}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subst}\left(\int \frac{\sqrt[4]{1-x^4}\,\sqrt[4]{1-kx^4}\,(3-}{x^4\left(-1+(1+k)x^4\right.}\right.}{((1-x)x(1-kx))^{3/4}}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subst}\left(\int\left(-\frac{3\sqrt[4]{1-x^4}\,\sqrt[4]{1-kx}}{x^4}\right.\right.}{((1-x)x(1-}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subst}\left(\int \frac{\sqrt[4]{1-x^4}\,\sqrt[4]{1-kx^4}\,(-}{1-(1+k)x^4+}\right.}{((1-x)x(1-kx))^{3/4}}$$

$$= \frac{4(1-x)^{3/4}(1-kx)^{3/4}F_1\left(-\frac{3}{4};-\frac{1}{4},-\frac{1}{4};\frac{1}{4};x,kx\right)}{((1-x)x(1-kx))^{3/4}} + \frac{(4(}{}$$

$$= \frac{4(1-x)^{3/4}(1-kx)^{3/4}F_1\left(-\frac{3}{4};-\frac{1}{4},-\frac{1}{4};\frac{1}{4};x,kx\right)}{((1-x)x(1-kx))^{3/4}} + \frac{(12}{}$$

Mathematica [F] time = 1.70, size = 0, normalized size = 0.00

$$\int \frac{(-1+x)(-1+kx)\left(3-2(1+k)x+kx^2\right)}{x((1-x)x(1-kx))^{3/4}\left(-1+(1+k)x-kx^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x)*(-1 + k*x)*(3 - 2*(1 + k)*x + k*x^2))/(x*((1 - x)*x*(1
- k*x))^(3/4)*(-1 + (1 + k)*x - k*x^2 + d*x^3)),x]
```

```
[Out] Integrate[((-1 + x)*(-1 + k*x)*(3 - 2*(1 + k)*x + k*x^2))/(x*((1 - x)*x*(1
- k*x))^(3/4)*(-1 + (1 + k)*x - k*x^2 + d*x^3)), x]
```

IntegrateAlgebraic [A] time = 4.04, size = 96, normalized size = 1.00

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{kx^3+(-k-1)x^2+x}}\right)-2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{kx^3+(-k-1)x^2+x}}\right)+\frac{4\sqrt[4]{kx^3+(-k-1)x^2+x}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)*(-1 + k*x)*(3 - 2*(1 + k)*x + k*x^2))/(x*((1 -
 x)*x*(1 - k*x))^(3/4)*(-1 + (1 + k)*x - k*x^2 + d*x^3)),x]
```

```
[Out] (4*(x + (-1 - k)*x^2 + k*x^3)^(1/4))/x + 2*d^(1/4)*ArcTan[(d^(1/4)*x)/(x +
(-1 - k)*x^2 + k*x^3)^(1/4)] - 2*d^(1/4)*ArcTanh[(d^(1/4)*x)/(x + (-1 - k)*
x^2 + k*x^3)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(k*x-1)*(3-2*(1+k)*x+k*x^2)/x/((1-x)*x*(-k*x+1))^(3/4)/(-1
+(1+k)*x-k*x^2+d*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(kx^2 - 2(k+1)x + 3\right)(kx-1)(x-1)}{\left(dx^3 - kx^2 + (k+1)x - 1\right)\left((kx-1)(x-1)x\right)^{\frac{3}{4}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(k*x-1)*(3-2*(1+k)*x+k*x^2)/x/((1-x)*x*(-k*x+1))^(3/4)/(-1
+(1+k)*x-k*x^2+d*x^3),x, algorithm="giac")
```

```
[Out] integrate((k*x^2 - 2*(k + 1)*x + 3)*(k*x - 1)*(x - 1)/((d*x^3 - k*x^2 + (k
+ 1)*x - 1)*((k*x - 1)*(x - 1)*x)^(3/4)*x), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(-1+x)(kx-1)\left(3-2(1+k)x+kx^2\right)}{x\left((1-x)x(-kx+1)\right)^{\frac{3}{4}}\left(-1+(1+k)x-kx^2+dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)*(k*x-1)*(3-2*(1+k)*x+k*x^2)/x/((1-x)*x*(-k*x+1))^(3/4)/(-1+(1+k)
*x-k*x^2+d*x^3),x)
```

```
[Out] int((-1+x)*(k*x-1)*(3-2*(1+k)*x+k*x^2)/x/((1-x)*x*(-k*x+1))^(3/4)/(-1+(1+k)
*x-k*x^2+d*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(kx^2 - 2(k+1)x + 3\right)(kx-1)(x-1)}{\left(dx^3 - kx^2 + (k+1)x - 1\right)\left((kx-1)(x-1)x\right)^{\frac{3}{4}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(k*x-1)*(3-2*(1+k)*x+k*x^2)/x/((1-x)*x*(-k*x+1))^(3/4)/(-1
+(1+k)*x-k*x^2+d*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k*x^2 - 2*(k + 1)*x + 3)*(k*x - 1)*(x - 1)/((d*x^3 - k*x^2 + (k
+ 1)*x - 1)*((k*x - 1)*(x - 1)*x)^(3/4)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(k\,x-1)\,(x-1)\,\left(k\,x^2-2\,x\,(k+1)+3\right)}{x\,{\left(x\,(k\,x-1)\,(x-1)\right)}^{3/4}\,\left(d\,x^3-k\,x^2+(k+1)\,x-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((k*x - 1)*(x - 1)*(k*x^2 - 2*x*(k + 1) + 3))/(x*(x*(k*x - 1)*(x - 1))^
(3/4)*(d*x^3 + x*(k + 1) - k*x^2 - 1)),x)
```

```
[Out] int(((k*x - 1)*(x - 1)*(k*x^2 - 2*x*(k + 1) + 3))/(x*(x*(k*x - 1)*(x - 1))^
(3/4)*(d*x^3 + x*(k + 1) - k*x^2 - 1)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(k*x-1)*(3-2*(1+k)*x+k*x**2)/x/((1-x)*x*(-k*x+1))**(3/4)/(
-1+(1+k)*x-k*x**2+d*x**3),x)
```

```
[Out] Timed out
```

3.1187 $\int \frac{(1-2x+x^2)(-2+(-1+k)(1+k)x+2k^2x^2)}{((1-x^2)(1-k^2x^2))^{3/4}(-1+d-(1+3d)x+(3d+k^2)x^2+(-d+k^2)x^3)}\,dx$

Optimal. Leaf size=96

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x-\sqrt[4]{d}}{\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}\right)}{d^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,x-\sqrt[4]{d}}{\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}\right)}{d^{3/4}}$$

Rubi [F] time = 180.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$Aborted

Verification is not applicable to the result.

```
[In] Int[((1 - 2*x + x^2)*(-2 + (-1 + k)*(1 + k)*x + 2*k^2*x^2))/(((1 - x^2)*(1
- k^2*x^2))^(3/4)*(-1 + d - (1 + 3*d)*x + (3*d + k^2)*x^2 + (-d + k^2)*x^3)
),x]
```

```
[Out] $Aborted
```

Rubi steps

Aborted

Mathematica [F] time = 2.13, size = 0, normalized size = 0.00

$$\int \frac{(1-2x+x^2)(-2+(-1+k)(1+k)x+2k^2x^2)}{((1-x^2)(1-k^2x^2))^{3/4}(-1+d-(1+3d)x+(3d+k^2)x^2+(-d+k^2)x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 - 2*x + x^2)*(-2 + (-1 + k)*(1 + k)*x + 2*k^2*x^2))/(((1 - x^
2)*(1 - k^2*x^2))^(3/4)*(-1 + d - (1 + 3*d)*x + (3*d + k^2)*x^2 + (-d + k^2
)*x^3)),x]
```

```
[Out] Integrate[((1 - 2*x + x^2)*(-2 + (-1 + k)*(1 + k)*x + 2*k^2*x^2))/(((1 - x^
2)*(1 - k^2*x^2))^(3/4)*(-1 + d - (1 + 3*d)*x + (3*d + k^2)*x^2 + (-d + k^2
)*x^3)), x]
```

IntegrateAlgebraic [A] time = 15.70, size = 96, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x-\sqrt[4]{d}}{\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}\right)}{d^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,x-\sqrt[4]{d}}{\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 - 2*x + x^2)*(-2 + (-1 + k)*(1 + k)*x + 2*k^2*x^2))/
(((1 - x^2)*(1 - k^2*x^2))^(3/4)*(-1 + d - (1 + 3*d)*x + (3*d + k^2)*x^2 +
(-d + k^2)*x^3)),x]
```

```
[Out] -(ArcTan[(-d^(1/4) + d^(1/4)*x)/(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4)]/d^(3/
4)) + ArcTanh[(-d^(1/4) + d^(1/4)*x)/(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4)]/
d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x+1)*(-2+(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))
^(3/4)/(-1+d-(1+3*d)*x+(k^2+3*d)*x^2+(k^2-d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,k^2x^2+(k+1)(k-1)x-2\right)\left(x^2-2\,x+1\right)}{\left(\left(k^2-d\right)x^3+\left(k^2+3\,d\right)x^2-(3\,d+1)x+d-1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x+1)*(-2+(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))
^(3/4)/(-1+d-(1+3*d)*x+(k^2+3*d)*x^2+(k^2-d)*x^3),x, algorithm="giac")
```

```
[Out] integrate((2*k^2*x^2 + (k + 1)*(k - 1)*x - 2)*(x^2 - 2*x + 1)/(((k^2 - d)*x
^3 + (k^2 + 3*d)*x^2 - (3*d + 1)*x + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(3/4)
), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-2x+1\right)\left(-2+(-1+k)\,(1+k)\,x+2k^2x^2\right)}{\left(\left(-x^2+1\right)\left(-k^2x^2+1\right)\right)^{\frac{3}{4}}\left(-1+d-(1+3d)\,x+\left(k^2+3d\right)x^2+\left(k^2-d\right)x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x+1)*(-2+(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))^(3/4)
/(-1+d-(1+3*d)*x+(k^2+3*d)*x^2+(k^2-d)*x^3),x)
```

```
[Out] int((x^2-2*x+1)*(-2+(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))^(3/4)
/(-1+d-(1+3*d)*x+(k^2+3*d)*x^2+(k^2-d)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,k^2x^2+(k+1)(k-1)x-2\right)\left(x^2-2\,x+1\right)}{\left(\left(k^2-d\right)x^3+\left(k^2+3\,d\right)x^2-(3\,d+1)x+d-1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x+1)*(-2+(-1+k)*(1+k)*x+2*k^2*x^2)/((-x^2+1)*(-k^2*x^2+1))
^(3/4)/(-1+d-(1+3*d)*x+(k^2+3*d)*x^2+(k^2-d)*x^3),x, algorithm="maxima")
```

```
[Out] integrate((2*k^2*x^2 + (k + 1)*(k - 1)*x - 2)*(x^2 - 2*x + 1)/(((k^2 - d)*x
^3 + (k^2 + 3*d)*x^2 - (3*d + 1)*x + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(3/4)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(x^2-2\,x+1\right)\,\left(2\,k^2\,x^2+x\,\left(k-1\right)\,\left(k+1\right)-2\right)}{{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)}^{3/4}\,\left(\left(d-k^2\right)\,x^3+\left(-k^2-3\,d\right)\,x^2+\left(3\,d+1\right)\,x-d+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^2 - 2*x + 1)*(2*k^2*x^2 + x*(k - 1)*(k + 1) - 2))/(((x^2 - 1)*(k^2
*x^2 - 1))^(3/4)*(x^3*(d - k^2) - d - x^2*(3*d + k^2) + x*(3*d + 1) + 1)),x
)
```

```
[Out] -int(((x^2 - 2*x + 1)*(2*k^2*x^2 + x*(k - 1)*(k + 1) - 2))/(((x^2 - 1)*(k^2
*x^2 - 1))^(3/4)*(x^3*(d - k^2) - d - x^2*(3*d + k^2) + x*(3*d + 1) + 1)),
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x+1)*(-2+(-1+k)*(1+k)*x+2*k**2*x**2)/((-x**2+1)*(-k**2*x*
*2+1))**(3/4)/(-1+d-(1+3*d)*x+(k**2+3*d)*x**2+(k**2-d)*x**3),x)
```

```
[Out] Timed out
```

3.1188 $\int \frac{x^2}{\left(-b+ax^2\right)\sqrt[4]{-bx^2+ax^4}}\,dx$

Optimal. Leaf size=96

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{a^{5/4}}-\frac{2\left(ax^4-bx^2\right)^{3/4}}{ax\left(ax^2-b\right)}$$

Rubi [A] time = 0.33, antiderivative size = 153, normalized size of antiderivative = 1.59, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2056, 288, 329, 240, 212, 206, 203}

$$\frac{\sqrt{x}\sqrt[4]{ax^2-b}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2-b}}\right)}{a^{5/4}\sqrt[4]{ax^4-bx^2}}+\frac{\sqrt{x}\sqrt[4]{ax^2-b}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2-b}}\right)}{a^{5/4}\sqrt[4]{ax^4-bx^2}}-\frac{2x}{a\sqrt[4]{ax^4-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4)),x]
```

```
[Out] (-2*x)/(a*(-(b*x^2) + a*x^4)^(1/4)) + (Sqrt[x]*(-b + a*x^2)^(1/4)*ArcTan[(a
^(1/4)*Sqrt[x])/(-b + a*x^2)^(1/4)])/(a^(5/4)*(-(b*x^2) + a*x^4)^(1/4)) + (
Sqrt[x]*(-b + a*x^2)^(1/4)*ArcTanh[(a^(1/4)*Sqrt[x])/(-b + a*x^2)^(1/4)])/(
a^(5/4)*(-(b*x^2) + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{x^2}{\left(-b+ax^2\right)\sqrt[4]{-bx^2+ax^4}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{-b+ax^2}\right)\int \frac{x^{3/2}}{\left(-b+ax^2\right)^{5/4}}\,dx}{\sqrt[4]{-bx^2+ax^4}}$$

$$= -\frac{2x}{a\sqrt[4]{-bx^2+ax^4}} + \frac{\left(\sqrt{x}\sqrt[4]{-b+ax^2}\right)\int \frac{1}{\sqrt{x}\sqrt[4]{-b+ax^2}}\,dx}{a\sqrt[4]{-bx^2+ax^4}}$$

$$= -\frac{2x}{a\sqrt[4]{-bx^2+ax^4}} + \frac{\left(2\sqrt{x}\sqrt[4]{-b+ax^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-b+ax^4}}\,dx, x, \sqrt{x}\right)}{a\sqrt[4]{-bx^2+ax^4}}$$

$$= -\frac{2x}{a\sqrt[4]{-bx^2+ax^4}} + \frac{\left(2\sqrt{x}\sqrt[4]{-b+ax^2}\right)\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{-b+ax^2}}\right)}{a\sqrt[4]{-bx^2+ax^4}}$$

$$= -\frac{2x}{a\sqrt[4]{-bx^2+ax^4}} + \frac{\left(\sqrt{x}\sqrt[4]{-b+ax^2}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{-b+ax^2}}\right)}{a\sqrt[4]{-bx^2+ax^4}} + \frac{\left(\sqrt{x}\sqrt[4]{\right.}}{}$$

$$= -\frac{2x}{a\sqrt[4]{-bx^2+ax^4}} + \frac{\sqrt{x}\sqrt[4]{-b+ax^2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{-b+ax^2}}\right)}{a^{5/4}\sqrt[4]{-bx^2+ax^4}} + \frac{\sqrt{x}\sqrt[4]{-b+ax^2}\tanh^{-1}\left(\right.}{a^{5/4}\sqrt[4]{-bx^2+ax^{4}}}$$

Mathematica [A] time = 0.19, size = 136, normalized size = 1.42

$$\frac{\left(ax^4-bx^2\right)^{3/4}\left(-2\sqrt[4]{a}\sqrt{x}\left(ax^2-b\right)^{3/4}+\left(ax^2-b\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2-b}}\right)+\left(ax^2-b\right)\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2-b}}\right)\right)}{a^{5/4}x^{3/2}\left(ax^2-b\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4)),x]
```

```
[Out] ((-(b*x^2) + a*x^4)^(3/4)*(-2*a^(1/4)*Sqrt[x]*(-b + a*x^2)^(3/4) + (-b + a*
x^2)*ArcTan[(a^(1/4)*Sqrt[x])/(-b + a*x^2)^(1/4)] + (-b + a*x^2)*ArcTanh[(a
^(1/4)*Sqrt[x])/(-b + a*x^2)^(1/4)]))/(a^(5/4)*x^(3/2)*(-b + a*x^2)^(7/4))
```

IntegrateAlgebraic [A] time = 0.28, size = 96, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{a^{5/4}}-\frac{2\left(ax^4-bx^2\right)^{3/4}}{ax\left(ax^2-b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4)),x]
```

```
[Out] (-2*(-(b*x^2) + a*x^4)^(3/4))/(a*x*(-b + a*x^2)) + ArcTan[(a^(1/4)*x)/(-(b*
x^2) + a*x^4)^(1/4)]/a^(5/4) + ArcTanh[(a^(1/4)*x)/(-(b*x^2) + a*x^4)^(1/4)
]/a^(5/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2-b)/(a*x^4-b*x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.22, size = 207, normalized size = 2.16

$$\frac{\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{2a^2}+\frac{\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{2a^2}-\frac{\sqrt{2}(-a)^{\frac{3}{4}}\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^2}}\right)}{4a^2}+\frac{\sqrt{2}(-a)^{\frac{3}{4}}\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^2}}\right)}{4a^2}-\frac{2}{\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2-b)/(a*x^4-b*x^2)^(1/4),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a - b/x^
2)^(1/4))/(-a)^(1/4))/a^2 + 1/2*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(-a)^(1/4) - 2*(a - b/x^2)^(1/4))/(-a)^(1/4))/a^2 - 1/4*sqrt(2)*(-a)^(
3/4)*log(sqrt(2)*(-a)^(1/4)*(a - b/x^2)^(1/4) + sqrt(-a) + sqrt(a - b/x^2))
/a^2 + 1/4*sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a - b/x^2)^(1/4) + s
qrt(-a) + sqrt(a - b/x^2))/a^2 - 2/((a - b/x^2)^(1/4)*a)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a\,x^2-b\right)\left(a\,x^4-b\,x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2-b)/(a*x^4-b*x^2)^(1/4),x)
```

```
[Out] int(x^2/(a*x^2-b)/(a*x^4-b*x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax^4-bx^2\right)^{\frac{1}{4}}\left(ax^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2-b)/(a*x^4-b*x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a*x^4 - b*x^2)^(1/4)*(a*x^2 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{\left(b-a\,x^2\right)\,\left(a\,x^4-b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((b - a*x^2)*(a*x^4 - b*x^2)^(1/4)),x)
[Out] -int(x^2/((b - a*x^2)*(a*x^4 - b*x^2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[4]{x^2\left(ax^2-b\right)}\left(ax^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**2-b)/(a*x**4-b*x**2)**(1/4),x)
[Out] Integral(x**2/((x**2*(a*x**2 - b))**(1/4)*(a*x**2 - b)), x)
```

3.1189 $\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)\left(aq+bx^2+apx^5\right)}{x^4\left(cq+dx^2+cpx^5\right)}\,dx$

Optimal. Leaf size=96

$$\frac{2\left(bc\sqrt{d}-ad^{3/2}\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}\,\sqrt{px^5+q}}\right)}{c^{5/2}}+\frac{2\sqrt{px^5+q}\,\left(acpx^5+acq-3adx^2+3bcx^2\right)}{3c^2x^3}$$

Rubi [F] time = 5.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{q+px^5}\,\left(-2q+3px^5\right)\left(aq+bx^2+apx^5\right)}{x^4\left(cq+dx^2+cpx^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5)*(a*q + b*x^2 + a*p*x^5))/(x^4*(c*q +
d*x^2 + c*p*x^5)),x]
```

```
[Out] (2*a*q*Sqrt[q + p*x^5])/(3*c*x^3) + (2*(b*c - a*d)*Sqrt[q + p*x^5])/(c^2*x)
 + (2*a*p*x^2*Sqrt[q + p*x^5])/(3*c) - (5*(b*c - a*d)*p*x^4*Sqrt[1 + (p*x^5
)/q]*Hypergeometric2F1[1/2, 4/5, 9/5, -((p*x^5)/q)])/(4*c^2*Sqrt[q + p*x^5]
) + (2*d*(b*c - a*d)*Defer[Int][Sqrt[q + p*x^5]/(c*q + d*x^2 + c*p*x^5), x]
)/c^2 + (5*(b*c - a*d)*p*Defer[Int][(x^3*Sqrt[q + p*x^5])/(c*q + d*x^2 + c*
p*x^5), x])/c
```

Rubi steps

$$\int \frac{\sqrt{q+px^5}\,\left(-2q+3px^5\right)\left(aq+bx^2+apx^5\right)}{x^4\left(cq+dx^2+cpx^5\right)}\,dx=\int\left(-\frac{2aq\sqrt{q+px^5}}{cx^4}-\frac{2(bc-ad)\sqrt{q+px^5}}{c^2x^2}+\frac{3apx\sqrt{q+px}\ldots}{c}\right.$$

$$=\frac{(bc-ad)\int\frac{\left(2d+5cpx^3\right)\sqrt{q+px^5}}{cq+dx^2+cpx^5}\,dx}{c^2}-\frac{(2(bc-ad))\int\frac{\sqrt{q+px^5}}{x^2}\,dx}{c^2}\ldots$$

$$=\frac{2aq\sqrt{q+px^5}}{3cx^3}+\frac{2(bc-ad)\sqrt{q+px^5}}{c^2x}+\frac{2apx^2\sqrt{q+px^5}}{3c}+\ldots$$

$$=\frac{2aq\sqrt{q+px^5}}{3cx^3}+\frac{2(bc-ad)\sqrt{q+px^5}}{c^2x}+\frac{2apx^2\sqrt{q+px^5}}{3c}+\ldots$$

$$=\frac{2aq\sqrt{q+px^5}}{3cx^3}+\frac{2(bc-ad)\sqrt{q+px^5}}{c^2x}+\frac{2apx^2\sqrt{q+px^5}}{3c}-\ldots$$

Mathematica [F] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{q+px^5}\,\left(-2q+3px^5\right)\left(aq+bx^2+apx^5\right)}{x^4\left(cq+dx^2+cpx^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5)*(a*q + b*x^2 + a*p*x^5))/(x^4*(
c*q + d*x^2 + c*p*x^5)),x]
```

```
[Out] Integrate[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5)*(a*q + b*x^2 + a*p*x^5))/(x^4*(
c*q + d*x^2 + c*p*x^5)), x]
```

IntegrateAlgebraic [A] time = 12.89, size = 96, normalized size = 1.00

$$\frac{2\left(bc\sqrt{d}-ad^{3/2}\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}\,\sqrt{px^5+q}}\right)}{c^{5/2}}+\frac{2\sqrt{px^5+q}\,\left(acpx^5+acq-3adx^2+3bcx^2\right)}{3c^2x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5)*(a*q + b*x^2 + a*p*x^5
))/(x^4*(c*q + d*x^2 + c*p*x^5)),x]
```

```
[Out] (2*Sqrt[q + p*x^5]*(a*c*q + 3*b*c*x^2 - 3*a*d*x^2 + a*c*p*x^5))/(3*c^2*x^3)
 + (2*(b*c*Sqrt[d] - a*d^(3/2))*ArcTan[(Sqrt[d]*x)/(Sqrt[c]*Sqrt[q + p*x^5]
)])/c^(5/2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^5+q)^(1/2)*(3*p*x^5-2*q)*(a*p*x^5+b*x^2+a*q)/x^4/(c*p*x^5+d*
x^2+c*q),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(apx^5+bx^2+aq\right)\left(3\,px^5-2\,q\right)\sqrt{px^5+q}}{\left(cpx^5+dx^2+cq\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^5+q)^(1/2)*(3*p*x^5-2*q)*(a*p*x^5+b*x^2+a*q)/x^4/(c*p*x^5+d*
x^2+c*q),x, algorithm="giac")
```

```
[Out] integrate((a*p*x^5 + b*x^2 + a*q)*(3*p*x^5 - 2*q)*sqrt(p*x^5 + q)/((c*p*x^5
 + d*x^2 + c*q)*x^4), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p\,x^5+q}\,\left(3p\,x^5-2q\right)\left(ap\,x^5+b\,x^2+aq\right)}{x^4\left(cp\,x^5+d\,x^2+cq\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^5+q)^(1/2)*(3*p*x^5-2*q)*(a*p*x^5+b*x^2+a*q)/x^4/(c*p*x^5+d*x^2+c*
q),x)
```

```
[Out] int((p*x^5+q)^(1/2)*(3*p*x^5-2*q)*(a*p*x^5+b*x^2+a*q)/x^4/(c*p*x^5+d*x^2+c*
q),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(apx^5+bx^2+aq\right)\left(3\,px^5-2\,q\right)\sqrt{px^5+q}}{\left(cpx^5+dx^2+cq\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^5+q)^(1/2)*(3*p*x^5-2*q)*(a*p*x^5+b*x^2+a*q)/x^4/(c*p*x^5+d*
x^2+c*q),x, algorithm="maxima")
```

```
[Out] integrate((a*p*x^5 + b*x^2 + a*q)*(3*p*x^5 - 2*q)*sqrt(p*x^5 + q)/((c*p*x^5
 + d*x^2 + c*q)*x^4), x)
```

mupad [B] time = 8.38, size = 184, normalized size = 1.92

$$\frac{2\,a\,{\left(p\,x^5+q\right)}^{3/2}}{3\,c\,x^3}+\frac{2\,b\,\sqrt{p\,x^5+q}}{c\,x}-\frac{2\,a\,d\,\sqrt{p\,x^5+q}}{c^2\,x}+\frac{a\,d^{3/2}\,\ln\left(\frac{c\,q-d\,x^2+c\,p\,x^5+\sqrt{c}\,\sqrt{d}\,x\,\sqrt{p\,x^5+q}\,2{}\mathrm{i}}{c\,p\,x^5+d\,x^2+c\,q}\right)\,1{}\mathrm{i}}{c^{5/2}}-\frac{b\,\sqrt{d}\,\ln\left(\frac{c\,q-d\,x^2+c\,p\,x^5+\sqrt{c}\,\sqrt{d}\,x\,\sqrt{p\,x^5+q}\,2{}\mathrm{i}}{c\,p\,x^5+d\,x^2+c\,q}\right)\,1{}\mathrm{i}}{c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q + p*x^5)^(1/2)*(2*q - 3*p*x^5)*(a*q + b*x^2 + a*p*x^5))/(x^4*(c*q
+ d*x^2 + c*p*x^5)),x)
```

```
[Out] (a*d^(3/2)*log((c*q - d*x^2 + c*p*x^5 + c^(1/2)*d^(1/2)*x*(q + p*x^5)^(1/2)
*2i)/(c*q + d*x^2 + c*p*x^5))*1i)/c^(5/2) - (b*d^(1/2)*log((c*q - d*x^2 + c
*p*x^5 + c^(1/2)*d^(1/2)*x*(q + p*x^5)^(1/2)*2i)/(c*q + d*x^2 + c*p*x^5))*1
i)/c^(3/2) + (2*a*(q + p*x^5)^(3/2))/(3*c*x^3) + (2*b*(q + p*x^5)^(1/2))/(c
*x) - (2*a*d*(q + p*x^5)^(1/2))/(c^2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^5+q}\,\left(3px^5-2q\right)\left(apx^5+aq+bx^2\right)}{x^4\left(cpx^5+cq+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**5+q)**(1/2)*(3*p*x**5-2*q)*(a*p*x**5+b*x**2+a*q)/x**4/(c*p*
x**5+d*x**2+c*q),x)
```

```
[Out] Integral(sqrt(p*x**5 + q)*(3*p*x**5 - 2*q)*(a*p*x**5 + a*q + b*x**2)/(x**4*
(c*p*x**5 + c*q + d*x**2)), x)
```

3.1190 $\int \frac{x}{\sqrt[3]{x^2+x^6}}\,dx$

Optimal. Leaf size=96

$$-\frac{1}{4}\log\left(\sqrt[3]{x^6+x^2}-x^2\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x^2}{x^2+2\sqrt[3]{x^6+x^2}}\right)+\frac{1}{8}\log\left(x^4+\sqrt[3]{x^6+x^2}\,x^2+\left(x^6+x^2\right)^{2/3}\right)$$

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2013, 2011, 329, 275, 239}

$$\frac{\sqrt{3}\sqrt[3]{x^2}\sqrt[3]{x^4+1}\tan^{-1}\left(\frac{\frac{2\left(x^2\right)^{2/3}}{\sqrt[3]{x^4+1}}+1}{\sqrt{3}}\right)}{4\sqrt[3]{x^6+x^2}}-\frac{3\sqrt[3]{x^2}\sqrt[3]{x^4+1}\log\left(\left(x^2\right)^{2/3}-\sqrt[3]{x^4+1}\right)}{8\sqrt[3]{x^6+x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(x^2 + x^6)^(1/3),x]
```

```
[Out] (Sqrt[3]*(x^2)^(1/3)*(1 + x^4)^(1/3)*ArcTan[(1 + (2*(x^2)^(2/3))/(1 + x^4)^
(1/3))/Sqrt[3]])/(4*(x^2 + x^6)^(1/3)) - (3*(x^2)^(1/3)*(1 + x^4)^(1/3)*Log
[(x^2)^(2/3) - (1 + x^4)^(1/3)])/(8*(x^2 + x^6)^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{x^2+x^6}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{x+x^3}}\,dx, x, x^2\right)$$
$$= \frac{\left(\sqrt[3]{x^2}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\sqrt[3]{1+x^2}}\,dx, x, x^2\right)}{2\sqrt[3]{x^2+x^6}}$$
$$= \frac{\left(3\sqrt[3]{x^2}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x^2}\right)}{2\sqrt[3]{x^2+x^6}}$$
$$= \frac{\left(3\sqrt[3]{x^2}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^3}}\,dx, x, \left(x^2\right)^{2/3}\right)}{4\sqrt[3]{x^2+x^6}}$$
$$= \frac{\sqrt{3}\sqrt[3]{x^2}\sqrt[3]{1+x^4}\tan^{-1}\left(\frac{1+\frac{2\left(x^2\right)^{2/3}}{\sqrt[3]{1+x^4}}}{\sqrt{3}}\right)}{4\sqrt[3]{x^2+x^6}} - \frac{3\sqrt[3]{x^2}\sqrt[3]{1+x^4}\log\left(\left(x^2\right)^{2/3}-\sqrt[3]{1+x^4}\right)}{8\sqrt[3]{x^2+x^6}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.41

$$\frac{3\left(x^6+x^2\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-x^4\right)}{4\left(x^4+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x^2 + x^6)^(1/3),x]
```

```
[Out] (3*(x^2 + x^6)^(2/3)*Hypergeometric2F1[1/3, 1/3, 4/3, -x^4])/(4*(1 + x^4)^(
2/3))
```

IntegrateAlgebraic [A] time = 0.32, size = 96, normalized size = 1.00

$$-\frac{1}{4}\log\left(\sqrt[3]{x^6+x^2}-x^2\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x^2}{x^2+2\sqrt[3]{x^6+x^2}}\right)+\frac{1}{8}\log\left(x^4+\sqrt[3]{x^6+x^2}\,x^2+\left(x^6+x^2\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(x^2 + x^6)^(1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(x^2 + x^6)^(1/3))])/4 - Log[-x^2 +
(x^2 + x^6)^(1/3)]/4 + Log[x^4 + x^2*(x^2 + x^6)^(1/3) + (x^2 + x^6)^(2/3)]
/8
```

fricas [A] time = 0.94, size = 92, normalized size = 0.96

$$\frac{1}{4}\sqrt{3}\arctan\left(-\frac{196\sqrt{3}\left(x^6+x^2\right)^{\frac{1}{3}}x^2-\sqrt{3}\left(539x^4+507\right)-1274\sqrt{3}\left(x^6+x^2\right)^{\frac{2}{3}}}{2205x^4+2197}\right)-\frac{1}{8}\log\left(3\left(x^6+x^2\right)^{\frac{1}{3}}x^2-3\left(x^6+x^2\right)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*arctan(-(196*sqrt(3)*(x^6 + x^2)^(1/3)*x^2 - sqrt(3)*(539*x^4 +
 507) - 1274*sqrt(3)*(x^6 + x^2)^(2/3))/(2205*x^4 + 2197)) - 1/8*log(3*(x^6
 + x^2)^(1/3)*x^2 - 3*(x^6 + x^2)^(2/3) + 1)
```

giac [A] time = 0.16, size = 55, normalized size = 0.57

$$-\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x^4}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{8}\log\left(\left(\frac{1}{x^4}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x^4}+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{4}\log\left(\left|\left(\frac{1}{x^4}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(1/x^4 + 1)^(1/3) + 1)) + 1/8*log((1/x^4
 + 1)^(2/3) + (1/x^4 + 1)^(1/3) + 1) - 1/4*log(abs((1/x^4 + 1)^(1/3) - 1))
```

maple [C] time = 0.36, size = 17, normalized size = 0.18

$$\frac{3x^{\frac{4}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6+x^2)^(1/3),x)
```

```
[Out] 3/4*x^(4/3)*hypergeom([1/3,1/3],[4/3],-x^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^6+x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^6 + x^2)^(1/3), x)
```

mupad [B] time = 0.94, size = 31, normalized size = 0.32

$$\frac{3\,x^2\left(x^4+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ -x^4\right)}{4\left(x^6+x^2\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 + x^6)^(1/3),x)
```

```
[Out] (3*x^2*(x^4 + 1)^(1/3)*hypergeom([1/3, 1/3], 4/3, -x^4))/(4*(x^2 + x^6)^(1/
3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{x^2\left(x^4+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6+x**2)**(1/3),x)
```

```
[Out] Integral(x/(x**2*(x**4 + 1))**(1/3), x)
```

3.1191 $\int \frac{-1+x^4}{\left(1+x^2+x^4\right)\sqrt[4]{x^2+x^6}}\,dx$

Optimal. Leaf size=96

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6+x^2}}{\sqrt{x^6+x^2}-x^2}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt{2}}+\frac{\sqrt{x^6+x^2}}{\sqrt{2}}}{x\sqrt[4]{x^6+x^2}}\right)}{\sqrt{2}}$$

Rubi [C] time = 1.07, antiderivative size = 319, normalized size of antiderivative = 3.32, number of steps used = 15, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2056, 6715, 6728, 245, 1438, 429, 510}

$$-\frac{2\sqrt[4]{x^4+1}\,xF_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};-\frac{2x^4}{1-i\sqrt{3}},-x^4\right)}{\sqrt[4]{x^6+x^2}}-\frac{2\sqrt[4]{x^4+1}\,xF_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};-\frac{2x^4}{1+i\sqrt{3}},-x^4\right)}{\sqrt[4]{x^6+x^2}}-\frac{2\left(-\sqrt{3}+i\right)\sqrt[4]{x^4+1}\,x^3F_1\left(\frac{5}{8};\frac{1}{4},1;\frac{13}{8};-x^4,-\frac{2x^4}{1-i\sqrt{3}}\right)}{5\left(\sqrt{3}+i\right)\sqrt[4]{x^6+x^2}}-\frac{2\left(\sqrt{3}+i\right)\sqrt[4]{x^4+1}\,x^3F_1\left(\frac{5}{8};\frac{1}{4},1;\frac{13}{8};-x^4,-\frac{2x^4}{1+i\sqrt{3}}\right)}{5\left(-\sqrt{3}+i\right)\sqrt[4]{x^6+x^2}}+\frac{2\sqrt[4]{x^4+1}\,x\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^4)/((1 + x^2 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, (-2*x^4)/(1 - I*Sqrt[3]),
-x^4])/(x^2 + x^6)^(1/4) - (2*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8,
(-2*x^4)/(1 + I*Sqrt[3]), -x^4])/(x^2 + x^6)^(1/4) - (2*(I - Sqrt[3])*x^3*(
1 + x^4)^(1/4)*AppellF1[5/8, 1/4, 1, 13/8, -x^4, (-2*x^4)/(1 - I*Sqrt[3])])
/(5*(I + Sqrt[3])*(x^2 + x^6)^(1/4)) - (2*(I + Sqrt[3])*x^3*(1 + x^4)^(1/4)
*AppellF1[5/8, 1/4, 1, 13/8, -x^4, (-2*x^4)/(1 + I*Sqrt[3])])/(5*(I - Sqrt[
3])*(x^2 + x^6)^(1/4)) + (2*x*(1 + x^4)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9
/8, -x^4])/(x^2 + x^6)^(1/4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-1+x^4}{\left(1+x^2+x^4\right)\sqrt[4]{x^2+x^6}}\,dx = \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^4}\right)\int \frac{-1+x^4}{\sqrt{x}\,\sqrt[4]{1+x^4}\left(1+x^2+x^4\right)}\,dx}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{-1+x^8}{\sqrt[4]{1+x^8}\left(1+x^4+x^8\right)}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[4]{1+x^8}}-\frac{2+x^4}{\sqrt[4]{1+x^8}\left(1+x^4+x^8\right)}\right)dx,x,\sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{2+x^4}{\sqrt[4]{1+x^8}\left(1+\cdots\right.}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int\left(\frac{1-i\sqrt{3}}{\left(1-i\sqrt{3}+2x^4\right)\sqrt[4]{1+x^8}}+\frac{}{(1\cdots}\right.\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(2\left(1-i\sqrt{3}\right)\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(1-i\sqrt{3}+2x^4\right)\sqrt[4]{\cdots}}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(2\left(1-i\sqrt{3}\right)\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int\left(\frac{i+\sqrt{3}\cdots}{2\left(-i+\sqrt{3}-2ix\cdots\right.}\right.\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(2\left(1-i\sqrt{3}\right)\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{1+x^8}\left(1+i\sqrt{3}\cdots\right.}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= -\frac{2x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};-\frac{2x^4}{1-i\sqrt{3}},-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{2x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};-\frac{2x^4}{1+i\sqrt{3}},-x^4\right)}{\sqrt[4]{x^2+x^6}}$$

Mathematica [F] time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{-1+x^4}{\left(1+x^2+x^4\right)\sqrt[4]{x^2+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^4)/((1 + x^2 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] Integrate[(-1 + x^4)/((1 + x^2 + x^4)*(x^2 + x^6)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.70, size = 96, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\,\sqrt[4]{x^6+x^2}}{\sqrt{x^6+x^2}-x^2}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt{2}}+\frac{\sqrt{x^6+x^2}}{\sqrt{2}}}{x\sqrt[4]{x^6+x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/((1 + x^2 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] -(ArcTan[(Sqrt[2]*x*(x^2 + x^6)^(1/4))/(-x^2 + Sqrt[x^2 + x^6])]/Sqrt[2]) -
 ArcTanh[(x^2/Sqrt[2] + Sqrt[x^2 + x^6]/Sqrt[2])/(x*(x^2 + x^6)^(1/4))]/Sqr
t[2]
```

fricas [B] time = 60.58, size = 678, normalized size = 7.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+x^2+1)/(x^6+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan((x^9 + 2*x^7 + 3*x^5 + 2*x^3 + 2*sqrt(2)*(x^6 + x^2)^(3
/4)*(x^4 - 3*x^2 + 1) + 2*sqrt(2)*(3*x^6 - x^4 + 3*x^2)*(x^6 + x^2)^(1/4) +
 4*sqrt(x^6 + x^2)*(x^5 + x^3 + x) + (16*(x^6 + x^2)^(3/4)*x^2 + 2*sqrt(2)*
sqrt(x^6 + x^2)*(x^5 - 3*x^3 + x) + sqrt(2)*(x^9 - 8*x^7 + x^5 - 8*x^3 + x)
 + 4*(x^6 + x^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt((x^5 + x^3 + 2*sqrt(2)*(x^6
+ x^2)^(1/4)*x^2 + 4*sqrt(x^6 + x^2)*x + 2*sqrt(2)*(x^6 + x^2)^(3/4) + x)/(
x^5 + x^3 + x)) + x)/(x^9 - 14*x^7 + 3*x^5 - 14*x^3 + x)) + 1/2*sqrt(2)*arc
tan((x^9 + 2*x^7 + 3*x^5 + 2*x^3 - 2*sqrt(2)*(x^6 + x^2)^(3/4)*(x^4 - 3*x^2
 + 1) - 2*sqrt(2)*(3*x^6 - x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 4*sqrt(x^6 + x^
2)*(x^5 + x^3 + x) + (16*(x^6 + x^2)^(3/4)*x^2 - 2*sqrt(2)*sqrt(x^6 + x^2)*
(x^5 - 3*x^3 + x) - sqrt(2)*(x^9 - 8*x^7 + x^5 - 8*x^3 + x) + 4*(x^6 + x^4
+ x^2)*(x^6 + x^2)^(1/4))*sqrt((x^5 + x^3 - 2*sqrt(2)*(x^6 + x^2)^(1/4)*x^2
 + 4*sqrt(x^6 + x^2)*x - 2*sqrt(2)*(x^6 + x^2)^(3/4) + x)/(x^5 + x^3 + x))
+ x)/(x^9 - 14*x^7 + 3*x^5 - 14*x^3 + x)) - 1/8*sqrt(2)*log(4*(x^5 + x^3 +
2*sqrt(2)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(x^6 + x^2)*x + 2*sqrt(2)*(x^6 + x^
2)^(3/4) + x)/(x^5 + x^3 + x)) + 1/8*sqrt(2)*log(4*(x^5 + x^3 - 2*sqrt(2)*(
x^6 + x^2)^(1/4)*x^2 + 4*sqrt(x^6 + x^2)*x - 2*sqrt(2)*(x^6 + x^2)^(3/4) +
x)/(x^5 + x^3 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+x^2+1)/(x^6+x^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 1)/((x^6 + x^2)^(1/4)*(x^4 + x^2 + 1)), x)
```

maple [C] time = 5.06, size = 238, normalized size = 2.48

$$\frac{\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{2\sqrt{x^6+x^2}\,\mathrm{RootOf}\left(\_Z^4+1\right)^3x-\mathrm{RootOf}\left(\_Z^4+1\right)x^5-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^2\left(x^6+x^2\right)^{\frac{1}{4}}x^2+\mathrm{RootOf}\left(\_Z^4+1\right)x^3+2\left(x^6+x^2\right)^{\frac{3}{4}}-\mathrm{RootOf}\left(\_Z^4+1\right)x}{x\left(x^2+x+1\right)\left(x^2-x+1\right)}\right)}{2}+\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^3x^5-\mathrm{RootOf}\left(\_Z^4+1\right)^3x^3-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^2\left(x^6+x^2\right)^{\frac{1}{4}}x^2+\mathrm{RootOf}\left(\_Z^4+1\right)^3x-2\sqrt{x^6+x^2}\,\mathrm{RootOf}\left(\_Z^4+1\right)x-2\left(x^6+x^2\right)^{\frac{3}{4}}}{x\left(x^2+x+1\right)\left(x^2-x+1\right)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/(x^4+x^2+1)/(x^6+x^2)^(1/4),x)
```

```
[Out] 1/2*RootOf(_Z^4+1)*ln(-(2*(x^6+x^2)^(1/2)*RootOf(_Z^4+1)^3*x-RootOf(_Z^4+1)
*x^5-2*RootOf(_Z^4+1)^2*(x^6+x^2)^(1/4)*x^2+RootOf(_Z^4+1)*x^3+2*(x^6+x^2)^
(3/4)-RootOf(_Z^4+1)*x)/x/(x^2+x+1)/(x^2-x+1))+1/2*RootOf(_Z^4+1)^3*ln((Roo
tOf(_Z^4+1)^3*x^5-RootOf(_Z^4+1)^3*x^3-2*RootOf(_Z^4+1)^2*(x^6+x^2)^(1/4)*x
^2+RootOf(_Z^4+1)^3*x-2*(x^6+x^2)^(1/2)*RootOf(_Z^4+1)*x-2*(x^6+x^2)^(3/4))
/x/(x^2+x+1)/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+x^2+1)/(x^6+x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 1)/((x^6 + x^2)^(1/4)*(x^4 + x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4-1}{\left(x^6+x^2\right)^{1/4}\,\left(x^4+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/((x^2 + x^6)^(1/4)*(x^2 + x^4 + 1)),x)
```

```
[Out] int((x^4 - 1)/((x^2 + x^6)^(1/4)*(x^2 + x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)\left(x^2+1\right)}{\sqrt[4]{x^2\left(x^4+1\right)}\,\left(x^2-x+1\right)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/(x**4+x**2+1)/(x**6+x**2)**(1/4),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)/((x**2*(x**4 + 1))**(1/4)*(x**2 - x + 1
)*(x**2 + x + 1)), x)
```

3.1192 $\int \frac{-1+x^4}{(1+x^2+x^4)\sqrt[4]{x^2+x^6}}\,dx$

Optimal. Leaf size=96

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6+x^2}}{\sqrt{x^6+x^2}-x^2}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt{2}}+\frac{\sqrt{x^6+x^2}}{\sqrt{2}}}{x\sqrt[4]{x^6+x^2}}\right)}{\sqrt{2}}$$

Rubi [C] time = 0.88, antiderivative size = 319, normalized size of antiderivative = 3.32, number of steps used = 15, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2056, 6715, 6728, 245, 1438, 429, 510}

$$-\frac{2\sqrt[4]{x^4+1}\,xF_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};-\frac{2x^4}{1-i\sqrt{3}},-x^4\right)}{\sqrt[4]{x^6+x^2}}-\frac{2\sqrt[4]{x^4+1}\,xF_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};-\frac{2x^4}{1+i\sqrt{3}},-x^4\right)}{\sqrt[4]{x^6+x^2}}-\frac{2\left(-\sqrt{3}+i\right)\sqrt[4]{x^4+1}\,x^3F_1\left(\frac{5}{8};\frac{1}{4},1;\frac{13}{8};-x^4,-\frac{2x^4}{1-i\sqrt{3}}\right)}{5\left(\sqrt{3}+i\right)\sqrt[4]{x^6+x^2}}-\frac{2\left(\sqrt{3}+i\right)\sqrt[4]{x^4+1}\,x^3F_1\left(\frac{5}{8};\frac{1}{4},1;\frac{13}{8};-x^4,-\frac{2x^4}{1+i\sqrt{3}}\right)}{5\left(-\sqrt{3}+i\right)\sqrt[4]{x^6+x^2}}+\frac{2\sqrt[4]{x^4+1}\,x\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^4)/((1 + x^2 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, (-2*x^4)/(1 - I*Sqrt[3]),
-x^4])/(x^2 + x^6)^(1/4) - (2*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8,
(-2*x^4)/(1 + I*Sqrt[3]), -x^4])/(x^2 + x^6)^(1/4) - (2*(I - Sqrt[3])*x^3*(
1 + x^4)^(1/4)*AppellF1[5/8, 1/4, 1, 13/8, -x^4, (-2*x^4)/(1 - I*Sqrt[3])])
/(5*(I + Sqrt[3])*(x^2 + x^6)^(1/4)) - (2*(I + Sqrt[3])*x^3*(1 + x^4)^(1/4)
*AppellF1[5/8, 1/4, 1, 13/8, -x^4, (-2*x^4)/(1 + I*Sqrt[3])])/(5*(I - Sqrt[
3])*(x^2 + x^6)^(1/4)) + (2*x*(1 + x^4)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9
/8, -x^4])/(x^2 + x^6)^(1/4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-1+x^4}{\left(1+x^2+x^4\right)\sqrt[4]{x^2+x^6}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{-1+x^4}{\sqrt{x}\sqrt[4]{1+x^4}\left(1+x^2+x^4\right)}\,dx}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{-1+x^8}{\sqrt[4]{1+x^8}\left(1+x^4+x^8\right)}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[4]{1+x^8}} - \frac{2+x^4}{\sqrt[4]{1+x^8}\left(1+x^4+x^8\right)}\right)dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{2+x^4}{\sqrt[4]{1+x^8}\left(1+\right.}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int\left(\frac{1-i\sqrt{3}}{\left(1-i\sqrt{3}+2x^4\right)\sqrt[4]{1+x^8}} + \frac{}{(1}\right.\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(2\left(1-i\sqrt{3}\right)\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(1-i\sqrt{3}+2x^4\right)\sqrt[4]{}}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(2\left(1-i\sqrt{3}\right)\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int\left(\frac{i+\sqrt{3}}{2\left(-i+\sqrt{3}-2ix\right.}\right.\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(2\left(1-i\sqrt{3}\right)\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{1+x^8}\left(1+i\sqrt{3}\right.}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= -\frac{2x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};-\frac{2x^4}{1-i\sqrt{3}},-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{2x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};-\frac{2x^4}{1+i\sqrt{3}},-x^4\right)}{\sqrt[4]{x^2+x^6}}$$

Mathematica [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{-1+x^4}{\left(1+x^2+x^4\right)\sqrt[4]{x^2+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^4)/((1 + x^2 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] Integrate[(-1 + x^4)/((1 + x^2 + x^4)*(x^2 + x^6)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.00, size = 96, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6+x^2}}{\sqrt{x^6+x^2}-x^2}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt{2}}+\frac{\sqrt{x^6+x^2}}{\sqrt{2}}}{x\sqrt[4]{x^6+x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/((1 + x^2 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] -(ArcTan[(Sqrt[2]*x*(x^2 + x^6)^(1/4))/(-x^2 + Sqrt[x^2 + x^6])]/Sqrt[2]) -
 ArcTanh[(x^2/Sqrt[2] + Sqrt[x^2 + x^6]/Sqrt[2])/(x*(x^2 + x^6)^(1/4))]/Sqr
t[2]
```

fricas [B] time = 58.13, size = 678, normalized size = 7.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+x^2+1)/(x^6+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan((x^9 + 2*x^7 + 3*x^5 + 2*x^3 + 2*sqrt(2)*(x^6 + x^2)^(3
/4)*(x^4 - 3*x^2 + 1) + 2*sqrt(2)*(3*x^6 - x^4 + 3*x^2)*(x^6 + x^2)^(1/4) +
 4*sqrt(x^6 + x^2)*(x^5 + x^3 + x) + (16*(x^6 + x^2)^(3/4)*x^2 + 2*sqrt(2)*
sqrt(x^6 + x^2)*(x^5 - 3*x^3 + x) + sqrt(2)*(x^9 - 8*x^7 + x^5 - 8*x^3 + x)
 + 4*(x^6 + x^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt((x^5 + x^3 + 2*sqrt(2)*(x^6
+ x^2)^(1/4)*x^2 + 4*sqrt(x^6 + x^2)*x + 2*sqrt(2)*(x^6 + x^2)^(3/4) + x)/(
x^5 + x^3 + x)) + x)/(x^9 - 14*x^7 + 3*x^5 - 14*x^3 + x)) + 1/2*sqrt(2)*arc
tan((x^9 + 2*x^7 + 3*x^5 + 2*x^3 - 2*sqrt(2)*(x^6 + x^2)^(3/4)*(x^4 - 3*x^2
 + 1) - 2*sqrt(2)*(3*x^6 - x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 4*sqrt(x^6 + x^
2)*(x^5 + x^3 + x) + (16*(x^6 + x^2)^(3/4)*x^2 - 2*sqrt(2)*sqrt(x^6 + x^2)*
(x^5 - 3*x^3 + x) - sqrt(2)*(x^9 - 8*x^7 + x^5 - 8*x^3 + x) + 4*(x^6 + x^4
+ x^2)*(x^6 + x^2)^(1/4))*sqrt((x^5 + x^3 - 2*sqrt(2)*(x^6 + x^2)^(1/4)*x^2
 + 4*sqrt(x^6 + x^2)*x - 2*sqrt(2)*(x^6 + x^2)^(3/4) + x)/(x^5 + x^3 + x))
+ x)/(x^9 - 14*x^7 + 3*x^5 - 14*x^3 + x)) - 1/8*sqrt(2)*log(4*(x^5 + x^3 +
2*sqrt(2)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(x^6 + x^2)*x + 2*sqrt(2)*(x^6 + x^
2)^(3/4) + x)/(x^5 + x^3 + x)) + 1/8*sqrt(2)*log(4*(x^5 + x^3 - 2*sqrt(2)*(
x^6 + x^2)^(1/4)*x^2 + 4*sqrt(x^6 + x^2)*x - 2*sqrt(2)*(x^6 + x^2)^(3/4) +
x)/(x^5 + x^3 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+x^2+1)/(x^6+x^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 1)/((x^6 + x^2)^(1/4)*(x^4 + x^2 + 1)), x)
```

maple [C] time = 4.61, size = 240, normalized size = 2.50

$$\frac{\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{2\sqrt{x^6+x^2}\,\mathrm{RootOf}\left(\_Z^4+1\right)^3x-\mathrm{RootOf}\left(\_Z^4+1\right)x^5-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^2\left(x^6+x^2\right)^{\frac{1}{4}}x^2+\mathrm{RootOf}\left(\_Z^4+1\right)x^3+2\left(x^6+x^2\right)^{\frac{3}{4}}-\mathrm{RootOf}\left(\_Z^4+1\right)x}{x\left(x^2+x+1\right)\left(x^2-x+1\right)}\right)}{2}+\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^4+1\right)^3x^5+\mathrm{RootOf}\left(\_Z^4+1\right)^3x^3+2\,\mathrm{RootOf}\left(\_Z^4+1\right)^2\left(x^6+x^2\right)^{\frac{1}{4}}x^2+2\sqrt{x^6+x^2}\,\mathrm{RootOf}\left(\_Z^4+1\right)x-\mathrm{RootOf}\left(\_Z^4+1\right)^3x+2\left(x^6+x^2\right)^{\frac{3}{4}}}{x\left(x^2+x+1\right)\left(x^2-x+1\right)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/(x^4+x^2+1)/(x^6+x^2)^(1/4),x)
```

```
[Out] 1/2*RootOf(_Z^4+1)*ln(-(2*(x^6+x^2)^(1/2)*RootOf(_Z^4+1)^3*x-RootOf(_Z^4+1)
*x^5-2*RootOf(_Z^4+1)^2*(x^6+x^2)^(1/4)*x^2+RootOf(_Z^4+1)*x^3+2*(x^6+x^2)^
(3/4)-RootOf(_Z^4+1)*x)/x/(x^2+x+1)/(x^2-x+1))+1/2*RootOf(_Z^4+1)^3*ln(-(-R
ootOf(_Z^4+1)^3*x^5+RootOf(_Z^4+1)^3*x^3+2*RootOf(_Z^4+1)^2*(x^6+x^2)^(1/4)
*x^2+2*(x^6+x^2)^(1/2)*RootOf(_Z^4+1)*x-RootOf(_Z^4+1)^3*x+2*(x^6+x^2)^(3/4
))/x/(x^2+x+1)/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+x^2+1)/(x^6+x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 1)/((x^6 + x^2)^(1/4)*(x^4 + x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4-1}{\left(x^6+x^2\right)^{1/4}\,\left(x^4+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/((x^2 + x^6)^(1/4)*(x^2 + x^4 + 1)),x)
```

```
[Out] int((x^4 - 1)/((x^2 + x^6)^(1/4)*(x^2 + x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)\left(x^2+1\right)}{\sqrt[4]{x^2\left(x^4+1\right)}\,\left(x^2-x+1\right)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/(x**4+x**2+1)/(x**6+x**2)**(1/4),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)/((x**2*(x**4 + 1))**(1/4)*(x**2 - x + 1
)*(x**2 + x + 1)), x)
```

3.1193 $\int \frac{x^2(2b+ax^6)}{(-b+ax^6)^{3/4}(-b-2cx^4+ax^6)}\,dx$

Optimal. Leaf size=96

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x(ax^6-b)^{3/4}}{b-ax^6}\right)}{2^{3/4}c^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x(ax^6-b)^{3/4}}{b-ax^6}\right)}{2^{3/4}c^{3/4}}$$

Rubi [F] time = 2.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(2b+ax^6\right)}{\left(-b+ax^6\right)^{3/4}\left(-b-2cx^4+ax^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(2*b + a*x^6))/((-b + a*x^6)^(3/4)*(-b - 2*c*x^4 + a*x^6)),x]
```

```
[Out] (Sqrt[(a*x^6)/(Sqrt[b] + Sqrt[-b + a*x^6])^2]*(Sqrt[b] + Sqrt[-b + a*x^6])*
EllipticF[2*ArcTan[(-b + a*x^6)^(1/4)/b^(1/4)], 1/2])/(3*a*b^(1/4)*x^3) + (
2*c*x*(1 - (a*x^6)/b)^(3/4)*Hypergeometric2F1[1/6, 3/4, 7/6, (a*x^6)/b])/(a
*(-b + a*x^6)^(3/4)) - (2*b*c*Defer[Int][1/((b + 2*c*x^4 - a*x^6)*(-b + a*x
^6)^(3/4)), x])/a - (4*c^2*Defer[Int][x^4/((b + 2*c*x^4 - a*x^6)*(-b + a*x^
6)^(3/4)), x])/a + 3*b*Defer[Int][x^2/((-b + a*x^6)^(3/4)*(-b - 2*c*x^4 + a
*x^6)), x]
```

Rubi steps

$$\int \frac{x^2\left(2b+ax^6\right)}{\left(-b+ax^6\right)^{3/4}\left(-b-2cx^4+ax^6\right)}\,dx=\int \left(\frac{2c}{a\left(-b+ax^6\right)^{3/4}}+\frac{x^2}{\left(-b+ax^6\right)^{3/4}}+\frac{2bc+3abx^2+4c^2x^4}{a\left(-b+ax^6\right)^{3/4}\left(-b-2cx^4+a\right.}\right.$$

$$=\frac{\int \frac{2bc+3abx^2+4c^2x^4}{\left(-b+ax^6\right)^{3/4}\left(-b-2cx^4+ax^6\right)}\,dx}{a}+\frac{(2c)\int \frac{1}{\left(-b+ax^6\right)^{3/4}}\,dx}{a}+\int \frac{x^2}{\left(-b+ax^6\right)^{3/4}}$$

$$=\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\left(-b+ax^2\right)^{3/4}}\,dx,x,x^3\right)+\frac{\int\left(-\frac{2bc}{\left(b+2cx^4-ax^6\right)\left(-b+ax^6\right)^{3/4}}-\frac{}{(b}\right.}{}$$

$$=\frac{2cx\left(1-\frac{ax^6}{b}\right)^{3/4}{}_2F_1\left(\frac{1}{6},\frac{3}{4};\frac{7}{6};\frac{ax^6}{b}\right)}{a\left(-b+ax^6\right)^{3/4}}+(3b)\int \frac{x^2}{\left(-b+ax^6\right)^{3/4}\left(-b-2cx^4\right.}$$

$$=\frac{\sqrt{\frac{ax^6}{\left(\sqrt{b}+\sqrt{-b+ax^6}\right)^2}}\left(\sqrt{b}+\sqrt{-b+ax^6}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-b+ax^6}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3a\sqrt[4]{b}\,x^3}+\frac{2cx}{}$$

Mathematica [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(2b+ax^6\right)}{\left(-b+ax^6\right)^{3/4}\left(-b-2cx^4+ax^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(2*b + a*x^6))/((-b + a*x^6)^(3/4)*(-b - 2*c*x^4 + a*x^6)),x
]
```

```
[Out] Integrate[(x^2*(2*b + a*x^6))/((-b + a*x^6)^(3/4)*(-b - 2*c*x^4 + a*x^6)),
x]
```

IntegrateAlgebraic [A] time = 15.65, size = 96, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x\left(ax^6-b\right)^{3/4}}{b-ax^6}\right)}{2^{3/4}c^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x\left(ax^6-b\right)^{3/4}}{b-ax^6}\right)}{2^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(2*b + a*x^6))/((-b + a*x^6)^(3/4)*(-b - 2*c*x^4 +
a*x^6)),x]
```

```
[Out] -(ArcTan[(2^(1/4)*c^(1/4)*x*(-b + a*x^6)^(3/4))/(b - a*x^6)]/(2^(3/4)*c^(3/
4))) + ArcTanh[(2^(1/4)*c^(1/4)*x*(-b + a*x^6)^(3/4))/(b - a*x^6)]/(2^(3/4)
*c^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^6+2*b)/(a*x^6-b)^(3/4)/(a*x^6-2*c*x^4-b),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^6+2\,b\right)x^2}{\left(ax^6-2\,cx^4-b\right)\left(ax^6-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^6+2*b)/(a*x^6-b)^(3/4)/(a*x^6-2*c*x^4-b),x, algorithm="g
iac")
```

```
[Out] integrate((a*x^6 + 2*b)*x^2/((a*x^6 - 2*c*x^4 - b)*(a*x^6 - b)^(3/4)), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a\,x^6+2b\right)}{\left(a\,x^6-b\right)^{\frac{3}{4}}\left(a\,x^6-2c\,x^4-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^6+2*b)/(a*x^6-b)^(3/4)/(a*x^6-2*c*x^4-b),x)
```

```
[Out] int(x^2*(a*x^6+2*b)/(a*x^6-b)^(3/4)/(a*x^6-2*c*x^4-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^6+2\,b\right)x^2}{\left(ax^6-2\,cx^4-b\right)\left(ax^6-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^6+2*b)/(a*x^6-b)^(3/4)/(a*x^6-2*c*x^4-b),x, algorithm="m
axima")
```

```
[Out] integrate((a*x^6 + 2*b)*x^2/((a*x^6 - 2*c*x^4 - b)*(a*x^6 - b)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^2\,\left(a\,x^6+2\,b\right)}{{\left(a\,x^6-b\right)}^{3/4}\,\left(-a\,x^6+2\,c\,x^4+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(2*b + a*x^6))/((a*x^6 - b)^(3/4)*(b - a*x^6 + 2*c*x^4)),x)
```

```
[Out] int(-(x^2*(2*b + a*x^6))/((a*x^6 - b)^(3/4)*(b - a*x^6 + 2*c*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x**6+2*b)/(a*x**6-b)**(3/4)/(a*x**6-2*c*x**4-b),x)
```

```
[Out] Timed out
```

3.1194 $\int \frac{1+bx+k^2x^2}{\sqrt{(1-x)x(1-k^2x)}\left(-1+k^2x^2\right)}\,dx$

Optimal. Leaf size=97

$$\frac{(-b-2k)\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^3+(-k^2-1)x^2+x}}\right)}{2(k-1)k}+\frac{(b-2k)\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^3+(-k^2-1)x^2+x}}\right)}{2k(k+1)}$$

Rubi [C] time = 5.29, antiderivative size = 269, normalized size of antiderivative = 2.77, number of steps used = 12, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {6718, 6725, 115, 168, 538, 537}

$$\frac{\sqrt{1-x}\sqrt{x}(b+2k)\sqrt{\frac{k^2(1-x)}{1-k^2}+1}\,\Pi\left(-\frac{k}{1-k};\sin^{-1}\left(\sqrt{1-x}\right)|-\frac{k^2}{1-k^2}\right)}{(1-k)k\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{\sqrt{1-x}\sqrt{x}\left(2-\frac{b}{k}\right)\sqrt{\frac{k^2(1-x)}{1-k^2}+1}\,\Pi\left(\frac{k}{k+1};\sin^{-1}\left(\sqrt{1-x}\right)|-\frac{k^2}{1-k^2}\right)}{(k+1)\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^2*x^2)),x]
```

```
[Out] (2*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticF[ArcSin[Sqrt[x]], k^2])/Sqr
t[(1 - x)*x*(1 - k^2*x)] + ((b + 2*k)*Sqrt[1 + (k^2*(1 - x))/(1 - k^2)]*Sqr
t[1 - x]*Sqrt[x]*EllipticPi[-(k/(1 - k)), ArcSin[Sqrt[1 - x]], -(k^2/(1 - k
^2))])/((1 - k)*k*Sqrt[(1 - x)*x*(1 - k^2*x)]) + ((2 - b/k)*Sqrt[1 + (k^2*(
1 - x))/(1 - k^2)]*Sqrt[1 - x]*Sqrt[x]*EllipticPi[k/(1 + k), ArcSin[Sqrt[1
- x]], -(k^2/(1 - k^2))])/((1 + k)*Sqrt[(1 - x)*x*(1 - k^2*x)])
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+bx+k^2x^2}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(-1+k^2x^2\right)}\,dx = \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{1+bx+k^2x^2}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\left(-1+k^2x^2\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \left(\frac{1}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}}+\frac{2+bx}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\left(-1+k^2x^2\right)}\right) \cdots}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right) \cdots}{\sqrt{(1-\cdots}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \cdots}{\cdots}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(\left(-2-\frac{b}{k}\right)\sqrt{1-x}\,\sqrt{x}\,\sqrt{1 \cdots}\right.}{2\sqrt{(1-\cdots}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\left(-2-\frac{b}{k}\right)\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-\cdots}\right.}{\cdots}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\left(-2-\frac{b}{k}\right)\sqrt{1+\frac{k^2(-1+x)}{-1+k^2}} \cdots\right.}{\cdots}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{(b+2k)\sqrt{1+\frac{k^2(1-x)}{1-k^2}}\,\sqrt{1-\cdots}}{(1-k)\cdots}$$

Mathematica [C] time = 2.02, size = 169, normalized size = 1.74

$$\frac{i\sqrt{\frac{1}{x-1}+1}\,(x-1)^{3/2}\sqrt{\frac{1-\frac{1}{k^2}}{x-1}+1}\left(2k\left(b+k^2+1\right)F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)-(k-1)(b-2k)\Pi\left(1+\frac{1}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)-(k+1)(b+2k)\Pi\left(\frac{k-1}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)}{k\left(k^2-1\right)\sqrt{(x-1)x\left(k^2x-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b*x + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^2*x^2)),
x]
```

```
[Out] (I*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (1 - k^(-2))/(-1 + x)]*(-1 + x)^(3/2)*(
2*k*(1 + b + k^2)*EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] - (b - 2
*k)*(-1 + k)*EllipticPi[1 + k^(-1), I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)]
- (1 + k)*(b + 2*k)*EllipticPi[(-1 + k)/k, I*ArcSinh[1/Sqrt[-1 + x]], 1 - k
^(-2)]))/(k*(-1 + k^2)*Sqrt[(-1 + x)*x*(-1 + k^2*x)])
```

IntegrateAlgebraic [A] time = 0.23, size = 97, normalized size = 1.00

$$\frac{(-b-2k)\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^3+(-k^2-1)x^2+x}}\right)}{2(k-1)k}+\frac{(b-2k)\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^3+(-k^2-1)x^2+x}}\right)}{2k(k+1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + b*x + k^2*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k
^2*x^2)),x]
```

```
[Out] ((-b - 2*k)*ArcTan[((-1 + k)*x)/Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3]])/(2*(-1
 + k)*k) + ((b - 2*k)*ArcTan[((1 + k)*x)/Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3]
])/(2*k*(1 + k))
```

fricas [B] time = 0.73, size = 197, normalized size = 2.03

$$-\frac{\left((b+2)k-2\,k^2-b\right)\arctan\left(\frac{\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\left(k^2x^2-2\left(k^2+k+1\right)x+1\right)}{2\left(\left(k^3+k^2\right)x^3-\left(k^3+k^2+k+1\right)x^2+(k+1)x\right)}\right)-\left((b+2)k+2\,k^2+b\right)\arctan\left(\frac{\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\left(k^2x^2-2\left(k^2-k+1\right)x+1\right)}{2\left(\left(k^3-k^2\right)x^3-\left(k^3-k^2+k-1\right)x^2+(k-1)x\right)}\right)}{4\left(k^3-k\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2+b*x+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-1),x, algorit
hm="fricas")
```

```
[Out] -1/4*(((b + 2)*k - 2*k^2 - b)*arctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*
(k^2*x^2 - 2*(k^2 + k + 1)*x + 1)/((k^3 + k^2)*x^3 - (k^3 + k^2 + k + 1)*x^
2 + (k + 1)*x)) - ((b + 2)*k + 2*k^2 + b)*arctan(1/2*sqrt(k^2*x^3 - (k^2 +
1)*x^2 + x)*(k^2*x^2 - 2*(k^2 - k + 1)*x + 1)/((k^3 - k^2)*x^3 - (k^3 - k^2
 + k - 1)*x^2 + (k - 1)*x)))/(k^3 - k)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^2+bx+1}{\left(k^2x^2-1\right)\sqrt{\left(k^2x-1\right)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2+b*x+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-1),x, algorit
hm="giac")
```

```
[Out] integrate((k^2*x^2 + b*x + 1)/((k^2*x^2 - 1)*sqrt((k^2*x - 1)*(x - 1)*x)),
x)
```

maple [C] time = 0.03, size = 333, normalized size = 3.43

$$-\frac{2\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\;\mathrm{EllipticF}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^2\sqrt{k^2x^3-k^2x^2-x^2+x}}-\frac{(b+2k)\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\;\mathrm{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}-\frac{1}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^4\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}-\frac{1}{k}\right)}-\frac{(b-2k)\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\;\mathrm{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}+\frac{1}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^4\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}+\frac{1}{k}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^2+b*x+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-1),x)
```

```
[Out] -2/k^2*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x
^3-k^2*x^2-x^2+x)^(1/2)*EllipticF((-(x-1/k^2)*k^2)^(1/2),(1/k^2/(1/k^2-1))^
(1/2))-(b+2*k)/k^4*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^
(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2-1/k)*EllipticPi((-(x-1/k^2)*k^2)
^(1/2),1/k^2/(1/k^2-1/k),(1/k^2/(1/k^2-1))^(1/2))-(b-2*k)/k^4*(-(x-1/k^2)*k
^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1
/2)/(1/k^2+1/k)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+1/k),(1/k^2/
(1/k^2-1))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^2 + bx + 1}{\left(k^2x^2 - 1\right)\sqrt{\left(k^2x - 1\right)(x - 1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^2+b*x+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-1),x, algorit
hm="maxima")
```

```
[Out] integrate((k^2*x^2 + b*x + 1)/((k^2*x^2 - 1)*sqrt((k^2*x - 1)*(x - 1)*x)),
x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^2 + b*x + 1)/((k^2*x^2 - 1)*(x*(k^2*x - 1)*(x - 1))^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx + k^2x^2 + 1}{\sqrt{x\,(x-1)\left(k^2x-1\right)}\,(kx-1)\,(kx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k**2*x**2+b*x+1)/((1-x)*x*(-k**2*x+1))**(1/2)/(k**2*x**2-1),x)
```

```
[Out] Integral((b*x + k**2*x**2 + 1)/(sqrt(x*(x - 1)*(k**2*x - 1))*(k*x - 1)*(k*x
 + 1)), x)
```

3.1195 $\int \frac{\sqrt[3]{-1+x^3}}{x^7} dx$

Optimal. Leaf size=97

$$\frac{1}{27}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{1}{54}\log\left(\left(x^3-1\right)^{2/3}-\sqrt[3]{x^3-1}+1\right)-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\sqrt[3]{x^3-1}\left(x^3-3\right)}{18x^6}$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {266, 47, 51, 58, 618, 204, 31}

$$\frac{\sqrt[3]{x^3-1}}{18x^3}+\frac{1}{18}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{9\sqrt{3}}-\frac{\sqrt[3]{x^3-1}}{6x^6}-\frac{\log(x)}{18}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)^(1/3)/x^7,x]
```

```
[Out] -1/6*(-1 + x^3)^(1/3)/x^6 + (-1 + x^3)^(1/3)/(18*x^3) - ArcTan[(1 - 2*(-1 +
 x^3)^(1/3))/Sqrt[3]]/(9*Sqrt[3]) - Log[x]/18 + Log[1 + (-1 + x^3)^(1/3)]/1
8
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{-1+x^3}}{x^7}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{-1+x}}{x^3}\,dx,x,x^3\right)\\
&= -\frac{\sqrt[3]{-1+x^3}}{6x^6}+\frac{1}{18}\text{Subst}\left(\int \frac{1}{(-1+x)^{2/3}x^2}\,dx,x,x^3\right)\\
&= -\frac{\sqrt[3]{-1+x^3}}{6x^6}+\frac{\sqrt[3]{-1+x^3}}{18x^3}+\frac{1}{27}\text{Subst}\left(\int \frac{1}{(-1+x)^{2/3}x}\,dx,x,x^3\right)\\
&= -\frac{\sqrt[3]{-1+x^3}}{6x^6}+\frac{\sqrt[3]{-1+x^3}}{18x^3}-\frac{\log(x)}{18}+\frac{1}{18}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt[3]{-1+x^3}\right)+\frac{1}{18}\text{Subst}\left(\int\right.\\
&= -\frac{\sqrt[3]{-1+x^3}}{6x^6}+\frac{\sqrt[3]{-1+x^3}}{18x^3}-\frac{\log(x)}{18}+\frac{1}{18}\log\left(1+\sqrt[3]{-1+x^3}\right)-\frac{1}{9}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,\right.\\
&= -\frac{\sqrt[3]{-1+x^3}}{6x^6}+\frac{\sqrt[3]{-1+x^3}}{18x^3}-\frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^3}}{\sqrt{3}}\right)}{9\sqrt{3}}-\frac{\log(x)}{18}+\frac{1}{18}\log\left(1+\sqrt[3]{-1+x^3}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.29

$$\frac{1}{4}\left(x^3-1\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},3;\frac{7}{3};1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)^(1/3)/x^7,x]
```

```
[Out] ((-1 + x^3)^(4/3)*Hypergeometric2F1[4/3, 3, 7/3, 1 - x^3])/4
```

IntegrateAlgebraic [A] time = 0.13, size = 97, normalized size = 1.00

$$\frac{1}{27}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{1}{54}\log\left(\left(x^3-1\right)^{2/3}-\sqrt[3]{x^3-1}+1\right)-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\sqrt[3]{x^3-1}\left(x^3-3\right)}{18x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)^(1/3)/x^7,x]
```

```
[Out] ((-3 + x^3)*(-1 + x^3)^(1/3))/(18*x^6) - ArcTan[1/Sqrt[3] - (2*(-1 + x^3)^(
1/3))/Sqrt[3]]/(9*Sqrt[3]) + Log[1 + (-1 + x^3)^(1/3)]/27 - Log[1 - (-1 + x
^3)^(1/3) + (-1 + x^3)^(2/3)]/54
```

fricas [A] time = 0.49, size = 86, normalized size = 0.89

$$\frac{2\sqrt{3}x^6\arctan\left(\frac{2}{3}\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-x^6\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+2x^6\log\left(\left(x^3-1\right)^{\frac{1}{3}}+1\right)+3\left(x^3-1\right)^{\frac{1}{3}}\left(x^3-3\right)}{54x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)/x^7,x, algorithm="fricas")
```

```
[Out] 1/54*(2*sqrt(3)*x^6*arctan(2/3*sqrt(3)*(x^3 - 1)^(1/3) - 1/3*sqrt(3)) - x^6
*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 2*x^6*log((x^3 - 1)^(1/3) + 1
) + 3*(x^3 - 1)^(1/3)*(x^3 - 3))/x^6
```

giac [A] time = 0.16, size = 79, normalized size = 0.81

$$\frac{1}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{\left(x^3-1\right)^{\frac{4}{3}}-2\left(x^3-1\right)^{\frac{1}{3}}}{18x^6}-\frac{1}{54}\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{27}\log\left(\left|\left(x^3-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)/x^7,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) + 1/18*((x^3 - 1)^
(4/3) - 2*(x^3 - 1)^(1/3))/x^6 - 1/54*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3)
 + 1) + 1/27*log(abs((x^3 - 1)^(1/3) + 1))
```

maple [C] time = 0.37, size = 89, normalized size = 0.92

$$\frac{x^6-4x^3+3}{18x^6\left(x^3-1\right)^{\frac{2}{3}}}+\frac{\left(-\operatorname{signum}\left(x^3-1\right)\right)^{\frac{2}{3}}\left(\frac{2\Gamma\left(\frac{2}{3}\right)x^3\operatorname{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],x^3\right)}{3}+\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)\right)}{27\Gamma\left(\frac{2}{3}\right)\operatorname{signum}\left(x^3-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/3)/x^7,x)
```

```
[Out] 1/18*(x^6-4*x^3+3)/x^6/(x^3-1)^(2/3)+1/27/GAMMA(2/3)/signum(x^3-1)^(2/3)*(-
signum(x^3-1))^(2/3)*(2/3*GAMMA(2/3)*x^3*hypergeom([1,1,5/3],[2,2],x^3)+(1/
6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*GAMMA(2/3))
```

maxima [A] time = 0.76, size = 91, normalized size = 0.94

$$\frac{1}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{\left(x^3-1\right)^{\frac{4}{3}}-2\left(x^3-1\right)^{\frac{1}{3}}}{18\left(2x^3+\left(x^3-1\right)^2-1\right)}-\frac{1}{54}\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{27}\log\left(\left(x^3-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)/x^7,x, algorithm="maxima")
```

```
[Out] 1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) + 1/18*((x^3 - 1)^
(4/3) - 2*(x^3 - 1)^(1/3))/(2*x^3 + (x^3 - 1)^2 - 1) - 1/54*log((x^3 - 1)^(
2/3) - (x^3 - 1)^(1/3) + 1) + 1/27*log((x^3 - 1)^(1/3) + 1)
```

mupad [B] time = 0.96, size = 107, normalized size = 1.10

$$\frac{\ln\left(\frac{\left(x^3-1\right)^{1/3}}{81}+\frac{1}{81}\right)}{27}-\frac{\frac{\left(x^3-1\right)^{1/3}}{9}-\frac{\left(x^3-1\right)^{4/3}}{18}}{\left(x^3-1\right)^2+2x^3-1}-\ln\left(\frac{1}{6}-\frac{\left(x^3-1\right)^{1/3}}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(\frac{1}{54}+\frac{\sqrt{3}\,1\mathrm{i}}{54}\right)+\ln\left(\frac{\left(x^3-1\right)^{1/3}}{3}-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(-\frac{1}{54}+\frac{\sqrt{3}\,1\mathrm{i}}{54}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)^(1/3)/x^7,x)
```

```
[Out] log((x^3 - 1)^(1/3)/81 + 1/81)/27 - ((x^3 - 1)^(1/3)/9 - (x^3 - 1)^(4/3)/18
)/((x^3 - 1)^2 + 2*x^3 - 1) - log((3^(1/2)*1i)/6 - (x^3 - 1)^(1/3)/3 + 1/6)
*((3^(1/2)*1i)/54 + 1/54) + log((3^(1/2)*1i)/6 + (x^3 - 1)^(1/3)/3 - 1/6)*(
(3^(1/2)*1i)/54 - 1/54)
```

sympy **[C]** time = 1.09, size = 34, normalized size = 0.35

$$-\frac{\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^{3}}\right)}{3x^{5}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/3)/x**7,x)
```

```
[Out] -gamma(5/3)*hyper((-1/3, 5/3), (8/3,), exp_polar(2*I*pi)/x**3)/(3*x**5*gamm
a(8/3))
```

3.1196 $\int \frac{\sqrt[3]{1+x^3}}{x^7} \, dx$

Optimal. Leaf size=97

$$-\frac{1}{27}\log\left(\sqrt[3]{x^3+1}-1\right)+\frac{1}{54}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\sqrt[3]{x^3+1}\left(-x^3-3\right)}{18x^6}$$

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {266, 47, 51, 57, 618, 204, 31}

$$-\frac{\sqrt[3]{x^3+1}}{18x^3}-\frac{1}{18}\log\left(1-\sqrt[3]{x^3+1}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{9\sqrt{3}}-\frac{\sqrt[3]{x^3+1}}{6x^6}+\frac{\log(x)}{18}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)^(1/3)/x^7,x]
```

```
[Out] -1/6*(1 + x^3)^(1/3)/x^6 - (1 + x^3)^(1/3)/(18*x^3) + ArcTan[(1 + 2*(1 + x^
3)^(1/3))/Sqrt[3]]/(9*Sqrt[3]) + Log[x]/18 - Log[1 - (1 + x^3)^(1/3)]/18
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{1+x^3}}{x^7}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{x^3}\,dx, x, x^3\right)\\
&= -\frac{\sqrt[3]{1+x^3}}{6x^6} + \frac{1}{18}\text{Subst}\left(\int \frac{1}{x^2(1+x)^{2/3}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt[3]{1+x^3}}{6x^6} - \frac{\sqrt[3]{1+x^3}}{18x^3} - \frac{1}{27}\text{Subst}\left(\int \frac{1}{x(1+x)^{2/3}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt[3]{1+x^3}}{6x^6} - \frac{\sqrt[3]{1+x^3}}{18x^3} + \frac{\log(x)}{18} + \frac{1}{18}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+x^3}\right) + \frac{1}{18}\text{Subst}\left(\int \frac{1}{1+x}\right.\\
&= -\frac{\sqrt[3]{1+x^3}}{6x^6} - \frac{\sqrt[3]{1+x^3}}{18x^3} + \frac{\log(x)}{18} - \frac{1}{18}\log\left(1-\sqrt[3]{1+x^3}\right) - \frac{1}{9}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\right.\\
&= -\frac{\sqrt[3]{1+x^3}}{6x^6} - \frac{\sqrt[3]{1+x^3}}{18x^3} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^3}}{\sqrt{3}}\right)}{9\sqrt{3}} + \frac{\log(x)}{18} - \frac{1}{18}\log\left(1-\sqrt[3]{1+x^3}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.27

$$-\frac{1}{4}\left(x^3+1\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},3;\frac{7}{3};x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)^(1/3)/x^7,x]
[Out] -1/4*((1 + x^3)^(4/3)*Hypergeometric2F1[4/3, 3, 7/3, 1 + x^3])
```

IntegrateAlgebraic [A] time = 0.11, size = 97, normalized size = 1.00

$$-\frac{1}{27}\log\left(\sqrt[3]{x^3+1}-1\right)+\frac{1}{54}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\sqrt[3]{x^3+1}\left(-x^3-3\right)}{18x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)^(1/3)/x^7,x]
[Out] ((-3 - x^3)*(1 + x^3)^(1/3))/(18*x^6) + ArcTan[1/Sqrt[3] + (2*(1 + x^3)^(1/
3))/Sqrt[3]]/(9*Sqrt[3]) - Log[-1 + (1 + x^3)^(1/3)]/27 + Log[1 + (1 + x^3)
^(1/3) + (1 + x^3)^(2/3)]/54
```

fricas [A] time = 0.48, size = 83, normalized size = 0.86

$$\frac{2\sqrt{3}x^6\arctan\left(\frac{2}{3}\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+x^6\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)-2x^6\log\left(\left(x^3+1\right)^{\frac{1}{3}}-1\right)-3\left(x^3+3\right)\left(x^3+1\right)^{\frac{1}{3}}}{54x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/3)/x^7,x, algorithm="fricas")
```

```
[Out] 1/54*(2*sqrt(3)*x^6*arctan(2/3*sqrt(3)*(x^3 + 1)^(1/3) + 1/3*sqrt(3)) + x^6
*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) - 2*x^6*log((x^3 + 1)^(1/3) - 1
) - 3*(x^3 + 3)*(x^3 + 1)^(1/3))/x^6
```

giac [A] time = 0.16, size = 77, normalized size = 0.79

$$\frac{1}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^3+1\right)^{\frac{4}{3}}+2\left(x^3+1\right)^{\frac{1}{3}}}{18x^6}+\frac{1}{54}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{27}\log\left(\left|\left(x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/3)/x^7,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) - 1/18*((x^3 + 1)^
(4/3) + 2*(x^3 + 1)^(1/3))/x^6 + 1/54*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3)
 + 1) - 1/27*log(abs((x^3 + 1)^(1/3) - 1))
```

maple [C] time = 0.32, size = 69, normalized size = 0.71

$$-\frac{x^6+4x^3+3}{18x^6\left(x^3+1\right)^{\frac{2}{3}}}-\frac{-\frac{2\Gamma\left(\frac{2}{3}\right)x^3\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],-x^3\right)}{3}+\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)\right)\Gamma\left(\frac{2}{3}\right)}{27\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(1/3)/x^7,x)
```

```
[Out] -1/18*(x^6+4*x^3+3)/x^6/(x^3+1)^(2/3)-1/27/GAMMA(2/3)*(-2/3*GAMMA(2/3)*x^3*
hypergeom([1,1,5/3],[2,2],-x^3)+(1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x))*GAMMA(2/
3))
```

maxima [A] time = 0.59, size = 91, normalized size = 0.94

$$\frac{1}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{\left(x^3+1\right)^{\frac{4}{3}}+2\left(x^3+1\right)^{\frac{1}{3}}}{18\left(2x^3-\left(x^3+1\right)^2+1\right)}+\frac{1}{54}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{27}\log\left(\left(x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/3)/x^7,x, algorithm="maxima")
```

```
[Out] 1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) + 1/18*((x^3 + 1)^
(4/3) + 2*(x^3 + 1)^(1/3))/(2*x^3 - (x^3 + 1)^2 + 1) + 1/54*log((x^3 + 1)^(
2/3) + (x^3 + 1)^(1/3) + 1) - 1/27*log((x^3 + 1)^(1/3) - 1)
```

mupad [B] time = 0.96, size = 108, normalized size = 1.11

$$\frac{\frac{\left(x^3+1\right)^{1/3}}{9}+\frac{\left(x^3+1\right)^{4/3}}{18}}{2x^3-\left(x^3+1\right)^2+1}-\frac{\ln\left(\frac{\left(x^3+1\right)^{1/3}}{81}-\frac{1}{81}\right)}{27}-\ln\left(\frac{\left(x^3+1\right)^{1/3}}{3}+\frac{1}{6}-\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(-\frac{1}{54}+\frac{\sqrt{3}\,1\mathrm{i}}{54}\right)+\ln\left(\frac{\left(x^3+1\right)^{1/3}}{3}+\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(\frac{1}{54}+\frac{\sqrt{3}\,1\mathrm{i}}{54}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)^(1/3)/x^7,x)
```

```
[Out] ((x^3 + 1)^(1/3)/9 + (x^3 + 1)^(4/3)/18)/(2*x^3 - (x^3 + 1)^2 + 1) - log((x
^3 + 1)^(1/3)/81 - 1/81)/27 - log((x^3 + 1)^(1/3)/3 - (3^(1/2)*1i)/6 + 1/6)
*((3^(1/2)*1i)/54 - 1/54) + log((3^(1/2)*1i)/6 + (x^3 + 1)^(1/3)/3 + 1/6)*(
(3^(1/2)*1i)/54 + 1/54)
```

sympy [C] time = 1.05, size = 32, normalized size = 0.33

$$-\frac{\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^3}\right)}{3x^5\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(1/3)/x**7,x)
```

```
[Out] -gamma(5/3)*hyper((-1/3, 5/3), (8/3,), exp_polar(I*pi)/x**3)/(3*x**5*gamma(
8/3))
```

3.1197 $\int \frac{(-1+x^3)\sqrt[3]{1+x^3}}{x^4}\,dx$

Optimal. Leaf size=97

$$\frac{\sqrt[3]{x^3+1}\,(3x^3+1)}{3x^3}+\frac{2}{9}\log\left(\sqrt[3]{x^3+1}-1\right)-\frac{1}{9}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {446, 78, 50, 57, 618, 204, 31}

$$\frac{\left(x^3+1\right)^{4/3}}{3x^3}+\frac{2}{3}\sqrt[3]{x^3+1}+\frac{1}{3}\log\left(1-\sqrt[3]{x^3+1}\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)*(1 + x^3)^(1/3))/x^4,x]
```

```
[Out] (2*(1 + x^3)^(1/3))/3 + (1 + x^3)^(4/3)/(3*x^3) - (2*ArcTan[(1 + 2*(1 + x^3
)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) - Log[x]/3 + Log[1 - (1 + x^3)^(1/3)]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+x^3)\sqrt[3]{1+x^3}}{x^4}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(-1+x)\sqrt[3]{1+x}}{x^2}\,dx,x,x^3\right)\\
&= \frac{(1+x^3)^{4/3}}{3x^3}+\frac{2}{9}\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{x}\,dx,x,x^3\right)\\
&= \frac{2}{3}\sqrt[3]{1+x^3}+\frac{(1+x^3)^{4/3}}{3x^3}+\frac{2}{9}\text{Subst}\left(\int \frac{1}{x(1+x)^{2/3}}\,dx,x,x^3\right)\\
&= \frac{2}{3}\sqrt[3]{1+x^3}+\frac{(1+x^3)^{4/3}}{3x^3}-\frac{\log(x)}{3}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1+x^3}\right)-\frac{1}{3}\text{Subst}\Big(\\
&= \frac{2}{3}\sqrt[3]{1+x^3}+\frac{(1+x^3)^{4/3}}{3x^3}-\frac{\log(x)}{3}+\frac{1}{3}\log\left(1-\sqrt[3]{1+x^3}\right)+\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.\\
&= \frac{2}{3}\sqrt[3]{1+x^3}+\frac{(1+x^3)^{4/3}}{3x^3}-\frac{2\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^3}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\log(x)}{3}+\frac{1}{3}\log\left(1-\sqrt[3]{1+x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 98, normalized size = 1.01

$$\frac{\sqrt[3]{x^3+1}}{3x^3}+\sqrt[3]{x^3+1}+\frac{2}{9}\log\left(1-\sqrt[3]{x^3+1}\right)-\frac{1}{9}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)*(1 + x^3)^(1/3))/x^4,x]
```

```
[Out] (1 + x^3)^(1/3) + (1 + x^3)^(1/3)/(3*x^3) - (2*ArcTan[(1 + 2*(1 + x^3)^(1/3
))/Sqrt[3]])/(3*Sqrt[3]) + (2*Log[1 - (1 + x^3)^(1/3)])/9 - Log[1 + (1 + x^
3)^(1/3) + (1 + x^3)^(2/3)]/9
```

IntegrateAlgebraic [A] time = 0.08, size = 97, normalized size = 1.00

$$\frac{\sqrt[3]{x^3+1}\left(3x^3+1\right)}{3x^3}+\frac{2}{9}\log\left(\sqrt[3]{x^3+1}-1\right)-\frac{1}{9}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)*(1 + x^3)^(1/3))/x^4,x]
```

```
[Out] ((1 + x^3)^(1/3)*(1 + 3*x^3))/(3*x^3) - (2*ArcTan[1/Sqrt[3] + (2*(1 + x^3)^
(1/3))/Sqrt[3]])/(3*Sqrt[3]) + (2*Log[-1 + (1 + x^3)^(1/3)])/9 - Log[1 + (1
 + x^3)^(1/3) + (1 + x^3)^(2/3)]/9
```

fricas [A] time = 0.51, size = 85, normalized size = 0.88

$$-\frac{2\sqrt{3}\,x^3\arctan\left(\frac{2}{3}\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+x^3\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)-2\,x^3\log\left(\left(x^3+1\right)^{\frac{1}{3}}-1\right)-3\left(3\,x^3+1\right)\left(x^3+1\right)^{\frac{1}{3}}}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(2*sqrt(3)*x^3*arctan(2/3*sqrt(3)*(x^3 + 1)^(1/3) + 1/3*sqrt(3)) + x^3
*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1) - 2*x^3*log((x^3 + 1)^(1/3) - 1
) - 3*(3*x^3 + 1)*(x^3 + 1)^(1/3))/x^3
```

giac [A] time = 0.18, size = 74, normalized size = 0.76

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\left(x^3+1\right)^{\frac{1}{3}}+\frac{\left(x^3+1\right)^{\frac{1}{3}}}{3\,x^3}-\frac{1}{9}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{2}{9}\log\left(\left|\left(x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x^4,x, algorithm="giac")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) + (x^3 + 1)^(1/3)
+ 1/3*(x^3 + 1)^(1/3)/x^3 - 1/9*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1)
+ 2/9*log(abs((x^3 + 1)^(1/3) - 1))
```

maple [C] time = 0.32, size = 71, normalized size = 0.73

$$\frac{3x^6+4x^3+1}{3x^3\left(x^3+1\right)^{\frac{2}{3}}}+\frac{-\frac{4\Gamma\left(\frac{2}{3}\right)x^3\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],-x^3\right)}{27}+\frac{2\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)\right)\Gamma\left(\frac{2}{3}\right)}{9}}{\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)*(x^3+1)^(1/3)/x^4,x)
```

```
[Out] 1/3*(3*x^6+4*x^3+1)/x^3/(x^3+1)^(2/3)+2/9/GAMMA(2/3)*(-2/3*GAMMA(2/3)*x^3*h
ypergeom([1,1,5/3],[2,2],-x^3)+(1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x))*GAMMA(2/3
))
```

maxima [A] time = 0.43, size = 73, normalized size = 0.75

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\left(x^3+1\right)^{\frac{1}{3}}+\frac{\left(x^3+1\right)^{\frac{1}{3}}}{3\,x^3}-\frac{1}{9}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{2}{9}\log\left(\left(x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x^4,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) + (x^3 + 1)^(1/3)
+ 1/3*(x^3 + 1)^(1/3)/x^3 - 1/9*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1)
+ 2/9*log((x^3 + 1)^(1/3) - 1)
```

mupad [B] time = 1.24, size = 152, normalized size = 1.57

$$\frac{\ln\left(\left(x^3+1\right)^{1/3}-1\right)}{3}-\frac{\ln\left(\frac{\left(x^3+1\right)^{1/3}}{9}-\frac{1}{9}\right)}{9}+\left(x^3+1\right)^{1/3}-\ln\left(\left(x^3+1\right)^{1/3}+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-\frac{1}{18}+\frac{\sqrt{3}\,1{}\mathrm{i}}{18}\right)+\ln\left(\left(x^3+1\right)^{1/3}+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{1}{18}+\frac{\sqrt{3}\,1{}\mathrm{i}}{18}\right)+\frac{\left(x^3+1\right)^{1/3}}{3\,x^3}+\ln\left(3\left(x^3+1\right)^{1/3}+\frac{3}{2}-\frac{\sqrt{3}\,3{}\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)-\ln\left(3\left(x^3+1\right)^{1/3}+\frac{3}{2}+\frac{\sqrt{3}\,3{}\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((x^3 - 1)*(x^3 + 1)^(1/3))/x^4,x)
```

```
[Out] log((x^3 + 1)^(1/3) - 1)/3 - log((x^3 + 1)^(1/3)/9 - 1/9)/9 + (x^3 + 1)^(1/
3) - log((x^3 + 1)^(1/3) - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/18 - 1/18) +
 log((3^(1/2)*1i)/2 + (x^3 + 1)^(1/3) + 1/2)*((3^(1/2)*1i)/18 + 1/18) + (x^
3 + 1)^(1/3)/(3*x^3) + log(3*(x^3 + 1)^(1/3) - (3^(1/2)*3i)/2 + 3/2)*((3^(1
/2)*1i)/6 - 1/6) - log((3^(1/2)*3i)/2 + 3*(x^3 + 1)^(1/3) + 3/2)*((3^(1/2)*
1i)/6 + 1/6)
```

sympy [C] time = 87.87, size = 65, normalized size = 0.67

$$-\frac{x\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^3}\right)}{3\Gamma\left(\frac{2}{3}\right)}+\frac{\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^3}\right)}{3x^2\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)*(x**3+1)**(1/3)/x**4,x)
```

```
[Out] -x*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), exp_polar(I*pi)/x**3)/(3*gamma(2
/3)) + gamma(2/3)*hyper((-1/3, 2/3), (5/3,), exp_polar(I*pi)/x**3)/(3*x**2*
gamma(5/3))
```

3.1198 $\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt{-x-x^2+x^3}}\,dx$

Optimal. Leaf size=97

$$-\sqrt{\frac{1}{5}+\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1-2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)-\sqrt{\frac{1}{5}-\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1+2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)$$

Rubi [C] time = 1.17, antiderivative size = 375, normalized size of antiderivative = 3.87, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2056, 6725, 716, 1098, 934, 168, 538, 537}

$$\frac{\sqrt{x}\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}\sqrt{\frac{\left(1+\sqrt{5}\right)x+2}{\left(1-\sqrt{5}\right)x+2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{\left(1-\sqrt{5}\right)x+2}}\sqrt{x^3-x^2-x}}-\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(-\frac{1}{2}i\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\sqrt{x^3-x^2-x}}-\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(\frac{1}{2}i\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\sqrt{x^3-x^2-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[-x - x^2 + x^3]),x]
```

```
[Out] (Sqrt[x]*Sqrt[-2 - (1 - Sqrt[5])*x]*Sqrt[(2 + (1 + Sqrt[5])*x)/(2 + (1 - Sq
rt[5])*x)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*Sqrt[x])/Sqrt[-2 - (1 - Sqrt[5
])*x]], (5 - Sqrt[5])/10])/(5^(1/4)*Sqrt[(2 + (1 - Sqrt[5])*x)^(-1)]*Sqrt[-
x - x^2 + x^3]) - (Sqrt[3 + Sqrt[5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[
1 - (2*x)/(1 + Sqrt[5])]*EllipticPi[(-1/2*I)*(1 + Sqrt[5]), ArcSin[Sqrt[2/(
1 + Sqrt[5])]*Sqrt[x]], (-3 - Sqrt[5])/2])/Sqrt[-x - x^2 + x^3] - (Sqrt[3 +
 Sqrt[5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1 + Sqrt[5])]*El
lipticPi[(I/2)*(1 + Sqrt[5]), ArcSin[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]], (-3 -
Sqrt[5])/2])/Sqrt[-x - x^2 + x^3]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 716

```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2,
Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt{-x-x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}\left(1+x^2\right)\sqrt{-1-x+x^2}}\,dx}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{-1-x+x^2}} - \frac{2}{\sqrt{x}\left(1+x^2\right)\sqrt{-1-x+x^2}}\right)dx}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-1-x+x^2}}\,dx}{\sqrt{-x-x^2+x^3}} - \frac{\left(2\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{1}{\sqrt{x}\left(1+x^2\right)\sqrt{-1-x+x^2}}}{\sqrt{-x-x^2+x^3}}$$

$$= -\frac{\left(2\sqrt{x}\sqrt{-1-x+x^2}\right)\int \left(\frac{i}{2(i-x)\sqrt{x}\sqrt{-1-x+x^2}} + \frac{i}{2\sqrt{x}(i+x)\sqrt{-1-x+x^2}}\right)dx}{\sqrt{-x-x^2+x^3}} + \frac{\left(2\sqrt{x}\sqrt{-1}\right.}{}$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - \frac{\left(i\sqrt{\right.}{}$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - \frac{\left(i\sqrt{\right.}{}$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} + \frac{\left(2i\right.}{}$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} + \frac{\left(2i\right.}{}$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - \frac{\sqrt{3}}{}$$

Mathematica [C] time = 0.66, size = 205, normalized size = 2.11

$$-\frac{2i\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{-\frac{1}{x^2}-\frac{1}{x}+1}\,x^{3/2}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{2}{1+\sqrt{5}}}}{\sqrt{x}}\right)|-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-\Pi\left(-\frac{1}{2}i\left(1+\sqrt{5}\right);i\sinh^{-1}\left(\frac{\sqrt{\frac{2}{1+\sqrt{5}}}}{\sqrt{x}}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)-\Pi\left(\frac{1}{2}i\left(1+\sqrt{5}\right);i\sinh^{-1}\left(\frac{\sqrt{\frac{2}{1+\sqrt{5}}}}{\sqrt{x}}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)\right)}{\sqrt{x\left(x^2-x-1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[-x - x^2 + x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 - x^(-2) - x^(-1)]*x^(3/2)*(EllipticF
[I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], -3/2 - Sqrt[5]/2] - EllipticPi[(
-1/2*I)*(1 + Sqrt[5]), I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], (-3 - Sqrt
[5])/2] - EllipticPi[(I/2)*(1 + Sqrt[5]), I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/S
qrt[x]], (-3 - Sqrt[5])/2]))/Sqrt[x*(-1 - x + x^2)]
```

IntegrateAlgebraic [A] time = 0.29, size = 97, normalized size = 1.00

$$-\sqrt{\frac{1}{5}+\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1-2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)-\sqrt{\frac{1}{5}-\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1+2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*Sqrt[-x - x^2 + x^3]),x]
```

```
[Out] -(Sqrt[1/5 + (2*I)/5]*ArcTan[(Sqrt[1 - 2*I]*Sqrt[-x - x^2 + x^3])/(-1 - x +
 x^2)]) - Sqrt[1/5 - (2*I)/5]*ArcTan[(Sqrt[1 + 2*I]*Sqrt[-x - x^2 + x^3])/(
-1 - x + x^2)]
```

fricas [B] time = 0.93, size = 2458, normalized size = 25.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3-x^2-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/80*5^(1/4)*(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 5)*log(5*(5*x^4 - 2
0*x^3 + 5^(1/4)*sqrt(x^3 - x^2 - x)*(sqrt(5)*sqrt(2)*(x^2 - 6*x - 1) - 5*sq
rt(2)*(x^2 - 2*x - 1))*sqrt(sqrt(5) + 5) + 30*x^2 + 20*sqrt(5)*(x^3 - x^2 -
 x) + 20*x + 5)/(x^4 + 2*x^2 + 1)) - 1/80*5^(1/4)*(sqrt(5)*sqrt(2) - sqrt(2
))*sqrt(sqrt(5) + 5)*log(5*(5*x^4 - 20*x^3 - 5^(1/4)*sqrt(x^3 - x^2 - x)*(s
qrt(5)*sqrt(2)*(x^2 - 6*x - 1) - 5*sqrt(2)*(x^2 - 2*x - 1))*sqrt(sqrt(5) +
5) + 30*x^2 + 20*sqrt(5)*(x^3 - x^2 - x) + 20*x + 5)/(x^4 + 2*x^2 + 1)) - 1
/10*5^(1/4)*sqrt(2)*sqrt(sqrt(5) + 5)*arctan(-1/200*(100*x^11 + 1300*x^10 -
 6700*x^9 - 4400*x^8 + 28400*x^7 + 1400*x^6 - 28400*x^5 - 4400*x^4 + 6700*x
^3 + 1300*x^2 + 5*sqrt(x^3 - x^2 - x)*(5^(3/4)*(sqrt(5)*sqrt(2)*(x^10 - 4*x
^9 - 17*x^8 + 56*x^7 + 78*x^6 - 136*x^5 - 78*x^4 + 56*x^3 + 17*x^2 - 4*x -
1) - sqrt(2)*(x^10 + 8*x^9 - 57*x^8 - 24*x^7 + 294*x^6 + 64*x^5 - 294*x^4 -
 24*x^3 + 57*x^2 + 8*x - 1)) + 4*5^(1/4)*(sqrt(5)*sqrt(2)*(3*x^9 + 7*x^8 +
14*x^7 - 81*x^6 - 10*x^5 + 81*x^4 + 14*x^3 - 7*x^2 + 3*x) - 5*sqrt(2)*(x^9
+ 9*x^8 - 18*x^7 - 15*x^6 + 26*x^5 + 15*x^4 - 18*x^3 - 9*x^2 + x)))*sqrt(sq
rt(5) + 5) - sqrt(5)*(240*x^10 + 160*x^9 - 1680*x^8 - 480*x^7 + 3200*x^6 +
480*x^5 - 1680*x^4 - 160*x^3 + 240*x^2 + sqrt(x^3 - x^2 - x)*(5^(3/4)*(sqrt
(5)*sqrt(2)*(x^10 - 6*x^9 - 25*x^8 + 120*x^7 - 58*x^6 - 196*x^5 + 58*x^4 +
120*x^3 + 25*x^2 - 6*x - 1) - sqrt(2)*(x^10 - 2*x^9 - 17*x^8 + 216*x^7 - 30
6*x^6 - 396*x^5 + 306*x^4 + 216*x^3 + 17*x^2 - 2*x - 1)) + 4*5^(1/4)*(sqrt(
5)*sqrt(2)*(3*x^9 - 3*x^8 - 56*x^7 + 69*x^6 + 90*x^5 - 69*x^4 - 56*x^3 + 3*
x^2 + 3*x) - 5*sqrt(2)*(x^9 + 3*x^8 - 28*x^7 + 11*x^6 + 54*x^5 - 11*x^4 - 2
8*x^3 - 3*x^2 + x)))*sqrt(sqrt(5) + 5) + 4*sqrt(5)*(5*x^11 - 25*x^10 - 105*
x^9 + 440*x^8 + 50*x^7 - 830*x^6 - 50*x^5 + 440*x^4 + 105*x^3 - 25*x^2 - sq
rt(5)*(x^11 + 3*x^10 - 37*x^9 + 96*x^8 - 6*x^7 - 166*x^6 + 6*x^5 + 96*x^4 +
 37*x^3 + 3*x^2 - x) - 5*x) - 80*sqrt(5)*(x^10 + 6*x^9 - 23*x^8 - 2*x^7 + 4
0*x^6 + 2*x^5 - 23*x^4 - 6*x^3 + x^2))*sqrt((5*x^4 - 20*x^3 + 5^(1/4)*sqrt(
x^3 - x^2 - x)*(sqrt(5)*sqrt(2)*(x^2 - 6*x - 1) - 5*sqrt(2)*(x^2 - 2*x - 1)
)*sqrt(sqrt(5) + 5) + 30*x^2 + 20*sqrt(5)*(x^3 - x^2 - x) + 20*x + 5)/(x^4
+ 2*x^2 + 1)) + 20*sqrt(5)*(5*x^11 - 15*x^10 - 15*x^9 + 20*x^8 - 20*x^7 + 7
0*x^6 + 20*x^5 + 20*x^4 + 15*x^3 - 15*x^2 - sqrt(5)*(x^11 + 13*x^10 - 67*x^
9 - 44*x^8 + 284*x^7 + 14*x^6 - 284*x^5 - 44*x^4 + 67*x^3 + 13*x^2 - x) - 5
*x) - 100*sqrt(5)*(x^11 - 3*x^10 - 3*x^9 + 4*x^8 - 4*x^7 + 14*x^6 + 4*x^5 +
 4*x^4 + 3*x^3 - 3*x^2 - x) - 100*x)/(x^11 - 9*x^10 - 45*x^9 + 180*x^8 + 18
*x^7 - 326*x^6 - 18*x^5 + 180*x^4 + 45*x^3 - 9*x^2 - x)) - 1/10*5^(1/4)*sqr
t(2)*sqrt(sqrt(5) + 5)*arctan(1/200*(100*x^11 + 1300*x^10 - 6700*x^9 - 4400
*x^8 + 28400*x^7 + 1400*x^6 - 28400*x^5 - 4400*x^4 + 6700*x^3 + 1300*x^2 -
5*sqrt(x^3 - x^2 - x)*(5^(3/4)*(sqrt(5)*sqrt(2)*(x^10 - 4*x^9 - 17*x^8 + 56
*x^7 + 78*x^6 - 136*x^5 - 78*x^4 + 56*x^3 + 17*x^2 - 4*x - 1) - sqrt(2)*(x^
10 + 8*x^9 - 57*x^8 - 24*x^7 + 294*x^6 + 64*x^5 - 294*x^4 - 24*x^3 + 57*x^2
 + 8*x - 1)) + 4*5^(1/4)*(sqrt(5)*sqrt(2)*(3*x^9 + 7*x^8 + 14*x^7 - 81*x^6
```

```
- 10*x^5 + 81*x^4 + 14*x^3 - 7*x^2 + 3*x) - 5*sqrt(2)*(x^9 + 9*x^8 - 18*x^7
 - 15*x^6 + 26*x^5 + 15*x^4 - 18*x^3 - 9*x^2 + x)))*sqrt(sqrt(5) + 5) - sqr
t(5)*(240*x^10 + 160*x^9 - 1680*x^8 - 480*x^7 + 3200*x^6 + 480*x^5 - 1680*x
^4 - 160*x^3 + 240*x^2 - sqrt(x^3 - x^2 - x)*(5^(3/4)*(sqrt(5)*sqrt(2)*(x^1
0 - 6*x^9 - 25*x^8 + 120*x^7 - 58*x^6 - 196*x^5 + 58*x^4 + 120*x^3 + 25*x^2
 - 6*x - 1) - sqrt(2)*(x^10 - 2*x^9 - 17*x^8 + 216*x^7 - 306*x^6 - 396*x^5
+ 306*x^4 + 216*x^3 + 17*x^2 - 2*x - 1)) + 4*5^(1/4)*(sqrt(5)*sqrt(2)*(3*x^
9 - 3*x^8 - 56*x^7 + 69*x^6 + 90*x^5 - 69*x^4 - 56*x^3 + 3*x^2 + 3*x) - 5*s
qrt(2)*(x^9 + 3*x^8 - 28*x^7 + 11*x^6 + 54*x^5 - 11*x^4 - 28*x^3 - 3*x^2 +
x)))*sqrt(sqrt(5) + 5) + 4*sqrt(5)*(5*x^11 - 25*x^10 - 105*x^9 + 440*x^8 +
50*x^7 - 830*x^6 - 50*x^5 + 440*x^4 + 105*x^3 - 25*x^2 - sqrt(5)*(x^11 + 3*
x^10 - 37*x^9 + 96*x^8 - 6*x^7 - 166*x^6 + 6*x^5 + 96*x^4 + 37*x^3 + 3*x^2
- x) - 5*x) - 80*sqrt(5)*(x^10 + 6*x^9 - 23*x^8 - 2*x^7 + 40*x^6 + 2*x^5 -
23*x^4 - 6*x^3 + x^2))*sqrt((5*x^4 - 20*x^3 - 5^(1/4)*sqrt(x^3 - x^2 - x)*(
sqrt(5)*sqrt(2)*(x^2 - 6*x - 1) - 5*sqrt(2)*(x^2 - 2*x - 1))*sqrt(sqrt(5) +
 5) + 30*x^2 + 20*sqrt(5)*(x^3 - x^2 - x) + 20*x + 5)/(x^4 + 2*x^2 + 1)) +
20*sqrt(5)*(5*x^11 - 15*x^10 - 15*x^9 + 20*x^8 - 20*x^7 + 70*x^6 + 20*x^5 +
 20*x^4 + 15*x^3 - 15*x^2 - sqrt(5)*(x^11 + 13*x^10 - 67*x^9 - 44*x^8 + 284
*x^7 + 14*x^6 - 284*x^5 - 44*x^4 + 67*x^3 + 13*x^2 - x) - 5*x) - 100*sqrt(5
)*(x^11 - 3*x^10 - 3*x^9 + 4*x^8 - 4*x^7 + 14*x^6 + 4*x^5 + 4*x^4 + 3*x^3 -
 3*x^2 - x) - 100*x)/(x^11 - 9*x^10 - 45*x^9 + 180*x^8 + 18*x^7 - 326*x^6 -
 18*x^5 + 180*x^4 + 45*x^3 - 9*x^2 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^3-x^2-x}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3-x^2-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^3 - x^2 - x)*(x^2 + 1)), x)
```

maple [C] time = 0.06, size = 735, normalized size = 7.58

$$\frac{2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\sqrt{-5\left(x-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\sqrt{5}}\sqrt{-\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}},\frac{\sqrt{5}\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)}{5\sqrt{x^3-x^2-x}}+\frac{i\sqrt{\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}-\frac{1}{2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}+\frac{\sqrt{5}}{\sqrt{5}-1}}\sqrt{-5x\sqrt{5}+\frac{5\sqrt{5}}{2}+\frac{25}{2}}\sqrt{-\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}},\frac{\frac{1}{2}-\frac{\sqrt{5}}{2}}{\frac{1}{2}-i-\frac{\sqrt{5}}{2}},\frac{\sqrt{5}\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)\sqrt{5}}{5\sqrt{x^3-x^2-x}\left(\frac{1}{2}-i-\frac{\sqrt{5}}{2}\right)}-\frac{i\sqrt{\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}-\frac{1}{2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}+\frac{\sqrt{5}}{\sqrt{5}-1}}\sqrt{-5x\sqrt{5}+\frac{5\sqrt{5}}{2}+\frac{25}{2}}\sqrt{-\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}},\frac{\frac{1}{2}-\frac{\sqrt{5}}{2}}{\frac{1}{2}-i-\frac{\sqrt{5}}{2}},\frac{\sqrt{5}\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)}{5\sqrt{x^3-x^2-x}\left(\frac{1}{2}-i-\frac{\sqrt{5}}{2}\right)}-\frac{i\sqrt{\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}-\frac{1}{2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}+\frac{\sqrt{5}}{\sqrt{5}-1}}\sqrt{-5x\sqrt{5}+\frac{5\sqrt{5}}{2}+\frac{25}{2}}\sqrt{-\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}},\frac{\frac{1}{2}-\frac{\sqrt{5}}{2}}{\frac{1}{2}+i-\frac{\sqrt{5}}{2}},\frac{\sqrt{5}\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)\sqrt{5}}{5\sqrt{x^3-x^2-x}\left(\frac{1}{2}+i-\frac{\sqrt{5}}{2}\right)}+\frac{i\sqrt{\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}-\frac{1}{2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}+\frac{\sqrt{5}}{\sqrt{5}-1}}\sqrt{-5x\sqrt{5}+\frac{5\sqrt{5}}{2}+\frac{25}{2}}\sqrt{-\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}},\frac{\frac{1}{2}-\frac{\sqrt{5}}{2}}{\frac{1}{2}+i-\frac{\sqrt{5}}{2}},\frac{\sqrt{5}\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)}{5\sqrt{x^3-x^2-x}\left(\frac{1}{2}+i-\frac{\sqrt{5}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^3-x^2-x)^(1/2),x)
```

```
[Out] 2/5*(1/2*5^(1/2)-1/2)*((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x-
1/2-1/2*5^(1/2))*5^(1/2))^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1
/2)*EllipticF(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),1/5*5^(1/2)*((1
/2*5^(1/2)-1/2)*5^(1/2))^(1/2))+1/5*I*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)
-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^
(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(1/2-I-1/2*5^(1/2))*El
lipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1
/2-I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))*5^(1/2)-1/
5*I*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2
))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)
/(x^3-x^2-x)^(1/2)/(1/2-I-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2
*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2-I-1/2*5^(1/2)),1/5*5^(1/2)*((1/
2*5^(1/2)-1/2)*5^(1/2))^(1/2))-1/5*I*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-
1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(
1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(1/2+I-1/2*5^(1/2))*Ell
ipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/
2+I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))*5^(1/2)+1/5
*I*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2)
)^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/
```

```
(x^3-x^2-x)^(1/2)/(1/2+I-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*
5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2+I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2
*5^(1/2)-1/2)*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^3-x^2-x}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3-x^2-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^3 - x^2 - x)*(x^2 + 1)), x)
```

mupad [B] time = 0.89, size = 210, normalized size = 2.16

$$-\frac{\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\left(\sqrt{5}+1\right)\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)+\Pi\left(-\frac{\sqrt{5}\,1\mathrm{i}}{2}-\frac{1}{2}\mathrm{i};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)+\Pi\left(\frac{\sqrt{5}\,1\mathrm{i}}{2}+\frac{1}{2}\mathrm{i};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)\right)}{\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + 1)*(x^3 - x^2 - x)^(1/2)),x)
```

```
[Out] -((x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/2 - 1/2)/(5^(1/2)/2 - 1/2))^(1/
2)*(5^(1/2) + 1)*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 + 1/2))^(1/2)*(ellipticP
i(- (5^(1/2)*1i)/2 - 1i/2, asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/2 +
 1/2)/(5^(1/2)/2 - 1/2)) - ellipticF(asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5
^(1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2)) + ellipticPi((5^(1/2)*1i)/2 + 1i/2, asin
((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2))))/(x^3
 - x^2 - x*(5^(1/2)/2 - 1/2)*(5^(1/2)/2 + 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{\sqrt{x\left(x^2-x-1\right)}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**3-x**2-x)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 - x - 1))*(x**2 + 1)), x)
```

3.1199 $\int \frac{\sqrt[3]{x^2+x^3}}{x}\,dx$

Optimal. Leaf size=97

$$\sqrt[3]{x^3+x^2}-\frac{1}{3}\log\left(\sqrt[3]{x^3+x^2}-x\right)+\frac{1}{6}\log\left(x^2+\sqrt[3]{x^3+x^2}\,x+\left(x^3+x^2\right)^{2/3}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3+x^2}+x}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 142, normalized size of antiderivative = 1.46, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2021, 2032, 59}

$$\sqrt[3]{x^3+x^2}-\frac{(x+1)^{2/3}x^{4/3}\log(x+1)}{6\left(x^3+x^2\right)^{2/3}}-\frac{(x+1)^{2/3}x^{4/3}\log\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x+1}}-1\right)}{2\left(x^3+x^2\right)^{2/3}}-\frac{(x+1)^{2/3}x^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{x+1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\left(x^3+x^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2 + x^3)^(1/3)/x,x]
```

```
[Out] (x^2 + x^3)^(1/3) - (x^(4/3)*(1 + x)^(2/3)*ArcTan[1/Sqrt[3] + (2*x^(1/3))/(
Sqrt[3]*(1 + x)^(1/3))])/(Sqrt[3]*(x^2 + x^3)^(2/3)) - (x^(4/3)*(1 + x)^(2/
3)*Log[1 + x])/(6*(x^2 + x^3)^(2/3)) - (x^(4/3)*(1 + x)^(2/3)*Log[-1 + x^(1
/3)/(1 + x)^(1/3)])/(2*(x^2 + x^3)^(2/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt[3]{x^2+x^3}}{x}\,dx = \sqrt[3]{x^2+x^3} + \frac{1}{3}\int \frac{x}{\left(x^2+x^3\right)^{2/3}}\,dx$$
$$= \sqrt[3]{x^2+x^3} + \frac{\left(x^{4/3}(1+x)^{2/3}\right)\int \frac{1}{\sqrt[3]{x}\,(1+x)^{2/3}}\,dx}{3\left(x^2+x^3\right)^{2/3}}$$
$$= \sqrt[3]{x^2+x^3} - \frac{x^{4/3}(1+x)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{\sqrt{3}\left(x^2+x^3\right)^{2/3}} - \frac{x^{4/3}(1+x)^{2/3}\log(1+x)}{6\left(x^2+x^3\right)^{2/3}} - \frac{x^{4/3}(1+x)^{2/3}}{2\left(x^2\right.}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.36

$$\frac{3\sqrt[3]{x^2(x+1)}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-x\right)}{2\sqrt[3]{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2 + x^3)^(1/3)/x,x]
```

```
[Out] (3*(x^2*(1 + x))^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/3, -x])/(2*(1 + x)^(1
/3))
```

IntegrateAlgebraic [A] time = 0.25, size = 97, normalized size = 1.00

$$\sqrt[3]{x^3+x^2} - \frac{1}{3}\log\left(\sqrt[3]{x^3+x^2}-x\right) + \frac{1}{6}\log\left(x^2+\sqrt[3]{x^3+x^2}\,x+\left(x^3+x^2\right)^{2/3}\right) - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x^2}+x}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2 + x^3)^(1/3)/x,x]
```

```
[Out] (x^2 + x^3)^(1/3) - ArcTan[(Sqrt[3]*x)/(x + 2*(x^2 + x^3)^(1/3))]/Sqrt[3] -
 Log[-x + (x^2 + x^3)^(1/3)]/3 + Log[x^2 + x*(x^2 + x^3)^(1/3) + (x^2 + x^3
)^(2/3)]/6
```

fricas [A] time = 0.40, size = 93, normalized size = 0.96

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+x^2\right)^{\frac{1}{3}}}{3x}\right) + \left(x^3+x^2\right)^{\frac{1}{3}} - \frac{1}{3}\log\left(-\frac{x-\left(x^3+x^2\right)^{\frac{1}{3}}}{x}\right) + \frac{1}{6}\log\left(\frac{x^2+\left(x^3+x^2\right)^{\frac{1}{3}}x+\left(x^3+x^2\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 + x^2)^(1/3))/x) + (x^3
+ x^2)^(1/3) - 1/3*log(-(x - (x^3 + x^2)^(1/3))/x) + 1/6*log((x^2 + (x^3 +
x^2)^(1/3)*x + (x^3 + x^2)^(2/3))/x^2)
```

giac [A] time = 0.33, size = 64, normalized size = 0.66

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right) + x\left(\frac{1}{x}+1\right)^{\frac{1}{3}} + \frac{1}{6}\log\left(\left(\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right) - \frac{1}{3}\log\left(\left|\left(\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/3)/x,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(1/x + 1)^(1/3) + 1)) + x*(1/x + 1)^(1/3)
 + 1/6*log((1/x + 1)^(2/3) + (1/x + 1)^(1/3) + 1) - 1/3*log(abs((1/x + 1)^(
1/3) - 1))
```

maple [C] time = 0.52, size = 443, normalized size = 4.57

$$\left(x^{2}(1+x)\right)^{\frac{1}{3}}+\frac{\left(-\frac{\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^{2}-2\_Z+4\right)^{2}x^{2}+48\operatorname{RootOf}\left(\_Z^{2}-2\_Z+4\right)\left(x^{3}+2x^{2}+x\right)^{\frac{2}{3}}-30\operatorname{RootOf}\left(\_Z^{2}-2\_Z+4\right)\left(x^{3}+2x^{2}+x\right)^{\frac{1}{3}}x-16\operatorname{RootOf}\left(\_Z^{2}-2\_Z+4\right)x^{2}-36\left(x^{3}+2x^{2}+x\right)^{\frac{2}{3}}-30\operatorname{RootOf}\left(\_Z^{2}-2\_Z+4\right)\left(x^{3}+2x^{2}+x\right)^{\frac{1}{3}}+96\left(x^{3}+2x^{2}+x\right)^{\frac{1}{3}}x+\operatorname{RootOf}\left(\_Z^{2}-2\_Z+4\right)^{2}-14\operatorname{RootOf}\left(\_Z^{2}-2\_Z+4\right)x-64x^{2}+96\left(x^{3}+2x^{2}+x\right)^{\frac{1}{3}}+2\operatorname{RootOf}\left(\_Z^{2}-2\_Z+4\right)-112x-48}{1+x}\right)}{3}+\frac{\operatorname{RootOf}\left(\_Z^{2}-2\_Z+4\right)\ln\left(-\frac{2\operatorname{RootOf}\left(\_Z^{2}-2\_Z+4\right)^{2}x^{2}+24\operatorname{RootOf}\left(\_Z^{2}-2\_Z+4\right)\left(x^{3}+2x^{2}+x\right)^{\frac{2}{3}}-9\operatorname{RootOf}\left(\_Z^{2}-2\_Z+4\right)\left(x^{3}+2x^{2}+x\right)^{\frac{1}{3}}x-19\operatorname{RootOf}\left(\_Z^{2}-2\_Z+4\right)x^{2}-30\left(x^{3}+2x^{2}+x\right)^{\frac{2}{3}}-9\operatorname{RootOf}\left(\_Z^{2}-2\_Z+4\right)\left(x^{3}+2x^{2}+x\right)^{\frac{1}{3}}+48\left(x^{3}+2x^{2}+x\right)^{\frac{1}{3}}x-2\operatorname{RootOf}\left(\_Z^{2}-2\_Z+4\right)^{2}-28\operatorname{RootOf}\left(\_Z^{2}-2\_Z+4\right)x-10x^{2}+48\left(x^{3}+2x^{2}+x\right)^{\frac{1}{3}}-9\operatorname{RootOf}\left(\_Z^{2}-2\_Z+4\right)-14x-4}{1+x}\right)}{6}\right)\left(x^{2}(1+x)\right)^{\frac{1}{3}}\left(x(1+x)^{2}\right)^{\frac{1}{3}}}{x(1+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2)^(1/3)/x,x)
```

```
[Out] (x^2*(1+x))^(1/3)+(-1/3*ln((-RootOf(_Z^2-2*_Z+4)^2*x^2+48*RootOf(_Z^2-2*_Z+
4)*(x^3+2*x^2+x)^(2/3)-30*RootOf(_Z^2-2*_Z+4)*(x^3+2*x^2+x)^(1/3)*x-16*Root
Of(_Z^2-2*_Z+4)*x^2-36*(x^3+2*x^2+x)^(2/3)-30*RootOf(_Z^2-2*_Z+4)*(x^3+2*x^
2+x)^(1/3)+96*(x^3+2*x^2+x)^(1/3)*x+RootOf(_Z^2-2*_Z+4)^2-14*RootOf(_Z^2-2*
_Z+4)*x-64*x^2+96*(x^3+2*x^2+x)^(1/3)+2*RootOf(_Z^2-2*_Z+4)-112*x-48)/(1+x)
)+1/6*RootOf(_Z^2-2*_Z+4)*ln(-(2*RootOf(_Z^2-2*_Z+4)^2*x^2+24*RootOf(_Z^2-2
*_Z+4)*(x^3+2*x^2+x)^(2/3)-9*RootOf(_Z^2-2*_Z+4)*(x^3+2*x^2+x)^(1/3)*x-19*R
ootOf(_Z^2-2*_Z+4)*x^2-30*(x^3+2*x^2+x)^(2/3)-9*RootOf(_Z^2-2*_Z+4)*(x^3+2*
x^2+x)^(1/3)+48*(x^3+2*x^2+x)^(1/3)*x-2*RootOf(_Z^2-2*_Z+4)^2-28*RootOf(_Z^
2-2*_Z+4)*x-10*x^2+48*(x^3+2*x^2+x)^(1/3)-9*RootOf(_Z^2-2*_Z+4)-14*x-4)/(1+
x)))*(x^2*(1+x))^(1/3)/x*(x*(1+x)^2)^(1/3)/(1+x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}+x^{2}\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] integrate((x^3 + x^2)^(1/3)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3+x^2\right)^{1/3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3)^(1/3)/x,x)
```

```
[Out] int((x^2 + x^3)^(1/3)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x^{2}\left(x+1\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2)**(1/3)/x,x)
```

```
[Out] Integral((x**2*(x + 1))**(1/3)/x, x)
```

3.1200 $\int \frac{(1+x^3)^{2/3}(2+x^3)}{x^6(1+2x^3)}\,dx$

Optimal. Leaf size=97

$$\log\left(\sqrt[3]{x^3+1}+x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}-x}\right)+\frac{(x^3+1)^{2/3}(11x^3-4)}{10x^5}-\frac{1}{2}\log\left(-\sqrt[3]{x^3+1}\,x+(x^3+1)^{2/3}+x^2\right)$$

Rubi [A] time = 0.14, antiderivative size = 107, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {580, 583, 12, 377, 200, 31, 634, 618, 204, 628}

$$\log\left(\frac{x}{\sqrt[3]{x^3+1}}+1\right)-\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{x^3+1}}}{\sqrt{3}}\right)-\frac{2(x^3+1)^{2/3}}{5x^5}+\frac{11(x^3+1)^{2/3}}{10x^2}-\frac{1}{2}\log\left(-\frac{x}{\sqrt[3]{x^3+1}}+\frac{x^2}{(x^3+1)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^3)^(2/3)*(2 + x^3))/(x^6*(1 + 2*x^3)),x]
```

```
[Out] (-2*(1 + x^3)^(2/3))/(5*x^5) + (11*(1 + x^3)^(2/3))/(10*x^2) - Sqrt[3]*ArcT
an[(1 - (2*x)/(1 + x^3)^(1/3))/Sqrt[3]] - Log[1 + x^2/(1 + x^3)^(2/3) - x/(
1 + x^3)^(1/3)]/2 + Log[1 + x/(1 + x^3)^(1/3)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
```

```
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(1+2x^3\right)}\,dx = -\frac{2\left(1+x^3\right)^{2/3}}{5x^5}+\frac{1}{5}\int \frac{-11-7x^3}{x^3\sqrt[3]{1+x^3}\left(1+2x^3\right)}\,dx$$
$$= -\frac{2\left(1+x^3\right)^{2/3}}{5x^5}+\frac{11\left(1+x^3\right)^{2/3}}{10x^2}-\frac{1}{10}\int -\frac{30}{\sqrt[3]{1+x^3}\left(1+2x^3\right)}\,dx$$
$$= -\frac{2\left(1+x^3\right)^{2/3}}{5x^5}+\frac{11\left(1+x^3\right)^{2/3}}{10x^2}+3\int \frac{1}{\sqrt[3]{1+x^3}\left(1+2x^3\right)}\,dx$$
$$= -\frac{2\left(1+x^3\right)^{2/3}}{5x^5}+\frac{11\left(1+x^3\right)^{2/3}}{10x^2}+3\,\text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, \frac{x}{\sqrt[3]{1+x^3}}\right)$$
$$= -\frac{2\left(1+x^3\right)^{2/3}}{5x^5}+\frac{11\left(1+x^3\right)^{2/3}}{10x^2}+\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x}{\sqrt[3]{1+x^3}}\right)+\text{Subst}\left(\int \frac{}{1}\right.$$
$$= -\frac{2\left(1+x^3\right)^{2/3}}{5x^5}+\frac{11\left(1+x^3\right)^{2/3}}{10x^2}+\log\left(1+\frac{x}{\sqrt[3]{1+x^3}}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx\right.$$
$$= -\frac{2\left(1+x^3\right)^{2/3}}{5x^5}+\frac{11\left(1+x^3\right)^{2/3}}{10x^2}-\frac{1}{2}\log\left(1+\frac{x^2}{\left(1+x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1+x^3}}\right)+\log\left(1+\right.$$
$$= -\frac{2\left(1+x^3\right)^{2/3}}{5x^5}+\frac{11\left(1+x^3\right)^{2/3}}{10x^2}-\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(1+\frac{x^2}{\left(1+x^3\right)^{2/3}}\right.$$

Mathematica [A] time = 0.11, size = 106, normalized size = 1.09

$$3\left(\frac{1}{3}\log\left(\frac{x}{\sqrt[3]{x^3+1}}+1\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(-\frac{x}{\sqrt[3]{x^3+1}}+\frac{x^2}{\left(x^3+1\right)^{2/3}}+1\right)\right)+\left(x^3+1\right)^{2/3}\left(\frac{11}{10x^2}-\frac{2}{5x^5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)^(2/3)*(2 + x^3))/(x^6*(1 + 2*x^3)),x]
```

```
[Out] (-2/(5*x^5) + 11/(10*x^2))*(1 + x^3)^(2/3) + 3*(ArcTan[(-1 + (2*x)/(1 + x^3
)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 + x^2/(1 + x^3)^(2/3) - x/(1 + x^3)^(1/3)
]/6 + Log[1 + x/(1 + x^3)^(1/3)]/3)
```

IntegrateAlgebraic [A] time = 0.20, size = 97, normalized size = 1.00

$$\log\left(\sqrt[3]{x^3+1}+x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}-x}\right)+\frac{\left(x^3+1\right)^{2/3}\left(11x^3-4\right)}{10x^5}-\frac{1}{2}\log\left(-\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)^(2/3)*(2 + x^3))/(x^6*(1 + 2*x^3)),x]
```

```
[Out] ((1 + x^3)^(2/3)*(-4 + 11*x^3))/(10*x^5) + Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x +
 2*(1 + x^3)^(1/3))] + Log[x + (1 + x^3)^(1/3)] - Log[x^2 - x*(1 + x^3)^(1/
3) + (1 + x^3)^(2/3)]/2
```

fricas [A] time = 0.84, size = 124, normalized size = 1.28

$$-\frac{10\sqrt{3}x^5\arctan\left(\frac{4\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}x^2+2\sqrt{3}\left(x^3+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(x^3+1\right)}{7x^3-1}\right)-5x^5\log\left(\frac{2x^3+3\left(x^3+1\right)^{\frac{1}{3}}x^2+3\left(x^3+1\right)^{\frac{2}{3}}x+1}{2x^3+1}\right)-\left(11x^3-4\right)\left(x^3+1\right)^{\frac{2}{3}}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^3+2)/x^6/(2*x^3+1),x, algorithm="fricas")
```

```
[Out] -1/10*(10*sqrt(3)*x^5*arctan((4*sqrt(3)*(x^3 + 1)^(1/3)*x^2 + 2*sqrt(3)*(x^
3 + 1)^(2/3)*x + sqrt(3)*(x^3 + 1))/(7*x^3 - 1)) - 5*x^5*log((2*x^3 + 3*(x^
3 + 1)^(1/3)*x^2 + 3*(x^3 + 1)^(2/3)*x + 1)/(2*x^3 + 1)) - (11*x^3 - 4)*(x^
3 + 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+2\right)\left(x^3+1\right)^{\frac{2}{3}}}{\left(2\,x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^3+2)/x^6/(2*x^3+1),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 2)*(x^3 + 1)^(2/3)/((2*x^3 + 1)*x^6), x)
```

maple [C] time = 1.21, size = 425, normalized size = 4.38

$$\frac{11x^6+7x^3-4}{10x^5\left(x^3+1\right)^{\frac{1}{3}}}-3\ln\left(\frac{-9\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3+3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3+1\right)^{\frac{2}{3}}x-3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3+1\right)^{\frac{1}{3}}x^2-6\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3+2x\left(x^3+1\right)^{\frac{2}{3}}-2x^2\left(x^3+1\right)^{\frac{1}{3}}-x^3-3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)-1}{2x^3+1}\right)\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)+3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\ln\left(-\frac{9\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3+3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3+1\right)^{\frac{2}{3}}x-3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3+1\right)^{\frac{1}{3}}x^2-x\left(x^3+1\right)^{\frac{2}{3}}+x^2\left(x^3+1\right)^{\frac{1}{3}}-3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)}{2x^3+1}\right)-\ln\left(\frac{-9\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3+3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3+1\right)^{\frac{2}{3}}x-3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3+1\right)^{\frac{1}{3}}x^2-6\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3+2x\left(x^3+1\right)^{\frac{2}{3}}-2x^2\left(x^3+1\right)^{\frac{1}{3}}-x^3-3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)-1}{2x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)*(x^3+2)/x^6/(2*x^3+1),x)
```

```
[Out] 1/10*(11*x^6+7*x^3-4)/x^5/(x^3+1)^(1/3)-3*ln((-9*RootOf(9*_Z^2+3*_Z+1)^2*x^
3+3*RootOf(9*_Z^2+3*_Z+1)*(x^3+1)^(2/3)*x-3*RootOf(9*_Z^2+3*_Z+1)*(x^3+1)^(
1/3)*x^2-6*RootOf(9*_Z^2+3*_Z+1)*x^3+2*x*(x^3+1)^(2/3)-2*x^2*(x^3+1)^(1/3)-
x^3-3*RootOf(9*_Z^2+3*_Z+1)-1)/(2*x^3+1))*RootOf(9*_Z^2+3*_Z+1)+3*RootOf(9*
_Z^2+3*_Z+1)*ln(-(9*RootOf(9*_Z^2+3*_Z+1)^2*x^3+3*RootOf(9*_Z^2+3*_Z+1)*(x^
3+1)^(2/3)*x-3*RootOf(9*_Z^2+3*_Z+1)*(x^3+1)^(1/3)*x^2-x*(x^3+1)^(2/3)+x^2*
(x^3+1)^(1/3)-3*RootOf(9*_Z^2+3*_Z+1))/(2*x^3+1))-ln((-9*RootOf(9*_Z^2+3*_Z
+1)^2*x^3+3*RootOf(9*_Z^2+3*_Z+1)*(x^3+1)^(2/3)*x-3*RootOf(9*_Z^2+3*_Z+1)*(
x^3+1)^(1/3)*x^2-6*RootOf(9*_Z^2+3*_Z+1)*x^3+2*x*(x^3+1)^(2/3)-2*x^2*(x^3+1
)^(1/3)-x^3-3*RootOf(9*_Z^2+3*_Z+1)-1)/(2*x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+2\right)\left(x^3+1\right)^{\frac{2}{3}}}{\left(2\,x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^3+2)/x^6/(2*x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 2)*(x^3 + 1)^(2/3)/((2*x^3 + 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(x^3+1\right)}^{2/3}\,\left(x^3+2\right)}{x^6\,\left(2\,x^3+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)^(2/3)*(x^3 + 2))/(x^6*(2*x^3 + 1)),x)
```

```
[Out] int(((x^3 + 1)^(2/3)*(x^3 + 2))/(x^6*(2*x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((x+1)\left(x^2-x+1\right)\right)^{\frac{2}{3}}\left(x^3+2\right)}{x^6\left(2x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)*(x**3+2)/x**6/(2*x**3+1),x)
[Out] Integral(((x + 1)*(x**2 - x + 1))**(2/3)*(x**3 + 2)/(x**6*(2*x**3 + 1)), x)
```

3.1201 $\int \frac{(-3+x^4)\sqrt[3]{1+x^4}}{x^9}\,dx$

Optimal. Leaf size=97

$$\frac{1}{6}\log\left(\sqrt[3]{x^4+1}-1\right)-\frac{1}{12}\log\left(\left(x^4+1\right)^{2/3}+\sqrt[3]{x^4+1}+1\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\sqrt[3]{x^4+1}\left(3-x^4\right)}{8x^8}$$

Rubi [A] time = 0.06, antiderivative size = 86, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {446, 78, 47, 57, 618, 204, 31}

$$-\frac{\sqrt[3]{x^4+1}}{2x^4}+\frac{1}{4}\log\left(1-\sqrt[3]{x^4+1}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3\left(x^4+1\right)^{4/3}}{8x^8}-\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + x^4)*(1 + x^4)^(1/3))/x^9,x]
```

```
[Out] -1/2*(1 + x^4)^(1/3)/x^4 + (3*(1 + x^4)^(4/3))/(8*x^8) - ArcTan[(1 + 2*(1 +
 x^4)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - Log[x]/3 + Log[1 - (1 + x^4)^(1/3)]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-3+x^4)\sqrt[3]{1+x^4}}{x^9}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{(-3+x)\sqrt[3]{1+x}}{x^3}\,dx, x, x^4\right)\\
&= \frac{3\left(1+x^4\right)^{4/3}}{8x^8} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt[3]{1+x}}{x^2}\,dx, x, x^4\right)\\
&= -\frac{\sqrt[3]{1+x^4}}{2x^4} + \frac{3\left(1+x^4\right)^{4/3}}{8x^8} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x(1+x)^{2/3}}\,dx, x, x^4\right)\\
&= -\frac{\sqrt[3]{1+x^4}}{2x^4} + \frac{3\left(1+x^4\right)^{4/3}}{8x^8} - \frac{\log(x)}{3} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+x^4}\right) - \frac{1}{4}\operatorname{Subs}\\
&= -\frac{\sqrt[3]{1+x^4}}{2x^4} + \frac{3\left(1+x^4\right)^{4/3}}{8x^8} - \frac{\log(x)}{3} + \frac{1}{4}\log\left(1-\sqrt[3]{1+x^4}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\right.\\
&= -\frac{\sqrt[3]{1+x^4}}{2x^4} + \frac{3\left(1+x^4\right)^{4/3}}{8x^8} - \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^4}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\log(x)}{3} + \frac{1}{4}\log\left(1-\sqrt[3]{1+x^4}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.36

$$\frac{3\left(x^4+1\right)^{4/3}\left(x^8\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};x^4+1\right)+1\right)}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + x^4)*(1 + x^4)^(1/3))/x^9,x]
```

```
[Out] (3*(1 + x^4)^(4/3)*(1 + x^8*Hypergeometric2F1[4/3, 2, 7/3, 1 + x^4]))/(8*x^
8)
```

IntegrateAlgebraic [A] time = 0.09, size = 97, normalized size = 1.00

$$\frac{1}{6}\log\left(\sqrt[3]{x^4+1}-1\right) - \frac{1}{12}\log\left(\left(x^4+1\right)^{2/3}+\sqrt[3]{x^4+1}+1\right) - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\sqrt[3]{x^4+1}\left(3-x^4\right)}{8x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 + x^4)^(1/3))/x^9,x]
```

```
[Out] ((3 - x^4)*(1 + x^4)^(1/3))/(8*x^8) - ArcTan[1/Sqrt[3] + (2*(1 + x^4)^(1/3)
)/Sqrt[3]]/(2*Sqrt[3]) + Log[-1 + (1 + x^4)^(1/3)]/6 - Log[1 + (1 + x^4)^(1
/3) + (1 + x^4)^(2/3)]/12
```

fricas [A] time = 0.41, size = 84, normalized size = 0.87

$$-\frac{4\sqrt{3}x^8\arctan\left(\frac{2}{3}\sqrt{3}\left(x^4+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+2x^8\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)-4x^8\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)+3\left(x^4+1\right)^{\frac{1}{3}}\left(x^4-3\right)}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(1/3)/x^9,x, algorithm="fricas")
```

```
[Out] -1/24*(4*sqrt(3)*x^8*arctan(2/3*sqrt(3)*(x^4 + 1)^(1/3) + 1/3*sqrt(3)) + 2*
x^8*log((x^4 + 1)^(2/3) + (x^4 + 1)^(1/3) + 1) - 4*x^8*log((x^4 + 1)^(1/3)
- 1) + 3*(x^4 + 1)^(1/3)*(x^4 - 3))/x^8
```

giac [A] time = 0.24, size = 76, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^4+1\right)^{\frac{4}{3}}-4\left(x^4+1\right)^{\frac{1}{3}}}{8x^8}-\frac{1}{12}\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(1/3)/x^9,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 + 1)^(1/3) + 1)) - 1/8*((x^4 + 1)^(
4/3) - 4*(x^4 + 1)^(1/3))/x^8 - 1/12*log((x^4 + 1)^(2/3) + (x^4 + 1)^(1/3)
+ 1) + 1/6*log((x^4 + 1)^(1/3) - 1)
```

maple [C] time = 0.35, size = 69, normalized size = 0.71

$$-\frac{x^8-2x^4-3}{8x^8\left(x^4+1\right)^{\frac{2}{3}}}+\frac{-\frac{2\Gamma\left(\frac{2}{3}\right)x^4\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],-x^4\right)}{3}+\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+4\ln(x)\right)\Gamma\left(\frac{2}{3}\right)}{6\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4+1)^(1/3)/x^9,x)
```

```
[Out] -1/8*(x^8-2*x^4-3)/x^8/(x^4+1)^(2/3)+1/6/GAMMA(2/3)*(-2/3*GAMMA(2/3)*x^4*hy
pergeom([1,1,5/3],[2,2],-x^4)+(1/6*Pi*3^(1/2)-3/2*ln(3)+4*ln(x))*GAMMA(2/3)
)
```

maxima [A] time = 0.41, size = 103, normalized size = 1.06

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^4+1\right)^{\frac{4}{3}}+2\left(x^4+1\right)^{\frac{1}{3}}}{8\left(2x^4-\left(x^4+1\right)^2+1\right)}-\frac{\left(x^4+1\right)^{\frac{1}{3}}}{4x^4}-\frac{1}{12}\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(1/3)/x^9,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 + 1)^(1/3) + 1)) - 1/8*((x^4 + 1)^(
4/3) + 2*(x^4 + 1)^(1/3))/(2*x^4 - (x^4 + 1)^2 + 1) - 1/4*(x^4 + 1)^(1/3)/x
^4 - 1/12*log((x^4 + 1)^(2/3) + (x^4 + 1)^(1/3) + 1) + 1/6*log((x^4 + 1)^(1
/3) - 1)
```

mupad [B] time = 1.27, size = 121, normalized size = 1.25

$$\frac{\ln\left(\frac{\left(x^4+1\right)^{1/3}}{16}-\frac{1}{16}\right)}{6}-\frac{\frac{\left(x^4+1\right)^{1/3}}{4}+\frac{\left(x^4+1\right)^{4/3}}{8}}{2x^4-\left(x^4+1\right)^2+1}-\frac{\left(x^4+1\right)^{1/3}}{4x^4}+\ln\left(\frac{3\left(x^4+1\right)^{1/3}}{4}+\frac{3}{8}-\frac{\sqrt{3}\,3\mathrm{i}}{8}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(\frac{3\left(x^4+1\right)^{1/3}}{4}+\frac{3}{8}+\frac{\sqrt{3}\,3\mathrm{i}}{8}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/3)*(x^4 - 3))/x^9,x)
```

```
[Out] log((x^4 + 1)^(1/3)/16 - 1/16)/6 - ((x^4 + 1)^(1/3)/4 + (x^4 + 1)^(4/3)/8)/
(2*x^4 - (x^4 + 1)^2 + 1) - (x^4 + 1)^(1/3)/(4*x^4) + log((3*(x^4 + 1)^(1/3
))/4 - (3^(1/2)*3i)/8 + 3/8)*((3^(1/2)*1i)/12 - 1/12) - log((3^(1/2)*3i)/8
+ (3*(x^4 + 1)^(1/3))/4 + 3/8)*((3^(1/2)*1i)/12 + 1/12)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4+1)**(1/3)/x**9,x)
```

```
[Out] Timed out
```

3.1202 $\int \frac{-b+ax^3}{x^3\sqrt[4]{bx+ax^4}}\,dx$

Optimal. Leaf size=97

$$\frac{2}{3}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)+\frac{2}{3}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)+\frac{4\left(ax^4+bx\right)^{3/4}}{9x^3}$$

Rubi [A] time = 0.18, antiderivative size = 143, normalized size of antiderivative = 1.47, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2038, 2011, 329, 275, 240, 212, 206, 203}

$$\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3+b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{ax^4+bx}}+\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3+b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{ax^4+bx}}+\frac{4\left(ax^4+bx\right)^{3/4}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^3)/(x^3*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*(b*x + a*x^4)^(3/4))/(9*x^3) + (2*a^(3/4)*x^(1/4)*(b + a*x^3)^(1/4)*ArcT
an[(a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)])/(3*(b*x + a*x^4)^(1/4)) + (2*a^(3/
4)*x^(1/4)*(b + a*x^3)^(1/4)*ArcTanh[(a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)])/
(3*(b*x + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-b+ax^3}{x^3\sqrt[4]{bx+ax^4}}\,dx &= \frac{4\left(bx+ax^4\right)^{3/4}}{9x^3}+a\int \frac{1}{\sqrt[4]{bx+ax^4}}\,dx\\
&= \frac{4\left(bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(a\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\int \frac{1}{\sqrt[4]{x}\,\sqrt[4]{b+ax^3}}\,dx}{\sqrt[4]{bx+ax^4}}\\
&= \frac{4\left(bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(4a\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt[4]{b+ax^{12}}}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^4}}\\
&= \frac{4\left(bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(4a\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx,x,x^{3/4}\right)}{3\sqrt[4]{bx+ax^4}}\\
&= \frac{4\left(bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(4a\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^4}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}\\
&= \frac{4\left(bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(2a\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}+\frac{\left(2a\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right.}{}\\
&= \frac{4\left(bx+ax^4\right)^{3/4}}{9x^3}+\frac{2a^{3/4}\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}+\frac{2a^{3/4}\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+a}}\right.}{3\sqrt[4]{bx+ax^4}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 66, normalized size = 0.68

$$\frac{4\left(3ax^3\sqrt[4]{\frac{ax^3}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{ax^3}{b}\right)+ax^3+b\right)}{9x^2\sqrt[4]{x\left(ax^3+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^3)/(x^3*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*(b + a*x^3 + 3*a*x^3*(1 + (a*x^3)/b)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5
/4, -((a*x^3)/b)]))/(9*x^2*(x*(b + a*x^3))^(1/4))
```

IntegrateAlgebraic [A] time = 0.33, size = 97, normalized size = 1.00

$$\frac{2}{3}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)+\frac{2}{3}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)+\frac{4\left(ax^4+bx\right)^{3/4}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^3)/(x^3*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*(b*x + a*x^4)^(3/4))/(9*x^3) + (2*a^(3/4)*ArcTan[(a^(1/4)*(b*x + a*x^4)^
(3/4))/(b + a*x^3)])/3 + (2*a^(3/4)*ArcTanh[(a^(1/4)*(b*x + a*x^4)^(3/4))/(
b + a*x^3)])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)/x^3/(a*x^4+b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.23, size = 185, normalized size = 1.91

$$\frac{1}{3}\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)+\frac{1}{3}\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)-\frac{1}{6}\sqrt{2}(-a)^{\frac{3}{4}}\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)+\frac{1}{6}\sqrt{2}(-a)^{\frac{3}{4}}\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)+\frac{4}{9}\left(a+\frac{b}{x^3}\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)/x^3/(a*x^4+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^
3)^(1/4))/(-a)^(1/4)) + 1/3*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(-a)^(1/4) - 2*(a + b/x^3)^(1/4))/(-a)^(1/4)) - 1/6*sqrt(2)*(-a)^(3/4)*log
(sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a) + sqrt(a + b/x^3)) + 1/6*s
qrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a) + sq
rt(a + b/x^3)) + 4/9*(a + b/x^3)^(3/4)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{a\,x^{3}-b}{x^{3}\left(a\,x^{4}+bx\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b)/x^3/(a*x^4+b*x)^(1/4),x)
```

```
[Out] int((a*x^3-b)/x^3/(a*x^4+b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{3}-b}{\left(ax^{4}+bx\right)^{\frac{1}{4}}x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)/x^3/(a*x^4+b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^3 - b)/((a*x^4 + b*x)^(1/4)*x^3), x)
```

mupad [B] time = 1.14, size = 58, normalized size = 0.60

$$\frac{4\left(a\,x^4+b\,x\right)^{3/4}}{9\,x^3}+\frac{4\,a\,x\,\left(\frac{a\,x^3}{b}+1\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{a\,x^3}{b}\right)}{3\,\left(a\,x^4+b\,x\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^3)/(x^3*(b*x + a*x^4)^(1/4)),x)
```

```
[Out] (4*(b*x + a*x^4)^(3/4))/(9*x^3) + (4*a*x*((a*x^3)/b + 1)^(1/4)*hypergeom([1
/4, 1/4], 5/4, -(a*x^3)/b))/(3*(b*x + a*x^4)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3-b}{x^3\sqrt[4]{x\left(ax^3+b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b)/x**3/(a*x**4+b*x)**(1/4),x)
```

```
[Out] Integral((a*x**3 - b)/(x**3*(x*(a*x**3 + b))**(1/4)), x)
```

3.1203 $\int \frac{b+ax^3}{x^3\sqrt[4]{bx+ax^4}}\,dx$

Optimal. Leaf size=97

$$\frac{2}{3}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)+\frac{2}{3}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)-\frac{4\left(ax^4+bx\right)^{3/4}}{9x^3}$$

Rubi [A] time = 0.23, antiderivative size = 143, normalized size of antiderivative = 1.47, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2052, 2011, 329, 275, 240, 212, 206, 203, 2014}

$$\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3+b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{ax^4+bx}}+\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3+b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{ax^4+bx}}-\frac{4\left(ax^4+bx\right)^{3/4}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^3)/(x^3*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(b*x + a*x^4)^(3/4))/(9*x^3) + (2*a^(3/4)*x^(1/4)*(b + a*x^3)^(1/4)*Arc
Tan[(a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)])/(3*(b*x + a*x^4)^(1/4)) + (2*a^(3
/4)*x^(1/4)*(b + a*x^3)^(1/4)*ArcTanh[(a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)])
/(3*(b*x + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{b+ax^3}{x^3\sqrt[4]{bx+ax^4}}\,dx &= \int \left(\frac{a}{\sqrt[4]{bx+ax^4}}+\frac{b}{x^3\sqrt[4]{bx+ax^4}}\right)dx\\
&= a\int \frac{1}{\sqrt[4]{bx+ax^4}}\,dx+b\int \frac{1}{x^3\sqrt[4]{bx+ax^4}}\,dx\\
&= -\frac{4\left(bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(a\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\int \frac{1}{\sqrt[4]{x}\,\sqrt[4]{b+ax^3}}\,dx}{\sqrt[4]{bx+ax^4}}\\
&= -\frac{4\left(bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(4a\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt[4]{b+ax^{12}}}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^4}}\\
&= -\frac{4\left(bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(4a\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx,x,x^{3/4}\right)}{3\sqrt[4]{bx+ax^4}}\\
&= -\frac{4\left(bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(4a\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^4}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}\\
&= -\frac{4\left(bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(2a\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}+\frac{\left(2a\sqrt[4]{x}\,\sqrt[4]{b+ax\right.}{}}{}\\
&= -\frac{4\left(bx+ax^4\right)^{3/4}}{9x^3}+\frac{2a^{3/4}\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}+\frac{2a^{3/4}\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}}{\sqrt[4]{b}}\right.}{3\sqrt[4]{bx+ax^4}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 53, normalized size = 0.55

$$-\frac{4\left(x\left(ax^3+b\right)\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};-\frac{ax^3}{b}\right)}{9x^3\left(\frac{ax^3}{b}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^3)/(x^3*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(x*(b + a*x^3))^(3/4)*Hypergeometric2F1[-3/4, -3/4, 1/4, -((a*x^3)/b)])
/(9*x^3*(1 + (a*x^3)/b)^(3/4))
```

IntegrateAlgebraic [A] time = 0.31, size = 97, normalized size = 1.00

$$\frac{2}{3}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)+\frac{2}{3}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)-\frac{4\left(ax^4+bx\right)^{3/4}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^3)/(x^3*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(b*x + a*x^4)^(3/4))/(9*x^3) + (2*a^(3/4)*ArcTan[(a^(1/4)*(b*x + a*x^4)
^(3/4))/(b + a*x^3)])/3 + (2*a^(3/4)*ArcTanh[(a^(1/4)*(b*x + a*x^4)^(3/4))/
(b + a*x^3)])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)/x^3/(a*x^4+b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.22, size = 185, normalized size = 1.91

$$\frac{1}{3}\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)+\frac{1}{3}\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)-\frac{1}{6}\sqrt{2}(-a)^{\frac{3}{4}}\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)+\frac{1}{6}\sqrt{2}(-a)^{\frac{3}{4}}\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)-\frac{4}{9}\left(a+\frac{b}{x^3}\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)/x^3/(a*x^4+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^
3)^(1/4))/(-a)^(1/4)) + 1/3*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(-a)^(1/4) - 2*(a + b/x^3)^(1/4))/(-a)^(1/4)) - 1/6*sqrt(2)*(-a)^(3/4)*log
(sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a) + sqrt(a + b/x^3)) + 1/6*s
qrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a) + sq
rt(a + b/x^3)) - 4/9*(a + b/x^3)^(3/4)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a\,x^3+b}{x^3\left(a\,x^4+bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b)/x^3/(a*x^4+b*x)^(1/4),x)
```

[Out] int((a*x^3+b)/x^3/(a*x^4+b*x)^(1/4),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 + b}{\left(ax^4 + bx\right)^{\frac{1}{4}} x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^3+b)/x^3/(a*x^4+b*x)^(1/4),x, algorithm="maxima")

[Out] integrate((a*x^3 + b)/((a*x^4 + b*x)^(1/4)*x^3), x)

mupad [B] time = 0.96, size = 58, normalized size = 0.60

$$\frac{4\,a\,x\,{\left(\frac{a\,x^3}{b}+1\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{a\,x^3}{b}\right)}{3\,{\left(a\,x^4+b\,x\right)}^{1/4}}-\frac{4\,{\left(a\,x^4+b\,x\right)}^{3/4}}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + a*x^3)/(x^3*(b*x + a*x^4)^(1/4)),x)

[Out] (4*a*x*((a*x^3)/b + 1)^(1/4)*hypergeom([1/4, 1/4], 5/4, -(a*x^3)/b))/(3*(b*x + a*x^4)^(1/4)) - (4*(b*x + a*x^4)^(3/4))/(9*x^3)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 + b}{x^3 \sqrt[4]{x \left(ax^3 + b\right)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**3+b)/x**3/(a*x**4+b*x)**(1/4),x)

[Out] Integral((a*x**3 + b)/(x**3*(x*(a*x**3 + b))**(1/4)), x)

3.1204 $\int \frac{1}{x^{13}\sqrt[3]{1+x^6}}\,dx$

Optimal. Leaf size=97

$$\frac{1}{27}\log\left(\sqrt[3]{x^6+1}-1\right)-\frac{1}{54}\log\left(\left(x^6+1\right)^{2/3}+\sqrt[3]{x^6+1}+1\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^6+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\left(x^6+1\right)^{2/3}\left(4x^6-3\right)}{36x^{12}}$$

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 51, 55, 618, 204, 31}

$$\frac{\left(x^6+1\right)^{2/3}}{9x^6}+\frac{1}{18}\log\left(1-\sqrt[3]{x^6+1}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^6+1}+1}{\sqrt{3}}\right)}{9\sqrt{3}}-\frac{\left(x^6+1\right)^{2/3}}{12x^{12}}-\frac{\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*(1 + x^6)^(1/3)),x]
```

```
[Out] -1/12*(1 + x^6)^(2/3)/x^12 + (1 + x^6)^(2/3)/(9*x^6) + ArcTan[(1 + 2*(1 + x
^6)^(1/3))/Sqrt[3]]/(9*Sqrt[3]) - Log[x]/9 + Log[1 - (1 + x^6)^(1/3)]/18
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{13}\sqrt[3]{1+x^6}}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{x^3\sqrt[3]{1+x}}\,dx, x, x^6\right)\\
&= -\frac{\left(1+x^6\right)^{2/3}}{12x^{12}} - \frac{1}{9}\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{1+x}}\,dx, x, x^6\right)\\
&= -\frac{\left(1+x^6\right)^{2/3}}{12x^{12}} + \frac{\left(1+x^6\right)^{2/3}}{9x^6} + \frac{1}{27}\text{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx, x, x^6\right)\\
&= -\frac{\left(1+x^6\right)^{2/3}}{12x^{12}} + \frac{\left(1+x^6\right)^{2/3}}{9x^6} - \frac{\log(x)}{9} - \frac{1}{18}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+x^6}\right) + \frac{1}{18}\text{Subst}\left(\int\right.\\
&= -\frac{\left(1+x^6\right)^{2/3}}{12x^{12}} + \frac{\left(1+x^6\right)^{2/3}}{9x^6} - \frac{\log(x)}{9} + \frac{1}{18}\log\left(1-\sqrt[3]{1+x^6}\right) - \frac{1}{9}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x\right.\\
&= -\frac{\left(1+x^6\right)^{2/3}}{12x^{12}} + \frac{\left(1+x^6\right)^{2/3}}{9x^6} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^6}}{\sqrt{3}}\right)}{9\sqrt{3}} - \frac{\log(x)}{9} + \frac{1}{18}\log\left(1-\sqrt[3]{1+x^6}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.27

$$-\frac{1}{4}\left(x^6+1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},3;\frac{5}{3};x^6+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^13*(1 + x^6)^(1/3)),x]
[Out] -1/4*((1 + x^6)^(2/3)*Hypergeometric2F1[2/3, 3, 5/3, 1 + x^6])
```

IntegrateAlgebraic [A] time = 0.12, size = 97, normalized size = 1.00

$$\frac{1}{27}\log\left(\sqrt[3]{x^6+1}-1\right) - \frac{1}{54}\log\left(\left(x^6+1\right)^{2/3}+\sqrt[3]{x^6+1}+1\right) + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^6+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{9\sqrt{3}} + \frac{\left(x^6+1\right)^{2/3}\left(4x^6-3\right)}{36x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^13*(1 + x^6)^(1/3)),x]
[Out] ((1 + x^6)^(2/3)*(-3 + 4*x^6))/(36*x^12) + ArcTan[1/Sqrt[3] + (2*(1 + x^6)^
(1/3))/Sqrt[3]]/(9*Sqrt[3]) + Log[-1 + (1 + x^6)^(1/3)]/27 - Log[1 + (1 + x
^6)^(1/3) + (1 + x^6)^(2/3)]/54
```

fricas [A] time = 0.40, size = 86, normalized size = 0.89

$$\frac{4\sqrt{3}x^{12}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^6+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right) - 2x^{12}\log\left(\left(x^6+1\right)^{\frac{2}{3}}+\left(x^6+1\right)^{\frac{1}{3}}+1\right) + 4x^{12}\log\left(\left(x^6+1\right)^{\frac{1}{3}}-1\right) + 3\left(4x^6-3\right)\left(x^6+1\right)^{\frac{2}{3}}}{108x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^6+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/108*(4*sqrt(3)*x^12*arctan(2/3*sqrt(3)*(x^6 + 1)^(1/3) + 1/3*sqrt(3)) - 2
*x^12*log((x^6 + 1)^(2/3) + (x^6 + 1)^(1/3) + 1) + 4*x^12*log((x^6 + 1)^(1/
3) - 1) + 3*(4*x^6 - 3)*(x^6 + 1)^(2/3))/x^12
```

giac [A] time = 0.18, size = 78, normalized size = 0.80

$$\frac{1}{27}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(x^6+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{4\left(x^6+1\right)^{\frac{5}{3}}-7\left(x^6+1\right)^{\frac{2}{3}}}{36\,x^{12}}-\frac{1}{54}\,\log\left(\left(x^6+1\right)^{\frac{2}{3}}+\left(x^6+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{27}\,\log\left(\left(x^6+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^6+1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3) + 1)) + 1/36*(4*(x^6 + 1
)^(5/3) - 7*(x^6 + 1)^(2/3))/x^12 - 1/54*log((x^6 + 1)^(2/3) + (x^6 + 1)^(1
/3) + 1) + 1/27*log((x^6 + 1)^(1/3) - 1)
```

maple [C] time = 0.42, size = 86, normalized size = 0.89

$$\frac{4x^{12}+x^6-3}{36x^{12}\left(x^6+1\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^6\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],-x^6\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+6\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{54\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(x^6+1)^(1/3),x)
```

```
[Out] 1/36*(4*x^12+x^6-3)/x^12/(x^6+1)^(1/3)+1/54/Pi*3^(1/2)*GAMMA(2/3)*(-2/9*Pi*
3^(1/2)/GAMMA(2/3)*x^6*hypergeom([1,1,4/3],[2,2],-x^6)+2/3*(-1/6*Pi*3^(1/2)
-3/2*ln(3)+6*ln(x))*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.42, size = 93, normalized size = 0.96

$$\frac{1}{27}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(x^6+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{4\left(x^6+1\right)^{\frac{5}{3}}-7\left(x^6+1\right)^{\frac{2}{3}}}{36\left(2\,x^6-\left(x^6+1\right)^2+1\right)}-\frac{1}{54}\,\log\left(\left(x^6+1\right)^{\frac{2}{3}}+\left(x^6+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{27}\,\log\left(\left(x^6+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^6+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3) + 1)) - 1/36*(4*(x^6 + 1
)^(5/3) - 7*(x^6 + 1)^(2/3))/(2*x^6 - (x^6 + 1)^2 + 1) - 1/54*log((x^6 + 1)
^(2/3) + (x^6 + 1)^(1/3) + 1) + 1/27*log((x^6 + 1)^(1/3) - 1)
```

mupad [B] time = 0.99, size = 118, normalized size = 1.22

$$\frac{\ln\left(\frac{\left(x^6+1\right)^{1/3}}{81}-\frac{1}{81}\right)}{27}+\ln\left(\frac{\left(x^6+1\right)^{1/3}}{81}-9\left(-\frac{1}{54}+\frac{\sqrt{3}\,1{}\mathrm{i}}{54}\right)^2\right)\left(-\frac{1}{54}+\frac{\sqrt{3}\,1{}\mathrm{i}}{54}\right)-\ln\left(\frac{\left(x^6+1\right)^{1/3}}{81}-9\left(\frac{1}{54}+\frac{\sqrt{3}\,1{}\mathrm{i}}{54}\right)^2\right)\left(\frac{1}{54}+\frac{\sqrt{3}\,1{}\mathrm{i}}{54}\right)+\frac{\frac{7\left(x^6+1\right)^{2/3}}{36}-\frac{\left(x^6+1\right)^{5/3}}{9}}{2\,x^6-\left(x^6+1\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^13*(x^6 + 1)^(1/3)),x)
```

```
[Out] log((x^6 + 1)^(1/3)/81 - 1/81)/27 + log((x^6 + 1)^(1/3)/81 - 9*((3^(1/2)*1i
)/54 - 1/54)^2)*((3^(1/2)*1i)/54 - 1/54) - log((x^6 + 1)^(1/3)/81 - 9*((3^(
1/2)*1i)/54 + 1/54)^2)*((3^(1/2)*1i)/54 + 1/54) + ((7*(x^6 + 1)^(2/3))/36 -
 (x^6 + 1)^(5/3)/9)/(2*x^6 - (x^6 + 1)^2 + 1)
```

sympy [C] time = 1.49, size = 31, normalized size = 0.32

$$-\frac{\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^6}\right)}{6x^{14}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(x**6+1)**(1/3),x)
```

```
[Out] -gamma(7/3)*hyper((1/3, 7/3), (10/3,), exp_polar(I*pi)/x**6)/(6*x**14*gamma
(10/3))
```

3.1205 $\int \frac{-3+5x^8}{\left(1+x^8\right)\sqrt[3]{1-x^3+x^8}}\,dx$

Optimal. Leaf size=97

$$-\log\left(\sqrt[3]{x^8-x^3+1}+x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^8-x^3+1}-x}\right)+\frac{1}{2}\log\left(x^2-\sqrt[3]{x^8-x^3+1}\,x+\left(x^8-x^3+1\right)^{2/3}\right)$$

Rubi [F] time = 1.96, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3+5x^8}{\left(1+x^8\right)\sqrt[3]{1-x^3+x^8}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3 + 5*x^8)/((1 + x^8)*(1 - x^3 + x^8)^(1/3)),x]
```

```
[Out] 5*Defer[Int][(1 - x^3 + x^8)^(-1/3), x] - (-1)^(1/8)*Defer[Int][1/(((-1)^(1
/8) - x)*(1 - x^3 + x^8)^(1/3)), x] - (-1)^(3/8)*Defer[Int][1/(((-1)^(3/8)
- x)*(1 - x^3 + x^8)^(1/3)), x] + (-1)^(5/8)*Defer[Int][1/((-(-1)^(5/8) - x
)*(1 - x^3 + x^8)^(1/3)), x] + (-1)^(7/8)*Defer[Int][1/((-(-1)^(7/8) - x)*(
1 - x^3 + x^8)^(1/3)), x] - (-1)^(1/8)*Defer[Int][1/(((-1)^(1/8) + x)*(1 -
x^3 + x^8)^(1/3)), x] - (-1)^(3/8)*Defer[Int][1/(((-1)^(3/8) + x)*(1 - x^3
+ x^8)^(1/3)), x] + (-1)^(5/8)*Defer[Int][1/((-(-1)^(5/8) + x)*(1 - x^3 + x
^8)^(1/3)), x] + (-1)^(7/8)*Defer[Int][1/((-(-1)^(7/8) + x)*(1 - x^3 + x^8)
^(1/3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3+5x^8}{\left(1+x^8\right)\sqrt[3]{1-x^3+x^8}}\,dx &= \int \left(\frac{5}{\sqrt[3]{1-x^3+x^8}}-\frac{8}{\left(1+x^8\right)\sqrt[3]{1-x^3+x^8}}\right)dx\\
&= 5\int \frac{1}{\sqrt[3]{1-x^3+x^8}}\,dx-8\int \frac{1}{\left(1+x^8\right)\sqrt[3]{1-x^3+x^8}}\,dx\\
&= 5\int \frac{1}{\sqrt[3]{1-x^3+x^8}}\,dx-8\int \left(\frac{i}{2\left(i-x^4\right)\sqrt[3]{1-x^3+x^8}}+\frac{i}{2\left(i+x^4\right)\sqrt[3]{1-x^3+x^8}}\right.\\
&= -\left(4i\int \frac{1}{\left(i-x^4\right)\sqrt[3]{1-x^3+x^8}}\,dx\right)-4i\int \frac{1}{\left(i+x^4\right)\sqrt[3]{1-x^3+x^8}}\,dx+5\int \frac{1}{\sqrt[3]{1-x}}\\
&= -\left(4i\int \left(-\frac{(-1)^{3/4}}{2\left(\sqrt[4]{-1}-x^2\right)\sqrt[3]{1-x^3+x^8}}-\frac{(-1)^{3/4}}{2\left(\sqrt[4]{-1}+x^2\right)\sqrt[3]{1-x^3+x^8}}\right)dx\right)-4i\int\\
&= 5\int \frac{1}{\sqrt[3]{1-x^3+x^8}}\,dx-\left(2\sqrt[4]{-1}\right)\int \frac{1}{\left(\sqrt[4]{-1}-x^2\right)\sqrt[3]{1-x^3+x^8}}\,dx-\left(2\sqrt[4]{-1}\right)\int\\
&= 5\int \frac{1}{\sqrt[3]{1-x^3+x^8}}\,dx-\left(2\sqrt[4]{-1}\right)\int \left(-\frac{(-1)^{7/8}}{2\left(\sqrt[8]{-1}-x\right)\sqrt[3]{1-x^3+x^8}}-\frac{(-1}{2\left(\sqrt[8]{-1}+x\right)}\right.\\
&= 5\int \frac{1}{\sqrt[3]{1-x^3+x^8}}\,dx-\sqrt[8]{-1}\int \frac{1}{\left(\sqrt[8]{-1}-x\right)\sqrt[3]{1-x^3+x^8}}\,dx-\sqrt[8]{-1}\int \frac{}{\left(\sqrt[8]{-1}+x\right.}
\end{aligned}$$

Mathematica [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{-3+5x^8}{\left(1+x^8\right)\sqrt[3]{1-x^3+x^8}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(-3 + 5*x^8)/((1 + x^8)*(1 - x^3 + x^8)^(1/3)),x]

[Out] Integrate[(-3 + 5*x^8)/((1 + x^8)*(1 - x^3 + x^8)^(1/3)), x]

IntegrateAlgebraic [A] time = 2.85, size = 97, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^8-x^3+1}+x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^8-x^3+1}-x}\right)+\frac{1}{2}\log\left(x^2-\sqrt[3]{x^8-x^3+1}\,x+\left(x^8-x^3+1\right)^{2/3}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-3 + 5*x^8)/((1 + x^8)*(1 - x^3 + x^8)^(1/3)),x]

[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(1 - x^3 + x^8)^(1/3))]) - Log[x + (1 - x^3 + x^8)^(1/3)] + Log[x^2 - x*(1 - x^3 + x^8)^(1/3) + (1 - x^3 + x^8)^(2/3)]/2

fricas [A] time = 1.92, size = 121, normalized size = 1.25

$$-\sqrt{3}\arctan\left(\frac{4\sqrt{3}\left(x^8-x^3+1\right)^{\frac{1}{3}}x^2+2\sqrt{3}\left(x^8-x^3+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(x^8-x^3+1\right)}{x^8-9x^3+1}\right)-\frac{1}{2}\log\left(\frac{x^8+3\left(x^8-x^3+1\right)^{\frac{1}{3}}x^2+3\left(x^8-x^3+1\right)^{\frac{2}{3}}x+1}{x^8+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^8-3)/(x^8+1)/(x^8-x^3+1)^(1/3),x, algorithm="fricas")

[Out] -sqrt(3)*arctan((4*sqrt(3)*(x^8 - x^3 + 1)^(1/3)*x^2 + 2*sqrt(3)*(x^8 - x^3 + 1)^(2/3)*x + sqrt(3)*(x^8 - x^3 + 1))/(x^8 - 9*x^3 + 1)) - 1/2*log((x^8 + 3*(x^8 - x^3 + 1)^(1/3)*x^2 + 3*(x^8 - x^3 + 1)^(2/3)*x + 1)/(x^8 + 1))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^8-3}{\left(x^8-x^3+1\right)^{\frac{1}{3}}\left(x^8+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^8-3)/(x^8+1)/(x^8-x^3+1)^(1/3),x, algorithm="giac")

[Out] integrate((5*x^8 - 3)/((x^8 - x^3 + 1)^(1/3)*(x^8 + 1)), x)

maple [C] time = 2.28, size = 362, normalized size = 3.73

$$\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\ln\left(\frac{x^8+\left(x^8-x^3+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x-\left(x^8-x^3+1\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x^2+\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x^3-2\left(x^8-x^3+1\right)^{\frac{2}{3}}x+2\left(x^8-x^3+1\right)^{\frac{1}{3}}x^2-2x^3+1}{x^8+1}\right)-\ln\left(-\frac{-x^8+\left(x^8-x^3+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x-\left(x^8-x^3+1\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x^2+\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x^3+\left(x^8-x^3+1\right)^{\frac{2}{3}}x-\left(x^8-x^3+1\right)^{\frac{1}{3}}x^2+x^3-1}{x^8+1}\right)\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)+\ln\left(-\frac{-x^8+\left(x^8-x^3+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x-\left(x^8-x^3+1\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x^2+\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x^3+\left(x^8-x^3+1\right)^{\frac{2}{3}}x-\left(x^8-x^3+1\right)^{\frac{1}{3}}x^2+x^3-1}{x^8+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^8-3)/(x^8+1)/(x^8-x^3+1)^(1/3),x)

[Out] RootOf(_Z^2-_Z+1)*ln((x^8+(x^8-x^3+1)^(2/3)*RootOf(_Z^2-_Z+1)*x-(x^8-x^3+1)^(1/3)*RootOf(_Z^2-_Z+1)*x^2+RootOf(_Z^2-_Z+1)*x^3-2*(x^8-x^3+1)^(2/3)*x+2*(x^8-x^3+1)^(1/3)*x^2-2*x^3+1)/(x^8+1))-ln(-(-x^8+(x^8-x^3+1)^(2/3)*RootOf(_Z^2-_Z+1)*x-(x^8-x^3+1)^(1/3)*RootOf(_Z^2-_Z+1)*x^2+RootOf(_Z^2-_Z+1)*x^3+(x^8-x^3+1)^(2/3)*x-(x^8-x^3+1)^(1/3)*x^2+x^3-1)/(x^8+1))*RootOf(_Z^2-_Z+1)+ln(-(-x^8+(x^8-x^3+1)^(2/3)*RootOf(_Z^2-_Z+1)*x-(x^8-x^3+1)^(1/3)*RootOf(_Z^2-_Z+1)*x^2+RootOf(_Z^2-_Z+1)*x^3+(x^8-x^3+1)^(2/3)*x-(x^8-x^3+1)^(1/3)*x^2+x^3-1)/(x^8+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^8-3}{\left(x^8-x^3+1\right)^{\frac{1}{3}}\left(x^8+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^8-3)/(x^8+1)/(x^8-x^3+1)^(1/3),x, algorithm="maxima")
[Out] integrate((5*x^8 - 3)/((x^8 - x^3 + 1)^(1/3)*(x^8 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^8-3}{\left(x^8+1\right)\,{\left(x^8-x^3+1\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^8 - 3)/((x^8 + 1)*(x^8 - x^3 + 1)^(1/3)),x)
[Out] int((5*x^8 - 3)/((x^8 + 1)*(x^8 - x^3 + 1)^(1/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**8-3)/(x**8+1)/(x**8-x**3+1)**(1/3),x)
[Out] Timed out
```

3.1206 $\int \frac{\sqrt{x+\sqrt{1+x}}}{1+\sqrt{1+x}}\,dx$

Optimal. Leaf size=97

$$\sqrt{x+1}\sqrt{x+\sqrt{x+1}}-\frac{3}{2}\sqrt{x+\sqrt{x+1}}+\frac{1}{4}\log\left(2\sqrt{x+1}-2\sqrt{x+\sqrt{x+1}}+1\right)-4\tan^{-1}\left(\sqrt{x+1}-\sqrt{x+\sqrt{x}\right.$$

Rubi [A] time = 0.15, antiderivative size = 91, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {814, 843, 621, 206, 724, 204}

$$-\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(3-2\sqrt{x+1}\right)-2\tan^{-1}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)-\frac{1}{4}\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[1 + x]]/(1 + Sqrt[1 + x]),x]
```

```
[Out] -1/2*((3 - 2*Sqrt[1 + x])*Sqrt[x + Sqrt[1 + x]]) - 2*ArcTan[(3 + Sqrt[1 + x
])/(2*Sqrt[x + Sqrt[1 + x]])] - ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqr
t[1 + x]])]/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x+\sqrt{1+x}}}{1+\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x\sqrt{-1+x+x^2}}{1+x}\,dx, x, \sqrt{1+x}\right)\\
&= -\frac{1}{2}\left(3-2\sqrt{1+x}\right)\sqrt{x+\sqrt{1+x}} - \frac{1}{2}\,\text{Subst}\left(\int \frac{-\frac{7}{2}+\frac{x}{2}}{(1+x)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)\\
&= -\frac{1}{2}\left(3-2\sqrt{1+x}\right)\sqrt{x+\sqrt{1+x}} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right) + 2\,\text{Subst}\Bigg(\\
&= -\frac{1}{2}\left(3-2\sqrt{1+x}\right)\sqrt{x+\sqrt{1+x}} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{1+2\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right) - 4\,\text{Subst}\Bigg(\int\\
&= -\frac{1}{2}\left(3-2\sqrt{1+x}\right)\sqrt{x+\sqrt{1+x}} - 2\tan^{-1}\left(\frac{3+\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right) - \frac{1}{4}\tanh^{-1}\left(\frac{1+2\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right.\end{aligned}$$

Mathematica [A] time = 0.07, size = 93, normalized size = 0.96

$$\frac{1}{4}\left(2\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}-3\right)+8\tan^{-1}\left(\frac{-\sqrt{x+1}-3}{2\sqrt{x+\sqrt{x+1}}}\right)-\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[1 + x]]/(1 + Sqrt[1 + x]),x]
```

```
[Out] (2*Sqrt[x + Sqrt[1 + x]]*(-3 + 2*Sqrt[1 + x]) + 8*ArcTan[(-3 - Sqrt[1 + x])
/(2*Sqrt[x + Sqrt[1 + x]])] - ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[
1 + x]])])/4
```

IntegrateAlgebraic [A] time = 0.20, size = 87, normalized size = 0.90

$$\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}-3\right)+\frac{1}{4}\log\left(-2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}}-1\right)-4\tan^{-1}\left(\sqrt{x+1}-\sqrt{x+\sqrt{x+1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x + Sqrt[1 + x]]/(1 + Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[x + Sqrt[1 + x]]*(-3 + 2*Sqrt[1 + x]))/2 - 4*ArcTan[1 + Sqrt[1 + x] -
 Sqrt[x + Sqrt[1 + x]]] + Log[-1 - 2*Sqrt[1 + x] + 2*Sqrt[x + Sqrt[1 + x]]]
/4
```

fricas [A] time = 1.86, size = 82, normalized size = 0.85

$$\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}-3\right)+2\arctan\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}-3\right)}{x-8}\right)+\frac{1}{8}\log\left(4\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)-8x-8\sqrt{x+1}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(1+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) - 3) + 2*arctan(2*sqrt(x + sqrt(x
+ 1))*(sqrt(x + 1) - 3)/(x - 8)) + 1/8*log(4*sqrt(x + sqrt(x + 1))*(2*sqrt(
x + 1) + 1) - 8*x - 8*sqrt(x + 1) - 5)
```

giac [A] time = 0.17, size = 65, normalized size = 0.67

$$\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}-3\right)+4\arctan\left(\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}-1\right)+\frac{1}{4}\log\left(-2\sqrt{x+\sqrt{x+1}}+2\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(1+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) - 3) + 4*arctan(sqrt(x + sqrt(x +
1)) - sqrt(x + 1) - 1) + 1/4*log(-2*sqrt(x + sqrt(x + 1)) + 2*sqrt(x + 1) +
 1)
```

maple [A] time = 0.01, size = 125, normalized size = 1.29

$$\frac{\left(1+2\sqrt{1+x}\right)\sqrt{x+\sqrt{1+x}}}{2}-\frac{5\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}\right)}{4}-2\sqrt{\left(1+\sqrt{1+x}\right)^{2}-\sqrt{1+x}-2}+\ln\left(\sqrt{1+x}+\frac{1}{2}+\sqrt{\left(1+\sqrt{1+x}\right)^{2}-\sqrt{1+x}-2}\right)+2\arctan\left(\frac{-\sqrt{1+x}-3}{2\sqrt{\left(1+\sqrt{1+x}\right)^{2}-\sqrt{1+x}-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(1+x)^(1/2))^(1/2)/(1+(1+x)^(1/2)),x)
```

```
[Out] 1/2*(1+2*(1+x)^(1/2))*(x+(1+x)^(1/2))^(1/2)-5/4*ln(1/2+(1+x)^(1/2)+(x+(1+x)
^(1/2))^(1/2))-2*((1+(1+x)^(1/2))^2-(1+x)^(1/2)-2)^(1/2)+ln((1+x)^(1/2)+1/2
+((1+(1+x)^(1/2))^2-(1+x)^(1/2)-2)^(1/2))+2*arctan(1/2*(-(1+x)^(1/2)-3)/((1
+(1+x)^(1/2))^2-(1+x)^(1/2)-2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(1+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1))/(sqrt(x + 1) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x + 1)^(1/2))^(1/2)/((x + 1)^(1/2) + 1),x)
```

```
[Out] int((x + (x + 1)^(1/2))^(1/2)/((x + 1)^(1/2) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)**(1/2))**(1/2)/(1+(1+x)**(1/2)),x)
```

```
[Out] Integral(sqrt(x + sqrt(x + 1))/(sqrt(x + 1) + 1), x)
```

3.1207 $\int \frac{x+\sqrt{1+x^2}}{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=97

$$\sqrt{x^2+1}\left(\frac{1}{3}\sqrt{\sqrt{x^2+1}+x}-\frac{1}{2}\right)+\frac{1}{3}(x+3)\sqrt{\sqrt{x^2+1}+x}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-2\log\left(\sqrt{\sqrt{x^2+1}+x}+1\right)-\frac{x}{2}$$

Rubi [A] time = 0.86, antiderivative size = 91, normalized size of antiderivative = 0.94, number of steps used = 41, number of rules used = 22, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.710, Rules used = {6742, 195, 215, 2117, 14, 2119, 448, 2122, 270, 266, 50, 63, 207, 2120, 462, 459, 329, 298, 203, 206, 466, 461}

$$\frac{1}{3}\left(\sqrt{x^2+1}+x\right)^{3/2}+\sqrt{\sqrt{x^2+1}+x}-\frac{\sqrt{x^2+1}}{2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{x^2+1}\right)-2\tanh^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)-\frac{x}{2}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[1 + x^2])/(1 + Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/2*x - Sqrt[1 + x^2]/2 + Sqrt[x + Sqrt[1 + x^2]] + (x + Sqrt[1 + x^2])^(3
/2)/3 + ArcTanh[Sqrt[1 + x^2]]/2 - 2*ArcTanh[Sqrt[x + Sqrt[1 + x^2]]] - Log
[x]/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x+\sqrt{1+x^2}}{1+\sqrt{x+\sqrt{1+x^2}}}\,dx = \int \left(\frac{x}{1+\sqrt{x+\sqrt{1+x^2}}}+\frac{\sqrt{1+x^2}}{1+\sqrt{x+\sqrt{1+x^2}}}\right)dx$$

$$= \int \frac{x}{1+\sqrt{x+\sqrt{1+x^2}}}\,dx+\int \frac{\sqrt{1+x^2}}{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

$$= \int \left(-\frac{1}{2}+\frac{x}{2}-\frac{\sqrt{1+x^2}}{2}+\frac{1}{2}\sqrt{x+\sqrt{1+x^2}}-\frac{1}{2}x\sqrt{x+\sqrt{1+x^2}}+\frac{1}{2}\sqrt{1+x^2}\sqrt{x+\sqrt{\ldots}}\right.$$

$$= -\frac{x}{2}+\frac{x^2}{4}-\frac{1}{2}\int \frac{\sqrt{1+x^2}}{x}\,dx-\frac{1}{2}\int \frac{1+x^2}{x}\,dx+\frac{1}{2}\int \sqrt{x+\sqrt{1+x^2}}\,dx-\frac{1}{2}\int x\sqrt{x\ldots}$$

$$= -\frac{x}{2}+\frac{x^2}{4}-\frac{1}{8}\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)}{x^{5/2}}\,dx,x,x+\sqrt{1+x^2}\right)+\frac{1}{8}\text{Subst}\left(\int \frac{(1+\ldots}{x^{5/2}(-\ldots}\right.$$

$$= -\frac{x}{2}-\frac{\sqrt{1+x^2}}{2}+\frac{1}{2\sqrt{x+\sqrt{1+x^2}}}-\frac{\log(x)}{2}-\frac{1}{8}\text{Subst}\left(\int \left(-\frac{1}{x^{5/2}}+x^{3/2}\right)dx,x,x+\ldots\right.$$

$$= -\frac{x}{2}-\frac{\sqrt{1+x^2}}{2}-\frac{1}{12\left(x+\sqrt{1+x^2}\right)^{3/2}}+\frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2}-\frac{1}{20}\left(x+\sqrt{1+x^2}\right)^{5/2}-\ldots$$

$$= -\frac{x}{2}-\frac{\sqrt{1+x^2}}{2}+\sqrt{x+\sqrt{1+x^2}}+\frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{1+x^2}\right)-\frac{\log(\ldots}{2}$$

$$= -\frac{x}{2}-\frac{\sqrt{1+x^2}}{2}+\sqrt{x+\sqrt{1+x^2}}+\frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2}-\tan^{-1}\left(\sqrt{x+\sqrt{1+x^2}}\right)+\frac{1}{2}\ldots$$

$$= -\frac{x}{2}-\frac{\sqrt{1+x^2}}{2}+\sqrt{x+\sqrt{1+x^2}}+\frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{1+x^2}\right)-2\tan\ldots$$

Mathematica [C] time = 1.41, size = 304, normalized size = 3.13

$$\frac{1}{12}\left(-\frac{4\sqrt{x^2+1}\left(\sqrt{x^2+1}+x\right)^{9/2}\left(\left(4x^2+4\sqrt{x^2+1}x+2\right){}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\left(x+\sqrt{x^2+1}\right)^2\right)-x^2-\sqrt{x^2+1}x-2\right)}{16x^6+28x^4+13x^2+5\sqrt{x^2+1}x+16\sqrt{x^2+1}x^5+20\sqrt{x^2+1}x^3+1}-4\left(\sqrt{x^2+1}-2x\right)\sqrt{\sqrt{x^2+1}+x}-6\sqrt{x^2+1}+6\log\left(\sqrt{x^2+1}+1\right)+\frac{12\sqrt{x^2+1}\left(\sqrt{x^2+1}+x\right)\left(\sqrt{\sqrt{x^2+1}+x}-\tan^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)-\tanh^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)\right)}{x^2+\sqrt{x^2+1}x+1}-6x-12\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[1 + x^2])/(1 + Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] (-6*x - 6*Sqrt[1 + x^2] - 4*(-2*x + Sqrt[1 + x^2])*Sqrt[x + Sqrt[1 + x^2]]
+ (12*Sqrt[1 + x^2]*(x + Sqrt[1 + x^2])*(Sqrt[x + Sqrt[1 + x^2]] - ArcTan[S
qrt[x + Sqrt[1 + x^2]]] - ArcTanh[Sqrt[x + Sqrt[1 + x^2]]]))/(1 + x^2 + x*S
qrt[1 + x^2]) - (4*Sqrt[1 + x^2]*(x + Sqrt[1 + x^2])^(9/2)*(-2 - x^2 - x*Sq
rt[1 + x^2] + (2 + 4*x^2 + 4*x*Sqrt[1 + x^2])*Hypergeometric2F1[3/4, 1, 7/4
, (x + Sqrt[1 + x^2])^2]))/(1 + 13*x^2 + 28*x^4 + 16*x^6 + 5*x*Sqrt[1 + x^2
] + 20*x^3*Sqrt[1 + x^2] + 16*x^5*Sqrt[1 + x^2]) - 12*Log[x] + 6*Log[1 + Sq
rt[1 + x^2]])/12
```

IntegrateAlgebraic [A] time = 0.13, size = 97, normalized size = 1.00

$$\sqrt{x^2+1}\left(\frac{1}{3}\sqrt{\sqrt{x^2+1}+x}-\frac{1}{2}\right)+\frac{1}{3}(x+3)\sqrt{\sqrt{x^2+1}+x}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-2\log\left(\sqrt{\sqrt{x^2+1}+x}+1\right)-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + Sqrt[1 + x^2])/(1 + Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/2*x + ((3 + x)*Sqrt[x + Sqrt[1 + x^2]])/3 + Sqrt[1 + x^2]*(-1/2 + Sqrt[x
 + Sqrt[1 + x^2]]/3) + Log[x + Sqrt[1 + x^2]]/2 - 2*Log[1 + Sqrt[x + Sqrt[1
 + x^2]]]
```

fricas [A] time = 0.43, size = 64, normalized size = 0.66

$$\frac{1}{3}\left(x+\sqrt{x^2+1}+3\right)\sqrt{x+\sqrt{x^2+1}}-\frac{1}{2}x-\frac{1}{2}\sqrt{x^2+1}-2\log\left(\sqrt{x+\sqrt{x^2+1}}+1\right)+\log\left(\sqrt{x+\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+1)^(1/2))/(1+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="frica
s")
```

```
[Out] 1/3*(x + sqrt(x^2 + 1) + 3)*sqrt(x + sqrt(x^2 + 1)) - 1/2*x - 1/2*sqrt(x^2
+ 1) - 2*log(sqrt(x + sqrt(x^2 + 1)) + 1) + log(sqrt(x + sqrt(x^2 + 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{x^2+1}}{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+1)^(1/2))/(1+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="giac"
)
```

```
[Out] integrate((x + sqrt(x^2 + 1))/(sqrt(x + sqrt(x^2 + 1)) + 1), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{x^2+1}}{1+\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+1)^(1/2))/(1+(x+(x^2+1)^(1/2))^(1/2)),x)
```

```
[Out] int((x+(x^2+1)^(1/2))/(1+(x+(x^2+1)^(1/2))^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}x^2+\frac{1}{2}\int\sqrt{x^2+1}\,dx-\int\frac{2x^2+\sqrt{x^2+1}(2x-1)-x+1}{2\left(x+\sqrt{x^2+1}+2\sqrt{x+\sqrt{x^2+1}}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+1)^(1/2))/(1+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="maxim
a")
```

```
[Out] 1/4*x^2 + 1/2*integrate(sqrt(x^2 + 1), x) - integrate(1/2*(2*x^2 + sqrt(x^2
 + 1)*(2*x - 1) - x + 1)/(x + sqrt(x^2 + 1) + 2*sqrt(x + sqrt(x^2 + 1)) + 1
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+\sqrt{x^2+1}}{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x^2 + 1)^(1/2))/((x + (x^2 + 1)^(1/2))^(1/2) + 1),x)
```

```
[Out] int((x + (x^2 + 1)^(1/2))/((x + (x^2 + 1)^(1/2))^(1/2) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{x^2+1}}{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+1)**(1/2))/(1+(x+(x**2+1)**(1/2))**(1/2)),x)
```

```
[Out] Integral((x + sqrt(x**2 + 1))/(sqrt(x + sqrt(x**2 + 1)) + 1), x)
```

3.1208 $\int \frac{x^2}{(-2b+ax^2)(-b+ax^2)^{3/4}}\,dx$

Optimal. Leaf size=98

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}\,a^{3/2}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{a}\,x}\right)}{\sqrt{2}\,a^{3/2}\sqrt[4]{b}}$$

Rubi [A] time = 0.04, antiderivative size = 96, normalized size of antiderivative = 0.98, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}\,a^{3/2}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}\,a^{3/2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2*b + a*x^2)*(-b + a*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[a]*x)/(Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4))]/(Sqrt[2]*a^(3/2)*b
^(1/4)) - ArcTanh[(Sqrt[a]*x)/(Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4))]/(Sqrt[2
]*a^(3/2)*b^(1/4))
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2b+ax^2\right)\left(-b+ax^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^2}}\right)}{\sqrt{2}\,a^{3/2}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^2}}\right)}{\sqrt{2}\,a^{3/2}\sqrt[4]{b}}$$

Mathematica [C] time = 0.07, size = 68, normalized size = 0.69

$$-\frac{x^3\left(1-\frac{ax^2}{b}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{ax^2}{b},\frac{ax^2}{2b}\right)}{6b\left(ax^2-b\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2*b + a*x^2)*(-b + a*x^2)^(3/4)),x]
```

```
[Out] -1/6*(x^3*(1 - (a*x^2)/b)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, (a*x^2)/b, (a*x^
2)/(2*b)])/(b*(-b + a*x^2)^(3/4))
```

IntegrateAlgebraic [A] time = 2.41, size = 98, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}\,a^{3/2}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{a}\,x}\right)}{\sqrt{2}\,a^{3/2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-2*b + a*x^2)*(-b + a*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[a]*x)/(Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4))]/(Sqrt[2]*a^(3/2)*b
^(1/4)) - ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4))/(Sqrt[a]*x)]/(Sqrt[2
]*a^(3/2)*b^(1/4))
```

fricas [B] time = 0.43, size = 207, normalized size = 2.11

$$2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^6b}\right)^{\frac{1}{4}}\arctan\left(\frac{4\left(\sqrt{\frac{1}{2}}\left(\frac{1}{4}\right)^{\frac{3}{4}}a^4bx\sqrt{\frac{a^4x^2\sqrt{\frac{1}{a^6b}}+2\sqrt{ax^2-b}}{x^2}}\left(\frac{1}{a^6b}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(ax^2-b\right)^{\frac{1}{4}}a^4b\left(\frac{1}{a^6b}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^6b}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}a^2x\left(\frac{1}{a^6b}\right)^{\frac{1}{4}}+\left(ax^2-b\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^6b}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}a^2x\left(\frac{1}{a^6b}\right)^{\frac{1}{4}}-\left(ax^2-b\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2-2*b)/(a*x^2-b)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*(1/4)^(1/4)*(1/(a^6*b))^(1/4)*arctan(4*(sqrt(1/2)*(1/4)^(3/4)*a^4*b*x*sqr
t((a^4*x^2*sqrt(1/(a^6*b)) + 2*sqrt(a*x^2 - b))/x^2)*(1/(a^6*b))^(3/4) - (1
/4)^(3/4)*(a*x^2 - b)^(1/4)*a^4*b*(1/(a^6*b))^(3/4))/x) - 1/2*(1/4)^(1/4)*(
1/(a^6*b))^(1/4)*log(((1/4)^(1/4)*a^2*x*(1/(a^6*b))^(1/4) + (a*x^2 - b)^(1/
4))/x) + 1/2*(1/4)^(1/4)*(1/(a^6*b))^(1/4)*log(-((1/4)^(1/4)*a^2*x*(1/(a^6*
b))^(1/4) - (a*x^2 - b)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax^2-b\right)^{\frac{3}{4}}\left(ax^2-2\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2-2*b)/(a*x^2-b)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a*x^2 - b)^(3/4)*(a*x^2 - 2*b)), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a\,x^2-2b\right)\left(a\,x^2-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2-2*b)/(a*x^2-b)^(3/4),x)
```

```
[Out] int(x^2/(a*x^2-2*b)/(a*x^2-b)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax^2-b\right)^{\frac{3}{4}}\left(ax^2-2\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2-2*b)/(a*x^2-b)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a*x^2 - b)^(3/4)*(a*x^2 - 2*b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{\left(a\,x^2-b\right)^{3/4}\,\left(2\,b-a\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((a*x^2 - b)^(3/4)*(2*b - a*x^2)),x)
```

```
[Out] -int(x^2/((a*x^2 - b)^(3/4)*(2*b - a*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax^2-2b\right)\left(ax^2-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**2-2*b)/(a*x**2-b)**(3/4),x)
```

```
[Out] Integral(x**2/((a*x**2 - 2*b)*(a*x**2 - b)**(3/4)), x)
```

3.1209 $$\int \frac{\left(-2k-(-1+k)(1+k)x+2kx^2\right)\left(1-2kx+k^2x^2\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{3/4}\left(1-d+(1+3d)kx-\left(1+3dk^2\right)x^2+k\left(-1+dk^2\right)x^3\right)}\,dx$$

Optimal. Leaf size=98

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,kx-\sqrt[4]{d}}{\sqrt[4]{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,kx-\sqrt[4]{d}}{\sqrt[4]{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{d^{3/4}}$$

Rubi [F] time = 49.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-2k-(-1+k)(1+k)x+2kx^2\right)\left(1-2kx+k^2x^2\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{3/4}\left(1-d+(1+3d)kx-\left(1+3dk^2\right)x^2+k\left(-1+dk^2\right)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*k - (-1 + k)*(1 + k)*x + 2*k*x^2)*(1 - 2*k*x + k^2*x^2))/(((1 - x^
2)*(1 - k^2*x^2))^(3/4)*(1 - d + (1 + 3*d)*k*x - (1 + 3*d*k^2)*x^2 + k*(-1
+ d*k^2)*x^3)),x]
```

```
[Out] (Sqrt[2]*k^(3/2)*Sqrt[-1 + k^2]*Sqrt[(-1 - k^2 + 2*k^2*x^2)^2]*Sqrt[(1 + k^
2*(1 - 2*x^2))^2/((1 - k^2)^2*(1 - (2*k*Sqrt[(1 - x^2)*(1 - k^2*x^2)])/(1 -
 k^2))^2)]*(1 - (2*k*Sqrt[(1 - x^2)*(1 - k^2*x^2)])/(1 - k^2))*EllipticF[2*
ArcTan[(Sqrt[2]*Sqrt[k]*((1 - x^2)*(1 - k^2*x^2))^(1/4))/Sqrt[-1 + k^2]], 1
/2])/((1 - d*k^2)*(1 + k^2 - 2*k^2*x^2)*Sqrt[(-1 - k^2*(1 - 2*x^2))^2]) + (
k*(5 + (1 + 3*d)*k^2 - d*k^4)*x*((1 - x^2)/(1 - k^2*x^2))^(3/4)*(1 - k^2*x^
2)*Hypergeometric2F1[1/2, 3/4, 3/2, ((1 - k^2)*x^2)/(1 - k^2*x^2)])/((1 - d
*k^2)^2*((1 - x^2)*(1 - k^2*x^2))^(3/4)) - (k*(7 + k^2 - 3*d^2*k^2*(1 - k^2
) - d*(5 + 2*k^2 + k^4))*(1 - x^2)^(3/4)*(1 - k^2*x^2)^(3/4)*Defer[Int][1/(
(1 - x^2)^(3/4)*(1 - k^2*x^2)^(3/4)*(1 - d + (1 + 3*d)*k*x - (1 + 3*d*k^2)*
x^2 - k*(1 - d*k^2)*x^3)), x])/((1 - d*k^2)^2*((1 - x^2)*(1 - k^2*x^2))^(3/
4)) + ((1 - k^4 - 6*d^2*k^4*(1 - k^2) - d*k^2*(19 + 14*k^2 - k^4))*(1 - x^2
)^(3/4)*(1 - k^2*x^2)^(3/4)*Defer[Int][x/((1 - x^2)^(3/4)*(1 - k^2*x^2)^(3/
4)*(1 - d + (1 + 3*d)*k*x - (1 + 3*d*k^2)*x^2 - k*(1 - d*k^2)*x^3)), x])/((
1 - d*k^2)^2*((1 - x^2)*(1 - k^2*x^2))^(3/4)) + (k*(5 + 3*(1 + 8*d)*k^2 + 3
*d^2*k^4 - 3*d^2*k^6)*(1 - x^2)^(3/4)*(1 - k^2*x^2)^(3/4)*Defer[Int][x^2/((
1 - x^2)^(3/4)*(1 - k^2*x^2)^(3/4)*(1 - d + (1 + 3*d)*k*x - (1 + 3*d*k^2)*x
^2 - k*(1 - d*k^2)*x^3)), x])/((1 - d*k^2)^2*((1 - x^2)*(1 - k^2*x^2))^(3/4
))
```

Rubi steps

$$\int \frac{\left(-2k-(-1+k)(1+k)x+2kx^2\right)\left(1-2kx+k^2x^2\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{3/4}\left(1-d+(1+3d)kx-\left(1+3dk^2\right)x^2+k\left(-1+dk^2\right)x^3\right)}\,dx = \int \frac{(-1}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right)^{3}\right.}{}$$

$$= \frac{\left(\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right)^{3}\right.}{}$$

$$= \frac{\left(\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right)^{3}\right.}{}$$

$$= -\frac{\left(2k^2\left(1-x^2\right)^{3/4}\left(1-k\right.\right.}{\left(1-dk^2\right)\left(\left(\right.\right.}$$

$$= \frac{k\left(5+(1+3d)k^2-dk^4\right.}{\left(1-d\right.}$$

$$= \frac{k\left(5+(1+3d)k^2-dk^4\right.}{\left(1-d\right.}$$

$$= \frac{k\left(5+(1+3d)k^2-dk^4\right.}{\left(1-d\right.}$$

$$= \frac{\sqrt{2}\,k^{3/2}\sqrt{-1+k^2}\sqrt{(-1}}{}$$

<u>Mathematica</u> [F] time = 1.72, size = 0, normalized size = 0.00

$$\int \frac{\left(-2k-(-1+k)(1+k)x+2kx^2\right)\left(1-2kx+k^2x^2\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{3/4}\left(1-d+(1+3d)kx-\left(1+3dk^2\right)x^2+k\left(-1+dk^2\right)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2*k - (-1 + k)*(1 + k)*x + 2*k*x^2)*(1 - 2*k*x + k^2*x^2))/(((
1 - x^2)*(1 - k^2*x^2))^(3/4)*(1 - d + (1 + 3*d)*k*x - (1 + 3*d*k^2)*x^2 +
k*(-1 + d*k^2)*x^3)),x]
```

```
[Out] Integrate[((-2*k - (-1 + k)*(1 + k)*x + 2*k*x^2)*(1 - 2*k*x + k^2*x^2))/(((
1 - x^2)*(1 - k^2*x^2))^(3/4)*(1 - d + (1 + 3*d)*k*x - (1 + 3*d*k^2)*x^2 +
k*(-1 + d*k^2)*x^3)), x]
```

IntegrateAlgebraic [A] time = 15.75, size = 98, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,kx-\sqrt[4]{d}}{\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,kx-\sqrt[4]{d}}{\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*k - (-1 + k)*(1 + k)*x + 2*k*x^2)*(1 - 2*k*x + k^2*
x^2))/(((1 - x^2)*(1 - k^2*x^2))^(3/4)*(1 - d + (1 + 3*d)*k*x - (1 + 3*d*k^
2)*x^2 + k*(-1 + d*k^2)*x^3)),x]
```

```
[Out] ArcTan[(-d^(1/4) + d^(1/4)*k*x)/(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4)]/d^(3/
4) - ArcTanh[(-d^(1/4) + d^(1/4)*k*x)/(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4)]
/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*k-(-1+k)*(1+k)*x+2*k*x^2)*(k^2*x^2-2*k*x+1)/((-x^2+1)*(-k^2*x
^2+1))^(3/4)/(1-d+(1+3*d)*k*x-(3*d*k^2+1)*x^2+k*(d*k^2-1)*x^3),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(k^2x^2-2\,kx+1\right)\left((k+1)(k-1)x-2\,kx^2+2\,k\right)}{\left(\left(dk^2-1\right)kx^3+(3\,d+1)kx-\left(3\,dk^2+1\right)x^2-d+1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*k-(-1+k)*(1+k)*x+2*k*x^2)*(k^2*x^2-2*k*x+1)/((-x^2+1)*(-k^2*x
^2+1))^(3/4)/(1-d+(1+3*d)*k*x-(3*d*k^2+1)*x^2+k*(d*k^2-1)*x^3),x, algorithm
="giac")
```

```
[Out] integrate(-(k^2*x^2 - 2*k*x + 1)*((k + 1)*(k - 1)*x - 2*k*x^2 + 2*k)/(((d*k
^2 - 1)*k*x^3 + (3*d + 1)*k*x - (3*d*k^2 + 1)*x^2 - d + 1)*((k^2*x^2 - 1)*(
x^2 - 1))^(3/4)), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(-2k-(-1+k)\,(1+k)\,x+2k\,x^2\right)\left(k^2x^2-2kx+1\right)}{\left(\left(-x^2+1\right)\left(-k^2x^2+1\right)\right)^{\frac{3}{4}}\left(1-d+(1+3d)\,kx-\left(3d\,k^2+1\right)x^2+k\left(d\,k^2-1\right)x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*k-(-1+k)*(1+k)*x+2*k*x^2)*(k^2*x^2-2*k*x+1)/((-x^2+1)*(-k^2*x^2+1))
^(3/4)/(1-d+(1+3*d)*k*x-(3*d*k^2+1)*x^2+k*(d*k^2-1)*x^3),x)
```

```
[Out] int((-2*k-(-1+k)*(1+k)*x+2*k*x^2)*(k^2*x^2-2*k*x+1)/((-x^2+1)*(-k^2*x^2+1))
^(3/4)/(1-d+(1+3*d)*k*x-(3*d*k^2+1)*x^2+k*(d*k^2-1)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(k^2x^2-2\,kx+1\right)\left((k+1)(k-1)x-2\,kx^2+2\,k\right)}{\left(\left(dk^2-1\right)kx^3+(3\,d+1)kx-\left(3\,dk^2+1\right)x^2-d+1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*k-(-1+k)*(1+k)*x+2*k*x^2)*(k^2*x^2-2*k*x+1)/((-x^2+1)*(-k^2*x
^2+1))^(3/4)/(1-d+(1+3*d)*k*x-(3*d*k^2+1)*x^2+k*(d*k^2-1)*x^3),x, algorithm
="maxima")
```

```
[Out] -integrate((k^2*x^2 - 2*k*x + 1)*((k + 1)*(k - 1)*x - 2*k*x^2 + 2*k)/(((d*k
^2 - 1)*k*x^3 + (3*d + 1)*k*x - (3*d*k^2 + 1)*x^2 - d + 1)*((k^2*x^2 - 1)*(
x^2 - 1))^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(-2\,k\,x^2+\left(k-1\right)\,\left(k+1\right)\,x+2\,k\right)\,\left(k^2\,x^2-2\,k\,x+1\right)}{{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)}^{3/4}\,\left(k\,\left(d\,k^2-1\right)\,x^3+\left(-3\,d\,k^2-1\right)\,x^2+k\,\left(3\,d+1\right)\,x-d+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*k - 2*k*x^2 + x*(k - 1)*(k + 1))*(k^2*x^2 - 2*k*x + 1))/(((x^2 - 1
)*(k^2*x^2 - 1))^(3/4)*(k*x*(3*d + 1) - x^2*(3*d*k^2 + 1) - d + k*x^3*(d*k^
2 - 1) + 1)),x)
```

```
[Out] int(-((2*k - 2*k*x^2 + x*(k - 1)*(k + 1))*(k^2*x^2 - 2*k*x + 1))/(((x^2 - 1
)*(k^2*x^2 - 1))^(3/4)*(k*x*(3*d + 1) - x^2*(3*d*k^2 + 1) - d + k*x^3*(d*k^
2 - 1) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*k-(-1+k)*(1+k)*x+2*k*x**2)*(k**2*x**2-2*k*x+1)/((-x**2+1)*(-k
**2*x**2+1))**(3/4)/(1-d+(1+3*d)*k*x-(3*d*k**2+1)*x**2+k*(d*k**2-1)*x**3),x
)
```

```
[Out] Timed out
```

3.1210 $\int \frac{(-1+x^2)(1+x^2)\sqrt{1+3x^2+x^4}}{x^2(1+x+x^2)^2}\,dx$

Optimal. Leaf size=98

$$\frac{\sqrt{x^4+3x^2+1}\,(x^2+2x+1)}{x\,(x^2+x+1)}-2\log\left(x^2+\sqrt{x^4+3x^2+1}+1\right)-3\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,x}{x^2+\sqrt{x^4+3x^2+1}+x+1}\right)+2\log$$

Rubi [F] time = 6.89, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^2)\,(1+x^2)\,\sqrt{1+3x^2+x^4}}{x^2\,(1+x+x^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^2)*(1 + x^2)*Sqrt[1 + 3*x^2 + x^4])/(x^2*(1 + x + x^2)^2),x]
```

```
[Out] -((x*(3 + Sqrt[5] + 2*x^2))/Sqrt[1 + 3*x^2 + x^4]) + ((3 - I*Sqrt[3])*x*(3
+ Sqrt[5] + 2*x^2))/(6*Sqrt[1 + 3*x^2 + x^4]) + ((3 + I*Sqrt[3])*x*(3 + Sqr
t[5] + 2*x^2))/(6*Sqrt[1 + 3*x^2 + x^4]) + Sqrt[1 + 3*x^2 + x^4] - ((3 - I*
Sqrt[3])*Sqrt[1 + 3*x^2 + x^4])/6 - ((3 + I*Sqrt[3])*Sqrt[1 + 3*x^2 + x^4])
/6 + Sqrt[1 + 3*x^2 + x^4]/x + (Sqrt[(1 + I*Sqrt[3])/3]*(I + Sqrt[3])*ArcTa
n[(1 - (3*I)*Sqrt[3] + 2*(2 - I*Sqrt[3])*x^2)/(4*Sqrt[1 + I*Sqrt[3]]*Sqrt[1
 + 3*x^2 + x^4])])/2 - ((I - Sqrt[3])*Sqrt[(1 - I*Sqrt[3])/3]*ArcTan[(1 + (
3*I)*Sqrt[3] + 2*(2 + I*Sqrt[3])*x^2)/(4*Sqrt[1 - I*Sqrt[3]]*Sqrt[1 + 3*x^2
 + x^4])])/2 + (3*ArcTanh[(3 + 2*x^2)/(2*Sqrt[1 + 3*x^2 + x^4])])/2 - ((9 -
 I*Sqrt[3])*ArcTanh[(3 + 2*x^2)/(2*Sqrt[1 + 3*x^2 + x^4])])/12 - ((9 + I*Sq
rt[3])*ArcTanh[(3 + 2*x^2)/(2*Sqrt[1 + 3*x^2 + x^4])])/12 - ArcTanh[(2 + 3*
x^2)/(2*Sqrt[1 + 3*x^2 + x^4])] + ((I - Sqrt[3])*Sqrt[(3 + Sqrt[5])/6]*Sqrt
[(2 + (3 - Sqrt[5])*x^2)/(2 + (3 + Sqrt[5])*x^2)]*(2 + (3 + Sqrt[5])*x^2)*E
llipticE[ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/(2*Sqrt[1 +
3*x^2 + x^4]) - ((I + Sqrt[3])*Sqrt[(3 + Sqrt[5])/6]*Sqrt[(2 + (3 - Sqrt[5]
)*x^2)/(2 + (3 + Sqrt[5])*x^2)]*(2 + (3 + Sqrt[5])*x^2)*EllipticE[ArcTan[Sq
rt[(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/(2*Sqrt[1 + 3*x^2 + x^4]) + (S
qrt[(3 + Sqrt[5])/2]*Sqrt[(2 + (3 - Sqrt[5])*x^2)/(2 + (3 + Sqrt[5])*x^2)]*
(2 + (3 + Sqrt[5])*x^2)*EllipticE[ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 + 3*
Sqrt[5])/2])/Sqrt[1 + 3*x^2 + x^4] - ((2*I - Sqrt[3])*Sqrt[2/(3*(3 + Sqrt[5
]))]*Sqrt[(2 + (3 - Sqrt[5])*x^2)/(2 + (3 + Sqrt[5])*x^2)]*(2 + (3 + Sqrt[5
])*x^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/Sqr
t[1 + 3*x^2 + x^4] + ((2*I + Sqrt[3])*Sqrt[2/(3*(3 + Sqrt[5]))]*Sqrt[(2 + (
3 - Sqrt[5])*x^2)/(2 + (3 + Sqrt[5])*x^2)]*(2 + (3 + Sqrt[5])*x^2)*Elliptic
F[ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/Sqrt[1 + 3*x^2 + x^
4] + (2*(I + Sqrt[3])*Sqrt[2/(3*(3 + Sqrt[5]))]*Sqrt[(2 + (3 - Sqrt[5])*x^2
)/(2 + (3 + Sqrt[5])*x^2)]*(2 + (3 + Sqrt[5])*x^2)*EllipticF[ArcTan[Sqrt[(3
 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/((2 - I*Sqrt[3] - Sqrt[5])*Sqrt[1 +
 3*x^2 + x^4]) - (2*(I - Sqrt[3])*Sqrt[2/(3*(3 + Sqrt[5]))]*Sqrt[(2 + (3 -
Sqrt[5])*x^2)/(2 + (3 + Sqrt[5])*x^2)]*(2 + (3 + Sqrt[5])*x^2)*EllipticF[Ar
cTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/((2 + I*Sqrt[3] - Sqrt[
5])*Sqrt[1 + 3*x^2 + x^4]) - (3*Sqrt[(2 + (3 - Sqrt[5])*x^2)/(2 + (3 + Sqrt
[5])*x^2)]*(2 + (3 + Sqrt[5])*x^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt[5])/2]*x
], (-5 + 3*Sqrt[5])/2])/(Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 + 3*x^2 + x^4]) - (4*
(1 + I*Sqrt[3])*Sqrt[(9 - 4*Sqrt[5])/3]*(3 + Sqrt[5] + 2*x^2)*EllipticPi[1
- (2*(3 - Sqrt[5]))/(I - Sqrt[3])^2, ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 +
 3*Sqrt[5])/2])/((5*I + Sqrt[3] - I*Sqrt[5] - Sqrt[15])*Sqrt[(3 + Sqrt[5] +
 2*x^2)/(3 - Sqrt[5] + 2*x^2)]*Sqrt[1 + 3*x^2 + x^4]) + (4*(1 - I*Sqrt[3])*
```

```
Sqrt[(9 - 4*Sqrt[5])/3]*(3 + Sqrt[5] + 2*x^2)*EllipticPi[1 - (2*(3 - Sqrt[5
]))/(I + Sqrt[3])^2, ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/
((5*I - Sqrt[3] - I*Sqrt[5] + Sqrt[15])*Sqrt[(3 + Sqrt[5] + 2*x^2)/(3 - Sqr
t[5] + 2*x^2)]*Sqrt[1 + 3*x^2 + x^4]) + (4*Defer[Int][Sqrt[1 + 3*x^2 + x^4]
/(-1 + I*Sqrt[3] - 2*x)^2, x])/3 - (4*(1 - I*Sqrt[3])*Defer[Int][Sqrt[1 + 3
*x^2 + x^4]/(-1 + I*Sqrt[3] - 2*x)^2, x])/3 + (4*Defer[Int][Sqrt[1 + 3*x^2
+ x^4]/(1 + I*Sqrt[3] + 2*x)^2, x])/3 - (4*(1 + I*Sqrt[3])*Defer[Int][Sqrt[
1 + 3*x^2 + x^4]/(1 + I*Sqrt[3] + 2*x)^2, x])/3
```

Rubi steps

$$\int \frac{(-1+x^2)(1+x^2)\sqrt{1+3x^2+x^4}}{x^2(1+x+x^2)^2}\,dx = \int \left(-\frac{\sqrt{1+3x^2+x^4}}{x^2} + \frac{2\sqrt{1+3x^2+x^4}}{x} + \frac{(-1-2x)\sqrt{1+3x^2+x^4}}{(1+x+x^2)^2}\right.$$

$$= 2\int \frac{\sqrt{1+3x^2+x^4}}{x}\,dx - 2\int \frac{x\sqrt{1+3x^2+x^4}}{1+x+x^2}\,dx - \int \frac{\sqrt{1+3x^2+}}{x^2}$$

$$= \frac{\sqrt{1+3x^2+x^4}}{x} - 2\int \left(\frac{\left(1+\frac{i}{\sqrt{3}}\right)\sqrt{1+3x^2+x^4}}{1-i\sqrt{3}+2x} + \frac{\left(1-\frac{i}{\sqrt{3}}\right)\sqrt{1+}}{1+i\sqrt{3}+}\right.$$

$$= \sqrt{1+3x^2+x^4} + \frac{\sqrt{1+3x^2+x^4}}{x} - \frac{1}{2}\mathrm{Subst}\left(\int \frac{-2-3x}{x\sqrt{1+3x+x^2}}\,dx,\right.$$

$$= -\frac{x(3+\sqrt{5}+2x^2)}{\sqrt{1+3x^2+x^4}} + \sqrt{1+3x^2+x^4} + \frac{\sqrt{1+3x^2+x^4}}{x} + \frac{\sqrt{\frac{1}{2}}(3+}{}$$

$$= -\frac{x(3+\sqrt{5}+2x^2)}{\sqrt{1+3x^2+x^4}} + \sqrt{1+3x^2+x^4} + \frac{\sqrt{1+3x^2+x^4}}{x} + \frac{\sqrt{\frac{1}{2}}(3+}{}$$

$$= -\frac{x(3+\sqrt{5}+2x^2)}{\sqrt{1+3x^2+x^4}} + \sqrt{1+3x^2+x^4} - \frac{1}{6}\left(3-i\sqrt{3}\right)\sqrt{1+3x^2+x^4}$$

$$= -\frac{x(3+\sqrt{5}+2x^2)}{\sqrt{1+3x^2+x^4}} + \frac{\left(3-i\sqrt{3}\right)x(3+\sqrt{5}+2x^2)}{6\sqrt{1+3x^2+x^4}} + \frac{\left(3+i\sqrt{3}\right)x(}{6\sqrt{1+}}$$

$$= -\frac{x(3+\sqrt{5}+2x^2)}{\sqrt{1+3x^2+x^4}} + \frac{\left(3-i\sqrt{3}\right)x(3+\sqrt{5}+2x^2)}{6\sqrt{1+3x^2+x^4}} + \frac{\left(3+i\sqrt{3}\right)x(}{6\sqrt{1+}}$$

$$= -\frac{x(3+\sqrt{5}+2x^2)}{\sqrt{1+3x^2+x^4}} + \frac{\left(3-i\sqrt{3}\right)x(3+\sqrt{5}+2x^2)}{6\sqrt{1+3x^2+x^4}} + \frac{\left(3+i\sqrt{3}\right)x(}{6\sqrt{1+}}$$

Mathematica [C] time = 3.86, size = 880, normalized size = 8.98

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^2)*(1 + x^2)*Sqrt[1 + 3*x^2 + x^4])/(x^2*(1 + x + x^2)^2
),x]
```

```
[Out] (2*Sqrt[1 + 3*x^2 + x^4]*(x^(-1) + (1 + x + x^2)^(-1)) + (((3*I)/2)*(I + Sq
rt[3])*ArcTan[(1 - (3*I)*Sqrt[3] + (4 - (2*I)*Sqrt[3])*x^2)/(4*Sqrt[1 + I*S
qrt[3]]*Sqrt[1 + 3*x^2 + x^4])])/Sqrt[1 + I*Sqrt[3]] - (3*(1 + I*Sqrt[3])*A
rcTan[(1 + (3*I)*Sqrt[3] + (4 + (2*I)*Sqrt[3])*x^2)/(4*Sqrt[1 - I*Sqrt[3]]*
Sqrt[1 + 3*x^2 + x^4])])/(2*Sqrt[1 - I*Sqrt[3]]) - 2*ArcTanh[(3 + 2*x^2)/(2
*Sqrt[1 + 3*x^2 + x^4])] - 2*ArcTanh[(2 + 3*x^2)/(2*Sqrt[1 + 3*x^2 + x^4])]
 - ((3*I)*Sqrt[2]*Sqrt[(-3 + Sqrt[5] - 2*x^2)/(-3 + Sqrt[5])]*Sqrt[3 + Sqrt
[5] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(3 + Sqrt[5])]*x], 7/2 + (3*Sqrt[5]
)/2])/Sqrt[1 + 3*x^2 + x^4] + ((3*I)*Sqrt[2]*Sqrt[(-3 + Sqrt[5] - 2*x^2)/(-
3 + Sqrt[5])]*Sqrt[3 + Sqrt[5] + 2*x^2]*EllipticPi[((-1)^(1/3)*(3 + Sqrt[5]
))/2, I*ArcSinh[Sqrt[2/(3 + Sqrt[5])]*x], (7 + 3*Sqrt[5])/2])/((1 + (-1)^(1
/3))*Sqrt[1 + 3*x^2 + x^4]) + (3*(-1)^(5/6)*Sqrt[2]*Sqrt[(-3 + Sqrt[5] - 2*
x^2)/(-3 + Sqrt[5])]*Sqrt[3 + Sqrt[5] + 2*x^2]*EllipticPi[((-1)^(1/3)*(3 +
Sqrt[5]))/2, I*ArcSinh[Sqrt[2/(3 + Sqrt[5])]*x], (7 + 3*Sqrt[5])/2])/((1 +
(-1)^(1/3))*Sqrt[1 + 3*x^2 + x^4]) + ((3*I)*Sqrt[2]*Sqrt[(-3 + Sqrt[5] - 2*
x^2)/(-3 + Sqrt[5])]*Sqrt[3 + Sqrt[5] + 2*x^2]*EllipticPi[-1/2*((-1)^(2/3)*
(3 + Sqrt[5])), I*ArcSinh[Sqrt[2/(3 + Sqrt[5])]*x], (7 + 3*Sqrt[5])/2])/((1
 + (-1)^(1/3))*Sqrt[1 + 3*x^2 + x^4]) + (3*(-1)^(5/6)*Sqrt[2]*Sqrt[(-3 + Sq
rt[5] - 2*x^2)/(-3 + Sqrt[5])]*Sqrt[3 + Sqrt[5] + 2*x^2]*EllipticPi[-1/2*((
-1)^(2/3)*(3 + Sqrt[5])), I*ArcSinh[Sqrt[2/(3 + Sqrt[5])]*x], (7 + 3*Sqrt[5
])/2])/((1 + (-1)^(1/3))*Sqrt[1 + 3*x^2 + x^4]))/2
```

IntegrateAlgebraic [A] time = 0.98, size = 98, normalized size = 1.00

$$\frac{\sqrt{x^4+3x^2+1}\left(x^2+2x+1\right)}{x\left(x^2+x+1\right)}-2\log\left(x^2+\sqrt{x^4+3x^2+1}+1\right)-3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}x}{x^2+\sqrt{x^4+3x^2+1}+x+1}\right)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*(1 + x^2)*Sqrt[1 + 3*x^2 + x^4])/(x^2*(1 + x
 + x^2)^2),x]
```

```
[Out] ((1 + 2*x + x^2)*Sqrt[1 + 3*x^2 + x^4])/(x*(1 + x + x^2)) - 3*Sqrt[2]*ArcTa
nh[(Sqrt[2]*x)/(1 + x + x^2 + Sqrt[1 + 3*x^2 + x^4])] + 2*Log[x] - 2*Log[1
+ x^2 + Sqrt[1 + 3*x^2 + x^4]]
```

fricas [A] time = 0.51, size = 150, normalized size = 1.53

$$\frac{3\sqrt{2}\left(x^3+x^2+x\right)\log\left(\frac{3x^4-2x^3+2\sqrt{2}\sqrt{x^4+3x^2+1}\left(x^2-x+1\right)+9x^2-2x+3}{x^4+2x^3+3x^2+2x+1}\right)+8\left(x^3+x^2+x\right)\log\left(-\frac{x^2-\sqrt{x^4+3x^2+1}+1}{x}\right)+4\sqrt{x^4+3x^2+1}\left(x^2+2x+1\right)}{4\left(x^3+x^2+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^2+1)*(x^4+3*x^2+1)^(1/2)/x^2/(x^2+x+1)^2,x, algorithm=
"fricas")
```

```
[Out] 1/4*(3*sqrt(2)*(x^3 + x^2 + x)*log((3*x^4 - 2*x^3 + 2*sqrt(2)*sqrt(x^4 + 3*
x^2 + 1)*(x^2 - x + 1) + 9*x^2 - 2*x + 3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1))
+ 8*(x^3 + x^2 + x)*log(-(x^2 - sqrt(x^4 + 3*x^2 + 1) + 1)/x) + 4*sqrt(x^4
+ 3*x^2 + 1)*(x^2 + 2*x + 1))/(x^3 + x^2 + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+3x^2+1}\left(x^2+1\right)\left(x^2-1\right)}{\left(x^2+x+1\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^2+1)*(x^4+3*x^2+1)^(1/2)/x^2/(x^2+x+1)^2,x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 1)*(x^2 + 1)*(x^2 - 1)/((x^2 + x + 1)^2*x^2),
x)
```

maple [C]   time = 0.21, size = 637, normalized size = 6.50

$$\frac{\sqrt{x^4+3x^2+1}}{x^2+x+1}+\frac{3\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)x^2}\,\operatorname{EllipticF}\left(x\left(\frac{i\sqrt{5}}{2}-\frac{i}{2}\right),\frac{3}{2}+\frac{\sqrt{5}}{2}\right)}{\left(\frac{i\sqrt{5}}{2}-\frac{i}{2}\right)\sqrt{x^4+3x^2+1}}-\frac{5\ln\left(2x^2+3+2\sqrt{x^4+3x^2+1}\right)}{2}-\frac{3\,\operatorname{arctanh}\left(\frac{\left(-2+i\sqrt{3}\right)\left(7x^2+\frac{11}{2}-\frac{5i\sqrt{3}}{2}\right)}{14\sqrt{-1-i\sqrt{3}}\,\sqrt{x^4+3x^2+1}}\right)}{4\sqrt{-1-i\sqrt{3}}}-\frac{3\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)x^2}\,\operatorname{EllipticPi}\left(\sqrt{-\frac{3}{2}+\frac{\sqrt{5}}{2}}\,x,-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{5}}{2}+\frac{3}{4}-\frac{3i\sqrt{3}}{4},\frac{\sqrt{-\frac{3}{2}-\frac{\sqrt{5}}{2}}}{\sqrt{-\frac{3}{2}+\frac{\sqrt{5}}{2}}}\right)}{\sqrt{-\frac{3}{2}+\frac{\sqrt{5}}{2}}\,\sqrt{x^4+3x^2+1}}-\frac{3i\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(-2-i\sqrt{3}\right)\left(7x^2+\frac{11}{2}+\frac{5i\sqrt{3}}{2}\right)}{14\sqrt{i\sqrt{3}-1}\,\sqrt{x^4+3x^2+1}}\right)}{4\sqrt{i\sqrt{3}-1}}+\frac{3i\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(-2+i\sqrt{3}\right)\left(7x^2+\frac{11}{2}-\frac{5i\sqrt{3}}{2}\right)}{14\sqrt{-1-i\sqrt{3}}\,\sqrt{x^4+3x^2+1}}\right)}{4\sqrt{-1-i\sqrt{3}}}-\frac{3\,\operatorname{arctanh}\left(\frac{\left(-2-i\sqrt{3}\right)\left(7x^2+\frac{11}{2}+\frac{5i\sqrt{3}}{2}\right)}{14\sqrt{i\sqrt{3}-1}\,\sqrt{x^4+3x^2+1}}\right)}{4\sqrt{i\sqrt{3}-1}}-\frac{3\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)x^2}\,\operatorname{EllipticPi}\left(\sqrt{-\frac{3}{2}+\frac{\sqrt{5}}{2}}\,x,-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{5}}{2}+\frac{3}{4}+\frac{3i\sqrt{3}}{4},\frac{\sqrt{-\frac{3}{2}-\frac{\sqrt{5}}{2}}}{\sqrt{-\frac{3}{2}+\frac{\sqrt{5}}{2}}}\right)}{\sqrt{-\frac{3}{2}+\frac{\sqrt{5}}{2}}\,\sqrt{x^4+3x^2+1}}+\frac{\sqrt{x^4+3x^2+1}}{x}+\frac{3\ln\left(\frac{3}{2}+x^2+\sqrt{x^4+3x^2+1}\right)}{2}-\operatorname{arctanh}\left(\frac{3x^2+2}{2\sqrt{x^4+3x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(x^2+1)*(x^4+3*x^2+1)^(1/2)/x^2/(x^2+x+1)^2,x)
```

```
[Out] (x^4+3*x^2+1)^(1/2)/(x^2+x+1)+3/(1/2*I*5^(1/2)-1/2*I)*(1-(-3/2+1/2*5^(1/2))
*x^2)^(1/2)*(1-(-3/2-1/2*5^(1/2))*x^2)^(1/2)/(x^4+3*x^2+1)^(1/2)*EllipticF(
x*(1/2*I*5^(1/2)-1/2*I),3/2+1/2*5^(1/2))-5/2*ln(2*x^2+3+2*(x^4+3*x^2+1)^(1/
2))-3/4/(-1-I*3^(1/2))^(1/2)*arctanh(1/14*(-2+I*3^(1/2))*(7*x^2+11/2-5/2*I*
3^(1/2))/(-1-I*3^(1/2))^(1/2)/(x^4+3*x^2+1)^(1/2))-3/(-3/2+1/2*5^(1/2))^(1/
2)*(1-(-3/2+1/2*5^(1/2))*x^2)^(1/2)*(1-(-3/2-1/2*5^(1/2))*x^2)^(1/2)/(x^4+3
*x^2+1)^(1/2)*EllipticPi((-3/2+1/2*5^(1/2))^(1/2)*x,-1/2*(-1/2+1/2*I*3^(1/2
))*5^(1/2)+3/4-3/4*I*3^(1/2),(-3/2-1/2*5^(1/2))^(1/2)/(-3/2+1/2*5^(1/2))^(1
/2))-3/4*I*3^(1/2)/(I*3^(1/2)-1)^(1/2)*arctanh(1/14*(-2-I*3^(1/2))*(7*x^2+1
1/2+5/2*I*3^(1/2))/(I*3^(1/2)-1)^(1/2)/(x^4+3*x^2+1)^(1/2))+3/4*I*3^(1/2)/(
-1-I*3^(1/2))^(1/2)*arctanh(1/14*(-2+I*3^(1/2))*(7*x^2+11/2-5/2*I*3^(1/2))/
(-1-I*3^(1/2))^(1/2)/(x^4+3*x^2+1)^(1/2))-3/4/(I*3^(1/2)-1)^(1/2)*arctanh(1
/14*(-2-I*3^(1/2))*(7*x^2+11/2+5/2*I*3^(1/2))/(I*3^(1/2)-1)^(1/2)/(x^4+3*x^
2+1)^(1/2))-3/(-3/2+1/2*5^(1/2))^(1/2)*(1-(-3/2+1/2*5^(1/2))*x^2)^(1/2)*(1-
(-3/2-1/2*5^(1/2))*x^2)^(1/2)/(x^4+3*x^2+1)^(1/2)*EllipticPi((-3/2+1/2*5^(1
/2))^(1/2)*x,-1/2*(-1/2-1/2*I*3^(1/2))*5^(1/2)+3/4+3/4*I*3^(1/2),(-3/2-1/2*
5^(1/2))^(1/2)/(-3/2+1/2*5^(1/2))^(1/2))+1/x*(x^4+3*x^2+1)^(1/2)+3/2*ln(3/2
+x^2+(x^4+3*x^2+1)^(1/2))-arctanh(1/2*(3*x^2+2)/(x^4+3*x^2+1)^(1/2))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+3\,x^2+1}\,\left(x^2+1\right)\left(x^2-1\right)}{\left(x^2+x+1\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^2+1)*(x^4+3*x^2+1)^(1/2)/x^2/(x^2+x+1)^2,x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 1)*(x^2 + 1)*(x^2 - 1)/((x^2 + x + 1)^2*x^2),
x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2-1\right)\,\left(x^2+1\right)\,\sqrt{x^4+3\,x^2+1}}{x^2\,{\left(x^2+x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*(x^2 + 1)*(3*x^2 + x^4 + 1)^(1/2))/(x^2*(x + x^2 + 1)^2),x)
```

```
[Out] int(((x^2 - 1)*(x^2 + 1)*(3*x^2 + x^4 + 1)^(1/2))/(x^2*(x + x^2 + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2+1\right)\sqrt{x^4+3x^2+1}}{x^2\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**2+1)*(x**4+3*x**2+1)**(1/2)/x**2/(x**2+x+1)**2,x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)*sqrt(x**4 + 3*x**2 + 1)/(x**2*(x**2 + x
 + 1)**2), x)
```

3.1211 $\int \frac{(2+x^2)(-4+x+2x^2)\sqrt{8-7x^2+2x^4}}{x^4}\,dx$

Optimal. Leaf size=98

$$\frac{\log\left(\sqrt{2}x^2+\sqrt{2x^4-7x^2+8}-2\sqrt{2}\right)}{2\sqrt{2}}+\frac{\sqrt{2x^4-7x^2+8}\,(4x^4+3x^3-14x^2-6x+16)}{6x^3}-\frac{\log(x)}{2\sqrt{2}}$$

Rubi [A] time = 0.14, antiderivative size = 93, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1662, 1590, 1251, 812, 838, 206}

$$-\frac{(2-x^2)\sqrt{2x^4-7x^2+8}}{2x^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}(2-x^2)}{\sqrt{2x^4-7x^2+8}}\right)}{2\sqrt{2}}+\frac{(2x^4-7x^2+8)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x^2)*(-4 + x + 2*x^2)*Sqrt[8 - 7*x^2 + 2*x^4])/x^4,x]
```

```
[Out] -1/2*((2 - x^2)*Sqrt[8 - 7*x^2 + 2*x^4])/x^2 + (8 - 7*x^2 + 2*x^4)^(3/2)/(3
*x^3) - ArcTanh[(Sqrt[2]*(2 - x^2))/Sqrt[8 - 7*x^2 + 2*x^4]]/(2*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 838

```
Int[((f_) + (g_.)*(x_))/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_
.)*(x_)^2]), x_Symbol] :> Dist[(4*f*(a - d))/(b*d - a*e), Subst[Int[1/(4*(a
 - d) - x^2), x], x, (2*(a - d) + (b - e)*x)/Sqrt[a + b*x + c*x^2]], x] /;
FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[4*c*(a - d) - (b - e)^2, 0] && EqQ[e
*f*(b - e) - 2*g*(b*d - a*e), 0] && NeQ[b*d - a*e, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+x^2)(-4+x+2x^2)\sqrt{8-7x^2+2x^4}}{x^4}\,dx &= \int \frac{(2+x^2)\sqrt{8-7x^2+2x^4}}{x^3}\,dx + \int \frac{(-8+2x^4)\sqrt{8-7x^2+2x^4}}{x^4} \\ &= \frac{(8-7x^2+2x^4)^{3/2}}{3x^3} + \frac{1}{2}\text{Subst}\left(\int \frac{(2+x)\sqrt{8-7x+2x^2}}{x^2}\,dx, x, x \right. \\ &= -\frac{(2-x^2)\sqrt{8-7x^2+2x^4}}{2x^2} + \frac{(8-7x^2+2x^4)^{3/2}}{3x^3} - \frac{1}{4}\text{Subst}\left(\int \right. \\ &= -\frac{(2-x^2)\sqrt{8-7x^2+2x^4}}{2x^2} + \frac{(8-7x^2+2x^4)^{3/2}}{3x^3} - 2\,\text{Subst}\left(\int \right. \\ &= -\frac{(2-x^2)\sqrt{8-7x^2+2x^4}}{2x^2} + \frac{(8-7x^2+2x^4)^{3/2}}{3x^3} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}}{\sqrt{8-}}\right.}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.25, size = 105, normalized size = 1.07

$$\frac{\sinh^{-1}\left(\frac{4x^2-7}{\sqrt{15}}\right)-\tanh^{-1}\left(\frac{16-7x^2}{4\sqrt{2}\sqrt{2x^4-7x^2+8}}\right)}{4\sqrt{2}}+\sqrt{2x^4-7x^2+8}\left(\frac{8}{3x^3}-\frac{1}{x^2}+\frac{2x}{3}-\frac{7}{3x}+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x^2)*(-4 + x + 2*x^2)*Sqrt[8 - 7*x^2 + 2*x^4])/x^4,x]
```

```
[Out] (1/2 + 8/(3*x^3) - x^(-2) - 7/(3*x) + (2*x)/3)*Sqrt[8 - 7*x^2 + 2*x^4] + (A
rcSinh[(-7 + 4*x^2)/Sqrt[15]] - ArcTanh[(16 - 7*x^2)/(4*Sqrt[2]*Sqrt[8 - 7*
x^2 + 2*x^4])])/(4*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.61, size = 98, normalized size = 1.00

$$\frac{\log\left(\sqrt{2}x^2+\sqrt{2x^4-7x^2+8}-2\sqrt{2}\right)}{2\sqrt{2}}+\frac{\sqrt{2x^4-7x^2+8}\left(4x^4+3x^3-14x^2-6x+16\right)}{6x^3}-\frac{\log(x)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^2)*(-4 + x + 2*x^2)*Sqrt[8 - 7*x^2 + 2*x^4])/x^4
,x]
```

```
[Out] (Sqrt[8 - 7*x^2 + 2*x^4]*(16 - 6*x - 14*x^2 + 3*x^3 + 4*x^4))/(6*x^3) - Log
[x]/(2*Sqrt[2]) + Log[-2*Sqrt[2] + Sqrt[2]*x^2 + Sqrt[8 - 7*x^2 + 2*x^4]]/(
2*Sqrt[2])
```

fricas [A] time = 0.47, size = 91, normalized size = 0.93

$$\frac{3\sqrt{2}x^3\log\left(\frac{4x^4+2\sqrt{2}\sqrt{2x^4-7x^2+8}\left(x^2-2\right)-15x^2+16}{x^2}\right)+4\left(4x^4+3x^3-14x^2-6x+16\right)\sqrt{2x^4-7x^2+8}}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)*(2*x^2+x-4)*(2*x^4-7*x^2+8)^(1/2)/x^4,x, algorithm="frica
s")
```

```
[Out] 1/24*(3*sqrt(2)*x^3*log((4*x^4 + 2*sqrt(2)*sqrt(2*x^4 - 7*x^2 + 8)*(x^2 - 2
) - 15*x^2 + 16)/x^2) + 4*(4*x^4 + 3*x^3 - 14*x^2 - 6*x + 16)*sqrt(2*x^4 -
7*x^2 + 8))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{2x^4-7x^2+8}\left(2x^2+x-4\right)\left(x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)*(2*x^2+x-4)*(2*x^4-7*x^2+8)^(1/2)/x^4,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(2*x^4 - 7*x^2 + 8)*(2*x^2 + x - 4)*(x^2 + 2)/x^4, x)
```

maple [B] time = 0.39, size = 163, normalized size = 1.66

$$\frac{2x\sqrt{2x^4-7x^2+8}}{3}+\frac{8\sqrt{2x^4-7x^2+8}}{3x^3}-\frac{7\sqrt{2x^4-7x^2+8}}{3x}-\frac{\left(2x^4-7x^2+8\right)^{\frac{3}{2}}}{8x^2}+\frac{\sqrt{2x^4-7x^2+8}}{16}+\frac{\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{15}\left(x^2-\frac{7}{4}\right)}{15}\right)}{8}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-7x^2+16\right)\sqrt{2}}{8\sqrt{2x^4-7x^2+8}}\right)}{8}+\frac{\left(4x^2-7\right)\sqrt{2x^4-7x^2+8}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)*(2*x^2+x-4)*(2*x^4-7*x^2+8)^(1/2)/x^4,x)
```

```
[Out] 2/3*x*(2*x^4-7*x^2+8)^(1/2)+8/3/x^3*(2*x^4-7*x^2+8)^(1/2)-7/3/x*(2*x^4-7*x^
2+8)^(1/2)-1/8/x^2*(2*x^4-7*x^2+8)^(3/2)+1/16*(2*x^4-7*x^2+8)^(1/2)+1/8*2^(
1/2)*arcsinh(4/15*15^(1/2)*(x^2-7/4))-1/8*2^(1/2)*arctanh(1/8*(-7*x^2+16)*2
^(1/2)/(2*x^4-7*x^2+8)^(1/2))+1/16*(4*x^2-7)*(2*x^4-7*x^2+8)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{2x^4-7x^2+8}\left(2x^2+x-4\right)\left(x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)*(2*x^2+x-4)*(2*x^4-7*x^2+8)^(1/2)/x^4,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(2*x^4 - 7*x^2 + 8)*(2*x^2 + x - 4)*(x^2 + 2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(x^2+2\right)\,\left(2x^2+x-4\right)\,\sqrt{2x^4-7x^2+8}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 2)*(x + 2*x^2 - 4)*(2*x^4 - 7*x^2 + 8)^(1/2))/x^4,x)
[Out] int(((x^2 + 2)*(x + 2*x^2 - 4)*(2*x^4 - 7*x^2 + 8)^(1/2))/x^4, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+2\right)\left(2x^2+x-4\right)\sqrt{2x^4-7x^2+8}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)*(2*x**2+x-4)*(2*x**4-7*x**2+8)**(1/2)/x**4,x)
[Out] Integral((x**2 + 2)*(2*x**2 + x - 4)*sqrt(2*x**4 - 7*x**2 + 8)/x**4, x)
```

3.1212 $\int x^4 \left(-b+ax^4\right)^{3/4} dx$

Optimal. Leaf size=98

$$-\frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{64a^{5/4}}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{64a^{5/4}}+\frac{\left(ax^4-b\right)^{3/4}\left(4ax^5-3bx\right)}{32a}$$

Rubi [A] time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {279, 321, 240, 212, 206, 203}

$$-\frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{64a^{5/4}}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{64a^{5/4}}-\frac{3bx\left(ax^4-b\right)^{3/4}}{32a}+\frac{1}{8}x^5\left(ax^4-b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(-b + a*x^4)^(3/4),x]
```

```
[Out] (-3*b*x*(-b + a*x^4)^(3/4))/(32*a) + (x^5*(-b + a*x^4)^(3/4))/8 - (3*b^2*Ar
cTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(64*a^(5/4)) - (3*b^2*ArcTanh[(a^(1/4
)*x)/(-b + a*x^4)^(1/4)])/(64*a^(5/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int x^4\left(-b+ax^4\right)^{3/4}\,dx &= \frac{1}{8}x^5\left(-b+ax^4\right)^{3/4}-\frac{1}{8}(3b)\int\frac{x^4}{\sqrt[4]{-b+ax^4}}\,dx\\
&= -\frac{3bx\left(-b+ax^4\right)^{3/4}}{32a}+\frac{1}{8}x^5\left(-b+ax^4\right)^{3/4}-\frac{\left(3b^2\right)\int\frac{1}{\sqrt[4]{-b+ax^4}}\,dx}{32a}\\
&= -\frac{3bx\left(-b+ax^4\right)^{3/4}}{32a}+\frac{1}{8}x^5\left(-b+ax^4\right)^{3/4}-\frac{\left(3b^2\right)\operatorname{Subst}\left(\int\frac{1}{1-ax^4}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)}{32a}\\
&= -\frac{3bx\left(-b+ax^4\right)^{3/4}}{32a}+\frac{1}{8}x^5\left(-b+ax^4\right)^{3/4}-\frac{\left(3b^2\right)\operatorname{Subst}\left(\int\frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)}{64a}-\frac{(3b^{\cdots}}{}\\
&= -\frac{3bx\left(-b+ax^4\right)^{3/4}}{32a}+\frac{1}{8}x^5\left(-b+ax^4\right)^{3/4}-\frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{64a^{5/4}}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{64a^{5/4}}
\end{aligned}
$$

Mathematica [C] time = 0.08, size = 65, normalized size = 0.66

$$
\frac{x\left(ax^4-b\right)^{3/4}\left(\frac{b\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};\frac{ax^4}{b}\right)}{\left(1-\frac{ax^4}{b}\right)^{3/4}}+ax^4-b\right)}{8a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(-b + a*x^4)^(3/4),x]
```

```
[Out] (x*(-b + a*x^4)^(3/4)*(-b + a*x^4 + (b*Hypergeometric2F1[-3/4, 1/4, 5/4, (a
*x^4)/b])/(1 - (a*x^4)/b)^(3/4)))/(8*a)
```

IntegrateAlgebraic [A] time = 0.37, size = 98, normalized size = 1.00

$$
-\frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{64a^{5/4}}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{64a^{5/4}}+\frac{\left(ax^4-b\right)^{3/4}\left(4ax^5-3bx\right)}{32a}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(-b + a*x^4)^(3/4),x]
```

```
[Out] ((-b + a*x^4)^(3/4)*(-3*b*x + 4*a*x^5))/(32*a) - (3*b^2*ArcTan[(a^(1/4)*x)/
(-b + a*x^4)^(1/4)])/(64*a^(5/4)) - (3*b^2*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)
^(1/4)])/(64*a^(5/4))
```

fricas [B] time = 0.42, size = 228, normalized size = 2.33

$$
-\frac{12\left(\frac{b^8}{a^5}\right)^{\frac{1}{4}}a\arctan\left(-\frac{\left(ax^4-b\right)^{\frac{1}{4}}\left(\frac{b^8}{a^5}\right)^{\frac{1}{4}}ab^6-\left(\frac{b^8}{a^5}\right)^{\frac{1}{4}}ax\sqrt{\frac{\sqrt{\frac{b^8}{a^5}}a^3b^8x^2+\sqrt{ax^4-b}\,b^{12}}{x^2}}}{b^8x}\right)+3\left(\frac{b^8}{a^5}\right)^{\frac{1}{4}}a\log\left(\frac{27\left(\left(ax^4-b\right)^{\frac{1}{4}}b^6+\left(\frac{b^8}{a^5}\right)^{\frac{3}{4}}a^4x\right)}{x}\right)-3\left(\frac{b^8}{a^5}\right)^{\frac{1}{4}}a\log\left(\frac{27\left(\left(ax^4-b\right)^{\frac{1}{4}}b^6-\left(\frac{b^8}{a^5}\right)^{\frac{3}{4}}a^4x\right)}{x}\right)-4\left(4ax^5-3bx\right)\left(ax^4-b\right)^{\frac{3}{4}}}{128a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^4-b)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/128*(12*(b^8/a^5)^(1/4)*a*arctan(-((a*x^4 - b)^(1/4)*(b^8/a^5)^(1/4)*a*b
^6 - (b^8/a^5)^(1/4)*a*x*sqrt((sqrt(b^8/a^5)*a^3*b^8*x^2 + sqrt(a*x^4 - b)*
b^12)/x^2))/(b^8*x)) + 3*(b^8/a^5)^(1/4)*a*log(27*((a*x^4 - b)^(1/4)*b^6 +
(b^8/a^5)^(3/4)*a^4*x)/x) - 3*(b^8/a^5)^(1/4)*a*log(27*((a*x^4 - b)^(1/4)*b
^6 - (b^8/a^5)^(3/4)*a^4*x)/x) - 4*(4*a*x^5 - 3*b*x)*(a*x^4 - b)^(3/4))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^4 - b\right)^{\frac{3}{4}} x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^4-b)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^4 - b)^(3/4)*x^4, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int x^4 \left(a\,x^4 - b\right)^{\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a*x^4-b)^(3/4),x)
```

```
[Out] int(x^4*(a*x^4-b)^(3/4),x)
```

maxima [B] time = 0.42, size = 162, normalized size = 1.65

$$\frac{3\,b^2\left(\frac{2\arctan\left(\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}} + \frac{\log\left(-\frac{a^{\frac{1}{4}} - \frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}} + \frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right)}{128\,a} + \frac{\frac{\left(ax^4-b\right)^{\frac{3}{4}}ab^2}{x^3} + \frac{3\left(ax^4-b\right)^{\frac{7}{4}}b^2}{x^7}}{32\left(a^3 - \frac{2\left(ax^4-b\right)a^2}{x^4} + \frac{\left(ax^4-b\right)^2 a}{x^8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^4-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 3/128*b^2*(2*arctan((a*x^4 - b)^(1/4)/(a^(1/4)*x))/a^(1/4) + log(-(a^(1/4)
- (a*x^4 - b)^(1/4)/x)/(a^(1/4) + (a*x^4 - b)^(1/4)/x))/a^(1/4))/a + 1/32*(
(a*x^4 - b)^(3/4)*a*b^2/x^3 + 3*(a*x^4 - b)^(7/4)*b^2/x^7)/(a^3 - 2*(a*x^4
- b)*a^2/x^4 + (a*x^4 - b)^2*a/x^8)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(a\,x^4 - b\right)^{3/4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a*x^4 - b)^(3/4),x)
```

```
[Out] int(x^4*(a*x^4 - b)^(3/4), x)
```

sympy [C] time = 1.32, size = 42, normalized size = 0.43

$$\frac{b^{\frac{3}{4}}x^5e^{\frac{3i\pi}{4}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{ax^4}{b}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a*x**4-b)**(3/4),x)
```

```
[Out] b**(3/4)*x**5*exp(3*I*pi/4)*gamma(5/4)*hyper((-3/4, 5/4), (9/4,), a*x**4/b)
/(4*gamma(9/4))
```

3.1213 $\int \frac{(-2b+ax^2)\sqrt[4]{bx^2+ax^4}}{x^2}\,dx$

Optimal. Leaf size=98

$$\frac{\sqrt[4]{ax^4+bx^2}\,(ax^2+8b)}{2x}+\frac{7}{4}\sqrt[4]{a}\,b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)-\frac{7}{4}\sqrt[4]{a}\,b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)$$

Rubi [A] time = 0.22, antiderivative size = 170, normalized size of antiderivative = 1.73, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2038, 2004, 2032, 329, 331, 298, 203, 206}

$$-\frac{7}{2}ax\sqrt[4]{ax^4+bx^2}+\frac{7\sqrt[4]{a}\,bx^{3/2}\left(ax^2+b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{4\left(ax^4+bx^2\right)^{3/4}}-\frac{7\sqrt[4]{a}\,bx^{3/2}\left(ax^2+b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{4\left(ax^4+bx^2\right)^{3/4}}+\frac{4\left(ax^4+bx^2\right)^{5/4}}{x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-2*b + a*x^2)*(b*x^2 + a*x^4)^(1/4))/x^2,x]
```

```
[Out] (-7*a*x*(b*x^2 + a*x^4)^(1/4))/2 + (4*(b*x^2 + a*x^4)^(5/4))/x^3 + (7*a^(1/
4)*b*x^(3/2)*(b + a*x^2)^(3/4)*ArcTan[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])
/(4*(b*x^2 + a*x^4)^(3/4)) - (7*a^(1/4)*b*x^(3/2)*(b + a*x^2)^(3/4)*ArcTanh
[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(4*(b*x^2 + a*x^4)^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{\left(-2b+ax^2\right)\sqrt[4]{bx^2+ax^4}}{x^2}\,dx = \frac{4\left(bx^2+ax^4\right)^{5/4}}{x^3}-(7a)\int \sqrt[4]{bx^2+ax^4}\,dx$$

$$= -\frac{7}{2}ax\sqrt[4]{bx^2+ax^4}+\frac{4\left(bx^2+ax^4\right)^{5/4}}{x^3}-\frac{1}{4}(7ab)\int \frac{x^2}{\left(bx^2+ax^4\right)^{3/4}}\,dx$$

$$= -\frac{7}{2}ax\sqrt[4]{bx^2+ax^4}+\frac{4\left(bx^2+ax^4\right)^{5/4}}{x^3}-\frac{\left(7abx^{3/2}\left(b+ax^2\right)^{3/4}\right)\int \frac{\sqrt{x}}{\left(b+ax^2\right)^{3/4}}\,dx}{4\left(bx^2+ax^4\right)^{3/4}}$$

$$= -\frac{7}{2}ax\sqrt[4]{bx^2+ax^4}+\frac{4\left(bx^2+ax^4\right)^{5/4}}{x^3}-\frac{\left(7abx^{3/2}\left(b+ax^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{\left(b+ax^4\right)^{3/}}\right.}{2\left(bx^2+ax^4\right)^{3/4}}$$

$$= -\frac{7}{2}ax\sqrt[4]{bx^2+ax^4}+\frac{4\left(bx^2+ax^4\right)^{5/4}}{x^3}-\frac{\left(7abx^{3/2}\left(b+ax^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx\right.}{2\left(bx^2+ax^4\right)^{3/4}}$$

$$= -\frac{7}{2}ax\sqrt[4]{bx^2+ax^4}+\frac{4\left(bx^2+ax^4\right)^{5/4}}{x^3}-\frac{\left(7\sqrt{a}\,bx^{3/2}\left(b+ax^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}}\right.}{4\left(bx^2+ax^4\right)^{3/4}}$$

$$= -\frac{7}{2}ax\sqrt[4]{bx^2+ax^4}+\frac{4\left(bx^2+ax^4\right)^{5/4}}{x^3}+\frac{7\sqrt[4]{a}\,bx^{3/2}\left(b+ax^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{4\left(bx^2+ax^4\right)^{3/4}}-$$

Mathematica [C] time = 0.07, size = 71, normalized size = 0.72

$$\frac{2\sqrt[4]{x^2\left(ax^2+b\right)}\left(6\left(ax^2+b\right)-\frac{7ax^2\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{ax^2}{b}\right)}{\sqrt[4]{\frac{ax^2}{b}+1}}\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-2*b + a*x^2)*(b*x^2 + a*x^4)^(1/4))/x^2,x]
```

```
[Out] (2*(x^2*(b + a*x^2))^(1/4)*(6*(b + a*x^2) - (7*a*x^2*Hypergeometric2F1[-1/4
, 3/4, 7/4, -((a*x^2)/b)])/(1 + (a*x^2)/b)^(1/4)))/(3*x)
```

IntegrateAlgebraic [A] time = 0.32, size = 98, normalized size = 1.00

$$\frac{\sqrt[4]{ax^4+bx^2}\left(ax^2+8b\right)}{2x}+\frac{7}{4}\sqrt[4]{a}\,b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)-\frac{7}{4}\sqrt[4]{a}\,b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*b + a*x^2)*(b*x^2 + a*x^4)^(1/4))/x^2,x]
```

```
[Out] ((8*b + a*x^2)*(b*x^2 + a*x^4)^(1/4))/(2*x) + (7*a^(1/4)*b*ArcTan[(a^(1/4)*
x)/(b*x^2 + a*x^4)^(1/4)])/4 - (7*a^(1/4)*b*ArcTanh[(a^(1/4)*x)/(b*x^2 + a*
x^4)^(1/4)])/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-2*b)*(a*x^4+b*x^2)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.20, size = 221, normalized size = 2.26

$$\frac{8\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}abx^2-14\sqrt{2}\,(-a)^{\frac{1}{4}}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)-14\sqrt{2}\,(-a)^{\frac{1}{4}}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)-7\sqrt{2}\,(-a)^{\frac{1}{4}}b^2\log\left(\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)+7\sqrt{2}\,(-a)^{\frac{1}{4}}b^2\log\left(-\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)+64\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}b^2}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-2*b)*(a*x^4+b*x^2)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/16*(8*(a + b/x^2)^(1/4)*a*b*x^2 - 14*sqrt(2)*(-a)^(1/4)*b^2*arctan(1/2*sq
rt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^2)^(1/4))/(-a)^(1/4)) - 14*sqrt(2)*(
-a)^(1/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(a + b/x^2)^(1/4)
)/(-a)^(1/4)) - 7*sqrt(2)*(-a)^(1/4)*b^2*log(sqrt(2)*(-a)^(1/4)*(a + b/x^2)
^(1/4) + sqrt(-a) + sqrt(a + b/x^2)) + 7*sqrt(2)*(-a)^(1/4)*b^2*log(-sqrt(2
)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sqrt(a + b/x^2)) + 64*(a + b/x^
2)^(1/4)*b^2)/b
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^2-2b\right)\left(a\,x^4+b\,x^2\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-2*b)*(a*x^4+b*x^2)^(1/4)/x^2,x)
```

[Out] int((a*x^2-2*b)*(a*x^4+b*x^2)^(1/4)/x^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4 + bx^2\right)^{\frac{1}{4}}\left(ax^2 - 2\,b\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^2-2*b)*(a*x^4+b*x^2)^(1/4)/x^2,x, algorithm="maxima")

[Out] integrate((a*x^4 + b*x^2)^(1/4)*(a*x^2 - 2*b)/x^2, x)

mupad [B] time = 1.28, size = 89, normalized size = 0.91

$$\frac{2\,a\,x\,\left(a\,x^4+b\,x^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{a\,x^2}{b}\right)}{3\,\left(\frac{a\,x^2}{b}+1\right)^{1/4}}+\frac{4\,b\,\left(a\,x^4+b\,x^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ -\frac{a\,x^2}{b}\right)}{x\,\left(\frac{a\,x^2}{b}+1\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-((a*x^4 + b*x^2)^(1/4)*(2*b - a*x^2))/x^2,x)

[Out] (2*a*x*(a*x^4 + b*x^2)^(1/4)*hypergeom([-1/4, 3/4], 7/4, -(a*x^2)/b))/(3*((a*x^2)/b + 1)^(1/4)) + (4*b*(a*x^4 + b*x^2)^(1/4)*hypergeom([-1/4, -1/4], 3/4, -(a*x^2)/b))/(x*((a*x^2)/b + 1)^(1/4))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\left(ax^2 + b\right)}\left(ax^2 - 2b\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**2-2*b)*(a*x**4+b*x**2)**(1/4)/x**2,x)

[Out] Integral((x**2*(a*x**2 + b))**(1/4)*(a*x**2 - 2*b)/x**2, x)

3.1214 $\int \frac{(2b+ax^2)\sqrt[4]{bx^2+ax^4}}{x^2}\,dx$

Optimal. Leaf size=98

$$\frac{\sqrt[4]{ax^4+bx^2}\,(ax^2-8b)}{2x}-\frac{9}{4}\sqrt[4]{a}\,b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)+\frac{9}{4}\sqrt[4]{a}\,b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)$$

Rubi [A] time = 0.22, antiderivative size = 170, normalized size of antiderivative = 1.73, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2038, 2004, 2032, 329, 331, 298, 203, 206}

$$\frac{9}{2}ax\sqrt[4]{ax^4+bx^2}-\frac{9\sqrt[4]{a}\,bx^{3/2}\left(ax^2+b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{4\left(ax^4+bx^2\right)^{3/4}}+\frac{9\sqrt[4]{a}\,bx^{3/2}\left(ax^2+b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{4\left(ax^4+bx^2\right)^{3/4}}-\frac{4\left(ax^4+bx^2\right)^{5/4}}{x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((2*b + a*x^2)*(b*x^2 + a*x^4)^(1/4))/x^2,x]
```

```
[Out] (9*a*x*(b*x^2 + a*x^4)^(1/4))/2 - (4*(b*x^2 + a*x^4)^(5/4))/x^3 - (9*a^(1/4
)*b*x^(3/2)*(b + a*x^2)^(3/4)*ArcTan[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/
(4*(b*x^2 + a*x^4)^(3/4)) + (9*a^(1/4)*b*x^(3/2)*(b + a*x^2)^(3/4)*ArcTanh[
(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(4*(b*x^2 + a*x^4)^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2b+ax^2\right)\sqrt[4]{bx^2+ax^4}}{x^2}\,dx &= -\frac{4\left(bx^2+ax^4\right)^{5/4}}{x^3}+(9a)\int \sqrt[4]{bx^2+ax^4}\,dx\\
&= \frac{9}{2}ax\sqrt[4]{bx^2+ax^4}-\frac{4\left(bx^2+ax^4\right)^{5/4}}{x^3}+\frac{1}{4}(9ab)\int \frac{x^2}{\left(bx^2+ax^4\right)^{3/4}}\,dx\\
&= \frac{9}{2}ax\sqrt[4]{bx^2+ax^4}-\frac{4\left(bx^2+ax^4\right)^{5/4}}{x^3}+\frac{\left(9abx^{3/2}\left(b+ax^2\right)^{3/4}\right)\int \frac{\sqrt{x}}{\left(b+ax^2\right)^{3/4}}\,dx}{4\left(bx^2+ax^4\right)^{3/4}}\\
&= \frac{9}{2}ax\sqrt[4]{bx^2+ax^4}-\frac{4\left(bx^2+ax^4\right)^{5/4}}{x^3}+\frac{\left(9abx^{3/2}\left(b+ax^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{\left(b+ax^4\right)^{3/4}}\,d
\end{aligned}$$

$$= \frac{9}{2}ax\sqrt[4]{bx^2+ax^4}-\frac{4\left(bx^2+ax^4\right)^{5/4}}{x^3}+\frac{\left(9abx^{3/2}\left(b+ax^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx,x\right.}{2\left(bx^2+ax^4\right)^{3/4}}$$

$$= \frac{9}{2}ax\sqrt[4]{bx^2+ax^4}-\frac{4\left(bx^2+ax^4\right)^{5/4}}{x^3}+\frac{\left(9\sqrt{a}\,bx^{3/2}\left(b+ax^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,d\right.}{4\left(bx^2+ax^4\right)^{3/4}}$$

$$= \frac{9}{2}ax\sqrt[4]{bx^2+ax^4}-\frac{4\left(bx^2+ax^4\right)^{5/4}}{x^3}-\frac{9\sqrt[4]{a}\,bx^{3/2}\left(b+ax^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{4\left(bx^2+ax^4\right)^{3/4}}+\frac{9}{}$$

Mathematica [C] time = 0.07, size = 69, normalized size = 0.70

$$\frac{2\sqrt[4]{x^2\left(ax^2+b\right)}\left(\frac{3ax^2\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{ax^2}{b}\right)}{\sqrt[4]{\frac{ax^2}{b}+1}}-2\left(ax^2+b\right)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2*b + a*x^2)*(b*x^2 + a*x^4)^(1/4))/x^2,x]
```

```
[Out] (2*(x^2*(b + a*x^2))^(1/4)*(-2*(b + a*x^2) + (3*a*x^2*Hypergeometric2F1[-1/
4, 3/4, 7/4, -((a*x^2)/b)])/(1 + (a*x^2)/b)^(1/4)))/x
```

IntegrateAlgebraic [A] time = 0.33, size = 98, normalized size = 1.00

$$\frac{\sqrt[4]{ax^4+bx^2}\left(ax^2-8b\right)}{2x}-\frac{9}{4}\sqrt[4]{a}\,b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)+\frac{9}{4}\sqrt[4]{a}\,b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2*b + a*x^2)*(b*x^2 + a*x^4)^(1/4))/x^2,x]
```

```
[Out] ((-8*b + a*x^2)*(b*x^2 + a*x^4)^(1/4))/(2*x) - (9*a^(1/4)*b*ArcTan[(a^(1/4)
*x)/(b*x^2 + a*x^4)^(1/4)])/4 + (9*a^(1/4)*b*ArcTanh[(a^(1/4)*x)/(b*x^2 + a
*x^4)^(1/4)])/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b)*(a*x^4+b*x^2)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.18, size = 221, normalized size = 2.26

$$\frac{8\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}abx^2+18\sqrt{2}\,(-a)^{\frac{1}{4}}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)+18\sqrt{2}\,(-a)^{\frac{1}{4}}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)+9\sqrt{2}\,(-a)^{\frac{1}{4}}b^2\log\left(\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)-9\sqrt{2}\,(-a)^{\frac{1}{4}}b^2\log\left(-\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)-64\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}b^2}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b)*(a*x^4+b*x^2)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/16*(8*(a + b/x^2)^(1/4)*a*b*x^2 + 18*sqrt(2)*(-a)^(1/4)*b^2*arctan(1/2*sq
rt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^2)^(1/4))/(-a)^(1/4)) + 18*sqrt(2)*(
-a)^(1/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(a + b/x^2)^(1/4)
)/(-a)^(1/4)) + 9*sqrt(2)*(-a)^(1/4)*b^2*log(sqrt(2)*(-a)^(1/4)*(a + b/x^2)
^(1/4) + sqrt(-a) + sqrt(a + b/x^2)) - 9*sqrt(2)*(-a)^(1/4)*b^2*log(-sqrt(2
)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sqrt(a + b/x^2)) - 64*(a + b/x^
2)^(1/4)*b^2)/b
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^2+2b\right)\left(a\,x^4+b\,x^2\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+2*b)*(a*x^4+b*x^2)^(1/4)/x^2,x)
```

```
[Out] int((a*x^2+2*b)*(a*x^4+b*x^2)^(1/4)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4 + bx^2\right)^{\frac{1}{4}}\left(ax^2 + 2\,b\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b)*(a*x^4+b*x^2)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + b*x^2)^(1/4)*(a*x^2 + 2*b)/x^2, x)
```

mupad [B] time = 1.10, size = 89, normalized size = 0.91

$$\frac{2\,a\,x\,\left(a\,x^4+b\,x^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{a\,x^2}{b}\right)}{3\,{\left(\frac{a\,x^2}{b}+1\right)}^{1/4}}-\frac{4\,b\,\left(a\,x^4+b\,x^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ -\frac{a\,x^2}{b}\right)}{x\,{\left(\frac{a\,x^2}{b}+1\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^4 + b*x^2)^(1/4)*(2*b + a*x^2))/x^2,x)
```

```
[Out] (2*a*x*(a*x^4 + b*x^2)^(1/4)*hypergeom([-1/4, 3/4], 7/4, -(a*x^2)/b))/(3*((
a*x^2)/b + 1)^(1/4)) - (4*b*(a*x^4 + b*x^2)^(1/4)*hypergeom([-1/4, -1/4], 3
/4, -(a*x^2)/b))/(x*((a*x^2)/b + 1)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\left(ax^2 + b\right)}\,\left(ax^2 + 2b\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+2*b)*(a*x**4+b*x**2)**(1/4)/x**2,x)
```

```
[Out] Integral((x**2*(a*x**2 + b))**(1/4)*(a*x**2 + 2*b)/x**2, x)
```

3.1215 $\int \frac{(1-3k^2)x+2k^2x^3}{\sqrt[4]{(1-x^2)(1-k^2x^2)}\,(-1+d+(-d+3k^2)x^2-3k^4x^4+k^6x^6)}\,dx$

Optimal. Leaf size=98

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{k^2x^2-1}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{k^2x^2-1}\right)}{d^{3/4}}$$

Rubi [F] time = 10.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-3k^2)\,x+2k^2x^3}{\sqrt[4]{(1-x^2)\,(1-k^2x^2)}\,\left(-1+d+(-d+3k^2)\,x^2-3k^4x^4+k^6x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 - 3*k^2)*x + 2*k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/4)*(-1 + d +
(-d + 3*k^2)*x^2 - 3*k^4*x^4 + k^6*x^6)),x]
```

```
[Out] (-4*k^2*(1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*Defer[Subst][Defer[Int][x^2/((1
 - k^2 + k^2*x^4)^(1/4)*(-1 + d*x^4 - 3*k^2*(-1 + x^4) - 3*k^4*(-1 + x^4)^2
 - k^6*(-1 + x^4)^3)), x], x, (1 - x^2)^(1/4)])/((1 - x^2)*(1 - k^2*x^2))^(
1/4) + (2*(1 - 3*k^2)*(1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*Defer[Subst][Defe
r[Int][x^2/((1 - k^2 + k^2*x^4)^(1/4)*(1 - d*x^4 + 3*k^2*(-1 + x^4) + 3*k^4
*(-1 + x^4)^2 + k^6*(-1 + x^4)^3)), x], x, (1 - x^2)^(1/4)])/((1 - x^2)*(1
- k^2*x^2))^(1/4) - (4*k^2*(1 - x^2)^(1/4)*(1 - k^2*x^2)^(1/4)*Defer[Subst]
[Defer[Int][x^6/((1 - k^2 + k^2*x^4)^(1/4)*(1 - d*x^4 + 3*k^2*(-1 + x^4) +
3*k^4*(-1 + x^4)^2 + k^6*(-1 + x^4)^3)), x], x, (1 - x^2)^(1/4)])/((1 - x^2
)*(1 - k^2*x^2))^(1/4)
```

Rubi steps

$$\int \frac{\left(1-3k^2\right)x+2k^2x^3}{\sqrt[4]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d+\left(-d+3k^2\right)x^2-3k^4x^4+k^6x^6\right)}\,dx=\int \frac{x\left(1-3k^2+2k^2x\right.}{\sqrt[4]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d+\left(-d+\right.\right.}$$

$$=\frac{1}{2}\,\text{Subst}\left(\int \frac{1-3k}{\sqrt[4]{(1-x)\left(1-k^2x\right)}\left(-1+d+\right.}\right.$$

$$=\frac{\left(\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{1-x}\,\sqrt[4]{1-k}}\right.}{2\sqrt[4]{\left(1-x^2\right)\left(1\right.}}$$

$$=\frac{\left(\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{1-x}\,\sqrt[4]{1-k}}\right.}{2\sqrt[4]{\left(1-x^2\right)\left(1\right.}}$$

$$=\frac{\left(\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\text{Subst}\left(\int\left(\frac{}{\sqrt[4]{1-x}\,\sqrt[4]{1-}}\right.\right.}{}$$

$$=\frac{\left(k^2\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{1-x}\,\sqrt[4]{1}}\right.}{\sqrt[4]{\left(1-x^2\right)\left(1\right.}}$$

$$=-\frac{\left(4k^2\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{1+k}}\right.}{}$$

$$=-\frac{\left(4k^2\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{1-k}}\right.}{}$$

$$=-\frac{\left(4k^2\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\text{Subst}\left(\int\left(\frac{}{\sqrt[4]{1-}}\right.\right.}{}$$

$$=-\frac{\left(4k^2\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{1-k}}\right.}{}$$

$$=-\frac{\left(4k^2\sqrt[4]{1-x^2}\,\sqrt[4]{1-k^2x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{1-k}}\right.}{}$$

Mathematica [F] time = 1.95, size = 0, normalized size = 0.00

$$\int \frac{(1-3k^2)\,x+2k^2x^3}{\sqrt[4]{(1-x^2)\,(1-k^2x^2)}\,\left(-1+d+(-d+3k^2)\,x^2-3k^4x^4+k^6x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 - 3*k^2)*x + 2*k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/4)*(-1
+ d + (-d + 3*k^2)*x^2 - 3*k^4*x^4 + k^6*x^6)),x]
```

```
[Out] Integrate[((1 - 3*k^2)*x + 2*k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/4)*(-1
+ d + (-d + 3*k^2)*x^2 - 3*k^4*x^4 + k^6*x^6)), x]
```

IntegrateAlgebraic [A] time = 12.07, size = 98, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{k^2x^2-1}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{k^2x^4+(-k^2-1)x^2+1}}{k^2x^2-1}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 - 3*k^2)*x + 2*k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(
1/4)*(-1 + d + (-d + 3*k^2)*x^2 - 3*k^4*x^4 + k^6*x^6)),x]
```

```
[Out] ArcTan[(d^(1/4)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4))/(-1 + k^2*x^2)]/d^(3/
4) - ArcTanh[(d^(1/4)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4))/(-1 + k^2*x^2)]
/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((-3*k^2+1)*x+2*k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d+(3*k
^2-d)*x^2-3*k^4*x^4+k^6*x^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,k^2x^3-(3\,k^2-1)x}{(k^6x^6-3\,k^4x^4+(3\,k^2-d)x^2+d-1)\left((k^2x^2-1)(x^2-1)\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((-3*k^2+1)*x+2*k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d+(3*k
^2-d)*x^2-3*k^4*x^4+k^6*x^6),x, algorithm="giac")
```

```
[Out] integrate((2*k^2*x^3 - (3*k^2 - 1)*x)/((k^6*x^6 - 3*k^4*x^4 + (3*k^2 - d)*x
^2 + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/4)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(-3k^2+1)\,x+2k^2x^3}{\left((-x^2+1)\,(-k^2x^2+1)\right)^{\frac{1}{4}}\left(-1+d+(3k^2-d)\,x^2-3k^4x^4+k^6x^6\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-3*k^2+1)*x+2*k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d+(3*k^2-d)*
x^2-3*k^4*x^4+k^6*x^6),x)
```

```
[Out] int(((-3*k^2+1)*x+2*k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d+(3*k^2-d)*
x^2-3*k^4*x^4+k^6*x^6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,k^2x^3 - \left(3\,k^2 - 1\right)x}{\left(k^6x^6 - 3\,k^4x^4 + \left(3\,k^2 - d\right)x^2 + d - 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3*k^2+1)*x+2*k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/4)/(-1+d+(3*k
^2-d)*x^2-3*k^4*x^4+k^6*x^6),x, algorithm="maxima")
```

```
[Out] integrate((2*k^2*x^3 - (3*k^2 - 1)*x)/((k^6*x^6 - 3*k^4*x^4 + (3*k^2 - d)*x
^2 + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{2\,k^2\,x^3 - x\,\left(3\,k^2 - 1\right)}{\left(\left(x^2 - 1\right)\,\left(k^2\,x^2 - 1\right)\right)^{1/4}\,\left(3\,k^4\,x^4 - d - k^6\,x^6 + x^2\,\left(d - 3\,k^2\right) + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*k^2*x^3 - x*(3*k^2 - 1))/(((x^2 - 1)*(k^2*x^2 - 1))^(1/4)*(3*k^4*x^
4 - d - k^6*x^6 + x^2*(d - 3*k^2) + 1)),x)
```

```
[Out] int(-(2*k^2*x^3 - x*(3*k^2 - 1))/(((x^2 - 1)*(k^2*x^2 - 1))^(1/4)*(3*k^4*x^
4 - d - k^6*x^6 + x^2*(d - 3*k^2) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3*k**2+1)*x+2*k**2*x**3)/((-x**2+1)*(-k**2*x**2+1))**(1/4)/(-1
+d+(3*k**2-d)*x**2-3*k**4*x**4+k**6*x**6),x)
```

```
[Out] Timed out
```

3.1216 $\int \frac{1}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$

Optimal. Leaf size=98

$$\frac{x}{2\sqrt{\sqrt{a^2x^4+b}+ax^2}}+\frac{\log\left(\sqrt{a^2x^4+b}+\sqrt{2}\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}+ax^2\right)}{2\sqrt{2}\sqrt{a}}$$

Rubi [F] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] Defer[Int][1/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]], x]
```

Rubi steps

$$\int \frac{1}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx=\int \frac{1}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$$

Mathematica [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] Integrate[1/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]], x]
```

IntegrateAlgebraic [A] time = 0.57, size = 98, normalized size = 1.00

$$\frac{x}{2\sqrt{\sqrt{a^2x^4+b}+ax^2}}+\frac{\log\left(\sqrt{a^2x^4+b}+\sqrt{2}\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}+ax^2\right)}{2\sqrt{2}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] x/(2*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]) + Log[a*x^2 + Sqrt[b + a^2*x^4] + Sqr
t[2]*Sqrt[a]*x*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]]/(2*Sqrt[2]*Sqrt[a])
```

fricas [A] time = 3.01, size = 229, normalized size = 2.34

$$\left[\frac{\frac{\sqrt{2}\,b\log\left(4\,a^2x^4+4\,\sqrt{a^2x^4+b}\,ax^2+2\left(\sqrt{2}\,a^{\frac{3}{2}}x^3+\sqrt{2}\,\sqrt{a^2x^4+b}\,\sqrt{a}\,x\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}+b\right)}{\sqrt{a}}-4\left(ax^3-\sqrt{a^2x^4+b}\,x\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{8\,b},-\frac{\sqrt{2}\,b\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\,\sqrt{ax^2+\sqrt{a^2x^4+b}}\,\sqrt{-\frac{1}{a}}}{2\,x}\right)+2\left(ax^3-\sqrt{a^2x^4+b}\,x\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{4\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*b*log(4*a^2*x^4 + 4*sqrt(a^2*x^4 + b)*a*x^2 + 2*(sqrt(2)*a^(3
/2)*x^3 + sqrt(2)*sqrt(a^2*x^4 + b)*sqrt(a)*x)*sqrt(a*x^2 + sqrt(a^2*x^4 +
b)) + b)/sqrt(a) - 4*(a*x^3 - sqrt(a^2*x^4 + b)*x)*sqrt(a*x^2 + sqrt(a^2*x^
4 + b)))/b, -1/4*(sqrt(2)*b*sqrt(-1/a)*arctan(1/2*sqrt(2)*sqrt(a*x^2 + sqrt
(a^2*x^4 + b))*sqrt(-1/a)/x) + 2*(a*x^3 - sqrt(a^2*x^4 + b)*x)*sqrt(a*x^2 +
 sqrt(a^2*x^4 + b)))/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + \sqrt{a^2x^4 + b}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*x^2 + sqrt(a^2*x^4 + b)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\,x^2 + \sqrt{a^2x^4 + b}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x)
```

```
[Out] int(1/(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + \sqrt{a^2x^4 + b}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x^2 + sqrt(a^2*x^4 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sqrt{a^2\,x^4+b}+a\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b + a^2*x^4)^(1/2) + a*x^2)^(1/2),x)
```

```
[Out] int(1/((b + a^2*x^4)^(1/2) + a*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + \sqrt{a^2x^4 + b}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+(a**2*x**4+b)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + sqrt(a**2*x**4 + b)), x)
```

3.1217 $\int \frac{(-3b+2ax^2)(b^2+a^2x^2)^{3/4}}{x} dx$

Optimal. Leaf size=99

$$\frac{2\left(a^2x^2+b^2\right)^{3/4}\left(2a^2x^2-7ab+2b^2\right)}{7a}-3b^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{a^2x^2+b^2}}{\sqrt{b}}\right)+3b^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{a^2x^2+b^2}}{\sqrt{b}}\right)$$

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {446, 80, 50, 63, 298, 203, 206}

$$-2b\left(a^2x^2+b^2\right)^{3/4}+\frac{4\left(a^2x^2+b^2\right)^{7/4}}{7a}-3b^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{a^2x^2+b^2}}{\sqrt{b}}\right)+3b^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{a^2x^2+b^2}}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-3*b + 2*a*x^2)*(b^2 + a^2*x^2)^(3/4))/x,x]
```

```
[Out] -2*b*(b^2 + a^2*x^2)^(3/4) + (4*(b^2 + a^2*x^2)^(7/4))/(7*a) - 3*b^(5/2)*Ar
cTan[(b^2 + a^2*x^2)^(1/4)/Sqrt[b]] + 3*b^(5/2)*ArcTanh[(b^2 + a^2*x^2)^(1/
4)/Sqrt[b]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(-3b+2ax^2\right)\left(b^2+a^2x^2\right)^{3/4}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(-3b+2ax)\left(b^2+a^2x\right)^{3/4}}{x}\,dx, x, x^2\right)$$

$$= \frac{4\left(b^2+a^2x^2\right)^{7/4}}{7a} - \frac{1}{2}(3b)\,\text{Subst}\left(\int \frac{\left(b^2+a^2x\right)^{3/4}}{x}\,dx, x, x^2\right)$$

$$= -2b\left(b^2+a^2x^2\right)^{3/4} + \frac{4\left(b^2+a^2x^2\right)^{7/4}}{7a} - \frac{1}{2}\left(3b^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt[4]{b^2+a^2x}}\,dx, x\right.$$

$$= -2b\left(b^2+a^2x^2\right)^{3/4} + \frac{4\left(b^2+a^2x^2\right)^{7/4}}{7a} - \frac{\left(6b^3\right)\text{Subst}\left(\int \frac{x^2}{-\frac{b^2}{a^2}+\frac{x^4}{a^2}}\,dx, x, \sqrt[4]{b^2+\right.}{a^2}$$

$$= -2b\left(b^2+a^2x^2\right)^{3/4} + \frac{4\left(b^2+a^2x^2\right)^{7/4}}{7a} + \left(3b^3\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt[4]{b^2+\right.$$

$$= -2b\left(b^2+a^2x^2\right)^{3/4} + \frac{4\left(b^2+a^2x^2\right)^{7/4}}{7a} - 3b^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b^2+a^2x^2}}{\sqrt{b}}\right) + 3b^{5/2}\tan$$

Mathematica [A] time = 0.07, size = 99, normalized size = 1.00

$$\frac{2\left(a^2x^2+b^2\right)^{3/4}\left(2a^2x^2-7ab+2b^2\right)}{7a} - 3b^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{a^2x^2+b^2}}{\sqrt{b}}\right) + 3b^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{a^2x^2+b^2}}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3*b + 2*a*x^2)*(b^2 + a^2*x^2)^(3/4))/x,x]
```

```
[Out] (2*(b^2 + a^2*x^2)^(3/4)*(-7*a*b + 2*b^2 + 2*a^2*x^2))/(7*a) - 3*b^(5/2)*Ar
cTan[(b^2 + a^2*x^2)^(1/4)/Sqrt[b]] + 3*b^(5/2)*ArcTanh[(b^2 + a^2*x^2)^(1/
4)/Sqrt[b]]
```

IntegrateAlgebraic [A] time = 0.10, size = 99, normalized size = 1.00

$$\frac{2\left(a^2x^2+b^2\right)^{3/4}\left(2a^2x^2-7ab+2b^2\right)}{7a} - 3b^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{a^2x^2+b^2}}{\sqrt{b}}\right) + 3b^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{a^2x^2+b^2}}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3*b + 2*a*x^2)*(b^2 + a^2*x^2)^(3/4))/x,x]
```

```
[Out] (2*(b^2 + a^2*x^2)^(3/4)*(-7*a*b + 2*b^2 + 2*a^2*x^2))/(7*a) - 3*b^(5/2)*Ar
cTan[(b^2 + a^2*x^2)^(1/4)/Sqrt[b]] + 3*b^(5/2)*ArcTanh[(b^2 + a^2*x^2)^(1/
4)/Sqrt[b]]
```

fricas [A] time = 0.45, size = 299, normalized size = 3.02

$$\left[-\frac{42\,ab^{\frac{5}{2}}\arctan\left(\frac{\left(a^2x^2+b^2\right)^{\frac{1}{4}}}{\sqrt{b}}\right)-21\,ab^{\frac{5}{2}}\log\left(\frac{a^2x^2+2\,b^2+2\left(a^2x^2+b^2\right)^{\frac{1}{4}}b^{\frac{3}{2}}+2\,\sqrt{a^2x^2+b^2}\,b+2\left(a^2x^2+b^2\right)^{\frac{3}{4}}\sqrt{b}}{x^2}\right)-4\left(2\,a^2x^2-7\,ab+2\,b^2\right)\left(a^2x^2+b^2\right)^{\frac{3}{4}}}{14\,a},\,-\frac{42\,a\sqrt{-b}\,b^2\arctan\left(\frac{\left(a^2x^2+b^2\right)^{\frac{1}{4}}\sqrt{-b}}{b}\right)-21\,a\sqrt{-b}\,b^2\log\left(\frac{a^2x^2+2\,b^2+2\left(a^2x^2+b^2\right)^{\frac{1}{4}}\sqrt{-b}\,b-2\,\sqrt{a^2x^2+b^2}\,b-2\left(a^2x^2+b^2\right)^{\frac{3}{4}}\sqrt{-b}}{x^2}\right)-4\left(2\,a^2x^2-7\,ab+2\,b^2\right)\left(a^2x^2+b^2\right)^{\frac{3}{4}}}{14\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2-3*b)*(a^2*x^2+b^2)^(3/4)/x,x, algorithm="fricas")
```

```
[Out] [-1/14*(42*a*b^(5/2)*arctan((a^2*x^2 + b^2)^(1/4)/sqrt(b)) - 21*a*b^(5/2)*l
og((a^2*x^2 + 2*b^2 + 2*(a^2*x^2 + b^2)^(1/4)*b^(3/2) + 2*sqrt(a^2*x^2 + b^
2)*b + 2*(a^2*x^2 + b^2)^(3/4)*sqrt(b))/x^2) - 4*(2*a^2*x^2 - 7*a*b + 2*b^2
)*(a^2*x^2 + b^2)^(3/4))/a, -1/14*(42*a*sqrt(-b)*b^2*arctan((a^2*x^2 + b^2)
^(1/4)*sqrt(-b)/b) - 21*a*sqrt(-b)*b^2*log((a^2*x^2 + 2*b^2 + 2*(a^2*x^2 +
b^2)^(1/4)*sqrt(-b)*b - 2*sqrt(a^2*x^2 + b^2)*b - 2*(a^2*x^2 + b^2)^(3/4)*s
qrt(-b))/x^2) - 4*(2*a^2*x^2 - 7*a*b + 2*b^2)*(a^2*x^2 + b^2)^(3/4))/a]
```

giac [A] time = 0.30, size = 97, normalized size = 0.98

$$-\frac{3\,b^3\arctan\left(\frac{\left(a^2x^2+b^2\right)^{\frac{1}{4}}}{\sqrt{-b}}\right)}{\sqrt{-b}}-3\,b^{\frac{5}{2}}\arctan\left(\frac{\left(a^2x^2+b^2\right)^{\frac{1}{4}}}{\sqrt{b}}\right)-\frac{2\left(7\left(a^2x^2+b^2\right)^{\frac{3}{4}}a^7b-2\left(a^2x^2+b^2\right)^{\frac{7}{4}}a^6\right)}{7\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2-3*b)*(a^2*x^2+b^2)^(3/4)/x,x, algorithm="giac")
```

```
[Out] -3*b^3*arctan((a^2*x^2 + b^2)^(1/4)/sqrt(-b))/sqrt(-b) - 3*b^(5/2)*arctan((
a^2*x^2 + b^2)^(1/4)/sqrt(b)) - 2/7*(7*(a^2*x^2 + b^2)^(3/4)*a^7*b - 2*(a^2
*x^2 + b^2)^(7/4)*a^6)/a^7
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(2a\,x^2-3b\right)\left(a^2x^2+b^2\right)^{\frac{3}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x^2-3*b)*(a^2*x^2+b^2)^(3/4)/x,x)
```

```
[Out] int((2*a*x^2-3*b)*(a^2*x^2+b^2)^(3/4)/x,x)
```

maxima [A] time = 0.44, size = 107, normalized size = 1.08

$$-\frac{1}{2}\left(6\,b^{\frac{3}{2}}\arctan\left(\frac{\left(a^2x^2+b^2\right)^{\frac{1}{4}}}{\sqrt{b}}\right)+3\,b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\left(a^2x^2+b^2\right)^{\frac{1}{4}}}{\sqrt{b}+\left(a^2x^2+b^2\right)^{\frac{1}{4}}}\right)+4\left(a^2x^2+b^2\right)^{\frac{3}{4}}\right)b+\frac{4\left(a^2x^2+b^2\right)^{\frac{7}{4}}}{7\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2-3*b)*(a^2*x^2+b^2)^(3/4)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(6*b^(3/2)*arctan((a^2*x^2 + b^2)^(1/4)/sqrt(b)) + 3*b^(3/2)*log(-(sqr
t(b) - (a^2*x^2 + b^2)^(1/4))/(sqrt(b) + (a^2*x^2 + b^2)^(1/4))) + 4*(a^2*x
^2 + b^2)^(3/4))*b + 4/7*(a^2*x^2 + b^2)^(7/4)/a
```

mupad [B] time = 1.12, size = 81, normalized size = 0.82

$$3\,b^{5/2}\,\mathrm{atanh}\left(\frac{{\left(a^2\,x^2+b^2\right)}^{1/4}}{\sqrt{b}}\right)-3\,b^{5/2}\,\mathrm{atan}\left(\frac{{\left(a^2\,x^2+b^2\right)}^{1/4}}{\sqrt{b}}\right)-2\,b\,{\left(a^2\,x^2+b^2\right)}^{3/4}+\frac{4\,{\left(a^2\,x^2+b^2\right)}^{7/4}}{7\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b^2 + a^2*x^2)^(3/4)*(3*b - 2*a*x^2))/x,x)
```

```
[Out] 3*b^(5/2)*atanh((b^2 + a^2*x^2)^(1/4)/b^(1/2)) - 3*b^(5/2)*atan((b^2 + a^2*
x^2)^(1/4)/b^(1/2)) - 2*b*(b^2 + a^2*x^2)^(3/4) + (4*(b^2 + a^2*x^2)^(7/4))
/(7*a)
```

sympy [A] time = 9.77, size = 88, normalized size = 0.89

$$\frac{3a^{\frac{3}{2}}bx^{\frac{3}{2}}\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{b^2e^{i\pi}}{a^2x^2}\right)}{2\Gamma\left(\frac{1}{4}\right)}+2a\left(\begin{cases}\frac{x^2\left(b^2\right)^{\frac{3}{4}}}{2} & \text{for } a^2=0\\ \frac{2\left(a^2x^2+b^2\right)^{\frac{7}{4}}}{7a^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x**2-3*b)*(a**2*x**2+b**2)**(3/4)/x,x)
```

```
[Out] 3*a**(3/2)*b*x**(3/2)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), b**2*exp_pola
r(I*pi)/(a**2*x**2))/(2*gamma(1/4)) + 2*a*Piecewise((x**2*(b**2)**(3/4)/2,
Eq(a**2, 0)), (2*(a**2*x**2 + b**2)**(7/4)/(7*a**2), True))
```

3.1218 $\int \frac{-1+x}{(-2-2x+x^2)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=99

$$\frac{1}{6}\sqrt{3+2\sqrt{3}}\ \tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\ \sqrt{x^3-1}}{x^2+x+1}\right)-\frac{1}{6}\sqrt{2\sqrt{3}-3}\ \tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\ \sqrt{x^3-1}}{x^2+x+1}\right)$$

Rubi [A] time = 0.62, antiderivative size = 97, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6728, 2135, 219, 2140, 206, 203}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{x^3-1}}\right)}{2\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(1-x)}{\sqrt{x^3-1}}\right)}{2\sqrt{3\left(2\sqrt{3}-3\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/((-2 - 2*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]]/Sqrt[3*(-3 + 2*S
qrt[3])] + ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]]/(2*Sqrt[3*
(3 + 2*Sqrt[3])])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2135

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-
6*a*d^3)/(c*(b*c^3 - 28*a*d^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
```

```
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x}{\left(-2-2x+x^2\right)\sqrt{-1+x^3}}\,dx &= \int \left(\frac{1}{\left(-2-2\sqrt{3}+2x\right)\sqrt{-1+x^3}}+\frac{1}{\left(-2+2\sqrt{3}+2x\right)\sqrt{-1+x^3}}\right)dx \\
&= \int \frac{1}{\left(-2-2\sqrt{3}+2x\right)\sqrt{-1+x^3}}\,dx+\int \frac{1}{\left(-2+2\sqrt{3}+2x\right)\sqrt{-1+x^3}}\,dx \\
&= -\frac{\int \frac{96\left(1-\sqrt{3}\right)-96x}{\left(-2-2\sqrt{3}+2x\right)\sqrt{-1+x^3}}\,dx}{192\sqrt{3}}+\frac{\int \frac{96\left(1+\sqrt{3}\right)-96x}{\left(-2+2\sqrt{3}+2x\right)\sqrt{-1+x^3}}\,dx}{192\sqrt{3}} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)x^2}\,dx,x,\frac{1+\frac{2\left(1-\sqrt{3}\right)x}{-2+2\sqrt{3}}}{\sqrt{-1+x^3}}\right)}{2\sqrt{3}}+\frac{\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx,x,\frac{1+\frac{2\left(1+\sqrt{3}\right.}{-2-2\sqrt{}}}{\sqrt{-1+x^3}}\right.}{2\sqrt{3}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{2\sqrt{3\left(-3+2\sqrt{3}\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{2\sqrt{3\left(3+2\sqrt{3}\right)}}
\end{aligned}$$

Mathematica [C] time = 0.23, size = 220, normalized size = 2.22

$$-\frac{\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1}\left(\left(3+(2+i)\sqrt{3}\right)\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{\left(i+\sqrt{3}\right)x+2i}{-3i+\sqrt{3}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)+\left(3-(2-i)\sqrt{3}\right)\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{\left(i+\sqrt{3}\right)x+2i}{-3i+\sqrt{3}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{3\left(\sqrt{3}+i\right)\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/((-2 - 2*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] -1/3*(Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((3 + (2 + I)*Sqrt[3
])*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[-((2*I +
(I + Sqrt[3])*x)/(-3*I + Sqrt[3]))]], (1 + I*Sqrt[3])/2] + (3 - (2 - I)*Sqr
t[3])*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[-((2*I
+ (I + Sqrt[3])*x)/(-3*I + Sqrt[3]))]], (1 + I*Sqrt[3])/2]))/((I + Sqrt[3])
*Sqrt[-1 + x^3])
```

IntegrateAlgebraic [A] time = 1.29, size = 99, normalized size = 1.00

$$\frac{1}{6}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{x^3-1}}{x^2+x+1}\right)-\frac{1}{6}\sqrt{2\sqrt{3}-3}\,\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{x^3-1}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/((-2 - 2*x + x^2)*Sqrt[-1 + x^3]),x]
```

[Out] (Sqrt[3 + 2*Sqrt[3]]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[-1 + x^3])/(1 + x + x^2)])/6 - (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[-1 + x^3])/(1 + x + x^2)])/6

fricas [B] time = 0.49, size = 233, normalized size = 2.35

$$-\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(\frac{\sqrt{x^3-1}\sqrt{2\sqrt{3}+3}\left(\sqrt{3}-2\right)}{x^2+x+1}\right)-\frac{1}{24}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^4+2x^3+6x^2+2\sqrt{x^3-1}\left(2x^2+\sqrt{3}\left(x^2+2x\right)+2x+2\right)\sqrt{2\sqrt{3}-3}+4\sqrt{3}\left(x^3-1\right)-4x+4}{x^4-4x^3+8x+4}\right)+\frac{1}{24}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^4+2x^3+6x^2-2\sqrt{x^3-1}\left(2x^2+\sqrt{3}\left(x^2+2x\right)+2x+2\right)\sqrt{2\sqrt{3}-3}+4\sqrt{3}\left(x^3-1\right)-4x+4}{x^4-4x^3+8x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(x^2-2*x-2)/(x^3-1)^(1/2),x, algorithm="fricas")

[Out] -1/6*sqrt(2*sqrt(3) + 3)*arctan(sqrt(x^3 - 1)*sqrt(2*sqrt(3) + 3)*(sqrt(3) - 2)/(x^2 + x + 1)) - 1/24*sqrt(2*sqrt(3) - 3)*log((x^4 + 2*x^3 + 6*x^2 + 2*sqrt(x^3 - 1)*(2*x^2 + sqrt(3)*(x^2 + 2*x) + 2*x + 2)*sqrt(2*sqrt(3) - 3) + 4*sqrt(3)*(x^3 - 1) - 4*x + 4)/(x^4 - 4*x^3 + 8*x + 4)) + 1/24*sqrt(2*sqrt(3) - 3)*log((x^4 + 2*x^3 + 6*x^2 - 2*sqrt(x^3 - 1)*(2*x^2 + sqrt(3)*(x^2 + 2*x) + 2*x + 2)*sqrt(2*sqrt(3) - 3) + 4*sqrt(3)*(x^3 - 1) - 4*x + 4)/(x^4 - 4*x^3 + 8*x + 4))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x^3-1}\left(x^2-2x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(x^2-2*x-2)/(x^3-1)^(1/2),x, algorithm="giac")

[Out] integrate((x - 1)/(sqrt(x^3 - 1)*(x^2 - 2*x - 2)), x)

maple [C] time = 0.27, size = 702, normalized size = 7.09

$$\frac{\sqrt{\frac{x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}+\frac{1}{3-i\sqrt{3}}-\frac{i\sqrt{3}}{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\sqrt{\frac{x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}+\frac{1}{3+i\sqrt{3}}+\frac{i\sqrt{3}}{3+i\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{3}}{2\sqrt{x^3-1}}+\frac{i\sqrt{\frac{x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}+\frac{1}{3-i\sqrt{3}}-\frac{i\sqrt{3}}{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\sqrt{\frac{x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}+\frac{1}{3+i\sqrt{3}}+\frac{i\sqrt{3}}{3+i\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^3-1}}-\frac{\sqrt{\frac{x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}+\frac{1}{3-i\sqrt{3}}-\frac{i\sqrt{3}}{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\sqrt{\frac{x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}+\frac{1}{3+i\sqrt{3}}+\frac{i\sqrt{3}}{3+i\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{3}}{2\sqrt{x^3-1}}-\frac{i\sqrt{\frac{x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}+\frac{1}{3-i\sqrt{3}}-\frac{i\sqrt{3}}{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\sqrt{\frac{x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}+\frac{1}{3+i\sqrt{3}}+\frac{i\sqrt{3}}{3+i\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+x)/(x^2-2*x-2)/(x^3-1)^(1/2),x)

[Out] 1/2*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+1/2*I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/2*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-1/2*I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x^3-1}\left(x^2-2\,x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-2*x-2)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/(sqrt(x^3 - 1)*(x^2 - 2*x - 2)), x)
```

mupad [B] time = 0.23, size = 221, normalized size = 2.23

$$-\frac{\left(\Pi\left(\sqrt{3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right);\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(-\sqrt{3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right);\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(\sqrt{3}+1\mathrm{i}\right)}{2\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((x^3 - 1)^(1/2)*(2*x - x^2 + 2)),x)
```

```
[Out] -((ellipticPi(3^(1/2)*((3^(1/2)*1i)/6 + 1/2), asin((-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipti
cPi(-3^(1/2)*((3^(1/2)*1i)/6 + 1/2), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*(-(x - (3^(1/2)*1
i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3^(1/2) +
 1i))/(2*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{\left(x-1\right)\left(x^2+x+1\right)}\left(x^2-2x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**2-2*x-2)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x - 1)/(sqrt((x - 1)*(x**2 + x + 1))*(x**2 - 2*x - 2)), x)
```

3.1219 $\int \frac{3-x+x^2}{(-2-2x+x^2)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=99

$$-\frac{1}{6}\sqrt{15+14\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{x^3-1}}{x^2+x+1}\right)-\frac{1}{6}\sqrt{14\sqrt{3}-15}\,\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{x^3-1}}{x^2+x+1}\right)$$

Rubi [C] time = 0.75, antiderivative size = 457, normalized size of antiderivative = 4.62, number of steps used = 13, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6728, 219, 2135, 2140, 206, 203}

$$\frac{1}{6}\sqrt{15+14\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(1-x)}{\sqrt{x^3-1}}\right)+\frac{1}{6}\sqrt{14\sqrt{3}-15}\,\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}+\frac{\sqrt{14-5\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2\,3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}+\frac{\sqrt{38-21\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2\,3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(3 - x + x^2)/((-2 - 2*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (Sqrt[15 + 14*Sqrt[3]]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]
])/6 + (Sqrt[-15 + 14*Sqrt[3]]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-
1 + x^3]])/6 + (Sqrt[38 - 21*Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[
3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sq
rt[3]])/(2*3^(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (
Sqrt[14 - 5*Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Ellipt
icF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(2*3^(3/4
)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) - (2*Sqrt[2 - Sqrt[3
]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sq
rt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1
- Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2135

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-
6*a*d^3)/(c*(b*c^3 - 28*a*d^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{3-x+x^2}{\left(-2-2x+x^2\right)\sqrt{-1+x^3}}\,dx = \int \left(\frac{1}{\sqrt{-1+x^3}}+\frac{5+x}{\left(-2-2x+x^2\right)\sqrt{-1+x^3}}\right)dx$$

$$= \int \frac{1}{\sqrt{-1+x^3}}\,dx + \int \frac{5+x}{\left(-2-2x+x^2\right)\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \int \Bigg(\frac{}{(-2-2}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \left(1-2\sqrt{3}\right)$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{1}{576}\left(-6+\right.$$

$$= \frac{\sqrt{38-21\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{2\,3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{\sqrt{14-5\sqrt{}}}{}$$

$$= \frac{1}{6}\sqrt{15+14\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right) + \frac{1}{6}\sqrt{-15+14\sqrt{3}}\,\tanh^{-1}\left(\frac{\sqrt{3+}}{\sqrt{}}\right.$$

Mathematica [C] time = 0.80, size = 290, normalized size = 2.93

$$\frac{\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\sqrt{x^2+x+1}\left(\left((9+6i)+(4-i)\sqrt{3}\right)\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)-\left((-9+6i)+(4+i)\sqrt{3}\right)\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)\right)+\frac{6\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\left(\left(\sqrt{3}+i\right)x+2i\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}\right)}{3\left(\sqrt{3}+i\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x + x^2)/((-2 - 2*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((6*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/(1 + (
-1)^(1/3))]*(2*I + (I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x
)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))
] + Sqrt[1 + x + x^2]*(((9 + 6*I) + (4 - I)*Sqrt[3])*EllipticPi[(2*Sqrt[3])
/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3)
)]], (-1)^(1/3)] - ((-9 + 6*I) + (4 + I)*Sqrt[3])*EllipticPi[((2*I)*Sqrt[3]
)/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]],
 (-1)^(1/3)])))/(3*(I + Sqrt[3])*Sqrt[-1 + x^3])
```

IntegrateAlgebraic [A] time = 1.63, size = 99, normalized size = 1.00

$$-\frac{1}{6}\sqrt{15+14\sqrt{3}}\ \tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{x^3-1}}{x^2+x+1}\right)-\frac{1}{6}\sqrt{14\sqrt{3}-15}\ \tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{x^3-1}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x + x^2)/((-2 - 2*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] -1/6*(Sqrt[15 + 14*Sqrt[3]]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[-1 + x^3])/(1
 + x + x^2)]) - (Sqrt[-15 + 14*Sqrt[3]]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[-
1 + x^3])/(1 + x + x^2)])/6
```

fricas [B] time = 0.47, size = 246, normalized size = 2.48

$$-\frac{1}{6}\sqrt{14\sqrt{3}+15}\ \arctan\left(\frac{\sqrt{x^3-1}\sqrt{14\sqrt{3}+15}\left(3\sqrt{3}-4\right)}{11\left(x^2+x+1\right)}\right)-\frac{1}{24}\sqrt{14\sqrt{3}-15}\ \log\left(\frac{11x^4+22x^3+66x^2+2\sqrt{x^3-1}\left(4x^2+\sqrt{3}\left(3x^2+2x+4\right)+10x-2\right)\sqrt{14\sqrt{3}-15}+44\sqrt{3}\left(x^3-1\right)-44x+44}{x^4-4x^3+8x+4}\right)+\frac{1}{24}\sqrt{14\sqrt{3}-15}\ \log\left(\frac{11x^4+22x^3+66x^2-2\sqrt{x^3-1}\left(4x^2+\sqrt{3}\left(3x^2+2x+4\right)+10x-2\right)\sqrt{14\sqrt{3}-15}+44\sqrt{3}\left(x^3-1\right)-44x+44}{x^4-4x^3+8x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+3)/(x^2-2*x-2)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(14*sqrt(3) + 15)*arctan(1/11*sqrt(x^3 - 1)*sqrt(14*sqrt(3) + 15)*
(3*sqrt(3) - 4)/(x^2 + x + 1)) - 1/24*sqrt(14*sqrt(3) - 15)*log((11*x^4 + 2
2*x^3 + 66*x^2 + 2*sqrt(x^3 - 1)*(4*x^2 + sqrt(3)*(3*x^2 + 2*x + 4) + 10*x
- 2)*sqrt(14*sqrt(3) - 15) + 44*sqrt(3)*(x^3 - 1) - 44*x + 44)/(x^4 - 4*x^3
 + 8*x + 4)) + 1/24*sqrt(14*sqrt(3) - 15)*log((11*x^4 + 22*x^3 + 66*x^2 - 2
*sqrt(x^3 - 1)*(4*x^2 + sqrt(3)*(3*x^2 + 2*x + 4) + 10*x - 2)*sqrt(14*sqrt(
3) - 15) + 44*sqrt(3)*(x^3 - 1) - 44*x + 44)/(x^4 - 4*x^3 + 8*x + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-x+3}{\sqrt{x^3-1}\left(x^2-2x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+3)/(x^2-2*x-2)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 3)/(sqrt(x^3 - 1)*(x^2 - 2*x - 2)), x)
```

maple [C] time = 0.27, size = 1517, normalized size = 15.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+3)/(x^2-2*x-2)/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-
```

```
3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-
1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+
1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*Ellip
ticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),
((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+1/2*I*(1/(-3/2-1/2*I*3^(1/
2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I
*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x
+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(
1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2)
)*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+I*(1/(-3/2-1/2*I
*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2
-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1
/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^
3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3
^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+1/
2*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(
1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1
/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^
(1/2))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-
1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(
1/2))*3^(1/2)+3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/
(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2
*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(
1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*
I*3^(1/2)))^(1/2))-1/2*I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^
(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I
/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-
1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/
(3/2-1/2*I*3^(1/2)))^(1/2))+I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/
2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*
I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+
1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(
-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1
/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-1/2*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-
3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-
1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+
1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*Ellip
ticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),(
(3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - x + 3}{\sqrt{x^3 - 1}\left(x^2 - 2x - 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+3)/(x^2-2*x-2)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 3)/(sqrt(x^3 - 1)*(x^2 - 2*x - 2)), x)
```

mupad [B] time = 0.09, size = 505, normalized size = 5.10

$$-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(\sqrt{3}-6\right)\Pi\left(\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{3};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}+\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(\sqrt{3}+6\right)\Pi\left(-\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{3};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - x + 3)/((x^3 - 1)^(1/2)*(2*x - x^2 + 2)),x)
```

```
[Out] (((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2)
)^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3^(1/2) + 6)*ellipticPi(-(3^(1/2)*((3^(1/2)
*1i)/2 + 3/2))/3, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)
*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3
)^(1/2)) - (((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3^(1/2) - 6)*ellipticPi((3^(1/2)
*((3^(1/2)*1i)/2 + 3/2))/3, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)),
-((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(((3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
+ 1) + x^3)^(1/2)) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)
/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x -
 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-x+3}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x^2-2x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+3)/(x**2-2*x-2)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x**2 - x + 3)/(sqrt((x - 1)*(x**2 + x + 1))*(x**2 - 2*x - 2)), x)
```

3.1220 $\int \frac{2x+x^2}{(-2-2x+x^2)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=99

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\,\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{x^3-1}}{x^2+x+1}\right)-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{x^3-1}}{x^2+x+1}\right)$$

Rubi [C] time = 1.00, antiderivative size = 450, normalized size of antiderivative = 4.55, number of steps used = 14, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1593, 6728, 219, 2135, 2140, 206, 203}

$$-\frac{1}{3}\sqrt{2\sqrt{3}-3}\,\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(1-x)}{\sqrt{x^3-1}}\right)+\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{x^3-1}}\right)+\frac{\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}-\frac{\left(2-\sqrt{3}\right)^{3/2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^2)/((-2 - 2*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] -1/3*(Sqrt[-3 + 2*Sqrt[3]]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 +
x^3]]) + (Sqrt[3 + 2*Sqrt[3]]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1
 + x^3]])/3 - (2*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3]
- x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[
3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) - ((2 -
Sqrt[3])^(3/2)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[Ar
cSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-
((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (Sqrt[2 + Sqrt[3]]*(1 - x)
*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)
/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[3]
- x)^2)]*Sqrt[-1 + x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2135

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-
6*a*d^3)/(c*(b*c^3 - 28*a*d^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{2x+x^2}{\left(-2-2x+x^2\right)\sqrt{-1+x^3}}\,dx = \int \frac{x(2+x)}{\left(-2-2x+x^2\right)\sqrt{-1+x^3}}\,dx$$

$$= \int \left(\frac{1}{\sqrt{-1+x^3}} + \frac{2(1+2x)}{\left(-2-2x+x^2\right)\sqrt{-1+x^3}}\right)dx$$

$$= 2\int \frac{1+2x}{\left(-2-2x+x^2\right)\sqrt{-1+x^3}}\,dx + \int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + 2\int\Bigg(\frac{}{(-2-}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \left(2\left(2-\sqrt{3}\right)\right.$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{1}{288}\left(-3+\right.$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\left(2-\sqrt{3}\right)^{3/2}}{}$$

$$= -\frac{1}{3}\sqrt{-3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right) + \frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\sqrt{3+2}}{\sqrt{-1}}\right.$$

Mathematica [C] time = 0.73, size = 291, normalized size = 2.94

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{3\sqrt{\frac{(-1)^{2/3}x+\sqrt[3]{-1}}{1+\sqrt[3]{-1}}}\left(\left(\sqrt{3}+i\right)x+2i\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} - \sqrt{x^2+x+1}\left(\left((1+2i)\sqrt{3}-3i\right)\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right) + i\left(3+(2+i)\sqrt{3}\right)\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)\right)\right)}{3\left(\sqrt{3}+i\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*x + x^2)/((-2 - 2*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((3*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/(1 +
 (-1)^(1/3))]*(2*I + (I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)
*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3
))] - Sqrt[1 + x + x^2]*((-3*I + (1 + 2*I)*Sqrt[3])*EllipticPi[(2*Sqrt[3])/
(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))
]], (-1)^(1/3)] + I*(3 + (2 + I)*Sqrt[3])*EllipticPi[((2*I)*Sqrt[3])/(3 + (
2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1
/3)])))/(3*(I + Sqrt[3])*Sqrt[-1 + x^3])
```

IntegrateAlgebraic [A] time = 1.36, size = 99, normalized size = 1.00

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\,\tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{x^3-1}}{x^2+x+1}\right)-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{x^3-1}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*x + x^2)/((-2 - 2*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[-1 + x^3])/(1 + x +
 x^2)])/3 - (Sqrt[3 + 2*Sqrt[3]]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[-1 + x^3
])/(1 + x + x^2)])/3
```

fricas [B] time = 0.48, size = 218, normalized size = 2.20

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\sqrt{x^3-1}\sqrt{2\sqrt{3}-3}}{x^2+x+1}\right)-\frac{1}{12}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^4+2x^3+6x^2+2\sqrt{x^3-1}\left(x^2+2\sqrt{3}(x-1)-2x+4\right)\sqrt{2\sqrt{3}+3}+4\sqrt{3}\left(x^3-1\right)-4x+4}{x^4-4x^3+8x+4}\right)+\frac{1}{12}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^4+2x^3+6x^2-2\sqrt{x^3-1}\left(x^2+2\sqrt{3}(x-1)-2x+4\right)\sqrt{2\sqrt{3}+3}+4\sqrt{3}\left(x^3-1\right)-4x+4}{x^4-4x^3+8x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^2-2*x-2)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2*sqrt(3) - 3)*arctan(sqrt(x^3 - 1)*sqrt(2*sqrt(3) - 3)/(x^2 + x +
 1)) - 1/12*sqrt(2*sqrt(3) + 3)*log((x^4 + 2*x^3 + 6*x^2 + 2*sqrt(x^3 - 1)*
(x^2 + 2*sqrt(3)*(x - 1) - 2*x + 4)*sqrt(2*sqrt(3) + 3) + 4*sqrt(3)*(x^3 -
1) - 4*x + 4)/(x^4 - 4*x^3 + 8*x + 4)) + 1/12*sqrt(2*sqrt(3) + 3)*log((x^4
+ 2*x^3 + 6*x^2 - 2*sqrt(x^3 - 1)*(x^2 + 2*sqrt(3)*(x - 1) - 2*x + 4)*sqrt(
2*sqrt(3) + 3) + 4*sqrt(3)*(x^3 - 1) - 4*x + 4)/(x^4 - 4*x^3 + 8*x + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2\,x}{\sqrt{x^3-1}\left(x^2-2\,x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^2-2*x-2)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 2*x)/(sqrt(x^3 - 1)*(x^2 - 2*x - 2)), x)
```

maple [C] time = 0.27, size = 1517, normalized size = 15.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x)/(x^2-2*x-2)/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-
3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-
1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+
1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*Ellip
ticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),
((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*I*(1/(-3/2-1/2*I*3^(1/2)
)*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3
^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1
/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/
2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*
3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+I*(1/(-3/2-1/2*I*3
```

```
^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1
/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2
))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-
1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(
1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+2*(1
/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2)
)*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/
2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2
))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*
(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)
)*3^(1/2)+3*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2
-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2)
)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3
^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)
))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^
(1/2)))^(1/2))-2*I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*
(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*
3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+
1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*
3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1
/2*I*3^(1/2)))^(1/2))+I*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(
1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/
(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1
/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(
3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-2*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*
I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^
(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((
-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2
*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2\,x}{\sqrt{x^3-1}\,\left(x^2-2\,x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^2-2*x-2)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 2*x)/(sqrt(x^3 - 1)*(x^2 - 2*x - 2)), x)
```

mupad [B] time = 0.83, size = 509, normalized size = 5.14

$$-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\frac{\left(4\sqrt{3}-6\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{3};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}+\frac{\left(4\sqrt{3}+6\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(-\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{3};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + x^2)/((x^3 - 1)^(1/2)*(2*x - x^2 + 2)),x)
```

```
[Out] ((4*3^(1/2) + 6)*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1
/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-(3^(1/2)*((3^(1/
2)*1i)/2 + 3/2))/3, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/
2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x
^3)^(1/2)) - ((4*3^(1/2) - 6)*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2
+ 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*
1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(
```

```
1/2)*((3^(1/2)*1i)/2 + 3/2))/3, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(((3^(1/2)*1i)/2 -
1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) + 1) + x^3)^(1/2)) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 +
1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-
(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1
i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)
*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(x+2)}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x^2-2x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x)/(x**2-2*x-2)/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x*(x + 2)/(sqrt((x - 1)*(x**2 + x + 1))*(x**2 - 2*x - 2)), x)
```

3.1221 $\int \frac{\sqrt[3]{-1+x^3}\left(1+x^3\right)}{x^7} dx$

Optimal. Leaf size=99

$$\frac{4}{27}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{2}{27}\log\left(\left(x^3-1\right)^{2/3}-\sqrt[3]{x^3-1}+1\right)-\frac{4\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\sqrt[3]{x^3-1}\left(-5x^3-3\right)}{18x^6}$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {446, 78, 47, 58, 618, 204, 31}

$$-\frac{4\sqrt[3]{x^3-1}}{9x^3}+\frac{2}{9}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{4\tan^{-1}\left(\frac{1-2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\left(x^3-1\right)^{4/3}}{6x^6}-\frac{2\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(1/3)*(1 + x^3))/x^7,x]
```

```
[Out] (-4*(-1 + x^3)^(1/3))/(9*x^3) + (-1 + x^3)^(4/3)/(6*x^6) - (4*ArcTan[(1 - 2
*(-1 + x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]) - (2*Log[x])/9 + (2*Log[1 + (-1 +
x^3)^(1/3)])/9
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{-1+x^3}\,(1+x^3)}{x^7}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{-1+x}\,(1+x)}{x^3}\,dx, x, x^3\right)\\
&= \frac{(-1+x^3)^{4/3}}{6x^6} + \frac{4}{9}\text{Subst}\left(\int \frac{\sqrt[3]{-1+x}}{x^2}\,dx, x, x^3\right)\\
&= -\frac{4\sqrt[3]{-1+x^3}}{9x^3} + \frac{(-1+x^3)^{4/3}}{6x^6} + \frac{4}{27}\text{Subst}\left(\int \frac{1}{(-1+x)^{2/3}x}\,dx, x, x^3\right)\\
&= -\frac{4\sqrt[3]{-1+x^3}}{9x^3} + \frac{(-1+x^3)^{4/3}}{6x^6} - \frac{2\log(x)}{9} + \frac{2}{9}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[3]{-1+x^3}\right) + \frac{2}{9}\\
&= -\frac{4\sqrt[3]{-1+x^3}}{9x^3} + \frac{(-1+x^3)^{4/3}}{6x^6} - \frac{2\log(x)}{9} + \frac{2}{9}\log\left(1+\sqrt[3]{-1+x^3}\right) - \frac{4}{9}\text{Subst}\left(\int -\right.\\
&= -\frac{4\sqrt[3]{-1+x^3}}{9x^3} + \frac{(-1+x^3)^{4/3}}{6x^6} - \frac{4\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^3}}{\sqrt{3}}\right)}{9\sqrt{3}} - \frac{2\log(x)}{9} + \frac{2}{9}\log\left(1+\sqrt[3]{-1}\right.
\end{aligned}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.38

$$\frac{(x^3-1)^{4/3}\left(2x^6\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};1-x^3\right)+1\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)^(1/3)*(1 + x^3))/x^7,x]
```

```
[Out] ((-1 + x^3)^(4/3)*(1 + 2*x^6*Hypergeometric2F1[4/3, 2, 7/3, 1 - x^3]))/(6*x
^6)
```

IntegrateAlgebraic [A] time = 0.14, size = 99, normalized size = 1.00

$$\frac{4}{27}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{2}{27}\log\left((x^3-1)^{2/3}-\sqrt[3]{x^3-1}+1\right)-\frac{4\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\sqrt[3]{x^3-1}\,(-5x^3-3)}{18x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(1/3)*(1 + x^3))/x^7,x]
```

```
[Out] ((-3 - 5*x^3)*(-1 + x^3)^(1/3))/(18*x^6) - (4*ArcTan[1/Sqrt[3] - (2*(-1 + x
^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]) + (4*Log[1 + (-1 + x^3)^(1/3)])/27 - (2*Lo
g[1 - (-1 + x^3)^(1/3) + (-1 + x^3)^(2/3)])/27
```

fricas [A] time = 0.39, size = 88, normalized size = 0.89

$$\frac{8\sqrt{3}x^6\arctan\left(\frac{2}{3}\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-4x^6\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+8x^6\log\left(\left(x^3-1\right)^{\frac{1}{3}}+1\right)-3\left(5x^3+3\right)\left(x^3-1\right)^{\frac{1}{3}}}{54x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(x^3+1)/x^7,x, algorithm="fricas")
```

```
[Out] 1/54*(8*sqrt(3)*x^6*arctan(2/3*sqrt(3)*(x^3 - 1)^(1/3) - 1/3*sqrt(3)) - 4*x
^6*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 8*x^6*log((x^3 - 1)^(1/3) +
 1) - 3*(5*x^3 + 3)*(x^3 - 1)^(1/3))/x^6
```

giac [A] time = 0.18, size = 81, normalized size = 0.82

$$\frac{4}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3-1\right)^{\frac{1}{3}}-1\right)\right)-\frac{5\left(x^3-1\right)^{\frac{4}{3}}+8\left(x^3-1\right)^{\frac{1}{3}}}{18x^6}-\frac{2}{27}\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+\frac{4}{27}\log\left(\left|\left(x^3-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(x^3+1)/x^7,x, algorithm="giac")
```

```
[Out] 4/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) - 1/18*(5*(x^3 - 1
)^(4/3) + 8*(x^3 - 1)^(1/3))/x^6 - 2/27*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/
3) + 1) + 4/27*log(abs((x^3 - 1)^(1/3) + 1))
```

maple [C] time = 0.27, size = 91, normalized size = 0.92

$$-\frac{5x^6-2x^3-3}{18x^6\left(x^3-1\right)^{\frac{2}{3}}}+\frac{4\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{2}{3}}\left(\frac{2\Gamma\left(\frac{2}{3}\right)x^3\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],x^3\right)}{3}+\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)\right)}{27\Gamma\left(\frac{2}{3}\right)\mathrm{signum}\left(x^3-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/3)*(x^3+1)/x^7,x)
```

```
[Out] -1/18*(5*x^6-2*x^3-3)/x^6/(x^3-1)^(2/3)+4/27/GAMMA(2/3)/signum(x^3-1)^(2/3)
*(-signum(x^3-1))^(2/3)*(2/3*GAMMA(2/3)*x^3*hypergeom([1,1,5/3],[2,2],x^3)+
(1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*GAMMA(2/3))
```

maxima [A] time = 0.48, size = 103, normalized size = 1.04

$$\frac{4}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{\left(x^3-1\right)^{\frac{4}{3}}-2\left(x^3-1\right)^{\frac{1}{3}}}{18\left(2x^3+\left(x^3-1\right)^2-1\right)}-\frac{\left(x^3-1\right)^{\frac{1}{3}}}{3x^3}-\frac{2}{27}\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+\frac{4}{27}\log\left(\left(x^3-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(x^3+1)/x^7,x, algorithm="maxima")
```

```
[Out] 4/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) + 1/18*((x^3 - 1)^
(4/3) - 2*(x^3 - 1)^(1/3))/(2*x^3 + (x^3 - 1)^2 - 1) - 1/3*(x^3 - 1)^(1/3)/
x^3 - 2/27*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 4/27*log((x^3 - 1)^
(1/3) + 1)
```

mupad [B] time = 1.04, size = 186, normalized size = 1.88

$$\frac{\ln\left(\frac{\left(x^3-1\right)^{1/3}}{9}+\frac{1}{9}\right)}{9}+\frac{\ln\left(\frac{\left(x^3-1\right)^{1/3}}{81}+\frac{1}{81}\right)}{27}-\frac{\frac{\left(x^3-1\right)^{1/3}}{9}-\frac{\left(x^3-1\right)^{4/3}}{18}}{\left(x^3-1\right)^2+2x^3-1}+\ln\left(\left(x^3-1\right)^{1/3}-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)-\frac{\left(x^3-1\right)^{1/3}}{3x^3}-\ln\left(\frac{1}{2}-\left(x^3-1\right)^{1/3}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)-\ln\left(\frac{1}{6}-\frac{\left(x^3-1\right)^{1/3}}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(\frac{1}{54}+\frac{\sqrt{3}\,1\mathrm{i}}{54}\right)+\ln\left(\frac{\left(x^3-1\right)^{1/3}}{3}-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(-\frac{1}{54}+\frac{\sqrt{3}\,1\mathrm{i}}{54}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(1/3)*(x^3 + 1))/x^7,x)
```

```
[Out] log((x^3 - 1)^(1/3)/9 + 1/9)/9 + log((x^3 - 1)^(1/3)/81 + 1/81)/27 - ((x^3
- 1)^(1/3)/9 - (x^3 - 1)^(4/3)/18)/((x^3 - 1)^2 + 2*x^3 - 1) + log((3^(1/2)
*1i)/2 + (x^3 - 1)^(1/3) - 1/2)*((3^(1/2)*1i)/18 - 1/18) - (x^3 - 1)^(1/3)/
(3*x^3) - log((3^(1/2)*1i)/2 - (x^3 - 1)^(1/3) + 1/2)*((3^(1/2)*1i)/18 + 1/
18) - log((3^(1/2)*1i)/6 - (x^3 - 1)^(1/3)/3 + 1/6)*((3^(1/2)*1i)/54 + 1/54
) + log((3^(1/2)*1i)/6 + (x^3 - 1)^(1/3)/3 - 1/6)*((3^(1/2)*1i)/54 - 1/54)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/3)*(x**3+1)/x**7,x)
```

```
[Out] Timed out
```

3.1222 $\int \frac{\sqrt[3]{-1+x^3}\left(-1+2x^3\right)}{x^7}\,dx$

Optimal. Leaf size=99

$$\frac{5}{27}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{5}{54}\log\left(\left(x^3-1\right)^{2/3}-\sqrt[3]{x^3-1}+1\right)-\frac{5\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\sqrt[3]{x^3-1}\left(3-13x^3\right)}{18x^6}$$

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {446, 78, 47, 58, 618, 204, 31}

$$-\frac{5\sqrt[3]{x^3-1}}{9x^3}+\frac{5}{18}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{5\tan^{-1}\left(\frac{1-2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{9\sqrt{3}}-\frac{\left(x^3-1\right)^{4/3}}{6x^6}-\frac{5\log(x)}{18}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(1/3)*(-1 + 2*x^3))/x^7,x]
```

```
[Out] (-5*(-1 + x^3)^(1/3))/(9*x^3) - (-1 + x^3)^(4/3)/(6*x^6) - (5*ArcTan[(1 - 2
*(-1 + x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]) - (5*Log[x])/18 + (5*Log[1 + (-1 +
 x^3)^(1/3)])/18
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{-1+x^3}\left(-1+2x^3\right)}{x^7}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt[3]{-1+x}\,(-1+2x)}{x^3}\,dx,x,x^3\right)\\
&= -\frac{\left(-1+x^3\right)^{4/3}}{6x^6}+\frac{5}{9}\operatorname{Subst}\left(\int \frac{\sqrt[3]{-1+x}}{x^2}\,dx,x,x^3\right)\\
&= -\frac{5\sqrt[3]{-1+x^3}}{9x^3}-\frac{\left(-1+x^3\right)^{4/3}}{6x^6}+\frac{5}{27}\operatorname{Subst}\left(\int \frac{1}{(-1+x)^{2/3}x}\,dx,x,x^3\right)\\
&= -\frac{5\sqrt[3]{-1+x^3}}{9x^3}-\frac{\left(-1+x^3\right)^{4/3}}{6x^6}-\frac{5\log(x)}{18}+\frac{5}{18}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt[3]{-1+x^3}\right)\\
&= -\frac{5\sqrt[3]{-1+x^3}}{9x^3}-\frac{\left(-1+x^3\right)^{4/3}}{6x^6}-\frac{5\log(x)}{18}+\frac{5}{18}\log\left(1+\sqrt[3]{-1+x^3}\right)-\frac{5}{9}\operatorname{Subst}\left(\int\right.\\
&= -\frac{5\sqrt[3]{-1+x^3}}{9x^3}-\frac{\left(-1+x^3\right)^{4/3}}{6x^6}-\frac{5\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^3}}{\sqrt{3}}\right)}{9\sqrt{3}}-\frac{5\log(x)}{18}+\frac{5}{18}\log\left(1+\sqrt[3]{\right.\right.}\end{aligned}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.38

$$\frac{\left(x^3-1\right)^{4/3}\left(5x^6\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};1-x^3\right)-2\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)^(1/3)*(-1 + 2*x^3))/x^7,x]
```

```
[Out] ((-1 + x^3)^(4/3)*(-2 + 5*x^6*Hypergeometric2F1[4/3, 2, 7/3, 1 - x^3]))/(12
*x^6)
```

IntegrateAlgebraic [A] time = 0.11, size = 99, normalized size = 1.00

$$\frac{5}{27}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{5}{54}\log\left(\left(x^3-1\right)^{2/3}-\sqrt[3]{x^3-1}+1\right)-\frac{5\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\sqrt[3]{x^3-1}\left(3-13x^3\right)}{18x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(1/3)*(-1 + 2*x^3))/x^7,x]
```

```
[Out] ((3 - 13*x^3)*(-1 + x^3)^(1/3))/(18*x^6) - (5*ArcTan[1/Sqrt[3] - (2*(-1 + x
^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]) + (5*Log[1 + (-1 + x^3)^(1/3)])/27 - (5*Lo
g[1 - (-1 + x^3)^(1/3) + (-1 + x^3)^(2/3)])/54
```

fricas [A] time = 0.41, size = 88, normalized size = 0.89

$$\frac{10\sqrt{3}x^6\arctan\left(\frac{2}{3}\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-5x^6\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+10x^6\log\left(\left(x^3-1\right)^{\frac{1}{3}}+1\right)-3\left(13x^3-3\right)\left(x^3-1\right)^{\frac{1}{3}}}{54x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(2*x^3-1)/x^7,x, algorithm="fricas")
```

```
[Out] 1/54*(10*sqrt(3)*x^6*arctan(2/3*sqrt(3)*(x^3 - 1)^(1/3) - 1/3*sqrt(3)) - 5*
x^6*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 10*x^6*log((x^3 - 1)^(1/3)
 + 1) - 3*(13*x^3 - 3)*(x^3 - 1)^(1/3))/x^6
```

giac [A] time = 0.18, size = 81, normalized size = 0.82

$$\frac{5}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3-1\right)^{\frac{1}{3}}-1\right)\right)-\frac{13\left(x^3-1\right)^{\frac{4}{3}}+10\left(x^3-1\right)^{\frac{1}{3}}}{18x^6}-\frac{5}{54}\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+\frac{5}{27}\log\left(\left|\left(x^3-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(2*x^3-1)/x^7,x, algorithm="giac")
```

```
[Out] 5/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) - 1/18*(13*(x^3 -
1)^(4/3) + 10*(x^3 - 1)^(1/3))/x^6 - 5/54*log((x^3 - 1)^(2/3) - (x^3 - 1)^(
1/3) + 1) + 5/27*log(abs((x^3 - 1)^(1/3) + 1))
```

maple [C] time = 0.26, size = 91, normalized size = 0.92

$$-\frac{13x^6-16x^3+3}{18x^6\left(x^3-1\right)^{\frac{2}{3}}}+\frac{5\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{2}{3}}\left(\frac{2\Gamma\left(\frac{2}{3}\right)x^3\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],x^3\right)}{3}+\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)\right)}{27\Gamma\left(\frac{2}{3}\right)\mathrm{signum}\left(x^3-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/3)*(2*x^3-1)/x^7,x)
```

```
[Out] -1/18*(13*x^6-16*x^3+3)/x^6/(x^3-1)^(2/3)+5/27/GAMMA(2/3)/signum(x^3-1)^(2/
3)*(-signum(x^3-1))^(2/3)*(2/3*GAMMA(2/3)*x^3*hypergeom([1,1,5/3],[2,2],x^3
)+(1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*GAMMA(2/3))
```

maxima [A] time = 0.64, size = 103, normalized size = 1.04

$$\frac{5}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3-1\right)^{\frac{1}{3}}-1\right)\right)-\frac{\left(x^3-1\right)^{\frac{4}{3}}-2\left(x^3-1\right)^{\frac{1}{3}}}{18\left(2x^3+\left(x^3-1\right)^2-1\right)}-\frac{2\left(x^3-1\right)^{\frac{1}{3}}}{3x^3}-\frac{5}{54}\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+\frac{5}{27}\log\left(\left(x^3-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(2*x^3-1)/x^7,x, algorithm="maxima")
```

```
[Out] 5/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) - 1/18*((x^3 - 1)^
(4/3) - 2*(x^3 - 1)^(1/3))/(2*x^3 + (x^3 - 1)^2 - 1) - 2/3*(x^3 - 1)^(1/3)/
x^3 - 5/54*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 5/27*log((x^3 - 1)^
(1/3) + 1)
```

mupad [B] time = 1.20, size = 187, normalized size = 1.89

$$\frac{2\ln\left(\frac{4\left(x^3-1\right)^{1/3}}{9}+\frac{4}{9}\right)}{9}-\frac{\ln\left(\frac{\left(x^3-1\right)^{1/3}}{81}+\frac{1}{81}\right)}{27}+\frac{\frac{\left(x^3-1\right)^{1/3}}{9}-\frac{\left(x^3-1\right)^{4/3}}{18}}{\left(x^3-1\right)^2+2x^3-1}-\frac{2\left(x^3-1\right)^{1/3}}{3x^3}-\ln\left(1-2\left(x^3-1\right)^{1/3}+\sqrt{3}\,1i\right)\left(\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)+\ln\left(2\left(x^3-1\right)^{1/3}-1+\sqrt{3}\,1i\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)+\ln\left(\frac{1}{6}-\frac{\left(x^3-1\right)^{1/3}}{3}+\frac{\sqrt{3}\,1i}{6}\right)\left(\frac{1}{54}+\frac{\sqrt{3}\,1i}{54}\right)-\ln\left(\frac{\left(x^3-1\right)^{1/3}}{3}-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)\left(-\frac{1}{54}+\frac{\sqrt{3}\,1i}{54}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(1/3)*(2*x^3 - 1))/x^7,x)
```

```
[Out] (2*log((4*(x^3 - 1)^(1/3))/9 + 4/9))/9 - log((x^3 - 1)^(1/3)/81 + 1/81)/27
+ ((x^3 - 1)^(1/3)/9 - (x^3 - 1)^(4/3)/18)/((x^3 - 1)^2 + 2*x^3 - 1) - (2*(
x^3 - 1)^(1/3))/(3*x^3) - log(3^(1/2)*1i - 2*(x^3 - 1)^(1/3) + 1)*((3^(1/2)
*1i)/9 + 1/9) + log(3^(1/2)*1i + 2*(x^3 - 1)^(1/3) - 1)*((3^(1/2)*1i)/9 - 1
/9) + log((3^(1/2)*1i)/6 - (x^3 - 1)^(1/3)/3 + 1/6)*((3^(1/2)*1i)/54 + 1/54
) - log((3^(1/2)*1i)/6 + (x^3 - 1)^(1/3)/3 - 1/6)*((3^(1/2)*1i)/54 - 1/54)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/3)*(2*x**3-1)/x**7,x)
```

```
[Out] Timed out
```

3.1223 $\int \frac{\left(a^2-2ax+x^2\right)\left(-2abx+(3a-b)x^2\right)}{\left(x^2(-a+x)(-b+x)\right)^{3/4}\left(a^3d-3a^2dx+(-b+3ad)x^2+(1-d)x^3\right)}\,dx$

Optimal. Leaf size=99

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x^3(-a-b)+abx^2+x^4\right)^{3/4}}{x^2(x-b)}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x^3(-a-b)+abx^2+x^4\right)^{3/4}}{x^2(x-b)}\right)}{d^{3/4}}$$

Rubi [F] time = 13.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=$ 0.000, Rules used = {}

$$\int \frac{\left(a^2-2ax+x^2\right)\left(-2abx+(3a-b)x^2\right)}{\left(x^2(-a+x)(-b+x)\right)^{3/4}\left(a^3d-3a^2dx+(-b+3ad)x^2+(1-d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a^2 - 2*a*x + x^2)*(-2*a*b*x + (3*a - b)*x^2))/((x^2*(-a + x)*(-b + x
))^(3/4)*(a^3*d - 3*a^2*d*x + (-b + 3*a*d)*x^2 + (1 - d)*x^3)),x]
```

```
[Out] (4*a*b*x^(3/2)*(-a + x)^(3/4)*(-b + x)^(3/4)*Defer[Subst][Defer[Int][(-a +
x^2)^(5/4)/((-b + x^2)^(3/4)*(-(a^3*d) + 3*a^2*d*x^2 + b*(1 - (3*a*d)/b)*x^
4 - (1 - d)*x^6)), x], x, Sqrt[x]])/((a - x)*(b - x)*x^2)^(3/4) + (2*(3*a -
 b)*x^(3/2)*(-a + x)^(3/4)*(-b + x)^(3/4)*Defer[Subst][Defer[Int][(x^2*(-a
+ x^2)^(5/4))/((-b + x^2)^(3/4)*(a^3*d - 3*a^2*d*x^2 - b*(1 - (3*a*d)/b)*x^
4 + (1 - d)*x^6)), x], x, Sqrt[x]])/((a - x)*(b - x)*x^2)^(3/4)
```

Rubi steps

$$\int \frac{\left(a^2-2ax+x^2\right)\left(-2abx+(3a-b)x^2\right)}{\left(x^2(-a+x)(-b+x)\right)^{3/4}\left(a^3d-3a^2dx+(-b+3ad)x^2+(1-d)x^3\right)}\,dx=\int \frac{(-a+x)^2\left(-2abx\right.}{\left(x^2(-a+x)(-b+x)\right)^{3/4}\left(a^3d-3a^2\right.}$$

$$=\int \frac{x(-a+x)^2(-2ab}{\left(x^2(-a+x)(-b+x)\right)^{3/4}\left(a^3d-3a^2\right.}$$

$$=\frac{\left(x^{3/2}(-a+x)^{3/4}(-b+x)^{3/4}\right)\int \frac{}{\sqrt{x}\,(-b+x)}}{\left(x^2(-a+x)(-\right.}$$

$$=\frac{\left(2x^{3/2}(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\left(\int\right.}{\left(x^2(-a\right.}$$

$$=\frac{\left(2x^{3/2}(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\left(\int\right.}{}$$

$$=\frac{\left(2(3a-b)x^{3/2}(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{S}}{\left(x^2\right.}$$

Mathematica [F] time = 4.24, size = 0, normalized size = 0.00

$$\int \frac{(a^2-2ax+x^2)(-2abx+(3a-b)x^2)}{(x^2(-a+x)(-b+x))^{3/4}(a^3d-3a^2dx+(-b+3ad)x^2+(1-d)x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a^2 - 2*a*x + x^2)*(-2*a*b*x + (3*a - b)*x^2))/((x^2*(-a + x)*(
-b + x))^(3/4)*(a^3*d - 3*a^2*d*x + (-b + 3*a*d)*x^2 + (1 - d)*x^3)),x]
```

```
[Out] Integrate[((a^2 - 2*a*x + x^2)*(-2*a*b*x + (3*a - b)*x^2))/((x^2*(-a + x)*(
-b + x))^(3/4)*(a^3*d - 3*a^2*d*x + (-b + 3*a*d)*x^2 + (1 - d)*x^3)), x]
```

IntegrateAlgebraic [A] time = 5.91, size = 99, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,(x^3(-a-b)+abx^2+x^4)^{3/4}}{x^2(x-b)}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,(x^3(-a-b)+abx^2+x^4)^{3/4}}{x^2(x-b)}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a^2 - 2*a*x + x^2)*(-2*a*b*x + (3*a - b)*x^2))/((x^2*(
-a + x)*(-b + x))^(3/4)*(a^3*d - 3*a^2*d*x + (-b + 3*a*d)*x^2 + (1 - d)*x^3
)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(3/4))/(x^2*(-b + x))]])/
d^(3/4) + (2*ArcTanh[(d^(1/4)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(3/4))/(x^2*(-
b + x))]])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-2*a*x+x^2)*(-2*a*b*x+(3*a-b)*x^2)/(x^2*(-a+x)*(-b+x))^(3/4)/
(a^3*d-3*a^2*d*x+(3*a*d-b)*x^2+(1-d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(2\,abx-(3\,a-b)x^2)(a^2-2\,ax+x^2)}{(a^3d-3\,a^2dx-(d-1)x^3+(3\,ad-b)x^2)((a-x)(b-x)x^2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-2*a*x+x^2)*(-2*a*b*x+(3*a-b)*x^2)/(x^2*(-a+x)*(-b+x))^(3/4)/
(a^3*d-3*a^2*d*x+(3*a*d-b)*x^2+(1-d)*x^3),x, algorithm="giac")
```

```
[Out] integrate(-(2*a*b*x - (3*a - b)*x^2)*(a^2 - 2*a*x + x^2)/((a^3*d - 3*a^2*d*
x - (d - 1)*x^3 + (3*a*d - b)*x^2)*((a - x)*(b - x)*x^2)^(3/4)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(a^2-2ax+x^2)(-2abx+(3a-b)\,x^2)}{(x^2\,(-a+x)\,(-b+x))^{\frac{3}{4}}(a^3d-3a^2dx+(3ad-b)\,x^2+(1-d)\,x^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-2*a*x+x^2)*(-2*a*b*x+(3*a-b)*x^2)/(x^2*(-a+x)*(-b+x))^(3/4)/(a^3*d
-3*a^2*d*x+(3*a*d-b)*x^2+(1-d)*x^3),x)
```

```
[Out] int((a^2-2*a*x+x^2)*(-2*a*b*x+(3*a-b)*x^2)/(x^2*(-a+x)*(-b+x))^(3/4)/(a^3*d
-3*a^2*d*x+(3*a*d-b)*x^2+(1-d)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(2\,abx-(3\,a-b)x^2\right)\left(a^2-2\,ax+x^2\right)}{\left(a^3d-3\,a^2dx-(d-1)x^3+(3\,ad-b)x^2\right)\left((a-x)(b-x)x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-2*a*x+x^2)*(-2*a*b*x+(3*a-b)*x^2)/(x^2*(-a+x)*(-b+x))^(3/4)/
(a^3*d-3*a^2*d*x+(3*a*d-b)*x^2+(1-d)*x^3),x, algorithm="maxima")
```

```
[Out] -integrate((2*a*b*x - (3*a - b)*x^2)*(a^2 - 2*a*x + x^2)/((a^3*d - 3*a^2*d*
x - (d - 1)*x^3 + (3*a*d - b)*x^2)*((a - x)*(b - x)*x^2)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^2\,\left(3\,a-b\right)-2\,a\,b\,x\right)\,\left(a^2-2\,a\,x+x^2\right)}{{\left(x^2\,\left(a-x\right)\,\left(b-x\right)\right)}^{3/4}\,\left(x^2\,\left(b-3\,a\,d\right)-a^3\,d+x^3\,\left(d-1\right)+3\,a^2\,d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^2*(3*a - b) - 2*a*b*x)*(a^2 - 2*a*x + x^2))/((x^2*(a - x)*(b - x))
^(3/4)*(x^2*(b - 3*a*d) - a^3*d + x^3*(d - 1) + 3*a^2*d*x)),x)
```

```
[Out] int(-((x^2*(3*a - b) - 2*a*b*x)*(a^2 - 2*a*x + x^2))/((x^2*(a - x)*(b - x))
^(3/4)*(x^2*(b - 3*a*d) - a^3*d + x^3*(d - 1) + 3*a^2*d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-2*a*x+x**2)*(-2*a*b*x+(3*a-b)*x**2)/(x**2*(-a+x)*(-b+x))**(
3/4)/(a**3*d-3*a**2*d*x+(3*a*d-b)*x**2+(1-d)*x**3),x)
```

```
[Out] Timed out
```

3.1224 $\int \frac{1+x^2}{\sqrt{\frac{-2-x+2x^2}{-1+x+x^2}}\left(1-x^2+x^4\right)} dx$

Optimal. Leaf size=99

$$\sqrt{\frac{1}{5}-\frac{3i}{5}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{2x^2-x-2}{x^2+x-1}}}{\sqrt{-1-3i}}\right)+\sqrt{\frac{1}{5}+\frac{3i}{5}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{2x^2-x-2}{x^2+x-1}}}{\sqrt{-1+3i}}\right)$$

Rubi [F] time = 4.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x^2}{\sqrt{\frac{-2-x+2x^2}{-1+x+x^2}}\left(1-x^2+x^4\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^2)/(Sqrt[(-2 - x + 2*x^2)/(-1 + x + x^2)]*(1 - x^2 + x^4)),x]
```

```
[Out] -1/2*((1 + I*Sqrt[3])*Sqrt[-2 - x + 2*x^2]*Defer[Int][Sqrt[-1 + x + x^2]/((
Sqrt[1 - I*Sqrt[3]] - Sqrt[2]*x)*Sqrt[-2 - x + 2*x^2]), x])/(Sqrt[1 - I*Sqr
t[3]]*Sqrt[(2 + x - 2*x^2)/(1 - x - x^2)]*Sqrt[-1 + x + x^2]) - ((1 - I*Sqr
t[3])*Sqrt[-2 - x + 2*x^2]*Defer[Int][Sqrt[-1 + x + x^2]/((Sqrt[1 + I*Sqrt[
3]] - Sqrt[2]*x)*Sqrt[-2 - x + 2*x^2]), x])/(2*Sqrt[1 + I*Sqrt[3]]*Sqrt[(2
+ x - 2*x^2)/(1 - x - x^2)]*Sqrt[-1 + x + x^2]) - ((1 + I*Sqrt[3])*Sqrt[-2
- x + 2*x^2]*Defer[Int][Sqrt[-1 + x + x^2]/((Sqrt[1 - I*Sqrt[3]] + Sqrt[2]*
x)*Sqrt[-2 - x + 2*x^2]), x])/(2*Sqrt[1 - I*Sqrt[3]]*Sqrt[(2 + x - 2*x^2)/(
1 - x - x^2)]*Sqrt[-1 + x + x^2]) - ((1 - I*Sqrt[3])*Sqrt[-2 - x + 2*x^2]*D
efer[Int][Sqrt[-1 + x + x^2]/((Sqrt[1 + I*Sqrt[3]] + Sqrt[2]*x)*Sqrt[-2 - x
 + 2*x^2]), x])/(2*Sqrt[1 + I*Sqrt[3]]*Sqrt[(2 + x - 2*x^2)/(1 - x - x^2)]*
Sqrt[-1 + x + x^2])
```

Rubi steps

$$\int \frac{1+x^2}{\sqrt{\frac{-2-x+2x^2}{-1+x+x^2}}\left(1-x^2+x^4\right)} dx = \frac{\sqrt{-2-x+2x^2}\int \frac{\left(1+x^2\right)\sqrt{-1+x+x^2}}{\sqrt{-2-x+2x^2}\left(1-x^2+x^4\right)} dx}{\sqrt{-1+x+x^2}\sqrt{\frac{-2-x+2x^2}{-1+x+x^2}}}$$

$$= \frac{\sqrt{-2-x+2x^2}\int \left(\frac{\left(1-i\sqrt{3}\right)\sqrt{-1+x+x^2}}{\left(-1-i\sqrt{3}+2x^2\right)\sqrt{-2-x+2x^2}}+\frac{\left(1+i\sqrt{3}\right)\sqrt{-1+x+x^2}}{\left(-1+i\sqrt{3}+2x^2\right)\sqrt{-2-x+2x^2}}\right) dx}{\sqrt{-1+x+x^2}\sqrt{\frac{-2-x+2x^2}{-1+x+x^2}}}$$

$$= \frac{\left(\left(1-i\sqrt{3}\right)\sqrt{-2-x+2x^2}\right)\int \frac{\sqrt{-1+x+x^2}}{\left(-1-i\sqrt{3}+2x^2\right)\sqrt{-2-x+2x^2}} dx}{\sqrt{-1+x+x^2}\sqrt{\frac{-2-x+2x^2}{-1+x+x^2}}} + \frac{\left(\left(1+i\sqrt{3}\right)\sqrt{-2-\right.}{\sqrt{\ }}$$

$$= \frac{\left(\left(1-i\sqrt{3}\right)\sqrt{-2-x+2x^2}\right)\int \left(\frac{\sqrt{1+i\sqrt{3}}\sqrt{-1+x+x^2}}{2\left(-1-i\sqrt{3}\right)\left(\sqrt{1+i\sqrt{3}}-\sqrt{2}x\right)\sqrt{-2-x+2x^2}}+\frac{\sqrt{\ }}{2\left(-1-i\sqrt{3}\right)}\right.}{\sqrt{-1+x+x^2}\sqrt{\frac{-2-x+2x^2}{-1+x+x^2}}}$$

$$= -\frac{\left(\left(1-i\sqrt{3}\right)\sqrt{-2-x+2x^2}\right)\int \frac{\sqrt{-1+x+x^2}}{\left(\sqrt{1+i\sqrt{3}}-\sqrt{2}x\right)\sqrt{-2-x+2x^2}} dx}{2\sqrt{1+i\sqrt{3}}\sqrt{-1+x+x^2}\sqrt{\frac{-2-x+2x^2}{-1+x+x^2}}} - \frac{\left(\left(1-i\sqrt{3}\right)\sqrt{\right.}{2\sqrt{1}}$$

Mathematica [C] time = 6.72, size = 11770, normalized size = 118.89

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2)/(Sqrt[(-2 - x + 2*x^2)/(-1 + x + x^2)]*(1 - x^2 + x^4))
,x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.54, size = 97, normalized size = 0.98

$$\sqrt{\frac{1}{5}+\frac{3i}{5}}\tan^{-1}\left(\sqrt{-\frac{1}{5}-\frac{3i}{5}}\sqrt{\frac{2x^2-x-2}{x^2+x-1}}\right)+\sqrt{\frac{1}{5}-\frac{3i}{5}}\tan^{-1}\left(\sqrt{-\frac{1}{5}+\frac{3i}{5}}\sqrt{\frac{2x^2-x-2}{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/(Sqrt[(-2 - x + 2*x^2)/(-1 + x + x^2)]*(1 - x^
2 + x^4)),x]
```

```
[Out] Sqrt[1/5 + (3*I)/5]*ArcTan[Sqrt[-1/5 - (3*I)/5]*Sqrt[(-2 - x + 2*x^2)/(-1 +
 x + x^2)]] + Sqrt[1/5 - (3*I)/5]*ArcTan[Sqrt[-1/5 + (3*I)/5]*Sqrt[(-2 - x
+ 2*x^2)/(-1 + x + x^2)]]
```

fricas [B] time = 6.43, size = 5095, normalized size = 51.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/((2*x^2-x-2)/(x^2+x-1))^(1/2)/(x^4-x^2+1),x, algorithm="f
ricas")
```

```
[Out] -1/100*10^(3/4)*sqrt(5)*sqrt(2*sqrt(10) + 20)*arctan(1/900*(113784843418327
559700*x^24 - 163924417908538346160O*x^23 + 2545884655734590980200*x^22 + 5
082124295256931700640O*x^21 - 1588901310315976902759O0*x^20 - 4650502374349
08643152000*x^19 + 1504931893222349028684600*x^18 + 212960773639089404924
40
00*x^17 - 6223660771133927367315000*x^16 - 5602507922355181669879200*x^15 +
 1388348408397489484714740O*x^14 + 8971054092806414576313600*x^13 - 1801909
1872612617624668700*x^12 - 8971054092806414576313600*x^11 + 138834840839748
94847147400*x^10 + 5602507922355181669879200*x^9 - 622366077113392736731500
0*x^8 - 21296077363908940492440O0*x^7 + 1504931893222349028684600*x^6 + 465
0502374349086431520O0*x^5 - 1588901310315976902759O0*x^4 - 508212429525693l
7006400*x^3 + 2545884655734590980200*x^2 + 17*sqrt(2)*((10^(3/4)*(sqrt(10)*
sqrt(5)*(3353156233262299*x^24 - 49929701526135692*x^23 + 1054883762666313
9
8*x^22 + 1245603618916311800*x^21 - 3729015017341775649*x^20 - 125744656485
22249504*x^19 + 28773197469313801650*x^18 + 61934497275400333884*x^17 - 104
756202796838736914*x^16 - 169570606189065646500*x^15 + 2170840682645487759
6
6*x^14 + 276610136446380234320*x^13 - 274989879451094143625*x^12 - 27661013
6446380234320*x^11 + 217084068264548775966*x^10 + 169570606189065646500*x^9
 - 104756202796838736914*x^8 - 61934497275400333884*x^7 + 28773197469313801
650*x^6 + 12574465648522249504*x^5 - 3729015017341775649*x^4 - 124560361891
6311800*x^3 + 105488376266631398*x^2 + 49929701526135692*x + 33531562332622
99) + 10*sqrt(5)*(1603095924058903*x^24 - 28424949163733294*x^23 + 12988283
3582498276*x^22 + 237455573469526820*x^21 - 2351987676228007923*x^20 - 4789
83875485473448*x^19 + 15731365763207610420*x^18 - 12309554059894392
42*x^17
- 5475787111286254647
8*x^16 + 7544291365549818150*x^15 + 111831054251172443
652*x^14 - 149929021439116372
00*x^13 - 141157732738263834575*x^12 + 1499290
214391163720
0*x^11 + 111831054251172443652*x^10 - 7544291365549818150*x^9 -
 54757871112862546478*x^8 + 12309554059894392
42*x^7 + 15731365763207610420*
x^6 + 478983875485473448*x^5 - 2351987676228007923*x^4 - 237455573469526820
```

```
*x^3 + 129882833582498276*x^2 + 28424949163733294*x + 1603095924058903)) +
160*10^(1/4)*(2*sqrt(10)*sqrt(5)*(25826047493168*x^24 - 423236377284370*x^2
3 + 1395910997963665*x^22 + 7977814049523724*x^21 - 39442227900912726*x^20
- 55889144962836926*x^19 + 308168334219135075*x^18 + 191213417542285986*x^1
7 - 1166656659447404872*x^16 - 371314532640925644*x^15 + 249568309660601850
9*x^14 + 476597108839388734*x^13 - 3197719822327310998*x^12 - 4765971088393
88734*x^11 + 2495683096606018509*x^10 + 371314532640925644*x^9 - 1166656659
447404872*x^8 - 191213417542285986*x^7 + 308168334219135075*x^6 + 558891449
62836926*x^5 - 39442227900912726*x^4 - 7977814049523724*x^3 + 1395910997963
665*x^2 + 423236377284370*x + 25826047493168) + sqrt(5)*(167291197847878*x^
24 - 2821779357503405*x^23 + 10956936121559405*x^22 + 35249787921032759*x^2
1 - 204017758359407706*x^20 - 312436240830040741*x^19 + 1399334424550385115
*x^18 + 1735800349150666551*x^17 - 4987682548518497012*x^16 - 5368381759522
587789*x^15 + 10387543967266594209*x^14 + 9352242816540595199*x^13 - 132202
91544952190258*x^12 - 9352242816540595199*x^11 + 10387543967266594209*x^10
+ 5368381759522587789*x^9 - 4987682548518497012*x^8 - 1735800349150666551*x
^7 + 1399334424550385115*x^6 + 312436240830040741*x^5 - 204017758359407706*
x^4 - 35249787921032759*x^3 + 10956936121559405*x^2 + 2821779357503405*x +
167291197847878)))*sqrt(2*sqrt(10) + 20) - 40*(7575923063334080*x^24 - 1208
46800036516320*x^23 + 365161535455038400*x^22 + 2264233628233167520*x^21 -
8547268899540342720*x^20 - 24722670274627958240*x^19 + 64588595464717514880
*x^18 + 139472668675247371680*x^17 - 248284848145133462080*x^16 - 427713008
722866409440*x^15 + 544910109896139954240*x^14 + 743321346231612491680*x^13
 - 707219317504256947840*x^12 - 743321346231612491680*x^11 + 54491010989613
9954240*x^10 + 427713008722866409440*x^9 - 248284848145133462080*x^8 - 1394
72668675247371680*x^7 + 64588595464717514880*x^6 + 24722670274627958240*x^5
 - 8547268899540342720*x^4 - 2264233628233167520*x^3 + 365161535455038400*x
^2 + sqrt(10)*(3941811839020250*x^24 - 64348465775688550*x^23 + 22588564721
0950450*x^22 + 941020417723504900*x^21 - 4570387865727921300*x^20 - 7999664
382660162200*x^19 + 29582666125955145150*x^18 + 37508204562313810350*x^17 -
 97777371150634079200*x^16 - 100481781887576998050*x^15 + 19198949341361230
0650*x^14 + 162041691927783863200*x^13 - 238833086260082653000*x^12 - 16204
1691927783863200*x^11 + 191989493413612300650*x^10 + 100481781887576998050*
x^9 - 97777371150634079200*x^8 - 37508204562313810350*x^7 + 295826661259551
45150*x^6 + 7999664382660162200*x^5 - 4570387865727921300*x^4 - 94102041772
3504900*x^3 + 225885647210950450*x^2 + sqrt(10)*(1160866886007179*x^24 - 17
845297455339433*x^23 + 44598386983244311*x^22 + 407427605373345862*x^21 - 1
374776119098044166*x^20 - 4157698868742924356*x^19 + 10927521516291262449*x
^18 + 21652775358983721237*x^17 - 41131357142108902096*x^16 - 6226422468105
1657251*x^15 + 87175766672417705979*x^14 + 104283289775600539600*x^13 - 111
301173832927037212*x^12 - 104283289775600539600*x^11 + 87175766672417705979
*x^10 + 62264224681051657251*x^9 - 41131357142108902096*x^8 - 2165277535898
3721237*x^7 + 10927521516291262449*x^6 + 4157698868742924356*x^5 - 13747761
19098044166*x^4 - 407427605373345862*x^3 + 44598386983244311*x^2 + 17845297
455339433*x + 1160866886007179) + 64348465775688550*x + 3941811839020250) +
 462400*sqrt(10)*(5159465558*x^24 - 81725010367*x^23 + 233461919938*x^22 +
1698386911873*x^21 - 6557645525970*x^20 - 15828018471263*x^19 + 48718561372
260*x^18 + 76722347870157*x^17 - 172301271560638*x^16 - 208531808984739*x^1
5 + 348922649378334*x^14 + 338140378140253*x^13 - 438000414906964*x^12 - 33
8140378140253*x^11 + 348922649378334*x^10 + 208531808984739*x^9 - 172301271
560638*x^8 - 76722347870157*x^7 + 48718561372260*x^6 + 15828018471263*x^5 -
 6557645525970*x^4 - 1698386911873*x^3 + 233461919938*x^2 + 81725010367*x +
 5159465558) + 120846800036516320*x + 7575923063334080)*sqrt((2*x^2 - x - 2
)/(x^2 + x - 1)))*sqrt((400*x^4 + 200*x^3 + 2*10^(1/4)*(sqrt(10)*sqrt(5)*(2
*x^4 + x^3 - 5*x^2 - x + 2) + 5*sqrt(5)*(x^4 + 2*x^3 - x^2 - 2*x + 1))*sqrt
(2*sqrt(10) + 20)*sqrt((2*x^2 - x - 2)/(x^2 + x - 1)) - 1000*x^2 + 5*sqrt(1
0)*(17*x^4 + 4*x^3 - 29*x^2 - 4*x + 17) - 200*x + 400)/(x^4 - x^2 + 1)) + 8
6700*(10^(3/4)*(sqrt(10)*sqrt(5)*(2442716712885*x^24 - 13728698401842*x^23
- 293186299957362*x^22 + 2310137367621964*x^21 + 1451517102418989*x^20 - 33
```

418412281395816*x^19 - 7178198464024966*x^18 + 183000238657163274*x^17 + 32
979923853964410*x^16 - 518819784198533318*x^15 - 84251682815179386*x^14 + 8
55052654520890752*x^13 + 1146378258721282285*x^12 - 855052654520890752*x^11
- 84251682815179386*x^10 + 518819784198533318*x^9 + 32979923853964410*x^8 -
183000238657163274*x^7 - 7178198464024966*x^6 + 33418412281395816*x^5 + 14
51517102418989*x^4 - 2310137367621964*x^3 - 293186299957362*x^2 + 137286984
01842*x + 2442716712885) + 2*sqrt(5)*(14773174320516*x^24 - 268375520761317
*x^23 + 1309307163746259*x^22 + 1740271771554118*x^21 - 22961098057094619*x
^20 + 1995670027514196*x^19 + 159995361247050941*x^18 - 36026323622261007*x
^17 - 580081963181949234*x^16 + 111975250170214681*x^15 + 1216656271908709911*x^14 - 182294736937952040*x^13 - 1549896649785392773*x^12 + 182294736937
952040*x^11 + 1216656271908709911*x^10 - 111975250170214681*x^9 - 580081963
181949234*x^8 + 36026323622261007*x^7 + 159995361247050941*x^6 - 1995670027
514196*x^5 - 22961098057094619*x^4 - 1740271771554118*x^3 + 130930716374625
9*x^2 + 268375520761317*x + 14773174320516)) + 32*10^(1/4)*(sqrt(10)*sqrt(5
)*(353203013202*x^24 - 5252268148153*x^23 + 9599899316880*x^22 + 1573184873
99903*x^21 - 528463660887158*x^20 - 1352806046560081*x^19 + 475420415994074
2*x^18 + 5302733573770307*x^17 - 19750563646089182*x^16 - 12045907326764941
*x^15 + 44826810647943796*x^14 + 17772009847998347*x^13 - 58660810041591536
*x^12 - 17772009847998347*x^11 + 44826810647943796*x^10 + 12045907326764941
*x^9 - 19750563646089182*x^8 - 5302733573770307*x^7 + 4754204159940742*x^6
+ 1352806046560081*x^5 - 528463660887158*x^4 - 157318487399903*x^3 + 959989
9316880*x^2 + 5252268148153*x + 353203013202) + 5*sqrt(5)*(239278797030*x^2
4 - 3894104051149*x^23 + 14241545124129*x^22 + 43944733024733*x^21 - 190384
021067588*x^20 - 581008343105785*x^19 + 922766399143729*x^18 + 415468445331
5363*x^17 - 1978225802691326*x^16 - 14164309049142511*x^15 + 21659338955337
13*x^14 + 25503495102958511*x^13 - 1877306138593118*x^12 - 2550349510295851
1*x^11 + 2165933895533713*x^10 + 14164309049142511*x^9 - 1978225802691326*x
^8 - 4154684453315363*x^7 + 922766399143729*x^6 + 581008343105785*x^5 - 190
384021067588*x^4 - 43944733024733*x^3 + 14241545124129*x^2 + 3894104051149*
x + 239278797030)))*sqrt(2*sqrt(10) + 20)*sqrt((2*x^2 - x - 2)/(x^2 + x - 1
)) + 1300500*sqrt(10)*(33568388558203*x^24 - 575319820414644*x^23 + 2600060
853588106*x^22 + 3231189194065640*x^21 - 35687236548135733*x^20 - 764562410
1059968*x^19 + 182678402540331150*x^18 + 9974894492537668*x^17 - 5120392946
51286218*x^16 - 7794844652266140*x^15 + 909771845956721522*x^14 + 385622927
8651280*x^13 - 1094808769706185685*x^12 - 3856229278651280*x^11 + 909771845
956721522*x^10 + 7794844652266140*x^9 - 512039294651286218*x^8 - 9974894492
537668*x^7 + 182678402540331150*x^6 + 7645624101059968*x^5 - 35687236548135
733*x^4 - 3231189194065640*x^3 + 2600060853588106*x^2 + 575319820414644*x +
33568388558203) - 10404000*sqrt(10)*(3913944654568*x^24 - 68902030177800*x
^23 + 324474338021870*x^22 + 346500573850454*x^21 - 4441518281684956*x^20 -
571126785242366*x^19 + 22736146340724250*x^18 - 100140780129284*x^17 - 637
42636856310672*x^16 + 1930227608742956*x^15 + 113272298306222934*x^14 - 370
7342105075886*x^13 - 136316462990101068*x^12 + 3707342105075886*x^11 + 1132
72298306222934*x^10 - 1930227608742956*x^9 - 63742636856310672*x^8 + 100140
780129284*x^7 + 22736146340724250*x^6 + 571126785242366*x^5 - 4441518281684
956*x^4 - 346500573850454*x^3 + 324474338021870*x^2 + sqrt(10)*(10081328742
94*x^24 - 14888655494103*x^23 + 24952193442996*x^22 + 469321825791782*x^21
- 1529467312445276*x^20 - 4322697049104696*x^19 + 14475801363390484*x^18 +
19894689930581267*x^17 - 59851766151398456*x^16 - 52525242803693941*x^15 +
133501316828030084*x^14 + 84262553164312340*x^13 - 173264213780669712*x^12
- 84262553164312340*x^11 + 133501316828030084*x^10 + 52525242803693941*x^9
- 59851766151398456*x^8 - 19894689930581267*x^7 + 14475801363390484*x^6 + 4
322697049104696*x^5 - 1529467312445276*x^4 - 469321825791782*x^3 + 24952193
442996*x^2 + 14888655494103*x + 1008132874294) + 68902030177800*x + 3913944
654568) + 16392441790853834616 00*x + 11378484341832755970 0)/(10727617050837
1881*x^24 - 2441300271537795968*x^23 + 17569300516266316746*x^22 - 16092110
779172766528*x^21 - 288186279976289132707*x^20 + 672190931265926674240*x^19
+ 1940582906697052688958*x^18 - 4803572187578142625280*x^17 - 697185756548

4310566550*x^16 + 15648066394863653981184*x^15 + 14595578320760563174802*x^14 - 27544211233989398727872*x^13 - 18571557939452786508651*x^12 + 27544211233989398727872*x^11 + 14595578320760563174802*x^10 - 15648066394863653981184*x^9 - 6971857565484310566550*x^8 + 4803572187578142625280*x^7 + 1940582906697052688958*x^6 - 672190931265926674240*x^5 - 288186279976289132707*x^4 + 16092110779172766528*x^3 + 17569300516266316746*x^2 + 2441300271537795968*x + 107276170508371881)) - 1/100*10^(3/4)*sqrt(5)*sqrt(2*sqrt(10) + 20)*arctan(-1/900*(113784843418327559700*x^24 - 1639244179085383461600*x^23 + 2545884655734590980200*x^22 + 50821242952569317006400*x^21 - 158890131031597690275900*x^20 - 465050237434908643152000*x^19 + 1504931893222349028684600*x^18 + 2129607736390894049244000*x^17 - 6223660771133927367315000*x^16 - 5602507922355181669879200*x^15 + 13883484083974894847147400*x^14 + 8971054092806414576313600*x^13 - 18019091872612617624668700*x^12 - 8971054092806414576313600*x^11 + 13883484083974894847147400*x^10 + 5602507922355181669879200*x^9 - 6223660771133927367315000*x^8 - 2129607736390894049244000*x^7 + 1504931893222349028684600*x^6 + 465050237434908643152000*x^5 - 158890131031597690275900*x^4 - 50821242952569317006400*x^3 + 2545884655734590980200*x^2 - 17*sqrt(2)*((10^(3/4)*(sqrt(10)*sqrt(5)*(3353156233262299*x^24 - 49929701526135692*x^23 + 105488376266631398*x^22 + 1245603618916311800*x^21 - 3729015017341775649*x^20 - 12574465648522249504*x^19 + 28773197469313801650*x^18 + 61934497275400333884*x^17 - 104756202796838736914*x^16 - 169570606189065646500*x^15 + 217084068264548775966*x^14 + 276610136446380234320*x^13 - 274989879451094143625*x^12 - 276610136446380234320*x^11 + 217084068264548775966*x^10 + 169570606189065646500*x^9 - 104756202796838736914*x^8 - 61934497275400333884*x^7 + 28773197469313801650*x^6 + 12574465648522249504*x^5 - 3729015017341775649*x^4 - 1245603618916311800*x^3 + 105488376266631398*x^2 + 49929701526135692*x + 3353156233262299) + 10*sqrt(5)*(1603095924058903*x^24 - 28424949163733294*x^23 + 129882833582498276*x^22 + 237455573469526820*x^21 - 2351987676228007923*x^20 - 478983875485473448*x^19 + 15731365763207610420*x^18 - 1230955405989439242*x^17 - 54757871112862546478*x^16 + 7544291365549818150*x^15 + 111831054251172443652*x^14 - 14992902143911637200*x^13 - 141157732738263834575*x^12 + 14992902143911637200*x^11 + 111831054251172443652*x^10 - 7544291365549818150*x^9 - 54757871112862546478*x^8 + 1230955405989439242*x^7 + 15731365763207610420*x^6 + 478983875485473448*x^5 - 2351987676228007923*x^4 - 237455573469526820*x^3 + 129882833582498276*x^2 + 28424949163733294*x + 1603095924058903)) + 160*10^(1/4)*(2*sqrt(10)*sqrt(5)*(25826047493168*x^24 - 423236377284370*x^23 + 1395910997963665*x^22 + 7977814049523724*x^21 - 39442227900912726*x^20 - 55889144962836926*x^19 + 308168334219135075*x^18 + 191213417542285986*x^17 - 1166656659447404872*x^16 - 371314532640925644*x^15 + 2495683096606018509*x^14 + 476597108839388734*x^13 - 3197719822327310998*x^12 - 476597108839388734*x^11 + 2495683096606018509*x^10 + 371314532640925644*x^9 - 1166656659447404872*x^8 - 191213417542285986*x^7 + 308168334219135075*x^6 + 55889144962836926*x^5 - 39442227900912726*x^4 - 7977814049523724*x^3 + 1395910997963665*x^2 + 423236377284370*x + 25826047493168) + sqrt(5)*(167291197847878*x^24 - 2821779357503405*x^23 + 10956936121559405*x^22 + 35249787921032759*x^21 - 204017758359407706*x^20 - 312436240830040741*x^19 + 1399334424550385115*x^18 + 1735800349150666551*x^17 - 4987682548518497012*x^16 - 5368381759522587789*x^15 + 10387543967266594209*x^14 + 9352242816540595199*x^13 - 13220291544952190258*x^12 - 9352242816540595199*x^11 + 10387543967266594209*x^10 + 5368381759522587789*x^9 - 4987682548518497012*x^8 - 1735800349150666551*x^7 + 1399334424550385115*x^6 + 312436240830040741*x^5 - 204017758359407706*x^4 - 35249787921032759*x^3 + 10956936121559405*x^2 + 2821779357503405*x + 167291197847878)))*sqrt(2*sqrt(10) + 20) + 40*(7575923063334080*x^24 - 120846800036516320*x^23 + 365161535455038400*x^22 + 2264233628233167520*x^21 - 8547268899540342720*x^20 - 24722670274627958240*x^19 + 64588595464717514880*x^18 + 139472668675247371680*x^17 - 248284848145133462080*x^16 - 427713008722866409440*x^15 + 544910109896139954240*x^14 + 743321346231612491680*x^13 - 707219317504256947840*x^12 - 743321346231612491680*x^11 + 544910109896139954240*x^10 + 427713008722866409440*x^9 - 2482

84848145133462080*x^8 - 1394726686752473716 80*x^7 + 6458859546471751488 0*x^
6 + 24722670274627958240*x^5 - 8547268899540342720*x^4 - 226423362823316752
0*x^3 + 3651615354550384 00*x^2 + sqrt(10)*(3941811839020250*x^24 - 64348465
775688550*x^23 + 225885647210950450*x^22 + 941020417723504900*x^21 - 457038
7865727921300*x^20 - 7999664382660162200*x^19 + 29582666125955145150*x^18 +
37508204562313810350*x^17 - 97777371150634079200*x^16 - 100481781887576998
050*x^15 + 191989493413612300650*x^14 + 162041691927783863200*x^13 - 238833
086260082653000*x^12 - 162041691927783863200*x^11 + 191989493413612300650*x
^10 + 100481781887576998050*x^9 - 97777371150634079200*x^8 - 37508204562313
810350*x^7 + 29582666125955145150*x^6 + 7999664382660162200*x^5 - 457038786
5727921300*x^4 - 941020417723504900*x^3 + 225885647210950450*x^2 + sqrt(10)
*(1160866886007179*x^24 - 17845297455339433*x^23 + 44598386983244311*x^22 +
407427605373345862*x^21 - 1374776119098044166*x^20 - 4157698868742924356*x
^19 + 10927521516291262449*x^18 + 21652775358983721237*x^17 - 4113135714210
8902096*x^16 - 62264224681051657251*x^15 + 87175766672417705979*x^14 + 1042
83289775600539600*x^13 - 111301173832927037212*x^12 - 104283289775600539600
*x^11 + 87175766672417705979*x^10 + 62264224681051657251*x^9 - 411313571421
08902096*x^8 - 21652775358983721237*x^7 + 10927521516291262449*x^6 + 415769
8868742924356*x^5 - 1374776119098044166*x^4 - 407427605373345862*x^3 + 4459
8386983244311*x^2 + 17845297455339433*x + 1160866886007179) + 6434846577568
8550*x + 3941811839020250) + 462400*sqrt(10)*(5159465558*x^24 - 81725010367
*x^23 + 233461919938*x^22 + 1698386911873*x^21 - 6557645525970*x^20 - 15828
018471263*x^19 + 48718561372260*x^18 + 76722347870157*x^17 - 17230127156063
8*x^16 - 208531808984739*x^15 + 348922649378334*x^14 + 338140378140253*x^13
- 438000414906964*x^12 - 338140378140253*x^11 + 348922649378334*x^10 + 208
531808984739*x^9 - 172301271560638*x^8 - 76722347870157*x^7 + 4871856137226
0*x^6 + 15828018471263*x^5 - 6557645525970*x^4 - 1698386911873*x^3 + 233461
919938*x^2 + 81725010367*x + 5159465558) + 120846800036516320*x + 757592306
3334080)*sqrt((2*x^2 - x - 2)/(x^2 + x - 1)))*sqrt((400*x^4 + 200*x^3 - 2*1
0^(1/4)*(sqrt(10)*sqrt(5)*(2*x^4 + x^3 - 5*x^2 - x + 2) + 5*sqrt(5)*(x^4 +
2*x^3 - x^2 - 2*x + 1))*sqrt(2*sqrt(10) + 20)*sqrt((2*x^2 - x - 2)/(x^2 + x
- 1)) - 1000*x^2 + 5*sqrt(10)*(17*x^4 + 4*x^3 - 29*x^2 - 4*x + 17) - 200*x
+ 400)/(x^4 - x^2 + 1)) - 86700*(10^(3/4)*(sqrt(10)*sqrt(5)*(2442716712885
*x^24 - 13728698401842*x^23 - 293186299957362*x^22 + 2310137367621964*x^21
+ 1451517102418989*x^20 - 33418412281395816*x^19 - 7178198464024966*x^18 +
183000238657163274*x^17 + 32979923853964410*x^16 - 518819784198533318*x^15
- 84251682815179386*x^14 + 855052654520890752*x^13 + 114637825872128285*x^1
2 - 855052654520890752*x^11 - 84251682815179386*x^10 + 518819784198533318*x
^9 + 32979923853964410*x^8 - 183000238657163274*x^7 - 7178198464024966*x^6
+ 33418412281395816*x^5 + 1451517102418989*x^4 - 2310137367621964*x^3 - 293
186299957362*x^2 + 13728698401842*x + 2442716712885) + 2*sqrt(5)*(147731743
20516*x^24 - 268375520761317*x^23 + 1309307163746259*x^22 + 174027177155411
8*x^21 - 22961098057094619*x^20 + 1995670027514196*x^19 + 15999536124705094
1*x^18 - 36026323622261007*x^17 - 580081963181949234*x^16 + 111975250170214
681*x^15 + 1216656271908709911*x^14 - 182294736937952040*x^13 - 15498966497
85392773*x^12 + 182294736937952040*x^11 + 1216656271908709911*x^10 - 111975
250170214681*x^9 - 580081963181949234*x^8 + 36026323622261007*x^7 + 1599953
61247050941*x^6 - 1995670027514196*x^5 - 22961098057094619*x^4 - 1740271771
554118*x^3 + 1309307163746259*x^2 + 268375520761317*x + 14773174320516)) +
32*10^(1/4)*(sqrt(10)*sqrt(5)*(353203013202*x^24 - 5252268148153*x^23 + 959
9899316880*x^22 + 157318487399903*x^21 - 528463660887158*x^20 - 13528060465
60081*x^19 + 4754204159940742*x^18 + 5302733573770307*x^17 - 19750563646089
182*x^16 - 12045907326764941*x^15 + 44826810647943796*x^14 + 17772009847998
347*x^13 - 58660810041591536*x^12 - 17772009847998347*x^11 + 44826810647943
796*x^10 + 12045907326764941*x^9 - 19750563646089182*x^8 - 5302733573770307
*x^7 + 4754204159940742*x^6 + 1352806046560081*x^5 - 528463660887158*x^4 -
157318487399903*x^3 + 9599899316880*x^2 + 5252268148153*x + 353203013202) +
5*sqrt(5)*(239278797030*x^24 - 3894104051149*x^23 + 14241545124129*x^22 +
43944733024733*x^21 - 190384021067588*x^20 - 581008343105785*x^19 + 9227663

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99143729*x^18 + 4154684453315363*x^17 - 1978225802691326*x^16 - 14164309049
142511*x^15 + 2165933895533713*x^14 + 25503495102958511*x^13 - 187730613859
3118*x^12 - 25503495102958511*x^11 + 2165933895533713*x^10 + 14164309049142
511*x^9 - 1978225802691326*x^8 - 4154684453315363*x^7 + 922766399143729*x^6
 + 581008343105785*x^5 - 190384021067588*x^4 - 43944733024733*x^3 + 1424154
5124129*x^2 + 3894104051149*x + 239278797030)))*sqrt(2*sqrt(10) + 20)*sqrt(
(2*x^2 - x - 2)/(x^2 + x - 1)) + 1300500*sqrt(10)*(33568388558203*x^24 - 57
5319820414644*x^23 + 2600060853588106*x^22 + 3231189194065640*x^21 - 356872
36548135733*x^20 - 7645624101059968*x^19 + 182678402540331150*x^18 + 997489
4492537668*x^17 - 512039294651286218*x^16 - 7794844652266140*x^15 + 9097718
45956721522*x^14 + 3856229278651280*x^13 - 1094808769706185685*x^12 - 38562
29278651280*x^11 + 909771845956721522*x^10 + 7794844652266140*x^9 - 5120392
94651286218*x^8 - 9974894492537668*x^7 + 182678402540331150*x^6 + 764562410
1059968*x^5 - 35687236548135733*x^4 - 3231189194065640*x^3 + 26000608535881
06*x^2 + 575319820414644*x + 33568388558203) - 10404000*sqrt(10)*(391394465
4568*x^24 - 68902030177800*x^23 + 324474338021870*x^22 + 346500573850454*x^
21 - 4441518281684956*x^20 - 571126785242366*x^19 + 22736146340724250*x^18
- 100140780129284*x^17 - 63742636856310672*x^16 + 1930227608742956*x^15 + 1
13272298306222934*x^14 - 3707342105075886*x^13 - 136316462990101068*x^12 +
3707342105075886*x^11 + 113272298306222934*x^10 - 1930227608742956*x^9 - 63
742636856310672*x^8 + 100140780129284*x^7 + 22736146340724250*x^6 + 5711267
85242366*x^5 - 4441518281684956*x^4 - 346500573850454*x^3 + 324474338021870
*x^2 + sqrt(10)*(1008132874294*x^24 - 14888655494103*x^23 + 24952193442996*
x^22 + 469321825791782*x^21 - 1529467312445276*x^20 - 4322697049104696*x^19
 + 14475801363390484*x^18 + 19894689930581267*x^17 - 59851766151398456*x^16
 - 52525242803693941*x^15 + 133501316828030084*x^14 + 84262553164312340*x^1
3 - 173264213780669712*x^12 - 84262553164312340*x^11 + 133501316828030084*x
^10 + 52525242803693941*x^9 - 59851766151398456*x^8 - 19894689930581267*x^7
 + 14475801363390484*x^6 + 4322697049104696*x^5 - 1529467312445276*x^4 - 46
9321825791782*x^3 + 24952193442996*x^2 + 14888655494103*x + 1008132874294)
+ 68902030177800*x + 3913944654568) + 1639244179085383461600*x + 1137848434
18327559700)/(107276170508371881*x^24 - 2441300271537795968*x^23 + 17569300
516266316746*x^22 - 16092110779172766528*x^21 - 28818627997628913270 7*x^20
+ 67219093126592667424 0*x^19 + 194058290669705268895 8*x^18 - 48035721875781
42625280*x^17 - 6971857565484310566550*x^16 + 1564806639486365398118 4*x^15
+ 1459557832076056317480 2*x^14 - 2754421123398939872787 2*x^13 - 18571557939
452786508651*x^12 + 27544211233989398727872*x^11 + 14595578320760563174802*
x^10 - 15648066394863653981184*x^9 - 6971857565484310566550*x^8 + 480357218
7578142625280*x^7 + 1940582906697052688958*x^6 - 672190931265926674240*x^5
- 288186279976289132707*x^4 + 16092110779172766528*x^3 + 175693005162663167
46*x^2 + 2441300271537795968*x + 107276170508371881)) - 1/1200*10^(1/4)*(sq
rt(10)*sqrt(5) - 10*sqrt(5))*sqrt(2*sqrt(10) + 20)*log(14450*(400*x^4 + 200
*x^3 + 2*10^(1/4)*(sqrt(10)*sqrt(5)*(2*x^4 + x^3 - 5*x^2 - x + 2) + 5*sqrt(
5)*(x^4 + 2*x^3 - x^2 - 2*x + 1))*sqrt(2*sqrt(10) + 20)*sqrt((2*x^2 - x - 2
)/(x^2 + x - 1)) - 1000*x^2 + 5*sqrt(10)*(17*x^4 + 4*x^3 - 29*x^2 - 4*x + 1
7) - 200*x + 400)/(x^4 - x^2 + 1)) + 1/1200*10^(1/4)*(sqrt(10)*sqrt(5) - 10
*sqrt(5))*sqrt(2*sqrt(10) + 20)*log(14450*(400*x^4 + 200*x^3 - 2*10^(1/4)*(
sqrt(10)*sqrt(5)*(2*x^4 + x^3 - 5*x^2 - x + 2) + 5*sqrt(5)*(x^4 + 2*x^3 - x
^2 - 2*x + 1))*sqrt(2*sqrt(10) + 20)*sqrt((2*x^2 - x - 2)/(x^2 + x - 1)) -
1000*x^2 + 5*sqrt(10)*(17*x^4 + 4*x^3 - 29*x^2 - 4*x + 17) - 200*x + 400)/(
x^4 - x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^4-x^2+1\right)\sqrt{\frac{2x^2-x-2}{x^2+x-1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/((2*x^2-x-2)/(x^2+x-1))^(1/2)/(x^4-x^2+1),x, algorithm="g
```

```
iac")
```

```
[Out] integrate((x^2 + 1)/((x^4 - x^2 + 1)*sqrt((2*x^2 - x - 2)/(x^2 + x - 1))),
x)
```

maple [C] time = 0.44, size = 7672, normalized size = 77.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/((2*x^2-x-2)/(x^2+x-1))^(1/2)/(x^4-x^2+1),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^4-x^2+1\right)\sqrt{\frac{2\,x^2-x-2}{x^2+x-1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/((2*x^2-x-2)/(x^2+x-1))^(1/2)/(x^4-x^2+1),x, algorithm="m
axima")
```

```
[Out] integrate((x^2 + 1)/((x^4 - x^2 + 1)*sqrt((2*x^2 - x - 2)/(x^2 + x - 1))),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+1}{\sqrt{-\frac{-2\,x^2+x+2}{x^2+x-1}}\,\left(x^4-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((-(x - 2*x^2 + 2)/(x + x^2 - 1))^(1/2)*(x^4 - x^2 + 1)),x)
```

```
[Out] int((x^2 + 1)/((-(x - 2*x^2 + 2)/(x + x^2 - 1))^(1/2)*(x^4 - x^2 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/((2*x**2-x-2)/(x**2+x-1))**(1/2)/(x**4-x**2+1),x)
```

```
[Out] Timed out
```

3.1225 $\int \sqrt[4]{bx^3 + ax^4}\,dx$

Optimal. Leaf size=99

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{16a^{7/4}}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{16a^{7/4}}+\frac{(4ax+b)\sqrt[4]{ax^4+bx^3}}{8a}$$

Rubi [A] time = 0.20, antiderivative size = 168, normalized size of antiderivative = 1.70, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2004, 2024, 2032, 63, 331, 298, 203, 206}

$$\frac{3b^2x^{9/4}(ax+b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{16a^{7/4}\left(ax^4+bx^3\right)^{3/4}}-\frac{3b^2x^{9/4}(ax+b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{16a^{7/4}\left(ax^4+bx^3\right)^{3/4}}+\frac{1}{2}x\sqrt[4]{ax^4+bx^3}+\frac{b\sqrt[4]{ax^4+bx^3}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^3 + a*x^4)^(1/4),x]
```

```
[Out] (b*(b*x^3 + a*x^4)^(1/4))/(8*a) + (x*(b*x^3 + a*x^4)^(1/4))/2 + (3*b^2*x^(9
/4)*(b + a*x)^(3/4)*ArcTan[(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(16*a^(7/4)*
(b*x^3 + a*x^4)^(3/4)) - (3*b^2*x^(9/4)*(b + a*x)^(3/4)*ArcTanh[(a^(1/4)*x^
(1/4))/(b + a*x)^(1/4)])/(16*a^(7/4)*(b*x^3 + a*x^4)^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt[4]{bx^3+ax^4}\,dx &= \frac{1}{2}x\sqrt[4]{bx^3+ax^4}+\frac{1}{8}b\int \frac{x^3}{\left(bx^3+ax^4\right)^{3/4}}\,dx \\
&= \frac{b\sqrt[4]{bx^3+ax^4}}{8a}+\frac{1}{2}x\sqrt[4]{bx^3+ax^4}-\frac{\left(3b^2\right)\int \frac{x^2}{\left(bx^3+ax^4\right)^{3/4}}\,dx}{32a} \\
&= \frac{b\sqrt[4]{bx^3+ax^4}}{8a}+\frac{1}{2}x\sqrt[4]{bx^3+ax^4}-\frac{\left(3b^2x^{9/4}(b+ax)^{3/4}\right)\int \frac{1}{\sqrt[4]{x}\,(b+ax)^{3/4}}\,dx}{32a\left(bx^3+ax^4\right)^{3/4}} \\
&= \frac{b\sqrt[4]{bx^3+ax^4}}{8a}+\frac{1}{2}x\sqrt[4]{bx^3+ax^4}-\frac{\left(3b^2x^{9/4}(b+ax)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{\left(b+ax^4\right)^{3/4}}\,dx,x,\sqrt[4]{x}\right)}{8a\left(bx^3+ax^4\right)^{3/4}} \\
&= \frac{b\sqrt[4]{bx^3+ax^4}}{8a}+\frac{1}{2}x\sqrt[4]{bx^3+ax^4}-\frac{\left(3b^2x^{9/4}(b+ax)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx,x,\frac{\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)}{8a\left(bx^3+ax^4\right)^{3/4}} \\
&= \frac{b\sqrt[4]{bx^3+ax^4}}{8a}+\frac{1}{2}x\sqrt[4]{bx^3+ax^4}-\frac{\left(3b^2x^{9/4}(b+ax)^{3/4}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)}{16a^{3/2}\left(bx^3+ax^4\right)^{3/4}}+\frac{\left(3b^2\right.}{} \\
&= \frac{b\sqrt[4]{bx^3+ax^4}}{8a}+\frac{1}{2}x\sqrt[4]{bx^3+ax^4}+\frac{3b^2x^{9/4}(b+ax)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)}{16a^{7/4}\left(bx^3+ax^4\right)^{3/4}}-\frac{3b^2x^{9/4}(b+ax)^{3/4}\tanh}{16a^{7/4}\left(bx^3+a\right.}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.47

$$
\frac{4x\sqrt[4]{x^3(ax+b)}\;{}_2F_1\left(-\frac{1}{4},\frac{7}{4};\frac{11}{4};-\frac{ax}{b}\right)}{7\sqrt[4]{\frac{ax}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^3 + a*x^4)^(1/4),x]
```

```
[Out] (4*x*(x^3*(b + a*x))^(1/4)*Hypergeometric2F1[-1/4, 7/4, 11/4, -((a*x)/b)])/
(7*(1 + (a*x)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.43, size = 99, normalized size = 1.00

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{16a^{7/4}}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{16a^{7/4}}+\frac{(4ax+b)\sqrt[4]{ax^4+bx^3}}{8a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^3 + a*x^4)^(1/4),x]
```

```
[Out] ((b + 4*a*x)*(b*x^3 + a*x^4)^(1/4))/(8*a) + (3*b^2*ArcTan[(a^(1/4)*x)/(b*x^
3 + a*x^4)^(1/4)])/(16*a^(7/4)) - (3*b^2*ArcTanh[(a^(1/4)*x)/(b*x^3 + a*x^4
)^(1/4)])/(16*a^(7/4))
```

fricas [B] time = 0.51, size = 234, normalized size = 2.36

$$\frac{12\,a\left(\frac{b^8}{a^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^4+bx^3\right)^{\frac{1}{4}}a^5b^2\left(\frac{b^8}{a^7}\right)^{\frac{3}{4}}-a^5\left(\frac{b^8}{a^7}\right)^{\frac{3}{4}}x\sqrt{\frac{a^4\sqrt{\frac{b^8}{a^7}}\,x^2+\sqrt{ax^4+bx^3}\,b^4}{x^2}}}{b^8x}\right)-3\,a\left(\frac{b^8}{a^7}\right)^{\frac{1}{4}}\log\left(\frac{3\left(a^2\left(\frac{b^8}{a^7}\right)^{\frac{1}{4}}x+\left(ax^4+bx^3\right)^{\frac{1}{4}}b^2\right)}{x}\right)+3\,a\left(\frac{b^8}{a^7}\right)^{\frac{1}{4}}\log\left(-\frac{3\left(a^2\left(\frac{b^8}{a^7}\right)^{\frac{1}{4}}x-\left(ax^4+bx^3\right)^{\frac{1}{4}}b^2\right)}{x}\right)+4\left(ax^4+bx^3\right)^{\frac{1}{4}}(4\,ax+b)}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^3)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/32*(12*a*(b^8/a^7)^(1/4)*arctan(-((a*x^4 + b*x^3)^(1/4)*a^5*b^2*(b^8/a^7)
^(3/4) - a^5*(b^8/a^7)^(3/4)*x*sqrt((a^4*sqrt(b^8/a^7)*x^2 + sqrt(a*x^4 + b
*x^3)*b^4)/x^2))/(b^8*x)) - 3*a*(b^8/a^7)^(1/4)*log(3*(a^2*(b^8/a^7)^(1/4)*
x + (a*x^4 + b*x^3)^(1/4)*b^2)/x) + 3*a*(b^8/a^7)^(1/4)*log(-3*(a^2*(b^8/a^
7)^(1/4)*x - (a*x^4 + b*x^3)^(1/4)*b^2)/x) + 4*(a*x^4 + b*x^3)^(1/4)*(4*a*x
 + b))/a
```

giac [B] time = 0.19, size = 243, normalized size = 2.45

$$\frac{\frac{6\sqrt{2}\,b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}a}+\frac{6\sqrt{2}\,b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}a}+\frac{3\sqrt{2}\,b^3\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{(-a)^{\frac{3}{4}}a}+\frac{3\sqrt{2}(-a)^{\frac{1}{4}}b^3\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{a^2}+\frac{8\left(\left(a+\frac{b}{x}\right)^{\frac{5}{4}}b^3+3\left(a+\frac{b}{x}\right)^{\frac{1}{4}}ab^3\right)x^2}{ab^2}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^3)^(1/4),x, algorithm="giac")
```

```
[Out] 1/64*(6*sqrt(2)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x)^(1
/4))/(-a)^(1/4))/((-a)^(3/4)*a) + 6*sqrt(2)*b^3*arctan(-1/2*sqrt(2)*(sqrt(2
)*(-a)^(1/4) - 2*(a + b/x)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a) + 3*sqrt(2)*b^
3*log(sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a + b/x))/((-a)^
(3/4)*a) + 3*sqrt(2)*(-a)^(1/4)*b^3*log(-sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4)
 + sqrt(-a) + sqrt(a + b/x))/a^2 + 8*((a + b/x)^(5/4)*b^3 + 3*(a + b/x)^(1/
4)*a*b^3)*x^2/(a*b^2))/b
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int\left(a\,x^4+b\,x^3\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b*x^3)^(1/4),x)
```

```
[Out] int((a*x^4+b*x^3)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^4 + bx^3\right)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^3)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + b*x^3)^(1/4), x)
```

mupad [B] time = 0.86, size = 38, normalized size = 0.38

$$\frac{4\,x\,\left(a\,x^4+b\,x^3\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{7}{4};\ \frac{11}{4};\ -\frac{a\,x}{b}\right)}{7\,{\left(\frac{a\,x}{b}+1\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4 + b*x^3)^(1/4),x)
```

```
[Out] (4*x*(a*x^4 + b*x^3)^(1/4)*hypergeom([-1/4, 7/4], 11/4, -(a*x)/b))/(7*((a*x
)/b + 1)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{ax^4 + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b*x**3)**(1/4),x)
```

```
[Out] Integral((a*x**4 + b*x**3)**(1/4), x)
```

3.1226 $\int \frac{1}{x^{13}\sqrt[3]{-1+x^6}}\,dx$

Optimal. Leaf size=99

$$-\frac{1}{27}\log\left(\sqrt[3]{x^6-1}+1\right)+\frac{1}{54}\log\left(\left(x^6-1\right)^{2/3}-\sqrt[3]{x^6-1}+1\right)-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\left(x^6-1\right)^{2/3}\left(4x^6+3\right)}{36x^{12}}$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 51, 56, 618, 204, 31}

$$\frac{\left(x^6-1\right)^{2/3}}{9x^6}-\frac{1}{18}\log\left(\sqrt[3]{x^6-1}+1\right)-\frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\left(x^6-1\right)^{2/3}}{12x^{12}}+\frac{\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*(-1 + x^6)^(1/3)),x]
```

```
[Out] (-1 + x^6)^(2/3)/(12*x^12) + (-1 + x^6)^(2/3)/(9*x^6) - ArcTan[(1 - 2*(-1 +
 x^6)^(1/3))/Sqrt[3]]/(9*Sqrt[3]) + Log[x]/9 - Log[1 + (-1 + x^6)^(1/3)]/18
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{13}\sqrt[3]{-1+x^6}}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x^3}\,dx,x,x^6\right)\\
&= \frac{\left(-1+x^6\right)^{2/3}}{12x^{12}}+\frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x^2}\,dx,x,x^6\right)\\
&= \frac{\left(-1+x^6\right)^{2/3}}{12x^{12}}+\frac{\left(-1+x^6\right)^{2/3}}{9x^6}+\frac{1}{27}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x}\,dx,x,x^6\right)\\
&= \frac{\left(-1+x^6\right)^{2/3}}{12x^{12}}+\frac{\left(-1+x^6\right)^{2/3}}{9x^6}+\frac{\log(x)}{9}-\frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt[3]{-1+x^6}\right)+\frac{1}{18}\operatorname{Subst}\Big(\\
&= \frac{\left(-1+x^6\right)^{2/3}}{12x^{12}}+\frac{\left(-1+x^6\right)^{2/3}}{9x^6}+\frac{\log(x)}{9}-\frac{1}{18}\log\left(1+\sqrt[3]{-1+x^6}\right)-\frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.\\
&= \frac{\left(-1+x^6\right)^{2/3}}{12x^{12}}+\frac{\left(-1+x^6\right)^{2/3}}{9x^6}-\frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^6}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\log(x)}{9}-\frac{1}{18}\log\left(1+\sqrt[3]{-1+x^6}\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 28, normalized size = 0.28

$$\frac{1}{4}\left(x^6-1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},3;\frac{5}{3};1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^13*(-1 + x^6)^(1/3)),x]
[Out] ((-1 + x^6)^(2/3)*Hypergeometric2F1[2/3, 3, 5/3, 1 - x^6])/4
```

IntegrateAlgebraic [A] time = 0.19, size = 99, normalized size = 1.00

$$-\frac{1}{27}\log\left(\sqrt[3]{x^6-1}+1\right)+\frac{1}{54}\log\left(\left(x^6-1\right)^{2/3}-\sqrt[3]{x^6-1}+1\right)-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\left(x^6-1\right)^{2/3}\left(4x^6+3\right)}{36x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^13*(-1 + x^6)^(1/3)),x]
[Out] ((-1 + x^6)^(2/3)*(3 + 4*x^6))/(36*x^12) - ArcTan[1/Sqrt[3] - (2*(-1 + x^6)
^(1/3))/Sqrt[3]]/(9*Sqrt[3]) - Log[1 + (-1 + x^6)^(1/3)]/27 + Log[1 - (-1 +
 x^6)^(1/3) + (-1 + x^6)^(2/3)]/54
```

fricas [A] time = 0.44, size = 88, normalized size = 0.89

$$\frac{4\sqrt{3}x^{12}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+2x^{12}\log\left(\left(x^6-1\right)^{\frac{2}{3}}-\left(x^6-1\right)^{\frac{1}{3}}+1\right)-4x^{12}\log\left(\left(x^6-1\right)^{\frac{1}{3}}+1\right)+3\left(4x^6+3\right)\left(x^6-1\right)^{\frac{2}{3}}}{108x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^6-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/108*(4*sqrt(3)*x^12*arctan(2/3*sqrt(3)*(x^6 - 1)^(1/3) - 1/3*sqrt(3)) + 2
*x^12*log((x^6 - 1)^(2/3) - (x^6 - 1)^(1/3) + 1) - 4*x^12*log((x^6 - 1)^(1/
3) + 1) + 3*(4*x^6 + 3)*(x^6 - 1)^(2/3))/x^12
```

giac [A] time = 0.16, size = 81, normalized size = 0.82

$$\frac{1}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^6-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{4\left(x^6-1\right)^{\frac{5}{3}}+7\left(x^6-1\right)^{\frac{2}{3}}}{36\,x^{12}}+\frac{1}{54}\log\left(\left(x^6-1\right)^{\frac{2}{3}}-\left(x^6-1\right)^{\frac{1}{3}}+1\right)-\frac{1}{27}\log\left(\left|\left(x^6-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^6-1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3) - 1)) + 1/36*(4*(x^6 - 1
)^(5/3) + 7*(x^6 - 1)^(2/3))/x^12 + 1/54*log((x^6 - 1)^(2/3) - (x^6 - 1)^(1
/3) + 1) - 1/27*log(abs((x^6 - 1)^(1/3) + 1))
```

maple [C] time = 0.27, size = 108, normalized size = 1.09

$$\frac{4x^{12}-x^6-3}{36x^{12}\left(x^6-1\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\mathrm{signum}\left(x^6-1\right)\right)^{\frac{1}{3}}\left(\frac{2\pi\sqrt{3}\,x^6\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^6\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+6\ln(x)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{54\pi\,\mathrm{signum}\left(x^6-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(x^6-1)^(1/3),x)
```

```
[Out] 1/36*(4*x^12-x^6-3)/x^12/(x^6-1)^(1/3)+1/54/Pi*3^(1/2)*GAMMA(2/3)/signum(x^
6-1)^(1/3)*(-signum(x^6-1))^(1/3)*(2/9*Pi*3^(1/2)/GAMMA(2/3)*x^6*hypergeom(
[1,1,4/3],[2,2],x^6)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+6*ln(x)+I*Pi)*Pi*3^(1/2
)/GAMMA(2/3))
```

maxima [A] time = 0.42, size = 93, normalized size = 0.94

$$\frac{1}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^6-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{4\left(x^6-1\right)^{\frac{5}{3}}+7\left(x^6-1\right)^{\frac{2}{3}}}{36\left(2\,x^6+\left(x^6-1\right)^2-1\right)}+\frac{1}{54}\log\left(\left(x^6-1\right)^{\frac{2}{3}}-\left(x^6-1\right)^{\frac{1}{3}}+1\right)-\frac{1}{27}\log\left(\left(x^6-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^6-1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3) - 1)) + 1/36*(4*(x^6 - 1
)^(5/3) + 7*(x^6 - 1)^(2/3))/(2*x^6 + (x^6 - 1)^2 - 1) + 1/54*log((x^6 - 1)
^(2/3) - (x^6 - 1)^(1/3) + 1) - 1/27*log((x^6 - 1)^(1/3) + 1)
```

mupad [B] time = 0.99, size = 116, normalized size = 1.17

$$\frac{\frac{7\left(x^6-1\right)^{2/3}}{36}+\frac{\left(x^6-1\right)^{5/3}}{9}}{\left(x^6-1\right)^2+2\,x^6-1}-\ln\left(9\left(-\frac{1}{54}+\frac{\sqrt{3}\,1\mathrm{i}}{54}\right)^2+\frac{\left(x^6-1\right)^{1/3}}{81}\right)\left(-\frac{1}{54}+\frac{\sqrt{3}\,1\mathrm{i}}{54}\right)+\ln\left(9\left(\frac{1}{54}+\frac{\sqrt{3}\,1\mathrm{i}}{54}\right)^2+\frac{\left(x^6-1\right)^{1/3}}{81}\right)\left(\frac{1}{54}+\frac{\sqrt{3}\,1\mathrm{i}}{54}\right)-\frac{\ln\left(\frac{\left(x^6-1\right)^{1/3}}{81}+\frac{1}{81}\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^13*(x^6 - 1)^(1/3)),x)
```

```
[Out] log(9*((3^(1/2)*1i)/54 + 1/54)^2 + (x^6 - 1)^(1/3)/81)*((3^(1/2)*1i)/54 + 1
/54) - log(9*((3^(1/2)*1i)/54 - 1/54)^2 + (x^6 - 1)^(1/3)/81)*((3^(1/2)*1i)
/54 - 1/54) - log((x^6 - 1)^(1/3)/81 + 1/81)/27 + ((7*(x^6 - 1)^(2/3))/36 +
 (x^6 - 1)^(5/3)/9)/((x^6 - 1)^2 + 2*x^6 - 1)
```

sympy [C] time = 1.60, size = 32, normalized size = 0.32

$$-\frac{\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3}, \frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^{6}}\right)}{6x^{14}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(x**6-1)**(1/3),x)
```

```
[Out] -gamma(7/3)*hyper((1/3, 7/3), (10/3,), exp_polar(2*I*pi)/x**6)/(6*x**14*gam
ma(10/3))
```

3.1227

$$\int \frac{\left(-1+2x^2\right)\left(-1+4x-4x^2+4x^4\right)}{\sqrt{\frac{1-2x^2}{1+2x^2}}\left(1+2x^2\right)\left(-1-8x+32x^2-40x^3+46x^4-64x^5+56x^6-32x^7+8x^8\right)}\,dx$$

Optimal. Leaf size=99

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\sqrt{\frac{1-2x^2}{2x^2+1}}}{x-1}\right)}{2\sqrt[4]{2}\,3^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\sqrt{\frac{1-2x^2}{2x^2+1}}}{x-1}\right)}{2\sqrt[4]{2}\,3^{3/4}}$$

Rubi [F] time = 8.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1+2x^2\right)\left(-1+4x-4x^2+4x^4\right)}{\sqrt{\frac{1-2x^2}{1+2x^2}}\left(1+2x^2\right)\left(-1-8x+32x^2-40x^3+46x^4-64x^5+56x^6-32x^7+8x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + 2*x^2)*(-1 + 4*x - 4*x^2 + 4*x^4))/(Sqrt[(1 - 2*x^2)/(1 + 2*x^2)
]*(1 + 2*x^2)*(-1 - 8*x + 32*x^2 - 40*x^3 + 46*x^4 - 64*x^5 + 56*x^6 - 32*x
^7 + 8*x^8)),x]
```

```
[Out] -((Sqrt[1 - 2*x^2]*Defer[Int][Sqrt[1 - 2*x^2]/(Sqrt[1 + 2*x^2]*(1 + 8*x - 3
2*x^2 + 40*x^3 - 46*x^4 + 64*x^5 - 56*x^6 + 32*x^7 - 8*x^8)), x])/(Sqrt[(1
- 2*x^2)/(1 + 2*x^2)]*Sqrt[1 + 2*x^2])) - (4*Sqrt[1 - 2*x^2]*Defer[Int][(x*
Sqrt[1 - 2*x^2])/(Sqrt[1 + 2*x^2]*(-1 - 8*x + 32*x^2 - 40*x^3 + 46*x^4 - 64
*x^5 + 56*x^6 - 32*x^7 + 8*x^8)), x])/(Sqrt[(1 - 2*x^2)/(1 + 2*x^2)]*Sqrt[1
 + 2*x^2]) + (4*Sqrt[1 - 2*x^2]*Defer[Int][(x^2*Sqrt[1 - 2*x^2])/(Sqrt[1 +
2*x^2]*(-1 - 8*x + 32*x^2 - 40*x^3 + 46*x^4 - 64*x^5 + 56*x^6 - 32*x^7 + 8*
x^8)), x])/(Sqrt[(1 - 2*x^2)/(1 + 2*x^2)]*Sqrt[1 + 2*x^2]) - (4*Sqrt[1 - 2*
x^2]*Defer[Int][(x^4*Sqrt[1 - 2*x^2])/(Sqrt[1 + 2*x^2]*(-1 - 8*x + 32*x^2 -
 40*x^3 + 46*x^4 - 64*x^5 + 56*x^6 - 32*x^7 + 8*x^8)), x])/(Sqrt[(1 - 2*x^2
)/(1 + 2*x^2)]*Sqrt[1 + 2*x^2])
```

Rubi steps

$$\int \frac{\left(-1+2x^2\right)\left(-1+4x-4x^2+4x^4\right)}{\sqrt{\frac{1-2x^2}{1+2x^2}}\left(1+2x^2\right)\left(-1-8x+32x^2-40x^3+46x^4-64x^5+56x^6-32x^7+8x^8\right)}\,dx=\frac{\sqrt{1-2x^2}\int\frac{\cdots}{\sqrt{1-2x^2}\,\sqrt{1\cdots}}}{\cdots}$$

$$=-\frac{\sqrt{1-2x^2}\int\frac{\cdots}{\sqrt{1+2x^2}\,(\cdots}}{\cdots}$$

$$=-\frac{\sqrt{1-2x^2}\int\left(\frac{\cdots}{\sqrt{1+2x^2}}\cdots\right.}{\cdots}$$

$$=-\frac{\sqrt{1-2x^2}\int\frac{\cdots}{\sqrt{1+2x^2}\,(\cdots}}{\cdots}$$

Mathematica [C] time = 60.49, size = 64755, normalized size = 654.09

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + 2*x^2)*(-1 + 4*x - 4*x^2 + 4*x^4))/(Sqrt[(1 - 2*x^2)/(1 +
2*x^2)]*(1 + 2*x^2)*(-1 - 8*x + 32*x^2 - 40*x^3 + 46*x^4 - 64*x^5 + 56*x^6
- 32*x^7 + 8*x^8)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.91, size = 99, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\sqrt{\frac{1-2x^2}{2x^2+1}}}{x-1}\right)}{2\sqrt[4]{2}\,3^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\sqrt{\frac{1-2x^2}{2x^2+1}}}{x-1}\right)}{2\sqrt[4]{2}\,3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + 2*x^2)*(-1 + 4*x - 4*x^2 + 4*x^4))/(Sqrt[(1 - 2*x
^2)/(1 + 2*x^2)]*(1 + 2*x^2)*(-1 - 8*x + 32*x^2 - 40*x^3 + 46*x^4 - 64*x^5
+ 56*x^6 - 32*x^7 + 8*x^8)),x]
```

```
[Out] ArcTan[((3/2)^(1/4)*Sqrt[(1 - 2*x^2)/(1 + 2*x^2)])/(-1 + x)]/(2*2^(1/4)*3^(
3/4)) - ArcTanh[((3/2)^(1/4)*Sqrt[(1 - 2*x^2)/(1 + 2*x^2)])/(-1 + x)]/(2*2^
(1/4)*3^(3/4))
```

fricas [B] time = 0.94, size = 708, normalized size = 7.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)*(4*x^4-4*x^2+4*x-1)/((-2*x^2+1)/(2*x^2+1))^(1/2)/(2*x^2
+1)/(8*x^8-32*x^7+56*x^6-64*x^5+46*x^4-40*x^3+32*x^2-8*x-1),x, algorithm="f
ricas")
```

```
[Out] 1/108*54^(3/4)*arctan(1/90*(sqrt(2)*(54^(3/4)*(8*x^8 - 32*x^7 + 48*x^6 - 48
*x^5 + 62*x^4 - 40*x^3 + 10*x^2 - 12*x + 7) + 3*54^(1/4)*(8*x^8 - 32*x^7 +
8*x^6 + 32*x^5 + 22*x^4 - 40*x^3 + 20*x^2 - 32*x + 17))*sqrt(7*sqrt(6) - 12
) + 20*(54^(3/4)*(4*x^7 - 12*x^6 + 16*x^5 - 16*x^4 + 13*x^3 - 7*x^2 + 3*x -
 1) - 9*54^(1/4)*(4*x^5 - 4*x^4 - x + 1))*sqrt(-(2*x^2 - 1)/(2*x^2 + 1)))/(
8*x^8 - 32*x^7 + 56*x^6 - 64*x^5 + 46*x^4 - 40*x^3 + 32*x^2 - 8*x - 1)) - 1
/432*54^(3/4)*log(-(54^(3/4)*(8*x^8 - 32*x^7 + 8*x^6 + 32*x^5 + 22*x^4 - 40
*x^3 + 20*x^2 - 32*x + 17) + 36*(24*x^7 - 72*x^6 + 84*x^5 - 84*x^4 + 78*x^3
 - 42*x^2 + sqrt(6)*(4*x^7 - 12*x^6 + 4*x^5 - 4*x^4 + 13*x^3 - 7*x^2 + 6*x
- 4) + 21*x - 9)*sqrt(-(2*x^2 - 1)/(2*x^2 + 1)) + 18*54^(1/4)*(8*x^8 - 32*x
^7 + 48*x^6 - 48*x^5 + 62*x^4 - 40*x^3 + 10*x^2 - 12*x + 7))/(8*x^8 - 32*x^
7 + 56*x^6 - 64*x^5 + 46*x^4 - 40*x^3 + 32*x^2 - 8*x - 1)) + 1/432*54^(3/4)
*log((54^(3/4)*(8*x^8 - 32*x^7 + 8*x^6 + 32*x^5 + 22*x^4 - 40*x^3 + 20*x^2
- 32*x + 17) - 36*(24*x^7 - 72*x^6 + 84*x^5 - 84*x^4 + 78*x^3 - 42*x^2 + sq
rt(6)*(4*x^7 - 12*x^6 + 4*x^5 - 4*x^4 + 13*x^3 - 7*x^2 + 6*x - 4) + 21*x -
9)*sqrt(-(2*x^2 - 1)/(2*x^2 + 1)) + 18*54^(1/4)*(8*x^8 - 32*x^7 + 48*x^6 -
48*x^5 + 62*x^4 - 40*x^3 + 10*x^2 - 12*x + 7))/(8*x^8 - 32*x^7 + 56*x^6 - 6
4*x^5 + 46*x^4 - 40*x^3 + 32*x^2 - 8*x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^4-4\,x^2+4\,x-1\right)\left(2\,x^2-1\right)}{\left(8\,x^8-32\,x^7+56\,x^6-64\,x^5+46\,x^4-40\,x^3+32\,x^2-8\,x-1\right)\left(2\,x^2+1\right)\sqrt{-\frac{2\,x^2-1}{2\,x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)*(4*x^4-4*x^2+4*x-1)/((-2*x^2+1)/(2*x^2+1))^(1/2)/(2*x^2
+1)/(8*x^8-32*x^7+56*x^6-64*x^5+46*x^4-40*x^3+32*x^2-8*x-1),x, algorithm="g
iac")
```

```
[Out] integrate((4*x^4 - 4*x^2 + 4*x - 1)*(2*x^2 - 1)/((8*x^8 - 32*x^7 + 56*x^6 -
 64*x^5 + 46*x^4 - 40*x^3 + 32*x^2 - 8*x - 1)*(2*x^2 + 1)*sqrt(-(2*x^2 - 1)
/(2*x^2 + 1))), x)
```

maple [C] time = 0.08, size = 867, normalized size = 8.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-1)*(4*x^4-4*x^2+4*x-1)/((-2*x^2+1)/(2*x^2+1))^(1/2)/(2*x^2+1)/(8
*x^8-32*x^7+56*x^6-64*x^5+46*x^4-40*x^3+32*x^2-8*x-1),x)
```

```
[Out] 1/24*(2*x^2-1)*sum((2*_alpha^3-2*_alpha^2+_alpha-1)*(8*2^(1/2)*(-4*_alpha^4
+1)^(1/2)*(-2*x^2+1)^(1/2)*(2*x^2+1)^(1/2)*EllipticPi(2^(1/2)*x,-32*_alpha^
7+132*_alpha^6-240*_alpha^5+284*_alpha^4-216*_alpha^3+183*_alpha^2-148*_alp
ha+48,1/2*(-2)^(1/2)*2^(1/2))*_alpha^7-32*2^(1/2)*(-4*_alpha^4+1)^(1/2)*(-2
*x^2+1)^(1/2)*(2*x^2+1)^(1/2)*EllipticPi(2^(1/2)*x,-32*_alpha^7+132*_alpha^
6-240*_alpha^5+284*_alpha^4-216*_alpha^3+183*_alpha^2-148*_alpha+48,1/2*(-2
)^(1/2)*2^(1/2))*_alpha^6+56*2^(1/2)*(-4*_alpha^4+1)^(1/2)*(-2*x^2+1)^(1/2)
*(2*x^2+1)^(1/2)*EllipticPi(2^(1/2)*x,-32*_alpha^7+132*_alpha^6-240*_alpha^
5+284*_alpha^4-216*_alpha^3+183*_alpha^2-148*_alpha+48,1/2*(-2)^(1/2)*2^(1/
2))*_alpha^5-64*2^(1/2)*(-4*_alpha^4+1)^(1/2)*(-2*x^2+1)^(1/2)*(2*x^2+1)^(1
/2)*EllipticPi(2^(1/2)*x,-32*_alpha^7+132*_alpha^6-240*_alpha^5+284*_alpha^
4-216*_alpha^3+183*_alpha^2-148*_alpha+48,1/2*(-2)^(1/2)*2^(1/2))*_alpha^4+
46*2^(1/2)*(-4*_alpha^4+1)^(1/2)*(-2*x^2+1)^(1/2)*(2*x^2+1)^(1/2)*EllipticP
i(2^(1/2)*x,-32*_alpha^7+132*_alpha^6-240*_alpha^5+284*_alpha^4-216*_alpha^
3+183*_alpha^2-148*_alpha+48,1/2*(-2)^(1/2)*2^(1/2))*_alpha^3-40*2^(1/2)*(-
4*_alpha^4+1)^(1/2)*(-2*x^2+1)^(1/2)*(2*x^2+1)^(1/2)*EllipticPi(2^(1/2)*x,-
32*_alpha^7+132*_alpha^6-240*_alpha^5+284*_alpha^4-216*_alpha^3+183*_alpha^
2-148*_alpha+48,1/2*(-2)^(1/2)*2^(1/2))*_alpha^2+32*2^(1/2)*(-4*_alpha^4+1)
^(1/2)*(-2*x^2+1)^(1/2)*(2*x^2+1)^(1/2)*EllipticPi(2^(1/2)*x,-32*_alpha^7+1
32*_alpha^6-240*_alpha^5+284*_alpha^4-216*_alpha^3+183*_alpha^2-148*_alpha+
48,1/2*(-2)^(1/2)*2^(1/2))*_alpha-8*2^(1/2)*(-2*x^2+1)^(1/2)*(2*x^2+1)^(1/2
)*EllipticPi(2^(1/2)*x,-32*_alpha^7+132*_alpha^6-240*_alpha^5+284*_alpha^4-
216*_alpha^3+183*_alpha^2-148*_alpha+48,1/2*(-2)^(1/2)*2^(1/2))*(-4*_alpha^
4+1)^(1/2)-arctanh(2*_alpha^2*(32*_alpha^7-132*_alpha^6+240*_alpha^5-284*_a
lpha^4+216*_alpha^3-183*_alpha^2+2*x^2+148*_alpha-48)/(-4*_alpha^4+1)^(1/2)
/(-4*x^4+1)^(1/2))*(-4*x^4+1)^(1/2))/(-4*_alpha^4+1)^(1/2)/(-4*x^4+1)^(1/2)
,_alpha=RootOf(8*_Z^8-32*_Z^7+56*_Z^6-64*_Z^5+46*_Z^4-40*_Z^3+32*_Z^2-8*_Z-
1))/(-(2*x^2-1)/(2*x^2+1))^(1/2)/(-(2*x^2+1)*(2*x^2-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^4-4\,x^2+4\,x-1\right)\left(2\,x^2-1\right)}{\left(8\,x^8-32\,x^7+56\,x^6-64\,x^5+46\,x^4-40\,x^3+32\,x^2-8\,x-1\right)\left(2\,x^2+1\right)\sqrt{-\frac{2\,x^2-1}{2\,x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)*(4*x^4-4*x^2+4*x-1)/((-2*x^2+1)/(2*x^2+1))^(1/2)/(2*x^2
+1)/(8*x^8-32*x^7+56*x^6-64*x^5+46*x^4-40*x^3+32*x^2-8*x-1),x, algorithm="m
axima")
```

```
[Out] integrate((4*x^4 - 4*x^2 + 4*x - 1)*(2*x^2 - 1)/((8*x^8 - 32*x^7 + 56*x^6 -
 64*x^5 + 46*x^4 - 40*x^3 + 32*x^2 - 8*x - 1)*(2*x^2 + 1)*sqrt(-(2*x^2 - 1)
/(2*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(2\,x^2-1\right)\,\left(4\,x^4-4\,x^2+4\,x-1\right)}{\left(2\,x^2+1\right)\,\sqrt{-\frac{2\,x^2-1}{2\,x^2+1}}\,\left(-8\,x^8+32\,x^7-56\,x^6+64\,x^5-46\,x^4+40\,x^3-32\,x^2+8\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*x^2 - 1)*(4*x - 4*x^2 + 4*x^4 - 1))/((2*x^2 + 1)*(-(2*x^2 - 1)/(2*
x^2 + 1))^(1/2)*(8*x - 32*x^2 + 40*x^3 - 46*x^4 + 64*x^5 - 56*x^6 + 32*x^7
- 8*x^8 + 1)),x)
```

```
[Out] int(-((2*x^2 - 1)*(4*x - 4*x^2 + 4*x^4 - 1))/((2*x^2 + 1)*(-(2*x^2 - 1)/(2*
x^2 + 1))^(1/2)*(8*x - 32*x^2 + 40*x^3 - 46*x^4 + 64*x^5 - 56*x^6 + 32*x^7
- 8*x^8 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-1)*(4*x**4-4*x**2+4*x-1)/((-2*x**2+1)/(2*x**2+1))**(1/2)/
(2*x**2+1)/(8*x**8-32*x**7+56*x**6-64*x**5+46*x**4-40*x**3+32*x**2-8*x-1),x
)
```

```
[Out] Timed out
```

3.1228 $\int \frac{1+kx^2}{\left(-1+ckx+kx^2\right)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}\,dx$

Optimal. Leaf size=100

$$\frac{2\sqrt{-c^2k^2+k^2-2k+1}\,\tan^{-1}\left(\frac{x\sqrt{-c^2k^2+k^2-2k+1}}{ckx+\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2-1}\right)}{(ck-k+1)(ck+k-1)}$$

Rubi [C] time = 4.73, antiderivative size = 697, normalized size of antiderivative = 6.97, number of steps used = 16, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {6719, 6728, 419, 2113, 537, 571, 93, 205}

$$-\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi\left(\frac{4k}{\left(c\sqrt{k}-\sqrt{kc^2+4}\right)^2};\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}-\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi\left(\frac{4k}{\left(\sqrt{k}c+\sqrt{kc^2+4}\right)^2};\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}-\frac{\sqrt{1-x^2}\left(\sqrt{c^2k+4}+c\sqrt{k}\right)\sqrt{1-k^2x^2}\,\tan^{-1}\left(\frac{\sqrt{k}\sqrt{1-x^2}\sqrt{-ck^{3/2}\sqrt{c^2k+4}-c^2k^2-2k+2}}{\sqrt{c\sqrt{k}\sqrt{c^2k+4}-\left(2-c^2\right)k+2}\sqrt{1-k^2x^2}}\right)}{\sqrt{c\sqrt{k}\sqrt{c^2k+4}-\left(2-c^2\right)k+2}\sqrt{-ck^{3/2}\sqrt{c^2k+4}-c^2k^2-2k+2}\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}-\frac{\sqrt{1-x^2}\left(c\sqrt{k}-\sqrt{c^2k+4}\right)\sqrt{1-k^2x^2}\,\tan^{-1}\left(\frac{\sqrt{k}\sqrt{1-x^2}\sqrt{ck^{3/2}\sqrt{c^2k+4}-c^2k^2-2k+2}}{\sqrt{c\left(-\sqrt{k}\right)\sqrt{c^2k+4}-\left(2-c^2\right)k+2}\sqrt{1-k^2x^2}}\right)}{\sqrt{c\left(-\sqrt{k}\right)\sqrt{c^2k+4}-\left(2-c^2\right)k+2}\sqrt{ck^{3/2}\sqrt{c^2k+4}-c^2k^2-2k+2}\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}+\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + k*x^2)/((-1 + c*k*x + k*x^2)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),x]
```

```
[Out] -(((c*Sqrt[k] + Sqrt[4 + c^2*k])*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*ArcTan[(Sq
rt[k]*Sqrt[2 - 2*k - c^2*k^2 - c*k^(3/2)*Sqrt[4 + c^2*k]]*Sqrt[1 - x^2])/(S
qrt[2 - (2 - c^2)*k + c*Sqrt[k]*Sqrt[4 + c^2*k]]*Sqrt[1 - k^2*x^2])])/(Sqrt
[2 - (2 - c^2)*k + c*Sqrt[k]*Sqrt[4 + c^2*k]]*Sqrt[2 - 2*k - c^2*k^2 - c*k^
(3/2)*Sqrt[4 + c^2*k]]*Sqrt[(1 - x^2)*(1 - k^2*x^2)])) - ((c*Sqrt[k] - Sqrt
[4 + c^2*k])*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*ArcTan[(Sqrt[k]*Sqrt[2 - 2*k -
 c^2*k^2 + c*k^(3/2)*Sqrt[4 + c^2*k]]*Sqrt[1 - x^2])/(Sqrt[2 - (2 - c^2)*k
- c*Sqrt[k]*Sqrt[4 + c^2*k]]*Sqrt[1 - k^2*x^2])])/(Sqrt[2 - (2 - c^2)*k - c
*Sqrt[k]*Sqrt[4 + c^2*k]]*Sqrt[2 - 2*k - c^2*k^2 + c*k^(3/2)*Sqrt[4 + c^2*k
]]*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) + (Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*Ellipt
icF[ArcSin[x], k^2])/Sqrt[(1 - x^2)*(1 - k^2*x^2)] - (Sqrt[1 - x^2]*Sqrt[1
- k^2*x^2]*EllipticPi[(4*k)/(c*Sqrt[k] - Sqrt[4 + c^2*k])^2, ArcSin[x], k^2
])/Sqrt[(1 - x^2)*(1 - k^2*x^2)] - (Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*Ellipti
cPi[(4*k)/(c*Sqrt[k] + Sqrt[4 + c^2*k])^2, ArcSin[x], k^2])/Sqrt[(1 - x^2)*
(1 - k^2*x^2)]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
```

```
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1+kx^2}{\left(-1+ckx+kx^2\right)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}\,dx = \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1+kx^2}{\sqrt{1-x^2}\left(-1+ckx+kx^2\right)\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int\left(\frac{1}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}} + \frac{2-ckx}{\sqrt{1-x^2}\left(-1+ckx+kx^2\right)\sqrt{1-k}\ldots}\right.}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\ldots\right.}{\sqrt{(1\ldots}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int\left(\ldots\right.}{\ldots}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(\sqrt{k}\left(c\sqrt{k}-\sqrt{4+c^2k}\right)\ldots\right.}{\ldots}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(2k^{3/2}\left(c\sqrt{k}-\sqrt{4+c^2k}\right)\ldots\right.}{\ldots}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(\frac{\ldots}{(c\ldots}\right.}{\sqrt{\left(1-x^2\right)(\ldots}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(\frac{\ldots}{(c\ldots}\right.}{\sqrt{\left(1-x^2\right)(\ldots}}$$

$$= -\frac{\left(c\sqrt{k}+\sqrt{4+c^2k}\right)\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\tan^{-1}\left(\frac{\sqrt{k}\,\sqrt{2-2k-\ldots}}{\sqrt{2-\left(2-c^2\right)\ldots}}\right.}{\sqrt{2-\left(2-c^2\right)k+c\sqrt{k}\,\sqrt{4+c^2k}}\,\sqrt{2-2k-c^2k^2-ck^{3/2}\sqrt{4+\ldots}}}$$

Mathematica [C] time = 7.49, size = 2045, normalized size = 20.45

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(1 + k*x^2)/((-1 + c*k*x + k*x^2)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),
x]
```

```
[Out] -((-4*k^(3/2)*Sqrt[-(k*(2 + (-2 + c^2)*k + c*Sqrt[k]*Sqrt[4 + c^2*k]))]*Sqr
t[-(k^2*(-2 + 2*k + c^2*k^2 + c*k^(3/2)*Sqrt[4 + c^2*k]))]*Sqrt[-1 + x^2]*S
qrt[-1 + k^2*x^2]*ArcTanh[(Sqrt[-(k^2*(-2 + 2*k + c^2*k^2 - c*k^(3/2)*Sqrt[
4 + c^2*k]))]*Sqrt[-1 + x^2])/(Sqrt[-(k*(2 + (-2 + c^2)*k - c*Sqrt[k]*Sqrt[
4 + c^2*k]))]*Sqrt[-1 + k^2*x^2])] - c^2*k^(5/2)*Sqrt[-(k*(2 + (-2 + c^2)*k
```

```
 + c*Sqrt[k]*Sqrt[4 + c^2*k]))]*Sqrt[-(k^2*(-2 + 2*k + c^2*k^2 + c*k^(3/2)*
Sqrt[4 + c^2*k]))]*Sqrt[-1 + x^2]*Sqrt[-1 + k^2*x^2]*ArcTanh[(Sqrt[-(k^2*(-
2 + 2*k + c^2*k^2 - c*k^(3/2)*Sqrt[4 + c^2*k]))]*Sqrt[-1 + x^2])/(Sqrt[-(k*
(2 + (-2 + c^2)*k - c*Sqrt[k]*Sqrt[4 + c^2*k]))]*Sqrt[-1 + k^2*x^2])] + c*k
^2*Sqrt[4 + c^2*k]*Sqrt[-(k*(2 + (-2 + c^2)*k + c*Sqrt[k]*Sqrt[4 + c^2*k]))
]*Sqrt[-(k^2*(-2 + 2*k + c^2*k^2 + c*k^(3/2)*Sqrt[4 + c^2*k]))]*Sqrt[-1 + x
^2]*Sqrt[-1 + k^2*x^2]*ArcTanh[(Sqrt[-(k^2*(-2 + 2*k + c^2*k^2 - c*k^(3/2)*
Sqrt[4 + c^2*k]))]*Sqrt[-1 + x^2])/(Sqrt[-(k*(2 + (-2 + c^2)*k - c*Sqrt[k]*
Sqrt[4 + c^2*k]))]*Sqrt[-1 + k^2*x^2])] + 4*k^(3/2)*Sqrt[-(k*(2 + (-2 + c^2
)*k - c*Sqrt[k]*Sqrt[4 + c^2*k]))]*Sqrt[-(k^2*(-2 + 2*k + c^2*k^2 - c*k^(3/
2)*Sqrt[4 + c^2*k]))]*Sqrt[-1 + x^2]*Sqrt[-1 + k^2*x^2]*ArcTanh[(Sqrt[-(k^2
*(-2 + 2*k + c^2*k^2 + c*k^(3/2)*Sqrt[4 + c^2*k]))]*Sqrt[-1 + x^2])/(Sqrt[-
(k*(2 + (-2 + c^2)*k + c*Sqrt[k]*Sqrt[4 + c^2*k]))]*Sqrt[-1 + k^2*x^2])] +
c^2*k^(5/2)*Sqrt[-(k*(2 + (-2 + c^2)*k - c*Sqrt[k]*Sqrt[4 + c^2*k]))]*Sqrt[
-(k^2*(-2 + 2*k + c^2*k^2 - c*k^(3/2)*Sqrt[4 + c^2*k]))]*Sqrt[-1 + x^2]*Sqr
t[-1 + k^2*x^2]*ArcTanh[(Sqrt[-(k^2*(-2 + 2*k + c^2*k^2 + c*k^(3/2)*Sqrt[4
+ c^2*k]))]*Sqrt[-1 + x^2])/(Sqrt[-(k*(2 + (-2 + c^2)*k + c*Sqrt[k]*Sqrt[4
+ c^2*k]))]*Sqrt[-1 + k^2*x^2])] + c*k^2*Sqrt[4 + c^2*k]*Sqrt[-(k*(2 + (-2
+ c^2)*k - c*Sqrt[k]*Sqrt[4 + c^2*k]))]*Sqrt[-(k^2*(-2 + 2*k + c^2*k^2 - c*
k^(3/2)*Sqrt[4 + c^2*k]))]*Sqrt[-1 + x^2]*Sqrt[-1 + k^2*x^2]*ArcTanh[(Sqrt[
-(k^2*(-2 + 2*k + c^2*k^2 + c*k^(3/2)*Sqrt[4 + c^2*k]))]*Sqrt[-1 + x^2])/(S
qrt[-(k*(2 + (-2 + c^2)*k + c*Sqrt[k]*Sqrt[4 + c^2*k]))]*Sqrt[-1 + k^2*x^2]
)] - Sqrt[4 + c^2*k]*Sqrt[-(k*(2 + (-2 + c^2)*k - c*Sqrt[k]*Sqrt[4 + c^2*k]
))]*Sqrt[-(k*(2 + (-2 + c^2)*k + c*Sqrt[k]*Sqrt[4 + c^2*k]))]*Sqrt[-(k^2*(-
2 + 2*k + c^2*k^2 - c*k^(3/2)*Sqrt[4 + c^2*k]))]*Sqrt[-(k^2*(-2 + 2*k + c^2
*k^2 + c*k^(3/2)*Sqrt[4 + c^2*k]))]*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*Ellipti
cF[ArcSin[x], k^2] + Sqrt[4 + c^2*k]*Sqrt[-(k*(2 + (-2 + c^2)*k - c*Sqrt[k]
*Sqrt[4 + c^2*k]))]*Sqrt[-(k*(2 + (-2 + c^2)*k + c*Sqrt[k]*Sqrt[4 + c^2*k])
)]*Sqrt[-(k^2*(-2 + 2*k + c^2*k^2 - c*k^(3/2)*Sqrt[4 + c^2*k]))]*Sqrt[-(k^2
*(-2 + 2*k + c^2*k^2 + c*k^(3/2)*Sqrt[4 + c^2*k]))]*Sqrt[1 - x^2]*Sqrt[1 -
k^2*x^2]*EllipticPi[(2*k)/(2 + c^2*k - c*Sqrt[k]*Sqrt[4 + c^2*k]), ArcSin[x
], k^2] + Sqrt[4 + c^2*k]*Sqrt[-(k*(2 + (-2 + c^2)*k - c*Sqrt[k]*Sqrt[4 + c
^2*k]))]*Sqrt[-(k*(2 + (-2 + c^2)*k + c*Sqrt[k]*Sqrt[4 + c^2*k]))]*Sqrt[-(k
^2*(-2 + 2*k + c^2*k^2 - c*k^(3/2)*Sqrt[4 + c^2*k]))]*Sqrt[-(k^2*(-2 + 2*k
+ c^2*k^2 + c*k^(3/2)*Sqrt[4 + c^2*k]))]*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*El
lipticPi[(2*k)/(2 + c^2*k + c*Sqrt[k]*Sqrt[4 + c^2*k]), ArcSin[x], k^2])/(S
qrt[4 + c^2*k]*Sqrt[-(k*(2 + (-2 + c^2)*k - c*Sqrt[k]*Sqrt[4 + c^2*k]))]*Sq
rt[-(k*(2 + (-2 + c^2)*k + c*Sqrt[k]*Sqrt[4 + c^2*k]))]*Sqrt[-(k^2*(-2 + 2*
k + c^2*k^2 - c*k^(3/2)*Sqrt[4 + c^2*k]))]*Sqrt[-(k^2*(-2 + 2*k + c^2*k^2 +
 c*k^(3/2)*Sqrt[4 + c^2*k]))]*Sqrt[(-1 + x^2)*(-1 + k^2*x^2)]))
```

IntegrateAlgebraic [A] time = 4.10, size = 100, normalized size = 1.00

$$\frac{2\sqrt{-c^2k^2+k^2-2k+1}\,\tan^{-1}\left(\frac{x\sqrt{-c^2k^2+k^2-2k+1}}{ckx+\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2-1}\right)}{(ck-k+1)(ck+k-1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + k*x^2)/((-1 + c*k*x + k*x^2)*Sqrt[(1 - x^2)*(1 - k^
2*x^2)]),x]
```

```
[Out] (2*Sqrt[1 - 2*k + k^2 - c^2*k^2]*ArcTan[(Sqrt[1 - 2*k + k^2 - c^2*k^2]*x)/(
-1 + c*k*x + k*x^2 + Sqrt[1 + (-1 - k^2)*x^2 + k^2*x^4])])/((1 - k + c*k)*(
-1 + k + c*k))
```

fricas [A] time = 0.90, size = 344, normalized size = 3.44

$$\left[\frac{\log\left(-\frac{\left(\left(2c^2-1\right)k^4+2k^3-k^2\right)x^4+2\left(ck^4-2ck^3+ck^2\right)x^3+\left(2c^2-1\right)k^2-\left(\left(c^2-2\right)k^4+2\left(c^2+3\right)k^3+\left(c^2-8\right)k^2+6k-2\right)x^2+2\sqrt{k^2x^4-\left(k^2+1\right)x^2+1}\left(ck^2x^2-ck+\left(k^2-2k+1\right)x\right)\sqrt{\left(c^2-1\right)k^2+2k-1}-2\left(ck^3-2ck^2+ck\right)x+2k-1}{2ck^2x^3+k^2x^4-2ckx+\left(c^2k^2-2k\right)x^2+1}\right)}{2\sqrt{\left(c^2-1\right)k^2+2k-1}}, -\frac{\sqrt{-\left(c^2-1\right)k^2-2k+1}\arctan\left(\frac{\sqrt{k^2x^4-\left(k^2+1\right)x^2+1}\sqrt{-\left(c^2-1\right)k^2-2k+1}}{ck^2x^2-ck+\left(k^2-2k+1\right)x}\right)}{\left(c^2-1\right)k^2+2k-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^2+1)/(c*k*x+k*x^2-1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algori
thm="fricas")
```

```
[Out] [1/2*log(-(((2*c^2 - 1)*k^4 + 2*k^3 - k^2)*x^4 + 2*(c*k^4 - 2*c*k^3 + c*k^2
)*x^3 + (2*c^2 - 1)*k^2 - ((c^2 - 2)*k^4 + 2*(c^2 + 3)*k^3 + (c^2 - 8)*k^2
+ 6*k - 2)*x^2 + 2*sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)*(c*k^2*x^2 - c*k + (k^
2 - 2*k + 1)*x)*sqrt((c^2 - 1)*k^2 + 2*k - 1) - 2*(c*k^3 - 2*c*k^2 + c*k)*x
 + 2*k - 1)/(2*c*k^2*x^3 + k^2*x^4 - 2*c*k*x + (c^2*k^2 - 2*k)*x^2 + 1))/sq
rt((c^2 - 1)*k^2 + 2*k - 1), -sqrt(-(c^2 - 1)*k^2 - 2*k + 1)*arctan(sqrt(k^
2*x^4 - (k^2 + 1)*x^2 + 1)*sqrt(-(c^2 - 1)*k^2 - 2*k + 1)/(c*k^2*x^2 - c*k
+ (k^2 - 2*k + 1)*x))/((c^2 - 1)*k^2 + 2*k - 1)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx^2+1}{\left(ckx+kx^2-1\right)\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^2+1)/(c*k*x+k*x^2-1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((k*x^2 + 1)/((c*k*x + k*x^2 - 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))),
x)
```

maple [C] time = 0.09, size = 11680, normalized size = 116.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^2+1)/(c*k*x+k*x^2-1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx^2+1}{\left(ckx+kx^2-1\right)\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^2+1)/(c*k*x+k*x^2-1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((k*x^2 + 1)/((c*k*x + k*x^2 - 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{k\,x^2+1}{\sqrt{\left(x^2-1\right)\,\left(k^2\,x^2-1\right)}\,\left(k\,x^2+c\,k\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^2 + 1)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/2)*(k*x^2 + c*k*x - 1)),x)
```

```
[Out] int((k*x^2 + 1)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/2)*(k*x^2 + c*k*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx^2+1}{\sqrt{(x-1)(x+1)(kx-1)(kx+1)}\,\left(ckx+kx^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x**2+1)/(c*k*x+k*x**2-1)/((-x**2+1)*(-k**2*x**2+1))**(1/2),x)
```

```
[Out] Integral((k*x**2 + 1)/(sqrt((x - 1)*(x + 1)*(k*x - 1)*(k*x + 1))*(c*k*x + k
*x**2 - 1)), x)
```

3.1229 $\int \frac{-1+kx^2}{(1+ckx+kx^2)\sqrt{(1-x^2)(1-k^2x^2)}}\,dx$

Optimal. Leaf size=100

$$\frac{2\sqrt{-c^2k^2+k^2+2k+1}\tan^{-1}\left(\frac{x\sqrt{-c^2k^2+k^2+2k+1}}{ckx+\sqrt{k^2x^4+(-k^2-1)x^2+1}+kx^2+1}\right)}{(ck-k-1)(ck+k+1)}$$

Rubi [C] time = 5.12, antiderivative size = 691, normalized size of antiderivative = 6.91, number of steps used = 16, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {6719, 6728, 419, 2113, 537, 571, 93, 208}

$$-\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi\left(\frac{4k}{\left(c\sqrt{k}-\sqrt{c^2k-4}\right)^2};\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}-\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi\left(\frac{4k}{\left(\sqrt{k}c+\sqrt{c^2k-4}\right)^2};\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}+\frac{\sqrt{1-x^2}\left(\sqrt{c^2k-4}+c\sqrt{k}\right)\sqrt{1-k^2x^2}\tanh^{-1}\left(\frac{\sqrt{k}\sqrt{1-x^2}\sqrt{-ck^{3/2}\sqrt{c^2k-4}-c^2k^2+2k+2}}{\sqrt{c(-\sqrt{k})\sqrt{c^2k-4}+(2-c^2)k+2}\sqrt{1-k^2x^2}}\right)}{\sqrt{c(-\sqrt{k})\sqrt{c^2k-4}+(2-c^2)k+2}\sqrt{-ck^{3/2}\sqrt{c^2k-4}-c^2k^2+2k+2}\sqrt{(1-x^2)(1-k^2x^2)}}+\frac{\sqrt{1-x^2}\left(c\sqrt{k}-\sqrt{c^2k-4}\right)\sqrt{1-k^2x^2}\tanh^{-1}\left(\frac{\sqrt{k}\sqrt{1-x^2}\sqrt{ck^{3/2}\sqrt{c^2k-4}-c^2k^2+2k+2}}{\sqrt{c\sqrt{k}\sqrt{c^2k-4}+(2-c^2)k+2}\sqrt{1-k^2x^2}}\right)}{\sqrt{c\sqrt{k}\sqrt{c^2k-4}+(2-c^2)k+2}\sqrt{ck^{3/2}\sqrt{c^2k-4}-c^2k^2+2k+2}\sqrt{(1-x^2)(1-k^2x^2)}}+\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + k*x^2)/((1 + c*k*x + k*x^2)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),x]
```

```
[Out] ((c*Sqrt[k] + Sqrt[-4 + c^2*k])*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*ArcTanh[(Sq
rt[k]*Sqrt[2 + 2*k - c^2*k^2 - c*k^(3/2)*Sqrt[-4 + c^2*k]]*Sqrt[1 - x^2])/(
Sqrt[2 + (2 - c^2)*k - c*Sqrt[k]*Sqrt[-4 + c^2*k]]*Sqrt[1 - k^2*x^2])])/(Sq
rt[2 + (2 - c^2)*k - c*Sqrt[k]*Sqrt[-4 + c^2*k]]*Sqrt[2 + 2*k - c^2*k^2 - c
*k^(3/2)*Sqrt[-4 + c^2*k]]*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) + ((c*Sqrt[k] - S
qrt[-4 + c^2*k])*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*ArcTanh[(Sqrt[k]*Sqrt[2 +
2*k - c^2*k^2 + c*k^(3/2)*Sqrt[-4 + c^2*k]]*Sqrt[1 - x^2])/(Sqrt[2 + (2 - c
^2)*k + c*Sqrt[k]*Sqrt[-4 + c^2*k]]*Sqrt[1 - k^2*x^2])])/(Sqrt[2 + (2 - c^2
)*k + c*Sqrt[k]*Sqrt[-4 + c^2*k]]*Sqrt[2 + 2*k - c^2*k^2 + c*k^(3/2)*Sqrt[-
4 + c^2*k]]*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) + (Sqrt[1 - x^2]*Sqrt[1 - k^2*x^
2]*EllipticF[ArcSin[x], k^2])/Sqrt[(1 - x^2)*(1 - k^2*x^2)] - (Sqrt[1 - x^2
]*Sqrt[1 - k^2*x^2]*EllipticPi[(4*k)/(c*Sqrt[k] - Sqrt[-4 + c^2*k])^2, ArcS
in[x], k^2])/Sqrt[(1 - x^2)*(1 - k^2*x^2)] - (Sqrt[1 - x^2]*Sqrt[1 - k^2*x^
2]*EllipticPi[(4*k)/(c*Sqrt[k] + Sqrt[-4 + c^2*k])^2, ArcSin[x], k^2])/Sqrt
[(1 - x^2)*(1 - k^2*x^2)]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
```

```
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-1+kx^2}{\left(1+ckx+kx^2\right)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}\,dx = \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{-1+kx^2}{\sqrt{1-x^2}\left(1+ckx+kx^2\right)\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \left(\frac{1}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}} - \frac{2+ckx}{\sqrt{1-x^2}\left(1+ckx+kx^2\right)\sqrt{1-k^2x^2}}\right.}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right.}{\sqrt{(1-}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int\left(\frac{}{(c}\right.}{}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(\sqrt{k}\left(c\sqrt{k}-\sqrt{-4+c^2k}\right)\right.}{}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(2k^{3/2}\left(c\sqrt{k}-\sqrt{-4+c^2k}\right.\right.}{}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(\frac{}{\left(c\sqrt{k}\right.}\right.}{\sqrt{\left(1-x^2\right)(}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(\frac{}{\left(c\sqrt{k}\right.}\right.}{\sqrt{\left(1-x^2\right)(}}$$

$$= \frac{\left(c\sqrt{k}+\sqrt{-4+c^2k}\right)\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{k}\,\sqrt{2+2k-c}}{\sqrt{2+\left(2-c^2\right)}}\right.}{\sqrt{2+\left(2-c^2\right)k-c\sqrt{k}\,\sqrt{-4+c^2k}}\,\sqrt{2+2k-c^2k^2-ck^{3/2}\sqrt{-4+}}}$$

Mathematica [C] time = 8.33, size = 2156, normalized size = 21.56

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + k*x^2)/((1 + c*k*x + k*x^2)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),
x]

[Out] -((k^(3/2)*Sqrt[-1 + x^2]*Sqrt[-1 + k^2*x^2]*(((c*Sqrt[k] - Sqrt[-4 + c^2*k
])*ArcTanh[(Sqrt[k^2*(2 + 2*k - c^2*k^2 + c*k^(3/2)*Sqrt[-4 + c^2*k])]*Sqrt
[-1 + x^2])/(Sqrt[k*(2 - (-2 + c^2)*k + c*Sqrt[k]*Sqrt[-4 + c^2*k])]*Sqrt[-
1 + k^2*x^2])])/(Sqrt[k*(2 - (-2 + c^2)*k + c*Sqrt[k]*Sqrt[-4 + c^2*k])]*Sq
rt[k^2*(2 + 2*k - c^2*k^2 + c*k^(3/2)*Sqrt[-4 + c^2*k])]) + ((c*Sqrt[k] + S
```

```
qrt[-4 + c^2*k])*ArcTanh[(Sqrt[-(k^2*(-2 - 2*k + c^2*k^2 + c*k^(3/2)*Sqrt[-
4 + c^2*k]))]*Sqrt[-1 + x^2])/(Sqrt[-(k*(-2 + (-2 + c^2)*k + c*Sqrt[k]*Sqrt
[-4 + c^2*k]))]*Sqrt[-1 + k^2*x^2])])/(Sqrt[-(k*(-2 + (-2 + c^2)*k + c*Sqrt
[k]*Sqrt[-4 + c^2*k]))]*Sqrt[-(k^2*(-2 - 2*k + c^2*k^2 + c*k^(3/2)*Sqrt[-4
+ c^2*k]))]))))/Sqrt[(-1 + x^2)*(-1 + k^2*x^2)]) + (Sqrt[1 - x^2]*Sqrt[1 - k
^2*x^2]*EllipticF[ArcSin[x], k^2])/Sqrt[(-1 + x^2)*(-1 + k^2*x^2)] - (4*c^2
*k*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[(-2*k^2)/(2*k - c^2*k^2 - c*k
^(3/2)*Sqrt[-4 + c^2*k]), ArcSin[x], k^2])/((2*k - c^2*k^2 - c*k^(3/2)*Sqrt
[-4 + c^2*k])*(-1/2*(-2*k + c^2*k^2 - c*k^(3/2)*Sqrt[-4 + c^2*k])/k^2 + (-2
*k + c^2*k^2 + c*k^(3/2)*Sqrt[-4 + c^2*k])/(2*k^2))*Sqrt[(-1 + x^2)*(-1 + k
^2*x^2)]) + (c^4*k^2*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[(-2*k^2)/(2
*k - c^2*k^2 - c*k^(3/2)*Sqrt[-4 + c^2*k]), ArcSin[x], k^2])/((2*k - c^2*k^
2 - c*k^(3/2)*Sqrt[-4 + c^2*k])*(-1/2*(-2*k + c^2*k^2 - c*k^(3/2)*Sqrt[-4 +
 c^2*k])/k^2 + (-2*k + c^2*k^2 + c*k^(3/2)*Sqrt[-4 + c^2*k])/(2*k^2))*Sqrt[
(-1 + x^2)*(-1 + k^2*x^2)]) - (2*c*Sqrt[k]*Sqrt[-4 + c^2*k]*Sqrt[1 - x^2]*S
qrt[1 - k^2*x^2]*EllipticPi[(-2*k^2)/(2*k - c^2*k^2 - c*k^(3/2)*Sqrt[-4 + c
^2*k]), ArcSin[x], k^2])/((2*k - c^2*k^2 - c*k^(3/2)*Sqrt[-4 + c^2*k])*(-1/
2*(-2*k + c^2*k^2 - c*k^(3/2)*Sqrt[-4 + c^2*k])/k^2 + (-2*k + c^2*k^2 + c*k
^(3/2)*Sqrt[-4 + c^2*k])/(2*k^2))*Sqrt[(-1 + x^2)*(-1 + k^2*x^2)]) + (c^3*k
^(3/2)*Sqrt[-4 + c^2*k]*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[(-2*k^2)
/(2*k - c^2*k^2 - c*k^(3/2)*Sqrt[-4 + c^2*k]), ArcSin[x], k^2])/((2*k - c^2
*k^2 - c*k^(3/2)*Sqrt[-4 + c^2*k])*(-1/2*(-2*k + c^2*k^2 - c*k^(3/2)*Sqrt[-
4 + c^2*k])/k^2 + (-2*k + c^2*k^2 + c*k^(3/2)*Sqrt[-4 + c^2*k])/(2*k^2))*Sq
rt[(-1 + x^2)*(-1 + k^2*x^2)]) - (4*c^2*k*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*E
llipticPi[(-2*k^2)/(2*k - c^2*k^2 + c*k^(3/2)*Sqrt[-4 + c^2*k]), ArcSin[x],
 k^2])/((2*k - c^2*k^2 + c*k^(3/2)*Sqrt[-4 + c^2*k])*((-2*k + c^2*k^2 - c*k
^(3/2)*Sqrt[-4 + c^2*k])/(2*k^2) - (-2*k + c^2*k^2 + c*k^(3/2)*Sqrt[-4 + c^
2*k])/(2*k^2))*Sqrt[(-1 + x^2)*(-1 + k^2*x^2)]) + (c^4*k^2*Sqrt[1 - x^2]*Sq
rt[1 - k^2*x^2]*EllipticPi[(-2*k^2)/(2*k - c^2*k^2 + c*k^(3/2)*Sqrt[-4 + c^
2*k]), ArcSin[x], k^2])/((2*k - c^2*k^2 + c*k^(3/2)*Sqrt[-4 + c^2*k])*((-2*
k + c^2*k^2 - c*k^(3/2)*Sqrt[-4 + c^2*k])/(2*k^2) - (-2*k + c^2*k^2 + c*k^(
3/2)*Sqrt[-4 + c^2*k])/(2*k^2))*Sqrt[(-1 + x^2)*(-1 + k^2*x^2)]) + (2*c*Sqr
t[k]*Sqrt[-4 + c^2*k]*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[(-2*k^2)/(
2*k - c^2*k^2 + c*k^(3/2)*Sqrt[-4 + c^2*k]), ArcSin[x], k^2])/((2*k - c^2*k
^2 + c*k^(3/2)*Sqrt[-4 + c^2*k])*((-2*k + c^2*k^2 - c*k^(3/2)*Sqrt[-4 + c^2
*k])/(2*k^2) - (-2*k + c^2*k^2 + c*k^(3/2)*Sqrt[-4 + c^2*k])/(2*k^2))*Sqrt[
(-1 + x^2)*(-1 + k^2*x^2)]) - (c^3*k^(3/2)*Sqrt[-4 + c^2*k]*Sqrt[1 - x^2]*S
qrt[1 - k^2*x^2]*EllipticPi[(-2*k^2)/(2*k - c^2*k^2 + c*k^(3/2)*Sqrt[-4 + c
^2*k]), ArcSin[x], k^2])/((2*k - c^2*k^2 + c*k^(3/2)*Sqrt[-4 + c^2*k])*((-2
*k + c^2*k^2 - c*k^(3/2)*Sqrt[-4 + c^2*k])/(2*k^2) - (-2*k + c^2*k^2 + c*k^
(3/2)*Sqrt[-4 + c^2*k])/(2*k^2))*Sqrt[(-1 + x^2)*(-1 + k^2*x^2)])
```

IntegrateAlgebraic [A] time = 3.72, size = 100, normalized size = 1.00

$$\frac{2\sqrt{-c^2k^2+k^2+2k+1}\,\tan^{-1}\left(\frac{x\sqrt{-c^2k^2+k^2+2k+1}}{ckx+\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2+1}\right)}{(ck-k-1)(ck+k+1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + k*x^2)/((1 + c*k*x + k*x^2)*Sqrt[(1 - x^2)*(1 - k^
2*x^2)]),x]
```

```
[Out] (2*Sqrt[1 + 2*k + k^2 - c^2*k^2]*ArcTan[(Sqrt[1 + 2*k + k^2 - c^2*k^2]*x)/(
1 + c*k*x + k*x^2 + Sqrt[1 + (-1 - k^2)*x^2 + k^2*x^4]])])/((-1 - k + c*k)*(
1 + k + c*k))
```

fricas [A] time = 0.91, size = 342, normalized size = 3.42

$$\left[\frac{\log\left(-\frac{\left(\left(2c^2-1\right)k^4-2k^3-k^2\right)x^4+2\left(ck^4+2ck^3+ck^2\right)x^3+\left(2c^2-1\right)k^2-\left(\left(c^2-2\right)k^4-2\left(c^2+3\right)k^3+\left(c^2-8\right)k^2-6k-2\right)x^2+2\sqrt{k^2x^4-\left(k^2+1\right)x^2+1}\left(ck^2x^2+ck+\left(k^2+2k+1\right)x\right)\sqrt{\left(c^2-1\right)k^2-2k-1}+2\left(ck^3+2ck^2+ck\right)x-2k-1}{2ck^2x^3+k^2x^4+2ckx+\left(c^2k^2+2k\right)x^2+1}\right)}{2\sqrt{\left(c^2-1\right)k^2-2k-1}},\ -\frac{\sqrt{-\left(c^2-1\right)k^2+2k+1}\,\arctan\left(\frac{\sqrt{k^2x^4-\left(k^2+1\right)x^2+1}\sqrt{-\left(c^2-1\right)k^2+2k+1}}{ck^2x^2+ck+\left(k^2+2k+1\right)x}\right)}{\left(c^2-1\right)k^2-2k-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^2-1)/(c*k*x+k*x^2+1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algori
thm="fricas")
```

```
[Out] [1/2*log(-(((2*c^2 - 1)*k^4 - 2*k^3 - k^2)*x^4 + 2*(c*k^4 + 2*c*k^3 + c*k^2
)*x^3 + (2*c^2 - 1)*k^2 - ((c^2 - 2)*k^4 - 2*(c^2 + 3)*k^3 + (c^2 - 8)*k^2
- 6*k - 2)*x^2 + 2*sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)*(c*k^2*x^2 + c*k + (k^
2 + 2*k + 1)*x)*sqrt((c^2 - 1)*k^2 - 2*k - 1) + 2*(c*k^3 + 2*c*k^2 + c*k)*x
 - 2*k - 1)/(2*c*k^2*x^3 + k^2*x^4 + 2*c*k*x + (c^2*k^2 + 2*k)*x^2 + 1))/sq
rt((c^2 - 1)*k^2 - 2*k - 1), -sqrt(-(c^2 - 1)*k^2 + 2*k + 1)*arctan(sqrt(k^
2*x^4 - (k^2 + 1)*x^2 + 1)*sqrt(-(c^2 - 1)*k^2 + 2*k + 1)/(c*k^2*x^2 + c*k
+ (k^2 + 2*k + 1)*x))/((c^2 - 1)*k^2 - 2*k - 1)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx^2-1}{(ckx+kx^2+1)\sqrt{(k^2x^2-1)(x^2-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^2-1)/(c*k*x+k*x^2+1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((k*x^2 - 1)/((c*k*x + k*x^2 + 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))),
x)
```

maple [C] time = 0.09, size = 11374, normalized size = 113.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^2-1)/(c*k*x+k*x^2+1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^2-1)/(c*k*x+k*x^2+1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*k-4>0)', see `assume?` for more
 details)Is c^2*k-4 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{k\,x^2-1}{\sqrt{\left(x^2-1\right)\,\left(k^2\,x^2-1\right)}\,\left(k\,x^2+c\,k\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^2 - 1)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/2)*(k*x^2 + c*k*x + 1)),x)
```

```
[Out] int((k*x^2 - 1)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/2)*(k*x^2 + c*k*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx^2-1}{\sqrt{(x-1)(x+1)(kx-1)(kx+1)}\left(ckx+kx^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x**2-1)/(c*k*x+k*x**2+1)/((-x**2+1)*(-k**2*x**2+1))**(1/2),x)
```

```
[Out] Integral((k*x**2 - 1)/(sqrt((x - 1)*(x + 1)*(k*x - 1)*(k*x + 1))*(c*k*x + k
*x**2 + 1)), x)
```

3.1230 $\int x^2 \sqrt[3]{x+x^3}\, dx$

Optimal. Leaf size=100

$$\frac{1}{12}\sqrt[3]{x^3+x}\left(3x^3+x\right)+\frac{1}{18}\log\left(\sqrt[3]{x^3+x}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x}+x}\right)}{6\sqrt{3}}-\frac{1}{36}\log\left(\sqrt[3]{x^3+x}\,x+\left(x^3+x\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.18, antiderivative size = 194, normalized size of antiderivative = 1.94, number of steps used = 12, number of rules used = 12, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {2021, 2024, 2032, 329, 275, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{4}\sqrt[3]{x^3+x}\,x^3+\frac{1}{12}\sqrt[3]{x^3+x}\,x+\frac{\left(x^2+1\right)^{2/3}x^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2+1}}\right)}{18\left(x^3+x\right)^{2/3}}-\frac{\left(x^2+1\right)^{2/3}x^{2/3}\log\left(\frac{x^{4/3}}{\left(x^2+1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2+1}}+1\right)}{36\left(x^3+x\right)^{2/3}}+\frac{\left(x^2+1\right)^{2/3}x^{2/3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)}{6\sqrt{3}\left(x^3+x\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(x + x^3)^(1/3),x]
```

```
[Out] (x*(x + x^3)^(1/3))/12 + (x^3*(x + x^3)^(1/3))/4 + (x^(2/3)*(1 + x^2)^(2/3)
*ArcTan[(1 + (2*x^(2/3))/(1 + x^2)^(1/3))/Sqrt[3]])/(6*Sqrt[3]*(x + x^3)^(2
/3)) + (x^(2/3)*(1 + x^2)^(2/3)*Log[1 - x^(2/3)/(1 + x^2)^(1/3)])/(18*(x +
x^3)^(2/3)) - (x^(2/3)*(1 + x^2)^(2/3)*Log[1 + x^(4/3)/(1 + x^2)^(2/3) + x^
(2/3)/(1 + x^2)^(1/3)])/(36*(x + x^3)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^2\sqrt[3]{x+x^3}\,dx = \frac{1}{4}x^3\sqrt[3]{x+x^3}+\frac{1}{6}\int\frac{x^3}{(x+x^3)^{2/3}}\,dx$$

$$=\frac{1}{12}x\sqrt[3]{x+x^3}+\frac{1}{4}x^3\sqrt[3]{x+x^3}-\frac{1}{9}\int\frac{x}{(x+x^3)^{2/3}}\,dx$$

$$=\frac{1}{12}x\sqrt[3]{x+x^3}+\frac{1}{4}x^3\sqrt[3]{x+x^3}-\frac{\left(x^{2/3}(1+x^2)^{2/3}\right)\int\frac{\sqrt[3]{x}}{(1+x^2)^{2/3}}\,dx}{9(x+x^3)^{2/3}}$$

$$=\frac{1}{12}x\sqrt[3]{x+x^3}+\frac{1}{4}x^3\sqrt[3]{x+x^3}-\frac{\left(x^{2/3}(1+x^2)^{2/3}\right)\text{Subst}\left(\int\frac{x^3}{(1+x^6)^{2/3}}\,dx,x,\sqrt[3]{x}\right)}{3(x+x^3)^{2/3}}$$

$$=\frac{1}{12}x\sqrt[3]{x+x^3}+\frac{1}{4}x^3\sqrt[3]{x+x^3}-\frac{\left(x^{2/3}(1+x^2)^{2/3}\right)\text{Subst}\left(\int\frac{x}{(1+x^3)^{2/3}}\,dx,x,x^{2/3}\right)}{6(x+x^3)^{2/3}}$$

$$=\frac{1}{12}x\sqrt[3]{x+x^3}+\frac{1}{4}x^3\sqrt[3]{x+x^3}-\frac{\left(x^{2/3}(1+x^2)^{2/3}\right)\text{Subst}\left(\int\frac{x}{1-x^3}\,dx,x,\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{6(x+x^3)^{2/3}}$$

$$=\frac{1}{12}x\sqrt[3]{x+x^3}+\frac{1}{4}x^3\sqrt[3]{x+x^3}-\frac{\left(x^{2/3}(1+x^2)^{2/3}\right)\text{Subst}\left(\int\frac{1}{1-x}\,dx,x,\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{18(x+x^3)^{2/3}}+\frac{\left(x^{2/3}(1+x^2\right.}{}$$

$$=\frac{1}{12}x\sqrt[3]{x+x^3}+\frac{1}{4}x^3\sqrt[3]{x+x^3}+\frac{x^{2/3}(1+x^2)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{18(x+x^3)^{2/3}}-\frac{\left(x^{2/3}(1+x^2)^{2/3}\right)\text{Subst}\left(\right.}{36(x+}$$

$$=\frac{1}{12}x\sqrt[3]{x+x^3}+\frac{1}{4}x^3\sqrt[3]{x+x^3}+\frac{x^{2/3}(1+x^2)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{18(x+x^3)^{2/3}}-\frac{x^{2/3}(1+x^2)^{2/3}\log\left(1+\right.}{36(x+x^3}$$

$$=\frac{1}{12}x\sqrt[3]{x+x^3}+\frac{1}{4}x^3\sqrt[3]{x+x^3}+\frac{x^{2/3}(1+x^2)^{2/3}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{6\sqrt{3}(x+x^3)^{2/3}}+\frac{x^{2/3}(1+x^2)^{2/3}\log\left(1-\right.}{18(x+x^3)^{2/3}}$$

Mathematica [C] time = 0.02, size = 50, normalized size = 0.50

$$\frac{x\sqrt[3]{x^3+x}\left((x^2+1)^{4/3}-{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-x^2\right)\right)}{4\sqrt[3]{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(x + x^3)^(1/3),x]
```

```
[Out] (x*(x + x^3)^(1/3)*((1 + x^2)^(4/3) - Hypergeometric2F1[-1/3, 2/3, 5/3, -x^
2]))/(4*(1 + x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.24, size = 100, normalized size = 1.00

$$\frac{1}{12}\sqrt[3]{x^3+x}\,(3x^3+x)+\frac{1}{18}\log\left(\sqrt[3]{x^3+x}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x}+x}\right)}{6\sqrt{3}}-\frac{1}{36}\log\left(\sqrt[3]{x^3+x}\,x+(x^3+x)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(x + x^3)^(1/3),x]
```

```
[Out] ((x + x^3)^(1/3)*(x + 3*x^3))/12 + ArcTan[(Sqrt[3]*x)/(x + 2*(x + x^3)^(1/3
))]/(6*Sqrt[3]) + Log[-x + (x + x^3)^(1/3)]/18 - Log[x^2 + x*(x + x^3)^(1/3
) + (x + x^3)^(2/3)]/36
```

fricas [A] time = 0.58, size = 96, normalized size = 0.96

$$\frac{1}{18}\sqrt{3}\arctan\left(-\frac{196\sqrt{3}\left(x^3+x\right)^{\frac{1}{3}}x-\sqrt{3}\left(539x^2+507\right)-1274\sqrt{3}\left(x^3+x\right)^{\frac{2}{3}}}{2205x^2+2197}\right)+\frac{1}{12}\left(3x^3+x\right)\left(x^3+x\right)^{\frac{1}{3}}+\frac{1}{36}\log\left(3\left(x^3+x\right)^{\frac{1}{3}}x-3\left(x^3+x\right)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+x)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*arctan(-(196*sqrt(3)*(x^3 + x)^(1/3)*x - sqrt(3)*(539*x^2 + 50
7) - 1274*sqrt(3)*(x^3 + x)^(2/3))/(2205*x^2 + 2197)) + 1/12*(3*x^3 + x)*(x
^3 + x)^(1/3) + 1/36*log(3*(x^3 + x)^(1/3)*x - 3*(x^3 + x)^(2/3) + 1)
```

giac [A] time = 0.32, size = 77, normalized size = 0.77

$$\frac{1}{12}\left(\left(\frac{1}{x^2}+1\right)^{\frac{4}{3}}+2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right)x^4-\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{36}\log\left(\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{18}\log\left(\left|\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+x)^(1/3),x, algorithm="giac")
```

```
[Out] 1/12*((1/x^2 + 1)^(4/3) + 2*(1/x^2 + 1)^(1/3))*x^4 - 1/18*sqrt(3)*arctan(1/
3*sqrt(3)*(2*(1/x^2 + 1)^(1/3) + 1)) - 1/36*log((1/x^2 + 1)^(2/3) + (1/x^2
+ 1)^(1/3) + 1) + 1/18*log(abs((1/x^2 + 1)^(1/3) - 1))
```

maple [C] time = 2.10, size = 735, normalized size = 7.35

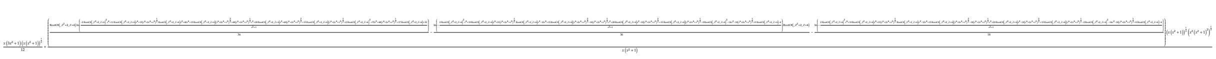

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3+x)^(1/3),x)
```

```
[Out] 1/12*x*(3*x^2+1)*(x*(x^2+1))^(1/3)+(1/36*RootOf(_Z^2+2*_Z+4)*ln((-2*RootOf(
_Z^2+2*_Z+4)^2*x^4+11*RootOf(_Z^2+2*_Z+4)*x^4+15*(x^6+2*x^4+x^2)^(1/3)*Root
Of(_Z^2+2*_Z+4)*x^2+40*x^4+15*RootOf(_Z^2+2*_Z+4)*(x^6+2*x^4+x^2)^(2/3)+48*
(x^6+2*x^4+x^2)^(1/3)*x^2+28*RootOf(_Z^2+2*_Z+4)*x^2+48*(x^6+2*x^4+x^2)^(2/
3)+15*RootOf(_Z^2+2*_Z+4)*(x^6+2*x^4+x^2)^(1/3)+2*RootOf(_Z^2+2*_Z+4)^2+70*
x^2+48*(x^6+2*x^4+x^2)^(1/3)+17*RootOf(_Z^2+2*_Z+4)+30)/(x^2+1))-1/36*ln(-(
2*RootOf(_Z^2+2*_Z+4)^2*x^4+19*RootOf(_Z^2+2*_Z+4)*x^4+15*(x^6+2*x^4+x^2)^(
1/3)*RootOf(_Z^2+2*_Z+4)*x^2-10*x^4+15*RootOf(_Z^2+2*_Z+4)*(x^6+2*x^4+x^2)^
(2/3)-18*(x^6+2*x^4+x^2)^(1/3)*x^2+28*RootOf(_Z^2+2*_Z+4)*x^2-18*(x^6+2*x^4
+x^2)^(2/3)+15*RootOf(_Z^2+2*_Z+4)*(x^6+2*x^4+x^2)^(1/3)-2*RootOf(_Z^2+2*_Z
+4)^2-14*x^2-18*(x^6+2*x^4+x^2)^(1/3)+9*RootOf(_Z^2+2*_Z+4)-4)/(x^2+1))*Roo
tOf(_Z^2+2*_Z+4)-1/18*ln(-(2*RootOf(_Z^2+2*_Z+4)^2*x^4+19*RootOf(_Z^2+2*_Z+
4)*x^4+15*(x^6+2*x^4+x^2)^(1/3)*RootOf(_Z^2+2*_Z+4)*x^2-10*x^4+15*RootOf(_Z
^2+2*_Z+4)*(x^6+2*x^4+x^2)^(2/3)-18*(x^6+2*x^4+x^2)^(1/3)*x^2+28*RootOf(_Z^
2+2*_Z+4)*x^2-18*(x^6+2*x^4+x^2)^(2/3)+15*RootOf(_Z^2+2*_Z+4)*(x^6+2*x^4+x^
2)^(1/3)-2*RootOf(_Z^2+2*_Z+4)^2-14*x^2-18*(x^6+2*x^4+x^2)^(1/3)+9*RootOf(_
Z^2+2*_Z+4)-4)/(x^2+1)))*(x*(x^2+1))^(1/3)/x*(x^2*(x^2+1)^2)^(1/3)/(x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^3+x\right)^{\frac{1}{3}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+x)^(1/3),x, algorithm="maxima")
[Out] integrate((x^3 + x)^(1/3)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(x^3 + x\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x + x^3)^(1/3),x)
[Out] int(x^2*(x + x^3)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt[3]{x\left(x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3+x)**(1/3),x)
[Out] Integral(x**2*(x*(x**2 + 1))**(1/3), x)
```

3.1231 $\int \frac{6+2x+x^2}{\sqrt[3]{2+x^2}\left(1+3x-2x^2+x^3\right)}\,dx$

Optimal. Leaf size=100

$$\log\left(\sqrt[3]{x^2+2}+x-1\right)-\frac{1}{2}\log\left(x^2+\left(x^2+2\right)^{2/3}+(1-x)\sqrt[3]{x^2+2}-2x+1\right)-\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2+2}}{\sqrt{3}}-\frac{2x}{\sqrt{3}}+\frac{2}{\sqrt{3}}}{\sqrt[3]{x^2+2}}\right)$$

Rubi [F] time = 0.59, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{6+2x+x^2}{\sqrt[3]{2+x^2}\left(1+3x-2x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(6 + 2*x + x^2)/((2 + x^2)^(1/3)*(1 + 3*x - 2*x^2 + x^3)),x]
```

```
[Out] 6*Defer[Int][1/((2 + x^2)^(1/3)*(1 + 3*x - 2*x^2 + x^3)), x] + 2*Defer[Int]
[x/((2 + x^2)^(1/3)*(1 + 3*x - 2*x^2 + x^3)), x] + Defer[Int][x^2/((2 + x^2
)^(1/3)*(1 + 3*x - 2*x^2 + x^3)), x]
```

Rubi steps

$$\int \frac{6+2x+x^2}{\sqrt[3]{2+x^2}\left(1+3x-2x^2+x^3\right)}\,dx=\int\left(\frac{6}{\sqrt[3]{2+x^2}\left(1+3x-2x^2+x^3\right)}+\frac{2x}{\sqrt[3]{2+x^2}\left(1+3x-2x^2+x^3\right)}+\frac{}{\sqrt[3]{2+x}}\right.$$
$$=2\int\frac{x}{\sqrt[3]{2+x^2}\left(1+3x-2x^2+x^3\right)}\,dx+6\int\frac{1}{\sqrt[3]{2+x^2}\left(1+3x-2x^2+x^3\right)}\,d$$

Mathematica [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{6+2x+x^2}{\sqrt[3]{2+x^2}\left(1+3x-2x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(6 + 2*x + x^2)/((2 + x^2)^(1/3)*(1 + 3*x - 2*x^2 + x^3)),x]
```

```
[Out] Integrate[(6 + 2*x + x^2)/((2 + x^2)^(1/3)*(1 + 3*x - 2*x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.13, size = 100, normalized size = 1.00

$$\log\left(\sqrt[3]{x^2+2}+x-1\right)-\frac{1}{2}\log\left(x^2+\left(x^2+2\right)^{2/3}+(1-x)\sqrt[3]{x^2+2}-2x+1\right)-\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2+2}}{\sqrt{3}}-\frac{2x}{\sqrt{3}}+\frac{2}{\sqrt{3}}}{\sqrt[3]{x^2+2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(6 + 2*x + x^2)/((2 + x^2)^(1/3)*(1 + 3*x - 2*x^2 + x^3)
),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(2/Sqrt[3] - (2*x)/Sqrt[3] + (2 + x^2)^(1/3)/Sqrt[3])/(2 +
 x^2)^(1/3)]) + Log[-1 + x + (2 + x^2)^(1/3)] - Log[1 - 2*x + x^2 + (1 - x)
*(2 + x^2)^(1/3) + (2 + x^2)^(2/3)]/2
```

fricas [A] time = 1.17, size = 138, normalized size = 1.38

$$-\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(x^2+2\right)^{\frac{2}{3}}(x-1)+2\sqrt{3}\left(x^2-2x+1\right)\left(x^2+2\right)^{\frac{1}{3}}+\sqrt{3}\left(x^3-2x^2+3x+1\right)}{3\left(x^3-4x^2+3x-3\right)}\right)+\frac{1}{2}\log\left(\frac{x^3-2x^2+3\left(x^2+2\right)^{\frac{2}{3}}(x-1)+3\left(x^2-2x+1\right)\left(x^2+2\right)^{\frac{1}{3}}+3x+1}{x^3-2x^2+3x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+6)/(x^2+2)^(1/3)/(x^3-2*x^2+3*x+1),x, algorithm="fricas"
)
```

```
[Out] -sqrt(3)*arctan(1/3*(2*sqrt(3)*(x^2 + 2)^(2/3)*(x - 1) + 2*sqrt(3)*(x^2 - 2
*x + 1)*(x^2 + 2)^(1/3) + sqrt(3)*(x^3 - 2*x^2 + 3*x + 1))/(x^3 - 4*x^2 + 3
*x - 3)) + 1/2*log((x^3 - 2*x^2 + 3*(x^2 + 2)^(2/3)*(x - 1) + 3*(x^2 - 2*x
+ 1)*(x^2 + 2)^(1/3) + 3*x + 1)/(x^3 - 2*x^2 + 3*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+2x+6}{\left(x^3-2x^2+3x+1\right)\left(x^2+2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+6)/(x^2+2)^(1/3)/(x^3-2*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 2*x + 6)/((x^3 - 2*x^2 + 3*x + 1)*(x^2 + 2)^(1/3)), x)
```

maple [C] time = 3.08, size = 772, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x+6)/(x^2+2)^(1/3)/(x^3-2*x^2+3*x+1),x)
```

```
[Out] RootOf(_Z^2+_Z+1)*ln(-(10*RootOf(_Z^2+_Z+1)^2*x^3+24*RootOf(_Z^2+_Z+1)*(x^2
+2)^(2/3)*x-24*RootOf(_Z^2+_Z+1)*(x^2+2)^(1/3)*x^2-25*RootOf(_Z^2+_Z+1)^2*x
^2+23*RootOf(_Z^2+_Z+1)*x^3-24*RootOf(_Z^2+_Z+1)*(x^2+2)^(2/3)+15*(x^2+2)^(
2/3)*x+48*RootOf(_Z^2+_Z+1)*(x^2+2)^(1/3)*x-15*(x^2+2)^(1/3)*x^2+30*RootOf(
_Z^2+_Z+1)^2*x-75*RootOf(_Z^2+_Z+1)*x^2+12*x^3-15*(x^2+2)^(2/3)-24*RootOf(_
Z^2+_Z+1)*(x^2+2)^(1/3)+30*(x^2+2)^(1/3)*x+69*RootOf(_Z^2+_Z+1)*x-44*x^2-15
*(x^2+2)^(1/3)-35*RootOf(_Z^2+_Z+1)+36*x-28)/(x^3-2*x^2+3*x+1))-ln((-188*Ro
otOf(_Z^2+_Z+1)^2*x^3+573*RootOf(_Z^2+_Z+1)*(x^2+2)^(2/3)*x-573*RootOf(_Z^2
+_Z+1)*(x^2+2)^(1/3)*x^2+470*RootOf(_Z^2+_Z+1)^2*x^2+300*RootOf(_Z^2+_Z+1)*
x^3-573*RootOf(_Z^2+_Z+1)*(x^2+2)^(2/3)-318*(x^2+2)^(2/3)*x+1146*RootOf(_Z^
2+_Z+1)*(x^2+2)^(1/3)*x+318*(x^2+2)^(1/3)*x^2-564*RootOf(_Z^2+_Z+1)^2*x-107
9*RootOf(_Z^2+_Z+1)*x^2-103*x^3+318*(x^2+2)^(2/3)-573*RootOf(_Z^2+_Z+1)*(x^
2+2)^(1/3)-636*(x^2+2)^(1/3)*x+900*RootOf(_Z^2+_Z+1)*x+618*x^2+318*(x^2+2)^
(1/3)-658*RootOf(_Z^2+_Z+1)-309*x+721)/(x^3-2*x^2+3*x+1))*RootOf(_Z^2+_Z+1)
-ln((-188*RootOf(_Z^2+_Z+1)^2*x^3+573*RootOf(_Z^2+_Z+1)*(x^2+2)^(2/3)*x-573
*RootOf(_Z^2+_Z+1)*(x^2+2)^(1/3)*x^2+470*RootOf(_Z^2+_Z+1)^2*x^2+300*RootOf
(_Z^2+_Z+1)*x^3-573*RootOf(_Z^2+_Z+1)*(x^2+2)^(2/3)-318*(x^2+2)^(2/3)*x+114
6*RootOf(_Z^2+_Z+1)*(x^2+2)^(1/3)*x+318*(x^2+2)^(1/3)*x^2-564*RootOf(_Z^2+_
Z+1)^2*x-1079*RootOf(_Z^2+_Z+1)*x^2-103*x^3+318*(x^2+2)^(2/3)-573*RootOf(_Z
^2+_Z+1)*(x^2+2)^(1/3)-636*(x^2+2)^(1/3)*x+900*RootOf(_Z^2+_Z+1)*x+618*x^2+
318*(x^2+2)^(1/3)-658*RootOf(_Z^2+_Z+1)-309*x+721)/(x^3-2*x^2+3*x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2+2x+6}{\left(x^3-2x^2+3x+1\right)\left(x^2+2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+6)/(x^2+2)^(1/3)/(x^3-2*x^2+3*x+1),x, algorithm="maxima"
)
```

```
[Out] integrate((x^2 + 2*x + 6)/((x^3 - 2*x^2 + 3*x + 1)*(x^2 + 2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+2x+6}{(x^2+2)^{1/3}\,(x^3-2x^2+3x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 + 6)/((x^2 + 2)^(1/3)*(3*x - 2*x^2 + x^3 + 1)),x)
```

```
[Out] int((2*x + x^2 + 6)/((x^2 + 2)^(1/3)*(3*x - 2*x^2 + x^3 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x+6)/(x**2+2)**(1/3)/(x**3-2*x**2+3*x+1),x)
```

```
[Out] Timed out
```

3.1232 $\int \frac{\sqrt[3]{x^2+x^3}}{x^2} \, dx$

Optimal. Leaf size=100

$$-\frac{3\sqrt[3]{x^3+x^2}}{x}-\log \left(\sqrt[3]{x^3+x^2}-x\right)+\frac{1}{2}\log \left(x^2+\sqrt[3]{x^3+x^2}\,x+\left(x^3+x^2\right)^{2/3}\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3+x^2}+x}\right)$$

Rubi [A] time = 0.06, antiderivative size = 147, normalized size of antiderivative = 1.47, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2020, 2032, 59}

$$-\frac{3\sqrt[3]{x^3+x^2}}{x}-\frac{(x+1)^{2/3}x^{4/3}\log(x+1)}{2\left(x^3+x^2\right)^{2/3}}-\frac{3(x+1)^{2/3}x^{4/3}\log\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x+1}}-1\right)}{2\left(x^3+x^2\right)^{2/3}}-\frac{\sqrt{3}\,(x+1)^{2/3}x^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{x+1}}+\frac{1}{\sqrt{3}}\right)}{\left(x^3+x^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2 + x^3)^(1/3)/x^2,x]
```

```
[Out] (-3*(x^2 + x^3)^(1/3))/x - (Sqrt[3]*x^(4/3)*(1 + x)^(2/3)*ArcTan[1/Sqrt[3]
 + (2*x^(1/3))/(Sqrt[3]*(1 + x)^(1/3))])/(x^2 + x^3)^(2/3) - (x^(4/3)*(1 + x
)^(2/3)*Log[1 + x])/(2*(x^2 + x^3)^(2/3)) - (3*x^(4/3)*(1 + x)^(2/3)*Log[-1
 + x^(1/3)/(1 + x)^(1/3)])/(2*(x^2 + x^3)^(2/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt[3]{x^2+x^3}}{x^2}\,dx = -\frac{3\sqrt[3]{x^2+x^3}}{x} + \int \frac{x}{\left(x^2+x^3\right)^{2/3}}\,dx$$
$$= -\frac{3\sqrt[3]{x^2+x^3}}{x} + \frac{\left(x^{4/3}(1+x)^{2/3}\right)\int \frac{1}{\sqrt[3]{x}\,(1+x)^{2/3}}\,dx}{\left(x^2+x^3\right)^{2/3}}$$
$$= -\frac{3\sqrt[3]{x^2+x^3}}{x} - \frac{\sqrt{3}\,x^{4/3}(1+x)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{\left(x^2+x^3\right)^{2/3}} - \frac{x^{4/3}(1+x)^{2/3}\log(1+x)}{2\left(x^2+x^3\right)^{2/3}} - \frac{3x^{4/3}(1+}{}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.36

$$-\frac{3\sqrt[3]{x^2(x+1)}\;{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-x\right)}{x\sqrt[3]{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2 + x^3)^(1/3)/x^2,x]
```

```
[Out] (-3*(x^2*(1 + x))^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, -x])/(x*(1 + x)^
(1/3))
```

IntegrateAlgebraic [A] time = 0.17, size = 100, normalized size = 1.00

$$-\frac{3\sqrt[3]{x^3+x^2}}{x} - \log\left(\sqrt[3]{x^3+x^2}-x\right) + \frac{1}{2}\log\left(x^2+\sqrt[3]{x^3+x^2}\,x+\left(x^3+x^2\right)^{2/3}\right) - \sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3+x^2}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2 + x^3)^(1/3)/x^2,x]
```

```
[Out] (-3*(x^2 + x^3)^(1/3))/x - Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(x^2 + x^3)^(1
/3))] - Log[-x + (x^2 + x^3)^(1/3)] + Log[x^2 + x*(x^2 + x^3)^(1/3) + (x^2
+ x^3)^(2/3)]/2
```

fricas [A] time = 0.39, size = 102, normalized size = 1.02

$$\frac{2\sqrt{3}\,x\arctan\left(\frac{\sqrt{3}\,x+2\sqrt{3}\left(x^3+x^2\right)^{\frac{1}{3}}}{3\,x}\right) - 2\,x\log\left(-\frac{x-\left(x^3+x^2\right)^{\frac{1}{3}}}{x}\right) + x\log\left(\frac{x^2+\left(x^3+x^2\right)^{\frac{1}{3}}x+\left(x^3+x^2\right)^{\frac{2}{3}}}{x^2}\right) - 6\left(x^3+x^2\right)^{\frac{1}{3}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*x*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 + x^2)^(1/3))/x) -
2*x*log(-(x - (x^3 + x^2)^(1/3))/x) + x*log((x^2 + (x^3 + x^2)^(1/3)*x + (x
^3 + x^2)^(2/3))/x^2) - 6*(x^3 + x^2)^(1/3))/x
```

giac [A] time = 0.32, size = 63, normalized size = 0.63

$$\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right) - 3\left(\frac{1}{x}+1\right)^{\frac{1}{3}} + \frac{1}{2}\log\left(\left(\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right) - \log\left(\left|\left(\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(1/x + 1)^(1/3) + 1)) - 3*(1/x + 1)^(1/3) + 1
/2*log((1/x + 1)^(2/3) + (1/x + 1)^(1/3) + 1) - log(abs((1/x + 1)^(1/3) - 1
))
```

maple [C] time = 0.51, size = 448, normalized size = 4.48

$$-\frac{3\left(x^2(1+x)\right)^{\frac{1}{3}}}{x}+\frac{\left(-\ln\left(\frac{-\mathrm{RootOf}(\_Z^2-2\_Z+4)^2x^2+48\,\mathrm{RootOf}(\_Z^2-2\_Z+4)(x^3+2x^2+x)^{\frac{2}{3}}-30\,\mathrm{RootOf}(\_Z^2-2\_Z+4)(x^3+2x^2+x)^{\frac{1}{3}}x-16\,\mathrm{RootOf}(\_Z^2-2\_Z+4)x^2-36(x^3+2x^2+x)^{\frac{2}{3}}-30\,\mathrm{RootOf}(\_Z^2-2\_Z+4)(x^3+2x^2+x)^{\frac{1}{3}}+96(x^3+2x^2+x)^{\frac{1}{3}}x+\mathrm{RootOf}(\_Z^2-2\_Z+4)^2-14\,\mathrm{RootOf}(\_Z^2-2\_Z+4)x-64x^2+96(x^3+2x^2+x)^{\frac{1}{3}}+2\,\mathrm{RootOf}(\_Z^2-2\_Z+4)-112x-48}{1+x}\right)+\frac{\mathrm{RootOf}(\_Z^2-2\_Z+4)\ln\left(-\frac{2\,\mathrm{RootOf}(\_Z^2-2\_Z+4)^2x^2+24\,\mathrm{RootOf}(\_Z^2-2\_Z+4)(x^3+2x^2+x)^{\frac{2}{3}}-9\,\mathrm{RootOf}(\_Z^2-2\_Z+4)(x^3+2x^2+x)^{\frac{1}{3}}x-19\,\mathrm{RootOf}(\_Z^2-2\_Z+4)x^2-30(x^3+2x^2+x)^{\frac{2}{3}}-9\,\mathrm{RootOf}(\_Z^2-2\_Z+4)(x^3+2x^2+x)^{\frac{1}{3}}+48(x^3+2x^2+x)^{\frac{1}{3}}x-2\,\mathrm{RootOf}(\_Z^2-2\_Z+4)^2-28\,\mathrm{RootOf}(\_Z^2-2\_Z+4)x-10x^2+48(x^3+2x^2+x)^{\frac{1}{3}}-9\,\mathrm{RootOf}(\_Z^2-2\_Z+4)-14x-4}{1+x}\right)}{2}\right)\left(x^2(1+x)\right)^{\frac{1}{3}}\left(x(1+x)^2\right)^{\frac{1}{3}}}{x(1+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2)^(1/3)/x^2,x)
```

```
[Out] -3*(x^2*(1+x))^(1/3)/x+(-ln((-RootOf(_Z^2-2*_Z+4)^2*x^2+48*RootOf(_Z^2-2*_Z
+4)*(x^3+2*x^2+x)^(2/3)-30*RootOf(_Z^2-2*_Z+4)*(x^3+2*x^2+x)^(1/3)*x-16*Roo
tOf(_Z^2-2*_Z+4)*x^2-36*(x^3+2*x^2+x)^(2/3)-30*RootOf(_Z^2-2*_Z+4)*(x^3+2*x
^2+x)^(1/3)+96*(x^3+2*x^2+x)^(1/3)*x+RootOf(_Z^2-2*_Z+4)^2-14*RootOf(_Z^2-2
*_Z+4)*x-64*x^2+96*(x^3+2*x^2+x)^(1/3)+2*RootOf(_Z^2-2*_Z+4)-112*x-48)/(1+x
))+1/2*RootOf(_Z^2-2*_Z+4)*ln(-(2*RootOf(_Z^2-2*_Z+4)^2*x^2+24*RootOf(_Z^2-
2*_Z+4)*(x^3+2*x^2+x)^(2/3)-9*RootOf(_Z^2-2*_Z+4)*(x^3+2*x^2+x)^(1/3)*x-19*
RootOf(_Z^2-2*_Z+4)*x^2-30*(x^3+2*x^2+x)^(2/3)-9*RootOf(_Z^2-2*_Z+4)*(x^3+2
*x^2+x)^(1/3)+48*(x^3+2*x^2+x)^(1/3)*x-2*RootOf(_Z^2-2*_Z+4)^2-28*RootOf(_Z
^2-2*_Z+4)*x-10*x^2+48*(x^3+2*x^2+x)^(1/3)-9*RootOf(_Z^2-2*_Z+4)-14*x-4)/(1
+x)))*(x^2*(1+x))^(1/3)/x*(x*(1+x)^2)^(1/3)/(1+x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+x^2\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x^3 + x^2)^(1/3)/x^2, x)
```

mupad [B] time = 1.03, size = 27, normalized size = 0.27

$$-\frac{3\left(x^2\,(x+1)\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},-\frac{1}{3};\ \frac{2}{3};\ -x\right)}{x\,(x+1)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3)^(1/3)/x^2,x)
```

```
[Out] -(3*(x^2*(x + 1))^(1/3)*hypergeom([-1/3, -1/3], 2/3, -x))/(x*(x + 1)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x^2\,(x+1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2)**(1/3)/x**2,x)
```

```
[Out] Integral((x**2*(x + 1))**(1/3)/x**2, x)
```

3.1233 $\int \frac{\sqrt[3]{1+x^4}\left(3+x^4\right)}{x^{13}}\,dx$

Optimal. Leaf size=100

$$\frac{1}{54}\log\left(\sqrt[3]{x^4+1}-1\right)-\frac{1}{108}\log\left(\left(x^4+1\right)^{2/3}+\sqrt[3]{x^4+1}+1\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{18\sqrt{3}}+\frac{\sqrt[3]{x^4+1}\left(x^8-6x^4-9\right)}{36x^{12}}$$

Rubi [A] time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {446, 78, 47, 51, 57, 618, 204, 31}

$$\frac{\sqrt[3]{x^4+1}}{36x^4}+\frac{1}{36}\log\left(1-\sqrt[3]{x^4+1}\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}+1}{\sqrt{3}}\right)}{18\sqrt{3}}-\frac{\left(x^4+1\right)^{4/3}}{4x^{12}}+\frac{\sqrt[3]{x^4+1}}{12x^8}-\frac{\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^4)^(1/3)*(3 + x^4))/x^13,x]
```

```
[Out] (1 + x^4)^(1/3)/(12*x^8) + (1 + x^4)^(1/3)/(36*x^4) - (1 + x^4)^(4/3)/(4*x^
12) - ArcTan[(1 + 2*(1 + x^4)^(1/3))/Sqrt[3]]/(18*Sqrt[3]) - Log[x]/27 + Lo
g[1 - (1 + x^4)^(1/3)]/36
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{1+x^4}\,\left(3+x^4\right)}{x^{13}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt[3]{1+x}\,(3+x)}{x^4}\,dx,x,x^4\right)\\
&= -\frac{\left(1+x^4\right)^{4/3}}{4x^{12}}-\frac{1}{6}\,\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{x^3}\,dx,x,x^4\right)\\
&= \frac{\sqrt[3]{1+x^4}}{12x^8}-\frac{\left(1+x^4\right)^{4/3}}{4x^{12}}-\frac{1}{36}\,\text{Subst}\left(\int \frac{1}{x^2(1+x)^{2/3}}\,dx,x,x^4\right)\\
&= \frac{\sqrt[3]{1+x^4}}{12x^8}+\frac{\sqrt[3]{1+x^4}}{36x^4}-\frac{\left(1+x^4\right)^{4/3}}{4x^{12}}+\frac{1}{54}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{2/3}}\,dx,x,x^4\right)\\
&= \frac{\sqrt[3]{1+x^4}}{12x^8}+\frac{\sqrt[3]{1+x^4}}{36x^4}-\frac{\left(1+x^4\right)^{4/3}}{4x^{12}}-\frac{\log(x)}{27}-\frac{1}{36}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1+x^4}\right)-\\
&= \frac{\sqrt[3]{1+x^4}}{12x^8}+\frac{\sqrt[3]{1+x^4}}{36x^4}-\frac{\left(1+x^4\right)^{4/3}}{4x^{12}}-\frac{\log(x)}{27}+\frac{1}{36}\log\left(1-\sqrt[3]{1+x^4}\right)+\frac{1}{18}\,\text{Subst}\Bigg(\int\\
&= \frac{\sqrt[3]{1+x^4}}{12x^8}+\frac{\sqrt[3]{1+x^4}}{36x^4}-\frac{\left(1+x^4\right)^{4/3}}{4x^{12}}-\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^4}}{\sqrt{3}}\right)}{18\sqrt{3}}-\frac{\log(x)}{27}+\frac{1}{36}\log\Big(1-\sqrt[3]{1}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.35

$$
\frac{\left(x^4+1\right)^{4/3}\left(x^{12}\,{}_2F_1\left(\frac{4}{3},3;\frac{7}{3};x^4+1\right)-2\right)}{8x^{12}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^4)^(1/3)*(3 + x^4))/x^13,x]
```

```
[Out] ((1 + x^4)^(4/3)*(-2 + x^12*Hypergeometric2F1[4/3, 3, 7/3, 1 + x^4]))/(8*x^
12)
```

IntegrateAlgebraic [A] time = 0.19, size = 100, normalized size = 1.00

$$\frac{1}{54}\log\left(\sqrt[3]{x^4+1}-1\right)-\frac{1}{108}\log\left(\left(x^4+1\right)^{2/3}+\sqrt[3]{x^4+1}+1\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{18\sqrt{3}}+\frac{\sqrt[3]{x^4+1}\left(x^8-6x^4-9\right)}{36x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^4)^(1/3)*(3 + x^4))/x^13,x]
```

```
[Out] ((1 + x^4)^(1/3)*(-9 - 6*x^4 + x^8))/(36*x^12) - ArcTan[1/Sqrt[3] + (2*(1 +
 x^4)^(1/3))/Sqrt[3]]/(18*Sqrt[3]) + Log[-1 + (1 + x^4)^(1/3)]/54 - Log[1 +
 (1 + x^4)^(1/3) + (1 + x^4)^(2/3)]/108
```

fricas [A] time = 0.39, size = 88, normalized size = 0.88

$$-\frac{2\sqrt{3}x^{12}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^4+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+x^{12}\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)-2x^{12}\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)-3\left(x^8-6x^4-9\right)\left(x^4+1\right)^{\frac{1}{3}}}{108x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/3)*(x^4+3)/x^13,x, algorithm="fricas")
```

```
[Out] -1/108*(2*sqrt(3)*x^12*arctan(2/3*sqrt(3)*(x^4 + 1)^(1/3) + 1/3*sqrt(3)) +
x^12*log((x^4 + 1)^(2/3) + (x^4 + 1)^(1/3) + 1) - 2*x^12*log((x^4 + 1)^(1/3
) - 1) - 3*(x^8 - 6*x^4 - 9)*(x^4 + 1)^(1/3))/x^12
```

giac [A] time = 0.16, size = 85, normalized size = 0.85

$$-\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{\left(x^4+1\right)^{\frac{7}{3}}-8\left(x^4+1\right)^{\frac{4}{3}}-2\left(x^4+1\right)^{\frac{1}{3}}}{36x^{12}}-\frac{1}{108}\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{54}\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/3)*(x^4+3)/x^13,x, algorithm="giac")
```

```
[Out] -1/54*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 + 1)^(1/3) + 1)) + 1/36*((x^4 + 1)
^(7/3) - 8*(x^4 + 1)^(4/3) - 2*(x^4 + 1)^(1/3))/x^12 - 1/108*log((x^4 + 1)^
(2/3) + (x^4 + 1)^(1/3) + 1) + 1/54*log((x^4 + 1)^(1/3) - 1)
```

maple [C] time = 0.25, size = 74, normalized size = 0.74

$$\frac{x^{12}-5x^8-15x^4-9}{36x^{12}\left(x^4+1\right)^{\frac{2}{3}}}+\frac{-\frac{2\Gamma\left(\frac{2}{3}\right)x^4\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],-x^4\right)}{3}+\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+4\ln(x)\right)\Gamma\left(\frac{2}{3}\right)}{54\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(1/3)*(x^4+3)/x^13,x)
```

```
[Out] 1/36*(x^12-5*x^8-15*x^4-9)/x^12/(x^4+1)^(2/3)+1/54/GAMMA(2/3)*(-2/3*GAMMA(2
/3)*x^4*hypergeom([1,1,5/3],[2,2],-x^4)+(1/6*Pi*3^(1/2)-3/2*ln(3)+4*ln(x))*
GAMMA(2/3))
```

maxima [A] time = 0.46, size = 146, normalized size = 1.46

$$-\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{5\left(x^4+1\right)^{\frac{7}{3}}-13\left(x^4+1\right)^{\frac{4}{3}}-10\left(x^4+1\right)^{\frac{1}{3}}}{72\left(3x^4+\left(x^4+1\right)^3-3\left(x^4+1\right)^2+2\right)}+\frac{\left(x^4+1\right)^{\frac{4}{3}}+2\left(x^4+1\right)^{\frac{1}{3}}}{24\left(2x^4-\left(x^4+1\right)^2+1\right)}-\frac{1}{108}\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{54}\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/3)*(x^4+3)/x^13,x, algorithm="maxima")
```

```
[Out] -1/54*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 + 1)^(1/3) + 1)) + 1/72*(5*(x^4 +
1)^(7/3) - 13*(x^4 + 1)^(4/3) - 10*(x^4 + 1)^(1/3))/(3*x^4 + (x^4 + 1)^3 -
3*(x^4 + 1)^2 + 2) + 1/24*((x^4 + 1)^(4/3) + 2*(x^4 + 1)^(1/3))/(2*x^4 - (x
^4 + 1)^2 + 1) - 1/108*log((x^4 + 1)^(2/3) + (x^4 + 1)^(1/3) + 1) + 1/54*lo
g((x^4 + 1)^(1/3) - 1)
```

mupad [B] time = 1.53, size = 232, normalized size = 2.32

$$\frac{5\ln\left(\frac{25\,{\left(x^4+1\right)}^{1/3}}{1296}-\frac{25}{1296}\right)}{108}-\frac{\ln\left(\frac{{\left(x^4+1\right)}^{1/3}}{144}-\frac{1}{144}\right)}{36}-\frac{\frac{5\,{\left(x^4+1\right)}^{1/3}}{36}+\frac{13\,{\left(x^4+1\right)}^{4/3}}{72}-\frac{5\,{\left(x^4+1\right)}^{7/3}}{72}}{{\left(x^4+1\right)}^3-3\,{\left(x^4+1\right)}^2+3\,x^4+2}+\frac{\frac{{\left(x^4+1\right)}^{1/3}}{12}+\frac{{\left(x^4+1\right)}^{4/3}}{24}}{2\,x^4-{\left(x^4+1\right)}^2+1}-\ln\left(\frac{{\left(x^4+1\right)}^{1/3}}{4}+\frac{1}{8}-\frac{\sqrt{3}\,1{}\mathrm{i}}{8}\right)\,\left(-\frac{1}{72}+\frac{\sqrt{3}\,1{}\mathrm{i}}{72}\right)+\ln\left(\frac{{\left(x^4+1\right)}^{1/3}}{4}+\frac{1}{8}+\frac{\sqrt{3}\,1{}\mathrm{i}}{8}\right)\,\left(\frac{1}{72}+\frac{\sqrt{3}\,1{}\mathrm{i}}{72}\right)+\ln\left(\frac{5\,{\left(x^4+1\right)}^{1/3}}{12}+\frac{5}{24}-\frac{\sqrt{3}\,5{}\mathrm{i}}{24}\right)\,\left(-\frac{5}{216}+\frac{\sqrt{3}\,5{}\mathrm{i}}{216}\right)-\ln\left(\frac{5\,{\left(x^4+1\right)}^{1/3}}{12}+\frac{5}{24}+\frac{\sqrt{3}\,5{}\mathrm{i}}{24}\right)\,\left(\frac{5}{216}+\frac{\sqrt{3}\,5{}\mathrm{i}}{216}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/3)*(x^4 + 3))/x^13,x)
```

```
[Out] (5*log((25*(x^4 + 1)^(1/3))/1296 - 25/1296))/108 - log((x^4 + 1)^(1/3)/144
- 1/144)/36 - ((5*(x^4 + 1)^(1/3))/36 + (13*(x^4 + 1)^(4/3))/72 - (5*(x^4 +
 1)^(7/3))/72)/((x^4 + 1)^3 - 3*(x^4 + 1)^2 + 3*x^4 + 2) + ((x^4 + 1)^(1/3)
/12 + (x^4 + 1)^(4/3)/24)/(2*x^4 - (x^4 + 1)^2 + 1) - log((x^4 + 1)^(1/3)/4
 - (3^(1/2)*1i)/8 + 1/8)*((3^(1/2)*1i)/72 - 1/72) + log((3^(1/2)*1i)/8 + (x
^4 + 1)^(1/3)/4 + 1/8)*((3^(1/2)*1i)/72 + 1/72) + log((5*(x^4 + 1)^(1/3))/1
2 - (3^(1/2)*5i)/24 + 5/24)*((3^(1/2)*5i)/216 - 5/216) - log((3^(1/2)*5i)/2
4 + (5*(x^4 + 1)^(1/3))/12 + 5/24)*((3^(1/2)*5i)/216 + 5/216)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(1/3)*(x**4+3)/x**13,x)
```

```
[Out] Timed out
```

3.1234 $\int \frac{(-1+x^4)^{2/3}(3+x^4)(-1+x^3+x^4)}{x^6(-1-x^3+x^4)}\,dx$

Optimal. Leaf size=100

$$2\log\left(\sqrt[3]{x^4-1}-x\right)-2\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^4-1}+x}\right)-\log\left(\sqrt[3]{x^4-1}\,x+\left(x^4-1\right)^{2/3}+x^2\right)+\frac{3\left(x^4-1\right)^{2/3}\left(x^4+5x^3-1\right)}{5x^5}$$

Rubi [F] time = 1.35, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)\left(-1+x^3+x^4\right)}{x^6\left(-1-x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^4)^(2/3)*(3 + x^4)*(-1 + x^3 + x^4))/(x^6*(-1 - x^3 + x^4)),x]
```

```
[Out] (3*(-1 + x^4)^(2/3))/x^2 - (12*x^2)/(1 + Sqrt[3] + (-1 + x^4)^(1/3)) + (6*3
^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + (-1 + x^4)^(1/3))*Sqrt[(1 - (-1 + x^4)^(1/3)
+ (-1 + x^4)^(2/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))^2]*EllipticE[ArcSin[(1
 - Sqrt[3] + (-1 + x^4)^(1/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))], -7 - 4*Sq
rt[3]])/(x^2*Sqrt[(1 + (-1 + x^4)^(1/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))^2
]) - (4*Sqrt[2]*3^(3/4)*(1 + (-1 + x^4)^(1/3))*Sqrt[(1 - (-1 + x^4)^(1/3) +
 (-1 + x^4)^(2/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))^2]*EllipticF[ArcSin[(1
- Sqrt[3] + (-1 + x^4)^(1/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))], -7 - 4*Sqr
t[3]])/(x^2*Sqrt[(1 + (-1 + x^4)^(1/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))^2]
) - (3*(-1 + x^4)^(2/3)*Hypergeometric2F1[-5/4, -2/3, -1/4, x^4])/(5*x^5*(1
 - x^4)^(2/3)) - ((-1 + x^4)^(2/3)*Hypergeometric2F1[-2/3, -1/4, 3/4, x^4])
/(x*(1 - x^4)^(2/3)) - 6*Defer[Int][(-1 + x^4)^(2/3)/(-1 - x^3 + x^4), x] +
 8*Defer[Int][(x*(-1 + x^4)^(2/3))/(-1 - x^3 + x^4), x]
```

Rubi steps

$$\int \frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)\left(-1+x^3+x^4\right)}{x^6\left(-1-x^3+x^4\right)}\,dx = \int \left(\frac{3\left(-1+x^4\right)^{2/3}}{x^6} - \frac{6\left(-1+x^4\right)^{2/3}}{x^3} + \frac{\left(-1+x^4\right)^{2/3}}{x^2} + \frac{2(-3+4x)}{-1-}\right.$$

$$= 2\int \frac{(-3+4x)\left(-1+x^4\right)^{2/3}}{-1-x^3+x^4}\,dx + 3\int \frac{\left(-1+x^4\right)^{2/3}}{x^6}\,dx - 6\int \frac{(-1}{}$$

$$= 2\int \left(-\frac{3\left(-1+x^4\right)^{2/3}}{-1-x^3+x^4} + \frac{4x\left(-1+x^4\right)^{2/3}}{-1-x^3+x^4}\right)dx - 3\,\text{Subst}\left(\int \frac{(-1+}{x}\right.$$

$$= \frac{3\left(-1+x^4\right)^{2/3}}{x^2} - \frac{3\left(-1+x^4\right)^{2/3}\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};x^4\right)}{5x^5\left(1-x^4\right)^{2/3}} - \frac{\left(-1+x^4\right)}{}$$

$$= \frac{3\left(-1+x^4\right)^{2/3}}{x^2} - \frac{3\left(-1+x^4\right)^{2/3}\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};x^4\right)}{5x^5\left(1-x^4\right)^{2/3}} - \frac{\left(-1+x^4\right)}{}$$

$$= \frac{3\left(-1+x^4\right)^{2/3}}{x^2} - \frac{3\left(-1+x^4\right)^{2/3}\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};x^4\right)}{5x^5\left(1-x^4\right)^{2/3}} - \frac{\left(-1+x^4\right)}{}$$

$$= \frac{3\left(-1+x^4\right)^{2/3}}{x^2} - \frac{12x^2}{1+\sqrt{3}+\sqrt[3]{-1+x^4}} + \frac{6\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1+\sqrt[3]{-}\right.}{}$$

Mathematica [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)\left(-1+x^3+x^4\right)}{x^6\left(-1-x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^4)^(2/3)*(3 + x^4)*(-1 + x^3 + x^4))/(x^6*(-1 - x^3 + x^
4)),x]
```

```
[Out] Integrate[((-1 + x^4)^(2/3)*(3 + x^4)*(-1 + x^3 + x^4))/(x^6*(-1 - x^3 + x^
4)), x]
```

IntegrateAlgebraic [A] time = 3.59, size = 100, normalized size = 1.00

$$2\log\left(\sqrt[3]{x^4-1}-x\right) - 2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^4-1}+x}\right) - \log\left(\sqrt[3]{x^4-1}\,x + \left(x^4-1\right)^{2/3} + x^2\right) + \frac{3\left(x^4-1\right)^{2/3}\left(x^4+5x^3-1\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)^(2/3)*(3 + x^4)*(-1 + x^3 + x^4))/(x^6*(-1 -
 x^3 + x^4)),x]
```

```
[Out] (3*(-1 + x^4)^(2/3)*(-1 + 5*x^3 + x^4))/(5*x^5) - 2*Sqrt[3]*ArcTan[(Sqrt[3]
*x)/(x + 2*(-1 + x^4)^(1/3))] + 2*Log[-x + (-1 + x^4)^(1/3)] - Log[x^2 + x*
(-1 + x^4)^(1/3) + (-1 + x^4)^(2/3)]
```

fricas [A] time = 4.86, size = 147, normalized size = 1.47

$$-\frac{10\sqrt{3}\,x^5\arctan\left(-\frac{14106128635054532\sqrt{3}\left(x^4-1\right)^{\frac{1}{3}}x^2-89654043956484782\sqrt{3}\left(x^4-1\right)^{\frac{2}{3}}x-\sqrt{3}\left(35416555940707109\,x^4+2357401720008016\,x^3-35416555940707109\right)}{3\left(51678794422160641\,x^4+201291873609016\,x^3-51678794422160641\right)}\right) - 5\,x^5\log\left(\frac{x^4-x^3+3\left(x^4-1\right)^{\frac{1}{3}}x^2-3\left(x^4-1\right)^{\frac{2}{3}}x-1}{x^4-x^3-1}\right) - 3\left(x^4+5\,x^3-1\right)\left(x^4-1\right)^{\frac{2}{3}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)*(x^4+x^3-1)/x^6/(x^4-x^3-1),x, algorithm="f
ricas")
```

```
[Out] -1/5*(10*sqrt(3)*x^5*arctan(-1/3*(14106128635054532*sqrt(3)*(x^4 - 1)^(1/3)
*x^2 - 89654043956484782*sqrt(3)*(x^4 - 1)^(2/3)*x - sqrt(3)*(3541655594070
7109*x^4 + 2357401720008016*x^3 - 35416555940707109))/(51678794422160641*x^
4 + 201291873609016*x^3 - 51678794422160641)) - 5*x^5*log((x^4 - x^3 + 3*(x
^4 - 1)^(1/3)*x^2 - 3*(x^4 - 1)^(2/3)*x - 1)/(x^4 - x^3 - 1)) - 3*(x^4 + 5*
x^3 - 1)*(x^4 - 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3-1\right)\left(x^4+3\right)\left(x^4-1\right)^{\frac{2}{3}}}{\left(x^4-x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)*(x^4+x^3-1)/x^6/(x^4-x^3-1),x, algorithm="g
iac")
```

```
[Out] integrate((x^4 + x^3 - 1)*(x^4 + 3)*(x^4 - 1)^(2/3)/((x^4 - x^3 - 1)*x^6),
x)
```

maple [C] time = 2.86, size = 321, normalized size = 3.21

$$\frac{\frac{3}{5}x^8-\frac{6}{5}x^4+3x^7-3x^3+\frac{3}{5}}{x^5\left(x^4-1\right)^{\frac{1}{3}}}+2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^4-1\right)^{\frac{2}{3}}x+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^4-1\right)^{\frac{1}{3}}x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3+x^4+2\left(x^4-1\right)^{\frac{2}{3}}x+2\left(x^4-1\right)^{\frac{1}{3}}x^2+x^3-1}{x^4-x^3-1}\right)-2\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^4-1\right)^{\frac{2}{3}}x+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^4-1\right)^{\frac{1}{3}}x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3-x^4-\left(x^4-1\right)^{\frac{2}{3}}x-\left(x^4-1\right)^{\frac{1}{3}}x^2+1}{x^4-x^3-1}\right)\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-2\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^4-1\right)^{\frac{2}{3}}x+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^4-1\right)^{\frac{1}{3}}x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3-x^4-\left(x^4-1\right)^{\frac{2}{3}}x-\left(x^4-1\right)^{\frac{1}{3}}x^2+1}{x^4-x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(2/3)*(x^4+3)*(x^4+x^3-1)/x^6/(x^4-x^3-1),x)
```

```
[Out] 3/5*(x^8+5*x^7-2*x^4-5*x^3+1)/x^5/(x^4-1)^(1/3)+2*RootOf(_Z^2+_Z+1)*ln((Roo
tOf(_Z^2+_Z+1)*(x^4-1)^(2/3)*x+RootOf(_Z^2+_Z+1)*(x^4-1)^(1/3)*x^2+RootOf(_
Z^2+_Z+1)*x^3+x^4+2*(x^4-1)^(2/3)*x+2*(x^4-1)^(1/3)*x^2+x^3-1)/(x^4-x^3-1))
-2*ln(-(RootOf(_Z^2+_Z+1)*(x^4-1)^(2/3)*x+RootOf(_Z^2+_Z+1)*(x^4-1)^(1/3)*x
^2+RootOf(_Z^2+_Z+1)*x^3-x^4-(x^4-1)^(2/3)*x-(x^4-1)^(1/3)*x^2+1)/(x^4-x^3-
1))*RootOf(_Z^2+_Z+1)-2*ln(-(RootOf(_Z^2+_Z+1)*(x^4-1)^(2/3)*x+RootOf(_Z^2+
_Z+1)*(x^4-1)^(1/3)*x^2+RootOf(_Z^2+_Z+1)*x^3-x^4-(x^4-1)^(2/3)*x-(x^4-1)^(
1/3)*x^2+1)/(x^4-x^3-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3-1\right)\left(x^4+3\right)\left(x^4-1\right)^{\frac{2}{3}}}{\left(x^4-x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)*(x^4+x^3-1)/x^6/(x^4-x^3-1),x, algorithm="m
axima")
```

```
[Out] integrate((x^4 + x^3 - 1)*(x^4 + 3)*(x^4 - 1)^(2/3)/((x^4 - x^3 - 1)*x^6),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^4-1\right)^{2/3}\,\left(x^4+3\right)\,\left(x^4+x^3-1\right)}{x^6\,\left(-x^4+x^3+1\right)} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^4 - 1)^(2/3)*(x^4 + 3)*(x^3 + x^4 - 1))/(x^6*(x^3 - x^4 + 1)),x)
[Out] int(-((x^4 - 1)^(2/3)*(x^4 + 3)*(x^3 + x^4 - 1))/(x^6*(x^3 - x^4 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(2/3)*(x**4+3)*(x**4+x**3-1)/x**6/(x**4-x**3-1),x)
[Out] Timed out
```

3.1235 $\int \frac{(-3+x^4)(1+x^4)^{2/3}(1+x^3+x^4)}{x^6(1-x^3+x^4)}\,dx$

Optimal. Leaf size=100

$$2\log\left(\sqrt[3]{x^4+1}-x\right)-2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+1}+x}\right)-\log\left(\sqrt[3]{x^4+1}\,x+\left(x^4+1\right)^{2/3}+x^2\right)+\frac{3\left(x^4+1\right)^{2/3}\left(x^4+5x^3+1\right.}{5x^5}$$

Rubi [F] time = 1.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}\left(1+x^3+x^4\right)}{x^6\left(1-x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + x^4)*(1 + x^4)^(2/3)*(1 + x^3 + x^4))/(x^6*(1 - x^3 + x^4)),x]
```

```
[Out] (3*(1 + x^4)^(2/3))/x^2 + (12*x^2)/(1 - Sqrt[3] - (1 + x^4)^(1/3)) - (6*3^(
1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 + x^4)^(1/3))*Sqrt[(1 + (1 + x^4)^(1/3) + (1
 + x^4)^(2/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqr
t[3] - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))], -7 + 4*Sqrt[3]])/
(x^2*Sqrt[-((1 - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2)]) + (4
*Sqrt[2]*3^(3/4)*(1 - (1 + x^4)^(1/3))*Sqrt[(1 + (1 + x^4)^(1/3) + (1 + x^4
)^(2/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
 (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))], -7 + 4*Sqrt[3]])/(x^2*S
qrt[-((1 - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2)]) + (3*Hyper
geometric2F1[-5/4, -2/3, -1/4, -x^4])/(5*x^5) - Hypergeometric2F1[-2/3, -1/
4, 3/4, -x^4]/x - 6*Defer[Int][(1 + x^4)^(2/3)/(1 - x^3 + x^4), x] + 8*Defe
r[Int][(x*(1 + x^4)^(2/3))/(1 - x^3 + x^4), x]
```

Rubi steps

$$\int \frac{(-3+x^4)(1+x^4)^{2/3}(1+x^3+x^4)}{x^6(1-x^3+x^4)}\,dx = \int \left(-\frac{3(1+x^4)^{2/3}}{x^6} - \frac{6(1+x^4)^{2/3}}{x^3} + \frac{(1+x^4)^{2/3}}{x^2} + \frac{2(-3+4x)(1+}{1-x^3+x}\right.$$

$$= 2\int \frac{(-3+4x)(1+x^4)^{2/3}}{1-x^3+x^4}\,dx - 3\int \frac{(1+x^4)^{2/3}}{x^6}\,dx - 6\int \frac{(1+x^4)^{2}}{x^3}$$

$$= \frac{3\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};-x^4\right)}{5x^5} - \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{4};\frac{3}{4};-x^4\right)}{x} + 2\int \left(-\frac{3(1+x^4}{1-x^3+}\right.$$

$$= \frac{3(1+x^4)^{2/3}}{x^2} + \frac{3\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};-x^4\right)}{5x^5} - \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{4};\frac{3}{4};-x^4\right)}{x} - 4$$

$$= \frac{3(1+x^4)^{2/3}}{x^2} + \frac{3\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};-x^4\right)}{5x^5} - \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{4};\frac{3}{4};-x^4\right)}{x} - 6$$

$$= \frac{3(1+x^4)^{2/3}}{x^2} + \frac{3\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};-x^4\right)}{5x^5} - \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{4};\frac{3}{4};-x^4\right)}{x} - 6$$

$$= \frac{3(1+x^4)^{2/3}}{x^2} + \frac{12x^2}{1-\sqrt{3}-\sqrt[3]{1+x^4}} - \frac{6\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1+x^4}\right.}{}$$

Mathematica [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(-3+x^4)(1+x^4)^{2/3}(1+x^3+x^4)}{x^6(1-x^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + x^4)*(1 + x^4)^(2/3)*(1 + x^3 + x^4))/(x^6*(1 - x^3 + x^4)
),x]
```

```
[Out] Integrate[((-3 + x^4)*(1 + x^4)^(2/3)*(1 + x^3 + x^4))/(x^6*(1 - x^3 + x^4)
), x]
```

IntegrateAlgebraic [A] time = 3.56, size = 100, normalized size = 1.00

$$2\log\left(\sqrt[3]{x^4+1}-x\right) - 2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+1}+x}\right) - \log\left(\sqrt[3]{x^4+1}\,x + (x^4+1)^{2/3} + x^2\right) + \frac{3(x^4+1)^{2/3}(x^4+5x^3+1)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 + x^4)^(2/3)*(1 + x^3 + x^4))/(x^6*(1 - x
^3 + x^4)),x]
```

```
[Out] (3*(1 + x^4)^(2/3)*(1 + 5*x^3 + x^4))/(5*x^5) - 2*Sqrt[3]*ArcTan[(Sqrt[3]*x
)/(x + 2*(1 + x^4)^(1/3))] + 2*Log[-x + (1 + x^4)^(1/3)] - Log[x^2 + x*(1 +
 x^4)^(1/3) + (1 + x^4)^(2/3)]
```

fricas [A] time = 3.00, size = 144, normalized size = 1.44

$$-\frac{10\sqrt{3}x^5\arctan\left(-\frac{13034\sqrt{3}(x^4+1)^{\frac{1}{3}}x^2-686\sqrt{3}(x^4+1)^{\frac{2}{3}}x+\sqrt{3}(37x^4+6137x^3+37)}{3(x^4+6859x^3+1)}\right) - 5x^5\log\left(\frac{x^4-x^3+3(x^4+1)^{\frac{1}{3}}x^2-3(x^4+1)^{\frac{2}{3}}x+1}{x^4-x^3+1}\right) - 3(x^4+5x^3+1)(x^4+1)^{\frac{2}{3}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)*(x^4+x^3+1)/x^6/(x^4-x^3+1),x, algorithm="f
ricas")
```

```
[Out] -1/5*(10*sqrt(3)*x^5*arctan(-1/3*(13034*sqrt(3)*(x^4 + 1)^(1/3)*x^2 - 686*s
qrt(3)*(x^4 + 1)^(2/3)*x + sqrt(3)*(37*x^4 + 6137*x^3 + 37))/(x^4 + 6859*x^
3 + 1)) - 5*x^5*log((x^4 - x^3 + 3*(x^4 + 1)^(1/3)*x^2 - 3*(x^4 + 1)^(2/3)*
x + 1)/(x^4 - x^3 + 1)) - 3*(x^4 + 5*x^3 + 1)*(x^4 + 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3+1\right)\left(x^4+1\right)^{\frac{2}{3}}\left(x^4-3\right)}{\left(x^4-x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)*(x^4+x^3+1)/x^6/(x^4-x^3+1),x, algorithm="g
iac")
```

```
[Out] integrate((x^4 + x^3 + 1)*(x^4 + 1)^(2/3)*(x^4 - 3)/((x^4 - x^3 + 1)*x^6),
x)
```

maple [C] time = 2.69, size = 320, normalized size = 3.20

$$\frac{\frac{3}{5}x^8+\frac{6}{5}x^4+3x^7+3x^3+\frac{3}{5}}{x^5\left(x^4+1\right)^{\frac{1}{3}}}+2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(-\frac{\left(x^4+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x+\left(x^4+1\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3+x^4+2\left(x^4+1\right)^{\frac{2}{3}}x+2x^2\left(x^4+1\right)^{\frac{1}{3}}+x^3+1}{x^4-x^3+1}\right)-2\ln\left(\frac{\left(x^4+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x+\left(x^4+1\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3-x^4-\left(x^4+1\right)^{\frac{2}{3}}x-x^2\left(x^4+1\right)^{\frac{1}{3}}-1}{x^4-x^3+1}\right)\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-2\ln\left(\frac{\left(x^4+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x+\left(x^4+1\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3-x^4-\left(x^4+1\right)^{\frac{2}{3}}x-x^2\left(x^4+1\right)^{\frac{1}{3}}-1}{x^4-x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4+1)^(2/3)*(x^4+x^3+1)/x^6/(x^4-x^3+1),x)
```

```
[Out] 3/5*(x^8+5*x^7+2*x^4+5*x^3+1)/x^5/(x^4+1)^(1/3)+2*RootOf(_Z^2+_Z+1)*ln(-((x
^4+1)^(2/3)*RootOf(_Z^2+_Z+1)*x+(x^4+1)^(1/3)*RootOf(_Z^2+_Z+1)*x^2+RootOf(
_Z^2+_Z+1)*x^3+x^4+2*(x^4+1)^(2/3)*x+2*x^2*(x^4+1)^(1/3)+x^3+1)/(x^4-x^3+1)
)-2*ln(((x^4+1)^(2/3)*RootOf(_Z^2+_Z+1)*x+(x^4+1)^(1/3)*RootOf(_Z^2+_Z+1)*x
^2+RootOf(_Z^2+_Z+1)*x^3-x^4-(x^4+1)^(2/3)*x-x^2*(x^4+1)^(1/3)-1)/(x^4-x^3+
1))*RootOf(_Z^2+_Z+1)-2*ln(((x^4+1)^(2/3)*RootOf(_Z^2+_Z+1)*x+(x^4+1)^(1/3)
*RootOf(_Z^2+_Z+1)*x^2+RootOf(_Z^2+_Z+1)*x^3-x^4-(x^4+1)^(2/3)*x-x^2*(x^4+1
)^(1/3)-1)/(x^4-x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3+1\right)\left(x^4+1\right)^{\frac{2}{3}}\left(x^4-3\right)}{\left(x^4-x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)*(x^4+x^3+1)/x^6/(x^4-x^3+1),x, algorithm="m
axima")
```

```
[Out] integrate((x^4 + x^3 + 1)*(x^4 + 1)^(2/3)*(x^4 - 3)/((x^4 - x^3 + 1)*x^6),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(x^4+1\right)}^{2/3}\,\left(x^4-3\right)\,\left(x^4+x^3+1\right)}{x^6\,\left(x^4-x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(2/3)*(x^4 - 3)*(x^3 + x^4 + 1))/(x^6*(x^4 - x^3 + 1)),x)
[Out] int(((x^4 + 1)^(2/3)*(x^4 - 3)*(x^3 + x^4 + 1))/(x^6*(x^4 - x^3 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4+1)**(2/3)*(x**4+x**3+1)/x**6/(x**4-x**3+1),x)
[Out] Timed out
```

3.1236 $\int \frac{\sqrt[3]{1+2x^7}\left(-3+8x^7\right)}{x^2\left(1+x^3+2x^7\right)}\,dx$

Optimal. Leaf size=100

$$\frac{3\sqrt[3]{2x^7+1}}{x}-\log\left(\sqrt[3]{2x^7+1}+x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2x^7+1}-x}\right)+\frac{1}{2}\log\left(-\sqrt[3]{2x^7+1}x+\left(2x^7+1\right)^{2/3}+x^2\right)$$

Rubi [F] time = 0.87, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{1+2x^7}\left(-3+8x^7\right)}{x^2\left(1+x^3+2x^7\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + 2*x^7)^(1/3)*(-3 + 8*x^7))/(x^2*(1 + x^3 + 2*x^7)),x]
```

```
[Out] (3*Hypergeometric2F1[-1/3, -1/7, 6/7, -2*x^7])/x + 3*Defer[Int][(x*(1 + 2*x
^7)^(1/3))/(1 + x^3 + 2*x^7), x] + 14*Defer[Int][(x^5*(1 + 2*x^7)^(1/3))/(1
 + x^3 + 2*x^7), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{1+2x^7}\left(-3+8x^7\right)}{x^2\left(1+x^3+2x^7\right)}\,dx &= \int \left(-\frac{3\sqrt[3]{1+2x^7}}{x^2}+\frac{x\left(3+14x^4\right)\sqrt[3]{1+2x^7}}{1+x^3+2x^7}\right)dx\\
&= -\left(3\int \frac{\sqrt[3]{1+2x^7}}{x^2}\,dx\right)+\int \frac{x\left(3+14x^4\right)\sqrt[3]{1+2x^7}}{1+x^3+2x^7}\,dx\\
&= \frac{3\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{7};\frac{6}{7};-2x^7\right)}{x}+\int \left(\frac{3x\sqrt[3]{1+2x^7}}{1+x^3+2x^7}+\frac{14x^5\sqrt[3]{1+2x^7}}{1+x^3+2x^7}\right)dx\\
&= \frac{3\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{7};\frac{6}{7};-2x^7\right)}{x}+3\int \frac{x\sqrt[3]{1+2x^7}}{1+x^3+2x^7}\,dx+14\int \frac{x^5\sqrt[3]{1+2x^7}}{1+x^3+2x^7}\,dx
\end{aligned}$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{1+2x^7}\left(-3+8x^7\right)}{x^2\left(1+x^3+2x^7\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + 2*x^7)^(1/3)*(-3 + 8*x^7))/(x^2*(1 + x^3 + 2*x^7)),x]
```

```
[Out] Integrate[((1 + 2*x^7)^(1/3)*(-3 + 8*x^7))/(x^2*(1 + x^3 + 2*x^7)), x]
```

IntegrateAlgebraic [A] time = 17.82, size = 100, normalized size = 1.00

$$\frac{3\sqrt[3]{2x^7+1}}{x}-\log\left(\sqrt[3]{2x^7+1}+x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2x^7+1}-x}\right)+\frac{1}{2}\log\left(-\sqrt[3]{2x^7+1}x+\left(2x^7+1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + 2*x^7)^(1/3)*(-3 + 8*x^7))/(x^2*(1 + x^3 + 2*x^7))
,x]
```

```
[Out] (3*(1 + 2*x^7)^(1/3))/x + Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(1 + 2*x^7)^(1
/3))] - Log[x + (1 + 2*x^7)^(1/3)] + Log[x^2 - x*(1 + 2*x^7)^(1/3) + (1 + 2
*x^7)^(2/3)]/2
```

fricas [A] time = 13.78, size = 141, normalized size = 1.41

$$\frac{2\sqrt{3}\,x\arctan\left(\frac{8377128467638\sqrt{3}\left(2x^7+1\right)^{\frac{1}{3}}x^2+15171948325814\sqrt{3}\left(2x^7+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(2102123379894x^7+4448471619035x^3+1051061689947\right)}{60468559237154x^7-5089335571601x^3+30234279618577}\right)-x\log\left(\frac{2x^7+x^3+3\left(2x^7+1\right)^{\frac{1}{3}}x^2+3\left(2x^7+1\right)^{\frac{2}{3}}x+1}{2x^7+x^3+1}\right)+6\left(2x^7+1\right)^{\frac{1}{3}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^7+1)^(1/3)*(8*x^7-3)/x^2/(2*x^7+x^3+1),x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*sqrt(3)*x*arctan((8377128467638*sqrt(3)*(2*x^7 + 1)^(1/3)*x^2 + 1517
1948325814*sqrt(3)*(2*x^7 + 1)^(2/3)*x + sqrt(3)*(2102123379894*x^7 + 44484
71619035*x^3 + 1051061689947))/(60468559237154*x^7 - 5089335571601*x^3 + 30
234279618577)) - x*log((2*x^7 + x^3 + 3*(2*x^7 + 1)^(1/3)*x^2 + 3*(2*x^7 +
1)^(2/3)*x + 1)/(2*x^7 + x^3 + 1)) + 6*(2*x^7 + 1)^(1/3))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(8x^7-3\right)\left(2x^7+1\right)^{\frac{1}{3}}}{\left(2x^7+x^3+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^7+1)^(1/3)*(8*x^7-3)/x^2/(2*x^7+x^3+1),x, algorithm="giac")
```

```
[Out] integrate((8*x^7 - 3)*(2*x^7 + 1)^(1/3)/((2*x^7 + x^3 + 1)*x^2), x)
```

maple [C] time = 6.00, size = 778, normalized size = 7.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^7+1)^(1/3)*(8*x^7-3)/x^2/(2*x^7+x^3+1),x)
```

```
[Out] 3*(2*x^7+1)^(1/3)/x+(RootOf(_Z^2-_Z+1)*ln((-8*x^14*RootOf(_Z^2-_Z+1)+4*x^14
+4*RootOf(_Z^2-_Z+1)^2*x^10-2*x^10*RootOf(_Z^2-_Z+1)+6*RootOf(_Z^2-_Z+1)*(4
*x^14+4*x^7+1)^(1/3)*x^8-6*(4*x^14+4*x^7+1)^(1/3)*x^8-8*x^7*RootOf(_Z^2-_Z+
1)+4*x^7+2*RootOf(_Z^2-_Z+1)^2*x^3-3*RootOf(_Z^2-_Z+1)*(4*x^14+4*x^7+1)^(2/
3)*x^2-RootOf(_Z^2-_Z+1)*x^3+3*(4*x^14+4*x^7+1)^(2/3)*x^2+3*RootOf(_Z^2-_Z+
1)*(4*x^14+4*x^7+1)^(1/3)*x-3*(4*x^14+4*x^7+1)^(1/3)*x-2*RootOf(_Z^2-_Z+1)+
1)/(2*x^7+1)/(2*x^7+x^3+1))-ln((8*x^14*RootOf(_Z^2-_Z+1)-4*x^14+4*RootOf(_Z
^2-_Z+1)^2*x^10-6*x^10*RootOf(_Z^2-_Z+1)-6*RootOf(_Z^2-_Z+1)*(4*x^14+4*x^7+
1)^(1/3)*x^8+2*x^10+8*x^7*RootOf(_Z^2-_Z+1)-4*x^7+2*RootOf(_Z^2-_Z+1)^2*x^3
+3*RootOf(_Z^2-_Z+1)*(4*x^14+4*x^7+1)^(2/3)*x^2-3*RootOf(_Z^2-_Z+1)*x^3-3*R
ootOf(_Z^2-_Z+1)*(4*x^14+4*x^7+1)^(1/3)*x+x^3+2*RootOf(_Z^2-_Z+1)-1)/(2*x^7
+1)/(2*x^7+x^3+1))*RootOf(_Z^2-_Z+1)+ln((8*x^14*RootOf(_Z^2-_Z+1)-4*x^14+4*
RootOf(_Z^2-_Z+1)^2*x^10-6*x^10*RootOf(_Z^2-_Z+1)-6*RootOf(_Z^2-_Z+1)*(4*x^
14+4*x^7+1)^(1/3)*x^8+2*x^10+8*x^7*RootOf(_Z^2-_Z+1)-4*x^7+2*RootOf(_Z^2-_Z
+1)^2*x^3+3*RootOf(_Z^2-_Z+1)*(4*x^14+4*x^7+1)^(2/3)*x^2-3*RootOf(_Z^2-_Z+1
)*x^3-3*RootOf(_Z^2-_Z+1)*(4*x^14+4*x^7+1)^(1/3)*x+x^3+2*RootOf(_Z^2-_Z+1)-
1)/(2*x^7+1)/(2*x^7+x^3+1)))/(2*x^7+1)^(2/3)*((2*x^7+1)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(8x^7-3\right)\left(2x^7+1\right)^{\frac{1}{3}}}{\left(2x^7+x^3+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^7+1)^(1/3)*(8*x^7-3)/x^2/(2*x^7+x^3+1),x, algorithm="maxima"
)
```

```
[Out] integrate((8*x^7 - 3)*(2*x^7 + 1)^(1/3)/((2*x^7 + x^3 + 1)*x^2), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^7+1\right)^{1/3}\,\left(8\,x^7-3\right)}{x^2\,\left(2\,x^7+x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^7 + 1)^(1/3)*(8*x^7 - 3))/(x^2*(x^3 + 2*x^7 + 1)),x)
```

```
[Out] int(((2*x^7 + 1)^(1/3)*(8*x^7 - 3))/(x^2*(x^3 + 2*x^7 + 1)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{2x^7+1}\,\left(8x^7-3\right)}{x^2\left(2x^7+x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**7+1)**(1/3)*(8*x**7-3)/x**2/(2*x**7+x**3+1),x)
```

```
[Out] Integral((2*x**7 + 1)**(1/3)*(8*x**7 - 3)/(x**2*(2*x**7 + x**3 + 1)), x)
```

3.1237 $$\int \frac{(-1+3x^4)\sqrt{1+x+2x^4+x^5+x^8}}{x^2(4+x+4x^4)}\,dx$$

Optimal. Leaf size=100

$$\frac{\sqrt{x^8+x^5+2x^4+x+1}}{4x}-\frac{1}{8}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{x^8+x^5+2x^4+x+1}}{x^4+x+1}\right)+\frac{1}{8}\tanh^{-1}\left(\frac{\sqrt{x^8+x^5+2x^4+x+1}}{x^4+x+1}\right)$$

Rubi [F] time = 1.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+3x^4)\sqrt{1+x+2x^4+x^5+x^8}}{x^2(4+x+4x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + 3*x^4)*Sqrt[1 + x + 2*x^4 + x^5 + x^8])/(x^2*(4 + x + 4*x^4)),x]
```

```
[Out] -1/4*Defer[Int][Sqrt[1 + x + 2*x^4 + x^5 + x^8]/x^2, x] + Defer[Int][Sqrt[1
 + x + 2*x^4 + x^5 + x^8]/x, x]/16 + Defer[Int][Sqrt[1 + x + 2*x^4 + x^5 +
x^8]/(-4 - x - 4*x^4), x]/16 + 4*Defer[Int][(x^2*Sqrt[1 + x + 2*x^4 + x^5 +
 x^8])/(4 + x + 4*x^4), x] - Defer[Int][(x^3*Sqrt[1 + x + 2*x^4 + x^5 + x^8
])/(4 + x + 4*x^4), x]/4
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+3x^4)\sqrt{1+x+2x^4+x^5+x^8}}{x^2(4+x+4x^4)}\,dx &= \int \left(-\frac{\sqrt{1+x+2x^4+x^5+x^8}}{4x^2}+\frac{\sqrt{1+x+2x^4+x^5+x^8}}{16x}+\frac{(-1+\cdots\right.\\
&= \frac{1}{16}\int \frac{\sqrt{1+x+2x^4+x^5+x^8}}{x}\,dx+\frac{1}{16}\int \frac{(-1+64x^2-4x^3)\sqrt{1\cdots}}{4+x+\cdots}\\
&= \frac{1}{16}\int \frac{\sqrt{1+x+2x^4+x^5+x^8}}{x}\,dx+\frac{1}{16}\int \left(\frac{\sqrt{1+x+2x^4+x^5+\cdots}}{-4-x-4x^4}\right.\\
&= \frac{1}{16}\int \frac{\sqrt{1+x+2x^4+x^5+x^8}}{x}\,dx+\frac{1}{16}\int \frac{\sqrt{1+x+2x^4+x^5+\cdots}}{-4-x-4x^4}
\end{aligned}$$

Mathematica [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(-1+3x^4)\sqrt{1+x+2x^4+x^5+x^8}}{x^2(4+x+4x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + 3*x^4)*Sqrt[1 + x + 2*x^4 + x^5 + x^8])/(x^2*(4 + x + 4*x^
4)),x]
```

```
[Out] Integrate[((-1 + 3*x^4)*Sqrt[1 + x + 2*x^4 + x^5 + x^8])/(x^2*(4 + x + 4*x^
4)), x]
```

IntegrateAlgebraic [A] time = 0.40, size = 100, normalized size = 1.00

$$\frac{\sqrt{x^8+x^5+2x^4+x+1}}{4x}-\frac{1}{8}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{x^8+x^5+2x^4+x+1}}{x^4+x+1}\right)+\frac{1}{8}\tanh^{-1}\left(\frac{\sqrt{x^8+x^5+2x^4+x+1}}{x^4+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + 3*x^4)*Sqrt[1 + x + 2*x^4 + x^5 + x^8])/(x^2*(4 +
 x + 4*x^4)),x]
```

```
[Out] Sqrt[1 + x + 2*x^4 + x^5 + x^8]/(4*x) - (Sqrt[3]*ArcTan[(Sqrt[3]*Sqrt[1 + x
 + 2*x^4 + x^5 + x^8])/(1 + x + x^4)])/8 + ArcTanh[Sqrt[1 + x + 2*x^4 + x^5
 + x^8]/(1 + x + x^4)]/8
```

fricas [A] time = 2.82, size = 94, normalized size = 0.94

$$-\frac{\sqrt{3}\,x\arctan\left(\frac{\sqrt{3}\left(2\,x^4-x+2\right)}{6\,\sqrt{x^8+x^5+2\,x^4+x+1}}\right)+x\log\left(\frac{2\,x^4+x-2\,\sqrt{x^8+x^5+2\,x^4+x+1}+2}{x}\right)-4\,\sqrt{x^8+x^5+2\,x^4+x+1}}{16\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)*(x^8+x^5+2*x^4+x+1)^(1/2)/x^2/(4*x^4+x+4),x, algorithm=
"fricas")
```

```
[Out] -1/16*(sqrt(3)*x*arctan(1/6*sqrt(3)*(2*x^4 - x + 2)/sqrt(x^8 + x^5 + 2*x^4
+ x + 1)) + x*log((2*x^4 + x - 2*sqrt(x^8 + x^5 + 2*x^4 + x + 1) + 2)/x) -
4*sqrt(x^8 + x^5 + 2*x^4 + x + 1))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^8+x^5+2\,x^4+x+1}\left(3\,x^4-1\right)}{\left(4\,x^4+x+4\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)*(x^8+x^5+2*x^4+x+1)^(1/2)/x^2/(4*x^4+x+4),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(x^8 + x^5 + 2*x^4 + x + 1)*(3*x^4 - 1)/((4*x^4 + x + 4)*x^2)
, x)
```

maple [C] time = 0.84, size = 125, normalized size = 1.25

$$\frac{\sqrt{x^8+x^5+2x^4+x+1}}{4x}+\frac{\ln\left(-\frac{2x^4+2\sqrt{x^8+x^5+2x^4+x+1}+x+2}{x}\right)}{16}+\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+3\right)x^4-\mathrm{RootOf}\left(\_Z^2+3\right)x+2\,\mathrm{RootOf}\left(\_Z^2+3\right)+6\sqrt{x^8+x^5+2x^4+x+1}}{4x^4+x+4}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4-1)*(x^8+x^5+2*x^4+x+1)^(1/2)/x^2/(4*x^4+x+4),x)
```

```
[Out] 1/4*(x^8+x^5+2*x^4+x+1)^(1/2)/x+1/16*ln(-(2*x^4+2*(x^8+x^5+2*x^4+x+1)^(1/2)
+x+2)/x)+1/16*RootOf(_Z^2+3)*ln(-(2*RootOf(_Z^2+3)*x^4-RootOf(_Z^2+3)*x+2*R
ootOf(_Z^2+3)+6*(x^8+x^5+2*x^4+x+1)^(1/2))/(4*x^4+x+4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^8+x^5+2\,x^4+x+1}\left(3\,x^4-1\right)}{\left(4\,x^4+x+4\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-1)*(x^8+x^5+2*x^4+x+1)^(1/2)/x^2/(4*x^4+x+4),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(x^8 + x^5 + 2*x^4 + x + 1)*(3*x^4 - 1)/((4*x^4 + x + 4)*x^2)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^4-1\right)\,\sqrt{x^8+x^5+2\,x^4+x+1}}{x^2\,\left(4\,x^4+x+4\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^4 - 1)*(x + 2*x^4 + x^5 + x^8 + 1)^(1/2))/(x^2*(x + 4*x^4 + 4)),x
)
```

```
[Out] int(((3*x^4 - 1)*(x + 2*x^4 + x^5 + x^8 + 1)^(1/2))/(x^2*(x + 4*x^4 + 4)),
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4-1)*(x**8+x**5+2*x**4+x+1)**(1/2)/x**2/(4*x**4+x+4),x)
```

```
[Out] Timed out
```

3.1238 $\int \frac{-b+cx^4+ax^8}{x^2\left(-b+ax^4\right)^{3/4}}\,dx$

Optimal. Leaf size=100

$$\frac{(-3b-4c)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{8a^{3/4}}+\frac{(3b+4c)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{8a^{3/4}}+\frac{\left(x^4-4\right)\sqrt[4]{ax^4-b}}{4x}$$

Rubi [A] time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1486, 451, 331, 298, 203, 206}

$$-\frac{(3b+4c)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{8a^{3/4}}+\frac{(3b+4c)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{8a^{3/4}}-\frac{\sqrt[4]{ax^4-b}}{x}+\frac{1}{4}x^3\sqrt[4]{ax^4-b}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + c*x^4 + a*x^8)/(x^2*(-b + a*x^4)^(3/4)),x]
```

```
[Out] -((-b + a*x^4)^(1/4)/x) + (x^3*(-b + a*x^4)^(1/4))/4 - ((3*b + 4*c)*ArcTan[
(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(8*a^(3/4)) + ((3*b + 4*c)*ArcTanh[(a^(1/4
)*x)/(-b + a*x^4)^(1/4)])/(8*a^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1486

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 2*n*p - n
+ 1)*(d + e*x^n)^(q + 1))/(e*f^(2*n*p - n + 1)*(m + 2*n*p + n*q + 1)), x] +
 Dist[1/(e*(m + 2*n*p + n*q + 1)), Int[(f*x)^m*(d + e*x^n)^q*ExpandToSum[e*
(m + 2*n*p + n*q + 1)*((a + b*x^n + c*x^(2*n))^p - c^p*x^(2*n*p)) - d*c^p*(
m + 2*n*p - n + 1)*x^(2*n*p - n), x], x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGtQ[p, 0]
&& GtQ[2*n*p, n - 1] &&  !IntegerQ[q] && NeQ[m + 2*n*p + n*q + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-b+cx^4+ax^8}{x^2\left(-b+ax^4\right)^{3/4}}\,dx &= \frac{1}{4}x^3\sqrt[4]{-b+ax^4}+\frac{\int \frac{-4ab+a(3b+4c)x^4}{x^2\left(-b+ax^4\right)^{3/4}}\,dx}{4a}\\
&= -\frac{\sqrt[4]{-b+ax^4}}{x}+\frac{1}{4}x^3\sqrt[4]{-b+ax^4}+\frac{1}{4}(3b+4c)\int \frac{x^2}{\left(-b+ax^4\right)^{3/4}}\,dx\\
&= -\frac{\sqrt[4]{-b+ax^4}}{x}+\frac{1}{4}x^3\sqrt[4]{-b+ax^4}+\frac{1}{4}(3b+4c)\,\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)\\
&= -\frac{\sqrt[4]{-b+ax^4}}{x}+\frac{1}{4}x^3\sqrt[4]{-b+ax^4}+\frac{(3b+4c)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)}{8\sqrt{a}}-\frac{(3b+\cdots}{\cdots}\\
&= -\frac{\sqrt[4]{-b+ax^4}}{x}+\frac{1}{4}x^3\sqrt[4]{-b+ax^4}-\frac{(3b+4c)\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{-b+ax^4}}\right)}{8a^{3/4}}+\frac{(3b+4c)\tanh^{-1}\left(\cdots\right)}{8a^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 99, normalized size = 0.99

$$\frac{2a^{3/4}\left(x^4-4\right)\sqrt[4]{ax^4-b}-x(3b+4c)\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)+x(3b+4c)\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{8a^{3/4}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + c*x^4 + a*x^8)/(x^2*(-b + a*x^4)^(3/4)),x]
```

```
[Out] (2*a^(3/4)*(-4 + x^4)*(-b + a*x^4)^(1/4) - (3*b + 4*c)*x*ArcTan[(a^(1/4)*x)
/(-b + a*x^4)^(1/4)] + (3*b + 4*c)*x*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)
])/(8*a^(3/4)*x)
```

IntegrateAlgebraic [A] time = 1.09, size = 100, normalized size = 1.00

$$\frac{(-3b-4c)\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{8a^{3/4}}+\frac{(3b+4c)\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{8a^{3/4}}+\frac{\left(x^4-4\right)\sqrt[4]{ax^4-b}}{4x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + c*x^4 + a*x^8)/(x^2*(-b + a*x^4)^(3/4)),x]
```

```
[Out] ((-4 + x^4)*(-b + a*x^4)^(1/4))/(4*x) + ((-3*b - 4*c)*ArcTan[(a^(1/4)*x)/(-
b + a*x^4)^(1/4)])/(8*a^(3/4)) + ((3*b + 4*c)*ArcTanh[(a^(1/4)*x)/(-b + a*x
^4)^(1/4)])/(8*a^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+c*x^4-b)/x^2/(a*x^4-b)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^8 + cx^4 - b}{\left(ax^4 - b\right)^{\frac{3}{4}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+c*x^4-b)/x^2/(a*x^4-b)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^8 + c*x^4 - b)/((a*x^4 - b)^(3/4)*x^2), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{a\,x^8 + c\,x^4 - b}{x^2 \left(a\,x^4 - b\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^8+c*x^4-b)/x^2/(a*x^4-b)^(3/4),x)
```

```
[Out] int((a*x^8+c*x^4-b)/x^2/(a*x^4-b)^(3/4),x)
```

maxima [B] time = 0.42, size = 217, normalized size = 2.17

$$\frac{1}{4}\,c\left(\frac{2\,\arctan\left(\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{3}{4}}} - \frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{3}{4}}}\right) + \frac{1}{16}\,a\left(\frac{3\left(\frac{2\,b\arctan\left(\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{3}{4}}} - \frac{b\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{3}{4}}}\right)}{a} + \frac{4\left(ax^4-b\right)^{\frac{1}{4}}b}{\left(a^2 - \frac{\left(ax^4-b\right)a}{x^4}\right)x}\right) - \frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+c*x^4-b)/x^2/(a*x^4-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/4*c*(2*arctan((a*x^4 - b)^(1/4)/(a^(1/4)*x))/a^(3/4) - log(-(a^(1/4) - (a
*x^4 - b)^(1/4)/x)/(a^(1/4) + (a*x^4 - b)^(1/4)/x))/a^(3/4)) + 1/16*a*(3*(2
*b*arctan((a*x^4 - b)^(1/4)/(a^(1/4)*x))/a^(3/4) - b*log(-(a^(1/4) - (a*x^4
 - b)^(1/4)/x)/(a^(1/4) + (a*x^4 - b)^(1/4)/x))/a^(3/4))/a + 4*(a*x^4 - b)^
(1/4)*b/((a^2 - (a*x^4 - b)*a/x^4)*x)) - (a*x^4 - b)^(1/4)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x^8 + c\,x^4 - b}{x^2\,{\left(a\,x^4 - b\right)}^{3/4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^8 - b + c*x^4)/(x^2*(a*x^4 - b)^(3/4)),x)
```

[Out] int((a*x^8 - b + c*x^4)/(x^2*(a*x^4 - b)^(3/4)), x)

sympy [C] time = 3.17, size = 168, normalized size = 1.68

$$-\frac{ax^7e^{\frac{i\pi}{4}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{ax^4}{b}\right)}{4b^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}-b\left(\begin{cases}-\frac{\sqrt[4]{a}\sqrt[4]{-1+\frac{b}{ax^4}}e^{\frac{i\pi}{4}}\Gamma\left(-\frac{1}{4}\right)}{4b\Gamma\left(\frac{3}{4}\right)} & \text{for } \left|\frac{b}{ax^4}\right|>1\\ -\frac{\sqrt[4]{a}\sqrt[4]{1-\frac{b}{ax^4}}\Gamma\left(-\frac{1}{4}\right)}{4b\Gamma\left(\frac{3}{4}\right)} & \text{otherwise}\end{cases}\right)+\frac{cx^3e^{-\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{ax^4}{b}\right)}{4b^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**8+c*x**4-b)/x**2/(a*x**4-b)**(3/4),x)

[Out] -a*x**7*exp(I*pi/4)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), a*x**4/b)/(4*b**(3/4)*gamma(11/4)) - b*Piecewise((-a**(1/4)*(-1 + b/(a*x**4))**(1/4)*exp(I*pi/4)*gamma(-1/4)/(4*b*gamma(3/4)), Abs(b/(a*x**4)) > 1), (-a**(1/4)*(1 - b/(a*x**4))**(1/4)*gamma(-1/4)/(4*b*gamma(3/4)), True)) + c*x**3*exp(-3*I*pi/4)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), a*x**4/b)/(4*b**(3/4)*gamma(7/4))

3.1239 $\int \frac{1}{(1+x)\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=100

$$\frac{2}{\sqrt{\sqrt{x^2+1}+x}}+2\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{1+\sqrt{2}}}\right)-2\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{\sqrt{2}-1}}\right)$$

Rubi [A] time = 0.15, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2119, 1628, 828, 826, 1166, 207, 203}

$$\frac{2}{\sqrt{\sqrt{x^2+1}+x}}+\frac{2\tan^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{1+\sqrt{2}}}-\frac{2\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{\sqrt{2}-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((1 + x)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] 2/Sqrt[x + Sqrt[1 + x^2]] + (2*ArcTan[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 +
x^2]]])/Sqrt[1 + Sqrt[2]] - (2*ArcTanh[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 +
x^2]]])/Sqrt[-1 + Sqrt[2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(1+x)\sqrt{x+\sqrt{1+x^2}}}\,dx &= \text{Subst}\left(\int \frac{1+x^2}{x^{3/2}\left(-1+2x+x^2\right)}\,dx, x, x+\sqrt{1+x^2}\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{x^{3/2}}+\frac{2(1-x)}{x^{3/2}\left(-1+2x+x^2\right)}\right)dx, x, x+\sqrt{1+x^2}\right)\\
&= -\frac{2}{\sqrt{x+\sqrt{1+x^2}}}+2\,\text{Subst}\left(\int \frac{1-x}{x^{3/2}\left(-1+2x+x^2\right)}\,dx, x, x+\sqrt{1+x^2}\right)\\
&= \frac{2}{\sqrt{x+\sqrt{1+x^2}}}-2\,\text{Subst}\left(\int \frac{-1-x}{\sqrt{x}\left(-1+2x+x^2\right)}\,dx, x, x+\sqrt{1+x^2}\right)\\
&= \frac{2}{\sqrt{x+\sqrt{1+x^2}}}-4\,\text{Subst}\left(\int \frac{-1-x^2}{-1+2x^2+x^4}\,dx, x, \sqrt{x+\sqrt{1+x^2}}\right)\\
&= \frac{2}{\sqrt{x+\sqrt{1+x^2}}}+2\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}+x^2}\,dx, x, \sqrt{x+\sqrt{1+x^2}}\right)+2\,\text{Subst}\Bigg(\\
&= \frac{2}{\sqrt{x+\sqrt{1+x^2}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{x+\sqrt{1+x^2}}}{\sqrt{1+\sqrt{2}}}\right)}{\sqrt{1+\sqrt{2}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{1+x^2}}}{\sqrt{-1+\sqrt{2}}}\right)}{\sqrt{-1+\sqrt{2}}}
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 100, normalized size = 1.00

$$
2\left(\frac{1}{\sqrt{\sqrt{x^2+1}+x}}-\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{1}{\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}}\right)-\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\frac{1}{\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] 2*(1/Sqrt[x + Sqrt[1 + x^2]] - Sqrt[-1 + Sqrt[2]]*ArcTan[1/(Sqrt[-1 + Sqrt[
2]]*Sqrt[x + Sqrt[1 + x^2]])] - Sqrt[1 + Sqrt[2]]*ArcTanh[1/(Sqrt[1 + Sqrt[
2]]*Sqrt[x + Sqrt[1 + x^2]])])
```

IntegrateAlgebraic [A] time = 0.21, size = 100, normalized size = 1.00

$$\frac{2}{\sqrt{\sqrt{x^2+1}+x}}+2\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\sqrt{\sqrt{2}-1}\,\sqrt{\sqrt{x^2+1}+x}\right)-2\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\,\sqrt{\sqrt{x^2+1}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] 2/Sqrt[x + Sqrt[1 + x^2]] + 2*Sqrt[-1 + Sqrt[2]]*ArcTan[Sqrt[-1 + Sqrt[2]]*
Sqrt[x + Sqrt[1 + x^2]]] - 2*Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[1 + Sqrt[2]]*Sq
rt[x + Sqrt[1 + x^2]]]
```

fricas [B] time = 0.44, size = 154, normalized size = 1.54

$$-2\sqrt{x+\sqrt{x^2+1}}\left(x-\sqrt{x^2+1}\right)-4\sqrt{\sqrt{2}-1}\arctan\left(\sqrt{x+\sqrt{2}+\sqrt{x^2+1}+1}\sqrt{\sqrt{2}-1}-\sqrt{x+\sqrt{x^2+1}}\sqrt{\sqrt{2}-1}\right)-\sqrt{\sqrt{2}+1}\log\left(2\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+2\sqrt{x+\sqrt{x^2+1}}\right)+\sqrt{\sqrt{2}+1}\log\left(-2\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+2\sqrt{x+\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(x + sqrt(x^2 + 1))*(x - sqrt(x^2 + 1)) - 4*sqrt(sqrt(2) - 1)*arctan
(sqrt(x + sqrt(2) + sqrt(x^2 + 1) + 1)*sqrt(sqrt(2) - 1) - sqrt(x + sqrt(x^
2 + 1))*sqrt(sqrt(2) - 1)) - sqrt(sqrt(2) + 1)*log(2*sqrt(sqrt(2) + 1)*(sqr
t(2) - 1) + 2*sqrt(x + sqrt(x^2 + 1))) + sqrt(sqrt(2) + 1)*log(-2*sqrt(sqrt
(2) + 1)*(sqrt(2) - 1) + 2*sqrt(x + sqrt(x^2 + 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+\sqrt{x^2+1}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x + sqrt(x^2 + 1))*(x + 1)), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(1/(1+x)/(x+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+\sqrt{x^2+1}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x + sqrt(x^2 + 1))*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{x+\sqrt{x^2+1}}\,\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + (x^2 + 1)^(1/2))^(1/2)*(x + 1)),x)
```

```
[Out] int(1/((x + (x^2 + 1)^(1/2))^(1/2)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/((x + 1)*sqrt(x + sqrt(x**2 + 1))), x)
```

3.1240 $\int \frac{1}{x\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$

Optimal. Leaf size=100

$$\frac{2}{\sqrt{\sqrt{a^2x^2+b^2}+ax}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.12, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2119, 453, 329, 298, 203, 206}

$$\frac{2}{\sqrt{\sqrt{a^2x^2+b^2}+ax}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

```
[Out] 2/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]] + (2*ArcTan[Sqrt[a*x + Sqrt[b^2 + a^2*x^2
]]/Sqrt[b]])/Sqrt[b] - (2*ArcTanh[Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]/Sqrt[b]])
/Sqrt[b]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx &= \text{Subst}\left(\int \frac{b^2+x^2}{x^{3/2}\left(-b^2+x^2\right)}\,dx, x, ax+\sqrt{b^2+a^2x^2}\right) \\
&= \frac{2}{\sqrt{ax+\sqrt{b^2+a^2x^2}}} + 2\,\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx, x, ax+\sqrt{b^2+a^2x^2}\right) \\
&= \frac{2}{\sqrt{ax+\sqrt{b^2+a^2x^2}}} + 4\,\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{ax+\sqrt{b^2+a^2x^2}}\right) \\
&= \frac{2}{\sqrt{ax+\sqrt{b^2+a^2x^2}}} - 2\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{ax+\sqrt{b^2+a^2x^2}}\right) + 2\,\text{Subst}\Bigg(\\
&= \frac{2}{\sqrt{ax+\sqrt{b^2+a^2x^2}}} + \frac{2\tan^{-1}\left(\frac{\sqrt{ax+\sqrt{b^2+a^2x^2}}}{\sqrt{b}}\right)}{\sqrt{b}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{ax+\sqrt{b^2+a^2x^2}}}{\sqrt{b}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [C] time = 2.19, size = 204, normalized size = 2.04

$$\frac{2}{3}\sqrt{\sqrt{a^2x^2+b^2}+ax}\left(\frac{\sqrt{a^2x^2+b^2}-2ax}{b^2}-\frac{\left(a^2x^2+b^2\right)\left(\sqrt{a^2x^2+b^2}+ax\right)\left(2\left(2ax\left(\sqrt{a^2x^2+b^2}+ax\right)+b^2\right){}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{\left(ax+\sqrt{b^2+a^2x^2}\right)^2}{b^2}\right)-ax\left(\sqrt{a^2x^2+b^2}+ax\right)-2b^2\right)}{\left(abx\left(\sqrt{a^2x^2+b^2}+ax\right)+b^3\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

```
[Out] (2*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]*((-2*a*x + Sqrt[b^2 + a^2*x^2])/b^2 - ((
b^2 + a^2*x^2)*(a*x + Sqrt[b^2 + a^2*x^2])*(-2*b^2 - a*x*(a*x + Sqrt[b^2 +
a^2*x^2]) + 2*(b^2 + 2*a*x*(a*x + Sqrt[b^2 + a^2*x^2]))*Hypergeometric2F1[3
/4, 1, 7/4, (a*x + Sqrt[b^2 + a^2*x^2])^2/b^2]))/(b^3 + a*b*x*(a*x + Sqrt[b
^2 + a^2*x^2]))^2))/3
```

IntegrateAlgebraic [A] time = 0.13, size = 100, normalized size = 1.00

$$\frac{2}{\sqrt{\sqrt{a^2x^2+b^2}+ax}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

```
[Out] 2/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]] + (2*ArcTan[Sqrt[a*x + Sqrt[b^2 + a^2*x^2
]]/Sqrt[b]])/Sqrt[b] - (2*ArcTanh[Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]/Sqrt[b]])
/Sqrt[b]
```

fricas [A] time = 0.44, size = 321, normalized size = 3.21

$$\left[\frac{2\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{ax+\sqrt{a^2x^2+b^2}}}{\sqrt{b}}\right)+b^{\frac{3}{2}}\log\left(\frac{b^2+\sqrt{ax+\sqrt{a^2x^2+b^2}}\left((ax-b)\sqrt{b}-\sqrt{a^2x^2+b^2}\,\sqrt{b}\right)+\sqrt{a^2x^2+b^2}\,b}{x}\right)-2\sqrt{ax+\sqrt{a^2x^2+b^2}}\left(ax-\sqrt{a^2x^2+b^2}\right)}{b^2}, \frac{2\sqrt{-b}\,b\arctan\left(\frac{\sqrt{ax+\sqrt{a^2x^2+b^2}}\,\sqrt{-b}}{b}\right)-\sqrt{-b}\,b\log\left(-\frac{b^2+\sqrt{ax+\sqrt{a^2x^2+b^2}}\left((ax+b)\sqrt{-b}-\sqrt{a^2x^2+b^2}\,\sqrt{-b}\right)-\sqrt{a^2x^2+b^2}\,b}{x}\right)-2\sqrt{ax+\sqrt{a^2x^2+b^2}}\left(ax-\sqrt{a^2x^2+b^2}\right)}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [(2*b^(3/2)*arctan(sqrt(a*x + sqrt(a^2*x^2 + b^2))/sqrt(b)) + b^(3/2)*log((
b^2 + sqrt(a*x + sqrt(a^2*x^2 + b^2))*((a*x - b)*sqrt(b) - sqrt(a^2*x^2 + b
^2)*sqrt(b)) + sqrt(a^2*x^2 + b^2)*b)/x) - 2*sqrt(a*x + sqrt(a^2*x^2 + b^2)
)*(a*x - sqrt(a^2*x^2 + b^2)))/b^2, (2*sqrt(-b)*b*arctan(sqrt(a*x + sqrt(a^
2*x^2 + b^2))*sqrt(-b)/b) - sqrt(-b)*b*log(-(b^2 + sqrt(a*x + sqrt(a^2*x^2
+ b^2))*((a*x + b)*sqrt(-b) - sqrt(a^2*x^2 + b^2)*sqrt(-b)) - sqrt(a^2*x^2
+ b^2)*b)/x) - 2*sqrt(a*x + sqrt(a^2*x^2 + b^2))*(a*x - sqrt(a^2*x^2 + b^2)
))/b^2]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + \sqrt{a^2x^2 + b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + sqrt(a^2*x^2 + b^2))*x), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{ax + \sqrt{a^2x^2 + b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/x/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + \sqrt{a^2x^2 + b^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + sqrt(a^2*x^2 + b^2))*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{a\,x + \sqrt{a^2\,x^2 + b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2)),x)
[Out] int(1/(x*(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{ax + \sqrt{a^2x^2 + b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x+(a**2*x**2+b**2)**(1/2))**(1/2),x)
[Out] Integral(1/(x*sqrt(a*x + sqrt(a**2*x**2 + b**2))), x)
```

3.1241 $\int \frac{(-3+x)^6\left(-1-x+x^2\right)^{3/2}}{-1+x}\,dx$

Optimal. Leaf size=101

$$\frac{19451047\log\left(2\sqrt{x^2-x-1}-2x+1\right)}{65536}+128\tan^{-1}\left(\sqrt{x^2-x-1}-x+1\right)+\frac{\sqrt{x^2-x-1}\left(1146880x^8-23296000x\right.}{}$$

Rubi [A] time = 0.37, antiderivative size = 199, normalized size of antiderivative = 1.97, number of steps used = 12, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1653, 814, 843, 621, 206, 724, 204}

$$\frac{1}{9}\left(x^2-x-1\right)^{5/2}(1-x)^4+\frac{229}{144}\left(x^2-x-1\right)^{5/2}(1-x)^3+\frac{19927\left(x^2-x-1\right)^{5/2}(1-x)^2}{2016}+\frac{281233\left(x^2-x-1\right)^{5/2}(1-x)}{8064}+\frac{6158183\left(x^2-x-1\right)^{5/2}}{80640}+\frac{(903871-1283454x)\left(x^2-x-1\right)^{3/2}}{12288}-\frac{(5567931-6941558x)\sqrt{x^2-x-1}}{32768}-64\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)+\frac{19451047\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)}{65536}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + x)^6*(-1 - x + x^2)^(3/2))/(-1 + x),x]
```

```
[Out] -1/32768*((5567931 - 6941558*x)*Sqrt[-1 - x + x^2]) + ((903871 - 1283454*x)
*(-1 - x + x^2)^(3/2))/12288 + (6158183*(-1 - x + x^2)^(5/2))/80640 + (2812
33*(1 - x)*(-1 - x + x^2)^(5/2))/8064 + (19927*(1 - x)^2*(-1 - x + x^2)^(5/
2))/2016 + (229*(1 - x)^3*(-1 - x + x^2)^(5/2))/144 + ((1 - x)^4*(-1 - x +
x^2)^(5/2))/9 - 64*ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])] + (19451047*ArcTa
nh[(1 - 2*x)/(2*Sqrt[-1 - x + x^2])])/65536
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
```

```
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(-3+x)^6\left(-1-x+x^2\right)^{3/2}}{-1+x}\,dx = \frac{1}{9}(1-x)^4\left(-1-x+x^2\right)^{5/2}+\frac{1}{9}\int \frac{\left(-1-x+x^2\right)^{3/2}\left(\frac{13125}{2}-\frac{26233x}{2}+10889x^2-\right.}{-1+x}$$

$$=\frac{229}{144}(1-x)^3\left(-1-x+x^2\right)^{5/2}+\frac{1}{9}(1-x)^4\left(-1-x+x^2\right)^{5/2}+\frac{1}{72}\int \frac{\left(-1-x+x\right.}{}$$

$$=\frac{19927(1-x)^2\left(-1-x+x^2\right)^{5/2}}{2016}+\frac{229}{144}(1-x)^3\left(-1-x+x^2\right)^{5/2}+\frac{1}{9}(1-x)^4\left(-1\right.$$

$$=\frac{281233(1-x)\left(-1-x+x^2\right)^{5/2}}{8064}+\frac{19927(1-x)^2\left(-1-x+x^2\right)^{5/2}}{2016}+\frac{229}{144}(1-x)$$

$$=\frac{6158183\left(-1-x+x^2\right)^{5/2}}{80640}+\frac{281233(1-x)\left(-1-x+x^2\right)^{5/2}}{8064}+\frac{19927(1-x)^2\left(\right.}{201}$$

$$=\frac{(903871-1283454x)\left(-1-x+x^2\right)^{3/2}}{12288}+\frac{6158183\left(-1-x+x^2\right)^{5/2}}{80640}+\frac{281233(}{}$$

$$=-\frac{(5567931-6941558x)\sqrt{-1-x+x^2}}{32768}+\frac{(903871-1283454x)\left(-1-x+x^2\right)^{3}}{12288}$$

$$=-\frac{(5567931-6941558x)\sqrt{-1-x+x^2}}{32768}+\frac{(903871-1283454x)\left(-1-x+x^2\right)^{3}}{12288}$$

$$=-\frac{(5567931-6941558x)\sqrt{-1-x+x^2}}{32768}+\frac{(903871-1283454x)\left(-1-x+x^2\right)^{3}}{12288}$$

$$=-\frac{(5567931-6941558x)\sqrt{-1-x+x^2}}{32768}+\frac{(903871-1283454x)\left(-1-x+x^2\right)^{3}}{12288}$$

Mathematica [A] time = 0.21, size = 107, normalized size = 1.06

$$-64\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)+\frac{19451047\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)}{65536}+\frac{\sqrt{x^2-x-1}\left(1146880x^8-23296000x^7+199009280x^6-910869760x^5+2304529024x^4-2700564848x^3-508033624x^2+4423205098x-1245336401\right)}{10321920}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + x)^6*(-1 - x + x^2)^(3/2))/(-1 + x),x]
```

```
[Out] (Sqrt[-1 - x + x^2]*(-1245336401 + 4423205098*x - 508033624*x^2 - 270056484
8*x^3 + 2304529024*x^4 - 910869760*x^5 + 199009280*x^6 - 23296000*x^7 + 114
6880*x^8))/10321920 - 64*ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])] + (19451047
*ArcTanh[(1 - 2*x)/(2*Sqrt[-1 - x + x^2])])/65536
```

IntegrateAlgebraic [A] time = 0.56, size = 101, normalized size = 1.00

$$\frac{19451047\log\left(2\sqrt{x^2-x-1}-2x+1\right)}{65536}+128\tan^{-1}\left(\sqrt{x^2-x-1}-x+1\right)+\frac{\sqrt{x^2-x-1}\left(1146880x^8-23296000x^7+199009280x^6-910869760x^5+2304529024x^4-2700564848x^3-508033624x^2+4423205098x-1245336401\right)}{10321920}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x)^6*(-1 - x + x^2)^(3/2))/(-1 + x),x]
```

```
[Out] (Sqrt[-1 - x + x^2]*(-1245336401 + 4423205098*x - 508033624*x^2 - 270056484
8*x^3 + 2304529024*x^4 - 910869760*x^5 + 199009280*x^6 - 23296000*x^7 + 114
6880*x^8))/10321920 + 128*ArcTan[1 - x + Sqrt[-1 - x + x^2]] + (19451047*Lo
g[1 - 2*x + 2*Sqrt[-1 - x + x^2]])/65536
```

fricas [A] time = 0.41, size = 91, normalized size = 0.90

$$\frac{1}{10321920}\left(1146880\,x^{8} - 23296000\,x^{7} + 199009280\,x^{6} - 910869760\,x^{5} + 2304529024\,x^{4} - 2700564848\,x^{3} - 508033624\,x^{2} + 4423205098\,x - 1245336401\right)\sqrt{x^{2} - x - 1} + 128\arctan\left(-x + \sqrt{x^{2} - x - 1} + 1\right) + \frac{19451047}{65536}\log\left(-2\,x + 2\sqrt{x^{2} - x - 1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)^6*(x^2-x-1)^(3/2)/(-1+x),x, algorithm="fricas")
```

```
[Out] 1/10321920*(1146880*x^8 - 23296000*x^7 + 199009280*x^6 - 910869760*x^5 + 23
04529024*x^4 - 2700564848*x^3 - 508033624*x^2 + 4423205098*x - 1245336401)*
sqrt(x^2 - x - 1) + 128*arctan(-x + sqrt(x^2 - x - 1) + 1) + 19451047/65536
*log(-2*x + 2*sqrt(x^2 - x - 1) + 1)
```

giac [A] time = 0.18, size = 92, normalized size = 0.91

$$\frac{1}{10321920}\left(2\left(4\left(2\left(8\left(10\left(4\left(14\left(16\,x - 325\right)x + 38869\right)x - 711617\right)x + 18004133\right)x - 168785303\right)x - 63504203\right)x + 2211602549\right)x - 1245336401\right)\sqrt{x^{2} - x - 1} + 128\arctan\left(-x + \sqrt{x^{2} - x - 1} + 1\right) + \frac{19451047}{65536}\log\left(\left|-2\,x + 2\sqrt{x^{2} - x - 1} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)^6*(x^2-x-1)^(3/2)/(-1+x),x, algorithm="giac")
```

```
[Out] 1/10321920*(2*(4*(2*(8*(10*(4*(14*(16*x - 325)*x + 38869)*x - 711617)*x + 1
8004133)*x - 168785303)*x - 63504203)*x + 2211602549)*x - 1245336401)*sqrt(
x^2 - x - 1) + 128*arctan(-x + sqrt(x^2 - x - 1) + 1) + 19451047/65536*log(
abs(-2*x + 2*sqrt(x^2 - x - 1) + 1))
```

maple [B] time = 0.03, size = 197, normalized size = 1.95

$$\frac{3208635(-1+2x)\sqrt{x^2-x-1}}{32768} + 8(-1+2x)\sqrt{(-1+x)^2+x-2} + \frac{x^4(x^2-x-1)^{\frac{5}{2}}}{9} - \frac{293x^3(x^2-x-1)^{\frac{5}{2}}}{144} + \frac{30889x^2(x^2-x-1)^{\frac{5}{2}}}{2016} - \frac{482705x(x^2-x-1)^{\frac{5}{2}}}{8064} - \frac{213909(-1+2x)(x^2-x-1)^{\frac{3}{2}}}{4096} + \frac{64\left((-1+x)^2+x-2\right)^{\frac{3}{2}}}{3} - 64\sqrt{(-1+x)^2+x-2} - 52\ln\left(x-\frac{1}{2}+\sqrt{(-1+x)^2+x-2}\right) + 64\arctan\left(\frac{-3+x}{2\sqrt{(-1+x)^2+x-2}}\right) - \frac{16043175\ln\left(x-\frac{1}{2}+\sqrt{x^2-x-1}\right)}{65536} + \frac{9904793(x^2-x-1)^{\frac{5}{2}}}{80640}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+x)^6*(x^2-x-1)^(3/2)/(-1+x),x)
```

```
[Out] 3208635/32768*(-1+2*x)*(x^2-x-1)^(1/2)+8*(-1+2*x)*((-1+x)^2+x-2)^(1/2)+1/9*
x^4*(x^2-x-1)^(5/2)-293/144*x^3*(x^2-x-1)^(5/2)+30889/2016*x^2*(x^2-x-1)^(5
/2)-482705/8064*x*(x^2-x-1)^(5/2)-213909/4096*(-1+2*x)*(x^2-x-1)^(3/2)+64/3
*((-1+x)^2+x-2)^(3/2)-64*((-1+x)^2+x-2)^(1/2)-52*ln(x-1/2+((-1+x)^2+x-2)^(1
/2))+64*arctan(1/2*(-3+x)/((-1+x)^2+x-2)^(1/2))-16043175/65536*ln(x-1/2+(x^
2-x-1)^(1/2))+9904793/80640*(x^2-x-1)^(5/2)
```

maxima [A] time = 0.42, size = 168, normalized size = 1.66

$$\frac{1}{9}\left(x^{2} - x - 1\right)^{\frac{5}{2}} x^{4} - \frac{293}{144}\left(x^{2} - x - 1\right)^{\frac{5}{2}} x^{3} + \frac{30889}{2016}\left(x^{2} - x - 1\right)^{\frac{5}{2}} x^{2} - \frac{482705}{8064}\left(x^{2} - x - 1\right)^{\frac{5}{2}} x + \frac{9904793}{80640}\left(x^{2} - x - 1\right)^{\frac{5}{2}} - \frac{213909}{2048}\left(x^{2} - x - 1\right)^{\frac{3}{2}} x + \frac{903871}{12288}\left(x^{2} - x - 1\right)^{\frac{3}{2}} + \frac{3470779}{16384}\sqrt{x^{2} - x - 1}\,x - \frac{5567931}{32768}\sqrt{x^{2} - x - 1} + 64\arcsin\left(\frac{\sqrt{5}\,x}{5\left|x - 1\right|} - \frac{3\sqrt{5}}{5\left|x - 1\right|}\right) - \frac{19451047}{65536}\log\left(2\,x + 2\sqrt{x^{2} - x - 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)^6*(x^2-x-1)^(3/2)/(-1+x),x, algorithm="maxima")
```

```
[Out] 1/9*(x^2 - x - 1)^(5/2)*x^4 - 293/144*(x^2 - x - 1)^(5/2)*x^3 + 30889/2016*
(x^2 - x - 1)^(5/2)*x^2 - 482705/8064*(x^2 - x - 1)^(5/2)*x + 9904793/80640
*(x^2 - x - 1)^(5/2) - 213909/2048*(x^2 - x - 1)^(3/2)*x + 903871/12288*(x^
2 - x - 1)^(3/2) + 3470779/16384*sqrt(x^2 - x - 1)*x - 5567931/32768*sqrt(x
^2 - x - 1) + 64*arcsin(1/5*sqrt(5)*x/abs(x - 1) - 3/5*sqrt(5)/abs(x - 1))
- 19451047/65536*log(2*x + 2*sqrt(x^2 - x - 1) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x-3)^6\left(x^2-x-1\right)^{3/2}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 3)^6*(x^2 - x - 1)^(3/2))/(x - 1),x)
[Out] int(((x - 3)^6*(x^2 - x - 1)^(3/2))/(x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-3)^6 \left(x^2-x-1\right)^{\frac{3}{2}}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)**6*(x**2-x-1)**(3/2)/(-1+x),x)
[Out] Integral((x - 3)**6*(x**2 - x - 1)**(3/2)/(x - 1), x)
```

3.1242 $\int \frac{(-1+x^3)\sqrt[3]{1+x^3}}{x^5}\,dx$

Optimal. Leaf size=101

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}+\frac{\sqrt[3]{x^3+1}\left(1-3x^3\right)}{4x^4}+\frac{1}{6}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {451, 277, 331, 292, 31, 634, 618, 204, 628}

$$-\frac{\sqrt[3]{x^3+1}}{x}-\frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{x^3+1}}\right)-\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\left(x^3+1\right)^{4/3}}{4x^4}+\frac{1}{6}\log\left(\frac{x}{\sqrt[3]{x^3+1}}+\frac{x^2}{\left(x^3+1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)*(1 + x^3)^(1/3))/x^5,x]
```

```
[Out] -((1 + x^3)^(1/3)/x) + (1 + x^3)^(4/3)/(4*x^4) - ArcTan[(1 + (2*x)/(1 + x^3
)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 - x/(1 + x^3)^(1/3)]/3 + Log[1 + x^2/(1 +
 x^3)^(2/3) + x/(1 + x^3)^(1/3)]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-1+x^3\right)\sqrt[3]{1+x^3}}{x^5}\,dx &= \frac{\left(1+x^3\right)^{4/3}}{4x^4}+\int \frac{\sqrt[3]{1+x^3}}{x^2}\,dx \\
&= -\frac{\sqrt[3]{1+x^3}}{x}+\frac{\left(1+x^3\right)^{4/3}}{4x^4}+\int \frac{x}{\left(1+x^3\right)^{2/3}}\,dx \\
&= -\frac{\sqrt[3]{1+x^3}}{x}+\frac{\left(1+x^3\right)^{4/3}}{4x^4}+\text{Subst}\left(\int \frac{x}{1-x^3}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right) \\
&= -\frac{\sqrt[3]{1+x^3}}{x}+\frac{\left(1+x^3\right)^{4/3}}{4x^4}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1-x}{1+x+\cdots}\right. \\
&= -\frac{\sqrt[3]{1+x^3}}{x}+\frac{\left(1+x^3\right)^{4/3}}{4x^4}-\frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\right)+\frac{1}{6}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,\frac{x}{\sqrt[3]{1-\cdots}}\right. \\
&= -\frac{\sqrt[3]{1+x^3}}{x}+\frac{\left(1+x^3\right)^{4/3}}{4x^4}-\frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\right)+\frac{1}{6}\log\left(1+\frac{x^2}{\left(1+x^3\right)^{2/3}}+\frac{x}{\sqrt[3]{1+x^3}}\cdots\right. \\
&= -\frac{\sqrt[3]{1+x^3}}{x}+\frac{\left(1+x^3\right)^{4/3}}{4x^4}-\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\right)+\frac{1}{6}\log\left(1+\frac{\cdots}{\left(1-\cdots\right.}\right.
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.37

$$
\frac{\left(x^3+1\right)^{4/3}}{4x^4}-\frac{{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-x^3\right)}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)*(1 + x^3)^(1/3))/x^5,x]
```

```
[Out] (1 + x^3)^(4/3)/(4*x^4) - Hypergeometric2F1[-1/3, -1/3, 2/3, -x^3]/x
```

IntegrateAlgebraic [A] time = 0.16, size = 101, normalized size = 1.00

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}+\frac{\sqrt[3]{x^3+1}\left(1-3x^3\right)}{4x^4}+\frac{1}{6}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)*(1 + x^3)^(1/3))/x^5,x]
```

```
[Out] ((1 - 3*x^3)*(1 + x^3)^(1/3))/(4*x^4) - ArcTan[(Sqrt[3]*x)/(x + 2*(1 + x^3)
^(1/3))]/Sqrt[3] - Log[-x + (1 + x^3)^(1/3)]/3 + Log[x^2 + x*(1 + x^3)^(1/3
) + (1 + x^3)^(2/3)]/6
```

fricas [A] time = 0.64, size = 112, normalized size = 1.11

$$-\frac{4\sqrt{3}x^4\arctan\left(-\frac{25382\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}x^2-13720\sqrt{3}\left(x^3+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(5831x^3+7200\right)}{58653x^3+8000}\right)+2x^4\log\left(3\left(x^3+1\right)^{\frac{1}{3}}x^2-3\left(x^3+1\right)^{\frac{2}{3}}x+1\right)+3\left(3x^3-1\right)\left(x^3+1\right)^{\frac{1}{3}}}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*x^4*arctan(-(25382*sqrt(3)*(x^3 + 1)^(1/3)*x^2 - 13720*sqr
t(3)*(x^3 + 1)^(2/3)*x + sqrt(3)*(5831*x^3 + 7200))/(58653*x^3 + 8000)) + 2
*x^4*log(3*(x^3 + 1)^(1/3)*x^2 - 3*(x^3 + 1)^(2/3)*x + 1) + 3*(3*x^3 - 1)*(
x^3 + 1)^(1/3))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^3+1\right)^{\frac{1}{3}}\left(x^3-1\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)^(1/3)*(x^3 - 1)/x^5, x)
```

maple [C] time = 0.24, size = 42, normalized size = 0.42

$$-\frac{3x^6+2x^3-1}{4x^4\left(x^3+1\right)^{\frac{2}{3}}}+\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)*(x^3+1)^(1/3)/x^5,x)
```

```
[Out] -1/4*(3*x^6+2*x^3-1)/x^4/(x^3+1)^(2/3)+1/2*x^2*hypergeom([2/3,2/3],[5/3],-x
^3)
```

maxima [A] time = 0.42, size = 93, normalized size = 0.92

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+1\right)^{\frac{4}{3}}}{4x^4}+\frac{1}{6}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)-\frac{1}{3}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x^5,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3)/x + 1)) - (x^3 + 1)^(1/3)
/x + 1/4*(x^3 + 1)^(4/3)/x^4 + 1/6*log((x^3 + 1)^(1/3)/x + (x^3 + 1)^(2/3)/
x^2 + 1) - 1/3*log((x^3 + 1)^(1/3)/x - 1)
```

mupad [B] time = 1.14, size = 40, normalized size = 0.40

$$\frac{\left(x^3+1\right)^{1/3}+x^3\left(x^3+1\right)^{1/3}}{4\,x^4}-\frac{{}_2\mathrm{F}_1\left(-\frac{1}{3},-\frac{1}{3};\ \frac{2}{3};\ -x^3\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)*(x^3 + 1)^(1/3))/x^5,x)
```

```
[Out] ((x^3 + 1)^(1/3) + x^3*(x^3 + 1)^(1/3))/(4*x^4) - hypergeom([-1/3, -1/3], 2
/3, -x^3)/x
```

sympy [C] time = 2.05, size = 87, normalized size = 0.86

$$-\frac{\sqrt[3]{1+\frac{1}{x^3}}\,\Gamma\left(-\frac{4}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)}+\frac{\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3x\Gamma\left(\frac{2}{3}\right)}-\frac{\sqrt[3]{1+\frac{1}{x^3}}\,\Gamma\left(-\frac{4}{3}\right)}{3x^3\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)*(x**3+1)**(1/3)/x**5,x)
```

```
[Out] -(1 + x**(-3))**(1/3)*gamma(-4/3)/(3*gamma(-1/3)) + gamma(-1/3)*hyper((-1/3
, -1/3), (2/3,), x**3*exp_polar(I*pi))/(3*x*gamma(2/3)) - (1 + x**(-3))**(1
/3)*gamma(-4/3)/(3*x**3*gamma(-1/3))
```

3.1243 $\int \frac{(-1+x^3)\sqrt[3]{1+x^3}}{x^2}\,dx$

Optimal. Leaf size=101

$$\frac{\sqrt[3]{x^3+1}\,(x^3+3)}{3x}+\frac{2}{9}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{2\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{3\sqrt{3}}-\frac{1}{9}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {453, 279, 331, 292, 31, 634, 618, 204, 628}

$$\frac{(x^3+1)^{4/3}}{x}+\frac{2}{9}\log\left(1-\frac{x}{\sqrt[3]{x^3+1}}\right)+\frac{2\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2}{3}\sqrt[3]{x^3+1}\,x^2-\frac{1}{9}\log\left(\frac{x}{\sqrt[3]{x^3+1}}+\frac{x^2}{(x^3+1)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)*(1 + x^3)^(1/3))/x^2,x]
```

```
[Out] (-2*x^2*(1 + x^3)^(1/3))/3 + (1 + x^3)^(4/3)/x + (2*ArcTan[(1 + (2*x)/(1 +
x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) + (2*Log[1 - x/(1 + x^3)^(1/3)])/9 - Log[
1 + x^2/(1 + x^3)^(2/3) + x/(1 + x^3)^(1/3)]/9
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(-1+x^3\right)\sqrt[3]{1+x^3}}{x^2}\,dx = \frac{\left(1+x^3\right)^{4/3}}{x} - 2\int x\sqrt[3]{1+x^3}\,dx$$
$$= -\frac{2}{3}x^2\sqrt[3]{1+x^3} + \frac{\left(1+x^3\right)^{4/3}}{x} - \frac{2}{3}\int \frac{x}{\left(1+x^3\right)^{2/3}}\,dx$$
$$= -\frac{2}{3}x^2\sqrt[3]{1+x^3} + \frac{\left(1+x^3\right)^{4/3}}{x} - \frac{2}{3}\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{x}{\sqrt[3]{1+x^3}}\right)$$
$$= -\frac{2}{3}x^2\sqrt[3]{1+x^3} + \frac{\left(1+x^3\right)^{4/3}}{x} - \frac{2}{9}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{x}{\sqrt[3]{1+x^3}}\right) + \frac{2}{9}\text{Subst}\left(\int \frac{1}{1+$$
$$= -\frac{2}{3}x^2\sqrt[3]{1+x^3} + \frac{\left(1+x^3\right)^{4/3}}{x} + \frac{2}{9}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\right) - \frac{1}{9}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x,$$
$$= -\frac{2}{3}x^2\sqrt[3]{1+x^3} + \frac{\left(1+x^3\right)^{4/3}}{x} + \frac{2}{9}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\right) - \frac{1}{9}\log\left(1+\frac{x^2}{\left(1+x^3\right)^{2/3}} + \frac{x}{\sqrt[3]{1+$$
$$= -\frac{2}{3}x^2\sqrt[3]{1+x^3} + \frac{\left(1+x^3\right)^{4/3}}{x} + \frac{2\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{2}{9}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\right) - \frac{1}{9}\log\left(1-$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.34

$$\frac{\left(x^3+1\right)^{4/3}}{x} - x^2\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)*(1 + x^3)^(1/3))/x^2,x]
```

```
[Out] (1 + x^3)^(4/3)/x - x^2*Hypergeometric2F1[-1/3, 2/3, 5/3, -x^3]
```

IntegrateAlgebraic [A] time = 0.19, size = 101, normalized size = 1.00

$$\frac{\sqrt[3]{x^3+1}\left(x^3+3\right)}{3x}+\frac{2}{9}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{2\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{3\sqrt{3}}-\frac{1}{9}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)*(1 + x^3)^(1/3))/x^2,x]
```

```
[Out] ((1 + x^3)^(1/3)*(3 + x^3))/(3*x) + (2*ArcTan[(Sqrt[3]*x)/(x + 2*(1 + x^3)^
(1/3))])/(3*Sqrt[3]) + (2*Log[-x + (1 + x^3)^(1/3)])/9 - Log[x^2 + x*(1 + x
^3)^(1/3) + (1 + x^3)^(2/3)]/9
```

fricas [A] time = 0.68, size = 105, normalized size = 1.04

$$\frac{2\sqrt{3}x\arctan\left(-\frac{25382\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}x^2-13720\sqrt{3}\left(x^3+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(5831x^3+7200\right)}{58653x^3+8000}\right)+x\log\left(3\left(x^3+1\right)^{\frac{1}{3}}x^2-3\left(x^3+1\right)^{\frac{2}{3}}x+1\right)+3\left(x^3+3\right)\left(x^3+1\right)^{\frac{1}{3}}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*(2*sqrt(3)*x*arctan(-(25382*sqrt(3)*(x^3 + 1)^(1/3)*x^2 - 13720*sqrt(3)
*(x^3 + 1)^(2/3)*x + sqrt(3)*(5831*x^3 + 7200))/(58653*x^3 + 8000)) + x*log
(3*(x^3 + 1)^(1/3)*x^2 - 3*(x^3 + 1)^(2/3)*x + 1) + 3*(x^3 + 3)*(x^3 + 1)^(
1/3))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^3+1\right)^{\frac{1}{3}}\left(x^3-1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)^(1/3)*(x^3 - 1)/x^2, x)
```

maple [C] time = 0.26, size = 40, normalized size = 0.40

$$\frac{x^6+4x^3+3}{3x\left(x^3+1\right)^{\frac{2}{3}}}-\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)*(x^3+1)^(1/3)/x^2,x)
```

```
[Out] 1/3*(x^6+4*x^3+3)/x/(x^3+1)^(2/3)-1/3*x^2*hypergeom([2/3,2/3],[5/3],-x^3)
```

maxima [A] time = 0.46, size = 105, normalized size = 1.04

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+1\right)^{\frac{1}{3}}}{x}+1\right)\right)+\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+1\right)^{\frac{1}{3}}}{3x\left(\frac{x^3+1}{x^3}-1\right)}-\frac{1}{9}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)+\frac{2}{9}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3)/x + 1)) + (x^3 + 1)^(1/3
)/x + 1/3*(x^3 + 1)^(1/3)/(x*((x^3 + 1)/x^3 - 1)) - 1/9*log((x^3 + 1)^(1/3)
/x + (x^3 + 1)^(2/3)/x^2 + 1) + 2/9*log((x^3 + 1)^(1/3)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^3-1)\,(x^3+1)^{1/3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)*(x^3 + 1)^(1/3))/x^2,x)
```

```
[Out] int(((x^3 - 1)*(x^3 + 1)^(1/3))/x^2, x)
```

sympy [C] time = 2.16, size = 65, normalized size = 0.64

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}-\frac{\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)*(x**3+1)**(1/3)/x**2,x)
```

```
[Out] x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5
/3)) - gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), x**3*exp_polar(I*pi))/(3*x*g
amma(2/3))
```

3.1244 $\int \frac{\sqrt[3]{-1+x^3}\,(1+x^3)}{x^5}\,dx$

Optimal. Leaf size=101

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}+\frac{\sqrt[3]{x^3-1}\,\left(-3x^3-1\right)}{4x^4}+\frac{1}{6}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {451, 277, 331, 292, 31, 634, 618, 204, 628}

$$-\frac{\sqrt[3]{x^3-1}}{x}-\frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{x^3-1}}\right)-\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\left(x^3-1\right)^{4/3}}{4x^4}+\frac{1}{6}\log\left(\frac{x}{\sqrt[3]{x^3-1}}+\frac{x^2}{\left(x^3-1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(1/3)*(1 + x^3))/x^5,x]
```

```
[Out] -((-1 + x^3)^(1/3)/x) + (-1 + x^3)^(4/3)/(4*x^4) - ArcTan[(1 + (2*x)/(-1 +
x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 - x/(-1 + x^3)^(1/3)]/3 + Log[1 + x^2/
(-1 + x^3)^(2/3) + x/(-1 + x^3)^(1/3)]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{-1+x^3}\,\left(1+x^3\right)}{x^5}\,dx &= \frac{\left(-1+x^3\right)^{4/3}}{4x^4}+\int \frac{\sqrt[3]{-1+x^3}}{x^2}\,dx\\
&= -\frac{\sqrt[3]{-1+x^3}}{x}+\frac{\left(-1+x^3\right)^{4/3}}{4x^4}+\int \frac{x}{\left(-1+x^3\right)^{2/3}}\,dx\\
&= -\frac{\sqrt[3]{-1+x^3}}{x}+\frac{\left(-1+x^3\right)^{4/3}}{4x^4}+\text{Subst}\left(\int \frac{x}{1-x^3}\,dx,x,\frac{x}{\sqrt[3]{-1+x^3}}\right)\\
&= -\frac{\sqrt[3]{-1+x^3}}{x}+\frac{\left(-1+x^3\right)^{4/3}}{4x^4}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\frac{x}{\sqrt[3]{-1+x^3}}\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\cdots}\right.\\
&= -\frac{\sqrt[3]{-1+x^3}}{x}+\frac{\left(-1+x^3\right)^{4/3}}{4x^4}-\frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{-1+x^3}}\right)+\frac{1}{6}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,\cdots\right.\\
&= -\frac{\sqrt[3]{-1+x^3}}{x}+\frac{\left(-1+x^3\right)^{4/3}}{4x^4}-\frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{-1+x^3}}\right)+\frac{1}{6}\log\left(1+\frac{x^2}{\left(-1+x^3\right)^{2/3}}+\cdots\right.\\
&= -\frac{\sqrt[3]{-1+x^3}}{x}+\frac{\left(-1+x^3\right)^{4/3}}{4x^4}-\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{-1+x^3}}\right)+\frac{1}{6}\log\left(1+\cdots\right.
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 50, normalized size = 0.50

$$\frac{\sqrt[3]{x^3-1}\left(-\frac{4x^3\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};x^3\right)}{\sqrt[3]{1-x^3}}+x^3-1\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)^(1/3)*(1 + x^3))/x^5,x]
```

```
[Out] ((-1 + x^3)^(1/3)*(-1 + x^3 - (4*x^3*Hypergeometric2F1[-1/3, -1/3, 2/3, x^3
])/(1 - x^3)^(1/3)))/(4*x^4)
```

IntegrateAlgebraic [A] time = 0.16, size = 101, normalized size = 1.00

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}+\frac{\sqrt[3]{x^3-1}\left(-3x^3-1\right)}{4x^4}+\frac{1}{6}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(1/3)*(1 + x^3))/x^5,x]
```

```
[Out] ((-1 - 3*x^3)*(-1 + x^3)^(1/3))/(4*x^4) - ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x
^3)^(1/3))]/Sqrt[3] - Log[-x + (-1 + x^3)^(1/3)]/3 + Log[x^2 + x*(-1 + x^3)
^(1/3) + (-1 + x^3)^(2/3)]/6
```

fricas [A] time = 0.64, size = 112, normalized size = 1.11

$$-\frac{4\sqrt{3}x^4\arctan\left(-\frac{25382\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}x^2-13720\sqrt{3}\left(x^3-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(5831x^3-7200\right)}{58653x^3-8000}\right)+2x^4\log\left(-3\left(x^3-1\right)^{\frac{1}{3}}x^2+3\left(x^3-1\right)^{\frac{2}{3}}x+1\right)+3\left(3x^3+1\right)\left(x^3-1\right)^{\frac{1}{3}}}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(x^3+1)/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*x^4*arctan(-(25382*sqrt(3)*(x^3 - 1)^(1/3)*x^2 - 13720*sqr
t(3)*(x^3 - 1)^(2/3)*x + sqrt(3)*(5831*x^3 - 7200))/(58653*x^3 - 8000)) + 2
*x^4*log(-3*(x^3 - 1)^(1/3)*x^2 + 3*(x^3 - 1)^(2/3)*x + 1) + 3*(3*x^3 + 1)*
(x^3 - 1)^(1/3))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^3+1\right)\left(x^3-1\right)^{\frac{1}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(x^3+1)/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)*(x^3 - 1)^(1/3)/x^5, x)
```

maple [C] time = 0.27, size = 58, normalized size = 0.57

$$-\frac{3x^6-2x^3-1}{4x^4\left(x^3-1\right)^{\frac{2}{3}}}+\frac{\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{2}{3}}x^2\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{2\mathrm{signum}\left(x^3-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/3)*(x^3+1)/x^5,x)
```

```
[Out] -1/4*(3*x^6-2*x^3-1)/x^4/(x^3-1)^(2/3)+1/2/signum(x^3-1)^(2/3)*(-signum(x^3
-1))^(2/3)*x^2*hypergeom([2/3,2/3],[5/3],x^3)
```

maxima [A] time = 0.44, size = 93, normalized size = 0.92

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3-1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3-1\right)^{\frac{4}{3}}}{4x^4}+\frac{1}{6}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3-1\right)^{\frac{2}{3}}}{x^2}+1\right)-\frac{1}{3}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(x^3+1)/x^5,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3)/x + 1)) - (x^3 - 1)^(1/3)
/x + 1/4*(x^3 - 1)^(4/3)/x^4 + 1/6*log((x^3 - 1)^(1/3)/x + (x^3 - 1)^(2/3)/
x^2 + 1) - 1/3*log((x^3 - 1)^(1/3)/x - 1)
```

mupad [B] time = 1.17, size = 55, normalized size = 0.54

$$-\frac{\left(x^3-1\right)^{1/3}-x^3\left(x^3-1\right)^{1/3}}{4\,x^4}-\frac{\left(x^3-1\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},-\frac{1}{3};\ \frac{2}{3};\ x^3\right)}{x\left(1-x^3\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(1/3)*(x^3 + 1))/x^5,x)
```

```
[Out] - ((x^3 - 1)^(1/3) - x^3*(x^3 - 1)^(1/3))/(4*x^4) - ((x^3 - 1)^(1/3)*hyperg
eom([-1/3, -1/3], 2/3, x^3))/(x*(1 - x^3)^(1/3))
```

sympy [C] time = 2.13, size = 165, normalized size = 1.63

$$\begin{cases} \frac{\sqrt[3]{-1+\frac{1}{x^3}}\,e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)}-\frac{\sqrt[3]{-1+\frac{1}{x^3}}\,e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{3x^3\Gamma\left(-\frac{1}{3}\right)} & \text{for}\ \frac{1}{|x^3|}>1 \\ -\frac{\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{4}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)}+\frac{\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{4}{3}\right)}{3x^3\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise} \end{cases}+\frac{e^{\frac{i\pi}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|x^3\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/3)*(x**3+1)/x**5,x)
```

```
[Out] Piecewise(((-1 + x**(-3))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(3*gamma(-1/3))
 - (-1 + x**(-3))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(3*x**3*gamma(-1/3)), 1
/Abs(x**3) > 1), (-(1 - 1/x**3)**(1/3)*gamma(-4/3)/(3*gamma(-1/3)) + (1 - 1
/x**3)**(1/3)*gamma(-4/3)/(3*x**3*gamma(-1/3)), True)) + exp(I*pi/3)*gamma(
-1/3)*hyper((-1/3, -1/3), (2/3,), x**3)/(3*x*gamma(2/3))
```

3.1245 $\int \frac{x}{(1+x^2)\sqrt{-x-x^2+x^3}}\,dx$

Optimal. Leaf size=101

$$-\frac{1}{2}\sqrt{-\frac{1}{5}-\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1-2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)-\frac{1}{2}\sqrt{-\frac{1}{5}+\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1+2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)$$

Rubi [C] time = 0.97, antiderivative size = 239, normalized size of antiderivative = 2.37, number of steps used = 17, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2056, 6725, 943, 716, 1098, 934, 168, 538, 537}

$$\frac{i\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\,\Pi\left(-\frac{1}{2}i\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{2\sqrt{x^3-x^2-x}}-\frac{i\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\,\Pi\left(\frac{1}{2}i\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{2\sqrt{x^3-x^2-x}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x^2)*Sqrt[-x - x^2 + x^3]),x]
```

```
[Out] ((I/2)*Sqrt[3 + Sqrt[5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1
 + Sqrt[5])]*EllipticPi[(-1/2*I)*(1 + Sqrt[5]), ArcSin[Sqrt[2/(1 + Sqrt[5])]
]*Sqrt[x]], (-3 - Sqrt[5])/2])/Sqrt[-x - x^2 + x^3] - ((I/2)*Sqrt[3 + Sqrt[
5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1 + Sqrt[5])]*Elliptic
Pi[(I/2)*(1 + Sqrt[5]), ArcSin[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]], (-3 - Sqrt[5
])/2])/Sqrt[-x - x^2 + x^3]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 716

```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2,
Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
```

```
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 943

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + b*x
+ c*x^2]), x], x] + Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt
[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x}{(1+x^2)\sqrt{-x-x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{\sqrt{x}}{(1+x^2)\sqrt{-1-x+x^2}}\,dx}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \left(\frac{i\sqrt{x}}{2(i-x)\sqrt{-1-x+x^2}}+\frac{i\sqrt{x}}{2(i+x)\sqrt{-1-x+x^2}}\right)dx}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(i\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{\sqrt{x}}{(i-x)\sqrt{-1-x+x^2}}\,dx}{2\sqrt{-x-x^2+x^3}} + \frac{\left(i\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{\sqrt{x}}{(i+x)\sqrt{-1-x+x}}}{2\sqrt{-x-x^2+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{1}{(i-x)\sqrt{x}\sqrt{-1-x+x^2}}\,dx}{2\sqrt{-x-x^2+x^3}} + \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{1}{\sqrt{x}(i+x)\sqrt{-}}}{2\sqrt{-x-x^2+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-1-\sqrt{5}+2x}\sqrt{-1+\sqrt{5}+2x}\right)\int \frac{1}{(i-x)\sqrt{x}\sqrt{-1-\sqrt{5}+2x}\sqrt{-1+\sqrt{5}+2x}}\,dx}{2\sqrt{-x-x^2+x^3}} +$$

$$= -\frac{\left(\sqrt{x}\sqrt{-1-\sqrt{5}+2x}\sqrt{-1+\sqrt{5}+2x}\right)\text{Subst}\left(\int \frac{1}{(-i-x^2)\sqrt{-1-\sqrt{5}+2x^2}\sqrt{-1+\sqrt{5}}}\right.}{\sqrt{-x-x^2+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-1+\sqrt{5}+2x}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\right)\text{Subst}\left(\int \frac{1}{(-i-x^2)\sqrt{-1+\sqrt{5}+2x^2}\sqrt{1+\frac{2x^2}{-1-\sqrt{5}}}}\,dx\right.}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{i\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{-1+\sqrt{5}+2x}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\,\Pi\left(-\frac{1}{2}i\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\right.\right.}{2\sqrt{-x-x^2+x^3}}$$

Mathematica [C] time = 0.64, size = 152, normalized size = 1.50

$$\frac{\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\sqrt{-x^2+x+1}\left(\Pi\left(\frac{1}{2}i\left(-1+\sqrt{5}\right);i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3+\sqrt{5}\right)\right)-\Pi\left(-\frac{1}{2}i\left(-1+\sqrt{5}\right);i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3+\sqrt{5}\right)\right)\right)}{\sqrt{x\left(x^2-x-1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 + x^2)*Sqrt[-x - x^2 + x^3]),x]
[Out] (Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]*Sqrt[1 + x - x^2]*(-EllipticPi[(-1/2*I)*(-1
+ Sqrt[5]), I*ArcSinh[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[x]], (-3 + Sqrt[5])/2] +
EllipticPi[(I/2)*(-1 + Sqrt[5]), I*ArcSinh[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[x]],
 (-3 + Sqrt[5])/2]))/Sqrt[x*(-1 - x + x^2)]
```

IntegrateAlgebraic [A] time = 0.33, size = 101, normalized size = 1.00

$$-\frac{1}{2}\sqrt{-\frac{1}{5}-\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1-2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)-\frac{1}{2}\sqrt{-\frac{1}{5}+\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1+2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((1 + x^2)*Sqrt[-x - x^2 + x^3]),x]
[Out] -1/2*(Sqrt[-1/5 - (2*I)/5]*ArcTan[(Sqrt[1 - 2*I]*Sqrt[-x - x^2 + x^3])/(-1
- x + x^2)]) - (Sqrt[-1/5 + (2*I)/5]*ArcTan[(Sqrt[1 + 2*I]*Sqrt[-x - x^2 +
x^3])/(-1 - x + x^2)])/2
```

fricas [B] time = 0.94, size = 2408, normalized size = 23.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^3-x^2-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/160*5^(1/4)*(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 5)*log(5*(5*x^4 -
 20*x^3 + 2*5^(1/4)*sqrt(x^3 - x^2 - x)*(sqrt(5)*sqrt(2)*(x^2 - x - 1) + 5*
sqrt(2)*x)*sqrt(-sqrt(5) + 5) + 30*x^2 + 20*sqrt(5)*(x^3 - x^2 - x) + 20*x
+ 5)/(x^4 + 2*x^2 + 1)) - 1/160*5^(1/4)*(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-s
qrt(5) + 5)*log(5*(5*x^4 - 20*x^3 - 2*5^(1/4)*sqrt(x^3 - x^2 - x)*(sqrt(5)*
sqrt(2)*(x^2 - x - 1) + 5*sqrt(2)*x)*sqrt(-sqrt(5) + 5) + 30*x^2 + 20*sqrt(
5)*(x^3 - x^2 - x) + 20*x + 5)/(x^4 + 2*x^2 + 1)) + 1/20*5^(1/4)*sqrt(2)*sq
rt(-sqrt(5) + 5)*arctan(-1/100*(50*x^11 + 650*x^10 - 3350*x^9 - 2200*x^8 +
14200*x^7 + 700*x^6 - 14200*x^5 - 2200*x^4 + 3350*x^3 + 650*x^2 + 5*sqrt(x^
3 - x^2 - x)*(5^(3/4)*(sqrt(5)*sqrt(2)*(3*x^9 - 10*x^8 - 20*x^7 + 54*x^6 +
50*x^5 - 54*x^4 - 20*x^3 + 10*x^2 + 3*x) - sqrt(2)*(x^10 - 7*x^9 - 7*x^8 +
76*x^7 + 24*x^6 - 186*x^5 - 24*x^4 + 76*x^3 + 7*x^2 - 7*x - 1)) + 2*5^(1/4)
*(sqrt(5)*sqrt(2)*(x^9 + 19*x^8 - 52*x^7 + 3*x^6 + 70*x^5 - 3*x^4 - 52*x^3
- 19*x^2 + x) - 5*sqrt(2)*(x^9 - x^8 + 16*x^7 - 33*x^6 - 18*x^5 + 33*x^4 +
16*x^3 + x^2 + x)))*sqrt(-sqrt(5) + 5) - sqrt(5)*(120*x^10 + 80*x^9 - 840*x
^8 - 240*x^7 + 1600*x^6 + 240*x^5 - 840*x^4 - 80*x^3 + 120*x^2 + sqrt(x^3 -
 x^2 - x)*(5^(3/4)*(sqrt(5)*sqrt(2)*(x^9 + 2*x^8 + 24*x^7 - 62*x^6 - 50*x^5
 + 62*x^4 + 24*x^3 - 2*x^2 + x) - sqrt(2)*(x^10 - 7*x^9 - 27*x^8 + 96*x^7 +
 4*x^6 - 146*x^5 - 4*x^4 + 96*x^3 + 27*x^2 - 7*x - 1)) + 2*5^(1/4)*(sqrt(5)
*sqrt(2)*(x^9 + 9*x^8 - 42*x^7 - 7*x^6 + 90*x^5 + 7*x^4 - 42*x^3 - 9*x^2 +
x) - 5*sqrt(2)*(x^9 - 3*x^8 - 14*x^7 + 29*x^6 + 18*x^5 - 29*x^4 - 14*x^3 +
3*x^2 + x)))*sqrt(-sqrt(5) + 5) + 2*sqrt(5)*(5*x^11 - 25*x^10 - 105*x^9 + 4
40*x^8 + 50*x^7 - 830*x^6 - 50*x^5 + 440*x^4 + 105*x^3 - 25*x^2 - sqrt(5)*(
x^11 + 3*x^10 - 37*x^9 + 96*x^8 - 6*x^7 - 166*x^6 + 6*x^5 + 96*x^4 + 37*x^3
 + 3*x^2 - x) - 5*x) - 40*sqrt(5)*(x^10 + 6*x^9 - 23*x^8 - 2*x^7 + 40*x^6 +
 2*x^5 - 23*x^4 - 6*x^3 + x^2))*sqrt((5*x^4 - 20*x^3 + 2*5^(1/4)*sqrt(x^3 -
 x^2 - x)*(sqrt(5)*sqrt(2)*(x^2 - x - 1) + 5*sqrt(2)*x)*sqrt(-sqrt(5) + 5)
+ 30*x^2 + 20*sqrt(5)*(x^3 - x^2 - x) + 20*x + 5)/(x^4 + 2*x^2 + 1)) + 10*s
qrt(5)*(5*x^11 - 15*x^10 - 15*x^9 + 20*x^8 - 20*x^7 + 70*x^6 + 20*x^5 + 20*
x^4 + 15*x^3 - 15*x^2 - sqrt(5)*(x^11 + 13*x^10 - 67*x^9 - 44*x^8 + 284*x^7
 + 14*x^6 - 284*x^5 - 44*x^4 + 67*x^3 + 13*x^2 - x) - 5*x) - 50*sqrt(5)*(x^
11 - 3*x^10 - 3*x^9 + 4*x^8 - 4*x^7 + 14*x^6 + 4*x^5 + 4*x^4 + 3*x^3 - 3*x^
2 - x) - 50*x)/(x^11 - 9*x^10 - 45*x^9 + 180*x^8 + 18*x^7 - 326*x^6 - 18*x^
5 + 180*x^4 + 45*x^3 - 9*x^2 - x)) + 1/20*5^(1/4)*sqrt(2)*sqrt(-sqrt(5) + 5
)*arctan(1/100*(50*x^11 + 650*x^10 - 3350*x^9 - 2200*x^8 + 14200*x^7 + 700*
x^6 - 14200*x^5 - 2200*x^4 + 3350*x^3 + 650*x^2 - 5*sqrt(x^3 - x^2 - x)*(5^
(3/4)*(sqrt(5)*sqrt(2)*(3*x^9 - 10*x^8 - 20*x^7 + 54*x^6 + 50*x^5 - 54*x^4
- 20*x^3 + 10*x^2 + 3*x) - sqrt(2)*(x^10 - 7*x^9 - 7*x^8 + 76*x^7 + 24*x^6
- 186*x^5 - 24*x^4 + 76*x^3 + 7*x^2 - 7*x - 1)) + 2*5^(1/4)*(sqrt(5)*sqrt(2
)*(x^9 + 19*x^8 - 52*x^7 + 3*x^6 + 70*x^5 - 3*x^4 - 52*x^3 - 19*x^2 + x) -
5*sqrt(2)*(x^9 - x^8 + 16*x^7 - 33*x^6 - 18*x^5 + 33*x^4 + 16*x^3 + x^2 + x
)))*sqrt(-sqrt(5) + 5) - sqrt(5)*(120*x^10 + 80*x^9 - 840*x^8 - 240*x^7 + 1
600*x^6 + 240*x^5 - 840*x^4 - 80*x^3 + 120*x^2 - sqrt(x^3 - x^2 - x)*(5^(3/
4)*(sqrt(5)*sqrt(2)*(x^9 + 2*x^8 + 24*x^7 - 62*x^6 - 50*x^5 + 62*x^4 + 24*x
^3 - 2*x^2 + x) - sqrt(2)*(x^10 - 7*x^9 - 27*x^8 + 96*x^7 + 4*x^6 - 146*x^5
 - 4*x^4 + 96*x^3 + 27*x^2 - 7*x - 1)) + 2*5^(1/4)*(sqrt(5)*sqrt(2)*(x^9 +
9*x^8 - 42*x^7 - 7*x^6 + 90*x^5 + 7*x^4 - 42*x^3 - 9*x^2 + x) - 5*sqrt(2)*(
x^9 - 3*x^8 - 14*x^7 + 29*x^6 + 18*x^5 - 29*x^4 - 14*x^3 + 3*x^2 + x)))*sqr
t(-sqrt(5) + 5) + 2*sqrt(5)*(5*x^11 - 25*x^10 - 105*x^9 + 440*x^8 + 50*x^7
- 830*x^6 - 50*x^5 + 440*x^4 + 105*x^3 - 25*x^2 - sqrt(5)*(x^11 + 3*x^10 -
37*x^9 + 96*x^8 - 6*x^7 - 166*x^6 + 6*x^5 + 96*x^4 + 37*x^3 + 3*x^2 - x) -
5*x) - 40*sqrt(5)*(x^10 + 6*x^9 - 23*x^8 - 2*x^7 + 40*x^6 + 2*x^5 - 23*x^4
- 6*x^3 + x^2))*sqrt((5*x^4 - 20*x^3 - 2*5^(1/4)*sqrt(x^3 - x^2 - x)*(sqrt(
```

```
5)*sqrt(2)*(x^2 - x - 1) + 5*sqrt(2)*x)*sqrt(-sqrt(5) + 5) + 30*x^2 + 20*sq
rt(5)*(x^3 - x^2 - x) + 20*x + 5)/(x^4 + 2*x^2 + 1)) + 10*sqrt(5)*(5*x^11 -
 15*x^10 - 15*x^9 + 20*x^8 - 20*x^7 + 70*x^6 + 20*x^5 + 20*x^4 + 15*x^3 - 1
5*x^2 - sqrt(5)*(x^11 + 13*x^10 - 67*x^9 - 44*x^8 + 284*x^7 + 14*x^6 - 284*
x^5 - 44*x^4 + 67*x^3 + 13*x^2 - x) - 5*x) - 50*sqrt(5)*(x^11 - 3*x^10 - 3*
x^9 + 4*x^8 - 4*x^7 + 14*x^6 + 4*x^5 + 4*x^4 + 3*x^3 - 3*x^2 - x) - 50*x)/(
x^11 - 9*x^10 - 45*x^9 + 180*x^8 + 18*x^7 - 326*x^6 - 18*x^5 + 180*x^4 + 45
*x^3 - 9*x^2 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3 - x^2 - x}\left(x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^3-x^2-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 - x^2 - x)*(x^2 + 1)), x)
```

maple [C] time = 0.05, size = 620, normalized size = 6.14

$$\frac{\sqrt{\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}-\frac{1}{2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}+\frac{\sqrt{5}}{\sqrt{5}-1}}\,\sqrt{-5x\sqrt{5}+\frac{5\sqrt{5}}{2}+\frac{25}{2}}\,\sqrt{-\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}, \frac{\frac{1}{2}-\frac{\sqrt{5}}{2}}{\frac{1}{2}-i-\frac{\sqrt{5}}{2}}, \frac{\sqrt{5}\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)\sqrt{5}}{10\sqrt{x^3-x^2-x}\left(\frac{1}{2}-i-\frac{\sqrt{5}}{2}\right)} - \frac{\sqrt{\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}-\frac{1}{2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}+\frac{\sqrt{5}}{\sqrt{5}-1}}\,\sqrt{-5x\sqrt{5}+\frac{5\sqrt{5}}{2}+\frac{25}{2}}\,\sqrt{-\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}, \frac{\frac{1}{2}-\frac{\sqrt{5}}{2}}{\frac{1}{2}-i-\frac{\sqrt{5}}{2}}, \frac{\sqrt{5}\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)}{10\sqrt{x^3-x^2-x}\left(\frac{1}{2}-i-\frac{\sqrt{5}}{2}\right)} + \frac{\sqrt{\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}-\frac{1}{2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}+\frac{\sqrt{5}}{\sqrt{5}-1}}\,\sqrt{-5x\sqrt{5}+\frac{5\sqrt{5}}{2}+\frac{25}{2}}\,\sqrt{-\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}, \frac{\frac{1}{2}-\frac{\sqrt{5}}{2}}{\frac{1}{2}+i-\frac{\sqrt{5}}{2}}, \frac{\sqrt{5}\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)\sqrt{5}}{10\sqrt{x^3-x^2-x}\left(\frac{1}{2}+i-\frac{\sqrt{5}}{2}\right)} - \frac{\sqrt{\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}-\frac{1}{2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}+\frac{\sqrt{5}}{\sqrt{5}-1}}\,\sqrt{-5x\sqrt{5}+\frac{5\sqrt{5}}{2}+\frac{25}{2}}\,\sqrt{-\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}, \frac{\frac{1}{2}-\frac{\sqrt{5}}{2}}{\frac{1}{2}+i-\frac{\sqrt{5}}{2}}, \frac{\sqrt{5}\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)}{10\sqrt{x^3-x^2-x}\left(\frac{1}{2}+i-\frac{\sqrt{5}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+1)/(x^3-x^2-x)^(1/2),x)
```

```
[Out] 1/10*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/
2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2
)/(x^3-x^2-x)^(1/2)/(1/2-I-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/
2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2-I-1/2*5^(1/2)),1/5*5^(1/2)*((1
/2*5^(1/2)-1/2)*5^(1/2))^(1/2))*5^(1/2)-1/10*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*
5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)
+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(1/2-I-1/2*5^(1
/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1
/2))/(1/2-I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))+1/1
0*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))
^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(
x^3-x^2-x)^(1/2)/(1/2+I-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5
^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2+I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*
5^(1/2)-1/2)*5^(1/2))^(1/2))*5^(1/2)-1/10*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(
1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25
/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(1/2+I-1/2*5^(1/2)
)*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2)
)/(1/2+I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3 - x^2 - x}\left(x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^3-x^2-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 - x^2 - x)*(x^2 + 1)), x)
```

mupad [B] time = 1.04, size = 178, normalized size = 1.76

$$\frac{\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\left(\sqrt{5}+1\right)\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\left(\Pi\left(-\frac{\sqrt{5}\,1\mathrm{i}}{2}-\frac{1}{2}\mathrm{i};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)-\Pi\left(\frac{\sqrt{5}\,1\mathrm{i}}{2}+\frac{1}{2}\mathrm{i};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)\right)1\mathrm{i}}{2\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 + 1)*(x^3 - x^2 - x)^(1/2)),x)
```

```
[Out] ((x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/2 - 1/2)/(5^(1/2)/2 - 1/2))^(1/2
)*(5^(1/2) + 1)*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 + 1/2))^(1/2)*(ellipticPi
(- (5^(1/2)*1i)/2 - 1i/2, asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/2 +
1/2)/(5^(1/2)/2 - 1/2)) - ellipticPi((5^(1/2)*1i)/2 + 1i/2, asin((x/(5^(1/2
)/2 + 1/2))^(1/2)), -(5^(1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2)))*1i)/(2*(x^3 - x^
2 - x*(5^(1/2)/2 - 1/2)*(5^(1/2)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x\left(x^2-x-1\right)}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+1)/(x**3-x**2-x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(x*(x**2 - x - 1))*(x**2 + 1)), x)
```

3.1246 $\int \frac{-1+x+x^2}{(1+x^2)\sqrt{-x-x^2+x^3}}\,dx$

Optimal. Leaf size=101

$$-\frac{1}{2}\sqrt{\frac{11}{5}+\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1-2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)-\frac{1}{2}\sqrt{\frac{11}{5}-\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1+2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)$$

Rubi [C] time = 1.20, antiderivative size = 383, normalized size of antiderivative = 3.79, number of steps used = 15, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2056, 6725, 716, 1098, 934, 168, 538, 537}

$$\frac{\sqrt{x}\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}\sqrt{\frac{\left(1+\sqrt{5}\right)x+2}{\left(1-\sqrt{5}\right)x+2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{\left(1-\sqrt{5}\right)x+2}}\sqrt{x^3-x^2-x}}-\frac{\left(1-\frac{i}{2}\right)\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(-\frac{1}{2}i\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\sqrt{x^3-x^2-x}}-\frac{\left(1+\frac{i}{2}\right)\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(\frac{1}{2}i\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{\sqrt{x^3-x^2-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x + x^2)/((1 + x^2)*Sqrt[-x - x^2 + x^3]),x]
```

```
[Out] (Sqrt[x]*Sqrt[-2 - (1 - Sqrt[5])*x]*Sqrt[(2 + (1 + Sqrt[5])*x)/(2 + (1 - Sq
rt[5])*x)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*Sqrt[x])/Sqrt[-2 - (1 - Sqrt[5
])*x]], (5 - Sqrt[5])/10])/(5^(1/4)*Sqrt[(2 + (1 - Sqrt[5])*x)^(-1)]*Sqrt[-
x - x^2 + x^3]) - ((1 - I/2)*Sqrt[3 + Sqrt[5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] +
2*x]*Sqrt[1 - (2*x)/(1 + Sqrt[5])]*EllipticPi[(-1/2*I)*(1 + Sqrt[5]), ArcSi
n[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]], (-3 - Sqrt[5])/2])/Sqrt[-x - x^2 + x^3] -
 ((1 + I/2)*Sqrt[3 + Sqrt[5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*
x)/(1 + Sqrt[5])]*EllipticPi[(I/2)*(1 + Sqrt[5]), ArcSin[Sqrt[2/(1 + Sqrt[5
])]*Sqrt[x]], (-3 - Sqrt[5])/2])/Sqrt[-x - x^2 + x^3]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 716

```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2,
Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+x+x^2}{\left(1+x^2\right)\sqrt{-x-x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{-1+x+x^2}{\sqrt{x}\left(1+x^2\right)\sqrt{-1-x+x^2}}\,dx}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{-1-x+x^2}} - \frac{2-x}{\sqrt{x}\left(1+x^2\right)\sqrt{-1-x+x^2}}\right)dx}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-1-x+x^2}}\,dx}{\sqrt{-x-x^2+x^3}} - \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{2-x}{\sqrt{x}\left(1+x^2\right)\sqrt{-1-x+}}}{\sqrt{-x-x^2+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \left(\frac{\frac{1}{2}+i}{(i-x)\sqrt{x}\sqrt{-1-x+x^2}} - \frac{\frac{1}{2}-i}{\sqrt{x}\,(i+x)\sqrt{-1-x+x^2}}\right)dx}{\sqrt{-x-x^2+x^3}} + \frac{\left(2\sqrt{x}\sqrt{-1\right.}{}$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - -$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - -$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - -$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - -$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - -\left(\right.$$

Mathematica [C] time = 0.63, size = 211, normalized size = 2.09

$$-\frac{i\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{-\frac{1}{x^2}-\frac{1}{x}+1}\,x^{3/2}\left(2F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{2}{1+\sqrt{5}}}}{\sqrt{x}}\right)|-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-(2-i)\Pi\left(-\frac{1}{2}i\left(1+\sqrt{5}\right);i\sinh^{-1}\left(\frac{\sqrt{\frac{2}{1+\sqrt{5}}}}{\sqrt{x}}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)-(2+i)\Pi\left(\frac{1}{2}i\left(1+\sqrt{5}\right);i\sinh^{-1}\left(\frac{\sqrt{\frac{2}{1+\sqrt{5}}}}{\sqrt{x}}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)\right)}{\sqrt{x\left(x^2-x-1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + x + x^2)/((1 + x^2)*Sqrt[-x - x^2 + x^3]),x]
```

```
[Out] ((-I)*Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 - x^(-2) - x^(-1)]*x^(3/2)*(2*EllipticF
[I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], -3/2 - Sqrt[5]/2] - (2 - I)*Elli
pticPi[(-1/2*I)*(1 + Sqrt[5]), I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], (-
3 - Sqrt[5])/2] - (2 + I)*EllipticPi[(I/2)*(1 + Sqrt[5]), I*ArcSinh[Sqrt[2/
(1 + Sqrt[5])]/Sqrt[x]], (-3 - Sqrt[5])/2]))/Sqrt[x*(-1 - x + x^2)]
```

IntegrateAlgebraic [A] time = 0.32, size = 101, normalized size = 1.00

$$-\frac{1}{2}\sqrt{\frac{11}{5}+\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1-2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)-\frac{1}{2}\sqrt{\frac{11}{5}-\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1+2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x + x^2)/((1 + x^2)*Sqrt[-x - x^2 + x^3]),x]
```

```
[Out] -1/2*(Sqrt[11/5 + (2*I)/5]*ArcTan[(Sqrt[1 - 2*I]*Sqrt[-x - x^2 + x^3])/(-1
- x + x^2)]) - (Sqrt[11/5 - (2*I)/5]*ArcTan[(Sqrt[1 + 2*I]*Sqrt[-x - x^2 +
x^3])/(-1 - x + x^2)])/2
```

fricas [B] time = 0.94, size = 2504, normalized size = 24.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(x^2+1)/(x^3-x^2-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/800*125^(1/4)*(5*sqrt(5)*sqrt(2) - 11*sqrt(2))*sqrt(55*sqrt(5) + 125)*log
(25*(25*x^4 - 100*x^3 + 2*125^(1/4)*sqrt(x^3 - x^2 - x)*(sqrt(5)*sqrt(2)*(2
*x^2 - 7*x - 2) - 5*sqrt(2)*(x^2 - 3*x - 1))*sqrt(55*sqrt(5) + 125) + 150*x
^2 + 100*sqrt(5)*(x^3 - x^2 - x) + 100*x + 25)/(x^4 + 2*x^2 + 1)) - 1/800*1
25^(1/4)*(5*sqrt(5)*sqrt(2) - 11*sqrt(2))*sqrt(55*sqrt(5) + 125)*log(25*(25
*x^4 - 100*x^3 - 2*125^(1/4)*sqrt(x^3 - x^2 - x)*(sqrt(5)*sqrt(2)*(2*x^2 -
7*x - 2) - 5*sqrt(2)*(x^2 - 3*x - 1))*sqrt(55*sqrt(5) + 125) + 150*x^2 + 10
0*sqrt(5)*(x^3 - x^2 - x) + 100*x + 25)/(x^4 + 2*x^2 + 1)) - 1/100*125^(1/4
)*sqrt(2)*sqrt(55*sqrt(5) + 125)*arctan(-1/2500*(1250*x^11 + 16250*x^10 - 8
3750*x^9 - 55000*x^8 + 355000*x^7 + 17500*x^6 - 355000*x^5 - 55000*x^4 + 83
750*x^3 + 16250*x^2 + 5*sqrt(x^3 - x^2 - x)*(125^(3/4)*(sqrt(5)*sqrt(2)*(x^
10 - x^9 - 27*x^8 + 36*x^7 + 132*x^6 - 86*x^5 - 132*x^4 + 36*x^3 + 27*x^2 -
 x - 1) - sqrt(2)*(2*x^10 + x^9 - 64*x^8 + 52*x^7 + 318*x^6 - 122*x^5 - 318
*x^4 + 52*x^3 + 64*x^2 + x - 2)) + 10*125^(1/4)*(sqrt(5)*sqrt(2)*(7*x^9 + 3
3*x^8 - 24*x^7 - 159*x^6 + 50*x^5 + 159*x^4 - 24*x^3 - 33*x^2 + 7*x) - 5*sq
rt(2)*(3*x^9 + 17*x^8 - 20*x^7 - 63*x^6 + 34*x^5 + 63*x^4 - 20*x^3 - 17*x^2
 + 3*x)))*sqrt(55*sqrt(5) + 125) + 250*sqrt(5)*(5*x^11 - 15*x^10 - 15*x^9 +
 20*x^8 - 20*x^7 + 70*x^6 + 20*x^5 + 20*x^4 + 15*x^3 - 15*x^2 - sqrt(5)*(x^
11 + 13*x^10 - 67*x^9 - 44*x^8 + 284*x^7 + 14*x^6 - 284*x^5 - 44*x^4 + 67*x
^3 + 13*x^2 - x) - 5*x) - 1250*sqrt(5)*(x^11 - 3*x^10 - 3*x^9 + 4*x^8 - 4*x
^7 + 14*x^6 + 4*x^5 + 4*x^4 + 3*x^3 - 3*x^2 - x) - (3000*x^10 + 2000*x^9 -
21000*x^8 - 6000*x^7 + 40000*x^6 + 6000*x^5 - 21000*x^4 - 2000*x^3 + 3000*x
^2 + sqrt(x^3 - x^2 - x)*(125^(3/4)*(sqrt(5)*sqrt(2)*(x^10 - 5*x^9 - 23*x^8
 + 144*x^7 - 120*x^6 - 246*x^5 + 120*x^4 + 144*x^3 + 23*x^2 - 5*x - 1) - sq
rt(2)*(2*x^10 - 9*x^9 - 44*x^8 + 312*x^7 - 302*x^6 - 542*x^5 + 302*x^4 + 31
2*x^3 + 44*x^2 - 9*x - 2)) + 10*125^(1/4)*(sqrt(5)*sqrt(2)*(7*x^9 + 3*x^8 -
 154*x^7 + 131*x^6 + 270*x^5 - 131*x^4 - 154*x^3 - 3*x^2 + 7*x) - 5*sqrt(2)
*(3*x^9 + 3*x^8 - 70*x^7 + 51*x^6 + 126*x^5 - 51*x^4 - 70*x^3 - 3*x^2 + 3*x
)))*sqrt(55*sqrt(5) + 125) + 50*sqrt(5)*(5*x^11 - 25*x^10 - 105*x^9 + 440*x
^8 + 50*x^7 - 830*x^6 - 50*x^5 + 440*x^4 + 105*x^3 - 25*x^2 - sqrt(5)*(x^11
 + 3*x^10 - 37*x^9 + 96*x^8 - 6*x^7 - 166*x^6 + 6*x^5 + 96*x^4 + 37*x^3 + 3
*x^2 - x) - 5*x) - 1000*sqrt(5)*(x^10 + 6*x^9 - 23*x^8 - 2*x^7 + 40*x^6 + 2
*x^5 - 23*x^4 - 6*x^3 + x^2))*sqrt((25*x^4 - 100*x^3 + 2*125^(1/4)*sqrt(x^3
 - x^2 - x)*(sqrt(5)*sqrt(2)*(2*x^2 - 7*x - 2) - 5*sqrt(2)*(x^2 - 3*x - 1))
*sqrt(55*sqrt(5) + 125) + 150*x^2 + 100*sqrt(5)*(x^3 - x^2 - x) + 100*x + 2
5)/(x^4 + 2*x^2 + 1)) - 1250*x)/(x^11 - 9*x^10 - 45*x^9 + 180*x^8 + 18*x^7
- 326*x^6 - 18*x^5 + 180*x^4 + 45*x^3 - 9*x^2 - x)) - 1/100*125^(1/4)*sqrt(
2)*sqrt(55*sqrt(5) + 125)*arctan(1/2500*(1250*x^11 + 16250*x^10 - 83750*x^9
 - 55000*x^8 + 355000*x^7 + 17500*x^6 - 355000*x^5 - 55000*x^4 + 83750*x^3
+ 16250*x^2 - 5*sqrt(x^3 - x^2 - x)*(125^(3/4)*(sqrt(5)*sqrt(2)*(x^10 - x^9
 - 27*x^8 + 36*x^7 + 132*x^6 - 86*x^5 - 132*x^4 + 36*x^3 + 27*x^2 - x - 1)
```

```
- sqrt(2)*(2*x^10 + x^9 - 64*x^8 + 52*x^7 + 318*x^6 - 122*x^5 - 318*x^4 + 5
2*x^3 + 64*x^2 + x - 2)) + 10*125^(1/4)*(sqrt(5)*sqrt(2)*(7*x^9 + 33*x^8 -
24*x^7 - 159*x^6 + 50*x^5 + 159*x^4 - 24*x^3 - 33*x^2 + 7*x) - 5*sqrt(2)*(3
*x^9 + 17*x^8 - 20*x^7 - 63*x^6 + 34*x^5 + 63*x^4 - 20*x^3 - 17*x^2 + 3*x))
)*sqrt(55*sqrt(5) + 125) + 250*sqrt(5)*(5*x^11 - 15*x^10 - 15*x^9 + 20*x^8
- 20*x^7 + 70*x^6 + 20*x^5 + 20*x^4 + 15*x^3 - 15*x^2 - sqrt(5)*(x^11 + 13*
x^10 - 67*x^9 - 44*x^8 + 284*x^7 + 14*x^6 - 284*x^5 - 44*x^4 + 67*x^3 + 13*
x^2 - x) - 5*x) - 1250*sqrt(5)*(x^11 - 3*x^10 - 3*x^9 + 4*x^8 - 4*x^7 + 14*
x^6 + 4*x^5 + 4*x^4 + 3*x^3 - 3*x^2 - x) - (3000*x^10 + 2000*x^9 - 21000*x^
8 - 6000*x^7 + 40000*x^6 + 6000*x^5 - 21000*x^4 - 2000*x^3 + 3000*x^2 - sqr
t(x^3 - x^2 - x)*(125^(3/4)*(sqrt(5)*sqrt(2)*(x^10 - 5*x^9 - 23*x^8 + 144*x
^7 - 120*x^6 - 246*x^5 + 120*x^4 + 144*x^3 + 23*x^2 - 5*x - 1) - sqrt(2)*(2
*x^10 - 9*x^9 - 44*x^8 + 312*x^7 - 302*x^6 - 542*x^5 + 302*x^4 + 312*x^3 +
44*x^2 - 9*x - 2)) + 10*125^(1/4)*(sqrt(5)*sqrt(2)*(7*x^9 + 3*x^8 - 154*x^7
 + 131*x^6 + 270*x^5 - 131*x^4 - 154*x^3 - 3*x^2 + 7*x) - 5*sqrt(2)*(3*x^9
+ 3*x^8 - 70*x^7 + 51*x^6 + 126*x^5 - 51*x^4 - 70*x^3 - 3*x^2 + 3*x)))*sqrt
(55*sqrt(5) + 125) + 50*sqrt(5)*(5*x^11 - 25*x^10 - 105*x^9 + 440*x^8 + 50*
x^7 - 830*x^6 - 50*x^5 + 440*x^4 + 105*x^3 - 25*x^2 - sqrt(5)*(x^11 + 3*x^1
0 - 37*x^9 + 96*x^8 - 6*x^7 - 166*x^6 + 6*x^5 + 96*x^4 + 37*x^3 + 3*x^2 - x
) - 5*x) - 1000*sqrt(5)*(x^10 + 6*x^9 - 23*x^8 - 2*x^7 + 40*x^6 + 2*x^5 - 2
3*x^4 - 6*x^3 + x^2))*sqrt((25*x^4 - 100*x^3 - 2*125^(1/4)*sqrt(x^3 - x^2 -
 x)*(sqrt(5)*sqrt(2)*(2*x^2 - 7*x - 2) - 5*sqrt(2)*(x^2 - 3*x - 1))*sqrt(55
*sqrt(5) + 125) + 150*x^2 + 100*sqrt(5)*(x^3 - x^2 - x) + 100*x + 25)/(x^4
+ 2*x^2 + 1)) - 1250*x)/(x^11 - 9*x^10 - 45*x^9 + 180*x^8 + 18*x^7 - 326*x^
6 - 18*x^5 + 180*x^4 + 45*x^3 - 9*x^2 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+x-1}{\sqrt{x^3-x^2-x}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(x^2+1)/(x^3-x^2-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + x - 1)/(sqrt(x^3 - x^2 - x)*(x^2 + 1)), x)
```

maple [C] time = 0.04, size = 1353, normalized size = 13.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-1)/(x^2+1)/(x^3-x^2-x)^(1/2),x)
```

```
[Out] 2/5*(1/2*5^(1/2)-1/2)*((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x-
1/2-1/2*5^(1/2))*5^(1/2))^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1
/2)*EllipticF(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),1/5*5^(1/2)*((1
/2*5^(1/2)-1/2)*5^(1/2))^(1/2))+1/10*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-
1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(
1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(1/2-I-1/2*5^(1/2))*Ell
ipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/
2-I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))*5^(1/2)+1/5
*I*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2)
)^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/
(x^3-x^2-x)^(1/2)/(1/2-I-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*
5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2-I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2
*5^(1/2)-1/2)*5^(1/2))^(1/2))*5^(1/2)-1/10*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^
(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+2
5/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(1/2-I-1/2*5^(1/2
))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2
))/(1/2-I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))-1/5*I
```

```
*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^
(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x
^3-x^2-x)^(1/2)/(1/2-I-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^
(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2-I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5
^(1/2)-1/2)*5^(1/2))^(1/2))+1/10*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)
+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)
*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(1/2+I-1/2*5^(1/2))*Ellipti
cPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2+I-
1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))*5^(1/2)-1/5*I*(
x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1
/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3
-x^2-x)^(1/2)/(1/2+I-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1
/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2+I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(
1/2)-1/2)*5^(1/2))^(1/2))*5^(1/2)-1/10*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2
)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)
^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(1/2+I-1/2*5^(1/2))*E
llipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(
1/2+I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))+1/5*I*(x/
(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2
)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x
^2-x)^(1/2)/(1/2+I-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2
)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2+I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/
2)-1/2)*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+x-1}{\sqrt{x^3-x^2-x}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(x^2+1)/(x^3-x^2-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + x - 1)/(sqrt(x^3 - x^2 - x)*(x^2 + 1)), x)
```

mupad [B] time = 0.84, size = 226, normalized size = 2.24

$$\frac{\left(\sqrt{5}\,\left(2+1\mathrm{i}\right)+2+1\mathrm{i}\right)\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\left(-2\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)+\Pi\left(-\frac{\sqrt{5}\,1\mathrm{i}}{2}-\frac{1}{2}\mathrm{i};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)\left(2-\mathrm{i}\right)+\Pi\left(\frac{\sqrt{5}\,1\mathrm{i}}{2}+\frac{1}{2}\mathrm{i};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)\left(2+1\mathrm{i}\right)\right)\left(-\frac{1}{5}+\frac{1}{10}\mathrm{i}\right)}{\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 - 1)/((x^2 + 1)*(x^3 - x^2 - x)^(1/2)),x)
```

```
[Out] ((5^(1/2)*(2 + 1i) + (2 + 1i))*(x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/2
- 1/2)/(5^(1/2)/2 - 1/2))^(1/2)*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 + 1/2))^(
1/2)*(ellipticPi(- (5^(1/2)*1i)/2 - 1i/2, asin((x/(5^(1/2)/2 + 1/2))^(1/2))
, -(5^(1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2))*(2 - 1i) - 2*ellipticF(asin((x/(5^(
1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2)) + ellipticPi((
5^(1/2)*1i)/2 + 1i/2, asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/2 + 1/2)
/(5^(1/2)/2 - 1/2))*(2 + 1i))*(- 1/5 + 1i/10))/(x^3 - x^2 - x*(5^(1/2)/2 -
1/2)*(5^(1/2)/2 + 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+x-1}{\sqrt{x\left(x^2-x-1\right)}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x-1)/(x**2+1)/(x**3-x**2-x)**(1/2),x)
```

```
[Out] Integral((x**2 + x - 1)/(sqrt(x*(x**2 - x - 1))*(x**2 + 1)), x)
```

3.1247 $\int \frac{x}{\sqrt[3]{x^2+x^3}}\,dx$

Optimal. Leaf size=101

$$\frac{\left(x^3+x^2\right)^{2/3}}{x}+\frac{1}{3}\log\left(\sqrt[3]{x^3+x^2}-x\right)-\frac{1}{6}\log\left(x^2+\sqrt[3]{x^3+x^2}\,x+\left(x^3+x^2\right)^{2/3}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x^2}+x}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 143, normalized size of antiderivative = 1.42, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2024, 2011, 59}

$$\frac{\left(x^3+x^2\right)^{2/3}}{x}+\frac{x^{2/3}\sqrt[3]{x+1}\log(x)}{6\sqrt[3]{x^3+x^2}}+\frac{x^{2/3}\sqrt[3]{x+1}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x}}-1\right)}{2\sqrt[3]{x^3+x^2}}+\frac{x^{2/3}\sqrt[3]{x+1}\tan^{-1}\left(\frac{2\sqrt[3]{x+1}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{x^3+x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(x^2 + x^3)^(1/3),x]
```

```
[Out] (x^2 + x^3)^(2/3)/x + (x^(2/3)*(1 + x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(1 + x)^
(1/3))/(Sqrt[3]*x^(1/3))])/(Sqrt[3]*(x^2 + x^3)^(1/3)) + (x^(2/3)*(1 + x)^(
1/3)*Log[x])/(6*(x^2 + x^3)^(1/3)) + (x^(2/3)*(1 + x)^(1/3)*Log[-1 + (1 + x
)^(1/3)/x^(1/3)])/(2*(x^2 + x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{x^2+x^3}}\,dx = \frac{\left(x^2+x^3\right)^{2/3}}{x} - \frac{1}{3}\int \frac{1}{\sqrt[3]{x^2+x^3}}\,dx$$
$$= \frac{\left(x^2+x^3\right)^{2/3}}{x} - \frac{\left(x^{2/3}\sqrt[3]{1+x}\right)\int \frac{1}{x^{2/3}\sqrt[3]{1+x}}\,dx}{3\sqrt[3]{x^2+x^3}}$$
$$= \frac{\left(x^2+x^3\right)^{2/3}}{x} + \frac{x^{2/3}\sqrt[3]{1+x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1+x}}{\sqrt{3}\sqrt[3]{x}}\right)}{\sqrt{3}\sqrt[3]{x^2+x^3}} + \frac{x^{2/3}\sqrt[3]{1+x}\log(x)}{6\sqrt[3]{x^2+x^3}} + \frac{x^{2/3}\sqrt[3]{1+x}\log\left(-1+\right.}{2\sqrt[3]{x^2+x^3}}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.35

$$\frac{3\left(x^2(x+1)\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{4}{3};\frac{7}{3};-x\right)}{4(x+1)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x^2 + x^3)^(1/3),x]
```

```
[Out] (3*(x^2*(1 + x))^(2/3)*Hypergeometric2F1[1/3, 4/3, 7/3, -x])/(4*(1 + x)^(2/
3))
```

IntegrateAlgebraic [A] time = 0.19, size = 101, normalized size = 1.00

$$\frac{\left(x^3+x^2\right)^{2/3}}{x} + \frac{1}{3}\log\left(\sqrt[3]{x^3+x^2}-x\right) - \frac{1}{6}\log\left(x^2+\sqrt[3]{x^3+x^2}\,x+\left(x^3+x^2\right)^{2/3}\right) - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x^2}+x}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(x^2 + x^3)^(1/3),x]
```

```
[Out] (x^2 + x^3)^(2/3)/x - ArcTan[(Sqrt[3]*x)/(x + 2*(x^2 + x^3)^(1/3))]/Sqrt[3]
 + Log[-x + (x^2 + x^3)^(1/3)]/3 - Log[x^2 + x*(x^2 + x^3)^(1/3) + (x^2 + x
^3)^(2/3)]/6
```

fricas [A] time = 0.41, size = 103, normalized size = 1.02

$$\frac{2\sqrt{3}\,x\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+x^2\right)^{\frac{1}{3}}}{3x}\right) + 2x\log\left(-\frac{x-\left(x^3+x^2\right)^{\frac{1}{3}}}{x}\right) - x\log\left(\frac{x^2+\left(x^3+x^2\right)^{\frac{1}{3}}x+\left(x^3+x^2\right)^{\frac{2}{3}}}{x^2}\right) + 6\left(x^3+x^2\right)^{\frac{2}{3}}}{6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*x*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 + x^2)^(1/3))/x) +
2*x*log(-(x - (x^3 + x^2)^(1/3))/x) - x*log((x^2 + (x^3 + x^2)^(1/3)*x + (x
^3 + x^2)^(2/3))/x^2) + 6*(x^3 + x^2)^(2/3))/x
```

giac [A] time = 0.23, size = 64, normalized size = 0.63

$$x\left(\frac{1}{x}+1\right)^{\frac{2}{3}} + \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{1}{6}\log\left(\left(\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right) + \frac{1}{3}\log\left(\left|\left(\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] x*(1/x + 1)^(2/3) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(1/x + 1)^(1/3) + 1))
 - 1/6*log((1/x + 1)^(2/3) + (1/x + 1)^(1/3) + 1) + 1/3*log(abs((1/x + 1)^(
1/3) - 1))
```

maple [C] time = 0.25, size = 30, normalized size = 0.30

$$\frac{x\left(1+x\right)}{\left(x^{2}\left(1+x\right)\right)^{\frac{1}{3}}}-x^{\frac{1}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3+x^2)^(1/3),x)
```

```
[Out] x*(1+x)/(x^2*(1+x))^(1/3)-x^(1/3)*hypergeom([1/3,1/3],[4/3],-x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^{3}+x^{2}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^3 + x^2)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(x^3+x^2\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 + x^3)^(1/3),x)
```

```
[Out] int(x/(x^2 + x^3)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{x^{2}\left(x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3+x**2)**(1/3),x)
```

```
[Out] Integral(x/(x**2*(x + 1))**(1/3), x)
```

3.1248 $\int \frac{\sqrt[3]{-1+x^3}\left(-1+2x^3\right)}{x^5}\,dx$

Optimal. Leaf size=101

$$-\frac{2}{3}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{2\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}+\frac{\sqrt[3]{x^3-1}\left(1-9x^3\right)}{4x^4}+\frac{1}{3}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {451, 277, 331, 292, 31, 634, 618, 204, 628}

$$-\frac{2\sqrt[3]{x^3-1}}{x}-\frac{2}{3}\log\left(1-\frac{x}{\sqrt[3]{x^3-1}}\right)-\frac{2\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\left(x^3-1\right)^{4/3}}{4x^4}+\frac{1}{3}\log\left(\frac{x}{\sqrt[3]{x^3-1}}+\frac{x^2}{\left(x^3-1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(1/3)*(-1 + 2*x^3))/x^5,x]
```

```
[Out] (-2*(-1 + x^3)^(1/3))/x - (-1 + x^3)^(4/3)/(4*x^4) - (2*ArcTan[(1 + (2*x)/(
-1 + x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - (2*Log[1 - x/(-1 + x^3)^(1/3)])/3 + Lo
g[1 + x^2/(-1 + x^3)^(2/3) + x/(-1 + x^3)^(1/3)]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{-1+x^3}\left(-1+2x^3\right)}{x^5}\,dx &= -\frac{\left(-1+x^3\right)^{4/3}}{4x^4}+2\int \frac{\sqrt[3]{-1+x^3}}{x^2}\,dx\\
&= -\frac{2\sqrt[3]{-1+x^3}}{x}-\frac{\left(-1+x^3\right)^{4/3}}{4x^4}+2\int \frac{x}{\left(-1+x^3\right)^{2/3}}\,dx\\
&= -\frac{2\sqrt[3]{-1+x^3}}{x}-\frac{\left(-1+x^3\right)^{4/3}}{4x^4}+2\,\text{Subst}\left(\int \frac{x}{1-x^3}\,dx,x,\frac{x}{\sqrt[3]{-1+x^3}}\right)\\
&= -\frac{2\sqrt[3]{-1+x^3}}{x}-\frac{\left(-1+x^3\right)^{4/3}}{4x^4}+\frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\frac{x}{\sqrt[3]{-1+x^3}}\right)-\frac{2}{3}\text{Subst}\Bigg(\\
&= -\frac{2\sqrt[3]{-1+x^3}}{x}-\frac{\left(-1+x^3\right)^{4/3}}{4x^4}-\frac{2}{3}\log\left(1-\frac{x}{\sqrt[3]{-1+x^3}}\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\right.\\
&= -\frac{2\sqrt[3]{-1+x^3}}{x}-\frac{\left(-1+x^3\right)^{4/3}}{4x^4}-\frac{2}{3}\log\left(1-\frac{x}{\sqrt[3]{-1+x^3}}\right)+\frac{1}{3}\log\left(1+\frac{x^2}{\left(-1+x^3\right)^{2/3}}\right.\\
&= -\frac{2\sqrt[3]{-1+x^3}}{x}-\frac{\left(-1+x^3\right)^{4/3}}{4x^4}-\frac{2\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log\left(1-\frac{x}{\sqrt[3]{-1+x^3}}\right)+
\end{aligned}$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.51

$$\frac{\sqrt[3]{x^3-1}\left(-\frac{8x^3\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};x^3\right)}{\sqrt[3]{1-x^3}}-x^3+1\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)^(1/3)*(-1 + 2*x^3))/x^5,x]
```

```
[Out] ((-1 + x^3)^(1/3)*(1 - x^3 - (8*x^3*Hypergeometric2F1[-1/3, -1/3, 2/3, x^3]
)/(1 - x^3)^(1/3)))/(4*x^4)
```

IntegrateAlgebraic [A] time = 0.16, size = 101, normalized size = 1.00

$$-\frac{2}{3}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{2\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}+\frac{\sqrt[3]{x^3-1}\left(1-9x^3\right)}{4x^4}+\frac{1}{3}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(1/3)*(-1 + 2*x^3))/x^5,x]
```

```
[Out] ((1 - 9*x^3)*(-1 + x^3)^(1/3))/(4*x^4) - (2*ArcTan[(Sqrt[3]*x)/(x + 2*(-1 +
 x^3)^(1/3))])/Sqrt[3] - (2*Log[-x + (-1 + x^3)^(1/3)])/3 + Log[x^2 + x*(-1
 + x^3)^(1/3) + (-1 + x^3)^(2/3)]/3
```

fricas [A] time = 0.65, size = 112, normalized size = 1.11

$$-\frac{8\sqrt{3}x^4\arctan\left(-\frac{25382\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}x^2-13720\sqrt{3}\left(x^3-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(5831x^3-7200\right)}{58653x^3-8000}\right)+4x^4\log\left(-3\left(x^3-1\right)^{\frac{1}{3}}x^2+3\left(x^3-1\right)^{\frac{2}{3}}x+1\right)+3\left(9x^3-1\right)\left(x^3-1\right)^{\frac{1}{3}}}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(2*x^3-1)/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(8*sqrt(3)*x^4*arctan(-(25382*sqrt(3)*(x^3 - 1)^(1/3)*x^2 - 13720*sqr
t(3)*(x^3 - 1)^(2/3)*x + sqrt(3)*(5831*x^3 - 7200))/(58653*x^3 - 8000)) + 4
*x^4*log(-3*(x^3 - 1)^(1/3)*x^2 + 3*(x^3 - 1)^(2/3)*x + 1) + 3*(9*x^3 - 1)*
(x^3 - 1)^(1/3))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^3-1\right)\left(x^3-1\right)^{\frac{1}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(2*x^3-1)/x^5,x, algorithm="giac")
```

```
[Out] integrate((2*x^3 - 1)*(x^3 - 1)^(1/3)/x^5, x)
```

maple [C] time = 0.27, size = 57, normalized size = 0.56

$$-\frac{9x^6-10x^3+1}{4x^4\left(x^3-1\right)^{\frac{2}{3}}}+\frac{\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{2}{3}}x^2\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{\mathrm{signum}\left(x^3-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/3)*(2*x^3-1)/x^5,x)
```

```
[Out] -1/4*(9*x^6-10*x^3+1)/x^4/(x^3-1)^(2/3)+1/signum(x^3-1)^(2/3)*(-signum(x^3-
1))^(2/3)*x^2*hypergeom([2/3,2/3],[5/3],x^3)
```

maxima [A] time = 0.44, size = 93, normalized size = 0.92

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3-1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{2\left(x^3-1\right)^{\frac{1}{3}}}{x}-\frac{\left(x^3-1\right)^{\frac{4}{3}}}{4x^4}+\frac{1}{3}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3-1\right)^{\frac{2}{3}}}{x^2}+1\right)-\frac{2}{3}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(2*x^3-1)/x^5,x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3)/x + 1)) - 2*(x^3 - 1)^(1/
3)/x - 1/4*(x^3 - 1)^(4/3)/x^4 + 1/3*log((x^3 - 1)^(1/3)/x + (x^3 - 1)^(2/3
)/x^2 + 1) - 2/3*log((x^3 - 1)^(1/3)/x - 1)
```

mupad [B] time = 1.12, size = 55, normalized size = 0.54

$$\frac{\left(x^3-1\right)^{1/3}-x^3\left(x^3-1\right)^{1/3}}{4\,x^4}-\frac{2\left(x^3-1\right)^{1/3}{}_2\mathrm{F}_1\left(-\frac{1}{3},-\frac{1}{3};\ \frac{2}{3};\ x^3\right)}{x\left(1-x^3\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(1/3)*(2*x^3 - 1))/x^5,x)
```

```
[Out] ((x^3 - 1)^(1/3) - x^3*(x^3 - 1)^(1/3))/(4*x^4) - (2*(x^3 - 1)^(1/3)*hyperg
eom([-1/3, -1/3], 2/3, x^3))/(x*(1 - x^3)^(1/3))
```

sympy [C] time = 2.35, size = 167, normalized size = 1.65

$$-\begin{cases}\frac{\sqrt[3]{-1+\frac{1}{x^3}}\,e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)}-\frac{\sqrt[3]{-1+\frac{1}{x^3}}\,e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{3x^3\Gamma\left(-\frac{1}{3}\right)} & \text{for}\ \frac{1}{|x^3|}>1\\ -\frac{\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{4}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)}+\frac{\sqrt[3]{1-\frac{1}{x^3}}\,\Gamma\left(-\frac{4}{3}\right)}{3x^3\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise}\end{cases}+\frac{2e^{\frac{i\pi}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|x^3\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/3)*(2*x**3-1)/x**5,x)
```

```
[Out] -Piecewise(((-1 + x**(-3))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(3*gamma(-1/3)
) - (-1 + x**(-3))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(3*x**3*gamma(-1/3)),
1/Abs(x**3) > 1), (-(1 - 1/x**3)**(1/3)*gamma(-4/3)/(3*gamma(-1/3)) + (1 -
1/x**3)**(1/3)*gamma(-4/3)/(3*x**3*gamma(-1/3)), True)) + 2*exp(I*pi/3)*gam
ma(-1/3)*hyper((-1/3, -1/3), (2/3,), x**3)/(3*x*gamma(2/3))
```

3.1249 $\int \frac{\left(-1+x^3\right)^{2/3}\left(-1+3x^3\right)}{x^6\left(-1+2x^3\right)}\,dx$

Optimal. Leaf size=101

$$\frac{1}{3}\log\left(\sqrt[3]{x^3-1}+x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}-x}\right)}{\sqrt{3}}+\frac{\left(x^3-1\right)^{2/3}\left(7x^3-2\right)}{10x^5}-\frac{1}{6}\log\left(-\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.13, antiderivative size = 111, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {580, 583, 12, 377, 200, 31, 634, 618, 204, 628}

$$\frac{1}{3}\log\left(\frac{x}{\sqrt[3]{x^3-1}}+1\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{x^3-1}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\left(x^3-1\right)^{2/3}}{5x^5}+\frac{7\left(x^3-1\right)^{2/3}}{10x^2}-\frac{1}{6}\log\left(-\frac{x}{\sqrt[3]{x^3-1}}+\frac{x^2}{\left(x^3-1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(2/3)*(-1 + 3*x^3))/(x^6*(-1 + 2*x^3)),x]
```

```
[Out] -1/5*(-1 + x^3)^(2/3)/x^5 + (7*(-1 + x^3)^(2/3))/(10*x^2) - ArcTan[(1 - (2*
x)/(-1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 + x^2/(-1 + x^3)^(2/3) - x/(-
1 + x^3)^(1/3)]/6 + Log[1 + x/(-1 + x^3)^(1/3)]/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
```

```
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(-1+3x^3\right)}{x^6\left(-1+2x^3\right)}\,dx = -\frac{\left(-1+x^3\right)^{2/3}}{5x^5} - \frac{1}{5}\int \frac{7-9x^3}{x^3\sqrt[3]{-1+x^3}\left(-1+2x^3\right)}\,dx$$
$$= -\frac{\left(-1+x^3\right)^{2/3}}{5x^5} + \frac{7\left(-1+x^3\right)^{2/3}}{10x^2} + \frac{1}{10}\int -\frac{10}{\sqrt[3]{-1+x^3}\left(-1+2x^3\right)}\,dx$$
$$= -\frac{\left(-1+x^3\right)^{2/3}}{5x^5} + \frac{7\left(-1+x^3\right)^{2/3}}{10x^2} - \int \frac{1}{\sqrt[3]{-1+x^3}\left(-1+2x^3\right)}\,dx$$
$$= -\frac{\left(-1+x^3\right)^{2/3}}{5x^5} + \frac{7\left(-1+x^3\right)^{2/3}}{10x^2} - \text{Subst}\left(\int \frac{1}{-1-x^3}\,dx, x, \frac{x}{\sqrt[3]{-1+x^3}}\right)$$
$$= -\frac{\left(-1+x^3\right)^{2/3}}{5x^5} + \frac{7\left(-1+x^3\right)^{2/3}}{10x^2} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1-x}\,dx, x, \frac{x}{\sqrt[3]{-1+x^3}}\right) - \frac{1}{3}\text{Sub}$$
$$= -\frac{\left(-1+x^3\right)^{2/3}}{5x^5} + \frac{7\left(-1+x^3\right)^{2/3}}{10x^2} + \frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{-1+x^3}}\right) - \frac{1}{6}\text{Subst}\left(\int \frac{-1+2}{1-x+}\right.$$
$$= -\frac{\left(-1+x^3\right)^{2/3}}{5x^5} + \frac{7\left(-1+x^3\right)^{2/3}}{10x^2} - \frac{1}{6}\log\left(1+\frac{x^2}{\left(-1+x^3\right)^{2/3}} - \frac{x}{\sqrt[3]{-1+x^3}}\right) + \frac{1}{3}\log$$
$$= -\frac{\left(-1+x^3\right)^{2/3}}{5x^5} + \frac{7\left(-1+x^3\right)^{2/3}}{10x^2} + \frac{\tan^{-1}\left(\frac{-1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{6}\log\left(1+\frac{x^2}{\left(-1+x^3\right)^{2/3}}\right.$$

Mathematica [A] time = 0.12, size = 111, normalized size = 1.10

$$\frac{1}{3}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right) + \frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{1-x^3}}-1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{6}\log\left(-\frac{x}{\sqrt[3]{1-x^3}} + \frac{x^2}{\left(1-x^3\right)^{2/3}} + 1\right) + \left(x^3-1\right)^{2/3}\left(\frac{7}{10x^2} - \frac{1}{5x^5}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^3)^(2/3)*(-1 + 3*x^3))/(x^6*(-1 + 2*x^3)),x]
```

```
[Out] (-1/5*1/x^5 + 7/(10*x^2))*(-1 + x^3)^(2/3) + ArcTan[(-1 + (2*x)/(1 - x^3)^(
1/3))/Sqrt[3]]/Sqrt[3] - Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)]/6
 + Log[1 + x/(1 - x^3)^(1/3)]/3
```

IntegrateAlgebraic [A] time = 0.20, size = 101, normalized size = 1.00

$$\frac{1}{3}\log\left(\sqrt[3]{x^3-1}+x\right) + \frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-1}-x}\right)}{\sqrt{3}} + \frac{\left(x^3-1\right)^{2/3}\left(7x^3-2\right)}{10x^5} - \frac{1}{6}\log\left(-\sqrt[3]{x^3-1}\,x + \left(x^3-1\right)^{2/3} + x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(2/3)*(-1 + 3*x^3))/(x^6*(-1 + 2*x^3)),x]
```

```
[Out] ((-1 + x^3)^(2/3)*(-2 + 7*x^3))/(10*x^5) + ArcTan[(Sqrt[3]*x)/(-x + 2*(-1 +
 x^3)^(1/3))]/Sqrt[3] + Log[x + (-1 + x^3)^(1/3)]/3 - Log[x^2 - x*(-1 + x^3
)^(1/3) + (-1 + x^3)^(2/3)]/6
```

fricas [A] time = 0.94, size = 124, normalized size = 1.23

$$-\frac{10\sqrt{3}x^5\arctan\left(\frac{4\sqrt{3}(x^3-1)^{\frac{1}{3}}x^2+2\sqrt{3}(x^3-1)^{\frac{2}{3}}x+\sqrt{3}(x^3-1)}{7x^3+1}\right)-5x^5\log\left(\frac{2x^3+3(x^3-1)^{\frac{1}{3}}x^2+3(x^3-1)^{\frac{2}{3}}x-1}{2x^3-1}\right)-3\left(7x^3-2\right)\left(x^3-1\right)^{\frac{2}{3}}}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(3*x^3-1)/x^6/(2*x^3-1),x, algorithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*x^5*arctan((4*sqrt(3)*(x^3 - 1)^(1/3)*x^2 + 2*sqrt(3)*(x^
3 - 1)^(2/3)*x + sqrt(3)*(x^3 - 1))/(7*x^3 + 1)) - 5*x^5*log((2*x^3 + 3*(x^
3 - 1)^(1/3)*x^2 + 3*(x^3 - 1)^(2/3)*x - 1)/(2*x^3 - 1)) - 3*(7*x^3 - 2)*(x
^3 - 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^3-1\right)\left(x^3-1\right)^{\frac{2}{3}}}{\left(2x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(3*x^3-1)/x^6/(2*x^3-1),x, algorithm="giac")
```

```
[Out] integrate((3*x^3 - 1)*(x^3 - 1)^(2/3)/((2*x^3 - 1)*x^6), x)
```

maple [C] time = 1.15, size = 393, normalized size = 3.89

$$\frac{7x^6-9x^3+2}{10x^5\left(x^3-1\right)^{\frac{1}{3}}}+\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\ln\left(\frac{9\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3-3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3-3x\left(x^3-1\right)^{\frac{2}{3}}+3x^2\left(x^3-1\right)^{\frac{1}{3}}+3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)-1}{2x^3-1}\right)-\frac{\ln\left(\frac{9\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3-3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3-1\right)^{\frac{2}{3}}x+3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3-1\right)^{\frac{1}{3}}x^2+6\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3-2x\left(x^3-1\right)^{\frac{2}{3}}+2x^2\left(x^3-1\right)^{\frac{1}{3}}+x^3-3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)-1}{2x^3-1}\right)}{3}-\ln\left(\frac{9\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3-3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3-1\right)^{\frac{2}{3}}x+3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3-1\right)^{\frac{1}{3}}x^2+6\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3-2x\left(x^3-1\right)^{\frac{2}{3}}+2x^2\left(x^3-1\right)^{\frac{1}{3}}+x^3-3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)-1}{2x^3-1}\right)\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)*(3*x^3-1)/x^6/(2*x^3-1),x)
```

```
[Out] 1/10*(7*x^6-9*x^3+2)/x^5/(x^3-1)^(1/3)+RootOf(9*_Z^2+3*_Z+1)*ln((9*RootOf(9
*_Z^2+3*_Z+1)^2*x^3-3*RootOf(9*_Z^2+3*_Z+1)*x^3-3*x*(x^3-1)^(2/3)+3*x^2*(x^
3-1)^(1/3)+3*RootOf(9*_Z^2+3*_Z+1)-1)/(2*x^3-1))-1/3*ln((9*RootOf(9*_Z^2+3*
_Z+1)^2*x^3-3*RootOf(9*_Z^2+3*_Z+1)*(x^3-1)^(2/3)*x+3*RootOf(9*_Z^2+3*_Z+1)
*(x^3-1)^(1/3)*x^2+6*RootOf(9*_Z^2+3*_Z+1)*x^3-2*x*(x^3-1)^(2/3)+2*x^2*(x^3
-1)^(1/3)+x^3-3*RootOf(9*_Z^2+3*_Z+1)-1)/(2*x^3-1))-ln((9*RootOf(9*_Z^2+3*_
Z+1)^2*x^3-3*RootOf(9*_Z^2+3*_Z+1)*(x^3-1)^(2/3)*x+3*RootOf(9*_Z^2+3*_Z+1)*
(x^3-1)^(1/3)*x^2+6*RootOf(9*_Z^2+3*_Z+1)*x^3-2*x*(x^3-1)^(2/3)+2*x^2*(x^3-
1)^(1/3)+x^3-3*RootOf(9*_Z^2+3*_Z+1)-1)/(2*x^3-1))*RootOf(9*_Z^2+3*_Z+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^3-1\right)\left(x^3-1\right)^{\frac{2}{3}}}{\left(2x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(3*x^3-1)/x^6/(2*x^3-1),x, algorithm="maxima")
```

```
[Out] integrate((3*x^3 - 1)*(x^3 - 1)^(2/3)/((2*x^3 - 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3-1\right)^{2/3}\,\left(3x^3-1\right)}{x^6\,\left(2x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(3*x^3 - 1))/(x^6*(2*x^3 - 1)),x)
```

```
[Out] int(((x^3 - 1)^(2/3)*(3*x^3 - 1))/(x^6*(2*x^3 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(3x^3-1\right)}{x^6\left(2x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)*(3*x**3-1)/x**6/(2*x**3-1),x)
```

```
[Out] Integral(((x - 1)*(x**2 + x + 1))**(2/3)*(3*x**3 - 1)/(x**6*(2*x**3 - 1)),
x)
```

3.1250 $\int \frac{x^4}{\left(-1+x^4\right)^2 \sqrt[4]{x^2+x^4}} \, dx$

Optimal. Leaf size=101

$$\frac{\left(x^4+x^2\right)^{3/4}\left(-13x^4-2x^2-5\right)}{80x\left(x^2-1\right)\left(x^2+1\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+x^2}}\right)}{32\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+x^2}}\right)}{32\sqrt[4]{2}}$$

Rubi [C] time = 9.82, antiderivative size = 109, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2056, 1254, 466, 510}

$$\frac{128x^5\Gamma\left(\frac{13}{4}\right)\left(17\left(-4x^4-9x^2+13\right){}_2F_1\left(1,2;\frac{17}{4};-\frac{2x^2}{1-x^2}\right)-64x^2\left(x^2+1\right){}_2F_1\left(2,3;\frac{21}{4};-\frac{2x^2}{1-x^2}\right)\right)}{89505\left(1-x^2\right)^3\left(x^2+1\right)\sqrt[4]{x^4+x^2}\,\Gamma\left(\frac{1}{4}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^4/((-1 + x^4)^2*(x^2 + x^4)^(1/4)),x]
```

```
[Out] (128*x^5*Gamma[13/4]*(17*(13 - 9*x^2 - 4*x^4)*Hypergeometric2F1[1, 2, 17/4,
 (-2*x^2)/(1 - x^2)] - 64*x^2*(1 + x^2)*Hypergeometric2F1[2, 3, 21/4, (-2*x
^2)/(1 - x^2)]))/(89505*(1 - x^2)^3*(1 + x^2)*(x^2 + x^4)^(1/4)*Gamma[1/4])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1254

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e)^p, x] /
; FreeQ[{a, c, d, e, f, q, m, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p
]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{x^4}{\left(-1+x^4\right)^2 \sqrt[4]{x^2+x^4}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{1+x^2}\right)\int \frac{x^{7/2}}{\sqrt[4]{1+x^2}\left(-1+x^4\right)^2}\,dx}{\sqrt[4]{x^2+x^4}}$$
$$= \frac{\left(\sqrt{x}\sqrt[4]{1+x^2}\right)\int \frac{x^{7/2}}{\left(-1+x^2\right)^2\left(1+x^2\right)^{9/4}}\,dx}{\sqrt[4]{x^2+x^4}}$$
$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^2}\right)\mathrm{Subst}\left(\int \frac{x^8}{\left(-1+x^4\right)^2\left(1+x^4\right)^{9/4}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^4}}$$
$$= \frac{128x^5\Gamma\left(\frac{13}{4}\right)\left(17\left(13-9x^2-4x^4\right)\,{}_2F_1\left(1,2;\frac{17}{4};-\frac{2x^2}{1-x^2}\right)-64x^2\left(1+x^2\right)\,{}_2F_1\left(2,3;\frac{21}{4};\right.\right.}{89505\left(1-x^2\right)^3\left(1+x^2\right)\sqrt[4]{x^2+x^4}\,\Gamma\left(\frac{1}{4}\right)}$$

Mathematica [A] time = 0.33, size = 112, normalized size = 1.11

$$\frac{x\left(-4\left(13x^4+2x^2+5\right)+5\,2^{3/4}\sqrt[4]{\frac{1}{x^2}+1}\left(x^4-1\right)\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{x^2}+1}}{\sqrt[4]{2}}\right)-5\,2^{3/4}\sqrt[4]{\frac{1}{x^2}+1}\left(x^4-1\right)\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{x^2}+1}}{\sqrt[4]{2}}\right)\right)}{320\left(x^4-1\right)\sqrt[4]{x^4+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((-1 + x^4)^2*(x^2 + x^4)^(1/4)),x]
```

```
[Out] (x*(-4*(5 + 2*x^2 + 13*x^4) + 5*2^(3/4)*(1 + x^(-2))^(1/4)*(-1 + x^4)*ArcTa
n[(1 + x^(-2))^(1/4)/2^(1/4)] - 5*2^(3/4)*(1 + x^(-2))^(1/4)*(-1 + x^4)*Arc
Tanh[(1 + x^(-2))^(1/4)/2^(1/4)]))/(320*(-1 + x^4)*(x^2 + x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.47, size = 101, normalized size = 1.00

$$\frac{\left(x^4+x^2\right)^{3/4}\left(-13x^4-2x^2-5\right)}{80x\left(x^2-1\right)\left(x^2+1\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^2}}\right)}{32\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^2}}\right)}{32\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((-1 + x^4)^2*(x^2 + x^4)^(1/4)),x]
```

```
[Out] ((-5 - 2*x^2 - 13*x^4)*(x^2 + x^4)^(3/4))/(80*x*(-1 + x^2)*(1 + x^2)^2) - A
rcTan[(2^(1/4)*x)/(x^2 + x^4)^(1/4)]/(32*2^(1/4)) - ArcTanh[(2^(1/4)*x)/(x^
2 + x^4)^(1/4)]/(32*2^(1/4))
```

fricas [B] time = 1.75, size = 311, normalized size = 3.08

$$\frac{20\cdot 2^{\frac{3}{4}}\left(x^7+x^5-x^3-x\right)\arctan\left(\frac{4\cdot 2^{\frac{3}{4}}\left(x^4+x^2\right)^{\frac{1}{4}}x^2+2^{\frac{3}{4}}\left(2\cdot 2^{\frac{3}{4}}\sqrt{x^4+x^2}\,x+2^{\frac{1}{4}}\left(3x^3+x\right)\right)+4\cdot 2^{\frac{1}{4}}\left(x^4+x^2\right)^{\frac{3}{4}}}{2\left(x^3-x\right)}\right)-5\cdot 2^{\frac{3}{4}}\left(x^7+x^5-x^3-x\right)\log\left(\frac{4\sqrt{2}\left(x^4+x^2\right)^{\frac{1}{4}}x^2+2^{\frac{3}{4}}\left(3x^3+x\right)+4\cdot 2^{\frac{1}{4}}\sqrt{x^4+x^2}\,x+4\left(x^4+x^2\right)^{\frac{3}{4}}}{x^3-x}\right)+5\cdot 2^{\frac{3}{4}}\left(x^7+x^5-x^3-x\right)\log\left(\frac{4\sqrt{2}\left(x^4+x^2\right)^{\frac{1}{4}}x^2-2^{\frac{3}{4}}\left(3x^3+x\right)-4\cdot 2^{\frac{1}{4}}\sqrt{x^4+x^2}\,x+4\left(x^4+x^2\right)^{\frac{3}{4}}}{x^3-x}\right)-16\left(13x^4+2x^2+5\right)\left(x^4+x^2\right)^{\frac{3}{4}}}{1280\left(x^7+x^5-x^3-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1)^2/(x^4+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/1280*(20*2^(3/4)*(x^7 + x^5 - x^3 - x)*arctan(1/2*(4*2^(3/4)*(x^4 + x^2)^
(1/4)*x^2 + 2^(3/4)*(2*2^(3/4)*sqrt(x^4 + x^2)*x + 2^(1/4)*(3*x^3 + x)) + 4
*2^(1/4)*(x^4 + x^2)^(3/4))/(x^3 - x)) - 5*2^(3/4)*(x^7 + x^5 - x^3 - x)*lo
g((4*sqrt(2)*(x^4 + x^2)^(1/4)*x^2 + 2^(3/4)*(3*x^3 + x) + 4*2^(1/4)*sqrt(x
```

```
^4 + x^2)*x + 4*(x^4 + x^2)^(3/4))/(x^3 - x)) + 5*2^(3/4)*(x^7 + x^5 - x^3
- x)*log((4*sqrt(2)*(x^4 + x^2)^(1/4)*x^2 - 2^(3/4)*(3*x^3 + x) - 4*2^(1/4)
*sqrt(x^4 + x^2)*x + 4*(x^4 + x^2)^(3/4))/(x^3 - x)) - 16*(13*x^4 + 2*x^2 +
 5)*(x^4 + x^2)^(3/4))/(x^7 + x^5 - x^3 - x)
```

giac [A] time = 0.36, size = 81, normalized size = 0.80

$$\frac{1}{64}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)-\frac{1}{128}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)+\frac{1}{128}\cdot 2^{\frac{3}{4}}\log\left(\left|-2^{\frac{1}{4}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right|\right)+\frac{\left(\frac{1}{x^2}+1\right)^{\frac{3}{4}}}{16\left(\frac{1}{x^2}-1\right)}-\frac{1}{10\left(\frac{1}{x^2}+1\right)^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1)^2/(x^4+x^2)^(1/4),x, algorithm="giac")
```

```
[Out] 1/64*2^(3/4)*arctan(1/2*2^(3/4)*(1/x^2 + 1)^(1/4)) - 1/128*2^(3/4)*log(2^(1
/4) + (1/x^2 + 1)^(1/4)) + 1/128*2^(3/4)*log(abs(-2^(1/4) + (1/x^2 + 1)^(1/
4))) + 1/16*(1/x^2 + 1)^(3/4)/(1/x^2 - 1) - 1/10/(1/x^2 + 1)^(5/4)
```

maple [C] time = 4.16, size = 268, normalized size = 2.65

$$-\frac{x\left(13x^4+2x^2+5\right)}{80\left(x^2+1\right)\left(x^2\left(x^2+1\right)\right)^{\frac{1}{4}}\left(x^2-1\right)}+\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\ln\left(\frac{\sqrt{x^4+x^2}\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\mathrm{RootOf}\left(\_Z^4-8\right)^2x-2\,\mathrm{RootOf}\left(\_Z^4-8\right)^2\left(x^4+x^2\right)^{\frac{1}{4}}x^2-3\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)x^3+4\left(x^4+x^2\right)^{\frac{3}{4}}-\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)x}{(1+x)x(-1+x)}\right)}{128}+\frac{\mathrm{RootOf}\left(\_Z^4-8\right)\ln\left(-\frac{\sqrt{x^4+x^2}\,\mathrm{RootOf}\left(\_Z^4-8\right)^3x-2\,\mathrm{RootOf}\left(\_Z^4-8\right)^2\left(x^4+x^2\right)^{\frac{1}{4}}x^2+3\,\mathrm{RootOf}\left(\_Z^4-8\right)x^3-4\left(x^4+x^2\right)^{\frac{3}{4}}+\mathrm{RootOf}\left(\_Z^4-8\right)x}{(1+x)x(-1+x)}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4-1)^2/(x^4+x^2)^(1/4),x)
```

```
[Out] -1/80*x*(13*x^4+2*x^2+5)/(x^2+1)/(x^2*(x^2+1))^(1/4)/(x^2-1)+1/128*RootOf(_
Z^2+RootOf(_Z^4-8)^2)*ln(((x^4+x^2)^(1/2)*RootOf(_Z^2+RootOf(_Z^4-8)^2)*Roo
tOf(_Z^4-8)^2*x-2*RootOf(_Z^4-8)^2*(x^4+x^2)^(1/4)*x^2-3*RootOf(_Z^2+RootOf
(_Z^4-8)^2)*x^3+4*(x^4+x^2)^(3/4)-RootOf(_Z^2+RootOf(_Z^4-8)^2)*x)/(1+x)/x/
(-1+x))+1/128*RootOf(_Z^4-8)*ln(-((x^4+x^2)^(1/2)*RootOf(_Z^4-8)^3*x-2*Root
Of(_Z^4-8)^2*(x^4+x^2)^(1/4)*x^2+3*RootOf(_Z^4-8)*x^3-4*(x^4+x^2)^(3/4)+Roo
tOf(_Z^4-8)*x)/(1+x)/x/(-1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(4x^5+x^3-3x\right)x^{\frac{7}{2}}}{21\left(x^8-2x^4+1\right)\left(x^2+1\right)^{\frac{1}{4}}}+\int\frac{16\left(4x^4+x^2-3\right)x^{\frac{7}{2}}}{21\left(x^{12}-3x^8+3x^4-1\right)\left(x^2+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1)^2/(x^4+x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] 2/21*(4*x^5 + x^3 - 3*x)*x^(7/2)/((x^8 - 2*x^4 + 1)*(x^2 + 1)^(1/4)) + inte
grate(16/21*(4*x^4 + x^2 - 3)*x^(7/2)/((x^12 - 3*x^8 + 3*x^4 - 1)*(x^2 + 1)
^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^4}{{\left(x^4+x^2\right)}^{1/4}\,{\left(x^4-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((x^2 + x^4)^(1/4)*(x^4 - 1)^2),x)
```

```
[Out] int(x^4/((x^2 + x^4)^(1/4)*(x^4 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt[4]{x^2\left(x^2+1\right)}\,(x-1)^2\,(x+1)^2\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4-1)**2/(x**4+x**2)**(1/4),x)
```

```
[Out] Integral(x**4/((x**2*(x**2 + 1))**(1/4)*(x - 1)**2*(x + 1)**2*(x**2 + 1)**2
), x)
```

3.1251 $\int \frac{(1+x^2)\sqrt[4]{x^3+x^4}}{x^2(-1+x^2)}\,dx$

Optimal. Leaf size=101

$$\frac{4\sqrt[4]{x^4+x^3}}{x}-2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)+2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^3}}\right)+2\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^3}}\right)$$

Rubi [A] time = 1.25, antiderivative size = 193, normalized size of antiderivative = 1.91, number of steps used = 44, number of rules used = 20, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.741, Rules used = {2056, 1586, 6733, 6725, 264, 331, 298, 203, 206, 1240, 410, 237, 335, 275, 231, 407, 409, 1213, 537, 494}

$$\frac{4\sqrt[4]{x^4+x^3}}{x}-\frac{2\sqrt[4]{x^4+x^3}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{x^{3/4}\sqrt[4]{x+1}}+\frac{2\sqrt[4]{2}\sqrt[4]{x^4+x^3}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{x^{3/4}\sqrt[4]{x+1}}+\frac{2\sqrt[4]{x^4+x^3}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{x^{3/4}\sqrt[4]{x+1}}-\frac{2\sqrt[4]{2}\sqrt[4]{x^4+x^3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{x^{3/4}\sqrt[4]{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(x^3 + x^4)^(1/4))/(x^2*(-1 + x^2)),x]
```

```
[Out] (4*(x^3 + x^4)^(1/4))/x - (2*(x^3 + x^4)^(1/4)*ArcTan[x^(1/4)/(1 + x)^(1/4)
])/(x^(3/4)*(1 + x)^(1/4)) + (2*2^(1/4)*(x^3 + x^4)^(1/4)*ArcTan[(2^(1/4)*x
^(1/4))/(1 + x)^(1/4)])/(x^(3/4)*(1 + x)^(1/4)) + (2*(x^3 + x^4)^(1/4)*ArcT
anh[x^(1/4)/(1 + x)^(1/4)])/(x^(3/4)*(1 + x)^(1/4)) - (2*2^(1/4)*(x^3 + x^4
)^(1/4)*ArcTanh[(2^(1/4)*x^(1/4))/(1 + x)^(1/4)])/(x^(3/4)*(1 + x)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^4)^(1/4)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
```

```
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{\left(1+x^2\right)\sqrt[4]{x^3+x^4}}{x^2\left(-1+x^2\right)}\,dx = \frac{\sqrt[4]{x^3+x^4}\int \frac{\sqrt[4]{1+x}\left(1+x^2\right)}{x^{5/4}\left(-1+x^2\right)}\,dx}{x^{3/4}\sqrt[4]{1+x}}$$

$$= \frac{\sqrt[4]{x^3+x^4}\int \frac{1+x^2}{(-1+x)x^{5/4}(1+x)^{3/4}}\,dx}{x^{3/4}\sqrt[4]{1+x}}$$

$$= \frac{\left(4\sqrt[4]{x^3+x^4}\right)\text{Subst}\left(\int \frac{1+x^8}{x^2\left(-1+x^4\right)\left(1+x^4\right)^{3/4}}\,dx, x, \sqrt[4]{x}\right)}{x^{3/4}\sqrt[4]{1+x}}$$

$$= \frac{\left(4\sqrt[4]{x^3+x^4}\right)\text{Subst}\left(\int\left(-\frac{1}{x^2\left(1+x^4\right)^{3/4}}+\frac{x^2}{\left(1+x^4\right)^{3/4}}+\frac{1}{\left(-1+x^2\right)\left(1+x^4\right)^{3/4}}+\frac{1}{\left(1+x^2\right)\left(1+x^4\right)^{3/4}}\right)dx, x\right.}{x^{3/4}\sqrt[4]{1+x}}$$

$$= -\frac{\left(4\sqrt[4]{x^3+x^4}\right)\text{Subst}\left(\int \frac{1}{x^2\left(1+x^4\right)^{3/4}}\,dx, x, \sqrt[4]{x}\right)}{x^{3/4}\sqrt[4]{1+x}} + \frac{\left(4\sqrt[4]{x^3+x^4}\right)\text{Subst}\left(\int \frac{x^2}{\left(1+x^4\right)^{3/4}}\,dx, x,\right.}{x^{3/4}\sqrt[4]{1+x}}$$

$$= \frac{4\sqrt[4]{x^3+x^4}}{x} + \frac{\left(4\sqrt[4]{x^3+x^4}\right)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}} + \frac{\left(4\sqrt[4]{x^3+x^4}\right)\text{Subst}\left(\int\left(\frac{}{(1-}\right.\right.}{}$$

$$= \frac{4\sqrt[4]{x^3+x^4}}{x} + \frac{\left(2\sqrt[4]{x^3+x^4}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}} - \frac{\left(2\sqrt[4]{x^3+x^4}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\right.}{x^{3/4}\sqrt[4]{1+x}}$$

$$= \frac{4\sqrt[4]{x^3+x^4}}{x} - \frac{2\sqrt[4]{x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}} + \frac{2\sqrt[4]{x^3+x^4}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}} + \frac{\left(2\sqrt[4]{x^3+x^4}\right)}{}$$

$$= \frac{4\sqrt[4]{x^3+x^4}}{x} - \frac{2\sqrt[4]{x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}} + \frac{2\sqrt[4]{x^3+x^4}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}} + 2\left(-\frac{\left(\sqrt{2}\sqrt[4]{x^3\right.}}{}\right.$$

$$= \frac{4\sqrt[4]{x^3+x^4}}{x} - \frac{2\sqrt[4]{x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}} + \frac{2\sqrt[4]{x^3+x^4}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}} + 2\left(\frac{\sqrt[4]{2}\sqrt[4]{x^3+}}{x^3}\right.$$

Mathematica [C] time = 0.06, size = 77, normalized size = 0.76

$$-\frac{4\sqrt[4]{x^3(x+1)}\left(3(x+1)\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};-x\right)+2\sqrt[4]{x+1}\left(x\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{2x}{x+1}\right)-3(x+1)\right)\right)}{3x(x+1)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(x^3 + x^4)^(1/4))/(x^2*(-1 + x^2)),x]
```

```
[Out] (-4*(x^3*(1 + x))^(1/4)*(3*(1 + x)*Hypergeometric2F1[-1/4, -1/4, 3/4, -x] +
 2*(1 + x)^(1/4)*(-3*(1 + x) + x*Hypergeometric2F1[3/4, 1, 7/4, (2*x)/(1 +
x)])))/(3*x*(1 + x)^(5/4))
```

IntegrateAlgebraic [A] time = 0.39, size = 101, normalized size = 1.00

$$\frac{4\sqrt[4]{x^4+x^3}}{x}-2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)+2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^3}}\right)+2\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^2)*(x^3 + x^4)^(1/4))/(x^2*(-1 + x^2)),x]
```

```
[Out] (4*(x^3 + x^4)^(1/4))/x - 2*ArcTan[x/(x^3 + x^4)^(1/4)] + 2*2^(1/4)*ArcTan[
(2^(1/4)*x)/(x^3 + x^4)^(1/4)] + 2*ArcTanh[x/(x^3 + x^4)^(1/4)] - 2*2^(1/4)
*ArcTanh[(2^(1/4)*x)/(x^3 + x^4)^(1/4)]
```

fricas [B] time = 0.44, size = 182, normalized size = 1.80

$$\frac{4 \cdot 2^{\frac{1}{4}} x \arctan\left(\frac{2^{\frac{3}{4}} x \sqrt{\frac{\sqrt{2} x^2 + \sqrt{x^4+x^3}}{x^2}} - 2^{\frac{3}{4}} \left(x^4+x^3\right)^{\frac{1}{4}}}{2 x}\right) - 2^{\frac{1}{4}} x \log\left(\frac{2^{\frac{1}{4}} x + \left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right) + 2^{\frac{1}{4}} x \log\left(-\frac{2^{\frac{1}{4}} x - \left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right) + 2 x \arctan\left(\frac{\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right) + x \log\left(\frac{x + \left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right) - x \log\left(-\frac{x - \left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right) + 4 \left(x^4 + x^3\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+x^3)^(1/4)/x^2/(x^2-1),x, algorithm="fricas")
```

```
[Out] (4*2^(1/4)*x*arctan(1/2*(2^(3/4)*x*sqrt((sqrt(2)*x^2 + sqrt(x^4 + x^3))/x^2
) - 2^(3/4)*(x^4 + x^3)^(1/4))/x) - 2^(1/4)*x*log((2^(1/4)*x + (x^4 + x^3)^
(1/4))/x) + 2^(1/4)*x*log(-(2^(1/4)*x - (x^4 + x^3)^(1/4))/x) + 2*x*arctan(
(x^4 + x^3)^(1/4)/x) + x*log((x + (x^4 + x^3)^(1/4))/x) - x*log(-(x - (x^4
+ x^3)^(1/4))/x) + 4*(x^4 + x^3)^(1/4))/x
```

giac [A] time = 0.26, size = 97, normalized size = 0.96

$$-2 \cdot 2^{\frac{1}{4}} \arctan\left(\frac{1}{2} \cdot 2^{\frac{3}{4}} \left(\frac{1}{x} + 1\right)^{\frac{1}{4}}\right) - 2^{\frac{1}{4}} \log\left(2^{\frac{1}{4}} + \left(\frac{1}{x} + 1\right)^{\frac{1}{4}}\right) + 2^{\frac{1}{4}} \log\left(\left|-2^{\frac{1}{4}} + \left(\frac{1}{x} + 1\right)^{\frac{1}{4}}\right|\right) + 4 \left(\frac{1}{x} + 1\right)^{\frac{1}{4}} + 2 \arctan\left(\left(\frac{1}{x} + 1\right)^{\frac{1}{4}}\right) + \log\left(\left(\frac{1}{x} + 1\right)^{\frac{1}{4}} + 1\right) - \log\left(\left|\left(\frac{1}{x} + 1\right)^{\frac{1}{4}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+x^3)^(1/4)/x^2/(x^2-1),x, algorithm="giac")
```

```
[Out] -2*2^(1/4)*arctan(1/2*2^(3/4)*(1/x + 1)^(1/4)) - 2^(1/4)*log(2^(1/4) + (1/x
 + 1)^(1/4)) + 2^(1/4)*log(abs(-2^(1/4) + (1/x + 1)^(1/4))) + 4*(1/x + 1)^(
1/4) + 2*arctan((1/x + 1)^(1/4)) + log((1/x + 1)^(1/4) + 1) - log(abs((1/x
+ 1)^(1/4) - 1))
```

maple [C] time = 2.23, size = 974, normalized size = 9.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+x^3)^(1/4)/x^2/(x^2-1),x)
```

```
[Out] 4*(x^3*(1+x))^(1/4)/x+(-ln((2*(x^4+3*x^3+3*x^2+x)^(3/4)-2*(x^4+3*x^3+3*x^2+
x)^(1/2)*x+2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2-2*x^3-2*(x^4+3*x^3+3*x^2+x)^(1/2
)+4*(x^4+3*x^3+3*x^2+x)^(1/4)*x-5*x^2+2*(x^4+3*x^3+3*x^2+x)^(1/4)-4*x-1)/(1
+x)^2)+RootOf(_Z^2+RootOf(_Z^4-2)^2)*ln((2*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf
(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x+2*(x^4+3*x^3+3*x^2+x)^(1/2)*Root
Of(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)-2*RootOf(_Z^4-2)^2*(x^4+3*x^3+3*
x^2+x)^(1/4)*x^2-4*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(_Z^4-2)^2*x-3*RootOf(_Z
^2+RootOf(_Z^4-2)^2)*x^3-2*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(_Z^4-2)^2-7*Roo
tOf(_Z^2+RootOf(_Z^4-2)^2)*x^2+4*(x^4+3*x^3+3*x^2+x)^(3/4)-5*RootOf(_Z^2+Ro
otOf(_Z^4-2)^2)*x-RootOf(_Z^2+RootOf(_Z^4-2)^2))/(-1+x)/(1+x)^2)-RootOf(_Z^
4-2)*ln((2*RootOf(_Z^4-2)^3*(x^4+3*x^3+3*x^2+x)^(1/2)*x+2*RootOf(_Z^4-2)^3*
(x^4+3*x^3+3*x^2+x)^(1/2)+2*RootOf(_Z^4-2)^2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2+
4*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(_Z^4-2)^2*x+3*RootOf(_Z^4-2)*x^3+2*(x^4+
3*x^3+3*x^2+x)^(1/4)*RootOf(_Z^4-2)^2+7*RootOf(_Z^4-2)*x^2+4*(x^4+3*x^3+3*x
^2+x)^(3/4)+5*RootOf(_Z^4-2)*x+RootOf(_Z^4-2))/(-1+x)/(1+x)^2)-1/2*RootOf(_
Z^4-2)^3*RootOf(_Z^2+RootOf(_Z^4-2)^2)*ln((2*RootOf(_Z^4-2)^3*(x^4+3*x^3+3*
x^2+x)^(1/2)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x-2*RootOf(_Z^4-2)^3*RootOf(_Z^2
+RootOf(_Z^4-2)^2)*x^3+2*RootOf(_Z^4-2)^3*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(
_Z^2+RootOf(_Z^4-2)^2)-5*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^3*x^2
```

```
-4*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^3*x-RootOf(_Z^4-2)^3*RootOf
(_Z^2+RootOf(_Z^4-2)^2)+4*(x^4+3*x^3+3*x^2+x)^(3/4)-4*(x^4+3*x^3+3*x^2+x)^(
1/4)*x^2-8*(x^4+3*x^3+3*x^2+x)^(1/4)*x-4*(x^4+3*x^3+3*x^2+x)^(1/4))/(1+x)^2
))*(x^3*(1+x))^(1/4)/x*(x*(1+x)^3)^(1/4)/(1+x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3\right)^{\frac{1}{4}}\left(x^2+1\right)}{\left(x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+x^3)^(1/4)/x^2/(x^2-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + x^3)^(1/4)*(x^2 + 1)/((x^2 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4+x^3\right)^{1/4}\,\left(x^2+1\right)}{x^2\,\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + x^4)^(1/4)*(x^2 + 1))/(x^2*(x^2 - 1)),x)
```

```
[Out] int(((x^3 + x^4)^(1/4)*(x^2 + 1))/(x^2*(x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\,(x+1)}\,\left(x^2+1\right)}{x^2\,(x-1)\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+x**3)**(1/4)/x**2/(x**2-1),x)
```

```
[Out] Integral((x**3*(x + 1))**(1/4)*(x**2 + 1)/(x**2*(x - 1)*(x + 1)), x)
```

3.1252 $\int \frac{(4+x^3)(-1-x^3+x^4)}{x^2(1+x^3)^{3/4}(1+x^3+x^4)}\,dx$

Optimal. Leaf size=101

$$\frac{4\sqrt[4]{x^3+1}}{x}-2\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{x^3+1}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{x^3+1}}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^3+1}}{\sqrt{x^3+1}+x^2}\right)$$

Rubi [F] time = 1.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(4+x^3)(-1-x^3+x^4)}{x^2(1+x^3)^{3/4}(1+x^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((4 + x^3)*(-1 - x^3 + x^4))/(x^2*(1 + x^3)^(3/4)*(1 + x^3 + x^4)),x]
```

```
[Out] (4*Hypergeometric2F1[-1/3, 3/4, 2/3, -x^3])/x - 2*x*Hypergeometric2F1[1/3,
3/4, 4/3, -x^3] + (x^2*Hypergeometric2F1[2/3, 3/4, 5/3, -x^3])/2 + 2*Defer[
Int][1/((1 + x^3)^(3/4)*(1 + x^3 + x^4)), x] - 2*Defer[Int][x/((1 + x^3)^(3
/4)*(1 + x^3 + x^4)), x] + 8*Defer[Int][x^2/((1 + x^3)^(3/4)*(1 + x^3 + x^4
)), x] + 2*Defer[Int][x^3/((1 + x^3)^(3/4)*(1 + x^3 + x^4)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(4+x^3)(-1-x^3+x^4)}{x^2(1+x^3)^{3/4}(1+x^3+x^4)}\,dx &= \int \left(-\frac{2}{(1+x^3)^{3/4}}-\frac{4}{x^2(1+x^3)^{3/4}}+\frac{x}{(1+x^3)^{3/4}}+\frac{2(1-x+4x^2+x^3)}{(1+x^3)^{3/4}(1+x^3+x^4)}\right.\\
&= -\left(2\int \frac{1}{(1+x^3)^{3/4}}\,dx\right)+2\int \frac{1-x+4x^2+x^3}{(1+x^3)^{3/4}(1+x^3+x^4)}\,dx-4\int \frac{1}{x^2(1+x}\\
&= \frac{4\,{}_2F_1\left(-\frac{1}{3},\frac{3}{4};\frac{2}{3};-x^3\right)}{x}-2x\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-x^3\right)+\frac{1}{2}x^2\,{}_2F_1\left(\frac{2}{3},\frac{3}{4};\frac{5}{3};-x^3\right)+2\\
&= \frac{4\,{}_2F_1\left(-\frac{1}{3},\frac{3}{4};\frac{2}{3};-x^3\right)}{x}-2x\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-x^3\right)+\frac{1}{2}x^2\,{}_2F_1\left(\frac{2}{3},\frac{3}{4};\frac{5}{3};-x^3\right)+2
\end{aligned}$$

Mathematica [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{(4+x^3)(-1-x^3+x^4)}{x^2(1+x^3)^{3/4}(1+x^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((4 + x^3)*(-1 - x^3 + x^4))/(x^2*(1 + x^3)^(3/4)*(1 + x^3 + x^4)
),x]
```

```
[Out] Integrate[((4 + x^3)*(-1 - x^3 + x^4))/(x^2*(1 + x^3)^(3/4)*(1 + x^3 + x^4)
), x]
```

IntegrateAlgebraic [A] time = 4.28, size = 101, normalized size = 1.00

$$\frac{4\sqrt[4]{x^3+1}}{x}-2\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{x^3+1}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{x^3+1}}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^3+1}}{\sqrt{x^3+1}+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((4 + x^3)*(-1 - x^3 + x^4))/(x^2*(1 + x^3)^(3/4)*(1 + x
^3 + x^4)),x]
```

```
[Out] (4*(1 + x^3)^(1/4))/x - 2*Sqrt[2]*ArcTan[(-(x^2/Sqrt[2]) + Sqrt[1 + x^3]/Sq
rt[2])/(x*(1 + x^3)^(1/4))] - 2*Sqrt[2]*ArcTanh[(Sqrt[2]*x*(1 + x^3)^(1/4))
/(x^2 + Sqrt[1 + x^3])]
```

fricas [B] time = 25.89, size = 709, normalized size = 7.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)*(x^4-x^3-1)/x^2/(x^3+1)^(3/4)/(x^4+x^3+1),x, algorithm="f
ricas")
```

```
[Out] 1/2*(4*sqrt(2)*x*arctan(-(x^8 + 2*x^7 + x^6 + 2*x^4 + 2*x^3 + 2*sqrt(2)*(3*
x^5 - x^4 - x)*(x^3 + 1)^(3/4) + 2*sqrt(2)*(x^7 - 3*x^6 - 3*x^3)*(x^3 + 1)^
(1/4) + 4*(x^6 + x^5 + x^2)*sqrt(x^3 + 1) - (16*(x^3 + 1)^(3/4)*x^5 + 2*sqr
t(2)*(3*x^6 - x^5 - x^2)*sqrt(x^3 + 1) + sqrt(2)*(x^8 + 8*x^7 - x^6 + 8*x^4
 - 2*x^3 - 1) + 4*(x^7 + x^6 + x^3)*(x^3 + 1)^(1/4))*sqrt((x^4 - 2*sqrt(2)*
(x^3 + 1)^(1/4)*x^3 + x^3 + 4*sqrt(x^3 + 1)*x^2 - 2*sqrt(2)*(x^3 + 1)^(3/4)
*x + 1)/(x^4 + x^3 + 1)) + 1)/(x^8 - 14*x^7 + x^6 - 14*x^4 + 2*x^3 + 1)) -
4*sqrt(2)*x*arctan(-(x^8 + 2*x^7 + x^6 + 2*x^4 + 2*x^3 - 2*sqrt(2)*(3*x^5 -
 x^4 - x)*(x^3 + 1)^(3/4) - 2*sqrt(2)*(x^7 - 3*x^6 - 3*x^3)*(x^3 + 1)^(1/4)
 + 4*(x^6 + x^5 + x^2)*sqrt(x^3 + 1) - (16*(x^3 + 1)^(3/4)*x^5 - 2*sqrt(2)*
(3*x^6 - x^5 - x^2)*sqrt(x^3 + 1) - sqrt(2)*(x^8 + 8*x^7 - x^6 + 8*x^4 - 2*
x^3 - 1) + 4*(x^7 + x^6 + x^3)*(x^3 + 1)^(1/4))*sqrt((x^4 + 2*sqrt(2)*(x^3
+ 1)^(1/4)*x^3 + x^3 + 4*sqrt(x^3 + 1)*x^2 + 2*sqrt(2)*(x^3 + 1)^(3/4)*x +
1)/(x^4 + x^3 + 1)) + 1)/(x^8 - 14*x^7 + x^6 - 14*x^4 + 2*x^3 + 1)) - sqrt(
2)*x*log(4*(x^4 + 2*sqrt(2)*(x^3 + 1)^(1/4)*x^3 + x^3 + 4*sqrt(x^3 + 1)*x^2
 + 2*sqrt(2)*(x^3 + 1)^(3/4)*x + 1)/(x^4 + x^3 + 1)) + sqrt(2)*x*log(4*(x^4
 - 2*sqrt(2)*(x^3 + 1)^(1/4)*x^3 + x^3 + 4*sqrt(x^3 + 1)*x^2 - 2*sqrt(2)*(x
^3 + 1)^(3/4)*x + 1)/(x^4 + x^3 + 1)) + 8*(x^3 + 1)^(1/4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^3-1\right)\left(x^3+4\right)}{\left(x^4+x^3+1\right)\left(x^3+1\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)*(x^4-x^3-1)/x^2/(x^3+1)^(3/4)/(x^4+x^3+1),x, algorithm="g
iac")
```

```
[Out] integrate((x^4 - x^3 - 1)*(x^3 + 4)/((x^4 + x^3 + 1)*(x^3 + 1)^(3/4)*x^2),
x)
```

maple [C] time = 2.18, size = 593, normalized size = 5.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+4)*(x^4-x^3-1)/x^2/(x^3+1)^(3/4)/(x^4+x^3+1),x)
```

```
[Out] 4*(x^3+1)^(1/4)/x+(-2*RootOf(_Z^4+1)^3*ln((-x^10*RootOf(_Z^4+1)^2+RootOf(_Z
^4+1)^2*x^9+2*RootOf(_Z^4+1)^3*(x^9+3*x^6+3*x^3+1)^(3/4)*x^3-2*RootOf(_Z^4+
1)^2*x^7-2*RootOf(_Z^4+1)*(x^9+3*x^6+3*x^3+1)^(1/4)*x^7+3*RootOf(_Z^4+1)^2*
x^6+2*(x^9+3*x^6+3*x^3+1)^(1/2)*x^5-RootOf(_Z^4+1)^2*x^4-4*RootOf(_Z^4+1)*(
x^9+3*x^6+3*x^3+1)^(1/4)*x^4+3*RootOf(_Z^4+1)^2*x^3+2*(x^9+3*x^6+3*x^3+1)^(
1/2)*x^2-2*RootOf(_Z^4+1)*(x^9+3*x^6+3*x^3+1)^(1/4)*x+RootOf(_Z^4+1)^2)/(x^
4+x^3+1)/(1+x)^2/(x^2-x+1)^2)-2*RootOf(_Z^4+1)*ln(-(-x^10*RootOf(_Z^4+1)^2+
2*RootOf(_Z^4+1)^3*(x^9+3*x^6+3*x^3+1)^(1/4)*x^7+RootOf(_Z^4+1)^2*x^9-2*Roo
tOf(_Z^4+1)^2*x^7+4*RootOf(_Z^4+1)^3*(x^9+3*x^6+3*x^3+1)^(1/4)*x^4+3*RootOf
(_Z^4+1)^2*x^6-2*RootOf(_Z^4+1)*(x^9+3*x^6+3*x^3+1)^(3/4)*x^3-2*(x^9+3*x^6+
3*x^3+1)^(1/2)*x^5-RootOf(_Z^4+1)^2*x^4+2*RootOf(_Z^4+1)^3*(x^9+3*x^6+3*x^3
+1)^(1/4)*x+3*RootOf(_Z^4+1)^2*x^3-2*(x^9+3*x^6+3*x^3+1)^(1/2)*x^2+RootOf(_
Z^4+1)^2)/(x^4+x^3+1)/(1+x)^2/(x^2-x+1)^2))/(x^3+1)^(3/4)*((x^3+1)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4 - x^3 - 1\right)\left(x^3 + 4\right)}{\left(x^4 + x^3 + 1\right)\left(x^3 + 1\right)^{\frac{3}{4}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)*(x^4-x^3-1)/x^2/(x^3+1)^(3/4)/(x^4+x^3+1),x, algorithm="m
axima")
```

```
[Out] integrate((x^4 - x^3 - 1)*(x^3 + 4)/((x^4 + x^3 + 1)*(x^3 + 1)^(3/4)*x^2),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^3 + 4\right)\,\left(-x^4 + x^3 + 1\right)}{x^2\,\left(x^3 + 1\right)^{3/4}\,\left(x^4 + x^3 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3 + 4)*(x^3 - x^4 + 1))/(x^2*(x^3 + 1)^(3/4)*(x^3 + x^4 + 1)),x)
```

```
[Out] int(-((x^3 + 4)*(x^3 - x^4 + 1))/(x^2*(x^3 + 1)^(3/4)*(x^3 + x^4 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+4)*(x**4-x**3-1)/x**2/(x**3+1)**(3/4)/(x**4+x**3+1),x)
```

```
[Out] Timed out
```

3.1253 $\int \frac{1+x^2+2x^4}{\sqrt[4]{1+x^2}\left(2+3x^2+x^4\right)} \, dx$

Optimal. Leaf size=101

$$\frac{4x}{\sqrt[4]{x^2+1}}+\frac{7}{4}\tan^{-1}\left(\frac{\sqrt[4]{x^2+1}-x}{\sqrt[4]{x^2+1}}\right)-\frac{7}{4}\tan^{-1}\left(\frac{\sqrt[4]{x^2+1}+x}{\sqrt[4]{x^2+1}}\right)-\frac{7}{4}\tanh^{-1}\left(\frac{2x\sqrt[4]{x^2+1}}{x^2+2\sqrt{x^2+1}}\right)$$

Rubi [A] time = 0.27, antiderivative size = 73, normalized size of antiderivative = 0.72, number of steps used = 9, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1688, 6725, 196, 285, 403, 397}

$$\frac{4x}{\sqrt[4]{x^2+1}}+\frac{7}{2}\tan^{-1}\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right)+\frac{7}{2}\tanh^{-1}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + 2*x^4)/((1 + x^2)^(1/4)*(2 + 3*x^2 + x^4)),x]
```

```
[Out] (4*x)/(1 + x^2)^(1/4) + (7*ArcTan[(1 + Sqrt[1 + x^2])/(x*(1 + x^2)^(1/4))])
/2 + (7*ArcTanh[(1 - Sqrt[1 + x^2])/(x*(1 + x^2)^(1/4))])/2
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rule 403

```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/(
b*c - a*d), Int[(a + b*x^2)^p, x], x] - Dist[d/(b*c - a*d), Int[(a + b*x^2)
^(p + 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && LtQ[p, -1] && EqQ[Denominator[p], 4] && (EqQ[p, -5/4] || EqQ[p, -7/4])
```

Rule 1688

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[Px*(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; F
reeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a
*e^2, 0] && IntegerQ[p] && (PolyQ[Px, x^2] || MatchQ[Px, ((f_) + (g_.)*x^2)
^(r_.) /; FreeQ[{f, g, r}, x]])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^2+2x^4}{\sqrt[4]{1+x^2}\left(2+3x^2+x^4\right)}\,dx &= \int \frac{1+x^2+2x^4}{\left(1+x^2\right)^{5/4}\left(2+x^2\right)}\,dx \\
&= \int \left(-\frac{3}{\left(1+x^2\right)^{5/4}}+\frac{2x^2}{\left(1+x^2\right)^{5/4}}+\frac{7}{\left(1+x^2\right)^{5/4}\left(2+x^2\right)}\right)dx \\
&= 2\int \frac{x^2}{\left(1+x^2\right)^{5/4}}\,dx-3\int \frac{1}{\left(1+x^2\right)^{5/4}}\,dx+7\int \frac{1}{\left(1+x^2\right)^{5/4}\left(2+x^2\right)}\,dx \\
&= \frac{4x}{\sqrt[4]{1+x^2}}-6E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)-4\int \frac{1}{\left(1+x^2\right)^{5/4}}\,dx+7\int \frac{1}{\left(1+x^2\right)^{5/4}}\,dx-7 \\
&= \frac{4x}{\sqrt[4]{1+x^2}}+\frac{7}{2}\tan^{-1}\left(\frac{1+\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right)+\frac{7}{2}\tanh^{-1}\left(\frac{1-\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.20, size = 127, normalized size = 1.26

$$
\frac{2x\left(\frac{21F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)}{\left(x^2+2\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)\right)-6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)\right)}+2\right)}{\sqrt[4]{x^2+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2 + 2*x^4)/((1 + x^2)^(1/4)*(2 + 3*x^2 + x^4)),x]
```

```
[Out] (2*x*(2 + (21*AppellF1[1/2, 1/4, 1, 3/2, -x^2, -1/2*x^2])/((2 + x^2)*(-6*Ap
pellF1[1/2, 1/4, 1, 3/2, -x^2, -1/2*x^2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2
, -x^2, -1/2*x^2] + AppellF1[3/2, 5/4, 1, 5/2, -x^2, -1/2*x^2])))))/(1 + x^
2)^(1/4)
```

IntegrateAlgebraic [A] time = 0.40, size = 101, normalized size = 1.00

$$
\frac{4x}{\sqrt[4]{x^2+1}}+\frac{7}{4}\tan^{-1}\left(\frac{\sqrt[4]{x^2+1}-x}{\sqrt[4]{x^2+1}}\right)-\frac{7}{4}\tan^{-1}\left(\frac{\sqrt[4]{x^2+1}+x}{\sqrt[4]{x^2+1}}\right)-\frac{7}{4}\tanh^{-1}\left(\frac{2x\sqrt[4]{x^2+1}}{x^2+2\sqrt{x^2+1}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2 + 2*x^4)/((1 + x^2)^(1/4)*(2 + 3*x^2 + x^4)),x]
```

```
[Out] (4*x)/(1 + x^2)^(1/4) + (7*ArcTan[(-x + (1 + x^2)^(1/4))/(1 + x^2)^(1/4)])/
4 - (7*ArcTan[(x + (1 + x^2)^(1/4))/(1 + x^2)^(1/4)])/4 - (7*ArcTanh[(2*x*(
1 + x^2)^(1/4))/(x^2 + 2*Sqrt[1 + x^2])])/4
```

fricas [A] time = 0.42, size = 137, normalized size = 1.36

$$
\frac{14\left(x^2+1\right)\arctan\left(\frac{x+2\left(x^2+1\right)^{\frac{1}{4}}}{x}\right)+14\left(x^2+1\right)\arctan\left(-\frac{x-2\left(x^2+1\right)^{\frac{1}{4}}}{x}\right)-7\left(x^2+1\right)\log\left(\frac{x^2+2\left(x^2+1\right)^{\frac{1}{4}}x+2\sqrt{x^2+1}}{x^2}\right)+7\left(x^2+1\right)\log\left(\frac{x^2-2\left(x^2+1\right)^{\frac{1}{4}}x+2\sqrt{x^2+1}}{x^2}\right)+32\left(x^2+1\right)^{\frac{3}{4}}x}{8\left(x^2+1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+x^2+1)/(x^2+1)^(1/4)/(x^4+3*x^2+2),x, algorithm="fricas")
```

```
[Out] 1/8*(14*(x^2 + 1)*arctan((x + 2*(x^2 + 1)^(1/4))/x) + 14*(x^2 + 1)*arctan(-
(x - 2*(x^2 + 1)^(1/4))/x) - 7*(x^2 + 1)*log((x^2 + 2*(x^2 + 1)^(1/4)*x + 2
*sqrt(x^2 + 1))/x^2) + 7*(x^2 + 1)*log((x^2 - 2*(x^2 + 1)^(1/4)*x + 2*sqrt(
x^2 + 1))/x^2) + 32*(x^2 + 1)^(3/4)*x)/(x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^{4}+x^{2}+1}{\left(x^{4}+3\,x^{2}+2\right)\left(x^{2}+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+x^2+1)/(x^2+1)^(1/4)/(x^4+3*x^2+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^4 + x^2 + 1)/((x^4 + 3*x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

maple [C] time = 1.38, size = 291, normalized size = 2.88

$$\frac{4x}{\left(x^{2}+1\right)^{\frac{1}{4}}}+7\,\mathrm{RootOf}\left(8\_Z^{2}+4\_Z+1\right)\ln\left(-\frac{4\,\mathrm{RootOf}\left(8\_Z^{2}+4\_Z+1\right)\left(x^{2}+1\right)^{\frac{3}{4}}-x\sqrt{x^{2}+1}+4\,\mathrm{RootOf}\left(8\_Z^{2}+4\_Z+1\right)\left(x^{2}+1\right)^{\frac{1}{4}}+4\,\mathrm{RootOf}\left(8\_Z^{2}+4\_Z+1\right)x+2\left(x^{2}+1\right)^{\frac{1}{4}}+x}{x^{2}+2}\right)-\frac{7\ln\left(\frac{4\,\mathrm{RootOf}\left(8\_Z^{2}+4\_Z+1\right)\left(x^{2}+1\right)^{\frac{3}{4}}+2\left(x^{2}+1\right)^{\frac{3}{4}}+x\sqrt{x^{2}+1}+4\,\mathrm{RootOf}\left(8\_Z^{2}+4\_Z+1\right)\left(x^{2}+1\right)^{\frac{1}{4}}+4\,\mathrm{RootOf}\left(8\_Z^{2}+4\_Z+1\right)x+x}{x^{2}+2}\right)}{2}-7\ln\left(\frac{4\,\mathrm{RootOf}\left(8\_Z^{2}+4\_Z+1\right)\left(x^{2}+1\right)^{\frac{3}{4}}+2\left(x^{2}+1\right)^{\frac{3}{4}}+x\sqrt{x^{2}+1}+4\,\mathrm{RootOf}\left(8\_Z^{2}+4\_Z+1\right)\left(x^{2}+1\right)^{\frac{1}{4}}+4\,\mathrm{RootOf}\left(8\_Z^{2}+4\_Z+1\right)x+x}{x^{2}+2}\right)\mathrm{RootOf}\left(8\_Z^{2}+4\_Z+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+x^2+1)/(x^2+1)^(1/4)/(x^4+3*x^2+2),x)
```

```
[Out] 4*x/(x^2+1)^(1/4)+7*RootOf(8*_Z^2+4*_Z+1)*ln(-(4*RootOf(8*_Z^2+4*_Z+1)*(x^2
+1)^(3/4)-x*(x^2+1)^(1/2)+4*RootOf(8*_Z^2+4*_Z+1)*(x^2+1)^(1/4)+4*RootOf(8*
_Z^2+4*_Z+1)*x+2*(x^2+1)^(1/4)+x)/(x^2+2))-7/2*ln((4*RootOf(8*_Z^2+4*_Z+1)*
(x^2+1)^(3/4)+2*(x^2+1)^(3/4)+x*(x^2+1)^(1/2)+4*RootOf(8*_Z^2+4*_Z+1)*(x^2+
1)^(1/4)+4*RootOf(8*_Z^2+4*_Z+1)*x+x)/(x^2+2))-7*ln((4*RootOf(8*_Z^2+4*_Z+1
)*(x^2+1)^(3/4)+2*(x^2+1)^(3/4)+x*(x^2+1)^(1/2)+4*RootOf(8*_Z^2+4*_Z+1)*(x^
2+1)^(1/4)+4*RootOf(8*_Z^2+4*_Z+1)*x+x)/(x^2+2))*RootOf(8*_Z^2+4*_Z+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^{4}+x^{2}+1}{\left(x^{4}+3\,x^{2}+2\right)\left(x^{2}+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+x^2+1)/(x^2+1)^(1/4)/(x^4+3*x^2+2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^4 + x^2 + 1)/((x^4 + 3*x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x^{4}+x^{2}+1}{{\left(x^{2}+1\right)}^{1/4}\,\left(x^{4}+3\,x^{2}+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 2*x^4 + 1)/((x^2 + 1)^(1/4)*(3*x^2 + x^4 + 2)),x)
```

```
[Out] int((x^2 + 2*x^4 + 1)/((x^2 + 1)^(1/4)*(3*x^2 + x^4 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^{4}+x^{2}+1}{\left(x^{2}+1\right)^{\frac{5}{4}}\left(x^{2}+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+x**2+1)/(x**2+1)**(1/4)/(x**4+3*x**2+2),x)
```

```
[Out] Integral((2*x**4 + x**2 + 1)/((x**2 + 1)**(5/4)*(x**2 + 2)), x)
```

3.1254 $\int \frac{(-b+ax^2)\sqrt[4]{-bx^2+ax^4}}{x^2}\,dx$

Optimal. Leaf size=101

$$\frac{\sqrt[4]{ax^4-bx^2}\,(ax^2+4b)}{2x}+\frac{5}{4}\sqrt[4]{a}\,b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)-\frac{5}{4}\sqrt[4]{a}\,b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)$$

Rubi [A] time = 0.47, antiderivative size = 183, normalized size of antiderivative = 1.81, number of steps used = 16, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2052, 2004, 2032, 329, 331, 298, 203, 206, 2020}

$$\frac{1}{2}ax\sqrt[4]{ax^4-bx^2}+\frac{2b\sqrt[4]{ax^4-bx^2}}{x}+\frac{5\sqrt[4]{a}\,bx^{3/2}\left(ax^2-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2-b}}\right)}{4\left(ax^4-bx^2\right)^{3/4}}-\frac{5\sqrt[4]{a}\,bx^{3/2}\left(ax^2-b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2-b}}\right)}{4\left(ax^4-bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4))/x^2,x]
```

```
[Out] (2*b*(-(b*x^2) + a*x^4)^(1/4))/x + (a*x*(-(b*x^2) + a*x^4)^(1/4))/2 + (5*a^
(1/4)*b*x^(3/2)*(-b + a*x^2)^(3/4)*ArcTan[(a^(1/4)*Sqrt[x])/(-b + a*x^2)^(1
/4)])/(4*(-(b*x^2) + a*x^4)^(3/4)) - (5*a^(1/4)*b*x^(3/2)*(-b + a*x^2)^(3/4
)*ArcTanh[(a^(1/4)*Sqrt[x])/(-b + a*x^2)^(1/4)])/(4*(-(b*x^2) + a*x^4)^(3/4
))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\left(-b+ax^2\right)\sqrt[4]{-bx^2+ax^4}}{x^2}\,dx = \int \left(a\sqrt[4]{-bx^2+ax^4} - \frac{b\sqrt[4]{-bx^2+ax^4}}{x^2}\right)dx$$

$$= a\int \sqrt[4]{-bx^2+ax^4}\,dx - b\int \frac{\sqrt[4]{-bx^2+ax^4}}{x^2}\,dx$$

$$= \frac{2b\sqrt[4]{-bx^2+ax^4}}{x} + \frac{1}{2}ax\sqrt[4]{-bx^2+ax^4} - \frac{1}{4}(ab)\int \frac{x^2}{\left(-bx^2+ax^4\right)^{3/4}}\,dx - (ab)\int \frac{}{(}$$

$$= \frac{2b\sqrt[4]{-bx^2+ax^4}}{x} + \frac{1}{2}ax\sqrt[4]{-bx^2+ax^4} - \frac{\left(abx^{3/2}\left(-b+ax^2\right)^{3/4}\right)\int \frac{\sqrt{x}}{\left(-b+ax^2\right)^{3/4}}\,dx}{4\left(-bx^2+ax^4\right)^{3/4}} -$$

$$= \frac{2b\sqrt[4]{-bx^2+ax^4}}{x} + \frac{1}{2}ax\sqrt[4]{-bx^2+ax^4} - \frac{\left(abx^{3/2}\left(-b+ax^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{\left(-b+ax^4\right.}\right.}{2\left(-bx^2+ax^4\right)^{3/4}}$$

$$= \frac{2b\sqrt[4]{-bx^2+ax^4}}{x} + \frac{1}{2}ax\sqrt[4]{-bx^2+ax^4} - \frac{\left(abx^{3/2}\left(-b+ax^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,d\right.}{2\left(-bx^2+ax^4\right)^{3/4}}$$

$$= \frac{2b\sqrt[4]{-bx^2+ax^4}}{x} + \frac{1}{2}ax\sqrt[4]{-bx^2+ax^4} - \frac{\left(\sqrt{a}\,bx^{3/2}\left(-b+ax^2\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}}\right.}{4\left(-bx^2+ax^4\right)^{3/4}}$$

$$= \frac{2b\sqrt[4]{-bx^2+ax^4}}{x} + \frac{1}{2}ax\sqrt[4]{-bx^2+ax^4} + \frac{5\sqrt[4]{a}\,bx^{3/2}\left(-b+ax^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{-b+ax^2}}\right.}{4\left(-bx^2+ax^4\right)^{3/4}}$$

Mathematica [C] time = 0.02, size = 55, normalized size = 0.54

$$\frac{2b\sqrt[4]{ax^4-bx^2}\ {}_2F_1\left(-\frac{5}{4},-\frac{1}{4};\frac{3}{4};\frac{ax^2}{b}\right)}{x\sqrt[4]{1-\frac{ax^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4))/x^2,x]
```

```
[Out] (2*b*(-(b*x^2) + a*x^4)^(1/4)*Hypergeometric2F1[-5/4, -1/4, 3/4, (a*x^2)/b]
)/(x*(1 - (a*x^2)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.36, size = 101, normalized size = 1.00

$$\frac{\sqrt[4]{ax^4-bx^2}\left(ax^2+4b\right)}{2x} + \frac{5}{4}\sqrt[4]{a}\,b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right) - \frac{5}{4}\sqrt[4]{a}\,b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4))/x^2,x]
```

```
[Out] ((4*b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4))/(2*x) + (5*a^(1/4)*b*ArcTan[(a^(1/
4)*x)/(-(b*x^2) + a*x^4)^(1/4)])/4 - (5*a^(1/4)*b*ArcTanh[(a^(1/4)*x)/(-(b*
x^2) + a*x^4)^(1/4)])/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(a*x^4-b*x^2)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.20, size = 229, normalized size = 2.27

$$\frac{8\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}} abx^2 - 10\sqrt{2}\,(-a)^{\frac{1}{4}} b^2 \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right) - 10\sqrt{2}\,(-a)^{\frac{1}{4}} b^2 \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right) - 5\sqrt{2}\,(-a)^{\frac{1}{4}} b^2 \log\left(\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^2}}\right) + 5\sqrt{2}\,(-a)^{\frac{1}{4}} b^2 \log\left(-\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^2}}\right) + 32\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}} b^2}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(a*x^4-b*x^2)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/16*(8*(a - b/x^2)^(1/4)*a*b*x^2 - 10*sqrt(2)*(-a)^(1/4)*b^2*arctan(1/2*sq
rt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a - b/x^2)^(1/4))/(-a)^(1/4)) - 10*sqrt(2)*(
-a)^(1/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(a - b/x^2)^(1/4)
)/(-a)^(1/4)) - 5*sqrt(2)*(-a)^(1/4)*b^2*log(sqrt(2)*(-a)^(1/4)*(a - b/x^2)
^(1/4) + sqrt(-a) + sqrt(a - b/x^2)) + 5*sqrt(2)*(-a)^(1/4)*b^2*log(-sqrt(2
)*(-a)^(1/4)*(a - b/x^2)^(1/4) + sqrt(-a) + sqrt(a - b/x^2)) + 32*(a - b/x^
2)^(1/4)*b^2)/b
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^2-b\right)\left(a\,x^4-b\,x^2\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)*(a*x^4-b*x^2)^(1/4)/x^2,x)
```

```
[Out] int((a*x^2-b)*(a*x^4-b*x^2)^(1/4)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4-bx^2\right)^{\frac{1}{4}}\left(ax^2-b\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(a*x^4-b*x^2)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 - b*x^2)^(1/4)*(a*x^2 - b)/x^2, x)
```

mupad [B] time = 1.36, size = 91, normalized size = 0.90

$$\frac{2\,a\,x\,\left(a\,x^4-b\,x^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ \frac{a\,x^2}{b}\right)}{3\left(1-\frac{a\,x^2}{b}\right)^{1/4}}+\frac{2\,b\,\left(a\,x^4-b\,x^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ \frac{a\,x^2}{b}\right)}{x\left(1-\frac{a\,x^2}{b}\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b - a*x^2)*(a*x^4 - b*x^2)^(1/4))/x^2,x)
```

```
[Out] (2*a*x*(a*x^4 - b*x^2)^(1/4)*hypergeom([-1/4, 3/4], 7/4, (a*x^2)/b))/(3*(1
- (a*x^2)/b)^(1/4)) + (2*b*(a*x^4 - b*x^2)^(1/4)*hypergeom([-1/4, -1/4], 3/
4, (a*x^2)/b))/(x*(1 - (a*x^2)/b)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\left(ax^2-b\right)}\left(ax^2-b\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)*(a*x**4-b*x**2)**(1/4)/x**2,x)
```

```
[Out] Integral((x**2*(a*x**2 - b))**(1/4)*(a*x**2 - b)/x**2, x)
```

3.1255 $\int \frac{1-x^4}{x^2\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

Optimal. Leaf size=101

$$-\frac{b\log\left(2\sqrt{a}\sqrt{ax^4+a+bx^3+bx+cx^2}-2ax^2-2a-bx\right)}{2a^{3/2}}+\frac{b\log(x)}{2a^{3/2}}-\frac{\sqrt{ax^4+a+bx^3+bx+cx^2}}{ax}$$

Rubi [F] time = 0.91, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1-x^4}{x^2\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - x^4)/(x^2*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] Defer[Int][1/(x^2*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x] - Defer[Int][x
^2/Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^4}{x^2\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx &= \int \left(\frac{1}{x^2\sqrt{a+bx+cx^2+bx^3+ax^4}}-\frac{x^2}{\sqrt{a+bx+cx^2+bx^3+ax^4}}\right)dx\\ &= \int \frac{1}{x^2\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx-\int \frac{x^2}{\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx\end{aligned}$$

Mathematica [C] time = 6.21, size = 3897, normalized size = 38.58

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(x^2*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] -((a + b*x + c*x^2 + b*x^3 + a*x^4)/(a*x*Sqrt[x*(b + c*x + b*x^2) + a*(1 +
x^4)])) + (b*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]*((2*(x - Root[a + b*#1 +
 c*#1^2 + b*#1^3 + a*#1^4 & , 2])^2*(-(EllipticF[ArcSin[Sqrt[((x - Root[a +
 b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 +
 a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((x - Ro
ot[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*
#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))]],
 -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1
^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & ,
1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((-Root[a + b*#1 + c
*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
 , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c
*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4
 & , 2]) + EllipticPi[(-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + R
oot[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])/(-Root[a + b*#1 + c*#1^2 +
b*#1^3 + a*#1^4 & , 2] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]),
ArcSin[Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a
+ b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3
+ a*#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(R
oot[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b
```

```
*#1^3 + a*#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & ,
 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#
1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & ,
 4]))/((-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 +
c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4
& , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*(-Root[a + b*#
1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#
1^4 & , 2]))*Sqrt[((-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root
[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(x - Root[a + b*#1 + c*#1^2 +
b*#1^3 + a*#1^4 & , 3]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & ,
 2])*(-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*
#1^2 + b*#1^3 + a*#1^4 & , 3]))]*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4
& , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])*Sqrt[((x - Root[a
 + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3
 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((x -
Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 +
b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))]
*Sqrt[((-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 +
c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#
1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(-Root[
a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^
3 + a*#1^4 & , 4]))])/(Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]*(-Root[a + b*#
1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#
1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#
1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])) + (2*(x - Root[a + b*#1 + c*#1^2 + b*
#1^3 + a*#1^4 & , 2])^2*(-(EllipticF[ArcSin[Sqrt[((x - Root[a + b*#1 + c*#1
^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & ,
2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((x - Root[a + b*#1
+ c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^
4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))]], -(((Root[a
+ b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3
+ a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a
+ b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((-Root[a + b*#1 + c*#1^2 + b*#1
^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root
[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1
^3 + a*#1^4 & , 4])))]*Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1]) - E
llipticPi[(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2]*(-Root[a + b*#1
+ c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^
4 & , 4]))/(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1]*(-Root[a + b*#1
 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1
^4 & , 4])), ArcSin[Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & ,
 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#
1^2 + b*#1^3 + a*#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1
^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1
 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3
 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a
 + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3
 + a*#1^4 & , 4]))/((-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Roo
t[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#
1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*(
-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 +
 b*#1^3 + a*#1^4 & , 2]))*Sqrt[((-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4
& , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(x - Root[a + b*#
1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3
 + a*#1^4 & , 2])*(-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[
a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3]))]*Sqrt[((x - Root[a + b*#1 + c*
#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
, 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((x - Root[a + b*#
1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#
```

```
1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))]*Sqrt[((-Roo
t[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#
1^3 + a*#1^4 & , 2])*(x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))
/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(-Root[a + b*#1 + c
*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
 , 4]))]*(-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1
+ c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/(Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]*
Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1]*Root[a + b*#1 + c*#1^2 + b*
#1^3 + a*#1^4 & , 2]*(-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Ro
ot[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*
#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))))/
(2*a*Sqrt[x*(b + c*x + b*x^2) + a*(1 + x^4)])
```

IntegrateAlgebraic [A] time = 0.34, size = 105, normalized size = 1.04

$$\frac{b\log(x)}{2a^{3/2}}-\frac{b\log\left(-2a^{3/2}\sqrt{ax^4+a+bx^3+bx+cx^2}+2a^2x^2+2a^2+abx\right)}{2a^{3/2}}-\frac{\sqrt{ax^4+a+bx^3+bx+cx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^4)/(x^2*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]/(a*x)) + (b*Log[x])/(2*a^(3/2)) - (
b*Log[2*a^2 + a*b*x + 2*a^2*x^2 - 2*a^(3/2)*Sqrt[a + b*x + c*x^2 + b*x^3 +
a*x^4]])/(2*a^(3/2))
```

fricas [A] time = 2.28, size = 216, normalized size = 2.14

$$\left[\frac{\sqrt{a}bx\log\left(\frac{8a^2x^4+8abx^3+8abx+(8a^2+b^2+4ac)x^2+4\sqrt{ax^4+bx^3+cx^2+bx+a}(2ax^2+bx+2a)\sqrt{a}+8a^2}{x^2}\right)-4\sqrt{ax^4+bx^3+cx^2+bx+a}\,a}{4a^2x},-\frac{\sqrt{-a}bx\arctan\left(\frac{2\sqrt{ax^4+bx^3+cx^2+bx+a}\sqrt{-a}}{2ax^2+bx+2a}\right)+2\sqrt{ax^4+bx^3+cx^2+bx+a}\,a}{2a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^2/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

```
[Out] [1/4*(sqrt(a)*b*x*log((8*a^2*x^4 + 8*a*b*x^3 + 8*a*b*x + (8*a^2 + b^2 + 4*a
*c)*x^2 + 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(2*a*x^2 + b*x + 2*a)*sqr
t(a) + 8*a^2)/x^2) - 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*a)/(a^2*x), -1
/2*(sqrt(-a)*b*x*arctan(2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(-a)/(2
*a*x^2 + b*x + 2*a)) + 2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*a)/(a^2*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^4-1}{\sqrt{ax^4+bx^3+cx^2+bx+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^2/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^4 - 1)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*x^2), x)
```

maple [C] time = 0.11, size = 3402, normalized size = 33.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/x^2/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2)/a/x+b/a*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b
+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*((RootOf(_Z^4*a+_Z
```

^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x
-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+
_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4
*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*
b+a,index=2))^2*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a
+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index
=3))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3)-RootOf(_Z^4*a+_Z^3*b+_Z^2
*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)*
((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_
Z*b+a,index=1))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4))/(RootOf(_Z^
4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=
1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)/(RootOf(_Z^4*a+_
Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))/(
RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*
b+a,index=1))/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2)*(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+
_Z*b+a,index=2)*EllipticF(((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-Roo
tOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z
*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_
Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2
)))^(1/2),((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*
b+_Z^2*c+_Z*b+a,index=3))*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+Roo
tOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*
b+a,index=3)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_
Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^
(1/2))+(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)+RootOf(_Z^4*a+_Z^3*b+_
Z^2*c+_Z*b+a,index=1))*EllipticPi(((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,inde
x=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_
Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(
_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a
,index=2)))^(1/2),(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*
a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=
4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)),((RootOf(_Z^4*a+_Z^3*b+_Z^2
*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))*(-RootOf(_Z
^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index
=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3)+RootOf(_Z^4*a+_Z^3*b+_Z^
2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_
Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)))-b/a*(-RootOf(_Z^4*a+_Z^3*b+_Z
^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*((RootOf(
_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,ind
ex=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*
b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-Ro
otOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)*(x-RootOf(_Z^4*a+_Z^3*b+_
Z^2*c+_Z*b+a,index=2))^2*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-Root
Of(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*
b+a,index=3))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3)-RootOf(_Z^4*a+_Z
^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)
))^(1/2)*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b
+_Z^2*c+_Z*b+a,index=1))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4))/(R
ootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b
+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)/(RootOf
(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,in
dex=2))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_
Z^2*c+_Z*b+a,index=1))/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2)/RootOf(_Z^4*a+_Z^3*b
+_Z^2*c+_Z*b+a,index=2)*(EllipticF(((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,ind
ex=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+
_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf
(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+
a,index=2)))^(1/2),((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^
4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,ind

```
ex=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_
Z^2*c+_Z*b+a,index=3)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf
(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,in
dex=2)))^(1/2))+(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+
_Z^3*b+_Z^2*c+_Z*b+a,index=1))/RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1)*
EllipticPi(((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3
*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/
(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z
*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2),RootO
f(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)*(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,
index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/RootOf(_Z^4*a+_Z^3*b+
_Z^2*c+_Z*b+a,index=1)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(
_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)),((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,
index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))*(-RootOf(_Z^4*a+_Z^3*
b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-Roo
tOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a
,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3
*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4-1}{\sqrt{ax^4+bx^3+cx^2+bx+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^2/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

```
[Out] -integrate((x^4 - 1)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^4-1}{x^2\,\sqrt{a\,x^4+b\,x^3+c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^2*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)),x)
```

```
[Out] -int((x^4 - 1)/(x^2*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{1}{x^2\sqrt{ax^4+a+bx^3+bx+cx^2}}\right)dx-\int \frac{x^2}{\sqrt{ax^4+a+bx^3+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/x**2/(a*x**4+b*x**3+c*x**2+b*x+a)**(1/2),x)
```

```
[Out] -Integral(-1/(x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2)), x) - Integral
(x**2/sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2), x)
```

3.1256 $\int \frac{-1+x^2}{\left(1+x^2\right) \sqrt[4]{x^2+x^6}} \, dx$

Optimal. Leaf size=101

$$\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{2^{3/4}}$$

Rubi [C] time = 0.64, antiderivative size = 127, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2056, 6715, 6725, 245, 1438, 429, 510}

$$-\frac{4\sqrt[4]{x^4+1}\,xF_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^6+x^2}}+\frac{4\sqrt[4]{x^4+1}\,x^3F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^6+x^2}}+\frac{2\sqrt[4]{x^4+1}\,x\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^2)/((1 + x^2)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-4*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, x^4, -x^4])/(x^2 + x^6)^(1
/4) + (4*x^3*(1 + x^4)^(1/4)*AppellF1[5/8, 1, 1/4, 13/8, x^4, -x^4])/(5*(x^
2 + x^6)^(1/4)) + (2*x*(1 + x^4)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -x^
4])/(x^2 + x^6)^(1/4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
```

```
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt[4]{x^2+x^6}}\,dx = \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^4}\right)\int \frac{-1+x^2}{\sqrt{x}\left(1+x^2\right)\sqrt[4]{1+x^4}}\,dx}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{-1+x^4}{\left(1+x^4\right)\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[4]{1+x^8}} - \frac{2}{\left(1+x^4\right)\sqrt[4]{1+x^8}}\right)dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(4\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(1+x^4\right)\sqrt[4]{1+x^8}}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(4\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\left(1-x^8\right)\sqrt[4]{1+x^8}} + \frac{x^4}{\left(-1+x^8\right)\sqrt[4]{1\cdot}}\right.\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(4\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(1-x^8\right)\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} -$$

$$= -\frac{4x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^2+x^6}} + \frac{4x^3\sqrt[4]{1+x^4}\,F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^2+x^6}} + \frac{2x\sqrt[4]{1+}}{}$$

Mathematica [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt[4]{x^2+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*(x^2 + x^6)^(1/4)),x]
[Out] Integrate[(-1 + x^2)/((1 + x^2)*(x^2 + x^6)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.50, size = 101, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}\,x^2-\sqrt{x^6+x^2}}\right)}{2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] ArcTan[(2^(3/4)*x*(x^2 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^2 + x^6])]/2^(3/
4) - ArcTanh[(x^2/2^(1/4) + Sqrt[x^2 + x^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))]
/2^(3/4)
```

fricas [B] time = 25.00, size = 526, normalized size = 5.21

$$\frac{1}{2}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{\left(4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2-\sqrt{2}\left(2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2-\sqrt{2}\left(x^5+2x^3+x\right)-4\sqrt{x^6+x^2}\,x+2\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}\right)\right)\sqrt{\frac{x^5+2x^3+4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2+4\sqrt{2}\sqrt{x^6+x^2}\,x+2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}+x}{x^5+2x^3+x}}+2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}}{2\left(x^5-2x^3+x\right)}\right)+\frac{1}{2}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{\left(4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2-\sqrt{2}\left(2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2+\sqrt{2}\left(x^5+2x^3+x\right)+4\sqrt{x^6+x^2}\,x+2\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}\right)\right)\sqrt{\frac{x^5+2x^3-4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2+4\sqrt{2}\sqrt{x^6+x^2}\,x-2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}+x}{x^5+2x^3+x}}+2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}}{2\left(x^5-2x^3+x\right)}\right)-\frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\frac{2\left(x^5+2x^3+4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2+4\sqrt{2}\sqrt{x^6+x^2}\,x+2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}+x\right)}{x^5+2x^3+x}\right)+\frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\frac{2\left(x^5+2x^3-4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2+4\sqrt{2}\sqrt{x^6+x^2}\,x-2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}+x\right)}{x^5+2x^3+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^6+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*2^(1/4)*arctan(-1/2*(4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(2*2^(3/
4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 + 2*x^3 + x) - 4*sqrt(x^6 + x^2)*x
+ 2*2^(1/4)*(x^6 + x^2)^(3/4))*sqrt((x^5 + 2*x^3 + 4*2^(1/4)*(x^6 + x^2)^(1
/4)*x^2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x + 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x
^5 + 2*x^3 + x)) + 2*2^(3/4)*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 1/2*2^
(1/4)*arctan(-1/2*(4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(2*2^(3/4)*(x^
6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 4*sqrt(x^6 + x^2)*x + 2*2^
(1/4)*(x^6 + x^2)^(3/4))*sqrt((x^5 + 2*x^3 - 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^
2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x - 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2
*x^3 + x)) + 2*2^(3/4)*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) - 1/8*2^(1/4)*
log(2*(x^5 + 2*x^3 + 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^6 +
 x^2)*x + 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x)) + 1/8*2^(1/4)
*log(2*(x^5 + 2*x^3 - 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^6
+ x^2)*x - 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^6+x^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/((x^6 + x^2)^(1/4)*(x^2 + 1)), x)
```

maple [F] time = 2.91, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^2+1\right)\left(x^6+x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^6+x^2)^(1/4),x)
```

```
[Out] int((x^2-1)/(x^2+1)/(x^6+x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^6+x^2)^(1/4),x, algorithm="maxima")
[Out] integrate((x^2 - 1)/((x^6 + x^2)^(1/4)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2-1}{\left(x^6+x^2\right)^{1/4}\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + x^6)^(1/4)*(x^2 + 1)),x)
[Out] int((x^2 - 1)/((x^2 + x^6)^(1/4)*(x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)}{\sqrt[4]{x^2\left(x^4+1\right)}\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**6+x**2)**(1/4),x)
[Out] Integral((x - 1)*(x + 1)/((x**2*(x**4 + 1))**(1/4)*(x**2 + 1)), x)
```

3.1257 $\int \frac{-1+x^2}{\left(1+x^2\right) \sqrt[4]{x^2+x^6}} \, dx$

Optimal. Leaf size=101

$$\frac{\tan^{-1}\left(\frac{2^{3/4} x \sqrt[4]{x^6+x^2}}{\sqrt{2} x^2-\sqrt{x^6+x^2}}\right)}{2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x \sqrt[4]{x^6+x^2}}\right)}{2^{3/4}}$$

Rubi [C] time = 0.51, antiderivative size = 127, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2056, 6715, 6725, 245, 1438, 429, 510}

$$-\frac{4 \sqrt[4]{x^4+1} x F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^6+x^2}}+\frac{4 \sqrt[4]{x^4+1} x^3 F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};x^4,-x^4\right)}{5 \sqrt[4]{x^6+x^2}}+\frac{2 \sqrt[4]{x^4+1} x\, _2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^2)/((1 + x^2)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-4*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, x^4, -x^4])/(x^2 + x^6)^(1
/4) + (4*x^3*(1 + x^4)^(1/4)*AppellF1[5/8, 1, 1/4, 13/8, x^4, -x^4])/(5*(x^
2 + x^6)^(1/4)) + (2*x*(1 + x^4)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -x^
4])/(x^2 + x^6)^(1/4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
```

```
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt[4]{x^2+x^6}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{-1+x^2}{\sqrt{x}\left(1+x^2\right)\sqrt[4]{1+x^4}}\,dx}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{-1+x^4}{\left(1+x^4\right)\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[4]{1+x^8}} - \frac{2}{\left(1+x^4\right)\sqrt[4]{1+x^8}}\right)dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(4\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(1+x^4\right)\sqrt[4]{1+x^8}}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(4\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\left(1-x^8\right)\sqrt[4]{1+x^8}} + \frac{x^4}{\left(-1+x^8\right)\sqrt[4]{1\right.}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(4\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(1-x^8\right)\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} -$$

$$= -\frac{4x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^2+x^6}} + \frac{4x^3\sqrt[4]{1+x^4}\,F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^2+x^6}} + \frac{2x\sqrt[4]{1+}}{}$$

Mathematica [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt[4]{x^2+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*(x^2 + x^6)^(1/4)),x]
[Out] Integrate[(-1 + x^2)/((1 + x^2)*(x^2 + x^6)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.00, size = 101, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}\,x^2-\sqrt{x^6+x^2}}\right)}{2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] ArcTan[(2^(3/4)*x*(x^2 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^2 + x^6])]/2^(3/
4) - ArcTanh[(x^2/2^(1/4) + Sqrt[x^2 + x^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))]
/2^(3/4)
```

fricas [B] time = 24.05, size = 526, normalized size = 5.21

$$\frac{1}{2}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2-\sqrt{2}\left(2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2-\sqrt{2}\left(x^5+2x^3+x\right)-4\sqrt{x^6+x^2}\,x+2\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}\right)\sqrt{\frac{x^5+2x^3+4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2+4\sqrt{2}\sqrt{x^6+x^2}\,x+2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}+x}{x^5+2x^3+x}}+2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}}{2\left(x^5-2x^3+x\right)}\right)+\frac{1}{2}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2-\sqrt{2}\left(2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2+\sqrt{2}\left(x^5+2x^3+x\right)+4\sqrt{x^6+x^2}\,x+2\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}\right)\sqrt{\frac{x^5+2x^3-4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2+4\sqrt{2}\sqrt{x^6+x^2}\,x-2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}+x}{x^5+2x^3+x}}+2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}}{2\left(x^5-2x^3+x\right)}\right)-\frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\frac{2\left(x^5+2x^3+4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2+4\sqrt{2}\sqrt{x^6+x^2}\,x+2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}+x\right)}{x^5+2x^3+x}\right)+\frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\frac{2\left(x^5+2x^3-4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2+4\sqrt{2}\sqrt{x^6+x^2}\,x-2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}+x\right)}{x^5+2x^3+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^6+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*2^(1/4)*arctan(-1/2*(4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(2*2^(3/
4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 + 2*x^3 + x) - 4*sqrt(x^6 + x^2)*x
+ 2*2^(1/4)*(x^6 + x^2)^(3/4))*sqrt((x^5 + 2*x^3 + 4*2^(1/4)*(x^6 + x^2)^(1
/4)*x^2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x + 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x
^5 + 2*x^3 + x)) + 2*2^(3/4)*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 1/2*2^
(1/4)*arctan(-1/2*(4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(2*2^(3/4)*(x^
6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 4*sqrt(x^6 + x^2)*x + 2*2^
(1/4)*(x^6 + x^2)^(3/4))*sqrt((x^5 + 2*x^3 - 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^
2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x - 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2
*x^3 + x)) + 2*2^(3/4)*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) - 1/8*2^(1/4)*
log(2*(x^5 + 2*x^3 + 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^6 +
 x^2)*x + 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x)) + 1/8*2^(1/4)
*log(2*(x^5 + 2*x^3 - 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^6
+ x^2)*x - 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^6+x^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/((x^6 + x^2)^(1/4)*(x^2 + 1)), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^2+1\right)\left(x^6+x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^6+x^2)^(1/4),x)
```

```
[Out] int((x^2-1)/(x^2+1)/(x^6+x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^6+x^2)^(1/4),x, algorithm="maxima")
[Out] integrate((x^2 - 1)/((x^6 + x^2)^(1/4)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2-1}{\left(x^6+x^2\right)^{1/4}\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + x^6)^(1/4)*(x^2 + 1)),x)
[Out] int((x^2 - 1)/((x^2 + x^6)^(1/4)*(x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)}{\sqrt[4]{x^2\left(x^4+1\right)}\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**6+x**2)**(1/4),x)
[Out] Integral((x - 1)*(x + 1)/((x**2*(x**4 + 1))**(1/4)*(x**2 + 1)), x)
```

3.1258 $\int \frac{\sqrt[4]{-x^3+x^4}\left(-1+x^8\right)}{x^4}\,dx$

Optimal. Leaf size=101

$$\frac{1463\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)}{32768}-\frac{1463\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)}{32768}+\frac{\sqrt[4]{x^4-x^3}\left(122880x^8-6144x^7-7296x^6-9120x^5-12540x^4-21\right.}{737280x^3}$$

Rubi [B] time = 0.48, antiderivative size = 242, normalized size of antiderivative = 2.40, number of steps used = 16, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2052, 2016, 2014, 2021, 2024, 2032, 63, 240, 212, 206, 203}

$$-\frac{1}{120}\sqrt[4]{x^4-x^3}\,x^4-\frac{19\sqrt[4]{x^4-x^3}\,x^3}{1920}-\frac{209\sqrt[4]{x^4-x^3}\,x}{12288}-\frac{1463\sqrt[4]{x^4-x^3}}{49152}-\frac{1463(x-1)^{3/4}x^{9/4}\tan^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{32768\left(x^4-x^3\right)^{3/4}}-\frac{1463(x-1)^{3/4}x^{9/4}\tanh^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{32768\left(x^4-x^3\right)^{3/4}}-\frac{4\left(x^4-x^3\right)^{5/4}}{9x^6}+\frac{1}{6}\sqrt[4]{x^4-x^3}\,x^5-\frac{16\left(x^4-x^3\right)^{5/4}}{45x^5}-\frac{19\sqrt[4]{x^4-x^3}\,x^2}{1536}$$

Antiderivative was successfully verified.

```
[In] Int[((-x^3 + x^4)^(1/4)*(-1 + x^8))/x^4,x]
```

```
[Out] (-1463*(-x^3 + x^4)^(1/4))/49152 - (209*x*(-x^3 + x^4)^(1/4))/12288 - (19*x
^2*(-x^3 + x^4)^(1/4))/1536 - (19*x^3*(-x^3 + x^4)^(1/4))/1920 - (x^4*(-x^3
 + x^4)^(1/4))/120 + (x^5*(-x^3 + x^4)^(1/4))/6 - (4*(-x^3 + x^4)^(5/4))/(9
*x^6) - (16*(-x^3 + x^4)^(5/4))/(45*x^5) - (1463*(-1 + x)^(3/4)*x^(9/4)*Arc
Tan[(-1 + x)^(1/4)/x^(1/4)])/(32768*(-x^3 + x^4)^(3/4)) - (1463*(-1 + x)^(3
/4)*x^(9/4)*ArcTanh[(-1 + x)^(1/4)/x^(1/4)])/(32768*(-x^3 + x^4)^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\sqrt[4]{-x^3+x^4}\left(-1+x^8\right)}{x^4}\,dx = \int \left(-\frac{\sqrt[4]{-x^3+x^4}}{x^4}+x^4\sqrt[4]{-x^3+x^4}\right)dx$$

$$= -\int \frac{\sqrt[4]{-x^3+x^4}}{x^4}\,dx + \int x^4\sqrt[4]{-x^3+x^4}\,dx$$

$$= \frac{1}{6}x^5\sqrt[4]{-x^3+x^4} - \frac{4\left(-x^3+x^4\right)^{5/4}}{9x^6} - \frac{1}{24}\int \frac{x^7}{\left(-x^3+x^4\right)^{3/4}}\,dx - \frac{4}{9}\int \frac{\sqrt[4]{-x^3+x^4}}{x^3}\,dx$$

$$= -\frac{1}{120}x^4\sqrt[4]{-x^3+x^4} + \frac{1}{6}x^5\sqrt[4]{-x^3+x^4} - \frac{4\left(-x^3+x^4\right)^{5/4}}{9x^6} - \frac{16\left(-x^3+x^4\right)^{5/4}}{45x^5} - \frac{19}{480}$$

$$= -\frac{19x^3\sqrt[4]{-x^3+x^4}}{1920} - \frac{1}{120}x^4\sqrt[4]{-x^3+x^4} + \frac{1}{6}x^5\sqrt[4]{-x^3+x^4} - \frac{4\left(-x^3+x^4\right)^{5/4}}{9x^6} - 16(-$$

$$= -\frac{19x^2\sqrt[4]{-x^3+x^4}}{1536} - \frac{19x^3\sqrt[4]{-x^3+x^4}}{1920} - \frac{1}{120}x^4\sqrt[4]{-x^3+x^4} + \frac{1}{6}x^5\sqrt[4]{-x^3+x^4} - 4(-$$

$$= -\frac{209x\sqrt[4]{-x^3+x^4}}{12288} - \frac{19x^2\sqrt[4]{-x^3+x^4}}{1536} - \frac{19x^3\sqrt[4]{-x^3+x^4}}{1920} - \frac{1}{120}x^4\sqrt[4]{-x^3+x^4} + \frac{1}{6}x$$

$$= -\frac{1463\sqrt[4]{-x^3+x^4}}{49152} - \frac{209x\sqrt[4]{-x^3+x^4}}{12288} - \frac{19x^2\sqrt[4]{-x^3+x^4}}{1536} - \frac{19x^3\sqrt[4]{-x^3+x^4}}{1920} - \frac{1}{120}$$

$$= -\frac{1463\sqrt[4]{-x^3+x^4}}{49152} - \frac{209x\sqrt[4]{-x^3+x^4}}{12288} - \frac{19x^2\sqrt[4]{-x^3+x^4}}{1536} - \frac{19x^3\sqrt[4]{-x^3+x^4}}{1920} - \frac{1}{120}$$

$$= -\frac{1463\sqrt[4]{-x^3+x^4}}{49152} - \frac{209x\sqrt[4]{-x^3+x^4}}{12288} - \frac{19x^2\sqrt[4]{-x^3+x^4}}{1536} - \frac{19x^3\sqrt[4]{-x^3+x^4}}{1920} - \frac{1}{120}$$

$$= -\frac{1463\sqrt[4]{-x^3+x^4}}{49152} - \frac{209x\sqrt[4]{-x^3+x^4}}{12288} - \frac{19x^2\sqrt[4]{-x^3+x^4}}{1536} - \frac{19x^3\sqrt[4]{-x^3+x^4}}{1920} - \frac{1}{120}$$

$$= -\frac{1463\sqrt[4]{-x^3+x^4}}{49152} - \frac{209x\sqrt[4]{-x^3+x^4}}{12288} - \frac{19x^2\sqrt[4]{-x^3+x^4}}{1536} - \frac{19x^3\sqrt[4]{-x^3+x^4}}{1920} - \frac{1}{120}$$

$$= -\frac{1463\sqrt[4]{-x^3+x^4}}{49152} - \frac{209x\sqrt[4]{-x^3+x^4}}{12288} - \frac{19x^2\sqrt[4]{-x^3+x^4}}{1536} - \frac{19x^3\sqrt[4]{-x^3+x^4}}{1920} - \frac{1}{120}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.47

$$\frac{4(x-1)^2\left(9x^{9/4}\,{}_2F_1\left(-\frac{19}{4},\frac{5}{4};\frac{9}{4};1-x\right)-4x-5\right)}{45\left((x-1)x^3\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-x^3 + x^4)^(1/4)*(-1 + x^8))/x^4,x]
```

```
[Out] (4*(-1 + x)^2*(-5 - 4*x + 9*x^(9/4)*Hypergeometric2F1[-19/4, 5/4, 9/4, 1 -
x]))/(45*((-1 + x)*x^3)^(3/4))
```

IntegrateAlgebraic [A] time = 0.63, size = 101, normalized size = 1.00

$$\frac{1463\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)}{32768}-\frac{1463\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)}{32768}+\frac{\sqrt[4]{x^4-x^3}\,(122880x^8-6144x^7-7296x^6-9120x^5-12540x^4-21945x^3-262144x^2-65536x+327680)}{737280x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-x^3 + x^4)^(1/4)*(-1 + x^8))/x^4,x]
```

```
[Out] ((-x^3 + x^4)^(1/4)*(327680 - 65536*x - 262144*x^2 - 21945*x^3 - 12540*x^4
- 9120*x^5 - 7296*x^6 - 6144*x^7 + 122880*x^8))/(737280*x^3) + (1463*ArcTan
[x/(-x^3 + x^4)^(1/4)])/32768 - (1463*ArcTanh[x/(-x^3 + x^4)^(1/4)])/32768
```

fricas [A] time = 0.41, size = 129, normalized size = 1.28

$$-\frac{131670\,x^3\arctan\left(\frac{(x^4-x^3)^{\frac{1}{4}}}{x}\right)+65835\,x^3\log\left(\frac{x+(x^4-x^3)^{\frac{1}{4}}}{x}\right)-65835\,x^3\log\left(-\frac{x-(x^4-x^3)^{\frac{1}{4}}}{x}\right)-4\,(122880\,x^8-6144\,x^7-7296\,x^6-9120\,x^5-12540\,x^4-21945\,x^3-262144\,x^2-65536\,x+327680)(x^4-x^3)^{\frac{1}{4}}}{2949120\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)*(x^8-1)/x^4,x, algorithm="fricas")
```

```
[Out] -1/2949120*(131670*x^3*arctan((x^4 - x^3)^(1/4)/x) + 65835*x^3*log((x + (x^
4 - x^3)^(1/4))/x) - 65835*x^3*log(-(x - (x^4 - x^3)^(1/4))/x) - 4*(122880*
x^8 - 6144*x^7 - 7296*x^6 - 9120*x^5 - 12540*x^4 - 21945*x^3 - 262144*x^2 -
 65536*x + 327680)*(x^4 - x^3)^(1/4))/x^3
```

giac [A] time = 0.27, size = 171, normalized size = 1.69

$$\frac{1}{245760}\left(7315\left(\frac{1}{x}-1\right)^5\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+40755\left(\frac{1}{x}-1\right)^4\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+92910\left(\frac{1}{x}-1\right)^3\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+109782\left(\frac{1}{x}-1\right)^2\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-69327\left(-\frac{1}{x}+1\right)^{\frac{5}{4}}-21945\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)x^6-\frac{4}{9}\left(\frac{1}{x}-1\right)^2\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+\frac{4}{5}\left(-\frac{1}{x}+1\right)^{\frac{5}{4}}+\frac{1463}{32768}\arctan\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)+\frac{1463}{65536}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right)-\frac{1463}{65536}\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)*(x^8-1)/x^4,x, algorithm="giac")
```

```
[Out] 1/245760*(7315*(1/x - 1)^5*(-1/x + 1)^(1/4) + 40755*(1/x - 1)^4*(-1/x + 1)^
(1/4) + 92910*(1/x - 1)^3*(-1/x + 1)^(1/4) + 109782*(1/x - 1)^2*(-1/x + 1)^
(1/4) - 69327*(-1/x + 1)^(5/4) - 21945*(-1/x + 1)^(1/4))*x^6 - 4/9*(1/x - 1
)^2*(-1/x + 1)^(1/4) + 4/5*(-1/x + 1)^(5/4) + 1463/32768*arctan((-1/x + 1)^
(1/4)) + 1463/65536*log((-1/x + 1)^(1/4) + 1) - 1463/65536*log(abs((-1/x +
1)^(1/4) - 1))
```

maple [C] time = 0.39, size = 445, normalized size = 4.41

$$\frac{(122880x^9-129024x^8-1152x^7-1824x^6-3420x^5-9405x^4-240199x^3+196608x^2+393216x-327680)(x^3(-1+x))^{\frac{1}{4}}}{737280x^3(-1+x)}+\frac{\left(\frac{1463\ln\left(\frac{2(x^4-3x^3+3x^2-x)^{\frac{3}{4}}-2\sqrt{x^4-3x^3+3x^2-x}\,x+2(x^4-3x^3+3x^2-x)^{\frac{1}{4}}x^2-2x^3+2\sqrt{x^4-3x^3+3x^2-x}-4(x^4-3x^3+3x^2-x)^{\frac{1}{4}}x+5x^2+2(x^4-3x^3+3x^2-x)^{\frac{1}{4}}-4x+1}{(-1+x)^2}\right)}{65536}+\frac{1463\,\mathrm{RootOf}(\_Z^2+1)\ln\left(\frac{2\sqrt{x^4-3x^3+3x^2-x}\,\mathrm{RootOf}(\_Z^2+1)x-2\,\mathrm{RootOf}(\_Z^2+1)x^3+2(x^4-3x^3+3x^2-x)^{\frac{3}{4}}-2\sqrt{x^4-3x^3+3x^2-x}\,\mathrm{RootOf}(\_Z^2+1)-2(x^4-3x^3+3x^2-x)^{\frac{1}{4}}x^2+5\,\mathrm{RootOf}(\_Z^2+1)x^2+4(x^4-3x^3+3x^2-x)^{\frac{1}{4}}x-4\,\mathrm{RootOf}(\_Z^2+1)x-2(x^4-3x^3+3x^2-x)^{\frac{1}{4}}+\mathrm{RootOf}(\_Z^2+1)}{(-1+x)^2}\right)}{65536}\right)(x^3(-1+x))^{\frac{1}{4}}(x(-1+x)^3)^{\frac{1}{4}}}{(-1+x)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3)^(1/4)*(x^8-1)/x^4,x)
```

```
[Out] 1/737280*(122880*x^9-129024*x^8-1152*x^7-1824*x^6-3420*x^5-9405*x^4-240199*
x^3+196608*x^2+393216*x-327680)/x^3*(x^3*(-1+x))^(1/4)/(-1+x)+(1463/65536*l
n((2*(x^4-3*x^3+3*x^2-x)^(3/4)-2*(x^4-3*x^3+3*x^2-x)^(1/2)*x+2*(x^4-3*x^3+3
*x^2-x)^(1/4)*x^2-2*x^3+2*(x^4-3*x^3+3*x^2-x)^(1/2)-4*(x^4-3*x^3+3*x^2-x)^(
1/4)*x+5*x^2+2*(x^4-3*x^3+3*x^2-x)^(1/4)-4*x+1)/(-1+x)^2)+1463/65536*RootOf
(_Z^2+1)*ln((2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+1)*x-2*RootOf(_Z^2+1)*
x^3+2*(x^4-3*x^3+3*x^2-x)^(3/4)-2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+1)-
2*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+5*RootOf(_Z^2+1)*x^2+4*(x^4-3*x^3+3*x^2-x)^
(1/4)*x-4*RootOf(_Z^2+1)*x-2*(x^4-3*x^3+3*x^2-x)^(1/4)+RootOf(_Z^2+1))/(-1+
x)^2))*(x^3*(-1+x))^(1/4)/(-1+x)/x*(x*(-1+x)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8-1\right)\left(x^4-x^3\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)*(x^8-1)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((x^8 - 1)*(x^4 - x^3)^(1/4)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^8-1\right)\,\left(x^4-x^3\right)^{1/4}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^8 - 1)*(x^4 - x^3)^(1/4))/x^4,x)
```

```
[Out] int(((x^8 - 1)*(x^4 - x^3)^(1/4))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\,(x-1)}\,(x-1)\,(x+1)\left(x^2+1\right)\left(x^4+1\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3)**(1/4)*(x**8-1)/x**4,x)
```

```
[Out] Integral((x**3*(x - 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)*(x**4 + 1)/x**4,
x)
```

3.1259 $\int \frac{\sqrt{1+2x^6}\left(-1+4x^6\right)}{2+x^4+8x^6+8x^{12}}\,dx$

Optimal. Leaf size=101

$$-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt{2x^6+1}}{2\sqrt{2}\,x^6-x^2+\sqrt{2}}\right)}{4\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{2^{3/4}x\sqrt{2x^6+1}}{2\sqrt{2}\,x^6+x^2+\sqrt{2}}\right)}{4\sqrt[4]{2}}$$

Rubi [F] time = 0.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+2x^6}\left(-1+4x^6\right)}{2+x^4+8x^6+8x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[1 + 2*x^6]*(-1 + 4*x^6))/(2 + x^4 + 8*x^6 + 8*x^12),x]
```

```
[Out] Defer[Int][Sqrt[1 + 2*x^6]/(-2 - x^4 - 8*x^6 - 8*x^12), x] + 4*Defer[Int][(
x^6*Sqrt[1 + 2*x^6])/(2 + x^4 + 8*x^6 + 8*x^12), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+2x^6}\left(-1+4x^6\right)}{2+x^4+8x^6+8x^{12}}\,dx &= \int \left(\frac{\sqrt{1+2x^6}}{-2-x^4-8x^6-8x^{12}}+\frac{4x^6\sqrt{1+2x^6}}{2+x^4+8x^6+8x^{12}}\right)dx\\ &= 4\int \frac{x^6\sqrt{1+2x^6}}{2+x^4+8x^6+8x^{12}}\,dx+\int \frac{\sqrt{1+2x^6}}{-2-x^4-8x^6-8x^{12}}\,dx\end{aligned}$$

Mathematica [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+2x^6}\left(-1+4x^6\right)}{2+x^4+8x^6+8x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[1 + 2*x^6]*(-1 + 4*x^6))/(2 + x^4 + 8*x^6 + 8*x^12),x]
```

```
[Out] Integrate[(Sqrt[1 + 2*x^6]*(-1 + 4*x^6))/(2 + x^4 + 8*x^6 + 8*x^12), x]
```

IntegrateAlgebraic [A] time = 5.29, size = 101, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt{2x^6+1}}{2\sqrt{2}\,x^6-x^2+\sqrt{2}}\right)}{4\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{2^{3/4}x\sqrt{2x^6+1}}{2\sqrt{2}\,x^6+x^2+\sqrt{2}}\right)}{4\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + 2*x^6]*(-1 + 4*x^6))/(2 + x^4 + 8*x^6 + 8*x^12
),x]
```

```
[Out] -1/4*ArcTan[(2^(3/4)*x*Sqrt[1 + 2*x^6])/(Sqrt[2] - x^2 + 2*Sqrt[2]*x^6)]/2^
(1/4) - ArcTanh[(2^(3/4)*x*Sqrt[1 + 2*x^6])/(Sqrt[2] + x^2 + 2*Sqrt[2]*x^6)
]/(4*2^(1/4))
```

fricas [B] time = 0.87, size = 1041, normalized size = 10.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+1)^(1/2)*(4*x^6-1)/(8*x^12+8*x^6+x^4+2),x, algorithm="fric
as")
```

```
[Out] 1/32*8^(3/4)*sqrt(2)*arctan(-1/2*(128*x^24 + 256*x^18 + 32*x^16 + 192*x^12
+ 32*x^10 + 2*x^8 + 64*x^6 + 8*x^4 + 8*sqrt(2)*(16*x^20 + 24*x^14 + 2*x^12
+ 12*x^8 + x^6 + 2*x^2) + sqrt(2*x^6 + 1)*(8^(3/4)*sqrt(2)*(24*x^15 + 24*x^
9 - x^7 + 6*x^3) + 4*8^(1/4)*sqrt(2)*(16*x^19 + 24*x^13 - 6*x^11 + 12*x^7 -
 3*x^5 + 2*x)) - (8^(3/4)*sqrt(2)*(16*x^20 + 24*x^14 - 6*x^12 + 12*x^8 - 3*
x^6 + 2*x^2) + 8^(1/4)*sqrt(2)*(64*x^24 + 128*x^18 - 64*x^16 + 96*x^12 - 64
*x^10 - x^8 + 32*x^6 - 16*x^4 + 4) + 8*(8*x^15 + 8*x^9 + x^7 + 2*x^3 + 4*sq
rt(2)*(2*x^11 + x^5))*sqrt(2*x^6 + 1))*sqrt((16*x^8 + 8*x^2 + sqrt(2)*(8*x^
12 + 8*x^6 + x^4 + 2) + sqrt(2*x^6 + 1)*(2*8^(1/4)*sqrt(2)*x^3 + 8^(3/4)*sq
rt(2)*(2*x^7 + x)))/(8*x^12 + 8*x^6 + x^4 + 2)) + 8)/(64*x^24 + 128*x^18 -
112*x^16 + 96*x^12 - 112*x^10 + x^8 + 32*x^6 - 28*x^4 + 4)) - 1/32*8^(3/4)*
sqrt(2)*arctan(-1/2*(128*x^24 + 256*x^18 + 32*x^16 + 192*x^12 + 32*x^10 + 2
*x^8 + 64*x^6 + 8*x^4 + 8*sqrt(2)*(16*x^20 + 24*x^14 + 2*x^12 + 12*x^8 + x^
6 + 2*x^2) - sqrt(2*x^6 + 1)*(8^(3/4)*sqrt(2)*(24*x^15 + 24*x^9 - x^7 + 6*x
^3) + 4*8^(1/4)*sqrt(2)*(16*x^19 + 24*x^13 - 6*x^11 + 12*x^7 - 3*x^5 + 2*x)
) + (8^(3/4)*sqrt(2)*(16*x^20 + 24*x^14 - 6*x^12 + 12*x^8 - 3*x^6 + 2*x^2)
+ 8^(1/4)*sqrt(2)*(64*x^24 + 128*x^18 - 64*x^16 + 96*x^12 - 64*x^10 - x^8 +
 32*x^6 - 16*x^4 + 4) - 8*(8*x^15 + 8*x^9 + x^7 + 2*x^3 + 4*sqrt(2)*(2*x^11
 + x^5))*sqrt(2*x^6 + 1))*sqrt((16*x^8 + 8*x^2 + sqrt(2)*(8*x^12 + 8*x^6 +
x^4 + 2) - sqrt(2*x^6 + 1)*(2*8^(1/4)*sqrt(2)*x^3 + 8^(3/4)*sqrt(2)*(2*x^7
+ x)))/(8*x^12 + 8*x^6 + x^4 + 2)) + 8)/(64*x^24 + 128*x^18 - 112*x^16 + 96
*x^12 - 112*x^10 + x^8 + 32*x^6 - 28*x^4 + 4)) - 1/128*8^(3/4)*sqrt(2)*log(
64*(16*x^8 + 8*x^2 + sqrt(2)*(8*x^12 + 8*x^6 + x^4 + 2) + sqrt(2*x^6 + 1)*(
2*8^(1/4)*sqrt(2)*x^3 + 8^(3/4)*sqrt(2)*(2*x^7 + x)))/(8*x^12 + 8*x^6 + x^4
 + 2)) + 1/128*8^(3/4)*sqrt(2)*log(64*(16*x^8 + 8*x^2 + sqrt(2)*(8*x^12 + 8
*x^6 + x^4 + 2) - sqrt(2*x^6 + 1)*(2*8^(1/4)*sqrt(2)*x^3 + 8^(3/4)*sqrt(2)*
(2*x^7 + x)))/(8*x^12 + 8*x^6 + x^4 + 2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^{6}-1\right)\sqrt{2\,x^{6}+1}}{8\,x^{12}+8\,x^{6}+x^{4}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+1)^(1/2)*(4*x^6-1)/(8*x^12+8*x^6+x^4+2),x, algorithm="giac
")
```

```
[Out] integrate((4*x^6 - 1)*sqrt(2*x^6 + 1)/(8*x^12 + 8*x^6 + x^4 + 2), x)
```

maple [C] time = 2.55, size = 187, normalized size = 1.85

$$\frac{\mathrm{RootOf}\left(\_Z^{4}+2\right)\ln\left(\frac{-4\,\mathrm{RootOf}\left(\_Z^{4}+2\right)x^{6}+\mathrm{RootOf}\left(\_Z^{4}+2\right)^{3}x^{2}+4\sqrt{2x^{6}+1}\,x-2\,\mathrm{RootOf}\left(\_Z^{4}+2\right)}{4x^{6}+x^{2}\,\mathrm{RootOf}\left(\_Z^{4}+2\right)^{2}+2}\right)}{8}+\frac{\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}+2\right)^{2}\right)\ln\left(\frac{-4\,\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}+2\right)^{2}\right)x^{6}-\mathrm{RootOf}\left(\_Z^{4}+2\right)^{2}\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}+2\right)^{2}\right)x^{2}+4\sqrt{2x^{6}+1}\,x-2\,\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}+2\right)^{2}\right)}{-4x^{6}+x^{2}\,\mathrm{RootOf}\left(\_Z^{4}+2\right)^{2}-2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6+1)^(1/2)*(4*x^6-1)/(8*x^12+8*x^6+x^4+2),x)
```

```
[Out] 1/8*RootOf(_Z^4+2)*ln((-4*RootOf(_Z^4+2)*x^6+RootOf(_Z^4+2)^3*x^2+4*(2*x^6+
1)^(1/2)*x-2*RootOf(_Z^4+2))/(4*x^6+x^2*RootOf(_Z^4+2)^2+2))+1/8*RootOf(_Z^
2+RootOf(_Z^4+2)^2)*ln((-4*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^6-RootOf(_Z^4+2)
^2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^2+4*(2*x^6+1)^(1/2)*x-2*RootOf(_Z^2+Root
Of(_Z^4+2)^2))/(-4*x^6+x^2*RootOf(_Z^4+2)^2-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^6-1\right)\sqrt{2\,x^6+1}}{8\,x^{12}+8\,x^6+x^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+1)^(1/2)*(4*x^6-1)/(8*x^12+8*x^6+x^4+2),x, algorithm="maxi
ma")
```

```
[Out] integrate((4*x^6 - 1)*sqrt(2*x^6 + 1)/(8*x^12 + 8*x^6 + x^4 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^6+1}\,\left(4\,x^6-1\right)}{8\,x^{12}+8\,x^6+x^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^6 + 1)^(1/2)*(4*x^6 - 1))/(x^4 + 8*x^6 + 8*x^12 + 2),x)
```

```
[Out] int(((2*x^6 + 1)^(1/2)*(4*x^6 - 1))/(x^4 + 8*x^6 + 8*x^12 + 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{3}-1\right)\left(2x^{3}+1\right)\sqrt{2x^{6}+1}}{8x^{12}+8x^{6}+x^{4}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**6+1)**(1/2)*(4*x**6-1)/(8*x**12+8*x**6+x**4+2),x)
```

```
[Out] Integral((2*x**3 - 1)*(2*x**3 + 1)*sqrt(2*x**6 + 1)/(8*x**12 + 8*x**6 + x**
4 + 2), x)
```

3.1260 $\int x^2\sqrt{x^2+\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=101

$$\frac{2\sqrt{x^4+1}\,x^4+\left(2x^4-1\right)x^2}{8x\sqrt{\sqrt{x^4+1}+x^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)}{4\sqrt{2}}$$

Rubi [F] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\sqrt{x^2+\sqrt{1+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*Sqrt[x^2 + Sqrt[1 + x^4]],x]
[Out] Defer[Int][x^2*Sqrt[x^2 + Sqrt[1 + x^4]], x]
```

Rubi steps

$$\int x^2\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int x^2\sqrt{x^2+\sqrt{1+x^4}}\,dx$$

Mathematica [B] time = 0.73, size = 250, normalized size = 2.48

$$\frac{x\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}\sqrt{x^2\left(\sqrt{x^4+1}+x^2\right)}\left(2x^4+2\sqrt{x^4+1}\,x^2+1\right)^2\left(\sqrt{2}\sqrt{-x^2\left(\sqrt{x^4+1}+x^2\right)}\left(2x^4+2\sqrt{x^4+1}\,x^2-1\right)-\left(\sqrt{x^4+1}+x^2\right)\sin^{-1}\left(\sqrt{x^4+1}+x^2\right)\right)}{8\sqrt{2}\sqrt{-x^4\left(2x^4+2\sqrt{x^4+1}\,x^2+1\right)}\left(16x^{12}+28x^8+13x^4+16\sqrt{x^4+1}\,x^{10}+20\sqrt{x^4+1}\,x^6+5\sqrt{x^4+1}\,x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[x^2 + Sqrt[1 + x^4]],x]
[Out] (x*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]*Sqrt[x^2*(x^2 + Sqrt[1 + x^4])]*
(1 + 2*x^4 + 2*x^2*Sqrt[1 + x^4])^2*(Sqrt[2]*Sqrt[-(x^2*(x^2 + Sqrt[1 + x^4
]))]*(-1 + 2*x^4 + 2*x^2*Sqrt[1 + x^4]) - (x^2 + Sqrt[1 + x^4])*ArcSin[x^2
+ Sqrt[1 + x^4]]))/(8*Sqrt[2]*Sqrt[-(x^4*(1 + 2*x^4 + 2*x^2*Sqrt[1 + x^4]))
]*(1 + 13*x^4 + 28*x^8 + 16*x^12 + 5*x^2*Sqrt[1 + x^4] + 20*x^6*Sqrt[1 + x^
4] + 16*x^10*Sqrt[1 + x^4]))
```

IntegrateAlgebraic [A] time = 0.38, size = 101, normalized size = 1.00

$$\frac{2\sqrt{x^4+1}\,x^4+\left(2x^4-1\right)x^2}{8x\sqrt{\sqrt{x^4+1}+x^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[x^2 + Sqrt[1 + x^4]],x]
[Out] (2*x^4*Sqrt[1 + x^4] + x^2*(-1 + 2*x^4))/(8*x*Sqrt[x^2 + Sqrt[1 + x^4]]) +
ArcTanh[(Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])]/(4
*Sqrt[2])
```

fricas [A] time = 0.66, size = 92, normalized size = 0.91

$$\frac{1}{8}\left(3x^3 - \sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}} + \frac{1}{32}\sqrt{2}\log\left(4x^4 + 4\sqrt{x^4+1}\,x^2 + 2\left(\sqrt{2}x^3 + \sqrt{2}\sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*x^3 - sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1/32*sqrt(2)*log(
4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*sqr
t(x^2 + sqrt(x^4 + 1)) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+\sqrt{x^4+1}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^2, x)
```

maple [C] time = 0.07, size = 62, normalized size = 0.61

$$\frac{\frac{5\sqrt{\pi}\,\sqrt{2}\,\mathrm{hypergeom}\left(\left[1,1,\frac{7}{4},\frac{9}{4}\right],\left[2,\frac{5}{2},3\right],-\frac{1}{x^4}\right)}{32x^4} - \frac{(1-4\ln(2)-4\ln(x))\sqrt{\pi}\,\sqrt{2}}{2} + 4\sqrt{\pi}\,\sqrt{2}\,x^4}{16\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] 1/16/Pi^(1/2)*(5/32*Pi^(1/2)*2^(1/2)/x^4*hypergeom([1,1,7/4,9/4],[2,5/2,3],
-1/x^4)-1/2*(1-4*ln(2)-4*ln(x))*Pi^(1/2)*2^(1/2)+4*Pi^(1/2)*2^(1/2)*x^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+\sqrt{x^4+1}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\sqrt{\sqrt{x^4+1}+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

```
[Out] int(x^2*((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

sympy [A] time = 1.25, size = 17, normalized size = 0.17

$$-\frac{G_{3,3}^{2,2}\left(\begin{matrix} 2,1 & \frac{3}{2} \\ \frac{3}{4},\frac{5}{4} & 0 \end{matrix}\middle| x^4\right)}{16\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] -meijerg(((2, 1), (3/2,)), ((3/4, 5/4), (0,)), x**4)/(16*sqrt(pi))
```

3.1261 $\int \frac{(-1+x^4)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=101

$$\frac{2\sqrt{x^4+1}\,x^4+(2x^4+3)\,x^2}{8x\sqrt{\sqrt{x^4+1}+x^2}}-\frac{11\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)}{4\sqrt{2}}$$

Rubi [C] time = 0.50, antiderivative size = 161, normalized size of antiderivative = 1.59, number of steps used = 11, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {6742, 2132, 206, 2133, 321, 216, 215}

$$\left(\frac{3}{16}-\frac{3i}{16}\right)\sqrt{1-ix^2}\,x+\left(\frac{3}{16}+\frac{3i}{16}\right)\sqrt{1+ix^2}\,x-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}+\left(\frac{1}{8}+\frac{i}{8}\right)\sqrt{1-ix^2}\,x^3+\left(\frac{1}{8}-\frac{i}{8}\right)\sqrt{1+ix^2}\,x^3+\frac{3i\sin^{-1}\left(\sqrt[4]{-1}\,x\right)}{8\sqrt{2}}-\frac{3\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)}{8\sqrt{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^4)*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

```
[Out] (3/16 - (3*I)/16)*x*Sqrt[1 - I*x^2] + (1/8 + I/8)*x^3*Sqrt[1 - I*x^2] + (3/
16 + (3*I)/16)*x*Sqrt[1 + I*x^2] + (1/8 - I/8)*x^3*Sqrt[1 + I*x^2] + (((3*I
)/8)*ArcSin[(-1)^(1/4)*x])/Sqrt[2] - (3*ArcSinh[(-1)^(1/4)*x])/(8*Sqrt[2])
- ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(-1+x^4\right)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \int \left(-\frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}+\frac{x^4\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\right)dx$$

$$= -\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx+\int \frac{x^4\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$$

$$= \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{x^4}{\sqrt{1-ix^2}}\,dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{x^4}{\sqrt{1+ix^2}}\,dx-\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x\right.$$

$$= \left(\frac{1}{8}+\frac{i}{8}\right)x^3\sqrt{1-ix^2}+\left(\frac{1}{8}-\frac{i}{8}\right)x^3\sqrt{1+ix^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}+\left(-\frac{3}{8}-\right.$$

$$= \left(\frac{3}{16}-\frac{3i}{16}\right)x\sqrt{1-ix^2}+\left(\frac{1}{8}+\frac{i}{8}\right)x^3\sqrt{1-ix^2}+\left(\frac{3}{16}+\frac{3i}{16}\right)x\sqrt{1+ix^2}+\left(\frac{1}{8}-\right.$$

$$= \left(\frac{3}{16}-\frac{3i}{16}\right)x\sqrt{1-ix^2}+\left(\frac{1}{8}+\frac{i}{8}\right)x^3\sqrt{1-ix^2}+\left(\frac{3}{16}+\frac{3i}{16}\right)x\sqrt{1+ix^2}+\left(\frac{1}{8}-\right.$$

Mathematica [C] time = 2.56, size = 286, normalized size = 2.83

$$\frac{x\left(x^4+\sqrt{x^4+1}\,x^2+1\right)\left(\frac{\left(2x^4+2\sqrt{x^4+1}\,x^2+1\right)^2{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};2x^4+2\sqrt{x^4+1}\,x^2+1\right)}{\sqrt{-x^2\left(\sqrt{x^4+1}+x^2\right)}\left(16x^{12}+28x^8+13x^4+16\sqrt{x^4+1}\,x^{10}+20\sqrt{x^4+1}\,x^6+5\sqrt{x^4+1}\,x^2+1\right)}+\frac{2\left(\log\left(1-\frac{\sqrt{x^2\left(\sqrt{x^4+1}+x^2\right)}}{\sqrt{2}x^2}\right)-\log\left(\frac{\sqrt{x^2\left(\sqrt{x^4+1}+x^2\right)}}{\sqrt{2}x^2}+1\right)\right)}{\sqrt{x^4+1}\sqrt{x^2\left(\sqrt{x^4+1}+x^2\right)}}\right)}{4\sqrt{2}\sqrt{\sqrt{x^4+1}+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^4)*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

```
[Out] (x*(1 + x^4 + x^2*Sqrt[1 + x^4])*(((1 + 2*x^4 + 2*x^2*Sqrt[1 + x^4])^2*Hype
rgeometric2F1[-3/2, -1/2, 1/2, 1 + 2*x^4 + 2*x^2*Sqrt[1 + x^4]])/(Sqrt[-(x^
2*(x^2 + Sqrt[1 + x^4]))]*(1 + 13*x^4 + 28*x^8 + 16*x^12 + 5*x^2*Sqrt[1 + x
^4] + 20*x^6*Sqrt[1 + x^4] + 16*x^10*Sqrt[1 + x^4])) + (2*(Log[1 - Sqrt[x^2
*(x^2 + Sqrt[1 + x^4])]/(Sqrt[2]*x^2)] - Log[1 + Sqrt[x^2*(x^2 + Sqrt[1 + x
^4])]/(Sqrt[2]*x^2)]))/(Sqrt[1 + x^4]*Sqrt[x^2*(x^2 + Sqrt[1 + x^4])])))/(4
*Sqrt[2]*Sqrt[x^2 + Sqrt[1 + x^4]])
```

IntegrateAlgebraic [A] time = 0.31, size = 114, normalized size = 1.13

$$\frac{2\sqrt{x^4+1}\,x^4+\left(2x^4+3\right)x^2}{8x\sqrt{\sqrt{x^4+1}+x^2}}-\frac{11\tanh^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

```
[Out] (2*x^4*Sqrt[1 + x^4] + x^2*(3 + 2*x^4))/(8*x*Sqrt[x^2 + Sqrt[1 + x^4]]) - (
11*ArcTanh[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2])/(x*Sqrt[x^2
 + Sqrt[1 + x^4]])])/(4*Sqrt[2])
```

fricas [A] time = 0.78, size = 90, normalized size = 0.89

$$-\frac{1}{8}\left(x^3-3\sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}}+\frac{11}{32}\sqrt{2}\log\left(4x^4+4\sqrt{x^4+1}\,x^2-2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/8*(x^3 - 3*sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 11/32*sqrt(2)*lo
g(4*x^4 + 4*sqrt(x^4 + 1)*x^2 - 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*s
qrt(x^2 + sqrt(x^4 + 1)) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((x^4 - 1)*sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

```
[Out] int((x^4-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((x^4 - 1)*sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-1\right)\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 + 1)^(1/2),x)
```

```
[Out] int(((x^4 - 1)*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 + 1)^(1/2), x)
```

sympy [A] time = 4.52, size = 32, normalized size = 0.32

$$-\frac{G_{3,3}^{2,2}\left(\begin{matrix} 1,1 & \frac{1}{2} \\ \frac{1}{4},\frac{3}{4} & 0 \end{matrix} \middle| x^4\right)}{4\sqrt{\pi}}+\frac{G_{3,3}^{2,2}\left(\begin{matrix} 2,1 & \frac{3}{2} \\ \frac{5}{4},\frac{7}{4} & 0 \end{matrix} \middle| x^4\right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

```
[Out] -meijerg(((1, 1), (1/2,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi)) + meijerg
(((2, 1), (3/2,)), ((5/4, 7/4), (0,)), x**4)/(4*sqrt(pi))
```

3.1262 $\int \sqrt{1+x^4} \sqrt{x^2+\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=101

$$\frac{2\sqrt{x^4+1}\,x^4+\left(2x^4+3\right)x^2}{8x\sqrt{\sqrt{x^4+1}+x^2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)}{4\sqrt{2}}$$

Rubi [F] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{1+x^4} \sqrt{x^2+\sqrt{1+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

```
[Out] Defer[Int][Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]], x]
```

Rubi steps

$$\int \sqrt{1+x^4} \sqrt{x^2+\sqrt{1+x^4}}\,dx=\int \sqrt{1+x^4} \sqrt{x^2+\sqrt{1+x^4}}\,dx$$

Mathematica [C] time = 0.39, size = 1439, normalized size = 14.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

```
[Out] (7*(x^2 + Sqrt[1 + x^4])^(7/2)*(1 + x^4 + x^2*Sqrt[1 + x^4])*(5*(1 + 5*x^4
+ 2*x^8 + 4*x^2*Sqrt[1 + x^4] + 2*x^6*Sqrt[1 + x^4])*Hypergeometric2F1[-1/2
, 1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] + 2*(1 + 5*x^4 + 4*x^8 + 3*x^2*Sqrt[1
+ x^4] + 4*x^6*Sqrt[1 + x^4])*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt[
1 + x^4])^2] + 2*(1 + x^4)*(1 + 8*x^4 + 8*x^8 + 4*x^2*Sqrt[1 + x^4] + 8*x^6
*Sqrt[1 + x^4])*HypergeometricPFQ[{1/2, 3/2, 2}, {1, 7/2}, (x^2 + Sqrt[1 +
x^4])^2]))/(2*x*(630*x^2*Hypergeometric2F1[-1/2, 1/2, 5/2, (x^2 + Sqrt[1 +
x^4])^2] + 3990*x^6*Hypergeometric2F1[-1/2, 1/2, 5/2, (x^2 + Sqrt[1 + x^4])
^2] + 6720*x^10*Hypergeometric2F1[-1/2, 1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2]
+ 3360*x^14*Hypergeometric2F1[-1/2, 1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] + 10
5*Sqrt[1 + x^4]*Hypergeometric2F1[-1/2, 1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2]
+ 1890*x^4*Sqrt[1 + x^4]*Hypergeometric2F1[-1/2, 1/2, 5/2, (x^2 + Sqrt[1 +
x^4])^2] + 5040*x^8*Sqrt[1 + x^4]*Hypergeometric2F1[-1/2, 1/2, 5/2, (x^2 +
Sqrt[1 + x^4])^2] + 3360*x^12*Sqrt[1 + x^4]*Hypergeometric2F1[-1/2, 1/2, 5/
2, (x^2 + Sqrt[1 + x^4])^2] + 140*x^2*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2
 + Sqrt[1 + x^4])^2] + 952*x^6*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt
[1 + x^4])^2] + 1232*x^10*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt[1 +
x^4])^2] - 448*x^14*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt[1 + x^4])^
2] - 896*x^18*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt[1 + x^4])^2] + 2
1*Sqrt[1 + x^4]*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt[1 + x^4])^2] +
 448*x^4*Sqrt[1 + x^4]*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt[1 + x^4
])^2] + 1120*x^8*Sqrt[1 + x^4]*Hypergeometric2F1[1/2, 3/2, 7/2, (x^2 + Sqrt
[1 + x^4])^2] - 896*x^16*Sqrt[1 + x^4]*Hypergeometric2F1[1/2, 3/2, 7/2, (x^
```

```
2 + Sqrt[1 + x^4])^2] + 54*x^2*Hypergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt
[1 + x^4])^2] + 720*x^6*Hypergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt[1 + x^
4])^2] + 2592*x^10*Hypergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt[1 + x^4])^2
] + 3456*x^14*Hypergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt[1 + x^4])^2] + 1
536*x^18*Hypergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt[1 + x^4])^2] + 6*Sqrt
[1 + x^4]*Hypergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt[1 + x^4])^2] + 240*x
^4*Sqrt[1 + x^4]*Hypergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt[1 + x^4])^2]
+ 1440*x^8*Sqrt[1 + x^4]*Hypergeometric2F1[3/2, 5/2, 9/2, (x^2 + Sqrt[1 + x
^4])^2] + 2688*x^12*Sqrt[1 + x^4]*Hypergeometric2F1[3/2, 5/2, 9/2, (x^2 + S
qrt[1 + x^4])^2] + 1536*x^16*Sqrt[1 + x^4]*Hypergeometric2F1[3/2, 5/2, 9/2,
 (x^2 + Sqrt[1 + x^4])^2] + 14*(26*x^2 + 328*x^6 + 1136*x^10 + 1472*x^14 +
640*x^18 + 3*Sqrt[1 + x^4] + 112*x^4*Sqrt[1 + x^4] + 640*x^8*Sqrt[1 + x^4]
+ 1152*x^12*Sqrt[1 + x^4] + 640*x^16*Sqrt[1 + x^4])*HypergeometricPFQ[{1/2,
 3/2, 2}, {1, 7/2}, (x^2 + Sqrt[1 + x^4])^2] + 12*(10*x^2 + 170*x^6 + 832*x
^10 + 1696*x^14 + 1536*x^18 + 512*x^22 + Sqrt[1 + x^4] + 50*x^4*Sqrt[1 + x^
4] + 400*x^8*Sqrt[1 + x^4] + 1120*x^12*Sqrt[1 + x^4] + 1280*x^16*Sqrt[1 + x
^4] + 512*x^20*Sqrt[1 + x^4])*HypergeometricPFQ[{3/2, 5/2, 3}, {2, 9/2}, (x
^2 + Sqrt[1 + x^4])^2]))
```

IntegrateAlgebraic [A] time = 0.43, size = 101, normalized size = 1.00

$$\frac{2\sqrt{x^4+1}\,x^4+\left(2x^4+3\right)x^2}{8x\sqrt{\sqrt{x^4+1}+x^2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

```
[Out] (2*x^4*Sqrt[1 + x^4] + x^2*(3 + 2*x^4))/(8*x*Sqrt[x^2 + Sqrt[1 + x^4]]) + (
5*ArcTanh[(Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])])
/(4*Sqrt[2])
```

fricas [A] time = 0.73, size = 90, normalized size = 0.89

$$-\frac{1}{8}\left(x^3-3\sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}}+\frac{5}{32}\sqrt{2}\log\left(4x^4+4\sqrt{x^4+1}\,x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(x^3 - 3*sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 5/32*sqrt(2)*log
(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*sq
rt(x^2 + sqrt(x^4 + 1)) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1)), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2),x)
[Out] int((x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2),x)
[Out] int((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+\sqrt{x^4+1}}\,\sqrt{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(1/2)*(x**2+(x**4+1)**(1/2))**(1/2),x)
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))*sqrt(x**4 + 1), x)
```

3.1263 $\int x^3\left(-1+x^3\right)^{2/3}\,dx$

Optimal. Leaf size=102

$$\frac{1}{27}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{9\sqrt{3}}+\frac{1}{18}\left(x^3-1\right)^{2/3}\left(3x^4-2x\right)-\frac{1}{54}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 0.77, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {279, 321, 239}

$$-\frac{1}{9}\left(x^3-1\right)^{2/3}x+\frac{1}{18}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{1}{6}\left(x^3-1\right)^{2/3}x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(-1 + x^3)^(2/3),x]
```

```
[Out] -1/9*(x*(-1 + x^3)^(2/3)) + (x^4*(-1 + x^3)^(2/3))/6 - ArcTan[(1 + (2*x)/(-
1 + x^3)^(1/3))/Sqrt[3]]/(9*Sqrt[3]) + Log[-x + (-1 + x^3)^(1/3)]/18
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^3\left(-1+x^3\right)^{2/3}\,dx&=\frac{1}{6}x^4\left(-1+x^3\right)^{2/3}-\frac{1}{3}\int\frac{x^3}{\sqrt[3]{-1+x^3}}\,dx\\&=-\frac{1}{9}x\left(-1+x^3\right)^{2/3}+\frac{1}{6}x^4\left(-1+x^3\right)^{2/3}-\frac{1}{9}\int\frac{1}{\sqrt[3]{-1+x^3}}\,dx\\&=-\frac{1}{9}x\left(-1+x^3\right)^{2/3}+\frac{1}{6}x^4\left(-1+x^3\right)^{2/3}-\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{1}{18}\log\left(-x+\sqrt[3]{-1+x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.02, size = 44, normalized size = 0.43

$$\frac{1}{6}x\left(x^3-1\right)^{2/3}\left(\frac{{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};x^3\right)}{\left(1-x^3\right)^{2/3}}+x^3-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(-1 + x^3)^(2/3),x]
```

```
[Out] (x*(-1 + x^3)^(2/3)*(-1 + x^3 + Hypergeometric2F1[-2/3, 1/3, 4/3, x^3]/(1 -
 x^3)^(2/3)))/6
```

IntegrateAlgebraic [A] time = 0.22, size = 102, normalized size = 1.00

$$\frac{1}{27}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{9\sqrt{3}}+\frac{1}{18}\left(x^3-1\right)^{2/3}\left(3x^4-2x\right)-\frac{1}{54}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(-1 + x^3)^(2/3),x]
```

```
[Out] ((-1 + x^3)^(2/3)*(-2*x + 3*x^4))/18 - ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x^3)
^(1/3))]/(9*Sqrt[3]) + Log[-x + (-1 + x^3)^(1/3)]/27 - Log[x^2 + x*(-1 + x^
3)^(1/3) + (-1 + x^3)^(2/3)]/54
```

fricas [A] time = 0.40, size = 94, normalized size = 0.92

$$\frac{1}{18}\left(3x^4-2x\right)\left(x^3-1\right)^{\frac{2}{3}}+\frac{1}{27}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{27}\log\left(-\frac{x-\left(x^3-1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{54}\log\left(\frac{x^2+\left(x^3-1\right)^{\frac{1}{3}}x+\left(x^3-1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^3-1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/18*(3*x^4 - 2*x)*(x^3 - 1)^(2/3) + 1/27*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2
*sqrt(3)*(x^3 - 1)^(1/3))/x) + 1/27*log(-(x - (x^3 - 1)^(1/3))/x) - 1/54*lo
g((x^2 + (x^3 - 1)^(1/3)*x + (x^3 - 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^3-1\right)^{\frac{2}{3}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^3-1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 1)^(2/3)*x^3, x)
```

maple [C] time = 0.27, size = 49, normalized size = 0.48

$$\frac{x\left(3x^3-2\right)\left(x^3-1\right)^{\frac{2}{3}}}{18}-\frac{\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{1}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^3\right)}{9\mathrm{signum}\left(x^3-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^3-1)^(2/3),x)
```

```
[Out] 1/18*x*(3*x^3-2)*(x^3-1)^(2/3)-1/9/signum(x^3-1)^(1/3)*(-signum(x^3-1))^(1/
3)*x*hypergeom([1/3,1/3],[4/3],x^3)
```

maxima [A] time = 0.42, size = 121, normalized size = 1.19

$$\frac{1}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3-1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\frac{\left(x^3-1\right)^{\frac{2}{3}}}{x^2}+\frac{2\left(x^3-1\right)^{\frac{5}{3}}}{x^5}}{18\left(\frac{2\left(x^3-1\right)}{x^3}-\frac{\left(x^3-1\right)^2}{x^6}-1\right)}-\frac{1}{54}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3-1\right)^{\frac{2}{3}}}{x^2}+1\right)+\frac{1}{27}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^3-1)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3)/x + 1)) - 1/18*((x^3 - 1
)^(2/3)/x^2 + 2*(x^3 - 1)^(5/3)/x^5)/(2*(x^3 - 1)/x^3 - (x^3 - 1)^2/x^6 - 1
) - 1/54*log((x^3 - 1)^(1/3)/x + (x^3 - 1)^(2/3)/x^2 + 1) + 1/27*log((x^3 -
 1)^(1/3)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(x^3-1\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^3 - 1)^(2/3),x)
```

```
[Out] int(x^3*(x^3 - 1)^(2/3), x)
```

sympy [C] time = 1.03, size = 34, normalized size = 0.33

$$\frac{x^4e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**3-1)**(2/3),x)
```

```
[Out] x**4*exp(2*I*pi/3)*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), x**3)/(3*gamma(7/3
))
```

3.1264 $\int x^4 \sqrt[3]{1+x^3}\, dx$

Optimal. Leaf size=102

$$\frac{1}{27}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{9\sqrt{3}}-\frac{1}{54}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)+\frac{1}{18}\sqrt[3]{x^3+1}\left(3x^5+x^2\right)$$

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {279, 321, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{27}\log\left(1-\frac{x}{\sqrt[3]{x^3+1}}\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{1}{6}\sqrt[3]{x^3+1}\,x^5+\frac{1}{18}\sqrt[3]{x^3+1}\,x^2-\frac{1}{54}\log\left(\frac{x}{\sqrt[3]{x^3+1}}+\frac{x^2}{\left(x^3+1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 + x^3)^(1/3),x]
```

```
[Out] (x^2*(1 + x^3)^(1/3))/18 + (x^5*(1 + x^3)^(1/3))/6 + ArcTan[(1 + (2*x)/(1 +
 x^3)^(1/3))/Sqrt[3]]/(9*Sqrt[3]) + Log[1 - x/(1 + x^3)^(1/3)]/27 - Log[1 +
 x^2/(1 + x^3)^(2/3) + x/(1 + x^3)^(1/3)]/54
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int x^4\sqrt[3]{1+x^3}\,dx &= \frac{1}{6}x^5\sqrt[3]{1+x^3}+\frac{1}{6}\int\frac{x^4}{\left(1+x^3\right)^{2/3}}\,dx\\
&= \frac{1}{18}x^2\sqrt[3]{1+x^3}+\frac{1}{6}x^5\sqrt[3]{1+x^3}-\frac{1}{9}\int\frac{x}{\left(1+x^3\right)^{2/3}}\,dx\\
&= \frac{1}{18}x^2\sqrt[3]{1+x^3}+\frac{1}{6}x^5\sqrt[3]{1+x^3}-\frac{1}{9}\,\mathrm{Subst}\left(\int\frac{x}{1-x^3}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right)\\
&= \frac{1}{18}x^2\sqrt[3]{1+x^3}+\frac{1}{6}x^5\sqrt[3]{1+x^3}-\frac{1}{27}\,\mathrm{Subst}\left(\int\frac{1}{1-x}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right)+\frac{1}{27}\,\mathrm{Subst}\left(\int\frac{1-x}{1+x+x
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{18}x^2\sqrt[3]{1+x^3}+\frac{1}{6}x^5\sqrt[3]{1+x^3}+\frac{1}{27}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\right)-\frac{1}{54}\,\mathrm{Subst}\left(\int\frac{1+2x}{1+x+x^2}\,dx,x,\frac{x}{\sqrt[3]{1+}}\right.\\
&= \frac{1}{18}x^2\sqrt[3]{1+x^3}+\frac{1}{6}x^5\sqrt[3]{1+x^3}+\frac{1}{27}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\right)-\frac{1}{54}\log\left(1+\frac{x^2}{\left(1+x^3\right)^{2/3}}+\frac{x}{\sqrt[3]{1+x^3}}\right)\\
&= \frac{1}{18}x^2\sqrt[3]{1+x^3}+\frac{1}{6}x^5\sqrt[3]{1+x^3}+\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{1}{27}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\right)-\frac{1}{54}\log\left(1+\frac{}{(1+}\right.
\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.33

$$\frac{1}{6}x^2\left(\left(x^3+1\right)^{4/3}-{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 + x^3)^(1/3),x]
```

[Out] (x^2*((1 + x^3)^(4/3) - Hypergeometric2F1[-1/3, 2/3, 5/3, -x^3]))/6

IntegrateAlgebraic [A] time = 0.20, size = 102, normalized size = 1.00

$$\frac{1}{27}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{9\sqrt{3}}-\frac{1}{54}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)+\frac{1}{18}\sqrt[3]{x^3+1}\left(3x^5+x^2\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4*(1 + x^3)^(1/3),x]

[Out] ((1 + x^3)^(1/3)*(x^2 + 3*x^5))/18 + ArcTan[(Sqrt[3]*x)/(x + 2*(1 + x^3)^(1/3))]/(9*Sqrt[3]) + Log[-x + (1 + x^3)^(1/3)]/27 - Log[x^2 + x*(1 + x^3)^(1/3) + (1 + x^3)^(2/3)]/54

fricas [A] time = 0.42, size = 94, normalized size = 0.92

$$-\frac{1}{27}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{18}\left(3x^5+x^2\right)\left(x^3+1\right)^{\frac{1}{3}}+\frac{1}{27}\log\left(-\frac{x-\left(x^3+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{54}\log\left(\frac{x^2+\left(x^3+1\right)^{\frac{1}{3}}x+\left(x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(x^3+1)^(1/3),x, algorithm="fricas")

[Out] -1/27*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 + 1)^(1/3))/x) + 1/18*(3*x^5 + x^2)*(x^3 + 1)^(1/3) + 1/27*log(-(x - (x^3 + 1)^(1/3))/x) - 1/54*log((x^2 + (x^3 + 1)^(1/3)*x + (x^3 + 1)^(2/3))/x^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^3+1\right)^{\frac{1}{3}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(x^3+1)^(1/3),x, algorithm="giac")

[Out] integrate((x^3 + 1)^(1/3)*x^4, x)

maple [C] time = 0.25, size = 37, normalized size = 0.36

$$\frac{x^2\left(3x^3+1\right)\left(x^3+1\right)^{\frac{1}{3}}}{18}-\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^3\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(x^3+1)^(1/3),x)

[Out] 1/18*x^2*(3*x^3+1)*(x^3+1)^(1/3)-1/18*x^2*hypergeom([2/3,2/3],[5/3],-x^3)

maxima [A] time = 0.42, size = 121, normalized size = 1.19

$$-\frac{1}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\frac{2\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+1\right)^{\frac{4}{3}}}{x^4}}{18\left(\frac{2\left(x^3+1\right)}{x^3}-\frac{\left(x^3+1\right)^2}{x^6}-1\right)}-\frac{1}{54}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)+\frac{1}{27}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(x^3+1)^(1/3),x, algorithm="maxima")

[Out] -1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3)/x + 1)) - 1/18*(2*(x^3 + 1)^(1/3)/x + (x^3 + 1)^(4/3)/x^4)/(2*(x^3 + 1)/x^3 - (x^3 + 1)^2/x^6 - 1)

```
- 1/54*log((x^3 + 1)^(1/3)/x + (x^3 + 1)^(2/3)/x^2 + 1) + 1/27*log((x^3 +
1)^(1/3)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(x^3+1\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^3 + 1)^(1/3),x)
```

```
[Out] int(x^4*(x^3 + 1)^(1/3), x)
```

sympy [C] time = 0.97, size = 31, normalized size = 0.30

$$\frac{x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**3+1)**(1/3),x)
```

```
[Out] x**5*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), x**3*exp_polar(I*pi))/(3*gamma(8
/3))
```

3.1265 $\int x^3\left(1+x^3\right)^{2/3}\,dx$

Optimal. Leaf size=102

$$\frac{1}{27}\log\left(\sqrt[3]{x^3+1}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{9\sqrt{3}}+\frac{1}{18}\left(x^3+1\right)^{2/3}\left(3x^4+2x\right)-\frac{1}{54}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 0.77, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {279, 321, 239}

$$\frac{1}{9}\left(x^3+1\right)^{2/3}x+\frac{1}{18}\log\left(\sqrt[3]{x^3+1}-x\right)-\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{1}{6}\left(x^3+1\right)^{2/3}x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x^3)^(2/3),x]
```

```
[Out] (x*(1 + x^3)^(2/3))/9 + (x^4*(1 + x^3)^(2/3))/6 - ArcTan[(1 + (2*x)/(1 + x^
3)^(1/3))/Sqrt[3]]/(9*Sqrt[3]) + Log[-x + (1 + x^3)^(1/3)]/18
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^3\left(1+x^3\right)^{2/3}\,dx&=\frac{1}{6}x^4\left(1+x^3\right)^{2/3}+\frac{1}{3}\int\frac{x^3}{\sqrt[3]{1+x^3}}\,dx\\&=\frac{1}{9}x\left(1+x^3\right)^{2/3}+\frac{1}{6}x^4\left(1+x^3\right)^{2/3}-\frac{1}{9}\int\frac{1}{\sqrt[3]{1+x^3}}\,dx\\&=\frac{1}{9}x\left(1+x^3\right)^{2/3}+\frac{1}{6}x^4\left(1+x^3\right)^{2/3}-\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{1}{18}\log\left(-x+\sqrt[3]{1+x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.31

$$\frac{1}{6}x\left(\left(x^3+1\right)^{5/3}-{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};-x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x^3)^(2/3),x]
```

```
[Out] (x*((1 + x^3)^(5/3) - Hypergeometric2F1[-2/3, 1/3, 4/3, -x^3]))/6
```

IntegrateAlgebraic [A] time = 0.22, size = 102, normalized size = 1.00

$$\frac{1}{27}\log\left(\sqrt[3]{x^3+1}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{9\sqrt{3}}+\frac{1}{18}\left(x^3+1\right)^{2/3}\left(3x^4+2x\right)-\frac{1}{54}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(1 + x^3)^(2/3),x]
```

```
[Out] ((1 + x^3)^(2/3)*(2*x + 3*x^4))/18 - ArcTan[(Sqrt[3]*x)/(x + 2*(1 + x^3)^(1
/3))]/(9*Sqrt[3]) + Log[-x + (1 + x^3)^(1/3)]/27 - Log[x^2 + x*(1 + x^3)^(1
/3) + (1 + x^3)^(2/3)]/54
```

fricas [A] time = 0.40, size = 94, normalized size = 0.92

$$\frac{1}{18}\left(3x^4+2x\right)\left(x^3+1\right)^{\frac{2}{3}}+\frac{1}{27}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{27}\log\left(-\frac{x-\left(x^3+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{54}\log\left(\frac{x^2+\left(x^3+1\right)^{\frac{1}{3}}x+\left(x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/18*(3*x^4 + 2*x)*(x^3 + 1)^(2/3) + 1/27*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2
*sqrt(3)*(x^3 + 1)^(1/3))/x) + 1/27*log(-(x - (x^3 + 1)^(1/3))/x) - 1/54*lo
g((x^2 + (x^3 + 1)^(1/3)*x + (x^3 + 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^3+1\right)^{\frac{2}{3}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^3+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)^(2/3)*x^3, x)
```

maple [C] time = 0.25, size = 33, normalized size = 0.32

$$\frac{x\left(3x^3+2\right)\left(x^3+1\right)^{\frac{2}{3}}}{18}-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x^3\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^3+1)^(2/3),x)
```

```
[Out] 1/18*x*(3*x^3+2)*(x^3+1)^(2/3)-1/9*x*hypergeom([1/3,1/3],[4/3],-x^3)
```

maxima [A] time = 0.41, size = 121, normalized size = 1.19

$$\frac{1}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\frac{\left(x^3+1\right)^{\frac{2}{3}}}{x^2}+\frac{2\left(x^3+1\right)^{\frac{5}{3}}}{x^5}}{18\left(\frac{2\left(x^3+1\right)}{x^3}-\frac{\left(x^3+1\right)^2}{x^6}-1\right)}-\frac{1}{54}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)+\frac{1}{27}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^3+1)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3)/x + 1)) - 1/18*((x^3 + 1
)^(2/3)/x^2 + 2*(x^3 + 1)^(5/3)/x^5)/(2*(x^3 + 1)/x^3 - (x^3 + 1)^2/x^6 - 1
) - 1/54*log((x^3 + 1)^(1/3)/x + (x^3 + 1)^(2/3)/x^2 + 1) + 1/27*log((x^3 +
 1)^(1/3)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(x^3+1\right)^{2/3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^3 + 1)^(2/3),x)
```

```
[Out] int(x^3*(x^3 + 1)^(2/3), x)
```

sympy [C] time = 1.00, size = 31, normalized size = 0.30

$$\frac{x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**3+1)**(2/3),x)
```

```
[Out] x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), x**3*exp_polar(I*pi))/(3*gamma(7
/3))
```

3.1266 $\int \frac{\left(-2+x^3\right)^{2/3}\left(4+x^3\right)}{x^6\left(-1+x^3\right)}\,dx$

Optimal. Leaf size=102

$$\frac{5}{3}\log\left(\sqrt[3]{x^3-2}+x\right)+\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-2}-x}\right)}{\sqrt{3}}+\frac{\left(x^3-2\right)^{2/3}\left(21x^3+8\right)}{10x^5}-\frac{5}{6}\log\left(-\sqrt[3]{x^3-2}\,x+\left(x^3-2\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.14, antiderivative size = 111, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {580, 583, 12, 377, 200, 31, 634, 618, 204, 628}

$$\frac{5}{3}\log\left(\frac{x}{\sqrt[3]{x^3-2}}+1\right)-\frac{5\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{x^3-2}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{4\left(x^3-2\right)^{2/3}}{5x^5}+\frac{21\left(x^3-2\right)^{2/3}}{10x^2}-\frac{5}{6}\log\left(-\frac{x}{\sqrt[3]{x^3-2}}+\frac{x^2}{\left(x^3-2\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-2 + x^3)^(2/3)*(4 + x^3))/(x^6*(-1 + x^3)),x]
```

```
[Out] (4*(-2 + x^3)^(2/3))/(5*x^5) + (21*(-2 + x^3)^(2/3))/(10*x^2) - (5*ArcTan[(
1 - (2*x)/(-2 + x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - (5*Log[1 + x^2/(-2 + x^3)^(
2/3) - x/(-2 + x^3)^(1/3)])/6 + (5*Log[1 + x/(-2 + x^3)^(1/3)])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
```

```
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(-2+x^3\right)^{2/3}\left(4+x^3\right)}{x^6\left(-1+x^3\right)}\,dx = \frac{4\left(-2+x^3\right)^{2/3}}{5x^5} - \frac{1}{5}\int \frac{42-17x^3}{x^3\sqrt[3]{-2+x^3}\left(-1+x^3\right)}\,dx$$

$$= \frac{4\left(-2+x^3\right)^{2/3}}{5x^5} + \frac{21\left(-2+x^3\right)^{2/3}}{10x^2} + \frac{1}{20}\int -\frac{100}{\sqrt[3]{-2+x^3}\left(-1+x^3\right)}\,dx$$

$$= \frac{4\left(-2+x^3\right)^{2/3}}{5x^5} + \frac{21\left(-2+x^3\right)^{2/3}}{10x^2} - 5\int \frac{1}{\sqrt[3]{-2+x^3}\left(-1+x^3\right)}\,dx$$

$$= \frac{4\left(-2+x^3\right)^{2/3}}{5x^5} + \frac{21\left(-2+x^3\right)^{2/3}}{10x^2} - 5\,\text{Subst}\left(\int \frac{1}{-1-x^3}\,dx, x, \frac{x}{\sqrt[3]{-2+x^3}}\right)$$

$$= \frac{4\left(-2+x^3\right)^{2/3}}{5x^5} + \frac{21\left(-2+x^3\right)^{2/3}}{10x^2} - \frac{5}{3}\text{Subst}\left(\int \frac{1}{-1-x}\,dx, x, \frac{x}{\sqrt[3]{-2+x^3}}\right) - \frac{5}{3}\text{Subs}$$

$$= \frac{4\left(-2+x^3\right)^{2/3}}{5x^5} + \frac{21\left(-2+x^3\right)^{2/3}}{10x^2} + \frac{5}{3}\log\left(1+\frac{x}{\sqrt[3]{-2+x^3}}\right) - \frac{5}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x}\right.$$

$$= \frac{4\left(-2+x^3\right)^{2/3}}{5x^5} + \frac{21\left(-2+x^3\right)^{2/3}}{10x^2} - \frac{5}{6}\log\left(1+\frac{x^2}{\left(-2+x^3\right)^{2/3}} - \frac{x}{\sqrt[3]{-2+x^3}}\right) + \frac{5}{3}\log($$

$$= \frac{4\left(-2+x^3\right)^{2/3}}{5x^5} + \frac{21\left(-2+x^3\right)^{2/3}}{10x^2} - \frac{5\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-2+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{5}{6}\log\left(1+\frac{x^2}{\left(-2+x^3\right)^{2/3}} -\right.$$

Mathematica [A] time = 0.19, size = 111, normalized size = 1.09

$$\frac{\left(x^3-2\right)^{2/3}\left(21x^3+8\right)}{10x^5} + \frac{5}{6}\left(2\log\left(\frac{x}{\sqrt[3]{1-2x^3}}+1\right) + 2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{1-2x^3}}-1}{\sqrt{3}}\right) - \log\left(-\frac{x}{\sqrt[3]{1-2x^3}} + \frac{x^2}{\left(1-2x^3\right)^{2/3}} + 1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-2 + x^3)^(2/3)*(4 + x^3))/(x^6*(-1 + x^3)),x]
```

```
[Out] ((-2 + x^3)^(2/3)*(8 + 21*x^3))/(10*x^5) + (5*(2*Sqrt[3]*ArcTan[(-1 + (2*x)
/(1 - 2*x^3)^(1/3))/Sqrt[3]] - Log[1 + x^2/(1 - 2*x^3)^(2/3) - x/(1 - 2*x^3
)^(1/3)] + 2*Log[1 + x/(1 - 2*x^3)^(1/3)]))/6
```

IntegrateAlgebraic [A] time = 0.20, size = 102, normalized size = 1.00

$$\frac{5}{3}\log\left(\sqrt[3]{x^3-2}+x\right) + \frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-2}-x}\right)}{\sqrt{3}} + \frac{\left(x^3-2\right)^{2/3}\left(21x^3+8\right)}{10x^5} - \frac{5}{6}\log\left(-\sqrt[3]{x^3-2}\,x + \left(x^3-2\right)^{2/3} + x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^3)^(2/3)*(4 + x^3))/(x^6*(-1 + x^3)),x]
```

```
[Out] ((-2 + x^3)^(2/3)*(8 + 21*x^3))/(10*x^5) + (5*ArcTan[(Sqrt[3]*x)/(-x + 2*(-
2 + x^3)^(1/3))])/Sqrt[3] + (5*Log[x + (-2 + x^3)^(1/3)])/3 - (5*Log[x^2 -
x*(-2 + x^3)^(1/3) + (-2 + x^3)^(2/3)])/6
```

fricas [A] time = 0.99, size = 122, normalized size = 1.20

$$-\frac{50\sqrt{3}x^5\arctan\left(\frac{4\sqrt{3}\left(x^3-2\right)^{\frac{1}{3}}x^2+2\sqrt{3}\left(x^3-2\right)^{\frac{2}{3}}x+\sqrt{3}\left(x^3-2\right)}{7x^3+2}\right) - 25x^5\log\left(\frac{2x^3+3\left(x^3-2\right)^{\frac{1}{3}}x^2+3\left(x^3-2\right)^{\frac{2}{3}}x-2}{x^3-1}\right) - 3\left(21x^3+8\right)\left(x^3-2\right)^{\frac{2}{3}}}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)^(2/3)*(x^3+4)/x^6/(x^3-1),x, algorithm="fricas")
```

```
[Out] -1/30*(50*sqrt(3)*x^5*arctan((4*sqrt(3)*(x^3 - 2)^(1/3)*x^2 + 2*sqrt(3)*(x^
3 - 2)^(2/3)*x + sqrt(3)*(x^3 - 2))/(7*x^3 + 2)) - 25*x^5*log((2*x^3 + 3*(x
^3 - 2)^(1/3)*x^2 + 3*(x^3 - 2)^(2/3)*x - 2)/(x^3 - 1)) - 3*(21*x^3 + 8)*(x
^3 - 2)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+4\right)\left(x^3-2\right)^{\frac{2}{3}}}{\left(x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)^(2/3)*(x^3+4)/x^6/(x^3-1),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 4)*(x^3 - 2)^(2/3)/((x^3 - 1)*x^6), x)
```

maple [C] time = 1.26, size = 268, normalized size = 2.63

$$\frac{21x^6-34x^3-16}{10x^5\left(x^3-2\right)^{\frac{1}{3}}}+\frac{5\ln\left(\frac{3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3-2\right)^{\frac{2}{3}}x+6\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3-2\right)^{\frac{1}{3}}x^2+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3-\left(x^3-2\right)^{\frac{2}{3}}x+\left(x^3-2\right)^{\frac{1}{3}}x^2+2}{(-1+x)\left(x^2+x+1\right)}\right)}{3}+5\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\ln\left(-\frac{9\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3-2\right)^{\frac{2}{3}}x-3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3-2\right)^{\frac{1}{3}}x^2+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3+2\left(x^3-2\right)^{\frac{2}{3}}x+\left(x^3-2\right)^{\frac{1}{3}}x^2-6\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)}{(-1+x)\left(x^2+x+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2)^(2/3)*(x^3+4)/x^6/(x^3-1),x)
```

```
[Out] 1/10*(21*x^6-34*x^3-16)/x^5/(x^3-2)^(1/3)+5/3*ln((3*RootOf(9*_Z^2+3*_Z+1)*(
x^3-2)^(2/3)*x+6*RootOf(9*_Z^2+3*_Z+1)*(x^3-2)^(1/3)*x^2+3*RootOf(9*_Z^2+3*
_Z+1)*x^3-(x^3-2)^(2/3)*x+(x^3-2)^(1/3)*x^2+2)/(-1+x)/(x^2+x+1))+5*RootOf(9
*_Z^2+3*_Z+1)*ln(-(9*RootOf(9*_Z^2+3*_Z+1)^2*x^3+3*RootOf(9*_Z^2+3*_Z+1)*(x
^3-2)^(2/3)*x-3*RootOf(9*_Z^2+3*_Z+1)*(x^3-2)^(1/3)*x^2+3*RootOf(9*_Z^2+3*_
Z+1)*x^3+2*(x^3-2)^(2/3)*x+(x^3-2)^(1/3)*x^2-6*RootOf(9*_Z^2+3*_Z+1))/(-1+x
)/(x^2+x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+4\right)\left(x^3-2\right)^{\frac{2}{3}}}{\left(x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)^(2/3)*(x^3+4)/x^6/(x^3-1),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 4)*(x^3 - 2)^(2/3)/((x^3 - 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(x^3-2\right)}^{2/3}\,\left(x^3+4\right)}{x^6\,\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 2)^(2/3)*(x^3 + 4))/(x^6*(x^3 - 1)),x)
```

```
[Out] int(((x^3 - 2)^(2/3)*(x^3 + 4))/(x^6*(x^3 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-2\right)^{\frac{2}{3}}\left(x^3+4\right)}{x^6\left(x-1\right)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2)**(2/3)*(x**3+4)/x**6/(x**3-1),x)
```

```
[Out] Integral((x**3 - 2)**(2/3)*(x**3 + 4)/(x**6*(x - 1)*(x**2 + x + 1)), x)
```

3.1267 $\int \frac{3b+ax^2}{(b+ax^2+x^3)\sqrt[4]{bx+ax^3}}\,dx$

Optimal. Leaf size=102

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^3+bx}}{\sqrt{ax^3+bx}+x^2}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3+bx}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3+bx}}\right)$$

Rubi [F] time = 2.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3b+ax^2}{(b+ax^2+x^3)\sqrt[4]{bx+ax^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3*b + a*x^2)/((b + a*x^2 + x^3)*(b*x + a*x^3)^(1/4)),x]
```

```
[Out] (12*b*x^(1/4)*(b + a*x^2)^(1/4)*Defer[Subst][Defer[Int][x^2/((b + a*x^8)^(1
/4)*(b + a*x^8 + x^12)), x], x, x^(1/4)])/(b*x + a*x^3)^(1/4) + (4*a*x^(1/4
)*(b + a*x^2)^(1/4)*Defer[Subst][Defer[Int][x^10/((b + a*x^8)^(1/4)*(b + a*
x^8 + x^12)), x], x, x^(1/4)])/(b*x + a*x^3)^(1/4)
```

Rubi steps

$$\begin{aligned}\int \frac{3b+ax^2}{(b+ax^2+x^3)\sqrt[4]{bx+ax^3}}\,dx &= \frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^2}\right)\int \frac{3b+ax^2}{\sqrt[4]{x}\sqrt[4]{b+ax^2}\,(b+ax^2+x^3)}\,dx}{\sqrt[4]{bx+ax^3}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{x^2(3b+ax^8)}{\sqrt[4]{b+ax^8}\,(b+ax^8+x^{12})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^3}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \left(\frac{3bx^2}{\sqrt[4]{b+ax^8}\,(b+ax^8+x^{12})}+\frac{ax^{10}}{\sqrt[4]{b+ax^8}\,(b+ax^8+x^{12})}\right)dx,x,\sqrt[4]{\right.}}{\sqrt[4]{bx+ax^3}}\\
&= \frac{\left(4a\sqrt[4]{x}\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{x^{10}}{\sqrt[4]{b+ax^8}\,(b+ax^8+x^{12})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^3}}+\frac{\left(12b\sqrt[4]{x}\sqrt[4]{b+a\right.}}{}\end{aligned}$$

Mathematica [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{3b+ax^2}{(b+ax^2+x^3)\sqrt[4]{bx+ax^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3*b + a*x^2)/((b + a*x^2 + x^3)*(b*x + a*x^3)^(1/4)),x]
```

```
[Out] Integrate[(3*b + a*x^2)/((b + a*x^2 + x^3)*(b*x + a*x^3)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.52, size = 102, normalized size = 1.00

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^3+bx}}{\sqrt{ax^3+bx}+x^2}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3+bx}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3+bx}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*b + a*x^2)/((b + a*x^2 + x^3)*(b*x + a*x^3)^(1/4)),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(-(x^2/Sqrt[2]) + Sqrt[b*x + a*x^3]/Sqrt[2])/(x*(b*x + a*x
^3)^(1/4))]) + Sqrt[2]*ArcTanh[(Sqrt[2]*x*(b*x + a*x^3)^(1/4))/(x^2 + Sqrt[
b*x + a*x^3])]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+3*b)/(a*x^2+x^3+b)/(a*x^3+b*x)^(1/4),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + 3b}{\left(ax^3 + bx\right)^{\frac{1}{4}}\left(ax^2 + x^3 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+3*b)/(a*x^2+x^3+b)/(a*x^3+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^2 + 3*b)/((a*x^3 + b*x)^(1/4)*(a*x^2 + x^3 + b)), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2 + 3b}{\left(a\,x^2 + x^3 + b\right)\left(a\,x^3 + bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+3*b)/(a*x^2+x^3+b)/(a*x^3+b*x)^(1/4),x)
```

```
[Out] int((a*x^2+3*b)/(a*x^2+x^3+b)/(a*x^3+b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + 3b}{\left(ax^3 + bx\right)^{\frac{1}{4}}\left(ax^2 + x^3 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+3*b)/(a*x^2+x^3+b)/(a*x^3+b*x)^(1/4),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^2 + 3*b)/((a*x^3 + b*x)^(1/4)*(a*x^2 + x^3 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x^2 + 3\,b}{{\left(a\,x^3 + b\,x\right)}^{1/4}\,\left(x^3 + a\,x^2 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*b + a*x^2)/((b*x + a*x^3)^(1/4)*(b + a*x^2 + x^3)),x)
```

```
[Out] int((3*b + a*x^2)/((b*x + a*x^3)^(1/4)*(b + a*x^2 + x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+3*b)/(a*x**2+x**3+b)/(a*x**3+b*x)**(1/4),x)
```

```
[Out] Timed out
```

3.1268 $\int \frac{(-3+x^4)\sqrt[3]{1+x^4}}{x^{13}}\,dx$

Optimal. Leaf size=102

$$-\frac{2}{27}\log\left(\sqrt[3]{x^4+1}-1\right)+\frac{1}{27}\log\left(\left(x^4+1\right)^{2/3}+\sqrt[3]{x^4+1}+1\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\sqrt[3]{x^4+1}\left(-4x^8-3x^4+9\right)}{36x^{12}}$$

Rubi [A] time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {446, 78, 47, 51, 57, 618, 204, 31}

$$-\frac{\sqrt[3]{x^4+1}}{9x^4}-\frac{1}{9}\log\left(1-\sqrt[3]{x^4+1}\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}+1}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\left(x^4+1\right)^{4/3}}{4x^{12}}-\frac{\sqrt[3]{x^4+1}}{3x^8}+\frac{4\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + x^4)*(1 + x^4)^(1/3))/x^13,x]
```

```
[Out] -1/3*(1 + x^4)^(1/3)/x^8 - (1 + x^4)^(1/3)/(9*x^4) + (1 + x^4)^(4/3)/(4*x^1
2) + (2*ArcTan[(1 + 2*(1 + x^4)^(1/3))/Sqrt[3]])/(9*Sqrt[3]) + (4*Log[x])/2
7 - Log[1 - (1 + x^4)^(1/3)]/9
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-3+x^4\right)\sqrt[3]{1+x^4}}{x^{13}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{(-3+x)\sqrt[3]{1+x}}{x^4}\,dx,x,x^4\right)\\
&= \frac{\left(1+x^4\right)^{4/3}}{4x^{12}}+\frac{2}{3}\operatorname{Subst}\left(\int \frac{\sqrt[3]{1+x}}{x^3}\,dx,x,x^4\right)\\
&= -\frac{\sqrt[3]{1+x^4}}{3x^8}+\frac{\left(1+x^4\right)^{4/3}}{4x^{12}}+\frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{x^2(1+x)^{2/3}}\,dx,x,x^4\right)\\
&= -\frac{\sqrt[3]{1+x^4}}{3x^8}-\frac{\sqrt[3]{1+x^4}}{9x^4}+\frac{\left(1+x^4\right)^{4/3}}{4x^{12}}-\frac{2}{27}\operatorname{Subst}\left(\int \frac{1}{x(1+x)^{2/3}}\,dx,x,x^4\right)\\
&= -\frac{\sqrt[3]{1+x^4}}{3x^8}-\frac{\sqrt[3]{1+x^4}}{9x^4}+\frac{\left(1+x^4\right)^{4/3}}{4x^{12}}+\frac{4\log(x)}{27}+\frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1+x^{\cdot}}\right.\\
&= -\frac{\sqrt[3]{1+x^4}}{3x^8}-\frac{\sqrt[3]{1+x^4}}{9x^4}+\frac{\left(1+x^4\right)^{4/3}}{4x^{12}}+\frac{4\log(x)}{27}-\frac{1}{9}\log\left(1-\sqrt[3]{1+x^4}\right)-\frac{2}{9}\operatorname{Subst}\\
&= -\frac{\sqrt[3]{1+x^4}}{3x^8}-\frac{\sqrt[3]{1+x^4}}{9x^4}+\frac{\left(1+x^4\right)^{4/3}}{4x^{12}}+\frac{2\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^4}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{4\log(x)}{27}-\frac{1}{9}\log\left(1\right.
\end{aligned}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.35

$$\frac{\left(x^4+1\right)^{4/3}\left(1-2x^{12}\,{}_2F_1\left(\frac{4}{3},3;\frac{7}{3};x^4+1\right)\right)}{4x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + x^4)*(1 + x^4)^(1/3))/x^13,x]
```

```
[Out] ((1 + x^4)^(4/3)*(1 - 2*x^12*Hypergeometric2F1[4/3, 3, 7/3, 1 + x^4]))/(4*x
^12)
```

IntegrateAlgebraic [A] time = 0.17, size = 102, normalized size = 1.00

$$-\frac{2}{27}\log\left(\sqrt[3]{x^4+1}-1\right)+\frac{1}{27}\log\left(\left(x^4+1\right)^{2/3}+\sqrt[3]{x^4+1}+1\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\sqrt[3]{x^4+1}\left(-4x^8-3x^4+9\right)}{36x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 + x^4)^(1/3))/x^13,x]
```

```
[Out] ((1 + x^4)^(1/3)*(9 - 3*x^4 - 4*x^8))/(36*x^12) + (2*ArcTan[1/Sqrt[3] + (2*
(1 + x^4)^(1/3))/Sqrt[3]])/(9*Sqrt[3]) - (2*Log[-1 + (1 + x^4)^(1/3)])/27 +
 Log[1 + (1 + x^4)^(1/3) + (1 + x^4)^(2/3)]/27
```

fricas [A] time = 0.45, size = 91, normalized size = 0.89

$$\frac{8\sqrt{3}x^{12}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^4+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+4x^{12}\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)-8x^{12}\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)-3\left(4x^8+3x^4-9\right)\left(x^4+1\right)^{\frac{1}{3}}}{108x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(1/3)/x^13,x, algorithm="fricas")
```

```
[Out] 1/108*(8*sqrt(3)*x^12*arctan(2/3*sqrt(3)*(x^4 + 1)^(1/3) + 1/3*sqrt(3)) + 4
*x^12*log((x^4 + 1)^(2/3) + (x^4 + 1)^(1/3) + 1) - 8*x^12*log((x^4 + 1)^(1/
3) - 1) - 3*(4*x^8 + 3*x^4 - 9)*(x^4 + 1)^(1/3))/x^12
```

giac [A] time = 0.27, size = 87, normalized size = 0.85

$$\frac{2}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{4\left(x^4+1\right)^{\frac{7}{3}}-5\left(x^4+1\right)^{\frac{4}{3}}-8\left(x^4+1\right)^{\frac{1}{3}}}{36x^{12}}+\frac{1}{27}\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)-\frac{2}{27}\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(1/3)/x^13,x, algorithm="giac")
```

```
[Out] 2/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 + 1)^(1/3) + 1)) - 1/36*(4*(x^4 + 1
)^(7/3) - 5*(x^4 + 1)^(4/3) - 8*(x^4 + 1)^(1/3))/x^12 + 1/27*log((x^4 + 1)^
(2/3) + (x^4 + 1)^(1/3) + 1) - 2/27*log((x^4 + 1)^(1/3) - 1)
```

maple [C] time = 0.25, size = 76, normalized size = 0.75

$$-\frac{4x^{12}+7x^8-6x^4-9}{36x^{12}\left(x^4+1\right)^{\frac{2}{3}}}-\frac{2\left(-\frac{2\Gamma\left(\frac{2}{3}\right)x^4\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],-x^4\right)}{3}+\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+4\ln(x)\right)\Gamma\left(\frac{2}{3}\right)\right)}{27\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4+1)^(1/3)/x^13,x)
```

```
[Out] -1/36*(4*x^12+7*x^8-6*x^4-9)/x^12/(x^4+1)^(2/3)-2/27/GAMMA(2/3)*(-2/3*GAMMA
(2/3)*x^4*hypergeom([1,1,5/3],[2,2],-x^4)+(1/6*Pi*3^(1/2)-3/2*ln(3)+4*ln(x)
)*GAMMA(2/3))
```

maxima [A] time = 0.45, size = 146, normalized size = 1.43

$$\frac{2}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{5\left(x^4+1\right)^{\frac{7}{3}}-13\left(x^4+1\right)^{\frac{4}{3}}-10\left(x^4+1\right)^{\frac{1}{3}}}{72\left(3x^4+\left(x^4+1\right)^3-3\left(x^4+1\right)^2+2\right)}+\frac{\left(x^4+1\right)^{\frac{4}{3}}+2\left(x^4+1\right)^{\frac{1}{3}}}{24\left(2x^4-\left(x^4+1\right)^2+1\right)}+\frac{1}{27}\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)-\frac{2}{27}\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(1/3)/x^13,x, algorithm="maxima")
```

```
[Out] 2/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 + 1)^(1/3) + 1)) - 1/72*(5*(x^4 + 1
)^(7/3) - 13*(x^4 + 1)^(4/3) - 10*(x^4 + 1)^(1/3))/(3*x^4 + (x^4 + 1)^3 - 3
*(x^4 + 1)^2 + 2) + 1/24*((x^4 + 1)^(4/3) + 2*(x^4 + 1)^(1/3))/(2*x^4 - (x^
4 + 1)^2 + 1) + 1/27*log((x^4 + 1)^(2/3) + (x^4 + 1)^(1/3) + 1) - 2/27*log(
(x^4 + 1)^(1/3) - 1)
```

mupad [B] time = 1.28, size = 231, normalized size = 2.26

$$\frac{\frac{5\left(x^4+1\right)^{1/3}}{36}+\frac{13\left(x^4+1\right)^{4/3}}{72}-\frac{5\left(x^4+1\right)^{7/3}}{72}}{\left(x^4+1\right)^3-3\left(x^4+1\right)^2+3x^4+2}-\frac{5\ln\left(\frac{25\left(x^4+1\right)^{1/3}}{1296}-\frac{25}{1296}\right)}{108}-\frac{\ln\left(\frac{\left(x^4+1\right)^{1/3}}{144}-\frac{1}{144}\right)}{36}+\frac{\frac{\left(x^4+1\right)^{1/3}}{12}+\frac{\left(x^4+1\right)^{4/3}}{24}}{2x^4-\left(x^4+1\right)^2+1}-\ln\left(\frac{\left(x^4+1\right)^{1/3}}{4}+\frac{1}{8}-\frac{\sqrt{3}\,1\mathrm{i}}{8}\right)\left(-\frac{1}{72}+\frac{\sqrt{3}\,1\mathrm{i}}{72}\right)+\ln\left(\frac{\left(x^4+1\right)^{1/3}}{4}+\frac{1}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8}\right)\left(\frac{1}{72}+\frac{\sqrt{3}\,1\mathrm{i}}{72}\right)-\ln\left(\frac{5\left(x^4+1\right)^{1/3}}{12}+\frac{5}{24}-\frac{\sqrt{3}\,5\mathrm{i}}{24}\right)\left(-\frac{5}{216}+\frac{\sqrt{3}\,5\mathrm{i}}{216}\right)+\ln\left(\frac{5\left(x^4+1\right)^{1/3}}{12}+\frac{5}{24}+\frac{\sqrt{3}\,5\mathrm{i}}{24}\right)\left(\frac{5}{216}+\frac{\sqrt{3}\,5\mathrm{i}}{216}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/3)*(x^4 - 3))/x^13,x)
```

```
[Out] ((5*(x^4 + 1)^(1/3))/36 + (13*(x^4 + 1)^(4/3))/72 - (5*(x^4 + 1)^(7/3))/72)
/((x^4 + 1)^3 - 3*(x^4 + 1)^2 + 3*x^4 + 2) - (5*log((25*(x^4 + 1)^(1/3))/12
96 - 25/1296))/108 - log((x^4 + 1)^(1/3)/144 - 1/144)/36 + ((x^4 + 1)^(1/3)
/12 + (x^4 + 1)^(4/3)/24)/(2*x^4 - (x^4 + 1)^2 + 1) - log((x^4 + 1)^(1/3)/4
 - (3^(1/2)*1i)/8 + 1/8)*((3^(1/2)*1i)/72 - 1/72) + log((3^(1/2)*1i)/8 + (x
^4 + 1)^(1/3)/4 + 1/8)*((3^(1/2)*1i)/72 + 1/72) - log((5*(x^4 + 1)^(1/3))/1
2 - (3^(1/2)*5i)/24 + 5/24)*((3^(1/2)*5i)/216 - 5/216) + log((3^(1/2)*5i)/2
4 + (5*(x^4 + 1)^(1/3))/12 + 5/24)*((3^(1/2)*5i)/216 + 5/216)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4+1)**(1/3)/x**13,x)
```

```
[Out] Timed out
```

3.1269 $\int \frac{-2b+ax^2}{\sqrt[4]{-b+ax^2}\left(-b+ax^2+x^4\right)}\,dx$

Optimal. Leaf size=102

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}+x^2}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}}$$

Rubi [C] time = 11.76, antiderivative size = 2432, normalized size of antiderivative = 23.84, number of steps used = 18, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1692, 399, 490, 1217, 220, 1707}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(-2*b + a*x^2)/((-b + a*x^2)^(1/4)*(-b + a*x^2 + x^4)),x]
```

```
[Out] -1/2*(Sqrt[b]*Sqrt[-a - Sqrt[a^2 + 4*b]]*Sqrt[(a*x^2)/b]*ArcTan[(Sqrt[a]*Sq
rt[-a - Sqrt[a^2 + 4*b]]*(-b + a*x^2)^(1/4))/(2^(1/4)*Sqrt[b]*(-a^2 - 2*b -
 a*Sqrt[a^2 + 4*b])^(1/4)*Sqrt[(a*x^2)/b])])/(2^(1/4)*Sqrt[a]*(-a^2 - 2*b -
 a*Sqrt[a^2 + 4*b])^(1/4)*x) - (Sqrt[b]*Sqrt[a + Sqrt[a^2 + 4*b]]*Sqrt[(a*x
^2)/b]*ArcTan[(Sqrt[a]*Sqrt[a + Sqrt[a^2 + 4*b]]*(-b + a*x^2)^(1/4))/(2^(1/
4)*Sqrt[b]*(-a^2 - 2*b - a*Sqrt[a^2 + 4*b])^(1/4)*Sqrt[(a*x^2)/b])])/(2*2^(
1/4)*Sqrt[a]*(-a^2 - 2*b - a*Sqrt[a^2 + 4*b])^(1/4)*x) - (Sqrt[b]*Sqrt[a -
Sqrt[a^2 + 4*b]]*Sqrt[(a*x^2)/b]*ArcTan[(Sqrt[a]*Sqrt[a - Sqrt[a^2 + 4*b]]*
(-b + a*x^2)^(1/4))/(2^(1/4)*Sqrt[b]*(-a^2 - 2*b + a*Sqrt[a^2 + 4*b])^(1/4)
*Sqrt[(a*x^2)/b])])/(2*2^(1/4)*Sqrt[a]*(-a^2 - 2*b + a*Sqrt[a^2 + 4*b])^(1/
4)*x) - (Sqrt[b]*Sqrt[-a + Sqrt[a^2 + 4*b]]*Sqrt[(a*x^2)/b]*ArcTan[(Sqrt[a]
*Sqrt[-a + Sqrt[a^2 + 4*b]]*(-b + a*x^2)^(1/4))/(2^(1/4)*Sqrt[b]*(-a^2 - 2*
b + a*Sqrt[a^2 + 4*b])^(1/4)*Sqrt[(a*x^2)/b])])/(2*2^(1/4)*Sqrt[a]*(-a^2 -
2*b + a*Sqrt[a^2 + 4*b])^(1/4)*x) - ((a + Sqrt[a^2 + 4*b])*(2*Sqrt[b] - Sqr
t[2]*Sqrt[-a^2 - 2*b - a*Sqrt[a^2 + 4*b]])*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[-b
+ a*x^2])^2]*(Sqrt[b] + Sqrt[-b + a*x^2])*EllipticF[2*ArcTan[(-b + a*x^2)^(
1/4)/b^(1/4)], 1/2])/(4*b^(1/4)*(a^2 + 4*b + a*Sqrt[a^2 + 4*b])*x) - ((a +
Sqrt[a^2 + 4*b])*(2*Sqrt[b] + Sqrt[2]*Sqrt[-a^2 - 2*b - a*Sqrt[a^2 + 4*b]])
*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[-b + a*x^2])^2]*(Sqrt[b] + Sqrt[-b + a*x^2])*
EllipticF[2*ArcTan[(-b + a*x^2)^(1/4)/b^(1/4)], 1/2])/(4*b^(1/4)*(a^2 + 4*b
 + a*Sqrt[a^2 + 4*b])*x) - ((a - Sqrt[a^2 + 4*b])*(2*Sqrt[b] - Sqrt[-2*a^2
- 4*b + 2*a*Sqrt[a^2 + 4*b]])*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[-b + a*x^2])^2]*
(Sqrt[b] + Sqrt[-b + a*x^2])*EllipticF[2*ArcTan[(-b + a*x^2)^(1/4)/b^(1/4)]
, 1/2])/(4*b^(1/4)*(a^2 + 4*b - a*Sqrt[a^2 + 4*b])*x) - ((a - Sqrt[a^2 + 4*
b])*(2*Sqrt[b] + Sqrt[-2*a^2 - 4*b + 2*a*Sqrt[a^2 + 4*b]])*Sqrt[(a*x^2)/(Sq
rt[b] + Sqrt[-b + a*x^2])^2]*(Sqrt[b] + Sqrt[-b + a*x^2])*EllipticF[2*ArcTa
n[(-b + a*x^2)^(1/4)/b^(1/4)], 1/2])/(4*b^(1/4)*(a^2 + 4*b - a*Sqrt[a^2 + 4
*b])*x) + ((a + Sqrt[a^2 + 4*b])*(Sqrt[2]*Sqrt[b] + Sqrt[-a^2 - 2*b - a*Sqr
t[a^2 + 4*b]])^2*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[-b + a*x^2])^2]*(Sqrt[b] + Sq
rt[-b + a*x^2])*EllipticPi[-1/4*(Sqrt[2]*Sqrt[b] - Sqrt[-a^2 - 2*b - a*Sqrt
[a^2 + 4*b]])^2/(Sqrt[2]*Sqrt[b]*Sqrt[-a^2 - 2*b - a*Sqrt[a^2 + 4*b]]), 2*A
rcTan[(-b + a*x^2)^(1/4)/b^(1/4)], 1/2])/(4*Sqrt[2]*b^(1/4)*Sqrt[-a^2 - 2*b
 - a*Sqrt[a^2 + 4*b]]*(a^2 + 4*b + a*Sqrt[a^2 + 4*b])*x) - ((a + Sqrt[a^2 +
 4*b])*(Sqrt[2]*Sqrt[b] - Sqrt[-a^2 - 2*b - a*Sqrt[a^2 + 4*b]])^2*Sqrt[(a*x
^2)/(Sqrt[b] + Sqrt[-b + a*x^2])^2]*(Sqrt[b] + Sqrt[-b + a*x^2])*EllipticPi
[(Sqrt[2]*Sqrt[b] + Sqrt[-a^2 - 2*b - a*Sqrt[a^2 + 4*b]])^2/(4*Sqrt[2]*Sqrt
[b]*Sqrt[-a^2 - 2*b - a*Sqrt[a^2 + 4*b]]), 2*ArcTan[(-b + a*x^2)^(1/4)/b^(1
/4)], 1/2])/(4*Sqrt[2]*b^(1/4)*Sqrt[-a^2 - 2*b - a*Sqrt[a^2 + 4*b]]*(a^2 +
4*b + a*Sqrt[a^2 + 4*b])*x) + ((a - Sqrt[a^2 + 4*b])*(Sqrt[2]*Sqrt[b] + Sqr
t[-a^2 - 2*b + a*Sqrt[a^2 + 4*b]])^2*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[-b + a*x^
```

```
2])^2]*(Sqrt[b] + Sqrt[-b + a*x^2])*EllipticPi[-1/4*(Sqrt[2]*Sqrt[b] - Sqrt
[-a^2 - 2*b + a*Sqrt[a^2 + 4*b]])^2/(Sqrt[2]*Sqrt[b]*Sqrt[-a^2 - 2*b + a*Sq
rt[a^2 + 4*b]]), 2*ArcTan[(-b + a*x^2)^(1/4)/b^(1/4)], 1/2])/(4*Sqrt[2]*b^(
1/4)*(a^2 + 4*b - a*Sqrt[a^2 + 4*b])*Sqrt[-a^2 - 2*b + a*Sqrt[a^2 + 4*b]]*x
) - ((a - Sqrt[a^2 + 4*b])*(Sqrt[2]*Sqrt[b] - Sqrt[-a^2 - 2*b + a*Sqrt[a^2
+ 4*b]])^2*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[-b + a*x^2])^2]*(Sqrt[b] + Sqrt[-b
+ a*x^2])*EllipticPi[(Sqrt[2]*Sqrt[b] + Sqrt[-a^2 - 2*b + a*Sqrt[a^2 + 4*b]
])^2/(4*Sqrt[2]*Sqrt[b]*Sqrt[-a^2 - 2*b + a*Sqrt[a^2 + 4*b]]), 2*ArcTan[(-b
 + a*x^2)^(1/4)/b^(1/4)], 1/2])/(4*Sqrt[2]*b^(1/4)*(a^2 + 4*b - a*Sqrt[a^2
+ 4*b])*Sqrt[-a^2 - 2*b + a*Sqrt[a^2 + 4*b]]*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{-2b+ax^2}{\sqrt[4]{-b+ax^2}\left(-b+ax^2+x^4\right)}dx = \int \left(\frac{a-\sqrt{a^2+4b}}{\left(a-\sqrt{a^2+4b}+2x^2\right)\sqrt[4]{-b+ax^2}} + \frac{a+\sqrt{a^2+4b}}{\left(a+\sqrt{a^2+4b}+2x^2\right)\sqrt[4]{-b+ax^2}}\right.$$

$$= \left(a-\sqrt{a^2+4b}\right)\int \frac{1}{\left(a-\sqrt{a^2+4b}+2x^2\right)\sqrt[4]{-b+ax^2}}dx + \left(a+\sqrt{a^2+4b}\right)\int$$

$$= \frac{\left(2\left(a-\sqrt{a^2+4b}\right)\sqrt{\frac{ax^2}{b}}\right)\text{Subst}\left(\int \frac{x^2}{\left(2b+a\left(a-\sqrt{a^2+4b}\right)+2x^4\right)\sqrt{1+\frac{x^4}{b}}}dx, x, \sqrt[4]{-b+\right.}{x}$$

$$= -\frac{\left(\left(a-\sqrt{a^2+4b}\right)\sqrt{\frac{ax^2}{b}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a^2-2b+a\sqrt{a^2+4b}}-\sqrt{2}x^2\right)\sqrt{1+\frac{x^4}{b}}}dx, x, \sqrt[4]{-\right.}{\sqrt{2}x}$$

$$= -\frac{\left(\left(a+\sqrt{a^2+4b}\right)\left(\sqrt{2}\sqrt{b}-\sqrt{-a^2-2b-a\sqrt{a^2+4b}}\right)\sqrt{\frac{ax^2}{b}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.}{\sqrt{2}\left(a^2+4b+a\sqrt{a^2+4b}\right)x}$$

$$= \frac{\sqrt{b}\left(a^3+4ab+\left(a^2+2b\right)\sqrt{a^2+4b}\right)\sqrt{\frac{ax^2}{b}}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{-a-\sqrt{a^2+4b}}\sqrt[4]{-b+ax^2}}{\sqrt[4]{2}\sqrt{b}\sqrt[4]{-a^2-2b-a\sqrt{a^2+4b}}\sqrt{\frac{a}{b}}}\right.}{\sqrt[4]{2}\sqrt{a}\sqrt{-a-\sqrt{a^2+4b}}\sqrt[4]{-a^2-2b-a\sqrt{a^2+4b}}\left(a^2+4b+a\sqrt{a^2+4b}\right)}$$

Mathematica [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{-2b+ax^2}{\sqrt[4]{-b+ax^2}\left(-b+ax^2+x^4\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*b + a*x^2)/((-b + a*x^2)^(1/4)*(-b + a*x^2 + x^4)),x]
```

```
[Out] Integrate[(-2*b + a*x^2)/((-b + a*x^2)^(1/4)*(-b + a*x^2 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.31, size = 102, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}+x^2}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*b + a*x^2)/((-b + a*x^2)^(1/4)*(-b + a*x^2 + x^4)),x
]
```

```
[Out] -(ArcTan[(-(x^2/Sqrt[2]) + Sqrt[-b + a*x^2]/Sqrt[2])/(x*(-b + a*x^2)^(1/4))
]/Sqrt[2]) + ArcTanh[(Sqrt[2]*x*(-b + a*x^2)^(1/4))/(x^2 + Sqrt[-b + a*x^2]
)]/Sqrt[2]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-2*b)/(a*x^2-b)^(1/4)/(x^4+a*x^2-b),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2-2b}{\left(x^4+ax^2-b\right)\left(ax^2-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-2*b)/(a*x^2-b)^(1/4)/(x^4+a*x^2-b),x, algorithm="giac")
[Out] integrate((a*x^2 - 2*b)/((x^4 + a*x^2 - b)*(a*x^2 - b)^(1/4)), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2-2b}{\left(a\,x^2-b\right)^{\frac{1}{4}}\left(x^4+a\,x^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-2*b)/(a*x^2-b)^(1/4)/(x^4+a*x^2-b),x)
[Out] int((a*x^2-2*b)/(a*x^2-b)^(1/4)/(x^4+a*x^2-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2-2b}{\left(x^4+ax^2-b\right)\left(ax^2-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-2*b)/(a*x^2-b)^(1/4)/(x^4+a*x^2-b),x, algorithm="maxima")
[Out] integrate((a*x^2 - 2*b)/((x^4 + a*x^2 - b)*(a*x^2 - b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{2\,b-a\,x^2}{{\left(a\,x^2-b\right)}^{1/4}\,\left(x^4+a\,x^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*b - a*x^2)/((a*x^2 - b)^(1/4)*(a*x^2 - b + x^4)),x)
[Out] int(-(2*b - a*x^2)/((a*x^2 - b)^(1/4)*(a*x^2 - b + x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2-2b}{\sqrt[4]{ax^2-b}\,\left(ax^2-b+x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-2*b)/(a*x**2-b)**(1/4)/(x**4+a*x**2-b),x)
[Out] Integral((a*x**2 - 2*b)/((a*x**2 - b)**(1/4)*(a*x**2 - b + x**4)), x)
```

3.1270 $\int \frac{-4b+ax^3}{\sqrt[4]{-b+ax^3}\left(-b+ax^3+x^4\right)}\,dx$

Optimal. Leaf size=102

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}+x^2}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3-b}}\right)$$

Rubi [F] time = 0.87, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-4b+ax^3}{\sqrt[4]{-b+ax^3}\left(-b+ax^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-4*b + a*x^3)/((-b + a*x^3)^(1/4)*(-b + a*x^3 + x^4)),x]
```

```
[Out] 4*b*Defer[Int][1/((-b + a*x^3)^(1/4)*(b - a*x^3 - x^4)), x] + a*Defer[Int][
x^3/((-b + a*x^3)^(1/4)*(-b + a*x^3 + x^4)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{-4b+ax^3}{\sqrt[4]{-b+ax^3}\left(-b+ax^3+x^4\right)}\,dx&=\int \left(\frac{4b}{\sqrt[4]{-b+ax^3}\left(b-ax^3-x^4\right)}+\frac{ax^3}{\sqrt[4]{-b+ax^3}\left(-b+ax^3+x^4\right)}\right)dx\\&=a\int \frac{x^3}{\sqrt[4]{-b+ax^3}\left(-b+ax^3+x^4\right)}\,dx+(4b)\int \frac{1}{\sqrt[4]{-b+ax^3}\left(b-ax^3-x^4\right)}\,dx\end{aligned}$$

Mathematica [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{-4b+ax^3}{\sqrt[4]{-b+ax^3}\left(-b+ax^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-4*b + a*x^3)/((-b + a*x^3)^(1/4)*(-b + a*x^3 + x^4)),x]
```

```
[Out] Integrate[(-4*b + a*x^3)/((-b + a*x^3)^(1/4)*(-b + a*x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.75, size = 102, normalized size = 1.00

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}+x^2}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3-b}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-4*b + a*x^3)/((-b + a*x^3)^(1/4)*(-b + a*x^3 + x^4)),x
]
```

```
[Out] -(Sqrt[2]*ArcTan[(-(x^2/Sqrt[2]) + Sqrt[-b + a*x^3]/Sqrt[2])/(x*(-b + a*x^3
)^(1/4))]) + Sqrt[2]*ArcTanh[(Sqrt[2]*x*(-b + a*x^3)^(1/4))/(x^2 + Sqrt[-b
+ a*x^3])]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-4*b)/(a*x^3-b)^(1/4)/(a*x^3+x^4-b),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 - 4\,b}{\left(ax^3 + x^4 - b\right)\left(ax^3 - b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-4*b)/(a*x^3-b)^(1/4)/(a*x^3+x^4-b),x, algorithm="giac")
[Out] integrate((a*x^3 - 4*b)/((a*x^3 + x^4 - b)*(a*x^3 - b)^(1/4)), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{a\,x^3 - 4b}{\left(a\,x^3 - b\right)^{\frac{1}{4}}\left(a\,x^3 + x^4 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-4*b)/(a*x^3-b)^(1/4)/(a*x^3+x^4-b),x)
[Out] int((a*x^3-4*b)/(a*x^3-b)^(1/4)/(a*x^3+x^4-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 - 4\,b}{\left(ax^3 + x^4 - b\right)\left(ax^3 - b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-4*b)/(a*x^3-b)^(1/4)/(a*x^3+x^4-b),x, algorithm="maxima")
[Out] integrate((a*x^3 - 4*b)/((a*x^3 + x^4 - b)*(a*x^3 - b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{4\,b - a\,x^3}{\left(a\,x^3 - b\right)^{1/4}\,\left(x^4 + a\,x^3 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*b - a*x^3)/((a*x^3 - b)^(1/4)*(a*x^3 - b + x^4)),x)
[Out] int(-(4*b - a*x^3)/((a*x^3 - b)^(1/4)*(a*x^3 - b + x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-4*b)/(a*x**3-b)**(1/4)/(a*x**3+x**4-b),x)
[Out] Timed out
```

3.1271 $\int \frac{(-1+x^3)(1+x^3)^3(1+x^6)^{2/3}}{x^6(1-x^3+x^6)}\,dx$

Optimal. Leaf size=102

$$\log\left(\sqrt[3]{x^6+1}-x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^6+1}+x}\right)-\frac{1}{2}\log\left(\sqrt[3]{x^6+1}\,x+\left(x^6+1\right)^{2/3}+x^2\right)+\frac{\left(x^6+1\right)^{2/3}\left(2x^6+15x^3+2\right.}{10x^5}$$

Rubi [C] time = 1.29, antiderivative size = 288, normalized size of antiderivative = 2.82, number of steps used = 18, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {6728, 245, 364, 275, 1438, 429, 465, 510}

$$\frac{3\left(-\sqrt{3}+i\right)xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};-\frac{2x^6}{1-i\sqrt{3}},-x^6\right)}{\sqrt{3}+i}+\frac{3\left(\sqrt{3}+i\right)xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};-\frac{2x^6}{1+i\sqrt{3}},-x^6\right)}{-\sqrt{3}+i}+\frac{3x^4F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-x^6,-\frac{2x^6}{1-i\sqrt{3}}\right)}{2\left(1-i\sqrt{3}\right)}+\frac{3x^4F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-x^6,-\frac{2x^6}{1+i\sqrt{3}}\right)}{2\left(1+i\sqrt{3}\right)}+x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};-x^6\right)+\frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};-x^6\right)}{5x^5}+\frac{3\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};-x^6\right)}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^3)*(1 + x^3)^3*(1 + x^6)^(2/3))/(x^6*(1 - x^3 + x^6)),x]
```

```
[Out] (3*(I - Sqrt[3])*x*AppellF1[1/6, 1, -2/3, 7/6, (-2*x^6)/(1 - I*Sqrt[3]), -x
^6])/(I + Sqrt[3]) + (3*(I + Sqrt[3])*x*AppellF1[1/6, 1, -2/3, 7/6, (-2*x^6
)/(1 + I*Sqrt[3]), -x^6])/(I - Sqrt[3]) + (3*x^4*AppellF1[2/3, -2/3, 1, 5/3
, -x^6, (-2*x^6)/(1 - I*Sqrt[3])])/(2*(1 - I*Sqrt[3])) + (3*x^4*AppellF1[2/
3, -2/3, 1, 5/3, -x^6, (-2*x^6)/(1 + I*Sqrt[3])])/(2*(1 + I*Sqrt[3])) + Hyp
ergeometric2F1[-5/6, -2/3, 1/6, -x^6]/(5*x^5) + (3*Hypergeometric2F1[-2/3,
-1/3, 2/3, -x^6])/(2*x^2) + x*Hypergeometric2F1[-2/3, 1/6, 7/6, -x^6]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
```

```
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(-1+x^3\right)\left(1+x^3\right)^3\left(1+x^6\right)^{2/3}}{x^6\left(1-x^3+x^6\right)}\,dx = \int \left(\left(1+x^6\right)^{2/3} - \frac{\left(1+x^6\right)^{2/3}}{x^6} - \frac{3\left(1+x^6\right)^{2/3}}{x^3} + \frac{3\left(-1+2x^3\right)\left(1+x^6\right)^{2/3}}{1-x^3+x^6}\right.$$

$$= -\left(3\int \frac{\left(1+x^6\right)^{2/3}}{x^3}\,dx\right) + 3\int \frac{\left(-1+2x^3\right)\left(1+x^6\right)^{2/3}}{1-x^3+x^6}\,dx + \int \left(1+x^6\right)^{2/}$$

$$= \frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};-x^6\right)}{5x^5} + x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};-x^6\right) - \frac{3}{2}\text{Subst}\left(\int \frac{\left(1+x^3\right)^{2/}}{x^2}\right.$$

$$= \frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};-x^6\right)}{5x^5} + \frac{3\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};-x^6\right)}{2x^2} + x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};-x^6\right)$$

$$= \frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};-x^6\right)}{5x^5} + \frac{3\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};-x^6\right)}{2x^2} + x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};-x^6\right)$$

$$= \frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};-x^6\right)}{5x^5} + \frac{3\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};-x^6\right)}{2x^2} + x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};-x^6\right)$$

$$= \frac{3\left(i-\sqrt{3}\right)xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};-\frac{2x^6}{1-i\sqrt{3}},-x^6\right)}{i+\sqrt{3}} + \frac{3\left(i+\sqrt{3}\right)xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6}\right.}{i-\sqrt{3}}$$

$$= \frac{3\left(i-\sqrt{3}\right)xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};-\frac{2x^6}{1-i\sqrt{3}},-x^6\right)}{i+\sqrt{3}} + \frac{3\left(i+\sqrt{3}\right)xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6}\right.}{i-\sqrt{3}}$$

Mathematica [F] time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^3\right)\left(1+x^3\right)^3\left(1+x^6\right)^{2/3}}{x^6\left(1-x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^3)*(1 + x^3)^3*(1 + x^6)^(2/3))/(x^6*(1 - x^3 + x^6)),x]
```

```
[Out] Integrate[((-1 + x^3)*(1 + x^3)^3*(1 + x^6)^(2/3))/(x^6*(1 - x^3 + x^6)), x
]
```

IntegrateAlgebraic [A] time = 2.34, size = 102, normalized size = 1.00

$$\log\left(\sqrt[3]{x^6+1}-x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^6+1}+x}\right)-\frac{1}{2}\log\left(\sqrt[3]{x^6+1}\,x+\left(x^6+1\right)^{2/3}+x^2\right)+\frac{\left(x^6+1\right)^{2/3}\left(2x^6+15x^3+2\right)}{10x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)*(1 + x^3)^3*(1 + x^6)^(2/3))/(x^6*(1 - x^3 +
 x^6)),x]
```

```
[Out] ((1 + x^6)^(2/3)*(2 + 15*x^3 + 2*x^6))/(10*x^5) - Sqrt[3]*ArcTan[(Sqrt[3]*x
)/(x + 2*(1 + x^6)^(1/3))] + Log[-x + (1 + x^6)^(1/3)] - Log[x^2 + x*(1 + x
^6)^(1/3) + (1 + x^6)^(2/3)]/2
```

fricas [A] time = 9.14, size = 147, normalized size = 1.44

$$-\frac{10\,\sqrt{3}\,x^{5}\arctan\left(\frac{1078\,\sqrt{3}\left(x^{6}+1\right)^{\frac{1}{3}}x^{2}+196\,\sqrt{3}\left(x^{6}+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(32\,x^{6}+605\,x^{3}+32\right)}{8\,x^{6}-1331\,x^{3}+8}\right)-5\,x^{5}\log\left(\frac{x^{6}-x^{3}+3\left(x^{6}+1\right)^{\frac{1}{3}}x^{2}-3\left(x^{6}+1\right)^{\frac{2}{3}}x+1}{x^{6}-x^{3}+1}\right)-\left(2\,x^{6}+15\,x^{3}+2\right)\left(x^{6}+1\right)^{\frac{2}{3}}}{10\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^3*(x^6+1)^(2/3)/x^6/(x^6-x^3+1),x, algorithm="fri
cas")
```

```
[Out] -1/10*(10*sqrt(3)*x^5*arctan((1078*sqrt(3)*(x^6 + 1)^(1/3)*x^2 + 196*sqrt(3
)*(x^6 + 1)^(2/3)*x + sqrt(3)*(32*x^6 + 605*x^3 + 32))/(8*x^6 - 1331*x^3 +
8)) - 5*x^5*log((x^6 - x^3 + 3*(x^6 + 1)^(1/3)*x^2 - 3*(x^6 + 1)^(2/3)*x +
1)/(x^6 - x^3 + 1)) - (2*x^6 + 15*x^3 + 2)*(x^6 + 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+1\right)^{\frac{2}{3}}\left(x^3+1\right)^3\left(x^3-1\right)}{\left(x^6-x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^3*(x^6+1)^(2/3)/x^6/(x^6-x^3+1),x, algorithm="gia
c")
```

```
[Out] integrate((x^6 + 1)^(2/3)*(x^3 + 1)^3*(x^3 - 1)/((x^6 - x^3 + 1)*x^6), x)
```

maple [C] time = 1.75, size = 496, normalized size = 4.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)*(x^3+1)^3*(x^6+1)^(2/3)/x^6/(x^6-x^3+1),x)
```

```
[Out] 1/10*(2*x^12+15*x^9+4*x^6+15*x^3+2)/x^5/(x^6+1)^(1/3)+3*RootOf(9*_Z^2+3*_Z+
1)*ln(-(6*RootOf(9*_Z^2+3*_Z+1)*x^6+x^6+18*RootOf(9*_Z^2+3*_Z+1)^2*x^3+9*Ro
otOf(9*_Z^2+3*_Z+1)*(x^6+1)^(2/3)*x+9*RootOf(9*_Z^2+3*_Z+1)*(x^6+1)^(1/3)*x
^2+9*RootOf(9*_Z^2+3*_Z+1)*x^3+x^3+6*RootOf(9*_Z^2+3*_Z+1)+1)/(x^6-x^3+1))-
3*ln(-(-6*RootOf(9*_Z^2+3*_Z+1)*x^6-x^6+18*RootOf(9*_Z^2+3*_Z+1)^2*x^3-9*Ro
```

```
otOf(9*_Z^2+3*_Z+1)*(x^6+1)^(2/3)*x-9*RootOf(9*_Z^2+3*_Z+1)*(x^6+1)^(1/3)*x
^2+3*RootOf(9*_Z^2+3*_Z+1)*x^3-3*x*(x^6+1)^(2/3)-3*x^2*(x^6+1)^(1/3)-6*Root
Of(9*_Z^2+3*_Z+1)-1)/(x^6-x^3+1))*RootOf(9*_Z^2+3*_Z+1)-ln(-(-6*RootOf(9*_Z
^2+3*_Z+1)*x^6-x^6+18*RootOf(9*_Z^2+3*_Z+1)^2*x^3-9*RootOf(9*_Z^2+3*_Z+1)*(
x^6+1)^(2/3)*x-9*RootOf(9*_Z^2+3*_Z+1)*(x^6+1)^(1/3)*x^2+3*RootOf(9*_Z^2+3*
_Z+1)*x^3-3*x*(x^6+1)^(2/3)-3*x^2*(x^6+1)^(1/3)-6*RootOf(9*_Z^2+3*_Z+1)-1)/
(x^6-x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^6+1)^{\frac{2}{3}}(x^3+1)^3(x^3-1)}{(x^6-x^3+1)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^3+1)^3*(x^6+1)^(2/3)/x^6/(x^6-x^3+1),x, algorithm="max
ima")
```

```
[Out] integrate((x^6 + 1)^(2/3)*(x^3 + 1)^3*(x^3 - 1)/((x^6 - x^3 + 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^3-1)\,(x^3+1)^3\,(x^6+1)^{2/3}}{x^6\,(x^6-x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)*(x^3 + 1)^3*(x^6 + 1)^(2/3))/(x^6*(x^6 - x^3 + 1)),x)
```

```
[Out] int(((x^3 - 1)*(x^3 + 1)^3*(x^6 + 1)^(2/3))/(x^6*(x^6 - x^3 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)*(x**3+1)**3*(x**6+1)**(2/3)/x**6/(x**6-x**3+1),x)
```

```
[Out] Timed out
```

3.1272 $\int \frac{\left(-1+x^6\right)^{2/3} \left(1+x^6\right) \left(-2+x^3+2 x^6\right)}{x^6 \left(-1-x^3+x^6\right)} \, dx$

Optimal. Leaf size=102

$$\log \left(\sqrt[3]{x^6-1}-x\right)-\sqrt{3} \tan^{-1}\left(\frac{\sqrt{3} x}{2 \sqrt[3]{x^6-1}+x}\right)-\frac{1}{2} \log \left(\sqrt[3]{x^6-1}\, x+\left(x^6-1\right)^{2/3}+x^2\right)+\frac{\left(x^6-1\right)^{2/3} \left(4 x^6+15 x^3-4\right)}{10 x^5}$$

Rubi [C] time = 1.29, antiderivative size = 389, normalized size of antiderivative = 3.81, number of steps used = 25, number of rules used = 12, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6728, 246, 245, 365, 364, 275, 1438, 430, 429, 465, 511, 510}

$$-\frac{3 \left(1-\sqrt{5}\right) \left(x^6-1\right)^{2/3} x F_1\left(\frac{1}{6};-\frac{2}{3},1;\frac{7}{6};x^6,\frac{2 x^6}{3-\sqrt{5}}\right)}{\left(3-\sqrt{5}\right) \left(1-x^6\right)^{2/3}}-\frac{3 \left(1+\sqrt{5}\right) \left(x^6-1\right)^{2/3} x F_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};\frac{2 x^6}{3+\sqrt{5}},x^6\right)}{\left(3+\sqrt{5}\right) \left(1-x^6\right)^{2/3}}-\frac{3 \left(x^6-1\right)^{2/3} x^4 F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^6,\frac{2 x^6}{3-\sqrt{5}}\right)}{2 \left(3-\sqrt{5}\right) \left(1-x^6\right)^{2/3}}-\frac{3 \left(x^6-1\right)^{2/3} x^4 F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^6,\frac{2 x^6}{3+\sqrt{5}}\right)}{2 \left(3+\sqrt{5}\right) \left(1-x^6\right)^{2/3}}+\frac{2 \left(x^6-1\right)^{2/3} x \, _2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};x^6\right)}{\left(1-x^6\right)^{2/3}}-\frac{2 \left(x^6-1\right)^{2/3} \, _2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5 \left(1-x^6\right)^{2/3} x^5}+\frac{3 \left(x^6-1\right)^{2/3} \, _2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};x^6\right)}{2 \left(1-x^6\right)^{2/3} x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^6)^(2/3)*(1 + x^6)*(-2 + x^3 + 2*x^6))/(x^6*(-1 - x^3 + x^6)),
x]
```

```
[Out] (-3*(1 - Sqrt[5])*x*(-1 + x^6)^(2/3)*AppellF1[1/6, -2/3, 1, 7/6, x^6, (2*x^
6)/(3 - Sqrt[5])])/((3 - Sqrt[5])*(1 - x^6)^(2/3)) - (3*(1 + Sqrt[5])*x*(-1
 + x^6)^(2/3)*AppellF1[1/6, 1, -2/3, 7/6, (2*x^6)/(3 + Sqrt[5]), x^6])/((3
+ Sqrt[5])*(1 - x^6)^(2/3)) - (3*x^4*(-1 + x^6)^(2/3)*AppellF1[2/3, -2/3, 1
, 5/3, x^6, (2*x^6)/(3 - Sqrt[5])])/(2*(3 - Sqrt[5])*(1 - x^6)^(2/3)) - (3*
x^4*(-1 + x^6)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, x^6, (2*x^6)/(3 + Sqrt[5])
])/(2*(3 + Sqrt[5])*(1 - x^6)^(2/3)) - (2*(-1 + x^6)^(2/3)*Hypergeometric2F
1[-5/6, -2/3, 1/6, x^6])/(5*x^5*(1 - x^6)^(2/3)) + (3*(-1 + x^6)^(2/3)*Hype
rgeometric2F1[-2/3, -1/3, 2/3, x^6])/(2*x^2*(1 - x^6)^(2/3)) + (2*x*(-1 + x
^6)^(2/3)*Hypergeometric2F1[-2/3, 1/6, 7/6, x^6])/(1 - x^6)^(2/3)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[q, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(-1+x^6\right)^{2/3}\left(1+x^6\right)\left(-2+x^3+2x^6\right)}{x^6\left(-1-x^3+x^6\right)}\,dx = \int \left(2\left(-1+x^6\right)^{2/3}+\frac{2\left(-1+x^6\right)^{2/3}}{x^6}-\frac{3\left(-1+x^6\right)^{2/3}}{x^3}+\frac{3\left(-1+2x^3 \right.}{-1-}\right.$$

$$= 2\int \left(-1+x^6\right)^{2/3}dx+2\int \frac{\left(-1+x^6\right)^{2/3}}{x^6}dx-3\int \frac{\left(-1+x^6\right)^{2/3}}{x^3}dx$$

$$= -\left(\frac{3}{2}\operatorname{Subst}\left(\int \frac{\left(-1+x^3\right)^{2/3}}{x^2}dx,x,x^2\right)\right)+3\int \left(\frac{2\left(-1+x^6\right)^{2/3}}{-1-\sqrt{5}+2x^3}+\right.$$

$$= -\frac{2\left(-1+x^6\right)^{2/3}{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5\left(1-x^6\right)^{2/3}}+\frac{2x\left(-1+x^6\right)^{2/3}{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6}\right.}{\left(1-x^6\right)^{2/3}}$$

$$= -\frac{2\left(-1+x^6\right)^{2/3}{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5\left(1-x^6\right)^{2/3}}+\frac{3\left(-1+x^6\right)^{2/3}{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\right.}{2x^2\left(1-x^6\right)^{2/3}}$$

$$= -\frac{2\left(-1+x^6\right)^{2/3}{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5\left(1-x^6\right)^{2/3}}+\frac{3\left(-1+x^6\right)^{2/3}{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\right.}{2x^2\left(1-x^6\right)^{2/3}}$$

$$= -\frac{2\left(-1+x^6\right)^{2/3}{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5\left(1-x^6\right)^{2/3}}+\frac{3\left(-1+x^6\right)^{2/3}{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\right.}{2x^2\left(1-x^6\right)^{2/3}}$$

$$= -\frac{3\left(1-\sqrt{5}\right)x\left(-1+x^6\right)^{2/3}F_1\left(\frac{1}{6};-\frac{2}{3},1;\frac{7}{6};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{\left(3-\sqrt{5}\right)\left(1-x^6\right)^{2/3}}-\frac{3\left(1+\sqrt{5}\right)}{}$$

$$= -\frac{3\left(1-\sqrt{5}\right)x\left(-1+x^6\right)^{2/3}F_1\left(\frac{1}{6};-\frac{2}{3},1;\frac{7}{6};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{\left(3-\sqrt{5}\right)\left(1-x^6\right)^{2/3}}-\frac{3\left(1+\sqrt{5}\right)}{}$$

Mathematica [F] time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^6\right)^{2/3}\left(1+x^6\right)\left(-2+x^3+2x^6\right)}{x^6\left(-1-x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^6)^(2/3)*(1 + x^6)*(-2 + x^3 + 2*x^6))/(x^6*(-1 - x^3 + x^6)),x]
```

```
[Out] Integrate[((-1 + x^6)^(2/3)*(1 + x^6)*(-2 + x^3 + 2*x^6))/(x^6*(-1 - x^3 + x^6)), x]
```

IntegrateAlgebraic [A] time = 2.34, size = 102, normalized size = 1.00

$$\log\left(\sqrt[3]{x^6-1}-x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^6-1}+x}\right)-\frac{1}{2}\log\left(\sqrt[3]{x^6-1}x+\left(x^6-1\right)^{2/3}+x^2\right)+\frac{\left(x^6-1\right)^{2/3}\left(4x^6+15x^3-4\right)}{10x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^6)^(2/3)*(1 + x^6)*(-2 + x^3 + 2*x^6))/(x^6*(-1
 - x^3 + x^6)),x]
```

[Out] ((-1 + x^6)^(2/3)*(-4 + 15*x^3 + 4*x^6))/(10*x^5) - Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x^6)^(1/3))] + Log[-x + (-1 + x^6)^(1/3)] - Log[x^2 + x*(-1 + x^6)^(1/3) + (-1 + x^6)^(2/3)]/2

fricas [A] time = 12.77, size = 148, normalized size = 1.45

$$-\frac{10\sqrt{3}x^5\arctan\left(\frac{47399638863594863345242891761429898599688622451126011503668045351488814414825 0\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}x^2+1932503148048922825567426596644883596781892608764360018412309996536651589 2788\sqrt{3}\left(x^6-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(771225779807741020855977802972631216428368740202755221603971931588718036144x^6+245889484278411189833195613987401279765924206559249102388797804808538611984375x^3-771225779807741020855977802972631216428368740202755221603971931588718036144\right)}{3\left(154075137855386652020330175695521646369068967401499860028038247124026691 44x^6-227351086091515241263579358841494627179170556108548407412281480599473216796875x^3-154075137855386652020330175695521646369068967401499860028038247124026691 44\right)}\right)-5x^5\log\left(\frac{x^6-x^3+3\left(x^6-1\right)^{\frac{1}{3}}x^2-3\left(x^6-1\right)^{\frac{2}{3}}x-1}{x^6-x^3-1}\right)-\left(4x^6+15x^3-4\right)\left(x^6-1\right)^{\frac{2}{3}}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6-1)^(2/3)*(x^6+1)*(2*x^6+x^3-2)/x^6/(x^6-x^3-1),x, algorithm="fricas")

[Out] -1/10*(10*sqrt(3)*x^5*arctan(1/3*(47399638863594863345242891761429898599688622451126011503668045351488814414825 0*sqrt(3)*(x^6 - 1)^(1/3)*x^2 + 1932503148048922825567426596644883596781892608764360018412309996536651589 2788*sqrt(3)*(x^6 - 1)^(2/3)*x + sqrt(3)*(771225779807741020855977802972631216428368740202755221603971931588718036144*x^6 + 245889484278411189833195613987401279765924206559249102388797804808538611984375*x^3 - 771225779807741020855977802972631216428368740202755221603971931588718036144))/(154075137855386652020330175695521646369068967401499860028038247124026691 44*x^6 - 227351086091515241263579358841494627179170556108548407412281480599473216796875*x^3 - 154075137855386652020330175695521646369068967401499860028038247124026691 44)) - 5*x^5*log((x^6 - x^3 + 3*(x^6 - 1)^(1/3)*x^2 - 3*(x^6 - 1)^(2/3)*x - 1)/(x^6 - x^3 - 1)) - (4*x^6 + 15*x^3 - 4)*(x^6 - 1)^(2/3))/x^5

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^6+x^3-2\right)\left(x^6+1\right)\left(x^6-1\right)^{\frac{2}{3}}}{\left(x^6-x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6-1)^(2/3)*(x^6+1)*(2*x^6+x^3-2)/x^6/(x^6-x^3-1),x, algorithm="giac")

[Out] integrate((2*x^6 + x^3 - 2)*(x^6 + 1)*(x^6 - 1)^(2/3)/((x^6 - x^3 - 1)*x^6), x)

maple [C] time = 1.92, size = 490, normalized size = 4.80

$$\frac{4x^{12}+15x^9-8x^6-15x^3+4}{10x^5\left(x^6-1\right)^{\frac{1}{3}}}-3\ln\left(\frac{6\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)x^6+x^6-18\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3+9\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^6-1\right)^{\frac{2}{3}}x+9\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^6-1\right)^{\frac{1}{3}}x^2-3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3+3x\left(x^6-1\right)^{\frac{2}{3}}+3x^2\left(x^6-1\right)^{\frac{1}{3}}-6\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)-1}{x^6-x^3-1}\right)\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)+3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\ln\left(-\frac{6\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)x^6+x^6+18\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3+9\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^6-1\right)^{\frac{2}{3}}x+9\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^6-1\right)^{\frac{1}{3}}x^2+9\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3+x^3-6\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)-1}{x^6-x^3-1}\right)-\ln\left(\frac{6\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)x^6+x^6-18\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3+9\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^6-1\right)^{\frac{2}{3}}x+9\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^6-1\right)^{\frac{1}{3}}x^2-3\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3+3x\left(x^6-1\right)^{\frac{2}{3}}+3x^2\left(x^6-1\right)^{\frac{1}{3}}-6\operatorname{RootOf}\left(9\_Z^2+3\_Z+1\right)-1}{x^6-x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6-1)^(2/3)*(x^6+1)*(2*x^6+x^3-2)/x^6/(x^6-x^3-1),x)

[Out] 1/10*(4*x^12+15*x^9-8*x^6-15*x^3+4)/x^5/(x^6-1)^(1/3)-3*ln((6*RootOf(9*_Z^2+3*_Z+1)*x^6+x^6-18*RootOf(9*_Z^2+3*_Z+1)^2*x^3+9*RootOf(9*_Z^2+3*_Z+1)*(x^6-1)^(2/3)*x+9*RootOf(9*_Z^2+3*_Z+1)*(x^6-1)^(1/3)*x^2-3*RootOf(9*_Z^2+3*_Z+1)*x^3+3*x*(x^6-1)^(2/3)+3*x^2*(x^6-1)^(1/3)-6*RootOf(9*_Z^2+3*_Z+1)-1)/(x^6-x^3-1))*RootOf(9*_Z^2+3*_Z+1)+3*RootOf(9*_Z^2+3*_Z+1)*ln(-(6*RootOf(9*_Z^2+3*_Z+1)*x^6+x^6+18*RootOf(9*_Z^2+3*_Z+1)^2*x^3+9*RootOf(9*_Z^2+3*_Z+1)*(x^6-1)^(2/3)*x+9*RootOf(9*_Z^2+3*_Z+1)*(x^6-1)^(1/3)*x^2+9*RootOf(9*_Z^2+3*_Z+1)*x^3+x^3-6*RootOf(9*_Z^2+3*_Z+1)-1)/(x^6-x^3-1))-ln((6*RootOf(9*_Z^2+3*_Z+1)*x^6+x^6-18*RootOf(9*_Z^2+3*_Z+1)^2*x^3+9*RootOf(9*_Z^2+3*_Z+1)*(x^6-1)^(2/3)*x+9*RootOf(9*_Z^2+3*_Z+1)*(x^6-1)^(1/3)*x^2-3*RootOf(9*_Z^2+3*_Z+1)*x^3+3*x*(x^6-1)^(2/3)+3*x^2*(x^6-1)^(1/3)-6*RootOf(9*_Z^2+3*_Z+1)-1)/(x^6-x^3-1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 x^{6}+x^{3}-2\right)\left(x^{6}+1\right)\left(x^{6}-1\right)^{\frac{2}{3}}}{\left(x^{6}-x^{3}-1\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(2/3)*(x^6+1)*(2*x^6+x^3-2)/x^6/(x^6-x^3-1),x, algorithm=
"maxima")
```

```
[Out] integrate((2*x^6 + x^3 - 2)*(x^6 + 1)*(x^6 - 1)^(2/3)/((x^6 - x^3 - 1)*x^6)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^{6}-1\right)^{2/3}\left(x^{6}+1\right)\left(2 x^{6}+x^{3}-2\right)}{x^{6}\left(-x^{6}+x^{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^6 - 1)^(2/3)*(x^6 + 1)*(x^3 + 2*x^6 - 2))/(x^6*(x^3 - x^6 + 1)),x)
```

```
[Out] int(-((x^6 - 1)^(2/3)*(x^6 + 1)*(x^3 + 2*x^6 - 2))/(x^6*(x^3 - x^6 + 1)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(2/3)*(x**6+1)*(2*x**6+x**3-2)/x**6/(x**6-x**3-1),x)
```

```
[Out] Timed out
```

3.1273 $\int \frac{\sqrt{cx^2-x\sqrt{-bx+ax^2}}}{x^3}\,dx$

Optimal. Leaf size=102

$$-\frac{4\left(-3ax+3b+2c^2x\right)\sqrt{-x\left(\sqrt{ax^2-bx}-cx\right)}}{15bx^2}-\frac{4c\sqrt{ax^2-bx}\sqrt{-x\left(\sqrt{ax^2-bx}-cx\right)}}{15bx^2}$$

Rubi [F] time = 0.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{cx^2-x\sqrt{-bx+ax^2}}}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c*x^2 - x*Sqrt[-(b*x) + a*x^2]]/x^3,x]
```

```
[Out] Defer[Int][Sqrt[c*x^2 - x*Sqrt[-(b*x) + a*x^2]]/x^3, x]
```

Rubi steps

$$\int \frac{\sqrt{cx^2-x\sqrt{-bx+ax^2}}}{x^3}\,dx=\int \frac{\sqrt{cx^2-x\sqrt{-bx+ax^2}}}{x^3}\,dx$$

Mathematica [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2-x\sqrt{-bx+ax^2}}}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c*x^2 - x*Sqrt[-(b*x) + a*x^2]]/x^3,x]
```

```
[Out] Integrate[Sqrt[c*x^2 - x*Sqrt[-(b*x) + a*x^2]]/x^3, x]
```

IntegrateAlgebraic [A] time = 7.13, size = 102, normalized size = 1.00

$$-\frac{4\left(-3ax+3b+2c^2x\right)\sqrt{-x\left(\sqrt{ax^2-bx}-cx\right)}}{15bx^2}-\frac{4c\sqrt{ax^2-bx}\sqrt{-x\left(\sqrt{ax^2-bx}-cx\right)}}{15bx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2 - x*Sqrt[-(b*x) + a*x^2]]/x^3,x]
```

```
[Out] (-4*(3*b - 3*a*x + 2*c^2*x)*Sqrt[-(x*(-(c*x) + Sqrt[-(b*x) + a*x^2]))])/(15
*b*x^2) - (4*c*Sqrt[-(b*x) + a*x^2]*Sqrt[-(x*(-(c*x) + Sqrt[-(b*x) + a*x^2]
))])/(15*b*x^2)
```

fricas [A] time = 0.78, size = 60, normalized size = 0.59

$$-\frac{4\sqrt{cx^2-\sqrt{ax^2-bx}\,x}\left(\left(2\,c^2-3\,a\right)x+\sqrt{ax^2-bx}\,c+3\,b\right)}{15\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2-x*(a*x^2-b*x)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -4/15*sqrt(c*x^2 - sqrt(a*x^2 - b*x)*x)*((2*c^2 - 3*a)*x + sqrt(a*x^2 - b*x
)*c + 3*b)/(b*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 - \sqrt{ax^2 - bx}\,x}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2-x*(a*x^2-b*x)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 - sqrt(a*x^2 - b*x)*x)/x^3, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^2 - x\sqrt{a\,x^2 - bx}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2-x*(a*x^2-b*x)^(1/2))^(1/2)/x^3,x)
```

```
[Out] int((c*x^2-x*(a*x^2-b*x)^(1/2))^(1/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 - \sqrt{ax^2 - bx}\,x}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2-x*(a*x^2-b*x)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 - sqrt(a*x^2 - b*x)*x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2 - x\,\sqrt{a\,x^2 - b\,x}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2 - x*(a*x^2 - b*x)^(1/2))^(1/2)/x^3,x)
```

```
[Out] int((c*x^2 - x*(a*x^2 - b*x)^(1/2))^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(cx - \sqrt{ax^2 - bx}\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2-x*(a*x**2-b*x)**(1/2))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x*(c*x - sqrt(a*x**2 - b*x)))/x**3, x)
```

3.1274 $\int \frac{1+x}{\left(-2+2x+x^2\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=103

$$\frac{1}{6}\sqrt{2\sqrt{3}-3}\,\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{x^3+1}}{x^2-x+1}\right)-\frac{1}{6}\sqrt{3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{x^3+1}}{x^2-x+1}\right)$$

Rubi [A] time = 0.65, antiderivative size = 93, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6728, 2135, 218, 2140, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)}{2\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{x^3+1}}\right)}{2\sqrt{3\left(2\sqrt{3}-3\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-2 + 2*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/(2*Sqrt[3*(3 + 2*Sqrt[3
])]) - ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/(2*Sqrt[3*(-3
+ 2*Sqrt[3])])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2135

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-
6*a*d^3)/(c*(b*c^3 - 28*a*d^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
```

```
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1+x}{\left(-2+2x+x^2\right)\sqrt{1+x^3}}\,dx = \int \left(\frac{1}{\left(2-2\sqrt{3}+2x\right)\sqrt{1+x^3}}+\frac{1}{\left(2+2\sqrt{3}+2x\right)\sqrt{1+x^3}}\right)dx$$

$$= \int \frac{1}{\left(2-2\sqrt{3}+2x\right)\sqrt{1+x^3}}\,dx + \int \frac{1}{\left(2+2\sqrt{3}+2x\right)\sqrt{1+x^3}}\,dx$$

$$= -\frac{\int \frac{96\left(1-\sqrt{3}\right)+96x}{\left(2+2\sqrt{3}+2x\right)\sqrt{1+x^3}}\,dx}{192\sqrt{3}} + \frac{\int \frac{96\left(1+\sqrt{3}\right)+96x}{\left(2-2\sqrt{3}+2x\right)\sqrt{1+x^3}}\,dx}{192\sqrt{3}}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1+\frac{2\left(1-\sqrt{3}\right)x}{2-2\sqrt{3}}}{\sqrt{1+x^3}}\right)}{2\sqrt{3}} + \frac{\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1+\frac{2\left(1+\sqrt{3}\right)x}{2+2\sqrt{3}}}{\sqrt{1+x^3}}\right)}{2\sqrt{3}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{2\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{2\sqrt{3\left(-3+2\sqrt{3}\right)}}$$

Mathematica [C] time = 0.21, size = 218, normalized size = 2.12

$$-\frac{\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1}\left(\left(3+(2+i)\sqrt{3}\right)\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)x-2i}{-3i+\sqrt{3}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)+\left(3-(2-i)\sqrt{3}\right)\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\left(i+\sqrt{3}\right)x-2i}{-3i+\sqrt{3}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{3\left(\sqrt{3}+i\right)\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-2 + 2*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] -1/3*(Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((3 + (2 + I)*Sqrt[3
])*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[(-2*I + (
I + Sqrt[3])*x)/(-3*I + Sqrt[3])]], (1 + I*Sqrt[3])/2] + (3 - (2 - I)*Sqrt[
3])*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(-2*I + (
I + Sqrt[3])*x)/(-3*I + Sqrt[3])]], (1 + I*Sqrt[3])/2]))/((I + Sqrt[3])*Sqr
t[1 + x^3])
```

IntegrateAlgebraic [A] time = 1.26, size = 103, normalized size = 1.00

$$\frac{1}{6}\sqrt{2\sqrt{3}-3}\,\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{x^3+1}}{x^2-x+1}\right)-\frac{1}{6}\sqrt{3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{x^3+1}}{x^2-x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((-2 + 2*x + x^2)*Sqrt[1 + x^3]),x]
```

[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[1 + x^3])/(1 - x + x
^2)])/6 - (Sqrt[3 + 2*Sqrt[3]]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[1 + x^3])
/(1 - x + x^2)])/6

fricas [B] time = 0.48, size = 237, normalized size = 2.30

$$\frac{1}{6}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\sqrt{x^3+1}\sqrt{2\sqrt{3}-3}\left(\sqrt{3}+2\right)}{x^2-x+1}\right)-\frac{1}{24}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^4-2x^3+6x^2+2\sqrt{x^3+1}\left(2x^2-\sqrt{3}\left(x^2-2x\right)-2x+2\right)\sqrt{2\sqrt{3}+3}+4\sqrt{3}\left(x^3+1\right)+4x+4}{x^4+4x^3-8x+4}\right)+\frac{1}{24}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^4-2x^3+6x^2-2\sqrt{x^3+1}\left(2x^2-\sqrt{3}\left(x^2-2x\right)-2x+2\right)\sqrt{2\sqrt{3}+3}+4\sqrt{3}\left(x^3+1\right)+4x+4}{x^4+4x^3-8x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^2+2*x-2)/(x^3+1)^(1/2),x, algorithm="fricas")

[Out] 1/6*sqrt(2*sqrt(3) - 3)*arctan(sqrt(x^3 + 1)*sqrt(2*sqrt(3) - 3)*(sqrt(3) +
 2)/(x^2 - x + 1)) - 1/24*sqrt(2*sqrt(3) + 3)*log((x^4 - 2*x^3 + 6*x^2 + 2*
sqrt(x^3 + 1)*(2*x^2 - sqrt(3)*(x^2 - 2*x) - 2*x + 2)*sqrt(2*sqrt(3) + 3) +
 4*sqrt(3)*(x^3 + 1) + 4*x + 4)/(x^4 + 4*x^3 - 8*x + 4)) + 1/24*sqrt(2*sqrt
(3) + 3)*log((x^4 - 2*x^3 + 6*x^2 - 2*sqrt(x^3 + 1)*(2*x^2 - sqrt(3)*(x^2 -
 2*x) - 2*x + 2)*sqrt(2*sqrt(3) + 3) + 4*sqrt(3)*(x^3 + 1) + 4*x + 4)/(x^4
+ 4*x^3 - 8*x + 4))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^3+1}\left(x^2+2x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^2+2*x-2)/(x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x^2 + 2*x - 2)), x)

maple [C] time = 0.27, size = 694, normalized size = 6.74

$$-\frac{\sqrt{\frac{x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}+\frac{1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}-\frac{1}{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}-\frac{i\sqrt{3}}{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\sqrt{\frac{x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}-\frac{1}{2\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)}+\frac{i\sqrt{3}}{-3+i\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{3}}{2\sqrt{x^3+1}}+\frac{i\sqrt{\frac{x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}+\frac{1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}-\frac{1}{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}-\frac{i\sqrt{3}}{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\sqrt{\frac{x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}-\frac{1}{2\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)}+\frac{i\sqrt{3}}{-3+i\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^3+1}}+\frac{\sqrt{\frac{x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}+\frac{1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}-\frac{1}{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}-\frac{i\sqrt{3}}{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\sqrt{\frac{x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}-\frac{1}{2\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)}+\frac{i\sqrt{3}}{-3+i\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{3}}{2\sqrt{x^3+1}}-\frac{i\sqrt{\frac{x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}+\frac{1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}-\frac{1}{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}-\frac{i\sqrt{3}}{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\sqrt{\frac{x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}-\frac{1}{2\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)}+\frac{i\sqrt{3}}{-3+i\sqrt{3}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)/(x^2+2*x-2)/(x^3+1)^(1/2),x)

[Out] -1/2*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3
^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1
/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1
/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(
1/2)))^(1/2))*3^(1/2)+1/2*I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))
^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))
+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/
(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+1/2*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/
2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*
I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1
/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*Ellip
ticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((
-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-1/2*I*(1/(3/2-1/2*
I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3
/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*
3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2
*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^3+1}\,\left(x^2+2\,x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x-2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x^2 + 2*x - 2)), x)
```

mupad [B] time = 0.21, size = 220, normalized size = 2.14

$$-\frac{\left(\Pi\left(\sqrt{3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right);\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(-\sqrt{3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right);\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(\sqrt{3}+1\mathrm{i}\right)\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{2\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^3 + 1)^(1/2)*(2*x + x^2 - 2)),x)
```

```
[Out] -((ellipticPi(3^(1/2)*((3^(1/2)*1i)/6 + 1/2), asin(((x + 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - elliptic
Pi(-3^(1/2)*((3^(1/2)*1i)/6 + 1/2), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((x + (3^(1/2)*1i)/
2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)*(3^(1/2) + 1i)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
)/(2*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\,\left(x^2+2x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+2*x-2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x**2 + 2*x - 2)), x)
```

3.1275 $\int \frac{3+x+x^2}{\left(-2+2x+x^2\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=103

$$-\frac{1}{6}\sqrt{14\sqrt{3}-15}\,\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{x^3+1}}{x^2-x+1}\right)-\frac{1}{6}\sqrt{15+14\sqrt{3}}\,\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{x^3+1}}{x^2-x+1}\right)$$

Rubi [C] time = 0.79, antiderivative size = 406, normalized size of antiderivative = 3.94, number of steps used = 13, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6728, 218, 2135, 2140, 206, 203}

$$-\frac{1}{6}\sqrt{14\sqrt{3}-15}\,\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)-\frac{1}{6}\sqrt{15+14\sqrt{3}}\,\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{x^3+1}}\right)-\frac{\sqrt{38+21\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\,3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}-\frac{\sqrt{14+5\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\,3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}+\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(3 + x + x^2)/((-2 + 2*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] -1/6*(Sqrt[-15 + 14*Sqrt[3]]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 +
x^3]]) - (Sqrt[15 + 14*Sqrt[3]]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt
[1 + x^3]])/6 + (2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3
] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqr
t[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (Sqrt[14
 + 5*Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[Arc
Sin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*3^(3/4)*Sqrt[
(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (Sqrt[38 + 21*Sqrt[3]]*(1 + x
)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x
)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3]
 + x)^2]*Sqrt[1 + x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2135

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-
6*a*d^3)/(c*(b*c^3 - 28*a*d^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{3+x+x^2}{\left(-2+2x+x^2\right)\sqrt{1+x^3}}\,dx &= \int \left(\frac{1}{\sqrt{1+x^3}}+\frac{5-x}{\left(-2+2x+x^2\right)\sqrt{1+x^3}}\right)dx\\
&= \int \frac{1}{\sqrt{1+x^3}}\,dx+\int \frac{5-x}{\left(-2+2x+x^2\right)\sqrt{1+x^3}}\,dx\\
&= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+\int\Bigg(\frac{-1}{\left(2-2\sqrt{3}\right.}\cdots\\
&= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+\left(-1-2\sqrt{3}\right)\int\cdots\\
&= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+\frac{1}{576}\left(6-\sqrt{3}\right)\cdots\\
&= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}-\frac{\sqrt{14+5\sqrt{3}}\,(1\cdots}{\cdots}\\
&= -\frac{1}{6}\sqrt{-15+14\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)-\frac{1}{6}\sqrt{15+14\sqrt{3}}\,\tanh^{-1}\left(\frac{\sqrt{-3+\cdots}}{\sqrt{\cdots}}\right.
\end{aligned}$$

Mathematica [C] time = 0.82, size = 286, normalized size = 2.78

$$\frac{\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(\sqrt{x^2-x+1}\left(\left((-9-6i)-(4-i)\sqrt{3}\right)\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)+\left((-9+6i)+(4+i)\sqrt{3}\right)\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)\right)+\frac{6\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(\left(\sqrt{3}+i\right)x-2i\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}\right)}{3\left(\sqrt{3}+i\right)\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 + x + x^2)/((-2 + 2*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((6*Sqrt[((-1)^(1/3) - (-1)^(2/3)*x)/(1 + (
-1)^(1/3))]*(-2*I + (I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3)
)] + Sqrt[1 - x + x^2]*(((-9 - 6*I) - (4 - I)*Sqrt[3])*EllipticPi[(2*Sqrt[3
])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)] + ((-9 + 6*I) + (4 + I)*Sqrt[3])*EllipticPi[((2*I)*Sqrt[
3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
], (-1)^(1/3)])))/(3*(I + Sqrt[3])*Sqrt[1 + x^3])
```

IntegrateAlgebraic [A] time = 1.56, size = 103, normalized size = 1.00

$$-\frac{1}{6}\sqrt{14\sqrt{3}-15}\,\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{x^3+1}}{x^2-x+1}\right)-\frac{1}{6}\sqrt{15+14\sqrt{3}}\,\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{x^3+1}}{x^2-x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + x + x^2)/((-2 + 2*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] -1/6*(Sqrt[-15 + 14*Sqrt[3]]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[1 + x^3])/(1
- x + x^2)]) - (Sqrt[15 + 14*Sqrt[3]]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[1
+ x^3])/(1 - x + x^2)])/6
```

fricas [B] time = 0.49, size = 250, normalized size = 2.43

$$-\frac{1}{6}\sqrt{14\sqrt{3}-15}\arctan\left(\frac{\sqrt{x^3+1}\sqrt{14\sqrt{3}-15}\left(3\sqrt{3}+4\right)}{11\left(x^2-x+1\right)}\right)+\frac{1}{24}\sqrt{14\sqrt{3}+15}\log\left(\frac{11x^4-22x^3+66x^2+2\sqrt{x^3+1}\left(4x^2-\sqrt{3}\left(3x^2-2x+4\right)-10x-2\right)\sqrt{14\sqrt{3}+15}+44\sqrt{3}\left(x^3+1\right)+44x+44}{x^4+4x^3-8x+4}\right)-\frac{1}{24}\sqrt{14\sqrt{3}+15}\log\left(\frac{11x^4-22x^3+66x^2-2\sqrt{x^3+1}\left(4x^2-\sqrt{3}\left(3x^2-2x+4\right)-10x-2\right)\sqrt{14\sqrt{3}+15}+44\sqrt{3}\left(x^3+1\right)+44x+44}{x^4+4x^3-8x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+3)/(x^2+2*x-2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(14*sqrt(3) - 15)*arctan(1/11*sqrt(x^3 + 1)*sqrt(14*sqrt(3) - 15)*
(3*sqrt(3) + 4)/(x^2 - x + 1)) + 1/24*sqrt(14*sqrt(3) + 15)*log((11*x^4 - 2
2*x^3 + 66*x^2 + 2*sqrt(x^3 + 1)*(4*x^2 - sqrt(3)*(3*x^2 - 2*x + 4) - 10*x
- 2)*sqrt(14*sqrt(3) + 15) + 44*sqrt(3)*(x^3 + 1) + 44*x + 44)/(x^4 + 4*x^3
 - 8*x + 4)) - 1/24*sqrt(14*sqrt(3) + 15)*log((11*x^4 - 22*x^3 + 66*x^2 - 2
*sqrt(x^3 + 1)*(4*x^2 - sqrt(3)*(3*x^2 - 2*x + 4) - 10*x - 2)*sqrt(14*sqrt(
3) + 15) + 44*sqrt(3)*(x^3 + 1) + 44*x + 44)/(x^4 + 4*x^3 - 8*x + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+x+3}{\sqrt{x^3+1}\left(x^2+2x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+3)/(x^2+2*x-2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + x + 3)/(sqrt(x^3 + 1)*(x^2 + 2*x - 2)), x)
```

maple [C] time = 0.26, size = 1501, normalized size = 14.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+3)/(x^2+2*x-2)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+1/2*(1/(3/2-1/2*I*3^(1/2))*x+1/(3
```

```
/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))
-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-
3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*
EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1
/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(1/(3/2-1/
2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(
-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*
I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+
1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-1
/2*I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3
^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1
/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1
/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(
1/2)))^(1/2))+I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-
3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2
+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3
^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/
2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^
(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3
^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+1/2*I*(1/(3/2-1/2*I*3^(1/2))*x+1
/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/
2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2
/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/
2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^
(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+I*(1/(3/2-1/2*I*3^
(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1
/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1
/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(
x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3
^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-
1/2*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^
(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)
*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/
2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/
2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/
2)))^(1/2))*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+x+3}{\sqrt{x^3+1}\left(x^2+2\,x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+3)/(x^2+2*x-2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + x + 3)/(sqrt(x^3 + 1)*(x^2 + 2*x - 2)), x)
```

mupad [B] time = 0.08, size = 505, normalized size = 4.90

$$\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}+\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\left(\sqrt{3}-6\right)\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{3};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\left(\sqrt{3}+6\right)\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(-\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{3};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 3)/((x^3 + 1)^(1/2)*(2*x + x^2 - 2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2))^(1/2) + (((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 -
 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*
(3^(1/2) - 6)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ell
ipticPi((3^(1/2)*((3^(1/2)*1i)/2 + 3/2))/3, asin(((x + 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(x^3 - x
*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*
1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)*(3^(1/2) + 6)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*ellipticPi(-(3^(1/2)*((3^(1/2)*1i)/2 + 3/2))/3, asin(((x + 1)/((3^(1/2)
*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*
(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + x + 3}{\sqrt{(x+1)\left(x^2 - x + 1\right)}\left(x^2 + 2x - 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+3)/(x**2+2*x-2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x**2 + x + 3)/(sqrt((x + 1)*(x**2 - x + 1))*(x**2 + 2*x - 2)), x)
```

3.1276 $\int \frac{3-x+2x^2}{\left(-2+2x+x^2\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=103

$$-\frac{1}{6}\sqrt{21+26\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{x^3+1}}{x^2-x+1}\right)-\frac{1}{6}\sqrt{26\sqrt{3}-21}\,\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{x^3+1}}{x^2-x+1}\right)$$

Rubi [C] time = 0.89, antiderivative size = 406, normalized size of antiderivative = 3.94, number of steps used = 13, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6728, 218, 2135, 2140, 206, 203}

$$-\frac{1}{6}\sqrt{21+26\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)-\frac{1}{6}\sqrt{26\sqrt{3}-21}\,\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{x^3+1}}\right)-\frac{\sqrt{266+153\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle|-7-4\sqrt{3}\right)}{2\,3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}+\frac{4\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}-\frac{\sqrt{26-7\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle|-7-4\sqrt{3}\right)}{2\,3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(3 - x + 2*x^2)/((-2 + 2*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] -1/6*(Sqrt[21 + 26*Sqrt[3]]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x
^3]]) - (Sqrt[-21 + 26*Sqrt[3]]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt
[1 + x^3]])/6 - (Sqrt[26 - 7*Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[
3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sq
rt[3]])/(2*3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (4*Sq
rt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[A
rcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[
(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (Sqrt[266 + 153*Sqrt[3]]*(1 +
 x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] +
 x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[
3] + x)^2]*Sqrt[1 + x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2135

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-
6*a*d^3)/(c*(b*c^3 - 28*a*d^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{3-x+2x^2}{\left(-2+2x+x^2\right)\sqrt{1+x^3}}\,dx = \int \left(\frac{2}{\sqrt{1+x^3}}+\frac{7-5x}{\left(-2+2x+x^2\right)\sqrt{1+x^3}}\right)dx$$

$$= 2\int \frac{1}{\sqrt{1+x^3}}\,dx + \int \frac{7-5x}{\left(-2+2x+x^2\right)\sqrt{1+x^3}}\,dx$$

$$= \frac{4\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \int \Bigg(\frac{\cdots}{(2-2\cdots}$$

$$= \frac{4\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \left(-5-4\sqrt{3}\cdots\right.$$

$$= \frac{4\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{576}\left(12-5\cdots\right.$$

$$= -\frac{\sqrt{26-7\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\,3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4\sqrt{2+\cdots}}{\cdots}$$

$$= -\frac{1}{6}\sqrt{21+26\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right) - \frac{1}{6}\sqrt{-21+26\sqrt{3}}\,\tanh^{-1}\left(\frac{\sqrt{-\cdots}}{\cdots}\right.$$

Mathematica [C] time = 0.81, size = 286, normalized size = 2.78

$$\frac{\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(\sqrt{x^2-x+1}\left(\left((-9-12i)-(2-5i)\sqrt{3}\right)\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)+\left((-9+12i)+(2+5i)\sqrt{3}\right)\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)\right)+\frac{12\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(\left(\sqrt{3}+i\right)x-2i\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}\right)}{3\left(\sqrt{3}+i\right)\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x + 2*x^2)/((-2 + 2*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((12*Sqrt[((-1)^(1/3) - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]*(-2*I + (I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)
*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3
))] + Sqrt[1 - x + x^2]*(((-9 - 12*I) - (2 - 5*I)*Sqrt[3])*EllipticPi[(2*Sq
rt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)
^(1/3))]], (-1)^(1/3)] + ((-9 + 12*I) + (2 + 5*I)*Sqrt[3])*EllipticPi[((2*I
)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^
(1/3))]], (-1)^(1/3)])))/(3*(I + Sqrt[3])*Sqrt[1 + x^3])
```

IntegrateAlgebraic [A] time = 1.54, size = 103, normalized size = 1.00

$$-\frac{1}{6}\sqrt{21+26\sqrt{3}}\ \tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\ \sqrt{x^3+1}}{x^2-x+1}\right)-\frac{1}{6}\sqrt{26\sqrt{3}-21}\ \tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\ \sqrt{x^3+1}}{x^2-x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)/((-2 + 2*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] -1/6*(Sqrt[21 + 26*Sqrt[3]]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[1 + x^3])/(1 -
 x + x^2)]) - (Sqrt[-21 + 26*Sqrt[3]]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[1
+ x^3])/(1 - x + x^2)])/6
```

fricas [B] time = 0.48, size = 250, normalized size = 2.43

$$-\frac{1}{6}\sqrt{26\sqrt{3}+21}\arctan\left(\frac{\sqrt{x^3+1}\sqrt{26\sqrt{3}+21}\left(3\sqrt{3}+2\right)}{23\left(x^2-x+1\right)}\right)+\frac{1}{24}\sqrt{26\sqrt{3}-21}\log\left(\frac{23x^4-46x^3+138x^2+2\sqrt{x^3+1}\left(2x^2-\sqrt{3}\left(3x^2+2x+8\right)-14x-10\right)\sqrt{26\sqrt{3}-21}+92\sqrt{3}\left(x^3+1\right)+92x+92}{x^4+4x^3-8x+4}\right)-\frac{1}{24}\sqrt{26\sqrt{3}-21}\log\left(\frac{23x^4-46x^3+138x^2-2\sqrt{x^3+1}\left(2x^2-\sqrt{3}\left(3x^2+2x+8\right)-14x-10\right)\sqrt{26\sqrt{3}-21}+92\sqrt{3}\left(x^3+1\right)+92x+92}{x^4+4x^3-8x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(x^2+2*x-2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(26*sqrt(3) + 21)*arctan(1/23*sqrt(x^3 + 1)*sqrt(26*sqrt(3) + 21)*
(3*sqrt(3) + 2)/(x^2 - x + 1)) + 1/24*sqrt(26*sqrt(3) - 21)*log((23*x^4 - 4
6*x^3 + 138*x^2 + 2*sqrt(x^3 + 1)*(2*x^2 - sqrt(3)*(3*x^2 + 2*x + 8) - 14*x
 - 10)*sqrt(26*sqrt(3) - 21) + 92*sqrt(3)*(x^3 + 1) + 92*x + 92)/(x^4 + 4*x
^3 - 8*x + 4)) - 1/24*sqrt(26*sqrt(3) - 21)*log((23*x^4 - 46*x^3 + 138*x^2
- 2*sqrt(x^3 + 1)*(2*x^2 - sqrt(3)*(3*x^2 + 2*x + 8) - 14*x - 10)*sqrt(26*s
qrt(3) - 21) + 92*sqrt(3)*(x^3 + 1) + 92*x + 92)/(x^4 + 4*x^3 - 8*x + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2-x+3}{\sqrt{x^3+1}\left(x^2+2\,x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(x^2+2*x-2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 - x + 3)/(sqrt(x^3 + 1)*(x^2 + 2*x - 2)), x)
```

maple [C] time = 0.29, size = 1501, normalized size = 14.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)/(x^2+2*x-2)/(x^3+1)^(1/2),x)
```

```
[Out] 4*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+5/2*(1/(3/2-1/2*I*3^(1/2))*x+1/(3
```

```
/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))
-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-
3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*
EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1
/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-6*(1/(3/2-1/
2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(
-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*
I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+
1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-5
/2*I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3
^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1
/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1
/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(
1/2)))^(1/2))+2*I*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/
(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*
3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3
/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I
*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-
3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-6*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*
3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-
3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I
*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*EllipticP
i(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+5/2*I*(1/(3/2-1/2*I*3^(1/2))*x
+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(
1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1
/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(
1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*
3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*I*(1/(3/2-1/2*
I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3
/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*
3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2
*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1
/2)-5/2*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*
I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3
^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))
^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3
^(1/2)))^(1/2))*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^2 - x + 3}{\sqrt{x^3+1}\,\left(x^2+2\,x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(x^2+2*x-2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x + 3)/(sqrt(x^3 + 1)*(x^2 + 2*x - 2)), x)
```

mupad [B] time = 0.84, size = 509, normalized size = 4.94

$$\frac{4\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}+\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(5\sqrt{3}-12\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{3};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(5\sqrt{3}+12\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(-\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{3};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)/((x^3 + 1)^(1/2)*(2*x + x^2 - 2)),x)
```

```
[Out] (4*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2))^(1/2) + (((3^(1/2)*1i)/2 + 3/2)*(5*3^(1/2) - 12)*((x +
 (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*
ellipticPi((3^(1/2)*((3^(1/2)*1i)/2 + 3/2))/3, asin(((x + 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(x^3
- x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (((3^(1/2)*1i)/2 + 3/2)*(5*3^(1/2) +
 12)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*ellipticPi(-(3^(1/2)*((3^(1/2)*1i)/2 + 3/2))/3, asin(((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
))/(3*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^2 - x + 3}{\sqrt{(x+1)\left(x^2 - x + 1\right)}\left(x^2 + 2x - 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)/(x**2+2*x-2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((2*x**2 - x + 3)/(sqrt((x + 1)*(x**2 - x + 1))*(x**2 + 2*x - 2)),
x)
```

3.1277 $\int \frac{(-1+x^3)^{2/3}(1+x^3)}{x^3}\,dx$

Optimal. Leaf size=103

$$-\frac{1}{9}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{3\sqrt{3}}+\frac{(x^3-1)^{2/3}(2x^3-3)}{6x^2}+\frac{1}{18}\log\left(\sqrt[3]{x^3-1}\,x+(x^3-1)^{2/3}+x^2\right)$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 0.77, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {453, 195, 239}

$$-\frac{1}{6}x(x^3-1)^{2/3}-\frac{1}{6}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{(x^3-1)^{5/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(2/3)*(1 + x^3))/x^3,x]
```

```
[Out] -1/6*(x*(-1 + x^3)^(2/3)) + (-1 + x^3)^(5/3)/(2*x^2) + ArcTan[(1 + (2*x)/(-
1 + x^3)^(1/3))/Sqrt[3]]/(3*Sqrt[3]) - Log[-x + (-1 + x^3)^(1/3)]/6
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+x^3)^{2/3}(1+x^3)}{x^3}\,dx &= \frac{(-1+x^3)^{5/3}}{2x^2}-\frac{1}{2}\int (-1+x^3)^{2/3}\,dx\\ &= -\frac{1}{6}x(-1+x^3)^{2/3}+\frac{(-1+x^3)^{5/3}}{2x^2}+\frac{1}{3}\int \frac{1}{\sqrt[3]{-1+x^3}}\,dx\\ &= -\frac{1}{6}x(-1+x^3)^{2/3}+\frac{(-1+x^3)^{5/3}}{2x^2}+\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(-x+\sqrt[3]{-1+x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.02, size = 50, normalized size = 0.49

$$\frac{\left(x^3-1\right)^{2/3}\left(-\frac{x^3\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};x^3\right)}{\left(1-x^3\right)^{2/3}}+x^3-1\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)^(2/3)*(1 + x^3))/x^3,x]
```

```
[Out] ((-1 + x^3)^(2/3)*(-1 + x^3 - (x^3*Hypergeometric2F1[-2/3, 1/3, 4/3, x^3])/
(1 - x^3)^(2/3)))/(2*x^2)
```

IntegrateAlgebraic [A] time = 0.21, size = 103, normalized size = 1.00

$$-\frac{1}{9}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{3\sqrt{3}}+\frac{\left(x^3-1\right)^{2/3}\left(2x^3-3\right)}{6x^2}+\frac{1}{18}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(2/3)*(1 + x^3))/x^3,x]
```

```
[Out] ((-1 + x^3)^(2/3)*(-3 + 2*x^3))/(6*x^2) + ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x
^3)^(1/3))]/(3*Sqrt[3]) - Log[-x + (-1 + x^3)^(1/3)]/9 + Log[x^2 + x*(-1 +
x^3)^(1/3) + (-1 + x^3)^(2/3)]/18
```

fricas [A] time = 0.85, size = 112, normalized size = 1.09

$$\frac{2\sqrt{3}x^2\arctan\left(-\frac{25382\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}x^2-13720\sqrt{3}\left(x^3-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(5831x^3-7200\right)}{58653x^3-8000}\right)-x^2\log\left(-3\left(x^3-1\right)^{\frac{1}{3}}x^2+3\left(x^3-1\right)^{\frac{2}{3}}x+1\right)+3\left(2x^3-3\right)\left(x^3-1\right)^{\frac{2}{3}}}{18x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^3+1)/x^3,x, algorithm="fricas")
```

```
[Out] 1/18*(2*sqrt(3)*x^2*arctan(-(25382*sqrt(3)*(x^3 - 1)^(1/3)*x^2 - 13720*sqrt
(3)*(x^3 - 1)^(2/3)*x + sqrt(3)*(5831*x^3 - 7200))/(58653*x^3 - 8000)) - x^
2*log(-3*(x^3 - 1)^(1/3)*x^2 + 3*(x^3 - 1)^(2/3)*x + 1) + 3*(2*x^3 - 3)*(x^
3 - 1)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^3+1\right)\left(x^3-1\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^3+1)/x^3,x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)*(x^3 - 1)^(2/3)/x^3, x)
```

maple [C] time = 0.27, size = 56, normalized size = 0.54

$$\frac{2x^6-5x^3+3}{6x^2\left(x^3-1\right)^{\frac{1}{3}}}+\frac{\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{1}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^3\right)}{3\mathrm{signum}\left(x^3-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)*(x^3+1)/x^3,x)
```

[Out] 1/6*(2*x^6-5*x^3+3)/x^2/(x^3-1)^(1/3)+1/3/signum(x^3-1)^(1/3)*(-signum(x^3-1))^(1/3)*x*hypergeom([1/3,1/3],[4/3],x^3)

maxima [A] time = 0.56, size = 106, normalized size = 1.03

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3-1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\left(x^3-1\right)^{\frac{2}{3}}}{2x^2}-\frac{\left(x^3-1\right)^{\frac{2}{3}}}{3x^2\left(\frac{x^3-1}{x^3}-1\right)}+\frac{1}{18}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3-1\right)^{\frac{2}{3}}}{x^2}+1\right)-\frac{1}{9}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)^(2/3)*(x^3+1)/x^3,x, algorithm="maxima")

[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3)/x + 1)) - 1/2*(x^3 - 1)^(2/3)/x^2 - 1/3*(x^3 - 1)^(2/3)/(x^2*((x^3 - 1)/x^3 - 1)) + 1/18*log((x^3 - 1)^(1/3)/x + (x^3 - 1)^(2/3)/x^2 + 1) - 1/9*log((x^3 - 1)^(1/3)/x - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3-1\right)^{2/3}\,\left(x^3+1\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^3 - 1)^(2/3)*(x^3 + 1))/x^3,x)

[Out] int(((x^3 - 1)^(2/3)*(x^3 + 1))/x^3, x)

sympy [C] time = 2.26, size = 70, normalized size = 0.68

$$-\frac{xe^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\\frac{1}{3}\end{matrix}\middle|x^3\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-1)**(2/3)*(x**3+1)/x**3,x)

[Out] -x*exp(-I*pi/3)*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), x**3)/(3*gamma(4/3)) + exp(2*I*pi/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), x**3)/(3*x**2*gamma(1/3))

3.1278 $\int x^8 \left(b+ax^4\right)^{3/4} dx$

Optimal. Leaf size=103

$$\frac{5b^3\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{256a^{9/4}}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{256a^{9/4}}+\frac{\left(ax^4+b\right)^{3/4}\left(32a^2x^9+12abx^5-15b^2x\right)}{384a^2}$$

Rubi [A] time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {279, 321, 240, 212, 206, 203}

$$\frac{5b^3\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{256a^{9/4}}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{256a^{9/4}}-\frac{5b^2x\left(ax^4+b\right)^{3/4}}{128a^2}+\frac{1}{12}x^9\left(ax^4+b\right)^{3/4}+\frac{bx^5\left(ax^4+b\right)^{3/4}}{32a}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(b + a*x^4)^(3/4),x]
```

```
[Out] (-5*b^2*x*(b + a*x^4)^(3/4))/(128*a^2) + (b*x^5*(b + a*x^4)^(3/4))/(32*a) +
 (x^9*(b + a*x^4)^(3/4))/12 + (5*b^3*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)])
/(256*a^(9/4)) + (5*b^3*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(256*a^(9/4
))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^8 \left(b+ax^4\right)^{3/4} dx = \frac{1}{12}x^9 \left(b+ax^4\right)^{3/4} + \frac{1}{4}b \int \frac{x^8}{\sqrt[4]{b+ax^4}} dx$$
$$= \frac{bx^5 \left(b+ax^4\right)^{3/4}}{32a} + \frac{1}{12}x^9 \left(b+ax^4\right)^{3/4} - \frac{\left(5b^2\right) \int \frac{x^4}{\sqrt[4]{b+ax^4}} dx}{32a}$$
$$= -\frac{5b^2x \left(b+ax^4\right)^{3/4}}{128a^2} + \frac{bx^5 \left(b+ax^4\right)^{3/4}}{32a} + \frac{1}{12}x^9 \left(b+ax^4\right)^{3/4} + \frac{\left(5b^3\right) \int \frac{1}{\sqrt[4]{b+ax^4}} dx}{128a^2}$$
$$= -\frac{5b^2x \left(b+ax^4\right)^{3/4}}{128a^2} + \frac{bx^5 \left(b+ax^4\right)^{3/4}}{32a} + \frac{1}{12}x^9 \left(b+ax^4\right)^{3/4} + \frac{\left(5b^3\right) \text{Subst}\left(\int \frac{1}{1-ax^4} dx, x\right.}{128a^2}$$
$$= -\frac{5b^2x \left(b+ax^4\right)^{3/4}}{128a^2} + \frac{bx^5 \left(b+ax^4\right)^{3/4}}{32a} + \frac{1}{12}x^9 \left(b+ax^4\right)^{3/4} + \frac{\left(5b^3\right) \text{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2} dx\right.}{256a^2}$$
$$= -\frac{5b^2x \left(b+ax^4\right)^{3/4}}{128a^2} + \frac{bx^5 \left(b+ax^4\right)^{3/4}}{32a} + \frac{1}{12}x^9 \left(b+ax^4\right)^{3/4} + \frac{5b^3 \tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b+ax^4}}\right)}{256a^{9/4}} + \frac{5b}{}$$

Mathematica [C] time = 0.05, size = 94, normalized size = 0.91

$$\frac{x \left(ax^4+b\right)^{3/4} \left(\left(\frac{ax^4}{b}+1\right)^{3/4} \left(8a^2x^8+3abx^4-5b^2\right)+5b^2 \, {}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};-\frac{ax^4}{b}\right)\right)}{96a^2 \left(\frac{ax^4}{b}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(b + a*x^4)^(3/4),x]
```

```
[Out] (x*(b + a*x^4)^(3/4)*((1 + (a*x^4)/b)^(3/4)*(-5*b^2 + 3*a*b*x^4 + 8*a^2*x^8
) + 5*b^2*Hypergeometric2F1[-3/4, 1/4, 5/4, -((a*x^4)/b)]))/(96*a^2*(1 + (a
*x^4)/b)^(3/4))
```

IntegrateAlgebraic [A] time = 0.43, size = 103, normalized size = 1.00

$$\frac{5b^3 \tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{256a^{9/4}} + \frac{5b^3 \tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{256a^{9/4}} + \frac{\left(ax^4+b\right)^{3/4} \left(32a^2x^9+12abx^5-15b^2x\right)}{384a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8*(b + a*x^4)^(3/4),x]
```

```
[Out] ((b + a*x^4)^(3/4)*(-15*b^2*x + 12*a*b*x^5 + 32*a^2*x^9))/(384*a^2) + (5*b^
3*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(256*a^(9/4)) + (5*b^3*ArcTanh[(a^
(1/4)*x)/(b + a*x^4)^(1/4)])/(256*a^(9/4))
```

fricas [B] time = 0.43, size = 239, normalized size = 2.32

$$\frac{60 \left(\frac{b^{12}}{a^9}\right)^{\frac{1}{4}} a^2 \arctan\left(-\frac{\left(ax^4+b\right)^{\frac{1}{4}} \left(\frac{b^{12}}{a^9}\right)^{\frac{1}{4}} a^2 b^9 - \left(\frac{b^{12}}{a^9}\right)^{\frac{1}{4}} a^2 x \sqrt{\frac{\sqrt{\frac{b^{12}}{a^9}} a^5 b^{12} x^2 + \sqrt{ax^4+b}\, b^{18}}{x^2}}}{b^{12} x}\right) + 15 \left(\frac{b^{12}}{a^9}\right)^{\frac{1}{4}} a^2 \log\left(\frac{125 \left(\left(ax^4+b\right)^{\frac{1}{4}} b^9 + \left(\frac{b^{12}}{a^9}\right)^{\frac{3}{4}} a^7 x\right)}{x}\right) - 15 \left(\frac{b^{12}}{a^9}\right)^{\frac{1}{4}} a^2 \log\left(\frac{125 \left(\left(ax^4+b\right)^{\frac{1}{4}} b^9 - \left(\frac{b^{12}}{a^9}\right)^{\frac{3}{4}} a^7 x\right)}{x}\right) + 4 \left(32 a^2 x^9 + 12 abx^5 - 15 b^2 x\right) \left(ax^4+b\right)^{\frac{3}{4}}}{1536 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a*x^4+b)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/1536*(60*(b^12/a^9)^(1/4)*a^2*arctan(-((a*x^4 + b)^(1/4)*(b^12/a^9)^(1/4)
*a^2*b^9 - (b^12/a^9)^(1/4)*a^2*x*sqrt((sqrt(b^12/a^9)*a^5*b^12*x^2 + sqrt(
a*x^4 + b)*b^18)/x^2))/(b^12*x)) + 15*(b^12/a^9)^(1/4)*a^2*log(125*((a*x^4
+ b)^(1/4)*b^9 + (b^12/a^9)^(3/4)*a^7*x)/x) - 15*(b^12/a^9)^(1/4)*a^2*log(1
25*((a*x^4 + b)^(1/4)*b^9 - (b^12/a^9)^(3/4)*a^7*x)/x) + 4*(32*a^2*x^9 + 12
*a*b*x^5 - 15*b^2*x)*(a*x^4 + b)^(3/4))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^4+b\right)^{\frac{3}{4}}x^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a*x^4+b)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^4 + b)^(3/4)*x^8, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int x^8\left(a\,x^4+b\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a*x^4+b)^(3/4),x)
```

```
[Out] int(x^8*(a*x^4+b)^(3/4),x)
```

maxima [B] time = 0.51, size = 189, normalized size = 1.83

$$-\frac{5\,b^3\left(\frac{2\arctan\left(\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right)}{512\,a^2}-\frac{\frac{5\left(ax^4+b\right)^{\frac{3}{4}}a^2b^3}{x^3}+\frac{42\left(ax^4+b\right)^{\frac{7}{4}}ab^3}{x^7}-\frac{15\left(ax^4+b\right)^{\frac{11}{4}}b^3}{x^{11}}}{384\left(a^5-\frac{3\left(ax^4+b\right)a^4}{x^4}+\frac{3\left(ax^4+b\right)^2a^3}{x^8}-\frac{\left(ax^4+b\right)^3a^2}{x^{12}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a*x^4+b)^(3/4),x, algorithm="maxima")
```

```
[Out] -5/512*b^3*(2*arctan((a*x^4 + b)^(1/4)/(a^(1/4)*x))/a^(1/4) + log(-(a^(1/4)
 - (a*x^4 + b)^(1/4)/x)/(a^(1/4) + (a*x^4 + b)^(1/4)/x))/a^(1/4))/a^2 - 1/3
84*(5*(a*x^4 + b)^(3/4)*a^2*b^3/x^3 + 42*(a*x^4 + b)^(7/4)*a*b^3/x^7 - 15*(
a*x^4 + b)^(11/4)*b^3/x^11)/(a^5 - 3*(a*x^4 + b)*a^4/x^4 + 3*(a*x^4 + b)^2*
a^3/x^8 - (a*x^4 + b)^3*a^2/x^12)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^8\,{\left(a\,x^4+b\right)}^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b + a*x^4)^(3/4),x)
```

```
[Out] int(x^8*(b + a*x^4)^(3/4), x)
```

sympy [C] time = 1.75, size = 39, normalized size = 0.38

$$\frac{b^{\frac{3}{4}}x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| \frac{ax^4e^{i\pi}}{b}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a*x**4+b)**(3/4),x)
```

```
[Out] b**(3/4)*x**9*gamma(9/4)*hyper((-3/4, 9/4), (13/4,), a*x**4*exp_polar(I*pi)
/b)/(4*gamma(13/4))
```

3.1279 $\int \frac{1}{(-b+ax^3)\sqrt[4]{bx+ax^4}}\,dx$

Optimal. Leaf size=103

$$-\frac{2^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}\,b}-\frac{2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}\,b}$$

Rubi [A] time = 0.21, antiderivative size = 149, normalized size of antiderivative = 1.45, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2056, 466, 465, 377, 212, 206, 203}

$$-\frac{2^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3+b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{a}\,b\sqrt[4]{ax^4+bx}}-\frac{2^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3+b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{a}\,b\sqrt[4]{ax^4+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] -1/3*(2^(3/4)*x^(1/4)*(b + a*x^3)^(1/4)*ArcTan[(2^(1/4)*a^(1/4)*x^(3/4))/(b
 + a*x^3)^(1/4)])/(a^(1/4)*b*(b*x + a*x^4)^(1/4)) - (2^(3/4)*x^(1/4)*(b + a
*x^3)^(1/4)*ArcTanh[(2^(1/4)*a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)])/(3*a^(1/4
)*b*(b*x + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{(-b+ax^3)\sqrt[4]{bx+ax^4}}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\int \frac{1}{\sqrt[4]{x}(-b+ax^3)\sqrt[4]{b+ax^3}}\,dx}{\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int \frac{x^2}{(-b+ax^{12})\sqrt[4]{b+ax^{12}}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int \frac{1}{(-b+ax^4)\sqrt[4]{b+ax^4}}\,dx, x, x^{3/4}\right)}{3\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int \frac{1}{-b+2abx^4}\,dx, x, \frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}$$

$$= -\frac{\left(2\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\sqrt{a}x^2}\,dx, x, \frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3b\sqrt[4]{bx+ax^4}} - \frac{\left(2\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\mathrm{Subs}}{3b\sqrt[4]{}}$$

$$= -\frac{2^{3/4}\sqrt[4]{x}\sqrt[4]{b+ax^3}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{a}b\sqrt[4]{bx+ax^4}} - \frac{2^{3/4}\sqrt[4]{x}\sqrt[4]{b+ax^3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{a}b\sqrt[4]{bx+ax^4}}$$

Mathematica [C] time = 0.02, size = 44, normalized size = 0.43

$$-\frac{4x\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{2ax^3}{ax^3+b}\right)}{3b\sqrt[4]{x(ax^3+b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (-4*x*Hypergeometric2F1[1/4, 1, 5/4, (2*a*x^3)/(b + a*x^3)])/(3*b*(x*(b + a
*x^3))^(1/4))
```

IntegrateAlgebraic [A] time = 0.40, size = 103, normalized size = 1.00

$$-\frac{2^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}(ax^4+bx)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}b} - \frac{2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}(ax^4+bx)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] -1/3*(2^(3/4)*ArcTan[(2^(1/4)*a^(1/4)*(b*x + a*x^4)^(3/4))/(b + a*x^3)])/(a
^(1/4)*b) - (2^(3/4)*ArcTanh[(2^(1/4)*a^(1/4)*(b*x + a*x^4)^(3/4))/(b + a*x
^3)])/(3*a^(1/4)*b)
```

fricas [B] time = 107.33, size = 422, normalized size = 4.10

$$\frac{2}{3}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(ax^4+bx\right)^{\frac{3}{4}}ab^3\left(\frac{1}{ab^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(ax^4+bx\right)^{\frac{1}{4}}abx^2\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}+\left(2\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{ax^4+bx}\,abx\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}+\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(3a^2b^3x^3+ab^4\right)\left(\frac{1}{ab^4}\right)^{\frac{3}{4}}\right)\sqrt{\sqrt{\frac{1}{2}}b^2\sqrt{\frac{1}{ab^4}}}\right)}{ax^3-b}\right)-\frac{1}{6}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{ax^4+bx}\,ab^3x\left(\frac{1}{ab^4}\right)^{\frac{3}{4}}+4\sqrt{\frac{1}{2}}\left(ax^4+bx\right)^{\frac{1}{4}}ab^2x^2\sqrt{\frac{1}{ab^4}}+\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(3abx^3+b^2\right)\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}+2\left(ax^4+bx\right)^{\frac{3}{4}}}{ax^3-b}\right)+\frac{1}{6}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{ax^4+bx}\,ab^3x\left(\frac{1}{ab^4}\right)^{\frac{3}{4}}-4\sqrt{\frac{1}{2}}\left(ax^4+bx\right)^{\frac{1}{4}}ab^2x^2\sqrt{\frac{1}{ab^4}}+\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(3abx^3+b^2\right)\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}-2\left(ax^4+bx\right)^{\frac{3}{4}}}{ax^3-b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3-b)/(a*x^4+b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/3*(1/2)^(1/4)*(1/(a*b^4))^(1/4)*arctan(2*(2*(1/2)^(3/4)*(a*x^4 + b*x)^(3/
4)*a*b^3*(1/(a*b^4))^(3/4) + 2*(1/2)^(1/4)*(a*x^4 + b*x)^(1/4)*a*b*x^2*(1/(
a*b^4))^(1/4) + (2*(1/2)^(1/4)*sqrt(a*x^4 + b*x)*a*b*x*(1/(a*b^4))^(1/4) +
(1/2)^(3/4)*(3*a^2*b^3*x^3 + a*b^4)*(1/(a*b^4))^(3/4))*sqrt(sqrt(1/2)*b^2*s
qrt(1/(a*b^4))))/(a*x^3 - b)) - 1/6*(1/2)^(1/4)*(1/(a*b^4))^(1/4)*log((4*(1
/2)^(3/4)*sqrt(a*x^4 + b*x)*a*b^3*x*(1/(a*b^4))^(3/4) + 4*sqrt(1/2)*(a*x^4
+ b*x)^(1/4)*a*b^2*x^2*sqrt(1/(a*b^4)) + (1/2)^(1/4)*(3*a*b*x^3 + b^2)*(1/(
a*b^4))^(1/4) + 2*(a*x^4 + b*x)^(3/4))/(a*x^3 - b)) + 1/6*(1/2)^(1/4)*(1/(a
*b^4))^(1/4)*log(-(4*(1/2)^(3/4)*sqrt(a*x^4 + b*x)*a*b^3*x*(1/(a*b^4))^(3/4
) - 4*sqrt(1/2)*(a*x^4 + b*x)^(1/4)*a*b^2*x^2*sqrt(1/(a*b^4)) + (1/2)^(1/4)
*(3*a*b*x^3 + b^2)*(1/(a*b^4))^(1/4) - 2*(a*x^4 + b*x)^(3/4))/(a*x^3 - b))
```

giac [B] time = 0.42, size = 203, normalized size = 1.97

$$-\frac{2^{\frac{1}{4}}(-a)^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{3ab}-\frac{2^{\frac{1}{4}}(-a)^{\frac{3}{4}}\arctan\left(-\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{3ab}-\frac{2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)}{6(-a)^{\frac{1}{4}}b}-\frac{2^{\frac{1}{4}}(-a)^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)}{6ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3-b)/(a*x^4+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] -1/3*2^(1/4)*(-a)^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4)*(-a)^(1/4) + 2*(a + b/x
^3)^(1/4))/(-a)^(1/4))/(a*b) - 1/3*2^(1/4)*(-a)^(3/4)*arctan(-1/2*2^(1/4)*(
2^(3/4)*(-a)^(1/4) - 2*(a + b/x^3)^(1/4))/(-a)^(1/4))/(a*b) - 1/6*2^(1/4)*l
og(2^(3/4)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(2)*sqrt(-a) + sqrt(a + b/x^3
))/((-a)^(1/4)*b) - 1/6*2^(1/4)*(-a)^(3/4)*log(-2^(3/4)*(-a)^(1/4)*(a + b/x
^3)^(1/4) + sqrt(2)*sqrt(-a) + sqrt(a + b/x^3))/(a*b)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,x^{3}-b\right)\left(a\,x^{4}+bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3-b)/(a*x^4+b*x)^(1/4),x)
```

```
[Out] int(1/(a*x^3-b)/(a*x^4+b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^{4}+bx\right)^{\frac{1}{4}}\left(ax^{3}-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3-b)/(a*x^4+b*x)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((a*x^4 + b*x)^(1/4)*(a*x^3 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(a\,x^4+b\,x\right)^{1/4}\,\left(b-a\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((b*x + a*x^4)^(1/4)*(b - a*x^3)),x)
[Out] -int(1/((b*x + a*x^4)^(1/4)*(b - a*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x\left(ax^3+b\right)}\,\left(ax^3-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**3-b)/(a*x**4+b*x)**(1/4),x)
[Out] Integral(1/((x*(a*x**3 + b))**(1/4)*(a*x**3 - b)), x)
```

3.1280 $\int \frac{x^2(-2+x^4)}{\sqrt[3]{-x+x^5}\,(-1+x^4+x^8)}\,dx$

Optimal. Leaf size=103

$$\frac{1}{4}\log\left(\sqrt[3]{x^5-x}+x^3\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^5-x}}{\sqrt[3]{x^5-x}-2x^3}\right)-\frac{1}{8}\log\left(x^6+\left(x^5-x\right)^{2/3}-\sqrt[3]{x^5-x}\,x^3\right)$$

Rubi [C] time = 0.77, antiderivative size = 117, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2056, 6728, 466, 465, 511, 510}

$$\frac{3\sqrt[3]{1-x^4}\,x^3F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^4,-\frac{2x^4}{1-\sqrt{5}}\right)}{8\sqrt[3]{x^5-x}}+\frac{3\sqrt[3]{1-x^4}\,x^3F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^4,-\frac{2x^4}{1+\sqrt{5}}\right)}{8\sqrt[3]{x^5-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^2*(-2 + x^4))/((-x + x^5)^(1/3)*(-1 + x^4 + x^8)),x]
```

```
[Out] (3*x^3*(1 - x^4)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, x^4, (-2*x^4)/(1 - Sqrt[5
])])/(8*(-x + x^5)^(1/3)) + (3*x^3*(1 - x^4)^(1/3)*AppellF1[2/3, 1/3, 1, 5/
3, x^4, (-2*x^4)/(1 + Sqrt[5])])/(8*(-x + x^5)^(1/3))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
```

```
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^2\left(-2+x^4\right)}{\sqrt[3]{-x+x^5}\left(-1+x^4+x^8\right)}\,dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\int \frac{x^{5/3}\left(-2+x^4\right)}{\sqrt[3]{-1+x^4}\left(-1+x^4+x^8\right)}\,dx}{\sqrt[3]{-x+x^5}}$$

$$= \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\int \left(\frac{\left(1-\sqrt{5}\right)x^{5/3}}{\sqrt[3]{-1+x^4}\left(1-\sqrt{5}+2x^4\right)}+\frac{\left(1+\sqrt{5}\right)x^{5/3}}{\sqrt[3]{-1+x^4}\left(1+\sqrt{5}+2x^4\right)}\right)dx}{\sqrt[3]{-x+x^5}}$$

$$= \frac{\left(\left(1-\sqrt{5}\right)\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\int \frac{x^{5/3}}{\sqrt[3]{-1+x^4}\left(1-\sqrt{5}+2x^4\right)}\,dx}{\sqrt[3]{-x+x^5}} + \frac{\left(\left(1+\sqrt{5}\right)\sqrt[3]{x}\sqrt[3]{-1+x\ldots}\right.}{\sqrt[3]{-\ldots}}$$

$$= \frac{\left(3\left(1-\sqrt{5}\right)\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \frac{x^7}{\sqrt[3]{-1+x^{12}}\left(1-\sqrt{5}+2x^{12}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x+x^5}} + \frac{\left(3\left(1\ldots\right.\right.}{\ldots}$$

$$= \frac{\left(3\left(1-\sqrt{5}\right)\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{-1+x^3}\left(1-\sqrt{5}+2x^3\right)}\,dx,x,x^{4/3}\right)}{4\sqrt[3]{-x+x^5}} + \frac{\left(3\left(1-\ldots\right.\right.}{\ldots}$$

$$= \frac{\left(3\left(1-\sqrt{5}\right)\sqrt[3]{x}\sqrt[3]{1-x^4}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x^3}\left(1-\sqrt{5}+2x^3\right)}\,dx,x,x^{4/3}\right)}{4\sqrt[3]{-x+x^5}} + \frac{\left(3\left(1+\ldots\right.\right.}{\ldots}$$

$$= \frac{3x^3\sqrt[3]{1-x^4}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^4,-\frac{2x^4}{1-\sqrt{5}}\right)}{8\sqrt[3]{-x+x^5}} + \frac{3x^3\sqrt[3]{1-x^4}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^4,-\frac{2x^4}{1+\sqrt{5}\ldots}\right)}{8\sqrt[3]{-x+x^5}}$$

Mathematica [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-2+x^4\right)}{\sqrt[3]{-x+x^5}\left(-1+x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(-2 + x^4))/((-x + x^5)^(1/3)*(-1 + x^4 + x^8)),x]
[Out] Integrate[(x^2*(-2 + x^4))/((-x + x^5)^(1/3)*(-1 + x^4 + x^8)), x]
```

IntegrateAlgebraic [A] time = 0.52, size = 103, normalized size = 1.00

$$\frac{1}{4}\log\left(\sqrt[3]{x^5-x}+x^3\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^5-x}}{\sqrt[3]{x^5-x}-2x^3}\right)-\frac{1}{8}\log\left(x^6+\left(x^5-x\right)^{2/3}-\sqrt[3]{x^5-x}\,x^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(-2 + x^4))/((-x + x^5)^(1/3)*(-1 + x^4 + x^8)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(-x + x^5)^(1/3))/(-2*x^3 + (-x + x^5)^(1/3))])/4
+ Log[x^3 + (-x + x^5)^(1/3)]/4 - Log[x^6 - x^3*(-x + x^5)^(1/3) + (-x + x^
5)^(2/3)]/8
```

fricas [A] time = 3.73, size = 112, normalized size = 1.09

$$-\frac{1}{4}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^8+2\sqrt{3}\left(x^5-x\right)^{\frac{1}{3}}x^5+4\sqrt{3}\left(x^5-x\right)^{\frac{2}{3}}x^2}{x^8-8x^4+8}\right)+\frac{1}{8}\log\left(\frac{x^8+3\left(x^5-x\right)^{\frac{1}{3}}x^5+x^4+3\left(x^5-x\right)^{\frac{2}{3}}x^2-1}{x^8+x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4-2)/(x^5-x)^(1/3)/(x^8+x^4-1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(3)*arctan((sqrt(3)*x^8 + 2*sqrt(3)*(x^5 - x)^(1/3)*x^5 + 4*sqrt(3
)*(x^5 - x)^(2/3)*x^2)/(x^8 - 8*x^4 + 8)) + 1/8*log((x^8 + 3*(x^5 - x)^(1/3
)*x^5 + x^4 + 3*(x^5 - x)^(2/3)*x^2 - 1)/(x^8 + x^4 - 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4-2)/(x^5-x)^(1/3)/(x^8+x^4-1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [C] time = 44.10, size = 613, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4-2)/(x^5-x)^(1/3)/(x^8+x^4-1),x)
```

```
[Out] 1/4*ln((4552326368968648991039692719756000*RootOf(16*_Z^2+4*_Z+1)^2*x^8+2199
310112028943683020577141723220*RootOf(16*_Z^2+4*_Z+1)*x^8+10964658969698991
50977315191298816*x^8-10105152304592546264409879974272 4*RootOf(16*_Z^2+4*_Z
+1)*(x^5-x)^(1/3)*x^5-7769303669706494278041075575050240*RootOf(16*_Z^2+4*_
Z+1)^2*x^4+1589389345365026959344463317732671*(x^5-x)^(1/3)*x^5-64586089045
06033300021952070673408*RootOf(16*_Z^2+4*_Z+1)*(x^5-x)^(2/3)*x^2-6214381346
082003948643537340963924*RootOf(16*_Z^2+4*_Z+1)*x^4-252628807614813656610 24
699935681*(x^5-x)^(2/3)*x^2-10322198483193191225997381293086 51*x^4+77693036
69706494278041075575050240*RootOf(16*_Z^2+4*_Z+1)^2+621438134608200394864 35
37340963924*RootOf(16*_Z^2+4*_Z+1)+10322198483193191225997381293086 51)/(x^8
+x^4-1))-1/4*ln((5727041415124155549372637198670 40*RootOf(16*_Z^2+4*_Z+1)^2
*x^8-2043377440472549032154367693505284*RootOf(16*_Z^2+4*_Z+1)*x^8-10322198
4831931912259973812930865 1*x^8+6357557381460107837377853270930684*RootOf(16
*_Z^2+4*_Z+1)*(x^5-x)^(1/3)*x^5-9774150681811892137595967485730816*RootOf(1
6*_Z^2+4*_Z+1)^2*x^4-2526288076148136566102469993568 1*(x^5-x)^(1/3)*x^5+645
8608904506033300021952070673408*RootOf(16*_Z^2+4*_Z+1)*(x^5-x)^(2/3)*x^2-45
29039623257700492643576695162024*RootOf(16*_Z^2+4*_Z+1)*x^4+158938934536502
6959344463317732671*(x^5-x)^(2/3)*x^2-6424604865058002837757706199016 5*x^4+
9774150681811892137595967485730816*RootOf(16*_Z^2+4*_Z+1)^2+452903962325770
0492643576695162024*RootOf(16*_Z^2+4*_Z+1)+6424604865058002837757706199016 5
)/(x^8+x^4-1))-ln((5727041415124155549372637198670 40*RootOf(16*_Z^2+4*_Z+1)
^2*x^8-2043377440472549032154367693505284*RootOf(16*_Z^2+4*_Z+1)*x^8-103221
98483193191225997381293086 51*x^8+6357557381460107837377853270930684*RootOf(
16*_Z^2+4*_Z+1)*(x^5-x)^(1/3)*x^5-9774150681811892137595967485730816*RootOf
(16*_Z^2+4*_Z+1)^2*x^4-2526288076148136566102469993568 1*(x^5-x)^(1/3)*x^5+6
```

```
4586089045060333000219520706734O8*RootOf(16*_Z^2+4*_Z+1)*(x^5-x)^(2/3)*x^2-
4529039623257700492643576695162024*RootOf(16*_Z^2+4*_Z+1)*x^4+1589389345365
026959344463317732671*(x^5-x)^(2/3)*x^2-642460486505800283775770619901 65*x^
4+977415068181189213759596748573O816*RootOf(16*_Z^2+4*_Z+1)^2+4529039623257
700492643576695162024*RootOf(16*_Z^2+4*_Z+1)+6424604865058002837757706199O1
65)/(x^8+x^4-1))*RootOf(16*_Z^2+4*_Z+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4-2)x^2}{(x^8+x^4-1)(x^5-x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4-2)/(x^5-x)^(1/3)/(x^8+x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 2)*x^2/((x^8 + x^4 - 1)*(x^5 - x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,(x^4-2)}{(x^5-x)^{1/3}\,(x^8+x^4-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^4 - 2))/((x^5 - x)^(1/3)*(x^4 + x^8 - 1)),x)
```

```
[Out] int((x^2*(x^4 - 2))/((x^5 - x)^(1/3)*(x^4 + x^8 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**4-2)/(x**5-x)**(1/3)/(x**8+x**4-1),x)
```

```
[Out] Timed out
```

3.1281 $\int \frac{1}{x^2\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$

Optimal. Leaf size=103

$$-\frac{1}{x\sqrt{\sqrt{a^2x^2+b^2}+ax}}+\frac{a\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{b^{3/2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{b^{3/2}}$$

Rubi [A] time = 0.21, antiderivative size = 130, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2119, 457, 329, 212, 206, 203}

$$\frac{2a\sqrt{\sqrt{a^2x^2+b^2}+ax}}{b^2-\left(\sqrt{a^2x^2+b^2}+ax\right)^2}+\frac{a\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{b^{3/2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

```
[Out] (2*a*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/(b^2 - (a*x + Sqrt[b^2 + a^2*x^2])^2)
 + (a*ArcTan[Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]/Sqrt[b]])/b^(3/2) + (a*ArcTanh
[Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]/Sqrt[b]])/b^(3/2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx = (2a)\,\text{Subst}\left(\int \frac{b^2+x^2}{\sqrt{x}\left(-b^2+x^2\right)^2}\,dx, x, ax+\sqrt{b^2+a^2x^2}\right)$$

$$= \frac{2a\sqrt{ax+\sqrt{b^2+a^2x^2}}}{b^2-\left(ax+\sqrt{b^2+a^2x^2}\right)^2} - a\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-b^2+x^2\right)}\,dx, x, ax+\sqrt{b^2+a^2x^2}\right)$$

$$= \frac{2a\sqrt{ax+\sqrt{b^2+a^2x^2}}}{b^2-\left(ax+\sqrt{b^2+a^2x^2}\right)^2} - (2a)\,\text{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx, x, \sqrt{ax+\sqrt{b^2+a^2x^2}}\right)$$

$$= \frac{2a\sqrt{ax+\sqrt{b^2+a^2x^2}}}{b^2-\left(ax+\sqrt{b^2+a^2x^2}\right)^2} + \frac{a\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{ax+\sqrt{b^2+a^2x^2}}\right)}{b} + \frac{a\,\text{Sul}}{}$$

$$= \frac{2a\sqrt{ax+\sqrt{b^2+a^2x^2}}}{b^2-\left(ax+\sqrt{b^2+a^2x^2}\right)^2} + \frac{a\tan^{-1}\left(\frac{\sqrt{ax+\sqrt{b^2+a^2x^2}}}{\sqrt{b}}\right)}{b^{3/2}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{ax+\sqrt{b^2+a^2x^2}}}{\sqrt{b}}\right)}{b^{3/2}}$$

Mathematica [C] time = 23.02, size = 9150, normalized size = 88.83

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.18, size = 103, normalized size = 1.00

$$-\frac{1}{x\sqrt{\sqrt{a^2x^2+b^2}+ax}} + \frac{a\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{b^{3/2}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

```
[Out] -(1/(x*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])) + (a*ArcTan[Sqrt[a*x + Sqrt[b^2 +
a^2*x^2]]/Sqrt[b]])/b^(3/2) + (a*ArcTanh[Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]/Sq
rt[b]])/b^(3/2)
```

fricas [A] time = 0.45, size = 335, normalized size = 3.25

$$\left[\frac{2\,a\sqrt{b}\,x\arctan\left(\frac{\sqrt{ax+\sqrt{a^2x^2+b^2}}}{\sqrt{b}}\right)+a\sqrt{b}\,x\log\left(\frac{b^2-\sqrt{ax+\sqrt{a^2x^2+b^2}}\left((ax-b)\sqrt{b}-\sqrt{a^2x^2+b^2}\,\sqrt{b}\right)+\sqrt{a^2x^2+b^2}\,b}{x}\right)+2\sqrt{ax+\sqrt{a^2x^2+b^2}}\left(ax-\sqrt{a^2x^2+b^2}\right)}{2\,b^2x}, -\frac{2\,a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{ax+\sqrt{a^2x^2+b^2}}\,\sqrt{-b}}{b}\right)+a\sqrt{-b}\,x\log\left(-\frac{b^2+\sqrt{ax+\sqrt{a^2x^2+b^2}}\left((ax+b)\sqrt{-b}-\sqrt{a^2x^2+b^2}\,\sqrt{-b}\right)-\sqrt{a^2x^2+b^2}\,b}{x}\right)-2\sqrt{ax+\sqrt{a^2x^2+b^2}}\left(ax-\sqrt{a^2x^2+b^2}\right)}{2\,b^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*sqrt(b)*x*arctan(sqrt(a*x + sqrt(a^2*x^2 + b^2))/sqrt(b)) + a*sqr
t(b)*x*log((b^2 - sqrt(a*x + sqrt(a^2*x^2 + b^2))*((a*x - b)*sqrt(b) - sqrt
(a^2*x^2 + b^2)*sqrt(b)) + sqrt(a^2*x^2 + b^2)*b)/x) + 2*sqrt(a*x + sqrt(a^
2*x^2 + b^2))*(a*x - sqrt(a^2*x^2 + b^2)))/(b^2*x), -1/2*(2*a*sqrt(-b)*x*ar
ctan(sqrt(a*x + sqrt(a^2*x^2 + b^2))*sqrt(-b)/b) + a*sqrt(-b)*x*log(-(b^2 +
 sqrt(a*x + sqrt(a^2*x^2 + b^2))*((a*x + b)*sqrt(-b) - sqrt(a^2*x^2 + b^2)*
sqrt(-b)) - sqrt(a^2*x^2 + b^2)*b)/x) - 2*sqrt(a*x + sqrt(a^2*x^2 + b^2))*(
a*x - sqrt(a^2*x^2 + b^2)))/(b^2*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + \sqrt{a^2x^2 + b^2}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + sqrt(a^2*x^2 + b^2))*x^2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{ax + \sqrt{a^2x^2 + b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/x^2/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + \sqrt{a^2x^2 + b^2}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + sqrt(a^2*x^2 + b^2))*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{a\,x + \sqrt{a^2\,x^2 + b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2)),x)
[Out] int(1/(x^2*(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*x+(a**2*x**2+b**2)**(1/2))**(1/2),x)
[Out] Integral(1/(x**2*sqrt(a*x + sqrt(a**2*x**2 + b**2))), x)
```

3.1282 $\int \frac{\sqrt[3]{-1+x^3}}{x^{10}}\,dx$

Optimal. Leaf size=104

$$\frac{5}{243}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{5}{486}\log\left(\left(x^3-1\right)^{2/3}-\sqrt[3]{x^3-1}+1\right)-\frac{5\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{\sqrt[3]{x^3-1}\left(5x^6+3x^3-18\right)}{162x^9}$$

Rubi [A] time = 0.11, antiderivative size = 100, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {266, 47, 51, 58, 618, 204, 31}

$$\frac{5\sqrt[3]{x^3-1}}{162x^3}+\frac{5}{162}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{5\tan^{-1}\left(\frac{1-2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{81\sqrt{3}}-\frac{\sqrt[3]{x^3-1}}{9x^9}+\frac{\sqrt[3]{x^3-1}}{54x^6}-\frac{5\log(x)}{162}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)^(1/3)/x^10,x]
```

```
[Out] -1/9*(-1 + x^3)^(1/3)/x^9 + (-1 + x^3)^(1/3)/(54*x^6) + (5*(-1 + x^3)^(1/3)
)/(162*x^3) - (5*ArcTan[(1 - 2*(-1 + x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]) - (
5*Log[x])/162 + (5*Log[1 + (-1 + x^3)^(1/3)])/162
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{-1+x^3}}{x^{10}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt[3]{-1+x}}{x^4}\,dx,x,x^3\right)\\
&= -\frac{\sqrt[3]{-1+x^3}}{9x^9}+\frac{1}{27}\operatorname{Subst}\left(\int \frac{1}{(-1+x)^{2/3}x^3}\,dx,x,x^3\right)\\
&= -\frac{\sqrt[3]{-1+x^3}}{9x^9}+\frac{\sqrt[3]{-1+x^3}}{54x^6}+\frac{5}{162}\operatorname{Subst}\left(\int \frac{1}{(-1+x)^{2/3}x^2}\,dx,x,x^3\right)\\
&= -\frac{\sqrt[3]{-1+x^3}}{9x^9}+\frac{\sqrt[3]{-1+x^3}}{54x^6}+\frac{5\sqrt[3]{-1+x^3}}{162x^3}+\frac{5}{243}\operatorname{Subst}\left(\int \frac{1}{(-1+x)^{2/3}x}\,dx,x,x^3\right)\\
&= -\frac{\sqrt[3]{-1+x^3}}{9x^9}+\frac{\sqrt[3]{-1+x^3}}{54x^6}+\frac{5\sqrt[3]{-1+x^3}}{162x^3}-\frac{5\log(x)}{162}+\frac{5}{162}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt[3]{-1+x^3}\right.\\
&= -\frac{\sqrt[3]{-1+x^3}}{9x^9}+\frac{\sqrt[3]{-1+x^3}}{54x^6}+\frac{5\sqrt[3]{-1+x^3}}{162x^3}-\frac{5\log(x)}{162}+\frac{5}{162}\log\left(1+\sqrt[3]{-1+x^3}\right)-\frac{5}{81}\operatorname{Subst}\\
&= -\frac{\sqrt[3]{-1+x^3}}{9x^9}+\frac{\sqrt[3]{-1+x^3}}{54x^6}+\frac{5\sqrt[3]{-1+x^3}}{162x^3}-\frac{5\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^3}}{\sqrt{3}}\right)}{81\sqrt{3}}-\frac{5\log(x)}{162}+\frac{5}{162}\log\left(1+\right.
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.27

$$\frac{1}{4}\left(x^3-1\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},4;\frac{7}{3};1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)^(1/3)/x^10,x]
```

```
[Out] ((-1 + x^3)^(4/3)*Hypergeometric2F1[4/3, 4, 7/3, 1 - x^3])/4
```

IntegrateAlgebraic [A] time = 0.19, size = 104, normalized size = 1.00

$$\frac{5}{243}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{5}{486}\log\left(\left(x^3-1\right)^{2/3}-\sqrt[3]{x^3-1}+1\right)-\frac{5\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{\sqrt[3]{x^3-1}\left(5x^6+3x^3-18\right)}{162x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)^(1/3)/x^10,x]
```

```
[Out] ((-1 + x^3)^(1/3)*(-18 + 3*x^3 + 5*x^6))/(162*x^9) - (5*ArcTan[1/Sqrt[3] -
(2*(-1 + x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]) + (5*Log[1 + (-1 + x^3)^(1/3)])
/243 - (5*Log[1 - (-1 + x^3)^(1/3) + (-1 + x^3)^(2/3)])/486
```

fricas [A] time = 0.43, size = 93, normalized size = 0.89

$$\frac{10\sqrt{3}x^9\arctan\left(\frac{2}{3}\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-5x^9\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+10x^9\log\left(\left(x^3-1\right)^{\frac{1}{3}}+1\right)+3\left(5x^6+3x^3-18\right)\left(x^3-1\right)^{\frac{1}{3}}}{486x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)/x^10,x, algorithm="fricas")
```

```
[Out] 1/486*(10*sqrt(3)*x^9*arctan(2/3*sqrt(3)*(x^3 - 1)^(1/3) - 1/3*sqrt(3)) - 5
*x^9*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 10*x^9*log((x^3 - 1)^(1/3
) + 1) + 3*(5*x^6 + 3*x^3 - 18)*(x^3 - 1)^(1/3))/x^9
```

giac [A] time = 0.24, size = 90, normalized size = 0.87

$$\frac{5}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{5\left(x^3-1\right)^{\frac{7}{3}}+13\left(x^3-1\right)^{\frac{4}{3}}-10\left(x^3-1\right)^{\frac{1}{3}}}{162x^9}-\frac{5}{486}\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+\frac{5}{243}\log\left(\left|\left(x^3-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)/x^10,x, algorithm="giac")
```

```
[Out] 5/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) + 1/162*(5*(x^3 -
 1)^(7/3) + 13*(x^3 - 1)^(4/3) - 10*(x^3 - 1)^(1/3))/x^9 - 5/486*log((x^3 -
 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 5/243*log(abs((x^3 - 1)^(1/3) + 1))
```

maple [C] time = 0.28, size = 96, normalized size = 0.92

$$\frac{5x^9-2x^6-21x^3+18}{162x^9\left(x^3-1\right)^{\frac{2}{3}}}+\frac{5\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{2}{3}}\left(\frac{2\Gamma\left(\frac{2}{3}\right)x^3\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],x^3\right)}{3}+\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)\right)}{243\Gamma\left(\frac{2}{3}\right)\mathrm{signum}\left(x^3-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/3)/x^10,x)
```

```
[Out] 1/162*(5*x^9-2*x^6-21*x^3+18)/x^9/(x^3-1)^(2/3)+5/243/GAMMA(2/3)/signum(x^3
-1)^(2/3)*(-signum(x^3-1))^(2/3)*(2/3*GAMMA(2/3)*x^3*hypergeom([1,1,5/3],[2
,2],x^3)+(1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*GAMMA(2/3))
```

maxima [A] time = 0.41, size = 111, normalized size = 1.07

$$\frac{5}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{5\left(x^3-1\right)^{\frac{7}{3}}+13\left(x^3-1\right)^{\frac{4}{3}}-10\left(x^3-1\right)^{\frac{1}{3}}}{162\left(\left(x^3-1\right)^3+3x^3+3\left(x^3-1\right)^2-2\right)}-\frac{5}{486}\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+\frac{5}{243}\log\left(\left(x^3-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)/x^10,x, algorithm="maxima")
```

```
[Out] 5/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) + 1/162*(5*(x^3 -
 1)^(7/3) + 13*(x^3 - 1)^(4/3) - 10*(x^3 - 1)^(1/3))/((x^3 - 1)^3 + 3*x^3 +
 3*(x^3 - 1)^2 - 2) - 5/486*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 5/
243*log((x^3 - 1)^(1/3) + 1)
```

mupad [B] time = 0.99, size = 124, normalized size = 1.19

$$\frac{5\ln\left(\frac{25\left(x^3-1\right)^{1/3}}{6561}+\frac{25}{6561}\right)}{243}+\frac{\frac{13\left(x^3-1\right)^{4/3}}{162}-\frac{5\left(x^3-1\right)^{1/3}}{81}+\frac{5\left(x^3-1\right)^{7/3}}{162}}{3\left(x^3-1\right)^2+\left(x^3-1\right)^3+3x^3-2}-\ln\left(\frac{5}{54}-\frac{5\left(x^3-1\right)^{1/3}}{27}+\frac{\sqrt{3}\,5\mathrm{i}}{54}\right)\left(\frac{5}{486}+\frac{\sqrt{3}\,5\mathrm{i}}{486}\right)+\ln\left(\frac{5\left(x^3-1\right)^{1/3}}{27}-\frac{5}{54}+\frac{\sqrt{3}\,5\mathrm{i}}{54}\right)\left(-\frac{5}{486}+\frac{\sqrt{3}\,5\mathrm{i}}{486}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)^(1/3)/x^10,x)
```

```
[Out] (5*log((25*(x^3 - 1)^(1/3))/6561 + 25/6561))/243 + ((13*(x^3 - 1)^(4/3))/16
2 - (5*(x^3 - 1)^(1/3))/81 + (5*(x^3 - 1)^(7/3))/162)/(3*(x^3 - 1)^2 + (x^3
 - 1)^3 + 3*x^3 - 2) - log((3^(1/2)*5i)/54 - (5*(x^3 - 1)^(1/3))/27 + 5/54)
*((3^(1/2)*5i)/486 + 5/486) + log((3^(1/2)*5i)/54 + (5*(x^3 - 1)^(1/3))/27
- 5/54)*((3^(1/2)*5i)/486 - 5/486)
```

sympy [C] time = 1.42, size = 34, normalized size = 0.33

$$-\frac{\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^3}\right)}{3x^8\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/3)/x**10,x)
```

```
[Out] -gamma(8/3)*hyper((-1/3, 8/3), (11/3,), exp_polar(2*I*pi)/x**3)/(3*x**8*gam
ma(11/3))
```

3.1283 $\int x^4\sqrt[3]{-1+x^3}\,dx$

Optimal. Leaf size=104

$$\frac{1}{27}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-1}+x}\right)}{9\sqrt{3}}-\frac{1}{54}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)+\frac{1}{18}\sqrt[3]{x^3-1}\left(3x^5-x^2\right)$$

Rubi [A] time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {279, 321, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{27}\log\left(1-\frac{x}{\sqrt[3]{x^3-1}}\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{1}{6}\sqrt[3]{x^3-1}\,x^5-\frac{1}{18}\sqrt[3]{x^3-1}\,x^2-\frac{1}{54}\log\left(\frac{x}{\sqrt[3]{x^3-1}}+\frac{x^2}{\left(x^3-1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(-1 + x^3)^(1/3),x]
```

```
[Out] -1/18*(x^2*(-1 + x^3)^(1/3)) + (x^5*(-1 + x^3)^(1/3))/6 + ArcTan[(1 + (2*x)
/(-1 + x^3)^(1/3))/Sqrt[3]]/(9*Sqrt[3]) + Log[1 - x/(-1 + x^3)^(1/3)]/27 -
Log[1 + x^2/(-1 + x^3)^(2/3) + x/(-1 + x^3)^(1/3)]/54
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int x^4\sqrt[3]{-1+x^3}\,dx &= \frac{1}{6}x^5\sqrt[3]{-1+x^3}-\frac{1}{6}\int\frac{x^4}{\left(-1+x^3\right)^{2/3}}\,dx\\
&= -\frac{1}{18}x^2\sqrt[3]{-1+x^3}+\frac{1}{6}x^5\sqrt[3]{-1+x^3}-\frac{1}{9}\int\frac{x}{\left(-1+x^3\right)^{2/3}}\,dx\\
&= -\frac{1}{18}x^2\sqrt[3]{-1+x^3}+\frac{1}{6}x^5\sqrt[3]{-1+x^3}-\frac{1}{9}\mathrm{Subst}\left(\int\frac{x}{1-x^3}\,dx,x,\frac{x}{\sqrt[3]{-1+x^3}}\right)\\
&= -\frac{1}{18}x^2\sqrt[3]{-1+x^3}+\frac{1}{6}x^5\sqrt[3]{-1+x^3}-\frac{1}{27}\mathrm{Subst}\left(\int\frac{1}{1-x}\,dx,x,\frac{x}{\sqrt[3]{-1+x^3}}\right)+\frac{1}{27}\mathrm{Subst}\Bigg(\int\\
&= -\frac{1}{18}x^2\sqrt[3]{-1+x^3}+\frac{1}{6}x^5\sqrt[3]{-1+x^3}+\frac{1}{27}\log\left(1-\frac{x}{\sqrt[3]{-1+x^3}}\right)-\frac{1}{54}\mathrm{Subst}\left(\int\frac{1+2x}{1+x+x^2}\,d\right.\\
&= -\frac{1}{18}x^2\sqrt[3]{-1+x^3}+\frac{1}{6}x^5\sqrt[3]{-1+x^3}+\frac{1}{27}\log\left(1-\frac{x}{\sqrt[3]{-1+x^3}}\right)-\frac{1}{54}\log\left(1+\frac{x^2}{\left(-1+x^3\right)^{2/3}}\right.\\
&= -\frac{1}{18}x^2\sqrt[3]{-1+x^3}+\frac{1}{6}x^5\sqrt[3]{-1+x^3}+\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{1}{27}\log\left(1-\frac{x}{\sqrt[3]{-1+x^3}}\right)-\frac{1}{54}\mathrm{lo}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.52

$$\frac{x^2\sqrt[3]{x^3-1}\left({}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\left(1-x^3\right)^{4/3}\right)}{6\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(-1 + x^3)^(1/3),x]
```

[Out] (x^2*(-1 + x^3)^(1/3)*(-(1 - x^3)^(4/3) + Hypergeometric2F1[-1/3, 2/3, 5/3, x^3]))/(6*(1 - x^3)^(1/3))

IntegrateAlgebraic [A] time = 0.18, size = 104, normalized size = 1.00

$$\frac{1}{27}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{9\sqrt{3}}-\frac{1}{54}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)+\frac{1}{18}\sqrt[3]{x^3-1}\left(3x^5-x^2\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4*(-1 + x^3)^(1/3),x]

[Out] ((-1 + x^3)^(1/3)*(-x^2 + 3*x^5))/18 + ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x^3)^(1/3))]/(9*Sqrt[3]) + Log[-x + (-1 + x^3)^(1/3)]/27 - Log[x^2 + x*(-1 + x^3)^(1/3) + (-1 + x^3)^(2/3)]/54

fricas [A] time = 0.41, size = 96, normalized size = 0.92

$$-\frac{1}{27}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{18}\left(3x^5-x^2\right)\left(x^3-1\right)^{\frac{1}{3}}+\frac{1}{27}\log\left(-\frac{x-\left(x^3-1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{54}\log\left(\frac{x^2+\left(x^3-1\right)^{\frac{1}{3}}x+\left(x^3-1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(x^3-1)^(1/3),x, algorithm="fricas")

[Out] -1/27*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 - 1)^(1/3))/x) + 1/18*(3*x^5 - x^2)*(x^3 - 1)^(1/3) + 1/27*log(-(x - (x^3 - 1)^(1/3))/x) - 1/54*log((x^2 + (x^3 - 1)^(1/3)*x + (x^3 - 1)^(2/3))/x^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^3-1\right)^{\frac{1}{3}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(x^3-1)^(1/3),x, algorithm="giac")

[Out] integrate((x^3 - 1)^(1/3)*x^4, x)

maple [C] time = 0.27, size = 53, normalized size = 0.51

$$\frac{x^2\left(3x^3-1\right)\left(x^3-1\right)^{\frac{1}{3}}}{18}-\frac{\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{2}{3}}x^2\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{18\mathrm{signum}\left(x^3-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(x^3-1)^(1/3),x)

[Out] 1/18*x^2*(3*x^3-1)*(x^3-1)^(1/3)-1/18/signum(x^3-1)^(2/3)*(-signum(x^3-1))^(2/3)*x^2*hypergeom([2/3,2/3],[5/3],x^3)

maxima [A] time = 0.46, size = 121, normalized size = 1.16

$$-\frac{1}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3-1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\frac{2\left(x^3-1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3-1\right)^{\frac{4}{3}}}{x^4}}{18\left(\frac{2\left(x^3-1\right)}{x^3}-\frac{\left(x^3-1\right)^2}{x^6}-1\right)}-\frac{1}{54}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3-1\right)^{\frac{2}{3}}}{x^2}+1\right)+\frac{1}{27}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^3-1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3)/x + 1)) - 1/18*(2*(x^3
- 1)^(1/3)/x + (x^3 - 1)^(4/3)/x^4)/(2*(x^3 - 1)/x^3 - (x^3 - 1)^2/x^6 - 1)
 - 1/54*log((x^3 - 1)^(1/3)/x + (x^3 - 1)^(2/3)/x^2 + 1) + 1/27*log((x^3 -
1)^(1/3)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(x^3-1\right)^{1/3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^3 - 1)^(1/3),x)
```

```
[Out] int(x^4*(x^3 - 1)^(1/3), x)
```

sympy [C] time = 1.00, size = 32, normalized size = 0.31

$$\frac{x^5 e^{\frac{i\pi}{3}} \Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**3-1)**(1/3),x)
```

```
[Out] x**5*exp(I*pi/3)*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), x**3)/(3*gamma(8/3))
```

3.1284 $\int \sqrt[3]{-x+x^3}\,dx$

Optimal. Leaf size=104

$$\frac{1}{2}\sqrt[3]{x^3-x}\,x+\frac{1}{6}\log\left(\sqrt[3]{x^3-x}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x}+x}\right)}{2\sqrt{3}}-\frac{1}{12}\log\left(\sqrt[3]{x^3-x}\,x+\left(x^3-x\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.16, antiderivative size = 186, normalized size of antiderivative = 1.79, number of steps used = 11, number of rules used = 11, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2004, 2032, 329, 275, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{2}\sqrt[3]{x^3-x}\,x+\frac{\left(x^2-1\right)^{2/3}x^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2-1}}\right)}{6\left(x^3-x\right)^{2/3}}-\frac{\left(x^2-1\right)^{2/3}x^{2/3}\log\left(\frac{x^{4/3}}{\left(x^2-1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2-1}}+1\right)}{12\left(x^3-x\right)^{2/3}}+\frac{\left(x^2-1\right)^{2/3}x^{2/3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1}{\sqrt{3}}\right)}{2\sqrt{3}\left(x^3-x\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)^(1/3),x]
```

```
[Out] (x*(-x + x^3)^(1/3))/2 + (x^(2/3)*(-1 + x^2)^(2/3)*ArcTan[(1 + (2*x^(2/3))/
(-1 + x^2)^(1/3))/Sqrt[3]])/(2*Sqrt[3]*(-x + x^3)^(2/3)) + (x^(2/3)*(-1 + x
^2)^(2/3)*Log[1 - x^(2/3)/(-1 + x^2)^(1/3)])/(6*(-x + x^3)^(2/3)) - (x^(2/3
)*(-1 + x^2)^(2/3)*Log[1 + x^(4/3)/(-1 + x^2)^(2/3) + x^(2/3)/(-1 + x^2)^(1
/3)])/(12*(-x + x^3)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \sqrt[3]{-x+x^3}\,dx = \frac{1}{2}x\sqrt[3]{-x+x^3} - \frac{1}{3}\int \frac{x}{\left(-x+x^3\right)^{2/3}}\,dx$$

$$= \frac{1}{2}x\sqrt[3]{-x+x^3} - \frac{\left(x^{2/3}\left(-1+x^2\right)^{2/3}\right)\int \frac{\sqrt[3]{x}}{\left(-1+x^2\right)^{2/3}}\,dx}{3\left(-x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}x\sqrt[3]{-x+x^3} - \frac{\left(x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x^3}{\left(-1+x^6\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{\left(-x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}x\sqrt[3]{-x+x^3} - \frac{\left(x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{\left(-1+x^3\right)^{2/3}}\,dx, x, x^{2/3}\right)}{2\left(-x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}x\sqrt[3]{-x+x^3} - \frac{\left(x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2\left(-x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}x\sqrt[3]{-x+x^3} - \frac{\left(x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{6\left(-x+x^3\right)^{2/3}} + \frac{\left(x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \right.}{6\left(-x+x^3\right.}$$

$$= \frac{1}{2}x\sqrt[3]{-x+x^3} + \frac{x^{2/3}\left(-1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{6\left(-x+x^3\right)^{2/3}} - \frac{\left(x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x,\right.}{12\left(-x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}x\sqrt[3]{-x+x^3} + \frac{x^{2/3}\left(-1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{6\left(-x+x^3\right)^{2/3}} - \frac{x^{2/3}\left(-1+x^2\right)^{2/3}\log\left(1+\frac{x^{4/3}}{\left(-1+x^2\right)^{2/3}}+\frac{x^{2}}{\sqrt[3]{-1}}\right.}{12\left(-x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}x\sqrt[3]{-x+x^3} + \frac{x^{2/3}\left(-1+x^2\right)^{2/3}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\left(-x+x^3\right)^{2/3}} + \frac{x^{2/3}\left(-1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{6\left(-x+x^3\right)^{2/3}} - \frac{x^{2/3}}{}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.38

$$\frac{3x\sqrt[3]{x\left(x^2-1\right)}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^2\right)}{4\sqrt[3]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)^(1/3),x]
```

```
[Out] (3*x*(x*(-1 + x^2))^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/3, x^2])/(4*(1 - x
^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.18, size = 104, normalized size = 1.00

$$\frac{1}{2}\sqrt[3]{x^3-x}\,x + \frac{1}{6}\log\left(\sqrt[3]{x^3-x}-x\right) + \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x}+x}\right)}{2\sqrt{3}} - \frac{1}{12}\log\left(\sqrt[3]{x^3-x}\,x+\left(x^3-x\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x + x^3)^(1/3),x]
```

```
[Out] (x*(-x + x^3)^(1/3))/2 + ArcTan[(Sqrt[3]*x)/(x + 2*(-x + x^3)^(1/3))]/(2*Sq
rt[3]) + Log[-x + (-x + x^3)^(1/3)]/6 - Log[x^2 + x*(-x + x^3)^(1/3) + (-x
+ x^3)^(2/3)]/12
```

fricas [A] time = 0.60, size = 99, normalized size = 0.95

$$\frac{1}{6}\sqrt{3}\arctan\left(-\frac{44032959556\sqrt{3}\left(x^3-x\right)^{\frac{1}{3}}x+\sqrt{3}\left(16754327161x^2-2707204793\right)-10524305234\sqrt{3}\left(x^3-x\right)^{\frac{2}{3}}}{81835897185x^2-1102302937}\right)+\frac{1}{2}\left(x^3-x\right)^{\frac{1}{3}}x+\frac{1}{12}\log\left(-3\left(x^3-x\right)^{\frac{1}{3}}x+3\left(x^3-x\right)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(-(44032959556*sqrt(3)*(x^3 - x)^(1/3)*x + sqrt(3)*(16754
327161*x^2 - 2707204793) - 10524305234*sqrt(3)*(x^3 - x)^(2/3))/(8183589718
5*x^2 - 1102302937)) + 1/2*(x^3 - x)^(1/3)*x + 1/12*log(-3*(x^3 - x)^(1/3)*
x + 3*(x^3 - x)^(2/3) + 1)
```

giac [A] time = 0.34, size = 77, normalized size = 0.74

$$\frac{1}{2}x^2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{12}\log\left(\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(\left|\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*x^2*(-1/x^2 + 1)^(1/3) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-1/x^2 + 1)
^(1/3) + 1)) - 1/12*log((-1/x^2 + 1)^(2/3) + (-1/x^2 + 1)^(1/3) + 1) + 1/6*
log(abs((-1/x^2 + 1)^(1/3) - 1))
```

maple [C] time = 0.29, size = 33, normalized size = 0.32

$$\frac{3\mathrm{signum}\left(x^2-1\right)^{\frac{1}{3}}x^{\frac{4}{3}}\mathrm{hypergeom}\left(\left[-\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^2\right)}{4\left(-\mathrm{signum}\left(x^2-1\right)\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x)^(1/3),x)
```

```
[Out] 3/4*signum(x^2-1)^(1/3)/(-signum(x^2-1))^(1/3)*x^(4/3)*hypergeom([-1/3,2/3]
,[5/3],x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^3-x\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - x)^(1/3), x)
```

mupad [B] time = 0.93, size = 29, normalized size = 0.28

$$\frac{3\,x\,\left(x^3-x\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{2}{3};\ \frac{5}{3};\ x^2\right)}{4\,\left(1-x^2\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - x)^(1/3),x)
[Out] (3*x*(x^3 - x)^(1/3)*hypergeom([-1/3, 2/3], 5/3, x^2))/(4*(1 - x^2)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{x^3 - x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x)**(1/3),x)
[Out] Integral((x**3 - x)**(1/3), x)
```

3.1285 $\int \frac{\sqrt[3]{-1+x^3}\left(-1+2x^3\right)}{x^{10}}\,dx$

Optimal. Leaf size=104

$$\frac{13}{243}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{13}{486}\log\left(\left(x^3-1\right)^{2/3}-\sqrt[3]{x^3-1}+1\right)-\frac{13\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{\sqrt[3]{x^3-1}\left(13x^6-57x^3\right.}{162x^9}$$

Rubi [A] time = 0.08, antiderivative size = 100, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {446, 78, 47, 51, 58, 618, 204, 31}

$$\frac{13\sqrt[3]{x^3-1}}{162x^3}+\frac{13}{162}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{13\tan^{-1}\left(\frac{1-2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{81\sqrt{3}}-\frac{\left(x^3-1\right)^{4/3}}{9x^9}-\frac{13\sqrt[3]{x^3-1}}{54x^6}-\frac{13\log(x)}{162}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(1/3)*(-1 + 2*x^3))/x^10,x]
```

```
[Out] (-13*(-1 + x^3)^(1/3))/(54*x^6) + (13*(-1 + x^3)^(1/3))/(162*x^3) - (-1 + x
^3)^(4/3)/(9*x^9) - (13*ArcTan[(1 - 2*(-1 + x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[
3]) - (13*Log[x])/162 + (13*Log[1 + (-1 + x^3)^(1/3)])/162
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{-1+x^3}\left(-1+2x^3\right)}{x^{10}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{-1+x}\,(-1+2x)}{x^4}\,dx,x,x^3\right)\\
&= -\frac{\left(-1+x^3\right)^{4/3}}{9x^9}+\frac{13}{27}\text{Subst}\left(\int \frac{\sqrt[3]{-1+x}}{x^3}\,dx,x,x^3\right)\\
&= -\frac{13\sqrt[3]{-1+x^3}}{54x^6}-\frac{\left(-1+x^3\right)^{4/3}}{9x^9}+\frac{13}{162}\text{Subst}\left(\int \frac{1}{(-1+x)^{2/3}x^2}\,dx,x,x^3\right)\\
&= -\frac{13\sqrt[3]{-1+x^3}}{54x^6}+\frac{13\sqrt[3]{-1+x^3}}{162x^3}-\frac{\left(-1+x^3\right)^{4/3}}{9x^9}+\frac{13}{243}\text{Subst}\left(\int \frac{1}{(-1+x)^{2/3}x}\,dx,x,
\right.\\
&= -\frac{13\sqrt[3]{-1+x^3}}{54x^6}+\frac{13\sqrt[3]{-1+x^3}}{162x^3}-\frac{\left(-1+x^3\right)^{4/3}}{9x^9}-\frac{13\log(x)}{162}+\frac{13}{162}\text{Subst}\left(\int \frac{1}{1+x}\right.\\
&= -\frac{13\sqrt[3]{-1+x^3}}{54x^6}+\frac{13\sqrt[3]{-1+x^3}}{162x^3}-\frac{\left(-1+x^3\right)^{4/3}}{9x^9}-\frac{13\log(x)}{162}+\frac{13}{162}\log\left(1+\sqrt[3]{-1+x\right.\\
&= -\frac{13\sqrt[3]{-1+x^3}}{54x^6}+\frac{13\sqrt[3]{-1+x^3}}{162x^3}-\frac{\left(-1+x^3\right)^{4/3}}{9x^9}-\frac{13\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^3}}{\sqrt{3}}\right)}{81\sqrt{3}}-\frac{13\log(x)}{162}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 38, normalized size = 0.37

$$\frac{\left(x^3-1\right)^{4/3}\left(13x^9\,{}_2F_1\left(\frac{4}{3},3;\frac{7}{3};1-x^3\right)-4\right)}{36x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)^(1/3)*(-1 + 2*x^3))/x^10,x]
```

```
[Out] ((-1 + x^3)^(4/3)*(-4 + 13*x^9*Hypergeometric2F1[4/3, 3, 7/3, 1 - x^3]))/(3
6*x^9)
```

IntegrateAlgebraic [A] time = 0.17, size = 104, normalized size = 1.00

$$\frac{13}{243}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{13}{486}\log\left(\left(x^3-1\right)^{2/3}-\sqrt[3]{x^3-1}+1\right)-\frac{13\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{\sqrt[3]{x^3-1}\left(13x^6-57x^3+18\right)}{162x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(1/3)*(-1 + 2*x^3))/x^10,x]
```

```
[Out] ((-1 + x^3)^(1/3)*(18 - 57*x^3 + 13*x^6))/(162*x^9) - (13*ArcTan[1/Sqrt[3]
- (2*(-1 + x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]) + (13*Log[1 + (-1 + x^3)^(1/3
)])/243 - (13*Log[1 - (-1 + x^3)^(1/3) + (-1 + x^3)^(2/3)])/486
```

fricas [A] time = 0.41, size = 93, normalized size = 0.89

$$\frac{26\sqrt{3}x^9\arctan\left(\frac{2}{3}\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-13x^9\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+26x^9\log\left(\left(x^3-1\right)^{\frac{1}{3}}+1\right)+3\left(13x^6-57x^3+18\right)\left(x^3-1\right)^{\frac{1}{3}}}{486x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(2*x^3-1)/x^10,x, algorithm="fricas")
```

```
[Out] 1/486*(26*sqrt(3)*x^9*arctan(2/3*sqrt(3)*(x^3 - 1)^(1/3) - 1/3*sqrt(3)) - 1
3*x^9*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 26*x^9*log((x^3 - 1)^(1/
3) + 1) + 3*(13*x^6 - 57*x^3 + 18)*(x^3 - 1)^(1/3))/x^9
```

giac [A] time = 0.18, size = 90, normalized size = 0.87

$$\frac{13}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{13\left(x^3-1\right)^{\frac{7}{3}}-31\left(x^3-1\right)^{\frac{4}{3}}-26\left(x^3-1\right)^{\frac{1}{3}}}{162x^9}-\frac{13}{486}\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+\frac{13}{243}\log\left(\left|\left(x^3-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(2*x^3-1)/x^10,x, algorithm="giac")
```

```
[Out] 13/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) + 1/162*(13*(x^3
 - 1)^(7/3) - 31*(x^3 - 1)^(4/3) - 26*(x^3 - 1)^(1/3))/x^9 - 13/486*log((x^
3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 13/243*log(abs((x^3 - 1)^(1/3) + 1))
```

maple [C] time = 0.29, size = 96, normalized size = 0.92

$$\frac{13x^9-70x^6+75x^3-18}{162x^9\left(x^3-1\right)^{\frac{2}{3}}}+\frac{13\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{2}{3}}\left(\frac{2\Gamma\left(\frac{2}{3}\right)x^3\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],x^3\right)}{3}+\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)\right)}{243\Gamma\left(\frac{2}{3}\right)\mathrm{signum}\left(x^3-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/3)*(2*x^3-1)/x^10,x)
```

```
[Out] 1/162*(13*x^9-70*x^6+75*x^3-18)/x^9/(x^3-1)^(2/3)+13/243/GAMMA(2/3)/signum(
x^3-1)^(2/3)*(-signum(x^3-1))^(2/3)*(2/3*GAMMA(2/3)*x^3*hypergeom([1,1,5/3]
,[2,2],x^3)+(1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*GAMMA(2/3))
```

maxima [A] time = 0.41, size = 146, normalized size = 1.40

$$\frac{13}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3-1\right)^{\frac{1}{3}}-1\right)\right)-\frac{5\left(x^3-1\right)^{\frac{7}{3}}+13\left(x^3-1\right)^{\frac{4}{3}}-10\left(x^3-1\right)^{\frac{1}{3}}}{162\left(\left(x^3-1\right)^3+3x^3+3\left(x^3-1\right)^2-2\right)}+\frac{\left(x^3-1\right)^{\frac{4}{3}}-2\left(x^3-1\right)^{\frac{1}{3}}}{9\left(2x^3+\left(x^3-1\right)^2-1\right)}-\frac{13}{486}\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+\frac{13}{243}\log\left(\left(x^3-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(2*x^3-1)/x^10,x, algorithm="maxima")
```

```
[Out] 13/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) - 1/162*(5*(x^3
- 1)^(7/3) + 13*(x^3 - 1)^(4/3) - 10*(x^3 - 1)^(1/3))/((x^3 - 1)^3 + 3*x^3
+ 3*(x^3 - 1)^2 - 2) + 1/9*((x^3 - 1)^(4/3) - 2*(x^3 - 1)^(1/3))/(2*x^3 + (
x^3 - 1)^2 - 1) - 13/486*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 13/24
3*log((x^3 - 1)^(1/3) + 1)
```

mupad [B] time = 1.38, size = 231, normalized size = 2.22

$$\frac{2\ln\left(\frac{4\,{\left(x^3-1\right)}^{1/3}}{81}+\frac{4}{81}\right)}{27}-\frac{5\ln\left(\frac{25\,{\left(x^3-1\right)}^{1/3}}{6561}+\frac{25}{6561}\right)}{243}-\frac{\frac{13\,{\left(x^3-1\right)}^{4/3}}{162}-\frac{5\,{\left(x^3-1\right)}^{1/3}}{81}+\frac{5\,{\left(x^3-1\right)}^{7/3}}{162}}{3\,{\left(x^3-1\right)}^2+{\left(x^3-1\right)}^3+3\,x^3-2}-\frac{\frac{2\,{\left(x^3-1\right)}^{1/3}}{9}-\frac{{\left(x^3-1\right)}^{4/3}}{9}}{{\left(x^3-1\right)}^2+2\,x^3-1}-\ln\left(\frac{1}{3}-\frac{2\,{\left(x^3-1\right)}^{1/3}}{3}+\frac{\sqrt{3}\,1{}\mathrm{i}}{3}\right)\,\left(\frac{1}{27}+\frac{\sqrt{3}\,1{}\mathrm{i}}{27}\right)+\ln\left(\frac{2\,{\left(x^3-1\right)}^{1/3}}{3}-\frac{1}{3}+\frac{\sqrt{3}\,1{}\mathrm{i}}{3}\right)\,\left(-\frac{1}{27}+\frac{\sqrt{3}\,1{}\mathrm{i}}{27}\right)+\ln\left(\frac{5}{54}-\frac{5\,{\left(x^3-1\right)}^{1/3}}{27}+\frac{\sqrt{3}\,5{}\mathrm{i}}{54}\right)\,\left(\frac{5}{486}+\frac{\sqrt{3}\,5{}\mathrm{i}}{486}\right)-\ln\left(\frac{5\,{\left(x^3-1\right)}^{1/3}}{27}-\frac{5}{54}+\frac{\sqrt{3}\,5{}\mathrm{i}}{54}\right)\,\left(-\frac{5}{486}+\frac{\sqrt{3}\,5{}\mathrm{i}}{486}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(1/3)*(2*x^3 - 1))/x^10,x)
```

```
[Out] (2*log((4*(x^3 - 1)^(1/3))/81 + 4/81))/27 - (5*log((25*(x^3 - 1)^(1/3))/656
1 + 25/6561))/243 - ((13*(x^3 - 1)^(4/3))/162 - (5*(x^3 - 1)^(1/3))/81 + (5
*(x^3 - 1)^(7/3))/162)/(3*(x^3 - 1)^2 + (x^3 - 1)^3 + 3*x^3 - 2) - ((2*(x^3
 - 1)^(1/3))/9 - (x^3 - 1)^(4/3)/9)/((x^3 - 1)^2 + 2*x^3 - 1) - log((3^(1/2
)*1i)/3 - (2*(x^3 - 1)^(1/3))/3 + 1/3)*((3^(1/2)*1i)/27 + 1/27) + log((3^(1
/2)*1i)/3 + (2*(x^3 - 1)^(1/3))/3 - 1/3)*((3^(1/2)*1i)/27 - 1/27) + log((3^
(1/2)*5i)/54 - (5*(x^3 - 1)^(1/3))/27 + 5/54)*((3^(1/2)*5i)/486 + 5/486) -
log((3^(1/2)*5i)/54 + (5*(x^3 - 1)^(1/3))/27 - 5/54)*((3^(1/2)*5i)/486 - 5/
486)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/3)*(2*x**3-1)/x**10,x)
```

```
[Out] Timed out
```

3.1286 $\int \frac{1+k^3x^3}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(-1+k^3x^3\right)}\,dx$

Optimal. Leaf size=104

$$-\frac{2\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}\right)}{3(k-1)}-\frac{4\tan^{-1}\left(\frac{\sqrt{k^2+k+1}\,\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{(x-1)\left(k^2x-1\right)}\right)}{3\sqrt{k^2+k+1}}$$

Rubi [C] time = 4.13, antiderivative size = 362, normalized size of antiderivative = 3.48, number of steps used = 19, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6718, 6725, 115, 6688, 934, 12, 168, 537}

$$\frac{4(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,\Pi\left(\frac{1}{k};\sin^{-1}\left(\sqrt{-k^2}\,\sqrt{-x}\right)|\frac{1}{k^2}\right)}{3\sqrt{-k^2}\,\sqrt{x-x^2}\,\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{4(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,\Pi\left(-\frac{\sqrt[3]{-1}}{k};\sin^{-1}\left(\sqrt{-k^2}\,\sqrt{-x}\right)|\frac{1}{k^2}\right)}{3\sqrt{-k^2}\,\sqrt{x-x^2}\,\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{4(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,\Pi\left(\frac{(-1)^{2/3}}{k};\sin^{-1}\left(\sqrt{-k^2}\,\sqrt{-x}\right)|\frac{1}{k^2}\right)}{3\sqrt{-k^2}\,\sqrt{x-x^2}\,\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + k^3*x^3)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^3*x^3)),x]
```

```
[Out] (2*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticF[ArcSin[Sqrt[x]], k^2])/Sqr
t[(1 - x)*x*(1 - k^2*x)] + (4*(1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^2*x]*Elli
pticPi[k^(-1), ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/(3*Sqrt[-k^2]*Sqrt[(1
- x)*x*(1 - k^2*x)]*Sqrt[x - x^2]) + (4*(1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k
^2*x]*EllipticPi[-((-1)^(1/3)/k), ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/(3*
Sqrt[-k^2]*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2]) + (4*(1 - x)*Sqrt[-x]
*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticPi[(-1)^(2/3)/k, ArcSin[Sqrt[-k^2]*Sqrt[-x
]], k^(-2)])/(3*Sqrt[-k^2]*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+k^3x^3}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(-1+k^3x^3\right)}\,dx = \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{1+k^3x^3}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\left(-1+k^3x^3\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int\left(\frac{1}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}}+\frac{2}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\left(-1+k^3x^3\right)}\right)\,d}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int\frac{1}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{\left(2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right.}{\sqrt{(1-}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{\left(2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)}{\sqrt{(1-x)x}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{\left(2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)}{}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}}-\frac{\left(2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int}{3\sqrt{(1-x)x\,(}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}}-\frac{\left(2\sqrt{2}\,\sqrt{2-2x}\,\sqrt{1-x}\,\sqrt{-}\right.}{3\sqrt{(}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}}-\frac{\left(2\sqrt{2-2x}\,\sqrt{1-x}\,\sqrt{-x}\,\sqrt{}\right.}{3\sqrt{(1-}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{\left(4\sqrt{2-2x}\,\sqrt{1-x}\,\sqrt{-x}\,\sqrt{}\right.}{}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{4(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-k^2}}{3\sqrt{-k^2}\,\sqrt{(1-x}}$$

Mathematica [C] time = 6.62, size = 310, normalized size = 2.98

$$-\frac{2\sqrt{\frac{1}{x-1}+1}\,(x-1)^{3/2}\sqrt{\frac{1-\frac{1}{k^2}}{x-1}+1}\,\sqrt{k^2x-1}\left(2i\sqrt{3}\left(k^2+k+1\right)\Pi\left(\frac{k-1}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+(k-1)\left(\left(\left(3-i\sqrt{3}\right)k-2i\sqrt{3}\right)\Pi\left(\frac{2\left(k^2+k+1\right)}{k\left(2k-i\sqrt{3}+1\right)};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)-i\left(2\sqrt{3}+\left(\sqrt{3}-3i\right)k\right)\Pi\left(\frac{2\left(k^2+k+1\right)}{k\left(2k+i\sqrt{3}+1\right)};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)-3i\sqrt{3}\left(k^3+1\right)F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)}{3\left(k^3-1\right)\sqrt{(x-1)x\left(k^2x-1\right)}\,\sqrt{3k^2x-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + k^3*x^3)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^3*x^3)),x]
```

```
[Out] (-2*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (1 - k^(-2))/(-1 + x)]*(-1 + x)^(3/2)*
Sqrt[-1 + k^2*x]*((-3*I)*Sqrt[3]*(1 + k^3)*EllipticF[I*ArcSinh[1/Sqrt[-1 +
x]], 1 - k^(-2)] + (2*I)*Sqrt[3]*(1 + k + k^2)*EllipticPi[(-1 + k)/k, I*Arc
Sinh[1/Sqrt[-1 + x]], 1 - k^(-2)] + (-1 + k)*(((-2*I)*Sqrt[3] + (3 - I*Sqrt
[3])*k)*EllipticPi[(2*(1 + k + k^2))/(k*(1 - I*Sqrt[3] + 2*k)), I*ArcSinh[1
```

```
/Sqrt[-1 + x]], 1 - k^(-2)] - I*(2*Sqrt[3] + (-3*I + Sqrt[3])*k)*EllipticPi
[(2*(1 + k + k^2))/(k*(1 + I*Sqrt[3] + 2*k)), I*ArcSinh[1/Sqrt[-1 + x]], 1
- k^(-2)]))))/(3*(-1 + k^3)*Sqrt[(-1 + x)*x*(-1 + k^2*x)]*Sqrt[-3 + 3*k^2*x]
)
```

IntegrateAlgebraic [A] time = 0.36, size = 104, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}\right)}{3(k-1)}-\frac{4\tan^{-1}\left(\frac{\sqrt{k^2+k+1}\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{(x-1)\left(k^2x-1\right)}\right)}{3\sqrt{k^2+k+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + k^3*x^3)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^3*x^3
)),x]
```

```
[Out] (-2*ArcTan[((-1 + k)*x)/Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3]])/(3*(-1 + k)) -
 (4*ArcTan[(Sqrt[1 + k + k^2]*Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3])/((-1 + x)
*(-1 + k^2*x))])/(3*Sqrt[1 + k + k^2])
```

fricas [B] time = 0.51, size = 207, normalized size = 1.99

$$\frac{2\sqrt{k^2+k+1}\,(k-1)\arctan\left(\frac{\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\,\left(k^2x^2-\left(2k^2+k+2\right)x+1\right)\sqrt{k^2+k+1}}{2\left(\left(k^4+k^3+k^2\right)x^3-\left(k^4+k^3+2k^2+k+1\right)x^2+\left(k^2+k+1\right)x\right)}\right)+\left(k^2+k+1\right)\arctan\left(\frac{\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\,\left(k^2x^2-2\left(k^2-k+1\right)x+1\right)}{2\left(\left(k^3-k^2\right)x^3-\left(k^3-k^2+k-1\right)x^2+(k-1)x\right)}\right)}{3\left(k^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^3*x^3+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^3*x^3-1),x, algorithm="
fricas")
```

```
[Out] 1/3*(2*sqrt(k^2 + k + 1)*(k - 1)*arctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 +
x)*(k^2*x^2 - (2*k^2 + k + 2)*x + 1)*sqrt(k^2 + k + 1)/((k^4 + k^3 + k^2)*x
^3 - (k^4 + k^3 + 2*k^2 + k + 1)*x^2 + (k^2 + k + 1)*x)) + (k^2 + k + 1)*ar
ctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2 - 2*(k^2 - k + 1)*x + 1
)/((k^3 - k^2)*x^3 - (k^3 - k^2 + k - 1)*x^2 + (k - 1)*x)))/(k^3 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^3x^3+1}{\left(k^3x^3-1\right)\sqrt{\left(k^2x-1\right)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^3*x^3+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^3*x^3-1),x, algorithm="
giac")
```

```
[Out] integrate((k^3*x^3 + 1)/((k^3*x^3 - 1)*sqrt((k^2*x - 1)*(x - 1)*x)), x)
```

maple [C] time = 0.07, size = 363, normalized size = 3.49

$$-\frac{2\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\mathrm{EllipticF}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^2\sqrt{k^2x^3-k^2x^2-x^2+x}}-\frac{4\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\mathrm{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}-\frac{1}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{3k^3\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}-\frac{1}{k}\right)}-\frac{4\left(\sum_{\_\alpha=\mathrm{RootOf}\left(k^2\_Z^2+k\_Z+1\right)}\frac{\left(-\_\alpha k-2\right)\left(\_\alpha k^2+k+1\right)\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\mathrm{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{\_\alpha k^2+k+1}{k^2+k+1},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{\left(2\_\alpha k+1\right)\left(k^2+k+1\right)\sqrt{x\left(k^2x^2-k^2x-x+1\right)}}\right)}{3k}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^3*x^3+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^3*x^3-1),x)
```

```
[Out] -2/k^2*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x
^3-k^2*x^2-x^2+x)^(1/2)*EllipticF((-(x-1/k^2)*k^2)^(1/2),(1/k^2/(1/k^2-1))^
(1/2))-4/3/k^3*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2
```

```
)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2-1/k)*EllipticPi((-(x-1/k^2)*k^2)^(1/
2),1/k^2/(1/k^2-1/k),(1/k^2/(1/k^2-1))^(1/2))-4/3/k*sum((-_alpha*k-2)/(2*_a
lpha*k+1)*(_alpha*k^2+k+1)/(k^2+k+1)*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-
1))^(1/2)*(k^2*x)^(1/2)/(x*(k^2*x^2-k^2*x-x+1))^(1/2)*EllipticPi((-(x-1/k^2
)*k^2)^(1/2),(_alpha*k^2+k+1)/(k^2+k+1),(1/k^2/(1/k^2-1))^(1/2)),_alpha=Roo
tOf(_Z^2*k^2+_Z*k+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^3x^3+1}{\left(k^3x^3-1\right)\sqrt{\left(k^2x-1\right)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^3*x^3+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^3*x^3-1),x, algorithm="
maxima")
```

```
[Out] integrate((k^3*x^3 + 1)/((k^3*x^3 - 1)*sqrt((k^2*x - 1)*(x - 1)*x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^3*x^3 + 1)/((k^3*x^3 - 1)*(x*(k^2*x - 1)*(x - 1))^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k**3*x**3+1)/((1-x)*x*(-k**2*x+1))**(1/2)/(k**3*x**3-1),x)
```

```
[Out] Timed out
```

3.1287 $\int \frac{(-3+x^4)(1+x^4)^{2/3}(1+2x^3+x^4)}{x^6(1-x^3+x^4)}\,dx$

Optimal. Leaf size=104

$$3\log\left(\sqrt[3]{x^4+1}-x\right)-3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+1}+x}\right)-\frac{3}{2}\log\left(\sqrt[3]{x^4+1}\,x+\left(x^4+1\right)^{2/3}+x^2\right)+\frac{3\left(x^4+1\right)^{2/3}\left(2x^4+15x^3\right.}{10x^5}$$

Rubi [F] time = 1.41, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}\left(1+2x^3+x^4\right)}{x^6\left(1-x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + x^4)*(1 + x^4)^(2/3)*(1 + 2*x^3 + x^4))/(x^6*(1 - x^3 + x^4)),x]
```

```
[Out] (9*(1 + x^4)^(2/3))/(2*x^2) + (18*x^2)/(1 - Sqrt[3] - (1 + x^4)^(1/3)) - (9
*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 + x^4)^(1/3))*Sqrt[(1 + (1 + x^4)^(1/3)
+ (1 + x^4)^(2/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2]*EllipticE[ArcSin[(1 +
 Sqrt[3] - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))], -7 + 4*Sqrt[3
]])/(x^2*Sqrt[-((1 - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2)])
+ (6*Sqrt[2]*3^(3/4)*(1 - (1 + x^4)^(1/3))*Sqrt[(1 + (1 + x^4)^(1/3) + (1 +
 x^4)^(2/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[
3] - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))], -7 + 4*Sqrt[3]])/(x
^2*Sqrt[-((1 - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2)]) + (3*H
ypergeometric2F1[-5/4, -2/3, -1/4, -x^4])/(5*x^5) - Hypergeometric2F1[-2/3,
 -1/4, 3/4, -x^4]/x - 9*Defer[Int][(1 + x^4)^(2/3)/(1 - x^3 + x^4), x] + 12
*Defer[Int][(x*(1 + x^4)^(2/3))/(1 - x^3 + x^4), x]
```

Rubi steps

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}\left(1+2x^3+x^4\right)}{x^6\left(1-x^3+x^4\right)}\,dx = \int \left(-\frac{3\left(1+x^4\right)^{2/3}}{x^6} - \frac{9\left(1+x^4\right)^{2/3}}{x^3} + \frac{\left(1+x^4\right)^{2/3}}{x^2} + \frac{3(-3+4x)\left(1 \right.}{1-x^3+}\right.$$

$$= -\left(3\int \frac{\left(1+x^4\right)^{2/3}}{x^6}\,dx\right) + 3\int \frac{(-3+4x)\left(1+x^4\right)^{2/3}}{1-x^3+x^4}\,dx - 9\int \frac{(1}{}$$

$$= \frac{3\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};-x^4\right)}{5x^5} - \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{4};\frac{3}{4};-x^4\right)}{x} + 3\int \left(-\frac{3\left(1+x\right.}{1-x^3}\right.$$

$$= \frac{9\left(1+x^4\right)^{2/3}}{2x^2} + \frac{3\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};-x^4\right)}{5x^5} - \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{4};\frac{3}{4};-x^4\right)}{x} -$$

$$= \frac{9\left(1+x^4\right)^{2/3}}{2x^2} + \frac{3\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};-x^4\right)}{5x^5} - \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{4};\frac{3}{4};-x^4\right)}{x} -$$

$$= \frac{9\left(1+x^4\right)^{2/3}}{2x^2} + \frac{3\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};-x^4\right)}{5x^5} - \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{4};\frac{3}{4};-x^4\right)}{x} -$$

$$= \frac{9\left(1+x^4\right)^{2/3}}{2x^2} + \frac{18x^2}{1-\sqrt{3}-\sqrt[3]{1+x^4}} - \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1+x^{}}\right.}{}$$

Mathematica [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}\left(1+2x^3+x^4\right)}{x^6\left(1-x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + x^4)*(1 + x^4)^(2/3)*(1 + 2*x^3 + x^4))/(x^6*(1 - x^3 + x^
4)),x]
```

```
[Out] Integrate[((-3 + x^4)*(1 + x^4)^(2/3)*(1 + 2*x^3 + x^4))/(x^6*(1 - x^3 + x^
4)), x]
```

IntegrateAlgebraic [A] time = 3.60, size = 104, normalized size = 1.00

$$3\log\left(\sqrt[3]{x^4+1}-x\right) - 3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+1}+x}\right) - \frac{3}{2}\log\left(\sqrt[3]{x^4+1}\,x+\left(x^4+1\right)^{2/3}+x^2\right) + \frac{3\left(x^4+1\right)^{2/3}\left(2x^4+15x^3+2\right)}{10x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 + x^4)^(2/3)*(1 + 2*x^3 + x^4))/(x^6*(1 -
 x^3 + x^4)),x]
```

```
[Out] (3*(1 + x^4)^(2/3)*(2 + 15*x^3 + 2*x^4))/(10*x^5) - 3*Sqrt[3]*ArcTan[(Sqrt[
3]*x)/(x + 2*(1 + x^4)^(1/3))] + 3*Log[-x + (1 + x^4)^(1/3)] - (3*Log[x^2 +
 x*(1 + x^4)^(1/3) + (1 + x^4)^(2/3)])/2
```

fricas [A] time = 3.25, size = 146, normalized size = 1.40

$$-\frac{3\left(10\sqrt{3}x^5\arctan\left(-\frac{13034\sqrt{3}\left(x^4+1\right)^{\frac{1}{3}}x^2-686\sqrt{3}\left(x^4+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(37x^4+6137x^3+37\right)}{3\left(x^4+6859x^3+1\right)}\right)-5x^5\log\left(\frac{x^4-x^3+3\left(x^4+1\right)^{\frac{1}{3}}x^2-3\left(x^4+1\right)^{\frac{2}{3}}x+1}{x^4-x^3+1}\right)-\left(2x^4+15x^3+2\right)\left(x^4+1\right)^{\frac{2}{3}}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)*(x^4+2*x^3+1)/x^6/(x^4-x^3+1),x, algorithm=
"fricas")
```

```
[Out] -3/10*(10*sqrt(3)*x^5*arctan(-1/3*(13034*sqrt(3)*(x^4 + 1)^(1/3)*x^2 - 686*
sqrt(3)*(x^4 + 1)^(2/3)*x + sqrt(3)*(37*x^4 + 6137*x^3 + 37))/(x^4 + 6859*x
^3 + 1)) - 5*x^5*log((x^4 - x^3 + 3*(x^4 + 1)^(1/3)*x^2 - 3*(x^4 + 1)^(2/3)
*x + 1)/(x^4 - x^3 + 1)) - (2*x^4 + 15*x^3 + 2)*(x^4 + 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}+2 x^{3}+1\right)\left(x^{4}+1\right)^{\frac{2}{3}}\left(x^{4}-3\right)}{\left(x^{4}-x^{3}+1\right) x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)*(x^4+2*x^3+1)/x^6/(x^4-x^3+1),x, algorithm=
"giac")
```

```
[Out] integrate((x^4 + 2*x^3 + 1)*(x^4 + 1)^(2/3)*(x^4 - 3)/((x^4 - x^3 + 1)*x^6)
, x)
```

maple [C] time = 2.91, size = 292, normalized size = 2.81

$$\frac{\frac{3}{5} x^{8}+\frac{6}{5} x^{4}+\frac{9}{2} x^{7}+\frac{9}{2} x^{3}+\frac{3}{5}}{x^{5} \left(x^{4}+1\right)^{\frac{1}{3}}}+3 \ln \left(\frac{\mathit{RootOf}\left(\_Z^{2}+\_Z+1\right)^{2} x^{3}+\mathit{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{4}-\left(x^{4}+1\right)^{\frac{2}{3}} \mathit{RootOf}\left(\_Z^{2}+\_Z+1\right) x-\left(x^{4}+1\right)^{\frac{1}{3}} \mathit{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{2}+2 \mathit{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{3}+x^{4}-2 \left(x^{4}+1\right)^{\frac{2}{3}} x+x^{2} \left(x^{4}+1\right)^{\frac{1}{3}}+x^{3}+\mathit{RootOf}\left(\_Z^{2}+\_Z+1\right)+1}{x^{4}-x^{3}+1}\right)+3 \mathit{RootOf}\left(\_Z^{2}+\_Z+1\right) \ln \left(-\frac{\mathit{RootOf}\left(\_Z^{2}+\_Z+1\right)^{2} x^{3}-\mathit{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{4}-\left(x^{4}+1\right)^{\frac{2}{3}} \mathit{RootOf}\left(\_Z^{2}+\_Z+1\right) x+2 \left(x^{4}+1\right)^{\frac{1}{3}} \mathit{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{2}+\mathit{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{3}-x^{4}+\left(x^{4}+1\right)^{\frac{2}{3}} x+x^{2} \left(x^{4}+1\right)^{\frac{1}{3}}-\mathit{RootOf}\left(\_Z^{2}+\_Z+1\right)-1}{x^{4}-x^{3}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4+1)^(2/3)*(x^4+2*x^3+1)/x^6/(x^4-x^3+1),x)
```

```
[Out] 3/10*(2*x^8+15*x^7+4*x^4+15*x^3+2)/x^5/(x^4+1)^(1/3)+3*ln((RootOf(_Z^2+_Z+1
)^2*x^3+RootOf(_Z^2+_Z+1)*x^4-(x^4+1)^(2/3)*RootOf(_Z^2+_Z+1)*x-(x^4+1)^(1/
3)*RootOf(_Z^2+_Z+1)*x^2+2*RootOf(_Z^2+_Z+1)*x^3+x^4-2*(x^4+1)^(2/3)*x+x^2*
(x^4+1)^(1/3)+x^3+RootOf(_Z^2+_Z+1)+1)/(x^4-x^3+1))+3*RootOf(_Z^2+_Z+1)*ln(
-(RootOf(_Z^2+_Z+1)^2*x^3-RootOf(_Z^2+_Z+1)*x^4-(x^4+1)^(2/3)*RootOf(_Z^2+_
Z+1)*x+2*(x^4+1)^(1/3)*RootOf(_Z^2+_Z+1)*x^2+RootOf(_Z^2+_Z+1)*x^3-x^4+(x^4
+1)^(2/3)*x+x^2*(x^4+1)^(1/3)-RootOf(_Z^2+_Z+1)-1)/(x^4-x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}+2 x^{3}+1\right)\left(x^{4}+1\right)^{\frac{2}{3}}\left(x^{4}-3\right)}{\left(x^{4}-x^{3}+1\right) x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)*(x^4+2*x^3+1)/x^6/(x^4-x^3+1),x, algorithm=
"maxima")
```

```
[Out] integrate((x^4 + 2*x^3 + 1)*(x^4 + 1)^(2/3)*(x^4 - 3)/((x^4 - x^3 + 1)*x^6)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(x^4+1\right)}^{2/3}\,\left(x^4-3\right)\,\left(x^4+2\,x^3+1\right)}{x^6\,\left(x^4-x^3+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(2/3)*(x^4 - 3)*(2*x^3 + x^4 + 1))/(x^6*(x^4 - x^3 + 1)),x)
```

```
[Out] int(((x^4 + 1)^(2/3)*(x^4 - 3)*(2*x^3 + x^4 + 1))/(x^6*(x^4 - x^3 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4+1)**(2/3)*(x**4+2*x**3+1)/x**6/(x**4-x**3+1),x)
[Out] Timed out
```

3.1288 $\int \frac{\left(b+ax^4\right)^{3/4}}{x^4\left(2b+ax^4\right)}\,dx$

Optimal. Leaf size=104

$$\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{2}\,\sqrt[4]{ax^4+b}}\right)}{4\,2^{3/4}b}+\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{2}\,\sqrt[4]{ax^4+b}}\right)}{4\,2^{3/4}b}-\frac{\left(ax^4+b\right)^{3/4}}{6bx^3}$$

Rubi [C] time = 0.07, antiderivative size = 46, normalized size of antiderivative = 0.44, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$-\frac{\left(ax^4+b\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\frac{ax^4}{2\left(ax^4+b\right)}\right)}{6bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^4)^(3/4)/(x^4*(2*b + a*x^4)),x]
```

```
[Out] -1/6*((b + a*x^4)^(3/4)*Hypergeometric2F1[-3/4, 1, 1/4, (a*x^4)/(2*(b + a*x
^4))])/(b*x^3)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(b+ax^4\right)^{3/4}}{x^4\left(2b+ax^4\right)}\,dx&=\frac{\left(b+ax^4\right)^{3/4}\int \frac{\left(1+\frac{ax^4}{b}\right)^{3/4}}{x^4\left(2b+ax^4\right)}\,dx}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\\&=-\frac{\left(b+ax^4\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\frac{ax^4}{2\left(b+ax^4\right)}\right)}{6bx^3}\end{aligned}$$

Mathematica [C] time = 0.04, size = 77, normalized size = 0.74

$$-\frac{\left(ax^4+b\right)^{3/4}\left(\frac{ax^4}{b}+2\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};-\frac{ax^4}{ax^4+2b}\right)}{6bx^3\left(\frac{2ax^4}{b}+2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^4)^(3/4)/(x^4*(2*b + a*x^4)),x]
```

```
[Out] -1/6*((b + a*x^4)^(3/4)*(2 + (a*x^4)/b)^(3/4)*Hypergeometric2F1[-3/4, -3/4,
 1/4, -((a*x^4)/(2*b + a*x^4))])/(b*x^3*(2 + (2*a*x^4)/b)^(3/4))
```

IntegrateAlgebraic [A] time = 0.35, size = 104, normalized size = 1.00

$$\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{2}\,\sqrt[4]{ax^4+b}}\right)}{4\,2^{3/4}b}+\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{2}\,\sqrt[4]{ax^4+b}}\right)}{4\,2^{3/4}b}-\frac{\left(ax^4+b\right)^{3/4}}{6bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^4)^(3/4)/(x^4*(2*b + a*x^4)),x]
```

```
[Out] -1/6*(b + a*x^4)^(3/4)/(b*x^3) + (a^(3/4)*ArcTan[(a^(1/4)*x)/(2^(1/4)*(b +
a*x^4)^(1/4))])/(4*2^(3/4)*b) + (a^(3/4)*ArcTanh[(a^(1/4)*x)/(2^(1/4)*(b +
a*x^4)^(1/4))])/(4*2^(3/4)*b)
```

fricas [B] time = 60.44, size = 474, normalized size = 4.56

$$-\frac{12\left(\frac{1}{8}\right)^{\frac{1}{4}}bx^3\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{4\left(\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(ax^4+b\right)^{\frac{1}{4}}a^4bx^3\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}+4\left(\frac{1}{8}\right)^{\frac{3}{4}}\left(ax^4+b\right)^{\frac{3}{4}}a^2b^3x\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}-2\sqrt{\frac{1}{2}}\left(\left(\frac{1}{8}\right)^{\frac{1}{4}}\sqrt{ax^4+b}\,a^2bx^2\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}+\left(\frac{1}{8}\right)^{\frac{3}{4}}\left(3ab^3x^4+2b^4\right)\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}\right)\right)\sqrt{\sqrt{\frac{1}{2}}a^2b^2\sqrt{\frac{a^3}{b^4}}}}{a^5x^4+2a^4b}\right)-3\left(\frac{1}{8}\right)^{\frac{1}{4}}bx^3\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}\log\left(\frac{2\sqrt{\frac{1}{2}}\left(ax^4+b\right)^{\frac{1}{4}}ab^2x^3\sqrt{\frac{a^3}{b^4}}+8\left(\frac{1}{8}\right)^{\frac{3}{4}}\sqrt{ax^4+b}\,b^3x^2\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}+2\left(ax^4+b\right)^{\frac{3}{4}}a^2x+\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(3a^2bx^4+2ab^2\right)\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}}{2\left(ax^4+2b\right)}\right)+3\left(\frac{1}{8}\right)^{\frac{1}{4}}bx^3\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}\log\left(\frac{2\sqrt{\frac{1}{2}}\left(ax^4+b\right)^{\frac{1}{4}}ab^2x^3\sqrt{\frac{a^3}{b^4}}-8\left(\frac{1}{8}\right)^{\frac{3}{4}}\sqrt{ax^4+b}\,b^3x^2\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}+2\left(ax^4+b\right)^{\frac{3}{4}}a^2x-\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(3a^2bx^4+2ab^2\right)\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}}{2\left(ax^4+2b\right)}\right)+8\left(ax^4+b\right)^{\frac{3}{4}}}{48\,bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4)/x^4/(a*x^4+2*b),x, algorithm="fricas")
```

```
[Out] -1/48*(12*(1/8)^(1/4)*b*x^3*(a^3/b^4)^(1/4)*arctan(-4*((1/8)^(1/4)*(a*x^4 +
 b)^(1/4)*a^4*b*x^3*(a^3/b^4)^(1/4) + 4*(1/8)^(3/4)*(a*x^4 + b)^(3/4)*a^2*b
^3*x*(a^3/b^4)^(3/4) - 2*sqrt(1/2)*((1/8)^(1/4)*sqrt(a*x^4 + b)*a^2*b*x^2*(
a^3/b^4)^(1/4) + (1/8)^(3/4)*(3*a*b^3*x^4 + 2*b^4)*(a^3/b^4)^(3/4))*sqrt(sq
rt(1/2)*a^2*b^2*sqrt(a^3/b^4)))/(a^5*x^4 + 2*a^4*b)) - 3*(1/8)^(1/4)*b*x^3*
(a^3/b^4)^(1/4)*log(1/2*(2*sqrt(1/2)*(a*x^4 + b)^(1/4)*a*b^2*x^3*sqrt(a^3/b
^4) + 8*(1/8)^(3/4)*sqrt(a*x^4 + b)*b^3*x^2*(a^3/b^4)^(3/4) + 2*(a*x^4 + b)
^(3/4)*a^2*x + (1/8)^(1/4)*(3*a^2*b*x^4 + 2*a*b^2)*(a^3/b^4)^(1/4))/(a*x^4
+ 2*b)) + 3*(1/8)^(1/4)*b*x^3*(a^3/b^4)^(1/4)*log(1/2*(2*sqrt(1/2)*(a*x^4 +
 b)^(1/4)*a*b^2*x^3*sqrt(a^3/b^4) - 8*(1/8)^(3/4)*sqrt(a*x^4 + b)*b^3*x^2*(
a^3/b^4)^(3/4) + 2*(a*x^4 + b)^(3/4)*a^2*x - (1/8)^(1/4)*(3*a^2*b*x^4 + 2*a
*b^2)*(a^3/b^4)^(1/4))/(a*x^4 + 2*b)) + 8*(a*x^4 + b)^(3/4))/(b*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4+b\right)^{\frac{3}{4}}}{\left(ax^4+2\,b\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4)/x^4/(a*x^4+2*b),x, algorithm="giac")
```

```
[Out] integrate((a*x^4 + b)^(3/4)/((a*x^4 + 2*b)*x^4), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^4+b\right)^{\frac{3}{4}}}{x^4\left(a\,x^4+2b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b)^(3/4)/x^4/(a*x^4+2*b),x)
[Out] int((a*x^4+b)^(3/4)/x^4/(a*x^4+2*b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4+b\right)^{\frac{3}{4}}}{\left(ax^4+2\,b\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4)/x^4/(a*x^4+2*b),x, algorithm="maxima")
[Out] integrate((a*x^4 + b)^(3/4)/((a*x^4 + 2*b)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^4+b\right)^{3/4}}{x^4\,\left(a\,x^4+2\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^4)^(3/4)/(x^4*(2*b + a*x^4)),x)
[Out] int((b + a*x^4)^(3/4)/(x^4*(2*b + a*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4+b\right)^{\frac{3}{4}}}{x^4\left(ax^4+2b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b)**(3/4)/x**4/(a*x**4+2*b),x)
[Out] Integral((a*x**4 + b)**(3/4)/(x**4*(a*x**4 + 2*b)), x)
```

3.1289 $\int \frac{-b+ax^3}{x^3 \sqrt[4]{-bx+ax^4}} \, dx$

Optimal. Leaf size=104

$$\frac{2}{3}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{2}{3}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)-\frac{4\left(ax^4-bx\right)^{3/4}}{9x^3}$$

Rubi [A] time = 0.26, antiderivative size = 154, normalized size of antiderivative = 1.48, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2052, 2011, 329, 275, 240, 212, 206, 203, 2014}

$$\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3-b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3\sqrt[4]{ax^4-bx}}+\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3-b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3\sqrt[4]{ax^4-bx}}-\frac{4\left(ax^4-bx\right)^{3/4}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^3)/(x^3*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(-(b*x) + a*x^4)^(3/4))/(9*x^3) + (2*a^(3/4)*x^(1/4)*(-b + a*x^3)^(1/4)
*ArcTan[(a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)])/(3*(-(b*x) + a*x^4)^(1/4)) +
 (2*a^(3/4)*x^(1/4)*(-b + a*x^3)^(1/4)*ArcTanh[(a^(1/4)*x^(3/4))/(-b + a*x^
3)^(1/4)])/(3*(-(b*x) + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-b+ax^3}{x^3\sqrt[4]{-bx+ax^4}}\,dx &= \int \left(\frac{a}{\sqrt[4]{-bx+ax^4}}-\frac{b}{x^3\sqrt[4]{-bx+ax^4}}\right)dx\\
&= a\int \frac{1}{\sqrt[4]{-bx+ax^4}}\,dx-b\int \frac{1}{x^3\sqrt[4]{-bx+ax^4}}\,dx\\
&= -\frac{4\left(-bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\int \frac{1}{\sqrt[4]{x}\sqrt[4]{-b+ax^3}}\,dx}{\sqrt[4]{-bx+ax^4}}\\
&= -\frac{4\left(-bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(4a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{-b+ax^{12}}}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^4}}\\
&= -\frac{4\left(-bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(4a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-b+ax^4}}\,dx,x,x^{3/4}\right)}{3\sqrt[4]{-bx+ax^4}}\\
&= -\frac{4\left(-bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(4a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{-b+ax^3}}\right)}{3\sqrt[4]{-bx+ax^4}}\\
&= -\frac{4\left(-bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(2a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{-b+ax^3}}\right)}{3\sqrt[4]{-bx+ax^4}}+\frac{\left(2a\sqrt[4]{x}\sqrt[4]{-b\right.}{}}{}\\
&= -\frac{4\left(-bx+ax^4\right)^{3/4}}{9x^3}+\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{-b+ax^3}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{-b+ax^3}}\right)}{3\sqrt[4]{-bx+ax^4}}+\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{-b+ax^3}\tanh^{-1}}{3\sqrt[4]{-bx+ax^4}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.52

$$-\frac{4\left(ax^4-bx\right)^{3/4}{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};\frac{ax^3}{b}\right)}{9x^3\left(1-\frac{ax^3}{b}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^3)/(x^3*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(-(b*x) + a*x^4)^(3/4)*Hypergeometric2F1[-3/4, -3/4, 1/4, (a*x^3)/b])/(
9*x^3*(1 - (a*x^3)/b)^(3/4))
```

IntegrateAlgebraic [A] time = 0.33, size = 104, normalized size = 1.00

$$\frac{2}{3}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{2}{3}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)-\frac{4\left(ax^4-bx\right)^{3/4}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^3)/(x^3*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(-(b*x) + a*x^4)^(3/4))/(9*x^3) + (2*a^(3/4)*ArcTan[(a^(1/4)*(-(b*x) +
a*x^4)^(3/4))/(-b + a*x^3)])/3 + (2*a^(3/4)*ArcTanh[(a^(1/4)*(-(b*x) + a*x^
4)^(3/4))/(-b + a*x^3)])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)/x^3/(a*x^4-b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.21, size = 192, normalized size = 1.85

$$\frac{1}{3}\sqrt{2}\left(-a\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}+2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)+\frac{1}{3}\sqrt{2}\left(-a\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}-2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)-\frac{1}{6}\sqrt{2}\left(-a\right)^{\frac{3}{4}}\log\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^3}}\right)+\frac{1}{6}\sqrt{2}\left(-a\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\left(-a\right)^{\frac{1}{4}}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^3}}\right)-\frac{4}{9}\left(a-\frac{b}{x^3}\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)/x^3/(a*x^4-b*x)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a - b/x^
3)^(1/4))/(-a)^(1/4)) + 1/3*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(-a)^(1/4) - 2*(a - b/x^3)^(1/4))/(-a)^(1/4)) - 1/6*sqrt(2)*(-a)^(3/4)*log
(sqrt(2)*(-a)^(1/4)*(a - b/x^3)^(1/4) + sqrt(-a) + sqrt(a - b/x^3)) + 1/6*s
qrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a - b/x^3)^(1/4) + sqrt(-a) + sq
rt(a - b/x^3)) - 4/9*(a - b/x^3)^(3/4)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{a\,x^3-b}{x^3\left(a\,x^4-bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b)/x^3/(a*x^4-b*x)^(1/4),x)
```

```
[Out] int((a*x^3-b)/x^3/(a*x^4-b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 - b}{\left(ax^4 - bx\right)^{\frac{1}{4}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)/x^3/(a*x^4-b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^3 - b)/((a*x^4 - b*x)^(1/4)*x^3), x)
```

mupad [B] time = 1.21, size = 60, normalized size = 0.58

$$\frac{4\,a\,x\,{\left(1-\frac{a\,x^3}{b}\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \frac{a\,x^3}{b}\right)}{3\,{\left(a\,x^4-b\,x\right)}^{1/4}}-\frac{4\,{\left(a\,x^4-b\,x\right)}^{3/4}}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^3)/(x^3*(a*x^4 - b*x)^(1/4)),x)
```

```
[Out] (4*a*x*(1 - (a*x^3)/b)^(1/4)*hypergeom([1/4, 1/4], 5/4, (a*x^3)/b))/(3*(a*x
^4 - b*x)^(1/4)) - (4*(a*x^4 - b*x)^(3/4))/(9*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 - b}{x^3 \sqrt[4]{x\left(ax^3 - b\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b)/x**3/(a*x**4-b*x)**(1/4),x)
```

```
[Out] Integral((a*x**3 - b)/(x**3*(x*(a*x**3 - b))**(1/4)), x)
```

3.1290 $\int \frac{b+ax^3}{x^3 \sqrt[4]{-bx+ax^4}} \, dx$

Optimal. Leaf size=104

$$\frac{2}{3}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{2}{3}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{4\left(ax^4-bx\right)^{3/4}}{9x^3}$$

Rubi [A] time = 0.19, antiderivative size = 154, normalized size of antiderivative = 1.48, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2038, 2011, 329, 275, 240, 212, 206, 203}

$$\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3-b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3\sqrt[4]{ax^4-bx}}+\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3-b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3\sqrt[4]{ax^4-bx}}+\frac{4\left(ax^4-bx\right)^{3/4}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^3)/(x^3*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (4*(-(b*x) + a*x^4)^(3/4))/(9*x^3) + (2*a^(3/4)*x^(1/4)*(-b + a*x^3)^(1/4)*
ArcTan[(a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)])/(3*(-(b*x) + a*x^4)^(1/4)) +
(2*a^(3/4)*x^(1/4)*(-b + a*x^3)^(1/4)*ArcTanh[(a^(1/4)*x^(3/4))/(-b + a*x^3
)^(1/4)])/(3*(-(b*x) + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{b+ax^3}{x^3\sqrt[4]{-bx+ax^4}}\,dx = \frac{4\left(-bx+ax^4\right)^{3/4}}{9x^3}+a\int \frac{1}{\sqrt[4]{-bx+ax^4}}\,dx$$
$$= \frac{4\left(-bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\int \frac{1}{\sqrt[4]{x}\sqrt[4]{-b+ax^3}}\,dx}{\sqrt[4]{-bx+ax^4}}$$
$$= \frac{4\left(-bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(4a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{-b+ax^{12}}}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^4}}$$
$$= \frac{4\left(-bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(4a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-b+ax^4}}\,dx,x,x^{3/4}\right)}{3\sqrt[4]{-bx+ax^4}}$$
$$= \frac{4\left(-bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(4a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{-b+ax^3}}\right)}{3\sqrt[4]{-bx+ax^4}}$$
$$= \frac{4\left(-bx+ax^4\right)^{3/4}}{9x^3}+\frac{\left(2a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{-b+ax^3}}\right)}{3\sqrt[4]{-bx+ax^4}}+\frac{\left(2a\sqrt[4]{x}\sqrt[4]{-b+\right.}{}$$
$$= \frac{4\left(-bx+ax^4\right)^{3/4}}{9x^3}+\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{-b+ax^3}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{-b+ax^3}}\right)}{3\sqrt[4]{-bx+ax^4}}+\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{-b+ax^3}\tanh^{-1}\left(\right.}{3\sqrt[4]{-bx+ax^4}}$$

Mathematica [C] time = 0.03, size = 69, normalized size = 0.66

$$\frac{4\left(3ax^3\sqrt[4]{1-\frac{ax^3}{b}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{ax^3}{b}\right)+ax^3-b\right)}{9x^2\sqrt[4]{ax^4-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^3)/(x^3*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (4*(-b + a*x^3 + 3*a*x^3*(1 - (a*x^3)/b)^(1/4)*Hypergeometric2F1[1/4, 1/4,
5/4, (a*x^3)/b]))/(9*x^2*(-(b*x) + a*x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.35, size = 104, normalized size = 1.00

$$\frac{2}{3}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{2}{3}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{4\left(ax^4-bx\right)^{3/4}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^3)/(x^3*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (4*(-(b*x) + a*x^4)^(3/4))/(9*x^3) + (2*a^(3/4)*ArcTan[(a^(1/4)*(-(b*x) + a
*x^4)^(3/4))/(-b + a*x^3)])/3 + (2*a^(3/4)*ArcTanh[(a^(1/4)*(-(b*x) + a*x^4
)^(3/4))/(-b + a*x^3)])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)/x^3/(a*x^4-b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.30, size = 192, normalized size = 1.85

$$\frac{1}{3}\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)+\frac{1}{3}\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)-\frac{1}{6}\sqrt{2}(-a)^{\frac{3}{4}}\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^3}}\right)+\frac{1}{6}\sqrt{2}(-a)^{\frac{3}{4}}\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^3}}\right)+\frac{4}{9}\left(a-\frac{b}{x^3}\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)/x^3/(a*x^4-b*x)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a - b/x^
3)^(1/4))/(-a)^(1/4)) + 1/3*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(-a)^(1/4) - 2*(a - b/x^3)^(1/4))/(-a)^(1/4)) - 1/6*sqrt(2)*(-a)^(3/4)*log
(sqrt(2)*(-a)^(1/4)*(a - b/x^3)^(1/4) + sqrt(-a) + sqrt(a - b/x^3)) + 1/6*s
qrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a - b/x^3)^(1/4) + sqrt(-a) + sq
rt(a - b/x^3)) + 4/9*(a - b/x^3)^(3/4)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{a\,x^{3}+b}{x^{3}\left(a\,x^{4}-b x\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b)/x^3/(a*x^4-b*x)^(1/4),x)
```

```
[Out] int((a*x^3+b)/x^3/(a*x^4-b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{3}+b}{\left(ax^{4}-bx\right)^{\frac{1}{4}}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)/x^3/(a*x^4-b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^3 + b)/((a*x^4 - b*x)^(1/4)*x^3), x)
```

mupad [B] time = 0.96, size = 60, normalized size = 0.58

$$\frac{4\left(a\,x^4-b\,x\right)^{3/4}}{9\,x^3}+\frac{4\,a\,x\left(1-\frac{a\,x^3}{b}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \frac{a\,x^3}{b}\right)}{3\left(a\,x^4-b\,x\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^3)/(x^3*(a*x^4 - b*x)^(1/4)),x)
```

```
[Out] (4*(a*x^4 - b*x)^(3/4))/(9*x^3) + (4*a*x*(1 - (a*x^3)/b)^(1/4)*hypergeom([1
/4, 1/4], 5/4, (a*x^3)/b))/(3*(a*x^4 - b*x)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3+b}{x^3\sqrt[4]{x\left(ax^3-b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b)/x**3/(a*x**4-b*x)**(1/4),x)
```

```
[Out] Integral((a*x**3 + b)/(x**3*(x*(a*x**3 - b))**(1/4)), x)
```

3.1291 $\int \frac{\sqrt[4]{-bx+ax^4}}{x^2}\,dx$

Optimal. Leaf size=104

$$-\frac{4\sqrt[4]{ax^4-bx}}{3x}-\frac{2}{3}\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{2}{3}\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)$$

Rubi [A] time = 0.17, antiderivative size = 154, normalized size of antiderivative = 1.48, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2020, 2032, 329, 275, 331, 298, 203, 206}

$$-\frac{4\sqrt[4]{ax^4-bx}}{3x}-\frac{2\sqrt[4]{a}x^{3/4}\left(ax^3-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3\left(ax^4-bx\right)^{3/4}}+\frac{2\sqrt[4]{a}x^{3/4}\left(ax^3-b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3\left(ax^4-bx\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-(b*x) + a*x^4)^(1/4)/x^2,x]
```

```
[Out] (-4*(-(b*x) + a*x^4)^(1/4))/(3*x) - (2*a^(1/4)*x^(3/4)*(-b + a*x^3)^(3/4)*A
rcTan[(a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)])/(3*(-(b*x) + a*x^4)^(3/4)) + (
2*a^(1/4)*x^(3/4)*(-b + a*x^3)^(3/4)*ArcTanh[(a^(1/4)*x^(3/4))/(-b + a*x^3)
^(1/4)])/(3*(-(b*x) + a*x^4)^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{-bx+ax^4}}{x^2}\,dx &= -\frac{4\sqrt[4]{-bx+ax^4}}{3x}+a\int \frac{x^2}{\left(-bx+ax^4\right)^{3/4}}\,dx\\
&= -\frac{4\sqrt[4]{-bx+ax^4}}{3x}+\frac{\left(ax^{3/4}\left(-b+ax^3\right)^{3/4}\right)\int \frac{x^{5/4}}{\left(-b+ax^3\right)^{3/4}}\,dx}{\left(-bx+ax^4\right)^{3/4}}\\
&= -\frac{4\sqrt[4]{-bx+ax^4}}{3x}+\frac{\left(4ax^{3/4}\left(-b+ax^3\right)^{3/4}\right)\mathrm{Subst}\left(\int \frac{x^8}{\left(-b+ax^{12}\right)^{3/4}}\,dx,x,\sqrt[4]{x}\right)}{\left(-bx+ax^4\right)^{3/4}}\\
&= -\frac{4\sqrt[4]{-bx+ax^4}}{3x}+\frac{\left(4ax^{3/4}\left(-b+ax^3\right)^{3/4}\right)\mathrm{Subst}\left(\int \frac{x^2}{\left(-b+ax^4\right)^{3/4}}\,dx,x,x^{3/4}\right)}{3\left(-bx+ax^4\right)^{3/4}}\\
&= -\frac{4\sqrt[4]{-bx+ax^4}}{3x}+\frac{\left(4ax^{3/4}\left(-b+ax^3\right)^{3/4}\right)\mathrm{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{-b+ax^3}}\right)}{3\left(-bx+ax^4\right)^{3/4}}\\
&= -\frac{4\sqrt[4]{-bx+ax^4}}{3x}+\frac{\left(2\sqrt{a}\,x^{3/4}\left(-b+ax^3\right)^{3/4}\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{-b+ax^3}}\right)}{3\left(-bx+ax^4\right)^{3/4}}-\frac{\left(2\sqrt{a}\,x^{3/4}\left(-\right.\right.}{}\\
&= -\frac{4\sqrt[4]{-bx+ax^4}}{3x}-\frac{2\sqrt[4]{a}\,x^{3/4}\left(-b+ax^3\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{-b+ax^3}}\right)}{3\left(-bx+ax^4\right)^{3/4}}+\frac{2\sqrt[4]{a}\,x^{3/4}\left(-b+ax^3\right)^{3/4}\tanh^{-1}}{3\left(-bx+ax^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.52

$$-\frac{4\sqrt[4]{ax^4-bx}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};\frac{ax^3}{b}\right)}{3x\sqrt[4]{1-\frac{ax^3}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-(b*x) + a*x^4)^(1/4)/x^2,x]
```

```
[Out] (-4*(-(b*x) + a*x^4)^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, (a*x^3)/b])/(
3*x*(1 - (a*x^3)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.30, size = 104, normalized size = 1.00

$$-\frac{4\sqrt[4]{ax^4-bx}}{3x}-\frac{2}{3}\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{2}{3}\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(b*x) + a*x^4)^(1/4)/x^2,x]
```

```
[Out] (-4*(-(b*x) + a*x^4)^(1/4))/(3*x) - (2*a^(1/4)*ArcTan[(a^(1/4)*(-(b*x) + a*
x^4)^(3/4))/(-b + a*x^3)])/3 + (2*a^(1/4)*ArcTanh[(a^(1/4)*(-(b*x) + a*x^4)
^(3/4))/(-b + a*x^3)])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.32, size = 192, normalized size = 1.85

$$\frac{1}{3}\sqrt{2}\left(-a\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}+2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)+\frac{1}{3}\sqrt{2}\left(-a\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}-2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)+\frac{1}{6}\sqrt{2}\left(-a\right)^{\frac{1}{4}}\log\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^3}}\right)-\frac{1}{6}\sqrt{2}\left(-a\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\left(-a\right)^{\frac{1}{4}}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^3}}\right)-\frac{4}{3}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(-a)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a - b/x^
3)^(1/4))/(-a)^(1/4)) + 1/3*sqrt(2)*(-a)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(-a)^(1/4) - 2*(a - b/x^3)^(1/4))/(-a)^(1/4)) + 1/6*sqrt(2)*(-a)^(1/4)*log
(sqrt(2)*(-a)^(1/4)*(a - b/x^3)^(1/4) + sqrt(-a) + sqrt(a - b/x^3)) - 1/6*s
qrt(2)*(-a)^(1/4)*log(-sqrt(2)*(-a)^(1/4)*(a - b/x^3)^(1/4) + sqrt(-a) + sq
rt(a - b/x^3)) - 4/3*(a - b/x^3)^(1/4)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{4}-b x \right)^{\frac{1}{4}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b*x)^(1/4)/x^2,x)
```

```
[Out] int((a*x^4-b*x)^(1/4)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(a x^{4} - b x\right)}^{\frac{1}{4}}}{x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 - b*x)^(1/4)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^4-b\,x\right)^{1/4}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4 - b*x)^(1/4)/x^2,x)
```

```
[Out] int((a*x^4 - b*x)^(1/4)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x\left(ax^3-b\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b*x)**(1/4)/x**2,x)
```

```
[Out] Integral((x*(a*x**3 - b))**(1/4)/x**2, x)
```

3.1292 $\int \frac{-b+ax^4}{x^4 \sqrt[4]{-b+2ax^4}} \, dx$

Optimal. Leaf size=104

$$\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{2ax^4-b}}\right)}{2\sqrt[4]{2}}+\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{2ax^4-b}}\right)}{2\sqrt[4]{2}}-\frac{\left(2ax^4-b\right)^{3/4}}{3x^3}$$

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {451, 240, 212, 206, 203}

$$\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{2ax^4-b}}\right)}{2\sqrt[4]{2}}+\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{2ax^4-b}}\right)}{2\sqrt[4]{2}}-\frac{\left(2ax^4-b\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^4)/(x^4*(-b + 2*a*x^4)^(1/4)),x]
```

```
[Out] -1/3*(-b + 2*a*x^4)^(3/4)/x^3 + (a^(3/4)*ArcTan[(2^(1/4)*a^(1/4)*x)/(-b + 2
*a*x^4)^(1/4)])/(2*2^(1/4)) + (a^(3/4)*ArcTanh[(2^(1/4)*a^(1/4)*x)/(-b + 2*
a*x^4)^(1/4)])/(2*2^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{-b+ax^4}{x^4\sqrt[4]{-b+2ax^4}}\,dx &= -\frac{\left(-b+2ax^4\right)^{3/4}}{3x^3}+a\int\frac{1}{\sqrt[4]{-b+2ax^4}}\,dx\\
&= -\frac{\left(-b+2ax^4\right)^{3/4}}{3x^3}+a\,\text{Subst}\left(\int\frac{1}{1-2ax^4}\,dx,x,\frac{x}{\sqrt[4]{-b+2ax^4}}\right)\\
&= -\frac{\left(-b+2ax^4\right)^{3/4}}{3x^3}+\frac{1}{2}a\,\text{Subst}\left(\int\frac{1}{1-\sqrt{2}\sqrt{a}x^2}\,dx,x,\frac{x}{\sqrt[4]{-b+2ax^4}}\right)+\frac{1}{2}a\,\text{Subst}\left(\int\frac{1}{1+\sqrt{}}\right.\\
&= -\frac{\left(-b+2ax^4\right)^{3/4}}{3x^3}+\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{-b+2ax^4}}\right)}{2\sqrt[4]{2}}+\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{-b+2ax^4}}\right)}{2\sqrt[4]{2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 104, normalized size = 1.00

$$\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{2ax^4-b}}\right)}{2\sqrt[4]{2}}+\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{2ax^4-b}}\right)}{2\sqrt[4]{2}}-\frac{\left(2ax^4-b\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^4)/(x^4*(-b + 2*a*x^4)^(1/4)),x]
```

```
[Out] -1/3*(-b + 2*a*x^4)^(3/4)/x^3 + (a^(3/4)*ArcTan[(2^(1/4)*a^(1/4)*x)/(-b + 2
*a*x^4)^(1/4)])/(2*2^(1/4)) + (a^(3/4)*ArcTanh[(2^(1/4)*a^(1/4)*x)/(-b + 2*
a*x^4)^(1/4)])/(2*2^(1/4))
```

IntegrateAlgebraic [A] time = 0.38, size = 104, normalized size = 1.00

$$\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{2ax^4-b}}\right)}{2\sqrt[4]{2}}+\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{2ax^4-b}}\right)}{2\sqrt[4]{2}}-\frac{\left(2ax^4-b\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^4)/(x^4*(-b + 2*a*x^4)^(1/4)),x]
```

```
[Out] -1/3*(-b + 2*a*x^4)^(3/4)/x^3 + (a^(3/4)*ArcTan[(2^(1/4)*a^(1/4)*x)/(-b + 2
*a*x^4)^(1/4)])/(2*2^(1/4)) + (a^(3/4)*ArcTanh[(2^(1/4)*a^(1/4)*x)/(-b + 2*
a*x^4)^(1/4)])/(2*2^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)/x^4/(2*a*x^4-b)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4-b}{\left(2\,ax^4-b\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)/x^4/(2*a*x^4-b)^(1/4),x, algorithm="giac")
[Out] integrate((a*x^4 - b)/((2*a*x^4 - b)^(1/4)*x^4), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{a\,x^{4}-b}{x^{4}\left(2 a\,x^{4}-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b)/x^4/(2*a*x^4-b)^(1/4),x)
[Out] int((a*x^4-b)/x^4/(2*a*x^4-b)^(1/4),x)
```

maxima [A] time = 0.41, size = 115, normalized size = 1.11

$$-\frac{1}{8}\left(\frac{2\cdot 2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}\left(2\,ax^{4}-b\right)^{\frac{1}{4}}}{2\,a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}}+\frac{2^{\frac{3}{4}}\log\left(-\frac{2^{\frac{1}{4}}a^{\frac{1}{4}}-\frac{\left(2\,ax^{4}-b\right)^{\frac{1}{4}}}{x}}{2^{\frac{1}{4}}a^{\frac{1}{4}}+\frac{\left(2\,ax^{4}-b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right)a-\frac{\left(2\,ax^{4}-b\right)^{\frac{3}{4}}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)/x^4/(2*a*x^4-b)^(1/4),x, algorithm="maxima")
[Out] -1/8*(2*2^(3/4)*arctan(1/2*2^(3/4)*(2*a*x^4 - b)^(1/4)/(a^(1/4)*x))/a^(1/4)
 + 2^(3/4)*log(-(2^(1/4)*a^(1/4) - (2*a*x^4 - b)^(1/4)/x)/(2^(1/4)*a^(1/4)
+ (2*a*x^4 - b)^(1/4)/x))/a^(1/4))*a - 1/3*(2*a*x^4 - b)^(3/4)/x^3
```

mupad [B] time = 1.17, size = 60, normalized size = 0.58

$$\frac{a\,x\left(1-\frac{2\,a\,x^4}{b}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \frac{2\,a\,x^4}{b}\right)}{\left(2\,a\,x^4-b\right)^{1/4}}-\frac{\left(2\,a\,x^4-b\right)^{3/4}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^4)/(x^4*(2*a*x^4 - b)^(1/4)),x)
[Out] (a*x*(1 - (2*a*x^4)/b)^(1/4)*hypergeom([1/4, 1/4], 5/4, (2*a*x^4)/b))/(2*a*
x^4 - b)^(1/4) - (2*a*x^4 - b)^(3/4)/(3*x^3)
```

sympy [C] time = 2.18, size = 141, normalized size = 1.36

$$\frac{axe^{-\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{4}\\\frac{5}{4}\end{matrix}\right|\frac{2ax^{4}}{b}\right)}{4\sqrt[4]{b}\,\Gamma\left(\frac{5}{4}\right)}-b\left(\begin{cases}-\frac{2^{\frac{3}{4}}a^{\frac{3}{4}}\left(-1+\frac{b}{2ax^{4}}\right)^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\Gamma\left(-\frac{3}{4}\right)}{4b\Gamma\left(\frac{1}{4}\right)} & \text{for}\ \frac{\left|\frac{b}{ax^{4}}\right|}{2}>1\\-\frac{2^{\frac{3}{4}}a^{\frac{3}{4}}\left(1-\frac{b}{2ax^{4}}\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{4b\Gamma\left(\frac{1}{4}\right)} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b)/x**4/(2*a*x**4-b)**(1/4),x)
```

```
[Out] a*x*exp(-I*pi/4)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), 2*a*x**4/b)/(4*b**(1/
4)*gamma(5/4)) - b*Piecewise((-2**(3/4)*a**(3/4)*(-1 + b/(2*a*x**4))**(3/4)
*exp(3*I*pi/4)*gamma(-3/4)/(4*b*gamma(1/4)), Abs(b/(a*x**4))/2 > 1), (-2**(
3/4)*a**(3/4)*(1 - b/(2*a*x**4))**(3/4)*gamma(-3/4)/(4*b*gamma(1/4)), True)
)
```

3.1293 $\int \frac{x^4(4b+ax^5)}{(-b+ax^5)^2 \sqrt[4]{-b+cx^4+ax^5}}\,dx$

Optimal. Leaf size=104

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^5-b+cx^4}}\right)}{2c^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^5-b+cx^4}}\right)}{2c^{5/4}}+\frac{x\left(ax^5-b+cx^4\right)^{3/4}}{c\left(b-ax^5\right)}$$

Rubi [F] time = 2.76, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4\left(4b+ax^5\right)}{\left(-b+ax^5\right)^2 \sqrt[4]{-b+cx^4+ax^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(4*b + a*x^5))/((-b + a*x^5)^2*(-b + c*x^4 + a*x^5)^(1/4)),x]
```

```
[Out] -1/5*Defer[Int][1/((b^(1/5) - a^(1/5)*x)*(-b + c*x^4 + a*x^5)^(1/4)), x]/a^
(4/5) - Defer[Int][1/((-((-1)^(1/5)*b^(1/5)) - a^(1/5)*x)*(-b + c*x^4 + a*x
^5)^(1/4)), x]/(5*a^(4/5)) - Defer[Int][1/(((-1)^(2/5)*b^(1/5) - a^(1/5)*x)
*(-b + c*x^4 + a*x^5)^(1/4)), x]/(5*a^(4/5)) - Defer[Int][1/((-((-1)^(3/5)*
b^(1/5)) - a^(1/5)*x)*(-b + c*x^4 + a*x^5)^(1/4)), x]/(5*a^(4/5)) - Defer[I
nt][1/(((-1)^(4/5)*b^(1/5) - a^(1/5)*x)*(-b + c*x^4 + a*x^5)^(1/4)), x]/(5*
a^(4/5)) + 5*b*Defer[Int][x^4/((b - a*x^5)^2*(-b + c*x^4 + a*x^5)^(1/4)), x
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(4b+ax^5\right)}{\left(-b+ax^5\right)^2 \sqrt[4]{-b+cx^4+ax^5}}\,dx &= \int \left(\frac{5bx^4}{\left(b-ax^5\right)^2 \sqrt[4]{-b+cx^4+ax^5}}+\frac{x^4}{\left(-b+ax^5\right) \sqrt[4]{-b+cx^4+ax^5}}\right) dx \\
&= (5b)\int \frac{x^4}{\left(b-ax^5\right)^2 \sqrt[4]{-b+cx^4+ax^5}}\,dx+\int \frac{x^4}{\left(-b+ax^5\right) \sqrt[4]{-b+cx^4+a}} \\
&= (5b)\int \frac{x^4}{\left(b-ax^5\right)^2 \sqrt[4]{-b+cx^4+ax^5}}\,dx+\int \left(-\frac{1}{5a^{4/5}\left(\sqrt[5]{b}-\sqrt[5]{a}\,x\right)\sqrt[4]{-b}}\right. \\
&= -\frac{\int \frac{1}{\left(\sqrt[5]{b}-\sqrt[5]{a}\,x\right)\sqrt[4]{-b+cx^4+ax^5}}\,dx}{5a^{4/5}}-\frac{\int \frac{1}{\left(-\sqrt[5]{-1}\,\sqrt[5]{b}-\sqrt[5]{a}\,x\right)\sqrt[4]{-b+cx^4+ax^5}}\,dx}{5a^{4/5}}-\frac{\int \frac{}{((-1}}{}
\end{aligned}$$

Mathematica [F] time = 2.97, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(4b+ax^5\right)}{\left(-b+ax^5\right)^2 \sqrt[4]{-b+cx^4+ax^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(4*b + a*x^5))/((-b + a*x^5)^2*(-b + c*x^4 + a*x^5)^(1/4)),x
]
```

```
[Out] Integrate[(x^4*(4*b + a*x^5))/((-b + a*x^5)^2*(-b + c*x^4 + a*x^5)^(1/4)),
x]
```

IntegrateAlgebraic [A] time = 3.10, size = 104, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{ax^5-b+cx^4}}\right)}{2c^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{ax^5-b+cx^4}}\right)}{2c^{5/4}}+\frac{x\left(ax^5-b+cx^4\right)^{3/4}}{c\left(b-ax^5\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(4*b + a*x^5))/((-b + a*x^5)^2*(-b + c*x^4 + a*x^5)
^(1/4)),x]
```

```
[Out] (x*(-b + c*x^4 + a*x^5)^(3/4))/(c*(b - a*x^5)) + ArcTan[(c^(1/4)*x)/(-b + c
*x^4 + a*x^5)^(1/4)]/(2*c^(5/4)) + ArcTanh[(c^(1/4)*x)/(-b + c*x^4 + a*x^5)
^(1/4)]/(2*c^(5/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^5+4*b)/(a*x^5-b)^2/(a*x^5+c*x^4-b)^(1/4),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^5+4\,b\right)x^4}{\left(ax^5+cx^4-b\right)^{\frac{1}{4}}\left(ax^5-b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^5+4*b)/(a*x^5-b)^2/(a*x^5+c*x^4-b)^(1/4),x, algorithm="g
iac")
```

```
[Out] integrate((a*x^5 + 4*b)*x^4/((a*x^5 + c*x^4 - b)^(1/4)*(a*x^5 - b)^2), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a\,x^5+4b\right)}{\left(a\,x^5-b\right)^2\left(a\,x^5+c\,x^4-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a*x^5+4*b)/(a*x^5-b)^2/(a*x^5+c*x^4-b)^(1/4),x)
```

```
[Out] int(x^4*(a*x^5+4*b)/(a*x^5-b)^2/(a*x^5+c*x^4-b)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^5+4\,b\right)x^4}{\left(ax^5+cx^4-b\right)^{\frac{1}{4}}\left(ax^5-b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^5+4*b)/(a*x^5-b)^2/(a*x^5+c*x^4-b)^(1/4),x, algorithm="m
axima")
```

```
[Out] integrate((a*x^5 + 4*b)*x^4/((a*x^5 + c*x^4 - b)^(1/4)*(a*x^5 - b)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(a\,x^5+4\,b\right)}{{\left(b-a\,x^5\right)}^2\,{\left(a\,x^5+c\,x^4-b\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(4*b + a*x^5))/((b - a*x^5)^2*(a*x^5 - b + c*x^4)^(1/4)),x)
```

```
[Out] int((x^4*(4*b + a*x^5))/((b - a*x^5)^2*(a*x^5 - b + c*x^4)^(1/4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a*x**5+4*b)/(a*x**5-b)**2/(a*x**5+c*x**4-b)**(1/4),x)
```

```
[Out] Timed out
```

3.1294 $\int \frac{1}{x^{19}\sqrt[3]{-1+x^6}}\,dx$

Optimal. Leaf size=104

$$-\frac{7}{243}\log\left(\sqrt[3]{x^6-1}+1\right)+\frac{7}{486}\log\left(\left(x^6-1\right)^{2/3}-\sqrt[3]{x^6-1}+1\right)-\frac{7\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{\left(x^6-1\right)^{2/3}\left(28x^{12}+21x\right.}{324x^{18}}$$

Rubi [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 51, 56, 618, 204, 31}

$$\frac{7\left(x^6-1\right)^{2/3}}{81x^6}-\frac{7}{162}\log\left(\sqrt[3]{x^6-1}+1\right)-\frac{7\tan^{-1}\left(\frac{1-2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{\left(x^6-1\right)^{2/3}}{18x^{18}}+\frac{7\left(x^6-1\right)^{2/3}}{108x^{12}}+\frac{7\log(x)}{81}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^19*(-1 + x^6)^(1/3)),x]
```

```
[Out] (-1 + x^6)^(2/3)/(18*x^18) + (7*(-1 + x^6)^(2/3))/(108*x^12) + (7*(-1 + x^6
)^(2/3))/(81*x^6) - (7*ArcTan[(1 - 2*(-1 + x^6)^(1/3))/Sqrt[3]])/(81*Sqrt[3
]) + (7*Log[x])/81 - (7*Log[1 + (-1 + x^6)^(1/3)])/162
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{19}\sqrt[3]{-1+x^6}}\,dx = \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x^4}\,dx,x,x^6\right)$$
$$= \frac{\left(-1+x^6\right)^{2/3}}{18x^{18}} + \frac{7}{54}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x^3}\,dx,x,x^6\right)$$
$$= \frac{\left(-1+x^6\right)^{2/3}}{18x^{18}} + \frac{7\left(-1+x^6\right)^{2/3}}{108x^{12}} + \frac{7}{81}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x^2}\,dx,x,x^6\right)$$
$$= \frac{\left(-1+x^6\right)^{2/3}}{18x^{18}} + \frac{7\left(-1+x^6\right)^{2/3}}{108x^{12}} + \frac{7\left(-1+x^6\right)^{2/3}}{81x^6} + \frac{7}{243}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x}\,dx,x,x^6\right)$$
$$= \frac{\left(-1+x^6\right)^{2/3}}{18x^{18}} + \frac{7\left(-1+x^6\right)^{2/3}}{108x^{12}} + \frac{7\left(-1+x^6\right)^{2/3}}{81x^6} + \frac{7\log(x)}{81} - \frac{7}{162}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,\right.$$
$$= \frac{\left(-1+x^6\right)^{2/3}}{18x^{18}} + \frac{7\left(-1+x^6\right)^{2/3}}{108x^{12}} + \frac{7\left(-1+x^6\right)^{2/3}}{81x^6} + \frac{7\log(x)}{81} - \frac{7}{162}\log\left(1+\sqrt[3]{-1+x^6}\right) -$$
$$= \frac{\left(-1+x^6\right)^{2/3}}{18x^{18}} + \frac{7\left(-1+x^6\right)^{2/3}}{108x^{12}} + \frac{7\left(-1+x^6\right)^{2/3}}{81x^6} - \frac{7\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^6}}{\sqrt{3}}\right)}{81\sqrt{3}} + \frac{7\log(x)}{81} - \frac{7}{162}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.27

$$\frac{1}{4}\left(x^6-1\right)^{2/3} {}_2F_1\left(\frac{2}{3},4;\frac{5}{3};1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^19*(-1 + x^6)^(1/3)),x]
[Out] ((-1 + x^6)^(2/3)*Hypergeometric2F1[2/3, 4, 5/3, 1 - x^6])/4
```

IntegrateAlgebraic [A] time = 0.20, size = 104, normalized size = 1.00

$$-\frac{7}{243}\log\left(\sqrt[3]{x^6-1}+1\right)+\frac{7}{486}\log\left(\left(x^6-1\right)^{2/3}-\sqrt[3]{x^6-1}+1\right)-\frac{7\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^6-1}}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{\left(x^6-1\right)^{2/3}\left(28x^{12}+21x^6+18\right)}{324x^{18}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^19*(-1 + x^6)^(1/3)),x]
[Out] ((-1 + x^6)^(2/3)*(18 + 21*x^6 + 28*x^12))/(324*x^18) - (7*ArcTan[1/Sqrt[3]
 - (2*(-1 + x^6)^(1/3))/Sqrt[3]])/(81*Sqrt[3]) - (7*Log[1 + (-1 + x^6)^(1/3
)])/243 + (7*Log[1 - (-1 + x^6)^(1/3) + (-1 + x^6)^(2/3)])/486
```

fricas [A] time = 0.65, size = 93, normalized size = 0.89

$$\frac{28\sqrt{3}x^{18}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+14x^{18}\log\left(\left(x^6-1\right)^{\frac{2}{3}}-\left(x^6-1\right)^{\frac{1}{3}}+1\right)-28x^{18}\log\left(\left(x^6-1\right)^{\frac{1}{3}}+1\right)+3\left(28x^{12}+21x^6+18\right)\left(x^6-1\right)^{\frac{2}{3}}}{972x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^19/(x^6-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/972*(28*sqrt(3)*x^18*arctan(2/3*sqrt(3)*(x^6 - 1)^(1/3) - 1/3*sqrt(3)) +
14*x^18*log((x^6 - 1)^(2/3) - (x^6 - 1)^(1/3) + 1) - 28*x^18*log((x^6 - 1)^
(1/3) + 1) + 3*(28*x^12 + 21*x^6 + 18)*(x^6 - 1)^(2/3))/x^18
```

giac [A] time = 0.35, size = 90, normalized size = 0.87

$$\frac{7}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^6-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{28\left(x^6-1\right)^{\frac{8}{3}}+77\left(x^6-1\right)^{\frac{5}{3}}+67\left(x^6-1\right)^{\frac{2}{3}}}{324\,x^{18}}+\frac{7}{486}\log\left(\left(x^6-1\right)^{\frac{2}{3}}-\left(x^6-1\right)^{\frac{1}{3}}+1\right)-\frac{7}{243}\log\left(\left|\left(x^6-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^19/(x^6-1)^(1/3),x, algorithm="giac")
```

```
[Out] 7/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3) - 1)) + 1/324*(28*(x^6
- 1)^(8/3) + 77*(x^6 - 1)^(5/3) + 67*(x^6 - 1)^(2/3))/x^18 + 7/486*log((x^6
 - 1)^(2/3) - (x^6 - 1)^(1/3) + 1) - 7/243*log(abs((x^6 - 1)^(1/3) + 1))
```

maple [C] time = 0.29, size = 113, normalized size = 1.09

$$\frac{28x^{18}-7x^{12}-3x^{6}-18}{324x^{18}\left(x^6-1\right)^{\frac{1}{3}}}+\frac{7\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\mathrm{signum}\left(x^6-1\right)\right)^{\frac{1}{3}}\left(\frac{2\pi\sqrt{3}\,x^6\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^6\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+6\ln(x)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{486\pi\,\mathrm{signum}\left(x^6-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^19/(x^6-1)^(1/3),x)
```

```
[Out] 1/324*(28*x^18-7*x^12-3*x^6-18)/x^18/(x^6-1)^(1/3)+7/486/Pi*3^(1/2)*GAMMA(2
/3)/signum(x^6-1)^(1/3)*(-signum(x^6-1))^(1/3)*(2/9*Pi*3^(1/2)/GAMMA(2/3)*x
^6*hypergeom([1,1,4/3],[2,2],x^6)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+6*ln(x)+I*
Pi)*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.43, size = 111, normalized size = 1.07

$$\frac{7}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^6-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{28\left(x^6-1\right)^{\frac{8}{3}}+77\left(x^6-1\right)^{\frac{5}{3}}+67\left(x^6-1\right)^{\frac{2}{3}}}{324\left(3x^6+\left(x^6-1\right)^3+3\left(x^6-1\right)^2-2\right)}+\frac{7}{486}\log\left(\left(x^6-1\right)^{\frac{2}{3}}-\left(x^6-1\right)^{\frac{1}{3}}+1\right)-\frac{7}{243}\log\left(\left(x^6-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^19/(x^6-1)^(1/3),x, algorithm="maxima")
```

```
[Out] 7/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3) - 1)) + 1/324*(28*(x^6
- 1)^(8/3) + 77*(x^6 - 1)^(5/3) + 67*(x^6 - 1)^(2/3))/(3*x^6 + (x^6 - 1)^3
+ 3*(x^6 - 1)^2 - 2) + 7/486*log((x^6 - 1)^(2/3) - (x^6 - 1)^(1/3) + 1) - 7
/243*log((x^6 - 1)^(1/3) + 1)
```

mupad [B] time = 1.06, size = 134, normalized size = 1.29

$$\frac{\frac{67\left(x^6-1\right)^{2/3}}{324}+\frac{77\left(x^6-1\right)^{5/3}}{324}+\frac{7\left(x^6-1\right)^{8/3}}{81}}{3\left(x^6-1\right)^2+\left(x^6-1\right)^3+3x^6-2}-\ln\left(9\left(-\frac{7}{486}+\frac{\sqrt{3}\,7\mathrm{i}}{486}\right)^2+\frac{49\left(x^6-1\right)^{1/3}}{6561}\right)\left(-\frac{7}{486}+\frac{\sqrt{3}\,7\mathrm{i}}{486}\right)+\ln\left(9\left(\frac{7}{486}+\frac{\sqrt{3}\,7\mathrm{i}}{486}\right)^2+\frac{49\left(x^6-1\right)^{1/3}}{6561}\right)\left(\frac{7}{486}+\frac{\sqrt{3}\,7\mathrm{i}}{486}\right)-\frac{7\ln\left(\frac{49\left(x^6-1\right)^{1/3}}{6561}+\frac{49}{6561}\right)}{243}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^19*(x^6 - 1)^(1/3)),x)
```

```
[Out] log(9*((3^(1/2)*7i)/486 + 7/486)^2 + (49*(x^6 - 1)^(1/3))/6561)*((3^(1/2)*7
i)/486 + 7/486) - log(9*((3^(1/2)*7i)/486 - 7/486)^2 + (49*(x^6 - 1)^(1/3))
/6561)*((3^(1/2)*7i)/486 - 7/486) - (7*log((49*(x^6 - 1)^(1/3))/6561 + 49/6
561))/243 + ((67*(x^6 - 1)^(2/3))/324 + (77*(x^6 - 1)^(5/3))/324 + (7*(x^6
- 1)^(8/3))/81)/(3*(x^6 - 1)^2 + (x^6 - 1)^3 + 3*x^6 - 2)
```

sympy [C] time = 2.41, size = 32, normalized size = 0.31

$$-\frac{\Gamma\left(\frac{10}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{10}{3} \\ \frac{13}{3} \end{matrix}\middle| \frac{e^{2i\pi}}{x^6}\right)}{6x^{20}\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**19/(x**6-1)**(1/3),x)
```

```
[Out] -gamma(10/3)*hyper((1/3, 10/3), (13/3,), exp_polar(2*I*pi)/x**6)/(6*x**20*g
amma(13/3))
```

3.1295 $\int \frac{x^7}{\sqrt[3]{-1+x^6}}\,dx$

Optimal. Leaf size=104

$$\frac{1}{6}\left(x^6-1\right)^{2/3}x^2-\frac{1}{18}\log\left(\sqrt[3]{x^6-1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{6\sqrt{3}}+\frac{1}{36}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)$$

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {275, 321, 239}

$$\frac{1}{6}\left(x^6-1\right)^{2/3}x^2-\frac{1}{12}\log\left(x^2-\sqrt[3]{x^6-1}\right)+\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6-1}}+1}{\sqrt{3}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(-1 + x^6)^(1/3),x]
```

```
[Out] (x^2*(-1 + x^6)^(2/3))/6 + ArcTan[(1 + (2*x^2)/(-1 + x^6)^(1/3))/Sqrt[3]]/(
6*Sqrt[3]) - Log[x^2 - (-1 + x^6)^(1/3)]/12
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\sqrt[3]{-1+x^6}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{\sqrt[3]{-1+x^3}}\,dx,x,x^2\right)\\ &= \frac{1}{6}x^2\left(-1+x^6\right)^{2/3}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x^3}}\,dx,x,x^2\right)\\ &= \frac{1}{6}x^2\left(-1+x^6\right)^{2/3}+\frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{-1+x^6}}}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{1}{12}\log\left(x^2-\sqrt[3]{-1+x^6}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 0.93

$$\frac{1}{36}\left(6\left(x^6-1\right)^{2/3}x^2-2\log\left(1-\frac{x^2}{\sqrt[3]{x^6-1}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6-1}}+1}{\sqrt{3}}\right)+\log\left(\frac{x^4}{\left(x^6-1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6-1}}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(-1 + x^6)^(1/3),x]
```

```
[Out] (6*x^2*(-1 + x^6)^(2/3) + 2*Sqrt[3]*ArcTan[(1 + (2*x^2)/(-1 + x^6)^(1/3))/S
qrt[3]] - 2*Log[1 - x^2/(-1 + x^6)^(1/3)] + Log[1 + x^4/(-1 + x^6)^(2/3) +
x^2/(-1 + x^6)^(1/3)])/36
```

IntegrateAlgebraic [A] time = 1.01, size = 104, normalized size = 1.00

$$\frac{1}{6}\left(x^6-1\right)^{2/3}x^2-\frac{1}{18}\log\left(\sqrt[3]{x^6-1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{6\sqrt{3}}+\frac{1}{36}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/(-1 + x^6)^(1/3),x]
```

```
[Out] (x^2*(-1 + x^6)^(2/3))/6 + ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(-1 + x^6)^(1/3))]
/(6*Sqrt[3]) - Log[-x^2 + (-1 + x^6)^(1/3)]/18 + Log[x^4 + x^2*(-1 + x^6)^(
1/3) + (-1 + x^6)^(2/3)]/36
```

fricas [A] time = 0.63, size = 94, normalized size = 0.90

$$\frac{1}{6}\left(x^6-1\right)^{\frac{2}{3}}x^2-\frac{1}{18}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}}{3x^2}\right)-\frac{1}{18}\log\left(-\frac{x^2-\left(x^6-1\right)^{\frac{1}{3}}}{x^2}\right)+\frac{1}{36}\log\left(\frac{x^4+\left(x^6-1\right)^{\frac{1}{3}}x^2+\left(x^6-1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*(x^6 - 1)^(2/3)*x^2 - 1/18*sqrt(3)*arctan(1/3*(sqrt(3)*x^2 + 2*sqrt(3)*
(x^6 - 1)^(1/3))/x^2) - 1/18*log(-(x^2 - (x^6 - 1)^(1/3))/x^2) + 1/36*log((
x^4 + (x^6 - 1)^(1/3)*x^2 + (x^6 - 1)^(2/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^7}{\left(x^6-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^7/(x^6 - 1)^(1/3), x)
```

maple [C] time = 0.28, size = 46, normalized size = 0.44

$$\frac{x^2\left(x^6-1\right)^{\frac{2}{3}}}{6}+\frac{\left(-\mathrm{signum}\left(x^6-1\right)\right)^{\frac{1}{3}}x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^6\right)}{6\mathrm{signum}\left(x^6-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^6-1)^(1/3),x)
```

```
[Out] 1/6*x^2*(x^6-1)^(2/3)+1/6/signum(x^6-1)^(1/3)*(-signum(x^6-1))^(1/3)*x^2*hy
pergeom([1/3,1/3],[4/3],x^6)
```

maxima [A] time = 0.43, size = 94, normalized size = 0.90

$$-\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)-\frac{\left(x^6-1\right)^{\frac{2}{3}}}{6\,x^4\left(\frac{x^6-1}{x^6}-1\right)}+\frac{1}{36}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6-1\right)^{\frac{2}{3}}}{x^4}+1\right)-\frac{1}{18}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6-1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3)/x^2 + 1)) - 1/6*(x^6 -
1)^(2/3)/(x^4*((x^6 - 1)/x^6 - 1)) + 1/36*log((x^6 - 1)^(1/3)/x^2 + (x^6 -
1)^(2/3)/x^4 + 1) - 1/18*log((x^6 - 1)^(1/3)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\left(x^6-1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^6 - 1)^(1/3),x)
```

```
[Out] int(x^7/(x^6 - 1)^(1/3), x)
```

sympy [C] time = 0.94, size = 34, normalized size = 0.33

$$-\frac{x^8 e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle| x^6\right)}{6\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**6-1)**(1/3),x)
```

```
[Out] -x**8*exp(2*I*pi/3)*gamma(4/3)*hyper((1/3, 4/3), (7/3,), x**6)/(6*gamma(7/3
))
```

3.1296 $\int x^3 \sqrt[3]{-1+x^6}\, dx$

Optimal. Leaf size=104

$$\frac{1}{6}\sqrt[3]{x^6-1}\,x^4+\frac{1}{18}\log\left(\sqrt[3]{x^6-1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{6\sqrt{3}}-\frac{1}{36}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)$$

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {275, 279, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{6}\sqrt[3]{x^6-1}\,x^4+\frac{1}{18}\log\left(1-\frac{x^2}{\sqrt[3]{x^6-1}}\right)+\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6-1}}+1}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{1}{36}\log\left(\frac{x^4}{\left(x^6-1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6-1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(-1 + x^6)^(1/3),x]
```

```
[Out] (x^4*(-1 + x^6)^(1/3))/6 + ArcTan[(1 + (2*x^2)/(-1 + x^6)^(1/3))/Sqrt[3]]/(
6*Sqrt[3]) + Log[1 - x^2/(-1 + x^6)^(1/3)]/18 - Log[1 + x^4/(-1 + x^6)^(2/3
) + x^2/(-1 + x^6)^(1/3)]/36
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt[3]{-1+x^6}\,dx &= \frac{1}{2}\text{Subst}\left(\int x\sqrt[3]{-1+x^3}\,dx,x,x^2\right)\\
&= \frac{1}{6}x^4\sqrt[3]{-1+x^6}-\frac{1}{6}\text{Subst}\left(\int\frac{x}{\left(-1+x^3\right)^{2/3}}\,dx,x,x^2\right)\\
&= \frac{1}{6}x^4\sqrt[3]{-1+x^6}-\frac{1}{6}\text{Subst}\left(\int\frac{x}{1-x^3}\,dx,x,\frac{x^2}{\sqrt[3]{-1+x^6}}\right)\\
&= \frac{1}{6}x^4\sqrt[3]{-1+x^6}-\frac{1}{18}\text{Subst}\left(\int\frac{1}{1-x}\,dx,x,\frac{x^2}{\sqrt[3]{-1+x^6}}\right)+\frac{1}{18}\text{Subst}\left(\int\frac{1-x}{1+x+x^2}\,dx,x,\frac{}{\sqrt[3]{-1}}\right.\\
&= \frac{1}{6}x^4\sqrt[3]{-1+x^6}+\frac{1}{18}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right)-\frac{1}{36}\text{Subst}\left(\int\frac{1+2x}{1+x+x^2}\,dx,x,\frac{x^2}{\sqrt[3]{-1+x^6}}\right)+\frac{1}{12}\\
&= \frac{1}{6}x^4\sqrt[3]{-1+x^6}+\frac{1}{18}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right)-\frac{1}{36}\log\left(1+\frac{x^4}{\left(-1+x^6\right)^{2/3}}+\frac{x^2}{\sqrt[3]{-1+x^6}}\right)-\frac{1}{6}\text{Subs}\\
&= \frac{1}{6}x^4\sqrt[3]{-1+x^6}+\frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{-1+x^6}}}{\sqrt{3}}\right)}{6\sqrt{3}}+\frac{1}{18}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right)-\frac{1}{36}\log\left(1+\frac{x^4}{\left(-1+x^6\right)^{2/3}}+\right.
\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.38

$$\frac{x^4\sqrt[3]{x^6-1}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^6\right)}{4\sqrt[3]{1-x^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(-1 + x^6)^(1/3),x]
```

[Out] (x^4*(-1 + x^6)^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/3, x^6])/(4*(1 - x^6)^(1/3))

IntegrateAlgebraic [A] time = 0.64, size = 104, normalized size = 1.00

$$\frac{1}{6}\sqrt[3]{x^6-1}\,x^4+\frac{1}{18}\log\left(\sqrt[3]{x^6-1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{6\sqrt{3}}-\frac{1}{36}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*(-1 + x^6)^(1/3),x]

[Out] (x^4*(-1 + x^6)^(1/3))/6 + ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(-1 + x^6)^(1/3))]/(6*Sqrt[3]) + Log[-x^2 + (-1 + x^6)^(1/3)]/18 - Log[x^4 + x^2*(-1 + x^6)^(1/3) + (-1 + x^6)^(2/3)]/36

fricas [A] time = 0.49, size = 94, normalized size = 0.90

$$\frac{1}{6}\left(x^6-1\right)^{\frac{1}{3}}x^4-\frac{1}{18}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}}{3x^2}\right)+\frac{1}{18}\log\left(-\frac{x^2-\left(x^6-1\right)^{\frac{1}{3}}}{x^2}\right)-\frac{1}{36}\log\left(\frac{x^4+\left(x^6-1\right)^{\frac{1}{3}}x^2+\left(x^6-1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(x^6-1)^(1/3),x, algorithm="fricas")

[Out] 1/6*(x^6 - 1)^(1/3)*x^4 - 1/18*sqrt(3)*arctan(1/3*(sqrt(3)*x^2 + 2*sqrt(3)*(x^6 - 1)^(1/3))/x^2) + 1/18*log(-(x^2 - (x^6 - 1)^(1/3))/x^2) - 1/36*log((x^4 + (x^6 - 1)^(1/3)*x^2 + (x^6 - 1)^(2/3))/x^4)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^6-1\right)^{\frac{1}{3}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(x^6-1)^(1/3),x, algorithm="giac")

[Out] integrate((x^6 - 1)^(1/3)*x^3, x)

maple [C] time = 0.28, size = 46, normalized size = 0.44

$$\frac{x^4\left(x^6-1\right)^{\frac{1}{3}}}{6}-\frac{\left(-\mathrm{signum}\left(x^6-1\right)\right)^{\frac{2}{3}}x^4\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^6\right)}{12\mathrm{signum}\left(x^6-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(x^6-1)^(1/3),x)

[Out] 1/6*x^4*(x^6-1)^(1/3)-1/12/signum(x^6-1)^(2/3)*(-signum(x^6-1))^(2/3)*x^4*hypergeom([2/3,2/3],[5/3],x^6)

maxima [A] time = 0.42, size = 94, normalized size = 0.90

$$-\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)-\frac{\left(x^6-1\right)^{\frac{1}{3}}}{6x^2\left(\frac{x^6-1}{x^6}-1\right)}-\frac{1}{36}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6-1\right)^{\frac{2}{3}}}{x^4}+1\right)+\frac{1}{18}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^6-1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3)/x^2 + 1)) - 1/6*(x^6 -
1)^(1/3)/(x^2*((x^6 - 1)/x^6 - 1)) - 1/36*log((x^6 - 1)^(1/3)/x^2 + (x^6 -
1)^(2/3)/x^4 + 1) + 1/18*log((x^6 - 1)^(1/3)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(x^6-1\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^6 - 1)^(1/3),x)
```

```
[Out] int(x^3*(x^6 - 1)^(1/3), x)
```

sympy [C] time = 0.91, size = 36, normalized size = 0.35

$$-\frac{x^4 e^{-\frac{2i\pi}{3}} \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\right| x^6\right)}{6\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**6-1)**(1/3),x)
```

```
[Out] -x**4*exp(-2*I*pi/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), x**6)/(6*gamma(5
/3))
```

3.1297 $\int x\left(-1+x^6\right)^{2/3}\,dx$

Optimal. Leaf size=104

$$\frac{1}{6}\left(x^6-1\right)^{2/3}x^2+\frac{1}{9}\log\left(\sqrt[3]{x^6-1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{3\sqrt{3}}-\frac{1}{18}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}x^2\right)$$

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {275, 195, 239}

$$\frac{1}{6}\left(x^6-1\right)^{2/3}x^2+\frac{1}{6}\log\left(x^2-\sqrt[3]{x^6-1}\right)-\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6-1}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(-1 + x^6)^(2/3),x]
```

```
[Out] (x^2*(-1 + x^6)^(2/3))/6 - ArcTan[(1 + (2*x^2)/(-1 + x^6)^(1/3))/Sqrt[3]]/(
3*Sqrt[3]) + Log[x^2 - (-1 + x^6)^(1/3)]/6
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x\left(-1+x^6\right)^{2/3}\,dx &= \frac{1}{2}\text{Subst}\left(\int\left(-1+x^3\right)^{2/3}\,dx,x,x^2\right)\\
&= \frac{1}{6}x^2\left(-1+x^6\right)^{2/3}-\frac{1}{3}\text{Subst}\left(\int\frac{1}{\sqrt[3]{-1+x^3}}\,dx,x,x^2\right)\\
&= \frac{1}{6}x^2\left(-1+x^6\right)^{2/3}-\frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{-1+x^6}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(x^2-\sqrt[3]{-1+x^6}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.38

$$\frac{x^2 \left(x^6-1\right)^{2/3} {}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};x^6\right)}{2 \left(1-x^6\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-1 + x^6)^(2/3),x]
```

```
[Out] (x^2*(-1 + x^6)^(2/3)*Hypergeometric2F1[-2/3, 1/3, 4/3, x^6])/(2*(1 - x^6)^
(2/3))
```

IntegrateAlgebraic [A] time = 0.61, size = 104, normalized size = 1.00

$$\frac{1}{6} \left(x^6-1\right)^{2/3} x^2+\frac{1}{9} \log \left(\sqrt[3]{x^6-1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3} x^2}{2 \sqrt[3]{x^6-1}+x^2}\right)}{3 \sqrt{3}}-\frac{1}{18} \log \left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1} x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(-1 + x^6)^(2/3),x]
```

```
[Out] (x^2*(-1 + x^6)^(2/3))/6 - ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(-1 + x^6)^(1/3))]
/(3*Sqrt[3]) + Log[-x^2 + (-1 + x^6)^(1/3)]/9 - Log[x^4 + x^2*(-1 + x^6)^(1
/3) + (-1 + x^6)^(2/3)]/18
```

fricas [A] time = 0.40, size = 94, normalized size = 0.90

$$\frac{1}{6} \left(x^6-1\right)^{\frac{2}{3}} x^2+\frac{1}{9} \sqrt{3} \arctan\left(\frac{\sqrt{3} x^2+2 \sqrt{3} \left(x^6-1\right)^{\frac{1}{3}}}{3 x^2}\right)+\frac{1}{9} \log\left(-\frac{x^2-\left(x^6-1\right)^{\frac{1}{3}}}{x^2}\right)-\frac{1}{18} \log\left(\frac{x^4+\left(x^6-1\right)^{\frac{1}{3}} x^2+\left(x^6-1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^6-1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/6*(x^6 - 1)^(2/3)*x^2 + 1/9*sqrt(3)*arctan(1/3*(sqrt(3)*x^2 + 2*sqrt(3)*(
x^6 - 1)^(1/3))/x^2) + 1/9*log(-(x^2 - (x^6 - 1)^(1/3))/x^2) - 1/18*log((x^
4 + (x^6 - 1)^(1/3)*x^2 + (x^6 - 1)^(2/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^6-1\right)^{\frac{2}{3}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^6-1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((x^6 - 1)^(2/3)*x, x)
```

maple [C] time = 0.28, size = 46, normalized size = 0.44

$$\frac{x^2 \left(x^6-1\right)^{\frac{2}{3}}}{6}-\frac{\left(-\mathrm{signum}\left(x^6-1\right)\right)^{\frac{1}{3}} x^2 \,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^6\right)}{3 \mathrm{signum}\left(x^6-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^6-1)^(2/3),x)
```

[Out] 1/6*x^2*(x^6-1)^(2/3)-1/3/signum(x^6-1)^(1/3)*(-signum(x^6-1))^(1/3)*x^2*hypergeom([1/3,1/3],[4/3],x^6)

maxima [A] time = 0.42, size = 94, normalized size = 0.90

$$\frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(\frac{2\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)-\frac{\left(x^6-1\right)^{\frac{2}{3}}}{6\,x^4\left(\frac{x^6-1}{x^6}-1\right)}-\frac{1}{18}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6-1\right)^{\frac{2}{3}}}{x^4}+1\right)+\frac{1}{9}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x^6-1)^(2/3),x, algorithm="maxima")

[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3)/x^2 + 1)) - 1/6*(x^6 - 1)^(2/3)/(x^4*((x^6 - 1)/x^6 - 1)) - 1/18*log((x^6 - 1)^(1/3)/x^2 + (x^6 - 1)^(2/3)/x^4 + 1) + 1/9*log((x^6 - 1)^(1/3)/x^2 - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(x^6-1\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(x^6 - 1)^(2/3),x)

[Out] int(x*(x^6 - 1)^(2/3), x)

sympy [C] time = 0.90, size = 34, normalized size = 0.33

$$-\frac{x^2e^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|x^6\right)}{6\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x**6-1)**(2/3),x)

[Out] -x**2*exp(-I*pi/3)*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), x**6)/(6*gamma(4/3))

3.1298 $\int \frac{x^7}{\sqrt[3]{1+x^6}}\,dx$

Optimal. Leaf size=104

$$\frac{1}{6}\left(x^6+1\right)^{2/3}x^2+\frac{1}{18}\log\left(\sqrt[3]{x^6+1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{6\sqrt{3}}-\frac{1}{36}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right)$$

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {275, 321, 239}

$$\frac{1}{6}\left(x^6+1\right)^{2/3}x^2+\frac{1}{12}\log\left(x^2-\sqrt[3]{x^6+1}\right)-\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6+1}}+1}{\sqrt{3}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + x^6)^(1/3),x]
```

```
[Out] (x^2*(1 + x^6)^(2/3))/6 - ArcTan[(1 + (2*x^2)/(1 + x^6)^(1/3))/Sqrt[3]]/(6*
Sqrt[3]) + Log[x^2 - (1 + x^6)^(1/3)]/12
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\sqrt[3]{1+x^6}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{1+x^3}}\,dx,x,x^2\right)\\ &= \frac{1}{6}x^2\left(1+x^6\right)^{2/3}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^3}}\,dx,x,x^2\right)\\ &= \frac{1}{6}x^2\left(1+x^6\right)^{2/3}-\frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{1+x^6}}}{\sqrt{3}}\right)}{6\sqrt{3}}+\frac{1}{12}\log\left(x^2-\sqrt[3]{1+x^6}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 99, normalized size = 0.95

$$\frac{1}{36}\left(6\left(x^6+1\right)^{2/3}x^2+2\log\left(1-\frac{x^2}{\sqrt[3]{x^6+1}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6+1}}+1}{\sqrt{3}}\right)-\log\left(\frac{x^4}{\left(x^6+1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6+1}}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + x^6)^(1/3),x]
```

```
[Out] (6*x^2*(1 + x^6)^(2/3) - 2*Sqrt[3]*ArcTan[(1 + (2*x^2)/(1 + x^6)^(1/3))/Sqr
t[3]] + 2*Log[1 - x^2/(1 + x^6)^(1/3)] - Log[1 + x^4/(1 + x^6)^(2/3) + x^2/
(1 + x^6)^(1/3)])/36
```

IntegrateAlgebraic [A] time = 0.97, size = 104, normalized size = 1.00

$$\frac{1}{6}\left(x^6+1\right)^{2/3}x^2+\frac{1}{18}\log\left(\sqrt[3]{x^6+1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{6\sqrt{3}}-\frac{1}{36}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/(1 + x^6)^(1/3),x]
```

```
[Out] (x^2*(1 + x^6)^(2/3))/6 - ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(1 + x^6)^(1/3))]/(
6*Sqrt[3]) + Log[-x^2 + (1 + x^6)^(1/3)]/18 - Log[x^4 + x^2*(1 + x^6)^(1/3)
 + (1 + x^6)^(2/3)]/36
```

fricas [A] time = 0.40, size = 94, normalized size = 0.90

$$\frac{1}{6}\left(x^6+1\right)^{\frac{2}{3}}x^2+\frac{1}{18}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6+1\right)^{\frac{1}{3}}}{3x^2}\right)+\frac{1}{18}\log\left(-\frac{x^2-\left(x^6+1\right)^{\frac{1}{3}}}{x^2}\right)-\frac{1}{36}\log\left(\frac{x^4+\left(x^6+1\right)^{\frac{1}{3}}x^2+\left(x^6+1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*(x^6 + 1)^(2/3)*x^2 + 1/18*sqrt(3)*arctan(1/3*(sqrt(3)*x^2 + 2*sqrt(3)*
(x^6 + 1)^(1/3))/x^2) + 1/18*log(-(x^2 - (x^6 + 1)^(1/3))/x^2) - 1/36*log((
x^4 + (x^6 + 1)^(1/3)*x^2 + (x^6 + 1)^(2/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^7}{\left(x^6+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^7/(x^6 + 1)^(1/3), x)
```

maple [C] time = 0.26, size = 30, normalized size = 0.29

$$\frac{x^2\left(x^6+1\right)^{\frac{2}{3}}}{6}-\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x^6\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^6+1)^(1/3),x)
```

[Out] 1/6*x^2*(x^6+1)^(2/3)-1/6*x^2*hypergeom([1/3,1/3],[4/3],-x^6)

maxima [A] time = 0.42, size = 94, normalized size = 0.90

$$\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)+\frac{\left(x^6+1\right)^{\frac{2}{3}}}{6x^4\left(\frac{x^6+1}{x^6}-1\right)}-\frac{1}{36}\log\left(\frac{\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6+1\right)^{\frac{2}{3}}}{x^4}+1\right)+\frac{1}{18}\log\left(\frac{\left(x^6+1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^6+1)^(1/3),x, algorithm="maxima")

[Out] 1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3)/x^2 + 1)) + 1/6*(x^6 + 1
)^(2/3)/(x^4*((x^6 + 1)/x^6 - 1)) - 1/36*log((x^6 + 1)^(1/3)/x^2 + (x^6 + 1
)^(2/3)/x^4 + 1) + 1/18*log((x^6 + 1)^(1/3)/x^2 - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\left(x^6+1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(x^6 + 1)^(1/3),x)

[Out] int(x^7/(x^6 + 1)^(1/3), x)

sympy [C] time = 0.93, size = 29, normalized size = 0.28

$$\frac{x^8\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle| x^6e^{i\pi}\right)}{6\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(x**6+1)**(1/3),x)

[Out] x**8*gamma(4/3)*hyper((1/3, 4/3), (7/3,), x**6*exp_polar(I*pi))/(6*gamma(7/
3))

3.1299 $\int x^3 \sqrt[3]{1+x^6}\,dx$

Optimal. Leaf size=104

$$\frac{1}{6}\sqrt[3]{x^6+1}\,x^4-\frac{1}{18}\log\left(\sqrt[3]{x^6+1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{6\sqrt{3}}+\frac{1}{36}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right)$$

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {275, 279, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{6}\sqrt[3]{x^6+1}\,x^4-\frac{1}{18}\log\left(1-\frac{x^2}{\sqrt[3]{x^6+1}}\right)-\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6+1}}+1}{\sqrt{3}}\right)}{6\sqrt{3}}+\frac{1}{36}\log\left(\frac{x^4}{\left(x^6+1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6+1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x^6)^(1/3),x]
```

```
[Out] (x^4*(1 + x^6)^(1/3))/6 - ArcTan[(1 + (2*x^2)/(1 + x^6)^(1/3))/Sqrt[3]]/(6*
Sqrt[3]) - Log[1 - x^2/(1 + x^6)^(1/3)]/18 + Log[1 + x^4/(1 + x^6)^(2/3) +
x^2/(1 + x^6)^(1/3)]/36
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt[3]{1+x^6}\,dx &= \frac{1}{2}\text{Subst}\left(\int x\sqrt[3]{1+x^3}\,dx,x,x^2\right)\\
&= \frac{1}{6}x^4\sqrt[3]{1+x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{x}{\left(1+x^3\right)^{2/3}}\,dx,x,x^2\right)\\
&= \frac{1}{6}x^4\sqrt[3]{1+x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{x}{1-x^3}\,dx,x,\frac{x^2}{\sqrt[3]{1+x^6}}\right)\\
&= \frac{1}{6}x^4\sqrt[3]{1+x^6}+\frac{1}{18}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\frac{x^2}{\sqrt[3]{1+x^6}}\right)-\frac{1}{18}\text{Subst}\left(\int \frac{1-x}{1+x+x^2}\,dx,x,\frac{x^2}{\sqrt[3]{1+x^6}}\right.\\
&= \frac{1}{6}x^4\sqrt[3]{1+x^6}-\frac{1}{18}\log\left(1-\frac{x^2}{\sqrt[3]{1+x^6}}\right)+\frac{1}{36}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,\frac{x^2}{\sqrt[3]{1+x^6}}\right)-\frac{1}{12}\text{Subst}\\
&= \frac{1}{6}x^4\sqrt[3]{1+x^6}-\frac{1}{18}\log\left(1-\frac{x^2}{\sqrt[3]{1+x^6}}\right)+\frac{1}{36}\log\left(1+\frac{x^4}{\left(1+x^6\right)^{2/3}}+\frac{x^2}{\sqrt[3]{1+x^6}}\right)+\frac{1}{6}\text{Subst}\left(\int -\right.\\
&= \frac{1}{6}x^4\sqrt[3]{1+x^6}-\frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{1+x^6}}}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{1}{18}\log\left(1-\frac{x^2}{\sqrt[3]{1+x^6}}\right)+\frac{1}{36}\log\left(1+\frac{x^4}{\left(1+x^6\right)^{2/3}}+\frac{x^2}{\sqrt[3]{1+x^6}}\right.
\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.21

$$\frac{1}{4}x^4\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x^6)^(1/3),x]
```

[Out] (x^4*Hypergeometric2F1[-1/3, 2/3, 5/3, -x^6])/4

IntegrateAlgebraic [A] time = 0.61, size = 104, normalized size = 1.00

$$\frac{1}{6}\sqrt[3]{x^6+1}\,x^4-\frac{1}{18}\log\left(\sqrt[3]{x^6+1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{6\sqrt{3}}+\frac{1}{36}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(1 + x^6)^(1/3),x]
```

```
[Out] (x^4*(1 + x^6)^(1/3))/6 - ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(1 + x^6)^(1/3))]/(
6*Sqrt[3]) - Log[-x^2 + (1 + x^6)^(1/3)]/18 + Log[x^4 + x^2*(1 + x^6)^(1/3)
 + (1 + x^6)^(2/3)]/36
```

fricas [A] time = 0.41, size = 94, normalized size = 0.90

$$\frac{1}{6}\left(x^6+1\right)^{\frac{1}{3}}x^4+\frac{1}{18}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6+1\right)^{\frac{1}{3}}}{3x^2}\right)-\frac{1}{18}\log\left(-\frac{x^2-\left(x^6+1\right)^{\frac{1}{3}}}{x^2}\right)+\frac{1}{36}\log\left(\frac{x^4+\left(x^6+1\right)^{\frac{1}{3}}x^2+\left(x^6+1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^6+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*(x^6 + 1)^(1/3)*x^4 + 1/18*sqrt(3)*arctan(1/3*(sqrt(3)*x^2 + 2*sqrt(3)*
(x^6 + 1)^(1/3))/x^2) - 1/18*log(-(x^2 - (x^6 + 1)^(1/3))/x^2) + 1/36*log((
x^4 + (x^6 + 1)^(1/3)*x^2 + (x^6 + 1)^(2/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^6+1\right)^{\frac{1}{3}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^6+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 1)^(1/3)*x^3, x)
```

maple [C] time = 0.26, size = 30, normalized size = 0.29

$$\frac{x^4\left(x^6+1\right)^{\frac{1}{3}}}{6}+\frac{x^4\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^6\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^6+1)^(1/3),x)
```

```
[Out] 1/6*x^4*(x^6+1)^(1/3)+1/12*x^4*hypergeom([2/3,2/3],[5/3],-x^6)
```

maxima [A] time = 0.41, size = 94, normalized size = 0.90

$$\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)+\frac{\left(x^6+1\right)^{\frac{1}{3}}}{6x^2\left(\frac{x^6+1}{x^6}-1\right)}+\frac{1}{36}\log\left(\frac{\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6+1\right)^{\frac{2}{3}}}{x^4}+1\right)-\frac{1}{18}\log\left(\frac{\left(x^6+1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^6+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3)/x^2 + 1)) + 1/6*(x^6 + 1
)^(1/3)/(x^2*((x^6 + 1)/x^6 - 1)) + 1/36*log((x^6 + 1)^(1/3)/x^2 + (x^6 + 1
)^(2/3)/x^4 + 1) - 1/18*log((x^6 + 1)^(1/3)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(x^6+1\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^6 + 1)^(1/3),x)
```

```
[Out] int(x^3*(x^6 + 1)^(1/3), x)
```

sympy [C] time = 0.86, size = 31, normalized size = 0.30

$$\frac{x^4\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| x^6 e^{i\pi}\right)}{6\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**6+1)**(1/3),x)
```

```
[Out] x**4*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), x**6*exp_polar(I*pi))/(6*gamma(5
/3))
```

3.1300 $\int x\left(1+x^6\right)^{2/3}\,dx$

Optimal. Leaf size=104

$$\frac{1}{6}\left(x^6+1\right)^{2/3}x^2-\frac{1}{9}\log\left(\sqrt[3]{x^6+1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{3\sqrt{3}}+\frac{1}{18}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right)$$

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {275, 195, 239}

$$\frac{1}{6}\left(x^6+1\right)^{2/3}x^2-\frac{1}{6}\log\left(x^2-\sqrt[3]{x^6+1}\right)+\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6+1}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + x^6)^(2/3),x]
```

```
[Out] (x^2*(1 + x^6)^(2/3))/6 + ArcTan[(1 + (2*x^2)/(1 + x^6)^(1/3))/Sqrt[3]]/(3*
Sqrt[3]) - Log[x^2 - (1 + x^6)^(1/3)]/6
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x\left(1+x^6\right)^{2/3}\,dx&=\frac{1}{2}\text{Subst}\left(\int\left(1+x^3\right)^{2/3}\,dx,x,x^2\right)\\&=\frac{1}{6}x^2\left(1+x^6\right)^{2/3}+\frac{1}{3}\text{Subst}\left(\int\frac{1}{\sqrt[3]{1+x^3}}\,dx,x,x^2\right)\\&=\frac{1}{6}x^2\left(1+x^6\right)^{2/3}+\frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{1+x^6}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^2-\sqrt[3]{1+x^6}\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.21

$$\frac{1}{2}x^2\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + x^6)^(2/3),x]
```

```
[Out] (x^2*Hypergeometric2F1[-2/3, 1/3, 4/3, -x^6])/2
```

IntegrateAlgebraic [A] time = 0.60, size = 104, normalized size = 1.00

$$\frac{1}{6}\left(x^6+1\right)^{2/3}x^2-\frac{1}{9}\log\left(\sqrt[3]{x^6+1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{3\sqrt{3}}+\frac{1}{18}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(1 + x^6)^(2/3),x]
```

```
[Out] (x^2*(1 + x^6)^(2/3))/6 + ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(1 + x^6)^(1/3))]/(
3*Sqrt[3]) - Log[-x^2 + (1 + x^6)^(1/3)]/9 + Log[x^4 + x^2*(1 + x^6)^(1/3)
+ (1 + x^6)^(2/3)]/18
```

fricas [A] time = 0.41, size = 94, normalized size = 0.90

$$\frac{1}{6}\left(x^6+1\right)^{\frac{2}{3}}x^2-\frac{1}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6+1\right)^{\frac{1}{3}}}{3x^2}\right)-\frac{1}{9}\log\left(-\frac{x^2-\left(x^6+1\right)^{\frac{1}{3}}}{x^2}\right)+\frac{1}{18}\log\left(\frac{x^4+\left(x^6+1\right)^{\frac{1}{3}}x^2+\left(x^6+1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^6+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/6*(x^6 + 1)^(2/3)*x^2 - 1/9*sqrt(3)*arctan(1/3*(sqrt(3)*x^2 + 2*sqrt(3)*(
x^6 + 1)^(1/3))/x^2) - 1/9*log(-(x^2 - (x^6 + 1)^(1/3))/x^2) + 1/18*log((x^
4 + (x^6 + 1)^(1/3)*x^2 + (x^6 + 1)^(2/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^6+1\right)^{\frac{2}{3}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^6+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 1)^(2/3)*x, x)
```

maple [C] time = 0.26, size = 30, normalized size = 0.29

$$\frac{x^2\left(x^6+1\right)^{\frac{2}{3}}}{6}+\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x^6\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^6+1)^(2/3),x)
```

```
[Out] 1/6*x^2*(x^6+1)^(2/3)+1/3*x^2*hypergeom([1/3,1/3],[4/3],-x^6)
```

maxima [A] time = 0.42, size = 94, normalized size = 0.90

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)+\frac{\left(x^6+1\right)^{\frac{2}{3}}}{6x^4\left(\frac{x^6+1}{x^6}-1\right)}+\frac{1}{18}\log\left(\frac{\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6+1\right)^{\frac{2}{3}}}{x^4}+1\right)-\frac{1}{9}\log\left(\frac{\left(x^6+1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^6+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3)/x^2 + 1)) + 1/6*(x^6 + 1
)^(2/3)/(x^4*((x^6 + 1)/x^6 - 1)) + 1/18*log((x^6 + 1)^(1/3)/x^2 + (x^6 + 1
)^(2/3)/x^4 + 1) - 1/9*log((x^6 + 1)^(1/3)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(x^6+1\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^6 + 1)^(2/3),x)
```

```
[Out] int(x*(x^6 + 1)^(2/3), x)
```

sympy [C] time = 0.88, size = 31, normalized size = 0.30

$$\frac{x^2\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|x^6e^{i\pi}\right)}{6\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**6+1)**(2/3),x)
```

```
[Out] x**2*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), x**6*exp_polar(I*pi))/(6*gamma(4
/3))
```

3.1301 $\int \frac{(1-x^3+x^5)(-3+2x^5)}{x^3(1+x^3+x^5)\sqrt[4]{x+x^6}}\,dx$

Optimal. Leaf size=104

$$\frac{4(x^6+x)^{3/4}}{3x^3}+2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6+x}}{\sqrt{x^6+x}-x^2}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{x^6+x}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{x^6+x}}\right)$$

Rubi [F] time = 2.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-x^3+x^5)(-3+2x^5)}{x^3(1+x^3+x^5)\sqrt[4]{x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 - x^3 + x^5)*(-3 + 2*x^5))/(x^3*(1 + x^3 + x^5)*(x + x^6)^(1/4)),x]
```

```
[Out] (4*(1 + x^5)^(1/4)*Hypergeometric2F1[-9/20, 1/4, 11/20, -x^5])/(3*x^2*(x +
x^6)^(1/4)) - (16*x*(1 + x^5)^(1/4)*Hypergeometric2F1[3/20, 1/4, 23/20, -x^
5])/(3*(x + x^6)^(1/4)) + (8*x^3*(1 + x^5)^(1/4)*Hypergeometric2F1[1/4, 11/
20, 31/20, -x^5])/(11*(x + x^6)^(1/4)) + (40*x^(1/4)*(1 + x^5)^(1/4)*Defer[
Subst][Defer[Int][x^2/((1 + x^20)^(1/4)*(1 + x^12 + x^20)), x], x, x^(1/4)]
)/(x + x^6)^(1/4) + (16*x^(1/4)*(1 + x^5)^(1/4)*Defer[Subst][Defer[Int][x^1
4/((1 + x^20)^(1/4)*(1 + x^12 + x^20)), x], x, x^(1/4)])/(x + x^6)^(1/4)
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x^3+x^5)(-3+2x^5)}{x^3(1+x^3+x^5)\sqrt[4]{x+x^6}}\,dx &= \frac{\left(\sqrt[4]{x}\sqrt[4]{1+x^5}\right)\int \frac{(1-x^3+x^5)(-3+2x^5)}{x^{13/4}\sqrt[4]{1+x^5}(1+x^3+x^5)}\,dx}{\sqrt[4]{x+x^6}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x^5}\right)\text{Subst}\left(\int \frac{(1-x^{12}+x^{20})(-3+2x^{20})}{x^{10}\sqrt[4]{1+x^{20}}(1+x^{12}+x^{20})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{x+x^6}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{1+x^5}\right)\text{Subst}\left(\int \left(-\frac{3}{x^{10}\sqrt[4]{1+x^{20}}}-\frac{4x^2}{\sqrt[4]{1+x^{20}}}+\frac{2x^{10}}{\sqrt[4]{1+x^{20}}}+\frac{2x^2(5+2x^{12})}{\sqrt[4]{1+x^{20}}(1+x^{12}+x^{20})}\right)\right.}{\sqrt[4]{x+x^6}}\\
&= \frac{\left(8\sqrt[4]{x}\sqrt[4]{1+x^5}\right)\text{Subst}\left(\int \frac{x^{10}}{\sqrt[4]{1+x^{20}}}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{x+x^6}}+\frac{\left(8\sqrt[4]{x}\sqrt[4]{1+x^5}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{1+x^{20}}}\right.}{\sqrt[4]{x+x^6}}\\
&= \frac{4\sqrt[4]{1+x^5}\,{}_2F_1\left(-\frac{9}{20},\frac{1}{4};\frac{11}{20};-x^5\right)}{3x^2\sqrt[4]{x+x^6}}-\frac{16x\sqrt[4]{1+x^5}\,{}_2F_1\left(\frac{3}{20},\frac{1}{4};\frac{23}{20};-x^5\right)}{3\sqrt[4]{x+x^6}}+\frac{8x^3\sqrt[4]{1+x}}{}\\
&= \frac{4\sqrt[4]{1+x^5}\,{}_2F_1\left(-\frac{9}{20},\frac{1}{4};\frac{11}{20};-x^5\right)}{3x^2\sqrt[4]{x+x^6}}-\frac{16x\sqrt[4]{1+x^5}\,{}_2F_1\left(\frac{3}{20},\frac{1}{4};\frac{23}{20};-x^5\right)}{3\sqrt[4]{x+x^6}}+\frac{8x^3\sqrt[4]{1+x}}{}
\end{aligned}$$

Mathematica [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(1-x^3+x^5)(-3+2x^5)}{x^3(1+x^3+x^5)\sqrt[4]{x+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 - x^3 + x^5)*(-3 + 2*x^5))/(x^3*(1 + x^3 + x^5)*(x + x^6)^(1/
4)),x]
```

```
[Out] Integrate[((1 - x^3 + x^5)*(-3 + 2*x^5))/(x^3*(1 + x^3 + x^5)*(x + x^6)^(1/
4)), x]
```

IntegrateAlgebraic [A] time = 2.83, size = 104, normalized size = 1.00

$$\frac{4\left(x^6+x\right)^{3/4}}{3x^3}+2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6+x}}{\sqrt{x^6+x}-x^2}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{x^6+x}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{x^6+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 - x^3 + x^5)*(-3 + 2*x^5))/(x^3*(1 + x^3 + x^5)*(x +
 x^6)^(1/4)),x]
```

```
[Out] (4*(x + x^6)^(3/4))/(3*x^3) + 2*Sqrt[2]*ArcTan[(Sqrt[2]*x*(x + x^6)^(1/4))/
(-x^2 + Sqrt[x + x^6])] + 2*Sqrt[2]*ArcTanh[(x^2/Sqrt[2] + Sqrt[x + x^6]/Sq
rt[2])/(x*(x + x^6)^(1/4))]
```

fricas [B] time = 129.22, size = 682, normalized size = 6.56

$$\frac{12\sqrt{2}x^3\arctan\left(\frac{x^{10}+2x^8+x^6+2x^5+2x^3+2\sqrt{2}\left(x^6+x\right)^{\frac{3}{4}}\left(x^5-3x^3+1\right)+2\sqrt{2}\left(3x^7-x^5+3x^2\right)\left(x^6+x\right)^{\frac{1}{4}}+4\left(x^6+x^4+x\right)\sqrt{x^6+x}+\left(16\left(x^6+x\right)^{\frac{3}{4}}x^3+2\sqrt{2}\left(x^6-3x^4+x\right)\sqrt{x^6+x}+\sqrt{2}\left(x^{10}-8x^8-x^6+2x^5-8x^3+1\right)+4\left(x^7+x^5+x^2\right)\left(x^6+x\right)^{\frac{1}{4}}\right)\sqrt{\frac{x^5+x^3+2\sqrt{2}\left(x^6+x\right)^{\frac{1}{4}}x^2+4\sqrt{x^6+x}\,x+2\sqrt{2}\left(x^6+x\right)^{\frac{3}{4}}+1}{x^5+x^3+1}}+1}{x^{10}-14x^8+x^6+2x^5-14x^3+1}\right)-12\sqrt{2}x^3\arctan\left(\frac{x^{10}+2x^8+x^6+2x^5+2x^3-2\sqrt{2}\left(x^6+x\right)^{\frac{3}{4}}\left(x^5-3x^3+1\right)-2\sqrt{2}\left(3x^7-x^5+3x^2\right)\left(x^6+x\right)^{\frac{1}{4}}+4\left(x^6+x^4+x\right)\sqrt{x^6+x}+\left(16\left(x^6+x\right)^{\frac{3}{4}}x^3-2\sqrt{2}\left(x^6-3x^4+x\right)\sqrt{x^6+x}-\sqrt{2}\left(x^{10}-8x^8-x^6+2x^5-8x^3+1\right)+4\left(x^7+x^5+x^2\right)\left(x^6+x\right)^{\frac{1}{4}}\right)\sqrt{\frac{x^5+x^3-2\sqrt{2}\left(x^6+x\right)^{\frac{1}{4}}x^2+4\sqrt{x^6+x}\,x-2\sqrt{2}\left(x^6+x\right)^{\frac{3}{4}}+1}{x^5+x^3+1}}+1}{x^{10}-14x^8+x^6+2x^5-14x^3+1}\right)+3\sqrt{2}x^3\log\left(\frac{4\left(x^5+x^3+2\sqrt{2}\left(x^6+x\right)^{\frac{1}{4}}x^2+4\sqrt{x^6+x}\,x+2\sqrt{2}\left(x^6+x\right)^{\frac{3}{4}}+1\right)}{x^5+x^3+1}\right)-3\sqrt{2}x^3\log\left(\frac{4\left(x^5+x^3-2\sqrt{2}\left(x^6+x\right)^{\frac{1}{4}}x^2+4\sqrt{x^6+x}\,x-2\sqrt{2}\left(x^6+x\right)^{\frac{3}{4}}+1\right)}{x^5+x^3+1}\right)+8\left(x^6+x\right)^{\frac{3}{4}}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^3+1)*(2*x^5-3)/x^3/(x^5+x^3+1)/(x^6+x)^(1/4),x, algorithm=
"fricas")
```

```
[Out] 1/6*(12*sqrt(2)*x^3*arctan((x^10 + 2*x^8 + x^6 + 2*x^5 + 2*x^3 + 2*sqrt(2)*
(x^6 + x)^(3/4)*(x^5 - 3*x^3 + 1) + 2*sqrt(2)*(3*x^7 - x^5 + 3*x^2)*(x^6 +
x)^(1/4) + 4*(x^6 + x^4 + x)*sqrt(x^6 + x) + (16*(x^6 + x)^(3/4)*x^3 + 2*sq
rt(2)*(x^6 - 3*x^4 + x)*sqrt(x^6 + x) + sqrt(2)*(x^10 - 8*x^8 - x^6 + 2*x^5
 - 8*x^3 + 1) + 4*(x^7 + x^5 + x^2)*(x^6 + x)^(1/4))*sqrt((x^5 + x^3 + 2*sq
rt(2)*(x^6 + x)^(1/4)*x^2 + 4*sqrt(x^6 + x)*x + 2*sqrt(2)*(x^6 + x)^(3/4) +
 1)/(x^5 + x^3 + 1)) + 1)/(x^10 - 14*x^8 + x^6 + 2*x^5 - 14*x^3 + 1)) - 12*
sqrt(2)*x^3*arctan((x^10 + 2*x^8 + x^6 + 2*x^5 + 2*x^3 - 2*sqrt(2)*(x^6 + x
)^(3/4)*(x^5 - 3*x^3 + 1) - 2*sqrt(2)*(3*x^7 - x^5 + 3*x^2)*(x^6 + x)^(1/4)
 + 4*(x^6 + x^4 + x)*sqrt(x^6 + x) + (16*(x^6 + x)^(3/4)*x^3 - 2*sqrt(2)*(x
^6 - 3*x^4 + x)*sqrt(x^6 + x) - sqrt(2)*(x^10 - 8*x^8 - x^6 + 2*x^5 - 8*x^3
 + 1) + 4*(x^7 + x^5 + x^2)*(x^6 + x)^(1/4))*sqrt((x^5 + x^3 - 2*sqrt(2)*(x
^6 + x)^(1/4)*x^2 + 4*sqrt(x^6 + x)*x - 2*sqrt(2)*(x^6 + x)^(3/4) + 1)/(x^5
 + x^3 + 1)) + 1)/(x^10 - 14*x^8 + x^6 + 2*x^5 - 14*x^3 + 1)) + 3*sqrt(2)*x
^3*log(4*(x^5 + x^3 + 2*sqrt(2)*(x^6 + x)^(1/4)*x^2 + 4*sqrt(x^6 + x)*x + 2
*sqrt(2)*(x^6 + x)^(3/4) + 1)/(x^5 + x^3 + 1)) - 3*sqrt(2)*x^3*log(4*(x^5 +
 x^3 - 2*sqrt(2)*(x^6 + x)^(1/4)*x^2 + 4*sqrt(x^6 + x)*x - 2*sqrt(2)*(x^6 +
 x)^(3/4) + 1)/(x^5 + x^3 + 1)) + 8*(x^6 + x)^(3/4))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^5-3\right)\left(x^5-x^3+1\right)}{\left(x^6+x\right)^{\frac{1}{4}}\left(x^5+x^3+1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^3+1)*(2*x^5-3)/x^3/(x^5+x^3+1)/(x^6+x)^(1/4),x, algorithm=
"giac")
```

```
[Out] integrate((2*x^5 - 3)*(x^5 - x^3 + 1)/((x^6 + x)^(1/4)*(x^5 + x^3 + 1)*x^3)
, x)
```

maple [C] time = 9.51, size = 220, normalized size = 2.12

$$\frac{\frac{4x^5}{3}+\frac{4}{3}}{x^2\left(x\left(x^5+1\right)\right)^{\frac{1}{4}}}-2\,\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{2\sqrt{x^6+x}\,\mathrm{RootOf}\left(\_Z^4+1\right)^3x-\mathrm{RootOf}\left(\_Z^4+1\right)x^5-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^2\left(x^6+x\right)^{\frac{1}{4}}x^2+\mathrm{RootOf}\left(\_Z^4+1\right)x^3+2\left(x^6+x\right)^{\frac{3}{4}}-\mathrm{RootOf}\left(\_Z^4+1\right)}{x^5+x^3+1}\right)+2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^3x^5-\mathrm{RootOf}\left(\_Z^4+1\right)^3x^3+2\,\mathrm{RootOf}\left(\_Z^4+1\right)^2\left(x^6+x\right)^{\frac{1}{4}}x^2-2\sqrt{x^6+x}\,\mathrm{RootOf}\left(\_Z^4+1\right)x+\mathrm{RootOf}\left(\_Z^4+1\right)^3+2\left(x^6+x\right)^{\frac{3}{4}}}{x^5+x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-x^3+1)*(2*x^5-3)/x^3/(x^5+x^3+1)/(x^6+x)^(1/4),x)
```

```
[Out] 4/3*(x^5+1)/x^2/(x*(x^5+1))^(1/4)-2*RootOf(_Z^4+1)*ln(-(2*(x^6+x)^(1/2)*Roo
tOf(_Z^4+1)^3*x-RootOf(_Z^4+1)*x^5-2*RootOf(_Z^4+1)^2*(x^6+x)^(1/4)*x^2+Roo
tOf(_Z^4+1)*x^3+2*(x^6+x)^(3/4)-RootOf(_Z^4+1))/(x^5+x^3+1))+2*RootOf(_Z^4+
1)^3*ln((RootOf(_Z^4+1)^3*x^5-RootOf(_Z^4+1)^3*x^3+2*RootOf(_Z^4+1)^2*(x^6+
x)^(1/4)*x^2-2*(x^6+x)^(1/2)*RootOf(_Z^4+1)*x+RootOf(_Z^4+1)^3+2*(x^6+x)^(3
/4))/(x^5+x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^5-3\right)\left(x^5-x^3+1\right)}{\left(x^6+x\right)^{\frac{1}{4}}\left(x^5+x^3+1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^3+1)*(2*x^5-3)/x^3/(x^5+x^3+1)/(x^6+x)^(1/4),x, algorithm=
"maxima")
```

```
[Out] integrate((2*x^5 - 3)*(x^5 - x^3 + 1)/((x^6 + x)^(1/4)*(x^5 + x^3 + 1)*x^3)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^5-3\right)\,\left(x^5-x^3+1\right)}{x^3\,{\left(x^6+x\right)}^{1/4}\,\left(x^5+x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^5 - 3)*(x^5 - x^3 + 1))/(x^3*(x + x^6)^(1/4)*(x^3 + x^5 + 1)),x)
```

```
[Out] int(((2*x^5 - 3)*(x^5 - x^3 + 1))/(x^3*(x + x^6)^(1/4)*(x^3 + x^5 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{5}-3\right)\left(x^{5}-x^{3}+1\right)}{x^{3}\sqrt[4]{x\left(x+1\right)\left(x^{4}-x^{3}+x^{2}-x+1\right)}\left(x^{5}+x^{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-x**3+1)*(2*x**5-3)/x**3/(x**5+x**3+1)/(x**6+x)**(1/4),x)
```

```
[Out] Integral((2*x**5 - 3)*(x**5 - x**3 + 1)/(x**3*(x*(x + 1)*(x**4 - x**3 + x**
2 - x + 1))**(1/4)*(x**5 + x**3 + 1)), x)
```

3.1302 $\int \frac{x}{\sqrt[3]{-x^2+x^6}}\,dx$

Optimal. Leaf size=104

$$-\frac{1}{4}\log\left(\sqrt[3]{x^6-x^2}-x^2\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x^2}{x^2+2\sqrt[3]{x^6-x^2}}\right)+\frac{1}{8}\log\left(x^4+\sqrt[3]{x^6-x^2}\,x^2+\left(x^6-x^2\right)^{2/3}\right)$$

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2013, 2011, 329, 275, 239}

$$\frac{\sqrt{3}\sqrt[3]{x^2}\sqrt[3]{x^4-1}\tan^{-1}\left(\frac{\frac{2\left(x^2\right)^{2/3}}{\sqrt[3]{x^4-1}}+1}{\sqrt{3}}\right)}{4\sqrt[3]{x^6-x^2}}-\frac{3\sqrt[3]{x^2}\sqrt[3]{x^4-1}\log\left(\left(x^2\right)^{2/3}-\sqrt[3]{x^4-1}\right)}{8\sqrt[3]{x^6-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(-x^2 + x^6)^(1/3),x]
```

```
[Out] (Sqrt[3]*(x^2)^(1/3)*(-1 + x^4)^(1/3)*ArcTan[(1 + (2*(x^2)^(2/3))/(-1 + x^4
)^(1/3))/Sqrt[3]])/(4*(-x^2 + x^6)^(1/3)) - (3*(x^2)^(1/3)*(-1 + x^4)^(1/3)
*Log[(x^2)^(2/3) - (-1 + x^4)^(1/3)])/(8*(-x^2 + x^6)^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{-x^2+x^6}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{-x+x^3}}\,dx, x, x^2\right)$$

$$= \frac{\left(\sqrt[3]{x^2}\,\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,\sqrt[3]{-1+x^2}}\,dx, x, x^2\right)}{2\sqrt[3]{-x^2+x^6}}$$

$$= \frac{\left(3\sqrt[3]{x^2}\,\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{-1+x^6}}\,dx, x, \sqrt[3]{x^2}\right)}{2\sqrt[3]{-x^2+x^6}}$$

$$= \frac{\left(3\sqrt[3]{x^2}\,\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x^3}}\,dx, x, \left(x^2\right)^{2/3}\right)}{4\sqrt[3]{-x^2+x^6}}$$

$$= \frac{\sqrt{3}\,\sqrt[3]{x^2}\,\sqrt[3]{-1+x^4}\,\tan^{-1}\left(\frac{1+\frac{2\left(x^2\right)^{2/3}}{\sqrt[3]{-1+x^4}}}{\sqrt{3}}\right)}{4\sqrt[3]{-x^2+x^6}} - \frac{3\sqrt[3]{x^2}\,\sqrt[3]{-1+x^4}\,\log\left(\left(x^2\right)^{2/3}-\sqrt[3]{-1+x^4}\right)}{8\sqrt[3]{-x^2+x^6}}$$

Mathematica [C] time = 0.01, size = 44, normalized size = 0.42

$$\frac{3x^2\sqrt[3]{1-x^4}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};x^4\right)}{4\sqrt[3]{x^2\left(x^4-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-x^2 + x^6)^(1/3),x]
```

```
[Out] (3*x^2*(1 - x^4)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, x^4])/(4*(x^2*(-1 +
 x^4))^(1/3))
```

IntegrateAlgebraic [A] time = 0.34, size = 104, normalized size = 1.00

$$-\frac{1}{4}\log\left(\sqrt[3]{x^6-x^2}-x^2\right)+\frac{1}{4}\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,x^2}{x^2+2\sqrt[3]{x^6-x^2}}\right)+\frac{1}{8}\log\left(x^4+\sqrt[3]{x^6-x^2}\,x^2+\left(x^6-x^2\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(-x^2 + x^6)^(1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(-x^2 + x^6)^(1/3))])/4 - Log[-x^2 +
 (-x^2 + x^6)^(1/3)]/4 + Log[x^4 + x^2*(-x^2 + x^6)^(1/3) + (-x^2 + x^6)^(2
/3)]/8
```

fricas [A] time = 0.74, size = 99, normalized size = 0.95

$$\frac{1}{4}\sqrt{3}\arctan\left(-\frac{44032959556\sqrt{3}\left(x^6-x^2\right)^{\frac{1}{3}}x^2+\sqrt{3}\left(16754327161\,x^4-2707204793\right)-10524305234\sqrt{3}\left(x^6-x^2\right)^{\frac{2}{3}}}{81835897185\,x^4-1102302937}\right)-\frac{1}{8}\log\left(-3\left(x^6-x^2\right)^{\frac{1}{3}}x^2+3\left(x^6-x^2\right)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*arctan(-(44032959556*sqrt(3)*(x^6 - x^2)^(1/3)*x^2 + sqrt(3)*(1
6754327161*x^4 - 2707204793) - 10524305234*sqrt(3)*(x^6 - x^2)^(2/3))/(8183
5897185*x^4 - 1102302937)) - 1/8*log(-3*(x^6 - x^2)^(1/3)*x^2 + 3*(x^6 - x^
2)^(2/3) + 1)
```

giac [A] time = 0.17, size = 63, normalized size = 0.61

$$-\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x^4}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{8}\log\left(\left(-\frac{1}{x^4}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x^4}+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{4}\log\left(\left|\left(-\frac{1}{x^4}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-x^2)^(1/3),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-1/x^4 + 1)^(1/3) + 1)) + 1/8*log((-1/x
^4 + 1)^(2/3) + (-1/x^4 + 1)^(1/3) + 1) - 1/4*log(abs((-1/x^4 + 1)^(1/3) -
1))
```

maple [C] time = 0.28, size = 33, normalized size = 0.32

$$\frac{3\left(-\mathrm{signum}\left(x^4-1\right)\right)^{\frac{1}{3}}x^{\frac{4}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^4\right)}{4\mathrm{signum}\left(x^4-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6-x^2)^(1/3),x)
```

```
[Out] 3/4/signum(x^4-1)^(1/3)*(-signum(x^4-1))^(1/3)*x^(4/3)*hypergeom([1/3,1/3],
[4/3],x^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(x^6-x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^6 - x^2)^(1/3), x)
```

mupad [B] time = 1.12, size = 33, normalized size = 0.32

$$\frac{3\,x^2\,\left(1-x^4\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ x^4\right)}{4\,\left(x^6-x^2\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6 - x^2)^(1/3),x)
```

```
[Out] (3*x^2*(1 - x^4)^(1/3)*hypergeom([1/3, 1/3], 4/3, x^4))/(4*(x^6 - x^2)^(1/3
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt[3]{x^2\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6-x**2)**(1/3),x)
```

```
[Out] Integral(x/(x**2*(x - 1)*(x + 1)*(x**2 + 1))**(1/3), x)
```

3.1303 $\int \frac{(-3+2x)\left(1-x+x^3\right)^{2/3}}{1-2x+x^2+2x^3-2x^4+2x^6}\,dx$

Optimal. Leaf size=104

$$-\tan^{-1}\left(\frac{x}{\sqrt[3]{x^3-x+1}}\right)-\frac{1}{2}\tan^{-1}\left(\frac{x\sqrt[3]{x^3-x+1}}{\left(x^3-x+1\right)^{2/3}-x^2}\right)-\frac{1}{2}\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\,x\sqrt[3]{x^3-x+1}}{\left(x^3-x+1\right)^{2/3}+x^2}\right)$$

Rubi [F] time = 0.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-3+2x)\left(1-x+x^3\right)^{2/3}}{1-2x+x^2+2x^3-2x^4+2x^6}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + 2*x)*(1 - x + x^3)^(2/3))/(1 - 2*x + x^2 + 2*x^3 - 2*x^4 + 2*x^6
),x]
```

```
[Out] -3*Defer[Int][(1 - x + x^3)^(2/3)/(1 - 2*x + x^2 + 2*x^3 - 2*x^4 + 2*x^6),
x] + 2*Defer[Int][(x*(1 - x + x^3)^(2/3))/(1 - 2*x + x^2 + 2*x^3 - 2*x^4 +
2*x^6), x]
```

Rubi steps

$$\int \frac{(-3+2x)\left(1-x+x^3\right)^{2/3}}{1-2x+x^2+2x^3-2x^4+2x^6}\,dx=\int\left(-\frac{3\left(1-x+x^3\right)^{2/3}}{1-2x+x^2+2x^3-2x^4+2x^6}+\frac{2x\left(1-x+x^3\right)^{2/3}}{1-2x+x^2+2x^3-2x^4+2x^6}\right)dx$$
$$=2\int\frac{x\left(1-x+x^3\right)^{2/3}}{1-2x+x^2+2x^3-2x^4+2x^6}\,dx-3\int\frac{\left(1-x+x^3\right)^{2/3}}{1-2x+x^2+2x^3-2x^4+2x^6}\,dx$$

Mathematica [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(-3+2x)\left(1-x+x^3\right)^{2/3}}{1-2x+x^2+2x^3-2x^4+2x^6}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + 2*x)*(1 - x + x^3)^(2/3))/(1 - 2*x + x^2 + 2*x^3 - 2*x^4 +
 2*x^6),x]
```

```
[Out] Integrate[((-3 + 2*x)*(1 - x + x^3)^(2/3))/(1 - 2*x + x^2 + 2*x^3 - 2*x^4 +
 2*x^6), x]
```

IntegrateAlgebraic [C] time = 0.43, size = 106, normalized size = 1.02

$$-\tan^{-1}\left(\frac{x}{\sqrt[3]{x^3-x+1}}\right)-\frac{1}{2}i\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{\left(1-i\sqrt{3}\right)x}{2\sqrt[3]{x^3-x+1}}\right)+\frac{1}{2}i\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{\left(1+i\sqrt{3}\right)x}{2\sqrt[3]{x^3-x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + 2*x)*(1 - x + x^3)^(2/3))/(1 - 2*x + x^2 + 2*x^3
- 2*x^4 + 2*x^6),x]
```

```
[Out] -ArcTan[x/(1 - x + x^3)^(1/3)] - (I/2)*(-I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3
])*x)/(2*(1 - x + x^3)^(1/3))] + (I/2)*(I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3]
)*x)/(2*(1 - x + x^3)^(1/3))]
```

fricas [B] time = 5.14, size = 2090, normalized size = 20.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^3-x+1)^(2/3)/(2*x^6-2*x^4+2*x^3+x^2-2*x+1),x, algorit
hm="fricas")
```

```
[Out] -1/8*sqrt(3)*log(8*(2*x^6 - 2*x^4 + 2*x^3 + x^2 + 2*(3*x^4 + sqrt(3)*(x^4 -
 x^2 + x))*(x^3 - x + 1)^(2/3) + 4*sqrt(3)*(x^6 - x^4 + x^3) + 2*(sqrt(3)*x
^5 + 3*x^5 - 3*x^3 + 3*x^2)*(x^3 - x + 1)^(1/3) - 2*x + 1)/(2*x^6 - 2*x^4 +
 2*x^3 + x^2 - 2*x + 1)) + 1/8*sqrt(3)*log(8*(2*x^6 - 2*x^4 + 2*x^3 + x^2 +
 2*(3*x^4 - sqrt(3)*(x^4 - x^2 + x))*(x^3 - x + 1)^(2/3) - 4*sqrt(3)*(x^6 -
 x^4 + x^3) - 2*(sqrt(3)*x^5 - 3*x^5 + 3*x^3 - 3*x^2)*(x^3 - x + 1)^(1/3) -
 2*x + 1)/(2*x^6 - 2*x^4 + 2*x^3 + x^2 - 2*x + 1)) - 1/2*arctan((24*x^12 -
16*x^10 + 16*x^9 - 32*x^8 + 64*x^7 - 8*x^6 - 72*x^5 + 70*x^4 - 16*x^3 - 12*
x^2 - sqrt(2)*(116*x^12 - 300*x^10 + 300*x^9 + 240*x^8 - 480*x^7 + 186*x^6
+ 162*x^5 - 163*x^4 + 58*x^3 - 6*x^2 + 4*(22*x^10 - 30*x^8 + 30*x^7 + 13*x^
6 - 26*x^5 + 11*x^4 + 6*x^3 - 6*x^2 - sqrt(3)*(14*x^10 - 20*x^8 + 20*x^7 +
3*x^6 - 6*x^5 + 4*x^4 - 3*x^3 + 3*x^2 - x) + 2*x)*(x^3 - x + 1)^(2/3) - sqr
t(3)*(36*x^12 - 36*x^10 + 36*x^9 - 4*x^8 + 8*x^7 - 2*x^6 - 6*x^5 + 7*x^4 -
6*x^3 + 6*x^2 - 4*x + 1) + 2*(28*x^11 - 2*x^9 + 2*x^8 - 40*x^7 + 80*x^6 - 2
5*x^5 - 45*x^4 + 45*x^3 - 15*x^2 - sqrt(3)*(32*x^11 - 70*x^9 + 70*x^8 + 46*
x^7 - 92*x^6 + 37*x^5 + 27*x^4 - 27*x^3 + 9*x^2))*(x^3 - x + 1)^(1/3) + 4*x
 - 1)*sqrt((2*x^6 - 2*x^4 + 2*x^3 + x^2 + 2*(3*x^4 + sqrt(3)*(x^4 - x^2 + x
))*(x^3 - x + 1)^(2/3) + 4*sqrt(3)*(x^6 - x^4 + x^3) + 2*(sqrt(3)*x^5 + 3*x
^5 - 3*x^3 + 3*x^2)*(x^3 - x + 1)^(1/3) - 2*x + 1)/(2*x^6 - 2*x^4 + 2*x^3 +
 x^2 - 2*x + 1)) + 4*(18*x^10 - 46*x^8 + 46*x^7 + 23*x^6 - 46*x^5 + 21*x^4
+ 6*x^3 - 6*x^2 - sqrt(3)*(2*x^10 + 4*x^8 - 4*x^7 - 3*x^6 + 6*x^5 - 2*x^4 -
 3*x^3 + 3*x^2 - x) + 2*x)*(x^3 - x + 1)^(2/3) - sqrt(3)*(20*x^12 - 40*x^10
 + 40*x^9 + 24*x^8 - 48*x^7 + 20*x^6 + 12*x^5 - 11*x^4 + 6*x^2 - 4*x + 1) +
 4*(2*x^11 + 16*x^9 - 16*x^8 - 27*x^7 + 54*x^6 - 20*x^5 - 21*x^4 + 21*x^3 -
 7*x^2 - sqrt(3)*(6*x^11 - 14*x^9 + 14*x^8 + 13*x^7 - 26*x^6 + 9*x^5 + 12*x
^4 - 12*x^3 + 4*x^2))*(x^3 - x + 1)^(1/3) + 8*x - 2)/(52*x^12 - 232*x^10 +
232*x^9 + 248*x^8 - 496*x^7 + 180*x^6 + 204*x^5 - 203*x^4 + 64*x^3 + 6*x^2
- 4*x + 1)) + 1/2*arctan(-(24*x^12 - 16*x^10 + 16*x^9 - 32*x^8 + 64*x^7 - 8
*x^6 - 72*x^5 + 70*x^4 - 16*x^3 - 12*x^2 - sqrt(2)*(116*x^12 - 300*x^10 + 3
00*x^9 + 240*x^8 - 480*x^7 + 186*x^6 + 162*x^5 - 163*x^4 + 58*x^3 - 6*x^2 +
 4*(22*x^10 - 30*x^8 + 30*x^7 + 13*x^6 - 26*x^5 + 11*x^4 + 6*x^3 - 6*x^2 +
sqrt(3)*(14*x^10 - 20*x^8 + 20*x^7 + 3*x^6 - 6*x^5 + 4*x^4 - 3*x^3 + 3*x^2
- x) + 2*x)*(x^3 - x + 1)^(2/3) + sqrt(3)*(36*x^12 - 36*x^10 + 36*x^9 - 4*x
^8 + 8*x^7 - 2*x^6 - 6*x^5 + 7*x^4 - 6*x^3 + 6*x^2 - 4*x + 1) + 2*(28*x^11
- 2*x^9 + 2*x^8 - 40*x^7 + 80*x^6 - 25*x^5 - 45*x^4 + 45*x^3 - 15*x^2 + sqr
t(3)*(32*x^11 - 70*x^9 + 70*x^8 + 46*x^7 - 92*x^6 + 37*x^5 + 27*x^4 - 27*x^
3 + 9*x^2))*(x^3 - x + 1)^(1/3) + 4*x - 1)*sqrt((2*x^6 - 2*x^4 + 2*x^3 + x^
2 + 2*(3*x^4 - sqrt(3)*(x^4 - x^2 + x))*(x^3 - x + 1)^(2/3) - 4*sqrt(3)*(x^
6 - x^4 + x^3) - 2*(sqrt(3)*x^5 - 3*x^5 + 3*x^3 - 3*x^2)*(x^3 - x + 1)^(1/3
) - 2*x + 1)/(2*x^6 - 2*x^4 + 2*x^3 + x^2 - 2*x + 1)) + 4*(18*x^10 - 46*x^8
 + 46*x^7 + 23*x^6 - 46*x^5 + 21*x^4 + 6*x^3 - 6*x^2 + sqrt(3)*(2*x^10 + 4*
x^8 - 4*x^7 - 3*x^6 + 6*x^5 - 2*x^4 - 3*x^3 + 3*x^2 - x) + 2*x)*(x^3 - x +
1)^(2/3) + sqrt(3)*(20*x^12 - 40*x^10 + 40*x^9 + 24*x^8 - 48*x^7 + 20*x^6 +
 12*x^5 - 11*x^4 + 6*x^2 - 4*x + 1) + 4*(2*x^11 + 16*x^9 - 16*x^8 - 27*x^7
+ 54*x^6 - 20*x^5 - 21*x^4 + 21*x^3 - 7*x^2 + sqrt(3)*(6*x^11 - 14*x^9 + 14
*x^8 + 13*x^7 - 26*x^6 + 9*x^5 + 12*x^4 - 12*x^3 + 4*x^2))*(x^3 - x + 1)^(1
/3) + 8*x - 2)/(52*x^12 - 232*x^10 + 232*x^9 + 248*x^8 - 496*x^7 + 180*x^6
+ 204*x^5 - 203*x^4 + 64*x^3 + 6*x^2 - 4*x + 1)) - 1/2*arctan((6*x^6 - 4*x^
```

```
4 + 4*x^3 - x^2 + 4*(3*x^4 - x^2 + x)*(x^3 - x + 1)^(2/3) - 4*(x^5 - 2*x^3
+ 2*x^2)*(x^3 - x + 1)^(1/3) + 2*x - 1)/(14*x^6 - 16*x^4 + 16*x^3 + x^2 - 2
*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-x+1\right)^{\frac{2}{3}}(2x-3)}{2x^6-2x^4+2x^3+x^2-2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^3-x+1)^(2/3)/(2*x^6-2*x^4+2*x^3+x^2-2*x+1),x, algorit
hm="giac")
```

```
[Out] integrate((x^3 - x + 1)^(2/3)*(2*x - 3)/(2*x^6 - 2*x^4 + 2*x^3 + x^2 - 2*x
+ 1), x)
```

maple [C] time = 40.64, size = 3488, normalized size = 33.54

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*x)*(x^3-x+1)^(2/3)/(2*x^6-2*x^4+2*x^3+x^2-2*x+1),x)
```

```
[Out] 1/2*RootOf(_Z^2+1)*ln((-3+6*x-224*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)^2*x
^4-6*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*RootOf(_Z^2+1)-26*RootOf(_Z^2+1)
*x^4+26*RootOf(_Z^2+1)*x^6+7*RootOf(_Z^2+1)*x^2+26*RootOf(_Z^2+1)*x^3-6*x^6
-3*x^2-6*x^3+6*x^4-28*RootOf(_Z^2+1)^2*x^6+224*RootOf(-2*_Z*RootOf(_Z^2+1)+
4*_Z^2-1)^2*x^6+44*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*RootOf(_Z^2+1)*x^6
-96*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)^2*RootOf(_Z^2+1)*x^3+24*RootOf(-2
*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*RootOf(_Z^2+1)^2*x^3-20*x^6*RootOf(-2*_Z*RootO
f(_Z^2+1)+4*_Z^2-1)-28*RootOf(_Z^2+1)^2*x^3+28*RootOf(_Z^2+1)^2*x^4+7*RootO
f(_Z^2+1)-14*RootOf(_Z^2+1)*x+14*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)+20*R
ootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*x^4-20*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_
Z^2-1)*x^3+14*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*x^2-28*RootOf(-2*_Z*Roo
tOf(_Z^2+1)+4*_Z^2-1)*x+224*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)^2*x^3-44*
RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*RootOf(_Z^2+1)*x^4+44*RootOf(-2*_Z*Ro
otOf(_Z^2+1)+4*_Z^2-1)*RootOf(_Z^2+1)*x^3-6*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_
Z^2-1)*RootOf(_Z^2+1)*x^2+12*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*RootOf(_
Z^2+1)*x-42*(x^3-x+1)^(2/3)*x^4+42*(x^3-x+1)^(2/3)*x^2-42*(x^3-x+1)^(2/3)*x
-96*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)^2*RootOf(_Z^2+1)*x^6+24*RootOf(-2
*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*RootOf(_Z^2+1)^2*x^6+96*RootOf(-2*_Z*RootOf(_Z
^2+1)+4*_Z^2-1)^2*RootOf(_Z^2+1)*x^4-24*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-
1)*RootOf(_Z^2+1)^2*x^4-104*(x^3-x+1)^(1/3)*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_
Z^2-1)*RootOf(_Z^2+1)*x^5+48*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)^2*(x^3-x
+1)^(1/3)*RootOf(_Z^2+1)*x^5-12*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*(x^3-
x+1)^(1/3)*RootOf(_Z^2+1)^2*x^5-36*(x^3-x+1)^(1/3)*RootOf(-2*_Z*RootOf(_Z^2
+1)+4*_Z^2-1)*RootOf(_Z^2+1)*x^3+36*(x^3-x+1)^(1/3)*RootOf(-2*_Z*RootOf(_Z^
2+1)+4*_Z^2-1)*RootOf(_Z^2+1)*x^2-48*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*
RootOf(_Z^2+1)*(x^3-x+1)^(2/3)*x^4+6*(x^3-x+1)^(2/3)*RootOf(_Z^2+1)^2*x^4+2
8*(x^3-x+1)^(1/3)*RootOf(_Z^2+1)^2*x^5-144*(x^3-x+1)^(2/3)*RootOf(-2*_Z*Roo
tOf(_Z^2+1)+4*_Z^2-1)*x^4+84*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*(x^3-x+1
)^(1/3)*x^5+36*RootOf(_Z^2+1)*(x^3-x+1)^(2/3)*x^4-84*(x^3-x+1)^(1/3)*RootOf
(_Z^2+1)*x^5-24*(x^3-x+1)^(2/3)*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*x^2-8
4*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*(x^3-x+1)^(1/3)*x^3+6*RootOf(_Z^2+1
)*(x^3-x+1)^(2/3)*x^2+84*(x^3-x+1)^(1/3)*RootOf(_Z^2+1)*x^3+24*(x^3-x+1)^(2
/3)*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*x+84*RootOf(-2*_Z*RootOf(_Z^2+1)+
4*_Z^2-1)*(x^3-x+1)^(1/3)*x^2-6*RootOf(_Z^2+1)*(x^3-x+1)^(2/3)*x-84*(x^3-x+
1)^(1/3)*RootOf(_Z^2+1)*x^2+96*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)^2*(x^3
-x+1)^(2/3)*x^4+112*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)^2*(x^3-x+1)^(1/3)
```

```
*x^5)/(2*x^6-2*x^4+2*x^3+x^2-2*x+1))+1/2*ln((3-6*x+448*RootOf(-2*_Z*RootOf(
_Z^2+1)+4*_Z^2-1)^2*x^4-2*RootOf(_Z^2+1)*x^4+2*RootOf(_Z^2+1)*x^6+7*RootOf(
_Z^2+1)*x^2+2*RootOf(_Z^2+1)*x^3+6*x^6+3*x^2+6*x^3-6*x^4-28*RootOf(_Z^2+1)^
2*x^6-448*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)^2*x^6+224*RootOf(-2*_Z*Root
Of(_Z^2+1)+4*_Z^2-1)*RootOf(_Z^2+1)*x^6-8*x^6*RootOf(-2*_Z*RootOf(_Z^2+1)+4
*_Z^2-1)-28*RootOf(_Z^2+1)^2*x^3+28*RootOf(_Z^2+1)^2*x^4+7*RootOf(_Z^2+1)-1
4*RootOf(_Z^2+1)*x-28*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)+8*RootOf(-2*_Z*
RootOf(_Z^2+1)+4*_Z^2-1)*x^4-8*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*x^3-28
*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*x^2+56*RootOf(-2*_Z*RootOf(_Z^2+1)+4
*_Z^2-1)*x-448*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)^2*x^3-224*RootOf(-2*_Z
*RootOf(_Z^2+1)+4*_Z^2-1)*RootOf(_Z^2+1)*x^4+224*RootOf(-2*_Z*RootOf(_Z^2+1
)+4*_Z^2-1)*RootOf(_Z^2+1)*x^3-42*(x^3-x+1)^(2/3)*x^4+42*(x^3-x+1)^(2/3)*x^
2-42*(x^3-x+1)^(2/3)*x+18*(x^3-x+1)^(1/3)*x^5-18*(x^3-x+1)^(1/3)*x^3+18*(x^
3-x+1)^(1/3)*x^2+112*(x^3-x+1)^(1/3)*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*
RootOf(_Z^2+1)*x^5-48*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*RootOf(_Z^2+1)*
(x^3-x+1)^(2/3)*x^4+6*(x^3-x+1)^(2/3)*RootOf(_Z^2+1)^2*x^4-14*(x^3-x+1)^(1/
3)*RootOf(_Z^2+1)^2*x^5-144*(x^3-x+1)^(2/3)*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_
Z^2-1)*x^4-144*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*(x^3-x+1)^(1/3)*x^5+36
*RootOf(_Z^2+1)*(x^3-x+1)^(2/3)*x^4+36*(x^3-x+1)^(1/3)*RootOf(_Z^2+1)*x^5-2
4*(x^3-x+1)^(2/3)*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*x^2+168*RootOf(-2*_
Z*RootOf(_Z^2+1)+4*_Z^2-1)*(x^3-x+1)^(1/3)*x^3+6*RootOf(_Z^2+1)*(x^3-x+1)^(
2/3)*x^2-42*(x^3-x+1)^(1/3)*RootOf(_Z^2+1)*x^3+24*(x^3-x+1)^(2/3)*RootOf(-2
*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*x-168*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*(x
^3-x+1)^(1/3)*x^2-6*RootOf(_Z^2+1)*(x^3-x+1)^(2/3)*x+42*(x^3-x+1)^(1/3)*Roo
tOf(_Z^2+1)*x^2+96*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)^2*(x^3-x+1)^(2/3)*
x^4-224*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)^2*(x^3-x+1)^(1/3)*x^5)/(2*x^6
-2*x^4+2*x^3+x^2-2*x+1))*RootOf(_Z^2+1)-ln((3-6*x+448*RootOf(-2*_Z*RootOf(_
Z^2+1)+4*_Z^2-1)^2*x^4-2*RootOf(_Z^2+1)*x^4+2*RootOf(_Z^2+1)*x^6+7*RootOf(_
Z^2+1)*x^2+2*RootOf(_Z^2+1)*x^3+6*x^6+3*x^2+6*x^3-6*x^4-28*RootOf(_Z^2+1)^2
*x^6-448*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)^2*x^6+224*RootOf(-2*_Z*RootO
f(_Z^2+1)+4*_Z^2-1)*RootOf(_Z^2+1)*x^6-8*x^6*RootOf(-2*_Z*RootOf(_Z^2+1)+4*
_Z^2-1)-28*RootOf(_Z^2+1)^2*x^3+28*RootOf(_Z^2+1)^2*x^4+7*RootOf(_Z^2+1)-14
*RootOf(_Z^2+1)*x-28*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)+8*RootOf(-2*_Z*R
ootOf(_Z^2+1)+4*_Z^2-1)*x^4-8*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*x^3-28*
RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*x^2+56*RootOf(-2*_Z*RootOf(_Z^2+1)+4*
_Z^2-1)*x-448*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)^2*x^3-224*RootOf(-2*_Z*
RootOf(_Z^2+1)+4*_Z^2-1)*RootOf(_Z^2+1)*x^4+224*RootOf(-2*_Z*RootOf(_Z^2+1)
+4*_Z^2-1)*RootOf(_Z^2+1)*x^3-42*(x^3-x+1)^(2/3)*x^4+42*(x^3-x+1)^(2/3)*x^2
-42*(x^3-x+1)^(2/3)*x+18*(x^3-x+1)^(1/3)*x^5-18*(x^3-x+1)^(1/3)*x^3+18*(x^3
-x+1)^(1/3)*x^2+112*(x^3-x+1)^(1/3)*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*R
ootOf(_Z^2+1)*x^5-48*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*RootOf(_Z^2+1)*(
x^3-x+1)^(2/3)*x^4+6*(x^3-x+1)^(2/3)*RootOf(_Z^2+1)^2*x^4-14*(x^3-x+1)^(1/3
)*RootOf(_Z^2+1)^2*x^5-144*(x^3-x+1)^(2/3)*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z
^2-1)*x^4-144*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*(x^3-x+1)^(1/3)*x^5+36*
RootOf(_Z^2+1)*(x^3-x+1)^(2/3)*x^4+36*(x^3-x+1)^(1/3)*RootOf(_Z^2+1)*x^5-24
*(x^3-x+1)^(2/3)*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*x^2+168*RootOf(-2*_Z
*RootOf(_Z^2+1)+4*_Z^2-1)*(x^3-x+1)^(1/3)*x^3+6*RootOf(_Z^2+1)*(x^3-x+1)^(2
/3)*x^2-42*(x^3-x+1)^(1/3)*RootOf(_Z^2+1)*x^3+24*(x^3-x+1)^(2/3)*RootOf(-2*
_Z*RootOf(_Z^2+1)+4*_Z^2-1)*x-168*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)*(x^
3-x+1)^(1/3)*x^2-6*RootOf(_Z^2+1)*(x^3-x+1)^(2/3)*x+42*(x^3-x+1)^(1/3)*Root
Of(_Z^2+1)*x^2+96*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)^2*(x^3-x+1)^(2/3)*x
^4-224*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)^2*(x^3-x+1)^(1/3)*x^5)/(2*x^6-
2*x^4+2*x^3+x^2-2*x+1))*RootOf(-2*_Z*RootOf(_Z^2+1)+4*_Z^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-x+1\right)^{\frac{2}{3}}(2x-3)}{2x^6-2x^4+2x^3+x^2-2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^3-x+1)^(2/3)/(2*x^6-2*x^4+2*x^3+x^2-2*x+1),x, algorit
hm="maxima")
```

```
[Out] integrate((x^3 - x + 1)^(2/3)*(2*x - 3)/(2*x^6 - 2*x^4 + 2*x^3 + x^2 - 2*x
+ 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x-3)\,\left(x^3-x+1\right)^{2/3}}{2\,x^6-2\,x^4+2\,x^3+x^2-2\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x - 3)*(x^3 - x + 1)^(2/3))/(x^2 - 2*x + 2*x^3 - 2*x^4 + 2*x^6 + 1)
,x)
```

```
[Out] int(((2*x - 3)*(x^3 - x + 1)^(2/3))/(x^2 - 2*x + 2*x^3 - 2*x^4 + 2*x^6 + 1)
, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x-3)\left(x^3-x+1\right)^{\frac{2}{3}}}{2x^6-2x^4+2x^3+x^2-2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x**3-x+1)**(2/3)/(2*x**6-2*x**4+2*x**3+x**2-2*x+1),x)
```

```
[Out] Integral((2*x - 3)*(x**3 - x + 1)**(2/3)/(2*x**6 - 2*x**4 + 2*x**3 + x**2 -
 2*x + 1), x)
```

3.1304 $\int \frac{(-2+x^6)(4+x^6)\sqrt[4]{-2+2x^4+x^6}}{x^6(-4-x^4+2x^6)}\,dx$

Optimal. Leaf size=104

$$\frac{1}{4}\sqrt[4]{\frac{5}{2}}\tan^{-1}\left(\frac{\sqrt[4]{\frac{5}{2}}x}{\sqrt[4]{x^6+2x^4-2}}\right)-\frac{1}{4}\sqrt[4]{\frac{5}{2}}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{5}{2}}x}{\sqrt[4]{x^6+2x^4-2}}\right)+\frac{\sqrt[4]{x^6+2x^4-2}\,(2x^6+9x^4-4)}{10x^5}$$

Rubi [F] time = 1.73, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+x^6)(4+x^6)\sqrt[4]{-2+2x^4+x^6}}{x^6(-4-x^4+2x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + x^6)*(4 + x^6)*(-2 + 2*x^4 + x^6)^(1/4))/(x^6*(-4 - x^4 + 2*x^6)
),x]
```

```
[Out] Defer[Int][(-2 + 2*x^4 + x^6)^(1/4), x]/2 + 2*Defer[Int][(-2 + 2*x^4 + x^6)
^(1/4)/x^6, x] - Defer[Int][(-2 + 2*x^4 + x^6)^(1/4)/x^2, x]/2 - Defer[Int]
[(x^2*(-2 + 2*x^4 + x^6)^(1/4))/(-4 - x^4 + 2*x^6), x]/2 + (3*Defer[Int][(x
^4*(-2 + 2*x^4 + x^6)^(1/4))/(-4 - x^4 + 2*x^6), x])/2
```

Rubi steps

$$\int \frac{(-2+x^6)(4+x^6)\sqrt[4]{-2+2x^4+x^6}}{x^6(-4-x^4+2x^6)}\,dx = \int \left(\frac{1}{2}\sqrt[4]{-2+2x^4+x^6}+\frac{2\sqrt[4]{-2+2x^4+x^6}}{x^6}-\frac{\sqrt[4]{-2+2x^4+x^6}}{2x^2}+\right.$$

$$=\frac{1}{2}\int \sqrt[4]{-2+2x^4+x^6}\,dx-\frac{1}{2}\int \frac{\sqrt[4]{-2+2x^4+x^6}}{x^2}\,dx+\frac{1}{2}\int \frac{x^2(1}{}$$

$$=\frac{1}{2}\int \sqrt[4]{-2+2x^4+x^6}\,dx-\frac{1}{2}\int \frac{\sqrt[4]{-2+2x^4+x^6}}{x^2}\,dx+\frac{1}{2}\int\left(-\frac{x^2}{}\right.$$

$$=\frac{1}{2}\int \sqrt[4]{-2+2x^4+x^6}\,dx-\frac{1}{2}\int \frac{\sqrt[4]{-2+2x^4+x^6}}{x^2}\,dx-\frac{1}{2}\int \frac{x^2\sqrt[4]{-}}{-4}$$

Mathematica [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{(-2+x^6)(4+x^6)\sqrt[4]{-2+2x^4+x^6}}{x^6(-4-x^4+2x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + x^6)*(4 + x^6)*(-2 + 2*x^4 + x^6)^(1/4))/(x^6*(-4 - x^4 +
2*x^6)),x]
```

```
[Out] Integrate[((-2 + x^6)*(4 + x^6)*(-2 + 2*x^4 + x^6)^(1/4))/(x^6*(-4 - x^4 +
2*x^6)), x]
```

IntegrateAlgebraic [A] time = 2.80, size = 104, normalized size = 1.00

$$\frac{1}{4}\sqrt[4]{\frac{5}{2}}\tan^{-1}\left(\frac{\sqrt[4]{\frac{5}{2}}x}{\sqrt[4]{x^6+2x^4-2}}\right)-\frac{1}{4}\sqrt[4]{\frac{5}{2}}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{5}{2}}x}{\sqrt[4]{x^6+2x^4-2}}\right)+\frac{\sqrt[4]{x^6+2x^4-2}\,(2x^6+9x^4-4)}{10x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^6)*(4 + x^6)*(-2 + 2*x^4 + x^6)^(1/4))/(x^6*(-4
 - x^4 + 2*x^6)),x]
```

```
[Out] ((-2 + 2*x^4 + x^6)^(1/4)*(-4 + 9*x^4 + 2*x^6))/(10*x^5) + ((5/2)^(1/4)*Arc
Tan[((5/2)^(1/4)*x)/(-2 + 2*x^4 + x^6)^(1/4)])/4 - ((5/2)^(1/4)*ArcTanh[((5
/2)^(1/4)*x)/(-2 + 2*x^4 + x^6)^(1/4)])/4
```

fricas [B] time = 133.64, size = 380, normalized size = 3.65

$$\frac{20 \cdot 5^{\frac{1}{4}} 2^{\frac{3}{4}} x^5 \arctan\left(\frac{20 \cdot 5^{\frac{3}{4}} 2^{\frac{1}{4}} \left(x^6+2x^4-2\right)^{\frac{1}{4}} x^3+20 \cdot 5^{\frac{1}{4}} 2^{\frac{3}{4}} \left(x^6+2x^4-2\right)^{\frac{3}{4}} x+\sqrt{5}\left(4 \cdot 5^{\frac{3}{4}} 2^{\frac{1}{4}} \sqrt{x^6+2x^4-2}\, x^2+5^{\frac{1}{4}} 2^{\frac{3}{4}} \left(2x^6+9x^4-4\right)\right)\sqrt{\sqrt{5}\sqrt{2}}}{10\left(2x^6-x^4-4\right)}\right)-5 \cdot 5^{\frac{1}{4}} 2^{\frac{3}{4}} x^5 \log\left(-\frac{10\sqrt{5}\sqrt{2}\left(x^6+2x^4-2\right)^{\frac{1}{4}} x^3+10 \cdot 5^{\frac{1}{4}} 2^{\frac{3}{4}} \sqrt{x^6+2x^4-2}\, x^2+5^{\frac{3}{4}} 2^{\frac{1}{4}}\left(2x^6+9x^4-4\right)+20\left(x^6+2x^4-2\right)^{\frac{3}{4}} x}{2x^6-x^4-4}\right)+5 \cdot 5^{\frac{1}{4}} 2^{\frac{3}{4}} x^5 \log\left(-\frac{10\sqrt{5}\sqrt{2}\left(x^6+2x^4-2\right)^{\frac{1}{4}} x^3-10 \cdot 5^{\frac{1}{4}} 2^{\frac{3}{4}} \sqrt{x^6+2x^4-2}\, x^2-5^{\frac{3}{4}} 2^{\frac{1}{4}}\left(2x^6+9x^4-4\right)+20\left(x^6+2x^4-2\right)^{\frac{3}{4}} x}{2x^6-x^4-4}\right)+16\left(2x^6+9x^4-4\right)\left(x^6+2x^4-2\right)^{\frac{1}{4}}}{160\, x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6+4)*(x^6+2*x^4-2)^(1/4)/x^6/(2*x^6-x^4-4),x, algorith
m="fricas")
```

```
[Out] 1/160*(20*5^(1/4)*2^(3/4)*x^5*arctan(1/10*(20*5^(3/4)*2^(1/4)*(x^6 + 2*x^4
- 2)^(1/4)*x^3 + 20*5^(1/4)*2^(3/4)*(x^6 + 2*x^4 - 2)^(3/4)*x + sqrt(5)*(4*
5^(3/4)*2^(1/4)*sqrt(x^6 + 2*x^4 - 2)*x^2 + 5^(1/4)*2^(3/4)*(2*x^6 + 9*x^4
- 4))*sqrt(sqrt(5)*sqrt(2)))/(2*x^6 - x^4 - 4)) - 5*5^(1/4)*2^(3/4)*x^5*log
(-(10*sqrt(5)*sqrt(2)*(x^6 + 2*x^4 - 2)^(1/4)*x^3 + 10*5^(1/4)*2^(3/4)*sqrt
(x^6 + 2*x^4 - 2)*x^2 + 5^(3/4)*2^(1/4)*(2*x^6 + 9*x^4 - 4) + 20*(x^6 + 2*x
^4 - 2)^(3/4)*x)/(2*x^6 - x^4 - 4)) + 5*5^(1/4)*2^(3/4)*x^5*log(-(10*sqrt(5
)*sqrt(2)*(x^6 + 2*x^4 - 2)^(1/4)*x^3 - 10*5^(1/4)*2^(3/4)*sqrt(x^6 + 2*x^4
 - 2)*x^2 - 5^(3/4)*2^(1/4)*(2*x^6 + 9*x^4 - 4) + 20*(x^6 + 2*x^4 - 2)^(3/4
)*x)/(2*x^6 - x^4 - 4)) + 16*(2*x^6 + 9*x^4 - 4)*(x^6 + 2*x^4 - 2)^(1/4))/x
^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+2x^4-2\right)^{\frac{1}{4}}\left(x^6+4\right)\left(x^6-2\right)}{\left(2x^6-x^4-4\right)x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6+4)*(x^6+2*x^4-2)^(1/4)/x^6/(2*x^6-x^4-4),x, algorith
m="giac")
```

```
[Out] integrate((x^6 + 2*x^4 - 2)^(1/4)*(x^6 + 4)*(x^6 - 2)/((2*x^6 - x^4 - 4)*x^
6), x)
```

maple [C] time = 4.31, size = 1513, normalized size = 14.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-2)*(x^6+4)*(x^6+2*x^4-2)^(1/4)/x^6/(2*x^6-x^4-4),x)
```

```
[Out] 1/10*(2*x^12+13*x^10+18*x^8-8*x^6-26*x^4+8)/x^5/(x^6+2*x^4-2)^(3/4)+(-1/16*
RootOf(_Z^2+RootOf(_Z^4-40)^2)*ln(-(-2*RootOf(_Z^4-40)^2*x^18-17*RootOf(_Z^
4-40)^2*x^16-2*(x^18+6*x^16+12*x^14+2*x^12-24*x^10-24*x^8+12*x^6+24*x^4-8)^
(1/4)*RootOf(_Z^2+RootOf(_Z^4-40)^2)*RootOf(_Z^4-40)^2*x^13-44*x^14*RootOf(
_Z^4-40)^2-8*(x^18+6*x^16+12*x^14+2*x^12-24*x^10-24*x^8+12*x^6+24*x^4-8)^(1
/4)*RootOf(_Z^2+RootOf(_Z^4-40)^2)*RootOf(_Z^4-40)^2*x^11-24*x^12*RootOf(_Z
^4-40)^2-8*(x^18+6*x^16+12*x^14+2*x^12-24*x^10-24*x^8+12*x^6+24*x^4-8)^(1/4
)*RootOf(_Z^2+RootOf(_Z^4-40)^2)*RootOf(_Z^4-40)^2*x^9+68*RootOf(_Z^4-40)^2
*x^10+8*(x^18+6*x^16+12*x^14+2*x^12-24*x^10-24*x^8+12*x^6+24*x^4-8)^(1/4)*R
ootOf(_Z^2+RootOf(_Z^4-40)^2)*RootOf(_Z^4-40)^2*x^7+40*(x^18+6*x^16+12*x^14
+2*x^12-24*x^10-24*x^8+12*x^6+24*x^4-8)^(1/2)*x^8+88*RootOf(_Z^4-40)^2*x^8+
```

```
16*(x^18+6*x^16+12*x^14+2*x^12-24*x^10-24*x^8+12*x^6+24*x^4-8)^(1/4)*RootOf
(_Z^2+RootOf(_Z^4-40)^2)*RootOf(_Z^4-40)^2*x^5+80*(x^18+6*x^16+12*x^14+2*x^
12-24*x^10-24*x^8+12*x^6+24*x^4-8)^(1/2)*x^6-24*RootOf(_Z^4-40)^2*x^6+20*Ro
otOf(_Z^2+RootOf(_Z^4-40)^2)*(x^18+6*x^16+12*x^14+2*x^12-24*x^10-24*x^8+12*
x^6+24*x^4-8)^(3/4)*x^3-68*RootOf(_Z^4-40)^2*x^4-8*(x^18+6*x^16+12*x^14+2*x
^12-24*x^10-24*x^8+12*x^6+24*x^4-8)^(1/4)*RootOf(_Z^2+RootOf(_Z^4-40)^2)*Ro
otOf(_Z^4-40)^2*x-80*(x^18+6*x^16+12*x^14+2*x^12-24*x^10-24*x^8+12*x^6+24*x
^4-8)^(1/2)*x^2+16*RootOf(_Z^4-40)^2)/(2*x^6-x^4-4)/(x^6+2*x^4-2)^2)+1/16*R
ootOf(_Z^4-40)*ln((-2*RootOf(_Z^4-40)^2*x^18-17*RootOf(_Z^4-40)^2*x^16+2*(x
^18+6*x^16+12*x^14+2*x^12-24*x^10-24*x^8+12*x^6+24*x^4-8)^(1/4)*RootOf(_Z^4
-40)^3*x^13-44*x^14*RootOf(_Z^4-40)^2+8*(x^18+6*x^16+12*x^14+2*x^12-24*x^10
-24*x^8+12*x^6+24*x^4-8)^(1/4)*RootOf(_Z^4-40)^3*x^11-24*x^12*RootOf(_Z^4-4
0)^2+8*(x^18+6*x^16+12*x^14+2*x^12-24*x^10-24*x^8+12*x^6+24*x^4-8)^(1/4)*Ro
otOf(_Z^4-40)^3*x^9+68*RootOf(_Z^4-40)^2*x^10-8*(x^18+6*x^16+12*x^14+2*x^12
-24*x^10-24*x^8+12*x^6+24*x^4-8)^(1/4)*RootOf(_Z^4-40)^3*x^7-40*(x^18+6*x^1
6+12*x^14+2*x^12-24*x^10-24*x^8+12*x^6+24*x^4-8)^(1/2)*x^8+88*RootOf(_Z^4-4
0)^2*x^8-16*(x^18+6*x^16+12*x^14+2*x^12-24*x^10-24*x^8+12*x^6+24*x^4-8)^(1/
4)*RootOf(_Z^4-40)^3*x^5-80*(x^18+6*x^16+12*x^14+2*x^12-24*x^10-24*x^8+12*x
^6+24*x^4-8)^(1/2)*x^6-24*RootOf(_Z^4-40)^2*x^6+20*(x^18+6*x^16+12*x^14+2*x
^12-24*x^10-24*x^8+12*x^6+24*x^4-8)^(3/4)*RootOf(_Z^4-40)*x^3-68*RootOf(_Z^
4-40)^2*x^4+8*(x^18+6*x^16+12*x^14+2*x^12-24*x^10-24*x^8+12*x^6+24*x^4-8)^(
1/4)*RootOf(_Z^4-40)^3*x+80*(x^18+6*x^16+12*x^14+2*x^12-24*x^10-24*x^8+12*x
^6+24*x^4-8)^(1/2)*x^2+16*RootOf(_Z^4-40)^2)/(2*x^6-x^4-4)/(x^6+2*x^4-2)^2)
)/(x^6+2*x^4-2)^(3/4)*((x^6+2*x^4-2)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+2\,x^4-2\right)^{\frac{1}{4}}\left(x^6+4\right)\left(x^6-2\right)}{\left(2\,x^6-x^4-4\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6+4)*(x^6+2*x^4-2)^(1/4)/x^6/(2*x^6-x^4-4),x, algorith
m="maxima")
```

```
[Out] integrate((x^6 + 2*x^4 - 2)^(1/4)*(x^6 + 4)*(x^6 - 2)/((2*x^6 - x^4 - 4)*x^
6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^6-2\right)\,\left(x^6+4\right)\,\left(x^6+2\,x^4-2\right)^{1/4}}{x^6\,\left(-2\,x^6+x^4+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^6 - 2)*(x^6 + 4)*(2*x^4 + x^6 - 2)^(1/4))/(x^6*(x^4 - 2*x^6 + 4)),
x)
```

```
[Out] int(-((x^6 - 2)*(x^6 + 4)*(2*x^4 + x^6 - 2)^(1/4))/(x^6*(x^4 - 2*x^6 + 4)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-2)*(x**6+4)*(x**6+2*x**4-2)**(1/4)/x**6/(2*x**6-x**4-4),x)
```

```
[Out] Timed out
```

3.1305 $\int \frac{(1+x^3)^{2/3}(-1+3x^6)}{x^9(1+2x^3)}\,dx$

Optimal. Leaf size=104

$$\frac{1}{3}\log\left(\sqrt[3]{x^3+1}+x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}-x}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(-\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)+\frac{\left(x^3+1\right)^{2/3}\left(x^6-14x^3+5\right)}{40x^8}$$

Rubi [C] time = 0.40, antiderivative size = 119, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6725, 271, 264, 277, 239, 429}

$$2xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-x^3,-2x^3\right)+\frac{1}{2}\log\left(\sqrt[3]{x^3+1}-x\right)-\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\left(x^3+1\right)^{5/3}}{8x^8}-\frac{19\left(x^3+1\right)^{5/3}}{40x^5}+\frac{\left(x^3+1\right)^{2/3}}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((1 + x^3)^(2/3)*(-1 + 3*x^6))/(x^9*(1 + 2*x^3)),x]
```

```
[Out] (1 + x^3)^(2/3)/(2*x^2) + (1 + x^3)^(5/3)/(8*x^8) - (19*(1 + x^3)^(5/3))/(4
0*x^5) + 2*x*AppellF1[1/3, -2/3, 1, 4/3, -x^3, -2*x^3] - ArcTan[(1 + (2*x)/
(1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[-x + (1 + x^3)^(1/3)]/2
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(1+x^3\right)^{2/3}\left(-1+3x^6\right)}{x^9\left(1+2x^3\right)}\,dx = \int \left(-\frac{\left(1+x^3\right)^{2/3}}{x^9}+\frac{2\left(1+x^3\right)^{2/3}}{x^6}-\frac{\left(1+x^3\right)^{2/3}}{x^3}+\frac{2\left(1+x^3\right)^{2/3}}{1+2x^3}\right)dx$$

$$= 2\int \frac{\left(1+x^3\right)^{2/3}}{x^6}\,dx + 2\int \frac{\left(1+x^3\right)^{2/3}}{1+2x^3}\,dx - \int \frac{\left(1+x^3\right)^{2/3}}{x^9}\,dx - \int \frac{\left(1+x^3\right)^{2/3}}{x^3}\,dx$$

$$= \frac{\left(1+x^3\right)^{2/3}}{2x^2}+\frac{\left(1+x^3\right)^{5/3}}{8x^8}-\frac{2\left(1+x^3\right)^{5/3}}{5x^5}+2xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-x^3,-2x^3\right)+\frac{3}{8}\int$$

$$= \frac{\left(1+x^3\right)^{2/3}}{2x^2}+\frac{\left(1+x^3\right)^{5/3}}{8x^8}-\frac{19\left(1+x^3\right)^{5/3}}{40x^5}+2xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-x^3,-2x^3\right)-$$

Mathematica [A] time = 0.12, size = 104, normalized size = 1.00

$$\frac{1}{3}\log\left(\frac{x}{\sqrt[3]{x^3+1}}+1\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(-\frac{x}{\sqrt[3]{x^3+1}}+\frac{x^2}{\left(x^3+1\right)^{2/3}}+1\right)+\frac{\left(x^3+1\right)^{2/3}\left(x^6-14x^3+5\right)}{40x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)^(2/3)*(-1 + 3*x^6))/(x^9*(1 + 2*x^3)),x]
```

```
[Out] ((1 + x^3)^(2/3)*(5 - 14*x^3 + x^6))/(40*x^8) + ArcTan[(-1 + (2*x)/(1 + x^3
)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 + x^2/(1 + x^3)^(2/3) - x/(1 + x^3)^(1/3)
]/6 + Log[1 + x/(1 + x^3)^(1/3)]/3
```

IntegrateAlgebraic [A] time = 0.20, size = 104, normalized size = 1.00

$$\frac{1}{3}\log\left(\sqrt[3]{x^3+1}+x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}-x}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(-\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)+\frac{\left(x^3+1\right)^{2/3}\left(x^6-14x^3+5\right)}{40x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)^(2/3)*(-1 + 3*x^6))/(x^9*(1 + 2*x^3)),x]
```

```
[Out] ((1 + x^3)^(2/3)*(5 - 14*x^3 + x^6))/(40*x^8) + ArcTan[(Sqrt[3]*x)/(-x + 2*
(1 + x^3)^(1/3))]/Sqrt[3] + Log[x + (1 + x^3)^(1/3)]/3 - Log[x^2 - x*(1 + x
^3)^(1/3) + (1 + x^3)^(2/3)]/6
```

fricas [A] time = 0.94, size = 127, normalized size = 1.22

$$-\frac{40\sqrt{3}x^8\arctan\left(\frac{4\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}x^2+2\sqrt{3}\left(x^3+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(x^3+1\right)}{7x^3-1}\right)-20x^8\log\left(\frac{2x^3+3\left(x^3+1\right)^{\frac{1}{3}}x^2+3\left(x^3+1\right)^{\frac{2}{3}}x+1}{2x^3+1}\right)-3\left(x^6-14x^3+5\right)\left(x^3+1\right)^{\frac{2}{3}}}{120x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(3*x^6-1)/x^9/(2*x^3+1),x, algorithm="fricas")
```

```
[Out] -1/120*(40*sqrt(3)*x^8*arctan((4*sqrt(3)*(x^3 + 1)^(1/3)*x^2 + 2*sqrt(3)*(x
^3 + 1)^(2/3)*x + sqrt(3)*(x^3 + 1))/(7*x^3 - 1)) - 20*x^8*log((2*x^3 + 3*(
x^3 + 1)^(1/3)*x^2 + 3*(x^3 + 1)^(2/3)*x + 1)/(2*x^3 + 1)) - 3*(x^6 - 14*x^
3 + 5)*(x^3 + 1)^(2/3))/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^6-1\right)\left(x^3+1\right)^{\frac{2}{3}}}{\left(2\,x^3+1\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(3*x^6-1)/x^9/(2*x^3+1),x, algorithm="giac")
```

```
[Out] integrate((3*x^6 - 1)*(x^3 + 1)^(2/3)/((2*x^3 + 1)*x^9), x)
```

maple [C] time = 1.08, size = 425, normalized size = 4.09

$$\frac{x^9-13x^6-9x^3+5}{40x^8\left(x^3+1\right)^{\frac{1}{3}}}-\frac{\ln\left(-\frac{9\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3-3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3+1\right)^{\frac{2}{3}}x+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3+1\right)^{\frac{1}{3}}x^2+6\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3-2x\left(x^3+1\right)^{\frac{2}{3}}+2x^2\left(x^3+1\right)^{\frac{1}{3}}+x^3+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)+1}{2x^3+1}\right)}{3}-\ln\left(-\frac{9\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3-3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3+1\right)^{\frac{2}{3}}x+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3+1\right)^{\frac{1}{3}}x^2+6\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3-2x\left(x^3+1\right)^{\frac{2}{3}}+2x^2\left(x^3+1\right)^{\frac{1}{3}}+x^3+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)+1}{2x^3+1}\right)\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)+\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\ln\left(-\frac{9\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3+1\right)^{\frac{2}{3}}x-3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3+1\right)^{\frac{1}{3}}x^2-x\left(x^3+1\right)^{\frac{2}{3}}+x^2\left(x^3+1\right)^{\frac{1}{3}}-3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)}{2x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)*(3*x^6-1)/x^9/(2*x^3+1),x)
```

```
[Out] 1/40*(x^9-13*x^6-9*x^3+5)/x^8/(x^3+1)^(1/3)-1/3*ln(-(9*RootOf(9*_Z^2+3*_Z+1
)^2*x^3-3*RootOf(9*_Z^2+3*_Z+1)*(x^3+1)^(2/3)*x+3*RootOf(9*_Z^2+3*_Z+1)*(x^
3+1)^(1/3)*x^2+6*RootOf(9*_Z^2+3*_Z+1)*x^3-2*x*(x^3+1)^(2/3)+2*x^2*(x^3+1)^
(1/3)+x^3+3*RootOf(9*_Z^2+3*_Z+1)+1)/(2*x^3+1))-ln(-(9*RootOf(9*_Z^2+3*_Z+1
)^2*x^3-3*RootOf(9*_Z^2+3*_Z+1)*(x^3+1)^(2/3)*x+3*RootOf(9*_Z^2+3*_Z+1)*(x^
3+1)^(1/3)*x^2+6*RootOf(9*_Z^2+3*_Z+1)*x^3-2*x*(x^3+1)^(2/3)+2*x^2*(x^3+1)^
(1/3)+x^3+3*RootOf(9*_Z^2+3*_Z+1)+1)/(2*x^3+1))*RootOf(9*_Z^2+3*_Z+1)+RootO
f(9*_Z^2+3*_Z+1)*ln(-(9*RootOf(9*_Z^2+3*_Z+1)^2*x^3+3*RootOf(9*_Z^2+3*_Z+1)
*(x^3+1)^(2/3)*x-3*RootOf(9*_Z^2+3*_Z+1)*(x^3+1)^(1/3)*x^2-x*(x^3+1)^(2/3)+
x^2*(x^3+1)^(1/3)-3*RootOf(9*_Z^2+3*_Z+1))/(2*x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^6-1\right)\left(x^3+1\right)^{\frac{2}{3}}}{\left(2\,x^3+1\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(3*x^6-1)/x^9/(2*x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((3*x^6 - 1)*(x^3 + 1)^(2/3)/((2*x^3 + 1)*x^9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(x^3+1\right)}^{2/3}\,\left(3\,x^6-1\right)}{x^9\,\left(2\,x^3+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)^(2/3)*(3*x^6 - 1))/(x^9*(2*x^3 + 1)),x)
```

```
[Out] int(((x^3 + 1)^(2/3)*(3*x^6 - 1))/(x^9*(2*x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x + 1\right) \left(x^{2} - x + 1\right)\right)^{\frac{2}{3}} \left(3 x^{6} - 1\right)}{x^{9} \left(2 x^{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)*(3*x**6-1)/x**9/(2*x**3+1),x)
```

```
[Out] Integral(((x + 1)*(x**2 - x + 1))**(2/3)*(3*x**6 - 1)/(x**9*(2*x**3 + 1)),
x)
```

3.1306 $\int \frac{\sqrt{1-x^4}\left(1+x^4\right)}{1-x^4+x^8} \, dx$

Optimal. Leaf size=104

$$\frac{\tan^{-1}\left(\frac{\frac{x^4}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{1-x^4}}\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^4}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{1-x^4}}\right)}{2\sqrt{2}}$$

Rubi [C] time = 0.47, antiderivative size = 139, normalized size of antiderivative = 1.34, number of steps used = 16, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6728, 406, 221, 409, 1213, 537}

$$-\frac{1}{2}\left(1+i\sqrt{3}\right)F\left(\sin^{-1}(x)|-1\right)-\frac{1}{2}\left(1-i\sqrt{3}\right)F\left(\sin^{-1}(x)|-1\right)+\frac{1}{2}\Pi\left(\frac{1}{2}\left(-i-\sqrt{3}\right);\sin^{-1}(x)|-1\right)+\frac{1}{2}\Pi\left(\frac{1}{2}\left(i-\sqrt{3}\right);\sin^{-1}(x)|-1\right)+\frac{1}{2}\Pi\left(\frac{1}{\sqrt{\frac{1}{2}\left(1-i\sqrt{3}\right)}};\sin^{-1}(x)|-1\right)+\frac{1}{2}\Pi\left(\frac{1}{\sqrt{\frac{1}{2}\left(1+i\sqrt{3}\right)}};\sin^{-1}(x)|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x^4]*(1 + x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -1/2*((1 - I*Sqrt[3])*EllipticF[ArcSin[x], -1]) - ((1 + I*Sqrt[3])*Elliptic
F[ArcSin[x], -1])/2 + EllipticPi[(-I - Sqrt[3])/2, ArcSin[x], -1]/2 + Ellip
ticPi[(I - Sqrt[3])/2, ArcSin[x], -1]/2 + EllipticPi[1/Sqrt[(1 - I*Sqrt[3])
/2], ArcSin[x], -1]/2 + EllipticPi[1/Sqrt[(1 + I*Sqrt[3])/2], ArcSin[x], -1
]/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-x^4}\left(1+x^4\right)}{1-x^4+x^8}\,dx = \int \left(\frac{\left(1-i\sqrt{3}\right)\sqrt{1-x^4}}{-1-i\sqrt{3}+2x^4}+\frac{\left(1+i\sqrt{3}\right)\sqrt{1-x^4}}{-1+i\sqrt{3}+2x^4}\right)dx$$

$$= \left(1-i\sqrt{3}\right)\int \frac{\sqrt{1-x^4}}{-1-i\sqrt{3}+2x^4}\,dx+\left(1+i\sqrt{3}\right)\int \frac{\sqrt{1-x^4}}{-1+i\sqrt{3}+2x^4}\,dx$$

$$= \frac{1}{2}\left(-1-i\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^4}}\,dx+\left(-1-i\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^4}\left(-1-i\sqrt{3}+2x^4\right)}\,dx+\frac{1}{2}\Big(-$$

$$= -\frac{1}{2}\left(1-i\sqrt{3}\right)F\left(\sin^{-1}(x)\middle|-1\right)-\frac{1}{2}\left(1+i\sqrt{3}\right)F\left(\sin^{-1}(x)\middle|-1\right)+\frac{1}{2}\int \frac{1}{\left(1-\frac{x^2}{\sqrt{\frac{1}{2}\left(1-i\sqrt{\right.}\right.}}\right.}$$

$$= -\frac{1}{2}\left(1-i\sqrt{3}\right)F\left(\sin^{-1}(x)\middle|-1\right)-\frac{1}{2}\left(1+i\sqrt{3}\right)F\left(\sin^{-1}(x)\middle|-1\right)+\frac{1}{2}\int \frac{}{\sqrt{1-x^2}\sqrt{1+}}$$

$$= -\frac{1}{2}\left(1-i\sqrt{3}\right)F\left(\sin^{-1}(x)\middle|-1\right)-\frac{1}{2}\left(1+i\sqrt{3}\right)F\left(\sin^{-1}(x)\middle|-1\right)+\frac{1}{2}\Pi\left(\frac{1}{2}\left(-i-\sqrt{3}\right)\right.$$

Mathematica [C] time = 0.23, size = 51, normalized size = 0.49

$$\frac{1}{2}\left(-2F\left(\sin^{-1}(x)\middle|-1\right)+\Pi\left(-\sqrt[6]{-1};\sin^{-1}(x)\middle|-1\right)+\Pi\left(\sqrt[6]{-1};\sin^{-1}(x)\middle|-1\right)+\Pi\left(-(-1)^{5/6};\sin^{-1}(x)\middle|-1\right)+\Pi\left((-1)^{5/6};\sin^{-1}(x)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x^4]*(1 + x^4))/(1 - x^4 + x^8),x]
```

```
[Out] (-2*EllipticF[ArcSin[x], -1] + EllipticPi[-(-1)^(1/6), ArcSin[x], -1] + Ell
ipticPi[(-1)^(1/6), ArcSin[x], -1] + EllipticPi[-(-1)^(5/6), ArcSin[x], -1]
 + EllipticPi[(-1)^(5/6), ArcSin[x], -1])/2
```

IntegrateAlgebraic [A] time = 0.33, size = 71, normalized size = 0.68

$$\frac{1}{2}(-1)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{-1}\,x\sqrt{1-x^4}}{x^4-1}\right)+\frac{1}{2}\sqrt[4]{-1}\,\tanh^{-1}\left(\frac{(-1)^{3/4}x\sqrt{1-x^4}}{x^4-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 - x^4]*(1 + x^4))/(1 - x^4 + x^8),x]
```

```
[Out] ((-1)^(3/4)*ArcTanh[((-1)^(1/4)*x*Sqrt[1 - x^4])/(-1 + x^4)])/2 + ((-1)^(1/
4)*ArcTanh[((-1)^(3/4)*x*Sqrt[1 - x^4])/(-1 + x^4)])/2
```

fricas [B] time = 0.54, size = 466, normalized size = 4.48

$$\frac{1}{4}\sqrt{2}\arctan\left(-\frac{x^8-x^4+2\sqrt{2}\left(x^5+x^3-x\right)\sqrt{-x^4+1}-\left(4\sqrt{-x^4+1}\,x^3-\sqrt{2}\left(x^8+2x^6-3x^4-2x^2+1\right)\right)\sqrt{\frac{x^8-4x^6-x^4+2\sqrt{2}\left(x^5-x^3-x\right)\sqrt{-x^4+1}+4x^2+1}{x^8-x^4+1}}+1}{x^8+4x^6-x^4-4x^2+1}\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{x^8-x^4-2\sqrt{2}\left(x^5+x^3-x\right)\sqrt{-x^4+1}-\left(4\sqrt{-x^4+1}\,x^3+\sqrt{2}\left(x^8+2x^6-3x^4-2x^2+1\right)\right)\sqrt{\frac{x^8-4x^6-x^4-2\sqrt{2}\left(x^5-x^3-x\right)\sqrt{-x^4+1}+4x^2+1}{x^8-x^4+1}}+1}{x^8+4x^6-x^4-4x^2+1}\right)-\frac{1}{16}\sqrt{2}\log\left(\frac{4\left(x^8-4x^6-x^4+2\sqrt{2}\left(x^5-x^3-x\right)\sqrt{-x^4+1}+4x^2+1\right)}{x^8-x^4+1}\right)+\frac{1}{16}\sqrt{2}\log\left(\frac{4\left(x^8-4x^6-x^4-2\sqrt{2}\left(x^5-x^3-x\right)\sqrt{-x^4+1}+4x^2+1\right)}{x^8-x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)*(x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(-(x^8 - x^4 + 2*sqrt(2)*(x^5 + x^3 - x)*sqrt(-x^4 + 1) -
 (4*sqrt(-x^4 + 1)*x^3 - sqrt(2)*(x^8 + 2*x^6 - 3*x^4 - 2*x^2 + 1))*sqrt((x
^8 - 4*x^6 - x^4 + 2*sqrt(2)*(x^5 - x^3 - x)*sqrt(-x^4 + 1) + 4*x^2 + 1)/(x
^8 - x^4 + 1)) + 1)/(x^8 + 4*x^6 - x^4 - 4*x^2 + 1)) - 1/4*sqrt(2)*arctan(-
(x^8 - x^4 - 2*sqrt(2)*(x^5 + x^3 - x)*sqrt(-x^4 + 1) - (4*sqrt(-x^4 + 1)*x
^3 + sqrt(2)*(x^8 + 2*x^6 - 3*x^4 - 2*x^2 + 1))*sqrt((x^8 - 4*x^6 - x^4 - 2
*sqrt(2)*(x^5 - x^3 - x)*sqrt(-x^4 + 1) + 4*x^2 + 1)/(x^8 - x^4 + 1)) + 1)/
(x^8 + 4*x^6 - x^4 - 4*x^2 + 1)) - 1/16*sqrt(2)*log(4*(x^8 - 4*x^6 - x^4 +
2*sqrt(2)*(x^5 - x^3 - x)*sqrt(-x^4 + 1) + 4*x^2 + 1)/(x^8 - x^4 + 1)) + 1/
16*sqrt(2)*log(4*(x^8 - 4*x^6 - x^4 - 2*sqrt(2)*(x^5 - x^3 - x)*sqrt(-x^4 +
 1) + 4*x^2 + 1)/(x^8 - x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)\sqrt{-x^4+1}}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)*(x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)*sqrt(-x^4 + 1)/(x^8 - x^4 + 1), x)
```

maple [A] time = 0.04, size = 122, normalized size = 1.17

$$\frac{\sqrt{2}\,\ln\left(\frac{-x^4+1}{x^2}+\frac{\sqrt{-x^4+1}\,\sqrt{2}}{x}+1\right)}{8}-\frac{\sqrt{2}\,\arctan\left(1+\frac{\sqrt{-x^4+1}\,\sqrt{2}}{x}\right)}{4}-\frac{\sqrt{2}\,\ln\left(\frac{-x^4+1}{x^2}-\frac{\sqrt{-x^4+1}\,\sqrt{2}}{x}+1\right)}{8}+\frac{\sqrt{2}\,\arctan\left(1-\frac{\sqrt{-x^4+1}\,\sqrt{2}}{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^(1/2)*(x^4+1)/(x^8-x^4+1),x)
```

```
[Out] 1/8*2^(1/2)*ln((-x^4+1)/x^2+(-x^4+1)^(1/2)*2^(1/2)/x+1)-1/4*2^(1/2)*arctan(
1+(-x^4+1)^(1/2)*2^(1/2)/x)-1/8*2^(1/2)*ln((-x^4+1)/x^2-(-x^4+1)^(1/2)*2^(1
/2)/x+1)+1/4*2^(1/2)*arctan(1-(-x^4+1)^(1/2)*2^(1/2)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)\sqrt{-x^4+1}}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)*(x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)*sqrt(-x^4 + 1)/(x^8 - x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-x^4}\,\left(x^4+1\right)}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - x^4)^(1/2)*(x^4 + 1))/(x^8 - x^4 + 1),x)
```

```
[Out] int(((1 - x^4)^(1/2)*(x^4 + 1))/(x^8 - x^4 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}\,\left(x^4+1\right)}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(1/2)*(x**4+1)/(x**8-x**4+1),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))*(x**4 + 1)/(x**8 - x**4 + 1), x)
```

3.1307 $\int \frac{x^2}{\left(b+ax^4\right)^{3/4}\left(-b^2+a^2x^8\right)}\,dx$

Optimal. Leaf size=104

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{4\,2^{3/4}a^{3/4}b^2}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{4\,2^{3/4}a^{3/4}b^2}-\frac{x^3}{6b^2\left(ax^4+b\right)^{3/4}}$$

Rubi [C] time = 4.06, antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1479, 511, 510}

$$-\frac{4x^3\Gamma\left(\frac{7}{4}\right)\left(11\left(-4a^2x^8-3abx^4+7b^2\right){}_2F_1\left(1,1;\frac{11}{4};-\frac{2ax^4}{b-ax^4}\right)-32ax^4\left(ax^4+b\right){}_2F_1\left(2,2;\frac{15}{4};-\frac{2ax^4}{b-ax^4}\right)\right)}{693b^2\Gamma\left(\frac{3}{4}\right)\left(b-ax^4\right)^2\left(ax^4+b\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2/((b + a*x^4)^(3/4)*(-b^2 + a^2*x^8)),x]
```

```
[Out] (-4*x^3*Gamma[7/4]*(11*(7*b^2 - 3*a*b*x^4 - 4*a^2*x^8)*Hypergeometric2F1[1,
 1, 11/4, (-2*a*x^4)/(b - a*x^4)] - 32*a*x^4*(b + a*x^4)*Hypergeometric2F1[
2, 2, 15/4, (-2*a*x^4)/(b - a*x^4)]))/(693*b^2*(b - a*x^4)^2*(b + a*x^4)^(3
/4)*Gamma[3/4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1479

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n
2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*(a/d + (c*x^n)/e)^
p, x] /; FreeQ[{a, c, d, e, f, q, m, n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2
 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2}{\left(b+ax^4\right)^{3/4}\left(-b^2+a^2x^8\right)}\,dx=\int \frac{x^2}{\left(-b+ax^4\right)\left(b+ax^4\right)^{7/4}}\,dx$$
$$=\frac{\left(1+\frac{ax^4}{b}\right)^{3/4}\int \frac{x^2}{\left(-b+ax^4\right)\left(1+\frac{ax^4}{b}\right)^{7/4}}\,dx}{b\left(b+ax^4\right)^{3/4}}$$
$$=-\frac{4x^3\Gamma\left(\frac{7}{4}\right)\left(11\left(7b^2-3abx^4-4a^2x^8\right){}_2F_1\left(1,1;\frac{11}{4};-\frac{2ax^4}{b-ax^4}\right)-32ax^4\left(b+ax^4\right.\right.}{693b^2\left(b-ax^4\right)^2\left(b+ax^4\right)^{3/4}\Gamma\left(\frac{3}{4}\right)}$$

Mathematica [C] time = 0.12, size = 93, normalized size = 0.89

$$-\frac{x^3\left(\left(\frac{ax^4}{b}+1\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{2ax^4}{b-ax^4}\right)+\left(1-\frac{ax^4}{b}\right)^{3/4}\right)}{6b^2\left(ax^4+b\right)^{3/4}\left(1-\frac{ax^4}{b}\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((b + a*x^4)^(3/4)*(-b^2 + a^2*x^8)),x]
```

```
[Out] -1/6*(x^3*((1 - (a*x^4)/b)^(3/4) + (1 + (a*x^4)/b)^(3/4)*Hypergeometric2F1[
3/4, 3/4, 7/4, (-2*a*x^4)/(b - a*x^4)]))/(b^2*(b + a*x^4)^(3/4)*(1 - (a*x^4
)/b)^(3/4))
```

IntegrateAlgebraic [A] time = 0.52, size = 104, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{4\,2^{3/4}a^{3/4}b^2}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{4\,2^{3/4}a^{3/4}b^2}-\frac{x^3}{6b^2\left(ax^4+b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((b + a*x^4)^(3/4)*(-b^2 + a^2*x^8)),x]
```

```
[Out] -1/6*x^3/(b^2*(b + a*x^4)^(3/4)) + ArcTan[(2^(1/4)*a^(1/4)*x)/(b + a*x^4)^(
1/4)]/(4*2^(3/4)*a^(3/4)*b^2) - ArcTanh[(2^(1/4)*a^(1/4)*x)/(b + a*x^4)^(1/
4)]/(4*2^(3/4)*a^(3/4)*b^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+b)^(3/4)/(a^2*x^8-b^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a^2x^8-b^2\right)\left(ax^4+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+b)^(3/4)/(a^2*x^8-b^2),x, algorithm="giac")
[Out] integrate(x^2/((a^2*x^8 - b^2)*(a*x^4 + b)^(3/4)), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(a\,x^{4}+b\right)^{\frac{3}{4}}\left(a^{2}x^{8}-b^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^4+b)^(3/4)/(a^2*x^8-b^2),x)
[Out] int(x^2/(a*x^4+b)^(3/4)/(a^2*x^8-b^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(a^{2}x^{8}-b^{2}\right)\left(ax^{4}+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+b)^(3/4)/(a^2*x^8-b^2),x, algorithm="maxima")
[Out] integrate(x^2/((a^2*x^8 - b^2)*(a*x^4 + b)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^{2}}{\left(b^{2}-a^{2}\,x^{8}\right)\,\left(a\,x^{4}+b\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((b^2 - a^2*x^8)*(b + a*x^4)^(3/4)),x)
[Out] -int(x^2/((b^2 - a^2*x^8)*(b + a*x^4)^(3/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(ax^{4}-b\right)\left(ax^{4}+b\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**4+b)**(3/4)/(a**2*x**8-b**2),x)
[Out] Integral(x**2/((a*x**4 - b)*(a*x**4 + b)**(7/4)), x)
```

3.1308 $\int \frac{\sqrt{1+\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=104

$$-\frac{2}{15}\sqrt{\sqrt{x^2+1}+1}\,(3x^2+1)+\sqrt{x^2+1}\left(\frac{8x}{15\sqrt{\sqrt{x^2+1}+1}}-\frac{2}{15}\sqrt{\sqrt{x^2+1}+1}\right)+\frac{2x(3x^2+11)}{15\sqrt{\sqrt{x^2+1}+1}}$$

Rubi [F] time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + Sqrt[1 + x^2]]/(x + Sqrt[1 + x^2]),x]
```

```
[Out] (2*(1 + Sqrt[1 + x^2])^(3/2))/3 - (2*(1 + Sqrt[1 + x^2])^(5/2))/5 + Defer[I
nt][Sqrt[1 + x^2]*Sqrt[1 + Sqrt[1 + x^2]], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}\,dx &= -\int x\sqrt{1+\sqrt{1+x^2}}\,dx+\int \sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}\,dx\\
&= -\left(\frac{1}{2}\,\text{Subst}\left(\int \sqrt{1+\sqrt{x}}\,dx,x,1+x^2\right)\right)+\int \sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}\,dx\\
&= \int \sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}\,dx-\text{Subst}\left(\int x\sqrt{1+x}\,dx,x,\sqrt{1+x^2}\right)\\
&= \int \sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}\,dx-\text{Subst}\left(\int \left(-\sqrt{1+x}+(1+x)^{3/2}\right)dx,x,\sqrt{1+x^2}\right)\\
&= \frac{2}{3}\left(1+\sqrt{1+x^2}\right)^{3/2}-\frac{2}{5}\left(1+\sqrt{1+x^2}\right)^{5/2}+\int \sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}\,dx
\end{aligned}$$

Mathematica [A] time = 0.13, size = 72, normalized size = 0.69

$$\frac{2\sqrt{\sqrt{x^2+1}+1}\left(-3x^3+\left(3\sqrt{x^2+1}+1\right)x^2-\left(\sqrt{x^2+1}+1\right)x+7\left(\sqrt{x^2+1}-1\right)\right)}{15x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + x^2]]/(x + Sqrt[1 + x^2]),x]
```

```
[Out] (2*Sqrt[1 + Sqrt[1 + x^2]]*(-3*x^3 + 7*(-1 + Sqrt[1 + x^2]) - x*(1 + Sqrt[1
 + x^2]) + x^2*(1 + 3*Sqrt[1 + x^2])))/(15*x)
```

IntegrateAlgebraic [A] time = 0.39, size = 104, normalized size = 1.00

$$-\frac{2}{15}\sqrt{\sqrt{x^2+1}+1}\,(3x^2+1)+\sqrt{x^2+1}\left(\frac{8x}{15\sqrt{\sqrt{x^2+1}+1}}-\frac{2}{15}\sqrt{\sqrt{x^2+1}+1}\right)+\frac{2x(3x^2+11)}{15\sqrt{\sqrt{x^2+1}+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 + x^2]]/(x + Sqrt[1 + x^2]),x]
```

```
[Out] (2*x*(11 + 3*x^2))/(15*Sqrt[1 + Sqrt[1 + x^2]]) - (2*(1 + 3*x^2)*Sqrt[1 + S
qrt[1 + x^2]])/15 + Sqrt[1 + x^2]*((8*x)/(15*Sqrt[1 + Sqrt[1 + x^2]]) - (2*
Sqrt[1 + Sqrt[1 + x^2]])/15)
```

fricas [A] time = 0.42, size = 48, normalized size = 0.46

$$-\frac{2\left(3x^3 - x^2 - \left(3x^2 - x + 7\right)\sqrt{x^2+1} + x + 7\right)\sqrt{\sqrt{x^2+1}+1}}{15x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/(x+(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2/15*(3*x^3 - x^2 - (3*x^2 - x + 7)*sqrt(x^2 + 1) + x + 7)*sqrt(sqrt(x^2 +
 1) + 1)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{x+\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/(x+(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1)/(x + sqrt(x^2 + 1)), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\sqrt{x^2+1}}}{x+\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x^2+1)^(1/2))^(1/2)/(x+(x^2+1)^(1/2)),x)
```

```
[Out] int((1+(x^2+1)^(1/2))^(1/2)/(x+(x^2+1)^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{x+\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2)/(x+(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1)/(x + sqrt(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{x+\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)^(1/2) + 1)^(1/2)/(x + (x^2 + 1)^(1/2)),x)
[Out] int(((x^2 + 1)^(1/2) + 1)^(1/2)/(x + (x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x^2+1}+1}}{x+\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x**2+1)**(1/2))**(1/2)/(x+(x**2+1)**(1/2)),x)
[Out] Integral(sqrt(sqrt(x**2 + 1) + 1)/(x + sqrt(x**2 + 1)), x)
```

3.1309 $\int \frac{(2+x)^2}{x\left(4-2x+x^2\right)\sqrt[3]{1+x+x^2}}\,dx$

Optimal. Leaf size=105

$$\log\left(\sqrt[3]{x^2+x+1}+x-1\right)-\frac{1}{2}\log\left(x^2+\left(x^2+x+1\right)^{2/3}+(1-x)\sqrt[3]{x^2+x+1}-2x+1\right)-\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2+x+1}}{\sqrt{3}}-\frac{2x}{\sqrt{3}}\ \cdots}{\sqrt[3]{x^2+x-\cdots}}\right)$$

Rubi [F] time = 0.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(2+x)^2}{x\left(4-2x+x^2\right)\sqrt[3]{1+x+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2 + x)^2/(x*(4 - 2*x + x^2)*(1 + x + x^2)^(1/3)),x]
```

```
[Out] (-3*((1 - I*Sqrt[3] + 2*x)/x)^(1/3)*((1 + I*Sqrt[3] + 2*x)/x)^(1/3)*AppellF
1[2/3, 1/3, 1/3, 5/3, -1/2*(1 - I*Sqrt[3])/x, -1/2*(1 + I*Sqrt[3])/x])/(2*2
^(2/3)*(1 + x + x^2)^(1/3)) + 6*Defer[Int][1/((4 - 2*x + x^2)*(1 + x + x^2)
^(1/3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+x)^2}{x\left(4-2x+x^2\right)\sqrt[3]{1+x+x^2}}\,dx &= \int \left(\frac{1}{x\sqrt[3]{1+x+x^2}}+\frac{6}{\left(4-2x+x^2\right)\sqrt[3]{1+x+x^2}}\right)dx\\
&= 6\int \frac{1}{\left(4-2x+x^2\right)\sqrt[3]{1+x+x^2}}\,dx+\int \frac{1}{x\sqrt[3]{1+x+x^2}}\,dx\\
&= 6\int \frac{1}{\left(4-2x+x^2\right)\sqrt[3]{1+x+x^2}}\,dx-\frac{\left(\sqrt[3]{\frac{1-i\sqrt{3}+2x}{x}}\sqrt[3]{\frac{1+i\sqrt{3}+2x}{x}}\right)\text{Subst}\left(\int \frac{\cdots}{\sqrt[3]{\cdots}}\cdots\right)}{2^{2/3}\left(\frac{1}{x}\right)^{2/3}\sqrt[3]{\cdots}}\\
&= -\frac{3\sqrt[3]{\frac{1-i\sqrt{3}+2x}{x}}\sqrt[3]{\frac{1+i\sqrt{3}+2x}{x}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{1-i\sqrt{3}}{2x},-\frac{1+i\sqrt{3}}{2x}\right)}{2\,2^{2/3}\sqrt[3]{1+x+x^2}}+6\int \frac{\cdots}{\left(4-2x+\cdots\right.}
\end{aligned}$$

Mathematica [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(2+x)^2}{x\left(4-2x+x^2\right)\sqrt[3]{1+x+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2 + x)^2/(x*(4 - 2*x + x^2)*(1 + x + x^2)^(1/3)),x]
```

```
[Out] Integrate[(2 + x)^2/(x*(4 - 2*x + x^2)*(1 + x + x^2)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.11, size = 105, normalized size = 1.00

$$\log\left(\sqrt[3]{x^2+x+1}+x-1\right)-\frac{1}{2}\log\left(x^2+\left(x^2+x+1\right)^{2/3}+(1-x)\sqrt[3]{x^2+x+1}-2x+1\right)-\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2+x+1}}{\sqrt{3}}-\frac{2x}{\sqrt{3}}+\frac{2}{\sqrt{3}}}{\sqrt[3]{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)^2/(x*(4 - 2*x + x^2)*(1 + x + x^2)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(2/Sqrt[3] - (2*x)/Sqrt[3] + (1 + x + x^2)^(1/3)/Sqrt[3])/
(1 + x + x^2)^(1/3)]) + Log[-1 + x + (1 + x + x^2)^(1/3)] - Log[1 - 2*x + x
^2 + (1 - x)*(1 + x + x^2)^(1/3) + (1 + x + x^2)^(2/3)]/2
```

fricas [A] time = 1.09, size = 139, normalized size = 1.32

$$-\sqrt{3}\arctan\left(\frac{4\sqrt{3}\left(x^{2}+x+1\right)^{\frac{2}{3}}(x-1)+2\sqrt{3}\left(x^{2}+x+1\right)^{\frac{1}{3}}\left(x^{2}-2x+1\right)+\sqrt{3}\left(x^{3}-3x^{2}+3x-1\right)}{x^{3}-11x^{2}-5x-9}\right)+\frac{1}{2}\log\left(\frac{x^{3}-2x^{2}+3\left(x^{2}+x+1\right)^{\frac{2}{3}}(x-1)+3\left(x^{2}+x+1\right)^{\frac{1}{3}}\left(x^{2}-2x+1\right)+4x}{x^{3}-2x^{2}+4x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)^2/x/(x^2-2*x+4)/(x^2+x+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan((4*sqrt(3)*(x^2 + x + 1)^(2/3)*(x - 1) + 2*sqrt(3)*(x^2 + x
 + 1)^(1/3)*(x^2 - 2*x + 1) + sqrt(3)*(x^3 - 3*x^2 + 3*x - 1))/(x^3 - 11*x^
2 - 5*x - 9)) + 1/2*log((x^3 - 2*x^2 + 3*(x^2 + x + 1)^(2/3)*(x - 1) + 3*(x
^2 + x + 1)^(1/3)*(x^2 - 2*x + 1) + 4*x)/(x^3 - 2*x^2 + 4*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+2)^{2}}{\left(x^{2}+x+1\right)^{\frac{1}{3}}\left(x^{2}-2x+4\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)^2/x/(x^2-2*x+4)/(x^2+x+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x + 2)^2/((x^2 + x + 1)^(1/3)*(x^2 - 2*x + 4)*x), x)
```

maple [C] time = 1.91, size = 641, normalized size = 6.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)^2/x/(x^2-2*x+4)/(x^2+x+1)^(1/3),x)
```

```
[Out] RootOf(_Z^2+_Z+1)*ln((RootOf(_Z^2+_Z+1)*(x^2+x+1)^(2/3)*x-(x^2+x+1)^(1/3)*R
ootOf(_Z^2+_Z+1)*x^2+RootOf(_Z^2+_Z+1)*x^3-RootOf(_Z^2+_Z+1)*(x^2+x+1)^(2/3
)+2*x*(x^2+x+1)^(2/3)+2*(x^2+x+1)^(1/3)*RootOf(_Z^2+_Z+1)*x-2*(x^2+x+1)^(1/
3)*x^2-3*RootOf(_Z^2+_Z+1)*x^2+x^3-2*(x^2+x+1)^(2/3)-(x^2+x+1)^(1/3)*RootOf
(_Z^2+_Z+1)+4*(x^2+x+1)^(1/3)*x+3*RootOf(_Z^2+_Z+1)*x-4*x^2-2*(x^2+x+1)^(1/
3)-RootOf(_Z^2+_Z+1)+2*x-2)/(x^2-2*x+4)/x)-ln(-(RootOf(_Z^2+_Z+1)*(x^2+x+1)
^(2/3)*x-(x^2+x+1)^(1/3)*RootOf(_Z^2+_Z+1)*x^2+RootOf(_Z^2+_Z+1)*x^3-RootOf
(_Z^2+_Z+1)*(x^2+x+1)^(2/3)-x*(x^2+x+1)^(2/3)+2*(x^2+x+1)^(1/3)*RootOf(_Z^2
+_Z+1)*x+(x^2+x+1)^(1/3)*x^2-3*RootOf(_Z^2+_Z+1)*x^2+(x^2+x+1)^(2/3)-(x^2+x
+1)^(1/3)*RootOf(_Z^2+_Z+1)-2*(x^2+x+1)^(1/3)*x+3*RootOf(_Z^2+_Z+1)*x+x^2+(
x^2+x+1)^(1/3)-RootOf(_Z^2+_Z+1)+x+1)/(x^2-2*x+4)/x)*RootOf(_Z^2+_Z+1)-ln(-
(RootOf(_Z^2+_Z+1)*(x^2+x+1)^(2/3)*x-(x^2+x+1)^(1/3)*RootOf(_Z^2+_Z+1)*x^2+
RootOf(_Z^2+_Z+1)*x^3-RootOf(_Z^2+_Z+1)*(x^2+x+1)^(2/3)-x*(x^2+x+1)^(2/3)+2
*(x^2+x+1)^(1/3)*RootOf(_Z^2+_Z+1)*x+(x^2+x+1)^(1/3)*x^2-3*RootOf(_Z^2+_Z+1
)*x^2+(x^2+x+1)^(2/3)-(x^2+x+1)^(1/3)*RootOf(_Z^2+_Z+1)-2*(x^2+x+1)^(1/3)*x
+3*RootOf(_Z^2+_Z+1)*x+x^2+(x^2+x+1)^(1/3)-RootOf(_Z^2+_Z+1)+x+1)/(x^2-2*x+
4)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+2)^{2}}{\left(x^{2}+x+1\right)^{\frac{1}{3}}\left(x^{2}-2x+4\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)^2/x/(x^2-2*x+4)/(x^2+x+1)^(1/3),x, algorithm="maxima")
[Out] integrate((x + 2)^2/((x^2 + x + 1)^(1/3)*(x^2 - 2*x + 4)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x+2)^2}{x\left(x^2-2x+4\right)\left(x^2+x+1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)^2/(x*(x^2 - 2*x + 4)*(x + x^2 + 1)^(1/3)),x)
[Out] int((x + 2)^2/(x*(x^2 - 2*x + 4)*(x + x^2 + 1)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+2)^2}{x\left(x^2-2x+4\right)\sqrt[3]{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)**2/x/(x**2-2*x+4)/(x**2+x+1)**(1/3),x)
[Out] Integral((x + 2)**2/(x*(x**2 - 2*x + 4)*(x**2 + x + 1)**(1/3)), x)
```

3.1310 $\int \frac{1}{\left(-2b+ax^2\right)\sqrt[4]{-b+ax^2}}\,dx$

Optimal. Leaf size=105

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{a}\,x}\right)}{2\sqrt{2}\sqrt{a}\,b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{a}\,x}\right)}{2\sqrt{2}\sqrt{a}\,b^{3/4}}$$

Rubi [A] time = 0.03, antiderivative size = 101, normalized size of antiderivative = 0.96, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}\right)}{2\sqrt{2}\sqrt{a}\,b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}\right)}{2\sqrt{2}\sqrt{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2*b + a*x^2)*(-b + a*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[a]*x)/(Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4))]/(Sqrt[2]*Sqrt
[a]*b^(3/4)) - ArcTanh[(Sqrt[a]*x)/(Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4))]/(2
*Sqrt[2]*Sqrt[a]*b^(3/4))
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\left(-2b+ax^2\right)\sqrt[4]{-b+ax^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^2}}\right)}{2\sqrt{2}\sqrt{a}\,b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^2}}\right)}{2\sqrt{2}\sqrt{a}\,b^{3/4}}$$

Mathematica [C] time = 0.15, size = 163, normalized size = 1.55

$$-\frac{6bxF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{ax^2}{b},\frac{ax^2}{2b}\right)}{\left(2b-ax^2\right)\sqrt[4]{ax^2-b}\left(ax^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{ax^2}{b},\frac{ax^2}{2b}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{ax^2}{b},\frac{ax^2}{2b}\right)\right)+6bF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{ax^2}{b},\frac{ax^2}{2b}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2*b + a*x^2)*(-b + a*x^2)^(1/4)),x]
```

```
[Out] (-6*b*x*AppellF1[1/2, 1/4, 1, 3/2, (a*x^2)/b, (a*x^2)/(2*b)])/((2*b - a*x^2
)*(-b + a*x^2)^(1/4)*(6*b*AppellF1[1/2, 1/4, 1, 3/2, (a*x^2)/b, (a*x^2)/(2*
b)] + a*x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (a*x^2)/b, (a*x^2)/(2*b)] + Appel
lF1[3/2, 5/4, 1, 5/2, (a*x^2)/b, (a*x^2)/(2*b)])))
```

IntegrateAlgebraic [A] time = 0.18, size = 105, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{a}\,x}\right)}{2\sqrt{2}\sqrt{a}\,b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{a}\,x}\right)}{2\sqrt{2}\sqrt{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2*b + a*x^2)*(-b + a*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4))/(Sqrt[a]*x)]/(2*Sqrt[2]*Sqrt[a]
*b^(3/4)) - ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4))/(Sqrt[a]*x)]/(2*Sq
rt[2]*Sqrt[a]*b^(3/4))
```

fricas [B] time = 113.08, size = 338, normalized size = 3.22

$$-\left(\frac{1}{4}\right)^{\frac{1}{4}} \left(\frac{1}{a^2 b^3}\right)^{\frac{1}{4}} \arctan\left(\frac{2 \left(\sqrt{\frac{1}{2}} \left(2 \left(\frac{1}{4}\right)^{\frac{3}{4}} a b^3 \left(\frac{1}{a^2 b^3}\right)^{\frac{3}{4}} + \left(\frac{1}{4}\right)^{\frac{1}{4}} \sqrt{a x^2 - b}\, b \left(\frac{1}{a^2 b^3}\right)^{\frac{1}{4}}\right) \sqrt{a b \sqrt{\frac{1}{a^2 b^3}}} - \left(\frac{1}{4}\right)^{\frac{1}{4}} \left(a x^2 - b\right)^{\frac{1}{4}} b \left(\frac{1}{a^2 b^3}\right)^{\frac{1}{4}}\right)}{x}\right) - \frac{1}{4} \left(\frac{1}{4}\right)^{\frac{1}{4}} \left(\frac{1}{a^2 b^3}\right)^{\frac{1}{4}} \log\left(\frac{2 \left(\frac{1}{4}\right)^{\frac{3}{4}} \sqrt{a x^2 - b}\, a^2 b^2 x \left(\frac{1}{a^2 b^3}\right)^{\frac{3}{4}} + \left(a x^2 - b\right)^{\frac{1}{4}} a b^2 \sqrt{\frac{1}{a^2 b^3}} + \left(\frac{1}{4}\right)^{\frac{1}{4}} a b x \left(\frac{1}{a^2 b^3}\right)^{\frac{1}{4}} + \left(a x^2 - b\right)^{\frac{3}{4}}}{a x^2 - 2 b}\right) + \frac{1}{4} \left(\frac{1}{4}\right)^{\frac{1}{4}} \left(\frac{1}{a^2 b^3}\right)^{\frac{1}{4}} \log\left(-\frac{2 \left(\frac{1}{4}\right)^{\frac{3}{4}} \sqrt{a x^2 - b}\, a^2 b^2 x \left(\frac{1}{a^2 b^3}\right)^{\frac{3}{4}} - \left(a x^2 - b\right)^{\frac{1}{4}} a b^2 \sqrt{\frac{1}{a^2 b^3}} + \left(\frac{1}{4}\right)^{\frac{1}{4}} a b x \left(\frac{1}{a^2 b^3}\right)^{\frac{1}{4}} - \left(a x^2 - b\right)^{\frac{3}{4}}}{a x^2 - 2 b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2-2*b)/(a*x^2-b)^(1/4),x, algorithm="fricas")
```

```
[Out] -(1/4)^(1/4)*(1/(a^2*b^3))^(1/4)*arctan(2*(sqrt(1/2)*(2*(1/4)^(3/4)*a*b^3*(
1/(a^2*b^3))^(3/4) + (1/4)^(1/4)*sqrt(a*x^2 - b)*b*(1/(a^2*b^3))^(1/4))*sqr
t(a*b*sqrt(1/(a^2*b^3))) - (1/4)^(1/4)*(a*x^2 - b)^(1/4)*b*(1/(a^2*b^3))^(1
/4))/x) - 1/4*(1/4)^(1/4)*(1/(a^2*b^3))^(1/4)*log((2*(1/4)^(3/4)*sqrt(a*x^2
 - b)*a^2*b^2*x*(1/(a^2*b^3))^(3/4) + (a*x^2 - b)^(1/4)*a*b^2*sqrt(1/(a^2*b
^3)) + (1/4)^(1/4)*a*b*x*(1/(a^2*b^3))^(1/4) + (a*x^2 - b)^(3/4))/(a*x^2 -
2*b)) + 1/4*(1/4)^(1/4)*(1/(a^2*b^3))^(1/4)*log(-(2*(1/4)^(3/4)*sqrt(a*x^2
- b)*a^2*b^2*x*(1/(a^2*b^3))^(3/4) - (a*x^2 - b)^(1/4)*a*b^2*sqrt(1/(a^2*b^
3)) + (1/4)^(1/4)*a*b*x*(1/(a^2*b^3))^(1/4) - (a*x^2 - b)^(3/4))/(a*x^2 - 2
*b))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a x^2 - b\right)^{\frac{1}{4}} \left(a x^2 - 2 b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2-2*b)/(a*x^2-b)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x^2 - b)^(1/4)*(a*x^2 - 2*b)), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,x^2 - 2b\right) \left(a\,x^2 - b\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2-2*b)/(a*x^2-b)^(1/4),x)
```

```
[Out] int(1/(a*x^2-2*b)/(a*x^2-b)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a x^2 - b\right)^{\frac{1}{4}} \left(a x^2 - 2 b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2-2*b)/(a*x^2-b)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x^2 - b)^(1/4)*(a*x^2 - 2*b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(a\,x^2-b\right)^{1/4}\,\left(2\,b-a\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a*x^2 - b)^(1/4)*(2*b - a*x^2)),x)
[Out] -int(1/((a*x^2 - b)^(1/4)*(2*b - a*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^2-2b\right)\sqrt[4]{ax^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2-2*b)/(a*x**2-b)**(1/4),x)
[Out] Integral(1/((a*x**2 - 2*b)*(a*x**2 - b)**(1/4)), x)
```

3.1311 $\int \frac{-1+x}{x^7 \sqrt[3]{1+x^3}} \, dx$

Optimal. Leaf size=105

$$-\frac{2}{27} \log\left(\sqrt[3]{x^3+1}-1\right)+\frac{1}{27} \log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)-\frac{2 \tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\left(x^3+1\right)^{2/3}\left(27x^4-20x^3-\right.}{90x^6}$$

Rubi [A] time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1844, 266, 51, 55, 618, 204, 31, 271, 264}

$$-\frac{2\left(x^3+1\right)^{2/3}}{9x^3}-\frac{1}{9}\log\left(1-\sqrt[3]{x^3+1}\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\left(x^3+1\right)^{2/3}}{6x^6}-\frac{\left(x^3+1\right)^{2/3}}{5x^5}+\frac{3\left(x^3+1\right)^{2/3}}{10x^2}+\frac{\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(x^7*(1 + x^3)^(1/3)),x]
```

```
[Out] (1 + x^3)^(2/3)/(6*x^6) - (1 + x^3)^(2/3)/(5*x^5) - (2*(1 + x^3)^(2/3))/(9*
x^3) + (3*(1 + x^3)^(2/3))/(10*x^2) - (2*ArcTan[(1 + 2*(1 + x^3)^(1/3))/Sqr
t[3]])/(9*Sqrt[3]) + Log[x]/9 - Log[1 - (1 + x^3)^(1/3)]/9
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rule steps

$$\begin{aligned}
\int \frac{-1+x}{x^7\sqrt[3]{1+x^3}}\,dx &= \int \left(-\frac{1}{x^7\sqrt[3]{1+x^3}}+\frac{1}{x^6\sqrt[3]{1+x^3}}\right)dx\\
&= -\int \frac{1}{x^7\sqrt[3]{1+x^3}}\,dx+\int \frac{1}{x^6\sqrt[3]{1+x^3}}\,dx\\
&= -\frac{\left(1+x^3\right)^{2/3}}{5x^5}-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt[3]{1+x}}\,dx,x,x^3\right)-\frac{3}{5}\int \frac{1}{x^3\sqrt[3]{1+x^3}}\,dx\\
&= \frac{\left(1+x^3\right)^{2/3}}{6x^6}-\frac{\left(1+x^3\right)^{2/3}}{5x^5}+\frac{3\left(1+x^3\right)^{2/3}}{10x^2}+\frac{2}{9}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt[3]{1+x}}\,dx,x,x^3\right)\\
&= \frac{\left(1+x^3\right)^{2/3}}{6x^6}-\frac{\left(1+x^3\right)^{2/3}}{5x^5}-\frac{2\left(1+x^3\right)^{2/3}}{9x^3}+\frac{3\left(1+x^3\right)^{2/3}}{10x^2}-\frac{2}{27}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx,x,x^3\right.\\
&= \frac{\left(1+x^3\right)^{2/3}}{6x^6}-\frac{\left(1+x^3\right)^{2/3}}{5x^5}-\frac{2\left(1+x^3\right)^{2/3}}{9x^3}+\frac{3\left(1+x^3\right)^{2/3}}{10x^2}+\frac{\log(x)}{9}+\frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx,\right.\\
&= \frac{\left(1+x^3\right)^{2/3}}{6x^6}-\frac{\left(1+x^3\right)^{2/3}}{5x^5}-\frac{2\left(1+x^3\right)^{2/3}}{9x^3}+\frac{3\left(1+x^3\right)^{2/3}}{10x^2}+\frac{\log(x)}{9}-\frac{1}{9}\log\left(1-\sqrt[3]{1+x^3}\right)-\\
&= \frac{\left(1+x^3\right)^{2/3}}{6x^6}-\frac{\left(1+x^3\right)^{2/3}}{5x^5}-\frac{2\left(1+x^3\right)^{2/3}}{9x^3}+\frac{3\left(1+x^3\right)^{2/3}}{10x^2}-\frac{2\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^3}}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\log(x)}{9}-
\end{aligned}$$

Mathematica [C] time = 0.03, size = 41, normalized size = 0.39

$$\frac{\left(x^3+1\right)^{2/3}\left(5x^5\,{}_2F_1\left(\frac{2}{3},3;\frac{5}{3};x^3+1\right)+3x^3-2\right)}{10x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(x^7*(1 + x^3)^(1/3)),x]
```

```
[Out] ((1 + x^3)^(2/3)*(-2 + 3*x^3 + 5*x^5*Hypergeometric2F1[2/3, 3, 5/3, 1 + x^3
]))/(10*x^5)
```

IntegrateAlgebraic [A] time = 15.64, size = 105, normalized size = 1.00

$$-\frac{2}{27}\log\left(\sqrt[3]{x^3+1}-1\right)+\frac{1}{27}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{9\sqrt{3}}+\frac{\left(x^3+1\right)^{2/3}\left(27x^4-20x^3-18x+15\right)}{90x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/(x^7*(1 + x^3)^(1/3)),x]
```

```
[Out] ((1 + x^3)^(2/3)*(15 - 18*x - 20*x^3 + 27*x^4))/(90*x^6) - (2*ArcTan[1/Sqrt
[3] + (2*(1 + x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]) - (2*Log[-1 + (1 + x^3)^(1/
3)])/27 + Log[1 + (1 + x^3)^(1/3) + (1 + x^3)^(2/3)]/27
```

fricas [A] time = 0.76, size = 114, normalized size = 1.09

$$\frac{20\sqrt{3}\,x^6\arctan\left(-\frac{\sqrt{3}\left(x^3+1\right)-2\sqrt{3}\left(x^3+1\right)^{\frac{2}{3}}+4\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}}{x^3+9}\right)-10\,x^6\log\left(\frac{x^3-3\left(x^3+1\right)^{\frac{2}{3}}+3\left(x^3+1\right)^{\frac{1}{3}}}{x^3}\right)+3\left(27\,x^4-20\,x^3-18\,x+15\right)\left(x^3+1\right)^{\frac{2}{3}}}{270\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x^7/(x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/270*(20*sqrt(3)*x^6*arctan(-(sqrt(3)*(x^3 + 1) - 2*sqrt(3)*(x^3 + 1)^(2/3
) + 4*sqrt(3)*(x^3 + 1)^(1/3))/(x^3 + 9)) - 10*x^6*log((x^3 - 3*(x^3 + 1)^(
2/3) + 3*(x^3 + 1)^(1/3))/x^3) + 3*(27*x^4 - 20*x^3 - 18*x + 15)*(x^3 + 1)^
(2/3))/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^3+1\right)^{\frac{1}{3}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x^7/(x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/((x^3 + 1)^(1/3)*x^7), x)
```

maple [C] time = 0.26, size = 101, normalized size = 0.96

$$\frac{27x^7-20x^6+9x^4-5x^3-18x+15}{90x^6\left(x^3+1\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^3\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],-x^3\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{27\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/x^7/(x^3+1)^(1/3),x)
```

```
[Out] 1/90*(27*x^7-20*x^6+9*x^4-5*x^3-18*x+15)/x^6/(x^3+1)^(1/3)-1/27/Pi*3^(1/2)*
GAMMA(2/3)*(-2/9*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1,1,4/3],[2,2],-x^3)+
2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x))*Pi*3^(1/2)/GAMMA(2/3))
```

maxima [A] time = 0.43, size = 117, normalized size = 1.11

$$-\frac{2}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{4\left(x^3+1\right)^{\frac{5}{3}}-7\left(x^3+1\right)^{\frac{2}{3}}}{18\left(2x^3-\left(x^3+1\right)^2+1\right)}+\frac{\left(x^3+1\right)^{\frac{2}{3}}}{2x^2}-\frac{\left(x^3+1\right)^{\frac{5}{3}}}{5x^5}+\frac{1}{27}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)-\frac{2}{27}\log\left(\left(x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x^7/(x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -2/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) + 1/18*(4*(x^3 +
1)^(5/3) - 7*(x^3 + 1)^(2/3))/(2*x^3 - (x^3 + 1)^2 + 1) + 1/2*(x^3 + 1)^(2/
3)/x^2 - 1/5*(x^3 + 1)^(5/3)/x^5 + 1/27*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/
3) + 1) - 2/27*log((x^3 + 1)^(1/3) - 1)
```

mupad [B] time = 1.15, size = 146, normalized size = 1.39

$$-\frac{2\ln\left(\frac{4\left(x^3+1\right)^{1/3}}{81}-\frac{4}{81}\right)}{27}-\ln\left(\frac{4\left(x^3+1\right)^{1/3}}{81}-9\left(-\frac{1}{27}+\frac{\sqrt{3}\,1\mathrm{i}}{27}\right)^2\right)\left(-\frac{1}{27}+\frac{\sqrt{3}\,1\mathrm{i}}{27}\right)+\ln\left(\frac{4\left(x^3+1\right)^{1/3}}{81}-9\left(\frac{1}{27}+\frac{\sqrt{3}\,1\mathrm{i}}{27}\right)^2\right)\left(\frac{1}{27}+\frac{\sqrt{3}\,1\mathrm{i}}{27}\right)-\frac{2\left(x^3+1\right)^{2/3}-3\,x^3\left(x^3+1\right)^{2/3}}{10\,x^5}-\frac{\frac{7\left(x^3+1\right)^{2/3}}{18}-\frac{2\left(x^3+1\right)^{5/3}}{9}}{2\,x^3-\left(x^3+1\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(x^7*(x^3 + 1)^(1/3)),x)
```

```
[Out] log((4*(x^3 + 1)^(1/3))/81 - 9*((3^(1/2)*1i)/27 + 1/27)^2)*((3^(1/2)*1i)/27
 + 1/27) - log((4*(x^3 + 1)^(1/3))/81 - 9*((3^(1/2)*1i)/27 - 1/27)^2)*((3^(
1/2)*1i)/27 - 1/27) - (2*log((4*(x^3 + 1)^(1/3))/81 - 4/81))/27 - (2*(x^3 +
 1)^(2/3) - 3*x^3*(x^3 + 1)^(2/3))/(10*x^5) - ((7*(x^3 + 1)^(2/3))/18 - (2*
(x^3 + 1)^(5/3))/9)/(2*x^3 - (x^3 + 1)^2 + 1)
```

sympy [C] time = 2.16, size = 82, normalized size = 0.78

$$\frac{\left(x^3+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}-\frac{2\left(x^3+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{9x^5\Gamma\left(\frac{1}{3}\right)}+\frac{\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{e^{i\pi}}{x^3}\right)}{3x^7\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x**7/(x**3+1)**(1/3),x)
```

```
[Out] (x**3 + 1)**(2/3)*gamma(-5/3)/(3*x**2*gamma(1/3)) - 2*(x**3 + 1)**(2/3)*gam
ma(-5/3)/(9*x**5*gamma(1/3)) + gamma(7/3)*hyper((1/3, 7/3), (10/3,), exp_po
lar(I*pi)/x**3)/(3*x**7*gamma(10/3))
```

3.1312 $\int \frac{3c+2bx+ax^2}{\sqrt[3]{c+bx+ax^2}\left(c+bx+ax^2+x^3\right)}\,dx$

Optimal. Leaf size=105

$$\log\left(\sqrt[3]{ax^2+bx+c}+x\right)-\frac{1}{2}\log\left(-x\sqrt[3]{ax^2+bx+c}+\left(ax^2+bx+c\right)^{2/3}+x^2\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{ax^2+bx+c}}{\sqrt[3]{ax^2+bx+c}-2x}\right)$$

Rubi [F] time = 1.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3c+2bx+ax^2}{\sqrt[3]{c+bx+ax^2}\left(c+bx+ax^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3*c + 2*b*x + a*x^2)/((c + b*x + a*x^2)^(1/3)*(c + b*x + a*x^2 + x^3))
,x]
```

```
[Out] 3*c*Defer[Int][1/((c + b*x + a*x^2)^(1/3)*(c + b*x + a*x^2 + x^3)), x] + 2*
b*Defer[Int][x/((c + b*x + a*x^2)^(1/3)*(c + b*x + a*x^2 + x^3)), x] + a*De
fer[Int][x^2/((c + b*x + a*x^2)^(1/3)*(c + b*x + a*x^2 + x^3)), x]
```

Rubi steps

$$\int \frac{3c+2bx+ax^2}{\sqrt[3]{c+bx+ax^2}\left(c+bx+ax^2+x^3\right)}\,dx=\int \left(\frac{3c}{\sqrt[3]{c+bx+ax^2}\left(c+bx+ax^2+x^3\right)}+\frac{2bx}{\sqrt[3]{c+bx+ax^2}\left(c+bx+a\right.}\right.$$
$$=a\int \frac{x^2}{\sqrt[3]{c+bx+ax^2}\left(c+bx+ax^2+x^3\right)}\,dx+(2b)\int \frac{}{\sqrt[3]{c+bx+ax^2}\left(\right.}$$

Mathematica [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{3c+2bx+ax^2}{\sqrt[3]{c+bx+ax^2}\left(c+bx+ax^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3*c + 2*b*x + a*x^2)/((c + b*x + a*x^2)^(1/3)*(c + b*x + a*x^2 +
 x^3)),x]
```

```
[Out] Integrate[(3*c + 2*b*x + a*x^2)/((c + b*x + a*x^2)^(1/3)*(c + b*x + a*x^2 +
 x^3)), x]
```

IntegrateAlgebraic [A] time = 0.54, size = 105, normalized size = 1.00

$$\log\left(\sqrt[3]{ax^2+bx+c}+x\right)-\frac{1}{2}\log\left(-x\sqrt[3]{ax^2+bx+c}+\left(ax^2+bx+c\right)^{2/3}+x^2\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{ax^2+bx+c}}{\sqrt[3]{ax^2+bx+c}-2x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*c + 2*b*x + a*x^2)/((c + b*x + a*x^2)^(1/3)*(c + b*x
+ a*x^2 + x^3)),x]
```

```
[Out] Sqrt[3]*ArcTan[(Sqrt[3]*(c + b*x + a*x^2)^(1/3))/(-2*x + (c + b*x + a*x^2)^
(1/3))] + Log[x + (c + b*x + a*x^2)^(1/3)] - Log[x^2 - x*(c + b*x + a*x^2)^
(1/3) + (c + b*x + a*x^2)^(2/3)]/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b*x+3*c)/(a*x^2+b*x+c)^(1/3)/(a*x^2+x^3+b*x+c),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + 2\,bx + 3\,c}{\left(ax^2 + x^3 + bx + c\right)\left(ax^2 + bx + c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b*x+3*c)/(a*x^2+b*x+c)^(1/3)/(a*x^2+x^3+b*x+c),x, algori
thm="giac")
```

```
[Out] integrate((a*x^2 + 2*b*x + 3*c)/((a*x^2 + x^3 + b*x + c)*(a*x^2 + b*x + c)^
(1/3)), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2 + 2bx + 3c}{\left(a\,x^2 + bx + c\right)^{\frac{1}{3}}\left(a\,x^2 + x^3 + bx + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+2*b*x+3*c)/(a*x^2+b*x+c)^(1/3)/(a*x^2+x^3+b*x+c),x)
```

```
[Out] int((a*x^2+2*b*x+3*c)/(a*x^2+b*x+c)^(1/3)/(a*x^2+x^3+b*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + 2\,bx + 3\,c}{\left(ax^2 + x^3 + bx + c\right)\left(ax^2 + bx + c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b*x+3*c)/(a*x^2+b*x+c)^(1/3)/(a*x^2+x^3+b*x+c),x, algori
thm="maxima")
```

```
[Out] integrate((a*x^2 + 2*b*x + 3*c)/((a*x^2 + x^3 + b*x + c)*(a*x^2 + b*x + c)^
(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x^2 + 2\,b\,x + 3\,c}{\left(a\,x^2 + b\,x + c\right)^{1/3}\,\left(x^3 + a\,x^2 + b\,x + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*c + 2*b*x + a*x^2)/((c + b*x + a*x^2)^(1/3)*(c + b*x + a*x^2 + x^3))
,x)
```

```
[Out] int((3*c + 2*b*x + a*x^2)/((c + b*x + a*x^2)^(1/3)*(c + b*x + a*x^2 + x^3))
, x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+2*b*x+3*c)/(a*x**2+b*x+c)**(1/3)/(a*x**2+x**3+b*x+c),x)
[Out] Timed out
```

3.1313 $\int \frac{x(3+5x^2)}{\sqrt[3]{1+x^2}\left(-1+x^3+x^5\right)}\,dx$

Optimal. Leaf size=105

$$\frac{1}{2}\log\left(x^2\left(x^2+1\right)^{2/3}\right)-\frac{1}{2}\log\left(\left(x^2+1\right)^{2/3}x^2+\sqrt[3]{x^2+1}\,x+1\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x\sqrt[3]{x^2+1}}{\sqrt[3]{x^2+1}\,x+2}\right)+2\tanh^{-1}\left(1-2x\sqrt[3]{x$$

Rubi [F] time = 0.55, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\left(3+5x^2\right)}{\sqrt[3]{1+x^2}\left(-1+x^3+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(3 + 5*x^2))/((1 + x^2)^(1/3)*(-1 + x^3 + x^5)),x]
```

```
[Out] 3*Defer[Int][x/((1 + x^2)^(1/3)*(-1 + x^3 + x^5)), x] + 5*Defer[Int][x^3/((
1 + x^2)^(1/3)*(-1 + x^3 + x^5)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(3+5x^2\right)}{\sqrt[3]{1+x^2}\left(-1+x^3+x^5\right)}\,dx &= \int \left(\frac{3x}{\sqrt[3]{1+x^2}\left(-1+x^3+x^5\right)}+\frac{5x^3}{\sqrt[3]{1+x^2}\left(-1+x^3+x^5\right)}\right)dx\\ &= 3\int \frac{x}{\sqrt[3]{1+x^2}\left(-1+x^3+x^5\right)}\,dx+5\int \frac{x^3}{\sqrt[3]{1+x^2}\left(-1+x^3+x^5\right)}\,dx\end{aligned}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x\left(3+5x^2\right)}{\sqrt[3]{1+x^2}\left(-1+x^3+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(3 + 5*x^2))/((1 + x^2)^(1/3)*(-1 + x^3 + x^5)),x]
```

```
[Out] Integrate[(x*(3 + 5*x^2))/((1 + x^2)^(1/3)*(-1 + x^3 + x^5)), x]
```

IntegrateAlgebraic [A] time = 2.56, size = 105, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2\left(x^2+1\right)^{2/3}\right)-\frac{1}{2}\log\left(\left(x^2+1\right)^{2/3}x^2+\sqrt[3]{x^2+1}\,x+1\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x\sqrt[3]{x^2+1}}{\sqrt[3]{x^2+1}\,x+2}\right)+2\tanh^{-1}\left(1-2x\sqrt[3]{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(3 + 5*x^2))/((1 + x^2)^(1/3)*(-1 + x^3 + x^5)),x]
```

```
[Out] Sqrt[3]*ArcTan[(Sqrt[3]*x*(1 + x^2)^(1/3))/(2 + x*(1 + x^2)^(1/3))] + 2*Arc
Tanh[1 - 2*x*(1 + x^2)^(1/3)] + Log[x^2*(1 + x^2)^(2/3)]/2 - Log[1 + x*(1 +
 x^2)^(1/3) + x^2*(1 + x^2)^(2/3)]/2
```

fricas [A] time = 2.80, size = 103, normalized size = 0.98

$$-\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(x^2+1\right)^{\frac{2}{3}}x^2-4\sqrt{3}\left(x^2+1\right)^{\frac{1}{3}}x-\sqrt{3}\left(x^5+x^3\right)}{x^5+x^3+8}\right)+\frac{1}{2}\log\left(\frac{x^5+x^3-3\left(x^2+1\right)^{\frac{2}{3}}x^2+3\left(x^2+1\right)^{\frac{1}{3}}x-1}{x^5+x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(5*x^2+3)/(x^2+1)^(1/3)/(x^5+x^3-1),x, algorithm="fricas")

[Out] -sqrt(3)*arctan((2*sqrt(3)*(x^2 + 1)^(2/3)*x^2 - 4*sqrt(3)*(x^2 + 1)^(1/3)*x - sqrt(3)*(x^5 + x^3))/(x^5 + x^3 + 8)) + 1/2*log((x^5 + x^3 - 3*(x^2 + 1)^(2/3)*x^2 + 3*(x^2 + 1)^(1/3)*x - 1)/(x^5 + x^3 - 1))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^{2}+3\right)x}{\left(x^{5}+x^{3}-1\right)\left(x^{2}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(5*x^2+3)/(x^2+1)^(1/3)/(x^5+x^3-1),x, algorithm="giac")

[Out] integrate((5*x^2 + 3)*x/((x^5 + x^3 - 1)*(x^2 + 1)^(1/3)), x)

maple [C] time = 3.62, size = 282, normalized size = 2.69

$$\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)^{2} x^{5}-\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{5}+\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)\left(x^{2}+1\right)^{\frac{2}{3}} x^{2}-\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)^{2} x^{3}-x^{2}\left(x^{2}+1\right)^{\frac{2}{3}}-\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{3}-2\,\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)\left(x^{2}+1\right)^{\frac{1}{3}} x-x\left(x^{2}+1\right)^{\frac{1}{3}}+\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)+1}{x^{5}+x^{3}-1}\right)+\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)^{2} x^{5}-2\,\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{5}+\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)\left(x^{2}+1\right)^{\frac{2}{3}} x^{2}-\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)^{2} x^{3}-x^{5}+2 x^{2}\left(x^{2}+1\right)^{\frac{2}{3}}-2\,\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{3}+\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)\left(x^{2}+1\right)^{\frac{1}{3}} x-x^{3}-x\left(x^{2}+1\right)^{\frac{1}{3}}-\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)-1}{x^{5}+x^{3}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(5*x^2+3)/(x^2+1)^(1/3)/(x^5+x^3-1),x)

[Out] ln(-(-RootOf(_Z^2+_Z+1)^2*x^5-RootOf(_Z^2+_Z+1)*x^5+RootOf(_Z^2+_Z+1)*(x^2+1)^(2/3)*x^2-RootOf(_Z^2+_Z+1)^2*x^3-x^2*(x^2+1)^(2/3)-RootOf(_Z^2+_Z+1)*x^3-2*RootOf(_Z^2+_Z+1)*(x^2+1)^(1/3)*x-x*(x^2+1)^(1/3)+RootOf(_Z^2+_Z+1)+1)/(x^5+x^3-1))+RootOf(_Z^2+_Z+1)*ln((-RootOf(_Z^2+_Z+1)^2*x^5-2*RootOf(_Z^2+_Z+1)*x^5+RootOf(_Z^2+_Z+1)*(x^2+1)^(2/3)*x^2-RootOf(_Z^2+_Z+1)^2*x^3-x^5+2*x^2*(x^2+1)^(2/3)-2*RootOf(_Z^2+_Z+1)*x^3+RootOf(_Z^2+_Z+1)*(x^2+1)^(1/3)*x-x^3-x*(x^2+1)^(1/3)-RootOf(_Z^2+_Z+1)-1)/(x^5+x^3-1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^{2}+3\right)x}{\left(x^{5}+x^{3}-1\right)\left(x^{2}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(5*x^2+3)/(x^2+1)^(1/3)/(x^5+x^3-1),x, algorithm="maxima")

[Out] integrate((5*x^2 + 3)*x/((x^5 + x^3 - 1)*(x^2 + 1)^(1/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(5\,x^2+3\right)}{{\left(x^2+1\right)}^{1/3}\,\left(x^5+x^3-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(5*x^2 + 3))/((x^2 + 1)^(1/3)*(x^3 + x^5 - 1)),x)

[Out] int((x*(5*x^2 + 3))/((x^2 + 1)^(1/3)*(x^3 + x^5 - 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(5 x^{2} + 3\right)}{\sqrt[3]{x^{2} + 1} \left(x^{5} + x^{3} - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x**2+3)/(x**2+1)**(1/3)/(x**5+x**3-1),x)
```

```
[Out] Integral(x*(5*x**2 + 3)/((x**2 + 1)**(1/3)*(x**5 + x**3 - 1)), x)
```

3.1314 $\int \frac{\left(-1+x^5\right)^{2/3}\left(3+2x^5\right)\left(-2+x^3+2x^5\right)}{x^6\left(-1+x^3+x^5\right)}\,dx$

Optimal. Leaf size=105

$$-\log\left(\sqrt[3]{x^5-1}+x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^5-1}-x}\right)+\frac{3\left(x^5-1\right)^{2/3}\left(4x^5-5x^3-4\right)}{10x^5}+\frac{1}{2}\log\left(-\sqrt[3]{x^5-1}\,x+\left(x^5-1\right)^{2/3}+\right.$$

Rubi [F] time = 1.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1+x^5\right)^{2/3}\left(3+2x^5\right)\left(-2+x^3+2x^5\right)}{x^6\left(-1+x^3+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^5)^(2/3)*(3 + 2*x^5)*(-2 + x^3 + 2*x^5))/(x^6*(-1 + x^3 + x^5)
),x]
```

```
[Out] (6*(-1 + x^5)^(2/3))/5 - (6*(-1 + x^5)^(2/3))/(5*x^5) - (3*(-1 + x^5)^(2/3)
*Hypergeometric2F1[-2/3, -2/5, 3/5, x^5])/(2*x^2*(1 - x^5)^(2/3)) - 3*Defer
[Int][(-1 + x^5)^(2/3)/(-1 + x^3 + x^5), x] - 5*Defer[Int][(x^2*(-1 + x^5)^
(2/3))/(-1 + x^3 + x^5), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^5\right)^{2/3}\left(3+2x^5\right)\left(-2+x^3+2x^5\right)}{x^6\left(-1+x^3+x^5\right)}\,dx &= \int \left(\frac{6\left(-1+x^5\right)^{2/3}}{x^6}+\frac{3\left(-1+x^5\right)^{2/3}}{x^3}+\frac{4\left(-1+x^5\right)^{2/3}}{x}+\frac{\left(-3-5x^2\right.}{-1+}\right.\\
&= 3\int \frac{\left(-1+x^5\right)^{2/3}}{x^3}\,dx+4\int \frac{\left(-1+x^5\right)^{2/3}}{x}\,dx+6\int \frac{\left(-1+x^5\right)^{2/3}}{x^6}\,d\\
&= \frac{4}{5}\text{Subst}\left(\int \frac{(-1+x)^{2/3}}{x}\,dx,x,x^5\right)+\frac{6}{5}\text{Subst}\left(\int \frac{(-1+x)^{2/3}}{x^2}\,dx,x\right.\\
&= \frac{6}{5}\left(-1+x^5\right)^{2/3}-\frac{6\left(-1+x^5\right)^{2/3}}{5x^5}-\frac{3\left(-1+x^5\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{5};\frac{3}{5};x\right.}{2x^2\left(1-x^5\right)^{2/3}}
\end{aligned}$$

Mathematica [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^5\right)^{2/3}\left(3+2x^5\right)\left(-2+x^3+2x^5\right)}{x^6\left(-1+x^3+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^5)^(2/3)*(3 + 2*x^5)*(-2 + x^3 + 2*x^5))/(x^6*(-1 + x^3
+ x^5)),x]
```

```
[Out] Integrate[((-1 + x^5)^(2/3)*(3 + 2*x^5)*(-2 + x^3 + 2*x^5))/(x^6*(-1 + x^3
+ x^5)), x]
```

IntegrateAlgebraic [A] time = 3.50, size = 105, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^5-1}+x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^5-1}-x}\right)+\frac{3\left(x^5-1\right)^{2/3}\left(4x^5-5x^3-4\right)}{10x^5}+\frac{1}{2}\log\left(-\sqrt[3]{x^5-1}\,x+\left(x^5-1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^5)^(2/3)*(3 + 2*x^5)*(-2 + x^3 + 2*x^5))/(x^6*(
-1 + x^3 + x^5)),x]
```

```
[Out] (3*(-1 + x^5)^(2/3)*(-4 - 5*x^3 + 4*x^5))/(10*x^5) - Sqrt[3]*ArcTan[(Sqrt[3
]*x)/(-x + 2*(-1 + x^5)^(1/3))] - Log[x + (-1 + x^5)^(1/3)] + Log[x^2 - x*(
-1 + x^5)^(1/3) + (-1 + x^5)^(2/3)]/2
```

fricas [A] time = 6.14, size = 144, normalized size = 1.37

$$-\frac{10\sqrt{3}x^5\arctan\left(\frac{1092\sqrt{3}\left(x^5-1\right)^{\frac{1}{3}}x^2+2002\sqrt{3}\left(x^5-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(121x^5+576x^3-121\right)}{3\left(1331x^5-216x^3-1331\right)}\right)+5x^5\log\left(\frac{x^5+x^3+3\left(x^5-1\right)^{\frac{1}{3}}x^2+3\left(x^5-1\right)^{\frac{2}{3}}x-1}{x^5+x^3-1}\right)-3\left(4x^5-5x^3-4\right)\left(x^5-1\right)^{\frac{2}{3}}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(2/3)*(2*x^5+3)*(2*x^5+x^3-2)/x^6/(x^5+x^3-1),x, algorith
m="fricas")
```

```
[Out] -1/10*(10*sqrt(3)*x^5*arctan(1/3*(1092*sqrt(3)*(x^5 - 1)^(1/3)*x^2 + 2002*s
qrt(3)*(x^5 - 1)^(2/3)*x + sqrt(3)*(121*x^5 + 576*x^3 - 121))/(1331*x^5 - 2
16*x^3 - 1331)) + 5*x^5*log((x^5 + x^3 + 3*(x^5 - 1)^(1/3)*x^2 + 3*(x^5 - 1
)^(2/3)*x - 1)/(x^5 + x^3 - 1)) - 3*(4*x^5 - 5*x^3 - 4)*(x^5 - 1)^(2/3))/x^
5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^5+x^3-2\right)\left(2x^5+3\right)\left(x^5-1\right)^{\frac{2}{3}}}{\left(x^5+x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(2/3)*(2*x^5+3)*(2*x^5+x^3-2)/x^6/(x^5+x^3-1),x, algorith
m="giac")
```

```
[Out] integrate((2*x^5 + x^3 - 2)*(2*x^5 + 3)*(x^5 - 1)^(2/3)/((x^5 + x^3 - 1)*x^
6), x)
```

maple [C] time = 3.53, size = 326, normalized size = 3.10

$$\frac{-\frac{3}{2}x^8+\frac{3}{2}x^3-\frac{12}{5}x^5+\frac{6}{5}+\frac{6}{5}x^{10}}{x^5\left(x^5-1\right)^{\frac{1}{3}}}+\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x^5+\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x^3+x^5-3\left(x^5-1\right)^{\frac{2}{3}}x+3\left(x^5-1\right)^{\frac{1}{3}}x^2-x^3-\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)-1}{x^5+x^3-1}\right)-\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x^5+\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x^3+2x^5-2\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x^3-3\left(x^5-1\right)^{\frac{2}{3}}x+3\left(x^5-1\right)^{\frac{1}{3}}x^2+\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)-2}{x^5+x^3-1}\right)\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)+\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x^5+\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x^3+2x^5-2\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)x^3-3\left(x^5-1\right)^{\frac{2}{3}}x+3\left(x^5-1\right)^{\frac{1}{3}}x^2+\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)-2}{x^5+x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)^(2/3)*(2*x^5+3)*(2*x^5+x^3-2)/x^6/(x^5+x^3-1),x)
```

```
[Out] 3/10*(4*x^10-5*x^8-8*x^5+5*x^3+4)/x^5/(x^5-1)^(1/3)+RootOf(_Z^2-_Z+1)*ln((R
ootOf(_Z^2-_Z+1)*x^5+RootOf(_Z^2-_Z+1)^2*x^3+x^5-3*(x^5-1)^(2/3)*x+3*(x^5-1
)^(1/3)*x^2-x^3-RootOf(_Z^2-_Z+1)-1)/(x^5+x^3-1))-ln((-RootOf(_Z^2-_Z+1)*x^
5+RootOf(_Z^2-_Z+1)^2*x^3+2*x^5-2*RootOf(_Z^2-_Z+1)*x^3-3*(x^5-1)^(2/3)*x+3
*(x^5-1)^(1/3)*x^2+RootOf(_Z^2-_Z+1)-2)/(x^5+x^3-1))*RootOf(_Z^2-_Z+1)+ln((
-RootOf(_Z^2-_Z+1)*x^5+RootOf(_Z^2-_Z+1)^2*x^3+2*x^5-2*RootOf(_Z^2-_Z+1)*x^
3-3*(x^5-1)^(2/3)*x+3*(x^5-1)^(1/3)*x^2+RootOf(_Z^2-_Z+1)-2)/(x^5+x^3-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{5}+x^{3}-2\right)\left(2\,x^{5}+3\right)\left(x^{5}-1\right)^{\frac{2}{3}}}{\left(x^{5}+x^{3}-1\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)^(2/3)*(2*x^5+3)*(2*x^5+x^3-2)/x^6/(x^5+x^3-1),x, algorith
m="maxima")
```

```
[Out] integrate((2*x^5 + x^3 - 2)*(2*x^5 + 3)*(x^5 - 1)^(2/3)/((x^5 + x^3 - 1)*x^
6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(x^5-1\right)}^{2/3}\,\left(2\,x^5+3\right)\,\left(2\,x^5+x^3-2\right)}{x^6\,\left(x^5+x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 - 1)^(2/3)*(2*x^5 + 3)*(x^3 + 2*x^5 - 2))/(x^6*(x^3 + x^5 - 1)),x
)
```

```
[Out] int(((x^5 - 1)^(2/3)*(2*x^5 + 3)*(x^3 + 2*x^5 - 2))/(x^6*(x^3 + x^5 - 1)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x-1\right)\left(x^{4}+x^{3}+x^{2}+x+1\right)\right)^{\frac{2}{3}}\left(2x^{5}+3\right)\left(2x^{5}+x^{3}-2\right)}{x^{6}\left(x^{5}+x^{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)**(2/3)*(2*x**5+3)*(2*x**5+x**3-2)/x**6/(x**5+x**3-1),x)
```

```
[Out] Integral(((x - 1)*(x**4 + x**3 + x**2 + x + 1))**(2/3)*(2*x**5 + 3)*(2*x**5
 + x**3 - 2)/(x**6*(x**5 + x**3 - 1)), x)
```

3.1315 $\int \frac{(1+x^3)^{2/3}(-2+x^3+x^6)}{x^9}\,dx$

Optimal. Leaf size=105

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)+\frac{\left(x^3+1\right)^{2/3}\left(-17x^6-2x^3+5\right.}{20x^8}$$

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1488, 271, 264, 277, 239}

$$-\frac{1}{2}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\left(x^3+1\right)^{5/3}}{4x^8}-\frac{7\left(x^3+1\right)^{5/3}}{20x^5}-\frac{\left(x^3+1\right)^{2/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^3)^(2/3)*(-2 + x^3 + x^6))/x^9,x]
```

```
[Out] -1/2*(1 + x^3)^(2/3)/x^2 + (1 + x^3)^(5/3)/(4*x^8) - (7*(1 + x^3)^(5/3))/(2
0*x^5) + ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[-x + (1
+ x^3)^(1/3)]/2
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(1+x^3\right)^{2/3}\left(-2+x^3+x^6\right)}{x^9}\,dx = \int \left(-\frac{2\left(1+x^3\right)^{2/3}}{x^9}+\frac{\left(1+x^3\right)^{2/3}}{x^6}+\frac{\left(1+x^3\right)^{2/3}}{x^3}\right)dx$$

$$= -\left(2\int \frac{\left(1+x^3\right)^{2/3}}{x^9}\,dx\right)+\int \frac{\left(1+x^3\right)^{2/3}}{x^6}\,dx+\int \frac{\left(1+x^3\right)^{2/3}}{x^3}\,dx$$

$$= -\frac{\left(1+x^3\right)^{2/3}}{2x^2}+\frac{\left(1+x^3\right)^{5/3}}{4x^8}-\frac{\left(1+x^3\right)^{5/3}}{5x^5}+\frac{3}{4}\int \frac{\left(1+x^3\right)^{2/3}}{x^6}\,dx+\int \frac{1}{\sqrt[3]{1+x^3}}\, d$$

$$= -\frac{\left(1+x^3\right)^{2/3}}{2x^2}+\frac{\left(1+x^3\right)^{5/3}}{4x^8}-\frac{7\left(1+x^3\right)^{5/3}}{20x^5}+\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(-x+\sqrt[3]{\right.$$

Mathematica [C] time = 0.02, size = 50, normalized size = 0.48

$$\frac{\left(x^3+1\right)^{2/3}\left(-7x^6-2x^3+5\right)-10x^6\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-x^3\right)}{20x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)^(2/3)*(-2 + x^3 + x^6))/x^9,x]
```

```
[Out] ((1 + x^3)^(2/3)*(5 - 2*x^3 - 7*x^6) - 10*x^6*Hypergeometric2F1[-2/3, -2/3,
 1/3, -x^3])/(20*x^8)
```

IntegrateAlgebraic [A] time = 0.18, size = 105, normalized size = 1.00

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)+\frac{\left(x^3+1\right)^{2/3}\left(-17x^6-2x^3+5\right)}{20x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)^(2/3)*(-2 + x^3 + x^6))/x^9,x]
```

```
[Out] ((1 + x^3)^(2/3)*(5 - 2*x^3 - 17*x^6))/(20*x^8) + ArcTan[(Sqrt[3]*x)/(x + 2
*(1 + x^3)^(1/3))]/Sqrt[3] - Log[-x + (1 + x^3)^(1/3)]/3 + Log[x^2 + x*(1 +
 x^3)^(1/3) + (1 + x^3)^(2/3)]/6
```

fricas [A] time = 0.73, size = 117, normalized size = 1.11

$$\frac{20\sqrt{3}x^8\arctan\left(-\frac{25382\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}x^2-13720\sqrt{3}\left(x^3+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(5831x^3+7200\right)}{58653x^3+8000}\right)-10x^8\log\left(3\left(x^3+1\right)^{\frac{1}{3}}x^2-3\left(x^3+1\right)^{\frac{2}{3}}x+1\right)-3\left(17x^6+2x^3-5\right)\left(x^3+1\right)^{\frac{2}{3}}}{60x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^6+x^3-2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/60*(20*sqrt(3)*x^8*arctan(-(25382*sqrt(3)*(x^3 + 1)^(1/3)*x^2 - 13720*sqr
t(3)*(x^3 + 1)^(2/3)*x + sqrt(3)*(5831*x^3 + 7200))/(58653*x^3 + 8000)) - 1
0*x^8*log(3*(x^3 + 1)^(1/3)*x^2 - 3*(x^3 + 1)^(2/3)*x + 1) - 3*(17*x^6 + 2*
x^3 - 5)*(x^3 + 1)^(2/3))/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^3-2\right)\left(x^3+1\right)^{\frac{2}{3}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^6+x^3-2)/x^9,x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^3 - 2)*(x^3 + 1)^(2/3)/x^9, x)
```

maple [C] time = 0.26, size = 44, normalized size = 0.42

$$-\frac{17x^9+19x^6-3x^3-5}{20x^8\left(x^3+1\right)^{\frac{1}{3}}}+x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)*(x^6+x^3-2)/x^9,x)
```

```
[Out] -1/20*(17*x^9+19*x^6-3*x^3-5)/x^8/(x^3+1)^(1/3)+x*hypergeom([1/3,1/3],[4/3]
,-x^3)
```

maxima [A] time = 0.41, size = 105, normalized size = 1.00

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\left(x^3+1\right)^{\frac{2}{3}}}{2x^2}-\frac{3\left(x^3+1\right)^{\frac{5}{3}}}{5x^5}+\frac{\left(x^3+1\right)^{\frac{8}{3}}}{4x^8}+\frac{1}{6}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)-\frac{1}{3}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^6+x^3-2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3)/x + 1)) - 1/2*(x^3 + 1)^
(2/3)/x^2 - 3/5*(x^3 + 1)^(5/3)/x^5 + 1/4*(x^3 + 1)^(8/3)/x^8 + 1/6*log((x^
3 + 1)^(1/3)/x + (x^3 + 1)^(2/3)/x^2 + 1) - 1/3*log((x^3 + 1)^(1/3)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(x^3+1\right)^{2/3}\left(x^6+x^3-2\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)^(2/3)*(x^3 + x^6 - 2))/x^9,x)
```

```
[Out] int(((x^3 + 1)^(2/3)*(x^3 + x^6 - 2))/x^9, x)
```

sympy [C] time = 3.22, size = 175, normalized size = 1.67

$$\frac{\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}-\frac{2\left(x^3+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{3x^2\Gamma\left(-\frac{2}{3}\right)}+\frac{\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\\frac{1}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}+\frac{\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)}+\frac{4\left(x^3+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{10\left(x^3+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^8\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)*(x**6+x**3-2)/x**9,x)
```

```
[Out] (1 + x**(-3))**(2/3)*gamma(-5/3)/(3*gamma(-2/3)) - 2*(x**3 + 1)**(2/3)*gamm
a(-8/3)/(3*x**2*gamma(-2/3)) + gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), x**3
*exp_polar(I*pi))/(3*x**2*gamma(1/3)) + (1 + x**(-3))**(2/3)*gamma(-5/3)/(3
*x**3*gamma(-2/3)) + 4*(x**3 + 1)**(2/3)*gamma(-8/3)/(9*x**5*gamma(-2/3)) +
 10*(x**3 + 1)**(2/3)*gamma(-8/3)/(9*x**8*gamma(-2/3))
```

3.1316 $\int \frac{\left(-1+x^6\right)^{2/3}\left(1+x^6\right)\left(-1-x^3+x^6\right)}{x^6\left(-1+x^3+x^6\right)}\,dx$

Optimal. Leaf size=105

$$-\frac{2}{3}\log\left(\sqrt[3]{x^6-1}+x\right)-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^6-1}-x}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(-\sqrt[3]{x^6-1}\,x+\left(x^6-1\right)^{2/3}+x^2\right)+\frac{\left(x^6-1\right)^{2/3}\left(x^6-5x^3-1\right)}{5x^5}$$

Rubi [C] time = 1.19, antiderivative size = 380, normalized size of antiderivative = 3.62, number of steps used = 25, number of rules used = 12, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6728, 246, 245, 365, 364, 275, 1438, 430, 429, 465, 511, 510}

$$-\frac{2\left(1-\sqrt{5}\right)\left(x^6-1\right)^{2/3}xF_1\left(\frac{1}{6};-\frac{2}{3},1;\frac{7}{6};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{\left(3-\sqrt{5}\right)\left(1-x^6\right)^{2/3}}-\frac{2\left(1+\sqrt{5}\right)\left(x^6-1\right)^{2/3}xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};\frac{2x^6}{3+\sqrt{5}},x^6\right)}{\left(3+\sqrt{5}\right)\left(1-x^6\right)^{2/3}}+\frac{\left(x^6-1\right)^{2/3}x^4F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{\left(3-\sqrt{5}\right)\left(1-x^6\right)^{2/3}}+\frac{\left(x^6-1\right)^{2/3}x^4F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^6,\frac{2x^6}{3+\sqrt{5}}\right)}{\left(3+\sqrt{5}\right)\left(1-x^6\right)^{2/3}}+\frac{\left(x^6-1\right)^{2/3}x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};x^6\right)}{\left(1-x^6\right)^{2/3}}-\frac{\left(x^6-1\right)^{2/3}\,{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5\left(1-x^6\right)^{2/3}x^5}-\frac{\left(x^6-1\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};x^6\right)}{\left(1-x^6\right)^{2/3}x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^6)^(2/3)*(1 + x^6)*(-1 - x^3 + x^6))/(x^6*(-1 + x^3 + x^6)),x]
```

```
[Out] (-2*(1 - Sqrt[5])*x*(-1 + x^6)^(2/3)*AppellF1[1/6, -2/3, 1, 7/6, x^6, (2*x^
6)/(3 - Sqrt[5])])/((3 - Sqrt[5])*(1 - x^6)^(2/3)) - (2*(1 + Sqrt[5])*x*(-1
 + x^6)^(2/3)*AppellF1[1/6, 1, -2/3, 7/6, (2*x^6)/(3 + Sqrt[5]), x^6])/((3
+ Sqrt[5])*(1 - x^6)^(2/3)) + (x^4*(-1 + x^6)^(2/3)*AppellF1[2/3, -2/3, 1,
5/3, x^6, (2*x^6)/(3 - Sqrt[5])])/((3 - Sqrt[5])*(1 - x^6)^(2/3)) + (x^4*(-
1 + x^6)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, x^6, (2*x^6)/(3 + Sqrt[5])])/((3
 + Sqrt[5])*(1 - x^6)^(2/3)) - ((-1 + x^6)^(2/3)*Hypergeometric2F1[-5/6, -2
/3, 1/6, x^6])/(5*x^5*(1 - x^6)^(2/3)) - ((-1 + x^6)^(2/3)*Hypergeometric2F
1[-2/3, -1/3, 2/3, x^6])/(x^2*(1 - x^6)^(2/3)) + (x*(-1 + x^6)^(2/3)*Hyperg
eometric2F1[-2/3, 1/6, 7/6, x^6])/(1 - x^6)^(2/3)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(-1+x^6\right)^{2/3}\left(1+x^6\right)\left(-1-x^3+x^6\right)}{x^6\left(-1+x^3+x^6\right)}\,dx = \int \left(\left(-1+x^6\right)^{2/3} + \frac{\left(-1+x^6\right)^{2/3}}{x^6} + \frac{2\left(-1+x^6\right)^{2/3}}{x^3} - \frac{2\left(1+2x^3\right)\left(-1+\right.}{-1+x^3+x}\right.$$

$$= 2\int \frac{\left(-1+x^6\right)^{2/3}}{x^3}\,dx - 2\int \frac{\left(1+2x^3\right)\left(-1+x^6\right)^{2/3}}{-1+x^3+x^6}\,dx + \int \left(-1+x^6\right)$$

$$= -\left(2\int \left(\frac{2\left(-1+x^6\right)^{2/3}}{1-\sqrt{5}+2x^3} + \frac{2\left(-1+x^6\right)^{2/3}}{1+\sqrt{5}+2x^3}\right)dx\right) + \frac{\left(-1+x^6\right)^{2/3}\int\left(1-\right.}{\left(1-x^6\right)^{2/3}}$$

$$= -\frac{\left(-1+x^6\right)^{2/3}\,{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5\left(1-x^6\right)^{2/3}} + \frac{x\left(-1+x^6\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};x^6\right)}{\left(1-x^6\right)^{2/3}}$$

$$= -\frac{\left(-1+x^6\right)^{2/3}\,{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5\left(1-x^6\right)^{2/3}} - \frac{\left(-1+x^6\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};x^6\right)}{x^2\left(1-x^6\right)^{2/3}}$$

$$= -\frac{\left(-1+x^6\right)^{2/3}\,{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5\left(1-x^6\right)^{2/3}} - \frac{\left(-1+x^6\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};x^6\right)}{x^2\left(1-x^6\right)^{2/3}}$$

$$= -\frac{\left(-1+x^6\right)^{2/3}\,{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5\left(1-x^6\right)^{2/3}} - \frac{\left(-1+x^6\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};x^6\right)}{x^2\left(1-x^6\right)^{2/3}}$$

$$= -\frac{2\left(1-\sqrt{5}\right)x\left(-1+x^6\right)^{2/3}F_1\left(\frac{1}{6};-\frac{2}{3},1;\frac{7}{6};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{\left(3-\sqrt{5}\right)\left(1-x^6\right)^{2/3}} - \frac{2\left(1+\sqrt{5}\right)x}{}$$

$$= -\frac{2\left(1-\sqrt{5}\right)x\left(-1+x^6\right)^{2/3}F_1\left(\frac{1}{6};-\frac{2}{3},1;\frac{7}{6};x^6,\frac{2x^6}{3-\sqrt{5}}\right)}{\left(3-\sqrt{5}\right)\left(1-x^6\right)^{2/3}} - \frac{2\left(1+\sqrt{5}\right)x}{}$$

Mathematica [F] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^6\right)^{2/3}\left(1+x^6\right)\left(-1-x^3+x^6\right)}{x^6\left(-1+x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^6)^(2/3)*(1 + x^6)*(-1 - x^3 + x^6))/(x^6*(-1 + x^3 + x^
6)),x]
```

```
[Out] Integrate[((-1 + x^6)^(2/3)*(1 + x^6)*(-1 - x^3 + x^6))/(x^6*(-1 + x^3 + x^
6)), x]
```

IntegrateAlgebraic [A] time = 2.30, size = 105, normalized size = 1.00

$$-\frac{2}{3}\log\left(\sqrt[3]{x^6-1}+x\right) - \frac{2\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^6-1}-x}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(-\sqrt[3]{x^6-1}\,x+\left(x^6-1\right)^{2/3}+x^2\right) + \frac{\left(x^6-1\right)^{2/3}\left(x^6-5x^3-1\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^6)^(2/3)*(1 + x^6)*(-1 - x^3 + x^6))/(x^6*(-1 +
 x^3 + x^6)),x]
```

[Out] ((-1 + x^6)^(2/3)*(-1 - 5*x^3 + x^6))/(5*x^5) - (2*ArcTan[(Sqrt[3]*x)/(-x + 2*(-1 + x^6)^(1/3))])/Sqrt[3] - (2*Log[x + (-1 + x^6)^(1/3)])/3 + Log[x^2 - x*(-1 + x^6)^(1/3) + (-1 + x^6)^(2/3)]/3

fricas [A] time = 17.23, size = 142, normalized size = 1.35

$$-\frac{10\sqrt{3}x^5\arctan\left(\frac{17707979315346691547103487078601066282657059082726673278841963389300888497059669011634\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}x^2+18779074824464902023518972945875034013564452605964125877184864112405550428883609929964\sqrt{3}\left(x^6-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(8791266734992875261237504664599259772605087326251698970792557525513888268399719816592x^6+9326814489551980499445247598236243638058784087870425269964007887066219234311690275757x^3-8791266734992875261237504664599259772605087326251698970792557525513888268399719816592\right)}{3\left(9923243904393545413458713816471868889492119646716071835561526356358143878603884871272x^6-8320283165512251371852516195766181258618636197629327742451887394495075584367754599527x^3-9923243904393545413458713816471868889492119646716071835561526356358143878603884871272\right)}\right)+5x^5\log\left(\frac{x^6+x^3+3\left(x^6-1\right)^{\frac{1}{3}}x^2+3\left(x^6-1\right)^{\frac{2}{3}}x-1}{x^6+x^3-1}\right)-3\left(x^6-5x^3-1\right)\left(x^6-1\right)^{\frac{2}{3}}}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6-1)^(2/3)*(x^6+1)*(x^6-x^3-1)/x^6/(x^6+x^3-1),x, algorithm="fricas")

[Out] -1/15*(10*sqrt(3)*x^5*arctan(1/3*(17707979315346691547103487078601066282657059082726673278841963389300888497059669011634*sqrt(3)*(x^6 - 1)^(1/3)*x^2 + 18779074824464902023518972945875034013564452605964125877184864112405550428883609929964*sqrt(3)*(x^6 - 1)^(2/3)*x + sqrt(3)*(8791266734992875261237504664599259772605087326251698970792557525513888268399719816592*x^6 + 9326814489551980499445247598236243638058784087870425269964007887066219234311690275757*x^3 - 8791266734992875261237504664599259772605087326251698970792557525513888268399719816592))/(9923243904393545413458713816471868889492119646716071835561526356358143878603884871272*x^6 - 8320283165512251371852516195766181258618636197629327742451887394495075584367754599527*x^3 - 9923243904393545413458713816471868889492119646716071835561526356358143878603884871272)) + 5*x^5*log((x^6 + x^3 + 3*(x^6 - 1)^(1/3)*x^2 + 3*(x^6 - 1)^(2/3)*x - 1)/(x^6 + x^3 - 1)) - 3*(x^6 - 5*x^3 - 1)*(x^6 - 1)^(2/3))/x^5

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6-x^3-1\right)\left(x^6+1\right)\left(x^6-1\right)^{\frac{2}{3}}}{\left(x^6+x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6-1)^(2/3)*(x^6+1)*(x^6-x^3-1)/x^6/(x^6+x^3-1),x, algorithm="giac")

[Out] integrate((x^6 - x^3 - 1)*(x^6 + 1)*(x^6 - 1)^(2/3)/((x^6 + x^3 - 1)*x^6), x)

maple [C] time = 2.01, size = 483, normalized size = 4.60

$$\frac{x^{12}-5x^9-2x^6+5x^3+1}{5x^5\left(x^6-1\right)^{\frac{1}{3}}}+2\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\ln\left(\frac{6\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)x^6-x^6+18\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)^2x^3-9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(x^6-1\right)^{\frac{2}{3}}x+9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(x^6-1\right)^{\frac{1}{3}}x^2-9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)x^3+x^3-6\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)+1}{x^6+x^3-1}\right)+\frac{2\ln\left(\frac{-6\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)x^6+x^6+18\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)^2x^3+9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(x^6-1\right)^{\frac{2}{3}}x-9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(x^6-1\right)^{\frac{1}{3}}x^2-3\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)x^3-3x\left(x^6-1\right)^{\frac{2}{3}}+3x^2\left(x^6-1\right)^{\frac{1}{3}}+6\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)-1}{x^6+x^3-1}\right)}{3}-2\ln\left(\frac{-6\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)x^6+x^6+18\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)^2x^3+9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(x^6-1\right)^{\frac{2}{3}}x-9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(x^6-1\right)^{\frac{1}{3}}x^2-3\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)x^3-3x\left(x^6-1\right)^{\frac{2}{3}}+3x^2\left(x^6-1\right)^{\frac{1}{3}}+6\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)-1}{x^6+x^3-1}\right)\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6-1)^(2/3)*(x^6+1)*(x^6-x^3-1)/x^6/(x^6+x^3-1),x)

[Out] 1/5*(x^12-5*x^9-2*x^6+5*x^3+1)/x^5/(x^6-1)^(1/3)+2*RootOf(9*_Z^2-3*_Z+1)*ln((6*RootOf(9*_Z^2-3*_Z+1)*x^6-x^6+18*RootOf(9*_Z^2-3*_Z+1)^2*x^3-9*RootOf(9*_Z^2-3*_Z+1)*(x^6-1)^(2/3)*x+9*RootOf(9*_Z^2-3*_Z+1)*(x^6-1)^(1/3)*x^2-9*RootOf(9*_Z^2-3*_Z+1)*x^3+x^3-6*RootOf(9*_Z^2-3*_Z+1)+1)/(x^6+x^3-1))+2/3*ln((-6*RootOf(9*_Z^2-3*_Z+1)*x^6+x^6+18*RootOf(9*_Z^2-3*_Z+1)^2*x^3+9*RootOf(9*_Z^2-3*_Z+1)*(x^6-1)^(2/3)*x-9*RootOf(9*_Z^2-3*_Z+1)*(x^6-1)^(1/3)*x^2-3*RootOf(9*_Z^2-3*_Z+1)*x^3-3*x*(x^6-1)^(2/3)+3*x^2*(x^6-1)^(1/3)+6*RootOf(9*_Z^2-3*_Z+1)-1)/(x^6+x^3-1))-2*ln((-6*RootOf(9*_Z^2-3*_Z+1)*x^6+x^6+18*RootOf(9*_Z^2-3*_Z+1)^2*x^3+9*RootOf(9*_Z^2-3*_Z+1)*(x^6-1)^(2/3)*x-9*RootOf(9*_Z^2-3*_Z+1)*(x^6-1)^(1/3)*x^2-3*RootOf(9*_Z^2-3*_Z+1)*x^3-3*x*(x^6-1)^(2/3)+3*x^2*(x^6-1)^(1/3)+6*RootOf(9*_Z^2-3*_Z+1)-1)/(x^6+x^3-1))*RootOf(9*_Z^2-3*_Z+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6 - x^3 - 1\right)\left(x^6 + 1\right)\left(x^6 - 1\right)^{\frac{2}{3}}}{\left(x^6 + x^3 - 1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(2/3)*(x^6+1)*(x^6-x^3-1)/x^6/(x^6+x^3-1),x, algorithm="m
axima")
```

```
[Out] integrate((x^6 - x^3 - 1)*(x^6 + 1)*(x^6 - 1)^(2/3)/((x^6 + x^3 - 1)*x^6),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^6 - 1\right)^{2/3}\,\left(x^6 + 1\right)\,\left(-x^6 + x^3 + 1\right)}{x^6\,\left(x^6 + x^3 - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^6 - 1)^(2/3)*(x^6 + 1)*(x^3 - x^6 + 1))/(x^6*(x^3 + x^6 - 1)),x)
```

```
[Out] int(-((x^6 - 1)^(2/3)*(x^6 + 1)*(x^3 - x^6 + 1))/(x^6*(x^3 + x^6 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(2/3)*(x**6+1)*(x**6-x**3-1)/x**6/(x**6+x**3-1),x)
```

```
[Out] Timed out
```

3.1317 $\int \frac{(-1+x^3)^{2/3}(-2+2x^3+x^6)}{x^9}\,dx$

Optimal. Leaf size=105

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)+\frac{\left(x^3-1\right)^{2/3}\left(-x^6-2x^3+1\right)}{4x^8}$$

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1488, 271, 264, 277, 239}

$$-\frac{1}{2}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\left(x^3-1\right)^{5/3}}{4x^8}+\frac{\left(x^3-1\right)^{5/3}}{4x^5}-\frac{\left(x^3-1\right)^{2/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(2/3)*(-2 + 2*x^3 + x^6))/x^9,x]
```

```
[Out] -1/2*(-1 + x^3)^(2/3)/x^2 - (-1 + x^3)^(5/3)/(4*x^8) + (-1 + x^3)^(5/3)/(4*
x^5) + ArcTan[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[-x + (-1
+ x^3)^(1/3)]/2
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1488

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(-2+2x^3+x^6\right)}{x^9}\,dx = \int \left(-\frac{2\left(-1+x^3\right)^{2/3}}{x^9}+\frac{2\left(-1+x^3\right)^{2/3}}{x^6}+\frac{\left(-1+x^3\right)^{2/3}}{x^3}\right)dx$$
$$= -\left(2\int \frac{\left(-1+x^3\right)^{2/3}}{x^9}\,dx\right)+2\int \frac{\left(-1+x^3\right)^{2/3}}{x^6}\,dx+\int \frac{\left(-1+x^3\right)^{2/3}}{x^3}\,dx$$
$$= -\frac{\left(-1+x^3\right)^{2/3}}{2x^2}-\frac{\left(-1+x^3\right)^{5/3}}{4x^8}+\frac{2\left(-1+x^3\right)^{5/3}}{5x^5}-\frac{3}{4}\int \frac{\left(-1+x^3\right)^{2/3}}{x^6}\,dx+\int$$
$$= -\frac{\left(-1+x^3\right)^{2/3}}{2x^2}-\frac{\left(-1+x^3\right)^{5/3}}{4x^8}+\frac{\left(-1+x^3\right)^{5/3}}{4x^5}+\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\Big($$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.54

$$\frac{\left(x^3-1\right)^{2/3}\left(\left(1-x^3\right)^{8/3}-2x^6\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};x^3\right)\right)}{4x^8\left(1-x^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)^(2/3)*(-2 + 2*x^3 + x^6))/x^9,x]
```

```
[Out] ((-1 + x^3)^(2/3)*((1 - x^3)^(8/3) - 2*x^6*Hypergeometric2F1[-2/3, -2/3, 1/
3, x^3]))/(4*x^8*(1 - x^3)^(2/3))
```

IntegrateAlgebraic [A] time = 0.16, size = 105, normalized size = 1.00

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)+\frac{\left(x^3-1\right)^{2/3}\left(-x^6-2x^3+1\right)}{4x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(2/3)*(-2 + 2*x^3 + x^6))/x^9,x]
```

```
[Out] ((-1 + x^3)^(2/3)*(1 - 2*x^3 - x^6))/(4*x^8) + ArcTan[(Sqrt[3]*x)/(x + 2*(-
1 + x^3)^(1/3))]/Sqrt[3] - Log[-x + (-1 + x^3)^(1/3)]/3 + Log[x^2 + x*(-1 +
 x^3)^(1/3) + (-1 + x^3)^(2/3)]/6
```

fricas [A] time = 0.72, size = 115, normalized size = 1.10

$$\frac{4\sqrt{3}x^8\arctan\left(-\frac{25382\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}x^2-13720\sqrt{3}\left(x^3-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(5831x^3-7200\right)}{58653x^3-8000}\right)-2x^8\log\left(-3\left(x^3-1\right)^{\frac{1}{3}}x^2+3\left(x^3-1\right)^{\frac{2}{3}}x+1\right)-3\left(x^6+2x^3-1\right)\left(x^3-1\right)^{\frac{2}{3}}}{12x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+2*x^3-2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(3)*x^8*arctan(-(25382*sqrt(3)*(x^3 - 1)^(1/3)*x^2 - 13720*sqrt
(3)*(x^3 - 1)^(2/3)*x + sqrt(3)*(5831*x^3 - 7200))/(58653*x^3 - 8000)) - 2*
x^8*log(-3*(x^3 - 1)^(1/3)*x^2 + 3*(x^3 - 1)^(2/3)*x + 1) - 3*(x^6 + 2*x^3
- 1)*(x^3 - 1)^(2/3))/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+2\,x^3-2\right)\left(x^3-1\right)^{\frac{2}{3}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+2*x^3-2)/x^9,x, algorithm="giac")
```

```
[Out] integrate((x^6 + 2*x^3 - 2)*(x^3 - 1)^(2/3)/x^9, x)
```

maple [C] time = 0.28, size = 56, normalized size = 0.53

$$-\frac{x^9+x^6-3x^3+1}{4x^8\left(x^3-1\right)^{\frac{1}{3}}}+\frac{\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{1}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^3\right)}{\mathrm{signum}\left(x^3-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)*(x^6+2*x^3-2)/x^9,x)
```

```
[Out] -1/4*(x^9+x^6-3*x^3+1)/x^8/(x^3-1)^(1/3)+1/signum(x^3-1)^(1/3)*(-signum(x^3
-1))^(1/3)*x*hypergeom([1/3,1/3],[4/3],x^3)
```

maxima [A] time = 0.42, size = 93, normalized size = 0.89

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3-1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\left(x^3-1\right)^{\frac{2}{3}}}{2\,x^2}+\frac{\left(x^3-1\right)^{\frac{8}{3}}}{4\,x^8}+\frac{1}{6}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3-1\right)^{\frac{2}{3}}}{x^2}+1\right)-\frac{1}{3}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+2*x^3-2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3)/x + 1)) - 1/2*(x^3 - 1)^
(2/3)/x^2 + 1/4*(x^3 - 1)^(8/3)/x^8 + 1/6*log((x^3 - 1)^(1/3)/x + (x^3 - 1)
^(2/3)/x^2 + 1) - 1/3*log((x^3 - 1)^(1/3)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(x^3-1\right)}^{2/3}\,\left(x^6+2\,x^3-2\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(2*x^3 + x^6 - 2))/x^9,x)
```

```
[Out] int(((x^3 - 1)^(2/3)*(2*x^3 + x^6 - 2))/x^9, x)
```

sympy [C] time = 3.58, size = 461, normalized size = 4.39

$$2\left(\begin{cases}\frac{\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}}e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{5}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}-\frac{\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}}e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)} & \text{for}\ \frac{1}{|x^3|}>1\\ -\frac{\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}+\frac{\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)} & \text{otherwise}\end{cases}\right)-2\left(\begin{cases}\frac{\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{8}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}+\frac{2\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^3\Gamma\left(-\frac{2}{3}\right)}-\frac{5\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^6\Gamma\left(-\frac{2}{3}\right)} & \text{for}\ \frac{1}{|x^3|}>1\\ \frac{3x^6\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^6\Gamma\left(-\frac{2}{3}\right)-9x^3\Gamma\left(-\frac{2}{3}\right)}-\frac{x^3\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^6\Gamma\left(-\frac{2}{3}\right)-9x^3\Gamma\left(-\frac{2}{3}\right)}+\frac{5\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^9\Gamma\left(-\frac{2}{3}\right)-9x^6\Gamma\left(-\frac{2}{3}\right)}-\frac{7\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^6\Gamma\left(-\frac{2}{3}\right)-9x^3\Gamma\left(-\frac{2}{3}\right)} & \text{otherwise}\end{cases}\right)+\frac{e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\right|x^3\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)*(x**6+2*x**3-2)/x**9,x)
```

```
[Out] 2*Piecewise(((-1 + x**(-3))**(2/3)*exp(-I*pi/3)*gamma(-5/3)/(3*gamma(-2/3))
 - (-1 + x**(-3))**(2/3)*exp(-I*pi/3)*gamma(-5/3)/(3*x**3*gamma(-2/3)), 1/A
```

```
bs(x**3) > 1), (-(1 - 1/x**3)**(2/3)*gamma(-5/3)/(3*gamma(-2/3)) + (1 - 1/x
**3)**(2/3)*gamma(-5/3)/(3*x**3*gamma(-2/3)), True)) - 2*Piecewise(((-1 + x
**(-3))**(2/3)*exp(2*I*pi/3)*gamma(-8/3)/(3*gamma(-2/3)) + 2*(-1 + x**(-3))
**(2/3)*exp(2*I*pi/3)*gamma(-8/3)/(9*x**3*gamma(-2/3)) - 5*(-1 + x**(-3))**
(2/3)*exp(2*I*pi/3)*gamma(-8/3)/(9*x**6*gamma(-2/3)), 1/Abs(x**3) > 1), (3*
x**6*(1 - 1/x**3)**(2/3)*gamma(-8/3)/(9*x**6*gamma(-2/3) - 9*x**3*gamma(-2/
3)) - x**3*(1 - 1/x**3)**(2/3)*gamma(-8/3)/(9*x**6*gamma(-2/3) - 9*x**3*gam
ma(-2/3)) + 5*(1 - 1/x**3)**(2/3)*gamma(-8/3)/(9*x**9*gamma(-2/3) - 9*x**6*
gamma(-2/3)) - 7*(1 - 1/x**3)**(2/3)*gamma(-8/3)/(9*x**6*gamma(-2/3) - 9*x*
*3*gamma(-2/3)), True)) + exp(2*I*pi/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/
3,), x**3)/(3*x**2*gamma(1/3))
```

3.1318 $\int \frac{-2+x^4}{\sqrt[4]{1+x^4}\left(-2+x^4+x^8\right)}\,dx$

Optimal. Leaf size=105

$$\frac{1}{3}\sqrt[4]{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{6\sqrt[4]{2}}+\frac{1}{3}\sqrt[4]{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{6\sqrt[4]{2}}$$

Rubi [A] time = 0.20, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6728, 377, 212, 206, 203}

$$\frac{1}{3}\sqrt[4]{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{6\sqrt[4]{2}}+\frac{1}{3}\sqrt[4]{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{6\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^4)/((1 + x^4)^(1/4)*(-2 + x^4 + x^8)),x]
```

```
[Out] (2^(1/4)*ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))])/3 + ArcTan[(2^(1/4)*x)/(1 + x
^4)^(1/4)]/(6*2^(1/4)) + (2^(1/4)*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))])/3 +
 ArcTanh[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(6*2^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-2+x^4}{\sqrt[4]{1+x^4}\left(-2+x^4+x^8\right)}\,dx = \int \left(-\frac{2}{3\sqrt[4]{1+x^4}\left(-2+2x^4\right)}+\frac{8}{3\sqrt[4]{1+x^4}\left(4+2x^4\right)}\right)dx$$
$$= -\left(\frac{2}{3}\int \frac{1}{\sqrt[4]{1+x^4}\left(-2+2x^4\right)}\,dx\right)+\frac{8}{3}\int \frac{1}{\sqrt[4]{1+x^4}\left(4+2x^4\right)}\,dx$$
$$= -\left(\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-2+4x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)\right)+\frac{8}{3}\operatorname{Subst}\left(\int \frac{1}{4-2x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right.$$
$$= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right.$$
$$= \frac{1}{3}\sqrt[4]{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{6\sqrt[4]{2}}+\frac{1}{3}\sqrt[4]{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)+\cdots$$

Mathematica [A] time = 0.06, size = 94, normalized size = 0.90

$$\frac{4\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)+\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)+4\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{6\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^4)/((1 + x^4)^(1/4)*(-2 + x^4 + x^8)),x]
```

```
[Out] (4*ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))] + Sqrt[2]*ArcTan[(2^(1/4)*x)/(1 + x^
4)^(1/4)] + 4*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))] + Sqrt[2]*ArcTanh[(2^(1/
4)*x)/(1 + x^4)^(1/4)])/(6*2^(3/4))
```

IntegrateAlgebraic [A] time = 0.41, size = 105, normalized size = 1.00

$$\frac{1}{3}\sqrt[4]{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{6\sqrt[4]{2}}+\frac{1}{3}\sqrt[4]{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{6\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x^4)/((1 + x^4)^(1/4)*(-2 + x^4 + x^8)),x]
```

```
[Out] (2^(1/4)*ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))])/3 + ArcTan[(2^(1/4)*x)/(1 + x
^4)^(1/4)]/(6*2^(1/4)) + (2^(1/4)*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))])/3 +
 ArcTanh[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(6*2^(1/4))
```

fricas [B] time = 9.75, size = 416, normalized size = 3.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2)/(x^4+1)^(1/4)/(x^8+x^4-2),x, algorithm="fricas")
```

```
[Out] -1/12*8^(3/4)*arctan(-1/2*(8^(3/4)*(x^4 + 1)^(1/4)*x^3 + 4*8^(1/4)*(x^4 + 1
)^(3/4)*x - 2^(1/4)*(8^(3/4)*sqrt(x^4 + 1)*x^2 + 8^(1/4)*(3*x^4 + 2)))/(x^4
 + 2)) - 1/12*2^(3/4)*arctan(1/2*(4*2^(3/4)*(x^4 + 1)^(1/4)*x^3 + 4*2^(1/4)
*(x^4 + 1)^(3/4)*x + 2^(3/4)*(2*2^(3/4)*sqrt(x^4 + 1)*x^2 + 2^(1/4)*(3*x^4
+ 1)))/(x^4 - 1)) + 1/48*8^(3/4)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 8*8^(
1/4)*sqrt(x^4 + 1)*x^2 + 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 +
 2)) - 1/48*8^(3/4)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 - 8*8^(1/4)*sqrt(x^4
```

```
 + 1)*x^2 - 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2)) + 1/48*2
^(3/4)*log((4*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 4*2^(1/4)*sqrt(x^4 + 1)*x^2 + 2
^(3/4)*(3*x^4 + 1) + 4*(x^4 + 1)^(3/4)*x)/(x^4 - 1)) - 1/48*2^(3/4)*log((4*
sqrt(2)*(x^4 + 1)^(1/4)*x^3 - 4*2^(1/4)*sqrt(x^4 + 1)*x^2 - 2^(3/4)*(3*x^4
+ 1) + 4*(x^4 + 1)^(3/4)*x)/(x^4 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-2}{\left(x^8+x^4-2\right)\left(x^4+1\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2)/(x^4+1)^(1/4)/(x^8+x^4-2),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 2)/((x^8 + x^4 - 2)*(x^4 + 1)^(1/4)), x)
```

maple [C] time = 2.94, size = 456, normalized size = 4.34

$$-\frac{\mathrm{RootOf}\left(\_Z^4-8\right)\ln\left(\frac{\sqrt{x^4+1}\,\mathrm{RootOf}\left(\_Z^4-8\right)^3x^2-2\left(x^4+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-8\right)^2x^3+3\,\mathrm{RootOf}\left(\_Z^4-8\right)x^4-4\left(x^4+1\right)^{\frac{3}{4}}x+\mathrm{RootOf}\left(\_Z^4-8\right)}{(-1+x)(1+x)\left(x^2+1\right)}\right)}{24}-\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\ln\left(-\frac{\sqrt{x^4+1}\,\mathrm{RootOf}\left(\_Z^4-8\right)^2\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)x^2-2\left(x^4+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-8\right)^2x^3-3\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)x^4+4\left(x^4+1\right)^{\frac{3}{4}}x-\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)}{(-1+x)(1+x)\left(x^2+1\right)}\right)}{24}-\frac{\mathrm{RootOf}\left(\_Z^4-8\right)^2\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\ln\left(-\frac{3\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\mathrm{RootOf}\left(\_Z^4-8\right)^2x^4+4\left(x^4+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-8\right)^2x^3-8\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\sqrt{x^4+1}\,x^2-16\left(x^4+1\right)^{\frac{3}{4}}x+2\,\mathrm{RootOf}\left(\_Z^4-8\right)^2\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)}{x^4+2}\right)}{24}-\frac{\mathrm{RootOf}\left(\_Z^4-8\right)^3\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^4-8\right)^3x^4-4\left(x^4+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-8\right)^2x^3+8\sqrt{x^4+1}\,\mathrm{RootOf}\left(\_Z^4-8\right)x^2-16\left(x^4+1\right)^{\frac{3}{4}}x+2\,\mathrm{RootOf}\left(\_Z^4-8\right)^3}{x^4+2}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2)/(x^4+1)^(1/4)/(x^8+x^4-2),x)
```

```
[Out] -1/24*RootOf(_Z^4-8)*ln(((x^4+1)^(1/2)*RootOf(_Z^4-8)^3*x^2-2*(x^4+1)^(1/4)
*RootOf(_Z^4-8)^2*x^3+3*RootOf(_Z^4-8)*x^4-4*(x^4+1)^(3/4)*x+RootOf(_Z^4-8)
)/(-1+x)/(1+x)/(x^2+1))-1/24*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln(-((x^4+1)^(1/
2)*RootOf(_Z^4-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^2-2*(x^4+1)^(1/4)*RootO
f(_Z^4-8)^2*x^3-3*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^4+4*(x^4+1)^(3/4)*x-RootO
f(_Z^2+RootOf(_Z^4-8)^2))/(-1+x)/(1+x)/(x^2+1))-1/24*RootOf(_Z^4-8)^2*RootO
f(_Z^2+RootOf(_Z^4-8)^2)*ln(-(3*RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8
)^2*x^4+4*(x^4+1)^(1/4)*RootOf(_Z^4-8)^2*x^3-8*RootOf(_Z^2+RootOf(_Z^4-8)^2
)*(x^4+1)^(1/2)*x^2-16*(x^4+1)^(3/4)*x+2*RootOf(_Z^4-8)^2*RootOf(_Z^2+RootO
f(_Z^4-8)^2))/(x^4+2))-1/24*RootOf(_Z^4-8)^3*ln((3*RootOf(_Z^4-8)^3*x^4-4*(
x^4+1)^(1/4)*RootOf(_Z^4-8)^2*x^3+8*(x^4+1)^(1/2)*RootOf(_Z^4-8)*x^2-16*(x^
4+1)^(3/4)*x+2*RootOf(_Z^4-8)^3)/(x^4+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-2}{\left(x^8+x^4-2\right)\left(x^4+1\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2)/(x^4+1)^(1/4)/(x^8+x^4-2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 2)/((x^8 + x^4 - 2)*(x^4 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4-2}{{\left(x^4+1\right)}^{1/4}\,\left(x^8+x^4-2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 2)/((x^4 + 1)^(1/4)*(x^4 + x^8 - 2)),x)
```

```
[Out] int((x^4 - 2)/((x^4 + 1)^(1/4)*(x^4 + x^8 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-2}{(x-1)(x+1)\left(x^2+1\right)\sqrt[4]{x^4+1}\left(x^4+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2)/(x**4+1)**(1/4)/(x**8+x**4-2),x)
```

```
[Out] Integral((x**4 - 2)/((x - 1)*(x + 1)*(x**2 + 1)*(x**4 + 1)**(1/4)*(x**4 + 2
)), x)
```

3.1319 $\int \frac{-2b-2ax^4+x^8}{x^4\sqrt[4]{b+ax^4}}\,dx$

Optimal. Leaf size=105

$$\frac{(-8a^2-b)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{5/4}}+\frac{(-8a^2-b)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{5/4}}+\frac{(8a+3x^4)(ax^4+b)^{3/4}}{12ax^3}$$

Rubi [A] time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1486, 451, 240, 212, 206, 203}

$$-\frac{(8a^2+b)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{5/4}}-\frac{(8a^2+b)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{8a^{5/4}}+\frac{x(ax^4+b)^{3/4}}{4a}+\frac{2(ax^4+b)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*b - 2*a*x^4 + x^8)/(x^4*(b + a*x^4)^(1/4)),x]
```

```
[Out] (2*(b + a*x^4)^(3/4))/(3*x^3) + (x*(b + a*x^4)^(3/4))/(4*a) - ((8*a^2 + b)*
ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)])/(8*a^(5/4)) - ((8*a^2 + b)*ArcTanh[(
a^(1/4)*x)/(b + a*x^4)^(1/4)])/(8*a^(5/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1486

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 2*n*p - n
+ 1)*(d + e*x^n)^(q + 1))/(e*f^(2*n*p - n + 1)*(m + 2*n*p + n*q + 1)), x] +
 Dist[1/(e*(m + 2*n*p + n*q + 1)), Int[(f*x)^m*(d + e*x^n)^q*ExpandToSum[e*
(m + 2*n*p + n*q + 1)*((a + b*x^n + c*x^(2*n))^p - c^p*x^(2*n*p)) - d*c^p*(
m + 2*n*p - n + 1)*x^(2*n*p - n), x], x], x] /; FreeQ[{a, b, c, d, e, f, m,
 q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGtQ[p, 0]
&& GtQ[2*n*p, n - 1] &&  !IntegerQ[q] && NeQ[m + 2*n*p + n*q + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2b-2ax^4+x^8}{x^4\sqrt[4]{b+ax^4}}\,dx &= \frac{x\left(b+ax^4\right)^{3/4}}{4a}+\frac{\int \frac{-8ab-\left(8a^2+b\right)x^4}{x^4\sqrt[4]{b+ax^4}}\,dx}{4a}\\
&= \frac{2\left(b+ax^4\right)^{3/4}}{3x^3}+\frac{x\left(b+ax^4\right)^{3/4}}{4a}+\frac{\left(-8a^2-b\right)\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx}{4a}\\
&= \frac{2\left(b+ax^4\right)^{3/4}}{3x^3}+\frac{x\left(b+ax^4\right)^{3/4}}{4a}+\frac{\left(-8a^2-b\right)\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)}{4a}\\
&= \frac{2\left(b+ax^4\right)^{3/4}}{3x^3}+\frac{x\left(b+ax^4\right)^{3/4}}{4a}+\frac{\left(-8a^2-b\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)}{8a}+\frac{\left(-8a^2-\right.}{}\\
&= \frac{2\left(b+ax^4\right)^{3/4}}{3x^3}+\frac{x\left(b+ax^4\right)^{3/4}}{4a}-\frac{\left(8a^2+b\right)\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b+ax^4}}\right)}{8a^{5/4}}-\frac{\left(8a^2+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b+ax^4}}\right)}{8a^{5/4}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 102, normalized size = 0.97

$$\frac{-3x^3\left(8a^2+b\right)\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)-3x^3\left(8a^2+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)+2\sqrt[4]{a}\left(8a+3x^4\right)\left(ax^4+b\right)^{3/4}}{24a^{5/4}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*b - 2*a*x^4 + x^8)/(x^4*(b + a*x^4)^(1/4)),x]
```

```
[Out] (2*a^(1/4)*(8*a + 3*x^4)*(b + a*x^4)^(3/4) - 3*(8*a^2 + b)*x^3*ArcTan[(a^(1
/4)*x)/(b + a*x^4)^(1/4)] - 3*(8*a^2 + b)*x^3*ArcTanh[(a^(1/4)*x)/(b + a*x^
4)^(1/4)])/(24*a^(5/4)*x^3)
```

IntegrateAlgebraic [A] time = 0.50, size = 105, normalized size = 1.00

$$\frac{\left(-8a^2-b\right)\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{8a^{5/4}}+\frac{\left(-8a^2-b\right)\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{8a^{5/4}}+\frac{\left(8a+3x^4\right)\left(ax^4+b\right)^{3/4}}{12ax^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*b - 2*a*x^4 + x^8)/(x^4*(b + a*x^4)^(1/4)),x]
```

```
[Out] ((8*a + 3*x^4)*(b + a*x^4)^(3/4))/(12*a*x^3) + ((-8*a^2 - b)*ArcTan[(a^(1/4
)*x)/(b + a*x^4)^(1/4)])/(8*a^(5/4)) + ((-8*a^2 - b)*ArcTanh[(a^(1/4)*x)/(b
 + a*x^4)^(1/4)])/(8*a^(5/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-2*a*x^4-2*b)/x^4/(a*x^4+b)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 - 2\,ax^4 - 2\,b}{\left(ax^4 + b\right)^{\frac{1}{4}} x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-2*a*x^4-2*b)/x^4/(a*x^4+b)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^8 - 2*a*x^4 - 2*b)/((a*x^4 + b)^(1/4)*x^4), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x^8 - 2a\,x^4 - 2b}{x^4\left(a\,x^4 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-2*a*x^4-2*b)/x^4/(a*x^4+b)^(1/4),x)
```

```
[Out] int((x^8-2*a*x^4-2*b)/x^4/(a*x^4+b)^(1/4),x)
```

maxima [B] time = 0.42, size = 192, normalized size = 1.83

$$\frac{1}{2}\,a\left(\frac{2\,\arctan\left(\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}} + \frac{\log\left(-\frac{a^{\frac{1}{4}} - \frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}} + \frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right) + \frac{b\left(\frac{2\,\arctan\left(\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}} + \frac{\log\left(-\frac{a^{\frac{1}{4}} - \frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}} + \frac{\left(ax^4+b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right)}{16\,a} + \frac{2\left(ax^4 + b\right)^{\frac{3}{4}}}{3\,x^3} - \frac{\left(ax^4 + b\right)^{\frac{3}{4}} b}{4\left(a^2 - \frac{\left(ax^4+b\right)a}{x^4}\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-2*a*x^4-2*b)/x^4/(a*x^4+b)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*arctan((a*x^4 + b)^(1/4)/(a^(1/4)*x))/a^(1/4) + log(-(a^(1/4) - (a
*x^4 + b)^(1/4)/x)/(a^(1/4) + (a*x^4 + b)^(1/4)/x))/a^(1/4)) + 1/16*b*(2*ar
ctan((a*x^4 + b)^(1/4)/(a^(1/4)*x))/a^(1/4) + log(-(a^(1/4) - (a*x^4 + b)^(
1/4)/x)/(a^(1/4) + (a*x^4 + b)^(1/4)/x))/a^(1/4))/a + 2/3*(a*x^4 + b)^(3/4)
/x^3 - 1/4*(a*x^4 + b)^(3/4)*b/((a^2 - (a*x^4 + b)*a/x^4)*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{-x^8 + 2\,a\,x^4 + 2\,b}{x^4\left(a\,x^4 + b\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*b + 2*a*x^4 - x^8)/(x^4*(b + a*x^4)^(1/4)),x)
```

```
[Out] int(-(2*b + 2*a*x^4 - x^8)/(x^4*(b + a*x^4)^(1/4)), x)
```

sympy [C] time = 2.95, size = 107, normalized size = 1.02

$$-\frac{a^{\frac{3}{4}}\left(1+\frac{b}{ax^4}\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2\Gamma\left(\frac{1}{4}\right)}-\frac{ax\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{ax^4e^{i\pi}}{b}\right)}{2\sqrt[4]{b}\,\Gamma\left(\frac{5}{4}\right)}+\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{ax^4e^{i\pi}}{b}\right)}{4\sqrt[4]{b}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-2*a*x**4-2*b)/x**4/(a*x**4+b)**(1/4),x)
```

```
[Out] -a**(3/4)*(1 + b/(a*x**4))**(3/4)*gamma(-3/4)/(2*gamma(1/4)) - a*x*gamma(1/
4)*hyper((1/4, 1/4), (5/4,), a*x**4*exp_polar(I*pi)/b)/(2*b**(1/4)*gamma(5/
4)) + x**5*gamma(5/4)*hyper((1/4, 5/4), (9/4,), a*x**4*exp_polar(I*pi)/b)/(
4*b**(1/4)*gamma(9/4))
```

3.1320 $\int \frac{1-2x^4+2x^8}{\sqrt[4]{1+x^4}\left(-2-x^4+x^8\right)}\,dx$

Optimal. Leaf size=105

$$-\frac{5x}{3\sqrt[4]{x^4+1}}+\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{5\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\,x}{\sqrt[4]{x^4+1}}\right)}{6\,2^{3/4}\sqrt[4]{3}}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\,x}{\sqrt[4]{x^4+1}}\right)}{6\,2^{3/4}\sqrt[4]{3}}$$

Rubi [A] time = 0.25, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6728, 240, 212, 206, 203, 1403, 382, 377}

$$-\frac{5x}{3\sqrt[4]{x^4+1}}+\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{5\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\,x}{\sqrt[4]{x^4+1}}\right)}{6\,2^{3/4}\sqrt[4]{3}}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\,x}{\sqrt[4]{x^4+1}}\right)}{6\,2^{3/4}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^4 + 2*x^8)/((1 + x^4)^(1/4)*(-2 - x^4 + x^8)),x]
```

```
[Out] (-5*x)/(3*(1 + x^4)^(1/4)) + ArcTan[x/(1 + x^4)^(1/4)] - (5*ArcTan[((3/2)^(
1/4)*x)/(1 + x^4)^(1/4)])/(6*2^(3/4)*3^(1/4)) + ArcTanh[x/(1 + x^4)^(1/4)]
- (5*ArcTanh[((3/2)^(1/4)*x)/(1 + x^4)^(1/4)])/(6*2^(3/4)*3^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 1403

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_))^(p_.), x_Symbol] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /;
FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1-2x^4+2x^8}{\sqrt[4]{1+x^4}\left(-2-x^4+x^8\right)}\,dx = \int \left(\frac{2}{\sqrt[4]{1+x^4}}+\frac{5}{\sqrt[4]{1+x^4}\left(-2-x^4+x^8\right)}\right)dx$$
$$= 2\int \frac{1}{\sqrt[4]{1+x^4}}\,dx + 5\int \frac{1}{\sqrt[4]{1+x^4}\left(-2-x^4+x^8\right)}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right) + 5\int \frac{1}{\left(-2+x^4\right)\left(1+x^4\right)^{5/4}}\,dx$$
$$= -\frac{5x}{3\sqrt[4]{1+x^4}} + \frac{5}{3}\int \frac{1}{\left(-2+x^4\right)\sqrt[4]{1+x^4}}\,dx + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right) +$$
$$= -\frac{5x}{3\sqrt[4]{1+x^4}} + \tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) + \tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) + \frac{5}{3}\text{Subst}\left(\int \frac{1}{-2+3x^4}\,d\right.$$
$$= -\frac{5x}{3\sqrt[4]{1+x^4}} + \tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) + \tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) - \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{3}x^2}\,dx,\right.}{6\sqrt{2}}$$
$$= -\frac{5x}{3\sqrt[4]{1+x^4}} + \tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) - \frac{5\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}x}{\sqrt[4]{1+x^4}}\right)}{6\,2^{3/4}\sqrt[4]{3}} + \tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) - \frac{5\tan}{6}$$

Mathematica [C] time = 0.33, size = 132, normalized size = 1.26

$$-\frac{1}{5}x^5F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-x^4,\frac{x^4}{2}\right)-\frac{5x}{3\sqrt[4]{x^4+1}}+\frac{7\left(-\log\left(2-\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt[4]{x^4+1}}\right)+\log\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt[4]{x^4+1}}+2\right)+2\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}x}{\sqrt[4]{x^4+1}}\right)\right)}{12\,2^{3/4}\sqrt[4]{3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - 2*x^4 + 2*x^8)/((1 + x^4)^(1/4)*(-2 - x^4 + x^8)),x]
```

```
[Out] (-5*x)/(3*(1 + x^4)^(1/4)) - (x^5*AppellF1[5/4, 1/4, 1, 9/4, -x^4, x^4/2])/
5 + (7*(2*ArcTan[((3/2)^(1/4)*x)/(1 + x^4)^(1/4)] - Log[2 - (2^(3/4)*3^(1/4
```

```
)*x)/(1 + x^4)^(1/4)] + Log[2 + (2^(3/4)*3^(1/4)*x)/(1 + x^4)^(1/4)]))/(12*
2^(3/4)*3^(1/4))
```

IntegrateAlgebraic [A] time = 0.47, size = 105, normalized size = 1.00

$$-\frac{5x}{3\sqrt[4]{x^4+1}}+\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{5\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\,x}{\sqrt[4]{x^4+1}}\right)}{6\,2^{3/4}\sqrt[4]{3}}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\,x}{\sqrt[4]{x^4+1}}\right)}{6\,2^{3/4}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - 2*x^4 + 2*x^8)/((1 + x^4)^(1/4)*(-2 - x^4 + x^8)),x
]
```

```
[Out] (-5*x)/(3*(1 + x^4)^(1/4)) + ArcTan[x/(1 + x^4)^(1/4)] - (5*ArcTan[((3/2)^(
1/4)*x)/(1 + x^4)^(1/4)])/(6*2^(3/4)*3^(1/4)) + ArcTanh[x/(1 + x^4)^(1/4)]
- (5*ArcTanh[((3/2)^(1/4)*x)/(1 + x^4)^(1/4)])/(6*2^(3/4)*3^(1/4))
```

fricas [B] time = 0.82, size = 205, normalized size = 1.95

$$-\frac{20\cdot 24^{\frac{3}{4}}\left(x^4+1\right)\arctan\left(\frac{24^{\frac{3}{4}}\sqrt{2}\,x\sqrt{\frac{\sqrt{6}x^2+2\sqrt{x^4+1}}{x^2}}-2\cdot 24^{\frac{3}{4}}\left(x^4+1\right)^{\frac{1}{4}}}{24\,x}\right)+5\cdot 24^{\frac{3}{4}}\left(x^4+1\right)\log\left(\frac{24^{\frac{1}{4}}x+2\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-5\cdot 24^{\frac{3}{4}}\left(x^4+1\right)\log\left(-\frac{24^{\frac{1}{4}}x-2\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+288\left(x^4+1\right)\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-144\left(x^4+1\right)\log\left(\frac{x+\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+144\left(x^4+1\right)\log\left(-\frac{x-\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+480\left(x^4+1\right)^{\frac{3}{4}}x}{288\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-2*x^4+1)/(x^4+1)^(1/4)/(x^8-x^4-2),x, algorithm="fricas")
```

```
[Out] -1/288*(20*24^(3/4)*(x^4 + 1)*arctan(1/24*(24^(3/4)*sqrt(2)*x*sqrt((sqrt(6)
*x^2 + 2*sqrt(x^4 + 1))/x^2) - 2*24^(3/4)*(x^4 + 1)^(1/4))/x) + 5*24^(3/4)*
(x^4 + 1)*log((24^(1/4)*x + 2*(x^4 + 1)^(1/4))/x) - 5*24^(3/4)*(x^4 + 1)*lo
g(-(24^(1/4)*x - 2*(x^4 + 1)^(1/4))/x) + 288*(x^4 + 1)*arctan((x^4 + 1)^(1/
4)/x) - 144*(x^4 + 1)*log((x + (x^4 + 1)^(1/4))/x) + 144*(x^4 + 1)*log(-(x
- (x^4 + 1)^(1/4))/x) + 480*(x^4 + 1)^(3/4)*x)/(x^4 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^8-2\,x^4+1}{\left(x^8-x^4-2\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-2*x^4+1)/(x^4+1)^(1/4)/(x^8-x^4-2),x, algorithm="giac")
```

```
[Out] integrate((2*x^8 - 2*x^4 + 1)/((x^8 - x^4 - 2)*(x^4 + 1)^(1/4)), x)
```

maple [F] time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{2x^8-2x^4+1}{\left(x^4+1\right)^{\frac{1}{4}}\left(x^8-x^4-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8-2*x^4+1)/(x^4+1)^(1/4)/(x^8-x^4-2),x)
```

```
[Out] int((2*x^8-2*x^4+1)/(x^4+1)^(1/4)/(x^8-x^4-2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^8-2\,x^4+1}{\left(x^8-x^4-2\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-2*x^4+1)/(x^4+1)^(1/4)/(x^8-x^4-2),x, algorithm="maxima")
[Out] integrate((2*x^8 - 2*x^4 + 1)/((x^8 - x^4 - 2)*(x^4 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{2x^8 - 2x^4 + 1}{\left(x^4 + 1\right)^{1/4}\left(-x^8 + x^4 + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^8 - 2*x^4 + 1)/((x^4 + 1)^(1/4)*(x^4 - x^8 + 2)),x)
[Out] int(-(2*x^8 - 2*x^4 + 1)/((x^4 + 1)^(1/4)*(x^4 - x^8 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^8 - 2x^4 + 1}{\left(x^4 - 2\right)\left(x^4 + 1\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8-2*x**4+1)/(x**4+1)**(1/4)/(x**8-x**4-2),x)
[Out] Integral((2*x**8 - 2*x**4 + 1)/((x**4 - 2)*(x**4 + 1)**(5/4)), x)
```

3.1321 $\int \frac{(1+x^3+x^8)^{2/3}(-3+5x^8)}{x^3(1+x^8)}\,dx$

Optimal. Leaf size=105

$$\log\left(\sqrt[3]{x^8+x^3+1}-x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^8+x^3+1}+x}\right)+\frac{3\left(x^8+x^3+1\right)^{2/3}}{2x^2}-\frac{1}{2}\log\left(x^2+\sqrt[3]{x^8+x^3+1}\,x+\left(x^8\right.\right.$$

Rubi [F] time = 3.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+x^3+x^8)^{2/3}(-3+5x^8)}{x^3(1+x^8)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^3 + x^8)^(2/3)*(-3 + 5*x^8))/(x^3*(1 + x^8)),x]
```

```
[Out] (-1)^(3/4)*Defer[Int][(1 + x^3 + x^8)^(2/3)/((-1)^(1/8) - x), x] + (-1)^(1/
4)*Defer[Int][(1 + x^3 + x^8)^(2/3)/((-1)^(3/8) - x), x] - (-1)^(3/4)*Defer
[Int][(1 + x^3 + x^8)^(2/3)/(-(-1)^(5/8) - x), x] - (-1)^(1/4)*Defer[Int][(
1 + x^3 + x^8)^(2/3)/(-(-1)^(7/8) - x), x] - 3*Defer[Int][(1 + x^3 + x^8)^(
2/3)/x^3, x] - (-1)^(3/4)*Defer[Int][(1 + x^3 + x^8)^(2/3)/((-1)^(1/8) + x)
, x] - (-1)^(1/4)*Defer[Int][(1 + x^3 + x^8)^(2/3)/((-1)^(3/8) + x), x] + (
-1)^(3/4)*Defer[Int][(1 + x^3 + x^8)^(2/3)/(-(-1)^(5/8) + x), x] + (-1)^(1/
4)*Defer[Int][(1 + x^3 + x^8)^(2/3)/(-(-1)^(7/8) + x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^3+x^8)^{2/3}(-3+5x^8)}{x^3(1+x^8)}\,dx &= \int \left(-\frac{3(1+x^3+x^8)^{2/3}}{x^3}+\frac{8x^5(1+x^3+x^8)^{2/3}}{1+x^8}\right)dx\\
&= -\left(3\int \frac{(1+x^3+x^8)^{2/3}}{x^3}\,dx\right)+8\int \frac{x^5(1+x^3+x^8)^{2/3}}{1+x^8}\,dx\\
&= -\left(3\int \frac{(1+x^3+x^8)^{2/3}}{x^3}\,dx\right)+8\int \left(\frac{x(1+x^3+x^8)^{2/3}}{2(-i+x^4)}+\frac{x(1+x^3+x^8)^{2/3}}{2(i+x^4)}\right.\\
&= -\left(3\int \frac{(1+x^3+x^8)^{2/3}}{x^3}\,dx\right)+4\int \frac{x(1+x^3+x^8)^{2/3}}{-i+x^4}\,dx+4\int \frac{x(1+x^3\cdots}{i+x\cdots}\\
&= -\left(3\int \frac{(1+x^3+x^8)^{2/3}}{x^3}\,dx\right)+4\int \left(-\frac{(-1)^{3/4}x(1+x^3+x^8)^{2/3}}{2(-\sqrt[4]{-1}+x^2)}+\frac{(-1)^{3/4}x\cdots}{2(\cdots}\right.\\
&= -\left(3\int \frac{(1+x^3+x^8)^{2/3}}{x^3}\,dx\right)-\left(2\sqrt[4]{-1}\right)\int \frac{x(1+x^3+x^8)^{2/3}}{-(-1)^{3/4}+x^2}\,dx+\left(2\sqrt[4]{-1}\right)\cdots\\
&= -\left(3\int \frac{(1+x^3+x^8)^{2/3}}{x^3}\,dx\right)-\left(2\sqrt[4]{-1}\right)\int \left(-\frac{(1+x^3+x^8)^{2/3}}{2((-1)^{3/8}-x)}+\frac{(1+x^3+\cdots}{2((-1)^{3/8}\cdots}\right.\\
&= -\left(3\int \frac{(1+x^3+x^8)^{2/3}}{x^3}\,dx\right)+\sqrt[4]{-1}\int \frac{(1+x^3+x^8)^{2/3}}{(-1)^{3/8}-x}\,dx-\sqrt[4]{-1}\int \frac{(1+\cdots}{-(\cdots}
\end{aligned}$$

Mathematica [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{\left(1+x^3+x^8\right)^{2/3} \left(-3+5 x^8\right)}{x^3 \left(1+x^8\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^3 + x^8)^(2/3)*(-3 + 5*x^8))/(x^3*(1 + x^8)),x]
[Out] Integrate[((1 + x^3 + x^8)^(2/3)*(-3 + 5*x^8))/(x^3*(1 + x^8)), x]
```

IntegrateAlgebraic [A] time = 2.69, size = 105, normalized size = 1.00

$$\log \left(\sqrt[3]{x^8+x^3+1}-x\right)-\sqrt{3} \tan ^{-1}\left(\frac{\sqrt{3} x}{2 \sqrt[3]{x^8+x^3+1}+x}\right)+\frac{3 \left(x^8+x^3+1\right)^{2/3}}{2 x^2}-\frac{1}{2} \log \left(x^2+\sqrt[3]{x^8+x^3+1} x+\left(x^8+x^3+1\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3 + x^8)^(2/3)*(-3 + 5*x^8))/(x^3*(1 + x^8)),x]
[Out] (3*(1 + x^3 + x^8)^(2/3))/(2*x^2) - Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(1 +
x^3 + x^8)^(1/3))] + Log[-x + (1 + x^3 + x^8)^(1/3)] - Log[x^2 + x*(1 + x^3
 + x^8)^(1/3) + (1 + x^3 + x^8)^(2/3)]/2
```

fricas [A] time = 17.14, size = 142, normalized size = 1.35

$$-\frac{2 \sqrt{3} x^{2} \arctan\left(-\frac{13787342107591362396272309184971387780386423854858791195768 \sqrt{3} \left(x^{8}+x^{3}+1\right)^{\frac{1}{3}} x^{2}-40425837525224298530820324142657092670161985796530402690554 6 \sqrt{3} \left(x^{8}+x^{3}+1\right)^{\frac{2}{3}} x-\sqrt{3} \left(82882407811392064917283059085655866224123024545593970500905 x^{8}+133192477088164680672740074788428524444887780970835805747392 9 x^{3}+82882407811392064917283059085655866224123024545593970500905\right)}{3 \left(26072296167104691046225677129692552015748975560524824210828 9 x^{8}+27106589816407830463546316663814240225274204825694596943161 7 x^{3}+26072296167104691046225677129692552015748975560524824210828 9\right)}\right)-x^{2} \log\left(\frac{x^{8}+3 \left(x^{8}+x^{3}+1\right)^{\frac{1}{3}} x^{2}-3 \left(x^{8}+x^{3}+1\right)^{\frac{2}{3}} x+1}{x^{8}+1}\right)-3 \left(x^{8}+x^{3}+1\right)^{\frac{2}{3}}}{2 x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+x^3+1)^(2/3)*(5*x^8-3)/x^3/(x^8+1),x, algorithm="fricas")
[Out] -1/2*(2*sqrt(3)*x^2*arctan(-1/3*(137873421075913623962723091849713877803864
238548587911957688*sqrt(3)*(x^8 + x^3 + 1)^(1/3)*x^2 - 40425837525224298530
8203241426570926701619857965304026905546*sqrt(3)*(x^8 + x^3 + 1)^(2/3)*x -
sqrt(3)*(82882407811392064917283059085655866224123024545593970500905*x^8 +
1331924770881646806727400747884285244448877809708358057473929*x^3 + 82882407
811392064917283059085655866224123024545593970500905))/(26072296167104691046
2256771296925520157489755605248242108289*x^8 + 27106589816407830463546316666
38142402252742048256945969431617*x^3 + 260722961671046910462256771296925520
157489755605248242108289)) - x^2*log((x^8 + 3*(x^8 + x^3 + 1)^(1/3)*x^2 - 3
*(x^8 + x^3 + 1)^(2/3)*x + 1)/(x^8 + 1)) - 3*(x^8 + x^3 + 1)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(5 \, x^{8} - 3\right)} {\left(x^{8} + x^{3} + 1\right)}^{\frac{2}{3}}}{{\left(x^{8} + 1\right)} x^{3}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+x^3+1)^(2/3)*(5*x^8-3)/x^3/(x^8+1),x, algorithm="giac")
[Out] integrate((5*x^8 - 3)*(x^8 + x^3 + 1)^(2/3)/((x^8 + 1)*x^3), x)
```

maple [C] time = 2.11, size = 336, normalized size = 3.20

$$\frac{3 \left(x^{8}+x^{3}+1\right)^{\frac{2}{3}}}{2 x^{2}}+\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right) \ln \left(\frac{x^{8}+\left(x^{8}+x^{3}+1\right)^{\frac{2}{3}} \operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right) x+\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right) \left(x^{8}+x^{3}+1\right)^{\frac{1}{3}} x^{2}+\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{3}+2 \left(x^{8}+x^{3}+1\right)^{\frac{2}{3}} x+2 \left(x^{8}+x^{3}+1\right)^{\frac{1}{3}} x^{2}+2 x^{3}+1}{x^{8}+1}\right)-\ln \left(-\frac{-x^{8}+\left(x^{8}+x^{3}+1\right)^{\frac{2}{3}} \operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right) x+\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right) \left(x^{8}+x^{3}+1\right)^{\frac{1}{3}} x^{2}+\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{3}-\left(x^{8}+x^{3}+1\right)^{\frac{2}{3}} x-\left(x^{8}+x^{3}+1\right)^{\frac{1}{3}} x^{2}-x^{3}-1}{x^{8}+1}\right) \operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right)-\ln \left(-\frac{-x^{8}+\left(x^{8}+x^{3}+1\right)^{\frac{2}{3}} \operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right) x+\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right) \left(x^{8}+x^{3}+1\right)^{\frac{1}{3}} x^{2}+\operatorname{RootOf}\left(\_Z^{2}+\_Z+1\right) x^{3}-\left(x^{8}+x^{3}+1\right)^{\frac{2}{3}} x-\left(x^{8}+x^{3}+1\right)^{\frac{1}{3}} x^{2}-x^{3}-1}{x^{8}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8+x^3+1)^(2/3)*(5*x^8-3)/x^3/(x^8+1),x)
```

```
[Out] 3/2*(x^8+x^3+1)^(2/3)/x^2+RootOf(_Z^2+_Z+1)*ln((x^8+(x^8+x^3+1)^(2/3)*RootO
f(_Z^2+_Z+1)*x+RootOf(_Z^2+_Z+1)*(x^8+x^3+1)^(1/3)*x^2+RootOf(_Z^2+_Z+1)*x^
3+2*(x^8+x^3+1)^(2/3)*x+2*(x^8+x^3+1)^(1/3)*x^2+2*x^3+1)/(x^8+1))-ln(-(-x^8
+(x^8+x^3+1)^(2/3)*RootOf(_Z^2+_Z+1)*x+RootOf(_Z^2+_Z+1)*(x^8+x^3+1)^(1/3)*
x^2+RootOf(_Z^2+_Z+1)*x^3-(x^8+x^3+1)^(2/3)*x-(x^8+x^3+1)^(1/3)*x^2-x^3-1)/
(x^8+1))*RootOf(_Z^2+_Z+1)-ln(-(-x^8+(x^8+x^3+1)^(2/3)*RootOf(_Z^2+_Z+1)*x+
RootOf(_Z^2+_Z+1)*(x^8+x^3+1)^(1/3)*x^2+RootOf(_Z^2+_Z+1)*x^3-(x^8+x^3+1)^(
2/3)*x-(x^8+x^3+1)^(1/3)*x^2-x^3-1)/(x^8+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^8-3\right)\left(x^8+x^3+1\right)^{\frac{2}{3}}}{\left(x^8+1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+x^3+1)^(2/3)*(5*x^8-3)/x^3/(x^8+1),x, algorithm="maxima")
[Out] integrate((5*x^8 - 3)*(x^8 + x^3 + 1)^(2/3)/((x^8 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^8-3\right)\,\left(x^8+x^3+1\right)^{2/3}}{x^3\,\left(x^8+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x^8 - 3)*(x^3 + x^8 + 1)^(2/3))/(x^3*(x^8 + 1)),x)
[Out] int(((5*x^8 - 3)*(x^3 + x^8 + 1)^(2/3))/(x^3*(x^8 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^8-3\right)\left(x^8+x^3+1\right)^{\frac{2}{3}}}{x^3\left(x^8+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8+x**3+1)**(2/3)*(5*x**8-3)/x**3/(x**8+1),x)
[Out] Integral((5*x**8 - 3)*(x**8 + x**3 + 1)**(2/3)/(x**3*(x**8 + 1)), x)
```

3.1322 $\int \frac{\sqrt{-1-x^2+x^6}\,(1+2x^6)}{8-x^4-16x^6+8x^{12}}\,dx$

Optimal. Leaf size=105

$$\frac{1}{8}\sqrt{\frac{1}{2}\left(4-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{4-\sqrt{2}}\,x}{2\sqrt{x^6-x^2-1}}\right)-\frac{1}{8}\sqrt{\frac{1}{2}\left(4+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{4+\sqrt{2}}\,x}{2\sqrt{x^6-x^2-1}}\right)$$

Rubi [F] time = 0.61, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-1-x^2+x^6}\,(1+2x^6)}{8-x^4-16x^6+8x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-1 - x^2 + x^6]*(1 + 2*x^6))/(8 - x^4 - 16*x^6 + 8*x^12),x]
```

```
[Out] Defer[Int][Sqrt[-1 - x^2 + x^6]/(8 - x^4 - 16*x^6 + 8*x^12), x] + 2*Defer[I
nt][(x^6*Sqrt[-1 - x^2 + x^6])/(8 - x^4 - 16*x^6 + 8*x^12), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1-x^2+x^6}\,(1+2x^6)}{8-x^4-16x^6+8x^{12}}\,dx&=\int\left(\frac{\sqrt{-1-x^2+x^6}}{8-x^4-16x^6+8x^{12}}+\frac{2x^6\sqrt{-1-x^2+x^6}}{8-x^4-16x^6+8x^{12}}\right)dx\\&=2\int\frac{x^6\sqrt{-1-x^2+x^6}}{8-x^4-16x^6+8x^{12}}\,dx+\int\frac{\sqrt{-1-x^2+x^6}}{8-x^4-16x^6+8x^{12}}\,dx\end{aligned}$$

Mathematica [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-1-x^2+x^6}\,(1+2x^6)}{8-x^4-16x^6+8x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[-1 - x^2 + x^6]*(1 + 2*x^6))/(8 - x^4 - 16*x^6 + 8*x^12),x]
```

```
[Out] Integrate[(Sqrt[-1 - x^2 + x^6]*(1 + 2*x^6))/(8 - x^4 - 16*x^6 + 8*x^12), x
]
```

IntegrateAlgebraic [A] time = 2.22, size = 105, normalized size = 1.00

$$\frac{1}{8}\sqrt{\frac{1}{2}\left(4-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{4-\sqrt{2}}\,x}{2\sqrt{x^6-x^2-1}}\right)-\frac{1}{8}\sqrt{\frac{1}{2}\left(4+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{4+\sqrt{2}}\,x}{2\sqrt{x^6-x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 - x^2 + x^6]*(1 + 2*x^6))/(8 - x^4 - 16*x^6 + 8
*x^12),x]
```

```
[Out] (Sqrt[(4 - Sqrt[2])/2]*ArcTan[(Sqrt[4 - Sqrt[2]]*x)/(2*Sqrt[-1 - x^2 + x^6]
)])/8 - (Sqrt[(4 + Sqrt[2])/2]*ArcTan[(Sqrt[4 + Sqrt[2]]*x)/(2*Sqrt[-1 - x^
2 + x^6])])/8
```

fricas [B] time = 0.65, size = 336, normalized size = 3.20

$$-\frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}+4}\arctan\left(\frac{196\left(4x^7+\sqrt{2}x^3-8x^3-4x\right)\sqrt{x^6-x^2-1}\sqrt{\sqrt{2}+4}-\left(72x^{12}-176x^8-144x^6+41x^4+176x^2-4\sqrt{2}\left(8x^{12}-25x^8-16x^6+10x^4+25x^2+8\right)+72\right)\sqrt{50\sqrt{2}+88}\sqrt{\sqrt{2}+4}}{98\left(8x^{12}-32x^8-16x^6+31x^4+32x^2+8\right)}\right)-\frac{1}{16}\sqrt{2}\sqrt{-\sqrt{2}+4}\arctan\left(-\frac{196\left(4x^7-\sqrt{2}x^3-8x^3-4x\right)\sqrt{x^6-x^2-1}\sqrt{-\sqrt{2}+4}-\left(72x^{12}-176x^8-144x^6+41x^4+176x^2+4\sqrt{2}\left(8x^{12}-25x^8-16x^6+10x^4+25x^2+8\right)+72\right)\sqrt{-\sqrt{2}+4}\sqrt{-50\sqrt{2}+88}}{98\left(8x^{12}-32x^8-16x^6+31x^4+32x^2+8\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2-1)^(1/2)*(2*x^6+1)/(8*x^12-16*x^6-x^4+8),x, algorithm="f
ricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(sqrt(2) + 4)*arctan(1/98*(196*(4*x^7 + sqrt(2)*x^3 - 8*x
^3 - 4*x)*sqrt(x^6 - x^2 - 1)*sqrt(sqrt(2) + 4) - (72*x^12 - 176*x^8 - 144*
x^6 + 41*x^4 + 176*x^2 - 4*sqrt(2)*(8*x^12 - 25*x^8 - 16*x^6 + 10*x^4 + 25*
x^2 + 8) + 72)*sqrt(50*sqrt(2) + 88)*sqrt(sqrt(2) + 4))/(8*x^12 - 32*x^8 -
16*x^6 + 31*x^4 + 32*x^2 + 8)) - 1/16*sqrt(2)*sqrt(-sqrt(2) + 4)*arctan(-1/
98*(196*(4*x^7 - sqrt(2)*x^3 - 8*x^3 - 4*x)*sqrt(x^6 - x^2 - 1)*sqrt(-sqrt(
2) + 4) - (72*x^12 - 176*x^8 - 144*x^6 + 41*x^4 + 176*x^2 + 4*sqrt(2)*(8*x^
12 - 25*x^8 - 16*x^6 + 10*x^4 + 25*x^2 + 8) + 72)*sqrt(-sqrt(2) + 4)*sqrt(-
50*sqrt(2) + 88))/(8*x^12 - 32*x^8 - 16*x^6 + 31*x^4 + 32*x^2 + 8))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^6+1\right)\sqrt{x^6-x^2-1}}{8x^{12}-16x^6-x^4+8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2-1)^(1/2)*(2*x^6+1)/(8*x^12-16*x^6-x^4+8),x, algorithm="g
iac")
```

```
[Out] integrate((2*x^6 + 1)*sqrt(x^6 - x^2 - 1)/(8*x^12 - 16*x^6 - x^4 + 8), x)
```

maple [C] time = 3.84, size = 580, normalized size = 5.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-x^2-1)^(1/2)*(2*x^6+1)/(8*x^12-16*x^6-x^4+8),x)
```

```
[Out] -RootOf(131072*_Z^4+2048*_Z^2+7)*ln(-(16384*RootOf(131072*_Z^4+2048*_Z^2+7)
^3*x^6+2097152*RootOf(131072*_Z^4+2048*_Z^2+7)^5*x^2+112*RootOf(131072*_Z^4
+2048*_Z^2+7)*x^6-2048*RootOf(131072*_Z^4+2048*_Z^2+7)^3*x^2+2048*(x^6-x^2-
1)^(1/2)*RootOf(131072*_Z^4+2048*_Z^2+7)^2*x-16384*RootOf(131072*_Z^4+2048*
_Z^2+7)^3-112*RootOf(131072*_Z^4+2048*_Z^2+7)*x^2+7*(x^6-x^2-1)^(1/2)*x-112
*RootOf(131072*_Z^4+2048*_Z^2+7))/(-x^6+128*x^2*RootOf(131072*_Z^4+2048*_Z^
2+7)^2+x^2+1))-1/8*RootOf(_Z^2+64*RootOf(131072*_Z^4+2048*_Z^2+7)^2+1)*ln(-
(-2048*RootOf(_Z^2+64*RootOf(131072*_Z^4+2048*_Z^2+7)^2+1)*RootOf(131072*_Z
^4+2048*_Z^2+7)^2*x^6+262144*RootOf(_Z^2+64*RootOf(131072*_Z^4+2048*_Z^2+7)
^2+1)*RootOf(131072*_Z^4+2048*_Z^2+7)^4*x^2-18*RootOf(_Z^2+64*RootOf(131072
*_Z^4+2048*_Z^2+7)^2+1)*x^6+8448*RootOf(131072*_Z^4+2048*_Z^2+7)^2*RootOf(_
Z^2+64*RootOf(131072*_Z^4+2048*_Z^2+7)^2+1)*x^2-2048*(x^6-x^2-1)^(1/2)*Root
Of(131072*_Z^4+2048*_Z^2+7)^2*x+2048*RootOf(131072*_Z^4+2048*_Z^2+7)^2*Root
Of(_Z^2+64*RootOf(131072*_Z^4+2048*_Z^2+7)^2+1)+54*RootOf(_Z^2+64*RootOf(13
1072*_Z^4+2048*_Z^2+7)^2+1)*x^2-25*(x^6-x^2-1)^(1/2)*x+18*RootOf(_Z^2+64*Ro
otOf(131072*_Z^4+2048*_Z^2+7)^2+1))/(x^6+128*x^2*RootOf(131072*_Z^4+2048*_Z
^2+7)^2+x^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^6+1\right)\sqrt{x^6-x^2-1}}{8x^{12}-16x^6-x^4+8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2-1)^(1/2)*(2*x^6+1)/(8*x^12-16*x^6-x^4+8),x, algorithm="m
axima")
```

```
[Out] integrate((2*x^6 + 1)*sqrt(x^6 - x^2 - 1)/(8*x^12 - 16*x^6 - x^4 + 8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(2\,x^6+1\right)\,\sqrt{x^6-x^2-1}}{-8\,x^{12}+16\,x^6+x^4-8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*x^6 + 1)*(x^6 - x^2 - 1)^(1/2))/(x^4 + 16*x^6 - 8*x^12 - 8),x)
```

```
[Out] int(-((2*x^6 + 1)*(x^6 - x^2 - 1)^(1/2))/(x^4 + 16*x^6 - 8*x^12 - 8), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{6}+1\right)\sqrt{x^{6}-x^{2}-1}}{8x^{12}-16x^{6}-x^{4}+8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-x**2-1)**(1/2)*(2*x**6+1)/(8*x**12-16*x**6-x**4+8),x)
```

```
[Out] Integral((2*x**6 + 1)*sqrt(x**6 - x**2 - 1)/(8*x**12 - 16*x**6 - x**4 + 8),
 x)
```

3.1323 $\int \frac{1}{(1+x^2)\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=105

$$\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{x^2+1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2+1}+x}}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{x^2+1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2+1}+x}}\right)$$

Rubi [A] time = 0.15, antiderivative size = 145, normalized size of antiderivative = 1.38, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2122, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(\sqrt{x^2+1}-\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right)}{\sqrt{2}}+\frac{\log\left(\sqrt{x^2+1}+\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right)}{\sqrt{2}}-\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\sqrt{x^2+1}+x}\right)+\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x + Sqrt[1 + x^2]]]) + Sqrt[2]*ArcTan[1 +
 Sqrt[2]*Sqrt[x + Sqrt[1 + x^2]]] - Log[1 + x + Sqrt[1 + x^2] - Sqrt[2]*Sqr
t[x + Sqrt[1 + x^2]]]/Sqrt[2] + Log[1 + x + Sqrt[1 + x^2] + Sqrt[2]*Sqrt[x
+ Sqrt[1 + x^2]]]/Sqrt[2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{1}{(1+x^2)\sqrt{x+\sqrt{1+x^2}}}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx, x, x+\sqrt{1+x^2}\right)$$

$$= 4\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x+\sqrt{1+x^2}}\right)$$

$$= 2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x+\sqrt{1+x^2}}\right) + 2\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x+\sqrt{1+\cdots}}\right.$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{x+\sqrt{1+x^2}}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{x+\cdots}\right.}{\sqrt{2}}$$

$$= -\frac{\log\left(1+x+\sqrt{1+x^2}-\sqrt{2}\sqrt{x+\sqrt{1+x^2}}\right)}{\sqrt{2}} + \frac{\log\left(1+x+\sqrt{1+x^2}+\sqrt{2}\sqrt{x\cdots}\right.}{\sqrt{2}}$$

$$= -\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x+\sqrt{1+x^2}}\right) + \sqrt{2}\tan^{-1}\left(1+\sqrt{2}\sqrt{x+\sqrt{1+x^2}}\right) - \frac{\log\cdots}{}$$

Mathematica [A] time = 0.04, size = 131, normalized size = 1.25

$$\frac{-\log\left(\sqrt{x^2+1}-\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right)+\log\left(\sqrt{x^2+1}+\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right)-2\tan^{-1}\left(1-\sqrt{2}\sqrt{\sqrt{x^2+1}+x}\right)+2\tan^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*Sqrt[x + Sqrt[1 + x^2]]] + 2*ArcTan[1 + Sqrt[2]*Sqrt
[x + Sqrt[1 + x^2]]] - Log[1 + x + Sqrt[1 + x^2] - Sqrt[2]*Sqrt[x + Sqrt[1
+ x^2]]] + Log[1 + x + Sqrt[1 + x^2] + Sqrt[2]*Sqrt[x + Sqrt[1 + x^2]]])/Sq
rt[2]
```

IntegrateAlgebraic [A] time = 0.14, size = 105, normalized size = 1.00

$$\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{x^2+1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2+1}+x}}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{x^2+1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2+1}+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x^2)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] Sqrt[2]*ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2] + Sqrt[1 + x^2]/Sqrt[2])/Sqrt[x +
Sqrt[1 + x^2]]] + Sqrt[2]*ArcTanh[(1/Sqrt[2] + x/Sqrt[2] + Sqrt[1 + x^2]/Sq
rt[2])/Sqrt[x + Sqrt[1 + x^2]]]
```

fricas [B] time = 0.43, size = 189, normalized size = 1.80

$$-2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+x+\sqrt{x^2+1}+1}-\sqrt{2}\sqrt{x+\sqrt{x^2+1}}-1\right)-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+4x+4\sqrt{x^2+1}+4}-\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+1\right)+\frac{1}{2}\sqrt{2}\log\left(4\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+4x+4\sqrt{x^2+1}+4\right)-\frac{1}{2}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+4x+4\sqrt{x^2+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x + sqrt(x^2 + 1)) + x + sqrt(x
^2 + 1) + 1) - sqrt(2)*sqrt(x + sqrt(x^2 + 1)) - 1) - 2*sqrt(2)*arctan(1/2*
sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x + sqrt(x^2 + 1)) + 4*x + 4*sqrt(x^2 + 1) + 4
) - sqrt(2)*sqrt(x + sqrt(x^2 + 1)) + 1) + 1/2*sqrt(2)*log(4*sqrt(2)*sqrt(x
 + sqrt(x^2 + 1)) + 4*x + 4*sqrt(x^2 + 1) + 4) - 1/2*sqrt(2)*log(-4*sqrt(2)
*sqrt(x + sqrt(x^2 + 1)) + 4*x + 4*sqrt(x^2 + 1) + 4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2+1)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 1)*sqrt(x + sqrt(x^2 + 1))), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(x+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(1/(x^2+1)/(x+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2+1)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 1)*sqrt(x + sqrt(x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2+1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(x + (x^2 + 1)^(1/2))^(1/2)),x)
[Out] int(1/((x^2 + 1)*(x + (x^2 + 1)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+\sqrt{x^2+1}}\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(x+(x**2+1)**(1/2))**(1/2),x)
[Out] Integral(1/(sqrt(x + sqrt(x**2 + 1))*(x**2 + 1)), x)
```

3.1324 $\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{x^2}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{\sqrt{a^2x^4+b}+ax^2}}{x}+\frac{\sqrt{a}\log\left(i\sqrt{a^2x^4+b}+i\sqrt{2}\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}+iax^2\right)}{\sqrt{2}}$$

Rubi [F] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]/x^2,x]
[Out] Defer[Int][Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]/x^2, x]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{x^2}\,dx=\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{x^2}\,dx$$

Mathematica [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]/x^2,x]
[Out] Integrate[Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]/x^2, x]
```

IntegrateAlgebraic [A] time = 0.55, size = 105, normalized size = 1.00

$$-\frac{\sqrt{\sqrt{a^2x^4+b}+ax^2}}{x}+\frac{\sqrt{a}\log\left(i\sqrt{a^2x^4+b}+i\sqrt{2}\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}+iax^2\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]/x^2,x]
[Out] -(Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]/x) + (Sqrt[a]*Log[I*a*x^2 + I*Sqrt[b + a^
2*x^4] + I*Sqrt[2]*Sqrt[a]*x*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]])/Sqrt[2]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2 + \sqrt{a^2x^4 + b}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^4 + b))/x^2, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\,x^2 + \sqrt{a^2x^4 + b}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/x^2,x)
[Out] int((a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2 + \sqrt{a^2x^4 + b}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^4 + b))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{a^2\,x^4 + b} + a\,x^2}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a^2*x^4)^(1/2) + a*x^2)^(1/2)/x^2,x)
[Out] int(((b + a^2*x^4)^(1/2) + a*x^2)^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2 + \sqrt{a^2x^4 + b}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+(a**2*x**4+b)**(1/2))**(1/2)/x**2,x)
[Out] Integral(sqrt(a*x**2 + sqrt(a**2*x**4 + b))/x**2, x)
```

3.1325 $\int \frac{-1+x}{(1+x)\sqrt{1+\sqrt{1+\sqrt{1+x}}}}\,dx$

Optimal. Leaf size=105

$$\frac{-8(-15x-128)+\sqrt{x+1}\left(8-24\sqrt{\sqrt{x+1}+1}\right)+64\sqrt{\sqrt{x+1}+1}}{105\sqrt{\sqrt{\sqrt{x+1}+1}+1}}+4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{\sqrt{2}}\right)$$

Rubi [A] time = 0.66, antiderivative size = 124, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1629, 63, 206}

$$\frac{8}{7}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{7/2}-\frac{24}{5}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{5/2}+\frac{16}{3}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{3/2}+\frac{8}{\sqrt{\sqrt{\sqrt{x+1}+1}+1}}+4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/((1 + x)*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]),x]
```

```
[Out] 8/Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]] + (16*(1 + Sqrt[1 + Sqrt[1 + x]])^(3/2))/
3 - (24*(1 + Sqrt[1 + Sqrt[1 + x]])^(5/2))/5 + (8*(1 + Sqrt[1 + Sqrt[1 + x]
])^(7/2))/7 + 4*Sqrt[2]*ArcTanh[Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]/Sqrt[2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{-1+x}{(1+x)\sqrt{1+\sqrt{1+\sqrt{1+x}}}}\,dx = 2\,\text{Subst}\left(\int \frac{-2+x^2}{x\sqrt{1+\sqrt{1+x}}}\,dx, x, \sqrt{1+x}\right)$$

$$= 4\,\text{Subst}\left(\int \frac{x\left(-2+\left(-1+x^2\right)^2\right)}{\sqrt{1+x}\left(-1+x^2\right)}\,dx, x, \sqrt{1+\sqrt{1+x}}\right)$$

$$= 4\,\text{Subst}\left(\int\left(-\frac{1}{(1+x)^{3/2}}+\frac{1}{(1-x)\sqrt{1+x}}+2\sqrt{1+x}-3(1+x)^{3/2}+(1+x)\right.\right.$$

$$= \frac{8}{\sqrt{1+\sqrt{1+\sqrt{1+x}}}}+\frac{16}{3}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{3/2}-\frac{24}{5}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{5}$$

$$= \frac{8}{\sqrt{1+\sqrt{1+\sqrt{1+x}}}}+\frac{16}{3}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{3/2}-\frac{24}{5}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{5}$$

$$= \frac{8}{\sqrt{1+\sqrt{1+\sqrt{1+x}}}}+\frac{16}{3}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{3/2}-\frac{24}{5}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{5}$$

Mathematica [A] time = 0.22, size = 106, normalized size = 1.01

$$\frac{8\left(15x+\sqrt{x+1}-3\sqrt{x+1}\sqrt{\sqrt{x+1}+1}+8\sqrt{\sqrt{x+1}+1}+128\right)}{105\sqrt{\sqrt{\sqrt{x+1}+1}+1}}+4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/((1 + x)*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]),x]
```

```
[Out] (8*(128 + 15*x + Sqrt[1 + x] + 8*Sqrt[1 + Sqrt[1 + x]] - 3*Sqrt[1 + x]*Sqrt
[1 + Sqrt[1 + x]]))/(105*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]) + 4*Sqrt[2]*ArcTa
nh[Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]/Sqrt[2]]
```

IntegrateAlgebraic [A] time = 0.15, size = 102, normalized size = 0.97

$$\frac{-8\sqrt{\sqrt{x+1}+1}\left(3\sqrt{x+1}-8\right)-8\left(-15(x+1)-\sqrt{x+1}-113\right)}{105\sqrt{\sqrt{\sqrt{x+1}+1}+1}}+4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/((1 + x)*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]),x]
```

```
[Out] (-8*Sqrt[1 + Sqrt[1 + x]]*(-8 + 3*Sqrt[1 + x]) - 8*(-113 - Sqrt[1 + x] - 15
*(1 + x)))/(105*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]) + 4*Sqrt[2]*ArcTanh[Sqrt[1
 + Sqrt[1 + Sqrt[1 + x]]]/Sqrt[2]]
```

fricas [A] time = 0.42, size = 150, normalized size = 1.43

$$\frac{2\left(105\sqrt{2}(x+1)\log\left(\frac{2\left(\sqrt{2}\sqrt{x+1}\sqrt{\sqrt{x+1}+1}+\sqrt{2}\sqrt{x+1}\right)\sqrt{\sqrt{\sqrt{x+1}+1}+1}+x+4\sqrt{x+1}\sqrt{\sqrt{x+1}+1}+4\sqrt{x+1}+1}{x+1}\right)-4\left(3(6x+41)\sqrt{x+1}-\left(15(x+8)\sqrt{x+1}+4x+4\right)\sqrt{\sqrt{x+1}+1}-4x-4\right)\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)}{105(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(1+(1+(1+x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 2/105*(105*sqrt(2)*(x + 1)*log((2*(sqrt(2)*sqrt(x + 1)*sqrt(sqrt(x + 1) + 1
) + sqrt(2)*sqrt(x + 1))*sqrt(sqrt(sqrt(x + 1) + 1) + 1) + x + 4*sqrt(x + 1
)*sqrt(sqrt(x + 1) + 1) + 4*sqrt(x + 1) + 1)/(x + 1)) - 4*(3*(6*x + 41)*sqr
t(x + 1) - (15*(x + 8)*sqrt(x + 1) + 4*x + 4)*sqrt(sqrt(x + 1) + 1) - 4*x -
 4)*sqrt(sqrt(sqrt(x + 1) + 1) + 1))/(x + 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(1+(1+(1+x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-2,[1]%%%}+%%%{-4,[0]%%%},%%%{-4,[1]%%%}+%%%{
-4,[0]%%%},%%%{1,[2]%%%}+%%%{-1,[1]%%%}+%%%{-1,[0]%%%}] at parameters value
s [-23]Warning, choosing root of [1,0,%%%{-2,[1]%%%}+%%%{-4,[0]%%%},%%%{-4,
[1]%%%}+%%%{-4,[0]%%%},%%%{1,[2]%%%}+%%%{-1,[1]%%%}+%%%{-1,[0]%%%}] at para
meters values [65]Warning, need to choose a branch for the root of a polyno
mial with parameters. This might be wrong.The choice was done assuming [x]=
[95]8/7*sqrt(sqrt(sqrt(x+1)+1)+1)*(sqrt(sqrt(sqrt(x+1)+1)+1)^3-24/5*sqrt(sqrt(sq
rt(x+1)+1)+1)*(sqrt(sqrt(x+1)+1)+1)^2+16/3*sqrt(sqrt(sqrt(x+1)+1)+1)*(sqrt(
sqrt(x+1)+1)+1)+8/sqrt(sqrt(sqrt(x+1)+1)+1)-4/sqrt(2)*ln(abs(2*sqrt(sqrt(sq
rt(x+1)+1)+1)-2*sqrt(2))/(2*sqrt(sqrt(sqrt(x+1)+1)+1)+2*sqrt(2)))
```

maple [A] time = 0.02, size = 86, normalized size = 0.82

$$\frac{8\left(1+\sqrt{1+\sqrt{1+x}}\right)^{\frac{7}{2}}}{7}-\frac{24\left(1+\sqrt{1+\sqrt{1+x}}\right)^{\frac{5}{2}}}{5}+\frac{16\left(1+\sqrt{1+\sqrt{1+x}}\right)^{\frac{3}{2}}}{3}+4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\sqrt{1+\sqrt{1+x}}}\,\sqrt{2}}{2}\right)+\frac{8}{\sqrt{1+\sqrt{1+\sqrt{1+x}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(1+x)/(1+(1+(1+x)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 8/7*(1+(1+(1+x)^(1/2))^(1/2))^(7/2)-24/5*(1+(1+(1+x)^(1/2))^(1/2))^(5/2)+16
/3*(1+(1+(1+x)^(1/2))^(1/2))^(3/2)+4*2^(1/2)*arctanh(1/2*(1+(1+(1+x)^(1/2))
^(1/2))^(1/2)*2^(1/2))+8/(1+(1+(1+x)^(1/2))^(1/2))^(1/2)
```

maxima [A] time = 0.41, size = 107, normalized size = 1.02

$$\frac{8}{7}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{7}{2}}-\frac{24}{5}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{5}{2}}+\frac{16}{3}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{3}{2}}-2\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{\sqrt{2}+\sqrt{\sqrt{\sqrt{x+1}+1}+1}}\right)+\frac{8}{\sqrt{\sqrt{\sqrt{x+1}+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(1+(1+(1+x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 8/7*(sqrt(sqrt(x + 1) + 1) + 1)^(7/2) - 24/5*(sqrt(sqrt(x + 1) + 1) + 1)^(5
/2) + 16/3*(sqrt(sqrt(x + 1) + 1) + 1)^(3/2) - 2*sqrt(2)*log(-(sqrt(2) - sq
rt(sqrt(sqrt(x + 1) + 1) + 1))/(sqrt(2) + sqrt(sqrt(sqrt(x + 1) + 1) + 1)))
 + 8/sqrt(sqrt(sqrt(x + 1) + 1) + 1)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x-1}{\sqrt{\sqrt{\sqrt{x+1}+1}+1}\;(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((((x + 1)^(1/2) + 1)^(1/2) + 1)^(1/2)*(x + 1)),x)
[Out] int((x - 1)/((((x + 1)^(1/2) + 1)^(1/2) + 1)^(1/2)*(x + 1)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{(x+1)\sqrt{\sqrt{\sqrt{x+1}+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(1+(1+(1+x)**(1/2))**(1/2))**(1/2),x)
[Out] Integral((x - 1)/((x + 1)*sqrt(sqrt(sqrt(x + 1) + 1) + 1)), x)
```

3.1326 $\int \frac{1}{(-2+x)\sqrt[3]{1+2x+x^2}}\,dx$

Optimal. Leaf size=106

$$\frac{\left((x+1)^2\right)^{2/3}\left(\frac{\log\left(3^{2/3}\sqrt[3]{x+1}-3\right)}{3^{2/3}}-\frac{\log\left(\sqrt[3]{3}(x+1)^{2/3}+3^{2/3}\sqrt[3]{x+1}+3\right)}{2\,3^{2/3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x+1}}{3^{5/6}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[6]{3}}\right)}{(x+1)^{4/3}}$$

Rubi [A] time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {646, 57, 617, 204, 31}

$$-\frac{(x+1)^{2/3}\log(2-x)}{2\,3^{2/3}\sqrt[3]{x^2+2x+1}}+\frac{\sqrt[3]{3}(x+1)^{2/3}\log\left(\sqrt[3]{3}-\sqrt[3]{x+1}\right)}{2\sqrt[3]{x^2+2x+1}}-\frac{(x+1)^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{x+1}+\sqrt[3]{3}}{3^{5/6}}\right)}{\sqrt[6]{3}\sqrt[3]{x^2+2x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + x)*(1 + 2*x + x^2)^(1/3)),x]
```

```
[Out] -(((1 + x)^(2/3)*ArcTan[(3^(1/3) + 2*(1 + x)^(1/3))/3^(5/6)])/(3^(1/6)*(1 + 
 2*x + x^2)^(1/3))) - ((1 + x)^(2/3)*Log[2 - x])/(2*3^(2/3)*(1 + 2*x + x^2)
^(1/3)) + (3^(1/3)*(1 + x)^(2/3)*Log[3^(1/3) - (1 + x)^(1/3)])/(2*(1 + 2*x
+ x^2)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, 
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
```

0]

Rubi steps

$$\int \frac{1}{(-2+x)\sqrt[3]{1+2x+x^2}}\,dx = \frac{(1+x)^{2/3}\int \frac{1}{(-2+x)(1+x)^{2/3}}\,dx}{\sqrt[3]{1+2x+x^2}}$$

$$= -\frac{(1+x)^{2/3}\log(2-x)}{2\,3^{2/3}\sqrt[3]{1+2x+x^2}} - \frac{\left(\sqrt[3]{3}\,(1+x)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{3}-x}\,dx, x, \sqrt[3]{1+x}\right)}{2\sqrt[3]{1+2x+x^2}} - \frac{\left(3^{2/3}(1+\right.}{}$$

$$= -\frac{(1+x)^{2/3}\log(2-x)}{2\,3^{2/3}\sqrt[3]{1+2x+x^2}} + \frac{\sqrt[3]{3}\,(1+x)^{2/3}\log\left(\sqrt[3]{3}-\sqrt[3]{1+x}\right)}{2\sqrt[3]{1+2x+x^2}} + \frac{\left(\sqrt[3]{3}\,(1+x)^{2/3}\right)\text{Subs}}{\sqrt[3]{1}}$$

$$= -\frac{(1+x)^{2/3}\tan^{-1}\left(\frac{1}{3}\left(\sqrt{3}+2\sqrt[6]{3}\sqrt[3]{1+x}\right)\right)}{\sqrt[6]{3}\sqrt[3]{1+2x+x^2}} - \frac{(1+x)^{2/3}\log(2-x)}{2\,3^{2/3}\sqrt[3]{1+2x+x^2}} + \frac{\sqrt[3]{3}\,(1+x)^{2/3}}{2\sqrt[3]{1}}$$

Mathematica [A] time = 0.05, size = 100, normalized size = 0.94

$$-\frac{(x+1)^{2/3}\left(-2\log\left(\sqrt[3]{3}-\sqrt[3]{x+1}\right)+\log\left((x+1)^{2/3}+\sqrt[3]{3}\sqrt[3]{x+1}+3^{2/3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{x+1}+\sqrt[3]{3}}{3^{5/6}}\right)\right)}{2\,3^{2/3}\sqrt[3]{(x+1)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + x)*(1 + 2*x + x^2)^(1/3)),x]
```

```
[Out] -1/2*((1 + x)^(2/3)*(2*Sqrt[3]*ArcTan[(3^(1/3) + 2*(1 + x)^(1/3))/3^(5/6)]
 - 2*Log[3^(1/3) - (1 + x)^(1/3)] + Log[3^(2/3) + 3^(1/3)*(1 + x)^(1/3) + (1
 + x)^(2/3)]))/(3^(2/3)*((1 + x)^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.22, size = 153, normalized size = 1.44

$$\frac{\log\left(-3\sqrt[3]{x^2+2x+1}+3^{2/3}x+3^{2/3}\right)}{3^{2/3}} - \frac{\log\left(\sqrt[3]{3}\left(x^2+2x+1\right)^{2/3}+3\sqrt[3]{x^2+2x+1}+3^{2/3}x+3^{2/3}\right)}{2\,3^{2/3}} + \frac{\tan^{-1}\left(\frac{3^{5/6}\sqrt[3]{x^2+2x+1}}{\sqrt[3]{3}\sqrt[3]{x^2+2x+1}+2x+2}\right)}{\sqrt[6]{3}} - \frac{\log(x+1)}{3\,3^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2 + x)*(1 + 2*x + x^2)^(1/3)),x]
```

```
[Out] ArcTan[(3^(5/6)*(1 + 2*x + x^2)^(1/3))/(2 + 2*x + 3^(1/3)*(1 + 2*x + x^2)^(
1/3))]/3^(1/6) - Log[1 + x]/(3*3^(2/3)) + Log[3^(2/3) + 3^(2/3)*x - 3*(1 +
2*x + x^2)^(1/3)]/3^(2/3) - Log[3^(2/3) + 3^(2/3)*x + 3*(1 + 2*x + x^2)^(1/
3) + 3^(1/3)*(1 + 2*x + x^2)^(2/3)]/(2*3^(2/3))
```

fricas [A] time = 0.43, size = 139, normalized size = 1.31

$$\frac{1}{3}\cdot 9^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{9^{\frac{1}{6}}\left(9^{\frac{1}{3}}\sqrt{3}(x+1)+6\sqrt{3}\left(x^2+2x+1\right)^{\frac{1}{3}}\right)}{9(x+1)}\right) - \frac{1}{18}\cdot 9^{\frac{2}{3}}\log\left(\frac{9^{\frac{2}{3}}\left(x^2+2x+1\right)+3\cdot 9^{\frac{1}{3}}\left(x^2+2x+1\right)^{\frac{1}{3}}(x+1)+9\left(x^2+2x+1\right)^{\frac{2}{3}}}{x^2+2x+1}\right) + \frac{1}{9}\cdot 9^{\frac{2}{3}}\log\left(-\frac{9^{\frac{1}{3}}(x+1)-3\left(x^2+2x+1\right)^{\frac{1}{3}}}{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(x^2+2*x+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*9^(1/6)*sqrt(3)*arctan(1/9*9^(1/6)*(9^(1/3)*sqrt(3)*(x + 1) + 6*sqrt(3)
*(x^2 + 2*x + 1)^(1/3))/(x + 1)) - 1/18*9^(2/3)*log((9^(2/3)*(x^2 + 2*x + 1
) + 3*9^(1/3)*(x^2 + 2*x + 1)^(1/3)*(x + 1) + 9*(x^2 + 2*x + 1)^(2/3))/(x^2
```

```
 + 2*x + 1)) + 1/9*9^(2/3)*log(-(9^(1/3)*(x + 1) - 3*(x^2 + 2*x + 1)^(1/3))
/(x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+2\,x+1\right)^{\frac{1}{3}}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(x^2+2*x+1)^(1/3),x, algorithm="giac")
[Out] integrate(1/((x^2 + 2*x + 1)^(1/3)*(x - 2)), x)
```

maple [C] time = 2.69, size = 942, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2+x)/(x^2+2*x+1)^(1/3),x)
[Out] RootOf(RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+9*_Z^2)*ln(-(63*RootOf(RootOf(_
Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+9*_Z^2)^2*RootOf(_Z^3-3)^2*x^2+6*RootOf(RootOf
(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+9*_Z^2)*RootOf(_Z^3-3)^3*x^2+63*RootOf(RootO
f(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+9*_Z^2)^2*RootOf(_Z^3-3)^2*x+63*(x^2+2*x+1)
^(2/3)*RootOf(RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+9*_Z^2)*RootOf(_Z^3-3)^2
+6*RootOf(RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+9*_Z^2)*RootOf(_Z^3-3)^3*x+7
2*(x^2+2*x+1)^(1/3)*RootOf(RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+9*_Z^2)*Roo
tOf(_Z^3-3)*x-21*(x^2+2*x+1)^(1/3)*RootOf(_Z^3-3)^2*x+72*(x^2+2*x+1)^(1/3)*
RootOf(RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+9*_Z^2)*RootOf(_Z^3-3)-21*RootO
f(RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+9*_Z^2)*x^2-21*(x^2+2*x+1)^(1/3)*Roo
tOf(_Z^3-3)^2-2*RootOf(_Z^3-3)*x^2-294*RootOf(RootOf(_Z^3-3)^2+3*_Z*RootOf(
_Z^3-3)+9*_Z^2)*x+135*(x^2+2*x+1)^(2/3)-28*RootOf(_Z^3-3)*x-273*RootOf(Root
Of(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+9*_Z^2)-26*RootOf(_Z^3-3))/(-2+x)/(1+x))+1
/3*RootOf(_Z^3-3)*ln((18*RootOf(RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+9*_Z^2
)^2*RootOf(_Z^3-3)^2*x^2+21*RootOf(RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+9*_
Z^2)*RootOf(_Z^3-3)^3*x^2+18*RootOf(RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+9*
_Z^2)^2*RootOf(_Z^3-3)^2*x+63*(x^2+2*x+1)^(2/3)*RootOf(RootOf(_Z^3-3)^2+3*_
Z*RootOf(_Z^3-3)+9*_Z^2)*RootOf(_Z^3-3)^2+21*RootOf(RootOf(_Z^3-3)^2+3*_Z*R
ootOf(_Z^3-3)+9*_Z^2)*RootOf(_Z^3-3)^3*x-135*(x^2+2*x+1)^(1/3)*RootOf(RootO
f(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+9*_Z^2)*RootOf(_Z^3-3)*x-21*(x^2+2*x+1)^(1/
3)*RootOf(_Z^3-3)^2*x-135*(x^2+2*x+1)^(1/3)*RootOf(RootOf(_Z^3-3)^2+3*_Z*Ro
otOf(_Z^3-3)+9*_Z^2)*RootOf(_Z^3-3)+24*RootOf(RootOf(_Z^3-3)^2+3*_Z*RootOf(
_Z^3-3)+9*_Z^2)*x^2-21*(x^2+2*x+1)^(1/3)*RootOf(_Z^3-3)^2+28*RootOf(_Z^3-3)
*x^2+102*RootOf(RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+9*_Z^2)*x-72*(x^2+2*x+
1)^(2/3)+119*RootOf(_Z^3-3)*x+78*RootOf(RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3
)+9*_Z^2)+91*RootOf(_Z^3-3))/(-2+x)/(1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+2\,x+1\right)^{\frac{1}{3}}(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(x^2+2*x+1)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 2*x + 1)^(1/3)*(x - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x-2)\,\left(x^2+2\,x+1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)*(2*x + x^2 + 1)^(1/3)),x)
[Out] int(1/((x - 2)*(2*x + x^2 + 1)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x-2)\,\sqrt[3]{(x+1)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(x**2+2*x+1)**(1/3),x)
[Out] Integral(1/((x - 2)*((x + 1)**2)**(1/3)), x)
```

3.1327 $\int \frac{\sqrt[3]{-x+x^3}}{x^2}\,dx$

Optimal. Leaf size=106

$$-\frac{3\sqrt[3]{x^3-x}}{2x}-\frac{1}{2}\log\left(\sqrt[3]{x^3-x}-x\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-x}+x}\right)+\frac{1}{4}\log\left(\sqrt[3]{x^3-x}\,x+\left(x^3-x\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.15, antiderivative size = 188, normalized size of antiderivative = 1.77, number of steps used = 11, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {2020, 2032, 329, 275, 331, 292, 31, 634, 618, 204, 628}

$$-\frac{3\sqrt[3]{x^3-x}}{2x}-\frac{x^{2/3}\left(x^2-1\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2-1}}\right)}{2\left(x^3-x\right)^{2/3}}+\frac{x^{2/3}\left(x^2-1\right)^{2/3}\log\left(\frac{x^{4/3}}{\left(x^2-1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2-1}}+1\right)}{4\left(x^3-x\right)^{2/3}}-\frac{\sqrt{3}\,x^{2/3}\left(x^2-1\right)^{2/3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1}{\sqrt{3}}\right)}{2\left(x^3-x\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)^(1/3)/x^2,x]
```

```
[Out] (-3*(-x + x^3)^(1/3))/(2*x) - (Sqrt[3]*x^(2/3)*(-1 + x^2)^(2/3)*ArcTan[(1 +
 (2*x^(2/3))/(-1 + x^2)^(1/3))/Sqrt[3]])/(2*(-x + x^3)^(2/3)) - (x^(2/3)*(-
1 + x^2)^(2/3)*Log[1 - x^(2/3)/(-1 + x^2)^(1/3)])/(2*(-x + x^3)^(2/3)) + (x
^(2/3)*(-1 + x^2)^(2/3)*Log[1 + x^(4/3)/(-1 + x^2)^(2/3) + x^(2/3)/(-1 + x^
2)^(1/3)])/(4*(-x + x^3)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt[3]{-x+x^3}}{x^2}\,dx = -\frac{3\sqrt[3]{-x+x^3}}{2x} + \int \frac{x}{\left(-x+x^3\right)^{2/3}}\,dx$$

$$= -\frac{3\sqrt[3]{-x+x^3}}{2x} + \frac{\left(x^{2/3}\left(-1+x^2\right)^{2/3}\right)\int \frac{\sqrt[3]{x}}{\left(-1+x^2\right)^{2/3}}\,dx}{\left(-x+x^3\right)^{2/3}}$$

$$= -\frac{3\sqrt[3]{-x+x^3}}{2x} + \frac{\left(3x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x^3}{\left(-1+x^6\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{\left(-x+x^3\right)^{2/3}}$$

$$= -\frac{3\sqrt[3]{-x+x^3}}{2x} + \frac{\left(3x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{\left(-1+x^3\right)^{2/3}}\,dx, x, x^{2/3}\right)}{2\left(-x+x^3\right)^{2/3}}$$

$$= -\frac{3\sqrt[3]{-x+x^3}}{2x} + \frac{\left(3x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2\left(-x+x^3\right)^{2/3}}$$

$$= -\frac{3\sqrt[3]{-x+x^3}}{2x} + \frac{\left(x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2\left(-x+x^3\right)^{2/3}} - \frac{\left(x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subs}}{2\left(-x\right.}$$

$$= -\frac{3\sqrt[3]{-x+x^3}}{2x} - \frac{x^{2/3}\left(-1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2\left(-x+x^3\right)^{2/3}} + \frac{\left(x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,d\right.}{4\left(-x+x^3\right)^{2/3}}$$

$$= -\frac{3\sqrt[3]{-x+x^3}}{2x} - \frac{x^{2/3}\left(-1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2\left(-x+x^3\right)^{2/3}} + \frac{x^{2/3}\left(-1+x^2\right)^{2/3}\log\left(1+\frac{x^{4/3}}{\left(-1+x^2\right)^{2/3}}+\right.}{4\left(-x+x^3\right)^{2/3}}$$

$$= -\frac{3\sqrt[3]{-x+x^3}}{2x} - \frac{\sqrt{3}\,x^{2/3}\left(-1+x^2\right)^{2/3}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}}{\sqrt{3}}\right)}{2\left(-x+x^3\right)^{2/3}} - \frac{x^{2/3}\left(-1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2\left(-x+x^3\right)^{2/3}}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.40

$$-\frac{3\sqrt[3]{x\left(x^2-1\right)}\ {}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};x^2\right)}{2x\sqrt[3]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)^(1/3)/x^2,x]
```

```
[Out] (-3*(x*(-1 + x^2))^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, x^2])/(2*x*(1 -
 x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.20, size = 106, normalized size = 1.00

$$-\frac{3\sqrt[3]{x^3-x}}{2x} - \frac{1}{2}\log\left(\sqrt[3]{x^3-x}-x\right) - \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x}+x}\right) + \frac{1}{4}\log\left(\sqrt[3]{x^3-x}\,x+\left(x^3-x\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x + x^3)^(1/3)/x^2,x]
```

```
[Out] (-3*(-x + x^3)^(1/3))/(2*x) - (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-x + x^3)
^(1/3))])/2 - Log[-x + (-x + x^3)^(1/3)]/2 + Log[x^2 + x*(-x + x^3)^(1/3) +
 (-x + x^3)^(2/3)]/4
```

fricas [A] time = 0.60, size = 104, normalized size = 0.98

$$-\frac{2\sqrt{3}\,x\arctan\left(-\frac{44032959556\sqrt{3}\left(x^3-x\right)^{\frac{1}{3}}x+\sqrt{3}\left(16754327161\,x^2-2707204793\right)-10524305234\sqrt{3}\left(x^3-x\right)^{\frac{2}{3}}}{81835897185\,x^2-1102302937}\right)+x\log\left(-3\left(x^3-x\right)^{\frac{1}{3}}x+3\left(x^3-x\right)^{\frac{2}{3}}+1\right)+6\left(x^3-x\right)^{\frac{1}{3}}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*x*arctan(-(44032959556*sqrt(3)*(x^3 - x)^(1/3)*x + sqrt(3)*
(16754327161*x^2 - 2707204793) - 10524305234*sqrt(3)*(x^3 - x)^(2/3))/(8183
5897185*x^2 - 1102302937)) + x*log(-3*(x^3 - x)^(1/3)*x + 3*(x^3 - x)^(2/3)
 + 1) + 6*(x^3 - x)^(1/3))/x
```

giac [A] time = 0.19, size = 74, normalized size = 0.70

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{3}{2}\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+\frac{1}{4}\log\left(\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{2}\log\left(\left|\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-1/x^2 + 1)^(1/3) + 1)) - 3/2*(-1/x^2 +
1)^(1/3) + 1/4*log((-1/x^2 + 1)^(2/3) + (-1/x^2 + 1)^(1/3) + 1) - 1/2*log(a
bs((-1/x^2 + 1)^(1/3) - 1))
```

maple [C] time = 1.93, size = 787, normalized size = 7.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x)^(1/3)/x^2,x)
```

```
[Out] -3/2*(x*(x^2-1))^(1/3)/x+(1/12*RootOf(_Z^2-6*_Z+36)*ln((-47*RootOf(_Z^2-6*_
Z+36)^2*x^4+3207*RootOf(_Z^2-6*_Z+36)*x^4+2925*(x^6-2*x^4+x^2)^(1/3)*RootOf
(_Z^2-6*_Z+36)*x^2+235*RootOf(_Z^2-6*_Z+36)^2*x^2-6930*x^4+2925*RootOf(_Z^2
-6*_Z+36)*(x^6-2*x^4+x^2)^(2/3)-5238*(x^6-2*x^4+x^2)^(1/3)*x^2-5601*RootOf(
_Z^2-6*_Z+36)*x^2-5238*(x^6-2*x^4+x^2)^(2/3)-2925*RootOf(_Z^2-6*_Z+36)*(x^6
-2*x^4+x^2)^(1/3)-188*RootOf(_Z^2-6*_Z+36)^2+11340*x^2+5238*(x^6-2*x^4+x^2)
^(1/3)+2394*RootOf(_Z^2-6*_Z+36)-4410)/(-1+x)/(1+x))-1/12*ln(-(47*RootOf(_Z
^2-6*_Z+36)^2*x^4+2643*RootOf(_Z^2-6*_Z+36)*x^4+2925*(x^6-2*x^4+x^2)^(1/3)*
RootOf(_Z^2-6*_Z+36)*x^2-235*RootOf(_Z^2-6*_Z+36)^2*x^2-10620*x^4+2925*Root
Of(_Z^2-6*_Z+36)*(x^6-2*x^4+x^2)^(2/3)-12312*(x^6-2*x^4+x^2)^(1/3)*x^2-2781
*RootOf(_Z^2-6*_Z+36)*x^2-12312*(x^6-2*x^4+x^2)^(2/3)-2925*RootOf(_Z^2-6*_Z
+36)*(x^6-2*x^4+x^2)^(1/3)+188*RootOf(_Z^2-6*_Z+36)^2+13806*x^2+12312*(x^6-
2*x^4+x^2)^(1/3)+138*RootOf(_Z^2-6*_Z+36)-3186)/(-1+x)/(1+x))*RootOf(_Z^2-6
*_Z+36)+1/2*ln(-(47*RootOf(_Z^2-6*_Z+36)^2*x^4+2643*RootOf(_Z^2-6*_Z+36)*x^
4+2925*(x^6-2*x^4+x^2)^(1/3)*RootOf(_Z^2-6*_Z+36)*x^2-235*RootOf(_Z^2-6*_Z+
36)^2*x^2-10620*x^4+2925*RootOf(_Z^2-6*_Z+36)*(x^6-2*x^4+x^2)^(2/3)-12312*(
x^6-2*x^4+x^2)^(1/3)*x^2-2781*RootOf(_Z^2-6*_Z+36)*x^2-12312*(x^6-2*x^4+x^2
)^(2/3)-2925*RootOf(_Z^2-6*_Z+36)*(x^6-2*x^4+x^2)^(1/3)+188*RootOf(_Z^2-6*_
Z+36)^2+13806*x^2+12312*(x^6-2*x^4+x^2)^(1/3)+138*RootOf(_Z^2-6*_Z+36)-3186
)/(-1+x)/(1+x)))*(x*(x^2-1))^(1/3)/x*(x^2*(x^2-1)^2)^(1/3)/(x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-x\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)^(1/3)/x^2,x, algorithm="maxima")
[Out] integrate((x^3 - x)^(1/3)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3-x\right)^{1/3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - x)^(1/3)/x^2,x)
[Out] int((x^3 - x)^(1/3)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x\left(x-1\right)\left(x+1\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x)**(1/3)/x**2,x)
[Out] Integral((x*(x - 1)*(x + 1))**(1/3)/x**2, x)
```

3.1328 $\int \frac{\sqrt[3]{-x^2+x^3}}{x}\,dx$

Optimal. Leaf size=106

$$\sqrt[3]{x^3-x^2}+\frac{1}{3}\log\left(\sqrt[3]{x^3-x^2}-x\right)-\frac{1}{6}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-x^2}+x}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.07, antiderivative size = 149, normalized size of antiderivative = 1.41, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2021, 2032, 59}

$$\sqrt[3]{x^3-x^2}+\frac{(x-1)^{2/3}x^{4/3}\log\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x-1}}-1\right)}{2\left(x^3-x^2\right)^{2/3}}+\frac{(x-1)^{2/3}x^{4/3}\log(x-1)}{6\left(x^3-x^2\right)^{2/3}}+\frac{(x-1)^{2/3}x^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{x-1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\left(x^3-x^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + x^3)^(1/3)/x,x]
```

```
[Out] (-x^2 + x^3)^(1/3) + ((-1 + x)^(2/3)*x^(4/3)*ArcTan[1/Sqrt[3] + (2*x^(1/3))
/(Sqrt[3]*(-1 + x)^(1/3))])/(Sqrt[3]*(-x^2 + x^3)^(2/3)) + ((-1 + x)^(2/3)*
x^(4/3)*Log[-1 + x^(1/3)/(-1 + x)^(1/3)])/(2*(-x^2 + x^3)^(2/3)) + ((-1 + x
)^(2/3)*x^(4/3)*Log[-1 + x])/(6*(-x^2 + x^3)^(2/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt[3]{-x^2+x^3}}{x}\,dx = \sqrt[3]{-x^2+x^3} - \frac{1}{3}\int \frac{x}{\left(-x^2+x^3\right)^{2/3}}\,dx$$
$$= \sqrt[3]{-x^2+x^3} - \frac{\left((-1+x)^{2/3}x^{4/3}\right)\int \frac{1}{(-1+x)^{2/3}\sqrt[3]{x}}\,dx}{3\left(-x^2+x^3\right)^{2/3}}$$
$$= \sqrt[3]{-x^2+x^3} + \frac{(-1+x)^{2/3}x^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{-1+x}}\right)}{\sqrt{3}\left(-x^2+x^3\right)^{2/3}} + \frac{(-1+x)^{2/3}x^{4/3}\log\left(-1+\frac{\sqrt[3]{x}}{\sqrt[3]{-1+x}}\right)}{2\left(-x^2+x^3\right)^{2/3}}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.33

$$\frac{3\left((x-1)x^2\right)^{4/3}\ {}_2F_1\left(\frac{1}{3},\frac{4}{3};\frac{7}{3};1-x\right)}{4x^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + x^3)^(1/3)/x,x]
```

```
[Out] (3*((-1 + x)*x^2)^(4/3)*Hypergeometric2F1[1/3, 4/3, 7/3, 1 - x])/(4*x^(8/3)
)
```

IntegrateAlgebraic [A] time = 0.21, size = 106, normalized size = 1.00

$$\sqrt[3]{x^3-x^2} + \frac{1}{3}\log\left(\sqrt[3]{x^3-x^2}-x\right) - \frac{1}{6}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right) + \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x^2}+x}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^2 + x^3)^(1/3)/x,x]
```

```
[Out] (-x^2 + x^3)^(1/3) + ArcTan[(Sqrt[3]*x)/(x + 2*(-x^2 + x^3)^(1/3))]/Sqrt[3]
 + Log[-x + (-x^2 + x^3)^(1/3)]/3 - Log[x^2 + x*(-x^2 + x^3)^(1/3) + (-x^2
+ x^3)^(2/3)]/6
```

fricas [A] time = 0.42, size = 103, normalized size = 0.97

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-x^2\right)^{\frac{1}{3}}}{3x}\right)+\left(x^3-x^2\right)^{\frac{1}{3}}+\frac{1}{3}\log\left(-\frac{x-\left(x^3-x^2\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{6}\log\left(\frac{x^2+\left(x^3-x^2\right)^{\frac{1}{3}}x+\left(x^3-x^2\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 - x^2)^(1/3))/x) + (x^3
 - x^2)^(1/3) + 1/3*log(-(x - (x^3 - x^2)^(1/3))/x) - 1/6*log((x^2 + (x^3 -
 x^2)^(1/3)*x + (x^3 - x^2)^(2/3))/x^2)
```

giac [A] time = 0.14, size = 74, normalized size = 0.70

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right)+x\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x^2)^(1/3)/x,x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-1/x + 1)^(1/3) + 1)) + x*(-1/x + 1)^(1/3) - 1/6*log((-1/x + 1)^(2/3) + (-1/x + 1)^(1/3) + 1) + 1/3*log(abs((-1/x + 1)^(1/3) - 1))

maple [C] time = 0.50, size = 434, normalized size = 4.09

$$\left((-1+x)x^2\right)^{\frac{1}{3}}+\frac{\left(\frac{\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^2+15\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^3-2x^2+x\right)^{\frac{2}{3}}-24\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^3-2x^2+x\right)^{\frac{1}{3}}x-3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x+10\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2-9\left(x^3-2x^2+x\right)^{\frac{2}{3}}+24\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^3-2x^2+x\right)^{\frac{1}{3}}-15\left(x^3-2x^2+x\right)^{\frac{1}{3}}x+2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2-23\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x+25x^2+15\left(x^3-2x^2+x\right)^{\frac{1}{3}}+13\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-40x+15}{-1+x}\right)}{3}+\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(\frac{5\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^2+15\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^3-2x^2+x\right)^{\frac{2}{3}}+9\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^3-2x^2+x\right)^{\frac{1}{3}}x-15\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x-19\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+24\left(x^3-2x^2+x\right)^{\frac{2}{3}}-9\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^3-2x^2+x\right)^{\frac{1}{3}}-15\left(x^3-2x^2+x\right)^{\frac{1}{3}}x+10\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2+22\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x-4x^2+15\left(x^3-2x^2+x\right)^{\frac{1}{3}}-3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)+5x-1}{-1+x}\right)}{3}\right)\left((-1+x)x^2\right)^{\frac{1}{3}}\left((-1+x)^2x\right)^{\frac{1}{3}}}{x(-1+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-x^2)^(1/3)/x,x)

[Out] ((-1+x)*x^2)^(1/3)+(1/3*ln(-(RootOf(_Z^2+_Z+1)^2*x^2+15*RootOf(_Z^2+_Z+1)*(x^3-2*x^2+x)^(2/3)-24*RootOf(_Z^2+_Z+1)*(x^3-2*x^2+x)^(1/3)*x-3*RootOf(_Z^2+_Z+1)^2*x+10*RootOf(_Z^2+_Z+1)*x^2-9*(x^3-2*x^2+x)^(2/3)+24*RootOf(_Z^2+_Z+1)*(x^3-2*x^2+x)^(1/3)-15*(x^3-2*x^2+x)^(1/3)*x+2*RootOf(_Z^2+_Z+1)^2-23*RootOf(_Z^2+_Z+1)*x+25*x^2+15*(x^3-2*x^2+x)^(1/3)+13*RootOf(_Z^2+_Z+1)-40*x+15)/(-1+x))+1/3*RootOf(_Z^2+_Z+1)*ln((5*RootOf(_Z^2+_Z+1)^2*x^2+15*RootOf(_Z^2+_Z+1)*(x^3-2*x^2+x)^(2/3)+9*RootOf(_Z^2+_Z+1)*(x^3-2*x^2+x)^(1/3)*x-15*RootOf(_Z^2+_Z+1)^2*x-19*RootOf(_Z^2+_Z+1)*x^2+24*(x^3-2*x^2+x)^(2/3)-9*RootOf(_Z^2+_Z+1)*(x^3-2*x^2+x)^(1/3)-15*(x^3-2*x^2+x)^(1/3)*x+10*RootOf(_Z^2+_Z+1)^2+22*RootOf(_Z^2+_Z+1)*x-4*x^2+15*(x^3-2*x^2+x)^(1/3)-3*RootOf(_Z^2+_Z+1)+5*x-1)/(-1+x)))*((-1+x)*x^2)^(1/3)/x*((-1+x)^2*x)^(1/3)/(-1+x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3 - x^2\right)^{\frac{1}{3}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x^2)^(1/3)/x,x, algorithm="maxima")

[Out] integrate((x^3 - x^2)^(1/3)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3 - x^2\right)^{1/3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 - x^2)^(1/3)/x,x)

[Out] int((x^3 - x^2)^(1/3)/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x^2\,(x-1)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-x**2)**(1/3)/x,x)

[Out] Integral((x**2*(x - 1))**(1/3)/x, x)

3.1329 $\int \frac{-3b+ax^2}{(-b+ax^2+x^3)\sqrt[4]{-bx+ax^3}}\,dx$

Optimal. Leaf size=106

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^3-bx}}{\sqrt{ax^3-bx}+x^2}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-bx}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3-bx}}\right)$$

Rubi [F] time = 2.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3b+ax^2}{(-b+ax^2+x^3)\sqrt[4]{-bx+ax^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3*b + a*x^2)/((-b + a*x^2 + x^3)*(-(b*x) + a*x^3)^(1/4)),x]
```

```
[Out] (12*b*x^(1/4)*(-b + a*x^2)^(1/4)*Defer[Subst][Defer[Int][x^2/((-b + a*x^8)^
(1/4)*(b - a*x^8 - x^12)), x], x, x^(1/4)])/(-(b*x) + a*x^3)^(1/4) + (4*a*x
^(1/4)*(-b + a*x^2)^(1/4)*Defer[Subst][Defer[Int][x^10/((-b + a*x^8)^(1/4)*
(-b + a*x^8 + x^12)), x], x, x^(1/4)])/(-(b*x) + a*x^3)^(1/4)
```

Rubi steps

$$\int \frac{-3b+ax^2}{(-b+ax^2+x^3)\sqrt[4]{-bx+ax^3}}\,dx=\frac{\left(\sqrt[4]{x}\sqrt[4]{-b+ax^2}\right)\int \frac{-3b+ax^2}{\sqrt[4]{x}\sqrt[4]{-b+ax^2}(-b+ax^2+x^3)}\,dx}{\sqrt[4]{-bx+ax^3}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^2}\right)\text{Subst}\left(\int \frac{x^2(-3b+ax^8)}{\sqrt[4]{-b+ax^8}(-b+ax^8+x^{12})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^3}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^2}\right)\text{Subst}\left(\int\left(\frac{3bx^2}{\sqrt[4]{-b+ax^8}(b-ax^8-x^{12})}+\frac{ax^{10}}{\sqrt[4]{-b+ax^8}(-b+ax^8+x^{12})}\right)\right.}{\sqrt[4]{-bx+ax^3}}$$

$$=\frac{\left(4a\sqrt[4]{x}\sqrt[4]{-b+ax^2}\right)\text{Subst}\left(\int \frac{x^{10}}{\sqrt[4]{-b+ax^8}(-b+ax^8+x^{12})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^3}}+\frac{\left(12b\sqrt[4]{x}\right.}{}$$

Mathematica [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{-3b+ax^2}{(-b+ax^2+x^3)\sqrt[4]{-bx+ax^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3*b + a*x^2)/((-b + a*x^2 + x^3)*(-(b*x) + a*x^3)^(1/4)),x]
```

```
[Out] Integrate[(-3*b + a*x^2)/((-b + a*x^2 + x^3)*(-(b*x) + a*x^3)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.55, size = 106, normalized size = 1.00

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^3-bx}}{\sqrt{ax^3-bx}+x^2}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-bx}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3-bx}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*b + a*x^2)/((-b + a*x^2 + x^3)*(-(b*x) + a*x^3)^(1/4
)),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(-(x^2/Sqrt[2]) + Sqrt[-(b*x) + a*x^3]/Sqrt[2])/(x*(-(b*x)
 + a*x^3)^(1/4))]) + Sqrt[2]*ArcTanh[(Sqrt[2]*x*(-(b*x) + a*x^3)^(1/4))/(x^
2 + Sqrt[-(b*x) + a*x^3])]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-3*b)/(a*x^2+x^3-b)/(a*x^3-b*x)^(1/4),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - 3b}{\left(ax^3 - bx\right)^{\frac{1}{4}}\left(ax^2 + x^3 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-3*b)/(a*x^2+x^3-b)/(a*x^3-b*x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^2 - 3*b)/((a*x^3 - b*x)^(1/4)*(a*x^2 + x^3 - b)), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2 - 3b}{\left(a\,x^2 + x^3 - b\right)\left(a\,x^3 - bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-3*b)/(a*x^2+x^3-b)/(a*x^3-b*x)^(1/4),x)
```

```
[Out] int((a*x^2-3*b)/(a*x^2+x^3-b)/(a*x^3-b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - 3b}{\left(ax^3 - bx\right)^{\frac{1}{4}}\left(ax^2 + x^3 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-3*b)/(a*x^2+x^3-b)/(a*x^3-b*x)^(1/4),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^2 - 3*b)/((a*x^3 - b*x)^(1/4)*(a*x^2 + x^3 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{3\,b - a\,x^2}{\left(a\,x^3 - b\,x\right)^{1/4}\,\left(x^3 + a\,x^2 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*b - a*x^2)/((a*x^3 - b*x)^(1/4)*(a*x^2 - b + x^3)),x)
[Out] int(-(3*b - a*x^2)/((a*x^3 - b*x)^(1/4)*(a*x^2 - b + x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-3*b)/(a*x**2+x**3-b)/(a*x**3-b*x)**(1/4),x)
[Out] Timed out
```

3.1330 $\int \frac{1}{(-2b+ax)\sqrt[4]{-bx^2+ax^3}}\,dx$

Optimal. Leaf size=106

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-bx^2}}{\sqrt{a}\,x}\right)}{\sqrt{2}\sqrt{a}\,b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-bx^2}}{\sqrt{a}\,x}\right)}{\sqrt{2}\sqrt{a}\,b^{3/4}}$$

Rubi [A] time = 0.36, antiderivative size = 170, normalized size of antiderivative = 1.60, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {2056, 107, 106, 490, 1211, 220, 1699, 203, 206}

$$\frac{\sqrt{\frac{ax}{b}}\sqrt[4]{ax-b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax-b}}{\sqrt[4]{b}\sqrt{\frac{ax}{b}}}\right)}{\sqrt{2}\,a\sqrt[4]{b}\sqrt[4]{ax^3-bx^2}}-\frac{\sqrt{\frac{ax}{b}}\sqrt[4]{ax-b}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax-b}}{\sqrt[4]{b}\sqrt{\frac{ax}{b}}}\right)}{\sqrt{2}\,a\sqrt[4]{b}\sqrt[4]{ax^3-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2*b + a*x)*(-(b*x^2) + a*x^3)^(1/4)),x]
```

```
[Out] (Sqrt[(a*x)/b]*(-b + a*x)^(1/4)*ArcTan[(Sqrt[2]*(-b + a*x)^(1/4))/(b^(1/4)*
Sqrt[(a*x)/b])])/(Sqrt[2]*a*b^(1/4)*(-(b*x^2) + a*x^3)^(1/4)) - (Sqrt[(a*x)
/b]*(-b + a*x)^(1/4)*ArcTanh[(Sqrt[2]*(-b + a*x)^(1/4))/(b^(1/4)*Sqrt[(a*x)
/b])])/(Sqrt[2]*a*b^(1/4)*(-(b*x^2) + a*x^3)^(1/4))
```

Rule 106

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
1/4)), x_Symbol] :> Dist[-4, Subst[Int[x^2/((b*e - a*f - b*x^4)*Sqrt[c - (d
*e)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f
}, x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 107

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
1/4)), x_Symbol] :> Dist[Sqrt[-((f*(c + d*x))/(d*e - c*f))]/Sqrt[c + d*x],
Int[1/((a + b*x)*Sqrt[-((c*f)/(d*e - c*f)) - (d*f*x)/(d*e - c*f)]*(e + f*x)
^(1/4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[-(f/(d*e - c*f)),
0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{(-2b+ax)\sqrt[4]{-bx^2+ax^3}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{-b+ax}\right)\int \frac{1}{\sqrt{x}(-2b+ax)\sqrt[4]{-b+ax}}\,dx}{\sqrt[4]{-bx^2+ax^3}}$$
$$= \frac{\left(\sqrt{\frac{ax}{b}}\sqrt[4]{-b+ax}\right)\int \frac{1}{\sqrt{\frac{ax}{b}}(-2b+ax)\sqrt[4]{-b+ax}}\,dx}{\sqrt[4]{-bx^2+ax^3}}$$
$$= -\frac{\left(4\sqrt{\frac{ax}{b}}\sqrt[4]{-b+ax}\right)\text{Subst}\left(\int \frac{x^2}{(ab-ax^4)\sqrt{1+\frac{x^4}{b}}}\,dx, x, \sqrt[4]{-b+ax}\right)}{\sqrt[4]{-bx^2+ax^3}}$$
$$= -\frac{\left(2\sqrt{\frac{ax}{b}}\sqrt[4]{-b+ax}\right)\text{Subst}\left(\int \frac{1}{(\sqrt{b}-x^2)\sqrt{1+\frac{x^4}{b}}}\,dx, x, \sqrt[4]{-b+ax}\right)}{a\sqrt[4]{-bx^2+ax^3}} + \frac{\left(2\sqrt{\frac{ax}{b}}\sqrt[4]{-b+\right.}{}$$
$$= \frac{\left(\sqrt{\frac{ax}{b}}\sqrt[4]{-b+ax}\right)\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{(\sqrt{b}+x^2)\sqrt{1+\frac{x^4}{b}}}\,dx, x, \sqrt[4]{-b+ax}\right)}{a\sqrt{b}\sqrt[4]{-bx^2+ax^3}} - \frac{\left(\sqrt{\frac{ax}{b}}\sqrt[4]{-b+ax}\right)}{}$$
$$= -\frac{\left(\sqrt{\frac{ax}{b}}\sqrt[4]{-b+ax}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-2x^2}\,dx, x, \frac{\sqrt[4]{-b+ax}}{\sqrt{\frac{ax}{b}}}\right)}{a\sqrt[4]{-bx^2+ax^3}} + \frac{\left(\sqrt{\frac{ax}{b}}\sqrt[4]{-b+ax}\right)\text{Subst}\left(\right.}{a\sqrt[4]{-bx^2}}$$
$$= \frac{\sqrt{\frac{ax}{b}}\sqrt[4]{-b+ax}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-b+ax}}{\sqrt[4]{b}\sqrt{\frac{ax}{b}}}\right)}{\sqrt{2}a\sqrt[4]{b}\sqrt[4]{-bx^2+ax^3}} - \frac{\sqrt{\frac{ax}{b}}\sqrt[4]{-b+ax}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-b+ax}}{\sqrt[4]{b}\sqrt{\frac{ax}{b}}}\right)}{\sqrt{2}a\sqrt[4]{b}\sqrt[4]{-bx^2+ax^3}}$$

Mathematica [C] time = 0.05, size = 61, normalized size = 0.58

$$-\frac{x\sqrt[4]{\frac{b-ax}{b}}F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{ax}{b},\frac{ax}{2b}\right)}{b\sqrt[4]{x^2(ax-b)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2*b + a*x)*(-(b*x^2) + a*x^3)^(1/4)),x]
```

```
[Out] -((x*((b - a*x)/b)^(1/4)*AppellF1[1/2, 1/4, 1, 3/2, (a*x)/b, (a*x)/(2*b)])/
(b*(x^2*(-b + a*x))^(1/4)))
```

IntegrateAlgebraic [A] time = 0.32, size = 106, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-bx^2}}{\sqrt{a}x}\right)}{\sqrt{2}\sqrt{a}b^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-bx^2}}{\sqrt{a}x}\right)}{\sqrt{2}\sqrt{a}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2*b + a*x)*(-(b*x^2) + a*x^3)^(1/4)),x]
```

```
[Out] ArcTan[(Sqrt[2]*b^(1/4)*(-(b*x^2) + a*x^3)^(1/4))/(Sqrt[a]*x)]/(Sqrt[2]*Sqr
t[a]*b^(3/4)) - ArcTanh[(Sqrt[2]*b^(1/4)*(-(b*x^2) + a*x^3)^(1/4))/(Sqrt[a]
*x)]/(Sqrt[2]*Sqrt[a]*b^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-2*b)/(a*x^3-b*x^2)^(1/4),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^3 - bx^2\right)^{\frac{1}{4}}(ax - 2\,b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-2*b)/(a*x^3-b*x^2)^(1/4),x, algorithm="giac")
[Out] integrate(1/((a*x^3 - b*x^2)^(1/4)*(a*x - 2*b)), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax - 2b)\left(a\,x^3 - b\,x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-2*b)/(a*x^3-b*x^2)^(1/4),x)
[Out] int(1/(a*x-2*b)/(a*x^3-b*x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^3 - bx^2\right)^{\frac{1}{4}}(ax - 2\,b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-2*b)/(a*x^3-b*x^2)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((a*x^3 - b*x^2)^(1/4)*(a*x - 2*b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{(2\,b - a\,x)\,\left(a\,x^3 - b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((2*b - a*x)*(a*x^3 - b*x^2)^(1/4)),x)
[Out] -int(1/((2*b - a*x)*(a*x^3 - b*x^2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^2\,(ax - b)}\,(ax - 2b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-2*b)/(a*x**3-b*x**2)**(1/4),x)
[Out] Integral(1/((x**2*(a*x - b))**(1/4)*(a*x - 2*b)), x)
```

3.1331 $\int \frac{b^3+a^3x^3}{\left(-b^3+a^3x^3\right)\sqrt{b^4+a^4x^4}}\,dx$

Optimal. Leaf size=106

$$-\frac{4\tan^{-1}\left(\frac{abx}{\sqrt{a^4x^4+b^4}+a^2x^2+abx+b^2}\right)}{3ab}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}abx}{\sqrt{a^4x^4+b^4}+a^2x^2-2abx+b^2}\right)}{3ab}$$

Rubi [C] time = 2.86, antiderivative size = 662, normalized size of antiderivative = 6.25, number of steps used = 29, number of rules used = 13, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {6725, 220, 2074, 1725, 1211, 1699, 208, 1248, 725, 206, 6728, 1217, 1707}

$$-\frac{2\tan^{-1}\left(\frac{abx}{\sqrt{a^4x^4+b^4}}\right)}{3ab}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}abx}{\sqrt{a^4x^4+b^4}}\right)}{3\sqrt{2}ab}-\frac{\tanh^{-1}\left(\frac{a^2x^2+b^2}{\sqrt{2}\sqrt{a^4x^4+b^4}}\right)}{3\sqrt{2}ab}-\frac{\left(a-\sqrt{3}\sqrt{-a^2}\right)\left(a^2x^2+b^2\right)\sqrt{\frac{a^4x^4+b^4}{\left(a^2x^2+b^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{6a^2b\sqrt{a^4x^4+b^4}}-\frac{\left(\sqrt{3}\sqrt{-a^2}+a\right)\left(a^2x^2+b^2\right)\sqrt{\frac{a^4x^4+b^4}{\left(a^2x^2+b^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{6a^2b\sqrt{a^4x^4+b^4}}+\frac{\left(a^2x^2+b^2\right)\sqrt{\frac{a^4x^4+b^4}{\left(a^2x^2+b^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{3ab\sqrt{a^4x^4+b^4}}+\frac{\left(a-\sqrt{3}\sqrt{-a^2}\right)\tanh^{-1}\left(\frac{\sqrt{a}\left(\left(a-\sqrt{3}\sqrt{-a^2}\right)^2x^2+4b^2\right)}{2\sqrt{2}\sqrt{\sqrt{3}\sqrt{-a^2}+a}\sqrt{a^4x^4+b^4}}\right)}{3\sqrt{2}a^{3/2}\sqrt{\sqrt{3}\sqrt{-a^2}+a}\,b}+\frac{\left(\sqrt{3}\sqrt{-a^2}+a\right)\tanh^{-1}\left(\frac{\sqrt{a}\left(\left(\sqrt{3}\sqrt{-a^2}+a\right)^2x^2+4b^2\right)}{2\sqrt{2}\sqrt{a-\sqrt{3}\sqrt{-a^2}}\sqrt{a^4x^4+b^4}}\right)}{3\sqrt{2}a^{3/2}\sqrt{a-\sqrt{3}\sqrt{-a^2}}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(b^3 + a^3*x^3)/((-b^3 + a^3*x^3)*Sqrt[b^4 + a^4*x^4]),x]
```

```
[Out] (-2*ArcTan[(a*b*x)/Sqrt[b^4 + a^4*x^4]])/(3*a*b) - ArcTanh[(Sqrt[2]*a*b*x)/
Sqrt[b^4 + a^4*x^4]]/(3*Sqrt[2]*a*b) - ArcTanh[(b^2 + a^2*x^2)/(Sqrt[2]*Sqr
t[b^4 + a^4*x^4])]/(3*Sqrt[2]*a*b) + ((a - Sqrt[3]*Sqrt[-a^2])*ArcTanh[(Sqr
t[a]*(4*b^2 + (a - Sqrt[3]*Sqrt[-a^2])^2*x^2))/(2*Sqrt[2]*Sqrt[a + Sqrt[3]*
Sqrt[-a^2]]*Sqrt[b^4 + a^4*x^4])])/(3*Sqrt[2]*a^(3/2)*Sqrt[a + Sqrt[3]*Sqrt
[-a^2]]*b) + ((a + Sqrt[3]*Sqrt[-a^2])*ArcTanh[(Sqrt[a]*(4*b^2 + (a + Sqrt[
3]*Sqrt[-a^2])^2*x^2))/(2*Sqrt[2]*Sqrt[a - Sqrt[3]*Sqrt[-a^2]]*Sqrt[b^4 + a
^4*x^4])])/(3*Sqrt[2]*a^(3/2)*Sqrt[a - Sqrt[3]*Sqrt[-a^2]]*b) + ((b^2 + a^2
*x^2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b],
1/2])/(3*a*b*Sqrt[b^4 + a^4*x^4]) - ((a - Sqrt[3]*Sqrt[-a^2])*(b^2 + a^2*x^
2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2
])/(6*a^2*b*Sqrt[b^4 + a^4*x^4]) - ((a + Sqrt[3]*Sqrt[-a^2])*(b^2 + a^2*x^2
)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2]
)/(6*a^2*b*Sqrt[b^4 + a^4*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
 + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[d,
 Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Dist[e, Int[x/((d^2 - e^
2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{b^3+a^3x^3}{\left(-b^3+a^3x^3\right)\sqrt{b^4+a^4x^4}}\,dx &= \int \left(\frac{1}{\sqrt{b^4+a^4x^4}}+\frac{2b^3}{\left(-b^3+a^3x^3\right)\sqrt{b^4+a^4x^4}}\right)dx \\
&= \left(2b^3\right)\int \frac{1}{\left(-b^3+a^3x^3\right)\sqrt{b^4+a^4x^4}}\,dx+\int \frac{1}{\sqrt{b^4+a^4x^4}}\,dx \\
&= \frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{2ab\sqrt{b^4+a^4x^4}}+\left(2b^3\right)\int\left(-\frac{1}{3b^2(b-ax)\sqrt{b^4+a^4x}} \right. \\
&= \frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{2ab\sqrt{b^4+a^4x^4}}-\frac{1}{3}(2b)\int\frac{1}{(b-ax)\sqrt{b^4+a^4x^4}}\,dx \\
&= \frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{2ab\sqrt{b^4+a^4x^4}}+\frac{1}{3}(2b)\int\left(\frac{-a+\sqrt{3}\sqrt{-}}{\left(ab-\sqrt{3}\sqrt{-a^2}\,b+2a^2\right.} \right. \\
&= \frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{2ab\sqrt{b^4+a^4x^4}}-\frac{1}{3}\int\frac{1}{\sqrt{b^4+a^4x^4}}\,dx-\frac{1}{3}\int\frac{}{\left(b^2-\right.} \\
&= \frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{3ab\sqrt{b^4+a^4x^4}}+\frac{1}{3}(ab)\,\text{Subst}\left(\int\frac{1}{2a^4b^4-x^2}\,dx,x,\right. \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}-\frac{\tanh^{-1}\left(\frac{b^2+a^2x^2}{\sqrt{2}\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}+\frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan\right.}{3ab\sqrt{b^4+a^4x^4}} \\
&= -\frac{2\tan^{-1}\left(\frac{abx}{\sqrt{b^4+a^4x^4}}\right)}{3ab}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}-\frac{\tanh^{-1}\left(\frac{b^2+a^2x^2}{\sqrt{2}\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}+\frac{\left(b^2+a^2x\right.}{} \\
&= -\frac{2\tan^{-1}\left(\frac{abx}{\sqrt{b^4+a^4x^4}}\right)}{3ab}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}-\frac{\tanh^{-1}\left(\frac{b^2+a^2x^2}{\sqrt{2}\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}+\frac{\left(a-\sqrt{3}\right.}{}
\end{aligned}
$$

Mathematica [C] time = 3.87, size = 566, normalized size = 5.34

$$
\frac{a^3\left(18ia\sqrt[4]{a^4}\,b\sqrt{\frac{a^4x^4}{b^4}+1}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}\,x\right)\middle|-1\right)-12ia\sqrt[4]{a^4}\,b\sqrt{\frac{a^4x^4}{b^4}+1}\,\Pi\left(-\frac{i}{2}-\frac{\sqrt{3}}{2};i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}\,x\right)\middle|-1\right)-12ia\sqrt[4]{a^4}\,b\sqrt{\frac{a^4x^4}{b^4}+1}\,\Pi\left(\frac{1}{2}\left(-i+\sqrt{3}\right);i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}\,x\right)\middle|-1\right)-\frac{3}{2}\sqrt{\frac{ia^2}{b^2}}\left((4+4i)ab\sqrt[4]{b^4}\sqrt{\frac{2a^4x^4}{b^4}+2}\,\Pi\left(\frac{i\sqrt{a^4}\sqrt{b^4}}{a^2b^2};i\sinh^{-1}\left(\frac{(1+i)\sqrt[4]{a^4}\,x}{\sqrt{2}\sqrt[4]{b^4}}\right)\middle|-1\right)-\sqrt[4]{a^4}\sqrt{a^4x^4+b^4}\left(\left(1-i\sqrt{3}+\sqrt{6+6i\sqrt{3}}\right)\tan^{-1}\left(\frac{\left(\sqrt{3}+i\right)b^2-2ia^2x^2}{\sqrt{2-2i\sqrt{3}}\sqrt{a^4x^4+b^4}}\right)+\left(1+i\sqrt{3}+\sqrt{6-6i\sqrt{3}}\right)\tan^{-1}\left(\frac{2ia^2x^2+\left(\sqrt{3}-i\right)b^2}{\sqrt{2+2i\sqrt{3}}\sqrt{a^4x^4+b^4}}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{a^2x^2+b^2}{\sqrt{2}\sqrt{a^4x^4+b^4}}\right)\right)\right)\right)}{18\sqrt[4]{a^4}\,b^5\left(\frac{ia^2}{b^2}\right)^{5/2}\sqrt{a^4x^4+b^4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b^3 + a^3*x^3)/((-b^3 + a^3*x^3)*Sqrt[b^4 + a^4*x^4]),x]
```

```
[Out] (a^3*((18*I)*a*(a^4)^(1/4)*b*Sqrt[1 + (a^4*x^4)/b^4]*EllipticF[I*ArcSinh[Sq
rt[(I*a^2)/b^2]*x], -1] - (12*I)*a*(a^4)^(1/4)*b*Sqrt[1 + (a^4*x^4)/b^4]*El
lipticPi[-1/2*I - Sqrt[3]/2, I*ArcSinh[Sqrt[(I*a^2)/b^2]*x], -1] - (12*I)*a
*(a^4)^(1/4)*b*Sqrt[1 + (a^4*x^4)/b^4]*EllipticPi[(-I + Sqrt[3])/2, I*ArcSi
nh[Sqrt[(I*a^2)/b^2]*x], -1] - (3*Sqrt[(I*a^2)/b^2]*(-((a^4)^(1/4)*Sqrt[b^4
 + a^4*x^4]*((1 - I*Sqrt[3] + Sqrt[6 + (6*I)*Sqrt[3]])*ArcTan[((I + Sqrt[3]
)*b^2 - (2*I)*a^2*x^2)/(Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[b^4 + a^4*x^4])] + (1
+ I*Sqrt[3] + Sqrt[6 - (6*I)*Sqrt[3]])*ArcTan[((-I + Sqrt[3])*b^2 + (2*I)*a
^2*x^2)/(Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[b^4 + a^4*x^4])] + 2*Sqrt[2]*ArcTanh[
(b^2 + a^2*x^2)/(Sqrt[2]*Sqrt[b^4 + a^4*x^4])])) + (4 + 4*I)*a*b*(b^4)^(1/4
)*Sqrt[2 + (2*a^4*x^4)/b^4]*EllipticPi[(I*Sqrt[a^4]*Sqrt[b^4])/(a^2*b^2), I
*ArcSinh[((1 + I)*(a^4)^(1/4)*x)/(Sqrt[2]*(b^4)^(1/4))], -1]))/2))/(18*(a^4
)^(1/4)*((I*a^2)/b^2)^(5/2)*b^5*Sqrt[b^4 + a^4*x^4])
```

IntegrateAlgebraic [A] time = 2.30, size = 106, normalized size = 1.00

$$-\frac{4\tan^{-1}\left(\frac{abx}{\sqrt{a^4x^4+b^4}+a^2x^2+abx+b^2}\right)}{3ab}-\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{a^4x^4+b^4}+a^2x^2-2abx+b^2}\right)}{3ab}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^3 + a^3*x^3)/((-b^3 + a^3*x^3)*Sqrt[b^4 + a^4*x^4]),x
]
```

```
[Out] (-4*ArcTan[(a*b*x)/(b^2 + a*b*x + a^2*x^2 + Sqrt[b^4 + a^4*x^4])])/(3*a*b)
- (Sqrt[2]*ArcTanh[(Sqrt[2]*a*b*x)/(b^2 - 2*a*b*x + a^2*x^2 + Sqrt[b^4 + a^
4*x^4])])/(3*a*b)
```

fricas [A] time = 0.88, size = 166, normalized size = 1.57

$$\frac{\sqrt{2}\log\left(-\frac{3\,a^4x^4-4\,a^3bx^3+6\,a^2b^2x^2-4\,ab^3x+3\,b^4-2\,\sqrt{2}\,\sqrt{a^4x^4+b^4}\,\left(a^2x^2-abx+b^2\right)}{a^4x^4-4\,a^3bx^3+6\,a^2b^2x^2-4\,ab^3x+b^4}\right)+8\,\arctan\left(\frac{\sqrt{a^4x^4+b^4}}{a^2x^2+2\,abx+b^2}\right)}{12\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3+b^3)/(a^3*x^3-b^3)/(a^4*x^4+b^4)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/12*(sqrt(2)*log(-(3*a^4*x^4 - 4*a^3*b*x^3 + 6*a^2*b^2*x^2 - 4*a*b^3*x + 3
*b^4 - 2*sqrt(2)*sqrt(a^4*x^4 + b^4)*(a^2*x^2 - a*b*x + b^2))/(a^4*x^4 - 4*
a^3*b*x^3 + 6*a^2*b^2*x^2 - 4*a*b^3*x + b^4)) + 8*arctan(sqrt(a^4*x^4 + b^4
)/(a^2*x^2 + 2*a*b*x + b^2)))/(a*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^3x^3+b^3}{\sqrt{a^4x^4+b^4}\left(a^3x^3-b^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3+b^3)/(a^3*x^3-b^3)/(a^4*x^4+b^4)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((a^3*x^3 + b^3)/(sqrt(a^4*x^4 + b^4)*(a^3*x^3 - b^3)), x)
```

maple [C] time = 0.07, size = 443, normalized size = 4.18

$$\frac{\sqrt{1-\frac{ia^2x^2}{b^2}}\,\sqrt{1+\frac{ia^2x^2}{b^2}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{ia^2}{b^2}},i\right)}{\sqrt{\frac{ia^2}{b^2}}\,\sqrt{a^4x^4+b^4}}+\frac{b\left(\sum_{\_\alpha=\mathrm{RootOf}\left(a^2\_Z^2+ab\_Z+b^2\right)}\frac{\left(-\_\alpha a-2b\right)\left(\frac{\mathrm{arctanh}\left(\frac{\left(\_\alpha a+b\right)ab\left(a\,x^2+b\_\alpha\right)}{\sqrt{b^3\left(\_\alpha a+b\right)}\,\sqrt{a^4x^4+b^4}}\right)}{\sqrt{b^3\left(\_\alpha a+b\right)}}+\frac{2a\left(\_\alpha a+b\right)\sqrt{1-\frac{ia^2x^2}{b^2}}\,\sqrt{1+\frac{ia^2x^2}{b^2}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{ia^2}{b^2}},-\frac{i\_\alpha a}{b},\frac{\sqrt{-\frac{ia^2}{b^2}}}{\sqrt{\frac{ia^2}{b^2}}}\right)}{\sqrt{\frac{ia^2}{b^2}}\,b^2\sqrt{a^4x^4+b^4}}\right)}{2\_\alpha a+b}\right)}{3a}+\frac{2b\left(-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\left(2a^2b^2x^2+2b^4\right)\sqrt{2}}{4\sqrt{b^4}\,\sqrt{a^4x^4+b^4}}\right)}{4\sqrt{b^4}}-\frac{a\sqrt{1-\frac{ia^2x^2}{b^2}}\,\sqrt{1+\frac{ia^2x^2}{b^2}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{ia^2}{b^2}},-i,\frac{\sqrt{-\frac{ia^2}{b^2}}}{\sqrt{\frac{ia^2}{b^2}}}\right)}{\sqrt{\frac{ia^2}{b^2}}\,b\sqrt{a^4x^4+b^4}}\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^3*x^3+b^3)/(a^3*x^3-b^3)/(a^4*x^4+b^4)^(1/2),x)
```

```
[Out] 1/(I*a^2/b^2)^(1/2)*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/b^2*x^2)^(1/2)/(a^4*x^
4+b^4)^(1/2)*EllipticF(x*(I*a^2/b^2)^(1/2),I)+1/3*b/a*sum((-_alpha*a-2*b)/(
2*_alpha*a+b)*(1/(b^3*(_alpha*a+b))^(1/2)*arctanh((_alpha*a+b)*a*b*(a*x^2+_
alpha*b)/(b^3*(_alpha*a+b))^(1/2)/(a^4*x^4+b^4)^(1/2))+2/(I*a^2/b^2)^(1/2)*
a*(_alpha*a+b)/b^2*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/b^2*x^2)^(1/2)/(a^4*x^4
+b^4)^(1/2)*EllipticPi(x*(I*a^2/b^2)^(1/2),-I*_alpha*a/b,(-I*a^2/b^2)^(1/2)
/(I*a^2/b^2)^(1/2))),_alpha=RootOf(_Z^2*a^2+_Z*a*b+b^2))+2/3*b/a*(-1/4*2^(1
/2)/(b^4)^(1/2)*arctanh(1/4*(2*a^2*b^2*x^2+2*b^4)*2^(1/2)/(b^4)^(1/2)/(a^4*
x^4+b^4)^(1/2))-1/(I*a^2/b^2)^(1/2)/b*a*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/b^
2*x^2)^(1/2)/(a^4*x^4+b^4)^(1/2)*EllipticPi(x*(I*a^2/b^2)^(1/2),-I,(-I*a^2/
b^2)^(1/2)/(I*a^2/b^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^3x^3 + b^3}{\sqrt{a^4x^4 + b^4}\left(a^3x^3 - b^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3+b^3)/(a^3*x^3-b^3)/(a^4*x^4+b^4)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((a^3*x^3 + b^3)/(sqrt(a^4*x^4 + b^4)*(a^3*x^3 - b^3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a^3\,x^3 + b^3}{\left(b^3 - a^3\,x^3\right)\,\sqrt{a^4\,x^4 + b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^3 + a^3*x^3)/((b^3 - a^3*x^3)*(b^4 + a^4*x^4)^(1/2)),x)
```

```
[Out] int(-(b^3 + a^3*x^3)/((b^3 - a^3*x^3)*(b^4 + a^4*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax + b)\left(a^2x^2 - abx + b^2\right)}{(ax - b)\sqrt{a^4x^4 + b^4}\,\left(a^2x^2 + abx + b^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**3*x**3+b**3)/(a**3*x**3-b**3)/(a**4*x**4+b**4)**(1/2),x)
```

```
[Out] Integral((a*x + b)*(a**2*x**2 - a*b*x + b**2)/((a*x - b)*sqrt(a**4*x**4 + b
**4)*(a**2*x**2 + a*b*x + b**2)), x)
```

3.1332 $\int \frac{(1+x^6)(-1+2x^6)(-1+x^4+2x^6)^{5/4}}{x^{10}(-1-x^4+2x^6)}\,dx$

Optimal. Leaf size=106

$$\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{2x^6+x^4-1}}\right)-\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{2x^6+x^4-1}}\right)+\frac{\sqrt[4]{2x^6+x^4-1}\,\left(20x^{12}+38x^{10}+104x^8-20x^6-19\right.}{45x^9}$$

Rubi [F] time = 2.88, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+x^6)(-1+2x^6)(-1+x^4+2x^6)^{5/4}}{x^{10}(-1-x^4+2x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^6)*(-1 + 2*x^6)*(-1 + x^4 + 2*x^6)^(5/4))/(x^10*(-1 - x^4 + 2*x
^6)),x]
```

```
[Out] ((3 + I*Sqrt[7])*Defer[Int][(-1 + x^4 + 2*x^6)^(5/4)/(Sqrt[-1 - I*Sqrt[7]]
 - 2*x), x])/(2*Sqrt[-1 - I*Sqrt[7]]) + ((3 - I*Sqrt[7])*Defer[Int][(-1 + x^
4 + 2*x^6)^(5/4)/(Sqrt[-1 + I*Sqrt[7]] - 2*x), x])/(2*Sqrt[-1 + I*Sqrt[7]])
 + Defer[Int][(-1 + x^4 + 2*x^6)^(5/4)/(-1 + x), x]/4 + Defer[Int][(-1 + x^
4 + 2*x^6)^(5/4)/x^10, x] - Defer[Int][(-1 + x^4 + 2*x^6)^(5/4)/x^6, x] + D
efer[Int][(-1 + x^4 + 2*x^6)^(5/4)/x^4, x] + Defer[Int][(-1 + x^4 + 2*x^6)^
(5/4)/x^2, x] - Defer[Int][(-1 + x^4 + 2*x^6)^(5/4)/(1 + x), x]/4 + ((3 + I
*Sqrt[7])*Defer[Int][(-1 + x^4 + 2*x^6)^(5/4)/(Sqrt[-1 - I*Sqrt[7]] + 2*x),
 x])/(2*Sqrt[-1 - I*Sqrt[7]]) + ((3 - I*Sqrt[7])*Defer[Int][(-1 + x^4 + 2*x
^6)^(5/4)/(Sqrt[-1 + I*Sqrt[7]] + 2*x), x])/(2*Sqrt[-1 + I*Sqrt[7]])
```

Rubi steps

$$\int \frac{(1+x^6)(-1+2x^6)(-1+x^4+2x^6)^{5/4}}{x^{10}(-1-x^4+2x^6)}\,dx = \int \left(\frac{(-1+x^4+2x^6)^{5/4}}{x^{10}}-\frac{(-1+x^4+2x^6)^{5/4}}{x^6}+\frac{(-1+x^4+2x^6}{x^4}\right.$$

$$=\frac{1}{2}\int \frac{(-1+x^4+2x^6)^{5/4}}{-1+x^2}\,dx+\frac{1}{2}\int \frac{(-5-6x^2)(-1+x^4+2x^6)}{1+x^2+2x^4}$$

$$=\frac{1}{2}\int \left(\frac{(-1+x^4+2x^6)^{5/4}}{2(-1+x)}-\frac{(-1+x^4+2x^6)^{5/4}}{2(1+x)}\right)dx+\frac{1}{2}\int\left(\left(-\right.\right.$$

$$=\frac{1}{4}\int \frac{(-1+x^4+2x^6)^{5/4}}{-1+x}\,dx-\frac{1}{4}\int \frac{(-1+x^4+2x^6)^{5/4}}{1+x}\,dx+\left(-3\right.$$

$$=\frac{1}{4}\int \frac{(-1+x^4+2x^6)^{5/4}}{-1+x}\,dx-\frac{1}{4}\int \frac{(-1+x^4+2x^6)^{5/4}}{1+x}\,dx+\left(-3\right.$$

$$=\frac{1}{4}\int \frac{(-1+x^4+2x^6)^{5/4}}{-1+x}\,dx-\frac{1}{4}\int \frac{(-1+x^4+2x^6)^{5/4}}{1+x}\,dx+\frac{(3}{}$$

Mathematica [F] time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{(1+x^6)(-1+2x^6)(-1+x^4+2x^6)^{5/4}}{x^{10}(-1-x^4+2x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^6)*(-1 + 2*x^6)*(-1 + x^4 + 2*x^6)^(5/4))/(x^10*(-1 - x^4
 + 2*x^6)),x]
```

```
[Out] Integrate[((1 + x^6)*(-1 + 2*x^6)*(-1 + x^4 + 2*x^6)^(5/4))/(x^10*(-1 - x^4
 + 2*x^6)), x]
```

IntegrateAlgebraic [A] time = 2.76, size = 106, normalized size = 1.00

$$\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{2x^6+x^4-1}}\right)-\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{2x^6+x^4-1}}\right)+\frac{\sqrt[4]{2x^6+x^4-1}\,(20x^{12}+38x^{10}+104x^8-20x^6-19x^4+5)}{45x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^6)*(-1 + 2*x^6)*(-1 + x^4 + 2*x^6)^(5/4))/(x^10*
(-1 - x^4 + 2*x^6)),x]
```

```
[Out] ((-1 + x^4 + 2*x^6)^(1/4)*(5 - 19*x^4 - 20*x^6 + 104*x^8 + 38*x^10 + 20*x^1
2))/(45*x^9) + 2^(1/4)*ArcTan[(2^(1/4)*x)/(-1 + x^4 + 2*x^6)^(1/4)] - 2^(1/
4)*ArcTanh[(2^(1/4)*x)/(-1 + x^4 + 2*x^6)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)*(2*x^6-1)*(2*x^6+x^4-1)^(5/4)/x^10/(2*x^6-x^4-1),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^6+x^4-1)^{\frac{5}{4}}(2x^6-1)(x^6+1)}{(2x^6-x^4-1)x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)*(2*x^6-1)*(2*x^6+x^4-1)^(5/4)/x^10/(2*x^6-x^4-1),x, algor
ithm="giac")
```

```
[Out] integrate((2*x^6 + x^4 - 1)^(5/4)*(2*x^6 - 1)*(x^6 + 1)/((2*x^6 - x^4 - 1)*
x^10), x)
```

maple [C] time = 3.45, size = 1584, normalized size = 14.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)*(2*x^6-1)*(2*x^6+x^4-1)^(5/4)/x^10/(2*x^6-x^4-1),x)
```

```
[Out] 1/45*(40*x^18+96*x^16+246*x^14+44*x^12-96*x^10-123*x^8+30*x^6+24*x^4-5)/x^9
/(2*x^6+x^4-1)^(3/4)+(-1/2*RootOf(_Z^4-2)*ln(-(8*RootOf(_Z^4-2)^2*x^18+20*R
ootOf(_Z^4-2)^2*x^16+8*(8*x^18+12*x^16+6*x^14-11*x^12-12*x^10-3*x^8+6*x^6+3
```

```
*x^4-1)^(1/4)*RootOf(_Z^4-2)^3*x^13+14*RootOf(_Z^4-2)^2*x^14+8*(8*x^18+12*x
^16+6*x^14-11*x^12-12*x^10-3*x^8+6*x^6+3*x^4-1)^(1/4)*RootOf(_Z^4-2)^3*x^11
-9*x^12*RootOf(_Z^4-2)^2+2*(8*x^18+12*x^16+6*x^14-11*x^12-12*x^10-3*x^8+6*x
^6+3*x^4-1)^(1/4)*RootOf(_Z^4-2)^3*x^9-20*x^10*RootOf(_Z^4-2)^2-8*(8*x^18+1
2*x^16+6*x^14-11*x^12-12*x^10-3*x^8+6*x^6+3*x^4-1)^(1/4)*RootOf(_Z^4-2)^3*x
^7+8*(8*x^18+12*x^16+6*x^14-11*x^12-12*x^10-3*x^8+6*x^6+3*x^4-1)^(1/2)*x^8-
7*x^8*RootOf(_Z^4-2)^2-4*(8*x^18+12*x^16+6*x^14-11*x^12-12*x^10-3*x^8+6*x^6
+3*x^4-1)^(1/4)*RootOf(_Z^4-2)^3*x^5+4*(8*x^18+12*x^16+6*x^14-11*x^12-12*x^
10-3*x^8+6*x^6+3*x^4-1)^(1/2)*x^6+6*RootOf(_Z^4-2)^2*x^6+4*(8*x^18+12*x^16+
6*x^14-11*x^12-12*x^10-3*x^8+6*x^6+3*x^4-1)^(3/4)*RootOf(_Z^4-2)*x^3+5*Root
Of(_Z^4-2)^2*x^4+2*(8*x^18+12*x^16+6*x^14-11*x^12-12*x^10-3*x^8+6*x^6+3*x^4
-1)^(1/4)*RootOf(_Z^4-2)^3*x-4*(8*x^18+12*x^16+6*x^14-11*x^12-12*x^10-3*x^8
+6*x^6+3*x^4-1)^(1/2)*x^2-RootOf(_Z^4-2)^2)/(2*x^6+x^4-1)^2/(-1+x)/(1+x)/(2
*x^4+x^2+1))+1/2*RootOf(_Z^2+RootOf(_Z^4-2)^2)*ln((8*RootOf(_Z^4-2)^2*x^18+
20*RootOf(_Z^4-2)^2*x^16-8*(8*x^18+12*x^16+6*x^14-11*x^12-12*x^10-3*x^8+6*x
^6+3*x^4-1)^(1/4)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2*x^13+14*Ro
otOf(_Z^4-2)^2*x^14-8*(8*x^18+12*x^16+6*x^14-11*x^12-12*x^10-3*x^8+6*x^6+3*
x^4-1)^(1/4)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2*x^11-9*x^12*Roo
tOf(_Z^4-2)^2-2*(8*x^18+12*x^16+6*x^14-11*x^12-12*x^10-3*x^8+6*x^6+3*x^4-1)
^(1/4)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2*x^9-20*x^10*RootOf(_Z
^4-2)^2+8*(8*x^18+12*x^16+6*x^14-11*x^12-12*x^10-3*x^8+6*x^6+3*x^4-1)^(1/4)
*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2*x^7-8*(8*x^18+12*x^16+6*x^1
4-11*x^12-12*x^10-3*x^8+6*x^6+3*x^4-1)^(1/2)*x^8-7*x^8*RootOf(_Z^4-2)^2+4*(
8*x^18+12*x^16+6*x^14-11*x^12-12*x^10-3*x^8+6*x^6+3*x^4-1)^(1/4)*RootOf(_Z^
2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2*x^5-4*(8*x^18+12*x^16+6*x^14-11*x^12-1
2*x^10-3*x^8+6*x^6+3*x^4-1)^(1/2)*x^6+6*RootOf(_Z^4-2)^2*x^6+4*RootOf(_Z^2+
RootOf(_Z^4-2)^2)*(8*x^18+12*x^16+6*x^14-11*x^12-12*x^10-3*x^8+6*x^6+3*x^4-
1)^(3/4)*x^3+5*RootOf(_Z^4-2)^2*x^4-2*(8*x^18+12*x^16+6*x^14-11*x^12-12*x^1
0-3*x^8+6*x^6+3*x^4-1)^(1/4)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2
*x+4*(8*x^18+12*x^16+6*x^14-11*x^12-12*x^10-3*x^8+6*x^6+3*x^4-1)^(1/2)*x^2-
RootOf(_Z^4-2)^2)/(2*x^6+x^4-1)^2/(-1+x)/(1+x)/(2*x^4+x^2+1)))/(2*x^6+x^4-1
)^(3/4)*((2*x^6+x^4-1)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^6+x^4-1\right)^{\frac{5}{4}}\left(2\,x^6-1\right)\left(x^6+1\right)}{\left(2\,x^6-x^4-1\right)x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)*(2*x^6-1)*(2*x^6+x^4-1)^(5/4)/x^10/(2*x^6-x^4-1),x, algor
ithm="maxima")
```

```
[Out] integrate((2*x^6 + x^4 - 1)^(5/4)*(2*x^6 - 1)*(x^6 + 1)/((2*x^6 - x^4 - 1)*
x^10), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^6+1\right)\,\left(2\,x^6-1\right)\,\left(2\,x^6+x^4-1\right)^{5/4}}{x^{10}\,\left(-2\,x^6+x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^6 + 1)*(2*x^6 - 1)*(x^4 + 2*x^6 - 1)^(5/4))/(x^10*(x^4 - 2*x^6 + 1
)),x)
```

```
[Out] int(-((x^6 + 1)*(2*x^6 - 1)*(x^4 + 2*x^6 - 1)^(5/4))/(x^10*(x^4 - 2*x^6 + 1
)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)*(2*x**6-1)*(2*x**6+x**4-1)**(5/4)/x**10/(2*x**6-x**4-1),
x)
```

```
[Out] Timed out
```

3.1333 $$\int \frac{\left(\left(1-3k^2\right)x+2k^2x^3\right)\left(1-2k^2x^2+k^4x^4\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{3/4}\left(1-d+\left(-1+3dk^2\right)x^2-3dk^4x^4+dk^6x^6\right)}\,dx$$

Optimal. Leaf size=106

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,k^2x^2-\sqrt[4]{d}}{\sqrt[4]{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,k^2x^2-\sqrt[4]{d}}{\sqrt[4]{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{d^{3/4}}$$

Rubi [F] time = 12.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(\left(1-3k^2\right)x+2k^2x^3\right)\left(1-2k^2x^2+k^4x^4\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{3/4}\left(1-d+\left(-1+3dk^2\right)x^2-3dk^4x^4+dk^6x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(((1 - 3*k^2)*x + 2*k^2*x^3)*(1 - 2*k^2*x^2 + k^4*x^4))/(((1 - x^2)*(1
- k^2*x^2))^(3/4)*(1 - d + (-1 + 3*d*k^2)*x^2 - 3*d*k^4*x^4 + d*k^6*x^6)),x
]
```

```
[Out] (-4*k^2*(1 - x^2)^(3/4)*(1 - k^2*x^2)^(3/4)*Defer[Subst][Defer[Int][(1 - k^
2 + k^2*x^4)^(5/4)/(x^4 - d*(1 + k^2*(-1 + x^4))^3), x], x, (1 - x^2)^(1/4)
])/((1 - x^2)*(1 - k^2*x^2))^(3/4) + (2*(1 - 3*k^2)*(1 - x^2)^(3/4)*(1 - k^
2*x^2)^(3/4)*Defer[Subst][Defer[Int][(1 - k^2 + k^2*x^4)^(5/4)/(-x^4 + d*(1
 + k^2*(-1 + x^4))^3), x], x, (1 - x^2)^(1/4)])/((1 - x^2)*(1 - k^2*x^2))^(
3/4) - (4*k^2*(1 - x^2)^(3/4)*(1 - k^2*x^2)^(3/4)*Defer[Subst][Defer[Int][(
x^4*(1 - k^2 + k^2*x^4)^(5/4))/(-x^4 + d*(1 + k^2*(-1 + x^4))^3), x], x, (1
 - x^2)^(1/4)])/((1 - x^2)*(1 - k^2*x^2))^(3/4)
```

Rubi steps

$$\int \frac{\left(\left(1-3k^2\right)x+2k^2x^3\right)\left(1-2k^2x^2+k^4x^4\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{3/4}\left(1-d+\left(-1+3dk^2\right)x^2-3dk^4x^4+dk^6x^6\right)}dx = \frac{\int \frac{\left(-k^2+k^4x^2\right)^2\left(\left(1-3k^2\right)x+2k^2x^3\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{3/4}\left(1-d+\left(-1+3dk^2\right)x^2-3dk^4}}{k^4}$$

$$= \frac{\int \frac{x\left(1-3k^2+2k^2x^2\right)\left(-k^2+k^4x^2\right)^2}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{3/4}\left(1-d+\left(-1+3dk^2\right)x^2-3dk^4}}{k^4}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(1-3k^2+2k^2x\right)\left(-k^2+k^4\right.}{\left(\left(1-x\right)\left(1-k^2x\right)\right)^{3/4}\left(1-d+\left(-1+3dk^2\right)x\right.}\right.}{2k^4}$$

$$= \frac{\left(\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right)^{3/4}\right)\text{Subst}\left(\int \right.}{2k^4\left(\left(1\right.\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right)^{3/4}\right)\text{Subst}\left(\int \right.}{2\left(\left(1-x^2\right)\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right)^{3/4}\right)\text{Subst}\left(\int \right.}{2\left(\left(1-x^2\right)\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right)^{3/4}\right)\text{Subst}\left(\int \left(\right.\right.}{}$$

$$= \frac{\left(k^2\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right)^{3/4}\right)\text{Subst}\left(\int\right.}{\left(\left(1-x^2\right)\right.}$$

$$= -\frac{\left(4k^2\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right)^{3/4}\right)\text{Subst}}{\left(\left(1-x^2\right)\right.}$$

$$= -\frac{\left(4k^2\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right)^{3/4}\right)\text{Subst}}{\left(\left(1-x^2\right)\left(1\right.\right.}$$

$$= -\frac{\left(4k^2\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right)^{3/4}\right)\text{Subst}}{}$$

$$= -\frac{\left(4k^2\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right)^{3/4}\right)\text{Subst}}{}$$

$$= -\frac{\left(4k^2\left(1-x^2\right)^{3/4}\left(1-k^2x^2\right)^{3/4}\right)\text{Subst}}{\left(\left(1-x^2\right)\left(1\right.\right.}$$

Mathematica [F] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(1-3k^2\right)x+2k^2x^3\right)\left(1-2k^2x^2+k^4x^4\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{3/4}\left(1-d+\left(-1+3dk^2\right)x^2-3dk^4x^4+dk^6x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(((1 - 3*k^2)*x + 2*k^2*x^3)*(1 - 2*k^2*x^2 + k^4*x^4))/(((1 - x^
2)*(1 - k^2*x^2))^(3/4)*(1 - d + (-1 + 3*d*k^2)*x^2 - 3*d*k^4*x^4 + d*k^6*x
^6)),x]
```

```
[Out] Integrate[(((1 - 3*k^2)*x + 2*k^2*x^3)*(1 - 2*k^2*x^2 + k^4*x^4))/(((1 - x^
2)*(1 - k^2*x^2))^(3/4)*(1 - d + (-1 + 3*d*k^2)*x^2 - 3*d*k^4*x^4 + d*k^6*x
^6)), x]
```

IntegrateAlgebraic [A] time = 15.95, size = 106, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{d}\,k^2x^2-\sqrt[4]{d}}{\sqrt[4]{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{d^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{d}\,k^2x^2-\sqrt[4]{d}}{\sqrt[4]{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(((1 - 3*k^2)*x + 2*k^2*x^3)*(1 - 2*k^2*x^2 + k^4*x^4))/
(((1 - x^2)*(1 - k^2*x^2))^(3/4)*(1 - d + (-1 + 3*d*k^2)*x^2 - 3*d*k^4*x^4
+ d*k^6*x^6)),x]
```

```
[Out] ArcTan[(-d^(1/4) + d^(1/4)*k^2*x^2)/(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/4)]/d
^(3/4) - ArcTanh[(-d^(1/4) + d^(1/4)*k^2*x^2)/(1 + (-1 - k^2)*x^2 + k^2*x^4
)^(1/4)]/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3*k^2+1)*x+2*k^2*x^3)*(k^4*x^4-2*k^2*x^2+1)/((-x^2+1)*(-k^2*x^
2+1))^(3/4)/(1-d+(3*d*k^2-1)*x^2-3*d*k^4*x^4+d*k^6*x^6),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(k^4x^4-2\,k^2x^2+1\right)\left(2\,k^2x^3-\left(3\,k^2-1\right)x\right)}{\left(dk^6x^6-3\,dk^4x^4+\left(3\,dk^2-1\right)x^2-d+1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3*k^2+1)*x+2*k^2*x^3)*(k^4*x^4-2*k^2*x^2+1)/((-x^2+1)*(-k^2*x^
2+1))^(3/4)/(1-d+(3*d*k^2-1)*x^2-3*d*k^4*x^4+d*k^6*x^6),x, algorithm="giac"
)
```

```
[Out] integrate((k^4*x^4 - 2*k^2*x^2 + 1)*(2*k^2*x^3 - (3*k^2 - 1)*x)/((d*k^6*x^6
 - 3*d*k^4*x^4 + (3*d*k^2 - 1)*x^2 - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(3/4)
), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(-3k^2+1\right)x+2k^2x^3\right)\left(k^4x^4-2k^2x^2+1\right)}{\left(\left(-x^2+1\right)\left(-k^2x^2+1\right)\right)^{\frac{3}{4}}\left(1-d+\left(3d\,k^2-1\right)x^2-3d\,k^4x^4+d\,k^6x^6\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-3*k^2+1)*x+2*k^2*x^3)*(k^4*x^4-2*k^2*x^2+1)/((-x^2+1)*(-k^2*x^2+1))^
(3/4)/(1-d+(3*d*k^2-1)*x^2-3*d*k^4*x^4+d*k^6*x^6),x)
```

```
[Out] int(((-3*k^2+1)*x+2*k^2*x^3)*(k^4*x^4-2*k^2*x^2+1)/((-x^2+1)*(-k^2*x^2+1))^
(3/4)/(1-d+(3*d*k^2-1)*x^2-3*d*k^4*x^4+d*k^6*x^6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(k^{4}x^{4} - 2\,k^{2}x^{2} + 1\right)\left(2\,k^{2}x^{3} - \left(3\,k^{2} - 1\right)x\right)}{\left(dk^{6}x^{6} - 3\,dk^{4}x^{4} + \left(3\,dk^{2} - 1\right)x^{2} - d + 1\right)\left(\left(k^{2}x^{2} - 1\right)\left(x^{2} - 1\right)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3*k^2+1)*x+2*k^2*x^3)*(k^4*x^4-2*k^2*x^2+1)/((-x^2+1)*(-k^2*x^
2+1))^(3/4)/(1-d+(3*d*k^2-1)*x^2-3*d*k^4*x^4+d*k^6*x^6),x, algorithm="maxim
a")
```

```
[Out] integrate((k^4*x^4 - 2*k^2*x^2 + 1)*(2*k^2*x^3 - (3*k^2 - 1)*x)/((d*k^6*x^6
 - 3*d*k^4*x^4 + (3*d*k^2 - 1)*x^2 - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(3/4)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,k^2\,x^3-x\,\left(3\,k^2-1\right)\right)\,\left(k^4\,x^4-2\,k^2\,x^2+1\right)}{{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)}^{3/4}\,\left(x^2\,\left(3\,d\,k^2-1\right)-d-3\,d\,k^4\,x^4+d\,k^6\,x^6+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*k^2*x^3 - x*(3*k^2 - 1))*(k^4*x^4 - 2*k^2*x^2 + 1))/(((x^2 - 1)*(k^
2*x^2 - 1))^(3/4)*(x^2*(3*d*k^2 - 1) - d - 3*d*k^4*x^4 + d*k^6*x^6 + 1)),x)
```

```
[Out] int(((2*k^2*x^3 - x*(3*k^2 - 1))*(k^4*x^4 - 2*k^2*x^2 + 1))/(((x^2 - 1)*(k^
2*x^2 - 1))^(3/4)*(x^2*(3*d*k^2 - 1) - d - 3*d*k^4*x^4 + d*k^6*x^6 + 1)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3*k**2+1)*x+2*k**2*x**3)*(k**4*x**4-2*k**2*x**2+1)/((-x**2+1)*
(-k**2*x**2+1))**(3/4)/(1-d+(3*d*k**2-1)*x**2-3*d*k**4*x**4+d*k**6*x**6),x)
```

```
[Out] Timed out
```

3.1334 $\int \frac{\sqrt[4]{-1+2x^4}\left(-1+x^4+x^8\right)}{x^6\left(-1+x^4\right)}\,dx$

Optimal. Leaf size=106

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{2x^4-1}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{2x^4-1}}\right)}{2^{3/4}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{2x^4-1}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{2x^4-1}}\right)}{2^{3/4}}+\frac{\left(2x^4-1\right)^{5/4}}{5x^5}$$

Rubi [C] time = 0.65, antiderivative size = 67, normalized size of antiderivative = 0.63, number of steps used = 23, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6725, 264, 1240, 407, 409, 1213, 537, 511, 510}

$$\frac{\left(2x^4-1\right)^{5/4}}{5x^5}-\frac{x^3\sqrt[4]{2x^4-1}\,F_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};2x^4,x^4\right)}{3\sqrt[4]{1-2x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + 2*x^4)^(1/4)*(-1 + x^4 + x^8))/(x^6*(-1 + x^4)),x]
```

```
[Out] (-1 + 2*x^4)^(5/4)/(5*x^5) - (x^3*(-1 + 2*x^4)^(1/4)*AppellF1[3/4, -1/4, 1,
 7/4, 2*x^4, x^4])/(3*(1 - 2*x^4)^(1/4))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !
IntegerQ[p] && ILtQ[q, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{-1+2x^4}\left(-1+x^4+x^8\right)}{x^6\left(-1+x^4\right)}\,dx &= \int \left(\frac{\sqrt[4]{-1+2x^4}}{x^6}+\frac{\sqrt[4]{-1+2x^4}}{2\left(-1+x^2\right)}+\frac{\sqrt[4]{-1+2x^4}}{2\left(1+x^2\right)}\right)dx \\
&= \frac{1}{2}\int \frac{\sqrt[4]{-1+2x^4}}{-1+x^2}\,dx+\frac{1}{2}\int \frac{\sqrt[4]{-1+2x^4}}{1+x^2}\,dx+\int \frac{\sqrt[4]{-1+2x^4}}{x^6}\,dx \\
&= \frac{\left(-1+2x^4\right)^{5/4}}{5x^5}+\frac{1}{2}\int \left(\frac{\sqrt[4]{-1+2x^4}}{1-x^4}+\frac{x^2\sqrt[4]{-1+2x^4}}{-1+x^4}\right)dx+\frac{1}{2}\int \left(\frac{\sqrt[4]{-1+2x^4}}{-1+x^4}\right. \\
&= \frac{\left(-1+2x^4\right)^{5/4}}{5x^5}+\frac{1}{2}\int \frac{\sqrt[4]{-1+2x^4}}{1-x^4}\,dx+\frac{1}{2}\int \frac{\sqrt[4]{-1+2x^4}}{-1+x^4}\,dx+2\left(\frac{1}{2}\int \frac{x^2\sqrt[4]{-1}}{-1+}\right. \\
&= \frac{\left(-1+2x^4\right)^{5/4}}{5x^5}+2\frac{\sqrt[4]{-1+2x^4}\int \frac{x^2\sqrt[4]{1-2x^4}}{-1+x^4}\,dx}{2\sqrt[4]{1-2x^4}}+\frac{1}{2}\left(\sqrt{-\frac{1}{-1+2x^4}}\sqrt{-1+2x^4}\right. \\
&= \frac{\left(-1+2x^4\right)^{5/4}}{5x^5}-\frac{x^3\sqrt[4]{-1+2x^4}\,F_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};2x^4,x^4\right)}{3\sqrt[4]{1-2x^4}}
\end{aligned}
$$

Mathematica [C] time = 0.14, size = 67, normalized size = 0.63

$$\frac{\sqrt[4]{2x^4-1}\left(-5x^8F_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};2x^4,x^4\right)-3\left(1-2x^4\right)^{5/4}\right)}{15x^5\sqrt[4]{1-2x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + 2*x^4)^(1/4)*(-1 + x^4 + x^8))/(x^6*(-1 + x^4)),x]
```

```
[Out] ((-1 + 2*x^4)^(1/4)*(-3*(1 - 2*x^4)^(5/4) - 5*x^8*AppellF1[3/4, -1/4, 1, 7/
4, 2*x^4, x^4]))/(15*x^5*(1 - 2*x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.34, size = 106, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{2x^4-1}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{2x^4-1}}\right)}{2^{3/4}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{2x^4-1}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{2x^4-1}}\right)}{2^{3/4}}+\frac{\left(2x^4-1\right)^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + 2*x^4)^(1/4)*(-1 + x^4 + x^8))/(x^6*(-1 + x^4)),x
]
```

```
[Out] (-1 + 2*x^4)^(5/4)/(5*x^5) + ArcTan[x/(-1 + 2*x^4)^(1/4)]/2 - ArcTan[(2^(1/
4)*x)/(-1 + 2*x^4)^(1/4)]/2^(3/4) - ArcTanh[x/(-1 + 2*x^4)^(1/4)]/2 + ArcTa
nh[(2^(1/4)*x)/(-1 + 2*x^4)^(1/4)]/2^(3/4)
```

fricas [B] time = 24.81, size = 330, normalized size = 3.11

$$\frac{20\cdot 2^{\frac{1}{4}}x^5\arctan\left(2\cdot 2^{\frac{3}{4}}\left(2x^4-1\right)^{\frac{1}{4}}x^3+2\cdot 2^{\frac{1}{4}}\left(2x^4-1\right)^{\frac{3}{4}}x+\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(2\cdot 2^{\frac{3}{4}}\sqrt{2x^4-1}\,x^2+2^{\frac{1}{4}}\left(4x^4-1\right)\right)\right)+5\cdot 2^{\frac{1}{4}}x^5\log\left(4\sqrt{2}\left(2x^4-1\right)^{\frac{1}{4}}x^3+4\cdot 2^{\frac{1}{4}}\sqrt{2x^4-1}\,x^2+2^{\frac{3}{4}}\left(4x^4-1\right)+4\left(2x^4-1\right)^{\frac{3}{4}}x\right)-5\cdot 2^{\frac{1}{4}}x^5\log\left(4\sqrt{2}\left(2x^4-1\right)^{\frac{1}{4}}x^3-4\cdot 2^{\frac{1}{4}}\sqrt{2x^4-1}\,x^2-2^{\frac{3}{4}}\left(4x^4-1\right)+4\left(2x^4-1\right)^{\frac{3}{4}}x\right)+10x^5\arctan\left(\frac{2\left(\left(2x^4-1\right)^{\frac{1}{4}}x^3+\left(2x^4-1\right)^{\frac{3}{4}}x\right)}{x^4-1}\right)+10x^5\log\left(-\frac{3x^4-2\left(2x^4-1\right)^{\frac{1}{4}}x^3+2\sqrt{2x^4-1}\,x^2-2\left(2x^4-1\right)^{\frac{3}{4}}x-1}{x^4-1}\right)+8\left(2x^4-1\right)^{\frac{5}{4}}}{40x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)^(1/4)*(x^8+x^4-1)/x^6/(x^4-1),x, algorithm="fricas")
```

```
[Out] 1/40*(20*2^(1/4)*x^5*arctan(2*2^(3/4)*(2*x^4 - 1)^(1/4)*x^3 + 2*2^(1/4)*(2*
x^4 - 1)^(3/4)*x + 1/2*2^(3/4)*(2*2^(3/4)*sqrt(2*x^4 - 1)*x^2 + 2^(1/4)*(4*
x^4 - 1))) + 5*2^(1/4)*x^5*log(4*sqrt(2)*(2*x^4 - 1)^(1/4)*x^3 + 4*2^(1/4)*
sqrt(2*x^4 - 1)*x^2 + 2^(3/4)*(4*x^4 - 1) + 4*(2*x^4 - 1)^(3/4)*x) - 5*2^(1
/4)*x^5*log(4*sqrt(2)*(2*x^4 - 1)^(1/4)*x^3 - 4*2^(1/4)*sqrt(2*x^4 - 1)*x^2
 - 2^(3/4)*(4*x^4 - 1) + 4*(2*x^4 - 1)^(3/4)*x) + 10*x^5*arctan(2*((2*x^4 -
 1)^(1/4)*x^3 + (2*x^4 - 1)^(3/4)*x)/(x^4 - 1)) + 10*x^5*log(-(3*x^4 - 2*(2
*x^4 - 1)^(1/4)*x^3 + 2*sqrt(2*x^4 - 1)*x^2 - 2*(2*x^4 - 1)^(3/4)*x - 1)/(x
^4 - 1)) + 8*(2*x^4 - 1)^(5/4))/x^5
```

giac [A] time = 0.22, size = 143, normalized size = 1.35

$$-\frac{1}{2}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x^4-1\right)^{\frac{1}{4}}}{2x}\right)-\frac{1}{4}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+\frac{\left(2x^4-1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{4}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}-\frac{\left(2x^4-1\right)^{\frac{1}{4}}}{x}\right)+\frac{\left(2x^4-1\right)^{\frac{1}{4}}\left(\frac{1}{x^4}-2\right)}{5x}+\frac{1}{2}\arctan\left(\frac{\left(2x^4-1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{4}\log\left(\frac{\left(2x^4-1\right)^{\frac{1}{4}}}{x}+1\right)-\frac{1}{4}\log\left(\left|\frac{\left(2x^4-1\right)^{\frac{1}{4}}}{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)^(1/4)*(x^8+x^4-1)/x^6/(x^4-1),x, algorithm="giac")
```

```
[Out] -1/2*2^(1/4)*arctan(1/2*2^(3/4)*(2*x^4 - 1)^(1/4)/x) - 1/4*2^(1/4)*log(2^(1
/4) + (2*x^4 - 1)^(1/4)/x) + 1/4*2^(1/4)*log(2^(1/4) - (2*x^4 - 1)^(1/4)/x)
 + 1/5*(2*x^4 - 1)^(1/4)*(1/x^4 - 2)/x + 1/2*arctan((2*x^4 - 1)^(1/4)/x) +
1/4*log((2*x^4 - 1)^(1/4)/x + 1) - 1/4*log(abs((2*x^4 - 1)^(1/4)/x - 1))
```

maple [F] time = 4.21, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^4-1\right)^{\frac{1}{4}}\left(x^8+x^4-1\right)}{x^6\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-1)^(1/4)*(x^8+x^4-1)/x^6/(x^4-1),x)
```

```
[Out] int((2*x^4-1)^(1/4)*(x^8+x^4-1)/x^6/(x^4-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8 + x^4 - 1\right)\left(2\,x^4 - 1\right)^{\frac{1}{4}}}{\left(x^4 - 1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)^(1/4)*(x^8+x^4-1)/x^6/(x^4-1),x, algorithm="maxima")
[Out] integrate((x^8 + x^4 - 1)*(2*x^4 - 1)^(1/4)/((x^4 - 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(2\,x^4-1\right)}^{1/4}\,\left(x^8+x^4-1\right)}{x^6\,\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^4 - 1)^(1/4)*(x^4 + x^8 - 1))/(x^6*(x^4 - 1)),x)
[Out] int(((2*x^4 - 1)^(1/4)*(x^4 + x^8 - 1))/(x^6*(x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{2x^4 - 1}\,\left(x^8 + x^4 - 1\right)}{x^6\,(x-1)\,(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-1)**(1/4)*(x**8+x**4-1)/x**6/(x**4-1),x)
[Out] Integral((2*x**4 - 1)**(1/4)*(x**8 + x**4 - 1)/(x**6*(x - 1)*(x + 1)*(x**2
+ 1)), x)
```

3.1335 $\int x^6 \left(-1+x^3\right)^{2/3} dx$

Optimal. Leaf size=107

$$\frac{4}{243}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{4\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{81\sqrt{3}}-\frac{2}{243}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)+\frac{1}{81}\left(x^3-1\right)^{2/3}\left(9x^7-3x^4\right.$$

Rubi [A] time = 0.03, antiderivative size = 95, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {279, 321, 239}

$$-\frac{4}{81}\left(x^3-1\right)^{2/3}x+\frac{2}{81}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{4\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{1}{9}\left(x^3-1\right)^{2/3}x^7-\frac{1}{27}\left(x^3-1\right)^{2/3}x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(-1 + x^3)^(2/3),x]
```

```
[Out] (-4*x*(-1 + x^3)^(2/3))/81 - (x^4*(-1 + x^3)^(2/3))/27 + (x^7*(-1 + x^3)^(2
/3))/9 - (4*ArcTan[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]) + (2
*Log[-x + (-1 + x^3)^(1/3)])/81
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^6\left(-1+x^3\right)^{2/3}dx &= \frac{1}{9}x^7\left(-1+x^3\right)^{2/3}-\frac{2}{9}\int\frac{x^6}{\sqrt[3]{-1+x^3}}dx\\ &= -\frac{1}{27}x^4\left(-1+x^3\right)^{2/3}+\frac{1}{9}x^7\left(-1+x^3\right)^{2/3}-\frac{4}{27}\int\frac{x^3}{\sqrt[3]{-1+x^3}}dx\\ &= -\frac{4}{81}x\left(-1+x^3\right)^{2/3}-\frac{1}{27}x^4\left(-1+x^3\right)^{2/3}+\frac{1}{9}x^7\left(-1+x^3\right)^{2/3}-\frac{4}{81}\int\frac{1}{\sqrt[3]{-1+x^3}}dx\\ &= -\frac{4}{81}x\left(-1+x^3\right)^{2/3}-\frac{1}{27}x^4\left(-1+x^3\right)^{2/3}+\frac{1}{9}x^7\left(-1+x^3\right)^{2/3}-\frac{4\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{2}{81}\log\end{aligned}$$

Mathematica [C] time = 0.02, size = 65, normalized size = 0.61

$$\frac{x\left(x^3-1\right)^{2/3}\left(2\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};x^3\right)+\left(1-x^3\right)^{2/3}\left(3x^6-x^3-2\right)\right)}{27\left(1-x^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(-1 + x^3)^(2/3),x]
```

```
[Out] (x*(-1 + x^3)^(2/3)*((1 - x^3)^(2/3)*(-2 - x^3 + 3*x^6) + 2*Hypergeometric2
F1[-2/3, 1/3, 4/3, x^3]))/(27*(1 - x^3)^(2/3))
```

IntegrateAlgebraic [A] time = 0.27, size = 107, normalized size = 1.00

$$\frac{4}{243}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{4\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{81\sqrt{3}}-\frac{2}{243}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)+\frac{1}{81}\left(x^3-1\right)^{2/3}\left(9x^7-3x^4-4x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6*(-1 + x^3)^(2/3),x]
```

```
[Out] ((-1 + x^3)^(2/3)*(-4*x - 3*x^4 + 9*x^7))/81 - (4*ArcTan[(Sqrt[3]*x)/(x + 2
*(-1 + x^3)^(1/3))])/(81*Sqrt[3]) + (4*Log[-x + (-1 + x^3)^(1/3)])/243 - (2
*Log[x^2 + x*(-1 + x^3)^(1/3) + (-1 + x^3)^(2/3)])/243
```

fricas [A] time = 0.42, size = 99, normalized size = 0.93

$$\frac{1}{81}\left(9x^7-3x^4-4x\right)\left(x^3-1\right)^{\frac{2}{3}}+\frac{4}{243}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}}{3x}\right)+\frac{4}{243}\log\left(-\frac{x-\left(x^3-1\right)^{\frac{1}{3}}}{x}\right)-\frac{2}{243}\log\left(\frac{x^2+\left(x^3-1\right)^{\frac{1}{3}}x+\left(x^3-1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^3-1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/81*(9*x^7 - 3*x^4 - 4*x)*(x^3 - 1)^(2/3) + 4/243*sqrt(3)*arctan(1/3*(sqrt
(3)*x + 2*sqrt(3)*(x^3 - 1)^(1/3))/x) + 4/243*log(-(x - (x^3 - 1)^(1/3))/x)
 - 2/243*log((x^2 + (x^3 - 1)^(1/3)*x + (x^3 - 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^3-1\right)^{\frac{2}{3}}x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^3-1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 1)^(2/3)*x^6, x)
```

maple [C] time = 0.28, size = 54, normalized size = 0.50

$$\frac{x\left(9x^{6}-3x^{3}-4\right)\left(x^{3}-1\right)^{\frac{2}{3}}}{81}-\frac{4\left(-\operatorname{signum}\left(x^{3}-1\right)\right)^{\frac{1}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^{3}\right)}{81\operatorname{signum}\left(x^{3}-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^3-1)^(2/3),x)
```

```
[Out] 1/81*x*(9*x^6-3*x^3-4)*(x^3-1)^(2/3)-4/81/signum(x^3-1)^(1/3)*(-signum(x^3-
1))^(1/3)*x*hypergeom([1/3,1/3],[4/3],x^3)
```

maxima [A] time = 0.41, size = 145, normalized size = 1.36

$$\frac{4}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^{3}-1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\frac{2\left(x^{3}-1\right)^{\frac{2}{3}}}{x^{2}}+\frac{11\left(x^{3}-1\right)^{\frac{5}{3}}}{x^{5}}-\frac{4\left(x^{3}-1\right)^{\frac{8}{3}}}{x^{8}}}{81\left(\frac{3\left(x^{3}-1\right)}{x^{3}}-\frac{3\left(x^{3}-1\right)^{2}}{x^{6}}+\frac{\left(x^{3}-1\right)^{3}}{x^{9}}-1\right)}-\frac{2}{243}\log\left(\frac{\left(x^{3}-1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^{3}-1\right)^{\frac{2}{3}}}{x^{2}}+1\right)+\frac{4}{243}\log\left(\frac{\left(x^{3}-1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^3-1)^(2/3),x, algorithm="maxima")
```

```
[Out] 4/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3)/x + 1)) - 1/81*(2*(x^3
- 1)^(2/3)/x^2 + 11*(x^3 - 1)^(5/3)/x^5 - 4*(x^3 - 1)^(8/3)/x^8)/(3*(x^3 -
1)/x^3 - 3*(x^3 - 1)^2/x^6 + (x^3 - 1)^3/x^9 - 1) - 2/243*log((x^3 - 1)^(1/
3)/x + (x^3 - 1)^(2/3)/x^2 + 1) + 4/243*log((x^3 - 1)^(1/3)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{6}\left(x^{3}-1\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^3 - 1)^(2/3),x)
```

```
[Out] int(x^6*(x^3 - 1)^(2/3), x)
```

sympy [C] time = 1.32, size = 34, normalized size = 0.32

$$-\frac{x^{7}e^{-\frac{i\pi}{3}}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|x^{3}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(x**3-1)**(2/3),x)
```

```
[Out] -x**7*exp(-I*pi/3)*gamma(7/3)*hyper((-2/3, 7/3), (10/3,), x**3)/(3*gamma(10
/3))
```

3.1336 $\int x^6 \left(1+x^3\right)^{2/3} \, dx$

Optimal. Leaf size=107

$$-\frac{4}{243}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{4\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{81\sqrt{3}}+\frac{2}{243}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)+\frac{1}{81}\left(x^3+1\right)^{2/3}\left(9x^7+3x^4\right.$$

Rubi [A] time = 0.03, antiderivative size = 95, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {279, 321, 239}

$$-\frac{4}{81}\left(x^3+1\right)^{2/3}x-\frac{2}{81}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{4\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{1}{9}\left(x^3+1\right)^{2/3}x^7+\frac{1}{27}\left(x^3+1\right)^{2/3}x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(1 + x^3)^(2/3),x]
```

```
[Out] (-4*x*(1 + x^3)^(2/3))/81 + (x^4*(1 + x^3)^(2/3))/27 + (x^7*(1 + x^3)^(2/3)
)/9 + (4*ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]) - (2*Log
[-x + (1 + x^3)^(1/3)])/81
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^6\left(1+x^3\right)^{2/3}dx &= \frac{1}{9}x^7\left(1+x^3\right)^{2/3}+\frac{2}{9}\int\frac{x^6}{\sqrt[3]{1+x^3}}dx\\ &= \frac{1}{27}x^4\left(1+x^3\right)^{2/3}+\frac{1}{9}x^7\left(1+x^3\right)^{2/3}-\frac{4}{27}\int\frac{x^3}{\sqrt[3]{1+x^3}}dx\\ &= -\frac{4}{81}x\left(1+x^3\right)^{2/3}+\frac{1}{27}x^4\left(1+x^3\right)^{2/3}+\frac{1}{9}x^7\left(1+x^3\right)^{2/3}+\frac{4}{81}\int\frac{1}{\sqrt[3]{1+x^3}}dx\\ &= -\frac{4}{81}x\left(1+x^3\right)^{2/3}+\frac{1}{27}x^4\left(1+x^3\right)^{2/3}+\frac{1}{9}x^7\left(1+x^3\right)^{2/3}+\frac{4\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{81\sqrt{3}}-\frac{2}{81}\log\left(-\right.\end{aligned}$$

Mathematica [C] time = 0.02, size = 43, normalized size = 0.40

$$\frac{1}{27}x\left(2\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};-x^3\right)+\left(x^3+1\right)^{2/3}\left(3x^6+x^3-2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(1 + x^3)^(2/3),x]
[Out] (x*((1 + x^3)^(2/3)*(-2 + x^3 + 3*x^6) + 2*Hypergeometric2F1[-2/3, 1/3, 4/3
, -x^3]))/27
```

IntegrateAlgebraic [A] time = 0.24, size = 107, normalized size = 1.00

$$-\frac{4}{243}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{4\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{81\sqrt{3}}+\frac{2}{243}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)+\frac{1}{81}\left(x^3+1\right)^{2/3}\left(9x^7+3x^4-4x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6*(1 + x^3)^(2/3),x]
[Out] ((1 + x^3)^(2/3)*(-4*x + 3*x^4 + 9*x^7))/81 + (4*ArcTan[(Sqrt[3]*x)/(x + 2*
(1 + x^3)^(1/3))])/(81*Sqrt[3]) - (4*Log[-x + (1 + x^3)^(1/3)])/243 + (2*Lo
g[x^2 + x*(1 + x^3)^(1/3) + (1 + x^3)^(2/3)])/243
```

fricas [A] time = 0.42, size = 99, normalized size = 0.93

$$\frac{1}{81}\left(9x^7+3x^4-4x\right)\left(x^3+1\right)^{\frac{2}{3}}-\frac{4}{243}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}}{3x}\right)-\frac{4}{243}\log\left(-\frac{x-\left(x^3+1\right)^{\frac{1}{3}}}{x}\right)+\frac{2}{243}\log\left(\frac{x^2+\left(x^3+1\right)^{\frac{1}{3}}x+\left(x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^3+1)^(2/3),x, algorithm="fricas")
[Out] 1/81*(9*x^7 + 3*x^4 - 4*x)*(x^3 + 1)^(2/3) - 4/243*sqrt(3)*arctan(1/3*(sqrt
(3)*x + 2*sqrt(3)*(x^3 + 1)^(1/3))/x) - 4/243*log(-(x - (x^3 + 1)^(1/3))/x)
 + 2/243*log((x^2 + (x^3 + 1)^(1/3)*x + (x^3 + 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^3+1\right)^{\frac{2}{3}}x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^3+1)^(2/3),x, algorithm="giac")
```

[Out] integrate((x^3 + 1)^(2/3)*x^6, x)

maple [C] time = 0.27, size = 38, normalized size = 0.36

$$\frac{x\left(9x^{6}+3x^{3}-4\right)\left(x^{3}+1\right)^{\frac{2}{3}}}{81}+\frac{4x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x^{3}\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(x^3+1)^(2/3),x)

[Out] 1/81*x*(9*x^6+3*x^3-4)*(x^3+1)^(2/3)+4/81*x*hypergeom([1/3,1/3],[4/3],-x^3)

maxima [A] time = 0.41, size = 145, normalized size = 1.36

$$-\frac{4}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^{3}+1\right)^{\frac{1}{3}}}{x}+1\right)\right)+\frac{\frac{2\left(x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}+\frac{11\left(x^{3}+1\right)^{\frac{5}{3}}}{x^{5}}-\frac{4\left(x^{3}+1\right)^{\frac{8}{3}}}{x^{8}}}{81\left(\frac{3\left(x^{3}+1\right)}{x^{3}}-\frac{3\left(x^{3}+1\right)^{2}}{x^{6}}+\frac{\left(x^{3}+1\right)^{3}}{x^{9}}-1\right)}+\frac{2}{243}\log\left(\frac{\left(x^{3}+1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}+1\right)-\frac{4}{243}\log\left(\frac{\left(x^{3}+1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(x^3+1)^(2/3),x, algorithm="maxima")

[Out] -4/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3)/x + 1)) + 1/81*(2*(x^3 + 1)^(2/3)/x^2 + 11*(x^3 + 1)^(5/3)/x^5 - 4*(x^3 + 1)^(8/3)/x^8)/(3*(x^3 + 1)/x^3 - 3*(x^3 + 1)^2/x^6 + (x^3 + 1)^3/x^9 - 1) + 2/243*log((x^3 + 1)^(1/3)/x + (x^3 + 1)^(2/3)/x^2 + 1) - 4/243*log((x^3 + 1)^(1/3)/x - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{6}\left(x^{3}+1\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(x^3 + 1)^(2/3),x)

[Out] int(x^6*(x^3 + 1)^(2/3), x)

sympy [C] time = 1.24, size = 31, normalized size = 0.29

$$\frac{x^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(x**3+1)**(2/3),x)

[Out] x**7*gamma(7/3)*hyper((-2/3, 7/3), (10/3,), x**3*exp_polar(I*pi))/(3*gamma(10/3))

3.1337 $\int x^4 \sqrt[3]{x+x^3}\,dx$

Optimal. Leaf size=107

$$-\frac{5}{162}\log\left(\sqrt[3]{x^3+x}-x\right)-\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x}+x}\right)}{54\sqrt{3}}+\frac{1}{108}\sqrt[3]{x^3+x}\left(18x^5+3x^3-5x\right)+\frac{5}{324}\log\left(\sqrt[3]{x^3+x}\,x+\left(x^3+x\right.\right.$$

Rubi [A] time = 0.23, antiderivative size = 210, normalized size of antiderivative = 1.96, number of steps used = 13, number of rules used = 12, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {2021, 2024, 2032, 329, 275, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{36}\sqrt[3]{x^3+x}\,x^3-\frac{5}{108}\sqrt[3]{x^3+x}\,x+\frac{1}{6}\sqrt[3]{x^3+x}\,x^5-\frac{5\left(x^2+1\right)^{2/3}x^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2+1}}\right)}{162\left(x^3+x\right)^{2/3}}+\frac{5\left(x^2+1\right)^{2/3}x^{2/3}\log\left(\frac{x^{4/3}}{\left(x^2+1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2+1}}+1\right)}{324\left(x^3+x\right)^{2/3}}-\frac{5\left(x^2+1\right)^{2/3}x^{2/3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)}{54\sqrt{3}\left(x^3+x\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(x + x^3)^(1/3),x]
```

```
[Out] (-5*x*(x + x^3)^(1/3))/108 + (x^3*(x + x^3)^(1/3))/36 + (x^5*(x + x^3)^(1/3
))/6 - (5*x^(2/3)*(1 + x^2)^(2/3)*ArcTan[(1 + (2*x^(2/3))/(1 + x^2)^(1/3))/
Sqrt[3]])/(54*Sqrt[3]*(x + x^3)^(2/3)) - (5*x^(2/3)*(1 + x^2)^(2/3)*Log[1 -
 x^(2/3)/(1 + x^2)^(1/3)])/(162*(x + x^3)^(2/3)) + (5*x^(2/3)*(1 + x^2)^(2/
3)*Log[1 + x^(4/3)/(1 + x^2)^(2/3) + x^(2/3)/(1 + x^2)^(1/3)])/(324*(x + x^
3)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^4\sqrt[3]{x+x^3}\,dx = \frac{1}{6}x^5\sqrt[3]{x+x^3} + \frac{1}{9}\int \frac{x^5}{\left(x+x^3\right)^{2/3}}\,dx$$

$$= \frac{1}{36}x^3\sqrt[3]{x+x^3} + \frac{1}{6}x^5\sqrt[3]{x+x^3} - \frac{5}{54}\int \frac{x^3}{\left(x+x^3\right)^{2/3}}\,dx$$

$$= -\frac{5}{108}x\sqrt[3]{x+x^3} + \frac{1}{36}x^3\sqrt[3]{x+x^3} + \frac{1}{6}x^5\sqrt[3]{x+x^3} + \frac{5}{81}\int \frac{x}{\left(x+x^3\right)^{2/3}}\,dx$$

$$= -\frac{5}{108}x\sqrt[3]{x+x^3} + \frac{1}{36}x^3\sqrt[3]{x+x^3} + \frac{1}{6}x^5\sqrt[3]{x+x^3} + \frac{\left(5x^{2/3}\left(1+x^2\right)^{2/3}\right)\int \frac{\sqrt[3]{x}}{\left(1+x^2\right)^{2/3}}\,dx}{81\left(x+x^3\right)^{2/3}}$$

$$= -\frac{5}{108}x\sqrt[3]{x+x^3} + \frac{1}{36}x^3\sqrt[3]{x+x^3} + \frac{1}{6}x^5\sqrt[3]{x+x^3} + \frac{\left(5x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x^3}{\left(1+x^6\right)^{2/3}}\,dx, \ldots\right.}{27\left(x+x^3\right)^{2/3}}$$

$$= -\frac{5}{108}x\sqrt[3]{x+x^3} + \frac{1}{36}x^3\sqrt[3]{x+x^3} + \frac{1}{6}x^5\sqrt[3]{x+x^3} + \frac{\left(5x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{\left(1+x^3\right)^{2/3}}\,dx, \ldots\right.}{54\left(x+x^3\right)^{2/3}}$$

$$= -\frac{5}{108}x\sqrt[3]{x+x^3} + \frac{1}{36}x^3\sqrt[3]{x+x^3} + \frac{1}{6}x^5\sqrt[3]{x+x^3} + \frac{\left(5x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \ldots\right.}{54\left(x+x^3\right)^{2/3}}$$

$$= -\frac{5}{108}x\sqrt[3]{x+x^3} + \frac{1}{36}x^3\sqrt[3]{x+x^3} + \frac{1}{6}x^5\sqrt[3]{x+x^3} + \frac{\left(5x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \ldots\right.}{162\left(x+x^3\right)^{2/3}}$$

$$= -\frac{5}{108}x\sqrt[3]{x+x^3} + \frac{1}{36}x^3\sqrt[3]{x+x^3} + \frac{1}{6}x^5\sqrt[3]{x+x^3} - \frac{5x^{2/3}\left(1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{162\left(x+x^3\right)^{2/3}} + \ldots$$

$$= -\frac{5}{108}x\sqrt[3]{x+x^3} + \frac{1}{36}x^3\sqrt[3]{x+x^3} + \frac{1}{6}x^5\sqrt[3]{x+x^3} - \frac{5x^{2/3}\left(1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{162\left(x+x^3\right)^{2/3}} + \ldots$$

$$= -\frac{5}{108}x\sqrt[3]{x+x^3} + \frac{1}{36}x^3\sqrt[3]{x+x^3} + \frac{1}{6}x^5\sqrt[3]{x+x^3} - \frac{5x^{2/3}\left(1+x^2\right)^{2/3}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{54\sqrt{3}\left(x+x^3\right)^{2/3}} - \ldots$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.57

$$\frac{x\sqrt[3]{x^3+x}\left(5\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-x^2\right)+\sqrt[3]{x^2+1}\left(6x^4+x^2-5\right)\right)}{36\sqrt[3]{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(x + x^3)^(1/3),x]
```

```
[Out] (x*(x + x^3)^(1/3)*((1 + x^2)^(1/3)*(-5 + x^2 + 6*x^4) + 5*Hypergeometric2F
1[-1/3, 2/3, 5/3, -x^2]))/(36*(1 + x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.33, size = 107, normalized size = 1.00

$$-\frac{5}{162}\log\left(\sqrt[3]{x^3+x}-x\right)-\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x}+x}\right)}{54\sqrt{3}}+\frac{1}{108}\sqrt[3]{x^3+x}\left(18x^5+3x^3-5x\right)+\frac{5}{324}\log\left(\sqrt[3]{x^3+x}\,x+\left(x^3+x\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(x + x^3)^(1/3),x]
```

```
[Out] ((x + x^3)^(1/3)*(-5*x + 3*x^3 + 18*x^5))/108 - (5*ArcTan[(Sqrt[3]*x)/(x +
2*(x + x^3)^(1/3))])/(54*Sqrt[3]) - (5*Log[-x + (x + x^3)^(1/3)])/162 + (5*
Log[x^2 + x*(x + x^3)^(1/3) + (x + x^3)^(2/3)])/324
```

fricas [A] time = 0.64, size = 103, normalized size = 0.96

$$-\frac{5}{162}\sqrt{3}\arctan\left(-\frac{196\sqrt{3}\left(x^3+x\right)^{\frac{1}{3}}x-\sqrt{3}\left(539x^2+507\right)-1274\sqrt{3}\left(x^3+x\right)^{\frac{2}{3}}}{2205x^2+2197}\right)+\frac{1}{108}\left(18x^5+3x^3-5x\right)\left(x^3+x\right)^{\frac{1}{3}}-\frac{5}{324}\log\left(3\left(x^3+x\right)^{\frac{1}{3}}x-3\left(x^3+x\right)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^3+x)^(1/3),x, algorithm="fricas")
```

```
[Out] -5/162*sqrt(3)*arctan(-(196*sqrt(3)*(x^3 + x)^(1/3)*x - sqrt(3)*(539*x^2 +
507) - 1274*sqrt(3)*(x^3 + x)^(2/3))/(2205*x^2 + 2197)) + 1/108*(18*x^5 + 3
*x^3 - 5*x)*(x^3 + x)^(1/3) - 5/324*log(3*(x^3 + x)^(1/3)*x - 3*(x^3 + x)^(
2/3) + 1)
```

giac [A] time = 0.41, size = 88, normalized size = 0.82

$$-\frac{1}{108}\left(5\left(\frac{1}{x^2}+1\right)^{\frac{7}{3}}-13\left(\frac{1}{x^2}+1\right)^{\frac{4}{3}}-10\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right)x^6+\frac{5}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{5}{324}\log\left(\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)-\frac{5}{162}\log\left(\left|\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^3+x)^(1/3),x, algorithm="giac")
```

```
[Out] -1/108*(5*(1/x^2 + 1)^(7/3) - 13*(1/x^2 + 1)^(4/3) - 10*(1/x^2 + 1)^(1/3))*
x^6 + 5/162*sqrt(3)*arctan(1/3*sqrt(3)*(2*(1/x^2 + 1)^(1/3) + 1)) + 5/324*l
og((1/x^2 + 1)^(2/3) + (1/x^2 + 1)^(1/3) + 1) - 5/162*log(abs((1/x^2 + 1)^(
1/3) - 1))
```

maple [C] time = 2.35, size = 508, normalized size = 4.75

$$\frac{x\left(18x^4+3x^2-5\right)\left(x\left(x^2+1\right)\right)^{\frac{1}{3}}}{108}+\frac{\left(-\frac{5\ln\left(-\frac{-5\operatorname{RootOf}\left(\_Z^2-2\_Z+4\right)^2x^4-38\operatorname{RootOf}\left(\_Z^2-2\_Z+4\right)x^4+18\left(x^6+2x^4+x^2\right)^{\frac{1}{3}}\operatorname{RootOf}\left(\_Z^2-2\_Z+4\right)x^2+16x^4+30\operatorname{RootOf}\left(\_Z^2-2\_Z+4\right)\left(x^6+2x^4+x^2\right)^{\frac{2}{3}}+60\left(x^6+2x^4+x^2\right)^{\frac{1}{3}}x^2-70\operatorname{RootOf}\left(\_Z^2-2\_Z+4\right)x^2-96\left(x^6+2x^4+x^2\right)^{\frac{2}{3}}+18\operatorname{RootOf}\left(\_Z^2-2\_Z+4\right)\left(x^6+2x^4+x^2\right)^{\frac{1}{3}}+5\operatorname{RootOf}\left(\_Z^2-2\_Z+4\right)^2+28x^2+60\left(x^6+2x^4+x^2\right)^{\frac{1}{3}}-32\operatorname{RootOf}\left(\_Z^2-2\_Z+4\right)+12}{x^2+1}\right)}{162}+\frac{5\operatorname{RootOf}\left(\_Z^2-2\_Z+4\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^2-2\_Z+4\right)^2x^4+20\operatorname{RootOf}\left(\_Z^2-2\_Z+4\right)x^4-48\left(x^6+2x^4+x^2\right)^{\frac{1}{3}}\operatorname{RootOf}\left(\_Z^2-2\_Z+4\right)x^2-100x^4+30\operatorname{RootOf}\left(\_Z^2-2\_Z+4\right)\left(x^6+2x^4+x^2\right)^{\frac{2}{3}}+60\left(x^6+2x^4+x^2\right)^{\frac{1}{3}}x^2+14\operatorname{RootOf}\left(\_Z^2-2\_Z+4\right)x^2+36\left(x^6+2x^4+x^2\right)^{\frac{2}{3}}-48\operatorname{RootOf}\left(\_Z^2-2\_Z+4\right)\left(x^6+2x^4+x^2\right)^{\frac{1}{3}}+\operatorname{RootOf}\left(\_Z^2-2\_Z+4\right)^2-140x^2+60\left(x^6+2x^4+x^2\right)^{\frac{1}{3}}-6\operatorname{RootOf}\left(\_Z^2-2\_Z+4\right)-40}{x^2+1}\right)}{324}\right)\left(x\left(x^2+1\right)\right)^{\frac{1}{3}}\left(x^2\left(x^2+1\right)^2\right)^{\frac{1}{3}}}{x\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^3+x)^(1/3),x)
```

```
[Out] 1/108*x*(18*x^4+3*x^2-5)*(x*(x^2+1))^(1/3)+(-5/162*ln(-(-5*RootOf(_Z^2-2*_Z
+4)^2*x^4-38*RootOf(_Z^2-2*_Z+4)*x^4+18*(x^6+2*x^4+x^2)^(1/3)*RootOf(_Z^2-2
*_Z+4)*x^2+16*x^4+30*RootOf(_Z^2-2*_Z+4)*(x^6+2*x^4+x^2)^(2/3)+60*(x^6+2*x^
4+x^2)^(1/3)*x^2-70*RootOf(_Z^2-2*_Z+4)*x^2-96*(x^6+2*x^4+x^2)^(2/3)+18*Roo
tOf(_Z^2-2*_Z+4)*(x^6+2*x^4+x^2)^(1/3)+5*RootOf(_Z^2-2*_Z+4)^2+28*x^2+60*(x
^6+2*x^4+x^2)^(1/3)-32*RootOf(_Z^2-2*_Z+4)+12)/(x^2+1))+5/324*RootOf(_Z^2-2
*_Z+4)*ln((-RootOf(_Z^2-2*_Z+4)^2*x^4+20*RootOf(_Z^2-2*_Z+4)*x^4-48*(x^6+2*
x^4+x^2)^(1/3)*RootOf(_Z^2-2*_Z+4)*x^2-100*x^4+30*RootOf(_Z^2-2*_Z+4)*(x^6+
2*x^4+x^2)^(2/3)+60*(x^6+2*x^4+x^2)^(1/3)*x^2+14*RootOf(_Z^2-2*_Z+4)*x^2+36
*(x^6+2*x^4+x^2)^(2/3)-48*RootOf(_Z^2-2*_Z+4)*(x^6+2*x^4+x^2)^(1/3)+RootOf(
_Z^2-2*_Z+4)^2-140*x^2+60*(x^6+2*x^4+x^2)^(1/3)-6*RootOf(_Z^2-2*_Z+4)-40)/(
x^2+1)))*(x*(x^2+1))^(1/3)/x*(x^2*(x^2+1)^2)^(1/3)/(x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^3+x\right)^{\frac{1}{3}} x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^3+x)^(1/3),x, algorithm="maxima")
[Out] integrate((x^3 + x)^(1/3)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(x^3+x\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x + x^3)^(1/3),x)
[Out] int(x^4*(x + x^3)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\sqrt[3]{x\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**3+x)**(1/3),x)
[Out] Integral(x**4*(x*(x**2 + 1))**(1/3), x)
```

3.1338 $\int \frac{x^2(3ab^2-2b(2a+b)x+(a+2b)x^2)}{(x(-a+x)(-b+x)^2)^{3/4}(ab^2-b(2a+b)x+(a+2b)x^2+(-1+d)x^3)}\,dx$

Optimal. Leaf size=107

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}}\right)}{d^{3/4}}$$

Rubi [F] time = 19.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(3ab^2-2b(2a+b)x+(a+2b)x^2\right)}{\left(x(-a+x)(-b+x)^2\right)^{3/4}\left(ab^2-b(2a+b)x+(a+2b)x^2+(-1+d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(3*a*b^2 - 2*b*(2*a + b)*x + (a + 2*b)*x^2))/((x*(-a + x)*(-b + x)
^2)^(3/4)*(a*b^2 - b*(2*a + b)*x + (a + 2*b)*x^2 + (-1 + d)*x^3)),x]
```

```
[Out] (4*(a + 2*b)*((b*(a - x))/(a*(b - x)))^(3/4)*(b - x)^2*x*Hypergeometric2F1[
1/4, 3/4, 5/4, -(((a - b)*x)/(a*(b - x)))])/(b*(1 - d)*(-((a - x)*(b - x)^2
*x))^(3/4)) + (4*a*b^2*(a + 2*b)*x^(3/4)*(-a + x)^(3/4)*(-b + x)^(3/2)*Defe
r[Subst][Defer[Int][1/((-a + x^4)^(3/4)*Sqrt[-b + x^4]*(-(b^2*x^4) + 2*b*x^
8 + (-1 + d)*x^12 + a*(b - x^4)^2)), x], x, x^(1/4)])/((1 - d)*(-((a - x)*(
b - x)^2*x))^(3/4)) - (4*b*(2*a + b)*(a + 2*b)*x^(3/4)*(-a + x)^(3/4)*(-b +
 x)^(3/2)*Defer[Subst][Defer[Int][x^4/((-a + x^4)^(3/4)*Sqrt[-b + x^4]*(-(b
^2*x^4) + 2*b*x^8 + (-1 + d)*x^12 + a*(b - x^4)^2)), x], x, x^(1/4)])/((1 -
 d)*(-((a - x)*(b - x)^2*x))^(3/4)) + (4*(a^2 + 4*b^2 + a*(b + 3*b*d))*x^(3
/4)*(-a + x)^(3/4)*(-b + x)^(3/2)*Defer[Subst][Defer[Int][x^8/((-a + x^4)^(
3/4)*Sqrt[-b + x^4]*(-(b^2*x^4) + 2*b*x^8 + (-1 + d)*x^12 + a*(b - x^4)^2))
, x], x, x^(1/4)])/((1 - d)*(-((a - x)*(b - x)^2*x))^(3/4))
```

Rubi steps

$$\int \frac{x^2\left(3ab^2-2b(2a+b)x+(a+2b)x^2\right)}{\left(x(-a+x)(-b+x)^2\right)^{3/4}\left(ab^2-b(2a+b)x+(a+2b)x^2+(-1+d)x^3\right)}\,dx = \frac{\left(x^{3/4}(-a+x)^{3/4}(-b+x)^{3/2}\right)\int \frac{}{(-}}{\left(x(-a\right.}$$

$$= \frac{\left(x^{3/4}(-a+x)^{3/4}(-b+x)^{3/2}\right)\int \frac{}{(-}}{\left(x(-a\right.}$$

$$= \frac{\left(4x^{3/4}(-a+x)^{3/4}(-b+x)^{3/2}\right)\mathrm{Su}}{}$$

$$= \frac{\left(4x^{3/4}(-a+x)^{3/4}(-b+x)^{3/2}\right)\mathrm{Su}}{}$$

$$= -\frac{\left(4(a+2b)x^{3/4}(-a+x)^{3/4}(-b+\right.}{(1-d)\left(x\right.}$$

$$= \frac{4(a+2b)\left(\frac{b(a-x)}{a(b-x)}\right)^{3/4}(b-x)^2x\,{}_2F}{b(1-d)\left(-\left((a-x)(b-\right.\right.}$$

$$= \frac{4(a+2b)\left(\frac{b(a-x)}{a(b-x)}\right)^{3/4}(b-x)^2x\,{}_2F}{b(1-d)\left(-\left((a-x)(b-\right.\right.}$$

$$= \frac{4(a+2b)\left(\frac{b(a-x)}{a(b-x)}\right)^{3/4}(b-x)^2x\,{}_2F}{b(1-d)\left(-\left((a-x)(b-\right.\right.}$$

Mathematica [F] time = 4.75, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(3ab^2-2b(2a+b)x+(a+2b)x^2\right)}{\left(x(-a+x)(-b+x)^2\right)^{3/4}\left(ab^2-b(2a+b)x+(a+2b)x^2+(-1+d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(3*a*b^2 - 2*b*(2*a + b)*x + (a + 2*b)*x^2))/((x*(-a + x)*(-
b + x)^2)^(3/4)*(a*b^2 - b*(2*a + b)*x + (a + 2*b)*x^2 + (-1 + d)*x^3)),x]
```

```
[Out] Integrate[(x^2*(3*a*b^2 - 2*b*(2*a + b)*x + (a + 2*b)*x^2))/((x*(-a + x)*(-
b + x)^2)^(3/4)*(a*b^2 - b*(2*a + b)*x + (a + 2*b)*x^2 + (-1 + d)*x^3)), x]
```

IntegrateAlgebraic [A] time = 3.65, size = 107, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(3*a*b^2 - 2*b*(2*a + b)*x + (a + 2*b)*x^2))/((x*(-
a + x)*(-b + x)^2)^(3/4)*(a*b^2 - b*(2*a + b)*x + (a + 2*b)*x^2 + (-1 + d)*
x^3)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*x)/(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x
^4)^(1/4)]])/d^(3/4) + (2*ArcTanh[(d^(1/4)*x)/(-(a*b^2*x) + (2*a*b + b^2)*x^
2 + (-a - 2*b)*x^3 + x^4)^(1/4)]])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*a*b^2-2*b*(2*a+b)*x+(a+2*b)*x^2)/(x*(-a+x)*(-b+x)^2)^(3/4)
/(a*b^2-b*(2*a+b)*x+(a+2*b)*x^2+(-1+d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,ab^2 - 2\,(2\,a + b)bx + (a + 2\,b)x^2\right)x^2}{\left(-(a - x)(b - x)^2x\right)^{\frac{3}{4}}\left((d - 1)x^3 + ab^2 - (2\,a + b)bx + (a + 2\,b)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*a*b^2-2*b*(2*a+b)*x+(a+2*b)*x^2)/(x*(-a+x)*(-b+x)^2)^(3/4)
/(a*b^2-b*(2*a+b)*x+(a+2*b)*x^2+(-1+d)*x^3),x, algorithm="giac")
```

```
[Out] integrate((3*a*b^2 - 2*(2*a + b)*b*x + (a + 2*b)*x^2)*x^2/((-(a - x)*(b - x
)^2*x)^(3/4)*((d - 1)*x^3 + a*b^2 - (2*a + b)*b*x + (a + 2*b)*x^2)), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(3a\,b^2 - 2b\,(2a + b)\,x + (a + 2b)\,x^2\right)}{\left(x\,(-a + x)\,(-b + x)^2\right)^{\frac{3}{4}}\left(a\,b^2 - b\,(2a + b)\,x + (a + 2b)\,x^2 + (-1 + d)\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*a*b^2-2*b*(2*a+b)*x+(a+2*b)*x^2)/(x*(-a+x)*(-b+x)^2)^(3/4)/(a*b^
2-b*(2*a+b)*x+(a+2*b)*x^2+(-1+d)*x^3),x)
```

```
[Out] int(x^2*(3*a*b^2-2*b*(2*a+b)*x+(a+2*b)*x^2)/(x*(-a+x)*(-b+x)^2)^(3/4)/(a*b^
2-b*(2*a+b)*x+(a+2*b)*x^2+(-1+d)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,ab^2 - 2\,(2\,a + b)bx + (a + 2\,b)x^2\right)x^2}{\left(-(a - x)(b - x)^2x\right)^{\frac{3}{4}}\left((d - 1)x^3 + ab^2 - (2\,a + b)bx + (a + 2\,b)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*a*b^2-2*b*(2*a+b)*x+(a+2*b)*x^2)/(x*(-a+x)*(-b+x)^2)^(3/4)
/(a*b^2-b*(2*a+b)*x+(a+2*b)*x^2+(-1+d)*x^3),x, algorithm="maxima")
```

```
[Out] integrate((3*a*b^2 - 2*(2*a + b)*b*x + (a + 2*b)*x^2)*x^2/((-(a - x)*(b - x
)^2*x)^(3/4)*((d - 1)*x^3 + a*b^2 - (2*a + b)*b*x + (a + 2*b)*x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(3\,a\,b^2 + x^2\,\left(a + 2\,b\right) - 2\,b\,x\,\left(2\,a + b\right)\right)}{{\left(-x\,\left(a - x\right)\,{\left(b - x\right)}^2\right)}^{3/4}\,\left(a\,b^2 + x^2\,\left(a + 2\,b\right) + x^3\,\left(d - 1\right) - b\,x\,\left(2\,a + b\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*a*b^2 + x^2*(a + 2*b) - 2*b*x*(2*a + b)))/((-x*(a - x)*(b - x)^
2)^(3/4)*(a*b^2 + x^2*(a + 2*b) + x^3*(d - 1) - b*x*(2*a + b))),x)
```

```
[Out] int((x^2*(3*a*b^2 + x^2*(a + 2*b) - 2*b*x*(2*a + b)))/((-x*(a - x)*(b - x)^
2)^(3/4)*(a*b^2 + x^2*(a + 2*b) + x^3*(d - 1) - b*x*(2*a + b))), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*a*b**2-2*b*(2*a+b)*x+(a+2*b)*x**2)/(x*(-a+x)*(-b+x)**2)**
(3/4)/(a*b**2-b*(2*a+b)*x+(a+2*b)*x**2+(-1+d)*x**3),x)
```

```
[Out] Timed out
```

3.1339 $\int \frac{\sqrt[3]{1+x^4}\left(3+x^4\right)}{x^{17}}\,dx$

Optimal. Leaf size=107

$$-\frac{5}{324}\log\left(\sqrt[3]{x^4+1}-1\right)+\frac{5}{648}\log\left(\left(x^4+1\right)^{2/3}+\sqrt[3]{x^4+1}+1\right)+\frac{5\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{108\sqrt{3}}+\frac{\sqrt[3]{x^4+1}\left(-10x^{12}+6x^8\right.}{432x^{16}}$$

Rubi [A] time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {446, 78, 47, 51, 57, 618, 204, 31}

$$-\frac{5\sqrt[3]{x^4+1}}{216x^4}-\frac{5}{216}\log\left(1-\sqrt[3]{x^4+1}\right)+\frac{5\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}+1}{\sqrt{3}}\right)}{108\sqrt{3}}-\frac{3\left(x^4+1\right)^{4/3}}{16x^{16}}+\frac{\sqrt[3]{x^4+1}}{12x^{12}}+\frac{\sqrt[3]{x^4+1}}{72x^8}+\frac{5\log(x)}{162}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^4)^(1/3)*(3 + x^4))/x^17,x]
```

```
[Out] (1 + x^4)^(1/3)/(12*x^12) + (1 + x^4)^(1/3)/(72*x^8) - (5*(1 + x^4)^(1/3))/
(216*x^4) - (3*(1 + x^4)^(4/3))/(16*x^16) + (5*ArcTan[(1 + 2*(1 + x^4)^(1/3
))/Sqrt[3]])/(108*Sqrt[3]) + (5*Log[x])/162 - (5*Log[1 - (1 + x^4)^(1/3)])/
216
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{1+x^4}\left(3+x^4\right)}{x^{17}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{\sqrt[3]{1+x}\,(3+x)}{x^5}\,dx,x,x^4\right)\\
&= -\frac{3\left(1+x^4\right)^{4/3}}{16x^{16}}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{\sqrt[3]{1+x}}{x^4}\,dx,x,x^4\right)\\
&= \frac{\sqrt[3]{1+x^4}}{12x^{12}}-\frac{3\left(1+x^4\right)^{4/3}}{16x^{16}}-\frac{1}{36}\operatorname{Subst}\left(\int \frac{1}{x^3(1+x)^{2/3}}\,dx,x,x^4\right)\\
&= \frac{\sqrt[3]{1+x^4}}{12x^{12}}+\frac{\sqrt[3]{1+x^4}}{72x^8}-\frac{3\left(1+x^4\right)^{4/3}}{16x^{16}}+\frac{5}{216}\operatorname{Subst}\left(\int \frac{1}{x^2(1+x)^{2/3}}\,dx,x,x^4\right)\\
&= \frac{\sqrt[3]{1+x^4}}{12x^{12}}+\frac{\sqrt[3]{1+x^4}}{72x^8}-\frac{5\sqrt[3]{1+x^4}}{216x^4}-\frac{3\left(1+x^4\right)^{4/3}}{16x^{16}}-\frac{5}{324}\operatorname{Subst}\left(\int \frac{1}{x(1+x)^{2/3}}\,dx,x,\right.\\
&= \frac{\sqrt[3]{1+x^4}}{12x^{12}}+\frac{\sqrt[3]{1+x^4}}{72x^8}-\frac{5\sqrt[3]{1+x^4}}{216x^4}-\frac{3\left(1+x^4\right)^{4/3}}{16x^{16}}+\frac{5\log(x)}{162}+\frac{5}{216}\operatorname{Subst}\left(\int \frac{1}{1-x}\right.\\
&= \frac{\sqrt[3]{1+x^4}}{12x^{12}}+\frac{\sqrt[3]{1+x^4}}{72x^8}-\frac{5\sqrt[3]{1+x^4}}{216x^4}-\frac{3\left(1+x^4\right)^{4/3}}{16x^{16}}+\frac{5\log(x)}{162}-\frac{5}{216}\log\left(1-\sqrt[3]{1+x}\right.\\
&= \frac{\sqrt[3]{1+x^4}}{12x^{12}}+\frac{\sqrt[3]{1+x^4}}{72x^8}-\frac{5\sqrt[3]{1+x^4}}{216x^4}-\frac{3\left(1+x^4\right)^{4/3}}{16x^{16}}+\frac{5\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^4}}{\sqrt{3}}\right)}{108\sqrt{3}}+\frac{5\log(x)}{162}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.33

$$
-\frac{3\left(x^4+1\right)^{4/3}\left(x^{16}\,{}_2F_1\left(\frac{4}{3},4;\frac{7}{3};x^4+1\right)+1\right)}{16x^{16}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^4)^(1/3)*(3 + x^4))/x^17,x]
```

```
[Out] (-3*(1 + x^4)^(4/3)*(1 + x^16*Hypergeometric2F1[4/3, 4, 7/3, 1 + x^4]))/(16
*x^16)
```

IntegrateAlgebraic [A] time = 0.25, size = 107, normalized size = 1.00

$$-\frac{5}{324}\log\left(\sqrt[3]{x^4+1}-1\right)+\frac{5}{648}\log\left(\left(x^4+1\right)^{2/3}+\sqrt[3]{x^4+1}+1\right)+\frac{5\tan^{-1}\left(\frac{2\sqrt[3]{x^4+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{108\sqrt{3}}+\frac{\sqrt[3]{x^4+1}\left(-10x^{12}+6x^8-45x^4-81\right)}{432x^{16}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^4)^(1/3)*(3 + x^4))/x^17,x]
```

```
[Out] ((1 + x^4)^(1/3)*(-81 - 45*x^4 + 6*x^8 - 10*x^12))/(432*x^16) + (5*ArcTan[1
/Sqrt[3] + (2*(1 + x^4)^(1/3))/Sqrt[3]])/(108*Sqrt[3]) - (5*Log[-1 + (1 + x
^4)^(1/3)])/324 + (5*Log[1 + (1 + x^4)^(1/3) + (1 + x^4)^(2/3)])/648
```

fricas [A] time = 0.43, size = 96, normalized size = 0.90

$$\frac{20\sqrt{3}x^{16}\arctan\left(\frac{2}{3}\sqrt{3}\left(x^4+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+10x^{16}\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)-20x^{16}\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)-3\left(10x^{12}-6x^8+45x^4+81\right)\left(x^4+1\right)^{\frac{1}{3}}}{1296x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/3)*(x^4+3)/x^17,x, algorithm="fricas")
```

```
[Out] 1/1296*(20*sqrt(3)*x^16*arctan(2/3*sqrt(3)*(x^4 + 1)^(1/3) + 1/3*sqrt(3)) +
 10*x^16*log((x^4 + 1)^(2/3) + (x^4 + 1)^(1/3) + 1) - 20*x^16*log((x^4 + 1)
^(1/3) - 1) - 3*(10*x^12 - 6*x^8 + 45*x^4 + 81)*(x^4 + 1)^(1/3))/x^16
```

giac [A] time = 0.13, size = 96, normalized size = 0.90

$$\frac{5}{324}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{10\left(x^4+1\right)^{\frac{10}{3}}-36\left(x^4+1\right)^{\frac{7}{3}}+87\left(x^4+1\right)^{\frac{4}{3}}+20\left(x^4+1\right)^{\frac{1}{3}}}{432x^{16}}+\frac{5}{648}\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)-\frac{5}{324}\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/3)*(x^4+3)/x^17,x, algorithm="giac")
```

```
[Out] 5/324*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 + 1)^(1/3) + 1)) - 1/432*(10*(x^4
+ 1)^(10/3) - 36*(x^4 + 1)^(7/3) + 87*(x^4 + 1)^(4/3) + 20*(x^4 + 1)^(1/3))
/x^16 + 5/648*log((x^4 + 1)^(2/3) + (x^4 + 1)^(1/3) + 1) - 5/324*log((x^4 +
 1)^(1/3) - 1)
```

maple [C] time = 0.28, size = 81, normalized size = 0.76

$$-\frac{10x^{16}+4x^{12}+39x^8+126x^4+81}{432x^{16}\left(x^4+1\right)^{\frac{2}{3}}}-\frac{5\left(-\frac{2\Gamma\left(\frac{2}{3}\right)x^4\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],-x^4\right)}{3}+\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+4\ln(x)\right)\Gamma\left(\frac{2}{3}\right)\right)}{324\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(1/3)*(x^4+3)/x^17,x)
```

```
[Out] -1/432*(10*x^16+4*x^12+39*x^8+126*x^4+81)/x^16/(x^4+1)^(2/3)-5/324/GAMMA(2/
3)*(-2/3*GAMMA(2/3)*x^4*hypergeom([1,1,5/3],[2,2],-x^4)+(1/6*Pi*3^(1/2)-3/2
*ln(3)+4*ln(x))*GAMMA(2/3))
```

maxima [B] time = 0.43, size = 182, normalized size = 1.70

$$\frac{5}{324}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^4+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{20\left(x^4+1\right)^{\frac{10}{3}}-72\left(x^4+1\right)^{\frac{7}{3}}+93\left(x^4+1\right)^{\frac{4}{3}}+40\left(x^4+1\right)^{\frac{1}{3}}}{432\left(\left(x^4+1\right)^4-4x^4-4\left(x^4+1\right)^3+6\left(x^4+1\right)^2-3\right)}+\frac{5\left(x^4+1\right)^{\frac{7}{3}}-13\left(x^4+1\right)^{\frac{4}{3}}-10\left(x^4+1\right)^{\frac{1}{3}}}{216\left(3x^4+\left(x^4+1\right)^3-3\left(x^4+1\right)^2+2\right)}+\frac{5}{648}\log\left(\left(x^4+1\right)^{\frac{2}{3}}+\left(x^4+1\right)^{\frac{1}{3}}+1\right)-\frac{5}{324}\log\left(\left(x^4+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/3)*(x^4+3)/x^17,x, algorithm="maxima")
```

```
[Out] 5/324*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^4 + 1)^(1/3) + 1)) - 1/432*(20*(x^4
+ 1)^(10/3) - 72*(x^4 + 1)^(7/3) + 93*(x^4 + 1)^(4/3) + 40*(x^4 + 1)^(1/3))
/((x^4 + 1)^4 - 4*x^4 - 4*(x^4 + 1)^3 + 6*(x^4 + 1)^2 - 3) + 1/216*(5*(x^4
+ 1)^(7/3) - 13*(x^4 + 1)^(4/3) - 10*(x^4 + 1)^(1/3))/(3*x^4 + (x^4 + 1)^3
- 3*(x^4 + 1)^2 + 2) + 5/648*log((x^4 + 1)^(2/3) + (x^4 + 1)^(1/3) + 1) - 5
/324*log((x^4 + 1)^(1/3) - 1)
```

mupad [B] time = 1.82, size = 268, normalized size = 2.50

$$\frac{5\ln\left(\frac{25\left(x^4+1\right)^{1/3}}{11664}-\frac{25}{11664}\right)}{324}-\frac{5\ln\left(\frac{25\left(x^4+1\right)^{1/3}}{2916}-\frac{25}{2916}\right)}{162}-\frac{\frac{5\left(x^4+1\right)^{1/3}}{108}+\frac{13\left(x^4+1\right)^{4/3}}{216}-\frac{5\left(x^4+1\right)^{7/3}}{216}}{\left(x^4+1\right)^3-3\left(x^4+1\right)^2+3x^4+2}+\frac{\frac{5\left(x^4+1\right)^{1/3}}{54}+\frac{31\left(x^4+1\right)^{4/3}}{144}-\frac{\left(x^4+1\right)^{7/3}}{6}+\frac{5\left(x^4+1\right)^{10/3}}{108}}{4\left(x^4+1\right)^3-6\left(x^4+1\right)^2-\left(x^4+1\right)^4+4x^4+3}-\ln\left(\frac{5\left(x^4+1\right)^{1/3}}{18}+\frac{5}{36}-\frac{\sqrt{3}\,5\mathrm{i}}{36}\right)\left(-\frac{5}{324}+\frac{\sqrt{3}\,5\mathrm{i}}{324}\right)+\ln\left(\frac{5\left(x^4+1\right)^{1/3}}{18}+\frac{5}{36}+\frac{\sqrt{3}\,5\mathrm{i}}{36}\right)\left(\frac{5}{324}+\frac{\sqrt{3}\,5\mathrm{i}}{324}\right)+\ln\left(\frac{5\left(x^4+1\right)^{1/3}}{36}+\frac{5}{72}-\frac{\sqrt{3}\,5\mathrm{i}}{72}\right)\left(-\frac{5}{648}+\frac{\sqrt{3}\,5\mathrm{i}}{648}\right)-\ln\left(\frac{5\left(x^4+1\right)^{1/3}}{36}+\frac{5}{72}+\frac{\sqrt{3}\,5\mathrm{i}}{72}\right)\left(\frac{5}{648}+\frac{\sqrt{3}\,5\mathrm{i}}{648}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/3)*(x^4 + 3))/x^17,x)
```

```
[Out] (5*log((25*(x^4 + 1)^(1/3))/11664 - 25/11664))/324 - (5*log((25*(x^4 + 1)^(
1/3))/2916 - 25/2916))/162 - ((5*(x^4 + 1)^(1/3))/108 + (13*(x^4 + 1)^(4/3)
)/216 - (5*(x^4 + 1)^(7/3))/216)/((x^4 + 1)^3 - 3*(x^4 + 1)^2 + 3*x^4 + 2)
+ ((5*(x^4 + 1)^(1/3))/54 + (31*(x^4 + 1)^(4/3))/144 - (x^4 + 1)^(7/3)/6 +
(5*(x^4 + 1)^(10/3))/108)/(4*(x^4 + 1)^3 - 6*(x^4 + 1)^2 - (x^4 + 1)^4 + 4*
x^4 + 3) - log((5*(x^4 + 1)^(1/3))/18 - (3^(1/2)*5i)/36 + 5/36)*((3^(1/2)*5
i)/324 - 5/324) + log((3^(1/2)*5i)/36 + (5*(x^4 + 1)^(1/3))/18 + 5/36)*((3^
(1/2)*5i)/324 + 5/324) + log((5*(x^4 + 1)^(1/3))/36 - (3^(1/2)*5i)/72 + 5/7
2)*((3^(1/2)*5i)/648 - 5/648) - log((3^(1/2)*5i)/72 + (5*(x^4 + 1)^(1/3))/3
6 + 5/72)*((3^(1/2)*5i)/648 + 5/648)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(1/3)*(x**4+3)/x**17,x)
```

```
[Out] Timed out
```

3.1340 $\int \frac{1}{(-1+x^4)^2 \sqrt[4]{-x^2+x^4}} \, dx$

Optimal. Leaf size=107

$$\frac{(x^4-x^2)^{3/4}(67x^4+2x^2-85)}{80x(x^2-1)^2(x^2+1)}+\frac{15\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^2}}\right)}{32\sqrt[4]{2}}+\frac{15\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^2}}\right)}{32\sqrt[4]{2}}$$

Rubi [A] time = 0.16, antiderivative size = 194, normalized size of antiderivative = 1.81, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2056, 1254, 466, 414, 527, 12, 377, 212, 206, 203}

$$-\frac{x}{40(1-x^2)\sqrt[4]{x^4-x^2}}+\frac{x}{4(1-x^2)(x^2+1)\sqrt[4]{x^4-x^2}}+\frac{67x}{80\sqrt[4]{x^4-x^2}}+\frac{15\sqrt[4]{x^2-1}\sqrt{x}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{32\sqrt[4]{2}\sqrt[4]{x^4-x^2}}+\frac{15\sqrt[4]{x^2-1}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{32\sqrt[4]{2}\sqrt[4]{x^4-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x^4)^2*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] (67*x)/(80*(-x^2 + x^4)^(1/4)) - x/(40*(1 - x^2)*(-x^2 + x^4)^(1/4)) + x/(4
*(1 - x^2)*(1 + x^2)*(-x^2 + x^4)^(1/4)) + (15*Sqrt[x]*(-1 + x^2)^(1/4)*Arc
Tan[(2^(1/4)*Sqrt[x])/(-1 + x^2)^(1/4)])/(32*2^(1/4)*(-x^2 + x^4)^(1/4)) +
(15*Sqrt[x]*(-1 + x^2)^(1/4)*ArcTanh[(2^(1/4)*Sqrt[x])/(-1 + x^2)^(1/4)])/(
32*2^(1/4)*(-x^2 + x^4)^(1/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1254

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e)^p, x] /
; FreeQ[{a, c, d, e, f, q, m, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p
]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{\left(-1+x^4\right)^2 \sqrt[4]{-x^2+x^4}}\,dx = \frac{\left(\sqrt{x}\,\sqrt[4]{-1+x^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt[4]{-1+x^2}\left(-1+x^4\right)^2}\,dx}{\sqrt[4]{-x^2+x^4}}$$

$$= \frac{\left(\sqrt{x}\,\sqrt[4]{-1+x^2}\right)\int \frac{1}{\sqrt{x}\left(-1+x^2\right)^{9/4}\left(1+x^2\right)^2}\,dx}{\sqrt[4]{-x^2+x^4}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\left(-1+x^4\right)^{9/4}\left(1+x^4\right)^2}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^4}}$$

$$= \frac{x}{4\left(1-x^2\right)\left(1+x^2\right)\sqrt[4]{-x^2+x^4}} + \frac{\left(\sqrt{x}\,\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{7-8x^4}{\left(-1+x^4\right)^{9/4}\left(1+x^4\right)}\,dx, x, \sqrt{}\right.}{4\sqrt[4]{-x^2+x^4}}$$

$$= -\frac{x}{40\left(1-x^2\right)\sqrt[4]{-x^2+x^4}} + \frac{x}{4\left(1-x^2\right)\left(1+x^2\right)\sqrt[4]{-x^2+x^4}} + \frac{\left(\sqrt{x}\,\sqrt[4]{-1+x^2}\right)\text{Subs}}{40}$$

$$= \frac{67x}{80\sqrt[4]{-x^2+x^4}} - \frac{x}{40\left(1-x^2\right)\sqrt[4]{-x^2+x^4}} + \frac{x}{4\left(1-x^2\right)\left(1+x^2\right)\sqrt[4]{-x^2+x^4}} + \frac{\left(\sqrt{x}\right.}{}$$

$$= \frac{67x}{80\sqrt[4]{-x^2+x^4}} - \frac{x}{40\left(1-x^2\right)\sqrt[4]{-x^2+x^4}} + \frac{x}{4\left(1-x^2\right)\left(1+x^2\right)\sqrt[4]{-x^2+x^4}} + \frac{\left(15\sqrt{}\right.}{}$$

$$= \frac{67x}{80\sqrt[4]{-x^2+x^4}} - \frac{x}{40\left(1-x^2\right)\sqrt[4]{-x^2+x^4}} + \frac{x}{4\left(1-x^2\right)\left(1+x^2\right)\sqrt[4]{-x^2+x^4}} + \frac{\left(15\sqrt{}\right.}{}$$

$$= \frac{67x}{80\sqrt[4]{-x^2+x^4}} - \frac{x}{40\left(1-x^2\right)\sqrt[4]{-x^2+x^4}} + \frac{x}{4\left(1-x^2\right)\left(1+x^2\right)\sqrt[4]{-x^2+x^4}} + \frac{\left(15\sqrt{}\right.}{}$$

$$= \frac{67x}{80\sqrt[4]{-x^2+x^4}} - \frac{x}{40\left(1-x^2\right)\sqrt[4]{-x^2+x^4}} + \frac{x}{4\left(1-x^2\right)\left(1+x^2\right)\sqrt[4]{-x^2+x^4}} + \frac{15\sqrt{x}}{}$$

Mathematica [C] time = 0.56, size = 92, normalized size = 0.86

$$\frac{x^3\left(75\sqrt[4]{1-x^2}\left(x^4-1\right){}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{2x^2}{x^2+1}\right)+\sqrt[4]{x^2+1}\left(67x^4+2x^2-85\right)\right)}{80\left(x^2\left(x^2-1\right)\right)^{5/4}\left(x^2+1\right)^{5/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-1 + x^4)^2*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] (x^3*((1 + x^2)^(1/4)*(-85 + 2*x^2 + 67*x^4) + 75*(1 - x^2)^(1/4)*(-1 + x^4
)*Hypergeometric2F1[1/4, 1/4, 5/4, (2*x^2)/(1 + x^2)]))/(80*(x^2*(-1 + x^2)
)^(5/4)*(1 + x^2)^(5/4))
```

IntegrateAlgebraic [A] time = 0.42, size = 107, normalized size = 1.00

$$\frac{\left(x^4-x^2\right)^{3/4}\left(67x^4+2x^2-85\right)}{80x\left(x^2-1\right)^2\left(x^2+1\right)}+\frac{15\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-x^2}}\right)}{32\sqrt[4]{2}}+\frac{15\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-x^2}}\right)}{32\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x^4)^2*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] ((-x^2 + x^4)^(3/4)*(-85 + 2*x^2 + 67*x^4))/(80*x*(-1 + x^2)^2*(1 + x^2)) +
 (15*ArcTan[(2^(1/4)*x)/(-x^2 + x^4)^(1/4)])/(32*2^(1/4)) + (15*ArcTanh[(2^
(1/4)*x)/(-x^2 + x^4)^(1/4)])/(32*2^(1/4))
```

fricas [B] time = 2.30, size = 331, normalized size = 3.09

$$-\frac{300\cdot 2^{\frac{3}{4}}\left(x^7-x^5-x^3+x\right)\arctan\left(\frac{4\cdot 2^{\frac{3}{4}}\left(x^4-x^2\right)^{\frac{1}{4}}x^2+2^{\frac{3}{4}}\left(2\cdot 2^{\frac{3}{4}}\sqrt{x^4-x^2}\,x+2^{\frac{1}{4}}\left(3x^3-x\right)\right)+4\cdot 2^{\frac{1}{4}}\left(x^4-x^2\right)^{\frac{3}{4}}}{2\left(x^3+x\right)}\right)-75\cdot 2^{\frac{3}{4}}\left(x^7-x^5-x^3+x\right)\log\left(\frac{4\sqrt{2}\left(x^4-x^2\right)^{\frac{1}{4}}x^2+2^{\frac{3}{4}}\left(3x^3-x\right)+4\cdot 2^{\frac{1}{4}}\sqrt{x^4-x^2}\,x+4\left(x^4-x^2\right)^{\frac{3}{4}}}{x^3+x}\right)+75\cdot 2^{\frac{3}{4}}\left(x^7-x^5-x^3+x\right)\log\left(\frac{4\sqrt{2}\left(x^4-x^2\right)^{\frac{1}{4}}x^2-2^{\frac{3}{4}}\left(3x^3-x\right)-4\cdot 2^{\frac{1}{4}}\sqrt{x^4-x^2}\,x+4\left(x^4-x^2\right)^{\frac{3}{4}}}{x^3+x}\right)-16\left(67x^4+2x^2-85\right)\left(x^4-x^2\right)^{\frac{3}{4}}}{1280\left(x^7-x^5-x^3+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)^2/(x^4-x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/1280*(300*2^(3/4)*(x^7 - x^5 - x^3 + x)*arctan(1/2*(4*2^(3/4)*(x^4 - x^2
)^(1/4)*x^2 + 2^(3/4)*(2*2^(3/4)*sqrt(x^4 - x^2)*x + 2^(1/4)*(3*x^3 - x)) +
 4*2^(1/4)*(x^4 - x^2)^(3/4))/(x^3 + x)) - 75*2^(3/4)*(x^7 - x^5 - x^3 + x)
*log((4*sqrt(2)*(x^4 - x^2)^(1/4)*x^2 + 2^(3/4)*(3*x^3 - x) + 4*2^(1/4)*sqr
t(x^4 - x^2)*x + 4*(x^4 - x^2)^(3/4))/(x^3 + x)) + 75*2^(3/4)*(x^7 - x^5 -
x^3 + x)*log((4*sqrt(2)*(x^4 - x^2)^(1/4)*x^2 - 2^(3/4)*(3*x^3 - x) - 4*2^(
1/4)*sqrt(x^4 - x^2)*x + 4*(x^4 - x^2)^(3/4))/(x^3 + x)) - 16*(67*x^4 + 2*x
^2 - 85)*(x^4 - x^2)^(3/4))/(x^7 - x^5 - x^3 + x)
```

giac [A] time = 0.32, size = 104, normalized size = 0.97

$$\frac{15}{64}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)-\frac{15}{128}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)+\frac{15}{128}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}-\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)-\frac{\frac{10}{x^2}-9}{10\left(\frac{1}{x^2}-1\right)\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}}+\frac{\left(-\frac{1}{x^2}+1\right)^{\frac{3}{4}}}{16\left(\frac{1}{x^2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)^2/(x^4-x^2)^(1/4),x, algorithm="giac")
```

```
[Out] 15/64*2^(3/4)*arctan(1/2*2^(3/4)*(-1/x^2 + 1)^(1/4)) - 15/128*2^(3/4)*log(2
^(1/4) + (-1/x^2 + 1)^(1/4)) + 15/128*2^(3/4)*log(2^(1/4) - (-1/x^2 + 1)^(1
/4)) - 1/10*(10/x^2 - 9)/((1/x^2 - 1)*(-1/x^2 + 1)^(1/4)) + 1/16*(-1/x^2 +
1)^(3/4)/(1/x^2 + 1)
```

maple [C] time = 3.86, size = 273, normalized size = 2.55

$$\frac{x\left(67x^4+2x^2-85\right)}{80\left(x^2+1\right)\left(x^2\left(x^2-1\right)\right)^{\frac{1}{4}}\left(x^2-1\right)}+\frac{15\,\mathrm{RootOf}\left(\_Z^4-8\right)\ln\left(\frac{\sqrt{x^4-x^2}\,\mathrm{RootOf}\left(\_Z^4-8\right)^3x+2\left(x^4-x^2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-8\right)^2x^2+3\,\mathrm{RootOf}\left(\_Z^4-8\right)x^3+4\left(x^4-x^2\right)^{\frac{3}{4}}-\mathrm{RootOf}\left(\_Z^4-8\right)x}{x\left(x^2+1\right)}\right)}{128}-\frac{15\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\ln\left(\frac{\sqrt{x^4-x^2}\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)\mathrm{RootOf}\left(\_Z^4-8\right)^2x-2\left(x^4-x^2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-8\right)^2x^2-3\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)x^3+4\left(x^4-x^2\right)^{\frac{3}{4}}+\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-8\right)^2\right)x}{x\left(x^2+1\right)}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-1)^2/(x^4-x^2)^(1/4),x)
```

```
[Out] 1/80*x*(67*x^4+2*x^2-85)/(x^2+1)/(x^2*(x^2-1))^(1/4)/(x^2-1)+15/128*RootOf(
_Z^4-8)*ln(((x^4-x^2)^(1/2)*RootOf(_Z^4-8)^3*x+2*(x^4-x^2)^(1/4)*RootOf(_Z^
4-8)^2*x^2+3*RootOf(_Z^4-8)*x^3+4*(x^4-x^2)^(3/4)-RootOf(_Z^4-8)*x)/x/(x^2+
1))-15/128*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln(((x^4-x^2)^(1/2)*RootOf(_Z^2+Ro
otOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x-2*(x^4-x^2)^(1/4)*RootOf(_Z^4-8)^2*x^2-3
*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^3+4*(x^4-x^2)^(3/4)+RootOf(_Z^2+RootOf(_Z^
4-8)^2)*x)/x/(x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4-x^2\right)^{\frac{1}{4}}\left(x^4-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)^2/(x^4-x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 - x^2)^(1/4)*(x^4 - 1)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^4-1\right)^2\left(x^4-x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^4 - 1)^2*(x^4 - x^2)^(1/4)),x)
```

```
[Out] int(1/((x^4 - 1)^2*(x^4 - x^2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^2\,(x-1)\,(x+1)}\,(x-1)^2\,(x+1)^2\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-1)**2/(x**4-x**2)**(1/4),x)
```

```
[Out] Integral(1/((x**2*(x - 1)*(x + 1))**(1/4)*(x - 1)**2*(x + 1)**2*(x**2 + 1)*
*2), x)
```

3.1341 $\int \frac{(-1+x^2)(1-x+x^2-x^3+x^4)}{(1-x+x^2)^2(1+x+x^2)\sqrt{1+3x^2+x^4}}\,dx$

Optimal. Leaf size=107

$$\frac{\sqrt{x^4+3x^2+1}}{4(x^2-x+1)}-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{x^2+\sqrt{x^4+3x^2+1}-x+1}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{x^2+\sqrt{x^4+3x^2+1}+x+1}\right)}{2\sqrt{2}}$$

Rubi [C] time = 10.76, antiderivative size = 5419, normalized size of antiderivative = 50.64, number of steps used = 136, number of rules used = 18, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6688, 6728, 1099, 6742, 1726, 1741, 12, 1247, 724, 204, 1716, 1189, 1135, 1214, 1456, 539, 1724, 2}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^2)*(1 - x + x^2 - x^3 + x^4))/((1 - x + x^2)^2*(1 + x + x^2)*S
qrt[1 + 3*x^2 + x^4]),x]
```

```
[Out] (x*(3 + Sqrt[5] + 2*x^2))/(12*(1 - I*Sqrt[3])*Sqrt[1 + 3*x^2 + x^4]) + (x*(
3 + Sqrt[5] + 2*x^2))/(12*(1 + I*Sqrt[3])*Sqrt[1 + 3*x^2 + x^4]) + ((I - Sq
rt[3])*x*(3 + Sqrt[5] + 2*x^2))/(24*(I + Sqrt[3])*Sqrt[1 + 3*x^2 + x^4]) +
((I + Sqrt[3])*x*(3 + Sqrt[5] + 2*x^2))/(24*(I - Sqrt[3])*Sqrt[1 + 3*x^2 +
x^4]) + Sqrt[1 + 3*x^2 + x^4]/(3*(1 + I*Sqrt[3])*(1 - I*Sqrt[3] - 2*x)) + (
(I + Sqrt[3])*Sqrt[1 + 3*x^2 + x^4])/(6*(I - Sqrt[3])*(1 - I*Sqrt[3] - 2*x)
) + Sqrt[1 + 3*x^2 + x^4]/(3*(1 - I*Sqrt[3])*(1 + I*Sqrt[3] - 2*x)) + ((I -
 Sqrt[3])*Sqrt[1 + 3*x^2 + x^4])/(6*(I + Sqrt[3])*(1 + I*Sqrt[3] - 2*x)) -
((1 - I*Sqrt[3])*ArcTan[(1 - (3*I)*Sqrt[3] + 2*(2 - I*Sqrt[3])*x^2)/(4*Sqrt
[1 + I*Sqrt[3]]*Sqrt[1 + 3*x^2 + x^4])]])/(16*Sqrt[1 + I*Sqrt[3]]) + ((I/4)*
ArcTan[(1 - (3*I)*Sqrt[3] + 2*(2 - I*Sqrt[3])*x^2)/(4*Sqrt[1 + I*Sqrt[3]]*S
qrt[1 + 3*x^2 + x^4])]])/Sqrt[3*(1 + I*Sqrt[3])] + ((1 + (3*I)*Sqrt[3])*ArcT
an[(1 - (3*I)*Sqrt[3] + 2*(2 - I*Sqrt[3])*x^2)/(4*Sqrt[1 + I*Sqrt[3]]*Sqrt[
1 + 3*x^2 + x^4])]])/(24*(1 + I*Sqrt[3])^(3/2)) + ((1 - (5*I)*Sqrt[3])*ArcTa
n[(1 - (3*I)*Sqrt[3] + 2*(2 - I*Sqrt[3])*x^2)/(4*Sqrt[1 + I*Sqrt[3]]*Sqrt[1
 + 3*x^2 + x^4])]])/(16*Sqrt[1 + I*Sqrt[3]]) - ((I - 3*Sqrt[3])*(I + Sqrt[3]
)*ArcTan[(1 - (3*I)*Sqrt[3] + 2*(2 - I*Sqrt[3])*x^2)/(4*Sqrt[1 + I*Sqrt[3]]
*Sqrt[1 + 3*x^2 + x^4])]])/(48*(1 + I*Sqrt[3])^(3/2)) - ((I/4)*ArcTan[(1 + (
3*I)*Sqrt[3] + 2*(2 + I*Sqrt[3])*x^2)/(4*Sqrt[1 - I*Sqrt[3]]*Sqrt[1 + 3*x^2
 + x^4])]])/Sqrt[3*(1 - I*Sqrt[3])] + ((5 - I*Sqrt[3])*ArcTan[(1 + (3*I)*Sqr
t[3] + 2*(2 + I*Sqrt[3])*x^2)/(4*Sqrt[1 - I*Sqrt[3]]*Sqrt[1 + 3*x^2 + x^4])
]])/(24*(1 - I*Sqrt[3])^(3/2)) - ((1 + I*Sqrt[3])*ArcTan[(1 + (3*I)*Sqrt[3]
 + 2*(2 + I*Sqrt[3])*x^2)/(4*Sqrt[1 - I*Sqrt[3]]*Sqrt[1 + 3*x^2 + x^4])]])/(1
6*Sqrt[1 - I*Sqrt[3]]) + ((1 - (3*I)*Sqrt[3])*ArcTan[(1 + (3*I)*Sqrt[3] + 2
*(2 + I*Sqrt[3])*x^2)/(4*Sqrt[1 - I*Sqrt[3]]*Sqrt[1 + 3*x^2 + x^4])]])/(24*(
1 - I*Sqrt[3])^(3/2)) + ((1 + (5*I)*Sqrt[3])*ArcTan[(1 + (3*I)*Sqrt[3] + 2*
(2 + I*Sqrt[3])*x^2)/(4*Sqrt[1 - I*Sqrt[3]]*Sqrt[1 + 3*x^2 + x^4])]])/(16*Sq
rt[1 - I*Sqrt[3]]) - (Sqrt[(3 + Sqrt[5])/2]*Sqrt[(2 + (3 - Sqrt[5])*x^2)/(2
 + (3 + Sqrt[5])*x^2)]*(2 + (3 + Sqrt[5])*x^2)*EllipticE[ArcTan[Sqrt[(3 + S
qrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/(12*(1 - I*Sqrt[3])*Sqrt[1 + 3*x^2 + x^
4]) - (Sqrt[(3 + Sqrt[5])/2]*Sqrt[(2 + (3 - Sqrt[5])*x^2)/(2 + (3 + Sqrt[5]
)*x^2)]*(2 + (3 + Sqrt[5])*x^2)*EllipticE[ArcTan[Sqrt[(3 + Sqrt[5])/2]*x],
(-5 + 3*Sqrt[5])/2])/(12*(1 + I*Sqrt[3])*Sqrt[1 + 3*x^2 + x^4]) - ((I - Sqr
t[3])*Sqrt[(3 + Sqrt[5])/2]*Sqrt[(2 + (3 - Sqrt[5])*x^2)/(2 + (3 + Sqrt[5])
*x^2)]*(2 + (3 + Sqrt[5])*x^2)*EllipticE[ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (
-5 + 3*Sqrt[5])/2])/(24*(I + Sqrt[3])*Sqrt[1 + 3*x^2 + x^4]) - ((I + Sqrt[3
])*Sqrt[(3 + Sqrt[5])/2]*Sqrt[(2 + (3 - Sqrt[5])*x^2)/(2 + (3 + Sqrt[5])*x^
2)]*(2 + (3 + Sqrt[5])*x^2)*EllipticE[ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5
+ 3*Sqrt[5])/2])/(24*(I - Sqrt[3])*Sqrt[1 + 3*x^2 + x^4]) + (7*Sqrt[(2 + (3
```

```
 - Sqrt[5])*x^2)/(2 + (3 + Sqrt[5])*x^2)]*(2 + (3 + Sqrt[5])*x^2)*EllipticF
[ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/(6*Sqrt[2*(3 + Sqrt[
5])]*Sqrt[1 + 3*x^2 + x^4]) + ((1 - I*Sqrt[3])*Sqrt[(2 + (3 - Sqrt[5])*x^2)
/(2 + (3 + Sqrt[5])*x^2)]*(2 + (3 + Sqrt[5])*x^2)*EllipticF[ArcTan[Sqrt[(3
+ Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/(24*Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 + 3
*x^2 + x^4]) + ((1 + I*Sqrt[3])*Sqrt[(2 + (3 - Sqrt[5])*x^2)/(2 + (3 + Sqrt
[5])*x^2)]*(2 + (3 + Sqrt[5])*x^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt[5])/2]*x
], (-5 + 3*Sqrt[5])/2])/(24*Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 + 3*x^2 + x^4]) -
((5*I - Sqrt[3])*Sqrt[(2 + (3 - Sqrt[5])*x^2)/(2 + (3 + Sqrt[5])*x^2)]*(2 +
 (3 + Sqrt[5])*x^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt
[5])/2])/(6*(I - Sqrt[3])*(2 - I*Sqrt[3] - Sqrt[5])*Sqrt[2*(3 + Sqrt[5])]*S
qrt[1 + 3*x^2 + x^4]) - ((2*I + Sqrt[3])*Sqrt[(2 + (3 - Sqrt[5])*x^2)/(2 +
(3 + Sqrt[5])*x^2)]*(2 + (3 + Sqrt[5])*x^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt
[5])/2]*x], (-5 + 3*Sqrt[5])/2])/(3*(I - Sqrt[3])*(2 - I*Sqrt[3] - Sqrt[5])
*Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 + 3*x^2 + x^4]) - ((2*I - Sqrt[3])*Sqrt[(2 +
(3 - Sqrt[5])*x^2)/(2 + (3 + Sqrt[5])*x^2)]*(2 + (3 + Sqrt[5])*x^2)*Ellipti
cF[ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/(3*(I + Sqrt[3])*(
2 + I*Sqrt[3] - Sqrt[5])*Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 + 3*x^2 + x^4]) + ((I
 + Sqrt[3])*Sqrt[(2 + (3 - Sqrt[5])*x^2)/(2 + (3 + Sqrt[5])*x^2)]*(2 + (3 +
 Sqrt[5])*x^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/
2])/(4*(2*I - Sqrt[3] - I*Sqrt[5])*Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 + 3*x^2 + x
^4]) + ((7*I + 3*Sqrt[3])*Sqrt[(2 + (3 - Sqrt[5])*x^2)/(2 + (3 + Sqrt[5])*x
^2)]*(2 + (3 + Sqrt[5])*x^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5
 + 3*Sqrt[5])/2])/(4*(2*I - Sqrt[3] - I*Sqrt[5])*Sqrt[2*(3 + Sqrt[5])]*Sqrt
[1 + 3*x^2 + x^4]) + ((7*I - 3*Sqrt[3])*Sqrt[(2 + (3 - Sqrt[5])*x^2)/(2 + (
3 + Sqrt[5])*x^2)]*(2 + (3 + Sqrt[5])*x^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt[
5])/2]*x], (-5 + 3*Sqrt[5])/2])/(4*(2*I + Sqrt[3] - I*Sqrt[5])*Sqrt[2*(3 +
Sqrt[5])]*Sqrt[1 + 3*x^2 + x^4]) + ((I - Sqrt[3])*Sqrt[(2 + (3 - Sqrt[5])*x
^2)/(2 + (3 + Sqrt[5])*x^2)]*(2 + (3 + Sqrt[5])*x^2)*EllipticF[ArcTan[Sqrt[
(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/(4*(2*I + Sqrt[3] - I*Sqrt[5])*Sq
rt[2*(3 + Sqrt[5])]*Sqrt[1 + 3*x^2 + x^4]) - ((I + Sqrt[3])*Sqrt[(2 + (3 -
Sqrt[5])*x^2)/(2 + (3 + Sqrt[5])*x^2)]*(2 + (3 + Sqrt[5])*x^2)*EllipticF[Ar
cTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/(2*(2 - I*Sqrt[3] - Sqr
t[5])*Sqrt[6*(3 + Sqrt[5])]*Sqrt[1 + 3*x^2 + x^4]) + ((I - Sqrt[3])*Sqrt[(2
 + (3 - Sqrt[5])*x^2)/(2 + (3 + Sqrt[5])*x^2)]*(2 + (3 + Sqrt[5])*x^2)*Elli
pticF[ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/(2*(2 + I*Sqrt[
3] - Sqrt[5])*Sqrt[6*(3 + Sqrt[5])]*Sqrt[1 + 3*x^2 + x^4]) - ((5*I + Sqrt[3
])*Sqrt[(2 + (3 - Sqrt[5])*x^2)/(2 + (3 + Sqrt[5])*x^2)]*(2 + (3 + Sqrt[5])
*x^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/(6*Sq
rt[2*(3 + Sqrt[5])]*(5*I + Sqrt[3] - I*Sqrt[5] - Sqrt[15])*Sqrt[1 + 3*x^2 +
 x^4]) + ((1 + I*Sqrt[3])*Sqrt[(9 - 4*Sqrt[5])/3]*(3 + Sqrt[5] + 2*x^2)*Ell
ipticPi[1 - (2*(3 - Sqrt[5]))/(I - Sqrt[3])^2, ArcTan[Sqrt[(3 + Sqrt[5])/2]
*x], (-5 + 3*Sqrt[5])/2])/(2*(5*I + Sqrt[3] - I*Sqrt[5] - Sqrt[15])*Sqrt[(3
 + Sqrt[5] + 2*x^2)/(3 - Sqrt[5] + 2*x^2)]*Sqrt[1 + 3*x^2 + x^4]) + ((I - S
qrt[3])*(1 + (5*I)*Sqrt[3])*Sqrt[9 - 4*Sqrt[5]]*(3 + Sqrt[5] + 2*x^2)*Ellip
ticPi[1 - (2*(3 - Sqrt[5]))/(I - Sqrt[3])^2, ArcTan[Sqrt[(3 + Sqrt[5])/2]*x
], (-5 + 3*Sqrt[5])/2])/(8*(5*I + Sqrt[3] - I*Sqrt[5] - Sqrt[15])*Sqrt[(3 +
 Sqrt[5] + 2*x^2)/(3 - Sqrt[5] + 2*x^2)]*Sqrt[1 + 3*x^2 + x^4]) - ((I + Sqr
t[3])*Sqrt[9 - 4*Sqrt[5]]*(3 + Sqrt[5] + 2*x^2)*EllipticPi[1 - (2*(3 - Sqrt
[5]))/(I - Sqrt[3])^2, ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2]
)/(4*(5*I + Sqrt[3] - I*Sqrt[5] - Sqrt[15])*Sqrt[(3 + Sqrt[5] + 2*x^2)/(3 -
 Sqrt[5] + 2*x^2)]*Sqrt[1 + 3*x^2 + x^4]) - ((I - Sqrt[3])*(I + 3*Sqrt[3])*
Sqrt[9 - 4*Sqrt[5]]*(3 + Sqrt[5] + 2*x^2)*EllipticPi[1 - (2*(3 - Sqrt[5]))/
(I - Sqrt[3])^2, ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/(24*
(1 - (3*I)*Sqrt[3] + Sqrt[5] + I*Sqrt[15])*Sqrt[(3 + Sqrt[5] + 2*x^2)/(3 -
Sqrt[5] + 2*x^2)]*Sqrt[1 + 3*x^2 + x^4]) + ((I + Sqrt[3])*(I + 3*Sqrt[3])*S
qrt[9 - 4*Sqrt[5]]*(3 + Sqrt[5] + 2*x^2)*EllipticPi[1 - (2*(3 - Sqrt[5]))/(
I - Sqrt[3])^2, ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/(24*(
1 - (3*I)*Sqrt[3] + Sqrt[5] + I*Sqrt[15])*Sqrt[(3 + Sqrt[5] + 2*x^2)/(3 - S
```

```
qrt[5] + 2*x^2)]*Sqrt[1 + 3*x^2 + x^4]) - ((1 - I*Sqrt[3])*Sqrt[(9 - 4*Sqrt
[5])/3]*(3 + Sqrt[5] + 2*x^2)*EllipticPi[1 - (2*(3 - Sqrt[5]))/(I + Sqrt[3]
)^2, ArcTan[Sqrt[(3 + Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/(2*(5*I - Sqrt[3
] - I*Sqrt[5] + Sqrt[15])*Sqrt[(3 + Sqrt[5] + 2*x^2)/(3 - Sqrt[5] + 2*x^2)]
*Sqrt[1 + 3*x^2 + x^4]) - ((I - Sqrt[3])*Sqrt[9 - 4*Sqrt[5]]*(3 + Sqrt[5] +
 2*x^2)*EllipticPi[1 - (2*(3 - Sqrt[5]))/(I + Sqrt[3])^2, ArcTan[Sqrt[(3 +
Sqrt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/(4*(5*I - Sqrt[3] - I*Sqrt[5] + Sqrt[1
5])*Sqrt[(3 + Sqrt[5] + 2*x^2)/(3 - Sqrt[5] + 2*x^2)]*Sqrt[1 + 3*x^2 + x^4]
) + ((1 - (5*I)*Sqrt[3])*(I + Sqrt[3])*Sqrt[9 - 4*Sqrt[5]]*(3 + Sqrt[5] + 2
*x^2)*EllipticPi[1 - (2*(3 - Sqrt[5]))/(I + Sqrt[3])^2, ArcTan[Sqrt[(3 + Sq
rt[5])/2]*x], (-5 + 3*Sqrt[5])/2])/(8*(5*I - Sqrt[3] - I*Sqrt[5] + Sqrt[15]
)*Sqrt[(3 + Sqrt[5] + 2*x^2)/(3 - Sqrt[5] + 2*x^2)]*Sqrt[1 + 3*x^2 + x^4])
- ((I - Sqrt[3])*(1 + (3*I)*Sqrt[3])*Sqrt[9 - 4*Sqrt[5]]*(3 + Sqrt[5] + 2*x
^2)*EllipticPi[1 - (2*(3 - Sqrt[5]))/(I + Sqrt[3])^2, ArcTan[Sqrt[(3 + Sqrt
[5])/2]*x], (-5 + 3*Sqrt[5])/2])/(24*(I - 3*Sqrt[3] + I*Sqrt[5] + Sqrt[15])
*Sqrt[(3 + Sqrt[5] + 2*x^2)/(3 - Sqrt[5] + 2*x^2)]*Sqrt[1 + 3*x^2 + x^4]) +
 ((1 + (3*I)*Sqrt[3])*(I + Sqrt[3])*Sqrt[9 - 4*Sqrt[5]]*(3 + Sqrt[5] + 2*x^
2)*EllipticPi[1 - (2*(3 - Sqrt[5]))/(I + Sqrt[3])^2, ArcTan[Sqrt[(3 + Sqrt[
5])/2]*x], (-5 + 3*Sqrt[5])/2])/(24*(I - 3*Sqrt[3] + I*Sqrt[5] + Sqrt[15])*
Sqrt[(3 + Sqrt[5] + 2*x^2)/(3 - Sqrt[5] + 2*x^2)]*Sqrt[1 + 3*x^2 + x^4])
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 1724

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Sym
bol] :> Dist[d, Int[1/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] - D
ist[e, Int[x/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a,
 b, c, d, e}, x]
```

Rule 1726

```
Int[((d_) + (e_.)*(x_))^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Sy
mbol] :> Simp[(e^3*(d + e*x)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/((q + 1)*(c*d
^4 + b*d^2*e^2 + a*e^4)), x] + Dist[1/((q + 1)*(c*d^4 + b*d^2*e^2 + a*e^4))
, Int[((d + e*x)^(q + 1)*Simp[d*(q + 1)*(c*d^2 + b*e^2) - e*(c*d^2*(q + 1)
+ b*e^2*(q + 2))*x + c*d*e^2*(q + 1)*x^2 - c*e^3*(q + 3)*x^3, x])/Sqrt[a +
b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c*d^4 + b*d^2*e^
2 + a*e^4, 0] && ILtQ[q, -1]
```

Rule 1741

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x
_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x
, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e
^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4
)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e},
x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4
, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(-1+x^2)(1-x+x^2-x^3+x^4)}{(1-x+x^2)^2(1+x+x^2)\sqrt{1+3x^2+x^4}}\,dx = \int \frac{-1+x-x^5+x^6}{(1-x+x^2)^2(1+x+x^2)\sqrt{1+3x^2+x^4}}\,dx$$

$$= \int \left(\frac{1}{\sqrt{1+3x^2+x^4}} + \frac{1+x}{2(1-x+x^2)^2\sqrt{1+3x^2+x^4}} + \frac{\cdots}{4(1\cdots}\right.$$

$$= \frac{1}{4}\int \frac{-8+x}{(1-x+x^2)\sqrt{1+3x^2+x^4}}\,dx + \frac{1}{4}\int \frac{-2-\cdots}{(1+x+x^2)\sqrt{1\cdots}}$$

$$= \frac{\sqrt{\frac{2+(3-\sqrt{5})x^2}{2+(3+\sqrt{5})x^2}}\,(2+(3+\sqrt{5})x^2)\,F\left(\tan^{-1}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}\,x\right)\cdots\right.}{\sqrt{2(3+\sqrt{5})}\sqrt{1+3x^2+x^4}}$$

= rest of steps removed due to Latex formating problem

Mathematica [C] time = 1.71, size = 767, normalized size = 7.17

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^2)*(1 - x + x^2 - x^3 + x^4))/((1 - x + x^2)^2*(1 + x +
x^2)*Sqrt[1 + 3*x^2 + x^4]),x]
```

```
[Out] (8*(1 + (-1)^(1/3))*Sqrt[6 - 2*Sqrt[5]]*(1 + 3*x^2 + x^4) + 3*(1 + (-1)^(1/
3))*Sqrt[6 - 2*Sqrt[5]]*(1 - x + x^2)*Sqrt[1 + 3*x^2 + x^4]*((1 - I*Sqrt[3]
)^(3/2)*ArcTan[(1 - (3*I)*Sqrt[3] + (4 - (2*I)*Sqrt[3])*x^2)/(4*Sqrt[1 + I*
Sqrt[3]]*Sqrt[1 + 3*x^2 + x^4])] + (1 + I*Sqrt[3])^(3/2)*ArcTan[(1 + (3*I)*
Sqrt[3] + (4 + (2*I)*Sqrt[3])*x^2)/(4*Sqrt[1 - I*Sqrt[3]]*Sqrt[1 + 3*x^2 +
x^4])]) - (40*I)*(1 + (-1)^(1/3))*(1 - x + x^2)*Sqrt[3 - Sqrt[5] + 2*x^2]*S
qrt[3 + Sqrt[5] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(3 + Sqrt[5])]*x], 7/2
+ (3*Sqrt[5])/2] + (40*I)*(1 - x + x^2)*Sqrt[3 - Sqrt[5] + 2*x^2]*Sqrt[3 +
Sqrt[5] + 2*x^2]*EllipticPi[((-1)^(1/3)*(3 + Sqrt[5]))/2, I*ArcSinh[Sqrt[2/
(3 + Sqrt[5])]*x], (7 + 3*Sqrt[5])/2] + 40*(-1)^(5/6)*(1 - x + x^2)*Sqrt[3
- Sqrt[5] + 2*x^2]*Sqrt[3 + Sqrt[5] + 2*x^2]*EllipticPi[((-1)^(1/3)*(3 + Sq
rt[5]))/2, I*ArcSinh[Sqrt[2/(3 + Sqrt[5])]*x], (7 + 3*Sqrt[5])/2] + (40*I)*
(1 - x + x^2)*Sqrt[3 - Sqrt[5] + 2*x^2]*Sqrt[3 + Sqrt[5] + 2*x^2]*EllipticP
i[-1/2*((-1)^(2/3)*(3 + Sqrt[5])), I*ArcSinh[Sqrt[2/(3 + Sqrt[5])]*x], (7 +
 3*Sqrt[5])/2] + 40*(-1)^(5/6)*(1 - x + x^2)*Sqrt[3 - Sqrt[5] + 2*x^2]*Sqrt
[3 + Sqrt[5] + 2*x^2]*EllipticPi[-1/2*((-1)^(2/3)*(3 + Sqrt[5])), I*ArcSinh
[Sqrt[2/(3 + Sqrt[5])]*x], (7 + 3*Sqrt[5])/2])/(32*(1 + (-1)^(1/3))*Sqrt[6
- 2*Sqrt[5]]*(1 - x + x^2)*Sqrt[1 + 3*x^2 + x^4])
```

IntegrateAlgebraic [A] time = 2.16, size = 107, normalized size = 1.00

$$\frac{\sqrt{x^4+3x^2+1}}{4\left(x^2-x+1\right)}-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x}{x^2+\sqrt{x^4+3x^2+1}-x+1}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{x^2+\sqrt{x^4+3x^2+1}+x+1}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*(1 - x + x^2 - x^3 + x^4))/((1 - x + x^2)^2*
(1 + x + x^2)*Sqrt[1 + 3*x^2 + x^4]),x]
```

```
[Out] Sqrt[1 + 3*x^2 + x^4]/(4*(1 - x + x^2)) - Sqrt[2]*ArcTanh[(Sqrt[2]*x)/(1 -
x + x^2 + Sqrt[1 + 3*x^2 + x^4])] - ArcTanh[(Sqrt[2]*x)/(1 + x + x^2 + Sqrt
[1 + 3*x^2 + x^4])]/(2*Sqrt[2])
```

fricas [B] time = 0.50, size = 184, normalized size = 1.72

$$\frac{\sqrt{2}\left(x^2-x+1\right)\log\left(\frac{3x^4-2x^3+2\sqrt{2}\sqrt{x^4+3x^2+1}\left(x^2-x+1\right)+9x^2-2x+3}{x^4+2x^3+3x^2+2x+1}\right)+4\sqrt{2}\left(x^2-x+1\right)\log\left(\frac{3x^4+2x^3-2\sqrt{2}\sqrt{x^4+3x^2+1}\left(x^2+x+1\right)+9x^2+2x+3}{x^4-2x^3+3x^2-2x+1}\right)+4\sqrt{x^4+3x^2+1}}{16\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4-x^3+x^2-x+1)/(x^2-x+1)^2/(x^2+x+1)/(x^4+3*x^2+1)^(1/
2),x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(2)*(x^2 - x + 1)*log((3*x^4 - 2*x^3 + 2*sqrt(2)*sqrt(x^4 + 3*x^2
 + 1)*(x^2 - x + 1) + 9*x^2 - 2*x + 3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)) + 4
*sqrt(2)*(x^2 - x + 1)*log((3*x^4 + 2*x^3 - 2*sqrt(2)*sqrt(x^4 + 3*x^2 + 1)
*(x^2 + x + 1) + 9*x^2 + 2*x + 3)/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 4*sqrt
(x^4 + 3*x^2 + 1))/(x^2 - x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^3+x^2-x+1\right)\left(x^2-1\right)}{\sqrt{x^4+3\,x^2+1}\left(x^2+x+1\right)\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4-x^3+x^2-x+1)/(x^2-x+1)^2/(x^2+x+1)/(x^4+3*x^2+1)^(1/
2),x, algorithm="giac")
```

[Out] integrate((x^4 - x^3 + x^2 - x + 1)*(x^2 - 1)/(sqrt(x^4 + 3*x^2 + 1)*(x^2 + x + 1)*(x^2 - x + 1)^2), x)

maple [C] time = 0.14, size = 1206, normalized size = 11.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-1)*(x^4-x^3+x^2-x+1)/(x^2-x+1)^2/(x^2+x+1)/(x^4+3*x^2+1)^(1/2),x)

[Out] 5/4/(1/2*I*5^(1/2)-1/2*I)*(1-(-3/2+1/2*5^(1/2))*x^2)^(1/2)*(1-(-3/2-1/2*5^(1/2))*x^2)^(1/2)/(x^4+3*x^2+1)^(1/2)*EllipticF(x*(1/2*I*5^(1/2)-1/2*I),3/2+1/2*5^(1/2))+1/4*(-1/2+1/2*I*3^(1/2))*(1/2/(-1-I*3^(1/2))^(1/2)*arctanh(1/14*(-2+I*3^(1/2))*(7*x^2+11/2-5/2*I*3^(1/2))/(-1-I*3^(1/2))^(1/2)/(x^4+3*x^2+1)^(1/2))-1/(-3/2+1/2*5^(1/2))^(1/2)*(-1/2-1/2*I*3^(1/2))*(1-(-3/2+1/2*5^(1/2))*x^2)^(1/2)*(1-(-3/2-1/2*5^(1/2))*x^2)^(1/2)/(x^4+3*x^2+1)^(1/2)*EllipticPi((-3/2+1/2*5^(1/2))^(1/2)*x,-1/2*(-1/2+1/2*I*3^(1/2))*5^(1/2)+3/4-3/4*I*3^(1/2),(-3/2-1/2*5^(1/2))^(1/2)/(-3/2+1/2*5^(1/2))^(1/2)))+1/4*(-1/2-1/2*I*3^(1/2))*(1/2/(I*3^(1/2)-1)^(1/2)*arctanh(1/14*(-2-I*3^(1/2))*(7*x^2+11/2+5/2*I*3^(1/2))/(I*3^(1/2)-1)^(1/2)/(x^4+3*x^2+1)^(1/2))-1/(-3/2+1/2*5^(1/2))^(1/2)*(-1/2+1/2*I*3^(1/2))*(1-(-3/2+1/2*5^(1/2))*x^2)^(1/2)*(1-(-3/2-1/2*5^(1/2))*x^2)^(1/2)/(x^4+3*x^2+1)^(1/2)*EllipticPi((-3/2+1/2*5^(1/2))^(1/2)*x,-1/2*(-1/2-1/2*I*3^(1/2))*5^(1/2)+3/4+3/4*I*3^(1/2),(-3/2-1/2*5^(1/2))^(1/2)/(-3/2+1/2*5^(1/2))^(1/2)))+1/4*(1/2+5/2*I*3^(1/2))*(-1/2/(I*3^(1/2)-1)^(1/2)*arctanh(1/14*(2+I*3^(1/2))*(7*x^2+11/2+5/2*I*3^(1/2))/(I*3^(1/2)-1)^(1/2)/(x^4+3*x^2+1)^(1/2))-1/(-3/2+1/2*5^(1/2))^(1/2)*(1/2-1/2*I*3^(1/2))*(1-(-3/2+1/2*5^(1/2))*x^2)^(1/2)*(1-(-3/2-1/2*5^(1/2))*x^2)^(1/2)/(x^4+3*x^2+1)^(1/2)*EllipticPi((-3/2+1/2*5^(1/2))^(1/2)*x,1/2*(1/2+1/2*I*3^(1/2))*5^(1/2)+3/4+3/4*I*3^(1/2),(-3/2-1/2*5^(1/2))^(1/2)/(-3/2+1/2*5^(1/2))^(1/2)))+1/4*(1/2-5/2*I*3^(1/2))*(-1/2/(-1-I*3^(1/2))^(1/2)*arctanh(1/14*(2-I*3^(1/2))*(7*x^2+11/2-5/2*I*3^(1/2))/(-1-I*3^(1/2))^(1/2)/(x^4+3*x^2+1)^(1/2))-1/(-3/2+1/2*5^(1/2))^(1/2)*(1/2+1/2*I*3^(1/2))*(1-(-3/2+1/2*5^(1/2))*x^2)^(1/2)*(1-(-3/2-1/2*5^(1/2))*x^2)^(1/2)/(x^4+3*x^2+1)^(1/2)*EllipticPi((-3/2+1/2*5^(1/2))^(1/2)*x,1/2*(1/2-1/2*I*3^(1/2))*5^(1/2)+3/4-3/4*I*3^(1/2),(-3/2-1/2*5^(1/2))^(1/2)/(-3/2+1/2*5^(1/2))^(1/2)))+1/4*(x^4+3*x^2+1)^(1/2)/(x^2-x+1)+1/2*(3/4-1/4*I*3^(1/2))*(-1/2/(I*3^(1/2)-1)^(1/2)*arctanh(1/14*(2+I*3^(1/2))*(7*x^2+11/2+5/2*I*3^(1/2))/(I*3^(1/2)-1)^(1/2)/(x^4+3*x^2+1)^(1/2))-1/(-3/2+1/2*5^(1/2))^(1/2)*(1/2-1/2*I*3^(1/2))*(1-(-3/2+1/2*5^(1/2))*x^2)^(1/2)*(1-(-3/2-1/2*5^(1/2))*x^2)^(1/2)/(x^4+3*x^2+1)^(1/2)*EllipticPi((-3/2+1/2*5^(1/2))^(1/2)*x,1/2*(1/2+1/2*I*3^(1/2))*5^(1/2)+3/4+3/4*I*3^(1/2),(-3/2-1/2*5^(1/2))^(1/2)/(-3/2+1/2*5^(1/2))^(1/2)))+1/2*(3/4+1/4*I*3^(1/2))*(-1/2/(-1-I*3^(1/2))^(1/2)*arctanh(1/14*(2-I*3^(1/2))*(7*x^2+11/2-5/2*I*3^(1/2))/(-1-I*3^(1/2))^(1/2)/(x^4+3*x^2+1)^(1/2))-1/(-3/2+1/2*5^(1/2))^(1/2)*(1/2+1/2*I*3^(1/2))*(1-(-3/2+1/2*5^(1/2))*x^2)^(1/2)*(1-(-3/2-1/2*5^(1/2))*x^2)^(1/2)/(x^4+3*x^2+1)^(1/2)*EllipticPi((-3/2+1/2*5^(1/2))^(1/2)*x,1/2*(1/2-1/2*I*3^(1/2))*5^(1/2)+3/4-3/4*I*3^(1/2),(-3/2-1/2*5^(1/2))^(1/2)/(-3/2+1/2*5^(1/2))^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4 - x^3 + x^2 - x + 1\right)\left(x^2 - 1\right)}{\sqrt{x^4 + 3x^2 + 1}\left(x^2 + x + 1\right)\left(x^2 - x + 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)*(x^4-x^3+x^2-x+1)/(x^2-x+1)^2/(x^2+x+1)/(x^4+3*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate((x^4 - x^3 + x^2 - x + 1)*(x^2 - 1)/(sqrt(x^4 + 3*x^2 + 1)*(x^2 + x + 1)*(x^2 - x + 1)^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2-1\right)\,\left(x^4-x^3+x^2-x+1\right)}{{\left(x^2-x+1\right)}^2\,\sqrt{x^4+3\,x^2+1}\,\left(x^2+x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*(x^2 - x - x^3 + x^4 + 1))/((x^2 - x + 1)^2*(3*x^2 + x^4 + 1
)^(1/2)*(x + x^2 + 1)),x)
```

```
[Out] int(((x^2 - 1)*(x^2 - x - x^3 + x^4 + 1))/((x^2 - x + 1)^2*(3*x^2 + x^4 + 1
)^(1/2)*(x + x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^4-x^3+x^2-x+1\right)}{\left(x^2-x+1\right)^2\left(x^2+x+1\right)\sqrt{x^4+3x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**4-x**3+x**2-x+1)/(x**2-x+1)**2/(x**2+x+1)/(x**4+3*x*
*2+1)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**4 - x**3 + x**2 - x + 1)/((x**2 - x + 1)**2*(x
**2 + x + 1)*sqrt(x**4 + 3*x**2 + 1)), x)
```

3.1342 $\int \frac{(b+ax^3)(b+2ax^3)}{x^6\sqrt[4]{bx+ax^4}}\,dx$

Optimal. Leaf size=107

$$\frac{4}{3}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)+\frac{4}{3}a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)-\frac{4\left(17ax^3+3b\right)\left(ax^4+bx\right)^{3/4}}{63x^6}$$

Rubi [A] time = 0.35, antiderivative size = 165, normalized size of antiderivative = 1.54, number of steps used = 12, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2052, 2011, 329, 275, 240, 212, 206, 203, 2016, 2014}

$$\frac{4a^{7/4}\sqrt[4]{x}\sqrt[4]{ax^3+b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{ax^4+bx}}+\frac{4a^{7/4}\sqrt[4]{x}\sqrt[4]{ax^3+b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{ax^4+bx}}-\frac{4b\left(ax^4+bx\right)^{3/4}}{21x^6}-\frac{68a\left(ax^4+bx\right)^{3/4}}{63x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((b + a*x^3)*(b + 2*a*x^3))/(x^6*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (-4*b*(b*x + a*x^4)^(3/4))/(21*x^6) - (68*a*(b*x + a*x^4)^(3/4))/(63*x^3) +
 (4*a^(7/4)*x^(1/4)*(b + a*x^3)^(1/4)*ArcTan[(a^(1/4)*x^(3/4))/(b + a*x^3)^
(1/4)])/(3*(b*x + a*x^4)^(1/4)) + (4*a^(7/4)*x^(1/4)*(b + a*x^3)^(1/4)*ArcT
anh[(a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)])/(3*(b*x + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\left(b+ax^3\right)\left(b+2ax^3\right)}{x^6\sqrt[4]{bx+ax^4}}\,dx = \int \left(\frac{2a^2}{\sqrt[4]{bx+ax^4}}+\frac{b^2}{x^6\sqrt[4]{bx+ax^4}}+\frac{3ab}{x^3\sqrt[4]{bx+ax^4}}\right)dx$$

$$= \left(2a^2\right)\int \frac{1}{\sqrt[4]{bx+ax^4}}\,dx+(3ab)\int \frac{1}{x^3\sqrt[4]{bx+ax^4}}\,dx+b^2\int \frac{1}{x^6\sqrt[4]{bx+ax^4}}\,dx$$

$$= -\frac{4b\left(bx+ax^4\right)^{3/4}}{21x^6}-\frac{4a\left(bx+ax^4\right)^{3/4}}{3x^3}-\frac{1}{7}(4ab)\int \frac{1}{x^3\sqrt[4]{bx+ax^4}}\,dx+\frac{\left(2a^2\sqrt[4]{x}\,\sqrt[4]{b \ldots\right.}{\ldots}$$

$$= -\frac{4b\left(bx+ax^4\right)^{3/4}}{21x^6}-\frac{68a\left(bx+ax^4\right)^{3/4}}{63x^3}+\frac{\left(8a^2\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{b+ax^{12}}}\,d\ldots\right.}{\sqrt[4]{bx+ax^4}}$$

$$= -\frac{4b\left(bx+ax^4\right)^{3/4}}{21x^6}-\frac{68a\left(bx+ax^4\right)^{3/4}}{63x^3}+\frac{\left(8a^2\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx\ldots\right.}{3\sqrt[4]{bx+ax^4}}$$

$$= -\frac{4b\left(bx+ax^4\right)^{3/4}}{21x^6}-\frac{68a\left(bx+ax^4\right)^{3/4}}{63x^3}+\frac{\left(8a^2\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx,x\ldots\right.}{3\sqrt[4]{bx+ax^4}}$$

$$= -\frac{4b\left(bx+ax^4\right)^{3/4}}{21x^6}-\frac{68a\left(bx+ax^4\right)^{3/4}}{63x^3}+\frac{\left(4a^2\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx\ldots\right.}{3\sqrt[4]{bx+ax^4}}$$

$$= -\frac{4b\left(bx+ax^4\right)^{3/4}}{21x^6}-\frac{68a\left(bx+ax^4\right)^{3/4}}{63x^3}+\frac{4a^{7/4}\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}+\frac{4a\ldots}{\ldots}$$

Mathematica [C] time = 0.08, size = 69, normalized size = 0.64

$$\frac{4\left(x\left(ax^3+b\right)\right)^{3/4}\left(-\frac{14ax^3\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};-\frac{ax^3}{b}\right)}{\left(\frac{ax^3}{b}+1\right)^{3/4}}-3\left(ax^3+b\right)\right)}{63x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b + a*x^3)*(b + 2*a*x^3))/(x^6*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*(x*(b + a*x^3))^(3/4)*(-3*(b + a*x^3) - (14*a*x^3*Hypergeometric2F1[-3/4
, -3/4, 1/4, -((a*x^3)/b)])/(1 + (a*x^3)/b)^(3/4)))/(63*x^6)
```

IntegrateAlgebraic [A] time = 0.41, size = 107, normalized size = 1.00

$$\frac{4}{3}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)+\frac{4}{3}a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)-\frac{4\left(17ax^3+3b\right)\left(ax^4+bx\right)^{3/4}}{63x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b + a*x^3)*(b + 2*a*x^3))/(x^6*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(3*b + 17*a*x^3)*(b*x + a*x^4)^(3/4))/(63*x^6) + (4*a^(7/4)*ArcTan[(a^(
1/4)*(b*x + a*x^4)^(3/4))/(b + a*x^3)])/3 + (4*a^(7/4)*ArcTanh[(a^(1/4)*(b*
x + a*x^4)^(3/4))/(b + a*x^3)])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)*(2*a*x^3+b)/x^6/(a*x^4+b*x)^(1/4),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [B] time = 0.28, size = 201, normalized size = 1.88

$$\frac{2}{3}\,\sqrt{2}\,(-a)^{\frac{3}{4}}\,a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)+\frac{2}{3}\,\sqrt{2}\,(-a)^{\frac{3}{4}}\,a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)-\frac{1}{3}\,\sqrt{2}\,(-a)^{\frac{3}{4}}\,a\log\left(\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)+\frac{1}{3}\,\sqrt{2}\,(-a)^{\frac{3}{4}}\,a\log\left(-\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)-\frac{4}{21}\left(a+\frac{b}{x^3}\right)^{\frac{7}{4}}-\frac{8}{9}\left(a+\frac{b}{x^3}\right)^{\frac{3}{4}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)*(2*a*x^3+b)/x^6/(a*x^4+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(2)*(-a)^(3/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/
x^3)^(1/4))/(-a)^(1/4)) + 2/3*sqrt(2)*(-a)^(3/4)*a*arctan(-1/2*sqrt(2)*(sqr
t(2)*(-a)^(1/4) - 2*(a + b/x^3)^(1/4))/(-a)^(1/4)) - 1/3*sqrt(2)*(-a)^(3/4)
*a*log(sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a) + sqrt(a + b/x^3)) +
 1/3*sqrt(2)*(-a)^(3/4)*a*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(
-a) + sqrt(a + b/x^3)) - 4/21*(a + b/x^3)^(7/4) - 8/9*(a + b/x^3)^(3/4)*a
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{3}+b\right)\left(2a\,x^{3}+b\right)}{x^{6}\left(a\,x^{4}+bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b)*(2*a*x^3+b)/x^6/(a*x^4+b*x)^(1/4),x)
```

```
[Out] int((a*x^3+b)*(2*a*x^3+b)/x^6/(a*x^4+b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,ax^{3}+b\right)\left(ax^{3}+b\right)}{\left(ax^{4}+bx\right)^{\frac{1}{4}}x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)*(2*a*x^3+b)/x^6/(a*x^4+b*x)^(1/4),x, algorithm="maxima"
)
```

```
[Out] integrate((2*a*x^3 + b)*(a*x^3 + b)/((a*x^4 + b*x)^(1/4)*x^6), x)
```

mupad [B] time = 1.81, size = 104, normalized size = 0.97

$$-\frac{12\,b\,\left(a\,x^4+b\,x\right)+68\,a\,x^3\,\left(a\,x^4+b\,x\right)-168\,a^2\,x^7\,{\left(\frac{a\,x^3}{b}+1\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{a\,x^3}{b}\right)}{{\left(a\,x^4+b\,x\right)}^{1/4}\,\left(\frac{63\,x^2\,\left(a\,x^4+b\,x\right)}{a}-\frac{63\,b\,x^3}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a*x^3)*(b + 2*a*x^3))/(x^6*(b*x + a*x^4)^(1/4)),x)
```

```
[Out] -(12*b*(b*x + a*x^4) + 68*a*x^3*(b*x + a*x^4) - 168*a^2*x^7*((a*x^3)/b + 1)
^(1/4)*hypergeom([1/4, 1/4], 5/4, -(a*x^3)/b))/((b*x + a*x^4)^(1/4)*((63*x^
2*(b*x + a*x^4))/a - (63*b*x^3)/a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^3+b\right)\left(2ax^3+b\right)}{x^6\sqrt[4]{x\left(ax^3+b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b)*(2*a*x**3+b)/x**6/(a*x**4+b*x)**(1/4),x)
[Out] Integral((a*x**3 + b)*(2*a*x**3 + b)/(x**6*(x*(a*x**3 + b))**(1/4)), x)
```

3.1343 $\int \frac{-b^3+a^3x^3}{(b^3+a^3x^3)\sqrt{b^4+a^4x^4}}\,dx$

Optimal. Leaf size=107

$$-\frac{4\tan^{-1}\left(\frac{abx}{\sqrt{a^4x^4+b^4}+a^2x^2-abx+b^2}\right)}{3ab}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{a^4x^4+b^4}+a^2x^2+2abx+b^2}\right)}{3ab}$$

Rubi [C] time = 2.66, antiderivative size = 662, normalized size of antiderivative = 6.19, number of steps used = 29, number of rules used = 13, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {6725, 220, 2074, 1725, 1211, 1699, 208, 1248, 725, 206, 6728, 1217, 1707}

$$-\frac{2\tan^{-1}\left(\frac{abx}{\sqrt{a^4x^4+b^4}}\right)}{3ab}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{a^4x^4+b^4}}\right)}{3\sqrt{2}\,ab}+\frac{\tanh^{-1}\left(\frac{a^2x^2+b^2}{\sqrt{2}\sqrt{a^4x^4+b^4}}\right)}{3\sqrt{2}\,ab}-\frac{\left(a-\sqrt{3}\sqrt{-a^2}\right)(a^2x^2+b^2)\sqrt{\frac{a^4x^4+b^4}{(a^2x^2+b^2)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{6a^2b\sqrt{a^4x^4+b^4}}-\frac{\left(\sqrt{3}\sqrt{-a^2}+a\right)(a^2x^2+b^2)\sqrt{\frac{a^4x^4+b^4}{(a^2x^2+b^2)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{6a^2b\sqrt{a^4x^4+b^4}}+\frac{(a^2x^2+b^2)\sqrt{\frac{a^4x^4+b^4}{(a^2x^2+b^2)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{3ab\sqrt{a^4x^4+b^4}}-\frac{\left(a-\sqrt{3}\sqrt{-a^2}\right)\tanh^{-1}\left(\frac{\sqrt{a}\left(\left(a-\sqrt{3}\sqrt{-a^2}\right)^2x^2+4b^2\right)}{2\sqrt{2}\sqrt{\sqrt{3}\sqrt{-a^2}+a}\sqrt{a^4x^4+b^4}}\right)}{3\sqrt{2}\,a^{3/2}\sqrt{\sqrt{3}\sqrt{-a^2}+a}\,b}-\frac{\left(\sqrt{3}\sqrt{-a^2}+a\right)\tanh^{-1}\left(\frac{\sqrt{a}\left(\left(\sqrt{3}\sqrt{-a^2}+a\right)^2x^2+4b^2\right)}{2\sqrt{2}\sqrt{a-\sqrt{3}\sqrt{-a^2}}\sqrt{a^4x^4+b^4}}\right)}{3\sqrt{2}\,a^{3/2}\sqrt{a-\sqrt{3}\sqrt{-a^2}}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(-b^3 + a^3*x^3)/((b^3 + a^3*x^3)*Sqrt[b^4 + a^4*x^4]),x]
```

```
[Out] (-2*ArcTan[(a*b*x)/Sqrt[b^4 + a^4*x^4]])/(3*a*b) - ArcTanh[(Sqrt[2]*a*b*x)/
Sqrt[b^4 + a^4*x^4]]/(3*Sqrt[2]*a*b) + ArcTanh[(b^2 + a^2*x^2)/(Sqrt[2]*Sqr
t[b^4 + a^4*x^4])]/(3*Sqrt[2]*a*b) - ((a - Sqrt[3]*Sqrt[-a^2])*ArcTanh[(Sqr
t[a]*(4*b^2 + (a - Sqrt[3]*Sqrt[-a^2])^2*x^2))/(2*Sqrt[2]*Sqrt[a + Sqrt[3]*
Sqrt[-a^2]]*Sqrt[b^4 + a^4*x^4])])/(3*Sqrt[2]*a^(3/2)*Sqrt[a + Sqrt[3]*Sqrt
[-a^2]]*b) - ((a + Sqrt[3]*Sqrt[-a^2])*ArcTanh[(Sqrt[a]*(4*b^2 + (a + Sqrt[
3]*Sqrt[-a^2])^2*x^2))/(2*Sqrt[2]*Sqrt[a - Sqrt[3]*Sqrt[-a^2]]*Sqrt[b^4 + a
^4*x^4])])/(3*Sqrt[2]*a^(3/2)*Sqrt[a - Sqrt[3]*Sqrt[-a^2]]*b) + ((b^2 + a^2
*x^2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b],
1/2])/(3*a*b*Sqrt[b^4 + a^4*x^4]) - ((a - Sqrt[3]*Sqrt[-a^2])*(b^2 + a^2*x^
2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2
])/(6*a^2*b*Sqrt[b^4 + a^4*x^4]) - ((a + Sqrt[3]*Sqrt[-a^2])*(b^2 + a^2*x^2
)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2]
)/(6*a^2*b*Sqrt[b^4 + a^4*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[d,
 Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Dist[e, Int[x/((d^2 - e^
2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-b^3+a^3x^3}{\left(b^3+a^3x^3\right)\sqrt{b^4+a^4x^4}}\,dx=\int\left(\frac{1}{\sqrt{b^4+a^4x^4}}-\frac{2b^3}{\left(b^3+a^3x^3\right)\sqrt{b^4+a^4x^4}}\right)dx$$

$$=-\left(\left(2b^3\right)\int\frac{1}{\left(b^3+a^3x^3\right)\sqrt{b^4+a^4x^4}}\,dx\right)+\int\frac{1}{\sqrt{b^4+a^4x^4}}\,dx$$

$$=\frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{2ab\sqrt{b^4+a^4x^4}}-\left(2b^3\right)\int\left(\frac{1}{3b^2(b+ax)\sqrt{b^4+a^4x^4}}+\right.$$

$$=\frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{2ab\sqrt{b^4+a^4x^4}}-\frac{1}{3}(2b)\int\frac{1}{(b+ax)\sqrt{b^4+a^4x^4}}\,dx-$$

$$=\frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{2ab\sqrt{b^4+a^4x^4}}-\frac{1}{3}(2b)\int\left(\frac{-a-\sqrt{3}\sqrt{-}}{\left(-ab-\sqrt{3}\sqrt{-a^2}\,b+2a^2\right.}\right.$$

$$=\frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{2ab\sqrt{b^4+a^4x^4}}-\frac{1}{3}\int\frac{1}{\sqrt{b^4+a^4x^4}}\,dx-\frac{1}{3}\int\frac{}{\left(b^2-a\right.}$$

$$=\frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{3ab\sqrt{b^4+a^4x^4}}-\frac{1}{3}(ab)\,\mathrm{Subst}\left(\int\frac{1}{2a^4b^4-x^2}\,dx,x,\right.$$

$$=-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}+\frac{\tanh^{-1}\left(\frac{b^2+a^2x^2}{\sqrt{2}\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}+\frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}F\left(2\tan\right.}{3ab\sqrt{b^4+a^4x^4}}$$

$$=-\frac{2\tan^{-1}\left(\frac{abx}{\sqrt{b^4+a^4x^4}}\right)}{3ab}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}+\frac{\tanh^{-1}\left(\frac{b^2+a^2x^2}{\sqrt{2}\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}+\frac{\left(b^2+a^2x^2\right.}{}$$

$$=-\frac{2\tan^{-1}\left(\frac{abx}{\sqrt{b^4+a^4x^4}}\right)}{3ab}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}+\frac{\tanh^{-1}\left(\frac{b^2+a^2x^2}{\sqrt{2}\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}-\frac{\left(a-\sqrt{3}\sqrt{-}\right.}{}$$

Mathematica [C] time = 3.40, size = 562, normalized size = 5.25

$$\frac{a^3\left(18ia\sqrt[4]{a^4}\,b\sqrt{\frac{a^4x^4}{b^4}+1}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}x\right)|-1\right)-\frac{3}{2}\left(8ia\sqrt[4]{a^4}\,b\sqrt{\frac{a^4x^4}{b^4}+1}\,\Pi\left(-\frac{i}{2}-\frac{\sqrt{3}}{2};i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}x\right)|-1\right)+8ia\sqrt[4]{a^4}\,b\sqrt{\frac{a^4x^4}{b^4}+1}\,\Pi\left(\frac{1}{2}\left(-i+\sqrt{3}\right);i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}x\right)|-1\right)+\sqrt{2}\sqrt{\frac{ia^2}{b^2}}\left((4+4i)ab\sqrt[4]{b^4}\sqrt{\frac{a^4x^4}{b^4}+1}\,\Pi\left(\frac{i\sqrt{a^4}\sqrt{b^4}}{a^2b^2};i\sinh^{-1}\left(\frac{(1+i)\sqrt[4]{a^4}\,x}{\sqrt{2}\sqrt[4]{b^4}}\right)|-1\right)+\sqrt[4]{a^4}\sqrt{a^4x^4+b^4}\left(\sqrt{1+i\sqrt{3}}\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{\left(\sqrt{3}+i\right)b^2-2ia^2x^2}{\sqrt{2-2i\sqrt{3}}\sqrt{a^4x^4+b^4}}\right)+\sqrt{1-i\sqrt{3}}\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{2ia^2x^2+\left(\sqrt{3}-i\right)b^2}{\sqrt{2+2i\sqrt{3}}\sqrt{a^4x^4+b^4}}\right)+2\tanh^{-1}\left(\frac{a^2x^2+b^2}{\sqrt{2}\sqrt{a^4x^4+b^4}}\right)\right)\right)\right)\right)}{18\sqrt[4]{a^4}\,b^5\left(\frac{ia^2}{b^2}\right)^{5/2}\sqrt{a^4x^4+b^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b^3 + a^3*x^3)/((b^3 + a^3*x^3)*Sqrt[b^4 + a^4*x^4]),x]
```

```
[Out] (a^3*((18*I)*a*(a^4)^(1/4)*b*Sqrt[1 + (a^4*x^4)/b^4]*EllipticF[I*ArcSinh[Sq
rt[(I*a^2)/b^2]*x], -1] - (3*((8*I)*a*(a^4)^(1/4)*b*Sqrt[1 + (a^4*x^4)/b^4]
*EllipticPi[-1/2*I - Sqrt[3]/2, I*ArcSinh[Sqrt[(I*a^2)/b^2]*x], -1] + (8*I)
*a*(a^4)^(1/4)*b*Sqrt[1 + (a^4*x^4)/b^4]*EllipticPi[(-I + Sqrt[3])/2, I*Arc
Sinh[Sqrt[(I*a^2)/b^2]*x], -1] + Sqrt[2]*Sqrt[(I*a^2)/b^2]*((a^4)^(1/4)*Sqr
t[b^4 + a^4*x^4]*(Sqrt[1 + I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((I + Sqrt[3])*
b^2 - (2*I)*a^2*x^2)/(Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[b^4 + a^4*x^4])] + Sqrt[
1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*b^2 + (2*I)*a^2*x^2)/(S
qrt[2 + (2*I)*Sqrt[3]]*Sqrt[b^4 + a^4*x^4])] + 2*ArcTanh[(b^2 + a^2*x^2)/(S
qrt[2]*Sqrt[b^4 + a^4*x^4])]]) + (4 + 4*I)*a*b*(b^4)^(1/4)*Sqrt[1 + (a^4*x^4
)/b^4]*EllipticPi[(I*Sqrt[a^4]*Sqrt[b^4])/(a^2*b^2), I*ArcSinh[((1 + I)*(a^
4)^(1/4)*x)/(Sqrt[2]*(b^4)^(1/4))], -1]))/2))/(18*(a^4)^(1/4)*((I*a^2)/b^2
)^(5/2)*b^5*Sqrt[b^4 + a^4*x^4])
```

IntegrateAlgebraic [A] time = 2.45, size = 107, normalized size = 1.00

$$-\frac{4\tan^{-1}\left(\frac{abx}{\sqrt{a^4x^4+b^4}+a^2x^2-abx+b^2}\right)}{3ab}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{a^4x^4+b^4}+a^2x^2+2abx+b^2}\right)}{3ab}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b^3 + a^3*x^3)/((b^3 + a^3*x^3)*Sqrt[b^4 + a^4*x^4]),x
]
```

```
[Out] (-4*ArcTan[(a*b*x)/(b^2 - a*b*x + a^2*x^2 + Sqrt[b^4 + a^4*x^4])])/(3*a*b)
- (Sqrt[2]*ArcTanh[(Sqrt[2]*a*b*x)/(b^2 + 2*a*b*x + a^2*x^2 + Sqrt[b^4 + a^
4*x^4])])/(3*a*b)
```

fricas [A] time = 0.57, size = 165, normalized size = 1.54

$$\frac{\sqrt{2}\log\left(-\frac{3\,a^4x^4+4\,a^3bx^3+6\,a^2b^2x^2+4\,ab^3x+3\,b^4+2\,\sqrt{2}\,\sqrt{a^4x^4+b^4}\left(a^2x^2+abx+b^2\right)}{a^4x^4+4\,a^3bx^3+6\,a^2b^2x^2+4\,ab^3x+b^4}\right)-8\,\arctan\left(\frac{\sqrt{a^4x^4+b^4}}{a^2x^2-2\,abx+b^2}\right)}{12\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3-b^3)/(a^3*x^3+b^3)/(a^4*x^4+b^4)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/12*(sqrt(2)*log(-(3*a^4*x^4 + 4*a^3*b*x^3 + 6*a^2*b^2*x^2 + 4*a*b^3*x + 3
*b^4 + 2*sqrt(2)*sqrt(a^4*x^4 + b^4)*(a^2*x^2 + a*b*x + b^2))/(a^4*x^4 + 4*
a^3*b*x^3 + 6*a^2*b^2*x^2 + 4*a*b^3*x + b^4)) - 8*arctan(sqrt(a^4*x^4 + b^4
)/(a^2*x^2 - 2*a*b*x + b^2)))/(a*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^3x^3-b^3}{\sqrt{a^4x^4+b^4}\left(a^3x^3+b^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3-b^3)/(a^3*x^3+b^3)/(a^4*x^4+b^4)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((a^3*x^3 - b^3)/(sqrt(a^4*x^4 + b^4)*(a^3*x^3 + b^3)), x)
```

maple [C] time = 0.07, size = 457, normalized size = 4.27

$$\frac{\sqrt{1-\frac{i a^{2} x^{2}}{b^{2}}}\,\sqrt{1+\frac{i a^{2} x^{2}}{b^{2}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i a^{2}}{b^{2}}}, i\right)}{\sqrt{\frac{i a^{2}}{b^{2}}}\,\sqrt{a^{4} x^{4}+b^{4}}}-\frac{b\left(\sum_{\_\alpha=\mathrm{RootOf}\left(a^{2}\_Z^{2}-a b\_Z+b^{2}\right)}\frac{\left(-\_\alpha a+2 b\right)\left(-\frac{\mathrm{arctanh}\left(\frac{\left(\_\alpha a-b\right) a b\left(a x^{2}-b\_\alpha\right)}{\sqrt{-b^{3}\left(\_\alpha a-b\right)}\,\sqrt{a^{4} x^{4}+b^{4}}}\right)}{\sqrt{-b^{3}\left(\_\alpha a-b\right)}}+\frac{2 a\left(\_\alpha a-b\right)\sqrt{1-\frac{i a^{2} x^{2}}{b^{2}}}\,\sqrt{1+\frac{i a^{2} x^{2}}{b^{2}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i a^{2}}{b^{2}}}, \frac{i\_\alpha a}{b}, \frac{\sqrt{-\frac{i a^{2}}{b^{2}}}}{\sqrt{\frac{i a^{2}}{b^{2}}}}\right)}{\sqrt{\frac{i a^{2}}{b^{2}}}\,b^{2}\sqrt{a^{4} x^{4}+b^{4}}}\right)}{2\_\alpha a-b}\right)}{3 a}-\frac{2 b\left(-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\left(2 a^{2} b^{2} x^{2}+2 b^{4}\right)\sqrt{2}}{4\sqrt{b^{4}}\,\sqrt{a^{4} x^{4}+b^{4}}}\right)}{4\sqrt{b^{4}}}+\frac{a\sqrt{1-\frac{i a^{2} x^{2}}{b^{2}}}\,\sqrt{1+\frac{i a^{2} x^{2}}{b^{2}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i a^{2}}{b^{2}}}, -i, \frac{\sqrt{-\frac{i a^{2}}{b^{2}}}}{\sqrt{\frac{i a^{2}}{b^{2}}}}\right)}{\sqrt{\frac{i a^{2}}{b^{2}}}\,b\sqrt{a^{4} x^{4}+b^{4}}}\right)}{3 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^3*x^3-b^3)/(a^3*x^3+b^3)/(a^4*x^4+b^4)^(1/2),x)
```

```
[Out] 1/(I*a^2/b^2)^(1/2)*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/b^2*x^2)^(1/2)/(a^4*x^
4+b^4)^(1/2)*EllipticF(x*(I*a^2/b^2)^(1/2),I)-1/3*b/a*sum((-_alpha*a+2*b)/(
2*_alpha*a-b)*(-1/(-b^3*(_alpha*a-b))^(1/2)*arctanh((_alpha*a-b)*a*b*(a*x^2
-_alpha*b)/(-b^3*(_alpha*a-b))^(1/2)/(a^4*x^4+b^4)^(1/2))+2/(I*a^2/b^2)^(1/
2)*a*(_alpha*a-b)/b^2*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/b^2*x^2)^(1/2)/(a^4*
x^4+b^4)^(1/2)*EllipticPi(x*(I*a^2/b^2)^(1/2),I*_alpha*a/b,(-I*a^2/b^2)^(1/
2)/(I*a^2/b^2)^(1/2))),_alpha=RootOf(_Z^2*a^2-_Z*a*b+b^2))-2/3*b/a*(-1/4*2^
(1/2)/(b^4)^(1/2)*arctanh(1/4*(2*a^2*b^2*x^2+2*b^4)*2^(1/2)/(b^4)^(1/2)/(a^
4*x^4+b^4)^(1/2))+1/(I*a^2/b^2)^(1/2)/b*a*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/
b^2*x^2)^(1/2)/(a^4*x^4+b^4)^(1/2)*EllipticPi(x*(I*a^2/b^2)^(1/2),-I,(-I*a^
2/b^2)^(1/2)/(I*a^2/b^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^{3} x^{3}-b^{3}}{\sqrt{a^{4} x^{4}+b^{4}}\left(a^{3} x^{3}+b^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3-b^3)/(a^3*x^3+b^3)/(a^4*x^4+b^4)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((a^3*x^3 - b^3)/(sqrt(a^4*x^4 + b^4)*(a^3*x^3 + b^3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{b^{3}-a^{3}\,x^{3}}{\left(a^{3}\,x^{3}+b^{3}\right)\,\sqrt{a^{4}\,x^{4}+b^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^3 - a^3*x^3)/((b^3 + a^3*x^3)*(b^4 + a^4*x^4)^(1/2)),x)
```

```
[Out] int(-(b^3 - a^3*x^3)/((b^3 + a^3*x^3)*(b^4 + a^4*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a x-b\right)\left(a^{2} x^{2}+a b x+b^{2}\right)}{\left(a x+b\right)\sqrt{a^{4} x^{4}+b^{4}}\left(a^{2} x^{2}-a b x+b^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**3*x**3-b**3)/(a**3*x**3+b**3)/(a**4*x**4+b**4)**(1/2),x)
```

```
[Out] Integral((a*x - b)*(a**2*x**2 + a*b*x + b**2)/((a*x + b)*sqrt(a**4*x**4 + b
**4)*(a**2*x**2 - a*b*x + b**2)), x)
```

3.1344 $\int \frac{x^4(-2b+ax^2)}{(-b+ax^2)^2\sqrt[4]{-b+ax^2+cx^4}}\,dx$

Optimal. Leaf size=107

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^2-b+cx^4}}\right)}{4c^{5/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^2-b+cx^4}}\right)}{4c^{5/4}}-\frac{x\left(ax^2-b+cx^4\right)^{3/4}}{2c\left(b-ax^2\right)}$$

Rubi [F] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4\left(-2b+ax^2\right)}{\left(-b+ax^2\right)^2\sqrt[4]{-b+ax^2+cx^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(-2*b + a*x^2))/((-b + a*x^2)^2*(-b + a*x^2 + c*x^4)^(1/4)),x]
```

```
[Out] Defer[Int][(x^4*(-2*b + a*x^2))/((-b + a*x^2)^2*(-b + a*x^2 + c*x^4)^(1/4))
, x]
```

Rubi steps

$$\int \frac{x^4\left(-2b+ax^2\right)}{\left(-b+ax^2\right)^2\sqrt[4]{-b+ax^2+cx^4}}\,dx=\int \frac{x^4\left(-2b+ax^2\right)}{\left(-b+ax^2\right)^2\sqrt[4]{-b+ax^2+cx^4}}\,dx$$

Mathematica [F] time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(-2b+ax^2\right)}{\left(-b+ax^2\right)^2\sqrt[4]{-b+ax^2+cx^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(-2*b + a*x^2))/((-b + a*x^2)^2*(-b + a*x^2 + c*x^4)^(1/4)),
x]
```

```
[Out] Integrate[(x^4*(-2*b + a*x^2))/((-b + a*x^2)^2*(-b + a*x^2 + c*x^4)^(1/4)),
 x]
```

IntegrateAlgebraic [A] time = 1.04, size = 107, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^2-b+cx^4}}\right)}{4c^{5/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^2-b+cx^4}}\right)}{4c^{5/4}}-\frac{x\left(ax^2-b+cx^4\right)^{3/4}}{2c\left(b-ax^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(-2*b + a*x^2))/((-b + a*x^2)^2*(-b + a*x^2 + c*x^4
)^(1/4)),x]
```

```
[Out] -1/2*(x*(-b + a*x^2 + c*x^4)^(3/4))/(c*(b - a*x^2)) - ArcTan[(c^(1/4)*x)/(-
b + a*x^2 + c*x^4)^(1/4)]/(4*c^(5/4)) - ArcTanh[(c^(1/4)*x)/(-b + a*x^2 + c
*x^4)^(1/4)]/(4*c^(5/4))
```

fricas [B] time = 0.51, size = 228, normalized size = 2.13

$$-\frac{4 \left(acx^{2}-bc\right) \left(\frac{1}{c^{5}}\right)^{\frac{1}{4}} \arctan\left(\frac{cx \sqrt{\frac{c^{3} \sqrt{\frac{1}{c^{5}}} x^{2}+\sqrt{cx^{4}+ax^{2}-b}}{x^{2}}} \left(\frac{1}{c^{5}}\right)^{\frac{1}{4}}-\left(cx^{4}+ax^{2}-b\right)^{\frac{1}{4}} c \left(\frac{1}{c^{5}}\right)^{\frac{1}{4}}}{x}\right)+\left(acx^{2}-bc\right) \left(\frac{1}{c^{5}}\right)^{\frac{1}{4}} \log\left(\frac{c^{4} \left(\frac{1}{c^{5}}\right)^{\frac{3}{4}} x+\left(cx^{4}+ax^{2}-b\right)^{\frac{1}{4}}}{x}\right)-\left(acx^{2}-bc\right) \left(\frac{1}{c^{5}}\right)^{\frac{1}{4}} \log\left(-\frac{c^{4} \left(\frac{1}{c^{5}}\right)^{\frac{3}{4}} x-\left(cx^{4}+ax^{2}-b\right)^{\frac{1}{4}}}{x}\right)-4 \left(cx^{4}+ax^{2}-b\right)^{\frac{3}{4}} x}{8 \left(acx^{2}-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^2-2*b)/(a*x^2-b)^2/(c*x^4+a*x^2-b)^(1/4),x, algorithm="f
ricas")
```

```
[Out] -1/8*(4*(a*c*x^2 - b*c)*(c^(-5))^(1/4)*arctan((c*x*sqrt((c^3*sqrt(c^(-5))*x
^2 + sqrt(c*x^4 + a*x^2 - b))/x^2)*(c^(-5))^(1/4) - (c*x^4 + a*x^2 - b)^(1/
4)*c*(c^(-5))^(1/4))/x) + (a*c*x^2 - b*c)*(c^(-5))^(1/4)*log((c^4*(c^(-5))^
(3/4)*x + (c*x^4 + a*x^2 - b)^(1/4))/x) - (a*c*x^2 - b*c)*(c^(-5))^(1/4)*lo
g(-(c^4*(c^(-5))^(3/4)*x - (c*x^4 + a*x^2 - b)^(1/4))/x) - 4*(c*x^4 + a*x^2
 - b)^(3/4)*x)/(a*c*x^2 - b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{2}-2\,b\right)x^{4}}{\left(cx^{4}+ax^{2}-b\right)^{\frac{1}{4}}\left(ax^{2}-b\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^2-2*b)/(a*x^2-b)^2/(c*x^4+a*x^2-b)^(1/4),x, algorithm="g
iac")
```

```
[Out] integrate((a*x^2 - 2*b)*x^4/((c*x^4 + a*x^2 - b)^(1/4)*(a*x^2 - b)^2), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x^{4}\left(a\,x^{2}-2b\right)}{\left(a\,x^{2}-b\right)^{2}\left(c\,x^{4}+a\,x^{2}-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a*x^2-2*b)/(a*x^2-b)^2/(c*x^4+a*x^2-b)^(1/4),x)
```

```
[Out] int(x^4*(a*x^2-2*b)/(a*x^2-b)^2/(c*x^4+a*x^2-b)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{2}-2\,b\right)x^{4}}{\left(cx^{4}+ax^{2}-b\right)^{\frac{1}{4}}\left(ax^{2}-b\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^2-2*b)/(a*x^2-b)^2/(c*x^4+a*x^2-b)^(1/4),x, algorithm="m
axima")
```

```
[Out] integrate((a*x^2 - 2*b)*x^4/((c*x^4 + a*x^2 - b)^(1/4)*(a*x^2 - b)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^{4}\left(2\,b-a\,x^{2}\right)}{\left(b-a\,x^{2}\right)^{2}\left(c\,x^{4}+a\,x^{2}-b\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(2*b - a*x^2))/((b - a*x^2)^2*(a*x^2 - b + c*x^4)^(1/4)),x)
[Out] -int((x^4*(2*b - a*x^2))/((b - a*x^2)^2*(a*x^2 - b + c*x^4)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(ax^2 - 2b\right)}{\left(ax^2 - b\right)^2 \sqrt[4]{ax^2 - b + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a*x**2-2*b)/(a*x**2-b)**2/(c*x**4+a*x**2-b)**(1/4),x)
[Out] Integral(x**4*(a*x**2 - 2*b)/((a*x**2 - b)**2*(a*x**2 - b + c*x**4)**(1/4))
, x)
```

3.1345 $\int \frac{(-1-2x+x^2)(-1+2x+x^2)}{(1-x+2x^2+x^3+x^4)\sqrt[3]{-x+x^5}}\,dx$

Optimal. Leaf size=107

$$\log\left(\sqrt[3]{x^5-x}+x^2+1\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^5-x}}{\sqrt[3]{x^5-x}-2x^2-2}\right)-\frac{1}{2}\log\left(\left(x^5-x\right)^{2/3}+x^4+2x^2+\left(-x^2-1\right)\sqrt[3]{x^5-x}+1\right)$$

Rubi [F] time = 2.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1-2x+x^2\right)\left(-1+2x+x^2\right)}{\left(1-x+2x^2+x^3+x^4\right)\sqrt[3]{-x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 - 2*x + x^2)*(-1 + 2*x + x^2))/((1 - x + 2*x^2 + x^3 + x^4)*(-x +
x^5)^(1/3)),x]
```

```
[Out] (3*x*(1 - x^4)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, x^4])/(2*(-x + x^5)^(
1/3)) + (3*x^(1/3)*(-1 + x^4)^(1/3)*Defer[Subst][Defer[Int][x^4/((-1 + x^12
)^(1/3)*(1 - x^3 + 2*x^6 + x^9 + x^12)), x], x, x^(1/3)])/(-x + x^5)^(1/3)
- (24*x^(1/3)*(-1 + x^4)^(1/3)*Defer[Subst][Defer[Int][x^7/((-1 + x^12)^(1/
3)*(1 - x^3 + 2*x^6 + x^9 + x^12)), x], x, x^(1/3)])/(-x + x^5)^(1/3) - (3*
x^(1/3)*(-1 + x^4)^(1/3)*Defer[Subst][Defer[Int][x^10/((-1 + x^12)^(1/3)*(1
 - x^3 + 2*x^6 + x^9 + x^12)), x], x, x^(1/3)])/(-x + x^5)^(1/3)
```

Rubi steps

$$\int \frac{\left(-1-2x+x^2\right)\left(-1+2x+x^2\right)}{\left(1-x+2x^2+x^3+x^4\right)\sqrt[3]{-x+x^5}}\,dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\int \frac{\left(-1-2x+x^2\right)\left(-1+2x+x^2\right)}{\sqrt[3]{x}\sqrt[3]{-1+x^4}\left(1-x+2x^2+x^3+x^4\right)}\,dx}{\sqrt[3]{-x+x^5}}$$
$$= \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\int \frac{1-6x^2+x^4}{\sqrt[3]{x}\sqrt[3]{-1+x^4}\left(1-x+2x^2+x^3+x^4\right)}\,dx}{\sqrt[3]{-x+x^5}}$$
$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \frac{x\left(1-6x^6+x^{12}\right)}{\sqrt[3]{-1+x^{12}}\left(1-x^3+2x^6+x^9+x^{12}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x+x^5}}$$
$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \left(\frac{x}{\sqrt[3]{-1+x^{12}}}+\frac{x^4\left(1-8x^3-x^6\right)}{\sqrt[3]{-1+x^{12}}\left(1-x^3+2x^6+x^9+x^{12}\right)}\right)d}{\sqrt[3]{-x+x^5}}$$
$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{-1+x^{12}}}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x+x^5}} + \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)}{}$$
$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x^6}}\,dx,x,x^{2/3}\right)}{2\sqrt[3]{-x+x^5}} + \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)}{}$$
$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1-x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x^6}}\,dx,x,x^{2/3}\right)}{2\sqrt[3]{-x+x^5}} + \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\text{Su}}{}$$
$$= \frac{3x\sqrt[3]{1-x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^4\right)}{2\sqrt[3]{-x+x^5}} + \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^4}\right)\text{Subst}\left(\int \frac{}{\sqrt[3]{-1+x^{12}}\,(}\right.}{\sqrt[3]{-x+x^5}}$$

Mathematica [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\left(-1-2x+x^2\right)\left(-1+2x+x^2\right)}{\left(1-x+2x^2+x^3+x^4\right)\sqrt[3]{-x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 - 2*x + x^2)*(-1 + 2*x + x^2))/((1 - x + 2*x^2 + x^3 + x^4)*
(-x + x^5)^(1/3)),x]
```

```
[Out] Integrate[((-1 - 2*x + x^2)*(-1 + 2*x + x^2))/((1 - x + 2*x^2 + x^3 + x^4)*
(-x + x^5)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 2.33, size = 107, normalized size = 1.00

$$\log\left(\sqrt[3]{x^5-x}+x^2+1\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^5-x}}{\sqrt[3]{x^5-x}-2x^2-2}\right)-\frac{1}{2}\log\left(\left(x^5-x\right)^{2/3}+x^4+2x^2+\left(-x^2-1\right)\sqrt[3]{x^5-x}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 - 2*x + x^2)*(-1 + 2*x + x^2))/((1 - x + 2*x^2 + x^
3 + x^4)*(-x + x^5)^(1/3)),x]
```

```
[Out] Sqrt[3]*ArcTan[(Sqrt[3]*(-x + x^5)^(1/3))/(-2 - 2*x^2 + (-x + x^5)^(1/3))]
+ Log[1 + x^2 + (-x + x^5)^(1/3)] - Log[1 + 2*x^2 + x^4 + (-1 - x^2)*(-x +
x^5)^(1/3) + (-x + x^5)^(2/3)]/2
```

fricas [A] time = 2.04, size = 154, normalized size = 1.44

$$-\sqrt{3}\arctan\left(\frac{541310\,\sqrt{3}\left(x^{5}-x\right)^{\frac{1}{3}}\left(x^{2}+1\right)+\sqrt{3}\left(311575\,x^{4}+193471\,x^{3}+623150\,x^{2}-193471\,x+311575\right)+777518\,\sqrt{3}\left(x^{5}-x\right)^{\frac{2}{3}}}{3\left(166375\,x^{4}-493039\,x^{3}+332750\,x^{2}+493039\,x+166375\right)}\right)+\frac{1}{2}\log\left(\frac{x^{4}+x^{3}+2\,x^{2}+3\left(x^{5}-x\right)^{\frac{1}{3}}\left(x^{2}+1\right)-x+3\left(x^{5}-x\right)^{\frac{2}{3}}+1}{x^{4}+x^{3}+2\,x^{2}-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)*(x^2+2*x-1)/(x^4+x^3+2*x^2-x+1)/(x^5-x)^(1/3),x, algo
rithm="fricas")
```

```
[Out] -sqrt(3)*arctan(1/3*(541310*sqrt(3)*(x^5 - x)^(1/3)*(x^2 + 1) + sqrt(3)*(31
1575*x^4 + 193471*x^3 + 623150*x^2 - 193471*x + 311575) + 777518*sqrt(3)*(x
^5 - x)^(2/3))/(166375*x^4 - 493039*x^3 + 332750*x^2 + 493039*x + 166375))
+ 1/2*log((x^4 + x^3 + 2*x^2 + 3*(x^5 - x)^(1/3)*(x^2 + 1) - x + 3*(x^5 - x
)^(2/3) + 1)/(x^4 + x^3 + 2*x^2 - x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}+2\,x-1\right)\left(x^{2}-2\,x-1\right)}{\left(x^{5}-x\right)^{\frac{1}{3}}\left(x^{4}+x^{3}+2\,x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)*(x^2+2*x-1)/(x^4+x^3+2*x^2-x+1)/(x^5-x)^(1/3),x, algo
rithm="giac")
```

```
[Out] integrate((x^2 + 2*x - 1)*(x^2 - 2*x - 1)/((x^5 - x)^(1/3)*(x^4 + x^3 + 2*x
^2 - x + 1)), x)
```

maple [C] time = 5.92, size = 783, normalized size = 7.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x-1)*(x^2+2*x-1)/(x^4+x^3+2*x^2-x+1)/(x^5-x)^(1/3),x)
```

```
[Out] RootOf(_Z^2+_Z+1)*ln(-(215628532170574272*RootOf(_Z^2+_Z+1)^2*x^4-898452217
377392800*RootOf(_Z^2+_Z+1)^2*x^3-812973192552633838*RootOf(_Z^2+_Z+1)*x^4+
431257064341148544*RootOf(_Z^2+_Z+1)^2*x^2+2253715023870923763*RootOf(_Z^2+
_Z+1)*(x^5-x)^(1/3)*x^2+326661081770322853*RootOf(_Z^2+_Z+1)*x^3+4121401065
95081815*x^4+898452217377392800*RootOf(_Z^2+_Z+1)^2*x-2253715023870923763*R
ootOf(_Z^2+_Z+1)*(x^5-x)^(2/3)-1625946385105267676*RootOf(_Z^2+_Z+1)*x^2+83
2090150298700567*(x^5-x)^(1/3)*x^2+130149507345815310*x^3+21562853217057427
2*RootOf(_Z^2+_Z+1)^2+2253715023870923763*RootOf(_Z^2+_Z+1)*(x^5-x)^(1/3)-3
26661081770322853*RootOf(_Z^2+_Z+1)*x-832090150298700567*(x^5-x)^(2/3)+8242
80213190163630*x^2-812973192552633838*RootOf(_Z^2+_Z+1)+832090150298700567*
(x^5-x)^(1/3)-130149507345815310*x+412140106595081815)/(x^4+x^3+2*x^2-x+1))
-ln(-(215628532170574272*RootOf(_Z^2+_Z+1)^2*x^4-898452217377392800*RootOf(
_Z^2+_Z+1)^2*x^3+1244230256893782382*RootOf(_Z^2+_Z+1)*x^4+4312570643411485
44*RootOf(_Z^2+_Z+1)^2*x^2-2253715023870923763*RootOf(_Z^2+_Z+1)*(x^5-x)^(1
/3)*x^2-2123565516525108453*RootOf(_Z^2+_Z+1)*x^3+1440741831318289925*x^4+8
98452217377392800*RootOf(_Z^2+_Z+1)^2*x+2253715023870923763*RootOf(_Z^2+_Z+
1)*(x^5-x)^(2/3)+2488460513787564764*RootOf(_Z^2+_Z+1)*x^2-1421624873572223
196*(x^5-x)^(1/3)*x^2-1094963791801900343*x^3+215628532170574272*RootOf(_Z^
2+_Z+1)^2-2253715023870923763*RootOf(_Z^2+_Z+1)*(x^5-x)^(1/3)+2123565516525
108453*RootOf(_Z^2+_Z+1)*x+1421624873572223196*(x^5-x)^(2/3)+28814836626365
79850*x^2+1244230256893782382*RootOf(_Z^2+_Z+1)-1421624873572223196*(x^5-x)
^(1/3)+1094963791801900343*x+1440741831318289925)/(x^4+x^3+2*x^2-x+1))*Root
Of(_Z^2+_Z+1)-ln(-(215628532170574272*RootOf(_Z^2+_Z+1)^2*x^4-8984522173773
92800*RootOf(_Z^2+_Z+1)^2*x^3+1244230256893782382*RootOf(_Z^2+_Z+1)*x^4+431
257064341148544*RootOf(_Z^2+_Z+1)^2*x^2-2253715023870923763*RootOf(_Z^2+_Z+
```

```
1)*(x^5-x)^(1/3)*x^2-2123565516525108453*RootOf(_Z^2+_Z+1)*x^3+144074183131
8289925*x^4+898452217377392800*RootOf(_Z^2+_Z+1)^2*x+2253715023870923763*Ro
otOf(_Z^2+_Z+1)*(x^5-x)^(2/3)+2488460513787564764*RootOf(_Z^2+_Z+1)*x^2-142
1624873572223196*(x^5-x)^(1/3)*x^2-1094963791801900343*x^3+2156285321705742
72*RootOf(_Z^2+_Z+1)^2-2253715023870923763*RootOf(_Z^2+_Z+1)*(x^5-x)^(1/3)+
2123565516525108453*RootOf(_Z^2+_Z+1)*x+1421624873572223196*(x^5-x)^(2/3)+2
881483662636579850*x^2+1244230256893782382*RootOf(_Z^2+_Z+1)-14216248735722
23196*(x^5-x)^(1/3)+1094963791801900343*x+1440741831318289925)/(x^4+x^3+2*x
^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+2x-1\right)\left(x^2-2x-1\right)}{\left(x^5-x\right)^{\frac{1}{3}}\left(x^4+x^3+2x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)*(x^2+2*x-1)/(x^4+x^3+2*x^2-x+1)/(x^5-x)^(1/3),x, algo
rithm="maxima")
```

```
[Out] integrate((x^2 + 2*x - 1)*(x^2 - 2*x - 1)/((x^5 - x)^(1/3)*(x^4 + x^3 + 2*x
^2 - x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(x^2+2x-1\right)\,\left(-x^2+2x+1\right)}{\left(x^5-x\right)^{1/3}\,\left(x^4+x^3+2x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*x + x^2 - 1)*(2*x - x^2 + 1))/((x^5 - x)^(1/3)*(2*x^2 - x + x^3 +
x^4 + 1)),x)
```

```
[Out] -int(((2*x + x^2 - 1)*(2*x - x^2 + 1))/((x^5 - x)^(1/3)*(2*x^2 - x + x^3 +
x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-2x-1\right)\left(x^2+2x-1\right)}{\sqrt[3]{x\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}\,\left(x^4+x^3+2x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x-1)*(x**2+2*x-1)/(x**4+x**3+2*x**2-x+1)/(x**5-x)**(1/3),
x)
```

```
[Out] Integral((x**2 - 2*x - 1)*(x**2 + 2*x - 1)/((x*(x - 1)*(x + 1)*(x**2 + 1))*
*(1/3)*(x**4 + x**3 + 2*x**2 - x + 1)), x)
```

3.1346 $\int \frac{-b-ax^3+x^6}{x^6\sqrt[4]{bx+ax^4}}\,dx$

Optimal. Leaf size=107

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}+\frac{4\left(ax^4+bx\right)^{3/4}\left(ax^3+b\right)}{21bx^6}$$

Rubi [A] time = 0.28, antiderivative size = 167, normalized size of antiderivative = 1.56, number of steps used = 12, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2052, 2011, 329, 275, 240, 212, 206, 203, 2016, 2014}

$$\frac{4\left(ax^4+bx\right)^{3/4}}{21x^6}+\frac{4a\left(ax^4+bx\right)^{3/4}}{21bx^3}+\frac{2\sqrt[4]{x}\sqrt[4]{ax^3+b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{a}\sqrt[4]{ax^4+bx}}+\frac{2\sqrt[4]{x}\sqrt[4]{ax^3+b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{a}\sqrt[4]{ax^4+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b - a*x^3 + x^6)/(x^6*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*(b*x + a*x^4)^(3/4))/(21*x^6) + (4*a*(b*x + a*x^4)^(3/4))/(21*b*x^3) + (
2*x^(1/4)*(b + a*x^3)^(1/4)*ArcTan[(a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)])/(3
*a^(1/4)*(b*x + a*x^4)^(1/4)) + (2*x^(1/4)*(b + a*x^3)^(1/4)*ArcTanh[(a^(1/
4)*x^(3/4))/(b + a*x^3)^(1/4)])/(3*a^(1/4)*(b*x + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{-b-ax^3+x^6}{x^6\sqrt[4]{bx+ax^4}}\,dx = \int\left(\frac{1}{\sqrt[4]{bx+ax^4}}-\frac{b}{x^6\sqrt[4]{bx+ax^4}}-\frac{a}{x^3\sqrt[4]{bx+ax^4}}\right)dx$$
$$= -\left(a\int\frac{1}{x^3\sqrt[4]{bx+ax^4}}\,dx\right)-b\int\frac{1}{x^6\sqrt[4]{bx+ax^4}}\,dx+\int\frac{1}{\sqrt[4]{bx+ax^4}}\,dx$$
$$= \frac{4\left(bx+ax^4\right)^{3/4}}{21x^6}+\frac{4a\left(bx+ax^4\right)^{3/4}}{9bx^3}+\frac{1}{7}(4a)\int\frac{1}{x^3\sqrt[4]{bx+ax^4}}\,dx+\frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\int\frac{1}{\sqrt[4]{x}\sqrt[4]{b}\cdots}}{\sqrt[4]{bx+ax^4}}$$
$$= \frac{4\left(bx+ax^4\right)^{3/4}}{21x^6}+\frac{4a\left(bx+ax^4\right)^{3/4}}{21bx^3}+\frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int\frac{x^2}{\sqrt[4]{b+ax^{12}}}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^4}}$$
$$= \frac{4\left(bx+ax^4\right)^{3/4}}{21x^6}+\frac{4a\left(bx+ax^4\right)^{3/4}}{21bx^3}+\frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt[4]{b+ax^4}}\,dx,x,x^{3/4}\right)}{3\sqrt[4]{bx+ax^4}}$$
$$= \frac{4\left(bx+ax^4\right)^{3/4}}{21x^6}+\frac{4a\left(bx+ax^4\right)^{3/4}}{21bx^3}+\frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int\frac{1}{1-ax^4}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}$$
$$= \frac{4\left(bx+ax^4\right)^{3/4}}{21x^6}+\frac{4a\left(bx+ax^4\right)^{3/4}}{21bx^3}+\frac{\left(2\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int\frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}+\cdots$$
$$= \frac{4\left(bx+ax^4\right)^{3/4}}{21x^6}+\frac{4a\left(bx+ax^4\right)^{3/4}}{21bx^3}+\frac{2\sqrt[4]{x}\sqrt[4]{b+ax^3}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{a}\sqrt[4]{bx+ax^4}}+\frac{2\sqrt[4]{x}\sqrt[4]{b+ax^3}\tan\cdots}{3\sqrt[4]{a}\sqrt[4]{bx+\cdots}}$$

Mathematica [C] time = 0.05, size = 72, normalized size = 0.67

$$\frac{4\left(7bx^6\sqrt[4]{\frac{ax^3}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{ax^3}{b}\right)+\left(ax^3+b\right)^2\right)}{21bx^5\sqrt[4]{x\left(ax^3+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b - a*x^3 + x^6)/(x^6*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*((b + a*x^3)^2 + 7*b*x^6*(1 + (a*x^3)/b)^(1/4)*Hypergeometric2F1[1/4, 1/
4, 5/4, -((a*x^3)/b)]))/(21*b*x^5*(x*(b + a*x^3))^(1/4))
```

IntegrateAlgebraic [A] time = 0.42, size = 107, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}+\frac{4\left(ax^4+bx\right)^{3/4}\left(ax^3+b\right)}{21bx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b - a*x^3 + x^6)/(x^6*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*(b + a*x^3)*(b*x + a*x^4)^(3/4))/(21*b*x^6) + (2*ArcTan[(a^(1/4)*(b*x +
a*x^4)^(3/4))/(b + a*x^3)])/(3*a^(1/4)) + (2*ArcTanh[(a^(1/4)*(b*x + a*x^4)
^(3/4))/(b + a*x^3)])/(3*a^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-a*x^3-b)/x^6/(a*x^4+b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.19, size = 191, normalized size = 1.79

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{3(-a)^{\frac{1}{4}}}-\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{3(-a)^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)}{6(-a)^{\frac{1}{4}}}+\frac{\sqrt{2}(-a)^{\frac{3}{4}}\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)}{6a}+\frac{4\left(a+\frac{b}{x^3}\right)^{\frac{7}{4}}}{21b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-a*x^3-b)/x^6/(a*x^4+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^3)^(1/4))/
(-a)^(1/4))/(-a)^(1/4) - 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4
) - 2*(a + b/x^3)^(1/4))/(-a)^(1/4))/(-a)^(1/4) + 1/6*sqrt(2)*log(sqrt(2)*(
-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a) + sqrt(a + b/x^3))/(-a)^(1/4) + 1/6*
sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a) + s
qrt(a + b/x^3))/a + 4/21*(a + b/x^3)^(7/4)/b
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^{6}-a\,x^{3}-b}{x^{6}\left(a\,x^{4}+bx\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-a*x^3-b)/x^6/(a*x^4+b*x)^(1/4),x)
```

```
[Out] int((x^6-a*x^3-b)/x^6/(a*x^4+b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}-ax^{3}-b}{\left(ax^{4}+bx\right)^{\frac{1}{4}}x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-a*x^3-b)/x^6/(a*x^4+b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^6 - a*x^3 - b)/((a*x^4 + b*x)^(1/4)*x^6), x)
```

mupad [B] time = 1.31, size = 77, normalized size = 0.72

$$\frac{4\left(a\,x^4+b\,x\right)^{3/4}}{21\,x^6}+\frac{4\,x\,\left(\frac{a\,x^3}{b}+1\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{a\,x^3}{b}\right)}{3\left(a\,x^4+b\,x\right)^{1/4}}+\frac{4\,a\left(a\,x^4+b\,x\right)^{3/4}}{21\,b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x^3 - x^6)/(x^6*(b*x + a*x^4)^(1/4)),x)
```

```
[Out] (4*(b*x + a*x^4)^(3/4))/(21*x^6) + (4*x*((a*x^3)/b + 1)^(1/4)*hypergeom([1/
4, 1/4], 5/4, -(a*x^3)/b))/(3*(b*x + a*x^4)^(1/4)) + (4*a*(b*x + a*x^4)^(3/
4))/(21*b*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-ax^{3}-b+x^{6}}{x^{6}\sqrt[4]{x\left(ax^{3}+b\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-a*x**3-b)/x**6/(a*x**4+b*x)**(1/4),x)
```

```
[Out] Integral((-a*x**3 - b + x**6)/(x**6*(x*(a*x**3 + b))**(1/4)), x)
```

3.1347 $\int \frac{x^4(-4+x^3)}{\sqrt[4]{-1+x^3}\,(-1+2x^3-x^6+x^8)}\,dx$

Optimal. Leaf size=107

$$\tan^{-1}\left(\frac{\sqrt[4]{x^3-1}}{x}\right)-\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^3-1}}\right)+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\,\sqrt[4]{x^3-1}}{\sqrt{x^3-1}-x^2}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x\,\sqrt[4]{x^3-1}}{\sqrt{x^3-1}+x^2}\right)}{\sqrt{2}}$$

Rubi [F] time = 1.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4\left(-4+x^3\right)}{\sqrt[4]{-1+x^3}\left(-1+2x^3-x^6+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(-4 + x^3))/((-1 + x^3)^(1/4)*(-1 + 2*x^3 - x^6 + x^8)),x]
```

```
[Out] -2*Defer[Int][1/((-1 + x^3)^(1/4)*(1 - x^3 + x^4)), x] + Defer[Int][x^3/((-
1 + x^3)^(1/4)*(1 - x^3 + x^4)), x]/2 - 2*Defer[Int][1/((-1 + x^3)^(1/4)*(-
1 + x^3 + x^4)), x] + Defer[Int][x^3/((-1 + x^3)^(1/4)*(-1 + x^3 + x^4)), x
]/2
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(-4+x^3\right)}{\sqrt[4]{-1+x^3}\left(-1+2x^3-x^6+x^8\right)}\,dx &= \int \left(\frac{-4+x^3}{2\sqrt[4]{-1+x^3}\left(1-x^3+x^4\right)}+\frac{-4+x^3}{2\sqrt[4]{-1+x^3}\left(-1+x^3+x^4\right)}\right)dx\\
&= \frac{1}{2}\int \frac{-4+x^3}{\sqrt[4]{-1+x^3}\left(1-x^3+x^4\right)}\,dx+\frac{1}{2}\int \frac{-4+x^3}{\sqrt[4]{-1+x^3}\left(-1+x^3+x^4\right)}\,dx\\
&= \frac{1}{2}\int \left(-\frac{4}{\sqrt[4]{-1+x^3}\left(1-x^3+x^4\right)}+\frac{x^3}{\sqrt[4]{-1+x^3}\left(1-x^3+x^4\right)}\right)dx+\frac{1}{2}\int\\
&= \frac{1}{2}\int \frac{x^3}{\sqrt[4]{-1+x^3}\left(1-x^3+x^4\right)}\,dx+\frac{1}{2}\int \frac{x^3}{\sqrt[4]{-1+x^3}\left(-1+x^3+x^4\right)}\,dx
\end{aligned}$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(-4+x^3\right)}{\sqrt[4]{-1+x^3}\left(-1+2x^3-x^6+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(-4 + x^3))/((-1 + x^3)^(1/4)*(-1 + 2*x^3 - x^6 + x^8)),x]
```

```
[Out] Integrate[(x^4*(-4 + x^3))/((-1 + x^3)^(1/4)*(-1 + 2*x^3 - x^6 + x^8)), x]
```

IntegrateAlgebraic [A] time = 1.17, size = 107, normalized size = 1.00

$$\tan^{-1}\left(\frac{\sqrt[4]{x^3-1}}{x}\right)-\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^3-1}}\right)+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\,\sqrt[4]{x^3-1}}{\sqrt{x^3-1}-x^2}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x\,\sqrt[4]{x^3-1}}{\sqrt{x^3-1}+x^2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(-4 + x^3))/((-1 + x^3)^(1/4)*(-1 + 2*x^3 - x^6 + x
^8)),x]
```

```
[Out] ArcTan[(-1 + x^3)^(1/4)/x] + ArcTan[(Sqrt[2]*x*(-1 + x^3)^(1/4))/(-x^2 + Sq
rt[-1 + x^3])]/Sqrt[2] - ArcTanh[x/(-1 + x^3)^(1/4)] + ArcTanh[(Sqrt[2]*x*(
-1 + x^3)^(1/4))/(x^2 + Sqrt[-1 + x^3])]/Sqrt[2]
```

fricas [B] time = 62.91, size = 784, normalized size = 7.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^3-4)/(x^3-1)^(1/4)/(x^8-x^6+2*x^3-1),x, algorithm="fricas"
)
```

```
[Out] -1/2*sqrt(2)*arctan((x^8 + 2*x^7 + x^6 - 2*x^4 - 2*x^3 + 2*sqrt(2)*(3*x^5 -
 x^4 + x)*(x^3 - 1)^(3/4) + 2*sqrt(2)*(x^7 - 3*x^6 + 3*x^3)*(x^3 - 1)^(1/4)
 + 4*(x^6 + x^5 - x^2)*sqrt(x^3 - 1) + (16*(x^3 - 1)^(3/4)*x^5 + 2*sqrt(2)*
(3*x^6 - x^5 + x^2)*sqrt(x^3 - 1) + sqrt(2)*(x^8 + 8*x^7 - x^6 - 8*x^4 + 2*
x^3 - 1) + 4*(x^7 + x^6 - x^3)*(x^3 - 1)^(1/4))*sqrt((x^4 - 2*sqrt(2)*(x^3
- 1)^(1/4)*x^3 + x^3 + 4*sqrt(x^3 - 1)*x^2 - 2*sqrt(2)*(x^3 - 1)^(3/4)*x -
1)/(x^4 + x^3 - 1)) + 1)/(x^8 - 14*x^7 + x^6 + 14*x^4 - 2*x^3 + 1)) + 1/2*s
qrt(2)*arctan((x^8 + 2*x^7 + x^6 - 2*x^4 - 2*x^3 - 2*sqrt(2)*(3*x^5 - x^4 +
 x)*(x^3 - 1)^(3/4) - 2*sqrt(2)*(x^7 - 3*x^6 + 3*x^3)*(x^3 - 1)^(1/4) + 4*(
x^6 + x^5 - x^2)*sqrt(x^3 - 1) + (16*(x^3 - 1)^(3/4)*x^5 - 2*sqrt(2)*(3*x^6
 - x^5 + x^2)*sqrt(x^3 - 1) - sqrt(2)*(x^8 + 8*x^7 - x^6 - 8*x^4 + 2*x^3 -
1) + 4*(x^7 + x^6 - x^3)*(x^3 - 1)^(1/4))*sqrt((x^4 + 2*sqrt(2)*(x^3 - 1)^(
1/4)*x^3 + x^3 + 4*sqrt(x^3 - 1)*x^2 + 2*sqrt(2)*(x^3 - 1)^(3/4)*x - 1)/(x^
4 + x^3 - 1)) + 1)/(x^8 - 14*x^7 + x^6 + 14*x^4 - 2*x^3 + 1)) + 1/8*sqrt(2)
*log(4*(x^4 + 2*sqrt(2)*(x^3 - 1)^(1/4)*x^3 + x^3 + 4*sqrt(x^3 - 1)*x^2 + 2
*sqrt(2)*(x^3 - 1)^(3/4)*x - 1)/(x^4 + x^3 - 1)) - 1/8*sqrt(2)*log(4*(x^4 -
 2*sqrt(2)*(x^3 - 1)^(1/4)*x^3 + x^3 + 4*sqrt(x^3 - 1)*x^2 - 2*sqrt(2)*(x^3
 - 1)^(3/4)*x - 1)/(x^4 + x^3 - 1)) + 1/2*arctan(2*((x^3 - 1)^(1/4)*x^3 + (
x^3 - 1)^(3/4)*x)/(x^4 - x^3 + 1)) + 1/2*log((x^4 - 2*(x^3 - 1)^(1/4)*x^3 +
 x^3 + 2*sqrt(x^3 - 1)*x^2 - 2*(x^3 - 1)^(3/4)*x - 1)/(x^4 - x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}-4\right)x^{4}}{\left(x^{8}-x^{6}+2\,x^{3}-1\right)\left(x^{3}-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^3-4)/(x^3-1)^(1/4)/(x^8-x^6+2*x^3-1),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 4)*x^4/((x^8 - x^6 + 2*x^3 - 1)*(x^3 - 1)^(1/4)), x)
```

maple [C] time = 3.16, size = 369, normalized size = 3.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^3-4)/(x^3-1)^(1/4)/(x^8-x^6+2*x^3-1),x)
```

```
[Out] -1/2*RootOf(_Z^4+1)^3*ln(-(RootOf(_Z^4+1)^3*x^4+2*(x^3-1)^(1/4)*RootOf(_Z^4
+1)^2*x^3-RootOf(_Z^4+1)^3*x^3+2*(x^3-1)^(1/2)*RootOf(_Z^4+1)*x^2+2*(x^3-1)
^(3/4)*x+RootOf(_Z^4+1)^3)/(x^4+x^3-1))+1/2*RootOf(_Z^4+1)*ln((2*(x^3-1)^(1
```

```
/2)*RootOf(_Z^4+1)^3*x^2+2*(x^3-1)^(1/4)*RootOf(_Z^4+1)^2*x^3+RootOf(_Z^4+1
)*x^4-RootOf(_Z^4+1)*x^3-2*(x^3-1)^(3/4)*x+RootOf(_Z^4+1))/(x^4+x^3-1))+1/2
*ln((2*(x^3-1)^(3/4)*x-2*x^2*(x^3-1)^(1/2)+2*(x^3-1)^(1/4)*x^3-x^4-x^3+1)/(
x^4-x^3+1))-1/2*RootOf(_Z^4+1)^2*ln(-(2*RootOf(_Z^4+1)^2*(x^3-1)^(1/2)*x^2-
RootOf(_Z^4+1)^2*x^4-RootOf(_Z^4+1)^2*x^3-2*(x^3-1)^(3/4)*x+2*(x^3-1)^(1/4)
*x^3+RootOf(_Z^4+1)^2)/(x^4-x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^3-4)x^4}{(x^8-x^6+2x^3-1)(x^3-1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^3-4)/(x^3-1)^(1/4)/(x^8-x^6+2*x^3-1),x, algorithm="maxima"
)
```

```
[Out] integrate((x^3 - 4)*x^4/((x^8 - x^6 + 2*x^3 - 1)*(x^3 - 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,(x^3-4)}{(x^3-1)^{1/4}\,(x^8-x^6+2x^3-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x^3 - 4))/((x^3 - 1)^(1/4)*(2*x^3 - x^6 + x^8 - 1)),x)
```

```
[Out] int((x^4*(x^3 - 4))/((x^3 - 1)^(1/4)*(2*x^3 - x^6 + x^8 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**3-4)/(x**3-1)**(1/4)/(x**8-x**6+2*x**3-1),x)
```

```
[Out] Timed out
```

3.1348 $\int \frac{x^6}{\left(b+ax^4\right)^{3/4}\left(-b^2+a^2x^8\right)}\,dx$

Optimal. Leaf size=107

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{4\,2^{3/4}a^{7/4}b}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{4\,2^{3/4}a^{7/4}b}+\frac{x^3}{6ab\left(ax^4+b\right)^{3/4}}$$

Rubi [C] time = 0.09, antiderivative size = 44, normalized size of antiderivative = 0.41, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1479, 511, 510}

$$-\frac{x^7\,{}_2F_1\left(1,\frac{7}{4};\frac{11}{4};\frac{2ax^4}{ax^4+b}\right)}{7b\left(ax^4+b\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((b + a*x^4)^(3/4)*(-b^2 + a^2*x^8)),x]
```

```
[Out] -1/7*(x^7*Hypergeometric2F1[1, 7/4, 11/4, (2*a*x^4)/(b + a*x^4)])/(b*(b + a
*x^4)^(7/4))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1479

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n
2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*(a/d + (c*x^n)/e)^
p, x] /; FreeQ[{a, c, d, e, f, q, m, n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2
 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^6}{\left(b+ax^4\right)^{3/4}\left(-b^2+a^2x^8\right)}\,dx = \int \frac{x^6}{\left(-b+ax^4\right)\left(b+ax^4\right)^{7/4}}\,dx$$
$$= \frac{\left(1+\frac{ax^4}{b}\right)^{3/4}\int \frac{x^6}{\left(-b+ax^4\right)\left(1+\frac{ax^4}{b}\right)^{7/4}}\,dx}{b\left(b+ax^4\right)^{3/4}}$$
$$= -\frac{x^7\,{}_2F_1\left(1,\frac{7}{4};\frac{11}{4};\frac{2ax^4}{b+ax^4}\right)}{7b\left(b+ax^4\right)^{7/4}}$$

Mathematica [C] time = 0.13, size = 97, normalized size = 0.91

$$\frac{x^3\left(\left(1-\frac{ax^4}{b}\right)^{3/4}-\left(\frac{ax^4}{b}+1\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{2ax^4}{b-ax^4}\right)\right)}{6ab\left(ax^4+b\right)^{3/4}\left(1-\frac{ax^4}{b}\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((b + a*x^4)^(3/4)*(-b^2 + a^2*x^8)),x]
```

```
[Out] (x^3*((1 - (a*x^4)/b)^(3/4) - (1 + (a*x^4)/b)^(3/4)*Hypergeometric2F1[3/4,
3/4, 7/4, (-2*a*x^4)/(b - a*x^4)]))/(6*a*b*(b + a*x^4)^(3/4)*(1 - (a*x^4)/b
)^(3/4))
```

IntegrateAlgebraic [A] time = 0.68, size = 107, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{4\,2^{3/4}a^{7/4}b}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{4\,2^{3/4}a^{7/4}b}+\frac{x^3}{6ab\left(ax^4+b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/((b + a*x^4)^(3/4)*(-b^2 + a^2*x^8)),x]
```

```
[Out] x^3/(6*a*b*(b + a*x^4)^(3/4)) + ArcTan[(2^(1/4)*a^(1/4)*x)/(b + a*x^4)^(1/4
)]/(4*2^(3/4)*a^(7/4)*b) - ArcTanh[(2^(1/4)*a^(1/4)*x)/(b + a*x^4)^(1/4)]/(
4*2^(3/4)*a^(7/4)*b)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a*x^4+b)^(3/4)/(a^2*x^8-b^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a^2x^8-b^2\right)\left(ax^4+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a*x^4+b)^(3/4)/(a^2*x^8-b^2),x, algorithm="giac")
```

```
[Out] integrate(x^6/((a^2*x^8 - b^2)*(a*x^4 + b)^(3/4)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(a\,x^{4}+b\right)^{\frac{3}{4}} \left(a^{2} x^{8}-b^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^4+b)^(3/4)/(a^2*x^8-b^2),x)
```

```
[Out] int(x^6/(a*x^4+b)^(3/4)/(a^2*x^8-b^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(a^{2} x^{8}-b^{2}\right) \left(a x^{4}+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a*x^4+b)^(3/4)/(a^2*x^8-b^2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((a^2*x^8 - b^2)*(a*x^4 + b)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^{6}}{\left(b^{2}-a^{2}\,x^{8}\right)\,\left(a\,x^{4}+b\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^6/((b^2 - a^2*x^8)*(b + a*x^4)^(3/4)),x)
```

```
[Out] -int(x^6/((b^2 - a^2*x^8)*(b + a*x^4)^(3/4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(a*x**4+b)**(3/4)/(a**2*x**8-b**2),x)
```

```
[Out] Timed out
```

3.1349 $\int \frac{(-b^2+ax^2)\sqrt{b+\sqrt{b^2+ax^2}}}{b^2+ax^2}\,dx$

Optimal. Leaf size=107

$$\frac{4bx}{3\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{2x\sqrt{ax^2+b^2}}{3\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{4b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}$$

Rubi [F] time = 0.94, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-b^2+ax^2)\sqrt{b+\sqrt{b^2+ax^2}}}{b^2+ax^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]])/(b^2 + a*x^2),x]
```

```
[Out] (2*a*x^3)/(3*(b + Sqrt[b^2 + a*x^2])^(3/2)) + (2*b*x)/Sqrt[b + Sqrt[b^2 + a
*x^2]] - b*Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b - Sqrt[-a]*x), x] - b*
Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b + Sqrt[-a]*x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-b^2+ax^2)\sqrt{b+\sqrt{b^2+ax^2}}}{b^2+ax^2}\,dx &= \int \left(\sqrt{b+\sqrt{b^2+ax^2}}-\frac{2b^2\sqrt{b+\sqrt{b^2+ax^2}}}{b^2+ax^2}\right)dx\\
&= -\left((2b^2)\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b^2+ax^2}\,dx\right)+\int \sqrt{b+\sqrt{b^2+ax^2}}\,dx\\
&= \frac{2ax^3}{3\left(b+\sqrt{b^2+ax^2}\right)^{3/2}}+\frac{2bx}{\sqrt{b+\sqrt{b^2+ax^2}}}-(2b^2)\int \left(\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{2b\left(b-\sqrt{-a}\,x\right)}\right.\\
&= \frac{2ax^3}{3\left(b+\sqrt{b^2+ax^2}\right)^{3/2}}+\frac{2bx}{\sqrt{b+\sqrt{b^2+ax^2}}}-b\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b-\sqrt{-a}\,x}\,dx-
\end{aligned}$$

Mathematica [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(-b^2+ax^2)\sqrt{b+\sqrt{b^2+ax^2}}}{b^2+ax^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]])/(b^2 + a*x^2),x]
```

```
[Out] Integrate[((-b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]])/(b^2 + a*x^2), x]
```

IntegrateAlgebraic [A] time = 0.24, size = 107, normalized size = 1.00

$$\frac{4bx}{3\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{2x\sqrt{ax^2+b^2}}{3\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{4b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]])/(b^2 + a*x^
2),x]
```

```
[Out] (4*b*x)/(3*Sqrt[b + Sqrt[b^2 + a*x^2]]) + (2*x*Sqrt[b^2 + a*x^2])/(3*Sqrt[b
 + Sqrt[b^2 + a*x^2]]) - (4*b^(3/2)*ArcTan[(Sqrt[a]*x)/(Sqrt[b]*Sqrt[b + Sq
rt[b^2 + a*x^2]])])/Sqrt[a]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b^2)*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2-b^2\right)\sqrt{b+\sqrt{ax^2+b^2}}}{ax^2+b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b^2)*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x^2 - b^2)*sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 + b^2), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^2-b^2\right)\sqrt{b+\sqrt{a\,x^2+b^2}}}{a\,x^2+b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b^2)*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2),x)
```

```
[Out] int((a*x^2-b^2)*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2-b^2\right)\sqrt{b+\sqrt{ax^2+b^2}}}{ax^2+b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b^2)*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x^2 - b^2)*sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 + b^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^2-b^2\right)\,\sqrt{b+\sqrt{b^2+a\,x^2}}}{b^2+a\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^2 - b^2)*(b + (a*x^2 + b^2)^(1/2))^(1/2))/(a*x^2 + b^2),x)
[Out] int(((a*x^2 - b^2)*(b + (a*x^2 + b^2)^(1/2))^(1/2))/(a*x^2 + b^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}\,\left(ax^2-b^2\right)}{ax^2+b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b**2)*(b+(a*x**2+b**2)**(1/2))**(1/2)/(a*x**2+b**2),x)
[Out] Integral(sqrt(b + sqrt(a*x**2 + b**2))*(a*x**2 - b**2)/(a*x**2 + b**2), x)
```

3.1350 $\int \sqrt[3]{x^2+x^3}\, dx$

Optimal. Leaf size=108

$$\frac{1}{6}\sqrt[3]{x^3+x^2}\,(3x+1)+\frac{1}{9}\log\left(\sqrt[3]{x^3+x^2}-x\right)-\frac{1}{18}\log\left(x^2+\sqrt[3]{x^3+x^2}\,x+\left(x^3+x^2\right)^{2/3}\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x^2}+x}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.09, antiderivative size = 164, normalized size of antiderivative = 1.52, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2004, 2024, 2032, 59}

$$\frac{1}{2}\sqrt[3]{x^3+x^2}\,x+\frac{1}{6}\sqrt[3]{x^3+x^2}+\frac{(x+1)^{2/3}x^{4/3}\log(x+1)}{18\left(x^3+x^2\right)^{2/3}}+\frac{(x+1)^{2/3}x^{4/3}\log\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x+1}}-1\right)}{6\left(x^3+x^2\right)^{2/3}}+\frac{(x+1)^{2/3}x^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{x+1}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\left(x^3+x^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2 + x^3)^(1/3),x]
```

```
[Out] (x^2 + x^3)^(1/3)/6 + (x*(x^2 + x^3)^(1/3))/2 + (x^(4/3)*(1 + x)^(2/3)*ArcT
an[1/Sqrt[3] + (2*x^(1/3))/(Sqrt[3]*(1 + x)^(1/3))])/(3*Sqrt[3]*(x^2 + x^3)
^(2/3)) + (x^(4/3)*(1 + x)^(2/3)*Log[1 + x])/(18*(x^2 + x^3)^(2/3)) + (x^(4
/3)*(1 + x)^(2/3)*Log[-1 + x^(1/3)/(1 + x)^(1/3)])/(6*(x^2 + x^3)^(2/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \sqrt[3]{x^2+x^3}\,dx = \frac{1}{2}x\sqrt[3]{x^2+x^3} + \frac{1}{6}\int \frac{x^2}{\left(x^2+x^3\right)^{2/3}}\,dx$$
$$= \frac{1}{6}\sqrt[3]{x^2+x^3} + \frac{1}{2}x\sqrt[3]{x^2+x^3} - \frac{1}{9}\int \frac{x}{\left(x^2+x^3\right)^{2/3}}\,dx$$
$$= \frac{1}{6}\sqrt[3]{x^2+x^3} + \frac{1}{2}x\sqrt[3]{x^2+x^3} - \frac{\left(x^{4/3}(1+x)^{2/3}\right)\int \frac{1}{\sqrt[3]{x}(1+x)^{2/3}}\,dx}{9\left(x^2+x^3\right)^{2/3}}$$
$$= \frac{1}{6}\sqrt[3]{x^2+x^3} + \frac{1}{2}x\sqrt[3]{x^2+x^3} + \frac{x^{4/3}(1+x)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{3\sqrt{3}\left(x^2+x^3\right)^{2/3}} + \frac{x^{4/3}(1+x)^{2/3}\log(1+x}{18\left(x^2+x^3\right)^{2/3}}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.33

$$\frac{3x\sqrt[3]{x^2(x+1)}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{3};\frac{8}{3};-x\right)}{5\sqrt[3]{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2 + x^3)^(1/3),x]
```

```
[Out] (3*x*(x^2*(1 + x))^(1/3)*Hypergeometric2F1[-1/3, 5/3, 8/3, -x])/(5*(1 + x)^
(1/3))
```

IntegrateAlgebraic [A] time = 0.20, size = 108, normalized size = 1.00

$$\frac{1}{6}\sqrt[3]{x^3+x^2}\,(3x+1) + \frac{1}{9}\log\left(\sqrt[3]{x^3+x^2} - x\right) - \frac{1}{18}\log\left(x^2 + \sqrt[3]{x^3+x^2}\,x + \left(x^3+x^2\right)^{2/3}\right) + \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x^2}+x}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2 + x^3)^(1/3),x]
```

```
[Out] ((1 + 3*x)*(x^2 + x^3)^(1/3))/6 + ArcTan[(Sqrt[3]*x)/(x + 2*(x^2 + x^3)^(1/
3))]/(3*Sqrt[3]) + Log[-x + (x^2 + x^3)^(1/3)]/9 - Log[x^2 + x*(x^2 + x^3)^
(1/3) + (x^2 + x^3)^(2/3)]/18
```

fricas [A] time = 0.56, size = 100, normalized size = 0.93

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}x + 2\sqrt{3}\left(x^3+x^2\right)^{\frac{1}{3}}}{3x}\right) + \frac{1}{6}\left(x^3+x^2\right)^{\frac{1}{3}}(3x+1) + \frac{1}{9}\log\left(-\frac{x-\left(x^3+x^2\right)^{\frac{1}{3}}}{x}\right) - \frac{1}{18}\log\left(\frac{x^2+\left(x^3+x^2\right)^{\frac{1}{3}}x+\left(x^3+x^2\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 + x^2)^(1/3))/x) + 1/6*
(x^3 + x^2)^(1/3)*(3*x + 1) + 1/9*log(-(x - (x^3 + x^2)^(1/3))/x) - 1/18*lo
g((x^2 + (x^3 + x^2)^(1/3)*x + (x^3 + x^2)^(2/3))/x^2)
```

giac [A] time = 0.18, size = 77, normalized size = 0.71

$$\frac{1}{6}\left(\left(\frac{1}{x}+1\right)^{\frac{4}{3}} + 2\left(\frac{1}{x}+1\right)^{\frac{1}{3}}\right)x^2 - \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{1}{18}\log\left(\left(\frac{1}{x}+1\right)^{\frac{2}{3}} + \left(\frac{1}{x}+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{9}\log\left(\left|\left(\frac{1}{x}+1\right)^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] 1/6*((1/x + 1)^(4/3) + 2*(1/x + 1)^(1/3))*x^2 - 1/9*sqrt(3)*arctan(1/3*sqrt
(3)*(2*(1/x + 1)^(1/3) + 1)) - 1/18*log((1/x + 1)^(2/3) + (1/x + 1)^(1/3) +
 1) + 1/9*log(abs((1/x + 1)^(1/3) - 1))
```

maple [C] time = 0.30, size = 15, normalized size = 0.14

$$\frac{3x^{\frac{5}{3}} \operatorname{hypergeom}\left(\left[-\frac{1}{3}, \frac{5}{3}\right], \left[\frac{8}{3}\right], -x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2)^(1/3),x)
```

```
[Out] 3/5*x^(5/3)*hypergeom([-1/3,5/3],[8/3],-x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^3 + x^2\right)^{\frac{1}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + x^2)^(1/3), x)
```

mupad [B] time = 1.13, size = 25, normalized size = 0.23

$$\frac{3\,x\,\left(x^3+x^2\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{5}{3};\ \frac{8}{3};\ -x\right)}{5\,{\left(x+1\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3)^(1/3),x)
```

```
[Out] (3*x*(x^2 + x^3)^(1/3)*hypergeom([-1/3, 5/3], 8/3, -x))/(5*(x + 1)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{x^3 + x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2)**(1/3),x)
```

```
[Out] Integral((x**3 + x**2)**(1/3), x)
```

3.1351 $\int \frac{-1+k^3x^3}{\sqrt{(1-x)x(1-k^2x)}\,(1+k^3x^3)}\,dx$

Optimal. Leaf size=108

$$-\frac{2\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^3+(-k^2-1)x^2+x}}\right)}{3(k+1)}-\frac{4\tan^{-1}\left(\frac{\sqrt{k^2-k+1}\,\sqrt{k^2x^3+(-k^2-1)x^2+x}}{(x-1)(k^2x-1)}\right)}{3\sqrt{k^2-k+1}}$$

Rubi [C] time = 2.56, antiderivative size = 364, normalized size of antiderivative = 3.37, number of steps used = 20, number of rules used = 9, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {6718, 6688, 6725, 714, 115, 934, 12, 168, 537}

$$\frac{4(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,\Pi\left(-\frac{1}{k};\sin^{-1}\left(\sqrt{-k^2}\,\sqrt{-x}\right)|\frac{1}{k^2}\right)}{3\sqrt{-k^2}\,\sqrt{x-x^2}\,\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{4(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,\Pi\left(\frac{\sqrt[3]{-1}}{k};\sin^{-1}\left(\sqrt{-k^2}\,\sqrt{-x}\right)|\frac{1}{k^2}\right)}{3\sqrt{-k^2}\,\sqrt{x-x^2}\,\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{4(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,\Pi\left(-\frac{(-1)^{2/3}}{k};\sin^{-1}\left(\sqrt{-k^2}\,\sqrt{-x}\right)|\frac{1}{k^2}\right)}{3\sqrt{-k^2}\,\sqrt{x-x^2}\,\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + k^3*x^3)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(1 + k^3*x^3)),x]
```

```
[Out] (2*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticF[ArcSin[Sqrt[x]], k^2])/Sqr
t[(1 - x)*x*(1 - k^2*x)] + (4*(1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^2*x]*Elli
pticPi[-k^(-1), ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/(3*Sqrt[-k^2]*Sqrt[(1
 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2]) + (4*(1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 -
k^2*x]*EllipticPi[(-1)^(1/3)/k, ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/(3*Sq
rt[-k^2]*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2]) + (4*(1 - x)*Sqrt[-x]*S
qrt[x]*Sqrt[1 - k^2*x]*EllipticPi[-((-1)^(2/3)/k), ArcSin[Sqrt[-k^2]*Sqrt[-
x]], k^(-2)])/(3*Sqrt[-k^2]*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 714

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c*x)/b]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0]
 && RationalQ[b]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !FreeQ[w, x] &&  !Fr
eeQ[z, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+k^3x^3}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(1+k^3x^3\right)}\,dx = \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{-1+k^3x^3}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\left(1+k^3x^3\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{-1+k^3x^3}{\sqrt{1-k^2x}\,\sqrt{x-x^2}\left(1+k^3x^3\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \left(\frac{1}{\sqrt{1-k^2x}\,\sqrt{x-x^2}}-\frac{2}{\sqrt{1-k^2x}\,\sqrt{x-x^2}\left(1+k^3x^3\right)}\right)dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-k^2x}\,\sqrt{x-x^2}}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int}{\sqrt{(1-x)x\left(1\right.}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)}{}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int}{3\sqrt{(1-x)x\left(1\right.}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\sqrt{2}\,\sqrt{2-2x}\,\sqrt{1-x}\,\sqrt{-x}\right.}{3\sqrt{(1}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\sqrt{2-2x}\,\sqrt{1-x}\,\sqrt{-x}\,\sqrt{x}\right.}{3\sqrt{(1-x}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(4\sqrt{2-2x}\,\sqrt{1-x}\,\sqrt{-x}\,\sqrt{x}\right.}{3}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{4(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-k^2x}}{3\sqrt{-k^2}\,\sqrt{(1-x)}}$$

Mathematica [C] time = 7.66, size = 314, normalized size = 2.91

$$-\frac{2\sqrt{\frac{1}{x-1}+1}\,(x-1)^{3/2}\sqrt{\frac{1-\frac{1}{k^2}}{x-1}+1}\,\sqrt{k^2x-1}\left(-2i\sqrt{3}\left(k^2-k+1\right)\Pi\left(1+\frac{1}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+(k+1)\left(\left(\left(3+i\sqrt{3}\right)k-2i\sqrt{3}\right)\Pi\left(\frac{2\left(k^2-k+1\right)}{k\left(2k-i\sqrt{3}-1\right)};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+i\left(-2\sqrt{3}+\left(\sqrt{3}+3i\right)k\right)\Pi\left(\frac{2\left(k^2-k+1\right)}{k\left(2k+i\sqrt{3}-1\right)};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)-3i\sqrt{3}\left(k^3-1\right)F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)}{3\left(k^3+1\right)\sqrt{(x-1)x\left(k^2x-1\right)}\,\sqrt{3k^2x-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + k^3*x^3)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(1 + k^3*x^3)),x]

[Out] (-2*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (1 - k^(-2))/(-1 + x)]*(-1 + x)^(3/2)*
Sqrt[-1 + k^2*x]*((-3*I)*Sqrt[3]*(-1 + k^3)*EllipticF[I*ArcSinh[1/Sqrt[-1 +
 x]], 1 - k^(-2)] - (2*I)*Sqrt[3]*(1 - k + k^2)*EllipticPi[1 + k^(-1), I*Ar
cSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] + (1 + k)*(((-2*I)*Sqrt[3] + (3 + I*Sqrt
[3])*k)*EllipticPi[(2*(1 - k + k^2))/(k*(-1 - I*Sqrt[3] + 2*k)), I*ArcSinh[
```

```
1/Sqrt[-1 + x]], 1 - k^(-2)] + I*(-2*Sqrt[3] + (3*I + Sqrt[3])*k)*EllipticP
i[(2*(1 - k + k^2))/(k*(-1 + I*Sqrt[3] + 2*k)), I*ArcSinh[1/Sqrt[-1 + x]],
1 - k^(-2)])))/(3*(1 + k^3)*Sqrt[(-1 + x)*x*(-1 + k^2*x)]*Sqrt[-3 + 3*k^2*x
])
```

IntegrateAlgebraic [A] time = 0.41, size = 108, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}\right)}{3(k+1)}-\frac{4\tan^{-1}\left(\frac{\sqrt{k^2-k+1}\,\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{(x-1)\left(k^2x-1\right)}\right)}{3\sqrt{k^2-k+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + k^3*x^3)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(1 + k^3*x^3
)),x]
```

```
[Out] (-2*ArcTan[((1 + k)*x)/Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3]])/(3*(1 + k)) - (
4*ArcTan[(Sqrt[1 - k + k^2]*Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3])/((-1 + x)*(
-1 + k^2*x))])/(3*Sqrt[1 - k + k^2])
```

fricas [B] time = 0.80, size = 217, normalized size = 2.01

$$\frac{2\sqrt{k^2-k+1}\,(k+1)\arctan\left(\frac{\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\,\left(k^2x^2-\left(2k^2-k+2\right)x+1\right)\sqrt{k^2-k+1}}{2\left(\left(k^4-k^3+k^2\right)x^3-\left(k^4-k^3+2k^2-k+1\right)x^2+\left(k^2-k+1\right)x\right)}\right)+\left(k^2-k+1\right)\arctan\left(\frac{\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\,\left(k^2x^2-2\left(k^2+k+1\right)x+1\right)}{2\left(\left(k^3+k^2\right)x^3-\left(k^3+k^2+k+1\right)x^2+(k+1)x\right)}\right)}{3\left(k^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^3*x^3-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^3*x^3+1),x, algorithm="
fricas")
```

```
[Out] 1/3*(2*sqrt(k^2 - k + 1)*(k + 1)*arctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 +
x)*(k^2*x^2 - (2*k^2 - k + 2)*x + 1)*sqrt(k^2 - k + 1)/((k^4 - k^3 + k^2)*x
^3 - (k^4 - k^3 + 2*k^2 - k + 1)*x^2 + (k^2 - k + 1)*x)) + (k^2 - k + 1)*ar
ctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2 - 2*(k^2 + k + 1)*x + 1
)/((k^3 + k^2)*x^3 - (k^3 + k^2 + k + 1)*x^2 + (k + 1)*x)))/(k^3 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^3x^3-1}{\left(k^3x^3+1\right)\sqrt{\left(k^2x-1\right)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^3*x^3-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^3*x^3+1),x, algorithm="
giac")
```

```
[Out] integrate((k^3*x^3 - 1)/((k^3*x^3 + 1)*sqrt((k^2*x - 1)*(x - 1)*x)), x)
```

maple [C] time = 0.08, size = 368, normalized size = 3.41

$$-\frac{2\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\mathrm{EllipticF}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^2\sqrt{k^2x^3-k^2x^2-x^2+x}}+\frac{4\left(\sum_{\_\alpha=\mathrm{RootOf}\left(k^2\_Z^2-k\_Z+1\right)}\frac{\left(-\_\alpha k+2\right)\left(\_\alpha k^2-k+1\right)\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\mathrm{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{-\_\alpha k^2-k+1}{k^2-k+1},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{\left(2\_\alpha k-1\right)\left(k^2-k+1\right)\sqrt{x\left(k^2x^2-k^2x-x+1\right)}}\right)}{3k}+\frac{4\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\mathrm{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}+\frac{1}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{3k^3\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}+\frac{1}{k}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^3*x^3-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^3*x^3+1),x)
```

```
[Out] -2/k^2*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x
^3-k^2*x^2-x^2+x)^(1/2)*EllipticF((-(x-1/k^2)*k^2)^(1/2),(1/k^2/(1/k^2-1))^
(1/2))+4/3/k*sum((-_alpha*k+2)/(2*_alpha*k-1)*(_alpha*k^2-k+1)/(k^2-k+1)*(-
```

```
(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(x*(k^2*x^2-k^2
*x-x+1))^(1/2)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),(_alpha*k^2-k+1)/(k^2-k+1)
,(1/k^2/(1/k^2-1))^(1/2)),_alpha=RootOf(_Z^2*k^2-_Z*k+1))+4/3/k^3*(-(x-1/k^
2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x
)^(1/2)/(1/k^2+1/k)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+1/k),(1/
k^2/(1/k^2-1))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^3x^3 - 1}{(k^3x^3 + 1)\sqrt{(k^2x - 1)(x - 1)x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^3*x^3-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^3*x^3+1),x, algorithm="
maxima")
```

```
[Out] integrate((k^3*x^3 - 1)/((k^3*x^3 + 1)*sqrt((k^2*x - 1)*(x - 1)*x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^3*x^3 - 1)/((k^3*x^3 + 1)*(x*(k^2*x - 1)*(x - 1))^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k**3*x**3-1)/((1-x)*x*(-k**2*x+1))**(1/2)/(k**3*x**3+1),x)
```

```
[Out] Timed out
```

3.1352 $\int \frac{c+bx^2+ck^2x^4}{\sqrt{(1-x^2)(1-k^2x^2)}\,(-1+k^2x^4)}\,dx$

Optimal. Leaf size=108

$$\frac{(-b-2ck)\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}}\right)}{4(k-1)k}+\frac{(b-2ck)\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}+kx^2+1}\right)}{2k(k+1)}$$

Rubi [C] time = 1.62, antiderivative size = 192, normalized size of antiderivative = 1.78, number of steps used = 8, number of rules used = 4, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.082, Rules used = {6719, 6725, 419, 537}

$$\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}(b-2ck)\Pi\left(-k;\sin^{-1}(x)|k^2\right)}{2k\sqrt{(1-x^2)(1-k^2x^2)}}-\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}(b+2ck)\Pi\left(k;\sin^{-1}(x)|k^2\right)}{2k\sqrt{(1-x^2)(1-k^2x^2)}}+\frac{c\sqrt{1-x^2}\sqrt{1-k^2x^2}F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + b*x^2 + c*k^2*x^4)/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(-1 + k^2*x^4)),
x]
```

```
[Out] (c*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticF[ArcSin[x], k^2])/Sqrt[(1 - x^2
)*(1 - k^2*x^2)] + ((b - 2*c*k)*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[
-k, ArcSin[x], k^2])/(2*k*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) - ((b + 2*c*k)*Sqr
t[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[k, ArcSin[x], k^2])/(2*k*Sqrt[(1 -
x^2)*(1 - k^2*x^2)])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{c+bx^2+ck^2x^4}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+k^2x^4\right)}\,dx = \frac{\left(\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \frac{c+bx^2+ck^2x^4}{\sqrt{1-x^2}\sqrt{1-k^2x^2}\left(-1+k^2x^4\right)}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \left(\frac{c}{\sqrt{1-x^2}\sqrt{1-k^2x^2}}+\frac{2c+bx^2}{\sqrt{1-x^2}\sqrt{1-k^2x^2}\left(-1+k^2x^4\right)}\right)dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \frac{2c+bx^2}{\sqrt{1-x^2}\sqrt{1-k^2x^2}\left(-1+k^2x^4\right)}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(c\sqrt{1-x^2}\sqrt{1-\cdots\right.}{\sqrt{\left(1-\cdots\right.}}$$

$$= \frac{c\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int\left(-\frac{\cdots}{2k\cdots}\right.}{\sqrt{\cdots}}$$

$$= \frac{c\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left((b-2ck)\sqrt{1-x^2}\sqrt{1-k^2x^2}\cdots\right.}{2k\sqrt{\left(1-x^2\right)\left(\cdots\right.}}$$

$$= \frac{c\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{(b-2ck)\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi\cdots}{2k\sqrt{\left(1-x^2\right)\left(1-\cdots\right.}}$$

Mathematica [C] time = 0.51, size = 93, normalized size = 0.86

$$\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\left((b-2ck)\Pi\left(-k;\sin^{-1}(x)|k^2\right)-(b+2ck)\Pi\left(k;\sin^{-1}(x)|k^2\right)+2ckF\left(\sin^{-1}(x)|k^2\right)\right)}{2k\sqrt{\left(x^2-1\right)\left(k^2x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + b*x^2 + c*k^2*x^4)/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(-1 + k^2*
x^4)),x]
```

```
[Out] (Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*(2*c*k*EllipticF[ArcSin[x], k^2] + (b - 2*
c*k)*EllipticPi[-k, ArcSin[x], k^2] - (b + 2*c*k)*EllipticPi[k, ArcSin[x],
k^2]))/(2*k*Sqrt[(-1 + x^2)*(-1 + k^2*x^2)])
```

IntegrateAlgebraic [A] time = 3.27, size = 108, normalized size = 1.00

$$\frac{(-b-2ck)\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{4(k-1)k}+\frac{(b-2ck)\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2+1}\right)}{2k(k+1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + b*x^2 + c*k^2*x^4)/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(
-1 + k^2*x^4)),x]
```

```
[Out] ((-b - 2*c*k)*ArcTan[((-1 + k)*x)/Sqrt[1 + (-1 - k^2)*x^2 + k^2*x^4]])/(4*(
-1 + k)*k) + ((b - 2*c*k)*ArcTan[((1 + k)*x)/(1 + k*x^2 + Sqrt[1 + (-1 - k^
2)*x^2 + k^2*x^4])])/(2*k*(1 + k))
```

fricas [A] time = 1.24, size = 107, normalized size = 0.99

$$\frac{\left(2\,ck^2-(b+2\,c)k+b\right)\arctan\left(\frac{\sqrt{k^2x^4-\left(k^2+1\right)x^2+1}}{(k+1)x}\right)+\left(2\,ck^2+(b+2\,c)k+b\right)\arctan\left(\frac{\sqrt{k^2x^4-\left(k^2+1\right)x^2+1}}{(k-1)x}\right)}{4\left(k^3-k\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*k^2*x^4+b*x^2+c)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^4-1),x,
algorithm="fricas")
```

```
[Out] 1/4*((2*c*k^2 - (b + 2*c)*k + b)*arctan(sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)/(
(k + 1)*x)) + (2*c*k^2 + (b + 2*c)*k + b)*arctan(sqrt(k^2*x^4 - (k^2 + 1)*x
^2 + 1)/((k - 1)*x)))/(k^3 - k)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ck^2x^4+bx^2+c}{\left(k^2x^4-1\right)\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*k^2*x^4+b*x^2+c)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^4-1),x,
algorithm="giac")
```

```
[Out] integrate((c*k^2*x^4 + b*x^2 + c)/((k^2*x^4 - 1)*sqrt((k^2*x^2 - 1)*(x^2 -
1))), x)
```

maple [C] time = 0.04, size = 174, normalized size = 1.61

$$\frac{c\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\mathrm{EllipticF}(x,k)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}-\frac{(2ck+b)\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\mathrm{EllipticPi}(x,k,k)}{2k\sqrt{k^2x^4-k^2x^2-x^2+1}}+\frac{(-2ck+b)\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\mathrm{EllipticPi}(x,-k,k)}{2k\sqrt{k^2x^4-k^2x^2-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*k^2*x^4+b*x^2+c)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^4-1),x)
```

```
[Out] c*(-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*EllipticF
(x,k)-1/2*(2*c*k+b)/k*(-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^
2+1)^(1/2)*EllipticPi(x,k,k)+1/2*(-2*c*k+b)/k*(-x^2+1)^(1/2)*(-k^2*x^2+1)^(
1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*EllipticPi(x,-k,k)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ck^2x^4+bx^2+c}{\left(k^2x^4-1\right)\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*k^2*x^4+b*x^2+c)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^4-1),x,
algorithm="maxima")
```

```
[Out] integrate((c*k^2*x^4 + b*x^2 + c)/((k^2*x^4 - 1)*sqrt((k^2*x^2 - 1)*(x^2 -
1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{c\,k^2\,x^4+b\,x^2+c}{\left(k^2\,x^4-1\right)\,\sqrt{\left(x^2-1\right)\,\left(k^2\,x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + b*x^2 + c*k^2*x^4)/((k^2*x^4 - 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2))
,x)
```

```
[Out] int((c + b*x^2 + c*k^2*x^4)/((k^2*x^4 - 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2))
, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^2 + ck^2x^4 + c}{\sqrt{(x-1)(x+1)(kx-1)(kx+1)}\,(kx^2-1)(kx^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*k**2*x**4+b*x**2+c)/((-x**2+1)*(-k**2*x**2+1))**(1/2)/(k**2*x*
*4-1),x)
```

```
[Out] Integral((b*x**2 + c*k**2*x**4 + c)/(sqrt((x - 1)*(x + 1)*(k*x - 1)*(k*x +
1))*(k*x**2 - 1)*(k*x**2 + 1)), x)
```

3.1353 $\int \frac{3b+ax^4}{\left(-b+x^3+ax^4\right)\sqrt[4]{-bx+ax^5}}\,dx$

Optimal. Leaf size=108

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^5-bx}}{\sqrt{ax^5-bx}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5-bx}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5-bx}}\right)$$

Rubi [F] time = 2.62, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3b+ax^4}{\left(-b+x^3+ax^4\right)\sqrt[4]{-bx+ax^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3*b + a*x^4)/((-b + x^3 + a*x^4)*(-(b*x) + a*x^5)^(1/4)),x]
```

```
[Out] (4*x*(1 - (a*x^4)/b)^(1/4)*Hypergeometric2F1[3/16, 1/4, 19/16, (a*x^4)/b])/
(3*(-(b*x) + a*x^5)^(1/4)) - (16*b*x^(1/4)*(-b + a*x^4)^(1/4)*Defer[Subst][
Defer[Int][x^2/((b - x^12 - a*x^16)*(-b + a*x^16)^(1/4)), x], x, x^(1/4)])/
(-(b*x) + a*x^5)^(1/4) - (4*x^(1/4)*(-b + a*x^4)^(1/4)*Defer[Subst][Defer[I
nt][x^14/((-b + a*x^16)^(1/4)*(-b + x^12 + a*x^16)), x], x, x^(1/4)])/(-(b*
x) + a*x^5)^(1/4)
```

Rubi steps

$$\int \frac{3b+ax^4}{\left(-b+x^3+ax^4\right)\sqrt[4]{-bx+ax^5}}\,dx=\frac{\left(\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\int \frac{3b+ax^4}{\sqrt[4]{x}\sqrt[4]{-b+ax^4}\left(-b+x^3+ax^4\right)}\,dx}{\sqrt[4]{-bx+ax^5}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\text{Subst}\left(\int \frac{x^2\left(3b+ax^{16}\right)}{\sqrt[4]{-b+ax^{16}}\left(-b+x^{12}+ax^{16}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^5}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\text{Subst}\left(\int \left(\frac{x^2}{\sqrt[4]{-b+ax^{16}}}+\frac{x^2\left(4b-x^{12}\right)}{\sqrt[4]{-b+ax^{16}}\left(-b+x^{12}+ax^{16}\right)}\right)dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^5}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{-b+ax^{16}}}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^5}}+\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\text{Sub}}{}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\text{Subst}\left(\int \left(-\frac{4bx^2}{\left(b-x^{12}-ax^{16}\right)\sqrt[4]{-b+ax^{16}}}-\frac{x^{14}}{\sqrt[4]{-b+ax^{16}}\left(-b+x^{12}+ax^{16}\right)}\right)\right.}{\sqrt[4]{-bx+ax^5}}$$

$$=\frac{4x\sqrt[4]{1-\frac{ax^4}{b}}\,{}_2F_1\left(\frac{3}{16},\frac{1}{4};\frac{19}{16};\frac{ax^4}{b}\right)}{3\sqrt[4]{-bx+ax^5}}-\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^4}\right)\text{Subst}\left(\int \frac{x^{14}}{\sqrt[4]{-b+ax^{16}}\left(-b+\right.}\right.}{\sqrt[4]{-bx+ax^5}}$$

Mathematica [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{3b+ax^4}{\left(-b+x^3+ax^4\right)\sqrt[4]{-bx+ax^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3*b + a*x^4)/((-b + x^3 + a*x^4)*(-(b*x) + a*x^5)^(1/4)),x]
```

```
[Out] Integrate[(3*b + a*x^4)/((-b + x^3 + a*x^4)*(-(b*x) + a*x^5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 1.24, size = 108, normalized size = 1.00

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^5-bx}}{\sqrt{ax^5-bx}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5-bx}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5-bx}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*b + a*x^4)/((-b + x^3 + a*x^4)*(-(b*x) + a*x^5)^(1/4)
),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(Sqrt[2]*x*(-(b*x) + a*x^5)^(1/4))/(-x^2 + Sqrt[-(b*x) + a
*x^5])]) - Sqrt[2]*ArcTanh[(x^2/Sqrt[2] + Sqrt[-(b*x) + a*x^5]/Sqrt[2])/(x*
(-(b*x) + a*x^5)^(1/4))]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+3*b)/(a*x^4+x^3-b)/(a*x^5-b*x)^(1/4),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4+3\,b}{\left(ax^5-bx\right)^{\frac{1}{4}}\left(ax^4+x^3-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+3*b)/(a*x^4+x^3-b)/(a*x^5-b*x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^4 + 3*b)/((a*x^5 - b*x)^(1/4)*(a*x^4 + x^3 - b)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{a\,x^4+3b}{\left(a\,x^4+x^3-b\right)\left(a\,x^5-bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+3*b)/(a*x^4+x^3-b)/(a*x^5-b*x)^(1/4),x)
```

```
[Out] int((a*x^4+3*b)/(a*x^4+x^3-b)/(a*x^5-b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4+3\,b}{\left(ax^5-bx\right)^{\frac{1}{4}}\left(ax^4+x^3-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+3*b)/(a*x^4+x^3-b)/(a*x^5-b*x)^(1/4),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^4 + 3*b)/((a*x^5 - b*x)^(1/4)*(a*x^4 + x^3 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x^4+3\,b}{{\left(a\,x^5-b\,x\right)}^{1/4}\,\left(a\,x^4+x^3-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*b + a*x^4)/((a*x^5 - b*x)^(1/4)*(a*x^4 - b + x^3)),x)
```

```
[Out] int((3*b + a*x^4)/((a*x^5 - b*x)^(1/4)*(a*x^4 - b + x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+3*b)/(a*x**4+x**3-b)/(a*x**5-b*x)**(1/4),x)
```

```
[Out] Timed out
```

3.1354 $\int \frac{-1+x^6}{x^6 \sqrt[3]{x+x^3}} \, dx$

Optimal. Leaf size=108

$$-\frac{1}{2}\log\left(\sqrt[3]{x^3+x}-x\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x}+x}\right)+\frac{1}{4}\log\left(\sqrt[3]{x^3+x}x+\left(x^3+x\right)^{2/3}+x^2\right)+\frac{3\left(x^3+x\right)^{2/3}\left(9x^4\right.}{80x^6}$$

Rubi [A] time = 0.19, antiderivative size = 151, normalized size of antiderivative = 1.40, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2052, 2011, 329, 275, 239, 2016, 2014}

$$\frac{3\left(x^3+x\right)^{2/3}}{16x^6}-\frac{9\left(x^3+x\right)^{2/3}}{40x^4}+\frac{27\left(x^3+x\right)^{2/3}}{80x^2}-\frac{3\sqrt[3]{x}\sqrt[3]{x^2+1}\log\left(x^{2/3}-\sqrt[3]{x^2+1}\right)}{4\sqrt[3]{x^3+x}}+\frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{x^2+1}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)/(x^6*(x + x^3)^(1/3)),x]
```

```
[Out] (3*(x + x^3)^(2/3))/(16*x^6) - (9*(x + x^3)^(2/3))/(40*x^4) + (27*(x + x^3)
^(2/3))/(80*x^2) + (Sqrt[3]*x^(1/3)*(1 + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3))
/(1 + x^2)^(1/3))/Sqrt[3]])/(2*(x + x^3)^(1/3)) - (3*x^(1/3)*(1 + x^2)^(1/3
)*Log[x^(2/3) - (1 + x^2)^(1/3)])/(4*(x + x^3)^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x^6}{x^6\sqrt[3]{x+x^3}}\,dx &= \int \left(\frac{1}{\sqrt[3]{x+x^3}}-\frac{1}{x^6\sqrt[3]{x+x^3}}\right)dx \\
&= \int \frac{1}{\sqrt[3]{x+x^3}}\,dx-\int \frac{1}{x^6\sqrt[3]{x+x^3}}\,dx \\
&= \frac{3\left(x+x^3\right)^{2/3}}{16x^6}+\frac{3}{4}\int \frac{1}{x^4\sqrt[3]{x+x^3}}\,dx+\frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\int \frac{1}{\sqrt[3]{x}\sqrt[3]{1+x^2}}\,dx}{\sqrt[3]{x+x^3}} \\
&= \frac{3\left(x+x^3\right)^{2/3}}{16x^6}-\frac{9\left(x+x^3\right)^{2/3}}{40x^4}-\frac{9}{20}\int \frac{1}{x^2\sqrt[3]{x+x^3}}\,dx+\frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{1+x^6}}\,dx,x,
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3\left(x+x^3\right)^{2/3}}{16x^6}-\frac{9\left(x+x^3\right)^{2/3}}{40x^4}+\frac{27\left(x+x^3\right)^{2/3}}{80x^2}+\frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^3}}\,dx,x,x^{2/3}\right)}{2\sqrt[3]{x+x^3}} \\
&= \frac{3\left(x+x^3\right)^{2/3}}{16x^6}-\frac{9\left(x+x^3\right)^{2/3}}{40x^4}+\frac{27\left(x+x^3\right)^{2/3}}{80x^2}+\frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{1+x^2}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{x+x^3}}-\frac{3\sqrt[3]{x}\sqrt[3]{1
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 163, normalized size = 1.51

$$
\frac{27x^6+9x^4-3x^2-40\sqrt[3]{x^2+1}\,x^{16/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2+1}}\right)+20\sqrt[3]{x^2+1}\,x^{16/3}\log\left(\frac{x^{4/3}}{\left(x^2+1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2+1}}+1\right)+40\sqrt{3}\sqrt[3]{x^2+1}\,x^{16/3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)+15}{80x^5\sqrt[3]{x^3+x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)/(x^6*(x + x^3)^(1/3)),x]
```

```
[Out] (15 - 3*x^2 + 9*x^4 + 27*x^6 + 40*Sqrt[3]*x^(16/3)*(1 + x^2)^(1/3)*ArcTan[(
1 + (2*x^(2/3))/(1 + x^2)^(1/3))/Sqrt[3]] - 40*x^(16/3)*(1 + x^2)^(1/3)*Log
[1 - x^(2/3)/(1 + x^2)^(1/3)] + 20*x^(16/3)*(1 + x^2)^(1/3)*Log[1 + x^(4/3)
/(1 + x^2)^(2/3) + x^(2/3)/(1 + x^2)^(1/3)])/(80*x^5*(x + x^3)^(1/3))
```

IntegrateAlgebraic [A] time = 0.36, size = 108, normalized size = 1.00

$$
-\frac{1}{2}\log\left(\sqrt[3]{x^3+x}-x\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x}+x}\right)+\frac{1}{4}\log\left(\sqrt[3]{x^3+x}\,x+\left(x^3+x\right)^{2/3}+x^2\right)+\frac{3\left(x^3+x\right)^{2/3}\left(9x^4-6x^2+5\right)}{80x^6}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^6)/(x^6*(x + x^3)^(1/3)),x]
```

```
[Out] (3*(x + x^3)^(2/3)*(5 - 6*x^2 + 9*x^4))/(80*x^6) + (Sqrt[3]*ArcTan[(Sqrt[3]
*x)/(x + 2*(x + x^3)^(1/3))])/2 - Log[-x + (x + x^3)^(1/3)]/2 + Log[x^2 + x
*(x + x^3)^(1/3) + (x + x^3)^(2/3)]/4
```

fricas [A] time = 0.64, size = 112, normalized size = 1.04

$$\frac{40\sqrt{3}x^6\arctan\left(-\frac{196\sqrt{3}\left(x^3+x\right)^{\frac{1}{3}}x-\sqrt{3}\left(539x^2+507\right)-1274\sqrt{3}\left(x^3+x\right)^{\frac{2}{3}}}{2205x^2+2197}\right)-20x^6\log\left(3\left(x^3+x\right)^{\frac{1}{3}}x-3\left(x^3+x\right)^{\frac{2}{3}}+1\right)+3\left(9x^4-6x^2+5\right)\left(x^3+x\right)^{\frac{2}{3}}}{80x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/x^6/(x^3+x)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/80*(40*sqrt(3)*x^6*arctan(-(196*sqrt(3)*(x^3 + x)^(1/3)*x - sqrt(3)*(539*
x^2 + 507) - 1274*sqrt(3)*(x^3 + x)^(2/3))/(2205*x^2 + 2197)) - 20*x^6*log(
3*(x^3 + x)^(1/3)*x - 3*(x^3 + x)^(2/3) + 1) + 3*(9*x^4 - 6*x^2 + 5)*(x^3 +
 x)^(2/3))/x^6
```

giac [A] time = 0.39, size = 82, normalized size = 0.76

$$\frac{3}{16}\left(\frac{1}{x^2}+1\right)^{\frac{8}{3}}-\frac{3}{5}\left(\frac{1}{x^2}+1\right)^{\frac{5}{3}}-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{3}{4}\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\frac{1}{4}\log\left(\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{2}\log\left(\left|\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/x^6/(x^3+x)^(1/3),x, algorithm="giac")
```

```
[Out] 3/16*(1/x^2 + 1)^(8/3) - 3/5*(1/x^2 + 1)^(5/3) - 1/2*sqrt(3)*arctan(1/3*sqr
t(3)*(2*(1/x^2 + 1)^(1/3) + 1)) + 3/4*(1/x^2 + 1)^(2/3) + 1/4*log((1/x^2 +
1)^(2/3) + (1/x^2 + 1)^(1/3) + 1) - 1/2*log(abs((1/x^2 + 1)^(1/3) - 1))
```

maple [C] time = 0.29, size = 49, normalized size = 0.45

$$\frac{-\frac{3}{80}x^2+\frac{3}{16}+\frac{9}{80}x^4+\frac{27}{80}x^6}{x^5\left(x\left(x^2+1\right)\right)^{\frac{1}{3}}}+\frac{3x^{\frac{2}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)/x^6/(x^3+x)^(1/3),x)
```

```
[Out] 3/80*(9*x^6+3*x^4-x^2+5)/x^5/(x*(x^2+1))^(1/3)+3/2*x^(2/3)*hypergeom([1/3,1
/3],[4/3],-x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6-1}{\left(x^3+x\right)^{\frac{1}{3}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/x^6/(x^3+x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^6 - 1)/((x^3 + x)^(1/3)*x^6), x)
```

mupad [B] time = 1.19, size = 52, normalized size = 0.48

$$\frac{3\left(x^3+x\right)^{2/3}\left(9x^4-6x^2+5\right)}{80x^6}+\frac{3x\left(x^2+1\right)^{1/3}{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ -x^2\right)}{2\left(x^3+x\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)/(x^6*(x + x^3)^(1/3)),x)
```

```
[Out] (3*(x + x^3)^(2/3)*(9*x^4 - 6*x^2 + 5))/(80*x^6) + (3*x*(x^2 + 1)^(1/3)*hyp
ergeom([1/3, 1/3], 4/3, -x^2))/(2*(x + x^3)^(1/3))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}{x^6\sqrt[3]{x\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)/x**6/(x**3+x)**(1/3),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1)/(x**6*(x*(x**2 + 1))
**(1/3)), x)
```

3.1355 $\int \frac{(-1+x^3)^{2/3}(2+x^6)}{x^6}\,dx$

Optimal. Leaf size=108

$$\frac{2}{9}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{2\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{3\sqrt{3}}-\frac{1}{9}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)+\frac{\left(x^3-1\right)^{2/3}\left(5x^6+6x^3-6\right)}{15x^5}$$

Rubi [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1487, 451, 277, 239}

$$\frac{1}{3}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{2\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2\left(x^3-1\right)^{5/3}}{5x^5}+\frac{\left(x^3-1\right)^{5/3}}{3x^2}+\frac{\left(x^3-1\right)^{2/3}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(2/3)*(2 + x^6))/x^6,x]
```

```
[Out] (-1 + x^3)^(2/3)/(3*x^2) + (2*(-1 + x^3)^(5/3))/(5*x^5) + (-1 + x^3)^(5/3)/
(3*x^2) - (2*ArcTan[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) + Lo
g[-x + (-1 + x^3)^(1/3)]/3
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1487

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 2*n*p - n + 1)*(d + e*x^n)^(
q + 1))/(e*f^(2*n*p - n + 1)*(m + 2*n*p + n*q + 1)), x] + Dist[1/(e*(m + 2*
n*p + n*q + 1)), Int[(f*x)^m*(d + e*x^n)^q*ExpandToSum[e*(m + 2*n*p + n*q +
 1)*((a + c*x^(2*n))^p - c^p*x^(2*n*p)) - d*c^p*(m + 2*n*p - n + 1)*x^(2*n*
p - n), x], x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IG
tQ[n, 0] && IGtQ[p, 0] && GtQ[2*n*p, n - 1] &&  !IntegerQ[q] && NeQ[m + 2*n
*p + n*q + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-1+x^3\right)^{2/3}\left(2+x^6\right)}{x^6}\,dx &= \frac{\left(-1+x^3\right)^{5/3}}{3x^2}+\frac{1}{3}\int \frac{\left(6-2x^3\right)\left(-1+x^3\right)^{2/3}}{x^6}\,dx\\ &= \frac{2\left(-1+x^3\right)^{5/3}}{5x^5}+\frac{\left(-1+x^3\right)^{5/3}}{3x^2}-\frac{2}{3}\int \frac{\left(-1+x^3\right)^{2/3}}{x^3}\,dx\\ &= \frac{\left(-1+x^3\right)^{2/3}}{3x^2}+\frac{2\left(-1+x^3\right)^{5/3}}{5x^5}+\frac{\left(-1+x^3\right)^{5/3}}{3x^2}-\frac{2}{3}\int \frac{1}{\sqrt[3]{-1+x^3}}\,dx\\ &= \frac{\left(-1+x^3\right)^{2/3}}{3x^2}+\frac{2\left(-1+x^3\right)^{5/3}}{5x^5}+\frac{\left(-1+x^3\right)^{5/3}}{3x^2}-\frac{2\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(-x+\right.\end{aligned}$$

Mathematica [C] time = 0.04, size = 52, normalized size = 0.48

$$\frac{x\left(x^3-1\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};x^3\right)}{\left(1-x^3\right)^{2/3}}+\frac{2\left(x^3-1\right)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)^(2/3)*(2 + x^6))/x^6,x]
```

```
[Out] (2*(-1 + x^3)^(5/3))/(5*x^5) + (x*(-1 + x^3)^(2/3)*Hypergeometric2F1[-2/3,
1/3, 4/3, x^3])/(1 - x^3)^(2/3)
```

IntegrateAlgebraic [A] time = 0.21, size = 108, normalized size = 1.00

$$\frac{2}{9}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{2\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{3\sqrt{3}}-\frac{1}{9}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)+\frac{\left(x^3-1\right)^{2/3}\left(5x^6+6x^3-6\right)}{15x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(2/3)*(2 + x^6))/x^6,x]
```

```
[Out] ((-1 + x^3)^(2/3)*(-6 + 6*x^3 + 5*x^6))/(15*x^5) - (2*ArcTan[(Sqrt[3]*x)/(x
 + 2*(-1 + x^3)^(1/3))])/(3*Sqrt[3]) + (2*Log[-x + (-1 + x^3)^(1/3)])/9 - L
og[x^2 + x*(-1 + x^3)^(1/3) + (-1 + x^3)^(2/3)]/9
```

fricas [A] time = 0.85, size = 117, normalized size = 1.08

$$-\frac{10\sqrt{3}x^5\arctan\left(-\frac{25382\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}x^2-13720\sqrt{3}\left(x^3-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(5831x^3-7200\right)}{58653x^3-8000}\right)-5x^5\log\left(-3\left(x^3-1\right)^{\frac{1}{3}}x^2+3\left(x^3-1\right)^{\frac{2}{3}}x+1\right)-3\left(5x^6+6x^3-6\right)\left(x^3-1\right)^{\frac{2}{3}}}{45x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/45*(10*sqrt(3)*x^5*arctan(-(25382*sqrt(3)*(x^3 - 1)^(1/3)*x^2 - 13720*sq
rt(3)*(x^3 - 1)^(2/3)*x + sqrt(3)*(5831*x^3 - 7200))/(58653*x^3 - 8000)) -
5*x^5*log(-3*(x^3 - 1)^(1/3)*x^2 + 3*(x^3 - 1)^(2/3)*x + 1) - 3*(5*x^6 + 6*
x^3 - 6)*(x^3 - 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+2\right)\left(x^3-1\right)^{\frac{2}{3}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((x^6 + 2)*(x^3 - 1)^(2/3)/x^6, x)
```

maple [C] time = 0.31, size = 59, normalized size = 0.55

$$\frac{5x^9+x^6-12x^3+6}{15x^5\left(x^3-1\right)^{\frac{1}{3}}}-\frac{2\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{1}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^3\right)}{3\mathrm{signum}\left(x^3-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)*(x^6+2)/x^6,x)
```

```
[Out] 1/15*(5*x^9+x^6-12*x^3+6)/x^5/(x^3-1)^(1/3)-2/3/signum(x^3-1)^(1/3)*(-signu
m(x^3-1))^(1/3)*x*hypergeom([1/3,1/3],[4/3],x^3)
```

maxima [A] time = 0.41, size = 106, normalized size = 0.98

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3-1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\left(x^3-1\right)^{\frac{2}{3}}}{3x^2\left(\frac{x^3-1}{x^3}-1\right)}+\frac{2\left(x^3-1\right)^{\frac{5}{3}}}{5x^5}-\frac{1}{9}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3-1\right)^{\frac{2}{3}}}{x^2}+1\right)+\frac{2}{9}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+2)/x^6,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3)/x + 1)) - 1/3*(x^3 - 1)^(
2/3)/(x^2*((x^3 - 1)/x^3 - 1)) + 2/5*(x^3 - 1)^(5/3)/x^5 - 1/9*log((x^3 - 1
)^(1/3)/x + (x^3 - 1)^(2/3)/x^2 + 1) + 2/9*log((x^3 - 1)^(1/3)/x - 1)
```

mupad [B] time = 1.13, size = 54, normalized size = 0.50

$$\frac{x\left(x^3-1\right)^{2/3}{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{1}{3};\ \frac{4}{3};\ x^3\right)}{\left(1-x^3\right)^{2/3}}-\frac{2\left(x^3-1\right)^{2/3}-2x^3\left(x^3-1\right)^{2/3}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(x^6 + 2))/x^6,x)
```

```
[Out] (x*(x^3 - 1)^(2/3)*hypergeom([-2/3, 1/3], 4/3, x^3))/(1 - x^3)^(2/3) - (2*(
x^3 - 1)^(2/3) - 2*x^3*(x^3 - 1)^(2/3))/(5*x^5)
```

sympy [C] time = 2.52, size = 160, normalized size = 1.48

$$-\frac{xe^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}+2\left(\begin{cases}\frac{\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}}e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{5}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}-\frac{\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}}e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)} & \text{for}\ \frac{1}{|x^3|}>1\\-\frac{\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}+\frac{\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)*(x**6+2)/x**6,x)
```

```
[Out] -x*exp(-I*pi/3)*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), x**3)/(3*gamma(4/3))
+ 2*Piecewise(((-1 + x**(-3))**(2/3)*exp(-I*pi/3)*gamma(-5/3)/(3*gamma(-2/3
)) - (-1 + x**(-3))**(2/3)*exp(-I*pi/3)*gamma(-5/3)/(3*x**3*gamma(-2/3)), 1
/Abs(x**3) > 1), (-(1 - 1/x**3)**(2/3)*gamma(-5/3)/(3*gamma(-2/3)) + (1 - 1
/x**3)**(2/3)*gamma(-5/3)/(3*x**3*gamma(-2/3)), True))
```

3.1356 $\int \frac{\left(-1+x^3\right)^{2/3}\left(-2+x^3+x^6\right)}{x^6}\,dx$

Optimal. Leaf size=108

$$-\frac{1}{9}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{3\sqrt{3}}+\frac{1}{18}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)+\frac{\left(x^3-1\right)^{2/3}\left(10x^6-27x^3+12\right)}{30x^5}$$

Rubi [A] time = 0.04, antiderivative size = 95, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1478, 453, 277, 195, 239}

$$-\frac{1}{6}x\left(x^3-1\right)^{2/3}-\frac{1}{6}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2\left(x^3-1\right)^{8/3}}{5x^5}+\frac{\left(x^3-1\right)^{5/3}}{10x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(2/3)*(-2 + x^3 + x^6))/x^6,x]
```

```
[Out] -1/6*(x*(-1 + x^3)^(2/3)) + (-1 + x^3)^(5/3)/(10*x^2) + (2*(-1 + x^3)^(8/3)
)/(5*x^5) + ArcTan[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]]/(3*Sqrt[3]) - Log[
-x + (-1 + x^3)^(1/3)]/6
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1478

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n
_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*
```

```
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-1+x^3)^{2/3}(-2+x^3+x^6)}{x^6}\,dx &= \int \frac{(-1+x^3)^{5/3}(2+x^3)}{x^6}\,dx \\
&= \frac{2(-1+x^3)^{8/3}}{5x^5} - \frac{1}{5}\int \frac{(-1+x^3)^{5/3}}{x^3}\,dx \\
&= \frac{(-1+x^3)^{5/3}}{10x^2} + \frac{2(-1+x^3)^{8/3}}{5x^5} - \frac{1}{2}\int (-1+x^3)^{2/3}\,dx \\
&= -\frac{1}{6}x(-1+x^3)^{2/3} + \frac{(-1+x^3)^{5/3}}{10x^2} + \frac{2(-1+x^3)^{8/3}}{5x^5} + \frac{1}{3}\int \frac{1}{\sqrt[3]{-1+x^3}}\,dx \\
&= -\frac{1}{6}x(-1+x^3)^{2/3} + \frac{(-1+x^3)^{5/3}}{10x^2} + \frac{2(-1+x^3)^{8/3}}{5x^5} + \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \cdots
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 77, normalized size = 0.71

$$
\frac{(x^3-1)^{2/3}\left(-5x^3\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};x^3\right)+10x^6\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};x^3\right)+4(1-x^3)^{5/3}\right)}{10x^5(1-x^3)^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)^(2/3)*(-2 + x^3 + x^6))/x^6,x]
```

```
[Out] ((-1 + x^3)^(2/3)*(4*(1 - x^3)^(5/3) - 5*x^3*Hypergeometric2F1[-2/3, -2/3,
1/3, x^3] + 10*x^6*Hypergeometric2F1[-2/3, 1/3, 4/3, x^3]))/(10*x^5*(1 - x^
3)^(2/3))
```

IntegrateAlgebraic [A] time = 0.22, size = 108, normalized size = 1.00

$$
-\frac{1}{9}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{3\sqrt{3}}+\frac{1}{18}\log\left(\sqrt[3]{x^3-1}\,x+(x^3-1)^{2/3}+x^2\right)+\frac{(x^3-1)^{2/3}(10x^6-27x^3+12)}{30x^5}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(2/3)*(-2 + x^3 + x^6))/x^6,x]
```

```
[Out] ((-1 + x^3)^(2/3)*(12 - 27*x^3 + 10*x^6))/(30*x^5) + ArcTan[(Sqrt[3]*x)/(x
+ 2*(-1 + x^3)^(1/3))]/(3*Sqrt[3]) - Log[-x + (-1 + x^3)^(1/3)]/9 + Log[x^2
 + x*(-1 + x^3)^(1/3) + (-1 + x^3)^(2/3)]/18
```

fricas [A] time = 0.86, size = 117, normalized size = 1.08

$$
\frac{10\sqrt{3}x^5\arctan\left(-\frac{25382\sqrt{3}(x^3-1)^{\frac{1}{3}}x^2-13720\sqrt{3}(x^3-1)^{\frac{2}{3}}x+\sqrt{3}(5831x^3-7200)}{58653x^3-8000}\right)-5x^5\log\left(-3(x^3-1)^{\frac{1}{3}}x^2+3(x^3-1)^{\frac{2}{3}}x+1\right)+3(10x^6-27x^3+12)(x^3-1)^{\frac{2}{3}}}{90x^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+x^3-2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/90*(10*sqrt(3)*x^5*arctan(-(25382*sqrt(3)*(x^3 - 1)^(1/3)*x^2 - 13720*sqr
t(3)*(x^3 - 1)^(2/3)*x + sqrt(3)*(5831*x^3 - 7200))/(58653*x^3 - 8000)) - 5
*x^5*log(-3*(x^3 - 1)^(1/3)*x^2 + 3*(x^3 - 1)^(2/3)*x + 1) + 3*(10*x^6 - 27
*x^3 + 12)*(x^3 - 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^3-2\right)\left(x^3-1\right)^{\frac{2}{3}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+x^3-2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^3 - 2)*(x^3 - 1)^(2/3)/x^6, x)
```

maple [C] time = 0.30, size = 61, normalized size = 0.56

$$\frac{10x^9-37x^6+39x^3-12}{30x^5\left(x^3-1\right)^{\frac{1}{3}}}+\frac{\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{1}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^3\right)}{3\mathrm{signum}\left(x^3-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)*(x^6+x^3-2)/x^6,x)
```

```
[Out] 1/30*(10*x^9-37*x^6+39*x^3-12)/x^5/(x^3-1)^(1/3)+1/3/signum(x^3-1)^(1/3)*(-
signum(x^3-1))^(1/3)*x*hypergeom([1/3,1/3],[4/3],x^3)
```

maxima [A] time = 0.42, size = 118, normalized size = 1.09

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3-1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\left(x^3-1\right)^{\frac{2}{3}}}{2x^2}-\frac{\left(x^3-1\right)^{\frac{2}{3}}}{3x^2\left(\frac{x^3-1}{x^3}-1\right)}-\frac{2\left(x^3-1\right)^{\frac{5}{3}}}{5x^5}+\frac{1}{18}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3-1\right)^{\frac{2}{3}}}{x^2}+1\right)-\frac{1}{9}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+x^3-2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3)/x + 1)) - 1/2*(x^3 - 1)^
(2/3)/x^2 - 1/3*(x^3 - 1)^(2/3)/(x^2*((x^3 - 1)/x^3 - 1)) - 2/5*(x^3 - 1)^(
5/3)/x^5 + 1/18*log((x^3 - 1)^(1/3)/x + (x^3 - 1)^(2/3)/x^2 + 1) - 1/9*log(
(x^3 - 1)^(1/3)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(x^3-1\right)}^{2/3}\,\left(x^6+x^3-2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(x^3 + x^6 - 2))/x^6,x)
```

```
[Out] int(((x^3 - 1)^(2/3)*(x^3 + x^6 - 2))/x^6, x)
```

sympy [C] time = 3.30, size = 199, normalized size = 1.84

$$-\frac{xe^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}-2\left(\begin{cases}\frac{\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}}e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{5}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}-\frac{\left(-1+\frac{1}{x^3}\right)^{\frac{2}{3}}e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)} & \text{for}\ \frac{1}{|x^3|}>1\\ -\frac{\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}+\frac{\left(1-\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)} & \text{otherwise}\end{cases}\right)+\frac{e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\right|x^3\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)*(x**6+x**3-2)/x**6,x)
```

```
[Out] -x*exp(-I*pi/3)*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), x**3)/(3*gamma(4/3))
- 2*Piecewise(((-1 + x**(-3))**(2/3)*exp(-I*pi/3)*gamma(-5/3)/(3*gamma(-2/3
)) - (-1 + x**(-3))**(2/3)*exp(-I*pi/3)*gamma(-5/3)/(3*x**3*gamma(-2/3)), 1
/Abs(x**3) > 1), (-(1 - 1/x**3)**(2/3)*gamma(-5/3)/(3*gamma(-2/3)) + (1 - 1
/x**3)**(2/3)*gamma(-5/3)/(3*x**3*gamma(-2/3)), True)) + exp(2*I*pi/3)*gamm
a(-2/3)*hyper((-2/3, -2/3), (1/3,), x**3)/(3*x**2*gamma(1/3))
```

3.1357 $\int \frac{\left(1+x^3\right)^{2/3}\left(1+2x^6\right)}{x^6}\,dx$

Optimal. Leaf size=108

$$-\frac{4}{9}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{4\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{3\sqrt{3}}+\frac{2}{9}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)+\frac{\left(x^3+1\right)^{2/3}\left(10x^6-3x^3-3\right)}{15x^5}$$

Rubi [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1487, 451, 277, 239}

$$-\frac{2}{3}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{4\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\left(x^3+1\right)^{5/3}}{5x^5}+\frac{2\left(x^3+1\right)^{5/3}}{3x^2}-\frac{2\left(x^3+1\right)^{2/3}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^3)^(2/3)*(1 + 2*x^6))/x^6,x]
```

```
[Out] (-2*(1 + x^3)^(2/3))/(3*x^2) - (1 + x^3)^(5/3)/(5*x^5) + (2*(1 + x^3)^(5/3)
)/(3*x^2) + (4*ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) - (
2*Log[-x + (1 + x^3)^(1/3)])/3
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1487

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 2*n*p - n + 1)*(d + e*x^n)^(
q + 1))/(e*f^(2*n*p - n + 1)*(m + 2*n*p + n*q + 1)), x] + Dist[1/(e*(m + 2*
n*p + n*q + 1)), Int[(f*x)^m*(d + e*x^n)^q*ExpandToSum[e*(m + 2*n*p + n*q +
 1)*((a + c*x^(2*n))^p - c^p*x^(2*n*p)) - d*c^p*(m + 2*n*p - n + 1)*x^(2*n*
p - n), x], x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IG
tQ[n, 0] && IGtQ[p, 0] && GtQ[2*n*p, n - 1] &&  !IntegerQ[q] && NeQ[m + 2*n
*p + n*q + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+x^3\right)^{2/3}\left(1+2x^6\right)}{x^6}\,dx &= \frac{2\left(1+x^3\right)^{5/3}}{3x^2}+\frac{1}{3}\int \frac{\left(1+x^3\right)^{2/3}\left(3+4x^3\right)}{x^6}\,dx\\
&= -\frac{\left(1+x^3\right)^{5/3}}{5x^5}+\frac{2\left(1+x^3\right)^{5/3}}{3x^2}+\frac{4}{3}\int \frac{\left(1+x^3\right)^{2/3}}{x^3}\,dx\\
&= -\frac{2\left(1+x^3\right)^{2/3}}{3x^2}-\frac{\left(1+x^3\right)^{5/3}}{5x^5}+\frac{2\left(1+x^3\right)^{5/3}}{3x^2}+\frac{4}{3}\int \frac{1}{\sqrt[3]{1+x^3}}\,dx\\
&= -\frac{2\left(1+x^3\right)^{2/3}}{3x^2}-\frac{\left(1+x^3\right)^{5/3}}{5x^5}+\frac{2\left(1+x^3\right)^{5/3}}{3x^2}+\frac{4\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2}{3}\log\left(-x+\right.
\end{aligned}$$

Mathematica [C] time = 0.03, size = 35, normalized size = 0.32

$$2x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};-x^3\right)-\frac{\left(x^3+1\right)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)^(2/3)*(1 + 2*x^6))/x^6,x]
```

```
[Out] -1/5*(1 + x^3)^(5/3)/x^5 + 2*x*Hypergeometric2F1[-2/3, 1/3, 4/3, -x^3]
```

IntegrateAlgebraic [A] time = 0.21, size = 108, normalized size = 1.00

$$-\frac{4}{9}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{4\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3+1}+x}\right)}{3\sqrt{3}}+\frac{2}{9}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)+\frac{\left(x^3+1\right)^{2/3}\left(10x^6-3x^3-3\right)}{15x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)^(2/3)*(1 + 2*x^6))/x^6,x]
```

```
[Out] ((1 + x^3)^(2/3)*(-3 - 3*x^3 + 10*x^6))/(15*x^5) + (4*ArcTan[(Sqrt[3]*x)/(x
 + 2*(1 + x^3)^(1/3))])/(3*Sqrt[3]) - (4*Log[-x + (1 + x^3)^(1/3)])/9 + (2*
Log[x^2 + x*(1 + x^3)^(1/3) + (1 + x^3)^(2/3)])/9
```

fricas [A] time = 0.86, size = 117, normalized size = 1.08

$$\frac{20\sqrt{3}\,x^5\arctan\left(-\frac{25382\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}x^2-13720\sqrt{3}\left(x^3+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(5831x^3+7200\right)}{58653x^3+8000}\right)-10x^5\log\left(3\left(x^3+1\right)^{\frac{1}{3}}x^2-3\left(x^3+1\right)^{\frac{2}{3}}x+1\right)+3\left(10x^6-3x^3-3\right)\left(x^3+1\right)^{\frac{2}{3}}}{45x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6+1)/x^6,x, algorithm="fricas")
```

```
[Out] 1/45*(20*sqrt(3)*x^5*arctan(-(25382*sqrt(3)*(x^3 + 1)^(1/3)*x^2 - 13720*sqr
t(3)*(x^3 + 1)^(2/3)*x + sqrt(3)*(5831*x^3 + 7200))/(58653*x^3 + 8000)) - 1
0*x^5*log(3*(x^3 + 1)^(1/3)*x^2 - 3*(x^3 + 1)^(2/3)*x + 1) + 3*(10*x^6 - 3*
x^3 - 3)*(x^3 + 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^6+1\right)\left(x^3+1\right)^{\frac{2}{3}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6+1)/x^6,x, algorithm="giac")
```

```
[Out] integrate((2*x^6 + 1)*(x^3 + 1)^(2/3)/x^6, x)
```

maple [C] time = 0.28, size = 45, normalized size = 0.42

$$\frac{10x^9+7x^6-6x^3-3}{15x^5\left(x^3+1\right)^{\frac{1}{3}}}+\frac{4x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)*(2*x^6+1)/x^6,x)
```

```
[Out] 1/15*(10*x^9+7*x^6-6*x^3-3)/x^5/(x^3+1)^(1/3)+4/3*x*hypergeom([1/3,1/3],[4/
3],-x^3)
```

maxima [A] time = 0.41, size = 106, normalized size = 0.98

$$-\frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+1\right)^{\frac{1}{3}}}{x}+1\right)\right)+\frac{2\left(x^3+1\right)^{\frac{2}{3}}}{3x^2\left(\frac{x^3+1}{x^3}-1\right)}-\frac{\left(x^3+1\right)^{\frac{5}{3}}}{5x^5}+\frac{2}{9}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)-\frac{4}{9}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6+1)/x^6,x, algorithm="maxima")
```

```
[Out] -4/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3)/x + 1)) + 2/3*(x^3 + 1)^
(2/3)/(x^2*((x^3 + 1)/x^3 - 1)) - 1/5*(x^3 + 1)^(5/3)/x^5 + 2/9*log((x^3 +
1)^(1/3)/x + (x^3 + 1)^(2/3)/x^2 + 1) - 4/9*log((x^3 + 1)^(1/3)/x - 1)
```

mupad [B] time = 1.12, size = 38, normalized size = 0.35

$$2\,x\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{1}{3};\ \frac{4}{3};\ -x^3\right)-\frac{\left(x^3+1\right)^{2/3}+x^3\left(x^3+1\right)^{2/3}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)^(2/3)*(2*x^6 + 1))/x^6,x)
```

```
[Out] 2*x*hypergeom([-2/3, 1/3], 4/3, -x^3) - ((x^3 + 1)^(2/3) + x^3*(x^3 + 1)^(2
/3))/(5*x^5)
```

sympy [C] time = 2.39, size = 85, normalized size = 0.79

$$\frac{2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}+\frac{\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)*(2*x**6+1)/x**6,x)
```

```
[Out] 2*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/
3)) + (1 + x**(-3))**(2/3)*gamma(-5/3)/(3*gamma(-2/3)) + (1 + x**(-3))**(2/
3)*gamma(-5/3)/(3*x**3*gamma(-2/3))
```

3.1358 $\int \frac{3b+2ax^5}{\left(-b+x^3+ax^5\right)\sqrt[4]{-bx+ax^6}}\,dx$

Optimal. Leaf size=108

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^6-bx}}{\sqrt{ax^6-bx}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6-bx}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^6-bx}}\right)$$

Rubi [F] time = 2.62, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3b+2ax^5}{\left(-b+x^3+ax^5\right)\sqrt[4]{-bx+ax^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3*b + 2*a*x^5)/((-b + x^3 + a*x^5)*(-(b*x) + a*x^6)^(1/4)),x]
```

```
[Out] (8*x*(1 - (a*x^5)/b)^(1/4)*Hypergeometric2F1[3/20, 1/4, 23/20, (a*x^5)/b])/
(3*(-(b*x) + a*x^6)^(1/4)) - (20*b*x^(1/4)*(-b + a*x^5)^(1/4)*Defer[Subst][
Defer[Int][x^2/((b - x^12 - a*x^20)*(-b + a*x^20)^(1/4)), x], x, x^(1/4)])/
(-(b*x) + a*x^6)^(1/4) - (8*x^(1/4)*(-b + a*x^5)^(1/4)*Defer[Subst][Defer[I
nt][x^14/((-b + a*x^20)^(1/4)*(-b + x^12 + a*x^20)), x], x, x^(1/4)])/(-(b*
x) + a*x^6)^(1/4)
```

Rubi steps

$$\int \frac{3b+2ax^5}{\left(-b+x^3+ax^5\right)\sqrt[4]{-bx+ax^6}}\,dx=\frac{\left(\sqrt[4]{x}\sqrt[4]{-b+ax^5}\right)\int \frac{3b+2ax^5}{\sqrt[4]{x}\sqrt[4]{-b+ax^5}\left(-b+x^3+ax^5\right)}\,dx}{\sqrt[4]{-bx+ax^6}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^5}\right)\text{Subst}\left(\int \frac{x^2\left(3b+2ax^{20}\right)}{\sqrt[4]{-b+ax^{20}}\left(-b+x^{12}+ax^{20}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^6}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^5}\right)\text{Subst}\left(\int\left(\frac{2x^2}{\sqrt[4]{-b+ax^{20}}}+\frac{x^2\left(5b-2x^{12}\right)}{\sqrt[4]{-b+ax^{20}}\left(-b+x^{12}+ax^{20}\right)}\right)dx,x,\sqrt[4]{\cdots}\right.}{\sqrt[4]{-bx+ax^6}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^5}\right)\text{Subst}\left(\int \frac{x^2\left(5b-2x^{12}\right)}{\sqrt[4]{-b+ax^{20}}\left(-b+x^{12}+ax^{20}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^6}}+\frac{\left(8\sqrt[4]{x}\sqrt[4]{\cdots}\right.}{\cdots}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^5}\right)\text{Subst}\left(\int\left(-\frac{5bx^2}{\left(b-x^{12}-ax^{20}\right)\sqrt[4]{-b+ax^{20}}}-\frac{2x^{14}}{\sqrt[4]{-b+ax^{20}}\left(-b+x^{12}+ax^{2\cdots}\right.}\right.\right.}{\sqrt[4]{-bx+ax^6}}$$

$$=\frac{8x\sqrt[4]{1-\frac{ax^5}{b}}\,{}_2F_1\left(\frac{3}{20},\frac{1}{4};\frac{23}{20};\frac{ax^5}{b}\right)}{3\sqrt[4]{-bx+ax^6}}-\frac{\left(8\sqrt[4]{x}\sqrt[4]{-b+ax^5}\right)\text{Subst}\left(\int \frac{\cdots}{\sqrt[4]{-b+ax^{20}}\,(\cdots}\right.}{\sqrt[4]{-bx+ax^6}}$$

Mathematica [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{3b+2ax^5}{\left(-b+x^3+ax^5\right)\sqrt[4]{-bx+ax^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3*b + 2*a*x^5)/((-b + x^3 + a*x^5)*(-(b*x) + a*x^6)^(1/4)),x]
```

```
[Out] Integrate[(3*b + 2*a*x^5)/((-b + x^3 + a*x^5)*(-(b*x) + a*x^6)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 3.06, size = 108, normalized size = 1.00

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^6-bx}}{\sqrt{ax^6-bx}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6-bx}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^6-bx}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*b + 2*a*x^5)/((-b + x^3 + a*x^5)*(-(b*x) + a*x^6)^(1/
4)),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(Sqrt[2]*x*(-(b*x) + a*x^6)^(1/4))/(-x^2 + Sqrt[-(b*x) + a
*x^6])]) - Sqrt[2]*ArcTanh[(x^2/Sqrt[2] + Sqrt[-(b*x) + a*x^6]/Sqrt[2])/(x*
(-(b*x) + a*x^6)^(1/4))]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^5+3*b)/(a*x^5+x^3-b)/(a*x^6-b*x)^(1/4),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax^5+3\,b}{\left(ax^6-bx\right)^{\frac{1}{4}}\left(ax^5+x^3-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^5+3*b)/(a*x^5+x^3-b)/(a*x^6-b*x)^(1/4),x, algorithm="giac"
)
```

```
[Out] integrate((2*a*x^5 + 3*b)/((a*x^6 - b*x)^(1/4)*(a*x^5 + x^3 - b)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{2a\,x^5+3b}{\left(a\,x^5+x^3-b\right)\left(a\,x^6-bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x^5+3*b)/(a*x^5+x^3-b)/(a*x^6-b*x)^(1/4),x)
```

```
[Out] int((2*a*x^5+3*b)/(a*x^5+x^3-b)/(a*x^6-b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax^5+3\,b}{\left(ax^6-bx\right)^{\frac{1}{4}}\left(ax^5+x^3-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^5+3*b)/(a*x^5+x^3-b)/(a*x^6-b*x)^(1/4),x, algorithm="maxim
a")
```

```
[Out] integrate((2*a*x^5 + 3*b)/((a*x^6 - b*x)^(1/4)*(a*x^5 + x^3 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,a\,x^5+3\,b}{{\left(a\,x^6-b\,x\right)}^{1/4}\,\left(a\,x^5+x^3-b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*b + 2*a*x^5)/((a*x^6 - b*x)^(1/4)*(a*x^5 - b + x^3)),x)
```

```
[Out] int((3*b + 2*a*x^5)/((a*x^6 - b*x)^(1/4)*(a*x^5 - b + x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2ax^{5} + 3b}{\sqrt[4]{x \left(ax^{5} - b\right)} \left(ax^{5} - b + x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x**5+3*b)/(a*x**5+x**3-b)/(a*x**6-b*x)**(1/4),x)
```

```
[Out] Integral((2*a*x**5 + 3*b)/((x*(a*x**5 - b))**(1/4)*(a*x**5 - b + x**3)), x)
```

3.1359 $\int \frac{\sqrt{-bx+a^2x^2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{4\left(3a^2x+b\right)\sqrt{a^2x^2-bx}\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{3b^2x^2}-\frac{4\left(3a^3x+5ab\right)\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{3b^2x}$$

Rubi [F] time = 3.59, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-bx+a^2x^2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[-(b*x) + a^2*x^2]/(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2),x]
```

```
[Out] (2*Sqrt[-(b*x) + a^2*x^2]*Defer[Subst][Defer[Int][(x^2*Sqrt[-b + a^2*x^2])/
(a*x^4 + x^2*Sqrt[-(b*x^2) + a^2*x^4])^(3/2), x], x, Sqrt[x]])/(Sqrt[x]*Sqr
t[-b + a^2*x])
```

Rubi steps

$$\int \frac{\sqrt{-bx+a^2x^2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx=\frac{\sqrt{-bx+a^2x^2}\int \frac{\sqrt{x}\sqrt{-b+a^2x}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx}{\sqrt{x}\sqrt{-b+a^2x}}$$

$$=\frac{\left(2\sqrt{-bx+a^2x^2}\right)\text{Subst}\left(\int \frac{x^2\sqrt{-b+a^2x^2}}{\left(ax^4+x^2\sqrt{-bx^2+a^2x^4}\right)^{3/2}}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{-b+a^2x}}$$

Mathematica [F] time = 1.77, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx+a^2x^2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[-(b*x) + a^2*x^2]/(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2),x
]
```

```
[Out] Integrate[Sqrt[-(b*x) + a^2*x^2]/(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2),
x]
```

IntegrateAlgebraic [A] time = 4.93, size = 108, normalized size = 1.00

$$\frac{4\left(3a^2x+b\right)\sqrt{a^2x^2-bx}\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{3b^2x^2}-\frac{4\left(3a^3x+5ab\right)\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{3b^2x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-(b*x) + a^2*x^2]/(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2]
)^(3/2),x]
```

```
[Out] (-4*(5*a*b + 3*a^3*x)*Sqrt[x*(a*x + Sqrt[-(b*x) + a^2*x^2])])/(3*b^2*x) + (
4*(b + 3*a^2*x)*Sqrt[-(b*x) + a^2*x^2]*Sqrt[x*(a*x + Sqrt[-(b*x) + a^2*x^2]
)])/(3*b^2*x^2)
```

fricas [A] time = 0.41, size = 70, normalized size = 0.65

$$-\frac{4\left(3\,a^{3}x^{2}+5\,abx-\sqrt{a^{2}x^{2}-bx}\left(3\,a^{2}x+b\right)\right)\sqrt{ax^{2}+\sqrt{a^{2}x^{2}-bx}\,x}}{3\,b^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algori
thm="fricas")
```

```
[Out] -4/3*(3*a^3*x^2 + 5*a*b*x - sqrt(a^2*x^2 - b*x)*(3*a^2*x + b))*sqrt(a*x^2 +
 sqrt(a^2*x^2 - b*x)*x)/(b^2*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^{2}x^{2}-bx}}{\left(ax^{2}+\sqrt{a^{2}x^{2}-bx}\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(a^2*x^2 - b*x)/(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^{2}x^{2}-bx}}{\left(a\,x^{2}+x\sqrt{a^{2}x^{2}-bx}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

```
[Out] int((a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^{2}x^{2}-bx}}{\left(ax^{2}+\sqrt{a^{2}x^{2}-bx}\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 - b*x)/(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a^2\,x^2-b\,x}}{{\left(a\,x^2+x\,\sqrt{a^2\,x^2-b\,x}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 - b*x)^(1/2)/(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2),x)
[Out] int((a^2*x^2 - b*x)^(1/2)/(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(a^{2}x-b\right)}}{\left(x\left(ax+\sqrt{a^{2}x^{2}-bx}\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b*x)**(1/2)/(a*x**2+x*(a**2*x**2-b*x)**(1/2))**(3/2),x
)
[Out] Integral(sqrt(x*(a**2*x - b))/(x*(a*x + sqrt(a**2*x**2 - b*x)))**(3/2), x)
```

3.1360 $\int x^7 \sqrt[3]{-1+x^3}\, dx$

Optimal. Leaf size=109

$$\frac{5}{243}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{81\sqrt{3}}-\frac{5}{486}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)+\frac{1}{162}\sqrt[3]{x^3-1}\,\left(18x^8-3x^5\right.$$

Rubi [A] time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {279, 321, 331, 292, 31, 634, 618, 204, 628}

$$\frac{5}{243}\log\left(1-\frac{x}{\sqrt[3]{x^3-1}}\right)+\frac{5\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{1}{9}\sqrt[3]{x^3-1}\,x^8-\frac{1}{54}\sqrt[3]{x^3-1}\,x^5-\frac{5}{162}\sqrt[3]{x^3-1}\,x^2-\frac{5}{486}\log\left(\frac{x}{\sqrt[3]{x^3-1}}+\frac{x^2}{\left(x^3-1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(-1 + x^3)^(1/3),x]
```

```
[Out] (-5*x^2*(-1 + x^3)^(1/3))/162 - (x^5*(-1 + x^3)^(1/3))/54 + (x^8*(-1 + x^3)
^(1/3))/9 + (5*ArcTan[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]) +
 (5*Log[1 - x/(-1 + x^3)^(1/3)])/243 - (5*Log[1 + x^2/(-1 + x^3)^(2/3) + x/
(-1 + x^3)^(1/3)])/486
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int x^7\sqrt[3]{-1+x^3}\,dx &= \frac{1}{9}x^8\sqrt[3]{-1+x^3}-\frac{1}{9}\int\frac{x^7}{\left(-1+x^3\right)^{2/3}}\,dx\\
&= -\frac{1}{54}x^5\sqrt[3]{-1+x^3}+\frac{1}{9}x^8\sqrt[3]{-1+x^3}-\frac{5}{54}\int\frac{x^4}{\left(-1+x^3\right)^{2/3}}\,dx\\
&= -\frac{5}{162}x^2\sqrt[3]{-1+x^3}-\frac{1}{54}x^5\sqrt[3]{-1+x^3}+\frac{1}{9}x^8\sqrt[3]{-1+x^3}-\frac{5}{81}\int\frac{x}{\left(-1+x^3\right)^{2/3}}\,dx\\
&= -\frac{5}{162}x^2\sqrt[3]{-1+x^3}-\frac{1}{54}x^5\sqrt[3]{-1+x^3}+\frac{1}{9}x^8\sqrt[3]{-1+x^3}-\frac{5}{81}\text{Subst}\left(\int\frac{x}{1-x^3}\,dx,x,\frac{x}{\sqrt[3]{-1+x^3}}\right.\\
&= -\frac{5}{162}x^2\sqrt[3]{-1+x^3}-\frac{1}{54}x^5\sqrt[3]{-1+x^3}+\frac{1}{9}x^8\sqrt[3]{-1+x^3}-\frac{5}{243}\text{Subst}\left(\int\frac{1}{1-x}\,dx,x,\frac{x}{\sqrt[3]{-1+x^3}}\right.\\
&= -\frac{5}{162}x^2\sqrt[3]{-1+x^3}-\frac{1}{54}x^5\sqrt[3]{-1+x^3}+\frac{1}{9}x^8\sqrt[3]{-1+x^3}+\frac{5}{243}\log\left(1-\frac{x}{\sqrt[3]{-1+x^3}}\right)-\frac{5}{486}\text{Sub}\\
&= -\frac{5}{162}x^2\sqrt[3]{-1+x^3}-\frac{1}{54}x^5\sqrt[3]{-1+x^3}+\frac{1}{9}x^8\sqrt[3]{-1+x^3}+\frac{5}{243}\log\left(1-\frac{x}{\sqrt[3]{-1+x^3}}\right)-\frac{5}{486}\log\\
&= -\frac{5}{162}x^2\sqrt[3]{-1+x^3}-\frac{1}{54}x^5\sqrt[3]{-1+x^3}+\frac{1}{9}x^8\sqrt[3]{-1+x^3}+\frac{5\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{5}{243}\log\left(1-\right.
\end{aligned}$$

Mathematica [C] time = 0.02, size = 67, normalized size = 0.61

$$\frac{x^2\sqrt[3]{x^3-1}\left(5\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\sqrt[3]{1-x^3}\left(6x^6-x^3-5\right)\right)}{54\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(-1 + x^3)^(1/3),x]
```

```
[Out] (x^2*(-1 + x^3)^(1/3)*((1 - x^3)^(1/3)*(-5 - x^3 + 6*x^6) + 5*Hypergeometri
c2F1[-1/3, 2/3, 5/3, x^3]))/(54*(1 - x^3)^(1/3))
```

IntegrateAlgebraic [A] time = 0.24, size = 109, normalized size = 1.00

$$\frac{5}{243}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{81\sqrt{3}}-\frac{5}{486}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)+\frac{1}{162}\sqrt[3]{x^3-1}\left(18x^8-3x^5-5x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(-1 + x^3)^(1/3),x]
```

```
[Out] ((-1 + x^3)^(1/3)*(-5*x^2 - 3*x^5 + 18*x^8))/162 + (5*ArcTan[(Sqrt[3]*x)/(x
 + 2*(-1 + x^3)^(1/3))])/(81*Sqrt[3]) + (5*Log[-x + (-1 + x^3)^(1/3)])/243
- (5*Log[x^2 + x*(-1 + x^3)^(1/3) + (-1 + x^3)^(2/3)])/486
```

fricas [A] time = 0.41, size = 101, normalized size = 0.93

$$-\frac{5}{243}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{162}\left(18x^8-3x^5-5x^2\right)\left(x^3-1\right)^{\frac{1}{3}}+\frac{5}{243}\log\left(-\frac{x-\left(x^3-1\right)^{\frac{1}{3}}}{x}\right)-\frac{5}{486}\log\left(\frac{x^2+\left(x^3-1\right)^{\frac{1}{3}}x+\left(x^3-1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^3-1)^(1/3),x, algorithm="fricas")
```

```
[Out] -5/243*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 - 1)^(1/3))/x) + 1/16
2*(18*x^8 - 3*x^5 - 5*x^2)*(x^3 - 1)^(1/3) + 5/243*log(-(x - (x^3 - 1)^(1/3
))/x) - 5/486*log((x^2 + (x^3 - 1)^(1/3)*x + (x^3 - 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^3-1\right)^{\frac{1}{3}}x^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^3-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 1)^(1/3)*x^7, x)
```

maple [C] time = 0.30, size = 58, normalized size = 0.53

$$\frac{x^2\left(18x^6-3x^3-5\right)\left(x^3-1\right)^{\frac{1}{3}}}{162}-\frac{5\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{2}{3}}x^2\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{162\mathrm{signum}\left(x^3-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(x^3-1)^(1/3),x)
```

```
[Out] 1/162*x^2*(18*x^6-3*x^3-5)*(x^3-1)^(1/3)-5/162/signum(x^3-1)^(2/3)*(-signum
(x^3-1))^(2/3)*x^2*hypergeom([2/3,2/3],[5/3],x^3)
```

maxima [A] time = 0.42, size = 145, normalized size = 1.33

$$-\frac{5}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3-1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\frac{10\left(x^3-1\right)^{\frac{1}{3}}}{x}+\frac{13\left(x^3-1\right)^{\frac{4}{3}}}{x^4}-\frac{5\left(x^3-1\right)^{\frac{7}{3}}}{x^7}}{162\left(\frac{3\left(x^3-1\right)}{x^3}-\frac{3\left(x^3-1\right)^2}{x^6}+\frac{\left(x^3-1\right)^3}{x^9}-1\right)}-\frac{5}{486}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3-1\right)^{\frac{2}{3}}}{x^2}+1\right)+\frac{5}{243}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^3-1)^(1/3),x, algorithm="maxima")
```

```
[Out] -5/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3)/x + 1)) - 1/162*(10*(x
^3 - 1)^(1/3)/x + 13*(x^3 - 1)^(4/3)/x^4 - 5*(x^3 - 1)^(7/3)/x^7)/(3*(x^3 -
 1)/x^3 - 3*(x^3 - 1)^2/x^6 + (x^3 - 1)^3/x^9 - 1) - 5/486*log((x^3 - 1)^(1
/3)/x + (x^3 - 1)^(2/3)/x^2 + 1) + 5/243*log((x^3 - 1)^(1/3)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^7 \left(x^3 - 1\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(x^3 - 1)^(1/3),x)
```

```
[Out] int(x^7*(x^3 - 1)^(1/3), x)
```

sympy [C] time = 1.24, size = 36, normalized size = 0.33

$$-\frac{x^8 e^{-\frac{2i\pi}{3}} \Gamma\left(\frac{8}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{8}{3} \\ \frac{11}{3} \end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(x**3-1)**(1/3),x)
```

```
[Out] -x**8*exp(-2*I*pi/3)*gamma(8/3)*hyper((-1/3, 8/3), (11/3,), x**3)/(3*gamma(
11/3))
```

3.1361 $\int x^7 \sqrt[3]{1+x^3}\, dx$

Optimal. Leaf size=109

$$-\frac{5}{243}\log\left(\sqrt[3]{x^3+1}-x\right)-\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{81\sqrt{3}}+\frac{5}{486}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)+\frac{1}{162}\sqrt[3]{x^3+1}\left(18x^8+3\right.$$

Rubi [A] time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {279, 321, 331, 292, 31, 634, 618, 204, 628}

$$-\frac{5}{243}\log\left(1-\frac{x}{\sqrt[3]{x^3+1}}\right)-\frac{5\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{1}{9}\sqrt[3]{x^3+1}\,x^8+\frac{1}{54}\sqrt[3]{x^3+1}\,x^5-\frac{5}{162}\sqrt[3]{x^3+1}\,x^2+\frac{5}{486}\log\left(\frac{x}{\sqrt[3]{x^3+1}}+\frac{x^2}{\left(x^3+1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(1 + x^3)^(1/3),x]
```

```
[Out] (-5*x^2*(1 + x^3)^(1/3))/162 + (x^5*(1 + x^3)^(1/3))/54 + (x^8*(1 + x^3)^(1
/3))/9 - (5*ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]) - (5*
Log[1 - x/(1 + x^3)^(1/3)])/243 + (5*Log[1 + x^2/(1 + x^3)^(2/3) + x/(1 + x
^3)^(1/3)])/486
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int x^7\sqrt[3]{1+x^3}\,dx = \frac{1}{9}x^8\sqrt[3]{1+x^3}+\frac{1}{9}\int\frac{x^7}{\left(1+x^3\right)^{2/3}}\,dx$$
$$=\frac{1}{54}x^5\sqrt[3]{1+x^3}+\frac{1}{9}x^8\sqrt[3]{1+x^3}-\frac{5}{54}\int\frac{x^4}{\left(1+x^3\right)^{2/3}}\,dx$$
$$=-\frac{5}{162}x^2\sqrt[3]{1+x^3}+\frac{1}{54}x^5\sqrt[3]{1+x^3}+\frac{1}{9}x^8\sqrt[3]{1+x^3}+\frac{5}{81}\int\frac{x}{\left(1+x^3\right)^{2/3}}\,dx$$
$$=-\frac{5}{162}x^2\sqrt[3]{1+x^3}+\frac{1}{54}x^5\sqrt[3]{1+x^3}+\frac{1}{9}x^8\sqrt[3]{1+x^3}+\frac{5}{81}\text{Subst}\left(\int\frac{x}{1-x^3}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right)$$
$$=-\frac{5}{162}x^2\sqrt[3]{1+x^3}+\frac{1}{54}x^5\sqrt[3]{1+x^3}+\frac{1}{9}x^8\sqrt[3]{1+x^3}+\frac{5}{243}\text{Subst}\left(\int\frac{1}{1-x}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right)-\frac{5}{24\ldots}$$
$$=-\frac{5}{162}x^2\sqrt[3]{1+x^3}+\frac{1}{54}x^5\sqrt[3]{1+x^3}+\frac{1}{9}x^8\sqrt[3]{1+x^3}-\frac{5}{243}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\right)+\frac{5}{486}\text{Subst}\left(\int\ldots\right.$$
$$=-\frac{5}{162}x^2\sqrt[3]{1+x^3}+\frac{1}{54}x^5\sqrt[3]{1+x^3}+\frac{1}{9}x^8\sqrt[3]{1+x^3}-\frac{5}{243}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\right)+\frac{5}{486}\log\left(1+\ldots\right.$$
$$=-\frac{5}{162}x^2\sqrt[3]{1+x^3}+\frac{1}{54}x^5\sqrt[3]{1+x^3}+\frac{1}{9}x^8\sqrt[3]{1+x^3}-\frac{5\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{81\sqrt{3}}-\frac{5}{243}\log\left(1-\frac{x}{\sqrt[3]{1+x^3}}\ldots\right.$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.41

$$\frac{1}{54}x^2\left(5\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-x^3\right)+\sqrt[3]{x^3+1}\,\left(6x^6+x^3-5\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(1 + x^3)^(1/3),x]
```

```
[Out] (x^2*((1 + x^3)^(1/3)*(-5 + x^3 + 6*x^6) + 5*Hypergeometric2F1[-1/3, 2/3, 5
/3, -x^3]))/54
```

IntegrateAlgebraic [A] time = 0.24, size = 109, normalized size = 1.00

$$-\frac{5}{243}\log\left(\sqrt[3]{x^3+1}-x\right)-\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{81\sqrt{3}}+\frac{5}{486}\log\left(\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)+\frac{1}{162}\sqrt[3]{x^3+1}\left(18x^8+3x^5-5x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(1 + x^3)^(1/3),x]
```

```
[Out] ((1 + x^3)^(1/3)*(-5*x^2 + 3*x^5 + 18*x^8))/162 - (5*ArcTan[(Sqrt[3]*x)/(x
+ 2*(1 + x^3)^(1/3))])/(81*Sqrt[3]) - (5*Log[-x + (1 + x^3)^(1/3)])/243 + (
5*Log[x^2 + x*(1 + x^3)^(1/3) + (1 + x^3)^(2/3)])/486
```

fricas [A] time = 0.40, size = 101, normalized size = 0.93

$$\frac{5}{243}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{162}\left(18x^8+3x^5-5x^2\right)\left(x^3+1\right)^{\frac{1}{3}}-\frac{5}{243}\log\left(-\frac{x-\left(x^3+1\right)^{\frac{1}{3}}}{x}\right)+\frac{5}{486}\log\left(\frac{x^2+\left(x^3+1\right)^{\frac{1}{3}}x+\left(x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 5/243*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 + 1)^(1/3))/x) + 1/162
*(18*x^8 + 3*x^5 - 5*x^2)*(x^3 + 1)^(1/3) - 5/243*log(-(x - (x^3 + 1)^(1/3)
)/x) + 5/486*log((x^2 + (x^3 + 1)^(1/3)*x + (x^3 + 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^3+1\right)^{\frac{1}{3}}x^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)^(1/3)*x^7, x)
```

maple [C] time = 0.32, size = 42, normalized size = 0.39

$$\frac{x^2\left(18x^6+3x^3-5\right)\left(x^3+1\right)^{\frac{1}{3}}}{162}+\frac{5x^2\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^3\right)}{162}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(x^3+1)^(1/3),x)
```

```
[Out] 1/162*x^2*(18*x^6+3*x^3-5)*(x^3+1)^(1/3)+5/162*x^2*hypergeom([2/3,2/3],[5/3
],-x^3)
```

maxima [A] time = 0.43, size = 145, normalized size = 1.33

$$\frac{5}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+1\right)^{\frac{1}{3}}}{x}+1\right)\right)+\frac{\frac{10\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{13\left(x^3+1\right)^{\frac{4}{3}}}{x^4}-\frac{5\left(x^3+1\right)^{\frac{7}{3}}}{x^7}}{162\left(\frac{3\left(x^3+1\right)}{x^3}-\frac{3\left(x^3+1\right)^2}{x^6}+\frac{\left(x^3+1\right)^3}{x^9}-1\right)}+\frac{5}{486}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)-\frac{5}{243}\log\left(\frac{\left(x^3+1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 5/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3)/x + 1)) + 1/162*(10*(x^
3 + 1)^(1/3)/x + 13*(x^3 + 1)^(4/3)/x^4 - 5*(x^3 + 1)^(7/3)/x^7)/(3*(x^3 +
1)/x^3 - 3*(x^3 + 1)^2/x^6 + (x^3 + 1)^3/x^9 - 1) + 5/486*log((x^3 + 1)^(1/
3)/x + (x^3 + 1)^(2/3)/x^2 + 1) - 5/243*log((x^3 + 1)^(1/3)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^7 \left(x^3+1\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(x^3 + 1)^(1/3),x)
```

```
[Out] int(x^7*(x^3 + 1)^(1/3), x)
```

sympy [C] time = 1.20, size = 31, normalized size = 0.28

$$\frac{x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(x**3+1)**(1/3),x)
```

```
[Out] x**8*gamma(8/3)*hyper((-1/3, 8/3), (11/3,), x**3*exp_polar(I*pi))/(3*gamma(
11/3))
```

3.1362 $\int \frac{\sqrt[3]{-1+x^3}\,(1+x^3)}{x^{13}}\,dx$

Optimal. Leaf size=109

$$\frac{25}{729}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{25\log\left(\left(x^3-1\right)^{2/3}-\sqrt[3]{x^3-1}+1\right)}{1458}-\frac{25\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{243\sqrt{3}}+\frac{\sqrt[3]{x^3-1}\left(50x^9+30x^6\right.}{972x^{12}}$$

Rubi [A] time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {446, 78, 47, 51, 58, 618, 204, 31}

$$\frac{25\sqrt[3]{x^3-1}}{486x^3}+\frac{25}{486}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{25\tan^{-1}\left(\frac{1-2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{243\sqrt{3}}+\frac{\left(x^3-1\right)^{4/3}}{12x^{12}}-\frac{5\sqrt[3]{x^3-1}}{27x^9}+\frac{5\sqrt[3]{x^3-1}}{162x^6}-\frac{25\log(x)}{486}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(1/3)*(1 + x^3))/x^13,x]
```

```
[Out] (-5*(-1 + x^3)^(1/3))/(27*x^9) + (5*(-1 + x^3)^(1/3))/(162*x^6) + (25*(-1 +
 x^3)^(1/3))/(486*x^3) + (-1 + x^3)^(4/3)/(12*x^12) - (25*ArcTan[(1 - 2*(-1
 + x^3)^(1/3))/Sqrt[3]])/(243*Sqrt[3]) - (25*Log[x])/486 + (25*Log[1 + (-1
+ x^3)^(1/3)])/486
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{-1+x^3}\left(1+x^3\right)}{x^{13}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt[3]{-1+x}\,(1+x)}{x^5}\,dx,x,x^3\right)\\
&= \frac{\left(-1+x^3\right)^{4/3}}{12x^{12}}+\frac{5}{9}\operatorname{Subst}\left(\int \frac{\sqrt[3]{-1+x}}{x^4}\,dx,x,x^3\right)\\
&= -\frac{5\sqrt[3]{-1+x^3}}{27x^9}+\frac{\left(-1+x^3\right)^{4/3}}{12x^{12}}+\frac{5}{81}\operatorname{Subst}\left(\int \frac{1}{(-1+x)^{2/3}x^3}\,dx,x,x^3\right)\\
&= -\frac{5\sqrt[3]{-1+x^3}}{27x^9}+\frac{5\sqrt[3]{-1+x^3}}{162x^6}+\frac{\left(-1+x^3\right)^{4/3}}{12x^{12}}+\frac{25}{486}\operatorname{Subst}\left(\int \frac{1}{(-1+x)^{2/3}x^2}\,dx,x,x^3\right)\\
&= -\frac{5\sqrt[3]{-1+x^3}}{27x^9}+\frac{5\sqrt[3]{-1+x^3}}{162x^6}+\frac{25\sqrt[3]{-1+x^3}}{486x^3}+\frac{\left(-1+x^3\right)^{4/3}}{12x^{12}}+\frac{25}{729}\operatorname{Subst}\left(\int \frac{1}{(-1+x
\right.\right.\\
&= -\frac{5\sqrt[3]{-1+x^3}}{27x^9}+\frac{5\sqrt[3]{-1+x^3}}{162x^6}+\frac{25\sqrt[3]{-1+x^3}}{486x^3}+\frac{\left(-1+x^3\right)^{4/3}}{12x^{12}}-\frac{25\log(x)}{486}+\frac{25}{486}\operatorname{Subst}\\
&= -\frac{5\sqrt[3]{-1+x^3}}{27x^9}+\frac{5\sqrt[3]{-1+x^3}}{162x^6}+\frac{25\sqrt[3]{-1+x^3}}{486x^3}+\frac{\left(-1+x^3\right)^{4/3}}{12x^{12}}-\frac{25\log(x)}{486}+\frac{25}{486}\log\left(1\right.\\
&= -\frac{5\sqrt[3]{-1+x^3}}{27x^9}+\frac{5\sqrt[3]{-1+x^3}}{162x^6}+\frac{25\sqrt[3]{-1+x^3}}{486x^3}+\frac{\left(-1+x^3\right)^{4/3}}{12x^{12}}-\frac{25\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x^3}}{\sqrt{3}}\right)}{243\sqrt{3}}-
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.35

$$\frac{\left(x^3-1\right)^{4/3}\left(5x^{12}\,{}_2F_1\left(\frac{4}{3},4;\frac{7}{3};1-x^3\right)+1\right)}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)^(1/3)*(1 + x^3))/x^13,x]
```

```
[Out] ((-1 + x^3)^(4/3)*(1 + 5*x^12*Hypergeometric2F1[4/3, 4, 7/3, 1 - x^3]))/(12
*x^12)
```

IntegrateAlgebraic [A] time = 0.19, size = 109, normalized size = 1.00

$$\frac{25}{729}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{25\log\left(\left(x^3-1\right)^{2/3}-\sqrt[3]{x^3-1}+1\right)}{1458}-\frac{25\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{243\sqrt{3}}+\frac{\sqrt[3]{x^3-1}\left(50x^9+30x^6-99x^3-81\right)}{972x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(1/3)*(1 + x^3))/x^13,x]
```

```
[Out] ((-1 + x^3)^(1/3)*(-81 - 99*x^3 + 30*x^6 + 50*x^9))/(972*x^12) - (25*ArcTan
[1/Sqrt[3] - (2*(-1 + x^3)^(1/3))/Sqrt[3]])/(243*Sqrt[3]) + (25*Log[1 + (-1
 + x^3)^(1/3)])/729 - (25*Log[1 - (-1 + x^3)^(1/3) + (-1 + x^3)^(2/3)])/145
8
```

fricas [A] time = 0.42, size = 98, normalized size = 0.90

$$\frac{100\,\sqrt{3}x^{12}\arctan\left(\frac{2}{3}\,\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)-50\,x^{12}\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+100\,x^{12}\log\left(\left(x^3-1\right)^{\frac{1}{3}}+1\right)+3\left(50\,x^9+30\,x^6-99\,x^3-81\right)\left(x^3-1\right)^{\frac{1}{3}}}{2916\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(x^3+1)/x^13,x, algorithm="fricas")
```

```
[Out] 1/2916*(100*sqrt(3)*x^12*arctan(2/3*sqrt(3)*(x^3 - 1)^(1/3) - 1/3*sqrt(3))
- 50*x^12*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 100*x^12*log((x^3 -
1)^(1/3) + 1) + 3*(50*x^9 + 30*x^6 - 99*x^3 - 81)*(x^3 - 1)^(1/3))/x^12
```

giac [A] time = 0.35, size = 99, normalized size = 0.91

$$\frac{25}{729}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(x^3-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{50\left(x^3-1\right)^{\frac{10}{3}}+180\left(x^3-1\right)^{\frac{7}{3}}+111\left(x^3-1\right)^{\frac{4}{3}}-100\left(x^3-1\right)^{\frac{1}{3}}}{972\,x^{12}}-\frac{25}{1458}\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+\frac{25}{729}\log\left(\left|\left(x^3-1\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(x^3+1)/x^13,x, algorithm="giac")
```

```
[Out] 25/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) + 1/972*(50*(x^3
 - 1)^(10/3) + 180*(x^3 - 1)^(7/3) + 111*(x^3 - 1)^(4/3) - 100*(x^3 - 1)^(1
/3))/x^12 - 25/1458*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1) + 25/729*log
(abs((x^3 - 1)^(1/3) + 1))
```

maple [C] time = 0.30, size = 101, normalized size = 0.93

$$\frac{50x^{12}-20x^9-129x^6+18x^3+81}{972x^{12}\left(x^3-1\right)^{\frac{2}{3}}}+\frac{25\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{2}{3}}\left(\frac{2\Gamma\left(\frac{2}{3}\right)x^3\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],x^3\right)}{3}+\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)\right)}{729\Gamma\left(\frac{2}{3}\right)\mathrm{signum}\left(x^3-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(1/3)*(x^3+1)/x^13,x)
```

```
[Out] 1/972*(50*x^12-20*x^9-129*x^6+18*x^3+81)/x^12/(x^3-1)^(2/3)+25/729/GAMMA(2/
3)/signum(x^3-1)^(2/3)*(-signum(x^3-1))^(2/3)*(2/3*GAMMA(2/3)*x^3*hypergeom
([1,1,5/3],[2,2],x^3)+(1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*GAMMA(2/3))
```

maxima [B] time = 0.42, size = 184, normalized size = 1.69

$$\frac{25}{729}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(x^3-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{20\left(x^3-1\right)^{\frac{10}{3}}+72\left(x^3-1\right)^{\frac{7}{3}}+93\left(x^3-1\right)^{\frac{4}{3}}-40\left(x^3-1\right)^{\frac{1}{3}}}{972\left(\left(x^3-1\right)^4+4\left(x^3-1\right)^3+4x^3+6\left(x^3-1\right)^2-3\right)}+\frac{5\left(x^3-1\right)^{\frac{7}{3}}+13\left(x^3-1\right)^{\frac{4}{3}}-10\left(x^3-1\right)^{\frac{1}{3}}}{162\left(\left(x^3-1\right)^3+3x^3+3\left(x^3-1\right)^2-2\right)}-\frac{25}{1458}\log\left(\left(x^3-1\right)^{\frac{2}{3}}-\left(x^3-1\right)^{\frac{1}{3}}+1\right)+\frac{25}{729}\log\left(\left(x^3-1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(1/3)*(x^3+1)/x^13,x, algorithm="maxima")
```

```
[Out] 25/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) + 1/972*(20*(x^3
 - 1)^(10/3) + 72*(x^3 - 1)^(7/3) + 93*(x^3 - 1)^(4/3) - 40*(x^3 - 1)^(1/3)
)/((x^3 - 1)^4 + 4*(x^3 - 1)^3 + 4*x^3 + 6*(x^3 - 1)^2 - 3) + 1/162*(5*(x^3
 - 1)^(7/3) + 13*(x^3 - 1)^(4/3) - 10*(x^3 - 1)^(1/3))/((x^3 - 1)^3 + 3*x^3
 + 3*(x^3 - 1)^2 - 2) - 25/1458*log((x^3 - 1)^(2/3) - (x^3 - 1)^(1/3) + 1)
+ 25/729*log((x^3 - 1)^(1/3) + 1)
```

mupad [B] time = 1.61, size = 265, normalized size = 2.43

$$\frac{5\ln\left(\frac{25\left(x^3-1\right)^{1/3}}{6561}+\frac{25}{6561}\right)}{243}+\frac{10\ln\left(\frac{100\left(x^3-1\right)^{1/3}}{59049}+\frac{100}{59049}\right)}{729}+\frac{\frac{31\left(x^3-1\right)^{4/3}}{324}-\frac{10\left(x^3-1\right)^{1/3}}{243}+\frac{2\left(x^3-1\right)^{7/3}}{27}+\frac{5\left(x^3-1\right)^{10/3}}{243}}{6\left(x^3-1\right)^2+4\left(x^3-1\right)^3+\left(x^3-1\right)^4+4x^3-3}+\frac{\frac{13\left(x^3-1\right)^{4/3}}{162}-\frac{5\left(x^3-1\right)^{1/3}}{81}+\frac{5\left(x^3-1\right)^{7/3}}{162}}{3\left(x^3-1\right)^2+\left(x^3-1\right)^3+3x^3-2}-\ln\left(\frac{5}{54}-\frac{5\left(x^3-1\right)^{1/3}}{27}+\frac{\sqrt{3}\,5{}\mathrm{i}}{54}\right)\left(\frac{5}{486}+\frac{\sqrt{3}\,5{}\mathrm{i}}{486}\right)+\ln\left(\frac{5\left(x^3-1\right)^{1/3}}{27}-\frac{5}{54}+\frac{\sqrt{3}\,5{}\mathrm{i}}{54}\right)\left(-\frac{5}{486}+\frac{\sqrt{3}\,5{}\mathrm{i}}{486}\right)-\ln\left(\frac{5}{81}-\frac{10\left(x^3-1\right)^{1/3}}{81}+\frac{\sqrt{3}\,5{}\mathrm{i}}{81}\right)\left(\frac{5}{729}+\frac{\sqrt{3}\,5{}\mathrm{i}}{729}\right)+\ln\left(\frac{10\left(x^3-1\right)^{1/3}}{81}-\frac{5}{81}+\frac{\sqrt{3}\,5{}\mathrm{i}}{81}\right)\left(-\frac{5}{729}+\frac{\sqrt{3}\,5{}\mathrm{i}}{729}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(1/3)*(x^3 + 1))/x^13,x)
```

```
[Out] (5*log((25*(x^3 - 1)^(1/3))/6561 + 25/6561))/243 + (10*log((100*(x^3 - 1)^(
1/3))/59049 + 100/59049))/729 + ((31*(x^3 - 1)^(4/3))/324 - (10*(x^3 - 1)^(
1/3))/243 + (2*(x^3 - 1)^(7/3))/27 + (5*(x^3 - 1)^(10/3))/243)/(6*(x^3 - 1)
^2 + 4*(x^3 - 1)^3 + (x^3 - 1)^4 + 4*x^3 - 3) + ((13*(x^3 - 1)^(4/3))/162 -
 (5*(x^3 - 1)^(1/3))/81 + (5*(x^3 - 1)^(7/3))/162)/(3*(x^3 - 1)^2 + (x^3 -
1)^3 + 3*x^3 - 2) - log((3^(1/2)*5i)/54 - (5*(x^3 - 1)^(1/3))/27 + 5/54)*((
3^(1/2)*5i)/486 + 5/486) + log((3^(1/2)*5i)/54 + (5*(x^3 - 1)^(1/3))/27 - 5
/54)*((3^(1/2)*5i)/486 - 5/486) - log((3^(1/2)*5i)/81 - (10*(x^3 - 1)^(1/3)
)/81 + 5/81)*((3^(1/2)*5i)/729 + 5/729) + log((3^(1/2)*5i)/81 + (10*(x^3 -
1)^(1/3))/81 - 5/81)*((3^(1/2)*5i)/729 - 5/729)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(1/3)*(x**3+1)/x**13,x)
```

```
[Out] Timed out
```

3.1363 $\int \frac{(-b+x)(a-3b+2x)}{(-a+x)((-a+x)(-b+x))^{3/4}\left(b-a^3d-\left(1-3a^2d\right)x-3adx^2+dx^3\right)}\,dx$

Optimal. Leaf size=109

$$-2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x(-a-b)+ab+x^2\right)^{3/4}}{b-x}\right)+2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{x(-a-b)+ab+x^2}}{\sqrt[4]{d}\,(a-x)}\right)-\frac{4(b-x)}{\left(ab-ax-bx+x^2\right)^{3/4}}$$

Rubi [F] time = 10.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-b+x)(a-3b+2x)}{(-a+x)((-a+x)(-b+x))^{3/4}\left(b-a^3d-\left(1-3a^2d\right)x-3adx^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-b + x)*(a - 3*b + 2*x))/((-a + x)*((-a + x)*(-b + x))^(3/4)*(b - a^3
*d - (1 - 3*a^2*d)*x - 3*a*d*x^2 + d*x^3)),x]
```

```
[Out] (-8*a*(b - x))/(3*(a - b)*((a - x)*(b - x))^(3/4)) - (4*(a - 3*b)*(b - x))/
(3*(a - b)*((a - x)*(b - x))^(3/4)) + (8*a*(1 - (a - b)/(a - x))^(3/4)*(-a
+ x)^(3/2)*EllipticF[ArcCot[Sqrt[-a + x]/Sqrt[a - b]]/2, 2])/(3*(a - b)^(3/
2)*((a - x)*(b - x))^(3/4)) + (4*(a - 3*b)*(1 - (a - b)/(a - x))^(3/4)*(-a
+ x)^(3/2)*EllipticF[ArcCot[Sqrt[-a + x]/Sqrt[a - b]]/2, 2])/(3*(a - b)^(3/
2)*((a - x)*(b - x))^(3/4)) + (8*b*(-a + x)^(3/4)*(-b + x)^(3/4)*Defer[Subs
t][Defer[Int][(a - b + x^4)^(1/4)/(a*(1 - b/a) + x^4 - d*x^12), x], x, (-a
+ x)^(1/4)])/((a - b)*((a - x)*(b - x))^(3/4)) - (4*(a - 3*b)*d*(-a + x)^(3
/4)*(-b + x)^(3/4)*Defer[Subst][Defer[Int][(x^8*(a - b + x^4)^(1/4))/(a*(1
- b/a) + x^4 - d*x^12), x], x, (-a + x)^(1/4)])/((a - b)*((a - x)*(b - x))^
(3/4)) - (4*(a - 3*b)*(-a + x)^(3/4)*(-b + x)^(3/4)*Defer[Subst][Defer[Int]
[(a - b + x^4)^(1/4)/(-(a*(1 - b/a)) - x^4 + d*x^12), x], x, (-a + x)^(1/4)
])/((a - b)*((a - x)*(b - x))^(3/4)) + (8*a*d*(-a + x)^(3/4)*(-b + x)^(3/4)
*Defer[Subst][Defer[Int][(x^8*(a - b + x^4)^(1/4))/(-(a*(1 - b/a)) - x^4 +
d*x^12), x], x, (-a + x)^(1/4)])/((a - b)*((a - x)*(b - x))^(3/4))
```

Rubi steps

$$\int \frac{(-b+x)(a-3b+2x)}{(-a+x)((-a+x)(-b+x))^{3/4}\left(b-a^3d-\left(1-3a^2d\right)x-3adx^2+dx^3\right)}dx = \frac{\left((-a+x)^{3/4}(-b+x)^{3/4}\right)\int \frac{}{(-a+x)^{7/4}(\ldots}}{((-a+x)(-b+\ldots}$$

$$= \frac{\left((-a+x)^{3/4}(-b+x)^{3/4}\right)\int \left(\frac{}{(-a+x)^{7/4}\ldots}\right.}{\ldots}$$

$$= \frac{\left(2(-a+x)^{3/4}(-b+x)^{3/4}\right)\int \frac{}{(-a+x)^{7/4}\ldots}}{((-a+x)(-b\ldots}$$

$$= \frac{\left(8(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\left(\int \ldots\right.}{((-a+x)(\ldots}$$

$$= \frac{\left(8(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\left(\int \ldots\right.}{((-a+x)(\ldots}$$

$$= \frac{\left(8(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\left(\int \ldots\right.}{((\ldots}$$

$$= \frac{\left(8(-a+x)^{3/4}(-b+x)^{3/4}\right)\text{Subst}\left(\int \ldots\right.}{(a-b)((-a+x\ldots}$$

$$= -\frac{8a(b-x)}{3(a-b)((a-x)(b-x))^{3/4}} - \frac{}{3(a-\ldots}$$

$$= -\frac{8a(b-x)}{3(a-b)((a-x)(b-x))^{3/4}} - \frac{}{3(a-\ldots}$$

$$= -\frac{8a(b-x)}{3(a-b)((a-x)(b-x))^{3/4}} - \frac{}{3(a-\ldots}$$

$$= -\frac{8a(b-x)}{3(a-b)((a-x)(b-x))^{3/4}} - \frac{}{3(a-\ldots}$$

$$= -\frac{8a(b-x)}{3(a-b)((a-x)(b-x))^{3/4}} - \frac{}{3(a-\ldots}$$

Mathematica [F] time = 3.83, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)(a-3b+2x)}{(-a+x)((-a+x)(-b+x))^{3/4}\left(b-a^3d-\left(1-3a^2d\right)x-3adx^2+dx^3\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-b + x)*(a - 3*b + 2*x))/((-a + x)*((-a + x)*(-b + x))^(3/4)*(b
 - a^3*d - (1 - 3*a^2*d)*x - 3*a*d*x^2 + d*x^3)),x]
```

```
[Out] Integrate[((-b + x)*(a - 3*b + 2*x))/((-a + x)*((-a + x)*(-b + x))^(3/4)*(b
 - a^3*d - (1 - 3*a^2*d)*x - 3*a*d*x^2 + d*x^3)), x]
```

IntegrateAlgebraic [A] time = 7.28, size = 109, normalized size = 1.00

$$-2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x(-a-b)+ab+x^2\right)^{3/4}}{b-x}\right)+2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{x(-a-b)+ab+x^2}}{\sqrt[4]{d}(a-x)}\right)-\frac{4(b-x)}{\left(ab-ax-bx+x^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + x)*(a - 3*b + 2*x))/((-a + x)*((-a + x)*(-b + x))
^(3/4)*(b - a^3*d - (1 - 3*a^2*d)*x - 3*a*d*x^2 + d*x^3)),x]
```

```
[Out] (-4*(b - x))/(a*b - a*x - b*x + x^2)^(3/4) - 2*d^(1/4)*ArcTan[(d^(1/4)*(a*b
 + (-a - b)*x + x^2)^(3/4))/(b - x)] + 2*d^(1/4)*ArcTanh[(a*b + (-a - b)*x
+ x^2)^(1/4)/(d^(1/4)*(a - x))]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(a-3*b+2*x)/(-a+x)/((-a+x)*(-b+x))^(3/4)/(b-a^3*d-(-3*a^2*
d+1)*x-3*a*d*x^2+d*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(a-3\,b+2\,x)(b-x)}{\left(a^3d+3\,adx^2-dx^3-\left(3\,a^2d-1\right)x-b\right)\left((a-x)(b-x)\right)^{\frac{3}{4}}(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(a-3*b+2*x)/(-a+x)/((-a+x)*(-b+x))^(3/4)/(b-a^3*d-(-3*a^2*
d+1)*x-3*a*d*x^2+d*x^3),x, algorithm="giac")
```

```
[Out] integrate(-(a - 3*b + 2*x)*(b - x)/((a^3*d + 3*a*d*x^2 - d*x^3 - (3*a^2*d -
 1)*x - b)*((a - x)*(b - x))^(3/4)*(a - x)), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)(a-3b+2x)}{(-a+x)\left((-a+x)(-b+x)\right)^{\frac{3}{4}}\left(b-a^3d-\left(-3a^2d+1\right)x-3ad\,x^2+d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b+x)*(a-3*b+2*x)/(-a+x)/((-a+x)*(-b+x))^(3/4)/(b-a^3*d-(-3*a^2*d+1)*x
-3*a*d*x^2+d*x^3),x)
```

```
[Out] int((-b+x)*(a-3*b+2*x)/(-a+x)/((-a+x)*(-b+x))^(3/4)/(b-a^3*d-(-3*a^2*d+1)*x
-3*a*d*x^2+d*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(a-3\,b+2\,x)(b-x)}{\left(a^3d+3\,adx^2-dx^3-\left(3\,a^2d-1\right)x-b\right)\left((a-x)(b-x)\right)^{\frac{3}{4}}(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(a-3*b+2*x)/(-a+x)/((-a+x)*(-b+x))^(3/4)/(b-a^3*d-(-3*a^2*
d+1)*x-3*a*d*x^2+d*x^3),x, algorithm="maxima")
```

```
[Out] -integrate((a - 3*b + 2*x)*(b - x)/((a^3*d + 3*a*d*x^2 - d*x^3 - (3*a^2*d -
 1)*x - b)*((a - x)*(b - x))^(3/4)*(a - x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b-x)\,(a-3\,b+2\,x)}{((a-x)\,(b-x))^{3/4}\,(a-x)\,\left(b-a^3\,d+d\,x^3+x\,\left(3\,a^2\,d-1\right)-3\,a\,d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b - x)*(a - 3*b + 2*x))/(((a - x)*(b - x))^(3/4)*(a - x)*(b - a^3*d +
 d*x^3 + x*(3*a^2*d - 1) - 3*a*d*x^2)),x)
```

```
[Out] int(((b - x)*(a - 3*b + 2*x))/(((a - x)*(b - x))^(3/4)*(a - x)*(b - a^3*d +
 d*x^3 + x*(3*a^2*d - 1) - 3*a*d*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(a-3*b+2*x)/(-a+x)/((-a+x)*(-b+x))**(3/4)/(b-a**3*d-(-3*a*
*2*d+1)*x-3*a*d*x**2+d*x**3),x)
```

```
[Out] Timed out
```

3.1364 $\int x^8 \left(-b+ax^4\right)^{3/4} dx$

Optimal. Leaf size=109

$$-\frac{5b^3\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{256a^{9/4}}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{256a^{9/4}}+\frac{\left(ax^4-b\right)^{3/4}\left(32a^2x^9-12abx^5-15b^2x\right)}{384a^2}$$

Rubi [A] time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {279, 321, 240, 212, 206, 203}

$$-\frac{5b^3\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{256a^{9/4}}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{256a^{9/4}}-\frac{5b^2x\left(ax^4-b\right)^{3/4}}{128a^2}+\frac{1}{12}x^9\left(ax^4-b\right)^{3/4}-\frac{bx^5\left(ax^4-b\right)^{3/4}}{32a}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(-b + a*x^4)^(3/4),x]
```

```
[Out] (-5*b^2*x*(-b + a*x^4)^(3/4))/(128*a^2) - (b*x^5*(-b + a*x^4)^(3/4))/(32*a)
 + (x^9*(-b + a*x^4)^(3/4))/12 - (5*b^3*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/
4)])/(256*a^(9/4)) - (5*b^3*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(256*a
^(9/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^8\left(-b+ax^4\right)^{3/4}\,dx=\frac{1}{12}x^9\left(-b+ax^4\right)^{3/4}-\frac{1}{4}b\int\frac{x^8}{\sqrt[4]{-b+ax^4}}\,dx$$
$$=-\frac{bx^5\left(-b+ax^4\right)^{3/4}}{32a}+\frac{1}{12}x^9\left(-b+ax^4\right)^{3/4}-\frac{\left(5b^2\right)\int\frac{x^4}{\sqrt[4]{-b+ax^4}}\,dx}{32a}$$
$$=-\frac{5b^2x\left(-b+ax^4\right)^{3/4}}{128a^2}-\frac{bx^5\left(-b+ax^4\right)^{3/4}}{32a}+\frac{1}{12}x^9\left(-b+ax^4\right)^{3/4}-\frac{\left(5b^3\right)\int\frac{1}{\sqrt[4]{-b+ax^4}}\,dx}{128a^2}$$
$$=-\frac{5b^2x\left(-b+ax^4\right)^{3/4}}{128a^2}-\frac{bx^5\left(-b+ax^4\right)^{3/4}}{32a}+\frac{1}{12}x^9\left(-b+ax^4\right)^{3/4}-\frac{\left(5b^3\right)\text{Subst}\left(\int\frac{1}{1-ax^4}\,d\right.}{128a^2}$$
$$=-\frac{5b^2x\left(-b+ax^4\right)^{3/4}}{128a^2}-\frac{bx^5\left(-b+ax^4\right)^{3/4}}{32a}+\frac{1}{12}x^9\left(-b+ax^4\right)^{3/4}-\frac{\left(5b^3\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{a}\,x^2}\right.}{256a^2}$$
$$=-\frac{5b^2x\left(-b+ax^4\right)^{3/4}}{128a^2}-\frac{bx^5\left(-b+ax^4\right)^{3/4}}{32a}+\frac{1}{12}x^9\left(-b+ax^4\right)^{3/4}-\frac{5b^3\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{256a^{9/4}}-$$

Mathematica [C] time = 0.06, size = 97, normalized size = 0.89

$$\frac{x\left(ax^4-b\right)^{3/4}\left(\left(1-\frac{ax^4}{b}\right)^{3/4}\left(8a^2x^8-3abx^4-5b^2\right)+5b^2\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};\frac{ax^4}{b}\right)\right)}{96a^2\left(1-\frac{ax^4}{b}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(-b + a*x^4)^(3/4),x]
```

```
[Out] (x*(-b + a*x^4)^(3/4)*((1 - (a*x^4)/b)^(3/4)*(-5*b^2 - 3*a*b*x^4 + 8*a^2*x^
8) + 5*b^2*Hypergeometric2F1[-3/4, 1/4, 5/4, (a*x^4)/b]))/(96*a^2*(1 - (a*x
^4)/b)^(3/4))
```

IntegrateAlgebraic [A] time = 0.43, size = 109, normalized size = 1.00

$$-\frac{5b^3\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{256a^{9/4}}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{256a^{9/4}}+\frac{\left(ax^4-b\right)^{3/4}\left(32a^2x^9-12abx^5-15b^2x\right)}{384a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8*(-b + a*x^4)^(3/4),x]
```

```
[Out] ((-b + a*x^4)^(3/4)*(-15*b^2*x - 12*a*b*x^5 + 32*a^2*x^9))/(384*a^2) - (5*b
^3*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(256*a^(9/4)) - (5*b^3*ArcTanh[(
a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(256*a^(9/4))
```

fricas [B] time = 0.46, size = 249, normalized size = 2.28

$$-\frac{60\left(\frac{b^{12}}{a^9}\right)^{\frac{1}{4}}a^2\arctan\left(-\frac{\left(ax^4-b\right)^{\frac{1}{4}}\left(\frac{b^{12}}{a^9}\right)^{\frac{1}{4}}a^2b^9-\left(\frac{b^{12}}{a^9}\right)^{\frac{1}{4}}a^2x\sqrt{\frac{\sqrt{\frac{b^{12}}{a^9}}a^5b^{12}x^2+\sqrt{ax^4-b}\,b^{18}}{x^2}}}{b^{12}x}\right)+15\left(\frac{b^{12}}{a^9}\right)^{\frac{1}{4}}a^2\log\left(\frac{125\left(\left(ax^4-b\right)^{\frac{1}{4}}b^9+\left(\frac{b^{12}}{a^9}\right)^{\frac{3}{4}}a^7x\right)}{x}\right)-15\left(\frac{b^{12}}{a^9}\right)^{\frac{1}{4}}a^2\log\left(\frac{125\left(\left(ax^4-b\right)^{\frac{1}{4}}b^9-\left(\frac{b^{12}}{a^9}\right)^{\frac{3}{4}}a^7x\right)}{x}\right)-4\left(32a^2x^9-12abx^5-15b^2x\right)\left(ax^4-b\right)^{\frac{3}{4}}}{1536a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a*x^4-b)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/1536*(60*(b^12/a^9)^(1/4)*a^2*arctan(-((a*x^4 - b)^(1/4)*(b^12/a^9)^(1/4
)*a^2*b^9 - (b^12/a^9)^(1/4)*a^2*x*sqrt((sqrt(b^12/a^9)*a^5*b^12*x^2 + sqrt
(a*x^4 - b)*b^18)/x^2))/(b^12*x)) + 15*(b^12/a^9)^(1/4)*a^2*log(125*((a*x^4
 - b)^(1/4)*b^9 + (b^12/a^9)^(3/4)*a^7*x)/x) - 15*(b^12/a^9)^(1/4)*a^2*log(
125*((a*x^4 - b)^(1/4)*b^9 - (b^12/a^9)^(3/4)*a^7*x)/x) - 4*(32*a^2*x^9 - 1
2*a*b*x^5 - 15*b^2*x)*(a*x^4 - b)^(3/4))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^4-b\right)^{\frac{3}{4}} x^8\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a*x^4-b)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^4 - b)^(3/4)*x^8, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int x^8 \left(a\,x^4-b\right)^{\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a*x^4-b)^(3/4),x)
```

```
[Out] int(x^8*(a*x^4-b)^(3/4),x)
```

maxima [B] time = 0.41, size = 207, normalized size = 1.90

$$\frac{5\,b^3\left(\frac{2\arctan\left(\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{\left(ax^4-b\right)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right)}{512\,a^2}+\frac{\frac{5\left(ax^4-b\right)^{\frac{3}{4}}a^2b^3}{x^3}+\frac{42\left(ax^4-b\right)^{\frac{7}{4}}ab^3}{x^7}-\frac{15\left(ax^4-b\right)^{\frac{11}{4}}b^3}{x^{11}}}{384\left(a^5-\frac{3\left(ax^4-b\right)a^4}{x^4}+\frac{3\left(ax^4-b\right)^2a^3}{x^8}-\frac{\left(ax^4-b\right)^3a^2}{x^{12}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a*x^4-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 5/512*b^3*(2*arctan((a*x^4 - b)^(1/4)/(a^(1/4)*x))/a^(1/4) + log(-(a^(1/4)
- (a*x^4 - b)^(1/4)/x)/(a^(1/4) + (a*x^4 - b)^(1/4)/x))/a^(1/4))/a^2 + 1/38
4*(5*(a*x^4 - b)^(3/4)*a^2*b^3/x^3 + 42*(a*x^4 - b)^(7/4)*a*b^3/x^7 - 15*(a
*x^4 - b)^(11/4)*b^3/x^11)/(a^5 - 3*(a*x^4 - b)*a^4/x^4 + 3*(a*x^4 - b)^2*a
^3/x^8 - (a*x^4 - b)^3*a^2/x^12)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^8\,{\left(a\,x^4-b\right)}^{3/4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a*x^4 - b)^(3/4),x)
```

```
[Out] int(x^8*(a*x^4 - b)^(3/4), x)
```

sympy [C] time = 1.74, size = 42, normalized size = 0.39

$$-\frac{b^{\frac{3}{4}}x^{9}e^{-\frac{i\pi}{4}}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{ax^{4}}{b}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a*x**4-b)**(3/4),x)
```

```
[Out] -b**(3/4)*x**9*exp(-I*pi/4)*gamma(9/4)*hyper((-3/4, 9/4), (13/4,), a*x**4/b
)/(4*gamma(13/4))
```

3.1365 $\int \frac{1}{(b+ax^3)\sqrt[4]{-bx+ax^4}}\,dx$

Optimal. Leaf size=109

$$\frac{2^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3\sqrt[4]{a}\,b}+\frac{2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3\sqrt[4]{a}\,b}$$

Rubi [A] time = 0.21, antiderivative size = 159, normalized size of antiderivative = 1.46, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2056, 466, 465, 377, 212, 206, 203}

$$\frac{2^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3\sqrt[4]{a}\,b\sqrt[4]{ax^4-bx}}+\frac{2^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3-b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3\sqrt[4]{a}\,b\sqrt[4]{ax^4-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b + a*x^3)*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (2^(3/4)*x^(1/4)*(-b + a*x^3)^(1/4)*ArcTan[(2^(1/4)*a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)])/(3*a^(1/4)*b*(-(b*x) + a*x^4)^(1/4)) + (2^(3/4)*x^(1/4)*(-b + a*x^3)^(1/4)*ArcTanh[(2^(1/4)*a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)])/(3*a^(1/4)*b*(-(b*x) + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{\left(b+ax^3\right)\sqrt[4]{-bx+ax^4}}\,dx=\frac{\left(\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\int \frac{1}{\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\left(b+ax^3\right)}\,dx}{\sqrt[4]{-bx+ax^4}}$$

$$=\frac{\left(4\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{-b+ax^{12}}\left(b+ax^{12}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^4}}$$

$$=\frac{\left(4\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-b+ax^4}\left(b+ax^4\right)}\,dx,x,x^{3/4}\right)}{3\sqrt[4]{-bx+ax^4}}$$

$$=\frac{\left(4\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{b-2abx^4}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{-b+ax^3}}\right)}{3\sqrt[4]{-bx+ax^4}}$$

$$=\frac{\left(2\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,\sqrt{a}\,x^2}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{-b+ax^3}}\right)}{3b\sqrt[4]{-bx+ax^4}}+\frac{\left(2\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\text{Subs}}{3b\sqrt[4]{}}$$

$$=\frac{2^{3/4}\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{-b+ax^3}}\right)}{3\sqrt[4]{a}\,b\sqrt[4]{-bx+ax^4}}+\frac{2^{3/4}\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{-b+ax^3}}\right)}{3\sqrt[4]{a}\,b\sqrt[4]{-bx+ax^4}}$$

Mathematica [C] time = 0.04, size = 76, normalized size = 0.70

$$\frac{4x\sqrt[4]{1-\frac{ax^3}{b}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{2ax^3}{ax^3+b}\right)}{3b\sqrt[4]{\frac{ax^3}{b}+1}\,\sqrt[4]{ax^4-bx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((b + a*x^3)*(-(b*x) + a*x^4)^(1/4)),x]
[Out] (4*x*(1 - (a*x^3)/b)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (2*a*x^3)/(b +
a*x^3)])/(3*b*(1 + (a*x^3)/b)^(1/4)*(-(b*x) + a*x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.40, size = 109, normalized size = 1.00

$$\frac{2^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3\sqrt[4]{a}\,b}+\frac{2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3\sqrt[4]{a}\,b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((b + a*x^3)*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (2^(3/4)*ArcTan[(2^(1/4)*a^(1/4)*(-(b*x) + a*x^4)^(3/4))/(-b + a*x^3)])/(3*
a^(1/4)*b) + (2^(3/4)*ArcTanh[(2^(1/4)*a^(1/4)*(-(b*x) + a*x^4)^(3/4))/(-b
+ a*x^3)])/(3*a^(1/4)*b)
```

fricas [B] time = 106.37, size = 430, normalized size = 3.94

$$-\frac{2}{3}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(2\left(\frac{1}{2}\right)^{\frac{3}{4}}(ax^4-bx)^{\frac{3}{4}}ab^3\left(\frac{1}{ab^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{2}\right)^{\frac{1}{4}}(ax^4-bx)^{\frac{1}{4}}abx^2\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}+\left(2\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{ax^4-bx}\,abx\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}+\left(\frac{1}{2}\right)^{\frac{3}{4}}(3a^2b^3x^3-ab^4)\left(\frac{1}{ab^4}\right)^{\frac{3}{4}}\right)\sqrt{\sqrt{\frac{1}{2}}b^2\sqrt{\frac{1}{ab^4}}}\right)}{ax^3+b}\right)+\frac{1}{6}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{ax^4-bx}\,ab^3x\left(\frac{1}{ab^4}\right)^{\frac{3}{4}}+4\sqrt{\frac{1}{2}}(ax^4-bx)^{\frac{1}{4}}ab^2x^2\sqrt{\frac{1}{ab^4}}+\left(\frac{1}{2}\right)^{\frac{1}{4}}(3abx^3-b^2)\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}+2(ax^4-bx)^{\frac{3}{4}}}{ax^3+b}\right)-\frac{1}{6}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{ax^4-bx}\,ab^3x\left(\frac{1}{ab^4}\right)^{\frac{3}{4}}-4\sqrt{\frac{1}{2}}(ax^4-bx)^{\frac{1}{4}}ab^2x^2\sqrt{\frac{1}{ab^4}}+\left(\frac{1}{2}\right)^{\frac{1}{4}}(3abx^3-b^2)\left(\frac{1}{ab^4}\right)^{\frac{1}{4}}-2(ax^4-bx)^{\frac{3}{4}}}{ax^3+b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3+b)/(a*x^4-b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] -2/3*(1/2)^(1/4)*(1/(a*b^4))^(1/4)*arctan(2*(2*(1/2)^(3/4)*(a*x^4 - b*x)^(3
/4)*a*b^3*(1/(a*b^4))^(3/4) + 2*(1/2)^(1/4)*(a*x^4 - b*x)^(1/4)*a*b*x^2*(1/
(a*b^4))^(1/4) + (2*(1/2)^(1/4)*sqrt(a*x^4 - b*x)*a*b*x*(1/(a*b^4))^(1/4) +
 (1/2)^(3/4)*(3*a^2*b^3*x^3 - a*b^4)*(1/(a*b^4))^(3/4))*sqrt(sqrt(1/2)*b^2*
sqrt(1/(a*b^4))))/(a*x^3 + b)) + 1/6*(1/2)^(1/4)*(1/(a*b^4))^(1/4)*log((4*(
1/2)^(3/4)*sqrt(a*x^4 - b*x)*a*b^3*x*(1/(a*b^4))^(3/4) + 4*sqrt(1/2)*(a*x^4
 - b*x)^(1/4)*a*b^2*x^2*sqrt(1/(a*b^4)) + (1/2)^(1/4)*(3*a*b*x^3 - b^2)*(1/
(a*b^4))^(1/4) + 2*(a*x^4 - b*x)^(3/4))/(a*x^3 + b)) - 1/6*(1/2)^(1/4)*(1/(
a*b^4))^(1/4)*log(-(4*(1/2)^(3/4)*sqrt(a*x^4 - b*x)*a*b^3*x*(1/(a*b^4))^(3/
4) - 4*sqrt(1/2)*(a*x^4 - b*x)^(1/4)*a*b^2*x^2*sqrt(1/(a*b^4)) + (1/2)^(1/4
)*(3*a*b*x^3 - b^2)*(1/(a*b^4))^(1/4) - 2*(a*x^4 - b*x)^(3/4))/(a*x^3 + b))
```

giac [B] time = 0.24, size = 209, normalized size = 1.92

$$\frac{2^{\frac{1}{4}}(-a)^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{3ab}+\frac{2^{\frac{1}{4}}(-a)^{\frac{3}{4}}\arctan\left(-\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{3ab}+\frac{2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a-\frac{b}{x^3}}\right)}{6(-a)^{\frac{1}{4}}b}+\frac{2^{\frac{1}{4}}(-a)^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a-\frac{b}{x^3}}\right)}{6ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3+b)/(a*x^4-b*x)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3*2^(1/4)*(-a)^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4)*(-a)^(1/4) + 2*(a - b/x^
3)^(1/4))/(-a)^(1/4))/(a*b) + 1/3*2^(1/4)*(-a)^(3/4)*arctan(-1/2*2^(1/4)*(2
^(3/4)*(-a)^(1/4) - 2*(a - b/x^3)^(1/4))/(-a)^(1/4))/(a*b) + 1/6*2^(1/4)*lo
g(2^(3/4)*(-a)^(1/4)*(a - b/x^3)^(1/4) + sqrt(2)*sqrt(-a) + sqrt(a - b/x^3)
)/((-a)^(1/4)*b) + 1/6*2^(1/4)*(-a)^(3/4)*log(-2^(3/4)*(-a)^(1/4)*(a - b/x^
3)^(1/4) + sqrt(2)*sqrt(-a) + sqrt(a - b/x^3))/(a*b)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,x^{3}+b\right)\left(a\,x^{4}-bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3+b)/(a*x^4-b*x)^(1/4),x)
```

```
[Out] int(1/(a*x^3+b)/(a*x^4-b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^{4}-bx\right)^{\frac{1}{4}}\left(ax^{3}+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3+b)/(a*x^4-b*x)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((a*x^4 - b*x)^(1/4)*(a*x^3 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\,x^4-b\,x\right)^{1/4}\,\left(a\,x^3+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x^4 - b*x)^(1/4)*(b + a*x^3)),x)
[Out] int(1/((a*x^4 - b*x)^(1/4)*(b + a*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x\left(ax^3-b\right)}\,\left(ax^3+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**3+b)/(a*x**4-b*x)**(1/4),x)
[Out] Integral(1/((x*(a*x**3 - b))**(1/4)*(a*x**3 + b)), x)
```

3.1366 $\int \frac{\sqrt[3]{-1+x^6}\left(1+x^6\right)}{x^3}\,dx$

Optimal. Leaf size=109

$$\frac{\sqrt[3]{x^6-1}\left(x^6-3\right)}{6x^2}-\frac{1}{9}\log\left(\sqrt[3]{x^6-1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{3\sqrt{3}}+\frac{1}{18}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)$$

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {453, 275, 279, 331, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{3}\sqrt[3]{x^6-1}\,x^4+\frac{\left(x^6-1\right)^{4/3}}{2x^2}-\frac{1}{9}\log\left(1-\frac{x^2}{\sqrt[3]{x^6-1}}\right)-\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6-1}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{18}\log\left(\frac{x^4}{\left(x^6-1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6-1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^6)^(1/3)*(1 + x^6))/x^3,x]
```

```
[Out] -1/3*(x^4*(-1 + x^6)^(1/3)) + (-1 + x^6)^(4/3)/(2*x^2) - ArcTan[(1 + (2*x^2
)/(-1 + x^6)^(1/3))/Sqrt[3]]/(3*Sqrt[3]) - Log[1 - x^2/(-1 + x^6)^(1/3)]/9
+ Log[1 + x^4/(-1 + x^6)^(2/3) + x^2/(-1 + x^6)^(1/3)]/18
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[3]{-1+x^6}\left(1+x^6\right)}{x^3}\,dx = \frac{\left(-1+x^6\right)^{4/3}}{2x^2} - 2\int x^3\sqrt[3]{-1+x^6}\,dx$$
$$= \frac{\left(-1+x^6\right)^{4/3}}{2x^2} - \text{Subst}\left(\int x\sqrt[3]{-1+x^3}\,dx, x, x^2\right)$$
$$= -\frac{1}{3}x^4\sqrt[3]{-1+x^6} + \frac{\left(-1+x^6\right)^{4/3}}{2x^2} + \frac{1}{3}\text{Subst}\left(\int \frac{x}{\left(-1+x^3\right)^{2/3}}\,dx, x, x^2\right)$$
$$= -\frac{1}{3}x^4\sqrt[3]{-1+x^6} + \frac{\left(-1+x^6\right)^{4/3}}{2x^2} + \frac{1}{3}\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{x^2}{\sqrt[3]{-1+x^6}}\right)$$
$$= -\frac{1}{3}x^4\sqrt[3]{-1+x^6} + \frac{\left(-1+x^6\right)^{4/3}}{2x^2} + \frac{1}{9}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{x^2}{\sqrt[3]{-1+x^6}}\right) - \frac{1}{9}\text{Subst}\Bigg($$
$$= -\frac{1}{3}x^4\sqrt[3]{-1+x^6} + \frac{\left(-1+x^6\right)^{4/3}}{2x^2} - \frac{1}{9}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right) + \frac{1}{18}\text{Subst}\left(\int \frac{1+2x}{1+x+x\right.$$
$$= -\frac{1}{3}x^4\sqrt[3]{-1+x^6} + \frac{\left(-1+x^6\right)^{4/3}}{2x^2} - \frac{1}{9}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right) + \frac{1}{18}\log\left(1+\frac{x^4}{\left(-1+x^6\right)^{2}}\right.$$
$$= -\frac{1}{3}x^4\sqrt[3]{-1+x^6} + \frac{\left(-1+x^6\right)^{4/3}}{2x^2} - \frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{-1+x^6}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{9}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right) + \frac{1}{18}$$

Mathematica [C] time = 0.03, size = 50, normalized size = 0.46

$$\frac{\sqrt[3]{x^6-1}\left(-\frac{x^6\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^6\right)}{\sqrt[3]{1-x^6}}+x^6-1\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^6)^(1/3)*(1 + x^6))/x^3,x]
```

```
[Out] ((-1 + x^6)^(1/3)*(-1 + x^6 - (x^6*Hypergeometric2F1[-1/3, 2/3, 5/3, x^6])/
(1 - x^6)^(1/3)))/(2*x^2)
```

IntegrateAlgebraic [A] time = 0.48, size = 109, normalized size = 1.00

$$\frac{\sqrt[3]{x^6-1}\left(x^6-3\right)}{6x^2} - \frac{1}{9}\log\left(\sqrt[3]{x^6-1}-x^2\right) - \frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{3\sqrt{3}} + \frac{1}{18}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^6)^(1/3)*(1 + x^6))/x^3,x]
```

```
[Out] ((-3 + x^6)*(-1 + x^6)^(1/3))/(6*x^2) - ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(-1 +
 x^6)^(1/3))]/(3*Sqrt[3]) - Log[-x^2 + (-1 + x^6)^(1/3)]/9 + Log[x^4 + x^2*
(-1 + x^6)^(1/3) + (-1 + x^6)^(2/3)]/18
```

fricas [A] time = 0.91, size = 113, normalized size = 1.04

$$-\frac{2\sqrt{3}x^2\arctan\left(-\frac{25382\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}x^4-13720\sqrt{3}\left(x^6-1\right)^{\frac{2}{3}}x^2+\sqrt{3}\left(5831x^6-7200\right)}{58653x^6-8000}\right)+x^2\log\left(-3\left(x^6-1\right)^{\frac{1}{3}}x^4+3\left(x^6-1\right)^{\frac{2}{3}}x^2+1\right)-3\left(x^6-1\right)^{\frac{1}{3}}\left(x^6-3\right)}{18x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(x^6+1)/x^3,x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*x^2*arctan(-(25382*sqrt(3)*(x^6 - 1)^(1/3)*x^4 - 13720*sqr
t(3)*(x^6 - 1)^(2/3)*x^2 + sqrt(3)*(5831*x^6 - 7200))/(58653*x^6 - 8000)) +
 x^2*log(-3*(x^6 - 1)^(1/3)*x^4 + 3*(x^6 - 1)^(2/3)*x^2 + 1) - 3*(x^6 - 1)^
(1/3)*(x^6 - 3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^6+1)(x^6-1)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(x^6+1)/x^3,x, algorithm="giac")
```

```
[Out] integrate((x^6 + 1)*(x^6 - 1)^(1/3)/x^3, x)
```

maple [C] time = 0.30, size = 56, normalized size = 0.51

$$\frac{x^{12}-4x^6+3}{6x^2\left(x^6-1\right)^{\frac{2}{3}}}+\frac{\left(-\mathrm{signum}\left(x^6-1\right)\right)^{\frac{2}{3}} x^4\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^6\right)}{6\mathrm{signum}\left(x^6-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/3)*(x^6+1)/x^3,x)
```

```
[Out] 1/6*(x^12-4*x^6+3)/x^2/(x^6-1)^(2/3)+1/6/signum(x^6-1)^(2/3)*(-signum(x^6-1
))^(2/3)*x^4*hypergeom([2/3,2/3],[5/3],x^6)
```

maxima [A] time = 0.42, size = 106, normalized size = 0.97

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)-\frac{\left(x^6-1\right)^{\frac{1}{3}}}{2x^2}-\frac{\left(x^6-1\right)^{\frac{1}{3}}}{6x^2\left(\frac{x^6-1}{x^6}-1\right)}+\frac{1}{18}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6-1\right)^{\frac{2}{3}}}{x^4}+1\right)-\frac{1}{9}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(x^6+1)/x^3,x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3)/x^2 + 1)) - 1/2*(x^6 - 1)
^(1/3)/x^2 - 1/6*(x^6 - 1)^(1/3)/(x^2*((x^6 - 1)/x^6 - 1)) + 1/18*log((x^6
- 1)^(1/3)/x^2 + (x^6 - 1)^(2/3)/x^4 + 1) - 1/9*log((x^6 - 1)^(1/3)/x^2 - 1
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^6-1)^{1/3}\,(x^6+1)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/3)*(x^6 + 1))/x^3,x)
```

```
[Out] int(((x^6 - 1)^(1/3)*(x^6 + 1))/x^3, x)
```

sympy [C] time = 3.25, size = 71, normalized size = 0.65

$$-\frac{x^4e^{-\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^6\right)}{6\Gamma\left(\frac{5}{3}\right)}+\frac{e^{\frac{i\pi}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|x^6\right)}{6x^2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/3)*(x**6+1)/x**3,x)
```

```
[Out] -x**4*exp(-2*I*pi/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), x**6)/(6*gamma(5
/3)) + exp(I*pi/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), x**6)/(6*x**2*ga
mma(2/3))
```

3.1367 $\int \frac{\sqrt[3]{-1+x^6}\left(-1+2x^6\right)}{x^9}\,dx$

Optimal. Leaf size=109

$$\frac{\sqrt[3]{x^6-1}\left(1-9x^6\right)}{8x^8}-\frac{1}{3}\log\left(\sqrt[3]{x^6-1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}x^2\right)$$

Rubi [A] time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {451, 275, 277, 331, 292, 31, 634, 618, 204, 628}

$$-\frac{\left(x^6-1\right)^{4/3}}{8x^8}-\frac{\sqrt[3]{x^6-1}}{x^2}-\frac{1}{3}\log\left(1-\frac{x^2}{\sqrt[3]{x^6-1}}\right)-\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6-1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(\frac{x^4}{\left(x^6-1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6-1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^6)^(1/3)*(-1 + 2*x^6))/x^9,x]
```

```
[Out] -((-1 + x^6)^(1/3)/x^2) - (-1 + x^6)^(4/3)/(8*x^8) - ArcTan[(1 + (2*x^2)/(-
1 + x^6)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 - x^2/(-1 + x^6)^(1/3)]/3 + Log[1
+ x^4/(-1 + x^6)^(2/3) + x^2/(-1 + x^6)^(1/3)]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[3]{-1+x^6}\left(-1+2x^6\right)}{x^9}\,dx = -\frac{\left(-1+x^6\right)^{4/3}}{8x^8} + 2\int \frac{\sqrt[3]{-1+x^6}}{x^3}\,dx$$
$$= -\frac{\left(-1+x^6\right)^{4/3}}{8x^8} + \text{Subst}\left(\int \frac{\sqrt[3]{-1+x^3}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt[3]{-1+x^6}}{x^2} - \frac{\left(-1+x^6\right)^{4/3}}{8x^8} + \text{Subst}\left(\int \frac{x}{\left(-1+x^3\right)^{2/3}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt[3]{-1+x^6}}{x^2} - \frac{\left(-1+x^6\right)^{4/3}}{8x^8} + \text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{x^2}{\sqrt[3]{-1+x^6}}\right)$$
$$= -\frac{\sqrt[3]{-1+x^6}}{x^2} - \frac{\left(-1+x^6\right)^{4/3}}{8x^8} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{x^2}{\sqrt[3]{-1+x^6}}\right) - \frac{1}{3}\text{Subst}\left(\int \frac{}{1}\right.$$
$$= -\frac{\sqrt[3]{-1+x^6}}{x^2} - \frac{\left(-1+x^6\right)^{4/3}}{8x^8} - \frac{1}{3}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right) + \frac{1}{6}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,\right.$$
$$= -\frac{\sqrt[3]{-1+x^6}}{x^2} - \frac{\left(-1+x^6\right)^{4/3}}{8x^8} - \frac{1}{3}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right) + \frac{1}{6}\log\left(1+\frac{x^4}{\left(-1+x^6\right)^{2/3}}+\right.$$
$$= -\frac{\sqrt[3]{-1+x^6}}{x^2} - \frac{\left(-1+x^6\right)^{4/3}}{8x^8} - \frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{-1+x^6}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right) + \frac{1}{6}\log\left(\right.$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.48

$$\frac{\sqrt[3]{x^6-1}\left(-\frac{8x^6\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};x^6\right)}{\sqrt[3]{1-x^6}}-x^6+1\right)}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^6)^(1/3)*(-1 + 2*x^6))/x^9,x]
```

```
[Out] ((-1 + x^6)^(1/3)*(1 - x^6 - (8*x^6*Hypergeometric2F1[-1/3, -1/3, 2/3, x^6]
)/(1 - x^6)^(1/3)))/(8*x^8)
```

IntegrateAlgebraic [A] time = 0.73, size = 109, normalized size = 1.00

$$\frac{\sqrt[3]{x^6-1}\left(1-9x^6\right)}{8x^8} - \frac{1}{3}\log\left(\sqrt[3]{x^6-1}-x^2\right) - \frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{\sqrt{3}} + \frac{1}{6}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^6)^(1/3)*(-1 + 2*x^6))/x^9,x]
```

```
[Out] ((1 - 9*x^6)*(-1 + x^6)^(1/3))/(8*x^8) - ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(-1
+ x^6)^(1/3))]/Sqrt[3] - Log[-x^2 + (-1 + x^6)^(1/3)]/3 + Log[x^4 + x^2*(-1
 + x^6)^(1/3) + (-1 + x^6)^(2/3)]/6
```

fricas [A] time = 0.63, size = 116, normalized size = 1.06

$$-\frac{8\sqrt{3}x^8\arctan\left(-\frac{25382\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}x^4-13720\sqrt{3}\left(x^6-1\right)^{\frac{2}{3}}x^2+\sqrt{3}\left(5831x^6-7200\right)}{58653x^6-8000}\right)+4x^8\log\left(-3\left(x^6-1\right)^{\frac{1}{3}}x^4+3\left(x^6-1\right)^{\frac{2}{3}}x^2+1\right)+3\left(9x^6-1\right)\left(x^6-1\right)^{\frac{1}{3}}}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(2*x^6-1)/x^9,x, algorithm="fricas")
```

```
[Out] -1/24*(8*sqrt(3)*x^8*arctan(-(25382*sqrt(3)*(x^6 - 1)^(1/3)*x^4 - 13720*sqr
t(3)*(x^6 - 1)^(2/3)*x^2 + sqrt(3)*(5831*x^6 - 7200))/(58653*x^6 - 8000)) +
 4*x^8*log(-3*(x^6 - 1)^(1/3)*x^4 + 3*(x^6 - 1)^(2/3)*x^2 + 1) + 3*(9*x^6 -
 1)*(x^6 - 1)^(1/3))/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{6}-1\right)\left(x^{6}-1\right)^{\frac{1}{3}}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(2*x^6-1)/x^9,x, algorithm="giac")
```

```
[Out] integrate((2*x^6 - 1)*(x^6 - 1)^(1/3)/x^9, x)
```

maple [C] time = 0.30, size = 58, normalized size = 0.53

$$-\frac{9x^{12}-10x^{6}+1}{8x^{8}\left(x^{6}-1\right)^{\frac{2}{3}}}+\frac{\left(-\mathrm{signum}\left(x^{6}-1\right)\right)^{\frac{2}{3}}x^{4}\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^{6}\right)}{2\mathrm{signum}\left(x^{6}-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/3)*(2*x^6-1)/x^9,x)
```

```
[Out] -1/8*(9*x^12-10*x^6+1)/x^8/(x^6-1)^(2/3)+1/2/signum(x^6-1)^(2/3)*(-signum(x
^6-1))^(2/3)*x^4*hypergeom([2/3,2/3],[5/3],x^6)
```

maxima [A] time = 0.42, size = 93, normalized size = 0.85

$$\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(\frac{2\left(x^{6}-1\right)^{\frac{1}{3}}}{x^{2}}+1\right)\right)-\frac{\left(x^{6}-1\right)^{\frac{1}{3}}}{x^{2}}-\frac{\left(x^{6}-1\right)^{\frac{4}{3}}}{8\,x^{8}}+\frac{1}{6}\,\log\left(\frac{\left(x^{6}-1\right)^{\frac{1}{3}}}{x^{2}}+\frac{\left(x^{6}-1\right)^{\frac{2}{3}}}{x^{4}}+1\right)-\frac{1}{3}\,\log\left(\frac{\left(x^{6}-1\right)^{\frac{1}{3}}}{x^{2}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(2*x^6-1)/x^9,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3)/x^2 + 1)) - (x^6 - 1)^(1/
3)/x^2 - 1/8*(x^6 - 1)^(4/3)/x^8 + 1/6*log((x^6 - 1)^(1/3)/x^2 + (x^6 - 1)^
(2/3)/x^4 + 1) - 1/3*log((x^6 - 1)^(1/3)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^{6}-1\right)^{1/3}\,\left(2\,x^{6}-1\right)}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/3)*(2*x^6 - 1))/x^9,x)
```

```
[Out] int(((x^6 - 1)^(1/3)*(2*x^6 - 1))/x^9, x)
```

sympy [C] time = 3.72, size = 167, normalized size = 1.53

$$-\begin{cases} \frac{\sqrt[3]{-1+\frac{1}{x^6}}\,e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{6\Gamma\left(-\frac{1}{3}\right)} - \frac{\sqrt[3]{-1+\frac{1}{x^6}}\,e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{6x^6\Gamma\left(-\frac{1}{3}\right)} & \text{for } \frac{1}{|x^6|} > 1 \\ -\frac{\sqrt[3]{1-\frac{1}{x^6}}\,\Gamma\left(-\frac{4}{3}\right)}{6\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt[3]{1-\frac{1}{x^6}}\,\Gamma\left(-\frac{4}{3}\right)}{6x^6\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise} \end{cases} + \frac{e^{\frac{i\pi}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle| x^6\right)}{3x^2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/3)*(2*x**6-1)/x**9,x)
```

```
[Out] -Piecewise(((-1 + x**(-6))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(6*gamma(-1/3)
) - (-1 + x**(-6))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(6*x**6*gamma(-1/3)),
1/Abs(x**6) > 1), (-(1 - 1/x**6)**(1/3)*gamma(-4/3)/(6*gamma(-1/3)) + (1 -
1/x**6)**(1/3)*gamma(-4/3)/(6*x**6*gamma(-1/3)), True)) + exp(I*pi/3)*gamma
(-1/3)*hyper((-1/3, -1/3), (2/3,), x**6)/(3*x**2*gamma(2/3))
```

3.1368 $\int \frac{\sqrt[4]{-1+x^4}\left(2-x^4+2x^8\right)}{x^{10}\left(-1+2x^4\right)}\,dx$

Optimal. Leaf size=109

$$2\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}-x^2}\right)-2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+x^2}\right)+\frac{\sqrt[4]{x^4-1}\,\left(65x^8+5x^4+2\right)}{9x^9}$$

Rubi [C] time = 0.38, antiderivative size = 121, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6725, 271, 264, 277, 331, 298, 203, 206, 511, 510}

$$-\frac{16\sqrt[4]{x^4-1}\,x^3F_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};x^4,2x^4\right)}{3\sqrt[4]{1-x^4}}+\frac{8\sqrt[4]{x^4-1}}{x}+4\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-4\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{2\left(x^4-1\right)^{5/4}}{9x^9}-\frac{7\left(x^4-1\right)^{5/4}}{9x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^4)^(1/4)*(2 - x^4 + 2*x^8))/(x^10*(-1 + 2*x^4)),x]
```

```
[Out] (8*(-1 + x^4)^(1/4))/x - (2*(-1 + x^4)^(5/4))/(9*x^9) - (7*(-1 + x^4)^(5/4)
)/(9*x^5) - (16*x^3*(-1 + x^4)^(1/4)*AppellF1[3/4, -1/4, 1, 7/4, x^4, 2*x^4
])/(3*(1 - x^4)^(1/4)) + 4*ArcTan[x/(-1 + x^4)^(1/4)] - 4*ArcTanh[x/(-1 + x
^4)^(1/4)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{-1+x^4}\left(2-x^4+2x^8\right)}{x^{10}\left(-1+2x^4\right)}\,dx = \int \left(-\frac{2\sqrt[4]{-1+x^4}}{x^{10}}-\frac{3\sqrt[4]{-1+x^4}}{x^6}-\frac{8\sqrt[4]{-1+x^4}}{x^2}+\frac{16x^2\sqrt[4]{-1+x^4}}{-1+2x^4}\right)dx$$

$$= -\left(2\int \frac{\sqrt[4]{-1+x^4}}{x^{10}}\,dx\right)-3\int \frac{\sqrt[4]{-1+x^4}}{x^6}\,dx-8\int \frac{\sqrt[4]{-1+x^4}}{x^2}\,dx+16\int \frac{x^2\sqrt[4]{-1}}{-1+}$$

$$= \frac{8\sqrt[4]{-1+x^4}}{x}-\frac{2\left(-1+x^4\right)^{5/4}}{9x^9}-\frac{3\left(-1+x^4\right)^{5/4}}{5x^5}-\frac{8}{9}\int \frac{\sqrt[4]{-1+x^4}}{x^6}\,dx-8\int \frac{}{(-1}$$

$$= \frac{8\sqrt[4]{-1+x^4}}{x}-\frac{2\left(-1+x^4\right)^{5/4}}{9x^9}-\frac{7\left(-1+x^4\right)^{5/4}}{9x^5}-\frac{16x^3\sqrt[4]{-1+x^4}\,F_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};x\right.}{3\sqrt[4]{1-x^4}}$$

$$= \frac{8\sqrt[4]{-1+x^4}}{x}-\frac{2\left(-1+x^4\right)^{5/4}}{9x^9}-\frac{7\left(-1+x^4\right)^{5/4}}{9x^5}-\frac{16x^3\sqrt[4]{-1+x^4}\,F_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};x\right.}{3\sqrt[4]{1-x^4}}$$

$$= \frac{8\sqrt[4]{-1+x^4}}{x}-\frac{2\left(-1+x^4\right)^{5/4}}{9x^9}-\frac{7\left(-1+x^4\right)^{5/4}}{9x^5}-\frac{16x^3\sqrt[4]{-1+x^4}\,F_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};x\right.}{3\sqrt[4]{1-x^4}}$$

Mathematica [C] time = 0.09, size = 97, normalized size = 0.89

$$\frac{-24\sqrt[4]{1-2x^4}\left(1-x^4\right)^{3/4}x^{12}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{x^4}{2x^4-1}\right)+130x^{16}-185x^{12}+54x^8-x^4+2}{9x^9\left(x^4-1\right)^{3/4}\left(2x^4-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^4)^(1/4)*(2 - x^4 + 2*x^8))/(x^10*(-1 + 2*x^4)),x]
```

```
[Out] (2 - x^4 + 54*x^8 - 185*x^12 + 130*x^16 - 24*x^12*(1 - 2*x^4)^(1/4)*(1 - x^
4)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, x^4/(-1 + 2*x^4)])/(9*x^9*(-1 + x
^4)^(3/4)*(-1 + 2*x^4))
```

IntegrateAlgebraic [A] time = 0.31, size = 109, normalized size = 1.00

$$2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}-x^2}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+x^2}\right)+\frac{\sqrt[4]{x^4-1}\left(65x^8+5x^4+2\right)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)^(1/4)*(2 - x^4 + 2*x^8))/(x^10*(-1 + 2*x^4))
,x]
```

```
[Out] ((-1 + x^4)^(1/4)*(2 + 5*x^4 + 65*x^8))/(9*x^9) + 2*Sqrt[2]*ArcTan[(Sqrt[2]
*x*(-1 + x^4)^(1/4))/(-x^2 + Sqrt[-1 + x^4])] - 2*Sqrt[2]*ArcTanh[(Sqrt[2]*
x*(-1 + x^4)^(1/4))/(x^2 + Sqrt[-1 + x^4])]
```

fricas [B] time = 2.61, size = 444, normalized size = 4.07

$$-\frac{36\sqrt{2}x^9\arctan\left(-\frac{\sqrt{2}\left(x^4-1\right)^{\frac{3}{4}}x^2-\sqrt{2}\left(x^4-1\right)^{\frac{5}{4}}+\left(2x^5-\sqrt{2}\left(x^4-1\right)^{\frac{3}{4}}x^2-\sqrt{2}\left(x^4-1\right)^{\frac{5}{4}}-2x\right)\sqrt{\frac{2x^4+2\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}x^3+4\sqrt{x^4-1}\,x^2+2\sqrt{2}\left(x^4-1\right)^{\frac{3}{4}}x-1}{2x^4-1}}}{2\left(x^5-x\right)}\right)+36\sqrt{2}x^9\arctan\left(-\frac{\sqrt{2}\left(x^4-1\right)^{\frac{3}{4}}x^2-\sqrt{2}\left(x^4-1\right)^{\frac{5}{4}}-\left(2x^5+\sqrt{2}\left(x^4-1\right)^{\frac{3}{4}}x^2+\sqrt{2}\left(x^4-1\right)^{\frac{5}{4}}-2x\right)\sqrt{\frac{2x^4-2\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}x^3+4\sqrt{x^4-1}\,x^2-2\sqrt{2}\left(x^4-1\right)^{\frac{3}{4}}x-1}{2x^4-1}}}{2\left(x^5-x\right)}\right)+9\sqrt{2}x^9\log\left(\frac{2x^4+2\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}x^3+4\sqrt{x^4-1}\,x^2+2\sqrt{2}\left(x^4-1\right)^{\frac{3}{4}}x-1}{2x^4-1}\right)-9\sqrt{2}x^9\log\left(\frac{2x^4-2\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}x^3+4\sqrt{x^4-1}\,x^2-2\sqrt{2}\left(x^4-1\right)^{\frac{3}{4}}x-1}{2x^4-1}\right)-2\left(65x^8+5x^4+2\right)\left(x^4-1\right)^{\frac{1}{4}}}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/4)*(2*x^8-x^4+2)/x^10/(2*x^4-1),x, algorithm="fricas")
```

```
[Out] -1/18*(36*sqrt(2)*x^9*arctan(-1/2*(sqrt(2)*(x^4 - 1)^(3/4)*x^2 - sqrt(2)*(x
^4 - 1)^(5/4) + (2*x^5 - sqrt(2)*(x^4 - 1)^(3/4)*x^2 - sqrt(2)*(x^4 - 1)^(5
/4) - 2*x)*sqrt((2*x^4 + 2*sqrt(2)*(x^4 - 1)^(1/4)*x^3 + 4*sqrt(x^4 - 1)*x^
2 + 2*sqrt(2)*(x^4 - 1)^(3/4)*x - 1)/(2*x^4 - 1)))/(x^5 - x)) + 36*sqrt(2)*
x^9*arctan(-1/2*(sqrt(2)*(x^4 - 1)^(3/4)*x^2 - sqrt(2)*(x^4 - 1)^(5/4) - (2
*x^5 + sqrt(2)*(x^4 - 1)^(3/4)*x^2 + sqrt(2)*(x^4 - 1)^(5/4) - 2*x)*sqrt((2
*x^4 - 2*sqrt(2)*(x^4 - 1)^(1/4)*x^3 + 4*sqrt(x^4 - 1)*x^2 - 2*sqrt(2)*(x^4
 - 1)^(3/4)*x - 1)/(2*x^4 - 1)))/(x^5 - x)) + 9*sqrt(2)*x^9*log((2*x^4 + 2*
sqrt(2)*(x^4 - 1)^(1/4)*x^3 + 4*sqrt(x^4 - 1)*x^2 + 2*sqrt(2)*(x^4 - 1)^(3/
4)*x - 1)/(2*x^4 - 1)) - 9*sqrt(2)*x^9*log((2*x^4 - 2*sqrt(2)*(x^4 - 1)^(1/
4)*x^3 + 4*sqrt(x^4 - 1)*x^2 - 2*sqrt(2)*(x^4 - 1)^(3/4)*x - 1)/(2*x^4 - 1)
) - 2*(65*x^8 + 5*x^4 + 2)*(x^4 - 1)^(1/4))/x^9
```

giac [A] time = 0.34, size = 172, normalized size = 1.58

$$2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)\right)+\sqrt{2}\log\left(\frac{\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}}{x}+\frac{\sqrt{x^4-1}}{x^2}+1\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}}{x}+\frac{\sqrt{x^4-1}}{x^2}+1\right)-\frac{\left(x^4-1\right)^{\frac{1}{4}}\left(\frac{1}{x^4}-1\right)}{x}-\frac{8\left(x^4-1\right)^{\frac{1}{4}}}{x}-\frac{2\left(x^8-2x^4+1\right)\left(x^4-1\right)^{\frac{1}{4}}}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/4)*(2*x^8-x^4+2)/x^10/(2*x^4-1),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^4 - 1)^(1/4)/x)) + 2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^4 - 1)^(1/4)/x)) + sqrt(2)*log(sqrt(2)*(
x^4 - 1)^(1/4)/x + sqrt(x^4 - 1)/x^2 + 1) - sqrt(2)*log(-sqrt(2)*(x^4 - 1)^
```

```
(1/4)/x + sqrt(x^4 - 1)/x^2 + 1) - (x^4 - 1)^(1/4)*(1/x^4 - 1)/x - 8*(x^4 -
 1)^(1/4)/x - 2/9*(x^8 - 2*x^4 + 1)*(x^4 - 1)^(1/4)/x^9
```

maple [C] time = 1.82, size = 508, normalized size = 4.66

$$\frac{65x^{12}-60x^{8}-3x^{4}-2}{9x^{9}\left(x^{4}-1\right)^{\frac{3}{4}}}+\frac{\left(-2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)\ln\left(-\frac{-2\left(x^{12}-3x^{8}+3x^{4}-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}x^{9}+x^{8}\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}+4\left(x^{12}-3x^{8}+3x^{4}-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}x^{5}+2\sqrt{x^{12}-3x^{8}+3x^{4}-1}\,x^{6}+2\left(x^{12}-3x^{8}+3x^{4}-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^{4}+1\right)x^{3}-2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}x^{4}-2\left(x^{12}-3x^{8}+3x^{4}-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}x-2\sqrt{x^{12}-3x^{8}+3x^{4}-1}\,x^{2}+\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}}{\left(-1+x\right)^{2}\left(1+x\right)^{2}\left(x^{2}+1\right)^{2}\left(2x^{4}-1\right)}\right)+2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}\ln\left(\frac{-2\left(x^{12}-3x^{8}+3x^{4}-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{4}+1\right)x^{9}+x^{8}\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}+2\left(x^{12}-3x^{8}+3x^{4}-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}x^{3}-2\sqrt{x^{12}-3x^{8}+3x^{4}-1}\,x^{6}+4\left(x^{12}-3x^{8}+3x^{4}-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{4}+1\right)x^{5}-2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}x^{4}+2\sqrt{x^{12}-3x^{8}+3x^{4}-1}\,x^{2}-2\left(x^{12}-3x^{8}+3x^{4}-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{4}+1\right)x+\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}}{\left(-1+x\right)^{2}\left(1+x\right)^{2}\left(x^{2}+1\right)^{2}\left(2x^{4}-1\right)}\right)\right)\left(\left(x^{4}-1\right)^{3}\right)^{\frac{1}{4}}}{\left(x^{4}-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(1/4)*(2*x^8-x^4+2)/x^10/(2*x^4-1),x)
```

```
[Out] 1/9*(65*x^12-60*x^8-3*x^4-2)/x^9/(x^4-1)^(3/4)+(-2*RootOf(_Z^4+1)*ln(-(-2*(
x^12-3*x^8+3*x^4-1)^(1/4)*RootOf(_Z^4+1)^3*x^9+x^8*RootOf(_Z^4+1)^2+4*(x^12
-3*x^8+3*x^4-1)^(1/4)*RootOf(_Z^4+1)^3*x^5+2*(x^12-3*x^8+3*x^4-1)^(1/2)*x^6
+2*(x^12-3*x^8+3*x^4-1)^(3/4)*RootOf(_Z^4+1)*x^3-2*RootOf(_Z^4+1)^2*x^4-2*(
x^12-3*x^8+3*x^4-1)^(1/4)*RootOf(_Z^4+1)^3*x-2*(x^12-3*x^8+3*x^4-1)^(1/2)*x
^2+RootOf(_Z^4+1)^2)/(-1+x)^2/(1+x)^2/(x^2+1)^2/(2*x^4-1))+2*RootOf(_Z^4+1)
^3*ln((-2*(x^12-3*x^8+3*x^4-1)^(1/4)*RootOf(_Z^4+1)*x^9+x^8*RootOf(_Z^4+1)^
2+2*(x^12-3*x^8+3*x^4-1)^(3/4)*RootOf(_Z^4+1)^3*x^3-2*(x^12-3*x^8+3*x^4-1)^
(1/2)*x^6+4*(x^12-3*x^8+3*x^4-1)^(1/4)*RootOf(_Z^4+1)*x^5-2*RootOf(_Z^4+1)^
2*x^4+2*(x^12-3*x^8+3*x^4-1)^(1/2)*x^2-2*(x^12-3*x^8+3*x^4-1)^(1/4)*RootOf(
_Z^4+1)*x+RootOf(_Z^4+1)^2)/(-1+x)^2/(1+x)^2/(x^2+1)^2/(2*x^4-1)))/(x^4-1)^
(3/4)*((x^4-1)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{8}-x^{4}+2\right)\left(x^{4}-1\right)^{\frac{1}{4}}}{\left(2\,x^{4}-1\right)x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(1/4)*(2*x^8-x^4+2)/x^10/(2*x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^8 - x^4 + 2)*(x^4 - 1)^(1/4)/((2*x^4 - 1)*x^10), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(x^4-1\right)}^{1/4}\,\left(2\,x^8-x^4+2\right)}{x^{10}\,\left(2\,x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)^(1/4)*(2*x^8 - x^4 + 2))/(x^10*(2*x^4 - 1)),x)
```

```
[Out] int(((x^4 - 1)^(1/4)*(2*x^8 - x^4 + 2))/(x^10*(2*x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{\left(x-1\right)\left(x+1\right)\left(x^{2}+1\right)}\left(2x^{8}-x^{4}+2\right)}{x^{10}\left(2x^{4}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(1/4)*(2*x**8-x**4+2)/x**10/(2*x**4-1),x)
```

```
[Out] Integral(((x - 1)*(x + 1)*(x**2 + 1))**(1/4)*(2*x**8 - x**4 + 2)/(x**10*(2*
x**4 - 1)), x)
```

3.1369 $\int \frac{(2+x^6)(1-2x^6+x^8+x^{12})}{x^8\sqrt[4]{-1+x^6}(-1+x^4+x^6)}\,dx$

Optimal. Leaf size=109

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6-1}}{\sqrt{x^6-1}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6-1}}{\sqrt{x^6-1}+x^2}\right)+\frac{2(x^6-1)^{3/4}(3x^6-7x^4-3)}{21x^7}$$

Rubi [F] time = 1.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(2+x^6)(1-2x^6+x^8+x^{12})}{x^8\sqrt[4]{-1+x^6}(-1+x^4+x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((2 + x^6)*(1 - 2*x^6 + x^8 + x^12))/(x^8*(-1 + x^6)^(1/4)*(-1 + x^4 +
x^6)),x]
```

```
[Out] (-2*(-1 + x^6)^(3/4))/(3*x^3) + (2*(1 - x^6)^(1/4)*Hypergeometric2F1[-7/6,
1/4, -1/6, x^6])/(7*x^7*(-1 + x^6)^(1/4)) - ((1 - x^6)^(1/4)*Hypergeometric
2F1[-1/6, 1/4, 5/6, x^6])/(x*(-1 + x^6)^(1/4)) + (2*x*(1 - x^6)^(1/4)*Hyper
geometric2F1[1/6, 1/4, 7/6, x^6])/(-1 + x^6)^(1/4) + (x^5*(1 - x^6)^(1/4)*H
ypergeometric2F1[1/4, 5/6, 11/6, x^6])/(5*(-1 + x^6)^(1/4)) + 6*Defer[Int][
1/((-1 + x^6)^(1/4)*(-1 + x^4 + x^6)), x] - 2*Defer[Int][x^4/((-1 + x^6)^(1
/4)*(-1 + x^4 + x^6)), x]
```

Rubi steps

$$\int \frac{(2+x^6)(1-2x^6+x^8+x^{12})}{x^8\sqrt[4]{-1+x^6}(-1+x^4+x^6)}\,dx=\int\left(\frac{2}{\sqrt[4]{-1+x^6}}-\frac{2}{x^8\sqrt[4]{-1+x^6}}-\frac{2}{x^4\sqrt[4]{-1+x^6}}+\frac{1}{x^2\sqrt[4]{-1+x^6}}-\frac{x^2}{\sqrt[4]{-1+}}\right.$$

$$=2\int\frac{1}{\sqrt[4]{-1+x^6}}\,dx-2\int\frac{1}{x^8\sqrt[4]{-1+x^6}}\,dx-2\int\frac{1}{x^4\sqrt[4]{-1+x^6}}\,dx-2\int$$

$$=-\left(\frac{1}{3}\operatorname{Subst}\left(\int\frac{1}{\sqrt[4]{-1+x^2}}\,dx,x,x^3\right)\right)-\frac{2}{3}\operatorname{Subst}\left(\int\frac{1}{x^2\sqrt[4]{-1+x^2}}\,dx,x,\right.$$

$$=-\frac{2(-1+x^6)^{3/4}}{3x^3}+\frac{2\sqrt[4]{1-x^6}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{4};-\frac{1}{6};x^6\right)}{7x^7\sqrt[4]{-1+x^6}}-\frac{\sqrt[4]{1-x^6}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{4};\frac{5}{6}\right.}{x\sqrt[4]{-1+x^6}}$$

$$=-\frac{2(-1+x^6)^{3/4}}{3x^3}+\frac{2\sqrt[4]{1-x^6}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{4};-\frac{1}{6};x^6\right)}{7x^7\sqrt[4]{-1+x^6}}-\frac{\sqrt[4]{1-x^6}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{4};\frac{5}{6}\right.}{x\sqrt[4]{-1+x^6}}$$

$$=-\frac{2(-1+x^6)^{3/4}}{3x^3}-\frac{2x^3\sqrt[4]{-1+x^6}}{3\left(1+\sqrt{-1+x^6}\right)}+\frac{2\sqrt{\frac{x^6}{\left(1+\sqrt{-1+x^6}\right)^2}}\left(1+\sqrt{-1+x^6}\right)}{3x^3}$$

$$=-\frac{2(-1+x^6)^{3/4}}{3x^3}+\frac{2\sqrt[4]{1-x^6}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{4};-\frac{1}{6};x^6\right)}{7x^7\sqrt[4]{-1+x^6}}-\frac{\sqrt[4]{1-x^6}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{4};\frac{5}{6}\right.}{x\sqrt[4]{-1+x^6}}$$

Mathematica [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{(2+x^6)(1-2x^6+x^8+x^{12})}{x^8\sqrt[4]{-1+x^6}\,(-1+x^4+x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((2 + x^6)*(1 - 2*x^6 + x^8 + x^12))/(x^8*(-1 + x^6)^(1/4)*(-1 +
x^4 + x^6)),x]
```

```
[Out] Integrate[((2 + x^6)*(1 - 2*x^6 + x^8 + x^12))/(x^8*(-1 + x^6)^(1/4)*(-1 +
x^4 + x^6)), x]
```

IntegrateAlgebraic [A] time = 15.41, size = 109, normalized size = 1.00

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6-1}}{\sqrt{x^6-1}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6-1}}{\sqrt{x^6-1}+x^2}\right)+\frac{2\left(x^6-1\right)^{3/4}\left(3x^6-7x^4-3\right)}{21x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^6)*(1 - 2*x^6 + x^8 + x^12))/(x^8*(-1 + x^6)^(1/
4)*(-1 + x^4 + x^6)),x]
```

```
[Out] (2*(-1 + x^6)^(3/4)*(-3 - 7*x^4 + 3*x^6))/(21*x^7) - Sqrt[2]*ArcTan[(Sqrt[2
]*x*(-1 + x^6)^(1/4))/(-x^2 + Sqrt[-1 + x^6])] - Sqrt[2]*ArcTanh[(Sqrt[2]*x
*(-1 + x^6)^(1/4))/(x^2 + Sqrt[-1 + x^6])]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)*(x^12+x^8-2*x^6+1)/x^8/(x^6-1)^(1/4)/(x^6+x^4-1),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{12}+x^8-2\,x^6+1\right)\left(x^6+2\right)}{\left(x^6+x^4-1\right)\left(x^6-1\right)^{\frac{1}{4}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)*(x^12+x^8-2*x^6+1)/x^8/(x^6-1)^(1/4)/(x^6+x^4-1),x, algor
ithm="giac")
```

```
[Out] integrate((x^12 + x^8 - 2*x^6 + 1)*(x^6 + 2)/((x^6 + x^4 - 1)*(x^6 - 1)^(1/
4)*x^8), x)
```

maple [C] time = 15.64, size = 239, normalized size = 2.19

$$\frac{\frac{2}{7}x^{12}-\frac{4}{7}x^6+\frac{2}{7}-\frac{2}{3}x^{10}+\frac{2}{3}x^4}{x^7\left(x^6-1\right)^{\frac{1}{4}}}+\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^3x^6-\mathrm{RootOf}\left(\_Z^4+1\right)^3x^4-2\left(x^6-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+1\right)^2x^3-2\,\mathrm{RootOf}\left(\_Z^4+1\right)\sqrt{x^6-1}\,x^2-2\left(x^6-1\right)^{\frac{3}{4}}x-\mathrm{RootOf}\left(\_Z^4+1\right)^3}{x^6+x^4-1}\right)-\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3\sqrt{x^6-1}\,x^2-\mathrm{RootOf}\left(\_Z^4+1\right)x^6+2\left(x^6-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+1\right)^2x^3+\mathrm{RootOf}\left(\_Z^4+1\right)x^4-2\left(x^6-1\right)^{\frac{3}{4}}x+\mathrm{RootOf}\left(\_Z^4+1\right)}{x^6+x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+2)*(x^12+x^8-2*x^6+1)/x^8/(x^6-1)^(1/4)/(x^6+x^4-1),x)
```

```
[Out] 2/21*(3*x^12-7*x^10-6*x^6+7*x^4+3)/x^7/(x^6-1)^(1/4)+RootOf(_Z^4+1)^3*ln((R
ootOf(_Z^4+1)^3*x^6-RootOf(_Z^4+1)^3*x^4-2*(x^6-1)^(1/4)*RootOf(_Z^4+1)^2*x
```

```
^3-2*RootOf(_Z^4+1)*(x^6-1)^(1/2)*x^2-2*(x^6-1)^(3/4)*x-RootOf(_Z^4+1)^3)/(
x^6+x^4-1))-RootOf(_Z^4+1)*ln((2*RootOf(_Z^4+1)^3*(x^6-1)^(1/2)*x^2-RootOf(
_Z^4+1)*x^6+2*(x^6-1)^(1/4)*RootOf(_Z^4+1)^2*x^3+RootOf(_Z^4+1)*x^4-2*(x^6-
1)^(3/4)*x+RootOf(_Z^4+1))/(x^6+x^4-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{12}+x^{8}-2\,x^{6}+1\right)\left(x^{6}+2\right)}{\left(x^{6}+x^{4}-1\right)\left(x^{6}-1\right)^{\frac{1}{4}}x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)*(x^12+x^8-2*x^6+1)/x^8/(x^6-1)^(1/4)/(x^6+x^4-1),x, algor
ithm="maxima")
```

```
[Out] integrate((x^12 + x^8 - 2*x^6 + 1)*(x^6 + 2)/((x^6 + x^4 - 1)*(x^6 - 1)^(1/
4)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6+2\right)\,\left(x^{12}+x^8-2\,x^6+1\right)}{x^8\,{\left(x^6-1\right)}^{1/4}\,\left(x^6+x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 + 2)*(x^8 - 2*x^6 + x^12 + 1))/(x^8*(x^6 - 1)^(1/4)*(x^4 + x^6 -
1)),x)
```

```
[Out] int(((x^6 + 2)*(x^8 - 2*x^6 + x^12 + 1))/(x^8*(x^6 - 1)^(1/4)*(x^4 + x^6 -
1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+2)*(x**12+x**8-2*x**6+1)/x**8/(x**6-1)**(1/4)/(x**6+x**4-1)
,x)
```

```
[Out] Timed out
```

3.1370 $\int \frac{\sqrt{x+x^2}}{x\sqrt{x^2+x\sqrt{x+x^2}}}\,dx$

Optimal. Leaf size=109

$$\sqrt{x\left(\sqrt{x^2+x}+x\right)}\left(\frac{\sqrt{2}\sqrt{\sqrt{x^2+x}-x}\tanh^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+x}-x}\right)}{x}+\frac{2(x-2)}{x}\right)-\frac{2\sqrt{x^2+x}\sqrt{x\left(\sqrt{x^2+x}+x\right)}}{x}$$

Rubi [F] time = 1.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{x+x^2}}{x\sqrt{x^2+x\sqrt{x+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[x + x^2]/(x*Sqrt[x^2 + x*Sqrt[x + x^2]]),x]
```

```
[Out] (2*Sqrt[x + x^2]*Defer[Subst][Defer[Int][Sqrt[1 + x^2]/Sqrt[x^4 + x^2*Sqrt[
x^2 + x^4]], x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + x])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x+x^2}}{x\sqrt{x^2+x\sqrt{x+x^2}}}\,dx &= \frac{\sqrt{x+x^2}\int \frac{\sqrt{1+x}}{\sqrt{x}\sqrt{x^2+x\sqrt{x+x^2}}}\,dx}{\sqrt{x}\sqrt{1+x}}\\
&= \frac{\left(2\sqrt{x+x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{x^4+x^2\sqrt{x^2+x^4}}}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1+x}}
\end{aligned}$$

Mathematica [C] time = 0.48, size = 97, normalized size = 0.89

$$-\frac{2\sqrt{x(x+1)}\left(\left(x+\sqrt{x(x+1)}\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{1}{2\left(x+\sqrt{x(x+1)}\right)}\right)+4x+4\sqrt{x(x+1)}+2\right)}{\sqrt{x\left(x+\sqrt{x(x+1)}\right)}\left(x+\sqrt{x(x+1)}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + x^2]/(x*Sqrt[x^2 + x*Sqrt[x + x^2]]),x]
```

```
[Out] (-2*Sqrt[x*(1 + x)]*(2 + 4*x + 4*Sqrt[x*(1 + x)] + (x + Sqrt[x*(1 + x)])*Hy
pergeometric2F1[-1/2, 1, 1/2, 1 + 1/(2*(x + Sqrt[x*(1 + x)]))]))/(Sqrt[x*(x
 + Sqrt[x*(1 + x)])]*(1 + x + Sqrt[x*(1 + x)]))
```

IntegrateAlgebraic [A] time = 4.22, size = 109, normalized size = 1.00

$$\sqrt{x\left(\sqrt{x^2+x}+x\right)}\left(\frac{\sqrt{2}\sqrt{\sqrt{x^2+x}-x}\tanh^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+x}-x}\right)}{x}+\frac{2(x-2)}{x}\right)-\frac{2\sqrt{x^2+x}\sqrt{x\left(\sqrt{x^2+x}+x\right)}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x + x^2]/(x*Sqrt[x^2 + x*Sqrt[x + x^2]]),x]
```

```
[Out] (-2*Sqrt[x + x^2]*Sqrt[x*(x + Sqrt[x + x^2])])/x + Sqrt[x*(x + Sqrt[x + x^2
])]*((2*(-2 + x))/x + (Sqrt[2]*Sqrt[-x + Sqrt[x + x^2]]*ArcTanh[Sqrt[2]*Sqr
t[-x + Sqrt[x + x^2]]])/x)
```

fricas [A] time = 0.43, size = 96, normalized size = 0.88

$$\frac{\sqrt{2}\,x\log\left(\frac{4\,x^2+2\,\sqrt{x^2+\sqrt{x^2+x}\,x}\left(\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2+x}\right)+4\,\sqrt{x^2+x}\,x+x}{x}\right)+4\,\sqrt{x^2+\sqrt{x^2+x}\,x}\left(x-\sqrt{x^2+x}\,-2\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(1/2)/x/(x^2+x*(x^2+x)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(sqrt(2)*x*log((4*x^2 + 2*sqrt(x^2 + sqrt(x^2 + x)*x)*(sqrt(2)*x + sqrt
(2)*sqrt(x^2 + x)) + 4*sqrt(x^2 + x)*x + x)/x) + 4*sqrt(x^2 + sqrt(x^2 + x)
*x)*(x - sqrt(x^2 + x) - 2))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+x}}{\sqrt{x^2+\sqrt{x^2+x}\,x}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(1/2)/x/(x^2+x*(x^2+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + x)/(sqrt(x^2 + sqrt(x^2 + x)*x)*x), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+x}}{x\sqrt{x^2+x\sqrt{x^2+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)^(1/2)/x/(x^2+x*(x^2+x)^(1/2))^(1/2),x)
```

```
[Out] int((x^2+x)^(1/2)/x/(x^2+x*(x^2+x)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+x}}{\sqrt{x^2+\sqrt{x^2+x}\,x}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(1/2)/x/(x^2+x*(x^2+x)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(x^2 + x)/(sqrt(x^2 + sqrt(x^2 + x)*x)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^2+x}}{x\,\sqrt{x^2+x\,\sqrt{x^2+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2)^(1/2)/(x*(x^2 + x*(x + x^2)^(1/2))^(1/2)),x)
```

```
[Out] int((x + x^2)^(1/2)/(x*(x^2 + x*(x + x^2)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\,(x+1)}}{x\sqrt{x\left(x+\sqrt{x^2+x}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)**(1/2)/x/(x**2+x*(x**2+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x*(x + 1))/(x*sqrt(x*(x + sqrt(x**2 + x)))), x)
```

3.1371 $\int \frac{\sqrt{x^2+x\sqrt{x+x^2}}}{\sqrt{x+x^2}}\,dx$

Optimal. Leaf size=109

$$\frac{3\sqrt{x^2+x}\sqrt{x\left(\sqrt{x^2+x}+x\right)}}{2x}+\sqrt{x\left(\sqrt{x^2+x}+x\right)}\left(-\frac{3\sqrt{\sqrt{x^2+x}-x}\,\tanh^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+x}-x}\right)}{2\sqrt{2}\,x}-\frac{1}{2}\right)$$

Rubi [F] time = 0.77, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{x^2+x\sqrt{x+x^2}}}{\sqrt{x+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[x^2 + x*Sqrt[x + x^2]]/Sqrt[x + x^2],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 + x]*Defer[Subst][Defer[Int][Sqrt[x^4 + x^2*Sqrt[x^2 + x^
4]]/Sqrt[1 + x^2], x], x, Sqrt[x]])/Sqrt[x + x^2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x^2+x\sqrt{x+x^2}}}{\sqrt{x+x^2}}\,dx &= \frac{\left(\sqrt{x}\sqrt{1+x}\right)\int \frac{\sqrt{x^2+x\sqrt{x+x^2}}}{\sqrt{x}\sqrt{1+x}}\,dx}{\sqrt{x+x^2}}\\ &= \frac{\left(2\sqrt{x}\sqrt{1+x}\right)\operatorname{Subst}\left(\int \frac{\sqrt{x^4+x^2\sqrt{x^2+x^4}}}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)}{\sqrt{x+x^2}}\end{aligned}$$

Mathematica [A] time = 0.36, size = 140, normalized size = 1.28

$$\frac{(x+1)\sqrt{x\left(x+\sqrt{x(x+1)}\right)}\left(2\sqrt{x+\sqrt{x(x+1)}}\left(2x+2\sqrt{x(x+1)}+3\right)-3\sqrt{4x+4\sqrt{x(x+1)}+2}\,\sinh^{-1}\left(\sqrt{2}\sqrt{x+\sqrt{x(x+1)}}\right)\right)}{4\sqrt{x(x+1)}\sqrt{x+\sqrt{x(x+1)}}\left(x+\sqrt{x(x+1)}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + x*Sqrt[x + x^2]]/Sqrt[x + x^2],x]
```

```
[Out] ((1 + x)*Sqrt[x*(x + Sqrt[x*(1 + x)])]*(2*Sqrt[x + Sqrt[x*(1 + x)]]*(3 + 2*
x + 2*Sqrt[x*(1 + x)]) - 3*Sqrt[2 + 4*x + 4*Sqrt[x*(1 + x)]]*ArcSinh[Sqrt[2
]*Sqrt[x + Sqrt[x*(1 + x)]]]))/(4*Sqrt[x*(1 + x)]*Sqrt[x + Sqrt[x*(1 + x)]]
*(1 + x + Sqrt[x*(1 + x)]))
```

IntegrateAlgebraic [A] time = 4.91, size = 109, normalized size = 1.00

$$\frac{3\sqrt{x^2+x}\sqrt{x\left(\sqrt{x^2+x}+x\right)}}{2x}+\sqrt{x\left(\sqrt{x^2+x}+x\right)}\left(-\frac{3\sqrt{\sqrt{x^2+x}-x}\,\tanh^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+x}-x}\right)}{2\sqrt{2}\,x}-\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2 + x*Sqrt[x + x^2]]/Sqrt[x + x^2],x]
```

```
[Out] (3*Sqrt[x + x^2]*Sqrt[x*(x + Sqrt[x + x^2])])/(2*x) + Sqrt[x*(x + Sqrt[x +
x^2])]*(-1/2 - (3*Sqrt[-x + Sqrt[x + x^2]]*ArcTanh[Sqrt[2]*Sqrt[-x + Sqrt[x
 + x^2]]])/(2*Sqrt[2]*x))
```

fricas [A] time = 0.41, size = 96, normalized size = 0.88

$$\frac{3\sqrt{2}\,x\log\left(\frac{4\,x^2-2\,\sqrt{x^2+\sqrt{x^2+x}\,x}\left(\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2+x}\right)+4\,\sqrt{x^2+x}\,x+x}{x}\right)-4\,\sqrt{x^2+\sqrt{x^2+x}\,x}\left(x-3\,\sqrt{x^2+x}\right)}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x*(x^2+x)^(1/2))^(1/2)/(x^2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(2)*x*log((4*x^2 - 2*sqrt(x^2 + sqrt(x^2 + x)*x)*(sqrt(2)*x + sq
rt(2)*sqrt(x^2 + x)) + 4*sqrt(x^2 + x)*x + x)/x) - 4*sqrt(x^2 + sqrt(x^2 +
x)*x)*(x - 3*sqrt(x^2 + x)))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^2+x}\,x}}{\sqrt{x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x*(x^2+x)^(1/2))^(1/2)/(x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^2 + x)*x)/sqrt(x^2 + x), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+x\sqrt{x^2+x}}}{\sqrt{x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x*(x^2+x)^(1/2))^(1/2)/(x^2+x)^(1/2),x)
```

```
[Out] int((x^2+x*(x^2+x)^(1/2))^(1/2)/(x^2+x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^2+x}\,x}}{\sqrt{x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x*(x^2+x)^(1/2))^(1/2)/(x^2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^2 + x)*x)/sqrt(x^2 + x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^2+x\,\sqrt{x^2+x}}}{\sqrt{x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x*(x + x^2)^(1/2))^(1/2)/(x + x^2)^(1/2),x)
[Out] int((x^2 + x*(x + x^2)^(1/2))^(1/2)/(x + x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(x+\sqrt{x^2+x}\right)}}{\sqrt{x\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x*(x**2+x)**(1/2))**(1/2)/(x**2+x)**(1/2),x)
[Out] Integral(sqrt(x*(x + sqrt(x**2 + x)))/sqrt(x*(x + 1)), x)
```

3.1372 $\int \frac{1}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$

Optimal. Leaf size=109

$$\frac{2x}{\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{2\sqrt{2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{a}}$$

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] Defer[Int][1/Sqrt[b + Sqrt[b^2 + a*x^2]], x]
```

Rubi steps

$$\int \frac{1}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx=\int \frac{1}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Mathematica [C] time = 0.39, size = 149, normalized size = 1.37

$$\frac{\sqrt{\sqrt{ax^2+b^2}+b}\left(-2b\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b-\sqrt{b^2+ax^2}}{2b}\right)+4\sqrt{ax^2+b^2}-\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}-b}\tan^{-1}\left(\frac{\sqrt{\sqrt{ax^2+b^2}-b}}{\sqrt{2}\sqrt{b}}\right)-2b\right)}{2ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 + a*x^2]]*(-2*b + 4*Sqrt[b^2 + a*x^2] - Sqrt[2]*Sqrt[b]*
Sqrt[-b + Sqrt[b^2 + a*x^2]]*ArcTan[Sqrt[-b + Sqrt[b^2 + a*x^2]]/(Sqrt[2]*S
qrt[b])] - 2*b*Hypergeometric2F1[-1/2, 1, 1/2, (b - Sqrt[b^2 + a*x^2])/(2*b
)]))/(2*a*x)
```

IntegrateAlgebraic [A] time = 0.16, size = 77, normalized size = 0.71

$$\frac{2x}{\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] (2*x)/Sqrt[b + Sqrt[b^2 + a*x^2]] - (Sqrt[2]*Sqrt[b]*ArcTan[(Sqrt[a]*x)/(Sq
rt[2]*Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/Sqrt[a]
```

fricas [A] time = 46.12, size = 239, normalized size = 2.19

$$\left[\frac{\sqrt{2}\,ax\sqrt{-\frac{b}{a}}\log\left(-\frac{ax^3+4\,b^2x-4\,\sqrt{ax^2+b^2}\,bx+2\left(2\,\sqrt{2}\,\sqrt{ax^2+b^2}\,b\sqrt{-\frac{b}{a}}-\sqrt{2}\left(ax^2+2\,b^2\right)\sqrt{-\frac{b}{a}}\right)\sqrt{b+\sqrt{ax^2+b^2}}}{x^3}\right)-4\,\sqrt{b+\sqrt{ax^2+b^2}}\left(b-\sqrt{ax^2+b^2}\right)}{2\,ax},\ \frac{\sqrt{2}\,ax\sqrt{\frac{b}{a}}\arctan\left(\frac{\sqrt{2}\,\sqrt{b+\sqrt{ax^2+b^2}}\,\sqrt{\frac{b}{a}}}{x}\right)-2\,\sqrt{b+\sqrt{ax^2+b^2}}\left(b-\sqrt{ax^2+b^2}\right)}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*x*sqrt(-b/a)*log(-(a*x^3 + 4*b^2*x - 4*sqrt(a*x^2 + b^2)*b*
x + 2*(2*sqrt(2)*sqrt(a*x^2 + b^2)*b*sqrt(-b/a) - sqrt(2)*(a*x^2 + 2*b^2)*s
qrt(-b/a))*sqrt(b + sqrt(a*x^2 + b^2)))/x^3) - 4*sqrt(b + sqrt(a*x^2 + b^2)
)*(b - sqrt(a*x^2 + b^2)))/(a*x), (sqrt(2)*a*x*sqrt(b/a)*arctan(sqrt(2)*sqr
t(b + sqrt(a*x^2 + b^2))*sqrt(b/a)/x) - 2*sqrt(b + sqrt(a*x^2 + b^2))*(b -
sqrt(a*x^2 + b^2)))/(a*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b+\sqrt{ax^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b + sqrt(a*x^2 + b^2)), x)
```

maple [C] time = 0.04, size = 29, normalized size = 0.27

$$\frac{\sqrt{2}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2},\frac{3}{2}\right],-\frac{x^2a}{b^2}\right)}{2\left(b^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] 1/2/(b^2)^(1/4)*2^(1/2)*x*hypergeom([1/4,1/2,3/4],[3/2,3/2],-x^2*a/b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b+\sqrt{ax^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b + sqrt(a*x^2 + b^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b+\sqrt{b^2+a\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b + (a*x^2 + b^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/(b + (a*x^2 + b^2)^(1/2))^(1/2), x)
```

sympy [C] time = 0.81, size = 42, normalized size = 0.39

$$\frac{x\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_3F_2\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2},\frac{3}{4}\\\frac{3}{2},\frac{3}{2}\end{matrix}\right|\frac{ax^2e^{i\pi}}{b^2}\right)}{2\pi\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b+(a*x**2+b**2)**(1/2))**(1/2),x)
```

```
[Out] x*gamma(1/4)*gamma(3/4)*hyper((1/4, 1/2, 3/4), (3/2, 3/2), a*x**2*exp_polar
(I*pi)/b**2)/(2*pi*sqrt(b))
```

3.1373 $\int \left(b^2+ax^2\right)^2 \sqrt{b+\sqrt{b^2+ax^2}}\,dx$

Optimal. Leaf size=109

$$\frac{2x\sqrt{ax^2+b^2}\left(63a^2x^4+206ab^2x^2+271b^4\right)}{693\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{4x\left(35a^2bx^4+118ab^3x^2+211b^5\right)}{693\sqrt{\sqrt{ax^2+b^2}+b}}$$

Rubi [F] time = 0.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(b^2+ax^2\right)^2 \sqrt{b+\sqrt{b^2+ax^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] (2*a*b^4*x^3)/(3*(b + Sqrt[b^2 + a*x^2])^(3/2)) + (2*b^5*x)/Sqrt[b + Sqrt[b
^2 + a*x^2]] + 2*a*b^2*Defer[Int][x^2*Sqrt[b + Sqrt[b^2 + a*x^2]], x] + a^2
*Defer[Int][x^4*Sqrt[b + Sqrt[b^2 + a*x^2]], x]
```

Rubi steps

$$\begin{aligned}\int \left(b^2+ax^2\right)^2 \sqrt{b+\sqrt{b^2+ax^2}}\,dx &= \int \left(b^4\sqrt{b+\sqrt{b^2+ax^2}}+2ab^2x^2\sqrt{b+\sqrt{b^2+ax^2}}+a^2x^4\sqrt{b+\sqrt{b^2+a\right.\\ &= a^2\int x^4\sqrt{b+\sqrt{b^2+ax^2}}\,dx+\left(2ab^2\right)\int x^2\sqrt{b+\sqrt{b^2+ax^2}}\,dx+b^4\int \\ &= \frac{2ab^4x^3}{3\left(b+\sqrt{b^2+ax^2}\right)^{3/2}}+\frac{2b^5x}{\sqrt{b+\sqrt{b^2+ax^2}}}+a^2\int x^4\sqrt{b+\sqrt{b^2+ax^2}}\,dx\end{aligned}$$

Mathematica [A] time = 0.29, size = 144, normalized size = 1.32

$$\frac{2x\left(63a^3x^6\sqrt{ax^2+b^2}+196a^3bx^6+914a^2b^3x^4+472a^2b^2x^4\sqrt{ax^2+b^2}+1848ab^5x^2+1386b^6\sqrt{ax^2+b^2}+1155ab^4x^2\sqrt{ax^2+b^2}+1386b^7\right)}{693\left(\sqrt{ax^2+b^2}+b\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] (2*x*(1386*b^7 + 1848*a*b^5*x^2 + 914*a^2*b^3*x^4 + 196*a^3*b*x^6 + 1386*b^
6*Sqrt[b^2 + a*x^2] + 1155*a*b^4*x^2*Sqrt[b^2 + a*x^2] + 472*a^2*b^2*x^4*Sq
rt[b^2 + a*x^2] + 63*a^3*x^6*Sqrt[b^2 + a*x^2]))/(693*(b + Sqrt[b^2 + a*x^2
])^(5/2))
```

IntegrateAlgebraic [A] time = 0.23, size = 109, normalized size = 1.00

$$\frac{2x\sqrt{ax^2+b^2}\left(63a^2x^4+206ab^2x^2+271b^4\right)}{693\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{4x\left(35a^2bx^4+118ab^3x^2+211b^5\right)}{693\sqrt{\sqrt{ax^2+b^2}+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] (2*x*Sqrt[b^2 + a*x^2]*(271*b^4 + 206*a*b^2*x^2 + 63*a^2*x^4))/(693*Sqrt[b
+ Sqrt[b^2 + a*x^2]]) + (4*x*(211*b^5 + 118*a*b^3*x^2 + 35*a^2*b*x^4))/(693
*Sqrt[b + Sqrt[b^2 + a*x^2]])
```

fricas [A] time = 0.51, size = 93, normalized size = 0.85

$$\frac{2\left(63\,a^{3}x^{6}+199\,a^{2}b^{2}x^{4}+241\,ab^{4}x^{2}-151\,b^{6}+\left(7\,a^{2}bx^{4}+30\,ab^{3}x^{2}+151\,b^{5}\right)\sqrt{ax^{2}+b^{2}}\right)\sqrt{b+\sqrt{ax^{2}+b^{2}}}}{693\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^2*(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*a^3*x^6 + 199*a^2*b^2*x^4 + 241*a*b^4*x^2 - 151*b^6 + (7*a^2*b*x^
4 + 30*a*b^3*x^2 + 151*b^5)*sqrt(a*x^2 + b^2))*sqrt(b + sqrt(a*x^2 + b^2))/
(a*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax^{2}+b^{2}\right)^{2}\sqrt{b+\sqrt{ax^{2}+b^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^2*(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^2 + b^2)^2*sqrt(b + sqrt(a*x^2 + b^2)), x)
```

maple [C] time = 0.33, size = 189, normalized size = 1.73

$$\frac{\left(b^{2}\right)^{\frac{1}{4}}a^{2}\sqrt{2}\,x^{5}\operatorname{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4},\frac{5}{2}\right],\left[\frac{1}{2},\frac{7}{2}\right],-\frac{x^{2}a}{b^{2}}\right)}{5}+\frac{2b^{2}\left(b^{2}\right)^{\frac{1}{4}}a\sqrt{2}\,x^{3}\operatorname{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4},\frac{3}{2}\right],\left[\frac{1}{2},\frac{5}{2}\right],-\frac{x^{2}a}{b^{2}}\right)}{3}-\frac{b^{4}\left(b^{2}\right)^{\frac{1}{4}}\left(-\frac{32\sqrt{\pi}\,\sqrt{2}\,x^{3}\sqrt{\frac{a}{b^{2}}}\,a\cosh\left(\frac{3\operatorname{arcsinh}\left(\frac{x\sqrt{a}}{b}\right)}{2}\right)}{3b^{2}}-\frac{8\sqrt{\pi}\,\sqrt{2}\,\sqrt{\frac{a}{b^{2}}}\left(-\frac{4x^{4}a^{2}}{3b^{4}}-\frac{2x^{2}a}{3b^{2}}+\frac{2}{3}\right)\sinh\left(\frac{3\operatorname{arcsinh}\left(\frac{x\sqrt{a}}{b}\right)}{2}\right)b}{\sqrt{a}\,\sqrt{\frac{x^{2}a}{b^{2}}+1}}\right)}{8\sqrt{\pi}\,\sqrt{\frac{a}{b^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b^2)^2*(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] 1/5*(b^2)^(1/4)*a^2*2^(1/2)*x^5*hypergeom([-1/4,1/4,5/2],[1/2,7/2],-x^2*a/b
^2)+2/3*b^2*(b^2)^(1/4)*a*2^(1/2)*x^3*hypergeom([-1/4,1/4,3/2],[1/2,5/2],-x
^2*a/b^2)-1/8*b^4*(b^2)^(1/4)/Pi^(1/2)/(a/b^2)^(1/2)*(-32/3*Pi^(1/2)*2^(1/2
)*x^3*(a/b^2)^(1/2)*a/b^2*cosh(3/2*arcsinh(x*a^(1/2)/b))-8*Pi^(1/2)*2^(1/2)
*(a/b^2)^(1/2)*(-4/3*x^4*a^2/b^4-2/3*x^2*a/b^2+2/3)*sinh(3/2*arcsinh(x*a^(1
/2)/b))/a^(1/2)*b/(x^2*a/b^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax^{2}+b^{2}\right)^{2}\sqrt{b+\sqrt{ax^{2}+b^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^2*(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 + b^2)^2*sqrt(b + sqrt(a*x^2 + b^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(b^{2}+a\,x^{2}\right)}^{2}\,\sqrt{b+\sqrt{b^{2}+a\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b^2)^2*(b + (a*x^2 + b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x^2 + b^2)^2*(b + (a*x^2 + b^2)^(1/2))^(1/2), x)
```

sympy [B] time = 7.33, size = 1100, normalized size = 10.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b**2)**2*(b+(a*x**2+b**2)**(1/2))**(1/2),x)
```

```
[Out] -315*sqrt(2)*a**4*b**(17/2)*x**7*gamma(-1/4)*gamma(1/4)/(13860*pi*a*b**10*s
qrt(a*x**2/b**2 + 1)*sqrt(sqrt(a*x**2/b**2 + 1) + 1) + 13860*pi*a*b**10*sqr
t(sqrt(a*x**2/b**2 + 1) + 1)) - 665*sqrt(2)*a**3*b**(21/2)*x**5*sqrt(a*x**2
/b**2 + 1)*gamma(-1/4)*gamma(1/4)/(13860*pi*a*b**10*sqrt(a*x**2/b**2 + 1)*s
qrt(sqrt(a*x**2/b**2 + 1) + 1) + 13860*pi*a*b**10*sqrt(sqrt(a*x**2/b**2 + 1
) + 1)) - 705*sqrt(2)*a**3*b**(21/2)*x**5*gamma(-1/4)*gamma(1/4)/(13860*pi*
a*b**10*sqrt(a*x**2/b**2 + 1)*sqrt(sqrt(a*x**2/b**2 + 1) + 1) + 13860*pi*a*
b**10*sqrt(sqrt(a*x**2/b**2 + 1) + 1)) - 32*sqrt(2)*a**2*b**(25/2)*x**3*sqr
t(a*x**2/b**2 + 1)*gamma(-1/4)*gamma(1/4)/(13860*pi*a*b**10*sqrt(a*x**2/b**
2 + 1)*sqrt(sqrt(a*x**2/b**2 + 1) + 1) + 13860*pi*a*b**10*sqrt(sqrt(a*x**2/
b**2 + 1) + 1)) + 32*sqrt(2)*a**2*b**(25/2)*x**3*gamma(-1/4)*gamma(1/4)/(13
860*pi*a*b**10*sqrt(a*x**2/b**2 + 1)*sqrt(sqrt(a*x**2/b**2 + 1) + 1) + 1386
0*pi*a*b**10*sqrt(sqrt(a*x**2/b**2 + 1) + 1)) - 30*sqrt(2)*a**2*b**(5/2)*x*
*5*gamma(-1/4)*gamma(1/4)/(420*pi*b**2*sqrt(a*x**2/b**2 + 1)*sqrt(sqrt(a*x*
*2/b**2 + 1) + 1) + 420*pi*b**2*sqrt(sqrt(a*x**2/b**2 + 1) + 1)) - 66*sqrt(
2)*a*b**(9/2)*x**3*sqrt(a*x**2/b**2 + 1)*gamma(-1/4)*gamma(1/4)/(420*pi*b**
2*sqrt(a*x**2/b**2 + 1)*sqrt(sqrt(a*x**2/b**2 + 1) + 1) + 420*pi*b**2*sqrt(
sqrt(a*x**2/b**2 + 1) + 1)) - 74*sqrt(2)*a*b**(9/2)*x**3*gamma(-1/4)*gamma(
1/4)/(420*pi*b**2*sqrt(a*x**2/b**2 + 1)*sqrt(sqrt(a*x**2/b**2 + 1) + 1) + 4
20*pi*b**2*sqrt(sqrt(a*x**2/b**2 + 1) + 1)) - sqrt(2)*a*b**(9/2)*x**3*gamma
(-1/4)*gamma(1/4)/(12*pi*b**2*sqrt(a*x**2/b**2 + 1)*sqrt(sqrt(a*x**2/b**2 +
 1) + 1) + 12*pi*b**2*sqrt(sqrt(a*x**2/b**2 + 1) + 1)) - 3*sqrt(2)*b**(13/2
)*x*sqrt(a*x**2/b**2 + 1)*gamma(-1/4)*gamma(1/4)/(12*pi*b**2*sqrt(a*x**2/b*
*2 + 1)*sqrt(sqrt(a*x**2/b**2 + 1) + 1) + 12*pi*b**2*sqrt(sqrt(a*x**2/b**2
+ 1) + 1)) - 3*sqrt(2)*b**(13/2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*b**2*sqrt(
a*x**2/b**2 + 1)*sqrt(sqrt(a*x**2/b**2 + 1) + 1) + 12*pi*b**2*sqrt(sqrt(a*x
**2/b**2 + 1) + 1))
```

3.1374 $\int \frac{1}{(1-3x)\sqrt[3]{-x+x^3}}\,dx$

Optimal. Leaf size=110

$$\frac{1}{4}\log\left(2\sqrt[3]{x^3-x}+x+1\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^3-x}}{\sqrt[3]{x^3-x}-x-1}\right)-\frac{1}{8}\log\left(4\left(x^3-x\right)^{2/3}+(-2x-2)\sqrt[3]{x^3-x}+x^2+2x+$$

Rubi [C] time = 0.22, antiderivative size = 220, normalized size of antiderivative = 2.00, number of steps used = 14, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {2056, 959, 466, 465, 377, 200, 31, 634, 618, 204, 628, 511, 510}

$$\frac{9\sqrt[3]{1-x^2}\,x^2F_1\left(\frac{5}{6};1,\frac{1}{3};\frac{11}{6};9x^2,x^2\right)}{5\sqrt[3]{x^3-x}}-\frac{\sqrt[3]{x^2-1}\,\sqrt[3]{x}\,\log\left(\frac{4x^{4/3}}{\left(x^2-1\right)^{2/3}}-\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1\right)}{8\sqrt[3]{x^3-x}}+\frac{\sqrt[3]{x^2-1}\,\sqrt[3]{x}\,\log\left(\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1\right)}{4\sqrt[3]{x^3-x}}-\frac{\sqrt{3}\sqrt[3]{x^2-1}\,\sqrt[3]{x}\,\tan^{-1}\left(\frac{1-\frac{4x^{2/3}}{\sqrt[3]{x^2-1}}}{\sqrt{3}}\right)}{4\sqrt[3]{x^3-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((1 - 3*x)*(-x + x^3)^(1/3)),x]
```

```
[Out] (9*x^2*(1 - x^2)^(1/3)*AppellF1[5/6, 1, 1/3, 11/6, 9*x^2, x^2])/(5*(-x + x^
3)^(1/3)) - (Sqrt[3]*x^(1/3)*(-1 + x^2)^(1/3)*ArcTan[(1 - (4*x^(2/3))/(-1 +
 x^2)^(1/3))/Sqrt[3]])/(4*(-x + x^3)^(1/3)) - (x^(1/3)*(-1 + x^2)^(1/3)*Log
[1 + (4*x^(4/3))/(-1 + x^2)^(2/3) - (2*x^(2/3))/(-1 + x^2)^(1/3)])/(8*(-x +
 x^3)^(1/3)) + (x^(1/3)*(-1 + x^2)^(1/3)*Log[1 + (2*x^(2/3))/(-1 + x^2)^(1/
3)])/(4*(-x + x^3)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{(1-3x)\sqrt[3]{-x+x^3}}\,dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\int \frac{1}{(1-3x)\sqrt[3]{x}\sqrt[3]{-1+x^2}}\,dx}{\sqrt[3]{-x+x^3}}$$

$$= \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\int \frac{1}{\sqrt[3]{x}\left(1-9x^2\right)\sqrt[3]{-1+x^2}}\,dx}{\sqrt[3]{-x+x^3}} + \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\int \frac{x^{2/3}}{\left(1-9x^2\right)\sqrt[3]{-1+x^2}}\,dx}{\sqrt[3]{-x+x^3}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{x}{\left(1-9x^6\right)\sqrt[3]{-1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-x+x^3}} + \frac{\left(9\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{}{(1-}\right.}{\sqrt[3]{-x+x^3}}$$

$$= \frac{\left(9\sqrt[3]{x}\sqrt[3]{1-x^2}\right)\text{Subst}\left(\int \frac{x^4}{\left(1-9x^6\right)\sqrt[3]{1-x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-x+x^3}} + \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{}{(1-9x^3)}\right.}{2\sqrt[3]{-x+x^3}}$$

$$= \frac{9x^2\sqrt[3]{1-x^2}\,F_1\left(\frac{5}{6};1,\frac{1}{3};\frac{11}{6};9x^2,x^2\right)}{5\sqrt[3]{-x+x^3}} + \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1+8x^3}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2\sqrt[3]{-x+x^3}}$$

$$= \frac{9x^2\sqrt[3]{1-x^2}\,F_1\left(\frac{5}{6};1,\frac{1}{3};\frac{11}{6};9x^2,x^2\right)}{5\sqrt[3]{-x+x^3}} + \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1+2x}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2\sqrt[3]{-x+x^3}} +$$

$$= \frac{9x^2\sqrt[3]{1-x^2}\,F_1\left(\frac{5}{6};1,\frac{1}{3};\frac{11}{6};9x^2,x^2\right)}{5\sqrt[3]{-x+x^3}} + \frac{\sqrt[3]{x}\sqrt[3]{-1+x^2}\,\log\left(1+\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{4\sqrt[3]{-x+x^3}} - \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x}\right.}{}$$

$$= \frac{9x^2\sqrt[3]{1-x^2}\,F_1\left(\frac{5}{6};1,\frac{1}{3};\frac{11}{6};9x^2,x^2\right)}{5\sqrt[3]{-x+x^3}} - \frac{\sqrt[3]{x}\sqrt[3]{-1+x^2}\,\log\left(1+\frac{4x^{4/3}}{\left(-1+x^2\right)^{2/3}}-\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{8\sqrt[3]{-x+x^3}} + \frac{\sqrt[3]{}}{}$$

$$= \frac{9x^2\sqrt[3]{1-x^2}\,F_1\left(\frac{5}{6};1,\frac{1}{3};\frac{11}{6};9x^2,x^2\right)}{5\sqrt[3]{-x+x^3}} - \frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{-1+x^2}\,\tan^{-1}\left(\frac{1-\frac{4x^{2/3}}{\sqrt[3]{-1+x^2}}}{\sqrt{3}}\right)}{4\sqrt[3]{-x+x^3}} - \frac{\sqrt[3]{x}\sqrt[3]{-1+}}{}$$

Mathematica [C] time = 0.30, size = 81, normalized size = 0.74

$$\frac{3\sqrt[3]{\frac{\frac{1}{x}-1}{\frac{1}{x}-3}}\sqrt[3]{\frac{\frac{1}{x}+1}{\frac{1}{x}-3}}\,xF_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{4}{\frac{1}{x}-3},-\frac{2}{\frac{1}{x}-3}\right)}{2\sqrt[3]{x\left(x^2-1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - 3*x)*(-x + x^3)^(1/3)),x]
```

```
[Out] (3*((-1 + x^(-1))/(-3 + x^(-1)))^(1/3)*((1 + x^(-1))/(-3 + x^(-1)))^(1/3)*x
*AppellF1[2/3, 1/3, 1/3, 5/3, -4/(-3 + x^(-1)), -2/(-3 + x^(-1))])/(2*(x*(-
1 + x^2))^(1/3))
```

IntegrateAlgebraic [A] time = 0.30, size = 110, normalized size = 1.00

$$\frac{1}{4}\log\left(2\sqrt[3]{x^3-x}+x+1\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^3-x}}{\sqrt[3]{x^3-x}-x-1}\right)-\frac{1}{8}\log\left(4\left(x^3-x\right)^{2/3}+(-2x-2)\sqrt[3]{x^3-x}+x^2+2x+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - 3*x)*(-x + x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(-x + x^3)^(1/3))/(-1 - x + (-x + x^3)^(1/3))])/4
+ Log[1 + x + 2*(-x + x^3)^(1/3)]/4 - Log[1 + 2*x + x^2 + (-2 - 2*x)*(-x +
x^3)^(1/3) + 4*(-x + x^3)^(2/3)]/8
```

fricas [A] time = 0.83, size = 117, normalized size = 1.06

$$\frac{1}{4}\sqrt{3}\arctan\left(\frac{286273\sqrt{3}\left(x^3-x\right)^{\frac{1}{3}}(x+1)+\sqrt{3}\left(635653x^2-434719x+66978\right)+539695\sqrt{3}\left(x^3-x\right)^{\frac{2}{3}}}{1293894x^2-1974837x-226981}\right)+\frac{1}{8}\log\left(\frac{9x^2+6\left(x^3-x\right)^{\frac{1}{3}}(x+1)-6x+12\left(x^3-x\right)^{\frac{2}{3}}+1}{9x^2-6x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3*x)/(x^3-x)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*arctan((286273*sqrt(3)*(x^3 - x)^(1/3)*(x + 1) + sqrt(3)*(63565
3*x^2 - 434719*x + 66978) + 539695*sqrt(3)*(x^3 - x)^(2/3))/(1293894*x^2 -
1974837*x - 226981)) + 1/8*log((9*x^2 + 6*(x^3 - x)^(1/3)*(x + 1) - 6*x + 1
2*(x^3 - x)^(2/3) + 1)/(9*x^2 - 6*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(x^3-x\right)^{\frac{1}{3}}(3x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3*x)/(x^3-x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^3 - x)^(1/3)*(3*x - 1)), x)
```

maple [C] time = 2.15, size = 667, normalized size = 6.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-3*x)/(x^3-x)^(1/3),x)
```

```
[Out] 1/2*RootOf(4*_Z^2+2*_Z+1)*ln(-(366800*RootOf(4*_Z^2+2*_Z+1)^2*x^2+15994800*
RootOf(4*_Z^2+2*_Z+1)*(x^3-x)^(2/3)-7997400*RootOf(4*_Z^2+2*_Z+1)*(x^3-x)^(
1/3)*x-681200*RootOf(4*_Z^2+2*_Z+1)^2*x-8546141*RootOf(4*_Z^2+2*_Z+1)*x^2+7
509306*(x^3-x)^(2/3)-7997400*RootOf(4*_Z^2+2*_Z+1)*(x^3-x)^(1/3)-3754653*x*
(x^3-x)^(1/3)-104800*RootOf(4*_Z^2+2*_Z+1)^2+16142294*RootOf(4*_Z^2+2*_Z+1)
*x-9661003*x^2-3754653*(x^3-x)^(1/3)+2532051*RootOf(4*_Z^2+2*_Z+1)+11337706
*x+558901)/(-1+3*x)^2)-1/4*ln((-733600*RootOf(4*_Z^2+2*_Z+1)^2*x^2+31989600
*RootOf(4*_Z^2+2*_Z+1)*(x^3-x)^(2/3)-15994800*RootOf(4*_Z^2+2*_Z+1)*(x^3-x)
^(1/3)*x+1362400*RootOf(4*_Z^2+2*_Z+1)^2*x-17825882*RootOf(4*_Z^2+2*_Z+1)*x
^2+976188*(x^3-x)^(2/3)-15994800*RootOf(4*_Z^2+2*_Z+1)*(x^3-x)^(1/3)-488094
*x*(x^3-x)^(1/3)+209600*RootOf(4*_Z^2+2*_Z+1)^2+33646988*RootOf(4*_Z^2+2*_Z
+1)*x+10592465*x^2-488094*(x^3-x)^(1/3)+5273702*RootOf(4*_Z^2+2*_Z+1)-61925
18*x+1466649)/(-1+3*x)^2)-1/2*ln((-733600*RootOf(4*_Z^2+2*_Z+1)^2*x^2+31989
600*RootOf(4*_Z^2+2*_Z+1)*(x^3-x)^(2/3)-15994800*RootOf(4*_Z^2+2*_Z+1)*(x^3
-x)^(1/3)*x+1362400*RootOf(4*_Z^2+2*_Z+1)^2*x-17825882*RootOf(4*_Z^2+2*_Z+1
)*x^2+976188*(x^3-x)^(2/3)-15994800*RootOf(4*_Z^2+2*_Z+1)*(x^3-x)^(1/3)-488
094*x*(x^3-x)^(1/3)+209600*RootOf(4*_Z^2+2*_Z+1)^2+33646988*RootOf(4*_Z^2+2
*_Z+1)*x+10592465*x^2-488094*(x^3-x)^(1/3)+5273702*RootOf(4*_Z^2+2*_Z+1)-61
92518*x+1466649)/(-1+3*x)^2)*RootOf(4*_Z^2+2*_Z+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(x^3-x\right)^{\frac{1}{3}}(3x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3*x)/(x^3-x)^(1/3),x, algorithm="maxima")
[Out] -integrate(1/((x^3 - x)^(1/3)*(3*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(x^3-x\right)^{1/3}\,\left(3\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^3 - x)^(1/3)*(3*x - 1)),x)
[Out] -int(1/((x^3 - x)^(1/3)*(3*x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x\sqrt[3]{x^3-x}-\sqrt[3]{x^3-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3*x)/(x**3-x)**(1/3),x)
[Out] -Integral(1/(3*x*(x**3 - x)**(1/3) - (x**3 - x)**(1/3)), x)
```

3.1375 $\int \frac{1}{(1+3x)\sqrt[3]{-x+x^3}}\,dx$

Optimal. Leaf size=110

$$\frac{1}{4}\log\left(2\sqrt[3]{x^3-x}+x-1\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^3-x}}{\sqrt[3]{x^3-x}-x+1}\right)-\frac{1}{8}\log\left(4\left(x^3-x\right)^{2/3}+(2-2x)\sqrt[3]{x^3-x}+x^2-2x+$$

Rubi [C] time = 0.21, antiderivative size = 220, normalized size of antiderivative = 2.00, number of steps used = 14, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {2056, 959, 466, 465, 377, 200, 31, 634, 618, 204, 628, 511, 510}

$$-\frac{9\sqrt[3]{1-x^2}\,x^2F_1\left(\frac{5}{6};1,\frac{1}{3};\frac{11}{6};9x^2,x^2\right)}{5\sqrt[3]{x^3-x}}-\frac{\sqrt[3]{x^2-1}\,\sqrt[3]{x}\log\left(\frac{4x^{4/3}}{\left(x^2-1\right)^{2/3}}-\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1\right)}{8\sqrt[3]{x^3-x}}+\frac{\sqrt[3]{x^2-1}\,\sqrt[3]{x}\log\left(\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1\right)}{4\sqrt[3]{x^3-x}}-\frac{\sqrt{3}\sqrt[3]{x^2-1}\,\sqrt[3]{x}\tan^{-1}\left(\frac{1-\frac{4x^{2/3}}{\sqrt[3]{x^2-1}}}{\sqrt{3}}\right)}{4\sqrt[3]{x^3-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((1 + 3*x)*(-x + x^3)^(1/3)),x]
```

```
[Out] (-9*x^2*(1 - x^2)^(1/3)*AppellF1[5/6, 1, 1/3, 11/6, 9*x^2, x^2])/(5*(-x + x
^3)^(1/3)) - (Sqrt[3]*x^(1/3)*(-1 + x^2)^(1/3)*ArcTan[(1 - (4*x^(2/3))/(-1
+ x^2)^(1/3))/Sqrt[3]])/(4*(-x + x^3)^(1/3)) - (x^(1/3)*(-1 + x^2)^(1/3)*Lo
g[1 + (4*x^(4/3))/(-1 + x^2)^(2/3) - (2*x^(2/3))/(-1 + x^2)^(1/3)])/(8*(-x
+ x^3)^(1/3)) + (x^(1/3)*(-1 + x^2)^(1/3)*Log[1 + (2*x^(2/3))/(-1 + x^2)^(1
/3)])/(4*(-x + x^3)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{(1+3x)\sqrt[3]{-x+x^3}}\,dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\int \frac{1}{\sqrt[3]{x}(1+3x)\sqrt[3]{-1+x^2}}\,dx}{\sqrt[3]{-x+x^3}}$$
$$= \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\int \frac{1}{\sqrt[3]{x}\left(1-9x^2\right)\sqrt[3]{-1+x^2}}\,dx}{\sqrt[3]{-x+x^3}} - \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\int \frac{x^{2/3}}{\left(1-9x^2\right)\sqrt[3]{-1+x^2}}\,dx}{\sqrt[3]{-x+x^3}}$$
$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{x}{\left(1-9x^6\right)\sqrt[3]{-1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-x+x^3}} - \frac{\left(9\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int\right.}{\sqrt[3]{-x+}}$$
$$= -\frac{\left(9\sqrt[3]{x}\sqrt[3]{1-x^2}\right)\text{Subst}\left(\int \frac{x^4}{\left(1-9x^6\right)\sqrt[3]{1-x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-x+x^3}} + \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int\right.}{2\sqrt[3]{-x+}}$$
$$= -\frac{9x^2\sqrt[3]{1-x^2}\,F_1\left(\frac{5}{6};1,\frac{1}{3};\frac{11}{6};9x^2,x^2\right)}{5\sqrt[3]{-x+x^3}} + \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1+8x^3}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{-1+}}\right.}{2\sqrt[3]{-x+x^3}}$$
$$= -\frac{9x^2\sqrt[3]{1-x^2}\,F_1\left(\frac{5}{6};1,\frac{1}{3};\frac{11}{6};9x^2,x^2\right)}{5\sqrt[3]{-x+x^3}} + \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1+2x}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right.}{2\sqrt[3]{-x+x^3}}$$
$$= -\frac{9x^2\sqrt[3]{1-x^2}\,F_1\left(\frac{5}{6};1,\frac{1}{3};\frac{11}{6};9x^2,x^2\right)}{5\sqrt[3]{-x+x^3}} + \frac{\sqrt[3]{x}\sqrt[3]{-1+x^2}\log\left(1+\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{4\sqrt[3]{-x+x^3}} - \frac{\left(\sqrt[3]{x}\sqrt[3]{-1}\right.}{}$$
$$= -\frac{9x^2\sqrt[3]{1-x^2}\,F_1\left(\frac{5}{6};1,\frac{1}{3};\frac{11}{6};9x^2,x^2\right)}{5\sqrt[3]{-x+x^3}} - \frac{\sqrt[3]{x}\sqrt[3]{-1+x^2}\log\left(1+\frac{4x^{4/3}}{\left(-1+x^2\right)^{2/3}}-\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{8\sqrt[3]{-x+x^3}}$$
$$= -\frac{9x^2\sqrt[3]{1-x^2}\,F_1\left(\frac{5}{6};1,\frac{1}{3};\frac{11}{6};9x^2,x^2\right)}{5\sqrt[3]{-x+x^3}} - \frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{-1+x^2}\tan^{-1}\left(\frac{1-\frac{4x^{2/3}}{\sqrt[3]{-1+x^2}}}{\sqrt{3}}\right)}{4\sqrt[3]{-x+x^3}} - \frac{\sqrt[3]{x}\sqrt[3]{}}{}$$

Mathematica [C] time = 0.27, size = 81, normalized size = 0.74

$$\frac{3\sqrt[3]{\frac{\frac{1}{x}-1}{\frac{1}{x}+3}}\sqrt[3]{\frac{\frac{1}{x}+1}{\frac{1}{x}+3}}\,xF_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{2}{3+\frac{1}{x}},\frac{4}{3+\frac{1}{x}}\right)}{2\sqrt[3]{x\left(x^2-1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + 3*x)*(-x + x^3)^(1/3)),x]
```

```
[Out] (3*((-1 + x^(-1))/(3 + x^(-1)))^(1/3)*((1 + x^(-1))/(3 + x^(-1)))^(1/3)*x*A
ppellF1[2/3, 1/3, 1/3, 5/3, 2/(3 + x^(-1)), 4/(3 + x^(-1))])/(2*(x*(-1 + x^
2))^(1/3))
```

IntegrateAlgebraic [A] time = 0.32, size = 110, normalized size = 1.00

$$\frac{1}{4}\log\left(2\sqrt[3]{x^3-x}+x-1\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^3-x}}{\sqrt[3]{x^3-x}-x+1}\right)-\frac{1}{8}\log\left(4\left(x^3-x\right)^{2/3}+(2-2x)\sqrt[3]{x^3-x}+x^2-2x+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + 3*x)*(-x + x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(-x + x^3)^(1/3))/(1 - x + (-x + x^3)^(1/3))])/4 +
 Log[-1 + x + 2*(-x + x^3)^(1/3)]/4 - Log[1 - 2*x + x^2 + (2 - 2*x)*(-x + x
^3)^(1/3) + 4*(-x + x^3)^(2/3)]/8
```

fricas [A] time = 0.81, size = 117, normalized size = 1.06

$$\frac{1}{4} \sqrt{3} \arctan\left(\frac{286273 \sqrt{3} \left(x^{3}-x\right)^{\frac{1}{3}} (x-1) + \sqrt{3} \left(635653 x^{2} + 434719 x + 66978\right) + 539695 \sqrt{3} \left(x^{3}-x\right)^{\frac{2}{3}}}{1293894 x^{2} + 1974837 x - 226981}\right) + \frac{1}{8} \log\left(\frac{9 x^{2} + 6 \left(x^{3}-x\right)^{\frac{1}{3}} (x-1) + 6 x + 12 \left(x^{3}-x\right)^{\frac{2}{3}} + 1}{9 x^{2} + 6 x + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+3*x)/(x^3-x)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*arctan((286273*sqrt(3)*(x^3 - x)^(1/3)*(x - 1) + sqrt(3)*(63565
3*x^2 + 434719*x + 66978) + 539695*sqrt(3)*(x^3 - x)^(2/3))/(1293894*x^2 +
1974837*x - 226981)) + 1/8*log((9*x^2 + 6*(x^3 - x)^(1/3)*(x - 1) + 6*x + 1
2*(x^3 - x)^(2/3) + 1)/(9*x^2 + 6*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{3}-x\right)^{\frac{1}{3}} (3 x + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+3*x)/(x^3-x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 - x)^(1/3)*(3*x + 1)), x)
```

maple [C] time = 2.58, size = 668, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+3*x)/(x^3-x)^(1/3),x)
```

```
[Out] 1/2*RootOf(4*_Z^2+2*_Z+1)*ln((-2308880*RootOf(4*_Z^2+2*_Z+1)^2*x^2+3629136*
RootOf(4*_Z^2+2*_Z+1)*(x^3-x)^(2/3)-1814568*RootOf(4*_Z^2+2*_Z+1)*(x^3-x)^(
1/3)*x+6003088*RootOf(4*_Z^2+2*_Z+1)^2*x+11550557*RootOf(4*_Z^2+2*_Z+1)*x^2
+11494278*(x^3-x)^(2/3)+1814568*RootOf(4*_Z^2+2*_Z+1)*(x^3-x)^(1/3)-5747139
*x*(x^3-x)^(1/3)-2770656*RootOf(4*_Z^2+2*_Z+1)^2+9052118*RootOf(4*_Z^2+2*_Z
+1)*x-7685285*x^2+5747139*(x^3-x)^(1/3)+832813*RootOf(4*_Z^2+2*_Z+1)-109007
90*x+1071835)/(1+3*x)^2)-1/4*ln(-(4617760*RootOf(4*_Z^2+2*_Z+1)^2*x^2+72582
72*RootOf(4*_Z^2+2*_Z+1)*(x^3-x)^(2/3)-3629136*RootOf(4*_Z^2+2*_Z+1)*(x^3-x
)^(1/3)*x-12006176*RootOf(4*_Z^2+2*_Z+1)^2*x+27718874*RootOf(4*_Z^2+2*_Z+1)
*x^2-19359420*(x^3-x)^(2/3)+3629136*RootOf(4*_Z^2+2*_Z+1)*(x^3-x)^(1/3)+967
9710*x*(x^3-x)^(1/3)+5541312*RootOf(4*_Z^2+2*_Z+1)^2+6098060*RootOf(4*_Z^2+
2*_Z+1)*x+28075567*x^2-9679710*(x^3-x)^(1/3)+7206938*RootOf(4*_Z^2+2*_Z+1)+
27852154*x+74471)/(1+3*x)^2)-1/2*ln(-(4617760*RootOf(4*_Z^2+2*_Z+1)^2*x^2+7
258272*RootOf(4*_Z^2+2*_Z+1)*(x^3-x)^(2/3)-3629136*RootOf(4*_Z^2+2*_Z+1)*(x
^3-x)^(1/3)*x-12006176*RootOf(4*_Z^2+2*_Z+1)^2*x+27718874*RootOf(4*_Z^2+2*_
Z+1)*x^2-19359420*(x^3-x)^(2/3)+3629136*RootOf(4*_Z^2+2*_Z+1)*(x^3-x)^(1/3)
+9679710*x*(x^3-x)^(1/3)+5541312*RootOf(4*_Z^2+2*_Z+1)^2+6098060*RootOf(4*_
Z^2+2*_Z+1)*x+28075567*x^2-9679710*(x^3-x)^(1/3)+7206938*RootOf(4*_Z^2+2*_Z
+1)+27852154*x+74471)/(1+3*x)^2)*RootOf(4*_Z^2+2*_Z+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{3}-x\right)^{\frac{1}{3}} (3 x + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+3*x)/(x^3-x)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^3 - x)^(1/3)*(3*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^3-x\right)^{1/3}\,\left(3\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 - x)^(1/3)*(3*x + 1)),x)
[Out] int(1/((x^3 - x)^(1/3)*(3*x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x\left(x-1\right)\left(x+1\right)}\,\left(3x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+3*x)/(x**3-x)**(1/3),x)
[Out] Integral(1/((x*(x - 1)*(x + 1))**(1/3)*(3*x + 1)), x)
```

3.1376 $\int \frac{x}{\sqrt[3]{-x^2+x^3}}\,dx$

Optimal. Leaf size=110

$$\frac{\left(x^3-x^2\right)^{2/3}}{x}-\frac{1}{3}\log\left(\sqrt[3]{x^3-x^2}-x\right)+\frac{1}{6}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-x^2}+x}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 152, normalized size of antiderivative = 1.38, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2024, 2011, 59}

$$\frac{\left(x^3-x^2\right)^{2/3}}{x}-\frac{\sqrt[3]{x-1}\,x^{2/3}\log\left(\frac{\sqrt[3]{x-1}}{\sqrt[3]{x}}-1\right)}{2\sqrt[3]{x^3-x^2}}-\frac{\sqrt[3]{x-1}\,x^{2/3}\log(x)}{6\sqrt[3]{x^3-x^2}}-\frac{\sqrt[3]{x-1}\,x^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{x-1}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{x^3-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(-x^2 + x^3)^(1/3),x]
```

```
[Out] (-x^2 + x^3)^(2/3)/x - ((-1 + x)^(1/3)*x^(2/3)*ArcTan[1/Sqrt[3] + (2*(-1 +
x)^(1/3))/(Sqrt[3]*x^(1/3))])/(Sqrt[3]*(-x^2 + x^3)^(1/3)) - ((-1 + x)^(1/3
)*x^(2/3)*Log[-1 + (-1 + x)^(1/3)/x^(1/3)])/(2*(-x^2 + x^3)^(1/3)) - ((-1 +
 x)^(1/3)*x^(2/3)*Log[x])/(6*(-x^2 + x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{-x^2+x^3}}\,dx = \frac{\left(-x^2+x^3\right)^{2/3}}{x} + \frac{1}{3}\int \frac{1}{\sqrt[3]{-x^2+x^3}}\,dx$$
$$= \frac{\left(-x^2+x^3\right)^{2/3}}{x} + \frac{\left(\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{1}{\sqrt[3]{-1+x}\,x^{2/3}}\,dx}{3\sqrt[3]{-x^2+x^3}}$$
$$= \frac{\left(-x^2+x^3\right)^{2/3}}{x} - \frac{\sqrt[3]{-1+x}\,x^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-1+x}}{\sqrt{3}\sqrt[3]{x}}\right)}{\sqrt{3}\sqrt[3]{-x^2+x^3}} - \frac{\sqrt[3]{-1+x}\,x^{2/3}\log\left(-1+\frac{\sqrt[3]{-1+x}}{\sqrt[3]{x}}\right)}{2\sqrt[3]{-x^2+x^3}} -$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.32

$$\frac{3\left((x-1)x^2\right)^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};1-x\right)}{2x^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-x^2 + x^3)^(1/3),x]
```

```
[Out] (3*((-1 + x)*x^2)^(2/3)*Hypergeometric2F1[-1/3, 2/3, 5/3, 1 - x])/(2*x^(4/3
))
```

IntegrateAlgebraic [A] time = 0.27, size = 110, normalized size = 1.00

$$\frac{\left(x^3-x^2\right)^{2/3}}{x} - \frac{1}{3}\log\left(\sqrt[3]{x^3-x^2}-x\right) + \frac{1}{6}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right) + \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x^2}+x}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(-x^2 + x^3)^(1/3),x]
```

```
[Out] (-x^2 + x^3)^(2/3)/x + ArcTan[(Sqrt[3]*x)/(x + 2*(-x^2 + x^3)^(1/3))]/Sqrt[
3] - Log[-x + (-x^2 + x^3)^(1/3)]/3 + Log[x^2 + x*(-x^2 + x^3)^(1/3) + (-x^
2 + x^3)^(2/3)]/6
```

fricas [A] time = 0.41, size = 113, normalized size = 1.03

$$-\frac{2\sqrt{3}\,x\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-x^2\right)^{\frac{1}{3}}}{3x}\right)+2x\log\left(-\frac{x-\left(x^3-x^2\right)^{\frac{1}{3}}}{x}\right)-x\log\left(\frac{x^2+\left(x^3-x^2\right)^{\frac{1}{3}}x+\left(x^3-x^2\right)^{\frac{2}{3}}}{x^2}\right)-6\left(x^3-x^2\right)^{\frac{2}{3}}}{6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 - x^2)^(1/3))/x) +
 2*x*log(-(x - (x^3 - x^2)^(1/3))/x) - x*log((x^2 + (x^3 - x^2)^(1/3)*x + (
x^3 - x^2)^(2/3))/x^2) - 6*(x^3 - x^2)^(2/3))/x
```

giac [A] time = 0.17, size = 74, normalized size = 0.67

$$x\left(-\frac{1}{x}+1\right)^{\frac{2}{3}} - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right) + \frac{1}{6}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right) - \frac{1}{3}\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-x^2)^(1/3),x, algorithm="giac")
```

```
[Out] x*(-1/x + 1)^(2/3) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-1/x + 1)^(1/3) + 1
)) + 1/6*log((-1/x + 1)^(2/3) + (-1/x + 1)^(1/3) + 1) - 1/3*log(abs((-1/x +
 1)^(1/3) - 1))
```

maple [C] time = 0.32, size = 41, normalized size = 0.37

$$\frac{x(-1+x)}{\left((-1+x)x^{2}\right)^{\frac{1}{3}}}+\frac{\left(-\mathrm{signum}\left(-1+x\right)\right)^{\frac{1}{3}} x^{\frac{1}{3}} \mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x\right)}{\mathrm{signum}\left(-1+x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3-x^2)^(1/3),x)
```

```
[Out] x*(-1+x)/((-1+x)*x^2)^(1/3)+1/signum(-1+x)^(1/3)*(-signum(-1+x))^(1/3)*x^(1
/3)*hypergeom([1/3,1/3],[4/3],x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^{3}-x^{2}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^3 - x^2)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(x^3-x^2\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3 - x^2)^(1/3),x)
```

```
[Out] int(x/(x^3 - x^2)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{x^{2}\left(x-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3-x**2)**(1/3),x)
```

```
[Out] Integral(x/(x**2*(x - 1))**(1/3), x)
```

3.1377 $\int \frac{(1-x^3)^{2/3}(-1+x^3)}{x^6(-1+2x^3)}\,dx$

Optimal. Leaf size=110

$$-\frac{1}{3}\log\left(\sqrt[3]{1-x^3}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{1-x^3}+x}\right)}{\sqrt{3}}+\frac{(1-x^3)^{2/3}(-3x^3-2)}{10x^5}+\frac{1}{6}\log\left(\sqrt[3]{1-x^3}\,x+(1-x^3)^{2/3}+x^2\right)$$

Rubi [C] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 0.38, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {21, 510}

$$-\frac{(1-2x^3)^{5/3}\,{}_2F_1\left(-\frac{5}{3},-\frac{5}{3};-\frac{2}{3};-\frac{x^3}{1-2x^3}\right)}{5x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((1 - x^3)^(2/3)*(-1 + x^3))/(x^6*(-1 + 2*x^3)),x]
```

```
[Out] -1/5*((1 - 2*x^3)^(5/3)*Hypergeometric2F1[-5/3, -5/3, -2/3, -(x^3/(1 - 2*x^
3))])/x^5
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x^3)^{2/3}(-1+x^3)}{x^6(-1+2x^3)}\,dx &= -\int \frac{(1-x^3)^{5/3}}{x^6(-1+2x^3)}\,dx\\ &= -\frac{(1-2x^3)^{5/3}\,{}_2F_1\left(-\frac{5}{3},-\frac{5}{3};-\frac{2}{3};-\frac{x^3}{1-2x^3}\right)}{5x^5}\end{aligned}$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.38

$$-\frac{(1-2x^3)^{5/3}\,{}_2F_1\left(-\frac{5}{3},-\frac{5}{3};-\frac{2}{3};-\frac{x^3}{1-2x^3}\right)}{5x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - x^3)^(2/3)*(-1 + x^3))/(x^6*(-1 + 2*x^3)),x]
```

```
[Out] -1/5*((1 - 2*x^3)^(5/3)*Hypergeometric2F1[-5/3, -5/3, -2/3, -(x^3/(1 - 2*x^
3))])/x^5
```

IntegrateAlgebraic [A] time = 0.26, size = 110, normalized size = 1.00

$$-\frac{1}{3}\log\left(\sqrt[3]{1-x^3}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{1-x^3}+x}\right)}{\sqrt{3}}+\frac{\left(1-x^3\right)^{2/3}\left(-3x^3-2\right)}{10x^5}+\frac{1}{6}\log\left(\sqrt[3]{1-x^3}\,x+\left(1-x^3\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 - x^3)^(2/3)*(-1 + x^3))/(x^6*(-1 + 2*x^3)),x]
```

```
[Out] ((-2 - 3*x^3)*(1 - x^3)^(2/3))/(10*x^5) + ArcTan[(Sqrt[3]*x)/(x + 2*(1 - x^
3)^(1/3))]/Sqrt[3] - Log[-x + (1 - x^3)^(1/3)]/3 + Log[x^2 + x*(1 - x^3)^(1
/3) + (1 - x^3)^(2/3)]/6
```

fricas [A] time = 0.94, size = 136, normalized size = 1.24

$$\frac{10\sqrt{3}x^5\arctan\left(-\frac{4\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}x^2-2\sqrt{3}\left(-x^3+1\right)^{\frac{2}{3}}x-\sqrt{3}\left(x^3-1\right)}{7x^3+1}\right)-5x^5\log\left(\frac{2x^3-3\left(-x^3+1\right)^{\frac{1}{3}}x^2+3\left(-x^3+1\right)^{\frac{2}{3}}x-1}{2x^3-1}\right)-3\left(3x^3+2\right)\left(-x^3+1\right)^{\frac{2}{3}}}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)*(x^3-1)/x^6/(2*x^3-1),x, algorithm="fricas")
```

```
[Out] 1/30*(10*sqrt(3)*x^5*arctan(-(4*sqrt(3)*(-x^3 + 1)^(1/3)*x^2 - 2*sqrt(3)*(-
x^3 + 1)^(2/3)*x - sqrt(3)*(x^3 - 1))/(7*x^3 + 1)) - 5*x^5*log((2*x^3 - 3*(
-x^3 + 1)^(1/3)*x^2 + 3*(-x^3 + 1)^(2/3)*x - 1)/(2*x^3 - 1)) - 3*(3*x^3 + 2
)*(-x^3 + 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-1\right)\left(-x^3+1\right)^{\frac{2}{3}}}{\left(2x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)*(x^3-1)/x^6/(2*x^3-1),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 1)*(-x^3 + 1)^(2/3)/((2*x^3 - 1)*x^6), x)
```

maple [C] time = 1.29, size = 278, normalized size = 2.53

$$\frac{3x^6-x^3-2}{10x^5\left(-x^3+1\right)^{\frac{1}{3}}}-\frac{\ln\left(-\frac{3\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x-6\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^2+3\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)x^3+\left(-x^3+1\right)^{\frac{2}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}x^2-1}{2x^3-1}\right)}{3}+\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\ln\left(-\frac{9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)^2x^3-3\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x-3\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^2-3\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)x^3+2\left(-x^3+1\right)^{\frac{2}{3}}x-\left(-x^3+1\right)^{\frac{1}{3}}x^2+3\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)}{2x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)*(x^3-1)/x^6/(2*x^3-1),x)
```

```
[Out] 1/10*(3*x^6-x^3-2)/x^5/(-x^3+1)^(1/3)-1/3*ln(-(3*RootOf(9*_Z^2-3*_Z+1)*(-x^
3+1)^(2/3)*x-6*RootOf(9*_Z^2-3*_Z+1)*(-x^3+1)^(1/3)*x^2+3*RootOf(9*_Z^2-3*_
Z+1)*x^3+(-x^3+1)^(2/3)*x+(-x^3+1)^(1/3)*x^2-1)/(2*x^3-1))+RootOf(9*_Z^2-3*
_Z+1)*ln(-(9*RootOf(9*_Z^2-3*_Z+1)^2*x^3-3*RootOf(9*_Z^2-3*_Z+1)*(-x^3+1)^(
2/3)*x-3*RootOf(9*_Z^2-3*_Z+1)*(-x^3+1)^(1/3)*x^2-3*RootOf(9*_Z^2-3*_Z+1)*x
^3+2*(-x^3+1)^(2/3)*x-(-x^3+1)^(1/3)*x^2+3*RootOf(9*_Z^2-3*_Z+1))/(2*x^3-1)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-1\right)\left(-x^3+1\right)^{\frac{2}{3}}}{\left(2x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)*(x^3-1)/x^6/(2*x^3-1),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 1)*(-x^3 + 1)^(2/3)/((2*x^3 - 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(1-x^3\right)^{5/3}}{x^6\,\left(2\,x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1 - x^3)^(5/3)/(x^6*(2*x^3 - 1)),x)
```

```
[Out] -int((1 - x^3)^(5/3)/(x^6*(2*x^3 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x-1)\left(x^2+x+1\right)}{x^6\left(2x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)*(x**3-1)/x**6/(2*x**3-1),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)*(x - 1)*(x**2 + x + 1)/(x**6*(2*x
**3 - 1)), x)
```

3.1378 $\int \frac{2b+ax}{\left(b+ax+x^2\right)\sqrt[4]{bx^2+ax^3}}\,dx$

Optimal. Leaf size=110

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^3+bx^2}}{\sqrt{ax^3+bx^2}+x^2}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3+bx^2}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3+bx^2}}\right)$$

Rubi [C] time = 1.39, antiderivative size = 573, normalized size of antiderivative = 5.21, number of steps used = 13, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2056, 6728, 107, 106, 490, 1218}

$$\frac{\sqrt{2}\sqrt[4]{b}\left(\sqrt{a^2-4b}+a\right)\sqrt{-\frac{ax}{b}}\sqrt[4]{ax+b}\,\Pi\left(-\frac{\sqrt{2}\sqrt{b}}{\sqrt{-a^2-\sqrt{a^2-4b}\,a+2b}};\sin^{-1}\left(\frac{\sqrt[4]{b+ax}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt{-a\sqrt{a^2-4b}-a^2+2b}\sqrt[4]{ax^3+bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\left(\sqrt{a^2-4b}+a\right)\sqrt{-\frac{ax}{b}}\sqrt[4]{ax+b}\,\Pi\left(\frac{\sqrt{2}\sqrt{b}}{\sqrt{-a^2-\sqrt{a^2-4b}\,a+2b}};\sin^{-1}\left(\frac{\sqrt[4]{b+ax}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt{-a\sqrt{a^2-4b}-a^2+2b}\sqrt[4]{ax^3+bx^2}}+\frac{\sqrt{2}\sqrt[4]{b}\left(a-\sqrt{a^2-4b}\right)\sqrt{-\frac{ax}{b}}\sqrt[4]{ax+b}\,\Pi\left(-\frac{\sqrt{2}\sqrt{b}}{\sqrt{-a^2+\sqrt{a^2-4b}\,a+2b}};\sin^{-1}\left(\frac{\sqrt[4]{b+ax}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt{a\sqrt{a^2-4b}-a^2+2b}\sqrt[4]{ax^3+bx^2}}-\frac{\sqrt{2}\sqrt[4]{b}\left(a-\sqrt{a^2-4b}\right)\sqrt{-\frac{ax}{b}}\sqrt[4]{ax+b}\,\Pi\left(\frac{\sqrt{2}\sqrt{b}}{\sqrt{-a^2+\sqrt{a^2-4b}\,a+2b}};\sin^{-1}\left(\frac{\sqrt[4]{b+ax}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt{a\sqrt{a^2-4b}-a^2+2b}\sqrt[4]{ax^3+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*b + a*x)/((b + a*x + x^2)*(b*x^2 + a*x^3)^(1/4)),x]
```

```
[Out] (Sqrt[2]*(a + Sqrt[a^2 - 4*b])*b^(1/4)*Sqrt[-((a*x)/b)]*(b + a*x)^(1/4)*Ell
ipticPi[-((Sqrt[2]*Sqrt[b])/Sqrt[-a^2 - a*Sqrt[a^2 - 4*b] + 2*b]), ArcSin[(
b + a*x)^(1/4)/b^(1/4)], -1])/(Sqrt[-a^2 - a*Sqrt[a^2 - 4*b] + 2*b]*(b*x^2
+ a*x^3)^(1/4)) - (Sqrt[2]*(a + Sqrt[a^2 - 4*b])*b^(1/4)*Sqrt[-((a*x)/b)]*(
b + a*x)^(1/4)*EllipticPi[(Sqrt[2]*Sqrt[b])/Sqrt[-a^2 - a*Sqrt[a^2 - 4*b] +
 2*b], ArcSin[(b + a*x)^(1/4)/b^(1/4)], -1])/(Sqrt[-a^2 - a*Sqrt[a^2 - 4*b]
 + 2*b]*(b*x^2 + a*x^3)^(1/4)) + (Sqrt[2]*(a - Sqrt[a^2 - 4*b])*b^(1/4)*Sqr
t[-((a*x)/b)]*(b + a*x)^(1/4)*EllipticPi[-((Sqrt[2]*Sqrt[b])/Sqrt[-a^2 + a*
Sqrt[a^2 - 4*b] + 2*b]), ArcSin[(b + a*x)^(1/4)/b^(1/4)], -1])/(Sqrt[-a^2 +
 a*Sqrt[a^2 - 4*b] + 2*b]*(b*x^2 + a*x^3)^(1/4)) - (Sqrt[2]*(a - Sqrt[a^2 -
 4*b])*b^(1/4)*Sqrt[-((a*x)/b)]*(b + a*x)^(1/4)*EllipticPi[(Sqrt[2]*Sqrt[b]
)/Sqrt[-a^2 + a*Sqrt[a^2 - 4*b] + 2*b], ArcSin[(b + a*x)^(1/4)/b^(1/4)], -1
])/(Sqrt[-a^2 + a*Sqrt[a^2 - 4*b] + 2*b]*(b*x^2 + a*x^3)^(1/4))
```

Rule 106

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
1/4)), x_Symbol] :> Dist[-4, Subst[Int[x^2/((b*e - a*f - b*x^4)*Sqrt[c - (d
*e)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f
}, x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 107

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
1/4)), x_Symbol] :> Dist[Sqrt[-((f*(c + d*x))/(d*e - c*f))]/Sqrt[c + d*x],
Int[1/((a + b*x)*Sqrt[-((c*f)/(d*e - c*f)) - (d*f*x)/(d*e - c*f)]*(e + f*x)
^(1/4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[-(f/(d*e - c*f)),
0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
```

```
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{2b+ax}{\left(b+ax+x^2\right)\sqrt[4]{bx^2+ax^3}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{b+ax}\right)\int \frac{2b+ax}{\sqrt{x}\sqrt[4]{b+ax}\left(b+ax+x^2\right)}\,dx}{\sqrt[4]{bx^2+ax^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt[4]{b+ax}\right)\int \left(\frac{a-\sqrt{a^2-4b}}{\sqrt{x}\left(a-\sqrt{a^2-4b}+2x\right)\sqrt[4]{b+ax}}+\frac{a+\sqrt{a^2-4b}}{\sqrt{x}\left(a+\sqrt{a^2-4b}+2x\right)\sqrt[4]{b+ax}}\right)dx}{\sqrt[4]{bx^2+ax^3}}$$

$$= \frac{\left(\left(a-\sqrt{a^2-4b}\right)\sqrt{x}\sqrt[4]{b+ax}\right)\int \frac{1}{\sqrt{x}\left(a-\sqrt{a^2-4b}+2x\right)\sqrt[4]{b+ax}}\,dx}{\sqrt[4]{bx^2+ax^3}} + \frac{\left(\left(a+\sqrt{a^2-4\right.\right.}{}$$

$$= \frac{\left(\left(a-\sqrt{a^2-4b}\right)\sqrt{-\frac{ax}{b}}\sqrt[4]{b+ax}\right)\int \frac{1}{\sqrt{-\frac{ax}{b}}\left(a-\sqrt{a^2-4b}+2x\right)\sqrt[4]{b+ax}}\,dx}{\sqrt[4]{bx^2+ax^3}} + \frac{\left(\left(a+\sqrt{\right.\right.}{}$$

$$= -\frac{\left(4\left(a-\sqrt{a^2-4b}\right)\sqrt{-\frac{ax}{b}}\sqrt[4]{b+ax}\right)\text{Subst}\left(\int \frac{x^2}{\left(-a\left(a-\sqrt{a^2-4b}\right)+2b-2x^4\right)\sqrt{1-\frac{x^4}{b}}}\right.}{\sqrt[4]{bx^2+ax^3}}$$

$$= -\frac{\left(\sqrt{2}\left(a-\sqrt{a^2-4b}\right)\sqrt{-\frac{ax}{b}}\sqrt[4]{b+ax}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a^2+a\sqrt{a^2-4b}+2b}-\sqrt{2}x^2\right)\sqrt{\right.}}\right.}{\sqrt[4]{bx^2+ax^3}}$$

$$= \frac{\sqrt{2}\left(a+\sqrt{a^2-4b}\right)\sqrt[4]{b}\sqrt{-\frac{ax}{b}}\sqrt[4]{b+ax}\,\Pi\left(-\frac{\sqrt{2}\sqrt{b}}{\sqrt{-a^2-a\sqrt{a^2-4b}+2b}};\sin^{-1}\left(\frac{\sqrt[4]{b+ax}}{\sqrt[4]{b}}\right.\right.}{\sqrt{-a^2-a\sqrt{a^2-4b}+2b}\sqrt[4]{bx^2+ax^3}}$$

Mathematica [C] time = 7.21, size = 585, normalized size = 5.32

$$\frac{i\sqrt{2}a(ax+b)^{3/4}\sqrt{1-\frac{b}{ax+b}}\left(\left(a^2+\sqrt{a^4-4a^2b}-4b\right)\sqrt{\frac{-a^2+\sqrt{a^4-4a^2b}+2b}{b^2}}\,\Pi\left(-\frac{\sqrt{2}}{\sqrt{b}\sqrt{-\frac{a^2-2b+\sqrt{a^4-4a^2b}}{b^2}}};i\sinh^{-1}\left(\frac{\sqrt{-\sqrt{b}}}{\sqrt[4]{b+ax}}\right)\middle|-1\right)+\left(-a^2-\sqrt{a^4-4a^2b}+4b\right)\sqrt{\frac{-a^2+\sqrt{a^4-4a^2b}+2b}{b^2}}\,\Pi\left(\frac{\sqrt{2}}{\sqrt{b}\sqrt{-\frac{a^2-2b+\sqrt{a^4-4a^2b}}{b^2}}};i\sinh^{-1}\left(\frac{\sqrt{-\sqrt{b}}}{\sqrt[4]{b+ax}}\right)\middle|-1\right)+\sqrt{-\frac{a^2+\sqrt{a^4-4a^2b}-2b}{b^2}}\left(-a^2+\sqrt{a^4-4a^2b}+4b\right)\left(\Pi\left(-\frac{\sqrt{2}}{\sqrt{b}\sqrt{\frac{-a^2+2b+\sqrt{a^4-4a^2b}}{b^2}}};i\sinh^{-1}\left(\frac{\sqrt{-\sqrt{b}}}{\sqrt[4]{b+ax}}\right)\middle|-1\right)-\Pi\left(\frac{\sqrt{2}}{\sqrt{b}\sqrt{\frac{-a^2+2b+\sqrt{a^4-4a^2b}}{b^2}}};i\sinh^{-1}\left(\frac{\sqrt{-\sqrt{b}}}{\sqrt[4]{b+ax}}\right)\middle|-1\right)\right)\right)}{\sqrt{-\sqrt{b}}\,b\sqrt{a^4-4a^2b}\sqrt{-\frac{a^2+\sqrt{a^4-4a^2b}-2b}{b^2}}\sqrt{\frac{-a^2+\sqrt{a^4-4a^2b}+2b}{b^2}}\sqrt[4]{x^2(ax+b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*b + a*x)/((b + a*x + x^2)*(b*x^2 + a*x^3)^(1/4)),x]
```

```
[Out] (I*Sqrt[2]*a*(b + a*x)^(3/4)*Sqrt[1 - b/(b + a*x)]*((a^2 - 4*b + Sqrt[a^4 -
 4*a^2*b])*Sqrt[(-a^2 + 2*b + Sqrt[a^4 - 4*a^2*b])/b^2]*EllipticPi[-(Sqrt[2
```

```
]/(Sqrt[b]*Sqrt[-((a^2 - 2*b + Sqrt[a^4 - 4*a^2*b])/b^2)])), I*ArcSinh[Sqrt
[-Sqrt[b]]/(b + a*x)^(1/4)], -1] + (-a^2 + 4*b - Sqrt[a^4 - 4*a^2*b])*Sqrt[
(-a^2 + 2*b + Sqrt[a^4 - 4*a^2*b])/b^2]*EllipticPi[Sqrt[2]/(Sqrt[b]*Sqrt[-(
(a^2 - 2*b + Sqrt[a^4 - 4*a^2*b])/b^2)]), I*ArcSinh[Sqrt[-Sqrt[b]]/(b + a*x
)^(1/4)], -1] + Sqrt[-((a^2 - 2*b + Sqrt[a^4 - 4*a^2*b])/b^2)]*(-a^2 + 4*b
+ Sqrt[a^4 - 4*a^2*b])*(EllipticPi[-(Sqrt[2]/(Sqrt[b]*Sqrt[(-a^2 + 2*b + Sq
rt[a^4 - 4*a^2*b])/b^2])), I*ArcSinh[Sqrt[-Sqrt[b]]/(b + a*x)^(1/4)], -1] -
 EllipticPi[Sqrt[2]/(Sqrt[b]*Sqrt[(-a^2 + 2*b + Sqrt[a^4 - 4*a^2*b])/b^2]),
 I*ArcSinh[Sqrt[-Sqrt[b]]/(b + a*x)^(1/4)], -1])))/(Sqrt[-Sqrt[b]]*b*Sqrt[a
^4 - 4*a^2*b]*Sqrt[-((a^2 - 2*b + Sqrt[a^4 - 4*a^2*b])/b^2)]*Sqrt[(-a^2 + 2
*b + Sqrt[a^4 - 4*a^2*b])/b^2]*(x^2*(b + a*x))^(1/4))
```

IntegrateAlgebraic [A] time = 0.65, size = 110, normalized size = 1.00

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^3+bx^2}}{\sqrt{ax^3+bx^2}+x^2}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3+bx^2}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*b + a*x)/((b + a*x + x^2)*(b*x^2 + a*x^3)^(1/4)),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(-(x^2/Sqrt[2]) + Sqrt[b*x^2 + a*x^3]/Sqrt[2])/(x*(b*x^2 +
 a*x^3)^(1/4))]) + Sqrt[2]*ArcTanh[(Sqrt[2]*x*(b*x^2 + a*x^3)^(1/4))/(x^2 +
 Sqrt[b*x^2 + a*x^3])]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+2*b)/(a*x+x^2+b)/(a*x^3+b*x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+2\,b}{\left(ax^3+bx^2\right)^{\frac{1}{4}}\left(ax+x^2+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+2*b)/(a*x+x^2+b)/(a*x^3+b*x^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((a*x + 2*b)/((a*x^3 + b*x^2)^(1/4)*(a*x + x^2 + b)), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{ax+2b}{\left(ax+x^2+b\right)\left(a\,x^3+b\,x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+2*b)/(a*x+x^2+b)/(a*x^3+b*x^2)^(1/4),x)
```

```
[Out] int((a*x+2*b)/(a*x+x^2+b)/(a*x^3+b*x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+2\,b}{\left(ax^3+bx^2\right)^{\frac{1}{4}}\left(ax+x^2+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+2*b)/(a*x+x^2+b)/(a*x^3+b*x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 2*b)/((a*x^3 + b*x^2)^(1/4)*(a*x + x^2 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,b+a\,x}{{\left(a\,x^3+b\,x^2\right)}^{1/4}\,\left(x^2+a\,x+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b + a*x)/((a*x^3 + b*x^2)^(1/4)*(b + a*x + x^2)),x)
```

```
[Out] int((2*b + a*x)/((a*x^3 + b*x^2)^(1/4)*(b + a*x + x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+2b}{\sqrt[4]{x^2\,(ax+b)}\,\left(ax+b+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+2*b)/(a*x+x**2+b)/(a*x**3+b*x**2)**(1/4),x)
```

```
[Out] Integral((a*x + 2*b)/((x**2*(a*x + b))**(1/4)*(a*x + b + x**2)), x)
```

3.1379 $\int \frac{x^3\sqrt{x+x^4}}{-b+ax^3}\,dx$

Optimal. Leaf size=110

$$\frac{2\sqrt{b}\,\sqrt{-a-b}\,\tan^{-1}\left(\frac{x\sqrt{x^4+x}\,\sqrt{-a-b}}{\sqrt{b}\,(x+1)(x^2-x+1)}\right)}{3a^2}+\frac{(a+2b)\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)}{3a^2}+\frac{\sqrt{x^4+x}\,x}{3a}$$

Rubi [A] time = 0.26, antiderivative size = 129, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2042, 466, 465, 478, 523, 215, 377, 208}

$$\frac{\sqrt{x^4+x}\,(a+2b)\sinh^{-1}\left(x^{3/2}\right)}{3a^2\sqrt{x^3+1}\,\sqrt{x}}-\frac{2\sqrt{b}\,\sqrt{x^4+x}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{x^{3/2}\sqrt{a+b}}{\sqrt{b}\,\sqrt{x^3+1}}\right)}{3a^2\sqrt{x^3+1}\,\sqrt{x}}+\frac{\sqrt{x^4+x}\,x}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[x + x^4])/(-b + a*x^3),x]
```

```
[Out] (x*Sqrt[x + x^4])/(3*a) + ((a + 2*b)*Sqrt[x + x^4]*ArcSinh[x^(3/2)])/(3*a^2
*Sqrt[x]*Sqrt[1 + x^3]) - (2*Sqrt[b]*Sqrt[a + b]*Sqrt[x + x^4]*ArcTanh[(Sqr
t[a + b]*x^(3/2))/(Sqrt[b]*Sqrt[1 + x^3])])/(3*a^2*Sqrt[x]*Sqrt[1 + x^3])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
```

```
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] && !IntegerQ[p]
&& NeQ[b*c - a*d, 0] && !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sqrt{x+x^4}}{-b+ax^3}\,dx &= \frac{\sqrt{x+x^4}\int \frac{x^{7/2}\sqrt{1+x^3}}{-b+ax^3}\,dx}{\sqrt{x}\sqrt{1+x^3}}\\
&= \frac{\left(2\sqrt{x+x^4}\right)\text{Subst}\left(\int \frac{x^8\sqrt{1+x^6}}{-b+ax^6}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1+x^3}}\\
&= \frac{\left(2\sqrt{x+x^4}\right)\text{Subst}\left(\int \frac{x^2\sqrt{1+x^2}}{-b+ax^2}\,dx,x,x^{3/2}\right)}{3\sqrt{x}\sqrt{1+x^3}}\\
&= \frac{x\sqrt{x+x^4}}{3a}-\frac{\sqrt{x+x^4}\,\text{Subst}\left(\int \frac{-b+(-a-2b)x^2}{\sqrt{1+x^2}\left(-b+ax^2\right)}\,dx,x,x^{3/2}\right)}{3a\sqrt{x}\sqrt{1+x^3}}\\
&= \frac{x\sqrt{x+x^4}}{3a}-\frac{\left((-a-2b)\sqrt{x+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,x^{3/2}\right)}{3a^2\sqrt{x}\sqrt{1+x^3}}+\frac{\left(2b(a+b)\sqrt{x+x^4}\right)\text{Subs}}{3a^2\sqrt{x}}\\
&= \frac{x\sqrt{x+x^4}}{3a}+\frac{(a+2b)\sqrt{x+x^4}\,\sinh^{-1}\left(x^{3/2}\right)}{3a^2\sqrt{x}\sqrt{1+x^3}}+\frac{\left(2b(a+b)\sqrt{x+x^4}\right)\text{Subst}\left(\int \frac{1}{-b-(-a-b)x^2}\,dx\right.}{3a^2\sqrt{x}\sqrt{1+x^3}}\\
&= \frac{x\sqrt{x+x^4}}{3a}+\frac{(a+2b)\sqrt{x+x^4}\,\sinh^{-1}\left(x^{3/2}\right)}{3a^2\sqrt{x}\sqrt{1+x^3}}-\frac{2\sqrt{b}\sqrt{a+b}\sqrt{x+x^4}\,\tanh^{-1}\left(\frac{\sqrt{a+b}\,x^{3/2}}{\sqrt{b}\sqrt{1+x^3}}\right)}{3a^2\sqrt{x}\sqrt{1+x^3}}
\end{aligned}$$

Mathematica [C] time = 0.19, size = 156, normalized size = 1.42

$$-\frac{x\sqrt{x^4+x}\left(x^3(a+2b)\sqrt{-\frac{x^3(a+b)}{b}}\,F_1\left(\frac{3}{2};\frac{1}{2},1;\frac{5}{2};-x^3,\frac{ax^3}{b}\right)-3b\left(\sqrt{x^3+1}\sqrt{-\frac{x^3(a+b)}{b}}-\sin^{-1}\left(\frac{\sqrt{-\frac{x^3(a+b)}{b}}}{\sqrt{1-\frac{ax^3}{b}}}\right)\right)\right)}{9ab\sqrt{x^3+1}\sqrt{-\frac{x^3(a+b)}{b}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sqrt[x + x^4])/(-b + a*x^3),x]
```

```
[Out] -1/9*(x*Sqrt[x + x^4]*((a + 2*b)*x^3*Sqrt[-(((a + b)*x^3)/b)]*AppellF1[3/2,
 1/2, 1, 5/2, -x^3, (a*x^3)/b] - 3*b*(Sqrt[-(((a + b)*x^3)/b)]*Sqrt[1 + x^3
] - ArcSin[Sqrt[-(((a + b)*x^3)/b)]/Sqrt[1 - (a*x^3)/b]])))/(a*b*Sqrt[-(((a
 + b)*x^3)/b)]*Sqrt[1 + x^3])
```

IntegrateAlgebraic [A] time = 0.66, size = 110, normalized size = 1.00

$$\frac{2\sqrt{b}\sqrt{-a-b}\tan^{-1}\left(\frac{x\sqrt{x^4+x}\sqrt{-a-b}}{\sqrt{b}(x+1)\left(x^2-x+1\right)}\right)}{3a^2}+\frac{(a+2b)\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)}{3a^2}+\frac{\sqrt{x^4+x}\,x}{3a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[x + x^4])/(-b + a*x^3),x]
```

```
[Out] (x*Sqrt[x + x^4])/(3*a) + (2*Sqrt[-a - b]*Sqrt[b]*ArcTan[(Sqrt[-a - b]*x*Sq
rt[x + x^4])/(Sqrt[b]*(1 + x)*(1 - x + x^2))])/(3*a^2) + ((a + 2*b)*ArcTanh
[x^2/Sqrt[x + x^4]])/(3*a^2)
```

fricas [A] time = 1.72, size = 233, normalized size = 2.12

$$\left[\frac{2\sqrt{x^4+x}\,ax+(a+2b)\log\left(-2x^3-2\sqrt{x^4+x}\,x-1\right)+\sqrt{ab+b^2}\log\left(-\frac{\left(a^2+8ab+8b^2\right)x^6+2\left(3ab+4b^2\right)x^3-4\left((a+2b)x^4+bx\right)\sqrt{x^4+x}\sqrt{ab+b^2}+b^2}{a^2x^6-2abx^3+b^2}\right)}{6a^2},\ \frac{2\sqrt{x^4+x}\,ax+(a+2b)\log\left(-2x^3-2\sqrt{x^4+x}\,x-1\right)+2\sqrt{-ab-b^2}\arctan\left(\frac{2\sqrt{x^4+x}\sqrt{-ab-b^2}\,x}{(a+2b)x^3+b}\right)}{6a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+x)^(1/2)/(a*x^3-b),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(x^4 + x)*a*x + (a + 2*b)*log(-2*x^3 - 2*sqrt(x^4 + x)*x - 1) +
 sqrt(a*b + b^2)*log(-((a^2 + 8*a*b + 8*b^2)*x^6 + 2*(3*a*b + 4*b^2)*x^3 -
4*((a + 2*b)*x^4 + b*x)*sqrt(x^4 + x)*sqrt(a*b + b^2) + b^2)/(a^2*x^6 - 2*a
*b*x^3 + b^2)))/a^2, 1/6*(2*sqrt(x^4 + x)*a*x + (a + 2*b)*log(-2*x^3 - 2*sq
rt(x^4 + x)*x - 1) + 2*sqrt(-a*b - b^2)*arctan(2*sqrt(x^4 + x)*sqrt(-a*b -
b^2)*x/((a + 2*b)*x^3 + b)))/a^2]
```

giac [A] time = 0.33, size = 101, normalized size = 0.92

$$\frac{\sqrt{x^4+x}\,x}{3a}+\frac{(a+2b)\log\left(\sqrt{\frac{1}{x^3}+1}+1\right)}{6a^2}-\frac{(a+2b)\log\left(\left|\sqrt{\frac{1}{x^3}+1}-1\right|\right)}{6a^2}+\frac{2\left(ab+b^2\right)\arctan\left(\frac{b\sqrt{\frac{1}{x^3}+1}}{\sqrt{-ab-b^2}}\right)}{3\sqrt{-ab-b^2}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+x)^(1/2)/(a*x^3-b),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(x^4 + x)*x/a + 1/6*(a + 2*b)*log(sqrt(1/x^3 + 1) + 1)/a^2 - 1/6*(a
 + 2*b)*log(abs(sqrt(1/x^3 + 1) - 1))/a^2 + 2/3*(a*b + b^2)*arctan(b*sqrt(1
/x^3 + 1)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a^2)
```

maple [C] time = 0.39, size = 963, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4+x)^(1/2)/(a*x^3-b),x)
```

```
[Out] 1/a*(1/3*x*(x^4+x)^(1/2)-(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1
/2*I*3^(1/2))/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/
2))/(1+x))^(1/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(
3/2+1/2*I*3^(1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1
```

```
/2)*(-EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-
3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2))+EllipticPi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))
^(1/2),(1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-
1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))+b/a*(-2/
a*(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1
/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))/(1+x))^(1/2)*(-(x-1
/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(3/2+1/2*I*3^(1/2))/(x*(
1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1/2)*(-EllipticF(((3/2+1
/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/
2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+Elliptic
Pi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),(1/2+1/2*I*3^(1/
2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))-2/3/a*4^(1/2)*sum((-a-b)/_alpha*
(1+x)^2*(_alpha^2-_alpha+1)/(a+b)*(-1-I*3^(1/2))*(x/(1+x)*(I*3^(1/2)+3)/(1+
I*3^(1/2)))^(1/2)*(-1/(1+x)*(-1+2*x+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)*(-1/(1+
x)*(-1+2*x-I*3^(1/2))/(1+I*3^(1/2)))^(1/2)/(I*3^(1/2)+3)/(x*(1+x)*(-1+2*x+I
*3^(1/2))*(-1+2*x-I*3^(1/2)))^(1/2)*(EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+
1/2*I*3^(1/2))/(1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/
2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+_alpha^2*a/b*EllipticPi(((3/2
+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),1/6*(I*_alpha^2*3^(1/2)*
a+3*_alpha^2*a+I*3^(1/2)*b+3*b)/b,((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2)
)/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))),_alpha=RootOf(_Z^3*a-b
)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+x}\,x^3}{ax^3-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+x)^(1/2)/(a*x^3-b),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x)*x^3/(a*x^3 - b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^3\,\sqrt{x^4+x}}{b-a\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(x + x^4)^(1/2))/(b - a*x^3),x)
```

```
[Out] -int((x^3*(x + x^4)^(1/2))/(b - a*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{x\,(x+1)\left(x^2-x+1\right)}}{ax^3-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**4+x)**(1/2)/(a*x**3-b),x)
```

```
[Out] Integral(x**3*sqrt(x*(x + 1)*(x**2 - x + 1))/(a*x**3 - b), x)
```

3.1380 $\int x^9 \sqrt[3]{1+x^6}\, dx$

Optimal. Leaf size=110

$$\frac{1}{54}\log\left(\sqrt[3]{x^6+1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{18\sqrt{3}}+\frac{1}{36}\sqrt[3]{x^6+1}\left(3x^{10}+x^4\right)-\frac{1}{108}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right)$$

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {275, 279, 321, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{12}\sqrt[3]{x^6+1}\,x^{10}+\frac{1}{36}\sqrt[3]{x^6+1}\,x^4+\frac{1}{54}\log\left(1-\frac{x^2}{\sqrt[3]{x^6+1}}\right)+\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6+1}}+1}{\sqrt{3}}\right)}{18\sqrt{3}}-\frac{1}{108}\log\left(\frac{x^4}{\left(x^6+1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6+1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(1 + x^6)^(1/3),x]
```

```
[Out] (x^4*(1 + x^6)^(1/3))/36 + (x^10*(1 + x^6)^(1/3))/12 + ArcTan[(1 + (2*x^2)/
(1 + x^6)^(1/3))/Sqrt[3]]/(18*Sqrt[3]) + Log[1 - x^2/(1 + x^6)^(1/3)]/54 -
Log[1 + x^4/(1 + x^6)^(2/3) + x^2/(1 + x^6)^(1/3)]/108
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int x^9\sqrt[3]{1+x^6}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^4\sqrt[3]{1+x^3}\,dx,x,x^2\right)\\
&= \frac{1}{12}x^{10}\sqrt[3]{1+x^6}+\frac{1}{12}\operatorname{Subst}\left(\int \frac{x^4}{\left(1+x^3\right)^{2/3}}\,dx,x,x^2\right)\\
&= \frac{1}{36}x^4\sqrt[3]{1+x^6}+\frac{1}{12}x^{10}\sqrt[3]{1+x^6}-\frac{1}{18}\operatorname{Subst}\left(\int \frac{x}{\left(1+x^3\right)^{2/3}}\,dx,x,x^2\right)\\
&= \frac{1}{36}x^4\sqrt[3]{1+x^6}+\frac{1}{12}x^{10}\sqrt[3]{1+x^6}-\frac{1}{18}\operatorname{Subst}\left(\int \frac{x}{1-x^3}\,dx,x,\frac{x^2}{\sqrt[3]{1+x^6}}\right)\\
&= \frac{1}{36}x^4\sqrt[3]{1+x^6}+\frac{1}{12}x^{10}\sqrt[3]{1+x^6}-\frac{1}{54}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx,x,\frac{x^2}{\sqrt[3]{1+x^6}}\right)+\frac{1}{54}\operatorname{Subst}\left(\int \frac{1}{1+\cdots}\right.\\
&= \frac{1}{36}x^4\sqrt[3]{1+x^6}+\frac{1}{12}x^{10}\sqrt[3]{1+x^6}+\frac{1}{54}\log\left(1-\frac{x^2}{\sqrt[3]{1+x^6}}\right)-\frac{1}{108}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,\cdots\right.\\
&= \frac{1}{36}x^4\sqrt[3]{1+x^6}+\frac{1}{12}x^{10}\sqrt[3]{1+x^6}+\frac{1}{54}\log\left(1-\frac{x^2}{\sqrt[3]{1+x^6}}\right)-\frac{1}{108}\log\left(1+\frac{x^4}{\left(1+x^6\right)^{2/3}}+\frac{x\cdots}{\sqrt[3]{1\cdots}}\right.\\
&= \frac{1}{36}x^4\sqrt[3]{1+x^6}+\frac{1}{12}x^{10}\sqrt[3]{1+x^6}+\frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{1+x^6}}}{\sqrt{3}}\right)}{18\sqrt{3}}+\frac{1}{54}\log\left(1-\frac{x^2}{\sqrt[3]{1+x^6}}\right)-\frac{1}{108}\log\left(1-\cdots\right.
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.31

$$\frac{1}{12}x^4\left(\left(x^6+1\right)^{4/3}-{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-x^6\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(1 + x^6)^(1/3),x]
[Out] (x^4*((1 + x^6)^(4/3) - Hypergeometric2F1[-1/3, 2/3, 5/3, -x^6]))/12
```

IntegrateAlgebraic [A] time = 0.88, size = 110, normalized size = 1.00

$$\frac{1}{54}\log\left(\sqrt[3]{x^6+1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{18\sqrt{3}}+\frac{1}{36}\sqrt[3]{x^6+1}\left(3x^{10}+x^4\right)-\frac{1}{108}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9*(1 + x^6)^(1/3),x]
[Out] ((1 + x^6)^(1/3)*(x^4 + 3*x^10))/36 + ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(1 + x^
6)^(1/3))]/(18*Sqrt[3]) + Log[-x^2 + (1 + x^6)^(1/3)]/54 - Log[x^4 + x^2*(1
 + x^6)^(1/3) + (1 + x^6)^(2/3)]/108
```

fricas [A] time = 0.41, size = 100, normalized size = 0.91

$$-\frac{1}{54}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6+1\right)^{\frac{1}{3}}}{3x^2}\right)+\frac{1}{36}\left(3x^{10}+x^4\right)\left(x^6+1\right)^{\frac{1}{3}}+\frac{1}{54}\log\left(-\frac{x^2-\left(x^6+1\right)^{\frac{1}{3}}}{x^2}\right)-\frac{1}{108}\log\left(\frac{x^4+\left(x^6+1\right)^{\frac{1}{3}}x^2+\left(x^6+1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(x^6+1)^(1/3),x, algorithm="fricas")
[Out] -1/54*sqrt(3)*arctan(1/3*(sqrt(3)*x^2 + 2*sqrt(3)*(x^6 + 1)^(1/3))/x^2) + 1
/36*(3*x^10 + x^4)*(x^6 + 1)^(1/3) + 1/54*log(-(x^2 - (x^6 + 1)^(1/3))/x^2)
 - 1/108*log((x^4 + (x^6 + 1)^(1/3)*x^2 + (x^6 + 1)^(2/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^6+1\right)^{\frac{1}{3}}x^9\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(x^6+1)^(1/3),x, algorithm="giac")
[Out] integrate((x^6 + 1)^(1/3)*x^9, x)
```

maple [C] time = 0.29, size = 37, normalized size = 0.34

$$\frac{x^4\left(3x^6+1\right)\left(x^6+1\right)^{\frac{1}{3}}}{36}-\frac{x^4\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^6\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(x^6+1)^(1/3),x)
[Out] 1/36*x^4*(3*x^6+1)*(x^6+1)^(1/3)-1/36*x^4*hypergeom([2/3,2/3],[5/3],-x^6)
```

maxima [A] time = 0.42, size = 121, normalized size = 1.10

$$-\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)-\frac{\frac{2\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6+1\right)^{\frac{4}{3}}}{x^8}}{36\left(\frac{2\left(x^6+1\right)}{x^6}-\frac{\left(x^6+1\right)^2}{x^{12}}-1\right)}-\frac{1}{108}\log\left(\frac{\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6+1\right)^{\frac{2}{3}}}{x^4}+1\right)+\frac{1}{54}\log\left(\frac{\left(x^6+1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(x^6+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/54*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3)/x^2 + 1)) - 1/36*(2*(x^
6 + 1)^(1/3)/x^2 + (x^6 + 1)^(4/3)/x^8)/(2*(x^6 + 1)/x^6 - (x^6 + 1)^2/x^12
 - 1) - 1/108*log((x^6 + 1)^(1/3)/x^2 + (x^6 + 1)^(2/3)/x^4 + 1) + 1/54*log
((x^6 + 1)^(1/3)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^9 \left(x^6+1\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(x^6 + 1)^(1/3),x)
```

```
[Out] int(x^9*(x^6 + 1)^(1/3), x)
```

sympy [C] time = 1.18, size = 31, normalized size = 0.28

$$\frac{x^{10}\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle| x^6 e^{i\pi}\right)}{6\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(x**6+1)**(1/3),x)
```

```
[Out] x**10*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), x**6*exp_polar(I*pi))/(6*gamma(
8/3))
```

3.1381 $\int \frac{b-ax^3+x^6}{x^6 \sqrt[4]{bx+ax^4}}\,dx$

Optimal. Leaf size=110

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}+\frac{4\left(ax^4+bx\right)^{3/4}\left(11ax^3-3b\right)}{63bx^6}$$

Rubi [A] time = 0.27, antiderivative size = 167, normalized size of antiderivative = 1.52, number of steps used = 12, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2052, 2011, 329, 275, 240, 212, 206, 203, 2016, 2014}

$$-\frac{4\left(ax^4+bx\right)^{3/4}}{21x^6}+\frac{44a\left(ax^4+bx\right)^{3/4}}{63bx^3}+\frac{2\sqrt[4]{x}\sqrt[4]{ax^3+b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{a}\sqrt[4]{ax^4+bx}}+\frac{2\sqrt[4]{x}\sqrt[4]{ax^3+b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{a}\sqrt[4]{ax^4+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(b - a*x^3 + x^6)/(x^6*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(b*x + a*x^4)^(3/4))/(21*x^6) + (44*a*(b*x + a*x^4)^(3/4))/(63*b*x^3) +
 (2*x^(1/4)*(b + a*x^3)^(1/4)*ArcTan[(a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)])/
(3*a^(1/4)*(b*x + a*x^4)^(1/4)) + (2*x^(1/4)*(b + a*x^3)^(1/4)*ArcTanh[(a^(
1/4)*x^(3/4))/(b + a*x^3)^(1/4)])/(3*a^(1/4)*(b*x + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{b-ax^3+x^6}{x^6\sqrt[4]{bx+ax^4}}\,dx = \int \left(\frac{1}{\sqrt[4]{bx+ax^4}}+\frac{b}{x^6\sqrt[4]{bx+ax^4}}-\frac{a}{x^3\sqrt[4]{bx+ax^4}}\right)dx$$

$$= -\left(a\int \frac{1}{x^3\sqrt[4]{bx+ax^4}}\,dx\right)+b\int \frac{1}{x^6\sqrt[4]{bx+ax^4}}\,dx+\int \frac{1}{\sqrt[4]{bx+ax^4}}\,dx$$

$$= -\frac{4\left(bx+ax^4\right)^{3/4}}{21x^6}+\frac{4a\left(bx+ax^4\right)^{3/4}}{9bx^3}-\frac{1}{7}(4a)\int \frac{1}{x^3\sqrt[4]{bx+ax^4}}\,dx+\frac{\left(\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\int \frac{}{\sqrt[4]{x}\,\sqrt[4]{}}}{\sqrt[4]{bx+ax^4}}$$

$$= -\frac{4\left(bx+ax^4\right)^{3/4}}{21x^6}+\frac{44a\left(bx+ax^4\right)^{3/4}}{63bx^3}+\frac{\left(4\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt[4]{b+ax^{12}}}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^4}}$$

$$= -\frac{4\left(bx+ax^4\right)^{3/4}}{21x^6}+\frac{44a\left(bx+ax^4\right)^{3/4}}{63bx^3}+\frac{\left(4\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx,x,x^{3/4}\right)}{3\sqrt[4]{bx+ax^4}}$$

$$= -\frac{4\left(bx+ax^4\right)^{3/4}}{21x^6}+\frac{44a\left(bx+ax^4\right)^{3/4}}{63bx^3}+\frac{\left(4\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^4}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}$$

$$= -\frac{4\left(bx+ax^4\right)^{3/4}}{21x^6}+\frac{44a\left(bx+ax^4\right)^{3/4}}{63bx^3}+\frac{\left(2\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}+$$

$$= -\frac{4\left(bx+ax^4\right)^{3/4}}{21x^6}+\frac{44a\left(bx+ax^4\right)^{3/4}}{63bx^3}+\frac{2\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{a}\,\sqrt[4]{bx+ax^4}}+\frac{2\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\,\mathrm{ta}}{3\sqrt[4]{a}\,\sqrt[4]{b}}$$

Mathematica [C] time = 0.05, size = 83, normalized size = 0.75

$$\frac{4\left(11a^2x^6+21bx^6\sqrt[4]{\frac{ax^3}{b}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{ax^3}{b}\right)+8abx^3-3b^2\right)}{63bx^5\sqrt[4]{x\left(ax^3+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b - a*x^3 + x^6)/(x^6*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*(-3*b^2 + 8*a*b*x^3 + 11*a^2*x^6 + 21*b*x^6*(1 + (a*x^3)/b)^(1/4)*Hyperg
eometric2F1[1/4, 1/4, 5/4, -((a*x^3)/b)]))/(63*b*x^5*(x*(b + a*x^3))^(1/4))
```

IntegrateAlgebraic [A] time = 0.55, size = 110, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}+\frac{4\left(ax^4+bx\right)^{3/4}\left(11ax^3-3b\right)}{63bx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b - a*x^3 + x^6)/(x^6*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*(-3*b + 11*a*x^3)*(b*x + a*x^4)^(3/4))/(63*b*x^6) + (2*ArcTan[(a^(1/4)*(
b*x + a*x^4)^(3/4))/(b + a*x^3)])/(3*a^(1/4)) + (2*ArcTanh[(a^(1/4)*(b*x +
a*x^4)^(3/4))/(b + a*x^3)])/(3*a^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-a*x^3+b)/x^6/(a*x^4+b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.28, size = 212, normalized size = 1.93

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}+2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)}{3\left(-a\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}-2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)}{3\left(-a\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)}{6\left(-a\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\left(-a\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\left(-a\right)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)}{6a}-\frac{4\left(3\left(a+\frac{b}{x^3}\right)^{\frac{7}{4}}b^6-14\left(a+\frac{b}{x^3}\right)^{\frac{3}{4}}ab^6\right)}{63b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-a*x^3+b)/x^6/(a*x^4+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^3)^(1/4))/
(-a)^(1/4))/(-a)^(1/4) - 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4
) - 2*(a + b/x^3)^(1/4))/(-a)^(1/4))/(-a)^(1/4) + 1/6*sqrt(2)*log(sqrt(2)*(
-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a) + sqrt(a + b/x^3))/(-a)^(1/4) + 1/6*
sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a) + s
qrt(a + b/x^3))/a - 4/63*(3*(a + b/x^3)^(7/4)*b^6 - 14*(a + b/x^3)^(3/4)*a*
b^6)/b^7
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^{6}-a\,x^{3}+b}{x^{6}\left(a\,x^{4}+b x\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-a*x^3+b)/x^6/(a*x^4+b*x)^(1/4),x)
```

```
[Out] int((x^6-a*x^3+b)/x^6/(a*x^4+b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}-a x^{3}+b}{\left(a x^{4}+b x\right)^{\frac{1}{4}} x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-a*x^3+b)/x^6/(a*x^4+b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^6 - a*x^3 + b)/((a*x^4 + b*x)^(1/4)*x^6), x)
```

mupad [B] time = 1.20, size = 77, normalized size = 0.70

$$\frac{4\,x\,{\left(\frac{a\,x^3}{b}+1\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{a\,x^3}{b}\right)}{3\,{\left(a\,x^4+b\,x\right)}^{1/4}}-\frac{4\,{\left(a\,x^4+b\,x\right)}^{3/4}}{21\,x^6}+\frac{44\,a\,{\left(a\,x^4+b\,x\right)}^{3/4}}{63\,b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a*x^3 + x^6)/(x^6*(b*x + a*x^4)^(1/4)),x)
```

```
[Out] (4*x*((a*x^3)/b + 1)^(1/4)*hypergeom([1/4, 1/4], 5/4, -(a*x^3)/b))/(3*(b*x
+ a*x^4)^(1/4)) - (4*(b*x + a*x^4)^(3/4))/(21*x^6) + (44*a*(b*x + a*x^4)^(3
/4))/(63*b*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-ax^{3}+b+x^{6}}{x^{6}\sqrt[4]{x\left(ax^{3}+b\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-a*x**3+b)/x**6/(a*x**4+b*x)**(1/4),x)
```

```
[Out] Integral((-a*x**3 + b + x**6)/(x**6*(x*(a*x**3 + b))**(1/4)), x)
```

3.1382 $\int \frac{-b+ax^6}{x^6 \sqrt[4]{bx+ax^4}} \, dx$

Optimal. Leaf size=110

$$\frac{2}{3}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)+\frac{2}{3}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)-\frac{4\left(ax^4+bx\right)^{3/4}\left(4ax^3-3b\right)}{63bx^6}$$

Rubi [A] time = 0.25, antiderivative size = 167, normalized size of antiderivative = 1.52, number of steps used = 11, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2052, 2011, 329, 275, 240, 212, 206, 203, 2016, 2014}

$$\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3+b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{ax^4+bx}}+\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3+b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{ax^4+bx}}+\frac{4\left(ax^4+bx\right)^{3/4}}{21x^6}-\frac{16a\left(ax^4+bx\right)^{3/4}}{63bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^6)/(x^6*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*(b*x + a*x^4)^(3/4))/(21*x^6) - (16*a*(b*x + a*x^4)^(3/4))/(63*b*x^3) +
(2*a^(3/4)*x^(1/4)*(b + a*x^3)^(1/4)*ArcTan[(a^(1/4)*x^(3/4))/(b + a*x^3)^(
1/4)])/(3*(b*x + a*x^4)^(1/4)) + (2*a^(3/4)*x^(1/4)*(b + a*x^3)^(1/4)*ArcTa
nh[(a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)])/(3*(b*x + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{-b+ax^6}{x^6\sqrt[4]{bx+ax^4}}\,dx = \int \left(\frac{a}{\sqrt[4]{bx+ax^4}} - \frac{b}{x^6\sqrt[4]{bx+ax^4}}\right) dx$$
$$= a\int \frac{1}{\sqrt[4]{bx+ax^4}}\,dx - b\int \frac{1}{x^6\sqrt[4]{bx+ax^4}}\,dx$$
$$= \frac{4\left(bx+ax^4\right)^{3/4}}{21x^6} + \frac{1}{7}(4a)\int \frac{1}{x^3\sqrt[4]{bx+ax^4}}\,dx + \frac{\left(a\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\int \frac{1}{\sqrt[4]{x}\sqrt[4]{b+ax^3}}\,dx}{\sqrt[4]{bx+ax^4}}$$
$$= \frac{4\left(bx+ax^4\right)^{3/4}}{21x^6} - \frac{16a\left(bx+ax^4\right)^{3/4}}{63bx^3} + \frac{\left(4a\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{b+ax^{12}}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^4}}$$
$$= \frac{4\left(bx+ax^4\right)^{3/4}}{21x^6} - \frac{16a\left(bx+ax^4\right)^{3/4}}{63bx^3} + \frac{\left(4a\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx, x, x^{3/4}\right)}{3\sqrt[4]{bx+ax^4}}$$
$$= \frac{4\left(bx+ax^4\right)^{3/4}}{21x^6} - \frac{16a\left(bx+ax^4\right)^{3/4}}{63bx^3} + \frac{\left(4a\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx, x, \frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}$$
$$= \frac{4\left(bx+ax^4\right)^{3/4}}{21x^6} - \frac{16a\left(bx+ax^4\right)^{3/4}}{63bx^3} + \frac{\left(2a\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}$$
$$= \frac{4\left(bx+ax^4\right)^{3/4}}{21x^6} - \frac{16a\left(bx+ax^4\right)^{3/4}}{63bx^3} + \frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{b+ax^3}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}} + \frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{b}}{}$$

Mathematica [A] time = 0.13, size = 138, normalized size = 1.25

$$\frac{2\left(21a^{3/4}bx^{21/4}\sqrt[4]{ax^3+b}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3+b}}\right)+21a^{3/4}bx^{21/4}\sqrt[4]{ax^3+b}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3+b}}\right)-8a^2x^6-2abx^3+6b^2\right)}{63bx^5\sqrt[4]{x\left(ax^3+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^6)/(x^6*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (2*(6*b^2 - 2*a*b*x^3 - 8*a^2*x^6 + 21*a^(3/4)*b*x^(21/4)*(b + a*x^3)^(1/4)
*ArcTan[(a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)] + 21*a^(3/4)*b*x^(21/4)*(b + a
*x^3)^(1/4)*ArcTanh[(a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)]))/(63*b*x^5*(x*(b
+ a*x^3))^(1/4))
```

IntegrateAlgebraic [A] time = 0.71, size = 110, normalized size = 1.00

$$\frac{2}{3}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)+\frac{2}{3}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)-\frac{4\left(ax^4+bx\right)^{3/4}\left(4ax^3-3b\right)}{63bx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^6)/(x^6*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(-3*b + 4*a*x^3)*(b*x + a*x^4)^(3/4))/(63*b*x^6) + (2*a^(3/4)*ArcTan[(a
^(1/4)*(b*x + a*x^4)^(3/4))/(b + a*x^3)])/3 + (2*a^(3/4)*ArcTanh[(a^(1/4)*(
b*x + a*x^4)^(3/4))/(b + a*x^3)])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-b)/x^6/(a*x^4+b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.25, size = 209, normalized size = 1.90

$$\frac{1}{3}\sqrt{2}\left(-a\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}+2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)+\frac{1}{3}\sqrt{2}\left(-a\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}-2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)-\frac{1}{6}\sqrt{2}\left(-a\right)^{\frac{3}{4}}\log\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)+\frac{1}{6}\sqrt{2}\left(-a\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\left(-a\right)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)+\frac{4\left(3\left(a+\frac{b}{x^3}\right)^{\frac{7}{4}}b^6-7\left(a+\frac{b}{x^3}\right)^{\frac{3}{4}}ab^6\right)}{63\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-b)/x^6/(a*x^4+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^
3)^(1/4))/(-a)^(1/4)) + 1/3*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(-a)^(1/4) - 2*(a + b/x^3)^(1/4))/(-a)^(1/4)) - 1/6*sqrt(2)*(-a)^(3/4)*log
(sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a) + sqrt(a + b/x^3)) + 1/6*s
qrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a) + sq
rt(a + b/x^3)) + 4/63*(3*(a + b/x^3)^(7/4)*b^6 - 7*(a + b/x^3)^(3/4)*a*b^6)
/b^7
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{a\,x^6-b}{x^6\left(a\,x^4+bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6-b)/x^6/(a*x^4+b*x)^(1/4),x)
```

```
[Out] int((a*x^6-b)/x^6/(a*x^4+b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^6-b}{\left(ax^4+bx\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-b)/x^6/(a*x^4+b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^6 - b)/((a*x^4 + b*x)^(1/4)*x^6), x)
```

mupad [B] time = 1.30, size = 71, normalized size = 0.65

$$\frac{4\left(a\,x^4+b\,x\right)^{3/4}\,\left(3\,b-4\,a\,x^3\right)}{63\,b\,x^6}+\frac{4\,a\,x\,{\left(\frac{a\,x^3}{b}+1\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{a\,x^3}{b}\right)}{3\,{\left(a\,x^4+b\,x\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^6)/(x^6*(b*x + a*x^4)^(1/4)),x)
```

```
[Out] (4*(b*x + a*x^4)^(3/4)*(3*b - 4*a*x^3))/(63*b*x^6) + (4*a*x*((a*x^3)/b + 1)
^(1/4)*hypergeom([1/4, 1/4], 5/4, -(a*x^3)/b))/(3*(b*x + a*x^4)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^6-b}{x^6\sqrt[4]{x\left(ax^3+b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6-b)/x**6/(a*x**4+b*x)**(1/4),x)
```

```
[Out] Integral((a*x**6 - b)/(x**6*(x*(a*x**3 + b))**(1/4)), x)
```

3.1383 $\int \frac{b+ax^6}{x^6 \sqrt[4]{bx+ax^4}} dx$

Optimal. Leaf size=110

$$\frac{2}{3}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)+\frac{2}{3}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)+\frac{4\left(ax^4+bx\right)^{3/4}\left(4ax^3-3b\right)}{63bx^6}$$

Rubi [A] time = 0.26, antiderivative size = 167, normalized size of antiderivative = 1.52, number of steps used = 11, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2052, 2011, 329, 275, 240, 212, 206, 203, 2016, 2014}

$$\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3+b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{ax^4+bx}}+\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3+b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{ax^4+bx}}-\frac{4\left(ax^4+bx\right)^{3/4}}{21x^6}+\frac{16a\left(ax^4+bx\right)^{3/4}}{63bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^6)/(x^6*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(b*x + a*x^4)^(3/4))/(21*x^6) + (16*a*(b*x + a*x^4)^(3/4))/(63*b*x^3) +
 (2*a^(3/4)*x^(1/4)*(b + a*x^3)^(1/4)*ArcTan[(a^(1/4)*x^(3/4))/(b + a*x^3)^
(1/4)])/(3*(b*x + a*x^4)^(1/4)) + (2*a^(3/4)*x^(1/4)*(b + a*x^3)^(1/4)*ArcT
anh[(a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)])/(3*(b*x + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{b+ax^6}{x^6\sqrt[4]{bx+ax^4}}\,dx = \int \left(\frac{a}{\sqrt[4]{bx+ax^4}}+\frac{b}{x^6\sqrt[4]{bx+ax^4}}\right)dx$$
$$= a\int \frac{1}{\sqrt[4]{bx+ax^4}}\,dx + b\int \frac{1}{x^6\sqrt[4]{bx+ax^4}}\,dx$$
$$= -\frac{4\left(bx+ax^4\right)^{3/4}}{21x^6} - \frac{1}{7}(4a)\int \frac{1}{x^3\sqrt[4]{bx+ax^4}}\,dx + \frac{\left(a\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\int \frac{1}{\sqrt[4]{x}\,\sqrt[4]{b+ax^3}}\,dx}{\sqrt[4]{bx+ax^4}}$$
$$= -\frac{4\left(bx+ax^4\right)^{3/4}}{21x^6} + \frac{16a\left(bx+ax^4\right)^{3/4}}{63bx^3} + \frac{\left(4a\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{b+ax^{12}}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^4}}$$
$$= -\frac{4\left(bx+ax^4\right)^{3/4}}{21x^6} + \frac{16a\left(bx+ax^4\right)^{3/4}}{63bx^3} + \frac{\left(4a\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx, x, x^{3/4}\right)}{3\sqrt[4]{bx+ax^4}}$$
$$= -\frac{4\left(bx+ax^4\right)^{3/4}}{21x^6} + \frac{16a\left(bx+ax^4\right)^{3/4}}{63bx^3} + \frac{\left(4a\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx, x, \frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}$$
$$= -\frac{4\left(bx+ax^4\right)^{3/4}}{21x^6} + \frac{16a\left(bx+ax^4\right)^{3/4}}{63bx^3} + \frac{\left(2a\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}$$
$$= -\frac{4\left(bx+ax^4\right)^{3/4}}{21x^6} + \frac{16a\left(bx+ax^4\right)^{3/4}}{63bx^3} + \frac{2a^{3/4}\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}} + \frac{2a^{3/4}\sqrt[4]{x}\,\sqrt[4]{b-}}{3}$$

Mathematica [A] time = 0.08, size = 138, normalized size = 1.25

$$\frac{2\left(21a^{3/4}bx^{21/4}\sqrt[4]{ax^3+b}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3+b}}\right)+21a^{3/4}bx^{21/4}\sqrt[4]{ax^3+b}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3+b}}\right)+8a^2x^6+2abx^3-6b^2\right)}{63bx^5\sqrt[4]{x\left(ax^3+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^6)/(x^6*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (2*(-6*b^2 + 2*a*b*x^3 + 8*a^2*x^6 + 21*a^(3/4)*b*x^(21/4)*(b + a*x^3)^(1/4
)*ArcTan[(a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)] + 21*a^(3/4)*b*x^(21/4)*(b +
a*x^3)^(1/4)*ArcTanh[(a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)]))/(63*b*x^5*(x*(b
 + a*x^3))^(1/4))
```

IntegrateAlgebraic [A] time = 0.73, size = 110, normalized size = 1.00

$$\frac{2}{3}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)+\frac{2}{3}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)+\frac{4\left(ax^4+bx\right)^{3/4}\left(4ax^3-3b\right)}{63bx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^6)/(x^6*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*(-3*b + 4*a*x^3)*(b*x + a*x^4)^(3/4))/(63*b*x^6) + (2*a^(3/4)*ArcTan[(a^
(1/4)*(b*x + a*x^4)^(3/4))/(b + a*x^3)])/3 + (2*a^(3/4)*ArcTanh[(a^(1/4)*(b
*x + a*x^4)^(3/4))/(b + a*x^3)])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^6/(a*x^4+b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.29, size = 209, normalized size = 1.90

$$\frac{1}{3}\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)+\frac{1}{3}\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)-\frac{1}{6}\sqrt{2}(-a)^{\frac{3}{4}}\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)+\frac{1}{6}\sqrt{2}(-a)^{\frac{3}{4}}\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)-\frac{4\left(3\left(a+\frac{b}{x^3}\right)^{\frac{7}{4}}b^6-7\left(a+\frac{b}{x^3}\right)^{\frac{3}{4}}ab^6\right)}{63b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^6/(a*x^4+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^
3)^(1/4))/(-a)^(1/4)) + 1/3*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(-a)^(1/4) - 2*(a + b/x^3)^(1/4))/(-a)^(1/4)) - 1/6*sqrt(2)*(-a)^(3/4)*log
(sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a) + sqrt(a + b/x^3)) + 1/6*s
qrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a) + sq
rt(a + b/x^3)) - 4/63*(3*(a + b/x^3)^(7/4)*b^6 - 7*(a + b/x^3)^(3/4)*a*b^6)
/b^7
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{a\,x^{6}+b}{x^{6}\left(a\,x^{4}+bx\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6+b)/x^6/(a*x^4+b*x)^(1/4),x)
```

```
[Out] int((a*x^6+b)/x^6/(a*x^4+b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{6}+b}{\left(ax^{4}+bx\right)^{\frac{1}{4}}x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^6/(a*x^4+b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^6 + b)/((a*x^4 + b*x)^(1/4)*x^6), x)
```

mupad [B] time = 1.24, size = 71, normalized size = 0.65

$$\frac{4\,a\,x\,{\left(\frac{a\,x^3}{b}+1\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{a\,x^3}{b}\right)}{3\,{\left(a\,x^4+b\,x\right)}^{1/4}}-\frac{4\,{\left(a\,x^4+b\,x\right)}^{3/4}\,\left(3\,b-4\,a\,x^3\right)}{63\,b\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^6)/(x^6*(b*x + a*x^4)^(1/4)),x)
```

```
[Out] (4*a*x*((a*x^3)/b + 1)^(1/4)*hypergeom([1/4, 1/4], 5/4, -(a*x^3)/b))/(3*(b*
x + a*x^4)^(1/4)) - (4*(b*x + a*x^4)^(3/4)*(3*b - 4*a*x^3))/(63*b*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{6}+b}{x^{6}\sqrt[4]{x\left(ax^{3}+b\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6+b)/x**6/(a*x**4+b*x)**(1/4),x)
```

```
[Out] Integral((a*x**6 + b)/(x**6*(x*(a*x**3 + b))**(1/4)), x)
```

3.1384 $\int \frac{-3b+2ax^5}{(2b+x^3+2ax^5)\sqrt[4]{bx+ax^6}}\,dx$

Optimal. Leaf size=110

$$-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{ax^6+bx}}{\sqrt{2}\sqrt{ax^6+bx}-x^2}\right)}{\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6+bx}}{\sqrt[4]{2}}+\frac{x^2}{2^{3/4}}}{x\sqrt[4]{ax^6+bx}}\right)}{\sqrt[4]{2}}$$

Rubi [F] time = 2.41, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3b+2ax^5}{(2b+x^3+2ax^5)\sqrt[4]{bx+ax^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3*b + 2*a*x^5)/((2*b + x^3 + 2*a*x^5)*(b*x + a*x^6)^(1/4)),x]
```

```
[Out] (4*x*(1 + (a*x^5)/b)^(1/4)*Hypergeometric2F1[3/20, 1/4, 23/20, -((a*x^5)/b)
])/(3*(b*x + a*x^6)^(1/4)) - (20*b*x^(1/4)*(b + a*x^5)^(1/4)*Defer[Subst][D
efer[Int][x^2/((b + a*x^20)^(1/4)*(2*b + x^12 + 2*a*x^20)), x], x, x^(1/4)]
)/(b*x + a*x^6)^(1/4) - (4*x^(1/4)*(b + a*x^5)^(1/4)*Defer[Subst][Defer[Int
][x^14/((b + a*x^20)^(1/4)*(2*b + x^12 + 2*a*x^20)), x], x, x^(1/4)])/(b*x
+ a*x^6)^(1/4)
```

Rubi steps

$$\int \frac{-3b+2ax^5}{(2b+x^3+2ax^5)\sqrt[4]{bx+ax^6}}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^5}\right)\int \frac{-3b+2ax^5}{\sqrt[4]{x}\sqrt[4]{b+ax^5}(2b+x^3+2ax^5)}\,dx}{\sqrt[4]{bx+ax^6}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^5}\right)\text{Subst}\left(\int \frac{x^2(-3b+2ax^{20})}{\sqrt[4]{b+ax^{20}}(2b+x^{12}+2ax^{20})}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^6}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^5}\right)\text{Subst}\left(\int \left(\frac{x^2}{\sqrt[4]{b+ax^{20}}}+\frac{x^2(-5b-x^{12})}{\sqrt[4]{b+ax^{20}}(2b+x^{12}+2ax^{20})}\right)dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^6}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^5}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{b+ax^{20}}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^6}} + \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^5}\right)\text{Subst}}{}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^5}\right)\text{Subst}\left(\int \left(-\frac{5bx^2}{\sqrt[4]{b+ax^{20}}(2b+x^{12}+2ax^{20})}-\frac{x^{14}}{\sqrt[4]{b+ax^{20}}(2b+x^{12}+2ax^{20})}\right.\right.}{\sqrt[4]{bx+ax^6}}$$

$$= \frac{4x\sqrt[4]{1+\frac{ax^5}{b}}\,{}_2F_1\left(\frac{3}{20},\frac{1}{4};\frac{23}{20};-\frac{ax^5}{b}\right)}{3\sqrt[4]{bx+ax^6}} - \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^5}\right)\text{Subst}\left(\int \frac{x}{\sqrt[4]{b+ax^{20}}(2b}\right.}{\sqrt[4]{bx+ax^6}}$$

Mathematica [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{-3b+2ax^5}{(2b+x^3+2ax^5)\sqrt[4]{bx+ax^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3*b + 2*a*x^5)/((2*b + x^3 + 2*a*x^5)*(b*x + a*x^6)^(1/4)),x]
```

```
[Out] Integrate[(-3*b + 2*a*x^5)/((2*b + x^3 + 2*a*x^5)*(b*x + a*x^6)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 2.99, size = 110, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{ax^6+bx}}{\sqrt{2}\sqrt{ax^6+bx}-x^2}\right)}{\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6+bx}}{\sqrt[4]{2}}+\frac{x^2}{2^{3/4}}}{x\sqrt[4]{ax^6+bx}}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*b + 2*a*x^5)/((2*b + x^3 + 2*a*x^5)*(b*x + a*x^6)^(1
/4)),x]
```

```
[Out] -(ArcTan[(2^(3/4)*x*(b*x + a*x^6)^(1/4))/(-x^2 + Sqrt[2]*Sqrt[b*x + a*x^6])
]/2^(1/4)) - ArcTanh[(x^2/2^(3/4) + Sqrt[b*x + a*x^6]/2^(1/4))/(x*(b*x + a*
x^6)^(1/4))]/2^(1/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^5-3*b)/(2*a*x^5+x^3+2*b)/(a*x^6+b*x)^(1/4),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax^5-3\,b}{\left(ax^6+bx\right)^{\frac{1}{4}}\left(2\,ax^5+x^3+2\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^5-3*b)/(2*a*x^5+x^3+2*b)/(a*x^6+b*x)^(1/4),x, algorithm="g
iac")
```

```
[Out] integrate((2*a*x^5 - 3*b)/((a*x^6 + b*x)^(1/4)*(2*a*x^5 + x^3 + 2*b)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{2a\,x^5-3b}{\left(2a\,x^5+x^3+2b\right)\left(a\,x^6+bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x^5-3*b)/(2*a*x^5+x^3+2*b)/(a*x^6+b*x)^(1/4),x)
```

```
[Out] int((2*a*x^5-3*b)/(2*a*x^5+x^3+2*b)/(a*x^6+b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax^5-3\,b}{\left(ax^6+bx\right)^{\frac{1}{4}}\left(2\,ax^5+x^3+2\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^5-3*b)/(2*a*x^5+x^3+2*b)/(a*x^6+b*x)^(1/4),x, algorithm="m
axima")
```

```
[Out] integrate((2*a*x^5 - 3*b)/((a*x^6 + b*x)^(1/4)*(2*a*x^5 + x^3 + 2*b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{3\,b-2\,a\,x^5}{{\left(a\,x^6+b\,x\right)}^{1/4}\,\left(2\,a\,x^5+x^3+2\,b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*b - 2*a*x^5)/((b*x + a*x^6)^(1/4)*(2*b + 2*a*x^5 + x^3)),x)
```

```
[Out] int(-(3*b - 2*a*x^5)/((b*x + a*x^6)^(1/4)*(2*b + 2*a*x^5 + x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2ax^5-3b}{\sqrt[4]{x\left(ax^5+b\right)}\left(2ax^5+2b+x^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x**5-3*b)/(2*a*x**5+x**3+2*b)/(a*x**6+b*x)**(1/4),x)
```

```
[Out] Integral((2*a*x**5 - 3*b)/((x*(a*x**5 + b))**(1/4)*(2*a*x**5 + 2*b + x**3))
, x)
```

3.1385 $\int \frac{(-1+x^4)(1+x^2+3x^4+x^6+x^8)}{(1+x^2+x^4)^{3/2}(1+3x^2+5x^4+3x^6+x^8)}\,dx$

Optimal. Leaf size=110

$$-\frac{x}{\sqrt{x^4+x^2+1}}+\sqrt{\frac{1}{3}\left(-2+2i\sqrt{3}\right)}\tan^{-1}\left(\frac{2x}{\left(\sqrt{3}-i\right)\sqrt{x^4+x^2+1}}\right)+\sqrt{\frac{1}{3}\left(-2-2i\sqrt{3}\right)}\tan^{-1}\left(\frac{2x}{\left(\sqrt{3}+i\right)\sqrt{x^4+x^2}}\right.$$

Rubi [F] time = 1.62, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^4)(1+x^2+3x^4+x^6+x^8)}{(1+x^2+x^4)^{3/2}(1+3x^2+5x^4+3x^6+x^8)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^4)*(1 + x^2 + 3*x^4 + x^6 + x^8))/((1 + x^2 + x^4)^(3/2)*(1 +
3*x^2 + 5*x^4 + 3*x^6 + x^8)),x]
```

```
[Out] (x*(1 - x^2))/Sqrt[1 + x^2 + x^4] - (x*(2 + x^2))/(3*Sqrt[1 + x^2 + x^4]) -
 (2*x*(1 + 2*x^2))/(3*Sqrt[1 + x^2 + x^4]) + (8*x*Sqrt[1 + x^2 + x^4])/(3*(
1 + x^2)) - (8*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcT
an[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1
 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4] - 4*Defer[Int][
1/((1 + x^2 + x^4)^(3/2)*(1 + 3*x^2 + 5*x^4 + 3*x^6 + x^8)), x] - 8*Defer[I
nt][x^2/((1 + x^2 + x^4)^(3/2)*(1 + 3*x^2 + 5*x^4 + 3*x^6 + x^8)), x] - 12*
Defer[Int][x^4/((1 + x^2 + x^4)^(3/2)*(1 + 3*x^2 + 5*x^4 + 3*x^6 + x^8)), x
] - 2*Defer[Int][x^6/((1 + x^2 + x^4)^(3/2)*(1 + 3*x^2 + 5*x^4 + 3*x^6 + x^
8)), x]
```

Rubi steps

$$\int \frac{(-1+x^4)(1+x^2+3x^4+x^6+x^8)}{(1+x^2+x^4)^{3/2}(1+3x^2+5x^4+3x^6+x^8)}\,dx=\int\left(\frac{3}{(1+x^2+x^4)^{3/2}}-\frac{2x^2}{(1+x^2+x^4)^{3/2}}+\frac{x^4}{(1+x^2+x^4)^{3/2}}-\right.$$

$$=-\left(2\int\frac{x^2}{(1+x^2+x^4)^{3/2}}\,dx\right)-2\int\frac{2+4x^2+6x}{(1+x^2+x^4)^{3/2}(1+3x^2}$$

$$=\frac{x(1-x^2)}{\sqrt{1+x^2+x^4}}-\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}}-\frac{2x(1+2x^2)}{3\sqrt{1+x^2+x^4}}+\frac{1}{3}\int\frac{}{\sqrt{1}}$$

$$=\frac{x(1-x^2)}{\sqrt{1+x^2+x^4}}-\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}}-\frac{2x(1+2x^2)}{3\sqrt{1+x^2+x^4}}-\frac{1}{3}\int\frac{}{\sqrt{1}}$$

$$=\frac{x(1-x^2)}{\sqrt{1+x^2+x^4}}-\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}}-\frac{2x(1+2x^2)}{3\sqrt{1+x^2+x^4}}+\frac{8x\sqrt{1+}}{3(1}$$

Mathematica [C] time = 2.29, size = 1525, normalized size = 13.86

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^4)*(1 + x^2 + 3*x^4 + x^6 + x^8))/((1 + x^2 + x^4)^(3/2)
*(1 + 3*x^2 + 5*x^4 + 3*x^6 + x^8)),x]
```

```
[Out] (-x - 2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-((El
lipticPi[-((-1)^(1/3)/Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 1, 0]), I*
ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]*(-1 + Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 +
#1^4 & , 1, 0]^2))/((Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 1, 0] - Roo
t[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 2, 0])*(Root[1 + 3*#1 + 5*#1^2 + 3*
#1^3 + #1^4 & , 1, 0] - Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 3, 0])*(
Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 1, 0] - Root[1 + 3*#1 + 5*#1^2 +
 3*#1^3 + #1^4 & , 4, 0]))) + (EllipticPi[-((-1)^(1/3)/Root[1 + 3*#1 + 5*#1
^2 + 3*#1^3 + #1^4 & , 2, 0]), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]*(-1 + R
oot[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 2, 0]^2))/((Root[1 + 3*#1 + 5*#1^
2 + 3*#1^3 + #1^4 & , 1, 0] - Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 2,
 0])*(Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 2, 0] - Root[1 + 3*#1 + 5*
#1^2 + 3*#1^3 + #1^4 & , 3, 0])*(Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & ,
 2, 0] - Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 4, 0])) + EllipticPi[-(
(-1)^(1/3)/Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 3, 0]), I*ArcSinh[(-1
)^(5/6)*x], (-1)^(2/3)]/((-Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 1, 0]
 + Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 3, 0])*(-Root[1 + 3*#1 + 5*#1
^2 + 3*#1^3 + #1^4 & , 2, 0] + Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 3
, 0])*(Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 3, 0] - Root[1 + 3*#1 + 5
*#1^2 + 3*#1^3 + #1^4 & , 4, 0])) - (EllipticPi[-((-1)^(1/3)/Root[1 + 3*#1
+ 5*#1^2 + 3*#1^3 + #1^4 & , 3, 0]), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]*R
oot[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 3, 0]^2)/((-Root[1 + 3*#1 + 5*#1^
2 + 3*#1^3 + #1^4 & , 1, 0] + Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 3,
 0])*(-Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 2, 0] + Root[1 + 3*#1 + 5
*#1^2 + 3*#1^3 + #1^4 & , 3, 0])*(Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 &
, 3, 0] - Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 4, 0])) + EllipticPi[-
((-1)^(1/3)/Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 4, 0]), I*ArcSinh[(-
1)^(5/6)*x], (-1)^(2/3)]/((-Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 1, 0
] + Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 4, 0])*(-Root[1 + 3*#1 + 5*#
1^2 + 3*#1^3 + #1^4 & , 2, 0] + Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & ,
4, 0])*(-Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 3, 0] + Root[1 + 3*#1 +
 5*#1^2 + 3*#1^3 + #1^4 & , 4, 0])) - (EllipticPi[-((-1)^(1/3)/Root[1 + 3*#
1 + 5*#1^2 + 3*#1^3 + #1^4 & , 4, 0]), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]
*Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 4, 0]^2)/((-Root[1 + 3*#1 + 5*#
1^2 + 3*#1^3 + #1^4 & , 1, 0] + Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & ,
4, 0])*(-Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 2, 0] + Root[1 + 3*#1 +
 5*#1^2 + 3*#1^3 + #1^4 & , 4, 0])*(-Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4
 & , 3, 0] + Root[1 + 3*#1 + 5*#1^2 + 3*#1^3 + #1^4 & , 4, 0]))))/Sqrt[1 +
x^2 + x^4]
```

IntegrateAlgebraic [**A**] time = 1.21, size = 73, normalized size = 0.66

$$-\frac{x}{\sqrt{x^4+x^2+1}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x\sqrt{x^4+x^2+1}}{x^4+1}\right)}{\sqrt{3}}-\tanh^{-1}\left(\frac{x\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(1 + x^2 + 3*x^4 + x^6 + x^8))/((1 + x^2 + x
^4)^(3/2)*(1 + 3*x^2 + 5*x^4 + 3*x^6 + x^8)),x]
```

```
[Out] -(x/Sqrt[1 + x^2 + x^4]) + ArcTan[(Sqrt[3]*x*Sqrt[1 + x^2 + x^4])/(1 + x^4)
]/Sqrt[3] - ArcTanh[(x*Sqrt[1 + x^2 + x^4])/(1 + x^2)^2]
```

fricas [**A**] time = 0.55, size = 146, normalized size = 1.33

$$-\frac{2\sqrt{3}\left(x^4+x^2+1\right)\arctan\left(\frac{\sqrt{3}\sqrt{x^4+x^2+1}\left(x^4+1\right)}{3\left(x^5+x^3+x\right)}\right)-3\left(x^4+x^2+1\right)\log\left(\frac{x^8+5x^6+7x^4+5x^2-2\left(x^5+2x^3+x\right)\sqrt{x^4+x^2+1}+1}{x^8+3x^6+5x^4+3x^2+1}\right)+6\sqrt{x^4+x^2+1}\,x}{6\left(x^4+x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^8+x^6+3*x^4+x^2+1)/(x^4+x^2+1)^(3/2)/(x^8+3*x^6+5*x^4+
3*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(x^4 + x^2 + 1)*arctan(1/3*sqrt(3)*sqrt(x^4 + x^2 + 1)*(x^4
 + 1)/(x^5 + x^3 + x)) - 3*(x^4 + x^2 + 1)*log((x^8 + 5*x^6 + 7*x^4 + 5*x^2
 - 2*(x^5 + 2*x^3 + x)*sqrt(x^4 + x^2 + 1) + 1)/(x^8 + 3*x^6 + 5*x^4 + 3*x^
2 + 1)) + 6*sqrt(x^4 + x^2 + 1)*x)/(x^4 + x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+x^6+3x^4+x^2+1\right)\left(x^4-1\right)}{\left(x^8+3x^6+5x^4+3x^2+1\right)\left(x^4+x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^8+x^6+3*x^4+x^2+1)/(x^4+x^2+1)^(3/2)/(x^8+3*x^6+5*x^4+
3*x^2+1),x, algorithm="giac")
```

```
[Out] integrate((x^8 + x^6 + 3*x^4 + x^2 + 1)*(x^4 - 1)/((x^8 + 3*x^6 + 5*x^4 + 3
*x^2 + 1)*(x^4 + x^2 + 1)^(3/2)), x)
```

maple [C] time = 0.26, size = 356, normalized size = 3.24

$$-\frac{2\left(\frac{1}{6}x^3+\frac{1}{3}x\right)}{\sqrt{x^4+x^2+1}}+\frac{-\frac{4}{3}x^3-\frac{2}{3}x}{\sqrt{x^4+x^2+1}}-\frac{6\left(-\frac{1}{6}x+\frac{1}{6}x^3\right)}{\sqrt{x^4+x^2+1}}+\frac{\frac{8}{3}x^3-\frac{2}{3}x}{\sqrt{x^4+x^2+1}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^8+3\_Z^6+5\_Z^4+3\_Z^2+1\right)}\_\alpha\left(2\_\alpha^6+6\_\alpha^4+8\_\alpha^2+3\right)\left(-\frac{\operatorname{arctanh}\left(\frac{(2\_\alpha^2+1)(-12\_\alpha^6-30\_\alpha^4-45\_\alpha^2+7x^2-10)}{14\sqrt{\_\alpha^4+\_\alpha^2+1}\sqrt{x^4+x^2+1}}\right)}{\sqrt{\_\alpha^4+\_\alpha^2+1}}-\frac{\sqrt{2}\left(-\_\alpha^7-3\_\alpha^5-5\_\alpha^3-3\_\alpha\right)\sqrt{x^2+2-ix^2\sqrt{3}}\sqrt{x^2+2+ix^2\sqrt{3}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x,\frac{i\sqrt{3}\_\alpha^6}{2}+\frac{\_\alpha^6}{2}+\frac{3}{2}+\frac{3i\_\alpha^4\sqrt{3}}{2}+\frac{3\_\alpha^4}{2}+\frac{5i\_\alpha^2\sqrt{3}}{2}+\frac{5\_\alpha^2}{2}+\frac{3i\sqrt{3}}{2},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{i\sqrt{3}-1}\sqrt{x^4+x^2+1}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^8+x^6+3*x^4+x^2+1)/(x^4+x^2+1)^(3/2)/(x^8+3*x^6+5*x^4+3*x^2+
1),x)
```

```
[Out] -2*(1/6*x^3+1/3*x)/(x^4+x^2+1)^(1/2)+4*(-1/3*x^3-1/6*x)/(x^4+x^2+1)^(1/2)-6
*(-1/6*x+1/6*x^3)/(x^4+x^2+1)^(1/2)+4*(2/3*x^3-1/6*x)/(x^4+x^2+1)^(1/2)-1/6
*sum(_alpha*(2*_alpha^6+6*_alpha^4+8*_alpha^2+3)*(-1/(_alpha^4+_alpha^2+1)^
(1/2)*arctanh(1/14*(2*_alpha^2+1)*(-12*_alpha^6-30*_alpha^4-45*_alpha^2+7*x
^2-10)/(_alpha^4+_alpha^2+1)^(1/2)/(x^4+x^2+1)^(1/2))-2^(1/2)*(-_alpha^7-3*
_alpha^5-5*_alpha^3-3*_alpha)/(I*3^(1/2)-1)^(1/2)*(x^2+2-I*3^(1/2)*x^2)^(1/
2)*(x^2+2+I*3^(1/2)*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(
1/2))^(1/2)*x,1/2*I*_alpha^6*3^(1/2)+1/2*_alpha^6+3/2+3/2*I*_alpha^4*3^(1/2
)+3/2*_alpha^4+5/2*I*_alpha^2*3^(1/2)+5/2*_alpha^2+3/2*I*3^(1/2),(-1/2-1/2*
I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))),_alpha=RootOf(_Z^8+3*_Z^6+5*_
Z^4+3*_Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+x^6+3x^4+x^2+1\right)\left(x^4-1\right)}{\left(x^8+3x^6+5x^4+3x^2+1\right)\left(x^4+x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^8+x^6+3*x^4+x^2+1)/(x^4+x^2+1)^(3/2)/(x^8+3*x^6+5*x^4+
3*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^8 + x^6 + 3*x^4 + x^2 + 1)*(x^4 - 1)/((x^8 + 3*x^6 + 5*x^4 + 3
*x^2 + 1)*(x^4 + x^2 + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-1\right)\,\left(x^8+x^6+3\,x^4+x^2+1\right)}{{\left(x^4+x^2+1\right)}^{3/2}\,\left(x^8+3\,x^6+5\,x^4+3\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)*(x^2 + 3*x^4 + x^6 + x^8 + 1))/((x^2 + x^4 + 1)^(3/2)*(3*x^2
 + 5*x^4 + 3*x^6 + x^8 + 1)),x)
```

```
[Out] int(((x^4 - 1)*(x^2 + 3*x^4 + x^6 + x^8 + 1))/((x^2 + x^4 + 1)^(3/2)*(3*x^2
 + 5*x^4 + 3*x^6 + x^8 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**8+x**6+3*x**4+x**2+1)/(x**4+x**2+1)**(3/2)/(x**8+3*x
**6+5*x**4+3*x**2+1),x)
```

```
[Out] Timed out
```

3.1386 $\int \frac{(-3+2x+2x^5)\sqrt{x-x^2+x^6}}{1-2x+x^2-x^3+x^4+2x^5-3x^6-x^8+x^{10}}\,dx$

Optimal. Leaf size=110

$$\sqrt{\frac{1}{5}\left(2\sqrt{5}-2\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\sqrt{x^6-x^2+x}}{x^2}\right)-\sqrt{\frac{1}{5}\left(2+2\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\sqrt{x^6-x^2+x}}{x^2}\right)$$

Rubi [F] time = 2.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-3+2x+2x^5)\sqrt{x-x^2+x^6}}{1-2x+x^2-x^3+x^4+2x^5-3x^6-x^8+x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + 2*x + 2*x^5)*Sqrt[x - x^2 + x^6])/(1 - 2*x + x^2 - x^3 + x^4 + 2
*x^5 - 3*x^6 - x^8 + x^10),x]
```

```
[Out] (-6*Sqrt[x - x^2 + x^6]*Defer[Subst][Defer[Int][(x^2*Sqrt[1 - x^2 + x^10])/
(1 - 2*x^2 + x^4 - x^6 + x^8 + 2*x^10 - 3*x^12 - x^16 + x^20), x], x, Sqrt[
x]])/(Sqrt[x]*Sqrt[1 - x + x^5]) + (4*Sqrt[x - x^2 + x^6]*Defer[Subst][Defe
r[Int][(x^4*Sqrt[1 - x^2 + x^10])/(1 - 2*x^2 + x^4 - x^6 + x^8 + 2*x^10 - 3
*x^12 - x^16 + x^20), x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 - x + x^5]) + (4*Sqr
t[x - x^2 + x^6]*Defer[Subst][Defer[Int][(x^12*Sqrt[1 - x^2 + x^10])/(1 - 2
*x^2 + x^4 - x^6 + x^8 + 2*x^10 - 3*x^12 - x^16 + x^20), x], x, Sqrt[x]])/(
Sqrt[x]*Sqrt[1 - x + x^5])
```

Rubi steps

$$\int \frac{(-3+2x+2x^5)\sqrt{x-x^2+x^6}}{1-2x+x^2-x^3+x^4+2x^5-3x^6-x^8+x^{10}}\,dx = \frac{\sqrt{x-x^2+x^6}\int \frac{\sqrt{x}\sqrt{1-x+x^5}(-3+2x+2x^5)}{1-2x+x^2-x^3+x^4+2x^5-3x^6-x^8+x^{10}}\,dx}{\sqrt{x}\sqrt{1-x+x^5}}$$

$$= \frac{\left(2\sqrt{x-x^2+x^6}\right)\text{Subst}\left(\int \frac{x^2\sqrt{1-x^2+x^{10}}(-3+2x^2+2x^{10})}{1-2x^2+x^4-x^6+x^8+2x^{10}-3x^{12}-x^{16}+x^{20}}\,d\right.}{\sqrt{x}\sqrt{1-x+x^5}}$$

$$= \frac{\left(2\sqrt{x-x^2+x^6}\right)\text{Subst}\left(\int\left(-\frac{3x^2\sqrt{1-x^2+x^{10}}}{1-2x^2+x^4-x^6+x^8+2x^{10}-3x^{12}-x^{16}+x^2}\right.\right.}{}$$

$$= \frac{\left(4\sqrt{x-x^2+x^6}\right)\text{Subst}\left(\int \frac{x^4\sqrt{1-x^2+x^{10}}}{1-2x^2+x^4-x^6+x^8+2x^{10}-3x^{12}-x^{16}+x^{20}}\,d\right.}{\sqrt{x}\sqrt{1-x+x^5}}$$

Mathematica [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{(-3+2x+2x^5)\sqrt{x-x^2+x^6}}{1-2x+x^2-x^3+x^4+2x^5-3x^6-x^8+x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + 2*x + 2*x^5)*Sqrt[x - x^2 + x^6])/(1 - 2*x + x^2 - x^3 + x
^4 + 2*x^5 - 3*x^6 - x^8 + x^10),x]
```

```
[Out] Integrate[((-3 + 2*x + 2*x^5)*Sqrt[x - x^2 + x^6])/(1 - 2*x + x^2 - x^3 + x
^4 + 2*x^5 - 3*x^6 - x^8 + x^10), x]
```

IntegrateAlgebraic [A] time = 9.32, size = 110, normalized size = 1.00

$$\sqrt{\frac{1}{5}\left(2\sqrt{5}-2\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\sqrt{x^6-x^2+x}}{x^2}\right)-\sqrt{\frac{1}{5}\left(2+2\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\sqrt{x^6-x^2+x}}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + 2*x + 2*x^5)*Sqrt[x - x^2 + x^6])/(1 - 2*x + x^2
- x^3 + x^4 + 2*x^5 - 3*x^6 - x^8 + x^10),x]
```

```
[Out] Sqrt[(-2 + 2*Sqrt[5])/5]*ArcTan[(Sqrt[1/2 + Sqrt[5]/2]*Sqrt[x - x^2 + x^6])
/x^2] - Sqrt[(2 + 2*Sqrt[5])/5]*ArcTanh[(Sqrt[-1/2 + Sqrt[5]/2]*Sqrt[x - x^
2 + x^6])/x^2]
```

fricas [B] time = 0.84, size = 566, normalized size = 5.15

$$-\frac{1}{5}\sqrt{5}\sqrt{2\sqrt{5}-2}\arctan\left(\frac{2\left(2x^6+\sqrt{5}x^4+x^4-2x^2+2x\right)\sqrt{x^6-x^2+x}\sqrt{2\sqrt{5}-2}+\left(3x^{10}+5x^8-3x^6+6x^5-5x^4+5x^3+3x^2+\sqrt{5}\left(x^{10}+3x^8-x^6+2x^5-3x^4+3x^3+x^2-2x+1\right)-6x+3\right)\sqrt{2\sqrt{5}-2}\sqrt{\sqrt{5}-2}}{4\left(x^{10}+x^8-3x^6+2x^5-x^4+x^3+x^2-2x+1\right)}\right)-\frac{1}{20}\sqrt{5}\sqrt{2\sqrt{5}+2}\log\left(-\frac{4\left(3x^6+x^4-3x^2+\sqrt{5}\left(x^6+x^4-x^2+x\right)+3x\right)\sqrt{x^6-x^2+x}+\left(x^{10}+5x^8-x^6+2x^5-5x^4+5x^3+x^2+\sqrt{5}\left(x^{10}+x^8-x^6+2x^5-x^4+x^3+x^2-2x+1\right)-2x+1\right)\sqrt{2\sqrt{5}+2}}{x^{10}-x^8-3x^6+2x^5+x^4-x^3+x^2-2x+1}\right)+\frac{1}{20}\sqrt{5}\sqrt{2\sqrt{5}+2}\log\left(-\frac{4\left(3x^6+x^4-3x^2+\sqrt{5}\left(x^6+x^4-x^2+x\right)+3x\right)\sqrt{x^6-x^2+x}-\left(x^{10}+5x^8-x^6+2x^5-5x^4+5x^3+x^2+\sqrt{5}\left(x^{10}+x^8-x^6+2x^5-x^4+x^3+x^2-2x+1\right)-2x+1\right)\sqrt{2\sqrt{5}+2}}{x^{10}-x^8-3x^6+2x^5+x^4-x^3+x^2-2x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^5+2*x-3)*(x^6-x^2+x)^(1/2)/(x^10-x^8-3*x^6+2*x^5+x^4-x^3+x^2
-2*x+1),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(5)*sqrt(2*sqrt(5) - 2)*arctan(1/4*(2*(2*x^6 + sqrt(5)*x^4 + x^4 -
 2*x^2 + 2*x)*sqrt(x^6 - x^2 + x)*sqrt(2*sqrt(5) - 2) + (3*x^10 + 5*x^8 - 3
*x^6 + 6*x^5 - 5*x^4 + 5*x^3 + 3*x^2 + sqrt(5)*(x^10 + 3*x^8 - x^6 + 2*x^5
- 3*x^4 + 3*x^3 + x^2 - 2*x + 1) - 6*x + 3)*sqrt(2*sqrt(5) - 2)*sqrt(sqrt(5
) - 2))/(x^10 + x^8 - 3*x^6 + 2*x^5 - x^4 + x^3 + x^2 - 2*x + 1)) - 1/20*sq
rt(5)*sqrt(2*sqrt(5) + 2)*log(-(4*(3*x^6 + x^4 - 3*x^2 + sqrt(5)*(x^6 + x^4
 - x^2 + x) + 3*x)*sqrt(x^6 - x^2 + x) + (x^10 + 5*x^8 - x^6 + 2*x^5 - 5*x^
4 + 5*x^3 + x^2 + sqrt(5)*(x^10 + x^8 - x^6 + 2*x^5 - x^4 + x^3 + x^2 - 2*x
 + 1) - 2*x + 1)*sqrt(2*sqrt(5) + 2))/(x^10 - x^8 - 3*x^6 + 2*x^5 + x^4 - x
^3 + x^2 - 2*x + 1)) + 1/20*sqrt(5)*sqrt(2*sqrt(5) + 2)*log(-(4*(3*x^6 + x^
4 - 3*x^2 + sqrt(5)*(x^6 + x^4 - x^2 + x) + 3*x)*sqrt(x^6 - x^2 + x) - (x^1
0 + 5*x^8 - x^6 + 2*x^5 - 5*x^4 + 5*x^3 + x^2 + sqrt(5)*(x^10 + x^8 - x^6 +
 2*x^5 - x^4 + x^3 + x^2 - 2*x + 1) - 2*x + 1)*sqrt(2*sqrt(5) + 2))/(x^10 -
 x^8 - 3*x^6 + 2*x^5 + x^4 - x^3 + x^2 - 2*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^6-x^2+x}\left(2x^5+2x-3\right)}{x^{10}-x^8-3x^6+2x^5+x^4-x^3+x^2-2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^5+2*x-3)*(x^6-x^2+x)^(1/2)/(x^10-x^8-3*x^6+2*x^5+x^4-x^3+x^2
-2*x+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^6 - x^2 + x)*(2*x^5 + 2*x - 3)/(x^10 - x^8 - 3*x^6 + 2*x^5
 + x^4 - x^3 + x^2 - 2*x + 1), x)
```

maple [C] time = 8.33, size = 941, normalized size = 8.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^5+2*x-3)*(x^6-x^2+x)^(1/2)/(x^10-x^8-3*x^6+2*x^5+x^4-x^3+x^2-2*x+1
),x)
```

```
[Out] 1/5*RootOf(_Z^2+25*RootOf(25*_Z^4-5*_Z^2-1)^2-5)*ln((225*RootOf(25*_Z^4-5*_
Z^2-1)^4*RootOf(_Z^2+25*RootOf(25*_Z^4-5*_Z^2-1)^2-5)*x^5-225*RootOf(25*_Z^
4-5*_Z^2-1)^4*RootOf(_Z^2+25*RootOf(25*_Z^4-5*_Z^2-1)^2-5)*x^3+85*RootOf(25
*_Z^4-5*_Z^2-1)^2*RootOf(_Z^2+25*RootOf(25*_Z^4-5*_Z^2-1)^2-5)*x^5-225*Root
Of(25*_Z^4-5*_Z^2-1)^4*RootOf(_Z^2+25*RootOf(25*_Z^4-5*_Z^2-1)^2-5)*x-40*Ro
otOf(25*_Z^4-5*_Z^2-1)^2*RootOf(_Z^2+25*RootOf(25*_Z^4-5*_Z^2-1)^2-5)*x^3+8
*RootOf(_Z^2+25*RootOf(25*_Z^4-5*_Z^2-1)^2-5)*x^5+225*RootOf(25*_Z^4-5*_Z^2
-1)^4*RootOf(_Z^2+25*RootOf(25*_Z^4-5*_Z^2-1)^2-5)-85*RootOf(25*_Z^4-5*_Z^2
-1)^2*RootOf(_Z^2+25*RootOf(25*_Z^4-5*_Z^2-1)^2-5)*x-350*(x^6-x^2+x)^(1/2)*
RootOf(25*_Z^4-5*_Z^2-1)^2*x+85*RootOf(25*_Z^4-5*_Z^2-1)^2*RootOf(_Z^2+25*R
ootOf(25*_Z^4-5*_Z^2-1)^2-5)-8*RootOf(_Z^2+25*RootOf(25*_Z^4-5*_Z^2-1)^2-5)
*x-50*(x^6-x^2+x)^(1/2)*x+8*RootOf(_Z^2+25*RootOf(25*_Z^4-5*_Z^2-1)^2-5))/(
5*x^5*RootOf(25*_Z^4-5*_Z^2-1)^2-5*x^3*RootOf(25*_Z^4-5*_Z^2-1)^2-x^5-5*x*R
ootOf(25*_Z^4-5*_Z^2-1)^2+2*x^3+5*RootOf(25*_Z^4-5*_Z^2-1)^2+x-1))-RootOf(2
5*_Z^4-5*_Z^2-1)*ln((-225*RootOf(25*_Z^4-5*_Z^2-1)^5*x^5+225*RootOf(25*_Z^4
-5*_Z^2-1)^5*x^3+175*RootOf(25*_Z^4-5*_Z^2-1)^3*x^5+225*RootOf(25*_Z^4-5*_Z
^2-1)^5*x-130*RootOf(25*_Z^4-5*_Z^2-1)^3*x^3-34*RootOf(25*_Z^4-5*_Z^2-1)*x^
5-225*RootOf(25*_Z^4-5*_Z^2-1)^5+70*(x^6-x^2+x)^(1/2)*RootOf(25*_Z^4-5*_Z^2
-1)^2*x-175*RootOf(25*_Z^4-5*_Z^2-1)^3*x+17*RootOf(25*_Z^4-5*_Z^2-1)*x^3+17
5*RootOf(25*_Z^4-5*_Z^2-1)^3-24*(x^6-x^2+x)^(1/2)*x+34*RootOf(25*_Z^4-5*_Z^
2-1)*x-34*RootOf(25*_Z^4-5*_Z^2-1))/(5*x^5*RootOf(25*_Z^4-5*_Z^2-1)^2-5*x^3
*RootOf(25*_Z^4-5*_Z^2-1)^2-5*x*RootOf(25*_Z^4-5*_Z^2-1)^2-x^3+5*RootOf(25*
_Z^4-5*_Z^2-1)^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^6 - x^2 + x}\left(2\,x^5 + 2\,x - 3\right)}{x^{10} - x^8 - 3\,x^6 + 2\,x^5 + x^4 - x^3 + x^2 - 2\,x + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^5+2*x-3)*(x^6-x^2+x)^(1/2)/(x^10-x^8-3*x^6+2*x^5+x^4-x^3+x^2
-2*x+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^6 - x^2 + x)*(2*x^5 + 2*x - 3)/(x^10 - x^8 - 3*x^6 + 2*x^5
 + x^4 - x^3 + x^2 - 2*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^5 + 2\,x - 3\right)\,\sqrt{x^6 - x^2 + x}}{x^{10} - x^8 - 3\,x^6 + 2\,x^5 + x^4 - x^3 + x^2 - 2\,x + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 2*x^5 - 3)*(x - x^2 + x^6)^(1/2))/(x^2 - 2*x - x^3 + x^4 + 2*x^
5 - 3*x^6 - x^8 + x^10 + 1),x)
```

```
[Out] int(((2*x + 2*x^5 - 3)*(x - x^2 + x^6)^(1/2))/(x^2 - 2*x - x^3 + x^4 + 2*x^
5 - 3*x^6 - x^8 + x^10 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(x^5 - x + 1\right)}\left(2x^5 + 2x - 3\right)}{x^{10} - x^8 - 3x^6 + 2x^5 + x^4 - x^3 + x^2 - 2x + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**5+2*x-3)*(x**6-x**2+x)**(1/2)/(x**10-x**8-3*x**6+2*x**5+x**
4-x**3+x**2-2*x+1),x)
```

```
[Out] Integral(sqrt(x*(x**5 - x + 1))*(2*x**5 + 2*x - 3)/(x**10 - x**8 - 3*x**6 +
 2*x**5 + x**4 - x**3 + x**2 - 2*x + 1), x)
```

3.1387 $\int \frac{\sqrt{x+\sqrt{1+x}}}{x^2\sqrt{1+x}}\,dx$

Optimal. Leaf size=110

$$-\frac{\sqrt{x+1}\sqrt{x+\sqrt{x+1}}}{x}+\frac{3}{2}\tan^{-1}\left(\frac{\sqrt{x+1}-\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}-\sqrt{x+\sqrt{x+1}}+2}\right)-\frac{1}{2}\tanh^{-1}\left(-\sqrt{x+1}+\sqrt{x+\sqrt{x+1}}+1\right)$$

Rubi [A] time = 0.37, antiderivative size = 96, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {976, 1021, 1033, 724, 206, 204}

$$-\frac{\left(x+\sqrt{x+1}\right)^{3/2}}{x}+\sqrt{x+\sqrt{x+1}}+\frac{3}{4}\tan^{-1}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)+\frac{1}{4}\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[1 + x]]/(x^2*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[x + Sqrt[1 + x]] - (x + Sqrt[1 + x])^(3/2)/x + (3*ArcTan[(3 + Sqrt[1 +
 x])/(2*Sqrt[x + Sqrt[1 + x]])])/4 + ArcTanh[(1 - 3*Sqrt[1 + x])/(2*Sqrt[x
+ Sqrt[1 + x]])]/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 976

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p +
1)*(d + e*x + f*x^2)^(q + 1))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p +
1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
+ q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*(-(c*e*(2*p
 + q + 4))))*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ
[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[
q, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x+\sqrt{1+x}}}{x^2\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{\sqrt{-1+x+x^2}}{\left(-1+x^2\right)^2}\,dx, x, \sqrt{1+x}\right)\\
&= -\frac{\left(x+\sqrt{1+x}\right)^{3/2}}{x} - \frac{1}{2}\text{Subst}\left(\int \frac{(1-2x)\sqrt{-1+x+x^2}}{-1+x^2}\,dx, x, \sqrt{1+x}\right)\\
&= \sqrt{x+\sqrt{1+x}} - \frac{\left(x+\sqrt{1+x}\right)^{3/2}}{x} + \frac{1}{2}\text{Subst}\left(\int \frac{2-x}{\left(-1+x^2\right)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)\\
&= \sqrt{x+\sqrt{1+x}} - \frac{\left(x+\sqrt{1+x}\right)^{3/2}}{x} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right) - \frac{3}{4}\ldots\\
&= \sqrt{x+\sqrt{1+x}} - \frac{\left(x+\sqrt{1+x}\right)^{3/2}}{x} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-1+3\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right) + \frac{3}{2}\text{Subst}\Big(\ldots\\
&= \sqrt{x+\sqrt{1+x}} - \frac{\left(x+\sqrt{1+x}\right)^{3/2}}{x} + \frac{3}{4}\tan^{-1}\left(\frac{3+\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right) + \frac{1}{4}\tanh^{-1}\left(\frac{1-3\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\ldots\right.
\end{aligned}$$

Mathematica [A] time = 0.13, size = 88, normalized size = 0.80

$$\frac{1}{4}\left(-\frac{4\sqrt{x+1}\sqrt{x+\sqrt{x+1}}}{x}+3\tan^{-1}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)+\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[1 + x]]/(x^2*Sqrt[1 + x]),x]
```

```
[Out] ((-4*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]])/x + 3*ArcTan[(3 + Sqrt[1 + x])/(2*S
qrt[x + Sqrt[1 + x]])] + ArcTanh[(1 - 3*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x
]])])/4
```

IntegrateAlgebraic [A] time = 0.27, size = 84, normalized size = 0.76

$$-\frac{\sqrt{x+1}\sqrt{x+\sqrt{x+1}}}{x}+\frac{3}{2}\tan^{-1}\left(\sqrt{x+1}-\sqrt{x+\sqrt{x+1}}+1\right)-\frac{1}{2}\tanh^{-1}\left(-\sqrt{x+1}+\sqrt{x+\sqrt{x+1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x + Sqrt[1 + x]]/(x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]])/x) + (3*ArcTan[1 + Sqrt[1 + x] - Sqrt
[x + Sqrt[1 + x]]])/2 - ArcTanh[1 - Sqrt[1 + x] + Sqrt[x + Sqrt[1 + x]]]/2
```

fricas [A] time = 2.29, size = 87, normalized size = 0.79

$$-\frac{3\,x\arctan\left(\frac{2\,\sqrt{x+\sqrt{x+1}}\,\left(\sqrt{x+1}-3\right)}{x-8}\right)-x\log\left(\frac{2\,\sqrt{x+\sqrt{x+1}}\,\left(\sqrt{x+1}+1\right)-3\,x-2\,\sqrt{x+1}-2}{x}\right)+4\,\sqrt{x+\sqrt{x+1}}\,\sqrt{x+1}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/x^2/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(3*x*arctan(2*sqrt(x + sqrt(x + 1))*(sqrt(x + 1) - 3)/(x - 8)) - x*log
((2*sqrt(x + sqrt(x + 1))*(sqrt(x + 1) + 1) - 3*x - 2*sqrt(x + 1) - 2)/x) +
 4*sqrt(x + sqrt(x + 1))*sqrt(x + 1))/x
```

giac [B] time = 0.43, size = 188, normalized size = 1.71

$$-\frac{\left(\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}\right)^{3}-2\left(\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}\right)^{2}+5\sqrt{x+\sqrt{x+1}}-5\sqrt{x+1}+1}{\left(\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}\right)^{4}-2\left(\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}\right)^{2}+4\sqrt{x+\sqrt{x+1}}-4\sqrt{x+1}}-\frac{3}{2}\arctan\left(\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}-1\right)-\frac{1}{4}\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}+2\right|\right)+\frac{1}{4}\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/x^2/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -((sqrt(x + sqrt(x + 1)) - sqrt(x + 1))^3 - 2*(sqrt(x + sqrt(x + 1)) - sqrt
(x + 1))^2 + 5*sqrt(x + sqrt(x + 1)) - 5*sqrt(x + 1) + 1)/((sqrt(x + sqrt(x
 + 1)) - sqrt(x + 1))^4 - 2*(sqrt(x + sqrt(x + 1)) - sqrt(x + 1))^2 + 4*sqr
t(x + sqrt(x + 1)) - 4*sqrt(x + 1)) - 3/2*arctan(sqrt(x + sqrt(x + 1)) - sq
rt(x + 1) - 1) - 1/4*log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1) + 2)) + 1/
4*log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1)))
```

maple [B] time = 0.02, size = 298, normalized size = 2.71

$$\frac{\sqrt{\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2}}{4}-\frac{\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2}\right)}{4}-\frac{\operatorname{arctanh}\left(\frac{-1+3\sqrt{1+x}}{2\sqrt{\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2}}\right)}{4}+\frac{3\sqrt{\left(1+\sqrt{1+x}\right)^{2}-\sqrt{1+x}-2}}{4}+\frac{\ln\left(\sqrt{1+x}+\frac{1}{2}+\sqrt{\left(1+\sqrt{1+x}\right)^{2}-\sqrt{1+x}-2}\right)}{4}-\frac{3\arctan\left(\frac{-\sqrt{1+x}-3}{2\sqrt{\left(1+\sqrt{1+x}\right)^{2}-\sqrt{1+x}-2}}\right)}{4}+\frac{\left(\left(1+\sqrt{1+x}\right)^{2}-\sqrt{1+x}-2\right)^{\frac{3}{2}}}{2+2\sqrt{1+x}}-\frac{\left(1+2\sqrt{1+x}\right)\sqrt{\left(1+\sqrt{1+x}\right)^{2}-\sqrt{1+x}-2}}{4}-\frac{\left(\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2\right)^{\frac{3}{2}}}{2\left(-1+\sqrt{1+x}\right)}+\frac{\left(1+2\sqrt{1+x}\right)\sqrt{\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(1+x)^(1/2))^(1/2)/x^2/(1+x)^(1/2),x)
```

```
[Out] 1/4*((-1+(1+x)^(1/2))^2+3*(1+x)^(1/2)-2)^(1/2)-1/4*ln(1/2+(1+x)^(1/2)+((-1+
(1+x)^(1/2))^2+3*(1+x)^(1/2)-2)^(1/2))-1/4*arctanh(1/2*(-1+3*(1+x)^(1/2))/(
(-1+(1+x)^(1/2))^2+3*(1+x)^(1/2)-2)^(1/2))+3/4*((1+(1+x)^(1/2))^2-(1+x)^(1/
2)-2)^(1/2)+1/4*ln((1+x)^(1/2)+1/2+((1+(1+x)^(1/2))^2-(1+x)^(1/2)-2)^(1/2))
-3/4*arctan(1/2*(-(1+x)^(1/2)-3)/((1+(1+x)^(1/2))^2-(1+x)^(1/2)-2)^(1/2))+1
/2/(1+(1+x)^(1/2))*((1+(1+x)^(1/2))^2-(1+x)^(1/2)-2)^(3/2)-1/4*(1+2*(1+x)^(
1/2))*((1+(1+x)^(1/2))^2-(1+x)^(1/2)-2)^(1/2)-1/2/(-1+(1+x)^(1/2))*((-1+(1+
x)^(1/2))^2+3*(1+x)^(1/2)-2)^(3/2)+1/4*(1+2*(1+x)^(1/2))*((-1+(1+x)^(1/2))^
2+3*(1+x)^(1/2)-2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}\,x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/x^2/(1+x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x + sqrt(x + 1))/(sqrt(x + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{x^2\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x + 1)^(1/2))^(1/2)/(x^2*(x + 1)^(1/2)),x)
[Out] int((x + (x + 1)^(1/2))^(1/2)/(x^2*(x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{x^2\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)**(1/2))**(1/2)/x**2/(1+x)**(1/2),x)
[Out] Integral(sqrt(x + sqrt(x + 1))/(x**2*sqrt(x + 1)), x)
```

3.1388 $\int \frac{\sqrt{x+\sqrt{1+x}}}{x\sqrt{1+x}}\,dx$

Optimal. Leaf size=110

$$-2\log\left(2\sqrt{x+1}-2\sqrt{x+\sqrt{x+1}}+1\right)-2\tan^{-1}\left(\frac{\sqrt{x+1}-\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}-\sqrt{x+\sqrt{x+1}}+2}\right)-2\tanh^{-1}\left(-\sqrt{x+1}+\sqrt{x+\right.$$

Rubi [A] time = 0.26, antiderivative size = 90, normalized size of antiderivative = 0.82, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {990, 621, 206, 1033, 724, 204}

$$-\tan^{-1}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)+\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)+2\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[1 + x]]/(x*Sqrt[1 + x]),x]
```

```
[Out] -ArcTan[(3 + Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] + ArcTanh[(1 - 3*Sqrt[
1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] + 2*ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[
x + Sqrt[1 + x]])]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 990

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x+\sqrt{1+x}}}{x\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{\sqrt{-1+x+x^2}}{-1+x^2}\,dx, x, \sqrt{1+x}\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right) + 2\,\text{Subst}\left(\int \frac{x}{\left(-1+x^2\right)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1\right.\\
&= 4\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{1+2\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right) + \text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x\right.\\
&= 2\tanh^{-1}\left(\frac{1+2\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right) - 2\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3-\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right) - 2\,\text{Subst}\left(\int \frac{}{4-}\right.\\
&= -\tan^{-1}\left(\frac{3+\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right) + \tanh^{-1}\left(\frac{1-3\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right) + 2\tanh^{-1}\left(\frac{1+2\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 90, normalized size = 0.82

$$-\tan^{-1}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)+\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)+2\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[1 + x]]/(x*Sqrt[1 + x]),x]
```

```
[Out] -ArcTan[(3 + Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] + ArcTanh[(1 - 3*Sqrt[
1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] + 2*ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[
x + Sqrt[1 + x]])]
```

IntegrateAlgebraic [A] time = 0.18, size = 84, normalized size = 0.76

$$-2\log\left(-2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}}-1\right)-2\tan^{-1}\left(\sqrt{x+1}-\sqrt{x+\sqrt{x+1}}+1\right)-2\tanh^{-1}\left(-\sqrt{x+1}+\sqrt{x+\sqrt{x+1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x + Sqrt[1 + x]]/(x*Sqrt[1 + x]),x]
```

```
[Out] -2*ArcTan[1 + Sqrt[1 + x] - Sqrt[x + Sqrt[1 + x]]] - 2*ArcTanh[1 - Sqrt[1 +
 x] + Sqrt[x + Sqrt[1 + x]]] - 2*Log[-1 - 2*Sqrt[1 + x] + 2*Sqrt[x + Sqrt[1
 + x]]]
```

fricas [A] time = 2.69, size = 74, normalized size = 0.67

$$\arctan\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}-3\right)}{x-8}\right)+\log\left(\frac{8x^2+2\left((4x-1)\sqrt{x+1}-2x-1\right)\sqrt{x+\sqrt{x+1}}-x+2\sqrt{x+1}+2}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/x/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(2*sqrt(x + sqrt(x + 1))*(sqrt(x + 1) - 3)/(x - 8)) + log((8*x^2 + 2*
((4*x - 1)*sqrt(x + 1) - 2*x - 1)*sqrt(x + sqrt(x + 1)) - x + 2*sqrt(x + 1)
 + 2)/x)
```

giac [A] time = 0.43, size = 86, normalized size = 0.78

$$2 \arctan\left(\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}-1\right)-2 \log\left(-2 \sqrt{x+\sqrt{x+1}}+2 \sqrt{x+1}+1\right)-\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}+2\right|\right)+\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/x/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x + sqrt(x + 1)) - sqrt(x + 1) - 1) - 2*log(-2*sqrt(x + sqrt(
x + 1)) + 2*sqrt(x + 1) + 1) - log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1)
+ 2)) + log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1)))
```

maple [A] time = 0.02, size = 170, normalized size = 1.55

$$\sqrt{\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2}+\frac{3\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2}\right)}{2}-\operatorname{arctanh}\left(\frac{-1+3\sqrt{1+x}}{2\sqrt{\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2}}\right)-\sqrt{\left(1+\sqrt{1+x}\right)^{2}-\sqrt{1+x}-2}+\frac{\ln\left(\sqrt{1+x}+\frac{1}{2}+\sqrt{\left(1+\sqrt{1+x}\right)^{2}-\sqrt{1+x}-2}\right)}{2}+\arctan\left(\frac{-\sqrt{1+x}-3}{2\sqrt{\left(1+\sqrt{1+x}\right)^{2}-\sqrt{1+x}-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(1+x)^(1/2))^(1/2)/x/(1+x)^(1/2),x)
```

```
[Out] ((-1+(1+x)^(1/2))^2+3*(1+x)^(1/2)-2)^(1/2)+3/2*ln(1/2+(1+x)^(1/2)+((-1+(1+x
)^(1/2))^2+3*(1+x)^(1/2)-2)^(1/2))-arctanh(1/2*(-1+3*(1+x)^(1/2))/((-1+(1+x
)^(1/2))^2+3*(1+x)^(1/2)-2)^(1/2))-((1+(1+x)^(1/2))^2-(1+x)^(1/2)-2)^(1/2)+
1/2*ln((1+x)^(1/2)+1/2+((1+(1+x)^(1/2))^2-(1+x)^(1/2)-2)^(1/2))+arctan(1/2*
(-(1+x)^(1/2)-3)/((1+(1+x)^(1/2))^2-(1+x)^(1/2)-2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/x/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1))/(sqrt(x + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{x\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x + 1)^(1/2))^(1/2)/(x*(x + 1)^(1/2)),x)
```

```
[Out] int((x + (x + 1)^(1/2))^(1/2)/(x*(x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{x\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)**(1/2))**(1/2)/x/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x + sqrt(x + 1))/(x*sqrt(x + 1)), x)
```

3.1389 $\int \frac{\sqrt{ax+\sqrt{-b+ax}}}{\sqrt{-b+ax}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{ax-b}\sqrt{\sqrt{ax-b}+ax}}{a}+\frac{\sqrt{\sqrt{ax-b}+ax}}{2a}+\frac{(1-4b)\log\left(2\sqrt{ax-b}-2\sqrt{\sqrt{ax-b}+ax}+1\right)}{4a}$$

Rubi [A] time = 0.19, antiderivative size = 93, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {612, 621, 206}

$$\frac{\sqrt{\sqrt{ax-b}+ax}\left(2\sqrt{ax-b}+1\right)}{2a}-\frac{(1-4b)\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{2\sqrt{\sqrt{ax-b}+ax}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + Sqrt[-b + a*x]]/Sqrt[-b + a*x],x]
```

```
[Out] (Sqrt[a*x + Sqrt[-b + a*x]]*(1 + 2*Sqrt[-b + a*x]))/(2*a) - ((1 - 4*b)*ArcT
anh[(1 + 2*Sqrt[-b + a*x])/(2*Sqrt[a*x + Sqrt[-b + a*x]])])/(4*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax+\sqrt{-b+ax}}}{\sqrt{-b+ax}}\,dx &= \frac{2\,\text{Subst}\left(\int \sqrt{b+x+x^2}\,dx,x,\sqrt{-b+ax}\right)}{a}\\
&= \frac{\sqrt{ax+\sqrt{-b+ax}}\left(1+2\sqrt{-b+ax}\right)}{2a}-\frac{(1-4b)\,\text{Subst}\left(\int \frac{1}{\sqrt{b+x+x^2}}\,dx,x,\sqrt{-b+ax}\right)}{4a}\\
&= \frac{\sqrt{ax+\sqrt{-b+ax}}\left(1+2\sqrt{-b+ax}\right)}{2a}-\frac{(1-4b)\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{1+2\sqrt{-b+ax}}{\sqrt{ax+\sqrt{-b+ax}}}\right)}{2a}\\
&= \frac{\sqrt{ax+\sqrt{-b+ax}}\left(1+2\sqrt{-b+ax}\right)}{2a}-\frac{(1-4b)\tanh^{-1}\left(\frac{1+2\sqrt{-b+ax}}{2\sqrt{ax+\sqrt{-b+ax}}}\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 89, normalized size = 0.81

$$\frac{2\sqrt{\sqrt{ax-b}+ax}\left(2\sqrt{ax-b}+1\right)+(4b-1)\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{2\sqrt{\sqrt{ax-b}+ax}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + Sqrt[-b + a*x]]/Sqrt[-b + a*x],x]
```

```
[Out] (2*Sqrt[a*x + Sqrt[-b + a*x]]*(1 + 2*Sqrt[-b + a*x]) + (-1 + 4*b)*ArcTanh[(
1 + 2*Sqrt[-b + a*x])/(2*Sqrt[a*x + Sqrt[-b + a*x]])])/(4*a)
```

IntegrateAlgebraic [A] time = 0.17, size = 95, normalized size = 0.86

$$\frac{\sqrt{\sqrt{ax-b}+ax}\left(2\sqrt{ax-b}+1\right)}{2a}+\frac{(1-4b)\log\left(a\left(-2\sqrt{ax-b}-1\right)+2a\sqrt{\sqrt{ax-b}+ax}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x + Sqrt[-b + a*x]]/Sqrt[-b + a*x],x]
```

```
[Out] (Sqrt[a*x + Sqrt[-b + a*x]]*(1 + 2*Sqrt[-b + a*x]))/(2*a) + ((1 - 4*b)*Log[
a*(-1 - 2*Sqrt[-b + a*x]) + 2*a*Sqrt[a*x + Sqrt[-b + a*x]]])/(4*a)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a*x-b)^(1/2))^(1/2)/(a*x-b)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.76, size = 74, normalized size = 0.67

$$-\frac{(4b-1)\log\left(\left|-2\sqrt{ax-b}+2\sqrt{ax+\sqrt{ax-b}}-1\right|\right)-2\sqrt{ax+\sqrt{ax-b}}\left(2\sqrt{ax-b}+1\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a*x-b)^(1/2))^(1/2)/(a*x-b)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*((4*b - 1)*log(abs(-2*sqrt(a*x - b) + 2*sqrt(a*x + sqrt(a*x - b)) - 1)
) - 2*sqrt(a*x + sqrt(a*x - b))*(2*sqrt(a*x - b) + 1))/a
```

maple [A] time = 0.01, size = 114, normalized size = 1.04

$$\frac{\sqrt{ax-b}\sqrt{ax+\sqrt{ax-b}}}{a}+\frac{\sqrt{ax+\sqrt{ax-b}}}{2a}+\frac{\ln\left(\frac{1}{2}+\sqrt{ax-b}+\sqrt{ax+\sqrt{ax-b}}\right)b}{a}-\frac{\ln\left(\frac{1}{2}+\sqrt{ax-b}+\sqrt{ax+\sqrt{ax-b}}\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+(a*x-b)^(1/2))^(1/2)/(a*x-b)^(1/2),x)
```

```
[Out] (a*x-b)^(1/2)*(a*x+(a*x-b)^(1/2))^(1/2)/a+1/2*(a*x+(a*x-b)^(1/2))^(1/2)/a+1
/a*ln(1/2+(a*x-b)^(1/2)+(a*x+(a*x-b)^(1/2))^(1/2))*b-1/4/a*ln(1/2+(a*x-b)^(
1/2)+(a*x+(a*x-b)^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+\sqrt{ax-b}}}{\sqrt{ax-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a*x-b)^(1/2))^(1/2)/(a*x-b)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x + sqrt(a*x - b))/sqrt(a*x - b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x+\sqrt{a\,x-b}}}{\sqrt{a\,x-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + (a*x - b)^(1/2))^(1/2)/(a*x - b)^(1/2),x)
[Out] int((a*x + (a*x - b)^(1/2))^(1/2)/(a*x - b)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+\sqrt{ax-b}}}{\sqrt{ax-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a*x-b)**(1/2))**(1/2)/(a*x-b)**(1/2),x)
[Out] Integral(sqrt(a*x + sqrt(a*x - b))/sqrt(a*x - b), x)
```

3.1390 $\int \frac{1-ax+b\sqrt{a+bx}}{\sqrt{a+bx}\left(x+ab\sqrt{a+bx}\right)}\,dx$

Optimal. Leaf size=110

$$\frac{2\left(a^3b^3+ab^3-2\right)\tanh^{-1}\left(\frac{ab^2+2\sqrt{a+bx}}{\sqrt{a}\sqrt{ab^4+4}}\right)}{\sqrt{a}\sqrt{ab^4+4}}+\left(a^2b+b\right)\log\left(ab^2\sqrt{a+bx}+bx\right)-\frac{2a\sqrt{a+bx}}{b}$$

Rubi [A] time = 0.56, antiderivative size = 104, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {1984, 1657, 634, 618, 206, 628}

$$-\frac{2\left(2-a\left(a^2+1\right)b^3\right)\tanh^{-1}\left(\frac{ab^2+2\sqrt{a+bx}}{\sqrt{a}\sqrt{ab^4+4}}\right)}{\sqrt{a}\sqrt{ab^4+4}}+\left(a^2+1\right)b\log\left(ab\sqrt{a+bx}+x\right)-\frac{2a\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a*x + b*Sqrt[a + b*x])/(Sqrt[a + b*x]*(x + a*b*Sqrt[a + b*x])),x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/b - (2*(2 - a*(1 + a^2)*b^3)*ArcTanh[(a*b^2 + 2*Sqrt[a
 + b*x])/(Sqrt[a]*Sqrt[4 + a*b^4])])/(Sqrt[a]*Sqrt[4 + a*b^4]) + (1 + a^2)*
b*Log[x + a*b*Sqrt[a + b*x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1984

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && QuadraticQ[{u, v}, x] &&  !QuadraticMat
```

```
chQ[{u, v}, x]
```

Rubi steps

$$\int \frac{1-ax+b\sqrt{a+bx}}{\sqrt{a+bx}\left(x+ab\sqrt{a+bx}\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{a^2+b+b^2x-ax^2}{x^2+a\left(-1+b^2x\right)}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{a^2+b+b^2x-ax^2}{-a+ab^2x+x^2}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int \left(-a+\frac{b+\left(1+a^2\right)b^2x}{-a+ab^2x+x^2}\right)dx, x, \sqrt{a+bx}\right)}{b}$$
$$= -\frac{2a\sqrt{a+bx}}{b} + \frac{2\,\text{Subst}\left(\int \frac{b+\left(1+a^2\right)b^2x}{-a+ab^2x+x^2}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= -\frac{2a\sqrt{a+bx}}{b} + \left(\left(1+a^2\right)b\right)\text{Subst}\left(\int \frac{ab^2+2x}{-a+ab^2x+x^2}\,dx, x, \sqrt{a+bx}\right) + (2$$
$$= -\frac{2a\sqrt{a+bx}}{b} + \left(1+a^2\right)b\log\left(x+ab\sqrt{a+bx}\right) - \left(2\left(2-a\left(1+a^2\right)b^3\right)\right)\text{Su}$$
$$= -\frac{2a\sqrt{a+bx}}{b} - \frac{2\left(2-a\left(1+a^2\right)b^3\right)\tanh^{-1}\left(\frac{ab^2+2\sqrt{a+bx}}{\sqrt{a}\sqrt{4+ab^4}}\right)}{\sqrt{a}\sqrt{4+ab^4}} + \left(1+a^2\right)b\log\left(\right.$$

Mathematica [A] time = 0.21, size = 106, normalized size = 0.96

$$-\frac{2\left(a^3b^3+ab^3-2\right)\tanh^{-1}\left(\frac{-ab^2-2\sqrt{a+bx}}{\sqrt{a}\sqrt{ab^4+4}}\right)}{\sqrt{a}\sqrt{ab^4+4}} + \left(a^2+1\right)b\log\left(ab\sqrt{a+bx}+x\right) - \frac{2a\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a*x + b*Sqrt[a + b*x])/(Sqrt[a + b*x]*(x + a*b*Sqrt[a + b*x]
)),x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/b - (2*(-2 + a*b^3 + a^3*b^3)*ArcTanh[(-(a*b^2) - 2*Sq
rt[a + b*x])/(Sqrt[a]*Sqrt[4 + a*b^4])])/(Sqrt[a]*Sqrt[4 + a*b^4]) + (1 + a
^2)*b*Log[x + a*b*Sqrt[a + b*x]]
```

IntegrateAlgebraic [A] time = 0.12, size = 124, normalized size = 1.13

$$\frac{2\left(a^3b^3+ab^3-2\right)\tanh^{-1}\left(\frac{2\sqrt{a+bx}}{\sqrt{a}\sqrt{ab^4+4}}+\frac{\sqrt{a}\,b^2}{\sqrt{ab^4+4}}\right)}{\sqrt{a}\sqrt{ab^4+4}} + \left(a^2b+b\right)\log\left(ab^2\sqrt{a+bx}+bx\right) - \frac{2a\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - a*x + b*Sqrt[a + b*x])/(Sqrt[a + b*x]*(x + a*b*Sqrt
[a + b*x])),x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/b + (2*(-2 + a*b^3 + a^3*b^3)*ArcTanh[(Sqrt[a]*b^2)/Sq
rt[4 + a*b^4] + (2*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[4 + a*b^4])])/(Sqrt[a]*Sqrt
[4 + a*b^4]) + (b + a^2*b)*Log[b*x + a*b^2*Sqrt[a + b*x]]
```

fricas [A] time = 0.44, size = 392, normalized size = 3.56

$$\left[\frac{\sqrt{a^2b^4+4a}\left((a^3+a)b^4-2b\right)\log\left(\frac{2a^3b^3-2bx^2+(a^2b^4-4a)x+\sqrt{a^2b^4+4a}\,(ab^2x+2a^2b)+\left(a^3b^5+4a^2b+\sqrt{a^2b^4+4a}\,(a^2b^3-2x)\right)\sqrt{bx+a}}{a^2b^3x+a^3b^2-x^2}\right)+\left((a^4+a^2)b^6+4(a^3+a)b^2\right)\log\left(\sqrt{bx+a}\,ab+x\right)-2\left(a^3b^4+4a^2\right)\sqrt{bx+a}}{a^2b^5+4ab}, -\frac{2\sqrt{-a^2b^4-4a}\left((a^3+a)b^4-2b\right)\arctan\left(\frac{\sqrt{-a^2b^4-4a}\,ab^2+2\sqrt{-a^2b^4-4a}\,\sqrt{bx+a}}{a^2b^4+4a}\right)-\left((a^4+a^2)b^6+4(a^3+a)b^2\right)\log\left(\sqrt{bx+a}\,ab+x\right)+2\left(a^3b^4+4a^2\right)\sqrt{bx+a}}{a^2b^5+4ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a*x+b*(b*x+a)^(1/2))/(b*x+a)^(1/2)/(x+a*b*(b*x+a)^(1/2)),x, al
gorithm="fricas")
```

```
[Out] [(sqrt(a^2*b^4 + 4*a)*((a^3 + a)*b^4 - 2*b)*log((2*a^3*b^3 - 2*b*x^2 + (a^2
*b^4 - 4*a)*x + sqrt(a^2*b^4 + 4*a)*(a*b^2*x + 2*a^2*b) + (a^3*b^5 + 4*a^2*
b + sqrt(a^2*b^4 + 4*a)*(a^2*b^3 - 2*x))*sqrt(b*x + a))/(a^2*b^3*x + a^3*b^
2 - x^2)) + ((a^4 + a^2)*b^6 + 4*(a^3 + a)*b^2)*log(sqrt(b*x + a)*a*b + x)
- 2*(a^3*b^4 + 4*a^2)*sqrt(b*x + a))/(a^2*b^5 + 4*a*b), -(2*sqrt(-a^2*b^4 -
 4*a)*((a^3 + a)*b^4 - 2*b)*arctan((sqrt(-a^2*b^4 - 4*a)*a*b^2 + 2*sqrt(-a^
2*b^4 - 4*a)*sqrt(b*x + a))/(a^2*b^4 + 4*a)) - ((a^4 + a^2)*b^6 + 4*(a^3 +
a)*b^2)*log(sqrt(b*x + a)*a*b + x) + 2*(a^3*b^4 + 4*a^2)*sqrt(b*x + a))/(a^
2*b^5 + 4*a*b)]
```

giac [A] time = 0.36, size = 100, normalized size = 0.91

$$\left(a^2b+b\right)\log\left(\sqrt{bx+a}\,ab^2+bx\right)-\frac{2\left(a^3b^3+ab^3-2\right)\arctan\left(\frac{ab^2+2\sqrt{bx+a}}{\sqrt{-a^2b^4-4a}}\right)}{\sqrt{-a^2b^4-4a}}-\frac{2\sqrt{bx+a}\,a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a*x+b*(b*x+a)^(1/2))/(b*x+a)^(1/2)/(x+a*b*(b*x+a)^(1/2)),x, al
gorithm="giac")
```

```
[Out] (a^2*b + b)*log(sqrt(b*x + a)*a*b^2 + b*x) - 2*(a^3*b^3 + a*b^3 - 2)*arctan
((a*b^2 + 2*sqrt(b*x + a))/sqrt(-a^2*b^4 - 4*a))/sqrt(-a^2*b^4 - 4*a) - 2*s
qrt(b*x + a)*a/b
```

maple [B] time = 0.01, size = 201, normalized size = 1.83

$$-\frac{2a\sqrt{bx+a}}{b}+\ln\left(bx+a\,b^2\sqrt{bx+a}\right)a^2b+\ln\left(bx+a\,b^2\sqrt{bx+a}\right)b+\frac{2\,\mathrm{arctanh}\left(\frac{a\,b^2+2\sqrt{bx+a}}{\sqrt{a^2b^4+4a}}\right)a^3b^3}{\sqrt{a^2b^4+4a}}+\frac{2\,\mathrm{arctanh}\left(\frac{a\,b^2+2\sqrt{bx+a}}{\sqrt{a^2b^4+4a}}\right)a\,b^3}{\sqrt{a^2b^4+4a}}-\frac{4\,\mathrm{arctanh}\left(\frac{a\,b^2+2\sqrt{bx+a}}{\sqrt{a^2b^4+4a}}\right)}{\sqrt{a^2b^4+4a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-a*x+b*(b*x+a)^(1/2))/(b*x+a)^(1/2)/(x+a*b*(b*x+a)^(1/2)),x)
```

```
[Out] -2*a*(b*x+a)^(1/2)/b+ln(b*x+a*b^2*(b*x+a)^(1/2))*a^2*b+ln(b*x+a*b^2*(b*x+a)
^(1/2))*b+2/(a^2*b^4+4*a)^(1/2)*arctanh((a*b^2+2*(b*x+a)^(1/2))/(a^2*b^4+4*
a)^(1/2))*a^3*b^3+2/(a^2*b^4+4*a)^(1/2)*arctanh((a*b^2+2*(b*x+a)^(1/2))/(a^
2*b^4+4*a)^(1/2))*a*b^3-4/(a^2*b^4+4*a)^(1/2)*arctanh((a*b^2+2*(b*x+a)^(1/2
))/(a^2*b^4+4*a)^(1/2))
```

maxima [A] time = 0.31, size = 125, normalized size = 1.14

$$\frac{\left(a^2+1\right)b^2\log\left(\sqrt{bx+a}\,ab^2+bx\right)-2\sqrt{bx+a}\,a-\frac{\left((a^3+a)b^4-2b\right)\log\left(\frac{ab^2-\sqrt{(ab^4+4)a}+2\sqrt{bx+a}}{ab^2+\sqrt{(ab^4+4)a}+2\sqrt{bx+a}}\right)}{\sqrt{(ab^4+4)a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a*x+b*(b*x+a)^(1/2))/(b*x+a)^(1/2)/(x+a*b*(b*x+a)^(1/2)),x, al
gorithm="maxima")
```

```
[Out] ((a^2 + 1)*b^2*log(sqrt(b*x + a)*a*b^2 + b*x) - 2*sqrt(b*x + a)*a - ((a^3 +
 a)*b^4 - 2*b)*log((a*b^2 - sqrt((a*b^4 + 4)*a) + 2*sqrt(b*x + a))/(a*b^2 +
 sqrt((a*b^4 + 4)*a) + 2*sqrt(b*x + a)))/sqrt((a*b^4 + 4)*a))/b
```

mupad [B] time = 1.19, size = 279, normalized size = 2.54

$$\frac{\ln\left(4\,a+2\,\sqrt{a\,\left(a\,b^4+4\right)}\,\sqrt{a+b\,x}+a^2\,b^4+a\,b^2\,\sqrt{a\,\left(a\,b^4+4\right)}\right)\,\left(a\,b\,\left(a\,b^4+4\right)-2\,\sqrt{a\,\left(a\,b^4+4\right)}+a^3\,b\,\left(a\,b^4+4\right)+a\,b^3\,\sqrt{a\,\left(a\,b^4+4\right)}+a^3\,b^3\,\sqrt{a\,\left(a\,b^4+4\right)}\right)}{a\,\left(a\,b^4+4\right)}-\frac{2\,a\,\sqrt{a+b\,x}}{b}+\frac{\ln\left(4\,a-2\,\sqrt{a\,\left(a\,b^4+4\right)}\,\sqrt{a+b\,x}+a^2\,b^4-a\,b^2\,\sqrt{a\,\left(a\,b^4+4\right)}\right)\,\left(2\,\sqrt{a\,\left(a\,b^4+4\right)}+a\,b\,\left(a\,b^4+4\right)+a^3\,b\,\left(a\,b^4+4\right)-a\,b^3\,\sqrt{a\,\left(a\,b^4+4\right)}-a^3\,b^3\,\sqrt{a\,\left(a\,b^4+4\right)}\right)}{a\,\left(a\,b^4+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(a + b*x)^(1/2) - a*x + 1)/((x + a*b*(a + b*x)^(1/2))*(a + b*x)^(1/2
)),x)
```

```
[Out] (log(4*a + 2*(a*(a*b^4 + 4))^(1/2)*(a + b*x)^(1/2) + a^2*b^4 + a*b^2*(a*(a*
b^4 + 4))^(1/2))*(a*b*(a*b^4 + 4) - 2*(a*(a*b^4 + 4))^(1/2) + a^3*b*(a*b^4
+ 4) + a*b^3*(a*(a*b^4 + 4))^(1/2) + a^3*b^3*(a*(a*b^4 + 4))^(1/2)))/(a*(a*
b^4 + 4)) - (2*a*(a + b*x)^(1/2))/b + (log(4*a - 2*(a*(a*b^4 + 4))^(1/2)*(a
 + b*x)^(1/2) + a^2*b^4 - a*b^2*(a*(a*b^4 + 4))^(1/2))*(2*(a*(a*b^4 + 4))^(
1/2) + a*b*(a*b^4 + 4) + a^3*b*(a*b^4 + 4) - a*b^3*(a*(a*b^4 + 4))^(1/2) -
a^3*b^3*(a*(a*b^4 + 4))^(1/2)))/(a*(a*b^4 + 4))
```

sympy [A] time = 92.60, size = 144, normalized size = 1.31

$$-\frac{2a\sqrt{a+bx}}{b}-2b\left(a^2+1\right)\log\left(\frac{1}{\sqrt{a+bx}}\right)-\frac{\left(-a^3b-ab\right)\log\left(\frac{ab^2}{\sqrt{a+bx}}-\frac{a}{a+bx}+1\right)}{a}-\frac{4\left(a^3b^3+ab^3+\frac{b^2\left(-a^3b-ab\right)}{2}-1\right)\operatorname{atan}\left(\frac{2\left(-\frac{b^2}{2}+\frac{1}{\sqrt{a+bx}}\right)}{\sqrt{-\frac{ab^4+4}{a}}}\right)}{a\sqrt{-\frac{ab^4+4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a*x+b*(b*x+a)**(1/2))/(b*x+a)**(1/2)/(x+a*b*(b*x+a)**(1/2)),x)
```

```
[Out] -2*a*sqrt(a + b*x)/b - 2*b*(a**2 + 1)*log(1/sqrt(a + b*x)) - (-a**3*b - a*b
)*log(a*b**2/sqrt(a + b*x) - a/(a + b*x) + 1)/a - 4*(a**3*b**3 + a*b**3 + b
**2*(-a**3*b - a*b)/2 - 1)*atan(2*(-b**2/2 + 1/sqrt(a + b*x))/sqrt(-(a*b**4
 + 4)/a))/(a*sqrt(-(a*b**4 + 4)/a))
```

3.1391 $\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^2}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{2}\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}}-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{x}$$

Rubi **[F]** time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b + Sqrt[b^2 + a*x^2]]/x^2,x]
```

```
[Out] Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/x^2, x]
```

Rubi steps

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^2}\,dx=\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^2}\,dx$$

Mathematica **[A]** time = 0.26, size = 99, normalized size = 0.90

$$\frac{\sqrt{\sqrt{ax^2+b^2}+b}\left(\sqrt{2}\sqrt{\sqrt{ax^2+b^2}-b}\tan^{-1}\left(\frac{\sqrt{\sqrt{ax^2+b^2}-b}}{\sqrt{2}\sqrt{b}}\right)-2\sqrt{b}\right)}{2\sqrt{b}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/x^2,x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 + a*x^2]]*(-2*Sqrt[b] + Sqrt[2]*Sqrt[-b + Sqrt[b^2 + a*x
^2]]*ArcTan[Sqrt[-b + Sqrt[b^2 + a*x^2]]/(Sqrt[2]*Sqrt[b])]))/(2*Sqrt[b]*x)
```

IntegrateAlgebraic **[A]** time = 0.19, size = 78, normalized size = 0.71

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{2}\sqrt{b}}-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + Sqrt[b^2 + a*x^2]]/x^2,x]
```

```
[Out] -(Sqrt[b + Sqrt[b^2 + a*x^2]]/x) + (Sqrt[a]*ArcTan[(Sqrt[a]*x)/(Sqrt[2]*Sqr
t[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/(Sqrt[2]*Sqrt[b])
```

fricas **[A]** time = 39.35, size = 213, normalized size = 1.94

$$\left[\frac{\sqrt{2}\,x\sqrt{-\frac{a}{b}}\log\left(-\frac{a^2x^3+4\,ab^2x-4\,\sqrt{ax^2+b^2}\,abx-2\left(2\,\sqrt{2}\,\sqrt{ax^2+b^2}\,b^2\sqrt{-\frac{a}{b}}-\sqrt{2}\left(abx^2+2\,b^3\right)\sqrt{-\frac{a}{b}}\right)\sqrt{b+\sqrt{ax^2+b^2}}}{x^3}\right)-4\sqrt{b+\sqrt{ax^2+b^2}}}{4\,x},-\frac{\sqrt{2}\,x\sqrt{\frac{a}{b}}\arctan\left(\frac{\sqrt{2}\,\sqrt{b+\sqrt{ax^2+b^2}}\,b\sqrt{\frac{a}{b}}}{ax}\right)+2\sqrt{b+\sqrt{ax^2+b^2}}}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*x*sqrt(-a/b)*log(-(a^2*x^3 + 4*a*b^2*x - 4*sqrt(a*x^2 + b^2)*
a*b*x - 2*(2*sqrt(2)*sqrt(a*x^2 + b^2)*b^2*sqrt(-a/b) - sqrt(2)*(a*b*x^2 +
2*b^3)*sqrt(-a/b))*sqrt(b + sqrt(a*x^2 + b^2)))/x^3) - 4*sqrt(b + sqrt(a*x^
2 + b^2)))/x, -1/2*(sqrt(2)*x*sqrt(a/b)*arctan(sqrt(2)*sqrt(b + sqrt(a*x^2
+ b^2))*b*sqrt(a/b)/(a*x)) + 2*sqrt(b + sqrt(a*x^2 + b^2)))/x]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/x^2, x)
```

maple [C] time = 0.04, size = 31, normalized size = 0.28

$$-\frac{\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\ \mathrm{hypergeom}\left(\left[-\frac{1}{2},-\frac{1}{4},\frac{1}{4}\right],\left[\frac{1}{2},\frac{1}{2}\right],-\frac{x^2a}{b^2}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+(a*x^2+b^2)^(1/2))^(1/2)/x^2,x)
```

```
[Out] -(b^2)^(1/4)*2^(1/2)/x*hypergeom([-1/2,-1/4,1/4],[1/2,1/2],-x^2*a/b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b+\sqrt{b^2+a\,x^2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/x^2, x)
```

sympy [C] time = 0.91, size = 48, normalized size = 0.44

$$\frac{\sqrt{b}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_{3}F_{2}\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{1}{2}\end{matrix}\right|\frac{ax^2e^{i\pi}}{b^2}\right)}{4\pi x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x**2+b**2)**(1/2))**(1/2)/x**2,x)
```

```
[Out] sqrt(b)*gamma(-1/4)*gamma(1/4)*hyper((-1/2, -1/4, 1/4), (1/2, 1/2), a*x**2*
exp_polar(I*pi)/b**2)/(4*pi*x)
```

3.1392 $\int \frac{1}{\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=110

$$\frac{4\left(32a^3x+39ab\right)\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{105b^3x^2}-\frac{4\left(15b-32a^2x\right)\sqrt{a^2x^2-bx}\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{105b^3x^3}$$

Rubi [F] time = 2.58, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2)),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[-b + a^2*x]*Defer[Subst][Defer[Int][1/(Sqrt[-b + a^2*x^2]*(
a*x^4 + x^2*Sqrt[-(b*x^2) + a^2*x^4])^(3/2)), x], x, Sqrt[x]])/Sqrt[-(b*x)
+ a^2*x^2]
```

Rubi steps

$$\int \frac{1}{\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx=\frac{\left(\sqrt{x}\sqrt{-b+a^2x}\right)\int \frac{1}{\sqrt{x}\sqrt{-b+a^2x}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx}{\sqrt{-bx+a^2x^2}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{-b+a^2x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+a^2x^2}\left(ax^4+x^2\sqrt{-bx^2+a^2x^4}\right)^{3/2}}\,dx,x\right.}{\sqrt{-bx+a^2x^2}}$$

Mathematica [F] time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2
)),x]
```

```
[Out] Integrate[1/(Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2
)), x]
```

IntegrateAlgebraic [A] time = 5.26, size = 110, normalized size = 1.00

$$\frac{4\left(32a^3x+39ab\right)\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{105b^3x^2}-\frac{4\left(15b-32a^2x\right)\sqrt{a^2x^2-bx}\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{105b^3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x
^2])^(3/2)),x]
```

```
[Out] (4*(39*a*b + 32*a^3*x)*Sqrt[x*(a*x + Sqrt[-(b*x) + a^2*x^2])])/(105*b^3*x^2
) - (4*(15*b - 32*a^2*x)*Sqrt[-(b*x) + a^2*x^2]*Sqrt[x*(a*x + Sqrt[-(b*x) +
 a^2*x^2])])/(105*b^3*x^3)
```

fricas [A] time = 0.42, size = 71, normalized size = 0.65

$$\frac{4\left(32\,a^3x^2+39\,abx+\sqrt{a^2x^2-bx}\left(32\,a^2x-15\,b\right)\right)\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}}{105\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algo
rithm="fricas")
```

```
[Out] 4/105*(32*a^3*x^2 + 39*a*b*x + sqrt(a^2*x^2 - b*x)*(32*a^2*x - 15*b))*sqrt(
a*x^2 + sqrt(a^2*x^2 - b*x)*x)/(b^3*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2-bx}\left(ax^2+\sqrt{a^2x^2-bx}\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algo
rithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 - b*x)*(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2)), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2-bx}\,\left(a\,x^2+x\sqrt{a^2x^2-bx}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

```
[Out] int(1/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2-bx}\left(ax^2+\sqrt{a^2x^2-bx}\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algo
rithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 - b*x)*(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2\,x^2-b\,x}\,\left(a\,x^2+x\,\sqrt{a^2\,x^2-b\,x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2*x^2 - b*x)^(1/2)*(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2)),x)
```

```
[Out] int(1/((a^2*x^2 - b*x)^(1/2)*(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x\left(ax+\sqrt{a^2x^2-bx}\right)\right)^{\frac{3}{2}}\sqrt{x\left(a^2x-b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*x**2-b*x)**(1/2)/(a*x**2+x*(a**2*x**2-b*x)**(1/2))**(3/2)
,x)
```

```
[Out] Integral(1/((x*(a*x + sqrt(a**2*x**2 - b*x)))**(3/2)*sqrt(x*(a**2*x - b))),
 x)
```

3.1393 $\int \frac{1}{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{x^2+1}\left(2\sqrt{\sqrt{x^2+1}+x}+5\right)+(2x+2)\sqrt{\sqrt{x^2+1}+x}+5x-1}{2\sqrt{x^2+1}+2x}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-2\log\left(\sqrt{\sqrt{x^2+1}+x}\right.$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2117, 1821, 1620}

$$\sqrt{\sqrt{x^2+1}+x}+\frac{1}{\sqrt{\sqrt{x^2+1}+x}}-\frac{1}{2\left(\sqrt{x^2+1}+x\right)}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-2\log\left(\sqrt{\sqrt{x^2+1}+x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x + Sqrt[1 + x^2]])^(-1),x]
```

```
[Out] -1/2*1/(x + Sqrt[1 + x^2]) + 1/Sqrt[x + Sqrt[1 + x^2]] + Sqrt[x + Sqrt[1 +
x^2]] + Log[x + Sqrt[1 + x^2]]/2 - 2*Log[1 + Sqrt[x + Sqrt[1 + x^2]]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+\sqrt{x+\sqrt{1+x^2}}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x^2}{\left(1+\sqrt{x}\right)x^2}\,dx,x,x+\sqrt{1+x^2}\right)\\
&= \operatorname{Subst}\left(\int \frac{1+x^4}{x^3(1+x)}\,dx,x,\sqrt{x+\sqrt{1+x^2}}\right)\\
&= \operatorname{Subst}\left(\int \left(1+\frac{1}{x^3}-\frac{1}{x^2}+\frac{1}{x}-\frac{2}{1+x}\right)dx,x,\sqrt{x+\sqrt{1+x^2}}\right)\\
&= -\frac{1}{2\left(x+\sqrt{1+x^2}\right)}+\frac{1}{\sqrt{x+\sqrt{1+x^2}}}+\sqrt{x+\sqrt{1+x^2}}+\frac{1}{2}\log\left(x+\sqrt{1+x^2}\right)-2\log
\end{aligned}$$

Mathematica [A] time = 0.04, size = 84, normalized size = 0.76

$$\sqrt{\sqrt{x^2+1}+x}+\frac{1}{\sqrt{\sqrt{x^2+1}+x}}-\frac{1}{2\left(\sqrt{x^2+1}+x\right)}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-2\log\left(\sqrt{\sqrt{x^2+1}+x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x + Sqrt[1 + x^2]])^(-1),x]
```

```
[Out] -1/2*1/(x + Sqrt[1 + x^2]) + 1/Sqrt[x + Sqrt[1 + x^2]] + Sqrt[x + Sqrt[1 +
x^2]] + Log[x + Sqrt[1 + x^2]]/2 - 2*Log[1 + Sqrt[x + Sqrt[1 + x^2]]]
```

IntegrateAlgebraic [A] time = 0.12, size = 110, normalized size = 1.00

$$\frac{\sqrt{x^2+1}\left(2\sqrt{\sqrt{x^2+1}+x}+5\right)+(2x+2)\sqrt{\sqrt{x^2+1}+x}+5x-1}{2\sqrt{x^2+1}+2x}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-2\log\left(\sqrt{\sqrt{x^2+1}+x}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[x + Sqrt[1 + x^2]])^(-1),x]
```

```
[Out] (-1 + 5*x + (2 + 2*x)*Sqrt[x + Sqrt[1 + x^2]] + Sqrt[1 + x^2]*(5 + 2*Sqrt[x
 + Sqrt[1 + x^2]]))/(2*x + 2*Sqrt[1 + x^2]) + Log[x + Sqrt[1 + x^2]]/2 - 2*
Log[1 + Sqrt[x + Sqrt[1 + x^2]]]
```

fricas [A] time = 0.41, size = 66, normalized size = 0.60

$$-\sqrt{x+\sqrt{x^2+1}}\left(x-\sqrt{x^2+1}-1\right)+\frac{1}{2}x-\frac{1}{2}\sqrt{x^2+1}-2\log\left(\sqrt{x+\sqrt{x^2+1}}+1\right)+\log\left(\sqrt{x+\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(x + sqrt(x^2 + 1))*(x - sqrt(x^2 + 1) - 1) + 1/2*x - 1/2*sqrt(x^2 + 1
) - 2*log(sqrt(x + sqrt(x^2 + 1)) + 1) + log(sqrt(x + sqrt(x^2 + 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x + sqrt(x^2 + 1)) + 1), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{1+\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(x+(x^2+1)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(1+(x+(x^2+1)^(1/2))^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x + sqrt(x^2 + 1)) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + (x^2 + 1)^(1/2))^(1/2) + 1),x)
[Out] int(1/((x + (x^2 + 1)^(1/2))^(1/2) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x+(x**2+1)**(1/2))**(1/2)),x)
[Out] Integral(1/(sqrt(x + sqrt(x**2 + 1)) + 1), x)
```

3.1394 $\int \frac{-2+x}{\left(2+x^2\right)\sqrt[3]{-1+x+2x^2}}\,dx$

Optimal. Leaf size=111

$$\frac{1}{2}\log\left(\sqrt[3]{2x^2+x-1}-x-1\right)-\frac{1}{4}\log\left(x^2+\left(2x^2+x-1\right)^{2/3}+(x+1)\sqrt[3]{2x^2+x-1}+2x+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\cdots}{\sqrt[3]{2\cdots}}\right.$$

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2+x}{\left(2+x^2\right)\sqrt[3]{-1+x+2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2 + x)/((2 + x^2)*(-1 + x + 2*x^2)^(1/3)),x]
```

```
[Out] Defer[Int][(-2 + x)/((2 + x^2)*(-1 + x + 2*x^2)^(1/3)), x]
```

Rubi steps

$$\int \frac{-2+x}{\left(2+x^2\right)\sqrt[3]{-1+x+2x^2}}\,dx=\int \frac{-2+x}{\left(2+x^2\right)\sqrt[3]{-1+x+2x^2}}\,dx$$

Mathematica [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{-2+x}{\left(2+x^2\right)\sqrt[3]{-1+x+2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2 + x)/((2 + x^2)*(-1 + x + 2*x^2)^(1/3)),x]
```

```
[Out] Integrate[(-2 + x)/((2 + x^2)*(-1 + x + 2*x^2)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.12, size = 111, normalized size = 1.00

$$\frac{1}{2}\log\left(\sqrt[3]{2x^2+x-1}-x-1\right)-\frac{1}{4}\log\left(x^2+\left(2x^2+x-1\right)^{2/3}+(x+1)\sqrt[3]{2x^2+x-1}+2x+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{2x^2+x-1}}{\sqrt[3]{2x^2+x-1}+2x+2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x)/((2 + x^2)*(-1 + x + 2*x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(-1 + x + 2*x^2)^(1/3))/(2 + 2*x + (-1 + x + 2*x^2
)^(1/3))])/2 + Log[-1 - x + (-1 + x + 2*x^2)^(1/3)]/2 - Log[1 + 2*x + x^2 +
 (1 + x)*(-1 + x + 2*x^2)^(1/3) + (-1 + x + 2*x^2)^(2/3)]/4
```

fricas [A] time = 1.05, size = 107, normalized size = 0.96

$$-\frac{1}{2}\sqrt{3}\arctan\left(-\frac{4\sqrt{3}\left(2x^2+x-1\right)^{\frac{1}{3}}(x+1)+\sqrt{3}(2x-1)-2\sqrt{3}\left(2x^2+x-1\right)^{\frac{2}{3}}}{8x^2+18x+7}\right)+\frac{1}{4}\log\left(\frac{x^2-3\left(2x^2+x-1\right)^{\frac{1}{3}}(x+1)+3\left(2x^2+x-1\right)^{\frac{2}{3}}+2}{x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(x^2+2)/(2*x^2+x-1)^(1/3),x, algorithm="fricas")
```

[Out] -1/2*sqrt(3)*arctan(-(4*sqrt(3)*(2*x^2 + x - 1)^(1/3)*(x + 1) + sqrt(3)*(2*
x - 1) - 2*sqrt(3)*(2*x^2 + x - 1)^(2/3))/(8*x^2 + 18*x + 7)) + 1/4*log((x^
2 - 3*(2*x^2 + x - 1)^(1/3)*(x + 1) + 3*(2*x^2 + x - 1)^(2/3) + 2)/(x^2 + 2
))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-2}{\left(2x^2+x-1\right)^{\frac{1}{3}}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+x)/(x^2+2)/(2*x^2+x-1)^(1/3),x, algorithm="giac")

[Out] integrate((x - 2)/((2*x^2 + x - 1)^(1/3)*(x^2 + 2)), x)

maple [C] time = 2.32, size = 552, normalized size = 4.97

$$\frac{\ln\left(-\frac{4\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)^2x^2+16\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)^2x-6\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)\left(2x^2+x-1\right)^{\frac{1}{3}}x+6\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)x^2-6\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)\left(2x^2+x-1\right)^{\frac{1}{3}}+20\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)x+3\left(2x^2+x-1\right)^{\frac{2}{3}}-3x\left(2x^2+x-1\right)^{\frac{1}{3}}+2x^2+2\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)-3\left(2x^2+x-1\right)^{\frac{1}{3}}+4x+2}{x^2+2}\right)}{2}-\frac{\ln\left(-\frac{4\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)^2x^2+16\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)^2x-6\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)\left(2x^2+x-1\right)^{\frac{1}{3}}x+4\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)x^2-6\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)\left(2x^2+x-1\right)^{\frac{1}{3}}+20\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)x+3\left(2x^2+x-1\right)^{\frac{2}{3}}-3x\left(2x^2+x-1\right)^{\frac{1}{3}}-2\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)-3\left(2x^2+x-1\right)^{\frac{1}{3}}+4x-2}{x^2+2}\right)}{2}-\ln\left(-\frac{4\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)^2x^2+16\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)^2x-6\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)\left(2x^2+x-1\right)^{\frac{1}{3}}x+4\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)x^2-6\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)\left(2x^2+x-1\right)^{\frac{1}{3}}+20\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)x+3\left(2x^2+x-1\right)^{\frac{2}{3}}-3x\left(2x^2+x-1\right)^{\frac{1}{3}}-2\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)-3\left(2x^2+x-1\right)^{\frac{1}{3}}+4x-2}{x^2+2}\right)\mathrm{RootOf}\left(4\_Z^2+2\_Z+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2+x)/(x^2+2)/(2*x^2+x-1)^(1/3),x)

[Out] 1/2*ln(-(4*RootOf(4*_Z^2+2*_Z+1)^2*x^2+16*RootOf(4*_Z^2+2*_Z+1)^2*x-6*RootO
f(4*_Z^2+2*_Z+1)*(2*x^2+x-1)^(1/3)*x+6*RootOf(4*_Z^2+2*_Z+1)*x^2-6*RootOf(4
*_Z^2+2*_Z+1)*(2*x^2+x-1)^(1/3)+20*RootOf(4*_Z^2+2*_Z+1)*x+3*(2*x^2+x-1)^(2
/3)-3*x*(2*x^2+x-1)^(1/3)+2*x^2+2*RootOf(4*_Z^2+2*_Z+1)-3*(2*x^2+x-1)^(1/3)
+4*x+2)/(x^2+2))-1/2*ln(-(4*RootOf(4*_Z^2+2*_Z+1)^2*x^2+16*RootOf(4*_Z^2+2*
_Z+1)^2*x-6*RootOf(4*_Z^2+2*_Z+1)*(2*x^2+x-1)^(1/3)*x+4*RootOf(4*_Z^2+2*_Z+
1)*x^2-6*RootOf(4*_Z^2+2*_Z+1)*(2*x^2+x-1)^(1/3)+20*RootOf(4*_Z^2+2*_Z+1)*x
+3*(2*x^2+x-1)^(2/3)-3*x*(2*x^2+x-1)^(1/3)-2*RootOf(4*_Z^2+2*_Z+1)-3*(2*x^2
+x-1)^(1/3)+4*x-2)/(x^2+2))-ln(-(4*RootOf(4*_Z^2+2*_Z+1)^2*x^2+16*RootOf(4*
_Z^2+2*_Z+1)^2*x-6*RootOf(4*_Z^2+2*_Z+1)*(2*x^2+x-1)^(1/3)*x+4*RootOf(4*_Z^
2+2*_Z+1)*x^2-6*RootOf(4*_Z^2+2*_Z+1)*(2*x^2+x-1)^(1/3)+20*RootOf(4*_Z^2+2*
_Z+1)*x+3*(2*x^2+x-1)^(2/3)-3*x*(2*x^2+x-1)^(1/3)-2*RootOf(4*_Z^2+2*_Z+1)-3
*(2*x^2+x-1)^(1/3)+4*x-2)/(x^2+2))*RootOf(4*_Z^2+2*_Z+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-2}{\left(2x^2+x-1\right)^{\frac{1}{3}}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+x)/(x^2+2)/(2*x^2+x-1)^(1/3),x, algorithm="maxima")

[Out] integrate((x - 2)/((2*x^2 + x - 1)^(1/3)*(x^2 + 2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x-2}{\left(x^2+2\right)\left(2x^2+x-1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - 2)/((x^2 + 2)*(x + 2*x^2 - 1)^(1/3)),x)

[Out] int((x - 2)/((x^2 + 2)*(x + 2*x^2 - 1)^(1/3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-2}{\sqrt[3]{(x+1)(2x-1)}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(x**2+2)/(2*x**2+x-1)**(1/3),x)
```

```
[Out] Integral((x - 2)/(((x + 1)*(2*x - 1))**(1/3)*(x**2 + 2)), x)
```

3.1395 $\int \frac{(-a+x)(-b+x)(-2ab+(a+b)x)}{(x^2(-a+x)(-b+x))^{3/4}(-ab+(a+b)x+(-1+d)x^2)}\,dx$

Optimal. Leaf size=111

$$-2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{x^3(-a-b)+abx^2+x^4}}\right)+2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{x^3(-a-b)+abx^2+x^4}}\right)-\frac{4\sqrt[4]{x^3(-a-b)+abx^2+x^4}}{x}$$

Rubi [F] time = 6.94, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-a+x)(-b+x)(-2ab+(a+b)x)}{(x^2(-a+x)(-b+x))^{3/4}(-ab+(a+b)x+(-1+d)x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-a + x)*(-b + x)*(-2*a*b + (a + b)*x))/((x^2*(-a + x)*(-b + x))^(3/4)
*(-(a*b) + (a + b)*x + (-1 + d)*x^2)),x]
```

```
[Out] ((a + b - Sqrt[a^2 - 2*a*b + b^2 + 4*a*b*d])*x^(3/2)*(-a + x)^(3/4)*(-b + x
)^(3/4)*Defer[Int][((-a + x)^(1/4)*(-b + x)^(1/4))/(x^(3/2)*(a + b - Sqrt[a
^2 - 2*a*b + b^2 + 4*a*b*d] + 2*(-1 + d)*x)), x])/((a - x)*(b - x)*x^2)^(3/
4) + ((a + b + Sqrt[a^2 - 2*a*b + b^2 + 4*a*b*d])*x^(3/2)*(-a + x)^(3/4)*(-
b + x)^(3/4)*Defer[Int][((-a + x)^(1/4)*(-b + x)^(1/4))/(x^(3/2)*(a + b + S
qrt[a^2 - 2*a*b + b^2 + 4*a*b*d] + 2*(-1 + d)*x)), x])/((a - x)*(b - x)*x^2
)^(3/4)
```

Rubi steps

$$\int \frac{(-a+x)(-b+x)(-2ab+(a+b)x)}{(x^2(-a+x)(-b+x))^{3/4}(-ab+(a+b)x+(-1+d)x^2)}\,dx = \frac{\left(x^{3/2}(-a+x)^{3/4}(-b+x)^{3/4}\right)\int \frac{\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\,(-2ab+(a\cdots}{x^{3/2}(-ab+(a+b)x+(-1+d\cdots}}{(x^2(-a+x)(-b+x))^{3/4}}$$

$$= \frac{\left(x^{3/2}(-a+x)^{3/4}(-b+x)^{3/4}\right)\int \left(\frac{\left(a+b-\sqrt{a^2-2ab+b^2+4ab\cdots}\right.}{x^{3/2}\left(a+b-\sqrt{a^2-2ab+b^2\cdots}\right.}\right.}{(x^2(-a+\cdots}$$

$$= \frac{\left(\left(a+b-\sqrt{a^2-2ab+b^2+4abd}\right)x^{3/2}(-a+x)^{3/4}\cdots\right.}{(x^2(-a+x)\cdots}$$

Mathematica [F] time = 24.68, size = 0, normalized size = 0.00

$$\int \frac{(-a+x)(-b+x)(-2ab+(a+b)x)}{(x^2(-a+x)(-b+x))^{3/4}(-ab+(a+b)x+(-1+d)x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-a + x)*(-b + x)*(-2*a*b + (a + b)*x))/((x^2*(-a + x)*(-b + x))
^(3/4)*(-(a*b) + (a + b)*x + (-1 + d)*x^2)),x]
```

```
[Out] Integrate[((-a + x)*(-b + x)*(-2*a*b + (a + b)*x))/((x^2*(-a + x)*(-b + x))
^(3/4)*(-(a*b) + (a + b)*x + (-1 + d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.77, size = 111, normalized size = 1.00

$$-2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{x^3(-a-b)+abx^2+x^4}}\right)+2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{x^3(-a-b)+abx^2+x^4}}\right)-\frac{4\sqrt[4]{x^3(-a-b)+abx^2+x^4}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-a + x)*(-b + x)*(-2*a*b + (a + b)*x))/((x^2*(-a + x)*
(-b + x))^(3/4)*(-(a*b) + (a + b)*x + (-1 + d)*x^2)),x]
```

```
[Out] (-4*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/4))/x - 2*d^(1/4)*ArcTan[(d^(1/4)*x)/
(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/4)] + 2*d^(1/4)*ArcTanh[(d^(1/4)*x)/(a*b*
x^2 + (-a - b)*x^3 + x^4)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-2*a*b+(a+b)*x)/(x^2*(-a+x)*(-b+x))^(3/4)/(-a*b+(a
+b)*x+(-1+d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(2\,ab-(a+b)x)(a-x)(b-x)}{\left((a-x)(b-x)x^2\right)^{\frac{3}{4}}\left((d-1)x^2-ab+(a+b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-2*a*b+(a+b)*x)/(x^2*(-a+x)*(-b+x))^(3/4)/(-a*b+(a
+b)*x+(-1+d)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-(2*a*b - (a + b)*x)*(a - x)*(b - x)/(((a - x)*(b - x)*x^2)^(3/4)
*((d - 1)*x^2 - a*b + (a + b)*x)), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(-a+x)\,(-b+x)\,(-2ab+(a+b)\,x)}{\left(x^2\,(-a+x)\,(-b+x)\right)^{\frac{3}{4}}\left(-ab+(a+b)\,x+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x)*(-b+x)*(-2*a*b+(a+b)*x)/(x^2*(-a+x)*(-b+x))^(3/4)/(-a*b+(a+b)*x+
(-1+d)*x^2),x)
```

```
[Out] int((-a+x)*(-b+x)*(-2*a*b+(a+b)*x)/(x^2*(-a+x)*(-b+x))^(3/4)/(-a*b+(a+b)*x+
(-1+d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(2\,ab-(a+b)x)(a-x)(b-x)}{\left((a-x)(b-x)x^2\right)^{\frac{3}{4}}\left((d-1)x^2-ab+(a+b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-2*a*b+(a+b)*x)/(x^2*(-a+x)*(-b+x))^(3/4)/(-a*b+(a
+b)*x+(-1+d)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((2*a*b - (a + b)*x)*(a - x)*(b - x)/(((a - x)*(b - x)*x^2)^(3/4)
*((d - 1)*x^2 - a*b + (a + b)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(2\,a\,b-x\,(a+b))\,(a-x)\,(b-x)}{{\left(x^2\,(a-x)\,(b-x)\right)}^{3/4}\,\left((d-1)\,x^2+(a+b)\,x-a\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*a*b - x*(a + b))*(a - x)*(b - x))/((x^2*(a - x)*(b - x))^(3/4)*(x*
(a + b) - a*b + x^2*(d - 1))),x)
```

```
[Out] int(-((2*a*b - x*(a + b))*(a - x)*(b - x))/((x^2*(a - x)*(b - x))^(3/4)*(x*
(a + b) - a*b + x^2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-2*a*b+(a+b)*x)/(x**2*(-a+x)*(-b+x))**(3/4)/(-a*b+
(a+b)*x+(-1+d)*x**2),x)
```

```
[Out] Timed out
```

3.1396 $\int \frac{1-x+x^2}{\left(-2+2x+x^2\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=111

$$-\frac{1}{2}\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{x^3+1}}{x^2-x+1}\right)-\frac{1}{2}\sqrt{\frac{1}{3}\left(2\sqrt{3}-3\right)}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{x^3+1}}{x^2-x+1}\right)$$

Rubi [C] time = 0.92, antiderivative size = 412, normalized size of antiderivative = 3.71, number of steps used = 13, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6728, 218, 2135, 2140, 206, 203}

$$-\frac{1}{2}\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)-\frac{1}{2}\sqrt{\frac{1}{3}\left(2\sqrt{3}-3\right)}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{x^3+1}}\right)-\frac{\left(2+\sqrt{3}\right)^{3/2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x + x^2)/((-2 + 2*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] -1/2*(Sqrt[(3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 +
 x^3]]) - (Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/
Sqrt[1 + x^3]])/2 - (Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt
[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*S
qrt[3]])/(2*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*S
qrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[
ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt
[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - ((2 + Sqrt[3])^(3/2)*(1 + x)
*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)
/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3]
+ x)^2]*Sqrt[1 + x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2135

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-
6*a*d^3)/(c*(b*c^3 - 28*a*d^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2140

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-x+x^2}{\left(-2+2x+x^2\right)\sqrt{1+x^3}}\,dx &= \int \left(\frac{1}{\sqrt{1+x^3}}+\frac{3(1-x)}{\left(-2+2x+x^2\right)\sqrt{1+x^3}}\right)dx \\
&= 3\int \frac{1-x}{\left(-2+2x+x^2\right)\sqrt{1+x^3}}\,dx+\int \frac{1}{\sqrt{1+x^3}}\,dx \\
&= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+3\int\Bigg(\frac{\cdots}{\left(2-2\sqrt{\cdots}\right.} \cdots \\
&= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+\left(-3+2\sqrt{3}\right)\int \cdots \\
&= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+\frac{1}{192}\left(2-\sqrt{3}\right) \cdots \\
&= -\frac{\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+\frac{2\sqrt{2+\sqrt{3}}\,(1 \cdots}{\cdots} \\
&= -\frac{1}{2}\sqrt{1+\frac{2}{\sqrt{3}}}\,\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)-\frac{1}{2}\sqrt{-1+\frac{2}{\sqrt{3}}}\,\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}} \cdots}{\sqrt{1+x \cdots}}\right. \cdots
\end{aligned}
$$

Mathematica [C] time = 0.91, size = 278, normalized size = 2.50

$$
\frac{\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\left(i\sqrt{x^2-x+1}\left(\left(\sqrt{3}+(-2+i)\right)\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)+\left(\sqrt{3}+(2+i)\right)\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)\right)+\frac{2\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(\left(\sqrt{3}+i\right)x-2i\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}}\right)}{\left(\sqrt{3}+i\right)\sqrt{x^3+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x + x^2)/((-2 + 2*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((2*Sqrt[((-1)^(1/3) - (-1)^(2/3)*x)/(1 + (
-1)^(1/3))]*(-2*I + (I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3)
)] + I*Sqrt[1 - x + x^2]*(((-2 + I) + Sqrt[3])*EllipticPi[(2*Sqrt[3])/(-3*I
 + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (
-1)^(1/3)] + ((2 + I) + Sqrt[3])*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sq
rt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])))/(
(I + Sqrt[3])*Sqrt[1 + x^3])
```

IntegrateAlgebraic [A] time = 1.03, size = 111, normalized size = 1.00

$$-\frac{1}{2}\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{x^3+1}}{x^2-x+1}\right)-\frac{1}{2}\sqrt{\frac{1}{3}\left(2\sqrt{3}-3\right)}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{x^3+1}}{x^2-x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x + x^2)/((-2 + 2*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] -1/2*(Sqrt[(3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[1 + x^3])/(1
 - x + x^2)]) - (Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqr
t[1 + x^3])/(1 - x + x^2)])/2
```

fricas [B] time = 0.51, size = 229, normalized size = 2.06

$$-\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}+3}\arctan\left(\frac{\sqrt{x^3+1}\sqrt{2\sqrt{3}+3}}{x^2-x+1}\right)-\frac{1}{24}\sqrt{3}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^4-2x^3+6x^2+2\sqrt{x^3+1}\left(x^2+2\sqrt{3}(x+1)+2x+4\right)\sqrt{2\sqrt{3}-3}+4\sqrt{3}\left(x^3+1\right)+4x+4}{x^4+4x^3-8x+4}\right)+\frac{1}{24}\sqrt{3}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^4-2x^3+6x^2-2\sqrt{x^3+1}\left(x^2+2\sqrt{3}(x+1)+2x+4\right)\sqrt{2\sqrt{3}-3}+4\sqrt{3}\left(x^3+1\right)+4x+4}{x^4+4x^3-8x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2+2*x-2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2*sqrt(3) + 3)*arctan(sqrt(x^3 + 1)*sqrt(2*sqrt(3) + 3)/(
x^2 - x + 1)) - 1/24*sqrt(3)*sqrt(2*sqrt(3) - 3)*log((x^4 - 2*x^3 + 6*x^2 +
 2*sqrt(x^3 + 1)*(x^2 + 2*sqrt(3)*(x + 1) + 2*x + 4)*sqrt(2*sqrt(3) - 3) +
4*sqrt(3)*(x^3 + 1) + 4*x + 4)/(x^4 + 4*x^3 - 8*x + 4)) + 1/24*sqrt(3)*sqrt
(2*sqrt(3) - 3)*log((x^4 - 2*x^3 + 6*x^2 - 2*sqrt(x^3 + 1)*(x^2 + 2*sqrt(3)
*(x + 1) + 2*x + 4)*sqrt(2*sqrt(3) - 3) + 4*sqrt(3)*(x^3 + 1) + 4*x + 4)/(x
^4 + 4*x^3 - 8*x + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-x+1}{\sqrt{x^3+1}\left(x^2+2x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2+2*x-2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)/(sqrt(x^3 + 1)*(x^2 + 2*x - 2)), x)
```

maple [C] time = 0.32, size = 1501, normalized size = 13.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+1)/(x^2+2*x-2)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2
```

```
-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1
/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/
2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*El
lipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2
),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3/2*I*(1/(3/2-1/2*I*3^
(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1
/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1
/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(
x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*
3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+I*(1/(3
/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-
1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2
+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/
2))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(
-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/
2))*3^(1/2)+3/2*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-
3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2
+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3
^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/
2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^
(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3
^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+3/2*I*(1/(3/2-1/2*I*3^(1/2))*x+1
/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/
2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2
/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/
2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^
(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+I*(1/(3/2-1/2*I*3^
(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1
/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1
/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(
x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3
^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-
3/2*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^
(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)
*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/
2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/
2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/
2)))^(1/2))*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - x + 1}{\sqrt{x^3 + 1}\left(x^2 + 2\,x - 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2+2*x-2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)/(sqrt(x^3 + 1)*(x^2 + 2*x - 2)), x)
```

mupad [B] time = 0.97, size = 509, normalized size = 4.59

$$\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}+\frac{\left(3\sqrt{3}-6\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{\left(3\sqrt{3}+6\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x + 1)/((x^3 + 1)^(1/2)*(2*x + x^2 - 2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2))^(1/2) + ((3*3^(1/2) - 6)*((3^(1/2)*1i)/2 + 3/2)*((x +
(3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*e
llipticPi((3^(1/2)*((3^(1/2)*1i)/2 + 3/2))/3, asin(((x + 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(x^3 -
 x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 -
1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - ((3*3^(1/2) + 6)*((3^(1/2)*1i)/2 + 3/
2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*ellipticPi(-(3^(1/2)*((3^(1/2)*1i)/2 + 3/2))/3, asin(((x + 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))
/(3*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - x + 1}{\sqrt{(x+1)\left(x^2 - x + 1\right)}\left(x^2 + 2x - 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+1)/(x**2+2*x-2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x**2 - x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x**2 + 2*x - 2)), x)
```

3.1397 $\int \frac{-1+x}{x\sqrt[3]{-x^2+x^3}}\,dx$

Optimal. Leaf size=111

$$-\frac{3\left(x^3-x^2\right)^{2/3}}{2x^2}-\log\left(\sqrt[3]{x^3-x^2}-x\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-x^2}+x}\right)$$

Rubi [A] time = 0.11, antiderivative size = 155, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2052, 2011, 59, 2014}

$$-\frac{3\left(x^3-x^2\right)^{2/3}}{2x^2}-\frac{3\sqrt[3]{x-1}\,x^{2/3}\log\left(\frac{\sqrt[3]{x-1}}{\sqrt[3]{x}}-1\right)}{2\sqrt[3]{x^3-x^2}}-\frac{\sqrt[3]{x-1}\,x^{2/3}\log(x)}{2\sqrt[3]{x^3-x^2}}-\frac{\sqrt{3}\sqrt[3]{x-1}\,x^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{x-1}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{x^3-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(x*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (-3*(-x^2 + x^3)^(2/3))/(2*x^2) - (Sqrt[3]*(-1 + x)^(1/3)*x^(2/3)*ArcTan[1/
Sqrt[3] + (2*(-1 + x)^(1/3))/(Sqrt[3]*x^(1/3))])/(-x^2 + x^3)^(1/3) - (3*(-
1 + x)^(1/3)*x^(2/3)*Log[-1 + (-1 + x)^(1/3)/x^(1/3)])/(2*(-x^2 + x^3)^(1/3
)) - ((-1 + x)^(1/3)*x^(2/3)*Log[x])/(2*(-x^2 + x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{-1+x}{x\sqrt[3]{-x^2+x^3}}\,dx = \int \left(\frac{1}{\sqrt[3]{-x^2+x^3}} - \frac{1}{x\sqrt[3]{-x^2+x^3}}\right) dx$$
$$= \int \frac{1}{\sqrt[3]{-x^2+x^3}}\,dx - \int \frac{1}{x\sqrt[3]{-x^2+x^3}}\,dx$$
$$= -\frac{3\left(-x^2+x^3\right)^{2/3}}{2x^2} + \frac{\left(\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{1}{\sqrt[3]{-1+x}\,x^{2/3}}\,dx}{\sqrt[3]{-x^2+x^3}}$$
$$= -\frac{3\left(-x^2+x^3\right)^{2/3}}{2x^2} - \frac{\sqrt{3}\,\sqrt[3]{-1+x}\,x^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-1+x}}{\sqrt{3}\,\sqrt[3]{x}}\right)}{\sqrt[3]{-x^2+x^3}} - \frac{3\sqrt[3]{-1+x}\,x^{2/3}\log\left(-1+\frac{\sqrt[3]{}}{}\right.}{2\sqrt[3]{-x^2+x^3}}$$

Mathematica [C] time = 0.02, size = 35, normalized size = 0.32

$$\frac{3\left((x-1)x^2\right)^{5/3}\,{}_2F_1\left(\frac{5}{3},\frac{5}{3};\frac{8}{3};1-x\right)}{5x^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(x*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (3*((-1 + x)*x^2)^(5/3)*Hypergeometric2F1[5/3, 5/3, 8/3, 1 - x])/(5*x^(10/3
))
```

IntegrateAlgebraic [A] time = 0.23, size = 111, normalized size = 1.00

$$-\frac{3\left(x^3-x^2\right)^{2/3}}{2x^2} - \log\left(\sqrt[3]{x^3-x^2}-x\right) + \frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right) + \sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-x^2}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/(x*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (-3*(-x^2 + x^3)^(2/3))/(2*x^2) + Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-x^2 +
 x^3)^(1/3))] - Log[-x + (-x^2 + x^3)^(1/3)] + Log[x^2 + x*(-x^2 + x^3)^(1/
3) + (-x^2 + x^3)^(2/3)]/2
```

fricas [A] time = 0.43, size = 119, normalized size = 1.07

$$-\frac{2\sqrt{3}\,x^2\arctan\left(\frac{\sqrt{3}\,x+2\sqrt{3}\left(x^3-x^2\right)^{\frac{1}{3}}}{3x}\right)+2x^2\log\left(-\frac{x-\left(x^3-x^2\right)^{\frac{1}{3}}}{x}\right)-x^2\log\left(\frac{x^2+\left(x^3-x^2\right)^{\frac{1}{3}}x+\left(x^3-x^2\right)^{\frac{2}{3}}}{x^2}\right)+3\left(x^3-x^2\right)^{\frac{2}{3}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x/(x^3-x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*x^2*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 - x^2)^(1/3))/x)
 + 2*x^2*log(-(x - (x^3 - x^2)^(1/3))/x) - x^2*log((x^2 + (x^3 - x^2)^(1/3)
*x + (x^3 - x^2)^(2/3))/x^2) + 3*(x^3 - x^2)^(2/3))/x^2
```

giac [A] time = 0.21, size = 74, normalized size = 0.67

$$-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{3}{2}\left(-\frac{1}{x}+1\right)^{\frac{2}{3}} + \frac{1}{2}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right) - \log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x/(x^3-x^2)^(1/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*(-1/x + 1)^(1/3) + 1)) - 3/2*(-1/x + 1)^(2/3
) + 1/2*log((-1/x + 1)^(2/3) + (-1/x + 1)^(1/3) + 1) - log(abs((-1/x + 1)^(
1/3) - 1))
```

maple [C] time = 0.32, size = 42, normalized size = 0.38

$$-\frac{3\left(-1+x\right)}{2\left(\left(-1+x\right)x^{2}\right)^{\frac{1}{3}}}+\frac{3\left(-\operatorname{signum}\left(-1+x\right)\right)^{\frac{1}{3}}x^{\frac{1}{3}}\operatorname{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x\right)}{\operatorname{signum}\left(-1+x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/x/(x^3-x^2)^(1/3),x)
```

```
[Out] -3/2*(-1+x)/((-1+x)*x^2)^(1/3)+3/signum(-1+x)^(1/3)*(-signum(-1+x))^(1/3)*x
^(1/3)*hypergeom([1/3,1/3],[4/3],x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^{3}-x^{2}\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x/(x^3-x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/((x^3 - x^2)^(1/3)*x), x)
```

mupad [B] time = 1.17, size = 44, normalized size = 0.40

$$\frac{3\,x\,{\left(1-x\right)}^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ x\right)}{{\left(x^3-x^2\right)}^{1/3}}-\frac{3\,{\left(x^3-x^2\right)}^{2/3}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(x*(x^3 - x^2)^(1/3)),x)
```

```
[Out] (3*x*(1 - x)^(1/3)*hypergeom([1/3, 1/3], 4/3, x))/(x^3 - x^2)^(1/3) - (3*(x
^3 - x^2)^(2/3))/(2*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{x\sqrt[3]{x^{2}\left(x-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x/(x**3-x**2)**(1/3),x)
```

```
[Out] Integral((x - 1)/(x*(x**2*(x - 1))**(1/3)), x)
```

3.1398 $\int \frac{1+x}{x\sqrt[3]{-x^2+x^3}}\,dx$

Optimal. Leaf size=111

$$\frac{3\left(x^3-x^2\right)^{2/3}}{2x^2}-\log\left(\sqrt[3]{x^3-x^2}-x\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-x^2}+x}\right)$$

Rubi [A] time = 0.08, antiderivative size = 155, normalized size of antiderivative = 1.40, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2038, 2011, 59}

$$\frac{3\left(x^3-x^2\right)^{2/3}}{2x^2}-\frac{3\sqrt[3]{x-1}\,x^{2/3}\log\left(\frac{\sqrt[3]{x-1}}{\sqrt[3]{x}}-1\right)}{2\sqrt[3]{x^3-x^2}}-\frac{\sqrt[3]{x-1}\,x^{2/3}\log(x)}{2\sqrt[3]{x^3-x^2}}-\frac{\sqrt{3}\sqrt[3]{x-1}\,x^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{x-1}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{x^3-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(x*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (3*(-x^2 + x^3)^(2/3))/(2*x^2) - (Sqrt[3]*(-1 + x)^(1/3)*x^(2/3)*ArcTan[1/S
qrt[3] + (2*(-1 + x)^(1/3))/(Sqrt[3]*x^(1/3))])/(-x^2 + x^3)^(1/3) - (3*(-1
 + x)^(1/3)*x^(2/3)*Log[-1 + (-1 + x)^(1/3)/x^(1/3)])/(2*(-x^2 + x^3)^(1/3)
) - ((-1 + x)^(1/3)*x^(2/3)*Log[x])/(2*(-x^2 + x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{1+x}{x\sqrt[3]{-x^2+x^3}}\,dx = \frac{3\left(-x^2+x^3\right)^{2/3}}{2x^2} + \int \frac{1}{\sqrt[3]{-x^2+x^3}}\,dx$$
$$= \frac{3\left(-x^2+x^3\right)^{2/3}}{2x^2} + \frac{\left(\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{1}{\sqrt[3]{-1+x}\,x^{2/3}}\,dx}{\sqrt[3]{-x^2+x^3}}$$
$$= \frac{3\left(-x^2+x^3\right)^{2/3}}{2x^2} - \frac{\sqrt{3}\,\sqrt[3]{-1+x}\,x^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-1+x}}{\sqrt{3}\sqrt[3]{x}}\right)}{\sqrt[3]{-x^2+x^3}} - \frac{3\sqrt[3]{-1+x}\,x^{2/3}\log\left(-1+\frac{\sqrt[3]{-1+x}}{\sqrt[3]{x}}\right.}{2\sqrt[3]{-x^2+x^3}}$$

Mathematica [C] time = 0.02, size = 41, normalized size = 0.37

$$\frac{3\left((x-1)x^2\right)^{2/3}\left(x^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};1-x\right)+1\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(x*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (3*((-1 + x)*x^2)^(2/3)*(1 + x^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, 1 - x
]))/(2*x^2)
```

IntegrateAlgebraic [A] time = 0.23, size = 111, normalized size = 1.00

$$\frac{3\left(x^3-x^2\right)^{2/3}}{2x^2} - \log\left(\sqrt[3]{x^3-x^2}-x\right) + \frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right) + \sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-x^2}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/(x*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (3*(-x^2 + x^3)^(2/3))/(2*x^2) + Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-x^2 +
x^3)^(1/3))] - Log[-x + (-x^2 + x^3)^(1/3)] + Log[x^2 + x*(-x^2 + x^3)^(1/3
) + (-x^2 + x^3)^(2/3)]/2
```

fricas [A] time = 0.42, size = 119, normalized size = 1.07

$$-\frac{2\sqrt{3}\,x^2\arctan\left(\frac{\sqrt{3}\,x+2\sqrt{3}\left(x^3-x^2\right)^{\frac{1}{3}}}{3x}\right) + 2x^2\log\left(-\frac{x-\left(x^3-x^2\right)^{\frac{1}{3}}}{x}\right) - x^2\log\left(\frac{x^2+\left(x^3-x^2\right)^{\frac{1}{3}}x+\left(x^3-x^2\right)^{\frac{2}{3}}}{x^2}\right) - 3\left(x^3-x^2\right)^{\frac{2}{3}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/x/(x^3-x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*x^2*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 - x^2)^(1/3))/x)
 + 2*x^2*log(-(x - (x^3 - x^2)^(1/3))/x) - x^2*log((x^2 + (x^3 - x^2)^(1/3)
*x + (x^3 - x^2)^(2/3))/x^2) - 3*(x^3 - x^2)^(2/3))/x^2
```

giac [A] time = 0.18, size = 74, normalized size = 0.67

$$-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right) + \frac{3}{2}\left(-\frac{1}{x}+1\right)^{\frac{2}{3}} + \frac{1}{2}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right) - \log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/x/(x^3-x^2)^(1/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*(-1/x + 1)^(1/3) + 1)) + 3/2*(-1/x + 1)^(2/3
) + 1/2*log((-1/x + 1)^(2/3) + (-1/x + 1)^(1/3) + 1) - log(abs((-1/x + 1)^(
1/3) - 1))
```

maple [C] time = 0.32, size = 42, normalized size = 0.38

$$\frac{-\frac{3}{2}+\frac{3x}{2}}{\left((-1+x)\,x^2\right)^{\frac{1}{3}}}+\frac{3\left(-\mathrm{signum}\,(-1+x)\right)^{\frac{1}{3}} x^{\frac{1}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x\right)}{\mathrm{signum}\,(-1+x)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/x/(x^3-x^2)^(1/3),x)
```

```
[Out] 3/2*(-1+x)/((-1+x)*x^2)^(1/3)+3/signum(-1+x)^(1/3)*(-signum(-1+x))^(1/3)*x^
(1/3)*hypergeom([1/3,1/3],[4/3],x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(x^3-x^2\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/x/(x^3-x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/((x^3 - x^2)^(1/3)*x), x)
```

mupad [B] time = 0.89, size = 44, normalized size = 0.40

$$\frac{3\left(x^3-x^2\right)^{2/3}}{2\,x^2}+\frac{3\,x\,(1-x)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ x\right)}{\left(x^3-x^2\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x*(x^3 - x^2)^(1/3)),x)
```

```
[Out] (3*(x^3 - x^2)^(2/3))/(2*x^2) + (3*x*(1 - x)^(1/3)*hypergeom([1/3, 1/3], 4/
3, x))/(x^3 - x^2)^(1/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{x\sqrt[3]{x^2\,(x-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/x/(x**3-x**2)**(1/3),x)
```

```
[Out] Integral((x + 1)/(x*(x**2*(x - 1))**(1/3)), x)
```

3.1399 $\int \frac{x^2}{\sqrt[3]{x^2+x^3}}\,dx$

Optimal. Leaf size=111

$$\frac{(x^3+x^2)^{2/3}(3x-4)}{6x}-\frac{2}{9}\log\left(\sqrt[3]{x^3+x^2}-x\right)+\frac{1}{9}\log\left(x^2+\sqrt[3]{x^3+x^2}\,x+(x^3+x^2)^{2/3}\right)+\frac{2\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3+x^2}+x}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.08, antiderivative size = 164, normalized size of antiderivative = 1.48, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2024, 2011, 59}

$$-\frac{2(x^3+x^2)^{2/3}}{3x}+\frac{1}{2}(x^3+x^2)^{2/3}-\frac{x^{2/3}\sqrt[3]{x+1}\log(x)}{9\sqrt[3]{x^3+x^2}}-\frac{x^{2/3}\sqrt[3]{x+1}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x}}-1\right)}{3\sqrt[3]{x^3+x^2}}-\frac{2x^{2/3}\sqrt[3]{x+1}\tan^{-1}\left(\frac{2\sqrt[3]{x+1}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{x^3+x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(x^2 + x^3)^(1/3),x]
```

```
[Out] (x^2 + x^3)^(2/3)/2 - (2*(x^2 + x^3)^(2/3))/(3*x) - (2*x^(2/3)*(1 + x)^(1/3
)*ArcTan[1/Sqrt[3] + (2*(1 + x)^(1/3))/(Sqrt[3]*x^(1/3))])/(3*Sqrt[3]*(x^2
+ x^3)^(1/3)) - (x^(2/3)*(1 + x)^(1/3)*Log[x])/(9*(x^2 + x^3)^(1/3)) - (x^(
2/3)*(1 + x)^(1/3)*Log[-1 + (1 + x)^(1/3)/x^(1/3)])/(3*(x^2 + x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt[3]{x^2+x^3}}\,dx &= \frac{1}{2}\left(x^2+x^3\right)^{2/3}-\frac{2}{3}\int \frac{x}{\sqrt[3]{x^2+x^3}}\,dx\\ &= \frac{1}{2}\left(x^2+x^3\right)^{2/3}-\frac{2\left(x^2+x^3\right)^{2/3}}{3x}+\frac{2}{9}\int \frac{1}{\sqrt[3]{x^2+x^3}}\,dx\\ &= \frac{1}{2}\left(x^2+x^3\right)^{2/3}-\frac{2\left(x^2+x^3\right)^{2/3}}{3x}+\frac{\left(2x^{2/3}\sqrt[3]{1+x}\right)\int \frac{1}{x^{2/3}\sqrt[3]{1+x}}\,dx}{9\sqrt[3]{x^2+x^3}}\\ &= \frac{1}{2}\left(x^2+x^3\right)^{2/3}-\frac{2\left(x^2+x^3\right)^{2/3}}{3x}-\frac{2x^{2/3}\sqrt[3]{1+x}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1+x}}{\sqrt{3}\sqrt[3]{x}}\right)}{3\sqrt{3}\sqrt[3]{x^2+x^3}}-\frac{x^{2/3}\sqrt[3]{1+x}\log(x)}{9\sqrt[3]{x^2+x^3}}-\end{aligned}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.34

$$\frac{3x^3\sqrt[3]{x+1}\,{}_2F_1\left(\frac{1}{3},\frac{7}{3};\frac{10}{3};-x\right)}{7\sqrt[3]{x^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(x^2 + x^3)^(1/3),x]
```

```
[Out] (3*x^3*(1 + x)^(1/3)*Hypergeometric2F1[1/3, 7/3, 10/3, -x])/(7*(x^2*(1 + x)
)^(1/3))
```

IntegrateAlgebraic [A] time = 0.26, size = 111, normalized size = 1.00

$$\frac{\left(x^3+x^2\right)^{2/3}(3x-4)}{6x}-\frac{2}{9}\log\left(\sqrt[3]{x^3+x^2}-x\right)+\frac{1}{9}\log\left(x^2+\sqrt[3]{x^3+x^2}\,x+\left(x^3+x^2\right)^{2/3}\right)+\frac{2\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x^2}+x}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(x^2 + x^3)^(1/3),x]
```

```
[Out] ((-4 + 3*x)*(x^2 + x^3)^(2/3))/(6*x) + (2*ArcTan[(Sqrt[3]*x)/(x + 2*(x^2 +
x^3)^(1/3))])/(3*Sqrt[3]) - (2*Log[-x + (x^2 + x^3)^(1/3)])/9 + Log[x^2 + x
*(x^2 + x^3)^(1/3) + (x^2 + x^3)^(2/3)]/9
```

fricas [A] time = 0.43, size = 108, normalized size = 0.97

$$-\frac{4\sqrt{3}x\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+x^2\right)^{\frac{1}{3}}}{3x}\right)+4x\log\left(-\frac{x-\left(x^3+x^2\right)^{\frac{1}{3}}}{x}\right)-2x\log\left(\frac{x^2+\left(x^3+x^2\right)^{\frac{1}{3}}x+\left(x^3+x^2\right)^{\frac{2}{3}}}{x^2}\right)-3\left(x^3+x^2\right)^{\frac{2}{3}}(3x-4)}{18x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/18*(4*sqrt(3)*x*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 + x^2)^(1/3))/x)
+ 4*x*log(-(x - (x^3 + x^2)^(1/3))/x) - 2*x*log((x^2 + (x^3 + x^2)^(1/3)*x
+ (x^3 + x^2)^(2/3))/x^2) - 3*(x^3 + x^2)^(2/3)*(3*x - 4))/x
```

giac [A] time = 0.20, size = 79, normalized size = 0.71

$$-\frac{1}{6}\left(4\left(\frac{1}{x}+1\right)^{\frac{5}{3}}-7\left(\frac{1}{x}+1\right)^{\frac{2}{3}}\right)x^2-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{9}\log\left(\left(\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left|\left(\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] -1/6*(4*(1/x + 1)^(5/3) - 7*(1/x + 1)^(2/3))*x^2 - 2/9*sqrt(3)*arctan(1/3*s
qrt(3)*(2*(1/x + 1)^(1/3) + 1)) + 1/9*log((1/x + 1)^(2/3) + (1/x + 1)^(1/3)
 + 1) - 2/9*log(abs((1/x + 1)^(1/3) - 1))
```

maple [C] time = 0.29, size = 36, normalized size = 0.32

$$\frac{(-4+3x)\,x\,(1+x)}{6\left(x^2\,(1+x)\right)^{\frac{1}{3}}}+\frac{2x^{\frac{1}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3+x^2)^(1/3),x)
```

```
[Out] 1/6*(-4+3*x)*x*(1+x)/(x^2*(1+x))^(1/3)+2/3*x^(1/3)*hypergeom([1/3,1/3],[4/3
],-x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(x^3+x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^3 + x^2)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^2}{\left(x^3+x^2\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2 + x^3)^(1/3),x)
```

```
[Out] int(x^2/(x^2 + x^3)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt[3]{x^2\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**3+x**2)**(1/3),x)
```

```
[Out] Integral(x**2/(x**2*(x + 1))**(1/3), x)
```

3.1400 $\int \frac{(a-3b+2x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{(-b+x)\sqrt[4]{(-a+x)(-b+x)}\left(-a^3+bd-\left(-3a^2+d\right)x-3ax^2+x^3\right)}\,dx$

Optimal. Leaf size=111

$$-2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x(-a-b)+ab+x^2}}{a-x}\right)+2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x(-a-b)+ab+x^2}}{a-x}\right)-\frac{4\left(ab-ax-bx+x^2\right)^{3/4}}{b-x}$$

Rubi [F] time = 18.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a-3b+2x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{(-b+x)\sqrt[4]{(-a+x)(-b+x)}\left(-a^3+bd-\left(-3a^2+d\right)x-3ax^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a - 3*b + 2*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/((-b + x)*((-a + x)*
(-b + x))^(1/4)*(-a^3 + b*d - (-3*a^2 + d)*x - 3*a*x^2 + x^3)),x]
```

```
[Out] (-4*(a - x))/((a - x)*(b - x))^(1/4) - (8*a*(1 - (a - b)/(a - x))^(1/4)*Sqr
t[-a + x]*EllipticE[ArcCot[Sqrt[-a + x]/Sqrt[a - b]]/2, 2])/(Sqrt[a - b]*((
a - x)*(b - x))^(1/4)) - (4*(a - 3*b)*(1 - (a - b)/(a - x))^(1/4)*Sqrt[-a +
 x]*EllipticE[ArcCot[Sqrt[-a + x]/Sqrt[a - b]]/2, 2])/(Sqrt[a - b]*((a - x)
*(b - x))^(1/4)) + (12*Sqrt[a - b]*(1 - (a - b)/(a - x))^(1/4)*Sqrt[-a + x]
*EllipticE[ArcCot[Sqrt[-a + x]/Sqrt[a - b]]/2, 2])/((a - x)*(b - x))^(1/4)
+ (8*a*d*(-a + x)^(1/4)*(-b + x)^(1/4)*Defer[Subst][Defer[Int][x^2/((a - b
+ x^4)^(1/4)*(-(a*(1 - b/a)*d) - d*x^4 + x^12)), x], x, (-a + x)^(1/4)])/((
a - x)*(b - x))^(1/4) + (8*d*(-a + x)^(1/4)*(-b + x)^(1/4)*Defer[Subst][Def
er[Int][x^6/((a - b + x^4)^(1/4)*(-(a*(1 - b/a)*d) - d*x^4 + x^12)), x], x,
 (-a + x)^(1/4)])/((a - x)*(b - x))^(1/4) - (4*(a - 3*b)*d*(-a + x)^(1/4)*(
-b + x)^(1/4)*Defer[Subst][Defer[Int][x^2/((a - b + x^4)^(1/4)*(a*(1 - b/a)
*d + x^4*(d - x^8))), x], x, (-a + x)^(1/4)])/((a - x)*(b - x))^(1/4)
```

Rubi steps

$$\int \frac{(a-3b+2x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{(-b+x)\sqrt[4]{(-a+x)(-b+x)}\left(-a^3+bd-\left(-3a^2+d\right)x-3ax^2+x^3\right)}\,dx = \frac{\left(\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\right)\int \frac{(a-3b+2x}{\sqrt[4]{-a+x}\,(-b+x)^{5/4}(}}{\sqrt[4]{(-a+x)(-b}}$$

$$= \frac{\left(\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\right)\int \frac{(-a+x)^{3/4}(a-3}{(-b+x)^{5/4}\left(-a^3+bd\right.}}{\sqrt[4]{(-a+x)(-b+x}}$$

$$= \frac{\left(\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\right)\int \frac{(-a+x)^{1}}{(-b+x)^{5/4}\left(-a^3+bd\right.}}{\sqrt[4]{(-a+x)(-b+x}}$$

$$= \frac{\left(\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\right)\int \frac{(-a+3b-2x}{(-b+x)^{5/4}\left(a^3-bd-\right.}}{\sqrt[4]{(-a+x)(-b+x)}}$$

$$= \frac{\left(\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\right)\int \left(\frac{3\left(1-\frac{a}{3b}\right.}{(-b+x)^{5/4}\left(a^3-bd\right.}\right.}{}$$

$$= \frac{\left(2\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\right)\int \frac{x(}{(-b+x)^{5/4}\left(-a^3+b\right.}}{\sqrt[4]{(-a+x)(-b+x}}$$

$$= \frac{\left(8\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\right)\text{Subst}\left(\int \frac{}{(a-b+}\right.}{\sqrt[4]{(-a+x)}}$$

$$= \frac{\left(8\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\right)\text{Subst}\left(\int \frac{}{(a-b+}\right.}{\sqrt[4]{(-a+x}}$$

$$= \frac{\left(8\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\right)\text{Subst}\left(\int\left(\frac{a}{(a-b-}\right.\right.}{}$$

$$= \frac{\left(8\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\right)\text{Subst}\left(\int \frac{x^6}{(a-b+}\right.}{\sqrt[4]{(-a+x)(-b+x}}$$

$$= -\frac{4(a-x)}{\sqrt[4]{(a-x)(b-x)}} + \frac{\left(8\sqrt[4]{-a+x}\,\sqrt[4]{-b}\right.}{}$$

$$= -\frac{4(a-x)}{\sqrt[4]{(a-x)(b-x)}} + \frac{\left(8\sqrt[4]{-a+x}\,\sqrt[4]{-b}\right.}{}$$

$$= -\frac{4(a-x)}{\sqrt[4]{(a-x)(b-x)}} + \frac{\left(8d\sqrt[4]{-a+x}\,\sqrt[4]{-}\right.}{}$$

$$= -\frac{4(a-x)}{\sqrt[4]{(a-x)(b-x)}} - \frac{8a\sqrt[4]{1-\frac{a-b}{a-x}}\,\sqrt{-a}}{\sqrt{a-}}$$

$$= -\frac{4(a-x)}{\sqrt[4]{(a-x)(b-x)}} - \frac{8a\sqrt[4]{1-\frac{a-b}{a-x}}\,\sqrt{-a}}{\sqrt{a-}}$$

Mathematica [F] time = 4.76, size = 0, normalized size = 0.00

$$\int \frac{(a-3b+2x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{(-b+x)\sqrt[4]{(-a+x)(-b+x)}\left(-a^3+bd-\left(-3a^2+d\right)x-3ax^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a - 3*b + 2*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/((-b + x)*((-a
 + x)*(-b + x))^(1/4)*(-a^3 + b*d - (-3*a^2 + d)*x - 3*a*x^2 + x^3)),x]
```

```
[Out] Integrate[((a - 3*b + 2*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/((-b + x)*((-a
 + x)*(-b + x))^(1/4)*(-a^3 + b*d - (-3*a^2 + d)*x - 3*a*x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 1.04, size = 111, normalized size = 1.00

$$-2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x(-a-b)+ab+x^2}}{a-x}\right)+2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x(-a-b)+ab+x^2}}{a-x}\right)-\frac{4\left(ab-ax-bx+x^2\right)^{3/4}}{b-x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a - 3*b + 2*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/((-b
+ x)*((-a + x)*(-b + x))^(1/4)*(-a^3 + b*d - (-3*a^2 + d)*x - 3*a*x^2 + x^3
)),x]
```

```
[Out] (-4*(a*b - a*x - b*x + x^2)^(3/4))/(b - x) - 2*d^(1/4)*ArcTan[(d^(1/4)*(a*b
 + (-a - b)*x + x^2)^(1/4))/(a - x)] + 2*d^(1/4)*ArcTanh[(d^(1/4)*(a*b + (-
a - b)*x + x^2)^(1/4))/(a - x)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-3*b+2*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/(-b+x)/((-a+x)*(-b+x))^(1/
4)/(-a^3+b*d-(-3*a^2+d)*x-3*a*x^2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^3-3\,a^2x+3\,ax^2-x^3\right)(a-3\,b+2\,x)}{\left(a^3+3\,ax^2-x^3-bd-\left(3\,a^2-d\right)x\right)((a-x)(b-x))^{\frac{1}{4}}\,(b-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-3*b+2*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/(-b+x)/((-a+x)*(-b+x))^(1/
4)/(-a^3+b*d-(-3*a^2+d)*x-3*a*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate(-(a^3 - 3*a^2*x + 3*a*x^2 - x^3)*(a - 3*b + 2*x)/((a^3 + 3*a*x^2
- x^3 - b*d - (3*a^2 - d)*x)*((a - x)*(b - x))^(1/4)*(b - x)), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{(a-3b+2x)\left(-a^3+3a^2x-3a\,x^2+x^3\right)}{(-b+x)\left((-a+x)\,(-b+x)\right)^{\frac{1}{4}}\left(-a^3+bd-\left(-3a^2+d\right)x-3a\,x^2+x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-3*b+2*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/(-b+x)/((-a+x)*(-b+x))^(1/4)/(-a
^3+b*d-(-3*a^2+d)*x-3*a*x^2+x^3),x)
```

```
[Out] int((a-3*b+2*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/(-b+x)/((-a+x)*(-b+x))^(1/4)/(-a
^3+b*d-(-3*a^2+d)*x-3*a*x^2+x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^3-3\,a^2x+3\,ax^2-x^3\right)(a-3\,b+2\,x)}{\left(a^3+3\,ax^2-x^3-bd-\left(3\,a^2-d\right)x\right)((a-x)(b-x))^{\frac{1}{4}}\,(b-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-3*b+2*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/(-b+x)/((-a+x)*(-b+x))^(1/
4)/(-a^3+b*d-(-3*a^2+d)*x-3*a*x^2+x^3),x, algorithm="maxima")
```

```
[Out] -integrate((a^3 - 3*a^2*x + 3*a*x^2 - x^3)*(a - 3*b + 2*x)/((a^3 + 3*a*x^2
- x^3 - b*d - (3*a^2 - d)*x)*((a - x)*(b - x))^(1/4)*(b - x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(a-3\,b+2\,x)\,\left(a^3-3\,a^2\,x+3\,a\,x^2-x^3\right)}{((a-x)\,(b-x))^{1/4}\,(b-x)\,\left(3\,a\,x^2-b\,d+x\,\left(d-3\,a^2\right)+a^3-x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a - 3*b + 2*x)*(3*a*x^2 - 3*a^2*x + a^3 - x^3))/(((a - x)*(b - x))^(
1/4)*(b - x)*(3*a*x^2 - b*d + x*(d - 3*a^2) + a^3 - x^3)),x)
```

```
[Out] int(-((a - 3*b + 2*x)*(3*a*x^2 - 3*a^2*x + a^3 - x^3))/(((a - x)*(b - x))^(
1/4)*(b - x)*(3*a*x^2 - b*d + x*(d - 3*a^2) + a^3 - x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-3*b+2*x)*(-a**3+3*a**2*x-3*a*x**2+x**3)/(-b+x)/((-a+x)*(-b+x))
**(1/4)/(-a**3+b*d-(-3*a**2+d)*x-3*a*x**2+x**3),x)
```

```
[Out] Timed out
```

3.1401 $\int \frac{x^3(3-2(1+k)x+kx^2)}{(-1+x)\sqrt[4]{(1-x)x(1-kx)}(-1+kx)(-d+d(1+k)x-dkx^2+x^3)}\,dx$

Optimal. Leaf size=111

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{x}\right)-2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{x}\right)+\frac{4\left(kx^3-kx^2-x^2+x\right)^{3/4}}{(x-1)(kx-1)}$$

Rubi [F] time = 38.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3\left(3-2(1+k)x+kx^2\right)}{(-1+x)\sqrt[4]{(1-x)x(1-kx)}(-1+kx)\left(-d+d(1+k)x-dkx^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*(3 - 2*(1 + k)*x + k*x^2))/((-1 + x)*((1 - x)*x*(1 - k*x))^(1/4)*(
-1 + k*x)*(-d + d*(1 + k)*x - d*k*x^2 + x^3)),x]
```

```
[Out] (4*(3 + d^2*k^3 - 3*d*k*(1 + k))*(1 - x)^(1/4)*x*(1 - k*x)^(1/4)*AppellF1[3
/4, 5/4, 5/4, 7/4, x, k*x])/(3*((1 - x)*x*(1 - k*x))^(1/4)) - (4*(2 + 2*k -
 d*k^2)*(1 - x)^(1/4)*x^2*(1 - k*x)^(1/4)*AppellF1[7/4, 5/4, 5/4, 11/4, x,
k*x])/(7*((1 - x)*x*(1 - k*x))^(1/4)) + (4*k*(1 - x)^(1/4)*x^3*(1 - k*x)^(1
/4)*AppellF1[11/4, 5/4, 5/4, 15/4, x, k*x])/(11*((1 - x)*x*(1 - k*x))^(1/4)
) - (4*d*(3 + d^2*k^3 - 3*d*k*(1 + k))*(1 - x)^(1/4)*x^(1/4)*(1 - k*x)^(1/4
)*Defer[Subst][Defer[Int][x^2/((1 - x^4)^(5/4)*(1 - k*x^4)^(5/4)*(d - d*(1
+ k)*x^4 + d*k*x^8 - x^12)), x], x, x^(1/4)])/((1 - x)*x*(1 - k*x))^(1/4) +
 (4*d*(5 + (5 - 3*d)*k - 7*d*k^2 - (3 - d)*d*k^3 + d^2*k^4)*(1 - x)^(1/4)*x
^(1/4)*(1 - k*x)^(1/4)*Defer[Subst][Defer[Int][x^6/((1 - x^4)^(5/4)*(1 - k*
x^4)^(5/4)*(d - d*(1 + k)*x^4 + d*k*x^8 - x^12)), x], x, x^(1/4)])/((1 - x)
*x*(1 - k*x))^(1/4) - (4*d*(2 + 8*k + (2 - 4*d)*k^2 - 4*d*k^3 + d^2*k^4)*(1
 - x)^(1/4)*x^(1/4)*(1 - k*x)^(1/4)*Defer[Subst][Defer[Int][x^10/((1 - x^4)
^(5/4)*(1 - k*x^4)^(5/4)*(d - d*(1 + k)*x^4 + d*k*x^8 - x^12)), x], x, x^(1
/4)])/((1 - x)*x*(1 - k*x))^(1/4)
```

Rubi steps

$$\int \frac{x^3\left(3-2(1+k)x+kx^2\right)}{(-1+x)\sqrt[4]{(1-x)x(1-kx)}\,(-1+kx)\left(-d+d(1+k)x-dkx^2+x^3\right)}\,dx = \frac{\left(\sqrt[4]{1-x}\,\sqrt[4]{x}\,\sqrt[4]{1-kx}\right)\int \frac{x^{11}}{\sqrt[4]{1-x}\,(-1+x)\,\sqrt[4]{1-}}}{\sqrt[4]{(1-x)x(1-}}$$

$$= -\frac{\left(\sqrt[4]{1-x}\,\sqrt[4]{x}\,\sqrt[4]{1-kx}\right)\int \frac{x^{11}}{(1-x)^{5/4}\,\sqrt[4]{1-kx}\,(}}{\sqrt[4]{(1-x)x(1-}}$$

$$= \frac{\left(\sqrt[4]{1-x}\,\sqrt[4]{x}\,\sqrt[4]{1-kx}\right)\int \frac{x^{11/4}(3-}{(1-x)^{5/4}(1-kx)^{5/4}(}}{\sqrt[4]{(1-x)x(1-kx)}}$$

$$= \frac{\left(4\sqrt[4]{1-x}\,\sqrt[4]{x}\,\sqrt[4]{1-kx}\right)\text{Subst}\left(\int \frac{}{(1-x^4)}\right.}{\sqrt[4]{(1-x}}$$

$$= \frac{\left(4\sqrt[4]{1-x}\,\sqrt[4]{x}\,\sqrt[4]{1-kx}\right)\text{Subst}\left(\int\left(\frac{(3+d}{(1-}\right.\right.}{}$$

$$= \frac{\left(4\sqrt[4]{1-x}\,\sqrt[4]{x}\,\sqrt[4]{1-kx}\right)\text{Subst}\left(\int \frac{x^2(d(3}{}\right.}{}$$

$$= \frac{4\left(3+d^2k^3-3dk(1+k)\right)\sqrt[4]{1-x}\,x\sqrt[4]{1}}{3\sqrt[4]{(1-x)x(1-k}}$$

$$= \frac{4\left(3+d^2k^3-3dk(1+k)\right)\sqrt[4]{1-x}\,x\sqrt[4]{1}}{3\sqrt[4]{(1-x)x(1-k}}$$

Mathematica [F] time = 7.89, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(3-2(1+k)x+kx^2\right)}{(-1+x)\sqrt[4]{(1-x)x(1-kx)}\,(-1+kx)\left(-d+d(1+k)x-dkx^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(3 - 2*(1 + k)*x + k*x^2))/((-1 + x)*((1 - x)*x*(1 - k*x))^(
1/4)*(-1 + k*x)*(-d + d*(1 + k)*x - d*k*x^2 + x^3)),x]
```

```
[Out] Integrate[(x^3*(3 - 2*(1 + k)*x + k*x^2))/((-1 + x)*((1 - x)*x*(1 - k*x))^(
1/4)*(-1 + k*x)*(-d + d*(1 + k)*x - d*k*x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.88, size = 111, normalized size = 1.00

$$2\sqrt[4]{d}\,\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{kx^3+(-k-1)x^2+x}}{x}\right)-2\sqrt[4]{d}\,\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{kx^3+(-k-1)x^2+x}}{x}\right)+\frac{4\left(kx^3-kx^2-x^2+x\right)^{3/4}}{(x-1)(kx-1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(3 - 2*(1 + k)*x + k*x^2))/((-1 + x)*((1 - x)*x*(1
- k*x))^(1/4)*(-1 + k*x)*(-d + d*(1 + k)*x - d*k*x^2 + x^3)),x]
```

```
[Out] (4*(x - x^2 - k*x^2 + k*x^3)^(3/4))/((-1 + x)*(-1 + k*x)) + 2*d^(1/4)*ArcTa
n[(d^(1/4)*(x + (-1 - k)*x^2 + k*x^3)^(1/4))/x] - 2*d^(1/4)*ArcTanh[(d^(1/4
)*(x + (-1 - k)*x^2 + k*x^3)^(1/4))/x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3-2*(1+k)*x+k*x^2)/(-1+x)/((1-x)*x*(-k*x+1))^(1/4)/(k*x-1)/(
-d+d*(1+k)*x-d*k*x^2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.38, size = 312, normalized size = 2.81

$$-\frac{\sqrt{2}\left(-d^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{1}{d}\right)^{\frac{1}{4}}+2\left(\frac{k}{x}-\frac{k}{x^{2}}-\frac{1}{x^{2}}+\frac{1}{x^{3}}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{1}{d}\right)^{\frac{1}{4}}}\right)}{d^{2}}-\frac{\sqrt{2}\left(-d^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{1}{d}\right)^{\frac{1}{4}}-2\left(\frac{k}{x}-\frac{k}{x^{2}}-\frac{1}{x^{2}}+\frac{1}{x^{3}}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{1}{d}\right)^{\frac{1}{4}}}\right)}{d^{2}}+\frac{\sqrt{2}\left(-d^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\left(\frac{k}{x}-\frac{k}{x^{2}}-\frac{1}{x^{2}}+\frac{1}{x^{3}}\right)^{\frac{1}{4}}\left(-\frac{1}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{k}{x}-\frac{k}{x^{2}}-\frac{1}{x^{2}}+\frac{1}{x^{3}}}+\sqrt{-\frac{1}{d}}\right)}{2\,d^{2}}-\frac{\sqrt{2}\left(-d^{3}\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\left(\frac{k}{x}-\frac{k}{x^{2}}-\frac{1}{x^{2}}+\frac{1}{x^{3}}\right)^{\frac{1}{4}}\left(-\frac{1}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{k}{x}-\frac{k}{x^{2}}-\frac{1}{x^{2}}+\frac{1}{x^{3}}}+\sqrt{-\frac{1}{d}}\right)}{2\,d^{2}}+\frac{4}{\left(\frac{k}{x}-\frac{k}{x^{2}}-\frac{1}{x^{2}}+\frac{1}{x^{3}}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3-2*(1+k)*x+k*x^2)/(-1+x)/((1-x)*x*(-k*x+1))^(1/4)/(k*x-1)/(
-d+d*(1+k)*x-d*k*x^2+x^3),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(-d^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-1/d)^(1/4) + 2*(k/x - k
/x^2 - 1/x^2 + 1/x^3)^(1/4))/(-1/d)^(1/4))/d^2 - sqrt(2)*(-d^3)^(3/4)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(-1/d)^(1/4) - 2*(k/x - k/x^2 - 1/x^2 + 1/x^3)^(1/4
))/(-1/d)^(1/4))/d^2 + 1/2*sqrt(2)*(-d^3)^(3/4)*log(sqrt(2)*(k/x - k/x^2 -
1/x^2 + 1/x^3)^(1/4)*(-1/d)^(1/4) + sqrt(k/x - k/x^2 - 1/x^2 + 1/x^3) + sqr
t(-1/d))/d^2 - 1/2*sqrt(2)*(-d^3)^(3/4)*log(-sqrt(2)*(k/x - k/x^2 - 1/x^2 +
 1/x^3)^(1/4)*(-1/d)^(1/4) + sqrt(k/x - k/x^2 - 1/x^2 + 1/x^3) + sqrt(-1/d)
)/d^2 + 4/(k/x - k/x^2 - 1/x^2 + 1/x^3)^(1/4)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(3-2 \left(1+k \right) x +k \,x^{2}\right)}{\left(-1+x \right) \left(\left(1-x \right) x \left(-k x +1\right)\right)^{\frac{1}{4}} \left(k x -1\right) \left(-d +d \left(1+k \right) x -d k \,x^{2}+x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3-2*(1+k)*x+k*x^2)/(-1+x)/((1-x)*x*(-k*x+1))^(1/4)/(k*x-1)/(-d+d*(
1+k)*x-d*k*x^2+x^3),x)
```

```
[Out] int(x^3*(3-2*(1+k)*x+k*x^2)/(-1+x)/((1-x)*x*(-k*x+1))^(1/4)/(k*x-1)/(-d+d*(
1+k)*x-d*k*x^2+x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{{\left(k x^{2} - 2 \,{\left(k + 1\right)} x + 3\right)} x^{3}}{{\left(d k x^{2} - d {\left(k + 1\right)} x - x^{3} + d\right)} \left({\left(k x - 1\right)} {\left(x - 1\right)} x\right)^{\frac{1}{4}} {\left(k x - 1\right)} {\left(x - 1\right)}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3-2*(1+k)*x+k*x^2)/(-1+x)/((1-x)*x*(-k*x+1))^(1/4)/(k*x-1)/(
-d+d*(1+k)*x-d*k*x^2+x^3),x, algorithm="maxima")
```

```
[Out] -integrate((k*x^2 - 2*(k + 1)*x + 3)*x^3/((d*k*x^2 - d*(k + 1)*x - x^3 + d)
*((k*x - 1)*(x - 1)*x)^(1/4)*(k*x - 1)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^3\,\left(k\,x^2-2\,x\,\left(k+1\right)+3\right)}{\left(k\,x-1\right)\,\left(x-1\right)\,{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/4}\,\left(-x^3+d\,k\,x^2-d\,\left(k+1\right)\,x+d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(k*x^2 - 2*x*(k + 1) + 3))/((k*x - 1)*(x - 1)*(x*(k*x - 1)*(x - 1
))^(1/4)*(d - x^3 - d*x*(k + 1) + d*k*x^2)),x)
```

```
[Out] int(-(x^3*(k*x^2 - 2*x*(k + 1) + 3))/((k*x - 1)*(x - 1)*(x*(k*x - 1)*(x - 1
))^(1/4)*(d - x^3 - d*x*(k + 1) + d*k*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3-2*(1+k)*x+k*x**2)/(-1+x)/((1-x)*x*(-k*x+1))**(1/4)/(k*x-1
)/(-d+d*(1+k)*x-d*k*x**2+x**3),x)
```

```
[Out] Timed out
```

3.1402 $\int \frac{(2+x^3)(1+2x^3)^{2/3}}{x^6(1+x^3)}\,dx$

Optimal. Leaf size=111

$$\frac{1}{3}\log\left(\sqrt[3]{2x^3+1}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2x^3+1}+x}\right)}{\sqrt{3}}+\frac{\left(2x^3+1\right)^{2/3}\left(-3x^3-4\right)}{10x^5}-\frac{1}{6}\log\left(\sqrt[3]{2x^3+1}\,x+\left(2x^3+1\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {580, 583, 12, 377, 200, 31, 634, 618, 204, 628}

$$\frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{2x^3+1}}\right)-\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{2x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2\left(2x^3+1\right)^{2/3}}{5x^5}-\frac{3\left(2x^3+1\right)^{2/3}}{10x^2}-\frac{1}{6}\log\left(\frac{x}{\sqrt[3]{2x^3+1}}+\frac{x^2}{\left(2x^3+1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x^3)*(1 + 2*x^3)^(2/3))/(x^6*(1 + x^3)),x]
```

```
[Out] (-2*(1 + 2*x^3)^(2/3))/(5*x^5) - (3*(1 + 2*x^3)^(2/3))/(10*x^2) - ArcTan[(1
 + (2*x)/(1 + 2*x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[1 - x/(1 + 2*x^3)^(1/3)]
/3 - Log[1 + x^2/(1 + 2*x^3)^(2/3) + x/(1 + 2*x^3)^(1/3)]/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
```

```
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(2+x^3)(1+2x^3)^{2/3}}{x^6(1+x^3)}\,dx = -\frac{2(1+2x^3)^{2/3}}{5x^5} + \frac{1}{5}\int \frac{3-2x^3}{x^3(1+x^3)\sqrt[3]{1+2x^3}}\,dx$$
$$= -\frac{2(1+2x^3)^{2/3}}{5x^5} - \frac{3(1+2x^3)^{2/3}}{10x^2} - \frac{1}{10}\int \frac{10}{(1+x^3)\sqrt[3]{1+2x^3}}\,dx$$
$$= -\frac{2(1+2x^3)^{2/3}}{5x^5} - \frac{3(1+2x^3)^{2/3}}{10x^2} - \int \frac{1}{(1+x^3)\sqrt[3]{1+2x^3}}\,dx$$
$$= -\frac{2(1+2x^3)^{2/3}}{5x^5} - \frac{3(1+2x^3)^{2/3}}{10x^2} - \text{Subst}\left(\int \frac{1}{1-x^3}\,dx, x, \frac{x}{\sqrt[3]{1+2x^3}}\right)$$
$$= -\frac{2(1+2x^3)^{2/3}}{5x^5} - \frac{3(1+2x^3)^{2/3}}{10x^2} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{x}{\sqrt[3]{1+2x^3}}\right) - \frac{1}{3}\text{Subs}$$
$$= -\frac{2(1+2x^3)^{2/3}}{5x^5} - \frac{3(1+2x^3)^{2/3}}{10x^2} + \frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{1+2x^3}}\right) - \frac{1}{6}\text{Subst}\left(\int \frac{1+2}{1+x+}\right.$$
$$= -\frac{2(1+2x^3)^{2/3}}{5x^5} - \frac{3(1+2x^3)^{2/3}}{10x^2} + \frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{1+2x^3}}\right) - \frac{1}{6}\log\left(1+\frac{x^2}{(1+2x^3)}\right.$$
$$= -\frac{2(1+2x^3)^{2/3}}{5x^5} - \frac{3(1+2x^3)^{2/3}}{10x^2} - \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+2x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{1+2x^3}}\right) -$$

Mathematica [A] time = 0.22, size = 102, normalized size = 0.92

$$\frac{1}{30}\left(10\log\left(1-\frac{x}{\sqrt[3]{x^3+2}}\right) - 10\sqrt{3}\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right) - \frac{3(2x^3+1)^{2/3}(3x^3+4)}{x^5} - 5\log\left(\frac{x}{\sqrt[3]{x^3+2}} + \frac{x^2}{(x^3+2)^{2/3}} + 1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + x^3)*(1 + 2*x^3)^(2/3))/(x^6*(1 + x^3)),x]
```

```
[Out] ((-3*(1 + 2*x^3)^(2/3)*(4 + 3*x^3))/x^5 - 10*Sqrt[3]*ArcTan[(1 + (2*x)/(2 +
 x^3)^(1/3))/Sqrt[3]] + 10*Log[1 - x/(2 + x^3)^(1/3)] - 5*Log[1 + x^2/(2 +
x^3)^(2/3) + x/(2 + x^3)^(1/3)])/30
```

IntegrateAlgebraic [A] time = 0.25, size = 111, normalized size = 1.00

$$\frac{1}{3}\log\left(\sqrt[3]{2x^3+1}-x\right) - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2x^3+1}+x}\right)}{\sqrt{3}} + \frac{(2x^3+1)^{2/3}(-3x^3-4)}{10x^5} - \frac{1}{6}\log\left(\sqrt[3]{2x^3+1}\,x + (2x^3+1)^{2/3} + x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^3)*(1 + 2*x^3)^(2/3))/(x^6*(1 + x^3)),x]
```

```
[Out] ((-4 - 3*x^3)*(1 + 2*x^3)^(2/3))/(10*x^5) - ArcTan[(Sqrt[3]*x)/(x + 2*(1 +
2*x^3)^(1/3))]/Sqrt[3] + Log[-x + (1 + 2*x^3)^(1/3)]/3 - Log[x^2 + x*(1 + 2
*x^3)^(1/3) + (1 + 2*x^3)^(2/3)]/6
```

fricas [A] time = 1.38, size = 133, normalized size = 1.20

$$-\frac{10\sqrt{3}x^5\arctan\left(-\frac{4\sqrt{3}(2x^3+1)^{\frac{1}{3}}x^2 - 2\sqrt{3}(2x^3+1)^{\frac{2}{3}}x + \sqrt{3}(2x^3+1)}{10x^3+1}\right) - 5x^5\log\left(\frac{x^3+3(2x^3+1)^{\frac{1}{3}}x^2 - 3(2x^3+1)^{\frac{2}{3}}x + 1}{x^3+1}\right) + 3(3x^3+4)(2x^3+1)^{\frac{2}{3}}}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(2*x^3+1)^(2/3)/x^6/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*x^5*arctan(-(4*sqrt(3)*(2*x^3 + 1)^(1/3)*x^2 - 2*sqrt(3)*
(2*x^3 + 1)^(2/3)*x + sqrt(3)*(2*x^3 + 1))/(10*x^3 + 1)) - 5*x^5*log((x^3 +
 3*(2*x^3 + 1)^(1/3)*x^2 - 3*(2*x^3 + 1)^(2/3)*x + 1)/(x^3 + 1)) + 3*(3*x^3
 + 4)*(2*x^3 + 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^3+1\right)^{\frac{2}{3}}\left(x^3+2\right)}{\left(x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(2*x^3+1)^(2/3)/x^6/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 1)^(2/3)*(x^3 + 2)/((x^3 + 1)*x^6), x)
```

maple [C] time = 1.36, size = 478, normalized size = 4.31

$$-\frac{6x^6+11x^3+4}{10x^5\left(2x^3+1\right)^{\frac{1}{3}}}+\frac{\ln\left(-\frac{27\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3+15\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(2x^3+1\right)^{\frac{2}{3}}x-3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(2x^3+1\right)^{\frac{1}{3}}x^2-12\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3+\left(2x^3+1\right)^{\frac{2}{3}}x+4\left(2x^3+1\right)^{\frac{1}{3}}x^2-4x^3-9\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)-2}{(1+x)\left(x^2-x+1\right)}\right)}{3}-\frac{\ln\left(\frac{9\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3+15\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(2x^3+1\right)^{\frac{2}{3}}x-12\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(2x^3+1\right)^{\frac{1}{3}}x^2+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3+4\left(2x^3+1\right)^{\frac{2}{3}}x+\left(2x^3+1\right)^{\frac{1}{3}}x^2-6x^3+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)-2}{(1+x)\left(x^2-x+1\right)}\right)}{3}-\ln\left(\frac{9\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3+15\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(2x^3+1\right)^{\frac{2}{3}}x-12\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(2x^3+1\right)^{\frac{1}{3}}x^2+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3+4\left(2x^3+1\right)^{\frac{2}{3}}x+\left(2x^3+1\right)^{\frac{1}{3}}x^2-6x^3+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)-2}{(1+x)\left(x^2-x+1\right)}\right)\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2)*(2*x^3+1)^(2/3)/x^6/(x^3+1),x)
```

```
[Out] -1/10*(6*x^6+11*x^3+4)/x^5/(2*x^3+1)^(1/3)+1/3*ln(-(27*RootOf(9*_Z^2+3*_Z+1
)^2*x^3+15*RootOf(9*_Z^2+3*_Z+1)*(2*x^3+1)^(2/3)*x-3*RootOf(9*_Z^2+3*_Z+1)*
(2*x^3+1)^(1/3)*x^2-12*RootOf(9*_Z^2+3*_Z+1)*x^3+(2*x^3+1)^(2/3)*x+4*(2*x^3
+1)^(1/3)*x^2-4*x^3-9*RootOf(9*_Z^2+3*_Z+1)-2)/(1+x)/(x^2-x+1))-1/3*ln((9*R
ootOf(9*_Z^2+3*_Z+1)^2*x^3+15*RootOf(9*_Z^2+3*_Z+1)*(2*x^3+1)^(2/3)*x-12*Ro
otOf(9*_Z^2+3*_Z+1)*(2*x^3+1)^(1/3)*x^2+3*RootOf(9*_Z^2+3*_Z+1)*x^3+4*(2*x^
3+1)^(2/3)*x+(2*x^3+1)^(1/3)*x^2-6*x^3+3*RootOf(9*_Z^2+3*_Z+1)-2)/(1+x)/(x^
2-x+1))-ln((9*RootOf(9*_Z^2+3*_Z+1)^2*x^3+15*RootOf(9*_Z^2+3*_Z+1)*(2*x^3+1
)^(2/3)*x-12*RootOf(9*_Z^2+3*_Z+1)*(2*x^3+1)^(1/3)*x^2+3*RootOf(9*_Z^2+3*_Z
+1)*x^3+4*(2*x^3+1)^(2/3)*x+(2*x^3+1)^(1/3)*x^2-6*x^3+3*RootOf(9*_Z^2+3*_Z+
1)-2)/(1+x)/(x^2-x+1))*RootOf(9*_Z^2+3*_Z+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^3+1\right)^{\frac{2}{3}}\left(x^3+2\right)}{\left(x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(2*x^3+1)^(2/3)/x^6/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^3 + 1)^(2/3)*(x^3 + 2)/((x^3 + 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3+2\right)\,{\left(2\,x^3+1\right)}^{2/3}}{x^6\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 2)*(2*x^3 + 1)^(2/3))/(x^6*(x^3 + 1)),x)
```

```
[Out] int(((x^3 + 2)*(2*x^3 + 1)^(2/3))/(x^6*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+2\right)\left(2x^3+1\right)^{\frac{2}{3}}}{x^6\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2)*(2*x**3+1)**(2/3)/x**6/(x**3+1),x)
[Out] Integral((x**3 + 2)*(2*x**3 + 1)**(2/3)/(x**6*(x + 1)*(x**2 - x + 1)), x)
```

3.1403 $\int \frac{3ab^2-2b(2a+b)x+(a+2b)x^2}{\sqrt[4]{x(-a+x)(-b+x)^2}\left(-ab^2d+b(2a+b)dx-(a+2b)dx^2+(-1+d)x^3\right)}\,dx$

Optimal. Leaf size=111

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}}{x}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}}{x}\right)}{d^{3/4}}$$

Rubi [F] time = 17.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3ab^2-2b(2a+b)x+(a+2b)x^2}{\sqrt[4]{x(-a+x)(-b+x)^2}\left(-ab^2d+b(2a+b)dx-(a+2b)dx^2+(-1+d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3*a*b^2 - 2*b*(2*a + b)*x + (a + 2*b)*x^2)/((x*(-a + x)*(-b + x)^2)^(1
/4)*(-(a*b^2*d) + b*(2*a + b)*d*x - (a + 2*b)*d*x^2 + (-1 + d)*x^3)),x]
```

```
[Out] (12*a*b*x^(1/4)*(-a + x)^(1/4)*Sqrt[-b + x]*Defer[Subst][Defer[Int][(x^2*Sq
rt[-b + x^4])/((-a + x^4)^(1/4)*(a*b^2*d - 2*a*b*(1 + b/(2*a))*d*x^4 + a*(1
 + (2*b)/a)*d*x^8 + (1 - d)*x^12)), x], x, x^(1/4)])/(-((a - x)*(b - x)^2*x
))^(1/4) - (4*(a + 2*b)*x^(1/4)*(-a + x)^(1/4)*Sqrt[-b + x]*Defer[Subst][De
fer[Int][(x^6*Sqrt[-b + x^4])/((-a + x^4)^(1/4)*(a*b^2*d - 2*a*b*(1 + b/(2*
a))*d*x^4 + a*(1 + (2*b)/a)*d*x^8 + (1 - d)*x^12)), x], x, x^(1/4)])/(-((a
- x)*(b - x)^2*x))^(1/4)
```

Rubi steps

$$\int \frac{3ab^2-2b(2a+b)x+(a+2b)x^2}{\sqrt[4]{x(-a+x)(-b+x)^2}\left(-ab^2d+b(2a+b)dx-(a+2b)dx^2+(-1+d)x^3\right)}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt{-b+x}\right)\int \ldots}{\sqrt[4]{x(\ldots}}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt{-b+x}\right)\int \ldots}{\sqrt[4]{x(-a\ldots}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt{-b+x}\right)\text{Subst}\ldots}{\ldots}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt{-b+x}\right)\text{Subst}\ldots}{\ldots}$$

$$= \frac{\left(4(-a-2b)\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt{-b+\ldots}\right.}{\ldots}$$

Mathematica [F] time = 4.41, size = 0, normalized size = 0.00

$$\int \frac{3ab^2-2b(2a+b)x+(a+2b)x^2}{\sqrt[4]{x(-a+x)(-b+x)^2}\left(-ab^2d+b(2a+b)dx-(a+2b)dx^2+(-1+d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3*a*b^2 - 2*b*(2*a + b)*x + (a + 2*b)*x^2)/((x*(-a + x)*(-b + x)
^2)^(1/4)*(-(a*b^2*d) + b*(2*a + b)*d*x - (a + 2*b)*d*x^2 + (-1 + d)*x^3)),
x]
```

```
[Out] Integrate[(3*a*b^2 - 2*b*(2*a + b)*x + (a + 2*b)*x^2)/((x*(-a + x)*(-b + x)
^2)^(1/4)*(-(a*b^2*d) + b*(2*a + b)*d*x - (a + 2*b)*d*x^2 + (-1 + d)*x^3)),
 x]
```

IntegrateAlgebraic [A] time = 0.46, size = 111, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}}{x}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}}{x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*a*b^2 - 2*b*(2*a + b)*x + (a + 2*b)*x^2)/((x*(-a + x)
*(-b + x)^2)^(1/4)*(-(a*b^2*d) + b*(2*a + b)*d*x - (a + 2*b)*d*x^2 + (-1 +
d)*x^3)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^
(1/4))/x])/d^(3/4) - (2*ArcTanh[(d^(1/4)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 +
(-a - 2*b)*x^3 + x^4)^(1/4))/x])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*b^2-2*b*(2*a+b)*x+(a+2*b)*x^2)/(x*(-a+x)*(-b+x)^2)^(1/4)/(-a
*b^2*d+b*(2*a+b)*d*x-(a+2*b)*d*x^2+(-1+d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{3\,ab^2-2\,(2\,a+b)bx+(a+2\,b)x^2}{\left(ab^2d-(2\,a+b)bdx+(a+2\,b)dx^2-(d-1)x^3\right)\left(-(a-x)(b-x)^2x\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*b^2-2*b*(2*a+b)*x+(a+2*b)*x^2)/(x*(-a+x)*(-b+x)^2)^(1/4)/(-a
*b^2*d+b*(2*a+b)*d*x-(a+2*b)*d*x^2+(-1+d)*x^3),x, algorithm="giac")
```

```
[Out] integrate(-(3*a*b^2 - 2*(2*a + b)*b*x + (a + 2*b)*x^2)/((a*b^2*d - (2*a + b
)*b*d*x + (a + 2*b)*d*x^2 - (d - 1)*x^3)*(-(a - x)*(b - x)^2*x)^(1/4)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{3a\,b^2-2b\,(2a+b)\,x+(a+2b)\,x^2}{\left(x\,(-a+x)\,(-b+x)^2\right)^{\frac{1}{4}}\left(-a\,b^2d+b\,(2a+b)\,dx-(a+2b)\,d\,x^2+(-1+d)\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a*b^2-2*b*(2*a+b)*x+(a+2*b)*x^2)/(x*(-a+x)*(-b+x)^2)^(1/4)/(-a*b^2*d
+b*(2*a+b)*d*x-(a+2*b)*d*x^2+(-1+d)*x^3),x)
```

```
[Out] int((3*a*b^2-2*b*(2*a+b)*x+(a+2*b)*x^2)/(x*(-a+x)*(-b+x)^2)^(1/4)/(-a*b^2*d
+b*(2*a+b)*d*x-(a+2*b)*d*x^2+(-1+d)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3\,ab^2 - 2\,(2\,a+b)bx + (a+2\,b)x^2}{\left(ab^2d - (2\,a+b)bdx + (a+2\,b)dx^2 - (d-1)x^3\right)\left(-(a-x)(b-x)^2x\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*b^2-2*b*(2*a+b)*x+(a+2*b)*x^2)/(x*(-a+x)*(-b+x)^2)^(1/4)/(-a
*b^2*d+b*(2*a+b)*d*x-(a+2*b)*d*x^2+(-1+d)*x^3),x, algorithm="maxima")
```

```
[Out] -integrate((3*a*b^2 - 2*(2*a + b)*b*x + (a + 2*b)*x^2)/((a*b^2*d - (2*a + b
)*b*d*x + (a + 2*b)*d*x^2 - (d - 1)*x^3)*(-(a - x)*(b - x)^2*x)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,a\,b^2 + x^2\,(a+2\,b) - 2\,b\,x\,(2\,a+b)}{\left(-x\,(a-x)\,(b-x)^2\right)^{1/4}\,\left(x^3\,(d-1) - d\,x^2\,(a+2\,b) - a\,b^2\,d + b\,d\,x\,(2\,a+b)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a*b^2 + x^2*(a + 2*b) - 2*b*x*(2*a + b))/((-x*(a - x)*(b - x)^2)^(1/
4)*(x^3*(d - 1) - d*x^2*(a + 2*b) - a*b^2*d + b*d*x*(2*a + b))),x)
```

```
[Out] int((3*a*b^2 + x^2*(a + 2*b) - 2*b*x*(2*a + b))/((-x*(a - x)*(b - x)^2)^(1/
4)*(x^3*(d - 1) - d*x^2*(a + 2*b) - a*b^2*d + b*d*x*(2*a + b))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*a*b**2-2*b*(2*a+b)*x+(a+2*b)*x**2)/(x*(-a+x)*(-b+x)**2)**(1/4)
/(-a*b**2*d+b*(2*a+b)*d*x-(a+2*b)*d*x**2+(-1+d)*x**3),x)
```

```
[Out] Timed out
```

3.1404 $\int \frac{(d+cx)\sqrt[4]{-bx^3+ax^4}}{x^2}\,dx$

Optimal. Leaf size=111

$$\frac{(bc-4ad)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^3}}\right)}{2a^{3/4}}+\frac{(4ad-bc)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^3}}\right)}{2a^{3/4}}+\frac{\sqrt[4]{ax^4-bx^3}\,(cx-4d)}{x}$$

Rubi [A] time = 0.28, antiderivative size = 198, normalized size of antiderivative = 1.78, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2038, 2021, 2032, 63, 331, 298, 203, 206}

$$\frac{x^{9/4}(ax-b)^{3/4}(bc-4ad)\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax-b}}\right)}{2a^{3/4}\left(ax^4-bx^3\right)^{3/4}}-\frac{x^{9/4}(ax-b)^{3/4}(bc-4ad)\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax-b}}\right)}{2a^{3/4}\left(ax^4-bx^3\right)^{3/4}}+\frac{\sqrt[4]{ax^4-bx^3}\,(bc-4ad)}{b}+\frac{4d\left(ax^4-bx^3\right)^{5/4}}{bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*x)*(-(b*x^3) + a*x^4)^(1/4))/x^2,x]
```

```
[Out] ((b*c - 4*a*d)*(-(b*x^3) + a*x^4)^(1/4))/b + (4*d*(-(b*x^3) + a*x^4)^(5/4))
/(b*x^4) + ((b*c - 4*a*d)*x^(9/4)*(-b + a*x)^(3/4)*ArcTan[(a^(1/4)*x^(1/4))
/(-b + a*x)^(1/4)])/(2*a^(3/4)*(-(b*x^3) + a*x^4)^(3/4)) - ((b*c - 4*a*d)*x
^(9/4)*(-b + a*x)^(3/4)*ArcTanh[(a^(1/4)*x^(1/4))/(-b + a*x)^(1/4)])/(2*a^(
3/4)*(-(b*x^3) + a*x^4)^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{(d+cx)\sqrt[4]{-bx^3+ax^4}}{x^2}\,dx = \frac{4d\left(-bx^3+ax^4\right)^{5/4}}{bx^4} + \frac{\left(4\left(\frac{bc}{4}-ad\right)\right)\int \frac{\sqrt[4]{-bx^3+ax^4}}{x}\,dx}{b}$$

$$= \frac{(bc-4ad)\sqrt[4]{-bx^3+ax^4}}{b} + \frac{4d\left(-bx^3+ax^4\right)^{5/4}}{bx^4} + \frac{1}{4}(-bc+4ad)\int \frac{x^2}{\left(-bx^3+ax^4\right)^{3/4}}$$

$$= \frac{(bc-4ad)\sqrt[4]{-bx^3+ax^4}}{b} + \frac{4d\left(-bx^3+ax^4\right)^{5/4}}{bx^4} + \frac{\left((-bc+4ad)x^{9/4}(-b+ax)^{3/4}\right)\int}{4\left(-bx^3+ax^4\right)^{3/4}}$$

$$= \frac{(bc-4ad)\sqrt[4]{-bx^3+ax^4}}{b} + \frac{4d\left(-bx^3+ax^4\right)^{5/4}}{bx^4} + \frac{\left((-bc+4ad)x^{9/4}(-b+ax)^{3/4}\right)\text{Sul}}{\left(-bx^3+ax\right.}$$

$$= \frac{(bc-4ad)\sqrt[4]{-bx^3+ax^4}}{b} + \frac{4d\left(-bx^3+ax^4\right)^{5/4}}{bx^4} + \frac{\left((-bc+4ad)x^{9/4}(-b+ax)^{3/4}\right)\text{Sul}}{\left(-bx^3+ax\right.}$$

$$= \frac{(bc-4ad)\sqrt[4]{-bx^3+ax^4}}{b} + \frac{4d\left(-bx^3+ax^4\right)^{5/4}}{bx^4} + \frac{\left((-bc+4ad)x^{9/4}(-b+ax)^{3/4}\right)\text{Sul}}{2\sqrt{a}\left(-bx^3+\right.}$$

$$= \frac{(bc-4ad)\sqrt[4]{-bx^3+ax^4}}{b} + \frac{4d\left(-bx^3+ax^4\right)^{5/4}}{bx^4} + \frac{(bc-4ad)x^{9/4}(-b+ax)^{3/4}\tan^{-1}\left(\right.}{2a^{3/4}\left(-bx^3+ax^4\right)^{3/4}}$$

Mathematica [C] time = 0.05, size = 87, normalized size = 0.78

$$\frac{4\sqrt[4]{x^3(ax-b)}\left(x(bc-4ad)\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{ax}{b}\right)-3d(b-ax)\sqrt[4]{1-\frac{ax}{b}}\right)}{3bx\sqrt[4]{1-\frac{ax}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*x)*(-(b*x^3) + a*x^4)^(1/4))/x^2,x]
```

```
[Out] (4*(x^3*(-b + a*x))^(1/4)*(-3*d*(b - a*x)*(1 - (a*x)/b)^(1/4) + (b*c - 4*a*
d)*x*Hypergeometric2F1[-1/4, 3/4, 7/4, (a*x)/b]))/(3*b*x*(1 - (a*x)/b)^(1/4
))
```

IntegrateAlgebraic [A] time = 0.62, size = 111, normalized size = 1.00

$$\frac{(bc-4ad)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^3}}\right)}{2a^{3/4}}+\frac{(4ad-bc)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^3}}\right)}{2a^{3/4}}+\frac{\sqrt[4]{ax^4-bx^3}\,(cx-4d)}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((d + c*x)*(-(b*x^3) + a*x^4)^(1/4))/x^2,x]
```

```
[Out] ((-4*d + c*x)*(-(b*x^3) + a*x^4)^(1/4))/x + ((b*c - 4*a*d)*ArcTan[(a^(1/4)*
x)/(-(b*x^3) + a*x^4)^(1/4)])/(2*a^(3/4)) + ((-(b*c) + 4*a*d)*ArcTanh[(a^(1
/4)*x)/(-(b*x^3) + a*x^4)^(1/4)])/(2*a^(3/4))
```

fricas [B] time = 0.43, size = 693, normalized size = 6.24

$$-\frac{4x\left(\frac{b^4c^4-16ab^3c^3d+96a^2b^2c^2d^2-256a^3bcd^3+256a^4d^4}{a^3}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2x\sqrt{\frac{a^2x^2\sqrt{\frac{b^4c^4-16ab^3c^3d+96a^2b^2c^2d^2-256a^3bcd^3+256a^4d^4}{a^3}}+\sqrt{ax^4-bx^3}(b^2c^2-8abcd+16a^2d^2)}{x^2}}\left(\frac{b^4c^4-16ab^3c^3d+96a^2b^2c^2d^2-256a^3bcd^3+256a^4d^4}{a^3}\right)^{\frac{3}{4}}+(ax^4-bx^3)^{\frac{1}{4}}(a^2bc-4a^3d)\left(\frac{b^4c^4-16ab^3c^3d+96a^2b^2c^2d^2-256a^3bcd^3+256a^4d^4}{a^3}\right)^{\frac{3}{4}}}{(b^4c^4-16ab^3c^3d+96a^2b^2c^2d^2-256a^3bcd^3+256a^4d^4)x}\right)+x\left(\frac{b^4c^4-16ab^3c^3d+96a^2b^2c^2d^2-256a^3bcd^3+256a^4d^4}{a^3}\right)^{\frac{1}{4}}\log\left(-\frac{ax\left(\frac{b^4c^4-16ab^3c^3d+96a^2b^2c^2d^2-256a^3bcd^3+256a^4d^4}{a^3}\right)^{\frac{1}{4}}+(ax^4-bx^3)^{\frac{1}{4}}(bc-4ad)}{x}\right)-x\left(\frac{b^4c^4-16ab^3c^3d+96a^2b^2c^2d^2-256a^3bcd^3+256a^4d^4}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{ax\left(\frac{b^4c^4-16ab^3c^3d+96a^2b^2c^2d^2-256a^3bcd^3+256a^4d^4}{a^3}\right)^{\frac{1}{4}}-(ax^4-bx^3)^{\frac{1}{4}}(bc-4ad)}{x}\right)-4\left(ax^4-bx^3\right)^{\frac{1}{4}}(cx-4d)}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+d)*(a*x^4-b*x^3)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*x*((b^4*c^4 - 16*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 256*a^3*b*c*d^3
 + 256*a^4*d^4)/a^3)^(1/4)*arctan((a^2*x*sqrt((a^2*x^2*sqrt((b^4*c^4 - 16*a
*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 256*a^3*b*c*d^3 + 256*a^4*d^4)/a^3) + sqr
t(a*x^4 - b*x^3)*(b^2*c^2 - 8*a*b*c*d + 16*a^2*d^2))/x^2)*((b^4*c^4 - 16*a*
b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 256*a^3*b*c*d^3 + 256*a^4*d^4)/a^3)^(3/4)
+ (a*x^4 - b*x^3)^(1/4)*(a^2*b*c - 4*a^3*d)*((b^4*c^4 - 16*a*b^3*c^3*d + 96
*a^2*b^2*c^2*d^2 - 256*a^3*b*c*d^3 + 256*a^4*d^4)/a^3)^(3/4))/((b^4*c^4 - 1
6*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 256*a^3*b*c*d^3 + 256*a^4*d^4)*x)) + x
*((b^4*c^4 - 16*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 256*a^3*b*c*d^3 + 256*a^
4*d^4)/a^3)^(1/4)*log(-(a*x*((b^4*c^4 - 16*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2
 - 256*a^3*b*c*d^3 + 256*a^4*d^4)/a^3)^(1/4) + (a*x^4 - b*x^3)^(1/4)*(b*c -
 4*a*d))/x) - x*((b^4*c^4 - 16*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 256*a^3*b
*c*d^3 + 256*a^4*d^4)/a^3)^(1/4)*log((a*x*((b^4*c^4 - 16*a*b^3*c^3*d + 96*a
^2*b^2*c^2*d^2 - 256*a^3*b*c*d^3 + 256*a^4*d^4)/a^3)^(1/4) - (a*x^4 - b*x^3
)^(1/4)*(b*c - 4*a*d))/x) - 4*(a*x^4 - b*x^3)^(1/4)*(c*x - 4*d))/x
```

giac [B] time = 0.22, size = 257, normalized size = 2.32

$$\frac{8\left(a-\frac{b}{x}\right)^{\frac{1}{4}}bcx-32\left(a-\frac{b}{x}\right)^{\frac{1}{4}}bd+\frac{2\sqrt{2}\left(b^2c-4abd\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}}+\frac{2\sqrt{2}\left(b^2c-4abd\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}}+\frac{\sqrt{2}\left(b^2c-4abd\right)\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x}}\right)}{(-a)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^2c-4abd\right)\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x}}\right)}{(-a)^{\frac{3}{4}}}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+d)*(a*x^4-b*x^3)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*(a - b/x)^(1/4)*b*c*x - 32*(a - b/x)^(1/4)*b*d + 2*sqrt(2)*(b^2*c -
4*a*b*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a - b/x)^(1/4))/(-a)^(
1/4))/(-a)^(3/4) + 2*sqrt(2)*(b^2*c - 4*a*b*d)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(-a)^(1/4) - 2*(a - b/x)^(1/4))/(-a)^(1/4))/(-a)^(3/4) + sqrt(2)*(b^2*c -
4*a*b*d)*log(sqrt(2)*(-a)^(1/4)*(a - b/x)^(1/4) + sqrt(-a) + sqrt(a - b/x))
/(-a)^(3/4) - sqrt(2)*(b^2*c - 4*a*b*d)*log(-sqrt(2)*(-a)^(1/4)*(a - b/x)^(
1/4) + sqrt(-a) + sqrt(a - b/x))/(-a)^(3/4))/b
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(cx+d)\left(a\,x^{4}-b\,x^{3}\right)^{\frac{1}{4}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x+d)*(a*x^4-b*x^3)^(1/4)/x^2,x)
```

```
[Out] int((c*x+d)*(a*x^4-b*x^3)^(1/4)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{4}-bx^{3}\right)^{\frac{1}{4}}(cx+d)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+d)*(a*x^4-b*x^3)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 - b*x^3)^(1/4)*(c*x + d)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^4-b\,x^3\right)^{1/4}\,\left(d+c\,x\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^4 - b*x^3)^(1/4)*(d + c*x))/x^2,x)
```

```
[Out] int(((a*x^4 - b*x^3)^(1/4)*(d + c*x))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^{3}\left(ax-b\right)}\,\left(cx+d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+d)*(a*x**4-b*x**3)**(1/4)/x**2,x)
```

```
[Out] Integral((x**3*(a*x - b))**(1/4)*(c*x + d)/x**2, x)
```

3.1405 $\int \frac{(3+x^5)\sqrt[3]{-2+x^3+x^5}}{x^2(-2+x^5)}\,dx$

Optimal. Leaf size=111

$$\frac{3\sqrt[3]{x^5+x^3-2}}{2x}+\frac{1}{2}\log\left(\sqrt[3]{x^5+x^3-2}-x\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^5+x^3-2}+x}\right)-\frac{1}{4}\log\left(x^2+\sqrt[3]{x^5+x^3-2}\,x+(\right.$$

Rubi [F] time = 1.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(3+x^5)\sqrt[3]{-2+x^3+x^5}}{x^2(-2+x^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((3 + x^5)*(-2 + x^3 + x^5)^(1/3))/(x^2*(-2 + x^5)),x]
```

```
[Out] -1/2*Defer[Int][(-2 + x^3 + x^5)^(1/3)/(2^(1/5) - x), x]/2^(1/5) - (3*Defer
[Int][(-2 + x^3 + x^5)^(1/3)/x^2, x])/2 - ((-1)^(2/5)*Defer[Int][(-2 + x^3
+ x^5)^(1/3)/(2^(1/5) + (-1)^(1/5)*x), x])/(2*2^(1/5)) - ((-1)^(4/5)*Defer[
Int][(-2 + x^3 + x^5)^(1/3)/(2^(1/5) - (-1)^(2/5)*x), x])/(2*2^(1/5)) + ((-
1/2)^(1/5)*Defer[Int][(-2 + x^3 + x^5)^(1/3)/(2^(1/5) + (-1)^(3/5)*x), x])/
2 + ((-1)^(3/5)*Defer[Int][(-2 + x^3 + x^5)^(1/3)/(2^(1/5) - (-1)^(4/5)*x),
 x])/(2*2^(1/5))
```

Rubi steps

$$\begin{aligned}
\int \frac{(3+x^5)\sqrt[3]{-2+x^3+x^5}}{x^2(-2+x^5)}\,dx &= \int \left(-\frac{3\sqrt[3]{-2+x^3+x^5}}{2x^2}+\frac{5x^3\sqrt[3]{-2+x^3+x^5}}{2(-2+x^5)}\right)dx\\
&= -\left(\frac{3}{2}\int \frac{\sqrt[3]{-2+x^3+x^5}}{x^2}\,dx\right)+\frac{5}{2}\int \frac{x^3\sqrt[3]{-2+x^3+x^5}}{-2+x^5}\,dx\\
&= -\left(\frac{3}{2}\int \frac{\sqrt[3]{-2+x^3+x^5}}{x^2}\,dx\right)+\frac{5}{2}\int \left(-\frac{\sqrt[3]{-2+x^3+x^5}}{5\sqrt[5]{2}\left(\sqrt[5]{2}-x\right)}-\frac{(-1)^{2/5}\sqrt[3]{-2+x^3+x}}{5\sqrt[5]{2}\left(\sqrt[5]{2}+\sqrt[5]{-1}x\right.}\right.\\
&= -\left(\frac{3}{2}\int \frac{\sqrt[3]{-2+x^3+x^5}}{x^2}\,dx\right)+\frac{1}{2}\sqrt[5]{-\frac{1}{2}}\int \frac{\sqrt[3]{-2+x^3+x^5}}{\sqrt[5]{2}+(-1)^{3/5}x}\,dx-\frac{\int \frac{\sqrt[3]{-2+x^3+x^5}}{\sqrt[5]{2}-x}\,d}{2\sqrt[5]{2}}
\end{aligned}$$

Mathematica [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(3+x^5)\sqrt[3]{-2+x^3+x^5}}{x^2(-2+x^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((3 + x^5)*(-2 + x^3 + x^5)^(1/3))/(x^2*(-2 + x^5)),x]
```

```
[Out] Integrate[((3 + x^5)*(-2 + x^3 + x^5)^(1/3))/(x^2*(-2 + x^5)), x]
```

IntegrateAlgebraic [A] time = 2.18, size = 111, normalized size = 1.00

$$\frac{3\sqrt[3]{x^5+x^3-2}}{2x}+\frac{1}{2}\log\left(\sqrt[3]{x^5+x^3-2}-x\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^5+x^3-2}+x}\right)-\frac{1}{4}\log\left(x^2+\sqrt[3]{x^5+x^3-2}\,x+\left(x^5+x^3-2\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3 + x^5)*(-2 + x^3 + x^5)^(1/3))/(x^2*(-2 + x^5)),x]
```

```
[Out] (3*(-2 + x^3 + x^5)^(1/3))/(2*x) + (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-2 +
 x^3 + x^5)^(1/3))])/2 + Log[-x + (-2 + x^3 + x^5)^(1/3)]/2 - Log[x^2 + x*(
-2 + x^3 + x^5)^(1/3) + (-2 + x^3 + x^5)^(2/3)]/4
```

fricas [A] time = 5.32, size = 136, normalized size = 1.23

$$\frac{2\,\sqrt{3}\,x\arctan\left(-\frac{240779826\,\sqrt{3}\,{\left(x^{5}+x^{3}-2\right)}^{\frac{1}{3}}x^{2}-64389332\,\sqrt{3}\,{\left(x^{5}+x^{3}-2\right)}^{\frac{2}{3}}x+\sqrt{3}\,{\left(18550880\,x^{5}+88195247\,x^{3}-37101760\right)}}{3\,{\left(2863288\,x^{5}+152584579\,x^{3}-5726576\right)}}\right)+x\log\left(\frac{x^{5}+3\,{\left(x^{5}+x^{3}-2\right)}^{\frac{1}{3}}x^{2}-3\,{\left(x^{5}+x^{3}-2\right)}^{\frac{2}{3}}x-2}{x^{5}-2}\right)+6\,{\left(x^{5}+x^{3}-2\right)}^{\frac{1}{3}}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+3)*(x^5+x^3-2)^(1/3)/x^2/(x^5-2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*x*arctan(-1/3*(240779826*sqrt(3)*(x^5 + x^3 - 2)^(1/3)*x^2 -
 64389332*sqrt(3)*(x^5 + x^3 - 2)^(2/3)*x + sqrt(3)*(18550880*x^5 + 8819524
7*x^3 - 37101760))/(2863288*x^5 + 152584579*x^3 - 5726576)) + x*log((x^5 +
3*(x^5 + x^3 - 2)^(1/3)*x^2 - 3*(x^5 + x^3 - 2)^(2/3)*x - 2)/(x^5 - 2)) + 6
*(x^5 + x^3 - 2)^(1/3))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(x^{5}+x^{3}-2\right)}^{\frac{1}{3}}{\left(x^{5}+3\right)}}{{\left(x^{5}-2\right)}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+3)*(x^5+x^3-2)^(1/3)/x^2/(x^5-2),x, algorithm="giac")
```

```
[Out] integrate((x^5 + x^3 - 2)^(1/3)*(x^5 + 3)/((x^5 - 2)*x^2), x)
```

maple [C] time = 1.99, size = 1119, normalized size = 10.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+3)*(x^5+x^3-2)^(1/3)/x^2/(x^5-2),x)
```

```
[Out] 3/2*(x^5+x^3-2)^(1/3)/x+(1/2*RootOf(_Z^2+_Z+1)*ln(-(2*x^10*RootOf(_Z^2+_Z+1
)-2*RootOf(_Z^2+_Z+1)^2*x^8+x^10+3*RootOf(_Z^2+_Z+1)*x^8+3*RootOf(_Z^2+_Z+1
)*(x^10+2*x^8+x^6-4*x^5-4*x^3+4)^(1/3)*x^6-2*RootOf(_Z^2+_Z+1)^2*x^6+2*x^8+
3*(x^10+2*x^8+x^6-4*x^5-4*x^3+4)^(1/3)*x^6+RootOf(_Z^2+_Z+1)*x^6+3*RootOf(_
Z^2+_Z+1)*(x^10+2*x^8+x^6-4*x^5-4*x^3+4)^(1/3)*x^4-8*RootOf(_Z^2+_Z+1)*x^5+
x^6+3*RootOf(_Z^2+_Z+1)*(x^10+2*x^8+x^6-4*x^5-4*x^3+4)^(2/3)*x^2+3*(x^10+2*
x^8+x^6-4*x^5-4*x^3+4)^(1/3)*x^4+4*RootOf(_Z^2+_Z+1)^2*x^3-4*x^5+3*(x^10+2*
x^8+x^6-4*x^5-4*x^3+4)^(2/3)*x^2-6*RootOf(_Z^2+_Z+1)*x^3-6*RootOf(_Z^2+_Z+1
)*(x^10+2*x^8+x^6-4*x^5-4*x^3+4)^(1/3)*x-4*x^3-6*(x^10+2*x^8+x^6-4*x^5-4*x^
3+4)^(1/3)*x+8*RootOf(_Z^2+_Z+1)+4)/(x^5-2)/(x^4+x^3+2*x^2+2*x+2)/(-1+x))-1
/2*ln((2*x^10*RootOf(_Z^2+_Z+1)+2*RootOf(_Z^2+_Z+1)^2*x^8+x^10+7*RootOf(_Z^
2+_Z+1)*x^8+3*RootOf(_Z^2+_Z+1)*(x^10+2*x^8+x^6-4*x^5-4*x^3+4)^(1/3)*x^6+2*
RootOf(_Z^2+_Z+1)^2*x^6+3*x^8+5*RootOf(_Z^2+_Z+1)*x^6+3*RootOf(_Z^2+_Z+1)*(
x^10+2*x^8+x^6-4*x^5-4*x^3+4)^(1/3)*x^4-8*RootOf(_Z^2+_Z+1)*x^5+2*x^6+3*Roo
tOf(_Z^2+_Z+1)*(x^10+2*x^8+x^6-4*x^5-4*x^3+4)^(2/3)*x^2-4*RootOf(_Z^2+_Z+1)
^2*x^3-4*x^5-14*RootOf(_Z^2+_Z+1)*x^3-6*RootOf(_Z^2+_Z+1)*(x^10+2*x^8+x^6-4
*x^5-4*x^3+4)^(1/3)*x-6*x^3+8*RootOf(_Z^2+_Z+1)+4)/(x^5-2)/(x^4+x^3+2*x^2+2
*x+2)/(-1+x))*RootOf(_Z^2+_Z+1)-1/2*ln((2*x^10*RootOf(_Z^2+_Z+1)+2*RootOf(_
Z^2+_Z+1)^2*x^8+x^10+7*RootOf(_Z^2+_Z+1)*x^8+3*RootOf(_Z^2+_Z+1)*(x^10+2*x^
8+x^6-4*x^5-4*x^3+4)^(1/3)*x^6+2*RootOf(_Z^2+_Z+1)^2*x^6+3*x^8+5*RootOf(_Z^
```

```
2+_Z+1)*x^6+3*RootOf(_Z^2+_Z+1)*(x^10+2*x^8+x^6-4*x^5-4*x^3+4)^(1/3)*x^4-8*
RootOf(_Z^2+_Z+1)*x^5+2*x^6+3*RootOf(_Z^2+_Z+1)*(x^10+2*x^8+x^6-4*x^5-4*x^3
+4)^(2/3)*x^2-4*RootOf(_Z^2+_Z+1)^2*x^3-4*x^5-14*RootOf(_Z^2+_Z+1)*x^3-6*Ro
otOf(_Z^2+_Z+1)*(x^10+2*x^8+x^6-4*x^5-4*x^3+4)^(1/3)*x-6*x^3+8*RootOf(_Z^2+
_Z+1)+4)/(x^5-2)/(x^4+x^3+2*x^2+2*x+2)/(-1+x)))/(x^5+x^3-2)^(2/3)*((x^5+x^3
-2)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5+x^3-2\right)^{\frac{1}{3}}\left(x^5+3\right)}{\left(x^5-2\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+3)*(x^5+x^3-2)^(1/3)/x^2/(x^5-2),x, algorithm="maxima")
```

```
[Out] integrate((x^5 + x^3 - 2)^(1/3)*(x^5 + 3)/((x^5 - 2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^5+3\right)\,\left(x^5+x^3-2\right)^{1/3}}{x^2\,\left(x^5-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 + 3)*(x^3 + x^5 - 2)^(1/3))/(x^2*(x^5 - 2)),x)
```

```
[Out] int(((x^5 + 3)*(x^3 + x^5 - 2)^(1/3))/(x^2*(x^5 - 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+3)*(x**5+x**3-2)**(1/3)/x**2/(x**5-2),x)
```

```
[Out] Timed out
```

3.1406 $\int \frac{(-3+x^5)(2+x^3+x^5)^{2/3}}{x^3(2+x^5)} dx$

Optimal. Leaf size=111

$$\frac{1}{2}\log\left(\sqrt[3]{x^5+x^3+2}-x\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^5+x^3+2}+x}\right)+\frac{3(x^5+x^3+2)^{2/3}}{4x^2}-\frac{1}{4}\log\left(x^2+\sqrt[3]{x^5+x^3+2}x+(\right.$$

Rubi [F] time = 1.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-3+x^5)(2+x^3+x^5)^{2/3}}{x^3(2+x^5)} dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + x^5)*(2 + x^3 + x^5)^(2/3))/(x^3*(2 + x^5)),x]
```

```
[Out] (-3*Defer[Int][(2 + x^3 + x^5)^(2/3)/x^3, x])/2 + Defer[Int][(2 + x^3 + x^5
)^(2/3)/(2^(1/5) + x), x]/(2*2^(2/5)) - ((-1)^(3/5)*Defer[Int][(2 + x^3 + x
^5)^(2/3)/(2^(1/5) - (-1)^(1/5)*x), x])/(2*2^(2/5)) - ((-1)^(1/5)*Defer[Int
][(2 + x^3 + x^5)^(2/3)/(2^(1/5) + (-1)^(2/5)*x), x])/(2*2^(2/5)) + ((-1)^(
4/5)*Defer[Int][(2 + x^3 + x^5)^(2/3)/(2^(1/5) - (-1)^(3/5)*x), x])/(2*2^(2
/5)) + ((-1/2)^(2/5)*Defer[Int][(2 + x^3 + x^5)^(2/3)/(2^(1/5) + (-1)^(4/5)
*x), x])/2
```

Rubi steps

$$\begin{aligned}
\int \frac{(-3+x^5)(2+x^3+x^5)^{2/3}}{x^3(2+x^5)} dx &= \int \left(-\frac{3(2+x^3+x^5)^{2/3}}{2x^3}+\frac{5x^2(2+x^3+x^5)^{2/3}}{2(2+x^5)}\right) dx \\
&= -\left(\frac{3}{2}\int \frac{(2+x^3+x^5)^{2/3}}{x^3} dx\right)+\frac{5}{2}\int \frac{x^2(2+x^3+x^5)^{2/3}}{2+x^5} dx \\
&= -\left(\frac{3}{2}\int \frac{(2+x^3+x^5)^{2/3}}{x^3} dx\right)+\frac{5}{2}\int \left(\frac{(2+x^3+x^5)^{2/3}}{5\, 2^{2/5}(\sqrt[5]{2}+x)}-\frac{(-1)^{3/5}(2+x^3+x^5)^{2}}{5\, 2^{2/5}(\sqrt[5]{2}-\sqrt[5]{-1}x)}\right. \\
&= -\left(\frac{3}{2}\int \frac{(2+x^3+x^5)^{2/3}}{x^3} dx\right)+\frac{1}{2}\left(-\frac{1}{2}\right)^{2/5}\int \frac{(2+x^3+x^5)^{2/3}}{\sqrt[5]{2}+(-1)^{4/5}x} dx+\frac{\int \frac{(2+x^3+x^5)}{\sqrt[5]{2}+x}}{2\, 2^{2/5}}
\end{aligned}$$

Mathematica [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{(-3+x^5)(2+x^3+x^5)^{2/3}}{x^3(2+x^5)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + x^5)*(2 + x^3 + x^5)^(2/3))/(x^3*(2 + x^5)),x]
```

```
[Out] Integrate[((-3 + x^5)*(2 + x^3 + x^5)^(2/3))/(x^3*(2 + x^5)), x]
```

IntegrateAlgebraic [A] time = 2.65, size = 111, normalized size = 1.00

$$\frac{1}{2}\log\left(\sqrt[3]{x^5+x^3+2}-x\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^5+x^3+2}+x}\right)+\frac{3\left(x^5+x^3+2\right)^{2/3}}{4x^2}-\frac{1}{4}\log\left(x^2+\sqrt[3]{x^5+x^3+2}\,x+\left(x^5+x^3+2\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^5)*(2 + x^3 + x^5)^(2/3))/(x^3*(2 + x^5)),x]
```

```
[Out] (3*(2 + x^3 + x^5)^(2/3))/(4*x^2) - (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(2 +
 x^3 + x^5)^(1/3))])/2 + Log[-x + (2 + x^3 + x^5)^(1/3)]/2 - Log[x^2 + x*(2
 + x^3 + x^5)^(1/3) + (2 + x^3 + x^5)^(2/3)]/4
```

fricas [A] time = 7.36, size = 141, normalized size = 1.27

$$-\frac{2\sqrt{3}x^2\arctan\left(-\frac{240779826\sqrt{3}\left(x^5+x^3+2\right)^{\frac{1}{3}}x^2-64389332\sqrt{3}\left(x^5+x^3+2\right)^{\frac{2}{3}}x+\sqrt{3}\left(18550880x^5+88195247x^3+37101760\right)}{3\left(2863288x^5+152584579x^3+5726576\right)}\right)-x^2\log\left(\frac{x^5+3\left(x^5+x^3+2\right)^{\frac{1}{3}}x^2-3\left(x^5+x^3+2\right)^{\frac{2}{3}}x+2}{x^5+2}\right)-3\left(x^5+x^3+2\right)^{\frac{2}{3}}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-3)*(x^5+x^3+2)^(2/3)/x^3/(x^5+2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*x^2*arctan(-1/3*(240779826*sqrt(3)*(x^5 + x^3 + 2)^(1/3)*x^
2 - 64389332*sqrt(3)*(x^5 + x^3 + 2)^(2/3)*x + sqrt(3)*(18550880*x^5 + 8819
5247*x^3 + 37101760))/(2863288*x^5 + 152584579*x^3 + 5726576)) - x^2*log((x
^5 + 3*(x^5 + x^3 + 2)^(1/3)*x^2 - 3*(x^5 + x^3 + 2)^(2/3)*x + 2)/(x^5 + 2)
) - 3*(x^5 + x^3 + 2)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^5+x^3+2\right)^{\frac{2}{3}}\left(x^5-3\right)}{\left(x^5+2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-3)*(x^5+x^3+2)^(2/3)/x^3/(x^5+2),x, algorithm="giac")
```

```
[Out] integrate((x^5 + x^3 + 2)^(2/3)*(x^5 - 3)/((x^5 + 2)*x^3), x)
```

maple [C] time = 2.72, size = 442, normalized size = 3.98

$$\frac{3\left(x^5+x^3+2\right)^{\frac{2}{3}}}{4x^2}+\frac{\ln\left(\frac{-2\operatorname{RootOf}\left(4\_Z^2+2\_Z+1\right)x^5+4\operatorname{RootOf}\left(4\_Z^2+2\_Z+1\right)^2x^3+x^5+6\operatorname{RootOf}\left(4\_Z^2+2\_Z+1\right)\left(x^5+x^3+2\right)^{\frac{1}{3}}x^2-4x^3\operatorname{RootOf}\left(4\_Z^2+2\_Z+1\right)-3\left(x^5+x^3+2\right)^{\frac{2}{3}}x+3\left(x^5+x^3+2\right)^{\frac{1}{3}}x^2+x^3-4\operatorname{RootOf}\left(4\_Z^2+2\_Z+1\right)+2}{x^5+2}\right)}{2}-\frac{\ln\left(-\frac{4\operatorname{RootOf}\left(4\_Z^2+2\_Z+1\right)x^5+8\operatorname{RootOf}\left(4\_Z^2+2\_Z+1\right)^2x^3+x^5-6\operatorname{RootOf}\left(4\_Z^2+2\_Z+1\right)\left(x^5+x^3+2\right)^{\frac{1}{3}}x^2+10x^3\operatorname{RootOf}\left(4\_Z^2+2\_Z+1\right)+3\left(x^5+x^3+2\right)^{\frac{2}{3}}x-3\left(x^5+x^3+2\right)^{\frac{1}{3}}x^2+2x^3+8\operatorname{RootOf}\left(4\_Z^2+2\_Z+1\right)+2}{x^5+2}\right)}{2}-\ln\left(-\frac{4\operatorname{RootOf}\left(4\_Z^2+2\_Z+1\right)x^5+8\operatorname{RootOf}\left(4\_Z^2+2\_Z+1\right)^2x^3+x^5-6\operatorname{RootOf}\left(4\_Z^2+2\_Z+1\right)\left(x^5+x^3+2\right)^{\frac{1}{3}}x^2+10x^3\operatorname{RootOf}\left(4\_Z^2+2\_Z+1\right)+3\left(x^5+x^3+2\right)^{\frac{2}{3}}x-3\left(x^5+x^3+2\right)^{\frac{1}{3}}x^2+2x^3+8\operatorname{RootOf}\left(4\_Z^2+2\_Z+1\right)+2}{x^5+2}\right)\operatorname{RootOf}\left(4\_Z^2+2\_Z+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-3)*(x^5+x^3+2)^(2/3)/x^3/(x^5+2),x)
```

```
[Out] 3/4*(x^5+x^3+2)^(2/3)/x^2+1/2*ln((-2*RootOf(4*_Z^2+2*_Z+1)*x^5+4*RootOf(4*_
Z^2+2*_Z+1)^2*x^3+x^5+6*RootOf(4*_Z^2+2*_Z+1)*(x^5+x^3+2)^(1/3)*x^2-4*x^3*R
ootOf(4*_Z^2+2*_Z+1)-3*(x^5+x^3+2)^(2/3)*x+3*(x^5+x^3+2)^(1/3)*x^2+x^3-4*Ro
otOf(4*_Z^2+2*_Z+1)+2)/(x^5+2))-1/2*ln(-(4*RootOf(4*_Z^2+2*_Z+1)*x^5+8*Root
Of(4*_Z^2+2*_Z+1)^2*x^3+x^5-6*RootOf(4*_Z^2+2*_Z+1)*(x^5+x^3+2)^(1/3)*x^2+1
0*x^3*RootOf(4*_Z^2+2*_Z+1)+3*(x^5+x^3+2)^(2/3)*x-3*(x^5+x^3+2)^(1/3)*x^2+2
*x^3+8*RootOf(4*_Z^2+2*_Z+1)+2)/(x^5+2))-ln(-(4*RootOf(4*_Z^2+2*_Z+1)*x^5+8
*RootOf(4*_Z^2+2*_Z+1)^2*x^3+x^5-6*RootOf(4*_Z^2+2*_Z+1)*(x^5+x^3+2)^(1/3)*
x^2+10*x^3*RootOf(4*_Z^2+2*_Z+1)+3*(x^5+x^3+2)^(2/3)*x-3*(x^5+x^3+2)^(1/3)*
x^2+2*x^3+8*RootOf(4*_Z^2+2*_Z+1)+2)/(x^5+2))*RootOf(4*_Z^2+2*_Z+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^5+x^3+2\right)^{\frac{2}{3}}\left(x^5-3\right)}{\left(x^5+2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-3)*(x^5+x^3+2)^(2/3)/x^3/(x^5+2),x, algorithm="maxima")
```

```
[Out] integrate((x^5 + x^3 + 2)^(2/3)*(x^5 - 3)/((x^5 + 2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^5-3\right)\,\left(x^5+x^3+2\right)^{2/3}}{x^3\,\left(x^5+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 - 3)*(x^3 + x^5 + 2)^(2/3))/(x^3*(x^5 + 2)),x)
```

```
[Out] int(((x^5 - 3)*(x^3 + x^5 + 2)^(2/3))/(x^3*(x^5 + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-3)*(x**5+x**3+2)**(2/3)/x**3/(x**5+2),x)
```

```
[Out] Timed out
```

3.1407 $\int \frac{\sqrt[3]{-1+x^6}\left(1+x^6\right)}{x^9}\,dx$

Optimal. Leaf size=111

$$\frac{\sqrt[3]{x^6-1}\left(-3x^6-1\right)}{8x^8}-\frac{1}{6}\log\left(\sqrt[3]{x^6-1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)$$

Rubi [A] time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {451, 275, 277, 331, 292, 31, 634, 618, 204, 628}

$$\frac{\left(x^6-1\right)^{4/3}}{8x^8}-\frac{\sqrt[3]{x^6-1}}{2x^2}-\frac{1}{6}\log\left(1-\frac{x^2}{\sqrt[3]{x^6-1}}\right)-\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6-1}}+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(\frac{x^4}{\left(x^6-1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6-1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^6)^(1/3)*(1 + x^6))/x^9,x]
```

```
[Out] -1/2*(-1 + x^6)^(1/3)/x^2 + (-1 + x^6)^(4/3)/(8*x^8) - ArcTan[(1 + (2*x^2)/
(-1 + x^6)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - Log[1 - x^2/(-1 + x^6)^(1/3)]/6 +
Log[1 + x^4/(-1 + x^6)^(2/3) + x^2/(-1 + x^6)^(1/3)]/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[3]{-1+x^6}\left(1+x^6\right)}{x^9}\,dx = \frac{\left(-1+x^6\right)^{4/3}}{8x^8}+\int \frac{\sqrt[3]{-1+x^6}}{x^3}\,dx$$
$$= \frac{\left(-1+x^6\right)^{4/3}}{8x^8}+\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt[3]{-1+x^3}}{x^2}\,dx,x,x^2\right)$$
$$= -\frac{\sqrt[3]{-1+x^6}}{2x^2}+\frac{\left(-1+x^6\right)^{4/3}}{8x^8}+\frac{1}{2}\text{Subst}\left(\int \frac{x}{\left(-1+x^3\right)^{2/3}}\,dx,x,x^2\right)$$
$$= -\frac{\sqrt[3]{-1+x^6}}{2x^2}+\frac{\left(-1+x^6\right)^{4/3}}{8x^8}+\frac{1}{2}\text{Subst}\left(\int \frac{x}{1-x^3}\,dx,x,\frac{x^2}{\sqrt[3]{-1+x^6}}\right)$$
$$= -\frac{\sqrt[3]{-1+x^6}}{2x^2}+\frac{\left(-1+x^6\right)^{4/3}}{8x^8}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\frac{x^2}{\sqrt[3]{-1+x^6}}\right)-\frac{1}{6}\text{Subst}\left(\int \right.$$
$$= -\frac{\sqrt[3]{-1+x^6}}{2x^2}+\frac{\left(-1+x^6\right)^{4/3}}{8x^8}-\frac{1}{6}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right)+\frac{1}{12}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,d\right.$$
$$= -\frac{\sqrt[3]{-1+x^6}}{2x^2}+\frac{\left(-1+x^6\right)^{4/3}}{8x^8}-\frac{1}{6}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right)+\frac{1}{12}\log\left(1+\frac{x^4}{\left(-1+x^6\right)^{2/3}}+\right.$$
$$= -\frac{\sqrt[3]{-1+x^6}}{2x^2}+\frac{\left(-1+x^6\right)^{4/3}}{8x^8}-\frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{-1+x^6}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{6}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right)+\frac{1}{12}\log$$

Mathematica [C] time = 0.02, size = 50, normalized size = 0.45

$$\frac{\sqrt[3]{x^6-1}\left(-\frac{4x^6\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};x^6\right)}{\sqrt[3]{1-x^6}}+x^6-1\right)}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^6)^(1/3)*(1 + x^6))/x^9,x]
```

```
[Out] ((-1 + x^6)^(1/3)*(-1 + x^6 - (4*x^6*Hypergeometric2F1[-1/3, -1/3, 2/3, x^6
])/(1 - x^6)^(1/3)))/(8*x^8)
```

IntegrateAlgebraic [A] time = 0.49, size = 111, normalized size = 1.00

$$\frac{\sqrt[3]{x^6-1}\left(-3x^6-1\right)}{8x^8}-\frac{1}{6}\log\left(\sqrt[3]{x^6-1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^6)^(1/3)*(1 + x^6))/x^9,x]
```

```
[Out] ((-1 - 3*x^6)*(-1 + x^6)^(1/3))/(8*x^8) - ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(-1
 + x^6)^(1/3))]/(2*Sqrt[3]) - Log[-x^2 + (-1 + x^6)^(1/3)]/6 + Log[x^4 + x^
2*(-1 + x^6)^(1/3) + (-1 + x^6)^(2/3)]/12
```

fricas [A] time = 0.70, size = 116, normalized size = 1.05

$$-\frac{4\sqrt{3}x^8\arctan\left(-\frac{25382\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}x^4-13720\sqrt{3}\left(x^6-1\right)^{\frac{2}{3}}x^2+\sqrt{3}\left(5831x^6-7200\right)}{58653x^6-8000}\right)+2x^8\log\left(-3\left(x^6-1\right)^{\frac{1}{3}}x^4+3\left(x^6-1\right)^{\frac{2}{3}}x^2+1\right)+3\left(3x^6+1\right)\left(x^6-1\right)^{\frac{1}{3}}}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(x^6+1)/x^9,x, algorithm="fricas")
```

```
[Out] -1/24*(4*sqrt(3)*x^8*arctan(-(25382*sqrt(3)*(x^6 - 1)^(1/3)*x^4 - 13720*sqr
t(3)*(x^6 - 1)^(2/3)*x^2 + sqrt(3)*(5831*x^6 - 7200))/(58653*x^6 - 8000)) +
 2*x^8*log(-3*(x^6 - 1)^(1/3)*x^4 + 3*(x^6 - 1)^(2/3)*x^2 + 1) + 3*(3*x^6 +
 1)*(x^6 - 1)^(1/3))/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+1\right)\left(x^6-1\right)^{\frac{1}{3}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(x^6+1)/x^9,x, algorithm="giac")
```

```
[Out] integrate((x^6 + 1)*(x^6 - 1)^(1/3)/x^9, x)
```

maple [C] time = 0.32, size = 58, normalized size = 0.52

$$-\frac{3x^{12}-2x^6-1}{8x^8\left(x^6-1\right)^{\frac{2}{3}}}+\frac{\left(-\mathrm{signum}\left(x^6-1\right)\right)^{\frac{2}{3}} x^4 \,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^6\right)}{4\mathrm{signum}\left(x^6-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)^(1/3)*(x^6+1)/x^9,x)
```

```
[Out] -1/8*(3*x^12-2*x^6-1)/x^8/(x^6-1)^(2/3)+1/4/signum(x^6-1)^(2/3)*(-signum(x^
6-1))^(2/3)*x^4*hypergeom([2/3,2/3],[5/3],x^6)
```

maxima [A] time = 0.45, size = 93, normalized size = 0.84

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)-\frac{\left(x^6-1\right)^{\frac{1}{3}}}{2x^2}+\frac{\left(x^6-1\right)^{\frac{4}{3}}}{8x^8}+\frac{1}{12}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6-1\right)^{\frac{2}{3}}}{x^4}+1\right)-\frac{1}{6}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)^(1/3)*(x^6+1)/x^9,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3)/x^2 + 1)) - 1/2*(x^6 - 1)
^(1/3)/x^2 + 1/8*(x^6 - 1)^(4/3)/x^8 + 1/12*log((x^6 - 1)^(1/3)/x^2 + (x^6
- 1)^(2/3)/x^4 + 1) - 1/6*log((x^6 - 1)^(1/3)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6-1\right)^{1/3}\,\left(x^6+1\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)^(1/3)*(x^6 + 1))/x^9,x)
```

```
[Out] int(((x^6 - 1)^(1/3)*(x^6 + 1))/x^9, x)
```

sympy [C] time = 3.37, size = 167, normalized size = 1.50

$$\begin{cases} \frac{\sqrt[3]{-1+\frac{1}{x^6}}\, e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{6\Gamma\left(-\frac{1}{3}\right)} - \frac{\sqrt[3]{-1+\frac{1}{x^6}}\, e^{-\frac{2i\pi}{3}}\Gamma\left(-\frac{4}{3}\right)}{6x^6\Gamma\left(-\frac{1}{3}\right)} & \text{for } \frac{1}{|x^6|} > 1 \\ -\frac{\sqrt[3]{1-\frac{1}{x^6}}\,\Gamma\left(-\frac{4}{3}\right)}{6\Gamma\left(-\frac{1}{3}\right)} + \frac{\sqrt[3]{1-\frac{1}{x^6}}\,\Gamma\left(-\frac{4}{3}\right)}{6x^6\Gamma\left(-\frac{1}{3}\right)} & \text{otherwise} \end{cases} + \frac{e^{\frac{i\pi}{3}}\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| x^6\right)}{6x^2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)**(1/3)*(x**6+1)/x**9,x)
```

```
[Out] Piecewise(((-1 + x**(-6))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(6*gamma(-1/3))
 - (-1 + x**(-6))**(1/3)*exp(-2*I*pi/3)*gamma(-4/3)/(6*x**6*gamma(-1/3)), 1
/Abs(x**6) > 1), (-(1 - 1/x**6)**(1/3)*gamma(-4/3)/(6*gamma(-1/3)) + (1 - 1
/x**6)**(1/3)*gamma(-4/3)/(6*x**6*gamma(-1/3)), True)) + exp(I*pi/3)*gamma(
-1/3)*hyper((-1/3, -1/3), (2/3,), x**6)/(6*x**2*gamma(2/3))
```

3.1408 $\int \frac{1}{\sqrt[4]{-1-3x^4-2x^8+2x^{12}+3x^{16}+x^{20}}}\,dx$

Optimal. Leaf size=111

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{x^{20}+3x^{16}+2x^{12}-2x^8-3x^4-1}}{\sqrt[4]{2}\,x\left(x^4+1\right)}\right)}{2\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{x^{20}+3x^{16}+2x^{12}-2x^8-3x^4-1}}{\sqrt[4]{2}\,x\left(x^4+1\right)}\right)}{2\sqrt[4]{2}}$$

Rubi [A] time = 0.06, antiderivative size = 121, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6688, 6719, 377, 212, 206, 203}

$$\frac{\sqrt[4]{x^4-1}\left(x^4+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt[4]{2}\sqrt[4]{-\left(\left(1-x^4\right)\left(x^4+1\right)^4\right)}}+\frac{\sqrt[4]{x^4-1}\left(x^4+1\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt[4]{2}\sqrt[4]{-\left(\left(1-x^4\right)\left(x^4+1\right)^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 3*x^4 - 2*x^8 + 2*x^12 + 3*x^16 + x^20)^(-1/4),x]
```

```
[Out] ((-1 + x^4)^(1/4)*(1 + x^4)*ArcTan[(2^(1/4)*x)/(-1 + x^4)^(1/4)])/(2*2^(1/4
)*(-((1 - x^4)*(1 + x^4)^4))^(1/4)) + ((-1 + x^4)^(1/4)*(1 + x^4)*ArcTanh[(
2^(1/4)*x)/(-1 + x^4)^(1/4)])/(2*2^(1/4)*(-((1 - x^4)*(1 + x^4)^4))^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
```

```
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt[4]{-1-3x^4-2x^8+2x^{12}+3x^{16}+x^{20}}}\,dx &= \int \frac{1}{\sqrt[4]{\left(-1+x^4\right)\left(1+x^4\right)^4}}\,dx\\
&= \frac{\left(\sqrt[4]{-1+x^4}\,\left(1+x^4\right)\right)\int \frac{1}{\sqrt[4]{-1+x^4}\,\left(1+x^4\right)}\,dx}{\sqrt[4]{\left(-1+x^4\right)\left(1+x^4\right)^4}}\\
&= \frac{\left(\sqrt[4]{-1+x^4}\,\left(1+x^4\right)\right)\mathrm{Subst}\left(\int \frac{1}{1-2x^4}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt[4]{\left(-1+x^4\right)\left(1+x^4\right)^4}}\\
&= \frac{\left(\sqrt[4]{-1+x^4}\,\left(1+x^4\right)\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)}{2\sqrt[4]{\left(-1+x^4\right)\left(1+x^4\right)^4}} + \frac{\left(\sqrt[4]{-1 \cdots\right.}{\cdots}\\
&= \frac{\sqrt[4]{-1+x^4}\,\left(1+x^4\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{-1+x^4}}\right)}{2\sqrt[4]{2}\,\sqrt[4]{-\left(\left(1-x^4\right)\left(1+x^4\right)^4\right)}} + \frac{\sqrt[4]{-1+x^4}\,\left(1+x^4\right)\tanh^{\cdots}}{2\sqrt[4]{2}\,\sqrt[4]{-\left(\left(1-x^4\right)\left(1 \cdots\right.\right.}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 75, normalized size = 0.68

$$
\frac{\sqrt[4]{x^4-1}\,\left(x^4+1\right)\left(\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-1}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-1}}\right)\right)}{2\sqrt[4]{2}\,\sqrt[4]{\left(x^4-1\right)\left(x^4+1\right)^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 3*x^4 - 2*x^8 + 2*x^12 + 3*x^16 + x^20)^(-1/4),x]
```

```
[Out] ((-1 + x^4)^(1/4)*(1 + x^4)*(ArcTan[(2^(1/4)*x)/(-1 + x^4)^(1/4)] + ArcTanh
[(2^(1/4)*x)/(-1 + x^4)^(1/4)]))/(2*2^(1/4)*((-1 + x^4)*(1 + x^4)^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.43, size = 111, normalized size = 1.00

$$
\frac{\tanh^{-1}\left(\frac{\sqrt[4]{x^{20}+3x^{16}+2x^{12}-2x^8-3x^4-1}}{\sqrt[4]{2}\,x\left(x^4+1\right)}\right)}{2\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{x^{20}+3x^{16}+2x^{12}-2x^8-3x^4-1}}{\sqrt[4]{2}\,x\left(x^4+1\right)}\right)}{2\sqrt[4]{2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 3*x^4 - 2*x^8 + 2*x^12 + 3*x^16 + x^20)^(-1/4),x]
```

```
[Out] -1/2*ArcTan[(-1 - 3*x^4 - 2*x^8 + 2*x^12 + 3*x^16 + x^20)^(1/4)/(2^(1/4)*x*
(1 + x^4))]/2^(1/4) + ArcTanh[(-1 - 3*x^4 - 2*x^8 + 2*x^12 + 3*x^16 + x^20)
^(1/4)/(2^(1/4)*x*(1 + x^4))]/(2*2^(1/4))
```

fricas [B] time = 4.88, size = 503, normalized size = 4.53

$$
-\frac{1}{4}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{4\cdot 2^{\frac{3}{4}}\left(x^{20}+3x^{16}+2x^{12}-2x^8-3x^4-1\right)^{\frac{1}{4}}\left(x^{11}+2x^7+x^3\right)+4\cdot 2^{\frac{1}{4}}\left(x^{20}+3x^{16}+2x^{12}-2x^8-3x^4-1\right)^{\frac{3}{4}}x+2^{\frac{3}{4}}\left(2\cdot 2^{\frac{3}{4}}\sqrt{x^{20}+3x^{16}+2x^{12}-2x^8-3x^4-1}\left(x^6+x^2\right)+2^{\frac{1}{4}}\left(3x^{16}+8x^{12}+6x^8-1\right)\right)}{2\left(x^{16}+4x^{12}+6x^8+4x^4+1\right)}\right)+\frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(\frac{2^{\frac{3}{4}}\left(3x^{16}+8x^{12}+6x^8-1\right)+4\sqrt{2}\left(x^{20}+3x^{16}+2x^{12}-2x^8-3x^4-1\right)^{\frac{1}{4}}\left(x^{11}+2x^7+x^3\right)+4\cdot 2^{\frac{1}{4}}\sqrt{x^{20}+3x^{16}+2x^{12}-2x^8-3x^4-1}\left(x^6+x^2\right)+4\left(x^{20}+3x^{16}+2x^{12}-2x^8-3x^4-1\right)^{\frac{3}{4}}x}{x^{16}+4x^{12}+6x^8+4x^4+1}\right)-\frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(-\frac{2^{\frac{3}{4}}\left(3x^{16}+8x^{12}+6x^8-1\right)-4\sqrt{2}\left(x^{20}+3x^{16}+2x^{12}-2x^8-3x^4-1\right)^{\frac{1}{4}}\left(x^{11}+2x^7+x^3\right)+4\cdot 2^{\frac{1}{4}}\sqrt{x^{20}+3x^{16}+2x^{12}-2x^8-3x^4-1}\left(x^6+x^2\right)-4\left(x^{20}+3x^{16}+2x^{12}-2x^8-3x^4-1\right)^{\frac{3}{4}}x}{x^{16}+4x^{12}+6x^8+4x^4+1}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^20+3*x^16+2*x^12-2*x^8-3*x^4-1)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/4*2^(3/4)*arctan(1/2*(4*2^(3/4)*(x^20 + 3*x^16 + 2*x^12 - 2*x^8 - 3*x^4
- 1)^(1/4)*(x^11 + 2*x^7 + x^3) + 4*2^(1/4)*(x^20 + 3*x^16 + 2*x^12 - 2*x^8
 - 3*x^4 - 1)^(3/4)*x + 2^(3/4)*(2*2^(3/4)*sqrt(x^20 + 3*x^16 + 2*x^12 - 2*
x^8 - 3*x^4 - 1)*(x^6 + x^2) + 2^(1/4)*(3*x^16 + 8*x^12 + 6*x^8 - 1)))/(x^1
6 + 4*x^12 + 6*x^8 + 4*x^4 + 1)) + 1/16*2^(3/4)*log((2^(3/4)*(3*x^16 + 8*x^
12 + 6*x^8 - 1) + 4*sqrt(2)*(x^20 + 3*x^16 + 2*x^12 - 2*x^8 - 3*x^4 - 1)^(1
/4)*(x^11 + 2*x^7 + x^3) + 4*2^(1/4)*sqrt(x^20 + 3*x^16 + 2*x^12 - 2*x^8 -
3*x^4 - 1)*(x^6 + x^2) + 4*(x^20 + 3*x^16 + 2*x^12 - 2*x^8 - 3*x^4 - 1)^(3/
4)*x)/(x^16 + 4*x^12 + 6*x^8 + 4*x^4 + 1)) - 1/16*2^(3/4)*log(-(2^(3/4)*(3*
x^16 + 8*x^12 + 6*x^8 - 1) - 4*sqrt(2)*(x^20 + 3*x^16 + 2*x^12 - 2*x^8 - 3*
x^4 - 1)^(1/4)*(x^11 + 2*x^7 + x^3) + 4*2^(1/4)*sqrt(x^20 + 3*x^16 + 2*x^12
 - 2*x^8 - 3*x^4 - 1)*(x^6 + x^2) - 4*(x^20 + 3*x^16 + 2*x^12 - 2*x^8 - 3*x
^4 - 1)^(3/4)*x)/(x^16 + 4*x^12 + 6*x^8 + 4*x^4 + 1))
```

giac [A] time = 0.35, size = 65, normalized size = 0.59

$$\frac{1}{4}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}\left(x^{4}-1\right)^{\frac{1}{4}}}{2\,x}\right)-\frac{1}{8}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}+\frac{\left(x^{4}-1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{8}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}-\frac{\left(x^{4}-1\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^20+3*x^16+2*x^12-2*x^8-3*x^4-1)^(1/4),x, algorithm="giac")
```

```
[Out] 1/4*2^(3/4)*arctan(1/2*2^(3/4)*(x^4 - 1)^(1/4)/x) - 1/8*2^(3/4)*log(2^(1/4)
 + (x^4 - 1)^(1/4)/x) + 1/8*2^(3/4)*log(2^(1/4) - (x^4 - 1)^(1/4)/x)
```

maple [C] time = 2.14, size = 635, normalized size = 5.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^20+3*x^16+2*x^12-2*x^8-3*x^4-1)^(1/4),x)
```

```
[Out] -1/8*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln((-3*x^16*RootOf(_Z^2+RootOf(_Z^4-8)^2
)-2*(x^20+3*x^16+2*x^12-2*x^8-3*x^4-1)^(1/4)*RootOf(_Z^4-8)^2*x^11-8*RootOf
(_Z^2+RootOf(_Z^4-8)^2)*x^12+(x^20+3*x^16+2*x^12-2*x^8-3*x^4-1)^(1/2)*RootO
f(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x^6-4*(x^20+3*x^16+2*x^12-2*x^8-3
*x^4-1)^(1/4)*RootOf(_Z^4-8)^2*x^7-6*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^8+(x^2
0+3*x^16+2*x^12-2*x^8-3*x^4-1)^(1/2)*RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_
Z^4-8)^2*x^2-2*(x^20+3*x^16+2*x^12-2*x^8-3*x^4-1)^(1/4)*RootOf(_Z^4-8)^2*x^
3+4*(x^20+3*x^16+2*x^12-2*x^8-3*x^4-1)^(3/4)*x+RootOf(_Z^2+RootOf(_Z^4-8)^2
))/(x^4+1)^4)+1/8*RootOf(_Z^4-8)*ln((3*x^16*RootOf(_Z^4-8)+2*(x^20+3*x^16+2
*x^12-2*x^8-3*x^4-1)^(1/4)*RootOf(_Z^4-8)^2*x^11+8*RootOf(_Z^4-8)*x^12+(x^2
0+3*x^16+2*x^12-2*x^8-3*x^4-1)^(1/2)*RootOf(_Z^4-8)^3*x^6+4*(x^20+3*x^16+2*
x^12-2*x^8-3*x^4-1)^(1/4)*RootOf(_Z^4-8)^2*x^7+6*RootOf(_Z^4-8)*x^8+(x^20+3
*x^16+2*x^12-2*x^8-3*x^4-1)^(1/2)*RootOf(_Z^4-8)^3*x^2+2*(x^20+3*x^16+2*x^1
2-2*x^8-3*x^4-1)^(1/4)*RootOf(_Z^4-8)^2*x^3+4*(x^20+3*x^16+2*x^12-2*x^8-3*x
^4-1)^(3/4)*x-RootOf(_Z^4-8))/(x^4+1)^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{20}+3\,x^{16}+2\,x^{12}-2\,x^{8}-3\,x^{4}-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^20+3*x^16+2*x^12-2*x^8-3*x^4-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^20 + 3*x^16 + 2*x^12 - 2*x^8 - 3*x^4 - 1)^(-1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^{20}+3\,x^{16}+2\,x^{12}-2\,x^8-3\,x^4-1\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^12 - 2*x^8 - 3*x^4 + 3*x^16 + x^20 - 1)^(1/4),x)
[Out] int(1/(2*x^12 - 2*x^8 - 3*x^4 + 3*x^16 + x^20 - 1)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^{20}+3x^{16}+2x^{12}-2x^8-3x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**20+3*x**16+2*x**12-2*x**8-3*x**4-1)**(1/4),x)
[Out] Integral((x**20 + 3*x**16 + 2*x**12 - 2*x**8 - 3*x**4 - 1)**(-1/4), x)
```

3.1409 $\int \frac{1+x^2}{\sqrt{b+ax}\sqrt{c+\sqrt{b+ax}}}\,dx$

Optimal. Leaf size=111

$$\frac{4\left(35a^2x^2+315a^2-56abx+48ac^2x+224b^2-288bc^2+128c^4\right)\sqrt{\sqrt{ax+b}+c}}{315a^3}-\frac{32\sqrt{ax+b}\left(5acx-16bc+8c^3\right.}{315a^3}$$

Rubi [A] time = 0.47, antiderivative size = 138, normalized size of antiderivative = 1.24, number of steps used = 3, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1850}

$$-\frac{8\left(b-3c^2\right)\left(\sqrt{ax+b}+c\right)^{5/2}}{5a^3}+\frac{16c\left(b-c^2\right)\left(\sqrt{ax+b}+c\right)^{3/2}}{3a^3}+\frac{4\left(\sqrt{ax+b}+c\right)^{9/2}}{9a^3}-\frac{16c\left(\sqrt{ax+b}+c\right)^{7/2}}{7a^3}+\frac{4\left(a^2+\left(b-c^2\right)^2\right)\sqrt{\sqrt{ax+b}+c}}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]]),x]
```

```
[Out] (4*(a^2 + (b - c^2)^2)*Sqrt[c + Sqrt[b + a*x]])/a^3 + (16*c*(b - c^2)*(c +
Sqrt[b + a*x])^(3/2))/(3*a^3) - (8*(b - 3*c^2)*(c + Sqrt[b + a*x])^(5/2))/(
5*a^3) - (16*c*(c + Sqrt[b + a*x])^(7/2))/(7*a^3) + (4*(c + Sqrt[b + a*x])^
(9/2))/(9*a^3)
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{\sqrt{b+ax}\sqrt{c+\sqrt{b+ax}}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{a^2+\left(b-x^2\right)^2}{\sqrt{c+x}}\,dx,x,\sqrt{b+ax}\right)}{a^3}\\ &= \frac{2\,\text{Subst}\left(\int \left(\frac{a^2+b^2-2bc^2+c^4}{\sqrt{c+x}}-4c\left(-b+c^2\right)\sqrt{c+x}-2\left(b-3c^2\right)(c+x)^{3/2}-4c(c+\right.\right.}{a^3}\\ &= \frac{4\left(a^2+\left(b-c^2\right)^2\right)\sqrt{c+\sqrt{b+ax}}}{a^3}+\frac{16c\left(b-c^2\right)\left(c+\sqrt{b+ax}\right)^{3/2}}{3a^3}-\frac{8\left(b-3c^2\right)}{}\end{aligned}$$

Mathematica [A] time = 0.19, size = 102, normalized size = 0.92

$$\frac{4\sqrt{\sqrt{ax+b}+c}\left(35a^2\left(x^2+9\right)+32\left(-2c^3\left(\sqrt{ax+b}-2c\right)+bc\left(4\sqrt{ax+b}-9c\right)+7b^2\right)-8ax\left(c\left(5\sqrt{ax+b}-6c\right)+7b\right)\right)}{315a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]]),x]
```

```
[Out] (4*Sqrt[c + Sqrt[b + a*x]]*(35*a^2*(9 + x^2) + 32*(7*b^2 - 2*c^3*(-2*c + Sq
rt[b + a*x]) + b*c*(-9*c + 4*Sqrt[b + a*x])) - 8*a*x*(7*b + c*(-6*c + 5*Sqr
t[b + a*x]))))/(315*a^3)
```

IntegrateAlgebraic [A] time = 0.10, size = 110, normalized size = 0.99

$$\frac{4\sqrt{\sqrt{ax+b}+c}\left(315a^2-64c^3\sqrt{ax+b}+48c^2(ax+b)-40c(ax+b)^{3/2}+168bc\sqrt{ax+b}+35(ax+b)^2-126b(ax+b)+315b^2-336bc^2+128c^4\right)}{315a^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/(Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]]),x]
```

```
[Out] (4*Sqrt[c + Sqrt[b + a*x]]*(315*a^2 + 315*b^2 - 336*b*c^2 + 128*c^4 + 168*b
*c*Sqrt[b + a*x] - 64*c^3*Sqrt[b + a*x] - 126*b*(b + a*x) + 48*c^2*(b + a*x
) - 40*c*(b + a*x)^(3/2) + 35*(b + a*x)^2))/(315*a^3)
```

fricas [A] time = 0.45, size = 84, normalized size = 0.76

$$\frac{4\left(128\,c^4+35\,a^2x^2-288\,bc^2+315\,a^2+224\,b^2+8\left(6\,ac^2-7\,ab\right)x-8\left(8\,c^3+5\,acx-16\,bc\right)\sqrt{ax+b}\right)\sqrt{c+\sqrt{ax+b}}}{315\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(a*x+b)^(1/2)/(c+(a*x+b)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] 4/315*(128*c^4 + 35*a^2*x^2 - 288*b*c^2 + 315*a^2 + 224*b^2 + 8*(6*a*c^2 -
7*a*b)*x - 8*(8*c^3 + 5*a*c*x - 16*b*c)*sqrt(a*x + b))*sqrt(c + sqrt(a*x +
b))/a^3
```

giac [A] time = 0.18, size = 160, normalized size = 1.44

$$\frac{4\left(35\left(c+\sqrt{ax+b}\right)^{\frac{9}{2}}-180\left(c+\sqrt{ax+b}\right)^{\frac{7}{2}}c+378\left(c+\sqrt{ax+b}\right)^{\frac{5}{2}}c^2-420\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}c^3+315\sqrt{c+\sqrt{ax+b}}\,c^4+315\,a^2\sqrt{c+\sqrt{ax+b}}+315\,b^2\sqrt{c+\sqrt{ax+b}}-42\left(3\left(c+\sqrt{ax+b}\right)^{\frac{5}{2}}-10\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}c+15\sqrt{c+\sqrt{ax+b}}\,c^2\right)b\right)}{315\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(a*x+b)^(1/2)/(c+(a*x+b)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] 4/315*(35*(c + sqrt(a*x + b))^(9/2) - 180*(c + sqrt(a*x + b))^(7/2)*c + 378
*(c + sqrt(a*x + b))^(5/2)*c^2 - 420*(c + sqrt(a*x + b))^(3/2)*c^3 + 315*sq
rt(c + sqrt(a*x + b))*c^4 + 315*a^2*sqrt(c + sqrt(a*x + b)) + 315*b^2*sqrt(
c + sqrt(a*x + b)) - 42*(3*(c + sqrt(a*x + b))^(5/2) - 10*(c + sqrt(a*x + b
))^(3/2)*c + 15*sqrt(c + sqrt(a*x + b))*c^2)*b)/a^3
```

maple [A] time = 0.00, size = 160, normalized size = 1.44

$$\frac{\frac{4\left(c+\sqrt{ax+b}\right)^{\frac{9}{2}}}{9}-\frac{16\left(c+\sqrt{ax+b}\right)^{\frac{7}{2}}c}{7}+\frac{24\left(c+\sqrt{ax+b}\right)^{\frac{5}{2}}c^2}{5}-\frac{8\left(c+\sqrt{ax+b}\right)^{\frac{5}{2}}b}{5}-\frac{16\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}c^3}{3}+\frac{16\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}bc}{3}+4c^4\sqrt{c+\sqrt{ax+b}}-8b\,c^2\sqrt{c+\sqrt{ax+b}}+4\sqrt{c+\sqrt{ax+b}}\,a^2+4b^2\sqrt{c+\sqrt{ax+b}}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(a*x+b)^(1/2)/(c+(a*x+b)^(1/2))^(1/2),x)
```

```
[Out] 2/a^3*(2/9*(c+(a*x+b)^(1/2))^(9/2)-8/7*(c+(a*x+b)^(1/2))^(7/2)*c+12/5*(c+(a
*x+b)^(1/2))^(5/2)*c^2-4/5*(c+(a*x+b)^(1/2))^(5/2)*b-8/3*(c+(a*x+b)^(1/2))^
(3/2)*c^3+8/3*(c+(a*x+b)^(1/2))^(3/2)*b*c+2*c^4*(c+(a*x+b)^(1/2))^(1/2)-4*b
*c^2*(c+(a*x+b)^(1/2))^(1/2)+2*(c+(a*x+b)^(1/2))^(1/2)*a^2+2*b^2*(c+(a*x+b)
^(1/2))^(1/2))
```

maxima [A] time = 0.31, size = 162, normalized size = 1.46

$$\frac{4\left(315\sqrt{c+\sqrt{ax+b}}+\frac{35\left(c+\sqrt{ax+b}\right)^{\frac{9}{2}}-180\left(c+\sqrt{ax+b}\right)^{\frac{7}{2}}c+378\left(c+\sqrt{ax+b}\right)^{\frac{5}{2}}c^2-420\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}c^3+315\sqrt{c+\sqrt{ax+b}}\,c^4+315\,b^2\sqrt{c+\sqrt{ax+b}}-42\left(3\left(c+\sqrt{ax+b}\right)^{\frac{5}{2}}-10\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}c+15\sqrt{c+\sqrt{ax+b}}\,c^2\right)b}{a^2}\right)}{315\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(a*x+b)^(1/2)/(c+(a*x+b)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 4/315*(315*sqrt(c + sqrt(a*x + b)) + (35*(c + sqrt(a*x + b))^(9/2) - 180*(c
 + sqrt(a*x + b))^(7/2)*c + 378*(c + sqrt(a*x + b))^(5/2)*c^2 - 420*(c + sq
rt(a*x + b))^(3/2)*c^3 + 315*sqrt(c + sqrt(a*x + b))*c^4 + 315*b^2*sqrt(c +
 sqrt(a*x + b)) - 42*(3*(c + sqrt(a*x + b))^(5/2) - 10*(c + sqrt(a*x + b))^
(3/2)*c + 15*sqrt(c + sqrt(a*x + b))*c^2)*b)/a^2)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+1}{\sqrt{c+\sqrt{b+a\,x}}\,\sqrt{b+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((c + (b + a*x)^(1/2))^(1/2)*(b + a*x)^(1/2)),x)
```

```
[Out] int((x^2 + 1)/((c + (b + a*x)^(1/2))^(1/2)*(b + a*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{c+\sqrt{ax+b}}\,\sqrt{ax+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(a*x+b)**(1/2)/(c+(a*x+b)**(1/2))**(1/2),x)
```

```
[Out] Integral((x**2 + 1)/(sqrt(c + sqrt(a*x + b))*sqrt(a*x + b)), x)
```

3.1410 $\int \frac{x^2}{\sqrt{-b+a^2x^2}\,\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx$

Optimal. Leaf size=111

$$\frac{8\sqrt{a^2x^2-b}\,\left(a^2x^3-4bx\right)}{7a^2\left(\sqrt{a^2x^2-b}+ax\right)^{9/4}}+\frac{4\left(18a^4x^4-81a^2bx^2+32b^2\right)}{63a^3\left(\sqrt{a^2x^2-b}+ax\right)^{9/4}}$$

Rubi [A] time = 0.35, antiderivative size = 93, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2120, 270}

$$-\frac{b^2}{9a^3\left(\sqrt{a^2x^2-b}+ax\right)^{9/4}}-\frac{2b}{a^3\sqrt[4]{\sqrt{a^2x^2-b}+ax}}+\frac{\left(\sqrt{a^2x^2-b}+ax\right)^{7/4}}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4)),x]
```

```
[Out] -1/9*b^2/(a^3*(a*x + Sqrt[-b + a^2*x^2])^(9/4)) - (2*b)/(a^3*(a*x + Sqrt[-b
 + a^2*x^2])^(1/4)) + (a*x + Sqrt[-b + a^2*x^2])^(7/4)/(7*a^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{-b+a^2x^2}\,\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(b+x^2\right)^2}{x^{13/4}}\,dx,x,ax+\sqrt{-b+a^2x^2}\right)}{4a^3}\\
&= \frac{\text{Subst}\left(\int \left(\frac{b^2}{x^{13/4}}+\frac{2b}{x^{5/4}}+x^{3/4}\right)dx,x,ax+\sqrt{-b+a^2x^2}\right)}{4a^3}\\
&= -\frac{b^2}{9a^3\left(ax+\sqrt{-b+a^2x^2}\right)^{9/4}}-\frac{2b}{a^3\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}+\frac{\left(ax+\sqrt{-b+a^2x^2}\right)^{7/4}}{7a^3}
\end{aligned}$$

Mathematica [B] time = 2.52, size = 629, normalized size = 5.67

$$\frac{4\sqrt{a^2x^2-b}\left(294912a^{19}x^{19}-2506752a^{17}bx^{17}+7745536a^{15}b^2x^{15}-12199936a^{13}b^3x^{13}+10988800a^{11}b^4x^{11}-5854336a^9b^5x^9+1812000a^7b^6x^7-302768a^5b^7x^5+23064a^3b^8x^3-32b^9\sqrt{a^2x^2-b}+4225a^2b^8x^2\sqrt{a^2x^2-b}+294912a^{18}x^{18}\sqrt{a^2x^2-b}-2359296a^{16}bx^{16}\sqrt{a^2x^2-b}+6602752a^{14}b^2x^{14}\sqrt{a^2x^2-b}-9175040a^{12}b^3x^{12}\sqrt{a^2x^2-b}+7090688a^{10}b^4x^{10}\sqrt{a^2x^2-b}-3127296a^8b^5x^8\sqrt{a^2x^2-b}+760704a^6b^6x^6\sqrt{a^2x^2-b}-91648a^4b^7x^4\sqrt{a^2x^2-b}-520ab^9x\right)}{63a^3\left(\sqrt{a^2x^2-b}+ax\right)^{21/4}\left(512a^{11}x^{11}-1536a^9bx^9+1696a^7b^2x^7-832a^5b^3x^5+170a^3b^4x^3-b^5\sqrt{a^2x^2-b}+50a^2b^4x^2\sqrt{a^2x^2-b}+512a^{10}x^{10}\sqrt{a^2x^2-b}-1280a^8bx^8\sqrt{a^2x^2-b}+1120a^6b^2x^6\sqrt{a^2x^2-b}-400a^4b^3x^4\sqrt{a^2x^2-b}-10ab^5x\right)\left(ax\left(\sqrt{a^2x^2-b}+ax\right)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4)),x]
```

```
[Out] (4*Sqrt[-b + a^2*x^2]*(-520*a*b^9*x + 23064*a^3*b^8*x^3 - 302768*a^5*b^7*x^
5 + 1812000*a^7*b^6*x^7 - 5854336*a^9*b^5*x^9 + 10988800*a^11*b^4*x^11 - 12
199936*a^13*b^3*x^13 + 7745536*a^15*b^2*x^15 - 2506752*a^17*b*x^17 + 294912
*a^19*x^19 - 32*b^9*Sqrt[-b + a^2*x^2] + 4225*a^2*b^8*x^2*Sqrt[-b + a^2*x^2
] - 91648*a^4*b^7*x^4*Sqrt[-b + a^2*x^2] + 760704*a^6*b^6*x^6*Sqrt[-b + a^2
*x^2] - 3127296*a^8*b^5*x^8*Sqrt[-b + a^2*x^2] + 7090688*a^10*b^4*x^10*Sqrt
[-b + a^2*x^2] - 9175040*a^12*b^3*x^12*Sqrt[-b + a^2*x^2] + 6602752*a^14*b^
2*x^14*Sqrt[-b + a^2*x^2] - 2359296*a^16*b*x^16*Sqrt[-b + a^2*x^2] + 294912
*a^18*x^18*Sqrt[-b + a^2*x^2]))/(63*a^3*(a*x + Sqrt[-b + a^2*x^2])^(21/4)*(
-10*a*b^5*x + 170*a^3*b^4*x^3 - 832*a^5*b^3*x^5 + 1696*a^7*b^2*x^7 - 1536*a
^9*b*x^9 + 512*a^11*x^11 - b^5*Sqrt[-b + a^2*x^2] + 50*a^2*b^4*x^2*Sqrt[-b
+ a^2*x^2] - 400*a^4*b^3*x^4*Sqrt[-b + a^2*x^2] + 1120*a^6*b^2*x^6*Sqrt[-b
+ a^2*x^2] - 1280*a^8*b*x^8*Sqrt[-b + a^2*x^2] + 512*a^10*x^10*Sqrt[-b + a^
2*x^2])*(-b + a*x*(a*x + Sqrt[-b + a^2*x^2])))
```

IntegrateAlgebraic [A] time = 0.17, size = 111, normalized size = 1.00

$$\frac{8\sqrt{a^2x^2-b}\left(a^2x^3-4bx\right)}{7a^2\left(\sqrt{a^2x^2-b}+ax\right)^{9/4}}+\frac{4\left(18a^4x^4-81a^2bx^2+32b^2\right)}{63a^3\left(\sqrt{a^2x^2-b}+ax\right)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4)
),x]
```

```
[Out] (8*Sqrt[-b + a^2*x^2]*(-4*b*x + a^2*x^3))/(7*a^2*(a*x + Sqrt[-b + a^2*x^2])
^(9/4)) + (4*(32*b^2 - 81*a^2*b*x^2 + 18*a^4*x^4))/(63*a^3*(a*x + Sqrt[-b +
 a^2*x^2])^(9/4))
```

fricas [A] time = 0.42, size = 68, normalized size = 0.61

$$-\frac{4\left(7\,a^3x^3+24\,abx-\left(7\,a^2x^2+32\,b\right)\sqrt{a^2x^2-b}\right)\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{3}{4}}}{63\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4),x, algorithm=
"fricas")
```

```
[Out] -4/63*(7*a^3*x^3 + 24*a*b*x - (7*a^2*x^2 + 32*b)*sqrt(a^2*x^2 - b))*(a*x +
sqrt(a^2*x^2 - b))^(3/4)/(a^3*b)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a^2x^2-b}\,\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4),x)
```

```
[Out] int(x^2/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a^2x^2-b}\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4),x, algorithm=
"maxima")
```

```
[Out] integrate(x^2/(sqrt(a^2*x^2 - b)*(a*x + sqrt(a^2*x^2 - b))^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{1/4}\,\sqrt{a^2\,x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x + (a^2*x^2 - b)^(1/2))^(1/4)*(a^2*x^2 - b)^(1/2)),x)
```

```
[Out] int(x^2/((a*x + (a^2*x^2 - b)^(1/2))^(1/4)*(a^2*x^2 - b)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[4]{ax+\sqrt{a^2x^2-b}}\,\sqrt{a^2x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2*x**2-b)**(1/2)/(a*x+(a**2*x**2-b)**(1/2))**(1/4),x)
```

```
[Out] Integral(x**2/((a*x + sqrt(a**2*x**2 - b))**(1/4)*sqrt(a**2*x**2 - b)), x)
```

3.1411 $\int x^2 \sqrt[3]{-x+x^3}\,dx$

Optimal. Leaf size=112

$$\frac{1}{12}\sqrt[3]{x^3-x}\left(3x^3-x\right)+\frac{1}{18}\log\left(\sqrt[3]{x^3-x}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x}+x}\right)}{6\sqrt{3}}-\frac{1}{36}\log\left(\sqrt[3]{x^3-x}\,x+\left(x^3-x\right)^{2/3}+x^2\right)$$

Rubi [A] time = 0.19, antiderivative size = 204, normalized size of antiderivative = 1.82, number of steps used = 12, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {2021, 2024, 2032, 329, 275, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{4}\sqrt[3]{x^3-x}\,x^3-\frac{1}{12}\sqrt[3]{x^3-x}\,x+\frac{\left(x^2-1\right)^{2/3}x^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2-1}}\right)}{18\left(x^3-x\right)^{2/3}}-\frac{\left(x^2-1\right)^{2/3}x^{2/3}\log\left(\frac{x^{4/3}}{\left(x^2-1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2-1}}+1\right)}{36\left(x^3-x\right)^{2/3}}+\frac{\left(x^2-1\right)^{2/3}x^{2/3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1}{\sqrt{3}}\right)}{6\sqrt{3}\left(x^3-x\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(-x + x^3)^(1/3),x]
```

```
[Out] -1/12*(x*(-x + x^3)^(1/3)) + (x^3*(-x + x^3)^(1/3))/4 + (x^(2/3)*(-1 + x^2)
^(2/3)*ArcTan[(1 + (2*x^(2/3))/(-1 + x^2)^(1/3))/Sqrt[3]])/(6*Sqrt[3]*(-x +
 x^3)^(2/3)) + (x^(2/3)*(-1 + x^2)^(2/3)*Log[1 - x^(2/3)/(-1 + x^2)^(1/3)])
/(18*(-x + x^3)^(2/3)) - (x^(2/3)*(-1 + x^2)^(2/3)*Log[1 + x^(4/3)/(-1 + x^
2)^(2/3) + x^(2/3)/(-1 + x^2)^(1/3)])/(36*(-x + x^3)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^2\sqrt[3]{-x+x^3}\,dx = \frac{1}{4}x^3\sqrt[3]{-x+x^3} - \frac{1}{6}\int \frac{x^3}{\left(-x+x^3\right)^{2/3}}\,dx$$

$$= -\frac{1}{12}x\sqrt[3]{-x+x^3} + \frac{1}{4}x^3\sqrt[3]{-x+x^3} - \frac{1}{9}\int \frac{x}{\left(-x+x^3\right)^{2/3}}\,dx$$

$$= -\frac{1}{12}x\sqrt[3]{-x+x^3} + \frac{1}{4}x^3\sqrt[3]{-x+x^3} - \frac{\left(x^{2/3}\left(-1+x^2\right)^{2/3}\right)\int \frac{\sqrt[3]{x}}{\left(-1+x^2\right)^{2/3}}\,dx}{9\left(-x+x^3\right)^{2/3}}$$

$$= -\frac{1}{12}x\sqrt[3]{-x+x^3} + \frac{1}{4}x^3\sqrt[3]{-x+x^3} - \frac{\left(x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x^3}{\left(-1+x^6\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{3\left(-x+x^3\right)^{2/3}}$$

$$= -\frac{1}{12}x\sqrt[3]{-x+x^3} + \frac{1}{4}x^3\sqrt[3]{-x+x^3} - \frac{\left(x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{\left(-1+x^3\right)^{2/3}}\,dx, x, x^{2/3}\right)}{6\left(-x+x^3\right)^{2/3}}$$

$$= -\frac{1}{12}x\sqrt[3]{-x+x^3} + \frac{1}{4}x^3\sqrt[3]{-x+x^3} - \frac{\left(x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{6\left(-x+x^3\right)^{2/3}}$$

$$= -\frac{1}{12}x\sqrt[3]{-x+x^3} + \frac{1}{4}x^3\sqrt[3]{-x+x^3} - \frac{\left(x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{18\left(-x+x^3\right)^{2/3}} + \frac{\left(x^{2/3}\left(\cdots\right.\right.}{\cdots}$$

$$= -\frac{1}{12}x\sqrt[3]{-x+x^3} + \frac{1}{4}x^3\sqrt[3]{-x+x^3} + \frac{x^{2/3}\left(-1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{18\left(-x+x^3\right)^{2/3}} - \frac{\left(x^{2/3}\left(-1+x^2\right)^{2/3}\right)\cdots}{36\cdots}$$

$$= -\frac{1}{12}x\sqrt[3]{-x+x^3} + \frac{1}{4}x^3\sqrt[3]{-x+x^3} + \frac{x^{2/3}\left(-1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{18\left(-x+x^3\right)^{2/3}} - \frac{x^{2/3}\left(-1+x^2\right)^{2/3}\log\cdots}{36\left(\cdots\right.}$$

$$= -\frac{1}{12}x\sqrt[3]{-x+x^3} + \frac{1}{4}x^3\sqrt[3]{-x+x^3} + \frac{x^{2/3}\left(-1+x^2\right)^{2/3}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}}{\sqrt{3}}\right)}{6\sqrt{3}\left(-x+x^3\right)^{2/3}} + \frac{x^{2/3}\left(-1+x^2\right)^{2/3}\log\cdots}{18\left(-x+\cdots\right.}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.48

$$\frac{x\sqrt[3]{x\left(x^2-1\right)}\left({}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^2\right)-\left(1-x^2\right)^{4/3}\right)}{4\sqrt[3]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(-x + x^3)^(1/3),x]
```

```
[Out] (x*(x*(-1 + x^2))^(1/3)*(-(1 - x^2)^(4/3) + Hypergeometric2F1[-1/3, 2/3, 5/
3, x^2]))/(4*(1 - x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.23, size = 112, normalized size = 1.00

$$\frac{1}{12}\sqrt[3]{x^3-x}\left(3x^3-x\right)+\frac{1}{18}\log\left(\sqrt[3]{x^3-x}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x}+x}\right)}{6\sqrt{3}}-\frac{1}{36}\log\left(\sqrt[3]{x^3-x}\,x+\left(x^3-x\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(-x + x^3)^(1/3),x]
```

```
[Out] ((-x + x^3)^(1/3)*(-x + 3*x^3))/12 + ArcTan[(Sqrt[3]*x)/(x + 2*(-x + x^3)^(
1/3))]/(6*Sqrt[3]) + Log[-x + (-x + x^3)^(1/3)]/18 - Log[x^2 + x*(-x + x^3)
^(1/3) + (-x + x^3)^(2/3)]/36
```

fricas [A] time = 0.62, size = 107, normalized size = 0.96

$$\frac{1}{18}\sqrt{3}\arctan\left(-\frac{44032959556\sqrt{3}\left(x^3-x\right)^{\frac{1}{3}}x+\sqrt{3}\left(16754327161x^2-2707204793\right)-10524305234\sqrt{3}\left(x^3-x\right)^{\frac{2}{3}}}{81835897185x^2-1102302937}\right)+\frac{1}{12}\left(3x^3-x\right)\left(x^3-x\right)^{\frac{1}{3}}+\frac{1}{36}\log\left(-3\left(x^3-x\right)^{\frac{1}{3}}x+3\left(x^3-x\right)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-x)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*arctan(-(44032959556*sqrt(3)*(x^3 - x)^(1/3)*x + sqrt(3)*(1675
4327161*x^2 - 2707204793) - 10524305234*sqrt(3)*(x^3 - x)^(2/3))/(818358971
85*x^2 - 1102302937)) + 1/12*(3*x^3 - x)*(x^3 - x)^(1/3) + 1/36*log(-3*(x^3
 - x)^(1/3)*x + 3*(x^3 - x)^(2/3) + 1)
```

giac [A] time = 2.01, size = 89, normalized size = 0.79

$$\frac{1}{12}\left(\left(-\frac{1}{x^2}+1\right)^{\frac{4}{3}}+2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right)x^4-\frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{36}\log\left(\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{18}\log\left(\left|\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-x)^(1/3),x, algorithm="giac")
```

```
[Out] 1/12*((-1/x^2 + 1)^(4/3) + 2*(-1/x^2 + 1)^(1/3))*x^4 - 1/18*sqrt(3)*arctan(
1/3*sqrt(3)*(2*(-1/x^2 + 1)^(1/3) + 1)) - 1/36*log((-1/x^2 + 1)^(2/3) + (-1
/x^2 + 1)^(1/3) + 1) + 1/18*log(abs((-1/x^2 + 1)^(1/3) - 1))
```

maple [C] time = 2.18, size = 543, normalized size = 4.85

$$\frac{x\left(3x^2-1\right)\left(x\left(x^2-1\right)\right)^{\frac{1}{3}}}{12}+\frac{\left(\frac{\ln\left(-\frac{-35\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)^2x^4-1956\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)x^4-4104\left(x^6-2x^4+x^2\right)^{\frac{1}{3}}\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)x^2+175\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)^2x^2+23364x^4+5850\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)\left(x^6-2x^4+x^2\right)^{\frac{2}{3}}-35100\left(x^6-2x^4+x^2\right)^{\frac{1}{3}}x^2+2010\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)x^2+10476\left(x^6-2x^4+x^2\right)^{\frac{2}{3}}+4104\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)\left(x^6-2x^4+x^2\right)^{\frac{1}{3}}-140\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)^2-38232x^2+35100\left(x^6-2x^4+x^2\right)^{\frac{1}{3}}-54\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)+14868}{\left(-1+x\right)\left(1+x\right)}\right)}{18}+\frac{\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)\ln\left(\frac{59\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)^2x^4-3750\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)x^4-1746\left(x^6-2x^4+x^2\right)^{\frac{1}{3}}\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)x^2-295\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)^2x^2+12600x^4+5850\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)\left(x^6-2x^4+x^2\right)^{\frac{2}{3}}-35100\left(x^6-2x^4+x^2\right)^{\frac{1}{3}}x^2+5652\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)x^2+24624\left(x^6-2x^4+x^2\right)^{\frac{2}{3}}+1746\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)\left(x^6-2x^4+x^2\right)^{\frac{1}{3}}+236\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)^2-16380x^2+35100\left(x^6-2x^4+x^2\right)^{\frac{1}{3}}-1902\operatorname{RootOf}\left(\_Z^2+6\_Z+36\right)+3780}{\left(-1+x\right)\left(1+x\right)}\right)}{108}\right)\left(x\left(x^2-1\right)\right)^{\frac{1}{3}}\left(x^2\left(x^2-1\right)^2\right)^{\frac{1}{3}}}{x\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3-x)^(1/3),x)
```

```
[Out] 1/12*x*(3*x^2-1)*(x*(x^2-1))^(1/3)+(1/18*ln(-(-35*RootOf(_Z^2+6*_Z+36)^2*x^
4-1956*RootOf(_Z^2+6*_Z+36)*x^4-4104*(x^6-2*x^4+x^2)^(1/3)*RootOf(_Z^2+6*_Z
+36)*x^2+175*RootOf(_Z^2+6*_Z+36)^2*x^2+23364*x^4+5850*RootOf(_Z^2+6*_Z+36)
*(x^6-2*x^4+x^2)^(2/3)-35100*(x^6-2*x^4+x^2)^(1/3)*x^2+2010*RootOf(_Z^2+6*_
Z+36)*x^2+10476*(x^6-2*x^4+x^2)^(2/3)+4104*RootOf(_Z^2+6*_Z+36)*(x^6-2*x^4+
x^2)^(1/3)-140*RootOf(_Z^2+6*_Z+36)^2-38232*x^2+35100*(x^6-2*x^4+x^2)^(1/3)
-54*RootOf(_Z^2+6*_Z+36)+14868)/(-1+x)/(1+x))+1/108*RootOf(_Z^2+6*_Z+36)*ln
((59*RootOf(_Z^2+6*_Z+36)^2*x^4-3750*RootOf(_Z^2+6*_Z+36)*x^4-1746*(x^6-2*x
^4+x^2)^(1/3)*RootOf(_Z^2+6*_Z+36)*x^2-295*RootOf(_Z^2+6*_Z+36)^2*x^2+12600
*x^4+5850*RootOf(_Z^2+6*_Z+36)*(x^6-2*x^4+x^2)^(2/3)-35100*(x^6-2*x^4+x^2)^
(1/3)*x^2+5652*RootOf(_Z^2+6*_Z+36)*x^2+24624*(x^6-2*x^4+x^2)^(2/3)+1746*Ro
otOf(_Z^2+6*_Z+36)*(x^6-2*x^4+x^2)^(1/3)+236*RootOf(_Z^2+6*_Z+36)^2-16380*x
^2+35100*(x^6-2*x^4+x^2)^(1/3)-1902*RootOf(_Z^2+6*_Z+36)+3780)/(-1+x)/(1+x)
))/x*(x*(x^2-1))^(1/3)*(x^2*(x^2-1)^2)^(1/3)/(x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^3-x\right)^{\frac{1}{3}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - x)^(1/3)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(x^3 - x\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3 - x)^(1/3),x)
```

```
[Out] int(x^2*(x^3 - x)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt[3]{x (x-1) (x+1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3-x)**(1/3),x)
```

```
[Out] Integral(x**2*(x*(x - 1)*(x + 1))**(1/3), x)
```

3.1412 $\int x^6 \sqrt[3]{x+x^3}\, dx$

Optimal. Leaf size=112

$$\frac{5}{243}\log\left(\sqrt[3]{x^3+x}-x\right)+\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x}+x}\right)}{81\sqrt{3}}-\frac{5}{486}\log\left(\sqrt[3]{x^3+x}\,x+\left(x^3+x\right)^{2/3}+x^2\right)+\frac{1}{648}\sqrt[3]{x^3+x}\,\left(81x^7+9\right.$$

Rubi [B] time = 0.24, antiderivative size = 226, normalized size of antiderivative = 2.02, number of steps used = 14, number of rules used = 12, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {2021, 2024, 2032, 329, 275, 331, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{54}\sqrt[3]{x^3+x}\,x^3+\frac{5}{162}\sqrt[3]{x^3+x}\,x+\frac{1}{8}\sqrt[3]{x^3+x}\,x^7+\frac{1}{72}\sqrt[3]{x^3+x}\,x^5+\frac{5\left(x^2+1\right)^{2/3}x^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2+1}}\right)}{243\left(x^3+x\right)^{2/3}}-\frac{5\left(x^2+1\right)^{2/3}x^{2/3}\log\left(\frac{x^{4/3}}{\left(x^2+1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2+1}}+1\right)}{486\left(x^3+x\right)^{2/3}}+\frac{5\left(x^2+1\right)^{2/3}x^{2/3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)}{81\sqrt{3}\left(x^3+x\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(x + x^3)^(1/3),x]
```

```
[Out] (5*x*(x + x^3)^(1/3))/162 - (x^3*(x + x^3)^(1/3))/54 + (x^5*(x + x^3)^(1/3)
)/72 + (x^7*(x + x^3)^(1/3))/8 + (5*x^(2/3)*(1 + x^2)^(2/3)*ArcTan[(1 + (2*
x^(2/3))/(1 + x^2)^(1/3))/Sqrt[3]])/(81*Sqrt[3]*(x + x^3)^(2/3)) + (5*x^(2/
3)*(1 + x^2)^(2/3)*Log[1 - x^(2/3)/(1 + x^2)^(1/3)])/(243*(x + x^3)^(2/3))
- (5*x^(2/3)*(1 + x^2)^(2/3)*Log[1 + x^(4/3)/(1 + x^2)^(2/3) + x^(2/3)/(1 +
 x^2)^(1/3)])/(486*(x + x^3)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^6\sqrt[3]{x+x^3}\,dx = \frac{1}{8}x^7\sqrt[3]{x+x^3} + \frac{1}{12}\int \frac{x^7}{\left(x+x^3\right)^{2/3}}\,dx$$

$$= \frac{1}{72}x^5\sqrt[3]{x+x^3} + \frac{1}{8}x^7\sqrt[3]{x+x^3} - \frac{2}{27}\int \frac{x^5}{\left(x+x^3\right)^{2/3}}\,dx$$

$$= -\frac{1}{54}x^3\sqrt[3]{x+x^3} + \frac{1}{72}x^5\sqrt[3]{x+x^3} + \frac{1}{8}x^7\sqrt[3]{x+x^3} + \frac{5}{81}\int \frac{x^3}{\left(x+x^3\right)^{2/3}}\,dx$$

$$= \frac{5}{162}x\sqrt[3]{x+x^3} - \frac{1}{54}x^3\sqrt[3]{x+x^3} + \frac{1}{72}x^5\sqrt[3]{x+x^3} + \frac{1}{8}x^7\sqrt[3]{x+x^3} - \frac{10}{243}\int \frac{x}{\left(x+x^3\right)^{2/3}}\,dx$$

$$= \frac{5}{162}x\sqrt[3]{x+x^3} - \frac{1}{54}x^3\sqrt[3]{x+x^3} + \frac{1}{72}x^5\sqrt[3]{x+x^3} + \frac{1}{8}x^7\sqrt[3]{x+x^3} - \frac{\left(10x^{2/3}\left(1+x^2\right)^{2/3}\right)\int \frac{}{(1+}}{243\left(x+x^3\right)^{2/3}}$$

$$= \frac{5}{162}x\sqrt[3]{x+x^3} - \frac{1}{54}x^3\sqrt[3]{x+x^3} + \frac{1}{72}x^5\sqrt[3]{x+x^3} + \frac{1}{8}x^7\sqrt[3]{x+x^3} - \frac{\left(10x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subs}}{81\left(x+\right.}$$

$$= \frac{5}{162}x\sqrt[3]{x+x^3} - \frac{1}{54}x^3\sqrt[3]{x+x^3} + \frac{1}{72}x^5\sqrt[3]{x+x^3} + \frac{1}{8}x^7\sqrt[3]{x+x^3} - \frac{\left(5x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}}{81\left(x+\right.}$$

$$= \frac{5}{162}x\sqrt[3]{x+x^3} - \frac{1}{54}x^3\sqrt[3]{x+x^3} + \frac{1}{72}x^5\sqrt[3]{x+x^3} + \frac{1}{8}x^7\sqrt[3]{x+x^3} - \frac{\left(5x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}}{81\left(x+\right.}$$

$$= \frac{5}{162}x\sqrt[3]{x+x^3} - \frac{1}{54}x^3\sqrt[3]{x+x^3} + \frac{1}{72}x^5\sqrt[3]{x+x^3} + \frac{1}{8}x^7\sqrt[3]{x+x^3} - \frac{\left(5x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}}{243\left(x+\right.}$$

$$= \frac{5}{162}x\sqrt[3]{x+x^3} - \frac{1}{54}x^3\sqrt[3]{x+x^3} + \frac{1}{72}x^5\sqrt[3]{x+x^3} + \frac{1}{8}x^7\sqrt[3]{x+x^3} + \frac{5x^{2/3}\left(1+x^2\right)^{2/3}\log\left(1-\right.}{243\left(x+x^3\right)^{2/3}}$$

$$= \frac{5}{162}x\sqrt[3]{x+x^3} - \frac{1}{54}x^3\sqrt[3]{x+x^3} + \frac{1}{72}x^5\sqrt[3]{x+x^3} + \frac{1}{8}x^7\sqrt[3]{x+x^3} + \frac{5x^{2/3}\left(1+x^2\right)^{2/3}\log\left(1-\right.}{243\left(x+x^3\right)^{2/3}}$$

$$= \frac{5}{162}x\sqrt[3]{x+x^3} - \frac{1}{54}x^3\sqrt[3]{x+x^3} + \frac{1}{72}x^5\sqrt[3]{x+x^3} + \frac{1}{8}x^7\sqrt[3]{x+x^3} + \frac{5x^{2/3}\left(1+x^2\right)^{2/3}\tan^{-1}\left(\right.}{81\sqrt{3}\left(x+x^3\right)^{2/3}}$$

Mathematica [C] time = 0.03, size = 68, normalized size = 0.61

$$\frac{x\sqrt[3]{x^3+x}\left(\sqrt[3]{x^2+1}\left(27x^6+3x^4-4x^2+20\right)-20\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-x^2\right)\right)}{216\sqrt[3]{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(x + x^3)^(1/3),x]
```

```
[Out] (x*(x + x^3)^(1/3)*((1 + x^2)^(1/3)*(20 - 4*x^2 + 3*x^4 + 27*x^6) - 20*Hype
rgeometric2F1[-1/3, 2/3, 5/3, -x^2]))/(216*(1 + x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.41, size = 112, normalized size = 1.00

$$\frac{5}{243}\log\left(\sqrt[3]{x^3+x}-x\right)+\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x}+x}\right)}{81\sqrt{3}}-\frac{5}{486}\log\left(\sqrt[3]{x^3+x}\,x+\left(x^3+x\right)^{2/3}+x^2\right)+\frac{1}{648}\sqrt[3]{x^3+x}\left(81x^7+9x^5-12x^3+20x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6*(x + x^3)^(1/3),x]
```

```
[Out] ((x + x^3)^(1/3)*(20*x - 12*x^3 + 9*x^5 + 81*x^7))/648 + (5*ArcTan[(Sqrt[3]
*x)/(x + 2*(x + x^3)^(1/3))])/(81*Sqrt[3]) + (5*Log[-x + (x + x^3)^(1/3)])/
243 - (5*Log[x^2 + x*(x + x^3)^(1/3) + (x + x^3)^(2/3)])/486
```

fricas [A] time = 0.65, size = 108, normalized size = 0.96

$$\frac{5}{243}\sqrt{3}\arctan\left(-\frac{196\sqrt{3}\left(x^3+x\right)^{\frac{1}{3}}x-\sqrt{3}\left(539x^2+507\right)-1274\sqrt{3}\left(x^3+x\right)^{\frac{2}{3}}}{2205x^2+2197}\right)+\frac{1}{648}\left(81x^7+9x^5-12x^3+20x\right)\left(x^3+x\right)^{\frac{1}{3}}+\frac{5}{486}\log\left(3\left(x^3+x\right)^{\frac{1}{3}}x-3\left(x^3+x\right)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^3+x)^(1/3),x, algorithm="fricas")
```

```
[Out] 5/243*sqrt(3)*arctan(-(196*sqrt(3)*(x^3 + x)^(1/3)*x - sqrt(3)*(539*x^2 + 5
07) - 1274*sqrt(3)*(x^3 + x)^(2/3))/(2205*x^2 + 2197)) + 1/648*(81*x^7 + 9*
x^5 - 12*x^3 + 20*x)*(x^3 + x)^(1/3) + 5/486*log(3*(x^3 + x)^(1/3)*x - 3*(x
^3 + x)^(2/3) + 1)
```

giac [A] time = 1.00, size = 97, normalized size = 0.87

$$\frac{1}{648}\left(20\left(\frac{1}{x^2}+1\right)^{\frac{10}{3}}-72\left(\frac{1}{x^2}+1\right)^{\frac{7}{3}}+93\left(\frac{1}{x^2}+1\right)^{\frac{4}{3}}+40\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right)x^8-\frac{5}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{5}{486}\log\left(\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)+\frac{5}{243}\log\left(\left|\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^3+x)^(1/3),x, algorithm="giac")
```

```
[Out] 1/648*(20*(1/x^2 + 1)^(10/3) - 72*(1/x^2 + 1)^(7/3) + 93*(1/x^2 + 1)^(4/3)
+ 40*(1/x^2 + 1)^(1/3))*x^8 - 5/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(1/x^2 +
1)^(1/3) + 1)) - 5/486*log((1/x^2 + 1)^(2/3) + (1/x^2 + 1)^(1/3) + 1) + 5/2
43*log(abs((1/x^2 + 1)^(1/3) - 1))
```

maple [C] time = 2.05, size = 745, normalized size = 6.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^3+x)^(1/3),x)
```

```
[Out] 1/648*x*(81*x^6+9*x^4-12*x^2+20)*(x*(x^2+1))^(1/3)+(5/486*RootOf(_Z^2+2*_Z+
4)*ln((-2*RootOf(_Z^2+2*_Z+4)^2*x^4+11*RootOf(_Z^2+2*_Z+4)*x^4+15*(x^6+2*x^
4+x^2)^(1/3)*RootOf(_Z^2+2*_Z+4)*x^2+40*x^4+15*RootOf(_Z^2+2*_Z+4)*(x^6+2*x
^4+x^2)^(2/3)+48*(x^6+2*x^4+x^2)^(1/3)*x^2+28*RootOf(_Z^2+2*_Z+4)*x^2+48*(x
^6+2*x^4+x^2)^(2/3)+15*RootOf(_Z^2+2*_Z+4)*(x^6+2*x^4+x^2)^(1/3)+2*RootOf(_
Z^2+2*_Z+4)^2+70*x^2+48*(x^6+2*x^4+x^2)^(1/3)+17*RootOf(_Z^2+2*_Z+4)+30)/(x
^2+1))-5/486*ln(-(2*RootOf(_Z^2+2*_Z+4)^2*x^4+19*RootOf(_Z^2+2*_Z+4)*x^4+15
*(x^6+2*x^4+x^2)^(1/3)*RootOf(_Z^2+2*_Z+4)*x^2-10*x^4+15*RootOf(_Z^2+2*_Z+4
)*(x^6+2*x^4+x^2)^(2/3)-18*(x^6+2*x^4+x^2)^(1/3)*x^2+28*RootOf(_Z^2+2*_Z+4)
*x^2-18*(x^6+2*x^4+x^2)^(2/3)+15*RootOf(_Z^2+2*_Z+4)*(x^6+2*x^4+x^2)^(1/3)-
2*RootOf(_Z^2+2*_Z+4)^2-14*x^2-18*(x^6+2*x^4+x^2)^(1/3)+9*RootOf(_Z^2+2*_Z+
4)-4)/(x^2+1))*RootOf(_Z^2+2*_Z+4)-5/243*ln(-(2*RootOf(_Z^2+2*_Z+4)^2*x^4+1
9*RootOf(_Z^2+2*_Z+4)*x^4+15*(x^6+2*x^4+x^2)^(1/3)*RootOf(_Z^2+2*_Z+4)*x^2-
```

```
10*x^4+15*RootOf(_Z^2+2*_Z+4)*(x^6+2*x^4+x^2)^(2/3)-18*(x^6+2*x^4+x^2)^(1/3
)*x^2+28*RootOf(_Z^2+2*_Z+4)*x^2-18*(x^6+2*x^4+x^2)^(2/3)+15*RootOf(_Z^2+2*
_Z+4)*(x^6+2*x^4+x^2)^(1/3)-2*RootOf(_Z^2+2*_Z+4)^2-14*x^2-18*(x^6+2*x^4+x^
2)^(1/3)+9*RootOf(_Z^2+2*_Z+4)-4)/(x^2+1)))*(x*(x^2+1))^(1/3)/x*(x^2*(x^2+1
)^2)^(1/3)/(x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^3+x\right)^{\frac{1}{3}} x^6\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^3+x)^(1/3),x, algorithm="maxima")
[Out] integrate((x^3 + x)^(1/3)*x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\left(x^3+x\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x + x^3)^(1/3),x)
[Out] int(x^6*(x + x^3)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \sqrt[3]{x\left(x^2+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(x**3+x)**(1/3),x)
[Out] Integral(x**6*(x*(x**2 + 1))**(1/3), x)
```

3.1413 $\int \frac{-3+2x}{x\sqrt[4]{-1+x^4}}\,dx$

Optimal. Leaf size=112

$$-\tan^{-1}\left(\frac{\sqrt[4]{x^4-1}}{x}\right)-\frac{3\tan^{-1}\left(\frac{\frac{\sqrt{x^4-1}}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{2}}+\tanh^{-1}\left(\frac{\sqrt[4]{x^4-1}}{x}\right)+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+1}\right)}{2\sqrt{2}}$$

Rubi [A] time = 0.14, antiderivative size = 153, normalized size of antiderivative = 1.37, number of steps used = 17, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {1833, 240, 212, 206, 203, 266, 63, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3\log\left(\sqrt{x^4-1}-\sqrt{2}\sqrt[4]{x^4-1}+1\right)}{4\sqrt{2}}+\frac{3\log\left(\sqrt{x^4-1}+\sqrt{2}\sqrt[4]{x^4-1}+1\right)}{4\sqrt{2}}+\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)+\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{x^4-1}\right)}{2\sqrt{2}}-\frac{3\tan^{-1}\left(\sqrt{2}\sqrt[4]{x^4-1}+1\right)}{2\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 2*x)/(x*(-1 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[x/(-1 + x^4)^(1/4)] + (3*ArcTan[1 - Sqrt[2]*(-1 + x^4)^(1/4)])/(2*Sq
rt[2]) - (3*ArcTan[1 + Sqrt[2]*(-1 + x^4)^(1/4)])/(2*Sqrt[2]) + ArcTanh[x/(
-1 + x^4)^(1/4)] - (3*Log[1 - Sqrt[2]*(-1 + x^4)^(1/4) + Sqrt[-1 + x^4]])/(
4*Sqrt[2]) + (3*Log[1 + Sqrt[2]*(-1 + x^4)^(1/4) + Sqrt[-1 + x^4]])/(4*Sqrt
[2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{-3+2x}{x\sqrt[4]{-1+x^4}}\,dx = \int \left(\frac{2}{\sqrt[4]{-1+x^4}} - \frac{3}{x\sqrt[4]{-1+x^4}}\right)dx$$
$$= 2\int \frac{1}{\sqrt[4]{-1+x^4}}\,dx - 3\int \frac{1}{x\sqrt[4]{-1+x^4}}\,dx$$
$$= -\left(\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-1+x}\,x}\,dx, x, x^4\right)\right) + 2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)$$
$$= -\left(3\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt[4]{-1+x^4}\right)\right) + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right) + \text{Subst}\left(\int \frac{}{1}\right.$$
$$= \tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right) + \tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right) + \frac{3}{2}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt[4]{-1+x^4}\right) - \frac{3}{2}\,\text{Subst}$$
$$= \tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right) + \tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right) - \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt[4]{-1+x^4}\right) - \frac{3}{4}$$
$$= \tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right) + \tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right) - \frac{3\log\left(1-\sqrt{2}\sqrt[4]{-1+x^4}+\sqrt{-1+x^4}\right)}{4\sqrt{2}} + \frac{3\log\left(\right.}{}$$
$$= \tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right) + \frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+x^4}\right)}{2\sqrt{2}} - \frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+x^4}\right)}{2\sqrt{2}} + \tanh^{-1}\left(\right.$$

Mathematica [C] time = 0.03, size = 51, normalized size = 0.46

$$-\left(x^4-1\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};1-x^4\right) + \tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right) + \tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 2*x)/(x*(-1 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[x/(-1 + x^4)^(1/4)] + ArcTanh[x/(-1 + x^4)^(1/4)] - (-1 + x^4)^(3/4)
*Hypergeometric2F1[3/4, 1, 7/4, 1 - x^4]
```

IntegrateAlgebraic [A] time = 4.40, size = 107, normalized size = 0.96

$$-\tan^{-1}\left(\frac{\sqrt[4]{x^4-1}}{x}\right) + \frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^4-1}}{\sqrt{x^4-1}-1}\right)}{2\sqrt{2}} + \tanh^{-1}\left(\frac{\sqrt[4]{x^4-1}}{x}\right) + \frac{3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+1}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 + 2*x)/(x*(-1 + x^4)^(1/4)),x]
```

```
[Out] -ArcTan[(-1 + x^4)^(1/4)/x] + (3*ArcTan[(Sqrt[2]*(-1 + x^4)^(1/4))/(-1 + Sq
rt[-1 + x^4])])/(2*Sqrt[2]) + ArcTanh[(-1 + x^4)^(1/4)/x] + (3*ArcTanh[(Sqr
t[2]*(-1 + x^4)^(1/4))/(1 + Sqrt[-1 + x^4])])/(2*Sqrt[2])
```

fricas [B] time = 18.68, size = 508, normalized size = 4.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/x/(x^4-1)^(1/4),x, algorithm="fricas")
```

```
[Out] -3/4*sqrt(2)*arctan(-(x^8 + 4*sqrt(x^4 - 1)*x^4 + 2*sqrt(2)*(x^4 - 1)^(3/4)
*(x^4 - 4) + 2*sqrt(2)*(3*x^4 - 4)*(x^4 - 1)^(1/4) - (4*(x^4 - 1)^(1/4)*x^4
```

```
 + 2*sqrt(2)*sqrt(x^4 - 1)*(x^4 - 4) + sqrt(2)*(x^8 - 10*x^4 + 8) + 16*(x^4
 - 1)^(3/4))*sqrt((x^4 + 2*sqrt(2)*(x^4 - 1)^(3/4) + 2*sqrt(2)*(x^4 - 1)^(1
/4) + 4*sqrt(x^4 - 1))/x^4))/(x^8 - 16*x^4 + 16)) + 3/4*sqrt(2)*arctan(-(x^
8 + 4*sqrt(x^4 - 1)*x^4 - 2*sqrt(2)*(x^4 - 1)^(3/4)*(x^4 - 4) - 2*sqrt(2)*(
3*x^4 - 4)*(x^4 - 1)^(1/4) - (4*(x^4 - 1)^(1/4)*x^4 - 2*sqrt(2)*sqrt(x^4 -
1)*(x^4 - 4) - sqrt(2)*(x^8 - 10*x^4 + 8) + 16*(x^4 - 1)^(3/4))*sqrt((x^4 -
 2*sqrt(2)*(x^4 - 1)^(3/4) - 2*sqrt(2)*(x^4 - 1)^(1/4) + 4*sqrt(x^4 - 1))/x
^4))/(x^8 - 16*x^4 + 16)) + 3/16*sqrt(2)*log(4*(x^4 + 2*sqrt(2)*(x^4 - 1)^(
3/4) + 2*sqrt(2)*(x^4 - 1)^(1/4) + 4*sqrt(x^4 - 1))/x^4) - 3/16*sqrt(2)*log
(4*(x^4 - 2*sqrt(2)*(x^4 - 1)^(3/4) - 2*sqrt(2)*(x^4 - 1)^(1/4) + 4*sqrt(x^
4 - 1))/x^4) - 1/2*arctan(2*(x^4 - 1)^(1/4)*x^3 + 2*(x^4 - 1)^(3/4)*x) + 1/
2*log(2*x^4 + 2*(x^4 - 1)^(1/4)*x^3 + 2*sqrt(x^4 - 1)*x^2 + 2*(x^4 - 1)^(3/
4)*x - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x-3}{\left(x^4-1\right)^{\frac{1}{4}}x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/x/(x^4-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((2*x - 3)/((x^4 - 1)^(1/4)*x), x)
```

maple [C] time = 0.33, size = 110, normalized size = 0.98

$$-\frac{3\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)\left(-\mathrm{signum}\left(x^4-1\right)\right)^{\frac{1}{4}}\left(\frac{\pi\sqrt{2}\,x^4\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{4}\right],[2,2],x^4\right)}{4\Gamma\left(\frac{3}{4}\right)}+\frac{\left(-3\ln(2)-\frac{\pi}{2}+4\ln(x)+i\pi\right)\pi\sqrt{2}}{\Gamma\left(\frac{3}{4}\right)}\right)}{8\pi\,\mathrm{signum}\left(x^4-1\right)^{\frac{1}{4}}}+\frac{2\left(-\mathrm{signum}\left(x^4-1\right)\right)^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{4}\right],\left[\frac{5}{4}\right],x^4\right)}{\mathrm{signum}\left(x^4-1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*x)/x/(x^4-1)^(1/4),x)
```

```
[Out] -3/8/Pi*2^(1/2)*GAMMA(3/4)/signum(x^4-1)^(1/4)*(-signum(x^4-1))^(1/4)*(1/4*
Pi*2^(1/2)/GAMMA(3/4)*x^4*hypergeom([1,1,5/4],[2,2],x^4)+(-3*ln(2)-1/2*Pi+4
*ln(x)+I*Pi)*Pi*2^(1/2)/GAMMA(3/4))+2/signum(x^4-1)^(1/4)*(-signum(x^4-1))^
(1/4)*x*hypergeom([1/4,1/4],[5/4],x^4)
```

maxima [A] time = 0.42, size = 148, normalized size = 1.32

$$-\frac{3}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(x^4-1\right)^{\frac{1}{4}}\right)\right)-\frac{3}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(x^4-1\right)^{\frac{1}{4}}\right)\right)+\frac{3}{8}\sqrt{2}\log\left(\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+\sqrt{x^4-1}+1\right)-\frac{3}{8}\sqrt{2}\log\left(-\sqrt{2}\left(x^4-1\right)^{\frac{1}{4}}+\sqrt{x^4-1}+1\right)-\arctan\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\log\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}+1\right)-\frac{1}{2}\log\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/x/(x^4-1)^(1/4),x, algorithm="maxima")
```

```
[Out] -3/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^4 - 1)^(1/4))) - 3/4*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^4 - 1)^(1/4))) + 3/8*sqrt(2)*log(sqrt
(2)*(x^4 - 1)^(1/4) + sqrt(x^4 - 1) + 1) - 3/8*sqrt(2)*log(-sqrt(2)*(x^4 -
1)^(1/4) + sqrt(x^4 - 1) + 1) - arctan((x^4 - 1)^(1/4)/x) + 1/2*log((x^4 -
1)^(1/4)/x + 1) - 1/2*log((x^4 - 1)^(1/4)/x - 1)
```

mupad [B] time = 1.14, size = 72, normalized size = 0.64

$$\frac{2x\left(1-x^4\right)^{1/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ x^4\right)}{\left(x^4-1\right)^{1/4}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(x^4-1\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{3}{4}+\frac{3}{4}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(x^4-1\right)^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{3}{4}-\frac{3}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 3)/(x*(x^4 - 1)^(1/4)),x)
```

```
[Out] (2*x*(1 - x^4)^(1/4)*hypergeom([1/4, 1/4], 5/4, x^4))/(x^4 - 1)^(1/4) - 2^(
1/2)*atan(2^(1/2)*(x^4 - 1)^(1/4)*(1/2 + 1i/2))*(3/4 + 3i/4) - 2^(1/2)*atan
(2^(1/2)*(x^4 - 1)^(1/4)*(1/2 - 1i/2))*(3/4 - 3i/4)
```

sympy [C] time = 2.53, size = 61, normalized size = 0.54

$$\frac{xe^{-\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|x^4\right)}{2\Gamma\left(\frac{5}{4}\right)}+\frac{3\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{e^{2i\pi}}{x^4}\right)}{4x\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/x/(x**4-1)**(1/4),x)
```

```
[Out] x*exp(-I*pi/4)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), x**4)/(2*gamma(5/4)) +
3*gamma(1/4)*hyper((1/4, 1/4), (5/4,), exp_polar(2*I*pi)/x**4)/(4*x*gamma(5
/4))
```

3.1414 $\int \frac{\sqrt[3]{-1+x^3-x^4}\left(-3+x^4\right)}{x^2\left(1+x^4\right)}\,dx$

Optimal. Leaf size=112

$$\frac{3\sqrt[3]{-x^4+x^3-1}}{x}+\log\left(\sqrt[3]{-x^4+x^3-1}-x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{-x^4+x^3-1}+x}\right)-\frac{1}{2}\log\left(x^2+\sqrt[3]{-x^4+x^3-1}\,x+\right.$$

Rubi [F] time = 1.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{-1+x^3-x^4}\left(-3+x^4\right)}{x^2\left(1+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^3 - x^4)^(1/3)*(-3 + x^4))/(x^2*(1 + x^4)),x]
```

```
[Out] (-1)^(3/4)*Defer[Int][(-1 + x^3 - x^4)^(1/3)/((-1)^(1/4) - x), x] - (-1)^(1
/4)*Defer[Int][(-1 + x^3 - x^4)^(1/3)/(-(-1)^(3/4) - x), x] - 3*Defer[Int][
(-1 + x^3 - x^4)^(1/3)/x^2, x] + (-1)^(3/4)*Defer[Int][(-1 + x^3 - x^4)^(1/
3)/((-1)^(1/4) + x), x] - (-1)^(1/4)*Defer[Int][(-1 + x^3 - x^4)^(1/3)/(-(-
1)^(3/4) + x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{-1+x^3-x^4}\left(-3+x^4\right)}{x^2\left(1+x^4\right)}\,dx &= \int \left(-\frac{3\sqrt[3]{-1+x^3-x^4}}{x^2}+\frac{4x^2\sqrt[3]{-1+x^3-x^4}}{1+x^4}\right)dx\\
&= -\left(3\int \frac{\sqrt[3]{-1+x^3-x^4}}{x^2}\,dx\right)+4\int \frac{x^2\sqrt[3]{-1+x^3-x^4}}{1+x^4}\,dx\\
&= -\left(3\int \frac{\sqrt[3]{-1+x^3-x^4}}{x^2}\,dx\right)+4\int \left(-\frac{\sqrt[3]{-1+x^3-x^4}}{2\left(i-x^2\right)}+\frac{\sqrt[3]{-1+x^3-x^4}}{2\left(i+x^2\right)}\right)dx\\
&= -\left(2\int \frac{\sqrt[3]{-1+x^3-x^4}}{i-x^2}\,dx\right)+2\int \frac{\sqrt[3]{-1+x^3-x^4}}{i+x^2}\,dx-3\int \frac{\sqrt[3]{-1+x^3-x^4}}{x^2}\\
&= -\left(2\int \left(-\frac{(-1)^{3/4}\sqrt[3]{-1+x^3-x^4}}{2\left(\sqrt[4]{-1}-x\right)}-\frac{(-1)^{3/4}\sqrt[3]{-1+x^3-x^4}}{2\left(\sqrt[4]{-1}+x\right)}\right)dx\right)+2\int \left(-\right.\\
&= -\left(3\int \frac{\sqrt[3]{-1+x^3-x^4}}{x^2}\,dx\right)-\sqrt[4]{-1}\int \frac{\sqrt[3]{-1+x^3-x^4}}{-(-1)^{3/4}-x}\,dx-\sqrt[4]{-1}\int \frac{\sqrt[3]{-1+}}{-(-1)^{3}}
\end{aligned}$$

Mathematica [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-1+x^3-x^4}\left(-3+x^4\right)}{x^2\left(1+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^3 - x^4)^(1/3)*(-3 + x^4))/(x^2*(1 + x^4)),x]
```

```
[Out] Integrate[((-1 + x^3 - x^4)^(1/3)*(-3 + x^4))/(x^2*(1 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.60, size = 112, normalized size = 1.00

$$\frac{3\sqrt[3]{-x^4+x^3-1}}{x}+\log\left(\sqrt[3]{-x^4+x^3-1}-x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{-x^4+x^3-1}+x}\right)-\frac{1}{2}\log\left(x^2+\sqrt[3]{-x^4+x^3-1}\,x+\left(-x^4+x^3-1\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3 - x^4)^(1/3)*(-3 + x^4))/(x^2*(1 + x^4)),x]
```

```
[Out] (3*(-1 + x^3 - x^4)^(1/3))/x + Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x^3
- x^4)^(1/3))] + Log[-x + (-1 + x^3 - x^4)^(1/3)] - Log[x^2 + x*(-1 + x^3 -
 x^4)^(1/3) + (-1 + x^3 - x^4)^(2/3)]/2
```

fricas [A] time = 2.10, size = 137, normalized size = 1.22

$$-\frac{2\sqrt{3}\,x\arctan\left(\frac{\sqrt{3}x^3-2\sqrt{3}\left(-x^4+x^3-1\right)^{\frac{1}{3}}x^2+4\sqrt{3}\left(-x^4+x^3-1\right)^{\frac{2}{3}}x}{8x^4-9x^3+8}\right)-x\log\left(\frac{x^4-3\left(-x^4+x^3-1\right)^{\frac{1}{3}}x^2+3\left(-x^4+x^3-1\right)^{\frac{2}{3}}x+1}{x^4+1}\right)-6\left(-x^4+x^3-1\right)^{\frac{1}{3}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^3-1)^(1/3)*(x^4-3)/x^2/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*x*arctan((sqrt(3)*x^3 - 2*sqrt(3)*(-x^4 + x^3 - 1)^(1/3)*x^
2 + 4*sqrt(3)*(-x^4 + x^3 - 1)^(2/3)*x)/(8*x^4 - 9*x^3 + 8)) - x*log((x^4 -
 3*(-x^4 + x^3 - 1)^(1/3)*x^2 + 3*(-x^4 + x^3 - 1)^(2/3)*x + 1)/(x^4 + 1))
- 6*(-x^4 + x^3 - 1)^(1/3))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^4-3\right)\left(-x^4+x^3-1\right)^{\frac{1}{3}}}{\left(x^4+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^3-1)^(1/3)*(x^4-3)/x^2/(x^4+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 3)*(-x^4 + x^3 - 1)^(1/3)/((x^4 + 1)*x^2), x)
```

maple [C] time = 2.58, size = 689, normalized size = 6.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^3-1)^(1/3)*(x^4-3)/x^2/(x^4+1),x)
```

```
[Out] -3*(x^4-x^3+1)/x/(-x^4+x^3-1)^(2/3)+(ln(-(RootOf(_Z^2+_Z+1)^2*x^7+RootOf(_Z
^2+_Z+1)*x^8-RootOf(_Z^2+_Z+1)^2*x^6-3*x^7*RootOf(_Z^2+_Z+1)-x^8+2*RootOf(_
Z^2+_Z+1)*x^6+2*x^7-3*(x^8-2*x^7+x^6+2*x^4-2*x^3+1)^(1/3)*x^5-x^6+RootOf(_Z
^2+_Z+1)^2*x^3-3*RootOf(_Z^2+_Z+1)*(x^8-2*x^7+x^6+2*x^4-2*x^3+1)^(2/3)*x^2+
2*RootOf(_Z^2+_Z+1)*x^4+3*(x^8-2*x^7+x^6+2*x^4-2*x^3+1)^(1/3)*x^4-3*RootOf(
_Z^2+_Z+1)*x^3-3*(x^8-2*x^7+x^6+2*x^4-2*x^3+1)^(2/3)*x^2-2*x^4+2*x^3-3*(x^8
-2*x^7+x^6+2*x^4-2*x^3+1)^(1/3)*x+RootOf(_Z^2+_Z+1)-1)/(x^4-x^3+1)/(x^4+1))
+RootOf(_Z^2+_Z+1)*ln((2*RootOf(_Z^2+_Z+1)^2*x^7-2*RootOf(_Z^2+_Z+1)*x^8-2*
RootOf(_Z^2+_Z+1)^2*x^6+7*x^7*RootOf(_Z^2+_Z+1)-x^8-5*RootOf(_Z^2+_Z+1)*x^6
+3*x^7+3*(x^8-2*x^7+x^6+2*x^4-2*x^3+1)^(1/3)*x^5-2*x^6+2*RootOf(_Z^2+_Z+1)^
2*x^3+3*RootOf(_Z^2+_Z+1)*(x^8-2*x^7+x^6+2*x^4-2*x^3+1)^(2/3)*x^2-4*RootOf(
_Z^2+_Z+1)*x^4-3*(x^8-2*x^7+x^6+2*x^4-2*x^3+1)^(1/3)*x^4+7*RootOf(_Z^2+_Z+1
)*x^3+3*(x^8-2*x^7+x^6+2*x^4-2*x^3+1)^(2/3)*x^2-2*x^4+3*x^3+3*(x^8-2*x^7+x^
6+2*x^4-2*x^3+1)^(1/3)*x-2*RootOf(_Z^2+_Z+1)-1)/(x^4-x^3+1)/(x^4+1)))/(-x^4
+x^3-1)^(2/3)*((x^4-x^3+1)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-3\right)\left(-x^4+x^3-1\right)^{\frac{1}{3}}}{\left(x^4+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^3-1)^(1/3)*(x^4-3)/x^2/(x^4+1),x, algorithm="maxima")
[Out] integrate((x^4 - 3)*(-x^4 + x^3 - 1)^(1/3)/((x^4 + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-3\right)\,\left(-x^4+x^3-1\right)^{1/3}}{x^2\,\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 3)*(x^3 - x^4 - 1)^(1/3))/(x^2*(x^4 + 1)),x)
[Out] int(((x^4 - 3)*(x^3 - x^4 - 1)^(1/3))/(x^2*(x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-3\right)\sqrt[3]{-x^4+x^3-1}}{x^2\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**3-1)**(1/3)*(x**4-3)/x**2/(x**4+1),x)
[Out] Integral((x**4 - 3)*(-x**4 + x**3 - 1)**(1/3)/(x**2*(x**4 + 1)), x)
```

3.1415 $\int \frac{x^3\sqrt{-x+x^4}}{-b+ax^3}\,dx$

Optimal. Leaf size=112

$$\frac{2\sqrt{b}\,\sqrt{a-b}\,\tan^{-1}\left(\frac{x\sqrt{x^4-x}\,\sqrt{a-b}}{\sqrt{b}\,(x-1)(x^2+x+1)}\right)}{3a^2}+\frac{(2b-a)\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)}{3a^2}+\frac{\sqrt{x^4-x}\,x}{3a}$$

Rubi [A] time = 0.26, antiderivative size = 149, normalized size of antiderivative = 1.33, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {2042, 466, 465, 478, 523, 217, 206, 377, 205}

$$\frac{2\sqrt{b}\,\sqrt{x^4-x}\,\sqrt{a-b}\,\tan^{-1}\left(\frac{x^{3/2}\sqrt{a-b}}{\sqrt{b}\,\sqrt{x^3-1}}\right)}{3a^2\sqrt{x^3-1}\,\sqrt{x}}-\frac{\sqrt{x^4-x}\,(a-2b)\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{x^3-1}}\right)}{3a^2\sqrt{x^3-1}\,\sqrt{x}}+\frac{\sqrt{x^4-x}\,x}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[-x + x^4])/(-b + a*x^3),x]
```

```
[Out] (x*Sqrt[-x + x^4])/(3*a) + (2*Sqrt[a - b]*Sqrt[b]*Sqrt[-x + x^4]*ArcTan[(Sq
rt[a - b]*x^(3/2))/(Sqrt[b]*Sqrt[-1 + x^3])])/(3*a^2*Sqrt[x]*Sqrt[-1 + x^3]
) - ((a - 2*b)*Sqrt[-x + x^4]*ArcTanh[x^(3/2)/Sqrt[-1 + x^3]])/(3*a^2*Sqrt[
x]*Sqrt[-1 + x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] && !IntegerQ[p]
&& NeQ[b*c - a*d, 0] && !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\sqrt{-x+x^4}}{-b+ax^3}\,dx &= \frac{\sqrt{-x+x^4}\int \frac{x^{7/2}\sqrt{-1+x^3}}{-b+ax^3}\,dx}{\sqrt{x}\sqrt{-1+x^3}} \\
&= \frac{\left(2\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{x^8\sqrt{-1+x^6}}{-b+ax^6}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{-1+x^3}} \\
&= \frac{\left(2\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{x^2\sqrt{-1+x^2}}{-b+ax^2}\,dx, x, x^{3/2}\right)}{3\sqrt{x}\sqrt{-1+x^3}} \\
&= \frac{x\sqrt{-x+x^4}}{3a} - \frac{\sqrt{-x+x^4}\,\text{Subst}\left(\int \frac{b+(a-2b)x^2}{\sqrt{-1+x^2}\left(-b+ax^2\right)}\,dx, x, x^{3/2}\right)}{3a\sqrt{x}\sqrt{-1+x^3}} \\
&= \frac{x\sqrt{-x+x^4}}{3a} - \frac{\left((a-2b)\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, x^{3/2}\right)}{3a^2\sqrt{x}\sqrt{-1+x^3}} - \frac{\left(2(a-b)b\sqrt{-x+x^4}\right)}{3a \cdots} \\
&= \frac{x\sqrt{-x+x^4}}{3a} - \frac{\left((a-2b)\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^{3/2}}{\sqrt{-1+x^3}}\right)}{3a^2\sqrt{x}\sqrt{-1+x^3}} - \frac{\left(2(a-b)b\sqrt{-x+x^4}\right)}{3a \cdots} \\
&= \frac{x\sqrt{-x+x^4}}{3a} + \frac{2\sqrt{a-b}\sqrt{b}\sqrt{-x+x^4}\tan^{-1}\left(\frac{\sqrt{a-b}\,x^{3/2}}{\sqrt{b}\sqrt{-1+x^3}}\right)}{3a^2\sqrt{x}\sqrt{-1+x^3}} - \frac{(a-2b)\sqrt{-x+x^4}\tanh^{-1}\left(\cdots\right)}{3a^2\sqrt{x}\sqrt{-1+x^3}}
\end{aligned}
$$

Mathematica [C] time = 0.54, size = 136, normalized size = 1.21

$$\frac{x^2\left(\frac{\sqrt{1-x^3}\,x^3(a-2b)F_1\left(\frac{3}{2};\frac{1}{2},1;\frac{5}{2};x^3,\frac{ax^3}{b}\right)}{b}+\frac{3\sqrt{1-x^3}\,\sin^{-1}\left(\frac{\sqrt{\frac{x^3(b-a)}{b}}}{\sqrt{1-\frac{ax^3}{b}}}\right)}{\sqrt{\frac{x^3(b-a)}{b}}}+3\left(x^3-1\right)\right)}{9a\sqrt{x\left(x^3-1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sqrt[-x + x^4])/(-b + a*x^3),x]
```

```
[Out] (x^2*(3*(-1 + x^3) + ((a - 2*b)*x^3*Sqrt[1 - x^3]*AppellF1[3/2, 1/2, 1, 5/2
, x^3, (a*x^3)/b])/b + (3*Sqrt[1 - x^3]*ArcSin[Sqrt[((-a + b)*x^3)/b]/Sqrt[
1 - (a*x^3)/b]])/Sqrt[((-a + b)*x^3)/b]))/(9*a*Sqrt[x*(-1 + x^3)])
```

IntegrateAlgebraic [A] time = 0.58, size = 112, normalized size = 1.00

$$\frac{2\sqrt{b}\,\sqrt{a-b}\,\tan^{-1}\left(\frac{x\sqrt{x^4-x}\,\sqrt{a-b}}{\sqrt{b}\,(x-1)\left(x^2+x+1\right)}\right)}{3a^2}+\frac{(2b-a)\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)}{3a^2}+\frac{\sqrt{x^4-x}\,x}{3a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[-x + x^4])/(-b + a*x^3),x]
```

```
[Out] (x*Sqrt[-x + x^4])/(3*a) + (2*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b]*x*Sqr
t[-x + x^4])/(Sqrt[b]*(-1 + x)*(1 + x + x^2))])/(3*a^2) + ((-a + 2*b)*ArcTa
nh[x^2/Sqrt[-x + x^4]])/(3*a^2)
```

fricas [A] time = 1.72, size = 247, normalized size = 2.21

$$\left[\frac{2\sqrt{x^4-x}\,ax-(a-2b)\log\left(-2x^3-2\sqrt{x^4-x}\,x+1\right)+\sqrt{-ab+b^2}\,\log\left(-\frac{\left(a^2-8ab+8b^2\right)x^6+2\left(3ab-4b^2\right)x^3+4\left((a-2b)x^4+bx\right)\sqrt{x^4-x}\,\sqrt{-ab+b^2}+b^2}{a^2x^6-2abx^3+b^2}\right)}{6a^2},\ \frac{2\sqrt{x^4-x}\,ax-(a-2b)\log\left(-2x^3-2\sqrt{x^4-x}\,x+1\right)+2\sqrt{ab-b^2}\,\arctan\left(-\frac{2\sqrt{x^4-x}\,\sqrt{ab-b^2}\,x}{(a-2b)x^3+b}\right)}{6a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4-x)^(1/2)/(a*x^3-b),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(x^4 - x)*a*x - (a - 2*b)*log(-2*x^3 - 2*sqrt(x^4 - x)*x + 1) +
 sqrt(-a*b + b^2)*log(-((a^2 - 8*a*b + 8*b^2)*x^6 + 2*(3*a*b - 4*b^2)*x^3 +
 4*((a - 2*b)*x^4 + b*x)*sqrt(x^4 - x)*sqrt(-a*b + b^2) + b^2)/(a^2*x^6 - 2
*a*b*x^3 + b^2)))/a^2, 1/6*(2*sqrt(x^4 - x)*a*x - (a - 2*b)*log(-2*x^3 - 2*
sqrt(x^4 - x)*x + 1) + 2*sqrt(a*b - b^2)*arctan(-2*sqrt(x^4 - x)*sqrt(a*b -
 b^2)*x/((a - 2*b)*x^3 + b)))/a^2]
```

giac [A] time = 0.56, size = 100, normalized size = 0.89

$$\frac{\sqrt{x^4-x}\,x}{3a}-\frac{(a-2b)\log\left(\sqrt{-\frac{1}{x^3}+1}+1\right)}{6a^2}+\frac{(a-2b)\log\left(\left|\sqrt{-\frac{1}{x^3}+1}-1\right|\right)}{6a^2}-\frac{2\sqrt{ab-b^2}\,\arctan\left(\frac{b\sqrt{-\frac{1}{x^3}+1}}{\sqrt{ab-b^2}}\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4-x)^(1/2)/(a*x^3-b),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(x^4 - x)*x/a - 1/6*(a - 2*b)*log(sqrt(-1/x^3 + 1) + 1)/a^2 + 1/6*(
a - 2*b)*log(abs(sqrt(-1/x^3 + 1) - 1))/a^2 - 2/3*sqrt(a*b - b^2)*arctan(b*
sqrt(-1/x^3 + 1)/sqrt(a*b - b^2))/a^2
```

maple [C] time = 0.02, size = 946, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4-x)^(1/2)/(a*x^3-b),x)
```

```
[Out] 1/a*(1/3*x*(x^4-x)^(1/2)-(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+
1/2*I*3^(1/2))/(-1+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^
(1/2))/(-1+x))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1
/2)/(-3/2+1/2*I*3^(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/
2)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^
(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2
*I*3^(1/2)))^(1/2))-EllipticPi(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))
/(-1+x))^(1/2),(-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)),((3/2+1/2*I*3^(1/2
))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))))+b/
a*(2/a*(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1
+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2))/(-1+x))^(1/
2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2)/(-3/2+1/2*I*3^
(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/2)))^(1/2)*(Ellipt
icF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*
3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)
)-EllipticPi(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),(-1
/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1
/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)))-2/3/a*4^(1/2)*sum(1/_
alpha*(-1+x)^2*(_alpha^2+_alpha+1)*(1-I*3^(1/2))*(x/(-1+x)*(-3+I*3^(1/2))/(
I*3^(1/2)-1))^(1/2)*(1/(-1+x)*(I*3^(1/2)+2*x+1)/(-1-I*3^(1/2)))^(1/2)*(1/(-
1+x)*(1+2*x-I*3^(1/2))/(I*3^(1/2)-1))^(1/2)/(-3+I*3^(1/2))/(x*(-1+x)*(I*3^(
1/2)+2*x+1)*(1+2*x-I*3^(1/2)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1
/2+1/2*I*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1
/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-_alpha^2*a/b*EllipticPi(((-3/
2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),1/6*(I*_alpha^2*3^(1/
2)*a-I*3^(1/2)*b-3*_alpha^2*a+3*b)/b,((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2
))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))),_alpha=RootOf(_Z^3*a-b)
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4 - x}\,x^3}{ax^3 - b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4-x)^(1/2)/(a*x^3-b),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 - x)*x^3/(a*x^3 - b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^3\,\sqrt{x^4 - x}}{b - a\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(x^4 - x)^(1/2))/(b - a*x^3),x)
```

```
[Out] -int((x^3*(x^4 - x)^(1/2))/(b - a*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{x\,(x-1)\left(x^2+x+1\right)}}{ax^3 - b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**4-x)**(1/2)/(a*x**3-b),x)
```

```
[Out] Integral(x**3*sqrt(x*(x - 1)*(x**2 + x + 1))/(a*x**3 - b), x)
```

3.1416 $\int \frac{1+2x^4}{(-1+x^4)\sqrt[4]{x^2+x^4}}\,dx$

Optimal. Leaf size=112

$$-\frac{3\left(x^4+x^2\right)^{3/4}}{x\left(x^2+1\right)}+2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^2}}\right)-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+x^2}}\right)}{2\sqrt[4]{2}}+2\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^2}}\right)-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+x^2}}\right)}{2\sqrt[4]{2}}$$

Rubi [A] time = 0.63, antiderivative size = 211, normalized size of antiderivative = 1.88, number of steps used = 14, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2056, 6715, 6725, 240, 212, 206, 203, 1404, 382, 377}

$$-\frac{3x}{\sqrt[4]{x^4+x^2}}+\frac{2\sqrt[4]{x^2+1}\sqrt{x}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2+1}}\right)}{\sqrt[4]{x^4+x^2}}-\frac{3\sqrt[4]{x^2+1}\sqrt{x}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{x^2+1}}\right)}{2\sqrt[4]{2}\sqrt[4]{x^4+x^2}}+\frac{2\sqrt[4]{x^2+1}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2+1}}\right)}{\sqrt[4]{x^4+x^2}}-\frac{3\sqrt[4]{x^2+1}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{x^2+1}}\right)}{2\sqrt[4]{2}\sqrt[4]{x^4+x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^4)/((-1 + x^4)*(x^2 + x^4)^(1/4)),x]
```

```
[Out] (-3*x)/(x^2 + x^4)^(1/4) + (2*Sqrt[x]*(1 + x^2)^(1/4)*ArcTan[Sqrt[x]/(1 + x
^2)^(1/4)])/(x^2 + x^4)^(1/4) - (3*Sqrt[x]*(1 + x^2)^(1/4)*ArcTan[(2^(1/4)*
Sqrt[x])/(1 + x^2)^(1/4)])/(2*2^(1/4)*(x^2 + x^4)^(1/4)) + (2*Sqrt[x]*(1 +
x^2)^(1/4)*ArcTanh[Sqrt[x]/(1 + x^2)^(1/4)])/(x^2 + x^4)^(1/4) - (3*Sqrt[x]
*(1 + x^2)^(1/4)*ArcTanh[(2^(1/4)*Sqrt[x])/(1 + x^2)^(1/4)])/(2*2^(1/4)*(x^
2 + x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 1404

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+2x^4}{\left(-1+x^4\right)\sqrt[4]{x^2+x^4}}\,dx = \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^2}\right)\int \frac{1+2x^4}{\sqrt{x}\,\sqrt[4]{1+x^2}\left(-1+x^4\right)}\,dx}{\sqrt[4]{x^2+x^4}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{1+2x^8}{\sqrt[4]{1+x^4}\left(-1+x^8\right)}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^4}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \left(\frac{2}{\sqrt[4]{1+x^4}}+\frac{3}{\sqrt[4]{1+x^4}\left(-1+x^8\right)}\right)dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^4}}$$

$$= \frac{\left(4\sqrt{x}\,\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^4}} + \frac{\left(6\sqrt{x}\,\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^4}\left(-1 \cdots\right.}\right.}{\sqrt[4]{x^2+x^4}}$$

$$= \frac{\left(4\sqrt{x}\,\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{\sqrt[4]{x^2+x^4}} + \frac{\left(6\sqrt{x}\,\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\left(-1+x^4\right)(1 \cdots}\right.}{\sqrt[4]{x^2+x^4}}$$

$$= -\frac{3x}{\sqrt[4]{x^2+x^4}} + \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{\sqrt[4]{x^2+x^4}} + \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^2}\right)\text{Sub}\cdots}{\cdots}$$

$$= -\frac{3x}{\sqrt[4]{x^2+x^4}} + \frac{2\sqrt{x}\,\sqrt[4]{1+x^2}\,\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{\sqrt[4]{x^2+x^4}} + \frac{2\sqrt{x}\,\sqrt[4]{1+x^2}\,\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{\sqrt[4]{x^2+x^4}} + \cdots$$

$$= -\frac{3x}{\sqrt[4]{x^2+x^4}} + \frac{2\sqrt{x}\,\sqrt[4]{1+x^2}\,\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{\sqrt[4]{x^2+x^4}} + \frac{2\sqrt{x}\,\sqrt[4]{1+x^2}\,\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{\sqrt[4]{x^2+x^4}} - \cdots$$

$$= -\frac{3x}{\sqrt[4]{x^2+x^4}} + \frac{2\sqrt{x}\,\sqrt[4]{1+x^2}\,\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{\sqrt[4]{x^2+x^4}} - \frac{3\sqrt{x}\,\sqrt[4]{1+x^2}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{2\sqrt[4]{2}\,\sqrt[4]{x^2+x^4}} + \frac{2\sqrt{\cdots}}{\cdots}$$

Mathematica [C] time = 0.44, size = 85, normalized size = 0.76

$$\frac{5\sqrt[4]{\frac{1}{x^2}+1}\,F_1\left(1;\frac{1}{4},1;2;-\frac{1}{x^2},\frac{1}{x^2}\right)-4x^2\left(4\sqrt[4]{x^2+1}\,x^2F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-x^2,x^2\right)+15\right)}{20x\sqrt[4]{x^4+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + 2*x^4)/((-1 + x^4)*(x^2 + x^4)^(1/4)),x]
```

```
[Out] (5*(1 + x^(-2))^(1/4)*AppellF1[1, 1/4, 1, 2, -x^(-2), x^(-2)] - 4*x^2*(15 +
 4*x^2*(1 + x^2)^(1/4)*AppellF1[5/4, 1/4, 1, 9/4, -x^2, x^2]))/(20*x*(x^2 +
 x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.33, size = 112, normalized size = 1.00

$$-\frac{3\left(x^4+x^2\right)^{3/4}}{x\left(x^2+1\right)}+2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^2}}\right)-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^2}}\right)}{2\sqrt[4]{2}}+2\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^2}}\right)-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^2}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x^4)/((-1 + x^4)*(x^2 + x^4)^(1/4)),x]
```

```
[Out] (-3*(x^2 + x^4)^(3/4))/(x*(1 + x^2)) + 2*ArcTan[x/(x^2 + x^4)^(1/4)] - (3*A
rcTan[(2^(1/4)*x)/(x^2 + x^4)^(1/4)])/(2*2^(1/4)) + 2*ArcTanh[x/(x^2 + x^4)
^(1/4)] - (3*ArcTanh[(2^(1/4)*x)/(x^2 + x^4)^(1/4)])/(2*2^(1/4))
```

fricas [B] time = 17.15, size = 351, normalized size = 3.13

$$\frac{12 \cdot 2^{\frac{3}{4}} \left(x^3 + x\right) \arctan\left(\frac{4 \cdot 2^{\frac{3}{4}} \left(x^4+x^2\right)^{\frac{1}{4}} x^2 + 2^{\frac{3}{4}} \left(2 \cdot 2^{\frac{3}{4}} \sqrt{x^4+x^2}\, x + 2^{\frac{1}{4}} \left(3 x^3+x\right)\right) + 4 \cdot 2^{\frac{1}{4}} \left(x^4+x^2\right)^{\frac{3}{4}}}{2 \left(x^3-x\right)}\right) - 3 \cdot 2^{\frac{3}{4}} \left(x^3 + x\right) \log\left(\frac{4 \sqrt{2} \left(x^4+x^2\right)^{\frac{1}{4}} x^2 + 2^{\frac{3}{4}} \left(3 x^3+x\right) + 4 \cdot 2^{\frac{1}{4}} \sqrt{x^4+x^2}\, x + 4 \left(x^4+x^2\right)^{\frac{3}{4}}}{x^3-x}\right) + 3 \cdot 2^{\frac{3}{4}} \left(x^3 + x\right) \log\left(\frac{4 \sqrt{2} \left(x^4+x^2\right)^{\frac{1}{4}} x^2 - 2^{\frac{3}{4}} \left(3 x^3+x\right) - 4 \cdot 2^{\frac{1}{4}} \sqrt{x^4+x^2}\, x + 4 \left(x^4+x^2\right)^{\frac{3}{4}}}{x^3-x}\right) + 16 \left(x^3 + x\right) \arctan\left(\frac{2 \left(\left(x^4+x^2\right)^{\frac{1}{4}} x^2 + \left(x^4+x^2\right)^{\frac{3}{4}}\right)}{x}\right) + 16 \left(x^3 + x\right) \log\left(\frac{2 x^3 + 2 \left(x^4+x^2\right)^{\frac{1}{4}} x^2 + 2 \sqrt{x^4+x^2}\, x + x + 2 \left(x^4+x^2\right)^{\frac{3}{4}}}{x}\right) - 48 \left(x^4 + x^2\right)^{\frac{3}{4}}}{16 \left(x^3 + x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+1)/(x^4-1)/(x^4+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/16*(12*2^(3/4)*(x^3 + x)*arctan(1/2*(4*2^(3/4)*(x^4 + x^2)^(1/4)*x^2 + 2^
(3/4)*(2*2^(3/4)*sqrt(x^4 + x^2)*x + 2^(1/4)*(3*x^3 + x)) + 4*2^(1/4)*(x^4
+ x^2)^(3/4))/(x^3 - x)) - 3*2^(3/4)*(x^3 + x)*log((4*sqrt(2)*(x^4 + x^2)^(
1/4)*x^2 + 2^(3/4)*(3*x^3 + x) + 4*2^(1/4)*sqrt(x^4 + x^2)*x + 4*(x^4 + x^2
)^(3/4))/(x^3 - x)) + 3*2^(3/4)*(x^3 + x)*log((4*sqrt(2)*(x^4 + x^2)^(1/4)*
x^2 - 2^(3/4)*(3*x^3 + x) - 4*2^(1/4)*sqrt(x^4 + x^2)*x + 4*(x^4 + x^2)^(3/
4))/(x^3 - x)) + 16*(x^3 + x)*arctan(2*((x^4 + x^2)^(1/4)*x^2 + (x^4 + x^2)
^(3/4))/x) + 16*(x^3 + x)*log((2*x^3 + 2*(x^4 + x^2)^(1/4)*x^2 + 2*sqrt(x^4
 + x^2)*x + x + 2*(x^4 + x^2)^(3/4))/x) - 48*(x^4 + x^2)^(3/4))/(x^3 + x)
```

giac [A] time = 0.44, size = 97, normalized size = 0.87

$$\frac{3}{4} \cdot 2^{\frac{3}{4}} \arctan\left(\frac{1}{2} \cdot 2^{\frac{3}{4}} \left(\frac{1}{x^2} + 1\right)^{\frac{1}{4}}\right) - \frac{3}{8} \cdot 2^{\frac{3}{4}} \log\left(2^{\frac{1}{4}} + \left(\frac{1}{x^2} + 1\right)^{\frac{1}{4}}\right) + \frac{3}{8} \cdot 2^{\frac{3}{4}} \log\left(\left|-2^{\frac{1}{4}} + \left(\frac{1}{x^2} + 1\right)^{\frac{1}{4}}\right|\right) - \frac{3}{\left(\frac{1}{x^2} + 1\right)^{\frac{1}{4}}} - 2 \arctan\left(\left(\frac{1}{x^2} + 1\right)^{\frac{1}{4}}\right) + \log\left(\left(\frac{1}{x^2} + 1\right)^{\frac{1}{4}} + 1\right) - \log\left(\left(\frac{1}{x^2} + 1\right)^{\frac{1}{4}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+1)/(x^4-1)/(x^4+x^2)^(1/4),x, algorithm="giac")
```

```
[Out] 3/4*2^(3/4)*arctan(1/2*2^(3/4)*(1/x^2 + 1)^(1/4)) - 3/8*2^(3/4)*log(2^(1/4)
 + (1/x^2 + 1)^(1/4)) + 3/8*2^(3/4)*log(abs(-2^(1/4) + (1/x^2 + 1)^(1/4)))
- 3/(1/x^2 + 1)^(1/4) - 2*arctan((1/x^2 + 1)^(1/4)) + log((1/x^2 + 1)^(1/4)
 + 1) - log((1/x^2 + 1)^(1/4) - 1)
```

maple [F] time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{2x^4 + 1}{\left(x^4 - 1\right) \left(x^4 + x^2\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+1)/(x^4-1)/(x^4+x^2)^(1/4),x)
```

```
[Out] int((2*x^4+1)/(x^4-1)/(x^4+x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\, x^4 + 1}{\left(x^4 + x^2\right)^{\frac{1}{4}} \left(x^4 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+1)/(x^4-1)/(x^4+x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((2*x^4 + 1)/((x^4 + x^2)^(1/4)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\, x^4 + 1}{{\left(x^4 + x^2\right)}^{1/4}\, \left(x^4 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4 + 1)/((x^2 + x^4)^(1/4)*(x^4 - 1)),x)
```

```
[Out] int((2*x^4 + 1)/((x^2 + x^4)^(1/4)*(x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^4 + 1}{\sqrt[4]{x^2 \left(x^2 + 1\right)} (x - 1) (x + 1) \left(x^2 + 1\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+1)/(x**4-1)/(x**4+x**2)**(1/4),x)
```

```
[Out] Integral((2*x**4 + 1)/((x**2*(x**2 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1))
, x)
```

3.1417 $\int \frac{b+ax^2}{x^2(-b+ax^2)\sqrt[4]{bx^2+ax^4}}\,dx$

Optimal. Leaf size=112

$$-\frac{2^{3/4}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4+bx^2}}\right)}{b}-\frac{2^{3/4}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4+bx^2}}\right)}{b}+\frac{2\left(ax^4+bx^2\right)^{3/4}}{3bx^3}$$

Rubi [C] time = 0.37, antiderivative size = 55, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2056, 466, 511, 510}

$$\frac{2\left(ax^2+b\right){}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\frac{2ax^2}{ax^2+b}\right)}{3bx\sqrt[4]{ax^4+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^2)/(x^2*(-b + a*x^2)*(b*x^2 + a*x^4)^(1/4)),x]
```

```
[Out] (2*(b + a*x^2)*Hypergeometric2F1[-3/4, 1, 1/4, (2*a*x^2)/(b + a*x^2)])/(3*b
*x*(b*x^2 + a*x^4)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{b+ax^2}{x^2\left(-b+ax^2\right)\sqrt[4]{bx^2+ax^4}}\,dx = \frac{\left(\sqrt{x}\,\sqrt[4]{b+ax^2}\right)\int \frac{\left(b+ax^2\right)^{3/4}}{x^{5/2}\left(-b+ax^2\right)}\,dx}{\sqrt[4]{bx^2+ax^4}}$$
$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{\left(b+ax^4\right)^{3/4}}{x^4\left(-b+ax^4\right)}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{bx^2+ax^4}}$$
$$= \frac{\left(2\sqrt{x}\,\left(b+ax^2\right)\right)\text{Subst}\left(\int \frac{\left(1+\frac{ax^4}{b}\right)^{3/4}}{x^4\left(-b+ax^4\right)}\,dx, x, \sqrt{x}\right)}{\left(1+\frac{ax^2}{b}\right)^{3/4}\sqrt[4]{bx^2+ax^4}}$$
$$= \frac{2\left(b+ax^2\right)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\frac{2ax^2}{b+ax^2}\right)}{3bx\sqrt[4]{bx^2+ax^4}}$$

Mathematica [C] time = 0.03, size = 48, normalized size = 0.43

$$\frac{2\left(x^2\left(ax^2+b\right)\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\frac{2ax^2}{ax^2+b}\right)}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^2)/(x^2*(-b + a*x^2)*(b*x^2 + a*x^4)^(1/4)),x]
```

```
[Out] (2*(x^2*(b + a*x^2))^(3/4)*Hypergeometric2F1[-3/4, 1, 1/4, (2*a*x^2)/(b + a
*x^2)])/(3*b*x^3)
```

IntegrateAlgebraic [A] time = 0.40, size = 112, normalized size = 1.00

$$-\frac{2^{3/4}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{b}-\frac{2^{3/4}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{b}+\frac{2\left(ax^4+bx^2\right)^{3/4}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^2)/(x^2*(-b + a*x^2)*(b*x^2 + a*x^4)^(1/4)),x]
```

```
[Out] (2*(b*x^2 + a*x^4)^(3/4))/(3*b*x^3) - (2^(3/4)*a^(3/4)*ArcTan[(2^(1/4)*a^(1
/4)*x)/(b*x^2 + a*x^4)^(1/4)])/b - (2^(3/4)*a^(3/4)*ArcTanh[(2^(1/4)*a^(1/4
)*x)/(b*x^2 + a*x^4)^(1/4)])/b
```

fricas [B] time = 79.94, size = 504, normalized size = 4.50

$$\frac{12\left(\frac{1}{2}\right)^{\frac{1}{4}}bx^3\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(2\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(ax^4+bx^2\right)^{\frac{1}{4}}a^4bx^2\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}+2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(ax^4+bx^2\right)^{\frac{3}{4}}a^2b^3\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}+\left(2\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{ax^4+bx^2}\,a^2bx\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}+\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(3ab^3x^3+b^4x\right)\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}\right)\sqrt{\sqrt{\frac{1}{2}}a^2b^2\sqrt{\frac{a^3}{b^4}}}\right)}{a^5x^3-a^4bx}\right)-3\left(\frac{1}{2}\right)^{\frac{1}{4}}bx^3\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}\log\left(\frac{4\sqrt{\frac{1}{2}}\left(ax^4+bx^2\right)^{\frac{1}{4}}ab^2x^2\sqrt{\frac{a^3}{b^4}}+4\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{ax^4+bx^2}\,b^3x\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}+2\left(ax^4+bx^2\right)^{\frac{3}{4}}a^2+\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(3a^2bx^3+ab^2x\right)\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}}{ax^3-bx}\right)+3\left(\frac{1}{2}\right)^{\frac{1}{4}}bx^3\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}\log\left(\frac{4\sqrt{\frac{1}{2}}\left(ax^4+bx^2\right)^{\frac{1}{4}}ab^2x^2\sqrt{\frac{a^3}{b^4}}-4\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{ax^4+bx^2}\,b^3x\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}+2\left(ax^4+bx^2\right)^{\frac{3}{4}}a^2-\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(3a^2bx^3+ab^2x\right)\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}}{ax^3-bx}\right)+4\left(ax^4+bx^2\right)^{\frac{3}{4}}}{6bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/x^2/(a*x^2-b)/(a*x^4+b*x^2)^(1/4),x, algorithm="fricas"
)
```

```
[Out] 1/6*(12*(1/2)^(1/4)*b*x^3*(a^3/b^4)^(1/4)*arctan(2*(2*(1/2)^(1/4)*(a*x^4 +
b*x^2)^(1/4)*a^4*b*x^2*(a^3/b^4)^(1/4) + 2*(1/2)^(3/4)*(a*x^4 + b*x^2)^(3/4
)*a^2*b^3*(a^3/b^4)^(3/4) + (2*(1/2)^(1/4)*sqrt(a*x^4 + b*x^2)*a^2*b*x*(a^3
/b^4)^(1/4) + (1/2)^(3/4)*(3*a*b^3*x^3 + b^4*x)*(a^3/b^4)^(3/4))*sqrt(sqrt(
```

```
1/2)*a^2*b^2*sqrt(a^3/b^4)))/(a^5*x^3 - a^4*b*x)) - 3*(1/2)^(1/4)*b*x^3*(a^
3/b^4)^(1/4)*log((4*sqrt(1/2)*(a*x^4 + b*x^2)^(1/4)*a*b^2*x^2*sqrt(a^3/b^4)
 + 4*(1/2)^(3/4)*sqrt(a*x^4 + b*x^2)*b^3*x*(a^3/b^4)^(3/4) + 2*(a*x^4 + b*x
^2)^(3/4)*a^2 + (1/2)^(1/4)*(3*a^2*b*x^3 + a*b^2*x)*(a^3/b^4)^(1/4))/(a*x^3
 - b*x)) + 3*(1/2)^(1/4)*b*x^3*(a^3/b^4)^(1/4)*log((4*sqrt(1/2)*(a*x^4 + b*
x^2)^(1/4)*a*b^2*x^2*sqrt(a^3/b^4) - 4*(1/2)^(3/4)*sqrt(a*x^4 + b*x^2)*b^3*
x*(a^3/b^4)^(3/4) + 2*(a*x^4 + b*x^2)^(3/4)*a^2 - (1/2)^(1/4)*(3*a^2*b*x^3
+ a*b^2*x)*(a^3/b^4)^(1/4))/(a*x^3 - b*x)) + 4*(a*x^4 + b*x^2)^(3/4))/(b*x^
3)
```

giac [B] time = 0.38, size = 208, normalized size = 1.86

$$-\frac{2^{\frac{1}{4}} (-a)^{\frac{3}{4}} \arctan\left(\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}} (-a)^{\frac{1}{4}} + 2\left(a + \frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2 (-a)^{\frac{1}{4}}}\right)}{b} - \frac{2^{\frac{1}{4}} (-a)^{\frac{3}{4}} \arctan\left(-\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}} (-a)^{\frac{1}{4}} - 2\left(a + \frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2 (-a)^{\frac{1}{4}}}\right)}{b} + \frac{2^{\frac{1}{4}} (-a)^{\frac{3}{4}} \log\left(2^{\frac{3}{4}} (-a)^{\frac{1}{4}} \left(a + \frac{b}{x^2}\right)^{\frac{1}{4}} + \sqrt{2}\sqrt{-a} + \sqrt{a + \frac{b}{x^2}}\right)}{2 b} - \frac{2^{\frac{1}{4}} (-a)^{\frac{3}{4}} \log\left(-2^{\frac{3}{4}} (-a)^{\frac{1}{4}} \left(a + \frac{b}{x^2}\right)^{\frac{1}{4}} + \sqrt{2}\sqrt{-a} + \sqrt{a + \frac{b}{x^2}}\right)}{2 b} + \frac{2\left(a + \frac{b}{x^2}\right)^{\frac{3}{4}}}{3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/x^2/(a*x^2-b)/(a*x^4+b*x^2)^(1/4),x, algorithm="giac")
```

```
[Out] -2^(1/4)*(-a)^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4)*(-a)^(1/4) + 2*(a + b/x^2)^
(1/4))/(-a)^(1/4))/b - 2^(1/4)*(-a)^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4)*(-a)
^(1/4) - 2*(a + b/x^2)^(1/4))/(-a)^(1/4))/b + 1/2*2^(1/4)*(-a)^(3/4)*log(2^
(3/4)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(2)*sqrt(-a) + sqrt(a + b/x^2))/b
- 1/2*2^(1/4)*(-a)^(3/4)*log(-2^(3/4)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(2
)*sqrt(-a) + sqrt(a + b/x^2))/b + 2/3*(a + b/x^2)^(3/4)/b
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{a\,x^{2}+b}{x^{2} \left(a\,x^{2}-b\right) \left(a\,x^{4}+b\,x^{2}\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)/x^2/(a*x^2-b)/(a*x^4+b*x^2)^(1/4),x)
```

```
[Out] int((a*x^2+b)/x^2/(a*x^2-b)/(a*x^4+b*x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{2}+b}{\left(ax^{4}+bx^{2}\right)^{\frac{1}{4}} \left(ax^{2}-b\right) x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/x^2/(a*x^2-b)/(a*x^4+b*x^2)^(1/4),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^2 + b)/((a*x^4 + b*x^2)^(1/4)*(a*x^2 - b)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a\,x^2+b}{x^2\,\left(b-a\,x^2\right)\,{\left(a\,x^4+b\,x^2\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x^2)/(x^2*(b - a*x^2)*(a*x^4 + b*x^2)^(1/4)),x)
```

```
[Out] -int((b + a*x^2)/(x^2*(b - a*x^2)*(a*x^4 + b*x^2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + b}{x^2 \sqrt[4]{x^2 (ax^2 + b)} (ax^2 - b)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b)/x**2/(a*x**2-b)/(a*x**4+b*x**2)**(1/4),x)
```

```
[Out] Integral((a*x**2 + b)/(x**2*(x**2*(a*x**2 + b))**(1/4)*(a*x**2 - b)), x)
```

3.1418 $\int x\sqrt[4]{bx^3+ax^4}\,dx$

Optimal. Leaf size=112

$$-\frac{7b^3\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{64a^{11/4}}+\frac{7b^3\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{64a^{11/4}}+\frac{\left(32a^2x^2+4abx-7b^2\right)\sqrt[4]{ax^4+bx^3}}{96a^2}$$

Rubi [A] time = 0.28, antiderivative size = 196, normalized size of antiderivative = 1.75, number of steps used = 9, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {2021, 2024, 2032, 63, 331, 298, 203, 206}

$$-\frac{7b^3x^{9/4}(ax+b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{64a^{11/4}\left(ax^4+bx^3\right)^{3/4}}+\frac{7b^3x^{9/4}(ax+b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{64a^{11/4}\left(ax^4+bx^3\right)^{3/4}}-\frac{7b^2\sqrt[4]{ax^4+bx^3}}{96a^2}+\frac{bx\sqrt[4]{ax^4+bx^3}}{24a}+\frac{1}{3}x^2\sqrt[4]{ax^4+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^3 + a*x^4)^(1/4),x]
```

```
[Out] (-7*b^2*(b*x^3 + a*x^4)^(1/4))/(96*a^2) + (b*x*(b*x^3 + a*x^4)^(1/4))/(24*a
) + (x^2*(b*x^3 + a*x^4)^(1/4))/3 - (7*b^3*x^(9/4)*(b + a*x)^(3/4)*ArcTan[(
a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(64*a^(11/4)*(b*x^3 + a*x^4)^(3/4)) + (7
*b^3*x^(9/4)*(b + a*x)^(3/4)*ArcTanh[(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(6
4*a^(11/4)*(b*x^3 + a*x^4)^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x\sqrt[4]{bx^3+ax^4}\,dx = \frac{1}{3}x^2\sqrt[4]{bx^3+ax^4}+\frac{1}{12}b\int\frac{x^4}{\left(bx^3+ax^4\right)^{3/4}}\,dx$$

$$= \frac{bx\sqrt[4]{bx^3+ax^4}}{24a}+\frac{1}{3}x^2\sqrt[4]{bx^3+ax^4}-\frac{\left(7b^2\right)\int\frac{x^3}{\left(bx^3+ax^4\right)^{3/4}}\,dx}{96a}$$

$$= -\frac{7b^2\sqrt[4]{bx^3+ax^4}}{96a^2}+\frac{bx\sqrt[4]{bx^3+ax^4}}{24a}+\frac{1}{3}x^2\sqrt[4]{bx^3+ax^4}+\frac{\left(7b^3\right)\int\frac{x^2}{\left(bx^3+ax^4\right)^{3/4}}\,dx}{128a^2}$$

$$= -\frac{7b^2\sqrt[4]{bx^3+ax^4}}{96a^2}+\frac{bx\sqrt[4]{bx^3+ax^4}}{24a}+\frac{1}{3}x^2\sqrt[4]{bx^3+ax^4}+\frac{\left(7b^3x^{9/4}(b+ax)^{3/4}\right)\int\frac{1}{\sqrt[4]{x}\,(b+ax)^{3}\cdots}}{128a^2\left(bx^3+ax^4\right)^{3/4}}$$

$$= -\frac{7b^2\sqrt[4]{bx^3+ax^4}}{96a^2}+\frac{bx\sqrt[4]{bx^3+ax^4}}{24a}+\frac{1}{3}x^2\sqrt[4]{bx^3+ax^4}+\frac{\left(7b^3x^{9/4}(b+ax)^{3/4}\right)\text{Subst}\left(\int\cdots\right.}{32a^2\left(bx^3+ax^4\cdots\right.}$$

$$= -\frac{7b^2\sqrt[4]{bx^3+ax^4}}{96a^2}+\frac{bx\sqrt[4]{bx^3+ax^4}}{24a}+\frac{1}{3}x^2\sqrt[4]{bx^3+ax^4}+\frac{\left(7b^3x^{9/4}(b+ax)^{3/4}\right)\text{Subst}\left(\int\frac{\cdots}{1\cdots}\right.}{32a^2\left(bx^3+ax^4\right)\cdots}$$

$$= -\frac{7b^2\sqrt[4]{bx^3+ax^4}}{96a^2}+\frac{bx\sqrt[4]{bx^3+ax^4}}{24a}+\frac{1}{3}x^2\sqrt[4]{bx^3+ax^4}+\frac{\left(7b^3x^{9/4}(b+ax)^{3/4}\right)\text{Subst}\left(\int\frac{\cdots}{1\cdots}\right.}{64a^{5/2}\left(bx^3+ax\cdots\right.}$$

$$= -\frac{7b^2\sqrt[4]{bx^3+ax^4}}{96a^2}+\frac{bx\sqrt[4]{bx^3+ax^4}}{24a}+\frac{1}{3}x^2\sqrt[4]{bx^3+ax^4}-\frac{7b^3x^{9/4}(b+ax)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{b+ax}}\cdots\right.}{64a^{11/4}\left(bx^3+ax^4\right)^{3/4}}$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.44

$$\frac{4x^2\sqrt[4]{x^3(ax+b)}\ {}_2F_1\left(-\frac{1}{4},\frac{11}{4};\frac{15}{4};-\frac{ax}{b}\right)}{11\sqrt[4]{\frac{ax}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^3 + a*x^4)^(1/4),x]
```

```
[Out] (4*x^2*(x^3*(b + a*x))^(1/4)*Hypergeometric2F1[-1/4, 11/4, 15/4, -((a*x)/b)
])/(11*(1 + (a*x)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.46, size = 112, normalized size = 1.00

$$-\frac{7b^3\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{64a^{11/4}}+\frac{7b^3\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{64a^{11/4}}+\frac{\left(32a^2x^2+4abx-7b^2\right)\sqrt[4]{ax^4+bx^3}}{96a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(b*x^3 + a*x^4)^(1/4),x]
```

```
[Out] ((-7*b^2 + 4*a*b*x + 32*a^2*x^2)*(b*x^3 + a*x^4)^(1/4))/(96*a^2) - (7*b^3*A
rcTan[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(64*a^(11/4)) + (7*b^3*ArcTanh[(a
^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(64*a^(11/4))
```

fricas [B] time = 0.44, size = 253, normalized size = 2.26

$$-\frac{84\,a^2\left(\frac{b^{12}}{a^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^4+bx^3\right)^{\frac{1}{4}}a^8b^3\left(\frac{b^{12}}{a^{11}}\right)^{\frac{3}{4}}-a^8\left(\frac{b^{12}}{a^{11}}\right)^{\frac{3}{4}}x\sqrt{\frac{a^6\sqrt{\frac{b^{12}}{a^{11}}}x^2+\sqrt{ax^4+bx^3}\,b^6}{x^2}}}{b^{12}x}\right)-21\,a^2\left(\frac{b^{12}}{a^{11}}\right)^{\frac{1}{4}}\log\left(\frac{7\left(a^3\left(\frac{b^{12}}{a^{11}}\right)^{\frac{1}{4}}x+\left(ax^4+bx^3\right)^{\frac{1}{4}}b^3\right)}{x}\right)+21\,a^2\left(\frac{b^{12}}{a^{11}}\right)^{\frac{1}{4}}\log\left(-\frac{7\left(a^3\left(\frac{b^{12}}{a^{11}}\right)^{\frac{1}{4}}x-\left(ax^4+bx^3\right)^{\frac{1}{4}}b^3\right)}{x}\right)-4\left(ax^4+bx^3\right)^{\frac{1}{4}}\left(32\,a^2x^2+4\,abx-7\,b^2\right)}{384\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x^4+b*x^3)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/384*(84*a^2*(b^12/a^11)^(1/4)*arctan(-((a*x^4 + b*x^3)^(1/4)*a^8*b^3*(b^
12/a^11)^(3/4) - a^8*(b^12/a^11)^(3/4)*x*sqrt((a^6*sqrt(b^12/a^11)*x^2 + sq
rt(a*x^4 + b*x^3)*b^6)/x^2))/(b^12*x)) - 21*a^2*(b^12/a^11)^(1/4)*log(7*(a^
3*(b^12/a^11)^(1/4)*x + (a*x^4 + b*x^3)^(1/4)*b^3)/x) + 21*a^2*(b^12/a^11)^
(1/4)*log(-7*(a^3*(b^12/a^11)^(1/4)*x - (a*x^4 + b*x^3)^(1/4)*b^3)/x) - 4*(
a*x^4 + b*x^3)^(1/4)*(32*a^2*x^2 + 4*a*b*x - 7*b^2))/a^2
```

giac [B] time = 0.38, size = 261, normalized size = 2.33

$$\frac{\frac{42\sqrt{2}(-a)^{\frac{1}{4}}b^4\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^3}+\frac{42\sqrt{2}(-a)^{\frac{1}{4}}b^4\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^3}+\frac{21\sqrt{2}(-a)^{\frac{1}{4}}b^4\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{a^3}+\frac{21\sqrt{2}b^4\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{(-a)^{\frac{3}{4}}a^2}-\frac{8\left(7\left(a+\frac{b}{x}\right)^{\frac{9}{4}}b^4-18\left(a+\frac{b}{x}\right)^{\frac{5}{4}}ab^4-21\left(a+\frac{b}{x}\right)^{\frac{1}{4}}a^2b^4\right)x^3}{a^2b^3}}{768\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x^4+b*x^3)^(1/4),x, algorithm="giac")
```

```
[Out] 1/768*(42*sqrt(2)*(-a)^(1/4)*b^4*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2
*(a + b/x)^(1/4))/(-a)^(1/4))/a^3 + 42*sqrt(2)*(-a)^(1/4)*b^4*arctan(-1/2*s
qrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(a + b/x)^(1/4))/(-a)^(1/4))/a^3 + 21*sqrt(2
)*(-a)^(1/4)*b^4*log(sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a
 + b/x))/a^3 + 21*sqrt(2)*b^4*log(-sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqr
t(-a) + sqrt(a + b/x))/((-a)^(3/4)*a^2) - 8*(7*(a + b/x)^(9/4)*b^4 - 18*(a
+ b/x)^(5/4)*a*b^4 - 21*(a + b/x)^(1/4)*a^2*b^4)*x^3/(a^2*b^3))/b
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x\left(a\,x^{4}+b\,x^{3}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^4+b*x^3)^(1/4),x)
[Out] int(x*(a*x^4+b*x^3)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^{4}+bx^{3}\right)^{\frac{1}{4}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x^4+b*x^3)^(1/4),x, algorithm="maxima")
[Out] integrate((a*x^4 + b*x^3)^(1/4)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a\,x^{4}+b\,x^{3}\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^4 + b*x^3)^(1/4),x)
[Out] int(x*(a*x^4 + b*x^3)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt[4]{x^{3}\left(ax+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x**4+b*x**3)**(1/4),x)
[Out] Integral(x*(x**3*(a*x + b))**(1/4), x)
```

3.1419 $\int \frac{-2b+ax^3}{\left(b+x^2+ax^3\right)\sqrt[4]{bx^2+ax^5}}\,dx$

Optimal. Leaf size=112

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^5+bx^2}}{\sqrt{ax^5+bx^2}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5+bx^2}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5+bx^2}}\right)$$

Rubi [F] time = 1.62, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2b+ax^3}{\left(b+x^2+ax^3\right)\sqrt[4]{bx^2+ax^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*b + a*x^3)/((b + x^2 + a*x^3)*(b*x^2 + a*x^5)^(1/4)),x]
```

```
[Out] (2*x*(1 + (a*x^3)/b)^(1/4)*Hypergeometric2F1[1/6, 1/4, 7/6, -((a*x^3)/b)])/
(b*x^2 + a*x^5)^(1/4) - (6*b*Sqrt[x]*(b + a*x^3)^(1/4)*Defer[Subst][Defer[I
nt][1/((b + a*x^6)^(1/4)*(b + x^4 + a*x^6)), x], x, Sqrt[x]])/(b*x^2 + a*x^
5)^(1/4) - (2*Sqrt[x]*(b + a*x^3)^(1/4)*Defer[Subst][Defer[Int][x^4/((b + a
*x^6)^(1/4)*(b + x^4 + a*x^6)), x], x, Sqrt[x]])/(b*x^2 + a*x^5)^(1/4)
```

Rubi steps

$$\int \frac{-2b+ax^3}{\left(b+x^2+ax^3\right)\sqrt[4]{bx^2+ax^5}}\,dx=\frac{\left(\sqrt{x}\sqrt[4]{b+ax^3}\right)\int \frac{-2b+ax^3}{\sqrt{x}\sqrt[4]{b+ax^3}\left(b+x^2+ax^3\right)}\,dx}{\sqrt[4]{bx^2+ax^5}}$$

$$=\frac{\left(2\sqrt{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{-2b+ax^6}{\sqrt[4]{b+ax^6}\left(b+x^4+ax^6\right)}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{bx^2+ax^5}}$$

$$=\frac{\left(2\sqrt{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[4]{b+ax^6}}-\frac{3b+x^4}{\sqrt[4]{b+ax^6}\left(b+x^4+ax^6\right)}\right)dx,x,\sqrt{x}\right)}{\sqrt[4]{bx^2+ax^5}}$$

$$=\frac{\left(2\sqrt{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{b+ax^6}}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{bx^2+ax^5}}-\frac{\left(2\sqrt{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \ldots\right.}{\sqrt[4]{bx^2+\ldots}}$$

$$=-\frac{\left(2\sqrt{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \left(\frac{3b}{\sqrt[4]{b+ax^6}\left(b+x^4+ax^6\right)}+\frac{x^4}{\sqrt[4]{b+ax^6}\left(b+x^4+ax^6\right)}\right)dx,x,\sqrt{x}\ldots\right.}{\sqrt[4]{bx^2+ax^5}}$$

$$=\frac{2x\sqrt[4]{1+\frac{ax^3}{b}}\,{}_2F_1\left(\frac{1}{6},\frac{1}{4};\frac{7}{6};-\frac{ax^3}{b}\right)}{\sqrt[4]{bx^2+ax^5}}-\frac{\left(2\sqrt{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{b+ax^6}\left(b+x^4+ax^6\right)}\ldots\right.}{\sqrt[4]{bx^2+ax^5}}$$

Mathematica [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{-2b+ax^3}{\left(b+x^2+ax^3\right)\sqrt[4]{bx^2+ax^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*b + a*x^3)/((b + x^2 + a*x^3)*(b*x^2 + a*x^5)^(1/4)),x]
```

```
[Out] Integrate[(-2*b + a*x^3)/((b + x^2 + a*x^3)*(b*x^2 + a*x^5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 2.92, size = 112, normalized size = 1.00

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^5+bx^2}}{\sqrt{ax^5+bx^2}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5+bx^2}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*b + a*x^3)/((b + x^2 + a*x^3)*(b*x^2 + a*x^5)^(1/4))
,x]
```

```
[Out] -(Sqrt[2]*ArcTan[(Sqrt[2]*x*(b*x^2 + a*x^5)^(1/4))/(-x^2 + Sqrt[b*x^2 + a*x
^5])]) - Sqrt[2]*ArcTanh[(x^2/Sqrt[2] + Sqrt[b*x^2 + a*x^5]/Sqrt[2])/(x*(b*
x^2 + a*x^5)^(1/4))]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*b)/(a*x^3+x^2+b)/(a*x^5+b*x^2)^(1/4),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3-2\,b}{\left(ax^5+bx^2\right)^{\frac{1}{4}}\left(ax^3+x^2+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*b)/(a*x^3+x^2+b)/(a*x^5+b*x^2)^(1/4),x, algorithm="giac"
)
```

```
[Out] integrate((a*x^3 - 2*b)/((a*x^5 + b*x^2)^(1/4)*(a*x^3 + x^2 + b)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{a\,x^3-2b}{\left(a\,x^3+x^2+b\right)\left(a\,x^5+b\,x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-2*b)/(a*x^3+x^2+b)/(a*x^5+b*x^2)^(1/4),x)
```

```
[Out] int((a*x^3-2*b)/(a*x^3+x^2+b)/(a*x^5+b*x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3-2\,b}{\left(ax^5+bx^2\right)^{\frac{1}{4}}\left(ax^3+x^2+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-2*b)/(a*x^3+x^2+b)/(a*x^5+b*x^2)^(1/4),x, algorithm="maxim
a")
```

```
[Out] integrate((a*x^3 - 2*b)/((a*x^5 + b*x^2)^(1/4)*(a*x^3 + x^2 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{2\,b-a\,x^3}{{\left(a\,x^5+b\,x^2\right)}^{1/4}\,\left(a\,x^3+x^2+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*b - a*x^3)/((a*x^5 + b*x^2)^(1/4)*(b + a*x^3 + x^2)),x)
```

```
[Out] int(-(2*b - a*x^3)/((a*x^5 + b*x^2)^(1/4)*(b + a*x^3 + x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-2*b)/(a*x**3+x**2+b)/(a*x**5+b*x**2)**(1/4),x)
```

```
[Out] Timed out
```

3.1420 $\int \frac{-b+ax^2}{(b+x+ax^2)\sqrt[4]{bx^3+ax^5}}\,dx$

Optimal. Leaf size=112

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^5+bx^3}}{\sqrt{ax^5+bx^3}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5+bx^3}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5+bx^3}}\right)$$

Rubi [C] time = 1.96, antiderivative size = 392, normalized size of antiderivative = 3.50, number of steps used = 21, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2056, 6728, 365, 364, 959, 466, 430, 429, 511, 510}

$$\frac{8ax^2\sqrt[4]{\frac{ax^2}{b}+1}\,F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};\frac{4a^2x^2}{\left(1-\sqrt{1-4ab}\right)^2},-\frac{ax^2}{b}\right)}{5\left(1-\sqrt{1-4ab}\right)\sqrt[4]{ax^5+bx^3}}+\frac{8ax^2\sqrt[4]{\frac{ax^2}{b}+1}\,F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};\frac{4a^2x^2}{\left(\sqrt{1-4ab}+1\right)^2},-\frac{ax^2}{b}\right)}{5\left(\sqrt{1-4ab}+1\right)\sqrt[4]{ax^5+bx^3}}-\frac{4x\sqrt[4]{\frac{ax^2}{b}+1}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\frac{4a^2x^2}{\left(1-\sqrt{1-4ab}\right)^2},-\frac{ax^2}{b}\right)}{\sqrt[4]{ax^5+bx^3}}-\frac{4x\sqrt[4]{\frac{ax^2}{b}+1}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\frac{4a^2x^2}{\left(\sqrt{1-4ab}+1\right)^2},-\frac{ax^2}{b}\right)}{\sqrt[4]{ax^5+bx^3}}+\frac{4x\sqrt[4]{\frac{ax^2}{b}+1}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{ax^2}{b}\right)}{\sqrt[4]{ax^5+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-b + a*x^2)/((b + x + a*x^2)*(b*x^3 + a*x^5)^(1/4)),x]
```

```
[Out] (-4*x*(1 + (a*x^2)/b)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, (4*a^2*x^2)/(1 - Sqr
t[1 - 4*a*b])^2, -((a*x^2)/b)])/(b*x^3 + a*x^5)^(1/4) - (4*x*(1 + (a*x^2)/b
)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, (4*a^2*x^2)/(1 + Sqrt[1 - 4*a*b])^2, -((
a*x^2)/b)])/(b*x^3 + a*x^5)^(1/4) + (8*a*x^2*(1 + (a*x^2)/b)^(1/4)*AppellF1
[5/8, 1, 1/4, 13/8, (4*a^2*x^2)/(1 - Sqrt[1 - 4*a*b])^2, -((a*x^2)/b)])/(5*
(1 - Sqrt[1 - 4*a*b])*(b*x^3 + a*x^5)^(1/4)) + (8*a*x^2*(1 + (a*x^2)/b)^(1/
4)*AppellF1[5/8, 1, 1/4, 13/8, (4*a^2*x^2)/(1 + Sqrt[1 - 4*a*b])^2, -((a*x^
2)/b)])/(5*(1 + Sqrt[1 - 4*a*b])*(b*x^3 + a*x^5)^(1/4)) + (4*x*(1 + (a*x^2)
/b)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -((a*x^2)/b)])/(b*x^3 + a*x^5)^(
1/4)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-b+ax^2}{(b+x+ax^2)\sqrt[4]{bx^3+ax^5}}\,dx = \frac{\left(x^{3/4}\sqrt[4]{b+ax^2}\right)\int \frac{-b+ax^2}{x^{3/4}\sqrt[4]{b+ax^2}(b+x+ax^2)}\,dx}{\sqrt[4]{bx^3+ax^5}}$$
$$= \frac{\left(x^{3/4}\sqrt[4]{b+ax^2}\right)\int \left(\frac{1}{x^{3/4}\sqrt[4]{b+ax^2}} - \frac{2b+x}{x^{3/4}\sqrt[4]{b+ax^2}(b+x+ax^2)}\right)dx}{\sqrt[4]{bx^3+ax^5}}$$
$$= \frac{\left(x^{3/4}\sqrt[4]{b+ax^2}\right)\int \frac{1}{x^{3/4}\sqrt[4]{b+ax^2}}\,dx}{\sqrt[4]{bx^3+ax^5}} - \frac{\left(x^{3/4}\sqrt[4]{b+ax^2}\right)\int \frac{2b+x}{x^{3/4}\sqrt[4]{b+ax^2}(b+x+ax^2)}\,dx}{\sqrt[4]{bx^3+ax^5}}$$
$$= -\frac{\left(x^{3/4}\sqrt[4]{b+ax^2}\right)\int \left(\frac{1-\sqrt{1-4ab}}{x^{3/4}\left(1-\sqrt{1-4ab}+2ax\right)\sqrt[4]{b+ax^2}} + \frac{1+\sqrt{1-4ab}}{x^{3/4}\left(1+\sqrt{1-4ab}+2ax\right)\sqrt[4]{b+ax^2}}\right) d}{\sqrt[4]{bx^3+ax^5}}$$
$$= \frac{4x\sqrt[4]{1+\frac{ax^2}{b}}\ {}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{ax^2}{b}\right)}{\sqrt[4]{bx^3+ax^5}} - \frac{\left(\left(1-\sqrt{1-4ab}\right)x^{3/4}\sqrt[4]{b+ax^2}\right)\int \frac{}{x^{3/4}(1-\sqrt{}}}{\sqrt[4]{bx^3+ax^5}}$$
$$= \frac{4x\sqrt[4]{1+\frac{ax^2}{b}}\ {}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{ax^2}{b}\right)}{\sqrt[4]{bx^3+ax^5}} + \frac{\left(2a\left(1-\sqrt{1-4ab}\right)x^{3/4}\sqrt[4]{b+ax^2}\right)\int \frac{}{\sqrt[4]{b+}}}{\sqrt[4]{bx^3+ax^5}}$$
$$= \frac{4x\sqrt[4]{1+\frac{ax^2}{b}}\ {}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{ax^2}{b}\right)}{\sqrt[4]{bx^3+ax^5}} + \frac{\left(8a\left(1-\sqrt{1-4ab}\right)x^{3/4}\sqrt[4]{b+ax^2}\right)\text{Subst}}{\sqrt[4]{bx^3 -}}$$
$$= \frac{4x\sqrt[4]{1+\frac{ax^2}{b}}\ {}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-\frac{ax^2}{b}\right)}{\sqrt[4]{bx^3+ax^5}} + \frac{\left(8a\left(1-\sqrt{1-4ab}\right)x^{3/4}\sqrt[4]{1+\frac{ax^2}{b}}\right)\text{Subst}}{\sqrt[4]{bx^3 -}}$$
$$= -\frac{4x\sqrt[4]{1+\frac{ax^2}{b}}\ F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\frac{4a^2x^2}{\left(1-\sqrt{1-4ab}\right)^2},-\frac{ax^2}{b}\right)}{\sqrt[4]{bx^3+ax^5}} - \frac{4x\sqrt[4]{1+\frac{ax^2}{b}}\ F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};-\right.}{\sqrt[4]{bx^3+ax}}$$

Mathematica [F] time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{-b+ax^2}{(b+x+ax^2)\sqrt[4]{bx^3+ax^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-b + a*x^2)/((b + x + a*x^2)*(b*x^3 + a*x^5)^(1/4)),x]
[Out] Integrate[(-b + a*x^2)/((b + x + a*x^2)*(b*x^3 + a*x^5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 2.87, size = 112, normalized size = 1.00

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^5+bx^3}}{\sqrt{ax^5+bx^3}-x^2}\right) - \sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5+bx^3}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5+bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^2)/((b + x + a*x^2)*(b*x^3 + a*x^5)^(1/4)),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(Sqrt[2]*x*(b*x^3 + a*x^5)^(1/4))/(-x^2 + Sqrt[b*x^3 + a*x
^5])]) - Sqrt[2]*ArcTanh[(x^2/Sqrt[2] + Sqrt[b*x^3 + a*x^5]/Sqrt[2])/(x*(b*
x^3 + a*x^5)^(1/4))]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(a*x^2+b+x)/(a*x^5+b*x^3)^(1/4),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - b}{\left(ax^5 + bx^3\right)^{\frac{1}{4}}\left(ax^2 + b + x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(a*x^2+b+x)/(a*x^5+b*x^3)^(1/4),x, algorithm="giac")
[Out] integrate((a*x^2 - b)/((a*x^5 + b*x^3)^(1/4)*(a*x^2 + b + x)), x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2 - b}{\left(a\,x^2 + b + x\right)\left(a\,x^5 + b\,x^3\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)/(a*x^2+b+x)/(a*x^5+b*x^3)^(1/4),x)
[Out] int((a*x^2-b)/(a*x^2+b+x)/(a*x^5+b*x^3)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - b}{\left(ax^5 + bx^3\right)^{\frac{1}{4}}\left(ax^2 + b + x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(a*x^2+b+x)/(a*x^5+b*x^3)^(1/4),x, algorithm="maxima")
[Out] integrate((a*x^2 - b)/((a*x^5 + b*x^3)^(1/4)*(a*x^2 + b + x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{b - a\,x^2}{\left(a\,x^5 + b\,x^3\right)^{1/4}\,\left(a\,x^2 + x + b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^2)/((a*x^5 + b*x^3)^(1/4)*(b + x + a*x^2)),x)
[Out] int(-(b - a*x^2)/((a*x^5 + b*x^3)^(1/4)*(b + x + a*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - b}{\sqrt[4]{x^3\left(ax^2 + b\right)}\left(ax^2 + b + x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)/(a*x**2+b+x)/(a*x**5+b*x**3)**(1/4),x)
```

```
[Out] Integral((a*x**2 - b)/((x**3*(a*x**2 + b))**(1/4)*(a*x**2 + b + x)), x)
```

3.1421 $\int \frac{x^{13}}{\sqrt[3]{-1+x^6}}\,dx$

Optimal. Leaf size=112

$$-\frac{1}{27}\log\left(\sqrt[3]{x^6-1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{9\sqrt{3}}+\frac{1}{36}\left(x^6-1\right)^{2/3}\left(3x^8+4x^2\right)+\frac{1}{54}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {275, 321, 239}

$$\frac{1}{12}\left(x^6-1\right)^{2/3}x^8+\frac{1}{9}\left(x^6-1\right)^{2/3}x^2-\frac{1}{18}\log\left(x^2-\sqrt[3]{x^6-1}\right)+\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6-1}}+1}{\sqrt{3}}\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(-1 + x^6)^(1/3),x]
```

```
[Out] (x^2*(-1 + x^6)^(2/3))/9 + (x^8*(-1 + x^6)^(2/3))/12 + ArcTan[(1 + (2*x^2)/
(-1 + x^6)^(1/3))/Sqrt[3]]/(9*Sqrt[3]) - Log[x^2 - (-1 + x^6)^(1/3)]/18
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13}}{\sqrt[3]{-1+x^6}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{\sqrt[3]{-1+x^3}}\,dx,x,x^2\right)\\
&= \frac{1}{12}x^8\left(-1+x^6\right)^{2/3}+\frac{1}{3}\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{-1+x^3}}\,dx,x,x^2\right)\\
&= \frac{1}{9}x^2\left(-1+x^6\right)^{2/3}+\frac{1}{12}x^8\left(-1+x^6\right)^{2/3}+\frac{1}{9}\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x^3}}\,dx,x,x^2\right)\\
&= \frac{1}{9}x^2\left(-1+x^6\right)^{2/3}+\frac{1}{12}x^8\left(-1+x^6\right)^{2/3}+\frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{-1+x^6}}}{\sqrt{3}}\right)}{9\sqrt{3}}-\frac{1}{18}\log\left(x^2-\sqrt[3]{-1+x^6}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 113, normalized size = 1.01

$$\frac{1}{108}\left(9\left(x^6-1\right)^{2/3}x^8+12\left(x^6-1\right)^{2/3}x^2-4\log\left(1-\frac{x^2}{\sqrt[3]{x^6-1}}\right)+4\sqrt{3}\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6-1}}+1}{\sqrt{3}}\right)+2\log\left(\frac{x^4}{\left(x^6-1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6-1}}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(-1 + x^6)^(1/3),x]
```

```
[Out] (12*x^2*(-1 + x^6)^(2/3) + 9*x^8*(-1 + x^6)^(2/3) + 4*Sqrt[3]*ArcTan[(1 + (
2*x^2)/(-1 + x^6)^(1/3))/Sqrt[3]] - 4*Log[1 - x^2/(-1 + x^6)^(1/3)] + 2*Log
[1 + x^4/(-1 + x^6)^(2/3) + x^2/(-1 + x^6)^(1/3)])/108
```

IntegrateAlgebraic [A] time = 2.76, size = 112, normalized size = 1.00

$$-\frac{1}{27}\log\left(\sqrt[3]{x^6-1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{9\sqrt{3}}+\frac{1}{36}\left(x^6-1\right)^{2/3}\left(3x^8+4x^2\right)+\frac{1}{54}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^13/(-1 + x^6)^(1/3),x]
```

```
[Out] ((-1 + x^6)^(2/3)*(4*x^2 + 3*x^8))/36 + ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(-1 +
 x^6)^(1/3))]/(9*Sqrt[3]) - Log[-x^2 + (-1 + x^6)^(1/3)]/27 + Log[x^4 + x^2
*(-1 + x^6)^(1/3) + (-1 + x^6)^(2/3)]/54
```

fricas [A] time = 0.67, size = 102, normalized size = 0.91

$$\frac{1}{36}\left(3x^8+4x^2\right)\left(x^6-1\right)^{\frac{2}{3}}-\frac{1}{27}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}}{3x^2}\right)-\frac{1}{27}\log\left(-\frac{x^2-\left(x^6-1\right)^{\frac{1}{3}}}{x^2}\right)+\frac{1}{54}\log\left(\frac{x^4+\left(x^6-1\right)^{\frac{1}{3}}x^2+\left(x^6-1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(x^6-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/36*(3*x^8 + 4*x^2)*(x^6 - 1)^(2/3) - 1/27*sqrt(3)*arctan(1/3*(sqrt(3)*x^2
 + 2*sqrt(3)*(x^6 - 1)^(1/3))/x^2) - 1/27*log(-(x^2 - (x^6 - 1)^(1/3))/x^2)
 + 1/54*log((x^4 + (x^6 - 1)^(1/3)*x^2 + (x^6 - 1)^(2/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(x^6-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(x^6-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^13/(x^6 - 1)^(1/3), x)
```

maple [C] time = 0.31, size = 53, normalized size = 0.47

$$\frac{x^2\left(3x^6+4\right)\left(x^6-1\right)^{\frac{2}{3}}}{36}+\frac{\left(-\mathrm{signum}\left(x^6-1\right)\right)^{\frac{1}{3}}x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^6\right)}{9\mathrm{signum}\left(x^6-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(x^6-1)^(1/3),x)
```

```
[Out] 1/36*x^2*(3*x^6+4)*(x^6-1)^(2/3)+1/9/signum(x^6-1)^(1/3)*(-signum(x^6-1))^(
1/3)*x^2*hypergeom([1/3,1/3],[4/3],x^6)
```

maxima [A] time = 0.41, size = 122, normalized size = 1.09

$$-\frac{1}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)-\frac{\frac{7\left(x^6-1\right)^{\frac{2}{3}}}{x^4}-\frac{4\left(x^6-1\right)^{\frac{5}{3}}}{x^{10}}}{36\left(\frac{2\left(x^6-1\right)}{x^6}-\frac{\left(x^6-1\right)^2}{x^{12}}-1\right)}+\frac{1}{54}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6-1\right)^{\frac{2}{3}}}{x^4}+1\right)-\frac{1}{27}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(x^6-1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3)/x^2 + 1)) - 1/36*(7*(x^
6 - 1)^(2/3)/x^4 - 4*(x^6 - 1)^(5/3)/x^10)/(2*(x^6 - 1)/x^6 - (x^6 - 1)^2/x
^12 - 1) + 1/54*log((x^6 - 1)^(1/3)/x^2 + (x^6 - 1)^(2/3)/x^4 + 1) - 1/27*l
og((x^6 - 1)^(1/3)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13}}{\left(x^6-1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(x^6 - 1)^(1/3),x)
```

```
[Out] int(x^13/(x^6 - 1)^(1/3), x)
```

sympy [C] time = 1.33, size = 31, normalized size = 0.28

$$\frac{x^{14}e^{-\frac{i\pi}{3}}\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|x^6\right)}{6\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(x**6-1)**(1/3),x)
```

```
[Out] x**14*exp(-I*pi/3)*gamma(7/3)*hyper((1/3, 7/3), (10/3,), x**6)/(6*gamma(10/
3))
```

3.1422 $\int x^9 \sqrt[3]{-1+x^6}\,dx$

Optimal. Leaf size=112

$$\frac{1}{54}\log\left(\sqrt[3]{x^6-1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{18\sqrt{3}}+\frac{1}{36}\sqrt[3]{x^6-1}\left(3x^{10}-x^4\right)-\frac{1}{108}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)$$

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {275, 279, 321, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{12}\sqrt[3]{x^6-1}\,x^{10}-\frac{1}{36}\sqrt[3]{x^6-1}\,x^4+\frac{1}{54}\log\left(1-\frac{x^2}{\sqrt[3]{x^6-1}}\right)+\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6-1}}+1}{\sqrt{3}}\right)}{18\sqrt{3}}-\frac{1}{108}\log\left(\frac{x^4}{\left(x^6-1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6-1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(-1 + x^6)^(1/3),x]
```

```
[Out] -1/36*(x^4*(-1 + x^6)^(1/3)) + (x^10*(-1 + x^6)^(1/3))/12 + ArcTan[(1 + (2*
x^2)/(-1 + x^6)^(1/3))/Sqrt[3]]/(18*Sqrt[3]) + Log[1 - x^2/(-1 + x^6)^(1/3)
]/54 - Log[1 + x^4/(-1 + x^6)^(2/3) + x^2/(-1 + x^6)^(1/3)]/108
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int x^9\sqrt[3]{-1+x^6}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^4\sqrt[3]{-1+x^3}\,dx,x,x^2\right)\\
&= \frac{1}{12}x^{10}\sqrt[3]{-1+x^6}-\frac{1}{12}\text{Subst}\left(\int \frac{x^4}{\left(-1+x^3\right)^{2/3}}\,dx,x,x^2\right)\\
&= -\frac{1}{36}x^4\sqrt[3]{-1+x^6}+\frac{1}{12}x^{10}\sqrt[3]{-1+x^6}-\frac{1}{18}\text{Subst}\left(\int \frac{x}{\left(-1+x^3\right)^{2/3}}\,dx,x,x^2\right)\\
&= -\frac{1}{36}x^4\sqrt[3]{-1+x^6}+\frac{1}{12}x^{10}\sqrt[3]{-1+x^6}-\frac{1}{18}\text{Subst}\left(\int \frac{x}{1-x^3}\,dx,x,\frac{x^2}{\sqrt[3]{-1+x^6}}\right)\\
&= -\frac{1}{36}x^4\sqrt[3]{-1+x^6}+\frac{1}{12}x^{10}\sqrt[3]{-1+x^6}-\frac{1}{54}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\frac{x^2}{\sqrt[3]{-1+x^6}}\right)+\frac{1}{54}\text{Subst}\Bigg(\int\\
&= -\frac{1}{36}x^4\sqrt[3]{-1+x^6}+\frac{1}{12}x^{10}\sqrt[3]{-1+x^6}+\frac{1}{54}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right)-\frac{1}{108}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,d\right.\\
&= -\frac{1}{36}x^4\sqrt[3]{-1+x^6}+\frac{1}{12}x^{10}\sqrt[3]{-1+x^6}+\frac{1}{54}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right)-\frac{1}{108}\log\left(1+\frac{x^4}{\left(-1+x^6\right)^{2/3}}\right.\\
&= -\frac{1}{36}x^4\sqrt[3]{-1+x^6}+\frac{1}{12}x^{10}\sqrt[3]{-1+x^6}+\frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{-1+x^6}}}{\sqrt{3}}\right)}{18\sqrt{3}}+\frac{1}{54}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right)-\frac{1}{108}\text{lo}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.48

$$\frac{x^4\sqrt[3]{x^6-1}\left({}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^6\right)-\left(1-x^6\right)^{4/3}\right)}{12\sqrt[3]{1-x^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(-1 + x^6)^(1/3),x]
[Out] (x^4*(-1 + x^6)^(1/3)*(-(1 - x^6)^(4/3) + Hypergeometric2F1[-1/3, 2/3, 5/3,
 x^6]))/(12*(1 - x^6)^(1/3))
```

IntegrateAlgebraic [A] time = 0.80, size = 112, normalized size = 1.00

$$\frac{1}{54}\log\left(\sqrt[3]{x^6-1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{18\sqrt{3}}+\frac{1}{36}\sqrt[3]{x^6-1}\left(3x^{10}-x^4\right)-\frac{1}{108}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9*(-1 + x^6)^(1/3),x]
[Out] ((-1 + x^6)^(1/3)*(-x^4 + 3*x^10))/36 + ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(-1 +
 x^6)^(1/3))]/(18*Sqrt[3]) + Log[-x^2 + (-1 + x^6)^(1/3)]/54 - Log[x^4 + x^
2*(-1 + x^6)^(1/3) + (-1 + x^6)^(2/3)]/108
```

fricas [A] time = 0.72, size = 102, normalized size = 0.91

$$-\frac{1}{54}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}}{3x^2}\right)+\frac{1}{36}\left(3x^{10}-x^4\right)\left(x^6-1\right)^{\frac{1}{3}}+\frac{1}{54}\log\left(-\frac{x^2-\left(x^6-1\right)^{\frac{1}{3}}}{x^2}\right)-\frac{1}{108}\log\left(\frac{x^4+\left(x^6-1\right)^{\frac{1}{3}}x^2+\left(x^6-1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(x^6-1)^(1/3),x, algorithm="fricas")
[Out] -1/54*sqrt(3)*arctan(1/3*(sqrt(3)*x^2 + 2*sqrt(3)*(x^6 - 1)^(1/3))/x^2) + 1
/36*(3*x^10 - x^4)*(x^6 - 1)^(1/3) + 1/54*log(-(x^2 - (x^6 - 1)^(1/3))/x^2)
 - 1/108*log((x^4 + (x^6 - 1)^(1/3)*x^2 + (x^6 - 1)^(2/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^6-1\right)^{\frac{1}{3}}x^9\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(x^6-1)^(1/3),x, algorithm="giac")
[Out] integrate((x^6 - 1)^(1/3)*x^9, x)
```

maple [C] time = 0.32, size = 53, normalized size = 0.47

$$\frac{x^4\left(3x^6-1\right)\left(x^6-1\right)^{\frac{1}{3}}}{36}-\frac{\left(-\mathrm{signum}\left(x^6-1\right)\right)^{\frac{2}{3}}x^4\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^6\right)}{36\mathrm{signum}\left(x^6-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(x^6-1)^(1/3),x)
[Out] 1/36*x^4*(3*x^6-1)*(x^6-1)^(1/3)-1/36/signum(x^6-1)^(2/3)*(-signum(x^6-1))^
(2/3)*x^4*hypergeom([2/3,2/3],[5/3],x^6)
```

maxima [A] time = 0.42, size = 121, normalized size = 1.08

$$-\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)-\frac{\frac{2\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6-1\right)^{\frac{4}{3}}}{x^8}}{36\left(\frac{2\left(x^6-1\right)}{x^6}-\frac{\left(x^6-1\right)^2}{x^{12}}-1\right)}-\frac{1}{108}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6-1\right)^{\frac{2}{3}}}{x^4}+1\right)+\frac{1}{54}\log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(x^6-1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/54*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3)/x^2 + 1)) - 1/36*(2*(x^
6 - 1)^(1/3)/x^2 + (x^6 - 1)^(4/3)/x^8)/(2*(x^6 - 1)/x^6 - (x^6 - 1)^2/x^12
 - 1) - 1/108*log((x^6 - 1)^(1/3)/x^2 + (x^6 - 1)^(2/3)/x^4 + 1) + 1/54*log
((x^6 - 1)^(1/3)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^9 \left(x^6-1\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(x^6 - 1)^(1/3),x)
```

```
[Out] int(x^9*(x^6 - 1)^(1/3), x)
```

sympy [C] time = 1.20, size = 32, normalized size = 0.29

$$\frac{x^{10}e^{\frac{i\pi}{3}}\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle| x^6\right)}{6\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(x**6-1)**(1/3),x)
```

```
[Out] x**10*exp(I*pi/3)*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), x**6)/(6*gamma(8/3)
)
```

3.1423 $\int x^7 \left(-1+x^6\right)^{2/3} \, dx$

Optimal. Leaf size=112

$$\frac{1}{54}\log\left(\sqrt[3]{x^6-1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{18\sqrt{3}}+\frac{1}{36}\left(x^6-1\right)^{2/3}\left(3x^8-2x^2\right)-\frac{1}{108}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {275, 279, 321, 239}

$$\frac{1}{12}\left(x^6-1\right)^{2/3}x^8-\frac{1}{18}\left(x^6-1\right)^{2/3}x^2+\frac{1}{36}\log\left(x^2-\sqrt[3]{x^6-1}\right)-\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6-1}}+1}{\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(-1 + x^6)^(2/3),x]
```

```
[Out] -1/18*(x^2*(-1 + x^6)^(2/3)) + (x^8*(-1 + x^6)^(2/3))/12 - ArcTan[(1 + (2*x
^2)/(-1 + x^6)^(1/3))/Sqrt[3]]/(18*Sqrt[3]) + Log[x^2 - (-1 + x^6)^(1/3)]/3
6
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^7\left(-1+x^6\right)^{2/3}dx=\frac{1}{2}\text{Subst}\left(\int x^3\left(-1+x^3\right)^{2/3}dx,x,x^2\right)$$
$$=\frac{1}{12}x^8\left(-1+x^6\right)^{2/3}-\frac{1}{6}\text{Subst}\left(\int\frac{x^3}{\sqrt[3]{-1+x^3}}dx,x,x^2\right)$$
$$=-\frac{1}{18}x^2\left(-1+x^6\right)^{2/3}+\frac{1}{12}x^8\left(-1+x^6\right)^{2/3}-\frac{1}{18}\text{Subst}\left(\int\frac{1}{\sqrt[3]{-1+x^3}}dx,x,x^2\right)$$
$$=-\frac{1}{18}x^2\left(-1+x^6\right)^{2/3}+\frac{1}{12}x^8\left(-1+x^6\right)^{2/3}-\frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{-1+x^6}}}{\sqrt{3}}\right)}{18\sqrt{3}}+\frac{1}{36}\log\left(x^2-\sqrt[3]{-1+x^6}\right)$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.48

$$\frac{x^2\left(x^6-1\right)^{2/3}\left({}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};x^6\right)-\left(1-x^6\right)^{5/3}\right)}{12\left(1-x^6\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(-1 + x^6)^(2/3),x]
```

```
[Out] (x^2*(-1 + x^6)^(2/3)*(-(1 - x^6)^(5/3) + Hypergeometric2F1[-2/3, 1/3, 4/3,
 x^6]))/(12*(1 - x^6)^(2/3))
```

IntegrateAlgebraic [A] time = 1.01, size = 112, normalized size = 1.00

$$\frac{1}{54}\log\left(\sqrt[3]{x^6-1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{18\sqrt{3}}+\frac{1}{36}\left(x^6-1\right)^{2/3}\left(3x^8-2x^2\right)-\frac{1}{108}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(-1 + x^6)^(2/3),x]
```

```
[Out] ((-1 + x^6)^(2/3)*(-2*x^2 + 3*x^8))/36 - ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(-1
+ x^6)^(1/3))]/(18*Sqrt[3]) + Log[-x^2 + (-1 + x^6)^(1/3)]/54 - Log[x^4 + x
^2*(-1 + x^6)^(1/3) + (-1 + x^6)^(2/3)]/108
```

fricas [A] time = 0.65, size = 102, normalized size = 0.91

$$\frac{1}{36}\left(3x^8-2x^2\right)\left(x^6-1\right)^{\frac{2}{3}}+\frac{1}{54}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}}{3x^2}\right)+\frac{1}{54}\log\left(-\frac{x^2-\left(x^6-1\right)^{\frac{1}{3}}}{x^2}\right)-\frac{1}{108}\log\left(\frac{x^4+\left(x^6-1\right)^{\frac{1}{3}}x^2+\left(x^6-1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^6-1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/36*(3*x^8 - 2*x^2)*(x^6 - 1)^(2/3) + 1/54*sqrt(3)*arctan(1/3*(sqrt(3)*x^2
 + 2*sqrt(3)*(x^6 - 1)^(1/3))/x^2) + 1/54*log(-(x^2 - (x^6 - 1)^(1/3))/x^2)
 - 1/108*log((x^4 + (x^6 - 1)^(1/3)*x^2 + (x^6 - 1)^(2/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^6-1\right)^{\frac{2}{3}}x^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^6-1)^(2/3),x, algorithm="giac")
[Out] integrate((x^6 - 1)^(2/3)*x^7, x)
```

maple [C] time = 0.32, size = 53, normalized size = 0.47

$$\frac{x^{2}\left(3x^{6}-2\right)\left(x^{6}-1\right)^{\frac{2}{3}}}{36}-\frac{\left(-\mathrm{signum}\left(x^{6}-1\right)\right)^{\frac{1}{3}}x^{2}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^{6}\right)}{18\mathrm{signum}\left(x^{6}-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(x^6-1)^(2/3),x)
[Out] 1/36*x^2*(3*x^6-2)*(x^6-1)^(2/3)-1/18/signum(x^6-1)^(1/3)*(-signum(x^6-1))^
(1/3)*x^2*hypergeom([1/3,1/3],[4/3],x^6)
```

maxima [A] time = 0.42, size = 121, normalized size = 1.08

$$\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^{6}-1\right)^{\frac{1}{3}}}{x^{2}}+1\right)\right)-\frac{\frac{\left(x^{6}-1\right)^{\frac{2}{3}}}{x^{4}}+\frac{2\left(x^{6}-1\right)^{\frac{5}{3}}}{x^{10}}}{36\left(\frac{2\left(x^{6}-1\right)}{x^{6}}-\frac{\left(x^{6}-1\right)^{2}}{x^{12}}-1\right)}-\frac{1}{108}\log\left(\frac{\left(x^{6}-1\right)^{\frac{1}{3}}}{x^{2}}+\frac{\left(x^{6}-1\right)^{\frac{2}{3}}}{x^{4}}+1\right)+\frac{1}{54}\log\left(\frac{\left(x^{6}-1\right)^{\frac{1}{3}}}{x^{2}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^6-1)^(2/3),x, algorithm="maxima")
[Out] 1/54*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3)/x^2 + 1)) - 1/36*((x^6 -
 1)^(2/3)/x^4 + 2*(x^6 - 1)^(5/3)/x^10)/(2*(x^6 - 1)/x^6 - (x^6 - 1)^2/x^12
 - 1) - 1/108*log((x^6 - 1)^(1/3)/x^2 + (x^6 - 1)^(2/3)/x^4 + 1) + 1/54*log
((x^6 - 1)^(1/3)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{7}\left(x^{6}-1\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(x^6 - 1)^(2/3),x)
[Out] int(x^7*(x^6 - 1)^(2/3), x)
```

sympy [C] time = 1.22, size = 34, normalized size = 0.30

$$\frac{x^{8}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|x^{6}\right)}{6\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(x**6-1)**(2/3),x)
[Out] x**8*exp(2*I*pi/3)*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), x**6)/(6*gamma(7/3
))
```

3.1424 $\int \frac{x^{13}}{\sqrt[3]{1+x^6}}\,dx$

Optimal. Leaf size=112

$$-\frac{1}{27}\log\left(\sqrt[3]{x^6+1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{9\sqrt{3}}+\frac{1}{36}\left(x^6+1\right)^{2/3}\left(3x^8-4x^2\right)+\frac{1}{54}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right)$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {275, 321, 239}

$$\frac{1}{12}\left(x^6+1\right)^{2/3}x^8-\frac{1}{9}\left(x^6+1\right)^{2/3}x^2-\frac{1}{18}\log\left(x^2-\sqrt[3]{x^6+1}\right)+\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6+1}}+1}{\sqrt{3}}\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(1 + x^6)^(1/3),x]
```

```
[Out] -1/9*(x^2*(1 + x^6)^(2/3)) + (x^8*(1 + x^6)^(2/3))/12 + ArcTan[(1 + (2*x^2)
/(1 + x^6)^(1/3))/Sqrt[3]]/(9*Sqrt[3]) - Log[x^2 - (1 + x^6)^(1/3)]/18
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13}}{\sqrt[3]{1+x^6}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{\sqrt[3]{1+x^3}}\,dx,x,x^2\right)\\
&= \frac{1}{12}x^8\left(1+x^6\right)^{2/3}-\frac{1}{3}\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{1+x^3}}\,dx,x,x^2\right)\\
&= -\frac{1}{9}x^2\left(1+x^6\right)^{2/3}+\frac{1}{12}x^8\left(1+x^6\right)^{2/3}+\frac{1}{9}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^3}}\,dx,x,x^2\right)\\
&= -\frac{1}{9}x^2\left(1+x^6\right)^{2/3}+\frac{1}{12}x^8\left(1+x^6\right)^{2/3}+\frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{1+x^6}}}{\sqrt{3}}\right)}{9\sqrt{3}}-\frac{1}{18}\log\left(x^2-\sqrt[3]{1+x^6}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 113, normalized size = 1.01

$$\frac{1}{108}\left(9\left(x^6+1\right)^{2/3}x^8-12\left(x^6+1\right)^{2/3}x^2-4\log\left(1-\frac{x^2}{\sqrt[3]{x^6+1}}\right)+4\sqrt{3}\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6+1}}+1}{\sqrt{3}}\right)+2\log\left(\frac{x^4}{\left(x^6+1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6+1}}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(1 + x^6)^(1/3),x]
```

```
[Out] (-12*x^2*(1 + x^6)^(2/3) + 9*x^8*(1 + x^6)^(2/3) + 4*Sqrt[3]*ArcTan[(1 + (2
*x^2)/(1 + x^6)^(1/3))/Sqrt[3]] - 4*Log[1 - x^2/(1 + x^6)^(1/3)] + 2*Log[1
+ x^4/(1 + x^6)^(2/3) + x^2/(1 + x^6)^(1/3)])/108
```

IntegrateAlgebraic [A] time = 2.52, size = 112, normalized size = 1.00

$$-\frac{1}{27}\log\left(\sqrt[3]{x^6+1}-x^2\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{9\sqrt{3}}+\frac{1}{36}\left(x^6+1\right)^{2/3}\left(3x^8-4x^2\right)+\frac{1}{54}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^13/(1 + x^6)^(1/3),x]
```

```
[Out] ((1 + x^6)^(2/3)*(-4*x^2 + 3*x^8))/36 + ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(1 +
x^6)^(1/3))]/(9*Sqrt[3]) - Log[-x^2 + (1 + x^6)^(1/3)]/27 + Log[x^4 + x^2*(
1 + x^6)^(1/3) + (1 + x^6)^(2/3)]/54
```

fricas [A] time = 0.79, size = 102, normalized size = 0.91

$$\frac{1}{36}\left(3x^8-4x^2\right)\left(x^6+1\right)^{\frac{2}{3}}-\frac{1}{27}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6+1\right)^{\frac{1}{3}}}{3x^2}\right)-\frac{1}{27}\log\left(-\frac{x^2-\left(x^6+1\right)^{\frac{1}{3}}}{x^2}\right)+\frac{1}{54}\log\left(\frac{x^4+\left(x^6+1\right)^{\frac{1}{3}}x^2+\left(x^6+1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(x^6+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/36*(3*x^8 - 4*x^2)*(x^6 + 1)^(2/3) - 1/27*sqrt(3)*arctan(1/3*(sqrt(3)*x^2
 + 2*sqrt(3)*(x^6 + 1)^(1/3))/x^2) - 1/27*log(-(x^2 - (x^6 + 1)^(1/3))/x^2)
 + 1/54*log((x^4 + (x^6 + 1)^(1/3)*x^2 + (x^6 + 1)^(2/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{13}}{\left(x^6+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(x^6+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^13/(x^6 + 1)^(1/3), x)
```

maple [C] time = 0.29, size = 37, normalized size = 0.33

$$\frac{x^2\left(3x^6-4\right)\left(x^6+1\right)^{\frac{2}{3}}}{36}+\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x^6\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(x^6+1)^(1/3),x)
```

```
[Out] 1/36*x^2*(3*x^6-4)*(x^6+1)^(2/3)+1/9*x^2*hypergeom([1/3,1/3],[4/3],-x^6)
```

maxima [A] time = 0.42, size = 122, normalized size = 1.09

$$-\frac{1}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)-\frac{\frac{7\left(x^6+1\right)^{\frac{2}{3}}}{x^4}-\frac{4\left(x^6+1\right)^{\frac{5}{3}}}{x^{10}}}{36\left(\frac{2\left(x^6+1\right)}{x^6}-\frac{\left(x^6+1\right)^2}{x^{12}}-1\right)}+\frac{1}{54}\log\left(\frac{\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6+1\right)^{\frac{2}{3}}}{x^4}+1\right)-\frac{1}{27}\log\left(\frac{\left(x^6+1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(x^6+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3)/x^2 + 1)) - 1/36*(7*(x^
6 + 1)^(2/3)/x^4 - 4*(x^6 + 1)^(5/3)/x^10)/(2*(x^6 + 1)/x^6 - (x^6 + 1)^2/x
^12 - 1) + 1/54*log((x^6 + 1)^(1/3)/x^2 + (x^6 + 1)^(2/3)/x^4 + 1) - 1/27*l
og((x^6 + 1)^(1/3)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13}}{\left(x^6+1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(x^6 + 1)^(1/3),x)
```

```
[Out] int(x^13/(x^6 + 1)^(1/3), x)
```

sympy [C] time = 1.31, size = 29, normalized size = 0.26

$$\frac{x^{14}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right| x^6 e^{i\pi}\right)}{6\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(x**6+1)**(1/3),x)
```

```
[Out] x**14*gamma(7/3)*hyper((1/3, 7/3), (10/3,), x**6*exp_polar(I*pi))/(6*gamma(
10/3))
```

3.1425 $\int x^7 \left(1+x^6\right)^{2/3} dx$

Optimal. Leaf size=112

$$\frac{1}{54}\log\left(\sqrt[3]{x^6+1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{18\sqrt{3}}+\frac{1}{36}\left(x^6+1\right)^{2/3}\left(3x^8+2x^2\right)-\frac{1}{108}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x\right.$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {275, 279, 321, 239}

$$\frac{1}{12}\left(x^6+1\right)^{2/3}x^8+\frac{1}{18}\left(x^6+1\right)^{2/3}x^2+\frac{1}{36}\log\left(x^2-\sqrt[3]{x^6+1}\right)-\frac{\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6+1}}+1}{\sqrt{3}}\right)}{18\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(1 + x^6)^(2/3),x]
```

```
[Out] (x^2*(1 + x^6)^(2/3))/18 + (x^8*(1 + x^6)^(2/3))/12 - ArcTan[(1 + (2*x^2)/(
1 + x^6)^(1/3))/Sqrt[3]]/(18*Sqrt[3]) + Log[x^2 - (1 + x^6)^(1/3)]/36
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int x^7\left(1+x^6\right)^{2/3}dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^3\left(1+x^3\right)^{2/3}dx,x,x^2\right)\\
&= \frac{1}{12}x^8\left(1+x^6\right)^{2/3}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{x^3}{\sqrt[3]{1+x^3}}dx,x,x^2\right)\\
&= \frac{1}{18}x^2\left(1+x^6\right)^{2/3}+\frac{1}{12}x^8\left(1+x^6\right)^{2/3}-\frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^3}}dx,x,x^2\right)\\
&= \frac{1}{18}x^2\left(1+x^6\right)^{2/3}+\frac{1}{12}x^8\left(1+x^6\right)^{2/3}-\frac{\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{1+x^6}}}{\sqrt{3}}\right)}{18\sqrt{3}}+\frac{1}{36}\log\left(x^2-\sqrt[3]{1+x^6}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.30

$$\frac{1}{12}x^2\left(\left(x^6+1\right)^{5/3}-{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};-x^6\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(1 + x^6)^(2/3),x]
```

```
[Out] (x^2*((1 + x^6)^(5/3) - Hypergeometric2F1[-2/3, 1/3, 4/3, -x^6]))/12
```

IntegrateAlgebraic [A] time = 1.01, size = 112, normalized size = 1.00

$$\frac{1}{54}\log\left(\sqrt[3]{x^6+1}-x^2\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{18\sqrt{3}}+\frac{1}{36}\left(x^6+1\right)^{2/3}\left(3x^8+2x^2\right)-\frac{1}{108}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(1 + x^6)^(2/3),x]
```

```
[Out] ((1 + x^6)^(2/3)*(2*x^2 + 3*x^8))/36 - ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*(1 + x
^6)^(1/3))]/(18*Sqrt[3]) + Log[-x^2 + (1 + x^6)^(1/3)]/54 - Log[x^4 + x^2*(
1 + x^6)^(1/3) + (1 + x^6)^(2/3)]/108
```

fricas [A] time = 0.42, size = 102, normalized size = 0.91

$$\frac{1}{36}\left(3x^8+2x^2\right)\left(x^6+1\right)^{\frac{2}{3}}+\frac{1}{54}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6+1\right)^{\frac{1}{3}}}{3x^2}\right)+\frac{1}{54}\log\left(-\frac{x^2-\left(x^6+1\right)^{\frac{1}{3}}}{x^2}\right)-\frac{1}{108}\log\left(\frac{x^4+\left(x^6+1\right)^{\frac{1}{3}}x^2+\left(x^6+1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^6+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/36*(3*x^8 + 2*x^2)*(x^6 + 1)^(2/3) + 1/54*sqrt(3)*arctan(1/3*(sqrt(3)*x^2
 + 2*sqrt(3)*(x^6 + 1)^(1/3))/x^2) + 1/54*log(-(x^2 - (x^6 + 1)^(1/3))/x^2)
 - 1/108*log((x^4 + (x^6 + 1)^(1/3)*x^2 + (x^6 + 1)^(2/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^6+1\right)^{\frac{2}{3}}x^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^6+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 1)^(2/3)*x^7, x)
```

maple [C] time = 0.30, size = 37, normalized size = 0.33

$$\frac{x^{2} \left(3 x^{6}+2\right) \left(x^{6}+1\right)^{\frac{2}{3}}}{36}-\frac{x^{2} \mathrm{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{3}\right], \left[\frac{4}{3}\right], -x^{6}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(x^6+1)^(2/3),x)
[Out] 1/36*x^2*(3*x^6+2)*(x^6+1)^(2/3)-1/18*x^2*hypergeom([1/3,1/3],[4/3],-x^6)
```

maxima [A] time = 0.41, size = 121, normalized size = 1.08

$$\frac{1}{54} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(\frac{2 \left(x^{6}+1\right)^{\frac{1}{3}}}{x^{2}}+1\right)\right)-\frac{\frac{\left(x^{6}+1\right)^{\frac{2}{3}}}{x^{4}}+\frac{2 \left(x^{6}+1\right)^{\frac{5}{3}}}{x^{10}}}{36 \left(\frac{2 \left(x^{6}+1\right)}{x^{6}}-\frac{\left(x^{6}+1\right)^{2}}{x^{12}}-1\right)}-\frac{1}{108} \log\left(\frac{\left(x^{6}+1\right)^{\frac{1}{3}}}{x^{2}}+\frac{\left(x^{6}+1\right)^{\frac{2}{3}}}{x^{4}}+1\right)+\frac{1}{54} \log\left(\frac{\left(x^{6}+1\right)^{\frac{1}{3}}}{x^{2}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^6+1)^(2/3),x, algorithm="maxima")
[Out] 1/54*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3)/x^2 + 1)) - 1/36*((x^6 +
 1)^(2/3)/x^4 + 2*(x^6 + 1)^(5/3)/x^10)/(2*(x^6 + 1)/x^6 - (x^6 + 1)^2/x^12
 - 1) - 1/108*log((x^6 + 1)^(1/3)/x^2 + (x^6 + 1)^(2/3)/x^4 + 1) + 1/54*log
((x^6 + 1)^(1/3)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{7} \left(x^{6}+1\right)^{2/3} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(x^6 + 1)^(2/3),x)
[Out] int(x^7*(x^6 + 1)^(2/3), x)
```

sympy [C] time = 1.17, size = 31, normalized size = 0.28

$$\frac{x^{8} \Gamma\left(\frac{4}{3}\right) {}_{2}F_{1}\left(\left.\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\right| x^{6} e^{i\pi}\right)}{6 \Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(x**6+1)**(2/3),x)
[Out] x**8*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), x**6*exp_polar(I*pi))/(6*gamma(7
/3))
```

3.1426 $\int \frac{\sqrt[3]{-1+2x^3+x^8}\left(3+5x^8\right)}{x^2\left(-1+x^3+x^8\right)} dx$

Optimal. Leaf size=112

$$\frac{3\sqrt[3]{x^8+2x^3-1}}{x}+\log\left(\sqrt[3]{x^8+2x^3-1}-x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^8+2x^3-1}+x}\right)-\frac{1}{2}\log\left(x^2+\sqrt[3]{x^8+2x^3-1}\,x+\left(x^8\right.\right.$$

Rubi [F] time = 0.99, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{-1+2x^3+x^8}\left(3+5x^8\right)}{x^2\left(-1+x^3+x^8\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + 2*x^3 + x^8)^(1/3)*(3 + 5*x^8))/(x^2*(-1 + x^3 + x^8)),x]
```

```
[Out] -3*Defer[Int][(-1 + 2*x^3 + x^8)^(1/3)/x^2, x] + 3*Defer[Int][(x*(-1 + 2*x^
3 + x^8)^(1/3))/(-1 + x^3 + x^8), x] + 8*Defer[Int][(x^6*(-1 + 2*x^3 + x^8)
^(1/3))/(-1 + x^3 + x^8), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{-1+2x^3+x^8}\left(3+5x^8\right)}{x^2\left(-1+x^3+x^8\right)} dx &= \int \left(-\frac{3\sqrt[3]{-1+2x^3+x^8}}{x^2}+\frac{x\left(3+8x^5\right)\sqrt[3]{-1+2x^3+x^8}}{-1+x^3+x^8}\right) dx \\
&= -\left(3\int \frac{\sqrt[3]{-1+2x^3+x^8}}{x^2}\,dx\right)+\int \frac{x\left(3+8x^5\right)\sqrt[3]{-1+2x^3+x^8}}{-1+x^3+x^8}\,dx \\
&= -\left(3\int \frac{\sqrt[3]{-1+2x^3+x^8}}{x^2}\,dx\right)+\int \left(\frac{3x\sqrt[3]{-1+2x^3+x^8}}{-1+x^3+x^8}+\frac{8x^6\sqrt[3]{-1+2x^3+x^8}}{-1+x^3+x^8}\right. \\
&= -\left(3\int \frac{\sqrt[3]{-1+2x^3+x^8}}{x^2}\,dx\right)+3\int \frac{x\sqrt[3]{-1+2x^3+x^8}}{-1+x^3+x^8}\,dx+8\int \frac{x^6\sqrt[3]{-1+2x^3}}{-1+x^3+}
\end{aligned}$$

Mathematica [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-1+2x^3+x^8}\left(3+5x^8\right)}{x^2\left(-1+x^3+x^8\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + 2*x^3 + x^8)^(1/3)*(3 + 5*x^8))/(x^2*(-1 + x^3 + x^8)),x]
```

```
[Out] Integrate[((-1 + 2*x^3 + x^8)^(1/3)*(3 + 5*x^8))/(x^2*(-1 + x^3 + x^8)), x]
```

IntegrateAlgebraic [A] time = 2.74, size = 112, normalized size = 1.00

$$\frac{3\sqrt[3]{x^8+2x^3-1}}{x}+\log\left(\sqrt[3]{x^8+2x^3-1}-x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^8+2x^3-1}+x}\right)-\frac{1}{2}\log\left(x^2+\sqrt[3]{x^8+2x^3-1}\,x+\left(x^8+2x^3-1\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + 2*x^3 + x^8)^(1/3)*(3 + 5*x^8))/(x^2*(-1 + x^3 + 
x^8)),x]
```

[Out] (3*(-1 + 2*x^3 + x^8)^(1/3))/x + Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + 2*
x^3 + x^8)^(1/3))] + Log[-x + (-1 + 2*x^3 + x^8)^(1/3)] - Log[x^2 + x*(-1 +
 2*x^3 + x^8)^(1/3) + (-1 + 2*x^3 + x^8)^(2/3)]/2

fricas [A] time = 17.21, size = 152, normalized size = 1.36

$$\frac{2\sqrt{3}\,x\arctan\left(\frac{23155756059884469826063290091369873601204942180224\,\sqrt{3}\left(x^{8}+2x^{3}-1\right)^{\frac{1}{3}}x^{2}+61059012875773331838678659685174425801373874951458\,\sqrt{3}\left(x^{8}+2x^{3}-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(35248398304721470575821713544519821387080907584081\,x^{8}+77355782772550371408192688432791971088370316149922\,x^{3}-35248398304721470575821713544519821387080907584081\right)}{3\left(20044909029062956675424368815298850195325332161233\,x^{8}+38996537437007387681732053612201126295409798546850\,x^{3}-20044909029062956675424368815298850195325332161233\right)}\right)+x\log\left(\frac{x^{8}+x^{3}+3\left(x^{8}+2x^{3}-1\right)^{\frac{1}{3}}x^{2}-3\left(x^{8}+2x^{3}-1\right)^{\frac{2}{3}}x-1}{x^{8}+x^{3}-1}\right)+6\left(x^{8}+2x^{3}-1\right)^{\frac{1}{3}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^8+2*x^3-1)^(1/3)*(5*x^8+3)/x^2/(x^8+x^3-1),x, algorithm="frica
s")

[Out] 1/2*(2*sqrt(3)*x*arctan(1/3*(2315575605988446982606329009136987360120494218
0224*sqrt(3)*(x^8 + 2*x^3 - 1)^(1/3)*x^2 + 61059012875773331838678659685174
425801373874951458*sqrt(3)*(x^8 + 2*x^3 - 1)^(2/3)*x + sqrt(3)*(35248398304
721470575821713544519821387080907584081*x^8 + 77355782772550371408192688432
791971088370316149922*x^3 - 35248398304721470575821713544519821387080907584
081))/(20044909029062956675424368815298850195325332161233*x^8 + 38996537437
007387681732053612201126295409798546850*x^3 - 20044909029062956675424368815
298850195325332161233)) + x*log((x^8 + x^3 + 3*(x^8 + 2*x^3 - 1)^(1/3)*x^2
- 3*(x^8 + 2*x^3 - 1)^(2/3)*x - 1)/(x^8 + x^3 - 1)) + 6*(x^8 + 2*x^3 - 1)^(
1/3))/x

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^{8}+3\right)\left(x^{8}+2\,x^{3}-1\right)^{\frac{1}{3}}}{\left(x^{8}+x^{3}-1\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^8+2*x^3-1)^(1/3)*(5*x^8+3)/x^2/(x^8+x^3-1),x, algorithm="giac"
)

[Out] integrate((5*x^8 + 3)*(x^8 + 2*x^3 - 1)^(1/3)/((x^8 + x^3 - 1)*x^2), x)

maple [C] time = 6.58, size = 1136, normalized size = 10.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8+2*x^3-1)^(1/3)*(5*x^8+3)/x^2/(x^8+x^3-1),x)

[Out] 3*(x^8+2*x^3-1)^(1/3)/x+(RootOf(_Z^2+_Z+1)*ln((-2*x^16*RootOf(_Z^2+_Z+1)-x^
16+2*RootOf(_Z^2+_Z+1)^2*x^11-7*x^11*RootOf(_Z^2+_Z+1)-3*(x^16+4*x^11-2*x^8
+4*x^6-4*x^3+1)^(1/3)*RootOf(_Z^2+_Z+1)*x^9-4*x^11-3*(x^16+4*x^11-2*x^8+4*x
^6-4*x^3+1)^(1/3)*x^9+4*RootOf(_Z^2+_Z+1)*x^8+4*RootOf(_Z^2+_Z+1)^2*x^6+2*x
^8-6*RootOf(_Z^2+_Z+1)*x^6-6*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(1/3)*RootOf
(_Z^2+_Z+1)*x^4-4*x^6-2*RootOf(_Z^2+_Z+1)^2*x^3-3*(x^16+4*x^11-2*x^8+4*x^6-
4*x^3+1)^(2/3)*RootOf(_Z^2+_Z+1)*x^2-6*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(1
/3)*x^4+7*RootOf(_Z^2+_Z+1)*x^3-3*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(2/3)*x
^2+3*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(1/3)*RootOf(_Z^2+_Z+1)*x+4*x^3+3*(x
^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(1/3)*x-2*RootOf(_Z^2+_Z+1)-1)/(x^8+2*x^3-1
)/(x^8+x^3-1))-ln((2*x^16*RootOf(_Z^2+_Z+1)+x^16+2*RootOf(_Z^2+_Z+1)^2*x^11
+11*x^11*RootOf(_Z^2+_Z+1)+3*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(1/3)*RootOf
(_Z^2+_Z+1)*x^9+5*x^11-4*RootOf(_Z^2+_Z+1)*x^8+4*RootOf(_Z^2+_Z+1)^2*x^6-2*
x^8+14*RootOf(_Z^2+_Z+1)*x^6+6*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(1/3)*Root
Of(_Z^2+_Z+1)*x^4+6*x^6-2*RootOf(_Z^2+_Z+1)^2*x^3+3*(x^16+4*x^11-2*x^8+4*x^
6-4*x^3+1)^(2/3)*RootOf(_Z^2+_Z+1)*x^2-11*RootOf(_Z^2+_Z+1)*x^3-3*(x^16+4*x
^11-2*x^8+4*x^6-4*x^3+1)^(1/3)*RootOf(_Z^2+_Z+1)*x-5*x^3+2*RootOf(_Z^2+_Z+1
)+1)/(x^8+2*x^3-1)/(x^8+x^3-1))*RootOf(_Z^2+_Z+1)-ln((2*x^16*RootOf(_Z^2+_Z

```
+1)+x^16+2*RootOf(_Z^2+_Z+1)^2*x^11+11*x^11*RootOf(_Z^2+_Z+1)+3*(x^16+4*x^1
1-2*x^8+4*x^6-4*x^3+1)^(1/3)*RootOf(_Z^2+_Z+1)*x^9+5*x^11-4*RootOf(_Z^2+_Z+
1)*x^8+4*RootOf(_Z^2+_Z+1)^2*x^6-2*x^8+14*RootOf(_Z^2+_Z+1)*x^6+6*(x^16+4*x
^11-2*x^8+4*x^6-4*x^3+1)^(1/3)*RootOf(_Z^2+_Z+1)*x^4+6*x^6-2*RootOf(_Z^2+_Z
+1)^2*x^3+3*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(2/3)*RootOf(_Z^2+_Z+1)*x^2-1
1*RootOf(_Z^2+_Z+1)*x^3-3*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(1/3)*RootOf(_Z
^2+_Z+1)*x-5*x^3+2*RootOf(_Z^2+_Z+1)+1)/(x^8+2*x^3-1)/(x^8+x^3-1)))/(x^8+2*
x^3-1)^(2/3)*((x^8+2*x^3-1)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^8+3\right)\left(x^8+2\,x^3-1\right)^{\frac{1}{3}}}{\left(x^8+x^3-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+2*x^3-1)^(1/3)*(5*x^8+3)/x^2/(x^8+x^3-1),x, algorithm="maxim
a")
```

```
[Out] integrate((5*x^8 + 3)*(x^8 + 2*x^3 - 1)^(1/3)/((x^8 + x^3 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^8+3\right)\,\left(x^8+2\,x^3-1\right)^{1/3}}{x^2\,\left(x^8+x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x^8 + 3)*(2*x^3 + x^8 - 1)^(1/3))/(x^2*(x^3 + x^8 - 1)),x)
```

```
[Out] int(((5*x^8 + 3)*(2*x^3 + x^8 - 1)^(1/3))/(x^2*(x^3 + x^8 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^8+3\right)\sqrt[3]{x^8+2x^3-1}}{x^2\left(x^8+x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8+2*x**3-1)**(1/3)*(5*x**8+3)/x**2/(x**8+x**3-1),x)
```

```
[Out] Integral((5*x**8 + 3)*(x**8 + 2*x**3 - 1)**(1/3)/(x**2*(x**8 + x**3 - 1)),
x)
```

3.1427 $\int \frac{\sqrt{b^2+a^2x^2}}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$

Optimal. Leaf size=112

$$\frac{4\sqrt{a^2x^2+b^2}\,\left(a^2x^3-b^2x\right)}{3\left(\sqrt{a^2x^2+b^2}+ax\right)^{5/2}}+\frac{2\left(10a^4x^4-5a^2b^2x^2-7b^4\right)}{15a\left(\sqrt{a^2x^2+b^2}+ax\right)^{5/2}}$$

Rubi [A] time = 0.13, antiderivative size = 95, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {2122, 270}

$$-\frac{b^2}{a\sqrt{\sqrt{a^2x^2+b^2}+ax}}+\frac{\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}{6a}-\frac{b^4}{10a\left(\sqrt{a^2x^2+b^2}+ax\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b^2 + a^2*x^2]/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] -1/10*b^4/(a*(a*x + Sqrt[b^2 + a^2*x^2])^(5/2)) - b^2/(a*Sqrt[a*x + Sqrt[b^
2 + a^2*x^2]]) + (a*x + Sqrt[b^2 + a^2*x^2])^(3/2)/(6*a)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b^2+a^2x^2}}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(b^2+x^2\right)^2}{x^{7/2}}\,dx,x,ax+\sqrt{b^2+a^2x^2}\right)}{4a}\\
&= \frac{\text{Subst}\left(\int \left(\frac{b^4}{x^{7/2}}+\frac{2b^2}{x^{3/2}}+\sqrt{x}\right)dx,x,ax+\sqrt{b^2+a^2x^2}\right)}{4a}\\
&= -\frac{b^4}{10a\left(ax+\sqrt{b^2+a^2x^2}\right)^{5/2}}-\frac{b^2}{a\sqrt{ax+\sqrt{b^2+a^2x^2}}}+\frac{\left(ax+\sqrt{b^2+a^2x^2}\right)^{3/2}}{6a}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 93, normalized size = 0.83

$$\frac{-\frac{4b^2}{\sqrt{\sqrt{a^2x^2+b^2}+ax}}+\frac{2}{3}\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}-\frac{2b^4}{5\left(\sqrt{a^2x^2+b^2}+ax\right)^{5/2}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b^2 + a^2*x^2]/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] ((-2*b^4)/(5*(a*x + Sqrt[b^2 + a^2*x^2])^(5/2)) - (4*b^2)/Sqrt[a*x + Sqrt[b
^2 + a^2*x^2]] + (2*(a*x + Sqrt[b^2 + a^2*x^2])^(3/2))/3)/(4*a)
```

IntegrateAlgebraic [A] time = 0.16, size = 112, normalized size = 1.00

$$\frac{4\sqrt{a^2x^2+b^2}\left(a^2x^3-b^2x\right)}{3\left(\sqrt{a^2x^2+b^2}+ax\right)^{5/2}}+\frac{2\left(10a^4x^4-5a^2b^2x^2-7b^4\right)}{15a\left(\sqrt{a^2x^2+b^2}+ax\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b^2 + a^2*x^2]/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] (4*Sqrt[b^2 + a^2*x^2]*(-(b^2*x) + a^2*x^3))/(3*(a*x + Sqrt[b^2 + a^2*x^2])
^(5/2)) + (2*(-7*b^4 - 5*a^2*b^2*x^2 + 10*a^4*x^4))/(15*a*(a*x + Sqrt[b^2 +
 a^2*x^2])^(5/2))
```

fricas [A] time = 0.41, size = 72, normalized size = 0.64

$$\frac{2\left(3\,a^3x^3+11\,ab^2x-\left(3\,a^2x^2+7\,b^2\right)\sqrt{a^2x^2+b^2}\right)\sqrt{ax+\sqrt{a^2x^2+b^2}}}{15\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b^2)^(1/2)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm=
"fricas")
```

```
[Out] 2/15*(3*a^3*x^3 + 11*a*b^2*x - (3*a^2*x^2 + 7*b^2)*sqrt(a^2*x^2 + b^2))*sqr
t(a*x + sqrt(a^2*x^2 + b^2))/(a*b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2+b^2}}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b^2)^(1/2)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(a^2*x^2 + b^2)/sqrt(a*x + sqrt(a^2*x^2 + b^2)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2+b^2}}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2+b^2)^(1/2)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a^2*x^2+b^2)^(1/2)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2+b^2}}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b^2)^(1/2)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 + b^2)/sqrt(a*x + sqrt(a^2*x^2 + b^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a^2\,x^2+b^2}}{\sqrt{a\,x+\sqrt{a^2\,x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2 + a^2*x^2)^(1/2)/(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int((b^2 + a^2*x^2)^(1/2)/(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2+b^2}}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2+b**2)**(1/2)/(a*x+(a**2*x**2+b**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a**2*x**2 + b**2)/sqrt(a*x + sqrt(a**2*x**2 + b**2)), x)
```

3.1428 $\int \frac{-2a+b+x}{\sqrt[4]{(-a+x)(-b+x)^2}\left(b^2+ad-(2b+d)x+x^2\right)}\,dx$

Optimal. Leaf size=113

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x(2ab+b^2)-ab^2+x^2(-a-2b)+x^3}}{b-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x(2ab+b^2)-ab^2+x^2(-a-2b)+x^3}}{b-x}\right)}{d^{3/4}}$$

Rubi [C] time = 65.52, antiderivative size = 3261, normalized size of antiderivative = 28.86, number of steps used = 21, number of rules used = 8, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6719, 6728, 107, 106, 490, 1217, 220, 1707}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(-2*a + b + x)/(((-a + x)*(-b + x)^2)^(1/4)*(b^2 + a*d - (2*b + d)*x +
x^2)),x]
```

```
[Out] (Sqrt[a - b]*Sqrt[Sqrt[d] - Sqrt[-4*a + 4*b + d]]*Sqrt[-((b - x)/(a - b))]*
(-a + x)^(1/4)*ArcTan[(d^(1/4)*Sqrt[Sqrt[d] - Sqrt[-4*a + 4*b + d]]*(-a + x
)^(1/4))/(2^(1/4)*Sqrt[a - b]*(-2*a + 2*b + d - Sqrt[d]*Sqrt[-4*a + 4*b + d
])^(1/4)*Sqrt[-((b - x)/(a - b))])])/(2^(1/4)*d^(3/4)*(-2*a + 2*b + d - Sqr
t[d]*Sqrt[-4*a + 4*b + d])^(1/4)*(-((a - x)*(b - x)^2))^(1/4)) + (Sqrt[a -
b]*Sqrt[-Sqrt[d] + Sqrt[-4*a + 4*b + d]]*Sqrt[-((b - x)/(a - b))]*(-a + x)^
(1/4)*ArcTan[(d^(1/4)*Sqrt[-Sqrt[d] + Sqrt[-4*a + 4*b + d]]*(-a + x)^(1/4))
/(2^(1/4)*Sqrt[a - b]*(-2*a + 2*b + d - Sqrt[d]*Sqrt[-4*a + 4*b + d])^(1/4)
*Sqrt[-((b - x)/(a - b))])])/(2^(1/4)*d^(3/4)*(-2*a + 2*b + d - Sqrt[d]*Sqr
t[-4*a + 4*b + d])^(1/4)*(-((a - x)*(b - x)^2))^(1/4)) + (Sqrt[a - b]*Sqrt[
Sqrt[d] + Sqrt[-4*a + 4*b + d]]*Sqrt[-((b - x)/(a - b))]*(-a + x)^(1/4)*Arc
Tan[(d^(1/4)*Sqrt[Sqrt[d] + Sqrt[-4*a + 4*b + d]]*(-a + x)^(1/4))/(2^(1/4)*
Sqrt[a - b]*(-2*a + 2*b + d + Sqrt[d]*Sqrt[-4*a + 4*b + d])^(1/4)*Sqrt[-((b
 - x)/(a - b))])])/(2^(1/4)*d^(3/4)*(-2*a + 2*b + d + Sqrt[d]*Sqrt[-4*a + 4
*b + d])^(1/4)*(-((a - x)*(b - x)^2))^(1/4)) - (Sqrt[-a + b]*Sqrt[Sqrt[d] +
 Sqrt[-4*a + 4*b + d]]*Sqrt[-((b - x)/(a - b))]*(-a + x)^(1/4)*ArcTan[(d^(1
/4)*Sqrt[Sqrt[d] + Sqrt[-4*a + 4*b + d]]*(-a + x)^(1/4))/(2^(1/4)*Sqrt[-a +
 b]*(-2*a + 2*b + d + Sqrt[d]*Sqrt[-4*a + 4*b + d])^(1/4)*Sqrt[-((b - x)/(a
 - b))])])/(2^(1/4)*d^(3/4)*(-2*a + 2*b + d + Sqrt[d]*Sqrt[-4*a + 4*b + d])
^(1/4)*(-((a - x)*(b - x)^2))^(1/4)) - ((1 - Sqrt[-4*a + 4*b + d]/Sqrt[d])*
(2*a - 2*b - Sqrt[2]*Sqrt[a - b]*Sqrt[-2*a + 2*b + d - Sqrt[d]*Sqrt[-4*a +
4*b + d]])*(-a + x)^(1/4)*Sqrt[-((b - x)/((a - b)*(1 + Sqrt[-a + x]/Sqrt[a
- b])^2))]*(1 + Sqrt[-a + x]/Sqrt[a - b])*EllipticF[2*ArcTan[(-a + x)^(1/4)
/(a - b)^(1/4)], 1/2])/(2*(a - b)^(1/4)*(4*a - 4*b - d + Sqrt[d]*Sqrt[-4*a
+ 4*b + d])*(-((a - x)*(b - x)^2))^(1/4)) - ((1 - Sqrt[-4*a + 4*b + d]/Sqrt
[d])*(2*a - 2*b + Sqrt[2]*Sqrt[a - b]*Sqrt[-2*a + 2*b + d - Sqrt[d]*Sqrt[-4
*a + 4*b + d]])*(-a + x)^(1/4)*Sqrt[-((b - x)/((a - b)*(1 + Sqrt[-a + x]/Sq
rt[a - b])^2))]*(1 + Sqrt[-a + x]/Sqrt[a - b])*EllipticF[2*ArcTan[(-a + x)^
(1/4)/(a - b)^(1/4)], 1/2])/(2*(a - b)^(1/4)*(4*a - 4*b - d + Sqrt[d]*Sqrt[
-4*a + 4*b + d])*(-((a - x)*(b - x)^2))^(1/4)) - ((1 + Sqrt[-4*a + 4*b + d]
/Sqrt[d])*(2*a - 2*b - Sqrt[2]*Sqrt[a - b]*Sqrt[-2*a + 2*b + d + Sqrt[d]*Sq
rt[-4*a + 4*b + d]])*(-a + x)^(1/4)*Sqrt[-((b - x)/((a - b)*(1 + Sqrt[-a +
x]/Sqrt[a - b])^2))]*(1 + Sqrt[-a + x]/Sqrt[a - b])*EllipticF[2*ArcTan[(-a
+ x)^(1/4)/(a - b)^(1/4)], 1/2])/(2*(a - b)^(1/4)*(4*a - 4*b - d - Sqrt[d]*
Sqrt[-4*a + 4*b + d])*(-((a - x)*(b - x)^2))^(1/4)) - ((1 + Sqrt[-4*a + 4*b
 + d]/Sqrt[d])*(2*a - 2*b + Sqrt[2]*Sqrt[a - b]*Sqrt[-2*a + 2*b + d + Sqrt[
d]*Sqrt[-4*a + 4*b + d]])*(-a + x)^(1/4)*Sqrt[-((b - x)/((a - b)*(1 + Sqrt[
-a + x]/Sqrt[a - b])^2))]*(1 + Sqrt[-a + x]/Sqrt[a - b])*EllipticF[2*ArcTan
[(-a + x)^(1/4)/(a - b)^(1/4)], 1/2])/(2*(a - b)^(1/4)*(4*a - 4*b - d - Sqr
t[d]*Sqrt[-4*a + 4*b + d])*(-((a - x)*(b - x)^2))^(1/4)) + ((a - b)^(1/4)*(
```

```
1 - Sqrt[-4*a + 4*b + d]/Sqrt[d])*(Sqrt[2]*Sqrt[a - b] + Sqrt[-2*a + 2*b +
d - Sqrt[d]*Sqrt[-4*a + 4*b + d]])^2*(-a + x)^(1/4)*Sqrt[-((b - x)/((a - b)
*(1 + Sqrt[-a + x]/Sqrt[a - b])^2))]*(1 + Sqrt[-a + x]/Sqrt[a - b])*Ellipti
cPi[-1/4*(Sqrt[2]*Sqrt[a - b] - Sqrt[-2*a + 2*b + d - Sqrt[d]*Sqrt[-4*a + 4
*b + d]])^2/(Sqrt[2]*Sqrt[a - b]*Sqrt[-2*a + 2*b + d - Sqrt[d]*Sqrt[-4*a +
4*b + d]]), 2*ArcTan[(-a + x)^(1/4)/(a - b)^(1/4)], 1/2])/(2*Sqrt[2]*Sqrt[-
2*a + 2*b + d - Sqrt[d]*Sqrt[-4*a + 4*b + d]]*(4*a - 4*b - d + Sqrt[d]*Sqrt
[-4*a + 4*b + d])*(-((a - x)*(b - x)^2))^(1/4)) - ((a - b)^(1/4)*(1 - Sqrt[
-4*a + 4*b + d]/Sqrt[d])*(Sqrt[2]*Sqrt[a - b] - Sqrt[-2*a + 2*b + d - Sqrt[
d]*Sqrt[-4*a + 4*b + d]])^2*(-a + x)^(1/4)*Sqrt[-((b - x)/((a - b)*(1 + Sqr
t[-a + x]/Sqrt[a - b])^2))]*(1 + Sqrt[-a + x]/Sqrt[a - b])*EllipticPi[(Sqrt
[2]*Sqrt[a - b] + Sqrt[-2*a + 2*b + d - Sqrt[d]*Sqrt[-4*a + 4*b + d]])^2/(4
*Sqrt[2]*Sqrt[a - b]*Sqrt[-2*a + 2*b + d - Sqrt[d]*Sqrt[-4*a + 4*b + d]]),
2*ArcTan[(-a + x)^(1/4)/(a - b)^(1/4)], 1/2])/(2*Sqrt[2]*Sqrt[-2*a + 2*b +
d - Sqrt[d]*Sqrt[-4*a + 4*b + d]]*(4*a - 4*b - d + Sqrt[d]*Sqrt[-4*a + 4*b
+ d])*(-((a - x)*(b - x)^2))^(1/4)) + ((a - b)^(1/4)*(1 + Sqrt[-4*a + 4*b +
 d]/Sqrt[d])*(Sqrt[2]*Sqrt[a - b] + Sqrt[-2*a + 2*b + d + Sqrt[d]*Sqrt[-4*a
 + 4*b + d]])^2*(-a + x)^(1/4)*Sqrt[-((b - x)/((a - b)*(1 + Sqrt[-a + x]/Sq
rt[a - b])^2))]*(1 + Sqrt[-a + x]/Sqrt[a - b])*EllipticPi[-1/4*(Sqrt[2]*Sqr
t[a - b] - Sqrt[-2*a + 2*b + d + Sqrt[d]*Sqrt[-4*a + 4*b + d]])^2/(Sqrt[2]*
Sqrt[a - b]*Sqrt[-2*a + 2*b + d + Sqrt[d]*Sqrt[-4*a + 4*b + d]]), 2*ArcTan[
(-a + x)^(1/4)/(a - b)^(1/4)], 1/2])/(2*Sqrt[2]*(4*a - 4*b - d - Sqrt[d]*Sq
rt[-4*a + 4*b + d])*Sqrt[-2*a + 2*b + d + Sqrt[d]*Sqrt[-4*a + 4*b + d]]*(-(
(a - x)*(b - x)^2))^(1/4)) - ((a - b)^(1/4)*(Sqrt[d] + Sqrt[-4*a + 4*b + d]
)*(Sqrt[2]*Sqrt[a - b] - Sqrt[-2*a + 2*b + d + Sqrt[d]*Sqrt[-4*a + 4*b + d]
])^2*(-a + x)^(1/4)*Sqrt[-((b - x)/((a - b)*(1 + Sqrt[-a + x]/Sqrt[a - b])^
2))]*(1 + Sqrt[-a + x]/Sqrt[a - b])*EllipticPi[(Sqrt[2]*Sqrt[a - b] + Sqrt[
-2*a + 2*b + d + Sqrt[d]*Sqrt[-4*a + 4*b + d]])^2/(4*Sqrt[2]*Sqrt[a - b]*Sq
rt[-2*a + 2*b + d + Sqrt[d]*Sqrt[-4*a + 4*b + d]]), 2*ArcTan[(-a + x)^(1/4)
/(a - b)^(1/4)], 1/2])/(2*Sqrt[2]*Sqrt[d]*(4*a - 4*b - d - Sqrt[d]*Sqrt[-4*
a + 4*b + d])*Sqrt[-2*a + 2*b + d + Sqrt[d]*Sqrt[-4*a + 4*b + d]]*(-((a - x
)*(b - x)^2))^(1/4))
```

Rule 106

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
1/4)), x_Symbol] :> Dist[-4, Subst[Int[x^2/((b*e - a*f - b*x^4)*Sqrt[c - (d
*e)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f
}, x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 107

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
1/4)), x_Symbol] :> Dist[Sqrt[-((f*(c + d*x))/(d*e - c*f))]/Sqrt[c + d*x],
Int[1/((a + b*x)*Sqrt[-((c*f)/(d*e - c*f)) - (d*f*x)/(d*e - c*f)]*(e + f*x)
^(1/4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[-(f/(d*e - c*f)),
0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
```

```
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-2a+b+x}{\sqrt[4]{(-a+x)(-b+x)^2}\left(b^2+ad-(2b+d)x+x^2\right)}\,dx = \frac{\left(\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\int \frac{-2a+b+x}{\sqrt[4]{-a+x}\,\sqrt{-b+x}\left(b^2+ad-(2b+d)x+x^2\right)}\,dx}{\sqrt[4]{(-a+x)(-b+x)^2}}$$

$$= \frac{\left(\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\int\left(\frac{1+\frac{\sqrt{-4a+4b+d}}{\sqrt{d}}}{\sqrt[4]{-a+x}\,\sqrt{-b+x}\left(-2b-d-\sqrt{d}\,\sqrt{-4a+4b+\cdots}\right.}\cdots\right.}{\sqrt[4]{(-a+x)(-\cdots}}$$

$$= \frac{\left(\left(1-\frac{\sqrt{-4a+4b+d}}{\sqrt{d}}\right)\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\int \frac{\cdots}{\sqrt[4]{-a+x}\,\sqrt{-b+x}\left(-\cdots\right.}}{\sqrt[4]{(-a+x)(-b+x)^2}}$$

$$= \frac{\left(\left(1-\frac{\sqrt{-4a+4b+d}}{\sqrt{d}}\right)\sqrt[4]{-a+x}\,\sqrt{-\frac{-b+x}{-a+b}}\right)\int \frac{\cdots}{\sqrt[4]{-a+x}\left(-2b-d+\cdots\right.}}{\sqrt[4]{(-a+x)(-b+x)^2}}$$

$$= -\frac{\left(4\left(1-\frac{\sqrt{-4a+4b+d}}{\sqrt{d}}\right)\sqrt[4]{-a+x}\,\sqrt{-\frac{-b+x}{-a+b}}\right)\text{Subst}\left(\int \frac{\cdots}{(-2a\cdots}\right.}{\sqrt[4]{(-a+\cdots}}$$

$$= -\frac{\left(\sqrt{2}\left(1-\frac{\sqrt{-4a+4b+d}}{\sqrt{d}}\right)\sqrt[4]{-a+x}\,\sqrt{-\frac{-b+x}{-a+b}}\right)\text{Subst}\left(\int \cdots\right.}{\sqrt[4]{(-\cdots}}$$

$$= -\frac{\left(\sqrt{2}\left(1-\frac{\sqrt{-4a+4b+d}}{\sqrt{d}}\right)\left(\sqrt{2}\,a-\sqrt{2}\,b-\sqrt{a-b}\,\sqrt{-2a\cdots}\right.\right.}{\sqrt{a-b}\,(4\cdots}$$

$$= \frac{\sqrt{\sqrt{d}-\sqrt{-4a+4b+d}}\left(\sqrt{2}\,\sqrt{a-b}-\sqrt{-2a+2b+\cdots}\right.}{2^{3/4}\cdots}$$

Mathematica [C] time = 25.69, size = 675, normalized size = 5.97

$$-\frac{i\sqrt{2}(x-a)^{3/4}\sqrt{\frac{b-x}{a-x}}\left(\left(\sqrt{d(-4a+4b+d)}+4a-4b-d\right)\sqrt{\frac{\sqrt{d(-4a+4b+d)}-2a+2b+d}{(a-b)^2}}\,\Pi\left(-\frac{\sqrt{2}}{\sqrt{b-a}\sqrt{\frac{-2a+2b+d-\sqrt{d(-4a+4b+d)}}{(a-b)^2}}};i\sinh^{-1}\left(\frac{\sqrt{-\sqrt{b-a}}}{\sqrt[4]{x-a}}\right)\middle|-1\right)-\left(\sqrt{d(-4a+4b+d)}+4a-4b-d\right)\sqrt{\frac{\sqrt{d(-4a+4b+d)}-2a+2b+d}{(a-b)^2}}\,\Pi\left(\frac{\sqrt{2}}{\sqrt{b-a}\sqrt{\frac{-2a+2b+d-\sqrt{d(-4a+4b+d)}}{(a-b)^2}}};i\sinh^{-1}\left(\frac{\sqrt{-\sqrt{b-a}}}{\sqrt[4]{x-a}}\right)\middle|-1\right)-\left(-\sqrt{d(-4a+4b+d)}+4a-4b-d\right)\sqrt{\frac{-\sqrt{d(-4a+4b+d)}-2a+2b+d}{(a-b)^2}}\left(\Pi\left(-\frac{\sqrt{2}}{\sqrt{b-a}\sqrt{\frac{-2a+2b+d+\sqrt{d(-4a+4b+d)}}{(a-b)^2}}};i\sinh^{-1}\left(\frac{\sqrt{-\sqrt{b-a}}}{\sqrt[4]{x-a}}\right)\middle|-1\right)-\Pi\left(\frac{\sqrt{2}}{\sqrt{b-a}\sqrt{\frac{-2a+2b+d+\sqrt{d(-4a+4b+d)}}{(a-b)^2}}};i\sinh^{-1}\left(\frac{\sqrt{-\sqrt{b-a}}}{\sqrt[4]{x-a}}\right)\middle|-1\right)\right)\right)}{(a-b)\sqrt{-\sqrt{b-a}}\,\sqrt{d(-4a+4b+d)}\,\sqrt{\frac{-\sqrt{d(-4a+4b+d)}-2a+2b+d}{(a-b)^2}}\,\sqrt{\frac{\sqrt{d(-4a+4b+d)}-2a+2b+d}{(a-b)^2}}\,\sqrt[4]{(x-a)(b-x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*a + b + x)/(((-a + x)*(-b + x)^2)^(1/4)*(b^2 + a*d - (2*b + d
)*x + x^2)),x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[(b - x)/(a - x)]*(-a + x)^(3/4)*((4*a - 4*b - d + Sqrt[d
*(-4*a + 4*b + d)])*Sqrt[(-2*a + 2*b + d + Sqrt[d*(-4*a + 4*b + d)])/(a - b
)^2]*EllipticPi[-(Sqrt[2]/(Sqrt[-a + b]*Sqrt[(-2*a + 2*b + d - Sqrt[d*(-4*a
 + 4*b + d)])/(a - b)^2])), I*ArcSinh[Sqrt[-Sqrt[-a + b]]/(-a + x)^(1/4)],
-1] - (4*a - 4*b - d + Sqrt[d*(-4*a + 4*b + d)])*Sqrt[(-2*a + 2*b + d + Sqr
t[d*(-4*a + 4*b + d)])/(a - b)^2]*EllipticPi[Sqrt[2]/(Sqrt[-a + b]*Sqrt[(-2
*a + 2*b + d - Sqrt[d*(-4*a + 4*b + d)])/(a - b)^2]), I*ArcSinh[Sqrt[-Sqrt[
-a + b]]/(-a + x)^(1/4)], -1] - (4*a - 4*b - d - Sqrt[d*(-4*a + 4*b + d)])*
Sqrt[(-2*a + 2*b + d - Sqrt[d*(-4*a + 4*b + d)])/(a - b)^2]*(EllipticPi[-(S
qrt[2]/(Sqrt[-a + b]*Sqrt[(-2*a + 2*b + d + Sqrt[d*(-4*a + 4*b + d)])/(a -
```

```
b)^2])), I*ArcSinh[Sqrt[-Sqrt[-a + b]]/(-a + x)^(1/4)], -1] - EllipticPi[Sq
rt[2]/(Sqrt[-a + b]*Sqrt[(-2*a + 2*b + d + Sqrt[d*(-4*a + 4*b + d)])/(a - b
)^2]), I*ArcSinh[Sqrt[-Sqrt[-a + b]]/(-a + x)^(1/4)], -1])))/((a - b)*Sqrt[
-Sqrt[-a + b]]*Sqrt[d*(-4*a + 4*b + d)]*Sqrt[(-2*a + 2*b + d - Sqrt[d*(-4*a
 + 4*b + d)])/(a - b)^2]*Sqrt[(-2*a + 2*b + d + Sqrt[d*(-4*a + 4*b + d)])/(
a - b)^2]*((b - x)^2*(-a + x))^(1/4))
```

IntegrateAlgebraic [A] time = 0.27, size = 113, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}}{b-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}}{b-x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*a + b + x)/(((-a + x)*(-b + x)^2)^(1/4)*(b^2 + a*d -
 (2*b + d)*x + x^2)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/
4))/(b - x)])/d^(3/4) + (2*ArcTanh[(d^(1/4)*(-(a*b^2) + (2*a*b + b^2)*x + (
-a - 2*b)*x^2 + x^3)^(1/4))/(b - x)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a+b+x)/((-a+x)*(-b+x)^2)^(1/4)/(b^2+a*d-(2*b+d)*x+x^2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,a-b-x}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{4}}\left(b^2+ad-(2\,b+d)x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a+b+x)/((-a+x)*(-b+x)^2)^(1/4)/(b^2+a*d-(2*b+d)*x+x^2),x, alg
orithm="giac")
```

```
[Out] integrate(-(2*a - b - x)/((-(a - x)*(b - x)^2)^(1/4)*(b^2 + a*d - (2*b + d)
*x + x^2)), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{-2a+b+x}{\left(\left(-a+x\right)\left(-b+x\right)^2\right)^{\frac{1}{4}}\left(b^2+ad-\left(2b+d\right)x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a+b+x)/((-a+x)*(-b+x)^2)^(1/4)/(b^2+a*d-(2*b+d)*x+x^2),x)
```

```
[Out] int((-2*a+b+x)/((-a+x)*(-b+x)^2)^(1/4)/(b^2+a*d-(2*b+d)*x+x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,a-b-x}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{4}}\left(b^2+ad-(2\,b+d)x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a+b+x)/((-a+x)*(-b+x)^2)^(1/4)/(b^2+a*d-(2*b+d)*x+x^2),x, alg
orithm="maxima")
```

```
[Out] -integrate((2*a - b - x)/((-(a - x)*(b - x)^2)^(1/4)*(b^2 + a*d - (2*b + d)
*x + x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b-2a+x}{\left(-(a-x)\,(b-x)^2\right)^{1/4}\,\left(a\,d-x\,(2\,b+d)+b^2+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - 2*a + x)/((-(a - x)*(b - x)^2)^(1/4)*(a*d - x*(2*b + d) + b^2 + x^
2)),x)
```

```
[Out] int((b - 2*a + x)/((-(a - x)*(b - x)^2)^(1/4)*(a*d - x*(2*b + d) + b^2 + x^
2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a+b+x)/((-a+x)*(-b+x)**2)**(1/4)/(b**2+a*d-(2*b+d)*x+x**2),x)
```

```
[Out] Timed out
```

3.1429 $\int x\sqrt[3]{x^2+x^3}\,dx$

Optimal. Leaf size=113

$$\frac{1}{54}\sqrt[3]{x^3+x^2}\left(18x^2+3x-5\right)-\frac{5}{81}\log\left(\sqrt[3]{x^3+x^2}-x\right)+\frac{5}{162}\log\left(x^2+\sqrt[3]{x^3+x^2}\,x+\left(x^3+x^2\right)^{2/3}\right)-\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x}\cdots}\right)}{27\sqrt{3}}$$

Rubi [A] time = 0.13, antiderivative size = 182, normalized size of antiderivative = 1.61, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2021, 2024, 2032, 59}

$$\frac{1}{3}\sqrt[3]{x^3+x^2}\,x^2+\frac{1}{18}\sqrt[3]{x^3+x^2}\,x-\frac{5}{54}\sqrt[3]{x^3+x^2}-\frac{5(x+1)^{2/3}x^{4/3}\log(x+1)}{162\left(x^3+x^2\right)^{2/3}}-\frac{5(x+1)^{2/3}x^{4/3}\log\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x+1}}-1\right)}{54\left(x^3+x^2\right)^{2/3}}-\frac{5(x+1)^{2/3}x^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{x+1}}+\frac{1}{\sqrt{3}}\right)}{27\sqrt{3}\left(x^3+x^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(x^2 + x^3)^(1/3),x]
```

```
[Out] (-5*(x^2 + x^3)^(1/3))/54 + (x*(x^2 + x^3)^(1/3))/18 + (x^2*(x^2 + x^3)^(1/
3))/3 - (5*x^(4/3)*(1 + x)^(2/3)*ArcTan[1/Sqrt[3] + (2*x^(1/3))/(Sqrt[3]*(1
 + x)^(1/3))])/(27*Sqrt[3]*(x^2 + x^3)^(2/3)) - (5*x^(4/3)*(1 + x)^(2/3)*Lo
g[1 + x])/(162*(x^2 + x^3)^(2/3)) - (5*x^(4/3)*(1 + x)^(2/3)*Log[-1 + x^(1/
3)/(1 + x)^(1/3)])/(54*(x^2 + x^3)^(2/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int x\sqrt[3]{x^2+x^3}\,dx &= \frac{1}{3}x^2\sqrt[3]{x^2+x^3}+\frac{1}{9}\int\frac{x^3}{\left(x^2+x^3\right)^{2/3}}\,dx\\
&= \frac{1}{18}x\sqrt[3]{x^2+x^3}+\frac{1}{3}x^2\sqrt[3]{x^2+x^3}-\frac{5}{54}\int\frac{x^2}{\left(x^2+x^3\right)^{2/3}}\,dx\\
&= -\frac{5}{54}\sqrt[3]{x^2+x^3}+\frac{1}{18}x\sqrt[3]{x^2+x^3}+\frac{1}{3}x^2\sqrt[3]{x^2+x^3}+\frac{5}{81}\int\frac{x}{\left(x^2+x^3\right)^{2/3}}\,dx\\
&= -\frac{5}{54}\sqrt[3]{x^2+x^3}+\frac{1}{18}x\sqrt[3]{x^2+x^3}+\frac{1}{3}x^2\sqrt[3]{x^2+x^3}+\frac{\left(5x^{4/3}(1+x)^{2/3}\right)\int\frac{1}{\sqrt[3]{x}(1+x)^{2/3}}\,dx}{81\left(x^2+x^3\right)^{2/3}}\\
&= -\frac{5}{54}\sqrt[3]{x^2+x^3}+\frac{1}{18}x\sqrt[3]{x^2+x^3}+\frac{1}{3}x^2\sqrt[3]{x^2+x^3}-\frac{5x^{4/3}(1+x)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{27\sqrt{3}\left(x^2+x^3\right)^{2/3}}-
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.31

$$
\frac{3\left(x^2(x+1)\right)^{4/3}\,{}_2F_1\left(-\frac{1}{3},\frac{8}{3};\frac{11}{3};-x\right)}{8(x+1)^{4/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(x^2 + x^3)^(1/3),x]
```

```
[Out] (3*(x^2*(1 + x))^(4/3)*Hypergeometric2F1[-1/3, 8/3, 11/3, -x])/(8*(1 + x)^(
4/3))
```

IntegrateAlgebraic [A] time = 0.26, size = 113, normalized size = 1.00

$$
\frac{1}{54}\sqrt[3]{x^3+x^2}\left(18x^2+3x-5\right)-\frac{5}{81}\log\left(\sqrt[3]{x^3+x^2}-x\right)+\frac{5}{162}\log\left(x^2+\sqrt[3]{x^3+x^2}\,x+\left(x^3+x^2\right)^{2/3}\right)-\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x^2}+x}\right)}{27\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(x^2 + x^3)^(1/3),x]
```

```
[Out] ((-5 + 3*x + 18*x^2)*(x^2 + x^3)^(1/3))/54 - (5*ArcTan[(Sqrt[3]*x)/(x + 2*(
x^2 + x^3)^(1/3))])/(27*Sqrt[3]) - (5*Log[-x + (x^2 + x^3)^(1/3)])/81 + (5*
Log[x^2 + x*(x^2 + x^3)^(1/3) + (x^2 + x^3)^(2/3)])/162
```

fricas [A] time = 0.42, size = 105, normalized size = 0.93

$$
\frac{5}{81}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+x^2\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{54}\left(x^3+x^2\right)^{\frac{1}{3}}\left(18x^2+3x-5\right)-\frac{5}{81}\log\left(-\frac{x-\left(x^3+x^2\right)^{\frac{1}{3}}}{x}\right)+\frac{5}{162}\log\left(\frac{x^2+\left(x^3+x^2\right)^{\frac{1}{3}}x+\left(x^3+x^2\right)^{\frac{2}{3}}}{x^2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^3+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] 5/81*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 + x^2)^(1/3))/x) + 1/54
*(x^3 + x^2)^(1/3)*(18*x^2 + 3*x - 5) - 5/81*log(-(x - (x^3 + x^2)^(1/3))/x
) + 5/162*log((x^2 + (x^3 + x^2)^(1/3)*x + (x^3 + x^2)^(2/3))/x^2)
```

giac [A] time = 0.18, size = 88, normalized size = 0.78

$$
-\frac{1}{54}\left(5\left(\frac{1}{x}+1\right)^{\frac{7}{3}}-13\left(\frac{1}{x}+1\right)^{\frac{4}{3}}-10\left(\frac{1}{x}+1\right)^{\frac{1}{3}}\right)x^3+\frac{5}{81}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{5}{162}\log\left(\left(\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)-\frac{5}{81}\log\left(\left|\left(\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^3+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] -1/54*(5*(1/x + 1)^(7/3) - 13*(1/x + 1)^(4/3) - 10*(1/x + 1)^(1/3))*x^3 + 5
/81*sqrt(3)*arctan(1/3*sqrt(3)*(2*(1/x + 1)^(1/3) + 1)) + 5/162*log((1/x +
1)^(2/3) + (1/x + 1)^(1/3) + 1) - 5/81*log(abs((1/x + 1)^(1/3) - 1))
```

maple [C] time = 0.45, size = 663, normalized size = 5.87

$$\frac{(18x^2+3x-5)\left(x^2(1+x)\right)^{\frac{1}{3}}}{54} + \text{[illegible]}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^3+x^2)^(1/3),x)
```

```
[Out] 1/54*(18*x^2+3*x-5)*(x^2*(1+x))^(1/3)+(5/162*RootOf(_Z^2-2*_Z+4)*ln((5*Root
Of(_Z^2-2*_Z+4)^2*x^2+48*RootOf(_Z^2-2*_Z+4)*(x^3+2*x^2+x)^(2/3)+48*RootOf(
_Z^2-2*_Z+4)*(x^3+2*x^2+x)^(1/3)*x+38*RootOf(_Z^2-2*_Z+4)*x^2-5*RootOf(_Z^2
-2*_Z+4)^2+48*RootOf(_Z^2-2*_Z+4)*(x^3+2*x^2+x)^(1/3)+70*RootOf(_Z^2-2*_Z+4
)*x-36*(x^3+2*x^2+x)^(2/3)-36*(x^3+2*x^2+x)^(1/3)*x-16*x^2+32*RootOf(_Z^2-2
*_Z+4)-36*(x^3+2*x^2+x)^(1/3)-28*x-12)/(1+x))-5/162*ln((5*RootOf(_Z^2-2*_Z+
4)^2*x^2-48*RootOf(_Z^2-2*_Z+4)*(x^3+2*x^2+x)^(2/3)-48*RootOf(_Z^2-2*_Z+4)*
(x^3+2*x^2+x)^(1/3)*x-58*RootOf(_Z^2-2*_Z+4)*x^2-5*RootOf(_Z^2-2*_Z+4)^2-48
*RootOf(_Z^2-2*_Z+4)*(x^3+2*x^2+x)^(1/3)-70*RootOf(_Z^2-2*_Z+4)*x+60*(x^3+2
*x^2+x)^(2/3)+60*(x^3+2*x^2+x)^(1/3)*x+80*x^2-12*RootOf(_Z^2-2*_Z+4)+60*(x^
3+2*x^2+x)^(1/3)+112*x+32)/(1+x))*RootOf(_Z^2-2*_Z+4)+5/81*ln((5*RootOf(_Z^
2-2*_Z+4)^2*x^2-48*RootOf(_Z^2-2*_Z+4)*(x^3+2*x^2+x)^(2/3)-48*RootOf(_Z^2-2
*_Z+4)*(x^3+2*x^2+x)^(1/3)*x-58*RootOf(_Z^2-2*_Z+4)*x^2-5*RootOf(_Z^2-2*_Z+
4)^2-48*RootOf(_Z^2-2*_Z+4)*(x^3+2*x^2+x)^(1/3)-70*RootOf(_Z^2-2*_Z+4)*x+60
*(x^3+2*x^2+x)^(2/3)+60*(x^3+2*x^2+x)^(1/3)*x+80*x^2-12*RootOf(_Z^2-2*_Z+4)
+60*(x^3+2*x^2+x)^(1/3)+112*x+32)/(1+x)))/x*(x^2*(1+x))^(1/3)*(x*(1+x)^2)^(
1/3)/(1+x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^3+x^2\right)^{\frac{1}{3}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^3+x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + x^2)^(1/3)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \left(x^3+x^2\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 + x^3)^(1/3),x)
```

```
[Out] int(x*(x^2 + x^3)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt[3]{x^2 (x+1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**3+x**2)**(1/3),x)
```

```
[Out] Integral(x*(x**2*(x + 1))**(1/3), x)
```

3.1430 $\int \frac{-((2a-3b)b)+2(a-2b)x+x^2}{\sqrt[4]{(-a+x)(-b+x)^2}\left(-a^3-b^2d+\left(3a^2+2bd\right)x-(3a+d)x^2+x^3\right)}\,dx$

Optimal. Leaf size=113

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}}{a-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}}{a-x}\right)}{d^{3/4}}$$

Rubi [F] time = 7.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-((2a-3b)b)+2(a-2b)x+x^2}{\sqrt[4]{(-a+x)(-b+x)^2}\left(-a^3-b^2d+\left(3a^2+2bd\right)x-(3a+d)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-((2*a - 3*b)*b) + 2*(a - 2*b)*x + x^2)/(((-a + x)*(-b + x)^2)^(1/4)*(
-a^3 - b^2*d + (3*a^2 + 2*b*d)*x - (3*a + d)*x^2 + x^3)),x]
```

```
[Out] (4*a*(-a + x)^(1/4)*Sqrt[-b + x]*Defer[Subst][Defer[Int][(x^2*Sqrt[a - b +
x^4])/(-(a^2*(1 + (b*(-2*a + b))/a^2)*d) - 2*a*(1 - b/a)*d*x^4 - d*x^8 + x^
12), x], x, (-a + x)^(1/4)]])/(-((a - x)*(b - x)^2))^(1/4) + (4*(-a + x)^(1/
4)*Sqrt[-b + x]*Defer[Subst][Defer[Int][(x^6*Sqrt[a - b + x^4])/(-(a^2*(1 +
 (b*(-2*a + b))/a^2)*d) - 2*a*(1 - b/a)*d*x^4 - d*x^8 + x^12), x], x, (-a +
 x)^(1/4)]])/(-((a - x)*(b - x)^2))^(1/4) - (4*(2*a - 3*b)*(-a + x)^(1/4)*Sq
rt[-b + x]*Defer[Subst][Defer[Int][(x^2*Sqrt[a - b + x^4])/(a^2*(1 + b^2/a^
2)*d - 2*b*d*x^4 + x^8*(d - x^4) + 2*a*d*(-b + x^4)), x], x, (-a + x)^(1/4)
])/(-((a - x)*(b - x)^2))^(1/4)
```

Rubi steps

$$\int \frac{-((2a-3b)b)+2(a-2b)x+x^2}{\sqrt[4]{(-a+x)(-b+x)^2}\left(-a^3-b^2d+\left(3a^2+2bd\right)x-(3a+d)x^2+x^3\right)}\,dx = \frac{\left(\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\int \frac{-((2 \cdots}{\sqrt[4]{-a+x}\,\sqrt{-b+x}\left(- \cdots\right.}}{\sqrt[4]{(-a+x)(-b \cdots}}$$

$$= \frac{\left(\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\int \frac{(2a- \cdots}{\sqrt[4]{-a+x}\left(-a^3-b^2d \cdots\right.}}{\sqrt[4]{(-a+x)(-b+ \cdots}}$$

$$= \frac{\left(\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\int\left(\frac{3(1 \cdots}{\sqrt[4]{-a+x}\left(a^3+b^2d \cdots\right.}\right.}{\cdots}$$

$$= \frac{\left(\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\int \frac{\cdots}{\sqrt[4]{-a+x}\left(-a^3-b^2d \cdots\right.}}{\sqrt[4]{(-a+x)(-b+ \cdots}}$$

$$= \frac{\left(4\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{\cdots}{-a^2d- \cdots}\right.}{\sqrt[4]{(-a+ \cdots}}$$

$$= \frac{\left(4\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{\cdots}{a^2\left(1+ \cdots\right.}\right.}{\sqrt[4]{(-a+ \cdots}}$$

$$= \frac{\left(4\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\text{Subst}\left(\int\left(\frac{\cdots}{-a^2(\cdots}\right.\right.}{\cdots}$$

$$= \frac{\left(4\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\text{Subst}\left(\int \frac{\cdots}{-a^2\left(1- \cdots\right.}\right.}{\sqrt[4]{(-a- \cdots}}$$

Mathematica [C] time = 7.34, size = 2458, normalized size = 21.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-((2*a - 3*b)*b) + 2*(a - 2*b)*x + x^2)/(((-a + x)*(-b + x)^2)^(
1/4)*(-a^3 - b^2*d + (3*a^2 + 2*b*d)*x - (3*a + d)*x^2 + x^3)),x]
```

```
[Out] ((-2*I)*(a - b)*(a - x)^(3/4)*Sqrt[(b - x)/(a - x)]*((-(EllipticPi[-(1/(Sqr
t[a - b]*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^
2*d)*#1^3 & , 2]])), I*ArcSinh[Sqrt[-Sqrt[a - b]]/(a - x)^(1/4)], -1]*Sqrt[
Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & ,
1]]*(1 - 4*(a - b)*Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d
 + b^2*d)*#1^3 & , 2] + 3*(a - b)^2*Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 +
 (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 2]^2)*(Root[1 + d*#1 + (-2*a*d + 2*b*d)
*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1] - Root[1 + d*#1 + (-2*a*d + 2
*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 3])) + EllipticPi[1/(Sqrt[a
 - b]*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d
)*#1^3 & , 2]]), I*ArcSinh[Sqrt[-Sqrt[a - b]]/(a - x)^(1/4)], -1]*Sqrt[Root
[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1]]*
(1 - 4*(a - b)*Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b
^2*d)*#1^3 & , 2] + 3*(a - b)^2*Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^
```

```
2*d - 2*a*b*d + b^2*d)*#1^3 & , 2]^2)*(Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^
2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1] - Root[1 + d*#1 + (-2*a*d + 2*b*d
)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 3]) + (EllipticPi[-(1/(Sqrt[a -
 b]*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*
#1^3 & , 1]])), I*ArcSinh[Sqrt[-Sqrt[a - b]]/(a - x)^(1/4)], -1] - Elliptic
Pi[1/(Sqrt[a - b]*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a
*b*d + b^2*d)*#1^3 & , 1]]), I*ArcSinh[Sqrt[-Sqrt[a - b]]/(a - x)^(1/4)], -
1])*(1 - 4*(a - b)*Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d
 + b^2*d)*#1^3 & , 1] + 3*(a - b)^2*Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 +
 (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1]^2)*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*
b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 2]]*(Root[1 + d*#1 + (-2*a*d
 + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 2] - Root[1 + d*#1 + (-
2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 3]))*Sqrt[Root[1 +
 d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 3]] + El
lipticPi[-(1/(Sqrt[a - b]*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2
*d - 2*a*b*d + b^2*d)*#1^3 & , 3]])), I*ArcSinh[Sqrt[-Sqrt[a - b]]/(a - x)^
(1/4)], -1]*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d +
 b^2*d)*#1^3 & , 1]]*(Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*
b*d + b^2*d)*#1^3 & , 1] - Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d -
 2*a*b*d + b^2*d)*#1^3 & , 2])*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 +
 (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 2]]*(1 - 4*(a - b)*Root[1 + d*#1 + (-2*
a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 3] + 3*(a - b)^2*Roo
t[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 3]^
2) - EllipticPi[1/(Sqrt[a - b]*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 +
 (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 3]]), I*ArcSinh[Sqrt[-Sqrt[a - b]]/(a -
 x)^(1/4)], -1]*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b
*d + b^2*d)*#1^3 & , 1]]*(Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d -
2*a*b*d + b^2*d)*#1^3 & , 1] - Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2
*d - 2*a*b*d + b^2*d)*#1^3 & , 2])*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1
^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 2]]*(1 - 4*(a - b)*Root[1 + d*#1 +
(-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 3] + 3*(a - b)^2
*Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & ,
 3]^2)))/(Sqrt[-Sqrt[a - b]]*((b - x)^2*(-a + x))^(1/4)*Sqrt[Root[1 + d*#1
+ (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1]]*Sqrt[Root[
1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 2]]*S
qrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3
& , 3]]*(-3*a + 3*b - 2*d + 2*(a - b)^3*d*Root[1 + d*#1 + (-2*a*d + 2*b*d)*
#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1]*Root[1 + d*#1 + (-2*a*d + 2*b*
d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 2]^2 + 2*(a - b)^3*d*Root[1 +
d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1]^2*Root
[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 3] +
 2*(a - b)^3*d*Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b
^2*d)*#1^3 & , 2]*Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d
+ b^2*d)*#1^3 & , 3]^2))
```

IntegrateAlgebraic [A] time = 0.31, size = 113, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{x(2ab+b^2)-ab^2+x^2(-a-2b)+x^3}}{a-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{x(2ab+b^2)-ab^2+x^2(-a-2b)+x^3}}{a-x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-((2*a - 3*b)*b) + 2*(a - 2*b)*x + x^2)/(((-a + x)*(-b
+ x)^2)^(1/4)*(-a^3 - b^2*d + (3*a^2 + 2*b*d)*x - (3*a + d)*x^2 + x^3)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/
4))/(a - x)])/d^(3/4) + (2*ArcTanh[(d^(1/4)*(-(a*b^2) + (2*a*b + b^2)*x + (
-a - 2*b)*x^2 + x^3)^(1/4))/(a - x)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2*a-3*b)*b+2*(a-2*b)*x+x^2)/((-a+x)*(-b+x)^2)^(1/4)/(-a^3-b^2*
d+(3*a^2+2*b*d)*x-(3*a+d)*x^2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2\,a-3\,b)b-2\,(a-2\,b)x-x^2}{\left(a^3+b^2d+(3\,a+d)x^2-x^3-\left(3\,a^2+2\,bd\right)x\right)\left(-(a-x)(b-x)^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2*a-3*b)*b+2*(a-2*b)*x+x^2)/((-a+x)*(-b+x)^2)^(1/4)/(-a^3-b^2*
d+(3*a^2+2*b*d)*x-(3*a+d)*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate(((2*a - 3*b)*b - 2*(a - 2*b)*x - x^2)/((a^3 + b^2*d + (3*a + d)*x
^2 - x^3 - (3*a^2 + 2*b*d)*x)*(-(a - x)*(b - x)^2)^(1/4)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{-\left(2a-3b\right)b+2\left(a-2b\right)x+x^2}{\left(\left(-a+x\right)\left(-b+x\right)^2\right)^{\frac{1}{4}}\left(-a^3-b^2d+\left(3a^2+2bd\right)x-\left(3a+d\right)x^2+x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(2*a-3*b)*b+2*(a-2*b)*x+x^2)/((-a+x)*(-b+x)^2)^(1/4)/(-a^3-b^2*d+(3*a
^2+2*b*d)*x-(3*a+d)*x^2+x^3),x)
```

```
[Out] int((-(2*a-3*b)*b+2*(a-2*b)*x+x^2)/((-a+x)*(-b+x)^2)^(1/4)/(-a^3-b^2*d+(3*a
^2+2*b*d)*x-(3*a+d)*x^2+x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2\,a-3\,b)b-2\,(a-2\,b)x-x^2}{\left(a^3+b^2d+(3\,a+d)x^2-x^3-\left(3\,a^2+2\,bd\right)x\right)\left(-(a-x)(b-x)^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2*a-3*b)*b+2*(a-2*b)*x+x^2)/((-a+x)*(-b+x)^2)^(1/4)/(-a^3-b^2*
d+(3*a^2+2*b*d)*x-(3*a+d)*x^2+x^3),x, algorithm="maxima")
```

```
[Out] integrate(((2*a - 3*b)*b - 2*(a - 2*b)*x - x^2)/((a^3 + b^2*d + (3*a + d)*x
^2 - x^3 - (3*a^2 + 2*b*d)*x)*(-(a - x)*(b - x)^2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{2\,x\,\left(a-2\,b\right)-b\,\left(2\,a-3\,b\right)+x^2}{{\left(-\left(a-x\right)\,{\left(b-x\right)}^2\right)}^{1/4}\,\left(b^2\,d-x\,\left(3\,a^2+2\,b\,d\right)+x^2\,\left(3\,a+d\right)+a^3-x^3\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x*(a - 2*b) - b*(2*a - 3*b) + x^2)/((-(a - x)*(b - x)^2)^(1/4)*(b^2
*d - x*(2*b*d + 3*a^2) + x^2*(3*a + d) + a^3 - x^3)),x)
```

```
[Out] int(-(2*x*(a - 2*b) - b*(2*a - 3*b) + x^2)/((-(a - x)*(b - x)^2)^(1/4)*(b^2
*d - x*(2*b*d + 3*a^2) + x^2*(3*a + d) + a^3 - x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2*a-3*b)*b+2*(a-2*b)*x+x**2)/((-a+x)*(-b+x)**2)**(1/4)/(-a**3-
b**2*d+(3*a**2+2*b*d)*x-(3*a+d)*x**2+x**3),x)
```

```
[Out] Timed out
```

3.1431 $\int \frac{(-1+x)^3\left(-1+2(-1+k)x+kx^2\right)}{x\sqrt[4]{(1-x)x(1-kx)}\,(-1+kx)\left(-1+(3+d)x-(3+dk)x^2+x^3\right)}\,dx$

Optimal. Leaf size=113

$$2\sqrt[4]{d}\,\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{kx^3+(-k-1)x^2+x}}{x-1}\right)-2\sqrt[4]{d}\,\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{kx^3+(-k-1)x^2+x}}{x-1}\right)+\frac{4\left(kx^3-kx^2-x^2+x\right)^{3/4}}{x(kx-1)}$$

Rubi [F] time = 26.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x)^3\left(-1+2(-1+k)x+kx^2\right)}{x\sqrt[4]{(1-x)x(1-kx)}\,(-1+kx)\left(-1+(3+d)x-(3+dk)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x)^3*(-1 + 2*(-1 + k)*x + k*x^2))/(x*((1 - x)*x*(1 - k*x))^(1/4)
*(-1 + k*x)*(-1 + (3 + d)*x - (3 + d*k)*x^2 + x^3)),x]
```

```
[Out] (-4*(1 - x)^(1/4)*(1 - k*x)^(1/4)*AppellF1[-1/4, -11/4, 5/4, 3/4, x, k*x])/
((1 - x)*x*(1 - k*x))^(1/4) + (4*(5 + d - 2*k)*(1 - x)^(1/4)*x^(1/4)*(1 - k
*x)^(1/4)*Defer[Subst][Defer[Int][(x^2*(1 - x^4)^(11/4))/((1 - k*x^4)^(5/4)
*(1 - 3*(1 + d/3)*x^4 + 3*(1 + (d*k)/3)*x^8 - x^12)), x], x, x^(1/4)])/((1
- x)*x*(1 - k*x))^(1/4) - (4*(3 + k + d*k)*(1 - x)^(1/4)*x^(1/4)*(1 - k*x)^
(1/4)*Defer[Subst][Defer[Int][(x^6*(1 - x^4)^(11/4))/((1 - k*x^4)^(5/4)*(1
- 3*(1 + d/3)*x^4 + 3*(1 + (d*k)/3)*x^8 - x^12)), x], x, x^(1/4)])/((1 - x)
*x*(1 - k*x))^(1/4) + (4*(1 - x)^(1/4)*x^(1/4)*(1 - k*x)^(1/4)*Defer[Subst]
[Defer[Int][(x^10*(1 - x^4)^(11/4))/((1 - k*x^4)^(5/4)*(1 - 3*(1 + d/3)*x^4
 + 3*(1 + (d*k)/3)*x^8 - x^12)), x], x, x^(1/4)])/((1 - x)*x*(1 - k*x))^(1/
4)
```

Rubi steps

$$\int \frac{(-1+x)^3\left(-1+2(-1+k)x+kx^2\right)}{x\sqrt[4]{(1-x)x(1-kx)}\,(-1+kx)\left(-1+(3+d)x-(3+dk)x^2+x^3\right)}dx = \frac{\left(\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\int \frac{(-1+}{\sqrt[4]{1-x}\,x^{5/4}\sqrt[4]{1-kx}}}{\sqrt[4]{(1-x)x(1}}$$

$$= -\frac{\left(\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\int \frac{(1-x)^{11}}{x^{5/4}\sqrt[4]{1-kx}(-1}}{\sqrt[4]{(1-x)x(1-}}$$

$$= \frac{\left(\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\int \frac{(1-x)^{11/4}(-}{x^{5/4}(1-kx)^{5/4}(-1}}{\sqrt[4]{(1-x)x(1-kx)}}$$

$$= \frac{\left(4\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\text{Subst}\left(\int \frac{(}{x^2(1-}\right.}{\sqrt[4]{(1-x)}}$$

$$= \frac{\left(4\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\text{Subst}\left(\int\left(\frac{(1-}{x^2(1}\right.\right.}{}$$

$$= \frac{\left(4\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\text{Subst}\left(\int \frac{(1-x}{x^2(1-}\right.}{\sqrt[4]{(1-x)x(1-kx)}}$$

$$= -\frac{4\sqrt[4]{1-x}\sqrt[4]{1-kx}\,F_1\left(-\frac{1}{4};-\frac{11}{4},\frac{5}{4};\frac{3}{4};x\right.}{\sqrt[4]{(1-x)x(1-kx)}}$$

$$= -\frac{4\sqrt[4]{1-x}\sqrt[4]{1-kx}\,F_1\left(-\frac{1}{4};-\frac{11}{4},\frac{5}{4};\frac{3}{4};x\right.}{\sqrt[4]{(1-x)x(1-kx)}}$$

Mathematica [F] time = 5.96, size = 0, normalized size = 0.00

$$\int \frac{(-1+x)^3\left(-1+2(-1+k)x+kx^2\right)}{x\sqrt[4]{(1-x)x(1-kx)}\,(-1+kx)\left(-1+(3+d)x-(3+dk)x^2+x^3\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x)^3*(-1 + 2*(-1 + k)*x + k*x^2))/(x*((1 - x)*x*(1 - k*x))
^(1/4)*(-1 + k*x)*(-1 + (3 + d)*x - (3 + d*k)*x^2 + x^3)),x]
```

```
[Out] Integrate[((-1 + x)^3*(-1 + 2*(-1 + k)*x + k*x^2))/(x*((1 - x)*x*(1 - k*x))
^(1/4)*(-1 + k*x)*(-1 + (3 + d)*x - (3 + d*k)*x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.89, size = 113, normalized size = 1.00

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{x-1}\right)-2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{x-1}\right)+\frac{4\left(kx^3-kx^2-x^2+x\right)^{3/4}}{x(kx-1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)^3*(-1 + 2*(-1 + k)*x + k*x^2))/(x*((1 - x)*x*(
1 - k*x))^(1/4)*(-1 + k*x)*(-1 + (3 + d)*x - (3 + d*k)*x^2 + x^3)),x]
```

```
[Out] (4*(x - x^2 - k*x^2 + k*x^3)^(3/4))/(x*(-1 + k*x)) + 2*d^(1/4)*ArcTan[(d^(1
/4)*(x + (-1 - k)*x^2 + k*x^3)^(1/4))/(-1 + x)] - 2*d^(1/4)*ArcTanh[(d^(1/4
)*(x + (-1 - k)*x^2 + k*x^3)^(1/4))/(-1 + x)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^3*(-1+2*(-1+k)*x+k*x^2)/x/((1-x)*x*(-k*x+1))^(1/4)/(k*x-1)
/(-1+(3+d)*x-(d*k+3)*x^2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(kx^2 + 2\,(k-1)x - 1\right)(x-1)^3}{((kx-1)(x-1)x)^{\frac{1}{4}}\left((dk+3)x^2 - x^3 - (d+3)x + 1\right)(kx-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^3*(-1+2*(-1+k)*x+k*x^2)/x/((1-x)*x*(-k*x+1))^(1/4)/(k*x-1)
/(-1+(3+d)*x-(d*k+3)*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate(-(k*x^2 + 2*(k - 1)*x - 1)*(x - 1)^3/(((k*x - 1)*(x - 1)*x)^(1/4)
*((d*k + 3)*x^2 - x^3 - (d + 3)*x + 1)*(k*x - 1)*x), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(-1+x)^3\left(-1+2\,(-1+k)\,x+k\,x^2\right)}{x\,((1-x)\,x\,(-kx+1))^{\frac{1}{4}}\,(kx-1)\left(-1+(3+d)\,x-(dk+3)\,x^2+x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^3*(-1+2*(-1+k)*x+k*x^2)/x/((1-x)*x*(-k*x+1))^(1/4)/(k*x-1)/(-1+(
3+d)*x-(d*k+3)*x^2+x^3),x)
```

```
[Out] int((-1+x)^3*(-1+2*(-1+k)*x+k*x^2)/x/((1-x)*x*(-k*x+1))^(1/4)/(k*x-1)/(-1+(
3+d)*x-(d*k+3)*x^2+x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(kx^2 + 2\,(k-1)x - 1\right)(x-1)^3}{((kx-1)(x-1)x)^{\frac{1}{4}}\left((dk+3)x^2 - x^3 - (d+3)x + 1\right)(kx-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^3*(-1+2*(-1+k)*x+k*x^2)/x/((1-x)*x*(-k*x+1))^(1/4)/(k*x-1)
/(-1+(3+d)*x-(d*k+3)*x^2+x^3),x, algorithm="maxima")
```

```
[Out] -integrate((k*x^2 + 2*(k - 1)*x - 1)*(x - 1)^3/(((k*x - 1)*(x - 1)*x)^(1/4)
*((d*k + 3)*x^2 - x^3 - (d + 3)*x + 1)*(k*x - 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x-1)^3\,\left(2\,x\,(k-1)+k\,x^2-1\right)}{x\,(k\,x-1)\,(x\,(k\,x-1)\,(x-1))^{1/4}\,\left(x^3+(-d\,k-3)\,x^2+(d+3)\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)^3*(2*x*(k - 1) + k*x^2 - 1))/(x*(k*x - 1)*(x*(k*x - 1)*(x - 1)
)^(1/4)*(x*(d + 3) - x^2*(d*k + 3) + x^3 - 1)),x)
```

```
[Out] int(((x - 1)^3*(2*x*(k - 1) + k*x^2 - 1))/(x*(k*x - 1)*(x*(k*x - 1)*(x - 1)
)^(1/4)*(x*(d + 3) - x^2*(d*k + 3) + x^3 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**3*(-1+2*(-1+k)*x+k*x**2)/x/((1-x)*x*(-k*x+1))**(1/4)/(k*x
-1)/(-1+(3+d)*x-(d*k+3)*x**2+x**3),x)
```

```
[Out] Timed out
```

3.1432 $\int \frac{b+ax^2}{\left(-b+ax^2\right)\sqrt{bx+ax^3}}\,dx$

Optimal. Leaf size=113

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}$$

Rubi [A] time = 0.24, antiderivative size = 159, normalized size of antiderivative = 1.41, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2056, 466, 404, 212, 206, 203}

$$-\frac{\sqrt{x}\sqrt{ax^2+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2+b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}-\frac{\sqrt{x}\sqrt{ax^2+b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2+b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^2)/((-b + a*x^2)*Sqrt[b*x + a*x^3]),x]
```

```
[Out] -((Sqrt[x]*Sqrt[b + a*x^2]*ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/Sqrt[b
+ a*x^2]])/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3])) - (Sqrt[x]*Sqrt[b +
 a*x^2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/Sqrt[b + a*x^2]])/(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 404

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[a/c,
 Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{b+ax^2}{\left(-b+ax^2\right)\sqrt{bx+ax^3}}\,dx &= \frac{\left(\sqrt{x}\,\sqrt{b+ax^2}\right)\int \frac{\sqrt{b+ax^2}}{\sqrt{x}\left(-b+ax^2\right)}\,dx}{\sqrt{bx+ax^3}}\\
&= \frac{\left(2\sqrt{x}\,\sqrt{b+ax^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{b+ax^4}}{-b+ax^4}\,dx, x, \sqrt{x}\right)}{\sqrt{bx+ax^3}}\\
&= -\frac{\left(2\sqrt{x}\,\sqrt{b+ax^2}\right)\operatorname{Subst}\left(\int \frac{1}{1-4abx^4}\,dx, x, \frac{\sqrt{x}}{\sqrt{b+ax^2}}\right)}{\sqrt{bx+ax^3}}\\
&= -\frac{\left(\sqrt{x}\,\sqrt{b+ax^2}\right)\operatorname{Subst}\left(\int \frac{1}{1-2\sqrt{a}\,\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b+ax^2}}\right)}{\sqrt{bx+ax^3}} - \frac{\left(\sqrt{x}\,\sqrt{b+ax^2}\right)\operatorname{Subst}\left(\cdots\right.}{\sqrt{b\cdots}}\\
&= -\frac{\sqrt{x}\,\sqrt{b+ax^2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{b+ax^2}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{bx+ax^3}} - \frac{\sqrt{x}\,\sqrt{b+ax^2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{b+ax^2}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{bx+ax^3}}
\end{aligned}
$$

Mathematica [C] time = 0.05, size = 60, normalized size = 0.53

$$
-\frac{2\sqrt{x\left(ax^2+b\right)}\,F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{ax^2}{b},\frac{ax^2}{b}\right)}{b\sqrt{\frac{ax^2}{b}+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b + a*x^2)/((-b + a*x^2)*Sqrt[b*x + a*x^3]),x]
```

```
[Out] (-2*Sqrt[x*(b + a*x^2)]*AppellF1[1/4, -1/2, 1, 5/4, -((a*x^2)/b), (a*x^2)/b
])/(b*Sqrt[1 + (a*x^2)/b])
```

IntegrateAlgebraic [A] time = 0.37, size = 113, normalized size = 1.00

$$
-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{ax^3+bx}}{ax^2+b}\right)}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{ax^3+bx}}{ax^2+b}\right)}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^2)/((-b + a*x^2)*Sqrt[b*x + a*x^3]),x]
```

```
[Out] -(ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3])/(b + a*x^2)]/(Sqrt[2]*
a^(1/4)*b^(1/4))) - ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3])/(b
+ a*x^2)]/(Sqrt[2]*a^(1/4)*b^(1/4))
```

fricas [B] time = 0.72, size = 326, normalized size = 2.88

$$
-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab}\right)^{\frac{1}{4}}\arctan\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{ax^3+bx}\,ab\left(\frac{1}{ab}\right)^{\frac{3}{4}}}{ax^2+b}\right) - \frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab}\right)^{\frac{1}{4}}\log\left(\frac{a^2x^4+6\,abx^2+b^2+8\sqrt{ax^3+bx}\left(\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{ab}\right)^{\frac{1}{4}}+\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(a^2bx^2+ab^2\right)\left(\frac{1}{ab}\right)^{\frac{3}{4}}\right)+4\left(a^2bx^3+ab^2x\right)\sqrt{\frac{1}{ab}}}{a^2x^4-2\,abx^2+b^2}\right) + \frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab}\right)^{\frac{1}{4}}\log\left(\frac{a^2x^4+6\,abx^2+b^2-8\sqrt{ax^3+bx}\left(\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{ab}\right)^{\frac{1}{4}}+\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(a^2bx^2+ab^2\right)\left(\frac{1}{ab}\right)^{\frac{3}{4}}\right)+4\left(a^2bx^3+ab^2x\right)\sqrt{\frac{1}{ab}}}{a^2x^4-2\,abx^2+b^2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a*x^2-b)/(a*x^3+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -(1/4)^(1/4)*(1/(a*b))^(1/4)*arctan(4*(1/4)^(3/4)*sqrt(a*x^3 + b*x)*a*b*(1/
(a*b))^(3/4)/(a*x^2 + b)) - 1/4*(1/4)^(1/4)*(1/(a*b))^(1/4)*log((a^2*x^4 +
6*a*b*x^2 + b^2 + 8*sqrt(a*x^3 + b*x)*((1/4)^(1/4)*a*b*x*(1/(a*b))^(1/4) +
(1/4)^(3/4)*(a^2*b*x^2 + a*b^2)*(1/(a*b))^(3/4)) + 4*(a^2*b*x^3 + a*b^2*x)*
sqrt(1/(a*b)))/(a^2*x^4 - 2*a*b*x^2 + b^2)) + 1/4*(1/4)^(1/4)*(1/(a*b))^(1/
4)*log((a^2*x^4 + 6*a*b*x^2 + b^2 - 8*sqrt(a*x^3 + b*x)*((1/4)^(1/4)*a*b*x*
(1/(a*b))^(1/4) + (1/4)^(3/4)*(a^2*b*x^2 + a*b^2)*(1/(a*b))^(3/4)) + 4*(a^2
*b*x^3 + a*b^2*x)*sqrt(1/(a*b)))/(a^2*x^4 - 2*a*b*x^2 + b^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + b}{\sqrt{ax^3 + bx}\,(ax^2 - b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a*x^2-b)/(a*x^3+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^2 + b)/(sqrt(a*x^3 + b*x)*(a*x^2 - b)), x)
```

maple [C] time = 0.05, size = 417, normalized size = 3.69

$$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a\,x^3+bx}}+2b\left(\frac{\sqrt{-ab}\,\sqrt{\frac{xa}{\sqrt{-ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{2\sqrt{ab}\,a\sqrt{a\,x^3+bx}\,\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)}-\frac{\sqrt{-ab}\,\sqrt{\frac{xa}{\sqrt{-ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{2\sqrt{ab}\,a\sqrt{a\,x^3+bx}\,\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)/(a*x^2-b)/(a*x^3+b*x)^(1/2),x)
```

```
[Out] 1/a*(-a*b)^(1/2)*((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x-1/a*(-a
*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2
)*EllipticF(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+2*b*(1
/2/(a*b)^(1/2)/a*(-a*b)^(1/2)*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(1/
2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(-a*b)^(1/2)-
1/a*(a*b)^(1/2))*EllipticPi(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),-1/
a*(-a*b)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2)),1/2*2^(1/2))-1/2/(a*b)^(
1/2)/a*(-a*b)^(1/2)*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2
)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(-a*b)^(1/2)+1/a*(a*b)^
(1/2))*EllipticPi(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),-1/a*(-a*b)^(
1/2)/(-1/a*(-a*b)^(1/2)+1/a*(a*b)^(1/2)),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + b}{\sqrt{ax^3 + bx}\,(ax^2 - b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a*x^2-b)/(a*x^3+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 + b)/(sqrt(a*x^3 + b*x)*(a*x^2 - b)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x^2)/((b*x + a*x^3)^(1/2)*(b - a*x^2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + b}{\sqrt{x(ax^2 + b)}\,(ax^2 - b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b)/(a*x**2-b)/(a*x**3+b*x)**(1/2),x)
```

```
[Out] Integral((a*x**2 + b)/(sqrt(x*(a*x**2 + b))*(a*x**2 - b)), x)
```

3.1433 $$\int \frac{(a^2-2ax+x^2)\left(-((2a-3b)b)+2(a-2b)x+x^2\right)}{\left((-a+x)(-b+x)^2\right)^{3/4}\left(-b^2-a^3d+(2b+3a^2d)x-(1+3ad)x^2+dx^3\right)}\,dx$$

Optimal. Leaf size=113

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x(2ab+b^2)-ab^2+x^2(-a-2b)+x^3\right)^{3/4}}{(b-x)^2}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x(2ab+b^2)-ab^2+x^2(-a-2b)+x^3\right)^{3/4}}{(b-x)^2}\right)}{d^{3/4}}$$

Rubi [F] time = 9.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a^2-2ax+x^2)\left(-((2a-3b)b)+2(a-2b)x+x^2\right)}{\left((-a+x)(-b+x)^2\right)^{3/4}\left(-b^2-a^3d+(2b+3a^2d)x-(1+3ad)x^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a^2 - 2*a*x + x^2)*(-((2*a - 3*b)*b) + 2*(a - 2*b)*x + x^2))/(((-a +
x)*(-b + x)^2)^(3/4)*(-b^2 - a^3*d + (2*b + 3*a^2*d)*x - (1 + 3*a*d)*x^2 +
d*x^3)),x]
```

```
[Out] (-2*(a - b)^(1/4)*(b - x)*(-a + x)^(3/4)*Sqrt[-((b - x)/((a - b)*(1 + Sqrt[
-a + x]/Sqrt[a - b])^2))]*(1 + Sqrt[-a + x]/Sqrt[a - b])*EllipticF[2*ArcTan
[(-a + x)^(1/4)/(a - b)^(1/4)], 1/2])/(d*(-((a - x)*(b - x)^2))^(3/4)) - (4
*(2*a - 3*b)*(-a + x)^(3/4)*(-b + x)^(3/2)*Defer[Subst][Defer[Int][x^8/(Sqr
t[a - b + x^4]*(a^2*(1 + (b*(-2*a + b))/a^2) + (2*a - 2*b)*x^4 + x^8 - d*x^
12)), x], x, (-a + x)^(1/4)])/(-((a - x)*(b - x)^2))^(3/4) - (4*(a - b)^2*(
-a + x)^(3/4)*(-b + x)^(3/2)*Defer[Subst][Defer[Int][1/(Sqrt[a - b + x^4]*(
a^2*(1 + (b*(-2*a + b))/a^2) + 2*a*(1 - b/a)*x^4 + x^8 - d*x^12)), x], x, (
-a + x)^(1/4)])/(d*(-((a - x)*(b - x)^2))^(3/4)) - (8*(a - b)*(-a + x)^(3/4
)*(-b + x)^(3/2)*Defer[Subst][Defer[Int][x^4/(Sqrt[a - b + x^4]*(a^2*(1 + (
b*(-2*a + b))/a^2) + 2*a*(1 - b/a)*x^4 + x^8 - d*x^12)), x], x, (-a + x)^(1
/4)])/(d*(-((a - x)*(b - x)^2))^(3/4)) - (4*(1 + a*d)*(-a + x)^(3/4)*(-b +
x)^(3/2)*Defer[Subst][Defer[Int][x^8/(Sqrt[a - b + x^4]*(a^2*(1 + (b*(-2*a
+ b))/a^2) + 2*a*(1 - b/a)*x^4 + x^8 - d*x^12)), x], x, (-a + x)^(1/4)])/(d
*(-((a - x)*(b - x)^2))^(3/4))
```

Rubi steps

$$\int \frac{\left(a^2-2ax+x^2\right)\left(-((2a-3b)b)+2(a-2b)x+x^2\right)}{\left((-a+x)(-b+x)^2\right)^{3/4}\left(-b^2-a^3d+\left(2b+3a^2d\right)x-(1+3ad)x^2+dx^3\right)}\,dx=\int \frac{(-a+x)^2\left(-\right.}{\left((-a+x)(-b+x)^2\right)^{3/4}\left(-b\right.}$$

$$=\frac{\left((-a+x)^{3/4}(-b+x)^{3/2}\right)\int \frac{}{(-b}}{\left((-a+\right.}$$

$$=\frac{\left((-a+x)^{3/4}(-b+x)^{3/2}\right)\int \frac{}{\sqrt{-}}}{\left((-a+\right.}$$

$$=\frac{\left((-a+x)^{3/4}(-b+x)^{3/2}\right)\int\left(\frac{}{\sqrt{}}\right.}{}$$

$$=\frac{\left((-a+x)^{3/4}(-b+x)^{3/2}\right)\int \frac{}{\sqrt{-}}}{\left((-a+\right.}$$

$$=-\frac{\left(4(-a+x)^{3/4}(-b+x)^{3/2}\right)\mathrm{Su}}{}$$

$$=-\frac{\left(4(-a+x)^{3/4}(-b+x)^{3/2}\right)\mathrm{Su}}{}$$

$$=-\frac{\left(4(-a+x)^{3/4}(-b+x)^{3/2}\right)\mathrm{Su}}{}$$

$$=-\frac{\left(4(-a+x)^{3/4}(-b+x)^{3/2}\right)\mathrm{Su}}{}$$

$$=\frac{\left(4(-2a+3b)(-a+x)^{3/4}(-b+\right.}{}$$

$$=-\frac{2\sqrt[4]{a-b}\,(b-x)(-a+x)^{3/4}}{}$$

$$=-\frac{2\sqrt[4]{a-b}\,(b-x)(-a+x)^{3/4}}{}$$

Mathematica [F] time = 3.55, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2-2ax+x^2\right)\left(-((2a-3b)b)+2(a-2b)x+x^2\right)}{\left((-a+x)(-b+x)^2\right)^{3/4}\left(-b^2-a^3d+\left(2b+3a^2d\right)x-(1+3ad)x^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a^2 - 2*a*x + x^2)*(-((2*a - 3*b)*b) + 2*(a - 2*b)*x + x^2))/((
(-a + x)*(-b + x)^2)^(3/4)*(-b^2 - a^3*d + (2*b + 3*a^2*d)*x - (1 + 3*a*d)*
x^2 + d*x^3)),x]
```

```
[Out] Integrate[((a^2 - 2*a*x + x^2)*(-((2*a - 3*b)*b) + 2*(a - 2*b)*x + x^2))/((
(-a + x)*(-b + x)^2)^(3/4)*(-b^2 - a^3*d + (2*b + 3*a^2*d)*x - (1 + 3*a*d)*
x^2 + d*x^3)), x]
```

IntegrateAlgebraic [A] time = 3.64, size = 113, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{3/4}}{(b-x)^2}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{3/4}}{(b-x)^2}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a^2 - 2*a*x + x^2)*(-((2*a - 3*b)*b) + 2*(a - 2*b)*x +
 x^2))/(((-a + x)*(-b + x)^2)^(3/4)*(-b^2 - a^3*d + (2*b + 3*a^2*d)*x - (1
+ 3*a*d)*x^2 + d*x^3)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(3/4
))/(b - x)^2])/d^(3/4) - (2*ArcTanh[(d^(1/4)*(-(a*b^2) + (2*a*b + b^2)*x +
(-a - 2*b)*x^2 + x^3)^(3/4))/(b - x)^2])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-2*a*x+x^2)*(-(2*a-3*b)*b+2*(a-2*b)*x+x^2)/((-a+x)*(-b+x)^2)^
(3/4)/(-b^2-a^3*d+(3*a^2*d+2*b)*x-(3*a*d+1)*x^2+d*x^3),x, algorithm="fricas
")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2-2\,ax+x^2\right)\left((2\,a-3\,b)b-2\,(a-2\,b)x-x^2\right)}{\left(a^3d-dx^3+(3\,ad+1)x^2+b^2-\left(3\,a^2d+2\,b\right)x\right)\left(-(a-x)(b-x)^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-2*a*x+x^2)*(-(2*a-3*b)*b+2*(a-2*b)*x+x^2)/((-a+x)*(-b+x)^2)^
(3/4)/(-b^2-a^3*d+(3*a^2*d+2*b)*x-(3*a*d+1)*x^2+d*x^3),x, algorithm="giac")
```

```
[Out] integrate((a^2 - 2*a*x + x^2)*((2*a - 3*b)*b - 2*(a - 2*b)*x - x^2)/((a^3*d
 - d*x^3 + (3*a*d + 1)*x^2 + b^2 - (3*a^2*d + 2*b)*x)*(-(a - x)*(b - x)^2)^
(3/4)), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2-2ax+x^2\right)\left(-(2a-3b)\,b+2\,(a-2b)\,x+x^2\right)}{\left((-a+x)\,(-b+x)^2\right)^{\frac{3}{4}}\left(-b^2-a^3d+\left(3a^2d+2b\right)x-(3ad+1)\,x^2+d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-2*a*x+x^2)*(-(2*a-3*b)*b+2*(a-2*b)*x+x^2)/((-a+x)*(-b+x)^2)^(3/4)/
(-b^2-a^3*d+(3*a^2*d+2*b)*x-(3*a*d+1)*x^2+d*x^3),x)
```

```
[Out] int((a^2-2*a*x+x^2)*(-(2*a-3*b)*b+2*(a-2*b)*x+x^2)/((-a+x)*(-b+x)^2)^(3/4)/
(-b^2-a^3*d+(3*a^2*d+2*b)*x-(3*a*d+1)*x^2+d*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2-2\,ax+x^2\right)\left((2\,a-3\,b)b-2\,(a-2\,b)x-x^2\right)}{\left(a^3d-dx^3+(3\,ad+1)x^2+b^2-\left(3\,a^2d+2\,b\right)x\right)\left(-(a-x)(b-x)^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-2*a*x+x^2)*(-(2*a-3*b)*b+2*(a-2*b)*x+x^2)/((-a+x)*(-b+x)^2)^
(3/4)/(-b^2-a^3*d+(3*a^2*d+2*b)*x-(3*a*d+1)*x^2+d*x^3),x, algorithm="maxima
")
```

```
[Out] integrate((a^2 - 2*a*x + x^2)*((2*a - 3*b)*b - 2*(a - 2*b)*x - x^2)/((a^3*d
 - d*x^3 + (3*a*d + 1)*x^2 + b^2 - (3*a^2*d + 2*b)*x)*(-(a - x)*(b - x)^2)^
(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(a^2-2\,a\,x+x^2\right)\,\left(2\,x\,\left(a-2\,b\right)-b\,\left(2\,a-3\,b\right)+x^2\right)}{{\left(-\left(a-x\right)\,{\left(b-x\right)}^2\right)}^{3/4}\,\left(a^3\,d-x\,\left(3\,d\,a^2+2\,b\right)-d\,x^3+x^2\,\left(3\,a\,d+1\right)+b^2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a^2 - 2*a*x + x^2)*(2*x*(a - 2*b) - b*(2*a - 3*b) + x^2))/((-(a - x)
*(b - x)^2)^(3/4)*(a^3*d - x*(2*b + 3*a^2*d) - d*x^3 + x^2*(3*a*d + 1) + b^
2)),x)
```

```
[Out] int(-((a^2 - 2*a*x + x^2)*(2*x*(a - 2*b) - b*(2*a - 3*b) + x^2))/((-(a - x)
*(b - x)^2)^(3/4)*(a^3*d - x*(2*b + 3*a^2*d) - d*x^3 + x^2*(3*a*d + 1) + b^
2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-2*a*x+x**2)*(-(2*a-3*b)*b+2*(a-2*b)*x+x**2)/((-a+x)*(-b+x)*
*2)**(3/4)/(-b**2-a**3*d+(3*a**2*d+2*b)*x-(3*a*d+1)*x**2+d*x**3),x)
```

```
[Out] Timed out
```

3.1434 $\int \frac{-b+ax^2}{x^2(b+ax^2)\sqrt[4]{-bx^2+ax^4}}\,dx$

Optimal. Leaf size=113

$$\frac{2^{3/4}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{b}+\frac{2^{3/4}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{b}-\frac{2\left(ax^4-bx^2\right)^{3/4}}{3bx^3}$$

Rubi [C] time = 0.38, antiderivative size = 58, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2056, 466, 511, 510}

$$\frac{2\left(b-ax^2\right){}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{2ax^2}{b-ax^2}\right)}{3bx\sqrt[4]{ax^4-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^2)/(x^2*(b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4)),x]
```

```
[Out] (2*(b - a*x^2)*Hypergeometric2F1[-3/4, 1, 1/4, (-2*a*x^2)/(b - a*x^2)])/(3*
b*x*(-(b*x^2) + a*x^4)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{-b+ax^2}{x^2\left(b+ax^2\right)\sqrt[4]{-bx^2+ax^4}}\,dx = \frac{\left(\sqrt{x}\,\sqrt[4]{-b+ax^2}\right)\int \frac{\left(-b+ax^2\right)^{3/4}}{x^{5/2}\left(b+ax^2\right)}\,dx}{\sqrt[4]{-bx^2+ax^4}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{-b+ax^2}\right)\text{Subst}\left(\int \frac{\left(-b+ax^4\right)^{3/4}}{x^4\left(b+ax^4\right)}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{-bx^2+ax^4}}$$

$$= \frac{\left(2\sqrt{x}\,\left(-b+ax^2\right)\right)\text{Subst}\left(\int \frac{\left(1-\frac{ax^4}{b}\right)^{3/4}}{x^4\left(b+ax^4\right)}\,dx, x, \sqrt{x}\right)}{\left(1-\frac{ax^2}{b}\right)^{3/4}\sqrt[4]{-bx^2+ax^4}}$$

$$= \frac{2\left(b-ax^2\right)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{2ax^2}{b-ax^2}\right)}{3bx\sqrt[4]{-bx^2+ax^4}}$$

Mathematica [**C**] time = 0.05, size = 80, normalized size = 0.71

$$-\frac{2\left(\frac{ax^2}{b}+1\right)^{3/4}\left(ax^4-bx^2\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};\frac{2ax^2}{ax^2+b}\right)}{3bx^3\left(1-\frac{ax^2}{b}\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-b + a*x^2)/(x^2*(b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4)),x]
```

```
[Out] (-2*(1 + (a*x^2)/b)^(3/4)*(-(b*x^2) + a*x^4)^(3/4)*Hypergeometric2F1[-3/4,
-3/4, 1/4, (2*a*x^2)/(b + a*x^2)])/(3*b*x^3*(1 - (a*x^2)/b)^(3/4))
```

IntegrateAlgebraic [**A**] time = 0.40, size = 113, normalized size = 1.00

$$\frac{2^{3/4}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{b}+\frac{2^{3/4}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{b}-\frac{2\left(ax^4-bx^2\right)^{3/4}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^2)/(x^2*(b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4)),
x]
```

```
[Out] (-2*(-(b*x^2) + a*x^4)^(3/4))/(3*b*x^3) + (2^(3/4)*a^(3/4)*ArcTan[(2^(1/4)*
a^(1/4)*x)/(-(b*x^2) + a*x^4)^(1/4)])/b + (2^(3/4)*a^(3/4)*ArcTanh[(2^(1/4)
*a^(1/4)*x)/(-(b*x^2) + a*x^4)^(1/4)])/b
```

fricas [**B**] time = 79.50, size = 514, normalized size = 4.55

$$-\frac{12\left(\frac{1}{2}\right)^{\frac{1}{4}}bx^3\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(2\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(ax^4-bx^2\right)^{\frac{1}{4}}a^4bx^2\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}+2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(ax^4-bx^2\right)^{\frac{3}{4}}a^2b^3\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}+\left(2\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{ax^4-bx^2}\,a^2bx\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}+\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(3ab^3x^3-b^4x\right)\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}\right)\sqrt{\sqrt{\frac{1}{2}}\,a^2b^2\sqrt{\frac{a^3}{b^4}}}\right)}{a^5x^3+a^4bx}\right)-3\left(\frac{1}{2}\right)^{\frac{1}{4}}bx^3\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}\log\left(\frac{4\sqrt{\frac{1}{2}}\left(ax^4-bx^2\right)^{\frac{1}{4}}ab^2x^2\sqrt{\frac{a^3}{b^4}}+4\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{ax^4-bx^2}\,b^3x\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}+2\left(ax^4-bx^2\right)^{\frac{3}{4}}a^2+\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(3a^2bx^3-ab^2x\right)\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}}{ax^3+bx}\right)+3\left(\frac{1}{2}\right)^{\frac{1}{4}}bx^3\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}\log\left(\frac{4\sqrt{\frac{1}{2}}\left(ax^4-bx^2\right)^{\frac{1}{4}}ab^2x^2\sqrt{\frac{a^3}{b^4}}-4\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{ax^4-bx^2}\,b^3x\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}+2\left(ax^4-bx^2\right)^{\frac{3}{4}}a^2-\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(3a^2bx^3-ab^2x\right)\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}}{ax^3+bx}\right)+4\left(ax^4-bx^2\right)^{\frac{3}{4}}}{6bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/x^2/(a*x^2+b)/(a*x^4-b*x^2)^(1/4),x, algorithm="fricas"
)
```

```
[Out] -1/6*(12*(1/2)^(1/4)*b*x^3*(a^3/b^4)^(1/4)*arctan(2*(2*(1/2)^(1/4)*(a*x^4 -
 b*x^2)^(1/4)*a^4*b*x^2*(a^3/b^4)^(1/4) + 2*(1/2)^(3/4)*(a*x^4 - b*x^2)^(3/
```

```
4)*a^2*b^3*(a^3/b^4)^(3/4) + (2*(1/2)^(1/4)*sqrt(a*x^4 - b*x^2)*a^2*b*x*(a^
3/b^4)^(1/4) + (1/2)^(3/4)*(3*a*b^3*x^3 - b^4*x)*(a^3/b^4)^(3/4))*sqrt(sqrt
(1/2)*a^2*b^2*sqrt(a^3/b^4)))/(a^5*x^3 + a^4*b*x)) - 3*(1/2)^(1/4)*b*x^3*(a
^3/b^4)^(1/4)*log((4*sqrt(1/2)*(a*x^4 - b*x^2)^(1/4)*a*b^2*x^2*sqrt(a^3/b^4
) + 4*(1/2)^(3/4)*sqrt(a*x^4 - b*x^2)*b^3*x*(a^3/b^4)^(3/4) + 2*(a*x^4 - b*
x^2)^(3/4)*a^2 + (1/2)^(1/4)*(3*a^2*b*x^3 - a*b^2*x)*(a^3/b^4)^(1/4))/(a*x^
3 + b*x)) + 3*(1/2)^(1/4)*b*x^3*(a^3/b^4)^(1/4)*log((4*sqrt(1/2)*(a*x^4 - b
*x^2)^(1/4)*a*b^2*x^2*sqrt(a^3/b^4) - 4*(1/2)^(3/4)*sqrt(a*x^4 - b*x^2)*b^3
*x*(a^3/b^4)^(3/4) + 2*(a*x^4 - b*x^2)^(3/4)*a^2 - (1/2)^(1/4)*(3*a^2*b*x^3
 - a*b^2*x)*(a^3/b^4)^(1/4))/(a*x^3 + b*x)) + 4*(a*x^4 - b*x^2)^(3/4))/(b*x
^3)
```

giac [B] time = 0.21, size = 213, normalized size = 1.88

$$\frac{2^{\frac{1}{4}}\left(-a\right)^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}\left(-a\right)^{\frac{1}{4}}+2\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)}{b}+\frac{2^{\frac{1}{4}}\left(-a\right)^{\frac{3}{4}}\arctan\left(-\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}\left(-a\right)^{\frac{1}{4}}-2\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)}{b}-\frac{2^{\frac{1}{4}}\left(-a\right)^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-a\right)^{\frac{1}{4}}\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a-\frac{b}{x^2}}\right)}{2\,b}+\frac{2^{\frac{1}{4}}\left(-a\right)^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-a\right)^{\frac{1}{4}}\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a-\frac{b}{x^2}}\right)}{2\,b}-\frac{2\left(a-\frac{b}{x^2}\right)^{\frac{3}{4}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/x^2/(a*x^2+b)/(a*x^4-b*x^2)^(1/4),x, algorithm="giac")
```

```
[Out] 2^(1/4)*(-a)^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4)*(-a)^(1/4) + 2*(a - b/x^2)^(
1/4))/(-a)^(1/4))/b + 2^(1/4)*(-a)^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4)*(-a)^
(1/4) - 2*(a - b/x^2)^(1/4))/(-a)^(1/4))/b - 1/2*2^(1/4)*(-a)^(3/4)*log(2^(
3/4)*(-a)^(1/4)*(a - b/x^2)^(1/4) + sqrt(2)*sqrt(-a) + sqrt(a - b/x^2))/b +
 1/2*2^(1/4)*(-a)^(3/4)*log(-2^(3/4)*(-a)^(1/4)*(a - b/x^2)^(1/4) + sqrt(2)
*sqrt(-a) + sqrt(a - b/x^2))/b - 2/3*(a - b/x^2)^(3/4)/b
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2-b}{x^2\left(a\,x^2+b\right)\left(a\,x^4-b\,x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)/x^2/(a*x^2+b)/(a*x^4-b*x^2)^(1/4),x)
```

```
[Out] int((a*x^2-b)/x^2/(a*x^2+b)/(a*x^4-b*x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2-b}{\left(ax^4-bx^2\right)^{\frac{1}{4}}\left(ax^2+b\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/x^2/(a*x^2+b)/(a*x^4-b*x^2)^(1/4),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^2 - b)/((a*x^4 - b*x^2)^(1/4)*(a*x^2 + b)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{b-a\,x^2}{x^2\,\left(a\,x^2+b\right)\,{\left(a\,x^4-b\,x^2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^2)/(x^2*(b + a*x^2)*(a*x^4 - b*x^2)^(1/4)),x)
```

```
[Out] int(-(b - a*x^2)/(x^2*(b + a*x^2)*(a*x^4 - b*x^2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - b}{x^2 \sqrt[4]{x^2 \left(ax^2 - b\right)} \left(ax^2 + b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)/x**2/(a*x**2+b)/(a*x**4-b*x**2)**(1/4),x)
```

```
[Out] Integral((a*x**2 - b)/(x**2*(x**2*(a*x**2 - b))**(1/4)*(a*x**2 + b)), x)
```

3.1435

$$\int \frac{(-b+x)(-6a+b+5x)}{\sqrt[4]{(-a+x)(-b+x)^2}\left(b^6+ad-\left(6b^5+d\right)x+15b^4x^2-20b^3x^3+15b^2x^4-6bx^5+x^6\right)}\,dx$$

Optimal. Leaf size=113

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x(2ab+b^2)-ab^2+x^2(-a-2b)+x^3}}{(b-x)^2}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x(2ab+b^2)-ab^2+x^2(-a-2b)+x^3}}{(b-x)^2}\right)}{d^{3/4}}$$

Rubi [F] time = 28.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-b+x)(-6a+b+5x)}{\sqrt[4]{(-a+x)(-b+x)^2}\left(b^6+ad-\left(6b^5+d\right)x+15b^4x^2-20b^3x^3+15b^2x^4-6bx^5+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-b + x)*(-6*a + b + 5*x))/(((-a + x)*(-b + x)^2)^(1/4)*(b^6 + a*d - (
6*b^5 + d)*x + 15*b^4*x^2 - 20*b^3*x^3 + 15*b^2*x^4 - 6*b*x^5 + x^6)),x]
```

```
[Out] (20*a*(-a + x)^(1/4)*Sqrt[-b + x]*Defer[Subst][Defer[Int][(x^2*Sqrt[a - b +
 x^4])/(a^6*(1 + b^6/a^6) - 6*b^5*(1 + d/(6*b^5))*x^4 + 15*b^4*x^8 - 20*b^3
*x^12 + 15*b^2*x^16 - 6*b*x^20 + x^24 - 6*a^5*(b - x^4) + 15*a^4*(b - x^4)^
2 - 20*a^3*(b - x^4)^3 + 15*a^2*(b - x^4)^4 - 6*a*(b - x^4)^5), x], x, (-a
+ x)^(1/4)]])/(-((a - x)*(b - x)^2))^(1/4) + (20*(-a + x)^(1/4)*Sqrt[-b + x]
*Defer[Subst][Defer[Int][(x^6*Sqrt[a - b + x^4])/(a^6*(1 + b^6/a^6) - 6*b^5
*(1 + d/(6*b^5))*x^4 + 15*b^4*x^8 - 20*b^3*x^12 + 15*b^2*x^16 - 6*b*x^20 +
x^24 - 6*a^5*(b - x^4) + 15*a^4*(b - x^4)^2 - 20*a^3*(b - x^4)^3 + 15*a^2*(
b - x^4)^4 - 6*a*(b - x^4)^5), x], x, (-a + x)^(1/4)]])/(-((a - x)*(b - x)^2
))^(1/4) - (4*(6*a - b)*(-a + x)^(1/4)*Sqrt[-b + x]*Defer[Subst][Defer[Int]
[(x^2*Sqrt[a - b + x^4])/(b^6*(1 + (a*d)/b^6) - (6*b^5 + d)*(a + x^4) + 15*
b^4*(a + x^4)^2 - 20*b^3*(a + x^4)^3 + 15*b^2*(a + x^4)^4 - 6*b*(a + x^4)^5
 + (a + x^4)^6), x], x, (-a + x)^(1/4)]])/(-((a - x)*(b - x)^2))^(1/4)
```

Rubi steps

$$\int \frac{(-b+x)(-6a+b+5x)}{\sqrt[4]{(-a+x)(-b+x)^2}\left(b^6+ad-\left(6b^5+d\right)x+15b^4x^2-20b^3x^3+15b^2x^4-6bx^5+x^6\right)}\,dx = \left(\sqrt[4]{-a+x}\,\sqrt{-b\cdots}\right.$$

$$= \left(\sqrt[4]{-a+x}\,\sqrt{-b\cdots}\right.$$

$$= \left(\sqrt[4]{-a+x}\,\sqrt{-b\cdots}\right.$$

$$= \left(5\sqrt[4]{-a+x}\,\sqrt{-b\cdots}\right.$$

$$= \left(20\sqrt[4]{-a+x}\,\sqrt{-\cdots}\right.$$

$$= \left(20\sqrt[4]{-a+x}\,\sqrt{-\cdots}\right.$$

$$= \left(20\sqrt[4]{-a+x}\,\sqrt{-\cdots}\right.$$

$$= \left(20\sqrt[4]{-a+x}\,\sqrt{-\cdots}\right.$$

$$= \left(20\sqrt[4]{-a+x}\,\sqrt{-\cdots}\right.$$

Mathematica [C] time = 22.59, size = 333050, normalized size = 2947.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-b + x)*(-6*a + b + 5*x))/(((-a + x)*(-b + x)^2)^(1/4)*(b^6 + a
*d - (6*b^5 + d)*x + 15*b^4*x^2 - 20*b^3*x^3 + 15*b^2*x^4 - 6*b*x^5 + x^6))
,x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.46, size = 113, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}}{(b-x)^2}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}}{(b-x)^2}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + x)*(-6*a + b + 5*x))/(((-a + x)*(-b + x)^2)^(1/4)
*(b^6 + a*d - (6*b^5 + d)*x + 15*b^4*x^2 - 20*b^3*x^3 + 15*b^2*x^4 - 6*b*x^
5 + x^6)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/4
))/(b - x)^2])/d^(3/4) - (2*ArcTanh[(d^(1/4)*(-(a*b^2) + (2*a*b + b^2)*x +
(-a - 2*b)*x^2 + x^3)^(1/4))/(b - x)^2])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-6*a+b+5*x)/((-a+x)*(-b+x)^2)^(1/4)/(b^6+a*d-(6*b^5+d)*x+
15*b^4*x^2-20*b^3*x^3+15*b^2*x^4-6*b*x^5+x^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(6\,a-b-5\,x)(b-x)}{\left(b^6+15\,b^4x^2-20\,b^3x^3+15\,b^2x^4-6\,bx^5+x^6+ad-\left(6\,b^5+d\right)x\right)\left(-(a-x)(b-x)^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-6*a+b+5*x)/((-a+x)*(-b+x)^2)^(1/4)/(b^6+a*d-(6*b^5+d)*x+
15*b^4*x^2-20*b^3*x^3+15*b^2*x^4-6*b*x^5+x^6),x, algorithm="giac")
```

```
[Out] integrate((6*a - b - 5*x)*(b - x)/((b^6 + 15*b^4*x^2 - 20*b^3*x^3 + 15*b^2*
x^4 - 6*b*x^5 + x^6 + a*d - (6*b^5 + d)*x)*(-(a - x)*(b - x)^2)^(1/4)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)\,(-6a+b+5x)}{\left((-a+x)\,(-b+x)^2\right)^{\frac{1}{4}}\left(b^6+ad-\left(6b^5+d\right)x+15b^4x^2-20b^3x^3+15b^2x^4-6b\,x^5+x^6\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b+x)*(-6*a+b+5*x)/((-a+x)*(-b+x)^2)^(1/4)/(b^6+a*d-(6*b^5+d)*x+15*b^4
*x^2-20*b^3*x^3+15*b^2*x^4-6*b*x^5+x^6),x)
```

```
[Out] int((-b+x)*(-6*a+b+5*x)/((-a+x)*(-b+x)^2)^(1/4)/(b^6+a*d-(6*b^5+d)*x+15*b^4
*x^2-20*b^3*x^3+15*b^2*x^4-6*b*x^5+x^6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(6\,a-b-5\,x)(b-x)}{\left(b^6+15\,b^4x^2-20\,b^3x^3+15\,b^2x^4-6\,bx^5+x^6+ad-\left(6\,b^5+d\right)x\right)\left(-(a-x)(b-x)^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-6*a+b+5*x)/((-a+x)*(-b+x)^2)^(1/4)/(b^6+a*d-(6*b^5+d)*x+
15*b^4*x^2-20*b^3*x^3+15*b^2*x^4-6*b*x^5+x^6),x, algorithm="maxima")
```

```
[Out] integrate((6*a - b - 5*x)*(b - x)/((b^6 + 15*b^4*x^2 - 20*b^3*x^3 + 15*b^2*
x^4 - 6*b*x^5 + x^6 + a*d - (6*b^5 + d)*x)*(-(a - x)*(b - x)^2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(b-x)\,(b-6\,a+5\,x)}{{\left(-(a-x)\,{(b-x)}^2\right)}^{1/4}\,\left(a\,d-6\,b\,x^5-x\,\left(6\,b^5+d\right)+b^6+x^6+15\,b^2\,x^4-20\,b^3\,x^3+15\,b^4\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b - x)*(b - 6*a + 5*x))/((-(a - x)*(b - x)^2)^(1/4)*(a*d - 6*b*x^5 -
 x*(d + 6*b^5) + b^6 + x^6 + 15*b^2*x^4 - 20*b^3*x^3 + 15*b^4*x^2)),x)
```

```
[Out] int(-((b - x)*(b - 6*a + 5*x))/((-(a - x)*(b - x)^2)^(1/4)*(a*d - 6*b*x^5 -
 x*(d + 6*b^5) + b^6 + x^6 + 15*b^2*x^4 - 20*b^3*x^3 + 15*b^4*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)\,(-6a+b+5x)}{\sqrt[4]{(-a+x)\,(-b+x)^2}\,\left(ad+b^6-6b^5x+15b^4x^2-20b^3x^3+15b^2x^4-6bx^5-dx+x^6\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-6*a+b+5*x)/((-a+x)*(-b+x)**2)**(1/4)/(b**6+a*d-(6*b**5+d
)*x+15*b**4*x**2-20*b**3*x**3+15*b**2*x**4-6*b*x**5+x**6),x)
```

```
[Out] Integral((-b + x)*(-6*a + b + 5*x)/(((-a + x)*(-b + x)**2)**(1/4)*(a*d + b*
*6 - 6*b**5*x + 15*b**4*x**2 - 20*b**3*x**3 + 15*b**2*x**4 - 6*b*x**5 - d*x
 + x**6)), x)
```

3.1436

$$\int \frac{(-6a+b+5x)\left(-b^5+5b^4x-10b^3x^2+10b^2x^3-5bx^4+x^5\right)}{\left((-a+x)(-b+x)^2\right)^{3/4}\left(a+b^6d-\left(1+6b^5d\right)x+15b^4dx^2-20b^3dx^3+15b^2dx^4-6bdx^5+dx^6\right)}\,dx$$

Optimal. Leaf size=113

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{3/4}}{a-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{3/4}}{a-x}\right)}{d^{3/4}}$$

Rubi [F] time = 52.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-6a+b+5x)\left(-b^5+5b^4x-10b^3x^2+10b^2x^3-5bx^4+x^5\right)}{\left((-a+x)(-b+x)^2\right)^{3/4}\left(a+b^6d-\left(1+6b^5d\right)x+15b^4dx^2-20b^3dx^3+15b^2dx^4-6bdx^5+dx^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-6*a + b + 5*x)*(-b^5 + 5*b^4*x - 10*b^3*x^2 + 10*b^2*x^3 - 5*b*x^4 +
 x^5))/(((-a + x)*(-b + x)^2)^(3/4)*(a + b^6*d - (1 + 6*b^5*d)*x + 15*b^4*d
*x^2 - 20*b^3*d*x^3 + 15*b^2*d*x^4 - 6*b*d*x^5 + d*x^6)),x]
```

```
[Out] (20*a*(-a + x)^(3/4)*(-b + x)^(3/2)*Defer[Subst][Defer[Int][(a - b + x^4)^(
7/2)/(a^6*(1 + (b*(-6*a^5 + 15*a^4*b - 20*a^3*b^2 + 15*a^2*b^3 - 6*a*b^4 +
b^5))/a^6)*d - (1 - 6*(a - b)^5*d)*x^4 + 15*a^4*(1 + (b*(-4*a^3 + 6*a^2*b -
 4*a*b^2 + b^3))/a^4)*d*x^8 + 20*a^3*(1 - (b*(3*a^2 - 3*a*b + b^2))/a^3)*d*
x^12 + 15*a^2*(1 + (b*(-2*a + b))/a^2)*d*x^16 + 6*a*(1 - b/a)*d*x^20 + d*x^
24), x], x, (-a + x)^(1/4)])/(-((a - x)*(b - x)^2))^(3/4) + (20*(-a + x)^(3
/4)*(-b + x)^(3/2)*Defer[Subst][Defer[Int][(x^4*(a - b + x^4)^(7/2))/(a^6*(
1 + (b*(-6*a^5 + 15*a^4*b - 20*a^3*b^2 + 15*a^2*b^3 - 6*a*b^4 + b^5))/a^6)*
d - (1 - 6*(a - b)^5*d)*x^4 + 15*a^4*(1 + (b*(-4*a^3 + 6*a^2*b - 4*a*b^2 +
b^3))/a^4)*d*x^8 + 20*a^3*(1 - (b*(3*a^2 - 3*a*b + b^2))/a^3)*d*x^12 + 15*a
^2*(1 + (b*(-2*a + b))/a^2)*d*x^16 + 6*a*(1 - b/a)*d*x^20 + d*x^24), x], x,
 (-a + x)^(1/4)])/(-((a - x)*(b - x)^2))^(3/4) - (4*(6*a - b)*(-a + x)^(3/4
)*(-b + x)^(3/2)*Defer[Subst][Defer[Int][(a - b + x^4)^(7/2)/(a*(1 + (b^6*d
)/a) - (1 + 6*b^5*d)*(a + x^4) + 15*b^4*d*(a + x^4)^2 - 20*b^3*d*(a + x^4)^
3 + 15*b^2*d*(a + x^4)^4 - 6*b*d*(a + x^4)^5 + d*(a + x^4)^6), x], x, (-a +
 x)^(1/4)])/(-((a - x)*(b - x)^2))^(3/4)
```

Rubi steps

$$\int \frac{(-6a+b+5x)\left(-b^5+5b^4x-10b^3x^2+10b^2x^3-5bx^4+x^5\right)}{\left((-a+x)(-b+x)^2\right)^{3/4}\left(a+b^6d-\left(1+6b^5d\right)x+15b^4dx^2-20b^3dx^3+15b^2dx^4-6bdx^5+dx^6\right)}\,dx = \frac{\left((-a+\right.}{}$$

$$= \frac{\left((-a+\right.}{}$$

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$$= \frac{\left(20(-\right.}{}$$

Mathematica [F] time = 2.35, size = 0, normalized size = 0.00

$$\int \frac{(-6a+b+5x)\left(-b^5+5b^4x-10b^3x^2+10b^2x^3-5bx^4+x^5\right)}{\left((-a+x)(-b+x)^2\right)^{3/4}\left(a+b^6d-\left(1+6b^5d\right)x+15b^4dx^2-20b^3dx^3+15b^2dx^4-6bdx^5+dx^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-6*a + b + 5*x)*(-b^5 + 5*b^4*x - 10*b^3*x^2 + 10*b^2*x^3 - 5*b
*x^4 + x^5))/(((-a + x)*(-b + x)^2)^(3/4)*(a + b^6*d - (1 + 6*b^5*d)*x + 15
*b^4*d*x^2 - 20*b^3*d*x^3 + 15*b^2*d*x^4 - 6*b*d*x^5 + d*x^6)),x]
```

```
[Out] Integrate[((-6*a + b + 5*x)*(-b^5 + 5*b^4*x - 10*b^3*x^2 + 10*b^2*x^3 - 5*b
*x^4 + x^5))/(((-a + x)*(-b + x)^2)^(3/4)*(a + b^6*d - (1 + 6*b^5*d)*x + 15
*b^4*d*x^2 - 20*b^3*d*x^3 + 15*b^2*d*x^4 - 6*b*d*x^5 + d*x^6)), x]
```

IntegrateAlgebraic [A] time = 0.48, size = 113, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{3/4}}{a-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{3/4}}{a-x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-6*a + b + 5*x)*(-b^5 + 5*b^4*x - 10*b^3*x^2 + 10*b^2*
x^3 - 5*b*x^4 + x^5))/(((-a + x)*(-b + x)^2)^(3/4)*(a + b^6*d - (1 + 6*b^5*
d)*x + 15*b^4*d*x^2 - 20*b^3*d*x^3 + 15*b^2*d*x^4 - 6*b*d*x^5 + d*x^6)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(3/
4))/(a - x)])/d^(3/4) + (2*ArcTanh[(d^(1/4)*(-(a*b^2) + (2*a*b + b^2)*x + (
-a - 2*b)*x^2 + x^3)^(3/4))/(a - x)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-6*a+b+5*x)*(-b^5+5*b^4*x-10*b^3*x^2+10*b^2*x^3-5*b*x^4+x^5)/((-
a+x)*(-b+x)^2)^(3/4)/(a+b^6*d-(6*b^5*d+1)*x+15*b^4*d*x^2-20*b^3*d*x^3+15*b^
2*d*x^4-6*b*d*x^5+d*x^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^5-5\,b^4x+10\,b^3x^2-10\,b^2x^3+5\,bx^4-x^5\right)(6\,a-b-5\,x)}{\left(b^6d+15\,b^4dx^2-20\,b^3dx^3+15\,b^2dx^4-6\,bdx^5+dx^6-\left(6\,b^5d+1\right)x+a\right)\left(-(a-x)(b-x)^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-6*a+b+5*x)*(-b^5+5*b^4*x-10*b^3*x^2+10*b^2*x^3-5*b*x^4+x^5)/((-
a+x)*(-b+x)^2)^(3/4)/(a+b^6*d-(6*b^5*d+1)*x+15*b^4*d*x^2-20*b^3*d*x^3+15*b^
2*d*x^4-6*b*d*x^5+d*x^6),x, algorithm="giac")
```

```
[Out] integrate((b^5 - 5*b^4*x + 10*b^3*x^2 - 10*b^2*x^3 + 5*b*x^4 - x^5)*(6*a -
b - 5*x)/((b^6*d + 15*b^4*d*x^2 - 20*b^3*d*x^3 + 15*b^2*d*x^4 - 6*b*d*x^5 +
 d*x^6 - (6*b^5*d + 1)*x + a)*(-(a - x)*(b - x)^2)^(3/4)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(-6a+b+5x)\left(-b^5+5b^4x-10b^3x^2+10b^2x^3-5b\,x^4+x^5\right)}{\left((-a+x)(-b+x)^2\right)^{\frac{3}{4}}\left(a+b^6d-\left(6b^5d+1\right)x+15b^4d\,x^2-20b^3d\,x^3+15b^2d\,x^4-6bd\,x^5+d\,x^6\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-6*a+b+5*x)*(-b^5+5*b^4*x-10*b^3*x^2+10*b^2*x^3-5*b*x^4+x^5)/((-a+x)*(
-b+x)^2)^(3/4)/(a+b^6*d-(6*b^5*d+1)*x+15*b^4*d*x^2-20*b^3*d*x^3+15*b^2*d*x^
4-6*b*d*x^5+d*x^6),x)
```

```
[Out] int((-6*a+b+5*x)*(-b^5+5*b^4*x-10*b^3*x^2+10*b^2*x^3-5*b*x^4+x^5)/((-a+x)*(
-b+x)^2)^(3/4)/(a+b^6*d-(6*b^5*d+1)*x+15*b^4*d*x^2-20*b^3*d*x^3+15*b^2*d*x^
4-6*b*d*x^5+d*x^6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^5 - 5\,b^4x + 10\,b^3x^2 - 10\,b^2x^3 + 5\,bx^4 - x^5\right)(6\,a - b - 5\,x)}{\left(b^6d + 15\,b^4dx^2 - 20\,b^3dx^3 + 15\,b^2dx^4 - 6\,bdx^5 + dx^6 - \left(6\,b^5d + 1\right)x + a\right)\left(-(a-x)(b-x)^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-6*a+b+5*x)*(-b^5+5*b^4*x-10*b^3*x^2+10*b^2*x^3-5*b*x^4+x^5)/((-
a+x)*(-b+x)^2)^(3/4)/(a+b^6*d-(6*b^5*d+1)*x+15*b^4*d*x^2-20*b^3*d*x^3+15*b^
2*d*x^4-6*b*d*x^5+d*x^6),x, algorithm="maxima")
```

```
[Out] integrate((b^5 - 5*b^4*x + 10*b^3*x^2 - 10*b^2*x^3 + 5*b*x^4 - x^5)*(6*a -
b - 5*x)/((b^6*d + 15*b^4*d*x^2 - 20*b^3*d*x^3 + 15*b^2*d*x^4 - 6*b*d*x^5 +
 d*x^6 - (6*b^5*d + 1)*x + a)*(-(a - x)*(b - x)^2)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(b-6\,a+5\,x)\,\left(b^5-5\,b^4\,x+10\,b^3\,x^2-10\,b^2\,x^3+5\,b\,x^4-x^5\right)}{\left(-(a-x)\,(b-x)^2\right)^{3/4}\,\left(a+b^6\,d+d\,x^6-x\,\left(6\,d\,b^5+1\right)+15\,b^2\,d\,x^4-20\,b^3\,d\,x^3+15\,b^4\,d\,x^2-6\,b\,d\,x^5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b - 6*a + 5*x)*(5*b*x^4 - 5*b^4*x + b^5 - x^5 - 10*b^2*x^3 + 10*b^3*
x^2))/((-(a - x)*(b - x)^2)^(3/4)*(a + b^6*d + d*x^6 - x*(6*b^5*d + 1) + 15
*b^2*d*x^4 - 20*b^3*d*x^3 + 15*b^4*d*x^2 - 6*b*d*x^5)),x)
```

```
[Out] int(-((b - 6*a + 5*x)*(5*b*x^4 - 5*b^4*x + b^5 - x^5 - 10*b^2*x^3 + 10*b^3*
x^2))/((-(a - x)*(b - x)^2)^(3/4)*(a + b^6*d + d*x^6 - x*(6*b^5*d + 1) + 15
*b^2*d*x^4 - 20*b^3*d*x^3 + 15*b^4*d*x^2 - 6*b*d*x^5)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-6*a+b+5*x)*(-b**5+5*b**4*x-10*b**3*x**2+10*b**2*x**3-5*b*x**4+x
**5)/((-a+x)*(-b+x)**2)**(3/4)/(a+b**6*d-(6*b**5*d+1)*x+15*b**4*d*x**2-20*b
**3*d*x**3+15*b**2*d*x**4-6*b*d*x**5+d*x**6),x)
```

```
[Out] Timed out
```

3.1437 $\int \frac{x^6(4+x^3)}{(1+x^3)^{3/4}(-1-2x^3-x^6+x^8)}\,dx$

Optimal. Leaf size=113

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^3+1}}\right)-\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^3+1}}\right)-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{x^3+1}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{x^3+1}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^3+1}}{\sqrt{x^3+1}+x^2}\right)}{\sqrt{2}}$$

Rubi [F] time = 2.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^6\left(4+x^3\right)}{\left(1+x^3\right)^{3/4}\left(-1-2x^3-x^6+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^6*(4 + x^3))/((1 + x^3)^(3/4)*(-1 - 2*x^3 - x^6 + x^8)),x]
```

```
[Out] (x^2*Hypergeometric2F1[2/3, 3/4, 5/3, -x^3])/2 + Defer[Int][1/((1 + x^3)^(3
/4)*(-1 - x^3 + x^4)), x]/2 + Defer[Int][x/((1 + x^3)^(3/4)*(-1 - x^3 + x^4
)), x]/2 + 2*Defer[Int][x^2/((1 + x^3)^(3/4)*(-1 - x^3 + x^4)), x] + Defer[
Int][x^3/((1 + x^3)^(3/4)*(-1 - x^3 + x^4)), x]/2 + Defer[Int][1/((1 + x^3)
^(3/4)*(1 + x^3 + x^4)), x]/2 - Defer[Int][x/((1 + x^3)^(3/4)*(1 + x^3 + x^
4)), x]/2 + 2*Defer[Int][x^2/((1 + x^3)^(3/4)*(1 + x^3 + x^4)), x] + Defer[
Int][x^3/((1 + x^3)^(3/4)*(1 + x^3 + x^4)), x]/2
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6\left(4+x^3\right)}{\left(1+x^3\right)^{3/4}\left(-1-2x^3-x^6+x^8\right)}\,dx &= \int \left(\frac{x}{\left(1+x^3\right)^{3/4}}+\frac{x\left(1+2x^3+4x^5+x^6\right)}{\left(1+x^3\right)^{3/4}\left(-1-2x^3-x^6+x^8\right)}\right)dx\\
&= \int \frac{x}{\left(1+x^3\right)^{3/4}}\,dx+\int \frac{x\left(1+2x^3+4x^5+x^6\right)}{\left(1+x^3\right)^{3/4}\left(-1-2x^3-x^6+x^8\right)}\,dx\\
&= \frac{1}{2}x^2\,{}_2F_1\left(\frac{2}{3},\frac{3}{4};\frac{5}{3};-x^3\right)+\int \left(\frac{1+x+4x^2+x^3}{2\left(1+x^3\right)^{3/4}\left(-1-x^3+x^4\right)}+\frac{1-x+}{2\left(1+x^3\right)^{3/4}}\right.\\
&= \frac{1}{2}x^2\,{}_2F_1\left(\frac{2}{3},\frac{3}{4};\frac{5}{3};-x^3\right)+\frac{1}{2}\int \frac{1+x+4x^2+x^3}{\left(1+x^3\right)^{3/4}\left(-1-x^3+x^4\right)}\,dx+\frac{1}{2}\int \frac{1}{(1+}\\
&= \frac{1}{2}x^2\,{}_2F_1\left(\frac{2}{3},\frac{3}{4};\frac{5}{3};-x^3\right)+\frac{1}{2}\int \left(\frac{1}{\left(1+x^3\right)^{3/4}\left(-1-x^3+x^4\right)}+\frac{}{\left(1+x^3\right)^{3/4}}\right.\\
&= \frac{1}{2}x^2\,{}_2F_1\left(\frac{2}{3},\frac{3}{4};\frac{5}{3};-x^3\right)+\frac{1}{2}\int \frac{1}{\left(1+x^3\right)^{3/4}\left(-1-x^3+x^4\right)}\,dx+\frac{1}{2}\int \frac{}{(1+}
\end{aligned}$$

Mathematica [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(4+x^3\right)}{\left(1+x^3\right)^{3/4}\left(-1-2x^3-x^6+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^6*(4 + x^3))/((1 + x^3)^(3/4)*(-1 - 2*x^3 - x^6 + x^8)),x]
```

```
[Out] Integrate[(x^6*(4 + x^3))/((1 + x^3)^(3/4)*(-1 - 2*x^3 - x^6 + x^8)), x]
```

IntegrateAlgebraic [A] time = 3.77, size = 113, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^3+1}}\right)-\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^3+1}}\right)-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{x^3+1}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{x^3+1}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt[4]{x^3+1}}{\sqrt{x^3+1}+x^2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^6*(4 + x^3))/((1 + x^3)^(3/4)*(-1 - 2*x^3 - x^6 + x^8
)),x]
```

```
[Out] ArcTan[x/(1 + x^3)^(1/4)] - ArcTan[(-(x^2/Sqrt[2]) + Sqrt[1 + x^3]/Sqrt[2])
/(x*(1 + x^3)^(1/4))]/Sqrt[2] - ArcTanh[x/(1 + x^3)^(1/4)] - ArcTanh[(Sqrt[
2]*x*(1 + x^3)^(1/4))/(x^2 + Sqrt[1 + x^3])]/Sqrt[2]
```

fricas [B] time = 1.30, size = 236, normalized size = 2.09

$$\sqrt{2}\arctan\left(\frac{\sqrt{2}x\sqrt{\frac{x^2+\sqrt{2}(x^3+1)^{\frac{1}{4}}x+\sqrt{x^3+1}}{x^2}}-x-\sqrt{2}(x^3+1)^{\frac{1}{4}}}{x}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{2}x\sqrt{\frac{x^2-\sqrt{2}(x^3+1)^{\frac{1}{4}}x+\sqrt{x^3+1}}{x^2}}+x-\sqrt{2}(x^3+1)^{\frac{1}{4}}}{x}\right)-\frac{1}{4}\sqrt{2}\log\left(\frac{4\left(x^2+\sqrt{2}(x^3+1)^{\frac{1}{4}}x+\sqrt{x^3+1}\right)}{x^2}\right)+\frac{1}{4}\sqrt{2}\log\left(\frac{4\left(x^2-\sqrt{2}(x^3+1)^{\frac{1}{4}}x+\sqrt{x^3+1}\right)}{x^2}\right)-\arctan\left(\frac{(x^3+1)^{\frac{1}{4}}}{x}\right)-\frac{1}{2}\log\left(\frac{x+(x^3+1)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\log\left(-\frac{x-(x^3+1)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^3+4)/(x^3+1)^(3/4)/(x^8-x^6-2*x^3-1),x, algorithm="fricas"
)
```

```
[Out] sqrt(2)*arctan((sqrt(2)*x*sqrt((x^2 + sqrt(2)*(x^3 + 1)^(1/4)*x + sqrt(x^3
+ 1))/x^2) - x - sqrt(2)*(x^3 + 1)^(1/4))/x) + sqrt(2)*arctan((sqrt(2)*x*sq
rt((x^2 - sqrt(2)*(x^3 + 1)^(1/4)*x + sqrt(x^3 + 1))/x^2) + x - sqrt(2)*(x^
3 + 1)^(1/4))/x) - 1/4*sqrt(2)*log(4*(x^2 + sqrt(2)*(x^3 + 1)^(1/4)*x + sqr
t(x^3 + 1))/x^2) + 1/4*sqrt(2)*log(4*(x^2 - sqrt(2)*(x^3 + 1)^(1/4)*x + sqr
t(x^3 + 1))/x^2) - arctan((x^3 + 1)^(1/4)/x) - 1/2*log((x + (x^3 + 1)^(1/4)
)/x) + 1/2*log(-(x - (x^3 + 1)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^3+4)x^6}{(x^8-x^6-2x^3-1)(x^3+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^3+4)/(x^3+1)^(3/4)/(x^8-x^6-2*x^3-1),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 4)*x^6/((x^8 - x^6 - 2*x^3 - 1)*(x^3 + 1)^(3/4)), x)
```

maple [C] time = 3.18, size = 425, normalized size = 3.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^3+4)/(x^3+1)^(3/4)/(x^8-x^6-2*x^3-1),x)
```

```
[Out] -1/2*ln(-(2*(x^3+1)^(3/4)*x+2*x^2*(x^3+1)^(1/2)+2*(x^3+1)^(1/4)*x^3+x^4+x^3
+1)/(x^4-x^3-1))+1/2*RootOf(_Z^2+1)*ln((2*RootOf(_Z^2+1)*(x^3+1)^(1/2)*x^2-
RootOf(_Z^2+1)*x^4-RootOf(_Z^2+1)*x^3-2*(x^3+1)^(3/4)*x+2*(x^3+1)^(1/4)*x^3
```

```
-RootOf(_Z^2+1))/(x^4-x^3-1))-1/2*RootOf(_Z^2+RootOf(_Z^2+1))*ln(-(RootOf(_
Z^2+RootOf(_Z^2+1))*RootOf(_Z^2+1)*x^4-RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_
Z^2+1)*x^3-2*RootOf(_Z^2+RootOf(_Z^2+1))*(x^3+1)^(1/2)*x^2+2*RootOf(_Z^2+1)
*(x^3+1)^(1/4)*x^3-2*(x^3+1)^(3/4)*x-RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2
+1)))/(x^4+x^3+1))-1/2*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))*ln((2*Roo
tOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))*(x^3+1)^(1/2)*x^2-RootOf(_Z^2+RootO
f(_Z^2+1))*x^4+2*RootOf(_Z^2+1)*(x^3+1)^(1/4)*x^3+RootOf(_Z^2+RootOf(_Z^2+1
))*x^3+2*(x^3+1)^(3/4)*x+RootOf(_Z^2+RootOf(_Z^2+1)))/(x^4+x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+4\right)x^6}{\left(x^8-x^6-2\,x^3-1\right)\left(x^3+1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^3+4)/(x^3+1)^(3/4)/(x^8-x^6-2*x^3-1),x, algorithm="maxima"
)
```

```
[Out] integrate((x^3 + 4)*x^6/((x^8 - x^6 - 2*x^3 - 1)*(x^3 + 1)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^6\,\left(x^3+4\right)}{{\left(x^3+1\right)}^{3/4}\,\left(-x^8+x^6+2\,x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^6*(x^3 + 4))/((x^3 + 1)^(3/4)*(2*x^3 + x^6 - x^8 + 1)),x)
```

```
[Out] int(-(x^6*(x^3 + 4))/((x^3 + 1)^(3/4)*(2*x^3 + x^6 - x^8 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(x**3+4)/(x**3+1)**(3/4)/(x**8-x**6-2*x**3-1),x)
```

```
[Out] Timed out
```

3.1438 $\int x^{10}\sqrt[3]{-1+x^3}\,dx$

Optimal. Leaf size=114

$$\frac{10}{729}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{10\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{243\sqrt{3}}-\frac{5}{729}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)+\frac{1}{972}\sqrt[3]{x^3-1}\left(81x^{11}-9\right.$$

Rubi [A] time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {279, 321, 331, 292, 31, 634, 618, 204, 628}

$$\frac{10}{729}\log\left(1-\frac{x}{\sqrt[3]{x^3-1}}\right)+\frac{10\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{243\sqrt{3}}+\frac{1}{12}\sqrt[3]{x^3-1}\,x^{11}-\frac{1}{108}\sqrt[3]{x^3-1}\,x^8-\frac{1}{81}\sqrt[3]{x^3-1}\,x^5-\frac{5}{243}\sqrt[3]{x^3-1}\,x^2-\frac{5}{729}\log\left(\frac{x}{\sqrt[3]{x^3-1}}+\frac{x^2}{\left(x^3-1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(-1 + x^3)^(1/3),x]
```

```
[Out] (-5*x^2*(-1 + x^3)^(1/3))/243 - (x^5*(-1 + x^3)^(1/3))/81 - (x^8*(-1 + x^3)
^(1/3))/108 + (x^11*(-1 + x^3)^(1/3))/12 + (10*ArcTan[(1 + (2*x)/(-1 + x^3)
^(1/3))/Sqrt[3]])/(243*Sqrt[3]) + (10*Log[1 - x/(-1 + x^3)^(1/3)])/729 - (5
*Log[1 + x^2/(-1 + x^3)^(2/3) + x/(-1 + x^3)^(1/3)])/729
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int x^{10}\sqrt[3]{-1+x^3}\,dx = \frac{1}{12}x^{11}\sqrt[3]{-1+x^3} - \frac{1}{12}\int \frac{x^{10}}{\left(-1+x^3\right)^{2/3}}\,dx$$

$$= -\frac{1}{108}x^8\sqrt[3]{-1+x^3} + \frac{1}{12}x^{11}\sqrt[3]{-1+x^3} - \frac{2}{27}\int \frac{x^7}{\left(-1+x^3\right)^{2/3}}\,dx$$

$$= -\frac{1}{81}x^5\sqrt[3]{-1+x^3} - \frac{1}{108}x^8\sqrt[3]{-1+x^3} + \frac{1}{12}x^{11}\sqrt[3]{-1+x^3} - \frac{5}{81}\int \frac{x^4}{\left(-1+x^3\right)^{2/3}}\,dx$$

$$= -\frac{5}{243}x^2\sqrt[3]{-1+x^3} - \frac{1}{81}x^5\sqrt[3]{-1+x^3} - \frac{1}{108}x^8\sqrt[3]{-1+x^3} + \frac{1}{12}x^{11}\sqrt[3]{-1+x^3} - \frac{10}{243}\int \frac{x}{(-1+}$$

$$= -\frac{5}{243}x^2\sqrt[3]{-1+x^3} - \frac{1}{81}x^5\sqrt[3]{-1+x^3} - \frac{1}{108}x^8\sqrt[3]{-1+x^3} + \frac{1}{12}x^{11}\sqrt[3]{-1+x^3} - \frac{10}{243}\text{Subst}\Big(\int$$

$$= -\frac{5}{243}x^2\sqrt[3]{-1+x^3} - \frac{1}{81}x^5\sqrt[3]{-1+x^3} - \frac{1}{108}x^8\sqrt[3]{-1+x^3} + \frac{1}{12}x^{11}\sqrt[3]{-1+x^3} - \frac{10}{729}\text{Subst}\Big(\int$$

$$= -\frac{5}{243}x^2\sqrt[3]{-1+x^3} - \frac{1}{81}x^5\sqrt[3]{-1+x^3} - \frac{1}{108}x^8\sqrt[3]{-1+x^3} + \frac{1}{12}x^{11}\sqrt[3]{-1+x^3} + \frac{10}{729}\log\Big(1-$$

$$= -\frac{5}{243}x^2\sqrt[3]{-1+x^3} - \frac{1}{81}x^5\sqrt[3]{-1+x^3} - \frac{1}{108}x^8\sqrt[3]{-1+x^3} + \frac{1}{12}x^{11}\sqrt[3]{-1+x^3} + \frac{10}{729}\log\Big(1-$$

$$= -\frac{5}{243}x^2\sqrt[3]{-1+x^3} - \frac{1}{81}x^5\sqrt[3]{-1+x^3} - \frac{1}{108}x^8\sqrt[3]{-1+x^3} + \frac{1}{12}x^{11}\sqrt[3]{-1+x^3} + \frac{10\tan^{-1}\Big(\cdots}{243\sqrt{3}}$$

Mathematica [C] time = 0.03, size = 72, normalized size = 0.63

$$\frac{x^2\sqrt[3]{x^3-1}\left(20\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\sqrt[3]{1-x^3}\left(27x^9-3x^6-4x^3-20\right)\right)}{324\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(-1 + x^3)^(1/3),x]
```

```
[Out] (x^2*(-1 + x^3)^(1/3)*((1 - x^3)^(1/3)*(-20 - 4*x^3 - 3*x^6 + 27*x^9) + 20*
Hypergeometric2F1[-1/3, 2/3, 5/3, x^3]))/(324*(1 - x^3)^(1/3))
```

IntegrateAlgebraic [A] time = 0.33, size = 114, normalized size = 1.00

$$\frac{10}{729}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{10\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{243\sqrt{3}}-\frac{5}{729}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)+\frac{1}{972}\sqrt[3]{x^3-1}\left(81x^{11}-9x^8-12x^5-20x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^10*(-1 + x^3)^(1/3),x]
```

```
[Out] ((-1 + x^3)^(1/3)*(-20*x^2 - 12*x^5 - 9*x^8 + 81*x^11))/972 + (10*ArcTan[(S
qrt[3]*x)/(x + 2*(-1 + x^3)^(1/3))])/(243*Sqrt[3]) + (10*Log[-x + (-1 + x^3
)^(1/3)])/729 - (5*Log[x^2 + x*(-1 + x^3)^(1/3) + (-1 + x^3)^(2/3)])/729
```

fricas [A] time = 0.44, size = 106, normalized size = 0.93

$$-\frac{10}{729}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{972}\left(81x^{11}-9x^8-12x^5-20x^2\right)\left(x^3-1\right)^{\frac{1}{3}}+\frac{10}{729}\log\left(-\frac{x-\left(x^3-1\right)^{\frac{1}{3}}}{x}\right)-\frac{5}{729}\log\left(\frac{x^2+\left(x^3-1\right)^{\frac{1}{3}}x+\left(x^3-1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(x^3-1)^(1/3),x, algorithm="fricas")
```

```
[Out] -10/729*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 - 1)^(1/3))/x) + 1/9
72*(81*x^11 - 9*x^8 - 12*x^5 - 20*x^2)*(x^3 - 1)^(1/3) + 10/729*log(-(x - (
x^3 - 1)^(1/3))/x) - 5/729*log((x^2 + (x^3 - 1)^(1/3)*x + (x^3 - 1)^(2/3))/
x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^3-1\right)^{\frac{1}{3}}x^{10}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(x^3-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 1)^(1/3)*x^10, x)
```

maple [C] time = 0.50, size = 63, normalized size = 0.55

$$\frac{x^2\left(81x^9-9x^6-12x^3-20\right)\left(x^3-1\right)^{\frac{1}{3}}}{972}-\frac{5\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{2}{3}}x^2\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{243\mathrm{signum}\left(x^3-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(x^3-1)^(1/3),x)
```

```
[Out] 1/972*x^2*(81*x^9-9*x^6-12*x^3-20)*(x^3-1)^(1/3)-5/243/signum(x^3-1)^(2/3)*
(-signum(x^3-1))^(2/3)*x^2*hypergeom([2/3,2/3],[5/3],x^3)
```

maxima [A] time = 0.43, size = 170, normalized size = 1.49

$$-\frac{10}{729}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(\frac{2\,{\left(x^{3}-1\right)}^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\frac{40\,{\left(x^{3}-1\right)}^{\frac{1}{3}}}{x}+\frac{93\,{\left(x^{3}-1\right)}^{\frac{4}{3}}}{x^{4}}-\frac{72\,{\left(x^{3}-1\right)}^{\frac{7}{3}}}{x^{7}}+\frac{20\,{\left(x^{3}-1\right)}^{\frac{10}{3}}}{x^{10}}}{972\,{\left(\frac{4\,{\left(x^{3}-1\right)}}{x^{3}}-\frac{6\,{\left(x^{3}-1\right)}^{2}}{x^{6}}+\frac{4\,{\left(x^{3}-1\right)}^{3}}{x^{9}}-\frac{{\left(x^{3}-1\right)}^{4}}{x^{12}}-1\right)}}-\frac{5}{729}\,\log\left(\frac{{\left(x^{3}-1\right)}^{\frac{1}{3}}}{x}+\frac{{\left(x^{3}-1\right)}^{\frac{2}{3}}}{x^{2}}+1\right)+\frac{10}{729}\,\log\left(\frac{{\left(x^{3}-1\right)}^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(x^3-1)^(1/3),x, algorithm="maxima")
```

```
[Out] -10/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3)/x + 1)) - 1/972*(40*(
x^3 - 1)^(1/3)/x + 93*(x^3 - 1)^(4/3)/x^4 - 72*(x^3 - 1)^(7/3)/x^7 + 20*(x^
3 - 1)^(10/3)/x^10)/(4*(x^3 - 1)/x^3 - 6*(x^3 - 1)^2/x^6 + 4*(x^3 - 1)^3/x^
9 - (x^3 - 1)^4/x^12 - 1) - 5/729*log((x^3 - 1)^(1/3)/x + (x^3 - 1)^(2/3)/x
^2 + 1) + 10/729*log((x^3 - 1)^(1/3)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{10}\,{\left(x^3-1\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(x^3 - 1)^(1/3),x)
```

```
[Out] int(x^10*(x^3 - 1)^(1/3), x)
```

sympy [C] time = 1.62, size = 32, normalized size = 0.28

$$\frac{x^{11} e^{\frac{i\pi}{3}} \Gamma\left(\frac{11}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{11}{3} \\ \frac{14}{3} \end{matrix}\middle| {x^{3}} \right)}{3 \Gamma\left(\frac{14}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(x**3-1)**(1/3),x)
```

```
[Out] x**11*exp(I*pi/3)*gamma(11/3)*hyper((-1/3, 11/3), (14/3,), x**3)/(3*gamma(1
4/3))
```

3.1439 $\int \frac{x}{(-1+x)\sqrt[3]{-x^2+x^3}}\,dx$

Optimal. Leaf size=114

$$-\frac{3\left(x^3-x^2\right)^{2/3}}{(x-1)x}-\log\left(\sqrt[3]{x^3-x^2}-x\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-x^2}+x}\right)$$

Rubi [A] time = 0.09, antiderivative size = 151, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2056, 47, 59}

$$-\frac{3x}{\sqrt[3]{x^3-x^2}}-\frac{3\sqrt[3]{x-1}\,x^{2/3}\log\left(\frac{\sqrt[3]{x-1}}{\sqrt[3]{x}}-1\right)}{2\sqrt[3]{x^3-x^2}}-\frac{\sqrt[3]{x-1}\,x^{2/3}\log(x)}{2\sqrt[3]{x^3-x^2}}-\frac{\sqrt{3}\sqrt[3]{x-1}\,x^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{x-1}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{x^3-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((-1 + x)*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (-3*x)/(-x^2 + x^3)^(1/3) - (Sqrt[3]*(-1 + x)^(1/3)*x^(2/3)*ArcTan[1/Sqrt[3
] + (2*(-1 + x)^(1/3))/(Sqrt[3]*x^(1/3))])/(-x^2 + x^3)^(1/3) - (3*(-1 + x)
^(1/3)*x^(2/3)*Log[-1 + (-1 + x)^(1/3)/x^(1/3)])/(2*(-x^2 + x^3)^(1/3)) - (
(-1 + x)^(1/3)*x^(2/3)*Log[x])/(2*(-x^2 + x^3)^(1/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{x}{(-1+x)\sqrt[3]{-x^2+x^3}}\,dx = \frac{\left(\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{\sqrt[3]{x}}{(-1+x)^{4/3}}\,dx}{\sqrt[3]{-x^2+x^3}}$$
$$= -\frac{3x}{\sqrt[3]{-x^2+x^3}} + \frac{\left(\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{1}{\sqrt[3]{-1+x}\,x^{2/3}}\,dx}{\sqrt[3]{-x^2+x^3}}$$
$$= -\frac{3x}{\sqrt[3]{-x^2+x^3}} - \frac{\sqrt{3}\,\sqrt[3]{-1+x}\,x^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-1+x}}{\sqrt{3}\sqrt[3]{x}}\right)}{\sqrt[3]{-x^2+x^3}} - \frac{3\sqrt[3]{-1+x}\,x^{2/3}\log\left(-1+\right.}{2\sqrt[3]{-x^2+x^3}}$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.29

$$-\frac{3x^{2/3}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};1-x\right)}{\sqrt[3]{(x-1)x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-1 + x)*(-x^2 + x^3)^(1/3)),x]
[Out] (-3*x^(2/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, 1 - x])/((-1 + x)*x^2)^(1/3)
```

IntegrateAlgebraic [A] time = 0.24, size = 114, normalized size = 1.00

$$-\frac{3\left(x^3-x^2\right)^{2/3}}{(x-1)x} - \log\left(\sqrt[3]{x^3-x^2}-x\right) + \frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right) + \sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x^2}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((-1 + x)*(-x^2 + x^3)^(1/3)),x]
[Out] (-3*(-x^2 + x^3)^(2/3))/((-1 + x)*x) + Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-
x^2 + x^3)^(1/3))] - Log[-x + (-x^2 + x^3)^(1/3)] + Log[x^2 + x*(-x^2 + x^3
)^(1/3) + (-x^2 + x^3)^(2/3)]/2
```

fricas [A] time = 0.71, size = 137, normalized size = 1.20

$$-\frac{2\sqrt{3}\left(x^2-x\right)\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-x^2\right)^{\frac{1}{3}}}{3x}\right)+2\left(x^2-x\right)\log\left(-\frac{x-\left(x^3-x^2\right)^{\frac{1}{3}}}{x}\right)-\left(x^2-x\right)\log\left(\frac{x^2+\left(x^3-x^2\right)^{\frac{1}{3}}x+\left(x^3-x^2\right)^{\frac{2}{3}}}{x^2}\right)+6\left(x^3-x^2\right)^{\frac{2}{3}}}{2\left(x^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)/(x^3-x^2)^(1/3),x, algorithm="fricas")
[Out] -1/2*(2*sqrt(3)*(x^2 - x)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 - x^2)^(1/
3))/x) + 2*(x^2 - x)*log(-(x - (x^3 - x^2)^(1/3))/x) - (x^2 - x)*log((x^2 +
 (x^3 - x^2)^(1/3)*x + (x^3 - x^2)^(2/3))/x^2) + 6*(x^3 - x^2)^(2/3))/(x^2
- x)
```

giac [A] time = 0.25, size = 74, normalized size = 0.65

$$-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{3}{\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}} + \frac{1}{2}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right) - \log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)/(x^3-x^2)^(1/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*(-1/x + 1)^(1/3) + 1)) - 3/(-1/x + 1)^(1/3)
+ 1/2*log((-1/x + 1)^(2/3) + (-1/x + 1)^(1/3) + 1) - log(abs((-1/x + 1)^(1/
3) - 1))
```

maple [C] time = 0.32, size = 40, normalized size = 0.35

$$-\frac{3x}{\left((-1+x)\,x^{2}\right)^{\frac{1}{3}}}+\frac{3\left(-\operatorname{signum}\left(-1+x\right)\right)^{\frac{1}{3}} x^{\frac{1}{3}} \operatorname{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x\right)}{\operatorname{signum}\left(-1+x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-1+x)/(x^3-x^2)^(1/3),x)
```

```
[Out] -3*x/((-1+x)*x^2)^(1/3)+3/signum(-1+x)^(1/3)*(-signum(-1+x))^(1/3)*x^(1/3)*
hypergeom([1/3,1/3],[4/3],x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^{3}-x^{2}\right)^{\frac{1}{3}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)/(x^3-x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 - x^2)^(1/3)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(x^{3}-x^{2}\right)^{1/3}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 - x^2)^(1/3)*(x - 1)),x)
```

```
[Out] int(x/((x^3 - x^2)^(1/3)*(x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{x^{2}\,(x-1)}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)/(x**3-x**2)**(1/3),x)
```

```
[Out] Integral(x/((x**2*(x - 1))**(1/3)*(x - 1)), x)
```

3.1440 $\int \frac{1+x}{(-1+x)\sqrt[3]{-x^2+x^3}}\,dx$

Optimal. Leaf size=114

$$-\frac{6\left(x^3-x^2\right)^{2/3}}{(x-1)x}-\log\left(\sqrt[3]{x^3-x^2}-x\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x^2}+x}\right)$$

Rubi [A] time = 0.09, antiderivative size = 151, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2056, 78, 59}

$$-\frac{6x}{\sqrt[3]{x^3-x^2}}-\frac{3\sqrt[3]{x-1}\,x^{2/3}\log\left(\frac{\sqrt[3]{x-1}}{\sqrt[3]{x}}-1\right)}{2\sqrt[3]{x^3-x^2}}-\frac{\sqrt[3]{x-1}\,x^{2/3}\log(x)}{2\sqrt[3]{x^3-x^2}}-\frac{\sqrt{3}\sqrt[3]{x-1}\,x^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{x-1}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{x^3-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-1 + x)*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (-6*x)/(-x^2 + x^3)^(1/3) - (Sqrt[3]*(-1 + x)^(1/3)*x^(2/3)*ArcTan[1/Sqrt[3
] + (2*(-1 + x)^(1/3))/(Sqrt[3]*x^(1/3))])/(-x^2 + x^3)^(1/3) - (3*(-1 + x)
^(1/3)*x^(2/3)*Log[-1 + (-1 + x)^(1/3)/x^(1/3)])/(2*(-x^2 + x^3)^(1/3)) - (
(-1 + x)^(1/3)*x^(2/3)*Log[x])/(2*(-x^2 + x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1+x}{(-1+x)\sqrt[3]{-x^2+x^3}}\,dx = \frac{\left(\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{1+x}{(-1+x)^{4/3}x^{2/3}}\,dx}{\sqrt[3]{-x^2+x^3}}$$
$$= -\frac{6x}{\sqrt[3]{-x^2+x^3}} + \frac{\left(\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{1}{\sqrt[3]{-1+x}\,x^{2/3}}\,dx}{\sqrt[3]{-x^2+x^3}}$$
$$= -\frac{6x}{\sqrt[3]{-x^2+x^3}} - \frac{\sqrt{3}\,\sqrt[3]{-1+x}\,x^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-1+x}}{\sqrt{3}\sqrt[3]{x}}\right)}{\sqrt[3]{-x^2+x^3}} - \frac{3\sqrt[3]{-1+x}\,x^{2/3}\log\left(-1+\right.}{2\sqrt[3]{-x^2+x^3}}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.32

$$-\frac{3\left(x^{2/3}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};1-x\right)+x\right)}{\sqrt[3]{(x-1)x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-1 + x)*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (-3*(x + x^(2/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, 1 - x]))/((-1 + x)*x^2)
^(1/3)
```

IntegrateAlgebraic [A] time = 0.25, size = 114, normalized size = 1.00

$$-\frac{6\left(x^3-x^2\right)^{2/3}}{(x-1)x} - \log\left(\sqrt[3]{x^3-x^2}-x\right) + \frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right) + \sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x^2}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((-1 + x)*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (-6*(-x^2 + x^3)^(2/3))/((-1 + x)*x) + Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-
x^2 + x^3)^(1/3))] - Log[-x + (-x^2 + x^3)^(1/3)] + Log[x^2 + x*(-x^2 + x^3
)^(1/3) + (-x^2 + x^3)^(2/3)]/2
```

fricas [A] time = 0.75, size = 137, normalized size = 1.20

$$-\frac{2\sqrt{3}\left(x^2-x\right)\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-x^2\right)^{\frac{1}{3}}}{3x}\right)+2\left(x^2-x\right)\log\left(-\frac{x-\left(x^3-x^2\right)^{\frac{1}{3}}}{x}\right)-\left(x^2-x\right)\log\left(\frac{x^2+\left(x^3-x^2\right)^{\frac{1}{3}}x+\left(x^3-x^2\right)^{\frac{2}{3}}}{x^2}\right)+12\left(x^3-x^2\right)^{\frac{2}{3}}}{2\left(x^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^3-x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(x^2 - x)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 - x^2)^(1/
3))/x) + 2*(x^2 - x)*log(-(x - (x^3 - x^2)^(1/3))/x) - (x^2 - x)*log((x^2 +
 (x^3 - x^2)^(1/3)*x + (x^3 - x^2)^(2/3))/x^2) + 12*(x^3 - x^2)^(2/3))/(x^2
 - x)
```

giac [A] time = 0.16, size = 74, normalized size = 0.65

$$-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{6}{\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}} + \frac{1}{2}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right) - \log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^3-x^2)^(1/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*(-1/x + 1)^(1/3) + 1)) - 6/(-1/x + 1)^(1/3)
+ 1/2*log((-1/x + 1)^(2/3) + (-1/x + 1)^(1/3) + 1) - log(abs((-1/x + 1)^(1/
3) - 1))
```

maple [C] time = 0.31, size = 40, normalized size = 0.35

$$-\frac{6x}{\left((-1+x)\,x^{2}\right)^{\frac{1}{3}}}+\frac{3\left(-\mathrm{signum}\left(-1+x\right)\right)^{\frac{1}{3}}x^{\frac{1}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x\right)}{\mathrm{signum}\left(-1+x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-1+x)/(x^3-x^2)^(1/3),x)
```

```
[Out] -6*x/((-1+x)*x^2)^(1/3)+3/signum(-1+x)^(1/3)*(-signum(-1+x))^(1/3)*x^(1/3)*
hypergeom([1/3,1/3],[4/3],x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(x^{3}-x^{2}\right)^{\frac{1}{3}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^3-x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/((x^3 - x^2)^(1/3)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+1}{\left(x^{3}-x^{2}\right)^{1/3}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^3 - x^2)^(1/3)*(x - 1)),x)
```

```
[Out] int((x + 1)/((x^3 - x^2)^(1/3)*(x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt[3]{x^{2}\,(x-1)}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x**3-x**2)**(1/3),x)
```

```
[Out] Integral((x + 1)/((x**2*(x - 1))**(1/3)*(x - 1)), x)
```

3.1441 $\int \frac{-2b+ax}{(-b+ax+x^2)\sqrt[4]{-bx^2+ax^3}}\,dx$

Optimal. Leaf size=114

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^3-bx^2}}{\sqrt{ax^3-bx^2}+x^2}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-bx^2}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3-bx^2}}\right)$$

Rubi [C] time = 14.51, antiderivative size = 2624, normalized size of antiderivative = 23.02, number of steps used = 21, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2056, 6728, 107, 106, 490, 1217, 220, 1707}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(-2*b + a*x)/((-b + a*x + x^2)*(-(b*x^2) + a*x^3)^(1/4)),x]
```

```
[Out] -((Sqrt[b]*Sqrt[-a - Sqrt[a^2 + 4*b]]*Sqrt[(a*x)/b]*(-b + a*x)^(1/4)*ArcTan
[(Sqrt[a]*Sqrt[-a - Sqrt[a^2 + 4*b]]*(-b + a*x)^(1/4))/(2^(1/4)*Sqrt[b]*(-a
^2 - 2*b - a*Sqrt[a^2 + 4*b])^(1/4)*Sqrt[(a*x)/b])])/(2^(1/4)*Sqrt[a]*(-a^2
 - 2*b - a*Sqrt[a^2 + 4*b])^(1/4)*(-(b*x^2) + a*x^3)^(1/4))) - (Sqrt[b]*Sqr
t[a + Sqrt[a^2 + 4*b]]*Sqrt[(a*x)/b]*(-b + a*x)^(1/4)*ArcTan[(Sqrt[a]*Sqrt[
a + Sqrt[a^2 + 4*b]]*(-b + a*x)^(1/4))/(2^(1/4)*Sqrt[b]*(-a^2 - 2*b - a*Sqr
t[a^2 + 4*b])^(1/4)*Sqrt[(a*x)/b])])/(2^(1/4)*Sqrt[a]*(-a^2 - 2*b - a*Sqrt[
a^2 + 4*b])^(1/4)*(-(b*x^2) + a*x^3)^(1/4)) - (Sqrt[b]*Sqrt[a - Sqrt[a^2 +
4*b]]*Sqrt[(a*x)/b]*(-b + a*x)^(1/4)*ArcTan[(Sqrt[a]*Sqrt[a - Sqrt[a^2 + 4*
b]]*(-b + a*x)^(1/4))/(2^(1/4)*Sqrt[b]*(-a^2 - 2*b + a*Sqrt[a^2 + 4*b])^(1/
4)*Sqrt[(a*x)/b])])/(2^(1/4)*Sqrt[a]*(-a^2 - 2*b + a*Sqrt[a^2 + 4*b])^(1/4)
*(-(b*x^2) + a*x^3)^(1/4)) - (Sqrt[b]*Sqrt[-a + Sqrt[a^2 + 4*b]]*Sqrt[(a*x)
/b]*(-b + a*x)^(1/4)*ArcTan[(Sqrt[a]*Sqrt[-a + Sqrt[a^2 + 4*b]]*(-b + a*x)^
(1/4))/(2^(1/4)*Sqrt[b]*(-a^2 - 2*b + a*Sqrt[a^2 + 4*b])^(1/4)*Sqrt[(a*x)/b
])])/(2^(1/4)*Sqrt[a]*(-a^2 - 2*b + a*Sqrt[a^2 + 4*b])^(1/4)*(-(b*x^2) + a*
x^3)^(1/4)) - ((a + Sqrt[a^2 + 4*b])*(2*Sqrt[b] - Sqrt[2]*Sqrt[-a^2 - 2*b -
 a*Sqrt[a^2 + 4*b]])*(-b + a*x)^(1/4)*Sqrt[(a*x)/(Sqrt[b] + Sqrt[-b + a*x])
^2]*(Sqrt[b] + Sqrt[-b + a*x])*EllipticF[2*ArcTan[(-b + a*x)^(1/4)/b^(1/4)]
, 1/2])/(2*b^(1/4)*(a^2 + 4*b + a*Sqrt[a^2 + 4*b])*(-(b*x^2) + a*x^3)^(1/4)
) - ((a + Sqrt[a^2 + 4*b])*(2*Sqrt[b] + Sqrt[2]*Sqrt[-a^2 - 2*b - a*Sqrt[a^
2 + 4*b]])*(-b + a*x)^(1/4)*Sqrt[(a*x)/(Sqrt[b] + Sqrt[-b + a*x])^2]*(Sqrt[
b] + Sqrt[-b + a*x])*EllipticF[2*ArcTan[(-b + a*x)^(1/4)/b^(1/4)], 1/2])/(2
*b^(1/4)*(a^2 + 4*b + a*Sqrt[a^2 + 4*b])*(-(b*x^2) + a*x^3)^(1/4)) - ((a -
Sqrt[a^2 + 4*b])*(2*Sqrt[b] - Sqrt[-2*a^2 - 4*b + 2*a*Sqrt[a^2 + 4*b]])*(-b
 + a*x)^(1/4)*Sqrt[(a*x)/(Sqrt[b] + Sqrt[-b + a*x])^2]*(Sqrt[b] + Sqrt[-b +
 a*x])*EllipticF[2*ArcTan[(-b + a*x)^(1/4)/b^(1/4)], 1/2])/(2*b^(1/4)*(a^2
+ 4*b - a*Sqrt[a^2 + 4*b])*(-(b*x^2) + a*x^3)^(1/4)) - ((a - Sqrt[a^2 + 4*b
])*(2*Sqrt[b] + Sqrt[-2*a^2 - 4*b + 2*a*Sqrt[a^2 + 4*b]])*(-b + a*x)^(1/4)*
Sqrt[(a*x)/(Sqrt[b] + Sqrt[-b + a*x])^2]*(Sqrt[b] + Sqrt[-b + a*x])*Ellipti
cF[2*ArcTan[(-b + a*x)^(1/4)/b^(1/4)], 1/2])/(2*b^(1/4)*(a^2 + 4*b - a*Sqrt
[a^2 + 4*b])*(-(b*x^2) + a*x^3)^(1/4)) + ((a + Sqrt[a^2 + 4*b])*(Sqrt[2]*Sq
rt[b] + Sqrt[-a^2 - 2*b - a*Sqrt[a^2 + 4*b]])^2*(-b + a*x)^(1/4)*Sqrt[(a*x)
/(Sqrt[b] + Sqrt[-b + a*x])^2]*(Sqrt[b] + Sqrt[-b + a*x])*EllipticPi[-1/4*(
Sqrt[2]*Sqrt[b] - Sqrt[-a^2 - 2*b - a*Sqrt[a^2 + 4*b]])^2/(Sqrt[2]*Sqrt[b]*
Sqrt[-a^2 - 2*b - a*Sqrt[a^2 + 4*b]]), 2*ArcTan[(-b + a*x)^(1/4)/b^(1/4)],
1/2])/(2*Sqrt[2]*b^(1/4)*Sqrt[-a^2 - 2*b - a*Sqrt[a^2 + 4*b]]*(a^2 + 4*b +
a*Sqrt[a^2 + 4*b])*(-(b*x^2) + a*x^3)^(1/4)) - ((a + Sqrt[a^2 + 4*b])*(Sqrt
[2]*Sqrt[b] - Sqrt[-a^2 - 2*b - a*Sqrt[a^2 + 4*b]])^2*(-b + a*x)^(1/4)*Sqrt
[(a*x)/(Sqrt[b] + Sqrt[-b + a*x])^2]*(Sqrt[b] + Sqrt[-b + a*x])*EllipticPi[
(Sqrt[2]*Sqrt[b] + Sqrt[-a^2 - 2*b - a*Sqrt[a^2 + 4*b]])^2/(4*Sqrt[2]*Sqrt[
b]*Sqrt[-a^2 - 2*b - a*Sqrt[a^2 + 4*b]]), 2*ArcTan[(-b + a*x)^(1/4)/b^(1/4)
```

```
], 1/2])/(2*Sqrt[2]*b^(1/4)*Sqrt[-a^2 - 2*b - a*Sqrt[a^2 + 4*b]]*(a^2 + 4*b
 + a*Sqrt[a^2 + 4*b])*(-(b*x^2) + a*x^3)^(1/4)) + ((a - Sqrt[a^2 + 4*b])*(S
qrt[2]*Sqrt[b] + Sqrt[-a^2 - 2*b + a*Sqrt[a^2 + 4*b]])^2*(-b + a*x)^(1/4)*S
qrt[(a*x)/(Sqrt[b] + Sqrt[-b + a*x])^2]*(Sqrt[b] + Sqrt[-b + a*x])*Elliptic
Pi[-1/4*(Sqrt[2]*Sqrt[b] - Sqrt[-a^2 - 2*b + a*Sqrt[a^2 + 4*b]])^2/(Sqrt[2]
*Sqrt[b]*Sqrt[-a^2 - 2*b + a*Sqrt[a^2 + 4*b]]), 2*ArcTan[(-b + a*x)^(1/4)/b
^(1/4)], 1/2])/(2*Sqrt[2]*b^(1/4)*(a^2 + 4*b - a*Sqrt[a^2 + 4*b])*Sqrt[-a^2
 - 2*b + a*Sqrt[a^2 + 4*b]]*(-(b*x^2) + a*x^3)^(1/4)) - ((a - Sqrt[a^2 + 4*
b])*(Sqrt[2]*Sqrt[b] - Sqrt[-a^2 - 2*b + a*Sqrt[a^2 + 4*b]])^2*(-b + a*x)^(
1/4)*Sqrt[(a*x)/(Sqrt[b] + Sqrt[-b + a*x])^2]*(Sqrt[b] + Sqrt[-b + a*x])*El
lipticPi[(Sqrt[2]*Sqrt[b] + Sqrt[-a^2 - 2*b + a*Sqrt[a^2 + 4*b]])^2/(4*Sqrt
[2]*Sqrt[b]*Sqrt[-a^2 - 2*b + a*Sqrt[a^2 + 4*b]]), 2*ArcTan[(-b + a*x)^(1/4
)/b^(1/4)], 1/2])/(2*Sqrt[2]*b^(1/4)*(a^2 + 4*b - a*Sqrt[a^2 + 4*b])*Sqrt[-
a^2 - 2*b + a*Sqrt[a^2 + 4*b]]*(-(b*x^2) + a*x^3)^(1/4))
```

Rule 106

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
1/4)), x_Symbol] :> Dist[-4, Subst[Int[x^2/((b*e - a*f - b*x^4)*Sqrt[c - (d
*e)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f
}, x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 107

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
1/4)), x_Symbol] :> Dist[Sqrt[-((f*(c + d*x))/(d*e - c*f))]/Sqrt[c + d*x],
Int[1/((a + b*x)*Sqrt[-((c*f)/(d*e - c*f)) - (d*f*x)/(d*e - c*f)]*(e + f*x)
^(1/4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[-(f/(d*e - c*f)),
0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
```

```
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-2b+ax}{\left(-b+ax+x^2\right)\sqrt[4]{-bx^2+ax^3}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{-b+ax}\right)\int \frac{-2b+ax}{\sqrt{x}\sqrt[4]{-b+ax}\left(-b+ax+x^2\right)}\,dx}{\sqrt[4]{-bx^2+ax^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt[4]{-b+ax}\right)\int\left(\frac{a-\sqrt{a^2+4b}}{\sqrt{x}\left(a-\sqrt{a^2+4b}+2x\right)\sqrt[4]{-b+ax}}+\frac{a+\sqrt{a^2+4b}}{\sqrt{x}\left(a+\sqrt{a^2+4b}+2x\right)\sqrt[4]{-b+ax}}\right)d\ldots}{\sqrt[4]{-bx^2+ax^3}}$$

$$= \frac{\left(\left(a-\sqrt{a^2+4b}\right)\sqrt{x}\sqrt[4]{-b+ax}\right)\int \frac{1}{\sqrt{x}\left(a-\sqrt{a^2+4b}+2x\right)\sqrt[4]{-b+ax}}\,dx}{\sqrt[4]{-bx^2+ax^3}} + \frac{\left(\left(a+\sqrt{\ldots}\right.\right.}{\ldots}$$

$$= \frac{\left(\left(a-\sqrt{a^2+4b}\right)\sqrt{\frac{ax}{b}}\sqrt[4]{-b+ax}\right)\int \frac{1}{\sqrt{\frac{ax}{b}}\left(a-\sqrt{a^2+4b}+2x\right)\sqrt[4]{-b+ax}}\,dx}{\sqrt[4]{-bx^2+ax^3}} + \frac{\left(\left(a+\ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(4\left(a-\sqrt{a^2+4b}\right)\sqrt{\frac{ax}{b}}\sqrt[4]{-b+ax}\right)\text{Subst}\left(\int \frac{x^2}{\left(-2b-a\left(a-\sqrt{a^2+4b}\right)-2x^4\right)\sqrt{1+\ldots}}\ldots\right.}{\sqrt[4]{-bx^2+ax^3}}$$

$$= -\frac{\left(\sqrt{2}\left(a-\sqrt{a^2+4b}\right)\sqrt{\frac{ax}{b}}\sqrt[4]{-b+ax}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a^2-2b+a\sqrt{a^2+4b}}-\sqrt{2}x^2\ldots\right.}\ldots\right.}{\sqrt[4]{-bx^2+ax^3}}$$

$$= -\frac{\left(\sqrt{2}\left(a+\sqrt{a^2+4b}\right)\left(\sqrt{2}\sqrt{b}-\sqrt{-a^2-2b-a\sqrt{a^2+4b}}\right)\sqrt{\frac{ax}{b}}\sqrt[4]{-b+\ldots}\right.}{\left(a^2+4b+a\sqrt{a^2+4b}\right)\sqrt[4]{-bx^2+\ldots}}$$

$$= \frac{2^{3/4}\sqrt{b}\left(a^3+4ab+\left(a^2+2b\right)\sqrt{a^2+4b}\right)\sqrt{\frac{ax}{b}}\sqrt[4]{-b+ax}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{-\ldots}}{\sqrt[4]{2}\sqrt{b}\sqrt[4]{\ldots}}\right.}{\sqrt{a}\sqrt{-a-\sqrt{a^2+4b}}\sqrt[4]{-a^2-2b-a\sqrt{a^2+4b}}\left(a^2+4b+a\sqrt{a^2+4b}\ldots\right.}$$

Mathematica [C] time = 7.43, size = 611, normalized size = 5.36

$$\frac{i\sqrt{2}a\sqrt{\frac{ax}{ax-b}}(ax-b)^{3/4}\left(\left(-a^2+\sqrt{a^4+4a^2b}-4b\right)\sqrt{-\frac{a^2+\sqrt{a^4+4a^2b}+2b}{b^2}}\,\Pi\left(-\frac{i\sqrt{2}}{\sqrt{b}\sqrt{\frac{-a^2-2b+\sqrt{a^4+4a^2b}}{b^2}}};i\sinh^{-1}\left(\frac{\sqrt{i\sqrt{b}}}{\sqrt[4]{ax-b}}\right)\middle|-1\right)+\left(a^2-\sqrt{a^4+4a^2b}+4b\right)\sqrt{-\frac{a^2+\sqrt{a^4+4a^2b}+2b}{b^2}}\,\Pi\left(\frac{i\sqrt{2}}{\sqrt{b}\sqrt{\frac{-a^2-2b+\sqrt{a^4+4a^2b}}{b^2}}};i\sinh^{-1}\left(\frac{\sqrt{i\sqrt{b}}}{\sqrt[4]{ax-b}}\right)\middle|-1\right)+\sqrt{\frac{-a^2+\sqrt{a^4+4a^2b}-2b}{b^2}}\left(a^2+\sqrt{a^4+4a^2b}+4b\right)\left(\Pi\left(-\frac{i\sqrt{2}}{\sqrt{b}\sqrt{-\frac{a^2+2b+\sqrt{a^4+4a^2b}}{b^2}}};i\sinh^{-1}\left(\frac{\sqrt{i\sqrt{b}}}{\sqrt[4]{ax-b}}\right)\middle|-1\right)-\Pi\left(\frac{i\sqrt{2}}{\sqrt{b}\sqrt{-\frac{a^2+2b+\sqrt{a^4+4a^2b}}{b^2}}};i\sinh^{-1}\left(\frac{\sqrt{i\sqrt{b}}}{\sqrt[4]{ax-b}}\right)\middle|-1\right)\right)\right)}{\left(i\sqrt{b}\right)^{5/2}\sqrt{a^4+4a^2b}\sqrt{\frac{-a^2+\sqrt{a^4+4a^2b}-2b}{b^2}}\sqrt{-\frac{a^2+\sqrt{a^4+4a^2b}+2b}{b^2}}\sqrt[4]{x^2(ax-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*b + a*x)/((-b + a*x + x^2)*(-(b*x^2) + a*x^3)^(1/4)),x]
```

```
[Out] (I*Sqrt[2]*a*Sqrt[(a*x)/(-b + a*x)]*(-b + a*x)^(3/4)*((-a^2 - 4*b + Sqrt[a^
4 + 4*a^2*b])*Sqrt[-((a^2 + 2*b + Sqrt[a^4 + 4*a^2*b])/b^2)]*EllipticPi[((-
I)*Sqrt[2])/(Sqrt[b]*Sqrt[(-a^2 - 2*b + Sqrt[a^4 + 4*a^2*b])/b^2]), I*ArcSi
nh[Sqrt[I*Sqrt[b]]/(-b + a*x)^(1/4)], -1] + (a^2 + 4*b - Sqrt[a^4 + 4*a^2*b
])*Sqrt[-((a^2 + 2*b + Sqrt[a^4 + 4*a^2*b])/b^2)]*EllipticPi[(I*Sqrt[2])/(S
qrt[b]*Sqrt[(-a^2 - 2*b + Sqrt[a^4 + 4*a^2*b])/b^2]), I*ArcSinh[Sqrt[I*Sqrt
[b]]/(-b + a*x)^(1/4)], -1] + Sqrt[(-a^2 - 2*b + Sqrt[a^4 + 4*a^2*b])/b^2]*
(a^2 + 4*b + Sqrt[a^4 + 4*a^2*b])*(EllipticPi[((-I)*Sqrt[2])/(Sqrt[b]*Sqrt[
-((a^2 + 2*b + Sqrt[a^4 + 4*a^2*b])/b^2)]), I*ArcSinh[Sqrt[I*Sqrt[b]]/(-b +
 a*x)^(1/4)], -1] - EllipticPi[(I*Sqrt[2])/(Sqrt[b]*Sqrt[-((a^2 + 2*b + Sqr
t[a^4 + 4*a^2*b])/b^2)]), I*ArcSinh[Sqrt[I*Sqrt[b]]/(-b + a*x)^(1/4)], -1])
))/((I*Sqrt[b])^(5/2)*Sqrt[a^4 + 4*a^2*b]*Sqrt[(-a^2 - 2*b + Sqrt[a^4 + 4*a
^2*b])/b^2]*Sqrt[-((a^2 + 2*b + Sqrt[a^4 + 4*a^2*b])/b^2)]*(x^2*(-b + a*x))
^(1/4))
```

IntegrateAlgebraic [A] time = 0.56, size = 114, normalized size = 1.00

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^3-bx^2}}{\sqrt{ax^3-bx^2}+x^2}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-bx^2}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3-bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*b + a*x)/((-b + a*x + x^2)*(-(b*x^2) + a*x^3)^(1/4))
,x]
```

```
[Out] -(Sqrt[2]*ArcTan[(-(x^2/Sqrt[2]) + Sqrt[-(b*x^2) + a*x^3]/Sqrt[2])/(x*(-(b*
x^2) + a*x^3)^(1/4))]) + Sqrt[2]*ArcTanh[(Sqrt[2]*x*(-(b*x^2) + a*x^3)^(1/4
))/(x^2 + Sqrt[-(b*x^2) + a*x^3])]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-2*b)/(a*x+x^2-b)/(a*x^3-b*x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-2b}{\left(ax^3-bx^2\right)^{\frac{1}{4}}\left(ax+x^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-2*b)/(a*x+x^2-b)/(a*x^3-b*x^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((a*x - 2*b)/((a*x^3 - b*x^2)^(1/4)*(a*x + x^2 - b)), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{ax-2b}{\left(ax+x^{2}-b\right)\left(a\,x^{3}-b\,x^{2}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-2*b)/(a*x+x^2-b)/(a*x^3-b*x^2)^(1/4),x)
[Out] int((a*x-2*b)/(a*x+x^2-b)/(a*x^3-b*x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-2\,b}{\left(ax^{3}-bx^{2}\right)^{\frac{1}{4}}\left(ax+x^{2}-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-2*b)/(a*x+x^2-b)/(a*x^3-b*x^2)^(1/4),x, algorithm="maxima")
[Out] integrate((a*x - 2*b)/((a*x^3 - b*x^2)^(1/4)*(a*x + x^2 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{2\,b-a\,x}{{\left(a\,x^3-b\,x^2\right)}^{1/4}\,\left(x^2+a\,x-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*b - a*x)/((a*x^3 - b*x^2)^(1/4)*(a*x - b + x^2)),x)
[Out] int(-(2*b - a*x)/((a*x^3 - b*x^2)^(1/4)*(a*x - b + x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-2b}{\sqrt[4]{x^{2}\left(ax-b\right)}\left(ax-b+x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-2*b)/(a*x+x**2-b)/(a*x**3-b*x**2)**(1/4),x)
[Out] Integral((a*x - 2*b)/((x**2*(a*x - b))**(1/4)*(a*x - b + x**2)), x)
```

3.1442 $\int \frac{-1+x}{\left(1+x+x^2\right)\sqrt[4]{1+x^4}}\,dx$

Optimal. Leaf size=114

$$\frac{\tan^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}-\sqrt{2}\,x-\frac{1}{\sqrt{2}}}{(x+1)\sqrt[4]{x^4+1}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt{2}\,x+\sqrt{2}\right)\sqrt[4]{x^4+1}}{\sqrt{x^4+1}+x^2+2x+1}\right)}{\sqrt{2}}$$

Rubi [C] time = 1.52, antiderivative size = 797, normalized size of antiderivative = 6.99, number of steps used = 46, number of rules used = 19, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.905, Rules used = {6728, 2153, 1240, 377, 212, 208, 205, 510, 1248, 746, 399, 490, 1213, 537, 444, 63, 298, 203, 206}

$$-\frac{1}{6}\left(1+i\sqrt{3}\right)F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1-i\sqrt{3}}\right)x^3-\frac{1}{6}\left(1-i\sqrt{3}\right)F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1+i\sqrt{3}}\right)x^3-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt[4]{x^4+1}}\right)}{2\left(-\frac{i-\sqrt{3}}{i+\sqrt{3}}\right)^{3/4}}-\frac{1}{2}\left(-\frac{i-\sqrt{3}}{i+\sqrt{3}}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x^4+1}}{\sqrt[4]{1-i\sqrt{3}}}\right)+\frac{1}{2}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x^4+1}}{\sqrt[4]{1+i\sqrt{3}}}\right)-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt[4]{x^4+1}}\right)}{2\left(-\frac{i-\sqrt{3}}{i+\sqrt{3}}\right)^{3/4}}-\frac{1}{2}\left(-\frac{i-\sqrt{3}}{i+\sqrt{3}}\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x^4+1}}{\sqrt[4]{1-i\sqrt{3}}}\right)-\frac{1}{2}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x^4+1}}{\sqrt[4]{1+i\sqrt{3}}}\right)+\frac{i\sqrt{-x^4}\,\Pi\left(\frac{1}{2}\left(-i-\sqrt{3}\right);\sin^{-1}\left(\sqrt[4]{x^4+1}\right)\middle|-1\right)}{2x^2}-\frac{i\sqrt{-x^4}\,\Pi\left(\frac{1}{2}\left(i-\sqrt{3}\right);\sin^{-1}\left(\sqrt[4]{x^4+1}\right)\middle|-1\right)}{2x^2}-\frac{i\sqrt{-x^4}\,\Pi\left(\frac{1}{\sqrt{\frac{1}{2}\left(1-i\sqrt{3}\right)}};\sin^{-1}\left(\sqrt[4]{x^4+1}\right)\middle|-1\right)}{2x^2}+\frac{i\sqrt{-x^4}\,\Pi\left(\frac{1}{\sqrt{\frac{1}{2}\left(1+i\sqrt{3}\right)}};\sin^{-1}\left(\sqrt[4]{x^4+1}\right)\middle|-1\right)}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x)/((1 + x + x^2)*(1 + x^4)^(1/4)),x]
```

```
[Out] -1/6*((1 + I*Sqrt[3])*x^3*AppellF1[3/4, 1/4, 1, 7/4, -x^4, (-2*x^4)/(1 - I*
Sqrt[3])]) - ((1 - I*Sqrt[3])*x^3*AppellF1[3/4, 1/4, 1, 7/4, -x^4, (-2*x^4)
/(1 + I*Sqrt[3])])/6 - ArcTan[x/((-((I - Sqrt[3])/(I + Sqrt[3])))^(1/4)*(1
+ x^4)^(1/4))]/(2*(-((I - Sqrt[3])/(I + Sqrt[3])))^(3/4)) - ((-((I - Sqrt[3
])/(I + Sqrt[3])))^(3/4)*ArcTan[((-((I - Sqrt[3])/(I + Sqrt[3])))^(1/4)*x)/
(1 + x^4)^(1/4)])/2 + (((1 - I*Sqrt[3])/2)^(3/4)*ArcTan[(2^(1/4)*(1 + x^4)^
(1/4))/(1 - I*Sqrt[3])^(1/4)])/2 + (((1 + I*Sqrt[3])/2)^(3/4)*ArcTan[(2^(1/
4)*(1 + x^4)^(1/4))/(1 + I*Sqrt[3])^(1/4)])/2 - ArcTanh[x/((-((I - Sqrt[3])
/(I + Sqrt[3])))^(1/4)*(1 + x^4)^(1/4))]/(2*(-((I - Sqrt[3])/(I + Sqrt[3]))
)^(3/4)) - ((-((I - Sqrt[3])/(I + Sqrt[3])))^(3/4)*ArcTanh[((-((I - Sqrt[3]
)/(I + Sqrt[3])))^(1/4)*x)/(1 + x^4)^(1/4)])/2 - (((1 - I*Sqrt[3])/2)^(3/4)
*ArcTanh[(2^(1/4)*(1 + x^4)^(1/4))/(1 - I*Sqrt[3])^(1/4)])/2 - (((1 + I*Sqr
t[3])/2)^(3/4)*ArcTanh[(2^(1/4)*(1 + x^4)^(1/4))/(1 + I*Sqrt[3])^(1/4)])/2
+ ((I/2)*Sqrt[-x^4]*EllipticPi[(-I - Sqrt[3])/2, ArcSin[(1 + x^4)^(1/4)], -
1])/x^2 - ((I/2)*Sqrt[-x^4]*EllipticPi[(I - Sqrt[3])/2, ArcSin[(1 + x^4)^(1
/4)], -1])/x^2 - ((I/2)*Sqrt[-x^4]*EllipticPi[1/Sqrt[(1 - I*Sqrt[3])/2], Ar
cSin[(1 + x^4)^(1/4)], -1])/x^2 + ((I/2)*Sqrt[-x^4]*EllipticPi[1/Sqrt[(1 +
I*Sqrt[3])/2], ArcSin[(1 + x^4)^(1/4)], -1])/x^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 2153

```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^nn)^p, (c/(c^2 - d^2*x^(2*n)) - (d*x^n)/
(c^2 - d^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !
IntegerQ[p] && ILtQ[q, 0] && IGtQ[Log[2, nn/n], 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-1+x}{\left(1+x+x^2\right)\sqrt[4]{1+x^4}}\,dx = \int \left(\frac{1+i\sqrt{3}}{\left(1-i\sqrt{3}+2x\right)\sqrt[4]{1+x^4}} + \frac{1-i\sqrt{3}}{\left(1+i\sqrt{3}+2x\right)\sqrt[4]{1+x^4}}\right)dx$$

$$= \left(1-i\sqrt{3}\right)\int \frac{1}{\left(1+i\sqrt{3}+2x\right)\sqrt[4]{1+x^4}}\,dx + \left(1+i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[4]{1+}}$$

$$= \left(1-i\sqrt{3}\right)\int \left(\frac{-i+\sqrt{3}}{2\left(i+\sqrt{3}+2ix^2\right)\sqrt[4]{1+x^4}} + \frac{x}{\left(1-i\sqrt{3}+2x^2\right)\sqrt[4]{1+x^4}}\right)dx + (1$$

$$= 2i\int \frac{1}{\left(-i+\sqrt{3}-2ix^2\right)\sqrt[4]{1+x^4}}\,dx - 2i\int \frac{1}{\left(i+\sqrt{3}+2ix^2\right)\sqrt[4]{1+x^4}}\,dx + (1-i$$

$$= 2i\int \left(\frac{1+i\sqrt{3}}{2\left(i+\sqrt{3}+2ix^4\right)\sqrt[4]{1+x^4}} + \frac{ix^2}{\sqrt[4]{1+x^4}\left(1-i\sqrt{3}+2x^4\right)}\right)dx - 2i\int \left(\frac{}{2(-}\right.$$

$$= -\left(2\int \frac{x^2}{\sqrt[4]{1+x^4}\left(1-i\sqrt{3}+2x^4\right)}\,dx\right) - 2\int \frac{x^2}{\sqrt[4]{1+x^4}\left(1+i\sqrt{3}+2x^4\right)}\,dx + (i-$$

$$= -\frac{2x^3F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1-i\sqrt{3}}\right)}{3\left(1-i\sqrt{3}\right)} - \frac{2x^3F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1+i\sqrt{3}}\right)}{3\left(1+i\sqrt{3}\right)} + \left(i-\sqrt{3}\right)\text{Su}$$

$$= -\frac{2x^3F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1-i\sqrt{3}}\right)}{3\left(1-i\sqrt{3}\right)} - \frac{2x^3F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1+i\sqrt{3}}\right)}{3\left(1+i\sqrt{3}\right)} - \left(2\left(1-i\sqrt{3}\right.\right.$$

$$= -\frac{2x^3F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1-i\sqrt{3}}\right)}{3\left(1-i\sqrt{3}\right)} - \frac{2x^3F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1+i\sqrt{3}}\right)}{3\left(1+i\sqrt{3}\right)} - \frac{\tan^{-1}\left(\frac{}{\sqrt[4]{-\frac{i}{i}}}\right.}{2\left(-\frac{i-}{i+}\right.}$$

$$= -\frac{2x^3F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1-i\sqrt{3}}\right)}{3\left(1-i\sqrt{3}\right)} - \frac{2x^3F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1+i\sqrt{3}}\right)}{3\left(1+i\sqrt{3}\right)} - \frac{\tan^{-1}\left(\frac{}{\sqrt[4]{-\frac{i}{i}}}\right.}{2\left(-\frac{i-}{i+}\right.}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{-1+x}{\left(1+x+x^2\right)\sqrt[4]{1+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x)/((1 + x + x^2)*(1 + x^4)^(1/4)),x]
[Out] Integrate[(-1 + x)/((1 + x + x^2)*(1 + x^4)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 1.55, size = 114, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}-\sqrt{2}x-\frac{1}{\sqrt{2}}}{(x+1)\sqrt[4]{x^4+1}}\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{\left(\sqrt{2}x+\sqrt{2}\right)\sqrt[4]{x^4+1}}{\sqrt{x^4+1}+x^2+2x+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/((1 + x + x^2)*(1 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[(-(1/Sqrt[2]) - Sqrt[2]*x - x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2])/((1
 + x)*(1 + x^4)^(1/4))]/Sqrt[2] - ArcTanh[((Sqrt[2] + Sqrt[2]*x)*(1 + x^4)^
(1/4))/(1 + 2*x + x^2 + Sqrt[1 + x^4])]/Sqrt[2]
```

fricas [B] time = 6.97, size = 1098, normalized size = 9.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2+x+1)/(x^4+1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(-(x^8 + 4*x^7 + 10*x^6 + 16*x^5 + 19*x^4 + 16*x^3 + sqrt
(2)*(x^5 + 7*x^4 + 15*x^3 + 15*x^2 + 7*x + 1)*(x^4 + 1)^(3/4) + 10*x^2 - sq
rt(2)*(x^7 + x^6 - 6*x^5 - 16*x^4 - 16*x^3 - 6*x^2 + x + 1)*(x^4 + 1)^(1/4)
 + 2*(x^6 + 4*x^5 + 8*x^4 + 10*x^3 + 8*x^2 + 4*x + 1)*sqrt(x^4 + 1) - (sqrt
(2)*(x^6 + 8*x^5 + 22*x^4 + 30*x^3 + 22*x^2 + 8*x + 1)*sqrt(x^4 + 1) + 4*(x
^5 + 5*x^4 + 10*x^3 + 10*x^2 + 5*x + 1)*(x^4 + 1)^(3/4) + sqrt(2)*(2*x^8 +
10*x^7 + 19*x^6 + 22*x^5 + 21*x^4 + 22*x^3 + 19*x^2 + 10*x + 2) + 2*(x^7 +
5*x^6 + 12*x^5 + 18*x^4 + 18*x^3 + 12*x^2 + 5*x + 1)*(x^4 + 1)^(1/4))*sqrt(
(x^4 + 2*x^3 - sqrt(2)*(x^4 + 1)^(3/4)*(x + 1) + 3*x^2 - sqrt(2)*(x^4 + 1)^
(1/4)*(x^3 + 3*x^2 + 3*x + 1) + 2*sqrt(x^4 + 1)*(x^2 + 2*x + 1) + 2*x + 1)/
(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)) + 4*x + 1)/(3*x^8 + 12*x^7 + 14*x^6 - 11*x
^4 + 14*x^2 + 12*x + 3)) - 1/2*sqrt(2)*arctan(-(x^8 + 4*x^7 + 10*x^6 + 16*x
^5 + 19*x^4 + 16*x^3 - sqrt(2)*(x^5 + 7*x^4 + 15*x^3 + 15*x^2 + 7*x + 1)*(x
^4 + 1)^(3/4) + 10*x^2 + sqrt(2)*(x^7 + x^6 - 6*x^5 - 16*x^4 - 16*x^3 - 6*x
^2 + x + 1)*(x^4 + 1)^(1/4) + 2*(x^6 + 4*x^5 + 8*x^4 + 10*x^3 + 8*x^2 + 4*x
 + 1)*sqrt(x^4 + 1) + (sqrt(2)*(x^6 + 8*x^5 + 22*x^4 + 30*x^3 + 22*x^2 + 8*
x + 1)*sqrt(x^4 + 1) - 4*(x^5 + 5*x^4 + 10*x^3 + 10*x^2 + 5*x + 1)*(x^4 + 1
)^(3/4) + sqrt(2)*(2*x^8 + 10*x^7 + 19*x^6 + 22*x^5 + 21*x^4 + 22*x^3 + 19*
x^2 + 10*x + 2) - 2*(x^7 + 5*x^6 + 12*x^5 + 18*x^4 + 18*x^3 + 12*x^2 + 5*x
+ 1)*(x^4 + 1)^(1/4))*sqrt((x^4 + 2*x^3 + sqrt(2)*(x^4 + 1)^(3/4)*(x + 1) +
 3*x^2 + sqrt(2)*(x^4 + 1)^(1/4)*(x^3 + 3*x^2 + 3*x + 1) + 2*sqrt(x^4 + 1)*
(x^2 + 2*x + 1) + 2*x + 1)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)) + 4*x + 1)/(3*x
^8 + 12*x^7 + 14*x^6 - 11*x^4 + 14*x^2 + 12*x + 3)) - 1/8*sqrt(2)*log(4*(x^
4 + 2*x^3 + sqrt(2)*(x^4 + 1)^(3/4)*(x + 1) + 3*x^2 + sqrt(2)*(x^4 + 1)^(1/
4)*(x^3 + 3*x^2 + 3*x + 1) + 2*sqrt(x^4 + 1)*(x^2 + 2*x + 1) + 2*x + 1)/(x^
4 + 2*x^3 + 3*x^2 + 2*x + 1)) + 1/8*sqrt(2)*log(4*(x^4 + 2*x^3 - sqrt(2)*(x
^4 + 1)^(3/4)*(x + 1) + 3*x^2 - sqrt(2)*(x^4 + 1)^(1/4)*(x^3 + 3*x^2 + 3*x
+ 1) + 2*sqrt(x^4 + 1)*(x^2 + 2*x + 1) + 2*x + 1)/(x^4 + 2*x^3 + 3*x^2 + 2*
x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^4+1\right)^{\frac{1}{4}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2+x+1)/(x^4+1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/((x^4 + 1)^(1/4)*(x^2 + x + 1)), x)
```

maple [C] time = 5.64, size = 390, normalized size = 3.42

$$\frac{\operatorname{RootOf}\left(\_Z^4+1\right)^3 \ln\left(\frac{2\operatorname{RootOf}\left(\_Z^4+1\right)^3 x^3+\left(x^4+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4+1\right)^2 x^3+3\operatorname{RootOf}\left(\_Z^4+1\right)^3 x^2+3\left(x^4+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4+1\right)^2 x^2+\sqrt{x^4+1}\,\operatorname{RootOf}\left(\_Z^4+1\right) x^2+2\operatorname{RootOf}\left(\_Z^4+1\right)^3 x+3\left(x^4+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4+1\right)^2 x+2\sqrt{x^4+1}\,\operatorname{RootOf}\left(\_Z^4+1\right) x+\left(x^4+1\right)^{\frac{3}{4}} x+\left(x^4+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4+1\right)^2+\sqrt{x^4+1}\,\operatorname{RootOf}\left(\_Z^4+1\right)+\left(x^4+1\right)^{\frac{3}{4}}}{\left(x^2+x+1\right)^2}\right)}{2}+\frac{\operatorname{RootOf}\left(\_Z^4+1\right) \ln\left(\frac{\sqrt{x^4+1}\,\operatorname{RootOf}\left(\_Z^4+1\right)^3 x^2+2\sqrt{x^4+1}\,\operatorname{RootOf}\left(\_Z^4+1\right)^3 x-\left(x^4+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4+1\right)^2 x^3+\sqrt{x^4+1}\,\operatorname{RootOf}\left(\_Z^4+1\right)^3-3\left(x^4+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4+1\right)^2 x^2-3\left(x^4+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4+1\right)^2 x+2\operatorname{RootOf}\left(\_Z^4+1\right) x^3+\left(x^4+1\right)^{\frac{3}{4}} x-\left(x^4+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4+1\right)^2+3\operatorname{RootOf}\left(\_Z^4+1\right) x^2+\left(x^4+1\right)^{\frac{3}{4}}+2\operatorname{RootOf}\left(\_Z^4+1\right) x}{\left(x^2+x+1\right)^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(x^2+x+1)/(x^4+1)^(1/4),x)
```

```
[Out] 1/2*RootOf(_Z^4+1)^3*ln((2*RootOf(_Z^4+1)^3*x^3+(x^4+1)^(1/4)*RootOf(_Z^4+1
)^2*x^3+3*RootOf(_Z^4+1)^3*x^2+3*(x^4+1)^(1/4)*RootOf(_Z^4+1)^2*x^2+(x^4+1)
^(1/2)*RootOf(_Z^4+1)*x^2+2*RootOf(_Z^4+1)^3*x+3*(x^4+1)^(1/4)*RootOf(_Z^4+
1)^2*x+2*(x^4+1)^(1/2)*RootOf(_Z^4+1)*x+(x^4+1)^(3/4)*x+(x^4+1)^(1/4)*RootO
f(_Z^4+1)^2+(x^4+1)^(1/2)*RootOf(_Z^4+1)+(x^4+1)^(3/4))/(x^2+x+1)^2)+1/2*Ro
otOf(_Z^4+1)*ln(((x^4+1)^(1/2)*RootOf(_Z^4+1)^3*x^2+2*(x^4+1)^(1/2)*RootOf(
_Z^4+1)^3*x-(x^4+1)^(1/4)*RootOf(_Z^4+1)^2*x^3+(x^4+1)^(1/2)*RootOf(_Z^4+1)
^3-3*(x^4+1)^(1/4)*RootOf(_Z^4+1)^2*x^2-3*(x^4+1)^(1/4)*RootOf(_Z^4+1)^2*x+
2*RootOf(_Z^4+1)*x^3+(x^4+1)^(3/4)*x-(x^4+1)^(1/4)*RootOf(_Z^4+1)^2+3*RootO
f(_Z^4+1)*x^2+(x^4+1)^(3/4)+2*RootOf(_Z^4+1)*x)/(x^2+x+1)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^4+1\right)^{\frac{1}{4}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2+x+1)/(x^4+1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/((x^4 + 1)^(1/4)*(x^2 + x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x-1}{\left(x^4+1\right)^{1/4}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x^4 + 1)^(1/4)*(x + x^2 + 1)),x)
```

```
[Out] int((x - 1)/((x^4 + 1)^(1/4)*(x + x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt[4]{x^4+1}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**2+x+1)/(x**4+1)**(1/4),x)
```

```
[Out] Integral((x - 1)/((x**4 + 1)**(1/4)*(x**2 + x + 1)), x)
```

3.1443 $\int \frac{x^2 \sqrt[4]{x^3+x^4}}{-1+x} \, dx$

Optimal. Leaf size=114

$$-\frac{155}{64} \tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)+2\sqrt[4]{2} \tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^3}}\right)+\frac{155}{64} \tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)-2\sqrt[4]{2} \tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^3}}\right)+\frac{1}{96}\sqrt[4]{x^4+$$

Rubi [B] time = 0.21, antiderivative size = 230, normalized size of antiderivative = 2.02, number of steps used = 27, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {2042, 101, 157, 50, 63, 331, 298, 203, 206, 105, 93}

$$\frac{13}{24}\sqrt[4]{x^4+x^3}\,x+\frac{101}{96}\sqrt[4]{x^4+x^3}-\frac{155\sqrt[4]{x^4+x^3}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{64\sqrt[4]{x+1}\,x^{3/4}}+\frac{2\sqrt[4]{2}\sqrt[4]{x^4+x^3}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{\sqrt[4]{x+1}\,x^{3/4}}+\frac{155\sqrt[4]{x^4+x^3}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{64\sqrt[4]{x+1}\,x^{3/4}}-\frac{2\sqrt[4]{2}\sqrt[4]{x^4+x^3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{\sqrt[4]{x+1}\,x^{3/4}}+\frac{1}{3}\sqrt[4]{x^4+x^3}\,x^2$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(x^3 + x^4)^(1/4))/(-1 + x),x]
```

```
[Out] (101*(x^3 + x^4)^(1/4))/96 + (13*x*(x^3 + x^4)^(1/4))/24 + (x^2*(x^3 + x^4)
^(1/4))/3 - (155*(x^3 + x^4)^(1/4)*ArcTan[x^(1/4)/(1 + x)^(1/4)])/(64*x^(3/
4)*(1 + x)^(1/4)) + (2*2^(1/4)*(x^3 + x^4)^(1/4)*ArcTan[(2^(1/4)*x^(1/4))/(
1 + x)^(1/4)])/(x^(3/4)*(1 + x)^(1/4)) + (155*(x^3 + x^4)^(1/4)*ArcTanh[x^(
1/4)/(1 + x)^(1/4)])/(64*x^(3/4)*(1 + x)^(1/4)) - (2*2^(1/4)*(x^3 + x^4)^(1
/4)*ArcTanh[(2^(1/4)*x^(1/4))/(1 + x)^(1/4)])/(x^(3/4)*(1 + x)^(1/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\int \frac{x^2\sqrt[4]{x^3+x^4}}{-1+x}\,dx = \frac{\sqrt[4]{x^3+x^4}\int \frac{x^{11/4}\sqrt[4]{1+x}}{-1+x}\,dx}{x^{3/4}\sqrt[4]{1+x}}$$

$$= \frac{1}{3}x^2\sqrt[4]{x^3+x^4} - \frac{\sqrt[4]{x^3+x^4}\int \frac{\left(-\frac{11}{4}-\frac{13x}{4}\right)x^{7/4}}{(-1+x)(1+x)^{3/4}}\,dx}{3x^{3/4}\sqrt[4]{1+x}}$$

$$= \frac{1}{3}x^2\sqrt[4]{x^3+x^4} + \frac{\left(13\sqrt[4]{x^3+x^4}\right)\int \frac{x^{7/4}}{(1+x)^{3/4}}\,dx}{12x^{3/4}\sqrt[4]{1+x}} + \frac{\left(2\sqrt[4]{x^3+x^4}\right)\int \frac{x^{7/4}}{(-1+x)(1+x)^{3/4}}\,dx}{x^{3/4}\sqrt[4]{1+x}}$$

$$= \frac{13}{24}x\sqrt[4]{x^3+x^4} + \frac{1}{3}x^2\sqrt[4]{x^3+x^4} - \frac{\left(91\sqrt[4]{x^3+x^4}\right)\int \frac{x^{3/4}}{(1+x)^{3/4}}\,dx}{96x^{3/4}\sqrt[4]{1+x}} + \frac{\left(2\sqrt[4]{x^3+x^4}\right)\int \frac{x^{3/4}}{(1+x)^{3/4}}\,dx}{x^{3/4}\sqrt[4]{1+x}} + \cdots$$

$$= \frac{101}{96}\sqrt[4]{x^3+x^4} + \frac{13}{24}x\sqrt[4]{x^3+x^4} + \frac{1}{3}x^2\sqrt[4]{x^3+x^4} + \frac{\left(91\sqrt[4]{x^3+x^4}\right)\int \frac{1}{\sqrt[4]{x}(1+x)^{3/4}}\,dx}{128x^{3/4}\sqrt[4]{1+x}} - \frac{\left(3\sqrt[4]{x^3+\cdots\right.}{2\cdots}$$

$$= \frac{101}{96}\sqrt[4]{x^3+x^4} + \frac{13}{24}x\sqrt[4]{x^3+x^4} + \frac{1}{3}x^2\sqrt[4]{x^3+x^4} + \frac{\left(91\sqrt[4]{x^3+x^4}\right)\text{Subst}\left(\int \frac{x^2}{\left(1+x^4\right)^{3/4}}\,dx, x, \sqrt[4]{x}\cdots\right.}{32x^{3/4}\sqrt[4]{1+x}}$$

$$= \frac{101}{96}\sqrt[4]{x^3+x^4} + \frac{13}{24}x\sqrt[4]{x^3+x^4} + \frac{1}{3}x^2\sqrt[4]{x^3+x^4} + \frac{\left(91\sqrt[4]{x^3+x^4}\right)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{32x^{3/4}\sqrt[4]{1+x}}$$

$$= \frac{101}{96}\sqrt[4]{x^3+x^4} + \frac{13}{24}x\sqrt[4]{x^3+x^4} + \frac{1}{3}x^2\sqrt[4]{x^3+x^4} + \frac{2\sqrt[4]{2}\sqrt[4]{x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}} - \frac{2\sqrt[4]{2}\sqrt[4]{x^3\cdots}}{\cdots}$$

$$= \frac{101}{96}\sqrt[4]{x^3+x^4} + \frac{13}{24}x\sqrt[4]{x^3+x^4} + \frac{1}{3}x^2\sqrt[4]{x^3+x^4} - \frac{155\sqrt[4]{x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{64x^{3/4}\sqrt[4]{1+x}} + \frac{2\sqrt[4]{2}\sqrt[4]{x^3+\cdots}}{x^{3\cdots}}$$

Mathematica [C] time = 0.07, size = 124, normalized size = 1.09

$$\frac{4\sqrt[4]{x^3(x+1)}\left(21(x+1)x^2\,{}_2F_1\left(-\frac{1}{4},\frac{11}{4};\frac{15}{4};-x\right)+33(x+1)x\,{}_2F_1\left(-\frac{1}{4},\frac{7}{4};\frac{11}{4};-x\right)+77(x+1)\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-x\right)+77(x+1)\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-x\right)-154\sqrt[4]{x+1}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{2x}{x+1}\right)\right)}{231(x+1)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(x^3 + x^4)^(1/4))/(-1 + x),x]
```

```
[Out] (4*(x^3*(1 + x))^(1/4)*(77*(1 + x)*Hypergeometric2F1[-1/4, 3/4, 7/4, -x] +
33*x*(1 + x)*Hypergeometric2F1[-1/4, 7/4, 11/4, -x] + 21*x^2*(1 + x)*Hyperg
eometric2F1[-1/4, 11/4, 15/4, -x] + 77*(1 + x)*Hypergeometric2F1[3/4, 3/4,
7/4, -x] - 154*(1 + x)^(1/4)*Hypergeometric2F1[3/4, 1, 7/4, (2*x)/(1 + x)])
)/(231*(1 + x)^(5/4))
```

IntegrateAlgebraic [A] time = 0.56, size = 114, normalized size = 1.00

$$-\frac{155}{64}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)+2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+x^3}}\right)+\frac{155}{64}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+x^3}}\right)+\frac{1}{96}\sqrt[4]{x^4+x^3}\left(32x^2+52x+101\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(x^3 + x^4)^(1/4))/(-1 + x),x]
```

```
[Out] ((101 + 52*x + 32*x^2)*(x^3 + x^4)^(1/4))/96 - (155*ArcTan[x/(x^3 + x^4)^(1
/4)])/64 + 2*2^(1/4)*ArcTan[(2^(1/4)*x)/(x^3 + x^4)^(1/4)] + (155*ArcTanh[x
/(x^3 + x^4)^(1/4)])/64 - 2*2^(1/4)*ArcTanh[(2^(1/4)*x)/(x^3 + x^4)^(1/4)]
```

fricas [B] time = 0.43, size = 183, normalized size = 1.61

$$\frac{1}{96}\left(x^4+x^3\right)^{\frac{1}{4}}\left(32\,x^2+52\,x+101\right)+4\cdot 2^{\frac{1}{4}}\arctan\left(\frac{2^{\frac{3}{4}}x\sqrt{\frac{\sqrt{2}x^2+\sqrt{x^4+x^3}}{x^2}}-2^{\frac{3}{4}}\left(x^4+x^3\right)^{\frac{1}{4}}}{2\,x}\right)-2^{\frac{1}{4}}\log\left(\frac{2^{\frac{1}{4}}x+\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)+2^{\frac{1}{4}}\log\left(-\frac{2^{\frac{1}{4}}x-\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{155}{64}\arctan\left(\frac{\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{155}{128}\log\left(\frac{x+\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)-\frac{155}{128}\log\left(-\frac{x-\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4+x^3)^(1/4)/(-1+x),x, algorithm="fricas")
```

```
[Out] 1/96*(x^4 + x^3)^(1/4)*(32*x^2 + 52*x + 101) + 4*2^(1/4)*arctan(1/2*(2^(3/4
)*x*sqrt((sqrt(2)*x^2 + sqrt(x^4 + x^3))/x^2) - 2^(3/4)*(x^4 + x^3)^(1/4))/
x) - 2^(1/4)*log((2^(1/4)*x + (x^4 + x^3)^(1/4))/x) + 2^(1/4)*log(-(2^(1/4)
*x - (x^4 + x^3)^(1/4))/x) + 155/64*arctan((x^4 + x^3)^(1/4)/x) + 155/128*l
og((x + (x^4 + x^3)^(1/4))/x) - 155/128*log(-(x - (x^4 + x^3)^(1/4))/x)
```

giac [A] time = 0.53, size = 123, normalized size = 1.08

$$\frac{1}{96}\left(101\left(\frac{1}{x}+1\right)^{\frac{9}{4}}-150\left(\frac{1}{x}+1\right)^{\frac{5}{4}}+81\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right)x^3-2\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right)-2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right)+2^{\frac{1}{4}}\log\left(\left|-2^{\frac{1}{4}}+\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right|\right)+\frac{155}{64}\arctan\left(\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right)+\frac{155}{128}\log\left(\left(\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right)-\frac{155}{128}\log\left(\left|\left(\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4+x^3)^(1/4)/(-1+x),x, algorithm="giac")
```

```
[Out] 1/96*(101*(1/x + 1)^(9/4) - 150*(1/x + 1)^(5/4) + 81*(1/x + 1)^(1/4))*x^3 -
 2*2^(1/4)*arctan(1/2*2^(3/4)*(1/x + 1)^(1/4)) - 2^(1/4)*log(2^(1/4) + (1/x
 + 1)^(1/4)) + 2^(1/4)*log(abs(-2^(1/4) + (1/x + 1)^(1/4))) + 155/64*arctan
((1/x + 1)^(1/4)) + 155/128*log((1/x + 1)^(1/4) + 1) - 155/128*log(abs((1/x
 + 1)^(1/4) - 1))
```

maple [C] time = 1.88, size = 878, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4+x^3)^(1/4)/(-1+x),x)
```

```
[Out] 1/96*(32*x^2+52*x+101)*(x^3*(1+x))^(1/4)+(-155/128*RootOf(_Z^2+1)*ln((2*(x^
4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^2+1)*x-2*RootOf(_Z^2+1)*x^3+2*(x^4+3*x^3+3
*x^2+x)^(3/4)+2*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^2+1)-2*(x^4+3*x^3+3*x^2
+x)^(1/4)*x^2-5*RootOf(_Z^2+1)*x^2-4*(x^4+3*x^3+3*x^2+x)^(1/4)*x-4*RootOf(_
Z^2+1)*x-2*(x^4+3*x^3+3*x^2+x)^(1/4)-RootOf(_Z^2+1))/(1+x)^2)-RootOf(_Z^4-2
)*ln((2*RootOf(_Z^4-2)^3*(x^4+3*x^3+3*x^2+x)^(1/2)*x+2*RootOf(_Z^4-2)^3*(x^
4+3*x^3+3*x^2+x)^(1/2)+2*RootOf(_Z^4-2)^2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2+4*(
x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(_Z^4-2)^2*x+3*RootOf(_Z^4-2)*x^3+2*(x^4+3*x
^3+3*x^2+x)^(1/4)*RootOf(_Z^4-2)^2+7*RootOf(_Z^4-2)*x^2+4*(x^4+3*x^3+3*x^2+
x)^(3/4)+5*RootOf(_Z^4-2)*x+RootOf(_Z^4-2))/(-1+x)/(1+x)^2)-RootOf(_Z^2+Roo
tOf(_Z^4-2)^2)*ln(-(2*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^4-2)^2*RootOf(_Z^
2+RootOf(_Z^4-2)^2)*x+2*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^4-2)^2*RootOf(_
Z^2+RootOf(_Z^4-2)^2)+2*RootOf(_Z^4-2)^2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2+4*(x
^4+3*x^3+3*x^2+x)^(1/4)*RootOf(_Z^4-2)^2*x-3*RootOf(_Z^2+RootOf(_Z^4-2)^2)*
x^3+2*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(_Z^4-2)^2-4*(x^4+3*x^3+3*x^2+x)^(3/4
)-7*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^2-5*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x-Roo
tOf(_Z^2+RootOf(_Z^4-2)^2))/(-1+x)/(1+x)^2)-155/128*ln((2*(x^4+3*x^3+3*x^2+
x)^(3/4)-2*(x^4+3*x^3+3*x^2+x)^(1/2)*x+2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2-2*x^
3-2*(x^4+3*x^3+3*x^2+x)^(1/2)+4*(x^4+3*x^3+3*x^2+x)^(1/4)*x-5*x^2+2*(x^4+3*
x^3+3*x^2+x)^(1/4)-4*x-1)/(1+x)^2))*(x^3*(1+x))^(1/4)/x*(x*(1+x)^3)^(1/4)/(
1+x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3\right)^{\frac{1}{4}}x^2}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4+x^3)^(1/4)/(-1+x),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + x^3)^(1/4)*x^2/(x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \left(x^4+x^3\right)^{1/4}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^3 + x^4)^(1/4))/(x - 1),x)
```

```
[Out] int((x^2*(x^3 + x^4)^(1/4))/(x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt[4]{x^3 (x+1)}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**4+x**3)**(1/4)/(-1+x),x)
```

```
[Out] Integral(x**2*(x**3*(x + 1))**(1/4)/(x - 1), x)
```

3.1444 $\int \frac{(-b+ax^3)(-b+2ax^3)}{x^6\sqrt[4]{-bx+ax^4}}\,dx$

Optimal. Leaf size=114

$$\frac{4}{3}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{4}{3}a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{4\left(3b-17ax^3\right)\left(ax^4-bx\right)^{3/4}}{63x^6}$$

Rubi [A] time = 0.40, antiderivative size = 177, normalized size of antiderivative = 1.55, number of steps used = 12, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {2052, 2011, 329, 275, 240, 212, 206, 203, 2016, 2014}

$$\frac{4a^{7/4}\sqrt[4]{x}\sqrt[4]{ax^3-b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3\sqrt[4]{ax^4-bx}}+\frac{4a^{7/4}\sqrt[4]{x}\sqrt[4]{ax^3-b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3\sqrt[4]{ax^4-bx}}+\frac{4b\left(ax^4-bx\right)^{3/4}}{21x^6}-\frac{68a\left(ax^4-bx\right)^{3/4}}{63x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a*x^3)*(-b + 2*a*x^3))/(x^6*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (4*b*(-(b*x) + a*x^4)^(3/4))/(21*x^6) - (68*a*(-(b*x) + a*x^4)^(3/4))/(63*x
^3) + (4*a^(7/4)*x^(1/4)*(-b + a*x^3)^(1/4)*ArcTan[(a^(1/4)*x^(3/4))/(-b +
a*x^3)^(1/4)])/(3*(-(b*x) + a*x^4)^(1/4)) + (4*a^(7/4)*x^(1/4)*(-b + a*x^3)
^(1/4)*ArcTanh[(a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)])/(3*(-(b*x) + a*x^4)^(
1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\left(-b+ax^3\right)\left(-b+2ax^3\right)}{x^6\sqrt[4]{-bx+ax^4}}\,dx = \int \left(\frac{2a^2}{\sqrt[4]{-bx+ax^4}}+\frac{b^2}{x^6\sqrt[4]{-bx+ax^4}}-\frac{3ab}{x^3\sqrt[4]{-bx+ax^4}}\right)dx$$

$$= \left(2a^2\right)\int \frac{1}{\sqrt[4]{-bx+ax^4}}\,dx-(3ab)\int \frac{1}{x^3\sqrt[4]{-bx+ax^4}}\,dx+b^2\int \frac{1}{x^6\sqrt[4]{-bx+ax^4}}$$

$$= \frac{4b\left(-bx+ax^4\right)^{3/4}}{21x^6}-\frac{4a\left(-bx+ax^4\right)^{3/4}}{3x^3}+\frac{1}{7}(4ab)\int \frac{1}{x^3\sqrt[4]{-bx+ax^4}}\,dx+\frac{\left(2a^2\right.}{}$$

$$= \frac{4b\left(-bx+ax^4\right)^{3/4}}{21x^6}-\frac{68a\left(-bx+ax^4\right)^{3/4}}{63x^3}+\frac{\left(8a^2\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{-b}}\right.}{\sqrt[4]{-bx+ax^4}}$$

$$= \frac{4b\left(-bx+ax^4\right)^{3/4}}{21x^6}-\frac{68a\left(-bx+ax^4\right)^{3/4}}{63x^3}+\frac{\left(8a^2\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{-b}}\right.}{3\sqrt[4]{-bx+ax^4}}$$

$$= \frac{4b\left(-bx+ax^4\right)^{3/4}}{21x^6}-\frac{68a\left(-bx+ax^4\right)^{3/4}}{63x^3}+\frac{\left(8a^2\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-ax}\right.}{3\sqrt[4]{-bx+ax^4}}$$

$$= \frac{4b\left(-bx+ax^4\right)^{3/4}}{21x^6}-\frac{68a\left(-bx+ax^4\right)^{3/4}}{63x^3}+\frac{\left(4a^2\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{}}\right.}{3\sqrt[4]{-bx+ax^4}}$$

$$= \frac{4b\left(-bx+ax^4\right)^{3/4}}{21x^6}-\frac{68a\left(-bx+ax^4\right)^{3/4}}{63x^3}+\frac{4a^{7/4}\sqrt[4]{x}\sqrt[4]{-b+ax^3}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{-b+ax}}\right.}{3\sqrt[4]{-bx+ax^4}}$$

Mathematica [C] time = 0.09, size = 71, normalized size = 0.62

$$\frac{4\left(ax^4-bx\right)^{3/4}\left(3\left(b-ax^3\right)-\frac{14ax^3\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};\frac{ax^3}{b}\right)}{\left(1-\frac{ax^3}{b}\right)^{3/4}}\right)}{63x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + a*x^3)*(-b + 2*a*x^3))/(x^6*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (4*(-(b*x) + a*x^4)^(3/4)*(3*(b - a*x^3) - (14*a*x^3*Hypergeometric2F1[-3/4
, -3/4, 1/4, (a*x^3)/b])/(1 - (a*x^3)/b)^(3/4)))/(63*x^6)
```

IntegrateAlgebraic [A] time = 0.47, size = 114, normalized size = 1.00

$$\frac{4}{3}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{4}{3}a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{4\left(3b-17ax^3\right)\left(ax^4-bx\right)^{3/4}}{63x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^3)*(-b + 2*a*x^3))/(x^6*(-(b*x) + a*x^4)^(1/4
)),x]
```

```
[Out] (4*(3*b - 17*a*x^3)*(-(b*x) + a*x^4)^(3/4))/(63*x^6) + (4*a^(7/4)*ArcTan[(a
^(1/4)*(-(b*x) + a*x^4)^(3/4))/(-b + a*x^3)])/3 + (4*a^(7/4)*ArcTanh[(a^(1/
4)*(-(b*x) + a*x^4)^(3/4))/(-b + a*x^3)])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(2*a*x^3-b)/x^6/(a*x^4-b*x)^(1/4),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [B] time = 0.30, size = 209, normalized size = 1.83

$$\frac{2}{3}\sqrt{2}(-a)^{\frac{3}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)+\frac{2}{3}\sqrt{2}(-a)^{\frac{3}{4}}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)-\frac{1}{3}\sqrt{2}(-a)^{\frac{3}{4}}a\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^3}}\right)+\frac{1}{3}\sqrt{2}(-a)^{\frac{3}{4}}a\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^3}}\right)-\frac{4}{21}\left(a-\frac{b}{x^3}\right)^{\frac{7}{4}}-\frac{8}{9}\left(a-\frac{b}{x^3}\right)^{\frac{3}{4}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(2*a*x^3-b)/x^6/(a*x^4-b*x)^(1/4),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(2)*(-a)^(3/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a - b/
x^3)^(1/4))/(-a)^(1/4)) + 2/3*sqrt(2)*(-a)^(3/4)*a*arctan(-1/2*sqrt(2)*(sqr
t(2)*(-a)^(1/4) - 2*(a - b/x^3)^(1/4))/(-a)^(1/4)) - 1/3*sqrt(2)*(-a)^(3/4)
*a*log(sqrt(2)*(-a)^(1/4)*(a - b/x^3)^(1/4) + sqrt(-a) + sqrt(a - b/x^3)) +
 1/3*sqrt(2)*(-a)^(3/4)*a*log(-sqrt(2)*(-a)^(1/4)*(a - b/x^3)^(1/4) + sqrt(
-a) + sqrt(a - b/x^3)) - 4/21*(a - b/x^3)^(7/4) - 8/9*(a - b/x^3)^(3/4)*a
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{3}-b\right)\left(2a\,x^{3}-b\right)}{x^{6}\left(a\,x^{4}-bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b)*(2*a*x^3-b)/x^6/(a*x^4-b*x)^(1/4),x)
```

```
[Out] int((a*x^3-b)*(2*a*x^3-b)/x^6/(a*x^4-b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,ax^{3}-b\right)\left(ax^{3}-b\right)}{\left(ax^{4}-bx\right)^{\frac{1}{4}}x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)*(2*a*x^3-b)/x^6/(a*x^4-b*x)^(1/4),x, algorithm="maxima"
)
```

```
[Out] integrate((2*a*x^3 - b)*(a*x^3 - b)/((a*x^4 - b*x)^(1/4)*x^6), x)
```

mupad [B] time = 1.88, size = 72, normalized size = 0.63

$$-\frac{4\left(3\,b^2+17\,a^2\,x^6-20\,a\,b\,x^3-42\,a^2\,x^6\left(1-\frac{a\,x^3}{b}\right)^{1/4}{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \frac{a\,x^3}{b}\right)\right)}{63\,x^5\left(a\,x^4-b\,x\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b - a*x^3)*(b - 2*a*x^3))/(x^6*(a*x^4 - b*x)^(1/4)),x)
```

```
[Out] -(4*(3*b^2 + 17*a^2*x^6 - 20*a*b*x^3 - 42*a^2*x^6*(1 - (a*x^3)/b)^(1/4)*hyp
ergeom([1/4, 1/4], 5/4, (a*x^3)/b)))/(63*x^5*(a*x^4 - b*x)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^3 - b\right)\left(2ax^3 - b\right)}{x^6 \sqrt[4]{x\left(ax^3 - b\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b)*(2*a*x**3-b)/x**6/(a*x**4-b*x)**(1/4),x)
```

```
[Out] Integral((a*x**3 - b)*(2*a*x**3 - b)/(x**6*(x*(a*x**3 - b))**(1/4)), x)
```

3.1445 $\int \frac{-b+ax^4+x^8}{x^8(-b+ax^4)\sqrt[4]{b+ax^4}}\,dx$

Optimal. Leaf size=114

$$\frac{(ax^4+b)^{3/4}(4ax^4-3b)}{21b^2x^7}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\sqrt[4]{2}\sqrt[4]{a}\,b}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\sqrt[4]{2}\sqrt[4]{a}\,b}$$

Rubi [A] time = 0.67, antiderivative size = 126, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {6725, 271, 264, 377, 212, 206, 203}

$$\frac{4a(ax^4+b)^{3/4}}{21b^2x^3}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\sqrt[4]{2}\sqrt[4]{a}\,b}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\sqrt[4]{2}\sqrt[4]{a}\,b}-\frac{(ax^4+b)^{3/4}}{7bx^7}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^4 + x^8)/(x^8*(-b + a*x^4)*(b + a*x^4)^(1/4)),x]
```

```
[Out] -1/7*(b + a*x^4)^(3/4)/(b*x^7) + (4*a*(b + a*x^4)^(3/4))/(21*b^2*x^3) - Arc
Tan[(2^(1/4)*a^(1/4)*x)/(b + a*x^4)^(1/4)]/(2*2^(1/4)*a^(1/4)*b) - ArcTanh[
(2^(1/4)*a^(1/4)*x)/(b + a*x^4)^(1/4)]/(2*2^(1/4)*a^(1/4)*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-b+ax^4+x^8}{x^8\left(-b+ax^4\right)\sqrt[4]{b+ax^4}}\,dx &= \int \left(\frac{1}{x^8\sqrt[4]{b+ax^4}}+\frac{1}{\left(-b+ax^4\right)\sqrt[4]{b+ax^4}}\right)dx\\ &= \int \frac{1}{x^8\sqrt[4]{b+ax^4}}\,dx+\int \frac{1}{\left(-b+ax^4\right)\sqrt[4]{b+ax^4}}\,dx\\ &= -\frac{\left(b+ax^4\right)^{3/4}}{7bx^7}-\frac{(4a)\int \frac{1}{x^4\sqrt[4]{b+ax^4}}\,dx}{7b}+\text{Subst}\left(\int \frac{1}{-b+2abx^4}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right.\\ &= -\frac{\left(b+ax^4\right)^{3/4}}{7bx^7}+\frac{4a\left(b+ax^4\right)^{3/4}}{21b^2x^3}-\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\sqrt{a}x^2}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)}{2b}-\text{Subs}\\ &= -\frac{\left(b+ax^4\right)^{3/4}}{7bx^7}+\frac{4a\left(b+ax^4\right)^{3/4}}{21b^2x^3}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{b+ax^4}}\right)}{2\sqrt[4]{2}\sqrt[4]{a}b}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{b+ax^4}}\right)}{2\sqrt[4]{2}\sqrt[4]{a}b}\end{aligned}$$

Mathematica [A] time = 0.12, size = 114, normalized size = 1.00

$$\frac{4\sqrt[4]{a}\left(ax^4+b\right)^{3/4}\left(4ax^4-3b\right)-21\,2^{3/4}bx^7\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)-21\,2^{3/4}bx^7\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{84\sqrt[4]{a}b^2x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^4 + x^8)/(x^8*(-b + a*x^4)*(b + a*x^4)^(1/4)),x]
```

```
[Out] (4*a^(1/4)*(b + a*x^4)^(3/4)*(-3*b + 4*a*x^4) - 21*2^(3/4)*b*x^7*ArcTan[(2^
(1/4)*a^(1/4)*x)/(b + a*x^4)^(1/4)] - 21*2^(3/4)*b*x^7*ArcTanh[(2^(1/4)*a^(
1/4)*x)/(b + a*x^4)^(1/4)])/(84*a^(1/4)*b^2*x^7)
```

IntegrateAlgebraic [A] time = 0.92, size = 114, normalized size = 1.00

$$\frac{\left(ax^4+b\right)^{3/4}\left(4ax^4-3b\right)}{21b^2x^7}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{2\sqrt[4]{2}\sqrt[4]{a}b}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{2\sqrt[4]{2}\sqrt[4]{a}b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^4 + x^8)/(x^8*(-b + a*x^4)*(b + a*x^4)^(1/4)),
x]
```

```
[Out] ((b + a*x^4)^(3/4)*(-3*b + 4*a*x^4))/(21*b^2*x^7) - ArcTan[(2^(1/4)*a^(1/4)
*x)/(b + a*x^4)^(1/4)]/(2*2^(1/4)*a^(1/4)*b) - ArcTanh[(2^(1/4)*a^(1/4)*x)/
(b + a*x^4)^(1/4)]/(2*2^(1/4)*a^(1/4)*b)
```

fricas [B] time = 166.00, size = 461, normalized size = 4.04

$$\frac{84 \left(\frac{1}{2}\right)^{\frac{1}{4}} b^{2} x^{7} \left(\frac{1}{a b^{4}}\right)^{\frac{1}{4}} \arctan\left(\frac{2 \left(2 \left(\frac{1}{2}\right)^{\frac{3}{4}} \left(a x^{4}+b\right)^{\frac{3}{4}} a b^{3} x \left(\frac{1}{a b^{4}}\right)^{\frac{3}{4}}+2 \left(\frac{1}{2}\right)^{\frac{1}{4}} \left(a x^{4}+b\right)^{\frac{1}{4}} a b x^{3} \left(\frac{1}{a b^{4}}\right)^{\frac{1}{4}}+\left(2 \left(\frac{1}{2}\right)^{\frac{1}{4}} \sqrt{a x^{4}+b}\, a b x^{2} \left(\frac{1}{a b^{4}}\right)^{\frac{1}{4}}+\left(\frac{1}{2}\right)^{\frac{3}{4}} \left(3 a^{2} b^{3} x^{4}+a b^{4}\right) \left(\frac{1}{a b^{4}}\right)^{\frac{3}{4}}\right) \sqrt{\sqrt{\frac{1}{2}}\, b^{2} \sqrt{\frac{1}{a b^{4}}}}\right)}{a x^{4}-b}\right) - 21 \left(\frac{1}{2}\right)^{\frac{1}{4}} b^{2} x^{7} \left(\frac{1}{a b^{4}}\right)^{\frac{1}{4}} \log\left(\frac{4 \left(\frac{1}{2}\right)^{\frac{3}{4}} \sqrt{a x^{4}+b}\, a b^{3} x^{2} \left(\frac{1}{a b^{4}}\right)^{\frac{3}{4}}+4 \sqrt{\frac{1}{2}} \left(a x^{4}+b\right)^{\frac{1}{4}} a b^{2} x^{3} \sqrt{\frac{1}{a b^{4}}}+2 \left(a x^{4}+b\right)^{\frac{3}{4}} x+\left(\frac{1}{2}\right)^{\frac{1}{4}} \left(3 a b x^{4}+b^{2}\right) \left(\frac{1}{a b^{4}}\right)^{\frac{1}{4}}}{2 \left(a x^{4}-b\right)}\right) + 21 \left(\frac{1}{2}\right)^{\frac{1}{4}} b^{2} x^{7} \left(\frac{1}{a b^{4}}\right)^{\frac{1}{4}} \log\left(-\frac{4 \left(\frac{1}{2}\right)^{\frac{3}{4}} \sqrt{a x^{4}+b}\, a b^{3} x^{2} \left(\frac{1}{a b^{4}}\right)^{\frac{3}{4}}-4 \sqrt{\frac{1}{2}} \left(a x^{4}+b\right)^{\frac{1}{4}} a b^{2} x^{3} \sqrt{\frac{1}{a b^{4}}}-2 \left(a x^{4}+b\right)^{\frac{3}{4}} x+\left(\frac{1}{2}\right)^{\frac{1}{4}} \left(3 a b x^{4}+b^{2}\right) \left(\frac{1}{a b^{4}}\right)^{\frac{1}{4}}}{2 \left(a x^{4}-b\right)}\right) + 8 \left(4 a x^{4}-3 b\right) \left(a x^{4}+b\right)^{\frac{3}{4}}}{168 b^{2} x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+a*x^4-b)/x^8/(a*x^4-b)/(a*x^4+b)^(1/4),x, algorithm="fricas"
)
```

```
[Out] 1/168*(84*(1/2)^(1/4)*b^2*x^7*(1/(a*b^4))^(1/4)*arctan(2*(2*(1/2)^(3/4)*(a*
x^4 + b)^(3/4)*a*b^3*x*(1/(a*b^4))^(3/4) + 2*(1/2)^(1/4)*(a*x^4 + b)^(1/4)*
a*b*x^3*(1/(a*b^4))^(1/4) + (2*(1/2)^(1/4)*sqrt(a*x^4 + b)*a*b*x^2*(1/(a*b^
4))^(1/4) + (1/2)^(3/4)*(3*a^2*b^3*x^4 + a*b^4)*(1/(a*b^4))^(3/4))*sqrt(sqr
t(1/2)*b^2*sqrt(1/(a*b^4))))/(a*x^4 - b)) - 21*(1/2)^(1/4)*b^2*x^7*(1/(a*b^
4))^(1/4)*log(1/2*(4*(1/2)^(3/4)*sqrt(a*x^4 + b)*a*b^3*x^2*(1/(a*b^4))^(3/4
) + 4*sqrt(1/2)*(a*x^4 + b)^(1/4)*a*b^2*x^3*sqrt(1/(a*b^4)) + 2*(a*x^4 + b)
^(3/4)*x + (1/2)^(1/4)*(3*a*b*x^4 + b^2)*(1/(a*b^4))^(1/4))/(a*x^4 - b)) +
21*(1/2)^(1/4)*b^2*x^7*(1/(a*b^4))^(1/4)*log(-1/2*(4*(1/2)^(3/4)*sqrt(a*x^4
 + b)*a*b^3*x^2*(1/(a*b^4))^(3/4) - 4*sqrt(1/2)*(a*x^4 + b)^(1/4)*a*b^2*x^3
*sqrt(1/(a*b^4)) - 2*(a*x^4 + b)^(3/4)*x + (1/2)^(1/4)*(3*a*b*x^4 + b^2)*(1
/(a*b^4))^(1/4))/(a*x^4 - b)) + 8*(4*a*x^4 - 3*b)*(a*x^4 + b)^(3/4))/(b^2*x
^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8} + a x^{4} - b}{{\left(a x^{4} + b\right)}^{\frac{1}{4}} {\left(a x^{4} - b\right)} x^{8}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+a*x^4-b)/x^8/(a*x^4-b)/(a*x^4+b)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^8 + a*x^4 - b)/((a*x^4 + b)^(1/4)*(a*x^4 - b)*x^8), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x^{8}+a \,x^{4}-b}{x^{8} \left(a \,x^{4}-b \right) \left(a \,x^{4}+b \right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8+a*x^4-b)/x^8/(a*x^4-b)/(a*x^4+b)^(1/4),x)
```

```
[Out] int((x^8+a*x^4-b)/x^8/(a*x^4-b)/(a*x^4+b)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8} + a x^{4} - b}{{\left(a x^{4} + b\right)}^{\frac{1}{4}} {\left(a x^{4} - b\right)} x^{8}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+a*x^4-b)/x^8/(a*x^4-b)/(a*x^4+b)^(1/4),x, algorithm="maxima"
)
```

```
[Out] integrate((x^8 + a*x^4 - b)/((a*x^4 + b)^(1/4)*(a*x^4 - b)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^8+a\,x^4-b}{x^8\,{\left(a\,x^4+b\right)}^{1/4}\,\left(b-a\,x^4\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x^4 - b + x^8)/(x^8*(b + a*x^4)^(1/4)*(b - a*x^4)),x)
[Out] -int((a*x^4 - b + x^8)/(x^8*(b + a*x^4)^(1/4)*(b - a*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4 - b + x^8}{x^8 \left(ax^4 - b\right) \sqrt[4]{ax^4 + b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8+a*x**4-b)/x**8/(a*x**4-b)/(a*x**4+b)**(1/4),x)
[Out] Integral((a*x**4 - b + x**8)/(x**8*(a*x**4 - b)*(a*x**4 + b)**(1/4)), x)
```

3.1446 $\int \frac{1+x^4}{x^4\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=114

$$-\frac{1}{8}\tan^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)-\frac{1}{8}\tanh^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)+\frac{384x^8+456x^6+48x^4-57x^2+\sqrt{x^2+1}\left(384x^7+264x\right.}{24x^3\left(\sqrt{x^2+1}+x\right)^{9/2}}$$

Rubi [A] time = 0.36, antiderivative size = 141, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6742, 2117, 14, 2119, 457, 288, 290, 329, 212, 206, 203}

$$\sqrt{\sqrt{x^2+1}+x}+\frac{1}{24x\sqrt{\sqrt{x^2+1}+x}}+\frac{1}{12x^2\left(\sqrt{x^2+1}+x\right)^{3/2}}-\frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}}-\frac{1}{8}\tan^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)-\frac{1}{8}\tanh^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)-\frac{1}{3x^3\sqrt{\sqrt{x^2+1}+x}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(x^4*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/3*1/(x + Sqrt[1 + x^2])^(3/2) + 1/(12*x^2*(x + Sqrt[1 + x^2])^(3/2)) - 1
/(3*x^3*Sqrt[x + Sqrt[1 + x^2]]) + 1/(24*x*Sqrt[x + Sqrt[1 + x^2]]) + Sqrt[
x + Sqrt[1 + x^2]] - ArcTan[Sqrt[x + Sqrt[1 + x^2]]]/8 - ArcTanh[Sqrt[x + S
qrt[1 + x^2]]]/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1+x^4}{x^4\sqrt{x+\sqrt{1+x^2}}}\,dx = \int \left(\frac{1}{\sqrt{x+\sqrt{1+x^2}}}+\frac{1}{x^4\sqrt{x+\sqrt{1+x^2}}}\right)dx$$

$$= \int \frac{1}{\sqrt{x+\sqrt{1+x^2}}}\,dx + \int \frac{1}{x^4\sqrt{x+\sqrt{1+x^2}}}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{x^{5/2}}\,dx, x, x+\sqrt{1+x^2}\right) + 8\,\text{Subst}\left(\int \frac{x^{3/2}(1+x^2)}{(-1+x^2)^4}\,dx, x, x+\sqrt{1+x^2}\right.$$

$$= -\frac{1}{3x^3\sqrt{x+\sqrt{1+x^2}}} + \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^{5/2}}+\frac{1}{\sqrt{x}}\right)dx, x, x+\sqrt{1+x^2}\right) - \frac{4}{3}\text{Subst}\left(\int \right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \frac{1}{12x^2\left(x+\sqrt{1+x^2}\right)^{3/2}} - \frac{1}{3x^3\sqrt{x+\sqrt{1+x^2}}} + \sqrt{x+\sqrt{1+x^2}}$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \frac{1}{12x^2\left(x+\sqrt{1+x^2}\right)^{3/2}} - \frac{1}{3x^3\sqrt{x+\sqrt{1+x^2}}} + \frac{1}{24x\sqrt{x+\sqrt{1+}}}$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \frac{1}{12x^2\left(x+\sqrt{1+x^2}\right)^{3/2}} - \frac{1}{3x^3\sqrt{x+\sqrt{1+x^2}}} + \frac{1}{24x\sqrt{x+\sqrt{1+}}}$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \frac{1}{12x^2\left(x+\sqrt{1+x^2}\right)^{3/2}} - \frac{1}{3x^3\sqrt{x+\sqrt{1+x^2}}} + \frac{1}{24x\sqrt{x+\sqrt{1+}}}$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \frac{1}{12x^2\left(x+\sqrt{1+x^2}\right)^{3/2}} - \frac{1}{3x^3\sqrt{x+\sqrt{1+x^2}}} + \frac{1}{24x\sqrt{x+\sqrt{1+}}}$$

Mathematica [C] time = 24.91, size = 9486, normalized size = 83.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^4)/(x^4*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.23, size = 114, normalized size = 1.00

$$-\frac{1}{8}\tan^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)-\frac{1}{8}\tanh^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)+\frac{384x^8+456x^6+48x^4-57x^2+\sqrt{x^2+1}\left(384x^7+264x^5-36x^3-30x\right)-8}{24x^3\left(\sqrt{x^2+1}+x\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)/(x^4*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] (-8 - 57*x^2 + 48*x^4 + 456*x^6 + 384*x^8 + Sqrt[1 + x^2]*(-30*x - 36*x^3 +
 264*x^5 + 384*x^7))/(24*x^3*(x + Sqrt[1 + x^2])^(9/2)) - ArcTan[Sqrt[x + S
qrt[1 + x^2]]]/8 - ArcTanh[Sqrt[x + Sqrt[1 + x^2]]]/8
```

fricas [A] time = 0.43, size = 109, normalized size = 0.96

$$-\frac{6x^3\arctan\left(\sqrt{x+\sqrt{x^2+1}}\right)+3x^3\log\left(\sqrt{x+\sqrt{x^2+1}}+1\right)-3x^3\log\left(\sqrt{x+\sqrt{x^2+1}}-1\right)+2\left(16x^5-19x^3-\left(16x^4-3x^2-8\right)\sqrt{x^2+1}-10x\right)\sqrt{x+\sqrt{x^2+1}}}{48x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x^4/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/48*(6*x^3*arctan(sqrt(x + sqrt(x^2 + 1))) + 3*x^3*log(sqrt(x + sqrt(x^2
+ 1)) + 1) - 3*x^3*log(sqrt(x + sqrt(x^2 + 1)) - 1) + 2*(16*x^5 - 19*x^3 -
(16*x^4 - 3*x^2 - 8)*sqrt(x^2 + 1) - 10*x)*sqrt(x + sqrt(x^2 + 1)))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\sqrt{x+\sqrt{x^2+1}}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x^4/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/(sqrt(x + sqrt(x^2 + 1))*x^4), x)
```

maple [C] time = 0.06, size = 84, normalized size = 0.74

$$-\frac{\sqrt{2}\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4},\frac{7}{4}\right],\left[\frac{3}{2},\frac{11}{4}\right],-\frac{1}{x^2}\right)}{7x^{\frac{7}{2}}}-\frac{-\frac{32\sqrt{\pi}\ \sqrt{2}\ \cosh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{1}{x}\right)}{2}\right)}{3x^{\frac{3}{2}}}-\frac{8\sqrt{\pi}\ \sqrt{2}\ x^{\frac{3}{2}}\left(-\frac{4}{3x^4}-\frac{2}{3x^2}+\frac{2}{3}\right)\sinh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{1}{x}\right)}{2}\right)}{\sqrt{1+\frac{1}{x^2}}}}{8\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/x^4/(x+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] -1/7*2^(1/2)/x^(7/2)*hypergeom([1/4,3/4,7/4],[3/2,11/4],-1/x^2)-1/8/Pi^(1/2
)*(-32/3*Pi^(1/2)*2^(1/2)/x^(3/2)*cosh(3/2*arcsinh(1/x))-8*Pi^(1/2)*2^(1/2)
*x^(3/2)*(-4/3/x^4-2/3/x^2+2/3)*sinh(3/2*arcsinh(1/x))/(1+1/x^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\sqrt{x+\sqrt{x^2+1}}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x^4/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(sqrt(x + sqrt(x^2 + 1))*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4+1}{x^4\,\sqrt{x+\sqrt{x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^4*(x + (x^2 + 1)^(1/2))^(1/2)),x)
```

```
[Out] int((x^4 + 1)/(x^4*(x + (x^2 + 1)^(1/2))^(1/2)), x)
```

sympy [C] time = 8.12, size = 90, normalized size = 0.79

$$\frac{4x}{3\sqrt{x+\sqrt{x^2+1}}}+\frac{2\sqrt{x^2+1}}{3\sqrt{x+\sqrt{x^2+1}}}-\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{7}{4}\right){}_{3}F_{2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{7}{4}\\ \frac{3}{2},\frac{11}{4}\end{matrix}\middle|\frac{e^{i\pi}}{x^2}\right)}{4\pi x^{\frac{7}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/x**4/(x+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] 4*x/(3*sqrt(x + sqrt(x**2 + 1))) + 2*sqrt(x**2 + 1)/(3*sqrt(x + sqrt(x**2 +
 1))) - gamma(1/4)*gamma(3/4)*gamma(7/4)*hyper((1/4, 3/4, 7/4), (3/2, 11/4)
, exp_polar(I*pi)/x**2)/(4*pi*x**(7/2)*gamma(11/4))
```

3.1447 $\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^2\sqrt{b^2+ax^2}}\,dx$

Optimal. Leaf size=114

$$-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{bx}-\frac{\sqrt{2}\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{\sqrt{2}\sqrt{b}}\right)}{b^{3/2}}$$

Rubi [F] time = 0.66, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^2\sqrt{b^2+ax^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b + Sqrt[b^2 + a*x^2]]/(x^2*Sqrt[b^2 + a*x^2]),x]
[Out] Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(x^2*Sqrt[b^2 + a*x^2]), x]
```

Rubi steps

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^2\sqrt{b^2+ax^2}}\,dx=\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^2\sqrt{b^2+ax^2}}\,dx$$

Mathematica [C] time = 0.24, size = 59, normalized size = 0.52

$$-\frac{\sqrt{\sqrt{ax^2+b^2}+b}\;{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b-\sqrt{b^2+ax^2}}{2b}\right)}{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(x^2*Sqrt[b^2 + a*x^2]),x]
[Out] -((Sqrt[b + Sqrt[b^2 + a*x^2]]*Hypergeometric2F1[-1/2, 1, 1/2, (b - Sqrt[b^
2 + a*x^2])/(2*b)])/(b*x))
```

IntegrateAlgebraic [A] time = 0.17, size = 82, normalized size = 0.72

$$-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{bx}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{2}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + Sqrt[b^2 + a*x^2]]/(x^2*Sqrt[b^2 + a*x^2]),x]
[Out] -(Sqrt[b + Sqrt[b^2 + a*x^2]]/(b*x)) - (Sqrt[a]*ArcTan[(Sqrt[a]*x)/(Sqrt[2]
*Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/(Sqrt[2]*b^(3/2))
```

fricas [A] time = 38.63, size = 219, normalized size = 1.92

$$\left[\frac{\sqrt{2}\,x\sqrt{-\frac{a}{b}}\log\left(-\frac{a^2x^3+4\,ab^2x-4\,\sqrt{ax^2+b^2}\,abx+2\left(2\,\sqrt{2}\,\sqrt{ax^2+b^2}\,b^2\sqrt{-\frac{a}{b}}-\sqrt{2}\left(abx^2+2\,b^3\right)\sqrt{-\frac{a}{b}}\right)\sqrt{b+\sqrt{ax^2+b^2}}}{x^3}\right)-4\sqrt{b+\sqrt{ax^2+b^2}}}{4\,bx},\frac{\sqrt{2}\,x\sqrt{\frac{a}{b}}\arctan\left(\frac{\sqrt{2}\sqrt{b+\sqrt{ax^2+b^2}}\,b\sqrt{\frac{a}{b}}}{ax}\right)-2\sqrt{b+\sqrt{ax^2+b^2}}}{2\,bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^2/(a*x^2+b^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/4*(sqrt(2)*x*sqrt(-a/b)*log(-(a^2*x^3 + 4*a*b^2*x - 4*sqrt(a*x^2 + b^2)*
a*b*x + 2*(2*sqrt(2)*sqrt(a*x^2 + b^2)*b^2*sqrt(-a/b) - sqrt(2)*(a*b*x^2 +
2*b^3)*sqrt(-a/b))*sqrt(b + sqrt(a*x^2 + b^2)))/x^3) - 4*sqrt(b + sqrt(a*x^
2 + b^2)))/(b*x), 1/2*(sqrt(2)*x*sqrt(a/b)*arctan(sqrt(2)*sqrt(b + sqrt(a*x
^2 + b^2))*b*sqrt(a/b)/(a*x)) - 2*sqrt(b + sqrt(a*x^2 + b^2)))/(b*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\sqrt{ax^2+b^2}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^2/(a*x^2+b^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(sqrt(a*x^2 + b^2)*x^2), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{x^2\sqrt{a\,x^2+b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+(a*x^2+b^2)^(1/2))^(1/2)/x^2/(a*x^2+b^2)^(1/2),x)
```

```
[Out] int((b+(a*x^2+b^2)^(1/2))^(1/2)/x^2/(a*x^2+b^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\sqrt{ax^2+b^2}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^2/(a*x^2+b^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(sqrt(a*x^2 + b^2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b+\sqrt{b^2+a\,x^2}}}{x^2\,\sqrt{b^2+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(x^2*(a*x^2 + b^2)^(1/2)),x)
```

```
[Out] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(x^2*(a*x^2 + b^2)^(1/2)), x)
```

sympy [C] time = 1.03, size = 44, normalized size = 0.39

$$-\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_{3}F_{2}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{1}{2}\end{matrix}\middle|\frac{ax^2e^{i\pi}}{b^2}\right)}{\pi\sqrt{b}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x**2+b**2)**(1/2))**(1/2)/x**2/(a*x**2+b**2)**(1/2),x)
```

```
[Out] -gamma(1/4)*gamma(3/4)*hyper((-1/2, 1/4, 3/4), (1/2, 1/2), a*x**2*exp_polar
(I*pi)/b**2)/(pi*sqrt(b)*x)
```

3.1448 $\int \frac{2+x^2}{x(2-2x+x^2)\sqrt[3]{1-x+x^2}}\,dx$

Optimal. Leaf size=115

$$\log\left(\sqrt[3]{x^2-x+1}+x-1\right)-\frac{1}{2}\log\left(x^2+\left(x^2-x+1\right)^{2/3}+(1-x)\sqrt[3]{x^2-x+1}-2x+1\right)-\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2-x+1}}{\sqrt{3}}-\frac{2x}{\sqrt{3}}}{\sqrt[3]{x^2-x+}}\right.$$

Rubi [F] time = 0.47, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2+x^2}{x\left(2-2x+x^2\right)\sqrt[3]{1-x+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2 + x^2)/(x*(2 - 2*x + x^2)*(1 - x + x^2)^(1/3)),x]
```

```
[Out] (-3*(-((1 - I*Sqrt[3] - 2*x)/x))^(1/3)*(-((1 + I*Sqrt[3] - 2*x)/x))^(1/3)*A
ppellF1[2/3, 1/3, 1/3, 5/3, (1 - I*Sqrt[3])/(2*x), (1 + I*Sqrt[3])/(2*x)])/
(2*2^(2/3)*(1 - x + x^2)^(1/3)) + 2*Defer[Int][1/((2 - 2*x + x^2)*(1 - x +
x^2)^(1/3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x^2}{x\left(2-2x+x^2\right)\sqrt[3]{1-x+x^2}}\,dx &= \int \left(\frac{1}{x\sqrt[3]{1-x+x^2}}+\frac{2}{\left(2-2x+x^2\right)\sqrt[3]{1-x+x^2}}\right)dx\\
&= 2\int \frac{1}{\left(2-2x+x^2\right)\sqrt[3]{1-x+x^2}}\,dx+\int \frac{1}{x\sqrt[3]{1-x+x^2}}\,dx\\
&= 2\int \frac{1}{\left(2-2x+x^2\right)\sqrt[3]{1-x+x^2}}\,dx-\frac{\left(\sqrt[3]{\frac{-1-i\sqrt{3}+2x}{x}}\sqrt[3]{\frac{-1+i\sqrt{3}+2x}{x}}\right)\text{Subst}\left(\int\right.}{2^{2/3}\left(\frac{1}{x}\right)^{2/3}\sqrt[3]{}}\\
&= -\frac{3\sqrt[3]{-\frac{1-i\sqrt{3}-2x}{x}}\sqrt[3]{-\frac{1+i\sqrt{3}-2x}{x}}\,F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{1-i\sqrt{3}}{2x},\frac{1+i\sqrt{3}}{2x}\right)}{2\,2^{2/3}\sqrt[3]{1-x+x^2}}+2\int \frac{}{\left(2-2x+x\right.}
\end{aligned}$$

Mathematica [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{2+x^2}{x\left(2-2x+x^2\right)\sqrt[3]{1-x+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2 + x^2)/(x*(2 - 2*x + x^2)*(1 - x + x^2)^(1/3)),x]
```

```
[Out] Integrate[(2 + x^2)/(x*(2 - 2*x + x^2)*(1 - x + x^2)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.13, size = 115, normalized size = 1.00

$$\log\left(\sqrt[3]{x^2-x+1}+x-1\right)-\frac{1}{2}\log\left(x^2+\left(x^2-x+1\right)^{2/3}+(1-x)\sqrt[3]{x^2-x+1}-2x+1\right)-\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2-x+1}}{\sqrt{3}}-\frac{2x}{\sqrt{3}}+\frac{2}{\sqrt{3}}}{\sqrt[3]{x^2-x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x^2)/(x*(2 - 2*x + x^2)*(1 - x + x^2)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(2/Sqrt[3] - (2*x)/Sqrt[3] + (1 - x + x^2)^(1/3)/Sqrt[3])/
(1 - x + x^2)^(1/3)]) + Log[-1 + x + (1 - x + x^2)^(1/3)] - Log[1 - 2*x + x
^2 + (1 - x)*(1 - x + x^2)^(1/3) + (1 - x + x^2)^(2/3)]/2
```

fricas [A] time = 1.07, size = 147, normalized size = 1.28

$$-\sqrt{3} \arctan\left(\frac{4 \sqrt{3} \left(x^{2} - x + 1\right)^{\frac{2}{3}} (x - 1) + 2 \sqrt{3} \left(x^{2} - x + 1\right)^{\frac{1}{3}} \left(x^{2} - 2 x + 1\right) + \sqrt{3} \left(x^{3} - 3 x^{2} + 3 x - 1\right)}{x^{3} - 11 x^{2} + 11 x - 9}\right) + \frac{1}{2} \log\left(\frac{x^{3} - 2 x^{2} + 3 \left(x^{2} - x + 1\right)^{\frac{2}{3}} (x - 1) + 3 \left(x^{2} - x + 1\right)^{\frac{1}{3}} \left(x^{2} - 2 x + 1\right) + 2 x}{x^{3} - 2 x^{2} + 2 x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/x/(x^2-2*x+2)/(x^2-x+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan((4*sqrt(3)*(x^2 - x + 1)^(2/3)*(x - 1) + 2*sqrt(3)*(x^2 - x
 + 1)^(1/3)*(x^2 - 2*x + 1) + sqrt(3)*(x^3 - 3*x^2 + 3*x - 1))/(x^3 - 11*x^
2 + 11*x - 9)) + 1/2*log((x^3 - 2*x^2 + 3*(x^2 - x + 1)^(2/3)*(x - 1) + 3*(
x^2 - x + 1)^(1/3)*(x^2 - 2*x + 1) + 2*x)/(x^3 - 2*x^2 + 2*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} + 2}{\left(x^{2} - x + 1\right)^{\frac{1}{3}} \left(x^{2} - 2 x + 2\right) x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/x/(x^2-2*x+2)/(x^2-x+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 2)/((x^2 - x + 1)^(1/3)*(x^2 - 2*x + 2)*x), x)
```

maple [C] time = 1.85, size = 615, normalized size = 5.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)/x/(x^2-2*x+2)/(x^2-x+1)^(1/3),x)
```

```
[Out] RootOf(_Z^2+_Z+1)*ln((RootOf(_Z^2+_Z+1)*(x^2-x+1)^(2/3)*x-(x^2-x+1)^(1/3)*R
ootOf(_Z^2+_Z+1)*x^2+RootOf(_Z^2+_Z+1)*x^3-RootOf(_Z^2+_Z+1)*(x^2-x+1)^(2/3
)+2*x*(x^2-x+1)^(2/3)+2*(x^2-x+1)^(1/3)*RootOf(_Z^2+_Z+1)*x-2*(x^2-x+1)^(1/
3)*x^2-3*RootOf(_Z^2+_Z+1)*x^2+x^3-2*(x^2-x+1)^(2/3)-(x^2-x+1)^(1/3)*RootOf
(_Z^2+_Z+1)+4*(x^2-x+1)^(1/3)*x+3*RootOf(_Z^2+_Z+1)*x-4*x^2-2*(x^2-x+1)^(1/
3)-RootOf(_Z^2+_Z+1)+4*x-2)/(x^2-2*x+2)/x)-ln((-RootOf(_Z^2+_Z+1)^2*x^3+3*R
ootOf(_Z^2+_Z+1)^2*x^2-2*RootOf(_Z^2+_Z+1)*x^3+3*x*(x^2-x+1)^(2/3)-3*(x^2-x
+1)^(1/3)*x^2-3*RootOf(_Z^2+_Z+1)^2*x+5*RootOf(_Z^2+_Z+1)*x^2-3*(x^2-x+1)^(
2/3)+6*(x^2-x+1)^(1/3)*x+RootOf(_Z^2+_Z+1)^2-5*RootOf(_Z^2+_Z+1)*x-2*x^2-3*
(x^2-x+1)^(1/3)+RootOf(_Z^2+_Z+1)+2*x-2)/(x^2-2*x+2)/x)*RootOf(_Z^2+_Z+1)-l
n((-RootOf(_Z^2+_Z+1)^2*x^3+3*RootOf(_Z^2+_Z+1)^2*x^2-2*RootOf(_Z^2+_Z+1)*x
^3+3*x*(x^2-x+1)^(2/3)-3*(x^2-x+1)^(1/3)*x^2-3*RootOf(_Z^2+_Z+1)^2*x+5*Root
Of(_Z^2+_Z+1)*x^2-3*(x^2-x+1)^(2/3)+6*(x^2-x+1)^(1/3)*x+RootOf(_Z^2+_Z+1)^2
-5*RootOf(_Z^2+_Z+1)*x-2*x^2-3*(x^2-x+1)^(1/3)+RootOf(_Z^2+_Z+1)+2*x-2)/(x^
2-2*x+2)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} + 2}{\left(x^{2} - x + 1\right)^{\frac{1}{3}} \left(x^{2} - 2 x + 2\right) x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/x/(x^2-2*x+2)/(x^2-x+1)^(1/3),x, algorithm="maxima")
[Out] integrate((x^2 + 2)/((x^2 - x + 1)^(1/3)*(x^2 - 2*x + 2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+2}{x\left(x^2-x+1\right)^{1/3}\left(x^2-2x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 2)/(x*(x^2 - x + 1)^(1/3)*(x^2 - 2*x + 2)),x)
[Out] int((x^2 + 2)/(x*(x^2 - x + 1)^(1/3)*(x^2 - 2*x + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2}{x\left(x^2-2x+2\right)\sqrt[3]{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)/x/(x**2-2*x+2)/(x**2-x+1)**(1/3),x)
[Out] Integral((x**2 + 2)/(x*(x**2 - 2*x + 2)*(x**2 - x + 1)**(1/3)), x)
```

3.1449 $\int \frac{-1+x}{x^{10}\sqrt[3]{1+x^3}}\,dx$

Optimal. Leaf size=115

$$\frac{14}{243}\log\left(\sqrt[3]{x^3+1}-1\right)-\frac{7}{243}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)+\frac{14\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{\left(x^3+1\right)^{2/3}\left(-729x^7\right.}{}$$

Rubi [A] time = 0.10, antiderivative size = 150, normalized size of antiderivative = 1.30, number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1844, 266, 51, 55, 618, 204, 31, 271, 264}

$$\frac{14\left(x^3+1\right)^{2/3}}{81x^3}+\frac{7}{81}\log\left(1-\sqrt[3]{x^3+1}\right)+\frac{14\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{\left(x^3+1\right)^{2/3}}{9x^9}-\frac{\left(x^3+1\right)^{2/3}}{8x^8}-\frac{7\left(x^3+1\right)^{2/3}}{54x^6}+\frac{3\left(x^3+1\right)^{2/3}}{20x^5}-\frac{9\left(x^3+1\right)^{2/3}}{40x^2}-\frac{7\log(x)}{81}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(x^10*(1 + x^3)^(1/3)),x]
```

```
[Out] (1 + x^3)^(2/3)/(9*x^9) - (1 + x^3)^(2/3)/(8*x^8) - (7*(1 + x^3)^(2/3))/(54
*x^6) + (3*(1 + x^3)^(2/3))/(20*x^5) + (14*(1 + x^3)^(2/3))/(81*x^3) - (9*(
1 + x^3)^(2/3))/(40*x^2) + (14*ArcTan[(1 + 2*(1 + x^3)^(1/3))/Sqrt[3]])/(81
*Sqrt[3]) - (7*Log[x])/81 + (7*Log[1 - (1 + x^3)^(1/3)])/81
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x}{x^{10}\sqrt[3]{1+x^3}}\,dx &= \int \left(-\frac{1}{x^{10}\sqrt[3]{1+x^3}}+\frac{1}{x^9\sqrt[3]{1+x^3}}\right)dx\\
&= -\int \frac{1}{x^{10}\sqrt[3]{1+x^3}}\,dx+\int \frac{1}{x^9\sqrt[3]{1+x^3}}\,dx\\
&= -\frac{\left(1+x^3\right)^{2/3}}{8x^8}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{x^4\sqrt[3]{1+x}}\,dx,x,x^3\right)-\frac{3}{4}\int \frac{1}{x^6\sqrt[3]{1+x^3}}\,dx\\
&= \frac{\left(1+x^3\right)^{2/3}}{9x^9}-\frac{\left(1+x^3\right)^{2/3}}{8x^8}+\frac{3\left(1+x^3\right)^{2/3}}{20x^5}+\frac{7}{27}\text{Subst}\left(\int \frac{1}{x^3\sqrt[3]{1+x}}\,dx,x,x^3\right)+\frac{9}{20}\int \frac{1}{x^3\sqrt[3]{1}}\\
&= \frac{\left(1+x^3\right)^{2/3}}{9x^9}-\frac{\left(1+x^3\right)^{2/3}}{8x^8}-\frac{7\left(1+x^3\right)^{2/3}}{54x^6}+\frac{3\left(1+x^3\right)^{2/3}}{20x^5}-\frac{9\left(1+x^3\right)^{2/3}}{40x^2}-\frac{14}{81}\text{Subst}\left(\int \frac{}{x^2\sqrt[3]{}}\right.\\
&= \frac{\left(1+x^3\right)^{2/3}}{9x^9}-\frac{\left(1+x^3\right)^{2/3}}{8x^8}-\frac{7\left(1+x^3\right)^{2/3}}{54x^6}+\frac{3\left(1+x^3\right)^{2/3}}{20x^5}+\frac{14\left(1+x^3\right)^{2/3}}{81x^3}-\frac{9\left(1+x^3\right)^{2/3}}{40x^2}+\frac{1}{2}\\
&= \frac{\left(1+x^3\right)^{2/3}}{9x^9}-\frac{\left(1+x^3\right)^{2/3}}{8x^8}-\frac{7\left(1+x^3\right)^{2/3}}{54x^6}+\frac{3\left(1+x^3\right)^{2/3}}{20x^5}+\frac{14\left(1+x^3\right)^{2/3}}{81x^3}-\frac{9\left(1+x^3\right)^{2/3}}{40x^2}-\frac{7}{}\\
&= \frac{\left(1+x^3\right)^{2/3}}{9x^9}-\frac{\left(1+x^3\right)^{2/3}}{8x^8}-\frac{7\left(1+x^3\right)^{2/3}}{54x^6}+\frac{3\left(1+x^3\right)^{2/3}}{20x^5}+\frac{14\left(1+x^3\right)^{2/3}}{81x^3}-\frac{9\left(1+x^3\right)^{2/3}}{40x^2}-\frac{7}{}\\
&= \frac{\left(1+x^3\right)^{2/3}}{9x^9}-\frac{\left(1+x^3\right)^{2/3}}{8x^8}-\frac{7\left(1+x^3\right)^{2/3}}{54x^6}+\frac{3\left(1+x^3\right)^{2/3}}{20x^5}+\frac{14\left(1+x^3\right)^{2/3}}{81x^3}-\frac{9\left(1+x^3\right)^{2/3}}{40x^2}+\frac{1}{}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 46, normalized size = 0.40

$$
-\frac{\left(x^3+1\right)^{2/3}\left(20x^8\,{}_2F_1\left(\frac{2}{3},4;\frac{5}{3};x^3+1\right)+9x^6-6x^3+5\right)}{40x^8}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(x^10*(1 + x^3)^(1/3)),x]
```

```
[Out] -1/40*((1 + x^3)^(2/3)*(5 - 6*x^3 + 9*x^6 + 20*x^8*Hypergeometric2F1[2/3, 4
, 5/3, 1 + x^3]))/x^8
```

IntegrateAlgebraic [A] time = 17.30, size = 115, normalized size = 1.00

$$\frac{14}{243}\log\left(\sqrt[3]{x^3+1}-1\right)-\frac{7}{243}\log\left(\left(x^3+1\right)^{2/3}+\sqrt[3]{x^3+1}+1\right)+\frac{14\tan^{-1}\left(\frac{2\sqrt[3]{x^3+1}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{\left(x^3+1\right)^{2/3}\left(-729x^7+560x^6+486x^4-420x^3-405x+360\right)}{3240x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/(x^10*(1 + x^3)^(1/3)),x]
```

```
[Out] ((1 + x^3)^(2/3)*(360 - 405*x - 420*x^3 + 486*x^4 + 560*x^6 - 729*x^7))/(32
40*x^9) + (14*ArcTan[1/Sqrt[3] + (2*(1 + x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3])
 + (14*Log[-1 + (1 + x^3)^(1/3)])/243 - (7*Log[1 + (1 + x^3)^(1/3) + (1 + x
^3)^(2/3)])/243
```

fricas [A] time = 0.84, size = 124, normalized size = 1.08

$$-\frac{560\,\sqrt{3}\,x^9\arctan\left(-\frac{\sqrt{3}\left(x^3+1\right)-2\,\sqrt{3}\left(x^3+1\right)^{\frac{2}{3}}+4\,\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}}{x^3+9}\right)-280\,x^9\log\left(\frac{x^3-3\left(x^3+1\right)^{\frac{2}{3}}+3\left(x^3+1\right)^{\frac{1}{3}}}{x^3}\right)+3\left(729\,x^7-560\,x^6-486\,x^4+420\,x^3+405\,x-360\right)\left(x^3+1\right)^{\frac{2}{3}}}{9720\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x^10/(x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/9720*(560*sqrt(3)*x^9*arctan(-(sqrt(3)*(x^3 + 1) - 2*sqrt(3)*(x^3 + 1)^(
2/3) + 4*sqrt(3)*(x^3 + 1)^(1/3))/(x^3 + 9)) - 280*x^9*log((x^3 - 3*(x^3 +
1)^(2/3) + 3*(x^3 + 1)^(1/3))/x^3) + 3*(729*x^7 - 560*x^6 - 486*x^4 + 420*x
^3 + 405*x - 360)*(x^3 + 1)^(2/3))/x^9
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^3+1\right)^{\frac{1}{3}}x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x^10/(x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/((x^3 + 1)^(1/3)*x^10), x)
```

maple [C] time = 0.30, size = 111, normalized size = 0.97

$$-\frac{729x^{10}-560x^9+243x^7-140x^6-81x^4+60x^3+405x-360}{3240x^9\left(x^3+1\right)^{\frac{1}{3}}}+\frac{7\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{2\pi\sqrt{3}\,x^3\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],-x^3\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{243\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/x^10/(x^3+1)^(1/3),x)
```

```
[Out] -1/3240*(729*x^10-560*x^9+243*x^7-140*x^6-81*x^4+60*x^3+405*x-360)/x^9/(x^3
+1)^(1/3)+7/243/Pi*3^(1/2)*GAMMA(2/3)*(-2/9*Pi*3^(1/2)/GAMMA(2/3)*x^3*hyper
geom([1,1,4/3],[2,2],-x^3)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x))*Pi*3^(1/
2)/GAMMA(2/3))
```

maxima [A] time = 0.43, size = 145, normalized size = 1.26

$$\frac{14}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{28\left(x^3+1\right)^{\frac{8}{3}}-77\left(x^3+1\right)^{\frac{5}{3}}+67\left(x^3+1\right)^{\frac{2}{3}}}{162\left(\left(x^3+1\right)^3+3x^3-3\left(x^3+1\right)^2+2\right)}-\frac{\left(x^3+1\right)^{\frac{2}{3}}}{2x^2}+\frac{2\left(x^3+1\right)^{\frac{5}{3}}}{5x^5}-\frac{\left(x^3+1\right)^{\frac{8}{3}}}{8x^8}-\frac{7}{243}\log\left(\left(x^3+1\right)^{\frac{2}{3}}+\left(x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{14}{243}\log\left(\left(x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x^10/(x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 14/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 1)^(1/3) + 1)) + 1/162*(28*(x^3
 + 1)^(8/3) - 77*(x^3 + 1)^(5/3) + 67*(x^3 + 1)^(2/3))/((x^3 + 1)^3 + 3*x^3
 - 3*(x^3 + 1)^2 + 2) - 1/2*(x^3 + 1)^(2/3)/x^2 + 2/5*(x^3 + 1)^(5/3)/x^5 -
 1/8*(x^3 + 1)^(8/3)/x^8 - 7/243*log((x^3 + 1)^(2/3) + (x^3 + 1)^(1/3) + 1)
 + 14/243*log((x^3 + 1)^(1/3) - 1)
```

mupad [B] time = 1.29, size = 158, normalized size = 1.37

$$\frac{14\ln\left(\frac{196\left(x^3+1\right)^{1/3}}{6561}-\frac{196}{6561}\right)}{243}+\ln\left(\frac{196\left(x^3+1\right)^{1/3}}{6561}-9\left(-\frac{7}{243}+\frac{\sqrt{3}\,7\mathrm{i}}{243}\right)^2\right)\left(-\frac{7}{243}+\frac{\sqrt{3}\,7\mathrm{i}}{243}\right)-\ln\left(\frac{196\left(x^3+1\right)^{1/3}}{6561}-9\left(\frac{7}{243}+\frac{\sqrt{3}\,7\mathrm{i}}{243}\right)^2\right)\left(\frac{7}{243}+\frac{\sqrt{3}\,7\mathrm{i}}{243}\right)+\frac{\frac{67\left(x^3+1\right)^{2/3}}{162}-\frac{77\left(x^3+1\right)^{5/3}}{162}+\frac{14\left(x^3+1\right)^{8/3}}{81}}{\left(x^3+1\right)^3-3\left(x^3+1\right)^2+3x^3+2}-\frac{\left(x^3+1\right)^{2/3}\left(9x^6-6x^3+5\right)}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(x^10*(x^3 + 1)^(1/3)),x)
```

```
[Out] (14*log((196*(x^3 + 1)^(1/3))/6561 - 196/6561))/243 + log((196*(x^3 + 1)^(1
/3))/6561 - 9*((3^(1/2)*7i)/243 - 7/243)^2)*((3^(1/2)*7i)/243 - 7/243) - lo
g((196*(x^3 + 1)^(1/3))/6561 - 9*((3^(1/2)*7i)/243 + 7/243)^2)*((3^(1/2)*7i
)/243 + 7/243) + ((67*(x^3 + 1)^(2/3))/162 - (77*(x^3 + 1)^(5/3))/162 + (14
*(x^3 + 1)^(8/3))/81)/((x^3 + 1)^3 - 3*(x^3 + 1)^2 + 3*x^3 + 2) - ((x^3 + 1
)^(2/3)*(9*x^6 - 6*x^3 + 5))/(40*x^8)
```

sympy [C] time = 2.85, size = 112, normalized size = 0.97

$$\frac{2\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{3\Gamma\left(\frac{1}{3}\right)}-\frac{4\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^3\Gamma\left(\frac{1}{3}\right)}+\frac{10\left(1+\frac{1}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{27x^6\Gamma\left(\frac{1}{3}\right)}+\frac{\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{10}{3}\\\frac{13}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^3}\right)}{3x^{10}\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x**10/(x**3+1)**(1/3),x)
```

```
[Out] 2*(1 + x**(-3))**(2/3)*gamma(-8/3)/(3*gamma(1/3)) - 4*(1 + x**(-3))**(2/3)*
gamma(-8/3)/(9*x**3*gamma(1/3)) + 10*(1 + x**(-3))**(2/3)*gamma(-8/3)/(27*x
**6*gamma(1/3)) + gamma(10/3)*hyper((1/3, 10/3), (13/3,), exp_polar(I*pi)/x
**3)/(3*x**10*gamma(13/3))
```

3.1450 $\int \frac{1}{x\sqrt[3]{b+ax^3}}\,dx$

Optimal. Leaf size=115

$$-\frac{\log\left(\sqrt[3]{b}\sqrt[3]{ax^3+b}+\left(ax^3+b\right)^{2/3}+b^{2/3}\right)}{6\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{ax^3+b}-\sqrt[3]{b}\right)}{3\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{ax^3+b}}{\sqrt{3}\sqrt[3]{b}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 0.72, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 55, 617, 204, 31}

$$\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{ax^3+b}\right)}{2\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{ax^3+b}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log(x)}{2\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b + a*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(b^(1/3) + 2*(b + a*x^3)^(1/3))/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*b^(1/3))
 - Log[x]/(2*b^(1/3)) + Log[b^(1/3) - (b + a*x^3)^(1/3)]/(2*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{b+ax^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{b+ax}}\,dx, x, x^3\right)$$

$$= -\frac{\log(x)}{2\sqrt[3]{b}} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{b^{2/3}+\sqrt[3]{b}\,x+x^2}\,dx, x, \sqrt[3]{b+ax^3}\right) - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{b}-x}\,dx, x, \sqrt[3]{b+ax^3}\right)}{2\sqrt[3]{b}}$$

$$= -\frac{\log(x)}{2\sqrt[3]{b}} + \frac{\log\left(\sqrt[3]{b}-\sqrt[3]{b+ax^3}\right)}{2\sqrt[3]{b}} - \frac{\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{b+ax^3}}{\sqrt[3]{b}}\right)}{\sqrt[3]{b}}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b+ax^3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{\log(x)}{2\sqrt[3]{b}} + \frac{\log\left(\sqrt[3]{b}-\sqrt[3]{b+ax^3}\right)}{2\sqrt[3]{b}}$$

Mathematica [A] time = 0.06, size = 70, normalized size = 0.61

$$\frac{3\log\left(\sqrt[3]{b}-\sqrt[3]{ax^3+b}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{ax^3+b}}{\sqrt[3]{b}}+1}{\sqrt{3}}\right)-3\log(x)}{6\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b + a*x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(b + a*x^3)^(1/3))/b^(1/3))/Sqrt[3]] - 3*Log[x] +
 3*Log[b^(1/3) - (b + a*x^3)^(1/3)])/(6*b^(1/3))
```

IntegrateAlgebraic [A] time = 0.10, size = 115, normalized size = 1.00

$$-\frac{\log\left(\sqrt[3]{b}\sqrt[3]{ax^3+b}+\left(ax^3+b\right)^{2/3}+b^{2/3}\right)}{6\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{ax^3+b}-\sqrt[3]{b}\right)}{3\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{ax^3+b}}{\sqrt{3}\sqrt[3]{b}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(b + a*x^3)^(1/3)),x]
```

```
[Out] ArcTan[1/Sqrt[3] + (2*(b + a*x^3)^(1/3))/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*b^(1/3
)) + Log[-b^(1/3) + (b + a*x^3)^(1/3)]/(3*b^(1/3)) - Log[b^(2/3) + b^(1/3)*
(b + a*x^3)^(1/3) + (b + a*x^3)^(2/3)]/(6*b^(1/3))
```

fricas [A] time = 0.43, size = 236, normalized size = 2.05

$$\left[\frac{3\sqrt{\frac{1}{3}}\,b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2ax^3+3\sqrt{\frac{1}{3}}\left(2\left(ax^3+b\right)^{\frac{2}{3}}b^{\frac{2}{3}}-\left(ax^3+b\right)^{\frac{1}{3}}b-b^{\frac{4}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-3\left(ax^3+b\right)^{\frac{1}{3}}b^{\frac{2}{3}}+3b}{x^3}\right)-b^{\frac{2}{3}}\log\left(\left(ax^3+b\right)^{\frac{2}{3}}+\left(ax^3+b\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)+2b^{\frac{2}{3}}\log\left(\left(ax^3+b\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{6b}, \frac{6\sqrt{\frac{1}{3}}\,b^{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(ax^3+b\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}}\right)-b^{\frac{2}{3}}\log\left(\left(ax^3+b\right)^{\frac{2}{3}}+\left(ax^3+b\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)+2b^{\frac{2}{3}}\log\left(\left(ax^3+b\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{6b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3+b)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*b*sqrt(-1/b^(2/3))*log((2*a*x^3 + 3*sqrt(1/3)*(2*(a*x^3 +
 b)^(2/3)*b^(2/3) - (a*x^3 + b)^(1/3)*b - b^(4/3))*sqrt(-1/b^(2/3)) - 3*(a*
x^3 + b)^(1/3)*b^(2/3) + 3*b)/x^3) - b^(2/3)*log((a*x^3 + b)^(2/3) + (a*x^3
 + b)^(1/3)*b^(1/3) + b^(2/3)) + 2*b^(2/3)*log((a*x^3 + b)^(1/3) - b^(1/3))
```

```
)/b, 1/6*(6*sqrt(1/3)*b^(2/3)*arctan(sqrt(1/3)*(2*(a*x^3 + b)^(1/3) + b^(1/
3))/b^(1/3)) - b^(2/3)*log((a*x^3 + b)^(2/3) + (a*x^3 + b)^(1/3)*b^(1/3) +
b^(2/3)) + 2*b^(2/3)*log((a*x^3 + b)^(1/3) - b^(1/3)))/b]
```

giac [A] time = 0.69, size = 87, normalized size = 0.76

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(ax^3+b\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{3\,b^{\frac{1}{3}}}-\frac{\log\left(\left(ax^3+b\right)^{\frac{2}{3}}+\left(ax^3+b\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{6\,b^{\frac{1}{3}}}+\frac{\log\left(\left|\left(ax^3+b\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right|\right)}{3\,b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3+b)^(1/3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(a*x^3 + b)^(1/3) + b^(1/3))/b^(1/3))/b^(
1/3) - 1/6*log((a*x^3 + b)^(2/3) + (a*x^3 + b)^(1/3)*b^(1/3) + b^(2/3))/b^(
1/3) + 1/3*log(abs((a*x^3 + b)^(1/3) - b^(1/3)))/b^(1/3)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a\,x^3+b\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^3+b)^(1/3),x)
```

```
[Out] int(1/x/(a*x^3+b)^(1/3),x)
```

maxima [A] time = 0.43, size = 86, normalized size = 0.75

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(ax^3+b\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{3\,b^{\frac{1}{3}}}-\frac{\log\left(\left(ax^3+b\right)^{\frac{2}{3}}+\left(ax^3+b\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{6\,b^{\frac{1}{3}}}+\frac{\log\left(\left(ax^3+b\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3+b)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(a*x^3 + b)^(1/3) + b^(1/3))/b^(1/3))/b^(
1/3) - 1/6*log((a*x^3 + b)^(2/3) + (a*x^3 + b)^(1/3)*b^(1/3) + b^(2/3))/b^(
1/3) + 1/3*log((a*x^3 + b)^(1/3) - b^(1/3))/b^(1/3)
```

mupad [B] time = 1.05, size = 100, normalized size = 0.87

$$\frac{\ln\left(\left(a\,x^3+b\right)^{1/3}-b^{1/3}\right)}{3\,b^{1/3}}+\frac{\ln\left(\left(a\,x^3+b\right)^{1/3}-\frac{b^{1/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,b^{1/3}}-\frac{\ln\left(\left(a\,x^3+b\right)^{1/3}-\frac{b^{1/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b + a*x^3)^(1/3)),x)
```

```
[Out] log((b + a*x^3)^(1/3) - b^(1/3))/(3*b^(1/3)) + (log((b + a*x^3)^(1/3) - (b^
(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(6*b^(1/3)) - (log((b + a*x^
3)^(1/3) - (b^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*b^(1/3))
```

sympy [C] time = 0.89, size = 37, normalized size = 0.32

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^{3}}\right)}{3\sqrt[3]{a}\,x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**3+b)**(1/3),x)
```

```
[Out] -gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*exp_polar(I*pi)/(a*x**3))/(3*a**(1/
3)*x*gamma(4/3))
```

3.1451 $\int \frac{\left(-1-2(-1+k)x+kx^2\right)\left(-1+3kx-3k^2x^2+k^3x^3\right)}{(-1+x)x\sqrt[4]{(1-x)x(1-kx)}\left(-1+(d+3k)x-\left(d+3k^2\right)x^2+k^3x^3\right)}\,dx$

Optimal. Leaf size=115

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{kx-1}\right)-2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{kx-1}\right)+\frac{4\left(kx^3-kx^2-x^2+x\right)^{3/4}}{(x-1)x}$$

Rubi [F] time = 38.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1-2(-1+k)x+kx^2\right)\left(-1+3kx-3k^2x^2+k^3x^3\right)}{(-1+x)x\sqrt[4]{(1-x)x(1-kx)}\left(-1+(d+3k)x-\left(d+3k^2\right)x^2+k^3x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 - 2*(-1 + k)*x + k*x^2)*(-1 + 3*k*x - 3*k^2*x^2 + k^3*x^3))/((-1 +
 x)*x*((1 - x)*x*(1 - k*x))^(1/4)*(-1 + (d + 3*k)*x - (d + 3*k^2)*x^2 + k^3
*x^3)),x]
```

```
[Out] (-4*(1 - x)^(1/4)*(1 - k*x)^(1/4)*AppellF1[-1/4, 5/4, -11/4, 3/4, x, k*x])/
((1 - x)*x*(1 - k*x))^(1/4) - (4*(2 - d - 5*k)*(1 - x)^(1/4)*x^(1/4)*(1 - k
*x)^(1/4)*Defer[Subst][Defer[Int][(x^2*(1 - k*x^4)^(11/4))/((1 - x^4)^(5/4)
*(-(-1 + k*x^4)^3 - d*(x^4 - x^8))), x], x, x^(1/4)])/((1 - x)*x*(1 - k*x))
^(1/4) - (4*(d + k + 3*k^2)*(1 - x)^(1/4)*x^(1/4)*(1 - k*x)^(1/4)*Defer[Sub
st][Defer[Int][(x^6*(1 - k*x^4)^(11/4))/((1 - x^4)^(5/4)*(-(-1 + k*x^4)^3 -
 d*(x^4 - x^8))), x], x, x^(1/4)])/((1 - x)*x*(1 - k*x))^(1/4) + (4*k^3*(1
- x)^(1/4)*x^(1/4)*(1 - k*x)^(1/4)*Defer[Subst][Defer[Int][(x^10*(1 - k*x^4
)^(11/4))/((1 - x^4)^(5/4)*(-(-1 + k*x^4)^3 - d*(x^4 - x^8))), x], x, x^(1/
4)])/((1 - x)*x*(1 - k*x))^(1/4)
```

Rubi steps

$$\int \frac{\left(-1-2(-1+k)x+kx^2\right)\left(-1+3kx-3k^2x^2+k^3x^3\right)}{(-1+x)x\sqrt[4]{(1-x)x(1-kx)}\left(-1+(d+3k)x-\left(d+3k^2\right)x^2+k^3x^3\right)}\,dx = \frac{\left(\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\int \frac{(-1-2(-1}{\sqrt[4]{1-x}(-1+x)x^{5/}}}{\sqrt[4]{(1-x)x(}}$$

$$= -\frac{\left(\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\int \frac{(-1-2(-1}{(1-x)^{5/4}x^{5/4}\sqrt[4]{}}}{\sqrt[4]{(1-x)x(}}$$

$$= -\frac{\left(\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\int \frac{(1-kx)^{3/4}(-1-}{(1-x)^{5/4}x^{5/4}(-}}{\sqrt[4]{(1-x)x(1-}}$$

$$= \frac{\left(\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\int \frac{(1-kx)^{3/4}(-1+}{(1-x)^{5/4}x^{5/4}(-1-}}{\sqrt[4]{(1-x)x(1-k}}$$

$$= \frac{\left(\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\int \frac{(1-kx)^{11/}}{(1-x)^{5/4}x^{5/4}(-1-}}{\sqrt[4]{(1-x)x(1-k}}$$

$$= \frac{\left(4\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\mathrm{Subst}\left(\int \frac{}{x^2(1}\right.}{\sqrt[4]{(1-}}$$

$$= \frac{\left(4\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\mathrm{Subst}\left(\int \frac{(1-}{x^2(1}\right.}{\sqrt[4]{(1-x)x}}$$

$$= \frac{\left(4\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\mathrm{Subst}\left(\int\left(\frac{(1-}{x^2(}\right.\right.}{}$$

$$= \frac{\left(4\sqrt[4]{1-x}\sqrt[4]{x}\sqrt[4]{1-kx}\right)\mathrm{Subst}\left(\int \frac{(1-k}{x^2(1}\right.}{\sqrt[4]{(1-x)x(1-kx)}}$$

$$= -\frac{4\sqrt[4]{1-x}\sqrt[4]{1-kx}\,F_1\left(-\frac{1}{4};\frac{5}{4},-\frac{11}{4};\frac{3}{4};\right.}{\sqrt[4]{(1-x)x(1-kx)}}$$

$$= -\frac{4\sqrt[4]{1-x}\sqrt[4]{1-kx}\,F_1\left(-\frac{1}{4};\frac{5}{4},-\frac{11}{4};\frac{3}{4};\right.}{\sqrt[4]{(1-x)x(1-kx)}}$$

$$= -\frac{4\sqrt[4]{1-x}\sqrt[4]{1-kx}\,F_1\left(-\frac{1}{4};\frac{5}{4},-\frac{11}{4};\frac{3}{4};\right.}{\sqrt[4]{(1-x)x(1-kx)}}$$

Mathematica [F] time = 7.09, size = 0, normalized size = 0.00

$$\int \frac{\left(-1-2(-1+k)x+kx^2\right)\left(-1+3kx-3k^2x^2+k^3x^3\right)}{(-1+x)x\sqrt[4]{(1-x)x(1-kx)}\left(-1+(d+3k)x-\left(d+3k^2\right)x^2+k^3x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 - 2*(-1 + k)*x + k*x^2)*(-1 + 3*k*x - 3*k^2*x^2 + k^3*x^3))/
((-1 + x)*x*((1 - x)*x*(1 - k*x))^(1/4)*(-1 + (d + 3*k)*x - (d + 3*k^2)*x^2
 + k^3*x^3)),x]
```

```
[Out] Integrate[((-1 - 2*(-1 + k)*x + k*x^2)*(-1 + 3*k*x - 3*k^2*x^2 + k^3*x^3))/
((-1 + x)*x*((1 - x)*x*(1 - k*x))^(1/4)*(-1 + (d + 3*k)*x - (d + 3*k^2)*x^2
 + k^3*x^3)), x]
```

IntegrateAlgebraic [A] time = 0.92, size = 115, normalized size = 1.00

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{kx-1}\right)-2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{kx^3+(-k-1)x^2+x}}{kx-1}\right)+\frac{4\left(kx^3-kx^2-x^2+x\right)^{3/4}}{(x-1)x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 - 2*(-1 + k)*x + k*x^2)*(-1 + 3*k*x - 3*k^2*x^2 + k
^3*x^3))/((-1 + x)*x*((1 - x)*x*(1 - k*x))^(1/4)*(-1 + (d + 3*k)*x - (d + 3
*k^2)*x^2 + k^3*x^3)),x]
```

```
[Out] (4*(x - x^2 - k*x^2 + k*x^3)^(3/4))/((-1 + x)*x) + 2*d^(1/4)*ArcTan[(d^(1/4
)*(x + (-1 - k)*x^2 + k*x^3)^(1/4))/(-1 + k*x)] - 2*d^(1/4)*ArcTanh[(d^(1/4
)*(x + (-1 - k)*x^2 + k*x^3)^(1/4))/(-1 + k*x)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-2*(-1+k)*x+k*x^2)*(k^3*x^3-3*k^2*x^2+3*k*x-1)/(-1+x)/x/((1-x)
*x*(-k*x+1))^(1/4)/(-1+(d+3*k)*x-(3*k^2+d)*x^2+k^3*x^3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(k^3x^3-3\,k^2x^2+3\,kx-1\right)\left(kx^2-2\,(k-1)x-1\right)}{\left(k^3x^3-\left(3\,k^2+d\right)x^2+(d+3\,k)x-1\right)((kx-1)(x-1)x)^{\frac{1}{4}}\,(x-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-2*(-1+k)*x+k*x^2)*(k^3*x^3-3*k^2*x^2+3*k*x-1)/(-1+x)/x/((1-x)
*x*(-k*x+1))^(1/4)/(-1+(d+3*k)*x-(3*k^2+d)*x^2+k^3*x^3),x, algorithm="giac"
)
```

```
[Out] integrate((k^3*x^3 - 3*k^2*x^2 + 3*k*x - 1)*(k*x^2 - 2*(k - 1)*x - 1)/((k^3
*x^3 - (3*k^2 + d)*x^2 + (d + 3*k)*x - 1)*((k*x - 1)*(x - 1)*x)^(1/4)*(x -
1)*x), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(-1-2\,(-1+k)\,x+k\,x^2\right)\left(k^3x^3-3k^2x^2+3kx-1\right)}{(-1+x)\,x\,((1-x)\,x\,(-kx+1))^{\frac{1}{4}}\left(-1+(d+3k)\,x-\left(3k^2+d\right)x^2+k^3x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-2*(-1+k)*x+k*x^2)*(k^3*x^3-3*k^2*x^2+3*k*x-1)/(-1+x)/x/((1-x)*x*(-k
*x+1))^(1/4)/(-1+(d+3*k)*x-(3*k^2+d)*x^2+k^3*x^3),x)
```

```
[Out] int((-1-2*(-1+k)*x+k*x^2)*(k^3*x^3-3*k^2*x^2+3*k*x-1)/(-1+x)/x/((1-x)*x*(-k
*x+1))^(1/4)/(-1+(d+3*k)*x-(3*k^2+d)*x^2+k^3*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(k^3x^3 - 3\,k^2x^2 + 3\,kx - 1\right)\left(kx^2 - 2\,(k-1)x - 1\right)}{\left(k^3x^3 - \left(3\,k^2 + d\right)x^2 + (d + 3\,k)x - 1\right)((kx-1)(x-1)x)^{\frac{1}{4}}\,(x-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-2*(-1+k)*x+k*x^2)*(k^3*x^3-3*k^2*x^2+3*k*x-1)/(-1+x)/x/((1-x)
*x*(-k*x+1))^(1/4)/(-1+(d+3*k)*x-(3*k^2+d)*x^2+k^3*x^3),x, algorithm="maxim
a")
```

```
[Out] integrate((k^3*x^3 - 3*k^2*x^2 + 3*k*x - 1)*(k*x^2 - 2*(k - 1)*x - 1)/((k^3
*x^3 - (3*k^2 + d)*x^2 + (d + 3*k)*x - 1)*((k*x - 1)*(x - 1)*x)^(1/4)*(x -
1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x\,(k-1) - k\,x^2 + 1\right)\,\left(-k^3\,x^3 + 3\,k^2\,x^2 - 3\,k\,x + 1\right)}{x\,(x-1)\,\left(x\,(k\,x-1)\,(x-1)\right)^{1/4}\,\left(k^3\,x^3 - x^2\,\left(3\,k^2 + d\right) + x\,(d + 3\,k) - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x*(k - 1) - k*x^2 + 1)*(3*k^2*x^2 - k^3*x^3 - 3*k*x + 1))/(x*(x - 1
)*(x*(k*x - 1)*(x - 1))^(1/4)*(k^3*x^3 - x^2*(d + 3*k^2) + x*(d + 3*k) - 1)
),x)
```

```
[Out] int(((2*x*(k - 1) - k*x^2 + 1)*(3*k^2*x^2 - k^3*x^3 - 3*k*x + 1))/(x*(x - 1
)*(x*(k*x - 1)*(x - 1))^(1/4)*(k^3*x^3 - x^2*(d + 3*k^2) + x*(d + 3*k) - 1)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-2*(-1+k)*x+k*x**2)*(k**3*x**3-3*k**2*x**2+3*k*x-1)/(-1+x)/x/(
(1-x)*x*(-k*x+1))**(1/4)/(-1+(d+3*k)*x-(3*k**2+d)*x**2+k**3*x**3),x)
```

```
[Out] Timed out
```

3.1452 $\int \frac{(3+x^4)(-1-x^3+x^4)^{2/3}}{x^3(-1+x^4)} dx$

Optimal. Leaf size=115

$$\log\left(\sqrt[3]{x^4-x^3-1}+x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4-x^3-1}-x}\right)+\frac{3\left(x^4-x^3-1\right)^{2/3}}{2x^2}-\frac{1}{2}\log\left(x^2-\sqrt[3]{x^4-x^3-1}\,x+(x^4$$

Rubi [F] time = 0.92, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(3+x^4)(-1-x^3+x^4)^{2/3}}{x^3(-1+x^4)} dx$$

Verification is not applicable to the result.

```
[In] Int[((3 + x^4)*(-1 - x^3 + x^4)^(2/3))/(x^3*(-1 + x^4)),x]
```

```
[Out] Defer[Int][(-1 - x^3 + x^4)^(2/3)/(I - x), x] + Defer[Int][(-1 - x^3 + x^4)
^(2/3)/(-1 + x), x] - 3*Defer[Int][(-1 - x^3 + x^4)^(2/3)/x^3, x] - Defer[I
nt][(-1 - x^3 + x^4)^(2/3)/(I + x), x] + Defer[Int][(-1 - x^3 + x^4)^(2/3)/
(1 + x), x]
```

Rubi steps

$$\int \frac{(3+x^4)(-1-x^3+x^4)^{2/3}}{x^3(-1+x^4)} dx = \int \left(\frac{(-1-x^3+x^4)^{2/3}}{-1+x}-\frac{3(-1-x^3+x^4)^{2/3}}{x^3}+\frac{(-1-x^3+x^4)^{2/3}}{1+x}-\frac{2x(-1-}{1}\right.$$

$$= -\left(2\int \frac{x(-1-x^3+x^4)^{2/3}}{1+x^2} dx\right)-3\int \frac{(-1-x^3+x^4)^{2/3}}{x^3} dx+\int \frac{(-1-x^3+}{-1+}$$

$$= -\left(2\int \left(-\frac{(-1-x^3+x^4)^{2/3}}{2(i-x)}+\frac{(-1-x^3+x^4)^{2/3}}{2(i+x)}\right) dx\right)-3\int \frac{(-1-x^3+x^4)}{x^3}$$

$$= -\left(3\int \frac{(-1-x^3+x^4)^{2/3}}{x^3} dx\right)+\int \frac{(-1-x^3+x^4)^{2/3}}{i-x} dx+\int \frac{(-1-x^3+x^4}{-1+x}$$

Mathematica [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(3+x^4)(-1-x^3+x^4)^{2/3}}{x^3(-1+x^4)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[((3 + x^4)*(-1 - x^3 + x^4)^(2/3))/(x^3*(-1 + x^4)),x]
```

```
[Out] Integrate[((3 + x^4)*(-1 - x^3 + x^4)^(2/3))/(x^3*(-1 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.57, size = 115, normalized size = 1.00

$$\log\left(\sqrt[3]{x^4-x^3-1}+x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4-x^3-1}-x}\right)+\frac{3\left(x^4-x^3-1\right)^{2/3}}{2x^2}-\frac{1}{2}\log\left(x^2-\sqrt[3]{x^4-x^3-1}\,x+\left(x^4-x^3-1\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3 + x^4)*(-1 - x^3 + x^4)^(2/3))/(x^3*(-1 + x^4)),x]
```

```
[Out] (3*(-1 - x^3 + x^4)^(2/3))/(2*x^2) + Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(-1
 - x^3 + x^4)^(1/3))] + Log[x + (-1 - x^3 + x^4)^(1/3)] - Log[x^2 - x*(-1 -
 x^3 + x^4)^(1/3) + (-1 - x^3 + x^4)^(2/3)]/2
```

fricas [A] time = 4.72, size = 150, normalized size = 1.30

$$\frac{2\sqrt{3}x^2\arctan\left(\frac{728574532\sqrt{3}\left(x^4-x^3-1\right)^{\frac{1}{3}}x^2+812477430\sqrt{3}\left(x^4-x^3-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(355231575x^4+41951449x^3-355231575\right)}{3\left(447697125x^4-770525981x^3-447697125\right)}\right)+x^2\log\left(\frac{x^4+3\left(x^4-x^3-1\right)^{\frac{1}{3}}x^2+3\left(x^4-x^3-1\right)^{\frac{2}{3}}x-1}{x^4-1}\right)+3\left(x^4-x^3-1\right)^{\frac{2}{3}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)*(x^4-x^3-1)^(2/3)/x^3/(x^4-1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*x^2*arctan(1/3*(728574532*sqrt(3)*(x^4 - x^3 - 1)^(1/3)*x^2
+ 812477430*sqrt(3)*(x^4 - x^3 - 1)^(2/3)*x + sqrt(3)*(355231575*x^4 + 4195
1449*x^3 - 355231575))/(447697125*x^4 - 770525981*x^3 - 447697125)) + x^2*l
og((x^4 + 3*(x^4 - x^3 - 1)^(1/3)*x^2 + 3*(x^4 - x^3 - 1)^(2/3)*x - 1)/(x^4
 - 1)) + 3*(x^4 - x^3 - 1)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^3-1\right)^{\frac{2}{3}}\left(x^4+3\right)}{\left(x^4-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)*(x^4-x^3-1)^(2/3)/x^3/(x^4-1),x, algorithm="giac")
```

```
[Out] integrate((x^4 - x^3 - 1)^(2/3)*(x^4 + 3)/((x^4 - 1)*x^3), x)
```

maple [C] time = 1.98, size = 433, normalized size = 3.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3)*(x^4-x^3-1)^(2/3)/x^3/(x^4-1),x)
```

```
[Out] 3/2*(x^4-x^3-1)^(2/3)/x^2+RootOf(_Z^2+_Z+1)*ln((RootOf(_Z^2+_Z+1)^2*x^3-Roo
tOf(_Z^2+_Z+1)*x^4+3*(x^4-x^3-1)^(2/3)*RootOf(_Z^2+_Z+1)*x-3*(x^4-x^3-1)^(1
/3)*RootOf(_Z^2+_Z+1)*x^2+4*RootOf(_Z^2+_Z+1)*x^3-2*x^4+3*(x^4-x^3-1)^(2/3)
*x-3*(x^4-x^3-1)^(1/3)*x^2+4*x^3+RootOf(_Z^2+_Z+1)+2)/(-1+x)/(1+x)/(x^2+1))
-ln(-(-RootOf(_Z^2+_Z+1)^2*x^3-RootOf(_Z^2+_Z+1)*x^4+3*(x^4-x^3-1)^(2/3)*Ro
otOf(_Z^2+_Z+1)*x-3*(x^4-x^3-1)^(1/3)*RootOf(_Z^2+_Z+1)*x^2+2*RootOf(_Z^2+_
Z+1)*x^3+x^4-x^3+RootOf(_Z^2+_Z+1)-1)/(-1+x)/(1+x)/(x^2+1))*RootOf(_Z^2+_Z+
1)-ln(-(-RootOf(_Z^2+_Z+1)^2*x^3-RootOf(_Z^2+_Z+1)*x^4+3*(x^4-x^3-1)^(2/3)*
RootOf(_Z^2+_Z+1)*x-3*(x^4-x^3-1)^(1/3)*RootOf(_Z^2+_Z+1)*x^2+2*RootOf(_Z^2
+_Z+1)*x^3+x^4-x^3+RootOf(_Z^2+_Z+1)-1)/(-1+x)/(1+x)/(x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^3-1\right)^{\frac{2}{3}}\left(x^4+3\right)}{\left(x^4-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3)*(x^4-x^3-1)^(2/3)/x^3/(x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^3 - 1)^(2/3)*(x^4 + 3)/((x^4 - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4+3\right)\,\left(x^4-x^3-1\right)^{2/3}}{x^3\,\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 3)*(x^4 - x^3 - 1)^(2/3))/(x^3*(x^4 - 1)),x)
[Out] int(((x^4 + 3)*(x^4 - x^3 - 1)^(2/3))/(x^3*(x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+3\right)\left(x^4-x^3-1\right)^{\frac{2}{3}}}{x^3\,(x-1)\,(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3)*(x**4-x**3-1)**(2/3)/x**3/(x**4-1),x)
[Out] Integral((x**4 + 3)*(x**4 - x**3 - 1)**(2/3)/(x**3*(x - 1)*(x + 1)*(x**2 +
1)), x)
```

3.1453 $\int \frac{2-x+x^2}{\sqrt[3]{-1+x^2}\,(3+4x+x^2)}\,dx$

Optimal. Leaf size=116

$$\frac{3(x^2-1)^{2/3}}{x+1}-\frac{7}{4}\log\left(2\sqrt[3]{x^2-1}+x-1\right)+\frac{7}{8}\log\left(x^2+4(x^2-1)^{2/3}+(2-2x)\sqrt[3]{x^2-1}-2x+1\right)-\frac{7}{4}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{}}{\sqrt[3]{x^2}}\right.$$

Rubi [F] time = 0.37, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2-x+x^2}{\sqrt[3]{-1+x^2}\,(3+4x+x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2 - x + x^2)/((-1 + x^2)^(1/3)*(3 + 4*x + x^2)),x]
```

```
[Out] (3*x)/(1 + Sqrt[3] + (-1 + x^2)^(1/3)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 +
(-1 + x^2)^(1/3))*Sqrt[(1 - (-1 + x^2)^(1/3) + (-1 + x^2)^(2/3))/(1 + Sqrt[
3] + (-1 + x^2)^(1/3))^2]*EllipticE[ArcSin[(1 - Sqrt[3] + (-1 + x^2)^(1/3))
/(1 + Sqrt[3] + (-1 + x^2)^(1/3))], -7 - 4*Sqrt[3]])/(2*x*Sqrt[(1 + (-1 + x
^2)^(1/3))/(1 + Sqrt[3] + (-1 + x^2)^(1/3))^2]) + (Sqrt[2]*3^(3/4)*(1 + (-1
 + x^2)^(1/3))*Sqrt[(1 - (-1 + x^2)^(1/3) + (-1 + x^2)^(2/3))/(1 + Sqrt[3]
+ (-1 + x^2)^(1/3))^2]*EllipticF[ArcSin[(1 - Sqrt[3] + (-1 + x^2)^(1/3))/(1
 + Sqrt[3] + (-1 + x^2)^(1/3))], -7 - 4*Sqrt[3]])/(x*Sqrt[(1 + (-1 + x^2)^(
1/3))/(1 + Sqrt[3] + (-1 + x^2)^(1/3))^2]) - Defer[Int][(1 + 5*x)/((-1 + x^
2)^(1/3)*(3 + 4*x + x^2)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{2-x+x^2}{\sqrt[3]{-1+x^2}\,(3+4x+x^2)}\,dx &= \int \left(\frac{1}{\sqrt[3]{-1+x^2}}-\frac{1+5x}{\sqrt[3]{-1+x^2}\,(3+4x+x^2)}\right)dx\\
&= \int \frac{1}{\sqrt[3]{-1+x^2}}\,dx-\int \frac{1+5x}{\sqrt[3]{-1+x^2}\,(3+4x+x^2)}\,dx\\
&= \frac{\left(3\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+x^3}}\,dx,x,\sqrt[3]{-1+x^2}\right)}{2x}-\int \frac{1+5x}{\sqrt[3]{-1+x^2}\,(3+4x+x^2)}\,dx\\
&= \frac{\left(3\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx,x,\sqrt[3]{-1+x^2}\right)}{2x}+\frac{\left(3\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\sqrt{x^2}\right)\text{Subst}\left(\int \right.}{x}\\
&= \frac{3x}{1+\sqrt{3}+\sqrt[3]{-1+x^2}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1+\sqrt[3]{-1+x^2}\right)\sqrt{\frac{1-\sqrt[3]{-1+x^2}+\left(-1+x^2\right)^{2/3}}{\left(1+\sqrt{3}+\sqrt[3]{-1+x^2}\right)^2}}\,E}{2x\sqrt{\frac{1+\sqrt[3]{-1+x^2}}{\left(1+\sqrt{3}+\sqrt[3]{-1+x^2}\right)^2}}}
\end{aligned}$$

Mathematica [C] time = 0.17, size = 71, normalized size = 0.61

$$\frac{3(x^2-1)^{2/3}}{x+1}-\frac{21(x^2-1)^{2/3}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};\frac{1-x}{2},\frac{1-x}{4}\right)}{8\sqrt[3]{2}(x+1)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 - x + x^2)/((-1 + x^2)^(1/3)*(3 + 4*x + x^2)),x]
```

```
[Out] (3*(-1 + x^2)^(2/3))/(1 + x) - (21*(-1 + x^2)^(2/3)*AppellF1[2/3, 1/3, 1, 5
/3, (1 - x)/2, (1 - x)/4])/(8*2^(1/3)*(1 + x)^(2/3))
```

IntegrateAlgebraic [A] time = 0.37, size = 116, normalized size = 1.00

$$\frac{3\left(x^2-1\right)^{2/3}}{x+1}-\frac{7}{4}\log\left(2\sqrt[3]{x^2-1}+x-1\right)+\frac{7}{8}\log\left(x^2+4\left(x^2-1\right)^{2/3}+(2-2x)\sqrt[3]{x^2-1}-2x+1\right)-\frac{7}{4}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^2-1}}{\sqrt[3]{x^2-1}-x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - x + x^2)/((-1 + x^2)^(1/3)*(3 + 4*x + x^2)),x]
```

```
[Out] (3*(-1 + x^2)^(2/3))/(1 + x) - (7*Sqrt[3]*ArcTan[(Sqrt[3]*(-1 + x^2)^(1/3))
/(1 - x + (-1 + x^2)^(1/3))])/4 - (7*Log[-1 + x + 2*(-1 + x^2)^(1/3)])/4 +
(7*Log[1 - 2*x + x^2 + (2 - 2*x)*(-1 + x^2)^(1/3) + 4*(-1 + x^2)^(2/3)])/8
```

fricas [A] time = 0.84, size = 127, normalized size = 1.09

$$\frac{14\sqrt{3}(x+1)\arctan\left(\frac{286273\sqrt{3}\left(x^2-1\right)^{\frac{1}{3}}(x-1)+\sqrt{3}\left(66978x^2+434719x+635653\right)+539695\sqrt{3}\left(x^2-1\right)^{\frac{2}{3}}}{226981x^2-1974837x-1293894}\right)-7(x+1)\log\left(\frac{x^2+6\left(x^2-1\right)^{\frac{1}{3}}(x-1)+6x+12\left(x^2-1\right)^{\frac{2}{3}}+9}{x^2+6x+9}\right)+24\left(x^2-1\right)^{\frac{2}{3}}}{8(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^2-1)^(1/3)/(x^2+4*x+3),x, algorithm="fricas")
```

```
[Out] 1/8*(14*sqrt(3)*(x + 1)*arctan((286273*sqrt(3)*(x^2 - 1)^(1/3)*(x - 1) + sq
rt(3)*(66978*x^2 + 434719*x + 635653) + 539695*sqrt(3)*(x^2 - 1)^(2/3))/(22
6981*x^2 - 1974837*x - 1293894)) - 7*(x + 1)*log((x^2 + 6*(x^2 - 1)^(1/3)*(
x - 1) + 6*x + 12*(x^2 - 1)^(2/3) + 9)/(x^2 + 6*x + 9)) + 24*(x^2 - 1)^(2/3
))/(x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-x+2}{\left(x^2+4x+3\right)\left(x^2-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^2-1)^(1/3)/(x^2+4*x+3),x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 2)/((x^2 + 4*x + 3)*(x^2 - 1)^(1/3)), x)
```

maple [C] time = 2.03, size = 594, normalized size = 5.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+2)/(x^2-1)^(1/3)/(x^2+4*x+3),x)
```

```
[Out] 3*(-1+x)/(x^2-1)^(1/3)+7/4*ln(-(96*RootOf(4*_Z^2-2*_Z+1)^2*x^2-288*RootOf(4
*_Z^2-2*_Z+1)^2*x-864*RootOf(4*_Z^2-2*_Z+1)*(x^2-1)^(2/3)+432*RootOf(4*_Z^2
-2*_Z+1)*(x^2-1)^(1/3)*x-278*RootOf(4*_Z^2-2*_Z+1)*x^2-432*RootOf(4*_Z^2-2*
_Z+1)*(x^2-1)^(1/3)+492*RootOf(4*_Z^2-2*_Z+1)*x-516*(x^2-1)^(2/3)+258*x*(x^
2-1)^(1/3)+17*x^2-342*RootOf(4*_Z^2-2*_Z+1)-258*(x^2-1)^(1/3)+918*x+969)/(3
+x)^2)-7/2*ln(-(96*RootOf(4*_Z^2-2*_Z+1)^2*x^2-288*RootOf(4*_Z^2-2*_Z+1)^2*
x-864*RootOf(4*_Z^2-2*_Z+1)*(x^2-1)^(2/3)+432*RootOf(4*_Z^2-2*_Z+1)*(x^2-1)
^(1/3)*x-278*RootOf(4*_Z^2-2*_Z+1)*x^2-432*RootOf(4*_Z^2-2*_Z+1)*(x^2-1)^(1
```

```
/3)+492*RootOf(4*_Z^2-2*_Z+1)*x-516*(x^2-1)^(2/3)+258*x*(x^2-1)^(1/3)+17*x^
2-342*RootOf(4*_Z^2-2*_Z+1)-258*(x^2-1)^(1/3)+918*x+969)/(3+x)^2)*RootOf(4*
_Z^2-2*_Z+1)+7/2*RootOf(4*_Z^2-2*_Z+1)*ln(-(48*RootOf(4*_Z^2-2*_Z+1)^2*x^2-
144*RootOf(4*_Z^2-2*_Z+1)^2*x+432*RootOf(4*_Z^2-2*_Z+1)*(x^2-1)^(2/3)-216*R
ootOf(4*_Z^2-2*_Z+1)*(x^2-1)^(1/3)*x+91*RootOf(4*_Z^2-2*_Z+1)*x^2+216*RootO
f(4*_Z^2-2*_Z+1)*(x^2-1)^(1/3)-102*RootOf(4*_Z^2-2*_Z+1)*x-474*(x^2-1)^(2/3
)+237*x*(x^2-1)^(1/3)-49*x^2+171*RootOf(4*_Z^2-2*_Z+1)-237*(x^2-1)^(1/3)+54
6*x+399)/(3+x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - x + 2}{\left(x^2 + 4x + 3\right)\left(x^2 - 1\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^2-1)^(1/3)/(x^2+4*x+3),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 2)/((x^2 + 4*x + 3)*(x^2 - 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 - x + 2}{\left(x^2 - 1\right)^{1/3}\,\left(x^2 + 4x + 3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x + 2)/((x^2 - 1)^(1/3)*(4*x + x^2 + 3)),x)
```

```
[Out] int((x^2 - x + 2)/((x^2 - 1)^(1/3)*(4*x + x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - x + 2}{\sqrt[3]{(x-1)(x+1)}\,(x+1)(x+3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+2)/(x**2-1)**(1/3)/(x**2+4*x+3),x)
```

```
[Out] Integral((x**2 - x + 2)/(((x - 1)*(x + 1))**(1/3)*(x + 1)*(x + 3)), x)
```

3.1454 $\int \frac{x(-1+kx)\left(-1+2(-1+k)x+kx^2\right)}{(-1+x)((1-x)x(1-kx))^{3/4}\left(-d+(1+3d)x-(3d+k)x^2+dx^3\right)}\,dx$

Optimal. Leaf size=116

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\,x-\sqrt[4]{d}}{\sqrt[4]{kx^3+(-k-1)x^2+x}}\right)-2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x-\sqrt[4]{d}}{\sqrt[4]{kx^3+(-k-1)x^2+x}}\right)+\frac{4\sqrt[4]{kx^3+(-k-1)x^2+x}}{x-1}$$

Rubi [F] time = 20.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(-1+kx)\left(-1+2(-1+k)x+kx^2\right)}{(-1+x)((1-x)x(1-kx))^{3/4}\left(-d+(1+3d)x-(3d+k)x^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-1 + k*x)*(-1 + 2*(-1 + k)*x + k*x^2))/((-1 + x)*((1 - x)*x*(1 - k*
x))^(3/4)*(-d + (1 + 3*d)*x - (3*d + k)*x^2 + d*x^3)),x]
```

```
[Out] (4*k*(1 - x)^(3/4)*x*(1 - k*x)^(3/4)*AppellF1[1/4, 7/4, -1/4, 5/4, x, k*x])
/(d*((1 - x)*x*(1 - k*x))^(3/4)) + (4*(d + k + 3*d*k)*(1 - x)^(3/4)*x^(3/4)
*(1 - k*x)^(3/4)*Defer[Subst][Defer[Int][(x^4*(1 - k*x^4)^(1/4))/((1 - x^4)
^(7/4)*(-x^4 + k*x^8 - d*(-1 + x^4)^3)), x], x, x^(1/4)])/(d*((1 - x)*x*(1
- k*x))^(3/4)) + (4*(d*(2 - 5*k) - k^2)*(1 - x)^(3/4)*x^(3/4)*(1 - k*x)^(3/
4)*Defer[Subst][Defer[Int][(x^8*(1 - k*x^4)^(1/4))/((1 - x^4)^(7/4)*(-x^4 +
 k*x^8 - d*(-1 + x^4)^3)), x], x, x^(1/4)])/(d*((1 - x)*x*(1 - k*x))^(3/4))
 + (4*k*(1 - x)^(3/4)*x^(3/4)*(1 - k*x)^(3/4)*Defer[Subst][Defer[Int][(1 -
k*x^4)^(1/4)/((1 - x^4)^(7/4)*(x^4 - k*x^8 + d*(-1 + x^4)^3)), x], x, x^(1/
4)])/((1 - x)*x*(1 - k*x))^(3/4)
```

Rubi steps

$$\int \frac{x(-1+kx)\left(-1+2(-1+k)x+kx^2\right)}{(-1+x)((1-x)x(1-kx))^{3/4}\left(-d+(1+3d)x-(3d+k)x^2+dx^3\right)}dx = \frac{\left((1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\int \frac{\sqrt[4]{x}}{(1-x)^{3/4}(-1+x}}{((1-x)x(1-}$$

$$= -\frac{\left((1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\int \frac{\sqrt[4]{x}(}{(1-x)^{7/4}(1-}}{((1-x)x(1-}$$

$$= \frac{\left((1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\int \frac{\sqrt[4]{x}\sqrt[4]{1-kx}}{(1-x)^{7/4}(-d+(}}{((1-x)x(1-kx))^{3/}}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subst}\left(\int \frac{}{(1-}\right.}{((1-x)x(}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subst}\left(\int \frac{x^4}{(1}\right.}{((1-x)x(1-}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subst}\left(\int\left(\frac{k}{d}\right.\right.}{(}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subst}\left(\int \frac{\sqrt[4]{1}}{}\right.}{d((1-x)}$$

$$= \frac{4k(1-x)^{3/4}x(1-kx)^{3/4}F_1\left(\frac{1}{4};\frac{7}{4},-\frac{1}{4};\frac{5}{4};\right.}{d((1-x)x(1-kx))^{3/4}}$$

$$= \frac{4k(1-x)^{3/4}x(1-kx)^{3/4}F_1\left(\frac{1}{4};\frac{7}{4},-\frac{1}{4};\frac{5}{4};\right.}{d((1-x)x(1-kx))^{3/4}}$$

$$= \frac{4k(1-x)^{3/4}x(1-kx)^{3/4}F_1\left(\frac{1}{4};\frac{7}{4},-\frac{1}{4};\frac{5}{4};\right.}{d((1-x)x(1-kx))^{3/4}}$$

Mathematica [F] time = 3.88, size = 0, normalized size = 0.00

$$\int \frac{x(-1+kx)\left(-1+2(-1+k)x+kx^2\right)}{(-1+x)((1-x)x(1-kx))^{3/4}\left(-d+(1+3d)x-(3d+k)x^2+dx^3\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(-1 + k*x)*(-1 + 2*(-1 + k)*x + k*x^2))/((-1 + x)*((1 - x)*x*(
1 - k*x))^(3/4)*(-d + (1 + 3*d)*x - (3*d + k)*x^2 + d*x^3)),x]
```

```
[Out] Integrate[(x*(-1 + k*x)*(-1 + 2*(-1 + k)*x + k*x^2))/((-1 + x)*((1 - x)*x*(
1 - k*x))^(3/4)*(-d + (1 + 3*d)*x - (3*d + k)*x^2 + d*x^3)), x]
```

IntegrateAlgebraic [A] time = 4.31, size = 116, normalized size = 1.00

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\,x-\sqrt[4]{d}}{\sqrt[4]{kx^3+(-k-1)x^2+x}}\right)-2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x-\sqrt[4]{d}}{\sqrt[4]{kx^3+(-k-1)x^2+x}}\right)+\frac{4\sqrt[4]{kx^3+(-k-1)x^2+x}}{x-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-1 + k*x)*(-1 + 2*(-1 + k)*x + k*x^2))/((-1 + x)*((1
 - x)*x*(1 - k*x))^(3/4)*(-d + (1 + 3*d)*x - (3*d + k)*x^2 + d*x^3)),x]
```

```
[Out] (4*(x + (-1 - k)*x^2 + k*x^3)^(1/4))/(-1 + x) + 2*d^(1/4)*ArcTan[(-d^(1/4)
+ d^(1/4)*x)/(x + (-1 - k)*x^2 + k*x^3)^(1/4)] - 2*d^(1/4)*ArcTanh[(-d^(1/4
) + d^(1/4)*x)/(x + (-1 - k)*x^2 + k*x^3)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(k*x-1)*(-1+2*(-1+k)*x+k*x^2)/(-1+x)/((1-x)*x*(-k*x+1))^(3/4)/(
-d+(1+3*d)*x-(3*d+k)*x^2+d*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(kx^2 + 2\,(k-1)x - 1\right)(kx-1)x}{\left(dx^3 - (3\,d+k)x^2 + (3\,d+1)x - d\right)\left((kx-1)(x-1)x\right)^{\frac{3}{4}}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(k*x-1)*(-1+2*(-1+k)*x+k*x^2)/(-1+x)/((1-x)*x*(-k*x+1))^(3/4)/(
-d+(1+3*d)*x-(3*d+k)*x^2+d*x^3),x, algorithm="giac")
```

```
[Out] integrate((k*x^2 + 2*(k - 1)*x - 1)*(k*x - 1)*x/((d*x^3 - (3*d + k)*x^2 + (
3*d + 1)*x - d)*((k*x - 1)*(x - 1)*x)^(3/4)*(x - 1)), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{x\,(kx-1)\left(-1+2\,(-1+k)\,x+k\,x^{2}\right)}{(-1+x)\,((1-x)\,x\,(-kx+1))^{\frac{3}{4}}\left(-d+(1+3d)\,x-(3d+k)\,x^{2}+d\,x^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(k*x-1)*(-1+2*(-1+k)*x+k*x^2)/(-1+x)/((1-x)*x*(-k*x+1))^(3/4)/(-d+(1+
3*d)*x-(3*d+k)*x^2+d*x^3),x)
```

```
[Out] int(x*(k*x-1)*(-1+2*(-1+k)*x+k*x^2)/(-1+x)/((1-x)*x*(-k*x+1))^(3/4)/(-d+(1+
3*d)*x-(3*d+k)*x^2+d*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(kx^2 + 2\,(k-1)x - 1\right)(kx-1)x}{\left(dx^3 - (3\,d+k)x^2 + (3\,d+1)x - d\right)\left((kx-1)(x-1)x\right)^{\frac{3}{4}}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(k*x-1)*(-1+2*(-1+k)*x+k*x^2)/(-1+x)/((1-x)*x*(-k*x+1))^(3/4)/(
-d+(1+3*d)*x-(3*d+k)*x^2+d*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k*x^2 + 2*(k - 1)*x - 1)*(k*x - 1)*x/((d*x^3 - (3*d + k)*x^2 + (
3*d + 1)*x - d)*((k*x - 1)*(x - 1)*x)^(3/4)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x\,\left(k\,x-1\right)\,\left(2\,x\,\left(k-1\right)+k\,x^2-1\right)}{\left(x-1\right)\,{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{3/4}\,\left(-d\,x^3+\left(3\,d+k\right)\,x^2+\left(-3\,d-1\right)\,x+d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(k*x - 1)*(2*x*(k - 1) + k*x^2 - 1))/((x - 1)*(x*(k*x - 1)*(x - 1))
^(3/4)*(d - d*x^3 + x^2*(3*d + k) - x*(3*d + 1))),x)
```

```
[Out] -int((x*(k*x - 1)*(2*x*(k - 1) + k*x^2 - 1))/((x - 1)*(x*(k*x - 1)*(x - 1))
^(3/4)*(d - d*x^3 + x^2*(3*d + k) - x*(3*d + 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(k*x-1)*(-1+2*(-1+k)*x+k*x**2)/(-1+x)/((1-x)*x*(-k*x+1))**(3/4)
/(-d+(1+3*d)*x-(3*d+k)*x**2+d*x**3),x)
```

```
[Out] Timed out
```

3.1455 $\int \frac{(-1+x)x\left(-1-2(-1+k)x+kx^2\right)}{((1-x)x(1-kx))^{3/4}(-1+kx)\left(-d+(1+3dk)x-\left(1+3dk^2\right)x^2+dk^3x^3\right)}\,dx$

Optimal. Leaf size=116

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\left(kx^3+(-k-1)x^2+x\right)^{3/4}}{(x-1)x}\right)-2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(kx^3+(-k-1)x^2+x\right)^{3/4}}{(x-1)x}\right)+\frac{4\left(x^2-x\right)}{\left(kx^3-kx^2-x^2+x\right.}$$

Rubi [F] time = 21.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x)x\left(-1-2(-1+k)x+kx^2\right)}{((1-x)x(1-kx))^{3/4}(-1+kx)\left(-d+(1+3dk)x-\left(1+3dk^2\right)x^2+dk^3x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x)*x*(-1 - 2*(-1 + k)*x + k*x^2))/(((1 - x)*x*(1 - k*x))^(3/4)*(
-1 + k*x)*(-d + (1 + 3*d*k)*x - (1 + 3*d*k^2)*x^2 + d*k^3*x^3)),x]
```

```
[Out] (4*(1 - x)^(3/4)*x*(1 - k*x)^(3/4)*AppellF1[1/4, -1/4, 7/4, 5/4, x, k*x])/(
d*k^2*((1 - x)*x*(1 - k*x))^(3/4)) + (4*(1 + d*k*(3 + k))*(1 - x)^(3/4)*x^(
3/4)*(1 - k*x)^(3/4)*Defer[Subst][Defer[Int][(x^4*(1 - x^4)^(1/4))/((1 - k*
x^4)^(7/4)*(-x^4 + x^8 - d*(-1 + k*x^4)^3)), x], x, x^(1/4)])/(d*k^2*((1 -
x)*x*(1 - k*x))^(3/4)) - (4*(5 + 1/(d*k^2) - 2*k)*(1 - x)^(3/4)*x^(3/4)*(1
- k*x)^(3/4)*Defer[Subst][Defer[Int][(x^8*(1 - x^4)^(1/4))/((1 - k*x^4)^(7/
4)*(-x^4 + x^8 - d*(-1 + k*x^4)^3)), x], x, x^(1/4)])/((1 - x)*x*(1 - k*x))
^(3/4) + (4*(1 - x)^(3/4)*x^(3/4)*(1 - k*x)^(3/4)*Defer[Subst][Defer[Int][(
1 - x^4)^(1/4)/((1 - k*x^4)^(7/4)*(x^4 - x^8 + d*(-1 + k*x^4)^3)), x], x, x
^(1/4)])/(k^2*((1 - x)*x*(1 - k*x))^(3/4))
```

Rubi steps

$$\int \frac{(-1+x)x\left(-1-2(-1+k)x+kx^2\right)}{((1-x)x(1-kx))^{3/4}(-1+kx)\left(-d+(1+3dk)x-\left(1+3dk^2\right)x^2+dk^3x^3\right)}\,dx = \frac{\left((1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\int \frac{}{(1-x}}{((1}$$

$$= -\frac{\left((1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\int \frac{}{(1-}}{((1-}$$

$$= \frac{\left((1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\int \frac{}{(1-k}}{((1-x)x}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subs}}{}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subs}}{((1-}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subs}}{}$$

$$= \frac{\left(4(1-x)^{3/4}x^{3/4}(1-kx)^{3/4}\right)\text{Subs}}{dk^2((1-x)x(1-}$$

$$= \frac{4(1-x)^{3/4}x(1-kx)^{3/4}F_1\left(\frac{1}{4};-\frac{1}{4}\right.}{dk^2((1-x)x(1-kx))^3}$$

$$= \frac{4(1-x)^{3/4}x(1-kx)^{3/4}F_1\left(\frac{1}{4};-\frac{1}{4}\right.}{dk^2((1-x)x(1-kx))^3}$$

$$= \frac{4(1-x)^{3/4}x(1-kx)^{3/4}F_1\left(\frac{1}{4};-\frac{1}{4}\right.}{dk^2((1-x)x(1-kx))^3}$$

Mathematica [F] time = 3.98, size = 0, normalized size = 0.00

$$\int \frac{(-1+x)x\left(-1-2(-1+k)x+kx^2\right)}{((1-x)x(1-kx))^{3/4}(-1+kx)\left(-d+(1+3dk)x-\left(1+3dk^2\right)x^2+dk^3x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x)*x*(-1 - 2*(-1 + k)*x + k*x^2))/(((1 - x)*x*(1 - k*x))^(
3/4)*(-1 + k*x)*(-d + (1 + 3*d*k)*x - (1 + 3*d*k^2)*x^2 + d*k^3*x^3)),x]
```

```
[Out] Integrate[((-1 + x)*x*(-1 - 2*(-1 + k)*x + k*x^2))/(((1 - x)*x*(1 - k*x))^(
3/4)*(-1 + k*x)*(-d + (1 + 3*d*k)*x - (1 + 3*d*k^2)*x^2 + d*k^3*x^3)), x]
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((-1 + x)*x*(-1 - 2*(-1 + k)*x + k*x^2))/(((1 - x)*x*(1
- k*x))^(3/4)*(-1 + k*x)*(-d + (1 + 3*d*k)*x - (1 + 3*d*k^2)*x^2 + d*k^3*x^
3)),x]
```

```
[Out] $Aborted
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*x*(-1-2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(3/4)/(k*x-1)/(
-d+(3*d*k+1)*x-(3*d*k^2+1)*x^2+d*k^3*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(kx^{2}-2\,(k-1)x-1\right)(x-1)x}{\left(dk^{3}x^{3}-\left(3\,dk^{2}+1\right)x^{2}+(3\,dk+1)x-d\right)((kx-1)(x-1)x)^{\frac{3}{4}}\,(kx-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*x*(-1-2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(3/4)/(k*x-1)/(
-d+(3*d*k+1)*x-(3*d*k^2+1)*x^2+d*k^3*x^3),x, algorithm="giac")
```

```
[Out] integrate((k*x^2 - 2*(k - 1)*x - 1)*(x - 1)*x/((d*k^3*x^3 - (3*d*k^2 + 1)*x
^2 + (3*d*k + 1)*x - d)*((k*x - 1)*(x - 1)*x)^(3/4)*(k*x - 1)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(-1+x)\,x\left(-1-2\,(-1+k)\,x+k\,x^{2}\right)}{((1-x)\,x\,(-kx+1))^{\frac{3}{4}}\,(kx-1)\left(-d+(3dk+1)\,x-\left(3d\,k^{2}+1\right)x^{2}+d\,k^{3}x^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)*x*(-1-2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(3/4)/(k*x-1)/(-d+(3*
d*k+1)*x-(3*d*k^2+1)*x^2+d*k^3*x^3),x)
```

```
[Out] int((-1+x)*x*(-1-2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(3/4)/(k*x-1)/(-d+(3*
d*k+1)*x-(3*d*k^2+1)*x^2+d*k^3*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(kx^{2}-2\,(k-1)x-1\right)(x-1)x}{\left(dk^{3}x^{3}-\left(3\,dk^{2}+1\right)x^{2}+(3\,dk+1)x-d\right)((kx-1)(x-1)x)^{\frac{3}{4}}\,(kx-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*x*(-1-2*(-1+k)*x+k*x^2)/((1-x)*x*(-k*x+1))^(3/4)/(k*x-1)/(
-d+(3*d*k+1)*x-(3*d*k^2+1)*x^2+d*k^3*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k*x^2 - 2*(k - 1)*x - 1)*(x - 1)*x/((d*k^3*x^3 - (3*d*k^2 + 1)*x
^2 + (3*d*k + 1)*x - d)*((k*x - 1)*(x - 1)*x)^(3/4)*(k*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(x-1)\,\left(2\,x\,(k-1)-k\,x^2+1\right)}{(k\,x-1)\,\left(x\,(k\,x-1)\,(x-1)\right)^{3/4}\,\left(d+x^2\,\left(3\,d\,k^2+1\right)-x\,(3\,d\,k+1)-d\,k^3\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x - 1)*(2*x*(k - 1) - k*x^2 + 1))/((k*x - 1)*(x*(k*x - 1)*(x - 1))^
(3/4)*(d + x^2*(3*d*k^2 + 1) - x*(3*d*k + 1) - d*k^3*x^3)),x)
```

```
[Out] int((x*(x - 1)*(2*x*(k - 1) - k*x^2 + 1))/((k*x - 1)*(x*(k*x - 1)*(x - 1))^
(3/4)*(d + x^2*(3*d*k^2 + 1) - x*(3*d*k + 1) - d*k^3*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*x*(-1-2*(-1+k)*x+k*x**2)/((1-x)*x*(-k*x+1))**(3/4)/(k*x-1)
/(-d+(3*d*k+1)*x-(3*d*k**2+1)*x**2+d*k**3*x**3),x)
```

```
[Out] Timed out
```

3.1456 $\int \frac{2b+ax^3}{(-b+x^2+ax^3)\sqrt[4]{-bx^2+ax^5}}\,dx$

Optimal. Leaf size=116

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^5-bx^2}}{\sqrt{ax^5-bx^2}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5-bx^2}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5-bx^2}}\right)$$

Rubi [F] time = 1.78, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2b+ax^3}{(-b+x^2+ax^3)\sqrt[4]{-bx^2+ax^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2*b + a*x^3)/((-b + x^2 + a*x^3)*(-(b*x^2) + a*x^5)^(1/4)),x]
```

```
[Out] (2*x*(1 - (a*x^3)/b)^(1/4)*Hypergeometric2F1[1/6, 1/4, 7/6, (a*x^3)/b])/(-(
b*x^2) + a*x^5)^(1/4) - (6*b*Sqrt[x]*(-b + a*x^3)^(1/4)*Defer[Subst][Defer[
Int][1/((b - x^4 - a*x^6)*(-b + a*x^6)^(1/4)), x], x, Sqrt[x]])/(-(b*x^2) +
 a*x^5)^(1/4) - (2*Sqrt[x]*(-b + a*x^3)^(1/4)*Defer[Subst][Defer[Int][x^4/(
(-b + a*x^6)^(1/4)*(-b + x^4 + a*x^6)), x], x, Sqrt[x]])/(-(b*x^2) + a*x^5)
^(1/4)
```

Rubi steps

$$\int \frac{2b+ax^3}{(-b+x^2+ax^3)\sqrt[4]{-bx^2+ax^5}}\,dx=\frac{\left(\sqrt{x}\sqrt[4]{-b+ax^3}\right)\int \frac{2b+ax^3}{\sqrt{x}\sqrt[4]{-b+ax^3}(-b+x^2+ax^3)}\,dx}{\sqrt[4]{-bx^2+ax^5}}$$

$$=\frac{\left(2\sqrt{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{2b+ax^6}{\sqrt[4]{-b+ax^6}(-b+x^4+ax^6)}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{-bx^2+ax^5}}$$

$$=\frac{\left(2\sqrt{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[4]{-b+ax^6}}+\frac{3b-x^4}{\sqrt[4]{-b+ax^6}(-b+x^4+ax^6)}\right)dx,x,\sqrt{x}\right)}{\sqrt[4]{-bx^2+ax^5}}$$

$$=\frac{\left(2\sqrt{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-b+ax^6}}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{-bx^2+ax^5}}+\frac{\left(2\sqrt{x}\sqrt[4]{-b+ax^3}\right)\text{S}}{}$$

$$=\frac{\left(2\sqrt{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \left(-\frac{3b}{(b-x^4-ax^6)\sqrt[4]{-b+ax^6}}-\frac{x^4}{\sqrt[4]{-b+ax^6}(-b+x^4+ax^6)}\right)\right.}{\sqrt[4]{-bx^2+ax^5}}$$

$$=\frac{2x\sqrt[4]{1-\frac{ax^3}{b}}\,{}_2F_1\left(\frac{1}{6},\frac{1}{4};\frac{7}{6};\frac{ax^3}{b}\right)}{\sqrt[4]{-bx^2+ax^5}}-\frac{\left(2\sqrt{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{-b+ax^6}(-}\right.}{\sqrt[4]{-bx^2+ax^5}}$$

Mathematica [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{2b+ax^3}{(-b+x^2+ax^3)\sqrt[4]{-bx^2+ax^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2*b + a*x^3)/((-b + x^2 + a*x^3)*(-(b*x^2) + a*x^5)^(1/4)),x]
```

```
[Out] Integrate[(2*b + a*x^3)/((-b + x^2 + a*x^3)*(-(b*x^2) + a*x^5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 2.95, size = 116, normalized size = 1.00

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^5-bx^2}}{\sqrt{ax^5-bx^2}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5-bx^2}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5-bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*b + a*x^3)/((-b + x^2 + a*x^3)*(-(b*x^2) + a*x^5)^(1/
4)),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(Sqrt[2]*x*(-(b*x^2) + a*x^5)^(1/4))/(-x^2 + Sqrt[-(b*x^2)
 + a*x^5])]) - Sqrt[2]*ArcTanh[(x^2/Sqrt[2] + Sqrt[-(b*x^2) + a*x^5]/Sqrt[2
])/(x*(-(b*x^2) + a*x^5)^(1/4))]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b)/(a*x^3+x^2-b)/(a*x^5-b*x^2)^(1/4),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3+2\,b}{\left(ax^5-bx^2\right)^{\frac{1}{4}}\left(ax^3+x^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b)/(a*x^3+x^2-b)/(a*x^5-b*x^2)^(1/4),x, algorithm="giac"
)
```

```
[Out] integrate((a*x^3 + 2*b)/((a*x^5 - b*x^2)^(1/4)*(a*x^3 + x^2 - b)), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{a\,x^3+2b}{\left(a\,x^3+x^2-b\right)\left(a\,x^5-b\,x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+2*b)/(a*x^3+x^2-b)/(a*x^5-b*x^2)^(1/4),x)
```

```
[Out] int((a*x^3+2*b)/(a*x^3+x^2-b)/(a*x^5-b*x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3+2\,b}{\left(ax^5-bx^2\right)^{\frac{1}{4}}\left(ax^3+x^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b)/(a*x^3+x^2-b)/(a*x^5-b*x^2)^(1/4),x, algorithm="maxim
a")
```

```
[Out] integrate((a*x^3 + 2*b)/((a*x^5 - b*x^2)^(1/4)*(a*x^3 + x^2 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x^3+2\,b}{{\left(a\,x^5-b\,x^2\right)}^{1/4}\,\left(a\,x^3+x^2-b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b + a*x^3)/((a*x^5 - b*x^2)^(1/4)*(a*x^3 - b + x^2)),x)
```

```
[Out] int((2*b + a*x^3)/((a*x^5 - b*x^2)^(1/4)*(a*x^3 - b + x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+2*b)/(a*x**3+x**2-b)/(a*x**5-b*x**2)**(1/4),x)
```

```
[Out] Timed out
```

3.1457 $\int \frac{b+ax^2}{\left(-b+x+ax^2\right)\sqrt[4]{-bx^3+ax^5}}\,dx$

Optimal. Leaf size=116

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^5-bx^3}}{\sqrt{ax^5-bx^3}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5-bx^3}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5-bx^3}}\right)$$

Rubi **[C]** time = 2.11, antiderivative size = 397, normalized size of antiderivative = 3.42, number of steps used = 21, number of rules used = 10, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2056, 6728, 365, 364, 959, 466, 430, 429, 511, 510}

$$\frac{8ax^2\sqrt[4]{1-\frac{ax^2}{b}}F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};\frac{4a^2x^2}{\left(1-\sqrt{4ab+1}\right)^2},\frac{ax^2}{b}\right)}{5\left(1-\sqrt{4ab+1}\right)\sqrt[4]{ax^5-bx^3}}+\frac{8ax^2\sqrt[4]{1-\frac{ax^2}{b}}F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};\frac{4a^2x^2}{\left(\sqrt{4ab+1}+1\right)^2},\frac{ax^2}{b}\right)}{5\left(\sqrt{4ab+1}+1\right)\sqrt[4]{ax^5-bx^3}}-\frac{4x\sqrt[4]{1-\frac{ax^2}{b}}F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\frac{4a^2x^2}{\left(1-\sqrt{4ab+1}\right)^2},\frac{ax^2}{b}\right)}{\sqrt[4]{ax^5-bx^3}}-\frac{4x\sqrt[4]{1-\frac{ax^2}{b}}F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\frac{4a^2x^2}{\left(\sqrt{4ab+1}+1\right)^2},\frac{ax^2}{b}\right)}{\sqrt[4]{ax^5-bx^3}}+\frac{4x\sqrt[4]{1-\frac{ax^2}{b}}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};\frac{ax^2}{b}\right)}{\sqrt[4]{ax^5-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(b + a*x^2)/((-b + x + a*x^2)*(-(b*x^3) + a*x^5)^(1/4)),x]
```

```
[Out] (-4*x*(1 - (a*x^2)/b)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, (4*a^2*x^2)/(1 - Sqr
t[1 + 4*a*b])^2, (a*x^2)/b])/(-(b*x^3) + a*x^5)^(1/4) - (4*x*(1 - (a*x^2)/b
)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, (4*a^2*x^2)/(1 + Sqrt[1 + 4*a*b])^2, (a*
x^2)/b])/(-(b*x^3) + a*x^5)^(1/4) + (8*a*x^2*(1 - (a*x^2)/b)^(1/4)*AppellF1
[5/8, 1, 1/4, 13/8, (4*a^2*x^2)/(1 - Sqrt[1 + 4*a*b])^2, (a*x^2)/b])/(5*(1
- Sqrt[1 + 4*a*b])*(-(b*x^3) + a*x^5)^(1/4)) + (8*a*x^2*(1 - (a*x^2)/b)^(1/
4)*AppellF1[5/8, 1, 1/4, 13/8, (4*a^2*x^2)/(1 + Sqrt[1 + 4*a*b])^2, (a*x^2)
/b])/(5*(1 + Sqrt[1 + 4*a*b])*(-(b*x^3) + a*x^5)^(1/4)) + (4*x*(1 - (a*x^2)
/b)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, (a*x^2)/b])/(-(b*x^3) + a*x^5)^(
1/4)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{b+ax^2}{\left(-b+x+ax^2\right)\sqrt[4]{-bx^3+ax^5}}\,dx = \frac{\left(x^{3/4}\sqrt[4]{-b+ax^2}\right)\int \frac{b+ax^2}{x^{3/4}\sqrt[4]{-b+ax^2}\left(-b+x+ax^2\right)}\,dx}{\sqrt[4]{-bx^3+ax^5}}$$

$$= \frac{\left(x^{3/4}\sqrt[4]{-b+ax^2}\right)\int \left(\frac{1}{x^{3/4}\sqrt[4]{-b+ax^2}}+\frac{2b-x}{x^{3/4}\sqrt[4]{-b+ax^2}\left(-b+x+ax^2\right)}\right)dx}{\sqrt[4]{-bx^3+ax^5}}$$

$$= \frac{\left(x^{3/4}\sqrt[4]{-b+ax^2}\right)\int \frac{1}{x^{3/4}\sqrt[4]{-b+ax^2}}\,dx}{\sqrt[4]{-bx^3+ax^5}} + \frac{\left(x^{3/4}\sqrt[4]{-b+ax^2}\right)\int \frac{2b-x}{x^{3/4}\sqrt[4]{-b+ax^2}\left(-b+x+a\right.}}{\sqrt[4]{-bx^3+ax^5}}$$

$$= \frac{\left(x^{3/4}\sqrt[4]{-b+ax^2}\right)\int \left(\frac{-1+\sqrt{1+4ab}}{x^{3/4}\left(1-\sqrt{1+4ab}+2ax\right)\sqrt[4]{-b+ax^2}}+\frac{-1-\sqrt{1+4ab}}{x^{3/4}\left(1+\sqrt{1+4ab}+2ax\right)\sqrt[4]{-b+ax^2}}\right.}{\sqrt[4]{-bx^3+ax^5}}$$

$$= \frac{4x\sqrt[4]{1-\frac{ax^2}{b}}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};\frac{ax^2}{b}\right)}{\sqrt[4]{-bx^3+ax^5}} + \frac{\left(\left(-1-\sqrt{1+4ab}\right)x^{3/4}\sqrt[4]{-b+ax^2}\right)\int \frac{}{x^{3/4}(1-}}{\sqrt[4]{-bx^3+ax^5}}$$

$$= \frac{4x\sqrt[4]{1-\frac{ax^2}{b}}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};\frac{ax^2}{b}\right)}{\sqrt[4]{-bx^3+ax^5}} - \frac{\left(2a\left(-1-\sqrt{1+4ab}\right)x^{3/4}\sqrt[4]{-b+ax^2}\right)\int \frac{}{\sqrt[4]{-}}}{\sqrt[4]{-bx^3+ax^5}}$$

$$= \frac{4x\sqrt[4]{1-\frac{ax^2}{b}}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};\frac{ax^2}{b}\right)}{\sqrt[4]{-bx^3+ax^5}} - \frac{\left(8a\left(-1-\sqrt{1+4ab}\right)x^{3/4}\sqrt[4]{-b+ax^2}\right)\text{Subs}}{\sqrt[4]{-bx^3}}$$

$$= \frac{4x\sqrt[4]{1-\frac{ax^2}{b}}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};\frac{ax^2}{b}\right)}{\sqrt[4]{-bx^3+ax^5}} - \frac{\left(8a\left(-1-\sqrt{1+4ab}\right)x^{3/4}\sqrt[4]{1-\frac{ax^2}{b}}\right)\text{Subst}}{\sqrt[4]{-bx^3}}$$

$$= -\frac{4x\sqrt[4]{1-\frac{ax^2}{b}}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\frac{4a^2x^2}{\left(1-\sqrt{1+4ab}\right)^2},\frac{ax^2}{b}\right)}{\sqrt[4]{-bx^3+ax^5}} - \frac{4x\sqrt[4]{1-\frac{ax^2}{b}}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\frac{}{(1+}\right.}{\sqrt[4]{-bx^3+ax^5}}$$

Mathematica [F] time = 1.23, size = 0, normalized size = 0.00

$$\int \frac{b+ax^2}{\left(-b+x+ax^2\right)\sqrt[4]{-bx^3+ax^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(b + a*x^2)/((-b + x + a*x^2)*(-(b*x^3) + a*x^5)^(1/4)),x]
[Out] Integrate[(b + a*x^2)/((-b + x + a*x^2)*(-(b*x^3) + a*x^5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 2.92, size = 116, normalized size = 1.00

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^5-bx^3}}{\sqrt{ax^5-bx^3}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5-bx^3}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5-bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^2)/((-b + x + a*x^2)*(-(b*x^3) + a*x^5)^(1/4)),
x]
```

```
[Out] -(Sqrt[2]*ArcTan[(Sqrt[2]*x*(-(b*x^3) + a*x^5)^(1/4))/(-x^2 + Sqrt[-(b*x^3)
 + a*x^5])]) - Sqrt[2]*ArcTanh[(x^2/Sqrt[2] + Sqrt[-(b*x^3) + a*x^5]/Sqrt[2
])/(x*(-(b*x^3) + a*x^5)^(1/4))]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a*x^2-b+x)/(a*x^5-b*x^3)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + b}{\left(ax^5 - bx^3\right)^{\frac{1}{4}}\left(ax^2 - b + x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a*x^2-b+x)/(a*x^5-b*x^3)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^2 + b)/((a*x^5 - b*x^3)^(1/4)*(a*x^2 - b + x)), x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2 + b}{\left(a\,x^2 - b + x\right)\left(a\,x^5 - b\,x^3\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)/(a*x^2-b+x)/(a*x^5-b*x^3)^(1/4),x)
```

```
[Out] int((a*x^2+b)/(a*x^2-b+x)/(a*x^5-b*x^3)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + b}{\left(ax^5 - bx^3\right)^{\frac{1}{4}}\left(ax^2 - b + x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a*x^2-b+x)/(a*x^5-b*x^3)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 + b)/((a*x^5 - b*x^3)^(1/4)*(a*x^2 - b + x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x^2 + b}{{\left(a\,x^5 - b\,x^3\right)}^{1/4}\,\left(a\,x^2 + x - b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^2)/((a*x^5 - b*x^3)^(1/4)*(x - b + a*x^2)),x)
```

```
[Out] int((b + a*x^2)/((a*x^5 - b*x^3)^(1/4)*(x - b + a*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + b}{\sqrt[4]{x^3 \left(ax^2 - b\right)} \left(ax^2 - b + x\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b)/(a*x**2-b+x)/(a*x**5-b*x**3)**(1/4),x)
[Out] Integral((a*x**2 + b)/((x**3*(a*x**2 - b))**(1/4)*(a*x**2 - b + x)), x)
```

3.1458 $\int \frac{b+ax^4}{\left(-b+x^2+ax^4\right)\sqrt[4]{-bx^2+ax^6}}\,dx$

Optimal. Leaf size=116

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^6-bx^2}}{\sqrt{ax^6-bx^2}-x^2}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6-bx^2}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^6-bx^2}}\right)}{\sqrt{2}}$$

Rubi [C] time = 2.17, antiderivative size = 447, normalized size of antiderivative = 3.85, number of steps used = 20, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2056, 6715, 6728, 246, 245, 1438, 430, 429, 511, 510}

$$-\frac{2x\sqrt[4]{1-\frac{ax^4}{b}}F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\frac{2a^2x^4}{2ab-\sqrt{4ab+1}+1},\frac{ax^4}{b}\right)}{\sqrt[4]{ax^6-bx^2}}-\frac{2x\sqrt[4]{1-\frac{ax^4}{b}}F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\frac{2a^2x^4}{2ab+\sqrt{4ab+1}+1},\frac{ax^4}{b}\right)}{\sqrt[4]{ax^6-bx^2}}+\frac{2ax^3\left(1-\sqrt{4ab+1}\right)\sqrt[4]{1-\frac{ax^4}{b}}F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};\frac{2a^2x^4}{2ab-\sqrt{4ab+1}+1},\frac{ax^4}{b}\right)}{5\left(2ab-\sqrt{4ab+1}+1\right)\sqrt[4]{ax^6-bx^2}}+\frac{2ax^3\left(\sqrt{4ab+1}+1\right)\sqrt[4]{1-\frac{ax^4}{b}}F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};\frac{2a^2x^4}{2ab+\sqrt{4ab+1}+1},\frac{ax^4}{b}\right)}{5\left(2ab+\sqrt{4ab+1}+1\right)\sqrt[4]{ax^6-bx^2}}+\frac{2x\sqrt[4]{1-\frac{ax^4}{b}}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};\frac{ax^4}{b}\right)}{\sqrt[4]{ax^6-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(b + a*x^4)/((-b + x^2 + a*x^4)*(-(b*x^2) + a*x^6)^(1/4)),x]
```

```
[Out] (-2*x*(1 - (a*x^4)/b)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, (2*a^2*x^4)/(1 + 2*a
*b - Sqrt[1 + 4*a*b]), (a*x^4)/b])/(-(b*x^2) + a*x^6)^(1/4) - (2*x*(1 - (a*
x^4)/b)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, (2*a^2*x^4)/(1 + 2*a*b + Sqrt[1 +
4*a*b]), (a*x^4)/b])/(-(b*x^2) + a*x^6)^(1/4) + (2*a*(1 - Sqrt[1 + 4*a*b])*
x^3*(1 - (a*x^4)/b)^(1/4)*AppellF1[5/8, 1, 1/4, 13/8, (2*a^2*x^4)/(1 + 2*a*
b - Sqrt[1 + 4*a*b]), (a*x^4)/b])/(5*(1 + 2*a*b - Sqrt[1 + 4*a*b])*(-(b*x^2
) + a*x^6)^(1/4)) + (2*a*(1 + Sqrt[1 + 4*a*b])*x^3*(1 - (a*x^4)/b)^(1/4)*Ap
pellF1[5/8, 1, 1/4, 13/8, (2*a^2*x^4)/(1 + 2*a*b + Sqrt[1 + 4*a*b]), (a*x^4
)/b])/(5*(1 + 2*a*b + Sqrt[1 + 4*a*b])*(-(b*x^2) + a*x^6)^(1/4)) + (2*x*(1
- (a*x^4)/b)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, (a*x^4)/b])/(-(b*x^2) +
 a*x^6)^(1/4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{b+ax^4}{\left(-b+x^2+ax^4\right)\sqrt[4]{-bx^2+ax^6}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{-b+ax^4}\right)\int \frac{b+ax^4}{\sqrt{x}\sqrt[4]{-b+ax^4}\left(-b+x^2+ax^4\right)}\,dx}{\sqrt[4]{-bx^2+ax^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{-b+ax^4}\right)\text{Subst}\left(\int \frac{b+ax^8}{\sqrt[4]{-b+ax^8}\left(-b+x^4+ax^8\right)}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{-bx^2+ax^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{-b+ax^4}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[4]{-b+ax^8}}+\frac{2b-x^4}{\sqrt[4]{-b+ax^8}\left(-b+x^4+ax^8\right)}\right)dx,x,\sqrt{x}\right.}{\sqrt[4]{-bx^2+ax^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{-b+ax^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-b+ax^8}}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{-bx^2+ax^6}} + \frac{\left(2\sqrt{x}\sqrt[4]{-b+ax^4}\right)\text{S}}{}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{-b+ax^4}\right)\text{Subst}\left(\int\left(\frac{-1+\sqrt{1+4ab}}{\left(1-\sqrt{1+4ab}+2ax^4\right)\sqrt[4]{-b+ax^8}}+\frac{-1-\sqrt{1+4a}}{\left(1+\sqrt{1+4ab}+2ax^4\right)}\right.\right.}{\sqrt[4]{-bx^2+ax^6}}$$

$$= \frac{2x\sqrt[4]{1-\frac{ax^4}{b}}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};\frac{ax^4}{b}\right)}{\sqrt[4]{-bx^2+ax^6}} + \frac{\left(2\left(-1-\sqrt{1+4ab}\right)\sqrt{x}\sqrt[4]{-b+ax^4}\right)\text{Su}}{\sqrt[4]{-b}}$$

$$= \frac{2x\sqrt[4]{1-\frac{ax^4}{b}}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};\frac{ax^4}{b}\right)}{\sqrt[4]{-bx^2+ax^6}} + \frac{\left(2\left(-1-\sqrt{1+4ab}\right)\sqrt{x}\sqrt[4]{-b+ax^4}\right)\text{Su}}{}$$

$$= \frac{2x\sqrt[4]{1-\frac{ax^4}{b}}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};\frac{ax^4}{b}\right)}{\sqrt[4]{-bx^2+ax^6}} + \frac{\left(2a\left(-1-\sqrt{1+4ab}\right)\sqrt{x}\sqrt[4]{-b+ax^4}\right)\text{S}}{\sqrt[4]{}}$$

$$= \frac{2x\sqrt[4]{1-\frac{ax^4}{b}}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};\frac{ax^4}{b}\right)}{\sqrt[4]{-bx^2+ax^6}} + \frac{\left(2a\left(-1-\sqrt{1+4ab}\right)\sqrt{x}\sqrt[4]{1-\frac{ax^4}{b}}\right)\text{Su}}{\sqrt[4]{}}$$

$$= -\frac{2x\sqrt[4]{1-\frac{ax^4}{b}}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};\frac{2a^2x^4}{1+2ab-\sqrt{1+4ab}},\frac{ax^4}{b}\right)}{\sqrt[4]{-bx^2+ax^6}} - \frac{2x\sqrt[4]{1-\frac{ax^4}{b}}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\right.}{\sqrt[4]{-bx^2}}$$

Mathematica [F] time = 1.97, size = 0, normalized size = 0.00

$$\int \frac{b+ax^4}{\left(-b+x^2+ax^4\right)\sqrt[4]{-bx^2+ax^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(b + a*x^4)/((-b + x^2 + a*x^4)*(-(b*x^2) + a*x^6)^(1/4)),x]
[Out] Integrate[(b + a*x^4)/((-b + x^2 + a*x^4)*(-(b*x^2) + a*x^6)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 1.51, size = 116, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^6-bx^2}}{\sqrt{ax^6-bx^2}-x^2}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6-bx^2}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^6-bx^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^4)/((-b + x^2 + a*x^4)*(-(b*x^2) + a*x^6)^(1/4)
),x]
```

```
[Out] -(ArcTan[(Sqrt[2]*x*(-(b*x^2) + a*x^6)^(1/4))/(-x^2 + Sqrt[-(b*x^2) + a*x^6
])]/Sqrt[2]) - ArcTanh[(x^2/Sqrt[2] + Sqrt[-(b*x^2) + a*x^6]/Sqrt[2])/(x*(-
(b*x^2) + a*x^6)^(1/4))]/Sqrt[2]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)/(a*x^4+x^2-b)/(a*x^6-b*x^2)^(1/4),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4 + b}{\left(ax^6 - bx^2\right)^{\frac{1}{4}}\left(ax^4 + x^2 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)/(a*x^4+x^2-b)/(a*x^6-b*x^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^4 + b)/((a*x^6 - b*x^2)^(1/4)*(a*x^4 + x^2 - b)), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{a\,x^4 + b}{\left(a\,x^4 + x^2 - b\right)\left(a\,x^6 - b\,x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b)/(a*x^4+x^2-b)/(a*x^6-b*x^2)^(1/4),x)
```

```
[Out] int((a*x^4+b)/(a*x^4+x^2-b)/(a*x^6-b*x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4 + b}{\left(ax^6 - bx^2\right)^{\frac{1}{4}}\left(ax^4 + x^2 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)/(a*x^4+x^2-b)/(a*x^6-b*x^2)^(1/4),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^4 + b)/((a*x^6 - b*x^2)^(1/4)*(a*x^4 + x^2 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x^4 + b}{\left(a\,x^6 - b\,x^2\right)^{1/4}\,\left(a\,x^4 + x^2 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^4)/((a*x^6 - b*x^2)^(1/4)*(a*x^4 - b + x^2)),x)
[Out] int((b + a*x^4)/((a*x^6 - b*x^2)^(1/4)*(a*x^4 - b + x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4 + b}{\sqrt[4]{x^2\left(ax^4 - b\right)}\left(ax^4 - b + x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b)/(a*x**4+x**2-b)/(a*x**6-b*x**2)**(1/4),x)
[Out] Integral((a*x**4 + b)/((x**2*(a*x**4 - b))**(1/4)*(a*x**4 - b + x**2)), x)
```

3.1459 $\int \frac{b+2ax^3}{(-b+x+ax^3)\sqrt[4]{-bx^3+ax^6}}\,dx$

Optimal. Leaf size=116

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^6-bx^3}}{\sqrt{ax^6-bx^3}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6-bx^3}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^6-bx^3}}\right)$$

Rubi [F] time = 1.77, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{b+2ax^3}{(-b+x+ax^3)\sqrt[4]{-bx^3+ax^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(b + 2*a*x^3)/((-b + x + a*x^3)*(-(b*x^3) + a*x^6)^(1/4)),x]
```

```
[Out] (8*x*(1 - (a*x^3)/b)^(1/4)*Hypergeometric2F1[1/12, 1/4, 13/12, (a*x^3)/b])/
(-(b*x^3) + a*x^6)^(1/4) - (12*b*x^(3/4)*(-b + a*x^3)^(1/4)*Defer[Subst][De
fer[Int][1/((b - x^4 - a*x^12)*(-b + a*x^12)^(1/4)), x], x, x^(1/4)])/(-(b*
x^3) + a*x^6)^(1/4) - (8*x^(3/4)*(-b + a*x^3)^(1/4)*Defer[Subst][Defer[Int]
[x^4/((-b + a*x^12)^(1/4)*(-b + x^4 + a*x^12)), x], x, x^(1/4)])/(-(b*x^3)
+ a*x^6)^(1/4)
```

Rubi steps

$$\int \frac{b+2ax^3}{(-b+x+ax^3)\sqrt[4]{-bx^3+ax^6}}\,dx=\frac{\left(x^{3/4}\sqrt[4]{-b+ax^3}\right)\int \frac{b+2ax^3}{x^{3/4}\sqrt[4]{-b+ax^3}\,(-b+x+ax^3)}\,dx}{\sqrt[4]{-bx^3+ax^6}}$$

$$=\frac{\left(4x^{3/4}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{b+2ax^{12}}{\sqrt[4]{-b+ax^{12}}\,(-b+x^4+ax^{12})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx^3+ax^6}}$$

$$=\frac{\left(4x^{3/4}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \left(\frac{2}{\sqrt[4]{-b+ax^{12}}}+\frac{3b-2x^4}{\sqrt[4]{-b+ax^{12}}\,(-b+x^4+ax^{12})}\right)dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx^3+ax^6}}$$

$$=\frac{\left(4x^{3/4}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{3b-2x^4}{\sqrt[4]{-b+ax^{12}}\,(-b+x^4+ax^{12})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx^3+ax^6}}+\frac{\left(8x^{3/4}\sqrt[4]{-b\right.}{}$$

$$=\frac{\left(4x^{3/4}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \left(-\frac{3b}{(b-x^4-ax^{12})\sqrt[4]{-b+ax^{12}}}-\frac{2x^4}{\sqrt[4]{-b+ax^{12}}\,(-b+x^4+ax^{12})}\right)\right.}{\sqrt[4]{-bx^3+ax^6}}$$

$$=\frac{8x\sqrt[4]{1-\frac{ax^3}{b}}\;{}_2F_1\left(\frac{1}{12},\frac{1}{4};\frac{13}{12};\frac{ax^3}{b}\right)}{\sqrt[4]{-bx^3+ax^6}}-\frac{\left(8x^{3/4}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{-b+ax^{12}}\,(-b+\right.}{\sqrt[4]{-bx^3+ax^6}}$$

Mathematica [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{b+2ax^3}{(-b+x+ax^3)\sqrt[4]{-bx^3+ax^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(b + 2*a*x^3)/((-b + x + a*x^3)*(-(b*x^3) + a*x^6)^(1/4)),x]
```

```
[Out] Integrate[(b + 2*a*x^3)/((-b + x + a*x^3)*(-(b*x^3) + a*x^6)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 2.93, size = 116, normalized size = 1.00

$$-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^6-bx^3}}{\sqrt{ax^6-bx^3}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6-bx^3}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^6-bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + 2*a*x^3)/((-b + x + a*x^3)*(-(b*x^3) + a*x^6)^(1/4)
),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(Sqrt[2]*x*(-(b*x^3) + a*x^6)^(1/4))/(-x^2 + Sqrt[-(b*x^3)
 + a*x^6])]) - Sqrt[2]*ArcTanh[(x^2/Sqrt[2] + Sqrt[-(b*x^3) + a*x^6]/Sqrt[2
])/(x*(-(b*x^3) + a*x^6)^(1/4))]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^3+b)/(a*x^3-b+x)/(a*x^6-b*x^3)^(1/4),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax^3+b}{\left(ax^6-bx^3\right)^{\frac{1}{4}}\left(ax^3-b+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^3+b)/(a*x^3-b+x)/(a*x^6-b*x^3)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((2*a*x^3 + b)/((a*x^6 - b*x^3)^(1/4)*(a*x^3 - b + x)), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{2a\,x^3+b}{\left(a\,x^3-b+x\right)\left(a\,x^6-b\,x^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x^3+b)/(a*x^3-b+x)/(a*x^6-b*x^3)^(1/4),x)
```

```
[Out] int((2*a*x^3+b)/(a*x^3-b+x)/(a*x^6-b*x^3)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax^3+b}{\left(ax^6-bx^3\right)^{\frac{1}{4}}\left(ax^3-b+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^3+b)/(a*x^3-b+x)/(a*x^6-b*x^3)^(1/4),x, algorithm="maxima"
)
```

```
[Out] integrate((2*a*x^3 + b)/((a*x^6 - b*x^3)^(1/4)*(a*x^3 - b + x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,a\,x^3+b}{{\left(a\,x^6-b\,x^3\right)}^{1/4}\,\left(a\,x^3+x-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*a*x^3)/((a*x^6 - b*x^3)^(1/4)*(x - b + a*x^3)),x)
```

```
[Out] int((b + 2*a*x^3)/((a*x^6 - b*x^3)^(1/4)*(x - b + a*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2ax^3+b}{\sqrt[4]{x^3\left(ax^3-b\right)}\left(ax^3-b+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x**3+b)/(a*x**3-b+x)/(a*x**6-b*x**3)**(1/4),x)
```

```
[Out] Integral((2*a*x**3 + b)/((x**3*(a*x**3 - b))**(1/4)*(a*x**3 - b + x)), x)
```

3.1460 $\int x^4 \sqrt[3]{-x+x^3}\,dx$

Optimal. Leaf size=117

$$\frac{5}{162}\log\left(\sqrt[3]{x^3-x}-x\right)+\frac{5\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-x}+x}\right)}{54\sqrt{3}}+\frac{1}{108}\sqrt[3]{x^3-x}\left(18x^5-3x^3-5x\right)-\frac{5}{324}\log\left(\sqrt[3]{x^3-x}\,x+\left(x^3-x\right)^{2/}\right.$$

Rubi [A] time = 0.23, antiderivative size = 222, normalized size of antiderivative = 1.90, number of steps used = 13, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {2021, 2024, 2032, 329, 275, 331, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{36}\sqrt[3]{x^3-x}\,x^3-\frac{5}{108}\sqrt[3]{x^3-x}\,x+\frac{1}{6}\sqrt[3]{x^3-x}\,x^5+\frac{5\left(x^2-1\right)^{2/3}x^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2-1}}\right)}{162\left(x^3-x\right)^{2/3}}-\frac{5\left(x^2-1\right)^{2/3}x^{2/3}\log\left(\frac{x^{4/3}}{\left(x^2-1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2-1}}+1\right)}{324\left(x^3-x\right)^{2/3}}+\frac{5\left(x^2-1\right)^{2/3}x^{2/3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1}{\sqrt{3}}\right)}{54\sqrt{3}\left(x^3-x\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(-x + x^3)^(1/3),x]
```

```
[Out] (-5*x*(-x + x^3)^(1/3))/108 - (x^3*(-x + x^3)^(1/3))/36 + (x^5*(-x + x^3)^(
1/3))/6 + (5*x^(2/3)*(-1 + x^2)^(2/3)*ArcTan[(1 + (2*x^(2/3))/(-1 + x^2)^(1
/3))/Sqrt[3]])/(54*Sqrt[3]*(-x + x^3)^(2/3)) + (5*x^(2/3)*(-1 + x^2)^(2/3)*
Log[1 - x^(2/3)/(-1 + x^2)^(1/3)])/(162*(-x + x^3)^(2/3)) - (5*x^(2/3)*(-1
+ x^2)^(2/3)*Log[1 + x^(4/3)/(-1 + x^2)^(2/3) + x^(2/3)/(-1 + x^2)^(1/3)])/
(324*(-x + x^3)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^4\sqrt[3]{-x+x^3}\,dx=\frac{1}{6}x^5\sqrt[3]{-x+x^3}-\frac{1}{9}\int\frac{x^5}{\left(-x+x^3\right)^{2/3}}\,dx$$

$$=-\frac{1}{36}x^3\sqrt[3]{-x+x^3}+\frac{1}{6}x^5\sqrt[3]{-x+x^3}-\frac{5}{54}\int\frac{x^3}{\left(-x+x^3\right)^{2/3}}\,dx$$

$$=-\frac{5}{108}x\sqrt[3]{-x+x^3}-\frac{1}{36}x^3\sqrt[3]{-x+x^3}+\frac{1}{6}x^5\sqrt[3]{-x+x^3}-\frac{5}{81}\int\frac{x}{\left(-x+x^3\right)^{2/3}}\,dx$$

$$=-\frac{5}{108}x\sqrt[3]{-x+x^3}-\frac{1}{36}x^3\sqrt[3]{-x+x^3}+\frac{1}{6}x^5\sqrt[3]{-x+x^3}-\frac{\left(5x^{2/3}\left(-1+x^2\right)^{2/3}\right)\int\frac{\sqrt[3]{x}}{\left(-1+x^2\right)^{2/3}}\,dx}{81\left(-x+x^3\right)^{2/3}}$$

$$=-\frac{5}{108}x\sqrt[3]{-x+x^3}-\frac{1}{36}x^3\sqrt[3]{-x+x^3}+\frac{1}{6}x^5\sqrt[3]{-x+x^3}-\frac{\left(5x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int\frac{x}{(-1+\cdots}\right.}{27\left(-x+x^3\right)^{2/3}}$$

$$=-\frac{5}{108}x\sqrt[3]{-x+x^3}-\frac{1}{36}x^3\sqrt[3]{-x+x^3}+\frac{1}{6}x^5\sqrt[3]{-x+x^3}-\frac{\left(5x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int\frac{\cdots}{(-1+\cdots}\right.}{54\left(-x+x^3\right)^{2/3}}$$

$$=-\frac{5}{108}x\sqrt[3]{-x+x^3}-\frac{1}{36}x^3\sqrt[3]{-x+x^3}+\frac{1}{6}x^5\sqrt[3]{-x+x^3}-\frac{\left(5x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int\frac{x}{1-x^3}\cdots\right.}{54\left(-x+x^3\right)^{2/3}}$$

$$=-\frac{5}{108}x\sqrt[3]{-x+x^3}-\frac{1}{36}x^3\sqrt[3]{-x+x^3}+\frac{1}{6}x^5\sqrt[3]{-x+x^3}-\frac{\left(5x^{2/3}\left(-1+x^2\right)^{2/3}\right)\text{Subst}\left(\int\frac{1}{1-x}\cdots\right.}{162\left(-x+x^3\right)^{2/3}}$$

$$=-\frac{5}{108}x\sqrt[3]{-x+x^3}-\frac{1}{36}x^3\sqrt[3]{-x+x^3}+\frac{1}{6}x^5\sqrt[3]{-x+x^3}+\frac{5x^{2/3}\left(-1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\cdots\right.}{162\left(-x+x^3\right)^{2/3}}$$

$$=-\frac{5}{108}x\sqrt[3]{-x+x^3}-\frac{1}{36}x^3\sqrt[3]{-x+x^3}+\frac{1}{6}x^5\sqrt[3]{-x+x^3}+\frac{5x^{2/3}\left(-1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\cdots\right.}{162\left(-x+x^3\right)^{2/3}}$$

$$=-\frac{5}{108}x\sqrt[3]{-x+x^3}-\frac{1}{36}x^3\sqrt[3]{-x+x^3}+\frac{1}{6}x^5\sqrt[3]{-x+x^3}+\frac{5x^{2/3}\left(-1+x^2\right)^{2/3}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}}{\sqrt{3}}\cdots\right.}{54\sqrt{3}\left(-x+x^3\right)^{2/3}}$$

Mathematica [C] time = 0.03, size = 67, normalized size = 0.57

$$\frac{x\sqrt[3]{x\left(x^2-1\right)}\left(5\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^2\right)+\sqrt[3]{1-x^2}\left(6x^4-x^2-5\right)\right)}{36\sqrt[3]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(-x + x^3)^(1/3),x]
```

```
[Out] (x*(x*(-1 + x^2))^(1/3)*((1 - x^2)^(1/3)*(-5 - x^2 + 6*x^4) + 5*Hypergeomet
ric2F1[-1/3, 2/3, 5/3, x^2]))/(36*(1 - x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.33, size = 117, normalized size = 1.00

$$\frac{5}{162}\log\left(\sqrt[3]{x^3-x}-x\right)+\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x}+x}\right)}{54\sqrt{3}}+\frac{1}{108}\sqrt[3]{x^3-x}\left(18x^5-3x^3-5x\right)-\frac{5}{324}\log\left(\sqrt[3]{x^3-x}\,x+\left(x^3-x\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(-x + x^3)^(1/3),x]
```

```
[Out] ((-x + x^3)^(1/3)*(-5*x - 3*x^3 + 18*x^5))/108 + (5*ArcTan[(Sqrt[3]*x)/(x +
 2*(-x + x^3)^(1/3))])/(54*Sqrt[3]) + (5*Log[-x + (-x + x^3)^(1/3)])/162 -
(5*Log[x^2 + x*(-x + x^3)^(1/3) + (-x + x^3)^(2/3)])/324
```

fricas [A] time = 0.82, size = 112, normalized size = 0.96

$$\frac{5}{162}\sqrt{3}\arctan\left(-\frac{44032959556\sqrt{3}\left(x^3-x\right)^{\frac{1}{3}}x+\sqrt{3}\left(16754327161x^2-2707204793\right)-10524305234\sqrt{3}\left(x^3-x\right)^{\frac{2}{3}}}{81835897185x^2-1102302937}\right)+\frac{1}{108}\left(18x^5-3x^3-5x\right)\left(x^3-x\right)^{\frac{1}{3}}+\frac{5}{324}\log\left(-3\left(x^3-x\right)^{\frac{1}{3}}x+3\left(x^3-x\right)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^3-x)^(1/3),x, algorithm="fricas")
```

```
[Out] 5/162*sqrt(3)*arctan(-(44032959556*sqrt(3)*(x^3 - x)^(1/3)*x + sqrt(3)*(167
54327161*x^2 - 2707204793) - 10524305234*sqrt(3)*(x^3 - x)^(2/3))/(81835897
185*x^2 - 1102302937)) + 1/108*(18*x^5 - 3*x^3 - 5*x)*(x^3 - x)^(1/3) + 5/3
24*log(-3*(x^3 - x)^(1/3)*x + 3*(x^3 - x)^(2/3) + 1)
```

giac [A] time = 0.18, size = 109, normalized size = 0.93

$$-\frac{1}{108}\left(5\left(\frac{1}{x^2}-1\right)^2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}-13\left(-\frac{1}{x^2}+1\right)^{\frac{4}{3}}-10\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right)x^6-\frac{5}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{5}{324}\log\left(\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)+\frac{5}{162}\log\left(\left|\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^3-x)^(1/3),x, algorithm="giac")
```

```
[Out] -1/108*(5*(1/x^2 - 1)^2*(-1/x^2 + 1)^(1/3) - 13*(-1/x^2 + 1)^(4/3) - 10*(-1
/x^2 + 1)^(1/3))*x^6 - 5/162*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-1/x^2 + 1)^(1/
3) + 1)) - 5/324*log((-1/x^2 + 1)^(2/3) + (-1/x^2 + 1)^(1/3) + 1) + 5/162*l
og(abs((-1/x^2 + 1)^(1/3) - 1))
```

maple [C] time = 1.95, size = 796, normalized size = 6.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^3-x)^(1/3),x)
```

```
[Out] 1/108*x*(18*x^4-3*x^2-5)*(x*(x^2-1))^(1/3)+(5/972*RootOf(_Z^2+6*_Z+36)*ln(-
(47*RootOf(_Z^2+6*_Z+36)^2*x^4+3207*RootOf(_Z^2+6*_Z+36)*x^4+2925*(x^6-2*x^
4+x^2)^(1/3)*RootOf(_Z^2+6*_Z+36)*x^2-235*RootOf(_Z^2+6*_Z+36)^2*x^2+6930*x
^4+2925*RootOf(_Z^2+6*_Z+36)*(x^6-2*x^4+x^2)^(2/3)+5238*(x^6-2*x^4+x^2)^(1/
3)*x^2-5601*RootOf(_Z^2+6*_Z+36)*x^2+5238*(x^6-2*x^4+x^2)^(2/3)-2925*RootOf
(_Z^2+6*_Z+36)*(x^6-2*x^4+x^2)^(1/3)+188*RootOf(_Z^2+6*_Z+36)^2-11340*x^2-5
238*(x^6-2*x^4+x^2)^(1/3)+2394*RootOf(_Z^2+6*_Z+36)+4410)/(-1+x)/(1+x))-5/9
72*ln((-47*RootOf(_Z^2+6*_Z+36)^2*x^4+2643*RootOf(_Z^2+6*_Z+36)*x^4+2925*(x
^6-2*x^4+x^2)^(1/3)*RootOf(_Z^2+6*_Z+36)*x^2+235*RootOf(_Z^2+6*_Z+36)^2*x^2
+10620*x^4+2925*RootOf(_Z^2+6*_Z+36)*(x^6-2*x^4+x^2)^(2/3)+12312*(x^6-2*x^4
+x^2)^(1/3)*x^2-2781*RootOf(_Z^2+6*_Z+36)*x^2+12312*(x^6-2*x^4+x^2)^(2/3)-2
925*RootOf(_Z^2+6*_Z+36)*(x^6-2*x^4+x^2)^(1/3)-188*RootOf(_Z^2+6*_Z+36)^2-1
3806*x^2-12312*(x^6-2*x^4+x^2)^(1/3)+138*RootOf(_Z^2+6*_Z+36)+3186)/(-1+x)/
(1+x))*RootOf(_Z^2+6*_Z+36)-5/162*ln((-47*RootOf(_Z^2+6*_Z+36)^2*x^4+2643*R
ootOf(_Z^2+6*_Z+36)*x^4+2925*(x^6-2*x^4+x^2)^(1/3)*RootOf(_Z^2+6*_Z+36)*x^2
```

```
+235*RootOf(_Z^2+6*_Z+36)^2*x^2+10620*x^4+2925*RootOf(_Z^2+6*_Z+36)*(x^6-2*
x^4+x^2)^(2/3)+12312*(x^6-2*x^4+x^2)^(1/3)*x^2-2781*RootOf(_Z^2+6*_Z+36)*x^
2+12312*(x^6-2*x^4+x^2)^(2/3)-2925*RootOf(_Z^2+6*_Z+36)*(x^6-2*x^4+x^2)^(1/
3)-188*RootOf(_Z^2+6*_Z+36)^2-13806*x^2-12312*(x^6-2*x^4+x^2)^(1/3)+138*Roo
tOf(_Z^2+6*_Z+36)+3186)/(-1+x)/(1+x)))/x*(x*(x^2-1))^(1/3)*(x^2*(x^2-1)^2)^
(1/3)/(x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^3-x\right)^{\frac{1}{3}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^3-x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - x)^(1/3)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(x^3-x\right)^{1/3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^3 - x)^(1/3),x)
```

```
[Out] int(x^4*(x^3 - x)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{4} \sqrt[3]{x \left(x - 1\right) \left(x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**3-x)**(1/3),x)
```

```
[Out] Integral(x**4*(x*(x - 1)*(x + 1))**(1/3), x)
```

3.1461 $\int \frac{x}{(-b+ax^2)\sqrt{bx+ax^3}}\,dx$

Optimal. Leaf size=117

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}$$

Rubi [A] time = 0.46, antiderivative size = 163, normalized size of antiderivative = 1.39, number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2042, 466, 490, 1211, 220, 1699, 205, 208}

$$\frac{\sqrt{x}\sqrt{ax^2+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2+b}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}\sqrt{ax^3+bx}}-\frac{\sqrt{x}\sqrt{ax^2+b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2+b}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}\sqrt{ax^3+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((-b + a*x^2)*Sqrt[b*x + a*x^3]),x]
```

```
[Out] (Sqrt[x]*Sqrt[b + a*x^2]*ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/Sqrt[b +
a*x^2]])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*Sqrt[b*x + a*x^3]) - (Sqrt[x]*Sqrt[b +
a*x^2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/Sqrt[b + a*x^2]])/(2*Sqrt[
2]*a^(3/4)*b^(3/4)*Sqrt[b*x + a*x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] && !IntegerQ[p]
&& NeQ[b*c - a*d, 0] && !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(-b+ax^2\right)\sqrt{bx+ax^3}}\,dx &= \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \frac{\sqrt{x}}{\left(-b+ax^2\right)\sqrt{b+ax^2}}\,dx}{\sqrt{bx+ax^3}}\\
&= \frac{\left(2\sqrt{x}\sqrt{b+ax^2}\right)\text{Subst}\left(\int \frac{x^2}{\left(-b+ax^4\right)\sqrt{b+ax^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{bx+ax^3}}\\
&= -\frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{b}-\sqrt{a}\,x^2\right)\sqrt{b+ax^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{a}\sqrt{bx+ax^3}} + \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\text{Subs}\cdots}{\cdots}\\
&= \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{a}\,x^2}{\left(\sqrt{b}+\sqrt{a}\,x^2\right)\sqrt{b+ax^4}}\,dx, x, \sqrt{x}\right)}{2\sqrt{a}\sqrt{b}\sqrt{bx+ax^3}} - \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\text{Subst}\cdots}{2\sqrt{a}\cdots}\\
&= -\frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-2\sqrt{a}\,bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b+ax^2}}\right)}{2\sqrt{a}\sqrt{bx+ax^3}} + \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\text{Subst}\left(\cdots\right.}{2\sqrt{a}\cdots}\\
&= \frac{\sqrt{x}\sqrt{b+ax^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{b+ax^2}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}\sqrt{bx+ax^3}} - \frac{\sqrt{x}\sqrt{b+ax^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{b+ax^2}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}\sqrt{bx+ax^3}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 66, normalized size = 0.56

$$-\frac{2x^2\sqrt{\frac{ax^2+b}{b}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{ax^2}{b},\frac{ax^2}{b}\right)}{3b\sqrt{x\left(ax^2+b\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((-b + a*x^2)*Sqrt[b*x + a*x^3]),x]
```

```
[Out] (-2*x^2*Sqrt[(b + a*x^2)/b]*AppellF1[3/4, 1/2, 1, 7/4, -((a*x^2)/b), (a*x^2
)/b])/(3*b*Sqrt[x*(b + a*x^2)])
```

IntegrateAlgebraic [A] time = 0.37, size = 117, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((-b + a*x^2)*Sqrt[b*x + a*x^3]),x]
```

```
[Out] ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3])/(b + a*x^2)]/(2*Sqrt[2]*
a^(3/4)*b^(3/4)) - ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3])/(b +
 a*x^2)]/(2*Sqrt[2]*a^(3/4)*b^(3/4))
```

fricas [B] time = 0.56, size = 344, normalized size = 2.94

$$\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{ax^3+bx}\,ab\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}}{ax^2+b}\right)-\frac{1}{8}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\log\left(\frac{a^2x^4+6\,abx^2+b^2+4\left(4\left(\frac{1}{4}\right)^{\frac{3}{4}}a^3b^3x\left(\frac{1}{a^3b^3}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(a^2bx^2+ab^2\right)\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\right)\sqrt{ax^3+bx}+4\left(a^3b^2x^3+a^2b^3x\right)\sqrt{\frac{1}{a^3b^3}}}{a^2x^4-2\,abx^2+b^2}\right)+\frac{1}{8}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\log\left(\frac{a^2x^4+6\,abx^2+b^2-4\left(4\left(\frac{1}{4}\right)^{\frac{3}{4}}a^3b^3x\left(\frac{1}{a^3b^3}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(a^2bx^2+ab^2\right)\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\right)\sqrt{ax^3+bx}+4\left(a^3b^2x^3+a^2b^3x\right)\sqrt{\frac{1}{a^3b^3}}}{a^2x^4-2\,abx^2+b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^2-b)/(a*x^3+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(1/4)^(1/4)*(1/(a^3*b^3))^(1/4)*arctan(2*(1/4)^(1/4)*sqrt(a*x^3 + b*x)*
a*b*(1/(a^3*b^3))^(1/4)/(a*x^2 + b)) - 1/8*(1/4)^(1/4)*(1/(a^3*b^3))^(1/4)*
log((a^2*x^4 + 6*a*b*x^2 + b^2 + 4*(4*(1/4)^(3/4)*a^3*b^3*x*(1/(a^3*b^3))^(
3/4) + (1/4)^(1/4)*(a^2*b*x^2 + a*b^2)*(1/(a^3*b^3))^(1/4))*sqrt(a*x^3 + b*
x) + 4*(a^3*b^2*x^3 + a^2*b^3*x)*sqrt(1/(a^3*b^3)))/(a^2*x^4 - 2*a*b*x^2 +
b^2)) + 1/8*(1/4)^(1/4)*(1/(a^3*b^3))^(1/4)*log((a^2*x^4 + 6*a*b*x^2 + b^2
- 4*(4*(1/4)^(3/4)*a^3*b^3*x*(1/(a^3*b^3))^(3/4) + (1/4)^(1/4)*(a^2*b*x^2 +
 a*b^2)*(1/(a^3*b^3))^(1/4))*sqrt(a*x^3 + b*x) + 4*(a^3*b^2*x^3 + a^2*b^3*x
)*sqrt(1/(a^3*b^3)))/(a^2*x^4 - 2*a*b*x^2 + b^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{ax^3+bx}\left(ax^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^2-b)/(a*x^3+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(a*x^3 + b*x)*(a*x^2 - b)), x)
```

maple [C] time = 0.04, size = 296, normalized size = 2.53

$$\frac{\sqrt{-ab}\,\sqrt{\frac{xa}{\sqrt{-ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{2a^2\sqrt{a\,x^3+bx}\,\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)}+\frac{\sqrt{-ab}\,\sqrt{\frac{xa}{\sqrt{-ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{2a^2\sqrt{a\,x^3+bx}\,\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2-b)/(a*x^3+b*x)^(1/2),x)
```

```
[Out] 1/2/a^2*(-a*b)^(1/2)*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/
2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)
^(1/2))*EllipticPi(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),-1/a*(-a*b)^
(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2)),1/2*2^(1/2))+1/2/a^2*(-a*b)^(1/2)
*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2
))^(1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(-a*b)^(1/2)+1/a*(a*b)^(1/2))*EllipticPi((
```

```
(x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2)/(-1/a*(-a*b)^(
1/2)+1/a*(a*b)^(1/2)),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{ax^3+bx}\left(ax^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^2-b)/(a*x^3+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(a*x^3 + b*x)*(a*x^2 - b)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((b*x + a*x^3)^(1/2)*(b - a*x^2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x\left(ax^2+b\right)}\left(ax^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x**2-b)/(a*x**3+b*x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(x*(a*x**2 + b))*(a*x**2 - b)), x)
```

3.1462 $\int \frac{\sqrt{bx+ax^3}}{-b^2+a^2x^4}\,dx$

Optimal. Leaf size=117

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}$$

Rubi [A] time = 0.38, antiderivative size = 163, normalized size of antiderivative = 1.39, number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2056, 1254, 466, 490, 1211, 220, 1699, 205, 208}

$$\frac{\sqrt{ax^3+bx}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2+b}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}\sqrt{x}\sqrt{ax^2+b}}-\frac{\sqrt{ax^3+bx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2+b}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}\sqrt{x}\sqrt{ax^2+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + a*x^3]/(-b^2 + a^2*x^4),x]
```

```
[Out] (Sqrt[b*x + a*x^3]*ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/Sqrt[b + a*x^2]
])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*Sqrt[x]*Sqrt[b + a*x^2]) - (Sqrt[b*x + a*x^3]
*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/Sqrt[b + a*x^2]])/(2*Sqrt[2]*a^(
3/4)*b^(3/4)*Sqrt[x]*Sqrt[b + a*x^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1254

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e)^p, x] /
; FreeQ[{a, c, d, e, f, q, m, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p
]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{bx+ax^3}}{-b^2+a^2x^4}\,dx &= \frac{\sqrt{bx+ax^3}\int \frac{\sqrt{x}\sqrt{b+ax^2}}{-b^2+a^2x^4}\,dx}{\sqrt{x}\sqrt{b+ax^2}}\\
&= \frac{\sqrt{bx+ax^3}\int \frac{\sqrt{x}}{\left(-b+ax^2\right)\sqrt{b+ax^2}}\,dx}{\sqrt{x}\sqrt{b+ax^2}}\\
&= \frac{\left(2\sqrt{bx+ax^3}\right)\operatorname{Subst}\left(\int \frac{x^2}{\left(-b+ax^4\right)\sqrt{b+ax^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{b+ax^2}}\\
&= -\frac{\sqrt{bx+ax^3}\operatorname{Subst}\left(\int \frac{1}{\left(\sqrt{b}-\sqrt{a}\,x^2\right)\sqrt{b+ax^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{a}\sqrt{x}\sqrt{b+ax^2}}+\frac{\sqrt{bx+ax^3}\operatorname{Subst}\left(\int \frac{1}{\left(\sqrt{b}+\sqrt{a}\,x^2\right)\sqrt{b+ax}}\right.}{\sqrt{a}\sqrt{x}\sqrt{b+ax^2}}\\
&= \frac{\sqrt{bx+ax^3}\operatorname{Subst}\left(\int \frac{\sqrt{b}-\sqrt{a}\,x^2}{\left(\sqrt{b}+\sqrt{a}\,x^2\right)\sqrt{b+ax^4}}\,dx,x,\sqrt{x}\right)}{2\sqrt{a}\sqrt{b}\sqrt{x}\sqrt{b+ax^2}}-\frac{\sqrt{bx+ax^3}\operatorname{Subst}\left(\int \frac{\sqrt{b}+\sqrt{a}\,x^2}{\left(\sqrt{b}-\sqrt{a}\,x^2\right)\sqrt{b+ax^4}}\right.}{2\sqrt{a}\sqrt{b}\sqrt{x}\sqrt{b+ax^2}}\\
&= -\frac{\sqrt{bx+ax^3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}-2\sqrt{a}\,bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{b+ax^2}}\right)}{2\sqrt{a}\sqrt{x}\sqrt{b+ax^2}}+\frac{\sqrt{bx+ax^3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}+2\sqrt{a}\,bx^2}\,dx,x,\right.}{2\sqrt{a}\sqrt{x}\sqrt{b+ax^2}}\\
&= \frac{\sqrt{bx+ax^3}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{b+ax^2}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}\sqrt{x}\sqrt{b+ax^2}}-\frac{\sqrt{bx+ax^3}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{b+ax^2}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}\sqrt{x}\sqrt{b+ax^2}}
\end{aligned}
$$

Mathematica [C] time = 0.91, size = 111, normalized size = 0.95

$$\frac{\sqrt{\frac{i\sqrt{a}\,x}{\sqrt{b}}}\sqrt{\frac{ax^2}{b}+1}\left(\Pi\left(i;i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{a}\,x}{\sqrt{b}}}\right)\middle|-1\right)-\Pi\left(-i;i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{a}\,x}{\sqrt{b}}}\right)\middle|-1\right)\right)}{a\sqrt{x\left(ax^2+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + a*x^3]/(-b^2 + a^2*x^4),x]
```

```
[Out] (Sqrt[(I*Sqrt[a]*x)/Sqrt[b]]*Sqrt[1 + (a*x^2)/b]*(-EllipticPi[-I, I*ArcSinh
[Sqrt[(I*Sqrt[a]*x)/Sqrt[b]]], -1] + EllipticPi[I, I*ArcSinh[Sqrt[(I*Sqrt[a
]*x)/Sqrt[b]]], -1]))/(a*Sqrt[x*(b + a*x^2)])
```

IntegrateAlgebraic [A] time = 0.35, size = 117, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b*x + a*x^3]/(-b^2 + a^2*x^4),x]
```

```
[Out] ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3])/(b + a*x^2)]/(2*Sqrt[2]*
a^(3/4)*b^(3/4)) - ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3])/(b +
 a*x^2)]/(2*Sqrt[2]*a^(3/4)*b^(3/4))
```

fricas [B] time = 0.53, size = 344, normalized size = 2.94

$$\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{ax^3+bx}\,ab\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}}{ax^2+b}\right)-\frac{1}{8}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\log\left(\frac{a^2x^4+6abx^2+b^2+4\left(4\left(\frac{1}{4}\right)^{\frac{3}{4}}a^3b^3x\left(\frac{1}{a^3b^3}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(a^2bx^2+ab^2\right)\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\right)\sqrt{ax^3+bx}+4\left(a^3b^2x^3+a^2b^3x\right)\sqrt{\frac{1}{a^3b^3}}}{a^2x^4-2abx^2+b^2}\right)+\frac{1}{8}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\log\left(\frac{a^2x^4+6abx^2+b^2-4\left(4\left(\frac{1}{4}\right)^{\frac{3}{4}}a^3b^3x\left(\frac{1}{a^3b^3}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(a^2bx^2+ab^2\right)\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\right)\sqrt{ax^3+bx}+4\left(a^3b^2x^3+a^2b^3x\right)\sqrt{\frac{1}{a^3b^3}}}{a^2x^4-2abx^2+b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x)^(1/2)/(a^2*x^4-b^2),x, algorithm="fricas")
```

```
[Out] 1/2*(1/4)^(1/4)*(1/(a^3*b^3))^(1/4)*arctan(2*(1/4)^(1/4)*sqrt(a*x^3 + b*x)*
a*b*(1/(a^3*b^3))^(1/4)/(a*x^2 + b)) - 1/8*(1/4)^(1/4)*(1/(a^3*b^3))^(1/4)*
log((a^2*x^4 + 6*a*b*x^2 + b^2 + 4*(4*(1/4)^(3/4)*a^3*b^3*x*(1/(a^3*b^3))^(
3/4) + (1/4)^(1/4)*(a^2*b*x^2 + a*b^2)*(1/(a^3*b^3))^(1/4))*sqrt(a*x^3 + b*
x) + 4*(a^3*b^2*x^3 + a^2*b^3*x)*sqrt(1/(a^3*b^3)))/(a^2*x^4 - 2*a*b*x^2 +
b^2)) + 1/8*(1/4)^(1/4)*(1/(a^3*b^3))^(1/4)*log((a^2*x^4 + 6*a*b*x^2 + b^2
- 4*(4*(1/4)^(3/4)*a^3*b^3*x*(1/(a^3*b^3))^(3/4) + (1/4)^(1/4)*(a^2*b*x^2 +
 a*b^2)*(1/(a^3*b^3))^(1/4))*sqrt(a*x^3 + b*x) + 4*(a^3*b^2*x^3 + a^2*b^3*x
)*sqrt(1/(a^3*b^3)))/(a^2*x^4 - 2*a*b*x^2 + b^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3+bx}}{a^2x^4-b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x)^(1/2)/(a^2*x^4-b^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^3 + b*x)/(a^2*x^4 - b^2), x)
```

maple [C] time = 0.06, size = 631, normalized size = 5.39

$$\frac{\frac{2b\sqrt{\frac{xa}{\sqrt{-ab}}+1}\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{ax^3+bx}}-\frac{b\sqrt{\frac{xa}{\sqrt{-ab}}+1}\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{ax^3+bx}}+\frac{b\sqrt{-ab}\sqrt{\frac{xa}{\sqrt{-ab}}+1}\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{a^2\sqrt{ax^3+bx}\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)}+\frac{b\sqrt{-ab}\sqrt{\frac{xa}{\sqrt{-ab}}+1}\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{a^2\sqrt{ax^3+bx}\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)}}{2b}-\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{xa}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a}+\frac{\sqrt{-ab}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a}\right)}{2ba\sqrt{ax^3+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b*x)^(1/2)/(a^2*x^4-b^2),x)
```

```
[Out] 1/2/b*(2/a*b*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x*a
/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)*EllipticE(((x+1/a*(-a*b)^(1/2))*a/(-
a*b)^(1/2))^(1/2),1/2*2^(1/2))-1/a*b*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a
*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)*EllipticF(((
x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+b/a^2*(-a*b)^(1/2)*(
x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))
^(1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2))*EllipticPi(((x
+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2)/(-1/a*(-a*b)^(1/
2)-1/a*(a*b)^(1/2)),1/2*2^(1/2))+b/a^2*(-a*b)^(1/2)*(x*a/(-a*b)^(1/2)+1)^(1
/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/
2)/(-1/a*(-a*b)^(1/2)+1/a*(a*b)^(1/2))*EllipticPi(((x+1/a*(-a*b)^(1/2))*a/(
-a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2)/(-1/a*(-a*b)^(1/2)+1/a*(a*b)^(1/2)),1/
2*2^(1/2)))-1/2/b/a*(-a*b)^(1/2)*((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2
)*(-2*(x-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/
(a*x^3+b*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x+1/a*(-a*b)^(1/2))*a/(-a*
b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x+1/a*(-a*b)^(1/2
))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3+bx}}{a^2x^4-b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x)^(1/2)/(a^2*x^4-b^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^3 + b*x)/(a^2*x^4 - b^2), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b*x + a*x^3)^(1/2)/(b^2 - a^2*x^4),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(ax^2+b\right)}}{\left(ax^2-b\right)\left(ax^2+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b*x)**(1/2)/(a**2*x**4-b**2),x)
```

```
[Out] Integral(sqrt(x*(a*x**2 + b))/((a*x**2 - b)*(a*x**2 + b)), x)
```

3.1463 $\int \frac{x^{19}}{\sqrt[3]{-1+x^6}}\,dx$

Optimal. Leaf size=117

$$-\frac{7}{243}\log\left(\sqrt[3]{x^6-1}-x^2\right)+\frac{7\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{81\sqrt{3}}+\frac{7}{486}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)+\frac{1}{324}\left(x^6-1\right)^{2/3}\left(18x^{14}\right.$$

Rubi [A] time = 0.05, antiderivative size = 101, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {275, 321, 239}

$$\frac{1}{18}\left(x^6-1\right)^{2/3}x^{14}+\frac{7}{108}\left(x^6-1\right)^{2/3}x^8+\frac{7}{81}\left(x^6-1\right)^{2/3}x^2-\frac{7}{162}\log\left(x^2-\sqrt[3]{x^6-1}\right)+\frac{7\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6-1}}+1}{\sqrt{3}}\right)}{81\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/(-1 + x^6)^(1/3),x]
```

```
[Out] (7*x^2*(-1 + x^6)^(2/3))/81 + (7*x^8*(-1 + x^6)^(2/3))/108 + (x^14*(-1 + x^
6)^(2/3))/18 + (7*ArcTan[(1 + (2*x^2)/(-1 + x^6)^(1/3))/Sqrt[3]])/(81*Sqrt[
3]) - (7*Log[x^2 - (-1 + x^6)^(1/3)])/162
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{19}}{\sqrt[3]{-1+x^6}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^9}{\sqrt[3]{-1+x^3}}\,dx,x,x^2\right)$$
$$= \frac{1}{18}x^{14}\left(-1+x^6\right)^{2/3}+\frac{7}{18}\text{Subst}\left(\int \frac{x^6}{\sqrt[3]{-1+x^3}}\,dx,x,x^2\right)$$
$$= \frac{7}{108}x^8\left(-1+x^6\right)^{2/3}+\frac{1}{18}x^{14}\left(-1+x^6\right)^{2/3}+\frac{7}{27}\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{-1+x^3}}\,dx,x,x^2\right)$$
$$= \frac{7}{81}x^2\left(-1+x^6\right)^{2/3}+\frac{7}{108}x^8\left(-1+x^6\right)^{2/3}+\frac{1}{18}x^{14}\left(-1+x^6\right)^{2/3}+\frac{7}{81}\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x^3}}\,dx\right.$$
$$= \frac{7}{81}x^2\left(-1+x^6\right)^{2/3}+\frac{7}{108}x^8\left(-1+x^6\right)^{2/3}+\frac{1}{18}x^{14}\left(-1+x^6\right)^{2/3}+\frac{7\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{-1+x^6}}}{\sqrt{3}}\right)}{81\sqrt{3}}-\frac{7}{162}\text{l}$$

Mathematica [A] time = 0.06, size = 127, normalized size = 1.09

$$\frac{1}{972}\left(54\left(x^6-1\right)^{2/3}x^{14}+63\left(x^6-1\right)^{2/3}x^8+84\left(x^6-1\right)^{2/3}x^2-28\log\left(1-\frac{x^2}{\sqrt[3]{x^6-1}}\right)+28\sqrt{3}\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6-1}}+1}{\sqrt{3}}\right)+14\log\left(\frac{x^4}{\left(x^6-1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6-1}}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/(-1 + x^6)^(1/3),x]
```

```
[Out] (84*x^2*(-1 + x^6)^(2/3) + 63*x^8*(-1 + x^6)^(2/3) + 54*x^14*(-1 + x^6)^(2/
3) + 28*Sqrt[3]*ArcTan[(1 + (2*x^2)/(-1 + x^6)^(1/3))/Sqrt[3]] - 28*Log[1 -
 x^2/(-1 + x^6)^(1/3)] + 14*Log[1 + x^4/(-1 + x^6)^(2/3) + x^2/(-1 + x^6)^(
1/3)])/972
```

IntegrateAlgebraic [A] time = 7.48, size = 117, normalized size = 1.00

$$-\frac{7}{243}\log\left(\sqrt[3]{x^6-1}-x^2\right)+\frac{7\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{81\sqrt{3}}+\frac{7}{486}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}x^2\right)+\frac{1}{324}\left(x^6-1\right)^{2/3}\left(18x^{14}+21x^8+28x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^19/(-1 + x^6)^(1/3),x]
```

```
[Out] ((-1 + x^6)^(2/3)*(28*x^2 + 21*x^8 + 18*x^14))/324 + (7*ArcTan[(Sqrt[3]*x^2
)/(x^2 + 2*(-1 + x^6)^(1/3))])/(81*Sqrt[3]) - (7*Log[-x^2 + (-1 + x^6)^(1/3
)])/243 + (7*Log[x^4 + x^2*(-1 + x^6)^(1/3) + (-1 + x^6)^(2/3)])/486
```

fricas [A] time = 0.42, size = 107, normalized size = 0.91

$$\frac{1}{324}\left(18x^{14}+21x^8+28x^2\right)\left(x^6-1\right)^{\frac{2}{3}}-\frac{7}{243}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}}{3x^2}\right)-\frac{7}{243}\log\left(-\frac{x^2-\left(x^6-1\right)^{\frac{1}{3}}}{x^2}\right)+\frac{7}{486}\log\left(\frac{x^4+\left(x^6-1\right)^{\frac{1}{3}}x^2+\left(x^6-1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(x^6-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/324*(18*x^14 + 21*x^8 + 28*x^2)*(x^6 - 1)^(2/3) - 7/243*sqrt(3)*arctan(1/
3*(sqrt(3)*x^2 + 2*sqrt(3)*(x^6 - 1)^(1/3))/x^2) - 7/243*log(-(x^2 - (x^6 -
 1)^(1/3))/x^2) + 7/486*log((x^4 + (x^6 - 1)^(1/3)*x^2 + (x^6 - 1)^(2/3))/x
^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{\left(x^6-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(x^6-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^19/(x^6 - 1)^(1/3), x)
```

maple [C] time = 0.38, size = 58, normalized size = 0.50

$$\frac{x^{2}\left(18x^{12}+21x^{6}+28\right)\left(x^{6}-1\right)^{\frac{2}{3}}}{324}+\frac{7\left(-\mathrm{signum}\left(x^{6}-1\right)\right)^{\frac{1}{3}}x^{2}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^{6}\right)}{81\mathrm{signum}\left(x^{6}-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(x^6-1)^(1/3),x)
```

```
[Out] 1/324*x^2*(18*x^12+21*x^6+28)*(x^6-1)^(2/3)+7/81/signum(x^6-1)^(1/3)*(-sign
um(x^6-1))^(1/3)*x^2*hypergeom([1/3,1/3],[4/3],x^6)
```

maxima [A] time = 0.41, size = 145, normalized size = 1.24

$$-\frac{7}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^{6}-1\right)^{\frac{1}{3}}}{x^{2}}+1\right)\right)-\frac{\frac{67\left(x^{6}-1\right)^{\frac{2}{3}}}{x^{4}}-\frac{77\left(x^{6}-1\right)^{\frac{5}{3}}}{x^{10}}+\frac{28\left(x^{6}-1\right)^{\frac{8}{3}}}{x^{16}}}{324\left(\frac{3\left(x^{6}-1\right)}{x^{6}}-\frac{3\left(x^{6}-1\right)^{2}}{x^{12}}+\frac{\left(x^{6}-1\right)^{3}}{x^{18}}-1\right)}+\frac{7}{486}\log\left(\frac{\left(x^{6}-1\right)^{\frac{1}{3}}}{x^{2}}+\frac{\left(x^{6}-1\right)^{\frac{2}{3}}}{x^{4}}+1\right)-\frac{7}{243}\log\left(\frac{\left(x^{6}-1\right)^{\frac{1}{3}}}{x^{2}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(x^6-1)^(1/3),x, algorithm="maxima")
```

```
[Out] -7/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3)/x^2 + 1)) - 1/324*(67*
(x^6 - 1)^(2/3)/x^4 - 77*(x^6 - 1)^(5/3)/x^10 + 28*(x^6 - 1)^(8/3)/x^16)/(3
*(x^6 - 1)/x^6 - 3*(x^6 - 1)^2/x^12 + (x^6 - 1)^3/x^18 - 1) + 7/486*log((x^
6 - 1)^(1/3)/x^2 + (x^6 - 1)^(2/3)/x^4 + 1) - 7/243*log((x^6 - 1)^(1/3)/x^2
 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{19}}{\left(x^{6}-1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(x^6 - 1)^(1/3),x)
```

```
[Out] int(x^19/(x^6 - 1)^(1/3), x)
```

sympy [C] time = 2.38, size = 34, normalized size = 0.29

$$-\frac{x^{20}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{10}{3}\\\frac{13}{3}\end{matrix}\right|x^{6}\right)}{6\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(x**6-1)**(1/3),x)
```

```
[Out] -x**20*exp(2*I*pi/3)*gamma(10/3)*hyper((1/3, 10/3), (13/3,), x**6)/(6*gamma
(13/3))
```

3.1464 $\int x^{15}\sqrt[3]{-1+x^6}\,dx$

Optimal. Leaf size=117

$$\frac{5}{486}\log\left(\sqrt[3]{x^6-1}-x^2\right)+\frac{5\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{162\sqrt{3}}-\frac{5}{972}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)+\frac{1}{324}\sqrt[3]{x^6-1}\,\left(18x^{16}-\right.$$

Rubi [A] time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {275, 279, 321, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{18}\sqrt[3]{x^6-1}\,x^{16}-\frac{1}{108}\sqrt[3]{x^6-1}\,x^{10}-\frac{5}{324}\sqrt[3]{x^6-1}\,x^4+\frac{5}{486}\log\left(1-\frac{x^2}{\sqrt[3]{x^6-1}}\right)+\frac{5\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6-1}}+1}{\sqrt{3}}\right)}{162\sqrt{3}}-\frac{5}{972}\log\left(\frac{x^4}{\left(x^6-1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6-1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^15*(-1 + x^6)^(1/3),x]
```

```
[Out] (-5*x^4*(-1 + x^6)^(1/3))/324 - (x^10*(-1 + x^6)^(1/3))/108 + (x^16*(-1 + x
^6)^(1/3))/18 + (5*ArcTan[(1 + (2*x^2)/(-1 + x^6)^(1/3))/Sqrt[3]])/(162*Sqr
t[3]) + (5*Log[1 - x^2/(-1 + x^6)^(1/3)])/486 - (5*Log[1 + x^4/(-1 + x^6)^(
2/3) + x^2/(-1 + x^6)^(1/3)])/972
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int x^{15}\sqrt[3]{-1+x^6}\,dx = \frac{1}{2}\text{Subst}\left(\int x^7\sqrt[3]{-1+x^3}\,dx,x,x^2\right)$$

$$= \frac{1}{18}x^{16}\sqrt[3]{-1+x^6} - \frac{1}{18}\text{Subst}\left(\int \frac{x^7}{\left(-1+x^3\right)^{2/3}}\,dx,x,x^2\right)$$

$$= -\frac{1}{108}x^{10}\sqrt[3]{-1+x^6} + \frac{1}{18}x^{16}\sqrt[3]{-1+x^6} - \frac{5}{108}\text{Subst}\left(\int \frac{x^4}{\left(-1+x^3\right)^{2/3}}\,dx,x,x^2\right)$$

$$= -\frac{5}{324}x^4\sqrt[3]{-1+x^6} - \frac{1}{108}x^{10}\sqrt[3]{-1+x^6} + \frac{1}{18}x^{16}\sqrt[3]{-1+x^6} - \frac{5}{162}\text{Subst}\left(\int \frac{x}{\left(-1+x^3\right)^{2/3}}\,d\right.$$

$$= -\frac{5}{324}x^4\sqrt[3]{-1+x^6} - \frac{1}{108}x^{10}\sqrt[3]{-1+x^6} + \frac{1}{18}x^{16}\sqrt[3]{-1+x^6} - \frac{5}{162}\text{Subst}\left(\int \frac{x}{1-x^3}\,dx,x,\right.$$

$$= -\frac{5}{324}x^4\sqrt[3]{-1+x^6} - \frac{1}{108}x^{10}\sqrt[3]{-1+x^6} + \frac{1}{18}x^{16}\sqrt[3]{-1+x^6} - \frac{5}{486}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\right.$$

$$= -\frac{5}{324}x^4\sqrt[3]{-1+x^6} - \frac{1}{108}x^{10}\sqrt[3]{-1+x^6} + \frac{1}{18}x^{16}\sqrt[3]{-1+x^6} + \frac{5}{486}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right) -$$

$$= -\frac{5}{324}x^4\sqrt[3]{-1+x^6} - \frac{1}{108}x^{10}\sqrt[3]{-1+x^6} + \frac{1}{18}x^{16}\sqrt[3]{-1+x^6} + \frac{5}{486}\log\left(1-\frac{x^2}{\sqrt[3]{-1+x^6}}\right) -$$

$$= -\frac{5}{324}x^4\sqrt[3]{-1+x^6} - \frac{1}{108}x^{10}\sqrt[3]{-1+x^6} + \frac{1}{18}x^{16}\sqrt[3]{-1+x^6} + \frac{5\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{-1+x^6}}}{\sqrt{3}}\right)}{162\sqrt{3}} + \frac{5}{486}\text{lo}$$

Mathematica [C] time = 0.03, size = 67, normalized size = 0.57

$$\frac{x^4\sqrt[3]{x^6-1}\left(5\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^6\right)+\sqrt[3]{1-x^6}\left(6x^{12}-x^6-5\right)\right)}{108\sqrt[3]{1-x^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15*(-1 + x^6)^(1/3),x]
```

```
[Out] (x^4*(-1 + x^6)^(1/3)*((1 - x^6)^(1/3)*(-5 - x^6 + 6*x^12) + 5*Hypergeometr
ic2F1[-1/3, 2/3, 5/3, x^6]))/(108*(1 - x^6)^(1/3))
```

IntegrateAlgebraic [A] time = 1.57, size = 117, normalized size = 1.00

$$\frac{5}{486}\log\left(\sqrt[3]{x^6-1}-x^2\right)+\frac{5\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6-1}+x^2}\right)}{162\sqrt{3}}-\frac{5}{972}\log\left(\left(x^6-1\right)^{2/3}+x^4+\sqrt[3]{x^6-1}\,x^2\right)+\frac{1}{324}\sqrt[3]{x^6-1}\left(18x^{16}-3x^{10}-5x^4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^15*(-1 + x^6)^(1/3),x]
```

```
[Out] ((-1 + x^6)^(1/3)*(-5*x^4 - 3*x^10 + 18*x^16))/324 + (5*ArcTan[(Sqrt[3]*x^2
)/(x^2 + 2*(-1 + x^6)^(1/3))])/(162*Sqrt[3]) + (5*Log[-x^2 + (-1 + x^6)^(1/
3)])/486 - (5*Log[x^4 + x^2*(-1 + x^6)^(1/3) + (-1 + x^6)^(2/3)])/972
```

fricas [A] time = 0.41, size = 107, normalized size = 0.91

$$-\frac{5}{486}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6-1\right)^{\frac{1}{3}}}{3x^2}\right)+\frac{1}{324}\left(18x^{16}-3x^{10}-5x^4\right)\left(x^6-1\right)^{\frac{1}{3}}+\frac{5}{486}\log\left(-\frac{x^2-\left(x^6-1\right)^{\frac{1}{3}}}{x^2}\right)-\frac{5}{972}\log\left(\frac{x^4+\left(x^6-1\right)^{\frac{1}{3}}x^2+\left(x^6-1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(x^6-1)^(1/3),x, algorithm="fricas")
```

```
[Out] -5/486*sqrt(3)*arctan(1/3*(sqrt(3)*x^2 + 2*sqrt(3)*(x^6 - 1)^(1/3))/x^2) +
1/324*(18*x^16 - 3*x^10 - 5*x^4)*(x^6 - 1)^(1/3) + 5/486*log(-(x^2 - (x^6 -
 1)^(1/3))/x^2) - 5/972*log((x^4 + (x^6 - 1)^(1/3)*x^2 + (x^6 - 1)^(2/3))/x
^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^6-1\right)^{\frac{1}{3}} x^{15}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(x^6-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^6 - 1)^(1/3)*x^15, x)
```

maple [C] time = 0.38, size = 58, normalized size = 0.50

$$\frac{x^4 \left(18x^{12}-3x^6-5\right) \left(x^6-1\right)^{\frac{1}{3}}}{324}-\frac{5 \left(-\mathrm{signum}\left(x^6-1\right)\right)^{\frac{2}{3}} x^4 \,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^6\right)}{324 \mathrm{signum}\left(x^6-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(x^6-1)^(1/3),x)
```

```
[Out] 1/324*x^4*(18*x^12-3*x^6-5)*(x^6-1)^(1/3)-5/324/signum(x^6-1)^(2/3)*(-signu
m(x^6-1))^(2/3)*x^4*hypergeom([2/3,2/3],[5/3],x^6)
```

maxima [A] time = 0.41, size = 145, normalized size = 1.24

$$-\frac{5}{486} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(\frac{2 \left(x^6-1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)-\frac{\frac{10 \left(x^6-1\right)^{\frac{1}{3}}}{x^2}+\frac{13 \left(x^6-1\right)^{\frac{4}{3}}}{x^8}-\frac{5 \left(x^6-1\right)^{\frac{7}{3}}}{x^{14}}}{324 \left(\frac{3 \left(x^6-1\right)}{x^6}-\frac{3 \left(x^6-1\right)^2}{x^{12}}+\frac{\left(x^6-1\right)^3}{x^{18}}-1\right)}-\frac{5}{972} \log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6-1\right)^{\frac{2}{3}}}{x^4}+1\right)+\frac{5}{486} \log\left(\frac{\left(x^6-1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(x^6-1)^(1/3),x, algorithm="maxima")
```

```
[Out] -5/486*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 - 1)^(1/3)/x^2 + 1)) - 1/324*(10*
(x^6 - 1)^(1/3)/x^2 + 13*(x^6 - 1)^(4/3)/x^8 - 5*(x^6 - 1)^(7/3)/x^14)/(3*(
x^6 - 1)/x^6 - 3*(x^6 - 1)^2/x^12 + (x^6 - 1)^3/x^18 - 1) - 5/972*log((x^6
- 1)^(1/3)/x^2 + (x^6 - 1)^(2/3)/x^4 + 1) + 5/486*log((x^6 - 1)^(1/3)/x^2 -
 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{15}\, \left(x^6-1\right)^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(x^6 - 1)^(1/3),x)
```

```
[Out] int(x^15*(x^6 - 1)^(1/3), x)
```

sympy [C] time = 1.91, size = 36, normalized size = 0.31

$$-\frac{x^{16}e^{-\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|x^6\right)}{6\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15*(x**6-1)**(1/3),x)
```

```
[Out] -x**16*exp(-2*I*pi/3)*gamma(8/3)*hyper((-1/3, 8/3), (11/3,), x**6)/(6*gamma
(11/3))
```

3.1465 $\int \frac{x^{19}}{\sqrt[3]{1+x^6}}\,dx$

Optimal. Leaf size=117

$$\frac{7}{243}\log\left(\sqrt[3]{x^6+1}-x^2\right)-\frac{7\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{81\sqrt{3}}-\frac{7}{486}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right)+\frac{1}{324}\left(x^6+1\right)^{2/3}\left(18x^{14}-\right.$$

Rubi [A] time = 0.05, antiderivative size = 101, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {275, 321, 239}

$$\frac{1}{18}\left(x^6+1\right)^{2/3}x^{14}-\frac{7}{108}\left(x^6+1\right)^{2/3}x^8+\frac{7}{81}\left(x^6+1\right)^{2/3}x^2+\frac{7}{162}\log\left(x^2-\sqrt[3]{x^6+1}\right)-\frac{7\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6+1}}+1}{\sqrt{3}}\right)}{81\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/(1 + x^6)^(1/3),x]
```

```
[Out] (7*x^2*(1 + x^6)^(2/3))/81 - (7*x^8*(1 + x^6)^(2/3))/108 + (x^14*(1 + x^6)^
(2/3))/18 - (7*ArcTan[(1 + (2*x^2)/(1 + x^6)^(1/3))/Sqrt[3]])/(81*Sqrt[3])
+ (7*Log[x^2 - (1 + x^6)^(1/3)])/162
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{19}}{\sqrt[3]{1+x^6}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^9}{\sqrt[3]{1+x^3}}\,dx,x,x^2\right)$$
$$= \frac{1}{18}x^{14}\left(1+x^6\right)^{2/3} - \frac{7}{18}\,\text{Subst}\left(\int \frac{x^6}{\sqrt[3]{1+x^3}}\,dx,x,x^2\right)$$
$$= -\frac{7}{108}x^8\left(1+x^6\right)^{2/3} + \frac{1}{18}x^{14}\left(1+x^6\right)^{2/3} + \frac{7}{27}\,\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{1+x^3}}\,dx,x,x^2\right)$$
$$= \frac{7}{81}x^2\left(1+x^6\right)^{2/3} - \frac{7}{108}x^8\left(1+x^6\right)^{2/3} + \frac{1}{18}x^{14}\left(1+x^6\right)^{2/3} - \frac{7}{81}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^3}}\,dx,x,x^2\right)$$
$$= \frac{7}{81}x^2\left(1+x^6\right)^{2/3} - \frac{7}{108}x^8\left(1+x^6\right)^{2/3} + \frac{1}{18}x^{14}\left(1+x^6\right)^{2/3} - \frac{7\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{1+x^6}}}{\sqrt{3}}\right)}{81\sqrt{3}} + \frac{7}{162}\log\left(x^2\right.$$

Mathematica [A] time = 0.05, size = 127, normalized size = 1.09

$$\frac{1}{972}\left(54\left(x^6+1\right)^{2/3}x^{14} - 63\left(x^6+1\right)^{2/3}x^8 + 84\left(x^6+1\right)^{2/3}x^2 + 28\log\left(1-\frac{x^2}{\sqrt[3]{x^6+1}}\right) - 28\sqrt{3}\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6+1}}+1}{\sqrt{3}}\right) - 14\log\left(\frac{x^4}{\left(x^6+1\right)^{2/3}} + \frac{x^2}{\sqrt[3]{x^6+1}} + 1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/(1 + x^6)^(1/3),x]
```

```
[Out] (84*x^2*(1 + x^6)^(2/3) - 63*x^8*(1 + x^6)^(2/3) + 54*x^14*(1 + x^6)^(2/3)
- 28*Sqrt[3]*ArcTan[(1 + (2*x^2)/(1 + x^6)^(1/3))/Sqrt[3]] + 28*Log[1 - x^2
/(1 + x^6)^(1/3)] - 14*Log[1 + x^4/(1 + x^6)^(2/3) + x^2/(1 + x^6)^(1/3)])/
972
```

IntegrateAlgebraic [A] time = 7.52, size = 117, normalized size = 1.00

$$\frac{7}{243}\log\left(\sqrt[3]{x^6+1}-x^2\right) - \frac{7\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{81\sqrt{3}} - \frac{7}{486}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right) + \frac{1}{324}\left(x^6+1\right)^{2/3}\left(18x^{14}-21x^8+28x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^19/(1 + x^6)^(1/3),x]
```

```
[Out] ((1 + x^6)^(2/3)*(28*x^2 - 21*x^8 + 18*x^14))/324 - (7*ArcTan[(Sqrt[3]*x^2)
/(x^2 + 2*(1 + x^6)^(1/3))])/(81*Sqrt[3]) + (7*Log[-x^2 + (1 + x^6)^(1/3)])
/243 - (7*Log[x^4 + x^2*(1 + x^6)^(1/3) + (1 + x^6)^(2/3)])/486
```

fricas [A] time = 0.41, size = 107, normalized size = 0.91

$$\frac{1}{324}\left(18x^{14}-21x^8+28x^2\right)\left(x^6+1\right)^{\frac{2}{3}} + \frac{7}{243}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6+1\right)^{\frac{1}{3}}}{3x^2}\right) + \frac{7}{243}\log\left(-\frac{x^2-\left(x^6+1\right)^{\frac{1}{3}}}{x^2}\right) - \frac{7}{486}\log\left(\frac{x^4+\left(x^6+1\right)^{\frac{1}{3}}x^2+\left(x^6+1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(x^6+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/324*(18*x^14 - 21*x^8 + 28*x^2)*(x^6 + 1)^(2/3) + 7/243*sqrt(3)*arctan(1/
3*(sqrt(3)*x^2 + 2*sqrt(3)*(x^6 + 1)^(1/3))/x^2) + 7/243*log(-(x^2 - (x^6 +
 1)^(1/3))/x^2) - 7/486*log((x^4 + (x^6 + 1)^(1/3)*x^2 + (x^6 + 1)^(2/3))/x
^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{\left(x^6+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(x^6+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^19/(x^6 + 1)^(1/3), x)
```

maple [C] time = 0.34, size = 42, normalized size = 0.36

$$\frac{x^{2}\left(18x^{12}-21x^{6}+28\right)\left(x^{6}+1\right)^{\frac{2}{3}}}{324}-\frac{7x^{2}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x^{6}\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(x^6+1)^(1/3),x)
```

```
[Out] 1/324*x^2*(18*x^12-21*x^6+28)*(x^6+1)^(2/3)-7/81*x^2*hypergeom([1/3,1/3],[4
/3],-x^6)
```

maxima [A] time = 0.41, size = 145, normalized size = 1.24

$$\frac{7}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^{6}+1\right)^{\frac{1}{3}}}{x^{2}}+1\right)\right)+\frac{\frac{67\left(x^{6}+1\right)^{\frac{2}{3}}}{x^{4}}-\frac{77\left(x^{6}+1\right)^{\frac{5}{3}}}{x^{10}}+\frac{28\left(x^{6}+1\right)^{\frac{8}{3}}}{x^{16}}}{324\left(\frac{3\left(x^{6}+1\right)}{x^{6}}-\frac{3\left(x^{6}+1\right)^{2}}{x^{12}}+\frac{\left(x^{6}+1\right)^{3}}{x^{18}}-1\right)}-\frac{7}{486}\log\left(\frac{\left(x^{6}+1\right)^{\frac{1}{3}}}{x^{2}}+\frac{\left(x^{6}+1\right)^{\frac{2}{3}}}{x^{4}}+1\right)+\frac{7}{243}\log\left(\frac{\left(x^{6}+1\right)^{\frac{1}{3}}}{x^{2}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(x^6+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 7/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3)/x^2 + 1)) + 1/324*(67*(
x^6 + 1)^(2/3)/x^4 - 77*(x^6 + 1)^(5/3)/x^10 + 28*(x^6 + 1)^(8/3)/x^16)/(3*
(x^6 + 1)/x^6 - 3*(x^6 + 1)^2/x^12 + (x^6 + 1)^3/x^18 - 1) - 7/486*log((x^6
 + 1)^(1/3)/x^2 + (x^6 + 1)^(2/3)/x^4 + 1) + 7/243*log((x^6 + 1)^(1/3)/x^2
- 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{19}}{\left(x^{6}+1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(x^6 + 1)^(1/3),x)
```

```
[Out] int(x^19/(x^6 + 1)^(1/3), x)
```

sympy [C] time = 2.30, size = 29, normalized size = 0.25

$$\frac{x^{20}\Gamma\left(\frac{10}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{10}{3}\\\frac{13}{3}\end{matrix}\right|x^{6}e^{i\pi}\right)}{6\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(x**6+1)**(1/3),x)
```

```
[Out] x**20*gamma(10/3)*hyper((1/3, 10/3), (13/3,), x**6*exp_polar(I*pi))/(6*gamm
a(13/3))
```

3.1466 $\int x^{15}\sqrt[3]{1+x^6}\,dx$

Optimal. Leaf size=117

$$-\frac{5}{486}\log\left(\sqrt[3]{x^6+1}-x^2\right)-\frac{5\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{162\sqrt{3}}+\frac{5}{972}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right)+\frac{1}{324}\sqrt[3]{x^6+1}\,(18x^{16}$$

Rubi [A] time = 0.10, antiderivative size = 135, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {275, 279, 321, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{18}\sqrt[3]{x^6+1}\,x^{16}+\frac{1}{108}\sqrt[3]{x^6+1}\,x^{10}-\frac{5}{324}\sqrt[3]{x^6+1}\,x^4-\frac{5}{486}\log\left(1-\frac{x^2}{\sqrt[3]{x^6+1}}\right)-\frac{5\tan^{-1}\left(\frac{\frac{2x^2}{\sqrt[3]{x^6+1}}+1}{\sqrt{3}}\right)}{162\sqrt{3}}+\frac{5}{972}\log\left(\frac{x^4}{\left(x^6+1\right)^{2/3}}+\frac{x^2}{\sqrt[3]{x^6+1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^15*(1 + x^6)^(1/3),x]
```

```
[Out] (-5*x^4*(1 + x^6)^(1/3))/324 + (x^10*(1 + x^6)^(1/3))/108 + (x^16*(1 + x^6)
^(1/3))/18 - (5*ArcTan[(1 + (2*x^2)/(1 + x^6)^(1/3))/Sqrt[3]])/(162*Sqrt[3]
) - (5*Log[1 - x^2/(1 + x^6)^(1/3)])/486 + (5*Log[1 + x^4/(1 + x^6)^(2/3) +
 x^2/(1 + x^6)^(1/3)])/972
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int x^{15}\sqrt[3]{1+x^6}\,dx = \frac{1}{2}\text{Subst}\left(\int x^7\sqrt[3]{1+x^3}\,dx,x,x^2\right)$$
$$= \frac{1}{18}x^{16}\sqrt[3]{1+x^6} + \frac{1}{18}\text{Subst}\left(\int \frac{x^7}{\left(1+x^3\right)^{2/3}}\,dx,x,x^2\right)$$
$$= \frac{1}{108}x^{10}\sqrt[3]{1+x^6} + \frac{1}{18}x^{16}\sqrt[3]{1+x^6} - \frac{5}{108}\text{Subst}\left(\int \frac{x^4}{\left(1+x^3\right)^{2/3}}\,dx,x,x^2\right)$$
$$= -\frac{5}{324}x^4\sqrt[3]{1+x^6} + \frac{1}{108}x^{10}\sqrt[3]{1+x^6} + \frac{1}{18}x^{16}\sqrt[3]{1+x^6} + \frac{5}{162}\text{Subst}\left(\int \frac{x}{\left(1+x^3\right)^{2/3}}\,dx,x,x^2\right.$$
$$= -\frac{5}{324}x^4\sqrt[3]{1+x^6} + \frac{1}{108}x^{10}\sqrt[3]{1+x^6} + \frac{1}{18}x^{16}\sqrt[3]{1+x^6} + \frac{5}{162}\text{Subst}\left(\int \frac{x}{1-x^3}\,dx,x,\frac{x^2}{\sqrt[3]{1+\right.$$
$$= -\frac{5}{324}x^4\sqrt[3]{1+x^6} + \frac{1}{108}x^{10}\sqrt[3]{1+x^6} + \frac{1}{18}x^{16}\sqrt[3]{1+x^6} + \frac{5}{486}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\frac{x^2}{\sqrt[3]{1+x\right.$$
$$= -\frac{5}{324}x^4\sqrt[3]{1+x^6} + \frac{1}{108}x^{10}\sqrt[3]{1+x^6} + \frac{1}{18}x^{16}\sqrt[3]{1+x^6} - \frac{5}{486}\log\left(1-\frac{x^2}{\sqrt[3]{1+x^6}}\right) + \frac{5}{972}\text{Sul}$$
$$= -\frac{5}{324}x^4\sqrt[3]{1+x^6} + \frac{1}{108}x^{10}\sqrt[3]{1+x^6} + \frac{1}{18}x^{16}\sqrt[3]{1+x^6} - \frac{5}{486}\log\left(1-\frac{x^2}{\sqrt[3]{1+x^6}}\right) + \frac{5}{972}\log$$
$$= -\frac{5}{324}x^4\sqrt[3]{1+x^6} + \frac{1}{108}x^{10}\sqrt[3]{1+x^6} + \frac{1}{18}x^{16}\sqrt[3]{1+x^6} - \frac{5\tan^{-1}\left(\frac{1+\frac{2x^2}{\sqrt[3]{1+x^6}}}{\sqrt{3}}\right)}{162\sqrt{3}} - \frac{5}{486}\log\left(1-\right.$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.38

$$\frac{1}{108}x^4\left(5\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-x^6\right)+\sqrt[3]{x^6+1}\left(6x^{12}+x^6-5\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15*(1 + x^6)^(1/3),x]
```

```
[Out] (x^4*((1 + x^6)^(1/3)*(-5 + x^6 + 6*x^12) + 5*Hypergeometric2F1[-1/3, 2/3,
5/3, -x^6]))/108
```

IntegrateAlgebraic [A] time = 1.56, size = 117, normalized size = 1.00

$$-\frac{5}{486}\log\left(\sqrt[3]{x^6+1}-x^2\right)-\frac{5\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{x^6+1}+x^2}\right)}{162\sqrt{3}}+\frac{5}{972}\log\left(\left(x^6+1\right)^{2/3}+x^4+\sqrt[3]{x^6+1}\,x^2\right)+\frac{1}{324}\sqrt[3]{x^6+1}\left(18x^{16}+3x^{10}-5x^4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^15*(1 + x^6)^(1/3),x]
```

```
[Out] ((1 + x^6)^(1/3)*(-5*x^4 + 3*x^10 + 18*x^16))/324 - (5*ArcTan[(Sqrt[3]*x^2)
/(x^2 + 2*(1 + x^6)^(1/3))])/(162*Sqrt[3]) - (5*Log[-x^2 + (1 + x^6)^(1/3)]
)/486 + (5*Log[x^4 + x^2*(1 + x^6)^(1/3) + (1 + x^6)^(2/3)])/972
```

fricas [A] time = 0.41, size = 107, normalized size = 0.91

$$\frac{5}{486}\sqrt{3}\arctan\left(\frac{\sqrt{3}x^2+2\sqrt{3}\left(x^6+1\right)^{\frac{1}{3}}}{3x^2}\right)+\frac{1}{324}\left(18x^{16}+3x^{10}-5x^4\right)\left(x^6+1\right)^{\frac{1}{3}}-\frac{5}{486}\log\left(-\frac{x^2-\left(x^6+1\right)^{\frac{1}{3}}}{x^2}\right)+\frac{5}{972}\log\left(\frac{x^4+\left(x^6+1\right)^{\frac{1}{3}}x^2+\left(x^6+1\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(x^6+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 5/486*sqrt(3)*arctan(1/3*(sqrt(3)*x^2 + 2*sqrt(3)*(x^6 + 1)^(1/3))/x^2) + 1
/324*(18*x^16 + 3*x^10 - 5*x^4)*(x^6 + 1)^(1/3) - 5/486*log(-(x^2 - (x^6 +
1)^(1/3))/x^2) + 5/972*log((x^4 + (x^6 + 1)^(1/3)*x^2 + (x^6 + 1)^(2/3))/x^
4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^6+1\right)^{\frac{1}{3}} x^{15}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(x^6+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 1)^(1/3)*x^15, x)
```

maple [C] time = 0.35, size = 42, normalized size = 0.36

$$\frac{x^4\left(18x^{12}+3x^6-5\right)\left(x^6+1\right)^{\frac{1}{3}}}{324}+\frac{5x^4\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^6\right)}{324}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(x^6+1)^(1/3),x)
```

```
[Out] 1/324*x^4*(18*x^12+3*x^6-5)*(x^6+1)^(1/3)+5/324*x^4*hypergeom([2/3,2/3],[5/
3],-x^6)
```

maxima [A] time = 0.41, size = 145, normalized size = 1.24

$$\frac{5}{486}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+1\right)\right)+\frac{\frac{10\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+\frac{13\left(x^6+1\right)^{\frac{4}{3}}}{x^8}-\frac{5\left(x^6+1\right)^{\frac{7}{3}}}{x^{14}}}{324\left(\frac{3\left(x^6+1\right)}{x^6}-\frac{3\left(x^6+1\right)^2}{x^{12}}+\frac{\left(x^6+1\right)^3}{x^{18}}-1\right)}+\frac{5}{972}\log\left(\frac{\left(x^6+1\right)^{\frac{1}{3}}}{x^2}+\frac{\left(x^6+1\right)^{\frac{2}{3}}}{x^4}+1\right)-\frac{5}{486}\log\left(\frac{\left(x^6+1\right)^{\frac{1}{3}}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(x^6+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 5/486*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^6 + 1)^(1/3)/x^2 + 1)) + 1/324*(10*(
x^6 + 1)^(1/3)/x^2 + 13*(x^6 + 1)^(4/3)/x^8 - 5*(x^6 + 1)^(7/3)/x^14)/(3*(x
^6 + 1)/x^6 - 3*(x^6 + 1)^2/x^12 + (x^6 + 1)^3/x^18 - 1) + 5/972*log((x^6 +
 1)^(1/3)/x^2 + (x^6 + 1)^(2/3)/x^4 + 1) - 5/486*log((x^6 + 1)^(1/3)/x^2 -
1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{15}\left(x^6+1\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(x^6 + 1)^(1/3),x)
```

```
[Out] int(x^15*(x^6 + 1)^(1/3), x)
```

sympy [C] time = 1.87, size = 31, normalized size = 0.26

$$\frac{x^{16}\Gamma\left(\frac{8}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{8}{3}\\\frac{11}{3}\end{matrix}\middle|x^6e^{i\pi}\right)}{6\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15*(x**6+1)**(1/3),x)
```

```
[Out] x**16*gamma(8/3)*hyper((-1/3, 8/3), (11/3,), x**6*exp_polar(I*pi))/(6*gamma
(11/3))
```

3.1467 $\int \frac{-b-ax^3+x^6}{x^6\sqrt[4]{-bx+ax^4}}\,dx$

Optimal. Leaf size=117

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3\sqrt[4]{a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3\sqrt[4]{a}}-\frac{4\left(ax^4-bx\right)^{3/4}\left(11ax^3+3b\right)}{63bx^6}$$

Rubi [A] time = 0.31, antiderivative size = 179, normalized size of antiderivative = 1.53, number of steps used = 12, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2052, 2011, 329, 275, 240, 212, 206, 203, 2016, 2014}

$$-\frac{4\left(ax^4-bx\right)^{3/4}}{21x^6}-\frac{44a\left(ax^4-bx\right)^{3/4}}{63bx^3}+\frac{2\sqrt[4]{x}\sqrt[4]{ax^3-b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3\sqrt[4]{a}\sqrt[4]{ax^4-bx}}+\frac{2\sqrt[4]{x}\sqrt[4]{ax^3-b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3\sqrt[4]{a}\sqrt[4]{ax^4-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b - a*x^3 + x^6)/(x^6*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(-(b*x) + a*x^4)^(3/4))/(21*x^6) - (44*a*(-(b*x) + a*x^4)^(3/4))/(63*b*
x^3) + (2*x^(1/4)*(-b + a*x^3)^(1/4)*ArcTan[(a^(1/4)*x^(3/4))/(-b + a*x^3)^
(1/4)])/(3*a^(1/4)*(-(b*x) + a*x^4)^(1/4)) + (2*x^(1/4)*(-b + a*x^3)^(1/4)*
ArcTanh[(a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)])/(3*a^(1/4)*(-(b*x) + a*x^4)^
(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{-b-ax^3+x^6}{x^6\sqrt[4]{-bx+ax^4}}\,dx = \int \left(\frac{1}{\sqrt[4]{-bx+ax^4}} - \frac{b}{x^6\sqrt[4]{-bx+ax^4}} - \frac{a}{x^3\sqrt[4]{-bx+ax^4}}\right) dx$$
$$= -\left(a\int \frac{1}{x^3\sqrt[4]{-bx+ax^4}}\,dx\right) - b\int \frac{1}{x^6\sqrt[4]{-bx+ax^4}}\,dx + \int \frac{1}{\sqrt[4]{-bx+ax^4}}\,dx$$
$$= -\frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6} - \frac{4a\left(-bx+ax^4\right)^{3/4}}{9bx^3} - \frac{1}{7}(4a)\int \frac{1}{x^3\sqrt[4]{-bx+ax^4}}\,dx + \frac{\left(\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)}{\sqrt[4]{-bx}}$$
$$= -\frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6} - \frac{44a\left(-bx+ax^4\right)^{3/4}}{63bx^3} + \frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{-b+ax^{12}}}\,dx, x, \sqrt[4]{\right.}{\sqrt[4]{-bx+ax^4}}$$
$$= -\frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6} - \frac{44a\left(-bx+ax^4\right)^{3/4}}{63bx^3} + \frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-b+ax^4}}\,dx, x, x^{3/4}\right.}{3\sqrt[4]{-bx+ax^4}}$$
$$= -\frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6} - \frac{44a\left(-bx+ax^4\right)^{3/4}}{63bx^3} + \frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx, x, \frac{x^{3/4}}{\sqrt[4]{-b+a}}\right.}{3\sqrt[4]{-bx+ax^4}}$$
$$= -\frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6} - \frac{44a\left(-bx+ax^4\right)^{3/4}}{63bx^3} + \frac{\left(2\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx, x, \frac{x^3}{\sqrt[4]{-b}}\right.}{3\sqrt[4]{-bx+ax^4}}$$
$$= -\frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6} - \frac{44a\left(-bx+ax^4\right)^{3/4}}{63bx^3} + \frac{2\sqrt[4]{x}\sqrt[4]{-b+ax^3}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{-b+ax^3}}\right)}{3\sqrt[4]{a}\sqrt[4]{-bx+ax^4}} + \frac{2\sqrt[4]{x}\sqrt[4]{-b}}{3}$$

Mathematica [C] time = 0.07, size = 84, normalized size = 0.72

$$\frac{4\left(-11a^2x^6+21bx^6\sqrt[4]{1-\frac{ax^3}{b}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{ax^3}{b}\right)+8abx^3+3b^2\right)}{63bx^5\sqrt[4]{ax^4-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b - a*x^3 + x^6)/(x^6*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (4*(3*b^2 + 8*a*b*x^3 - 11*a^2*x^6 + 21*b*x^6*(1 - (a*x^3)/b)^(1/4)*Hyperge
ometric2F1[1/4, 1/4, 5/4, (a*x^3)/b]))/(63*b*x^5*(-(b*x) + a*x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.57, size = 117, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3\sqrt[4]{a}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3\sqrt[4]{a}} - \frac{4\left(ax^4-bx\right)^{3/4}\left(11ax^3+3b\right)}{63bx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b - a*x^3 + x^6)/(x^6*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(3*b + 11*a*x^3)*(-(b*x) + a*x^4)^(3/4))/(63*b*x^6) + (2*ArcTan[(a^(1/4
)*(-(b*x) + a*x^4)^(3/4))/(-b + a*x^3)])/(3*a^(1/4)) + (2*ArcTanh[(a^(1/4)*
(-(b*x) + a*x^4)^(3/4))/(-b + a*x^3)])/(3*a^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-a*x^3-b)/x^6/(a*x^4-b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.31, size = 220, normalized size = 1.88

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{3(-a)^{\frac{1}{4}}}-\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{3(-a)^{\frac{1}{4}}}-\frac{\sqrt{2}(-a)^{\frac{3}{4}}\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^3}}\right)}{6a}-\frac{\sqrt{2}\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^3}}\right)}{6(-a)^{\frac{1}{4}}}+\frac{4\left(3\left(a-\frac{b}{x^3}\right)^{\frac{7}{4}}b^6-14\left(a-\frac{b}{x^3}\right)^{\frac{3}{4}}ab^6\right)}{63b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-a*x^3-b)/x^6/(a*x^4-b*x)^(1/4),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a - b/x^3)^(1/4))/
(-a)^(1/4))/(-a)^(1/4) - 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4
) - 2*(a - b/x^3)^(1/4))/(-a)^(1/4))/(-a)^(1/4) - 1/6*sqrt(2)*(-a)^(3/4)*lo
g(sqrt(2)*(-a)^(1/4)*(a - b/x^3)^(1/4) + sqrt(-a) + sqrt(a - b/x^3))/a - 1/
6*sqrt(2)*log(-sqrt(2)*(-a)^(1/4)*(a - b/x^3)^(1/4) + sqrt(-a) + sqrt(a - b
/x^3))/(-a)^(1/4) + 4/63*(3*(a - b/x^3)^(7/4)*b^6 - 14*(a - b/x^3)^(3/4)*a*
b^6)/b^7
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x^6 - a\,x^3 - b}{x^6 \left(a\,x^4 - bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-a*x^3-b)/x^6/(a*x^4-b*x)^(1/4),x)
```

```
[Out] int((x^6-a*x^3-b)/x^6/(a*x^4-b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 - ax^3 - b}{\left(ax^4 - bx\right)^{\frac{1}{4}} x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-a*x^3-b)/x^6/(a*x^4-b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^6 - a*x^3 - b)/((a*x^4 - b*x)^(1/4)*x^6), x)
```

mupad [B] time = 1.41, size = 80, normalized size = 0.68

$$\frac{4\,x\,\left(1-\frac{a\,x^3}{b}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \frac{a\,x^3}{b}\right)}{3\,\left(a\,x^4-b\,x\right)^{1/4}}-\frac{4\,\left(a\,x^4-b\,x\right)^{3/4}}{21\,x^6}-\frac{44\,a\,\left(a\,x^4-b\,x\right)^{3/4}}{63\,b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x^3 - x^6)/(x^6*(a*x^4 - b*x)^(1/4)),x)
```

```
[Out] (4*x*(1 - (a*x^3)/b)^(1/4)*hypergeom([1/4, 1/4], 5/4, (a*x^3)/b))/(3*(a*x^4
 - b*x)^(1/4)) - (4*(a*x^4 - b*x)^(3/4))/(21*x^6) - (44*a*(a*x^4 - b*x)^(3/
4))/(63*b*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-ax^3 - b + x^6}{x^6 \sqrt[4]{x\left(ax^3 - b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-a*x**3-b)/x**6/(a*x**4-b*x)**(1/4),x)
```

```
[Out] Integral((-a*x**3 - b + x**6)/(x**6*(x*(a*x**3 - b))**(1/4)), x)
```

3.1468 $\int \frac{-3-x^4+3x^6}{(1-x^4+x^6)\sqrt[3]{1-x^3-x^4+x^6}}\,dx$

Optimal. Leaf size=117

$$-\log\left(\sqrt[3]{x^6-x^4-x^3+1}+x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^6-x^4-x^3+1}-x}\right)+\frac{1}{2}\log\left(x^2-\sqrt[3]{x^6-x^4-x^3+1}\,x+\left(x^6-\right.\right.$$

Rubi [F] time = 0.89, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3-x^4+3x^6}{(1-x^4+x^6)\sqrt[3]{1-x^3-x^4+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3 - x^4 + 3*x^6)/((1 - x^4 + x^6)*(1 - x^3 - x^4 + x^6)^(1/3)),x]
```

```
[Out] 3*Defer[Int][(1 - x^3 - x^4 + x^6)^(-1/3), x] - 6*Defer[Int][1/((1 - x^4 +
x^6)*(1 - x^3 - x^4 + x^6)^(1/3)), x] + 2*Defer[Int][x^4/((1 - x^4 + x^6)*(
1 - x^3 - x^4 + x^6)^(1/3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3-x^4+3x^6}{(1-x^4+x^6)\sqrt[3]{1-x^3-x^4+x^6}}\,dx &= \int \left(\frac{3}{\sqrt[3]{1-x^3-x^4+x^6}}-\frac{2(3-x^4)}{(1-x^4+x^6)\sqrt[3]{1-x^3-x^4+x^6}}\right)dx\\
&= -\left(2\int \frac{3-x^4}{(1-x^4+x^6)\sqrt[3]{1-x^3-x^4+x^6}}\,dx\right)+3\int \frac{1}{\sqrt[3]{1-x^3-x^4+x^6}}\\
&= -\left(2\int \left(\frac{3}{(1-x^4+x^6)\sqrt[3]{1-x^3-x^4+x^6}}-\frac{x^4}{(1-x^4+x^6)\sqrt[3]{1-x^3-x}}\right.\right.\\
&= 2\int \frac{x^4}{(1-x^4+x^6)\sqrt[3]{1-x^3-x^4+x^6}}\,dx+3\int \frac{1}{\sqrt[3]{1-x^3-x^4+x^6}}\,dx
\end{aligned}$$

Mathematica [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{-3-x^4+3x^6}{(1-x^4+x^6)\sqrt[3]{1-x^3-x^4+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3 - x^4 + 3*x^6)/((1 - x^4 + x^6)*(1 - x^3 - x^4 + x^6)^(1/3)),
x]
```

```
[Out] Integrate[(-3 - x^4 + 3*x^6)/((1 - x^4 + x^6)*(1 - x^3 - x^4 + x^6)^(1/3)),
 x]
```

IntegrateAlgebraic [A] time = 0.43, size = 117, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^6-x^4-x^3+1}+x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^6-x^4-x^3+1}-x}\right)+\frac{1}{2}\log\left(x^2-\sqrt[3]{x^6-x^4-x^3+1}\,x+\left(x^6-x^4-x^3+1\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 - x^4 + 3*x^6)/((1 - x^4 + x^6)*(1 - x^3 - x^4 + x^6
)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(1 - x^3 - x^4 + x^6)^(1/3))]) - Log[x
 + (1 - x^3 - x^4 + x^6)^(1/3)] + Log[x^2 - x*(1 - x^3 - x^4 + x^6)^(1/3) +
 (1 - x^3 - x^4 + x^6)^(2/3)]/2
```

fricas [A] time = 2.94, size = 157, normalized size = 1.34

$$-\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(x^6-x^4-x^3+1\right)^{\frac{1}{3}}x^2+2\sqrt{3}\left(x^6-x^4-x^3+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(x^6-x^4+1\right)}{3\left(x^6-x^4-2x^3+1\right)}\right)-\frac{1}{2}\log\left(\frac{x^6-x^4+3\left(x^6-x^4-x^3+1\right)^{\frac{1}{3}}x^2+3\left(x^6-x^4-x^3+1\right)^{\frac{2}{3}}x+1}{x^6-x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-x^4-3)/(x^6-x^4+1)/(x^6-x^4-x^3+1)^(1/3),x, algorithm="fri
cas")
```

```
[Out] -sqrt(3)*arctan(1/3*(2*sqrt(3)*(x^6 - x^4 - x^3 + 1)^(1/3)*x^2 + 2*sqrt(3)*
(x^6 - x^4 - x^3 + 1)^(2/3)*x + sqrt(3)*(x^6 - x^4 + 1))/(x^6 - x^4 - 2*x^3
 + 1)) - 1/2*log((x^6 - x^4 + 3*(x^6 - x^4 - x^3 + 1)^(1/3)*x^2 + 3*(x^6 -
x^4 - x^3 + 1)^(2/3)*x + 1)/(x^6 - x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^6-x^4-3}{\left(x^6-x^4-x^3+1\right)^{\frac{1}{3}}\left(x^6-x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-x^4-3)/(x^6-x^4+1)/(x^6-x^4-x^3+1)^(1/3),x, algorithm="gia
c")
```

```
[Out] integrate((3*x^6 - x^4 - 3)/((x^6 - x^4 - x^3 + 1)^(1/3)*(x^6 - x^4 + 1)),
x)
```

maple [C] time = 3.38, size = 582, normalized size = 4.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^6-x^4-3)/(x^6-x^4+1)/(x^6-x^4-x^3+1)^(1/3),x)
```

```
[Out] RootOf(_Z^2-_Z+1)*ln((RootOf(_Z^2-_Z+1)*x^6-x^6+RootOf(_Z^2-_Z+1)^2*x^3-Roo
tOf(_Z^2-_Z+1)*x^4-2*RootOf(_Z^2-_Z+1)*(x^6-x^4-x^3+1)^(2/3)*x+2*RootOf(_Z^
2-_Z+1)*(x^6-x^4-x^3+1)^(1/3)*x^2-3*RootOf(_Z^2-_Z+1)*x^3+x^4+(x^6-x^4-x^3+
1)^(2/3)*x-(x^6-x^4-x^3+1)^(1/3)*x^2+2*x^3+RootOf(_Z^2-_Z+1)-1)/(x^6-x^4+1)
)-ln((-RootOf(_Z^2-_Z+1)*x^6+RootOf(_Z^2-_Z+1)^2*x^3+RootOf(_Z^2-_Z+1)*x^4+
2*RootOf(_Z^2-_Z+1)*(x^6-x^4-x^3+1)^(2/3)*x-2*RootOf(_Z^2-_Z+1)*(x^6-x^4-x^
3+1)^(1/3)*x^2+RootOf(_Z^2-_Z+1)*x^3-(x^6-x^4-x^3+1)^(2/3)*x+(x^6-x^4-x^3+1
)^(1/3)*x^2-RootOf(_Z^2-_Z+1))/(x^6-x^4+1))*RootOf(_Z^2-_Z+1)+ln((-RootOf(_
Z^2-_Z+1)*x^6+RootOf(_Z^2-_Z+1)^2*x^3+RootOf(_Z^2-_Z+1)*x^4+2*RootOf(_Z^2-_
Z+1)*(x^6-x^4-x^3+1)^(2/3)*x-2*RootOf(_Z^2-_Z+1)*(x^6-x^4-x^3+1)^(1/3)*x^2+
RootOf(_Z^2-_Z+1)*x^3-(x^6-x^4-x^3+1)^(2/3)*x+(x^6-x^4-x^3+1)^(1/3)*x^2-Roo
tOf(_Z^2-_Z+1))/(x^6-x^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^6-x^4-3}{\left(x^6-x^4-x^3+1\right)^{\frac{1}{3}}\left(x^6-x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-x^4-3)/(x^6-x^4+1)/(x^6-x^4-x^3+1)^(1/3),x, algorithm="max
ima")
```

```
[Out] integrate((3*x^6 - x^4 - 3)/((x^6 - x^4 - x^3 + 1)^(1/3)*(x^6 - x^4 + 1)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{-3\,x^6+x^4+3}{\left(x^6-x^4+1\right)\,{\left(x^6-x^4-x^3+1\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 3*x^6 + 3)/((x^6 - x^4 + 1)*(x^6 - x^4 - x^3 + 1)^(1/3)),x)
```

```
[Out] int(-(x^4 - 3*x^6 + 3)/((x^6 - x^4 + 1)*(x^6 - x^4 - x^3 + 1)^(1/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**6-x**4-3)/(x**6-x**4+1)/(x**6-x**4-x**3+1)**(1/3),x)
```

```
[Out] Timed out
```

3.1469 $\int \frac{-b+ax^6}{x^6\sqrt[4]{-bx+ax^4}}\,dx$

Optimal. Leaf size=117

$$\frac{2}{3}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{2}{3}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)-\frac{4\left(ax^4-bx\right)^{3/4}\left(4ax^3+3b\right)}{63bx^6}$$

Rubi [A] time = 0.28, antiderivative size = 179, normalized size of antiderivative = 1.53, number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2052, 2011, 329, 275, 240, 212, 206, 203, 2016, 2014}

$$\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3-b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3\sqrt[4]{ax^4-bx}}+\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3-b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3\sqrt[4]{ax^4-bx}}-\frac{4\left(ax^4-bx\right)^{3/4}}{21x^6}-\frac{16a\left(ax^4-bx\right)^{3/4}}{63bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^6)/(x^6*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(-(b*x) + a*x^4)^(3/4))/(21*x^6) - (16*a*(-(b*x) + a*x^4)^(3/4))/(63*b*
x^3) + (2*a^(3/4)*x^(1/4)*(-b + a*x^3)^(1/4)*ArcTan[(a^(1/4)*x^(3/4))/(-b +
 a*x^3)^(1/4)])/(3*(-(b*x) + a*x^4)^(1/4)) + (2*a^(3/4)*x^(1/4)*(-b + a*x^3
)^(1/4)*ArcTanh[(a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)])/(3*(-(b*x) + a*x^4)^
(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{-b+ax^6}{x^6\sqrt[4]{-bx+ax^4}}\,dx = \int \left(\frac{a}{\sqrt[4]{-bx+ax^4}} - \frac{b}{x^6\sqrt[4]{-bx+ax^4}}\right) dx$$
$$= a\int \frac{1}{\sqrt[4]{-bx+ax^4}}\,dx - b\int \frac{1}{x^6\sqrt[4]{-bx+ax^4}}\,dx$$
$$= -\frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6} - \frac{1}{7}(4a)\int \frac{1}{x^3\sqrt[4]{-bx+ax^4}}\,dx + \frac{\left(a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\int \frac{1}{\sqrt[4]{x}\sqrt[4]{-b+ax^3}}\,dx}{\sqrt[4]{-bx+ax^4}}$$
$$= -\frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6} - \frac{16a\left(-bx+ax^4\right)^{3/4}}{63bx^3} + \frac{\left(4a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{-b+ax^{12}}}\,dx, x, \sqrt[4]{\right.}{\sqrt[4]{-bx+ax^4}}$$
$$= -\frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6} - \frac{16a\left(-bx+ax^4\right)^{3/4}}{63bx^3} + \frac{\left(4a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-b+ax^4}}\,dx, x, x^3\right.}{3\sqrt[4]{-bx+ax^4}}$$
$$= -\frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6} - \frac{16a\left(-bx+ax^4\right)^{3/4}}{63bx^3} + \frac{\left(4a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx, x, \frac{x^{3/}}{\sqrt[4]{-b+}}\right.}{3\sqrt[4]{-bx+ax^4}}$$
$$= -\frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6} - \frac{16a\left(-bx+ax^4\right)^{3/4}}{63bx^3} + \frac{\left(2a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx, x, \right.}{3\sqrt[4]{-bx+ax^4}}$$
$$= -\frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6} - \frac{16a\left(-bx+ax^4\right)^{3/4}}{63bx^3} + \frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{-b+ax^3}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{-b+ax^3}}\right)}{3\sqrt[4]{-bx+ax^4}} + \frac{2a^{3/4}}{}$$

Mathematica [A] time = 0.15, size = 147, normalized size = 1.26

$$\frac{2\left(21a^{3/4}bx^{21/4}\sqrt[4]{ax^3-b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)+21a^{3/4}bx^{21/4}\sqrt[4]{ax^3-b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)-8a^2x^6+2abx^3+6b^2\right)}{63bx^5\sqrt[4]{ax^4-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^6)/(x^6*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (2*(6*b^2 + 2*a*b*x^3 - 8*a^2*x^6 + 21*a^(3/4)*b*x^(21/4)*(-b + a*x^3)^(1/4
)*ArcTan[(a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)] + 21*a^(3/4)*b*x^(21/4)*(-b
+ a*x^3)^(1/4)*ArcTanh[(a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)]))/(63*b*x^5*(-
(b*x) + a*x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.72, size = 117, normalized size = 1.00

$$\frac{2}{3}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{2}{3}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)-\frac{4\left(ax^4-bx\right)^{3/4}\left(4ax^3+3b\right)}{63bx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^6)/(x^6*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(3*b + 4*a*x^3)*(-(b*x) + a*x^4)^(3/4))/(63*b*x^6) + (2*a^(3/4)*ArcTan[
(a^(1/4)*(-(b*x) + a*x^4)^(3/4))/(-b + a*x^3)])/3 + (2*a^(3/4)*ArcTanh[(a^(
1/4)*(-(b*x) + a*x^4)^(3/4))/(-b + a*x^3)])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-b)/x^6/(a*x^4-b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.35, size = 217, normalized size = 1.85

$$\frac{1}{3}\,\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)+\frac{1}{3}\,\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)-\frac{1}{6}\,\sqrt{2}\,(-a)^{\frac{3}{4}}\log\left(\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^3}}\right)+\frac{1}{6}\,\sqrt{2}\,(-a)^{\frac{3}{4}}\log\left(-\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^3}}\right)+\frac{4\left(3\left(a-\frac{b}{x^3}\right)^{\frac{7}{4}}b^6-7\left(a-\frac{b}{x^3}\right)^{\frac{3}{4}}ab^6\right)}{63\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-b)/x^6/(a*x^4-b*x)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a - b/x^
3)^(1/4))/(-a)^(1/4)) + 1/3*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(-a)^(1/4) - 2*(a - b/x^3)^(1/4))/(-a)^(1/4)) - 1/6*sqrt(2)*(-a)^(3/4)*log
(sqrt(2)*(-a)^(1/4)*(a - b/x^3)^(1/4) + sqrt(-a) + sqrt(a - b/x^3)) + 1/6*s
qrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a - b/x^3)^(1/4) + sqrt(-a) + sq
rt(a - b/x^3)) + 4/63*(3*(a - b/x^3)^(7/4)*b^6 - 7*(a - b/x^3)^(3/4)*a*b^6)
/b^7
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{a\,x^6-b}{x^6\left(a\,x^4-bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6-b)/x^6/(a*x^4-b*x)^(1/4),x)
```

```
[Out] int((a*x^6-b)/x^6/(a*x^4-b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^6-b}{\left(ax^4-bx\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-b)/x^6/(a*x^4-b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^6 - b)/((a*x^4 - b*x)^(1/4)*x^6), x)
```

mupad [B] time = 1.37, size = 73, normalized size = 0.62

$$\frac{4\,a\,x\,{\left(1-\frac{a\,x^3}{b}\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \frac{a\,x^3}{b}\right)}{3\,{\left(a\,x^4-b\,x\right)}^{1/4}}-\frac{4\,{\left(a\,x^4-b\,x\right)}^{3/4}\,\left(4\,a\,x^3+3\,b\right)}{63\,b\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^6)/(x^6*(a*x^4 - b*x)^(1/4)),x)
```

```
[Out] (4*a*x*(1 - (a*x^3)/b)^(1/4)*hypergeom([1/4, 1/4], 5/4, (a*x^3)/b))/(3*(a*x
^4 - b*x)^(1/4)) - (4*(a*x^4 - b*x)^(3/4)*(3*b + 4*a*x^3))/(63*b*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^6-b}{x^6\sqrt[4]{x\left(ax^3-b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6-b)/x**6/(a*x**4-b*x)**(1/4),x)
```

```
[Out] Integral((a*x**6 - b)/(x**6*(x*(a*x**3 - b))**(1/4)), x)
```

3.1470 $\int \frac{b+ax^6}{x^6 \sqrt[4]{-bx+ax^4}} \, dx$

Optimal. Leaf size=117

$$\frac{2}{3}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{2}{3}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{4\left(ax^4-bx\right)^{3/4}\left(4ax^3+3b\right)}{63bx^6}$$

Rubi [A] time = 0.27, antiderivative size = 179, normalized size of antiderivative = 1.53, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2052, 2011, 329, 275, 240, 212, 206, 203, 2016, 2014}

$$\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3-b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3\sqrt[4]{ax^4-bx}}+\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{ax^3-b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3\sqrt[4]{ax^4-bx}}+\frac{4\left(ax^4-bx\right)^{3/4}}{21x^6}+\frac{16a\left(ax^4-bx\right)^{3/4}}{63bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^6)/(x^6*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (4*(-(b*x) + a*x^4)^(3/4))/(21*x^6) + (16*a*(-(b*x) + a*x^4)^(3/4))/(63*b*x
^3) + (2*a^(3/4)*x^(1/4)*(-b + a*x^3)^(1/4)*ArcTan[(a^(1/4)*x^(3/4))/(-b +
a*x^3)^(1/4)])/(3*(-(b*x) + a*x^4)^(1/4)) + (2*a^(3/4)*x^(1/4)*(-b + a*x^3)
^(1/4)*ArcTanh[(a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)])/(3*(-(b*x) + a*x^4)^(
1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{b+ax^6}{x^6\sqrt[4]{-bx+ax^4}}\,dx = \int \left(\frac{a}{\sqrt[4]{-bx+ax^4}}+\frac{b}{x^6\sqrt[4]{-bx+ax^4}}\right)dx$$
$$= a\int \frac{1}{\sqrt[4]{-bx+ax^4}}\,dx + b\int \frac{1}{x^6\sqrt[4]{-bx+ax^4}}\,dx$$
$$= \frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6}+\frac{1}{7}(4a)\int \frac{1}{x^3\sqrt[4]{-bx+ax^4}}\,dx+\frac{\left(a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\int \frac{1}{\sqrt[4]{x}\sqrt[4]{-b+ax^3}}\,dx}{\sqrt[4]{-bx+ax^4}}$$
$$= \frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6}+\frac{16a\left(-bx+ax^4\right)^{3/4}}{63bx^3}+\frac{\left(4a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{-b+ax^{12}}}\,dx,x,\right.}{\sqrt[4]{-bx+ax^4}}$$
$$= \frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6}+\frac{16a\left(-bx+ax^4\right)^{3/4}}{63bx^3}+\frac{\left(4a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-b+ax^4}}\,dx,x,\right.}{3\sqrt[4]{-bx+ax^4}}$$
$$= \frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6}+\frac{16a\left(-bx+ax^4\right)^{3/4}}{63bx^3}+\frac{\left(4a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx,x,\right.}{3\sqrt[4]{-bx+ax^4}}$$
$$= \frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6}+\frac{16a\left(-bx+ax^4\right)^{3/4}}{63bx^3}+\frac{\left(2a\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx,x,\right.}{3\sqrt[4]{-bx+ax^4}}$$
$$= \frac{4\left(-bx+ax^4\right)^{3/4}}{21x^6}+\frac{16a\left(-bx+ax^4\right)^{3/4}}{63bx^3}+\frac{2a^{3/4}\sqrt[4]{x}\sqrt[4]{-b+ax^3}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{-b+ax^3}}\right)}{3\sqrt[4]{-bx+ax^4}}+\frac{2a^{3/4}}{}$$

Mathematica [A] time = 0.10, size = 147, normalized size = 1.26

$$\frac{2\left(21a^{3/4}bx^{21/4}\sqrt[4]{ax^3-b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)+21a^{3/4}bx^{21/4}\sqrt[4]{ax^3-b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)+8a^2x^6-2abx^3-6b^2\right)}{63bx^5\sqrt[4]{ax^4-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^6)/(x^6*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (2*(-6*b^2 - 2*a*b*x^3 + 8*a^2*x^6 + 21*a^(3/4)*b*x^(21/4)*(-b + a*x^3)^(1/
4)*ArcTan[(a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)] + 21*a^(3/4)*b*x^(21/4)*(-b
 + a*x^3)^(1/4)*ArcTanh[(a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)]))/(63*b*x^5*(
-(b*x) + a*x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.74, size = 117, normalized size = 1.00

$$\frac{2}{3}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{2}{3}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)+\frac{4\left(ax^4-bx\right)^{3/4}\left(4ax^3+3b\right)}{63bx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^6)/(x^6*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (4*(3*b + 4*a*x^3)*(-(b*x) + a*x^4)^(3/4))/(63*b*x^6) + (2*a^(3/4)*ArcTan[(
a^(1/4)*(-(b*x) + a*x^4)^(3/4))/(-b + a*x^3)])/3 + (2*a^(3/4)*ArcTanh[(a^(1
/4)*(-(b*x) + a*x^4)^(3/4))/(-b + a*x^3)])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^6/(a*x^4-b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.29, size = 217, normalized size = 1.85

$$\frac{1}{3}\sqrt{2}\left(-a\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}+2\left(a-\frac{b}{x^{3}}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)+\frac{1}{3}\sqrt{2}\left(-a\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}-2\left(a-\frac{b}{x^{3}}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)-\frac{1}{6}\sqrt{2}\left(-a\right)^{\frac{3}{4}}\log\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}\left(a-\frac{b}{x^{3}}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^{3}}}\right)+\frac{1}{6}\sqrt{2}\left(-a\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\left(-a\right)^{\frac{1}{4}}\left(a-\frac{b}{x^{3}}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^{3}}}\right)-\frac{4\left(3\left(a-\frac{b}{x^{3}}\right)^{\frac{7}{4}}b^{6}-7\left(a-\frac{b}{x^{3}}\right)^{\frac{3}{4}}ab^{6}\right)}{63\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^6/(a*x^4-b*x)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a - b/x^
3)^(1/4))/(-a)^(1/4)) + 1/3*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(-a)^(1/4) - 2*(a - b/x^3)^(1/4))/(-a)^(1/4)) - 1/6*sqrt(2)*(-a)^(3/4)*log
(sqrt(2)*(-a)^(1/4)*(a - b/x^3)^(1/4) + sqrt(-a) + sqrt(a - b/x^3)) + 1/6*s
qrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a - b/x^3)^(1/4) + sqrt(-a) + sq
rt(a - b/x^3)) - 4/63*(3*(a - b/x^3)^(7/4)*b^6 - 7*(a - b/x^3)^(3/4)*a*b^6)
/b^7
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{a\,x^{6}+b}{x^{6} \left(a\,x^{4}-b x \right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6+b)/x^6/(a*x^4-b*x)^(1/4),x)
```

```
[Out] int((a*x^6+b)/x^6/(a*x^4-b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{6} + b}{{\left(ax^{4} - bx\right)}^{\frac{1}{4}} x^{6}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^6/(a*x^4-b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^6 + b)/((a*x^4 - b*x)^(1/4)*x^6), x)
```

mupad [B] time = 1.18, size = 73, normalized size = 0.62

$$\frac{4\,{\left(a\,x^4-b\,x\right)}^{3/4}\,\left(4\,a\,x^3+3\,b\right)}{63\,b\,x^6}+\frac{4\,a\,x\,{\left(1-\frac{a\,x^3}{b}\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \frac{a\,x^3}{b}\right)}{3\,{\left(a\,x^4-b\,x\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^6)/(x^6*(a*x^4 - b*x)^(1/4)),x)
```

```
[Out] (4*(a*x^4 - b*x)^(3/4)*(3*b + 4*a*x^3))/(63*b*x^6) + (4*a*x*(1 - (a*x^3)/b)
^(1/4)*hypergeom([1/4, 1/4], 5/4, (a*x^3)/b))/(3*(a*x^4 - b*x)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{6} + b}{x^{6} \sqrt[4]{x \left(ax^{3} - b\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6+b)/x**6/(a*x**4-b*x)**(1/4),x)
```

```
[Out] Integral((a*x**6 + b)/(x**6*(x*(a*x**3 - b))**(1/4)), x)
```

3.1471 $\int \frac{\sqrt[4]{x^2+x^4}\left(-1-x^4+x^8\right)}{-1+x^4}\,dx$

Optimal. Leaf size=117

$$-\frac{7}{128}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^2}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^2}}\right)}{2^{3/4}}+\frac{7}{128}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^2}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+x^2}}\right)}{2^{3/4}}+\frac{1}{192}\sqrt[4]{x^4+x^2}\left(32x^5+4\right.$$

Rubi [B] time = 0.58, antiderivative size = 248, normalized size of antiderivative = 2.12, number of steps used = 24, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2056, 1586, 6725, 321, 329, 331, 298, 203, 206, 466, 494}

$$-\frac{7}{192}\sqrt[4]{x^4+x^2}\,x-\frac{7\sqrt[4]{x^4+x^2}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2+1}}\right)}{128\sqrt[4]{x^2+1}\sqrt{x}}-\frac{\sqrt[4]{x^4+x^2}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{x^2+1}}\right)}{2^{3/4}\sqrt[4]{x^2+1}\sqrt{x}}+\frac{7\sqrt[4]{x^4+x^2}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2+1}}\right)}{128\sqrt[4]{x^2+1}\sqrt{x}}+\frac{\sqrt[4]{x^4+x^2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{x^2+1}}\right)}{2^{3/4}\sqrt[4]{x^2+1}\sqrt{x}}+\frac{1}{6}\sqrt[4]{x^4+x^2}\,x^5+\frac{1}{48}\sqrt[4]{x^4+x^2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[((x^2 + x^4)^(1/4)*(-1 - x^4 + x^8))/(-1 + x^4),x]
```

```
[Out] (-7*x*(x^2 + x^4)^(1/4))/192 + (x^3*(x^2 + x^4)^(1/4))/48 + (x^5*(x^2 + x^4
)^(1/4))/6 - (7*(x^2 + x^4)^(1/4)*ArcTan[Sqrt[x]/(1 + x^2)^(1/4)])/(128*Sqr
t[x]*(1 + x^2)^(1/4)) - ((x^2 + x^4)^(1/4)*ArcTan[(2^(1/4)*Sqrt[x])/(1 + x^
2)^(1/4)])/(2^(3/4)*Sqrt[x]*(1 + x^2)^(1/4)) + (7*(x^2 + x^4)^(1/4)*ArcTanh
[Sqrt[x]/(1 + x^2)^(1/4)])/(128*Sqrt[x]*(1 + x^2)^(1/4)) + ((x^2 + x^4)^(1/
4)*ArcTanh[(2^(1/4)*Sqrt[x])/(1 + x^2)^(1/4)])/(2^(3/4)*Sqrt[x]*(1 + x^2)^(
1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{x^2+x^4}\left(-1-x^4+x^8\right)}{-1+x^4}dx = \frac{\sqrt[4]{x^2+x^4}\int \frac{\sqrt{x}\sqrt[4]{1+x^2}\left(-1-x^4+x^8\right)}{-1+x^4}dx}{\sqrt{x}\sqrt[4]{1+x^2}}$$

$$= \frac{\sqrt[4]{x^2+x^4}\int \frac{\sqrt{x}\left(-1-x^4+x^8\right)}{\left(-1+x^2\right)\left(1+x^2\right)^{3/4}}dx}{\sqrt{x}\sqrt[4]{1+x^2}}$$

$$= \frac{\sqrt[4]{x^2+x^4}\int \left(\frac{x^{9/2}}{\left(1+x^2\right)^{3/4}}+\frac{x^{13/2}}{\left(1+x^2\right)^{3/4}}-\frac{\sqrt{x}}{\left(-1+x^2\right)\left(1+x^2\right)^{3/4}}\right)dx}{\sqrt{x}\sqrt[4]{1+x^2}}$$

$$= \frac{\sqrt[4]{x^2+x^4}\int \frac{x^{9/2}}{\left(1+x^2\right)^{3/4}}dx}{\sqrt{x}\sqrt[4]{1+x^2}} + \frac{\sqrt[4]{x^2+x^4}\int \frac{x^{13/2}}{\left(1+x^2\right)^{3/4}}dx}{\sqrt{x}\sqrt[4]{1+x^2}} - \frac{\sqrt[4]{x^2+x^4}\int \frac{\sqrt{x}}{\left(-1+x^2\right)\left(1+x^2\right)^{3/4}}}{\sqrt{x}\sqrt[4]{1+x^2}}$$

$$= \frac{1}{4}x^3\sqrt[4]{x^2+x^4} + \frac{1}{6}x^5\sqrt[4]{x^2+x^4} - \frac{\left(7\sqrt[4]{x^2+x^4}\right)\int \frac{x^{5/2}}{\left(1+x^2\right)^{3/4}}dx}{8\sqrt{x}\sqrt[4]{1+x^2}} - \frac{\left(11\sqrt[4]{x^2+x^4}\right)\int}{12\sqrt{x}\sqrt[4]{1}}$$

$$= -\frac{7}{16}x\sqrt[4]{x^2+x^4} + \frac{1}{48}x^3\sqrt[4]{x^2+x^4} + \frac{1}{6}x^5\sqrt[4]{x^2+x^4} + \frac{\left(21\sqrt[4]{x^2+x^4}\right)\int \frac{\sqrt{x}}{\left(1+x^2\right)^{3/4}}d}{32\sqrt{x}\sqrt[4]{1+x^2}}$$

$$= -\frac{7}{192}x\sqrt[4]{x^2+x^4} + \frac{1}{48}x^3\sqrt[4]{x^2+x^4} + \frac{1}{6}x^5\sqrt[4]{x^2+x^4} - \frac{\left(77\sqrt[4]{x^2+x^4}\right)\int \frac{\sqrt{x}}{\left(1+x^2\right)^{3/4}}}{128\sqrt{x}\sqrt[4]{1+x^2}}$$

$$= -\frac{7}{192}x\sqrt[4]{x^2+x^4} + \frac{1}{48}x^3\sqrt[4]{x^2+x^4} + \frac{1}{6}x^5\sqrt[4]{x^2+x^4} - \frac{\sqrt[4]{x^2+x^4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{2^{3/4}\sqrt{x}\sqrt[4]{1+x^2}}$$

$$= -\frac{7}{192}x\sqrt[4]{x^2+x^4} + \frac{1}{48}x^3\sqrt[4]{x^2+x^4} + \frac{1}{6}x^5\sqrt[4]{x^2+x^4} - \frac{\sqrt[4]{x^2+x^4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{2^{3/4}\sqrt{x}\sqrt[4]{1+x^2}}$$

$$= -\frac{7}{192}x\sqrt[4]{x^2+x^4} + \frac{1}{48}x^3\sqrt[4]{x^2+x^4} + \frac{1}{6}x^5\sqrt[4]{x^2+x^4} - \frac{21\sqrt[4]{x^2+x^4}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right.}{32\sqrt{x}\sqrt[4]{1+x^2}}$$

$$= -\frac{7}{192}x\sqrt[4]{x^2+x^4} + \frac{1}{48}x^3\sqrt[4]{x^2+x^4} + \frac{1}{6}x^5\sqrt[4]{x^2+x^4} - \frac{7\sqrt[4]{x^2+x^4}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{1+x^2}}\right)}{128\sqrt{x}\sqrt[4]{1+x^2}}$$

<u>**Mathematica**</u> [A] time = 0.59, size = 117, normalized size = 1.00

$$\frac{1}{384}x\sqrt[4]{x^4+x^2}\left(64x^4+8x^2+\frac{3\left(\frac{1}{x^2}+1\right)^{3/4}\left(7\tan^{-1}\left(\sqrt[4]{\frac{1}{x^2}+1}\right)+64\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{x^2}+1}}{\sqrt[4]{2}}\right)+7\tanh^{-1}\left(\sqrt[4]{\frac{1}{x^2}+1}\right)+64\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{x^2}+1}}{\sqrt[4]{2}}\right)\right)}{x^2+1}-14\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((x^2 + x^4)^(1/4)*(-1 - x^4 + x^8))/(-1 + x^4),x]
```

```
[Out] (x*(x^2 + x^4)^(1/4)*(-14 + 8*x^2 + 64*x^4 + (3*(1 + x^(-2))^(3/4)*(7*ArcTa
n[(1 + x^(-2))^(1/4)] + 64*2^(1/4)*ArcTan[(1 + x^(-2))^(1/4)/2^(1/4)] + 7*A
rcTanh[(1 + x^(-2))^(1/4)] + 64*2^(1/4)*ArcTanh[(1 + x^(-2))^(1/4)/2^(1/4)]
))/(1 + x^2)))/384
```

IntegrateAlgebraic [A] time = 0.51, size = 117, normalized size = 1.00

$$-\frac{7}{128}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^2}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+x^2}}\right)}{2^{3/4}}+\frac{7}{128}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^2}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+x^2}}\right)}{2^{3/4}}+\frac{1}{192}\sqrt[4]{x^4+x^2}\left(32x^5+4x^3-7x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((x^2 + x^4)^(1/4)*(-1 - x^4 + x^8))/(-1 + x^4),x]
```

```
[Out] ((x^2 + x^4)^(1/4)*(-7*x + 4*x^3 + 32*x^5))/192 - (7*ArcTan[x/(x^2 + x^4)^(
1/4)])/128 - ArcTan[(2^(1/4)*x)/(x^2 + x^4)^(1/4)]/2^(3/4) + (7*ArcTanh[x/(
x^2 + x^4)^(1/4)])/128 + ArcTanh[(2^(1/4)*x)/(x^2 + x^4)^(1/4)]/2^(3/4)
```

fricas [B] time = 8.95, size = 330, normalized size = 2.82

$$\frac{1}{8}\cdot 8^{\frac{3}{4}}\arctan\left(\frac{16\cdot 8^{\frac{1}{4}}\left(x^4+x^2\right)^{\frac{1}{4}}x^2+2^{\frac{3}{4}}\left(8^{\frac{3}{4}}\left(3x^3+x\right)+8\cdot 8^{\frac{1}{4}}\sqrt{x^4+x^2}\,x\right)+4\cdot 8^{\frac{3}{4}}\left(x^4+x^2\right)^{\frac{3}{4}}}{8\left(x^3-x\right)}\right)+\frac{1}{32}\cdot 8^{\frac{3}{4}}\log\left(\frac{4\sqrt{2}\left(x^4+x^2\right)^{\frac{1}{4}}x^2+8^{\frac{3}{4}}\sqrt{x^4+x^2}\,x+8^{\frac{1}{4}}\left(3x^3+x\right)+4\left(x^4+x^2\right)^{\frac{3}{4}}}{x^3-x}\right)-\frac{1}{32}\cdot 8^{\frac{3}{4}}\log\left(\frac{4\sqrt{2}\left(x^4+x^2\right)^{\frac{1}{4}}x^2-8^{\frac{3}{4}}\sqrt{x^4+x^2}\,x-8^{\frac{1}{4}}\left(3x^3+x\right)+4\left(x^4+x^2\right)^{\frac{3}{4}}}{x^3-x}\right)+\frac{1}{192}\left(32x^5+4x^3-7x\right)\left(x^4+x^2\right)^{\frac{1}{4}}-\frac{7}{256}\arctan\left(\frac{2\left(\left(x^4+x^2\right)^{\frac{1}{4}}x^2+\left(x^4+x^2\right)^{\frac{3}{4}}\right)}{x}\right)+\frac{7}{256}\log\left(\frac{2x^3+2\left(x^4+x^2\right)^{\frac{1}{4}}x^2+2\sqrt{x^4+x^2}\,x+x+2\left(x^4+x^2\right)^{\frac{3}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2)^(1/4)*(x^8-x^4-1)/(x^4-1),x, algorithm="fricas")
```

```
[Out] 1/8*8^(3/4)*arctan(1/8*(16*8^(1/4)*(x^4 + x^2)^(1/4)*x^2 + 2^(3/4)*(8^(3/4)
*(3*x^3 + x) + 8*8^(1/4)*sqrt(x^4 + x^2)*x) + 4*8^(3/4)*(x^4 + x^2)^(3/4))/
(x^3 - x)) + 1/32*8^(3/4)*log((4*sqrt(2)*(x^4 + x^2)^(1/4)*x^2 + 8^(3/4)*sq
rt(x^4 + x^2)*x + 8^(1/4)*(3*x^3 + x) + 4*(x^4 + x^2)^(3/4))/(x^3 - x)) - 1
/32*8^(3/4)*log((4*sqrt(2)*(x^4 + x^2)^(1/4)*x^2 - 8^(3/4)*sqrt(x^4 + x^2)*
x - 8^(1/4)*(3*x^3 + x) + 4*(x^4 + x^2)^(3/4))/(x^3 - x)) + 1/192*(32*x^5 +
 4*x^3 - 7*x)*(x^4 + x^2)^(1/4) - 7/256*arctan(2*((x^4 + x^2)^(1/4)*x^2 + (
x^4 + x^2)^(3/4))/x) + 7/256*log((2*x^3 + 2*(x^4 + x^2)^(1/4)*x^2 + 2*sqrt(
x^4 + x^2)*x + x + 2*(x^4 + x^2)^(3/4))/x)
```

giac [A] time = 0.20, size = 123, normalized size = 1.05

$$-\frac{1}{192}\left(7\left(\frac{1}{x^2}+1\right)^{\frac{9}{4}}-18\left(\frac{1}{x^2}+1\right)^{\frac{5}{4}}-21\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)x^6+\frac{1}{2}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)+\frac{1}{4}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)-\frac{1}{4}\cdot 2^{\frac{1}{4}}\log\left(\left|-2^{\frac{1}{4}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right|\right)+\frac{7}{128}\arctan\left(\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)+\frac{7}{256}\log\left(\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}+1\right)-\frac{7}{256}\log\left(\left(\frac{1}{x^2}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2)^(1/4)*(x^8-x^4-1)/(x^4-1),x, algorithm="giac")
```

```
[Out] -1/192*(7*(1/x^2 + 1)^(9/4) - 18*(1/x^2 + 1)^(5/4) - 21*(1/x^2 + 1)^(1/4))*
x^6 + 1/2*2^(1/4)*arctan(1/2*2^(3/4)*(1/x^2 + 1)^(1/4)) + 1/4*2^(1/4)*log(2
^(1/4) + (1/x^2 + 1)^(1/4)) - 1/4*2^(1/4)*log(abs(-2^(1/4) + (1/x^2 + 1)^(1
/4))) + 7/128*arctan((1/x^2 + 1)^(1/4)) + 7/256*log((1/x^2 + 1)^(1/4) + 1)
- 7/256*log((1/x^2 + 1)^(1/4) - 1)
```

maple [F] time = 1.64, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^2\right)^{\frac{1}{4}}\left(x^8-x^4-1\right)}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2)^(1/4)*(x^8-x^4-1)/(x^4-1),x)
```

```
[Out] int((x^4+x^2)^(1/4)*(x^8-x^4-1)/(x^4-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8-x^4-1\right)\left(x^4+x^2\right)^{\frac{1}{4}}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2)^(1/4)*(x^8-x^4-1)/(x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((x^8 - x^4 - 1)*(x^4 + x^2)^(1/4)/(x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^4+x^2\right)^{1/4}\left(-x^8+x^4+1\right)}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^2 + x^4)^(1/4)*(x^4 - x^8 + 1))/(x^4 - 1),x)
```

```
[Out] int(-((x^2 + x^4)^(1/4)*(x^4 - x^8 + 1))/(x^4 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\left(x^2+1\right)}\left(x^8-x^4-1\right)}{(x-1)(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2)**(1/4)*(x**8-x**4-1)/(x**4-1),x)
```

```
[Out] Integral((x**2*(x**2 + 1))**(1/4)*(x**8 - x**4 - 1)/((x - 1)*(x + 1)*(x**2
+ 1)), x)
```

3.1472 $\int \frac{\sqrt{cx^2-x\sqrt{-bx+ax^2}}}{x^3\sqrt{-bx+ax^2}}\,dx$

Optimal. Leaf size=117

$$\frac{4\sqrt{ax^2-bx}\,\left(20ax+15b+4c^2x\right)\sqrt{-x\left(\sqrt{ax^2-bx}-cx\right)}}{105b^2x^3}-\frac{4\left(32acx+3bc-8c^3x\right)\sqrt{-x\left(\sqrt{ax^2-bx}-cx\right)}}{105b^2x^2}$$

Rubi [F] time = 3.92, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{cx^2-x\sqrt{-bx+ax^2}}}{x^3\sqrt{-bx+ax^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c*x^2 - x*Sqrt[-(b*x) + a*x^2]]/(x^3*Sqrt[-(b*x) + a*x^2]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[-b + a*x]*Defer[Subst][Defer[Int][Sqrt[c*x^4 - x^2*Sqrt[-(b
*x^2) + a*x^4]]/(x^6*Sqrt[-b + a*x^2]), x], x, Sqrt[x]])/Sqrt[-(b*x) + a*x^
2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2-x\sqrt{-bx+ax^2}}}{x^3\sqrt{-bx+ax^2}}\,dx &= \frac{\left(\sqrt{x}\,\sqrt{-b+ax}\right)\int \frac{\sqrt{cx^2-x\sqrt{-bx+ax^2}}}{x^{7/2}\sqrt{-b+ax}}\,dx}{\sqrt{-bx+ax^2}}\\ &= \frac{\left(2\sqrt{x}\,\sqrt{-b+ax}\right)\text{Subst}\left(\int \frac{\sqrt{cx^4-x^2\sqrt{-bx^2+ax^4}}}{x^6\sqrt{-b+ax^2}}\,dx,x,\sqrt{x}\right)}{\sqrt{-bx+ax^2}}\end{aligned}$$

Mathematica [F] time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2-x\sqrt{-bx+ax^2}}}{x^3\sqrt{-bx+ax^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c*x^2 - x*Sqrt[-(b*x) + a*x^2]]/(x^3*Sqrt[-(b*x) + a*x^2]),x
]
```

```
[Out] Integrate[Sqrt[c*x^2 - x*Sqrt[-(b*x) + a*x^2]]/(x^3*Sqrt[-(b*x) + a*x^2]),
x]
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2 - x*Sqrt[-(b*x) + a*x^2]]/(x^3*Sqrt[-(b*x) +
a*x^2]),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.43, size = 78, normalized size = 0.67

$$-\frac{4\left(3\,bcx-8\left(c^3-4\,ac\right)x^2-\sqrt{ax^2-bx}\left(4\left(c^2+5\,a\right)x+15\,b\right)\right)\sqrt{cx^2-\sqrt{ax^2-bx}\,x}}{105\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2-x*(a*x^2-b*x)^(1/2))^(1/2)/x^3/(a*x^2-b*x)^(1/2),x, algori
thm="fricas")
```

```
[Out] -4/105*(3*b*c*x - 8*(c^3 - 4*a*c)*x^2 - sqrt(a*x^2 - b*x)*(4*(c^2 + 5*a)*x
+ 15*b))*sqrt(c*x^2 - sqrt(a*x^2 - b*x)*x)/(b^2*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2-\sqrt{ax^2-bx}\,x}}{\sqrt{ax^2-bx}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2-x*(a*x^2-b*x)^(1/2))^(1/2)/x^3/(a*x^2-b*x)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(c*x^2 - sqrt(a*x^2 - b*x)*x)/(sqrt(a*x^2 - b*x)*x^3), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^2-x\sqrt{a\,x^2-bx}}}{x^3\sqrt{a\,x^2-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2-x*(a*x^2-b*x)^(1/2))^(1/2)/x^3/(a*x^2-b*x)^(1/2),x)
```

```
[Out] int((c*x^2-x*(a*x^2-b*x)^(1/2))^(1/2)/x^3/(a*x^2-b*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2-\sqrt{ax^2-bx}\,x}}{\sqrt{ax^2-bx}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2-x*(a*x^2-b*x)^(1/2))^(1/2)/x^3/(a*x^2-b*x)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 - sqrt(a*x^2 - b*x)*x)/(sqrt(a*x^2 - b*x)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2-x\,\sqrt{a\,x^2-b\,x}}}{x^3\,\sqrt{a\,x^2-b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2 - x*(a*x^2 - b*x)^(1/2))^(1/2)/(x^3*(a*x^2 - b*x)^(1/2)),x)
```

```
[Out] int((c*x^2 - x*(a*x^2 - b*x)^(1/2))^(1/2)/(x^3*(a*x^2 - b*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(cx-\sqrt{ax^2-bx}\right)}}{x^3\sqrt{x\,(ax-b)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2-x*(a*x**2-b*x)**(1/2))**(1/2)/x**3/(a*x**2-b*x)**(1/2),x)
[Out] Integral(sqrt(x*(c*x - sqrt(a*x**2 - b*x)))/(x**3*sqrt(x*(a*x - b))), x)
```

3.1473 $\int \frac{b+x^3}{\sqrt[3]{a+x^3}}\,dx$

Optimal. Leaf size=118

$$\frac{1}{9}(a-3b)\log\left(\sqrt[3]{a+x^3}-x\right)+\frac{1}{9}\left(3\sqrt{3}\,b-\sqrt{3}\,a\right)\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{a+x^3}+x}\right)+\frac{1}{18}(3b-a)\log\left(x\sqrt[3]{a+x^3}+\left(a+x^3\right)^{2/3}+\right.$$

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 0.62, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {388, 239}

$$\frac{1}{6}(a-3b)\log\left(\sqrt[3]{a+x^3}-x\right)-\frac{(a-3b)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{a+x^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}x\left(a+x^3\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(b + x^3)/(a + x^3)^(1/3),x]
```

```
[Out] (x*(a + x^3)^(2/3))/3 - ((a - 3*b)*ArcTan[(1 + (2*x)/(a + x^3)^(1/3))/Sqrt[
3]])/(3*Sqrt[3]) + ((a - 3*b)*Log[-x + (a + x^3)^(1/3)])/6
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{b+x^3}{\sqrt[3]{a+x^3}}\,dx &= \frac{1}{3}x\left(a+x^3\right)^{2/3}-\frac{1}{3}(a-3b)\int \frac{1}{\sqrt[3]{a+x^3}}\,dx\\ &= \frac{1}{3}x\left(a+x^3\right)^{2/3}-\frac{(a-3b)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{a+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}(a-3b)\log\left(-x+\sqrt[3]{a+x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.09, size = 98, normalized size = 0.83

$$\frac{1}{3}\left(x\left(a+x^3\right)^{2/3}-\frac{1}{6}(a-3b)\left(-2\log\left(1-\frac{x}{\sqrt[3]{a+x^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{a+x^3}}+1}{\sqrt{3}}\right)+\log\left(\frac{x}{\sqrt[3]{a+x^3}}+\frac{x^2}{\left(a+x^3\right)^{2/3}}+1\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + x^3)/(a + x^3)^(1/3),x]
```

```
[Out] (x*(a + x^3)^(2/3) - ((a - 3*b)*(2*Sqrt[3]*ArcTan[(1 + (2*x)/(a + x^3)^(1/3
))/Sqrt[3]] - 2*Log[1 - x/(a + x^3)^(1/3)] + Log[1 + x^2/(a + x^3)^(2/3) +
x/(a + x^3)^(1/3)]))/6)/3
```

IntegrateAlgebraic [A] time = 0.28, size = 118, normalized size = 1.00

$$\frac{1}{9}(a-3b)\log\left(\sqrt[3]{a+x^3}-x\right)+\frac{1}{9}\left(3\sqrt{3}b-\sqrt{3}a\right)\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{a+x^3}+x}\right)+\frac{1}{18}(3b-a)\log\left(x\sqrt[3]{a+x^3}+\left(a+x^3\right)^{2/3}+x^2\right)+\frac{1}{3}x\left(a+x^3\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + x^3)/(a + x^3)^(1/3),x]
```

```
[Out] (x*(a + x^3)^(2/3))/3 + ((-(Sqrt[3]*a) + 3*Sqrt[3]*b)*ArcTan[(Sqrt[3]*x)/(x
 + 2*(a + x^3)^(1/3))])/9 + ((a - 3*b)*Log[-x + (a + x^3)^(1/3)])/9 + ((-a
+ 3*b)*Log[x^2 + x*(a + x^3)^(1/3) + (a + x^3)^(2/3)])/18
```

fricas [A] time = 0.41, size = 101, normalized size = 0.86

$$\frac{1}{9}\sqrt{3}(a-3b)\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+a\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{9}(a-3b)\log\left(-\frac{x-\left(x^3+a\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{18}(a-3b)\log\left(\frac{x^2+\left(x^3+a\right)^{\frac{1}{3}}x+\left(x^3+a\right)^{\frac{2}{3}}}{x^2}\right)+\frac{1}{3}\left(x^3+a\right)^{\frac{2}{3}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+b)/(x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*(a - 3*b)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 + a)^(1/3))/x)
 + 1/9*(a - 3*b)*log(-(x - (x^3 + a)^(1/3))/x) - 1/18*(a - 3*b)*log((x^2 +
(x^3 + a)^(1/3)*x + (x^3 + a)^(2/3))/x^2) + 1/3*(x^3 + a)^(2/3)*x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3+b}{\left(x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+b)/(x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 + b)/(x^3 + a)^(1/3), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int\frac{x^3+b}{\left(x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+b)/(x^3+a)^(1/3),x)
```

```
[Out] int((x^3+b)/(x^3+a)^(1/3),x)
```

maxima [A] time = 0.41, size = 170, normalized size = 1.44

$$\frac{1}{9}\sqrt{3}a\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+a\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{1}{6}\left(2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+a\right)^{\frac{1}{3}}}{x}+1\right)\right)-\log\left(\frac{\left(x^3+a\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+a\right)^{\frac{2}{3}}}{x^2}+1\right)+2\log\left(\frac{\left(x^3+a\right)^{\frac{1}{3}}}{x}-1\right)\right)b-\frac{1}{18}a\log\left(\frac{\left(x^3+a\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+a\right)^{\frac{2}{3}}}{x^2}+1\right)+\frac{1}{9}a\log\left(\frac{\left(x^3+a\right)^{\frac{1}{3}}}{x}-1\right)+\frac{\left(x^3+a\right)^{\frac{2}{3}}a}{3x^2\left(\frac{x^3+a}{x^3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+b)/(x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*(x^3 + a)^(1/3)/x + 1)) - 1/6*(2*sqrt(3
)*arctan(1/3*sqrt(3)*(2*(x^3 + a)^(1/3)/x + 1)) - log((x^3 + a)^(1/3)/x + (
x^3 + a)^(2/3)/x^2 + 1) + 2*log((x^3 + a)^(1/3)/x - 1))*b - 1/18*a*log((x^3
 + a)^(1/3)/x + (x^3 + a)^(2/3)/x^2 + 1) + 1/9*a*log((x^3 + a)^(1/3)/x - 1)
 + 1/3*(x^3 + a)^(2/3)*a/(x^2*((x^3 + a)/x^3 - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3+b}{\left(x^3+a\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + x^3)/(a + x^3)^(1/3),x)
```

```
[Out] int((b + x^3)/(a + x^3)^(1/3), x)
```

sympy [C] time = 1.90, size = 73, normalized size = 0.62

$$\frac{bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{x^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{x^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+b)/(x**3+a)**(1/3),x)
```

```
[Out] b*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_polar(I*pi)/a)/(3*a**(1/3
)*gamma(4/3)) + x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), x**3*exp_polar(I*
pi)/a)/(3*a**(1/3)*gamma(7/3))
```

3.1474 $\int \frac{b+ax^2}{\sqrt[3]{x+x^3}}\,dx$

Optimal. Leaf size=118

$$\frac{1}{6}(a-3b)\log\left(\sqrt[3]{x^3+x}-x\right)+\frac{1}{6}\left(3\sqrt{3}\,b-\sqrt{3}\,a\right)\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3+x}+x}\right)+\frac{1}{12}(3b-a)\log\left(\sqrt[3]{x^3+x}\,x+\left(x^3+x\right)^{2/3}\right)$$

Rubi [A] time = 0.16, antiderivative size = 223, normalized size of antiderivative = 1.89, number of steps used = 11, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2053, 2011, 329, 275, 239, 2024}

$$\frac{1}{2}a\left(x^3+x\right)^{2/3}+\frac{a\sqrt[3]{x}\,\sqrt[3]{x^2+1}\log\left(x^{2/3}-\sqrt[3]{x^2+1}\right)}{4\sqrt[3]{x^3+x}}-\frac{a\sqrt[3]{x}\,\sqrt[3]{x^2+1}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)}{2\sqrt{3}\,\sqrt[3]{x^3+x}}-\frac{3b\sqrt[3]{x}\,\sqrt[3]{x^2+1}\log\left(x^{2/3}-\sqrt[3]{x^2+1}\right)}{4\sqrt[3]{x^3+x}}+\frac{\sqrt{3}\,b\sqrt[3]{x}\,\sqrt[3]{x^2+1}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^2)/(x + x^3)^(1/3),x]
```

```
[Out] (a*(x + x^3)^(2/3))/2 - (a*x^(1/3)*(1 + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3))/
(1 + x^2)^(1/3))/Sqrt[3]])/(2*Sqrt[3]*(x + x^3)^(1/3)) + (Sqrt[3]*b*x^(1/3)
*(1 + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3))/(1 + x^2)^(1/3))/Sqrt[3]])/(2*(x +
 x^3)^(1/3)) + (a*x^(1/3)*(1 + x^2)^(1/3)*Log[x^(2/3) - (1 + x^2)^(1/3)])/(
4*(x + x^3)^(1/3)) - (3*b*x^(1/3)*(1 + x^2)^(1/3)*Log[x^(2/3) - (1 + x^2)^(
1/3)])/(4*(x + x^3)^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
```

```
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2053

```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Expan
dIntegrand[Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && (Po
lyQ[Pq, x] || PolyQ[Pq, x^n]) && !IntegerQ[p] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{b+ax^2}{\sqrt[3]{x+x^3}}\,dx &= \int \left(\frac{b}{\sqrt[3]{x+x^3}}+\frac{ax^2}{\sqrt[3]{x+x^3}}\right)dx \\
&= a\int \frac{x^2}{\sqrt[3]{x+x^3}}\,dx + b\int \frac{1}{\sqrt[3]{x+x^3}}\,dx \\
&= \frac{1}{2}a\left(x+x^3\right)^{2/3} - \frac{1}{3}a\int \frac{1}{\sqrt[3]{x+x^3}}\,dx + \frac{\left(b\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\int \frac{1}{\sqrt[3]{x}\sqrt[3]{1+x^2}}\,dx}{\sqrt[3]{x+x^3}} \\
&= \frac{1}{2}a\left(x+x^3\right)^{2/3} - \frac{\left(a\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\int \frac{1}{\sqrt[3]{x}\sqrt[3]{1+x^2}}\,dx}{3\sqrt[3]{x+x^3}} + \frac{\left(3b\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\operatorname{Subst}\left(\int \frac{x}{\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x+x^3}} \\
&= \frac{1}{2}a\left(x+x^3\right)^{2/3} - \frac{\left(a\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\operatorname{Subst}\left(\int \frac{x}{\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x+x^3}} + \frac{\left(3b\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^{\ldots}}}\right.}{2\sqrt[3]{x+x^3}} \\
&= \frac{1}{2}a\left(x+x^3\right)^{2/3} + \frac{\sqrt{3}\,b\sqrt[3]{x}\sqrt[3]{1+x^2}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{x+x^3}} - \frac{3b\sqrt[3]{x}\sqrt[3]{1+x^2}\log\left(x^{2/3}-\sqrt[3]{1+x^2}\right)}{4\sqrt[3]{x+x^3}} - \frac{(a\ldots}{} \\
&= \frac{1}{2}a\left(x+x^3\right)^{2/3} - \frac{a\sqrt[3]{x}\sqrt[3]{1+x^2}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\sqrt[3]{x+x^3}} + \frac{\sqrt{3}\,b\sqrt[3]{x}\sqrt[3]{1+x^2}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{x+x^3}} + \frac{a\sqrt[3]{x}\sqrt[3]{1\ldots}}{}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 136, normalized size = 1.15

$$\frac{\sqrt[3]{x}\left(ax^{2/3}\left(x^2+1\right)-\frac{1}{6}\sqrt[3]{x^2+1}\,(a-3b)\left(-2\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2+1}}\right)+\log\left(\frac{x^{4/3}}{\left(x^2+1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2+1}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)\right)\right)}{2\sqrt[3]{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^2)/(x + x^3)^(1/3),x]
```

```
[Out] (x^(1/3)*(a*x^(2/3)*(1 + x^2) - ((a - 3*b)*(1 + x^2)^(1/3)*(2*Sqrt[3]*ArcTa
n[(1 + (2*x^(2/3))/(1 + x^2)^(1/3))/Sqrt[3]] - 2*Log[1 - x^(2/3)/(1 + x^2)^
(1/3)] + Log[1 + x^(4/3)/(1 + x^2)^(2/3) + x^(2/3)/(1 + x^2)^(1/3)]))/6))/(
2*(x + x^3)^(1/3))
```

IntegrateAlgebraic [A] time = 0.34, size = 118, normalized size = 1.00

$$\frac{1}{6}(a-3b)\log\left(\sqrt[3]{x^3+x}-x\right)+\frac{1}{6}\left(3\sqrt{3}\,b-\sqrt{3}\,a\right)\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3+x}+x}\right)+\frac{1}{12}(3b-a)\log\left(\sqrt[3]{x^3+x}\,x+\left(x^3+x\right)^{2/3}+x^2\right)+\frac{1}{2}a\left(x^3+x\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^2)/(x + x^3)^(1/3),x]
```

```
[Out] (a*(x + x^3)^(2/3))/2 + ((-(Sqrt[3]*a) + 3*Sqrt[3]*b)*ArcTan[(Sqrt[3]*x)/(x
 + 2*(x + x^3)^(1/3))])/6 + ((a - 3*b)*Log[-x + (x + x^3)^(1/3)])/6 + ((-a
+ 3*b)*Log[x^2 + x*(x + x^3)^(1/3) + (x + x^3)^(2/3)])/12
```

fricas [A] time = 83.61, size = 100, normalized size = 0.85

$$-\frac{1}{6}\sqrt{3}(a-3b)\arctan\left(-\frac{196\sqrt{3}\left(x^3+x\right)^{\frac{1}{3}}x-\sqrt{3}\left(539x^2+507\right)-1274\sqrt{3}\left(x^3+x\right)^{\frac{2}{3}}}{2205x^2+2197}\right)+\frac{1}{12}(a-3b)\log\left(3\left(x^3+x\right)^{\frac{1}{3}}x-3\left(x^3+x\right)^{\frac{2}{3}}+1\right)+\frac{1}{2}\left(x^3+x\right)^{\frac{2}{3}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(x^3+x)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*(a - 3*b)*arctan(-(196*sqrt(3)*(x^3 + x)^(1/3)*x - sqrt(3)*(53
9*x^2 + 507) - 1274*sqrt(3)*(x^3 + x)^(2/3))/(2205*x^2 + 2197)) + 1/12*(a -
 3*b)*log(3*(x^3 + x)^(1/3)*x - 3*(x^3 + x)^(2/3) + 1) + 1/2*(x^3 + x)^(2/3
)*a
```

giac [A] time = 0.24, size = 83, normalized size = 0.70

$$\frac{1}{2}ax^2\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\frac{1}{6}\sqrt{3}(a-3b)\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{12}(a-3b)\log\left(\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}(a-3b)\log\left(\left|\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(x^3+x)^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*a*x^2*(1/x^2 + 1)^(2/3) + 1/6*sqrt(3)*(a - 3*b)*arctan(1/3*sqrt(3)*(2*(
1/x^2 + 1)^(1/3) + 1)) - 1/12*(a - 3*b)*log((1/x^2 + 1)^(2/3) + (1/x^2 + 1)
^(1/3) + 1) + 1/6*(a - 3*b)*log(abs((1/x^2 + 1)^(1/3) - 1))
```

maple [C] time = 0.30, size = 54, normalized size = 0.46

$$\frac{ax\left(x^2+1\right)}{2\left(x\left(x^2+1\right)\right)^{\frac{1}{3}}}+\frac{3b\,x^{\frac{2}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x^2\right)}{2}-\frac{a\,x^{\frac{2}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)/(x^3+x)^(1/3),x)
```

```
[Out] 1/2*a*x*(x^2+1)/(x*(x^2+1))^(1/3)+3/2*b*x^(2/3)*hypergeom([1/3,1/3],[4/3],-
x^2)-1/2*a*x^(2/3)*hypergeom([1/3,1/3],[4/3],-x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax^2+b}{\left(x^3+x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(x^3+x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 + b)/(x^3 + x)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a\,x^2+b}{{\left(x^3+x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^2)/(x + x^3)^(1/3),x)
```

```
[Out] int((b + a*x^2)/(x + x^3)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + b}{\sqrt[3]{x\left(x^2 + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b)/(x**3+x)**(1/3),x)
```

```
[Out] Integral((a*x**2 + b)/(x*(x**2 + 1))**(1/3), x)
```

3.1475 $\int \sqrt[3]{-x^2+x^3}\,dx$

Optimal. Leaf size=118

$$\frac{1}{6}\sqrt[3]{x^3-x^2}\,(3x-1)+\frac{1}{9}\log\left(\sqrt[3]{x^3-x^2}-x\right)-\frac{1}{18}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x^2}+x}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.09, antiderivative size = 174, normalized size of antiderivative = 1.47, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2004, 2024, 2032, 59}

$$\frac{1}{2}\sqrt[3]{x^3-x^2}\,x-\frac{1}{6}\sqrt[3]{x^3-x^2}+\frac{(x-1)^{2/3}x^{4/3}\log\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x-1}}-1\right)}{6\left(x^3-x^2\right)^{2/3}}+\frac{(x-1)^{2/3}x^{4/3}\log(x-1)}{18\left(x^3-x^2\right)^{2/3}}+\frac{(x-1)^{2/3}x^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{x-1}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\left(x^3-x^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + x^3)^(1/3),x]
```

```
[Out] -1/6*(-x^2 + x^3)^(1/3) + (x*(-x^2 + x^3)^(1/3))/2 + ((-1 + x)^(2/3)*x^(4/3
)*ArcTan[1/Sqrt[3] + (2*x^(1/3))/(Sqrt[3]*(-1 + x)^(1/3))])/(3*Sqrt[3]*(-x^
2 + x^3)^(2/3)) + ((-1 + x)^(2/3)*x^(4/3)*Log[-1 + x^(1/3)/(-1 + x)^(1/3)])
/(6*(-x^2 + x^3)^(2/3)) + ((-1 + x)^(2/3)*x^(4/3)*Log[-1 + x])/(18*(-x^2 +
x^3)^(2/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \sqrt[3]{-x^2+x^3}\,dx = \frac{1}{2}x\sqrt[3]{-x^2+x^3} - \frac{1}{6}\int \frac{x^2}{\left(-x^2+x^3\right)^{2/3}}\,dx$$
$$= -\frac{1}{6}\sqrt[3]{-x^2+x^3} + \frac{1}{2}x\sqrt[3]{-x^2+x^3} - \frac{1}{9}\int \frac{x}{\left(-x^2+x^3\right)^{2/3}}\,dx$$
$$= -\frac{1}{6}\sqrt[3]{-x^2+x^3} + \frac{1}{2}x\sqrt[3]{-x^2+x^3} - \frac{\left((-1+x)^{2/3}x^{4/3}\right)\int \frac{1}{(-1+x)^{2/3}\sqrt[3]{x}}\,dx}{9\left(-x^2+x^3\right)^{2/3}}$$
$$= -\frac{1}{6}\sqrt[3]{-x^2+x^3} + \frac{1}{2}x\sqrt[3]{-x^2+x^3} + \frac{(-1+x)^{2/3}x^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{-1+x}}\right)}{3\sqrt{3}\left(-x^2+x^3\right)^{2/3}} + \frac{(-1+x)^{2/3}x^{4/3}\,\mathrm{lo}}{6\left(-x^2+\right.}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.30

$$\frac{3\left((x-1)x^2\right)^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{4}{3};\frac{7}{3};1-x\right)}{4x^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + x^3)^(1/3),x]
```

```
[Out] (3*((-1 + x)*x^2)^(4/3)*Hypergeometric2F1[-2/3, 4/3, 7/3, 1 - x])/(4*x^(8/3
))
```

IntegrateAlgebraic [A] time = 0.20, size = 118, normalized size = 1.00

$$\frac{1}{6}\sqrt[3]{x^3-x^2}\,(3x-1)+\frac{1}{9}\log\left(\sqrt[3]{x^3-x^2}-x\right)-\frac{1}{18}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x^2}+x}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^2 + x^3)^(1/3),x]
```

```
[Out] ((-1 + 3*x)*(-x^2 + x^3)^(1/3))/6 + ArcTan[(Sqrt[3]*x)/(x + 2*(-x^2 + x^3)^
(1/3))]/(3*Sqrt[3]) + Log[-x + (-x^2 + x^3)^(1/3)]/9 - Log[x^2 + x*(-x^2 +
x^3)^(1/3) + (-x^2 + x^3)^(2/3)]/18
```

fricas [A] time = 0.40, size = 110, normalized size = 0.93

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-x^2\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{6}\left(x^3-x^2\right)^{\frac{1}{3}}(3x-1)+\frac{1}{9}\log\left(-\frac{x-\left(x^3-x^2\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{18}\log\left(\frac{x^2+\left(x^3-x^2\right)^{\frac{1}{3}}x+\left(x^3-x^2\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 - x^2)^(1/3))/x) + 1/6*
(x^3 - x^2)^(1/3)*(3*x - 1) + 1/9*log(-(x - (x^3 - x^2)^(1/3))/x) - 1/18*lo
g((x^2 + (x^3 - x^2)^(1/3)*x + (x^3 - x^2)^(2/3))/x^2)
```

giac [A] time = 0.18, size = 89, normalized size = 0.75

$$\frac{1}{6}\left(\left(-\frac{1}{x}+1\right)^{\frac{4}{3}}+2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}\right)x^2-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{18}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{9}\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)^(1/3),x, algorithm="giac")
```

```
[Out] 1/6*((-1/x + 1)^(4/3) + 2*(-1/x + 1)^(1/3))*x^2 - 1/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*(-1/x + 1)^(1/3) + 1)) - 1/18*log((-1/x + 1)^(2/3) + (-1/x + 1)^(1
/3) + 1) + 1/9*log(abs((-1/x + 1)^(1/3) - 1))
```

maple [C] time = 0.34, size = 27, normalized size = 0.23

$$\frac{3\mathrm{signum}\left(-1+x\right)^{\frac{1}{3}} x^{\frac{5}{3}} \mathrm{hypergeom}\left(\left[-\frac{1}{3},\frac{5}{3}\right],\left[\frac{8}{3}\right],x\right)}{5\left(-\mathrm{signum}\left(-1+x\right)\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x^2)^(1/3),x)
```

```
[Out] 3/5*signum(-1+x)^(1/3)/(-signum(-1+x))^(1/3)*x^(5/3)*hypergeom([-1/3,5/3],[
8/3],x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^3-x^2\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - x^2)^(1/3), x)
```

mupad [B] time = 1.03, size = 27, normalized size = 0.23

$$\frac{3\,x\,{\left(x^3-x^2\right)}^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{5}{3};\ \frac{8}{3};\ x\right)}{5\,{\left(1-x\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - x^2)^(1/3),x)
```

```
[Out] (3*x*(x^3 - x^2)^(1/3)*hypergeom([-1/3, 5/3], 8/3, x))/(5*(1 - x)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{x^3-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x**2)**(1/3),x)
```

```
[Out] Integral((x**3 - x**2)**(1/3), x)
```

3.1476 $\int x^2 \sqrt[3]{x^2+x^3}\,dx$

Optimal. Leaf size=118

$$\frac{1}{324}\sqrt[3]{x^3+x^2}\left(81x^3+9x^2-12x+20\right)+\frac{10}{243}\log\left(\sqrt[3]{x^3+x^2}-x\right)-\frac{5}{243}\log\left(x^2+\sqrt[3]{x^3+x^2}\,x+\left(x^3+x^2\right)^{2/3}\right)+\frac{10\,\mathrm{t}}{}$$

Rubi [A] time = 0.19, antiderivative size = 200, normalized size of antiderivative = 1.69, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2021, 2024, 2032, 59}

$$\frac{1}{4}\sqrt[3]{x^3+x^2}\,x^3+\frac{1}{36}\sqrt[3]{x^3+x^2}\,x^2-\frac{1}{27}\sqrt[3]{x^3+x^2}\,x+\frac{5}{81}\sqrt[3]{x^3+x^2}+\frac{5(x+1)^{2/3}x^{4/3}\log(x+1)}{243\left(x^3+x^2\right)^{2/3}}+\frac{5(x+1)^{2/3}x^{4/3}\log\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x+1}}-1\right)}{81\left(x^3+x^2\right)^{2/3}}+\frac{10(x+1)^{2/3}x^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{x+1}}+\frac{1}{\sqrt{3}}\right)}{81\sqrt{3}\left(x^3+x^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(x^2 + x^3)^(1/3),x]
```

```
[Out] (5*(x^2 + x^3)^(1/3))/81 - (x*(x^2 + x^3)^(1/3))/27 + (x^2*(x^2 + x^3)^(1/3
))/36 + (x^3*(x^2 + x^3)^(1/3))/4 + (10*x^(4/3)*(1 + x)^(2/3)*ArcTan[1/Sqrt
[3] + (2*x^(1/3))/(Sqrt[3]*(1 + x)^(1/3))])/(81*Sqrt[3]*(x^2 + x^3)^(2/3))
+ (5*x^(4/3)*(1 + x)^(2/3)*Log[1 + x])/(243*(x^2 + x^3)^(2/3)) + (5*x^(4/3)
*(1 + x)^(2/3)*Log[-1 + x^(1/3)/(1 + x)^(1/3)])/(81*(x^2 + x^3)^(2/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int x^2\sqrt[3]{x^2+x^3}\,dx &= \frac{1}{4}x^3\sqrt[3]{x^2+x^3} + \frac{1}{12}\int \frac{x^4}{\left(x^2+x^3\right)^{2/3}}\,dx \\
&= \frac{1}{36}x^2\sqrt[3]{x^2+x^3} + \frac{1}{4}x^3\sqrt[3]{x^2+x^3} - \frac{2}{27}\int \frac{x^3}{\left(x^2+x^3\right)^{2/3}}\,dx \\
&= -\frac{1}{27}x\sqrt[3]{x^2+x^3} + \frac{1}{36}x^2\sqrt[3]{x^2+x^3} + \frac{1}{4}x^3\sqrt[3]{x^2+x^3} + \frac{5}{81}\int \frac{x^2}{\left(x^2+x^3\right)^{2/3}}\,dx \\
&= \frac{5}{81}\sqrt[3]{x^2+x^3} - \frac{1}{27}x\sqrt[3]{x^2+x^3} + \frac{1}{36}x^2\sqrt[3]{x^2+x^3} + \frac{1}{4}x^3\sqrt[3]{x^2+x^3} - \frac{10}{243}\int \frac{x}{\left(x^2+x^3\right)^{2/3}}\,dx \\
&= \frac{5}{81}\sqrt[3]{x^2+x^3} - \frac{1}{27}x\sqrt[3]{x^2+x^3} + \frac{1}{36}x^2\sqrt[3]{x^2+x^3} + \frac{1}{4}x^3\sqrt[3]{x^2+x^3} - \frac{\left(10x^{4/3}(1+x)^{2/3}\right)\int \frac{}{\sqrt[3]{x}}}{243\left(x^2+x^3\right)^{2/}} \\
&= \frac{5}{81}\sqrt[3]{x^2+x^3} - \frac{1}{27}x\sqrt[3]{x^2+x^3} + \frac{1}{36}x^2\sqrt[3]{x^2+x^3} + \frac{1}{4}x^3\sqrt[3]{x^2+x^3} + \frac{10x^{4/3}(1+x)^{2/3}\tan^{-1}\left(\right.}{81\sqrt{3}\left(x^2+\right.}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.32

$$
\frac{3x^3\sqrt[3]{x^2(x+1)}\,{}_2F_1\left(-\frac{1}{3},\frac{11}{3};\frac{14}{3};-x\right)}{11\sqrt[3]{x+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(x^2 + x^3)^(1/3),x]
```

```
[Out] (3*x^3*(x^2*(1 + x))^(1/3)*Hypergeometric2F1[-1/3, 11/3, 14/3, -x])/(11*(1
+ x)^(1/3))
```

IntegrateAlgebraic [A] time = 0.35, size = 118, normalized size = 1.00

$$
\frac{1}{324}\sqrt[3]{x^3+x^2}\left(81x^3+9x^2-12x+20\right)+\frac{10}{243}\log\left(\sqrt[3]{x^3+x^2}-x\right)-\frac{5}{243}\log\left(x^2+\sqrt[3]{x^3+x^2}\,x+\left(x^3+x^2\right)^{2/3}\right)+\frac{10\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x^2}+x}\right)}{81\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(x^2 + x^3)^(1/3),x]
```

```
[Out] ((x^2 + x^3)^(1/3)*(20 - 12*x + 9*x^2 + 81*x^3))/324 + (10*ArcTan[(Sqrt[3]*
x)/(x + 2*(x^2 + x^3)^(1/3))])/(81*Sqrt[3]) + (10*Log[-x + (x^2 + x^3)^(1/3
)])/243 - (5*Log[x^2 + x*(x^2 + x^3)^(1/3) + (x^2 + x^3)^(2/3)])/243
```

fricas [A] time = 0.40, size = 110, normalized size = 0.93

$$
-\frac{10}{243}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+x^2\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{324}\left(81x^3+9x^2-12x+20\right)\left(x^3+x^2\right)^{\frac{1}{3}}+\frac{10}{243}\log\left(-\frac{x-\left(x^3+x^2\right)^{\frac{1}{3}}}{x}\right)-\frac{5}{243}\log\left(\frac{x^2+\left(x^3+x^2\right)^{\frac{1}{3}}x+\left(x^3+x^2\right)^{\frac{2}{3}}}{x^2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -10/243*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 + x^2)^(1/3))/x) + 1
/324*(81*x^3 + 9*x^2 - 12*x + 20)*(x^3 + x^2)^(1/3) + 10/243*log(-(x - (x^3
 + x^2)^(1/3))/x) - 5/243*log((x^2 + (x^3 + x^2)^(1/3)*x + (x^3 + x^2)^(2/3
))/x^2)
```

giac [A] time = 0.19, size = 97, normalized size = 0.82

$$\frac{1}{324}\left(20\left(\frac{1}{x}+1\right)^{\frac{10}{3}}-72\left(\frac{1}{x}+1\right)^{\frac{7}{3}}+93\left(\frac{1}{x}+1\right)^{\frac{4}{3}}+40\left(\frac{1}{x}+1\right)^{\frac{1}{3}}\right)x^4-\frac{10}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{5}{243}\log\left(\left(\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)+\frac{10}{243}\log\left(\left|\left(\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] 1/324*(20*(1/x + 1)^(10/3) - 72*(1/x + 1)^(7/3) + 93*(1/x + 1)^(4/3) + 40*(
1/x + 1)^(1/3))*x^4 - 10/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(1/x + 1)^(1/3)
+ 1)) - 5/243*log((1/x + 1)^(2/3) + (1/x + 1)^(1/3) + 1) + 10/243*log(abs((
1/x + 1)^(1/3) - 1))
```

maple [C] time = 0.50, size = 461, normalized size = 3.91

$$\frac{\left(81x^3+9x^2-12x+20\right)\left(x^2(1+x)\right)^{\frac{1}{3}}}{324}+\frac{\left(\frac{10\ln\left(-\frac{\operatorname{RootOf}(\_Z^2+2\_Z+4)^2x^2+48\operatorname{RootOf}(\_Z^2+2\_Z+4)(x^3+2x^2+x)^{\frac{2}{3}}-30\operatorname{RootOf}(\_Z^2+2\_Z+4)(x^3+2x^2+x)^{\frac{1}{3}}x-16\operatorname{RootOf}(\_Z^2+2\_Z+4)x^2+36(x^3+2x^2+x)^{\frac{2}{3}}-30\operatorname{RootOf}(\_Z^2+2\_Z+4)(x^3+2x^2+x)^{\frac{1}{3}}-96(x^3+2x^2+x)^{\frac{1}{3}}x-\operatorname{RootOf}(\_Z^2+2\_Z+4)^2-14\operatorname{RootOf}(\_Z^2+2\_Z+4)x+64x^2-96(x^3+2x^2+x)^{\frac{1}{3}}+2\operatorname{RootOf}(\_Z^2+2\_Z+4)+112x+48}{1+x}\right)}{243}+\frac{5\operatorname{RootOf}(\_Z^2+2\_Z+4)\ln\left(\frac{-2\operatorname{RootOf}(\_Z^2+2\_Z+4)^2x^2+24\operatorname{RootOf}(\_Z^2+2\_Z+4)(x^3+2x^2+x)^{\frac{2}{3}}-9\operatorname{RootOf}(\_Z^2+2\_Z+4)(x^3+2x^2+x)^{\frac{1}{3}}x-19\operatorname{RootOf}(\_Z^2+2\_Z+4)x^2+30(x^3+2x^2+x)^{\frac{2}{3}}-9\operatorname{RootOf}(\_Z^2+2\_Z+4)(x^3+2x^2+x)^{\frac{1}{3}}-48(x^3+2x^2+x)^{\frac{1}{3}}x+2\operatorname{RootOf}(\_Z^2+2\_Z+4)^2-28\operatorname{RootOf}(\_Z^2+2\_Z+4)x+10x^2-48(x^3+2x^2+x)^{\frac{1}{3}}-9\operatorname{RootOf}(\_Z^2+2\_Z+4)+14x+4}{1+x}\right)}{243}\right)\left(x^2(1+x)\right)^{\frac{1}{3}}\left(x(1+x)^2\right)^{\frac{1}{3}}}{x(1+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3+x^2)^(1/3),x)
```

```
[Out] 1/324*(81*x^3+9*x^2-12*x+20)*(x^2*(1+x))^(1/3)+(10/243*ln(-(RootOf(_Z^2+2*_
Z+4)^2*x^2+48*RootOf(_Z^2+2*_Z+4)*(x^3+2*x^2+x)^(2/3)-30*RootOf(_Z^2+2*_Z+4
)*(x^3+2*x^2+x)^(1/3)*x-16*RootOf(_Z^2+2*_Z+4)*x^2+36*(x^3+2*x^2+x)^(2/3)-3
0*RootOf(_Z^2+2*_Z+4)*(x^3+2*x^2+x)^(1/3)-96*(x^3+2*x^2+x)^(1/3)*x-RootOf(_
Z^2+2*_Z+4)^2-14*RootOf(_Z^2+2*_Z+4)*x+64*x^2-96*(x^3+2*x^2+x)^(1/3)+2*Root
Of(_Z^2+2*_Z+4)+112*x+48)/(1+x))+5/243*RootOf(_Z^2+2*_Z+4)*ln((-2*RootOf(_Z
^2+2*_Z+4)^2*x^2+24*RootOf(_Z^2+2*_Z+4)*(x^3+2*x^2+x)^(2/3)-9*RootOf(_Z^2+2
*_Z+4)*(x^3+2*x^2+x)^(1/3)*x-19*RootOf(_Z^2+2*_Z+4)*x^2+30*(x^3+2*x^2+x)^(2
/3)-9*RootOf(_Z^2+2*_Z+4)*(x^3+2*x^2+x)^(1/3)-48*(x^3+2*x^2+x)^(1/3)*x+2*Ro
otOf(_Z^2+2*_Z+4)^2-28*RootOf(_Z^2+2*_Z+4)*x+10*x^2-48*(x^3+2*x^2+x)^(1/3)-
9*RootOf(_Z^2+2*_Z+4)+14*x+4)/(1+x)))/x*(x^2*(1+x))^(1/3)*(x*(1+x)^2)^(1/3)
/(1+x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^3+x^2\right)^{\frac{1}{3}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + x^2)^(1/3)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(x^3+x^2\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2 + x^3)^(1/3),x)
```

```
[Out] int(x^2*(x^2 + x^3)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt[3]{x^2(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+x**2)**(1/3),x)
```

```
[Out] Integral(x**2*(x**2*(x + 1))**(1/3), x)
```

3.1477 $\int \frac{(-3+x^4)(1-x^3+x^4)(1+x^3+x^4)^{2/3}}{x^6(1+x^4)} dx$

Optimal. Leaf size=118

$$-\log\left(\sqrt[3]{x^4+x^3+1}-x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+x^3+1}+x}\right)+\frac{3\left(x^4+x^3+1\right)^{2/3}\left(2x^4-3x^3+2\right)}{10x^5}+\frac{1}{2}\log\left(x^2+\sqrt[3]{}\right.$$

Rubi [F] time = 1.63, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-3+x^4)(1-x^3+x^4)(1+x^3+x^4)^{2/3}}{x^6(1+x^4)} dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + x^4)*(1 - x^3 + x^4)*(1 + x^3 + x^4)^(2/3))/(x^6*(1 + x^4)),x]
```

```
[Out] (-I)*Defer[Int][(1 + x^3 + x^4)^(2/3)/((-1)^(1/4) - x), x] + I*Defer[Int][(
1 + x^3 + x^4)^(2/3)/(-(-1)^(3/4) - x), x] - 3*Defer[Int][(1 + x^3 + x^4)^(
2/3)/x^6, x] + 3*Defer[Int][(1 + x^3 + x^4)^(2/3)/x^3, x] + Defer[Int][(1 +
 x^3 + x^4)^(2/3)/x^2, x] + I*Defer[Int][(1 + x^3 + x^4)^(2/3)/((-1)^(1/4)
+ x), x] - I*Defer[Int][(1 + x^3 + x^4)^(2/3)/(-(-1)^(3/4) + x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-3+x^4)(1-x^3+x^4)(1+x^3+x^4)^{2/3}}{x^6(1+x^4)} dx &= \int \left(-\frac{3(1+x^3+x^4)^{2/3}}{x^6}+\frac{3(1+x^3+x^4)^{2/3}}{x^3}+\frac{(1+x^3+x^4)^{2/}}{x^2}\right. \\
&= -\left(3\int \frac{(1+x^3+x^4)^{2/3}}{x^6} dx\right)+3\int \frac{(1+x^3+x^4)^{2/3}}{x^3} dx-4\int \\
&= -\left(3\int \frac{(1+x^3+x^4)^{2/3}}{x^6} dx\right)+3\int \frac{(1+x^3+x^4)^{2/3}}{x^3} dx-4\int \\
&= 2i\int \frac{x(1+x^3+x^4)^{2/3}}{-i+x^2} dx-2i\int \frac{x(1+x^3+x^4)^{2/3}}{i+x^2} dx-3\int \\
&= 2i\int \left(-\frac{(1+x^3+x^4)^{2/3}}{2(\sqrt[4]{-1}-x)}+\frac{(1+x^3+x^4)^{2/3}}{2(\sqrt[4]{-1}+x)}\right) dx-2i\int \left(-\frac{(1-}{2(-}\right. \\
&= -\left(i\int \frac{(1+x^3+x^4)^{2/3}}{\sqrt[4]{-1}-x} dx\right)+i\int \frac{(1+x^3+x^4)^{2/3}}{-(-1)^{3/4}-x} dx+i\int
\end{aligned}$$

Mathematica [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{(-3+x^4)(1-x^3+x^4)(1+x^3+x^4)^{2/3}}{x^6(1+x^4)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + x^4)*(1 - x^3 + x^4)*(1 + x^3 + x^4)^(2/3))/(x^6*(1 + x^4)
),x]
```

```
[Out] Integrate[((-3 + x^4)*(1 - x^3 + x^4)*(1 + x^3 + x^4)^(2/3))/(x^6*(1 + x^4)
), x]
```

IntegrateAlgebraic [A] time = 2.00, size = 118, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^4+x^3+1}-x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+x^3+1}+x}\right)+\frac{3\left(x^4+x^3+1\right)^{2/3}\left(2x^4-3x^3+2\right)}{10x^5}+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^4+x^3+1}\,x+\left(x^4+x^3+1\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 - x^3 + x^4)*(1 + x^3 + x^4)^(2/3))/(x^6*
(1 + x^4)),x]
```

```
[Out] (3*(1 + x^3 + x^4)^(2/3)*(2 - 3*x^3 + 2*x^4))/(10*x^5) + Sqrt[3]*ArcTan[(Sq
rt[3]*x)/(x + 2*(1 + x^3 + x^4)^(1/3))] - Log[-x + (1 + x^3 + x^4)^(1/3)] +
 Log[x^2 + x*(1 + x^3 + x^4)^(1/3) + (1 + x^3 + x^4)^(2/3)]/2
```

fricas [A] time = 2.81, size = 153, normalized size = 1.30

$$\frac{10\sqrt{3}x^5\arctan\left(-\frac{7043582\sqrt{3}\left(x^4+x^3+1\right)^{\frac{1}{3}}x^2-984256\sqrt{3}\left(x^4+x^3+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(145408x^4+3029663x^3+145408\right)}{32768x^4+12041757x^3+32768}\right)-5x^5\log\left(\frac{x^4+3\left(x^4+x^3+1\right)^{\frac{1}{3}}x^2-3\left(x^4+x^3+1\right)^{\frac{2}{3}}x+1}{x^4+1}\right)+3\left(2x^4-3x^3+2\right)\left(x^4+x^3+1\right)^{\frac{2}{3}}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4-x^3+1)*(x^4+x^3+1)^(2/3)/x^6/(x^4+1),x, algorithm="f
ricas")
```

```
[Out] 1/10*(10*sqrt(3)*x^5*arctan(-(7043582*sqrt(3)*(x^4 + x^3 + 1)^(1/3)*x^2 - 9
84256*sqrt(3)*(x^4 + x^3 + 1)^(2/3)*x + sqrt(3)*(145408*x^4 + 3029663*x^3 +
 145408))/(32768*x^4 + 12041757*x^3 + 32768)) - 5*x^5*log((x^4 + 3*(x^4 + x
^3 + 1)^(1/3)*x^2 - 3*(x^4 + x^3 + 1)^(2/3)*x + 1)/(x^4 + 1)) + 3*(2*x^4 -
3*x^3 + 2)*(x^4 + x^3 + 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^4+x^3+1\right)^{\frac{2}{3}}\left(x^4-x^3+1\right)\left(x^4-3\right)}{\left(x^4+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4-x^3+1)*(x^4+x^3+1)^(2/3)/x^6/(x^4+1),x, algorithm="g
iac")
```

```
[Out] integrate((x^4 + x^3 + 1)^(2/3)*(x^4 - x^3 + 1)*(x^4 - 3)/((x^4 + 1)*x^6),
x)
```

maple [C] time = 2.07, size = 445, normalized size = 3.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4-x^3+1)*(x^4+x^3+1)^(2/3)/x^6/(x^4+1),x)
```

```
[Out] 3/10*(2*x^8-x^7-3*x^6+4*x^4-x^3+2)/x^5/(x^4+x^3+1)^(1/3)+RootOf(_Z^2-_Z+1)*
ln(-(RootOf(_Z^2-_Z+1)^2*x^3-RootOf(_Z^2-_Z+1)*x^4-3*(x^4+x^3+1)^(2/3)*Root
Of(_Z^2-_Z+1)*x-3*(x^4+x^3+1)^(1/3)*RootOf(_Z^2-_Z+1)*x^2-4*RootOf(_Z^2-_Z+
1)*x^3+2*x^4+3*(x^4+x^3+1)^(2/3)*x+3*(x^4+x^3+1)^(1/3)*x^2+4*x^3-RootOf(_Z^
2-_Z+1)+2)/(x^4+1))-ln(-(RootOf(_Z^2-_Z+1)^2*x^3+RootOf(_Z^2-_Z+1)*x^4+3*(x
```

```
^4+x^3+1)^(2/3)*RootOf(_Z^2-_Z+1)*x+3*(x^4+x^3+1)^(1/3)*RootOf(_Z^2-_Z+1)*x
^2+2*RootOf(_Z^2-_Z+1)*x^3+x^4+x^3+RootOf(_Z^2-_Z+1)+1)/(x^4+1))*RootOf(_Z^
2-_Z+1)+ln(-(RootOf(_Z^2-_Z+1)^2*x^3+RootOf(_Z^2-_Z+1)*x^4+3*(x^4+x^3+1)^(2
/3)*RootOf(_Z^2-_Z+1)*x+3*(x^4+x^3+1)^(1/3)*RootOf(_Z^2-_Z+1)*x^2+2*RootOf(
_Z^2-_Z+1)*x^3+x^4+x^3+RootOf(_Z^2-_Z+1)+1)/(x^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3+1\right)^{\frac{2}{3}}\left(x^4-x^3+1\right)\left(x^4-3\right)}{\left(x^4+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4-x^3+1)*(x^4+x^3+1)^(2/3)/x^6/(x^4+1),x, algorithm="m
axima")
```

```
[Out] integrate((x^4 + x^3 + 1)^(2/3)*(x^4 - x^3 + 1)*(x^4 - 3)/((x^4 + 1)*x^6),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-3\right)\,\left(x^4+x^3+1\right)^{2/3}\,\left(x^4-x^3+1\right)}{x^6\,\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 3)*(x^3 + x^4 + 1)^(2/3)*(x^4 - x^3 + 1))/(x^6*(x^4 + 1)),x)
```

```
[Out] int(((x^4 - 3)*(x^3 + x^4 + 1)^(2/3)*(x^4 - x^3 + 1))/(x^6*(x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-3\right)\left(x^4-x^3+1\right)\left(x^4+x^3+1\right)^{\frac{2}{3}}}{x^6\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4-x**3+1)*(x**4+x**3+1)**(2/3)/x**6/(x**4+1),x)
```

```
[Out] Integral((x**4 - 3)*(x**4 - x**3 + 1)*(x**4 + x**3 + 1)**(2/3)/(x**6*(x**4
+ 1)), x)
```

3.1478 $\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt[3]{x+x^5}}\,dx$

Optimal. Leaf size=118

$$\frac{\log\left(2^{2/3}\sqrt[3]{x^5+x}-2x\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^5+x}+x}\right)}{2\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{x^5+x}\,x+\sqrt[3]{2}\left(x^5+x\right)^{2/3}+2x^2\right)}{4\sqrt[3]{2}}$$

Rubi [C] time = 0.60, antiderivative size = 123, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2056, 6715, 6725, 245, 1438, 429, 465, 510}

$$-\frac{3\sqrt[3]{x^4+1}\,xF_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};x^4,-x^4\right)}{\sqrt[3]{x^5+x}}-\frac{3\sqrt[3]{x^4+1}\,x^3F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};x^4,-x^4\right)}{4\sqrt[3]{x^5+x}}+\frac{3\sqrt[3]{x^4+1}\,x\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x^5+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^2)/((-1 + x^2)*(x + x^5)^(1/3)),x]
```

```
[Out] (-3*x*(1 + x^4)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6, x^4, -x^4])/(x + x^5)^(1/3
) - (3*x^3*(1 + x^4)^(1/3)*AppellF1[2/3, 1, 1/3, 5/3, x^4, -x^4])/(4*(x + x
^5)^(1/3)) + (3*x*(1 + x^4)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -x^4])/(
2*(x + x^5)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
```

```
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt[3]{x+x^5}}\,dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\int \frac{1+x^2}{\sqrt[3]{x}\left(-1+x^2\right)\sqrt[3]{1+x^4}}\,dx}{\sqrt[3]{x+x^5}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1+x^3}{\left(-1+x^3\right)\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{x+x^5}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1+x^6}}+\frac{2}{\left(-1+x^3\right)\sqrt[3]{1+x^6}}\right)dx, x, x^{2/3}\right)}{2\sqrt[3]{x+x^5}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{x+x^5}} + \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(-1+x^3\right)\sqrt[3]{1-\right.}}\right.}{\sqrt[3]{x+x^5}}$$

$$= \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} + \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int\left(\frac{1}{\left(-1+x^6\right)\sqrt[3]{1+x^6}}+\frac{x^3}{\left(-1+x^6\right)\sqrt[3]{\right.}}\right.\right.}{\sqrt[3]{x+x^5}}$$

$$= \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} + \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(-1+x^6\right)\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{\sqrt[3]{x+x^5}}$$

$$= -\frac{3x\sqrt[3]{1+x^4}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};x^4,-x^4\right)}{\sqrt[3]{x+x^5}} + \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} + \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x\right.}}{}$$

$$= -\frac{3x\sqrt[3]{1+x^4}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};x^4,-x^4\right)}{\sqrt[3]{x+x^5}} - \frac{3x^3\sqrt[3]{1+x^4}\,F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};x^4,-x^4\right)}{4\sqrt[3]{x+x^5}} + \frac{3x\sqrt[3]{1+}}{}$$

Mathematica [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt[3]{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^2)/((-1 + x^2)*(x + x^5)^(1/3)),x]
```

```
[Out] Integrate[(1 + x^2)/((-1 + x^2)*(x + x^5)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.39, size = 118, normalized size = 1.00

$$\frac{\log \left(2^{2/3}\sqrt[3]{x^5+x}-2x\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^5+x}+x}\right)}{2\sqrt[3]{2}}-\frac{\log \left(2^{2/3}\sqrt[3]{x^5+x}\,x+\sqrt[3]{2}\left(x^5+x\right)^{2/3}+2x^2\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 + x^2)*(x + x^5)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(x + x^5)^(1/3))])/2^(1/3) +
Log[-2*x + 2^(2/3)*(x + x^5)^(1/3)]/(2*2^(1/3)) - Log[2*x^2 + 2^(2/3)*x*(x
+ x^5)^(1/3) + 2^(1/3)*(x + x^5)^(2/3)]/(4*2^(1/3))
```

fricas [B] time = 4.14, size = 294, normalized size = 2.49

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(-\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}\left(x^{12}-24x^{10}-57x^{8}-56x^{6}-57x^{4}-24x^{2}+1\right)+24\sqrt{2}\left(x^{9}-x^{7}-x^{3}+x\right)\left(x^{5}+x\right)^{\frac{1}{3}}-12\cdot 2^{\frac{1}{6}}\left(x^{8}+14x^{6}+6x^{4}+14x^{2}+1\right)\left(x^{5}+x\right)^{\frac{2}{3}}\right)}{6\left(x^{12}+48x^{10}+15x^{8}+88x^{6}+15x^{4}+48x^{2}+1\right)}\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}\left(x^{8}+14x^{6}+6x^{4}+14x^{2}+1\right)+12\cdot 2^{\frac{1}{3}}\left(x^{5}+x^{3}+x\right)\left(x^{5}+x\right)^{\frac{1}{3}}+6\left(x^{5}+x\right)^{\frac{2}{3}}\left(x^{4}+4x^{2}+1\right)}{x^{8}-4x^{6}+6x^{4}-4x^{2}+1}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{3\cdot 2^{\frac{2}{3}}\left(x^{5}+x\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(x^{4}-2x^{2}+1\right)-6\left(x^{5}+x\right)^{\frac{1}{3}}x}{x^{4}-2x^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^5+x)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(-1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^12 - 24*x^10
- 57*x^8 - 56*x^6 - 57*x^4 - 24*x^2 + 1) + 24*sqrt(2)*(x^9 - x^7 - x^3 + x)
*(x^5 + x)^(1/3) - 12*2^(1/6)*(x^8 + 14*x^6 + 6*x^4 + 14*x^2 + 1)*(x^5 + x)
^(2/3))/(x^12 + 48*x^10 + 15*x^8 + 88*x^6 + 15*x^4 + 48*x^2 + 1)) - 1/24*2^
(2/3)*log((2^(2/3)*(x^8 + 14*x^6 + 6*x^4 + 14*x^2 + 1) + 12*2^(1/3)*(x^5 +
x^3 + x)*(x^5 + x)^(1/3) + 6*(x^5 + x)^(2/3)*(x^4 + 4*x^2 + 1))/(x^8 - 4*x^
6 + 6*x^4 - 4*x^2 + 1)) + 1/12*2^(2/3)*log((3*2^(2/3)*(x^5 + x)^(2/3) - 2^(
1/3)*(x^4 - 2*x^2 + 1) - 6*(x^5 + x)^(1/3)*x)/(x^4 - 2*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^5+x\right)^{\frac{1}{3}}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^5+x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/((x^5 + x)^(1/3)*(x^2 - 1)), x)
```

maple [C] time = 20.51, size = 951, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-1)/(x^5+x)^(1/3),x)
```

```
[Out] 1/2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*ln(-(-2776889956931
2240*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2
*x^4-5367726430214212*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*R
ootOf(_Z^3-4)^3*x^4+118017823169577020*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(
_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+22812837328410401*RootOf(RootOf(_Z^3
-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x^2+58185583206163764*(x
^5+x)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3
```

```
-4)^2-173555622308201500*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2
)*x^4-33548290188838825*RootOf(_Z^3-4)*x^4+151047510129487038*(x^5+x)^(1/3)
*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x-58185
583206163764*RootOf(_Z^3-4)^2*(x^5+x)^(1/3)*x-27768899569312240*RootOf(Root
Of(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2-536772643021421
2*RootOf(_Z^3-4)^3*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)-1110
75598277248960*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-2147
0905720856848*RootOf(_Z^3-4)*x^2+267418676541814566*(x^5+x)^(2/3)-173555622
308201500*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)-3354829018883
8825*RootOf(_Z^3-4))/(1+x)^2/(-1+x)^2)+1/4*RootOf(_Z^3-4)*ln((-536772643021
4212*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2
*x^4-6942224892328060*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*R
ootOf(_Z^3-4)^3*x^4+22812837328410401*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_
Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+29504455792394255*RootOf(RootOf(_Z^3-
4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x^2+29092791603081882*(x^
5+x)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-
4)^2+22812837328410401*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*
x^4+29504455792394255*RootOf(_Z^3-4)*x^4-133709338270907283*(x^5+x)^(1/3)*R
ootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x-2909279
1603081882*RootOf(_Z^3-4)^2*(x^5+x)^(1/3)*x-5367726430214212*RootOf(RootOf(
_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2-6942224892328060*R
ootOf(_Z^3-4)^3*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+6709658
0377677650*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2+86777811
154100750*RootOf(_Z^3-4)*x^2-75523755064743519*(x^5+x)^(2/3)+22812837328410
401*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+29504455792394255*R
ootOf(_Z^3-4))/(1+x)^2/(-1+x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^5+x\right)^{\frac{1}{3}}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^5+x)^(1/3),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/((x^5 + x)^(1/3)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+1}{\left(x^2-1\right)\left(x^5+x\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((x^2 - 1)*(x + x^5)^(1/3)),x)
[Out] int((x^2 + 1)/((x^2 - 1)*(x + x^5)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt[3]{x\left(x^4+1\right)}\,(x-1)(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-1)/(x**5+x)**(1/3),x)
[Out] Integral((x**2 + 1)/((x*(x**4 + 1))**(1/3)*(x - 1)*(x + 1)), x)
```

3.1479 $\int \frac{(-1+x^2)\sqrt[4]{x^2+x^6}}{x^2(1+x^2)}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt[4]{x^6+x^2}}{x}-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{\sqrt[4]{2}}$$

Rubi [C] time = 0.59, antiderivative size = 147, normalized size of antiderivative = 1.25, number of steps used = 20, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2056, 6725, 277, 329, 364, 1312, 1336, 325, 1337, 466, 510}

$$\frac{8\sqrt[4]{x^6+x^2}\,xF_1\left(\frac{3}{8};1,\frac{3}{4};\frac{11}{8};x^4,-x^4\right)}{3\sqrt[4]{x^4+1}}-\frac{8\sqrt[4]{x^6+x^2}\,x^3F_1\left(\frac{7}{8};1,\frac{3}{4};\frac{15}{8};x^4,-x^4\right)}{7\sqrt[4]{x^4+1}}-\frac{4\sqrt[4]{x^6+x^2}\,x\,{}_2F_1\left(\frac{3}{8},\frac{3}{4};\frac{11}{8};-x^4\right)}{3\sqrt[4]{x^4+1}}+\frac{2\sqrt[4]{x^6+x^2}}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^2)*(x^2 + x^6)^(1/4))/(x^2*(1 + x^2)),x]
```

```
[Out] (2*(x^2 + x^6)^(1/4))/x + (8*x*(x^2 + x^6)^(1/4)*AppellF1[3/8, 1, 3/4, 11/8
, x^4, -x^4])/(3*(1 + x^4)^(1/4)) - (8*x^3*(x^2 + x^6)^(1/4)*AppellF1[7/8,
1, 3/4, 15/8, x^4, -x^4])/(7*(1 + x^4)^(1/4)) - (4*x*(x^2 + x^6)^(1/4)*Hype
rgeometric2F1[3/8, 3/4, 11/8, -x^4])/(3*(1 + x^4)^(1/4))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1312

```
Int[(((f_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^4)^(p_.))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a + c*x^4)^(p - 1)
, x], x] - Dist[(c*d^2 + a*e^2)/(d*e*f^2), Int[((f*x)^(m + 2)*(a + c*x^4)^(
p - 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && Lt
Q[m, 0]
```

Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 1337

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^4)^p, (d
/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4))^(-q), x], x], x] /; FreeQ[{a, c
, d, e, f, m, p}, x] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(-1+x^2\right)\sqrt[4]{x^2+x^6}}{x^2\left(1+x^2\right)}\,dx = \frac{\sqrt[4]{x^2+x^6}\int \frac{\left(-1+x^2\right)\sqrt[4]{1+x^4}}{x^{3/2}\left(1+x^2\right)}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\sqrt[4]{x^2+x^6}\int \left(\frac{\sqrt[4]{1+x^4}}{x^{3/2}}-\frac{2\sqrt[4]{1+x^4}}{x^{3/2}\left(1+x^2\right)}\right)dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\sqrt[4]{x^2+x^6}\int \frac{\sqrt[4]{1+x^4}}{x^{3/2}}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} - \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{\sqrt[4]{1+x^4}}{x^{3/2}\left(1+x^2\right)}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= -\frac{2\sqrt[4]{x^2+x^6}}{x} + \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{x^{5/2}}{\left(1+x^4\right)^{3/4}}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} - \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{1+x^2}{x^{3/2}\left(1+x^4\right)^{3/4}}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(4\sqrt[4]{x^2\cdots\right.}{\cdots}$$

$$= -\frac{2\sqrt[4]{x^2+x^6}}{x} - \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \left(\frac{1}{x^{3/2}\left(1+x^4\right)^{3/4}}+\frac{\sqrt{x}}{\left(1+x^4\right)^{3/4}}\right)dx}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(4\sqrt[4]{x^2+x^6}\right)\int \left(\frac{\cdots}{\left(1-x^4\right)\cdots}\right.}{\sqrt{x}\cdots}$$

$$= -\frac{2\sqrt[4]{x^2+x^6}}{x} + \frac{4x^3\sqrt[4]{x^2+x^6}\,{}_2F_1\left(\frac{3}{4},\frac{7}{8};\frac{15}{8};-x^4\right)}{7\sqrt[4]{1+x^4}} - \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{1}{x^{3/2}\left(1+x^4\right)^{3/4}}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} - \cdots$$

$$= \frac{2\sqrt[4]{x^2+x^6}}{x} + \frac{4x^3\sqrt[4]{x^2+x^6}\,{}_2F_1\left(\frac{3}{4},\frac{7}{8};\frac{15}{8};-x^4\right)}{7\sqrt[4]{1+x^4}} - \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{x^{5/2}}{\left(1+x^4\right)^{3/4}}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} - \frac{\left(4\sqrt[4]{x\cdots\right.}{\cdots}$$

$$= \frac{2\sqrt[4]{x^2+x^6}}{x} + \frac{8x\sqrt[4]{x^2+x^6}\,F_1\left(\frac{3}{8};1,\frac{3}{4};\frac{11}{8};x^4,-x^4\right)}{3\sqrt[4]{1+x^4}} - \frac{8x^3\sqrt[4]{x^2+x^6}\,F_1\left(\frac{7}{8};1,\frac{3}{4};\frac{15}{8};x^4,-\cdots\right.}{7\sqrt[4]{1+x^4}}$$

$$= \frac{2\sqrt[4]{x^2+x^6}}{x} + \frac{8x\sqrt[4]{x^2+x^6}\,F_1\left(\frac{3}{8};1,\frac{3}{4};\frac{11}{8};x^4,-x^4\right)}{3\sqrt[4]{1+x^4}} - \frac{8x^3\sqrt[4]{x^2+x^6}\,F_1\left(\frac{7}{8};1,\frac{3}{4};\frac{15}{8};x^4,-\cdots\right.}{7\sqrt[4]{1+x^4}}$$

Mathematica [F] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^2\right)\sqrt[4]{x^2+x^6}}{x^2\left(1+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^2)*(x^2 + x^6)^(1/4))/(x^2*(1 + x^2)),x]
```

```
[Out] Integrate[((-1 + x^2)*(x^2 + x^6)^(1/4))/(x^2*(1 + x^2)), x]
```

IntegrateAlgebraic [A] time = 0.48, size = 118, normalized size = 1.00

$$\frac{2\sqrt[4]{x^6+x^2}}{x} - \frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{\sqrt[4]{2}} - \frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*(x^2 + x^6)^(1/4))/(x^2*(1 + x^2)),x]
```

```
[Out] (2*(x^2 + x^6)^(1/4))/x - ArcTan[(2^(3/4)*x*(x^2 + x^6)^(1/4))/(Sqrt[2]*x^2
 - Sqrt[x^2 + x^6])]/2^(1/4) - ArcTanh[(x^2/2^(1/4) + Sqrt[x^2 + x^6]/2^(3/
4))/(x*(x^2 + x^6)^(1/4))]/2^(1/4)
```

fricas [B] time = 7.32, size = 545, normalized size = 4.62

$$-\frac{4\cdot 2^{\frac{3}{4}}x\arctan\left(-\frac{4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2-\sqrt{2}\left(2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2-\sqrt{2}\left(x^5+2x^3+x\right)-4\sqrt{x^6+x^2}\,x+2\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}\right)\sqrt{\frac{x^5+2x^3+4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2+4\sqrt{2}\sqrt{x^6+x^2}\,x+2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}+x}{x^5+2x^3+x}}+2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}}{2\left(x^5-2x^3+x\right)}\right)+4\cdot 2^{\frac{3}{4}}x\arctan\left(-\frac{4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2-\sqrt{2}\left(2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2+\sqrt{2}\left(x^5+2x^3+x\right)+4\sqrt{x^6+x^2}\,x+2\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}\right)\sqrt{\frac{x^5+2x^3-4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2+4\sqrt{2}\sqrt{x^6+x^2}\,x-2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}+x}{x^5+2x^3+x}}+2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}}{2\left(x^5-2x^3+x\right)}\right)+2^{\frac{3}{4}}x\log\left(\frac{2\left(x^5+2x^3+4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2+4\sqrt{2}\sqrt{x^6+x^2}\,x+2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}+x\right)}{x^5+2x^3+x}\right)-2^{\frac{3}{4}}x\log\left(\frac{2\left(x^5+2x^3-4\cdot 2^{\frac{1}{4}}\left(x^6+x^2\right)^{\frac{1}{4}}x^2+4\sqrt{2}\sqrt{x^6+x^2}\,x-2\cdot 2^{\frac{3}{4}}\left(x^6+x^2\right)^{\frac{3}{4}}+x\right)}{x^5+2x^3+x}\right)-16\left(x^6+x^2\right)^{\frac{1}{4}}}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^6+x^2)^(1/4)/x^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/8*(4*2^(3/4)*x*arctan(-1/2*(4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(2
*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 + 2*x^3 + x) - 4*sqrt(x^6 + x
^2)*x + 2*2^(1/4)*(x^6 + x^2)^(3/4))*sqrt((x^5 + 2*x^3 + 4*2^(1/4)*(x^6 + x
^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x + 2*2^(3/4)*(x^6 + x^2)^(3/4) +
 x)/(x^5 + 2*x^3 + x)) + 2*2^(3/4)*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) +
4*2^(3/4)*x*arctan(-1/2*(4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(2*2^(3/
4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 4*sqrt(x^6 + x^2)*x
+ 2*2^(1/4)*(x^6 + x^2)^(3/4))*sqrt((x^5 + 2*x^3 - 4*2^(1/4)*(x^6 + x^2)^(1
/4)*x^2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x - 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x
^5 + 2*x^3 + x)) + 2*2^(3/4)*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 2^(3/4
)*x*log(2*(x^5 + 2*x^3 + 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x
^6 + x^2)*x + 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x)) - 2^(3/4)
*x*log(2*(x^5 + 2*x^3 - 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^
6 + x^2)*x - 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x)) - 16*(x^6
+ x^2)^(1/4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^2-1\right)}{\left(x^2+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^6+x^2)^(1/4)/x^2/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^2)^(1/4)*(x^2 - 1)/((x^2 + 1)*x^2), x)
```

maple [F] time = 5.81, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-1\right)\left(x^6+x^2\right)^{\frac{1}{4}}}{x^2\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(x^6+x^2)^(1/4)/x^2/(x^2+1),x)
```

```
[Out] int((x^2-1)*(x^6+x^2)^(1/4)/x^2/(x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^2-1\right)}{\left(x^2+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^6+x^2)^(1/4)/x^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + x^2)^(1/4)*(x^2 - 1)/((x^2 + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6+x^2\right)^{1/4}\,\left(x^2-1\right)}{x^2\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + x^6)^(1/4)*(x^2 - 1))/(x^2*(x^2 + 1)),x)
[Out] int(((x^2 + x^6)^(1/4)*(x^2 - 1))/(x^2*(x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\left(x^4+1\right)}\,(x-1)\,(x+1)}{x^2\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**6+x**2)**(1/4)/x**2/(x**2+1),x)
[Out] Integral((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)/(x**2*(x**2 + 1)), x)
```

3.1480 $\int \frac{\left(-1+x^2\right) \sqrt[4]{x^2+x^6}}{x^2\left(1+x^2\right)} \, dx$

Optimal. Leaf size=118

$$\frac{2\sqrt[4]{x^6+x^2}}{x}-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{\sqrt[4]{2}}$$

Rubi [C] time = 0.55, antiderivative size = 147, normalized size of antiderivative = 1.25, number of steps used = 20, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2056, 6725, 277, 329, 364, 1312, 1336, 325, 1337, 466, 510}

$$\frac{8\sqrt[4]{x^6+x^2}\,xF_1\left(\frac{3}{8};1,\frac{3}{4};\frac{11}{8};x^4,-x^4\right)}{3\sqrt[4]{x^4+1}}-\frac{8\sqrt[4]{x^6+x^2}\,x^3F_1\left(\frac{7}{8};1,\frac{3}{4};\frac{15}{8};x^4,-x^4\right)}{7\sqrt[4]{x^4+1}}-\frac{4\sqrt[4]{x^6+x^2}\,x\,{}_2F_1\left(\frac{3}{8},\frac{3}{4};\frac{11}{8};-x^4\right)}{3\sqrt[4]{x^4+1}}+\frac{2\sqrt[4]{x^6+x^2}}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^2)*(x^2 + x^6)^(1/4))/(x^2*(1 + x^2)),x]
```

```
[Out] (2*(x^2 + x^6)^(1/4))/x + (8*x*(x^2 + x^6)^(1/4)*AppellF1[3/8, 1, 3/4, 11/8
, x^4, -x^4])/(3*(1 + x^4)^(1/4)) - (8*x^3*(x^2 + x^6)^(1/4)*AppellF1[7/8,
1, 3/4, 15/8, x^4, -x^4])/(7*(1 + x^4)^(1/4)) - (4*x*(x^2 + x^6)^(1/4)*Hype
rgeometric2F1[3/8, 3/4, 11/8, -x^4])/(3*(1 + x^4)^(1/4))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1312

```
Int[(((f_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^4)^(p_.))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a + c*x^4)^(p - 1)
, x], x] - Dist[(c*d^2 + a*e^2)/(d*e*f^2), Int[((f*x)^(m + 2)*(a + c*x^4)^(
p - 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && Lt
Q[m, 0]
```

Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 1337

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^4)^p, (d
/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4))^(-q), x], x], x] /; FreeQ[{a, c
, d, e, f, m, p}, x] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(-1+x^2)\sqrt[4]{x^2+x^6}}{x^2(1+x^2)}\,dx = \frac{\sqrt[4]{x^2+x^6}\int \frac{(-1+x^2)\sqrt[4]{1+x^4}}{x^{3/2}(1+x^2)}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$
$$= \frac{\sqrt[4]{x^2+x^6}\int \left(\frac{\sqrt[4]{1+x^4}}{x^{3/2}} - \frac{2\sqrt[4]{1+x^4}}{x^{3/2}(1+x^2)}\right)dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$
$$= \frac{\sqrt[4]{x^2+x^6}\int \frac{\sqrt[4]{1+x^4}}{x^{3/2}}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} - \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{\sqrt[4]{1+x^4}}{x^{3/2}(1+x^2)}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$
$$= -\frac{2\sqrt[4]{x^2+x^6}}{x} + \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{x^{5/2}}{(1+x^4)^{3/4}}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} - \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{1+x^2}{x^{3/2}(1+x^4)^{3/4}}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{(4}{}$$
$$= -\frac{2\sqrt[4]{x^2+x^6}}{x} - \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \left(\frac{1}{x^{3/2}(1+x^4)^{3/4}} + \frac{\sqrt{x}}{(1+x^4)^{3/4}}\right)dx}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(4\sqrt[4]{x^2+x^6}\right)\int \left(\frac{}{(1-}\right.}{}$$
$$= -\frac{2\sqrt[4]{x^2+x^6}}{x} + \frac{4x^3\sqrt[4]{x^2+x^6}\,{}_2F_1\left(\frac{3}{4},\frac{7}{8};\frac{15}{8};-x^4\right)}{7\sqrt[4]{1+x^4}} - \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{1}{x^{3/2}(1+x^4)^{3/4}}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$
$$= \frac{2\sqrt[4]{x^2+x^6}}{x} + \frac{4x^3\sqrt[4]{x^2+x^6}\,{}_2F_1\left(\frac{3}{4},\frac{7}{8};\frac{15}{8};-x^4\right)}{7\sqrt[4]{1+x^4}} - \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{x^{5/2}}{(1+x^4)^{3/4}}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} - \frac{(4}{}$$
$$= \frac{2\sqrt[4]{x^2+x^6}}{x} + \frac{8x\sqrt[4]{x^2+x^6}\,F_1\left(\frac{3}{8};1,\frac{3}{4};\frac{11}{8};x^4,-x^4\right)}{3\sqrt[4]{1+x^4}} - \frac{8x^3\sqrt[4]{x^2+x^6}\,F_1\left(\frac{7}{8};1,\frac{3}{4};\frac{15}{8};x\right.}{7\sqrt[4]{1+x^4}}$$
$$= \frac{2\sqrt[4]{x^2+x^6}}{x} + \frac{8x\sqrt[4]{x^2+x^6}\,F_1\left(\frac{3}{8};1,\frac{3}{4};\frac{11}{8};x^4,-x^4\right)}{3\sqrt[4]{1+x^4}} - \frac{8x^3\sqrt[4]{x^2+x^6}\,F_1\left(\frac{7}{8};1,\frac{3}{4};\frac{15}{8};x\right.}{7\sqrt[4]{1+x^4}}$$

Mathematica [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^2)\sqrt[4]{x^2+x^6}}{x^2(1+x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^2)*(x^2 + x^6)^(1/4))/(x^2*(1 + x^2)),x]
[Out] Integrate[((-1 + x^2)*(x^2 + x^6)^(1/4))/(x^2*(1 + x^2)), x]
```

IntegrateAlgebraic [A] time = 0.00, size = 118, normalized size = 1.00

$$\frac{2\sqrt[4]{x^6+x^2}}{x} - \frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{\sqrt[4]{2}} - \frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*(x^2 + x^6)^(1/4))/(x^2*(1 + x^2)),x]
```

```
[Out] (2*(x^2 + x^6)^(1/4))/x - ArcTan[(2^(3/4)*x*(x^2 + x^6)^(1/4))/(Sqrt[2]*x^2
 - Sqrt[x^2 + x^6])]/2^(1/4) - ArcTanh[(x^2/2^(1/4) + Sqrt[x^2 + x^6]/2^(3/
4))/(x*(x^2 + x^6)^(1/4))]/2^(1/4)
```

fricas [B] time = 7.40, size = 545, normalized size = 4.62

$$-\frac{4\cdot 2^{\frac{3}{4}} x \arctan\left(-\frac{4\cdot 2^{\frac{1}{4}}\left(x^{6}+x^{2}\right)^{\frac{1}{4}} x^{2}-\sqrt{2}\left(2\cdot 2^{\frac{3}{4}}\left(x^{6}+x^{2}\right)^{\frac{1}{4}} x^{2}-\sqrt{2}\left(x^{5}+2 x^{3}+x\right)-4 \sqrt{x^{6}+x^{2}}\, x+2\cdot 2^{\frac{1}{4}}\left(x^{6}+x^{2}\right)^{\frac{3}{4}}\right) \sqrt{\frac{x^{5}+2 x^{3}+4\cdot 2^{\frac{1}{4}}\left(x^{6}+x^{2}\right)^{\frac{1}{4}} x^{2}+4 \sqrt{2} \sqrt{x^{6}+x^{2}}\, x+2\cdot 2^{\frac{3}{4}}\left(x^{6}+x^{2}\right)^{\frac{3}{4}}+x}{x^{5}+2 x^{3}+x}}+2\cdot 2^{\frac{3}{4}}\left(x^{6}+x^{2}\right)^{\frac{3}{4}}}{2\left(x^{5}-2 x^{3}+x\right)}\right)+4\cdot 2^{\frac{3}{4}} x \arctan\left(-\frac{4\cdot 2^{\frac{1}{4}}\left(x^{6}+x^{2}\right)^{\frac{1}{4}} x^{2}-\sqrt{2}\left(2\cdot 2^{\frac{3}{4}}\left(x^{6}+x^{2}\right)^{\frac{1}{4}} x^{2}+\sqrt{2}\left(x^{5}+2 x^{3}+x\right)+4 \sqrt{x^{6}+x^{2}}\, x+2\cdot 2^{\frac{1}{4}}\left(x^{6}+x^{2}\right)^{\frac{3}{4}}\right) \sqrt{\frac{x^{5}+2 x^{3}-4\cdot 2^{\frac{1}{4}}\left(x^{6}+x^{2}\right)^{\frac{1}{4}} x^{2}+4 \sqrt{2} \sqrt{x^{6}+x^{2}}\, x-2\cdot 2^{\frac{3}{4}}\left(x^{6}+x^{2}\right)^{\frac{3}{4}}+x}{x^{5}+2 x^{3}+x}}+2\cdot 2^{\frac{3}{4}}\left(x^{6}+x^{2}\right)^{\frac{3}{4}}}{2\left(x^{5}-2 x^{3}+x\right)}\right)+2^{\frac{3}{4}} x \log\left(\frac{2\left(x^{5}+2 x^{3}+4\cdot 2^{\frac{1}{4}}\left(x^{6}+x^{2}\right)^{\frac{1}{4}} x^{2}+4 \sqrt{2} \sqrt{x^{6}+x^{2}}\, x+2\cdot 2^{\frac{3}{4}}\left(x^{6}+x^{2}\right)^{\frac{3}{4}}+x\right)}{x^{5}+2 x^{3}+x}\right)-2^{\frac{3}{4}} x \log\left(\frac{2\left(x^{5}+2 x^{3}-4\cdot 2^{\frac{1}{4}}\left(x^{6}+x^{2}\right)^{\frac{1}{4}} x^{2}+4 \sqrt{2} \sqrt{x^{6}+x^{2}}\, x-2\cdot 2^{\frac{3}{4}}\left(x^{6}+x^{2}\right)^{\frac{3}{4}}+x\right)}{x^{5}+2 x^{3}+x}\right)-16\left(x^{6}+x^{2}\right)^{\frac{1}{4}}}{8 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^6+x^2)^(1/4)/x^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/8*(4*2^(3/4)*x*arctan(-1/2*(4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(2
*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 + 2*x^3 + x) - 4*sqrt(x^6 + x
^2)*x + 2*2^(1/4)*(x^6 + x^2)^(3/4))*sqrt((x^5 + 2*x^3 + 4*2^(1/4)*(x^6 + x
^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x + 2*2^(3/4)*(x^6 + x^2)^(3/4) +
 x)/(x^5 + 2*x^3 + x)) + 2*2^(3/4)*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) +
4*2^(3/4)*x*arctan(-1/2*(4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(2*2^(3/
4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 4*sqrt(x^6 + x^2)*x
+ 2*2^(1/4)*(x^6 + x^2)^(3/4))*sqrt((x^5 + 2*x^3 - 4*2^(1/4)*(x^6 + x^2)^(1
/4)*x^2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x - 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x
^5 + 2*x^3 + x)) + 2*2^(3/4)*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 2^(3/4
)*x*log(2*(x^5 + 2*x^3 + 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x
^6 + x^2)*x + 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x)) - 2^(3/4)
*x*log(2*(x^5 + 2*x^3 - 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^
6 + x^2)*x - 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x)) - 16*(x^6
+ x^2)^(1/4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}+x^{2}\right)^{\frac{1}{4}}\left(x^{2}-1\right)}{\left(x^{2}+1\right) x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^6+x^2)^(1/4)/x^2/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^2)^(1/4)*(x^2 - 1)/((x^2 + 1)*x^2), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}-1\right)\left(x^{6}+x^{2}\right)^{\frac{1}{4}}}{x^{2}\left(x^{2}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(x^6+x^2)^(1/4)/x^2/(x^2+1),x)
```

```
[Out] int((x^2-1)*(x^6+x^2)^(1/4)/x^2/(x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}+x^{2}\right)^{\frac{1}{4}}\left(x^{2}-1\right)}{\left(x^{2}+1\right) x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^6+x^2)^(1/4)/x^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + x^2)^(1/4)*(x^2 - 1)/((x^2 + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6+x^2\right)^{1/4}\,\left(x^2-1\right)}{x^2\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + x^6)^(1/4)*(x^2 - 1))/(x^2*(x^2 + 1)),x)
[Out] int(((x^2 + x^6)^(1/4)*(x^2 - 1))/(x^2*(x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\left(x^4+1\right)}\,(x-1)\,(x+1)}{x^2\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**6+x**2)**(1/4)/x**2/(x**2+1),x)
[Out] Integral((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)/(x**2*(x**2 + 1)), x)
```

3.1481 $\int \frac{(-1+x^3)^{2/3}(1-5x^3+4x^6)}{x^6(-1+2x^3)^2}\,dx$

Optimal. Leaf size=118

$$\frac{7}{9}\log\left(\sqrt[3]{x^3-1}+x\right)+\frac{7\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}-x}\right)}{3\sqrt{3}}-\frac{7}{18}\log\left(-\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)+\frac{\left(x^3-1\right)^{2/3}\left(62x^6-33x^3+6\right)}{30x^5\left(2x^3-1\right)}$$

Rubi [C] time = 0.48, antiderivative size = 224, normalized size of antiderivative = 1.90, number of steps used = 15, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6742, 264, 277, 239, 378, 377, 200, 31, 634, 618, 204, 628, 430, 429}

$$-\frac{2x\left(x^3-1\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,2x^3\right)}{\left(1-x^3\right)^{2/3}}-\frac{2x\left(x^3-1\right)^{2/3}}{3\left(1-2x^3\right)}+\frac{4}{9}\log\left(\frac{x}{\sqrt[3]{x^3-1}}+1\right)+\frac{1}{2}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{4\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{x^3-1}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\left(x^3-1\right)^{5/3}}{5x^5}+\frac{\left(x^3-1\right)^{2/3}}{2x^2}-\frac{2}{9}\log\left(-\frac{x}{\sqrt[3]{x^3-1}}+\frac{x^2}{\left(x^3-1\right)^{2/3}}+1\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^3)^(2/3)*(1 - 5*x^3 + 4*x^6))/(x^6*(-1 + 2*x^3)^2),x]
```

```
[Out] (-1 + x^3)^(2/3)/(2*x^2) - (2*x*(-1 + x^3)^(2/3))/(3*(1 - 2*x^3)) + (-1 + x
^3)^(5/3)/(5*x^5) - (2*x*(-1 + x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, x^3,
2*x^3])/(1 - x^3)^(2/3) - (4*ArcTan[(1 - (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]])/
(3*Sqrt[3]) - ArcTan[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - (2*Log
[1 + x^2/(-1 + x^3)^(2/3) - x/(-1 + x^3)^(1/3)])/9 + (4*Log[1 + x/(-1 + x^3
)^(1/3)])/9 + Log[-x + (-1 + x^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(1-5x^3+4x^6\right)}{x^6\left(-1+2x^3\right)^2}\,dx = \int \left(\frac{\left(-1+x^3\right)^{2/3}}{x^6} - \frac{\left(-1+x^3\right)^{2/3}}{x^3} - \frac{2\left(-1+x^3\right)^{2/3}}{\left(-1+2x^3\right)^2} + \frac{2\left(-1+x^3\right)^{2/3}}{-1+2x^3}\right) dx$$

$$= -\left(2\int \frac{\left(-1+x^3\right)^{2/3}}{\left(-1+2x^3\right)^2}\,dx\right) + 2\int \frac{\left(-1+x^3\right)^{2/3}}{-1+2x^3}\,dx + \int \frac{\left(-1+x^3\right)^{2/3}}{x^6}\,dx - \int \frac{(}{}$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{2x^2} - \frac{2x\left(-1+x^3\right)^{2/3}}{3\left(1-2x^3\right)} + \frac{\left(-1+x^3\right)^{5/3}}{5x^5} - \frac{4}{3}\int \frac{1}{\sqrt[3]{-1+x^3}\left(-1+2x^3\right)}$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{2x^2} - \frac{2x\left(-1+x^3\right)^{2/3}}{3\left(1-2x^3\right)} + \frac{\left(-1+x^3\right)^{5/3}}{5x^5} - \frac{2x\left(-1+x^3\right)^{2/3} F_1\left(\frac{1}{3};-\frac{2}{3},1\right.}{\left(1-x^3\right)^{2/3}}$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{2x^2} - \frac{2x\left(-1+x^3\right)^{2/3}}{3\left(1-2x^3\right)} + \frac{\left(-1+x^3\right)^{5/3}}{5x^5} - \frac{2x\left(-1+x^3\right)^{2/3} F_1\left(\frac{1}{3};-\frac{2}{3},1\right.}{\left(1-x^3\right)^{2/3}}$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{2x^2} - \frac{2x\left(-1+x^3\right)^{2/3}}{3\left(1-2x^3\right)} + \frac{\left(-1+x^3\right)^{5/3}}{5x^5} - \frac{2x\left(-1+x^3\right)^{2/3} F_1\left(\frac{1}{3};-\frac{2}{3},1\right.}{\left(1-x^3\right)^{2/3}}$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{2x^2} - \frac{2x\left(-1+x^3\right)^{2/3}}{3\left(1-2x^3\right)} + \frac{\left(-1+x^3\right)^{5/3}}{5x^5} - \frac{2x\left(-1+x^3\right)^{2/3} F_1\left(\frac{1}{3};-\frac{2}{3},1\right.}{\left(1-x^3\right)^{2/3}}$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{2x^2} - \frac{2x\left(-1+x^3\right)^{2/3}}{3\left(1-2x^3\right)} + \frac{\left(-1+x^3\right)^{5/3}}{5x^5} - \frac{2x\left(-1+x^3\right)^{2/3} F_1\left(\frac{1}{3};-\frac{2}{3},1\right.}{\left(1-x^3\right)^{2/3}}$$

Mathematica [A] time = 0.22, size = 125, normalized size = 1.06

$$\frac{7}{18}\left(2\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{1-x^3}}-1}{\sqrt{3}}\right)-\log\left(-\frac{x}{\sqrt[3]{1-x^3}}+\frac{x^2}{\left(1-x^3\right)^{2/3}}+1\right)\right)+\frac{\left(x^3-1\right)^{2/3}\left(62x^6-33x^3+6\right)}{30x^5\left(2x^3-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^3)^(2/3)*(1 - 5*x^3 + 4*x^6))/(x^6*(-1 + 2*x^3)^2),x]
```

```
[Out] ((-1 + x^3)^(2/3)*(6 - 33*x^3 + 62*x^6))/(30*x^5*(-1 + 2*x^3)) + (7*(2*Sqrt
[3]*ArcTan[(-1 + (2*x)/(1 - x^3)^(1/3))/Sqrt[3]] - Log[1 + x^2/(1 - x^3)^(2
/3) - x/(1 - x^3)^(1/3)] + 2*Log[1 + x/(1 - x^3)^(1/3)]))/18
```

IntegrateAlgebraic [A] time = 0.27, size = 118, normalized size = 1.00

$$\frac{7}{9}\log\left(\sqrt[3]{x^3-1}+x\right)+\frac{7\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}-x}\right)}{3\sqrt{3}}-\frac{7}{18}\log\left(-\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)+\frac{\left(x^3-1\right)^{2/3}\left(62x^6-33x^3+6\right)}{30x^5\left(2x^3-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(2/3)*(1 - 5*x^3 + 4*x^6))/(x^6*(-1 + 2*x^3)
^2),x]
```

```
[Out] ((-1 + x^3)^(2/3)*(6 - 33*x^3 + 62*x^6))/(30*x^5*(-1 + 2*x^3)) + (7*ArcTan[
(Sqrt[3]*x)/(-x + 2*(-1 + x^3)^(1/3))])/(3*Sqrt[3]) + (7*Log[x + (-1 + x^3)
^(1/3)])/9 - (7*Log[x^2 - x*(-1 + x^3)^(1/3) + (-1 + x^3)^(2/3)])/18
```

fricas [A] time = 1.07, size = 155, normalized size = 1.31

$$-\frac{70\sqrt{3}\left(2x^8-x^5\right)\arctan\left(\frac{4\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}x^2+2\sqrt{3}\left(x^3-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(x^3-1\right)}{7x^3+1}\right)-35\left(2x^8-x^5\right)\log\left(\frac{2x^3+3\left(x^3-1\right)^{\frac{1}{3}}x^2+3\left(x^3-1\right)^{\frac{2}{3}}x-1}{2x^3-1}\right)-3\left(62x^6-33x^3+6\right)\left(x^3-1\right)^{\frac{2}{3}}}{90\left(2x^8-x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(4*x^6-5*x^3+1)/x^6/(2*x^3-1)^2,x, algorithm="frica
s")
```

```
[Out] -1/90*(70*sqrt(3)*(2*x^8 - x^5)*arctan((4*sqrt(3)*(x^3 - 1)^(1/3)*x^2 + 2*s
qrt(3)*(x^3 - 1)^(2/3)*x + sqrt(3)*(x^3 - 1))/(7*x^3 + 1)) - 35*(2*x^8 - x^
5)*log((2*x^3 + 3*(x^3 - 1)^(1/3)*x^2 + 3*(x^3 - 1)^(2/3)*x - 1)/(2*x^3 - 1
)) - 3*(62*x^6 - 33*x^3 + 6)*(x^3 - 1)^(2/3))/(2*x^8 - x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4x^6-5x^3+1\right)\left(x^3-1\right)^{\frac{2}{3}}}{\left(2x^3-1\right)^2x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(4*x^6-5*x^3+1)/x^6/(2*x^3-1)^2,x, algorithm="giac"
)
```

```
[Out] integrate((4*x^6 - 5*x^3 + 1)*(x^3 - 1)^(2/3)/((2*x^3 - 1)^2*x^6), x)
```

maple [C] time = 1.37, size = 407, normalized size = 3.45

$$\frac{62x^9-95x^6+39x^3-6}{30\left(2x^3-1\right)\left(x^3-1\right)^{\frac{1}{3}}x^5}+\frac{7\ln\left(-\frac{3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3-1\right)^{\frac{2}{3}}x+6\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3-1\right)^{\frac{1}{3}}x^2+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3+2x\left(x^3-1\right)^{\frac{2}{3}}+x^2\left(x^3-1\right)^{\frac{1}{3}}+x^3-1}{2x^3-1}\right)}{9}-\frac{7\ln\left(-\frac{9\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3-3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3-1\right)^{\frac{2}{3}}x+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3-1\right)^{\frac{1}{3}}x^2+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3+x\left(x^3-1\right)^{\frac{2}{3}}+2x^2\left(x^3-1\right)^{\frac{1}{3}}+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)+1}{2x^3-1}\right)}{9}-\frac{7\ln\left(-\frac{9\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)^2x^3-3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3-1\right)^{\frac{2}{3}}x+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(x^3-1\right)^{\frac{1}{3}}x^2+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)x^3+x\left(x^3-1\right)^{\frac{2}{3}}+2x^2\left(x^3-1\right)^{\frac{1}{3}}+3\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)+1}{2x^3-1}\right)\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)*(4*x^6-5*x^3+1)/x^6/(2*x^3-1)^2,x)
```

```
[Out] 1/30*(62*x^9-95*x^6+39*x^3-6)/(2*x^3-1)/(x^3-1)^(1/3)/x^5+7/9*ln(-(3*RootOf
(9*_Z^2+3*_Z+1)*(x^3-1)^(2/3)*x+6*RootOf(9*_Z^2+3*_Z+1)*(x^3-1)^(1/3)*x^2+3
*RootOf(9*_Z^2+3*_Z+1)*x^3+2*x*(x^3-1)^(2/3)+x^2*(x^3-1)^(1/3)+x^3-1)/(2*x^
3-1))-7/9*ln(-(9*RootOf(9*_Z^2+3*_Z+1)^2*x^3-3*RootOf(9*_Z^2+3*_Z+1)*(x^3-1
)^(2/3)*x+3*RootOf(9*_Z^2+3*_Z+1)*(x^3-1)^(1/3)*x^2+3*RootOf(9*_Z^2+3*_Z+1)
*x^3+x*(x^3-1)^(2/3)+2*x^2*(x^3-1)^(1/3)+3*RootOf(9*_Z^2+3*_Z+1)+1)/(2*x^3-
1))-7/3*ln(-(9*RootOf(9*_Z^2+3*_Z+1)^2*x^3-3*RootOf(9*_Z^2+3*_Z+1)*(x^3-1)^
(2/3)*x+3*RootOf(9*_Z^2+3*_Z+1)*(x^3-1)^(1/3)*x^2+3*RootOf(9*_Z^2+3*_Z+1)*x
^3+x*(x^3-1)^(2/3)+2*x^2*(x^3-1)^(1/3)+3*RootOf(9*_Z^2+3*_Z+1)+1)/(2*x^3-1)
)*RootOf(9*_Z^2+3*_Z+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4x^6-5x^3+1\right)\left(x^3-1\right)^{\frac{2}{3}}}{\left(2x^3-1\right)^2x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(4*x^6-5*x^3+1)/x^6/(2*x^3-1)^2,x, algorithm="maxim
a")
```

```
[Out] integrate((4*x^6 - 5*x^3 + 1)*(x^3 - 1)^(2/3)/((2*x^3 - 1)^2*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3-1\right)^{2/3}\,\left(4\,x^6-5\,x^3+1\right)}{x^6\,\left(2\,x^3-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(4*x^6 - 5*x^3 + 1))/(x^6*(2*x^3 - 1)^2),x)
```

```
[Out] int(((x^3 - 1)^(2/3)*(4*x^6 - 5*x^3 + 1))/(x^6*(2*x^3 - 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)*(4*x**6-5*x**3+1)/x**6/(2*x**3-1)**2,x)
```

```
[Out] Timed out
```

3.1482 $\int \frac{\sqrt{1+x}\sqrt{1+\sqrt{1+x}}}{x-\sqrt{1+x}}\,dx$

Optimal. Leaf size=118

$$\frac{4}{3}\sqrt{x+1}\sqrt{\sqrt{x+1}+1}+\frac{16}{3}\sqrt{\sqrt{x+1}+1}-\frac{4}{5}\left(2\sqrt{5}-5\right)\tanh^{-1}\left(\frac{2\sqrt{\sqrt{x+1}+1}}{\sqrt{5}-1}\right)-\frac{4}{5}\left(5+2\sqrt{5}\right)\tanh^{-1}\left(\frac{2\sqrt{\cdots}}{1\cdots}\right.$$

Rubi [A] time = 0.49, antiderivative size = 131, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {897, 1287, 1166, 207}

$$\frac{4}{3}\left(\sqrt{x+1}+1\right)^{3/2}+4\sqrt{\sqrt{x+1}+1}-4\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\sqrt{\sqrt{x+1}+1}\right)+4\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\sqrt{\sqrt{x+1}+1}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[1 + x]*Sqrt[1 + Sqrt[1 + x]])/(x - Sqrt[1 + x]),x]
```

```
[Out] 4*Sqrt[1 + Sqrt[1 + x]] + (4*(1 + Sqrt[1 + x])^(3/2))/3 - 4*Sqrt[(9 + 4*Sqr
t[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*Sqrt[1 + Sqrt[1 + x]]] + 4*Sqrt[(9 -
 4*Sqrt[5])/5]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*Sqrt[1 + Sqrt[1 + x]]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{1+x}\sqrt{1+\sqrt{1+x}}}{x-\sqrt{1+x}}\,dx = 2\,\text{Subst}\left(\int \frac{x^2\sqrt{1+x}}{-1-x+x^2}\,dx, x, \sqrt{1+x}\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x^2\left(-1+x^2\right)^2}{1-3x^2+x^4}\,dx, x, \sqrt{1+\sqrt{1+x}}\right)$$
$$= 4\,\text{Subst}\left(\int \left(1+x^2-\frac{1-3x^2}{1-3x^2+x^4}\right)dx, x, \sqrt{1+\sqrt{1+x}}\right)$$
$$= 4\sqrt{1+\sqrt{1+x}} + \frac{4}{3}\left(1+\sqrt{1+x}\right)^{3/2} - 4\,\text{Subst}\left(\int \frac{1-3x^2}{1-3x^2+x^4}\,dx, x, \sqrt{1+\sqrt{1+x}}\right.$$
$$= 4\sqrt{1+\sqrt{1+x}} + \frac{4}{3}\left(1+\sqrt{1+x}\right)^{3/2} + \frac{1}{5}\left(2\left(15-7\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\right.$$
$$= 4\sqrt{1+\sqrt{1+x}} + \frac{4}{3}\left(1+\sqrt{1+x}\right)^{3/2} - 4\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\sqrt{1+\right.$$

Mathematica [A] time = 0.26, size = 143, normalized size = 1.21

$$\frac{4}{3}\sqrt{\sqrt{x+1}+1}\left(\sqrt{x+1}+4\right)-\sqrt{\frac{2}{5}\left(3+\sqrt{5}\right)}\left(3\sqrt{5}-7\right)\tanh^{-1}\left(\sqrt{\frac{2}{3-\sqrt{5}}}\sqrt{\sqrt{x+1}+1}\right)-2\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}}\left(7+3\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\sqrt{\sqrt{x+1}+1}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 + x]*Sqrt[1 + Sqrt[1 + x]])/(x - Sqrt[1 + x]),x]
```

```
[Out] (4*Sqrt[1 + Sqrt[1 + x]]*(4 + Sqrt[1 + x]))/3 - Sqrt[(2*(3 + Sqrt[5]))/5]*(
-7 + 3*Sqrt[5])*ArcTanh[Sqrt[2/(3 - Sqrt[5])]*Sqrt[1 + Sqrt[1 + x]]] - 2*Sq
rt[2/(5*(3 + Sqrt[5]))]*(7 + 3*Sqrt[5])*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*Sqrt[
1 + Sqrt[1 + x]]]
```

IntegrateAlgebraic [A] time = 0.18, size = 103, normalized size = 0.87

$$\frac{4}{3}\sqrt{\sqrt{x+1}+1}\left(\sqrt{x+1}+4\right)-\frac{4}{5}\left(5+2\sqrt{5}\right)\tanh^{-1}\left(\frac{1}{2}\left(\sqrt{5}-1\right)\sqrt{\sqrt{x+1}+1}\right)-\frac{4}{5}\left(2\sqrt{5}-5\right)\tanh^{-1}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt{\sqrt{x+1}+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x]*Sqrt[1 + Sqrt[1 + x]])/(x - Sqrt[1 + x]),x]
```

```
[Out] (4*Sqrt[1 + Sqrt[1 + x]]*(4 + Sqrt[1 + x]))/3 - (4*(5 + 2*Sqrt[5])*ArcTanh[
((-1 + Sqrt[5])*Sqrt[1 + Sqrt[1 + x]])/2])/5 - (4*(-5 + 2*Sqrt[5])*ArcTanh[
((1 + Sqrt[5])*Sqrt[1 + Sqrt[1 + x]])/2])/5
```

fricas [B] time = 0.45, size = 243, normalized size = 2.06

$$\frac{4}{3}\left(\sqrt{x+1}+4\right)\sqrt{\sqrt{x+1}+1}+\frac{4}{5}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(3x+1)+\left(\sqrt{5}(x+2)+5x\right)\sqrt{x+1}-\left(\sqrt{5}(x+2)+\left(\sqrt{5}(2x-1)+5\right)\sqrt{x+1}+5x\right)\sqrt{\sqrt{x+1}+1}+3x+3}{x^2-x-1}\right)+\frac{4}{5}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}(3x+1)-\left(\sqrt{5}(x+2)-5x\right)\sqrt{x+1}-\left(\sqrt{5}(x+2)+\left(\sqrt{5}(2x-1)-5\right)\sqrt{x+1}-5x\right)\sqrt{\sqrt{x+1}+1}+3x+3}{x^2-x-1}\right)-2\log\left(\sqrt{x+1}+\sqrt{\sqrt{x+1}+1}\right)+2\log\left(\sqrt{x+1}-\sqrt{\sqrt{x+1}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(1+(1+x)^(1/2))^(1/2)/(x-(1+x)^(1/2)),x, algorithm="f
ricas")
```

```
[Out] 4/3*(sqrt(x + 1) + 4)*sqrt(sqrt(x + 1) + 1) + 4/5*sqrt(5)*log((2*x^2 + sqrt
(5)*(3*x + 1) + (sqrt(5)*(x + 2) + 5*x)*sqrt(x + 1) - (sqrt(5)*(x + 2) + (s
qrt(5)*(2*x - 1) + 5)*sqrt(x + 1) + 5*x)*sqrt(sqrt(x + 1) + 1) + 3*x + 3)/(
x^2 - x - 1)) + 4/5*sqrt(5)*log((2*x^2 - sqrt(5)*(3*x + 1) - (sqrt(5)*(x +
2) - 5*x)*sqrt(x + 1) - (sqrt(5)*(x + 2) + (sqrt(5)*(2*x - 1) - 5)*sqrt(x +
```

1) - 5*x)*sqrt(sqrt(x + 1) + 1) + 3*x + 3)/(x^2 - x - 1)) - 2*log(sqrt(x + 1) + sqrt(sqrt(x + 1) + 1)) + 2*log(sqrt(x + 1) - sqrt(sqrt(x + 1) + 1))

giac [A] time = 1.71, size = 149, normalized size = 1.26

$$\frac{4}{3}\left(\sqrt{x+1}+1\right)^{\frac{3}{2}}+\frac{4}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{\sqrt{x+1}+1}-1}{\sqrt{5}+2\sqrt{\sqrt{x+1}+1}+1}\right)+\frac{4}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{\sqrt{x+1}+1}-1\right|}{\left|\sqrt{5}+2\sqrt{\sqrt{x+1}+1}-1\right|}\right)+4\sqrt{\sqrt{x+1}+1}-2\log\left(\sqrt{x+1}+\sqrt{\sqrt{x+1}+1}\right)+2\log\left(\left|\sqrt{x+1}-\sqrt{\sqrt{x+1}+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(1/2)*(1+(1+x)^(1/2))^(1/2)/(x-(1+x)^(1/2)),x, algorithm="giac")

[Out] 4/3*(sqrt(x + 1) + 1)^(3/2) + 4/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(sqrt(x + 1) + 1) - 1)/(sqrt(5) + 2*sqrt(sqrt(x + 1) + 1) + 1)) + 4/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(sqrt(x + 1) + 1) - 1)/abs(sqrt(5) + 2*sqrt(sqrt(x + 1) + 1) - 1)) + 4*sqrt(sqrt(x + 1) + 1) - 2*log(sqrt(x + 1) + sqrt(sqrt(x + 1) + 1)) + 2*log(abs(sqrt(x + 1) - sqrt(sqrt(x + 1) + 1)))

maple [A] time = 0.01, size = 110, normalized size = 0.93

$$\frac{4\left(1+\sqrt{1+x}\right)^{\frac{3}{2}}}{3}+4\sqrt{1+\sqrt{1+x}}-2\ln\left(\sqrt{1+x}+\sqrt{1+\sqrt{1+x}}\right)-\frac{8\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(1+2\sqrt{1+\sqrt{1+x}}\right)\sqrt{5}}{5}\right)}{5}+2\ln\left(\sqrt{1+x}-\sqrt{1+\sqrt{1+x}}\right)-\frac{8\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(-1+2\sqrt{1+\sqrt{1+x}}\right)\sqrt{5}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)^(1/2)*(1+(1+x)^(1/2))^(1/2)/(x-(1+x)^(1/2)),x)

[Out] 4/3*(1+(1+x)^(1/2))^(3/2)+4*(1+(1+x)^(1/2))^(1/2)-2*ln((1+x)^(1/2)+(1+(1+x)^(1/2))^(1/2))-8/5*5^(1/2)*arctanh(1/5*(1+2*(1+(1+x)^(1/2))^(1/2))*5^(1/2))+2*ln((1+x)^(1/2)-(1+(1+x)^(1/2))^(1/2))-8/5*5^(1/2)*arctanh(1/5*(-1+2*(1+(1+x)^(1/2))^(1/2))*5^(1/2))

maxima [A] time = 0.44, size = 145, normalized size = 1.23

$$\frac{4}{3}\left(\sqrt{x+1}+1\right)^{\frac{3}{2}}+\frac{4}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{\sqrt{x+1}+1}+1}{\sqrt{5}+2\sqrt{\sqrt{x+1}+1}-1}\right)+\frac{4}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{\sqrt{x+1}+1}-1}{\sqrt{5}+2\sqrt{\sqrt{x+1}+1}+1}\right)+4\sqrt{\sqrt{x+1}+1}-2\log\left(\sqrt{x+1}+\sqrt{\sqrt{x+1}+1}\right)+2\log\left(\sqrt{x+1}-\sqrt{\sqrt{x+1}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(1/2)*(1+(1+x)^(1/2))^(1/2)/(x-(1+x)^(1/2)),x, algorithm="maxima")

[Out] 4/3*(sqrt(x + 1) + 1)^(3/2) + 4/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(sqrt(x + 1) + 1) + 1)/(sqrt(5) + 2*sqrt(sqrt(x + 1) + 1) - 1)) + 4/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(sqrt(x + 1) + 1) - 1)/(sqrt(5) + 2*sqrt(sqrt(x + 1) + 1) + 1)) + 4*sqrt(sqrt(x + 1) + 1) - 2*log(sqrt(x + 1) + sqrt(sqrt(x + 1) + 1)) + 2*log(sqrt(x + 1) - sqrt(sqrt(x + 1) + 1))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{x+1}+1}\,\sqrt{x+1}}{x-\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((x + 1)^(1/2) + 1)^(1/2)*(x + 1)^(1/2))/(x - (x + 1)^(1/2)),x)

[Out] int((((x + 1)^(1/2) + 1)^(1/2)*(x + 1)^(1/2))/(x - (x + 1)^(1/2)), x)

sympy [A] time = 18.31, size = 279, normalized size = 2.36

$$\frac{4\left(\sqrt{x+1}+1\right)^{\frac{3}{2}}}{3}+4\sqrt{\sqrt{x+1}+1}+16\left(\begin{cases}-\frac{\sqrt{5}\operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{\sqrt{x+1}+1}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for }\left(\sqrt{\sqrt{x+1}+1}-\frac{1}{2}\right)^{2}>\frac{5}{4}\\ -\frac{\sqrt{5}\operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{\sqrt{x+1}+1}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for }\left(\sqrt{\sqrt{x+1}+1}-\frac{1}{2}\right)^{2}<\frac{5}{4}\end{cases}\right)+16\left(\begin{cases}-\frac{\sqrt{5}\operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for }\left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)^{2}>\frac{5}{4}\\ -\frac{\sqrt{5}\operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for }\left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)^{2}<\frac{5}{4}\end{cases}\right)+2\log\left(\sqrt{x+1}-\sqrt{\sqrt{x+1}+1}\right)-2\log\left(\sqrt{x+1}+\sqrt{\sqrt{x+1}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(1+(1+x)**(1/2))**(1/2)/(x-(1+x)**(1/2)),x)
```

```
[Out] 4*(sqrt(x + 1) + 1)**(3/2)/3 + 4*sqrt(sqrt(x + 1) + 1) + 16*Piecewise((-sqr
t(5)*acoth(2*sqrt(5)*(sqrt(sqrt(x + 1) + 1) - 1/2)/5)/10, (sqrt(sqrt(x + 1)
 + 1) - 1/2)**2 > 5/4), (-sqrt(5)*atanh(2*sqrt(5)*(sqrt(sqrt(x + 1) + 1) -
1/2)/5)/10, (sqrt(sqrt(x + 1) + 1) - 1/2)**2 < 5/4)) + 16*Piecewise((-sqrt(
5)*acoth(2*sqrt(5)*(sqrt(sqrt(x + 1) + 1) + 1/2)/5)/10, (sqrt(sqrt(x + 1) +
 1) + 1/2)**2 > 5/4), (-sqrt(5)*atanh(2*sqrt(5)*(sqrt(sqrt(x + 1) + 1) + 1/
2)/5)/10, (sqrt(sqrt(x + 1) + 1) + 1/2)**2 < 5/4)) + 2*log(sqrt(x + 1) - sq
rt(sqrt(x + 1) + 1)) - 2*log(sqrt(x + 1) + sqrt(sqrt(x + 1) + 1))
```

3.1483 $\int \frac{\sqrt{b+ax}}{\sqrt{abx+\sqrt{b+ax}}}\,dx$

Optimal. Leaf size=118

$$\frac{\sqrt{b(ax+b)+\sqrt{ax+b}-b^2}\,\left(2b\sqrt{ax+b}-3\right)}{2ab^2}+\frac{\left(-4b^3-3\right)\log\left(2\sqrt{b}\,\sqrt{b(ax+b)+\sqrt{ax+b}-b^2}-2b\sqrt{ax+b}\right.}{4ab^{5/2}}$$

Rubi [A] time = 0.25, antiderivative size = 148, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {742, 640, 621, 206}

$$\frac{\sqrt{ax+b}\,\sqrt{b(ax+b)+\sqrt{ax+b}-b^2}}{ab}-\frac{3\sqrt{b(ax+b)+\sqrt{ax+b}-b^2}}{2ab^2}+\frac{\left(4b^3+3\right)\tanh^{-1}\left(\frac{2b\sqrt{ax+b}+1}{2\sqrt{b}\,\sqrt{b(ax+b)+\sqrt{ax+b}-b^2}}\right)}{4ab^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b + a*x]/Sqrt[a*b*x + Sqrt[b + a*x]],x]
```

```
[Out] (-3*Sqrt[-b^2 + Sqrt[b + a*x] + b*(b + a*x)])/(2*a*b^2) + (Sqrt[b + a*x]*Sq
rt[-b^2 + Sqrt[b + a*x] + b*(b + a*x)])/(a*b) + ((3 + 4*b^3)*ArcTanh[(1 + 2
*b*Sqrt[b + a*x])/(2*Sqrt[b]*Sqrt[-b^2 + Sqrt[b + a*x] + b*(b + a*x)])])/(4
*a*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{b+ax}}{\sqrt{abx+\sqrt{b+ax}}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-b^2+x+bx^2}}\,dx, x, \sqrt{b+ax}\right)}{a}$$

$$= \frac{\sqrt{b+ax}\sqrt{-b^2+\sqrt{b+ax}+b(b+ax)}}{ab} + \frac{\text{Subst}\left(\int \frac{b^2-\frac{3x}{2}}{\sqrt{-b^2+x+bx^2}}\,dx, x, \sqrt{b+ax}\right)}{ab}$$

$$= -\frac{3\sqrt{-b^2+\sqrt{b+ax}+b(b+ax)}}{2ab^2} + \frac{\sqrt{b+ax}\sqrt{-b^2+\sqrt{b+ax}+b(b+ax)}}{ab} + \frac{\left(3+4b^3\right. \cdots}{\cdots}$$

$$= -\frac{3\sqrt{-b^2+\sqrt{b+ax}+b(b+ax)}}{2ab^2} + \frac{\sqrt{b+ax}\sqrt{-b^2+\sqrt{b+ax}+b(b+ax)}}{ab} + \frac{\left(3+4b^3\right. \cdots}{\cdots}$$

$$= -\frac{3\sqrt{-b^2+\sqrt{b+ax}+b(b+ax)}}{2ab^2} + \frac{\sqrt{b+ax}\sqrt{-b^2+\sqrt{b+ax}+b(b+ax)}}{ab} + \frac{\left(3+4b^3\right. \cdots}{\cdots}$$

Mathematica [A] time = 0.13, size = 102, normalized size = 0.86

$$\frac{\left(4b^3+3\right)\tanh^{-1}\left(\frac{2b\sqrt{ax+b}+1}{2\sqrt{b}\sqrt{abx+\sqrt{ax+b}}}\right)+2\sqrt{b}\sqrt{abx+\sqrt{ax+b}}\left(2b\sqrt{ax+b}-3\right)}{4ab^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b + a*x]/Sqrt[a*b*x + Sqrt[b + a*x]],x]
```

```
[Out] (2*Sqrt[b]*Sqrt[a*b*x + Sqrt[b + a*x]]*(-3 + 2*b*Sqrt[b + a*x]) + (3 + 4*b^
3)*ArcTanh[(1 + 2*b*Sqrt[b + a*x])/(2*Sqrt[b]*Sqrt[a*b*x + Sqrt[b + a*x]])]
)/(4*a*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.26, size = 126, normalized size = 1.07

$$\frac{\sqrt{b(ax+b)+\sqrt{ax+b}-b^2}\left(2b\sqrt{ax+b}-3\right)}{2ab^2}+\frac{\left(-4b^3-3\right)\log\left(2ab^3\sqrt{ax+b}+ab^2-2ab^{5/2}\sqrt{b(ax+b)+\sqrt{ax+b}-b^2}\right)}{4ab^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + a*x]/Sqrt[a*b*x + Sqrt[b + a*x]],x]
```

```
[Out] ((-3 + 2*b*Sqrt[b + a*x])*Sqrt[-b^2 + Sqrt[b + a*x] + b*(b + a*x)])/(2*a*b^
2) + ((-3 - 4*b^3)*Log[a*b^2 + 2*a*b^3*Sqrt[b + a*x] - 2*a*b^(5/2)*Sqrt[-b^
2 + Sqrt[b + a*x] + b*(b + a*x)]])/(4*a*b^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^(1/2)/(a*b*x+(a*x+b)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^(1/2)/(a*b*x+(a*x+b)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 161, normalized size = 1.36

$$\frac{\sqrt{ax+b}\,\sqrt{-b^{2}+\sqrt{ax+b}+b\,(ax+b)}}{ab}-\frac{3\sqrt{-b^{2}+\sqrt{ax+b}+b\,(ax+b)}}{2a\,b^{2}}+\frac{3\ln\left(\frac{\frac{1}{2}+b\sqrt{ax+b}}{\sqrt{b}}+\sqrt{-b^{2}+\sqrt{ax+b}+b\,(ax+b)}\right)}{4a\,b^{\frac{5}{2}}}+\frac{\sqrt{b}\,\ln\left(\frac{\frac{1}{2}+b\sqrt{ax+b}}{\sqrt{b}}+\sqrt{-b^{2}+\sqrt{ax+b}+b\,(ax+b)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)^(1/2)/(a*b*x+(a*x+b)^(1/2))^(1/2),x)
```

```
[Out] 1/a*(a*x+b)^(1/2)/b*(-b^2+(a*x+b)^(1/2)+b*(a*x+b))^(1/2)-3/2/a/b^2*(-b^2+(a
*x+b)^(1/2)+b*(a*x+b))^(1/2)+3/4/a/b^(5/2)*ln((1/2+b*(a*x+b)^(1/2))/b^(1/2)
+(-b^2+(a*x+b)^(1/2)+b*(a*x+b))^(1/2))+1/a*b^(1/2)*ln((1/2+b*(a*x+b)^(1/2))
/b^(1/2)+(-b^2+(a*x+b)^(1/2)+b*(a*x+b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+b}}{\sqrt{abx+\sqrt{ax+b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^(1/2)/(a*b*x+(a*x+b)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b)/sqrt(a*b*x + sqrt(a*x + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b+a\,x}}{\sqrt{\sqrt{b+a\,x}+a\,b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x)^(1/2)/((b + a*x)^(1/2) + a*b*x)^(1/2),x)
```

```
[Out] int((b + a*x)^(1/2)/((b + a*x)^(1/2) + a*b*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+b}}{\sqrt{abx+\sqrt{ax+b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)**(1/2)/(a*b*x+(a*x+b)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a*x + b)/sqrt(a*b*x + sqrt(a*x + b)), x)
```

3.1484 $\int \frac{\sqrt{b+a^2x^4}}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$

Optimal. Leaf size=118

$$\frac{1}{4}x\sqrt{\sqrt{a^2x^4+b}+ax^2}+\frac{bx}{8\left(\sqrt{a^2x^4+b}+ax^2\right)^{3/2}}+\frac{5\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}}{\sqrt{b}}\right)}{8\sqrt{2}\sqrt{a}}$$

Rubi [F] time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{b+a^2x^4}}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b + a^2*x^4]/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] Defer[Int][Sqrt[b + a^2*x^4]/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]], x]
```

Rubi steps

$$\int \frac{\sqrt{b+a^2x^4}}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx=\int \frac{\sqrt{b+a^2x^4}}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+a^2x^4}}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[b + a^2*x^4]/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] Integrate[Sqrt[b + a^2*x^4]/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]], x]
```

IntegrateAlgebraic [A] time = 0.30, size = 118, normalized size = 1.00

$$\frac{1}{4}x\sqrt{\sqrt{a^2x^4+b}+ax^2}+\frac{bx}{8\left(\sqrt{a^2x^4+b}+ax^2\right)^{3/2}}+\frac{5\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}}{\sqrt{b}}\right)}{8\sqrt{2}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + a^2*x^4]/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] (b*x)/(8*(a*x^2 + Sqrt[b + a^2*x^4])^(3/2)) + (x*Sqrt[a*x^2 + Sqrt[b + a^2*
x^4]])/4 + (5*Sqrt[b]*ArcTan[(Sqrt[2]*Sqrt[a]*x*Sqrt[a*x^2 + Sqrt[b + a^2*x
^4]])/Sqrt[b]])/(8*Sqrt[2]*Sqrt[a])
```

fricas [A] time = 4.45, size = 319, normalized size = 2.70

$$\left[\frac{5\sqrt{\frac{1}{2}}\,b\sqrt{-\frac{b}{a}}\log\left(4\,a^2bx^4-4\,\sqrt{a^2x^4+b}\,abx^2+b^2+4\left(2\sqrt{\frac{1}{2}}\,\sqrt{a^2x^4+b}\,a^2x^3\sqrt{-\frac{b}{a}}-\sqrt{\frac{1}{2}}\left(2\,a^3x^5+abx\right)\sqrt{-\frac{b}{a}}\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}\right)+2\left(2\,a^2x^5-2\,\sqrt{a^2x^4+b}\,ax^3+3\,bx\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{16\,b},-\frac{5\sqrt{\frac{1}{2}}\,b\sqrt{\frac{b}{a}}\arctan\left(-\frac{\left(\sqrt{\frac{1}{2}}\,ax^2\sqrt{\frac{b}{a}}-\sqrt{\frac{1}{2}}\,\sqrt{a^2x^4+b}\,\sqrt{\frac{b}{a}}\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{bx}\right)-\left(2\,a^2x^5-2\,\sqrt{a^2x^4+b}\,ax^3+3\,bx\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{8\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^4+b)^(1/2)/(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/16*(5*sqrt(1/2)*b*sqrt(-b/a)*log(4*a^2*b*x^4 - 4*sqrt(a^2*x^4 + b)*a*b*x
^2 + b^2 + 4*(2*sqrt(1/2)*sqrt(a^2*x^4 + b)*a^2*x^3*sqrt(-b/a) - sqrt(1/2)*
(2*a^3*x^5 + a*b*x)*sqrt(-b/a))*sqrt(a*x^2 + sqrt(a^2*x^4 + b))) + 2*(2*a^2
*x^5 - 2*sqrt(a^2*x^4 + b)*a*x^3 + 3*b*x)*sqrt(a*x^2 + sqrt(a^2*x^4 + b)))/
b, -1/8*(5*sqrt(1/2)*b*sqrt(b/a)*arctan(-(sqrt(1/2)*a*x^2*sqrt(b/a) - sqrt(
1/2)*sqrt(a^2*x^4 + b)*sqrt(b/a))*sqrt(a*x^2 + sqrt(a^2*x^4 + b))/(b*x)) -
(2*a^2*x^5 - 2*sqrt(a^2*x^4 + b)*a*x^3 + 3*b*x)*sqrt(a*x^2 + sqrt(a^2*x^4 +
 b)))/b]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^4+b)^(1/2)/(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^4+b}}{\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^4+b)^(1/2)/(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x)
```

```
[Out] int((a^2*x^4+b)^(1/2)/(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^4+b}}{\sqrt{ax^2+\sqrt{a^2x^4+b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^4+b)^(1/2)/(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(a^2*x^4 + b)/sqrt(a*x^2 + sqrt(a^2*x^4 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a^2\,x^4+b}}{\sqrt{\sqrt{a^2\,x^4+b}+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a^2*x^4)^(1/2)/((b + a^2*x^4)^(1/2) + a*x^2)^(1/2),x)
```

```
[Out] int((b + a^2*x^4)^(1/2)/((b + a^2*x^4)^(1/2) + a*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^4+b}}{\sqrt{ax^2+\sqrt{a^2x^4+b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**4+b)**(1/2)/(a*x**2+(a**2*x**4+b)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a**2*x**4 + b)/sqrt(a*x**2 + sqrt(a**2*x**4 + b)), x)
```

3.1485 $\int \frac{-ab+(-a+2b)x}{\sqrt[4]{x(-a+x)(-b+x)^2}\left(-b^2+(2b-ad)x+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=119

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}}{b-x}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}}{b-x}\right)}{d^{3/4}}$$

Rubi [F] time = 3.91, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-ab+(-a+2b)x}{\sqrt[4]{x(-a+x)(-b+x)^2}\left(-b^2+(2b-ad)x+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a*b) + (-a + 2*b)*x)/((x*(-a + x)*(-b + x)^2)^(1/4)*(-b^2 + (2*b - a
*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] -(((a - 2*b + Sqrt[-4*a*b + 4*b^2 + a^2*d]/Sqrt[d])*x^(1/4)*(-a + x)^(1/4)*
Sqrt[-b + x]*Defer[Int][1/(x^(1/4)*(-a + x)^(1/4)*Sqrt[-b + x]*(2*b - a*d -
 Sqrt[d]*Sqrt[-4*a*b + 4*b^2 + a^2*d] + 2*(-1 + d)*x)), x])/(-((a - x)*(b -
 x)^2*x))^(1/4)) - ((a - 2*b - Sqrt[-4*a*b + 4*b^2 + a^2*d]/Sqrt[d])*x^(1/4
)*(-a + x)^(1/4)*Sqrt[-b + x]*Defer[Int][1/(x^(1/4)*(-a + x)^(1/4)*Sqrt[-b
+ x]*(2*b - a*d + Sqrt[d]*Sqrt[-4*a*b + 4*b^2 + a^2*d] + 2*(-1 + d)*x)), x]
)/(-((a - x)*(b - x)^2*x))^(1/4)
```

Rubi steps

$$\int \frac{-ab+(-a+2b)x}{\sqrt[4]{x(-a+x)(-b+x)^2}\left(-b^2+(2b-ad)x+(-1+d)x^2\right)}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt{-b+x}\right)\int \frac{-ab+(-a+}{\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt{-b+x}\left(-b^2+\right.}}{\sqrt[4]{x(-a+x)(-b+x)^2}}$$

$$=\frac{\left(\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt{-b+x}\right)\int\left(\frac{-a+2}{\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt{-b+x}\left(2b-\right.}\right.}{}$$

$$=\frac{\left(\left(-a+2b-\frac{\sqrt{-4ab+4b^2+a^2d}}{\sqrt{d}}\right)\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt{-b}\right.}{\sqrt[4]{x(-}}$$

Mathematica [F] time = 6.98, size = 0, normalized size = 0.00

$$\int \frac{-ab+(-a+2b)x}{\sqrt[4]{x(-a+x)(-b+x)^2}\left(-b^2+(2b-ad)x+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-(a*b) + (-a + 2*b)*x)/((x*(-a + x)*(-b + x)^2)^(1/4)*(-b^2 + (2
*b - a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] Integrate[(-(a*b) + (-a + 2*b)*x)/((x*(-a + x)*(-b + x)^2)^(1/4)*(-b^2 + (2
*b - a*d)*x + (-1 + d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.39, size = 119, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}}{b-x}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}}{b-x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*b) + (-a + 2*b)*x)/((x*(-a + x)*(-b + x)^2)^(1/4)*(
-b^2 + (2*b - a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^
(1/4))/(b - x)])/d^(3/4) - (2*ArcTanh[(d^(1/4)*(-(a*b^2*x) + (2*a*b + b^2)*
x^2 + (-a - 2*b)*x^3 + x^4)^(1/4))/(b - x)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+(-a+2*b)*x)/(x*(-a+x)*(-b+x)^2)^(1/4)/(-b^2+(-a*d+2*b)*x+(-
1+d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ab+(a-2b)x}{\left(-(a-x)(b-x)^2x\right)^{\frac{1}{4}}\left((d-1)x^2-b^2-(ad-2b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+(-a+2*b)*x)/(x*(-a+x)*(-b+x)^2)^(1/4)/(-b^2+(-a*d+2*b)*x+(-
1+d)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-(a*b + (a - 2*b)*x)/((-(a - x)*(b - x)^2*x)^(1/4)*((d - 1)*x^2 -
 b^2 - (a*d - 2*b)*x)), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{-ab+(-a+2b)\,x}{\left(x\,(-a+x)\,(-b+x)^2\right)^{\frac{1}{4}}\left(-b^2+(-ad+2b)\,x+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*b+(-a+2*b)*x)/(x*(-a+x)*(-b+x)^2)^(1/4)/(-b^2+(-a*d+2*b)*x+(-1+d)*x
^2),x)
```

```
[Out] int((-a*b+(-a+2*b)*x)/(x*(-a+x)*(-b+x)^2)^(1/4)/(-b^2+(-a*d+2*b)*x+(-1+d)*x
^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ab+(a-2b)x}{\left(-(a-x)(b-x)^2x\right)^{\frac{1}{4}}\left((d-1)x^2-b^2-(ad-2b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+(-a+2*b)*x)/(x*(-a+x)*(-b+x)^2)^(1/4)/(-b^2+(-a*d+2*b)*x+(-
1+d)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((a*b + (a - 2*b)*x)/((-(a - x)*(b - x)^2*x)^(1/4)*((d - 1)*x^2 -
 b^2 - (a*d - 2*b)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a\,b+x\,\left(a-2\,b\right)}{{\left(-x\,\left(a-x\right)\,{\left(b-x\right)}^2\right)}^{1/4}\,\left(x\,\left(2\,b-a\,d\right)-b^2+x^2\,\left(d-1\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*b + x*(a - 2*b))/((-x*(a - x)*(b - x)^2)^(1/4)*(x*(2*b - a*d) - b^2
 + x^2*(d - 1))),x)
```

```
[Out] -int((a*b + x*(a - 2*b))/((-x*(a - x)*(b - x)^2)^(1/4)*(x*(2*b - a*d) - b^2
 + x^2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+(-a+2*b)*x)/(x*(-a+x)*(-b+x)**2)**(1/4)/(-b**2+(-a*d+2*b)*x
+(-1+d)*x**2),x)
```

```
[Out] Timed out
```

3.1486 $\int x^{13}\sqrt[3]{-1+x^3}\,dx$

Optimal. Leaf size=119

$$\frac{22\log\left(\sqrt[3]{x^3-1}-x\right)}{2187}+\frac{22\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{729\sqrt{3}}-\frac{11\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)}{2187}+\frac{\sqrt[3]{x^3-1}\left(972x^{14}-81x^{11}-9\right.}{14580}$$

Rubi [A] time = 0.10, antiderivative size = 161, normalized size of antiderivative = 1.35, number of steps used = 12, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {279, 321, 331, 292, 31, 634, 618, 204, 628}

$$\frac{22\log\left(1-\frac{x}{\sqrt[3]{x^3-1}}\right)}{2187}+\frac{22\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{729\sqrt{3}}+\frac{1}{15}\sqrt[3]{x^3-1}\,x^{14}-\frac{1}{180}\sqrt[3]{x^3-1}\,x^{11}-\frac{11\sqrt[3]{x^3-1}\,x^8}{1620}-\frac{11\sqrt[3]{x^3-1}\,x^5}{1215}-\frac{11}{729}\sqrt[3]{x^3-1}\,x^2-\frac{11\log\left(\frac{x}{\sqrt[3]{x^3-1}}+\frac{x^2}{\left(x^3-1\right)^{2/3}}+1\right)}{2187}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(-1 + x^3)^(1/3),x]
```

```
[Out] (-11*x^2*(-1 + x^3)^(1/3))/729 - (11*x^5*(-1 + x^3)^(1/3))/1215 - (11*x^8*(
-1 + x^3)^(1/3))/1620 - (x^11*(-1 + x^3)^(1/3))/180 + (x^14*(-1 + x^3)^(1/3
))/15 + (22*ArcTan[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]])/(729*Sqrt[3]) + (
22*Log[1 - x/(-1 + x^3)^(1/3)])/2187 - (11*Log[1 + x^2/(-1 + x^3)^(2/3) + x
/(-1 + x^3)^(1/3)])/2187
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int x^{13}\sqrt[3]{-1+x^3}\,dx = \frac{1}{15}x^{14}\sqrt[3]{-1+x^3} - \frac{1}{15}\int \frac{x^{13}}{\left(-1+x^3\right)^{2/3}}\,dx$$

$$= -\frac{1}{180}x^{11}\sqrt[3]{-1+x^3} + \frac{1}{15}x^{14}\sqrt[3]{-1+x^3} - \frac{11}{180}\int \frac{x^{10}}{\left(-1+x^3\right)^{2/3}}\,dx$$

$$= -\frac{11x^8\sqrt[3]{-1+x^3}}{1620} - \frac{1}{180}x^{11}\sqrt[3]{-1+x^3} + \frac{1}{15}x^{14}\sqrt[3]{-1+x^3} - \frac{22}{405}\int \frac{x^7}{\left(-1+x^3\right)^{2/3}}\,dx$$

$$= -\frac{11x^5\sqrt[3]{-1+x^3}}{1215} - \frac{11x^8\sqrt[3]{-1+x^3}}{1620} - \frac{1}{180}x^{11}\sqrt[3]{-1+x^3} + \frac{1}{15}x^{14}\sqrt[3]{-1+x^3} - \frac{11}{243}\int \frac{x^4}{(-1+x\ldots}$$

$$= -\frac{11}{729}x^2\sqrt[3]{-1+x^3} - \frac{11x^5\sqrt[3]{-1+x^3}}{1215} - \frac{11x^8\sqrt[3]{-1+x^3}}{1620} - \frac{1}{180}x^{11}\sqrt[3]{-1+x^3} + \frac{1}{15}x^{14}\sqrt[3]{-1+x\ldots}$$

$$= -\frac{11}{729}x^2\sqrt[3]{-1+x^3} - \frac{11x^5\sqrt[3]{-1+x^3}}{1215} - \frac{11x^8\sqrt[3]{-1+x^3}}{1620} - \frac{1}{180}x^{11}\sqrt[3]{-1+x^3} + \frac{1}{15}x^{14}\sqrt[3]{-1+x\ldots}$$

$$= -\frac{11}{729}x^2\sqrt[3]{-1+x^3} - \frac{11x^5\sqrt[3]{-1+x^3}}{1215} - \frac{11x^8\sqrt[3]{-1+x^3}}{1620} - \frac{1}{180}x^{11}\sqrt[3]{-1+x^3} + \frac{1}{15}x^{14}\sqrt[3]{-1+x\ldots}$$

$$= -\frac{11}{729}x^2\sqrt[3]{-1+x^3} - \frac{11x^5\sqrt[3]{-1+x^3}}{1215} - \frac{11x^8\sqrt[3]{-1+x^3}}{1620} - \frac{1}{180}x^{11}\sqrt[3]{-1+x^3} + \frac{1}{15}x^{14}\sqrt[3]{-1+x\ldots}$$

$$= -\frac{11}{729}x^2\sqrt[3]{-1+x^3} - \frac{11x^5\sqrt[3]{-1+x^3}}{1215} - \frac{11x^8\sqrt[3]{-1+x^3}}{1620} - \frac{1}{180}x^{11}\sqrt[3]{-1+x^3} + \frac{1}{15}x^{14}\sqrt[3]{-1+x\ldots}$$

$$= -\frac{11}{729}x^2\sqrt[3]{-1+x^3} - \frac{11x^5\sqrt[3]{-1+x^3}}{1215} - \frac{11x^8\sqrt[3]{-1+x^3}}{1620} - \frac{1}{180}x^{11}\sqrt[3]{-1+x^3} + \frac{1}{15}x^{14}\sqrt[3]{-1+x\ldots}$$

Mathematica [C] time = 0.03, size = 77, normalized size = 0.65

$$\frac{x^2\sqrt[3]{x^3-1}\left(220\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\sqrt[3]{1-x^3}\left(324x^{12}-27x^9-33x^6-44x^3-220\right)\right)}{4860\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(-1 + x^3)^(1/3),x]
```

```
[Out] (x^2*(-1 + x^3)^(1/3)*((1 - x^3)^(1/3)*(-220 - 44*x^3 - 33*x^6 - 27*x^9 + 3
24*x^12) + 220*Hypergeometric2F1[-1/3, 2/3, 5/3, x^3]))/(4860*(1 - x^3)^(1/
3))
```

IntegrateAlgebraic [A] time = 0.35, size = 119, normalized size = 1.00

$$\frac{22\log\left(\sqrt[3]{x^3-1}-x\right)}{2187}+\frac{22\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{729\sqrt{3}}-\frac{11\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)}{2187}+\frac{\sqrt[3]{x^3-1}\left(972x^{14}-81x^{11}-99x^8-132x^5-220x^2\right)}{14580}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^13*(-1 + x^3)^(1/3),x]
```

```
[Out] ((-1 + x^3)^(1/3)*(-220*x^2 - 132*x^5 - 99*x^8 - 81*x^11 + 972*x^14))/14580
 + (22*ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x^3)^(1/3))])/(729*Sqrt[3]) + (22*Lo
```

```
g[-x + (-1 + x^3)^(1/3)])/2187 - (11*Log[x^2 + x*(-1 + x^3)^(1/3) + (-1 + x
^3)^(2/3)])/2187
```

fricas [A] time = 0.43, size = 111, normalized size = 0.93

$$-\frac{22}{2187}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{14580}\left(972x^{14}-81x^{11}-99x^{8}-132x^{5}-220x^{2}\right)\left(x^3-1\right)^{\frac{1}{3}}+\frac{22}{2187}\log\left(-\frac{x-\left(x^3-1\right)^{\frac{1}{3}}}{x}\right)-\frac{11}{2187}\log\left(\frac{x^2+\left(x^3-1\right)^{\frac{1}{3}}x+\left(x^3-1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(x^3-1)^(1/3),x, algorithm="fricas")
```

```
[Out] -22/2187*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 - 1)^(1/3))/x) + 1/
14580*(972*x^14 - 81*x^11 - 99*x^8 - 132*x^5 - 220*x^2)*(x^3 - 1)^(1/3) + 2
2/2187*log(-(x - (x^3 - 1)^(1/3))/x) - 11/2187*log((x^2 + (x^3 - 1)^(1/3)*x
 + (x^3 - 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^3-1\right)^{\frac{1}{3}}x^{13}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(x^3-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 1)^(1/3)*x^13, x)
```

maple [C] time = 0.32, size = 68, normalized size = 0.57

$$\frac{x^2\left(972x^{12}-81x^9-99x^6-132x^3-220\right)\left(x^3-1\right)^{\frac{1}{3}}}{14580}-\frac{11\left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{2}{3}}x^2\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{729\mathrm{signum}\left(x^3-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(x^3-1)^(1/3),x)
```

```
[Out] 1/14580*x^2*(972*x^12-81*x^9-99*x^6-132*x^3-220)*(x^3-1)^(1/3)-11/729/signu
m(x^3-1)^(2/3)*(-signum(x^3-1))^(2/3)*x^2*hypergeom([2/3,2/3],[5/3],x^3)
```

maxima [A] time = 0.42, size = 193, normalized size = 1.62

$$-\frac{22}{2187}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3-1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{\frac{440\left(x^3-1\right)^{\frac{1}{3}}}{x}+\frac{1555\left(x^3-1\right)^{\frac{4}{3}}}{x^4}-\frac{1815\left(x^3-1\right)^{\frac{7}{3}}}{x^7}+\frac{1012\left(x^3-1\right)^{\frac{10}{3}}}{x^{10}}-\frac{220\left(x^3-1\right)^{\frac{13}{3}}}{x^{13}}}{14580\left(\frac{5\left(x^3-1\right)}{x^3}-\frac{10\left(x^3-1\right)^2}{x^6}+\frac{10\left(x^3-1\right)^3}{x^9}-\frac{5\left(x^3-1\right)^4}{x^{12}}+\frac{\left(x^3-1\right)^5}{x^{15}}-1\right)}-\frac{11}{2187}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3-1\right)^{\frac{2}{3}}}{x^2}+1\right)+\frac{22}{2187}\log\left(\frac{\left(x^3-1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(x^3-1)^(1/3),x, algorithm="maxima")
```

```
[Out] -22/2187*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3)/x + 1)) - 1/14580*(4
40*(x^3 - 1)^(1/3)/x + 1555*(x^3 - 1)^(4/3)/x^4 - 1815*(x^3 - 1)^(7/3)/x^7
+ 1012*(x^3 - 1)^(10/3)/x^10 - 220*(x^3 - 1)^(13/3)/x^13)/(5*(x^3 - 1)/x^3
- 10*(x^3 - 1)^2/x^6 + 10*(x^3 - 1)^3/x^9 - 5*(x^3 - 1)^4/x^12 + (x^3 - 1)^
5/x^15 - 1) - 11/2187*log((x^3 - 1)^(1/3)/x + (x^3 - 1)^(2/3)/x^2 + 1) + 22
/2187*log((x^3 - 1)^(1/3)/x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{13}\,{\left(x^3-1\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(x^3 - 1)^(1/3),x)
```

```
[Out] int(x^13*(x^3 - 1)^(1/3), x)
```

sympy [C] time = 2.78, size = 36, normalized size = 0.30

$$-\frac{x^{14}e^{-\frac{2i\pi}{3}}\Gamma\left(\frac{14}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{14}{3}\\ \frac{17}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{17}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(x**3-1)**(1/3),x)
```

```
[Out] -x**14*exp(-2*I*pi/3)*gamma(14/3)*hyper((-1/3, 14/3), (17/3,), x**3)/(3*gam
ma(17/3))
```

3.1487 $\int \frac{ab^2-2(2a-b)bx+(3a-2b)x^2}{\sqrt[4]{x(-a+x)(-b+x)^2}\left(a^3+\left(-3a^2+b^2d\right)x+(3a-2bd)x^2+(-1+d)x^3\right)}\,dx$

Optimal. Leaf size=119

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}}{a-x}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}}{a-x}\right)}{d^{3/4}}$$

Rubi [F] time = 9.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ab^2-2(2a-b)bx+(3a-2b)x^2}{\sqrt[4]{x(-a+x)(-b+x)^2}\left(a^3+\left(-3a^2+b^2d\right)x+(3a-2bd)x^2+(-1+d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*b^2 - 2*(2*a - b)*b*x + (3*a - 2*b)*x^2)/((x*(-a + x)*(-b + x)^2)^(1
/4)*(a^3 + (-3*a^2 + b^2*d)*x + (3*a - 2*b*d)*x^2 + (-1 + d)*x^3)),x]
```

```
[Out] (4*(3*a - 2*b)*x^(1/4)*(-a + x)^(1/4)*Sqrt[-b + x]*Defer[Subst][Defer[Int][
(x^6*Sqrt[-b + x^4])/((-a + x^4)^(1/4)*(a^3 - 3*a^2*(1 - (b^2*d)/(3*a^2))*x
^4 + 3*a*(1 - (2*b*d)/(3*a))*x^8 - (1 - d)*x^12)), x], x, x^(1/4)])/(-((a -
 x)*(b - x)^2*x))^(1/4) + (4*a*b*x^(1/4)*(-a + x)^(1/4)*Sqrt[-b + x]*Defer[
Subst][Defer[Int][(x^2*Sqrt[-b + x^4])/((-a + x^4)^(1/4)*(-a^3 + 3*a^2*(1 -
 (b^2*d)/(3*a^2))*x^4 - 3*a*(1 - (2*b*d)/(3*a))*x^8 + (1 - d)*x^12)), x], x
, x^(1/4)])/(-((a - x)*(b - x)^2*x))^(1/4)
```

Rubi steps

$$\int \frac{ab^2-2(2a-b)bx+(3a-2b)x^2}{\sqrt[4]{x(-a+x)(-b+x)^2}\left(a^3+\left(-3a^2+b^2d\right)x+(3a-2bd)x^2+(-1+d)x^3\right)}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt{-b+x}\right)\int \cdots}{\cdots}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt{-b+x}\right)\int \cdots}{\sqrt[4]{x}\cdots}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt{-b+x}\right)\mathrm{Su}\cdots}{\cdots}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt{-b+x}\right)\mathrm{Su}\cdots}{\cdots}$$

$$= \frac{\left(4(3a-2b)\sqrt[4]{x}\sqrt[4]{-a+x}\sqrt{-\cdots}\right.}{\cdots}$$

Mathematica [F] time = 4.95, size = 0, normalized size = 0.00

$$\int \frac{ab^2-2(2a-b)bx+(3a-2b)x^2}{\sqrt[4]{x(-a+x)(-b+x)^2}\left(a^3+\left(-3a^2+b^2d\right)x+(3a-2bd)x^2+(-1+d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*b^2 - 2*(2*a - b)*b*x + (3*a - 2*b)*x^2)/((x*(-a + x)*(-b + x)
^2)^(1/4)*(a^3 + (-3*a^2 + b^2*d)*x + (3*a - 2*b*d)*x^2 + (-1 + d)*x^3)),x]
```

```
[Out] Integrate[(a*b^2 - 2*(2*a - b)*b*x + (3*a - 2*b)*x^2)/((x*(-a + x)*(-b + x)
^2)^(1/4)*(a^3 + (-3*a^2 + b^2*d)*x + (3*a - 2*b*d)*x^2 + (-1 + d)*x^3)), x
]
```

IntegrateAlgebraic [A] time = 0.44, size = 119, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}}{a-x}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}}{a-x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b^2 - 2*(2*a - b)*b*x + (3*a - 2*b)*x^2)/((x*(-a + x)
*(-b + x)^2)^(1/4)*(a^3 + (-3*a^2 + b^2*d)*x + (3*a - 2*b*d)*x^2 + (-1 + d)
*x^3)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^
(1/4))/(a - x)])/d^(3/4) - (2*ArcTanh[(d^(1/4)*(-(a*b^2*x) + (2*a*b + b^2)*
x^2 + (-a - 2*b)*x^3 + x^4)^(1/4))/(a - x)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b^2-2*(2*a-b)*b*x+(3*a-2*b)*x^2)/(x*(-a+x)*(-b+x)^2)^(1/4)/(a^
3+(b^2*d-3*a^2)*x+(-2*b*d+3*a)*x^2+(-1+d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab^2 - 2(2a-b)bx + (3a-2b)x^2}{\left(-(a-x)(b-x)^2x\right)^{\frac{1}{4}}\left((d-1)x^3 + a^3 - (2bd-3a)x^2 + \left(b^2d - 3a^2\right)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b^2-2*(2*a-b)*b*x+(3*a-2*b)*x^2)/(x*(-a+x)*(-b+x)^2)^(1/4)/(a^
3+(b^2*d-3*a^2)*x+(-2*b*d+3*a)*x^2+(-1+d)*x^3),x, algorithm="giac")
```

```
[Out] integrate((a*b^2 - 2*(2*a - b)*b*x + (3*a - 2*b)*x^2)/((-(a - x)*(b - x)^2*
x)^(1/4)*((d - 1)*x^3 + a^3 - (2*b*d - 3*a)*x^2 + (b^2*d - 3*a^2)*x)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{a\,b^2 - 2\,(2a-b)\,bx + (3a-2b)\,x^2}{\left(x\,(-a+x)\,(-b+x)^2\right)^{\frac{1}{4}}\left(a^3 + \left(b^2d - 3a^2\right)x + (-2bd+3a)\,x^2 + (-1+d)\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b^2-2*(2*a-b)*b*x+(3*a-2*b)*x^2)/(x*(-a+x)*(-b+x)^2)^(1/4)/(a^3+(b^2
*d-3*a^2)*x+(-2*b*d+3*a)*x^2+(-1+d)*x^3),x)
```

```
[Out] int((a*b^2-2*(2*a-b)*b*x+(3*a-2*b)*x^2)/(x*(-a+x)*(-b+x)^2)^(1/4)/(a^3+(b^2
*d-3*a^2)*x+(-2*b*d+3*a)*x^2+(-1+d)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab^2 - 2(2a - b)bx + (3a - 2b)x^2}{\left(-(a - x)(b - x)^2 x\right)^{\frac{1}{4}} \left((d - 1)x^3 + a^3 - (2bd - 3a)x^2 + \left(b^2 d - 3a^2\right)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b^2-2*(2*a-b)*b*x+(3*a-2*b)*x^2)/(x*(-a+x)*(-b+x)^2)^(1/4)/(a^
3+(b^2*d-3*a^2)*x+(-2*b*d+3*a)*x^2+(-1+d)*x^3),x, algorithm="maxima")
```

```
[Out] integrate((a*b^2 - 2*(2*a - b)*b*x + (3*a - 2*b)*x^2)/((-(a - x)*(b - x)^2*
x)^(1/4)*((d - 1)*x^3 + a^3 - (2*b*d - 3*a)*x^2 + (b^2*d - 3*a^2)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,(3\,a - 2\,b) + a\,b^2 - 2\,b\,x\,(2\,a - b)}{\left(-x\,(a - x)\,(b - x)^2\right)^{1/4}\,\left(x^2\,(3\,a - 2\,b\,d) + a^3 + x\,\left(b^2\,d - 3\,a^2\right) + x^3\,(d - 1)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*a - 2*b) + a*b^2 - 2*b*x*(2*a - b))/((-x*(a - x)*(b - x)^2)^(1/
4)*(x^2*(3*a - 2*b*d) + a^3 + x*(b^2*d - 3*a^2) + x^3*(d - 1))),x)
```

```
[Out] int((x^2*(3*a - 2*b) + a*b^2 - 2*b*x*(2*a - b))/((-x*(a - x)*(b - x)^2)^(1/
4)*(x^2*(3*a - 2*b*d) + a^3 + x*(b^2*d - 3*a^2) + x^3*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b**2-2*(2*a-b)*b*x+(3*a-2*b)*x**2)/(x*(-a+x)*(-b+x)**2)**(1/4)
/(a**3+(b**2*d-3*a**2)*x+(-2*b*d+3*a)*x**2+(-1+d)*x**3),x)
```

```
[Out] Timed out
```

3.1488 $\int \frac{(1+x^6)(-1-x^3+x^6)^{2/3}}{x^3(-1+x^6)}\,dx$

Optimal. Leaf size=119

$$\frac{1}{3}\log\left(\sqrt[3]{x^6-x^3-1}+x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^6-x^3-1}-x}\right)}{\sqrt{3}}+\frac{\left(x^6-x^3-1\right)^{2/3}}{2x^2}-\frac{1}{6}\log\left(x^2-\sqrt[3]{x^6-x^3-1}\,x+\left(x^6-x^3-1\right)^{2/3}\right)$$

Rubi [F] time = 1.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+x^6\right)\left(-1-x^3+x^6\right)^{2/3}}{x^3\left(-1+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^6)*(-1 - x^3 + x^6)^(2/3))/(x^3*(-1 + x^6)),x]
```

```
[Out] ((-1 - x^3 + x^6)^(2/3)*AppellF1[-2/3, -2/3, -2/3, 1/3, (2*x^3)/(1 + Sqrt[5
]), (2*x^3)/(1 - Sqrt[5])])/(2*x^2*(1 - (2*x^3)/(1 - Sqrt[5]))^(2/3)*(1 - (
2*x^3)/(1 + Sqrt[5]))^(2/3)) + Defer[Int][(-1 - x^3 + x^6)^(2/3)/(-1 + x),
x]/3 + Defer[Int][(-1 - x^3 + x^6)^(2/3)/(1 + x), x]/3 - ((1 + I*Sqrt[3])*D
efer[Int][(-1 - x^3 + x^6)^(2/3)/(-1 - I*Sqrt[3] + 2*x), x])/3 - ((1 - I*Sq
rt[3])*Defer[Int][(-1 - x^3 + x^6)^(2/3)/(1 - I*Sqrt[3] + 2*x), x])/3 - ((1
 - I*Sqrt[3])*Defer[Int][(-1 - x^3 + x^6)^(2/3)/(-1 + I*Sqrt[3] + 2*x), x])
/3 - ((1 + I*Sqrt[3])*Defer[Int][(-1 - x^3 + x^6)^(2/3)/(1 + I*Sqrt[3] + 2*
x), x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+x^6\right)\left(-1-x^3+x^6\right)^{2/3}}{x^3\left(-1+x^6\right)}\,dx &= \int \left(-\frac{\left(-1-x^3+x^6\right)^{2/3}}{x^3}+\frac{2x\left(-1-x^3+x^6\right)^{2/3}}{3\left(-1+x^2\right)}+\frac{(2-x)\left(-1-x^3+x^6\right)^{2/3}}{3\left(1-x+x^2\right)}+\ldots\right.\\
&= \frac{1}{3}\int \frac{(2-x)\left(-1-x^3+x^6\right)^{2/3}}{1-x+x^2}\,dx+\frac{1}{3}\int \frac{(-2-x)\left(-1-x^3+x^6\right)^{2/3}}{1+x+x^2}\,dx+\frac{2}{3}\int \ldots\\
&= \frac{1}{3}\int \left(\frac{\left(-1-i\sqrt{3}\right)\left(-1-x^3+x^6\right)^{2/3}}{-1-i\sqrt{3}+2x}+\frac{\left(-1+i\sqrt{3}\right)\left(-1-x^3+x^6\right)^{2/3}}{-1+i\sqrt{3}+2x}\right)dx+\frac{1}{3}\ldots\\
&= \frac{\left(-1-x^3+x^6\right)^{2/3}F_1\left(-\frac{2}{3};-\frac{2}{3},-\frac{2}{3};\frac{1}{3};\frac{2x^3}{1+\sqrt{5}},\frac{2x^3}{1-\sqrt{5}}\right)}{2x^2\left(1-\frac{2x^3}{1-\sqrt{5}}\right)^{2/3}\left(1-\frac{2x^3}{1+\sqrt{5}}\right)^{2/3}}+\frac{1}{3}\int \frac{\left(-1-x^3+x^6\right)^{2/3}}{-1+x}\,dx\ldots
\end{aligned}$$

Mathematica [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\left(1+x^6\right)\left(-1-x^3+x^6\right)^{2/3}}{x^3\left(-1+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^6)*(-1 - x^3 + x^6)^(2/3))/(x^3*(-1 + x^6)),x]
```

[Out] Integrate[((1 + x^6)*(-1 - x^3 + x^6)^(2/3))/(x^3*(-1 + x^6)), x]

IntegrateAlgebraic [A] time = 0.35, size = 119, normalized size = 1.00

$$\frac{1}{3}\log\left(\sqrt[3]{x^6-x^3-1}+x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^6-x^3-1}-x}\right)}{\sqrt{3}}+\frac{\left(x^6-x^3-1\right)^{2/3}}{2x^2}-\frac{1}{6}\log\left(x^2-\sqrt[3]{x^6-x^3-1}\,x+\left(x^6-x^3-1\right)^{2/3}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + x^6)*(-1 - x^3 + x^6)^(2/3))/(x^3*(-1 + x^6)),x]

[Out] (-1 - x^3 + x^6)^(2/3)/(2*x^2) + ArcTan[(Sqrt[3]*x)/(-x + 2*(-1 - x^3 + x^6
)^(1/3))]/Sqrt[3] + Log[x + (-1 - x^3 + x^6)^(1/3)]/3 - Log[x^2 - x*(-1 - x
^3 + x^6)^(1/3) + (-1 - x^3 + x^6)^(2/3)]/6

fricas [A] time = 17.56, size = 149, normalized size = 1.25

$$\frac{2\sqrt{3}x^2\arctan\left(\frac{377916639464896406983903892597481126726653448417603984366325734068057972584403925 14\sqrt{3}\left(x^6-x^3-1\right)^{\frac{1}{3}}x^2+426162825235527199042479104917729248073007919805353037206096056412855329005651 58554\sqrt{3}\left(x^6-x^3-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(183230471683433120927601559493133076475092570182205635516405557078015298 68232673857x^6+241230928853153960292876061601240606731772356938745264198851611723986782106238 3020x^3-183230471683433120927601559493133076475092570182205635516405557078015298 68232673857\right)}{710582473559489405933426903442308224224790895510954955244430133983133539872942708 91x^6-120611919705063540903957449627281556219949205233443553235863268572136995238508326602x^3-710582473559489405933426903442308224224790895510954955244430133983133539872942708 91}\right)+x^2\log\left(\frac{x^6+3\left(x^6-x^3-1\right)^{\frac{1}{3}}x^2+3\left(x^6-x^3-1\right)^{\frac{2}{3}}x-1}{x^6-1}\right)+3\left(x^6-x^3-1\right)^{\frac{2}{3}}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+1)*(x^6-x^3-1)^(2/3)/x^3/(x^6-1),x, algorithm="fricas")

[Out] 1/6*(2*sqrt(3)*x^2*arctan((377916639464896406983903892597481126726653448417
603984366325734068057972584403925 14*sqrt(3)*(x^6 - x^3 - 1)^(1/3)*x^2 + 426
162825235527199042479104917729248073007919805353037206096056412855329005651
58554*sqrt(3)*(x^6 - x^3 - 1)^(2/3)*x + sqrt(3)*(183230471683433120927601559
49313307647509257018220563551640555707801529868232673857*x^6 + 24123092885
31539602928760616012406067317723569387452641988516117239867821062383020*x^3
 - 183230471683433120927601559493133076475092570182205635516405557078015298
68232673857))/(710582473559489405933426903442308224224790895510954955244430
1339831335398729427089 1*x^6 - 120611919705063540903957449627281556219949205
233443553235863268572136995238508326602*x^3 - 7105824735594894059334269034 4
2308224224790895510954955244430133983133539872942708 91)) + x^2*log((x^6 + 3
*(x^6 - x^3 - 1)^(1/3)*x^2 + 3*(x^6 - x^3 - 1)^(2/3)*x - 1)/(x^6 - 1)) + 3*
(x^6 - x^3 - 1)^(2/3))/x^2

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6-x^3-1\right)^{\frac{2}{3}}\left(x^6+1\right)}{\left(x^6-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+1)*(x^6-x^3-1)^(2/3)/x^3/(x^6-1),x, algorithm="giac")

[Out] integrate((x^6 - x^3 - 1)^(2/3)*(x^6 + 1)/((x^6 - 1)*x^3), x)

maple [C] time = 1.84, size = 573, normalized size = 4.82

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6+1)*(x^6-x^3-1)^(2/3)/x^3/(x^6-1),x)

[Out] 1/2*(x^6-x^3-1)^(2/3)/x^2+RootOf(9*_Z^2+3*_Z+1)*ln((-3*RootOf(9*_Z^2+3*_Z+1
)*x^6+9*RootOf(9*_Z^2+3*_Z+1)^2*x^3+3*(x^6-x^3-1)^(2/3)*RootOf(9*_Z^2+3*_Z+
1)*x-3*RootOf(9*_Z^2+3*_Z+1)*(x^6-x^3-1)^(1/3)*x^2+6*RootOf(9*_Z^2+3*_Z+1)*
x^3-(x^6-x^3-1)^(2/3)*x+(x^6-x^3-1)^(1/3)*x^2+3*RootOf(9*_Z^2+3*_Z+1))/(-1+
x)/(x^2+x+1)/(1+x)/(x^2-x+1))-1/3*ln(-(-3*RootOf(9*_Z^2+3*_Z+1)*x^6-x^6-9*R

```
ootOf(9*_Z^2+3*_Z+1)^2*x^3+3*(x^6-x^3-1)^(2/3)*RootOf(9*_Z^2+3*_Z+1)*x-3*Ro
otOf(9*_Z^2+3*_Z+1)*(x^6-x^3-1)^(1/3)*x^2+2*(x^6-x^3-1)^(2/3)*x-2*(x^6-x^3-
1)^(1/3)*x^2+x^3+3*RootOf(9*_Z^2+3*_Z+1)+1)/(-1+x)/(x^2+x+1)/(1+x)/(x^2-x+1
))-ln(-(-3*RootOf(9*_Z^2+3*_Z+1)*x^6-x^6-9*RootOf(9*_Z^2+3*_Z+1)^2*x^3+3*(x
^6-x^3-1)^(2/3)*RootOf(9*_Z^2+3*_Z+1)*x-3*RootOf(9*_Z^2+3*_Z+1)*(x^6-x^3-1)
^(1/3)*x^2+2*(x^6-x^3-1)^(2/3)*x-2*(x^6-x^3-1)^(1/3)*x^2+x^3+3*RootOf(9*_Z^
2+3*_Z+1)+1)/(-1+x)/(x^2+x+1)/(1+x)/(x^2-x+1))*RootOf(9*_Z^2+3*_Z+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6 - x^3 - 1\right)^{\frac{2}{3}}\left(x^6 + 1\right)}{\left(x^6 - 1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)*(x^6-x^3-1)^(2/3)/x^3/(x^6-1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 - x^3 - 1)^(2/3)*(x^6 + 1)/((x^6 - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6 + 1\right)\,\left(x^6 - x^3 - 1\right)^{2/3}}{x^3\,\left(x^6 - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 + 1)*(x^6 - x^3 - 1)^(2/3))/(x^3*(x^6 - 1)),x)
```

```
[Out] int(((x^6 + 1)*(x^6 - x^3 - 1)^(2/3))/(x^3*(x^6 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2 + 1\right)\left(x^4 - x^2 + 1\right)\left(x^6 - x^3 - 1\right)^{\frac{2}{3}}}{x^3\left(x - 1\right)\left(x + 1\right)\left(x^2 - x + 1\right)\left(x^2 + x + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)*(x**6-x**3-1)**(2/3)/x**3/(x**6-1),x)
```

```
[Out] Integral((x**2 + 1)*(x**4 - x**2 + 1)*(x**6 - x**3 - 1)**(2/3)/(x**3*(x - 1
)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1)), x)
```

3.1489 $\int \frac{(4+x^6)\sqrt[4]{-2-x^4+x^6}}{x^2(-2+x^6)}\,dx$

Optimal. Leaf size=119

$$\frac{2\sqrt[4]{x^6-x^4-2}}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6-x^4-2}}{\sqrt{x^6-x^4-2}-x^2}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6-x^4-2}}{x^2+\sqrt{x^6-x^4-2}}\right)}{\sqrt{2}}$$

Rubi [F] time = 1.75, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(4+x^6)\sqrt[4]{-2-x^4+x^6}}{x^2(-2+x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((4 + x^6)*(-2 - x^4 + x^6)^(1/4))/(x^2*(-2 + x^6)),x]
```

```
[Out] -1/2*Defer[Int][(-2 - x^4 + x^6)^(1/4)/(2^(1/6) - x), x]/2^(1/6) - 2*Defer[
Int][(-2 - x^4 + x^6)^(1/4)/x^2, x] - Defer[Int][(-2 - x^4 + x^6)^(1/4)/(2^
(1/6) + x), x]/(2*2^(1/6)) - ((-1)^(2/3)*Defer[Int][(-2 - x^4 + x^6)^(1/4)/
(2^(1/6) - (-1)^(1/3)*x), x])/(2*2^(1/6)) - ((-1)^(2/3)*Defer[Int][(-2 - x^
4 + x^6)^(1/4)/(2^(1/6) + (-1)^(1/3)*x), x])/(2*2^(1/6)) + ((-1)^(1/3)*Defe
r[Int][(-2 - x^4 + x^6)^(1/4)/(2^(1/6) - (-1)^(2/3)*x), x])/(2*2^(1/6)) + (
(-1)^(1/3)*Defer[Int][(-2 - x^4 + x^6)^(1/4)/(2^(1/6) + (-1)^(2/3)*x), x])/
(2*2^(1/6))
```

Rubi steps

$$\begin{aligned}
\int \frac{(4+x^6)\sqrt[4]{-2-x^4+x^6}}{x^2(-2+x^6)}\,dx &= \int \left(-\frac{2\sqrt[4]{-2-x^4+x^6}}{x^2}+\frac{3x^4\sqrt[4]{-2-x^4+x^6}}{-2+x^6}\right)dx\\
&= -\left(2\int \frac{\sqrt[4]{-2-x^4+x^6}}{x^2}\,dx\right)+3\int \frac{x^4\sqrt[4]{-2-x^4+x^6}}{-2+x^6}\,dx\\
&= -\left(2\int \frac{\sqrt[4]{-2-x^4+x^6}}{x^2}\,dx\right)+3\int \left(\frac{x\sqrt[4]{-2-x^4+x^6}}{2\left(-\sqrt{2}+x^3\right)}+\frac{x\sqrt[4]{-2-x^4+x^6}}{2\left(\sqrt{2}+x^3\right)}\right)dx\\
&= \frac{3}{2}\int \frac{x\sqrt[4]{-2-x^4+x^6}}{-\sqrt{2}+x^3}\,dx+\frac{3}{2}\int \frac{x\sqrt[4]{-2-x^4+x^6}}{\sqrt{2}+x^3}\,dx-2\int \frac{\sqrt[4]{-2-x^4+x^6}}{x^2}\,d\\
&= \frac{3}{2}\int \left(-\frac{\sqrt[4]{-2-x^4+x^6}}{3\sqrt[6]{2}\left(\sqrt[6]{2}-x\right)}-\frac{(-1)^{2/3}\sqrt[4]{-2-x^4+x^6}}{3\sqrt[6]{2}\left(\sqrt[6]{2}+\sqrt[3]{-1}\,x\right)}+\frac{\sqrt[3]{-1}\sqrt[4]{-2-x^4+x^6}}{3\sqrt[6]{2}\left(\sqrt[6]{2}-(-1)^{2/3}x\right)}\right)dx\\
&= -\left(2\int \frac{\sqrt[4]{-2-x^4+x^6}}{x^2}\,dx\right)-\frac{\int \frac{\sqrt[4]{-2-x^4+x^6}}{\sqrt[6]{2}-x}\,dx}{2\sqrt[6]{2}}-\frac{\int \frac{\sqrt[4]{-2-x^4+x^6}}{\sqrt[6]{2}+x}\,dx}{2\sqrt[6]{2}}+\frac{\sqrt[3]{-1}\int \frac{\sqrt[4]{}}{\sqrt[6]{2}}}{2}
\end{aligned}$$

Mathematica [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(4+x^6)\sqrt[4]{-2-x^4+x^6}}{x^2(-2+x^6)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[((4 + x^6)*(-2 - x^4 + x^6)^(1/4))/(x^2*(-2 + x^6)),x]

[Out] Integrate[((4 + x^6)*(-2 - x^4 + x^6)^(1/4))/(x^2*(-2 + x^6)), x]

IntegrateAlgebraic [A] time = 1.03, size = 119, normalized size = 1.00

$$\frac{2\sqrt[4]{x^6-x^4-2}}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x\sqrt[4]{x^6-x^4-2}}{\sqrt{x^6-x^4-2}-x^2}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt[4]{x^6-x^4-2}}{x^2+\sqrt{x^6-x^4-2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((4 + x^6)*(-2 - x^4 + x^6)^(1/4))/(x^2*(-2 + x^6)),x]

[Out] (2*(-2 - x^4 + x^6)^(1/4))/x + ArcTan[(Sqrt[2]*x*(-2 - x^4 + x^6)^(1/4))/(-x^2 + Sqrt[-2 - x^4 + x^6])]/Sqrt[2] - ArcTanh[(Sqrt[2]*x*(-2 - x^4 + x^6)^(1/4))/(x^2 + Sqrt[-2 - x^4 + x^6])]/Sqrt[2]

fricas [B] time = 101.29, size = 780, normalized size = 6.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+4)*(x^6-x^4-2)^(1/4)/x^2/(x^6-2),x, algorithm="fricas")

[Out] -1/8*(4*sqrt(2)*x*arctan(-(x^12 - 4*x^6 + 2*sqrt(2)*(x^7 - 4*x^5 - 2*x)*(x^6 - x^4 - 2)^(3/4) + 2*sqrt(2)*(3*x^9 - 4*x^7 - 6*x^3)*(x^6 - x^4 - 2)^(1/4) + 4*(x^8 - 2*x^2)*sqrt(x^6 - x^4 - 2) - (16*(x^6 - x^4 - 2)^(3/4)*x^5 + 2*sqrt(2)*(x^8 - 4*x^6 - 2*x^2)*sqrt(x^6 - x^4 - 2) + sqrt(2)*(x^12 - 10*x^10 + 8*x^8 - 4*x^6 + 20*x^4 + 4) + 4*(x^9 - 2*x^3)*(x^6 - x^4 - 2)^(1/4))*sqrt((x^6 + 2*sqrt(2)*(x^6 - x^4 - 2)^(1/4)*x^3 + 4*sqrt(x^6 - x^4 - 2)*x^2 + 2*sqrt(2)*(x^6 - x^4 - 2)^(3/4)*x - 2)/(x^6 - 2)) + 4)/(x^12 - 16*x^10 + 16*x^8 - 4*x^6 + 32*x^4 + 4)) - 4*sqrt(2)*x*arctan(-(x^12 - 4*x^6 - 2*sqrt(2)*(x^7 - 4*x^5 - 2*x)*(x^6 - x^4 - 2)^(3/4) - 2*sqrt(2)*(3*x^9 - 4*x^7 - 6*x^3)*(x^6 - x^4 - 2)^(1/4) + 4*(x^8 - 2*x^2)*sqrt(x^6 - x^4 - 2) - (16*(x^6 - x^4 - 2)^(3/4)*x^5 - 2*sqrt(2)*(x^8 - 4*x^6 - 2*x^2)*sqrt(x^6 - x^4 - 2) - sqrt(2)*(x^12 - 10*x^10 + 8*x^8 - 4*x^6 + 20*x^4 + 4) + 4*(x^9 - 2*x^3)*(x^6 - x^4 - 2)^(1/4))*sqrt((x^6 - 2*sqrt(2)*(x^6 - x^4 - 2)^(1/4)*x^3 + 4*sqrt(x^6 - x^4 - 2)*x^2 - 2*sqrt(2)*(x^6 - x^4 - 2)^(3/4)*x - 2)/(x^6 - 2)) + 4)/(x^12 - 16*x^10 + 16*x^8 - 4*x^6 + 32*x^4 + 4)) + sqrt(2)*x*log(4*(x^6 + 2*sqrt(2)*(x^6 - x^4 - 2)^(1/4)*x^3 + 4*sqrt(x^6 - x^4 - 2)*x^2 + 2*sqrt(2)*(x^6 - x^4 - 2)^(3/4)*x - 2)/(x^6 - 2)) - sqrt(2)*x*log(4*(x^6 - 2*sqrt(2)*(x^6 - x^4 - 2)^(1/4)*x^3 + 4*sqrt(x^6 - x^4 - 2)*x^2 - 2*sqrt(2)*(x^6 - x^4 - 2)^(3/4)*x - 2)/(x^6 - 2)) - 16*(x^6 - x^4 - 2)^(1/4))/x

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6-x^4-2\right)^{\frac{1}{4}}\left(x^6+4\right)}{\left(x^6-2\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+4)*(x^6-x^4-2)^(1/4)/x^2/(x^6-2),x, algorithm="giac")

[Out] integrate((x^6 - x^4 - 2)^(1/4)*(x^6 + 4)/((x^6 - 2)*x^2), x)

maple [C] time = 2.81, size = 1372, normalized size = 11.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+4)*(x^6-x^4-2)^(1/4)/x^2/(x^6-2),x)
```

```
[Out] 2*(x^6-x^4-2)^(1/4)/x+(-1/2*RootOf(_Z^4+1)^3*ln(-(RootOf(_Z^4+1)^2*x^18-4*R
ootOf(_Z^4+1)^2*x^16+5*x^14*RootOf(_Z^4+1)^2-2*RootOf(_Z^4+1)*(x^18-3*x^16+
3*x^14-7*x^12+12*x^10-6*x^8+12*x^6-12*x^4-8)^(1/4)*x^13-8*RootOf(_Z^4+1)^2*
x^12+4*RootOf(_Z^4+1)*(x^18-3*x^16+3*x^14-7*x^12+12*x^10-6*x^8+12*x^6-12*x^
4-8)^(1/4)*x^11+16*x^10*RootOf(_Z^4+1)^2-2*RootOf(_Z^4+1)*(x^18-3*x^16+3*x^
14-7*x^12+12*x^10-6*x^8+12*x^6-12*x^4-8)^(1/4)*x^9+2*(x^18-3*x^16+3*x^14-7*
x^12+12*x^10-6*x^8+12*x^6-12*x^4-8)^(1/2)*x^8-10*x^8*RootOf(_Z^4+1)^2+2*Roo
tOf(_Z^4+1)^3*(x^18-3*x^16+3*x^14-7*x^12+12*x^10-6*x^8+12*x^6-12*x^4-8)^(3/
4)*x^3+8*RootOf(_Z^4+1)*(x^18-3*x^16+3*x^14-7*x^12+12*x^10-6*x^8+12*x^6-12*
x^4-8)^(1/4)*x^7-2*(x^18-3*x^16+3*x^14-7*x^12+12*x^10-6*x^8+12*x^6-12*x^4-8
)^(1/2)*x^6+12*RootOf(_Z^4+1)^2*x^6-8*RootOf(_Z^4+1)*(x^18-3*x^16+3*x^14-7*
x^12+12*x^10-6*x^8+12*x^6-12*x^4-8)^(1/4)*x^5-16*RootOf(_Z^4+1)^2*x^4-4*(x^
18-3*x^16+3*x^14-7*x^12+12*x^10-6*x^8+12*x^6-12*x^4-8)^(1/2)*x^2-8*RootOf(_
Z^4+1)*(x^18-3*x^16+3*x^14-7*x^12+12*x^10-6*x^8+12*x^6-12*x^4-8)^(1/4)*x-8*
RootOf(_Z^4+1)^2)/(x^6-2)/(x^6-x^4-2)^2)-1/2*RootOf(_Z^4+1)*ln(-(-RootOf(_Z
^4+1)^2*x^18+4*RootOf(_Z^4+1)^2*x^16-2*RootOf(_Z^4+1)^3*(x^18-3*x^16+3*x^14
-7*x^12+12*x^10-6*x^8+12*x^6-12*x^4-8)^(1/4)*x^13-5*x^14*RootOf(_Z^4+1)^2+4
*RootOf(_Z^4+1)^3*(x^18-3*x^16+3*x^14-7*x^12+12*x^10-6*x^8+12*x^6-12*x^4-8)
^(1/4)*x^11+8*RootOf(_Z^4+1)^2*x^12-2*RootOf(_Z^4+1)^3*(x^18-3*x^16+3*x^14-
7*x^12+12*x^10-6*x^8+12*x^6-12*x^4-8)^(1/4)*x^9-16*x^10*RootOf(_Z^4+1)^2+8*
RootOf(_Z^4+1)^3*(x^18-3*x^16+3*x^14-7*x^12+12*x^10-6*x^8+12*x^6-12*x^4-8)^
(1/4)*x^7+2*(x^18-3*x^16+3*x^14-7*x^12+12*x^10-6*x^8+12*x^6-12*x^4-8)^(1/2)
*x^8+10*x^8*RootOf(_Z^4+1)^2-8*RootOf(_Z^4+1)^3*(x^18-3*x^16+3*x^14-7*x^12+
12*x^10-6*x^8+12*x^6-12*x^4-8)^(1/4)*x^5-2*(x^18-3*x^16+3*x^14-7*x^12+12*x^
10-6*x^8+12*x^6-12*x^4-8)^(1/2)*x^6-12*RootOf(_Z^4+1)^2*x^6+2*RootOf(_Z^4+1
)*(x^18-3*x^16+3*x^14-7*x^12+12*x^10-6*x^8+12*x^6-12*x^4-8)^(3/4)*x^3+16*Ro
otOf(_Z^4+1)^2*x^4-8*RootOf(_Z^4+1)^3*(x^18-3*x^16+3*x^14-7*x^12+12*x^10-6*
x^8+12*x^6-12*x^4-8)^(1/4)*x-4*(x^18-3*x^16+3*x^14-7*x^12+12*x^10-6*x^8+12*
x^6-12*x^4-8)^(1/2)*x^2+8*RootOf(_Z^4+1)^2)/(x^6-2)/(x^6-x^4-2)^2))/(x^6-x^
4-2)^(3/4)*((x^6-x^4-2)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6 - x^4 - 2\right)^{\frac{1}{4}} \left(x^6 + 4\right)}{\left(x^6 - 2\right)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+4)*(x^6-x^4-2)^(1/4)/x^2/(x^6-2),x, algorithm="maxima")
```

```
[Out] integrate((x^6 - x^4 - 2)^(1/4)*(x^6 + 4)/((x^6 - 2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6 + 4\right)\,\left(x^6 - x^4 - 2\right)^{1/4}}{x^2\,\left(x^6 - 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 + 4)*(x^6 - x^4 - 2)^(1/4))/(x^2*(x^6 - 2)),x)
```

```
[Out] int(((x^6 + 4)*(x^6 - x^4 - 2)^(1/4))/(x^2*(x^6 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6 + 4\right) \sqrt[4]{x^6 - x^4 - 2}}{x^2 \left(x^6 - 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+4)*(x**6-x**4-2)**(1/4)/x**2/(x**6-2),x)
```

```
[Out] Integral((x**6 + 4)*(x**6 - x**4 - 2)**(1/4)/(x**2*(x**6 - 2)), x)
```

3.1490 $\int \sqrt[4]{bx^7+ax^8}\,dx$

Optimal. Leaf size=119

$$\frac{7b^3\tan^{-1}\left(\frac{\sqrt[4]{ax^8+bx^7}}{\sqrt[4]{a}\,x^2}\right)}{64a^{11/4}}+\frac{7b^3\tanh^{-1}\left(\frac{\sqrt[4]{ax^8+bx^7}}{\sqrt[4]{a}\,x^2}\right)}{64a^{11/4}}+\frac{\left(32a^2x^2+4abx-7b^2\right)\sqrt[4]{ax^8+bx^7}}{96a^2x}$$

Rubi [A] time = 0.25, antiderivative size = 196, normalized size of antiderivative = 1.65, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2004, 2024, 2032, 63, 331, 298, 203, 206}

$$-\frac{7b^3x^{21/4}(ax+b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{64a^{11/4}\left(ax^8+bx^7\right)^{3/4}}+\frac{7b^3x^{21/4}(ax+b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{64a^{11/4}\left(ax^8+bx^7\right)^{3/4}}-\frac{7b^2\sqrt[4]{ax^8+bx^7}}{96a^2x}+\frac{1}{3}x\sqrt[4]{ax^8+bx^7}+\frac{b\sqrt[4]{ax^8+bx^7}}{24a}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^7 + a*x^8)^(1/4),x]
```

```
[Out] (b*(b*x^7 + a*x^8)^(1/4))/(24*a) - (7*b^2*(b*x^7 + a*x^8)^(1/4))/(96*a^2*x)
 + (x*(b*x^7 + a*x^8)^(1/4))/3 - (7*b^3*x^(21/4)*(b + a*x)^(3/4)*ArcTan[(a^
(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(64*a^(11/4)*(b*x^7 + a*x^8)^(3/4)) + (7*b
^3*x^(21/4)*(b + a*x)^(3/4)*ArcTanh[(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(64
*a^(11/4)*(b*x^7 + a*x^8)^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \sqrt[4]{bx^7+ax^8}\,dx = \frac{1}{3}x\sqrt[4]{bx^7+ax^8} + \frac{1}{12}b\int \frac{x^7}{\left(bx^7+ax^8\right)^{3/4}}\,dx$$

$$= \frac{b\sqrt[4]{bx^7+ax^8}}{24a} + \frac{1}{3}x\sqrt[4]{bx^7+ax^8} - \frac{\left(7b^2\right)\int \frac{x^6}{\left(bx^7+ax^8\right)^{3/4}}\,dx}{96a}$$

$$= \frac{b\sqrt[4]{bx^7+ax^8}}{24a} - \frac{7b^2\sqrt[4]{bx^7+ax^8}}{96a^2x} + \frac{1}{3}x\sqrt[4]{bx^7+ax^8} + \frac{\left(7b^3\right)\int \frac{x^5}{\left(bx^7+ax^8\right)^{3/4}}\,dx}{128a^2}$$

$$= \frac{b\sqrt[4]{bx^7+ax^8}}{24a} - \frac{7b^2\sqrt[4]{bx^7+ax^8}}{96a^2x} + \frac{1}{3}x\sqrt[4]{bx^7+ax^8} + \frac{\left(7b^3x^{21/4}(b+ax)^{3/4}\right)\int \frac{1}{\sqrt[4]{x}\,(b+ax)^{3/4}}\,dx}{128a^2\left(bx^7+ax^8\right)^{3/4}}$$

$$= \frac{b\sqrt[4]{bx^7+ax^8}}{24a} - \frac{7b^2\sqrt[4]{bx^7+ax^8}}{96a^2x} + \frac{1}{3}x\sqrt[4]{bx^7+ax^8} + \frac{\left(7b^3x^{21/4}(b+ax)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{\left(b+ax^4\right)^{3/4}}\right.}{32a^2\left(bx^7+ax^8\right)^{3/4}}$$

$$= \frac{b\sqrt[4]{bx^7+ax^8}}{24a} - \frac{7b^2\sqrt[4]{bx^7+ax^8}}{96a^2x} + \frac{1}{3}x\sqrt[4]{bx^7+ax^8} + \frac{\left(7b^3x^{21/4}(b+ax)^{3/4}\right)\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx,\right.}{32a^2\left(bx^7+ax^8\right)^{3/4}}$$

$$= \frac{b\sqrt[4]{bx^7+ax^8}}{24a} - \frac{7b^2\sqrt[4]{bx^7+ax^8}}{96a^2x} + \frac{1}{3}x\sqrt[4]{bx^7+ax^8} + \frac{\left(7b^3x^{21/4}(b+ax)^{3/4}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,d\right.}{64a^{5/2}\left(bx^7+ax^8\right)^{3/4}}$$

$$= \frac{b\sqrt[4]{bx^7+ax^8}}{24a} - \frac{7b^2\sqrt[4]{bx^7+ax^8}}{96a^2x} + \frac{1}{3}x\sqrt[4]{bx^7+ax^8} - \frac{7b^3x^{21/4}(b+ax)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right)}{64a^{11/4}\left(bx^7+ax^8\right)^{3/4}} + \frac{7b}{}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.39

$$\frac{4x\sqrt[4]{x^7(ax+b)}\,{}_2F_1\left(-\frac{1}{4},\frac{11}{4};\frac{15}{4};-\frac{ax}{b}\right)}{11\sqrt[4]{\frac{ax}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^7 + a*x^8)^(1/4),x]
```

```
[Out] (4*x*(x^7*(b + a*x))^(1/4)*Hypergeometric2F1[-1/4, 11/4, 15/4, -((a*x)/b)])
/(11*(1 + (a*x)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.55, size = 119, normalized size = 1.00

$$\frac{7b^3\tan^{-1}\left(\frac{\sqrt[4]{ax^8+bx^7}}{\sqrt[4]{a}\,x^2}\right)}{64a^{11/4}}+\frac{7b^3\tanh^{-1}\left(\frac{\sqrt[4]{ax^8+bx^7}}{\sqrt[4]{a}\,x^2}\right)}{64a^{11/4}}+\frac{\left(32a^2x^2+4abx-7b^2\right)\sqrt[4]{ax^8+bx^7}}{96a^2x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b*x^7 + a*x^8)^(1/4),x]
```

```
[Out] ((-7*b^2 + 4*a*b*x + 32*a^2*x^2)*(b*x^7 + a*x^8)^(1/4))/(96*a^2*x) + (7*b^3
*ArcTan[(b*x^7 + a*x^8)^(1/4)/(a^(1/4)*x^2)])/(64*a^(11/4)) + (7*b^3*ArcTan
h[(b*x^7 + a*x^8)^(1/4)/(a^(1/4)*x^2)])/(64*a^(11/4))
```

fricas [B] time = 0.41, size = 265, normalized size = 2.23

$$-\frac{84\,a^2\left(\frac{b^{12}}{a^{11}}\right)^{\frac{1}{4}}x\arctan\left(-\frac{\left(ax^8+bx^7\right)^{\frac{1}{4}}a^8b^3\left(\frac{b^{12}}{a^{11}}\right)^{\frac{3}{4}}-a^8\left(\frac{b^{12}}{a^{11}}\right)^{\frac{3}{4}}x^2\sqrt{\frac{a^6\sqrt{\frac{b^{12}}{a^{11}}}x^4+\sqrt{ax^8+bx^7}\,b^6}{x^4}}}{b^{12}x^2}\right)-21\,a^2\left(\frac{b^{12}}{a^{11}}\right)^{\frac{1}{4}}x\log\left(\frac{7\left(a^3\left(\frac{b^{12}}{a^{11}}\right)^{\frac{1}{4}}x^2+\left(ax^8+bx^7\right)^{\frac{1}{4}}b^3\right)}{x^2}\right)+21\,a^2\left(\frac{b^{12}}{a^{11}}\right)^{\frac{1}{4}}x\log\left(-\frac{7\left(a^3\left(\frac{b^{12}}{a^{11}}\right)^{\frac{1}{4}}x^2-\left(ax^8+bx^7\right)^{\frac{1}{4}}b^3\right)}{x^2}\right)-4\left(ax^8+bx^7\right)^{\frac{1}{4}}\left(32\,a^2x^2+4\,abx-7\,b^2\right)}{384\,a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+b*x^7)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/384*(84*a^2*(b^12/a^11)^(1/4)*x*arctan(-((a*x^8 + b*x^7)^(1/4)*a^8*b^3*(
b^12/a^11)^(3/4) - a^8*(b^12/a^11)^(3/4)*x^2*sqrt((a^6*sqrt(b^12/a^11)*x^4
+ sqrt(a*x^8 + b*x^7)*b^6)/x^4))/(b^12*x^2)) - 21*a^2*(b^12/a^11)^(1/4)*x*l
og(7*(a^3*(b^12/a^11)^(1/4)*x^2 + (a*x^8 + b*x^7)^(1/4)*b^3)/x^2) + 21*a^2*
(b^12/a^11)^(1/4)*x*log(-7*(a^3*(b^12/a^11)^(1/4)*x^2 - (a*x^8 + b*x^7)^(1/
4)*b^3)/x^2) - 4*(a*x^8 + b*x^7)^(1/4)*(32*a^2*x^2 + 4*a*b*x - 7*b^2))/(a^2
*x)
```

giac [B] time = 0.34, size = 261, normalized size = 2.19

$$\frac{\frac{42\sqrt{2}(-a)^{\frac{1}{4}}b^4\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^3}+\frac{42\sqrt{2}(-a)^{\frac{1}{4}}b^4\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^3}+\frac{21\sqrt{2}(-a)^{\frac{1}{4}}b^4\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{a^3}+\frac{21\sqrt{2}b^4\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{(-a)^{\frac{3}{4}}a^2}-\frac{8\left(7\left(a+\frac{b}{x}\right)^{\frac{9}{4}}b^4-18\left(a+\frac{b}{x}\right)^{\frac{5}{4}}ab^4-21\left(a+\frac{b}{x}\right)^{\frac{1}{4}}a^2b^4\right)x^3}{a^2b^3}}{768\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+b*x^7)^(1/4),x, algorithm="giac")
```

```
[Out] 1/768*(42*sqrt(2)*(-a)^(1/4)*b^4*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2
*(a + b/x)^(1/4))/(-a)^(1/4))/a^3 + 42*sqrt(2)*(-a)^(1/4)*b^4*arctan(-1/2*s
qrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(a + b/x)^(1/4))/(-a)^(1/4))/a^3 + 21*sqrt(2
)*(-a)^(1/4)*b^4*log(sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a
 + b/x))/a^3 + 21*sqrt(2)*b^4*log(-sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqr
t(-a) + sqrt(a + b/x))/((-a)^(3/4)*a^2) - 8*(7*(a + b/x)^(9/4)*b^4 - 18*(a
+ b/x)^(5/4)*a*b^4 - 21*(a + b/x)^(1/4)*a^2*b^4)*x^3/(a^2*b^3))/b
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \left(a\,x^{8}+b\,x^{7}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^8+b*x^7)^(1/4),x)
```

```
[Out] int((a*x^8+b*x^7)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^{8}+bx^{7}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+b*x^7)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^8 + b*x^7)^(1/4), x)
```

mupad [B] time = 0.93, size = 38, normalized size = 0.32

$$\frac{4\,x\,\left(a\,x^{8}+b\,x^{7}\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{11}{4};\ \frac{15}{4};\ -\frac{a\,x}{b}\right)}{11\,\left(\frac{a\,x}{b}+1\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^8 + b*x^7)^(1/4),x)
```

```
[Out] (4*x*(a*x^8 + b*x^7)^(1/4)*hypergeom([-1/4, 11/4], 15/4, -(a*x)/b))/(11*((a
*x)/b + 1)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{ax^{8}+bx^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**8+b*x**7)**(1/4),x)
```

```
[Out] Integral((a*x**8 + b*x**7)**(1/4), x)
```

3.1491 $\int \frac{-1+x^{10}}{\sqrt{1+x^4}\left(1+x^{10}\right)}\,dx$

Optimal. Leaf size=119

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{5\sqrt{2}}-\frac{1}{5}\sqrt{2\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt{x^4+1}}\right)-\frac{1}{5}\sqrt{2\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt{x^4+1}}\right)$$

Rubi [F] time = 1.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+x^{10}}{\sqrt{1+x^4}\left(1+x^{10}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x^10)/(Sqrt[1 + x^4]*(1 + x^10)),x]
```

```
[Out] -1/5*ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2] + (2*(1 + x^2)*Sqrt[(1 + x^4
)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(5*Sqrt[1 + x^4]) - (8*Defer[In
t][1/(Sqrt[1 + x^4]*(1 - x^2 + x^4 - x^6 + x^8)), x])/5 + (6*Defer[Int][x^2
/(Sqrt[1 + x^4]*(1 - x^2 + x^4 - x^6 + x^8)), x])/5 - (4*Defer[Int][x^4/(Sq
rt[1 + x^4]*(1 - x^2 + x^4 - x^6 + x^8)), x])/5 + (2*Defer[Int][x^6/(Sqrt[1
 + x^4]*(1 - x^2 + x^4 - x^6 + x^8)), x])/5
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^{10}}{\sqrt{1+x^4}\left(1+x^{10}\right)}\,dx &= \int \left(\frac{1}{\sqrt{1+x^4}}-\frac{2}{\sqrt{1+x^4}\left(1+x^{10}\right)}\right)dx\\
&= -\left(2\int \frac{1}{\sqrt{1+x^4}\left(1+x^{10}\right)}\,dx\right)+\int \frac{1}{\sqrt{1+x^4}}\,dx\\
&= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}}-2\int \left(\frac{1}{5\left(1+x^2\right)\sqrt{1+x^4}}+\frac{4-3x^2+}{5\sqrt{1+x^4}\left(1-x^2\right.}\right.\\
&= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}}-\frac{2}{5}\int \frac{1}{\left(1+x^2\right)\sqrt{1+x^4}}\,dx-\frac{2}{5}\int \frac{4-3}{\sqrt{1+x^4}\left(1\right.}\\
&= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}}-\frac{1}{5}\int \frac{1}{\sqrt{1+x^4}}\,dx-\frac{1}{5}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^4}}\,dx\\
&= \frac{2\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{1+x^4}}-\frac{1}{5}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\frac{x}{\sqrt{1+x^4}}\right)+\frac{2}{5}\int\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{5\sqrt{2}}+\frac{2\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{1+x^4}}+\frac{2}{5}\int \frac{x^6}{\sqrt{1+x^4}\left(1-x^2+\right.}
\end{aligned}$$

Mathematica [C] time = 0.76, size = 375, normalized size = 3.15

$$\frac{\sqrt[4]{-1}\left(\left(-4-\sqrt[5]{-1}+(-1)^{2/5}-(-1)^{3/5}+(-1)^{4/5}\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)+2\left(\Pi\left(-i;i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)+\left(2-\sqrt[5]{-1}+(-1)^{2/5}-(-1)^{3/5}+(-1)^{4/5}\right)\Pi\left(-\sqrt[10]{-1};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)-(-1)^{4/5}\Pi\left((-1)^{3/10};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)+(-1)^{3/5}\Pi\left((-1)^{3/10};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)-(-1)^{2/5}\Pi\left((-1)^{3/10};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)+\sqrt[5]{-1}\Pi\left((-1)^{3/10};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)+\Pi\left((-1)^{7/10};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)-(-1)^{4/5}\Pi\left(-(-1)^{9/10};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)+(-1)^{3/5}\Pi\left(-(-1)^{9/10};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)-(-1)^{2/5}\Pi\left(-(-1)^{9/10};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)+\sqrt[5]{-1}\Pi\left(-(-1)^{9/10};i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)\right)\right)}{4+\sqrt[5]{-1}-(-1)^{2/5}+(-1)^{3/5}-(-1)^{4/5}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + x^10)/(Sqrt[1 + x^4]*(1 + x^10)),x]
```

```
[Out] ((-1)^(1/4)*((-4 - (-1)^(1/5) + (-1)^(2/5) - (-1)^(3/5) + (-1)^(4/5))*Ellip
ticF[I*ArcSinh[(-1)^(1/4)*x], -1] + 2*(EllipticPi[-I, I*ArcSinh[(-1)^(1/4)*
x], -1] + (2 - (-1)^(1/5) + (-1)^(2/5) - (-1)^(3/5) + (-1)^(4/5))*EllipticP
i[-(-1)^(1/10), I*ArcSinh[(-1)^(1/4)*x], -1] + (-1)^(1/5)*EllipticPi[(-1)^(
3/10), I*ArcSinh[(-1)^(1/4)*x], -1] - (-1)^(2/5)*EllipticPi[(-1)^(3/10), I*
ArcSinh[(-1)^(1/4)*x], -1] + (-1)^(3/5)*EllipticPi[(-1)^(3/10), I*ArcSinh[(
-1)^(1/4)*x], -1] - (-1)^(4/5)*EllipticPi[(-1)^(3/10), I*ArcSinh[(-1)^(1/4)
*x], -1] + EllipticPi[(-1)^(7/10), I*ArcSinh[(-1)^(1/4)*x], -1] + (-1)^(1/5
)*EllipticPi[-(-1)^(9/10), I*ArcSinh[(-1)^(1/4)*x], -1] - (-1)^(2/5)*Ellipt
icPi[-(-1)^(9/10), I*ArcSinh[(-1)^(1/4)*x], -1] + (-1)^(3/5)*EllipticPi[-(-
1)^(9/10), I*ArcSinh[(-1)^(1/4)*x], -1] - (-1)^(4/5)*EllipticPi[-(-1)^(9/10
), I*ArcSinh[(-1)^(1/4)*x], -1])))/(4 + (-1)^(1/5) - (-1)^(2/5) + (-1)^(3/5
) - (-1)^(4/5))
```

IntegrateAlgebraic [A] time = 2.26, size = 119, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{5\sqrt{2}}-\frac{1}{5}\sqrt{2\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt{x^4+1}}\right)-\frac{1}{5}\sqrt{2\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt{x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^10)/(Sqrt[1 + x^4]*(1 + x^10)),x]
```

```
[Out] -1/5*ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2] - (Sqrt[2*(1 + Sqrt[5])]*Arc
Tan[(Sqrt[-1/2 + Sqrt[5]/2]*x)/Sqrt[1 + x^4]])/5 - (Sqrt[2*(-1 + Sqrt[5])]*
ArcTanh[(Sqrt[1/2 + Sqrt[5]/2]*x)/Sqrt[1 + x^4]])/5
```

fricas [B] time = 0.56, size = 395, normalized size = 3.32

$$-\frac{1}{10}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)-\frac{1}{5}\sqrt{2\sqrt{5}+2}\arctan\left(-\frac{2\left(x^5-2x^3-\sqrt{5}\left(x^5+x\right)+x\right)\sqrt{x^4+1}\sqrt{2\sqrt{5}+2}-\left(x^8+5x^6+3x^4+5x^2+\sqrt{5}\left(x^8+x^6+3x^4+x^2+1\right)+1\right)\sqrt{2\sqrt{5}+2}\sqrt{\sqrt{5}-2}}{4\left(x^8+x^6+x^4+x^2+1\right)}\right)-\frac{1}{20}\sqrt{2\sqrt{5}-2}\log\left(-\frac{4\left(3x^5+x^3+\sqrt{5}\left(x^5+x^3+x\right)+3x\right)\sqrt{x^4+1}+\left(3x^8+5x^6+9x^4+5x^2+\sqrt{5}\left(x^8+3x^6+3x^4+3x^2+1\right)+3\right)\sqrt{2\sqrt{5}-2}}{x^8-x^6+x^4-x^2+1}\right)+\frac{1}{20}\sqrt{2\sqrt{5}-2}\log\left(-\frac{4\left(3x^5+x^3+\sqrt{5}\left(x^5+x^3+x\right)+3x\right)\sqrt{x^4+1}-\left(3x^8+5x^6+9x^4+5x^2+\sqrt{5}\left(x^8+3x^6+3x^4+3x^2+1\right)+3\right)\sqrt{2\sqrt{5}-2}}{x^8-x^6+x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^10-1)/(x^4+1)^(1/2)/(x^10+1),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) - 1/5*sqrt(2*sqrt(5) + 2)*arc
tan(-1/4*(2*(x^5 - 2*x^3 - sqrt(5)*(x^5 + x) + x)*sqrt(x^4 + 1)*sqrt(2*sqrt
(5) + 2) - (x^8 + 5*x^6 + 3*x^4 + 5*x^2 + sqrt(5)*(x^8 + x^6 + 3*x^4 + x^2
+ 1) + 1)*sqrt(2*sqrt(5) + 2)*sqrt(sqrt(5) - 2))/(x^8 + x^6 + x^4 + x^2 + 1
)) - 1/20*sqrt(2*sqrt(5) - 2)*log(-(4*(3*x^5 + x^3 + sqrt(5)*(x^5 + x^3 + x
) + 3*x)*sqrt(x^4 + 1) + (3*x^8 + 5*x^6 + 9*x^4 + 5*x^2 + sqrt(5)*(x^8 + 3*
x^6 + 3*x^4 + 3*x^2 + 1) + 3)*sqrt(2*sqrt(5) - 2))/(x^8 - x^6 + x^4 - x^2 +
 1)) + 1/20*sqrt(2*sqrt(5) - 2)*log(-(4*(3*x^5 + x^3 + sqrt(5)*(x^5 + x^3 +
 x) + 3*x)*sqrt(x^4 + 1) - (3*x^8 + 5*x^6 + 9*x^4 + 5*x^2 + sqrt(5)*(x^8 +
3*x^6 + 3*x^4 + 3*x^2 + 1) + 3)*sqrt(2*sqrt(5) - 2))/(x^8 - x^6 + x^4 - x^2
 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}-1}{\left(x^{10}+1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^10-1)/(x^4+1)^(1/2)/(x^10+1),x, algorithm="giac")
```

```
[Out] integrate((x^10 - 1)/((x^10 + 1)*sqrt(x^4 + 1)), x)
```

maple [C] time = 0.09, size = 260, normalized size = 2.18

$$\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{\sum_{\_\alpha=\operatorname{RootOf}(\_Z^8-\_Z^6+\_Z^4-\_Z^2+1)}\_\alpha\left(-\frac{\operatorname{arctanh}\left(\frac{\_\alpha^2(-\_\alpha^6+\_\alpha^4-\_\alpha^2+x^2+1)}{\sqrt{\_\alpha^4+1}\,\sqrt{x^4+1}}\right)}{\sqrt{\_\alpha^4+1}}+\frac{2(-1)^{\frac{3}{4}}(-\_\alpha^7+\_\alpha^5-\_\alpha^3+\_\alpha)\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticPi}\left((-1)^{\frac{1}{4}}x,i\_\alpha^6-i\_\alpha^4+i\_\alpha^2-i,i\right)}{\sqrt{x^4+1}}\right)}{10}+\frac{2(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticPi}\left((-1)^{\frac{1}{4}}x,i,-\sqrt{-i}\,(-1)^{\frac{3}{4}}\right)}{5\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10-1)/(x^4+1)^(1/2)/(x^10+1),x)
```

```
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+1/10*sum(_alpha*(-1/(_alpha^4+1
)^(1/2)*arctanh(_alpha^2*(-_alpha^6+_alpha^4-_alpha^2+x^2+1)/(_alpha^4+1)^(
1/2)/(x^4+1)^(1/2))+2*(-1)^(3/4)*(-_alpha^7+_alpha^5-_alpha^3+_alpha)*(1-I*
x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I*_alpha^6
-I*_alpha^4+I*_alpha^2-I,I)),_alpha=RootOf(_Z^8-_Z^6+_Z^4-_Z^2+1))+2/5*(-1)
^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*
x,I,(-I)^(1/2)/(-1)^(1/4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}-1}{\left(x^{10}+1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^10-1)/(x^4+1)^(1/2)/(x^10+1),x, algorithm="maxima")
```

```
[Out] integrate((x^10 - 1)/((x^10 + 1)*sqrt(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{10}-1}{\sqrt{x^4+1}\,\left(x^{10}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10 - 1)/((x^4 + 1)^(1/2)*(x^10 + 1)),x)
```

```
[Out] int((x^10 - 1)/((x^4 + 1)^(1/2)*(x^10 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**10-1)/(x**4+1)**(1/2)/(x**10+1),x)
```

```
[Out] Timed out
```

3.1492 $\int \frac{1+x^{10}}{\sqrt{1+x^4}\left(-1+x^{10}\right)}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{5}\sqrt{2\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt{x^4+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{5\sqrt{2}}-\frac{1}{5}\sqrt{2\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt{x^4+1}}\right)$$

Rubi [F] time = 1.88, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x^{10}}{\sqrt{1+x^4}\left(-1+x^{10}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^10)/(Sqrt[1 + x^4]*(-1 + x^10)),x]
```

```
[Out] -1/5*ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2] + (2*(1 + x^2)*Sqrt[(1 + x^
4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(5*Sqrt[1 + x^4]) - (4*Defer[I
nt][1/(Sqrt[1 + x^4]*(1 - x + x^2 - x^3 + x^4)), x])/5 + (3*Defer[Int][x/(S
qrt[1 + x^4]*(1 - x + x^2 - x^3 + x^4)), x])/5 - (2*Defer[Int][x^2/(Sqrt[1
+ x^4]*(1 - x + x^2 - x^3 + x^4)), x])/5 + Defer[Int][x^3/(Sqrt[1 + x^4]*(1
 - x + x^2 - x^3 + x^4)), x]/5 - (4*Defer[Int][1/(Sqrt[1 + x^4]*(1 + x + x^
2 + x^3 + x^4)), x])/5 - (3*Defer[Int][x/(Sqrt[1 + x^4]*(1 + x + x^2 + x^3
+ x^4)), x])/5 - (2*Defer[Int][x^2/(Sqrt[1 + x^4]*(1 + x + x^2 + x^3 + x^4)
), x])/5 - Defer[Int][x^3/(Sqrt[1 + x^4]*(1 + x + x^2 + x^3 + x^4)), x]/5
```

Rubi steps

$$\int \frac{1+x^{10}}{\sqrt{1+x^4}\left(-1+x^{10}\right)}\,dx = \int \left(\frac{1}{\sqrt{1+x^4}} + \frac{2}{\sqrt{1+x^4}\left(-1+x^{10}\right)}\right) dx$$

$$= 2\int \frac{1}{\sqrt{1+x^4}\left(-1+x^{10}\right)}\,dx + \int \frac{1}{\sqrt{1+x^4}}\,dx$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} + 2\int \left(\frac{1}{5\left(-1+x^2\right)\sqrt{1+x^4}} + \frac{-4+3x}{10\sqrt{1+x^4}\left(1-\right.}\right.$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} + \frac{1}{5}\int \frac{-4+3x-2x^2+x^3}{\sqrt{1+x^4}\left(1-x+x^2-x^3+x^4\right)}\,dx + \frac{1}{5}$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} - \frac{1}{5}\int \frac{1}{\sqrt{1+x^4}}\,dx - \frac{1}{5}\int \frac{-1-x^2}{\left(-1+x^2\right)\sqrt{1+x^4}}$$

$$= \frac{2\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{1+x^4}} + \frac{1}{5}\int \frac{x^3}{\sqrt{1+x^4}\left(1-x+x^2-x^3+x^4\right)}\,dx -$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{5\sqrt{2}} + \frac{2\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{1+x^4}} + \frac{1}{5}\int \frac{x^3}{\sqrt{1+x^4}\left(1-x\right.}$$

Mathematica [C] time = 0.66, size = 371, normalized size = 3.12

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^10)/(Sqrt[1 + x^4]*(-1 + x^10)),x]
```

```
[Out] ((-1)^(1/4)*((-4 - (-1)^(1/5) + (-1)^(2/5) - (-1)^(3/5) + (-1)^(4/5))*Ellip
ticF[I*ArcSinh[(-1)^(1/4)*x], -1] + 2*(EllipticPi[I, ArcSin[(-1)^(3/4)*x],
-1] + (2 - (-1)^(1/5) + (-1)^(2/5) - (-1)^(3/5) + (-1)^(4/5))*EllipticPi[(-
1)^(1/10), I*ArcSinh[(-1)^(1/4)*x], -1] + (-1)^(1/5)*EllipticPi[-(-1)^(3/10
), I*ArcSinh[(-1)^(1/4)*x], -1] - (-1)^(2/5)*EllipticPi[-(-1)^(3/10), I*Arc
Sinh[(-1)^(1/4)*x], -1] + (-1)^(3/5)*EllipticPi[-(-1)^(3/10), I*ArcSinh[(-1
)^(1/4)*x], -1] - (-1)^(4/5)*EllipticPi[-(-1)^(3/10), I*ArcSinh[(-1)^(1/4)*
x], -1] + EllipticPi[-(-1)^(7/10), I*ArcSinh[(-1)^(1/4)*x], -1] + (-1)^(1/5
)*EllipticPi[(-1)^(9/10), I*ArcSinh[(-1)^(1/4)*x], -1] - (-1)^(2/5)*Ellipti
cPi[(-1)^(9/10), I*ArcSinh[(-1)^(1/4)*x], -1] + (-1)^(3/5)*EllipticPi[(-1)^
(9/10), I*ArcSinh[(-1)^(1/4)*x], -1] - (-1)^(4/5)*EllipticPi[(-1)^(9/10), I
*ArcSinh[(-1)^(1/4)*x], -1])))/(4 + (-1)^(1/5) - (-1)^(2/5) + (-1)^(3/5) -
(-1)^(4/5))
```

IntegrateAlgebraic [A] time = 2.17, size = 119, normalized size = 1.00

$$-\frac{1}{5}\sqrt{2\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt{x^4+1}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{5\sqrt{2}} - \frac{1}{5}\sqrt{2\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt{x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^10)/(Sqrt[1 + x^4]*(-1 + x^10)),x]
```

```
[Out] -1/5*(Sqrt[2*(-1 + Sqrt[5])]*ArcTan[(Sqrt[1/2 + Sqrt[5]/2]*x)/Sqrt[1 + x^4]
]) - ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(5*Sqrt[2]) - (Sqrt[2*(1 + Sqrt[5])
]*ArcTanh[(Sqrt[-1/2 + Sqrt[5]/2]*x)/Sqrt[1 + x^4]])/5
```

fricas [B] time = 0.57, size = 431, normalized size = 3.62

$$-\frac{1}{5} \sqrt{2 \sqrt{5} - 2} \arctan\left(\frac{2 \left(x^{5} + 2 x^{3} + \sqrt{5} \left(x^{5} + x\right) + x\right) \sqrt{x^{4} + 1} \sqrt{2 \sqrt{5} - 2} - \left(x^{8} - 5 x^{6} + 3 x^{4} - 5 x^{2} - \sqrt{5} \left(x^{8} - x^{6} + 3 x^{4} - x^{2} + 1\right) + 1\right) \sqrt{2 \sqrt{5} - 2} \sqrt{\sqrt{5} + 2}}{4 \left(x^{8} - x^{6} + x^{4} - x^{2} + 1\right)}\right) + \frac{1}{20} \sqrt{2} \log\left(\frac{x^{4} - 2 \sqrt{2} \sqrt{x^{4} + 1} x + 2 x^{2} + 1}{x^{4} - 2 x^{2} + 1}\right) - \frac{1}{20} \sqrt{2 \sqrt{5} + 2} \log\left(-\frac{4 \left(3 x^{5} - x^{3} - \sqrt{5} \left(x^{5} - x^{3} + x\right) + 3 x\right) \sqrt{x^{4} + 1} + \left(3 x^{8} - 5 x^{6} + 9 x^{4} - 5 x^{2} - \sqrt{5} \left(x^{8} - 3 x^{6} + 3 x^{4} - 3 x^{2} + 1\right) + 3\right) \sqrt{2 \sqrt{5} + 2}}{x^{8} + x^{6} + x^{4} + x^{2} + 1}\right) + \frac{1}{20} \sqrt{2 \sqrt{5} + 2} \log\left(-\frac{4 \left(3 x^{5} - x^{3} - \sqrt{5} \left(x^{5} - x^{3} + x\right) + 3 x\right) \sqrt{x^{4} + 1} - \left(3 x^{8} - 5 x^{6} + 9 x^{4} - 5 x^{2} - \sqrt{5} \left(x^{8} - 3 x^{6} + 3 x^{4} - 3 x^{2} + 1\right) + 3\right) \sqrt{2 \sqrt{5} + 2}}{x^{8} + x^{6} + x^{4} + x^{2} + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^10+1)/(x^4+1)^(1/2)/(x^10-1),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(2*sqrt(5) - 2)*arctan(1/4*(2*(x^5 + 2*x^3 + sqrt(5)*(x^5 + x) + x
)*sqrt(x^4 + 1)*sqrt(2*sqrt(5) - 2) - (x^8 - 5*x^6 + 3*x^4 - 5*x^2 - sqrt(5
)*(x^8 - x^6 + 3*x^4 - x^2 + 1) + 1)*sqrt(2*sqrt(5) - 2)*sqrt(sqrt(5) + 2))
/(x^8 - x^6 + x^4 - x^2 + 1)) + 1/20*sqrt(2)*log((x^4 - 2*sqrt(2)*sqrt(x^4
+ 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1)) - 1/20*sqrt(2*sqrt(5) + 2)*log(-(4*(
3*x^5 - x^3 - sqrt(5)*(x^5 - x^3 + x) + 3*x)*sqrt(x^4 + 1) + (3*x^8 - 5*x^6
 + 9*x^4 - 5*x^2 - sqrt(5)*(x^8 - 3*x^6 + 3*x^4 - 3*x^2 + 1) + 3)*sqrt(2*sq
rt(5) + 2))/(x^8 + x^6 + x^4 + x^2 + 1)) + 1/20*sqrt(2*sqrt(5) + 2)*log(-(4
*(3*x^5 - x^3 - sqrt(5)*(x^5 - x^3 + x) + 3*x)*sqrt(x^4 + 1) - (3*x^8 - 5*x
^6 + 9*x^4 - 5*x^2 - sqrt(5)*(x^8 - 3*x^6 + 3*x^4 - 3*x^2 + 1) + 3)*sqrt(2*
sqrt(5) + 2))/(x^8 + x^6 + x^4 + x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10} + 1}{\left(x^{10} - 1\right) \sqrt{x^{4} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^10+1)/(x^4+1)^(1/2)/(x^10-1),x, algorithm="giac")
```

```
[Out] integrate((x^10 + 1)/((x^10 - 1)*sqrt(x^4 + 1)), x)
```

maple [C] time = 0.08, size = 377, normalized size = 3.17

$$\frac{\sqrt{-i x^{2} + 1}\, \sqrt{i x^{2} + 1}\, \mathrm{EllipticF}\left(x \left(\frac{\sqrt{2}}{2} + \frac{i \sqrt{2}}{2}\right), i\right)}{\left(\frac{\sqrt{2}}{2} + \frac{i \sqrt{2}}{2}\right) \sqrt{x^{4} + 1}} + \frac{2 (-1)^{\frac{3}{4}} \sqrt{-i x^{2} + 1}\, \sqrt{i x^{2} + 1}\, \mathrm{EllipticPi}\left((-1)^{\frac{1}{4}} x, -i, -\sqrt{-i}\, (-1)^{\frac{3}{4}}\right)}{5 \sqrt{x^{4} + 1}} + \frac{\left(\sum_{\_\alpha = \mathrm{RootOf}\left(\_Z^{4} + \_Z^{3} + \_Z^{2} + \_Z + 1\right)} \_\alpha \left(-\frac{\operatorname{arctanh}\left(\frac{\_\alpha^{2} \left(\_\alpha^{3} + x^{2}\right)}{\sqrt{-\_\alpha^{3} - \_\alpha^{2} - \_\alpha}\, \sqrt{x^{4} + 1}}\right)}{\sqrt{-\_\alpha^{3} - \_\alpha^{2} - \_\alpha}} + \frac{2 (-1)^{\frac{3}{4}} \left(-\_\alpha^{3} - \_\alpha^{2} - \_\alpha - 1\right) \sqrt{-i x^{2} + 1}\, \sqrt{i x^{2} + 1}\, \mathrm{EllipticPi}\left((-1)^{\frac{1}{4}} x, -i \_\alpha^{3}, i\right)}{\sqrt{x^{4} + 1}}\right)\right)}{10} + \frac{\left(\sum_{\_\alpha = \mathrm{RootOf}\left(\_Z^{4} - \_Z^{3} + \_Z^{2} - \_Z + 1\right)} \_\alpha \left(-\frac{\operatorname{arctanh}\left(\frac{\_\alpha^{2} \left(-\_\alpha^{3} + x^{2}\right)}{\sqrt{\_\alpha^{3} - \_\alpha^{2} + \_\alpha}\, \sqrt{x^{4} + 1}}\right)}{\sqrt{\_\alpha^{3} - \_\alpha^{2} + \_\alpha}} + \frac{2 (-1)^{\frac{3}{4}} \left(-\_\alpha^{3} + \_\alpha^{2} - \_\alpha + 1\right) \sqrt{-i x^{2} + 1}\, \sqrt{i x^{2} + 1}\, \mathrm{EllipticPi}\left((-1)^{\frac{1}{4}} x, i \_\alpha^{3}, i\right)}{\sqrt{x^{4} + 1}}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10+1)/(x^4+1)^(1/2)/(x^10-1),x)
```

```
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+2/5*(-1)^(3/4)*(1-I*x^2)^(1/2)*
(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1
/4))+1/10*sum(_alpha*(-1/(-_alpha^3-_alpha^2-_alpha)^(1/2)*arctanh(_alpha^2
*(_alpha^3+x^2)/(-_alpha^3-_alpha^2-_alpha)^(1/2)/(x^4+1)^(1/2))+2*(-1)^(3/
4)*(-_alpha^3-_alpha^2-_alpha-1)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1
/2)*EllipticPi((-1)^(1/4)*x,-I*_alpha^3,I)),_alpha=RootOf(_Z^4+_Z^3+_Z^2+_Z
+1))+1/10*sum(_alpha*(-1/(_alpha^3-_alpha^2+_alpha)^(1/2)*arctanh(_alpha^2*
(-_alpha^3+x^2)/(_alpha^3-_alpha^2+_alpha)^(1/2)/(x^4+1)^(1/2))+2*(-1)^(3/4
)*(-_alpha^3+_alpha^2-_alpha+1)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/
2)*EllipticPi((-1)^(1/4)*x,I*_alpha^3,I)),_alpha=RootOf(_Z^4-_Z^3+_Z^2-_Z+1
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10} + 1}{\left(x^{10} - 1\right) \sqrt{x^{4} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^10+1)/(x^4+1)^(1/2)/(x^10-1),x, algorithm="maxima")
[Out] integrate((x^10 + 1)/((x^10 - 1)*sqrt(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{10}+1}{\sqrt{x^4+1}\,\left(x^{10}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10 + 1)/((x^4 + 1)^(1/2)*(x^10 - 1)),x)
[Out] int((x^10 + 1)/((x^4 + 1)^(1/2)*(x^10 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)\left(x^8-x^6+x^4-x^2+1\right)}{(x-1)\,(x+1)\,\sqrt{x^4+1}\,\left(x^4-x^3+x^2-x+1\right)\left(x^4+x^3+x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**10+1)/(x**4+1)**(1/2)/(x**10-1),x)
[Out] Integral((x**2 + 1)*(x**8 - x**6 + x**4 - x**2 + 1)/((x - 1)*(x + 1)*sqrt(x
**4 + 1)*(x**4 - x**3 + x**2 - x + 1)*(x**4 + x**3 + x**2 + x + 1)), x)
```

3.1493 $\int \frac{1+x^{16}}{\sqrt{1+x^4}\left(-1+x^{16}\right)} \, dx$

Optimal. Leaf size=119

$$-\frac{x}{4\sqrt{x^4+1}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{x^4+1}}\right)}{4\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{8\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{x^4+1}}\right)}{4\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{8\sqrt{2}}$$

Rubi [C] time = 1.05, antiderivative size = 404, normalized size of antiderivative = 3.39, number of steps used = 37, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {6725, 220, 2073, 1222, 1179, 1196, 1211, 1699, 207, 1198, 203, 21, 1429, 409, 1217, 1707}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{x^4+1}}\right)}{4\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{8\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{x^4+1}}\right)}{4\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{8\sqrt{2}}-\frac{x\left(1-x^2\right)}{8\sqrt{x^4+1}}-\frac{x\left(x^2+1\right)}{8\sqrt{x^4+1}}+\frac{i\left(\sqrt{2}+(1+i)\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{16\sqrt{x^4+1}}-\frac{i\left(\sqrt{2}+(1-i)\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{16\sqrt{x^4+1}}+\frac{i\left(\sqrt{2}+(-1+i)\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{16\sqrt{x^4+1}}-\frac{i\left(\sqrt{2}+(-1-i)\right)\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{16\sqrt{x^4+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^16)/(Sqrt[1 + x^4]*(-1 + x^16)),x]
```

```
[Out] -1/8*(x*(1 - x^2))/Sqrt[1 + x^4] - (x*(1 + x^2))/(8*Sqrt[1 + x^4]) - ArcTan
[(2^(1/4)*x)/Sqrt[1 + x^4]]/(4*2^(1/4)) - ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]
/(8*Sqrt[2]) - ArcTanh[(2^(1/4)*x)/Sqrt[1 + x^4]]/(4*2^(1/4)) - ArcTanh[(Sq
rt[2]*x)/Sqrt[1 + x^4]]/(8*Sqrt[2]) + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2
]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4]) - ((I/16)*((-1 - I) + Sqrt
[2])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/Sqr
t[1 + x^4] + ((I/16)*((-1 + I) + Sqrt[2])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2
)^2]*EllipticF[2*ArcTan[x], 1/2])/Sqrt[1 + x^4] - ((I/16)*((1 - I) + Sqrt[2
])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/Sqrt[
1 + x^4] + ((I/16)*((1 + I) + Sqrt[2])*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2
]*EllipticF[2*ArcTan[x], 1/2])/Sqrt[1 + x^4]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
```

```
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1222

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/(
c*d^2 + a*e^2), Int[(c*d - c*e*x^2)*(a + c*x^4)^p, x], x] + Dist[e^2/(c*d^2
 + a*e^2), Int[(a + c*x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1429

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> W
ith[{r = Rt[-(a*c), 2]}, -Dist[c/(2*r), Int[(d + e*x^n)^q/(r - c*x^n), x],
x] - Dist[c/(2*r), Int[(d + e*x^n)^q/(r + c*x^n), x], x]] /; FreeQ[{a, c, d
, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
```

```
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^{16}}{\sqrt{1+x^4}\left(-1+x^{16}\right)}\,dx = \int\left(\frac{1}{\sqrt{1+x^4}}+\frac{2}{\sqrt{1+x^4}\left(-1+x^{16}\right)}\right)dx$$

$$= 2\int \frac{1}{\sqrt{1+x^4}\left(-1+x^{16}\right)}\,dx + \int \frac{1}{\sqrt{1+x^4}}\,dx$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} + 2\int\left(\frac{1}{4\left(-1+x^2\right)\left(1+x^4\right)^{3/2}} - \frac{1}{4\left(1+x^2\right)(1\ldots}\right.$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} + \frac{1}{2}\int \frac{1}{\left(-1+x^2\right)\left(1+x^4\right)^{3/2}}\,dx - \frac{1}{2}\int \frac{\ldots}{\left(1+x^2\ldots\right.}$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} + \frac{1}{4}\int \frac{-1-x^2}{\left(1+x^4\right)^{3/2}}\,dx - \frac{1}{4}\int \frac{1-x^2}{\left(1+x^4\right)^{3/2}}\,dx + \ldots$$

$$= -\frac{x\left(1-x^2\right)}{8\sqrt{1+x^4}} - \frac{x\left(1+x^2\right)}{8\sqrt{1+x^4}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} - \frac{1}{2}i\int \frac{1}{\left(i-x^4\right)\sqrt{1\ldots}}$$

$$= -\frac{x\left(1-x^2\right)}{8\sqrt{1+x^4}} - \frac{x\left(1+x^2\right)}{8\sqrt{1+x^4}} + \frac{x\sqrt{1+x^4}}{8\left(1+x^2\right)} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{8\sqrt{1+x^4}} + \frac{3(\ldots}{\ldots}$$

$$= -\frac{x\left(1-x^2\right)}{8\sqrt{1+x^4}} - \frac{x\left(1+x^2\right)}{8\sqrt{1+x^4}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{8\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{8\sqrt{2}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\ldots}{4\sqrt{1\ldots}}$$

$$= -\frac{x\left(1-x^2\right)}{8\sqrt{1+x^4}} - \frac{x\left(1+x^2\right)}{8\sqrt{1+x^4}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+x^4}}\right)}{4\sqrt[4]{2}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{8\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{1+x^4}}\right)}{4\sqrt[4]{2}} - \ldots$$

Mathematica [C] time = 1.18, size = 227, normalized size = 1.91

$$\left(\frac{1}{8}+\frac{i}{8}\right)\left(-\frac{(1-i)x}{\sqrt{x^4+1}} - 3\sqrt{2}F\left(i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)\middle|-1\right) + \sqrt{2}\Pi\left(-i;i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)\middle|-1\right) + \sqrt{2}\Pi\left(i;i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)\middle|-1\right) + \sqrt{2}\Pi\left(-\frac{1+i}{\sqrt{2}};i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)\middle|-1\right) + \sqrt{2}\Pi\left(-\frac{1-i}{\sqrt{2}};i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)\middle|-1\right) + \sqrt{2}\Pi\left(\frac{1-i}{\sqrt{2}};i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)\middle|-1\right) + \sqrt{2}\Pi\left(\frac{1+i}{\sqrt{2}};i\sinh^{-1}\left(\frac{(1+i)x}{\sqrt{2}}\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^16)/(Sqrt[1 + x^4]*(-1 + x^16)),x]
```

```
[Out] (1/8 + I/8)*(((-1 + I)*x)/Sqrt[1 + x^4] - 3*Sqrt[2]*EllipticF[I*ArcSinh[((1
 + I)*x)/Sqrt[2]], -1] + Sqrt[2]*EllipticPi[-I, I*ArcSinh[((1 + I)*x)/Sqrt[
2]], -1] + Sqrt[2]*EllipticPi[I, I*ArcSinh[((1 + I)*x)/Sqrt[2]], -1] + Sqrt
[2]*EllipticPi[(-1 - I)/Sqrt[2], I*ArcSinh[((1 + I)*x)/Sqrt[2]], -1] + Sqrt
[2]*EllipticPi[(-1 + I)/Sqrt[2], I*ArcSinh[((1 + I)*x)/Sqrt[2]], -1] + Sqrt
[2]*EllipticPi[(1 - I)/Sqrt[2], I*ArcSinh[((1 + I)*x)/Sqrt[2]], -1] + Sqrt[
2]*EllipticPi[(1 + I)/Sqrt[2], I*ArcSinh[((1 + I)*x)/Sqrt[2]], -1])
```

IntegrateAlgebraic [A] time = 0.62, size = 119, normalized size = 1.00

$$-\frac{x}{4\sqrt{x^4+1}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{x^4+1}}\right)}{4\sqrt[4]{2}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{8\sqrt{2}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt{x^4+1}}\right)}{4\sqrt[4]{2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^16)/(Sqrt[1 + x^4]*(-1 + x^16)),x]
```

```
[Out] -1/4*x/Sqrt[1 + x^4] - ArcTan[(2^(1/4)*x)/Sqrt[1 + x^4]]/(4*2^(1/4)) - ArcT
an[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(8*Sqrt[2]) - ArcTanh[(2^(1/4)*x)/Sqrt[1 + x^
4]]/(4*2^(1/4)) - ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(8*Sqrt[2])
```

fricas [B] time = 0.59, size = 302, normalized size = 2.54

$$-\frac{4 \cdot 2^{\frac{3}{4}} \left(x^{4}+1\right) \arctan\left(\frac{2^{\frac{3}{4}} \left(2 \cdot 2^{\frac{3}{4}} \left(x^{6}+x^{2}\right)+2^{\frac{1}{4}} \left(x^{8}+4 x^{4}+1\right)\right)+4 \sqrt{x^{4}+1} \left(2^{\frac{3}{4}} x^{3}+2^{\frac{1}{4}} \left(x^{5}+x\right)\right)}{2 \left(x^{8}+1\right)}\right)+2^{\frac{3}{4}} \left(x^{4}+1\right) \log\left(-\frac{2^{\frac{3}{4}} \left(x^{8}+4 x^{4}+1\right)+4 \left(x^{5}+\sqrt{2} x^{3}+x\right) \sqrt{x^{4}+1}+4 \cdot 2^{\frac{1}{4}} \left(x^{6}+x^{2}\right)}{x^{8}+1}\right)-2^{\frac{3}{4}} \left(x^{4}+1\right) \log\left(\frac{2^{\frac{3}{4}} \left(x^{8}+4 x^{4}+1\right)-4 \left(x^{5}+\sqrt{2} x^{3}+x\right) \sqrt{x^{4}+1}+4 \cdot 2^{\frac{1}{4}} \left(x^{6}+x^{2}\right)}{x^{8}+1}\right)+2 \sqrt{2} \left(x^{4}+1\right) \arctan\left(\frac{\sqrt{2} x}{\sqrt{x^{4}+1}}\right)-\sqrt{2} \left(x^{4}+1\right) \log\left(\frac{x^{4}-2 \sqrt{2} \sqrt{x^{4}+1} x+2 x^{2}+1}{x^{4}-2 x^{2}+1}\right)+8 \sqrt{x^{4}+1} x}{32 \left(x^{4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^16+1)/(x^4+1)^(1/2)/(x^16-1),x, algorithm="fricas")
```

```
[Out] -1/32*(4*2^(3/4)*(x^4 + 1)*arctan(1/2*(2^(3/4)*(2*2^(3/4)*(x^6 + x^2) + 2^(
1/4)*(x^8 + 4*x^4 + 1)) + 4*sqrt(x^4 + 1)*(2^(3/4)*x^3 + 2^(1/4)*(x^5 + x))
)/(x^8 + 1)) + 2^(3/4)*(x^4 + 1)*log(-(2^(3/4)*(x^8 + 4*x^4 + 1) + 4*(x^5 +
 sqrt(2)*x^3 + x)*sqrt(x^4 + 1) + 4*2^(1/4)*(x^6 + x^2))/(x^8 + 1)) - 2^(3/
4)*(x^4 + 1)*log((2^(3/4)*(x^8 + 4*x^4 + 1) - 4*(x^5 + sqrt(2)*x^3 + x)*sqr
t(x^4 + 1) + 4*2^(1/4)*(x^6 + x^2))/(x^8 + 1)) + 2*sqrt(2)*(x^4 + 1)*arctan
(sqrt(2)*x/sqrt(x^4 + 1)) - sqrt(2)*(x^4 + 1)*log((x^4 - 2*sqrt(2)*sqrt(x^4
 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1)) + 8*sqrt(x^4 + 1)*x)/(x^4 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{16}+1}{\left(x^{16}-1\right)\sqrt{x^{4}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^16+1)/(x^4+1)^(1/2)/(x^16-1),x, algorithm="giac")
```

```
[Out] integrate((x^16 + 1)/((x^16 - 1)*sqrt(x^4 + 1)), x)
```

maple [C] time = 0.04, size = 272, normalized size = 2.29

$$\frac{3 \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)}{4 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}-\frac{x}{4 \sqrt{x^{4}+1}}+\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticPi}\left(\left(-1\right)^{\frac{1}{4}} x, -i, -\sqrt{-i}\, \left(-1\right)^{\frac{3}{4}}\right)}{4 \sqrt{x^{4}+1}}+\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticPi}\left(\left(-1\right)^{\frac{1}{4}} x, i, -\sqrt{-i}\, \left(-1\right)^{\frac{3}{4}}\right)}{4 \sqrt{x^{4}+1}}+\frac{\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^{8}+1\right)}\_\alpha \left(-\frac{\operatorname{arctanh}\left(\frac{\_\alpha^{2}\left(-\_\alpha^{6}+x^{2}\right)}{\sqrt{\_\alpha^{4}+1}\, \sqrt{x^{4}+1}}\right)}{\sqrt{\_\alpha^{4}+1}}-\frac{2 \left(-1\right)^{\frac{3}{4}} \_\alpha^{7} \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticPi}\left(\left(-1\right)^{\frac{1}{4}} x, i \_\alpha^{6}, i\right)}{\sqrt{x^{4}+1}}\right)\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^16+1)/(x^4+1)^(1/2)/(x^16-1),x)
```

```
[Out] 3/4/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/
2)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-1/4*x/(x^4+1)^(1/2)+1/4*(-1)^
(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x
,-I,(-I)^(1/2)/(-1)^(1/4))+1/4*(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(
x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4))+1/16*sum(_alp
ha*(-1/(_alpha^4+1)^(1/2)*arctanh(_alpha^2*(-_alpha^6+x^2)/(_alpha^4+1)^(1/
2)/(x^4+1)^(1/2))-2*(-1)^(3/4)*_alpha^7*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^
4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I*_alpha^6,I)),_alpha=RootOf(_Z^8+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{16}+1}{\left(x^{16}-1\right)\sqrt{x^{4}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^16+1)/(x^4+1)^(1/2)/(x^16-1),x, algorithm="maxima")
```

```
[Out] integrate((x^16 + 1)/((x^16 - 1)*sqrt(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{16}+1}{\sqrt{x^4+1}\,\left(x^{16}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^16 + 1)/((x^4 + 1)^(1/2)*(x^16 - 1)),x)
```

```
[Out] int((x^16 + 1)/((x^4 + 1)^(1/2)*(x^16 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**16+1)/(x**4+1)**(1/2)/(x**16-1),x)
```

```
[Out] Timed out
```

3.1494 $\int \frac{\sqrt{b+ax}\sqrt{c+\sqrt{b+ax}}}{1-\sqrt{b+ax}}\,dx$

Optimal. Leaf size=119

$$-\frac{4\left(3ax+3b-2c^2+5c+15\right)\sqrt{\sqrt{ax+b}+c}}{15a}-\frac{4(c+5)\sqrt{ax+b}\sqrt{\sqrt{ax+b}+c}}{15a}-\frac{4\sqrt{-c-1}\,\tan^{-1}\left(\frac{\sqrt{-c-1}\sqrt{\sqrt{ax+b}+}}{c+1}\right.}{a}$$

Rubi [A] time = 0.12, antiderivative size = 106, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {513, 446, 88, 50, 63, 206}

$$-\frac{4\left(\sqrt{ax+b}+c\right)^{5/2}}{5a}-\frac{4(1-c)\left(\sqrt{ax+b}+c\right)^{3/2}}{3a}-\frac{4\sqrt{\sqrt{ax+b}+c}}{a}+\frac{4\sqrt{c+1}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{ax+b}+c}}{\sqrt{c+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]])/(1 - Sqrt[b + a*x]),x]
```

```
[Out] (-4*Sqrt[c + Sqrt[b + a*x]])/a - (4*(1 - c)*(c + Sqrt[b + a*x])^(3/2))/(3*a
) - (4*(c + Sqrt[b + a*x])^(5/2))/(5*a) + (4*Sqrt[1 + c]*ArcTanh[Sqrt[c + S
qrt[b + a*x]]/Sqrt[1 + c]])/a
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 513

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^(n_))^(p_.)*((c_.) + (d_.)*(v_)^(n_))^(q
_.), x_Symbol] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b
*x^n)^p*(c + d*x^n)^q, x], x, v], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
&& LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{\sqrt{b+ax}\sqrt{c+\sqrt{b+ax}}}{1-\sqrt{b+ax}}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{c+\sqrt{x}}\,\sqrt{x}}{1-\sqrt{x}}\,dx, x, b+ax\right)}{a}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x^2\sqrt{c+x}}{1-x}\,dx, x, \sqrt{b+ax}\right)}{a}$$

$$= \frac{2\,\text{Subst}\left(\int \left((-1+c)\sqrt{c+x}+\frac{\sqrt{c+x}}{1-x}-(c+x)^{3/2}\right)dx, x, \sqrt{b+ax}\right)}{a}$$

$$= -\frac{4(1-c)\left(c+\sqrt{b+ax}\right)^{3/2}}{3a}-\frac{4\left(c+\sqrt{b+ax}\right)^{5/2}}{5a}+\frac{2\,\text{Subst}\left(\int \frac{\sqrt{c+x}}{1-x}\,dx, x, \sqrt{b\right.}}{a}$$

$$= -\frac{4\sqrt{c+\sqrt{b+ax}}}{a}-\frac{4(1-c)\left(c+\sqrt{b+ax}\right)^{3/2}}{3a}-\frac{4\left(c+\sqrt{b+ax}\right)^{5/2}}{5a}+\frac{(2(1+}{}$$

$$= -\frac{4\sqrt{c+\sqrt{b+ax}}}{a}-\frac{4(1-c)\left(c+\sqrt{b+ax}\right)^{3/2}}{3a}-\frac{4\left(c+\sqrt{b+ax}\right)^{5/2}}{5a}+\frac{(4(1+}{}$$

$$= -\frac{4\sqrt{c+\sqrt{b+ax}}}{a}-\frac{4(1-c)\left(c+\sqrt{b+ax}\right)^{3/2}}{3a}-\frac{4\left(c+\sqrt{b+ax}\right)^{5/2}}{5a}+\frac{4\sqrt{1-}}{}$$

Mathematica [A] time = 0.20, size = 96, normalized size = 0.81

$$\frac{60\sqrt{c+1}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{ax+b}+c}}{\sqrt{c+1}}\right)-4\sqrt{\sqrt{ax+b}+c}\left(c\left(\sqrt{ax+b}+5\right)+5\sqrt{ax+b}+3ax+3b-2c^2+15\right)}{15a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]])/(1 - Sqrt[b + a*x]),x]
```

```
[Out] (-4*Sqrt[c + Sqrt[b + a*x]]*(15 + 3*b - 2*c^2 + 3*a*x + 5*Sqrt[b + a*x] + c
*(5 + Sqrt[b + a*x])) + 60*Sqrt[1 + c]*ArcTanh[Sqrt[c + Sqrt[b + a*x]]/Sqrt
[1 + c]])/(15*a)
```

IntegrateAlgebraic [A] time = 0.15, size = 108, normalized size = 0.91

$$\frac{4\sqrt{\sqrt{ax+b}+c}\left(-c\sqrt{ax+b}-3(ax+b)-5\sqrt{ax+b}+2c^2-5c-15\right)}{15a}-\frac{4\sqrt{-c-1}\,\tan^{-1}\left(\frac{\sqrt{-c-1}\sqrt{\sqrt{ax+b}+c}}{c+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]])/(1 - Sqrt[b + a*
x]),x]
```

```
[Out] (4*Sqrt[c + Sqrt[b + a*x]]*(-15 - 5*c + 2*c^2 - 5*Sqrt[b + a*x] - c*Sqrt[b
+ a*x] - 3*(b + a*x)))/(15*a) - (4*Sqrt[-1 - c]*ArcTan[(Sqrt[-1 - c]*Sqrt[c
 + Sqrt[b + a*x]])/(1 + c)])/a
```

fricas [A] time = 0.45, size = 200, normalized size = 1.68

$$\left[\frac{2\left(2\left(2c^2-3ax-\sqrt{ax+b}(c+5)-3b-5c-15\right)\sqrt{c+\sqrt{ax+b}}+15\sqrt{c+1}\log\left(\frac{ax+2\left(\sqrt{ax+b}\sqrt{c+1}+\sqrt{c+1}\right)\sqrt{c+\sqrt{ax+b}}+2\sqrt{ax+b}(c+1)+b+2c+1}{ax+b-1}\right)\right)}{15a}, \frac{4\left(\left(2c^2-3ax-\sqrt{ax+b}(c+5)-3b-5c-15\right)\sqrt{c+\sqrt{ax+b}}-15\sqrt{-c-1}\arctan\left(\frac{\sqrt{c+\sqrt{ax+b}}\sqrt{-c-1}}{c+1}\right)\right)}{15a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^(1/2)*(c+(a*x+b)^(1/2))^(1/2)/(1-(a*x+b)^(1/2)),x, algori
thm="fricas")
```

```
[Out] [2/15*(2*(2*c^2 - 3*a*x - sqrt(a*x + b)*(c + 5) - 3*b - 5*c - 15)*sqrt(c +
sqrt(a*x + b)) + 15*sqrt(c + 1)*log((a*x + 2*(sqrt(a*x + b)*sqrt(c + 1) + s
qrt(c + 1))*sqrt(c + sqrt(a*x + b)) + 2*sqrt(a*x + b)*(c + 1) + b + 2*c + 1
)/(a*x + b - 1)))/a, 4/15*((2*c^2 - 3*a*x - sqrt(a*x + b)*(c + 5) - 3*b - 5
*c - 15)*sqrt(c + sqrt(a*x + b)) - 15*sqrt(-c - 1)*arctan(sqrt(c + sqrt(a*x
 + b))*sqrt(-c - 1)/(c + 1)))/a]
```

giac [A] time = 0.15, size = 107, normalized size = 0.90

$$-\frac{4(c+1)\arctan\left(\frac{\sqrt{c+\sqrt{ax+b}}}{\sqrt{-c-1}}\right)}{a\sqrt{-c-1}}-\frac{4\left(3a^4\left(c+\sqrt{ax+b}\right)^{\frac{5}{2}}-5a^4\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}c+5a^4\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}+15a^4\sqrt{c+\sqrt{ax+b}}\right)}{15a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^(1/2)*(c+(a*x+b)^(1/2))^(1/2)/(1-(a*x+b)^(1/2)),x, algori
thm="giac")
```

```
[Out] -4*(c + 1)*arctan(sqrt(c + sqrt(a*x + b))/sqrt(-c - 1))/(a*sqrt(-c - 1)) -
4/15*(3*a^4*(c + sqrt(a*x + b))^(5/2) - 5*a^4*(c + sqrt(a*x + b))^(3/2)*c +
 5*a^4*(c + sqrt(a*x + b))^(3/2) + 15*a^4*sqrt(c + sqrt(a*x + b)))/a^5
```

maple [A] time = 0.01, size = 85, normalized size = 0.71

$$-\frac{2\left(\frac{2\left(c+\sqrt{ax+b}\right)^{\frac{5}{2}}}{5}-\frac{2c\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}}{3}+\frac{2\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}}{3}+2\sqrt{c+\sqrt{ax+b}}-2\sqrt{1+c}\,\operatorname{arctanh}\left(\frac{\sqrt{c+\sqrt{ax+b}}}{\sqrt{1+c}}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)^(1/2)*(c+(a*x+b)^(1/2))^(1/2)/(1-(a*x+b)^(1/2)),x)
```

```
[Out] -2/a*(2/5*(c+(a*x+b)^(1/2))^(5/2)-2/3*c*(c+(a*x+b)^(1/2))^(3/2)+2/3*(c+(a*x
+b)^(1/2))^(3/2)+2*(c+(a*x+b)^(1/2))^(1/2)-2*(1+c)^(1/2)*arctanh((c+(a*x+b)
^(1/2))^(1/2)/(1+c)^(1/2)))
```

maxima [A] time = 1.80, size = 95, normalized size = 0.80

$$-\frac{2\left(6\left(c+\sqrt{ax+b}\right)^{\frac{5}{2}}-10\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}(c-1)+15\sqrt{c+1}\log\left(\frac{\sqrt{c+\sqrt{ax+b}}-\sqrt{c+1}}{\sqrt{c+\sqrt{ax+b}}+\sqrt{c+1}}\right)+30\sqrt{c+\sqrt{ax+b}}\right)}{15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^(1/2)*(c+(a*x+b)^(1/2))^(1/2)/(1-(a*x+b)^(1/2)),x, algori
thm="maxima")
```

```
[Out] -2/15*(6*(c + sqrt(a*x + b))^(5/2) - 10*(c + sqrt(a*x + b))^(3/2)*(c - 1) +
 15*sqrt(c + 1)*log((sqrt(c + sqrt(a*x + b)) - sqrt(c + 1))/(sqrt(c + sqrt(
a*x + b)) + sqrt(c + 1))) + 30*sqrt(c + sqrt(a*x + b)))/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{c+\sqrt{b+a\,x}}\,\sqrt{b+a\,x}}{\sqrt{b+a\,x}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c + (b + a*x)^(1/2))^(1/2)*(b + a*x)^(1/2))/((b + a*x)^(1/2) - 1),x)
```

```
[Out] -int(((c + (b + a*x)^(1/2))^(1/2)*(b + a*x)^(1/2))/((b + a*x)^(1/2) - 1), x
)
```

sympy [A] time = 5.58, size = 90, normalized size = 0.76

$$\frac{2\left(\frac{2(c-1)\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}}{3}-\frac{2\left(c+\sqrt{ax+b}\right)^{\frac{5}{2}}}{5}-2\sqrt{c+\sqrt{ax+b}}-\frac{2(c+1)\operatorname{atan}\left(\frac{\sqrt{c+\sqrt{ax+b}}}{\sqrt{-c-1}}\right)}{\sqrt{-c-1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)**(1/2)*(c+(a*x+b)**(1/2))**(1/2)/(1-(a*x+b)**(1/2)),x)
```

```
[Out] 2*(2*(c - 1)*(c + sqrt(a*x + b))**(3/2)/3 - 2*(c + sqrt(a*x + b))**(5/2)/5
- 2*sqrt(c + sqrt(a*x + b)) - 2*(c + 1)*atan(sqrt(c + sqrt(a*x + b))/sqrt(-
c - 1))/sqrt(-c - 1))/a
```

3.1495 $\int x\sqrt{-1+x^2}\,\sqrt{x^2+x\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=119

$$\frac{1}{192}\sqrt{x^2-1}\,\sqrt{x^2+\sqrt{x^2-1}\,x}\,\left(56x^2-39\right)+\frac{13\log\left(\sqrt{x^2-1}-\sqrt{2}\,\sqrt{x^2+\sqrt{x^2-1}\,x}+x\right)}{64\sqrt{2}}+\frac{1}{192}\left(13x-8x^3\right)\sqrt{x^2+$$

Rubi [F] time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\sqrt{-1+x^2}\,\sqrt{x^2+x\sqrt{-1+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*Sqrt[-1 + x^2]*Sqrt[x^2 + x*Sqrt[-1 + x^2]],x]
```

```
[Out] Defer[Int][x*Sqrt[-1 + x^2]*Sqrt[x^2 + x*Sqrt[-1 + x^2]], x]
```

Rubi steps

$$\int x\sqrt{-1+x^2}\,\sqrt{x^2+x\sqrt{-1+x^2}}\,dx=\int x\sqrt{-1+x^2}\,\sqrt{x^2+x\sqrt{-1+x^2}}\,dx$$

Mathematica [A] time = 2.12, size = 238, normalized size = 2.00

$$\frac{\sqrt{x^2-1}\left(\sqrt{x^2-1}+x\right)^4\left(\sqrt{2}\sqrt{x\left(\sqrt{x^2-1}+x\right)}\left(192x^6-360x^4+212x^2+104\sqrt{x^2-1}\,x+192\sqrt{x^2-1}\,x^5-264\sqrt{x^2-1}\,x^3-39\right)-39\left(4x^3+4\sqrt{x^2-1}\,x^2-\sqrt{x^2-1}-3x\right)\sinh^{-1}\left(\sqrt{x^2-1}+x\right)\right)}{192\sqrt{2}\left(64x^8-144x^6+104x^4-25x^2-7\sqrt{x^2-1}\,x+64\sqrt{x^2-1}\,x^7-112\sqrt{x^2-1}\,x^5+56\sqrt{x^2-1}\,x^3+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[-1 + x^2]*Sqrt[x^2 + x*Sqrt[-1 + x^2]],x]
```

```
[Out] (Sqrt[-1 + x^2]*(x + Sqrt[-1 + x^2])^4*(Sqrt[2]*Sqrt[x*(x + Sqrt[-1 + x^2])
]*(-39 + 212*x^2 - 360*x^4 + 192*x^6 + 104*x*Sqrt[-1 + x^2] - 264*x^3*Sqrt[
-1 + x^2] + 192*x^5*Sqrt[-1 + x^2]) - 39*(-3*x + 4*x^3 - Sqrt[-1 + x^2] + 4
*x^2*Sqrt[-1 + x^2])*ArcSinh[x + Sqrt[-1 + x^2]]))/(192*Sqrt[2]*(1 - 25*x^2
 + 104*x^4 - 144*x^6 + 64*x^8 - 7*x*Sqrt[-1 + x^2] + 56*x^3*Sqrt[-1 + x^2]
- 112*x^5*Sqrt[-1 + x^2] + 64*x^7*Sqrt[-1 + x^2]))
```

IntegrateAlgebraic [A] time = 1.49, size = 119, normalized size = 1.00

$$\frac{1}{192}\sqrt{x^2-1}\sqrt{x^2+\sqrt{x^2-1}\,x}\,\left(56x^2-39\right)+\frac{13\log\left(\sqrt{x^2-1}-\sqrt{2}\sqrt{x^2+\sqrt{x^2-1}\,x}+x\right)}{64\sqrt{2}}+\frac{1}{192}\left(13x-8x^3\right)\sqrt{x^2+\sqrt{x^2-1}\,x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[-1 + x^2]*Sqrt[x^2 + x*Sqrt[-1 + x^2]],x]
```

```
[Out] (Sqrt[-1 + x^2]*(-39 + 56*x^2)*Sqrt[x^2 + x*Sqrt[-1 + x^2]])/192 + ((13*x -
 8*x^3)*Sqrt[x^2 + x*Sqrt[-1 + x^2]])/192 + (13*Log[x + Sqrt[-1 + x^2] - Sq
rt[2]*Sqrt[x^2 + x*Sqrt[-1 + x^2]]])/(64*Sqrt[2])
```

fricas [A] time = 0.99, size = 100, normalized size = 0.84

$$-\frac{1}{192}\left(8\,x^3-\left(56\,x^2-39\right)\sqrt{x^2-1}-13\,x\right)\sqrt{x^2+\sqrt{x^2-1}\,x}+\frac{13}{256}\sqrt{2}\log\left(-4\,x^2+2\sqrt{x^2+\sqrt{x^2-1}\,x}\left(\sqrt{2}\,x+\sqrt{2}\sqrt{x^2-1}\right)-4\sqrt{x^2-1}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(1/2)*(x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/192*(8*x^3 - (56*x^2 - 39)*sqrt(x^2 - 1) - 13*x)*sqrt(x^2 + sqrt(x^2 - 1
)*x) + 13/256*sqrt(2)*log(-4*x^2 + 2*sqrt(x^2 + sqrt(x^2 - 1)*x)*(sqrt(2)*x
 + sqrt(2)*sqrt(x^2 - 1)) - 4*sqrt(x^2 - 1)*x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2 + \sqrt{x^2 - 1}\,x}\,\sqrt{x^2 - 1}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(1/2)*(x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^2 - 1)*x)*sqrt(x^2 - 1)*x, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int x\sqrt{x^2-1}\,\sqrt{x^2+x\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2-1)^(1/2)*(x^2+x*(x^2-1)^(1/2))^(1/2),x)
```

```
[Out] int(x*(x^2-1)^(1/2)*(x^2+x*(x^2-1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2 + \sqrt{x^2 - 1}\,x}\,\sqrt{x^2 - 1}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^(1/2)*(x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^2 - 1)*x)*sqrt(x^2 - 1)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{x^2-1}\,\sqrt{x\,\sqrt{x^2-1}+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 - 1)^(1/2)*(x*(x^2 - 1)^(1/2) + x^2)^(1/2),x)
```

```
[Out] int(x*(x^2 - 1)^(1/2)*(x*(x^2 - 1)^(1/2) + x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x\left(x+\sqrt{x^2-1}\right)}\,\sqrt{(x-1)\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2-1)**(1/2)*(x**2+x*(x**2-1)**(1/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(x*(x + sqrt(x**2 - 1)))*sqrt((x - 1)*(x + 1)), x)
```

3.1496 $\int \frac{(-q+px^2)\sqrt{q^2+p^2x^4}\left(bx^3+a(q+px^2)^3\right)}{x^6}\,dx$

Optimal. Leaf size=119

$$\frac{\sqrt{p^2x^4+q^2}\,\left(6ap^4x^8+20ap^3qx^6+12ap^2q^2x^4+20apq^3x^2+6aq^4+15bpx^5+15bqx^3\right)}{30x^5}-bpq\log\left(\sqrt{p^2x^4+q^2}+px\right.$$

Rubi [A] time = 0.49, antiderivative size = 163, normalized size of antiderivative = 1.37, number of steps used = 16, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {1833, 1584, 1252, 813, 844, 217, 206, 266, 63, 208, 1835, 1586, 449}

$$\frac{ap^2(p^2x^4+q^2)^{3/2}}{5x}+\frac{aq^2(p^2x^4+q^2)^{3/2}}{5x^5}+\frac{2apq(p^2x^4+q^2)^{3/2}}{3x^3}-\frac{1}{2}bpq\tanh^{-1}\left(\frac{\sqrt{p^2x^4+q^2}}{q}\right)+\frac{b(px^2+q)\sqrt{p^2x^4+q^2}}{2x^2}-\frac{1}{2}bpq\tanh^{-1}\left(\frac{px^2}{\sqrt{p^2x^4+q^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-q + p*x^2)*Sqrt[q^2 + p^2*x^4]*(b*x^3 + a*(q + p*x^2)^3))/x^6,x]
```

```
[Out] (b*(q + p*x^2)*Sqrt[q^2 + p^2*x^4])/(2*x^2) + (a*q^2*(q^2 + p^2*x^4)^(3/2))
/(5*x^5) + (2*a*p*q*(q^2 + p^2*x^4)^(3/2))/(3*x^3) + (a*p^2*(q^2 + p^2*x^4)
^(3/2))/(5*x) - (b*p*q*ArcTanh[(p*x^2)/Sqrt[q^2 + p^2*x^4]])/2 - (b*p*q*Arc
Tanh[Sqrt[q^2 + p^2*x^4]/q])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
```

```
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1835

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[(Pq0*(c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
c*(m + 1)), x] + Dist[1/(2*a*c*(m + 1)), Int[(c*x)^(m + 1)*ExpandToSum[(2*a
*(m + 1)*(Pq - Pq0))/x - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*x
^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(-q+px^2\right)\sqrt{q^2+p^2x^4}\left(bx^3+a\left(q+px^2\right)^3\right)}{x^6}\,dx = \int\left(\frac{\left(-bqx^2+bpx^4\right)\sqrt{q^2+p^2x^4}}{x^5} + \frac{\sqrt{q^2+p^2x^4}\left(-aq^4-2ap\right.}{x}\right.$$

$$= \int \frac{\left(-bqx^2+bpx^4\right)\sqrt{q^2+p^2x^4}}{x^5}\,dx + \int \frac{\sqrt{q^2+p^2x^4}\left(-aq^4-\right.}{}$$

$$= \frac{aq^2\left(q^2+p^2x^4\right)^{3/2}}{5x^5} - \frac{\int \frac{\sqrt{q^2+p^2x^4}\left(20apq^5x+2ap^2q^4x^3-20ap^3q^3x^5-10a\right.}{x^5}}{10q^2}$$

$$= \frac{aq^2\left(q^2+p^2x^4\right)^{3/2}}{5x^5} + \frac{1}{2}\,\text{Subst}\left(\int \frac{(-bq+bpx)\sqrt{q^2+p^2x^2}}{x^2}\,d\right.$$

$$= \frac{b\left(q+px^2\right)\sqrt{q^2+p^2x^4}}{2x^2} + \frac{aq^2\left(q^2+p^2x^4\right)^{3/2}}{5x^5} + \frac{2apq\left(q^2+p\right.}{3x^3}$$

$$= \frac{b\left(q+px^2\right)\sqrt{q^2+p^2x^4}}{2x^2} + \frac{aq^2\left(q^2+p^2x^4\right)^{3/2}}{5x^5} + \frac{2apq\left(q^2+p\right.}{3x^3}$$

$$= \frac{b\left(q+px^2\right)\sqrt{q^2+p^2x^4}}{2x^2} + \frac{aq^2\left(q^2+p^2x^4\right)^{3/2}}{5x^5} + \frac{2apq\left(q^2+p\right.}{3x^3}$$

$$= \frac{b\left(q+px^2\right)\sqrt{q^2+p^2x^4}}{2x^2} + \frac{aq^2\left(q^2+p^2x^4\right)^{3/2}}{5x^5} + \frac{2apq\left(q^2+p\right.}{3x^3}$$

$$= \frac{b\left(q+px^2\right)\sqrt{q^2+p^2x^4}}{2x^2} + \frac{aq^2\left(q^2+p^2x^4\right)^{3/2}}{5x^5} + \frac{2apq\left(q^2+p\right.}{3x^3}$$

Mathematica [C] time = 0.27, size = 354, normalized size = 2.97

$$\frac{6aq^4\sqrt{p^2x^4+q^2}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{p^2x^4}{q^2}\right)+20apq^3x^2\sqrt{p^2x^4+q^2}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{p^2x^4}{q^2}\right)+10ap^4x^8\sqrt{p^2x^4+q^2}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{p^2x^4}{q^2}\right)+60ap^3qx^6\sqrt{p^2x^4+q^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{p^2x^4}{q^2}\right)+15bpx^5\sqrt{p^2x^4+q^2}\sqrt{\frac{p^2x^4}{q^2}+1}-15bpqx^5\sqrt{\frac{p^2x^4}{q^2}+1}\tanh^{-1}\left(\frac{\sqrt{p^2x^4+q^2}}{q}\right)+15bqx^3\sqrt{p^2x^4+q^2}\sqrt{\frac{p^2x^4}{q^2}+1}-15bpx^5\sqrt{p^2x^4+q^2}\sinh^{-1}\left(\frac{px^2}{q}\right)}{30x^5\sqrt{\frac{p^2x^4}{q^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-q + p*x^2)*Sqrt[q^2 + p^2*x^4]*(b*x^3 + a*(q + p*x^2)^3))/x^6,
x]
```

```
[Out] (15*b*q*x^3*Sqrt[q^2 + p^2*x^4]*Sqrt[1 + (p^2*x^4)/q^2] + 15*b*p*x^5*Sqrt[q
^2 + p^2*x^4]*Sqrt[1 + (p^2*x^4)/q^2] - 15*b*p*x^5*Sqrt[q^2 + p^2*x^4]*ArcS
inh[(p*x^2)/q] - 15*b*p*q*x^5*Sqrt[1 + (p^2*x^4)/q^2]*ArcTanh[Sqrt[q^2 + p^
2*x^4]/q] + 6*a*q^4*Sqrt[q^2 + p^2*x^4]*Hypergeometric2F1[-5/4, -1/2, -1/4,
 -((p^2*x^4)/q^2)] + 20*a*p*q^3*x^2*Sqrt[q^2 + p^2*x^4]*Hypergeometric2F1[-
3/4, -1/2, 1/4, -((p^2*x^4)/q^2)] + 60*a*p^3*q*x^6*Sqrt[q^2 + p^2*x^4]*Hype
rgeometric2F1[-1/2, 1/4, 5/4, -((p^2*x^4)/q^2)] + 10*a*p^4*x^8*Sqrt[q^2 + p
^2*x^4]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((p^2*x^4)/q^2)])/(30*x^5*Sqrt[1
 + (p^2*x^4)/q^2])
```

IntegrateAlgebraic [A] time = 3.27, size = 119, normalized size = 1.00

$$\frac{\sqrt{p^2x^4+q^2}\left(6ap^4x^8+20ap^3qx^6+12ap^2q^2x^4+20apq^3x^2+6aq^4+15bpx^5+15bqx^3\right)}{30x^5} - bpq\log\left(\sqrt{p^2x^4+q^2}+px^2+q\right)+bpq\log(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-q + p*x^2)*Sqrt[q^2 + p^2*x^4]*(b*x^3 + a*(q + p*x^2)
^3))/x^6,x]
```

```
[Out] (Sqrt[q^2 + p^2*x^4]*(6*a*q^4 + 20*a*p*q^3*x^2 + 15*b*q*x^3 + 12*a*p^2*q^2*
x^4 + 15*b*p*x^5 + 20*a*p^3*q*x^6 + 6*a*p^4*x^8))/(30*x^5) + b*p*q*Log[x] -
 b*p*q*Log[q + p*x^2 + Sqrt[q^2 + p^2*x^4]]
```

fricas [A] time = 0.69, size = 117, normalized size = 0.98

$$\frac{30\,bpqx^5\log\left(\frac{px^2+q-\sqrt{p^2x^4+q^2}}{x}\right)+\left(6\,ap^4x^8+20\,ap^3qx^6+12\,ap^2q^2x^4+20\,apq^3x^2+15\,bpx^5+6\,aq^4+15\,bqx^3\right)\sqrt{p^2x^4+q^2}}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^2-q)*(p^2*x^4+q^2)^(1/2)*(b*x^3+a*(p*x^2+q)^3)/x^6,x, algori
thm="fricas")
```

```
[Out] 1/30*(30*b*p*q*x^5*log((p*x^2 + q - sqrt(p^2*x^4 + q^2))/x) + (6*a*p^4*x^8
+ 20*a*p^3*q*x^6 + 12*a*p^2*q^2*x^4 + 20*a*p*q^3*x^2 + 15*b*p*x^5 + 6*a*q^4
 + 15*b*q*x^3)*sqrt(p^2*x^4 + q^2))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^4+q^2}\left(\left(px^2+q\right)^3a+bx^3\right)\left(px^2-q\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^2-q)*(p^2*x^4+q^2)^(1/2)*(b*x^3+a*(p*x^2+q)^3)/x^6,x, algori
thm="giac")
```

```
[Out] integrate(sqrt(p^2*x^4 + q^2)*((p*x^2 + q)^3*a + b*x^3)*(p*x^2 - q)/x^6, x)
```

maple [B] time = 0.06, size = 261, normalized size = 2.19

$$\frac{a\,p^4x^3\sqrt{p^2x^4+q^2}}{5}+\frac{2qa\,p^3x\sqrt{p^2x^4+q^2}}{3}+\frac{2q^3ap\sqrt{p^2x^4+q^2}}{3x^3}+\frac{b\left(p^2x^4+q^2\right)^{\frac{3}{2}}}{2q\,x^2}-\frac{b\,p^2x^2\sqrt{p^2x^4+q^2}}{2q}-\frac{qb\,p^2\ln\left(\frac{p^2x^2}{\sqrt{p^2}}+\sqrt{p^2x^4+q^2}\right)}{2\sqrt{p^2}}+\frac{a\,q^4\sqrt{p^2x^4+q^2}}{5x^5}+\frac{2a\,q^2p^2\sqrt{p^2x^4+q^2}}{5x}+\frac{pb\sqrt{p^2x^4+q^2}}{2}-\frac{pb\,q^2\ln\left(\frac{2q^2+2\sqrt{q^2}\,\sqrt{p^2x^4+q^2}}{x^2}\right)}{2\sqrt{q^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^2-q)*(p^2*x^4+q^2)^(1/2)*(b*x^3+a*(p*x^2+q)^3)/x^6,x)
```

```
[Out] 1/5*a*p^4*x^3*(p^2*x^4+q^2)^(1/2)+2/3*q*a*p^3*x*(p^2*x^4+q^2)^(1/2)+2/3*q^3
*a*p*(p^2*x^4+q^2)^(1/2)/x^3+1/2/q*b/x^2*(p^2*x^4+q^2)^(3/2)-1/2/q*b*p^2*x^
2*(p^2*x^4+q^2)^(1/2)-1/2*q*b*p^2*ln(p^2*x^2/(p^2)^(1/2)+(p^2*x^4+q^2)^(1/2
))/(p^2)^(1/2)+1/5*a*q^4*(p^2*x^4+q^2)^(1/2)/x^5+2/5*a*q^2*p^2*(p^2*x^4+q^2
)^(1/2)/x+1/2*p*b*(p^2*x^4+q^2)^(1/2)-1/2*p*b*q^2/(q^2)^(1/2)*ln((2*q^2+2*(
q^2)^(1/2)*(p^2*x^4+q^2)^(1/2))/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^4+q^2}\left(\left(px^2+q\right)^3a+bx^3\right)\left(px^2-q\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^2-q)*(p^2*x^4+q^2)^(1/2)*(b*x^3+a*(p*x^2+q)^3)/x^6,x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(p^2*x^4 + q^2)*((p*x^2 + q)^3*a + b*x^3)*(p*x^2 - q)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{p^2\,x^4+q^2}\,\left(q-p\,x^2\right)\,\left(a\,{\left(p\,x^2+q\right)}^3+b\,x^3\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((p^2*x^4 + q^2)^(1/2)*(q - p*x^2)*(a*(q + p*x^2)^3 + b*x^3))/x^6,x)
```

```
[Out] -int(((p^2*x^4 + q^2)^(1/2)*(q - p*x^2)*(a*(q + p*x^2)^3 + b*x^3))/x^6, x)
```

sympy [C] time = 8.30, size = 323, normalized size = 2.71

$$\frac{ap^4qx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{p^2x^4e^{i\pi}}{q^2}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ap^3q^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{p^2x^4e^{i\pi}}{q^2}\right)}{2\Gamma\left(\frac{5}{4}\right)}-\frac{apq^4\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{p^2x^4e^{i\pi}}{q^2}\right)}{2x^3\Gamma\left(\frac{1}{4}\right)}-\frac{aq^5\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\ -\frac{1}{4}\end{matrix}\middle|\frac{p^2x^4e^{i\pi}}{q^2}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}+\frac{bp^2x^2}{2\sqrt{\frac{p^2x^4}{q^2}+1}}+\frac{bp^2x^2}{2\sqrt{1+\frac{q^2}{p^2x^4}}}-\frac{bpq\operatorname{asinh}\left(\frac{q}{px^2}\right)}{2}-\frac{bpq\operatorname{asinh}\left(\frac{px^2}{q}\right)}{2}+\frac{bq^2}{2x^2\sqrt{\frac{p^2x^4}{q^2}+1}}+\frac{bq^2}{2x^2\sqrt{1+\frac{q^2}{p^2x^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**2-q)*(p**2*x**4+q**2)**(1/2)*(b*x**3+a*(p*x**2+q)**3)/x**6,
x)
```

```
[Out] a*p**4*q*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), p**2*x**4*exp_polar(I*p
i)/q**2)/(4*gamma(7/4)) + a*p**3*q**2*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,
), p**2*x**4*exp_polar(I*pi)/q**2)/(2*gamma(5/4)) - a*p*q**4*gamma(-3/4)*hy
per((-3/4, -1/2), (1/4,), p**2*x**4*exp_polar(I*pi)/q**2)/(2*x**3*gamma(1/4
)) - a*q**5*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), p**2*x**4*exp_polar(I*
pi)/q**2)/(4*x**5*gamma(-1/4)) + b*p**2*x**2/(2*sqrt(p**2*x**4/q**2 + 1)) +
 b*p**2*x**2/(2*sqrt(1 + q**2/(p**2*x**4))) - b*p*q*asinh(q/(p*x**2))/2 - b
*p*q*asinh(p*x**2/q)/2 + b*q**2/(2*x**2*sqrt(p**2*x**4/q**2 + 1)) + b*q**2/
(2*x**2*sqrt(1 + q**2/(p**2*x**4)))
```

3.1497 $\int \frac{(-1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=119

$$\frac{1}{2}\sqrt{\sqrt{x^4+1}+x^2}\,x-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)}{\sqrt{2}}-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)$$

Rubi [C] time = 0.48, antiderivative size = 113, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {6742, 2132, 206, 2133, 321, 216, 215}

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{1-ix^2}\,x+\left(\frac{1}{4}-\frac{i}{4}\right)\sqrt{1+ix^2}\,x-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}-\frac{\sin^{-1}\left(\sqrt[4]{-1}\,x\right)}{2\sqrt{2}}+\frac{i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)}{2\sqrt{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

```
[Out] (1/4 + I/4)*x*Sqrt[1 - I*x^2] + (1/4 - I/4)*x*Sqrt[1 + I*x^2] - ArcSin[(-1)
^(1/4)*x]/(2*Sqrt[2]) + ((I/2)*ArcSinh[(-1)^(1/4)*x])/Sqrt[2] - ArcTanh[(Sq
rt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx &= \int \left(-\frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}+\frac{x^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\right)dx\\
&= -\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx+\int \frac{x^2\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx\\
&= \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{x^2}{\sqrt{1-ix^2}}\,dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{x^2}{\sqrt{1+ix^2}}\,dx-\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x\right.\\
&= \left(\frac{1}{4}+\frac{i}{4}\right)x\sqrt{1-ix^2}+\left(\frac{1}{4}-\frac{i}{4}\right)x\sqrt{1+ix^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}+\left(-\frac{1}{4}-\frac{i}{4}\right.\\
&= \left(\frac{1}{4}+\frac{i}{4}\right)x\sqrt{1-ix^2}+\left(\frac{1}{4}-\frac{i}{4}\right)x\sqrt{1+ix^2}-\frac{\sin^{-1}\left(\sqrt[4]{-1}\,x\right)}{2\sqrt{2}}+\frac{i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right.}{2\sqrt{2}}
\end{aligned}$$

Mathematica [B] time = 1.76, size = 247, normalized size = 2.08

$$\frac{\sqrt{x^2\left(\sqrt{x^4+1}+x^2\right)}\left(\left(\left(\sqrt{x^4+1}+x^2\right)^2+1\right)^2\left(\log\left(1-\frac{\sqrt{x^2\left(\sqrt{x^4+1}+x^2\right)}}{\sqrt{2}x^2}\right)-\log\left(\frac{\sqrt{x^2\left(\sqrt{x^4+1}+x^2\right)}}{\sqrt{2}x^2}+1\right)\right)+4\left(x^4+1\right)\left(2x^4+2\sqrt{x^4+1}x^2+1\right)\left(\sqrt{2}\sqrt{x^2\left(\sqrt{x^4+1}+x^2\right)}-\tan^{-1}\left(\sqrt{\left(\sqrt{x^4+1}+x^2\right)^2-1}\right)\right)\right)}{8\sqrt{2}\sqrt{x^4+1}\left(\sqrt{x^4+1}+x^2\right)^{3/2}\left(x^5+\sqrt{x^4+1}x^3+x\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

```
[Out] (Sqrt[x^2*(x^2 + Sqrt[1 + x^4])]*(4*(1 + x^4)*(1 + 2*x^4 + 2*x^2*Sqrt[1 + x
^4])*(Sqrt[2]*Sqrt[x^2*(x^2 + Sqrt[1 + x^4])] - ArcTan[Sqrt[-1 + (x^2 + Sqr
t[1 + x^4])^2]]) + (1 + (x^2 + Sqrt[1 + x^4])^2)^2*(Log[1 - Sqrt[x^2*(x^2 +
 Sqrt[1 + x^4])]/(Sqrt[2]*x^2)] - Log[1 + Sqrt[x^2*(x^2 + Sqrt[1 + x^4])]/(
Sqrt[2]*x^2)])))/(8*Sqrt[2]*Sqrt[1 + x^4]*(x^2 + Sqrt[1 + x^4])^(3/2)*(x +
x^5 + x^3*Sqrt[1 + x^4]))
```

IntegrateAlgebraic [A] time = 0.40, size = 145, normalized size = 1.22

$$\frac{1}{2}\sqrt{\sqrt{x^4+1}+x^2}\,x-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}-\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/Sqrt[1 + x^4],x]
```

```
[Out] (x*Sqrt[x^2 + Sqrt[1 + x^4]])/2 - ArcTan[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt
[1 + x^4]/Sqrt[2])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])]/Sqrt[2] - Sqrt[2]*ArcTanh
[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2])/(x*Sqrt[x^2 + Sqrt[1
+ x^4]])]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}\,\left(x^2-1\right)}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - 1)/sqrt(x^4 + 1), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-1\right)\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}\,\left(x^2-1\right)}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - 1)/sqrt(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2-1\right)\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 + 1)^(1/2),x)
```

```
[Out] int(((x^2 - 1)*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 + 1)^(1/2), x)
```

sympy [A] time = 4.13, size = 32, normalized size = 0.27

$$-\frac{G_{3,3}^{2,2}\left(\begin{matrix} 1,1 & \frac{1}{2} \\ \frac{1}{4},\frac{3}{4} & 0 \end{matrix} \middle| x^4\right)}{4\sqrt{\pi}}+\frac{G_{3,3}^{2,2}\left(\begin{matrix} \frac{3}{2},1 & 1 \\ \frac{3}{4},\frac{5}{4} & 0 \end{matrix} \middle| x^4\right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
```

```
[Out] -meijerg(((1, 1), (1/2,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi)) + meijerg
(((3/2, 1), (1,)), ((3/4, 5/4), (0,)), x**4)/(4*sqrt(pi))
```

3.1498 $\int \frac{x\sqrt{1+x}}{x+\sqrt{1+\sqrt{1+x}}}\,dx$

Optimal. Leaf size=119

$$\sqrt{x+1}\left(\frac{2(x+1)}{3}-\frac{4}{3}\sqrt{\sqrt{x+1}+1}\right)-\frac{4}{3}\sqrt{\sqrt{x+1}+1}-\frac{2}{5}\left(\sqrt{5}-5\right)\log\left(-2\sqrt{\sqrt{x+1}+1}+\sqrt{5}-1\right)+\frac{2}{5}\left(5+\right.$$

Rubi [A] time = 0.69, antiderivative size = 127, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1628, 632, 31}

$$\frac{2}{3}\left(\sqrt{x+1}+1\right)^3-2\left(\sqrt{x+1}+1\right)^2-\frac{4}{3}\left(\sqrt{x+1}+1\right)^{3/2}+2\sqrt{x+1}+\frac{2}{5}\left(5-\sqrt{5}\right)\log\left(2\sqrt{\sqrt{x+1}+1}-\sqrt{5}+1\right)+\frac{2}{5}\left(5+\sqrt{5}\right)\log\left(2\sqrt{\sqrt{x+1}+1}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 + x])/(x + Sqrt[1 + Sqrt[1 + x]]),x]
```

```
[Out] 2*Sqrt[1 + x] - (4*(1 + Sqrt[1 + x])^(3/2))/3 - 2*(1 + Sqrt[1 + x])^2 + (2*
(1 + Sqrt[1 + x])^3)/3 + (2*(5 - Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 + Sqrt
[1 + x]]])/5 + (2*(5 + Sqrt[5])*Log[1 + Sqrt[5] + 2*Sqrt[1 + Sqrt[1 + x]]])
/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sqrt{1+x}}{x+\sqrt{1+\sqrt{1+x}}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^2\left(-1+x^2\right)}{-1+x^2+\sqrt{1+x}}\,dx, x, \sqrt{1+x}\right)\\
&= 4\,\text{Subst}\left(\int \frac{(-1+x)x^2(1+x)^2\left(-2+x^2\right)}{-1+x+x^2}\,dx, x, \sqrt{1+\sqrt{1+x}}\right)\\
&= 4\,\text{Subst}\left(\int \left(x-x^2-2x^3+x^5+\frac{x}{-1+x+x^2}\right)dx, x, \sqrt{1+\sqrt{1+x}}\right)\\
&= 2\sqrt{1+x}-\frac{4}{3}\left(1+\sqrt{1+x}\right)^{3/2}-2\left(1+\sqrt{1+x}\right)^2+\frac{2}{3}\left(1+\sqrt{1+x}\right)^3+4\,\text{Subst}\left(\int \frac{}{-1}\right.\\
&= 2\sqrt{1+x}-\frac{4}{3}\left(1+\sqrt{1+x}\right)^{3/2}-2\left(1+\sqrt{1+x}\right)^2+\frac{2}{3}\left(1+\sqrt{1+x}\right)^3+\frac{1}{5}\left(2\left(5-\sqrt{5}\right)\right)\\
&= 2\sqrt{1+x}-\frac{4}{3}\left(1+\sqrt{1+x}\right)^{3/2}-2\left(1+\sqrt{1+x}\right)^2+\frac{2}{3}\left(1+\sqrt{1+x}\right)^3+\frac{2}{5}\left(5-\sqrt{5}\right)\log
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 128, normalized size = 1.08

$$\frac{1}{15}\left(10\left(\sqrt{x+1}x+\sqrt{x+1}-2\sqrt{x+1}\sqrt{\sqrt{x+1}+1}-2\sqrt{\sqrt{x+1}+1}-2\right)-6\left(\sqrt{5}-5\right)\log\left(2\sqrt{\sqrt{x+1}+1}-\sqrt{5}+1\right)+6\left(5+\sqrt{5}\right)\log\left(2\sqrt{\sqrt{x+1}+1}+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 + x])/(x + Sqrt[1 + Sqrt[1 + x]]),x]
```

```
[Out] (10*(-2 + Sqrt[1 + x] + x*Sqrt[1 + x] - 2*Sqrt[1 + Sqrt[1 + x]] - 2*Sqrt[1
+ x]*Sqrt[1 + Sqrt[1 + x]]) - 6*(-5 + Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 +
 Sqrt[1 + x]]] + 6*(5 + Sqrt[5])*Log[1 + Sqrt[5] + 2*Sqrt[1 + Sqrt[1 + x]]]
)/15
```

IntegrateAlgebraic [A] time = 0.12, size = 99, normalized size = 0.83

$$-\frac{4}{3}\left(\sqrt{x+1}+1\right)^{3/2}+\frac{2}{3}\left((x+1)^{3/2}+1\right)-\frac{2}{5}\left(\sqrt{5}-5\right)\log\left(-2\sqrt{\sqrt{x+1}+1}+\sqrt{5}-1\right)+\frac{2}{5}\left(5+\sqrt{5}\right)\log\left(2\sqrt{\sqrt{x+1}+1}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[1 + x])/(x + Sqrt[1 + Sqrt[1 + x]]),x]
```

```
[Out] (-4*(1 + Sqrt[1 + x])^(3/2))/3 + (2*(1 + (1 + x)^(3/2)))/3 - (2*(-5 + Sqrt[
5])*Log[-1 + Sqrt[5] - 2*Sqrt[1 + Sqrt[1 + x]]])/5 + (2*(5 + Sqrt[5])*Log[1
 + Sqrt[5] + 2*Sqrt[1 + Sqrt[1 + x]]])/5
```

fricas [A] time = 0.43, size = 130, normalized size = 1.09

$$\frac{2}{3}(x+1)^{\frac{3}{2}}-\frac{4}{3}\left(\sqrt{x+1}+1\right)^{\frac{3}{2}}+\frac{2}{5}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}(3x+1)-\left(\sqrt{5}(x+2)-5x\right)\sqrt{x+1}+\left(\sqrt{5}(x+2)+\left(\sqrt{5}(2x-1)-5\right)\sqrt{x+1}-5x\right)\sqrt{\sqrt{x+1}+1}+3x+3}{x^2-x-1}\right)+2\log\left(\sqrt{x+1}+\sqrt{\sqrt{x+1}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)/(x+(1+(1+x)^(1/2))^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*(x + 1)^(3/2) - 4/3*(sqrt(x + 1) + 1)^(3/2) + 2/5*sqrt(5)*log((2*x^2 -
sqrt(5)*(3*x + 1) - (sqrt(5)*(x + 2) - 5*x)*sqrt(x + 1) + (sqrt(5)*(x + 2)
+ (sqrt(5)*(2*x - 1) - 5)*sqrt(x + 1) - 5*x)*sqrt(sqrt(x + 1) + 1) + 3*x +
3)/(x^2 - x - 1)) + 2*log(sqrt(x + 1) + sqrt(sqrt(x + 1) + 1))
```

giac [A] time = 0.16, size = 102, normalized size = 0.86

$$\frac{2}{3}\left(\sqrt{x+1}+1\right)^3-2\left(\sqrt{x+1}+1\right)^2-\frac{4}{3}\left(\sqrt{x+1}+1\right)^{\frac{3}{2}}-\frac{2}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{\sqrt{x+1}+1}-1}{\sqrt{5}+2\sqrt{\sqrt{x+1}+1}+1}\right)+2\sqrt{x+1}+2\log\left(\sqrt{x+1}+\sqrt{\sqrt{x+1}+1}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)/(x+(1+(1+x)^(1/2))^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*(sqrt(x + 1) + 1)^3 - 2*(sqrt(x + 1) + 1)^2 - 4/3*(sqrt(x + 1) + 1)^(3/
2) - 2/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(sqrt(x + 1) + 1) - 1)/(sqrt(5) + 2*
sqrt(sqrt(x + 1) + 1) + 1)) + 2*sqrt(x + 1) + 2*log(sqrt(x + 1) + sqrt(sqrt
(x + 1) + 1)) + 2
```

maple [A] time = 0.01, size = 63, normalized size = 0.53

$$\frac{2(1+x)^{\frac{3}{2}}}{3}+\frac{2}{3}-\frac{4\left(1+\sqrt{1+x}\right)^{\frac{3}{2}}}{3}+2\ln\left(\sqrt{1+x}+\sqrt{1+\sqrt{1+x}}\right)+\frac{4\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(1+2\sqrt{1+\sqrt{1+x}}\right)\sqrt{5}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^(1/2)/(x+(1+(1+x)^(1/2))^(1/2)),x)
```

```
[Out] 2/3*(1+x)^(3/2)+2/3-4/3*(1+(1+x)^(1/2))^(3/2)+2*ln((1+x)^(1/2)+(1+(1+x)^(1/
2))^(1/2))+4/5*5^(1/2)*arctanh(1/5*(1+2*(1+(1+x)^(1/2))^(1/2))*5^(1/2))
```

maxima [A] time = 0.86, size = 102, normalized size = 0.86

$$\frac{2}{3}\left(\sqrt{x+1}+1\right)^{3}-2\left(\sqrt{x+1}+1\right)^{2}-\frac{4}{3}\left(\sqrt{x+1}+1\right)^{\frac{3}{2}}-\frac{2}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{\sqrt{x+1}+1}-1}{\sqrt{5}+2\sqrt{\sqrt{x+1}+1}+1}\right)+2\sqrt{x+1}+2\log\left(\sqrt{x+1}+\sqrt{\sqrt{x+1}+1}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)/(x+(1+(1+x)^(1/2))^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*(sqrt(x + 1) + 1)^3 - 2*(sqrt(x + 1) + 1)^2 - 4/3*(sqrt(x + 1) + 1)^(3/
2) - 2/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(sqrt(x + 1) + 1) - 1)/(sqrt(5) + 2*
sqrt(sqrt(x + 1) + 1) + 1)) + 2*sqrt(x + 1) + 2*log(sqrt(x + 1) + sqrt(sqrt
(x + 1) + 1)) + 2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sqrt{x+1}}{x+\sqrt{\sqrt{x+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^(1/2))/(x + ((x + 1)^(1/2) + 1)^(1/2)),x)
```

```
[Out] int((x*(x + 1)^(1/2))/(x + ((x + 1)^(1/2) + 1)^(1/2)), x)
```

sympy [A] time = 138.85, size = 175, normalized size = 1.47

$$2\sqrt{x+1}-\frac{4\left(\sqrt{x+1}+1\right)^{\frac{3}{2}}}{3}+\frac{2\left(\sqrt{x+1}+1\right)^{3}}{3}-2\left(\sqrt{x+1}+1\right)^{2}-8\left(\begin{cases}-\frac{\sqrt{5}\operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)^{2}>\frac{5}{4}\\-\frac{\sqrt{5}\operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)^{2}<\frac{5}{4}\end{cases}\right)+2\log\left(\sqrt{x+1}+\sqrt{\sqrt{x+1}+1}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(1/2)/(x+(1+(1+x)**(1/2))**(1/2)),x)
```

```
[Out] 2*sqrt(x + 1) - 4*(sqrt(x + 1) + 1)**(3/2)/3 + 2*(sqrt(x + 1) + 1)**3/3 - 2
*(sqrt(x + 1) + 1)**2 - 8*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(sqrt(x
+ 1) + 1) + 1/2)/5)/10, (sqrt(sqrt(x + 1) + 1) + 1/2)**2 > 5/4), (-sqrt(5)*
atanh(2*sqrt(5)*(sqrt(sqrt(x + 1) + 1) + 1/2)/5)/10, (sqrt(sqrt(x + 1) + 1)
 + 1/2)**2 < 5/4)) + 2*log(sqrt(x + 1) + sqrt(sqrt(x + 1) + 1)) + 2
```

3.1499 $\int \frac{b+ax^8}{x^6(-b+ax^4)(b+ax^4)^{3/4}}\,dx$

Optimal. Leaf size=120

$$\frac{\left(a^{5/4}+\sqrt[4]{a}\,b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}b^2}-\frac{\left(a^{5/4}+\sqrt[4]{a}\,b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}b^2}+\frac{\left(ax^4+b\right)^{5/4}}{5b^2x^5}$$

Rubi [A] time = 0.74, antiderivative size = 132, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {6725, 271, 264, 494, 298, 203, 206}

$$\frac{a\sqrt[4]{ax^4+b}}{5b^2x}+\frac{\sqrt[4]{a}\,(a+b)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}b^2}-\frac{\sqrt[4]{a}\,(a+b)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}b^2}+\frac{\sqrt[4]{ax^4+b}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^8)/(x^6*(-b + a*x^4)*(b + a*x^4)^(3/4)),x]
```

```
[Out] (b + a*x^4)^(1/4)/(5*b*x^5) + (a*(b + a*x^4)^(1/4))/(5*b^2*x) + (a^(1/4)*(a
 + b)*ArcTan[(2^(1/4)*a^(1/4)*x)/(b + a*x^4)^(1/4)])/(2*2^(3/4)*b^2) - (a^(
1/4)*(a + b)*ArcTanh[(2^(1/4)*a^(1/4)*x)/(b + a*x^4)^(1/4)])/(2*2^(3/4)*b^2
)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{b+ax^8}{x^6\left(-b+ax^4\right)\left(b+ax^4\right)^{3/4}}\,dx = \int \left(-\frac{1}{x^6\left(b+ax^4\right)^{3/4}}-\frac{a}{bx^2\left(b+ax^4\right)^{3/4}}-\frac{a(a+b)x^2}{b\left(b-ax^4\right)\left(b+ax^4\right)^{3/4}}\right)dx$$

$$= -\frac{a\int \frac{1}{x^2\left(b+ax^4\right)^{3/4}}\,dx}{b}-\frac{(a(a+b))\int \frac{x^2}{\left(b-ax^4\right)\left(b+ax^4\right)^{3/4}}\,dx}{b}-\int \frac{1}{x^6\left(b+ax^4\right)^{3/4}}\,dx$$

$$= \frac{\sqrt[4]{b+ax^4}}{5bx^5}+\frac{a\sqrt[4]{b+ax^4}}{b^2x}+\frac{(4a)\int \frac{1}{x^2\left(b+ax^4\right)^{3/4}}\,dx}{5b}-\frac{(a(a+b))\,\mathrm{Subst}\left(\int \frac{x^2}{b-2ab\ldots}\right.}{b}$$

$$= \frac{\sqrt[4]{b+ax^4}}{5bx^5}+\frac{a\sqrt[4]{b+ax^4}}{5b^2x}-\frac{\left(\sqrt{a}\,(a+b)\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,\sqrt{a}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)}{2\sqrt{2}\,b^2}$$

$$= \frac{\sqrt[4]{b+ax^4}}{5bx^5}+\frac{a\sqrt[4]{b+ax^4}}{5b^2x}+\frac{\sqrt[4]{a}\,(a+b)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{2\,2^{3/4}b^2}-\frac{\sqrt[4]{a}\,(a+b)\tanh^{-1}\ldots}{2\,2^{3/4}b^2}$$

Mathematica [C] time = 0.12, size = 65, normalized size = 0.54

$$\frac{3\left(ax^4+b\right)^2-5ax^8(a+b)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{2ax^4}{ax^4+b}\right)}{15b^2x^5\left(ax^4+b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^8)/(x^6*(-b + a*x^4)*(b + a*x^4)^(3/4)),x]
```

```
[Out] (3*(b + a*x^4)^2 - 5*a*(a + b)*x^8*Hypergeometric2F1[3/4, 1, 7/4, (2*a*x^4)
/(b + a*x^4)])/(15*b^2*x^5*(b + a*x^4)^(3/4))
```

IntegrateAlgebraic [A] time = 1.07, size = 120, normalized size = 1.00

$$\frac{\left(a^{5/4}+\sqrt[4]{a}\,b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}b^2}-\frac{\left(a^{5/4}+\sqrt[4]{a}\,b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}b^2}+\frac{\left(ax^4+b\right)^{5/4}}{5b^2x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^8)/(x^6*(-b + a*x^4)*(b + a*x^4)^(3/4)),x]
```

```
[Out] (b + a*x^4)^(5/4)/(5*b^2*x^5) + ((a^(5/4) + a^(1/4)*b)*ArcTan[(2^(1/4)*a^(1
/4)*x)/(b + a*x^4)^(1/4)])/(2*2^(3/4)*b^2) - ((a^(5/4) + a^(1/4)*b)*ArcTanh
[(2^(1/4)*a^(1/4)*x)/(b + a*x^4)^(1/4)])/(2*2^(3/4)*b^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+b)/x^6/(a*x^4-b)/(a*x^4+b)^(3/4),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^8+b}{\left(ax^4+b\right)^{\frac{3}{4}}\left(ax^4-b\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+b)/x^6/(a*x^4-b)/(a*x^4+b)^(3/4),x, algorithm="giac")
[Out] integrate((a*x^8 + b)/((a*x^4 + b)^(3/4)*(a*x^4 - b)*x^6), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{a\,x^8+b}{x^6\left(a\,x^4-b\right)\left(a\,x^4+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^8+b)/x^6/(a*x^4-b)/(a*x^4+b)^(3/4),x)
[Out] int((a*x^8+b)/x^6/(a*x^4-b)/(a*x^4+b)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^8+b}{\left(ax^4+b\right)^{\frac{3}{4}}\left(ax^4-b\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8+b)/x^6/(a*x^4-b)/(a*x^4+b)^(3/4),x, algorithm="maxima")
[Out] integrate((a*x^8 + b)/((a*x^4 + b)^(3/4)*(a*x^4 - b)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a\,x^8+b}{x^6\left(a\,x^4+b\right)^{3/4}\left(b-a\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x^8)/(x^6*(b + a*x^4)^(3/4)*(b - a*x^4)),x)
[Out] -int((b + a*x^8)/(x^6*(b + a*x^4)^(3/4)*(b - a*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^8+b}{x^6\left(ax^4-b\right)\left(ax^4+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**8+b)/x**6/(a*x**4-b)/(a*x**4+b)**(3/4),x)
[Out] Integral((a*x**8 + b)/(x**6*(a*x**4 - b)*(a*x**4 + b)**(3/4)), x)
```

3.1500 $\int \frac{-ab-ac+3bc+2(a-b-c)x+x^2}{\sqrt[4]{(-a+x)(-b+x)(-c+x)}\left(-a^3-bcd+\left(3a^2+bd+cd\right)x-(3a+d)x^2+x^3\right)}\,dx$

Optimal. Leaf size=121

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}}{a-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}}{a-x}\right)}{d^{3/4}}$$

Rubi [F] time = 63.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-ab-ac+3bc+2(a-b-c)x+x^2}{\sqrt[4]{(-a+x)(-b+x)(-c+x)}\left(-a^3-bcd+\left(3a^2+bd+cd\right)x-(3a+d)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a*b) - a*c + 3*b*c + 2*(a - b - c)*x + x^2)/(((-a + x)*(-b + x)*(-c
+ x))^(1/4)*(-a^3 - b*c*d + (3*a^2 + b*d + c*d)*x - (3*a + d)*x^2 + x^3)),x
]
```

```
[Out] (-8*a*(a - b - c)*(-a + x)^(1/4)*(-b + x)^(1/4)*(-c + x)^(1/4)*Defer[Subst]
[Defer[Int][x^2/((a - b + x^4)^(1/4)*(a - c + x^4)^(1/4)*(a^2*(1 + (b*c - a
*(b + c))/a^2)*d + 2*a*(1 - (b + c)/(2*a))*d*x^4 + d*x^8 - x^12)), x], x, (
-a + x)^(1/4)])/(-((a - x)*(b - x)*(c - x)))^(1/4) - (8*(a - b - c)*(-a + x
)^(1/4)*(-b + x)^(1/4)*(-c + x)^(1/4)*Defer[Subst][Defer[Int][x^6/((a - b +
 x^4)^(1/4)*(a - c + x^4)^(1/4)*(a^2*(1 + (b*c - a*(b + c))/a^2)*d + 2*a*(1
 - (b + c)/(2*a))*d*x^4 + d*x^8 - x^12)), x], x, (-a + x)^(1/4)])/(-((a - x
)*(b - x)*(c - x)))^(1/4) + (4*a^2*(-a + x)^(1/4)*(-b + x)^(1/4)*(-c + x)^(
1/4)*Defer[Subst][Defer[Int][x^2/((a - b + x^4)^(1/4)*(a - c + x^4)^(1/4)*(
-(a^2*(1 + (b*c - a*(b + c))/a^2)*d) - 2*a*(1 - (b + c)/(2*a))*d*x^4 - d*x^
8 + x^12)), x], x, (-a + x)^(1/4)])/(-((a - x)*(b - x)*(c - x)))^(1/4) + (8
*a*(-a + x)^(1/4)*(-b + x)^(1/4)*(-c + x)^(1/4)*Defer[Subst][Defer[Int][x^6
/((a - b + x^4)^(1/4)*(a - c + x^4)^(1/4)*(-(a^2*(1 + (b*c - a*(b + c))/a^2
)*d) - 2*a*(1 - (b + c)/(2*a))*d*x^4 - d*x^8 + x^12)), x], x, (-a + x)^(1/4
)])/(-((a - x)*(b - x)*(c - x)))^(1/4) + (4*(-a + x)^(1/4)*(-b + x)^(1/4)*(
-c + x)^(1/4)*Defer[Subst][Defer[Int][x^10/((a - b + x^4)^(1/4)*(a - c + x^
4)^(1/4)*(-(a^2*(1 + (b*c - a*(b + c))/a^2)*d) - 2*a*(1 - (b + c)/(2*a))*d*
x^4 - d*x^8 + x^12)), x], x, (-a + x)^(1/4)])/(-((a - x)*(b - x)*(c - x)))^
(1/4) - (4*(3*b*c - a*(b + c))*(-a + x)^(1/4)*(-b + x)^(1/4)*(-c + x)^(1/4)
*Defer[Subst][Defer[Int][x^2/((a - b + x^4)^(1/4)*(a - c + x^4)^(1/4)*(a^3*
(1 + (b*c*d)/a^3) - (3*a^2 + (b + c)*d)*(a + x^4) + (3*a + d)*(a + x^4)^2 -
 (a + x^4)^3)), x], x, (-a + x)^(1/4)])/(-((a - x)*(b - x)*(c - x)))^(1/4)
```

Rubi steps

$$\int \frac{-ab-ac+3bc+2(a-b-c)x+x^2}{\sqrt[4]{(-a+x)(-b+x)(-c+x)}\left(-a^3-bcd+\left(3a^2+bd+cd\right)x-(3a+d)x^2+x^3\right)}\,dx = \frac{\left(\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\,\sqrt[4]{-c+\right.}{}$$

$$= \frac{\left(\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\,\sqrt[4]{-c+\right.}{}$$

$$= \frac{\left(\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\,\sqrt[4]{-c+\right.}{}$$

$$= \frac{\left(\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\,\sqrt[4]{-c+\right.}{}$$

$$= \frac{\left(4\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\,\sqrt[4]{-c\right.}{}$$

$$= \frac{\left(4\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\,\sqrt[4]{-c\right.}{}$$

$$= \frac{\left(4\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\,\sqrt[4]{-c\right.}{}$$

$$= \frac{\left(4\sqrt[4]{-a+x}\,\sqrt[4]{-b+x}\,\sqrt[4]{-c\right.}{}$$

Mathematica [F] time = 6.90, size = 0, normalized size = 0.00

$$\int \frac{-ab-ac+3bc+2(a-b-c)x+x^2}{\sqrt[4]{(-a+x)(-b+x)(-c+x)}\left(-a^3-bcd+\left(3a^2+bd+cd\right)x-(3a+d)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-(a*b) - a*c + 3*b*c + 2*(a - b - c)*x + x^2)/(((-a + x)*(-b + x
)*(-c + x))^(1/4)*(-a^3 - b*c*d + (3*a^2 + b*d + c*d)*x - (3*a + d)*x^2 + x
^3)),x]
```

```
[Out] Integrate[(-(a*b) - a*c + 3*b*c + 2*(a - b - c)*x + x^2)/(((-a + x)*(-b + x
)*(-c + x))^(1/4)*(-a^3 - b*c*d + (3*a^2 + b*d + c*d)*x - (3*a + d)*x^2 + x
^3)), x]
```

IntegrateAlgebraic [A] time = 0.45, size = 121, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}}{a-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}}{a-x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*b) - a*c + 3*b*c + 2*(a - b - c)*x + x^2)/(((-a + x
)*(-b + x)*(-c + x))^(1/4)*(-a^3 - b*c*d + (3*a^2 + b*d + c*d)*x - (3*a + d
)*x^2 + x^3)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(-(a*b*c) + (a*b + a*c + b*c)*x + (-a - b - c)*x^2 + x^
3)^(1/4))/(a - x)])/d^(3/4) + (2*ArcTanh[(d^(1/4)*(-(a*b*c) + (a*b + a*c +
b*c)*x + (-a - b - c)*x^2 + x^3)^(1/4))/(a - x)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b-a*c+3*b*c+2*(a-b-c)*x+x^2)/((-a+x)*(-b+x)*(-c+x))^(1/4)/(-a
^3-b*c*d+(3*a^2+b*d+c*d)*x-(3*a+d)*x^2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab + ac - 3\,bc - 2\,(a - b - c)x - x^2}{\left(a^3 + bcd + (3\,a + d)x^2 - x^3 - \left(3\,a^2 + bd + cd\right)x\right)(-(a - x)(b - x)(c - x))^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b-a*c+3*b*c+2*(a-b-c)*x+x^2)/((-a+x)*(-b+x)*(-c+x))^(1/4)/(-a
^3-b*c*d+(3*a^2+b*d+c*d)*x-(3*a+d)*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate((a*b + a*c - 3*b*c - 2*(a - b - c)*x - x^2)/((a^3 + b*c*d + (3*a
+ d)*x^2 - x^3 - (3*a^2 + b*d + c*d)*x)*(-(a - x)*(b - x)*(c - x))^(1/4)),
x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{-ab - ac + 3bc + 2\,(a - b - c)\,x + x^2}{\left((-a + x)\,(-b + x)\,(-c + x)\right)^{\frac{1}{4}}\left(-a^3 - bcd + \left(3a^2 + bd + cd\right)x - (3a + d)\,x^2 + x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*b-a*c+3*b*c+2*(a-b-c)*x+x^2)/((-a+x)*(-b+x)*(-c+x))^(1/4)/(-a^3-b*c
*d+(3*a^2+b*d+c*d)*x-(3*a+d)*x^2+x^3),x)
```

```
[Out] int((-a*b-a*c+3*b*c+2*(a-b-c)*x+x^2)/((-a+x)*(-b+x)*(-c+x))^(1/4)/(-a^3-b*c
*d+(3*a^2+b*d+c*d)*x-(3*a+d)*x^2+x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab + ac - 3\,bc - 2\,(a - b - c)x - x^2}{\left(a^3 + bcd + (3\,a + d)x^2 - x^3 - \left(3\,a^2 + bd + cd\right)x\right)(-(a - x)(b - x)(c - x))^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b-a*c+3*b*c+2*(a-b-c)*x+x^2)/((-a+x)*(-b+x)*(-c+x))^(1/4)/(-a
^3-b*c*d+(3*a^2+b*d+c*d)*x-(3*a+d)*x^2+x^3),x, algorithm="maxima")
```

```
[Out] integrate((a*b + a*c - 3*b*c - 2*(a - b - c)*x - x^2)/((a^3 + b*c*d + (3*a
+ d)*x^2 - x^3 - (3*a^2 + b*d + c*d)*x)*(-(a - x)*(b - x)*(c - x))^(1/4)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,b + a\,c - 3\,b\,c + 2\,x\,(b - a + c) - x^2}{(-(a - x)\,(b - x)\,(c - x))^{1/4}\,\left(x^2\,(3\,a + d) - x\,\left(3\,a^2 + b\,d + c\,d\right) + a^3 - x^3 + b\,c\,d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b + a*c - 3*b*c + 2*x*(b - a + c) - x^2)/((-(a - x)*(b - x)*(c - x))
^(1/4)*(x^2*(3*a + d) - x*(b*d + c*d + 3*a^2) + a^3 - x^3 + b*c*d)),x)
```

```
[Out] int((a*b + a*c - 3*b*c + 2*x*(b - a + c) - x^2)/((-(a - x)*(b - x)*(c - x))
^(1/4)*(x^2*(3*a + d) - x*(b*d + c*d + 3*a^2) + a^3 - x^3 + b*c*d)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b-a*c+3*b*c+2*(a-b-c)*x+x**2)/((-a+x)*(-b+x)*(-c+x))**(1/4)/(
-a**3-b*c*d+(3*a**2+b*d+c*d)*x-(3*a+d)*x**2+x**3),x)
```

```
[Out] Timed out
```

3.1501 $\int \frac{1}{(-b+ax)\sqrt[4]{b^2x+a^2x^3}}\,dx$

Optimal. Leaf size=121

$$\frac{\tan^{-1}\left(\frac{2^{3/4}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{a^2x^3+b^2x}}{ax+b}\right)}{2\sqrt[4]{2}\,a^{3/4}b^{3/4}}-\frac{\tanh^{-1}\left(\frac{2^{3/4}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{a^2x^3+b^2x}}{ax+b}\right)}{2\sqrt[4]{2}\,a^{3/4}b^{3/4}}$$

Rubi [C] time = 0.29, antiderivative size = 150, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2056, 959, 466, 511, 510}

$$-\frac{4ax^2\sqrt[4]{\frac{a^2x^2}{b^2}+1}\,F_1\left(\frac{7}{8};1,\frac{1}{4};\frac{15}{8};\frac{a^2x^2}{b^2},-\frac{a^2x^2}{b^2}\right)}{7b^2\sqrt[4]{a^2x^3+b^2x}}-\frac{4x\sqrt[4]{\frac{a^2x^2}{b^2}+1}\,F_1\left(\frac{3}{8};1,\frac{1}{4};\frac{11}{8};\frac{a^2x^2}{b^2},-\frac{a^2x^2}{b^2}\right)}{3b\sqrt[4]{a^2x^3+b^2x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((-b + a*x)*(b^2*x + a^2*x^3)^(1/4)),x]
```

```
[Out] (-4*x*(1 + (a^2*x^2)/b^2)^(1/4)*AppellF1[3/8, 1, 1/4, 11/8, (a^2*x^2)/b^2,
-((a^2*x^2)/b^2)])/(3*b*(b^2*x + a^2*x^3)^(1/4)) - (4*a*x^2*(1 + (a^2*x^2)/
b^2)^(1/4)*AppellF1[7/8, 1, 1/4, 15/8, (a^2*x^2)/b^2, -((a^2*x^2)/b^2)])/(7
*b^2*(b^2*x + a^2*x^3)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{(-b+ax)\sqrt[4]{b^2x+a^2x^3}}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{b^2+a^2x^2}\right)\int \frac{1}{\sqrt[4]{x}(-b+ax)\sqrt[4]{b^2+a^2x^2}}\,dx}{\sqrt[4]{b^2x+a^2x^3}}$$
$$= -\frac{\left(a\sqrt[4]{x}\sqrt[4]{b^2+a^2x^2}\right)\int \frac{x^{3/4}}{\left(b^2-a^2x^2\right)\sqrt[4]{b^2+a^2x^2}}\,dx}{\sqrt[4]{b^2x+a^2x^3}} - \frac{\left(b\sqrt[4]{x}\sqrt[4]{b^2+a^2x^2}\right)\int \frac{1}{\sqrt[4]{x}\left(b^2-a^2x^2\right)\sqrt[4]{b^2}}}{\sqrt[4]{b^2x+a^2x^3}}$$
$$= -\frac{\left(4a\sqrt[4]{x}\sqrt[4]{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{x^6}{\left(b^2-a^2x^8\right)\sqrt[4]{b^2+a^2x^8}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{b^2x+a^2x^3}} - \frac{\left(4b\sqrt[4]{x}\sqrt[4]{b^2+a^2x^2}\right.}{}$$
$$= -\frac{\left(4a\sqrt[4]{x}\sqrt[4]{1+\frac{a^2x^2}{b^2}}\right)\text{Subst}\left(\int \frac{x^6}{\left(b^2-a^2x^8\right)\sqrt[4]{1+\frac{a^2x^8}{b^2}}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{b^2x+a^2x^3}} - \frac{\left(4b\sqrt[4]{x}\sqrt[4]{1+\frac{a^2x^2}{b^2}}\right)\text{S}}{}$$
$$= -\frac{4x\sqrt[4]{1+\frac{a^2x^2}{b^2}}\,F_1\left(\frac{3}{8};1,\frac{1}{4};\frac{11}{8};\frac{a^2x^2}{b^2},-\frac{a^2x^2}{b^2}\right)}{3b\sqrt[4]{b^2x+a^2x^3}} - \frac{4ax^2\sqrt[4]{1+\frac{a^2x^2}{b^2}}\,F_1\left(\frac{7}{8};1,\frac{1}{4};\frac{15}{8};\frac{a^2x^2}{b^2},-\frac{a^2}{}\right.}{7b^2\sqrt[4]{b^2x+a^2x^3}}$$

Mathematica [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{(-b+ax)\sqrt[4]{b^2x+a^2x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((-b + a*x)*(b^2*x + a^2*x^3)^(1/4)),x]
[Out] Integrate[1/((-b + a*x)*(b^2*x + a^2*x^3)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.38, size = 121, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2^{3/4}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{a^2x^3+b^2x}}{ax+b}\right)}{2\sqrt[4]{2}\,a^{3/4}b^{3/4}} - \frac{\tanh^{-1}\left(\frac{2^{3/4}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{a^2x^3+b^2x}}{ax+b}\right)}{2\sqrt[4]{2}\,a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-b + a*x)*(b^2*x + a^2*x^3)^(1/4)),x]
[Out] ArcTan[(2^(3/4)*a^(1/4)*b^(1/4)*(b^2*x + a^2*x^3)^(1/4))/(b + a*x)]/(2*2^(1
/4)*a^(3/4)*b^(3/4)) - ArcTanh[(2^(3/4)*a^(1/4)*b^(1/4)*(b^2*x + a^2*x^3)^(
1/4))/(b + a*x)]/(2*2^(1/4)*a^(3/4)*b^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-b)/(a^2*x^3+b^2*x)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^3 + b^2x\right)^{\frac{1}{4}}(ax - b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-b)/(a^2*x^3+b^2*x)^(1/4),x, algorithm="giac")
[Out] integrate(1/((a^2*x^3 + b^2*x)^(1/4)*(a*x - b)), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax - b)\left(a^2x^3 + b^2x\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-b)/(a^2*x^3+b^2*x)^(1/4),x)
[Out] int(1/(a*x-b)/(a^2*x^3+b^2*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^3 + b^2x\right)^{\frac{1}{4}}(ax - b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-b)/(a^2*x^3+b^2*x)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((a^2*x^3 + b^2*x)^(1/4)*(a*x - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(a^2\,x^3 + b^2\,x\right)^{1/4}\,(b - a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((b^2*x + a^2*x^3)^(1/4)*(b - a*x)),x)
[Out] -int(1/((b^2*x + a^2*x^3)^(1/4)*(b - a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x\left(a^2x^2 + b^2\right)}\,(ax - b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-b)/(a**2*x**3+b**2*x)**(1/4),x)
[Out] Integral(1/((x*(a**2*x**2 + b**2))**(1/4)*(a*x - b)), x)
```

3.1502 $\int \frac{(2+x^2)(-2-2x+x^2)\sqrt{4-3x^2+x^4}}{x^2(-2+x^2)(-4+x+2x^2)}\,dx$

Optimal. Leaf size=121

$$\frac{\sqrt{x^4-3x^2+4}}{2x}-\frac{5}{4}\log\left(x^2+\sqrt{x^4-3x^2+4}-2\right)+4\tanh^{-1}\left(\frac{x}{x^2+\sqrt{x^4-3x^2+4}-2}\right)-\frac{5}{2}\sqrt{5}\tanh^{-1}\left(\frac{}{2x^2+2\sqrt{x^4}}\right.$$

Rubi [C] time = 3.65, antiderivative size = 917, normalized size of antiderivative = 7.58, number of steps used = 53, number of rules used = 21, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6725, 1117, 1197, 1103, 1195, 1114, 734, 843, 619, 215, 724, 206, 1208, 1210, 1698, 207, 6728, 1728, 1216, 1706, 1247}

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Int[((2 + x^2)*(-2 - 2*x + x^2)*Sqrt[4 - 3*x^2 + x^4])/(x^2*(-2 + x^2)*(-4
+ x + 2*x^2)),x]
```

```
[Out] (-5*Sqrt[4 - 3*x^2 + x^4])/16 + (5*(1 - Sqrt[33])*Sqrt[4 - 3*x^2 + x^4])/32
 + (5*(1 + Sqrt[33])*Sqrt[4 - 3*x^2 + x^4])/32 + Sqrt[4 - 3*x^2 + x^4]/(2*x
) - (15*ArcSinh[(3 - 2*x^2)/Sqrt[7]])/32 + (5*(7 - Sqrt[33])*ArcSinh[(3 - 2
*x^2)/Sqrt[7]])/64 + (5*(7 + Sqrt[33])*ArcSinh[(3 - 2*x^2)/Sqrt[7]])/64 + 2
*ArcTanh[x/Sqrt[4 - 3*x^2 + x^4]] - (5*Sqrt[5]*ArcTanh[(Sqrt[5]*x)/(2*Sqrt[
4 - 3*x^2 + x^4])])/4 + (5*ArcTanh[(8 - 3*x^2)/(4*Sqrt[4 - 3*x^2 + x^4])])/
8 - (5*Sqrt[5]*ArcTanh[(13 + 3*Sqrt[33] + 2*(5 - Sqrt[33])*x^2)/(2*Sqrt[10*
(17 - Sqrt[33])]*Sqrt[4 - 3*x^2 + x^4])])/8 + (5*Sqrt[5]*ArcTanh[(13 - 3*Sq
rt[33] + 2*(5 + Sqrt[33])*x^2)/(2*Sqrt[10*(17 + Sqrt[33])]*Sqrt[4 - 3*x^2 +
 x^4])])/8 - (5*(2 + x^2)*Sqrt[(4 - 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*A
rcTan[x/Sqrt[2]], 7/8])/(4*Sqrt[2]*Sqrt[4 - 3*x^2 + x^4]) + (5*(9 - Sqrt[33
])*(2 + x^2)*Sqrt[(4 - 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[
2]], 7/8])/(32*Sqrt[2]*Sqrt[4 - 3*x^2 + x^4]) - (25*(17 - Sqrt[33])*(2 + x^
2)*Sqrt[(4 - 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 7/8])
/(16*Sqrt[2]*(33 - Sqrt[33])*Sqrt[4 - 3*x^2 + x^4]) + (5*(9 + Sqrt[33])*(2
+ x^2)*Sqrt[(4 - 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 7
/8])/(32*Sqrt[2]*Sqrt[4 - 3*x^2 + x^4]) - (25*(17 + Sqrt[33])*(2 + x^2)*Sqr
t[(4 - 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 7/8])/(16*S
qrt[2]*(33 + Sqrt[33])*Sqrt[4 - 3*x^2 + x^4]) + (25*(17 - Sqrt[33])*(2 + x^
2)*Sqrt[(4 - 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[33/32, 2*ArcTan[x/Sqrt[2]
], 7/8])/(32*Sqrt[2]*(33 - 17*Sqrt[33])*Sqrt[4 - 3*x^2 + x^4]) + (25*(17 +
Sqrt[33])*(2 + x^2)*Sqrt[(4 - 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[33/32, 2
*ArcTan[x/Sqrt[2]], 7/8])/(32*Sqrt[2]*(33 + 17*Sqrt[33])*Sqrt[4 - 3*x^2 + x
^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1117

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^p)/(d*(m + 1)), x] - Dist[(2*p)
/(d^2*(m + 1)), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
```

```
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1210

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
```

```
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbo
l] :> Dist[d, Int[(a + b*x^2 + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, I
nt[(x*(a + b*x^2 + c*x^4)^p)/(d^2 - e^2*x^2), x], x] /; FreeQ[{a, b, c, d,
e}, x] && IntegerQ[p + 1/2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(2+x^2)(-2-2x+x^2)\sqrt{4-3x^2+x^4}}{x^2(-2+x^2)(-4+x+2x^2)}\,dx = \int \left(-\frac{\sqrt{4-3x^2+x^4}}{2x^2} - \frac{5\sqrt{4-3x^2+x^4}}{8x} - \frac{4\sqrt{4-3x^2+x^4}}{-2+x^2} + \frac{5(1\cdots}{\cdots}\right.$$

$$= -\left(\frac{1}{2}\int \frac{\sqrt{4-3x^2+x^4}}{x^2}\,dx\right) - \frac{5}{8}\int \frac{\sqrt{4-3x^2+x^4}}{x}\,dx + \frac{5}{8}\int \frac{(17\cdots}{\cdots}$$

$$= \frac{\sqrt{4-3x^2+x^4}}{2x} - \frac{5}{16}\,\text{Subst}\left(\int \frac{\sqrt{4-3x+x^2}}{x}\,dx, x, x^2\right) - \frac{1}{2}\int \frac{\cdots}{\sqrt{\cdots}}$$

$$= -\frac{5}{16}\sqrt{4-3x^2+x^4} + \frac{\sqrt{4-3x^2+x^4}}{2x} + \frac{5}{32}\,\text{Subst}\left(\int \frac{-8+3\cdots}{x\sqrt{4-3x\cdots}}\right.$$

$$= -\frac{5}{16}\sqrt{4-3x^2+x^4} + \frac{\sqrt{4-3x^2+x^4}}{2x} - \frac{5x\sqrt{4-3x^2+x^4}}{2+x^2} + \frac{5\sqrt{2\cdots}}{\cdots}$$

$$= -\frac{5}{16}\sqrt{4-3x^2+x^4} + \frac{\sqrt{4-3x^2+x^4}}{2x} - \frac{5x\sqrt{4-3x^2+x^4}}{2+x^2} + 2\tan\cdots$$

$$= -\frac{5}{16}\sqrt{4-3x^2+x^4} + \frac{5}{32}\left(1-\sqrt{33}\right)\sqrt{4-3x^2+x^4} + \frac{5}{32}\left(1+\sqrt{3\cdots}\right.$$

$$= -\frac{5}{16}\sqrt{4-3x^2+x^4} + \frac{5}{32}\left(1-\sqrt{33}\right)\sqrt{4-3x^2+x^4} + \frac{5}{32}\left(1+\sqrt{3\cdots}\right.$$

$$= -\frac{5}{16}\sqrt{4-3x^2+x^4} + \frac{5}{32}\left(1-\sqrt{33}\right)\sqrt{4-3x^2+x^4} + \frac{5}{32}\left(1+\sqrt{3\cdots}\right.$$

$$= -\frac{5}{16}\sqrt{4-3x^2+x^4} + \frac{5}{32}\left(1-\sqrt{33}\right)\sqrt{4-3x^2+x^4} + \frac{5}{32}\left(1+\sqrt{3\cdots}\right.$$

Mathematica [C] time = 1.60, size = 871, normalized size = 7.20

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + x^2)*(-2 - 2*x + x^2)*Sqrt[4 - 3*x^2 + x^4])/(x^2*(-2 + x^2
)*(-4 + x + 2*x^2)),x]
```

```
[Out] (32*Sqrt[(-I)/(3*I + Sqrt[7])] - 24*Sqrt[(-I)/(3*I + Sqrt[7])]*x^2 + 8*Sqrt
[(-I)/(3*I + Sqrt[7])]*x^4 + 10*Sqrt[(-I)/(3*I + Sqrt[7])]*x*Sqrt[4 - 3*x^2
```

```
 + x^4]*ArcSinh[(3 - 2*x^2)/Sqrt[7]] + 10*Sqrt[(-I)/(3*I + Sqrt[7])]*x*Sqrt
[4 - 3*x^2 + x^4]*ArcTanh[(8 - 3*x^2)/(4*Sqrt[4 - 3*x^2 + x^4])] + 10*Sqrt[
(-5*I)/(3*I + Sqrt[7])]*x*Sqrt[4 - 3*x^2 + x^4]*ArcTanh[(13 - 3*Sqrt[33] +
2*(5 + Sqrt[33])*x^2)/(2*Sqrt[10*(17 + Sqrt[33])]*Sqrt[4 - 3*x^2 + x^4])] -
 10*Sqrt[(-5*I)/(3*I + Sqrt[7])]*x*Sqrt[4 - 3*x^2 + x^4]*ArcTanh[(13 + 3*Sq
rt[33] - 2*(-5 + Sqrt[33])*x^2)/(2*Sqrt[10]*Sqrt[-((-17 + Sqrt[33])*(4 - 3*
x^2 + x^4))])] - (9*I)*x*Sqrt[1 + ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[2 - ((
4*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(3*I + Sqrt[7])]
*x], (3*I + Sqrt[7])/(3*I - Sqrt[7])] - (32*I)*x*Sqrt[1 + ((2*I)*x^2)/(-3*I
 + Sqrt[7])]*Sqrt[2 - ((4*I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[3/4 - (I/4)*S
qrt[7], I*ArcSinh[Sqrt[(-2*I)/(3*I + Sqrt[7])]*x], (3*I + Sqrt[7])/(3*I - S
qrt[7])] + (25*I)*x*Sqrt[1 + ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[2 - ((4*I)*
x^2)/(3*I + Sqrt[7])]*EllipticPi[((4*I)*(3*I + Sqrt[7]))/(-17 + Sqrt[33]),
I*ArcSinh[Sqrt[(-2*I)/(3*I + Sqrt[7])]*x], (3*I + Sqrt[7])/(3*I - Sqrt[7])]
 + (25*I)*x*Sqrt[1 + ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[2 - ((4*I)*x^2)/(3*
I + Sqrt[7])]*EllipticPi[(12 - (4*I)*Sqrt[7])/(17 + Sqrt[33]), I*ArcSinh[Sq
rt[(-2*I)/(3*I + Sqrt[7])]*x], (3*I + Sqrt[7])/(3*I - Sqrt[7])])/(16*Sqrt[(
-I)/(3*I + Sqrt[7])]*x*Sqrt[4 - 3*x^2 + x^4])
```

IntegrateAlgebraic [A] time = 1.12, size = 121, normalized size = 1.00

$$\frac{\sqrt{x^4-3x^2+4}}{2x}-\frac{5}{4}\log\left(x^2+\sqrt{x^4-3x^2+4}-2\right)+4\tanh^{-1}\left(\frac{x}{x^2+\sqrt{x^4-3x^2+4}-2}\right)-\frac{5}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5}\,x}{2x^2+2\sqrt{x^4-3x^2+4}+x-4}\right)+\frac{5\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^2)*(-2 - 2*x + x^2)*Sqrt[4 - 3*x^2 + x^4])/(x^2*
(-2 + x^2)*(-4 + x + 2*x^2)),x]
```

```
[Out] Sqrt[4 - 3*x^2 + x^4]/(2*x) + 4*ArcTanh[x/(-2 + x^2 + Sqrt[4 - 3*x^2 + x^4]
)] - (5*Sqrt[5]*ArcTanh[(Sqrt[5]*x)/(-4 + x + 2*x^2 + 2*Sqrt[4 - 3*x^2 + x^
4])])/2 + (5*Log[x])/4 - (5*Log[-2 + x^2 + Sqrt[4 - 3*x^2 + x^4]])/4
```

fricas [A] time = 0.81, size = 150, normalized size = 1.24

$$\frac{5\,\sqrt{5}\,x\log\left(\frac{6\,x^4-4\,x^3+2\,\sqrt{5}\,\sqrt{x^4-3\,x^2+4}\,\left(x^2-2\,x-2\right)-15\,x^2+8\,x+24}{4\,x^4+4\,x^3-15\,x^2-8\,x+16}\right)+16\,x\log\left(-\frac{x+\sqrt{x^4-3\,x^2+4}}{x^2-2}\right)+10\,x\log\left(-\frac{x^2-\sqrt{x^4-3\,x^2+4}-2}{x}\right)+4\,\sqrt{x^4-3\,x^2+4}}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)*(x^2-2*x-2)*(x^4-3*x^2+4)^(1/2)/x^2/(x^2-2)/(2*x^2+x-4),x
, algorithm="fricas")
```

```
[Out] 1/8*(5*sqrt(5)*x*log((6*x^4 - 4*x^3 + 2*sqrt(5)*sqrt(x^4 - 3*x^2 + 4)*(x^2
- 2*x - 2) - 15*x^2 + 8*x + 24)/(4*x^4 + 4*x^3 - 15*x^2 - 8*x + 16)) + 16*x
*log(-(x + sqrt(x^4 - 3*x^2 + 4))/(x^2 - 2)) + 10*x*log(-(x^2 - sqrt(x^4 -
3*x^2 + 4) - 2)/x) + 4*sqrt(x^4 - 3*x^2 + 4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4-3\,x^2+4}\left(x^2-2\,x-2\right)\left(x^2+2\right)}{\left(2\,x^2+x-4\right)\left(x^2-2\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)*(x^2-2*x-2)*(x^4-3*x^2+4)^(1/2)/x^2/(x^2-2)/(2*x^2+x-4),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 - 3*x^2 + 4)*(x^2 - 2*x - 2)*(x^2 + 2)/((2*x^2 + x - 4)*
(x^2 - 2)*x^2), x)
```

maple [C] time = 0.66, size = 1643, normalized size = 13.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)*(x^2-2*x-2)*(x^4-3*x^2+4)^(1/2)/x^2/(x^2-2)/(2*x^2+x-4),x)
```

```
[Out] -3/2/(6+2*I*7^(1/2))^(1/2)*(1-3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1-3/8*x^2+1
/8*I*x^2*7^(1/2))^(1/2)/(x^4-3*x^2+4)^(1/2)*EllipticF(1/4*x*(6+2*I*7^(1/2))
^(1/2),1/4*(2-6*I*7^(1/2))^(1/2))-35/32*ln(2*x^2-3+2*(x^4-3*x^2+4)^(1/2))-3
2/(6+2*I*7^(1/2))^(1/2)*(1-3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1-3/8*x^2+1/8*
I*x^2*7^(1/2))^(1/2)/(x^4-3*x^2+4)^(1/2)/(I*7^(1/2)-3)*EllipticF(1/4*x*(6+2
*I*7^(1/2))^(1/2),1/4*(2-6*I*7^(1/2))^(1/2))+32/(6+2*I*7^(1/2))^(1/2)*(1-3/
8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1-3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4-3*x
^2+4)^(1/2)/(I*7^(1/2)-3)*EllipticE(1/4*x*(6+2*I*7^(1/2))^(1/2),1/4*(2-6*I*
7^(1/2))^(1/2))+25/64*33^(1/2)/(85/32-5/32*33^(1/2))^(1/2)*arctanh(-5/8/(85
/32-5/32*33^(1/2))^(1/2)/(x^4-3*x^2+4)^(1/2)*x^2+1/8/(85/32-5/32*33^(1/2))^
(1/2)/(x^4-3*x^2+4)^(1/2)*x^2*33^(1/2)-13/16/(85/32-5/32*33^(1/2))^(1/2)/(x
^4-3*x^2+4)^(1/2)-3/16/(85/32-5/32*33^(1/2))^(1/2)/(x^4-3*x^2+4)^(1/2)*33^(
1/2))-25/32*33^(1/2)/(3/8+1/8*I*7^(1/2))^(1/2)/(-1/4+1/4*33^(1/2))*(1-3/8*x
^2-1/8*I*x^2*7^(1/2))^(1/2)*(1-3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4-3*x^2+
4)^(1/2)*EllipticPi((3/8+1/8*I*7^(1/2))^(1/2)*x,1/(3/8+1/8*I*7^(1/2))/(-1/4
+1/4*33^(1/2))^2,(3/8-1/8*I*7^(1/2))^(1/2)/(3/8+1/8*I*7^(1/2))^(1/2))-25/64
/(85/32-5/32*33^(1/2))^(1/2)*arctanh(-5/8/(85/32-5/32*33^(1/2))^(1/2)/(x^4-
3*x^2+4)^(1/2)*x^2+1/8/(85/32-5/32*33^(1/2))^(1/2)/(x^4-3*x^2+4)^(1/2)*x^2*
33^(1/2)-13/16/(85/32-5/32*33^(1/2))^(1/2)/(x^4-3*x^2+4)^(1/2)-3/16/(85/32-
5/32*33^(1/2))^(1/2)/(x^4-3*x^2+4)^(1/2)*33^(1/2))+25/32/(3/8+1/8*I*7^(1/2)
)^(1/2)/(-1/4+1/4*33^(1/2))*(1-3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1-3/8*x^2+
1/8*I*x^2*7^(1/2))^(1/2)/(x^4-3*x^2+4)^(1/2)*EllipticPi((3/8+1/8*I*7^(1/2))
^(1/2)*x,1/(3/8+1/8*I*7^(1/2))/(-1/4+1/4*33^(1/2))^2,(3/8-1/8*I*7^(1/2))^(1
/2)/(3/8+1/8*I*7^(1/2))^(1/2))+25/64*33^(1/2)/(85/32+5/32*33^(1/2))^(1/2)*a
rctanh(5/8/(85/32+5/32*33^(1/2))^(1/2)/(x^4-3*x^2+4)^(1/2)*x^2+1/8/(85/32+5
/32*33^(1/2))^(1/2)/(x^4-3*x^2+4)^(1/2)*x^2*33^(1/2)+13/16/(85/32+5/32*33^(
1/2))^(1/2)/(x^4-3*x^2+4)^(1/2)-3/16/(85/32+5/32*33^(1/2))^(1/2)/(x^4-3*x^2
+4)^(1/2)*33^(1/2))+25/32*33^(1/2)/(3/8+1/8*I*7^(1/2))^(1/2)/(-1/4-1/4*33^(
1/2))*(1-3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1-3/8*x^2+1/8*I*x^2*7^(1/2))^(1/
2)/(x^4-3*x^2+4)^(1/2)*EllipticPi((3/8+1/8*I*7^(1/2))^(1/2)*x,1/(3/8+1/8*I*
7^(1/2))/(-1/4-1/4*33^(1/2))^2,(3/8-1/8*I*7^(1/2))^(1/2)/(3/8+1/8*I*7^(1/2)
)^(1/2))+25/64/(85/32+5/32*33^(1/2))^(1/2)*arctanh(5/8/(85/32+5/32*33^(1/2)
)^(1/2)/(x^4-3*x^2+4)^(1/2)*x^2+1/8/(85/32+5/32*33^(1/2))^(1/2)/(x^4-3*x^2+
4)^(1/2)*x^2*33^(1/2)+13/16/(85/32+5/32*33^(1/2))^(1/2)/(x^4-3*x^2+4)^(1/2)
-3/16/(85/32+5/32*33^(1/2))^(1/2)/(x^4-3*x^2+4)^(1/2)*33^(1/2))+25/32/(3/8+
1/8*I*7^(1/2))^(1/2)/(-1/4-1/4*33^(1/2))*(1-3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2
)*(1-3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4-3*x^2+4)^(1/2)*EllipticPi((3/8+1
/8*I*7^(1/2))^(1/2)*x,1/(3/8+1/8*I*7^(1/2))/(-1/4-1/4*33^(1/2))^2,(3/8-1/8*
I*7^(1/2))^(1/2)/(3/8+1/8*I*7^(1/2))^(1/2))+4/(3/8+1/8*I*7^(1/2))^(1/2)*(1-
3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1-3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4-3
*x^2+4)^(1/2)*EllipticPi((3/8+1/8*I*7^(1/2))^(1/2)*x,1/2/(3/8+1/8*I*7^(1/2)
),(3/8-1/8*I*7^(1/2))^(1/2)/(3/8+1/8*I*7^(1/2))^(1/2))+1/2*(x^4-3*x^2+4)^(1
/2)/x+6/(6+2*I*7^(1/2))^(1/2)*(1-(3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(3/8-1/8
*I*7^(1/2))*x^2)^(1/2)/(x^4-3*x^2+4)^(1/2)*EllipticF(1/4*x*(6+2*I*7^(1/2))^
(1/2),1/4*(2-6*I*7^(1/2))^(1/2))+32/(6+2*I*7^(1/2))^(1/2)*(1-(3/8+1/8*I*7^(
1/2))*x^2)^(1/2)*(1-(3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4-3*x^2+4)^(1/2)/(I*7
^(1/2)-3)*(EllipticF(1/4*x*(6+2*I*7^(1/2))^(1/2),1/4*(2-6*I*7^(1/2))^(1/2))
-EllipticE(1/4*x*(6+2*I*7^(1/2))^(1/2),1/4*(2-6*I*7^(1/2))^(1/2)))+15/32*ar
csinh(2/7*7^(1/2)*(x^2-3/2))+5/8*arctanh(1/4*(-3*x^2+8)/(x^4-3*x^2+4)^(1/2)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4-3\,x^2+4}\,\left(x^2-2\,x-2\right)\left(x^2+2\right)}{\left(2\,x^2+x-4\right)\left(x^2-2\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)*(x^2-2*x-2)*(x^4-3*x^2+4)^(1/2)/x^2/(x^2-2)/(2*x^2+x-4),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 - 3*x^2 + 4)*(x^2 - 2*x - 2)*(x^2 + 2)/((2*x^2 + x - 4)*
(x^2 - 2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^2+2\right)\left(-x^2+2\,x+2\right)\sqrt{x^4-3\,x^2+4}}{x^2\left(x^2-2\right)\left(2\,x^2+x-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^2 + 2)*(2*x - x^2 + 2)*(x^4 - 3*x^2 + 4)^(1/2))/(x^2*(x^2 - 2)*(x
+ 2*x^2 - 4)),x)
```

```
[Out] int(-((x^2 + 2)*(2*x - x^2 + 2)*(x^4 - 3*x^2 + 4)^(1/2))/(x^2*(x^2 - 2)*(x
+ 2*x^2 - 4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+2\right)\left(x^2-2x-2\right)\sqrt{x^4-3x^2+4}}{x^2\left(x^2-2\right)\left(2x^2+x-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)*(x**2-2*x-2)*(x**4-3*x**2+4)**(1/2)/x**2/(x**2-2)/(2*x**
2+x-4),x)
```

```
[Out] Integral((x**2 + 2)*(x**2 - 2*x - 2)*sqrt(x**4 - 3*x**2 + 4)/(x**2*(x**2 -
2)*(2*x**2 + x - 4)), x)
```

3.1503 $\int \frac{1+x}{(-1+x)\sqrt[3]{x^2+x^4}}\,dx$

Optimal. Leaf size=121

$$\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}-2x\right)}{\sqrt[3]{2}}-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^4+x^2}\,x+\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}+x}\right)}{\sqrt[3]{2}}$$

Rubi [C] time = 0.65, antiderivative size = 127, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2056, 6733, 6725, 245, 1438, 429, 465, 510}

$$-\frac{3\sqrt[3]{x^2+1}\,x^2F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};x^2,-x^2\right)}{2\sqrt[3]{x^4+x^2}}-\frac{6\sqrt[3]{x^2+1}\,xF_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};x^2,-x^2\right)}{\sqrt[3]{x^4+x^2}}+\frac{3\sqrt[3]{x^2+1}\,x\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^4+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x)/((-1 + x)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] (-6*x*(1 + x^2)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6, x^2, -x^2])/(x^2 + x^4)^(1
/3) - (3*x^2*(1 + x^2)^(1/3)*AppellF1[2/3, 1, 1/3, 5/3, x^2, -x^2])/(2*(x^2
 + x^4)^(1/3)) + (3*x*(1 + x^2)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -x^2
])/(x^2 + x^4)^(1/3)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
```

```
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{1+x}{(-1+x)\sqrt[3]{x^2+x^4}}\,dx = \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{1+x}{(-1+x)x^{2/3}\sqrt[3]{1+x^2}}\,dx}{\sqrt[3]{x^2+x^4}}$$
$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1+x^3}{\left(-1+x^3\right)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}}$$
$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1+x^6}}+\frac{2}{\left(-1+x^3\right)\sqrt[3]{1+x^6}}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}}$$
$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(6x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\left(-1+x^3\right)\sqrt[3]{1+}}\right.}{\sqrt[3]{x^2+x^4}}$$
$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(6x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int\left(\frac{1}{\left(-1+x^6\right)\sqrt[3]{1+x^6}}+\frac{x^3}{\left(-1+x^6\right)\sqrt[3]{}}\right.\right.}{\sqrt[3]{x^2+x^4}}$$
$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(6x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\left(-1+x^6\right)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}}$$
$$= -\frac{6x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};x^2,-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(3x^{2/3}\sqrt[3]{1+x}\right.}{}$$
$$= -\frac{6x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};x^2,-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{3x^2\sqrt[3]{1+x^2}\,F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};x^2,-x^2\right)}{2\sqrt[3]{x^2+x^4}} + \frac{3x\sqrt[3]{1+}}{}$$

Mathematica [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1+x}{(-1+x)\sqrt[3]{x^2+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x)/((-1 + x)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] Integrate[(1 + x)/((-1 + x)*(x^2 + x^4)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.40, size = 121, normalized size = 1.00

$$\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}-2x\right)}{\sqrt[3]{2}}-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^4+x^2}\,x+\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}+x}\right)}{\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((-1 + x)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(x^2 + x^4)^(1/3))])/2^(1/3)) +
Log[-2*x + 2^(2/3)*(x^2 + x^4)^(1/3)]/2^(1/3) - Log[2*x^2 + 2^(2/3)*x*(x^2
+ x^4)^(1/3) + 2^(1/3)*(x^2 + x^4)^(2/3)]/(2*2^(1/3))
```

fricas [B] time = 5.34, size = 307, normalized size = 2.54

$$-\frac{1}{6}\cdot 4^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{3\cdot 4^{\frac{2}{3}}\sqrt{3}\left(x^4+2x^3-6x^2+2x+1\right)\left(x^4+x^2\right)^{\frac{2}{3}}+6\cdot 4^{\frac{1}{3}}\sqrt{3}\left(x^5+14x^4+6x^3+14x^2+x\right)\left(x^4+x^2\right)^{\frac{1}{3}}+\sqrt{3}\left(x^7+30x^6+51x^5+52x^4+51x^3+30x^2+x\right)}{3\left(x^7-6x^6-93x^5-20x^4-93x^3-6x^2+x\right)}\right)-\frac{1}{12}\cdot 4^{\frac{1}{3}}\log\left(\frac{6\cdot 4^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{2}{3}}\left(x^2+4x+1\right)+4^{\frac{2}{3}}\left(x^5+14x^4+6x^3+14x^2+x\right)+24\left(x^4+x^2\right)^{\frac{1}{3}}\left(x^3+x^2+x\right)}{x^5-4x^4+6x^3-4x^2+x}\right)+\frac{1}{6}\cdot 4^{\frac{1}{3}}\log\left(-\frac{3\cdot 4^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}x+4^{\frac{1}{3}}\left(x^3-2x^2+x\right)-6\left(x^4+x^2\right)^{\frac{2}{3}}}{x^3-2x^2+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^4+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/3)*sqrt(3)*arctan(1/3*(3*4^(2/3)*sqrt(3)*(x^4 + 2*x^3 - 6*x^2 + 2
*x + 1)*(x^4 + x^2)^(2/3) + 6*4^(1/3)*sqrt(3)*(x^5 + 14*x^4 + 6*x^3 + 14*x^
2 + x)*(x^4 + x^2)^(1/3) + sqrt(3)*(x^7 + 30*x^6 + 51*x^5 + 52*x^4 + 51*x^3
 + 30*x^2 + x))/(x^7 - 6*x^6 - 93*x^5 - 20*x^4 - 93*x^3 - 6*x^2 + x)) - 1/1
2*4^(1/3)*log((6*4^(1/3)*(x^4 + x^2)^(2/3)*(x^2 + 4*x + 1) + 4^(2/3)*(x^5 +
 14*x^4 + 6*x^3 + 14*x^2 + x) + 24*(x^4 + x^2)^(1/3)*(x^3 + x^2 + x))/(x^5
- 4*x^4 + 6*x^3 - 4*x^2 + x)) + 1/6*4^(1/3)*log(-(3*4^(2/3)*(x^4 + x^2)^(1/
3)*x + 4^(1/3)*(x^3 - 2*x^2 + x) - 6*(x^4 + x^2)^(2/3))/(x^3 - 2*x^2 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(x^4+x^2\right)^{\frac{1}{3}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^4+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/((x^4 + x^2)^(1/3)*(x - 1)), x)
```

maple [C] time = 9.50, size = 971, normalized size = 8.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-1+x)/(x^4+x^2)^(1/3),x)
```

```
[Out] 1/2*RootOf(_Z^3-4)*ln((11000*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+
_Z^2)*RootOf(_Z^3-4)^3*x^3+1334*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-
4)+_Z^2)^2*RootOf(_Z^3-4)^2*x^3-27500*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf
(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x^2-3335*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*Roo
tOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x^2+11000*RootOf(4*RootOf(_Z^3-4)^2+2*
_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x+1334*RootOf(4*RootOf(_Z^3-4)^2+2
*_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x-52824*(x^4+x^2)^(2/3)*RootOf(
4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2+211296*(x^4+x
```

```
^2)^(1/3)*RootOf(_Z^3-4)^2*x+151332*(x^4+x^2)^(1/3)*RootOf(4*RootOf(_Z^3-4)
^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)*x-110000*RootOf(_Z^3-4)*x^3-133
40*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*x^3-396000*RootOf(_Z
^3-4)*x^2-48024*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*x^2-110
000*RootOf(_Z^3-4)*x-13340*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z
^2)*x+182736*(x^4+x^2)^(2/3))/(-1+x)^2/x)+1/4*RootOf(4*RootOf(_Z^3-4)^2+2*_
Z*RootOf(_Z^3-4)+_Z^2)*ln(-(1334*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3
-4)+_Z^2)*RootOf(_Z^3-4)^3*x^3+2750*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_
Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x^3-3335*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*Roo
tOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x^2-6875*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*
RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x^2+1334*RootOf(4*RootOf(_Z^3-4)^2+
2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x+2750*RootOf(4*RootOf(_Z^3-4)^2
+2*_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x-26412*(x^4+x^2)^(2/3)*RootO
f(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2+105648*(x^4
+x^2)^(1/3)*RootOf(_Z^3-4)^2*x-22842*(x^4+x^2)^(1/3)*RootOf(4*RootOf(_Z^3-4
)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)*x+24012*RootOf(_Z^3-4)*x^3+495
00*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*x^3+21344*RootOf(_Z^
3-4)*x^2+44000*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*x^2+2401
2*RootOf(_Z^3-4)*x+49500*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2
)*x-302664*(x^4+x^2)^(2/3))/(-1+x)^2/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(x^4+x^2\right)^{\frac{1}{3}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x^4+x^2)^(1/3),x, algorithm="maxima")
[Out] integrate((x + 1)/((x^4 + x^2)^(1/3)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+1}{\left(x^4+x^2\right)^{1/3}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^2 + x^4)^(1/3)*(x - 1)),x)
[Out] int((x + 1)/((x^2 + x^4)^(1/3)*(x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt[3]{x^2\left(x^2+1\right)}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x**4+x**2)**(1/3),x)
[Out] Integral((x + 1)/((x**2*(x**2 + 1))**(1/3)*(x - 1)), x)
```

3.1504 $\int \frac{-b^2+a^4x^4}{\sqrt{bx+a^2x^3}\left(b^2+a^4x^4\right)}\,dx$

Optimal. Leaf size=121

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{a}\sqrt[4]{b}\sqrt{a^2x^3+bx}}{a^2x^2+b}\right)}{\sqrt[4]{2}\sqrt{a}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{a}\sqrt[4]{b}\sqrt{a^2x^3+bx}}{a^2x^2+b}\right)}{\sqrt[4]{2}\sqrt{a}\sqrt[4]{b}}$$

Rubi [C] time = 6.23, antiderivative size = 1906, normalized size of antiderivative = 15.75, number of steps used = 23, number of rules used = 9, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2056, 1586, 6715, 6725, 406, 220, 409, 1217, 1707}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(-b^2 + a^4*x^4)/(Sqrt[b*x + a^2*x^3]*(b^2 + a^4*x^4)),x]
```

```
[Out] ((a^2 - Sqrt[-a^4])^(3/2)*Sqrt[x]*Sqrt[b + a^2*x^2]*ArcTan[(Sqrt[a^2 - Sqrt
[-a^4]]*(-b)^(1/4)*Sqrt[x])/((-a^4)^(1/8)*Sqrt[b + a^2*x^2])]])/(4*a^2*(-a^4
)^(3/8)*(-b)^(1/4)*Sqrt[b*x + a^2*x^3]) - (a^2*(-a^2 + Sqrt[-a^4])^(3/2)*Sq
rt[x]*Sqrt[b + a^2*x^2]*ArcTan[(Sqrt[-a^2 + Sqrt[-a^4]]*(-b)^(1/4)*Sqrt[x])
/((-a^4)^(1/8)*Sqrt[b + a^2*x^2])]])/(4*(-a^4)^(11/8)*(-b)^(1/4)*Sqrt[b*x +
a^2*x^3]) - (a^3*(-a^2 + Sqrt[-a^4])^(3/2)*Sqrt[x]*Sqrt[b + a^2*x^2]*ArcTan
[((-a^4)^(1/8)*Sqrt[-a^2 + Sqrt[-a^4]]*b^(1/4)*Sqrt[x])/(a*Sqrt[b + a^2*x^2
])])/(4*(-a^4)^(13/8)*b^(1/4)*Sqrt[b*x + a^2*x^3]) - ((a^2 + Sqrt[-a^4])^(3
/2)*Sqrt[x]*Sqrt[b + a^2*x^2]*ArcTan[(Sqrt[a^2 + Sqrt[-a^4]]*b^(1/4)*Sqrt[x
])/((-a^4)^(1/8)*Sqrt[b + a^2*x^2])]])/(4*a^2*(-a^4)^(3/8)*b^(1/4)*Sqrt[b*x
+ a^2*x^3]) - ((a^2 - Sqrt[-a^4])*((-a^4)^(1/4)*Sqrt[-b] + a*Sqrt[b])*Sqrt[
x]*(Sqrt[b] + a*x)*Sqrt[(b + a^2*x^2)/(Sqrt[b] + a*x)^2]*EllipticF[2*ArcTan
[(Sqrt[a]*Sqrt[x])/b^(1/4)], 1/2])/(4*a^(7/2)*b^(3/4)*Sqrt[b*x + a^2*x^3])
+ ((a^2 - Sqrt[-a^4])*Sqrt[x]*(Sqrt[b] + a*x)*Sqrt[(b + a^2*x^2)/(Sqrt[b] +
 a*x)^2]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt[x])/b^(1/4)], 1/2])/(2*a^(5/2)*b^
(1/4)*Sqrt[b*x + a^2*x^3]) + ((a^2 + Sqrt[-a^4])*Sqrt[x]*(Sqrt[b] + a*x)*Sq
rt[(b + a^2*x^2)/(Sqrt[b] + a*x)^2]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt[x])/b^
(1/4)], 1/2])/(2*a^(5/2)*b^(1/4)*Sqrt[b*x + a^2*x^3]) - ((a - (-a^4)^(1/4))
*(a^2 + Sqrt[-a^4])*Sqrt[x]*(Sqrt[b] + a*x)*Sqrt[(b + a^2*x^2)/(Sqrt[b] + a
*x)^2]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt[x])/b^(1/4)], 1/2])/(4*a^(7/2)*b^(1
/4)*Sqrt[b*x + a^2*x^3]) - ((a + (-a^4)^(1/4))*(a^2 + Sqrt[-a^4])*Sqrt[x]*(
Sqrt[b] + a*x)*Sqrt[(b + a^2*x^2)/(Sqrt[b] + a*x)^2]*EllipticF[2*ArcTan[(Sq
rt[a]*Sqrt[x])/b^(1/4)], 1/2])/(4*a^(7/2)*b^(1/4)*Sqrt[b*x + a^2*x^3]) - ((
a^2 - Sqrt[-a^4])*(a + ((-a^4)^(1/4)*Sqrt[b])/Sqrt[-b])*Sqrt[x]*(Sqrt[b] +
a*x)*Sqrt[(b + a^2*x^2)/(Sqrt[b] + a*x)^2]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt
[x])/b^(1/4)], 1/2])/(4*a^(7/2)*b^(1/4)*Sqrt[b*x + a^2*x^3]) + ((a + (-a^4)
^(1/4))^2*(a^2 + Sqrt[-a^4])*Sqrt[x]*(Sqrt[b] + a*x)*Sqrt[(b + a^2*x^2)/(Sq
rt[b] + a*x)^2]*EllipticPi[(a^3*(a - (-a^4)^(1/4))^2)/(4*(-a^4)^(5/4)), 2*A
rcTan[(Sqrt[a]*Sqrt[x])/b^(1/4)], 1/2])/(8*a^(9/2)*b^(1/4)*Sqrt[b*x + a^2*x
^3]) + ((a - (-a^4)^(1/4))^2*(a^2 + Sqrt[-a^4])*Sqrt[x]*(Sqrt[b] + a*x)*Sqr
t[(b + a^2*x^2)/(Sqrt[b] + a*x)^2]*EllipticPi[(a + (-a^4)^(1/4))^2/(4*a*(-a
^4)^(1/4)), 2*ArcTan[(Sqrt[a]*Sqrt[x])/b^(1/4)], 1/2])/(8*a^(9/2)*b^(1/4)*S
qrt[b*x + a^2*x^3]) + ((a^2 - Sqrt[-a^4])*(a^2*b - Sqrt[-a^4]*b - 2*a*(-a^4
)^(1/4)*Sqrt[-b^2])*Sqrt[x]*(Sqrt[b] + a*x)*Sqrt[(b + a^2*x^2)/(Sqrt[b] + a
*x)^2]*EllipticPi[(a^3*(a*Sqrt[-b] - (-a^4)^(1/4)*Sqrt[b])^2)/(4*(-a^4)^(5/
4)*Sqrt[-b^2]), 2*ArcTan[(Sqrt[a]*Sqrt[x])/b^(1/4)], 1/2])/(8*a^(9/2)*b^(5/
4)*Sqrt[b*x + a^2*x^3]) + ((a^2 - Sqrt[-a^4])*(a^2*b - Sqrt[-a^4]*b + 2*a*(
-a^4)^(1/4)*Sqrt[-b^2])*Sqrt[x]*(Sqrt[b] + a*x)*Sqrt[(b + a^2*x^2)/(Sqrt[b]
 + a*x)^2]*EllipticPi[(a*Sqrt[-b] + (-a^4)^(1/4)*Sqrt[b])^2/(4*a*(-a^4)^(1/
4)*Sqrt[-b^2]), 2*ArcTan[(Sqrt[a]*Sqrt[x])/b^(1/4)], 1/2])/(8*a^(9/2)*b^(5/
4)*Sqrt[b*x + a^2*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

```
[n, 0]
```

Rubi steps

$$\int \frac{-b^2+a^4x^4}{\sqrt{bx+a^2x^3}\left(b^2+a^4x^4\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{b+a^2x^2}\right)\int \frac{-b^2+a^4x^4}{\sqrt{x}\sqrt{b+a^2x^2}\left(b^2+a^4x^4\right)}\,dx}{\sqrt{bx+a^2x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{b+a^2x^2}\right)\int \frac{\left(-b+a^2x^2\right)\sqrt{b+a^2x^2}}{\sqrt{x}\left(b^2+a^4x^4\right)}\,dx}{\sqrt{bx+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b+a^2x^2}\right)\mathrm{Subst}\left(\int \frac{\left(-b+a^2x^4\right)\sqrt{b+a^2x^4}}{b^2+a^4x^8}\,dx, x, \sqrt{x}\right)}{\sqrt{bx+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b+a^2x^2}\right)\mathrm{Subst}\left(\int\left(-\frac{\sqrt{-a^4}\left(a^2b-\sqrt{-a^4}\,b\right)\sqrt{b+a^2x^4}}{2a^4b\left(b-\sqrt{-a^4}\,x^4\right)}+\frac{\sqrt{-a^4}\left(a^2b+\sqrt{-a^4}\,b\right)\sqrt{b+a^2x^4}}{2a^4b\left(b+\sqrt{-a^4}\,x^4\right)}\cdots\right.\right.}{\sqrt{bx+a^2x^3}}$$

$$= -\frac{\left(\left(a^2+\sqrt{-a^4}\right)\sqrt{x}\sqrt{b+a^2x^2}\right)\mathrm{Subst}\left(\int \frac{\sqrt{b+a^2x^4}}{b-\sqrt{-a^4}\,x^4}\,dx, x, \sqrt{x}\right)}{a^2\sqrt{bx+a^2x^3}} + \frac{\left(\sqrt{-a^4}\left(a^2b+\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(\left(a^2+\sqrt{-a^4}\right)\sqrt{x}\sqrt{b+a^2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+a^2x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^4}\sqrt{bx+a^2x^3}} - \frac{\left(\left(a^2+\sqrt{-a^4}\right)\left(a^2\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(a^2+\sqrt{-a^4}\right)\sqrt{x}\left(\sqrt{b}+ax\right)\sqrt{\frac{b+a^2x^2}{\left(\sqrt{b}+ax\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{5/2}\sqrt[4]{b}\sqrt{bx+a^2x^3}} + \frac{\left(a^2+\sqrt{-a^4}\right)\cdots}{\cdots}$$

$$= \frac{\left(a^2+\sqrt{-a^4}\right)\sqrt{x}\left(\sqrt{b}+ax\right)\sqrt{\frac{b+a^2x^2}{\left(\sqrt{b}+ax\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{5/2}\sqrt[4]{b}\sqrt{bx+a^2x^3}} + \frac{\left(a^2+\sqrt{-a^4}\right)\cdots}{\cdots}$$

$$= -\frac{\sqrt[8]{-a^4}\sqrt{x}\sqrt{b+a^2x^2}\,\tan^{-1}\left(\frac{\sqrt{a^2-\sqrt{-a^4}}\,\sqrt[4]{-b}\,\sqrt{x}}{\sqrt[8]{-a^4}\sqrt{b+a^2x^2}}\right)}{2\sqrt{a^2-\sqrt{-a^4}}\,\sqrt[4]{-b}\sqrt{bx+a^2x^3}} + \frac{\sqrt[8]{-a^4}\,\sqrt[4]{-b}\sqrt{-b^2}\sqrt{x}\sqrt{b+a^2x^2}\cdots}{2\sqrt{-a^2+\sqrt{-a^4}}\cdots}$$

Mathematica [C] time = 1.29, size = 234, normalized size = 1.93

$$-\frac{ix^{3/2}\sqrt{\frac{b}{a^2x^2}+1}\left(2F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(-\sqrt[4]{-1};i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(\sqrt[4]{-1};i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(-(-1)^{3/4};i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left((-1)^{3/4};i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{a}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{\sqrt{\frac{i\sqrt{b}}{a}}\sqrt{x\left(a^2x^2+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b^2 + a^4*x^4)/(Sqrt[b*x + a^2*x^3]*(b^2 + a^4*x^4)),x]
```

```
[Out] ((-I)*Sqrt[1 + b/(a^2*x^2)]*x^(3/2)*(2*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])
/a]/Sqrt[x]], -1] - EllipticPi[-(-1)^(1/4), I*ArcSinh[Sqrt[(I*Sqrt[b])/a]/S
qrt[x]], -1] - EllipticPi[(-1)^(1/4), I*ArcSinh[Sqrt[(I*Sqrt[b])/a]/Sqrt[x]
```

```
], -1] - EllipticPi[-(-1)^(3/4), I*ArcSinh[Sqrt[(I*Sqrt[b])/a]/Sqrt[x]], -1
] - EllipticPi[(-1)^(3/4), I*ArcSinh[Sqrt[(I*Sqrt[b])/a]/Sqrt[x]], -1]))/(S
qrt[(I*Sqrt[b])/a]*Sqrt[x*(b + a^2*x^2)])
```

IntegrateAlgebraic [A] time = 0.40, size = 121, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{a}\sqrt[4]{b}\sqrt{a^2x^3+bx}}{a^2x^2+b}\right)}{\sqrt[4]{2}\sqrt{a}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{a}\sqrt[4]{b}\sqrt{a^2x^3+bx}}{a^2x^2+b}\right)}{\sqrt[4]{2}\sqrt{a}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b^2 + a^4*x^4)/(Sqrt[b*x + a^2*x^3]*(b^2 + a^4*x^4)),x
]
```

```
[Out] -(ArcTan[(2^(1/4)*Sqrt[a]*b^(1/4)*Sqrt[b*x + a^2*x^3])/(b + a^2*x^2)]/(2^(1
/4)*Sqrt[a]*b^(1/4))) - ArcTanh[(2^(1/4)*Sqrt[a]*b^(1/4)*Sqrt[b*x + a^2*x^3
])/(b + a^2*x^2)]/(2^(1/4)*Sqrt[a]*b^(1/4))
```

fricas [B] time = 0.94, size = 344, normalized size = 2.84

$$-\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{a^2b}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{a^2x^3+bx}\,a^2b\left(\frac{1}{a^2b}\right)^{\frac{3}{4}}}{a^2x^2+b}\right)-\frac{1}{4}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{a^2b}\right)^{\frac{1}{4}}\log\left(\frac{a^4x^4+4a^2bx^2+b^2+4\sqrt{\frac{1}{2}}\left(a^4bx^3+a^2b^2x\right)\sqrt{\frac{1}{a^2b}}+4\sqrt{a^2x^3+bx}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}a^2bx\left(\frac{1}{a^2b}\right)^{\frac{1}{4}}+\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(a^4bx^2+a^2b^2\right)\left(\frac{1}{a^2b}\right)^{\frac{3}{4}}\right)}{a^4x^4+b^2}\right)+\frac{1}{4}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(\frac{1}{a^2b}\right)^{\frac{1}{4}}\log\left(\frac{a^4x^4+4a^2bx^2+b^2+4\sqrt{\frac{1}{2}}\left(a^4bx^3+a^2b^2x\right)\sqrt{\frac{1}{a^2b}}-4\sqrt{a^2x^3+bx}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}a^2bx\left(\frac{1}{a^2b}\right)^{\frac{1}{4}}+\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(a^4bx^2+a^2b^2\right)\left(\frac{1}{a^2b}\right)^{\frac{3}{4}}\right)}{a^4x^4+b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^2)/(a^2*x^3+b*x)^(1/2)/(a^4*x^4+b^2),x, algorithm="fri
cas")
```

```
[Out] -(1/2)^(1/4)*(1/(a^2*b))^(1/4)*arctan(2*(1/2)^(3/4)*sqrt(a^2*x^3 + b*x)*a^2
*b*(1/(a^2*b))^(3/4)/(a^2*x^2 + b)) - 1/4*(1/2)^(1/4)*(1/(a^2*b))^(1/4)*log
((a^4*x^4 + 4*a^2*b*x^2 + b^2 + 4*sqrt(1/2)*(a^4*b*x^3 + a^2*b^2*x)*sqrt(1/
(a^2*b)) + 4*sqrt(a^2*x^3 + b*x)*((1/2)^(1/4)*a^2*b*x*(1/(a^2*b))^(1/4) + (
1/2)^(3/4)*(a^4*b*x^2 + a^2*b^2)*(1/(a^2*b))^(3/4)))/(a^4*x^4 + b^2)) + 1/4
*(1/2)^(1/4)*(1/(a^2*b))^(1/4)*log((a^4*x^4 + 4*a^2*b*x^2 + b^2 + 4*sqrt(1/
2)*(a^4*b*x^3 + a^2*b^2*x)*sqrt(1/(a^2*b)) - 4*sqrt(a^2*x^3 + b*x)*((1/2)^(
1/4)*a^2*b*x*(1/(a^2*b))^(1/4) + (1/2)^(3/4)*(a^4*b*x^2 + a^2*b^2)*(1/(a^2*
b))^(3/4)))/(a^4*x^4 + b^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^4x^4-b^2}{\left(a^4x^4+b^2\right)\sqrt{a^2x^3+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^2)/(a^2*x^3+b*x)^(1/2)/(a^4*x^4+b^2),x, algorithm="gia
c")
```

```
[Out] integrate((a^4*x^4 - b^2)/((a^4*x^4 + b^2)*sqrt(a^2*x^3 + b*x)), x)
```

maple [C] time = 0.08, size = 303, normalized size = 2.50

$$\frac{\sqrt{-b}\sqrt{\frac{\left(x+\frac{\sqrt{-b}}{a}\right)a}{\sqrt{-b}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-b}}{a}\right)a}{\sqrt{-b}}}\sqrt{-\frac{xa}{\sqrt{-b}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-b}}{a}\right)a}{\sqrt{-b}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a^2x^3+bx}}-\frac{\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(a^4\_Z^4+b^2\right)}\frac{\sqrt{-b}\sqrt{\frac{\left(x+\frac{\sqrt{-b}}{a}\right)a}{\sqrt{-b}}}\sqrt{-\frac{\left(x-\frac{\sqrt{-b}}{a}\right)a}{\sqrt{-b}}}\sqrt{-\frac{xa}{\sqrt{-b}}}\left(a\left(a^2\_\alpha^3-b\_\alpha\right)-a^2\sqrt{-b}\,\_\alpha^2+b\sqrt{-b}\right)\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-b}}{a}\right)a}{\sqrt{-b}}},-\frac{\_\alpha^3\sqrt{-b}\,a^3+\_\alpha^2a^2b-\_\alpha\sqrt{-b}\,ab-b^2}{2b^2},\frac{\sqrt{2}}{2}\right)}{\_\alpha^3\sqrt{x\left(a^2x^2+b\right)}}\right)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^4*x^4-b^2)/(a^2*x^3+b*x)^(1/2)/(a^4*x^4+b^2),x)
```

```
[Out] (-b)^(1/2)/a*((x+(-b)^(1/2)/a)/(-b)^(1/2)*a)^(1/2)*(-2*(x-(-b)^(1/2)/a)/(-b
)^(1/2)*a)^(1/2)*(-x/(-b)^(1/2)*a)^(1/2)/(a^2*x^3+b*x)^(1/2)*EllipticF(((x+
```

```
(-b)^(1/2)/a)/(-b)^(1/2)*a)^(1/2),1/2*2^(1/2))-1/4/a^4*2^(1/2)*sum(1/_alpha
^3*(-b)^(1/2)*((x+(-b)^(1/2)/a)/(-b)^(1/2)*a)^(1/2)*(-(x-(-b)^(1/2)/a)/(-b)
^(1/2)*a)^(1/2)*(-x/(-b)^(1/2)*a)^(1/2)/(x*(a^2*x^2+b))^(1/2)*(a*(_alpha^3*
a^2-_alpha*b)-a^2*(-b)^(1/2)*_alpha^2+b*(-b)^(1/2))*EllipticPi(((x+(-b)^(1/
2)/a)/(-b)^(1/2)*a)^(1/2),-1/2*(_alpha^3*(-b)^(1/2)*a^3+_alpha^2*a^2*b-_alp
ha*(-b)^(1/2)*a*b-b^2)/b^2,1/2*2^(1/2)),_alpha=RootOf(_Z^4*a^4+b^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^4x^4 - b^2}{\left(a^4x^4 + b^2\right)\sqrt{a^2x^3 + bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^2)/(a^2*x^3+b*x)^(1/2)/(a^4*x^4+b^2),x, algorithm="max
ima")
```

```
[Out] integrate((a^4*x^4 - b^2)/((a^4*x^4 + b^2)*sqrt(a^2*x^3 + b*x)), x)
```

mupad [B] time = 5.63, size = 164, normalized size = 1.36

$$\frac{2^{3/4}\,\ln\left(\frac{2^{3/4}\,b+2^{3/4}\,a^2\,x^2-4\,\sqrt{a}\,b^{1/4}\,\sqrt{a^2\,x^3+b\,x}+2\,2^{1/4}\,a\,\sqrt{b}\,x}{b+a^2\,x^2-\sqrt{2}\,a\,\sqrt{b}\,x}\right)}{4\,\sqrt{a}\,b^{1/4}}+\frac{2^{3/4}\,\ln\left(\frac{2^{3/4}\,b+2^{3/4}\,a^2\,x^2-2\,2^{1/4}\,a\,\sqrt{b}\,x+\sqrt{a}\,b^{1/4}\,\sqrt{a^2\,x^3+b\,x}\,4{}\mathrm{i}}{b+a^2\,x^2+\sqrt{2}\,a\,\sqrt{b}\,x}\right)\,1{}\mathrm{i}}{4\,\sqrt{a}\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^2 - a^4*x^4)/((b^2 + a^4*x^4)*(b*x + a^2*x^3)^(1/2)),x)
```

```
[Out] (2^(3/4)*log((2^(3/4)*b + 2^(3/4)*a^2*x^2 - 4*a^(1/2)*b^(1/4)*(b*x + a^2*x^
3)^(1/2) + 2*2^(1/4)*a*b^(1/2)*x)/(b + a^2*x^2 - 2^(1/2)*a*b^(1/2)*x)))/(4*
a^(1/2)*b^(1/4)) + (2^(3/4)*log((2^(3/4)*b + 2^(3/4)*a^2*x^2 + a^(1/2)*b^(1
/4)*(b*x + a^2*x^3)^(1/2)*4i - 2*2^(1/4)*a*b^(1/2)*x)/(b + a^2*x^2 + 2^(1/2
)*a*b^(1/2)*x))*1i)/(4*a^(1/2)*b^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2 - b\right)\left(a^2x^2 + b\right)}{\sqrt{x\left(a^2x^2 + b\right)}\left(a^4x^4 + b^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**4*x**4-b**2)/(a**2*x**3+b*x)**(1/2)/(a**4*x**4+b**2),x)
```

```
[Out] Integral((a**2*x**2 - b)*(a**2*x**2 + b)/(sqrt(x*(a**2*x**2 + b))*(a**4*x**
4 + b**2)), x)
```

3.1505 $\int \frac{-1+x^2}{(1+x^2)\sqrt[3]{x+x^5}}\,dx$

Optimal. Leaf size=121

$$-\frac{\log\left(2^{2/3}\sqrt[3]{x^5+x}+2x\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^5+x}-x}\right)}{2\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{x^5+x}\,x-\sqrt[3]{2}\left(x^5+x\right)^{2/3}-2x^2\right)}{4\sqrt[3]{2}}$$

Rubi [C] time = 0.62, antiderivative size = 123, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2056, 6715, 6725, 245, 1438, 429, 465, 510}

$$-\frac{3\sqrt[3]{x^4+1}\,xF_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};x^4,-x^4\right)}{\sqrt[3]{x^5+x}}+\frac{3\sqrt[3]{x^4+1}\,x^3F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};x^4,-x^4\right)}{4\sqrt[3]{x^5+x}}+\frac{3\sqrt[3]{x^4+1}\,x\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x^5+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^2)/((1 + x^2)*(x + x^5)^(1/3)),x]
```

```
[Out] (-3*x*(1 + x^4)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6, x^4, -x^4])/(x + x^5)^(1/3
) + (3*x^3*(1 + x^4)^(1/3)*AppellF1[2/3, 1, 1/3, 5/3, x^4, -x^4])/(4*(x + x
^5)^(1/3)) + (3*x*(1 + x^4)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -x^4])/(
2*(x + x^5)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
```

```
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt[3]{x+x^5}}\,dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\int \frac{-1+x^2}{\sqrt[3]{x}\left(1+x^2\right)\sqrt[3]{1+x^4}}\,dx}{\sqrt[3]{x+x^5}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{-1+x^3}{\left(1+x^3\right)\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{x+x^5}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1+x^6}}-\frac{2}{\left(1+x^3\right)\sqrt[3]{1+x^6}}\right)dx, x, x^{2/3}\right)}{2\sqrt[3]{x+x^5}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{x+x^5}} - \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(1+x^3\right)\sqrt[3]{1+x^6}}\,dx\right.}{\sqrt[3]{x+x^5}}$$

$$= \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} - \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\left(1-x^6\right)\sqrt[3]{1+x^6}}+\frac{x^3}{\left(-1+x^6\right)\sqrt[3]{1+x^6}}\right)\right.}{\sqrt[3]{x+x^5}}$$

$$= \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} - \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(1-x^6\right)\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{\sqrt[3]{x+x^5}} - \frac{(3\cdots}{}$$

$$= -\frac{3x\sqrt[3]{1+x^4}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};x^4,-x^4\right)}{\sqrt[3]{x+x^5}} + \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} - \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{S}\cdots}{}$$

$$= -\frac{3x\sqrt[3]{1+x^4}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};x^4,-x^4\right)}{\sqrt[3]{x+x^5}} + \frac{3x^3\sqrt[3]{1+x^4}\,F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};x^4,-x^4\right)}{4\sqrt[3]{x+x^5}} + \frac{3x\sqrt[3]{1+x^4}\cdots}{2\sqrt[3]{\cdots}}$$

Mathematica [**F**] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt[3]{x+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*(x + x^5)^(1/3)),x]
```

```
[Out] Integrate[(-1 + x^2)/((1 + x^2)*(x + x^5)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.38, size = 121, normalized size = 1.00

$$-\frac{\log\left(2^{2/3}\sqrt[3]{x^5+x}+2x\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^5+x}-x}\right)}{2\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{x^5+x}\,x-\sqrt[3]{2}\left(x^5+x\right)^{2/3}-2x^2\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*(x + x^5)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(x + x^5)^(1/3))])/2^(1/3) -
 Log[2*x + 2^(2/3)*(x + x^5)^(1/3)]/(2*2^(1/3)) + Log[-2*x^2 + 2^(2/3)*x*(x
 + x^5)^(1/3) - 2^(1/3)*(x + x^5)^(2/3)]/(4*2^(1/3))
```

fricas [B] time = 4.41, size = 320, normalized size = 2.64

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(12\cdot 2^{\frac{1}{6}}(-1)^{\frac{2}{3}}\left(x^8-14x^6+6x^4-14x^2+1\right)\left(x^5+x\right)^{\frac{2}{3}}-24\sqrt{2}(-1)^{\frac{1}{3}}\left(x^9+x^7+x^3+x\right)\left(x^5+x\right)^{\frac{1}{3}}+2^{\frac{5}{6}}\left(x^{12}+24x^{10}-57x^8+56x^6-57x^4+24x^2+1\right)\right)}{6\left(x^{12}-48x^{10}+15x^8-88x^6+15x^4-48x^2+1\right)}\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(\frac{12\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^5-x^3+x\right)\left(x^5+x\right)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^8-14x^6+6x^4-14x^2+1\right)-6\left(x^5+x\right)^{\frac{2}{3}}\left(x^4-4x^2+1\right)}{x^8+4x^6+6x^4+4x^2+1}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^4+2x^2+1\right)-3\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^5+x\right)^{\frac{2}{3}}+6\left(x^5+x\right)^{\frac{1}{3}}x}{x^4+2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^5+x)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(12*2^(1/6)*(-1)
^(2/3)*(x^8 - 14*x^6 + 6*x^4 - 14*x^2 + 1)*(x^5 + x)^(2/3) - 24*sqrt(2)*(-1
)^(1/3)*(x^9 + x^7 + x^3 + x)*(x^5 + x)^(1/3) + 2^(5/6)*(x^12 + 24*x^10 - 5
7*x^8 + 56*x^6 - 57*x^4 + 24*x^2 + 1))/(x^12 - 48*x^10 + 15*x^8 - 88*x^6 +
15*x^4 - 48*x^2 + 1)) - 1/24*2^(2/3)*(-1)^(1/3)*log((12*2^(1/3)*(-1)^(2/3)*
(x^5 - x^3 + x)*(x^5 + x)^(1/3) - 2^(2/3)*(-1)^(1/3)*(x^8 - 14*x^6 + 6*x^4
- 14*x^2 + 1) - 6*(x^5 + x)^(2/3)*(x^4 - 4*x^2 + 1))/(x^8 + 4*x^6 + 6*x^4 +
 4*x^2 + 1)) + 1/12*2^(2/3)*(-1)^(1/3)*log(-(2^(1/3)*(-1)^(2/3)*(x^4 + 2*x^
2 + 1) - 3*2^(2/3)*(-1)^(1/3)*(x^5 + x)^(2/3) + 6*(x^5 + x)^(1/3)*x)/(x^4 +
 2*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^5+x\right)^{\frac{1}{3}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^5+x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/((x^5 + x)^(1/3)*(x^2 + 1)), x)
```

maple [C] time = 16.71, size = 1135, normalized size = 9.38

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^5+x)^(1/3),x)
```

```
[Out] -1/4*ln((2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^
3+4)^2*x^4-RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+
4)^3*x^4-4*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^
3+4)^2*x^2+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^
```

```
3+4)^3*x^2+6*(x^5+x)^(2/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z
^2)*RootOf(_Z^3+4)^2-4*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*
x^4+2*RootOf(_Z^3+4)*x^4+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z
^2)^2*RootOf(_Z^3+4)^2-RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*
RootOf(_Z^3+4)^3+6*(x^5+x)^(1/3)*RootOf(_Z^3+4)^2*x-4*RootOf(RootOf(_Z^3+4)
^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)+2*RootOf(_Z^3+4))/(x^2+1)^2)*RootOf(_Z^3+4)-
1/2*ln((2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3
+4)^2*x^4-RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4
)^3*x^4-4*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3
+4)^2*x^2+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3
+4)^3*x^2+6*(x^5+x)^(2/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^
2)*RootOf(_Z^3+4)^2-4*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x
^4+2*RootOf(_Z^3+4)*x^4+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^
2)^2*RootOf(_Z^3+4)^2-RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*R
ootOf(_Z^3+4)^3+6*(x^5+x)^(1/3)*RootOf(_Z^3+4)^2*x-4*RootOf(RootOf(_Z^3+4)^
2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)+2*RootOf(_Z^3+4))/(x^2+1)^2)*RootOf(RootOf(_Z
^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)+1/4*RootOf(_Z^3+4)*ln(-(4*RootOf(RootOf
(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x^4+RootOf(RootOf
(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x^4-8*RootOf(RootOf
(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x^2-2*RootOf(Root
Of(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x^2+6*(x^5+x)^(2/
3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2+4*R
ootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2+RootO
f(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3-12*(x^5+x)^
(1/3)*RootOf(_Z^3+4)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+
16*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2+4*RootOf(_Z^3+4)
*x^2-12*(x^5+x)^(2/3))/(x^2+1)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^5+x\right)^{\frac{1}{3}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^5+x)^(1/3),x, algorithm="maxima")
[Out] integrate((x^2 - 1)/((x^5 + x)^(1/3)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2-1}{\left(x^2+1\right)\left(x^5+x\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + 1)*(x + x^5)^(1/3)),x)
[Out] int((x^2 - 1)/((x^2 + 1)*(x + x^5)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{\sqrt[3]{x\left(x^4+1\right)}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**5+x)**(1/3),x)
[Out] Integral((x - 1)*(x + 1)/((x*(x**4 + 1))**(1/3)*(x**2 + 1)), x)
```

3.1506 $\int \frac{\sqrt{-x+x^2}\sqrt{x^2-x\sqrt{-x+x^2}}}{x^3}\,dx$

Optimal. Leaf size=121

$$\sqrt{x\left(x-\sqrt{x^2-x}\right)}\left(\frac{4}{3x}-\frac{2\sqrt{2}\sqrt{\sqrt{x^2-x}+x}\,\tanh^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2-x}+x}\right)}{x}\right)-\frac{4\sqrt{x^2-x}\sqrt{x\left(x-\sqrt{x^2-x}\right)}}{3x^2}$$

Rubi [F] time = 2.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-x+x^2}\sqrt{x^2-x\sqrt{-x+x^2}}}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-x + x^2]*Sqrt[x^2 - x*Sqrt[-x + x^2]])/x^3,x]
```

```
[Out] (2*Sqrt[-x + x^2]*Defer[Subst][Defer[Int][(Sqrt[-1 + x^2]*Sqrt[x^4 - x^2*Sq
rt[-x^2 + x^4]])/x^4, x], x, Sqrt[x]])/(Sqrt[-1 + x]*Sqrt[x])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-x+x^2}\sqrt{x^2-x\sqrt{-x+x^2}}}{x^3}\,dx &= \frac{\sqrt{-x+x^2}\int \frac{\sqrt{-1+x}\sqrt{x^2-x\sqrt{-x+x^2}}}{x^{5/2}}\,dx}{\sqrt{-1+x}\sqrt{x}}\\ &= \frac{\left(2\sqrt{-x+x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{-1+x^2}\sqrt{x^4-x^2\sqrt{-x^2+x^4}}}{x^4}\,dx,x,\sqrt{x}\right)}{\sqrt{-1+x}\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.88, size = 175, normalized size = 1.45

$$\frac{2\sqrt{(x-1)x}\left(-16x^2+16\left(\sqrt{(x-1)x}+1\right)x-8\sqrt{(x-1)x}+3\sqrt{4x-4\sqrt{(x-1)x}-2}\left(x-\sqrt{(x-1)x}\right)^{3/2}\log\left(\sqrt{4x-4\sqrt{(x-1)x}-2}+2\sqrt{x-\sqrt{(x-1)x}}\right)-2\right)}{3\sqrt{x\left(x-\sqrt{(x-1)x}\right)}\left(\sqrt{(x-1)x}-x\right)\left(-x+\sqrt{(x-1)x}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-x + x^2]*Sqrt[x^2 - x*Sqrt[-x + x^2]])/x^3,x]
```

```
[Out] (2*Sqrt[(-1 + x)*x]*(-2 - 16*x^2 - 8*Sqrt[(-1 + x)*x] + 16*x*(1 + Sqrt[(-1
+ x)*x]) + 3*Sqrt[-2 + 4*x - 4*Sqrt[(-1 + x)*x]]*(x - Sqrt[(-1 + x)*x])^(3/
2)*Log[Sqrt[-2 + 4*x - 4*Sqrt[(-1 + x)*x]] + 2*Sqrt[x - Sqrt[(-1 + x)*x]]])
)/(3*Sqrt[x*(x - Sqrt[(-1 + x)*x])]*(-x + Sqrt[(-1 + x)*x])*(1 - x + Sqrt[(
-1 + x)*x]))
```

IntegrateAlgebraic [A] time = 4.11, size = 121, normalized size = 1.00

$$\sqrt{x\left(x-\sqrt{x^2-x}\right)}\left(\frac{4}{3x}-\frac{2\sqrt{2}\sqrt{\sqrt{x^2-x}+x}\,\tanh^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2-x}+x}\right)}{x}\right)-\frac{4\sqrt{x^2-x}\sqrt{x\left(x-\sqrt{x^2-x}\right)}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-x + x^2]*Sqrt[x^2 - x*Sqrt[-x + x^2]])/x^3,x]
```

```
[Out] (-4*Sqrt[-x + x^2]*Sqrt[x*(x - Sqrt[-x + x^2])])/(3*x^2) + Sqrt[x*(x - Sqrt
[-x + x^2])]*(4/(3*x) - (2*Sqrt[2]*Sqrt[x + Sqrt[-x + x^2]]*ArcTanh[Sqrt[2]
*Sqrt[x + Sqrt[-x + x^2]]])/x)
```

fricas [A] time = 0.79, size = 114, normalized size = 0.94

$$\frac{3\,\sqrt{2}\,x^{2}\log\left(-\frac{4\,x^{2}-2\,\sqrt{x^{2}-\sqrt{x^{2}-x}\,x}\left(\sqrt{2}\,x-\sqrt{2}\,\sqrt{x^{2}-x}\right)-4\,\sqrt{x^{2}-x}\,x-x}{x}\right)+4\,\sqrt{x^{2}-\sqrt{x^{2}-x}\,x}\left(x-\sqrt{x^{2}-x}\right)}{3\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)^(1/2)*(x^2-x*(x^2-x)^(1/2))^(1/2)/x^3,x, algorithm="frica
s")
```

```
[Out] 1/3*(3*sqrt(2)*x^2*log(-(4*x^2 - 2*sqrt(x^2 - sqrt(x^2 - x)*x)*(sqrt(2)*x -
 sqrt(2)*sqrt(x^2 - x)) - 4*sqrt(x^2 - x)*x - x)/x) + 4*sqrt(x^2 - sqrt(x^2
 - x)*x)*(x - sqrt(x^2 - x)))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{2}-\sqrt{x^{2}-x}\,x}\,\sqrt{x^{2}-x}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)^(1/2)*(x^2-x*(x^2-x)^(1/2))^(1/2)/x^3,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(x^2 - sqrt(x^2 - x)*x)*sqrt(x^2 - x)/x^3, x)
```

maple [F] time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{2}-x}\,\sqrt{x^{2}-x\sqrt{x^{2}-x}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x)^(1/2)*(x^2-x*(x^2-x)^(1/2))^(1/2)/x^3,x)
```

```
[Out] int((x^2-x)^(1/2)*(x^2-x*(x^2-x)^(1/2))^(1/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{2}-\sqrt{x^{2}-x}\,x}\,\sqrt{x^{2}-x}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)^(1/2)*(x^2-x*(x^2-x)^(1/2))^(1/2)/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(x^2 - sqrt(x^2 - x)*x)*sqrt(x^2 - x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^{2}-x}\,\sqrt{x^{2}-x\,\sqrt{x^{2}-x}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - x)^(1/2)*(x^2 - x*(x^2 - x)^(1/2))^(1/2))/x^3,x)
```

```
[Out] int(((x^2 - x)^(1/2)*(x^2 - x*(x^2 - x)^(1/2))^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x(x-1)}\sqrt{x\left(x-\sqrt{x^2-x}\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x)**(1/2)*(x**2-x*(x**2-x)**(1/2))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x*(x - 1))*sqrt(x*(x - sqrt(x**2 - x)))/x**3, x)
```

3.1507 $\int \frac{b+2ax}{\sqrt[4]{c+bx+ax^2}\left(5c+4bx+4ax^2\right)}\,dx$

Optimal. Leaf size=122

$$-\frac{\tan^{-1}\left(1-\frac{2\sqrt[4]{ax^2+bx+c}}{\sqrt[4]{c}}\right)}{2\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{2\sqrt[4]{ax^2+bx+c}}{\sqrt[4]{c}}+1\right)}{2\sqrt[4]{c}}-\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^2+bx+c}}{\sqrt[4]{c}}+\frac{\sqrt[4]{c}}{2}}{\sqrt[4]{ax^2+bx+c}}\right)}{2\sqrt[4]{c}}$$

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{b+2ax}{\sqrt[4]{c+bx+ax^2}\left(5c+4bx+4ax^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(b + 2*a*x)/((c + b*x + a*x^2)^(1/4)*(5*c + 4*b*x + 4*a*x^2)),x]
```

```
[Out] Defer[Int][(b + 2*a*x)/((c + b*x + a*x^2)^(1/4)*(5*c + 4*b*x + 4*a*x^2)), x
]
```

Rubi steps

$$\int \frac{b+2ax}{\sqrt[4]{c+bx+ax^2}\left(5c+4bx+4ax^2\right)}\,dx=\int \frac{b+2ax}{\sqrt[4]{c+bx+ax^2}\left(5c+4bx+4ax^2\right)}\,dx$$

Mathematica [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{b+2ax}{\sqrt[4]{c+bx+ax^2}\left(5c+4bx+4ax^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(b + 2*a*x)/((c + b*x + a*x^2)^(1/4)*(5*c + 4*b*x + 4*a*x^2)),x]
```

```
[Out] Integrate[(b + 2*a*x)/((c + b*x + a*x^2)^(1/4)*(5*c + 4*b*x + 4*a*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.30, size = 122, normalized size = 1.00

$$-\frac{\tan^{-1}\left(1-\frac{2\sqrt[4]{ax^2+bx+c}}{\sqrt[4]{c}}\right)}{2\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{2\sqrt[4]{ax^2+bx+c}}{\sqrt[4]{c}}+1\right)}{2\sqrt[4]{c}}-\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^2+bx+c}}{\sqrt[4]{c}}+\frac{\sqrt[4]{c}}{2}}{\sqrt[4]{ax^2+bx+c}}\right)}{2\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + 2*a*x)/((c + b*x + a*x^2)^(1/4)*(5*c + 4*b*x + 4*a*
x^2)),x]
```

```
[Out] -1/2*ArcTan[1 - (2*(c + b*x + a*x^2)^(1/4))/c^(1/4)]/c^(1/4) + ArcTan[1 + (
2*(c + b*x + a*x^2)^(1/4))/c^(1/4)]/(2*c^(1/4)) - ArcTanh[(c^(1/4)/2 + Sqrt
[c + b*x + a*x^2]/c^(1/4))/(c + b*x + a*x^2)^(1/4)]/(2*c^(1/4))
```

fricas [A] time = 0.80, size = 154, normalized size = 1.26

$$-2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{c}\right)^{\frac{1}{4}}\arctan\left(2\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{-\frac{1}{2}c\sqrt{-\frac{1}{c}}+\sqrt{ax^2+bx+c}}\left(-\frac{1}{c}\right)^{\frac{1}{4}}-2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(ax^2+bx+c\right)^{\frac{1}{4}}\left(-\frac{1}{c}\right)^{\frac{1}{4}}\right)+\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{c}\right)^{\frac{1}{4}}\log\left(2\left(\frac{1}{4}\right)^{\frac{3}{4}}c\left(-\frac{1}{c}\right)^{\frac{3}{4}}+\left(ax^2+bx+c\right)^{\frac{1}{4}}\right)-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{c}\right)^{\frac{1}{4}}\log\left(-2\left(\frac{1}{4}\right)^{\frac{3}{4}}c\left(-\frac{1}{c}\right)^{\frac{3}{4}}+\left(ax^2+bx+c\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+b)/(a*x^2+b*x+c)^(1/4)/(4*a*x^2+4*b*x+5*c),x, algorithm="f
ricas")
```

```
[Out] -2*(1/4)^(1/4)*(-1/c)^(1/4)*arctan(2*(1/4)^(1/4)*sqrt(-1/2*c*sqrt(-1/c) + s
qrt(a*x^2 + b*x + c))*(-1/c)^(1/4) - 2*(1/4)^(1/4)*(a*x^2 + b*x + c)^(1/4)*
(-1/c)^(1/4)) + 1/2*(1/4)^(1/4)*(-1/c)^(1/4)*log(2*(1/4)^(3/4)*c*(-1/c)^(3/
4) + (a*x^2 + b*x + c)^(1/4)) - 1/2*(1/4)^(1/4)*(-1/c)^(1/4)*log(-2*(1/4)^(
3/4)*c*(-1/c)^(3/4) + (a*x^2 + b*x + c)^(1/4))
```

giac [B] time = 0.41, size = 202, normalized size = 1.66

$$\frac{4^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}c^{\frac{1}{4}}+2\left(ax^2+bx+c\right)^{\frac{1}{4}}\right)}{c^{\frac{1}{4}}}\right)}{8c^{\frac{1}{4}}}+\frac{4^{\frac{3}{4}}\sqrt{2}\arctan\left(-\frac{2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}c^{\frac{1}{4}}-2\left(ax^2+bx+c\right)^{\frac{1}{4}}\right)}{c^{\frac{1}{4}}}\right)}{8c^{\frac{1}{4}}}-\frac{4^{\frac{3}{4}}\sqrt{2}\log\left(\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(ax^2+bx+c\right)^{\frac{1}{4}}c^{\frac{1}{4}}+\sqrt{ax^2+bx+c}+\frac{1}{2}\sqrt{c}\right)}{16c^{\frac{1}{4}}}+\frac{4^{\frac{3}{4}}\sqrt{2}\log\left(-\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(ax^2+bx+c\right)^{\frac{1}{4}}c^{\frac{1}{4}}+\sqrt{ax^2+bx+c}+\frac{1}{2}\sqrt{c}\right)}{16c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+b)/(a*x^2+b*x+c)^(1/4)/(4*a*x^2+4*b*x+5*c),x, algorithm="g
iac")
```

```
[Out] 1/8*4^(3/4)*sqrt(2)*arctan(2*sqrt(2)*(1/4)^(3/4)*(sqrt(2)*(1/4)^(1/4)*c^(1/
4) + 2*(a*x^2 + b*x + c)^(1/4))/c^(1/4))/c^(1/4) + 1/8*4^(3/4)*sqrt(2)*arct
an(-2*sqrt(2)*(1/4)^(3/4)*(sqrt(2)*(1/4)^(1/4)*c^(1/4) - 2*(a*x^2 + b*x + c
)^(1/4))/c^(1/4))/c^(1/4) - 1/16*4^(3/4)*sqrt(2)*log(sqrt(2)*(1/4)^(1/4)*(a
*x^2 + b*x + c)^(1/4)*c^(1/4) + sqrt(a*x^2 + b*x + c) + 1/2*sqrt(c))/c^(1/4
) + 1/16*4^(3/4)*sqrt(2)*log(-sqrt(2)*(1/4)^(1/4)*(a*x^2 + b*x + c)^(1/4)*c
^(1/4) + sqrt(a*x^2 + b*x + c) + 1/2*sqrt(c))/c^(1/4)
```

maple [F] time = 1.49, size = 0, normalized size = 0.00

$$\int \frac{2ax+b}{\left(a\,x^2+bx+c\right)^{\frac{1}{4}}\left(4a\,x^2+4bx+5c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x+b)/(a*x^2+b*x+c)^(1/4)/(4*a*x^2+4*b*x+5*c),x)
```

```
[Out] int((2*a*x+b)/(a*x^2+b*x+c)^(1/4)/(4*a*x^2+4*b*x+5*c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax+b}{\left(4\,ax^2+4\,bx+5\,c\right)\left(ax^2+bx+c\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+b)/(a*x^2+b*x+c)^(1/4)/(4*a*x^2+4*b*x+5*c),x, algorithm="m
axima")
```

```
[Out] integrate((2*a*x + b)/((4*a*x^2 + 4*b*x + 5*c)*(a*x^2 + b*x + c)^(1/4)), x)
```

mupad [B] time = 1.33, size = 57, normalized size = 0.47

$$\frac{\sqrt{2}\,\left(\mathrm{atan}\left(\frac{\sqrt{2}\,{\left(a\,x^2+b\,x+c\right)}^{1/4}}{{\left(-c\right)}^{1/4}}\right)-\mathrm{atanh}\left(\frac{\sqrt{2}\,{\left(a\,x^2+b\,x+c\right)}^{1/4}}{{\left(-c\right)}^{1/4}}\right)\right)}{2\,{\left(-c\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*a*x)/((c + b*x + a*x^2)^(1/4)*(5*c + 4*b*x + 4*a*x^2)),x)
```

```
[Out] (2^(1/2)*(atan((2^(1/2)*(c + b*x + a*x^2)^(1/4))/(-c)^(1/4)) - atanh((2^(1/
2)*(c + b*x + a*x^2)^(1/4))/(-c)^(1/4))))/(2*(-c)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2ax + b}{\sqrt[4]{ax^2 + bx + c}\,(4ax^2 + 4bx + 5c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+b)/(a*x**2+b*x+c)**(1/4)/(4*a*x**2+4*b*x+5*c),x)
```

```
[Out] Integral((2*a*x + b)/((a*x**2 + b*x + c)**(1/4)*(4*a*x**2 + 4*b*x + 5*c)),
x)
```

3.1508 $\int x^6 \sqrt[3]{-x+x^3}\, dx$

Optimal. Leaf size=122

$$\frac{5}{243}\log\left(\sqrt[3]{x^3-x}-x\right)+\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x}+x}\right)}{81\sqrt{3}}-\frac{5}{486}\log\left(\sqrt[3]{x^3-x}\,x+\left(x^3-x\right)^{2/3}+x^2\right)+\frac{1}{648}\sqrt[3]{x^3-x}\,\left(81x^7-9x\right.$$

Rubi [A] time = 0.26, antiderivative size = 240, normalized size of antiderivative = 1.97, number of steps used = 14, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {2021, 2024, 2032, 329, 275, 331, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{54}\sqrt[3]{x^3-x}\,x^3-\frac{5}{162}\sqrt[3]{x^3-x}\,x+\frac{1}{8}\sqrt[3]{x^3-x}\,x^7-\frac{1}{72}\sqrt[3]{x^3-x}\,x^5+\frac{5\left(x^2-1\right)^{2/3}x^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2-1}}\right)}{243\left(x^3-x\right)^{2/3}}-\frac{5\left(x^2-1\right)^{2/3}x^{2/3}\log\left(\frac{x^{4/3}}{\left(x^2-1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2-1}}+1\right)}{486\left(x^3-x\right)^{2/3}}+\frac{5\left(x^2-1\right)^{2/3}x^{2/3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1}{\sqrt{3}}\right)}{81\sqrt{3}\left(x^3-x\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(-x + x^3)^(1/3),x]
```

```
[Out] (-5*x*(-x + x^3)^(1/3))/162 - (x^3*(-x + x^3)^(1/3))/54 - (x^5*(-x + x^3)^(
1/3))/72 + (x^7*(-x + x^3)^(1/3))/8 + (5*x^(2/3)*(-1 + x^2)^(2/3)*ArcTan[(1
 + (2*x^(2/3))/(-1 + x^2)^(1/3))/Sqrt[3]])/(81*Sqrt[3]*(-x + x^3)^(2/3)) +
(5*x^(2/3)*(-1 + x^2)^(2/3)*Log[1 - x^(2/3)/(-1 + x^2)^(1/3)])/(243*(-x + x
^3)^(2/3)) - (5*x^(2/3)*(-1 + x^2)^(2/3)*Log[1 + x^(4/3)/(-1 + x^2)^(2/3) +
 x^(2/3)/(-1 + x^2)^(1/3)])/(486*(-x + x^3)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^6\sqrt[3]{-x+x^3}\,dx = \frac{1}{8}x^7\sqrt[3]{-x+x^3} - \frac{1}{12}\int \frac{x^7}{\left(-x+x^3\right)^{2/3}}\,dx$$

$$= -\frac{1}{72}x^5\sqrt[3]{-x+x^3} + \frac{1}{8}x^7\sqrt[3]{-x+x^3} - \frac{2}{27}\int \frac{x^5}{\left(-x+x^3\right)^{2/3}}\,dx$$

$$= -\frac{1}{54}x^3\sqrt[3]{-x+x^3} - \frac{1}{72}x^5\sqrt[3]{-x+x^3} + \frac{1}{8}x^7\sqrt[3]{-x+x^3} - \frac{5}{81}\int \frac{x^3}{\left(-x+x^3\right)^{2/3}}\,dx$$

$$= -\frac{5}{162}x\sqrt[3]{-x+x^3} - \frac{1}{54}x^3\sqrt[3]{-x+x^3} - \frac{1}{72}x^5\sqrt[3]{-x+x^3} + \frac{1}{8}x^7\sqrt[3]{-x+x^3} - \frac{10}{243}\int \frac{x}{\left(-x+x^{\cdots}\right.}$$

$$= -\frac{5}{162}x\sqrt[3]{-x+x^3} - \frac{1}{54}x^3\sqrt[3]{-x+x^3} - \frac{1}{72}x^5\sqrt[3]{-x+x^3} + \frac{1}{8}x^7\sqrt[3]{-x+x^3} - \frac{\left(10x^{2/3}\left(-1+x^{\cdots}\right.\right.}{243\left(\cdots\right.}$$

$$= -\frac{5}{162}x\sqrt[3]{-x+x^3} - \frac{1}{54}x^3\sqrt[3]{-x+x^3} - \frac{1}{72}x^5\sqrt[3]{-x+x^3} + \frac{1}{8}x^7\sqrt[3]{-x+x^3} - \frac{\left(10x^{2/3}\left(-1+x^{\cdots}\right.\right.}{\cdots}$$

$$= -\frac{5}{162}x\sqrt[3]{-x+x^3} - \frac{1}{54}x^3\sqrt[3]{-x+x^3} - \frac{1}{72}x^5\sqrt[3]{-x+x^3} + \frac{1}{8}x^7\sqrt[3]{-x+x^3} - \frac{\left(5x^{2/3}\left(-1+x^2\right)\cdots\right.}{\cdots}$$

$$= -\frac{5}{162}x\sqrt[3]{-x+x^3} - \frac{1}{54}x^3\sqrt[3]{-x+x^3} - \frac{1}{72}x^5\sqrt[3]{-x+x^3} + \frac{1}{8}x^7\sqrt[3]{-x+x^3} - \frac{\left(5x^{2/3}\left(-1+x^2\right)\cdots\right.}{\cdots}$$

$$= -\frac{5}{162}x\sqrt[3]{-x+x^3} - \frac{1}{54}x^3\sqrt[3]{-x+x^3} - \frac{1}{72}x^5\sqrt[3]{-x+x^3} + \frac{1}{8}x^7\sqrt[3]{-x+x^3} - \frac{\left(5x^{2/3}\left(-1+x^2\right)\cdots\right.}{\cdots}$$

$$= -\frac{5}{162}x\sqrt[3]{-x+x^3} - \frac{1}{54}x^3\sqrt[3]{-x+x^3} - \frac{1}{72}x^5\sqrt[3]{-x+x^3} + \frac{1}{8}x^7\sqrt[3]{-x+x^3} + \frac{5x^{2/3}\left(-1+x^2\right)^{\cdots}}{243\left(\cdots\right.}$$

$$= -\frac{5}{162}x\sqrt[3]{-x+x^3} - \frac{1}{54}x^3\sqrt[3]{-x+x^3} - \frac{1}{72}x^5\sqrt[3]{-x+x^3} + \frac{1}{8}x^7\sqrt[3]{-x+x^3} + \frac{5x^{2/3}\left(-1+x^2\right)^{\cdots}}{243\left(\cdots\right.}$$

$$= -\frac{5}{162}x\sqrt[3]{-x+x^3} - \frac{1}{54}x^3\sqrt[3]{-x+x^3} - \frac{1}{72}x^5\sqrt[3]{-x+x^3} + \frac{1}{8}x^7\sqrt[3]{-x+x^3} + \frac{5x^{2/3}\left(-1+x^2\right)^{\cdots}}{81\sqrt{3}\left(\cdots\right.}$$

Mathematica [C] time = 0.03, size = 72, normalized size = 0.59

$$\frac{x\sqrt[3]{x\left(x^2-1\right)}\left(20\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^2\right)+\sqrt[3]{1-x^2}\left(27x^6-3x^4-4x^2-20\right)\right)}{216\sqrt[3]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(-x + x^3)^(1/3),x]
```

```
[Out] (x*(x*(-1 + x^2))^(1/3)*((1 - x^2)^(1/3)*(-20 - 4*x^2 - 3*x^4 + 27*x^6) + 2
0*Hypergeometric2F1[-1/3, 2/3, 5/3, x^2]))/(216*(1 - x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.47, size = 122, normalized size = 1.00

$$\frac{5}{243}\log\left(\sqrt[3]{x^3-x}-x\right)+\frac{5\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-x}+x}\right)}{81\sqrt{3}}-\frac{5}{486}\log\left(\sqrt[3]{x^3-x}\,x+\left(x^3-x\right)^{2/3}+x^2\right)+\frac{1}{648}\sqrt[3]{x^3-x}\left(81x^7-9x^5-12x^3-20x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6*(-x + x^3)^(1/3),x]
```

```
[Out] ((-x + x^3)^(1/3)*(-20*x - 12*x^3 - 9*x^5 + 81*x^7))/648 + (5*ArcTan[(Sqrt[
3]*x)/(x + 2*(-x + x^3)^(1/3))])/(81*Sqrt[3]) + (5*Log[-x + (-x + x^3)^(1/3
)])/243 - (5*Log[x^2 + x*(-x + x^3)^(1/3) + (-x + x^3)^(2/3)])/486
```

fricas [A] time = 2.12, size = 117, normalized size = 0.96

$$\frac{5}{243}\sqrt{3}\arctan\left(-\frac{44032959556\sqrt{3}\left(x^3-x\right)^{\frac{1}{3}}x+\sqrt{3}\left(16754327161\,x^2-2707204793\right)-10524305234\sqrt{3}\left(x^3-x\right)^{\frac{2}{3}}}{81835897185\,x^2-1102302937}\right)+\frac{1}{648}\left(81\,x^7-9\,x^5-12\,x^3-20\,x\right)\left(x^3-x\right)^{\frac{1}{3}}+\frac{5}{486}\log\left(-3\left(x^3-x\right)^{\frac{1}{3}}x+3\left(x^3-x\right)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^3-x)^(1/3),x, algorithm="fricas")
```

```
[Out] 5/243*sqrt(3)*arctan(-(44032959556*sqrt(3)*(x^3 - x)^(1/3)*x + sqrt(3)*(167
54327161*x^2 - 2707204793) - 10524305234*sqrt(3)*(x^3 - x)^(2/3))/(81835897
185*x^2 - 1102302937)) + 1/648*(81*x^7 - 9*x^5 - 12*x^3 - 20*x)*(x^3 - x)^(
1/3) + 5/486*log(-3*(x^3 - x)^(1/3)*x + 3*(x^3 - x)^(2/3) + 1)
```

giac [A] time = 0.19, size = 127, normalized size = 1.04

$$-\frac{1}{648}\left(20\left(\frac{1}{x^2}-1\right)^3\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+72\left(\frac{1}{x^2}-1\right)^2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}-93\left(-\frac{1}{x^2}+1\right)^{\frac{4}{3}}-40\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right)x^8-\frac{5}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{5}{486}\log\left(\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)+\frac{5}{243}\log\left(\left|\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^3-x)^(1/3),x, algorithm="giac")
```

```
[Out] -1/648*(20*(1/x^2 - 1)^3*(-1/x^2 + 1)^(1/3) + 72*(1/x^2 - 1)^2*(-1/x^2 + 1)
^(1/3) - 93*(-1/x^2 + 1)^(4/3) - 40*(-1/x^2 + 1)^(1/3))*x^8 - 5/243*sqrt(3)
*arctan(1/3*sqrt(3)*(2*(-1/x^2 + 1)^(1/3) + 1)) - 5/486*log((-1/x^2 + 1)^(2
/3) + (-1/x^2 + 1)^(1/3) + 1) + 5/243*log(abs((-1/x^2 + 1)^(1/3) - 1))
```

maple [C] time = 1.65, size = 801, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^3-x)^(1/3),x)
```

```
[Out] 1/648*x*(81*x^6-9*x^4-12*x^2-20)*(x*(x^2-1))^(1/3)+(5/1458*RootOf(_Z^2+6*_Z
+36)*ln(-(47*RootOf(_Z^2+6*_Z+36)^2*x^4+3207*RootOf(_Z^2+6*_Z+36)*x^4+2925*
(x^6-2*x^4+x^2)^(1/3)*RootOf(_Z^2+6*_Z+36)*x^2-235*RootOf(_Z^2+6*_Z+36)^2*x
^2+6930*x^4+2925*RootOf(_Z^2+6*_Z+36)*(x^6-2*x^4+x^2)^(2/3)+5238*(x^6-2*x^4
+x^2)^(1/3)*x^2-5601*RootOf(_Z^2+6*_Z+36)*x^2+5238*(x^6-2*x^4+x^2)^(2/3)-29
25*RootOf(_Z^2+6*_Z+36)*(x^6-2*x^4+x^2)^(1/3)+188*RootOf(_Z^2+6*_Z+36)^2-11
340*x^2-5238*(x^6-2*x^4+x^2)^(1/3)+2394*RootOf(_Z^2+6*_Z+36)+4410)/(-1+x)/(
1+x))-5/1458*ln((-47*RootOf(_Z^2+6*_Z+36)^2*x^4+2643*RootOf(_Z^2+6*_Z+36)*x
^4+2925*(x^6-2*x^4+x^2)^(1/3)*RootOf(_Z^2+6*_Z+36)*x^2+235*RootOf(_Z^2+6*_Z
+36)^2*x^2+10620*x^4+2925*RootOf(_Z^2+6*_Z+36)*(x^6-2*x^4+x^2)^(2/3)+12312*
(x^6-2*x^4+x^2)^(1/3)*x^2-2781*RootOf(_Z^2+6*_Z+36)*x^2+12312*(x^6-2*x^4+x^
2)^(2/3)-2925*RootOf(_Z^2+6*_Z+36)*(x^6-2*x^4+x^2)^(1/3)-188*RootOf(_Z^2+6*
_Z+36)^2-13806*x^2-12312*(x^6-2*x^4+x^2)^(1/3)+138*RootOf(_Z^2+6*_Z+36)+318
6)/(-1+x)/(1+x))*RootOf(_Z^2+6*_Z+36)-5/243*ln((-47*RootOf(_Z^2+6*_Z+36)^2*
x^4+2643*RootOf(_Z^2+6*_Z+36)*x^4+2925*(x^6-2*x^4+x^2)^(1/3)*RootOf(_Z^2+6*
```

```
_Z+36)*x^2+235*RootOf(_Z^2+6*_Z+36)^2*x^2+10620*x^4+2925*RootOf(_Z^2+6*_Z+3
6)*(x^6-2*x^4+x^2)^(2/3)+12312*(x^6-2*x^4+x^2)^(1/3)*x^2-2781*RootOf(_Z^2+6
*_Z+36)*x^2+12312*(x^6-2*x^4+x^2)^(2/3)-2925*RootOf(_Z^2+6*_Z+36)*(x^6-2*x^
4+x^2)^(1/3)-188*RootOf(_Z^2+6*_Z+36)^2-13806*x^2-12312*(x^6-2*x^4+x^2)^(1/
3)+138*RootOf(_Z^2+6*_Z+36)+3186)/(-1+x)/(1+x)))/x*(x*(x^2-1))^(1/3)*(x^2*(
x^2-1)^2)^(1/3)/(x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^3 - x\right)^{\frac{1}{3}} x^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^3-x)^(1/3),x, algorithm="maxima")
[Out] integrate((x^3 - x)^(1/3)*x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6 \left(x^3 - x\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^3 - x)^(1/3),x)
[Out] int(x^6*(x^3 - x)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \sqrt[3]{x (x-1) (x+1)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(x**3-x)**(1/3),x)
[Out] Integral(x**6*(x*(x - 1)*(x + 1))**(1/3), x)
```

3.1509 $\int \frac{1}{\sqrt[3]{-1-x+x^2+x^3}}\,dx$

Optimal. Leaf size=122

$$-\log\left(\sqrt[3]{x^3+x^2-x-1}-x-1\right)+\frac{1}{2}\log\left(x^2+\left(x^3+x^2-x-1\right)^{2/3}+(x+1)\sqrt[3]{x^3+x^2-x-1}+2x+1\right)-\sqrt{3}\tan^{-}$$

Rubi [A] time = 0.14, antiderivative size = 167, normalized size of antiderivative = 1.37, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2067, 2064, 60}

$$-\frac{3(-x-1)^{2/3}\sqrt[3]{x-1}\log\left(\frac{\sqrt[3]{x-1}}{\sqrt[3]{-x-1}}+1\right)}{2\sqrt[3]{x^3+x^2-x-1}}-\frac{(-x-1)^{2/3}\sqrt[3]{x-1}\log\left(-\frac{8}{3}(x+1)\right)}{2\sqrt[3]{x^3+x^2-x-1}}-\frac{\sqrt{3}(-x-1)^{2/3}\sqrt[3]{x-1}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x-1}}{\sqrt{3}\sqrt[3]{-x-1}}\right)}{\sqrt[3]{x^3+x^2-x-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - x + x^2 + x^3)^(-1/3),x]
```

```
[Out] -((Sqrt[3]*(-1 - x)^(2/3)*(-1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2*(-1 + x)^(1/
3))/(Sqrt[3]*(-1 - x)^(1/3))])/(-1 - x + x^2 + x^3)^(1/3)) - (3*(-1 - x)^(2
/3)*(-1 + x)^(1/3)*Log[1 + (-1 + x)^(1/3)/(-1 - x)^(1/3)])/(2*(-1 - x + x^2
 + x^3)^(1/3)) - ((-1 - x)^(2/3)*(-1 + x)^(1/3)*Log[(-8*(1 + x))/3])/(2*(-1
 - x + x^2 + x^3)^(1/3))
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 2064

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-1-x+x^2+x^3}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt[3]{-\frac{16}{27}-\frac{4x}{3}+x^3}}\,dx, x, \frac{1}{3}+x\right)$$

$$= \frac{\left(4\ 2^{2/3}(-1-x)^{2/3}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\left(-\frac{16}{9}-\frac{8x}{3}\right)^{2/3}\sqrt[3]{-\frac{16}{9}+\frac{4x}{3}}}\,dx, x, \frac{1}{3}+x\right)}{3\sqrt[3]{-1-x+x^2+x^3}}$$

$$= -\frac{\sqrt{3}\,(-1-x)^{2/3}\sqrt[3]{-1+x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{-1+x}}{\sqrt{3}\sqrt[3]{-1-x}}\right)}{\sqrt[3]{-1-x+x^2+x^3}} - \frac{3(-1-x)^{2/3}\sqrt[3]{-1+x}\,\log\left(1+\right.}{2\sqrt[3]{-1-x+x^2+x^3}}$$

Mathematica [C] time = 0.02, size = 48, normalized size = 0.39

$$\frac{3\left((x-1)(x+1)^2\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1-x}{2}\right)}{2\ 2^{2/3}(x+1)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - x + x^2 + x^3)^(-1/3),x]
```

```
[Out] (3*((-1 + x)*(1 + x)^2)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (1 - x)/2])/
(2*2^(2/3)*(1 + x)^(4/3))
```

IntegrateAlgebraic [A] time = 0.29, size = 122, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^3+x^2-x-1}-x-1\right)+\frac{1}{2}\log\left(x^2+\left(x^3+x^2-x-1\right)^{2/3}+(x+1)\sqrt[3]{x^3+x^2-x-1}+2x+1\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^3+x^2-x-1}}{\sqrt[3]{x^3+x^2-x-1}+2x+2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - x + x^2 + x^3)^(-1/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*(-1 - x + x^2 + x^3)^(1/3))/(2 + 2*x + (-1 - x +
x^2 + x^3)^(1/3))]) - Log[-1 - x + (-1 - x + x^2 + x^3)^(1/3)] + Log[1 + 2*
x + x^2 + (1 + x)*(-1 - x + x^2 + x^3)^(1/3) + (-1 - x + x^2 + x^3)^(2/3)]/
2
```

fricas [A] time = 0.66, size = 120, normalized size = 0.98

$$-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x+1)+2\sqrt{3}\left(x^3+x^2-x-1\right)^{\frac{1}{3}}}{3(x+1)}\right)+\frac{1}{2}\log\left(\frac{x^2+\left(x^3+x^2-x-1\right)^{\frac{1}{3}}(x+1)+2x+\left(x^3+x^2-x-1\right)^{\frac{2}{3}}+1}{x^2+2x+1}\right)-\log\left(-\frac{x-\left(x^3+x^2-x-1\right)^{\frac{1}{3}}+1}{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2-x-1)^(1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(1/3*(sqrt(3)*(x + 1) + 2*sqrt(3)*(x^3 + x^2 - x - 1)^(1/3))
/(x + 1)) + 1/2*log((x^2 + (x^3 + x^2 - x - 1)^(1/3)*(x + 1) + 2*x + (x^3 +
 x^2 - x - 1)^(2/3) + 1)/(x^2 + 2*x + 1)) - log(-(x - (x^3 + x^2 - x - 1)^(
1/3) + 1)/(x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+x^2-x-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2-x-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 + x^2 - x - 1)^(-1/3), x)
```

maple [C] time = 0.39, size = 558, normalized size = 4.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+x^2-x-1)^(1/3),x)
```

```
[Out] RootOf(_Z^2-_Z+1)*ln(-(-2*RootOf(_Z^2-_Z+1)^2*x^2+3*RootOf(_Z^2-_Z+1)*(x^3+
x^2-x-1)^(2/3)+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)*x-2*RootOf(_Z^2-_Z+1
)^2*x+5*RootOf(_Z^2-_Z+1)*x^2+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)+4*Roo
tOf(_Z^2-_Z+1)*x-2*x^2-RootOf(_Z^2-_Z+1)+2)/(1+x))-ln((2*RootOf(_Z^2-_Z+1)^
2*x^2+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(2/3)+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-
x-1)^(1/3)*x+2*RootOf(_Z^2-_Z+1)^2*x+RootOf(_Z^2-_Z+1)*x^2-3*(x^3+x^2-x-1)^
(2/3)+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)-3*x*(x^3+x^2-x-1)^(1/3)-x^2-3
*(x^3+x^2-x-1)^(1/3)-RootOf(_Z^2-_Z+1)-2*x-1)/(1+x))*RootOf(_Z^2-_Z+1)+ln((
2*RootOf(_Z^2-_Z+1)^2*x^2+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(2/3)+3*RootOf(
_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)*x+2*RootOf(_Z^2-_Z+1)^2*x+RootOf(_Z^2-_Z+1)*
x^2-3*(x^3+x^2-x-1)^(2/3)+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)-3*x*(x^3+
x^2-x-1)^(1/3)-x^2-3*(x^3+x^2-x-1)^(1/3)-RootOf(_Z^2-_Z+1)-2*x-1)/(1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+x^2-x-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2-x-1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + x^2 - x - 1)^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^3+x^2-x-1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - x + x^3 - 1)^(1/3),x)
```

```
[Out] int(1/(x^2 - x + x^3 - 1)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x^3+x^2-x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+x**2-x-1)**(1/3),x)
```

```
[Out] Integral((x**3 + x**2 - x - 1)**(-1/3), x)
```

3.1510 $\int \frac{1+2x^4}{\left(-1+x^4\right)\sqrt[4]{-x^2+x^4}}\,dx$

Optimal. Leaf size=122

$$-\frac{3\left(x^4-x^2\right)^{3/4}}{x\left(x^2-1\right)}+2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^2}}\right)}{2\sqrt[4]{2}}+2\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^2}}\right)}{2\sqrt[4]{2}}$$

Rubi [A] time = 0.61, antiderivative size = 221, normalized size of antiderivative = 1.81, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2056, 6715, 6725, 240, 212, 206, 203, 1404, 382, 377}

$$-\frac{3x}{\sqrt[4]{x^4-x^2}}+\frac{2\sqrt[4]{x^2-1}\sqrt{x}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{\sqrt[4]{x^4-x^2}}-\frac{3\sqrt[4]{x^2-1}\sqrt{x}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{2\sqrt[4]{2}\sqrt[4]{x^4-x^2}}+\frac{2\sqrt[4]{x^2-1}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{\sqrt[4]{x^4-x^2}}-\frac{3\sqrt[4]{x^2-1}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{2\sqrt[4]{2}\sqrt[4]{x^4-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^4)/((-1 + x^4)*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] (-3*x)/(-x^2 + x^4)^(1/4) + (2*Sqrt[x]*(-1 + x^2)^(1/4)*ArcTan[Sqrt[x]/(-1
+ x^2)^(1/4)])/(-x^2 + x^4)^(1/4) - (3*Sqrt[x]*(-1 + x^2)^(1/4)*ArcTan[(2^(
1/4)*Sqrt[x])/(-1 + x^2)^(1/4)])/(2*2^(1/4)*(-x^2 + x^4)^(1/4)) + (2*Sqrt[x
]*(-1 + x^2)^(1/4)*ArcTanh[Sqrt[x]/(-1 + x^2)^(1/4)])/(-x^2 + x^4)^(1/4) -
(3*Sqrt[x]*(-1 + x^2)^(1/4)*ArcTanh[(2^(1/4)*Sqrt[x])/(-1 + x^2)^(1/4)])/(2
*2^(1/4)*(-x^2 + x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 1404

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+2x^4}{(-1+x^4)\sqrt[4]{-x^2+x^4}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{-1+x^2}\right)\int \frac{1+2x^4}{\sqrt{x}\sqrt[4]{-1+x^2}(-1+x^4)}\,dx}{\sqrt[4]{-x^2+x^4}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{1+2x^8}{\sqrt[4]{-1+x^4}(-1+x^8)}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^4}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \left(\frac{2}{\sqrt[4]{-1+x^4}} + \frac{3}{\sqrt[4]{-1+x^4}(-1+x^8)}\right)dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^4}}$$

$$= \frac{\left(4\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-1+x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^4}} + \frac{\left(6\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \cdots\right.}{\sqrt[4]{-x^2+x^4}}$$

$$= \frac{\left(4\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}} + \frac{\left(6\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \cdots\right.}{\sqrt[4]{-x^2+x^4}}$$

$$= -\frac{3x}{\sqrt[4]{-x^2+x^4}} + \frac{\left(2\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}} + \frac{\left(2\sqrt{x}\sqrt[4]{-1+\cdots\right.}{\cdots}$$

$$= -\frac{3x}{\sqrt[4]{-x^2+x^4}} + \frac{2\sqrt{x}\sqrt[4]{-1+x^2}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}} + \frac{2\sqrt{x}\sqrt[4]{-1+x^2}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+\cdots}}\right.}{\sqrt[4]{-x^2+x^4}}$$

$$= -\frac{3x}{\sqrt[4]{-x^2+x^4}} + \frac{2\sqrt{x}\sqrt[4]{-1+x^2}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}} + \frac{2\sqrt{x}\sqrt[4]{-1+x^2}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+\cdots}}\right.}{\sqrt[4]{-x^2+x^4}}$$

$$= -\frac{3x}{\sqrt[4]{-x^2+x^4}} + \frac{2\sqrt{x}\sqrt[4]{-1+x^2}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}} - \frac{3\sqrt{x}\sqrt[4]{-1+x^2}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{x}}{\sqrt[4]{-1+x^2}}\right.}{2\sqrt[4]{2}\sqrt[4]{-x^2+x^4}}$$

Mathematica [C] time = 0.30, size = 109, normalized size = 0.89

$$\frac{x\left(4\sqrt[4]{1-x^4}\,x^2F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};x^2,-x^2\right)+5\sqrt[4]{1-x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{2x^2}{x^2+1}\right)-15\sqrt[4]{x^2+1}\right)}{5\sqrt[4]{x^2(x^2-1)}\sqrt[4]{x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + 2*x^4)/((-1 + x^4)*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] (x*(-15*(1 + x^2)^(1/4) + 4*x^2*(1 - x^4)^(1/4)*AppellF1[5/4, 1/4, 1, 9/4,
x^2, -x^2] + 5*(1 - x^2)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (2*x^2)/(1
+ x^2)]))/(5*(x^2*(-1 + x^2))^(1/4)*(1 + x^2)^(1/4))
```

IntegrateAlgebraic [A] time = 0.38, size = 122, normalized size = 1.00

$$-\frac{3(x^4-x^2)^{3/4}}{x(x^2-1)}+2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^2}}\right)}{2\sqrt[4]{2}}+2\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^2}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x^4)/((-1 + x^4)*(-x^2 + x^4)^(1/4)),x]
```

[Out] (-3*(-x^2 + x^4)^(3/4))/(x*(-1 + x^2)) + 2*ArcTan[x/(-x^2 + x^4)^(1/4)] - (
3*ArcTan[(2^(1/4)*x)/(-x^2 + x^4)^(1/4)])/(2*2^(1/4)) + 2*ArcTanh[x/(-x^2 +
 x^4)^(1/4)] - (3*ArcTanh[(2^(1/4)*x)/(-x^2 + x^4)^(1/4)])/(2*2^(1/4))

fricas [B] time = 17.89, size = 395, normalized size = 3.24

$$\frac{12\cdot 2^{\frac{3}{4}}\left(x^{3}-x\right)\arctan\left(\frac{4\cdot 2^{\frac{3}{4}}\left(x^{4}-x^{2}\right)^{\frac{1}{4}}x^{2}+2^{\frac{3}{4}}\left(2\cdot 2^{\frac{3}{4}}\sqrt{x^{4}-x^{2}}\,x+2^{\frac{1}{4}}\left(3\,x^{3}-x\right)\right)+4\cdot 2^{\frac{1}{4}}\left(x^{4}-x^{2}\right)^{\frac{3}{4}}}{2\left(x^{3}+x\right)}\right)-3\cdot 2^{\frac{3}{4}}\left(x^{3}-x\right)\log\left(\frac{4\sqrt{2}\left(x^{4}-x^{2}\right)^{\frac{1}{4}}x^{2}+2^{\frac{3}{4}}\left(3\,x^{3}-x\right)+4\cdot 2^{\frac{1}{4}}\sqrt{x^{4}-x^{2}}\,x+4\left(x^{4}-x^{2}\right)^{\frac{3}{4}}}{x^{3}+x}\right)+3\cdot 2^{\frac{3}{4}}\left(x^{3}-x\right)\log\left(\frac{4\sqrt{2}\left(x^{4}-x^{2}\right)^{\frac{1}{4}}x^{2}-2^{\frac{3}{4}}\left(3\,x^{3}-x\right)-4\cdot 2^{\frac{1}{4}}\sqrt{x^{4}-x^{2}}\,x+4\left(x^{4}-x^{2}\right)^{\frac{3}{4}}}{x^{3}+x}\right)-16\left(x^{3}-x\right)\arctan\left(\frac{2\left(\left(x^{4}-x^{2}\right)^{\frac{1}{4}}x^{2}+\left(x^{4}-x^{2}\right)^{\frac{3}{4}}\right)}{x}\right)+16\left(x^{3}-x\right)\log\left(\frac{2\,x^{3}+2\left(x^{4}-x^{2}\right)^{\frac{1}{4}}x^{2}+2\sqrt{x^{4}-x^{2}}\,x-x+2\left(x^{4}-x^{2}\right)^{\frac{3}{4}}}{x}\right)-48\left(x^{4}-x^{2}\right)^{\frac{3}{4}}}{16\left(x^{3}-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^4+1)/(x^4-1)/(x^4-x^2)^(1/4),x, algorithm="fricas")

[Out] 1/16*(12*2^(3/4)*(x^3 - x)*arctan(1/2*(4*2^(3/4)*(x^4 - x^2)^(1/4)*x^2 + 2^
(3/4)*(2*2^(3/4)*sqrt(x^4 - x^2)*x + 2^(1/4)*(3*x^3 - x)) + 4*2^(1/4)*(x^4
- x^2)^(3/4))/(x^3 + x)) - 3*2^(3/4)*(x^3 - x)*log((4*sqrt(2)*(x^4 - x^2)^(
1/4)*x^2 + 2^(3/4)*(3*x^3 - x) + 4*2^(1/4)*sqrt(x^4 - x^2)*x + 4*(x^4 - x^2
)^(3/4))/(x^3 + x)) + 3*2^(3/4)*(x^3 - x)*log((4*sqrt(2)*(x^4 - x^2)^(1/4)*
x^2 - 2^(3/4)*(3*x^3 - x) - 4*2^(1/4)*sqrt(x^4 - x^2)*x + 4*(x^4 - x^2)^(3/
4))/(x^3 + x)) - 16*(x^3 - x)*arctan(2*((x^4 - x^2)^(1/4)*x^2 + (x^4 - x^2)
^(3/4))/x) + 16*(x^3 - x)*log((2*x^3 + 2*(x^4 - x^2)^(1/4)*x^2 + 2*sqrt(x^4
 - x^2)*x - x + 2*(x^4 - x^2)^(3/4))/x) - 48*(x^4 - x^2)^(3/4))/(x^3 - x)

giac [A] time = 0.36, size = 112, normalized size = 0.92

$$-\frac{3}{4}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(-\frac{1}{x^{2}}+1\right)^{\frac{1}{4}}\right)+\frac{3}{8}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}+\left(-\frac{1}{x^{2}}+1\right)^{\frac{1}{4}}\right)-\frac{3}{8}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}-\left(-\frac{1}{x^{2}}+1\right)^{\frac{1}{4}}\right)+\frac{3}{\left(-\frac{1}{x^{2}}+1\right)^{\frac{1}{4}}}+2\arctan\left(\left(-\frac{1}{x^{2}}+1\right)^{\frac{1}{4}}\right)-\log\left(\left(-\frac{1}{x^{2}}+1\right)^{\frac{1}{4}}+1\right)+\log\left(-\left(-\frac{1}{x^{2}}+1\right)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^4+1)/(x^4-1)/(x^4-x^2)^(1/4),x, algorithm="giac")

[Out] -3/4*2^(3/4)*arctan(1/2*2^(3/4)*(-1/x^2 + 1)^(1/4)) + 3/8*2^(3/4)*log(2^(1/
4) + (-1/x^2 + 1)^(1/4)) - 3/8*2^(3/4)*log(2^(1/4) - (-1/x^2 + 1)^(1/4)) +
3/(-1/x^2 + 1)^(1/4) + 2*arctan((-1/x^2 + 1)^(1/4)) - log((-1/x^2 + 1)^(1/4
) + 1) + log(-(-1/x^2 + 1)^(1/4) + 1)

maple [F] time = 2.00, size = 0, normalized size = 0.00

$$\int \frac{2x^{4}+1}{\left(x^{4}-1\right)\left(x^{4}-x^{2}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^4+1)/(x^4-1)/(x^4-x^2)^(1/4),x)

[Out] int((2*x^4+1)/(x^4-1)/(x^4-x^2)^(1/4),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^{4}+1}{\left(x^{4}-x^{2}\right)^{\frac{1}{4}}\left(x^{4}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^4+1)/(x^4-1)/(x^4-x^2)^(1/4),x, algorithm="maxima")

[Out] integrate((2*x^4 + 1)/((x^4 - x^2)^(1/4)*(x^4 - 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x^{4}+1}{\left(x^{4}-1\right)\,{\left(x^{4}-x^{2}\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4 + 1)/((x^4 - 1)*(x^4 - x^2)^(1/4)),x)
```

```
[Out] int((2*x^4 + 1)/((x^4 - 1)*(x^4 - x^2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^4+1}{\sqrt[4]{x^2(x-1)(x+1)}\,(x-1)(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+1)/(x**4-1)/(x**4-x**2)**(1/4),x)
```

```
[Out] Integral((2*x**4 + 1)/((x**2*(x - 1)*(x + 1))**(1/4)*(x - 1)*(x + 1)*(x**2
+ 1)), x)
```

3.1511 $\int \frac{(-b+ax^4)\sqrt[4]{bx^2+ax^4}}{x^2}\,dx$

Optimal. Leaf size=122

$$\frac{(32ab+3b^2)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{32a^{3/4}}+\frac{(-32ab-3b^2)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{32a^{3/4}}+\frac{\sqrt[4]{ax^4+bx^2}\,(4ax^4+bx^2+32b)}{16x}$$

Rubi [B] time = 0.49, antiderivative size = 321, normalized size of antiderivative = 2.63, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2052, 2020, 2032, 329, 331, 298, 203, 206, 2021, 2024}

$$\frac{3b^2x^{3/2}(ax^2+b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{32a^{3/4}(ax^4+bx^2)^{3/4}}-\frac{3b^2x^{3/2}(ax^2+b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{32a^{3/4}(ax^4+bx^2)^{3/4}}+\frac{1}{16}bx\sqrt[4]{ax^4+bx^2}+\frac{2b\sqrt[4]{ax^4+bx^2}}{x}+\frac{\sqrt[4]{a}\,bx^{3/2}(ax^2+b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{(ax^4+bx^2)^{3/4}}-\frac{\sqrt[4]{a}\,bx^{3/2}(ax^2+b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{(ax^4+bx^2)^{3/4}}+\frac{1}{4}ax^3\sqrt[4]{ax^4+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a*x^4)*(b*x^2 + a*x^4)^(1/4))/x^2,x]
```

```
[Out] (2*b*(b*x^2 + a*x^4)^(1/4))/x + (b*x*(b*x^2 + a*x^4)^(1/4))/16 + (a*x^3*(b*
x^2 + a*x^4)^(1/4))/4 + (a^(1/4)*b*x^(3/2)*(b + a*x^2)^(3/4)*ArcTan[(a^(1/4
)*Sqrt[x])/(b + a*x^2)^(1/4)])/(b*x^2 + a*x^4)^(3/4) + (3*b^2*x^(3/2)*(b +
a*x^2)^(3/4)*ArcTan[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(32*a^(3/4)*(b*x^
2 + a*x^4)^(3/4)) - (a^(1/4)*b*x^(3/2)*(b + a*x^2)^(3/4)*ArcTanh[(a^(1/4)*S
qrt[x])/(b + a*x^2)^(1/4)])/(b*x^2 + a*x^4)^(3/4) - (3*b^2*x^(3/2)*(b + a*x
^2)^(3/4)*ArcTanh[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(32*a^(3/4)*(b*x^2
+ a*x^4)^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{(-b+ax^4)\sqrt[4]{bx^2+ax^4}}{x^2}\,dx = \int \left(-\frac{b\sqrt[4]{bx^2+ax^4}}{x^2}+ax^2\sqrt[4]{bx^2+ax^4}\right)dx$$
$$= a\int x^2\sqrt[4]{bx^2+ax^4}\,dx - b\int \frac{\sqrt[4]{bx^2+ax^4}}{x^2}\,dx$$
$$= \frac{2b\sqrt[4]{bx^2+ax^4}}{x}+\frac{1}{4}ax^3\sqrt[4]{bx^2+ax^4}+\frac{1}{8}(ab)\int \frac{x^4}{(bx^2+ax^4)^{3/4}}\,dx-(ab)\int \frac{}{(bx^2 -}$$
$$= \frac{2b\sqrt[4]{bx^2+ax^4}}{x}+\frac{1}{16}bx\sqrt[4]{bx^2+ax^4}+\frac{1}{4}ax^3\sqrt[4]{bx^2+ax^4}-\frac{1}{32}(3b^2)\int \frac{x^2}{(bx^2+ax^4}$$
$$= \frac{2b\sqrt[4]{bx^2+ax^4}}{x}+\frac{1}{16}bx\sqrt[4]{bx^2+ax^4}+\frac{1}{4}ax^3\sqrt[4]{bx^2+ax^4}-\frac{\left(2abx^{3/2}(b+ax^2)^{3/4}\right)}{(bx}$$
$$= \frac{2b\sqrt[4]{bx^2+ax^4}}{x}+\frac{1}{16}bx\sqrt[4]{bx^2+ax^4}+\frac{1}{4}ax^3\sqrt[4]{bx^2+ax^4}-\frac{\left(2abx^{3/2}(b+ax^2)^{3/4}\right)}{(bx}$$
$$= \frac{2b\sqrt[4]{bx^2+ax^4}}{x}+\frac{1}{16}bx\sqrt[4]{bx^2+ax^4}+\frac{1}{4}ax^3\sqrt[4]{bx^2+ax^4}-\frac{\left(\sqrt{a}\,bx^{3/2}(b+ax^2)^{3/4}\right)}{(b}$$
$$= \frac{2b\sqrt[4]{bx^2+ax^4}}{x}+\frac{1}{16}bx\sqrt[4]{bx^2+ax^4}+\frac{1}{4}ax^3\sqrt[4]{bx^2+ax^4}+\frac{\sqrt[4]{a}\,bx^{3/2}(b+ax^2)^{3/4}\,\mathrm{t}}{(bx^2+ax^4)}$$
$$= \frac{2b\sqrt[4]{bx^2+ax^4}}{x}+\frac{1}{16}bx\sqrt[4]{bx^2+ax^4}+\frac{1}{4}ax^3\sqrt[4]{bx^2+ax^4}+\frac{\sqrt[4]{a}\,bx^{3/2}(b+ax^2)^{3/4}\,\mathrm{t}}{(bx^2+ax^4)}$$

Mathematica [C] time = 0.15, size = 108, normalized size = 0.89

$$\frac{\sqrt[4]{x^2(ax^2+b)}\left(x^2\left((ax^2+b)\sqrt[4]{\frac{ax^2}{b}+1}-b\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{ax^2}{b}\right)\right)+8b\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};-\frac{ax^2}{b}\right)\right)}{4x\sqrt[4]{\frac{ax^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + a*x^4)*(b*x^2 + a*x^4)^(1/4))/x^2,x]
```

```
[Out] ((x^2*(b + a*x^2))^(1/4)*(8*b*Hypergeometric2F1[-1/4, -1/4, 3/4, -((a*x^2)/
b)] + x^2*((b + a*x^2)*(1 + (a*x^2)/b)^(1/4) - b*Hypergeometric2F1[-1/4, 3/
4, 7/4, -((a*x^2)/b)])))/(4*x*(1 + (a*x^2)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.54, size = 122, normalized size = 1.00

$$\frac{(32ab+3b^2)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{32a^{3/4}}+\frac{(-32ab-3b^2)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{32a^{3/4}}+\frac{\sqrt[4]{ax^4+bx^2}\,(4ax^4+bx^2+32b)}{16x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^4)*(b*x^2 + a*x^4)^(1/4))/x^2,x]
```

[Out] ((b*x^2 + a*x^4)^(1/4)*(32*b + b*x^2 + 4*a*x^4))/(16*x) + ((32*a*b + 3*b^2)*ArcTan[(a^(1/4)*x)/(b*x^2 + a*x^4)^(1/4)])/(32*a^(3/4)) + ((-32*a*b - 3*b^2)*ArcTanh[(a^(1/4)*x)/(b*x^2 + a*x^4)^(1/4)])/(32*a^(3/4))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^4-b)*(a*x^4+b*x^2)^(1/4)/x^2,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 0.38, size = 277, normalized size = 2.27

$$\frac{\frac{8\left(\left(a+\frac{b}{x^2}\right)^{\frac{5}{4}}b^3+3\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}ab^3\right)x^4}{b^2}+256\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}b^2+\frac{2\sqrt{2}\left(32ab^2+3b^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}}+\frac{2\sqrt{2}\left(32ab^2+3b^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}}+\frac{\sqrt{2}\left(32ab^2+3b^3\right)\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)}{(-a)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(32ab^2+3b^3\right)\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)}{(-a)^{\frac{3}{4}}}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^4-b)*(a*x^4+b*x^2)^(1/4)/x^2,x, algorithm="giac")

[Out] 1/128*(8*((a + b/x^2)^(5/4)*b^3 + 3*(a + b/x^2)^(1/4)*a*b^3)*x^4/b^2 + 256*(a + b/x^2)^(1/4)*b^2 + 2*sqrt(2)*(32*a*b^2 + 3*b^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^2)^(1/4))/(-a)^(1/4))/(-a)^(3/4) + 2*sqrt(2)*(32*a*b^2 + 3*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(a + b/x^2)^(1/4))/(-a)^(1/4))/(-a)^(3/4) + sqrt(2)*(32*a*b^2 + 3*b^3)*log(sqrt(2)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sqrt(a + b/x^2))/(-a)^(3/4) - sqrt(2)*(32*a*b^2 + 3*b^3)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sqrt(a + b/x^2))/(-a)^(3/4))/b

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{4}-b\right)\left(a\,x^{4}+b\,x^{2}\right)^{\frac{1}{4}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^4-b)*(a*x^4+b*x^2)^(1/4)/x^2,x)

[Out] int((a*x^4-b)*(a*x^4+b*x^2)^(1/4)/x^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{4}+bx^{2}\right)^{\frac{1}{4}}\left(ax^{4}-b\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^4-b)*(a*x^4+b*x^2)^(1/4)/x^2,x, algorithm="maxima")

[Out] integrate((a*x^4 + b*x^2)^(1/4)*(a*x^4 - b)/x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(b-a\,x^4\right)\,{\left(a\,x^4+b\,x^2\right)}^{1/4}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b - a*x^4)*(a*x^4 + b*x^2)^(1/4))/x^2,x)
[Out] -int(((b - a*x^4)*(a*x^4 + b*x^2)^(1/4))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2 \left(ax^2 + b\right)} \left(ax^4 - b\right)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b)*(a*x**4+b*x**2)**(1/4)/x**2,x)
[Out] Integral((x**2*(a*x**2 + b))**(1/4)*(a*x**4 - b)/x**2, x)
```

3.1512 $\int \frac{(b+ax^4)\sqrt[4]{bx^2+ax^4}}{x^2}\,dx$

Optimal. Leaf size=122

$$\frac{(3b^2-32ab)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{32a^{3/4}}+\frac{(32ab-3b^2)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{32a^{3/4}}+\frac{\sqrt[4]{ax^4+bx^2}\,(4ax^4+bx^2-32b)}{16x}$$

Rubi [B] time = 0.51, antiderivative size = 321, normalized size of antiderivative = 2.63, number of steps used = 17, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2052, 2020, 2032, 329, 331, 298, 203, 206, 2021, 2024}

$$\frac{3b^2x^{3/2}(ax^2+b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{32a^{3/4}(ax^4+bx^2)^{3/4}}-\frac{3b^2x^{3/2}(ax^2+b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{32a^{3/4}(ax^4+bx^2)^{3/4}}+\frac{1}{16}bx\sqrt[4]{ax^4+bx^2}-\frac{2b\sqrt[4]{ax^4+bx^2}}{x}-\frac{\sqrt[4]{a}\,bx^{3/2}(ax^2+b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{(ax^4+bx^2)^{3/4}}+\frac{\sqrt[4]{a}\,bx^{3/2}(ax^2+b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{(ax^4+bx^2)^{3/4}}+\frac{1}{4}ax^3\sqrt[4]{ax^4+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((b + a*x^4)*(b*x^2 + a*x^4)^(1/4))/x^2,x]
```

```
[Out] (-2*b*(b*x^2 + a*x^4)^(1/4))/x + (b*x*(b*x^2 + a*x^4)^(1/4))/16 + (a*x^3*(b
*x^2 + a*x^4)^(1/4))/4 - (a^(1/4)*b*x^(3/2)*(b + a*x^2)^(3/4)*ArcTan[(a^(1/
4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(b*x^2 + a*x^4)^(3/4) + (3*b^2*x^(3/2)*(b +
 a*x^2)^(3/4)*ArcTan[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(32*a^(3/4)*(b*x
^2 + a*x^4)^(3/4)) + (a^(1/4)*b*x^(3/2)*(b + a*x^2)^(3/4)*ArcTanh[(a^(1/4)*
Sqrt[x])/(b + a*x^2)^(1/4)])/(b*x^2 + a*x^4)^(3/4) - (3*b^2*x^(3/2)*(b + a*
x^2)^(3/4)*ArcTanh[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(32*a^(3/4)*(b*x^2
 + a*x^4)^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\left(b+ax^4\right)\sqrt[4]{bx^2+ax^4}}{x^2}\,dx = \int \left(\frac{b\sqrt[4]{bx^2+ax^4}}{x^2} + ax^2\sqrt[4]{bx^2+ax^4}\right) dx$$

$$= a\int x^2\sqrt[4]{bx^2+ax^4}\,dx + b\int \frac{\sqrt[4]{bx^2+ax^4}}{x^2}\,dx$$

$$= -\frac{2b\sqrt[4]{bx^2+ax^4}}{x} + \frac{1}{4}ax^3\sqrt[4]{bx^2+ax^4} + \frac{1}{8}(ab)\int \frac{x^4}{\left(bx^2+ax^4\right)^{3/4}}\,dx + (ab)\int \frac{}{(b}$$

$$= -\frac{2b\sqrt[4]{bx^2+ax^4}}{x} + \frac{1}{16}bx\sqrt[4]{bx^2+ax^4} + \frac{1}{4}ax^3\sqrt[4]{bx^2+ax^4} - \frac{1}{32}\left(3b^2\right)\int \frac{x}{\left(bx^2+\right.}$$

$$= -\frac{2b\sqrt[4]{bx^2+ax^4}}{x} + \frac{1}{16}bx\sqrt[4]{bx^2+ax^4} + \frac{1}{4}ax^3\sqrt[4]{bx^2+ax^4} + \frac{\left(2abx^{3/2}\left(b+ax^2\right)^{3}\right.}{(}$$

$$= -\frac{2b\sqrt[4]{bx^2+ax^4}}{x} + \frac{1}{16}bx\sqrt[4]{bx^2+ax^4} + \frac{1}{4}ax^3\sqrt[4]{bx^2+ax^4} + \frac{\left(2abx^{3/2}\left(b+ax^2\right)^{3}\right.}{(}$$

$$= -\frac{2b\sqrt[4]{bx^2+ax^4}}{x} + \frac{1}{16}bx\sqrt[4]{bx^2+ax^4} + \frac{1}{4}ax^3\sqrt[4]{bx^2+ax^4} + \frac{\left(\sqrt{a}\,bx^{3/2}\left(b+ax^2\right)\right.}{}$$

$$= -\frac{2b\sqrt[4]{bx^2+ax^4}}{x} + \frac{1}{16}bx\sqrt[4]{bx^2+ax^4} + \frac{1}{4}ax^3\sqrt[4]{bx^2+ax^4} - \frac{\sqrt[4]{a}\,bx^{3/2}\left(b+ax^2\right)^{3}}{\left(bx^2+a\right.}$$

$$= -\frac{2b\sqrt[4]{bx^2+ax^4}}{x} + \frac{1}{16}bx\sqrt[4]{bx^2+ax^4} + \frac{1}{4}ax^3\sqrt[4]{bx^2+ax^4} - \frac{\sqrt[4]{a}\,bx^{3/2}\left(b+ax^2\right)^{3}}{\left(bx^2+a\right.}$$

Mathematica [C] time = 0.08, size = 108, normalized size = 0.89

$$\frac{\sqrt[4]{x^2\left(ax^2+b\right)}\left(x^2\left(\left(ax^2+b\right)\sqrt[4]{\frac{ax^2}{b}+1} - b\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{ax^2}{b}\right)\right) - 8b\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};-\frac{ax^2}{b}\right)\right)}{4x\sqrt[4]{\frac{ax^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b + a*x^4)*(b*x^2 + a*x^4)^(1/4))/x^2,x]
```

```
[Out] ((x^2*(b + a*x^2))^(1/4)*(-8*b*Hypergeometric2F1[-1/4, -1/4, 3/4, -((a*x^2)
/b)] + x^2*((b + a*x^2)*(1 + (a*x^2)/b)^(1/4) - b*Hypergeometric2F1[-1/4, 3
/4, 7/4, -((a*x^2)/b)])))/(4*x*(1 + (a*x^2)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.51, size = 122, normalized size = 1.00

$$\frac{\left(3b^2-32ab\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{32a^{3/4}} + \frac{\left(32ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{32a^{3/4}} + \frac{\sqrt[4]{ax^4+bx^2}\left(4ax^4+bx^2-32b\right)}{16x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b + a*x^4)*(b*x^2 + a*x^4)^(1/4))/x^2,x]
```

```
[Out] ((b*x^2 + a*x^4)^(1/4)*(-32*b + b*x^2 + 4*a*x^4))/(16*x) + ((-32*a*b + 3*b^
2)*ArcTan[(a^(1/4)*x)/(b*x^2 + a*x^4)^(1/4)])/(32*a^(3/4)) + ((32*a*b - 3*b
^2)*ArcTanh[(a^(1/4)*x)/(b*x^2 + a*x^4)^(1/4)])/(32*a^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)*(a*x^4+b*x^2)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.30, size = 277, normalized size = 2.27

$$\frac{\frac{8\left(\left(a+\frac{b}{x^2}\right)^{\frac{5}{4}}b^3+3\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}ab^3\right)x^4}{b^2}-256\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}b^2-\frac{2\sqrt{2}\left(32\,ab^2-3\,b^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}}-\frac{2\sqrt{2}\left(32\,ab^2-3\,b^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(32\,ab^2-3\,b^3\right)\log\left(\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)}{(-a)^{\frac{3}{4}}}+\frac{\sqrt{2}\left(32\,ab^2-3\,b^3\right)\log\left(-\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)}{(-a)^{\frac{3}{4}}}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)*(a*x^4+b*x^2)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/128*(8*((a + b/x^2)^(5/4)*b^3 + 3*(a + b/x^2)^(1/4)*a*b^3)*x^4/b^2 - 256*
(a + b/x^2)^(1/4)*b^2 - 2*sqrt(2)*(32*a*b^2 - 3*b^3)*arctan(1/2*sqrt(2)*(sq
rt(2)*(-a)^(1/4) + 2*(a + b/x^2)^(1/4))/(-a)^(1/4))/(-a)^(3/4) - 2*sqrt(2)*
(32*a*b^2 - 3*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(a + b/x^2)^
(1/4))/(-a)^(1/4))/(-a)^(3/4) - sqrt(2)*(32*a*b^2 - 3*b^3)*log(sqrt(2)*(-a)
^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sqrt(a + b/x^2))/(-a)^(3/4) + sqrt(2)
*(32*a*b^2 - 3*b^3)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) +
sqrt(a + b/x^2))/(-a)^(3/4))/b
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^4+b\right)\left(a\,x^4+b\,x^2\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b)*(a*x^4+b*x^2)^(1/4)/x^2,x)
```

```
[Out] int((a*x^4+b)*(a*x^4+b*x^2)^(1/4)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4+bx^2\right)^{\frac{1}{4}}\left(ax^4+b\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)*(a*x^4+b*x^2)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + b*x^2)^(1/4)*(a*x^4 + b)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^4+b\right)\,{\left(a\,x^4+b\,x^2\right)}^{1/4}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a*x^4)*(a*x^4 + b*x^2)^(1/4))/x^2,x)
[Out] int(((b + a*x^4)*(a*x^4 + b*x^2)^(1/4))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2 \left(ax^2+b\right)} \left(ax^4+b\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b)*(a*x**4+b*x**2)**(1/4)/x**2,x)
[Out] Integral((x**2*(a*x**2 + b))**(1/4)*(a*x**4 + b)/x**2, x)
```

3.1513 $\int \frac{-b-2ax^4+2x^8}{\sqrt[4]{-b+ax^4}}\,dx$

Optimal. Leaf size=122

$$\frac{(ax^4-b)^{3/4}(-8a^2x+4ax^5+5bx)}{16a^2}+\frac{(5b^2-24a^2b)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{32a^{9/4}}+\frac{(5b^2-24a^2b)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{32a^{9/4}}$$

Rubi [A] time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1411, 388, 240, 212, 206, 203}

$$-\frac{1}{16}x\left(8-\frac{5b}{a^2}\right)(ax^4-b)^{3/4}-\frac{b(24a^2-5b)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{32a^{9/4}}-\frac{b(24a^2-5b)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{32a^{9/4}}+\frac{x^5(ax^4-b)^{3/4}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(-b - 2*a*x^4 + 2*x^8)/(-b + a*x^4)^(1/4),x]
```

```
[Out] -1/16*((8 - (5*b)/a^2)*x*(-b + a*x^4)^(3/4)) + (x^5*(-b + a*x^4)^(3/4))/(4*
a) - ((24*a^2 - 5*b)*b*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(32*a^(9/4))
 - ((24*a^2 - 5*b)*b*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(32*a^(9/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
```

```
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\int \frac{-b-2ax^4+2x^8}{\sqrt[4]{-b+ax^4}}\,dx = \frac{x^5\left(-b+ax^4\right)^{3/4}}{4a} + \frac{\int \frac{-8ab-\left(16a^2-10b\right)x^4}{\sqrt[4]{-b+ax^4}}\,dx}{8a}$$

$$= -\frac{1}{16}\left(8-\frac{5b}{a^2}\right)x\left(-b+ax^4\right)^{3/4} + \frac{x^5\left(-b+ax^4\right)^{3/4}}{4a} - \frac{\left(32a^2b-b\left(-16a^2+10b\right)\right)\int \frac{}{\sqrt[4]{-b}}}{32a^2}$$

$$= -\frac{1}{16}\left(8-\frac{5b}{a^2}\right)x\left(-b+ax^4\right)^{3/4} + \frac{x^5\left(-b+ax^4\right)^{3/4}}{4a} - \frac{\left(32a^2b-b\left(-16a^2+10b\right)\right)\mathrm{Subst}}{32a^2}$$

$$= -\frac{1}{16}\left(8-\frac{5b}{a^2}\right)x\left(-b+ax^4\right)^{3/4} + \frac{x^5\left(-b+ax^4\right)^{3/4}}{4a} - \frac{\left(32a^2b-b\left(-16a^2+10b\right)\right)\mathrm{Subst}}{64a^2}$$

$$= -\frac{1}{16}\left(8-\frac{5b}{a^2}\right)x\left(-b+ax^4\right)^{3/4} + \frac{x^5\left(-b+ax^4\right)^{3/4}}{4a} - \frac{\left(24a^2-5b\right)b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{32a^{9/4}} -$$

Mathematica [A] time = 0.10, size = 112, normalized size = 0.92

$$\frac{2\sqrt[4]{a}\,x\left(ax^4-b\right)^{3/4}\left(-8a^2+4ax^4+5b\right)-b\left(24a^2-5b\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)-b\left(24a^2-5b\right)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{32a^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b - 2*a*x^4 + 2*x^8)/(-b + a*x^4)^(1/4),x]
```

```
[Out] (2*a^(1/4)*x*(-b + a*x^4)^(3/4)*(-8*a^2 + 5*b + 4*a*x^4) - (24*a^2 - 5*b)*b
*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)] - (24*a^2 - 5*b)*b*ArcTanh[(a^(1/4)
*x)/(-b + a*x^4)^(1/4)])/(32*a^(9/4))
```

IntegrateAlgebraic [A] time = 0.77, size = 122, normalized size = 1.00

$$\frac{\left(ax^4-b\right)^{3/4}\left(-8a^2x+4ax^5+5bx\right)}{16a^2} + \frac{\left(5b^2-24a^2b\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{32a^{9/4}} + \frac{\left(5b^2-24a^2b\right)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{32a^{9/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b - 2*a*x^4 + 2*x^8)/(-b + a*x^4)^(1/4),x]
```

```
[Out] ((-b + a*x^4)^(3/4)*(-8*a^2*x + 5*b*x + 4*a*x^5))/(16*a^2) + ((-24*a^2*b +
5*b^2)*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(32*a^(9/4)) + ((-24*a^2*b +
 5*b^2)*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(32*a^(9/4))
```

fricas [B] time = 0.63, size = 726, normalized size = 5.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-2*a*x^4-b)/(a*x^4-b)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/64*(4*a^2*((331776*a^8*b^4 - 276480*a^6*b^5 + 86400*a^4*b^6 - 12000*a^2*b
^7 + 625*b^8)/a^9)^(1/4)*arctan((a^2*x*sqrt(((331776*a^13*b^4 - 276480*a^11
*b^5 + 86400*a^9*b^6 - 12000*a^7*b^7 + 625*a^5*b^8)*x^2*sqrt((331776*a^8*b^
4 - 276480*a^6*b^5 + 86400*a^4*b^6 - 12000*a^2*b^7 + 625*b^8)/a^9) + (19110
2976*a^12*b^6 - 238878720*a^10*b^7 + 124416000*a^8*b^8 - 34560000*a^6*b^9 +
 5400000*a^4*b^10 - 450000*a^2*b^11 + 15625*b^12)*sqrt(a*x^4 - b))/x^2)*((3
31776*a^8*b^4 - 276480*a^6*b^5 + 86400*a^4*b^6 - 12000*a^2*b^7 + 625*b^8)/a
^9)^(1/4) + (13824*a^8*b^3 - 8640*a^6*b^4 + 1800*a^4*b^5 - 125*a^2*b^6)*(a*
x^4 - b)^(1/4)*((331776*a^8*b^4 - 276480*a^6*b^5 + 86400*a^4*b^6 - 12000*a^
2*b^7 + 625*b^8)/a^9)^(1/4))/((331776*a^8*b^4 - 276480*a^6*b^5 + 86400*a^4*
b^6 - 12000*a^2*b^7 + 625*b^8)*x)) - a^2*((331776*a^8*b^4 - 276480*a^6*b^5
+ 86400*a^4*b^6 - 12000*a^2*b^7 + 625*b^8)/a^9)^(1/4)*log(-(a^7*x*((331776*
a^8*b^4 - 276480*a^6*b^5 + 86400*a^4*b^6 - 12000*a^2*b^7 + 625*b^8)/a^9)^(3
/4) + (13824*a^6*b^3 - 8640*a^4*b^4 + 1800*a^2*b^5 - 125*b^6)*(a*x^4 - b)^(
1/4))/x) + a^2*((331776*a^8*b^4 - 276480*a^6*b^5 + 86400*a^4*b^6 - 12000*a^
2*b^7 + 625*b^8)/a^9)^(1/4)*log((a^7*x*((331776*a^8*b^4 - 276480*a^6*b^5 +
86400*a^4*b^6 - 12000*a^2*b^7 + 625*b^8)/a^9)^(3/4) - (13824*a^6*b^3 - 8640
*a^4*b^4 + 1800*a^2*b^5 - 125*b^6)*(a*x^4 - b)^(1/4))/x) + 4*(4*a*x^5 - (8*
a^2 - 5*b)*x)*(a*x^4 - b)^(3/4))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^8-2\,ax^4-b}{\left(ax^4-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-2*a*x^4-b)/(a*x^4-b)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((2*x^8 - 2*a*x^4 - b)/(a*x^4 - b)^(1/4), x)
```

maple [F] time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{2x^8-2a\,x^4-b}{\left(a\,x^4-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8-2*a*x^4-b)/(a*x^4-b)^(1/4),x)
```

```
[Out] int((2*x^8-2*a*x^4-b)/(a*x^4-b)^(1/4),x)
```

maxima [B] time = 0.73, size = 361, normalized size = 2.96

$$\frac{1}{8}a\left(\frac{b\left(\frac{2\arctan\left(\frac{(ax^4-b)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{(ax^4-b)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{(ax^4-b)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right)}{a}-\frac{4\left(ax^4-b\right)^{\frac{3}{4}}b}{\left(a^2-\frac{(ax^4-b)a}{x^4}\right)x^3}\right)+\frac{1}{4}b\left(\frac{2\arctan\left(\frac{(ax^4-b)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{(ax^4-b)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{(ax^4-b)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right)-\frac{5b^2\left(\frac{2\arctan\left(\frac{(ax^4-b)^{\frac{1}{4}}}{a^{\frac{1}{4}}x}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(-\frac{a^{\frac{1}{4}}-\frac{(ax^4-b)^{\frac{1}{4}}}{x}}{a^{\frac{1}{4}}+\frac{(ax^4-b)^{\frac{1}{4}}}{x}}\right)}{a^{\frac{1}{4}}}\right)}{64\,a^2}+\frac{\frac{9(ax^4-b)^{\frac{3}{4}}ab^2}{x^3}-\frac{5(ax^4-b)^{\frac{7}{4}}b^2}{x^7}}{16\left(a^4-\frac{2(ax^4-b)a^3}{x^4}+\frac{(ax^4-b)^2a^2}{x^8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-2*a*x^4-b)/(a*x^4-b)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/8*a*(b*(2*arctan((a*x^4 - b)^(1/4)/(a^(1/4)*x))/a^(1/4) + log(-(a^(1/4) -
 (a*x^4 - b)^(1/4)/x)/(a^(1/4) + (a*x^4 - b)^(1/4)/x))/a^(1/4))/a - 4*(a*x^
4 - b)^(3/4)*b/((a^2 - (a*x^4 - b)*a/x^4)*x^3)) + 1/4*b*(2*arctan((a*x^4 -
b)^(1/4)/(a^(1/4)*x))/a^(1/4) + log(-(a^(1/4) - (a*x^4 - b)^(1/4)/x)/(a^(1/
```

```
4) + (a*x^4 - b)^(1/4)/x))/a^(1/4)) - 5/64*b^2*(2*arctan((a*x^4 - b)^(1/4)/
(a^(1/4)*x))/a^(1/4) + log(-(a^(1/4) - (a*x^4 - b)^(1/4)/x)/(a^(1/4) + (a*x
^4 - b)^(1/4)/x))/a^(1/4))/a^2 + 1/16*(9*(a*x^4 - b)^(3/4)*a*b^2/x^3 - 5*(a
*x^4 - b)^(7/4)*b^2/x^7)/(a^4 - 2*(a*x^4 - b)*a^3/x^4 + (a*x^4 - b)^2*a^2/x
^8)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{-2\,x^8+2\,a\,x^4+b}{{\left(a\,x^4-b\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + 2*a*x^4 - 2*x^8)/(a*x^4 - b)^(1/4),x)
```

```
[Out] int(-(b + 2*a*x^4 - 2*x^8)/(a*x^4 - b)^(1/4), x)
```

sympy [C] time = 3.36, size = 122, normalized size = 1.00

$$\frac{ax^5e^{\frac{3i\pi}{4}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{ax^4}{b}\right)}{2\sqrt[4]{b}\Gamma\left(\frac{9}{4}\right)} - \frac{b^{\frac{3}{4}}xe^{-\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{ax^4}{b}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{x^9e^{-\frac{i\pi}{4}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{ax^4}{b}\right)}{2\sqrt[4]{b}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8-2*a*x**4-b)/(a*x**4-b)**(1/4),x)
```

```
[Out] a*x**5*exp(3*I*pi/4)*gamma(5/4)*hyper((1/4, 5/4), (9/4,), a*x**4/b)/(2*b**(
1/4)*gamma(9/4)) - b**(3/4)*x*exp(-I*pi/4)*gamma(1/4)*hyper((1/4, 1/4), (5/
4,), a*x**4/b)/(4*gamma(5/4)) + x**9*exp(-I*pi/4)*gamma(9/4)*hyper((1/4, 9/
4), (13/4,), a*x**4/b)/(2*b**(1/4)*gamma(13/4))
```

3.1514 $\int \frac{-1+x}{\left(-4-2x+x^2\right)\sqrt[3]{-2-2x+x^2}}\,dx$

Optimal. Leaf size=123

$$\frac{\log\left(2^{2/3}\sqrt[3]{x^2-2x-2}-2\right)}{2\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}\left(x^2-2x-2\right)^{2/3}+2^{2/3}\sqrt[3]{x^2-2x-2}+2\right)}{4\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{x^2-2x-2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}$$

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+x}{\left(-4-2x+x^2\right)\sqrt[3]{-2-2x+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x)/((-4 - 2*x + x^2)*(-2 - 2*x + x^2)^(1/3)),x]
```

```
[Out] Defer[Int][(-1 + x)/((-4 - 2*x + x^2)*(-2 - 2*x + x^2)^(1/3)), x]
```

Rubi steps

$$\int \frac{-1+x}{\left(-4-2x+x^2\right)\sqrt[3]{-2-2x+x^2}}\,dx=\int \frac{-1+x}{\left(-4-2x+x^2\right)\sqrt[3]{-2-2x+x^2}}\,dx$$

Mathematica [A] time = 0.08, size = 78, normalized size = 0.63

$$\frac{-\log\left(x^2-2x-4\right)+3\log\left(\sqrt[3]{2}-\sqrt[3]{x^2-2x-2}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{x^2-2x-2}+1}{\sqrt{3}}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/((-4 - 2*x + x^2)*(-2 - 2*x + x^2)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(-2 - 2*x + x^2)^(1/3))/Sqrt[3]] - Log[-4 - 
2*x + x^2] + 3*Log[2^(1/3) - (-2 - 2*x + x^2)^(1/3)])/(4*2^(1/3))
```

IntegrateAlgebraic [A] time = 0.23, size = 123, normalized size = 1.00

$$\frac{\log\left(2^{2/3}\sqrt[3]{x^2-2x-2}-2\right)}{2\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}\left(x^2-2x-2\right)^{2/3}+2^{2/3}\sqrt[3]{x^2-2x-2}+2\right)}{4\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{x^2-2x-2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/((-4 - 2*x + x^2)*(-2 - 2*x + x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2^(2/3)*(-2 - 2*x + x^2)^(1/3))/Sqrt[3]])/(2*2
^(1/3)) + Log[-2 + 2^(2/3)*(-2 - 2*x + x^2)^(1/3)]/(2*2^(1/3)) - Log[2 + 2^
(2/3)*(-2 - 2*x + x^2)^(1/3) + 2^(1/3)*(-2 - 2*x + x^2)^(2/3)]/(4*2^(1/3))
```

fricas [A] time = 0.67, size = 93, normalized size = 0.76

$$\frac{1}{4}\,\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}+2\,\sqrt{2}\left(x^2-2\,x-2\right)^{\frac{1}{3}}\right)\right)-\frac{1}{8}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(x^2-2\,x-2\right)^{\frac{1}{3}}+\left(x^2-2\,x-2\right)^{\frac{2}{3}}\right)+\frac{1}{4}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(x^2-2\,x-2\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-2*x-4)/(x^2-2*x-2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6) + 2*sqrt(2)*(x^2 -
2*x - 2)^(1/3))) - 1/8*2^(2/3)*log(2^(2/3) + 2^(1/3)*(x^2 - 2*x - 2)^(1/3)
+ (x^2 - 2*x - 2)^(2/3)) + 1/4*2^(2/3)*log(-2^(1/3) + (x^2 - 2*x - 2)^(1/3)
)
```

giac [A] time = 0.30, size = 84, normalized size = 0.68

$$\frac{1}{4}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{2}x^2-x-1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{8}\cdot 2^{\frac{2}{3}}\log\left(\left(\frac{1}{2}x^2-x-1\right)^{\frac{2}{3}}+\left(\frac{1}{2}x^2-x-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{4}\cdot 2^{\frac{2}{3}}\log\left(\left|\left(\frac{1}{2}x^2-x-1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-2*x-4)/(x^2-2*x-2)^(1/3),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(3)*2^(2/3)*arctan(1/3*sqrt(3)*(2*(1/2*x^2 - x - 1)^(1/3) + 1)) - 1
/8*2^(2/3)*log((1/2*x^2 - x - 1)^(2/3) + (1/2*x^2 - x - 1)^(1/3) + 1) + 1/4
*2^(2/3)*log(abs((1/2*x^2 - x - 1)^(1/3) - 1))
```

maple [C] time = 9.90, size = 1042, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(x^2-2*x-4)/(x^2-2*x-2)^(1/3),x)
```

```
[Out] RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)*ln((RootOf(RootOf(_Z^3
-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)*RootOf(_Z^3-4)^3*x^2+2*RootOf(RootOf(_Z^
3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)^2*RootOf(_Z^3-4)^2*x^2-2*RootOf(RootOf(
_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)*RootOf(_Z^3-4)^3*x+3*(x^2-2*x-2)^(2/
3)*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)*RootOf(_Z^3-4)^2-4*
RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)^2*RootOf(_Z^3-4)^2*x+6
*(x^2-2*x-2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)*Roo
tOf(_Z^3-4)+RootOf(_Z^3-4)*x^2+2*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4
)+16*_Z^2)*x^2-2*RootOf(_Z^3-4)*x-4*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^
3-4)+16*_Z^2)*x-2*RootOf(_Z^3-4)-4*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3
-4)+16*_Z^2))/(x^2-2*x-4))-1/4*ln(-(RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^
3-4)+16*_Z^2)*RootOf(_Z^3-4)^3*x^2-4*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z
^3-4)+16*_Z^2)^2*RootOf(_Z^3-4)^2*x^2-2*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf
(_Z^3-4)+16*_Z^2)*RootOf(_Z^3-4)^3*x+6*(x^2-2*x-2)^(2/3)*RootOf(RootOf(_Z^3
-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)*RootOf(_Z^3-4)^2+8*RootOf(RootOf(_Z^3-4)
^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)^2*RootOf(_Z^3-4)^2*x+3*(x^2-2*x-2)^(1/3)*Ro
otOf(_Z^3-4)^2+12*(x^2-2*x-2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^
3-4)+16*_Z^2)*RootOf(_Z^3-4)+6*(x^2-2*x-2)^(2/3)+2*RootOf(_Z^3-4)-8*RootOf(
RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2))/(x^2-2*x-4))*RootOf(_Z^3-4)-
ln(-(RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)*RootOf(_Z^3-4)^3*
x^2-4*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)^2*RootOf(_Z^3-4)
^2*x^2-2*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)*RootOf(_Z^3-4
)^3*x+6*(x^2-2*x-2)^(2/3)*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z
^2)*RootOf(_Z^3-4)^2+8*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)
^2*RootOf(_Z^3-4)^2*x+3*(x^2-2*x-2)^(1/3)*RootOf(_Z^3-4)^2+12*(x^2-2*x-2)^(
1/3)*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)*RootOf(_Z^3-4)+6*
(x^2-2*x-2)^(2/3)+2*RootOf(_Z^3-4)-8*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z
^3-4)+16*_Z^2))/(x^2-2*x-4))*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16
*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^2-2\,x-2\right)^{\frac{1}{3}}\left(x^2-2\,x-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-2*x-4)/(x^2-2*x-2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/((x^2 - 2*x - 2)^(1/3)*(x^2 - 2*x - 4)), x)
```

mupad [B] time = 1.19, size = 109, normalized size = 0.89

$$\frac{2^{2/3}\,\ln\left(\frac{9\,{\left(x^2-2\,x-2\right)}^{1/3}}{4}-\frac{9\,2^{1/3}}{4}\right)}{4}+\frac{2^{2/3}\,\ln\left(\frac{9\,{\left(x^2-2\,x-2\right)}^{1/3}}{4}-\frac{9\,2^{1/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{8}-\frac{2^{2/3}\,\ln\left(\frac{9\,{\left(x^2-2\,x-2\right)}^{1/3}}{4}-\frac{9\,2^{1/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((x^2 - 2*x - 2)^(1/3)*(2*x - x^2 + 4)),x)
```

```
[Out] (2^(2/3)*log((9*(x^2 - 2*x - 2)^(1/3))/4 - (9*2^(1/3))/4))/4 + (2^(2/3)*log
((9*(x^2 - 2*x - 2)^(1/3))/4 - (9*2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*
1i - 1))/8 - (2^(2/3)*log((9*(x^2 - 2*x - 2)^(1/3))/4 - (9*2^(1/3)*(3^(1/2)
*1i + 1)^2)/16)*(3^(1/2)*1i + 1))/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^2-2x-4\right)\sqrt[3]{x^2-2x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**2-2*x-4)/(x**2-2*x-2)**(1/3),x)
```

```
[Out] Integral((x - 1)/((x**2 - 2*x - 4)*(x**2 - 2*x - 2)**(1/3)), x)
```

3.1515 $\int \frac{1}{(1+x)\sqrt[3]{3+2x+x^2}}\,dx$

Optimal. Leaf size=123

$$\frac{\log\left(2^{2/3}\sqrt[3]{x^2+2x+3}-2\right)}{2\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}\left(x^2+2x+3\right)^{2/3}+2^{2/3}\sqrt[3]{x^2+2x+3}+2\right)}{4\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{x^2+2x+3}}{\sqrt{3}}+\ldots\right)}{2\sqrt[3]{2}}$$

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 0.67, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {694, 266, 55, 617, 204, 31}

$$-\frac{\log(x+1)}{2\sqrt[3]{2}}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{(x+1)^2+2}\right)}{4\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{(x+1)^2+2}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(3 + 2*x + x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2^(2/3)*(2 + (1 + x)^2)^(1/3))/Sqrt[3]])/(2*2^(1/3)) -
 Log[1 + x]/(2*2^(1/3)) + (3*Log[2^(1/3) - (2 + (1 + x)^2)^(1/3)])/(4*2^(1/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
```

```
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(1+x)\sqrt[3]{3+2x+x^2}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt[3]{2+x^2}}\,dx, x, 1+x\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt[3]{2+x}}\,dx, x, (1+x)^2\right)$$
$$= -\frac{\log(1+x)}{2\sqrt[3]{2}} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{2+(1+x)^2}\right) - \frac{3\,\text{Subst}\left(\int \frac{}{\sqrt[3]{2}}\right.}{}$$
$$= -\frac{\log(1+x)}{2\sqrt[3]{2}} + \frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{2+(1+x)^2}\right)}{4\sqrt[3]{2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{2+\right.}{2\sqrt[3]{2}}$$
$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{2+(1+x)^2}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{\log(1+x)}{2\sqrt[3]{2}} + \frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{2+(1+x)^2}\right)}{4\sqrt[3]{2}}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.59

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{x^2+2x+3}+1}{\sqrt{3}}\right)-2\log(x+1)+3\log\left(\sqrt[3]{2}-\sqrt[3]{(x+1)^2+2}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(3 + 2*x + x^2)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(3 + 2*x + x^2)^(1/3))/Sqrt[3]] - 2*Log[1 +
x] + 3*Log[2^(1/3) - (2 + (1 + x)^2)^(1/3)])/(4*2^(1/3))
```

IntegrateAlgebraic [A] time = 0.22, size = 123, normalized size = 1.00

$$\frac{\log\left(2^{2/3}\sqrt[3]{x^2+2x+3}-2\right)}{2\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}\left(x^2+2x+3\right)^{2/3}+2^{2/3}\sqrt[3]{x^2+2x+3}+2\right)}{4\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{x^2+2x+3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x)*(3 + 2*x + x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2^(2/3)*(3 + 2*x + x^2)^(1/3))/Sqrt[3]])/(2*2^
(1/3)) + Log[-2 + 2^(2/3)*(3 + 2*x + x^2)^(1/3)]/(2*2^(1/3)) - Log[2 + 2^(2
/3)*(3 + 2*x + x^2)^(1/3) + 2^(1/3)*(3 + 2*x + x^2)^(2/3)]/(4*2^(1/3))
```

fricas [A] time = 0.59, size = 93, normalized size = 0.76

$$\frac{1}{4}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}+2\sqrt{2}\left(x^2+2x+3\right)^{\frac{1}{3}}\right)\right)-\frac{1}{8}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(x^2+2x+3\right)^{\frac{1}{3}}+\left(x^2+2x+3\right)^{\frac{2}{3}}\right)+\frac{1}{4}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(x^2+2x+3\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x+3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6) + 2*sqrt(2)*(x^2 +
2*x + 3)^(1/3))) - 1/8*2^(2/3)*log(2^(2/3) + 2^(1/3)*(x^2 + 2*x + 3)^(1/3)
```

```
+ (x^2 + 2*x + 3)^(2/3)) + 1/4*2^(2/3)*log(-2^(1/3) + (x^2 + 2*x + 3)^(1/3)
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+2\,x+3\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x+3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 2*x + 3)^(1/3)*(x + 1)), x)
```

maple [C] time = 9.08, size = 805, normalized size = 6.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^2+2*x+3)^(1/3),x)
```

```
[Out] 1/2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*ln(-(-RootOf(RootOf
(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x^2-16*RootOf(RootO
f(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+48*(x^2+2*x+
3)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)
^2-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x
-32*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*
x-48*(x^2+2*x+3)^(1/3)*RootOf(_Z^3-4)^2+30*(x^2+2*x+3)^(1/3)*RootOf(RootOf(
_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)-5*RootOf(_Z^3-4)*x^2-8
0*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2+126*(x^2+2*x+3)^(
2/3)-10*RootOf(_Z^3-4)*x-160*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*
_Z^2)*x-19*RootOf(_Z^3-4)-304*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4
*_Z^2))/(1+x)^2)+1/4*RootOf(_Z^3-4)*ln((-4*RootOf(RootOf(_Z^3-4)^2+2*_Z*Roo
tOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x^2-RootOf(RootOf(_Z^3-4)^2+2*_Z*RootO
f(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+24*(x^2+2*x+3)^(2/3)*RootOf(RootOf
(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2-8*RootOf(RootOf(_Z^
3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x-2*RootOf(RootOf(_Z^3-
4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-24*(x^2+2*x+3)^(1/3)*
RootOf(_Z^3-4)^2-63*(x^2+2*x+3)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_
Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)+12*RootOf(_Z^3-4)*x^2+3*RootOf(RootOf(_Z^3-4)
^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-15*(x^2+2*x+3)^(2/3)+24*RootOf(_Z^3-4)*x
+6*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+76*RootOf(_Z^3-4)+
19*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(1+x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+2\,x+3\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x+3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 2*x + 3)^(1/3)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x+1)\,\left(x^2+2\,x+3\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)*(2*x + x^2 + 3)^(1/3)),x)
[Out] int(1/((x + 1)*(2*x + x^2 + 3)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)\sqrt[3]{x^2+2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+2*x+3)**(1/3),x)
[Out] Integral(1/((x + 1)*(x**2 + 2*x + 3)**(1/3)), x)
```

3.1516 $\int \frac{1}{x\left(-b+ax^2\right)^{3/4}}\,dx$

Optimal. Leaf size=123

$$\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}\,b^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}-\sqrt{b}}\right)}{\sqrt{2}\,b^{3/4}}$$

Rubi [A] time = 0.20, antiderivative size = 196, normalized size of antiderivative = 1.59, number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {266, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)}{2\sqrt{2}\,b^{3/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)}{2\sqrt{2}\,b^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-b + a*x^2)^(3/4)),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*(-b + a*x^2)^(1/4))/b^(1/4)]/(Sqrt[2]*b^(3/4))) + Arc
Tan[1 + (Sqrt[2]*(-b + a*x^2)^(1/4))/b^(1/4)]/(Sqrt[2]*b^(3/4)) - Log[Sqrt[
b] - Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4) + Sqrt[-b + a*x^2]]/(2*Sqrt[2]*b^(3
/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4) + Sqrt[-b + a*x^2]]
/(2*Sqrt[2]*b^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x\left(-b+ax^2\right)^{3/4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(-b+ax)^{3/4}}\,dx, x, x^2\right)$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^2}\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^2}\right)}{a\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^2}\right)}{a\sqrt{b}}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^2}\right)}{2\sqrt{2}\,b^{3/4}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}-2x}{-\sqrt{b}+\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^2}\right)}{2\sqrt{2}\,b^{3/4}}$$
$$= -\frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^2}+\sqrt{-b+ax^2}\right)}{2\sqrt{2}\,b^{3/4}} + \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^2}+\sqrt{-b+ax^2}\right)}{2\sqrt{2}\,b^{3/4}}$$
$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}} - \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^2}+\sqrt{-b+ax^2}\right)}{2\sqrt{2}\,b^{3/4}}$$

Mathematica [A] time = 0.07, size = 165, normalized size = 1.34

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)+\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}+1\right)}{2\sqrt{2}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-b + a*x^2)^(3/4)),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*(-b + a*x^2)^(1/4))/b^(1/4)] + 2*ArcTan[1 + (Sqrt[2
]*(-b + a*x^2)^(1/4))/b^(1/4)] - Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^2)
```

```
^(1/4) + Sqrt[-b + a*x^2]] + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/
4) + Sqrt[-b + a*x^2]])/(2*Sqrt[2]*b^(3/4))
```

IntegrateAlgebraic [A] time = 0.22, size = 121, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}\,b^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}+\sqrt{b}}\right)}{\sqrt{2}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-b + a*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a*x^2]/(Sqrt[2]*b^(1/4)))/(-b + a*x^
2)^(1/4)]/(Sqrt[2]*b^(3/4)) + ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4))/
(Sqrt[b] + Sqrt[-b + a*x^2])]/(Sqrt[2]*b^(3/4))
```

fricas [A] time = 0.62, size = 134, normalized size = 1.09

$$2\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^2\sqrt{-\frac{1}{b^3}}+\sqrt{ax^2-b}}\,b^2\left(-\frac{1}{b^3}\right)^{\frac{3}{4}}-\left(ax^2-b\right)^{\frac{1}{4}}b^2\left(-\frac{1}{b^3}\right)^{\frac{3}{4}}\right)+\frac{1}{2}\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}+\left(ax^2-b\right)^{\frac{1}{4}}\right)-\frac{1}{2}\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(-b\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}+\left(ax^2-b\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^2-b)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*(-1/b^3)^(1/4)*arctan(sqrt(b^2*sqrt(-1/b^3) + sqrt(a*x^2 - b))*b^2*(-1/b^
3)^(3/4) - (a*x^2 - b)^(1/4)*b^2*(-1/b^3)^(3/4)) + 1/2*(-1/b^3)^(1/4)*log(b
*(-1/b^3)^(1/4) + (a*x^2 - b)^(1/4)) - 1/2*(-1/b^3)^(1/4)*log(-b*(-1/b^3)^(
1/4) + (a*x^2 - b)^(1/4))
```

giac [A] time = 0.17, size = 162, normalized size = 1.32

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{2\,b^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{2\,b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)}{4\,b^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)}{4\,b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^2-b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^2 - b)^(1/4))/b^(1
/4))/b^(3/4) + 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^2
- b)^(1/4))/b^(1/4))/b^(3/4) + 1/4*sqrt(2)*log(sqrt(2)*(a*x^2 - b)^(1/4)*b^
(1/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(3/4) - 1/4*sqrt(2)*log(-sqrt(2)*(a*x^
2 - b)^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(3/4)
```

maple [F] time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a\,x^2-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^2-b)^(3/4),x)
```

```
[Out] int(1/x/(a*x^2-b)^(3/4),x)
```

maxima [A] time = 2.06, size = 162, normalized size = 1.32

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{2\,b^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{2\,b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)}{4\,b^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)}{4\,b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^2-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^2 - b)^(1/4))/b^(1
/4))/b^(3/4) + 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^2
- b)^(1/4))/b^(1/4))/b^(3/4) + 1/4*sqrt(2)*log(sqrt(2)*(a*x^2 - b)^(1/4)*b^
(1/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(3/4) - 1/4*sqrt(2)*log(-sqrt(2)*(a*x^
2 - b)^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(3/4)
```

mupad [B] time = 1.00, size = 44, normalized size = 0.36

$$-\frac{\operatorname{atan}\left(\frac{{\left(a\,x^2-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)+\operatorname{atanh}\left(\frac{{\left(a\,x^2-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{{\left(-b\right)}^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 - b)^(3/4)),x)
```

```
[Out] -(atan((a*x^2 - b)^(1/4)/(-b)^(1/4)) + atanh((a*x^2 - b)^(1/4)/(-b)^(1/4)))
/(-b)^(3/4)
```

sympy [C] time = 0.95, size = 42, normalized size = 0.34

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^2}\right)}{2a^{\frac{3}{4}}x^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**2-b)**(3/4),x)
```

```
[Out] -gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*exp_polar(2*I*pi)/(a*x**2))/(2*a**(
3/4)*x**(3/2)*gamma(7/4))
```

3.1517 $\int \frac{1}{x\sqrt[3]{-b+ax^3}}\,dx$

Optimal. Leaf size=123

$$\frac{\log\left(-\sqrt[3]{b}\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}+b^{2/3}\right)}{6\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{ax^3-b}+\sqrt[3]{b}\right)}{3\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {266, 56, 617, 204, 31}

$$-\frac{\log\left(\sqrt[3]{ax^3-b}+\sqrt[3]{b}\right)}{2\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{\log(x)}{2\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-b + a*x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[(b^(1/3) - 2*(-b + a*x^3)^(1/3))/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*b^(1/
3))) + Log[x]/(2*b^(1/3)) - Log[b^(1/3) + (-b + a*x^3)^(1/3)]/(2*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt[3]{-b+ax^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt[3]{-b+ax}}\,dx,x,x^3\right)\\
&= \frac{\log(x)}{2\sqrt[3]{b}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{b^{2/3}-\sqrt[3]{b}\,x+x^2}\,dx,x,\sqrt[3]{-b+ax^3}\right)-\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+x}\,dx,x,\sqrt[3]{-b+ax}\right.}{2\sqrt[3]{b}}\\
&= \frac{\log(x)}{2\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{-b+ax^3}\right)}{2\sqrt[3]{b}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{-b+ax^3}}{\sqrt[3]{b}}\right)}{\sqrt[3]{b}}\\
&= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-b+ax^3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{\log(x)}{2\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{-b+ax^3}\right)}{2\sqrt[3]{b}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.32

$$
\frac{\left(ax^3-b\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-\frac{ax^3}{b}\right)}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-b + a*x^3)^(1/3)),x]
```

```
[Out] ((-b + a*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, 1 - (a*x^3)/b])/(2*b)
```

IntegrateAlgebraic [A] time = 0.11, size = 123, normalized size = 1.00

$$
\frac{\log\left(-\sqrt[3]{b}\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}+b^{2/3}\right)}{6\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{ax^3-b}+\sqrt[3]{b}\right)}{3\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{b}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-b + a*x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] - (2*(-b + a*x^3)^(1/3))/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*b^(
1/3))) - Log[b^(1/3) + (-b + a*x^3)^(1/3)]/(3*b^(1/3)) + Log[b^(2/3) - b^(1
/3)*(-b + a*x^3)^(1/3) + (-b + a*x^3)^(2/3)]/(6*b^(1/3))
```

fricas [A] time = 0.68, size = 308, normalized size = 2.50

$$
\left[\frac{3\sqrt{\frac{1}{3}}\,b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(\frac{2ax^3+3\sqrt{\frac{1}{3}}\left(2\left(ax^3-b\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}+\left(ax^3-b\right)^{\frac{1}{3}}b+(-b)^{\frac{1}{3}}b\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}-3\left(ax^3-b\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}-3b}{x^3}\right)+(-b)^{\frac{2}{3}}\log\left(\left(ax^3-b\right)^{\frac{2}{3}}+\left(ax^3-b\right)^{\frac{1}{3}}(-b)^{\frac{1}{3}}+(-b)^{\frac{2}{3}}\right)-2(-b)^{\frac{2}{3}}\log\left(\left(ax^3-b\right)^{\frac{1}{3}}-(-b)^{\frac{1}{3}}\right)}{6b},\ \frac{6\sqrt{\frac{1}{3}}\,b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\left(ax^3-b\right)^{\frac{1}{3}}+(-b)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\right)+(-b)^{\frac{2}{3}}\log\left(\left(ax^3-b\right)^{\frac{2}{3}}+\left(ax^3-b\right)^{\frac{1}{3}}(-b)^{\frac{1}{3}}+(-b)^{\frac{2}{3}}\right)-2(-b)^{\frac{2}{3}}\log\left(\left(ax^3-b\right)^{\frac{1}{3}}-(-b)^{\frac{1}{3}}\right)}{6b}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3-b)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*b*sqrt((-b)^(1/3)/b)*log((2*a*x^3 + 3*sqrt(1/3)*(2*(a*x^3
 - b)^(2/3)*(-b)^(2/3) + (a*x^3 - b)^(1/3)*b + (-b)^(1/3)*b)*sqrt((-b)^(1/3
)/b) - 3*(a*x^3 - b)^(1/3)*(-b)^(2/3) - 3*b)/x^3) + (-b)^(2/3)*log((a*x^3 -
 b)^(2/3) + (a*x^3 - b)^(1/3)*(-b)^(1/3) + (-b)^(2/3)) - 2*(-b)^(2/3)*log((
a*x^3 - b)^(1/3) - (-b)^(1/3)))/b, 1/6*(6*sqrt(1/3)*b*sqrt(-(-b)^(1/3)/b)*a
rctan(sqrt(1/3)*(2*(a*x^3 - b)^(1/3) + (-b)^(1/3))*sqrt(-(-b)^(1/3)/b)) + (
-b)^(2/3)*log((a*x^3 - b)^(2/3) + (a*x^3 - b)^(1/3)*(-b)^(1/3) + (-b)^(2/3)
) - 2*(-b)^(2/3)*log((a*x^3 - b)^(1/3) - (-b)^(1/3)))/b]
```

giac [A] time = 2.06, size = 120, normalized size = 0.98

$$-\frac{\sqrt{3}\left(-b\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(ax^{3}-b\right)^{\frac{1}{3}}+\left(-b\right)^{\frac{1}{3}}\right)}{3\left(-b\right)^{\frac{1}{3}}}\right)}{3\,b}+\frac{\left(-b\right)^{\frac{2}{3}}\log\left(\left(ax^{3}-b\right)^{\frac{2}{3}}+\left(ax^{3}-b\right)^{\frac{1}{3}}\left(-b\right)^{\frac{1}{3}}+\left(-b\right)^{\frac{2}{3}}\right)}{6\,b}-\frac{\left(-b\right)^{\frac{2}{3}}\log\left(\left|\left(ax^{3}-b\right)^{\frac{1}{3}}-\left(-b\right)^{\frac{1}{3}}\right|\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3-b)^(1/3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(-b)^(2/3)*arctan(1/3*sqrt(3)*(2*(a*x^3 - b)^(1/3) + (-b)^(1/3
))/(-b)^(1/3))/b + 1/6*(-b)^(2/3)*log((a*x^3 - b)^(2/3) + (a*x^3 - b)^(1/3)
*(-b)^(1/3) + (-b)^(2/3))/b - 1/3*(-b)^(2/3)*log(abs((a*x^3 - b)^(1/3) - (-
b)^(1/3)))/b
```

maple [F] time = 1.15, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a\,x^{3}-b\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^3-b)^(1/3),x)
```

```
[Out] int(1/x/(a*x^3-b)^(1/3),x)
```

maxima [A] time = 1.45, size = 95, normalized size = 0.77

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(ax^{3}-b\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{3\,b^{\frac{1}{3}}}+\frac{\log\left(\left(ax^{3}-b\right)^{\frac{2}{3}}-\left(ax^{3}-b\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{6\,b^{\frac{1}{3}}}-\frac{\log\left(\left(ax^{3}-b\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3-b)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(a*x^3 - b)^(1/3) - b^(1/3))/b^(1/3))/b^(
1/3) + 1/6*log((a*x^3 - b)^(2/3) - (a*x^3 - b)^(1/3)*b^(1/3) + b^(2/3))/b^(
1/3) - 1/3*log((a*x^3 - b)^(1/3) + b^(1/3))/b^(1/3)
```

mupad [B] time = 1.04, size = 118, normalized size = 0.96

$$\frac{\ln\left(\left(a\,x^{3}-b\right)^{1/3}-\left(-b\right)^{1/3}\right)}{3\left(-b\right)^{1/3}}+\frac{\ln\left(\left(a\,x^{3}-b\right)^{1/3}-\frac{\left(-b\right)^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\left(-b\right)^{1/3}}-\frac{\ln\left(\left(a\,x^{3}-b\right)^{1/3}-\frac{\left(-b\right)^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\left(-b\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^3 - b)^(1/3)),x)
```

```
[Out] log((a*x^3 - b)^(1/3) - (-b)^(1/3))/(3*(-b)^(1/3)) + (log((a*x^3 - b)^(1/3)
 - ((-b)^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(6*(-b)^(1/3)) - (l
og((a*x^3 - b)^(1/3) - ((-b)^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))
/(6*(-b)^(1/3))
```

sympy [C] time = 0.89, size = 39, normalized size = 0.32

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^{3}}\right)}{3\sqrt[3]{a}\,x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**3-b)**(1/3),x)
```

```
[Out] -gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*exp_polar(2*I*pi)/(a*x**3))/(3*a**(
1/3)*x*gamma(4/3))
```

3.1518 $\int \frac{-b^3+a^3x^3}{\sqrt{b^2x+a^2x^3}\left(b^3+a^3x^3\right)}\,dx$

Optimal. Leaf size=123

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt{a}\sqrt{b}}-\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt{a}\sqrt{b}}$$

Rubi [C] time = 3.48, antiderivative size = 728, normalized size of antiderivative = 5.92, number of steps used = 19, number of rules used = 11, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2056, 6715, 6725, 220, 2073, 1211, 1699, 205, 6728, 1217, 1707}

$$-\frac{4\sqrt{x}\sqrt{a^2x^2+b^2}\tan^{-1}\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{x}}{\sqrt{a^2x^2+b^2}}\right)}{3\sqrt{-a}\sqrt{b}\sqrt{a^2x^3+b^2x}}-\frac{\sqrt{2}\sqrt{x}\sqrt{a^2x^2+b^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{x}}{\sqrt{a^2x^2+b^2}}\right)}{3\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}+\frac{2\sqrt{x}(ax+b)\sqrt{\frac{a^2x^2+b^2}{(ax+b)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{3\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}-\frac{2\left(a-\sqrt{3}\sqrt{-a^2}\right)\sqrt{x}(ax+b)\sqrt{\frac{a^2x^2+b^2}{(ax+b)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{3\sqrt{a}\left(3a-\sqrt{3}\sqrt{-a^2}\right)\sqrt{b}\sqrt{a^2x^3+b^2x}}-\frac{2\left(\sqrt{3}\sqrt{-a^2}+a\right)\sqrt{x}(ax+b)\sqrt{\frac{a^2x^2+b^2}{(ax+b)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{3\sqrt{a}\left(\sqrt{3}\sqrt{-a^2}+3a\right)\sqrt{b}\sqrt{a^2x^3+b^2x}}-\frac{\sqrt{a}\sqrt{x}(ax+b)\sqrt{\frac{a^2x^2+b^2}{(ax+b)^2}}\Pi\left(\frac{3}{4};2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{3\sqrt{3}\sqrt{-a^2}\sqrt{b}\sqrt{a^2x^3+b^2x}}-\frac{\sqrt{-a^2}\sqrt{x}(ax+b)\sqrt{\frac{a^2x^2+b^2}{(ax+b)^2}}\Pi\left(\frac{3}{4};2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{3\sqrt{3}a^{3/2}\sqrt{b}\sqrt{a^2x^3+b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b^3 + a^3*x^3)/(Sqrt[b^2*x + a^2*x^3]*(b^3 + a^3*x^3)),x]
```

```
[Out] (-4*Sqrt[x]*Sqrt[b^2 + a^2*x^2]*ArcTan[(Sqrt[-a]*Sqrt[b]*Sqrt[x])/Sqrt[b^2
+ a^2*x^2]])/(3*Sqrt[-a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) - (Sqrt[2]*Sqrt[x]*
Sqrt[b^2 + a^2*x^2]*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[x])/Sqrt[b^2 + a^2
*x^2]])/(3*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) + (2*Sqrt[x]*(b + a*x)*Sq
rt[(b^2 + a^2*x^2)/(b + a*x)^2]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b
]], 1/2])/(3*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) - (2*(a - Sqrt[3]*Sqrt[
-a^2])*Sqrt[x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2]*EllipticF[2*ArcT
an[(Sqrt[a]*Sqrt[x])/Sqrt[b]], 1/2])/(3*Sqrt[a]*(3*a - Sqrt[3]*Sqrt[-a^2])*
Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) - (2*(a + Sqrt[3]*Sqrt[-a^2])*Sqrt[x]*(b + a
*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt[x])/
Sqrt[b]], 1/2])/(3*Sqrt[a]*(3*a + Sqrt[3]*Sqrt[-a^2])*Sqrt[b]*Sqrt[b^2*x +
a^2*x^3]) - (Sqrt[a]*Sqrt[x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2]*El
lipticPi[3/4, 2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]], 1/2])/(3*Sqrt[3]*Sqrt[-a
^2]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) - (Sqrt[-a^2]*Sqrt[x]*(b + a*x)*Sqrt[(b^
2 + a^2*x^2)/(b + a*x)^2]*EllipticPi[3/4, 2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b
]], 1/2])/(3*Sqrt[3]*a^(3/2)*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
```

```
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-b^3+a^3x^3}{\sqrt{b^2x+a^2x^3}\left(b^3+a^3x^3\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{b^2+a^2x^2}\right)\int \frac{-b^3+a^3x^3}{\sqrt{x}\sqrt{b^2+a^2x^2}\left(b^3+a^3x^3\right)}\,dx}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{-b^3+a^3x^6}{\sqrt{b^2+a^2x^4}\left(b^3+a^3x^6\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt{b^2+a^2x^4}} - \frac{2b^3}{\sqrt{b^2+a^2x^4}\left(b^3+a^3x^6\right)}\right)dx, x, \sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2+a^2x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}} - \frac{\left(4b^3\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Su}}{\sqrt{}}$$

$$= \frac{\sqrt{x}\,(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\left(4b^3\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\Big(}{}$$

$$= \frac{\sqrt{x}\,(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\left(4b\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\Big(\int}{3\sqrt{b^2x+}}$$

$$= \frac{\sqrt{x}\,(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\Big(\int}{3\sqrt{b^2x+a^2}}$$

$$= \frac{2\sqrt{x}\,(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{3\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\left(2b\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\Big(}{3\sqrt{b^2x+}}$$

$$= -\frac{\sqrt{2}\sqrt{x}\sqrt{b^2+a^2x^2}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{x}}{\sqrt{b^2+a^2x^2}}\right)}{3\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} + \frac{2\sqrt{x}\,(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}\,F\Big(2\tan^{-}}{3\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}}$$

$$= -\frac{4\sqrt{x}\sqrt{b^2+a^2x^2}\,\tan^{-1}\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{x}}{\sqrt{b^2+a^2x^2}}\right)}{3\sqrt{-a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\sqrt{2}\sqrt{x}\sqrt{b^2+a^2x^2}\,\tan^{-1}\Big(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{}}{\sqrt{b^2+a^2x^2}}}{3\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}}$$

Mathematica [C] time = 1.22, size = 208, normalized size = 1.69

$$-\frac{(-1)^{5/6}x^{3/2}\sqrt{\frac{b^2}{a^2x^2}+1}\left(\left(3-3i\sqrt{3}\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)+4\left((-1)^{2/3}\Pi\left(-i;i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)+\left(\sqrt[3]{-1}-1\right)\Pi\left(\sqrt[6]{-1};i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)+(-1)^{2/3}\Pi\left((-1)^{5/6};i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)\right)\right)}{3\sqrt{\frac{ib}{a}}\sqrt{x\left(a^2x^2+b^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b^3 + a^3*x^3)/(Sqrt[b^2*x + a^2*x^3]*(b^3 + a^3*x^3)),x]
```

```
[Out] -1/3*((-1)^(5/6)*Sqrt[1 + b^2/(a^2*x^2)]*x^(3/2)*((3 - (3*I)*Sqrt[3])*Ellip
ticF[I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] + 4*((-1)^(2/3)*EllipticPi[-I, I
*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] + (-1 + (-1)^(1/3))*EllipticPi[(-1)^(1
/6), I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] + (-1)^(2/3)*EllipticPi[(-1)^(5/
6), I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1])))/(Sqrt[(I*b)/a]*Sqrt[x*(b^2 + a
^2*x^2)])
```

IntegrateAlgebraic [A] time = 0.53, size = 123, normalized size = 1.00

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt{a}\sqrt{b}}-\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b^3 + a^3*x^3)/(Sqrt[b^2*x + a^2*x^3]*(b^3 + a^3*x^3))
,x]
```

```
[Out] -1/3*(Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])/(b^2 +
 a^2*x^2)])/(Sqrt[a]*Sqrt[b]) - (4*ArcTanh[(Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^
2*x^3])/(b^2 + a^2*x^2)])/(3*Sqrt[a]*Sqrt[b])
```

fricas [B] time = 0.80, size = 421, normalized size = 3.42

$$\left[-\frac{\sqrt{2}ab\sqrt{\frac{1}{ab}}\arctan\left(\frac{2\sqrt{2}\sqrt{a^2x^3+b^2x}\,ab\sqrt{\frac{1}{ab}}}{a^2x^2-2abx+b^2}\right)-2\sqrt{ab}\log\left(\frac{a^4x^4+6a^3bx^3+3a^2b^2x^2+6ab^3x+b^4-4\sqrt{a^2x^3+b^2x}\left(a^2x^2+abx+b^2\right)\sqrt{ab}}{a^4x^4-2a^3bx^3+3a^2b^2x^2-2ab^3x+b^4}\right)}{6ab},\ \frac{\sqrt{2}ab\sqrt{-\frac{1}{ab}}\log\left(\frac{a^4x^4-12a^3bx^3+6a^2b^2x^2-12ab^3x+b^4+4\sqrt{2}\left(a^3bx^2-2a^2b^2x+ab^3\right)\sqrt{a^2x^3+b^2x}\sqrt{-\frac{1}{ab}}}{a^4x^4+4a^3bx^3+6a^2b^2x^2+4ab^3x+b^4}\right)+8\sqrt{-ab}\arctan\left(\frac{\sqrt{a^2x^3+b^2x}\left(a^2x^2+abx+b^2\right)\sqrt{-ab}}{2\left(a^3bx^3+ab^3x\right)}\right)}{12ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3-b^3)/(a^2*x^3+b^2*x)^(1/2)/(a^3*x^3+b^3),x, algorithm="f
ricas")
```

```
[Out] [-1/6*(sqrt(2)*a*b*sqrt(1/(a*b))*arctan(2*sqrt(2)*sqrt(a^2*x^3 + b^2*x)*a*b
*sqrt(1/(a*b))/(a^2*x^2 - 2*a*b*x + b^2)) - 2*sqrt(a*b)*log((a^4*x^4 + 6*a^
3*b*x^3 + 3*a^2*b^2*x^2 + 6*a*b^3*x + b^4 - 4*sqrt(a^2*x^3 + b^2*x)*(a^2*x^
2 + a*b*x + b^2)*sqrt(a*b))/(a^4*x^4 - 2*a^3*b*x^3 + 3*a^2*b^2*x^2 - 2*a*b^
3*x + b^4)))/(a*b), 1/12*(sqrt(2)*a*b*sqrt(-1/(a*b))*log((a^4*x^4 - 12*a^3*
b*x^3 + 6*a^2*b^2*x^2 - 12*a*b^3*x + b^4 + 4*sqrt(2)*(a^3*b*x^2 - 2*a^2*b^2
*x + a*b^3)*sqrt(a^2*x^3 + b^2*x)*sqrt(-1/(a*b)))/(a^4*x^4 + 4*a^3*b*x^3 +
6*a^2*b^2*x^2 + 4*a*b^3*x + b^4)) + 8*sqrt(-a*b)*arctan(1/2*sqrt(a^2*x^3 +
b^2*x)*(a^2*x^2 + a*b*x + b^2)*sqrt(-a*b)/(a^3*b*x^3 + a*b^3*x)))/(a*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a^3x^3-b^3}{\left(a^3x^3+b^3\right)\sqrt{a^2x^3+b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3-b^3)/(a^2*x^3+b^2*x)^(1/2)/(a^3*x^3+b^3),x, algorithm="g
iac")
```

```
[Out] integrate((a^3*x^3 - b^3)/((a^3*x^3 + b^3)*sqrt(a^2*x^3 + b^2*x)), x)
```

maple [C] time = 0.09, size = 391, normalized size = 3.18

$$\frac{ib\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\sqrt{2}\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\sqrt{\frac{ixa}{b}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a^2x^3+b^2x}}+\frac{2i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^2a^2-\_Zab+b^2\right)}\frac{\left(-\_\alpha a+2b\right)\left(i\_\alpha a-ib+b\right)\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\sqrt{\frac{ixa}{b}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},-\frac{\_\alpha a-b-ib}{b},\frac{\sqrt{2}}{2}\right)}{\left(2\_\alpha a-b\right)\sqrt{x\left(a^2x^2+b^2\right)}}\right)}{3a}-\frac{2ib^2\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\sqrt{2}\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\sqrt{\frac{ixa}{b}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},-\frac{ib}{a\left(-\frac{ib}{a}+\frac{b}{a}\right)},\frac{\sqrt{2}}{2}\right)}{3a^2\sqrt{a^2x^3+b^2x}\left(-\frac{ib}{a}+\frac{b}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^3*x^3-b^3)/(a^2*x^3+b^2*x)^(1/2)/(a^3*x^3+b^3),x)
```

```
[Out] I*b/a*(-I*(x+I*b/a)/b*a)^(1/2)*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1
/2)/(a^2*x^3+b^2*x)^(1/2)*EllipticF((-I*(x+I*b/a)/b*a)^(1/2),1/2*2^(1/2))+2
/3*I/a*2^(1/2)*sum((-_alpha*a+2*b)/(2*_alpha*a-b)*(I*_alpha*a-I*b+b)*(-I*(x
+I*b/a)/b*a)^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1/2)/(x*(a^2*x^2+b^2)
)^(1/2)*EllipticPi((-I*(x+I*b/a)/b*a)^(1/2),-(_alpha*a-b-I*b)/b,1/2*2^(1/2)
```

```
),_alpha=RootOf(_Z^2*a^2-_Z*a*b+b^2))-2/3*I*b^2/a^2*(-I*(x+I*b/a)/b*a)^(1/2
)*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1/2)/(a^2*x^3+b^2*x)^(1/2)/(-I
*b/a+b/a)*EllipticPi((-I*(x+I*b/a)/b*a)^(1/2),-I*b/a/(-I*b/a+b/a),1/2*2^(1/
2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^3x^3 - b^3}{\left(a^3x^3 + b^3\right)\sqrt{a^2x^3 + b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3-b^3)/(a^2*x^3+b^2*x)^(1/2)/(a^3*x^3+b^3),x, algorithm="m
axima")
```

```
[Out] integrate((a^3*x^3 - b^3)/((a^3*x^3 + b^3)*sqrt(a^2*x^3 + b^2*x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^3 - a^3*x^3)/((b^3 + a^3*x^3)*(b^2*x + a^2*x^3)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax - b)\left(a^2x^2 + abx + b^2\right)}{\sqrt{x\left(a^2x^2 + b^2\right)}\,(ax + b)\left(a^2x^2 - abx + b^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**3*x**3-b**3)/(a**2*x**3+b**2*x)**(1/2)/(a**3*x**3+b**3),x)
```

```
[Out] Integral((a*x - b)*(a**2*x**2 + a*b*x + b**2)/(sqrt(x*(a**2*x**2 + b**2))*(
a*x + b)*(a**2*x**2 - a*b*x + b**2)), x)
```

3.1519 $\int \frac{b^3+a^3x^3}{\sqrt{b^2x+a^2x^3}\left(-b^3+a^3x^3\right)}\,dx$

Optimal. Leaf size=123

$$-\frac{4\tan^{-1}\left(\frac{\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt{a}\sqrt{b}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt{a}\sqrt{b}}$$

Rubi [C] time = 3.39, antiderivative size = 747, normalized size of antiderivative = 6.07, number of steps used = 19, number of rules used = 11, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2056, 6715, 6725, 220, 2073, 1211, 1699, 208, 6728, 1217, 1707}

$$-\frac{4\sqrt{x}\sqrt{a^2x^2+b^2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{b}\sqrt{x}}{\sqrt{a^2x^2+b^2}}\right)}{3\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}-\frac{\sqrt{2}\sqrt{x}\sqrt{a^2x^2+b^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{x}}{\sqrt{a^2x^2+b^2}}\right)}{3\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}+\frac{2\sqrt{x}(ax+b)\sqrt{\frac{a^2x^2+b^2}{(ax+b)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{3\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}-\frac{4\sqrt{a}\sqrt{x}(ax+b)\sqrt{\frac{a^2x^2+b^2}{(ax+b)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{3}\left(3\sqrt{-a^2}+\sqrt{3}a\right)\sqrt{b}\sqrt{a^2x^3+b^2x}}-\frac{4\sqrt{-a^2}\sqrt{x}(ax+b)\sqrt{\frac{a^2x^2+b^2}{(ax+b)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{3}\sqrt{a}\left(\sqrt{3}\sqrt{-a^2}+3a\right)\sqrt{b}\sqrt{a^2x^3+b^2x}}+\frac{\left(\sqrt{3}\sqrt{-a^2}+a\right)\sqrt{x}(ax+b)\sqrt{\frac{a^2x^2+b^2}{(ax+b)^2}}\Pi\left(\frac{1}{4};2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\left(3a-\sqrt{3}\sqrt{-a^2}\right)\sqrt{b}\sqrt{a^2x^3+b^2x}}+\frac{\left(a-\sqrt{3}\sqrt{-a^2}\right)\sqrt{x}(ax+b)\sqrt{\frac{a^2x^2+b^2}{(ax+b)^2}}\Pi\left(\frac{1}{4};2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\left(\sqrt{3}\sqrt{-a^2}+3a\right)\sqrt{b}\sqrt{a^2x^3+b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b^3 + a^3*x^3)/(Sqrt[b^2*x + a^2*x^3]*(-b^3 + a^3*x^3)),x]
```

```
[Out] (-4*Sqrt[x]*Sqrt[b^2 + a^2*x^2]*ArcTan[(Sqrt[a]*Sqrt[b]*Sqrt[x])/Sqrt[b^2 +
 a^2*x^2]])/(3*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) - (Sqrt[2]*Sqrt[x]*Sq
rt[b^2 + a^2*x^2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[x])/Sqrt[b^2 + a^2*
x^2]])/(3*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) + (2*Sqrt[x]*(b + a*x)*Sqr
t[(b^2 + a^2*x^2)/(b + a*x)^2]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]
], 1/2])/(3*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) - (4*Sqrt[a]*Sqrt[x]*(b
+ a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt[x
])/Sqrt[b]], 1/2])/(Sqrt[3]*(Sqrt[3]*a + 3*Sqrt[-a^2])*Sqrt[b]*Sqrt[b^2*x +
 a^2*x^3]) - (4*Sqrt[-a^2]*Sqrt[x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)
^2]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]], 1/2])/(Sqrt[3]*Sqrt[a]*(
3*a + Sqrt[3]*Sqrt[-a^2])*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) + ((a + Sqrt[3]*Sq
rt[-a^2])*Sqrt[x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2]*EllipticPi[1/
4, 2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]], 1/2])/(Sqrt[a]*(3*a - Sqrt[3]*Sqrt[
-a^2])*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) + ((a - Sqrt[3]*Sqrt[-a^2])*Sqrt[x]*(
b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2]*EllipticPi[1/4, 2*ArcTan[(Sqrt[a
]*Sqrt[x])/Sqrt[b]], 1/2])/(Sqrt[a]*(3*a + Sqrt[3]*Sqrt[-a^2])*Sqrt[b]*Sqrt
[b^2*x + a^2*x^3])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
```

```
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{b^3+a^3x^3}{\sqrt{b^2x+a^2x^3}\left(-b^3+a^3x^3\right)}dx = \frac{\left(\sqrt{x}\sqrt{b^2+a^2x^2}\right)\int \frac{b^3+a^3x^3}{\sqrt{x}\sqrt{b^2+a^2x^2}\left(-b^3+a^3x^3\right)}dx}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{b^3+a^3x^6}{\sqrt{b^2+a^2x^4}\left(-b^3+a^3x^6\right)}dx,x,\sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt{b^2+a^2x^4}}+\frac{2b^3}{\sqrt{b^2+a^2x^4}\left(-b^3+a^3x^6\right)}\right)dx,x,\sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2+a^2x^4}}dx,x,\sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}} + \frac{\left(4b^3\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Sub}}{\sqrt{}}$$

$$= \frac{\sqrt{x}(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} + \frac{\left(4b^3\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{\sqrt{x}(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\left(4b\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int\right.}{3\sqrt{b^2x+}}$$

$$= \frac{\sqrt{x}(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int\right.}{3\sqrt{b^2x+a^2x^3}}$$

$$= \frac{2\sqrt{x}(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{3\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\left(2b\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int\right.}{3\sqrt{b^2x+}}$$

$$= -\frac{\sqrt{2}\sqrt{x}\sqrt{b^2+a^2x^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{x}}{\sqrt{b^2+a^2x^2}}\right)}{3\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} + \frac{2\sqrt{x}(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}F\left(2\tan^{-1}\right.}{3\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}}$$

$$= -\frac{4\sqrt{x}\sqrt{b^2+a^2x^2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{b}\sqrt{x}}{\sqrt{b^2+a^2x^2}}\right)}{3\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\sqrt{2}\sqrt{x}\sqrt{b^2+a^2x^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{x}}{\sqrt{b^2+a^2x^2}}\right.}{3\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}}$$

Mathematica [C] time = 1.06, size = 212, normalized size = 1.72

$$-\frac{(-1)^{5/6}x^{3/2}\sqrt{\frac{b^2}{a^2x^2}+1}\left(\left(3-3i\sqrt{3}\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)+4\left((-1)^{2/3}\Pi\left(i;i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)+\left(\sqrt[3]{-1}-1\right)\Pi\left(-\sqrt[6]{-1};i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)+(-1)^{2/3}\Pi\left(-(-1)^{5/6};i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)\right)\right)}{3\sqrt{\frac{ib}{a}}\sqrt{x\left(a^2x^2+b^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^3 + a^3*x^3)/(Sqrt[b^2*x + a^2*x^3]*(-b^3 + a^3*x^3)),x]
```

```
[Out] -1/3*((-1)^(5/6)*Sqrt[1 + b^2/(a^2*x^2)]*x^(3/2)*((3 - (3*I)*Sqrt[3])*Ellip
ticF[I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] + 4*((-1)^(2/3)*EllipticPi[I, I*
ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] + (-1 + (-1)^(1/3))*EllipticPi[-(-1)^(1
/6), I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] + (-1)^(2/3)*EllipticPi[-(-1)^(5
/6), I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1])))/(Sqrt[(I*b)/a]*Sqrt[x*(b^2 +
a^2*x^2)])
```

IntegrateAlgebraic [A] time = 0.49, size = 123, normalized size = 1.00

$$-\frac{4\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{b}\,\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt{a}\,\sqrt{b}}-\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^3 + a^3*x^3)/(Sqrt[b^2*x + a^2*x^3]*(-b^3 + a^3*x^3))
,x]
```

```
[Out] (-4*ArcTan[(Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])/(b^2 + a^2*x^2)])/(3*Sqr
t[a]*Sqrt[b]) - (Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*
x^3])/(b^2 + a^2*x^2)])/(3*Sqrt[a]*Sqrt[b])
```

fricas [B] time = 0.71, size = 423, normalized size = 3.44

$$\left[\frac{\sqrt{2}\,ab\sqrt{\frac{1}{ab}}\,\log\left(\frac{a^4x^4+12\,a^3bx^3+6\,a^2b^2x^2+12\,ab^3x+b^4-4\,\sqrt{2}\,(a^3bx^2+2\,a^2b^2x+ab^3)\sqrt{a^2x^3+b^2x}\,\sqrt{\frac{1}{ab}}}{a^4x^4-4\,a^3bx^3+6\,a^2b^2x^2-4\,ab^3x+b^4}\right)+8\,\sqrt{ab}\,\arctan\left(\frac{\sqrt{a^2x^3+b^2x}\,(a^2x^2-abx+b^2)\sqrt{ab}}{2\,(a^3bx^3+ab^3x)}\right)}{12\,ab},\ \frac{\sqrt{2}\,ab\sqrt{-\frac{1}{ab}}\,\arctan\left(\frac{2\,\sqrt{2}\,\sqrt{a^2x^3+b^2x}\,ab\sqrt{-\frac{1}{ab}}}{a^2x^2+2\,abx+b^2}\right)-2\,\sqrt{-ab}\,\log\left(\frac{a^4x^4-6\,a^3bx^3+3\,a^2b^2x^2-6\,ab^3x+b^4-4\,\sqrt{a^2x^3+b^2x}\,(a^2x^2-abx+b^2)\sqrt{-ab}}{a^4x^4+2\,a^3bx^3+3\,a^2b^2x^2+2\,ab^3x+b^4}\right)}{6\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3+b^3)/(a^2*x^3+b^2*x)^(1/2)/(a^3*x^3-b^3),x, algorithm="f
ricas")
```

```
[Out] [1/12*(sqrt(2)*a*b*sqrt(1/(a*b))*log((a^4*x^4 + 12*a^3*b*x^3 + 6*a^2*b^2*x^
2 + 12*a*b^3*x + b^4 - 4*sqrt(2)*(a^3*b*x^2 + 2*a^2*b^2*x + a*b^3)*sqrt(a^2
*x^3 + b^2*x)*sqrt(1/(a*b)))/(a^4*x^4 - 4*a^3*b*x^3 + 6*a^2*b^2*x^2 - 4*a*b
^3*x + b^4)) + 8*sqrt(a*b)*arctan(1/2*sqrt(a^2*x^3 + b^2*x)*(a^2*x^2 - a*b*
x + b^2)*sqrt(a*b)/(a^3*b*x^3 + a*b^3*x)))/(a*b), 1/6*(sqrt(2)*a*b*sqrt(-1/
(a*b))*arctan(2*sqrt(2)*sqrt(a^2*x^3 + b^2*x)*a*b*sqrt(-1/(a*b))/(a^2*x^2 +
 2*a*b*x + b^2)) - 2*sqrt(-a*b)*log((a^4*x^4 - 6*a^3*b*x^3 + 3*a^2*b^2*x^2
- 6*a*b^3*x + b^4 - 4*sqrt(a^2*x^3 + b^2*x)*(a^2*x^2 - a*b*x + b^2)*sqrt(-a
*b))/(a^4*x^4 + 2*a^3*b*x^3 + 3*a^2*b^2*x^2 + 2*a*b^3*x + b^4)))/(a*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^3x^3+b^3}{\left(a^3x^3-b^3\right)\sqrt{a^2x^3+b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3+b^3)/(a^2*x^3+b^2*x)^(1/2)/(a^3*x^3-b^3),x, algorithm="g
iac")
```

```
[Out] integrate((a^3*x^3 + b^3)/((a^3*x^3 - b^3)*sqrt(a^2*x^3 + b^2*x)), x)
```

maple [C] time = 0.09, size = 387, normalized size = 3.15

$$\frac{ib\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{2}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\operatorname{EllipticF}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a^2x^3+b^2x}}+\frac{2i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^2a^2+\_Zab+b^2\right)}\frac{\left(-\_\alpha a-2b\right)\left(i\_\alpha a+ib+b\right)\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\operatorname{EllipticPi}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},\frac{\_\alpha a-ib+b}{b},\frac{\sqrt{2}}{2}\right)}{\left(2\_\alpha a+b\right)\sqrt{x\left(a^2x^2+b^2\right)}}\right)}{3a}+\frac{2ib^2\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{2}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\operatorname{EllipticPi}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},-\frac{ib}{a\left(-\frac{ib}{a}-\frac{b}{a}\right)},\frac{\sqrt{2}}{2}\right)}{3a^2\sqrt{a^2x^3+b^2x}\,\left(-\frac{ib}{a}-\frac{b}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^3*x^3+b^3)/(a^2*x^3+b^2*x)^(1/2)/(a^3*x^3-b^3),x)
```

```
[Out] I*b/a*(-I*(x+I*b/a)/b*a)^(1/2)*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1
/2)/(a^2*x^3+b^2*x)^(1/2)*EllipticF((-I*(x+I*b/a)/b*a)^(1/2),1/2*2^(1/2))+2
/3*I/a*2^(1/2)*sum((-_alpha*a-2*b)/(2*_alpha*a+b)*(I*_alpha*a+I*b+b)*(-I*(x
+I*b/a)/b*a)^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1/2)/(x*(a^2*x^2+b^2)
)^(1/2)*EllipticPi((-I*(x+I*b/a)/b*a)^(1/2),(_alpha*a+b-I*b)/b,1/2*2^(1/2))
```

```
,_alpha=RootOf(_Z^2*a^2+_Z*a*b+b^2))+2/3*I*b^2/a^2*(-I*(x+I*b/a)/b*a)^(1/2)
*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1/2)/(a^2*x^3+b^2*x)^(1/2)/(-I*
b/a-b/a)*EllipticPi((-I*(x+I*b/a)/b*a)^(1/2),-I*b/a/(-I*b/a-b/a),1/2*2^(1/2
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^3x^3 + b^3}{\left(a^3x^3 - b^3\right)\sqrt{a^2x^3 + b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3+b^3)/(a^2*x^3+b^2*x)^(1/2)/(a^3*x^3-b^3),x, algorithm="m
axima")
```

```
[Out] integrate((a^3*x^3 + b^3)/((a^3*x^3 - b^3)*sqrt(a^2*x^3 + b^2*x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^3 + a^3*x^3)/((b^3 - a^3*x^3)*(b^2*x + a^2*x^3)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+b)\left(a^2x^2 - abx + b^2\right)}{\sqrt{x\left(a^2x^2 + b^2\right)}\,(ax-b)\left(a^2x^2 + abx + b^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**3*x**3+b**3)/(a**2*x**3+b**2*x)**(1/2)/(a**3*x**3-b**3),x)
```

```
[Out] Integral((a*x + b)*(a**2*x**2 - a*b*x + b**2)/(sqrt(x*(a**2*x**2 + b**2))*(
a*x - b)*(a**2*x**2 + a*b*x + b**2)), x)
```

3.1520 $\int \frac{(-4b+ax^3)(b-ax^3+x^4)}{x^4\sqrt[4]{-b+ax^3}\,(-b+ax^3+x^4)}\,dx$

Optimal. Leaf size=123

$$\frac{4\left(ax^3-b\right)^{3/4}}{3x^3}-2\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3-b}}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}+x^2}\right)$$

Rubi [F] time = 3.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-4b+ax^3)(b-ax^3+x^4)}{x^4\sqrt[4]{-b+ax^3}\,(-b+ax^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-4*b + a*x^3)*(b - a*x^3 + x^4))/(x^4*(-b + a*x^3)^(1/4)*(-b + a*x^3
+ x^4)),x]
```

```
[Out] (4*(-b + a*x^3)^(3/4))/(3*x^3) + 8*b*Defer[Int][1/((-b + a*x^3)^(1/4)*(b -
a*x^3 - x^4)), x] + 2*a*Defer[Int][x^3/((-b + a*x^3)^(1/4)*(-b + a*x^3 + x^
4)), x]
```

Rubi steps

$$\int \frac{(-4b+ax^3)(b-ax^3+x^4)}{x^4\sqrt[4]{-b+ax^3}\,(-b+ax^3+x^4)}\,dx=\int\left(\frac{4b}{x^4\sqrt[4]{-b+ax^3}}-\frac{a}{x\sqrt[4]{-b+ax^3}}+\frac{2(4b-ax^3)}{\sqrt[4]{-b+ax^3}\,(b-ax^3-x^4)}\right)dx$$

$$=2\int\frac{4b-ax^3}{\sqrt[4]{-b+ax^3}\,(b-ax^3-x^4)}\,dx-a\int\frac{1}{x\sqrt[4]{-b+ax^3}}\,dx+(4b)\int$$

$$=2\int\left(\frac{4b}{\sqrt[4]{-b+ax^3}\,(b-ax^3-x^4)}+\frac{ax^3}{\sqrt[4]{-b+ax^3}\,(-b+ax^3+x^4)}\right)dx-$$

$$=\frac{4\left(-b+ax^3\right)^{3/4}}{3x^3}-\frac{4}{3}\operatorname{Subst}\left(\int\frac{x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{-b+ax^3}\right)+\frac{1}{3}a\operatorname{Subst}($$

$$=\frac{4\left(-b+ax^3\right)^{3/4}}{3x^3}+\frac{2}{3}\operatorname{Subst}\left(\int\frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{-b+ax^3}\right)-\frac{2}{3}\operatorname{Subst}($$

$$=\frac{4\left(-b+ax^3\right)^{3/4}}{3x^3}-\frac{2}{3}\operatorname{Subst}\left(\int\frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{-b+ax^3}\right)+\frac{2}{3}\operatorname{Subst}($$

$$=\frac{4\left(-b+ax^3\right)^{3/4}}{3x^3}-\frac{a\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^3}+\sqrt{-b+ax^3}\right)}{3\sqrt{2}\sqrt[4]{b}}+\frac{a\log}{}$$

$$=\frac{4\left(-b+ax^3\right)^{3/4}}{3x^3}+\frac{\sqrt{2}\,a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)}{3\sqrt[4]{b}}-\frac{\sqrt{2}\,a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{}}{}\right)}{3\sqrt[4]{b}}$$

$$=\frac{4\left(-b+ax^3\right)^{3/4}}{3x^3}+(2a)\int\frac{x^3}{\sqrt[4]{-b+ax^3}\,(-b+ax^3+x^4)}\,dx+(8b)\int$$

Mathematica [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(-4b+ax^3\right)\left(b-ax^3+x^4\right)}{x^4\sqrt[4]{-b+ax^3}\left(-b+ax^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-4*b + a*x^3)*(b - a*x^3 + x^4))/(x^4*(-b + a*x^3)^(1/4)*(-b +
a*x^3 + x^4)),x]
```

```
[Out] Integrate[((-4*b + a*x^3)*(b - a*x^3 + x^4))/(x^4*(-b + a*x^3)^(1/4)*(-b +
a*x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 1.72, size = 123, normalized size = 1.00

$$\frac{4\left(ax^3-b\right)^{3/4}}{3x^3}-2\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3-b}}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4*b + a*x^3)*(b - a*x^3 + x^4))/(x^4*(-b + a*x^3)^(1/
4)*(-b + a*x^3 + x^4)),x]
```

```
[Out] (4*(-b + a*x^3)^(3/4))/(3*x^3) - 2*Sqrt[2]*ArcTan[(-(x^2/Sqrt[2]) + Sqrt[-b
 + a*x^3]/Sqrt[2])/(x*(-b + a*x^3)^(1/4))] + 2*Sqrt[2]*ArcTanh[(Sqrt[2]*x*(
-b + a*x^3)^(1/4))/(x^2 + Sqrt[-b + a*x^3])]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-4*b)*(-a*x^3+x^4+b)/x^4/(a*x^3-b)^(1/4)/(a*x^3+x^4-b),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(ax^3-x^4-b\right)\left(ax^3-4\,b\right)}{\left(ax^3+x^4-b\right)\left(ax^3-b\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-4*b)*(-a*x^3+x^4+b)/x^4/(a*x^3-b)^(1/4)/(a*x^3+x^4-b),x, a
lgorithm="giac")
```

```
[Out] integrate(-(a*x^3 - x^4 - b)*(a*x^3 - 4*b)/((a*x^3 + x^4 - b)*(a*x^3 - b)^(
1/4)*x^4), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^3-4b\right)\left(-a\,x^3+x^4+b\right)}{x^4\left(a\,x^3-b\right)^{\frac{1}{4}}\left(a\,x^3+x^4-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-4*b)*(-a*x^3+x^4+b)/x^4/(a*x^3-b)^(1/4)/(a*x^3+x^4-b),x)
```

```
[Out] int((a*x^3-4*b)*(-a*x^3+x^4+b)/x^4/(a*x^3-b)^(1/4)/(a*x^3+x^4-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(ax^3 - x^4 - b\right)\left(ax^3 - 4\,b\right)}{\left(ax^3 + x^4 - b\right)\left(ax^3 - b\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-4*b)*(-a*x^3+x^4+b)/x^4/(a*x^3-b)^(1/4)/(a*x^3+x^4-b),x, a
lgorithm="maxima")
```

```
[Out] -integrate((a*x^3 - x^4 - b)*(a*x^3 - 4*b)/((a*x^3 + x^4 - b)*(a*x^3 - b)^(
1/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(4\,b - a\,x^3\right)\,\left(x^4 - a\,x^3 + b\right)}{x^4\,{\left(a\,x^3 - b\right)}^{1/4}\,\left(x^4 + a\,x^3 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((4*b - a*x^3)*(b - a*x^3 + x^4))/(x^4*(a*x^3 - b)^(1/4)*(a*x^3 - b +
x^4)),x)
```

```
[Out] int(-((4*b - a*x^3)*(b - a*x^3 + x^4))/(x^4*(a*x^3 - b)^(1/4)*(a*x^3 - b +
x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-4*b)*(-a*x**3+x**4+b)/x**4/(a*x**3-b)**(1/4)/(a*x**3+x**4
-b),x)
```

```
[Out] Timed out
```

3.1521 $\int \frac{(-2+x^2)(-1+x^2)\sqrt[4]{-1+x^2+x^4}}{x^6(-1+x^2+2x^4)}\,dx$

Optimal. Leaf size=123

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^4+x^2-1}}{\sqrt{x^4+x^2-1}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^4+x^2-1}}{x^2+\sqrt{x^4+x^2-1}}\right)+\frac{2\sqrt[4]{x^4+x^2-1}\,(9x^4-x^2+1)}{5x^5}$$

Rubi [F] time = 1.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+x^2)(-1+x^2)\sqrt[4]{-1+x^2+x^4}}{x^6(-1+x^2+2x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + x^2)*(-1 + x^2)*(-1 + x^2 + x^4)^(1/4))/(x^6*(-1 + x^2 + 2*x^4))
,x]
```

```
[Out] (4*(-1 + x^2 + x^4)^(1/4)*AppellF1[-1/2, -1/4, -1/4, 1/2, (-2*x^2)/(1 - Sqr
t[5]), (-2*x^2)/(1 + Sqrt[5])])/(x*(1 + (2*x^2)/(1 - Sqrt[5]))^(1/4)*(1 + (
2*x^2)/(1 + Sqrt[5]))^(1/4)) - ((1 + (2*x^2)/(1 + Sqrt[5]))^(5/4)*(-1 + x^2
 + x^4)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, (-2*(x^2/(1 - Sqrt[5]) -
x^2/(1 + Sqrt[5])))/(1 + (2*x^2)/(1 + Sqrt[5]))])/(3*x^3*(1 + (2*x^2)/(1 -
Sqrt[5]))^(1/4)) - (4*(1 + (2*x^2)/(1 + Sqrt[5]))*(-1 + x^2 + x^4)^(1/4)*((
3*(1 + Sqrt[5]) - (13 + 3*Sqrt[5])*x^2 + 2*(1 + Sqrt[5])*x^4)*Gamma[-1/4]*H
ypergeometric2F1[-1/4, 1, -1/2, (-2*Sqrt[5]*x^2)/(2 - (1 + Sqrt[5])*x^2)] -
 4*x^2*(5 + Sqrt[5] + 2*Sqrt[5]*x^2)*Gamma[3/4]*Hypergeometric2F1[3/4, 2, 1
/2, (-2*Sqrt[5]*x^2)/(2 - (1 + Sqrt[5])*x^2)]))/(15*(3 + Sqrt[5])*x^5*(1 -
Sqrt[5] + 2*x^2)*Gamma[-1/4]) + 2*Defer[Int][(-1 + x^2 + x^4)^(1/4)/(1 + x^
2), x] + 4*Defer[Int][(-1 + x^2 + x^4)^(1/4)/(-1 + 2*x^2), x]
```

Rubi steps

$$\int \frac{(-2+x^2)(-1+x^2)\sqrt[4]{-1+x^2+x^4}}{x^6(-1+x^2+2x^4)}\,dx=\int \left(-\frac{2\sqrt[4]{-1+x^2+x^4}}{x^6}+\frac{\sqrt[4]{-1+x^2+x^4}}{x^4}-\frac{4\sqrt[4]{-1+x^2+x^4}}{x^2}+\frac{2\sqrt[4]{-}\right.$$

$$=-\left(2\int \frac{\sqrt[4]{-1+x^2+x^4}}{x^6}\,dx\right)+2\int \frac{\sqrt[4]{-1+x^2+x^4}}{1+x^2}\,dx-4\int \frac{\sqrt[4]{-1+}}{x}$$

$$=2\int \frac{\sqrt[4]{-1+x^2+x^4}}{1+x^2}\,dx+4\int \frac{\sqrt[4]{-1+x^2+x^4}}{-1+2x^2}\,dx+\frac{\sqrt[4]{-1+x^2+x^4}}{\sqrt[4]{1+\frac{2}{1-}}}$$

$$=\frac{4\sqrt[4]{-1+x^2+x^4}\,F_1\left(-\frac{1}{2};-\frac{1}{4},-\frac{1}{4};\frac{1}{2};-\frac{2x^2}{1-\sqrt{5}},-\frac{2x^2}{1+\sqrt{5}}\right)}{x\sqrt[4]{1+\frac{2x^2}{1-\sqrt{5}}}\sqrt[4]{1+\frac{2x^2}{1+\sqrt{5}}}}-\frac{\left(1+\frac{2x^2}{1+\sqrt{5}}\right)^{5/4}}{}$$

Mathematica [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{(-2+x^2)(-1+x^2)\sqrt[4]{-1+x^2+x^4}}{x^6(-1+x^2+2x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + x^2)*(-1 + x^2)*(-1 + x^2 + x^4)^(1/4))/(x^6*(-1 + x^2 + 2
*x^4)),x]
```

```
[Out] Integrate[((-2 + x^2)*(-1 + x^2)*(-1 + x^2 + x^4)^(1/4))/(x^6*(-1 + x^2 + 2
*x^4)), x]
```

IntegrateAlgebraic [A] time = 0.37, size = 123, normalized size = 1.00

$$\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^4+x^2-1}}{\sqrt{x^4+x^2-1}-x^2}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^4+x^2-1}}{x^2+\sqrt{x^4+x^2-1}}\right)+\frac{2\sqrt[4]{x^4+x^2-1}\,(9x^4-x^2+1)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^2)*(-1 + x^2)*(-1 + x^2 + x^4)^(1/4))/(x^6*(-1
+ x^2 + 2*x^4)),x]
```

```
[Out] (2*(-1 + x^2 + x^4)^(1/4)*(1 - x^2 + 9*x^4))/(5*x^5) + Sqrt[2]*ArcTan[(Sqrt
[2]*x*(-1 + x^2 + x^4)^(1/4))/(-x^2 + Sqrt[-1 + x^2 + x^4])] - Sqrt[2]*ArcT
anh[(Sqrt[2]*x*(-1 + x^2 + x^4)^(1/4))/(x^2 + Sqrt[-1 + x^2 + x^4])]
```

fricas [B] time = 8.64, size = 559, normalized size = 4.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)*(x^2-1)*(x^4+x^2-1)^(1/4)/x^6/(2*x^4+x^2-1),x, algorithm=
"fricas")
```

```
[Out] 1/20*(20*sqrt(2)*x^5*arctan((sqrt(2)*(x^4 + x^2 - 1)^(1/4)*x^3 + sqrt(2)*(x
^4 + x^2 - 1)^(3/4)*x - (2*x^4 - sqrt(2)*(x^4 + x^2 - 1)^(1/4)*x^3 + 2*sqrt
(x^4 + x^2 - 1)*x^2 - sqrt(2)*(x^4 + x^2 - 1)^(3/4)*x + x^2 - 1)*sqrt((2*x^
4 + 2*sqrt(2)*(x^4 + x^2 - 1)^(1/4)*x^3 + 4*sqrt(x^4 + x^2 - 1)*x^2 + 2*sqr
t(2)*(x^4 + x^2 - 1)^(3/4)*x + x^2 - 1)/(2*x^4 + x^2 - 1)))/(x^2 - 1)) + 20
*sqrt(2)*x^5*arctan((sqrt(2)*(x^4 + x^2 - 1)^(1/4)*x^3 + sqrt(2)*(x^4 + x^2
 - 1)^(3/4)*x + (2*x^4 + sqrt(2)*(x^4 + x^2 - 1)^(1/4)*x^3 + 2*sqrt(x^4 + x
^2 - 1)*x^2 + sqrt(2)*(x^4 + x^2 - 1)^(3/4)*x + x^2 - 1)*sqrt((2*x^4 - 2*sq
rt(2)*(x^4 + x^2 - 1)^(1/4)*x^3 + 4*sqrt(x^4 + x^2 - 1)*x^2 - 2*sqrt(2)*(x^
4 + x^2 - 1)^(3/4)*x + x^2 - 1)/(2*x^4 + x^2 - 1)))/(x^2 - 1)) - 5*sqrt(2)*
x^5*log((2*x^4 + 2*sqrt(2)*(x^4 + x^2 - 1)^(1/4)*x^3 + 4*sqrt(x^4 + x^2 - 1
)*x^2 + 2*sqrt(2)*(x^4 + x^2 - 1)^(3/4)*x + x^2 - 1)/(2*x^4 + x^2 - 1)) + 5
*sqrt(2)*x^5*log((2*x^4 - 2*sqrt(2)*(x^4 + x^2 - 1)^(1/4)*x^3 + 4*sqrt(x^4
+ x^2 - 1)*x^2 - 2*sqrt(2)*(x^4 + x^2 - 1)^(3/4)*x + x^2 - 1)/(2*x^4 + x^2
- 1)) + 8*(9*x^4 - x^2 + 1)*(x^4 + x^2 - 1)^(1/4))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^2-1\right)^{\frac{1}{4}}\left(x^2-1\right)\left(x^2-2\right)}{\left(2\,x^4+x^2-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)*(x^2-1)*(x^4+x^2-1)^(1/4)/x^6/(2*x^4+x^2-1),x, algorithm=
"giac")
```

```
[Out] integrate((x^4 + x^2 - 1)^(1/4)*(x^2 - 1)*(x^2 - 2)/((2*x^4 + x^2 - 1)*x^6)
, x)
```

maple [C] time = 2.22, size = 848, normalized size = 6.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2)*(x^2-1)*(x^4+x^2-1)^(1/4)/x^6/(2*x^4+x^2-1),x)
```

```
[Out] 2/5*(9*x^8+8*x^6-9*x^4+2*x^2-1)/x^5/(x^4+x^2-1)^(3/4)+(RootOf(_Z^4+1)^3*ln(
(-x^10*RootOf(_Z^4+1)^2-2*RootOf(_Z^4+1)*(x^12+3*x^10-5*x^6+3*x^2-1)^(1/4)*
x^9-x^8*RootOf(_Z^4+1)^2+2*(x^12+3*x^10-5*x^6+3*x^2-1)^(3/4)*RootOf(_Z^4+1)
^3*x^3-4*RootOf(_Z^4+1)*(x^12+3*x^10-5*x^6+3*x^2-1)^(1/4)*x^7-2*(x^12+3*x^1
0-5*x^6+3*x^2-1)^(1/2)*x^6+3*RootOf(_Z^4+1)^2*x^6+2*RootOf(_Z^4+1)*(x^12+3*
x^10-5*x^6+3*x^2-1)^(1/4)*x^5-2*(x^12+3*x^10-5*x^6+3*x^2-1)^(1/2)*x^4+RootO
f(_Z^4+1)^2*x^4+4*RootOf(_Z^4+1)*(x^12+3*x^10-5*x^6+3*x^2-1)^(1/4)*x^3+2*(x
^12+3*x^10-5*x^6+3*x^2-1)^(1/2)*x^2-3*RootOf(_Z^4+1)^2*x^2-2*RootOf(_Z^4+1)
*(x^12+3*x^10-5*x^6+3*x^2-1)^(1/4)*x+RootOf(_Z^4+1)^2)/(2*x^2-1)/(x^2+1)/(x
^4+x^2-1)^2)+RootOf(_Z^4+1)*ln((-2*RootOf(_Z^4+1)^3*(x^12+3*x^10-5*x^6+3*x^
2-1)^(1/4)*x^9+x^10*RootOf(_Z^4+1)^2-4*RootOf(_Z^4+1)^3*(x^12+3*x^10-5*x^6+
3*x^2-1)^(1/4)*x^7+x^8*RootOf(_Z^4+1)^2+2*RootOf(_Z^4+1)^3*(x^12+3*x^10-5*x
^6+3*x^2-1)^(1/4)*x^5-2*(x^12+3*x^10-5*x^6+3*x^2-1)^(1/2)*x^6-3*RootOf(_Z^4
+1)^2*x^6+2*(x^12+3*x^10-5*x^6+3*x^2-1)^(3/4)*RootOf(_Z^4+1)*x^3+4*RootOf(_
Z^4+1)^3*(x^12+3*x^10-5*x^6+3*x^2-1)^(1/4)*x^3-2*(x^12+3*x^10-5*x^6+3*x^2-1
)^(1/2)*x^4-RootOf(_Z^4+1)^2*x^4-2*RootOf(_Z^4+1)^3*(x^12+3*x^10-5*x^6+3*x^
2-1)^(1/4)*x+2*(x^12+3*x^10-5*x^6+3*x^2-1)^(1/2)*x^2+3*RootOf(_Z^4+1)^2*x^2
-RootOf(_Z^4+1)^2)/(2*x^2-1)/(x^2+1)/(x^4+x^2-1)^2))/(x^4+x^2-1)^(3/4)*((x^
4+x^2-1)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4 + x^2 - 1\right)^{\frac{1}{4}}\left(x^2 - 1\right)\left(x^2 - 2\right)}{\left(2\,x^4 + x^2 - 1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)*(x^2-1)*(x^4+x^2-1)^(1/4)/x^6/(2*x^4+x^2-1),x, algorithm=
"maxima")
```

```
[Out] integrate((x^4 + x^2 - 1)^(1/4)*(x^2 - 1)*(x^2 - 2)/((2*x^4 + x^2 - 1)*x^6)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2-1\right)\,\left(x^2-2\right)\,{\left(x^4+x^2-1\right)}^{1/4}}{x^6\,\left(2\,x^4+x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*(x^2 - 2)*(x^2 + x^4 - 1)^(1/4))/(x^6*(x^2 + 2*x^4 - 1)),x)
```

```
[Out] int(((x^2 - 1)*(x^2 - 2)*(x^2 + x^4 - 1)^(1/4))/(x^6*(x^2 + 2*x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x - 1\right) \left(x + 1\right) \left(x^{2} - 2\right) \sqrt[4]{x^{4} + x^{2} - 1}}{x^{6} \left(x^{2} + 1\right) \left(2 x^{2} - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2)*(x**2-1)*(x**4+x**2-1)**(1/4)/x**6/(2*x**4+x**2-1),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 - 2)*(x**4 + x**2 - 1)**(1/4)/(x**6*(x**2 +
1)*(2*x**2 - 1)), x)
```

3.1522 $\int x^2 \sqrt[4]{bx^3+ax^4}\, dx$

Optimal. Leaf size=123

$$\frac{77b^4\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{1024a^{15/4}}-\frac{77b^4\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{1024a^{15/4}}+\frac{\left(384a^3x^3+32a^2bx^2-44ab^2x+77b^3\right)\sqrt[4]{ax^4+bx^3}}{1536a^3}$$

Rubi [A] time = 0.35, antiderivative size = 224, normalized size of antiderivative = 1.82, number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2021, 2024, 2032, 63, 331, 298, 203, 206}

$$\frac{77b^4x^{9/4}(ax+b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{1024a^{15/4}\left(ax^4+bx^3\right)^{3/4}}-\frac{77b^4x^{9/4}(ax+b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{1024a^{15/4}\left(ax^4+bx^3\right)^{3/4}}+\frac{77b^3\sqrt[4]{ax^4+bx^3}}{1536a^3}-\frac{11b^2x\sqrt[4]{ax^4+bx^3}}{384a^2}+\frac{1}{4}x^3\sqrt[4]{ax^4+bx^3}+\frac{bx^2\sqrt[4]{ax^4+bx^3}}{48a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^3 + a*x^4)^(1/4),x]
```

```
[Out] (77*b^3*(b*x^3 + a*x^4)^(1/4))/(1536*a^3) - (11*b^2*x*(b*x^3 + a*x^4)^(1/4)
)/(384*a^2) + (b*x^2*(b*x^3 + a*x^4)^(1/4))/(48*a) + (x^3*(b*x^3 + a*x^4)^(
1/4))/4 + (77*b^4*x^(9/4)*(b + a*x)^(3/4)*ArcTan[(a^(1/4)*x^(1/4))/(b + a*x
)^(1/4)])/(1024*a^(15/4)*(b*x^3 + a*x^4)^(3/4)) - (77*b^4*x^(9/4)*(b + a*x)
^(3/4)*ArcTanh[(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(1024*a^(15/4)*(b*x^3 +
a*x^4)^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^2 \sqrt[4]{bx^3+ax^4}\,dx = \frac{1}{4}x^3\sqrt[4]{bx^3+ax^4} + \frac{1}{16}b\int \frac{x^5}{\left(bx^3+ax^4\right)^{3/4}}\,dx$$

$$= \frac{bx^2\sqrt[4]{bx^3+ax^4}}{48a} + \frac{1}{4}x^3\sqrt[4]{bx^3+ax^4} - \frac{\left(11b^2\right)\int \frac{x^4}{\left(bx^3+ax^4\right)^{3/4}}\,dx}{192a}$$

$$= -\frac{11b^2x\sqrt[4]{bx^3+ax^4}}{384a^2} + \frac{bx^2\sqrt[4]{bx^3+ax^4}}{48a} + \frac{1}{4}x^3\sqrt[4]{bx^3+ax^4} + \frac{\left(77b^3\right)\int \frac{x^3}{\left(bx^3+ax^4\right)^{3/4}}\,dx}{1536a^2}$$

$$= \frac{77b^3\sqrt[4]{bx^3+ax^4}}{1536a^3} - \frac{11b^2x\sqrt[4]{bx^3+ax^4}}{384a^2} + \frac{bx^2\sqrt[4]{bx^3+ax^4}}{48a} + \frac{1}{4}x^3\sqrt[4]{bx^3+ax^4} - \frac{\left(77b^4\right)\int}{2}$$

$$= \frac{77b^3\sqrt[4]{bx^3+ax^4}}{1536a^3} - \frac{11b^2x\sqrt[4]{bx^3+ax^4}}{384a^2} + \frac{bx^2\sqrt[4]{bx^3+ax^4}}{48a} + \frac{1}{4}x^3\sqrt[4]{bx^3+ax^4} - \frac{\left(77b^4x^{9/}\right.}{2}$$

$$= \frac{77b^3\sqrt[4]{bx^3+ax^4}}{1536a^3} - \frac{11b^2x\sqrt[4]{bx^3+ax^4}}{384a^2} + \frac{bx^2\sqrt[4]{bx^3+ax^4}}{48a} + \frac{1}{4}x^3\sqrt[4]{bx^3+ax^4} - \frac{\left(77b^4x^{9/}\right.}{}$$

$$= \frac{77b^3\sqrt[4]{bx^3+ax^4}}{1536a^3} - \frac{11b^2x\sqrt[4]{bx^3+ax^4}}{384a^2} + \frac{bx^2\sqrt[4]{bx^3+ax^4}}{48a} + \frac{1}{4}x^3\sqrt[4]{bx^3+ax^4} - \frac{\left(77b^4x^{9/}\right.}{}$$

$$= \frac{77b^3\sqrt[4]{bx^3+ax^4}}{1536a^3} - \frac{11b^2x\sqrt[4]{bx^3+ax^4}}{384a^2} + \frac{bx^2\sqrt[4]{bx^3+ax^4}}{48a} + \frac{1}{4}x^3\sqrt[4]{bx^3+ax^4} - \frac{\left(77b^4x^{9/}\right.}{}$$

$$= \frac{77b^3\sqrt[4]{bx^3+ax^4}}{1536a^3} - \frac{11b^2x\sqrt[4]{bx^3+ax^4}}{384a^2} + \frac{bx^2\sqrt[4]{bx^3+ax^4}}{48a} + \frac{1}{4}x^3\sqrt[4]{bx^3+ax^4} + \frac{77b^4x^{9/4}}{102}$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.40

$$\frac{4x^3\sqrt[4]{x^3(ax+b)}\,{}_2F_1\left(-\frac{1}{4},\frac{15}{4};\frac{19}{4};-\frac{ax}{b}\right)}{15\sqrt[4]{\frac{ax}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^3 + a*x^4)^(1/4),x]
```

```
[Out] (4*x^3*(x^3*(b + a*x))^(1/4)*Hypergeometric2F1[-1/4, 15/4, 19/4, -((a*x)/b)
])/(15*(1 + (a*x)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.52, size = 123, normalized size = 1.00

$$\frac{77b^4\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{1024a^{15/4}} - \frac{77b^4\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{1024a^{15/4}} + \frac{\left(384a^3x^3+32a^2bx^2-44ab^2x+77b^3\right)\sqrt[4]{ax^4+bx^3}}{1536a^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(b*x^3 + a*x^4)^(1/4),x]
```

```
[Out] ((77*b^3 - 44*a*b^2*x + 32*a^2*b*x^2 + 384*a^3*x^3)*(b*x^3 + a*x^4)^(1/4))/
(1536*a^3) + (77*b^4*ArcTan[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(1024*a^(15
/4)) - (77*b^4*ArcTanh[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(1024*a^(15/4))
```

fricas [B] time = 0.45, size = 264, normalized size = 2.15

$$\frac{924 a^3 \left(\frac{b^{16}}{a^{15}}\right)^{\frac14} \arctan\left(-\frac{(ax^4+bx^3)^{\frac14} a^{11} b^4 \left(\frac{b^{16}}{a^{15}}\right)^{\frac34} - a^{11}\left(\frac{b^{16}}{a^{15}}\right)^{\frac34} x \sqrt{\frac{a^8\sqrt{\frac{b^{16}}{a^{15}}}x^2+\sqrt{ax^4+bx^3}\,b^8}{x^2}}}{b^{16}x}\right) - 231 a^3 \left(\frac{b^{16}}{a^{15}}\right)^{\frac14}\log\left(\frac{77\left(a^4\left(\frac{b^{16}}{a^{15}}\right)^{\frac14}x+(ax^4+bx^3)^{\frac14}b^4\right)}{x}\right) + 231 a^3 \left(\frac{b^{16}}{a^{15}}\right)^{\frac14}\log\left(-\frac{77\left(a^4\left(\frac{b^{16}}{a^{15}}\right)^{\frac14}x-(ax^4+bx^3)^{\frac14}b^4\right)}{x}\right) + 4\left(384a^3x^3+32a^2bx^2-44ab^2x+77b^3\right)(ax^4+bx^3)^{\frac14}}{6144a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^4+b*x^3)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/6144*(924*a^3*(b^16/a^15)^(1/4)*arctan(-((a*x^4 + b*x^3)^(1/4)*a^11*b^4*(
b^16/a^15)^(3/4) - a^11*(b^16/a^15)^(3/4)*x*sqrt((a^8*sqrt(b^16/a^15)*x^2 +
 sqrt(a*x^4 + b*x^3)*b^8)/x^2))/(b^16*x)) - 231*a^3*(b^16/a^15)^(1/4)*log(7
7*(a^4*(b^16/a^15)^(1/4)*x + (a*x^4 + b*x^3)^(1/4)*b^4)/x) + 231*a^3*(b^16/
a^15)^(1/4)*log(-77*(a^4*(b^16/a^15)^(1/4)*x - (a*x^4 + b*x^3)^(1/4)*b^4)/x
) + 4*(384*a^3*x^3 + 32*a^2*b*x^2 - 44*a*b^2*x + 77*b^3)*(a*x^4 + b*x^3)^(1
/4))/a^3
```

giac [B] time = 0.27, size = 278, normalized size = 2.26

$$\frac{\frac{462\sqrt{2} b^5 \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac14}+2\left(a+\frac{b}{x}\right)^{\frac14}\right)}{2(-a)^{\frac14}}\right)}{(-a)^{\frac34}a^3} + \frac{462\sqrt{2} b^5\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac14}-2\left(a+\frac{b}{x}\right)^{\frac14}\right)}{2(-a)^{\frac14}}\right)}{(-a)^{\frac34}a^3} + \frac{231\sqrt{2} b^5\log\left(\sqrt{2}(-a)^{\frac14}\left(a+\frac{b}{x}\right)^{\frac14}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{(-a)^{\frac34}a^3} + \frac{231\sqrt{2}(-a)^{\frac14}b^5\log\left(-\sqrt{2}(-a)^{\frac14}\left(a+\frac{b}{x}\right)^{\frac14}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{a^4} + \frac{8\left(77\left(a+\frac{b}{x}\right)^{\frac{13}{4}}b^5-275\left(a+\frac{b}{x}\right)^{\frac94}ab^5+351\left(a+\frac{b}{x}\right)^{\frac54}a^2b^5+231\left(a+\frac{b}{x}\right)^{\frac14}a^3b^5\right)x^4}{a^3b^4}}{12288b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^4+b*x^3)^(1/4),x, algorithm="giac")
```

```
[Out] 1/12288*(462*sqrt(2)*b^5*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/
x)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a^3) + 462*sqrt(2)*b^5*arctan(-1/2*sqrt(2
)*(sqrt(2)*(-a)^(1/4) - 2*(a + b/x)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a^3) + 2
31*sqrt(2)*b^5*log(sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a +
 b/x))/((-a)^(3/4)*a^3) + 231*sqrt(2)*(-a)^(1/4)*b^5*log(-sqrt(2)*(-a)^(1/4
)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a + b/x))/a^4 + 8*(77*(a + b/x)^(13/4)*
b^5 - 275*(a + b/x)^(9/4)*a*b^5 + 351*(a + b/x)^(5/4)*a^2*b^5 + 231*(a + b/
x)^(1/4)*a^3*b^5)*x^4/(a^3*b^4))/b
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x^{2} \left(a\,x^{4}+b\,x^{3}\right)^{\frac{1}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^4+b*x^3)^(1/4),x)
```

```
[Out] int(x^2*(a*x^4+b*x^3)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(a x^{4} + b x^{3}\right)}^{\frac{1}{4}} x^{2}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^4+b*x^3)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + b*x^3)^(1/4)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(a\,x^4+b\,x^3\right)}^{1/4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^4 + b*x^3)^(1/4),x)
```

```
[Out] int(x^2*(a*x^4 + b*x^3)^(1/4), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt[4]{x^3 (ax+b)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x**4+b*x**3)**(1/4),x)
```

```
[Out] Integral(x**2*(x**3*(a*x + b))**(1/4), x)
```

3.1523 $\int \frac{1+x^6}{\sqrt{x+x^2+x^3}\left(1-x^6\right)} \, dx$

Optimal. Leaf size=123

$$\frac{2\sqrt{x^3+x^2+x}}{3\left(x^2+x+1\right)}+\frac{1}{3}\tan^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)+\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x^3+x^2+x}}{x^2+x+1}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x^3+x^2+x}}{x^2+x+1}\right)}{3\sqrt{3}}$$

Rubi [C] time = 2.51, antiderivative size = 785, normalized size of antiderivative = 6.38, number of steps used = 43, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {2056, 6725, 716, 1103, 934, 169, 538, 537, 849, 822, 839, 1197, 1195}

$$\frac{2x\left(-i\sqrt{3}x-(-1)^{2/3}+1\right)}{9\sqrt{x^3+x^2+x}}+\frac{2x\left(i\sqrt{3}x+\sqrt[3]{-1}+1\right)}{9\sqrt{x^3+x^2+x}}+\frac{\left(1+i\sqrt{3}\right)\sqrt{x}(x+1)\sqrt{\frac{x^2+x+1}{(x+1)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{6\sqrt{x^3+x^2+x}}+\frac{\left(1-i\sqrt{3}\right)\sqrt{x}(x+1)\sqrt{\frac{x^2+x+1}{(x+1)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{6\sqrt{x^3+x^2+x}}-\frac{\sqrt{x}(x+1)\sqrt{\frac{x^2+x+1}{(x+1)^2}}F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x^3+x^2+x}}+\frac{4\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\Pi\left(-1;\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{3\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}+\frac{4\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\Pi\left(\frac{1}{2}\left(1-i\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{3\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}+\frac{4\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\Pi\left(\frac{1}{2}\left(-1+i\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{3\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}+\frac{4\sqrt{x}\sqrt{1+\frac{2x}{1-i\sqrt{3}}}\sqrt{1+\frac{2x}{1+i\sqrt{3}}}\Pi\left(\frac{1}{2}\left(1+i\sqrt{3}\right);\sin^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{x}\right)|\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)}{3\left(1-i\sqrt{3}\right)\sqrt{x^3+x^2+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^6)/(Sqrt[x + x^2 + x^3]*(1 - x^6)),x]
```

```
[Out] (2*x*(1 - (-1)^(2/3) - I*Sqrt[3]*x))/(9*Sqrt[x + x^2 + x^3]) + (2*x*(1 + (-
1)^(1/3) + I*Sqrt[3]*x))/(9*Sqrt[x + x^2 + x^3]) - (Sqrt[x]*(1 + x)*Sqrt[(1
 + x + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/4])/Sqrt[x + x^2 + x^
3] + ((1 - I*Sqrt[3])*Sqrt[x]*(1 + x)*Sqrt[(1 + x + x^2)/(1 + x)^2]*Ellipti
cF[2*ArcTan[Sqrt[x]], 1/4])/(6*Sqrt[x + x^2 + x^3]) + ((1 + I*Sqrt[3])*Sqrt
[x]*(1 + x)*Sqrt[(1 + x + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/4]
)/(6*Sqrt[x + x^2 + x^3]) + (4*Sqrt[x]*Sqrt[1 + (2*x)/(1 - I*Sqrt[3])]*Sqrt
[1 + (2*x)/(1 + I*Sqrt[3])]*EllipticPi[-1, ArcSin[((1 - I*Sqrt[3])*Sqrt[x])
/2], (I + Sqrt[3])/(I - Sqrt[3])]])/(3*(1 - I*Sqrt[3])*Sqrt[x + x^2 + x^3])
 + (4*Sqrt[x]*Sqrt[1 + (2*x)/(1 - I*Sqrt[3])]*Sqrt[1 + (2*x)/(1 + I*Sqrt[3])
]*EllipticPi[(1 - I*Sqrt[3])/2, ArcSin[((1 - I*Sqrt[3])*Sqrt[x])/2], (I + S
qrt[3])/(I - Sqrt[3])]])/(3*(1 - I*Sqrt[3])*Sqrt[x + x^2 + x^3]) + (4*Sqrt[x
]*Sqrt[1 + (2*x)/(1 - I*Sqrt[3])]*Sqrt[1 + (2*x)/(1 + I*Sqrt[3])]*EllipticP
i[(-1 + I*Sqrt[3])/2, ArcSin[((1 - I*Sqrt[3])*Sqrt[x])/2], (I + Sqrt[3])/(I
 - Sqrt[3])]])/(3*(1 - I*Sqrt[3])*Sqrt[x + x^2 + x^3]) + (4*Sqrt[x]*Sqrt[1 +
 (2*x)/(1 - I*Sqrt[3])]*Sqrt[1 + (2*x)/(1 + I*Sqrt[3])]*EllipticPi[(1 + I*S
qrt[3])/2, ArcSin[((1 - I*Sqrt[3])*Sqrt[x])/2], (I + Sqrt[3])/(I - Sqrt[3])
]])/(3*(1 - I*Sqrt[3])*Sqrt[x + x^2 + x^3])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
```

```
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 716

```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2,
Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m^2, 1/4]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 839

```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Dist[2, Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 849

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + (c*x)/e)*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
```

```
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^6}{\sqrt{x+x^2+x^3}\left(1-x^6\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1+x^6}{\sqrt{x}\sqrt{1+x+x^2}\left(1-x^6\right)}\,dx}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \left(-\frac{1}{\sqrt{x}\sqrt{1+x+x^2}} + \frac{2}{\sqrt{x}\sqrt{1+x+x^2}\left(1-x^6\right)}\right)dx}{\sqrt{x+x^2+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+x+x^2}}\,dx}{\sqrt{x+x^2+x^3}} + \frac{\left(2\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+x+x^2}\left(1-x^6\right)}}{\sqrt{x+x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{1+x+x^2}\right)\int \left(\frac{1}{2\sqrt{x}\sqrt{1+x+x^2}\left(1-x^3\right)} + \frac{1}{2\sqrt{x}\sqrt{1+x+x^2}\left(1+x^3\right)}\right)dx}{\sqrt{x+x^2+x^3}} - \frac{\left(2\sqrt{x}\sqrt{1+\right.}{}$$

$$= -\frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} + \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{1+x+x^2}\left(1-\right.}}{\sqrt{x+x^2+x^3}}$$

$$= -\frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} + \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \left(-\frac{1}{3(-1-x)\sqrt{x}\sqrt{}}\right.}{}$$

$$= -\frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} - \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{1}{(-1-x)\sqrt{x}\sqrt{1+x+}}}{3\sqrt{x+x^2+x^3}}$$

$$= -\frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{4}\right)}{\sqrt{x+x^2+x^3}} - \frac{\left(\sqrt{x}\sqrt{1-i\sqrt{3}+2x}\sqrt{1+i\sqrt{3}+2}\right.}{3\sqrt{x-}}$$

$$= \frac{2x\left(1-(-1)^{2/3}-i\sqrt{3}\,x\right)}{9\sqrt{x+x^2+x^3}} + \frac{2x\left(1+\sqrt[3]{-1}+i\sqrt{3}\,x\right)}{9\sqrt{x+x^2+x^3}} - \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\,\mathrm{ta}\right.}{\sqrt{x+x^2+x^3}}$$

$$= \frac{2x\left(1-(-1)^{2/3}-i\sqrt{3}\,x\right)}{9\sqrt{x+x^2+x^3}} + \frac{2x\left(1+\sqrt[3]{-1}+i\sqrt{3}\,x\right)}{9\sqrt{x+x^2+x^3}} - \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\,\mathrm{ta}\right.}{\sqrt{x+x^2+x^3}}$$

$$= \frac{2x\left(1-(-1)^{2/3}-i\sqrt{3}\,x\right)}{9\sqrt{x+x^2+x^3}} + \frac{2x\left(1+\sqrt[3]{-1}+i\sqrt{3}\,x\right)}{9\sqrt{x+x^2+x^3}} - \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\,\mathrm{ta}\right.}{\sqrt{x+x^2+x^3}}$$

$$= \frac{2x\left(1-(-1)^{2/3}-i\sqrt{3}\,x\right)}{9\sqrt{x+x^2+x^3}} + \frac{2x\left(1+\sqrt[3]{-1}+i\sqrt{3}\,x\right)}{9\sqrt{x+x^2+x^3}} - \frac{\sqrt{x}(1+x)\sqrt{\frac{1+x+x^2}{(1+x)^2}}\,F\left(2\,\mathrm{ta}\right.}{\sqrt{x+x^2+x^3}}$$

Mathematica [C] time = 5.02, size = 939, normalized size = 7.63

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^6)/(Sqrt[x + x^2 + x^3]*(1 - x^6)),x]
```

```
[Out] (2*x*(1 + Sqrt[x]*(-2*(-1)^(2/3)*Sqrt[1 - (-1)^(2/3)/x]*Sqrt[((-1)^(1/3) +
x)/x]*EllipticF[I*ArcSinh[(-1)^(5/6)/Sqrt[x]], (-1)^(2/3)] + ((-1)^(2/3)*Sq
rt[(1 - (-1)^(1/3) + Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x]))]
*(-1 + (-1)^(1/3)*Sqrt[x])^2*Sqrt[(-1 + (-1)^(2/3)*Sqrt[x])/((1 + (-1)^(1/3
))*(-1 + (-1)^(1/3)*Sqrt[x]))]*Sqrt[-((1 + (-1)^(2/3)*Sqrt[x])/(-1 + (-1)^(
1/3) + Sqrt[x]))]*((1 + (-1)^(1/3))*EllipticF[ArcSin[Sqrt[(1 - (-1)^(1/3) +
 Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x]))]], -3] - 2*(-1)^(1/3
)*EllipticPi[-1, ArcSin[Sqrt[(1 - (-1)^(1/3) + Sqrt[x])/((1 + (-1)^(1/3))*(
-1 + (-1)^(1/3)*Sqrt[x]))]], -3]))/((1 + (-1)^(2/3))*x) - Sqrt[1 - (-1)^(2/
3)/x]*Sqrt[((-1)^(1/3) + x)/x]*EllipticPi[-1, I*ArcSinh[(-1)^(5/6)/Sqrt[x]]
, (-1)^(2/3)] + (-1)^(1/3)*Sqrt[1 - (-1)^(2/3)/x]*Sqrt[((-1)^(1/3) + x)/x]*
EllipticPi[-1, I*ArcSinh[(-1)^(5/6)/Sqrt[x]], (-1)^(2/3)] - ((-1)^(2/3)*Sqr
t[(1 - (-1)^(1/3) + Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x]))]*
(-1 + (-1)^(1/3)*Sqrt[x])^2*Sqrt[(-1 + (-1)^(2/3)*Sqrt[x])/((1 + (-1)^(1/3)
)*(-1 + (-1)^(1/3)*Sqrt[x]))]*Sqrt[-((1 + (-1)^(2/3)*Sqrt[x])/(-1 + (-1)^(1
/3) + Sqrt[x]))]*((-1 + (-1)^(1/3))*EllipticF[ArcSin[Sqrt[(1 - (-1)^(1/3) +
 Sqrt[x])/((1 + (-1)^(1/3))*(-1 + (-1)^(1/3)*Sqrt[x]))]], -3] - 2*(-1)^(1/3
)*EllipticPi[3, ArcSin[Sqrt[(1 - (-1)^(1/3) + Sqrt[x])/((1 + (-1)^(1/3))*(-
1 + (-1)^(1/3)*Sqrt[x]))]], -3]))/((1 + (-1)^(2/3))*x) + (-1)^(2/3)*Sqrt[1
- (-1)^(2/3)/x]*Sqrt[((-1)^(1/3) + x)/x]*EllipticPi[(-1)^(1/3), I*ArcSinh[(
-1)^(5/6)/Sqrt[x]], (-1)^(2/3)] - (3*(-1)^(1/3)*Sqrt[1 - (-1)^(2/3)/x]*Sqrt
[((-1)^(1/3) + x)/x]*EllipticPi[-(-1)^(2/3), I*ArcSinh[(-1)^(5/6)/Sqrt[x]],
 (-1)^(2/3)])/(1 + (-1)^(1/3))^2 + (3*(-1)^(2/3)*Sqrt[1 - (-1)^(2/3)/x]*Sqr
t[((-1)^(1/3) + x)/x]*EllipticPi[-(-1)^(2/3), I*ArcSinh[(-1)^(5/6)/Sqrt[x]]
, (-1)^(2/3)])/(1 + (-1)^(1/3))^2)))/(3*Sqrt[x*(1 + x + x^2)])
```

IntegrateAlgebraic [A] time = 0.22, size = 123, normalized size = 1.00

$$\frac{2\sqrt{x^3+x^2+x}}{3\left(x^2+x+1\right)}+\frac{1}{3}\tan^{-1}\left(\frac{\sqrt{x^3+x^2+x}}{x^2+x+1}\right)+\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x^3+x^2+x}}{x^2+x+1}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x^3+x^2+x}}{x^2+x+1}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^6)/(Sqrt[x + x^2 + x^3]*(1 - x^6)),x]
```

```
[Out] (2*Sqrt[x + x^2 + x^3])/(3*(1 + x + x^2)) + ArcTan[Sqrt[x + x^2 + x^3]/(1 +
 x + x^2)]/3 + (Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x + x^2 + x^3])/(1 + x + x^2)
])/3 + ArcTanh[(Sqrt[3]*Sqrt[x + x^2 + x^3])/(1 + x + x^2)]/(3*Sqrt[3])
```

fricas [A] time = 0.45, size = 196, normalized size = 1.59

$$\frac{3\sqrt{2}\left(x^2+x+1\right)\log\left(\frac{x^4+14x^3+4\sqrt{2}\sqrt{x^3+x^2+x}\left(x^2+3x+1\right)+19x^2+14x+1}{x^4-2x^3+3x^2-2x+1}\right)+\sqrt{3}\left(x^2+x+1\right)\log\left(\frac{x^4+20x^3+4\sqrt{3}\sqrt{x^3+x^2+x}\left(x^2+4x+1\right)+30x^2+20x+1}{x^4-4x^3+6x^2-4x+1}\right)-6\left(x^2+x+1\right)\arctan\left(\frac{x^2+1}{2\sqrt{x^3+x^2+x}}\right)+24\sqrt{x^3+x^2+x}}{36\left(x^2+x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^3+x^2+x)^(1/2)/(-x^6+1),x, algorithm="fricas")
```

```
[Out] 1/36*(3*sqrt(2)*(x^2 + x + 1)*log((x^4 + 14*x^3 + 4*sqrt(2)*sqrt(x^3 + x^2
+ x)*(x^2 + 3*x + 1) + 19*x^2 + 14*x + 1)/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1))
+ sqrt(3)*(x^2 + x + 1)*log((x^4 + 20*x^3 + 4*sqrt(3)*sqrt(x^3 + x^2 + x)*(
x^2 + 4*x + 1) + 30*x^2 + 20*x + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)) - 6*(x
^2 + x + 1)*arctan(1/2*(x^2 + 1)/sqrt(x^3 + x^2 + x)) + 24*sqrt(x^3 + x^2 +
 x))/(x^2 + x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^6+1}{\left(x^6-1\right)\sqrt{x^3+x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^3+x^2+x)^(1/2)/(-x^6+1),x, algorithm="giac")
[Out] integrate(-(x^6 + 1)/((x^6 - 1)*sqrt(x^3 + x^2 + x)), x)
```

maple [C] time = 0.30, size = 736, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/(x^3+x^2+x)^(1/2)/(-x^6+1),x)
[Out] -4/9*(1/2+1/2*I*3^(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*
3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1
/2)/(x^3+x^2+x)^(1/2)*EllipticF(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))
^(1/2),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))-2/9*(1/2+1/2*I*3
^(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2)*3^(1/2)*(I*(x+1/2
-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(
1/2)/(-3/2-1/2*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1
/2)))^(1/2),(-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2))+2/9*(1/2+1/2*I*3^(1/2))*((x+1/2+1/2*I*3^(1/2
))/(1/2+1/2*I*3^(1/2)))^(1/2)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2)*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2)/(1/2-1/2*I*3^(1/2))*Ell
ipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1/2),(-1/2-1/2*I*3^(1/
2))/(1/2-1/2*I*3^(1/2)),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))
+2/3*x/(x*(x^2+x+1))^(1/2)-2/9*(1/2+1/2*I*3^(1/2))^2*((x+1/2+1/2*I*3^(1/2))
/(1/2+1/2*I*3^(1/2)))^(1/2)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)
*(x/(-1/2-1/2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2)*(-1/4*I*(1/2+1/2*I*3^(1/2
))*3^(1/2)-5/8+3/8*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*
3^(1/2)))^(1/2),1/2,1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))-2/9
*(1/2-1/2*I*3^(1/2))*(1/2+1/2*I*3^(1/2))*((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*
3^(1/2)))^(1/2)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(x/(-1/2-1/
2*I*3^(1/2)))^(1/2)/(x^3+x^2+x)^(1/2)*(-1/4*I*(1/2-1/2*I*3^(1/2))*3^(1/2)-5
/8+1/8*I*3^(1/2))*EllipticPi(((x+1/2+1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))^(1
/2),1/2+1/2*I*3^(1/2),1/3*3^(1/2)*(I*(-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^6+1}{\left(x^6-1\right)\sqrt{x^3+x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^3+x^2+x)^(1/2)/(-x^6+1),x, algorithm="maxima")
[Out] -integrate((x^6 + 1)/((x^6 - 1)*sqrt(x^3 + x^2 + x)), x)
```

mupad [B] time = 0.88, size = 1195, normalized size = 9.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^6 + 1)/((x^6 - 1)*(x + x^2 + x^3)^(1/2)),x)
[Out] (2*((3^(1/2)*1i)/6 - 1/6)*(x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*
1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 + 1/2))^(1/2)*ellipticPi(-1, asin((x/((3^(1/2)*1i)/2 - 1/2))^(1
/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))/(x^2 + x^3 - x*((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)/2 - 1/2)
*(x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i
)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/2)
*ellipticF(asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/
((3^(1/2)*1i)/2 + 1/2)))/(x^2 + x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i
```

```
)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)/6 - 1/6)*(x/((3^(1/2)*1i)/2 - 1/2))^(1
/2)*((ellipticE(asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 -
1/2)/((3^(1/2)*1i)/2 + 1/2)) - ((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(x/((3^(1/
2)*1i)/2 + 1/2) + 1)^(1/2))/(1 - x/((3^(1/2)*1i)/2 - 1/2))^(1/2))/(((3^(1/2
)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2) + 1) - ellipticF(asin((x/((3^(1/2)*1i
)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))*(-(x -
 (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 +
 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/2))/(x^2 + x^3 - x*((3^(1/2)*1i)/2 - 1/2)*
((3^(1/2)*1i)/2 + 1/2))^(1/2) + (2*((3^(1/2)*1i)/2 - 1/2)*(x/((3^(1/2)*1i)/
2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 1/2))^(1/2)*
((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/2)*ellipticPi(1/2 -
(3^(1/2)*1i)/2, asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 -
1/2)/((3^(1/2)*1i)/2 + 1/2)))/(3*(x^2 + x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*1i)/2 + 1/2))^(1/2)) + (2*((3^(1/2)*1i)/2 - 1/2)*(x/((3^(1/2)*1i)/2 -
 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 1/2))^(1/2)*((x
 + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/2)*ellipticPi((3^(1/2)*
1i)/2 - 1/2, asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2
)/((3^(1/2)*1i)/2 + 1/2)))/(3*(x^2 + x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2))^(1/2)) + (2*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6)*
(x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/2)*
ellipticPi(((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2), asin((x/((3^(1/2)
*1i)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)))/(((
3^(1/2)*1i)/2 + 1/2)*(x^2 + x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2))^(1/2)) + (2*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6)*(x/((3^(1
/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 1/2)
)^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/2)*(elliptic
E(asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)
*1i)/2 + 1/2)) + (sin(2*asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)))*((3^(1/2)*1
i)/2 - 1/2))/(2*((3^(1/2)*1i)/2 + 1/2)*(x/((3^(1/2)*1i)/2 + 1/2) + 1)^(1/2)
)))/(((3^(1/2)*1i)/2 + 1/2)*(((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)
+ 1)*(x^2 + x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^6}{x^6\sqrt{x^3+x^2+x}-\sqrt{x^3+x^2+x}}\,dx-\int \frac{1}{x^6\sqrt{x^3+x^2+x}-\sqrt{x^3+x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/(x**3+x**2+x)**(1/2)/(-x**6+1),x)
```

```
[Out] -Integral(x**6/(x**6*sqrt(x**3 + x**2 + x) - sqrt(x**3 + x**2 + x)), x) - I
ntegral(1/(x**6*sqrt(x**3 + x**2 + x) - sqrt(x**3 + x**2 + x)), x)
```

3.1524 $\int \frac{1-3x^3+3x^6}{x^6\left(-1+2x^3\right)\sqrt[4]{-x+x^4}}\,dx$

Optimal. Leaf size=123

$$-\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^4-x}}{\sqrt{x^4-x}-x^2}\right)-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{x^4-x}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{x^4-x}}\right)+\frac{4\left(x^4-x\right)^{3/4}\left(x^3-1\right)}{21x^6}$$

Rubi [B] time = 0.82, antiderivative size = 338, normalized size of antiderivative = 2.75, number of steps used = 20, number of rules used = 14, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2056, 6725, 271, 264, 466, 465, 494, 461, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{x}\sqrt[4]{x^3-1}\log\left(\frac{x^{3/2}}{\sqrt{x^3-1}}-\frac{\sqrt{2}x^{3/4}}{\sqrt[4]{x^3-1}}+1\right)}{3\sqrt{2}\sqrt[4]{x^4-x}}-\frac{\sqrt[4]{x}\sqrt[4]{x^3-1}\log\left(\frac{x^{3/2}}{\sqrt{x^3-1}}+\frac{\sqrt{2}x^{3/4}}{\sqrt[4]{x^3-1}}+1\right)}{3\sqrt{2}\sqrt[4]{x^4-x}}+\frac{\sqrt{2}\sqrt[4]{x}\sqrt[4]{x^3-1}\tan^{-1}\left(1-\frac{\sqrt{2}x^{3/4}}{\sqrt[4]{x^3-1}}\right)}{3\sqrt[4]{x^4-x}}-\frac{\sqrt{2}\sqrt[4]{x}\sqrt[4]{x^3-1}\tan^{-1}\left(\frac{\sqrt{2}x^{3/4}}{\sqrt[4]{x^3-1}}+1\right)}{3\sqrt[4]{x^4-x}}+\frac{\left(1-x^3\right)^2}{21x^5\sqrt[4]{x^4-x}}+\frac{1-x^3}{7x^5\sqrt[4]{x^4-x}}-\frac{1-x^3}{7x^2\sqrt[4]{x^4-x}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x^3 + 3*x^6)/(x^6*(-1 + 2*x^3)*(-x + x^4)^(1/4)),x]
```

```
[Out] (1 - x^3)/(7*x^5*(-x + x^4)^(1/4)) - (1 - x^3)/(7*x^2*(-x + x^4)^(1/4)) + (
1 - x^3)^2/(21*x^5*(-x + x^4)^(1/4)) + (Sqrt[2]*x^(1/4)*(-1 + x^3)^(1/4)*Ar
cTan[1 - (Sqrt[2]*x^(3/4))/(-1 + x^3)^(1/4)])/(3*(-x + x^4)^(1/4)) - (Sqrt[
2]*x^(1/4)*(-1 + x^3)^(1/4)*ArcTan[1 + (Sqrt[2]*x^(3/4))/(-1 + x^3)^(1/4)])
/(3*(-x + x^4)^(1/4)) + (x^(1/4)*(-1 + x^3)^(1/4)*Log[1 + x^(3/2)/Sqrt[-1 +
 x^3] - (Sqrt[2]*x^(3/4))/(-1 + x^3)^(1/4)])/(3*Sqrt[2]*(-x + x^4)^(1/4)) -
 (x^(1/4)*(-1 + x^3)^(1/4)*Log[1 + x^(3/2)/Sqrt[-1 + x^3] + (Sqrt[2]*x^(3/4
))/(-1 + x^3)^(1/4)])/(3*Sqrt[2]*(-x + x^4)^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
```

```
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
```

```
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1-3x^3+3x^6}{x^6\left(-1+2x^3\right)\sqrt[4]{-x+x^4}}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\int \frac{1-3x^3+3x^6}{x^{25/4}\sqrt[4]{-1+x^3}\left(-1+2x^3\right)}\,dx}{\sqrt[4]{-x+x^4}}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\int \left(-\frac{3}{4x^{25/4}\sqrt[4]{-1+x^3}}+\frac{3}{2x^{13/4}\sqrt[4]{-1+x^3}}+\frac{1}{4x^{25/4}\sqrt[4]{-1+x^3}\left(-1+2x^3\right)}\right)dx}{\sqrt[4]{-x+x^4}}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\int \frac{1}{x^{25/4}\sqrt[4]{-1+x^3}\left(-1+2x^3\right)}\,dx}{4\sqrt[4]{-x+x^4}} - \frac{\left(3\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\int \frac{1}{x^{25/4}\sqrt[4]{-1+x^3}}\,dx}{4\sqrt[4]{-x+x^4}}$$

$$= \frac{1-x^3}{7x^5\sqrt[4]{-x+x^4}} - \frac{2\left(1-x^3\right)}{3x^2\sqrt[4]{-x+x^4}} - \frac{\left(3\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\int \frac{1}{x^{13/4}\sqrt[4]{-1+x^3}}\,dx}{7\sqrt[4]{-x+x^4}} + \frac{\left(\sqrt[4]{x}\right.}{}$$

$$= \frac{1-x^3}{7x^5\sqrt[4]{-x+x^4}} - \frac{10\left(1-x^3\right)}{21x^2\sqrt[4]{-x+x^4}} + \frac{\left(\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\text{Subst}\left(\int \frac{1}{x^8\sqrt[4]{-1+x^4}\left(-1+2x^4\right.}\right.}{3\sqrt[4]{-x+x^4}}$$

$$= \frac{1-x^3}{7x^5\sqrt[4]{-x+x^4}} - \frac{10\left(1-x^3\right)}{21x^2\sqrt[4]{-x+x^4}} + \frac{\left(\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\text{Subst}\left(\int \frac{\left(1-x^4\right)^2}{x^8\left(-1-x^4\right)}\,dx,x,\right.}{3\sqrt[4]{-x+x^4}}$$

$$= \frac{1-x^3}{7x^5\sqrt[4]{-x+x^4}} - \frac{10\left(1-x^3\right)}{21x^2\sqrt[4]{-x+x^4}} + \frac{\left(\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\text{Subst}\left(\int \left(-\frac{1}{x^8}+\frac{3}{x^4}-\frac{4}{1+}\right.\right.}{3\sqrt[4]{-x+x^4}}$$

$$= \frac{1-x^3}{7x^5\sqrt[4]{-x+x^4}} - \frac{1-x^3}{7x^2\sqrt[4]{-x+x^4}} + \frac{\left(1-x^3\right)^2}{21x^5\sqrt[4]{-x+x^4}} - \frac{\left(4\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\text{Subst}\left(\right.}{3\sqrt[4]{-x}}$$

$$= \frac{1-x^3}{7x^5\sqrt[4]{-x+x^4}} - \frac{1-x^3}{7x^2\sqrt[4]{-x+x^4}} + \frac{\left(1-x^3\right)^2}{21x^5\sqrt[4]{-x+x^4}} - \frac{\left(2\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\text{Subst}\left(\right.}{3\sqrt[4]{-x}}$$

$$= \frac{1-x^3}{7x^5\sqrt[4]{-x+x^4}} - \frac{1-x^3}{7x^2\sqrt[4]{-x+x^4}} + \frac{\left(1-x^3\right)^2}{21x^5\sqrt[4]{-x+x^4}} - \frac{\left(\sqrt[4]{x}\sqrt[4]{-1+x^3}\right)\text{Subst}\left(\right.}{3\sqrt[4]{-}}$$

$$= \frac{1-x^3}{7x^5\sqrt[4]{-x+x^4}} - \frac{1-x^3}{7x^2\sqrt[4]{-x+x^4}} + \frac{\left(1-x^3\right)^2}{21x^5\sqrt[4]{-x+x^4}} + \frac{\sqrt[4]{x}\sqrt[4]{-1+x^3}\log\left(1+\right.}{3\sqrt{2}\sqrt[4]{-x+}}$$

$$= \frac{1-x^3}{7x^5\sqrt[4]{-x+x^4}} - \frac{1-x^3}{7x^2\sqrt[4]{-x+x^4}} + \frac{\left(1-x^3\right)^2}{21x^5\sqrt[4]{-x+x^4}} + \frac{\sqrt{2}\sqrt[4]{x}\sqrt[4]{-1+x^3}\tan^{-1}\left(\right.}{3\sqrt[4]{-x+x^4}}$$

Mathematica [C] time = 1.37, size = 192, normalized size = 1.56

$$\frac{16\left(1-2x^3\right)^2x^3\,{}_3F_2\left(\frac{5}{4},2,2;1,\frac{9}{4};\frac{x^3}{1-x^3}\right)-8x^3\,{}_2F_1\left(\frac{5}{4},2;\frac{9}{4};\frac{x^3}{1-x^3}\right)+192x^9\,{}_2F_1\left(\frac{5}{4},2;\frac{9}{4};\frac{x^3}{1-x^3}\right)-80x^6\,{}_2F_1\left(\frac{5}{4},2;\frac{9}{4};\frac{x^3}{1-x^3}\right)-5\left(128x^9-144x^6+13x^3+3\right)\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{x^3}{1-x^3}\right)+150x^9-345x^6+240x^3-45}{315x^4\left(x\left(x^3-1\right)\right)^{5/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - 3*x^3 + 3*x^6)/(x^6*(-1 + 2*x^3)*(-x + x^4)^(1/4)),x]
```

```
[Out] (-45 + 240*x^3 - 345*x^6 + 150*x^9 - 5*(3 + 13*x^3 - 144*x^6 + 128*x^9)*Hyp
ergeometric2F1[1/4, 1, 5/4, x^3/(1 - x^3)] - 8*x^3*Hypergeometric2F1[5/4, 2
, 9/4, x^3/(1 - x^3)] - 80*x^6*Hypergeometric2F1[5/4, 2, 9/4, x^3/(1 - x^3)
] + 192*x^9*Hypergeometric2F1[5/4, 2, 9/4, x^3/(1 - x^3)] + 16*x^3*(1 - 2*x
^3)^2*HypergeometricPFQ[{5/4, 2, 2}, {1, 9/4}, x^3/(1 - x^3)])/(315*x^4*(x*
(-1 + x^3))^(5/4))
```

IntegrateAlgebraic [A] time = 0.68, size = 123, normalized size = 1.00

$$-\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^4-x}}{\sqrt{x^4-x}-x^2}\right)-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{x^4-x}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{x^4-x}}\right)+\frac{4\left(x^4-x\right)^{3/4}\left(x^3-1\right)}{21x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - 3*x^3 + 3*x^6)/(x^6*(-1 + 2*x^3)*(-x + x^4)^(1/4)),
x]
```

```
[Out] (4*(-1 + x^3)*(-x + x^4)^(3/4))/(21*x^6) - (Sqrt[2]*ArcTan[(Sqrt[2]*x*(-x +
 x^4)^(1/4))/(-x^2 + Sqrt[-x + x^4])])/3 - (Sqrt[2]*ArcTanh[(x^2/Sqrt[2] +
Sqrt[-x + x^4]/Sqrt[2])/(x*(-x + x^4)^(1/4))])/3
```

fricas [B] time = 2.02, size = 479, normalized size = 3.89

$$\frac{28\sqrt{2}x^6\arctan\left(-\frac{\sqrt{2}(x^4-x)^{\frac{3}{4}}x-\sqrt{2}(x^4-x)^{\frac{1}{4}}(x^3-1)+\left(2x^4-\sqrt{2}(x^4-x)^{\frac{3}{4}}x-\sqrt{2}(x^4-x)^{\frac{1}{4}}(x^3-1)-2x\right)\sqrt{\frac{2x^3+2\sqrt{2}(x^4-x)^{\frac{1}{4}}x^2+4\sqrt{x^4-x}\,x+2\sqrt{2}(x^4-x)^{\frac{3}{4}}-1}{2x^3-1}}}{2(x^4-x)}\right)+28\sqrt{2}x^6\arctan\left(-\frac{\sqrt{2}(x^4-x)^{\frac{3}{4}}x-\sqrt{2}(x^4-x)^{\frac{1}{4}}(x^3-1)-\left(2x^4+\sqrt{2}(x^4-x)^{\frac{3}{4}}x+\sqrt{2}(x^4-x)^{\frac{1}{4}}(x^3-1)-2x\right)\sqrt{\frac{2x^3-2\sqrt{2}(x^4-x)^{\frac{1}{4}}x^2+4\sqrt{x^4-x}\,x-2\sqrt{2}(x^4-x)^{\frac{3}{4}}-1}{2x^3-1}}}{2(x^4-x)}\right)-7\sqrt{2}x^6\log\left(\frac{2x^3+2\sqrt{2}(x^4-x)^{\frac{1}{4}}x^2+4\sqrt{x^4-x}\,x+2\sqrt{2}(x^4-x)^{\frac{3}{4}}-1}{2x^3-1}\right)+7\sqrt{2}x^6\log\left(\frac{2x^3-2\sqrt{2}(x^4-x)^{\frac{1}{4}}x^2+4\sqrt{x^4-x}\,x-2\sqrt{2}(x^4-x)^{\frac{3}{4}}-1}{2x^3-1}\right)+16(x^4-x)^{\frac{3}{4}}(x^3-1)}{84x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-3*x^3+1)/x^6/(2*x^3-1)/(x^4-x)^(1/4),x, algorithm="fricas"
)
```

```
[Out] 1/84*(28*sqrt(2)*x^6*arctan(-1/2*(sqrt(2)*(x^4 - x)^(3/4)*x - sqrt(2)*(x^4
- x)^(1/4)*(x^3 - 1) + (2*x^4 - sqrt(2)*(x^4 - x)^(3/4)*x - sqrt(2)*(x^4 -
x)^(1/4)*(x^3 - 1) - 2*x)*sqrt((2*x^3 + 2*sqrt(2)*(x^4 - x)^(1/4)*x^2 + 4*s
qrt(x^4 - x)*x + 2*sqrt(2)*(x^4 - x)^(3/4) - 1)/(2*x^3 - 1)))/(x^4 - x)) +
28*sqrt(2)*x^6*arctan(-1/2*(sqrt(2)*(x^4 - x)^(3/4)*x - sqrt(2)*(x^4 - x)^(
1/4)*(x^3 - 1) - (2*x^4 + sqrt(2)*(x^4 - x)^(3/4)*x + sqrt(2)*(x^4 - x)^(1/
4)*(x^3 - 1) - 2*x)*sqrt((2*x^3 - 2*sqrt(2)*(x^4 - x)^(1/4)*x^2 + 4*sqrt(x^
4 - x)*x - 2*sqrt(2)*(x^4 - x)^(3/4) - 1)/(2*x^3 - 1)))/(x^4 - x)) - 7*sqrt
(2)*x^6*log((2*x^3 + 2*sqrt(2)*(x^4 - x)^(1/4)*x^2 + 4*sqrt(x^4 - x)*x + 2*
sqrt(2)*(x^4 - x)^(3/4) - 1)/(2*x^3 - 1)) + 7*sqrt(2)*x^6*log((2*x^3 - 2*sq
rt(2)*(x^4 - x)^(1/4)*x^2 + 4*sqrt(x^4 - x)*x - 2*sqrt(2)*(x^4 - x)^(3/4) -
 1)/(2*x^3 - 1)) + 16*(x^4 - x)^(3/4)*(x^3 - 1))/x^6
```

giac [A] time = 0.24, size = 125, normalized size = 1.02

$$-\frac{4}{21}\left(-\frac{1}{x^3}+1\right)^{\frac{7}{4}}-\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{3}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}\right)\right)+\frac{1}{6}\sqrt{2}\log\left(\sqrt{2}\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}+\sqrt{-\frac{1}{x^3}+1}+1\right)-\frac{1}{6}\sqrt{2}\log\left(-\sqrt{2}\left(-\frac{1}{x^3}+1\right)^{\frac{1}{4}}+\sqrt{-\frac{1}{x^3}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-3*x^3+1)/x^6/(2*x^3-1)/(x^4-x)^(1/4),x, algorithm="giac")
```

```
[Out] -4/21*(-1/x^3 + 1)^(7/4) - 1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(-1/
x^3 + 1)^(1/4))) - 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(-1/x^3 + 1
)^(1/4))) + 1/6*sqrt(2)*log(sqrt(2)*(-1/x^3 + 1)^(1/4) + sqrt(-1/x^3 + 1) +
 1) - 1/6*sqrt(2)*log(-sqrt(2)*(-1/x^3 + 1)^(1/4) + sqrt(-1/x^3 + 1) + 1)
```

maple [C] time = 4.27, size = 187, normalized size = 1.52

$$\frac{\frac{4}{21}x^{6}-\frac{8}{21}x^{3}+\frac{4}{21}}{x^{5}\left(x\left(x^{3}-1\right)\right)^{\frac{1}{4}}}+\frac{\mathrm{RootOf}\left(\_Z^{4}+1\right)\ln\left(-\frac{2\sqrt{x^{4}-x}\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}x-2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}\left(x^{4}-x\right)^{\frac{1}{4}}x^{2}+2\left(x^{4}-x\right)^{\frac{3}{4}}+\mathrm{RootOf}\left(\_Z^{4}+1\right)}{2x^{3}-1}\right)}{3}+\frac{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}\left(x^{4}-x\right)^{\frac{1}{4}}x^{2}+2\sqrt{x^{4}-x}\,\mathrm{RootOf}\left(\_Z^{4}+1\right)x+\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}+2\left(x^{4}-x\right)^{\frac{3}{4}}}{2x^{3}-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^6-3*x^3+1)/x^6/(2*x^3-1)/(x^4-x)^(1/4),x)
```

```
[Out] 4/21*(x^6-2*x^3+1)/x^5/(x*(x^3-1))^(1/4)+1/3*RootOf(_Z^4+1)*ln(-(2*(x^4-x)^
(1/2)*RootOf(_Z^4+1)^3*x-2*RootOf(_Z^4+1)^2*(x^4-x)^(1/4)*x^2+2*(x^4-x)^(3/
4)+RootOf(_Z^4+1))/(2*x^3-1))+1/3*RootOf(_Z^4+1)^3*ln((2*RootOf(_Z^4+1)^2*(
x^4-x)^(1/4)*x^2+2*(x^4-x)^(1/2)*RootOf(_Z^4+1)*x+RootOf(_Z^4+1)^3+2*(x^4-x
)^(3/4))/(2*x^3-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^{6}-3\,x^{3}+1}{\left(x^{4}-x\right)^{\frac{1}{4}}\left(2\,x^{3}-1\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-3*x^3+1)/x^6/(2*x^3-1)/(x^4-x)^(1/4),x, algorithm="maxima"
)
```

```
[Out] integrate((3*x^6 - 3*x^3 + 1)/((x^4 - x)^(1/4)*(2*x^3 - 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x^{6}-3\,x^{3}+1}{x^{6}\,{\left(x^{4}-x\right)}^{1/4}\,\left(2\,x^{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^6 - 3*x^3 + 1)/(x^6*(x^4 - x)^(1/4)*(2*x^3 - 1)),x)
```

```
[Out] int((3*x^6 - 3*x^3 + 1)/(x^6*(x^4 - x)^(1/4)*(2*x^3 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^{6}-3x^{3}+1}{x^{6}\sqrt[4]{x\left(x-1\right)\left(x^{2}+x+1\right)}\left(2x^{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**6-3*x**3+1)/x**6/(2*x**3-1)/(x**4-x)**(1/4),x)
```

```
[Out] Integral((3*x**6 - 3*x**3 + 1)/(x**6*(x*(x - 1)*(x**2 + x + 1))**(1/4)*(2*x
**3 - 1)), x)
```

3.1525 $\int \frac{b+ax^6}{x^3(b+ax^3)\sqrt[4]{bx+ax^4}}\,dx$

Optimal. Leaf size=123

$$-\frac{4\left(ax^4+bx\right)^{3/4}\left(4ax^3+3bx^3+b\right)}{9bx^3\left(ax^3+b\right)}+\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}$$

Rubi [A] time = 0.37, antiderivative size = 183, normalized size of antiderivative = 1.49, number of steps used = 12, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2056, 1489, 271, 264, 288, 329, 275, 240, 212, 206, 203}

$$-\frac{16ax}{9b\sqrt[4]{ax^4+bx}}-\frac{4x}{3\sqrt[4]{ax^4+bx}}-\frac{4}{9x^2\sqrt[4]{ax^4+bx}}+\frac{2\sqrt[4]{x}\sqrt[4]{ax^3+b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{a}\sqrt[4]{ax^4+bx}}+\frac{2\sqrt[4]{x}\sqrt[4]{ax^3+b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{a}\sqrt[4]{ax^4+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^6)/(x^3*(b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] -4/(9*x^2*(b*x + a*x^4)^(1/4)) - (4*x)/(3*(b*x + a*x^4)^(1/4)) - (16*a*x)/(
9*b*(b*x + a*x^4)^(1/4)) + (2*x^(1/4)*(b + a*x^3)^(1/4)*ArcTan[(a^(1/4)*x^(
3/4))/(b + a*x^3)^(1/4)])/(3*a^(1/4)*(b*x + a*x^4)^(1/4)) + (2*x^(1/4)*(b +
 a*x^3)^(1/4)*ArcTanh[(a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)])/(3*a^(1/4)*(b*x
 + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1489

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IG
tQ[n, 0] && IGtQ[p, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{b+ax^6}{x^3\left(b+ax^3\right)\sqrt[4]{bx+ax^4}}\,dx = \frac{\left(\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\int \frac{b+ax^6}{x^{13/4}\left(b+ax^3\right)^{5/4}}\,dx}{\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\int \left(\frac{b}{x^{13/4}\left(b+ax^3\right)^{5/4}}+\frac{ax^{11/4}}{\left(b+ax^3\right)^{5/4}}\right)dx}{\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(a\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\int \frac{x^{11/4}}{\left(b+ax^3\right)^{5/4}}\,dx}{\sqrt[4]{bx+ax^4}} + \frac{\left(b\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\int \frac{1}{x^{13/4}\left(b+ax^3\right)^{5/4}}\,dx}{\sqrt[4]{bx+ax^4}}$$

$$= -\frac{4}{9x^2\sqrt[4]{bx+ax^4}} - \frac{4x}{3\sqrt[4]{bx+ax^4}} + \frac{\left(\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\int \frac{1}{\sqrt[4]{x}\,\sqrt[4]{b+ax^3}}\,dx}{\sqrt[4]{bx+ax^4}} - \frac{\left(4a\sqrt[4]{x}\,\sqrt[4]{b\right.}}{}$$

$$= -\frac{4}{9x^2\sqrt[4]{bx+ax^4}} - \frac{4x}{3\sqrt[4]{bx+ax^4}} - \frac{16ax}{9b\sqrt[4]{bx+ax^4}} + \frac{\left(4\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \right.}{\sqrt[4]{bx+ax^4}}$$

$$= -\frac{4}{9x^2\sqrt[4]{bx+ax^4}} - \frac{4x}{3\sqrt[4]{bx+ax^4}} - \frac{16ax}{9b\sqrt[4]{bx+ax^4}} + \frac{\left(4\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \right.}{3\sqrt[4]{bx+ax^4}}$$

$$= -\frac{4}{9x^2\sqrt[4]{bx+ax^4}} - \frac{4x}{3\sqrt[4]{bx+ax^4}} - \frac{16ax}{9b\sqrt[4]{bx+ax^4}} + \frac{\left(4\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \right.}{3\sqrt[4]{bx+ax^4}}$$

$$= -\frac{4}{9x^2\sqrt[4]{bx+ax^4}} - \frac{4x}{3\sqrt[4]{bx+ax^4}} - \frac{16ax}{9b\sqrt[4]{bx+ax^4}} + \frac{\left(2\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \right.}{3\sqrt[4]{bx+ax}}$$

$$= -\frac{4}{9x^2\sqrt[4]{bx+ax^4}} - \frac{4x}{3\sqrt[4]{bx+ax^4}} - \frac{16ax}{9b\sqrt[4]{bx+ax^4}} + \frac{2\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{b+ax^3}}\right.}{3\sqrt[4]{a}\,\sqrt[4]{bx+ax^4}}$$

Mathematica [C] time = 0.12, size = 73, normalized size = 0.59

$$\frac{4\left(3ax^6\sqrt[4]{\frac{ax^3}{b}+1}\;{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};-\frac{ax^3}{b}\right)-5\left(4ax^3+b\right)\right)}{45bx^2\sqrt[4]{x\left(ax^3+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^6)/(x^3*(b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*(-5*(b + 4*a*x^3) + 3*a*x^6*(1 + (a*x^3)/b)^(1/4)*Hypergeometric2F1[5/4,
 5/4, 9/4, -((a*x^3)/b)]))/(45*b*x^2*(x*(b + a*x^3))^(1/4))
```

IntegrateAlgebraic [A] time = 0.47, size = 123, normalized size = 1.00

$$-\frac{4\left(ax^4+bx\right)^{3/4}\left(4ax^3+3bx^3+b\right)}{9bx^3\left(ax^3+b\right)}+\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^6)/(x^3*(b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(b + 4*a*x^3 + 3*b*x^3)*(b*x + a*x^4)^(3/4))/(9*b*x^3*(b + a*x^3)) + (2
*ArcTan[(a^(1/4)*(b*x + a*x^4)^(3/4))/(b + a*x^3)])/(3*a^(1/4)) + (2*ArcTan
h[(a^(1/4)*(b*x + a*x^4)^(3/4))/(b + a*x^3)])/(3*a^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^3/(a*x^3+b)/(a*x^4+b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.39, size = 217, normalized size = 1.76

$$\frac{\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{3\,a}+\frac{\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{3\,a}-\frac{\sqrt{2}\,(-a)^{\frac{3}{4}}\log\left(\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)}{6\,a}+\frac{\sqrt{2}\,(-a)^{\frac{3}{4}}\log\left(-\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)}{6\,a}-\frac{4\,(a+b)}{3\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}b}-\frac{4\left(a+\frac{b}{x^3}\right)^{\frac{3}{4}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^3/(a*x^3+b)/(a*x^4+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^
3)^(1/4))/(-a)^(1/4))/a + 1/3*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(
2)*(-a)^(1/4) - 2*(a + b/x^3)^(1/4))/(-a)^(1/4))/a - 1/6*sqrt(2)*(-a)^(3/4)
*log(sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a) + sqrt(a + b/x^3))/a +
 1/6*sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a
) + sqrt(a + b/x^3))/a - 4/3*(a + b)/((a + b/x^3)^(1/4)*b) - 4/9*(a + b/x^3
)^(3/4)/b
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{a\,x^{6}+b}{x^{3}\left(a\,x^{3}+b\right)\left(a\,x^{4}+bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6+b)/x^3/(a*x^3+b)/(a*x^4+b*x)^(1/4),x)
```

```
[Out] int((a*x^6+b)/x^3/(a*x^3+b)/(a*x^4+b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{6}+b}{\left(ax^{4}+bx\right)^{\frac{1}{4}}\left(ax^{3}+b\right)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^3/(a*x^3+b)/(a*x^4+b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^6 + b)/((a*x^4 + b*x)^(1/4)*(a*x^3 + b)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x^6+b}{x^3\,{\left(a\,x^4+b\,x\right)}^{1/4}\,\left(a\,x^3+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^6)/(x^3*(b*x + a*x^4)^(1/4)*(b + a*x^3)),x)
```

```
[Out] int((b + a*x^6)/(x^3*(b*x + a*x^4)^(1/4)*(b + a*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^6 + b}{x^3 \sqrt[4]{x\left(ax^3 + b\right)}\left(ax^3 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6+b)/x**3/(a*x**3+b)/(a*x**4+b*x)**(1/4),x)
```

```
[Out] Integral((a*x**6 + b)/(x**3*(x*(a*x**3 + b))**(1/4)*(a*x**3 + b)), x)
```

3.1526 $\int \frac{(-1-x-x^2+x^4)(2+x+2x^4)}{\sqrt{-1-x+x^4}\,(4+8x+3x^2-x^3-7x^4-8x^5+x^6+4x^8)}\,dx$

Optimal. Leaf size=123

$$\frac{\sqrt[4]{-1}\sqrt{\sqrt{15}+7i}\,\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{i\sqrt{15}}{2}}\,x}{2\sqrt{x^4-x-1}}\right)}{\sqrt{10}}-(-1)^{3/4}\sqrt{\frac{1}{10}\left(\sqrt{15}-7i\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}-\frac{i\sqrt{15}}{2}}\,x}{2\sqrt{x^4-x-1}}\right)$$

Rubi [F] time = 3.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1-x-x^2+x^4)(2+x+2x^4)}{\sqrt{-1-x+x^4}\,(4+8x+3x^2-x^3-7x^4-8x^5+x^6+4x^8)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 - x - x^2 + x^4)*(2 + x + 2*x^4))/(Sqrt[-1 - x + x^4]*(4 + 8*x + 3
*x^2 - x^3 - 7*x^4 - 8*x^5 + x^6 + 4*x^8)),x]
```

```
[Out] Defer[Int][1/Sqrt[-1 - x + x^4], x]/2 - 4*Defer[Int][1/(Sqrt[-1 - x + x^4]*
(4 + 8*x + 3*x^2 - x^3 - 7*x^4 - 8*x^5 + x^6 + 4*x^8)), x] - 7*Defer[Int][x
/(Sqrt[-1 - x + x^4]*(4 + 8*x + 3*x^2 - x^3 - 7*x^4 - 8*x^5 + x^6 + 4*x^8))
, x] - (9*Defer[Int][x^2/(Sqrt[-1 - x + x^4]*(4 + 8*x + 3*x^2 - x^3 - 7*x^4
 - 8*x^5 + x^6 + 4*x^8)), x])/2 - Defer[Int][x^3/(Sqrt[-1 - x + x^4]*(4 + 8
*x + 3*x^2 - x^3 - 7*x^4 - 8*x^5 + x^6 + 4*x^8)), x]/2 + (7*Defer[Int][x^4/
(Sqrt[-1 - x + x^4]*(4 + 8*x + 3*x^2 - x^3 - 7*x^4 - 8*x^5 + x^6 + 4*x^8)),
 x])/2 + 3*Defer[Int][x^5/(Sqrt[-1 - x + x^4]*(4 + 8*x + 3*x^2 - x^3 - 7*x^
4 - 8*x^5 + x^6 + 4*x^8)), x] - (5*Defer[Int][x^6/(Sqrt[-1 - x + x^4]*(4 +
8*x + 3*x^2 - x^3 - 7*x^4 - 8*x^5 + x^6 + 4*x^8)), x])/2
```

Rubi steps

$$\int \frac{(-1-x-x^2+x^4)(2+x+2x^4)}{\sqrt{-1-x+x^4}\,(4+8x+3x^2-x^3-7x^4-8x^5+x^6+4x^8)}\,dx=\int\left(\frac{1}{2\sqrt{-1-x+x^4}}-\frac{8+14x+}{2\sqrt{-1-x+x^4}\,(4+}\right.$$

$$=\frac{1}{2}\int\frac{1}{\sqrt{-1-x+x^4}}\,dx-\frac{1}{2}\int\frac{8+}{\sqrt{-1-x+x}}$$

$$=\frac{1}{2}\int\frac{1}{\sqrt{-1-x+x^4}}\,dx-\frac{1}{2}\int\left(\frac{}{\sqrt{-1-x+}}\right.$$

$$=\frac{1}{2}\int\frac{1}{\sqrt{-1-x+x^4}}\,dx-\frac{1}{2}\int\frac{}{\sqrt{-1-x+x}}$$

Mathematica [C] time = 6.89, size = 109075, normalized size = 886.79

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 - x - x^2 + x^4)*(2 + x + 2*x^4))/(Sqrt[-1 - x + x^4]*(4 + 8
*x + 3*x^2 - x^3 - 7*x^4 - 8*x^5 + x^6 + 4*x^8)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 3.80, size = 117, normalized size = 0.95

$$\frac{\sqrt[4]{-1}\sqrt{\sqrt{15}+7i}\tan^{-1}\left(\frac{\sqrt{\frac{1}{8}+\frac{i\sqrt{15}}{8}}x}{\sqrt{x^4-x-1}}\right)}{\sqrt{10}}-(-1)^{3/4}\sqrt{\frac{1}{10}\left(\sqrt{15}-7i\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{8}-\frac{i\sqrt{15}}{8}}x}{\sqrt{x^4-x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 - x - x^2 + x^4)*(2 + x + 2*x^4))/(Sqrt[-1 - x + x^
4]*(4 + 8*x + 3*x^2 - x^3 - 7*x^4 - 8*x^5 + x^6 + 4*x^8)),x]
```

```
[Out] -((-1)^(3/4)*Sqrt[(-7*I + Sqrt[15])/10]*ArcTan[(Sqrt[1/8 - (I/8)*Sqrt[15]]*
x)/Sqrt[-1 - x + x^4]]) + ((-1)^(1/4)*Sqrt[7*I + Sqrt[15]]*ArcTan[(Sqrt[1/8
 + (I/8)*Sqrt[15]]*x)/Sqrt[-1 - x + x^4]])/Sqrt[10]
```

fricas [B] time = 0.96, size = 935, normalized size = 7.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2-x-1)*(2*x^4+x+2)/(x^4-x-1)^(1/2)/(4*x^8+x^6-8*x^5-7*x^4-
x^3+3*x^2+8*x+4),x, algorithm="fricas")
```

```
[Out] 1/300*sqrt(15)*sqrt(10)*sqrt(5)*sqrt(3)*sqrt(2)*arctan(1/375*(80*sqrt(15)*s
qrt(10)*sqrt(5)*sqrt(3)*sqrt(2)*(2*x^5 - x^3 - 2*x^2 - 2*x)*sqrt(x^4 - x -
1) + 225*sqrt(5)*sqrt(3)*(4*x^8 + x^6 - 8*x^5 - 7*x^4 - x^3 + 3*x^2 + 8*x +
 4) + 2*sqrt(10)*(sqrt(15)*sqrt(10)*sqrt(5)*sqrt(3)*sqrt(2)*(4*x^8 - 9*x^6
- 8*x^5 - 9*x^4 + 9*x^3 + 13*x^2 + 8*x + 4) + 120*sqrt(5)*sqrt(3)*(x^5 + x^
3 - x^2 - x)*sqrt(x^4 - x - 1))*sqrt((20*x^8 + 35*x^6 - 40*x^5 - 35*x^4 + s
qrt(15)*sqrt(10)*sqrt(2)*(2*x^5 + x^3 - 2*x^2 - 2*x)*sqrt(x^4 - x - 1) - 35
*x^3 - 15*x^2 + 40*x + 20)/(4*x^8 + x^6 - 8*x^5 - 7*x^4 - x^3 + 3*x^2 + 8*x
 + 4)))/(4*x^8 - 31*x^6 - 8*x^5 - 7*x^4 + 31*x^3 + 35*x^2 + 8*x + 4)) - 1/3
00*sqrt(15)*sqrt(10)*sqrt(5)*sqrt(3)*sqrt(2)*arctan(-1/375*(80*sqrt(15)*sqr
t(10)*sqrt(5)*sqrt(3)*sqrt(2)*(2*x^5 - x^3 - 2*x^2 - 2*x)*sqrt(x^4 - x - 1)
 - 225*sqrt(5)*sqrt(3)*(4*x^8 + x^6 - 8*x^5 - 7*x^4 - x^3 + 3*x^2 + 8*x + 4
) + 2*sqrt(10)*(sqrt(15)*sqrt(10)*sqrt(5)*sqrt(3)*sqrt(2)*(4*x^8 - 9*x^6 -
8*x^5 - 9*x^4 + 9*x^3 + 13*x^2 + 8*x + 4) - 120*sqrt(5)*sqrt(3)*(x^5 + x^3
- x^2 - x)*sqrt(x^4 - x - 1))*sqrt((20*x^8 + 35*x^6 - 40*x^5 - 35*x^4 - sqr
t(15)*sqrt(10)*sqrt(2)*(2*x^5 + x^3 - 2*x^2 - 2*x)*sqrt(x^4 - x - 1) - 35*x
^3 - 15*x^2 + 40*x + 20)/(4*x^8 + x^6 - 8*x^5 - 7*x^4 - x^3 + 3*x^2 + 8*x +
 4)))/(4*x^8 - 31*x^6 - 8*x^5 - 7*x^4 + 31*x^3 + 35*x^2 + 8*x + 4)) - 1/80*
sqrt(15)*sqrt(10)*sqrt(2)*log(640*(20*x^8 + 35*x^6 - 40*x^5 - 35*x^4 + sqrt
(15)*sqrt(10)*sqrt(2)*(2*x^5 + x^3 - 2*x^2 - 2*x)*sqrt(x^4 - x - 1) - 35*x^
3 - 15*x^2 + 40*x + 20)/(4*x^8 + x^6 - 8*x^5 - 7*x^4 - x^3 + 3*x^2 + 8*x +
4)) + 1/80*sqrt(15)*sqrt(10)*sqrt(2)*log(640*(20*x^8 + 35*x^6 - 40*x^5 - 35
*x^4 - sqrt(15)*sqrt(10)*sqrt(2)*(2*x^5 + x^3 - 2*x^2 - 2*x)*sqrt(x^4 - x -
 1) - 35*x^3 - 15*x^2 + 40*x + 20)/(4*x^8 + x^6 - 8*x^5 - 7*x^4 - x^3 + 3*x
^2 + 8*x + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^4+x+2\right)\left(x^4-x^2-x-1\right)}{\left(4x^8+x^6-8x^5-7x^4-x^3+3x^2+8x+4\right)\sqrt{x^4-x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2-x-1)*(2*x^4+x+2)/(x^4-x-1)^(1/2)/(4*x^8+x^6-8*x^5-7*x^4-
x^3+3*x^2+8*x+4),x, algorithm="giac")
```

```
[Out] integrate((2*x^4 + x + 2)*(x^4 - x^2 - x - 1)/((4*x^8 + x^6 - 8*x^5 - 7*x^4
 - x^3 + 3*x^2 + 8*x + 4)*sqrt(x^4 - x - 1)), x)
```

maple [C] time = 8.68, size = 8123, normalized size = 66.04

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^2-x-1)*(2*x^4+x+2)/(x^4-x-1)^(1/2)/(4*x^8+x^6-8*x^5-7*x^4-x^3+3*
x^2+8*x+4),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^4+x+2\right)\left(x^4-x^2-x-1\right)}{\left(4\,x^8+x^6-8\,x^5-7\,x^4-x^3+3\,x^2+8\,x+4\right)\sqrt{x^4-x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2-x-1)*(2*x^4+x+2)/(x^4-x-1)^(1/2)/(4*x^8+x^6-8*x^5-7*x^4-
x^3+3*x^2+8*x+4),x, algorithm="maxima")
```

```
[Out] integrate((2*x^4 + x + 2)*(x^4 - x^2 - x - 1)/((4*x^8 + x^6 - 8*x^5 - 7*x^4
 - x^3 + 3*x^2 + 8*x + 4)*sqrt(x^4 - x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(2\,x^4+x+2\right)\,\left(-x^4+x^2+x+1\right)}{\sqrt{x^4-x-1}\,\left(4\,x^8+x^6-8\,x^5-7\,x^4-x^3+3\,x^2+8\,x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + 2*x^4 + 2)*(x + x^2 - x^4 + 1))/((x^4 - x - 1)^(1/2)*(8*x + 3*x^
2 - x^3 - 7*x^4 - 8*x^5 + x^6 + 4*x^8 + 4)),x)
```

```
[Out] int(-((x + 2*x^4 + 2)*(x + x^2 - x^4 + 1))/((x^4 - x - 1)^(1/2)*(8*x + 3*x^
2 - x^3 - 7*x^4 - 8*x^5 + x^6 + 4*x^8 + 4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)\left(x^3-x^2-1\right)\left(2x^4+x+2\right)}{\sqrt{x^4-x-1}\,\left(4x^8+x^6-8x^5-7x^4-x^3+3x^2+8x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**2-x-1)*(2*x**4+x+2)/(x**4-x-1)**(1/2)/(4*x**8+x**6-8*x**
5-7*x**4-x**3+3*x**2+8*x+4),x)
```

```
[Out] Integral((x + 1)*(x**3 - x**2 - 1)*(2*x**4 + x + 2)/(sqrt(x**4 - x - 1)*(4*
x**8 + x**6 - 8*x**5 - 7*x**4 - x**3 + 3*x**2 + 8*x + 4)), x)
```

3.1527 $\int \frac{(1+x^6)^2(-1+2x^6)}{(1-x^2+x^6)^{3/2}(1-x^2-x^4+2x^6-x^8+x^{12})} dx$

Optimal. Leaf size=123

$$\frac{x}{\sqrt{x^6-x^2+1}}-\sqrt{\frac{2}{145+65\sqrt{5}}}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt{x^6-x^2+1}}\right)-\sqrt{\frac{1}{10}\left(29+13\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt{x^6-x^2+1}}\right)$$

Rubi [F] time = 2.97, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+x^6)^2(-1+2x^6)}{(1-x^2+x^6)^{3/2}(1-x^2-x^4+2x^6-x^8+x^{12})} dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^6)^2*(-1 + 2*x^6))/((1 - x^2 + x^6)^(3/2)*(1 - x^2 - x^4 + 2*x^
6 - x^8 + x^12)),x]
```

```
[Out] -(x/Sqrt[1 - x^2 + x^6]) + 2*Defer[Int][x^2/(1 - x^2 + x^6)^(3/2), x] - 3*D
efer[Int][x^2/((1 - x^2 + x^6)^(3/2)*(1 - x^2 - x^4 + 2*x^6 - x^8 + x^12)),
 x] + Defer[Int][x^4/((1 - x^2 + x^6)^(3/2)*(1 - x^2 - x^4 + 2*x^6 - x^8 +
x^12)), x] + 2*Defer[Int][x^6/((1 - x^2 + x^6)^(3/2)*(1 - x^2 - x^4 + 2*x^6
 - x^8 + x^12)), x] - 3*Defer[Int][x^8/((1 - x^2 + x^6)^(3/2)*(1 - x^2 - x^
4 + 2*x^6 - x^8 + x^12)), x] + 4*Defer[Int][x^10/((1 - x^2 + x^6)^(3/2)*(1
- x^2 - x^4 + 2*x^6 - x^8 + x^12)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^6)^2(-1+2x^6)}{(1-x^2+x^6)^{3/2}(1-x^2-x^4+2x^6-x^8+x^{12})} dx &= \int \left(-\frac{1}{(1-x^2+x^6)^{3/2}}+\frac{2x^2}{(1-x^2+x^6)^{3/2}}+\frac{2x^6}{(1-x^2+x^6)^{3/}}\right.\\
&= 2\int \frac{x^2}{(1-x^2+x^6)^{3/2}} dx + 2\int \frac{x^6}{(1-x^2+x^6)^{3/2}} dx - \int \frac{}{(1}\\
&= -\frac{x}{\sqrt{1-x^2+x^6}} + 2\int \frac{x^2}{(1-x^2+x^6)^{3/2}} dx + \int \left(-\frac{}{(1-x^2}\right.\\
&= -\frac{x}{\sqrt{1-x^2+x^6}} + 2\int \frac{x^2}{(1-x^2+x^6)^{3/2}} dx + 2\int \frac{}{(1-x^2+}
\end{aligned}$$

Mathematica [F] time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{(1+x^6)^2(-1+2x^6)}{(1-x^2+x^6)^{3/2}(1-x^2-x^4+2x^6-x^8+x^{12})} dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^6)^2*(-1 + 2*x^6))/((1 - x^2 + x^6)^(3/2)*(1 - x^2 - x^4
+ 2*x^6 - x^8 + x^12)),x]
```

[Out] Integrate[((1 + x^6)^2*(-1 + 2*x^6))/((1 - x^2 + x^6)^(3/2)*(1 - x^2 - x^4 + 2*x^6 - x^8 + x^12)), x]

IntegrateAlgebraic [A] time = 3.45, size = 123, normalized size = 1.00

$$\frac{x}{\sqrt{x^6-x^2+1}}-\sqrt{\frac{2}{145+65\sqrt{5}}}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt{x^6-x^2+1}}\right)-\sqrt{\frac{1}{10}\left(29+13\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt{x^6-x^2+1}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + x^6)^2*(-1 + 2*x^6))/((1 - x^2 + x^6)^(3/2)*(1 - x^2 - x^4 + 2*x^6 - x^8 + x^12)),x]

[Out] x/Sqrt[1 - x^2 + x^6] - Sqrt[2/(145 + 65*Sqrt[5])]*ArcTan[(Sqrt[1/2 + Sqrt[5]/2]*x)/Sqrt[1 - x^2 + x^6]] - Sqrt[(29 + 13*Sqrt[5])/10]*ArcTanh[(Sqrt[-1/2 + Sqrt[5]/2]*x)/Sqrt[1 - x^2 + x^6]]

fricas [B] time = 0.83, size = 555, normalized size = 4.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+1)^2*(2*x^6-1)/(x^6-x^2+1)^(3/2)/(x^12-x^8+2*x^6-x^4-x^2+1), x, algorithm="fricas")

[Out] -1/40*(4*sqrt(10)*(x^6 - x^2 + 1)*sqrt(13*sqrt(5) - 29)*arctan(1/20*(2*sqrt(10)*(15*x^7 - 5*x^3 + sqrt(5)*(7*x^7 - 3*x^3 + 7*x) + 15*x)*sqrt(x^6 - x^2 + 1)*sqrt(13*sqrt(5) - 29) + sqrt(10)*(5*x^12 + 5*x^8 + 10*x^6 - 5*x^4 + 5*x^2 + sqrt(5)*(3*x^12 - x^8 + 6*x^6 + x^4 - x^2 + 3) + 5)*sqrt(13*sqrt(5) - 29)*sqrt(sqrt(5) + 2))/(x^12 - 3*x^8 + 2*x^6 + x^4 - 3*x^2 + 1)) + sqrt(10)*(x^6 - x^2 + 1)*sqrt(13*sqrt(5) + 29)*log(-(sqrt(10)*(25*x^12 - 105*x^8 + 50*x^6 + 105*x^4 - 105*x^2 - sqrt(5)*(11*x^12 - 47*x^8 + 22*x^6 + 47*x^4 - 47*x^2 + 11) + 25)*sqrt(13*sqrt(5) + 29) + 20*(3*x^7 - 4*x^3 - sqrt(5)*(x^7 - 2*x^3 + x) + 3*x)*sqrt(x^6 - x^2 + 1))/(x^12 - x^8 + 2*x^6 - x^4 - x^2 + 1)) - sqrt(10)*(x^6 - x^2 + 1)*sqrt(13*sqrt(5) + 29)*log((sqrt(10)*(25*x^12 - 105*x^8 + 50*x^6 + 105*x^4 - 105*x^2 - sqrt(5)*(11*x^12 - 47*x^8 + 22*x^6 + 47*x^4 - 47*x^2 + 11) + 25)*sqrt(13*sqrt(5) + 29) - 20*(3*x^7 - 4*x^3 - sqrt(5)*(x^7 - 2*x^3 + x) + 3*x)*sqrt(x^6 - x^2 + 1))/(x^12 - x^8 + 2*x^6 - x^4 - x^2 + 1)) - 40*sqrt(x^6 - x^2 + 1)*x)/(x^6 - x^2 + 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^6-1\right)\left(x^6+1\right)^2}{\left(x^{12}-x^8+2\,x^6-x^4-x^2+1\right)\left(x^6-x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+1)^2*(2*x^6-1)/(x^6-x^2+1)^(3/2)/(x^12-x^8+2*x^6-x^4-x^2+1), x, algorithm="giac")

[Out] integrate((2*x^6 - 1)*(x^6 + 1)^2/((x^12 - x^8 + 2*x^6 - x^4 - x^2 + 1)*(x^6 - x^2 + 1)^(3/2)), x)

maple [C] time = 4.32, size = 599, normalized size = 4.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)^2*(2*x^6-1)/(x^6-x^2+1)^(3/2)/(x^12-x^8+2*x^6-x^4-x^2+1),x)
```

```
[Out] x/(x^6-x^2+1)^(1/2)-RootOf(400*_Z^4-580*_Z^2-1)*ln((1430*RootOf(400*_Z^4-58
0*_Z^2-1)^3*x^6+2200*RootOf(400*_Z^4-580*_Z^2-1)^5*x^2-2093*RootOf(400*_Z^4
-580*_Z^2-1)*x^6-6960*RootOf(400*_Z^4-580*_Z^2-1)^3*x^2+130*RootOf(400*_Z^4
-580*_Z^2-1)^2*(x^6-x^2+1)^(1/2)*x+1430*RootOf(400*_Z^4-580*_Z^2-1)^3+5474*
RootOf(400*_Z^4-580*_Z^2-1)*x^2-221*(x^6-x^2+1)^(1/2)*x-2093*RootOf(400*_Z^
4-580*_Z^2-1))/(-13*x^6+20*RootOf(400*_Z^4-580*_Z^2-1)^2*x^2-8*x^2-13))+1/1
0*RootOf(_Z^2+100*RootOf(400*_Z^4-580*_Z^2-1)^2-145)*ln(-(-2860*RootOf(_Z^2
+100*RootOf(400*_Z^4-580*_Z^2-1)^2-145)*RootOf(400*_Z^4-580*_Z^2-1)^2*x^6+4
400*RootOf(_Z^2+100*RootOf(400*_Z^4-580*_Z^2-1)^2-145)*RootOf(400*_Z^4-580*
_Z^2-1)^4*x^2-39*RootOf(_Z^2+100*RootOf(400*_Z^4-580*_Z^2-1)^2-145)*x^6+116
0*RootOf(_Z^2+100*RootOf(400*_Z^4-580*_Z^2-1)^2-145)*RootOf(400*_Z^4-580*_Z
^2-1)^2*x^2+2600*RootOf(400*_Z^4-580*_Z^2-1)^2*(x^6-x^2+1)^(1/2)*x-2860*Roo
tOf(400*_Z^4-580*_Z^2-1)^2*RootOf(_Z^2+100*RootOf(400*_Z^4-580*_Z^2-1)^2-14
5)+15*RootOf(_Z^2+100*RootOf(400*_Z^4-580*_Z^2-1)^2-145)*x^2+650*(x^6-x^2+1
)^(1/2)*x-39*RootOf(_Z^2+100*RootOf(400*_Z^4-580*_Z^2-1)^2-145))/(13*x^6+20
*RootOf(400*_Z^4-580*_Z^2-1)^2*x^2-21*x^2+13))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{6}-1\right)\left(x^{6}+1\right)^{2}}{\left(x^{12}-x^{8}+2\,x^{6}-x^{4}-x^{2}+1\right)\left(x^{6}-x^{2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)^2*(2*x^6-1)/(x^6-x^2+1)^(3/2)/(x^12-x^8+2*x^6-x^4-x^2+1),
x, algorithm="maxima")
```

```
[Out] integrate((2*x^6 - 1)*(x^6 + 1)^2/((x^12 - x^8 + 2*x^6 - x^4 - x^2 + 1)*(x^
6 - x^2 + 1)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^6+1\right)^2\,\left(2\,x^6-1\right)}{\left(x^6-x^2+1\right)^{3/2}\,\left(-x^{12}+x^8-2\,x^6+x^4+x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^6 + 1)^2*(2*x^6 - 1))/((x^6 - x^2 + 1)^(3/2)*(x^2 + x^4 - 2*x^6 +
x^8 - x^12 - 1)),x)
```

```
[Out] int(-((x^6 + 1)^2*(2*x^6 - 1))/((x^6 - x^2 + 1)^(3/2)*(x^2 + x^4 - 2*x^6 +
x^8 - x^12 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)**2*(2*x**6-1)/(x**6-x**2+1)**(3/2)/(x**12-x**8+2*x**6-x*
*4-x**2+1),x)
```

```
[Out] Timed out
```

3.1528 $\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx$

Optimal. Leaf size=123

$$\frac{\sqrt{x}\left(4-4\sqrt{x+1}\right)}{3x+\left(6\sqrt{x+1}-6\right)\sqrt{x}-3\sqrt{x+1}+3}-\frac{2}{9}\log\left(-\sqrt{x}+\sqrt{x+1}-1\right)-2\log\left(\sqrt{x}+\sqrt{x+1}-1\right)+\frac{10}{9}\log\left(x+\left(\right.\right.$$

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 0.60, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6742, 97, 157, 54, 215, 93, 207}

$$-\frac{4\sqrt{x}\sqrt{x+1}}{3(1-3x)}+\frac{8}{9(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] 8/(9*(1 - 3*x)) - (4*Sqrt[x]*Sqrt[1 + x])/(3*(1 - 3*x)) - (8*ArcSinh[Sqrt[x
]])/9 + (10*ArcTanh[(2*Sqrt[x])/Sqrt[1 + x]])/9 + (5*Log[1 - 3*x])/9
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx &= \int \left(\frac{8}{3(-1+3x)^2}-\frac{4\sqrt{x}\sqrt{1+x}}{(-1+3x)^2}+\frac{5}{3(-1+3x)}\right)dx \\
&= \frac{8}{9(1-3x)}+\frac{5}{9}\log(1-3x)-4\int \frac{\sqrt{x}\sqrt{1+x}}{(-1+3x)^2}\,dx \\
&= \frac{8}{9(1-3x)}-\frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{4}{3}\int \frac{\frac{1}{2}+x}{\sqrt{x}\sqrt{1+x}(-1+3x)}\,dx \\
&= \frac{8}{9(1-3x)}-\frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{4}{9}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx-\frac{10}{9}\int \frac{1}{\sqrt{x}\sqrt{1+x}(-
\\
&= \frac{8}{9(1-3x)}-\frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{8}{9}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{x}\right)-\frac{20}{9}\text{Su}
\\
&= \frac{8}{9(1-3x)}-\frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)}-\frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{1+x}}\right)+\frac{5}{9}\log(1-3x)
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 126, normalized size = 1.02

$$
\frac{12x^{3/2}+12\sqrt{x}-8\sqrt{x+1}+15\sqrt{x+1}\,x\log(1-3x)-5\sqrt{x+1}\log(1-3x)+10\sqrt{-x-1}(3x-1)\tan^{-1}\left(\frac{2\sqrt{x}}{\sqrt{-x-1}}\right)-8\sqrt{x+1}(3x-1)\sinh^{-1}\left(\sqrt{x}\right)}{9\sqrt{x+1}(3x-1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] (12*Sqrt[x] + 12*x^(3/2) - 8*Sqrt[1 + x] - 8*Sqrt[1 + x]*(-1 + 3*x)*ArcSinh
[Sqrt[x]] + 10*Sqrt[-1 - x]*(-1 + 3*x)*ArcTan[(2*Sqrt[x])/Sqrt[-1 - x]] - 5
*Sqrt[1 + x]*Log[1 - 3*x] + 15*x*Sqrt[1 + x]*Log[1 - 3*x])/(9*Sqrt[1 + x]*(
-1 + 3*x))
```

IntegrateAlgebraic [A] time = 0.10, size = 78, normalized size = 0.63

$$
\frac{4\sqrt{x}\sqrt{x+1}}{3(3x-1)}-\frac{8}{9(3x-1)}-\frac{2}{9}\log\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{10}{9}\log\left(-x+\sqrt{x+1}\sqrt{x}+1\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] -8/(9*(-1 + 3*x)) + (4*Sqrt[x]*Sqrt[1 + x])/(3*(-1 + 3*x)) - (2*Log[-Sqrt[x
] + Sqrt[1 + x]])/9 + (10*Log[1 - x + Sqrt[x]*Sqrt[1 + x]])/9
```

fricas [A] time = 0.42, size = 105, normalized size = 0.85

$$
-\frac{5(3x-1)\log\left(3\sqrt{x+1}\sqrt{x}-3x-1\right)-4(3x-1)\log\left(2\sqrt{x+1}\sqrt{x}-2x-1\right)-5(3x-1)\log\left(\sqrt{x+1}\sqrt{x}-x+1\right)-5(3x-1)\log(3x-1)-12\sqrt{x+1}\sqrt{x}-12x+12}{9(3x-1)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/9*(5*(3*x - 1)*log(3*sqrt(x + 1)*sqrt(x) - 3*x - 1) - 4*(3*x - 1)*log(2*
sqrt(x + 1)*sqrt(x) - 2*x - 1) - 5*(3*x - 1)*log(sqrt(x + 1)*sqrt(x) - x +
1) - 5*(3*x - 1)*log(3*x - 1) - 12*sqrt(x + 1)*sqrt(x) - 12*x + 12)/(3*x -
1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0,1] at parameters
values [-89]Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0,
1] at parameters values [63]2*((-5*(x+1)+4)/6/(3*(x+1)-4)+5/18*ln(abs(3*(x+
1)-4))+2*(1/9*ln((sqrt(x)-sqrt(x+1))^2)+5/36*ln(abs((sqrt(x)-sqrt(x+1))^2-3
))-5/36*ln(abs(3*(sqrt(x)-sqrt(x+1))^2-1))-(10*(sqrt(x)-sqrt(x+1))^2-6)/9/(
3*(sqrt(x)-sqrt(x+1))^4-10*(sqrt(x)-sqrt(x+1))^2+3)))
```

maple [A] time = 0.02, size = 115, normalized size = 0.93

$$-\frac{8}{9\left(-1+3x\right)}+\frac{5\ln\left(-1+3x\right)}{9}-\frac{\sqrt{x}\,\sqrt{1+x}\,\left(12\ln\left(x+\frac{1}{2}+\sqrt{x\left(1+x\right)}\right)x-15\,\mathrm{arctanh}\left(\frac{5x+1}{4\sqrt{x(1+x)}}\right)x-4\ln\left(x+\frac{1}{2}+\sqrt{x\left(1+x\right)}\right)+5\,\mathrm{arctanh}\left(\frac{5x+1}{4\sqrt{x(1+x)}}\right)-12\sqrt{x\left(1+x\right)}\right)}{9\sqrt{x\left(1+x\right)}\,\left(-1+3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^(1/2)+(1+x)^(1/2))^2,x)
```

```
[Out] -8/9/(-1+3*x)+5/9*ln(-1+3*x)-1/9*x^(1/2)*(1+x)^(1/2)*(12*ln(x+1/2+(x*(1+x))
^(1/2))*x-15*arctanh(1/4*(5*x+1)/(x*(1+x))^(1/2))*x-4*ln(x+1/2+(x*(1+x))^(1
/2))+5*arctanh(1/4*(5*x+1)/(x*(1+x))^(1/2))-12*(x*(1+x))^(1/2))/(x*(1+x))^(
1/2)/(-1+3*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sqrt{x+1}+2\sqrt{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(x + 1) + 2*sqrt(x))^(-2), x)
```

mupad [B] time = 2.19, size = 82, normalized size = 0.67

$$\frac{10\,\mathrm{atanh}\left(\frac{2662400\,\sqrt{x}}{81\,\left(\frac{665600\,x}{81\,{\left(\sqrt{x+1}-1\right)}^2}+\frac{665600}{81}\right)\,\left(\sqrt{x+1}-1\right)}\right)}{9}+\frac{5\,\ln\left(x-\frac{1}{3}\right)}{9}-\frac{16\,\mathrm{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)}{9}-\frac{8}{27\,\left(x-\frac{1}{3}\right)}+\frac{4\,\sqrt{x}\,\sqrt{x+1}}{3\,\left(3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(1/2) + 2*x^(1/2))^2,x)
```

```
[Out] (10*atanh((2662400*x^(1/2))/(81*((665600*x)/(81*((x + 1)^(1/2) - 1)^2) + 66
5600/81)*((x + 1)^(1/2) - 1))))/9 + (5*log(x - 1/3))/9 - (16*atanh(x^(1/2)/
```

```
((x + 1)^(1/2) - 1)))/9 - 8/(27*(x - 1/3)) + (4*x^(1/2)*(x + 1)^(1/2))/(3*(
3*x - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\sqrt{x} + \sqrt{x+1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Integral((2*sqrt(x) + sqrt(x + 1))**(-2), x)
```

3.1529 $\int \frac{1}{x^3\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$

Optimal. Leaf size=123

$$\frac{4(ax^2-2)\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{15x^2}-\frac{2b(2ax^2-3)\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{15ax^3}$$

Rubi [F] time = 0.41, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]),x]
[Out] Defer[Int][1/(x^3*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]), x]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx=\int \frac{1}{x^3\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$$

Mathematica [A] time = 4.69, size = 98, normalized size = 0.80

$$-\frac{2(ax^2-1)\left(5bx\sqrt{\frac{a(ax^2-1)}{b^2}}+5ax^2-3\right)}{15x^2\sqrt{x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(bx\sqrt{\frac{a(ax^2-1)}{b^2}}+ax^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]),x]
[Out] (-2*(-1 + a*x^2)*(-3 + 5*a*x^2 + 5*b*x*Sqrt[(a*(-1 + a*x^2))/b^2]))/(15*x^2
*Sqrt[x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*(-1 + a*x^2 + b*x*Sqrt[(a*(-1
 + a*x^2))/b^2])^2)
```

IntegrateAlgebraic [A] time = 2.99, size = 123, normalized size = 1.00

$$\frac{4(ax^2-2)\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{15x^2}-\frac{2b(2ax^2-3)\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{15ax^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]])
,x]
```

```
[Out] (4*(-2 + a*x^2)*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]])/(15*x^2)
- (2*b*(-3 + 2*a*x^2)*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*Sqrt[a*x^2 + b*x*Sqrt[
-(a/b^2) + (a^2*x^2)/b^2]])/(15*a*x^3)
```

fricas [A] time = 0.45, size = 79, normalized size = 0.64

$$\frac{2\left(2\,a^2x^3 - 4\,ax - \left(2\,abx^2 - 3\,b\right)\sqrt{\frac{a^2x^2-a}{b^2}}\right)\sqrt{ax^2 + bx\sqrt{\frac{a^2x^2-a}{b^2}}}}{15\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="
fricas")
```

```
[Out] 2/15*(2*a^2*x^3 - 4*a*x - (2*a*b*x^2 - 3*b)*sqrt((a^2*x^2 - a)/b^2))*sqrt(a
*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))/(a*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\,bx}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(1/(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)*x^3), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a\,x^2 + bx\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/x^3/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\,bx}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(1/(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{a\,x^2 + b\,x\,\sqrt{\frac{a^2\,x^2}{b^2} - \frac{a}{b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)),x)
[Out] int(1/(x^3*(a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{x\left(ax+b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*x**2+b*x*(-a/b**2+a**2*x**2/b**2)**(1/2))**(1/2),x)
[Out] Integral(1/(x**3*sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))), x)
```

3.1530 $\int \frac{1}{x\sqrt[4]{-b+ax^2}}\,dx$

Optimal. Leaf size=124

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}-\sqrt{b}}\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}\sqrt[4]{b}}$$

Rubi [A] time = 0.21, antiderivative size = 196, normalized size of antiderivative = 1.58, number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {266, 63, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)}{2\sqrt{2}\sqrt[4]{b}}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)}{2\sqrt{2}\sqrt[4]{b}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-b + a*x^2)^(1/4)),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*(-b + a*x^2)^(1/4))/b^(1/4)]/(Sqrt[2]*b^(1/4))) + Arc
Tan[1 + (Sqrt[2]*(-b + a*x^2)^(1/4))/b^(1/4)]/(Sqrt[2]*b^(1/4)) + Log[Sqrt[
b] - Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4) + Sqrt[-b + a*x^2]]/(2*Sqrt[2]*b^(1
/4)) - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4) + Sqrt[-b + a*x^2]]
/(2*Sqrt[2]*b^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x\sqrt[4]{-b+ax^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{-b+ax}}\,dx, x, x^2\right)$$

$$= \frac{2\operatorname{Subst}\left(\int \frac{x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^2}\right)}{a}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^2}\right)}{a} + \frac{\operatorname{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^2}\right)}{a}$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{2}\sqrt[4]{b}\,x+x^2}\,dx, x, \sqrt[4]{-b+ax^2}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{2}\sqrt[4]{b}\,x+x^2}\,d\right.$$

$$= \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^2}+\sqrt{-b+ax^2}\right)}{2\sqrt{2}\sqrt[4]{b}} - \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^2}+\sqrt{-b+\right.}}{2\sqrt{2}\sqrt[4]{b}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}} + \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^2}+\sqrt{\right.}}{2\sqrt{2}\sqrt[4]{b}}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.44

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^2-b}}{\sqrt[4]{-b}}\right)+\tanh^{-1}\left(\frac{b\sqrt[4]{ax^2-b}}{(-b)^{5/4}}\right)}{\sqrt[4]{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-b + a*x^2)^(1/4)),x]
```

```
[Out] (ArcTan[(-b + a*x^2)^(1/4)/(-b)^(1/4)] + ArcTanh[(b*(-b + a*x^2)^(1/4))/(-b
)^(5/4)])/(-b)^(1/4)
```

IntegrateAlgebraic [A] time = 0.21, size = 122, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}+\sqrt{b}}\right)}{\sqrt{2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-b + a*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a*x^2]/(Sqrt[2]*b^(1/4)))/(-b + a*x^
2)^(1/4)]/(Sqrt[2]*b^(1/4)) - ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4))/
(Sqrt[b] + Sqrt[-b + a*x^2])]/(Sqrt[2]*b^(1/4))
```

fricas [A] time = 0.42, size = 127, normalized size = 1.02

$$-2\left(-\frac{1}{b}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-b\sqrt{-\frac{1}{b}}+\sqrt{ax^2-b}}\left(-\frac{1}{b}\right)^{\frac{1}{4}}-\left(ax^2-b\right)^{\frac{1}{4}}\left(-\frac{1}{b}\right)^{\frac{1}{4}}\right)+\frac{1}{2}\left(-\frac{1}{b}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{1}{b}\right)^{\frac{3}{4}}+\left(ax^2-b\right)^{\frac{1}{4}}\right)-\frac{1}{2}\left(-\frac{1}{b}\right)^{\frac{1}{4}}\log\left(-b\left(-\frac{1}{b}\right)^{\frac{3}{4}}+\left(ax^2-b\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^2-b)^(1/4),x, algorithm="fricas")
```

```
[Out] -2*(-1/b)^(1/4)*arctan(sqrt(-b*sqrt(-1/b) + sqrt(a*x^2 - b))*(-1/b)^(1/4) -
 (a*x^2 - b)^(1/4)*(-1/b)^(1/4)) + 1/2*(-1/b)^(1/4)*log(b*(-1/b)^(3/4) + (a
*x^2 - b)^(1/4)) - 1/2*(-1/b)^(1/4)*log(-b*(-1/b)^(3/4) + (a*x^2 - b)^(1/4)
)
```

giac [A] time = 0.18, size = 162, normalized size = 1.31

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{2b^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{2b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)}{4b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)}{4b^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^2-b)^(1/4),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^2 - b)^(1/4))/b^(1
/4))/b^(1/4) + 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^2
- b)^(1/4))/b^(1/4))/b^(1/4) - 1/4*sqrt(2)*log(sqrt(2)*(a*x^2 - b)^(1/4)*b^
(1/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(1/4) + 1/4*sqrt(2)*log(-sqrt(2)*(a*x^
2 - b)^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(1/4)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a\,x^2-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^2-b)^(1/4),x)
```

```
[Out] int(1/x/(a*x^2-b)^(1/4),x)
```

maxima [A] time = 0.56, size = 162, normalized size = 1.31

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{2b^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{2b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)}{4b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)}{4b^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^2-b)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^2 - b)^(1/4))/b^(1
/4))/b^(1/4) + 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^2
- b)^(1/4))/b^(1/4))/b^(1/4) - 1/4*sqrt(2)*log(sqrt(2)*(a*x^2 - b)^(1/4)*b^
(1/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(1/4) + 1/4*sqrt(2)*log(-sqrt(2)*(a*x^
2 - b)^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(1/4)
```

mupad [B] time = 1.04, size = 45, normalized size = 0.36

$$\frac{\mathrm{atan}\left(\frac{{\left(a\,x^2-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)-\mathrm{atanh}\left(\frac{{\left(a\,x^2-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{{\left(-b\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 - b)^(1/4)),x)
```

```
[Out] (atan((a*x^2 - b)^(1/4)/(-b)^(1/4)) - atanh((a*x^2 - b)^(1/4)/(-b)^(1/4)))/
(-b)^(1/4)
```

sympy [C] time = 0.92, size = 42, normalized size = 0.34

$$-\frac{\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^2}\right)}{2\sqrt[4]{a}\,\sqrt{x}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**2-b)**(1/4),x)
```

```
[Out] -gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*exp_polar(2*I*pi)/(a*x**2))/(2*a**(
1/4)*sqrt(x)*gamma(5/4))
```

3.1531 $\int \frac{1+k^{3/2}x^3}{\sqrt{(1-x^2)(1-k^2x^2)}\left(-1+k^{3/2}x^3\right)}\,dx$

Optimal. Leaf size=124

$$-\frac{2\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}+kx^2-2\sqrt{k}\,x+1}\right)}{3(k-1)}-\frac{4\tan^{-1}\left(\frac{\sqrt{k^2+k+1}\,x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}+kx^2+\sqrt{k}\,x+1}\right)}{3\sqrt{k^2+k+1}}$$

Rubi [C] time = 3.22, antiderivative size = 610, normalized size of antiderivative = 4.92, number of steps used = 21, number of rules used = 9, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6719, 6725, 419, 2113, 537, 571, 93, 205, 208}

$$\frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\tan^{-1}\left(\frac{\sqrt{k}\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{3(1-k)\sqrt{(1-x^2)(1-k^2x^2)}}+\frac{2(-1)^{2/3}\sqrt{1-x^2}\sqrt{1-k^2x^2}\tanh^{-1}\left(\frac{\sqrt{k}\sqrt{k+\sqrt[3]{-1}}\sqrt{1-x^2}}{\sqrt{\sqrt[3]{-1}k+1}\sqrt{1-k^2x^2}}\right)}{3\sqrt{k+\sqrt[3]{-1}}\sqrt{\sqrt[3]{-1}k+1}\sqrt{(1-x^2)(1-k^2x^2)}}-\frac{2\sqrt[3]{-1}\sqrt{1-x^2}\sqrt{1-k^2x^2}\tanh^{-1}\left(\frac{\sqrt{k}\sqrt{k-(-1)^{2/3}}\sqrt{1-x^2}}{\sqrt{1-(-1)^{2/3}k}\sqrt{1-k^2x^2}}\right)}{3\sqrt{k-(-1)^{2/3}}\sqrt{1-(-1)^{2/3}k}\sqrt{(1-x^2)(1-k^2x^2)}}+\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}-\frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\Pi\left(k;\sin^{-1}(x)|k^2\right)}{3\sqrt{(1-x^2)(1-k^2x^2)}}-\frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\Pi\left(-\sqrt[3]{-1}k;\sin^{-1}(x)|k^2\right)}{3\sqrt{(1-x^2)(1-k^2x^2)}}-\frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\Pi\left((-1)^{2/3}k;\sin^{-1}(x)|k^2\right)}{3\sqrt{(1-x^2)(1-k^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + k^(3/2)*x^3)/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(-1 + k^(3/2)*x^3)),x]
```

```
[Out] (2*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*ArcTan[(Sqrt[k]*Sqrt[1 - x^2])/Sqrt[1 -
k^2*x^2]])/(3*(1 - k)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) + (2*(-1)^(2/3)*Sqrt[1
 - x^2]*Sqrt[1 - k^2*x^2]*ArcTanh[(Sqrt[k]*Sqrt[(-1)^(1/3) + k]*Sqrt[1 - x^
2])/(Sqrt[1 + (-1)^(1/3)*k]*Sqrt[1 - k^2*x^2])]])/(3*Sqrt[(-1)^(1/3) + k]*Sq
rt[1 + (-1)^(1/3)*k]*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) - (2*(-1)^(1/3)*Sqrt[1
- x^2]*Sqrt[1 - k^2*x^2]*ArcTanh[(Sqrt[k]*Sqrt[-(-1)^(2/3) + k]*Sqrt[1 - x^
2])/(Sqrt[1 - (-1)^(2/3)*k]*Sqrt[1 - k^2*x^2])]])/(3*Sqrt[-(-1)^(2/3) + k]*S
qrt[1 - (-1)^(2/3)*k]*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) + (Sqrt[1 - x^2]*Sqrt[
1 - k^2*x^2]*EllipticF[ArcSin[x], k^2])/Sqrt[(1 - x^2)*(1 - k^2*x^2)] - (2*
Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[k, ArcSin[x], k^2])/(3*Sqrt[(1 -
 x^2)*(1 - k^2*x^2)]) - (2*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[-((-1
)^(1/3)*k), ArcSin[x], k^2])/(3*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) - (2*Sqrt[1
- x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[(-1)^(2/3)*k, ArcSin[x], k^2])/(3*Sqrt[
(1 - x^2)*(1 - k^2*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+k^{3/2}x^3}{\sqrt{(1-x^2)(1-k^2x^2)}\left(-1+k^{3/2}x^3\right)}\,dx = \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1+k^{3/2}x^3}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\left(-1+k^{3/2}x^3\right)}\,dx}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

$$= \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \left(\frac{1}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}} + \frac{2}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\left(-1+k^{3/2}x^3\right)}\right)dx}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

$$= \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}}\,dx}{\sqrt{(1-x^2)(1-k^2x^2)}} + \frac{\left(2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \cdots}{\sqrt{(1-x^2)(\cdots}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}} + \frac{\left(2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int\left(-\frac{\cdots}{3(1-\cdots}\right.}{\cdots}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}} - \frac{\left(2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{\cdots}{(1-\sqrt{k}\,x)\cdots}}{3\sqrt{(1-x^2)(1-k^2\cdots}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}} - \frac{\left(2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{\cdots}{\sqrt{1-x^2}\,(1\cdots}}{3\sqrt{(1-x^2)(1-k^2x\cdots}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}} - \frac{2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(k;\sin^{-1}(\cdots\right.}{3\sqrt{(1-x^2)(1-k^2x^2)}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}} - \frac{2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(k;\sin^{-1}(\cdots\right.}{3\sqrt{(1-x^2)(1-k^2x^2)}}$$

$$= \frac{2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\tan^{-1}\left(\frac{\sqrt{k}\,\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{3(1-k)\sqrt{(1-x^2)(1-k^2x^2)}} + \frac{2(-1)^{2/3}\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\mathrm{t}\cdots}{3\sqrt{\sqrt[3]{-1}+k}\,\sqrt{1+\sqrt[3]{-1}\,k\cdots}}$$

Mathematica [C] time = 4.79, size = 444, normalized size = 3.58

$$\frac{2\sqrt{k}\,\sqrt{x^2-1}\,\sqrt{k^2x^2-1}\left(-\frac{\tanh^{-1}\left(\frac{\sqrt{(k-1)k}\,\sqrt{x^2-1}}{\sqrt{1-k}\,\sqrt{k^2x^2-1}}\right)i}{\sqrt{1-k}\,\sqrt{(k-1)k}} - \frac{(-1)^{2/3}\tanh^{-1}\left(\frac{\sqrt{k\left(k+\sqrt[3]{-1}\right)}\,\sqrt{x^2-1}}{\sqrt{\sqrt[3]{-1}\,k+1}\,\sqrt{k^2x^2-1}}\right)}{\sqrt{k\left(k+\sqrt[3]{-1}\right)}\,\sqrt{\sqrt[3]{-1}\,k+1}} + \frac{\sqrt[3]{-1}\tanh^{-1}\left(\frac{\sqrt{k\left(k-(-1)^{2/3}\right)}\,\sqrt{x^2-1}}{\sqrt{1-(-1)^{2/3}k}\,\sqrt{k^2x^2-1}}\right)}{\sqrt{k\left(k-(-1)^{2/3}\right)}\,\sqrt{1-(-1)^{2/3}k}}\right) + 3\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right) - 2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(k;\sin^{-1}(x)|k^2\right) - 2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(-\sqrt[3]{-1}\,k;\sin^{-1}(x)|k^2\right) - 2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(\frac{1}{2}i\left(i+\sqrt{3}\right)k;\sin^{-1}(x)|k^2\right)}{3\sqrt{(x^2-1)(k^2x^2-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + k^(3/2)*x^3)/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(-1 + k^(3/2)*x^
3)),x]
```

```
[Out] (2*Sqrt[k]*Sqrt[-1 + x^2]*Sqrt[-1 + k^2*x^2]*(-(ArcTanh[(Sqrt[(-1 + k)*k]*S
qrt[-1 + x^2])/(Sqrt[1 - k]*Sqrt[-1 + k^2*x^2])]/(Sqrt[1 - k]*Sqrt[(-1 + k)
*k])) - ((-1)^(2/3)*ArcTanh[(Sqrt[k*((-1)^(1/3) + k)]*Sqrt[-1 + x^2])/(Sqrt
[1 + (-1)^(1/3)*k]*Sqrt[-1 + k^2*x^2])])/(Sqrt[k*((-1)^(1/3) + k)]*Sqrt[1 +
 (-1)^(1/3)*k]) + ((-1)^(1/3)*ArcTanh[(Sqrt[k*(-(-1)^(2/3) + k)]*Sqrt[-1 +
x^2])/(Sqrt[1 - (-1)^(2/3)*k]*Sqrt[-1 + k^2*x^2])])/(Sqrt[k*(-(-1)^(2/3) +
```

```
k)]*Sqrt[1 - (-1)^(2/3)*k])) + 3*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticF[
ArcSin[x], k^2] - 2*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[k, ArcSin[x]
, k^2] - 2*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[-((-1)^(1/3)*k), ArcS
in[x], k^2] - 2*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[(I/2)*(I + Sqrt[
3])*k, ArcSin[x], k^2])/(3*Sqrt[(-1 + x^2)*(-1 + k^2*x^2)])
```

IntegrateAlgebraic [A] time = 4.20, size = 124, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2-2\sqrt{k}\,x+1}\right)}{3(k-1)}-\frac{4\tan^{-1}\left(\frac{\sqrt{k^2+k+1}\,x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2+\sqrt{k}\,x+1}\right)}{3\sqrt{k^2+k+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + k^(3/2)*x^3)/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(-1 + k
^(3/2)*x^3)),x]
```

```
[Out] (-2*ArcTan[((-1 + k)*x)/(1 - 2*Sqrt[k]*x + k*x^2 + Sqrt[1 + (-1 - k^2)*x^2
+ k^2*x^4])])/(3*(-1 + k)) - (4*ArcTan[(Sqrt[1 + k + k^2]*x)/(1 + Sqrt[k]*x
 + k*x^2 + Sqrt[1 + (-1 - k^2)*x^2 + k^2*x^4])])/(3*Sqrt[1 + k + k^2])
```

fricas [B] time = 0.82, size = 213, normalized size = 1.72

$$\frac{2\sqrt{k^2+k+1}\,(k-1)\arctan\left(-\frac{\sqrt{k^2x^4-\left(k^2+1\right)x^2+1}\,\sqrt{k^2+k+1}\left(\left(k^2+2k+1\right)x-\left(kx^2+1\right)\sqrt{k}\right)}{k^3x^4-\left(k^4+4k^3+4k^2+4k+1\right)x^2+k}\right)-\left(k^2+k+1\right)\arctan\left(\frac{\sqrt{k^2x^4-\left(k^2+1\right)x^2+1}\left(\left(k^3+k^2-k-1\right)x+2\left(\left(k^2-k\right)x^2+k-1\right)\sqrt{k}\right)}{4k^3x^4-\left(k^4+4k^3-2k^2+4k+1\right)x^2+4k}\right)}{3\left(k^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+k^(3/2)*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(-1+k^(3/2)*x^3),x,
 algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(k^2 + k + 1)*(k - 1)*arctan(-sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)*
sqrt(k^2 + k + 1)*((k^2 + 2*k + 1)*x - (k*x^2 + 1)*sqrt(k))/(k^3*x^4 - (k^4
 + 4*k^3 + 4*k^2 + 4*k + 1)*x^2 + k)) - (k^2 + k + 1)*arctan(sqrt(k^2*x^4 -
 (k^2 + 1)*x^2 + 1)*((k^3 + k^2 - k - 1)*x + 2*((k^2 - k)*x^2 + k - 1)*sqrt
(k))/(4*k^3*x^4 - (k^4 + 4*k^3 - 2*k^2 + 4*k + 1)*x^2 + 4*k)))/(k^3 - 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+k^(3/2)*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(-1+k^(3/2)*x^3),x,
 algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.31, size = 1772, normalized size = 14.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+k^(3/2)*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(-1+k^(3/2)*x^3),x)
```

```
[Out] (-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*EllipticF(x
,k)-1/3/(-(-1+k)^2)^(1/2)*ln((-2*k^2+4*k-2+(-k^3+2*k^2-k)*(x^2-1/k)+2*(-(-1
+k)^2)^(1/2)*(k^3*(x^2-1/k)^2+(-k^3+2*k^2-k)*(x^2-1/k)-k^2+2*k-1)^(1/2))/(x
^2-1/k))-2/3*(-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2
)*EllipticPi(x,k,k)+1/6/k^(1/2)*sum((-_alpha^2*k-2)/_alpha/(2*_alpha^2*k+1)
```

```
*(-1/(-k*_alpha^2*(k^2+k+1))^(1/2)*arctanh(1/2*k*(2*_alpha^2*k^2-k^2-1)/(k^
4+2*k^3+6*k^2+2*k+1)*(_alpha^2*k^4+k^4*x^2+2*k^3*x^2-2*_alpha^2*k^2+6*k^2*x
^2+2*k*x^2+_alpha^2-4*k^2+x^2-4*k-4)/(-k*_alpha^2*(k^2+k+1))^(1/2)/(k^3*x^4
-k^3*x^2-k*x^2+k)^(1/2))+2*k*_alpha*(_alpha^2*k+1)/(k^2)^(1/2)*(-k^2*x^2+1)
^(1/2)*(-x^2+1)^(1/2)/((k^2*x^4-k^2*x^2-x^2+1)*k)^(1/2)*EllipticPi(x*(k^2)^
(1/2),-(_alpha^2*k+1)/k,1/(k^2)^(1/2))),_alpha=RootOf(_Z^4*k^2+_Z^2*k+1))+1
/6/k^(1/2)*2^(1/2)/((-(-3*k^2)^(1/2)*k^2+k^3-(-3*k^2)^(1/2)*k+k^2-(-3*k^2)^
(1/2)+k)/k^2)^(1/2)*ln(((-(-3*k^2)^(1/2)*k^2+k^3-(-3*k^2)^(1/2)*k+k^2-(-3*k
^2)^(1/2)+k)/k^2+(-k^2-1-k+(-3*k^2)^(1/2))*(x^2-1/2*(-k+(-3*k^2)^(1/2))/k^2
)+1/2*2^(1/2)*((-(-3*k^2)^(1/2)*k^2+k^3-(-3*k^2)^(1/2)*k+k^2-(-3*k^2)^(1/2)
+k)/k^2)^(1/2)*(4*(x^2-1/2*(-k+(-3*k^2)^(1/2))/k^2)^2*k^2+4*(-k^2-1-k+(-3*k
^2)^(1/2))*(x^2-1/2*(-k+(-3*k^2)^(1/2))/k^2)+2*(-(-3*k^2)^(1/2)*k^2+k^3-(-3
*k^2)^(1/2)*k+k^2-(-3*k^2)^(1/2)+k)/k^2)^(1/2))/(x^2-1/2*(-k+(-3*k^2)^(1/2)
)/k^2))-1/2*k^(1/2)/(-3*k^2)^(1/2)*2^(1/2)/((-(-3*k^2)^(1/2)*k^2+k^3-(-3*k^
2)^(1/2)*k+k^2-(-3*k^2)^(1/2)+k)/k^2)^(1/2)*ln(((-(-3*k^2)^(1/2)*k^2+k^3-(-
3*k^2)^(1/2)*k+k^2-(-3*k^2)^(1/2)+k)/k^2+(-k^2-1-k+(-3*k^2)^(1/2))*(x^2-1/2
*(-k+(-3*k^2)^(1/2))/k^2)+1/2*2^(1/2)*((-(-3*k^2)^(1/2)*k^2+k^3-(-3*k^2)^(1
/2)*k+k^2-(-3*k^2)^(1/2)+k)/k^2)^(1/2)*(4*(x^2-1/2*(-k+(-3*k^2)^(1/2))/k^2)
^2*k^2+4*(-k^2-1-k+(-3*k^2)^(1/2))*(x^2-1/2*(-k+(-3*k^2)^(1/2))/k^2)+2*(-(-
3*k^2)^(1/2)*k^2+k^3-(-3*k^2)^(1/2)*k+k^2-(-3*k^2)^(1/2)+k)/k^2)^(1/2))/(x^
2-1/2*(-k+(-3*k^2)^(1/2))/k^2))+1/6/k^(1/2)*2^(1/2)/(((-3*k^2)^(1/2)*k^2+k^
3+(-3*k^2)^(1/2)*k+k^2+(-3*k^2)^(1/2)+k)/k^2)^(1/2)*ln((((-3*k^2)^(1/2)*k^2
+k^3+(-3*k^2)^(1/2)*k+k^2+(-3*k^2)^(1/2)+k)/k^2+(-k^2-1-(-3*k^2)^(1/2)-k)*(
x^2+1/2*((-3*k^2)^(1/2)+k)/k^2)+1/2*2^(1/2)*(((-3*k^2)^(1/2)*k^2+k^3+(-3*k^
2)^(1/2)*k+k^2+(-3*k^2)^(1/2)+k)/k^2)^(1/2)*(4*(x^2+1/2*((-3*k^2)^(1/2)+k)/
k^2)^2*k^2+4*(-k^2-1-(-3*k^2)^(1/2)-k)*(x^2+1/2*((-3*k^2)^(1/2)+k)/k^2)+2*(
(-3*k^2)^(1/2)*k^2+k^3+(-3*k^2)^(1/2)*k+k^2+(-3*k^2)^(1/2)+k)/k^2)^(1/2))/(
x^2+1/2*((-3*k^2)^(1/2)+k)/k^2))+1/2*k^(1/2)/(-3*k^2)^(1/2)*2^(1/2)/(((-3*k
^2)^(1/2)*k^2+k^3+(-3*k^2)^(1/2)*k+k^2+(-3*k^2)^(1/2)+k)/k^2)^(1/2)*ln((((-
3*k^2)^(1/2)*k^2+k^3+(-3*k^2)^(1/2)*k+k^2+(-3*k^2)^(1/2)+k)/k^2+(-k^2-1-(-3
*k^2)^(1/2)-k)*(x^2+1/2*((-3*k^2)^(1/2)+k)/k^2)+1/2*2^(1/2)*(((-3*k^2)^(1/2
)*k^2+k^3+(-3*k^2)^(1/2)*k+k^2+(-3*k^2)^(1/2)+k)/k^2)^(1/2)*(4*(x^2+1/2*((-
3*k^2)^(1/2)+k)/k^2)^2*k^2+4*(-k^2-1-(-3*k^2)^(1/2)-k)*(x^2+1/2*((-3*k^2)^(
1/2)+k)/k^2)+2*((-3*k^2)^(1/2)*k^2+k^3+(-3*k^2)^(1/2)*k+k^2+(-3*k^2)^(1/2)+
k)/k^2)^(1/2))/(x^2+1/2*((-3*k^2)^(1/2)+k)/k^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^{\frac{3}{2}}x^3+1}{\left(k^{\frac{3}{2}}x^3-1\right)\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+k^(3/2)*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(-1+k^(3/2)*x^3),x,
 algorithm="maxima")
```

```
[Out] integrate((k^(3/2)*x^3 + 1)/((k^(3/2)*x^3 - 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1)
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{k^{3/2}\,x^3+1}{\left(k^{3/2}\,x^3-1\right)\,\sqrt{\left(x^2-1\right)\,\left(k^2\,x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^(3/2)*x^3 + 1)/((k^(3/2)*x^3 - 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)),x
)
```

```
[Out] int((k^(3/2)*x^3 + 1)/((k^(3/2)*x^3 - 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{k}\,x+1\right)\left(-\sqrt{k}\,x+kx^2+1\right)}{\sqrt{(x-1)\,(x+1)\,(kx-1)\,(kx+1)}\,\left(\sqrt{k}\,x-1\right)\left(\sqrt{k}\,x+kx^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+k**(3/2)*x**3)/((-x**2+1)*(-k**2*x**2+1))**(1/2)/(-1+k**(3/2)*
x**3),x)
```

```
[Out] Integral((sqrt(k)*x + 1)*(-sqrt(k)*x + k*x**2 + 1)/(sqrt((x - 1)*(x + 1)*(k
*x - 1)*(k*x + 1))*(sqrt(k)*x - 1)*(sqrt(k)*x + k*x**2 + 1)), x)
```

3.1532 $\int \frac{(-d+2cx)\sqrt[4]{bx^3+ax^4}}{x}\,dx$

Optimal. Leaf size=124

$$\frac{(4abd+3b^2c)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{8a^{7/4}}+\frac{(-4abd-3b^2c)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{8a^{7/4}}+\frac{\sqrt[4]{ax^4+bx^3}\,(4acx-4ad+bc)}{4a}$$

Rubi [A] time = 0.29, antiderivative size = 193, normalized size of antiderivative = 1.56, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2039, 2021, 2032, 63, 331, 298, 203, 206}

$$\frac{bx^{9/4}(ax+b)^{3/4}(4ad+3bc)\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{8a^{7/4}\left(ax^4+bx^3\right)^{3/4}}-\frac{bx^{9/4}(ax+b)^{3/4}(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{8a^{7/4}\left(ax^4+bx^3\right)^{3/4}}-\frac{\sqrt[4]{ax^4+bx^3}\,(4ad+3bc)}{4a}+\frac{c\left(ax^4+bx^3\right)^{5/4}}{ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-d + 2*c*x)*(b*x^3 + a*x^4)^(1/4))/x,x]
```

```
[Out] -1/4*((3*b*c + 4*a*d)*(b*x^3 + a*x^4)^(1/4))/a + (c*(b*x^3 + a*x^4)^(5/4))/
(a*x^3) + (b*(3*b*c + 4*a*d)*x^(9/4)*(b + a*x)^(3/4)*ArcTan[(a^(1/4)*x^(1/4
))/(b + a*x)^(1/4)])/(8*a^(7/4)*(b*x^3 + a*x^4)^(3/4)) - (b*(3*b*c + 4*a*d)
*x^(9/4)*(b + a*x)^(3/4)*ArcTanh[(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(8*a^(
7/4)*(b*x^3 + a*x^4)^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2039

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(b*(m + n + p*(j + n) + 1)), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\int \frac{(-d+2cx)\sqrt[4]{bx^3+ax^4}}{x}\,dx = \frac{c\left(bx^3+ax^4\right)^{5/4}}{ax^3} - \frac{\left(\frac{3bc}{2}+2ad\right)\int \frac{\sqrt[4]{bx^3+ax^4}}{x}\,dx}{2a}$$

$$= -\frac{(3bc+4ad)\sqrt[4]{bx^3+ax^4}}{4a} + \frac{c\left(bx^3+ax^4\right)^{5/4}}{ax^3} - \frac{(b(3bc+4ad))\int \frac{x^2}{\left(bx^3+ax^4\right)^{3/4}}\,dx}{16a}$$

$$= -\frac{(3bc+4ad)\sqrt[4]{bx^3+ax^4}}{4a} + \frac{c\left(bx^3+ax^4\right)^{5/4}}{ax^3} - \frac{\left(b(3bc+4ad)x^{9/4}(b+ax)^{3/4}\right)\int}{16a\left(bx^3+ax^4\right)^{3/}}$$

$$= -\frac{(3bc+4ad)\sqrt[4]{bx^3+ax^4}}{4a} + \frac{c\left(bx^3+ax^4\right)^{5/4}}{ax^3} - \frac{\left(b(3bc+4ad)x^{9/4}(b+ax)^{3/4}\right)\mathrm{S}}{4a\left(bx^3+\right.}$$

$$= -\frac{(3bc+4ad)\sqrt[4]{bx^3+ax^4}}{4a} + \frac{c\left(bx^3+ax^4\right)^{5/4}}{ax^3} - \frac{\left(b(3bc+4ad)x^{9/4}(b+ax)^{3/4}\right)\mathrm{S}}{4a\left(bx^3+\right.}$$

$$= -\frac{(3bc+4ad)\sqrt[4]{bx^3+ax^4}}{4a} + \frac{c\left(bx^3+ax^4\right)^{5/4}}{ax^3} - \frac{\left(b(3bc+4ad)x^{9/4}(b+ax)^{3/4}\right)\mathrm{S}}{8a^{3/2}\left(bx^3\right.}$$

$$= -\frac{(3bc+4ad)\sqrt[4]{bx^3+ax^4}}{4a} + \frac{c\left(bx^3+ax^4\right)^{5/4}}{ax^3} + \frac{b(3bc+4ad)x^{9/4}(b+ax)^{3/4}\,\mathrm{tar}}{8a^{7/4}\left(bx^3+ax^4\right)^{3/4}}$$

Mathematica [C] time = 0.05, size = 81, normalized size = 0.65

$$\frac{\sqrt[4]{x^3(ax+b)}\left(3c(ax+b)\sqrt[4]{\frac{ax}{b}+1}-(4ad+3bc)\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{ax}{b}\right)\right)}{3a\sqrt[4]{\frac{ax}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-d + 2*c*x)*(b*x^3 + a*x^4)^(1/4))/x,x]
```

```
[Out] ((x^3*(b + a*x))^(1/4)*(3*c*(b + a*x)*(1 + (a*x)/b)^(1/4) - (3*b*c + 4*a*d)
*Hypergeometric2F1[-1/4, 3/4, 7/4, -((a*x)/b)]))/(3*a*(1 + (a*x)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.70, size = 124, normalized size = 1.00

$$\frac{\left(4abd+3b^2c\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{8a^{7/4}}+\frac{\left(-4abd-3b^2c\right)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{8a^{7/4}}+\frac{\sqrt[4]{ax^4+bx^3}\,(4acx-4ad+bc)}{4a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-d + 2*c*x)*(b*x^3 + a*x^4)^(1/4))/x,x]
```

```
[Out] ((b*c - 4*a*d + 4*a*c*x)*(b*x^3 + a*x^4)^(1/4))/(4*a) + ((3*b^2*c + 4*a*b*d
)*ArcTan[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(8*a^(7/4)) + ((-3*b^2*c - 4*a
*b*d)*ArcTanh[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(8*a^(7/4))
```

fricas [B] time = 0.44, size = 771, normalized size = 6.22

$$\frac{4a\left(\frac{81b^8c^4+432ab^7c^3d+864a^2b^6c^2d^2+768a^3b^5cd^3+256a^4b^4d^4}{a^7}\right)^{\frac14}\arctan\left(\frac{a^5x\sqrt{\frac{a^4x^2\sqrt{\frac{81b^8c^4+432ab^7c^3d+864a^2b^6c^2d^2+768a^3b^5cd^3+256a^4b^4d^4}{a^7}}+\left(9b^4c^2+24ab^3cd+16a^2b^2d^2\right)\sqrt{ax^4+bx^3}}{x^2}}\left(\frac{81b^8c^4+432ab^7c^3d+864a^2b^6c^2d^2+768a^3b^5cd^3+256a^4b^4d^4}{a^7}\right)^{\frac34}-\left(3a^5b^2c+4a^6bd\right)\left(ax^4+bx^3\right)^{\frac14}\left(\frac{81b^8c^4+432ab^7c^3d+864a^2b^6c^2d^2+768a^3b^5cd^3+256a^4b^4d^4}{a^7}\right)^{\frac34}}{\left(81b^8c^4+432ab^7c^3d+864a^2b^6c^2d^2+768a^3b^5cd^3+256a^4b^4d^4\right)x}\right)-a\left(\frac{81b^8c^4+432ab^7c^3d+864a^2b^6c^2d^2+768a^3b^5cd^3+256a^4b^4d^4}{a^7}\right)^{\frac14}\log\left(\frac{a^2x\left(\frac{81b^8c^4+432ab^7c^3d+864a^2b^6c^2d^2+768a^3b^5cd^3+256a^4b^4d^4}{a^7}\right)^{\frac14}+\left(ax^4+bx^3\right)^{\frac14}\left(3b^2c+4abd\right)}{x}\right)+a\left(\frac{81b^8c^4+432ab^7c^3d+864a^2b^6c^2d^2+768a^3b^5cd^3+256a^4b^4d^4}{a^7}\right)^{\frac14}\log\left(-\frac{a^2x\left(\frac{81b^8c^4+432ab^7c^3d+864a^2b^6c^2d^2+768a^3b^5cd^3+256a^4b^4d^4}{a^7}\right)^{\frac14}-\left(ax^4+bx^3\right)^{\frac14}\left(3b^2c+4abd\right)}{x}\right)+4\left(ax^4+bx^3\right)^{\frac14}(4acx+bc-4ad)}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x-d)*(a*x^4+b*x^3)^(1/4)/x,x, algorithm="fricas")
```

```
[Out] 1/16*(4*a*a*((81*b^8*c^4 + 432*a*b^7*c^3*d + 864*a^2*b^6*c^2*d^2 + 768*a^3*b^
5*c*d^3 + 256*a^4*b^4*d^4)/a^7)^(1/4)*arctan((a^5*x*sqrt((a^4*x^2*sqrt((81*
b^8*c^4 + 432*a*b^7*c^3*d + 864*a^2*b^6*c^2*d^2 + 768*a^3*b^5*c*d^3 + 256*a
^4*b^4*d^4)/a^7) + (9*b^4*c^2 + 24*a*b^3*c*d + 16*a^2*b^2*d^2)*sqrt(a*x^4 +
 b*x^3))/x^2)*((81*b^8*c^4 + 432*a*b^7*c^3*d + 864*a^2*b^6*c^2*d^2 + 768*a^
3*b^5*c*d^3 + 256*a^4*b^4*d^4)/a^7)^(3/4) - (3*a^5*b^2*c + 4*a^6*b*d)*(a*x^
4 + b*x^3)^(1/4)*((81*b^8*c^4 + 432*a*b^7*c^3*d + 864*a^2*b^6*c^2*d^2 + 768
*a^3*b^5*c*d^3 + 256*a^4*b^4*d^4)/a^7)^(3/4))/((81*b^8*c^4 + 432*a*b^7*c^3*
d + 864*a^2*b^6*c^2*d^2 + 768*a^3*b^5*c*d^3 + 256*a^4*b^4*d^4)*x)) - a*((81
*b^8*c^4 + 432*a*b^7*c^3*d + 864*a^2*b^6*c^2*d^2 + 768*a^3*b^5*c*d^3 + 256*
a^4*b^4*d^4)/a^7)^(1/4)*log((a^2*x*((81*b^8*c^4 + 432*a*b^7*c^3*d + 864*a^2
*b^6*c^2*d^2 + 768*a^3*b^5*c*d^3 + 256*a^4*b^4*d^4)/a^7)^(1/4) + (a*x^4 + b
*x^3)^(1/4)*(3*b^2*c + 4*a*b*d))/x) + a*((81*b^8*c^4 + 432*a*b^7*c^3*d + 86
4*a^2*b^6*c^2*d^2 + 768*a^3*b^5*c*d^3 + 256*a^4*b^4*d^4)/a^7)^(1/4)*log(-(a
^2*x*((81*b^8*c^4 + 432*a*b^7*c^3*d + 864*a^2*b^6*c^2*d^2 + 768*a^3*b^5*c*d
^3 + 256*a^4*b^4*d^4)/a^7)^(1/4) - (a*x^4 + b*x^3)^(1/4)*(3*b^2*c + 4*a*b*d
))/x) + 4*(a*x^4 + b*x^3)^(1/4)*(4*a*c*x + b*c - 4*a*d))/a
```

giac [B] time = 0.22, size = 322, normalized size = 2.60

$$\frac{\frac{2\sqrt{2}\left(3b^3c+4ab^2d\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac14}+2\left(a+\frac{b}{x}\right)^{\frac14}\right)}{2(-a)^{\frac14}}\right)}{(-a)^{\frac34}a}+\frac{2\sqrt{2}\left(3b^3c+4ab^2d\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac14}-2\left(a+\frac{b}{x}\right)^{\frac14}\right)}{2(-a)^{\frac14}}\right)}{(-a)^{\frac34}a}+\frac{\sqrt{2}\left(3b^3c+4ab^2d\right)\log\left(\sqrt{2}(-a)^{\frac14}\left(a+\frac{b}{x}\right)^{\frac14}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{(-a)^{\frac34}a}-\frac{\sqrt{2}\left(3b^3c+4ab^2d\right)\log\left(-\sqrt{2}(-a)^{\frac14}\left(a+\frac{b}{x}\right)^{\frac14}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{(-a)^{\frac34}a}+\frac{8\left(\left(a+\frac{b}{x}\right)^{\frac54}b^3c+3\left(a+\frac{b}{x}\right)^{\frac14}ab^3c-4\left(a+\frac{b}{x}\right)^{\frac54}ab^2d+4\left(a+\frac{b}{x}\right)^{\frac14}a^2b^2d\right)x^2}{ab^2}}{32b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x-d)*(a*x^4+b*x^3)^(1/4)/x,x, algorithm="giac")
```

```
[Out] 1/32*(2*sqrt(2)*(3*b^3*c + 4*a*b^2*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4
) + 2*(a + b/x)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a) + 2*sqrt(2)*(3*b^3*c + 4*
a*b^2*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(a + b/x)^(1/4))/(-a)^
(1/4))/((-a)^(3/4)*a) + sqrt(2)*(3*b^3*c + 4*a*b^2*d)*log(sqrt(2)*(-a)^(1/4
)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a + b/x))/((-a)^(3/4)*a) - sqrt(2)*(3*b
^3*c + 4*a*b^2*d)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt
(a + b/x))/((-a)^(3/4)*a) + 8*((a + b/x)^(5/4)*b^3*c + 3*(a + b/x)^(1/4)*a*
b^3*c - 4*(a + b/x)^(5/4)*a*b^2*d + 4*(a + b/x)^(1/4)*a^2*b^2*d)*x^2/(a*b^2
))/b
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(2cx-d)\left(a\,x^{4}+b\,x^{3}\right)^{\frac{1}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x-d)*(a*x^4+b*x^3)^(1/4)/x,x)
```

```
[Out] int((2*c*x-d)*(a*x^4+b*x^3)^(1/4)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{4}+bx^{3}\right)^{\frac{1}{4}}(2\,cx-d)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x-d)*(a*x^4+b*x^3)^(1/4)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + b*x^3)^(1/4)*(2*c*x - d)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(a\,x^4+b\,x^3\right)^{1/4}\,\left(d-2\,c\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*x^4 + b*x^3)^(1/4)*(d - 2*c*x))/x,x)
```

```
[Out] -int(((a*x^4 + b*x^3)^(1/4)*(d - 2*c*x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^{3}\left(ax+b\right)}\,\left(2cx-d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x-d)*(a*x**4+b*x**3)**(1/4)/x,x)
```

```
[Out] Integral((x**3*(a*x + b))**(1/4)*(2*c*x - d)/x, x)
```

3.1533 $\int \frac{-1+x^6}{\left(1+x^6\right)\sqrt[3]{1+a^3x^3+x^6}}\,dx$

Optimal. Leaf size=124

$$\frac{\log\left(\sqrt[3]{a^3x^3+x^6+1}-ax\right)}{3a}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,ax}{2\sqrt[3]{a^3x^3+x^6+1}+ax}\right)}{\sqrt{3}\,a}-\frac{\log\left(ax\sqrt[3]{a^3x^3+x^6+1}+\left(a^3x^3+x^6+1\right)^{2/3}+a^2x^2\right)}{6a}$$

Rubi [F] time = 2.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+x^6}{\left(1+x^6\right)\sqrt[3]{1+a^3x^3+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x^6)/((1 + x^6)*(1 + a^3*x^3 + x^6)^(1/3)),x]
```

```
[Out] (x*(1 + (2*x^3)/(a^3 - Sqrt[-4 + a^6]))^(1/3)*(1 + (2*x^3)/(a^3 + Sqrt[-4 +
 a^6]))^(1/3)*AppellF1[1/3, 1/3, 1/3, 4/3, (-2*x^3)/(a^3 - Sqrt[-4 + a^6]),
 (-2*x^3)/(a^3 + Sqrt[-4 + a^6])])/(1 + a^3*x^3 + x^6)^(1/3) - (I/3)*Defer[
Int][1/((I - x)*(1 + a^3*x^3 + x^6)^(1/3)), x] - (I/3)*Defer[Int][1/((I + x
)*(1 + a^3*x^3 + x^6)^(1/3)), x] - (Sqrt[1 - I*Sqrt[3]]*Defer[Int][1/((Sqrt
[1 - I*Sqrt[3]] - Sqrt[2]*x)*(1 + a^3*x^3 + x^6)^(1/3)), x])/3 - (Sqrt[1 +
I*Sqrt[3]]*Defer[Int][1/((Sqrt[1 + I*Sqrt[3]] - Sqrt[2]*x)*(1 + a^3*x^3 + x
^6)^(1/3)), x])/3 - (Sqrt[1 - I*Sqrt[3]]*Defer[Int][1/((Sqrt[1 - I*Sqrt[3]]
 + Sqrt[2]*x)*(1 + a^3*x^3 + x^6)^(1/3)), x])/3 - (Sqrt[1 + I*Sqrt[3]]*Defe
r[Int][1/((Sqrt[1 + I*Sqrt[3]] + Sqrt[2]*x)*(1 + a^3*x^3 + x^6)^(1/3)), x])
/3
```

Rubi steps

$$\int \frac{-1+x^6}{\left(1+x^6\right)\sqrt[3]{1+a^3x^3+x^6}}\,dx=\int \left(\frac{1}{\sqrt[3]{1+a^3x^3+x^6}}-\frac{2}{\left(1+x^6\right)\sqrt[3]{1+a^3x^3+x^6}}\right)dx$$

$$=-\left(2\int \frac{1}{\left(1+x^6\right)\sqrt[3]{1+a^3x^3+x^6}}\,dx\right)+\int \frac{1}{\sqrt[3]{1+a^3x^3+x^6}}\,dx$$

$$=-\left(2\int \left(\frac{1}{3\left(1+x^2\right)\sqrt[3]{1+a^3x^3+x^6}}+\frac{2-x^2}{3\left(1-x^2+x^4\right)\sqrt[3]{1+a^3x^3+x^6}}\right)dx\right)+$$

$$=\frac{x\sqrt[3]{1+\frac{2x^3}{a^3-\sqrt{-4+a^6}}}\sqrt[3]{1+\frac{2x^3}{a^3+\sqrt{-4+a^6}}}F_1\left(\frac{1}{3};\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{2x^3}{a^3-\sqrt{-4+a^6}},-\frac{2x^3}{a^3+\sqrt{-4+a^6}}\right)}{\sqrt[3]{1+a^3x^3+x^6}}-$$

$$=\frac{x\sqrt[3]{1+\frac{2x^3}{a^3-\sqrt{-4+a^6}}}\sqrt[3]{1+\frac{2x^3}{a^3+\sqrt{-4+a^6}}}F_1\left(\frac{1}{3};\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{2x^3}{a^3-\sqrt{-4+a^6}},-\frac{2x^3}{a^3+\sqrt{-4+a^6}}\right)}{\sqrt[3]{1+a^3x^3+x^6}}-$$

$$=\frac{x\sqrt[3]{1+\frac{2x^3}{a^3-\sqrt{-4+a^6}}}\sqrt[3]{1+\frac{2x^3}{a^3+\sqrt{-4+a^6}}}F_1\left(\frac{1}{3};\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{2x^3}{a^3-\sqrt{-4+a^6}},-\frac{2x^3}{a^3+\sqrt{-4+a^6}}\right)}{\sqrt[3]{1+a^3x^3+x^6}}-$$

$$=\frac{x\sqrt[3]{1+\frac{2x^3}{a^3-\sqrt{-4+a^6}}}\sqrt[3]{1+\frac{2x^3}{a^3+\sqrt{-4+a^6}}}F_1\left(\frac{1}{3};\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{2x^3}{a^3-\sqrt{-4+a^6}},-\frac{2x^3}{a^3+\sqrt{-4+a^6}}\right)}{\sqrt[3]{1+a^3x^3+x^6}}-$$

$$=\frac{x\sqrt[3]{1+\frac{2x^3}{a^3-\sqrt{-4+a^6}}}\sqrt[3]{1+\frac{2x^3}{a^3+\sqrt{-4+a^6}}}F_1\left(\frac{1}{3};\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{2x^3}{a^3-\sqrt{-4+a^6}},-\frac{2x^3}{a^3+\sqrt{-4+a^6}}\right)}{\sqrt[3]{1+a^3x^3+x^6}}-$$

Mathematica [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{-1+x^6}{\left(1+x^6\right)\sqrt[3]{1+a^3x^3+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^6)/((1 + x^6)*(1 + a^3*x^3 + x^6)^(1/3)),x]
[Out] Integrate[(-1 + x^6)/((1 + x^6)*(1 + a^3*x^3 + x^6)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 1.07, size = 128, normalized size = 1.03

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,ax}{2\sqrt[3]{a^3x^3+x^6+1}+ax}\right)}{\sqrt{3}\,a}+\frac{\log\left(a^2x-a\sqrt[3]{a^3x^3+x^6+1}\right)}{3a}-\frac{\log\left(ax\sqrt[3]{a^3x^3+x^6+1}+\left(a^3x^3+x^6+1\right)^{2/3}+a^2x^2\right)}{6a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^6)/((1 + x^6)*(1 + a^3*x^3 + x^6)^(1/3)),x]
[Out] -(ArcTan[(Sqrt[3]*a*x)/(a*x + 2*(1 + a^3*x^3 + x^6)^(1/3))]/(Sqrt[3]*a)) +
Log[a^2*x - a*(1 + a^3*x^3 + x^6)^(1/3)]/(3*a) - Log[a^2*x^2 + a*x*(1 + a^3
*x^3 + x^6)^(1/3) + (1 + a^3*x^3 + x^6)^(2/3)]/(6*a)
```

fricas [A] time = 4.77, size = 148, normalized size = 1.19

$$-\frac{2\sqrt{3}\arctan\left(-\frac{4\sqrt{3}\left(a^3x^3+x^6+1\right)^{\frac{1}{3}}a^2x^2-2\sqrt{3}\left(a^3x^3+x^6+1\right)^{\frac{2}{3}}ax+\sqrt{3}\left(a^3x^3+x^6+1\right)}{9a^3x^3+x^6+1}\right)-\log\left(\frac{x^6+3\left(a^3x^3+x^6+1\right)^{\frac{1}{3}}a^2x^2-3\left(a^3x^3+x^6+1\right)^{\frac{2}{3}}ax+1}{x^6+1}\right)}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^6+1)/(a^3*x^3+x^6+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(-(4*sqrt(3)*(a^3*x^3 + x^6 + 1)^(1/3)*a^2*x^2 - 2*sq
rt(3)*(a^3*x^3 + x^6 + 1)^(2/3)*a*x + sqrt(3)*(a^3*x^3 + x^6 + 1))/(9*a^3*x
^3 + x^6 + 1)) - log((x^6 + 3*(a^3*x^3 + x^6 + 1)^(1/3)*a^2*x^2 - 3*(a^3*x^
3 + x^6 + 1)^(2/3)*a*x + 1)/(x^6 + 1)))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6-1}{\left(a^3x^3+x^6+1\right)^{\frac{1}{3}}\left(x^6+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^6+1)/(a^3*x^3+x^6+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^6 - 1)/((a^3*x^3 + x^6 + 1)^(1/3)*(x^6 + 1)), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^6-1}{\left(x^6+1\right)\left(a^3x^3+x^6+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)/(x^6+1)/(a^3*x^3+x^6+1)^(1/3),x)
```

```
[Out] int((x^6-1)/(x^6+1)/(a^3*x^3+x^6+1)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6-1}{\left(a^3x^3+x^6+1\right)^{\frac{1}{3}}\left(x^6+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^6+1)/(a^3*x^3+x^6+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^6 - 1)/((a^3*x^3 + x^6 + 1)^(1/3)*(x^6 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6-1}{\left(x^6+1\right)\left(a^3\,x^3+x^6+1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)/((x^6 + 1)*(x^6 + a^3*x^3 + 1)^(1/3)),x)
```

```
[Out] int((x^6 - 1)/((x^6 + 1)*(x^6 + a^3*x^3 + 1)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}{\left(x^2+1\right)\left(x^4-x^2+1\right)\sqrt[3]{a^3x^3+x^6+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)/(x**6+1)/(a**3*x**3+x**6+1)**(1/3),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1)/((x**2 + 1)*(x**4 -
x**2 + 1)*(a**3*x**3 + x**6 + 1)**(1/3)), x)
```

3.1534 $\int \frac{(2+x^3+4x^6)\sqrt[3]{x+2x^3-x^4-x^7}}{(-1-2x^2+x^3+x^6)(-1-x^2+x^3+x^6)}\,dx$

Optimal. Leaf size=124

$$-\log\left(\sqrt[3]{-x^7-x^4+2x^3+x}-x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{-x^7-x^4+2x^3+x}+x}\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{-x^7-x^4+2x^3+x}\,x+\right.$$

Rubi [F] time = 4.66, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(2+x^3+4x^6)\sqrt[3]{x+2x^3-x^4-x^7}}{(-1-2x^2+x^3+x^6)(-1-x^2+x^3+x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((2 + x^3 + 4*x^6)*(x + 2*x^3 - x^4 - x^7)^(1/3))/((-1 - 2*x^2 + x^3 +
x^6)*(-1 - x^2 + x^3 + x^6)),x]
```

```
[Out] -(((x + 2*x^3 - x^4 - x^7)^(1/3)*Defer[Subst][Defer[Int][1/((-1 + x)*(1 + 2
*x^6 - x^9 - x^18)^(2/3)), x], x, x^(1/3)])/(x^(1/3)*(1 + 2*x^2 - x^3 - x^6
)^(1/3))) - (12*(x + 2*x^3 - x^4 - x^7)^(1/3)*Defer[Subst][Defer[Int][x^3/(
1 + 2*x^6 - x^9 - x^18)^(2/3), x], x, x^(1/3)])/(x^(1/3)*(1 + 2*x^2 - x^3 -
 x^6)^(1/3)) + ((1 - I*Sqrt[3])*(x + 2*x^3 - x^4 - x^7)^(1/3)*Defer[Subst][
Defer[Int][1/((1 - I*Sqrt[3] + 2*x)*(1 + 2*x^6 - x^9 - x^18)^(2/3)), x], x,
 x^(1/3)])/(x^(1/3)*(1 + 2*x^2 - x^3 - x^6)^(1/3)) + ((1 + I*Sqrt[3])*(x +
2*x^3 - x^4 - x^7)^(1/3)*Defer[Subst][Defer[Int][1/((1 + I*Sqrt[3] + 2*x)*(
1 + 2*x^6 - x^9 - x^18)^(2/3)), x], x, x^(1/3)])/(x^(1/3)*(1 + 2*x^2 - x^3
- x^6)^(1/3)) - (3*(x + 2*x^3 - x^4 - x^7)^(1/3)*Defer[Subst][Defer[Int][1/
((1 + x^3 + 2*x^6 + x^9 + x^12 + x^15)*(1 + 2*x^6 - x^9 - x^18)^(2/3)), x],
 x, x^(1/3)])/(x^(1/3)*(1 + 2*x^2 - x^3 - x^6)^(1/3)) + (12*(x + 2*x^3 - x^
4 - x^7)^(1/3)*Defer[Subst][Defer[Int][x^3/((1 + x^3 + 2*x^6 + x^9 + x^12 +
 x^15)*(1 + 2*x^6 - x^9 - x^18)^(2/3)), x], x, x^(1/3)])/(x^(1/3)*(1 + 2*x^
2 - x^3 - x^6)^(1/3)) + (6*(x + 2*x^3 - x^4 - x^7)^(1/3)*Defer[Subst][Defer
[Int][x^6/((1 + x^3 + 2*x^6 + x^9 + x^12 + x^15)*(1 + 2*x^6 - x^9 - x^18)^(
2/3)), x], x, x^(1/3)])/(x^(1/3)*(1 + 2*x^2 - x^3 - x^6)^(1/3)) + (15*(x +
2*x^3 - x^4 - x^7)^(1/3)*Defer[Subst][Defer[Int][x^9/((1 + x^3 + 2*x^6 + x^
9 + x^12 + x^15)*(1 + 2*x^6 - x^9 - x^18)^(2/3)), x], x, x^(1/3)])/(x^(1/3)
*(1 + 2*x^2 - x^3 - x^6)^(1/3)) + (3*(x + 2*x^3 - x^4 - x^7)^(1/3)*Defer[Su
bst][Defer[Int][x^12/((1 + x^3 + 2*x^6 + x^9 + x^12 + x^15)*(1 + 2*x^6 - x^
9 - x^18)^(2/3)), x], x, x^(1/3)])/(x^(1/3)*(1 + 2*x^2 - x^3 - x^6)^(1/3))
```

Rubi steps

$$\int \frac{\left(2+x^3+4x^6\right)\sqrt[3]{x+2x^3-x^4-x^7}}{\left(-1-2x^2+x^3+x^6\right)\left(-1-x^2+x^3+x^6\right)}\,dx = \frac{\sqrt[3]{x+2x^3-x^4-x^7}\int \frac{\sqrt[3]{x}\sqrt[3]{1+2x^2-x^3-x^6}\left(2+x^3+4x^6\right)}{\left(-1-2x^2+x^3+x^6\right)\left(-1-x^2+x^3+x^6\right)}\,dx}{\sqrt[3]{x}\sqrt[3]{1+2x^2-x^3-x^6}}$$

$$= -\frac{\sqrt[3]{x+2x^3-x^4-x^7}\int \frac{\sqrt[3]{x}\left(2+x^3+4x^6\right)}{\left(1+2x^2-x^3-x^6\right)^{2/3}\left(-1-x^2+x^3+x^6\right)}\,dx}{\sqrt[3]{x}\sqrt[3]{1+2x^2-x^3-x^6}}$$

$$= -\frac{\left(3\sqrt[3]{x+2x^3-x^4-x^7}\right)\text{Subst}\left(\int \frac{x^3\left(2+x^9+4x^{18}\right)}{\left(1+2x^6-x^9-x^{18}\right)^{2/3}\left(-1-x^6+x^9+\right.}\right.}{\sqrt[3]{x}\sqrt[3]{1+2x^2-x^3-x^6}}$$

$$= -\frac{\left(3\sqrt[3]{x+2x^3-x^4-x^7}\right)\text{Subst}\left(\int\left(\frac{4x^3}{\left(1+2x^6-x^9-x^{18}\right)^{2/3}}+\frac{}{\left(1+2x^6\right.}\right.\right.}{\sqrt[3]{x}\sqrt[3]{1+2x^2-x^3-x^6}}$$

$$= -\frac{\left(3\sqrt[3]{x+2x^3-x^4-x^7}\right)\text{Subst}\left(\int \frac{x^3\left(6+4x^6-3x^9\right)}{\left(1+2x^6-x^9-x^{18}\right)^{2/3}\left(-1-x^6+x^9+\right.}\right.}{\sqrt[3]{x}\sqrt[3]{1+2x^2-x^3-x^6}}$$

$$= -\frac{\left(3\sqrt[3]{x+2x^3-x^4-x^7}\right)\text{Subst}\left(\int\left(\frac{1}{3(-1+x)\left(1+2x^6-x^9-x^{18}\right)^{2/3}}+\right.\right.}{\sqrt[3]{x}}$$

$$= -\frac{\sqrt[3]{x+2x^3-x^4-x^7}\,\text{Subst}\left(\int \frac{1}{(-1+x)\left(1+2x^6-x^9-x^{18}\right)^{2/3}}\,dx, x, \sqrt[3]{}\right.}{\sqrt[3]{x}\sqrt[3]{1+2x^2-x^3-x^6}}$$

$$= -\frac{\sqrt[3]{x+2x^3-x^4-x^7}\,\text{Subst}\left(\int \frac{1}{(-1+x)\left(1+2x^6-x^9-x^{18}\right)^{2/3}}\,dx, x, \sqrt[3]{}\right.}{\sqrt[3]{x}\sqrt[3]{1+2x^2-x^3-x^6}}$$

$$= -\frac{\sqrt[3]{x+2x^3-x^4-x^7}\,\text{Subst}\left(\int \frac{1}{(-1+x)\left(1+2x^6-x^9-x^{18}\right)^{2/3}}\,dx, x, \sqrt[3]{}\right.}{\sqrt[3]{x}\sqrt[3]{1+2x^2-x^3-x^6}}$$

Mathematica [F] time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{\left(2+x^3+4x^6\right)\sqrt[3]{x+2x^3-x^4-x^7}}{\left(-1-2x^2+x^3+x^6\right)\left(-1-x^2+x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((2 + x^3 + 4*x^6)*(x + 2*x^3 - x^4 - x^7)^(1/3))/((-1 - 2*x^2 +
x^3 + x^6)*(-1 - x^2 + x^3 + x^6)),x]
```

```
[Out] Integrate[((2 + x^3 + 4*x^6)*(x + 2*x^3 - x^4 - x^7)^(1/3))/((-1 - 2*x^2 +
x^3 + x^6)*(-1 - x^2 + x^3 + x^6)), x]
```

IntegrateAlgebraic [A] time = 0.52, size = 124, normalized size = 1.00

$$-\log\left(\sqrt[3]{-x^7-x^4+2x^3+x}-x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{-x^7-x^4+2x^3+x}+x}\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{-x^7-x^4+2x^3+x}\,x+\left(-x^7-x^4+2x^3+x\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^3 + 4*x^6)*(x + 2*x^3 - x^4 - x^7)^(1/3))/((-1 -
 2*x^2 + x^3 + x^6)*(-1 - x^2 + x^3 + x^6)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(x + 2*x^3 - x^4 - x^7)^(1/3))]) - Log[
-x + (x + 2*x^3 - x^4 - x^7)^(1/3)] + Log[x^2 + x*(x + 2*x^3 - x^4 - x^7)^(
1/3) + (x + 2*x^3 - x^4 - x^7)^(2/3)]/2
```

fricas [A] time = 2.67, size = 174, normalized size = 1.40

$$\sqrt{3} \arctan\left(-\frac{70 \sqrt{3} \left(-x^{7} - x^{4} + 2 x^{3} + x\right)^{\frac{1}{3}} x - \sqrt{3} \left(32 x^{6} + 32 x^{3} - 39 x^{2} - 32\right) - 56 \sqrt{3} \left(-x^{7} - x^{4} + 2 x^{3} + x\right)^{\frac{2}{3}}}{64 x^{6} + 64 x^{3} - 253 x^{2} - 64}\right) - \frac{1}{2} \log\left(\frac{x^{6} + x^{3} - x^{2} - 3 \left(-x^{7} - x^{4} + 2 x^{3} + x\right)^{\frac{1}{3}} x + 3 \left(-x^{7} - x^{4} + 2 x^{3} + x\right)^{\frac{2}{3}} - 1}{x^{6} + x^{3} - x^{2} - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6+x^3+2)*(-x^7-x^4+2*x^3+x)^(1/3)/(x^6+x^3-2*x^2-1)/(x^6+x^3
-x^2-1),x, algorithm="fricas")
```

```
[Out] sqrt(3)*arctan(-(70*sqrt(3)*(-x^7 - x^4 + 2*x^3 + x)^(1/3)*x - sqrt(3)*(32*
x^6 + 32*x^3 - 39*x^2 - 32) - 56*sqrt(3)*(-x^7 - x^4 + 2*x^3 + x)^(2/3))/(6
4*x^6 + 64*x^3 - 253*x^2 - 64)) - 1/2*log((x^6 + x^3 - x^2 - 3*(-x^7 - x^4
+ 2*x^3 + x)^(1/3)*x + 3*(-x^7 - x^4 + 2*x^3 + x)^(2/3) - 1)/(x^6 + x^3 - x
^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^{7} - x^{4} + 2 x^{3} + x\right)^{\frac{1}{3}} \left(4 x^{6} + x^{3} + 2\right)}{\left(x^{6} + x^{3} - x^{2} - 1\right) \left(x^{6} + x^{3} - 2 x^{2} - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6+x^3+2)*(-x^7-x^4+2*x^3+x)^(1/3)/(x^6+x^3-2*x^2-1)/(x^6+x^3
-x^2-1),x, algorithm="giac")
```

```
[Out] integrate((-x^7 - x^4 + 2*x^3 + x)^(1/3)*(4*x^6 + x^3 + 2)/((x^6 + x^3 - x^
2 - 1)*(x^6 + x^3 - 2*x^2 - 1)), x)
```

maple [C] time = 23.04, size = 536, normalized size = 4.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^6+x^3+2)*(-x^7-x^4+2*x^3+x)^(1/3)/(x^6+x^3-2*x^2-1)/(x^6+x^3-x^2-1
),x)
```

```
[Out] -ln((3161575138941971602132907799321044*RootOf(_Z^2-_Z+1)^2*x^6+43232446774
2150168674553252691379 75*RootOf(_Z^2-_Z+1)*x^6-501616012463911978940747053 6
9999504*x^6+3161575138941971602132907799321044*RootOf(_Z^2-_Z+1)^2*x^3-5611
7958716219995937859113437948531*RootOf(_Z^2-_Z+1)^2*x^2+4323244677421501686
7455325269137975*RootOf(_Z^2-_Z+1)*x^3+99717198298490157965795846237779567*
RootOf(_Z^2-_Z+1)*(-x^7-x^4+2*x^3+x)^(2/3)-602523308575093505228269932701 82
077*RootOf(_Z^2-_Z+1)*(-x^7-x^4+2*x^3+x)^(1/3)*x-297409306379177999746979 72
598107978*RootOf(_Z^2-_Z+1)*x^2-5016160124639119789407470536999950 4*x^3-316
1575138941971602132907799321044*RootOf(_Z^2-_Z+1)^2-602523308575093505228 26
993270182077*(-x^7-x^4+2*x^3+x)^(2/3)-39464867440980807442968852967597490*x
*(-x^7-x^4+2*x^3+x)^(1/3)+96922415967603331524144345969151584*x^2-432324467
74215016867455325269137975*RootOf(_Z^2-_Z+1)+50161601246391197894074705369 9
99504)/(-1+x)/(x^5+x^4+x^3+2*x^2+x+1))+RootOf(_Z^2-_Z+1)*ln((23921138623709
266187215697152638 0*RootOf(_Z^2-_Z+1)^2*x^6-716836585841327368849153707238 7
909*RootOf(_Z^2-_Z+1)*x^6+53562387771570262158079770140846928*x^6+239211386
23709266187215697152638 0*RootOf(_Z^2-_Z+1)^2*x^3-42460021057083947482307862
44593245*RootOf(_Z^2-_Z+1)^2*x^2-716836585841327368849153707238 7909*RootOf(
_Z^2-_Z+1)*x^3-99717198298490157965795846237779567*RootOf(_Z^2-_Z+1)*(-x^7-
x^4+2*x^3+x)^(2/3)+39464867440980807442968852967597490*RootOf(_Z^2-_Z+1)*(-
x^7-x^4+2*x^3+x)^(1/3)*x+71427487435393926297677159615636851*RootOf(_Z^2-_Z
```

```
+1)*x^2+53562387771570262158079770140846928*x^3-239211386237092661872156971
526380*RootOf(_Z^2-_Z+1)^2+394648674409808074429688529675974 90*(-x^7-x^4+2*
x^3+x)^(2/3)+602523308575093505228269932701820 77*x*(-x^7-x^4+2*x^3+x)^(1/3)
-157286376789531722210234245651693360*x^2+716836585841327368849153707238790
9*RootOf(_Z^2-_Z+1)-53562387771570262158079770140846928)/(-1+x)/(x^5+x^4+x^
3+2*x^2+x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^7 - x^4 + 2\,x^3 + x\right)^{\frac{1}{3}}\left(4\,x^6 + x^3 + 2\right)}{\left(x^6 + x^3 - x^2 - 1\right)\left(x^6 + x^3 - 2\,x^2 - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6+x^3+2)*(-x^7-x^4+2*x^3+x)^(1/3)/(x^6+x^3-2*x^2-1)/(x^6+x^3
-x^2-1),x, algorithm="maxima")
```

```
[Out] integrate((-x^7 - x^4 + 2*x^3 + x)^(1/3)*(4*x^6 + x^3 + 2)/((x^6 + x^3 - x^
2 - 1)*(x^6 + x^3 - 2*x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(4\,x^6+x^3+2\right)\,{\left(-x^7-x^4+2\,x^3+x\right)}^{1/3}}{\left(-x^6-x^3+2\,x^2+1\right)\,\left(-x^6-x^3+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 4*x^6 + 2)*(x + 2*x^3 - x^4 - x^7)^(1/3))/((2*x^2 - x^3 - x^6 +
 1)*(x^2 - x^3 - x^6 + 1)),x)
```

```
[Out] int(((x^3 + 4*x^6 + 2)*(x + 2*x^3 - x^4 - x^7)^(1/3))/((2*x^2 - x^3 - x^6 +
 1)*(x^2 - x^3 - x^6 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{- x \left(x^{6} + x^{3} - 2 x^{2} - 1\right)} \left(4 x^{6} + x^{3} + 2\right)}{\left(x - 1\right) \left(x^{6} + x^{3} - 2 x^{2} - 1\right) \left(x^{5} + x^{4} + x^{3} + 2 x^{2} + x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**6+x**3+2)*(-x**7-x**4+2*x**3+x)**(1/3)/(x**6+x**3-2*x**2-1)
/(x**6+x**3-x**2-1),x)
```

```
[Out] Integral((-x*(x**6 + x**3 - 2*x**2 - 1))**(1/3)*(4*x**6 + x**3 + 2)/((x - 1
)*(x**6 + x**3 - 2*x**2 - 1)*(x**5 + x**4 + x**3 + 2*x**2 + x + 1)), x)
```

3.1535 $\int \frac{x^2}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$

Optimal. Leaf size=124

$$\frac{x\sqrt{\sqrt{a^2x^4+b}+ax^2}}{4a}-\frac{bx}{8a\left(\sqrt{a^2x^4+b}+ax^2\right)^{3/2}}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}}{\sqrt{b}}\right)}{8\sqrt{2}\,a^{3/2}}$$

Rubi [F] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] Defer[Int][x^2/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]], x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx=\int \frac{x^2}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] Integrate[x^2/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]], x]
```

IntegrateAlgebraic [A] time = 0.28, size = 124, normalized size = 1.00

$$\frac{x\sqrt{\sqrt{a^2x^4+b}+ax^2}}{4a}-\frac{bx}{8a\left(\sqrt{a^2x^4+b}+ax^2\right)^{3/2}}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}}{\sqrt{b}}\right)}{8\sqrt{2}\,a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] -1/8*(b*x)/(a*(a*x^2 + Sqrt[b + a^2*x^4])^(3/2)) + (x*Sqrt[a*x^2 + Sqrt[b +
 a^2*x^4]])/(4*a) - (Sqrt[b]*ArcTan[(Sqrt[2]*Sqrt[a]*x*Sqrt[a*x^2 + Sqrt[b
+ a^2*x^4]])/Sqrt[b]])/(8*Sqrt[2]*a^(3/2))
```

fricas [A] time = 3.10, size = 323, normalized size = 2.60

$$\left[\frac{\sqrt{\frac{1}{2}}\, b \sqrt{-\frac{b}{a}} \log\left(4\, a^{2} b x^{4} - 4\, \sqrt{a^{2} x^{4} + b}\, a b x^{2} + b^{2} - 4\, \left(2\, \sqrt{\frac{1}{2}}\, \sqrt{a^{2} x^{4} + b}\, a^{2} x^{3} \sqrt{-\frac{b}{a}} - \sqrt{\frac{1}{2}}\, \left(2\, a^{3} x^{5} + a b x\right)\sqrt{-\frac{b}{a}}\right)\sqrt{a x^{2} + \sqrt{a^{2} x^{4} + b}}\right) - 2\, \left(2\, a^{2} x^{5} - 2\, \sqrt{a^{2} x^{4} + b}\, a x^{3} - b x\right)\sqrt{a x^{2} + \sqrt{a^{2} x^{4} + b}}}{16\, a b}, \frac{\sqrt{\frac{1}{2}}\, b \sqrt{\frac{b}{a}} \arctan\left(-\frac{\left(\sqrt{\frac{1}{2}}\, a x^{2} \sqrt{\frac{b}{a}} - \sqrt{\frac{1}{2}}\, \sqrt{a^{2} x^{4} + b}\, \sqrt{\frac{b}{a}}\right)\sqrt{a x^{2} + \sqrt{a^{2} x^{4} + b}}}{b x}\right) - \left(2\, a^{2} x^{5} - 2\, \sqrt{a^{2} x^{4} + b}\, a x^{3} - b x\right)\sqrt{a x^{2} + \sqrt{a^{2} x^{4} + b}}}{8\, a b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(sqrt(1/2)*b*sqrt(-b/a)*log(4*a^2*b*x^4 - 4*sqrt(a^2*x^4 + b)*a*b*x^2
 + b^2 - 4*(2*sqrt(1/2)*sqrt(a^2*x^4 + b)*a^2*x^3*sqrt(-b/a) - sqrt(1/2)*(2
*a^3*x^5 + a*b*x)*sqrt(-b/a))*sqrt(a*x^2 + sqrt(a^2*x^4 + b))) - 2*(2*a^2*x
^5 - 2*sqrt(a^2*x^4 + b)*a*x^3 - b*x)*sqrt(a*x^2 + sqrt(a^2*x^4 + b)))/(a*b
), 1/8*(sqrt(1/2)*b*sqrt(b/a)*arctan(-(sqrt(1/2)*a*x^2*sqrt(b/a) - sqrt(1/2
)*sqrt(a^2*x^4 + b)*sqrt(b/a))*sqrt(a*x^2 + sqrt(a^2*x^4 + b))/(b*x)) - (2*
a^2*x^5 - 2*sqrt(a^2*x^4 + b)*a*x^3 - b*x)*sqrt(a*x^2 + sqrt(a^2*x^4 + b)))
/(a*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\sqrt{a x^{2} + \sqrt{a^{2} x^{4} + b}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(a*x^2 + sqrt(a^2*x^4 + b)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\sqrt{a\, x^{2}+\sqrt{a^{2} x^{4}+b}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x)
```

```
[Out] int(x^2/(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\sqrt{a x^{2} + \sqrt{a^{2} x^{4} + b}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a*x^2 + sqrt(a^2*x^4 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{\sqrt{a^2\,x^4+b}+a\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b + a^2*x^4)^(1/2) + a*x^2)^(1/2),x)
```

```
[Out] int(x^2/((b + a^2*x^4)^(1/2) + a*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{ax^2 + \sqrt{a^2x^4 + b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**2+(a**2*x**4+b)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*x**2 + sqrt(a**2*x**4 + b)), x)
```

3.1536 $\int \frac{1}{(-1+x)\sqrt[3]{-3-2x+x^2}}\,dx$

Optimal. Leaf size=125

$$-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^2-2x-3}+2\right)}{2\,2^{2/3}}+\frac{\log\left(2^{2/3}\left(x^2-2x-3\right)^{2/3}-2\sqrt[3]{2}\sqrt[3]{x^2-2x-3}+4\right)}{4\,2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2}\sqrt[3]{x^2-2x}}{\sqrt{3}}\right.}{2\,2^{2/3}}$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 0.67, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {694, 266, 56, 617, 204, 31}

$$-\frac{3\log\left(\sqrt[3]{(x-1)^2-4}+2^{2/3}\right)}{4\,2^{2/3}}+\frac{\log(1-x)}{2\,2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\sqrt[3]{2}\sqrt[3]{(x-1)^2-4}}{\sqrt{3}}\right)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)*(-3 - 2*x + x^2)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 - 2^(1/3)*(-4 + (-1 + x)^2)^(1/3))/Sqrt[3]])/2^(2/3
) - (3*Log[2^(2/3) + (-4 + (-1 + x)^2)^(1/3)])/(4*2^(2/3)) + Log[1 - x]/(2*
2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
```

```
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(-1+x)\sqrt[3]{-3-2x+x^2}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt[3]{-4+x^2}}\,dx, x, -1+x\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{-4+x}\,x}\,dx, x, (-1+x)^2\right)$$
$$= \frac{\log(1-x)}{2\,2^{2/3}} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}-2^{2/3}x+x^2}\,dx, x, \sqrt[3]{-4+(-1+x)^2}\right) - \frac{3\,\text{Subst}(}{}$$
$$= -\frac{3\log\left(2^{2/3}+\sqrt[3]{-4+(-1+x)^2}\right)}{4\,2^{2/3}} + \frac{\log(1-x)}{2\,2^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\sqrt[3]{2}\right.}{2\,2^{2/3}}$$
$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\sqrt[3]{2}\sqrt[3]{-4+(-1+x)^2}}{\sqrt{3}}\right)}{2\,2^{2/3}} - \frac{3\log\left(2^{2/3}+\sqrt[3]{-4+(-1+x)^2}\right)}{4\,2^{2/3}} + \frac{\log(1-x)}{2\,2^{2/3}}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.29

$$\frac{3}{16}\left((x-1)^2-4\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{1}{4}\left(4-(x-1)^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)*(-3 - 2*x + x^2)^(1/3)),x]
```

```
[Out] (3*(-4 + (-1 + x)^2)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (4 - (-1 + x)^2)/
4])/16
```

IntegrateAlgebraic [A] time = 0.22, size = 125, normalized size = 1.00

$$-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^2-2x-3}+2\right)}{2\,2^{2/3}} + \frac{\log\left(2^{2/3}\left(x^2-2x-3\right)^{2/3}-2\sqrt[3]{2}\sqrt[3]{x^2-2x-3}+4\right)}{4\,2^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2}\sqrt[3]{x^2-2x-3}}{\sqrt{3}}\right)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x)*(-3 - 2*x + x^2)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(1/3)*(-3 - 2*x + x^2)^(1/3))/Sqrt[3]])
/2^(2/3) - Log[2 + 2^(1/3)*(-3 - 2*x + x^2)^(1/3)]/(2*2^(2/3)) + Log[4 - 2*
2^(1/3)*(-3 - 2*x + x^2)^(1/3) + 2^(2/3)*(-3 - 2*x + x^2)^(2/3)]/(4*2^(2/3)
)
```

fricas [A] time = 0.40, size = 113, normalized size = 0.90

$$\frac{1}{4}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(2\,(-1)^{\frac{1}{3}}\left(x^2-2x-3\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\right)\right) - \frac{1}{16}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^2-2x-3\right)^{\frac{1}{3}}-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}+\left(x^2-2x-3\right)^{\frac{2}{3}}\right) + \frac{1}{8}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(4^{\frac{1}{3}}(-1)^{\frac{2}{3}}+\left(x^2-2x-3\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^2-2*x-3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*sqrt(3)*(2*(-1)^(1/3)*(x^
2 - 2*x - 3)^(1/3) + 4^(1/3))) - 1/16*4^(2/3)*(-1)^(1/3)*log(-4^(1/3)*(-1)^
(2/3)*(x^2 - 2*x - 3)^(1/3) - 4^(2/3)*(-1)^(1/3) + (x^2 - 2*x - 3)^(2/3)) +
 1/8*4^(2/3)*(-1)^(1/3)*log(4^(1/3)*(-1)^(2/3) + (x^2 - 2*x - 3)^(1/3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-2\,x-3\right)^{\frac{1}{3}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^2-2*x-3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - 2*x - 3)^(1/3)*(x - 1)), x)
```

maple [C] time = 8.00, size = 1199, normalized size = 9.59

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)/(x^2-2*x-3)^(1/3),x)
```

```
[Out] -1/4*ln(-(44*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^
3+2)^3*x^2+80*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(
_Z^3+2)^2*x^2-88*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf
(_Z^3+2)^3*x-160*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*Root
Of(_Z^3+2)^2*x+240*(x^2-2*x-3)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)
^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)+198*(x^2-2*x-3)^(1/3)*RootOf(_Z^3+2)^2-84*(x
^2-2*x-3)^(1/3)*RootOf(_Z^3+2)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+
4*_Z^2)+33*RootOf(_Z^3+2)*x^2+60*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2
)+4*_Z^2)*x^2-66*RootOf(_Z^3+2)*x-120*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_
Z^3+2)+4*_Z^2)*x-42*(x^2-2*x-3)^(2/3)-407*RootOf(_Z^3+2)-740*RootOf(RootOf(
_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2))/(-1+x)^2)*RootOf(_Z^3+2)-1/2*ln(-(44
*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^2+8
0*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^
2-88*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x
-160*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2
*x+240*(x^2-2*x-3)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+2*_Z*Root
Of(_Z^3+2)+4*_Z^2)+198*(x^2-2*x-3)^(1/3)*RootOf(_Z^3+2)^2-84*(x^2-2*x-3)^(1
/3)*RootOf(_Z^3+2)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)+33*R
ootOf(_Z^3+2)*x^2+60*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^
2-66*RootOf(_Z^3+2)*x-120*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^
2)*x-42*(x^2-2*x-3)^(2/3)-407*RootOf(_Z^3+2)-740*RootOf(RootOf(_Z^3+2)^2+2*
_Z*RootOf(_Z^3+2)+4*_Z^2))/(-1+x)^2)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z
^3+2)+4*_Z^2)+1/4*RootOf(_Z^3+2)*ln(-(44*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootO
f(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^2+8*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootO
f(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^2-88*RootOf(RootOf(_Z^3+2)^2+2*_Z*Ro
otOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x-16*RootOf(RootOf(_Z^3+2)^2+2*_Z*Roo
tOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x-240*(x^2-2*x-3)^(2/3)*RootOf(_Z^3+
2)^2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)-42*(x^2-2*x-3)^(1/
3)*RootOf(_Z^3+2)^2+396*(x^2-2*x-3)^(1/3)*RootOf(_Z^3+2)*RootOf(RootOf(_Z^3
+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)-77*RootOf(_Z^3+2)*x^2-14*RootOf(RootOf(_Z
^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^2+154*RootOf(_Z^3+2)*x+28*RootOf(Root
Of(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x+198*(x^2-2*x-3)^(2/3)+407*RootOf
(_Z^3+2)+74*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2))/(-1+x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-2\,x-3\right)^{\frac{1}{3}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^2-2*x-3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - 2*x - 3)^(1/3)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x-1)\left(x^2-2x-3\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)*(x^2 - 2*x - 3)^(1/3)),x)
```

```
[Out] int(1/((x - 1)*(x^2 - 2*x - 3)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(x-3)(x+1)}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x**2-2*x-3)**(1/3),x)
```

```
[Out] Integral(1/(((x - 3)*(x + 1))**(1/3)*(x - 1)), x)
```

3.1537 $\int \frac{1}{(-1+x)\sqrt[3]{-1-2x+x^2}}\,dx$

Optimal. Leaf size=125

$$-\frac{\log\left(2^{2/3}\sqrt[3]{x^2-2x-1}+2\right)}{2\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}\left(x^2-2x-1\right)^{2/3}+2^{2/3}\sqrt[3]{x^2-2x-1}-2\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\sqrt[3]{x^2-2}}{\sqrt{3}}\right.}{2\sqrt[3]{2}}$$

Rubi [A] time = 0.07, antiderivative size = 84, normalized size of antiderivative = 0.67, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {694, 266, 56, 617, 204, 31}

$$-\frac{3\log\left(\sqrt[3]{(x-1)^2-2}+\sqrt[3]{2}\right)}{4\sqrt[3]{2}}+\frac{\log(1-x)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2^{2/3}\sqrt[3]{(x-1)^2-2}}{\sqrt{3}}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)*(-1 - 2*x + x^2)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 - 2^(2/3)*(-2 + (-1 + x)^2)^(1/3))/Sqrt[3]])/2^(1/3
) - (3*Log[2^(1/3) + (-2 + (-1 + x)^2)^(1/3)])/(4*2^(1/3)) + Log[1 - x]/(2*
2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
```

```
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(-1+x)\sqrt[3]{-1-2x+x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{x\sqrt[3]{-2+x^2}}\,dx, x, -1+x\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{-2+x}\,x}\,dx, x, (-1+x)^2\right)\\
&= \frac{\log(1-x)}{2\sqrt[3]{2}} + \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{-2+(-1+x)^2}\right) - \frac{3\,\text{Subst}\left(\right.}{}\\
&= -\frac{3\log\left(\sqrt[3]{2}+\sqrt[3]{-2+(-1+x)^2}\right)}{4\sqrt[3]{2}} + \frac{\log(1-x)}{2\sqrt[3]{2}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-2^{2/3}\right.}{2\sqrt[3]{2}}\\
&= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2^{2/3}\sqrt[3]{-2+(-1+x)^2}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{3\log\left(\sqrt[3]{2}+\sqrt[3]{-2+(-1+x)^2}\right)}{4\sqrt[3]{2}} + \frac{\log(1-x)}{2\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.29

$$\frac{3}{8}\left((x-1)^2-2\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{1}{2}\left(2-(x-1)^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)*(-1 - 2*x + x^2)^(1/3)),x]
```

```
[Out] (3*(-2 + (-1 + x)^2)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (2 - (-1 + x)^2)/
2])/8
```

IntegrateAlgebraic [A] time = 0.23, size = 125, normalized size = 1.00

$$-\frac{\log\left(2^{2/3}\sqrt[3]{x^2-2x-1}+2\right)}{2\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}\left(x^2-2x-1\right)^{2/3}+2^{2/3}\sqrt[3]{x^2-2x-1}-2\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\sqrt[3]{x^2-2x-1}}{\sqrt{3}}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x)*(-1 - 2*x + x^2)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(-1 - 2*x + x^2)^(1/3))/Sqrt[3]])
/2^(1/3) - Log[2 + 2^(2/3)*(-1 - 2*x + x^2)^(1/3)]/(2*2^(1/3)) + Log[-2 + 2
^(2/3)*(-1 - 2*x + x^2)^(1/3) - 2^(1/3)*(-1 - 2*x + x^2)^(2/3)]/(4*2^(1/3))
```

fricas [A] time = 0.40, size = 116, normalized size = 0.93

$$\frac{1}{4}\sqrt{3}\,2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{1}{6}}\left(2\sqrt{2}(-1)^{\frac{1}{3}}\left(x^2-2x-1\right)^{\frac{1}{3}}+2^{\frac{5}{6}}\right)\right)-\frac{1}{8}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^2-2x-1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}+\left(x^2-2x-1\right)^{\frac{2}{3}}\right)+\frac{1}{4}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}+\left(x^2-2x-1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^2-2*x-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2*sqrt(2)*(-1)^(
1/3)*(x^2 - 2*x - 1)^(1/3) + 2^(5/6))) - 1/8*2^(2/3)*(-1)^(1/3)*log(-2^(1/3
)*(-1)^(2/3)*(x^2 - 2*x - 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (x^2 - 2*x - 1)^(
```

```
2/3)) + 1/4*2^(2/3)*(-1)^(1/3)*log(2^(1/3)*(-1)^(2/3) + (x^2 - 2*x - 1)^(1/
3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-2\,x-1\right)^{\frac{1}{3}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^2-2*x-1)^(1/3),x, algorithm="giac")
[Out] integrate(1/((x^2 - 2*x - 1)^(1/3)*(x - 1)), x)
```

maple [C] time = 7.95, size = 803, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)/(x^2-2*x-1)^(1/3),x)
[Out] 1/4*RootOf(_Z^3+4)*ln((2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2
)*RootOf(_Z^3+4)^3*x^2-RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^
2*RootOf(_Z^3+4)^2*x^2-4*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2
)*RootOf(_Z^3+4)^3*x+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^
2*RootOf(_Z^3+4)^2*x-12*(x^2-2*x-1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*Root
Of(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2+12*(x^2-2*x-1)^(1/3)*RootOf(_Z^3+4)^2+1
5*(x^2-2*x-1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*Roo
tOf(_Z^3+4)+2*RootOf(_Z^3+4)*x^2-RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4
)+4*_Z^2)*x^2-4*RootOf(_Z^3+4)*x+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3
+4)+4*_Z^2)*x+9*(x^2-2*x-1)^(2/3)-14*RootOf(_Z^3+4)+7*RootOf(RootOf(_Z^3+4)
^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(-1+x)^2)+1/2*RootOf(RootOf(_Z^3+4)^2+2*_Z*
RootOf(_Z^3+4)+4*_Z^2)*ln((RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z
^2)*RootOf(_Z^3+4)^3*x^2-8*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z
^2)^2*RootOf(_Z^3+4)^2*x^2-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*
_Z^2)*RootOf(_Z^3+4)^3*x+16*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_
Z^2)^2*RootOf(_Z^3+4)^2*x+24*(x^2-2*x-1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z
*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2-24*(x^2-2*x-1)^(1/3)*RootOf(_Z^3+4
)^2-18*(x^2-2*x-1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2
)*RootOf(_Z^3+4)-3*RootOf(_Z^3+4)*x^2+24*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootO
f(_Z^3+4)+4*_Z^2)*x^2+6*RootOf(_Z^3+4)*x-48*RootOf(RootOf(_Z^3+4)^2+2*_Z*Ro
otOf(_Z^3+4)+4*_Z^2)*x-30*(x^2-2*x-1)^(2/3)+7*RootOf(_Z^3+4)-56*RootOf(Root
Of(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(-1+x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-2\,x-1\right)^{\frac{1}{3}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^2-2*x-1)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^2 - 2*x - 1)^(1/3)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x-1)\,\left(x^2-2\,x-1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)*(x^2 - 2*x - 1)^(1/3)),x)
```

```
[Out] int(1/((x - 1)*(x^2 - 2*x - 1)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x-1)\sqrt[3]{x^2-2x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x**2-2*x-1)**(1/3),x)
```

```
[Out] Integral(1/((x - 1)*(x**2 - 2*x - 1)**(1/3)), x)
```

3.1538 $\int \frac{1}{(1+x)\sqrt[3]{-1+2x+x^2}}\,dx$

Optimal. Leaf size=125

$$-\frac{\log\left(2^{2/3}\sqrt[3]{x^2+2x-1}+2\right)}{2\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}\left(x^2+2x-1\right)^{2/3}+2^{2/3}\sqrt[3]{x^2+2x-1}-2\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\sqrt[3]{x^2+2}}{\sqrt{3}}\right.}{2\sqrt[3]{2}}$$

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 0.66, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {694, 266, 56, 617, 204, 31}

$$\frac{\log(x+1)}{2\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{(x+1)^2-2}+\sqrt[3]{2}\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2^{2/3}\sqrt[3]{(x+1)^2-2}}{\sqrt{3}}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(-1 + 2*x + x^2)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 - 2^(2/3)*(-2 + (1 + x)^2)^(1/3))/Sqrt[3]])/2^(1/3)
 + Log[1 + x]/(2*2^(1/3)) - (3*Log[2^(1/3) + (-2 + (1 + x)^2)^(1/3)])/(4*2^
(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
```

```
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(1+x)\sqrt[3]{-1+2x+x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{x\sqrt[3]{-2+x^2}}\,dx, x, 1+x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{-2+x}\,x}\,dx, x, (1+x)^2\right)\\
&= \frac{\log(1+x)}{2\sqrt[3]{2}} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{-2+(1+x)^2}\right) - \frac{3\,\text{Subst}\left(\int \right.}{}\\
&= \frac{\log(1+x)}{2\sqrt[3]{2}} - \frac{3\log\left(\sqrt[3]{2}+\sqrt[3]{-2+(1+x)^2}\right)}{4\sqrt[3]{2}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-2^{2/3}\sqrt[3]{-2}\right.}{2\sqrt[3]{2}}\\
&= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2^{2/3}\sqrt[3]{-2+(1+x)^2}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} + \frac{\log(1+x)}{2\sqrt[3]{2}} - \frac{3\log\left(\sqrt[3]{2}+\sqrt[3]{-2+(1+x)^2}\right)}{4\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.29

$$\frac{3}{8}\left((x+1)^2-2\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{1}{2}\left(2-(x+1)^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(-1 + 2*x + x^2)^(1/3)),x]
```

```
[Out] (3*(-2 + (1 + x)^2)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (2 - (1 + x)^2)/2]
)/8
```

IntegrateAlgebraic [A] time = 0.22, size = 125, normalized size = 1.00

$$-\frac{\log\left(2^{2/3}\sqrt[3]{x^2+2x-1}+2\right)}{2\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}\left(x^2+2x-1\right)^{2/3}+2^{2/3}\sqrt[3]{x^2+2x-1}-2\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\sqrt[3]{x^2+2x-1}}{\sqrt{3}}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x)*(-1 + 2*x + x^2)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(-1 + 2*x + x^2)^(1/3))/Sqrt[3]])
/2^(1/3) - Log[2 + 2^(2/3)*(-1 + 2*x + x^2)^(1/3)]/(2*2^(1/3)) + Log[-2 + 2
^(2/3)*(-1 + 2*x + x^2)^(1/3) - 2^(1/3)*(-1 + 2*x + x^2)^(2/3)]/(4*2^(1/3))
```

fricas [A] time = 0.40, size = 116, normalized size = 0.93

$$\frac{1}{4}\sqrt{3}\,2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{1}{6}}\left(2\sqrt{2}(-1)^{\frac{1}{3}}\left(x^2+2x-1\right)^{\frac{1}{3}}+2^{\frac{5}{6}}\right)\right)-\frac{1}{8}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^2+2x-1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}+\left(x^2+2x-1\right)^{\frac{2}{3}}\right)+\frac{1}{4}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}+\left(x^2+2x-1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2*sqrt(2)*(-1)^(
1/3)*(x^2 + 2*x - 1)^(1/3) + 2^(5/6))) - 1/8*2^(2/3)*(-1)^(1/3)*log(-2^(1/3
)*(-1)^(2/3)*(x^2 + 2*x - 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (x^2 + 2*x - 1)^(
```

```
2/3)) + 1/4*2^(2/3)*(-1)^(1/3)*log(2^(1/3)*(-1)^(2/3) + (x^2 + 2*x - 1)^(1/
3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+2x-1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 2*x - 1)^(1/3)*(x + 1)), x)
```

maple [C] time = 7.33, size = 802, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^2+2*x-1)^(1/3),x)
```

```
[Out] 1/4*RootOf(_Z^3+4)*ln(-(-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z
^2)*RootOf(_Z^3+4)^3*x^2+RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2
)^2*RootOf(_Z^3+4)^2*x^2+12*(x^2+2*x-1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*
RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2-4*RootOf(RootOf(_Z^3+4)^2+2*_Z*Root
Of(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf
(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x-12*(x^2+2*x-1)^(1/3)*RootOf(_Z^3+4)^2
-15*(x^2+2*x-1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*R
ootOf(_Z^3+4)-2*RootOf(_Z^3+4)*x^2+RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3
+4)+4*_Z^2)*x^2-9*(x^2+2*x-1)^(2/3)-4*RootOf(_Z^3+4)*x+2*RootOf(RootOf(_Z^3
+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+14*RootOf(_Z^3+4)-7*RootOf(RootOf(_Z^3+
4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(1+x)^2)+1/2*RootOf(RootOf(_Z^3+4)^2+2*_Z
*RootOf(_Z^3+4)+4*_Z^2)*ln((RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_
Z^2)*RootOf(_Z^3+4)^3*x^2-8*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_
Z^2)^2*RootOf(_Z^3+4)^2*x^2+24*(x^2+2*x-1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+2*
_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*R
ootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x-16*RootOf(RootOf(_Z^3+4)^2+2*_Z*Ro
otOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x-24*(x^2+2*x-1)^(1/3)*RootOf(_Z^3+
4)^2-18*(x^2+2*x-1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^
2)*RootOf(_Z^3+4)-3*RootOf(_Z^3+4)*x^2+24*RootOf(RootOf(_Z^3+4)^2+2*_Z*Root
Of(_Z^3+4)+4*_Z^2)*x^2-30*(x^2+2*x-1)^(2/3)-6*RootOf(_Z^3+4)*x+48*RootOf(Ro
otOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+7*RootOf(_Z^3+4)-56*RootOf(Roo
tOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(1+x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+2x-1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x-1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 2*x - 1)^(1/3)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x+1)\,\left(x^2+2\,x-1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)*(2*x + x^2 - 1)^(1/3)),x)
```

```
[Out] int(1/((x + 1)*(2*x + x^2 - 1)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)\sqrt[3]{x^2+2x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+2*x-1)**(1/3),x)
```

```
[Out] Integral(1/((x + 1)*(x**2 + 2*x - 1)**(1/3)), x)
```

3.1539 $\int \frac{-2abx^2+(a+b)x^3}{(-a+x)(-b+x)\sqrt[4]{x^2(-a+x)(-b+x)}\left(-abd+(a+b)dx+(1-d)x^2\right)}\,dx$

Optimal. Leaf size=125

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt[4]{x^3(-a-b)+abx^2+x^4}}\right)+2\sqrt[4]{d}\tanh^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt[4]{x^3(-a-b)+abx^2+x^4}}\right)-\frac{4\left(x^3(-a-b)+abx\right.}{x(x-a)(x-}$$

Rubi [F] time = 19.79, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2abx^2+(a+b)x^3}{(-a+x)(-b+x)\sqrt[4]{x^2(-a+x)(-b+x)}\left(-abd+(a+b)dx+(1-d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*a*b*x^2 + (a + b)*x^3)/((-a + x)*(-b + x)*(x^2*(-a + x)*(-b + x))^(
1/4)*(-(a*b*d) + (a + b)*d*x + (1 - d)*x^2)),x]
```

```
[Out] ((a + b - Sqrt[2*a*b*(2 - d) + a^2*d + b^2*d]/Sqrt[d])*Sqrt[x]*(-a + x)^(1/
4)*(-b + x)^(1/4)*Defer[Int][x^(3/2)/((-a + x)^(5/4)*(-b + x)^(5/4)*((a + b
)*d - Sqrt[d]*Sqrt[4*a*b + a^2*d - 2*a*b*d + b^2*d] + 2*(1 - d)*x)), x])/((
a - x)*(b - x)*x^2)^(1/4) + ((a + b + Sqrt[2*a*b*(2 - d) + a^2*d + b^2*d]/S
qrt[d])*Sqrt[x]*(-a + x)^(1/4)*(-b + x)^(1/4)*Defer[Int][x^(3/2)/((-a + x)^
(5/4)*(-b + x)^(5/4)*((a + b)*d + Sqrt[d]*Sqrt[4*a*b + a^2*d - 2*a*b*d + b^
2*d] + 2*(1 - d)*x)), x])/((a - x)*(b - x)*x^2)^(1/4)
```

Rubi steps

$$\int \frac{-2abx^2+(a+b)x^3}{(-a+x)(-b+x)\sqrt[4]{x^2(-a+x)(-b+x)}\left(-abd+(a+b)dx+(1-d)x^2\right)}\,dx = \int \frac{x^2(}{(-a+x)(-b+x)\sqrt[4]{x^2(-a+}}$$

$$= \frac{\left(\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\int \frac{}{(-a+}}{\sqrt[4]{x^2(-a+}}$$

$$= \frac{\left(\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\int\left(\frac{}{(-a+}\right.}{}$$

$$= \frac{\left(\left(a+b-\frac{\sqrt{2ab(2-d)+a^2d+b^2d}}{\sqrt{d}}\right)\sqrt{x}\right.}{}$$

Mathematica [F] time = 15.03, size = 0, normalized size = 0.00

$$\int \frac{-2abx^2+(a+b)x^3}{(-a+x)(-b+x)\sqrt[4]{x^2(-a+x)(-b+x)}\left(-abd+(a+b)dx+(1-d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*a*b*x^2 + (a + b)*x^3)/((-a + x)*(-b + x)*(x^2*(-a + x)*(-b +
 x))^(1/4)*(-(a*b*d) + (a + b)*d*x + (1 - d)*x^2)),x]
```

```
[Out] Integrate[(-2*a*b*x^2 + (a + b)*x^3)/((-a + x)*(-b + x)*(x^2*(-a + x)*(-b +
 x))^(1/4)*(-(a*b*d) + (a + b)*d*x + (1 - d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.73, size = 125, normalized size = 1.00

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt[4]{x^3(-a-b)+abx^2+x^4}}\right)+2\sqrt[4]{d}\tanh^{-1}\left(\frac{x}{\sqrt[4]{d}\sqrt[4]{x^3(-a-b)+abx^2+x^4}}\right)-\frac{4\left(x^3(-a-b)+abx^2+x^4\right)^{3/4}}{x(x-a)(x-b)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*a*b*x^2 + (a + b)*x^3)/((-a + x)*(-b + x)*(x^2*(-a +
 x)*(-b + x))^(1/4)*(-(a*b*d) + (a + b)*d*x + (1 - d)*x^2)),x]
```

```
[Out] (-4*(a*b*x^2 + (-a - b)*x^3 + x^4)^(3/4))/(x*(-a + x)*(-b + x)) + 2*d^(1/4)
*ArcTan[x/(d^(1/4)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/4))] + 2*d^(1/4)*ArcTa
nh[x/(d^(1/4)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/4))]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b*x^2+(a+b)*x^3)/(-a+x)/(-b+x)/(x^2*(-a+x)*(-b+x))^(1/4)/(-
a*b*d+(a+b)*d*x+(1-d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,abx^2-(a+b)x^3}{\left((a-x)(b-x)x^2\right)^{\frac{1}{4}}\left(abd-(a+b)dx+(d-1)x^2\right)(a-x)(b-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b*x^2+(a+b)*x^3)/(-a+x)/(-b+x)/(x^2*(-a+x)*(-b+x))^(1/4)/(-
a*b*d+(a+b)*d*x+(1-d)*x^2),x, algorithm="giac")
```

```
[Out] integrate((2*a*b*x^2 - (a + b)*x^3)/(((a - x)*(b - x)*x^2)^(1/4)*(a*b*d - (
a + b)*d*x + (d - 1)*x^2)*(a - x)*(b - x)), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{-2ab\,x^2+(a+b)\,x^3}{(-a+x)\,(-b+x)\left(x^2\,(-a+x)\,(-b+x)\right)^{\frac{1}{4}}\left(-abd+(a+b)\,dx+(1-d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a*b*x^2+(a+b)*x^3)/(-a+x)/(-b+x)/(x^2*(-a+x)*(-b+x))^(1/4)/(-a*b*d+
(a+b)*d*x+(1-d)*x^2),x)
```

```
[Out] int((-2*a*b*x^2+(a+b)*x^3)/(-a+x)/(-b+x)/(x^2*(-a+x)*(-b+x))^(1/4)/(-a*b*d+
(a+b)*d*x+(1-d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,abx^2-(a+b)x^3}{\left((a-x)(b-x)x^2\right)^{\frac{1}{4}}\left(abd-(a+b)dx+(d-1)x^2\right)(a-x)(b-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b*x^2+(a+b)*x^3)/(-a+x)/(-b+x)/(x^2*(-a+x)*(-b+x))^(1/4)/(-
a*b*d+(a+b)*d*x+(1-d)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((2*a*b*x^2 - (a + b)*x^3)/(((a - x)*(b - x)*x^2)^(1/4)*(a*b*d - (
a + b)*d*x + (d - 1)*x^2)*(a - x)*(b - x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^3\,\left(a+b\right)-2\,a\,b\,x^2}{\left(a-x\right)\,\left(b-x\right)\,\left(\left(d-1\right)\,x^2-d\,\left(a+b\right)\,x+a\,b\,d\right)\,{\left(x^2\,\left(a-x\right)\,\left(b-x\right)\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(a + b) - 2*a*b*x^2)/((a - x)*(b - x)*(x^2*(d - 1) - d*x*(a + b)
+ a*b*d)*(x^2*(a - x)*(b - x))^(1/4)),x)
```

```
[Out] int(-(x^3*(a + b) - 2*a*b*x^2)/((a - x)*(b - x)*(x^2*(d - 1) - d*x*(a + b)
+ a*b*d)*(x^2*(a - x)*(b - x))^(1/4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b*x**2+(a+b)*x**3)/(-a+x)/(-b+x)/(x**2*(-a+x)*(-b+x))**(1/4
)/(-a*b*d+(a+b)*d*x+(1-d)*x**2),x)
```

```
[Out] Timed out
```

3.1540 $\int \frac{(a^2-2ax+x^2)(ab^2-2(2a-b)bx+(3a-2b)x^2)}{(x(-a+x)(-b+x)^2)^{3/4}(a^3d+(b^2-3a^2d)x+(-2b+3ad)x^2+(1-d)x^3)}\,dx$

Optimal. Leaf size=125

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4\right)^{3/4}}{x(x-b)^2}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4\right)^{3/4}}{x(x-b)^2}\right)}{d^{3/4}}$$

Rubi [F] time = 8.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a^2-2ax+x^2)(ab^2-2(2a-b)bx+(3a-2b)x^2)}{(x(-a+x)(-b+x)^2)^{3/4}(a^3d+(b^2-3a^2d)x+(-2b+3ad)x^2+(1-d)x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a^2 - 2*a*x + x^2)*(a*b^2 - 2*(2*a - b)*b*x + (3*a - 2*b)*x^2))/((x*(
-a + x)*(-b + x)^2)^(3/4)*(a^3*d + (b^2 - 3*a^2*d)*x + (-2*b + 3*a*d)*x^2 +
 (1 - d)*x^3)),x]
```

```
[Out] (4*a*b*x^(3/4)*(-a + x)^(3/4)*(-b + x)^(3/2)*Defer[Subst][Defer[Int][(-a +
x^4)^(5/4)/(Sqrt[-b + x^4]*(-(a^3*d) - b^2*(1 - (3*a^2*d)/b^2)*x^4 + 2*b*(1
 - (3*a*d)/(2*b))*x^8 - (1 - d)*x^12)), x], x, x^(1/4)])/(-((a - x)*(b - x)
^2*x))^(3/4) + (4*(3*a - 2*b)*x^(3/4)*(-a + x)^(3/4)*(-b + x)^(3/2)*Defer[S
ubst][Defer[Int][(x^4*(-a + x^4)^(5/4))/(Sqrt[-b + x^4]*(a^3*d + b^2*(1 - (
3*a^2*d)/b^2)*x^4 - 2*b*(1 - (3*a*d)/(2*b))*x^8 + (1 - d)*x^12)), x], x, x^
(1/4)])/(-((a - x)*(b - x)^2*x))^(3/4)
```

Rubi steps

$$\int \frac{(a^2-2ax+x^2)(ab^2-2(2a-b)bx+(3a-2b)x^2)}{(x(-a+x)(-b+x)^2)^{3/4}(a^3d+(b^2-3a^2d)x+(-2b+3ad)x^2+(1-d)x^3)}\,dx = \int \frac{(-a+x)^2 (\cdots}{(x(-a+x)(-b+x)^2)^{3/4}(a\cdots}$$

$$= \frac{(x^{3/4}(-a+x)^{3/4}(-b+x)^{3/2})\int \cdots}{(x(\cdots}$$

$$= \frac{(x^{3/4}(-a+x)^{3/4}(-b+x)^{3/2})\int \cdots}{(x(-\cdots}$$

$$= \frac{(4x^{3/4}(-a+x)^{3/4}(-b+x)^{3/2})\cdots}{\cdots}$$

$$= \frac{(4x^{3/4}(-a+x)^{3/4}(-b+x)^{3/2})\cdots}{\cdots}$$

$$= \frac{(4(3a-2b)x^{3/4}(-a+x)^{3/4}(-b\cdots}{\cdots}$$

Mathematica [F] time = 5.48, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2-2ax+x^2\right)\left(ab^2-2(2a-b)bx+(3a-2b)x^2\right)}{\left(x(-a+x)(-b+x)^2\right)^{3/4}\left(a^3d+\left(b^2-3a^2d\right)x+(-2b+3ad)x^2+(1-d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a^2 - 2*a*x + x^2)*(a*b^2 - 2*(2*a - b)*b*x + (3*a - 2*b)*x^2))
/((x*(-a + x)*(-b + x)^2)^(3/4)*(a^3*d + (b^2 - 3*a^2*d)*x + (-2*b + 3*a*d)
*x^2 + (1 - d)*x^3)),x]
```

```
[Out] Integrate[((a^2 - 2*a*x + x^2)*(a*b^2 - 2*(2*a - b)*b*x + (3*a - 2*b)*x^2))
/((x*(-a + x)*(-b + x)^2)^(3/4)*(a^3*d + (b^2 - 3*a^2*d)*x + (-2*b + 3*a*d)
*x^2 + (1 - d)*x^3)), x]
```

IntegrateAlgebraic [A] time = 3.72, size = 125, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{3/4}}{x(x-b)^2}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{3/4}}{x(x-b)^2}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a^2 - 2*a*x + x^2)*(a*b^2 - 2*(2*a - b)*b*x + (3*a - 2
*b)*x^2))/((x*(-a + x)*(-b + x)^2)^(3/4)*(a^3*d + (b^2 - 3*a^2*d)*x + (-2*b
 + 3*a*d)*x^2 + (1 - d)*x^3)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)
^(3/4))/(x*(-b + x)^2)])/d^(3/4) + (2*ArcTanh[(d^(1/4)*(-(a*b^2*x) + (2*a*b
 + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(3/4))/(x*(-b + x)^2)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-2*a*x+x^2)*(a*b^2-2*(2*a-b)*b*x+(3*a-2*b)*x^2)/(x*(-a+x)*(-b
+x)^2)^(3/4)/(a^3*d+(-3*a^2*d+b^2)*x+(3*a*d-2*b)*x^2+(1-d)*x^3),x, algorith
m="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ab^2-2\,(2\,a-b)bx+(3\,a-2\,b)x^2\right)\left(a^2-2\,ax+x^2\right)}{\left(a^3d-(d-1)x^3+(3\,ad-2\,b)x^2-\left(3\,a^2d-b^2\right)x\right)\left(-(a-x)(b-x)^2x\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-2*a*x+x^2)*(a*b^2-2*(2*a-b)*b*x+(3*a-2*b)*x^2)/(x*(-a+x)*(-b
+x)^2)^(3/4)/(a^3*d+(-3*a^2*d+b^2)*x+(3*a*d-2*b)*x^2+(1-d)*x^3),x, algorith
m="giac")
```

```
[Out] integrate((a*b^2 - 2*(2*a - b)*b*x + (3*a - 2*b)*x^2)*(a^2 - 2*a*x + x^2)/(
(a^3*d - (d - 1)*x^3 + (3*a*d - 2*b)*x^2 - (3*a^2*d - b^2)*x)*(-(a - x)*(b
- x)^2*x)^(3/4)), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2-2ax+x^2\right)\left(a\,b^2-2\,(2a-b)\,bx+(3a-2b)\,x^2\right)}{\left(x\,(-a+x)\,(-b+x)^2\right)^{\frac{3}{4}}\left(a^3d+\left(-3a^2d+b^2\right)x+(3ad-2b)\,x^2+(1-d)\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-2*a*x+x^2)*(a*b^2-2*(2*a-b)*b*x+(3*a-2*b)*x^2)/(x*(-a+x)*(-b+x)^2)
^(3/4)/(a^3*d+(-3*a^2*d+b^2)*x+(3*a*d-2*b)*x^2+(1-d)*x^3),x)
```

```
[Out] int((a^2-2*a*x+x^2)*(a*b^2-2*(2*a-b)*b*x+(3*a-2*b)*x^2)/(x*(-a+x)*(-b+x)^2)
^(3/4)/(a^3*d+(-3*a^2*d+b^2)*x+(3*a*d-2*b)*x^2+(1-d)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ab^2 - 2\,(2\,a - b)bx + (3\,a - 2\,b)x^2\right)\left(a^2 - 2\,ax + x^2\right)}{\left(a^3d - (d-1)x^3 + (3\,ad - 2\,b)x^2 - \left(3\,a^2d - b^2\right)x\right)\left(-(a-x)(b-x)^2x\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-2*a*x+x^2)*(a*b^2-2*(2*a-b)*b*x+(3*a-2*b)*x^2)/(x*(-a+x)*(-b
+x)^2)^(3/4)/(a^3*d+(-3*a^2*d+b^2)*x+(3*a*d-2*b)*x^2+(1-d)*x^3),x, algorith
m="maxima")
```

```
[Out] integrate((a*b^2 - 2*(2*a - b)*b*x + (3*a - 2*b)*x^2)*(a^2 - 2*a*x + x^2)/(
(a^3*d - (d - 1)*x^3 + (3*a*d - 2*b)*x^2 - (3*a^2*d - b^2)*x)*(-(a - x)*(b
- x)^2*x)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(x^2\,\left(3\,a-2\,b\right)+a\,b^2-2\,b\,x\,\left(2\,a-b\right)\right)\,\left(a^2-2\,a\,x+x^2\right)}{{\left(-x\,\left(a-x\right)\,{\left(b-x\right)}^2\right)}^{3/4}\,\left(x^2\,\left(2\,b-3\,a\,d\right)-a^3\,d+x\,\left(3\,a^2\,d-b^2\right)+x^3\,\left(d-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^2*(3*a - 2*b) + a*b^2 - 2*b*x*(2*a - b))*(a^2 - 2*a*x + x^2))/((-x
*(a - x)*(b - x)^2)^(3/4)*(x^2*(2*b - 3*a*d) - a^3*d + x*(3*a^2*d - b^2) +
x^3*(d - 1))),x)
```

```
[Out] -int(((x^2*(3*a - 2*b) + a*b^2 - 2*b*x*(2*a - b))*(a^2 - 2*a*x + x^2))/((-x
*(a - x)*(b - x)^2)^(3/4)*(x^2*(2*b - 3*a*d) - a^3*d + x*(3*a^2*d - b^2) +
x^3*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-2*a*x+x**2)*(a*b**2-2*(2*a-b)*b*x+(3*a-2*b)*x**2)/(x*(-a+x)
*(-b+x)**2)**(3/4)/(a**3*d+(-3*a**2*d+b**2)*x+(3*a*d-2*b)*x**2+(1-d)*x**3),
x)
```

```
[Out] Timed out
```

3.1541 $\int \frac{-1+k^{3/2}x^3}{\sqrt{(1-x^2)(1-k^2x^2)}\,(1+k^{3/2}x^3)}\,dx$

Optimal. Leaf size=125

$$-\frac{4\tan^{-1}\left(\frac{\sqrt{k^2+k+1}\,x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}+kx^2-\sqrt{k}\,x+1}\right)}{3\sqrt{k^2+k+1}}-\frac{2\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}+kx^2+2\sqrt{k}\,x+1}\right)}{3(k-1)}$$

Rubi [C] time = 2.94, antiderivative size = 610, normalized size of antiderivative = 4.88, number of steps used = 21, number of rules used = 9, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6719, 6725, 419, 2113, 537, 571, 93, 205, 208}

$$-\frac{2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\tan^{-1}\left(\frac{\sqrt{k}\,\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{3(1-k)\sqrt{(1-x^2)(1-k^2x^2)}}-\frac{2(-1)^{2/3}\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{k}\,\sqrt{k+\sqrt[3]{-1}}\,\sqrt{1-x^2}}{\sqrt{\sqrt[3]{-1}k+1}\,\sqrt{1-k^2x^2}}\right)}{3\sqrt{k+\sqrt[3]{-1}}\,\sqrt{\sqrt[3]{-1}k+1}\,\sqrt{(1-x^2)(1-k^2x^2)}}+\frac{2\sqrt[3]{-1}\,\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{k}\,\sqrt{k-(-1)^{2/3}}\,\sqrt{1-x^2}}{\sqrt{1-(-1)^{2/3}k}\,\sqrt{1-k^2x^2}}\right)}{3\sqrt{k-(-1)^{2/3}}\,\sqrt{1-(-1)^{2/3}k}\,\sqrt{(1-x^2)(1-k^2x^2)}}+\frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}-\frac{2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(k;\sin^{-1}(x)|k^2\right)}{3\sqrt{(1-x^2)(1-k^2x^2)}}-\frac{2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(-\sqrt[3]{-1}k;\sin^{-1}(x)|k^2\right)}{3\sqrt{(1-x^2)(1-k^2x^2)}}-\frac{2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left((-1)^{2/3}k;\sin^{-1}(x)|k^2\right)}{3\sqrt{(1-x^2)(1-k^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + k^(3/2)*x^3)/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(1 + k^(3/2)*x^3)),x]
```

```
[Out] (-2*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*ArcTan[(Sqrt[k]*Sqrt[1 - x^2])/Sqrt[1 -
 k^2*x^2]])/(3*(1 - k)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) - (2*(-1)^(2/3)*Sqrt[
1 - x^2]*Sqrt[1 - k^2*x^2]*ArcTanh[(Sqrt[k]*Sqrt[(-1)^(1/3) + k]*Sqrt[1 - x
^2])/(Sqrt[1 + (-1)^(1/3)*k]*Sqrt[1 - k^2*x^2])]])/(3*Sqrt[(-1)^(1/3) + k]*S
qrt[1 + (-1)^(1/3)*k]*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) + (2*(-1)^(1/3)*Sqrt[1
 - x^2]*Sqrt[1 - k^2*x^2]*ArcTanh[(Sqrt[k]*Sqrt[-(-1)^(2/3) + k]*Sqrt[1 - x
^2])/(Sqrt[1 - (-1)^(2/3)*k]*Sqrt[1 - k^2*x^2])]])/(3*Sqrt[-(-1)^(2/3) + k]*
Sqrt[1 - (-1)^(2/3)*k]*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) + (Sqrt[1 - x^2]*Sqrt
[1 - k^2*x^2]*EllipticF[ArcSin[x], k^2])/Sqrt[(1 - x^2)*(1 - k^2*x^2)] - (2
*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[k, ArcSin[x], k^2])/(3*Sqrt[(1
- x^2)*(1 - k^2*x^2)]) - (2*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[-((-
1)^(1/3)*k), ArcSin[x], k^2])/(3*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) - (2*Sqrt[1
 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[(-1)^(2/3)*k, ArcSin[x], k^2])/(3*Sqrt
[(1 - x^2)*(1 - k^2*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+k^{3/2}x^3}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(1+k^{3/2}x^3\right)}\,dx = \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{-1+k^{3/2}x^3}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\left(1+k^{3/2}x^3\right)}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \left(\frac{1}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}} - \frac{2}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\left(1+k^{3/2}x^3\right)}\right) dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int}{\sqrt{\left(1-x^2\right)}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int\left(-\frac{}{3(-}\right.}{}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{}{(-1-\sqrt{}}}{3\sqrt{\left(1-x^2\right)\left(1-\right.}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{}{\sqrt{1-x^2}}}{3\sqrt{\left(1-x^2\right)\left(1-k\right.}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(k;\sin^{-}\right.}{3\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(k;\sin^{-}\right.}{3\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= -\frac{2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\tan^{-1}\left(\frac{\sqrt{k}\,\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{3(1-k)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{2(-1)^{2/3}\sqrt{1-x^2}\,\sqrt{1-k^2x}}{3\sqrt{\sqrt[3]{-1}+k}\,\sqrt{1+\sqrt[3]{-}}}$$

Mathematica [C] time = 4.92, size = 444, normalized size = 3.55

$$\frac{-2\sqrt{k}\sqrt{x^2-1}\sqrt{k^2x^2-1}\left(-\frac{\tanh^{-1}\left(\frac{\sqrt{(k-1)k}\sqrt{x^2-1}}{\sqrt{1-k}\sqrt{k^2x^2-1}}\right)}{\sqrt{1-k}\sqrt{(k-1)k}}-\frac{(-1)^{2/3}\tanh^{-1}\left(\frac{\sqrt{k\left(k+\sqrt[3]{-1}\right)}\sqrt{x^2-1}}{\sqrt{\sqrt[3]{-1}k+1}\sqrt{k^2x^2-1}}\right)}{\sqrt{k\left(k+\sqrt[3]{-1}\right)}\sqrt{\sqrt[3]{-1}k+1}}+\frac{\sqrt[3]{-1}\tanh^{-1}\left(\frac{\sqrt{k\left(k-(-1)^{2/3}\right)}\sqrt{x^2-1}}{\sqrt{1-(-1)^{2/3}k}\sqrt{k^2x^2-1}}\right)}{\sqrt{k\left(k-(-1)^{2/3}\right)}\sqrt{1-(-1)^{2/3}k}}\right)+3\sqrt{1-x^2}\sqrt{1-k^2x^2}F\left(\sin^{-1}(x)|k^2\right)-2\sqrt{1-x^2}\sqrt{1-k^2x^2}\Pi\left(k;\sin^{-1}(x)|k^2\right)-2\sqrt{1-x^2}\sqrt{1-k^2x^2}\Pi\left(-\sqrt[3]{-1}k;\sin^{-1}(x)|k^2\right)-2\sqrt{1-x^2}\sqrt{1-k^2x^2}\Pi\left(\frac{1}{2}i\left(i+\sqrt{3}\right)k;\sin^{-1}(x)|k^2\right)}{3\sqrt{\left(x^2-1\right)\left(k^2x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + k^(3/2)*x^3)/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(1 + k^(3/2)*x^
3)),x]
```

```
[Out] (-2*Sqrt[k]*Sqrt[-1 + x^2]*Sqrt[-1 + k^2*x^2]*(-(ArcTanh[(Sqrt[(-1 + k)*k]*
Sqrt[-1 + x^2])/(Sqrt[1 - k]*Sqrt[-1 + k^2*x^2])]/(Sqrt[1 - k]*Sqrt[(-1 + k
)*k])) - ((-1)^(2/3)*ArcTanh[(Sqrt[k*((-1)^(1/3) + k)]*Sqrt[-1 + x^2])/(Sqr
t[1 + (-1)^(1/3)*k]*Sqrt[-1 + k^2*x^2])])/(Sqrt[k*((-1)^(1/3) + k)]*Sqrt[1
+ (-1)^(1/3)*k]) + ((-1)^(1/3)*ArcTanh[(Sqrt[k*(-(-1)^(2/3) + k)]*Sqrt[-1 +
 x^2])/(Sqrt[1 - (-1)^(2/3)*k]*Sqrt[-1 + k^2*x^2])])/(Sqrt[k*(-(-1)^(2/3) +
```

```
 k)]*Sqrt[1 - (-1)^(2/3)*k])) + 3*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticF
[ArcSin[x], k^2] - 2*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[k, ArcSin[x
], k^2] - 2*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[-((-1)^(1/3)*k), Arc
Sin[x], k^2] - 2*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[(I/2)*(I + Sqrt
[3])*k, ArcSin[x], k^2])/(3*Sqrt[(-1 + x^2)*(-1 + k^2*x^2)])
```

IntegrateAlgebraic [A] time = 4.26, size = 125, normalized size = 1.00

$$-\frac{4\tan^{-1}\left(\frac{\sqrt{k^2+k+1}\,x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2-\sqrt{k}\,x+1}\right)}{3\sqrt{k^2+k+1}}-\frac{2\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2+2\sqrt{k}\,x+1}\right)}{3(k-1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + k^(3/2)*x^3)/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(1 + k
^(3/2)*x^3)),x]
```

```
[Out] (-4*ArcTan[(Sqrt[1 + k + k^2]*x)/(1 - Sqrt[k]*x + k*x^2 + Sqrt[1 + (-1 - k^
2)*x^2 + k^2*x^4])])/(3*Sqrt[1 + k + k^2]) - (2*ArcTan[((-1 + k)*x)/(1 + 2*
Sqrt[k]*x + k*x^2 + Sqrt[1 + (-1 - k^2)*x^2 + k^2*x^4])])/(3*(-1 + k))
```

fricas [A] time = 0.81, size = 212, normalized size = 1.70

$$-\frac{2\sqrt{k^2+k+1}\,(k-1)\arctan\left(\frac{\sqrt{k^2x^4-\left(k^2+1\right)x^2+1}\,\sqrt{k^2+k+1}\left(\left(k^2+2k+1\right)x+\left(kx^2+1\right)\sqrt{k}\right)}{k^3x^4-\left(k^4+4k^3+4k^2+4k+1\right)x^2+k}\right)-\left(k^2+k+1\right)\arctan\left(-\frac{\sqrt{k^2x^4-\left(k^2+1\right)x^2+1}\left(\left(k^3+k^2-k-1\right)x-2\left(\left(k^2-k\right)x^2+k-1\right)\sqrt{k}\right)}{4k^3x^4-\left(k^4+4k^3-2k^2+4k+1\right)x^2+4k}\right)}{3\left(k^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+k^(3/2)*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(1+k^(3/2)*x^3),x,
 algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(k^2 + k + 1)*(k - 1)*arctan(sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)*
sqrt(k^2 + k + 1)*((k^2 + 2*k + 1)*x + (k*x^2 + 1)*sqrt(k))/(k^3*x^4 - (k^4
 + 4*k^3 + 4*k^2 + 4*k + 1)*x^2 + k)) - (k^2 + k + 1)*arctan(-sqrt(k^2*x^4
- (k^2 + 1)*x^2 + 1)*((k^3 + k^2 - k - 1)*x - 2*((k^2 - k)*x^2 + k - 1)*sqr
t(k))/(4*k^3*x^4 - (k^4 + 4*k^3 - 2*k^2 + 4*k + 1)*x^2 + 4*k)))/(k^3 - 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+k^(3/2)*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(1+k^(3/2)*x^3),x,
 algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.26, size = 1771, normalized size = 14.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+k^(3/2)*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(1+k^(3/2)*x^3),x)
```

```
[Out] (-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*EllipticF(x
,k)-1/6/k^(1/2)*sum((_alpha^2*k+2)/_alpha/(2*_alpha^2*k+1)*(-1/(-k*_alpha^2
*(k^2+k+1))^(1/2)*arctanh(1/2*k*(2*_alpha^2*k^2-k^2-1)/(k^4+2*k^3+6*k^2+2*k
+1)*(_alpha^2*k^4+k^4*x^2+2*k^3*x^2-2*_alpha^2*k^2+6*k^2*x^2+2*k*x^2+_alpha
^2-4*k^2+x^2-4*k-4)/(-k*_alpha^2*(k^2+k+1))^(1/2)/(k^3*x^4-k^3*x^2-k*x^2+k)
```

```
^(1/2))+2*k*_alpha*(_alpha^2*k+1)/(k^2)^(1/2)*(-k^2*x^2+1)^(1/2)*(-x^2+1)^(
1/2)/((k^2*x^4-k^2*x^2-x^2+1)*k)^(1/2)*EllipticPi(x*(k^2)^(1/2),-(_alpha^2*
k+1)/k,1/(k^2)^(1/2))),_alpha=RootOf(_Z^4*k^2+_Z^2*k+1))-1/6/k^(1/2)*2^(1/2
)/((-(-3*k^2)^(1/2)*k^2+k^3-(-3*k^2)^(1/2)*k+k^2-(-3*k^2)^(1/2)+k)/k^2)^(1/
2)*ln(((-(-3*k^2)^(1/2)*k^2+k^3-(-3*k^2)^(1/2)*k+k^2-(-3*k^2)^(1/2)+k)/k^2+
(-k^2-1-k+(-3*k^2)^(1/2))*(x^2-1/2*(-k+(-3*k^2)^(1/2))/k^2)+1/2*2^(1/2)*((-
(-3*k^2)^(1/2)*k^2+k^3-(-3*k^2)^(1/2)*k+k^2-(-3*k^2)^(1/2)+k)/k^2)^(1/2)*(4
*(x^2-1/2*(-k+(-3*k^2)^(1/2))/k^2)^2*k^2+4*(-k^2-1-k+(-3*k^2)^(1/2))*(x^2-1
/2*(-k+(-3*k^2)^(1/2))/k^2)+2*(-(-3*k^2)^(1/2)*k^2+k^3-(-3*k^2)^(1/2)*k+k^2
-(-3*k^2)^(1/2)+k)/k^2)^(1/2))/(x^2-1/2*(-k+(-3*k^2)^(1/2))/k^2))+1/2*k^(1/
2)/(-3*k^2)^(1/2)*2^(1/2)/((-(-3*k^2)^(1/2)*k^2+k^3-(-3*k^2)^(1/2)*k+k^2-(-
3*k^2)^(1/2)+k)/k^2)^(1/2)*ln(((-(-3*k^2)^(1/2)*k^2+k^3-(-3*k^2)^(1/2)*k+k^
2-(-3*k^2)^(1/2)+k)/k^2+(-k^2-1-k+(-3*k^2)^(1/2))*(x^2-1/2*(-k+(-3*k^2)^(1/
2))/k^2)+1/2*2^(1/2)*((-(-3*k^2)^(1/2)*k^2+k^3-(-3*k^2)^(1/2)*k+k^2-(-3*k^2
)^(1/2)+k)/k^2)^(1/2)*(4*(x^2-1/2*(-k+(-3*k^2)^(1/2))/k^2)^2*k^2+4*(-k^2-1-
k+(-3*k^2)^(1/2))*(x^2-1/2*(-k+(-3*k^2)^(1/2))/k^2)+2*(-(-3*k^2)^(1/2)*k^2+
k^3-(-3*k^2)^(1/2)*k+k^2-(-3*k^2)^(1/2)+k)/k^2)^(1/2))/(x^2-1/2*(-k+(-3*k^2
)^(1/2))/k^2))-1/6/k^(1/2)*2^(1/2)/(((-3*k^2)^(1/2)*k^2+k^3+(-3*k^2)^(1/2)*
k+k^2+(-3*k^2)^(1/2)+k)/k^2)^(1/2)*ln((((-3*k^2)^(1/2)*k^2+k^3+(-3*k^2)^(1/
2)*k+k^2+(-3*k^2)^(1/2)+k)/k^2+(-k^2-1-(-3*k^2)^(1/2)-k)*(x^2+1/2*((-3*k^2)
^(1/2)+k)/k^2)+1/2*2^(1/2)*(((-3*k^2)^(1/2)*k^2+k^3+(-3*k^2)^(1/2)*k+k^2+(-
3*k^2)^(1/2)+k)/k^2)^(1/2)*(4*(x^2+1/2*((-3*k^2)^(1/2)+k)/k^2)^2*k^2+4*(-k^
2-1-(-3*k^2)^(1/2)-k)*(x^2+1/2*((-3*k^2)^(1/2)+k)/k^2)+2*((-3*k^2)^(1/2)*k^
2+k^3+(-3*k^2)^(1/2)*k+k^2+(-3*k^2)^(1/2)+k)/k^2)^(1/2))/(x^2+1/2*((-3*k^2)
^(1/2)+k)/k^2))-1/2*k^(1/2)/(-3*k^2)^(1/2)*2^(1/2)/(((-3*k^2)^(1/2)*k^2+k^3
+(-3*k^2)^(1/2)*k+k^2+(-3*k^2)^(1/2)+k)/k^2)^(1/2)*ln((((-3*k^2)^(1/2)*k^2+
k^3+(-3*k^2)^(1/2)*k+k^2+(-3*k^2)^(1/2)+k)/k^2+(-k^2-1-(-3*k^2)^(1/2)-k)*(x
^2+1/2*((-3*k^2)^(1/2)+k)/k^2)+1/2*2^(1/2)*(((-3*k^2)^(1/2)*k^2+k^3+(-3*k^2
)^(1/2)*k+k^2+(-3*k^2)^(1/2)+k)/k^2)^(1/2)*(4*(x^2+1/2*((-3*k^2)^(1/2)+k)/k
^2)^2*k^2+4*(-k^2-1-(-3*k^2)^(1/2)-k)*(x^2+1/2*((-3*k^2)^(1/2)+k)/k^2)+2*((
-3*k^2)^(1/2)*k^2+k^3+(-3*k^2)^(1/2)*k+k^2+(-3*k^2)^(1/2)+k)/k^2)^(1/2))/(x
^2+1/2*((-3*k^2)^(1/2)+k)/k^2))+1/3/(-(-1+k)^2)^(1/2)*ln((-2*k^2+4*k-2+(-k^
3+2*k^2-k)*(x^2-1/k)+2*(-(-1+k)^2)^(1/2)*(k^3*(x^2-1/k)^2+(-k^3+2*k^2-k)*(x
^2-1/k)-k^2+2*k-1)^(1/2))/(x^2-1/k))-2/3*(-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/
(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*EllipticPi(x,k,k)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^{\frac{3}{2}}x^3-1}{\left(k^{\frac{3}{2}}x^3+1\right)\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+k^(3/2)*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(1+k^(3/2)*x^3),x,
 algorithm="maxima")
```

```
[Out] integrate((k^(3/2)*x^3 - 1)/((k^(3/2)*x^3 + 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1)
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{k^{3/2}\,x^3-1}{\left(k^{3/2}\,x^3+1\right)\,\sqrt{\left(x^2-1\right)\,\left(k^2\,x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^(3/2)*x^3 - 1)/((k^(3/2)*x^3 + 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)),x
)
```

```
[Out] int((k^(3/2)*x^3 - 1)/((k^(3/2)*x^3 + 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{k}\,x-1\right)\left(\sqrt{k}\,x+kx^2+1\right)}{\sqrt{(x-1)\,(x+1)\,(kx-1)\,(kx+1)}\,\left(\sqrt{k}\,x+1\right)\left(-\sqrt{k}\,x+kx^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+k**(3/2)*x**3)/((-x**2+1)*(-k**2*x**2+1))**(1/2)/(1+k**(3/2)*
x**3),x)
```

```
[Out] Integral((sqrt(k)*x - 1)*(sqrt(k)*x + k*x**2 + 1)/(sqrt((x - 1)*(x + 1)*(k*
x - 1)*(k*x + 1))*(sqrt(k)*x + 1)*(-sqrt(k)*x + k*x**2 + 1)), x)
```

3.1542 $\int \frac{\sqrt{-x-x^2+x^3}}{-1+x^4}\,dx$

Optimal. Leaf size=125

$$\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x^3-x^2-x}}{x^2-x-1}\right)-\frac{1}{4}\sqrt{1-2i}\,\tan^{-1}\left(\frac{\sqrt{1-2i}\,\sqrt{x^3-x^2-x}}{x^2-x-1}\right)-\frac{1}{4}\sqrt{1+2i}\,\tan^{-1}\left(\frac{\sqrt{1+2i}\,\sqrt{x^3-x^2-x}}{x^2-x-1}\right)$$

Rubi [C] time = 5.08, antiderivative size = 1650, normalized size of antiderivative = 13.20, number of steps used = 55, number of rules used = 19, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.792, Rules used = {2056, 6725, 918, 6733, 1716, 1187, 1098, 1184, 1214, 1456, 540, 421, 419, 538, 537, 1712, 1700, 1698, 205}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[-x - x^2 + x^3]/(-1 + x^4),x]
```

```
[Out] -1/4*(Sqrt[1 - 2*I]*Sqrt[-x - x^2 + x^3]*ArcTan[(Sqrt[1 - 2*I]*Sqrt[x])/Sqr
t[-1 - x + x^2]])/(Sqrt[x]*Sqrt[-1 - x + x^2]) - (Sqrt[1 + 2*I]*Sqrt[-x - x
^2 + x^3]*ArcTan[(Sqrt[1 + 2*I]*Sqrt[x])/Sqrt[-1 - x + x^2]])/(4*Sqrt[x]*Sq
rt[-1 - x + x^2]) + ((1 + Sqrt[5])*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/
(1 + Sqrt[5])]*Sqrt[-x - x^2 + x^3]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[5])]*
Sqrt[x]], (-3 - Sqrt[5])/2])/(2*Sqrt[2]*(1 - Sqrt[5])*Sqrt[x]*(1 + x - x^2)
) + ((1 + Sqrt[5])*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1 + Sqrt[5])]*S
qrt[-x - x^2 + x^3]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]], (-3 -
Sqrt[5])/2])/(2*Sqrt[2]*(3 + Sqrt[5])*Sqrt[x]*(1 + x - x^2)) - (Sqrt[-2 - (
1 - Sqrt[5])*x]*Sqrt[(2 + (1 + Sqrt[5])*x)/(2 + (1 - Sqrt[5])*x)]*Sqrt[-x -
 x^2 + x^3]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*Sqrt[x])/Sqrt[-2 - (1 - Sqrt[
5])*x]], (5 - Sqrt[5])/10])/(12*5^(1/4)*Sqrt[x]*Sqrt[(2 + (1 - Sqrt[5])*x)^
(-1)]*(1 + x - x^2)) - (Sqrt[-2 - (1 - Sqrt[5])*x]*Sqrt[(2 + (1 + Sqrt[5])*
x)/(2 + (1 - Sqrt[5])*x)]*Sqrt[-x - x^2 + x^3]*EllipticF[ArcSin[(Sqrt[2]*5^
(1/4)*Sqrt[x])/Sqrt[-2 - (1 - Sqrt[5])*x]], (5 - Sqrt[5])/10])/(2*5^(1/4)*(
1 - Sqrt[5])*Sqrt[x]*Sqrt[(2 + (1 - Sqrt[5])*x)^(-1)]*(1 + x - x^2)) + ((1
- Sqrt[5])*Sqrt[-2 - (1 - Sqrt[5])*x]*Sqrt[(2 + (1 + Sqrt[5])*x)/(2 + (1 -
Sqrt[5])*x)]*Sqrt[-x - x^2 + x^3]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*Sqrt[x]
)/Sqrt[-2 - (1 - Sqrt[5])*x]], (5 - Sqrt[5])/10])/(4*5^(1/4)*Sqrt[x]*Sqrt[(
2 + (1 - Sqrt[5])*x)^(-1)]*(1 + x - x^2)) + ((1/8 - I/24)*((1 - 2*I) + Sqrt
[5])*Sqrt[-2 - (1 - Sqrt[5])*x]*Sqrt[(2 + (1 + Sqrt[5])*x)/(2 + (1 - Sqrt[5
])*x)]*Sqrt[-x - x^2 + x^3]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*Sqrt[x])/Sqrt
[-2 - (1 - Sqrt[5])*x]], (5 - Sqrt[5])/10])/(5^(1/4)*Sqrt[x]*Sqrt[(2 + (1 -
 Sqrt[5])*x)^(-1)]*(1 + x - x^2)) + ((1/8 + I/24)*((1 + 2*I) + Sqrt[5])*Sqr
t[-2 - (1 - Sqrt[5])*x]*Sqrt[(2 + (1 + Sqrt[5])*x)/(2 + (1 - Sqrt[5])*x)]*S
qrt[-x - x^2 + x^3]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*Sqrt[x])/Sqrt[-2 - (1
 - Sqrt[5])*x]], (5 - Sqrt[5])/10])/(5^(1/4)*Sqrt[x]*Sqrt[(2 + (1 - Sqrt[5]
)*x)^(-1)]*(1 + x - x^2)) - (Sqrt[-2 - (1 - Sqrt[5])*x]*Sqrt[(2 + (1 + Sqrt
[5])*x)/(2 + (1 - Sqrt[5])*x)]*Sqrt[-x - x^2 + x^3]*EllipticF[ArcSin[(Sqrt[
2]*5^(1/4)*Sqrt[x])/Sqrt[-2 - (1 - Sqrt[5])*x]], (5 - Sqrt[5])/10])/(2*5^(1
/4)*(3 + Sqrt[5])*Sqrt[x]*Sqrt[(2 + (1 - Sqrt[5])*x)^(-1)]*(1 + x - x^2)) -
 ((2 + Sqrt[5])*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1 + Sqrt[5])]*Sqrt
[-x - x^2 + x^3]*EllipticPi[(-1 - Sqrt[5])/2, ArcSin[Sqrt[2/(1 + Sqrt[5])]*
Sqrt[x]], (-3 - Sqrt[5])/2])/(Sqrt[2]*(3 + Sqrt[5])*Sqrt[x]*(1 + x - x^2))
+ (Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1 + Sqrt[5])]*Sqrt[-x - x^2 + x
^3]*EllipticPi[(1 + Sqrt[5])/2, ArcSin[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]], (-3
- Sqrt[5])/2])/(Sqrt[2]*(1 - Sqrt[5])*Sqrt[x]*(1 + x - x^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 540

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x
] + Dist[(b*c - a*d)/b, Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

Rule 918

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/(e*(2*m + 5)), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m*Simp[b*d*f - 3*a*e*f + a*d*g + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x -
 (c*e*f - 3*c*d*g + b*e*g)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e*x*(b + q + 2*c*x^2))/(2*c*Sqrt
[a + b*x^2 + c*x^4]), x] - Simp[(e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q
)*x^2)]*Sqrt[(2*a + (b + q)*x^2)/q]*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*
```

```
x^2)/(2*q)]], (b + q)/(2*q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b
 + q)*x^2)]), x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1187

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*d - e*(b - q))/(2*c), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[e/(2*c), Int[(b - q + 2*c*x^2)/Sqrt[
a + b*x^2 + c*x^4], x], x] /; NeQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1214

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/(2*c*d - e*(b - q)), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1456

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1700

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1712

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, -Dist[C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1716

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
```

```
P4x, x, 4]}, -Dist[(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{\sqrt{-x-x^2+x^3}}{-1+x^4}\,dx = \frac{\sqrt{-x-x^2+x^3}\int \frac{\sqrt{x}\sqrt{-1-x+x^2}}{-1+x^4}\,dx}{\sqrt{x}\sqrt{-1-x+x^2}}$$

$$= \frac{\sqrt{-x-x^2+x^3}\int \left(-\frac{\sqrt{x}\sqrt{-1-x+x^2}}{2\left(1-x^2\right)}-\frac{\sqrt{x}\sqrt{-1-x+x^2}}{2\left(1+x^2\right)}\right)dx}{\sqrt{x}\sqrt{-1-x+x^2}}$$

$$= -\frac{\sqrt{-x-x^2+x^3}\int \frac{\sqrt{x}\sqrt{-1-x+x^2}}{1-x^2}\,dx}{2\sqrt{x}\sqrt{-1-x+x^2}}-\frac{\sqrt{-x-x^2+x^3}\int \frac{\sqrt{x}\sqrt{-1-x+x^2}}{1+x^2}\,dx}{2\sqrt{x}\sqrt{-1-x+x^2}}$$

$$= -\frac{\sqrt{-x-x^2+x^3}\int \left(\frac{i\sqrt{x}\sqrt{-1-x+x^2}}{2(i-x)}+\frac{i\sqrt{x}\sqrt{-1-x+x^2}}{2(i+x)}\right)dx}{2\sqrt{x}\sqrt{-1-x+x^2}}-\frac{\sqrt{-x-x^2+x^3}\int \left(\frac{\sqrt{x}\sqrt{-1-x+x^2}}{2(1-x)}\right.}{2\sqrt{x}\sqrt{-1-x}}$$

$$= -\frac{\left(i\sqrt{-x-x^2+x^3}\right)\int \frac{\sqrt{x}\sqrt{-1-x+x^2}}{i-x}\,dx}{4\sqrt{x}\sqrt{-1-x+x^2}}-\frac{\left(i\sqrt{-x-x^2+x^3}\right)\int \frac{\sqrt{x}\sqrt{-1-x+x^2}}{i+x}\,dx}{4\sqrt{x}\sqrt{-1-x+x^2}}-\frac{\sqrt{-x-}}{}$$

$$= -\frac{\left(i\sqrt{-x-x^2+x^3}\right)\int \frac{-i-(2+2i)x-(1-3i)x^2}{(i-x)\sqrt{x}\sqrt{-1-x+x^2}}\,dx}{12\sqrt{x}\sqrt{-1-x+x^2}}+\frac{\left(i\sqrt{-x-x^2+x^3}\right)\int \frac{-i+(2-2i)x+(1+3i)x^2}{\sqrt{x}(i+x)\sqrt{-1-x+x^2}}\,dx}{12\sqrt{x}\sqrt{-1-x+x^2}}$$

$$= -\frac{\left(i\sqrt{-x-x^2+x^3}\right)\text{Subst}\left(\int \frac{-i-(2+2i)x^2-(1-3i)x^4}{\left(i-x^2\right)\sqrt{-1-x^2+x^4}}\,dx,x,\sqrt{x}\right)}{6\sqrt{x}\sqrt{-1-x+x^2}}+\frac{\left(i\sqrt{-x-x^2+x^3}\right)\text{Subst}}{6\sqrt{x}}$$

$$= -\frac{\left(i\sqrt{-x-x^2+x^3}\right)\text{Subst}\left(\int \frac{(1-4i)-(2+2i)x^2}{\left(i-x^2\right)\sqrt{-1-x^2+x^4}}\,dx,x,\sqrt{x}\right)}{6\sqrt{x}\sqrt{-1-x+x^2}}+\frac{\left(i\sqrt{-x-x^2+x^3}\right)\text{Subst}\left(\int\right.}{6\sqrt{x}\sqrt{-1}}$$

$$= -\frac{\left(\left(\frac{1}{4}-\frac{i}{12}\right)\sqrt{-x-x^2+x^3}\right)\text{Subst}\left(\int \frac{-1-\sqrt{5}+2x^2}{\sqrt{-1-x^2+x^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{-1-x+x^2}}+-\frac{\left(\left(\frac{1}{4}+\frac{i}{2}\right)\sqrt{-x-x^2+}\right.}{}$$

$$= -\frac{\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{12\sqrt[4]{5}\sqrt{x}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\left(1+x-x^2\right)}$$

$$= -\frac{\sqrt{1-2i}\sqrt{-x-x^2+x^3}\tan^{-1}\left(\frac{\sqrt{1-2i}\sqrt{x}}{\sqrt{-1-x+x^2}}\right)}{4\sqrt{x}\sqrt{-1-x+x^2}}-\frac{\sqrt{1+2i}\sqrt{-x-x^2+x^3}\tan^{-1}\left(\frac{\sqrt{1+2i}\sqrt{x}}{\sqrt{-1-x+x^2}}\right.}{4\sqrt{x}\sqrt{-1-x+x^2}}$$

$$= -\frac{\sqrt{1-2i}\sqrt{-x-x^2+x^3}\tan^{-1}\left(\frac{\sqrt{1-2i}\sqrt{x}}{\sqrt{-1-x+x^2}}\right)}{4\sqrt{x}\sqrt{-1-x+x^2}}-\frac{\sqrt{1+2i}\sqrt{-x-x^2+x^3}\tan^{-1}\left(\frac{\sqrt{1+2i}\sqrt{x}}{\sqrt{-1-x+x^2}}\right.}{4\sqrt{x}\sqrt{-1-x+x^2}}$$

$$= -\frac{\sqrt{1-2i}\sqrt{-x-x^2+x^3}\tan^{-1}\left(\frac{\sqrt{1-2i}\sqrt{x}}{\sqrt{-1-x+x^2}}\right)}{4\sqrt{x}\sqrt{-1-x+x^2}}-\frac{\sqrt{1+2i}\sqrt{-x-x^2+x^3}\tan^{-1}\left(\frac{\sqrt{1+2i}\sqrt{x}}{\sqrt{-1-x+x^2}}\right.}{4\sqrt{x}\sqrt{-1-x+x^2}}$$

Mathematica [C] time = 13.09, size = 2469, normalized size = 19.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-x - x^2 + x^3]/(-1 + x^4),x]
```

```
[Out] (2*(-1 + Sqrt[x])*(1 + Sqrt[x])*(1 + x)*(1 + x^2)*Sqrt[x*(-1 - x + x^2)]*(S
qrt[-1 - x + x^2]/(8*(-1 + Sqrt[x])) - Sqrt[-1 - x + x^2]/(8*(1 + Sqrt[x]))
 + Sqrt[-1 - x + x^2]/(4*(1 + x)) - (x*Sqrt[-1 - x + x^2])/(2*(1 + x^2)))*(
((1/2 + I/4)*Sqrt[(-1 + Sqrt[5])/2]*Sqrt[1 - (2*x)/(1 - Sqrt[5])]*Sqrt[1 -
(2*x)/(1 + Sqrt[5])]*EllipticPi[(-1/2*I)*(1 - Sqrt[5]), I*ArcSinh[Sqrt[2/(-
1 + Sqrt[5])]*Sqrt[x]], (1 - Sqrt[5])/(1 + Sqrt[5])])/Sqrt[-1 - x + x^2] -
((1/2 - I/4)*Sqrt[(-1 + Sqrt[5])/2]*Sqrt[1 - (2*x)/(1 - Sqrt[5])]*Sqrt[1 -
(2*x)/(1 + Sqrt[5])]*EllipticPi[(I/2)*(1 - Sqrt[5]), I*ArcSinh[Sqrt[2/(-1 +
 Sqrt[5])]*Sqrt[x]], (1 - Sqrt[5])/(1 + Sqrt[5])])/Sqrt[-1 - x + x^2] - ((I
/4)*Sqrt[(-1 + Sqrt[5])/2]*Sqrt[1 - (2*x)/(1 - Sqrt[5])]*Sqrt[1 - (2*x)/(1
+ Sqrt[5])]*EllipticPi[(-1 + Sqrt[5])/2, I*ArcSinh[Sqrt[2/(-1 + Sqrt[5])]*S
qrt[x]], (1 - Sqrt[5])/(1 + Sqrt[5])])/Sqrt[-1 - x + x^2] + ((I*(I*Sqrt[(-1
 + Sqrt[5])/2] + Sqrt[(1 + Sqrt[5])/2])*Sqrt[((Sqrt[-1 + Sqrt[5]] + I*Sqrt[
1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5])] - (2*I)*Sqrt[x]))/((Sqrt[-1 + Sqrt[5]
] - I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5])] + (2*I)*Sqrt[x]))]*((-I)*S
qrt[(-1 + Sqrt[5])/2] + Sqrt[x])^2*Sqrt[(I*(-Sqrt[(1 + Sqrt[5])/2] + Sqrt[x
]))/((I*Sqrt[(-1 + Sqrt[5])/2] + Sqrt[(1 + Sqrt[5])/2])*((-I)*Sqrt[(-1 + Sq
rt[5])/2] + Sqrt[x]))]*Sqrt[(I*(Sqrt[(1 + Sqrt[5])/2] + Sqrt[x]))/((I*Sqrt[
(-1 + Sqrt[5])/2] - Sqrt[(1 + Sqrt[5])/2])*((-I)*Sqrt[(-1 + Sqrt[5])/2] + S
qrt[x]))]*((1 + I*Sqrt[(-1 + Sqrt[5])/2])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 +
 Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5])] - (2*I)*Sqrt[x]))/
((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5])] + (2*I)
*Sqrt[x]))]], (Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])^2/(Sqrt[-1 + Sqrt[
5]] + I*Sqrt[1 + Sqrt[5]])^2] - I*Sqrt[2*(-1 + Sqrt[5])]*EllipticPi[((-1 +
I*Sqrt[(-1 + Sqrt[5])/2])*(I*Sqrt[(-1 + Sqrt[5])/2] + Sqrt[(1 + Sqrt[5])/2]
))/((-1 - I*Sqrt[(-1 + Sqrt[5])/2])*((-I)*Sqrt[(-1 + Sqrt[5])/2] + Sqrt[(1
+ Sqrt[5])/2])), ArcSin[Sqrt[((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(S
qrt[2*(-1 + Sqrt[5])] - (2*I)*Sqrt[x]))/((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + S
qrt[5]])*(Sqrt[2*(-1 + Sqrt[5])] + (2*I)*Sqrt[x]))]], (Sqrt[-1 + Sqrt[5]] -
 I*Sqrt[1 + Sqrt[5]])^2/(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])^2]))/((-
1 - I*Sqrt[(-1 + Sqrt[5])/2])*(1 - I*Sqrt[(-1 + Sqrt[5])/2])*(I*Sqrt[(-1 +
Sqrt[5])/2] - Sqrt[(1 + Sqrt[5])/2])*Sqrt[-1 - x + x^2]) - (I*(I*Sqrt[(-1 +
 Sqrt[5])/2] + Sqrt[(1 + Sqrt[5])/2])*Sqrt[((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1
+ Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5])] - (2*I)*Sqrt[x]))/((Sqrt[-1 + Sqrt[5]]
- I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5])] + (2*I)*Sqrt[x]))]*((-I)*Sqr
t[(-1 + Sqrt[5])/2] + Sqrt[x])^2*Sqrt[(I*(-Sqrt[(1 + Sqrt[5])/2] + Sqrt[x])
)/((I*Sqrt[(-1 + Sqrt[5])/2] + Sqrt[(1 + Sqrt[5])/2])*((-I)*Sqrt[(-1 + Sqrt
[5])/2] + Sqrt[x]))]*Sqrt[(I*(Sqrt[(1 + Sqrt[5])/2] + Sqrt[x]))/((I*Sqrt[(-
1 + Sqrt[5])/2] - Sqrt[(1 + Sqrt[5])/2])*((-I)*Sqrt[(-1 + Sqrt[5])/2] + Sqr
t[x]))]*((-1 + I*Sqrt[(-1 + Sqrt[5])/2])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 +
Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5])] - (2*I)*Sqrt[x]))/(
(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5])] + (2*I)*
Sqrt[x]))]], (Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])^2/(Sqrt[-1 + Sqrt[5
]] + I*Sqrt[1 + Sqrt[5]])^2] - I*Sqrt[2*(-1 + Sqrt[5])]*EllipticPi[((1 + I*
Sqrt[(-1 + Sqrt[5])/2])*(I*Sqrt[(-1 + Sqrt[5])/2] + Sqrt[(1 + Sqrt[5])/2]))
/((1 - I*Sqrt[(-1 + Sqrt[5])/2])*((-I)*Sqrt[(-1 + Sqrt[5])/2] + Sqrt[(1 + S
qrt[5])/2])), ArcSin[Sqrt[((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt
[2*(-1 + Sqrt[5])] - (2*I)*Sqrt[x]))/((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt
[5]])*(Sqrt[2*(-1 + Sqrt[5])] + (2*I)*Sqrt[x]))]], (Sqrt[-1 + Sqrt[5]] - I*
Sqrt[1 + Sqrt[5]])^2/(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])^2]))/((-1 -
 I*Sqrt[(-1 + Sqrt[5])/2])*(1 - I*Sqrt[(-1 + Sqrt[5])/2])*(I*Sqrt[(-1 + Sqr
t[5])/2] - Sqrt[(1 + Sqrt[5])/2])*Sqrt[-1 - x + x^2]))/4))/(x^(3/2)*(-1 - x
 + x^2))
```

IntegrateAlgebraic [A] time = 0.23, size = 125, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x^3-x^2-x}}{x^2-x-1}\right)-\frac{1}{4}\sqrt{1-2i}\,\tan^{-1}\left(\frac{\sqrt{1-2i}\,\sqrt{x^3-x^2-x}}{x^2-x-1}\right)-\frac{1}{4}\sqrt{1+2i}\,\tan^{-1}\left(\frac{\sqrt{1+2i}\,\sqrt{x^3-x^2-x}}{x^2-x-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-x - x^2 + x^3]/(-1 + x^4),x]
```

```
[Out] ArcTan[Sqrt[-x - x^2 + x^3]/(-1 - x + x^2)]/2 - (Sqrt[1 - 2*I]*ArcTan[(Sqrt
[1 - 2*I]*Sqrt[-x - x^2 + x^3])/(-1 - x + x^2)])/4 - (Sqrt[1 + 2*I]*ArcTan[
(Sqrt[1 + 2*I]*Sqrt[-x - x^2 + x^3])/(-1 - x + x^2)])/4
```

fricas [B] time = 0.93, size = 2484, normalized size = 19.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2-x)^(1/2)/(x^4-1),x, algorithm="fricas")
```

```
[Out] -1/40*5^(3/4)*sqrt(2)*sqrt(sqrt(5) + 5)*arctan(-1/40*(20*x^11 + 260*x^10 -
1340*x^9 - 880*x^8 + 5680*x^7 + 280*x^6 - 5680*x^5 - 880*x^4 + 1340*x^3 + 2
60*x^2 + sqrt(x^3 - x^2 - x)*(5^(3/4)*(sqrt(5)*sqrt(2)*(x^10 - 4*x^9 - 17*x
^8 + 56*x^7 + 78*x^6 - 136*x^5 - 78*x^4 + 56*x^3 + 17*x^2 - 4*x - 1) - sqrt
(2)*(x^10 + 8*x^9 - 57*x^8 - 24*x^7 + 294*x^6 + 64*x^5 - 294*x^4 - 24*x^3 +
 57*x^2 + 8*x - 1)) + 4*5^(1/4)*(sqrt(5)*sqrt(2)*(3*x^9 + 7*x^8 + 14*x^7 -
81*x^6 - 10*x^5 + 81*x^4 + 14*x^3 - 7*x^2 + 3*x) - 5*sqrt(2)*(x^9 + 9*x^8 -
 18*x^7 - 15*x^6 + 26*x^5 + 15*x^4 - 18*x^3 - 9*x^2 + x)))*sqrt(sqrt(5) + 5
) - sqrt(1/5)*(240*x^10 + 160*x^9 - 1680*x^8 - 480*x^7 + 3200*x^6 + 480*x^5
 - 1680*x^4 - 160*x^3 + 240*x^2 + sqrt(x^3 - x^2 - x)*(5^(3/4)*(sqrt(5)*sqr
t(2)*(x^10 - 6*x^9 - 25*x^8 + 120*x^7 - 58*x^6 - 196*x^5 + 58*x^4 + 120*x^3
 + 25*x^2 - 6*x - 1) - sqrt(2)*(x^10 - 2*x^9 - 17*x^8 + 216*x^7 - 306*x^6 -
 396*x^5 + 306*x^4 + 216*x^3 + 17*x^2 - 2*x - 1)) + 4*5^(1/4)*(sqrt(5)*sqrt
(2)*(3*x^9 - 3*x^8 - 56*x^7 + 69*x^6 + 90*x^5 - 69*x^4 - 56*x^3 + 3*x^2 + 3
*x) - 5*sqrt(2)*(x^9 + 3*x^8 - 28*x^7 + 11*x^6 + 54*x^5 - 11*x^4 - 28*x^3 -
 3*x^2 + x)))*sqrt(sqrt(5) + 5) + 4*sqrt(5)*(5*x^11 - 25*x^10 - 105*x^9 + 4
40*x^8 + 50*x^7 - 830*x^6 - 50*x^5 + 440*x^4 + 105*x^3 - 25*x^2 - sqrt(5)*(
x^11 + 3*x^10 - 37*x^9 + 96*x^8 - 6*x^7 - 166*x^6 + 6*x^5 + 96*x^4 + 37*x^3
 + 3*x^2 - x) - 5*x) - 80*sqrt(5)*(x^10 + 6*x^9 - 23*x^8 - 2*x^7 + 40*x^6 +
 2*x^5 - 23*x^4 - 6*x^3 + x^2))*sqrt((5*x^4 - 20*x^3 + 5^(1/4)*sqrt(x^3 - x
^2 - x)*(sqrt(5)*sqrt(2)*(x^2 - 6*x - 1) - 5*sqrt(2)*(x^2 - 2*x - 1))*sqrt(
sqrt(5) + 5) + 30*x^2 + 20*sqrt(5)*(x^3 - x^2 - x) + 20*x + 5)/(x^4 + 2*x^2
 + 1)) + 4*sqrt(5)*(5*x^11 - 15*x^10 - 15*x^9 + 20*x^8 - 20*x^7 + 70*x^6 +
20*x^5 + 20*x^4 + 15*x^3 - 15*x^2 - sqrt(5)*(x^11 + 13*x^10 - 67*x^9 - 44*x
^8 + 284*x^7 + 14*x^6 - 284*x^5 - 44*x^4 + 67*x^3 + 13*x^2 - x) - 5*x) - 20
*sqrt(5)*(x^11 - 3*x^10 - 3*x^9 + 4*x^8 - 4*x^7 + 14*x^6 + 4*x^5 + 4*x^4 +
3*x^3 - 3*x^2 - x) - 20*x)/(x^11 - 9*x^10 - 45*x^9 + 180*x^8 + 18*x^7 - 326
*x^6 - 18*x^5 + 180*x^4 + 45*x^3 - 9*x^2 - x)) - 1/40*5^(3/4)*sqrt(2)*sqrt(
sqrt(5) + 5)*arctan(1/40*(20*x^11 + 260*x^10 - 1340*x^9 - 880*x^8 + 5680*x^
7 + 280*x^6 - 5680*x^5 - 880*x^4 + 1340*x^3 + 260*x^2 - sqrt(x^3 - x^2 - x)
*(5^(3/4)*(sqrt(5)*sqrt(2)*(x^10 - 4*x^9 - 17*x^8 + 56*x^7 + 78*x^6 - 136*x
^5 - 78*x^4 + 56*x^3 + 17*x^2 - 4*x - 1) - sqrt(2)*(x^10 + 8*x^9 - 57*x^8 -
 24*x^7 + 294*x^6 + 64*x^5 - 294*x^4 - 24*x^3 + 57*x^2 + 8*x - 1)) + 4*5^(1
/4)*(sqrt(5)*sqrt(2)*(3*x^9 + 7*x^8 + 14*x^7 - 81*x^6 - 10*x^5 + 81*x^4 + 1
4*x^3 - 7*x^2 + 3*x) - 5*sqrt(2)*(x^9 + 9*x^8 - 18*x^7 - 15*x^6 + 26*x^5 +
15*x^4 - 18*x^3 - 9*x^2 + x)))*sqrt(sqrt(5) + 5) - sqrt(1/5)*(240*x^10 + 16
0*x^9 - 1680*x^8 - 480*x^7 + 3200*x^6 + 480*x^5 - 1680*x^4 - 160*x^3 + 240*
x^2 - sqrt(x^3 - x^2 - x)*(5^(3/4)*(sqrt(5)*sqrt(2)*(x^10 - 6*x^9 - 25*x^8
 + 120*x^7 - 58*x^6 - 196*x^5 + 58*x^4 + 120*x^3 + 25*x^2 - 6*x - 1) - sqrt(
2)*(x^10 - 2*x^9 - 17*x^8 + 216*x^7 - 306*x^6 - 396*x^5 + 306*x^4 + 216*x^3
 + 17*x^2 - 2*x - 1)) + 4*5^(1/4)*(sqrt(5)*sqrt(2)*(3*x^9 - 3*x^8 - 56*x^7
```

```
+ 69*x^6 + 90*x^5 - 69*x^4 - 56*x^3 + 3*x^2 + 3*x) - 5*sqrt(2)*(x^9 + 3*x^8
 - 28*x^7 + 11*x^6 + 54*x^5 - 11*x^4 - 28*x^3 - 3*x^2 + x)))*sqrt(sqrt(5) +
 5) + 4*sqrt(5)*(5*x^11 - 25*x^10 - 105*x^9 + 440*x^8 + 50*x^7 - 830*x^6 -
50*x^5 + 440*x^4 + 105*x^3 - 25*x^2 - sqrt(5)*(x^11 + 3*x^10 - 37*x^9 + 96*
x^8 - 6*x^7 - 166*x^6 + 6*x^5 + 96*x^4 + 37*x^3 + 3*x^2 - x) - 5*x) - 80*sq
rt(5)*(x^10 + 6*x^9 - 23*x^8 - 2*x^7 + 40*x^6 + 2*x^5 - 23*x^4 - 6*x^3 + x^
2))*sqrt((5*x^4 - 20*x^3 - 5^(1/4)*sqrt(x^3 - x^2 - x)*(sqrt(5)*sqrt(2)*(x^
2 - 6*x - 1) - 5*sqrt(2)*(x^2 - 2*x - 1))*sqrt(sqrt(5) + 5) + 30*x^2 + 20*s
qrt(5)*(x^3 - x^2 - x) + 20*x + 5)/(x^4 + 2*x^2 + 1)) + 4*sqrt(5)*(5*x^11 -
 15*x^10 - 15*x^9 + 20*x^8 - 20*x^7 + 70*x^6 + 20*x^5 + 20*x^4 + 15*x^3 - 1
5*x^2 - sqrt(5)*(x^11 + 13*x^10 - 67*x^9 - 44*x^8 + 284*x^7 + 14*x^6 - 284*
x^5 - 44*x^4 + 67*x^3 + 13*x^2 - x) - 5*x) - 20*sqrt(5)*(x^11 - 3*x^10 - 3*
x^9 + 4*x^8 - 4*x^7 + 14*x^6 + 4*x^5 + 4*x^4 + 3*x^3 - 3*x^2 - x) - 20*x)/(
x^11 - 9*x^10 - 45*x^9 + 180*x^8 + 18*x^7 - 326*x^6 - 18*x^5 + 180*x^4 + 45
*x^3 - 9*x^2 - x)) + 1/320*5^(1/4)*(sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(
5) + 5)*log(1/5*(5*x^4 - 20*x^3 + 5^(1/4)*sqrt(x^3 - x^2 - x)*(sqrt(5)*sqrt
(2)*(x^2 - 6*x - 1) - 5*sqrt(2)*(x^2 - 2*x - 1))*sqrt(sqrt(5) + 5) + 30*x^2
 + 20*sqrt(5)*(x^3 - x^2 - x) + 20*x + 5)/(x^4 + 2*x^2 + 1)) - 1/320*5^(1/4
)*(sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(5) + 5)*log(1/5*(5*x^4 - 20*x^3 -
 5^(1/4)*sqrt(x^3 - x^2 - x)*(sqrt(5)*sqrt(2)*(x^2 - 6*x - 1) - 5*sqrt(2)*(
x^2 - 2*x - 1))*sqrt(sqrt(5) + 5) + 30*x^2 + 20*sqrt(5)*(x^3 - x^2 - x) + 2
0*x + 5)/(x^4 + 2*x^2 + 1)) - 1/4*arctan(1/2*(x^2 - 2*x - 1)/sqrt(x^3 - x^2
 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^3 - x^2 - x}}{x^4 - 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2-x)^(1/2)/(x^4-1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^3 - x^2 - x)/(x^4 - 1), x)
```

maple [C] time = 0.08, size = 2289, normalized size = 18.31

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x^2-x)^(1/2)/(x^4-1),x)
```

```
[Out] -1/5*(1/2*5^(1/2)-1/2)*((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x
-1/2-1/2*5^(1/2))*5^(1/2))^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(
1/2)*EllipticF(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),1/5*5^(1/2)*((
1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))+1/5*(1/2*5^(1/2)-1/2)*((x-1/2+1/2*5^(1/2))
/(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x-1/2-1/2*5^(1/2))*5^(1/2))^(1/2)*(-x/(1/2*5
^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)*(-5^(1/2)*EllipticE(((x-1/2+1/2*5^(1/2
))/(1/2*5^(1/2)-1/2))^(1/2),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))+
(1/2+1/2*5^(1/2))*EllipticF(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),1
/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)))-1/10*(1/2*5^(1/2)-1/2)*((x-1
/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x-1/2-1/2*5^(1/2))*5^(1/2))^(
1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(-1/2-1/2*5^(1/2))*Elli
pticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(-1/
2-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))+1/10*(1/2*5^(
1/2)-1/2)*((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x-1/2-1/2*5^(1
/2))*5^(1/2))^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(3/2-1/2
*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2
*5^(1/2))/(3/2-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))+
1/10*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/
```

```
2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2
)/(x^3-x^2-x)^(1/2)*EllipticF(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2)
,1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))*5^(1/2)-3/10*(x/(1/2*5^(1/2
)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(
1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)
*EllipticF(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),1/5*5^(1/2)*((1/2*
5^(1/2)-1/2)*5^(1/2))^(1/2))+1/2*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)
+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)
*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)*EllipticE(((x-1/2+1/2*5^(1/
2))/(1/2*5^(1/2)-1/2))^(1/2),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))
-1/10*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1
/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/
2)/(x^3-x^2-x)^(1/2)*5^(1/2)*EllipticE(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/
2))^(1/2),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))+1/10*(x/(1/2*5^(1/
2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^
(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2
)/(1/2-I-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1
/2),(1/2-1/2*5^(1/2))/(1/2-I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^
(1/2))^(1/2))*5^(1/2)-1/20*I*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2
/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x
/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(1/2-I-1/2*5^(1/2))*EllipticPi(
((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2-I-1/2*
5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))-1/10*(x/(1/2*5^(1/2
)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(
1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)
/(1/2-I-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/
2),(1/2-1/2*5^(1/2))/(1/2-I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(
1/2))^(1/2))+1/20*I*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(
1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(
1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(1/2+I-1/2*5^(1/2))*EllipticPi(((x-1/2+1
/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2+I-1/2*5^(1/2)),
1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))+1/10*(x/(1/2*5^(1/2)-1/2)-1/
2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*
5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(1/2+I-1
/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1
/2*5^(1/2))/(1/2+I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/
2))*5^(1/2)-1/20*I*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1
/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1
/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(1/2+I-1/2*5^(1/2))*EllipticPi(((x-1/2+1/
2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2+I-1/2*5^(1/2)),1
/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))*5^(1/2)-1/10*(x/(1/2*5^(1/2)-
1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/
2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(
1/2+I-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2)
,(1/2-1/2*5^(1/2))/(1/2+I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/
2))^(1/2))+1/20*I*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/
2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/
2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(1/2-I-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2
*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2-I-1/2*5^(1/2)),1/
5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^3 - x^2 - x}}{x^4 - 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2-x)^(1/2)/(x^4-1),x, algorithm="maxima")
[Out] integrate(sqrt(x^3 - x^2 - x)/(x^4 - 1), x)
```

mupad [B] time = 0.07, size = 537, normalized size = 4.30

$$\frac{\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\Pi\left(-\frac{\sqrt{5}}{2}-\frac{1}{2};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)}{2\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}+\frac{\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\Pi\left(\frac{\sqrt{5}}{2}+\frac{1}{2};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)}{2\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}+\frac{\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\Pi\left(-\frac{\sqrt{5}\,1\mathrm{i}}{2}-\frac{1}{2}\mathrm{i};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)\left(-\frac{1}{2}+1\mathrm{i}\right)}{\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}+\frac{\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\Pi\left(\frac{\sqrt{5}\,1\mathrm{i}}{2}+\frac{1}{2}\mathrm{i};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)\left(-\frac{1}{2}-\mathrm{i}\right)}{\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - x^2 - x)^(1/2)/(x^4 - 1),x)
```

```
[Out] ((5^(1/2)/2 + 1/2)*(x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/2 - 1/2)/(5^(1
/2)/2 - 1/2))^(1/2)*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 + 1/2))^(1/2)*ellipti
cPi(- 5^(1/2)/2 - 1/2, asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/2 + 1/2
)/(5^(1/2)/2 - 1/2)))/(2*(x^3 - x^2 - x*(5^(1/2)/2 - 1/2)*(5^(1/2)/2 + 1/2)
)^(1/2)) + ((5^(1/2)/2 + 1/2)*(x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/2 -
 1/2)/(5^(1/2)/2 - 1/2))^(1/2)*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 + 1/2))^(1
/2)*ellipticPi(5^(1/2)/2 + 1/2, asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2
)/2 + 1/2)/(5^(1/2)/2 - 1/2)))/(2*(x^3 - x^2 - x*(5^(1/2)/2 - 1/2)*(5^(1/2)
/2 + 1/2))^(1/2)) - ((5^(1/2)/2 + 1/2)*(x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^
(1/2)/2 - 1/2)/(5^(1/2)/2 - 1/2))^(1/2)*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 +
 1/2))^(1/2)*ellipticPi(- (5^(1/2)*1i)/2 - 1i/2, asin((x/(5^(1/2)/2 + 1/2))
^(1/2)), -(5^(1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2))*(1/2 - 1i))/(x^3 - x^2 - x*(
5^(1/2)/2 - 1/2)*(5^(1/2)/2 + 1/2))^(1/2) - ((5^(1/2)/2 + 1/2)*(x/(5^(1/2)/
2 + 1/2))^(1/2)*((x + 5^(1/2)/2 - 1/2)/(5^(1/2)/2 - 1/2))^(1/2)*((5^(1/2)/2
 - x + 1/2)/(5^(1/2)/2 + 1/2))^(1/2)*ellipticPi((5^(1/2)*1i)/2 + 1i/2, asin
((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2))*(1/2 +
 1i))/(x^3 - x^2 - x*(5^(1/2)/2 - 1/2)*(5^(1/2)/2 + 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(x^2-x-1\right)}}{(x-1)\,(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x**2-x)**(1/2)/(x**4-1),x)
```

```
[Out] Integral(sqrt(x*(x**2 - x - 1))/((x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.1543 $\int \frac{(b+ax^3)\sqrt{x+x^4}}{-d+cx^3}\,dx$

Optimal. Leaf size=125

$$\frac{2\sqrt{-c-d}\,(ad+bc)\tan^{-1}\left(\frac{x\sqrt{x^4+x}\,\sqrt{-c-d}}{\sqrt{d}\,(x+1)(x^2-x+1)}\right)}{3c^2\sqrt{d}}+\frac{\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)(ac+2ad+2bc)}{3c^2}+\frac{a\sqrt{x^4+x}\,x}{3c}$$

Rubi [A] time = 0.35, antiderivative size = 144, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2056, 581, 584, 329, 275, 215, 466, 465, 377, 208}

$$\frac{\sqrt{x^4+x}\,\sinh^{-1}\left(x^{3/2}\right)(a(c+2d)+2bc)}{3c^2\sqrt{x^3+1}\,\sqrt{x}}-\frac{2\sqrt{x^4+x}\,\sqrt{c+d}\,(ad+bc)\tanh^{-1}\left(\frac{x^{3/2}\sqrt{c+d}}{\sqrt{d}\,\sqrt{x^3+1}}\right)}{3c^2\sqrt{d}\,\sqrt{x^3+1}\,\sqrt{x}}+\frac{a\sqrt{x^4+x}\,x}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[((b + a*x^3)*Sqrt[x + x^4])/(-d + c*x^3),x]
```

```
[Out] (a*x*Sqrt[x + x^4])/(3*c) + ((2*b*c + a*(c + 2*d))*Sqrt[x + x^4]*ArcSinh[x^
(3/2)])/(3*c^2*Sqrt[x]*Sqrt[1 + x^3]) - (2*Sqrt[c + d]*(b*c + a*d)*Sqrt[x +
 x^4]*ArcTanh[(Sqrt[c + d]*x^(3/2))/(Sqrt[d]*Sqrt[1 + x^3])])/(3*c^2*Sqrt[d
]*Sqrt[x]*Sqrt[1 + x^3])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
```

```
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\left(b+ax^3\right)\sqrt{x+x^4}}{-d+cx^3}\,dx = \frac{\sqrt{x+x^4}\int \frac{\sqrt{x}\sqrt{1+x^3}\left(b+ax^3\right)}{-d+cx^3}\,dx}{\sqrt{x}\sqrt{1+x^3}}$$

$$= \frac{ax\sqrt{x+x^4}}{3c} + \frac{\sqrt{x+x^4}\int \frac{\sqrt{x}\left(\frac{3}{2}(2bc+ad)+\frac{3}{2}(2bc+a(c+2d))x^3\right)}{\sqrt{1+x^3}\left(-d+cx^3\right)}\,dx}{3c\sqrt{x}\sqrt{1+x^3}}$$

$$= \frac{ax\sqrt{x+x^4}}{3c} + \frac{\sqrt{x+x^4}\int \left(\frac{3(2bc+a(c+2d))\sqrt{x}}{2c\sqrt{1+x^3}} + \frac{\left(\frac{3}{2}c(2bc+ad)+\frac{3}{2}d(2bc+a(c+2d))\right)\sqrt{x}}{c\sqrt{1+x^3}\left(-d+cx^3\right)}\right)dx}{3c\sqrt{x}\sqrt{1+x^3}}$$

$$= \frac{ax\sqrt{x+x^4}}{3c} + \frac{\left((c+d)(bc+ad)\sqrt{x+x^4}\right)\int \frac{\sqrt{x}}{\sqrt{1+x^3}\left(-d+cx^3\right)}\,dx}{c^2\sqrt{x}\sqrt{1+x^3}} + \frac{\left((2bc+a(c+2d))\right.}{2c^2\sqrt{x}}$$

$$= \frac{ax\sqrt{x+x^4}}{3c} + \frac{\left(2(c+d)(bc+ad)\sqrt{x+x^4}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^6}\left(-d+cx^6\right)}\,dx, x, \sqrt{x}\right)}{c^2\sqrt{x}\sqrt{1+x^3}} + \frac{(}{}$$

$$= \frac{ax\sqrt{x+x^4}}{3c} + \frac{\left(2(c+d)(bc+ad)\sqrt{x+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\left(-d+cx^2\right)}\,dx, x, x^{3/2}\right)}{3c^2\sqrt{x}\sqrt{1+x^3}} + \frac{(}{}$$

$$= \frac{ax\sqrt{x+x^4}}{3c} + \frac{(2bc+a(c+2d))\sqrt{x+x^4}\sinh^{-1}\left(x^{3/2}\right)}{3c^2\sqrt{x}\sqrt{1+x^3}} + \frac{\left(2(c+d)(bc+ad)\sqrt{x+x}\right.}{3c}$$

$$= \frac{ax\sqrt{x+x^4}}{3c} + \frac{(2bc+a(c+2d))\sqrt{x+x^4}\sinh^{-1}\left(x^{3/2}\right)}{3c^2\sqrt{x}\sqrt{1+x^3}} - \frac{2\sqrt{c+d}\,(bc+ad)\sqrt{x+x^4}}{3c^2\sqrt{d}\sqrt{x}\sqrt{}}$$

Mathematica [C] time = 0.35, size = 170, normalized size = 1.36

$$-\frac{x\sqrt{x^4+x}\left(x^3\sqrt{-\frac{x^3(c+d)}{d}}(a(c+2d)+2bc)F_1\left(\frac{3}{2};\frac{1}{2},1;\frac{5}{2};-x^3,\frac{cx^3}{d}\right)+3(ad+2bc)\sin^{-1}\left(\frac{\sqrt{-\frac{x^3(c+d)}{d}}}{\sqrt{1-\frac{cx^3}{d}}}\right)-3ad\sqrt{x^3+1}\sqrt{-\frac{x^3(c+d)}{d}}\right)}{9cd\sqrt{x^3+1}\sqrt{-\frac{x^3(c+d)}{d}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((b + a*x^3)*Sqrt[x + x^4])/(-d + c*x^3),x]
```

```
[Out] -1/9*(x*Sqrt[x + x^4]*(-3*a*d*Sqrt[-(((c + d)*x^3)/d)]*Sqrt[1 + x^3] + (2*b
*c + a*(c + 2*d))*x^3*Sqrt[-(((c + d)*x^3)/d)]*AppellF1[3/2, 1/2, 1, 5/2, -
x^3, (c*x^3)/d] + 3*(2*b*c + a*d)*ArcSin[Sqrt[-(((c + d)*x^3)/d)]/Sqrt[1 -
(c*x^3)/d]]))/(c*d*Sqrt[-(((c + d)*x^3)/d)]*Sqrt[1 + x^3])
```

IntegrateAlgebraic [A] time = 0.76, size = 125, normalized size = 1.00

$$\frac{2\sqrt{-c-d}\,(ad+bc)\tan^{-1}\left(\frac{x\sqrt{x^4+x}\sqrt{-c-d}}{\sqrt{d}\,(x+1)\left(x^2-x+1\right)}\right)}{3c^2\sqrt{d}} + \frac{\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)(ac+2ad+2bc)}{3c^2} + \frac{a\sqrt{x^4+x}\,x}{3c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b + a*x^3)*Sqrt[x + x^4])/(-d + c*x^3),x]
```

```
[Out] (a*x*Sqrt[x + x^4])/(3*c) + (2*Sqrt[-c - d]*(b*c + a*d)*ArcTan[(Sqrt[-c - d
]*x*Sqrt[x + x^4])/(Sqrt[d]*(1 + x)*(1 - x + x^2))])/(3*c^2*Sqrt[d]) + ((a*
c + 2*b*c + 2*a*d)*ArcTanh[x^2/Sqrt[x + x^4]])/(3*c^2)
```

fricas [A] time = 4.06, size = 266, normalized size = 2.13

$$\left[\frac{2\sqrt{x^4+x}\,acx+(bc+ad)\sqrt{\frac{c+d}{d}}\log\left(-\frac{(c^2+8cd+8d^2)x^6+2(3cd+4d^2)x^3+d^2-4\left((cd+2d^2)x^4+d^2x\right)\sqrt{x^4+x}\sqrt{\frac{c+d}{d}}}{c^2x^6-2cdx^3+d^2}\right)+((a+2b)c+2ad)\log\left(-2x^3-2\sqrt{x^4+x}\,x-1\right)}{6c^2}, \frac{2\sqrt{x^4+x}\,acx+2(bc+ad)\sqrt{-\frac{c+d}{d}}\arctan\left(\frac{2\sqrt{x^4+x}\,dx\sqrt{-\frac{c+d}{d}}}{(c+2d)x^3+d}\right)+((a+2b)c+2ad)\log\left(-2x^3-2\sqrt{x^4+x}\,x-1\right)}{6c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)*(x^4+x)^(1/2)/(c*x^3-d),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(x^4 + x)*a*c*x + (b*c + a*d)*sqrt((c + d)/d)*log(-((c^2 + 8*c*
d + 8*d^2)*x^6 + 2*(3*c*d + 4*d^2)*x^3 + d^2 - 4*((c*d + 2*d^2)*x^4 + d^2*x
)*sqrt(x^4 + x)*sqrt((c + d)/d))/(c^2*x^6 - 2*c*d*x^3 + d^2)) + ((a + 2*b)*
c + 2*a*d)*log(-2*x^3 - 2*sqrt(x^4 + x)*x - 1))/c^2, 1/6*(2*sqrt(x^4 + x)*a
*c*x + 2*(b*c + a*d)*sqrt(-(c + d)/d)*arctan(2*sqrt(x^4 + x)*d*x*sqrt(-(c +
 d)/d)/((c + 2*d)*x^3 + d)) + ((a + 2*b)*c + 2*a*d)*log(-2*x^3 - 2*sqrt(x^4
 + x)*x - 1))/c^2]
```

giac [A] time = 0.57, size = 128, normalized size = 1.02

$$\frac{\sqrt{x^4+x}\,ax}{3c}+\frac{(ac+2bc+2ad)\log\left(\sqrt{\frac{1}{x^3}+1}+1\right)}{6c^2}-\frac{(ac+2bc+2ad)\log\left(\left|\sqrt{\frac{1}{x^3}+1}-1\right|\right)}{6c^2}+\frac{2\left(bc^2+acd+bcd+ad^2\right)\arctan\left(\frac{d\sqrt{\frac{1}{x^3}+1}}{\sqrt{-cd-d^2}}\right)}{3\sqrt{-cd-d^2}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)*(x^4+x)^(1/2)/(c*x^3-d),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(x^4 + x)*a*x/c + 1/6*(a*c + 2*b*c + 2*a*d)*log(sqrt(1/x^3 + 1) + 1
)/c^2 - 1/6*(a*c + 2*b*c + 2*a*d)*log(abs(sqrt(1/x^3 + 1) - 1))/c^2 + 2/3*(
b*c^2 + a*c*d + b*c*d + a*d^2)*arctan(d*sqrt(1/x^3 + 1)/sqrt(-c*d - d^2))/(
sqrt(-c*d - d^2)*c^2)
```

maple [C] time = 0.39, size = 970, normalized size = 7.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b)*(x^4+x)^(1/2)/(c*x^3-d),x)
```

```
[Out] a/c*(1/3*x*(x^4+x)^(1/2)-(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1
/2*I*3^(1/2))/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/
2))/(1+x))^(1/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(
3/2+1/2*I*3^(1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1
/2)*(-EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-
3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2))+EllipticPi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))
^(1/2),(1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-
1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))+(a*d+b*c
)/c*(-2/c*(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(
1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))/(1+x))^(1/2
)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(3/2+1/2*I*3^(1/
2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1/2)*(-EllipticF
(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-3/2+1/2*I*3^(1/
2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+
EllipticPi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),(1/2+1/2
*I*3^(1/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/
(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))-2/3/c*4^(1/2)*sum((-c-d)
/_alpha*(1+x)^2*(_alpha^2-_alpha+1)/(c+d)*(-1-I*3^(1/2))*(x/(1+x)*(I*3^(1/2
```

```
)+3)/(1+I*3^(1/2)))^(1/2)*(-1/(1+x)*(-1+2*x+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)
*(-1/(1+x)*(-1+2*x-I*3^(1/2))/(1+I*3^(1/2)))^(1/2)/(I*3^(1/2)+3)/(x*(1+x)*(
-1+2*x+I*3^(1/2))*(-1+2*x-I*3^(1/2)))^(1/2)*(EllipticF(((3/2+1/2*I*3^(1/2))
*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/
2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+_alpha^2*c/d*Elliptic
Pi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),1/6*(I*_alpha^2*
3^(1/2)*c+3*_alpha^2*c+I*3^(1/2)*d+3*d)/d,((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I
*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))),_alpha=RootOf(
_Z^3*c-d)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^3+b\right)\sqrt{x^4+x}}{cx^3-d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)*(x^4+x)^(1/2)/(c*x^3-d),x, algorithm="maxima")
```

```
[Out] integrate((a*x^3 + b)*sqrt(x^4 + x)/(c*x^3 - d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(a\,x^3+b\right)\,\sqrt{x^4+x}}{d-c\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b + a*x^3)*(x + x^4)^(1/2))/(d - c*x^3),x)
```

```
[Out] int(-((b + a*x^3)*(x + x^4)^(1/2))/(d - c*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(x+1\right)\left(x^2-x+1\right)}\,\left(ax^3+b\right)}{cx^3-d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b)*(x**4+x)**(1/2)/(c*x**3-d),x)
```

```
[Out] Integral(sqrt(x*(x + 1)*(x**2 - x + 1))*(a*x**3 + b)/(c*x**3 - d), x)
```

3.1544 $\int \frac{1-2x+2x^4}{x\sqrt[4]{-1+x^4}}\,dx$

Optimal. Leaf size=125

$$\frac{2}{3}\left(x^4-1\right)^{3/4}+\tan^{-1}\left(\frac{\sqrt[4]{x^4-1}}{x}\right)+\frac{\tan^{-1}\left(\frac{\frac{\sqrt{x^4-1}}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{2}}-\tanh^{-1}\left(\frac{\sqrt[4]{x^4-1}}{x}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+1}\right)}{2\sqrt{2}}$$

Rubi [A] time = 0.14, antiderivative size = 170, normalized size of antiderivative = 1.36, number of steps used = 18, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {1833, 240, 212, 206, 203, 446, 80, 63, 297, 1162, 617, 204, 1165, 628}

$$\frac{2}{3}\left(x^4-1\right)^{3/4}+\frac{\log\left(\sqrt{x^4-1}-\sqrt{2}\sqrt[4]{x^4-1}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{x^4-1}+\sqrt{2}\sqrt[4]{x^4-1}+1\right)}{4\sqrt{2}}-\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{x^4-1}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt[4]{x^4-1}+1\right)}{2\sqrt{2}}-\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x + 2*x^4)/(x*(-1 + x^4)^(1/4)),x]
```

```
[Out] (2*(-1 + x^4)^(3/4))/3 - ArcTan[x/(-1 + x^4)^(1/4)] - ArcTan[1 - Sqrt[2]*(-
1 + x^4)^(1/4)]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*(-1 + x^4)^(1/4)]/(2*Sqrt[
2]) - ArcTanh[x/(-1 + x^4)^(1/4)] + Log[1 - Sqrt[2]*(-1 + x^4)^(1/4) + Sqrt
[-1 + x^4]]/(4*Sqrt[2]) - Log[1 + Sqrt[2]*(-1 + x^4)^(1/4) + Sqrt[-1 + x^4]
]/(4*Sqrt[2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-2x+2x^4}{x\sqrt[4]{-1+x^4}}\,dx &= \int \left(-\frac{2}{\sqrt[4]{-1+x^4}}+\frac{1+2x^4}{x\sqrt[4]{-1+x^4}}\right) dx\\
&= -\left(2\int \frac{1}{\sqrt[4]{-1+x^4}}\,dx\right)+\int \frac{1+2x^4}{x\sqrt[4]{-1+x^4}}\,dx\\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{1+2x}{\sqrt[4]{-1+x}\,x}\,dx,x,x^4\right)-2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)\\
&= \frac{2}{3}\left(-1+x^4\right)^{3/4}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-1+x}\,x}\,dx,x,x^4\right)-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)-\text{S}\\
&= \frac{2}{3}\left(-1+x^4\right)^{3/4}-\tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)-\tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)+\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt[4]{-1+}\right.\\
&= \frac{2}{3}\left(-1+x^4\right)^{3/4}-\tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)-\tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt[4]{-1}\right.\\
&= \frac{2}{3}\left(-1+x^4\right)^{3/4}-\tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)-\tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx\right.\\
&= \frac{2}{3}\left(-1+x^4\right)^{3/4}-\tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)-\tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)+\frac{\log\left(1-\sqrt{2}\sqrt[4]{-1+x^4}+\sqrt{-1}\right.}{4\sqrt{2}}\\
&= \frac{2}{3}\left(-1+x^4\right)^{3/4}-\tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+x^4}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+x^4}\right.}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 70, normalized size = 0.56

$$
\frac{1}{3}\left(x^4-1\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};1-x^4\right)+\frac{2}{3}\left(x^4-1\right)^{3/4}-\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x + 2*x^4)/(x*(-1 + x^4)^(1/4)),x]
```

```
[Out] (2*(-1 + x^4)^(3/4))/3 - ArcTan[x/(-1 + x^4)^(1/4)] - ArcTanh[x/(-1 + x^4)^
(1/4)] + ((-1 + x^4)^(3/4)*Hypergeometric2F1[3/4, 1, 7/4, 1 - x^4])/3
```

IntegrateAlgebraic [A] time = 4.73, size = 120, normalized size = 0.96

$$
\frac{2}{3}\left(x^4-1\right)^{3/4}+\tan^{-1}\left(\frac{\sqrt[4]{x^4-1}}{x}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^4-1}}{\sqrt{x^4-1}-1}\right)}{2\sqrt{2}}-\tanh^{-1}\left(\frac{\sqrt[4]{x^4-1}}{x}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+1}\right)}{2\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - 2*x + 2*x^4)/(x*(-1 + x^4)^(1/4)),x]
```

```
[Out] (2*(-1 + x^4)^(3/4))/3 + ArcTan[(-1 + x^4)^(1/4)/x] - ArcTan[(Sqrt[2]*(-1 +
 x^4)^(1/4))/(-1 + Sqrt[-1 + x^4])]/(2*Sqrt[2]) - ArcTanh[(-1 + x^4)^(1/4)/
x] - ArcTanh[(Sqrt[2]*(-1 + x^4)^(1/4))/(1 + Sqrt[-1 + x^4])]/(2*Sqrt[2])
```

fricas [B] time = 17.87, size = 517, normalized size = 4.14

$$\frac{1}{4}\sqrt{2}\arctan\left(-\frac{x^{8}+4\sqrt{x^{4}-1}x^{4}+2\sqrt{2}\left(x^{4}-1\right)^{\frac{3}{4}}\left(x^{4}-4\right)+2\sqrt{2}\left(3x^{4}-4\right)\left(x^{4}-1\right)^{\frac{1}{4}}-\left(4\left(x^{4}-1\right)^{\frac{1}{4}}x^{4}+2\sqrt{2}\sqrt{x^{4}-1}\left(x^{4}-4\right)+\sqrt{2}\left(x^{8}-10x^{4}+8\right)+16\left(x^{4}-1\right)^{\frac{3}{4}}\right)\sqrt{\frac{x^{4}+2\sqrt{2}\left(x^{4}-1\right)^{\frac{3}{4}}+2\sqrt{2}\left(x^{4}-1\right)^{\frac{1}{4}}+4\sqrt{x^{4}-1}}{x^{4}}}}{x^{8}-16x^{4}+16}\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{x^{8}+4\sqrt{x^{4}-1}x^{4}-2\sqrt{2}\left(x^{4}-1\right)^{\frac{3}{4}}\left(x^{4}-4\right)-2\sqrt{2}\left(3x^{4}-4\right)\left(x^{4}-1\right)^{\frac{1}{4}}-\left(4\left(x^{4}-1\right)^{\frac{1}{4}}x^{4}-2\sqrt{2}\sqrt{x^{4}-1}\left(x^{4}-4\right)-\sqrt{2}\left(x^{8}-10x^{4}+8\right)+16\left(x^{4}-1\right)^{\frac{3}{4}}\right)\sqrt{\frac{x^{4}-2\sqrt{2}\left(x^{4}-1\right)^{\frac{3}{4}}-2\sqrt{2}\left(x^{4}-1\right)^{\frac{1}{4}}+4\sqrt{x^{4}-1}}{x^{4}}}}{x^{8}-16x^{4}+16}\right)-\frac{1}{16}\sqrt{2}\log\left(\frac{4\left(x^{4}+2\sqrt{2}\left(x^{4}-1\right)^{\frac{3}{4}}+2\sqrt{2}\left(x^{4}-1\right)^{\frac{1}{4}}+4\sqrt{x^{4}-1}\right)}{x^{4}}\right)+\frac{1}{16}\sqrt{2}\log\left(\frac{4\left(x^{4}-2\sqrt{2}\left(x^{4}-1\right)^{\frac{3}{4}}-2\sqrt{2}\left(x^{4}-1\right)^{\frac{1}{4}}+4\sqrt{x^{4}-1}\right)}{x^{4}}\right)+\frac{2}{3}\left(x^{4}-1\right)^{\frac{3}{4}}+\frac{1}{2}\arctan\left(2\left(x^{4}-1\right)^{\frac{1}{4}}x^{3}+2\left(x^{4}-1\right)^{\frac{3}{4}}x\right)+\frac{1}{2}\log\left(-2x^{4}+2\left(x^{4}-1\right)^{\frac{1}{4}}x^{3}-2\sqrt{x^{4}-1}x^{2}+2\left(x^{4}-1\right)^{\frac{3}{4}}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x+1)/x/(x^4-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(-(x^8 + 4*sqrt(x^4 - 1)*x^4 + 2*sqrt(2)*(x^4 - 1)^(3/4)*
(x^4 - 4) + 2*sqrt(2)*(3*x^4 - 4)*(x^4 - 1)^(1/4) - (4*(x^4 - 1)^(1/4)*x^4
+ 2*sqrt(2)*sqrt(x^4 - 1)*(x^4 - 4) + sqrt(2)*(x^8 - 10*x^4 + 8) + 16*(x^4
- 1)^(3/4))*sqrt((x^4 + 2*sqrt(2)*(x^4 - 1)^(3/4) + 2*sqrt(2)*(x^4 - 1)^(1/
4) + 4*sqrt(x^4 - 1))/x^4))/(x^8 - 16*x^4 + 16)) - 1/4*sqrt(2)*arctan(-(x^8
 + 4*sqrt(x^4 - 1)*x^4 - 2*sqrt(2)*(x^4 - 1)^(3/4)*(x^4 - 4) - 2*sqrt(2)*(3
*x^4 - 4)*(x^4 - 1)^(1/4) - (4*(x^4 - 1)^(1/4)*x^4 - 2*sqrt(2)*sqrt(x^4 - 1
)*(x^4 - 4) - sqrt(2)*(x^8 - 10*x^4 + 8) + 16*(x^4 - 1)^(3/4))*sqrt((x^4 -
2*sqrt(2)*(x^4 - 1)^(3/4) - 2*sqrt(2)*(x^4 - 1)^(1/4) + 4*sqrt(x^4 - 1))/x^
4))/(x^8 - 16*x^4 + 16)) - 1/16*sqrt(2)*log(4*(x^4 + 2*sqrt(2)*(x^4 - 1)^(3
/4) + 2*sqrt(2)*(x^4 - 1)^(1/4) + 4*sqrt(x^4 - 1))/x^4) + 1/16*sqrt(2)*log(
4*(x^4 - 2*sqrt(2)*(x^4 - 1)^(3/4) - 2*sqrt(2)*(x^4 - 1)^(1/4) + 4*sqrt(x^4
 - 1))/x^4) + 2/3*(x^4 - 1)^(3/4) + 1/2*arctan(2*(x^4 - 1)^(1/4)*x^3 + 2*(x
^4 - 1)^(3/4)*x) + 1/2*log(-2*x^4 + 2*(x^4 - 1)^(1/4)*x^3 - 2*sqrt(x^4 - 1)
*x^2 + 2*(x^4 - 1)^(3/4)*x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^{4}-2\,x+1}{\left(x^{4}-1\right)^{\frac{1}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x+1)/x/(x^4-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((2*x^4 - 2*x + 1)/((x^4 - 1)^(1/4)*x), x)
```

maple [C] time = 0.32, size = 142, normalized size = 1.14

$$\frac{\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)\left(-\mathrm{signum}\left(x^{4}-1\right)\right)^{\frac{1}{4}}\left(\frac{\pi\sqrt{2}\,x^{4}\,\mathrm{hypergeom}\left(\left[1,1,\frac{5}{4}\right],[2,2],x^{4}\right)}{4\Gamma\left(\frac{3}{4}\right)}+\frac{\left(-3\ln(2)-\frac{\pi}{2}+4\ln(x)+i\pi\right)\pi\sqrt{2}}{\Gamma\left(\frac{3}{4}\right)}\right)}{8\pi\,\mathrm{signum}\left(x^{4}-1\right)^{\frac{1}{4}}}+\frac{\left(-\mathrm{signum}\left(x^{4}-1\right)\right)^{\frac{1}{4}}x^{4}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},1\right],[2],x^{4}\right)}{2\,\mathrm{signum}\left(x^{4}-1\right)^{\frac{1}{4}}}-\frac{2\left(-\mathrm{signum}\left(x^{4}-1\right)\right)^{\frac{1}{4}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{4}\right],\left[\frac{5}{4}\right],x^{4}\right)}{\mathrm{signum}\left(x^{4}-1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-2*x+1)/x/(x^4-1)^(1/4),x)
```

```
[Out] 1/8/Pi*2^(1/2)*GAMMA(3/4)/signum(x^4-1)^(1/4)*(-signum(x^4-1))^(1/4)*(1/4*P
i*2^(1/2)/GAMMA(3/4)*x^4*hypergeom([1,1,5/4],[2,2],x^4)+(-3*ln(2)-1/2*Pi+4*
ln(x)+I*Pi)*Pi*2^(1/2)/GAMMA(3/4))+1/2/signum(x^4-1)^(1/4)*(-signum(x^4-1))
^(1/4)*x^4*hypergeom([1/4,1],[2],x^4)-2/signum(x^4-1)^(1/4)*(-signum(x^4-1)
)^(1/4)*x*hypergeom([1/4,1/4],[5/4],x^4)
```

maxima [A] time = 0.43, size = 155, normalized size = 1.24

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(x^{4}-1\right)^{\frac{1}{4}}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(x^{4}-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}\left(x^{4}-1\right)^{\frac{1}{4}}+\sqrt{x^{4}-1}+1\right)+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}\left(x^{4}-1\right)^{\frac{1}{4}}+\sqrt{x^{4}-1}+1\right)+\frac{2}{3}\left(x^{4}-1\right)^{\frac{3}{4}}+\arctan\left(\frac{\left(x^{4}-1\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{2}\log\left(\frac{\left(x^{4}-1\right)^{\frac{1}{4}}}{x}+1\right)+\frac{1}{2}\log\left(\frac{\left(x^{4}-1\right)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-2*x+1)/x/(x^4-1)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(x^4 - 1)^(1/4))) + 1/4*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(x^4 - 1)^(1/4))) - 1/8*sqrt(2)*log(sqrt(
2)*(x^4 - 1)^(1/4) + sqrt(x^4 - 1) + 1) + 1/8*sqrt(2)*log(-sqrt(2)*(x^4 - 1
```

```
)^(1/4) + sqrt(x^4 - 1) + 1) + 2/3*(x^4 - 1)^(3/4) + arctan((x^4 - 1)^(1/4)
/x) - 1/2*log((x^4 - 1)^(1/4)/x + 1) + 1/2*log((x^4 - 1)^(1/4)/x - 1)
```

mupad [B] time = 1.30, size = 81, normalized size = 0.65

$$\frac{2\left(x^4-1\right)^{3/4}}{3}-\frac{2\,x\left(1-x^4\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ x^4\right)}{\left(x^4-1\right)^{1/4}}+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \left(x^4-1\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \left(x^4-1\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4 - 2*x + 1)/(x*(x^4 - 1)^(1/4)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*(x^4 - 1)^(1/4)*(1/2 - 1i/2))*(1/4 - 1i/4) + 2^(1/2)*a
tan(2^(1/2)*(x^4 - 1)^(1/4)*(1/2 + 1i/2))*(1/4 + 1i/4) + (2*(x^4 - 1)^(3/4)
)/3 - (2*x*(1 - x^4)^(1/4)*hypergeom([1/4, 1/4], 5/4, x^4))/(x^4 - 1)^(1/4)
```

sympy [C] time = 3.18, size = 71, normalized size = 0.57

$$-\frac{xe^{-\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|x^4\right)}{2\Gamma\left(\frac{5}{4}\right)}+\frac{2\left(x^4-1\right)^{\frac{3}{4}}}{3}-\frac{\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{e^{2i\pi}}{x^4}\right)}{4x\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-2*x+1)/x/(x**4-1)**(1/4),x)
```

```
[Out] -x*exp(-I*pi/4)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), x**4)/(2*gamma(5/4)) +
 2*(x**4 - 1)**(3/4)/3 - gamma(1/4)*hyper((1/4, 1/4), (5/4,), exp_polar(2*I
*pi)/x**4)/(4*x*gamma(5/4))
```

3.1545 $\int \frac{x^2(-4b+ax^5)}{(b+ax^5)^{3/4}(b+cx^4+ax^5)}\,dx$

Optimal. Leaf size=125

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5+b}}{\sqrt{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5+b}}\right)}{c^{3/4}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x\sqrt[4]{ax^5+b}}{\sqrt{ax^5+b}-\sqrt{c}x^2}\right)}{c^{3/4}}$$

Rubi [F] time = 2.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(-4b+ax^5\right)}{\left(b+ax^5\right)^{3/4}\left(b+cx^4+ax^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(-4*b + a*x^5))/((b + a*x^5)^(3/4)*(b + c*x^4 + a*x^5)),x]
```

```
[Out] (c^2*x*(1 + (a*x^5)/b)^(3/4)*Hypergeometric2F1[1/5, 3/4, 6/5, -((a*x^5)/b)]
)/(a^2*(b + a*x^5)^(3/4)) - (c*x^2*(1 + (a*x^5)/b)^(3/4)*Hypergeometric2F1[
2/5, 3/4, 7/5, -((a*x^5)/b)])/(2*a*(b + a*x^5)^(3/4)) + (x^3*(1 + (a*x^5)/b
)^(3/4)*Hypergeometric2F1[3/5, 3/4, 8/5, -((a*x^5)/b)])/(3*(b + a*x^5)^(3/4
)) - (b*c^2*Defer[Int][1/((b + a*x^5)^(3/4)*(b + c*x^4 + a*x^5)), x])/a^2 +
 (b*c*Defer[Int][x/((b + a*x^5)^(3/4)*(b + c*x^4 + a*x^5)), x])/a - 5*b*Def
er[Int][x^2/((b + a*x^5)^(3/4)*(b + c*x^4 + a*x^5)), x] - (c^3*Defer[Int][x
^4/((b + a*x^5)^(3/4)*(b + c*x^4 + a*x^5)), x])/a^2
```

Rubi steps

$$\int \frac{x^2\left(-4b+ax^5\right)}{\left(b+ax^5\right)^{3/4}\left(b+cx^4+ax^5\right)}\,dx=\int\left(\frac{c^2}{a^2\left(b+ax^5\right)^{3/4}}-\frac{cx}{a\left(b+ax^5\right)^{3/4}}+\frac{x^2}{\left(b+ax^5\right)^{3/4}}-\frac{bc^2-abcx+5a^2b}{a^2\left(b+ax^5\right)^{3/4}\left(b+\right.}\right.$$

$$=-\frac{\int\frac{bc^2-abcx+5a^2bx^2+c^3x^4}{\left(b+ax^5\right)^{3/4}\left(b+cx^4+ax^5\right)}\,dx}{a^2}-\frac{c\int\frac{x}{\left(b+ax^5\right)^{3/4}}\,dx}{a}+\frac{c^2\int\frac{1}{\left(b+ax^5\right)^{3/4}}\,dx}{a^2}+\int\frac{}{\left(b+\right.}$$

$$=-\frac{\int\left(\frac{bc^2}{\left(b+ax^5\right)^{3/4}\left(b+cx^4+ax^5\right)}-\frac{abcx}{\left(b+ax^5\right)^{3/4}\left(b+cx^4+ax^5\right)}+\frac{5a^2bx^2}{\left(b+ax^5\right)^{3/4}\left(b+cx^4+ax^5\right)}+\frac{}{\left(b+ax\right.}\right.}{a^2}$$

$$=\frac{c^2x\left(1+\frac{ax^5}{b}\right)^{3/4}{}_2F_1\left(\frac{1}{5},\frac{3}{4};\frac{6}{5};-\frac{ax^5}{b}\right)}{a^2\left(b+ax^5\right)^{3/4}}-\frac{cx^2\left(1+\frac{ax^5}{b}\right)^{3/4}{}_2F_1\left(\frac{2}{5},\frac{3}{4};\frac{7}{5};-\frac{ax^5}{b}\right)}{2a\left(b+ax^5\right)^{3/4}}-$$

Mathematica [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-4b+ax^5\right)}{\left(b+ax^5\right)^{3/4}\left(b+cx^4+ax^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(-4*b + a*x^5))/((b + a*x^5)^(3/4)*(b + c*x^4 + a*x^5)),x]
```

```
[Out] Integrate[(x^2*(-4*b + a*x^5))/((b + a*x^5)^(3/4)*(b + c*x^4 + a*x^5)), x]
```

IntegrateAlgebraic [A] time = 13.12, size = 125, normalized size = 1.00

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5+b}}{\sqrt{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5+b}}\right)}{c^{3/4}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x\sqrt[4]{ax^5+b}}{\sqrt{ax^5+b}-\sqrt{c}x^2}\right)}{c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(-4*b + a*x^5))/((b + a*x^5)^(3/4)*(b + c*x^4 + a*x
^5)),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[2]*c^(1/4)*x*(b + a*x^5)^(1/4))/(-(Sqrt[c]*x^2) + S
qrt[b + a*x^5])])/c^(3/4)) + (Sqrt[2]*ArcTanh[((c^(1/4)*x^2)/Sqrt[2] + Sqrt
[b + a*x^5]/(Sqrt[2]*c^(1/4)))/(x*(b + a*x^5)^(1/4))])/c^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^5-4*b)/(a*x^5+b)^(3/4)/(a*x^5+c*x^4+b),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^5-4\,b\right)x^2}{\left(ax^5+cx^4+b\right)\left(ax^5+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^5-4*b)/(a*x^5+b)^(3/4)/(a*x^5+c*x^4+b),x, algorithm="gia
c")
```

```
[Out] integrate((a*x^5 - 4*b)*x^2/((a*x^5 + c*x^4 + b)*(a*x^5 + b)^(3/4)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a\,x^5-4b\right)}{\left(a\,x^5+b\right)^{\frac{3}{4}}\left(a\,x^5+c\,x^4+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^5-4*b)/(a*x^5+b)^(3/4)/(a*x^5+c*x^4+b),x)
```

```
[Out] int(x^2*(a*x^5-4*b)/(a*x^5+b)^(3/4)/(a*x^5+c*x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^5-4\,b\right)x^2}{\left(ax^5+cx^4+b\right)\left(ax^5+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^5-4*b)/(a*x^5+b)^(3/4)/(a*x^5+c*x^4+b),x, algorithm="max
ima")
```

```
[Out] integrate((a*x^5 - 4*b)*x^2/((a*x^5 + c*x^4 + b)*(a*x^5 + b)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2\,\left(4\,b-a\,x^5\right)}{{\left(a\,x^5+b\right)}^{3/4}\,\left(a\,x^5+c\,x^4+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(4*b - a*x^5))/((b + a*x^5)^(3/4)*(b + a*x^5 + c*x^4)),x)
```

```
[Out] -int((x^2*(4*b - a*x^5))/((b + a*x^5)^(3/4)*(b + a*x^5 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a x^{5} - 4 b\right)}{\left(a x^{5} + b\right)^{\frac{3}{4}} \left(a x^{5} + b + c x^{4}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x**5-4*b)/(a*x**5+b)**(3/4)/(a*x**5+c*x**4+b),x)
```

```
[Out] Integral(x**2*(a*x**5 - 4*b)/((a*x**5 + b)**(3/4)*(a*x**5 + b + c*x**4)), x
)
```

3.1546 $\int \frac{1}{\sqrt[6]{1+2x-x^2-4x^3-x^4+2x^5+x^6}}\,dx$

Optimal. Leaf size=125

$$\frac{\sqrt[3]{x-1}\,(x+1)^{2/3}\left(-\log\left(\sqrt[3]{x-1}-\sqrt[3]{x+1}\right)+\frac{1}{2}\log\left((x-1)^{2/3}+\sqrt[3]{x+1}\,\sqrt[3]{x-1}+(x+1)^{2/3}\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{\ldots}}{2\sqrt[3]{x-1}\ldots}\right.\right.}{\sqrt[6]{(x-1)^2(x+1)^4}}$$

Rubi [A] time = 0.04, antiderivative size = 159, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {6688, 6719, 59}

$$-\frac{\sqrt[3]{x-1}\,(x+1)^{2/3}\log(x+1)}{2\sqrt[6]{(1-x)^2(x+1)^4}}-\frac{3\sqrt[3]{x-1}\,(x+1)^{2/3}\log\left(\frac{\sqrt[3]{x-1}}{\sqrt[3]{x+1}}-1\right)}{2\sqrt[6]{(1-x)^2(x+1)^4}}-\frac{\sqrt{3}\,\sqrt[3]{x-1}\,(x+1)^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{x-1}}{\sqrt{3}\,\sqrt[3]{x+1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[6]{(1-x)^2(x+1)^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x - x^2 - 4*x^3 - x^4 + 2*x^5 + x^6)^(-1/6),x]
```

```
[Out] -((Sqrt[3]*(-1 + x)^(1/3)*(1 + x)^(2/3)*ArcTan[1/Sqrt[3] + (2*(-1 + x)^(1/3
))/(Sqrt[3]*(1 + x)^(1/3))])/((1 - x)^2*(1 + x)^4)^(1/6)) - ((-1 + x)^(1/3)
*(1 + x)^(2/3)*Log[1 + x])/(2*((1 - x)^2*(1 + x)^4)^(1/6)) - (3*(-1 + x)^(1
/3)*(1 + x)^(2/3)*Log[-1 + (-1 + x)^(1/3)/(1 + x)^(1/3)])/(2*((1 - x)^2*(1
+ x)^4)^(1/6))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[6]{1+2x-x^2-4x^3-x^4+2x^5+x^6}}\,dx &= \int \frac{1}{\sqrt[6]{(-1+x)^2(1+x)^4}}\,dx\\ &= \frac{\left(\sqrt[3]{-1+x}\,(1+x)^{2/3}\right)\int \frac{1}{\sqrt[3]{-1+x}\,(1+x)^{2/3}}\,dx}{\sqrt[6]{(-1+x)^2(1+x)^4}}\\ &= -\frac{\sqrt{3}\,\sqrt[3]{-1+x}\,(1+x)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-1+x}}{\sqrt{3}\,\sqrt[3]{1+x}}\right)}{\sqrt[6]{(1-x)^2(1+x)^4}}-\frac{\sqrt[3]{-1+x}\,(1+x)^{2/3}\ldots}{2\sqrt[6]{(1-x)^2(1\ldots}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 53, normalized size = 0.42

$$\frac{3(x-1)(x+1)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1-x}{2}\right)}{2\,2^{2/3}\sqrt[6]{(x-1)^2(x+1)^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x - x^2 - 4*x^3 - x^4 + 2*x^5 + x^6)^(-1/6),x]
```

```
[Out] (3*(-1 + x)*(1 + x)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (1 - x)/2])/(2*2
^(2/3)*((-1 + x)^2*(1 + x)^4)^(1/6))
```

IntegrateAlgebraic [A] time = 11.53, size = 122, normalized size = 0.98

$$\frac{\sqrt[3]{x-1}\,(x+1)^{2/3}\left(-\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-1}}-1\right)+\frac{1}{2}\log\left(\frac{(x+1)^{2/3}}{(x-1)^{2/3}}+\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-1}}+1\right)+\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{x+1}}{\sqrt{3}\,\sqrt[3]{x-1}}+\frac{1}{\sqrt{3}}\right)\right)}{\sqrt[6]{(x-1)^2(x+1)^4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x - x^2 - 4*x^3 - x^4 + 2*x^5 + x^6)^(-1/6),x]
```

```
[Out] ((-1 + x)^(1/3)*(1 + x)^(2/3)*(Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 + x)^(1/3))
/(Sqrt[3]*(-1 + x)^(1/3))] - Log[-1 + (1 + x)^(1/3)/(-1 + x)^(1/3)] + Log[1
 + (1 + x)^(1/3)/(-1 + x)^(1/3) + (1 + x)^(2/3)/(-1 + x)^(2/3)]/2))/((-1 +
x)^2*(1 + x)^4)^(1/6)
```

fricas [A] time = 0.39, size = 188, normalized size = 1.50

$$-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x+1)+2\sqrt{3}\left(x^6+2x^5-x^4-4x^3-x^2+2x+1\right)^{\frac{1}{6}}}{3(x+1)}\right)+\frac{1}{2}\log\left(\frac{x^2+\left(x^6+2x^5-x^4-4x^3-x^2+2x+1\right)^{\frac{1}{6}}(x+1)+2x+\left(x^6+2x^5-x^4-4x^3-x^2+2x+1\right)^{\frac{1}{3}}+1}{x^2+2x+1}\right)-\log\left(-\frac{x-\left(x^6+2x^5-x^4-4x^3-x^2+2x+1\right)^{\frac{1}{6}}+1}{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2*x^5-x^4-4*x^3-x^2+2*x+1)^(1/6),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(1/3*(sqrt(3)*(x + 1) + 2*sqrt(3)*(x^6 + 2*x^5 - x^4 - 4*x^3
 - x^2 + 2*x + 1)^(1/6))/(x + 1)) + 1/2*log((x^2 + (x^6 + 2*x^5 - x^4 - 4*x
^3 - x^2 + 2*x + 1)^(1/6)*(x + 1) + 2*x + (x^6 + 2*x^5 - x^4 - 4*x^3 - x^2
+ 2*x + 1)^(1/3) + 1)/(x^2 + 2*x + 1)) - log(-(x - (x^6 + 2*x^5 - x^4 - 4*x
^3 - x^2 + 2*x + 1)^(1/6) + 1)/(x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+2\,x^5-x^4-4\,x^3-x^2+2\,x+1\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2*x^5-x^4-4*x^3-x^2+2*x+1)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 2*x^5 - x^4 - 4*x^3 - x^2 + 2*x + 1)^(-1/6), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+2x^5-x^4-4x^3-x^2+2x+1\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+2*x^5-x^4-4*x^3-x^2+2*x+1)^(1/6),x)
```

```
[Out] int(1/(x^6+2*x^5-x^4-4*x^3-x^2+2*x+1)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+2\,x^5-x^4-4\,x^3-x^2+2\,x+1\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2*x^5-x^4-4*x^3-x^2+2*x+1)^(1/6),x, algorithm="maxima")
[Out] integrate((x^6 + 2*x^5 - x^4 - 4*x^3 - x^2 + 2*x + 1)^(-1/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^6+2\,x^5-x^4-4\,x^3-x^2+2\,x+1\right)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x - x^2 - 4*x^3 - x^4 + 2*x^5 + x^6 + 1)^(1/6),x)
[Out] int(1/(2*x - x^2 - 4*x^3 - x^4 + 2*x^5 + x^6 + 1)^(1/6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{x^{6}+2x^{5}-x^{4}-4x^{3}-x^{2}+2x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+2*x**5-x**4-4*x**3-x**2+2*x+1)**(1/6),x)
[Out] Integral((x**6 + 2*x**5 - x**4 - 4*x**3 - x**2 + 2*x + 1)**(-1/6), x)
```

3.1547 $\displaystyle\int \frac{x^2(-2b+ax^6)}{(b+ax^6)^{3/4}(b+cx^4+ax^6)}\,dx$

Optimal. Leaf size=125

$$\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6+b}}{\sqrt{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^6+b}}\right)}{\sqrt{2}\,c^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x\sqrt[4]{ax^6+b}}{\sqrt{ax^6+b}-\sqrt{c}\,x^2}\right)}{\sqrt{2}\,c^{3/4}}$$

Rubi [F] time = 1.63, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(-2b+ax^6\right)}{\left(b+ax^6\right)^{3/4}\left(b+cx^4+ax^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(-2*b + a*x^6))/((b + a*x^6)^(3/4)*(b + c*x^4 + a*x^6)),x]
```

```
[Out] (2*Sqrt[b]*(1 + (a*x^6)/b)^(3/4)*EllipticF[ArcTan[(Sqrt[a]*x^3)/Sqrt[b]]/2,
 2])/(3*Sqrt[a]*(b + a*x^6)^(3/4)) - (c*x*(1 + (a*x^6)/b)^(3/4)*Hypergeomet
ric2F1[1/6, 3/4, 7/6, -((a*x^6)/b)])/(a*(b + a*x^6)^(3/4)) + (b*c*Defer[Int
][1/((b + a*x^6)^(3/4)*(b + c*x^4 + a*x^6)), x])/a - 3*b*Defer[Int][x^2/((b
 + a*x^6)^(3/4)*(b + c*x^4 + a*x^6)), x] + (c^2*Defer[Int][x^4/((b + a*x^6)
^(3/4)*(b + c*x^4 + a*x^6)), x])/a
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(-2b+ax^6\right)}{\left(b+ax^6\right)^{3/4}\left(b+cx^4+ax^6\right)}\,dx &= \int \left(-\frac{c}{a\left(b+ax^6\right)^{3/4}}+\frac{x^2}{\left(b+ax^6\right)^{3/4}}+\frac{bc-3abx^2+c^2x^4}{a\left(b+ax^6\right)^{3/4}\left(b+cx^4+ax^6\right)}\right)dx\\
&= \frac{\int \frac{bc-3abx^2+c^2x^4}{\left(b+ax^6\right)^{3/4}\left(b+cx^4+ax^6\right)}\,dx}{a}-\frac{c\int \frac{1}{\left(b+ax^6\right)^{3/4}}\,dx}{a}+\int \frac{x^2}{\left(b+ax^6\right)^{3/4}}\,dx\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\left(b+ax^2\right)^{3/4}}\,dx,x,x^3\right)+\frac{\int \left(\frac{bc}{\left(b+ax^6\right)^{3/4}\left(b+cx^4+ax^6\right)}-\frac{3abx^2}{\left(b+ax^6\right)^{3/4}\left(b+\right.}\right.}{a}\\
&= -\frac{cx\left(1+\frac{ax^6}{b}\right)^{3/4}{}_2F_1\left(\frac{1}{6},\frac{3}{4};\frac{7}{6};-\frac{ax^6}{b}\right)}{a\left(b+ax^6\right)^{3/4}}-(3b)\int \frac{x^2}{\left(b+ax^6\right)^{3/4}\left(b+cx^4+ax^6\right)}\\
&= \frac{2\sqrt{b}\left(1+\frac{ax^6}{b}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{a}\,x^3}{\sqrt{b}}\right)\middle|2\right)}{3\sqrt{a}\left(b+ax^6\right)^{3/4}}-\frac{cx\left(1+\frac{ax^6}{b}\right)^{3/4}{}_2F_1\left(\frac{1}{6},\frac{3}{4};\frac{7}{6};-\frac{ax}{b}\right.}{a\left(b+ax^6\right)^{3/4}}
\end{aligned}$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-2b+ax^6\right)}{\left(b+ax^6\right)^{3/4}\left(b+cx^4+ax^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(-2*b + a*x^6))/((b + a*x^6)^(3/4)*(b + c*x^4 + a*x^6)),x]
```

```
[Out] Integrate[(x^2*(-2*b + a*x^6))/((b + a*x^6)^(3/4)*(b + c*x^4 + a*x^6)), x]
```

IntegrateAlgebraic [A] time = 15.74, size = 125, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6+b}}{\sqrt{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}x^2}{\sqrt{2}}}{x\sqrt[4]{ax^6+b}}\right)}{\sqrt{2}c^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x\sqrt[4]{ax^6+b}}{\sqrt{ax^6+b}-\sqrt{c}x^2}\right)}{\sqrt{2}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(-2*b + a*x^6))/((b + a*x^6)^(3/4)*(b + c*x^4 + a*x
^6)),x]
```

```
[Out] -(ArcTan[(Sqrt[2]*c^(1/4)*x*(b + a*x^6)^(1/4))/(-(Sqrt[c]*x^2) + Sqrt[b + a
*x^6])]/(Sqrt[2]*c^(3/4))) + ArcTanh[((c^(1/4)*x^2)/Sqrt[2] + Sqrt[b + a*x^
6]/(Sqrt[2]*c^(1/4)))/(x*(b + a*x^6)^(1/4))]/(Sqrt[2]*c^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^6-2*b)/(a*x^6+b)^(3/4)/(a*x^6+c*x^4+b),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^6-2b)x^2}{(ax^6+cx^4+b)(ax^6+b)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^6-2*b)/(a*x^6+b)^(3/4)/(a*x^6+c*x^4+b),x, algorithm="gia
c")
```

```
[Out] integrate((a*x^6 - 2*b)*x^2/((a*x^6 + c*x^4 + b)*(a*x^6 + b)^(3/4)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^2(ax^6-2b)}{(ax^6+b)^{\frac{3}{4}}(ax^6+cx^4+b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^6-2*b)/(a*x^6+b)^(3/4)/(a*x^6+c*x^4+b),x)
```

```
[Out] int(x^2*(a*x^6-2*b)/(a*x^6+b)^(3/4)/(a*x^6+c*x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^6-2b)x^2}{(ax^6+cx^4+b)(ax^6+b)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^6-2*b)/(a*x^6+b)^(3/4)/(a*x^6+c*x^4+b),x, algorithm="max
ima")
```

```
[Out] integrate((a*x^6 - 2*b)*x^2/((a*x^6 + c*x^4 + b)*(a*x^6 + b)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2\,\left(2\,b-a\,x^6\right)}{{\left(a\,x^6+b\right)}^{3/4}\,\left(a\,x^6+c\,x^4+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(2*b - a*x^6))/((b + a*x^6)^(3/4)*(b + a*x^6 + c*x^4)),x)
```

```
[Out] -int((x^2*(2*b - a*x^6))/((b + a*x^6)^(3/4)*(b + a*x^6 + c*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x**6-2*b)/(a*x**6+b)**(3/4)/(a*x**6+c*x**4+b),x)
```

```
[Out] Timed out
```

3.1548 $\int \frac{(4+x^3)(1+2x^3+x^6+x^8)}{x^4\sqrt[4]{1+x^3}(-1-2x^3-x^6+x^8)}\,dx$

Optimal. Leaf size=125

$$\frac{4\left(x^3+1\right)^{3/4}}{3x^3}-2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^3+1}}\right)-2\tanh^{-1}\left(\frac{\sqrt[4]{x^3+1}}{x}\right)+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^3+1}}{\sqrt{x^3+1}-x^2}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^3+1}}{\sqrt{x^3+1}+x^2}\right)$$

Rubi [F] time = 1.59, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(4+x^3)(1+2x^3+x^6+x^8)}{x^4\sqrt[4]{1+x^3}(-1-2x^3-x^6+x^8)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((4 + x^3)*(1 + 2*x^3 + x^6 + x^8))/(x^4*(1 + x^3)^(1/4)*(-1 - 2*x^3 -
x^6 + x^8)),x]
```

```
[Out] (4*(1 + x^3)^(3/4))/(3*x^3) + 4*Defer[Int][1/((1 + x^3)^(1/4)*(-1 - x^3 + x
^4)), x] + Defer[Int][x^3/((1 + x^3)^(1/4)*(-1 - x^3 + x^4)), x] + 4*Defer[
Int][1/((1 + x^3)^(1/4)*(1 + x^3 + x^4)), x] + Defer[Int][x^3/((1 + x^3)^(1
/4)*(1 + x^3 + x^4)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(4+x^3)(1+2x^3+x^6+x^8)}{x^4\sqrt[4]{1+x^3}(-1-2x^3-x^6+x^8)}\,dx &= \int \left(-\frac{4}{x^4\sqrt[4]{1+x^3}}-\frac{1}{x\sqrt[4]{1+x^3}}+\frac{4+x^3}{\sqrt[4]{1+x^3}\left(-1-x^3+x^4\right)}+\frac{4+}{\sqrt[4]{1+x^3}\,(1-}\right.\\
&= -\left(4\int \frac{1}{x^4\sqrt[4]{1+x^3}}\,dx\right)-\int \frac{1}{x\sqrt[4]{1+x^3}}\,dx+\int \frac{4+x^3}{\sqrt[4]{1+x^3}\left(-1-x^3+x^4\right)}\\
&= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{1+x}}\,dx,x,x^3\right)\right)-\frac{4}{3}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt[4]{1+x}}\,dx,x,x^3\right)\\
&= \frac{4\left(1+x^3\right)^{3/4}}{3x^3}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{1+x}}\,dx,x,x^3\right)-\frac{4}{3}\operatorname{Subst}\left(\int \frac{x^2}{-1+x^4}\right.\\
&= \frac{4\left(1+x^3\right)^{3/4}}{3x^3}+\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[4]{1+x^3}\right)-\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\right.\\
&= \frac{4\left(1+x^3\right)^{3/4}}{3x^3}-\frac{2}{3}\tan^{-1}\left(\sqrt[4]{1+x^3}\right)+\frac{2}{3}\tanh^{-1}\left(\sqrt[4]{1+x^3}\right)-\frac{2}{3}\operatorname{Subst}\left(\int\right.\\
&= \frac{4\left(1+x^3\right)^{3/4}}{3x^3}+4\int \frac{1}{\sqrt[4]{1+x^3}\left(-1-x^3+x^4\right)}\,dx+4\int \frac{1}{\sqrt[4]{1+x^3}\,(1+x^3}
\end{aligned}$$

Mathematica [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{(4+x^3)(1+2x^3+x^6+x^8)}{x^4\sqrt[4]{1+x^3}(-1-2x^3-x^6+x^8)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((4 + x^3)*(1 + 2*x^3 + x^6 + x^8))/(x^4*(1 + x^3)^(1/4)*(-1 - 2*
x^3 - x^6 + x^8)),x]
```

```
[Out] Integrate[((4 + x^3)*(1 + 2*x^3 + x^6 + x^8))/(x^4*(1 + x^3)^(1/4)*(-1 - 2*
x^3 - x^6 + x^8)), x]
```

IntegrateAlgebraic [A] time = 14.71, size = 125, normalized size = 1.00

$$\frac{4\left(x^3+1\right)^{3/4}}{3x^3}-2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^3+1}}\right)-2\tanh^{-1}\left(\frac{\sqrt[4]{x^3+1}}{x}\right)+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^3+1}}{\sqrt{x^3+1}-x^2}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^3+1}}{\sqrt{x^3+1}+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((4 + x^3)*(1 + 2*x^3 + x^6 + x^8))/(x^4*(1 + x^3)^(1/4)
*(-1 - 2*x^3 - x^6 + x^8)),x]
```

```
[Out] (4*(1 + x^3)^(3/4))/(3*x^3) - 2*ArcTan[x/(1 + x^3)^(1/4)] + Sqrt[2]*ArcTan[
(Sqrt[2]*x*(1 + x^3)^(1/4))/(-x^2 + Sqrt[1 + x^3])] - 2*ArcTanh[(1 + x^3)^(
1/4)/x] + Sqrt[2]*ArcTanh[(Sqrt[2]*x*(1 + x^3)^(1/4))/(x^2 + Sqrt[1 + x^3])
]
```

fricas [B] time = 47.17, size = 816, normalized size = 6.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)*(x^8+x^6+2*x^3+1)/x^4/(x^3+1)^(1/4)/(x^8-x^6-2*x^3-1),x,
algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(2)*x^3*arctan((x^8 + 2*x^7 + x^6 + 2*x^4 + 2*x^3 + 2*sqrt(2)
*(3*x^5 - x^4 - x)*(x^3 + 1)^(3/4) + 2*sqrt(2)*(x^7 - 3*x^6 - 3*x^3)*(x^3 +
 1)^(1/4) + 4*(x^6 + x^5 + x^2)*sqrt(x^3 + 1) + (16*(x^3 + 1)^(3/4)*x^5 + 2
*sqrt(2)*(3*x^6 - x^5 - x^2)*sqrt(x^3 + 1) + sqrt(2)*(x^8 + 8*x^7 - x^6 + 8
*x^4 - 2*x^3 - 1) + 4*(x^7 + x^6 + x^3)*(x^3 + 1)^(1/4))*sqrt((x^4 - 2*sqrt
(2)*(x^3 + 1)^(1/4)*x^3 + x^3 + 4*sqrt(x^3 + 1)*x^2 - 2*sqrt(2)*(x^3 + 1)^(
3/4)*x + 1)/(x^4 + x^3 + 1)) + 1)/(x^8 - 14*x^7 + x^6 - 14*x^4 + 2*x^3 + 1)
) - 12*sqrt(2)*x^3*arctan((x^8 + 2*x^7 + x^6 + 2*x^4 + 2*x^3 - 2*sqrt(2)*(3
*x^5 - x^4 - x)*(x^3 + 1)^(3/4) - 2*sqrt(2)*(x^7 - 3*x^6 - 3*x^3)*(x^3 + 1)
^(1/4) + 4*(x^6 + x^5 + x^2)*sqrt(x^3 + 1) + (16*(x^3 + 1)^(3/4)*x^5 - 2*sq
rt(2)*(3*x^6 - x^5 - x^2)*sqrt(x^3 + 1) - sqrt(2)*(x^8 + 8*x^7 - x^6 + 8*x^
4 - 2*x^3 - 1) + 4*(x^7 + x^6 + x^3)*(x^3 + 1)^(1/4))*sqrt((x^4 + 2*sqrt(2)
*(x^3 + 1)^(1/4)*x^3 + x^3 + 4*sqrt(x^3 + 1)*x^2 + 2*sqrt(2)*(x^3 + 1)^(3/4
)*x + 1)/(x^4 + x^3 + 1)) + 1)/(x^8 - 14*x^7 + x^6 - 14*x^4 + 2*x^3 + 1)) -
 3*sqrt(2)*x^3*log(4*(x^4 + 2*sqrt(2)*(x^3 + 1)^(1/4)*x^3 + x^3 + 4*sqrt(x^
3 + 1)*x^2 + 2*sqrt(2)*(x^3 + 1)^(3/4)*x + 1)/(x^4 + x^3 + 1)) + 3*sqrt(2)*
x^3*log(4*(x^4 - 2*sqrt(2)*(x^3 + 1)^(1/4)*x^3 + x^3 + 4*sqrt(x^3 + 1)*x^2
- 2*sqrt(2)*(x^3 + 1)^(3/4)*x + 1)/(x^4 + x^3 + 1)) - 12*x^3*arctan(2*((x^3
 + 1)^(1/4)*x^3 + (x^3 + 1)^(3/4)*x)/(x^4 - x^3 - 1)) - 12*x^3*log((x^4 - 2
*(x^3 + 1)^(1/4)*x^3 + x^3 + 2*sqrt(x^3 + 1)*x^2 - 2*(x^3 + 1)^(3/4)*x + 1)
/(x^4 - x^3 - 1)) - 16*(x^3 + 1)^(3/4))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+x^6+2\,x^3+1\right)\left(x^3+4\right)}{\left(x^8-x^6-2\,x^3-1\right)\left(x^3+1\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)*(x^8+x^6+2*x^3+1)/x^4/(x^3+1)^(1/4)/(x^8-x^6-2*x^3-1),x,
algorithm="giac")
```

[Out] integrate((x^8 + x^6 + 2*x^3 + 1)*(x^3 + 4)/((x^8 - x^6 - 2*x^3 - 1)*(x^3 + 1)^(1/4)*x^4), x)

maple [C] time = 4.03, size = 436, normalized size = 3.49

$$\frac{4\left(x^{3}+1\right)^{\frac{3}{4}}}{3 x^{3}}+\operatorname{RootOf}\left(\_Z^{2}+1\right) \ln \left(-\frac{2 \operatorname{RootOf}\left(\_Z^{2}+1\right) \sqrt{x^{3}+1}\, x^{2}-\operatorname{RootOf}\left(\_Z^{2}+1\right) x^{4}+2\left(x^{3}+1\right)^{\frac{3}{4}} x-2\left(x^{3}+1\right)^{\frac{1}{4}} x^{3}-\operatorname{RootOf}\left(\_Z^{2}+1\right) x^{3}-\operatorname{RootOf}\left(\_Z^{2}+1\right)}{x^{4}-x^{3}-1}\right)-\operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\right) \ln \left(\frac{-2 \operatorname{RootOf}\left(\_Z^{2}+1\right) \operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\right) \sqrt{x^{3}+1}\, x^{2}+2 \operatorname{RootOf}\left(\_Z^{2}+1\right)\left(x^{3}+1\right)^{\frac{1}{4}} x^{3}+\operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\right) x^{4}+2\left(x^{3}+1\right)^{\frac{3}{4}} x-\operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\right) x^{3}-\operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\right)}{x^{4}+x^{3}+1}\right)+\operatorname{RootOf}\left(\_Z^{2}+1\right) \operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\right) \ln \left(\frac{-\operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\right) \operatorname{RootOf}\left(\_Z^{2}+1\right) x^{4}+2 \operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\right) \sqrt{x^{3}+1}\, x^{2}-2 \operatorname{RootOf}\left(\_Z^{2}+1\right)\left(x^{3}+1\right)^{\frac{1}{4}} x^{3}+\operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\right) \operatorname{RootOf}\left(\_Z^{2}+1\right) x^{3}+2\left(x^{3}+1\right)^{\frac{3}{4}} x+\operatorname{RootOf}\left(\_Z^{2}+1\right) \operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\right)}{x^{4}+x^{3}+1}\right)-\ln \left(-\frac{2\left(x^{3}+1\right)^{\frac{3}{4}} x+2 x^{2} \sqrt{x^{3}+1}+2\left(x^{3}+1\right)^{\frac{1}{4}} x^{3}+x^{4}+x^{3}+1}{x^{4}-x^{3}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+4)*(x^8+x^6+2*x^3+1)/x^4/(x^3+1)^(1/4)/(x^8-x^6-2*x^3-1),x)

[Out] 4/3*(x^3+1)^(3/4)/x^3+RootOf(_Z^2+1)*ln(-(2*RootOf(_Z^2+1)*(x^3+1)^(1/2)*x^2-RootOf(_Z^2+1)*x^4+2*(x^3+1)^(3/4)*x-2*(x^3+1)^(1/4)*x^3-RootOf(_Z^2+1)*x^3-RootOf(_Z^2+1))/(x^4-x^3-1))-RootOf(_Z^2+RootOf(_Z^2+1))*ln((-2*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))*(x^3+1)^(1/2)*x^2+2*RootOf(_Z^2+1)*(x^3+1)^(1/4)*x^3+RootOf(_Z^2+RootOf(_Z^2+1))*x^4+2*(x^3+1)^(3/4)*x-RootOf(_Z^2+RootOf(_Z^2+1))*x^3-RootOf(_Z^2+RootOf(_Z^2+1)))/(x^4+x^3+1))+RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))*ln((-RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2+1)*x^4+2*RootOf(_Z^2+RootOf(_Z^2+1))*(x^3+1)^(1/2)*x^2-2*RootOf(_Z^2+1)*(x^3+1)^(1/4)*x^3+RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2+1)*x^3+2*(x^3+1)^(3/4)*x+RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1)))/(x^4+x^3+1))-ln(-(2*(x^3+1)^(3/4)*x+2*x^2*(x^3+1)^(1/2)+2*(x^3+1)^(1/4)*x^3+x^4+x^3+1)/(x^4-x^3-1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{8}+x^{6}+2 x^{3}+1\right)\left(x^{3}+4\right)}{\left(x^{8}-x^{6}-2 x^{3}-1\right)\left(x^{3}+1\right)^{\frac{1}{4}} x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+4)*(x^8+x^6+2*x^3+1)/x^4/(x^3+1)^(1/4)/(x^8-x^6-2*x^3-1),x, algorithm="maxima")

[Out] integrate((x^8 + x^6 + 2*x^3 + 1)*(x^3 + 4)/((x^8 - x^6 - 2*x^3 - 1)*(x^3 + 1)^(1/4)*x^4), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^{3}+4\right)\,\left(x^{8}+x^{6}+2\,x^{3}+1\right)}{x^{4}\,{\left(x^{3}+1\right)}^{1/4}\,\left(-x^{8}+x^{6}+2\,x^{3}+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-((x^3 + 4)*(2*x^3 + x^6 + x^8 + 1))/(x^4*(x^3 + 1)^(1/4)*(2*x^3 + x^6 - x^8 + 1)),x)

[Out] int(-((x^3 + 4)*(2*x^3 + x^6 + x^8 + 1))/(x^4*(x^3 + 1)^(1/4)*(2*x^3 + x^6 - x^8 + 1)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+4)*(x**8+x**6+2*x**3+1)/x**4/(x**3+1)**(1/4)/(x**8-x**6-2*x**3-1),x)

[Out] Timed out

3.1549 $\int \frac{2-x^4+2x^8}{\sqrt[4]{1+x^4}\left(-2+x^4+x^8\right)}\,dx$

Optimal. Leaf size=125

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\sqrt[4]{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\sqrt[4]{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)-$$

Rubi [A] time = 0.35, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6728, 240, 212, 206, 203, 377}

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\sqrt[4]{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\sqrt[4]{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x^4 + 2*x^8)/((1 + x^4)^(1/4)*(-2 + x^4 + x^8)),x]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)] - 2^(1/4)*ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))] - A
rcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) + ArcTanh[x/(1 + x^4)^(1/4)]
 - 2^(1/4)*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))] - ArcTanh[(2^(1/4)*x)/(1 +
x^4)^(1/4)]/(2*2^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{2-x^4+2x^8}{\sqrt[4]{1+x^4}\left(-2+x^4+x^8\right)}\,dx = \int \left(\frac{2}{\sqrt[4]{1+x^4}} + \frac{3\left(2-x^4\right)}{\sqrt[4]{1+x^4}\left(-2+x^4+x^8\right)}\right)dx$$

$$= 2\int \frac{1}{\sqrt[4]{1+x^4}}\,dx + 3\int \frac{2-x^4}{\sqrt[4]{1+x^4}\left(-2+x^4+x^8\right)}\,dx$$

$$= 2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right) + 3\int \left(\frac{2}{3\sqrt[4]{1+x^4}\left(-2+2x^4\right)} - \frac{8}{3\sqrt[4]{1+x^4}\,(4}\right.$$

$$= 2\int \frac{1}{\sqrt[4]{1+x^4}\left(-2+2x^4\right)}\,dx - 8\int \frac{1}{\sqrt[4]{1+x^4}\left(4+2x^4\right)}\,dx + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx\right.$$

$$= \tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) + \tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) + 2\,\text{Subst}\left(\int \frac{1}{-2+4x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)$$

$$= \tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) + \tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right.$$

$$= \tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) - \sqrt[4]{2}\,\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{1+x^4}}\right) - \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt[4]{2}} + \tanh^{-1}\left(\frac{x}{\sqrt[4]{1+}}\right.$$

Mathematica [A] time = 0.10, size = 125, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right) - \sqrt[4]{2}\,\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{x^4+1}}\right) - \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}} + \tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right) - \sqrt[4]{2}\,\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{x^4+1}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x^4 + 2*x^8)/((1 + x^4)^(1/4)*(-2 + x^4 + x^8)),x]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)] - 2^(1/4)*ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))] - A
rcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) + ArcTanh[x/(1 + x^4)^(1/4)]
 - 2^(1/4)*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))] - ArcTanh[(2^(1/4)*x)/(1 +
x^4)^(1/4)]/(2*2^(1/4))
```

IntegrateAlgebraic [A] time = 0.49, size = 125, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right) - \sqrt[4]{2}\,\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{x^4+1}}\right) - \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}} + \tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right) - \sqrt[4]{2}\,\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{x^4+1}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - x^4 + 2*x^8)/((1 + x^4)^(1/4)*(-2 + x^4 + x^8)),x]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)] - 2^(1/4)*ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))] - A
rcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(2*2^(1/4)) + ArcTanh[x/(1 + x^4)^(1/4)]
 - 2^(1/4)*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))] - ArcTanh[(2^(1/4)*x)/(1 +
x^4)^(1/4)]/(2*2^(1/4))
```

fricas [B] time = 0.42, size = 255, normalized size = 2.04

$$-\frac{1}{2}\cdot 8^{\frac{3}{4}}\arctan\left(\frac{8^{\frac{3}{4}}\sqrt{2}\,x\sqrt{\frac{\sqrt{2}\,x^2+2\,\sqrt{x^4+1}}{x^2}} - 2\cdot 8^{\frac{3}{4}}\left(x^4+1\right)^{\frac{1}{4}}}{8\,x}\right) - \frac{1}{2}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}x\sqrt{\frac{\sqrt{2}\,x^2+\sqrt{x^4+1}}{x^2}} - 2^{\frac{3}{4}}\left(x^4+1\right)^{\frac{1}{4}}}{2\,x}\right) - \frac{1}{8}\cdot 8^{\frac{3}{4}}\log\left(\frac{8^{\frac{1}{4}}x+2\left(x^4+1\right)^{\frac{1}{4}}}{x}\right) + \frac{1}{8}\cdot 8^{\frac{3}{4}}\log\left(-\frac{8^{\frac{1}{4}}x-2\left(x^4+1\right)^{\frac{1}{4}}}{x}\right) - \frac{1}{8}\cdot 2^{\frac{3}{4}}\log\left(\frac{2^{\frac{1}{4}}x+\left(x^4+1\right)^{\frac{1}{4}}}{x}\right) + \frac{1}{8}\cdot 2^{\frac{3}{4}}\log\left(-\frac{2^{\frac{1}{4}}x-\left(x^4+1\right)^{\frac{1}{4}}}{x}\right) - \arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right) + \frac{1}{2}\log\left(\frac{x+\left(x^4+1\right)^{\frac{1}{4}}}{x}\right) - \frac{1}{2}\log\left(-\frac{x-\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-x^4+2)/(x^4+1)^(1/4)/(x^8+x^4-2),x, algorithm="fricas")
```

```
[Out] -1/2*8^(3/4)*arctan(1/8*(8^(3/4)*sqrt(2)*x*sqrt((sqrt(2)*x^2 + 2*sqrt(x^4 +
 1))/x^2) - 2*8^(3/4)*(x^4 + 1)^(1/4))/x) - 1/2*2^(3/4)*arctan(1/2*(2^(3/4)
*x*sqrt((sqrt(2)*x^2 + sqrt(x^4 + 1))/x^2) - 2^(3/4)*(x^4 + 1)^(1/4))/x) -
1/8*8^(3/4)*log((8^(1/4)*x + 2*(x^4 + 1)^(1/4))/x) + 1/8*8^(3/4)*log(-(8^(1
/4)*x - 2*(x^4 + 1)^(1/4))/x) - 1/8*2^(3/4)*log((2^(1/4)*x + (x^4 + 1)^(1/4
))/x) + 1/8*2^(3/4)*log(-(2^(1/4)*x - (x^4 + 1)^(1/4))/x) - arctan((x^4 + 1
)^(1/4)/x) + 1/2*log((x + (x^4 + 1)^(1/4))/x) - 1/2*log(-(x - (x^4 + 1)^(1/
4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 x^{8} - x^{4} + 2}{\left(x^{8} + x^{4} - 2\right) \left(x^{4} + 1\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-x^4+2)/(x^4+1)^(1/4)/(x^8+x^4-2),x, algorithm="giac")
```

```
[Out] integrate((2*x^8 - x^4 + 2)/((x^8 + x^4 - 2)*(x^4 + 1)^(1/4)), x)
```

maple [F] time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{2x^{8} - x^{4} + 2}{\left(x^{4} + 1\right)^{\frac{1}{4}} \left(x^{8} + x^{4} - 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8-x^4+2)/(x^4+1)^(1/4)/(x^8+x^4-2),x)
```

```
[Out] int((2*x^8-x^4+2)/(x^4+1)^(1/4)/(x^8+x^4-2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 x^{8} - x^{4} + 2}{\left(x^{8} + x^{4} - 2\right) \left(x^{4} + 1\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-x^4+2)/(x^4+1)^(1/4)/(x^8+x^4-2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^8 - x^4 + 2)/((x^8 + x^4 - 2)*(x^4 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2 x^{8} - x^{4} + 2}{\left(x^{4} + 1\right)^{1/4} \left(x^{8} + x^{4} - 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8 - x^4 + 2)/((x^4 + 1)^(1/4)*(x^4 + x^8 - 2)),x)
```

```
[Out] int((2*x^8 - x^4 + 2)/((x^4 + 1)^(1/4)*(x^4 + x^8 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^{8} - x^{4} + 2}{\left(x - 1\right) \left(x + 1\right) \left(x^{2} + 1\right) \sqrt[4]{x^{4} + 1} \left(x^{4} + 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8-x**4+2)/(x**4+1)**(1/4)/(x**8+x**4-2),x)
```

```
[Out] Integral((2*x**8 - x**4 + 2)/((x - 1)*(x + 1)*(x**2 + 1)*(x**4 + 1)**(1/4)*
(x**4 + 2)), x)
```

3.1550 $\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^2}\,dx$

Optimal. Leaf size=125

$$\frac{3x}{4b^2\sqrt{ax^2+b^2}\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{x}{2b\left(ax^2+b^2\right)\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{3\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{4\sqrt{a}\,b^{5/2}}$$

Rubi [F] time = 1.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^2,x]
```

```
[Out] Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b - Sqrt[-a]*x), x]/(4*b^3) + Defer
[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b + Sqrt[-a]*x), x]/(4*b^3) - (a*Defer[I
nt][Sqrt[b + Sqrt[b^2 + a*x^2]]/(Sqrt[-a]*b - a*x)^2, x])/(4*b^2) - (a*Defe
r[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(Sqrt[-a]*b + a*x)^2, x])/(4*b^2)
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^2}\,dx &= \int \left(-\frac{a\sqrt{b+\sqrt{b^2+ax^2}}}{4b^2\left(\sqrt{-a}\,b-ax\right)^2}-\frac{a\sqrt{b+\sqrt{b^2+ax^2}}}{4b^2\left(\sqrt{-a}\,b+ax\right)^2}-\frac{a\sqrt{b+\sqrt{b^2+ax^2}}}{2b^2\left(-ab^2-a^2x^2\right)}\right)dx\\
&= -\frac{a\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(\sqrt{-a}\,b-ax\right)^2}\,dx}{4b^2}-\frac{a\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(\sqrt{-a}\,b+ax\right)^2}\,dx}{4b^2}-\frac{a\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{-ab^2-a^2x^2}\,dx}{2b^2}\\
&= -\frac{a\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(\sqrt{-a}\,b-ax\right)^2}\,dx}{4b^2}-\frac{a\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(\sqrt{-a}\,b+ax\right)^2}\,dx}{4b^2}-\frac{a\int \left(-\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{2ab\left(b-\sqrt{-a}\,x\right)}-\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{2ab\left(b+\sqrt{-a}\,x\right)}\right)dx}{2b^2}\\
&= \frac{\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b-\sqrt{-a}\,x}\,dx}{4b^3}+\frac{\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b+\sqrt{-a}\,x}\,dx}{4b^3}-\frac{a\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(\sqrt{-a}\,b-ax\right)^2}\,dx}{4b^2}-\frac{a\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(\sqrt{-a}\,b+ax\right)^2}\,dx}{4b^2}
\end{aligned}$$

Mathematica [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^2,x]
```

```
[Out] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^2, x]
```

IntegrateAlgebraic [A] time = 0.28, size = 125, normalized size = 1.00

$$\frac{3x}{4b^2\sqrt{ax^2+b^2}\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{x}{2b\left(ax^2+b^2\right)\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{3\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{4\sqrt{a}\,b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^2,x]
```

```
[Out] x/(2*b*(b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]]) + (3*x)/(4*b^2*Sqrt[b^2 +
 a*x^2]*Sqrt[b + Sqrt[b^2 + a*x^2]]) + (3*ArcTan[(Sqrt[a]*x)/(Sqrt[b]*Sqrt[
b + Sqrt[b^2 + a*x^2]])])/(4*Sqrt[a]*b^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\left(ax^2+b^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 + b^2)^2, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{\left(a\,x^2+b^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^2,x)
```

```
[Out] int((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\left(ax^2+b^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 + b^2)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b+\sqrt{b^2+a\,x^2}}}{{\left(b^2+a\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(a*x^2 + b^2)^2,x)
```

```
[Out] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(a*x^2 + b^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\left(ax^2+b^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x**2+b**2)**(1/2))**(1/2)/(a*x**2+b**2)**2,x)
```

```
[Out] Integral(sqrt(b + sqrt(a*x**2 + b**2))/(a*x**2 + b**2)**2, x)
```

3.1551 $\int \frac{(b+ax^3)\sqrt{-x+x^4}}{-d+cx^3}\,dx$

Optimal. Leaf size=126

$$\frac{2\sqrt{c-d}\,(ad+bc)\tan^{-1}\left(\frac{x\sqrt{x^4-x}\,\sqrt{c-d}}{\sqrt{d}\,(x-1)(x^2+x+1)}\right)}{3c^2\sqrt{d}}+\frac{\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)(-ac+2ad+2bc)}{3c^2}+\frac{a\sqrt{x^4-x}\,x}{3c}$$

Rubi [A] time = 0.37, antiderivative size = 164, normalized size of antiderivative = 1.30, number of steps used = 12, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {2056, 581, 584, 329, 275, 217, 206, 466, 465, 377, 205}

$$\frac{2\sqrt{x^4-x}\,\sqrt{c-d}\,(ad+bc)\tan^{-1}\left(\frac{x^{3/2}\sqrt{c-d}}{\sqrt{d}\,\sqrt{x^3-1}}\right)}{3c^2\sqrt{d}\,\sqrt{x^3-1}\,\sqrt{x}}-\frac{\sqrt{x^4-x}\,\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{x^3-1}}\right)(ac-2ad-2bc)}{3c^2\sqrt{x^3-1}\,\sqrt{x}}+\frac{a\sqrt{x^4-x}\,x}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[((b + a*x^3)*Sqrt[-x + x^4])/(-d + c*x^3),x]
```

```
[Out] (a*x*Sqrt[-x + x^4])/(3*c) + (2*Sqrt[c - d]*(b*c + a*d)*Sqrt[-x + x^4]*ArcT
an[(Sqrt[c - d]*x^(3/2))/(Sqrt[d]*Sqrt[-1 + x^3])])/(3*c^2*Sqrt[d]*Sqrt[x]*
Sqrt[-1 + x^3]) - ((a*c - 2*b*c - 2*a*d)*Sqrt[-x + x^4]*ArcTanh[x^(3/2)/Sqr
t[-1 + x^3]])/(3*c^2*Sqrt[x]*Sqrt[-1 + x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\left(b+ax^3\right)\sqrt{-x+x^4}}{-d+cx^3}\,dx = \frac{\sqrt{-x+x^4}\int \frac{\sqrt{x}\sqrt{-1+x^3}\left(b+ax^3\right)}{-d+cx^3}dx}{\sqrt{x}\sqrt{-1+x^3}}$$

$$= \frac{ax\sqrt{-x+x^4}}{3c} + \frac{\sqrt{-x+x^4}\int \frac{\sqrt{x}\left(-\frac{3}{2}(2bc+ad)-\frac{3}{2}(ac-2bc-2ad)x^3\right)}{\sqrt{-1+x^3}\left(-d+cx^3\right)}dx}{3c\sqrt{x}\sqrt{-1+x^3}}$$

$$= \frac{ax\sqrt{-x+x^4}}{3c} + \frac{\sqrt{-x+x^4}\int \left(-\frac{3(ac-2bc-2ad)\sqrt{x}}{2c\sqrt{-1+x^3}} + \frac{\left(-\frac{3}{2}d(ac-2bc-2ad)-\frac{3}{2}c(2bc+ad)\right)\sqrt{x}}{c\sqrt{-1+x^3}\left(-d+cx^3\right)}\right)dx}{3c\sqrt{x}\sqrt{-1+x^3}}$$

$$= \frac{ax\sqrt{-x+x^4}}{3c} - \frac{\left((ac-2bc-2ad)\sqrt{-x+x^4}\right)\int \frac{\sqrt{x}}{\sqrt{-1+x^3}}dx}{2c^2\sqrt{x}\sqrt{-1+x^3}} - \frac{\left((c-d)(bc+ad)\sqrt{-x+\ldots\right.}{c^2\sqrt{x}\sqrt{\ldots}}$$

$$= \frac{ax\sqrt{-x+x^4}}{3c} - \frac{\left((ac-2bc-2ad)\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x^6}}dx, x, \sqrt{x}\right)}{c^2\sqrt{x}\sqrt{-1+x^3}} - \frac{\left(2(c-d\ldots\right.}{\ldots}$$

$$= \frac{ax\sqrt{-x+x^4}}{3c} - \frac{\left((ac-2bc-2ad)\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}dx, x, x^{3/2}\right)}{3c^2\sqrt{x}\sqrt{-1+x^3}} - \frac{\left(2(c-d\ldots\right.}{\ldots}$$

$$= \frac{ax\sqrt{-x+x^4}}{3c} - \frac{\left((ac-2bc-2ad)\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{1}{1-x^2}dx, x, \frac{x^{3/2}}{\sqrt{-1+x^3}}\right)}{3c^2\sqrt{x}\sqrt{-1+x^3}} - \frac{\left(2(c-\ldots\right.}{\ldots}$$

$$= \frac{ax\sqrt{-x+x^4}}{3c} + \frac{2\sqrt{c-d}\,(bc+ad)\sqrt{-x+x^4}\,\tan^{-1}\left(\frac{\sqrt{c-d}\,x^{3/2}}{\sqrt{d}\sqrt{-1+x^3}}\right)}{3c^2\sqrt{d}\sqrt{x}\sqrt{-1+x^3}} + \frac{(2bc-a(c-2d))\sqrt{\ldots}}{3c^2\sqrt{\ldots}}$$

Mathematica [C] time = 0.61, size = 186, normalized size = 1.48

$$\frac{\sqrt{1-x^3}\,x^5\sqrt{\frac{x^3(d-c)}{d}}\,(a(c-2d)-2bc)F_1\left(\frac{3}{2};\frac{1}{2},1;\frac{5}{2};x^3,\frac{cx^3}{d}\right)+3x^2\left(\sqrt{1-x^3}\,(ad+2bc)\sin^{-1}\left(\frac{\sqrt{\frac{x^3(d-c)}{d}}}{\sqrt{1-\frac{cx^3}{d}}}\right)+ad\left(x^3-1\right)\sqrt{\frac{x^3(d-c)}{d}}\right)}{9cd\sqrt{x\left(x^3-1\right)}\sqrt{\frac{x^3(d-c)}{d}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((b + a*x^3)*Sqrt[-x + x^4])/(-d + c*x^3),x]
```

```
[Out] ((-2*b*c + a*(c - 2*d))*x^5*Sqrt[((-c + d)*x^3)/d]*Sqrt[1 - x^3]*AppellF1[3
/2, 1/2, 1, 5/2, x^3, (c*x^3)/d] + 3*x^2*(a*d*Sqrt[((-c + d)*x^3)/d]*(-1 +
x^3) + (2*b*c + a*d)*Sqrt[1 - x^3]*ArcSin[Sqrt[((-c + d)*x^3)/d]/Sqrt[1 - (
c*x^3)/d]]))/(9*c*d*Sqrt[((-c + d)*x^3)/d]*Sqrt[x*(-1 + x^3)])
```

IntegrateAlgebraic [A] time = 0.70, size = 126, normalized size = 1.00

$$\frac{2\sqrt{c-d}\,(ad+bc)\tan^{-1}\left(\frac{x\sqrt{x^4-x}\,\sqrt{c-d}}{\sqrt{d}\,(x-1)\left(x^2+x+1\right)}\right)}{3c^2\sqrt{d}} + \frac{\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-x}}\right)(-ac+2ad+2bc)}{3c^2} + \frac{a\sqrt{x^4-x}\,x}{3c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b + a*x^3)*Sqrt[-x + x^4])/(-d + c*x^3),x]
```

```
[Out] (a*x*Sqrt[-x + x^4])/(3*c) + (2*Sqrt[c - d]*(b*c + a*d)*ArcTan[(Sqrt[c - d]
*x*Sqrt[-x + x^4])/(Sqrt[d]*(-1 + x)*(1 + x + x^2))]])/(3*c^2*Sqrt[d]) + ((-
(a*c) + 2*b*c + 2*a*d)*ArcTanh[x^2/Sqrt[-x + x^4]])/(3*c^2)
```

fricas [A] time = 3.99, size = 288, normalized size = 2.29

$$\left[\frac{2\sqrt{x^4-x}\,acx+(bc+ad)\sqrt{-\frac{c-d}{d}}\log\left(-\frac{(c^2-8cd+8d^2)x^6+2(3cd-4d^2)x^3+d^2+4((cd-2d^2)x^4+d^2x)\sqrt{x^4-x}\sqrt{-\frac{c-d}{d}}}{c^2x^6-2cdx^3+d^2}\right)-((a-2b)c-2ad)\log\left(-2x^3-2\sqrt{x^4-x}\,x+1\right)}{6c^2}, \frac{2\sqrt{x^4-x}\,acx+2(bc+ad)\sqrt{\frac{c-d}{d}}\arctan\left(-\frac{2\sqrt{x^4-x}\,dx\sqrt{\frac{c-d}{d}}}{(c-2d)x^3+d}\right)-((a-2b)c-2ad)\log\left(-2x^3-2\sqrt{x^4-x}\,x+1\right)}{6c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)*(x^4-x)^(1/2)/(c*x^3-d),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(x^4 - x)*a*c*x + (b*c + a*d)*sqrt(-(c - d)/d)*log(-((c^2 - 8*c
*d + 8*d^2)*x^6 + 2*(3*c*d - 4*d^2)*x^3 + d^2 + 4*((c*d - 2*d^2)*x^4 + d^2*
x)*sqrt(x^4 - x)*sqrt(-(c - d)/d))/(c^2*x^6 - 2*c*d*x^3 + d^2)) - ((a - 2*b
)*c - 2*a*d)*log(-2*x^3 - 2*sqrt(x^4 - x)*x + 1))/c^2, 1/6*(2*sqrt(x^4 - x)
*a*c*x + 2*(b*c + a*d)*sqrt((c - d)/d)*arctan(-2*sqrt(x^4 - x)*d*x*sqrt((c
- d)/d)/((c - 2*d)*x^3 + d)) - ((a - 2*b)*c - 2*a*d)*log(-2*x^3 - 2*sqrt(x^
4 - x)*x + 1))/c^2]
```

giac [A] time = 0.72, size = 136, normalized size = 1.08

$$\frac{\sqrt{x^4-x}\,ax}{3c}-\frac{(ac-2bc-2ad)\log\left(\sqrt{-\frac{1}{x^3}+1}+1\right)}{6c^2}+\frac{(ac-2bc-2ad)\log\left(\left|\sqrt{-\frac{1}{x^3}+1}-1\right|\right)}{6c^2}-\frac{2\left(bc^2+acd-bcd-ad^2\right)\arctan\left(\frac{d\sqrt{-\frac{1}{x^3}+1}}{\sqrt{cd-d^2}}\right)}{3\sqrt{cd-d^2}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)*(x^4-x)^(1/2)/(c*x^3-d),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(x^4 - x)*a*x/c - 1/6*(a*c - 2*b*c - 2*a*d)*log(sqrt(-1/x^3 + 1) +
1)/c^2 + 1/6*(a*c - 2*b*c - 2*a*d)*log(abs(sqrt(-1/x^3 + 1) - 1))/c^2 - 2/3
*(b*c^2 + a*c*d - b*c*d - a*d^2)*arctan(d*sqrt(-1/x^3 + 1)/sqrt(c*d - d^2))
/(sqrt(c*d - d^2)*c^2)
```

maple [C] time = 0.38, size = 953, normalized size = 7.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b)*(x^4-x)^(1/2)/(c*x^3-d),x)
```

```
[Out] a/c*(1/3*x*(x^4-x)^(1/2)-(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+
1/2*I*3^(1/2))/(-1+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^
(1/2))/(-1+x))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1
/2)/(-3/2+1/2*I*3^(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/
2)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^
(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2
*I*3^(1/2)))^(1/2))-EllipticPi(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))
/(-1+x))^(1/2),(-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)),((3/2+1/2*I*3^(1/2
))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))))+(a
*d+b*c)/c*(2/c*(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1
/2))/(-1+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2))/(-1
+x))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2)/(-3/2+
1/2*I*3^(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/2)))^(1/2)
*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),((3/
2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)
))^(1/2))-EllipticPi(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(
1/2),(-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)),((3/2+1/2*I*3^(1/2))*(1/2-1/
2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)))-2/3/c*4^(1/2)
*sum(1/_alpha*(-1+x)^2*(_alpha^2+_alpha+1)*(1-I*3^(1/2))*(x/(-1+x)*(-3+I*3^
```

```
(1/2))/(I*3^(1/2)-1))^(1/2)*(1/(-1+x)*(I*3^(1/2)+2*x+1)/(-1-I*3^(1/2)))^(1/
2)*(1/(-1+x)*(1+2*x-I*3^(1/2))/(I*3^(1/2)-1))^(1/2)/(-3+I*3^(1/2))/(x*(-1+x
)*(I*3^(1/2)+2*x+1)*(1+2*x-I*3^(1/2)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2
))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(
1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-_alpha^2*c/d*Elliptic
Pi(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),1/6*(I*_alpha
^2*3^(1/2)*c-I*3^(1/2)*d-3*_alpha^2*c+3*d)/d,((3/2+1/2*I*3^(1/2))*(1/2-1/2*
I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))),_alpha=RootOf(_
Z^3*c-d)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^3+b)\sqrt{x^4-x}}{cx^3-d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)*(x^4-x)^(1/2)/(c*x^3-d),x, algorithm="maxima")
```

```
[Out] integrate((a*x^3 + b)*sqrt(x^4 - x)/(c*x^3 - d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{x^4-x}\,\left(a\,x^3+b\right)}{d-c\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^4 - x)^(1/2)*(b + a*x^3))/(d - c*x^3),x)
```

```
[Out] int(-((x^4 - x)^(1/2)*(b + a*x^3))/(d - c*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x(x-1)\left(x^2+x+1\right)}\left(ax^3+b\right)}{cx^3-d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b)*(x**4-x)**(1/2)/(c*x**3-d),x)
```

```
[Out] Integral(sqrt(x*(x - 1)*(x**2 + x + 1))*(a*x**3 + b)/(c*x**3 - d), x)
```

3.1552 $\int \frac{b+ax^3}{x^3(-b+ax^3)\sqrt[4]{bx+ax^4}}\,dx$

Optimal. Leaf size=126

$$-\frac{2\,2^{3/4}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3b}-\frac{2\,2^{3/4}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3b}+\frac{4\left(ax^4+bx\right)^{3/4}}{9bx^3}$$

Rubi [C] time = 0.38, antiderivative size = 53, normalized size of antiderivative = 0.42, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2056, 466, 465, 511, 510}

$$\frac{4\left(ax^3+b\right)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\frac{2ax^3}{ax^3+b}\right)}{9bx^2\sqrt[4]{ax^4+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^3)/(x^3*(-b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*(b + a*x^3)*Hypergeometric2F1[-3/4, 1, 1/4, (2*a*x^3)/(b + a*x^3)])/(9*b
*x^2*(b*x + a*x^4)^(1/4))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
```

```
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{b+ax^3}{x^3\left(-b+ax^3\right)\sqrt[4]{bx+ax^4}}\,dx &= \frac{\left(\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\int \frac{\left(b+ax^3\right)^{3/4}}{x^{13/4}\left(-b+ax^3\right)}\,dx}{\sqrt[4]{bx+ax^4}}\\
&= \frac{\left(4\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\operatorname{Subst}\left(\int \frac{\left(b+ax^{12}\right)^{3/4}}{x^{10}\left(-b+ax^{12}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^4}}\\
&= \frac{\left(4\sqrt[4]{x}\,\sqrt[4]{b+ax^3}\right)\operatorname{Subst}\left(\int \frac{\left(b+ax^4\right)^{3/4}}{x^4\left(-b+ax^4\right)}\,dx,x,x^{3/4}\right)}{3\sqrt[4]{bx+ax^4}}\\
&= \frac{\left(4\sqrt[4]{x}\,\left(b+ax^3\right)\right)\operatorname{Subst}\left(\int \frac{\left(1+\frac{ax^4}{b}\right)^{3/4}}{x^4\left(-b+ax^4\right)}\,dx,x,x^{3/4}\right)}{3\left(1+\frac{ax^3}{b}\right)^{3/4}\sqrt[4]{bx+ax^4}}\\
&= \frac{4\left(b+ax^3\right)\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\frac{2ax^3}{b+ax^3}\right)}{9bx^2\sqrt[4]{bx+ax^4}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 46, normalized size = 0.37

$$\frac{4\left(x\left(ax^3+b\right)\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\frac{2ax^3}{ax^3+b}\right)}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^3)/(x^3*(-b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*(x*(b + a*x^3))^(3/4)*Hypergeometric2F1[-3/4, 1, 1/4, (2*a*x^3)/(b + a*x
^3)])/(9*b*x^3)
```

IntegrateAlgebraic [A] time = 0.45, size = 126, normalized size = 1.00

$$-\frac{2\,2^{3/4}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3b}-\frac{2\,2^{3/4}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3b}+\frac{4\left(ax^4+bx\right)^{3/4}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^3)/(x^3*(-b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*(b*x + a*x^4)^(3/4))/(9*b*x^3) - (2*2^(3/4)*a^(3/4)*ArcTan[(2^(1/4)*a^(1
/4)*(b*x + a*x^4)^(3/4))/(b + a*x^3)])/(3*b) - (2*2^(3/4)*a^(3/4)*ArcTanh[(
2^(1/4)*a^(1/4)*(b*x + a*x^4)^(3/4))/(b + a*x^3)])/(3*b)
```

fricas [B] time = 89.71, size = 479, normalized size = 3.80

$$\frac{12\cdot 8^{\frac{1}{4}}bx^3\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}\arctan\left(\frac{16\cdot 8^{\frac{1}{4}}\left(ax^4+bx\right)^{\frac{1}{4}}a^4bx^2\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}+4\cdot 8^{\frac{3}{4}}\left(ax^4+bx\right)^{\frac{3}{4}}a^2b^3\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}+\sqrt{2}\left(8\cdot 8^{\frac{1}{4}}\sqrt{ax^4+bx}\,a^2bx\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}+8^{\frac{3}{4}}\left(3\,ab^3x^3+b^4\right)\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}\right)\sqrt{\sqrt{2}\,a^2b^2\sqrt{\frac{a^3}{b^4}}}}{8\left(a^5x^3-a^4b\right)}\right)-3\cdot 8^{\frac{1}{4}}bx^3\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}\log\left(\frac{4\sqrt{2}\left(ax^4+bx\right)^{\frac{1}{4}}ab^2x^2\sqrt{\frac{a^3}{b^4}}+8^{\frac{3}{4}}\sqrt{ax^4+bx}\,b^3x\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}+4\left(ax^4+bx\right)^{\frac{3}{4}}a^2+8^{\frac{1}{4}}\left(3\,a^2bx^3+ab^2\right)\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}}{ax^3-b}\right)+3\cdot 8^{\frac{1}{4}}bx^3\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}\log\left(\frac{4\sqrt{2}\left(ax^4+bx\right)^{\frac{1}{4}}ab^2x^2\sqrt{\frac{a^3}{b^4}}-8^{\frac{3}{4}}\sqrt{ax^4+bx}\,b^3x\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}+4\left(ax^4+bx\right)^{\frac{3}{4}}a^2-8^{\frac{1}{4}}\left(3\,a^2bx^3+ab^2\right)\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}}{ax^3-b}\right)+8\left(ax^4+bx\right)^{\frac{3}{4}}}{18\,bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)/x^3/(a*x^3-b)/(a*x^4+b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/18*(12*8^(1/4)*b*x^3*(a^3/b^4)^(1/4)*arctan(1/8*(16*8^(1/4)*(a*x^4 + b*x)
^(1/4)*a^4*b*x^2*(a^3/b^4)^(1/4) + 4*8^(3/4)*(a*x^4 + b*x)^(3/4)*a^2*b^3*(a
^3/b^4)^(3/4) + sqrt(2)*(8*8^(1/4)*sqrt(a*x^4 + b*x)*a^2*b*x*(a^3/b^4)^(1/4
) + 8^(3/4)*(3*a*b^3*x^3 + b^4)*(a^3/b^4)^(3/4))*sqrt(sqrt(2)*a^2*b^2*sqrt(
a^3/b^4)))/(a^5*x^3 - a^4*b)) - 3*8^(1/4)*b*x^3*(a^3/b^4)^(1/4)*log((4*sqrt
(2)*(a*x^4 + b*x)^(1/4)*a*b^2*x^2*sqrt(a^3/b^4) + 8^(3/4)*sqrt(a*x^4 + b*x)
*b^3*x*(a^3/b^4)^(3/4) + 4*(a*x^4 + b*x)^(3/4)*a^2 + 8^(1/4)*(3*a^2*b*x^3 +
 a*b^2)*(a^3/b^4)^(1/4))/(a*x^3 - b)) + 3*8^(1/4)*b*x^3*(a^3/b^4)^(1/4)*log
((4*sqrt(2)*(a*x^4 + b*x)^(1/4)*a*b^2*x^2*sqrt(a^3/b^4) - 8^(3/4)*sqrt(a*x^
4 + b*x)*b^3*x*(a^3/b^4)^(3/4) + 4*(a*x^4 + b*x)^(3/4)*a^2 - 8^(1/4)*(3*a^2
*b*x^3 + a*b^2)*(a^3/b^4)^(1/4))/(a*x^3 - b)) + 8*(a*x^4 + b*x)^(3/4))/(b*x
^3)
```

giac [B] time = 0.40, size = 208, normalized size = 1.65

$$-\frac{2\cdot 2^{\frac{1}{4}}\left(-a\right)^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}\left(-a\right)^{\frac{1}{4}}+2\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)}{3\,b}-\frac{2\cdot 2^{\frac{1}{4}}\left(-a\right)^{\frac{3}{4}}\arctan\left(-\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}\left(-a\right)^{\frac{1}{4}}-2\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)}{3\,b}+\frac{2^{\frac{1}{4}}\left(-a\right)^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-a\right)^{\frac{1}{4}}\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a+\frac{b}{x^{3}}}\right)}{3\,b}-\frac{2^{\frac{1}{4}}\left(-a\right)^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-a\right)^{\frac{1}{4}}\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a+\frac{b}{x^{3}}}\right)}{3\,b}+\frac{4\left(a+\frac{b}{x^{3}}\right)^{\frac{3}{4}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)/x^3/(a*x^3-b)/(a*x^4+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] -2/3*2^(1/4)*(-a)^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4)*(-a)^(1/4) + 2*(a + b/x
^3)^(1/4))/(-a)^(1/4))/b - 2/3*2^(1/4)*(-a)^(3/4)*arctan(-1/2*2^(1/4)*(2^(3
/4)*(-a)^(1/4) - 2*(a + b/x^3)^(1/4))/(-a)^(1/4))/b + 1/3*2^(1/4)*(-a)^(3/4
)*log(2^(3/4)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(2)*sqrt(-a) + sqrt(a + b/
x^3))/b - 1/3*2^(1/4)*(-a)^(3/4)*log(-2^(3/4)*(-a)^(1/4)*(a + b/x^3)^(1/4)
+ sqrt(2)*sqrt(-a) + sqrt(a + b/x^3))/b + 4/9*(a + b/x^3)^(3/4)/b
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{a\,x^{3}+b}{x^{3}\left(a\,x^{3}-b\right)\left(a\,x^{4}+bx\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b)/x^3/(a*x^3-b)/(a*x^4+b*x)^(1/4),x)
```

```
[Out] int((a*x^3+b)/x^3/(a*x^3-b)/(a*x^4+b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{3}+b}{\left(ax^{4}+bx\right)^{\frac{1}{4}}\left(ax^{3}-b\right)x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)/x^3/(a*x^3-b)/(a*x^4+b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^3 + b)/((a*x^4 + b*x)^(1/4)*(a*x^3 - b)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a\,x^{3}+b}{x^{3}\,{\left(a\,x^{4}+b\,x\right)}^{1/4}\,\left(b-a\,x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x^3)/(x^3*(b*x + a*x^4)^(1/4)*(b - a*x^3)),x)
```

```
[Out] -int((b + a*x^3)/(x^3*(b*x + a*x^4)^(1/4)*(b - a*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 + b}{x^3 \sqrt[4]{x\left(ax^3 + b\right)}\left(ax^3 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b)/x**3/(a*x**3-b)/(a*x**4+b*x)**(1/4),x)
```

```
[Out] Integral((a*x**3 + b)/(x**3*(x*(a*x**3 + b))**(1/4)*(a*x**3 - b)), x)
```

3.1553 $\int \frac{(1+2x^6)\sqrt[3]{x+x^3-x^7}}{(-1+x^6)^2}\,dx$

Optimal. Leaf size=126

$$-\frac{1}{6}\log\left(\sqrt[3]{-x^7+x^3+x}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{-x^7+x^3+x}+x}\right)}{2\sqrt{3}}-\frac{\sqrt[3]{-x^7+x^3+x}\,x}{2\left(x^6-1\right)}+\frac{1}{12}\log\left(x^2+\sqrt[3]{-x^7+x^3+x}\,x+\left(-x^7\right.\right.$$

Rubi [F] time = 8.91, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+2x^6)\sqrt[3]{x+x^3-x^7}}{(-1+x^6)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + 2*x^6)*(x + x^3 - x^7)^(1/3))/(-1 + x^6)^2,x]
```

```
[Out] (2*(x + x^3 - x^7)^(1/3)*Defer[Subst][Defer[Int][(1 + x^3 - x^9)^(1/3)/(-1
+ I*Sqrt[3] - 2*x)^2, x], x, x^(2/3)])/(9*x^(1/3)*(1 + x^2 - x^6)^(1/3)) -
(((2*I)/9)*(x + x^3 - x^7)^(1/3)*Defer[Subst][Defer[Int][(1 + x^3 - x^9)^(1
/3)/(-1 + I*Sqrt[3] - 2*x), x], x, x^(2/3)])/(Sqrt[3]*x^(1/3)*(1 + x^2 - x^
6)^(1/3)) + ((x + x^3 - x^7)^(1/3)*Defer[Subst][Defer[Int][(1 + x^3 - x^9)^
(1/3)/(-1 + x)^2, x], x, x^(2/3)])/(18*x^(1/3)*(1 + x^2 - x^6)^(1/3)) - ((x
 + x^3 - x^7)^(1/3)*Defer[Subst][Defer[Int][(1 + x^3 - x^9)^(1/3)/(-1 + x),
 x], x, x^(2/3)])/(18*x^(1/3)*(1 + x^2 - x^6)^(1/3)) + ((3 - I*Sqrt[3])*(x
+ x^3 - x^7)^(1/3)*Defer[Subst][Defer[Int][(1 + x^3 - x^9)^(1/3)/(1 - I*Sqr
t[3] + 2*x), x], x, x^(2/3)])/(54*x^(1/3)*(1 + x^2 - x^6)^(1/3)) + (2*(x +
x^3 - x^7)^(1/3)*Defer[Subst][Defer[Int][(1 + x^3 - x^9)^(1/3)/(1 + I*Sqrt[
3] + 2*x)^2, x], x, x^(2/3)])/(9*x^(1/3)*(1 + x^2 - x^6)^(1/3)) - (((2*I)/9
)*(x + x^3 - x^7)^(1/3)*Defer[Subst][Defer[Int][(1 + x^3 - x^9)^(1/3)/(1 +
I*Sqrt[3] + 2*x), x], x, x^(2/3)])/(Sqrt[3]*x^(1/3)*(1 + x^2 - x^6)^(1/3))
+ ((3 + I*Sqrt[3])*(x + x^3 - x^7)^(1/3)*Defer[Subst][Defer[Int][(1 + x^3 -
 x^9)^(1/3)/(1 + I*Sqrt[3] + 2*x), x], x, x^(2/3)])/(54*x^(1/3)*(1 + x^2 -
x^6)^(1/3)) - (2*(x + x^3 - x^7)^(1/3)*Defer[Subst][Defer[Int][(x*(1 + x^3
- x^9)^(1/3))/(-1 + I*Sqrt[3] - 2*x^3)^2, x], x, x^(2/3)])/(x^(1/3)*(1 + x^
2 - x^6)^(1/3)) + ((1 - I*Sqrt[3])*(x + x^3 - x^7)^(1/3)*Defer[Subst][Defer
[Int][(x*(1 + x^3 - x^9)^(1/3))/(-1 + I*Sqrt[3] - 2*x^3)^2, x], x, x^(2/3)]
)/(x^(1/3)*(1 + x^2 - x^6)^(1/3)) - (2*(x + x^3 - x^7)^(1/3)*Defer[Subst][D
efer[Int][(x*(1 + x^3 - x^9)^(1/3))/(1 + I*Sqrt[3] + 2*x^3)^2, x], x, x^(2/
3)])/(x^(1/3)*(1 + x^2 - x^6)^(1/3)) + ((1 + I*Sqrt[3])*(x + x^3 - x^7)^(1/
3)*Defer[Subst][Defer[Int][(x*(1 + x^3 - x^9)^(1/3))/(1 + I*Sqrt[3] + 2*x^3
)^2, x], x, x^(2/3)])/(x^(1/3)*(1 + x^2 - x^6)^(1/3))
```

Rubi steps

$$\int \frac{\left(1+2x^6\right)\sqrt[3]{x+x^3-x^7}}{\left(-1+x^6\right)^2}\,dx = \frac{\sqrt[3]{x+x^3-x^7}\int \frac{\sqrt[3]{x}\sqrt[3]{1+x^2-x^6}\left(1+2x^6\right)}{\left(-1+x^6\right)^2}dx}{\sqrt[3]{x}\sqrt[3]{1+x^2-x^6}}$$

$$= \frac{\left(3\sqrt[3]{x+x^3-x^7}\right)\text{Subst}\left(\int \frac{x^3\sqrt[3]{1+x^6-x^{18}}\left(1+2x^{18}\right)}{\left(-1+x^{18}\right)^2}dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1+x^2-x^6}}$$

$$= \frac{\left(3\sqrt[3]{x+x^3-x^7}\right)\text{Subst}\left(\int \frac{x\sqrt[3]{1+x^3-x^9}\left(1+2x^9\right)}{\left(-1+x^9\right)^2}dx, x, x^{2/3}\right)}{2\sqrt[3]{x}\sqrt[3]{1+x^2-x^6}}$$

$$= \frac{\left(3\sqrt[3]{x+x^3-x^7}\right)\text{Subst}\left(\int\left(\frac{\sqrt[3]{1+x^3-x^9}}{27(-1+x)^2} - \frac{\sqrt[3]{1+x^3-x^9}}{27(-1+x)} - \frac{\sqrt[3]{1+x^3-x^9}}{9\left(1+x+x^2\right)^2} + \frac{(1+x)\sqrt[3]{1+x^3-x^9}}{27\left(1+x+x^2\right)} + \right.\right.}{2\sqrt[3]{x}\sqrt[3]{1+x^2-x^6}}$$

= rest of steps removed due to Latex formating problem

Mathematica [F] time = 2.67, size = 0, normalized size = 0.00

$$\int \frac{\left(1+2x^6\right)\sqrt[3]{x+x^3-x^7}}{\left(-1+x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + 2*x^6)*(x + x^3 - x^7)^(1/3))/(-1 + x^6)^2,x]
[Out] Integrate[((1 + 2*x^6)*(x + x^3 - x^7)^(1/3))/(-1 + x^6)^2, x]
```

IntegrateAlgebraic [A] time = 0.22, size = 126, normalized size = 1.00

$$-\frac{1}{6}\log\left(\sqrt[3]{-x^7+x^3+x}-x\right) - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{-x^7+x^3+x}+x}\right)}{2\sqrt{3}} - \frac{\sqrt[3]{-x^7+x^3+x}\,x}{2\left(x^6-1\right)} + \frac{1}{12}\log\left(x^2+\sqrt[3]{-x^7+x^3+x}\,x+\left(-x^7+x^3+x\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + 2*x^6)*(x + x^3 - x^7)^(1/3))/(-1 + x^6)^2,x]
[Out] -1/2*(x*(x + x^3 - x^7)^(1/3))/(-1 + x^6) - ArcTan[(Sqrt[3]*x)/(x + 2*(x +
x^3 - x^7)^(1/3))]/(2*Sqrt[3]) - Log[-x + (x + x^3 - x^7)^(1/3)]/6 + Log[x^
2 + x*(x + x^3 - x^7)^(1/3) + (x + x^3 - x^7)^(2/3)]/12
```

fricas [A] time = 1.08, size = 151, normalized size = 1.20

$$\frac{2\sqrt{3}\left(x^6-1\right)\arctan\left(-\frac{4\sqrt{3}\left(-x^7+x^3+x\right)^{\frac{1}{3}}x-\sqrt{3}\left(x^6-x^2-1\right)-2\sqrt{3}\left(-x^7+x^3+x\right)^{\frac{2}{3}}}{x^6-9x^2-1}\right)-\left(x^6-1\right)\log\left(\frac{x^6-3\left(-x^7+x^3+x\right)^{\frac{1}{3}}x+3\left(-x^7+x^3+x\right)^{\frac{2}{3}}-1}{x^6-1}\right)-6\left(-x^7+x^3+x\right)^{\frac{1}{3}}x}{12\left(x^6-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+1)*(-x^7+x^3+x)^(1/3)/(x^6-1)^2,x, algorithm="fricas")
[Out] 1/12*(2*sqrt(3)*(x^6 - 1)*arctan(-(4*sqrt(3)*(-x^7 + x^3 + x)^(1/3)*x - sqr
t(3)*(x^6 - x^2 - 1) - 2*sqrt(3)*(-x^7 + x^3 + x)^(2/3))/(x^6 - 9*x^2 - 1))
 - (x^6 - 1)*log((x^6 - 3*(-x^7 + x^3 + x)^(1/3)*x + 3*(-x^7 + x^3 + x)^(2/
3) - 1)/(x^6 - 1)) - 6*(-x^7 + x^3 + x)^(1/3)*x)/(x^6 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^7+x^3+x\right)^{\frac{1}{3}}\left(2x^6+1\right)}{\left(x^6-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+1)*(-x^7+x^3+x)^(1/3)/(x^6-1)^2,x, algorithm="giac")
```

```
[Out] integrate((-x^7 + x^3 + x)^(1/3)*(2*x^6 + 1)/(x^6 - 1)^2, x)
```

maple [C] time = 19.72, size = 1137, normalized size = 9.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6+1)*(-x^7+x^3+x)^(1/3)/(x^6-1)^2,x)
```

```
[Out] -1/2*x/(x^6-1)*(-x*(x^6-x^2-1))^(1/3)+(1/6*ln(-(10282164833688*(x^14-2*x^10
-2*x^8+x^6+2*x^4+x^2)^(1/3)*RootOf(_Z^2+59*_Z+3481)*x^2-10282164833688*(x^1
4-2*x^10-2*x^8+x^6+2*x^4+x^2)^(1/3)*RootOf(_Z^2+59*_Z+3481)*x^6+16299024656
7115*x^12-325980493134230*x^6+314126657020258*x^2-314126657020258*x^8+15113
6410453143*x^4-11413752794033*RootOf(_Z^2+59*_Z+3481)+13821682980*RootOf(_Z
^2+59*_Z+3481)^2*x^12+13821682980*RootOf(_Z^2+59*_Z+3481)^2+24975485320739*
RootOf(_Z^2+59*_Z+3481)*x^8+22827505588066*RootOf(_Z^2+59*_Z+3481)*x^6-1356
1732526706*RootOf(_Z^2+59*_Z+3481)*x^4-24975485320739*RootOf(_Z^2+59*_Z+348
1)*x^2-11413752794033*RootOf(_Z^2+59*_Z+3481)*x^12-231513189915*RootOf(_Z^2
+59*_Z+3481)^2*x^8-27643365960*RootOf(_Z^2+59*_Z+3481)^2*x^6+217691506935*R
ootOf(_Z^2+59*_Z+3481)^2*x^4+231513189915*RootOf(_Z^2+59*_Z+3481)^2*x^2-951
278629528797*(x^14-2*x^10-2*x^8+x^6+2*x^4+x^2)^(1/3)+1557926354716389*(x^14
-2*x^10-2*x^8+x^6+2*x^4+x^2)^(2/3)+951278629528797*(x^14-2*x^10-2*x^8+x^6+2
*x^4+x^2)^(1/3)*x^6+16123366602183*RootOf(_Z^2+59*_Z+3481)*(x^14-2*x^10-2*x
^8+x^6+2*x^4+x^2)^(2/3)-951278629528797*(x^14-2*x^10-2*x^8+x^6+2*x^4+x^2)^(
1/3)*x^2+10282164833688*RootOf(_Z^2+59*_Z+3481)*(x^14-2*x^10-2*x^8+x^6+2*x^
4+x^2)^(1/3)+162990246567115)/(x^6-x^2-1)/(-1+x)/(1+x)/(x^2+x+1)/(x^2-x+1))
+1/354*RootOf(_Z^2+59*_Z+3481)*ln((-26405531435871*(x^14-2*x^10-2*x^8+x^6+2
*x^4+x^2)^(1/3)*RootOf(_Z^2+59*_Z+3481)*x^2+26405531435871*(x^14-2*x^10-2*x
^8+x^6+2*x^4+x^2)^(1/3)*RootOf(_Z^2+59*_Z+3481)*x^6+709670857187355*x^12-14
19341714374710*x^6+2080899293108685*x^2-2080899293108685*x^8+13712284359213
30*x^4-2147979732673*RootOf(_Z^2+59*_Z+3481)-3405296212*RootOf(_Z^2+59*_Z+3
481)^2*x^12-3405296212*RootOf(_Z^2+59*_Z+3481)^2-4969813596014*RootOf(_Z^2+
59*_Z+3481)*x^8+4295959465346*RootOf(_Z^2+59*_Z+3481)*x^6+7117793328687*Roo
tOf(_Z^2+59*_Z+3481)*x^4+4969813596014*RootOf(_Z^2+59*_Z+3481)*x^2-21479797
32673*RootOf(_Z^2+59*_Z+3481)*x^12+57038711551*RootOf(_Z^2+59*_Z+3481)^2*x^
8+6810592424*RootOf(_Z^2+59*_Z+3481)^2*x^6-53633415339*RootOf(_Z^2+59*_Z+34
81)^2*x^4-57038711551*RootOf(_Z^2+59*_Z+3481)^2*x^2-951278629528797*(x^14-2
*x^10-2*x^8+x^6+2*x^4+x^2)^(1/3)-606647725187592*(x^14-2*x^10-2*x^8+x^6+2*x
^4+x^2)^(2/3)+951278629528797*(x^14-2*x^10-2*x^8+x^6+2*x^4+x^2)^(1/3)*x^6+1
6123366602183*RootOf(_Z^2+59*_Z+3481)*(x^14-2*x^10-2*x^8+x^6+2*x^4+x^2)^(2/
3)-951278629528797*(x^14-2*x^10-2*x^8+x^6+2*x^4+x^2)^(1/3)*x^2-264055314358
71*RootOf(_Z^2+59*_Z+3481)*(x^14-2*x^10-2*x^8+x^6+2*x^4+x^2)^(1/3)+70967085
7187355)/(x^6-x^2-1)/(-1+x)/(1+x)/(x^2+x+1)/(x^2-x+1)))*(-x*(x^6-x^2-1))^(1
/3)*(x^2*(x^6-x^2-1)^2)^(1/3)/x/(x^6-x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^7+x^3+x\right)^{\frac{1}{3}}\left(2x^6+1\right)}{\left(x^6-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+1)*(-x^7+x^3+x)^(1/3)/(x^6-1)^2,x, algorithm="maxima")
```

```
[Out] integrate((-x^7 + x^3 + x)^(1/3)*(2*x^6 + 1)/(x^6 - 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^6+1\right)\,{\left(-x^7+x^3+x\right)}^{1/3}}{{\left(x^6-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^6 + 1)*(x + x^3 - x^7)^(1/3))/(x^6 - 1)^2,x)
```

```
[Out] int(((2*x^6 + 1)*(x + x^3 - x^7)^(1/3))/(x^6 - 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-x\left(x^{6}-x^{2}-1\right)}\,\left(2x^{6}+1\right)}{\left(x-1\right)^{2}\left(x+1\right)^{2}\left(x^{2}-x+1\right)^{2}\left(x^{2}+x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**6+1)*(-x**7+x**3+x)**(1/3)/(x**6-1)**2,x)
```

```
[Out] Integral((-x*(x**6 - x**2 - 1))**(1/3)*(2*x**6 + 1)/((x - 1)**2*(x + 1)**2*
(x**2 - x + 1)**2*(x**2 + x + 1)**2), x)
```

3.1554 $\int \frac{-\left((2a-b)b^2\right)+(4a-b)bx-(2a+b)x^2+x^3}{\left((-a+x)(-b+x)^2\right)^{3/4}\left(a+b^2d-(1+2bd)x+dx^2\right)}\,dx$

Optimal. Leaf size=127

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{3/4}}{(x-a)(b-x)}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{3/4}}{(x-a)(b-x)}\right)}{d^{3/4}}$$

Rubi [C] time = 2.73, antiderivative size = 283, normalized size of antiderivative = 2.23, number of steps used = 8, number of rules used = 5, integrand size = 77, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {6688, 6719, 6728, 137, 136}

$$-\frac{4\left(1-\sqrt{-4ad+4bd+1}\right)\sqrt[4]{-\left((a-x)(b-x)^2\right)}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{a-x}{a-b},-\frac{2d(a-x)}{-2ad+2bd-\sqrt{-4ad+4bd+1}+1}\right)}{\left(-\sqrt{-4ad+4bd+1}-2ad+2bd+1\right)\sqrt{-\frac{b-x}{a-b}}}-\frac{4\left(\sqrt{-4ad+4bd+1}+1\right)\sqrt[4]{-\left((a-x)(b-x)^2\right)}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{a-x}{a-b},-\frac{2d(a-x)}{-2ad+2bd+\sqrt{-4ad+4bd+1}+1}\right)}{\left(\sqrt{-4ad+4bd+1}-2ad+2bd+1\right)\sqrt{-\frac{b-x}{a-b}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-((2*a - b)*b^2) + (4*a - b)*b*x - (2*a + b)*x^2 + x^3)/(((-a + x)*(-b
 + x)^2)^(3/4)*(a + b^2*d - (1 + 2*b*d)*x + d*x^2)),x]
```

```
[Out] (-4*(1 - Sqrt[1 - 4*a*d + 4*b*d])*(-((a - x)*(b - x)^2))^(1/4)*AppellF1[1/4
, -1/2, 1, 5/4, (a - x)/(a - b), (-2*d*(a - x))/(1 - 2*a*d + 2*b*d - Sqrt[1
 - 4*a*d + 4*b*d])])/((1 - 2*a*d + 2*b*d - Sqrt[1 - 4*a*d + 4*b*d])*Sqrt[-(
(b - x)/(a - b))]) - (4*(1 + Sqrt[1 - 4*a*d + 4*b*d])*(-((a - x)*(b - x)^2)
)^(1/4)*AppellF1[1/4, -1/2, 1, 5/4, (a - x)/(a - b), (-2*d*(a - x))/(1 - 2*
a*d + 2*b*d + Sqrt[1 - 4*a*d + 4*b*d])])/((1 - 2*a*d + 2*b*d + Sqrt[1 - 4*a
*d + 4*b*d])*Sqrt[-((b - x)/(a - b))])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-\left((2a-b)b^2\right)+(4a-b)bx-(2a+b)x^2+x^3}{\left((-a+x)(-b+x)^2\right)^{3/4}\left(a+b^2d-(1+2bd)x+dx^2\right)}\,dx=\int \frac{(2a-b-x)\sqrt[4]{-\left((a-x)(b-x)^2\right)}}{(a-x)\left(a+b^2d-(1+2bd)x+dx^2\right)}\,dx$$

$$=\frac{\sqrt[4]{-\left((a-x)(b-x)^2\right)}\int \frac{(2a-b-x)\sqrt{b-x}}{(a-x)^{3/4}\left(a+b^2d-(1+2bd)x+dx^2\right)}\,dx}{\sqrt[4]{a-x}\sqrt{b-x}}$$

$$=\frac{\sqrt[4]{-\left((a-x)(b-x)^2\right)}\int \left(\frac{\left(-1-\sqrt{1-4ad+4bd}\right)\sqrt{b-x}}{(a-x)^{3/4}\left(-1-2bd-\sqrt{1-4ad+4bd}+2d\right.}\right.}{\sqrt[4]{a-x}\sqrt{b-x}}$$

$$=\frac{\left(\left(-1-\sqrt{1-4ad+4bd}\right)\sqrt[4]{-\left((a-x)(b-x)^2\right)}\right)\int \frac{}{(a-}}{\sqrt[4]{a-x}\sqrt{b-x}}$$

$$=\frac{\left(\left(-1-\sqrt{1-4ad+4bd}\right)\sqrt[4]{-\left((a-x)(b-x)^2\right)}\right)\int \frac{}{(a-}}{\sqrt[4]{a-x}\sqrt{-\frac{b-x}{a-b}}}$$

$$=-\frac{4\left(1-\sqrt{1-4ad+4bd}\right)\sqrt[4]{-\left((a-x)(b-x)^2\right)}F_1\left(\frac{1}{4}\right.}{\left(1-2ad+2bd-\sqrt{1-4ad}\right.}$$

Mathematica [C] time = 1.55, size = 488, normalized size = 3.84

$$\frac{2\sqrt[4]{(x-a)(b-x)^2}\left(\sqrt[4]{a-b}\sqrt{\frac{x-b}{a-b}}\,\Pi\left(-\frac{\sqrt{2}\sqrt{a-b}\sqrt{d}}{\sqrt{2ad-2bd-\sqrt{-4ad+4bd+1}-1}};\sin^{-1}\left(\frac{\sqrt[4]{a-x}}{\sqrt[4]{a-b}}\right)\middle|-1\right)+\sqrt[4]{a-b}\sqrt{\frac{x-b}{a-b}}\,\Pi\left(\frac{\sqrt{2}\sqrt{a-b}\sqrt{d}}{\sqrt{2ad-2bd-\sqrt{-4ad+4bd+1}-1}};\sin^{-1}\left(\frac{\sqrt[4]{a-x}}{\sqrt[4]{a-b}}\right)\middle|-1\right)+\sqrt[4]{a-b}\sqrt{\frac{x-b}{a-b}}\,\Pi\left(-\frac{\sqrt{2}\sqrt{a-b}\sqrt{d}}{\sqrt{2ad-2bd+\sqrt{-4ad+4bd+1}-1}};\sin^{-1}\left(\frac{\sqrt[4]{a-x}}{\sqrt[4]{a-b}}\right)\middle|-1\right)+\sqrt[4]{a-b}\sqrt{\frac{x-b}{a-b}}\,\Pi\left(\frac{\sqrt{2}\sqrt{a-b}\sqrt{d}}{\sqrt{2ad-2bd+\sqrt{-4ad+4bd+1}-1}};\sin^{-1}\left(\frac{\sqrt[4]{a-x}}{\sqrt[4]{a-b}}\right)\middle|-1\right)+\frac{(x-b)\left(\frac{a-x}{a-b}\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{x-b}{a-b}\right)}{(a-x)^{3/4}}\right)}{d\sqrt[4]{a-x}\,(b-x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-((2*a - b)*b^2) + (4*a - b)*b*x - (2*a + b)*x^2 + x^3)/(((-a +
x)*(-b + x)^2)^(3/4)*(a + b^2*d - (1 + 2*b*d)*x + d*x^2)),x]
```

```
[Out] (2*((b - x)^2*(-a + x))^(1/4)*((a - b)^(1/4)*Sqrt[(-b + x)/(a - b)]*Ellipti
cPi[-((Sqrt[2]*Sqrt[a - b]*Sqrt[d])/Sqrt[-1 + 2*a*d - 2*b*d - Sqrt[1 - 4*a*
d + 4*b*d]]), ArcSin[(a - x)^(1/4)/(a - b)^(1/4)], -1] + (a - b)^(1/4)*Sqrt
[(-b + x)/(a - b)]*EllipticPi[(Sqrt[2]*Sqrt[a - b]*Sqrt[d])/Sqrt[-1 + 2*a*d
 - 2*b*d - Sqrt[1 - 4*a*d + 4*b*d]], ArcSin[(a - x)^(1/4)/(a - b)^(1/4)], -
1] + (a - b)^(1/4)*Sqrt[(-b + x)/(a - b)]*EllipticPi[-((Sqrt[2]*Sqrt[a - b]
*Sqrt[d])/Sqrt[-1 + 2*a*d - 2*b*d + Sqrt[1 - 4*a*d + 4*b*d]]), ArcSin[(a -
x)^(1/4)/(a - b)^(1/4)], -1] + (a - b)^(1/4)*Sqrt[(-b + x)/(a - b)]*Ellipti
cPi[(Sqrt[2]*Sqrt[a - b]*Sqrt[d])/Sqrt[-1 + 2*a*d - 2*b*d + Sqrt[1 - 4*a*d
+ 4*b*d]], ArcSin[(a - x)^(1/4)/(a - b)^(1/4)], -1] + (((a - x)/(a - b))^(3
/4)*(-b + x)*Hypergeometric2F1[1/2, 3/4, 3/2, (-b + x)/(a - b)])/(a - x)^(3
/4)))/(d*(a - x)^(1/4)*(b - x))
```

IntegrateAlgebraic [A] time = 3.56, size = 127, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{3/4}}{(x-a)(b-x)}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{3/4}}{(x-a)(b-x)}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-((2*a - b)*b^2) + (4*a - b)*b*x - (2*a + b)*x^2 + x^3)
/(((-a + x)*(-b + x)^2)^(3/4)*(a + b^2*d - (1 + 2*b*d)*x + d*x^2)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(3/
4))/((b - x)*(-a + x))]/d^(3/4) + (2*ArcTanh[(d^(1/4)*(-(a*b^2) + (2*a*b +
 b^2)*x + (-a - 2*b)*x^2 + x^3)^(3/4))/((b - x)*(-a + x))]/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2*a-b)*b^2+(4*a-b)*b*x-(2*a+b)*x^2+x^3)/((-a+x)*(-b+x)^2)^(3/4
)/(a+b^2*d-(2*b*d+1)*x+d*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(2\,a-b)b^2-(4\,a-b)bx+(2\,a+b)x^2-x^3}{\left(-(a-x)(b-x)^2\right)^{\frac{3}{4}}\left(b^2d+dx^2-(2\,bd+1)x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2*a-b)*b^2+(4*a-b)*b*x-(2*a+b)*x^2+x^3)/((-a+x)*(-b+x)^2)^(3/4
)/(a+b^2*d-(2*b*d+1)*x+d*x^2),x, algorithm="giac")
```

```
[Out] integrate(-((2*a - b)*b^2 - (4*a - b)*b*x + (2*a + b)*x^2 - x^3)/((-(a - x)
*(b - x)^2)^(3/4)*(b^2*d + d*x^2 - (2*b*d + 1)*x + a)), x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{-\left(2a-b\right)b^{2}+\left(4a-b\right)bx-\left(2a+b\right)x^{2}+x^{3}}{\left(\left(-a+x\right)\left(-b+x\right)^{2}\right)^{\frac{3}{4}}\left(a+b^{2}d-\left(2bd+1\right)x+d\,x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(2*a-b)*b^2+(4*a-b)*b*x-(2*a+b)*x^2+x^3)/((-a+x)*(-b+x)^2)^(3/4)/(a+b
^2*d-(2*b*d+1)*x+d*x^2),x)
```

```
[Out] int((-(2*a-b)*b^2+(4*a-b)*b*x-(2*a+b)*x^2+x^3)/((-a+x)*(-b+x)^2)^(3/4)/(a+b
^2*d-(2*b*d+1)*x+d*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(2\,a-b)b^2-(4\,a-b)bx+(2\,a+b)x^2-x^3}{\left(-(a-x)(b-x)^2\right)^{\frac{3}{4}}\left(b^2d+dx^2-(2\,bd+1)x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2*a-b)*b^2+(4*a-b)*b*x-(2*a+b)*x^2+x^3)/((-a+x)*(-b+x)^2)^(3/4
)/(a+b^2*d-(2*b*d+1)*x+d*x^2),x, algorithm="maxima")
```

```
[Out] -integrate(((2*a - b)*b^2 - (4*a - b)*b*x + (2*a + b)*x^2 - x^3)/((-(a - x)
*(b - x)^2)^(3/4)*(b^2*d + d*x^2 - (2*b*d + 1)*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{b^2\,\left(2\,a-b\right)+x^2\,\left(2\,a+b\right)-x^3-b\,x\,\left(4\,a-b\right)}{{\left(-\left(a-x\right)\,{\left(b-x\right)}^2\right)}^{3/4}\,\left(a-x\,\left(2\,b\,d+1\right)+b^2\,d+d\,x^2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^2*(2*a - b) + x^2*(2*a + b) - x^3 - b*x*(4*a - b))/((-(a - x)*(b -
x)^2)^(3/4)*(a - x*(2*b*d + 1) + b^2*d + d*x^2)),x)
```

```
[Out] int(-(b^2*(2*a - b) + x^2*(2*a + b) - x^3 - b*x*(4*a - b))/((-(a - x)*(b -
x)^2)^(3/4)*(a - x*(2*b*d + 1) + b^2*d + d*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2*a-b)*b**2+(4*a-b)*b*x-(2*a+b)*x**2+x**3)/((-a+x)*(-b+x)**2)*
*(3/4)/(a+b**2*d-(2*b*d+1)*x+d*x**2),x)
```

```
[Out] Timed out
```

3.1555 $\int \frac{\sqrt{x+\sqrt{1+x}}}{x^2}\,dx$

Optimal. Leaf size=127

$$-\frac{\sqrt{x+\sqrt{x+1}}}{x}-\frac{1}{4}\tan^{-1}\left(\frac{2x+\sqrt{x+1}\left(3-2\sqrt{x+\sqrt{x+1}}\right)-2\sqrt{x+\sqrt{x+1}}+1}{2\sqrt{x+1}-2\sqrt{x+\sqrt{x+1}}+2}\right)-\frac{3}{2}\tanh^{-1}\left(-\sqrt{x+1}+\sqrt{}\right.$$

Rubi [A] time = 0.14, antiderivative size = 83, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1014, 1033, 724, 206, 204}

$$-\frac{\sqrt{x+\sqrt{x+1}}}{x}-\frac{1}{4}\tan^{-1}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)+\frac{3}{4}\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[1 + x]]/x^2,x]
```

```
[Out] -(Sqrt[x + Sqrt[1 + x]]/x) - ArcTan[(3 + Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 +
x]])]/4 + (3*ArcTanh[(1 - 3*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1014

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a*h - g*c*x)*(a + c*x^2)^(p + 1)*(d +
 e*x + f*x^2)^q)/(2*a*c*(p + 1)), x] + Dist[2/(4*a*c*(p + 1)), Int[(a + c*x
^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[g*c*d*(2*p + 3) - a*(h*e*q) + (g
*c*e*(2*p + q + 3) - a*(2*h*f*q))*x + g*c*f*(2*p + 2*q + 3)*x^2, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] &&
 GtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
```

```
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{\sqrt{x+\sqrt{1+x}}}{x^2}\,dx = 2\,\text{Subst}\left(\int \frac{x\sqrt{-1+x+x^2}}{\left(-1+x^2\right)^2}\,dx, x, \sqrt{1+x}\right)$$
$$= -\frac{\sqrt{x+\sqrt{1+x}}}{x} + \text{Subst}\left(\int \frac{\frac{1}{2}+x}{\left(-1+x^2\right)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)$$
$$= -\frac{\sqrt{x+\sqrt{1+x}}}{x} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right) + \frac{3}{4}\text{Subst}\left(\int \frac{}{(-1+}\right.$$
$$= -\frac{\sqrt{x+\sqrt{1+x}}}{x} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3-\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x,\right.$$
$$= -\frac{\sqrt{x+\sqrt{1+x}}}{x} - \frac{1}{4}\tan^{-1}\left(\frac{3+\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right) + \frac{3}{4}\tanh^{-1}\left(\frac{1-3\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)$$

Mathematica [A] time = 0.05, size = 85, normalized size = 0.67

$$-\frac{\sqrt{x+\sqrt{x+1}}}{x} + \frac{1}{4}\tan^{-1}\left(\frac{-\sqrt{x+1}-3}{2\sqrt{x+\sqrt{x+1}}}\right) - \frac{3}{4}\tanh^{-1}\left(\frac{3\sqrt{x+1}-1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[1 + x]]/x^2,x]
```

```
[Out] -(Sqrt[x + Sqrt[1 + x]]/x) + ArcTan[(-3 - Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 +
 x]])]/4 - (3*ArcTanh[(-1 + 3*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/4
```

IntegrateAlgebraic [A] time = 0.28, size = 77, normalized size = 0.61

$$-\frac{\sqrt{x+\sqrt{x+1}}}{x} - \frac{1}{2}\tan^{-1}\left(\sqrt{x+1} - \sqrt{x+\sqrt{x+1}} + 1\right) - \frac{3}{2}\tanh^{-1}\left(-\sqrt{x+1} + \sqrt{x+\sqrt{x+1}} + 1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x + Sqrt[1 + x]]/x^2,x]
```

```
[Out] -(Sqrt[x + Sqrt[1 + x]]/x) - ArcTan[1 + Sqrt[1 + x] - Sqrt[x + Sqrt[1 + x]]
]/2 - (3*ArcTanh[1 - Sqrt[1 + x] + Sqrt[x + Sqrt[1 + x]]])/2
```

fricas [A] time = 5.97, size = 81, normalized size = 0.64

$$\frac{x\arctan\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}-3\right)}{x-8}\right) + 3\,x\log\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}+1\right)-3\,x-2\,\sqrt{x+1}-2}{x}\right) - 4\,\sqrt{x+\sqrt{x+1}}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(x*arctan(2*sqrt(x + sqrt(x + 1))*(sqrt(x + 1) - 3)/(x - 8)) + 3*x*log(
(2*sqrt(x + sqrt(x + 1))*(sqrt(x + 1) + 1) - 3*x - 2*sqrt(x + 1) - 2)/x) -
4*sqrt(x + sqrt(x + 1)))/x
```

giac [A] time = 1.03, size = 188, normalized size = 1.48

$$-\frac{2\left(\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}\right)^{3}-3\left(\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}\right)^{2}-\sqrt{x+\sqrt{x+1}}+\sqrt{x+1}+1}{\left(\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}\right)^{4}-2\left(\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}\right)^{2}+4\sqrt{x+\sqrt{x+1}}-4\sqrt{x+1}}+\frac{1}{2}\arctan\left(\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}-1\right)-\frac{3}{4}\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}+2\right|\right)+\frac{3}{4}\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -(2*(sqrt(x + sqrt(x + 1)) - sqrt(x + 1))^3 - 3*(sqrt(x + sqrt(x + 1)) - sq
rt(x + 1))^2 - sqrt(x + sqrt(x + 1)) + sqrt(x + 1) + 1)/((sqrt(x + sqrt(x +
 1)) - sqrt(x + 1))^4 - 2*(sqrt(x + sqrt(x + 1)) - sqrt(x + 1))^2 + 4*sqrt(
x + sqrt(x + 1)) - 4*sqrt(x + 1)) + 1/2*arctan(sqrt(x + sqrt(x + 1)) - sqrt
(x + 1) - 1) - 3/4*log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1) + 2)) + 3/4*
log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1)))
```

maple [B] time = 0.02, size = 298, normalized size = 2.35

$$-\frac{\left(\left(1+\sqrt{1+x}\right)^{2}-\sqrt{1+x}-2\right)^{\frac{3}{2}}}{2\left(1+\sqrt{1+x}\right)}-\frac{\sqrt{\left(1+\sqrt{1+x}\right)^{2}-\sqrt{1+x}-2}}{4}-\frac{\ln\left(\sqrt{1+x}+\frac{1}{2}+\sqrt{\left(1+\sqrt{1+x}\right)^{2}-\sqrt{1+x}-2}\right)}{2}+\frac{\arctan\left(\frac{-\sqrt{1+x}-3}{2\sqrt{\left(1+\sqrt{1+x}\right)^{2}-\sqrt{1+x}-2}}\right)}{4}+\frac{\left(1+2\sqrt{1+x}\right)\sqrt{\left(1+\sqrt{1+x}\right)^{2}-\sqrt{1+x}-2}}{4}-\frac{\left(\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2\right)^{\frac{3}{2}}}{2\left(-1+\sqrt{1+x}\right)}+\frac{3\sqrt{\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2}}{4}+\frac{\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2}\right)}{2}-\frac{3\,\mathrm{arctanh}\left(\frac{-1+3\sqrt{1+x}}{2\sqrt{\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2}}\right)}{4}+\frac{\left(1+2\sqrt{1+x}\right)\sqrt{\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(1+x)^(1/2))^(1/2)/x^2,x)
```

```
[Out] -1/2/(1+(1+x)^(1/2))*((1+(1+x)^(1/2))^2-(1+x)^(1/2)-2)^(3/2)-1/4*((1+(1+x)^
(1/2))^2-(1+x)^(1/2)-2)^(1/2)-1/2*ln((1+x)^(1/2)+1/2+((1+(1+x)^(1/2))^2-(1+
x)^(1/2)-2)^(1/2))+1/4*arctan(1/2*(-(1+x)^(1/2)-3)/((1+(1+x)^(1/2))^2-(1+x)
^(1/2)-2)^(1/2))+1/4*(1+2*(1+x)^(1/2))*((1+(1+x)^(1/2))^2-(1+x)^(1/2)-2)^(1
/2)-1/2/(-1+(1+x)^(1/2))*((-1+(1+x)^(1/2))^2+3*(1+x)^(1/2)-2)^(3/2)+3/4*((-
1+(1+x)^(1/2))^2+3*(1+x)^(1/2)-2)^(1/2)+1/2*ln(1/2+(1+x)^(1/2)+((-1+(1+x)^(
1/2))^2+3*(1+x)^(1/2)-2)^(1/2))-3/4*arctanh(1/2*(-1+3*(1+x)^(1/2))/((-1+(1+
x)^(1/2))^2+3*(1+x)^(1/2)-2)^(1/2))+1/4*(1+2*(1+x)^(1/2))*((-1+(1+x)^(1/2))
^2+3*(1+x)^(1/2)-2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x + 1)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int((x + (x + 1)^(1/2))^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x + sqrt(x + 1))/x**2, x)
```

3.1556 $\int \frac{(-1+x)\sqrt{x+\sqrt{1+x^2}}}{1+x}\,dx$

Optimal. Leaf size=127

$$\frac{2\sqrt{x^2+1}\,(x-6)+2\left(x^2-6x-1\right)}{3\sqrt{\sqrt{x^2+1}+x}}+4\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{1+\sqrt{2}}}\right)+4\sqrt{\sqrt{2}-1}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{\sqrt{2}-1}}\right)$$

Rubi [A] time = 0.32, antiderivative size = 136, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6742, 2117, 14, 2119, 1628, 826, 1166, 207, 203}

$$\frac{1}{3}\left(\sqrt{x^2+1}+x\right)^{3/2}-4\sqrt{\sqrt{x^2+1}+x}-\frac{1}{\sqrt{\sqrt{x^2+1}+x}}+4\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)+4\sqrt{\sqrt{2}-1}\,\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)*Sqrt[x + Sqrt[1 + x^2]])/(1 + x),x]
```

```
[Out] -(1/Sqrt[x + Sqrt[1 + x^2]]) - 4*Sqrt[x + Sqrt[1 + x^2]] + (x + Sqrt[1 + x^
2])^(3/2)/3 + 4*Sqrt[1 + Sqrt[2]]*ArcTan[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1
 + x^2]]] + 4*Sqrt[-1 + Sqrt[2]]*ArcTanh[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1
+ x^2]]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(-1+x)\sqrt{x+\sqrt{1+x^2}}}{1+x}\,dx = \int \left(\sqrt{x+\sqrt{1+x^2}} - \frac{2\sqrt{x+\sqrt{1+x^2}}}{1+x}\right)dx$$

$$= -\left(2\int \frac{\sqrt{x+\sqrt{1+x^2}}}{1+x}\,dx\right) + \int \sqrt{x+\sqrt{1+x^2}}\,dx$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x^2}{x^{3/2}}\,dx, x, x+\sqrt{1+x^2}\right) - 2\operatorname{Subst}\left(\int \frac{1+x^2}{\sqrt{x}\left(-1+2x+x^2\right)}\,dx, x, x\right.$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{x^{3/2}}+\sqrt{x}\right)dx, x, x+\sqrt{1+x^2}\right) - 2\operatorname{Subst}\left(\int \left(\frac{1}{\sqrt{x}} + \frac{2(1-}{\sqrt{x}\left(-1+2\right.}\right.\right.$$

$$= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}} - 4\sqrt{x+\sqrt{1+x^2}} + \frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2} - 4\operatorname{Subst}\left(\int \frac{}{\sqrt{x}\left(-1\right.}\right.$$

$$= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}} - 4\sqrt{x+\sqrt{1+x^2}} + \frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2} - 8\operatorname{Subst}\left(\int \frac{1-}{-1+2}\right.$$

$$= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}} - 4\sqrt{x+\sqrt{1+x^2}} + \frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2} + \left(4\left(1-\sqrt{2}\right)\right)\operatorname{Subst}$$

$$= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}} - 4\sqrt{x+\sqrt{1+x^2}} + \frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2} + 4\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\right.$$

Mathematica [A] time = 0.34, size = 150, normalized size = 1.18

$$\frac{1}{3}\left(\sqrt{x^2+1}+x\right)^{3/2} - 4\sqrt{\sqrt{x^2+1}+x} - \frac{1}{\sqrt{\sqrt{x^2+1}+x}} + 4\sqrt{\sqrt{2}-1}\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{1+\sqrt{2}}}\right) + 4\left(\sqrt{2}-1\right)\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{\sqrt{2}-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)*Sqrt[x + Sqrt[1 + x^2]])/(1 + x),x]
```

```
[Out] -(1/Sqrt[x + Sqrt[1 + x^2]]) - 4*Sqrt[x + Sqrt[1 + x^2]] + (x + Sqrt[1 + x^
2])^(3/2)/3 + 4*Sqrt[-1 + Sqrt[2]]*(1 + Sqrt[2])*ArcTan[Sqrt[x + Sqrt[1 + x
^2]]/Sqrt[1 + Sqrt[2]]] + 4*(-1 + Sqrt[2])*Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[x
 + Sqrt[1 + x^2]]/Sqrt[-1 + Sqrt[2]]]
```

IntegrateAlgebraic [A] time = 0.33, size = 127, normalized size = 1.00

$$\frac{2\sqrt{x^2+1}\,(x-6)+2\left(x^2-6x-1\right)}{3\sqrt{\sqrt{x^2+1}+x}}+4\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\sqrt{\sqrt{2}-1}\,\sqrt{\sqrt{x^2+1}+x}\right)+4\sqrt{\sqrt{2}-1}\,\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\,\sqrt{\sqrt{x^2+1}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)*Sqrt[x + Sqrt[1 + x^2]])/(1 + x),x]
```

```
[Out] (2*(-6 + x)*Sqrt[1 + x^2] + 2*(-1 - 6*x + x^2))/(3*Sqrt[x + Sqrt[1 + x^2]])
 + 4*Sqrt[1 + Sqrt[2]]*ArcTan[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]] +
 4*Sqrt[-1 + Sqrt[2]]*ArcTanh[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]]
```

fricas [A] time = 0.43, size = 158, normalized size = 1.24

$$\frac{2}{3}\left(2x-\sqrt{x^2+1}-6\right)\sqrt{x+\sqrt{x^2+1}}-8\sqrt{\sqrt{2}+1}\,\arctan\left(\sqrt{x+\sqrt{2}+\sqrt{x^2+1}+1}\,\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)-\sqrt{x+\sqrt{x^2+1}}\,\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)\right)+2\sqrt{\sqrt{2}-1}\,\log\left(4\sqrt{x+\sqrt{x^2+1}}+4\sqrt{\sqrt{2}-1}\right)-2\sqrt{\sqrt{2}-1}\,\log\left(4\sqrt{x+\sqrt{x^2+1}}-4\sqrt{\sqrt{2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(x+(x^2+1)^(1/2))^(1/2)/(1+x),x, algorithm="fricas")
```

```
[Out] 2/3*(2*x - sqrt(x^2 + 1) - 6)*sqrt(x + sqrt(x^2 + 1)) - 8*sqrt(sqrt(2) + 1)
*arctan(sqrt(x + sqrt(2) + sqrt(x^2 + 1) + 1)*sqrt(sqrt(2) + 1)*(sqrt(2) -
1) - sqrt(x + sqrt(x^2 + 1))*sqrt(sqrt(2) + 1)*(sqrt(2) - 1)) + 2*sqrt(sqrt
(2) - 1)*log(4*sqrt(x + sqrt(x^2 + 1)) + 4*sqrt(sqrt(2) - 1)) - 2*sqrt(sqrt
(2) - 1)*log(4*sqrt(x + sqrt(x^2 + 1)) - 4*sqrt(sqrt(2) - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x^2+1}}\,(x-1)}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(x+(x^2+1)^(1/2))^(1/2)/(1+x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + sqrt(x^2 + 1))*(x - 1)/(x + 1), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(-1+x)\sqrt{x+\sqrt{x^2+1}}}{1+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)*(x+(x^2+1)^(1/2))^(1/2)/(1+x),x)
```

```
[Out] int((-1+x)*(x+(x^2+1)^(1/2))^(1/2)/(1+x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x^2+1}}\,(x-1)}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(x+(x^2+1)^(1/2))^(1/2)/(1+x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x^2 + 1))*(x - 1)/(x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x+\sqrt{x^2+1}}\,\left(x-1\right)}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + (x^2 + 1)^(1/2))^(1/2)*(x - 1))/(x + 1),x)
```

```
[Out] int(((x + (x^2 + 1)^(1/2))^(1/2)*(x - 1))/(x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-1\right)\sqrt{x+\sqrt{x^2+1}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(x+(x**2+1)**(1/2))**(1/2)/(1+x),x)
```

```
[Out] Integral((x - 1)*sqrt(x + sqrt(x**2 + 1))/(x + 1), x)
```

3.1557 $\int \frac{(1+x)\sqrt{x+\sqrt{1+x^2}}}{-1+x}\,dx$

Optimal. Leaf size=127

$$\frac{2\sqrt{x^2+1}\,(x+6)+2\left(x^2+6x-1\right)}{3\sqrt{\sqrt{x^2+1}+x}}-4\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{\sqrt{2}-1}}\right)-4\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{1+\sqrt{2}}}\right)$$

Rubi [A] time = 0.27, antiderivative size = 136, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6742, 2117, 14, 2119, 1628, 826, 1166, 207, 203}

$$\frac{1}{3}\left(\sqrt{x^2+1}+x\right)^{3/2}+4\sqrt{\sqrt{x^2+1}+x}-\frac{1}{\sqrt{\sqrt{x^2+1}+x}}-4\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\sqrt{1+\sqrt{2}}\,\sqrt{\sqrt{x^2+1}+x}\right)-4\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\,\sqrt{\sqrt{x^2+1}+x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)*Sqrt[x + Sqrt[1 + x^2]])/(-1 + x),x]
```

```
[Out] -(1/Sqrt[x + Sqrt[1 + x^2]]) + 4*Sqrt[x + Sqrt[1 + x^2]] + (x + Sqrt[1 + x^
2])^(3/2)/3 - 4*Sqrt[-1 + Sqrt[2]]*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1
 + x^2]]] - 4*Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1
+ x^2]]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(1+x)\sqrt{x+\sqrt{1+x^2}}}{-1+x}\,dx = \int \left(\sqrt{x+\sqrt{1+x^2}} + \frac{2\sqrt{x+\sqrt{1+x^2}}}{-1+x}\right)dx$$

$$= 2\int \frac{\sqrt{x+\sqrt{1+x^2}}}{-1+x}\,dx + \int \sqrt{x+\sqrt{1+x^2}}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1+x^2}{x^{3/2}}\,dx, x, x+\sqrt{1+x^2}\right) + 2\,\text{Subst}\left(\int \frac{1+x^2}{\sqrt{x}\left(-1-2x+x^2\right)}\,dx, x, x +$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{x^{3/2}}+\sqrt{x}\right)dx, x, x+\sqrt{1+x^2}\right) + 2\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{x}} + \frac{2(1+x)}{\sqrt{x}\left(-1-2x\right.}\right.\right.$$

$$= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}} + 4\sqrt{x+\sqrt{1+x^2}} + \frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2} + 4\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(-1}\right.$$

$$= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}} + 4\sqrt{x+\sqrt{1+x^2}} + \frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2} + 8\,\text{Subst}\left(\int \frac{1+x}{-1-2x^2}\right.$$

$$= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}} + 4\sqrt{x+\sqrt{1+x^2}} + \frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2} + \left(4\left(1-\sqrt{2}\right)\right)\text{Subst}\left(\right.$$

$$= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}} + 4\sqrt{x+\sqrt{1+x^2}} + \frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2} - 4\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\right.$$

Mathematica [A] time = 0.31, size = 150, normalized size = 1.18

$$\frac{1}{3}\left(\sqrt{x^2+1}+x\right)^{3/2} + 4\sqrt{\sqrt{x^2+1}+x} - \frac{1}{\sqrt{\sqrt{x^2+1}+x}} - 4\left(\sqrt{2}-1\right)\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{\sqrt{2}-1}}\right) - 4\sqrt{\sqrt{2}-1}\left(1+\sqrt{2}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{1+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)*Sqrt[x + Sqrt[1 + x^2]])/(-1 + x),x]
```

```
[Out] -(1/Sqrt[x + Sqrt[1 + x^2]]) + 4*Sqrt[x + Sqrt[1 + x^2]] + (x + Sqrt[1 + x^
2])^(3/2)/3 - 4*(-1 + Sqrt[2])*Sqrt[1 + Sqrt[2]]*ArcTan[Sqrt[x + Sqrt[1 + x
^2]]/Sqrt[-1 + Sqrt[2]]] - 4*Sqrt[-1 + Sqrt[2]]*(1 + Sqrt[2])*ArcTanh[Sqrt[
x + Sqrt[1 + x^2]]/Sqrt[1 + Sqrt[2]]]
```

IntegrateAlgebraic [A] time = 0.28, size = 127, normalized size = 1.00

$$\frac{2\sqrt{x^2+1}\,(x+6)+2\left(x^2+6x-1\right)}{3\sqrt{\sqrt{x^2+1}+x}}-4\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\sqrt{1+\sqrt{2}}\,\sqrt{\sqrt{x^2+1}+x}\right)-4\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\,\sqrt{\sqrt{x^2+1}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x)*Sqrt[x + Sqrt[1 + x^2]])/(-1 + x),x]
```

```
[Out] (2*(6 + x)*Sqrt[1 + x^2] + 2*(-1 + 6*x + x^2))/(3*Sqrt[x + Sqrt[1 + x^2]])
- 4*Sqrt[-1 + Sqrt[2]]*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]] -
4*Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]]
```

fricas [A] time = 0.43, size = 158, normalized size = 1.24

$$\frac{2}{3}\left(2x-\sqrt{x^2+1}+6\right)\sqrt{x+\sqrt{x^2+1}}+8\sqrt{\sqrt{2}-1}\arctan\left(\sqrt{x+\sqrt{2}+\sqrt{x^2+1}-1}\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}-\sqrt{x+\sqrt{x^2+1}}\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}\right)-2\sqrt{\sqrt{2}+1}\log\left(4\sqrt{x+\sqrt{x^2+1}}+4\sqrt{\sqrt{2}+1}\right)+2\sqrt{\sqrt{2}+1}\log\left(4\sqrt{x+\sqrt{x^2+1}}-4\sqrt{\sqrt{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(x+(x^2+1)^(1/2))^(1/2)/(-1+x),x, algorithm="fricas")
```

```
[Out] 2/3*(2*x - sqrt(x^2 + 1) + 6)*sqrt(x + sqrt(x^2 + 1)) + 8*sqrt(sqrt(2) - 1)
*arctan(sqrt(x + sqrt(2) + sqrt(x^2 + 1) - 1)*(sqrt(2) + 1)*sqrt(sqrt(2) -
1) - sqrt(x + sqrt(x^2 + 1))*(sqrt(2) + 1)*sqrt(sqrt(2) - 1)) - 2*sqrt(sqrt
(2) + 1)*log(4*sqrt(x + sqrt(x^2 + 1)) + 4*sqrt(sqrt(2) + 1)) + 2*sqrt(sqrt
(2) + 1)*log(4*sqrt(x + sqrt(x^2 + 1)) - 4*sqrt(sqrt(2) + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x^2+1}}\,(x+1)}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(x+(x^2+1)^(1/2))^(1/2)/(-1+x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + sqrt(x^2 + 1))*(x + 1)/(x - 1), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(1+x)\sqrt{x+\sqrt{x^2+1}}}{-1+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)*(x+(x^2+1)^(1/2))^(1/2)/(-1+x),x)
```

```
[Out] int((1+x)*(x+(x^2+1)^(1/2))^(1/2)/(-1+x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x^2+1}}\,(x+1)}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(x+(x^2+1)^(1/2))^(1/2)/(-1+x),x, algorithm="maxima")
[Out] integrate(sqrt(x + sqrt(x^2 + 1))*(x + 1)/(x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x+\sqrt{x^2+1}}\,(x+1)}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + (x^2 + 1)^(1/2))^(1/2)*(x + 1))/(x - 1),x)
[Out] int(((x + (x^2 + 1)^(1/2))^(1/2)*(x + 1))/(x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)\sqrt{x+\sqrt{x^2+1}}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(x+(x**2+1)**(1/2))**(1/2)/(-1+x),x)
[Out] Integral((x + 1)*sqrt(x + sqrt(x**2 + 1))/(x - 1), x)
```

3.1558 $\int \frac{6+2x+x^2}{(1+x)\sqrt[3]{2+x+x^2}\left(2-x+2x^2\right)}\,dx$

Optimal. Leaf size=128

$$\frac{\log\left(\sqrt[3]{x^2+x+2}+\sqrt[3]{2}x\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{x^2+x+2}\,x+\left(x^2+x+2\right)^{2/3}\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2+x+2}}{\sqrt{3}}-\frac{2\sqrt[3]{2}x}{\sqrt{3}}}{\sqrt[3]{x^2+x+2}}\right)}{\sqrt[3]{2}}$$

Rubi [F] time = 0.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{6+2x+x^2}{(1+x)\sqrt[3]{2+x+x^2}\left(2-x+2x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(6 + 2*x + x^2)/((1 + x)*(2 + x + x^2)^(1/3)*(2 - x + 2*x^2)),x]
```

```
[Out] (-3*((1 - I*Sqrt[7] + 2*x)/(1 + x))^(1/3)*((1 + I*Sqrt[7] + 2*x)/(1 + x))^(
1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (1 - I*Sqrt[7])/(2*(1 + x)), (1 + I*Sqrt[
7])/(2*(1 + x))])/(2*2^(2/3)*(2 + x + x^2)^(1/3)) + Defer[Int][(4 - x)/((2
+ x + x^2)^(1/3)*(2 - x + 2*x^2)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{6+2x+x^2}{(1+x)\sqrt[3]{2+x+x^2}\left(2-x+2x^2\right)}\,dx &= \int \frac{6+2x+x^2}{\sqrt[3]{2+x+x^2}\left(2+x+x^2+2x^3\right)}\,dx\\
&= \int \left(\frac{1}{(1+x)\sqrt[3]{2+x+x^2}}+\frac{4-x}{\sqrt[3]{2+x+x^2}\left(2-x+2x^2\right)}\right)dx\\
&= \int \frac{1}{(1+x)\sqrt[3]{2+x+x^2}}\,dx+\int \frac{4-x}{\sqrt[3]{2+x+x^2}\left(2-x+2x^2\right)}\,dx\\
&= -\frac{\left(\sqrt[3]{\frac{1-i\sqrt{7}+2x}{1+x}}\sqrt[3]{\frac{1+i\sqrt{7}+2x}{1+x}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\sqrt[3]{1-\frac{1}{2}\left(1-i\sqrt{7}\right)x}\sqrt[3]{1-\frac{1}{2}\left(1+i\sqrt{7}\right)x}}\right.}{2^{2/3}\left(\frac{1}{1+x}\right)^{2/3}\sqrt[3]{2+x+x^2}}\\
&= -\frac{3\sqrt[3]{\frac{1-i\sqrt{7}+2x}{1+x}}\sqrt[3]{\frac{1+i\sqrt{7}+2x}{1+x}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{1-i\sqrt{7}}{2(1+x)},\frac{1+i\sqrt{7}}{2(1+x)}\right)}{2\,2^{2/3}\sqrt[3]{2+x+x^2}}+\int \frac{}{\sqrt[3]{2+x}}
\end{aligned}$$

Mathematica [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{6+2x+x^2}{(1+x)\sqrt[3]{2+x+x^2}\left(2-x+2x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(6 + 2*x + x^2)/((1 + x)*(2 + x + x^2)^(1/3)*(2 - x + 2*x^2)),x]
```

```
[Out] Integrate[(6 + 2*x + x^2)/((1 + x)*(2 + x + x^2)^(1/3)*(2 - x + 2*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.20, size = 128, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{x^2+x+2}+\sqrt[3]{2}\,x\right)}{\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2-\sqrt[3]{2}\,\sqrt[3]{x^2+x+2}\,x+\left(x^2+x+2\right)^{2/3}\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2+x+2}}{\sqrt{3}}-\frac{2\sqrt[3]{2}\,x}{\sqrt{3}}}{\sqrt[3]{x^2+x+2}}\right)}{\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(6 + 2*x + x^2)/((1 + x)*(2 + x + x^2)^(1/3)*(2 - x + 2*
x^2)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[((-2*2^(1/3)*x)/Sqrt[3] + (2 + x + x^2)^(1/3)/Sqrt[3])/(2
 + x + x^2)^(1/3)])/2^(1/3)) + Log[2^(1/3)*x + (2 + x + x^2)^(1/3)]/2^(1/3)
 - Log[2^(2/3)*x^2 - 2^(1/3)*x*(2 + x + x^2)^(1/3) + (2 + x + x^2)^(2/3)]/(
2*2^(1/3))
```

fricas [B] time = 17.30, size = 407, normalized size = 3.18

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}\left(8x^9+48x^8+18x^7+37x^6-147x^5-111x^4-107x^3+18x^2+12x+8\right)+12\sqrt{2}\left(4x^8-14x^7-13x^6-26x^5+5x^4+4x^3+4x^2\right)\left(x^2+x+2\right)^{\frac{1}{3}}+12\cdot 2^{\frac{1}{6}}\left(8x^7+2x^6+x^5+2x^4-5x^3-4x^2-4x\right)\left(x^2+x+2\right)^{\frac{2}{3}}\right)}{6\left(8x^9-96x^8-90x^7-179x^6+33x^5+33x^4+37x^3+18x^2+12x+8\right)}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{6\cdot 2^{\frac{1}{3}}\left(x^2+x+2\right)^{\frac{1}{3}}x^2+2^{\frac{2}{3}}\left(2x^3+x^2+x+2\right)+6\left(x^2+x+2\right)^{\frac{2}{3}}x}{2x^3+x^2+x+2}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{3\cdot 2^{\frac{2}{3}}\left(4x^4-x^3-x^2-2x\right)\left(x^2+x+2\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(4x^6-14x^5-13x^4-26x^3+5x^2+4x+4\right)-12\left(x^5-x^4-x^3-2x^2\right)\left(x^2+x+2\right)^{\frac{1}{3}}}{4x^6+4x^5+5x^4+10x^3+5x^2+4x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+6)/(1+x)/(x^2+x+2)^(1/3)/(2*x^2-x+2),x, algorithm="frica
s")
```

```
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(8*x^9 + 48*x^8 +
18*x^7 + 37*x^6 - 147*x^5 - 111*x^4 - 107*x^3 + 18*x^2 + 12*x + 8) + 12*sqr
t(2)*(4*x^8 - 14*x^7 - 13*x^6 - 26*x^5 + 5*x^4 + 4*x^3 + 4*x^2)*(x^2 + x +
2)^(1/3) + 12*2^(1/6)*(8*x^7 + 2*x^6 + x^5 + 2*x^4 - 5*x^3 - 4*x^2 - 4*x)*(
x^2 + x + 2)^(2/3))/(8*x^9 - 96*x^8 - 90*x^7 - 179*x^6 + 33*x^5 + 33*x^4 +
37*x^3 + 18*x^2 + 12*x + 8)) + 1/6*2^(2/3)*log((6*2^(1/3)*(x^2 + x + 2)^(1/
3)*x^2 + 2^(2/3)*(2*x^3 + x^2 + x + 2) + 6*(x^2 + x + 2)^(2/3)*x)/(2*x^3 +
x^2 + x + 2)) - 1/12*2^(2/3)*log((3*2^(2/3)*(4*x^4 - x^3 - x^2 - 2*x)*(x^2
+ x + 2)^(2/3) + 2^(1/3)*(4*x^6 - 14*x^5 - 13*x^4 - 26*x^3 + 5*x^2 + 4*x +
4) - 12*(x^5 - x^4 - x^3 - 2*x^2)*(x^2 + x + 2)^(1/3))/(4*x^6 + 4*x^5 + 5*x
^4 + 10*x^3 + 5*x^2 + 4*x + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2x+6}{\left(2x^2-x+2\right)\left(x^2+x+2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+6)/(1+x)/(x^2+x+2)^(1/3)/(2*x^2-x+2),x, algorithm="giac"
)
```

```
[Out] integrate((x^2 + 2*x + 6)/((2*x^2 - x + 2)*(x^2 + x + 2)^(1/3)*(x + 1)), x)
```

maple [C] time = 5.55, size = 671, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x+6)/(1+x)/(x^2+x+2)^(1/3)/(2*x^2-x+2),x)
```

```
[Out] RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*ln((RootOf(RootOf(_Z^3-
4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x^3+(x^2+x+2)^(2/3)*RootO
f(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-2*(x^2+x+
2)^(1/3)*RootOf(_Z^3-4)^2*x^2-2*(x^2+x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_
```

```
Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x^2+2*RootOf(_Z^3-4)*x^3-RootOf(_Z^
3-4)*x^2+4*(x^2+x+2)^(2/3)*x-RootOf(_Z^3-4)*x-2*RootOf(_Z^3-4))/(1+x)/(2*x^
2-x+2))-1/2*ln(-(RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf
(_Z^3-4)^3*x^3+(x^2+x+2)^(2/3)*RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+2*_
Z*RootOf(_Z^3-4)+4*_Z^2)*x+(x^2+x+2)^(1/3)*RootOf(_Z^3-4)^2*x^2-2*(x^2+x+2)
^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x
^2+RootOf(_Z^3-4)*x^2-2*(x^2+x+2)^(2/3)*x+RootOf(_Z^3-4)*x+2*RootOf(_Z^3-4)
)/(1+x)/(2*x^2-x+2))*RootOf(_Z^3-4)-ln(-(RootOf(RootOf(_Z^3-4)^2+2*_Z*RootO
f(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x^3+(x^2+x+2)^(2/3)*RootOf(_Z^3-4)^2*Roo
tOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+(x^2+x+2)^(1/3)*RootOf(_
Z^3-4)^2*x^2-2*(x^2+x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+
4*_Z^2)*RootOf(_Z^3-4)*x^2+RootOf(_Z^3-4)*x^2-2*(x^2+x+2)^(2/3)*x+RootOf(_Z
^3-4)*x+2*RootOf(_Z^3-4))/(1+x)/(2*x^2-x+2))*RootOf(RootOf(_Z^3-4)^2+2*_Z*R
ootOf(_Z^3-4)+4*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 2\,x + 6}{\left(2\,x^2 - x + 2\right)\left(x^2 + x + 2\right)^{\frac{1}{3}}(x + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+6)/(1+x)/(x^2+x+2)^(1/3)/(2*x^2-x+2),x, algorithm="maxim
a")
```

```
[Out] integrate((x^2 + 2*x + 6)/((2*x^2 - x + 2)*(x^2 + x + 2)^(1/3)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 + 2\,x + 6}{(x + 1)\,\left(2\,x^2 - x + 2\right)\,\left(x^2 + x + 2\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 + 6)/((x + 1)*(2*x^2 - x + 2)*(x + x^2 + 2)^(1/3)),x)
```

```
[Out] int((2*x + x^2 + 6)/((x + 1)*(2*x^2 - x + 2)*(x + x^2 + 2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 2x + 6}{(x + 1)\,\sqrt[3]{x^2 + x + 2}\,\left(2x^2 - x + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x+6)/(1+x)/(x**2+x+2)**(1/3)/(2*x**2-x+2),x)
```

```
[Out] Integral((x**2 + 2*x + 6)/((x + 1)*(x**2 + x + 2)**(1/3)*(2*x**2 - x + 2)),
 x)
```

3.1559 $\int \frac{1}{x\left(-b+ax^3\right)^{3/4}}\,dx$

Optimal. Leaf size=128

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{3b^{3/4}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}-\sqrt{b}}\right)}{3b^{3/4}}$$

Rubi [A] time = 0.21, antiderivative size = 201, normalized size of antiderivative = 1.57, number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {266, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{3\sqrt{2}b^{3/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{3\sqrt{2}b^{3/4}}-\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}\right)}{3b^{3/4}}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}+1\right)}{3b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-b + a*x^3)^(3/4)),x]
```

```
[Out] -1/3*(Sqrt[2]*ArcTan[1 - (Sqrt[2]*(-b + a*x^3)^(1/4))/b^(1/4)])/b^(3/4) + (
Sqrt[2]*ArcTan[1 + (Sqrt[2]*(-b + a*x^3)^(1/4))/b^(1/4)])/(3*b^(3/4)) - Log
[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4) + Sqrt[-b + a*x^3]]/(3*Sqrt[2
]*b^(3/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4) + Sqrt[-b + a
*x^3]]/(3*Sqrt[2]*b^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x\left(-b+ax^3\right)^{3/4}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(-b+ax)^{3/4}}\,dx, x, x^3\right)$$

$$= \frac{4\,\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{3a}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{3a\sqrt{b}} + \frac{2\,\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{3a\sqrt{b}}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{3\sqrt{2}\,b^{3/4}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}-2x}{-\sqrt{b}+\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+\ldots}\right.}{3\sqrt{2}\,b^{3/4}}$$

$$= -\frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^3}+\sqrt{-b+ax^3}\right)}{3\sqrt{2}\,b^{3/4}} + \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^3}+\sqrt{-\ldots}\right.}{3\sqrt{2}\,b^{3/4}}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)}{3b^{3/4}} + \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)}{3b^{3/4}} - \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b\ldots}\right.}{3\sqrt{2}\,\ldots}$$

Mathematica [A] time = 0.08, size = 165, normalized size = 1.29

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)+\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}+1\right)}{3\sqrt{2}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-b + a*x^3)^(3/4)),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*(-b + a*x^3)^(1/4))/b^(1/4)] + 2*ArcTan[1 + (Sqrt[2
]*(-b + a*x^3)^(1/4))/b^(1/4)] - Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^3)
```

```
^(1/4) + Sqrt[-b + a*x^3]] + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/
4) + Sqrt[-b + a*x^3]])/(3*Sqrt[2]*b^(3/4))
```

IntegrateAlgebraic [A] time = 0.22, size = 127, normalized size = 0.99

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{3b^{3/4}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}+\sqrt{b}}\right)}{3b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-b + a*x^3)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a*x^3]/(Sqrt[2]*b^(1/4)))/(
-b + a*x^3)^(1/4)])/(3*b^(3/4)) + (Sqrt[2]*ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a
*x^3)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^3])])/(3*b^(3/4))
```

fricas [A] time = 0.41, size = 134, normalized size = 1.05

$$\frac{4}{3}\left(-\frac{1}{b^3}\right)^{\frac14}\arctan\left(\sqrt{b^2\sqrt{-\frac{1}{b^3}}+\sqrt{ax^3-b}}\,b^2\left(-\frac{1}{b^3}\right)^{\frac34}-\left(ax^3-b\right)^{\frac14}b^2\left(-\frac{1}{b^3}\right)^{\frac34}\right)+\frac13\left(-\frac{1}{b^3}\right)^{\frac14}\log\left(b\left(-\frac{1}{b^3}\right)^{\frac14}+\left(ax^3-b\right)^{\frac14}\right)-\frac13\left(-\frac{1}{b^3}\right)^{\frac14}\log\left(-b\left(-\frac{1}{b^3}\right)^{\frac14}+\left(ax^3-b\right)^{\frac14}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3-b)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/3*(-1/b^3)^(1/4)*arctan(sqrt(b^2*sqrt(-1/b^3) + sqrt(a*x^3 - b))*b^2*(-1/
b^3)^(3/4) - (a*x^3 - b)^(1/4)*b^2*(-1/b^3)^(3/4)) + 1/3*(-1/b^3)^(1/4)*log
(b*(-1/b^3)^(1/4) + (a*x^3 - b)^(1/4)) - 1/3*(-1/b^3)^(1/4)*log(-b*(-1/b^3)
^(1/4) + (a*x^3 - b)^(1/4))
```

giac [A] time = 0.32, size = 162, normalized size = 1.27

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac14}+2\left(ax^3-b\right)^{\frac14}\right)}{2b^{\frac14}}\right)}{3b^{\frac34}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac14}-2\left(ax^3-b\right)^{\frac14}\right)}{2b^{\frac14}}\right)}{3b^{\frac34}}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^3-b\right)^{\frac14}b^{\frac14}+\sqrt{ax^3-b}+\sqrt{b}\right)}{6b^{\frac34}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^3-b\right)^{\frac14}b^{\frac14}+\sqrt{ax^3-b}+\sqrt{b}\right)}{6b^{\frac34}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3-b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1/4))/b^(1
/4))/b^(3/4) + 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^3
- b)^(1/4))/b^(1/4))/b^(3/4) + 1/6*sqrt(2)*log(sqrt(2)*(a*x^3 - b)^(1/4)*b^
(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(3/4) - 1/6*sqrt(2)*log(-sqrt(2)*(a*x^
3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(3/4)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a\,x^3-b\right)^{\frac34}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^3-b)^(3/4),x)
```

```
[Out] int(1/x/(a*x^3-b)^(3/4),x)
```

maxima [A] time = 0.43, size = 162, normalized size = 1.27

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac14}+2\left(ax^3-b\right)^{\frac14}\right)}{2b^{\frac14}}\right)}{3b^{\frac34}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac14}-2\left(ax^3-b\right)^{\frac14}\right)}{2b^{\frac14}}\right)}{3b^{\frac34}}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^3-b\right)^{\frac14}b^{\frac14}+\sqrt{ax^3-b}+\sqrt{b}\right)}{6b^{\frac34}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^3-b\right)^{\frac14}b^{\frac14}+\sqrt{ax^3-b}+\sqrt{b}\right)}{6b^{\frac34}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1/4))/b^(1
/4))/b^(3/4) + 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^3
- b)^(1/4))/b^(1/4))/b^(3/4) + 1/6*sqrt(2)*log(sqrt(2)*(a*x^3 - b)^(1/4)*b^
(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(3/4) - 1/6*sqrt(2)*log(-sqrt(2)*(a*x^
3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(3/4)
```

mupad [B] time = 1.02, size = 51, normalized size = 0.40

$$-\frac{2\,\mathrm{atan}\left(\frac{{\left(a\,x^3-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{3\,{\left(-b\right)}^{3/4}}-\frac{2\,\mathrm{atanh}\left(\frac{{\left(a\,x^3-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{3\,{\left(-b\right)}^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^3 - b)^(3/4)),x)
```

```
[Out] - (2*atan((a*x^3 - b)^(1/4)/(-b)^(1/4)))/(3*(-b)^(3/4)) - (2*atanh((a*x^3 -
 b)^(1/4)/(-b)^(1/4)))/(3*(-b)^(3/4))
```

sympy [C] time = 1.07, size = 42, normalized size = 0.33

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^3}\right)}{3a^{\frac{3}{4}}x^{\frac{9}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**3-b)**(3/4),x)
```

```
[Out] -gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*exp_polar(2*I*pi)/(a*x**3))/(3*a**(
3/4)*x**(9/4)*gamma(7/4))
```

3.1560 $\int \frac{1}{x\sqrt[4]{-b+ax^3}}\,dx$

Optimal. Leaf size=128

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}-\sqrt{b}}\right)}{3\sqrt[4]{b}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{3\sqrt[4]{b}}$$

Rubi [A] time = 0.19, antiderivative size = 201, normalized size of antiderivative = 1.57, number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {266, 63, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{3\sqrt{2}\sqrt[4]{b}}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{3\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}\right)}{3\sqrt[4]{b}}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}+1\right)}{3\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-b + a*x^3)^(1/4)),x]
```

```
[Out] -1/3*(Sqrt[2]*ArcTan[1 - (Sqrt[2]*(-b + a*x^3)^(1/4))/b^(1/4)])/b^(1/4) + (
Sqrt[2]*ArcTan[1 + (Sqrt[2]*(-b + a*x^3)^(1/4))/b^(1/4)])/(3*b^(1/4)) + Log
[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4) + Sqrt[-b + a*x^3]]/(3*Sqrt[2
]*b^(1/4)) - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4) + Sqrt[-b + a
*x^3]]/(3*Sqrt[2]*b^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x\sqrt[4]{-b+ax^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt[4]{-b+ax}}\,dx, x, x^3\right)$$

$$= \frac{4\,\text{Subst}\left(\int \frac{x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{3a}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{3a} + \frac{2\,\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{3a}$$

$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{2}\sqrt[4]{b}\,x+x^2}\,dx, x, \sqrt[4]{-b+ax^3}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{2}\sqrt[4]{b}\,x+x^2}\,d$$

$$= \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^3}+\sqrt{-b+ax^3}\right)}{3\sqrt{2}\sqrt[4]{b}} - \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^3}+\sqrt{-b+a}\right.}{3\sqrt{2}\sqrt[4]{b}}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)}{3\sqrt[4]{b}} + \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)}{3\sqrt[4]{b}} + \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+\right.}{3\sqrt{2}\sqrt[4]{b}}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 0.45

$$\frac{2\left(\tan^{-1}\left(\frac{\sqrt[4]{ax^3-b}}{\sqrt[4]{-b}}\right)+\tanh^{-1}\left(\frac{b\sqrt[4]{ax^3-b}}{(-b)^{5/4}}\right)\right)}{3\sqrt[4]{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-b + a*x^3)^(1/4)),x]
```

```
[Out] (2*(ArcTan[(-b + a*x^3)^(1/4)/(-b)^(1/4)] + ArcTanh[(b*(-b + a*x^3)^(1/4))/
(-b)^(5/4)]))/(3*(-b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.19, size = 127, normalized size = 0.99

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{3\sqrt[4]{b}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}+\sqrt{b}}\right)}{3\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-b + a*x^3)^(1/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a*x^3]/(Sqrt[2]*b^(1/4)))/(
-b + a*x^3)^(1/4)])/(3*b^(1/4)) - (Sqrt[2]*ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a
*x^3)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^3])])/(3*b^(1/4))
```

fricas [A] time = 0.41, size = 127, normalized size = 0.99

$$-\frac{4}{3}\left(-\frac{1}{b}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-b\sqrt{-\frac{1}{b}}+\sqrt{ax^3-b}}\left(-\frac{1}{b}\right)^{\frac{1}{4}}-\left(ax^3-b\right)^{\frac{1}{4}}\left(-\frac{1}{b}\right)^{\frac{1}{4}}\right)+\frac{1}{3}\left(-\frac{1}{b}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{1}{b}\right)^{\frac{3}{4}}+\left(ax^3-b\right)^{\frac{1}{4}}\right)-\frac{1}{3}\left(-\frac{1}{b}\right)^{\frac{1}{4}}\log\left(-b\left(-\frac{1}{b}\right)^{\frac{3}{4}}+\left(ax^3-b\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3-b)^(1/4),x, algorithm="fricas")
```

```
[Out] -4/3*(-1/b)^(1/4)*arctan(sqrt(-b*sqrt(-1/b) + sqrt(a*x^3 - b))*(-1/b)^(1/4)
 - (a*x^3 - b)^(1/4)*(-1/b)^(1/4)) + 1/3*(-1/b)^(1/4)*log(b*(-1/b)^(3/4) +
(a*x^3 - b)^(1/4)) - 1/3*(-1/b)^(1/4)*log(-b*(-1/b)^(3/4) + (a*x^3 - b)^(1/
4))
```

giac [A] time = 0.33, size = 162, normalized size = 1.27

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{3b^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{3b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{6b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{6b^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3-b)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1/4))/b^(1
/4))/b^(1/4) + 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^3
- b)^(1/4))/b^(1/4))/b^(1/4) - 1/6*sqrt(2)*log(sqrt(2)*(a*x^3 - b)^(1/4)*b^
(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(1/4) + 1/6*sqrt(2)*log(-sqrt(2)*(a*x^
3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(1/4)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a\,x^{3}-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^3-b)^(1/4),x)
```

```
[Out] int(1/x/(a*x^3-b)^(1/4),x)
```

maxima [A] time = 0.44, size = 162, normalized size = 1.27

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{3b^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{3b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{6b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{6b^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^3-b)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1/4))/b^(1
/4))/b^(1/4) + 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^3
- b)^(1/4))/b^(1/4))/b^(1/4) - 1/6*sqrt(2)*log(sqrt(2)*(a*x^3 - b)^(1/4)*b^
(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(1/4) + 1/6*sqrt(2)*log(-sqrt(2)*(a*x^
3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(1/4)
```

mupad [B] time = 0.96, size = 51, normalized size = 0.40

$$\frac{2\,\mathrm{atan}\left(\frac{{\left(a\,x^3-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{3\,{\left(-b\right)}^{1/4}}-\frac{2\,\mathrm{atanh}\left(\frac{{\left(a\,x^3-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{3\,{\left(-b\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^3 - b)^(1/4)),x)
```

```
[Out] (2*atan((a*x^3 - b)^(1/4)/(-b)^(1/4)))/(3*(-b)^(1/4)) - (2*atanh((a*x^3 - b
)^(1/4)/(-b)^(1/4)))/(3*(-b)^(1/4))
```

sympy [C] time = 1.02, size = 42, normalized size = 0.33

$$-\frac{\Gamma\left(\frac{1}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\,\frac{be^{2i\pi}}{ax^{3}}\right)}{3\sqrt[4]{a}\,x^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**3-b)**(1/4),x)
```

```
[Out] -gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*exp_polar(2*I*pi)/(a*x**3))/(3*a**(
1/4)*x**(3/4)*gamma(5/4))
```

3.1561 $\int \frac{(-3b+ax^2)(b-ax^2+x^3)}{x^3(-b+ax^2+x^3)\sqrt[4]{-bx+ax^3}}\,dx$

Optimal. Leaf size=128

$$\frac{4\left(ax^3-bx\right)^{3/4}}{3x^3}-2\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-bx}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3-bx}}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^3-bx}}{\sqrt{ax^3-bx}+x^2}\right)$$

Rubi [F] time = 5.97, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-3b+ax^2\right)\left(b-ax^2+x^3\right)}{x^3\left(-b+ax^2+x^3\right)\sqrt[4]{-bx+ax^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3*b + a*x^2)*(b - a*x^2 + x^3))/(x^3*(-b + a*x^2 + x^3)*(-(b*x) + a*
x^3)^(1/4)),x]
```

```
[Out] (-4*b*(1 - (a*x^2)/b)^(1/4)*Hypergeometric2F1[-9/8, 1/4, -1/8, (a*x^2)/b])/
(3*x^2*(-(b*x) + a*x^3)^(1/4)) + (4*a*(1 - (a*x^2)/b)^(1/4)*Hypergeometric2
F1[-1/8, 1/4, 7/8, (a*x^2)/b])/(-(b*x) + a*x^3)^(1/4) + (24*b*x^(1/4)*(-b +
 a*x^2)^(1/4)*Defer[Subst][Defer[Int][x^2/((-b + a*x^8)^(1/4)*(b - a*x^8 -
x^12)), x], x, x^(1/4)])/(-(b*x) + a*x^3)^(1/4) + (8*a*x^(1/4)*(-b + a*x^2)
^(1/4)*Defer[Subst][Defer[Int][x^10/((-b + a*x^8)^(1/4)*(-b + a*x^8 + x^12)
), x], x, x^(1/4)])/(-(b*x) + a*x^3)^(1/4)
```

Rubi steps

$$\int \frac{\left(-3b+ax^2\right)\left(b-ax^2+x^3\right)}{x^3\left(-b+ax^2+x^3\right)\sqrt[4]{-bx+ax^3}}\,dx=\frac{\left(\sqrt[4]{x}\sqrt[4]{-b+ax^2}\right)\int \frac{(-3b+ax^2)(b-ax^2+x^3)}{x^{13/4}\sqrt[4]{-b+ax^2}\left(-b+ax^2+x^3\right)}\,dx}{\sqrt[4]{-bx+ax^3}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^2}\right)\text{Subst}\left(\int \frac{(-3b+ax^8)(b-ax^8+x^{12})}{x^{10}\sqrt[4]{-b+ax^8}\left(-b+ax^8+x^{12}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^3}}$$

$$=\frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^2}\right)\text{Subst}\left(\int\left(\frac{3b}{x^{10}\sqrt[4]{-b+ax^8}}-\frac{a}{x^2\sqrt[4]{-b+ax^8}}+\frac{2x^2(3b-ax^8)}{\sqrt[4]{-b+ax^8}\left(b-ax^8-x^{12}\right)}\right.\right.}{\sqrt[4]{-bx+ax^3}}$$

$$=\frac{\left(8\sqrt[4]{x}\sqrt[4]{-b+ax^2}\right)\text{Subst}\left(\int \frac{x^2(3b-ax^8)}{\sqrt[4]{-b+ax^8}\left(b-ax^8-x^{12}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^3}}-\frac{\left(4a\sqrt[4]{x}\sqrt[4]{-b\right.}}{}$$

$$=\frac{\left(8\sqrt[4]{x}\sqrt[4]{-b+ax^2}\right)\text{Subst}\left(\int\left(\frac{3bx^2}{\sqrt[4]{-b+ax^8}\left(b-ax^8-x^{12}\right)}+\frac{ax^{10}}{\sqrt[4]{-b+ax^8}\left(-b+ax^8+x^{12}\right)}\right)\right.}{\sqrt[4]{-bx+ax^3}}$$

$$=-\frac{4b\sqrt[4]{1-\frac{ax^2}{b}}\,{}_2F_1\left(-\frac{9}{8},\frac{1}{4};-\frac{1}{8};\frac{ax^2}{b}\right)}{3x^2\sqrt[4]{-bx+ax^3}}+\frac{4a\sqrt[4]{1-\frac{ax^2}{b}}\,{}_2F_1\left(-\frac{1}{8},\frac{1}{4};\frac{7}{8};\frac{ax^2}{b}\right)}{\sqrt[4]{-bx+ax^3}}+\cdots$$

Mathematica [F] time = 2.22, size = 0, normalized size = 0.00

$$\int \frac{\left(-3b+ax^2\right)\left(b-ax^2+x^3\right)}{x^3\left(-b+ax^2+x^3\right)\sqrt[4]{-bx+ax^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3*b + a*x^2)*(b - a*x^2 + x^3))/(x^3*(-b + a*x^2 + x^3)*(-(b*x
) + a*x^3)^(1/4)),x]
```

```
[Out] Integrate[((-3*b + a*x^2)*(b - a*x^2 + x^3))/(x^3*(-b + a*x^2 + x^3)*(-(b*x
) + a*x^3)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 1.31, size = 128, normalized size = 1.00

$$\frac{4\left(ax^3-bx\right)^{3/4}}{3x^3}-2\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-bx}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3-bx}}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^3-bx}}{\sqrt{ax^3-bx}+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3*b + a*x^2)*(b - a*x^2 + x^3))/(x^3*(-b + a*x^2 + x^
3)*(-(b*x) + a*x^3)^(1/4)),x]
```

```
[Out] (4*(-(b*x) + a*x^3)^(3/4))/(3*x^3) - 2*Sqrt[2]*ArcTan[(-(x^2/Sqrt[2]) + Sqr
t[-(b*x) + a*x^3]/Sqrt[2])/(x*(-(b*x) + a*x^3)^(1/4))] + 2*Sqrt[2]*ArcTanh[
(Sqrt[2]*x*(-(b*x) + a*x^3)^(1/4))/(x^2 + Sqrt[-(b*x) + a*x^3])]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-3*b)*(-a*x^2+x^3+b)/x^3/(a*x^2+x^3-b)/(a*x^3-b*x)^(1/4),x,
 algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(ax^2-x^3-b\right)\left(ax^2-3\,b\right)}{\left(ax^3-bx\right)^{\frac{1}{4}}\left(ax^2+x^3-b\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-3*b)*(-a*x^2+x^3+b)/x^3/(a*x^2+x^3-b)/(a*x^3-b*x)^(1/4),x,
 algorithm="giac")
```

```
[Out] integrate(-(a*x^2 - x^3 - b)*(a*x^2 - 3*b)/((a*x^3 - b*x)^(1/4)*(a*x^2 + x^
3 - b)*x^3), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^2-3b\right)\left(-a\,x^2+x^3+b\right)}{x^3\left(a\,x^2+x^3-b\right)\left(a\,x^3-bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-3*b)*(-a*x^2+x^3+b)/x^3/(a*x^2+x^3-b)/(a*x^3-b*x)^(1/4),x)
```

```
[Out] int((a*x^2-3*b)*(-a*x^2+x^3+b)/x^3/(a*x^2+x^3-b)/(a*x^3-b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(ax^2 - x^3 - b\right)\left(ax^2 - 3\,b\right)}{\left(ax^3 - bx\right)^{\frac{1}{4}}\left(ax^2 + x^3 - b\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-3*b)*(-a*x^2+x^3+b)/x^3/(a*x^2+x^3-b)/(a*x^3-b*x)^(1/4),x,
 algorithm="maxima")
```

```
[Out] -integrate((a*x^2 - x^3 - b)*(a*x^2 - 3*b)/((a*x^3 - b*x)^(1/4)*(a*x^2 + x^
3 - b)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(3\,b - a\,x^2\right)\,\left(x^3 - a\,x^2 + b\right)}{x^3\,\left(a\,x^3 - b\,x\right)^{1/4}\,\left(x^3 + a\,x^2 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((3*b - a*x^2)*(b - a*x^2 + x^3))/(x^3*(a*x^3 - b*x)^(1/4)*(a*x^2 - b
+ x^3)),x)
```

```
[Out] int(-((3*b - a*x^2)*(b - a*x^2 + x^3))/(x^3*(a*x^3 - b*x)^(1/4)*(a*x^2 - b
+ x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-3*b)*(-a*x**2+x**3+b)/x**3/(a*x**2+x**3-b)/(a*x**3-b*x)**
(1/4),x)
```

```
[Out] Timed out
```

3.1562 $\int \frac{1}{x\left(-b+ax^4\right)^{3/4}}\,dx$

Optimal. Leaf size=128

$$\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^4-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^4-b}}\right)}{2\sqrt{2}\,b^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^4-b}}{\sqrt{ax^4-b}-\sqrt{b}}\right)}{2\sqrt{2}\,b^{3/4}}$$

Rubi [A] time = 0.19, antiderivative size = 201, normalized size of antiderivative = 1.57, number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {266, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^4-b}+\sqrt{ax^4-b}+\sqrt{b}\right)}{4\sqrt{2}\,b^{3/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^4-b}+\sqrt{ax^4-b}+\sqrt{b}\right)}{4\sqrt{2}\,b^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^4-b}}{\sqrt[4]{b}}\right)}{2\sqrt{2}\,b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^4-b}}{\sqrt[4]{b}}+1\right)}{2\sqrt{2}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-b + a*x^4)^(3/4)),x]
```

```
[Out] -1/2*ArcTan[1 - (Sqrt[2]*(-b + a*x^4)^(1/4))/b^(1/4)]/(Sqrt[2]*b^(3/4)) + A
rcTan[1 + (Sqrt[2]*(-b + a*x^4)^(1/4))/b^(1/4)]/(2*Sqrt[2]*b^(3/4)) - Log[S
qrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^4)^(1/4) + Sqrt[-b + a*x^4]]/(4*Sqrt[2]*
b^(3/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a*x^4)^(1/4) + Sqrt[-b + a*x
^4]]/(4*Sqrt[2]*b^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x\left(-b+ax^4\right)^{3/4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(-b+ax)^{3/4}}\,dx, x, x^4\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^4}\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^4}\right)}{2a\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^4}\right)}{2a\sqrt{b}}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}\,\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,x-x^2}\,dx, x, \sqrt[4]{-b+ax^4}\right)}{4\sqrt{2}\,b^{3/4}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\,\sqrt[4]{b}-2x}{-\sqrt{b}+\sqrt{2}\,\sqrt[4]{b}\,x-x^2}\,dx, x, \sqrt[4]{-b+ax\right.}{4\sqrt{2}\,b^{3/4}}$$

$$= -\frac{\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{-b+ax^4}+\sqrt{-b+ax^4}\right)}{4\sqrt{2}\,b^{3/4}} + \frac{\log\left(\sqrt{b}+\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{-b+ax^4}+\sqrt{-b+\right.}{4\sqrt{2}\,b^{3/4}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{-b+ax^4}}{\sqrt[4]{b}}\right)}{2\sqrt{2}\,b^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{-b+ax^4}}{\sqrt[4]{b}}\right)}{2\sqrt{2}\,b^{3/4}} - \frac{\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{-b+ax^4}+\sqrt{\right.}{4\sqrt{2}\,b^{3/4}}$$

Mathematica [A] time = 0.07, size = 165, normalized size = 1.29

$$\frac{-\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{ax^4-b}+\sqrt{ax^4-b}+\sqrt{b}\right)+\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{ax^4-b}+\sqrt{ax^4-b}+\sqrt{b}\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{ax^4-b}}{\sqrt[4]{b}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{ax^4-b}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-b + a*x^4)^(3/4)),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*(-b + a*x^4)^(1/4))/b^(1/4)] + 2*ArcTan[1 + (Sqrt[2
]*(-b + a*x^4)^(1/4))/b^(1/4)] - Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^4)
```

```
^(1/4) + Sqrt[-b + a*x^4]] + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a*x^4)^(1/
4) + Sqrt[-b + a*x^4]])/(4*Sqrt[2]*b^(3/4))
```

IntegrateAlgebraic [A] time = 0.20, size = 127, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\frac{\sqrt{ax^4-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^4-b}}\right)}{2\sqrt{2}\,b^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^4-b}}{\sqrt{ax^4-b}+\sqrt{b}}\right)}{2\sqrt{2}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-b + a*x^4)^(3/4)),x]
```

```
[Out] ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a*x^4]/(Sqrt[2]*b^(1/4)))/(-b + a*x^
4)^(1/4)]/(2*Sqrt[2]*b^(3/4)) + ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^4)^(1/4)
)/(Sqrt[b] + Sqrt[-b + a*x^4])]/(2*Sqrt[2]*b^(3/4))
```

fricas [A] time = 0.42, size = 133, normalized size = 1.04

$$\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^2\sqrt{-\frac{1}{b^3}}+\sqrt{ax^4-b}}\,b^2\left(-\frac{1}{b^3}\right)^{\frac{3}{4}}-\left(ax^4-b\right)^{\frac{1}{4}}b^2\left(-\frac{1}{b^3}\right)^{\frac{3}{4}}\right)+\frac{1}{4}\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}+\left(ax^4-b\right)^{\frac{1}{4}}\right)-\frac{1}{4}\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(-b\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}+\left(ax^4-b\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4-b)^(3/4),x, algorithm="fricas")
```

```
[Out] (-1/b^3)^(1/4)*arctan(sqrt(b^2*sqrt(-1/b^3) + sqrt(a*x^4 - b))*b^2*(-1/b^3)
^(3/4) - (a*x^4 - b)^(1/4)*b^2*(-1/b^3)^(3/4)) + 1/4*(-1/b^3)^(1/4)*log(b*(
-1/b^3)^(1/4) + (a*x^4 - b)^(1/4)) - 1/4*(-1/b^3)^(1/4)*log(-b*(-1/b^3)^(1/
4) + (a*x^4 - b)^(1/4))
```

giac [A] time = 0.28, size = 162, normalized size = 1.27

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{4\,b^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{4\,b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{8\,b^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{8\,b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4-b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^4 - b)^(1/4))/b^(1
/4))/b^(3/4) + 1/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^4
- b)^(1/4))/b^(1/4))/b^(3/4) + 1/8*sqrt(2)*log(sqrt(2)*(a*x^4 - b)^(1/4)*b^
(1/4) + sqrt(a*x^4 - b) + sqrt(b))/b^(3/4) - 1/8*sqrt(2)*log(-sqrt(2)*(a*x^
4 - b)^(1/4)*b^(1/4) + sqrt(a*x^4 - b) + sqrt(b))/b^(3/4)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a\,x^4-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^4-b)^(3/4),x)
```

```
[Out] int(1/x/(a*x^4-b)^(3/4),x)
```

maxima [A] time = 0.44, size = 162, normalized size = 1.27

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{4\,b^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{4\,b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{8\,b^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{8\,b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^4 - b)^(1/4))/b^(1
/4))/b^(3/4) + 1/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^4
- b)^(1/4))/b^(1/4))/b^(3/4) + 1/8*sqrt(2)*log(sqrt(2)*(a*x^4 - b)^(1/4)*b^
(1/4) + sqrt(a*x^4 - b) + sqrt(b))/b^(3/4) - 1/8*sqrt(2)*log(-sqrt(2)*(a*x^
4 - b)^(1/4)*b^(1/4) + sqrt(a*x^4 - b) + sqrt(b))/b^(3/4)
```

mupad [B] time = 1.00, size = 44, normalized size = 0.34

$$-\frac{\operatorname{atan}\left(\frac{\left(a\,x^4-b\right)^{1/4}}{(-b)^{1/4}}\right)+\operatorname{atanh}\left(\frac{\left(a\,x^4-b\right)^{1/4}}{(-b)^{1/4}}\right)}{2\,(-b)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^4 - b)^(3/4)),x)
```

```
[Out] -(atan((a*x^4 - b)^(1/4)/(-b)^(1/4)) + atanh((a*x^4 - b)^(1/4)/(-b)^(1/4)))
/(2*(-b)^(3/4))
```

sympy [C] time = 1.17, size = 41, normalized size = 0.32

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{be^{2i\pi}}{ax^4}\right)}{4a^{\frac{3}{4}}x^3\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**4-b)**(3/4),x)
```

```
[Out] -gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*exp_polar(2*I*pi)/(a*x**4))/(4*a**(
3/4)*x**3*gamma(7/4))
```

3.1563 $\int \frac{1}{x\sqrt[4]{-b+ax^4}}\,dx$

Optimal. Leaf size=128

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^4-b}}{\sqrt{ax^4-b}-\sqrt{b}}\right)}{2\sqrt{2}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^4-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^4-b}}\right)}{2\sqrt{2}\sqrt[4]{b}}$$

Rubi [A] time = 0.18, antiderivative size = 201, normalized size of antiderivative = 1.57, number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {266, 63, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^4-b}+\sqrt{ax^4-b}+\sqrt{b}\right)}{4\sqrt{2}\sqrt[4]{b}}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^4-b}+\sqrt{ax^4-b}+\sqrt{b}\right)}{4\sqrt{2}\sqrt[4]{b}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^4-b}}{\sqrt[4]{b}}\right)}{2\sqrt{2}\sqrt[4]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^4-b}}{\sqrt[4]{b}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-b + a*x^4)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[1 - (Sqrt[2]*(-b + a*x^4)^(1/4))/b^(1/4)]/(Sqrt[2]*b^(1/4)) + A
rcTan[1 + (Sqrt[2]*(-b + a*x^4)^(1/4))/b^(1/4)]/(2*Sqrt[2]*b^(1/4)) + Log[S
qrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^4)^(1/4) + Sqrt[-b + a*x^4]]/(4*Sqrt[2]*
b^(1/4)) - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a*x^4)^(1/4) + Sqrt[-b + a*x
^4]]/(4*Sqrt[2]*b^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt[4]{-b+ax^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{-b+ax}}\,dx, x, x^4\right)\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^4}\right)}{a}\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^4}\right)}{2a} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^4}\right)}{2a}\\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{2}\sqrt[4]{b}\,x+x^2}\,dx, x, \sqrt[4]{-b+ax^4}\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{2}\sqrt[4]{b}\,x+x^2}\,dx,\right.\\
&= \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^4}+\sqrt{-b+ax^4}\right)}{4\sqrt{2}\sqrt[4]{b}} - \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^4}+\sqrt{-b+ax^4}\right.}{4\sqrt{2}\sqrt[4]{b}}\\
&= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^4}}{\sqrt[4]{b}}\right)}{2\sqrt{2}\sqrt[4]{b}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^4}}{\sqrt[4]{b}}\right)}{2\sqrt{2}\sqrt[4]{b}} + \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^4}+\sqrt{-b}\right.}{4\sqrt{2}\sqrt[4]{b}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 0.45

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{ax^4-b}}{\sqrt[4]{-b}}\right)+\tanh^{-1}\left(\frac{b\sqrt[4]{ax^4-b}}{(-b)^{5/4}}\right)}{2\sqrt[4]{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-b + a*x^4)^(1/4)),x]
```

```
[Out] (ArcTan[(-b + a*x^4)^(1/4)/(-b)^(1/4)] + ArcTanh[(b*(-b + a*x^4)^(1/4))/(-b
)^(5/4)])/(2*(-b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.17, size = 127, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\frac{\sqrt{ax^4-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^4-b}}\right)}{2\sqrt{2}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^4-b}}{\sqrt{ax^4-b}+\sqrt{b}}\right)}{2\sqrt{2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-b + a*x^4)^(1/4)),x]
```

```
[Out] ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a*x^4]/(Sqrt[2]*b^(1/4)))/(-b + a*x^
4)^(1/4)]/(2*Sqrt[2]*b^(1/4)) - ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^4)^(1/4)
)/(Sqrt[b] + Sqrt[-b + a*x^4])]/(2*Sqrt[2]*b^(1/4))
```

fricas [A] time = 0.41, size = 127, normalized size = 0.99

$$-\left(-\frac{1}{b}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-b\sqrt{-\frac{1}{b}}+\sqrt{ax^4-b}}\left(-\frac{1}{b}\right)^{\frac{1}{4}}-\left(ax^4-b\right)^{\frac{1}{4}}\left(-\frac{1}{b}\right)^{\frac{1}{4}}\right)+\frac{1}{4}\left(-\frac{1}{b}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{1}{b}\right)^{\frac{3}{4}}+\left(ax^4-b\right)^{\frac{1}{4}}\right)-\frac{1}{4}\left(-\frac{1}{b}\right)^{\frac{1}{4}}\log\left(-b\left(-\frac{1}{b}\right)^{\frac{3}{4}}+\left(ax^4-b\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4-b)^(1/4),x, algorithm="fricas")
```

```
[Out] -(-1/b)^(1/4)*arctan(sqrt(-b*sqrt(-1/b) + sqrt(a*x^4 - b))*(-1/b)^(1/4) - (
a*x^4 - b)^(1/4)*(-1/b)^(1/4)) + 1/4*(-1/b)^(1/4)*log(b*(-1/b)^(3/4) + (a*x
^4 - b)^(1/4)) - 1/4*(-1/b)^(1/4)*log(-b*(-1/b)^(3/4) + (a*x^4 - b)^(1/4))
```

giac [A] time = 0.37, size = 162, normalized size = 1.27

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{4b^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{4b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{8b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{8b^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4-b)^(1/4),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^4 - b)^(1/4))/b^(1
/4))/b^(1/4) + 1/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^4
- b)^(1/4))/b^(1/4))/b^(1/4) - 1/8*sqrt(2)*log(sqrt(2)*(a*x^4 - b)^(1/4)*b^
(1/4) + sqrt(a*x^4 - b) + sqrt(b))/b^(1/4) + 1/8*sqrt(2)*log(-sqrt(2)*(a*x^
4 - b)^(1/4)*b^(1/4) + sqrt(a*x^4 - b) + sqrt(b))/b^(1/4)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int\frac{1}{x\left(a\,x^4-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^4-b)^(1/4),x)
```

```
[Out] int(1/x/(a*x^4-b)^(1/4),x)
```

maxima [A] time = 0.43, size = 162, normalized size = 1.27

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{4b^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{4b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{8b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{8b^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4-b)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^4 - b)^(1/4))/b^(1
/4))/b^(1/4) + 1/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^4
- b)^(1/4))/b^(1/4))/b^(1/4) - 1/8*sqrt(2)*log(sqrt(2)*(a*x^4 - b)^(1/4)*b^
(1/4) + sqrt(a*x^4 - b) + sqrt(b))/b^(1/4) + 1/8*sqrt(2)*log(-sqrt(2)*(a*x^
4 - b)^(1/4)*b^(1/4) + sqrt(a*x^4 - b) + sqrt(b))/b^(1/4)
```

mupad [B] time = 0.97, size = 46, normalized size = 0.36

$$\frac{\operatorname{atan}\left(\frac{(a\,x^4-b)^{1/4}}{(-b)^{1/4}}\right)-\operatorname{atanh}\left(\frac{(a\,x^4-b)^{1/4}}{(-b)^{1/4}}\right)}{2\,(-b)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^4 - b)^(1/4)),x)
```

```
[Out] (atan((a*x^4 - b)^(1/4)/(-b)^(1/4)) - atanh((a*x^4 - b)^(1/4)/(-b)^(1/4)))/
(2*(-b)^(1/4))
```

sympy [C] time = 1.02, size = 39, normalized size = 0.30

$$-\frac{\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^4}\right)}{4\sqrt[4]{a}\,x\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**4-b)**(1/4),x)
```

```
[Out] -gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*exp_polar(2*I*pi)/(a*x**4))/(4*a**(
1/4)*x*gamma(5/4))
```

3.1564 $\int \frac{1}{x\left(-b+ax^5\right)^{3/4}}\,dx$

Optimal. Leaf size=128

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^5-b}}\right)}{5b^{3/4}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^5-b}}{\sqrt{ax^5-b}-\sqrt{b}}\right)}{5b^{3/4}}$$

Rubi [A] time = 0.19, antiderivative size = 201, normalized size of antiderivative = 1.57, number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {266, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^5-b}+\sqrt{ax^5-b}+\sqrt{b}\right)}{5\sqrt{2}b^{3/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^5-b}+\sqrt{ax^5-b}+\sqrt{b}\right)}{5\sqrt{2}b^{3/4}}-\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^5-b}}{\sqrt[4]{b}}\right)}{5b^{3/4}}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^5-b}}{\sqrt[4]{b}}+1\right)}{5b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-b + a*x^5)^(3/4)),x]
```

```
[Out] -1/5*(Sqrt[2]*ArcTan[1 - (Sqrt[2]*(-b + a*x^5)^(1/4))/b^(1/4)])/b^(3/4) + (
Sqrt[2]*ArcTan[1 + (Sqrt[2]*(-b + a*x^5)^(1/4))/b^(1/4)])/(5*b^(3/4)) - Log
[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^5)^(1/4) + Sqrt[-b + a*x^5]]/(5*Sqrt[2
]*b^(3/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a*x^5)^(1/4) + Sqrt[-b + a
*x^5]]/(5*Sqrt[2]*b^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x\left(-b+ax^5\right)^{3/4}}\,dx = \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x(-b+ax)^{3/4}}\,dx, x, x^5\right)$$

$$= \frac{4\,\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^5}\right)}{5a}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^5}\right)}{5a\sqrt{b}} + \frac{2\,\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^5}\right)}{5a\sqrt{b}}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^5}\right)}{5\sqrt{2}\,b^{3/4}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}-2x}{-\sqrt{b}+\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^5}\right)}{5\sqrt{2}\,b^{3/4}}$$

$$= -\frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^5}+\sqrt{-b+ax^5}\right)}{5\sqrt{2}\,b^{3/4}} + \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^5}+\sqrt{-b+\dots}\right)}{5\sqrt{2}\,b^{3/4}}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^5}}{\sqrt[4]{b}}\right)}{5b^{3/4}} + \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^5}}{\sqrt[4]{b}}\right)}{5b^{3/4}} - \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+a\dots}\right.}{5\sqrt{2}\,b^{3/4}}$$

Mathematica [A] time = 0.07, size = 165, normalized size = 1.29

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^5-b}+\sqrt{ax^5-b}+\sqrt{b}\right)+\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^5-b}+\sqrt{ax^5-b}+\sqrt{b}\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^5-b}}{\sqrt[4]{b}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^5-b}}{\sqrt[4]{b}}+1\right)}{5\sqrt{2}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-b + a*x^5)^(3/4)),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*(-b + a*x^5)^(1/4))/b^(1/4)] + 2*ArcTan[1 + (Sqrt[2
]*(-b + a*x^5)^(1/4))/b^(1/4)] - Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^5)
```

```
^(1/4) + Sqrt[-b + a*x^5]] + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a*x^5)^(1/
4) + Sqrt[-b + a*x^5]])/(5*Sqrt[2]*b^(3/4))
```

IntegrateAlgebraic [A] time = 0.18, size = 127, normalized size = 0.99

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{ax^5-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^5-b}}\right)}{5b^{3/4}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^5-b}}{\sqrt{ax^5-b}+\sqrt{b}}\right)}{5b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-b + a*x^5)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a*x^5]/(Sqrt[2]*b^(1/4)))/(
-b + a*x^5)^(1/4)])/(5*b^(3/4)) + (Sqrt[2]*ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a
*x^5)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^5])])/(5*b^(3/4))
```

fricas [A] time = 0.41, size = 134, normalized size = 1.05

$$\frac{4}{5}\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^2\sqrt{-\frac{1}{b^3}}+\sqrt{ax^5-b}}\,b^2\left(-\frac{1}{b^3}\right)^{\frac{3}{4}}-\left(ax^5-b\right)^{\frac{1}{4}}b^2\left(-\frac{1}{b^3}\right)^{\frac{3}{4}}\right)+\frac{1}{5}\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}+\left(ax^5-b\right)^{\frac{1}{4}}\right)-\frac{1}{5}\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(-b\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}+\left(ax^5-b\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^5-b)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/5*(-1/b^3)^(1/4)*arctan(sqrt(b^2*sqrt(-1/b^3) + sqrt(a*x^5 - b))*b^2*(-1/
b^3)^(3/4) - (a*x^5 - b)^(1/4)*b^2*(-1/b^3)^(3/4)) + 1/5*(-1/b^3)^(1/4)*log
(b*(-1/b^3)^(1/4) + (a*x^5 - b)^(1/4)) - 1/5*(-1/b^3)^(1/4)*log(-b*(-1/b^3)
^(1/4) + (a*x^5 - b)^(1/4))
```

giac [A] time = 0.32, size = 162, normalized size = 1.27

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^5-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{5b^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^5-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{5b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^5-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^5-b}+\sqrt{b}\right)}{10b^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^5-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^5-b}+\sqrt{b}\right)}{10b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^5-b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^5 - b)^(1/4))/b^(1
/4))/b^(3/4) + 1/5*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^5
- b)^(1/4))/b^(1/4))/b^(3/4) + 1/10*sqrt(2)*log(sqrt(2)*(a*x^5 - b)^(1/4)*b
^(1/4) + sqrt(a*x^5 - b) + sqrt(b))/b^(3/4) - 1/10*sqrt(2)*log(-sqrt(2)*(a*
x^5 - b)^(1/4)*b^(1/4) + sqrt(a*x^5 - b) + sqrt(b))/b^(3/4)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a\,x^5-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^5-b)^(3/4),x)
```

```
[Out] int(1/x/(a*x^5-b)^(3/4),x)
```

maxima [A] time = 0.41, size = 162, normalized size = 1.27

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^5-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{5b^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^5-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{5b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^5-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^5-b}+\sqrt{b}\right)}{10b^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^5-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^5-b}+\sqrt{b}\right)}{10b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^5-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^5 - b)^(1/4))/b^(1
/4))/b^(3/4) + 1/5*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^5
- b)^(1/4))/b^(1/4))/b^(3/4) + 1/10*sqrt(2)*log(sqrt(2)*(a*x^5 - b)^(1/4)*b
^(1/4) + sqrt(a*x^5 - b) + sqrt(b))/b^(3/4) - 1/10*sqrt(2)*log(-sqrt(2)*(a*
x^5 - b)^(1/4)*b^(1/4) + sqrt(a*x^5 - b) + sqrt(b))/b^(3/4)
```

mupad [B] time = 1.02, size = 51, normalized size = 0.40

$$-\frac{2\,\mathrm{atan}\left(\frac{{\left(a\,x^5-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{5\,{\left(-b\right)}^{3/4}}-\frac{2\,\mathrm{atanh}\left(\frac{{\left(a\,x^5-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{5\,{\left(-b\right)}^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^5 - b)^(3/4)),x)
```

```
[Out] - (2*atan((a*x^5 - b)^(1/4)/(-b)^(1/4)))/(5*(-b)^(3/4)) - (2*atanh((a*x^5 -
 b)^(1/4)/(-b)^(1/4)))/(5*(-b)^(3/4))
```

sympy [C] time = 1.05, size = 42, normalized size = 0.33

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^5}\right)}{5a^{\frac{3}{4}}x^{\frac{15}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**5-b)**(3/4),x)
```

```
[Out] -gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*exp_polar(2*I*pi)/(a*x**5))/(5*a**(
3/4)*x**(15/4)*gamma(7/4))
```

3.1565 $\int \frac{1}{x\left(-b+ax^6\right)^{3/4}}\,dx$

Optimal. Leaf size=128

$$\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^6-b}}\right)}{3\sqrt{2}\,b^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}}{\sqrt{ax^6-b}-\sqrt{b}}\right)}{3\sqrt{2}\,b^{3/4}}$$

Rubi [A] time = 0.20, antiderivative size = 201, normalized size of antiderivative = 1.57, number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {266, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}+\sqrt{ax^6-b}+\sqrt{b}\right)}{6\sqrt{2}\,b^{3/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}+\sqrt{ax^6-b}+\sqrt{b}\right)}{6\sqrt{2}\,b^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^6-b}}{\sqrt[4]{b}}\right)}{3\sqrt{2}\,b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^6-b}}{\sqrt[4]{b}}+1\right)}{3\sqrt{2}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-b + a*x^6)^(3/4)),x]
```

```
[Out] -1/3*ArcTan[1 - (Sqrt[2]*(-b + a*x^6)^(1/4))/b^(1/4)]/(Sqrt[2]*b^(3/4)) + A
rcTan[1 + (Sqrt[2]*(-b + a*x^6)^(1/4))/b^(1/4)]/(3*Sqrt[2]*b^(3/4)) - Log[S
qrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^6)^(1/4) + Sqrt[-b + a*x^6]]/(6*Sqrt[2]*
b^(3/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a*x^6)^(1/4) + Sqrt[-b + a*x
^6]]/(6*Sqrt[2]*b^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x\left(-b+ax^6\right)^{3/4}}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x(-b+ax)^{3/4}}\,dx, x, x^6\right)$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^6}\right)}{3a}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^6}\right)}{3a\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^6}\right)}{3a\sqrt{b}}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^6}\right)}{6\sqrt{2}\,b^{3/4}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}-2x}{-\sqrt{b}+\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^{6}}\right.}{6\sqrt{2}\,b^{3/4}}$$

$$= -\frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^6}+\sqrt{-b+ax^6}\right)}{6\sqrt{2}\,b^{3/4}} + \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^6}+\sqrt{-b+\ldots}\right.}{6\sqrt{2}\,b^{3/4}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^6}}{\sqrt[4]{b}}\right)}{3\sqrt{2}\,b^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^6}}{\sqrt[4]{b}}\right)}{3\sqrt{2}\,b^{3/4}} - \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^6}+\sqrt{\ldots}\right.}{6\sqrt{2}\,b^{3/4}}$$

Mathematica [A] time = 0.07, size = 165, normalized size = 1.29

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}+\sqrt{ax^6-b}+\sqrt{b}\right)+\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}+\sqrt{ax^6-b}+\sqrt{b}\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^6-b}}{\sqrt[4]{b}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^6-b}}{\sqrt[4]{b}}+1\right)}{6\sqrt{2}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-b + a*x^6)^(3/4)),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*(-b + a*x^6)^(1/4))/b^(1/4)] + 2*ArcTan[1 + (Sqrt[2
]*(-b + a*x^6)^(1/4))/b^(1/4)] - Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^6)
```

```
^(1/4) + Sqrt[-b + a*x^6]] + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a*x^6)^(1/
4) + Sqrt[-b + a*x^6]])/(6*Sqrt[2]*b^(3/4))
```

IntegrateAlgebraic [A] time = 0.17, size = 127, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\frac{\sqrt{ax^6-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^6-b}}\right)}{3\sqrt{2}\,b^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}}{\sqrt{ax^6-b}+\sqrt{b}}\right)}{3\sqrt{2}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-b + a*x^6)^(3/4)),x]
```

```
[Out] ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a*x^6]/(Sqrt[2]*b^(1/4)))/(-b + a*x^
6)^(1/4)]/(3*Sqrt[2]*b^(3/4)) + ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^6)^(1/4)
)/(Sqrt[b] + Sqrt[-b + a*x^6])]/(3*Sqrt[2]*b^(3/4))
```

fricas [A] time = 0.40, size = 134, normalized size = 1.05

$$\frac{2}{3}\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^2\sqrt{-\frac{1}{b^3}}+\sqrt{ax^6-b}}\,b^2\left(-\frac{1}{b^3}\right)^{\frac{3}{4}}-\left(ax^6-b\right)^{\frac{1}{4}}b^2\left(-\frac{1}{b^3}\right)^{\frac{3}{4}}\right)+\frac{1}{6}\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}+\left(ax^6-b\right)^{\frac{1}{4}}\right)-\frac{1}{6}\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(-b\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}+\left(ax^6-b\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^6-b)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/3*(-1/b^3)^(1/4)*arctan(sqrt(b^2*sqrt(-1/b^3) + sqrt(a*x^6 - b))*b^2*(-1/
b^3)^(3/4) - (a*x^6 - b)^(1/4)*b^2*(-1/b^3)^(3/4)) + 1/6*(-1/b^3)^(1/4)*log
(b*(-1/b^3)^(1/4) + (a*x^6 - b)^(1/4)) - 1/6*(-1/b^3)^(1/4)*log(-b*(-1/b^3)
^(1/4) + (a*x^6 - b)^(1/4))
```

giac [A] time = 0.38, size = 162, normalized size = 1.27

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+2\left(ax^6-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{6\,b^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-2\left(ax^6-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{6\,b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^6-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^6-b}+\sqrt{b}\right)}{12\,b^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^6-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^6-b}+\sqrt{b}\right)}{12\,b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^6-b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^6 - b)^(1/4))/b^(1
/4))/b^(3/4) + 1/6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^6
- b)^(1/4))/b^(1/4))/b^(3/4) + 1/12*sqrt(2)*log(sqrt(2)*(a*x^6 - b)^(1/4)*b
^(1/4) + sqrt(a*x^6 - b) + sqrt(b))/b^(3/4) - 1/12*sqrt(2)*log(-sqrt(2)*(a*
x^6 - b)^(1/4)*b^(1/4) + sqrt(a*x^6 - b) + sqrt(b))/b^(3/4)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a\,x^6-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^6-b)^(3/4),x)
```

```
[Out] int(1/x/(a*x^6-b)^(3/4),x)
```

maxima [A] time = 0.41, size = 162, normalized size = 1.27

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+2\left(ax^6-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{6\,b^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-2\left(ax^6-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{6\,b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^6-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^6-b}+\sqrt{b}\right)}{12\,b^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^6-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^6-b}+\sqrt{b}\right)}{12\,b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^6-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^6 - b)^(1/4))/b^(1
/4))/b^(3/4) + 1/6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^6
- b)^(1/4))/b^(1/4))/b^(3/4) + 1/12*sqrt(2)*log(sqrt(2)*(a*x^6 - b)^(1/4)*b
^(1/4) + sqrt(a*x^6 - b) + sqrt(b))/b^(3/4) - 1/12*sqrt(2)*log(-sqrt(2)*(a*
x^6 - b)^(1/4)*b^(1/4) + sqrt(a*x^6 - b) + sqrt(b))/b^(3/4)
```

mupad [B] time = 1.02, size = 44, normalized size = 0.34

$$-\frac{\mathrm{atan}\left(\frac{{\left(a\,x^6-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)+\mathrm{atanh}\left(\frac{{\left(a\,x^6-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{3\,{\left(-b\right)}^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^6 - b)^(3/4)),x)
```

```
[Out] -(atan((a*x^6 - b)^(1/4)/(-b)^(1/4)) + atanh((a*x^6 - b)^(1/4)/(-b)^(1/4)))
/(3*(-b)^(3/4))
```

sympy [C] time = 1.05, size = 42, normalized size = 0.33

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^6}\right)}{6a^{\frac{3}{4}}x^{\frac{9}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**6-b)**(3/4),x)
```

```
[Out] -gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*exp_polar(2*I*pi)/(a*x**6))/(6*a**(
3/4)*x**(9/2)*gamma(7/4))
```

3.1566 $\int \frac{(1+x^4)(-1+x^2+x^4)^{3/2}}{(-1+x^4)(1+x^2-x^4-x^6+x^8)}\,dx$

Optimal. Leaf size=128

$$-\sqrt{\frac{1}{2}\left(3-i\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}\,x}{\sqrt{x^4+x^2-1}}\right)-\sqrt{\frac{1}{2}\left(3+i\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}\,x}{\sqrt{x^4+x^2-1}}\right)-\tanh^{-1}\left(\frac{x}{\sqrt{x^4+x^2-1}}\right)$$

Rubi [F] time = 2.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+x^4\right)\left(-1+x^2+x^4\right)^{3/2}}{\left(-1+x^4\right)\left(1+x^2-x^4-x^6+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^4)*(-1 + x^2 + x^4)^(3/2))/((-1 + x^4)*(1 + x^2 - x^4 - x^6 + x
^8)),x]
```

```
[Out] (4*x*(1 + Sqrt[5] + 2*x^2))/(3*Sqrt[-1 + x^2 + x^4]) - (x*(1 + 3*x^2)*Sqrt[
-1 + x^2 + x^4])/15 + (x*(11 + 3*x^2)*Sqrt[-1 + x^2 + x^4])/15 - (4*5^(1/4)
*Sqrt[(2 - (1 - Sqrt[5])*x^2)/(2 - (1 + Sqrt[5])*x^2)]*Sqrt[-2 + (1 + Sqrt[
5])*x^2]*EllipticE[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 + (1 + Sqrt[5])*x^2]]
, (5 + Sqrt[5])/10])/(3*Sqrt[(2 - (1 + Sqrt[5])*x^2)^(-1)]*Sqrt[-1 + x^2 +
x^4]) - ((1 - Sqrt[5])*Sqrt[1 + Sqrt[5] + 2*x^2]*Sqrt[1 + (2*x^2)/(1 - Sqrt
[5])]*EllipticF[ArcSin[Sqrt[2/(-1 + Sqrt[5])]*x], (-3 + Sqrt[5])/2])/(Sqrt[
2]*(3 - Sqrt[5])*Sqrt[-1 + x^2 + x^4]) - ((1 - Sqrt[5])*Sqrt[1 + Sqrt[5] +
2*x^2]*Sqrt[1 + (2*x^2)/(1 - Sqrt[5])]*EllipticF[ArcSin[Sqrt[2/(-1 + Sqrt[5
])]*x], (-3 + Sqrt[5])/2])/(Sqrt[2]*(1 + Sqrt[5])*Sqrt[-1 + x^2 + x^4]) + (
(1 - 4*Sqrt[5])*Sqrt[(2 - (1 - Sqrt[5])*x^2)/(2 - (1 + Sqrt[5])*x^2)]*Sqrt[
-2 + (1 + Sqrt[5])*x^2]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 + (1 +
 Sqrt[5])*x^2]], (5 + Sqrt[5])/10])/(6*5^(3/4)*Sqrt[(2 - (1 + Sqrt[5])*x^2)
^(-1)]*Sqrt[-1 + x^2 + x^4]) - ((1 - Sqrt[5])*Sqrt[(2 - (1 - Sqrt[5])*x^2)/
(2 - (1 + Sqrt[5])*x^2)]*Sqrt[-2 + (1 + Sqrt[5])*x^2]*EllipticF[ArcSin[(Sqr
t[2]*5^(1/4)*x)/Sqrt[-2 + (1 + Sqrt[5])*x^2]], (5 + Sqrt[5])/10])/(4*5^(1/4
)*Sqrt[(2 - (1 + Sqrt[5])*x^2)^(-1)]*Sqrt[-1 + x^2 + x^4]) - (Sqrt[(2 - (1
- Sqrt[5])*x^2)/(2 - (1 + Sqrt[5])*x^2)]*Sqrt[-2 + (1 + Sqrt[5])*x^2]*Ellip
ticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 + (1 + Sqrt[5])*x^2]], (5 + Sqrt[5]
)/10])/(5^(1/4)*(3 - Sqrt[5])*Sqrt[(2 - (1 + Sqrt[5])*x^2)^(-1)]*Sqrt[-1 +
x^2 + x^4]) + ((4 - Sqrt[5])*Sqrt[(2 - (1 - Sqrt[5])*x^2)/(2 - (1 + Sqrt[5]
)*x^2)]*Sqrt[-2 + (1 + Sqrt[5])*x^2]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/S
qrt[-2 + (1 + Sqrt[5])*x^2]], (5 + Sqrt[5])/10])/(6*5^(3/4)*Sqrt[(2 - (1 +
Sqrt[5])*x^2)^(-1)]*Sqrt[-1 + x^2 + x^4]) - (Sqrt[(2 - (1 - Sqrt[5])*x^2)/(
2 - (1 + Sqrt[5])*x^2)]*Sqrt[-2 + (1 + Sqrt[5])*x^2]*EllipticF[ArcSin[(Sqrt
[2]*5^(1/4)*x)/Sqrt[-2 + (1 + Sqrt[5])*x^2]], (5 + Sqrt[5])/10])/(5^(1/4)*(
1 + Sqrt[5])*Sqrt[(2 - (1 + Sqrt[5])*x^2)^(-1)]*Sqrt[-1 + x^2 + x^4]) + ((3
 + Sqrt[5])*Sqrt[(2 - (1 - Sqrt[5])*x^2)/(2 - (1 + Sqrt[5])*x^2)]*Sqrt[-2 +
 (1 + Sqrt[5])*x^2]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*x)/Sqrt[-2 + (1 + Sqr
t[5])*x^2]], (5 + Sqrt[5])/10])/(4*5^(1/4)*Sqrt[(2 - (1 + Sqrt[5])*x^2)^(-1
)]*Sqrt[-1 + x^2 + x^4]) - (Sqrt[2]*Sqrt[1 + Sqrt[5] + 2*x^2]*Sqrt[1 + (2*x
^2)/(1 - Sqrt[5])]*EllipticPi[(1 - Sqrt[5])/2, ArcSin[Sqrt[2/(-1 + Sqrt[5])
]*x], (-3 + Sqrt[5])/2])/((1 + Sqrt[5])*Sqrt[-1 + x^2 + x^4]) + (Sqrt[2]*(2
 - Sqrt[5])*Sqrt[1 + Sqrt[5] + 2*x^2]*Sqrt[1 + (2*x^2)/(1 - Sqrt[5])]*Ellip
ticPi[(-1 + Sqrt[5])/2, ArcSin[Sqrt[2/(-1 + Sqrt[5])]*x], (-3 + Sqrt[5])/2]
)/((3 - Sqrt[5])*Sqrt[-1 + x^2 + x^4]) + Defer[Int][(-1 + x^2 + x^4)^(3/2)/
```

```
(1 + x^2 - x^4 - x^6 + x^8), x] + 2*Defer[Int][(x^2*(-1 + x^2 + x^4)^(3/2))
/(1 + x^2 - x^4 - x^6 + x^8), x] - 2*Defer[Int][(x^4*(-1 + x^2 + x^4)^(3/2)
)/(1 + x^2 - x^4 - x^6 + x^8), x]
```

Rubi steps

$$\int \frac{(1+x^4)(-1+x^2+x^4)^{3/2}}{(-1+x^4)(1+x^2-x^4-x^6+x^8)}\,dx = \int \left(\frac{(-1+x^2+x^4)^{3/2}}{-1-x^2} + \frac{(-1+x^2+x^4)^{3/2}}{-1+x^2} + \frac{(1+2x^2-2x^4)(-1+x^2}{1+x^2-x^4-x^6+x}\right.$$

$$= \int \frac{(-1+x^2+x^4)^{3/2}}{-1-x^2}\,dx + \int \frac{(-1+x^2+x^4)^{3/2}}{-1+x^2}\,dx + \int \frac{(1+2x^2-2x^4}{1+x^2-}$$

$$= -\int x^2\sqrt{-1+x^2+x^4}\,dx - \int (-2-x^2)\sqrt{-1+x^2+x^4}\,dx - \int \frac{\sqrt{-1+}}{-1}$$

$$= -\frac{1}{15}x(1+3x^2)\sqrt{-1+x^2+x^4} + \frac{1}{15}x(11+3x^2)\sqrt{-1+x^2+x^4} - \frac{1}{15}$$

$$= -\frac{1}{15}x(1+3x^2)\sqrt{-1+x^2+x^4} + \frac{1}{15}x(11+3x^2)\sqrt{-1+x^2+x^4} + \frac{1}{15}$$

$$= \frac{x(1+\sqrt{5}+2x^2)}{3\sqrt{-1+x^2+x^4}} - \frac{1}{15}x(1+3x^2)\sqrt{-1+x^2+x^4} + \frac{1}{15}x(11+3x^2)\sqrt{-}$$

$$= \frac{x(1+\sqrt{5}+2x^2)}{3\sqrt{-1+x^2+x^4}} - \frac{1}{15}x(1+3x^2)\sqrt{-1+x^2+x^4} + \frac{1}{15}x(11+3x^2)\sqrt{-}$$

$$= \frac{x(1+\sqrt{5}+2x^2)}{3\sqrt{-1+x^2+x^4}} - \frac{1}{15}x(1+3x^2)\sqrt{-1+x^2+x^4} + \frac{1}{15}x(11+3x^2)\sqrt{-}$$

$$= \frac{x(1+\sqrt{5}+2x^2)}{3\sqrt{-1+x^2+x^4}} - \frac{1}{15}x(1+3x^2)\sqrt{-1+x^2+x^4} + \frac{1}{15}x(11+3x^2)\sqrt{-}$$

Mathematica [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{(1+x^4)(-1+x^2+x^4)^{3/2}}{(-1+x^4)(1+x^2-x^4-x^6+x^8)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^4)*(-1 + x^2 + x^4)^(3/2))/((-1 + x^4)*(1 + x^2 - x^4 - x
^6 + x^8)),x]
```

```
[Out] Integrate[((1 + x^4)*(-1 + x^2 + x^4)^(3/2))/((-1 + x^4)*(1 + x^2 - x^4 - x
^6 + x^8)), x]
```

IntegrateAlgebraic [A] time = 1.44, size = 128, normalized size = 1.00

$$-\sqrt{\frac{1}{2}\left(3-i\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}\,x}{\sqrt{x^4+x^2-1}}\right)-\sqrt{\frac{1}{2}\left(3+i\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}\,x}{\sqrt{x^4+x^2-1}}\right)-\tanh^{-1}\left(\frac{x}{\sqrt{x^4+x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^4)*(-1 + x^2 + x^4)^(3/2))/((-1 + x^4)*(1 + x^2
- x^4 - x^6 + x^8)),x]
```

```
[Out] -(Sqrt[(3 - I*Sqrt[3])/2]*ArcTan[(Sqrt[-3/2 - (I/2)*Sqrt[3]]*x)/Sqrt[-1 + x
^2 + x^4]]) - Sqrt[(3 + I*Sqrt[3])/2]*ArcTan[(Sqrt[-3/2 + (I/2)*Sqrt[3]]*x)
/Sqrt[-1 + x^2 + x^4]] - ArcTanh[x/Sqrt[-1 + x^2 + x^4]]
```

fricas [B] time = 4.23, size = 4669, normalized size = 36.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^4+x^2-1)^(3/2)/(x^4-1)/(x^8-x^6-x^4+x^2+1),x, algorith
m="fricas")
```

```
[Out] -1/16*12^(1/4)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(48*(12*x
^6 + 12*x^4 + 12^(1/4)*sqrt(x^4 + x^2 - 1)*(3*sqrt(2)*x^3 + sqrt(3)*sqrt(2)
*(x^5 + x^3 - x))*sqrt(sqrt(3) + 2) - 12*x^2 + sqrt(3)*(x^8 + 5*x^6 + 5*x^4
 - 5*x^2 + 1))/(x^8 - x^6 - x^4 + x^2 + 1)) + 1/16*12^(1/4)*(sqrt(3)*sqrt(2
) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(48*(12*x^6 + 12*x^4 - 12^(1/4)*sqrt(x^
4 + x^2 - 1)*(3*sqrt(2)*x^3 + sqrt(3)*sqrt(2)*(x^5 + x^3 - x))*sqrt(sqrt(3)
 + 2) - 12*x^2 + sqrt(3)*(x^8 + 5*x^6 + 5*x^4 - 5*x^2 + 1))/(x^8 - x^6 - x^
4 + x^2 + 1)) + 1/4*12^(1/4)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/36*(36*x^4
8 + 648*x^46 + 108*x^44 - 26208*x^42 - 80784*x^40 + 122472*x^38 + 679176*x^
36 - 158760*x^34 - 2555388*x^32 - 118872*x^30 + 5525496*x^28 + 511704*x^26
- 7137036*x^24 - 511704*x^22 + 5525496*x^20 + 118872*x^18 - 2555388*x^16 +
158760*x^14 + 679176*x^12 - 122472*x^10 - 80784*x^8 + 26208*x^6 + 108*x^4 -
 648*x^2 + 6*sqrt(x^4 + x^2 - 1)*(12^(3/4)*(sqrt(3)*sqrt(2)*(2*x^45 + 37*x^
43 + 31*x^41 - 895*x^39 - 1651*x^37 + 6780*x^35 + 13803*x^33 - 25689*x^31 -
 50211*x^29 + 55250*x^27 + 92893*x^25 - 70967*x^23 - 92893*x^21 + 55250*x^1
9 + 50211*x^17 - 25689*x^15 - 13803*x^13 + 6780*x^11 + 1651*x^9 - 895*x^7 -
 31*x^5 + 37*x^3 - 2*x) - 3*sqrt(2)*(x^45 + 20*x^43 + 35*x^41 - 323*x^39 -
542*x^37 + 2868*x^35 + 4170*x^33 - 13119*x^31 - 16182*x^29 + 31720*x^27 + 3
1130*x^25 - 42331*x^23 - 31130*x^21 + 31720*x^19 + 16182*x^17 - 13119*x^15
- 4170*x^13 + 2868*x^11 + 542*x^9 - 323*x^7 - 35*x^5 + 20*x^3 - x)) + 36*12
^(1/4)*(sqrt(3)*sqrt(2)*(2*x^43 + 37*x^41 + 60*x^39 - 464*x^37 - 953*x^35 +
 2094*x^33 + 4501*x^31 - 5092*x^29 - 10456*x^27 + 7721*x^25 + 13692*x^23 -
7721*x^21 - 10456*x^19 + 5092*x^17 + 4501*x^15 - 2094*x^13 - 953*x^11 + 464
*x^9 + 60*x^7 - 37*x^5 + 2*x^3) - 3*sqrt(2)*(x^43 + 21*x^41 + 55*x^39 - 205
*x^37 - 667*x^35 + 728*x^33 + 2835*x^31 - 1397*x^29 - 6231*x^27 + 1797*x^25
 + 8014*x^23 - 1797*x^21 - 6231*x^19 + 1397*x^17 + 2835*x^15 - 728*x^13 - 6
67*x^11 + 205*x^9 + 55*x^7 - 21*x^5 + x^3)))*sqrt(sqrt(3) + 2) - sqrt(3)*(2
4*(216*x^41 + 1944*x^39 + 4536*x^37 - 6696*x^35 - 35640*x^33 - 3888*x^31 +
105192*x^29 + 44712*x^27 - 171072*x^25 - 72144*x^23 + 171072*x^21 + 44712*x
^19 - 105192*x^17 - 3888*x^15 + 35640*x^13 - 6696*x^11 - 4536*x^9 + 1944*x^
7 - 216*x^5 + sqrt(3)*(6*x^43 + 72*x^41 + 858*x^39 + 3294*x^37 - 1152*x^35
- 23760*x^33 - 14400*x^31 + 68598*x^29 + 54414*x^27 - 110712*x^25 - 79452*x
^23 + 110712*x^21 + 54414*x^19 - 68598*x^17 - 14400*x^15 + 23760*x^13 - 115
2*x^11 - 3294*x^9 + 858*x^7 - 72*x^5 + 6*x^3 - sqrt(3)*(x^45 + 13*x^43 - 62
*x^41 - 832*x^39 - 1721*x^37 + 3372*x^35 + 12831*x^33 - 2676*x^31 - 37065*x
```

^29 - 6848*x^27 + 59641*x^25 + 13942*x^23 - 59641*x^21 - 6848*x^19 + 37065*
x^17 - 2676*x^15 - 12831*x^13 + 3372*x^11 + 1721*x^9 - 832*x^7 + 62*x^5 + 1
3*x^3 - x)) - 18*sqrt(3)*(x^43 + 10*x^41 - 67*x^39 - 355*x^37 + 176*x^35 +
2158*x^33 + 400*x^31 - 6023*x^29 - 2137*x^27 + 9674*x^25 + 3254*x^23 - 9674
*x^21 - 2137*x^19 + 6023*x^17 + 400*x^15 - 2158*x^13 + 176*x^11 + 355*x^9 -
67*x^7 - 10*x^5 + x^3))*sqrt(x^4 + x^2 - 1) + (12^(3/4)*(sqrt(3)*sqrt(2)*(
x^48 + 18*x^46 - 3*x^44 + 58*x^42 + 3342*x^40 + 8712*x^38 - 18214*x^36 - 67
788*x^34 + 36675*x^32 + 214984*x^30 - 35874*x^28 - 367752*x^26 + 28147*x^24
+ 367752*x^22 - 35874*x^20 - 214984*x^18 + 36675*x^16 + 67788*x^14 - 18214
*x^12 - 8712*x^10 + 3342*x^8 - 58*x^6 - 3*x^4 - 18*x^2 + 1) - 3*sqrt(2)*(x^
48 + 16*x^46 - 31*x^44 - 696*x^42 - 1640*x^40 + 1732*x^38 + 9026*x^36 + 310
8*x^34 - 16143*x^32 - 16316*x^30 + 9658*x^28 + 27416*x^26 - 1743*x^24 - 274
16*x^22 + 9658*x^20 + 16316*x^18 - 16143*x^16 - 3108*x^14 + 9026*x^12 - 173
2*x^10 - 1640*x^8 + 696*x^6 - 31*x^4 - 16*x^2 + 1)) + 36*12^(1/4)*(sqrt(3)*
sqrt(2)*(x^46 + 21*x^44 + 37*x^42 - 36*x^40 + 376*x^38 + 666*x^36 - 3516*x^
34 - 4890*x^32 + 10920*x^30 + 13636*x^28 - 18156*x^26 - 18794*x^24 + 18156*
x^22 + 13636*x^20 - 10920*x^18 - 4890*x^16 + 3516*x^14 + 666*x^12 - 376*x^1
0 - 36*x^8 - 37*x^6 + 21*x^4 - x^2) - 3*sqrt(2)*(x^46 + 15*x^44 - 29*x^42 -
438*x^40 - 338*x^38 + 2664*x^36 + 3204*x^34 - 7680*x^32 - 10338*x^30 + 133
66*x^28 + 17610*x^26 - 15854*x^24 - 17610*x^22 + 13366*x^20 + 10338*x^18 -
7680*x^16 - 3204*x^14 + 2664*x^12 + 338*x^10 - 438*x^8 + 29*x^6 + 15*x^4 -
x^2)))*sqrt(sqrt(3) + 2))*sqrt((12*x^6 + 12*x^4 - 12^(1/4)*sqrt(x^4 + x^2 -
1)*(3*sqrt(2)*x^3 + sqrt(3)*sqrt(2)*(x^5 + x^3 - x))*sqrt(sqrt(3) + 2) - 1
2*x^2 + sqrt(3)*(x^8 + 5*x^6 + 5*x^4 - 5*x^2 + 1))/(x^8 - x^6 - x^4 + x^2 +
1)) + 72*sqrt(3)*(3*x^46 + 51*x^44 + 21*x^42 - 1284*x^40 - 2328*x^38 + 813
6*x^36 + 18288*x^34 - 23832*x^32 - 60048*x^30 + 41052*x^28 + 104802*x^26 -
48246*x^24 - 104802*x^22 + 41052*x^20 + 60048*x^18 - 23832*x^16 - 18288*x^1
4 + 8136*x^12 + 2328*x^10 - 1284*x^8 - 21*x^6 + 51*x^4 - 3*x^2 - sqrt(3)*(x
^46 + 25*x^44 + 103*x^42 - 82*x^40 - 908*x^38 - 18*x^36 + 3858*x^34 + 522*x
^32 - 9636*x^30 - 1046*x^28 + 15022*x^26 + 1198*x^24 - 15022*x^22 - 1046*x^
20 + 9636*x^18 + 522*x^16 - 3858*x^14 - 18*x^12 + 908*x^10 - 82*x^8 - 103*x
^6 + 25*x^4 - x^2)) - 72*sqrt(3)*(x^46 + 35*x^44 + 383*x^42 - 14*x^40 - 466
4*x^38 - 1596*x^36 + 25464*x^34 + 10866*x^32 - 74448*x^30 - 30740*x^28 + 12
5294*x^26 + 42896*x^24 - 125294*x^22 - 30740*x^20 + 74448*x^18 + 10866*x^16
- 25464*x^14 - 1596*x^12 + 4664*x^10 - 14*x^8 - 383*x^6 + 35*x^4 - x^2) +
36)/(x^48 + 18*x^46 - 111*x^44 - 2552*x^42 - 3606*x^40 + 27594*x^38 + 53426
*x^36 - 113958*x^34 - 252837*x^32 + 250858*x^30 + 592002*x^28 - 353226*x^26
- 777749*x^24 + 353226*x^22 + 592002*x^20 - 250858*x^18 - 252837*x^16 + 11
3958*x^14 + 53426*x^12 - 27594*x^10 - 3606*x^8 + 2552*x^6 - 111*x^4 - 18*x^
2 + 1)) + 1/4*12^(1/4)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(1/36*(36*x^48 + 648
*x^46 + 108*x^44 - 26208*x^42 - 80784*x^40 + 122472*x^38 + 679176*x^36 - 15
8760*x^34 - 2555388*x^32 - 118872*x^30 + 5525496*x^28 + 511704*x^26 - 71370
36*x^24 - 511704*x^22 + 5525496*x^20 + 118872*x^18 - 2555388*x^16 + 158760*
x^14 + 679176*x^12 - 122472*x^10 - 80784*x^8 + 26208*x^6 + 108*x^4 - 648*x^
2 - 6*sqrt(x^4 + x^2 - 1)*(12^(3/4)*(sqrt(3)*sqrt(2)*(2*x^45 + 37*x^43 + 31
*x^41 - 895*x^39 - 1651*x^37 + 6780*x^35 + 13803*x^33 - 25689*x^31 - 50211*
x^29 + 55250*x^27 + 92893*x^25 - 70967*x^23 - 92893*x^21 + 55250*x^19 + 502
11*x^17 - 25689*x^15 - 13803*x^13 + 6780*x^11 + 1651*x^9 - 895*x^7 - 31*x^5
+ 37*x^3 - 2*x) - 3*sqrt(2)*(x^45 + 20*x^43 + 35*x^41 - 323*x^39 - 542*x^3
7 + 2868*x^35 + 4170*x^33 - 13119*x^31 - 16182*x^29 + 31720*x^27 + 31130*x^
25 - 42331*x^23 - 31130*x^21 + 31720*x^19 + 16182*x^17 - 13119*x^15 - 4170*
x^13 + 2868*x^11 + 542*x^9 - 323*x^7 - 35*x^5 + 20*x^3 - x)) + 36*12^(1/4)*
(sqrt(3)*sqrt(2)*(2*x^43 + 37*x^41 + 60*x^39 - 464*x^37 - 953*x^35 + 2094*x
^33 + 4501*x^31 - 5092*x^29 - 10456*x^27 + 7721*x^25 + 13692*x^23 - 7721*x^
21 - 10456*x^19 + 5092*x^17 + 4501*x^15 - 2094*x^13 - 953*x^11 + 464*x^9 +
60*x^7 - 37*x^5 + 2*x^3) - 3*sqrt(2)*(x^43 + 21*x^41 + 55*x^39 - 205*x^37 -
667*x^35 + 728*x^33 + 2835*x^31 - 1397*x^29 - 6231*x^27 + 1797*x^25 + 8014
*x^23 - 1797*x^21 - 6231*x^19 + 1397*x^17 + 2835*x^15 - 728*x^13 - 667*x^11
+ 205*x^9 + 55*x^7 - 21*x^5 + x^3)))*sqrt(sqrt(3) + 2) - sqrt(3)*(24*(216*

```
x^41 + 1944*x^39 + 4536*x^37 - 6696*x^35 - 35640*x^33 - 3888*x^31 + 105192*
x^29 + 44712*x^27 - 171072*x^25 - 72144*x^23 + 171072*x^21 + 44712*x^19 - 1
05192*x^17 - 3888*x^15 + 35640*x^13 - 6696*x^11 - 4536*x^9 + 1944*x^7 - 216
*x^5 + sqrt(3)*(6*x^43 + 72*x^41 + 858*x^39 + 3294*x^37 - 1152*x^35 - 23760
*x^33 - 14400*x^31 + 68598*x^29 + 54414*x^27 - 110712*x^25 - 79452*x^23 + 1
10712*x^21 + 54414*x^19 - 68598*x^17 - 14400*x^15 + 23760*x^13 - 1152*x^11
- 3294*x^9 + 858*x^7 - 72*x^5 + 6*x^3 - sqrt(3)*(x^45 + 13*x^43 - 62*x^41 -
 832*x^39 - 1721*x^37 + 3372*x^35 + 12831*x^33 - 2676*x^31 - 37065*x^29 - 6
848*x^27 + 59641*x^25 + 13942*x^23 - 59641*x^21 - 6848*x^19 + 37065*x^17 -
2676*x^15 - 12831*x^13 + 3372*x^11 + 1721*x^9 - 832*x^7 + 62*x^5 + 13*x^3 -
 x)) - 18*sqrt(3)*(x^43 + 10*x^41 - 67*x^39 - 355*x^37 + 176*x^35 + 2158*x^
33 + 400*x^31 - 6023*x^29 - 2137*x^27 + 9674*x^25 + 3254*x^23 - 9674*x^21 -
 2137*x^19 + 6023*x^17 + 400*x^15 - 2158*x^13 + 176*x^11 + 355*x^9 - 67*x^7
 - 10*x^5 + x^3))*sqrt(x^4 + x^2 - 1) - (12^(3/4)*(sqrt(3)*sqrt(2)*(x^48 +
18*x^46 - 3*x^44 + 58*x^42 + 3342*x^40 + 8712*x^38 - 18214*x^36 - 67788*x^3
4 + 36675*x^32 + 214984*x^30 - 35874*x^28 - 367752*x^26 + 28147*x^24 + 3677
52*x^22 - 35874*x^20 - 214984*x^18 + 36675*x^16 + 67788*x^14 - 18214*x^12 -
 8712*x^10 + 3342*x^8 - 58*x^6 - 3*x^4 - 18*x^2 + 1) - 3*sqrt(2)*(x^48 + 16
*x^46 - 31*x^44 - 696*x^42 - 1640*x^40 + 1732*x^38 + 9026*x^36 + 3108*x^34
- 16143*x^32 - 16316*x^30 + 9658*x^28 + 27416*x^26 - 1743*x^24 - 27416*x^22
 + 9658*x^20 + 16316*x^18 - 16143*x^16 - 3108*x^14 + 9026*x^12 - 1732*x^10
- 1640*x^8 + 696*x^6 - 31*x^4 - 16*x^2 + 1)) + 36*12^(1/4)*(sqrt(3)*sqrt(2)
*(x^46 + 21*x^44 + 37*x^42 - 36*x^40 + 376*x^38 + 666*x^36 - 3516*x^34 - 48
90*x^32 + 10920*x^30 + 13636*x^28 - 18156*x^26 - 18794*x^24 + 18156*x^22 +
13636*x^20 - 10920*x^18 - 4890*x^16 + 3516*x^14 + 666*x^12 - 376*x^10 - 36*
x^8 - 37*x^6 + 21*x^4 - x^2) - 3*sqrt(2)*(x^46 + 15*x^44 - 29*x^42 - 438*x^
40 - 338*x^38 + 2664*x^36 + 3204*x^34 - 7680*x^32 - 10338*x^30 + 13366*x^28
 + 17610*x^26 - 15854*x^24 - 17610*x^22 + 13366*x^20 + 10338*x^18 - 7680*x^
16 - 3204*x^14 + 2664*x^12 + 338*x^10 - 438*x^8 + 29*x^6 + 15*x^4 - x^2)))*
sqrt(sqrt(3) + 2))*sqrt((12*x^6 + 12*x^4 + 12^(1/4)*sqrt(x^4 + x^2 - 1)*(3*
sqrt(2)*x^3 + sqrt(3)*sqrt(2)*(x^5 + x^3 - x))*sqrt(sqrt(3) + 2) - 12*x^2 +
 sqrt(3)*(x^8 + 5*x^6 + 5*x^4 - 5*x^2 + 1))/(x^8 - x^6 - x^4 + x^2 + 1)) +
72*sqrt(3)*(3*x^46 + 51*x^44 + 21*x^42 - 1284*x^40 - 2328*x^38 + 8136*x^36
+ 18288*x^34 - 23832*x^32 - 60048*x^30 + 41052*x^28 + 104802*x^26 - 48246*x
^24 - 104802*x^22 + 41052*x^20 + 60048*x^18 - 23832*x^16 - 18288*x^14 + 813
6*x^12 + 2328*x^10 - 1284*x^8 - 21*x^6 + 51*x^4 - 3*x^2 - sqrt(3)*(x^46 + 2
5*x^44 + 103*x^42 - 82*x^40 - 908*x^38 - 18*x^36 + 3858*x^34 + 522*x^32 - 9
636*x^30 - 1046*x^28 + 15022*x^26 + 1198*x^24 - 15022*x^22 - 1046*x^20 + 96
36*x^18 + 522*x^16 - 3858*x^14 - 18*x^12 + 908*x^10 - 82*x^8 - 103*x^6 + 25
*x^4 - x^2)) - 72*sqrt(3)*(x^46 + 35*x^44 + 383*x^42 - 14*x^40 - 4664*x^38
- 1596*x^36 + 25464*x^34 + 10866*x^32 - 74448*x^30 - 30740*x^28 + 125294*x^
26 + 42896*x^24 - 125294*x^22 - 30740*x^20 + 74448*x^18 + 10866*x^16 - 2546
4*x^14 - 1596*x^12 + 4664*x^10 - 14*x^8 - 383*x^6 + 35*x^4 - x^2) + 36)/(x^
48 + 18*x^46 - 111*x^44 - 2552*x^42 - 3606*x^40 + 27594*x^38 + 53426*x^36 -
 113958*x^34 - 252837*x^32 + 250858*x^30 + 592002*x^28 - 353226*x^26 - 7777
49*x^24 + 353226*x^22 + 592002*x^20 - 250858*x^18 - 252837*x^16 + 113958*x^
14 + 53426*x^12 - 27594*x^10 - 3606*x^8 + 2552*x^6 - 111*x^4 - 18*x^2 + 1))
 + 1/2*log(-(x^4 + 2*x^2 - 2*sqrt(x^4 + x^2 - 1)*x - 1)/(x^4 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^2-1\right)^{\frac{3}{2}}\left(x^4+1\right)}{\left(x^8-x^6-x^4+x^2+1\right)\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^4+x^2-1)^(3/2)/(x^4-1)/(x^8-x^6-x^4+x^2+1),x, algorith
m="giac")
```

```
[Out] integrate((x^4 + x^2 - 1)^(3/2)*(x^4 + 1)/((x^8 - x^6 - x^4 + x^2 + 1)*(x^4
 - 1)), x)
```

maple [C] time = 0.17, size = 871, normalized size = 6.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)*(x^4+x^2-1)^(3/2)/(x^4-1)/(x^8-x^6-x^4+x^2+1),x)
```

```
[Out] 32/15/(2-2*5^(1/2))^(1/2)*(1-(1/2-1/2*5^(1/2))*x^2)^(1/2)*(1-(1/2+1/2*5^(1/
2))*x^2)^(1/2)/(x^4+x^2-1)^(1/2)*EllipticF(1/2*x*(2-2*5^(1/2))^(1/2),1/2*I+
1/2*I*5^(1/2))-1/(1/2-1/2*5^(1/2))^(1/2)*(1-(1/2-1/2*5^(1/2))*x^2)^(1/2)*(1
-(1/2+1/2*5^(1/2))*x^2)^(1/2)/(x^4+x^2-1)^(1/2)*EllipticPi((1/2-1/2*5^(1/2)
)^(1/2)*x,1/(1/2-1/2*5^(1/2)),(1/2+1/2*5^(1/2))^(1/2)/(1/2-1/2*5^(1/2))^(1/
2))-1/4*sum(_alpha*(2*_alpha^6-2*_alpha^4+1)*(-1/(_alpha^4+_alpha^2-1)^(1/2
)*arctanh(1/14*(2*_alpha^2+1)*(20*_alpha^6-30*_alpha^4-5*_alpha^2+7*x^2+26)
/(_alpha^4+_alpha^2-1)^(1/2)/(x^4+x^2-1)^(1/2))-2^(1/2)*(-_alpha^7+_alpha^5
+_alpha^3-_alpha)/(-5^(1/2)+1)^(1/2)*(-x^2+2+5^(1/2)*x^2)^(1/2)*(-x^2+2-5^(
1/2)*x^2)^(1/2)/(x^4+x^2-1)^(1/2)*EllipticPi((1/2-1/2*5^(1/2))^(1/2)*x,1/2*
_alpha^6*5^(1/2)+1/2*_alpha^6-1/2*_alpha^4*5^(1/2)-1/2*_alpha^4-1/2*5^(1/2)
*_alpha^2-1/2*_alpha^2+1/2*5^(1/2)+1/2,(1/2+1/2*5^(1/2))^(1/2)/(1/2-1/2*5^(
1/2))^(1/2))),_alpha=RootOf(_Z^8-_Z^6-_Z^4+_Z^2+1))-1/(1/2-1/2*5^(1/2))^(1/
2)*(1-1/2*x^2+1/2*5^(1/2)*x^2)^(1/2)*(1-1/2*x^2-1/2*5^(1/2)*x^2)^(1/2)/(x^4
+x^2-1)^(1/2)*EllipticPi((1/2-1/2*5^(1/2))^(1/2)*x,-1/(1/2-1/2*5^(1/2)),(1/
2+1/2*5^(1/2))^(1/2)/(1/2-1/2*5^(1/2))^(1/2))+92/15/(2-2*5^(1/2))^(1/2)*(1-
1/2*x^2+1/2*5^(1/2)*x^2)^(1/2)*(1-1/2*x^2-1/2*5^(1/2)*x^2)^(1/2)/(x^4+x^2-1
)^(1/2)/(5^(1/2)+1)*EllipticF(1/2*x*(2-2*5^(1/2))^(1/2),1/2*I+1/2*I*5^(1/2)
)-92/15/(2-2*5^(1/2))^(1/2)*(1-1/2*x^2+1/2*5^(1/2)*x^2)^(1/2)*(1-1/2*x^2-1/
2*5^(1/2)*x^2)^(1/2)/(x^4+x^2-1)^(1/2)/(5^(1/2)+1)*EllipticE(1/2*x*(2-2*5^(
1/2))^(1/2),1/2*I+1/2*I*5^(1/2))-92/15/(2-2*5^(1/2))^(1/2)*(1-(1/2-1/2*5^(1
/2))*x^2)^(1/2)*(1-(1/2+1/2*5^(1/2))*x^2)^(1/2)/(x^4+x^2-1)^(1/2)/(5^(1/2)+
1)*(EllipticF(1/2*x*(2-2*5^(1/2))^(1/2),1/2*I+1/2*I*5^(1/2))-EllipticE(1/2*
x*(2-2*5^(1/2))^(1/2),1/2*I+1/2*I*5^(1/2)))-2/15/(2-2*5^(1/2))^(1/2)*(1-1/2
*x^2+1/2*5^(1/2)*x^2)^(1/2)*(1-1/2*x^2-1/2*5^(1/2)*x^2)^(1/2)/(x^4+x^2-1)^(
1/2)*EllipticF(1/2*x*(2-2*5^(1/2))^(1/2),1/2*I+1/2*I*5^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^2-1\right)^{\frac{3}{2}}\left(x^4+1\right)}{\left(x^8-x^6-x^4+x^2+1\right)\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^4+x^2-1)^(3/2)/(x^4-1)/(x^8-x^6-x^4+x^2+1),x, algorith
m="maxima")
```

```
[Out] integrate((x^4 + x^2 - 1)^(3/2)*(x^4 + 1)/((x^8 - x^6 - x^4 + x^2 + 1)*(x^4
 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4+1\right)\,{\left(x^4+x^2-1\right)}^{3/2}}{\left(x^4-1\right)\,\left(x^8-x^6-x^4+x^2+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)*(x^2 + x^4 - 1)^(3/2))/((x^4 - 1)*(x^2 - x^4 - x^6 + x^8 + 1
)),x)
```

```
[Out] int(((x^4 + 1)*(x^2 + x^4 - 1)^(3/2))/((x^4 - 1)*(x^2 - x^4 - x^6 + x^8 + 1
)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)*(x**4+x**2-1)**(3/2)/(x**4-1)/(x**8-x**6-x**4+x**2+1),x)
```

```
[Out] Timed out
```

3.1567 $\int \frac{(-1+x^6)(1+x^3+x^6)^{2/3}}{1+x^6+x^{12}}\,dx$

Optimal. Leaf size=128

$$\frac{\log\left(2^{2/3}\sqrt[3]{x^6+x^3+1}-2x\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^6+x^3+1}+x}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^6+x^3+1}\,x+\sqrt[3]{2}\left(x^6+x^3+1\right)^{2/3}\right)}{6\sqrt[3]{2}}$$

Rubi [F] time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^6)(1+x^3+x^6)^{2/3}}{1+x^6+x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^6)*(1 + x^3 + x^6)^(2/3))/(1 + x^6 + x^12),x]
```

```
[Out] (x*(1 + (2*x^3)/(1 - I*Sqrt[3]))^(1/3)*(1 + (2*x^3)/(1 + I*Sqrt[3]))^(1/3)*
AppellF1[1/3, 1/3, 1/3, 4/3, (-2*x^3)/(1 - I*Sqrt[3]), (-2*x^3)/(1 + I*Sqrt
[3])])/(1 + x^3 + x^6)^(1/3) + (1 + I*Sqrt[3])*Defer[Int][1/((-1 - I*Sqrt[3
] + 2*x^3)*(1 + x^3 + x^6)^(1/3)), x] + (1 - I*Sqrt[3])*Defer[Int][1/((-1 +
 I*Sqrt[3] + 2*x^3)*(1 + x^3 + x^6)^(1/3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^6)(1+x^3+x^6)^{2/3}}{1+x^6+x^{12}}\,dx &= \int \frac{-1+x^6}{(1-x^3+x^6)\sqrt[3]{1+x^3+x^6}}\,dx\\
&= \int \left(\frac{1}{\sqrt[3]{1+x^3+x^6}}-\frac{2-x^3}{(1-x^3+x^6)\sqrt[3]{1+x^3+x^6}}\right)dx\\
&= \int \frac{1}{\sqrt[3]{1+x^3+x^6}}\,dx-\int \frac{2-x^3}{(1-x^3+x^6)\sqrt[3]{1+x^3+x^6}}\,dx\\
&= \frac{\left(\sqrt[3]{1+\frac{2x^3}{1-i\sqrt{3}}}\sqrt[3]{1+\frac{2x^3}{1+i\sqrt{3}}}\right)\int \frac{1}{\sqrt[3]{1+\frac{2x^3}{1-i\sqrt{3}}}\sqrt[3]{1+\frac{2x^3}{1+i\sqrt{3}}}}\,dx}{\sqrt[3]{1+x^3+x^6}}-\int\left(\frac{-1-i\text{v}}{(-1-i\sqrt{3}+2x^3)}\right.\\
&= \frac{x\sqrt[3]{1+\frac{2x^3}{1-i\sqrt{3}}}\sqrt[3]{1+\frac{2x^3}{1+i\sqrt{3}}}F_1\left(\frac{1}{3};\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{2x^3}{1-i\sqrt{3}},-\frac{2x^3}{1+i\sqrt{3}}\right)}{\sqrt[3]{1+x^3+x^6}}-\left(-1-i\sqrt{3}\right)\int \frac{}{(-}
\end{aligned}$$

Mathematica [F] time = 1.23, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^6)(1+x^3+x^6)^{2/3}}{1+x^6+x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^6)*(1 + x^3 + x^6)^(2/3))/(1 + x^6 + x^12),x]
```

```
[Out] Integrate[((-1 + x^6)*(1 + x^3 + x^6)^(2/3))/(1 + x^6 + x^12), x]
```

IntegrateAlgebraic [A] time = 0.55, size = 128, normalized size = 1.00

$$\frac{\log\left(2^{2/3}\sqrt[3]{x^6+x^3+1}-2x\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^6+x^3+1}+x}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^6+x^3+1}\,x+\sqrt[3]{2}\left(x^6+x^3+1\right)^{2/3}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^6)*(1 + x^3 + x^6)^(2/3))/(1 + x^6 + x^12),x]
```

```
[Out] -(ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(1 + x^3 + x^6)^(1/3))]/(2^(1/3)*Sqrt[3])
) + Log[-2*x + 2^(2/3)*(1 + x^3 + x^6)^(1/3)]/(3*2^(1/3)) - Log[2*x^2 + 2^(
2/3)*x*(1 + x^3 + x^6)^(1/3) + 2^(1/3)*(1 + x^3 + x^6)^(2/3)]/(6*2^(1/3))
```

fricas [B] time = 10.65, size = 322, normalized size = 2.52

$$-\frac{1}{18}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}\left(x^{13}+16x^{10}+21x^{7}+16x^{4}+x\right)\left(x^{6}+x^{3}+1\right)^{\frac{2}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(x^{18}-21x^{15}-102x^{12}-133x^{9}-102x^{6}-21x^{3}+1\right)-24\sqrt{6}\left(x^{14}+x^{11}+x^{5}+x^{2}\right)\left(x^{6}+x^{3}+1\right)^{\frac{1}{3}}\right)}{6\left(x^{18}+51x^{15}+114x^{12}+155x^{9}+114x^{6}+51x^{3}+1\right)}\right)+\frac{1}{18}\cdot2^{\frac{2}{3}}\log\left(-\frac{3\cdot2^{\frac{2}{3}}\left(x^{6}+x^{3}+1\right)^{\frac{2}{3}}x-6\left(x^{6}+x^{3}+1\right)^{\frac{1}{3}}x^{2}-2^{\frac{1}{3}}\left(x^{6}-x^{3}+1\right)}{x^{6}-x^{3}+1}\right)-\frac{1}{36}\cdot2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}\left(x^{12}+16x^{9}+21x^{6}+16x^{3}+1\right)+12\cdot2^{\frac{1}{3}}\left(x^{8}+2x^{5}+x^{2}\right)\left(x^{6}+x^{3}+1\right)^{\frac{1}{3}}+6\left(x^{7}+5x^{4}+x\right)\left(x^{6}+x^{3}+1\right)^{\frac{2}{3}}}{x^{12}-2x^{9}+3x^{6}-2x^{3}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)*(x^6+x^3+1)^(2/3)/(x^12+x^6+1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(x^13 + 16*x^10
 + 21*x^7 + 16*x^4 + x)*(x^6 + x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(x^18 - 21*
x^15 - 102*x^12 - 133*x^9 - 102*x^6 - 21*x^3 + 1) - 24*sqrt(6)*(x^14 + x^11
 + x^5 + x^2)*(x^6 + x^3 + 1)^(1/3))/(x^18 + 51*x^15 + 114*x^12 + 155*x^9 +
 114*x^6 + 51*x^3 + 1)) + 1/18*2^(2/3)*log(-(3*2^(2/3)*(x^6 + x^3 + 1)^(2/3
)*x - 6*(x^6 + x^3 + 1)^(1/3)*x^2 - 2^(1/3)*(x^6 - x^3 + 1))/(x^6 - x^3 + 1
)) - 1/36*2^(2/3)*log((2^(2/3)*(x^12 + 16*x^9 + 21*x^6 + 16*x^3 + 1) + 12*2
^(1/3)*(x^8 + 2*x^5 + x^2)*(x^6 + x^3 + 1)^(1/3) + 6*(x^7 + 5*x^4 + x)*(x^6
 + x^3 + 1)^(2/3))/(x^12 - 2*x^9 + 3*x^6 - 2*x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^6+x^3+1\right)^{\frac{2}{3}}\left(x^6-1\right)}{x^{12}+x^6+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)*(x^6+x^3+1)^(2/3)/(x^12+x^6+1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^3 + 1)^(2/3)*(x^6 - 1)/(x^12 + x^6 + 1), x)
```

maple [C] time = 7.69, size = 1037, normalized size = 8.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)*(x^6+x^3+1)^(2/3)/(x^12+x^6+1),x)
```

```
[Out] -1/6*ln((15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^
3-4)^3*x^3+72*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf
(_Z^3-4)^2*x^3+5*RootOf(_Z^3-4)*x^6+24*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(
_Z^3-4)+36*_Z^2)*x^6-18*RootOf(_Z^3-4)^2*(x^6+x^3+1)^(2/3)*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x-9*RootOf(_Z^3-4)^2*(x^6+x^3+1)^(1/3)
*x^2-18*(x^6+x^3+1)^(1/3)*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootO
f(_Z^3-4)+36*_Z^2)*x^2+15*RootOf(_Z^3-4)*x^3+72*RootOf(RootOf(_Z^3-4)^2+6*_
Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+6*(x^6+x^3+1)^(2/3)*x+5*RootOf(_Z^3-4)+24*Roo
tOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x^6-x^3+1))*RootOf(_Z^3
-4)-ln((15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3
-4)^3*x^3+72*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(
_Z^3-4)^2*x^3+5*RootOf(_Z^3-4)*x^6+24*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_
```

```
Z^3-4)+36*_Z^2)*x^6-18*RootOf(_Z^3-4)^2*(x^6+x^3+1)^(2/3)*RootOf(RootOf(_Z^
3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x-9*RootOf(_Z^3-4)^2*(x^6+x^3+1)^(1/3)*
x^2-18*(x^6+x^3+1)^(1/3)*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf
(_Z^3-4)+36*_Z^2)*x^2+15*RootOf(_Z^3-4)*x^3+72*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2)*x^3+6*(x^6+x^3+1)^(2/3)*x+5*RootOf(_Z^3-4)+24*Root
Of(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x^6-x^3+1))*RootOf(RootO
f(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+1/6*RootOf(_Z^3-4)*ln((15*RootOf(R
ootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-5*Roo
tOf(_Z^3-4)*x^6-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^6+
18*RootOf(_Z^3-4)^2*(x^6+x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_
Z^3-4)+36*_Z^2)*x-3*RootOf(_Z^3-4)^2*(x^6+x^3+1)^(1/3)*x^2-54*(x^6+x^3+1)^(
1/3)*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^
2-5*RootOf(_Z^3-4)*x^3-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^
2)*x^3+18*(x^6+x^3+1)^(2/3)*x-5*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*
_Z*RootOf(_Z^3-4)+36*_Z^2))/(x^6-x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^3+1\right)^{\frac{2}{3}}\left(x^6-1\right)}{x^{12}+x^6+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)*(x^6+x^3+1)^(2/3)/(x^12+x^6+1),x, algorithm="maxima")
[Out] integrate((x^6 + x^3 + 1)^(2/3)*(x^6 - 1)/(x^12 + x^6 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6-1\right)\,{\left(x^6+x^3+1\right)}^{2/3}}{x^{12}+x^6+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)*(x^3 + x^6 + 1)^(2/3))/(x^6 + x^12 + 1),x)
[Out] int(((x^6 - 1)*(x^3 + x^6 + 1)^(2/3))/(x^6 + x^12 + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)*(x**6+x**3+1)**(2/3)/(x**12+x**6+1),x)
[Out] Timed out
```

3.1568 $\int \frac{(-1+x^3)^3 (1+x^3) \sqrt{2+3x^6+2x^{12}}}{x^7 (1+x^6)} \, dx$

Optimal. Leaf size=128

$$-\frac{\log\left(\sqrt{2}\,x^6+\sqrt{2x^{12}+3x^6+2}+\sqrt{2}\right)}{6\sqrt{2}}+\frac{\sqrt{2x^{12}+3x^6+2}\,\left(x^6-4x^3+1\right)}{6x^6}-\frac{4}{3}\tan^{-1}\left(\frac{x^3}{\sqrt{2}\,x^6+\sqrt{2x^{12}+3x^6+2}\right.$$

Rubi [F] time = 2.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1+x^3\right)^3 \left(1+x^3\right) \sqrt{2+3x^6+2x^{12}}}{x^7 \left(1+x^6\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^3)^3*(1 + x^3)*Sqrt[2 + 3*x^6 + 2*x^12])/(x^7*(1 + x^6)),x]
```

```
[Out] Sqrt[2 + 3*x^6 + 2*x^12]/6 + Sqrt[2 + 3*x^6 + 2*x^12]/(6*x^6) - (2*Sqrt[2 +
 3*x^6 + 2*x^12])/(3*x^3) + (4*x^3*Sqrt[2 + 3*x^6 + 2*x^12])/(3*(1 + x^6))
- ArcSinh[(3 + 4*x^6)/Sqrt[7]]/(12*Sqrt[2]) - ArcTanh[(4 + 3*x^6)/(2*Sqrt[2
]*Sqrt[2 + 3*x^6 + 2*x^12])]/(12*Sqrt[2]) - (4*Sqrt[2]*(1 + x^6)*Sqrt[(2 +
3*x^6 + 2*x^12)/(1 + x^6)^2]*EllipticE[2*ArcTan[x^3], 1/8])/(3*Sqrt[2 + 3*x
^6 + 2*x^12]) + (7*(1 + x^6)*Sqrt[(2 + 3*x^6 + 2*x^12)/(1 + x^6)^2]*Ellipti
cF[2*ArcTan[x^3], 1/8])/(3*Sqrt[2]*Sqrt[2 + 3*x^6 + 2*x^12]) + ((2*I)/3)*De
fer[Int][Sqrt[2 + 3*x^6 + 2*x^12]/(I - x), x] + ((2*I)/3)*Defer[Int][Sqrt[2
 + 3*x^6 + 2*x^12]/(I + x), x] + (2*(1 + I*Sqrt[3])*Defer[Int][Sqrt[2 + 3*x
^6 + 2*x^12]/(Sqrt[1 - I*Sqrt[3]] - Sqrt[2]*x), x])/(3*Sqrt[1 - I*Sqrt[3]])
 + (2*(1 - I*Sqrt[3])*Defer[Int][Sqrt[2 + 3*x^6 + 2*x^12]/(Sqrt[1 + I*Sqrt[
3]] - Sqrt[2]*x), x])/(3*Sqrt[1 + I*Sqrt[3]]) + (2*(1 + I*Sqrt[3])*Defer[In
t][Sqrt[2 + 3*x^6 + 2*x^12]/(Sqrt[1 - I*Sqrt[3]] + Sqrt[2]*x), x])/(3*Sqrt[
1 - I*Sqrt[3]]) + (2*(1 - I*Sqrt[3])*Defer[Int][Sqrt[2 + 3*x^6 + 2*x^12]/(S
qrt[1 + I*Sqrt[3]] + Sqrt[2]*x), x])/(3*Sqrt[1 + I*Sqrt[3]])
```

Rubi steps

$$\int \frac{(-1+x^3)^3 (1+x^3)\sqrt{2+3x^6+2x^{12}}}{x^7(1+x^6)}\,dx = \int \left(-\frac{\sqrt{2+3x^6+2x^{12}}}{x^7} + \frac{2\sqrt{2+3x^6+2x^{12}}}{x^4} + \frac{\sqrt{2+3x^6+2x^{12}}}{x} + \right.$$

$$= \frac{4}{3}\int \frac{\sqrt{2+3x^6+2x^{12}}}{1+x^2}\,dx - \frac{4}{3}\int \frac{(1+x^2)\sqrt{2+3x^6+2x^{12}}}{1-x^2+x^4}\,dx + 2$$

$$= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{\sqrt{2+3x+2x^2}}{x^2}\,dx, x, x^6\right)\right) + \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt{2+3x}}{x}\right.$$

$$= \frac{1}{6}\sqrt{2+3x^6+2x^{12}} + \frac{\sqrt{2+3x^6+2x^{12}}}{6x^6} - \frac{2\sqrt{2+3x^6+2x^{12}}}{3x^3} + \frac{2}{3}i$$

$$= \frac{1}{6}\sqrt{2+3x^6+2x^{12}} + \frac{\sqrt{2+3x^6+2x^{12}}}{6x^6} - \frac{2\sqrt{2+3x^6+2x^{12}}}{3x^3} + \frac{2}{3}i$$

$$= \frac{1}{6}\sqrt{2+3x^6+2x^{12}} + \frac{\sqrt{2+3x^6+2x^{12}}}{6x^6} - \frac{2\sqrt{2+3x^6+2x^{12}}}{3x^3} + \frac{4x^3}{}$$

$$= \frac{1}{6}\sqrt{2+3x^6+2x^{12}} + \frac{\sqrt{2+3x^6+2x^{12}}}{6x^6} - \frac{2\sqrt{2+3x^6+2x^{12}}}{3x^3} + \frac{4x^3}{}$$

Mathematica [F] time = 1.16, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^3)^3 (1+x^3)\sqrt{2+3x^6+2x^{12}}}{x^7(1+x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^3)^3*(1 + x^3)*Sqrt[2 + 3*x^6 + 2*x^12])/(x^7*(1 + x^6))
,x]
```

```
[Out] Integrate[((-1 + x^3)^3*(1 + x^3)*Sqrt[2 + 3*x^6 + 2*x^12])/(x^7*(1 + x^6))
, x]
```

IntegrateAlgebraic [A] time = 3.85, size = 128, normalized size = 1.00

$$-\frac{\log\left(\sqrt{2}x^6+\sqrt{2x^{12}+3x^6+2}+\sqrt{2}\right)}{6\sqrt{2}} + \frac{\sqrt{2x^{12}+3x^6+2}\,(x^6-4x^3+1)}{6x^6} - \frac{4}{3}\tan^{-1}\left(\frac{x^3}{\sqrt{2}x^6+\sqrt{2x^{12}+3x^6+2}+\sqrt{2}}\right) + \frac{\log(x)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^3*(1 + x^3)*Sqrt[2 + 3*x^6 + 2*x^12])/(x^7*(
1 + x^6)),x]
```

```
[Out] ((1 - 4*x^3 + x^6)*Sqrt[2 + 3*x^6 + 2*x^12])/(6*x^6) - (4*ArcTan[x^3/(Sqrt[
2] + Sqrt[2]*x^6 + Sqrt[2 + 3*x^6 + 2*x^12])])/3 + Log[x]/(2*Sqrt[2]) - Log
[Sqrt[2] + Sqrt[2]*x^6 + Sqrt[2 + 3*x^6 + 2*x^12]]/(6*Sqrt[2])
```

fricas [A] time = 0.52, size = 105, normalized size = 0.82

$$\frac{\sqrt{2}\,x^6\log\left(-\frac{4\,x^{12}+7\,x^6-2\,\sqrt{2}\,\sqrt{2\,x^{12}+3\,x^6+2}\,(x^6+1)+4}{x^6}\right) - 16\,x^6\arctan\left(\frac{x^3}{\sqrt{2\,x^{12}+3\,x^6+2}}\right) + 4\,\sqrt{2\,x^{12}+3\,x^6+2}\,(x^6-4\,x^3+1)}{24\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^3*(x^3+1)*(2*x^12+3*x^6+2)^(1/2)/x^7/(x^6+1),x, algorithm
="fricas")
```

```
[Out] 1/24*(sqrt(2)*x^6*log(-(4*x^12 + 7*x^6 - 2*sqrt(2)*sqrt(2*x^12 + 3*x^6 + 2)
*(x^6 + 1) + 4)/x^6) - 16*x^6*arctan(x^3/sqrt(2*x^12 + 3*x^6 + 2)) + 4*sqrt
(2*x^12 + 3*x^6 + 2)*(x^6 - 4*x^3 + 1))/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^{12}+3\,x^{6}+2}\,\left(x^{3}+1\right)\left(x^{3}-1\right)^{3}}{\left(x^{6}+1\right)x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^3*(x^3+1)*(2*x^12+3*x^6+2)^(1/2)/x^7/(x^6+1),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(2*x^12 + 3*x^6 + 2)*(x^3 + 1)*(x^3 - 1)^3/((x^6 + 1)*x^7), x
)
```

maple [C] time = 1.51, size = 165, normalized size = 1.29

$$-\frac{8x^{15}-2x^{12}+12x^{9}-3x^{6}+8x^{3}-2}{6x^{6}\sqrt{2x^{12}+3x^{6}+2}}+\frac{\sqrt{2x^{12}+3x^{6}+2}}{6}+\frac{\mathrm{RootOf}\left(\_Z^{2}-2\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^{2}-2\right)x^{6}+\sqrt{2x^{12}+3x^{6}+2}-\mathrm{RootOf}\left(\_Z^{2}-2\right)}{x^{3}}\right)}{12}+\frac{2\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right)x^{3}+\sqrt{2x^{12}+3x^{6}+2}}{\left(x^{2}+1\right)\left(x^{4}-x^{2}+1\right)}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^3*(x^3+1)*(2*x^12+3*x^6+2)^(1/2)/x^7/(x^6+1),x)
```

```
[Out] -1/6*(8*x^15-2*x^12+12*x^9-3*x^6+8*x^3-2)/x^6/(2*x^12+3*x^6+2)^(1/2)+1/6*(2
*x^12+3*x^6+2)^(1/2)+1/12*RootOf(_Z^2-2)*ln((-RootOf(_Z^2-2)*x^6+(2*x^12+3*
x^6+2)^(1/2)-RootOf(_Z^2-2))/x^3)+2/3*RootOf(_Z^2+1)*ln((RootOf(_Z^2+1)*x^3
+(2*x^12+3*x^6+2)^(1/2))/(x^2+1)/(x^4-x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^{12}+3\,x^{6}+2}\,\left(x^{3}+1\right)\left(x^{3}-1\right)^{3}}{\left(x^{6}+1\right)x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^3*(x^3+1)*(2*x^12+3*x^6+2)^(1/2)/x^7/(x^6+1),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(2*x^12 + 3*x^6 + 2)*(x^3 + 1)*(x^3 - 1)^3/((x^6 + 1)*x^7), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(x^{3}-1\right)}^{3}\,\left(x^{3}+1\right)\,\sqrt{2\,x^{12}+3\,x^{6}+2}}{x^{7}\,\left(x^{6}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^3*(x^3 + 1)*(3*x^6 + 2*x^12 + 2)^(1/2))/(x^7*(x^6 + 1)),x)
```

```
[Out] int(((x^3 - 1)^3*(x^3 + 1)*(3*x^6 + 2*x^12 + 2)^(1/2))/(x^7*(x^6 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)^3 (x+1) \left(x^2-x+1\right) \left(x^2+x+1\right)^3 \sqrt{2x^{12}+3x^6+2}}{x^7 \left(x^2+1\right) \left(x^4-x^2+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**3*(x**3+1)*(2*x**12+3*x**6+2)**(1/2)/x**7/(x**6+1),x)
```

```
[Out] Integral((x - 1)**3*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1)**3*sqrt(2*x**12 +
 3*x**6 + 2)/(x**7*(x**2 + 1)*(x**4 - x**2 + 1)), x)
```

3.1569 $\int \frac{d+cx}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$

Optimal. Leaf size=128

$$\frac{c\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}{6a^2}-\frac{b^2d}{3a\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}+\frac{d\sqrt{\sqrt{a^2x^2+b^2}+ax}}{a}+\frac{b^4c}{10a^2\left(\sqrt{a^2x^2+b^2}+ax\right)^{5/2}}$$

Rubi [A] time = 0.11, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2119, 1628}

$$\frac{c\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}{6a^2}-\frac{b^2d}{3a\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}+\frac{d\sqrt{\sqrt{a^2x^2+b^2}+ax}}{a}+\frac{b^4c}{10a^2\left(\sqrt{a^2x^2+b^2}+ax\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*x)/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] (b^4*c)/(10*a^2*(a*x + Sqrt[b^2 + a^2*x^2])^(5/2)) - (b^2*d)/(3*a*(a*x + Sq
rt[b^2 + a^2*x^2])^(3/2)) + (d*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/a + (c*(a*x
 + Sqrt[b^2 + a^2*x^2])^(3/2))/(6*a^2)
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{d+cx}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx &= \frac{\text{Subst}\left(\int \frac{(b^2+x^2)(-b^2c+2adx+cx^2)}{x^{7/2}}\,dx, x, ax+\sqrt{b^2+a^2x^2}\right)}{4a^2}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{b^4c}{x^{7/2}}+\frac{2ab^2d}{x^{5/2}}+\frac{2ad}{\sqrt{x}}+c\sqrt{x}\right)dx, x, ax+\sqrt{b^2+a^2x^2}\right)}{4a^2}\\ &= \frac{b^4c}{10a^2\left(ax+\sqrt{b^2+a^2x^2}\right)^{5/2}}-\frac{b^2d}{3a\left(ax+\sqrt{b^2+a^2x^2}\right)^{3/2}}+\frac{d\sqrt{ax+\sqrt{b^2+a^2x^2}}}{a}+\end{aligned}$$

Mathematica [B] time = 5.09, size = 1053, normalized size = 8.23

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*x)/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] (2*(b^2 + a^2*x^2)^(3/2)*(2*b^26*c + 41943040*a^25*x^24*(3*d + c*x)*(a*x +
Sqrt[b^2 + a^2*x^2]) + 20971520*a^23*b^2*x^22*(a*x*(37*d + 13*c*x) + 2*(17*
d + 6*c*x)*Sqrt[b^2 + a^2*x^2]) + 1392640*a^13*b^12*x^12*(a*x*(707*d + 387*
c*x) + 14*(26*d + 15*c*x)*Sqrt[b^2 + a^2*x^2]) + 2288*a^5*b^20*x^4*(a*x*(20
5*d + 277*c*x) + 2*(20*d + 33*c*x)*Sqrt[b^2 + a^2*x^2]) + a*b^24*(2*a*x*(65
*d + 302*c*x) + (5*d + 49*c*x)*Sqrt[b^2 + a^2*x^2]) + 40*a^3*b^22*x^2*(a*x*
(361*d + 773*c*x) + 2*(21*d + 62*c*x)*Sqrt[b^2 + a^2*x^2]) + 3342336*a^15*b
^10*x^14*(a*x*(655*d + 317*c*x) + 2*(195*d + 98*c*x)*Sqrt[b^2 + a^2*x^2]) +
 9152*a^7*b^18*x^6*(a*x*(765*d + 749*c*x) + 2*(105*d + 118*c*x)*Sqrt[b^2 +
a^2*x^2]) + 18304*a^9*b^16*x^8*(a*x*(3175*d + 2441*c*x) + 5*(225*d + 191*c*
x)*Sqrt[b^2 + a^2*x^2]) + 524288*a^19*b^6*x^18*(a*x*(6275*d + 2519*c*x) + 1
0*(475*d + 194*c*x)*Sqrt[b^2 + a^2*x^2]) + 1245184*a^17*b^8*x^16*(2*a*x*(13
20*d + 577*c*x) + 3*(595*d + 267*c*x)*Sqrt[b^2 + a^2*x^2]) + 1048576*a^21*b
^4*x^20*(a*x*(2005*d + 749*c*x) + 2*(840*d + 317*c*x)*Sqrt[b^2 + a^2*x^2])
+ 106496*a^11*b^14*x^10*(a*x*(2785*d + 1773*c*x) + 2*(605*d + 414*c*x)*Sqrt
[b^2 + a^2*x^2])))/(15*a^2*(a*x + Sqrt[b^2 + a^2*x^2])^(9/2)*(10*a*b^10*x +
 170*a^3*b^8*x^3 + 832*a^5*b^6*x^5 + 1696*a^7*b^4*x^7 + 1536*a^9*b^2*x^9 +
512*a^11*x^11 + b^10*Sqrt[b^2 + a^2*x^2] + 50*a^2*b^8*x^2*Sqrt[b^2 + a^2*x^
2] + 400*a^4*b^6*x^4*Sqrt[b^2 + a^2*x^2] + 1120*a^6*b^4*x^6*Sqrt[b^2 + a^2*
x^2] + 1280*a^8*b^2*x^8*Sqrt[b^2 + a^2*x^2] + 512*a^10*x^10*Sqrt[b^2 + a^2*
x^2])*(b^2 + a*x*(a*x + Sqrt[b^2 + a^2*x^2]))*(b^10 + 256*a^9*x^9*(a*x + Sq
rt[b^2 + a^2*x^2]) + 40*a^3*b^6*x^3*(7*a*x + 3*Sqrt[b^2 + a^2*x^2]) + 64*a^
7*b^2*x^7*(11*a*x + 9*Sqrt[b^2 + a^2*x^2]) + a*b^8*x*(41*a*x + 9*Sqrt[b^2 +
 a^2*x^2]) + 16*a^5*b^4*x^5*(43*a*x + 27*Sqrt[b^2 + a^2*x^2])))
```

IntegrateAlgebraic [A] time = 0.21, size = 128, normalized size = 1.00

$$\frac{c\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}{6a^2}-\frac{b^2d}{3a\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}+\frac{d\sqrt{\sqrt{a^2x^2+b^2}+ax}}{a}+\frac{b^4c}{10a^2\left(\sqrt{a^2x^2+b^2}+ax\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + c*x)/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] (b^4*c)/(10*a^2*(a*x + Sqrt[b^2 + a^2*x^2])^(5/2)) - (b^2*d)/(3*a*(a*x + Sq
rt[b^2 + a^2*x^2])^(3/2)) + (d*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/a + (c*(a*x
 + Sqrt[b^2 + a^2*x^2])^(3/2))/(6*a^2)
```

fricas [A] time = 0.40, size = 98, normalized size = 0.77

$$-\frac{2\left(3\,a^3cx^3+5\,a^3dx^2+ab^2cx-5\,ab^2d-\left(3\,a^2cx^2+5\,a^2dx+2\,b^2c\right)\sqrt{a^2x^2+b^2}\right)\sqrt{ax+\sqrt{a^2x^2+b^2}}}{15\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*a^3*c*x^3 + 5*a^3*d*x^2 + a*b^2*c*x - 5*a*b^2*d - (3*a^2*c*x^2 + 5
*a^2*d*x + 2*b^2*c)*sqrt(a^2*x^2 + b^2))*sqrt(a*x + sqrt(a^2*x^2 + b^2))/(a
^2*b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx+d}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate((c*x + d)/sqrt(a*x + sqrt(a^2*x^2 + b^2)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{cx+d}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x+d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
[Out] int((c*x+d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx+d}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate((c*x + d)/sqrt(a*x + sqrt(a^2*x^2 + b^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d+c\,x}{\sqrt{a\,x+\sqrt{a^2\,x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + c*x)/(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2),x)
[Out] int((d + c*x)/(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx+d}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+d)/(a*x+(a**2*x**2+b**2)**(1/2))**(1/2),x)
[Out] Integral((c*x + d)/sqrt(a*x + sqrt(a**2*x**2 + b**2)), x)
```

3.1570 $\int \frac{1}{\sqrt{1+\sqrt{1+\sqrt{1+x}}}}\,dx$

Optimal. Leaf size=128

$$\sqrt{x+1}\left(\frac{8}{7}\sqrt{\sqrt{x+1}+1}\sqrt{\sqrt{\sqrt{x+1}+1}+1}-\frac{48}{35}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)+\frac{32}{105}\sqrt{\sqrt{x+1}+1}\sqrt{\sqrt{\sqrt{x+1}+1}+1}$$

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 0.55, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{8}{7}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{7/2}-\frac{24}{5}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{5/2}+\frac{16}{3}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]],x]
```

```
[Out] (16*(1 + Sqrt[1 + Sqrt[1 + x]])^(3/2))/3 - (24*(1 + Sqrt[1 + Sqrt[1 + x]])^
(5/2))/5 + (8*(1 + Sqrt[1 + Sqrt[1 + x]])^(7/2))/7
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\sqrt{1+\sqrt{1+x}}}}\,dx = 2\,\text{Subst}\left(\int \frac{x}{\sqrt{1+\sqrt{1+x}}}\,dx, x, \sqrt{1+x}\right)$$
$$= 2\,\text{Subst}\left(\int \frac{-1+x}{\sqrt{1+\sqrt{x}}}\,dx, x, 1+\sqrt{1+x}\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x\left(-1+x^2\right)}{\sqrt{1+x}}\,dx, x, \sqrt{1+\sqrt{1+x}}\right)$$
$$= 4\,\text{Subst}\left(\int \left(2\sqrt{1+x}-3(1+x)^{3/2}+(1+x)^{5/2}\right)dx, x, \sqrt{1+\sqrt{1+x}}\right)$$
$$= \frac{16}{3}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{3/2}-\frac{24}{5}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{5/2}+\frac{8}{7}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{7}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.38

$$\frac{8}{105}\left(15\sqrt{x+1}-33\sqrt{\sqrt{x+1}+1}+37\right)\left(\sqrt{\sqrt{x+1}+1}+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]],x]
```

```
[Out] (8*(37 + 15*Sqrt[1 + x] - 33*Sqrt[1 + Sqrt[1 + x]])*(1 + Sqrt[1 + Sqrt[1 +
x]])^(3/2))/105
```

IntegrateAlgebraic [A] time = 0.07, size = 82, normalized size = 0.64

$$\frac{8}{105}\sqrt{\sqrt{x+1}+1}\left(15\sqrt{x+1}+4\right)\sqrt{\sqrt{\sqrt{x+1}+1}+1}-\frac{16}{105}\left(9\sqrt{x+1}-2\right)\sqrt{\sqrt{\sqrt{x+1}+1}+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]],x]
```

```
[Out] (-16*(-2 + 9*Sqrt[1 + x])*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]])/105 + (8*Sqrt[1
+ Sqrt[1 + x]]*(4 + 15*Sqrt[1 + x])*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]])/105
```

fricas [A] time = 0.40, size = 43, normalized size = 0.34

$$\frac{8}{105}\left(\left(15\sqrt{x+1}+4\right)\sqrt{\sqrt{x+1}+1}-18\sqrt{x+1}+4\right)\sqrt{\sqrt{\sqrt{x+1}+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+(1+x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/105*((15*sqrt(x + 1) + 4)*sqrt(sqrt(x + 1) + 1) - 18*sqrt(x + 1) + 4)*sqr
t(sqrt(sqrt(x + 1) + 1) + 1)
```

giac [C] time = 1.32, size = 79, normalized size = 0.62

$$\frac{8\left(15\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{7}{2}}-63\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{5}{2}}+70\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{3}{2}}\right)}{105\,\text{sgn}\left(4\left(\sqrt{x+1}+1\right)^{2}-8\sqrt{x+1}-7\right)\text{sgn}\left(4x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+(1+x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/105*(15*(sqrt(sqrt(x + 1) + 1) + 1)^(7/2) - 63*(sqrt(sqrt(x + 1) + 1) + 1
)^(5/2) + 70*(sqrt(sqrt(x + 1) + 1) + 1)^(3/2))/(sgn(4*(sqrt(x + 1) + 1)^2
- 8*sqrt(x + 1) - 7)*sgn(4*x + 1))
```

maple [A] time = 0.01, size = 47, normalized size = 0.37

$$\frac{8\left(1+\sqrt{1+\sqrt{1+x}}\right)^{\frac{7}{2}}}{7}-\frac{24\left(1+\sqrt{1+\sqrt{1+x}}\right)^{\frac{5}{2}}}{5}+\frac{16\left(1+\sqrt{1+\sqrt{1+x}}\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(1+(1+x)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 8/7*(1+(1+(1+x)^(1/2))^(1/2))^(7/2)-24/5*(1+(1+(1+x)^(1/2))^(1/2))^(5/2)+16
/3*(1+(1+(1+x)^(1/2))^(1/2))^(3/2)
```

maxima [C] time = 0.32, size = 46, normalized size = 0.36

$$\frac{8}{7}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{7}{2}}-\frac{24}{5}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{5}{2}}+\frac{16}{3}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+(1+x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/7*(sqrt(sqrt(x + 1) + 1) + 1)^(7/2) - 24/5*(sqrt(sqrt(x + 1) + 1) + 1)^(5
/2) + 16/3*(sqrt(sqrt(x + 1) + 1) + 1)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x+1}+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x + 1)^(1/2) + 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int(1/(((x + 1)^(1/2) + 1)^(1/2) + 1)^(1/2), x)
```

sympy [B] time = 1.72, size = 445, normalized size = 3.48

$$-\frac{336(x+1)^{\frac{13}{4}}\sqrt[4]{\sqrt{x+1}+1}\sin\left(\frac{5\operatorname{atan}\left(\sqrt[4]{x+1}\right)}{2}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{105\pi(x+1)^{\frac{5}{2}}+105\pi(x+1)^2}-\frac{112(x+1)^{\frac{11}{4}}\sqrt[4]{\sqrt{x+1}+1}\sin\left(\frac{5\operatorname{atan}\left(\sqrt[4]{x+1}\right)}{2}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{105\pi(x+1)^{\frac{5}{2}}+105\pi(x+1)^2}+\frac{224(x+1)^{\frac{9}{4}}\sqrt[4]{\sqrt{x+1}+1}\sin\left(\frac{5\operatorname{atan}\left(\sqrt[4]{x+1}\right)}{2}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{105\pi(x+1)^{\frac{5}{2}}+105\pi(x+1)^2}-\frac{152(x+1)^{\frac{5}{2}}\left(\sqrt{x+1}+1\right)^{\frac{3}{4}}\cos\left(\frac{7\operatorname{atan}\left(\sqrt[4]{x+1}\right)}{2}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{105\pi(x+1)^{\frac{5}{2}}+105\pi(x+1)^2}-\frac{64(x+1)^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{105\pi(x+1)^{\frac{5}{2}}+105\pi(x+1)^2}-\frac{216(x+1)^3\left(\sqrt{x+1}+1\right)^{\frac{3}{4}}\cos\left(\frac{7\operatorname{atan}\left(\sqrt[4]{x+1}\right)}{2}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{105\pi(x+1)^{\frac{5}{2}}+105\pi(x+1)^2}+\frac{64(x+1)^2\left(\sqrt{x+1}+1\right)^{\frac{3}{4}}\cos\left(\frac{7\operatorname{atan}\left(\sqrt[4]{x+1}\right)}{2}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{105\pi(x+1)^{\frac{5}{2}}+105\pi(x+1)^2}-\frac{64(x+1)^2\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{105\pi(x+1)^{\frac{5}{2}}+105\pi(x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+(1+x)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] -336*(x + 1)**(13/4)*(sqrt(x + 1) + 1)**(1/4)*sin(5*atan((x + 1)**(1/4))/2)
*gamma(1/4)*gamma(3/4)/(105*pi*(x + 1)**(5/2) + 105*pi*(x + 1)**2) - 112*(x
 + 1)**(11/4)*(sqrt(x + 1) + 1)**(1/4)*sin(5*atan((x + 1)**(1/4))/2)*gamma(
1/4)*gamma(3/4)/(105*pi*(x + 1)**(5/2) + 105*pi*(x + 1)**2) + 224*(x + 1)**
(9/4)*(sqrt(x + 1) + 1)**(1/4)*sin(5*atan((x + 1)**(1/4))/2)*gamma(1/4)*gam
ma(3/4)/(105*pi*(x + 1)**(5/2) + 105*pi*(x + 1)**2) - 152*(x + 1)**(5/2)*(s
qrt(x + 1) + 1)**(3/4)*cos(7*atan((x + 1)**(1/4))/2)*gamma(1/4)*gamma(3/4)/
(105*pi*(x + 1)**(5/2) + 105*pi*(x + 1)**2) - 64*(x + 1)**(5/2)*gamma(1/4)*
gamma(3/4)/(105*pi*(x + 1)**(5/2) + 105*pi*(x + 1)**2) - 216*(x + 1)**3*(sq
```

```
rt(x + 1) + 1)**(3/4)*cos(7*atan((x + 1)**(1/4))/2)*gamma(1/4)*gamma(3/4)/(
105*pi*(x + 1)**(5/2) + 105*pi*(x + 1)**2) + 64*(x + 1)**2*(sqrt(x + 1) + 1
)**(3/4)*cos(7*atan((x + 1)**(1/4))/2)*gamma(1/4)*gamma(3/4)/(105*pi*(x + 1
)**(5/2) + 105*pi*(x + 1)**2) - 64*(x + 1)**2*gamma(1/4)*gamma(3/4)/(105*pi
*(x + 1)**(5/2) + 105*pi*(x + 1)**2)
```

3.1571 $\int \frac{\left(-b+ax^2\right)\sqrt[3]{x+x^3}}{x^2}\,dx$

Optimal. Leaf size=129

$$\frac{1}{6}(3b-a)\log\left(\sqrt[3]{x^3+x}-x\right)+\frac{1}{6}\left(3\sqrt{3}\,b-\sqrt{3}\,a\right)\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3+x}+x}\right)+\frac{\sqrt[3]{x^3+x}\left(ax^2+3b\right)}{2x}+\frac{1}{12}(a-3b)\log\left(\sqrt[3]{x^3\right.$$

Rubi [A] time = 0.21, antiderivative size = 215, normalized size of antiderivative = 1.67, number of steps used = 12, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2038, 2004, 2032, 329, 275, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{2}x\sqrt[3]{x^3+x}\,(a-3b)-\frac{x^{2/3}\left(x^2+1\right)^{2/3}(a-3b)\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2+1}}\right)}{6\left(x^3+x\right)^{2/3}}+\frac{x^{2/3}\left(x^2+1\right)^{2/3}(a-3b)\log\left(\frac{x^{4/3}}{\left(x^2+1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2+1}}+1\right)}{12\left(x^3+x\right)^{2/3}}-\frac{x^{2/3}\left(x^2+1\right)^{2/3}(a-3b)\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)}{2\sqrt{3}\left(x^3+x\right)^{2/3}}+\frac{3b\left(x^3+x\right)^{4/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a*x^2)*(x + x^3)^(1/3))/x^2,x]
```

```
[Out] ((a - 3*b)*x*(x + x^3)^(1/3))/2 + (3*b*(x + x^3)^(4/3))/(2*x^2) - ((a - 3*b
)*x^(2/3)*(1 + x^2)^(2/3)*ArcTan[(1 + (2*x^(2/3))/(1 + x^2)^(1/3))/Sqrt[3]]
)/(2*Sqrt[3]*(x + x^3)^(2/3)) - ((a - 3*b)*x^(2/3)*(1 + x^2)^(2/3)*Log[1 -
x^(2/3)/(1 + x^2)^(1/3)])/(6*(x + x^3)^(2/3)) + ((a - 3*b)*x^(2/3)*(1 + x^2
)^(2/3)*Log[1 + x^(4/3)/(1 + x^2)^(2/3) + x^(2/3)/(1 + x^2)^(1/3)])/(12*(x
+ x^3)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{\left(-b+ax^2\right)\sqrt[3]{x+x^3}}{x^2}\,dx = \frac{3b\left(x+x^3\right)^{4/3}}{2x^2} - (-a+3b)\int \sqrt[3]{x+x^3}\,dx$$
$$= \frac{1}{2}(a-3b)x\sqrt[3]{x+x^3} + \frac{3b\left(x+x^3\right)^{4/3}}{2x^2} - \frac{1}{3}(-a+3b)\int \frac{x}{\left(x+x^3\right)^{2/3}}\,dx$$
$$= \frac{1}{2}(a-3b)x\sqrt[3]{x+x^3} + \frac{3b\left(x+x^3\right)^{4/3}}{2x^2} - \frac{\left((-a+3b)x^{2/3}\left(1+x^2\right)^{2/3}\right)\int \frac{\sqrt[3]{x}}{\left(1+x^2\right)^{2/3}}\,dx}{3\left(x+x^3\right)^{2/3}}$$
$$= \frac{1}{2}(a-3b)x\sqrt[3]{x+x^3} + \frac{3b\left(x+x^3\right)^{4/3}}{2x^2} - \frac{\left((-a+3b)x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x^3}{\left(1+x^6\right)^{2/3}}\right.}{\left(x+x^3\right)^{2/3}}$$
$$= \frac{1}{2}(a-3b)x\sqrt[3]{x+x^3} + \frac{3b\left(x+x^3\right)^{4/3}}{2x^2} - \frac{\left((-a+3b)x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{\left(1+x^3\right)^{2/3}}\right.}{2\left(x+x^3\right)^{2/3}}$$
$$= \frac{1}{2}(a-3b)x\sqrt[3]{x+x^3} + \frac{3b\left(x+x^3\right)^{4/3}}{2x^2} - \frac{\left((-a+3b)x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{1-x^3}\,dx,\right.}{2\left(x+x^3\right)^{2/3}}$$
$$= \frac{1}{2}(a-3b)x\sqrt[3]{x+x^3} + \frac{3b\left(x+x^3\right)^{4/3}}{2x^2} - \frac{\left((-a+3b)x^{2/3}\left(1+x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-x}\,dx,\right.}{6\left(x+x^3\right)^{2/3}}$$
$$= \frac{1}{2}(a-3b)x\sqrt[3]{x+x^3} + \frac{3b\left(x+x^3\right)^{4/3}}{2x^2} - \frac{(a-3b)x^{2/3}\left(1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{6\left(x+x^3\right)^{2/3}} - \frac{((}{}$$
$$= \frac{1}{2}(a-3b)x\sqrt[3]{x+x^3} + \frac{3b\left(x+x^3\right)^{4/3}}{2x^2} - \frac{(a-3b)x^{2/3}\left(1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{6\left(x+x^3\right)^{2/3}} + \frac{(a}{}$$
$$= \frac{1}{2}(a-3b)x\sqrt[3]{x+x^3} + \frac{3b\left(x+x^3\right)^{4/3}}{2x^2} - \frac{(a-3b)x^{2/3}\left(1+x^2\right)^{2/3}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\left(x+x^3\right)^{2/3}} - \frac{(a}{}$$

Mathematica [C] time = 0.02, size = 62, normalized size = 0.48

$$\frac{3\sqrt[3]{x^3+x}\left(x^2(a-3b)\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-x^2\right)+2b\left(x^2+1\right)^{4/3}\right)}{4x\sqrt[3]{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + a*x^2)*(x + x^3)^(1/3))/x^2,x]
```

```
[Out] (3*(x + x^3)^(1/3)*(2*b*(1 + x^2)^(4/3) + (a - 3*b)*x^2*Hypergeometric2F1[-
1/3, 2/3, 5/3, -x^2]))/(4*x*(1 + x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.34, size = 129, normalized size = 1.00

$$\frac{1}{6}(3b-a)\log\left(\sqrt[3]{x^3+x}-x\right)+\frac{1}{6}\left(3\sqrt{3}\,b-\sqrt{3}\,a\right)\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3+x}+x}\right)+\frac{\sqrt[3]{x^3+x}\left(ax^2+3b\right)}{2x}+\frac{1}{12}(a-3b)\log\left(\sqrt[3]{x^3+x}\,x+\left(x^3+x\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((-b + a*x^2)*(x + x^3)^(1/3))/x^2,x]

[Out] ((3*b + a*x^2)*(x + x^3)^(1/3))/(2*x) + ((-(Sqrt[3]*a) + 3*Sqrt[3]*b)*ArcTan[(Sqrt[3]*x)/(x + 2*(x + x^3)^(1/3))])/6 + ((-a + 3*b)*Log[-x + (x + x^3)^(1/3)])/6 + ((a - 3*b)*Log[x^2 + x*(x + x^3)^(1/3) + (x + x^3)^(2/3)])/12

fricas [A] time = 69.45, size = 114, normalized size = 0.88

$$-\frac{2\sqrt{3}(a-3b)x\arctan\left(-\frac{196\sqrt{3}(x^3+x)^{\frac{1}{3}}x-\sqrt{3}(539x^2+507)-1274\sqrt{3}(x^3+x)^{\frac{2}{3}}}{2205x^2+2197}\right)+(a-3b)x\log\left(3(x^3+x)^{\frac{1}{3}}x-3(x^3+x)^{\frac{2}{3}}+1\right)-6(ax^2+3b)(x^3+x)^{\frac{1}{3}}}{12x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^2-b)*(x^3+x)^(1/3)/x^2,x, algorithm="fricas")

[Out] -1/12*(2*sqrt(3)*(a - 3*b)*x*arctan(-(196*sqrt(3)*(x^3 + x)^(1/3)*x - sqrt(3)*(539*x^2 + 507) - 1274*sqrt(3)*(x^3 + x)^(2/3))/(2205*x^2 + 2197)) + (a - 3*b)*x*log(3*(x^3 + x)^(1/3)*x - 3*(x^3 + x)^(2/3) + 1) - 6*(a*x^2 + 3*b)*(x^3 + x)^(1/3))/x

giac [A] time = 0.48, size = 93, normalized size = 0.72

$$\frac{1}{2}ax^2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+\frac{1}{6}\sqrt{3}(a-3b)\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{12}(a-3b)\log\left(\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{6}(a-3b)\log\left(\left|\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)+\frac{3}{2}b\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^2-b)*(x^3+x)^(1/3)/x^2,x, algorithm="giac")

[Out] 1/2*a*x^2*(1/x^2 + 1)^(1/3) + 1/6*sqrt(3)*(a - 3*b)*arctan(1/3*sqrt(3)*(2*(1/x^2 + 1)^(1/3) + 1)) + 1/12*(a - 3*b)*log((1/x^2 + 1)^(2/3) + (1/x^2 + 1)^(1/3) + 1) - 1/6*(a - 3*b)*log(abs((1/x^2 + 1)^(1/3) - 1)) + 3/2*b*(1/x^2 + 1)^(1/3)

maple [C] time = 1.94, size = 512, normalized size = 3.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2-b)*(x^3+x)^(1/3)/x^2,x)

[Out] 1/2*(a*x^2+3*b)*(x*(x^2+1))^(1/3)/x+1/12*(a-3*b)*(RootOf(_Z^2-2*_Z+4)*ln((-RootOf(_Z^2-2*_Z+4)^2*x^4+20*RootOf(_Z^2-2*_Z+4)*x^4-48*(x^6+2*x^4+x^2)^(1/3)*RootOf(_Z^2-2*_Z+4)*x^2-100*x^4+30*RootOf(_Z^2-2*_Z+4)*(x^6+2*x^4+x^2)^(2/3)+60*(x^6+2*x^4+x^2)^(1/3)*x^2+14*RootOf(_Z^2-2*_Z+4)*x^2+36*(x^6+2*x^4+x^2)^(2/3)-48*RootOf(_Z^2-2*_Z+4)*(x^6+2*x^4+x^2)^(1/3)+RootOf(_Z^2-2*_Z+4)^2-140*x^2+60*(x^6+2*x^4+x^2)^(1/3)-6*RootOf(_Z^2-2*_Z+4)-40)/(x^2+1))-2*ln(-(-5*RootOf(_Z^2-2*_Z+4)^2*x^4-38*RootOf(_Z^2-2*_Z+4)*x^4+18*(x^6+2*x^4+x^2)^(1/3)*RootOf(_Z^2-2*_Z+4)*x^2+16*x^4+30*RootOf(_Z^2-2*_Z+4)*(x^6+2*x^4+x^2)^(2/3)+60*(x^6+2*x^4+x^2)^(1/3)*x^2-70*RootOf(_Z^2-2*_Z+4)*x^2-96*(x^6+2*x^4+x^2)^(2/3)+18*RootOf(_Z^2-2*_Z+4)*(x^6+2*x^4+x^2)^(1/3)+5*RootOf(_Z^2-2*_Z+4)^2+28*x^2+60*(x^6+2*x^4+x^2)^(1/3)-32*RootOf(_Z^2-2*_Z+4)+12)/(x^2+1)))*(x*(x^2+1))^(1/3)/x*(x^2*(x^2+1)^2)^(1/3)/(x^2+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ax^2-b)(x^3+x)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(x^3+x)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 - b)*(x^3 + x)^(1/3)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(b-a\,x^2\right)\,{\left(x^3+x\right)}^{1/3}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b - a*x^2)*(x + x^3)^(1/3))/x^2,x)
```

```
[Out] -int(((b - a*x^2)*(x + x^3)^(1/3))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x \left(x^{2} + 1\right)} \left(a x^{2} - b\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)*(x**3+x)**(1/3)/x**2,x)
```

```
[Out] Integral((x*(x**2 + 1))**(1/3)*(a*x**2 - b)/x**2, x)
```

3.1572 $\int \frac{-1+x^4}{\sqrt{-x-x^2+x^3}\left(1+x^4\right)}\,dx$

Optimal. Leaf size=129

$$-\sqrt{\frac{1}{3}\left(1+i\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{1-i\sqrt{2}}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)-\sqrt{\frac{1}{3}\left(1-i\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{1+i\sqrt{2}}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)$$

Rubi [C] time = 1.98, antiderivative size = 625, normalized size of antiderivative = 4.84, number of steps used = 27, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2056, 6725, 716, 1098, 934, 168, 538, 537}

$$\frac{\sqrt{x}\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}\sqrt{\frac{\left(1+\sqrt{5}\right)x+2}{\left(1-\sqrt{5}\right)x+2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{\left(1-\sqrt{5}\right)x+2}}\sqrt{x^3-x^2-x}}-\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(-\frac{1}{2}\sqrt[4]{-1}\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{2\sqrt{x^3-x^2-x}}-\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(\frac{1}{2}\sqrt[4]{-1}\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{2\sqrt{x^3-x^2-x}}-\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(-\frac{1}{2}(-1)^{3/4}\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{2\sqrt{x^3-x^2-x}}-\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(\frac{1}{2}(-1)^{3/4}\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{2\sqrt{x^3-x^2-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^4)/(Sqrt[-x - x^2 + x^3]*(1 + x^4)),x]
```

```
[Out] (Sqrt[x]*Sqrt[-2 - (1 - Sqrt[5])*x]*Sqrt[(2 + (1 + Sqrt[5])*x)/(2 + (1 - Sq
rt[5])*x)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*Sqrt[x])/Sqrt[-2 - (1 - Sqrt[5
])*x]], (5 - Sqrt[5])/10])/(5^(1/4)*Sqrt[(2 + (1 - Sqrt[5])*x)^(-1)]*Sqrt[-
x - x^2 + x^3]) - (Sqrt[3 + Sqrt[5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[
1 - (2*x)/(1 + Sqrt[5])]*EllipticPi[-1/2*((-1)^(1/4)*(1 + Sqrt[5])), ArcSin
[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]], (-3 - Sqrt[5])/2])/(2*Sqrt[-x - x^2 + x^3]
) - (Sqrt[3 + Sqrt[5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1 +
 Sqrt[5])]*EllipticPi[((-1)^(1/4)*(1 + Sqrt[5]))/2, ArcSin[Sqrt[2/(1 + Sqrt
[5])]*Sqrt[x]], (-3 - Sqrt[5])/2])/(2*Sqrt[-x - x^2 + x^3]) - (Sqrt[3 + Sqr
t[5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1 + Sqrt[5])]*Ellipt
icPi[-1/2*((-1)^(3/4)*(1 + Sqrt[5])), ArcSin[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]]
, (-3 - Sqrt[5])/2])/(2*Sqrt[-x - x^2 + x^3]) - (Sqrt[3 + Sqrt[5]]*Sqrt[x]*
Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1 + Sqrt[5])]*EllipticPi[((-1)^(3/
4)*(1 + Sqrt[5]))/2, ArcSin[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]], (-3 - Sqrt[5])/
2])/(2*Sqrt[-x - x^2 + x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 716

```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2,
Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+x^4}{\sqrt{-x-x^2+x^3}\left(1+x^4\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{-1+x^4}{\sqrt{x}\sqrt{-1-x+x^2}\left(1+x^4\right)}\,dx}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{-1-x+x^2}} - \frac{2}{\sqrt{x}\sqrt{-1-x+x^2}\left(1+x^4\right)}\right)dx}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-1-x+x^2}}\,dx}{\sqrt{-x-x^2+x^3}} - \frac{\left(2\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-1-x+x^2}\,(1}}{\sqrt{-x-x^2+x^3}}$$

$$= -\frac{\left(2\sqrt{x}\sqrt{-1-x+x^2}\right)\int \left(\frac{i}{2\sqrt{x}\left(i-x^2\right)\sqrt{-1-x+x^2}} + \frac{i}{2\sqrt{x}\left(i+x^2\right)\sqrt{-1-x+x^2}}\right)dx}{\sqrt{-x-x^2+x^3}} + \frac{\left(2\sqrt{\right.}}{}$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} -$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} -$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} -$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} -$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} +$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} +$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} -$$

Mathematica [C] time = 1.43, size = 329, normalized size = 2.55

$$-\frac{i\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{-\frac{1}{x^2}-\frac{1}{x}+1}\,x^{3/2}\left(2F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{2}{1+\sqrt{5}}}}{\sqrt{x}}\right)|-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-\Pi\left(-\frac{1}{2}\sqrt[4]{-1}\left(1+\sqrt{5}\right);i\sinh^{-1}\left(\frac{\sqrt{\frac{2}{1+\sqrt{5}}}}{\sqrt{x}}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)-\Pi\left(\frac{1}{2}\sqrt[4]{-1}\left(1+\sqrt{5}\right);i\sinh^{-1}\left(\frac{\sqrt{\frac{2}{1+\sqrt{5}}}}{\sqrt{x}}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)-\Pi\left(-\frac{1}{2}(-1)^{3/4}\left(1+\sqrt{5}\right);i\sinh^{-1}\left(\frac{\sqrt{\frac{2}{1+\sqrt{5}}}}{\sqrt{x}}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)-\Pi\left(\frac{1}{2}(-1)^{3/4}\left(1+\sqrt{5}\right);i\sinh^{-1}\left(\frac{\sqrt{\frac{2}{1+\sqrt{5}}}}{\sqrt{x}}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)\right)}{\sqrt{x\left(x^2-x-1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + x^4)/(Sqrt[-x - x^2 + x^3]*(1 + x^4)),x]
```

```
[Out] ((-I)*Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 - x^(-2) - x^(-1)]*x^(3/2)*(2*EllipticF
[I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], -3/2 - Sqrt[5]/2] - EllipticPi[-
1/2*((-1)^(1/4)*(1 + Sqrt[5])), I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], (
-3 - Sqrt[5])/2] - EllipticPi[((-1)^(1/4)*(1 + Sqrt[5]))/2, I*ArcSinh[Sqrt[
2/(1 + Sqrt[5])]/Sqrt[x]], (-3 - Sqrt[5])/2] - EllipticPi[-1/2*((-1)^(3/4)*
(1 + Sqrt[5])), I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], (-3 - Sqrt[5])/2]
 - EllipticPi[((-1)^(3/4)*(1 + Sqrt[5]))/2, I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]
/Sqrt[x]], (-3 - Sqrt[5])/2]))/Sqrt[x*(-1 - x + x^2)]
```

IntegrateAlgebraic [A] time = 0.45, size = 129, normalized size = 1.00

$$-\sqrt{\frac{1}{3}\left(1+i\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{1-i\sqrt{2}}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)-\sqrt{\frac{1}{3}\left(1-i\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{1+i\sqrt{2}}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/(Sqrt[-x - x^2 + x^3]*(1 + x^4)),x]
```

```
[Out] -(Sqrt[(1 + I*Sqrt[2])/3]*ArcTan[(Sqrt[1 - I*Sqrt[2]]*Sqrt[-x - x^2 + x^3])
/(-1 - x + x^2)]) - Sqrt[(1 - I*Sqrt[2])/3]*ArcTan[(Sqrt[1 + I*Sqrt[2]]*Sqr
t[-x - x^2 + x^3])/(-1 - x + x^2)]
```

fricas [B] time = 0.83, size = 2368, normalized size = 18.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^3-x^2-x)^(1/2)/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*3^(1/4)*sqrt(sqrt(3) + 3)*(sqrt(3) - 1)*log(3*(3*x^4 - 12*x^3 + 2*3^(
1/4)*sqrt(x^3 - x^2 - x)*(3*x^2 - sqrt(3)*(x^2 - 4*x - 1) - 6*x - 3)*sqrt(s
qrt(3) + 3) + 12*x^2 + 12*sqrt(3)*(x^3 - x^2 - x) + 12*x + 3)/(x^4 + 1)) +
1/24*3^(1/4)*sqrt(sqrt(3) + 3)*(sqrt(3) - 1)*log(3*(3*x^4 - 12*x^3 - 2*3^(1
/4)*sqrt(x^3 - x^2 - x)*(3*x^2 - sqrt(3)*(x^2 - 4*x - 1) - 6*x - 3)*sqrt(sq
rt(3) + 3) + 12*x^2 + 12*sqrt(3)*(x^3 - x^2 - x) + 12*x + 3)/(x^4 + 1)) - 1
/6*3^(1/4)*sqrt(2)*sqrt(sqrt(3) + 3)*arctan(-1/36*(18*sqrt(3)*sqrt(2)*(x^11
 - 3*x^10 - 3*x^9 + 8*x^8 + 4*x^7 - 6*x^6 - 4*x^5 + 8*x^4 + 3*x^3 - 3*x^2 -
 x) + 3*sqrt(x^3 - x^2 - x)*(3^(3/4)*(sqrt(3)*sqrt(2)*(x^10 - 4*x^9 - 9*x^8
 + 36*x^7 + 26*x^6 - 72*x^5 - 26*x^4 + 36*x^3 + 9*x^2 - 4*x - 1) - sqrt(2)*
(x^10 + 2*x^9 - 29*x^8 + 12*x^7 + 110*x^6 - 28*x^5 - 110*x^4 + 12*x^3 + 29*
x^2 + 2*x - 1)) + 4*3^(1/4)*(sqrt(3)*sqrt(2)*(2*x^9 + 3*x^8 - 5*x^7 - 23*x^
6 + 12*x^5 + 23*x^4 - 5*x^3 - 3*x^2 + 2*x) - 3*sqrt(2)*(x^9 + 4*x^8 - 11*x^
7 - 8*x^6 + 18*x^5 + 8*x^4 - 11*x^3 - 4*x^2 + x)))*sqrt(sqrt(3) + 3) - sqrt
(3)*(24*sqrt(3)*sqrt(2)*(x^10 + 2*x^9 - 13*x^8 + 2*x^7 + 22*x^6 - 2*x^5 - 1
3*x^4 - 2*x^3 + x^2) + sqrt(x^3 - x^2 - x)*(3^(3/4)*(sqrt(3)*sqrt(2)*(x^10
- 6*x^9 - 9*x^8 + 68*x^7 - 38*x^6 - 108*x^5 + 38*x^4 + 68*x^3 + 9*x^2 - 6*x
 - 1) - sqrt(2)*(x^10 - 4*x^9 - 5*x^8 + 84*x^7 - 106*x^6 - 136*x^5 + 106*x^
4 + 84*x^3 + 5*x^2 - 4*x - 1)) + 4*3^(1/4)*(sqrt(3)*sqrt(2)*(2*x^9 - 3*x^8
- 23*x^7 + 31*x^6 + 36*x^5 - 31*x^4 - 23*x^3 + 3*x^2 + 2*x) - 3*sqrt(2)*(x^
9 - 15*x^7 + 12*x^6 + 26*x^5 - 12*x^4 - 15*x^3 + x)))*sqrt(sqrt(3) + 3) - 4
8*sqrt(2)*(x^10 - 7*x^8 + 2*x^7 + 12*x^6 - 2*x^5 - 7*x^4 + x^2) + 2*sqrt(3)
*(sqrt(3)*sqrt(2)*(x^11 - x^10 - 17*x^9 + 48*x^8 + 2*x^7 - 82*x^6 - 2*x^5 +
 48*x^4 + 17*x^3 - x^2 - x) - 3*sqrt(2)*(x^11 - 5*x^10 - 9*x^9 + 52*x^8 - 6
*x^7 - 90*x^6 + 6*x^5 + 52*x^4 + 9*x^3 - 5*x^2 - x)))*sqrt((3*x^4 - 12*x^3
+ 2*3^(1/4)*sqrt(x^3 - x^2 - x)*(3*x^2 - sqrt(3)*(x^2 - 4*x - 1) - 6*x - 3)
*sqrt(sqrt(3) + 3) + 12*x^2 + 12*sqrt(3)*(x^3 - x^2 - x) + 12*x + 3)/(x^4 +
 1)) - 18*sqrt(2)*(x^11 + 5*x^10 - 35*x^9 + 116*x^7 - 22*x^6 - 116*x^5 + 35
```

```
*x^3 + 5*x^2 - x) + 6*sqrt(3)*(sqrt(3)*sqrt(2)*(x^11 + 5*x^10 - 35*x^9 + 11
6*x^7 - 22*x^6 - 116*x^5 + 35*x^3 + 5*x^2 - x) - 3*sqrt(2)*(x^11 - 3*x^10 -
 3*x^9 + 8*x^8 + 4*x^7 - 6*x^6 - 4*x^5 + 8*x^4 + 3*x^3 - 3*x^2 - x)))/(x^11
 - 9*x^10 - 17*x^9 + 104*x^8 - 14*x^7 - 178*x^6 + 14*x^5 + 104*x^4 + 17*x^3
 - 9*x^2 - x)) - 1/6*3^(1/4)*sqrt(2)*sqrt(sqrt(3) + 3)*arctan(1/36*(18*sqrt
(3)*sqrt(2)*(x^11 - 3*x^10 - 3*x^9 + 8*x^8 + 4*x^7 - 6*x^6 - 4*x^5 + 8*x^4
+ 3*x^3 - 3*x^2 - x) - 3*sqrt(x^3 - x^2 - x)*(3^(3/4)*(sqrt(3)*sqrt(2)*(x^1
0 - 4*x^9 - 9*x^8 + 36*x^7 + 26*x^6 - 72*x^5 - 26*x^4 + 36*x^3 + 9*x^2 - 4*
x - 1) - sqrt(2)*(x^10 + 2*x^9 - 29*x^8 + 12*x^7 + 110*x^6 - 28*x^5 - 110*x
^4 + 12*x^3 + 29*x^2 + 2*x - 1)) + 4*3^(1/4)*(sqrt(3)*sqrt(2)*(2*x^9 + 3*x^
8 - 5*x^7 - 23*x^6 + 12*x^5 + 23*x^4 - 5*x^3 - 3*x^2 + 2*x) - 3*sqrt(2)*(x^
9 + 4*x^8 - 11*x^7 - 8*x^6 + 18*x^5 + 8*x^4 - 11*x^3 - 4*x^2 + x)))*sqrt(sq
rt(3) + 3) - sqrt(3)*(24*sqrt(3)*sqrt(2)*(x^10 + 2*x^9 - 13*x^8 + 2*x^7 + 2
2*x^6 - 2*x^5 - 13*x^4 - 2*x^3 + x^2) - sqrt(x^3 - x^2 - x)*(3^(3/4)*(sqrt(
3)*sqrt(2)*(x^10 - 6*x^9 - 9*x^8 + 68*x^7 - 38*x^6 - 108*x^5 + 38*x^4 + 68*
x^3 + 9*x^2 - 6*x - 1) - sqrt(2)*(x^10 - 4*x^9 - 5*x^8 + 84*x^7 - 106*x^6 -
 136*x^5 + 106*x^4 + 84*x^3 + 5*x^2 - 4*x - 1)) + 4*3^(1/4)*(sqrt(3)*sqrt(2
)*(2*x^9 - 3*x^8 - 23*x^7 + 31*x^6 + 36*x^5 - 31*x^4 - 23*x^3 + 3*x^2 + 2*x
) - 3*sqrt(2)*(x^9 - 15*x^7 + 12*x^6 + 26*x^5 - 12*x^4 - 15*x^3 + x)))*sqrt
(sqrt(3) + 3) - 48*sqrt(2)*(x^10 - 7*x^8 + 2*x^7 + 12*x^6 - 2*x^5 - 7*x^4 +
 x^2) + 2*sqrt(3)*(sqrt(3)*sqrt(2)*(x^11 - x^10 - 17*x^9 + 48*x^8 + 2*x^7 -
 82*x^6 - 2*x^5 + 48*x^4 + 17*x^3 - x^2 - x) - 3*sqrt(2)*(x^11 - 5*x^10 - 9
*x^9 + 52*x^8 - 6*x^7 - 90*x^6 + 6*x^5 + 52*x^4 + 9*x^3 - 5*x^2 - x)))*sqrt
((3*x^4 - 12*x^3 - 2*3^(1/4)*sqrt(x^3 - x^2 - x)*(3*x^2 - sqrt(3)*(x^2 - 4*
x - 1) - 6*x - 3)*sqrt(sqrt(3) + 3) + 12*x^2 + 12*sqrt(3)*(x^3 - x^2 - x) +
 12*x + 3)/(x^4 + 1)) - 18*sqrt(2)*(x^11 + 5*x^10 - 35*x^9 + 116*x^7 - 22*x
^6 - 116*x^5 + 35*x^3 + 5*x^2 - x) + 6*sqrt(3)*(sqrt(3)*sqrt(2)*(x^11 + 5*x
^10 - 35*x^9 + 116*x^7 - 22*x^6 - 116*x^5 + 35*x^3 + 5*x^2 - x) - 3*sqrt(2)
*(x^11 - 3*x^10 - 3*x^9 + 8*x^8 + 4*x^7 - 6*x^6 - 4*x^5 + 8*x^4 + 3*x^3 - 3
*x^2 - x)))/(x^11 - 9*x^10 - 17*x^9 + 104*x^8 - 14*x^7 - 178*x^6 + 14*x^5 +
 104*x^4 + 17*x^3 - 9*x^2 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\left(x^4+1\right)\sqrt{x^3-x^2-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^3-x^2-x)^(1/2)/(x^4+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 1)/((x^4 + 1)*sqrt(x^3 - x^2 - x)), x)
```

maple [C] time = 0.12, size = 288, normalized size = 2.23

$$\frac{2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\sqrt{-5\left(x-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\sqrt{5}}\,\sqrt{-\frac{x}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}},\frac{\sqrt{5}\,\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)}{5\sqrt{x^3-x^2-x}}+\frac{5^{\frac{3}{4}}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\_\alpha\left(\sqrt{5}-1\right)\sqrt{\frac{2x-1+\sqrt{5}}{\sqrt{5}-1}}\,\sqrt{-2x+1+\sqrt{5}}\,\sqrt{-\frac{x}{\sqrt{5}-1}}\left(3\_\alpha^3-\_\alpha^2+2\_\alpha+1+\sqrt{5}\left(\_\alpha^3-\_\alpha^2-1\right)\right)\mathrm{EllipticPi}\left(\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}},-\frac{\sqrt{5}\,\_\alpha^3}{6}-\frac{\_\alpha^3}{6}+\frac{\_\alpha^2}{3}+\frac{\_\alpha}{6}-\frac{\sqrt{5}}{6}+\frac{1}{2}-\frac{\_\alpha\sqrt{5}}{6},\frac{\sqrt{5}\,\sqrt{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\sqrt{5}}}{5}\right)}{\sqrt{x\left(x^2-x-1\right)}}\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/(x^3-x^2-x)^(1/2)/(x^4+1),x)
```

```
[Out] 2/5*(1/2*5^(1/2)-1/2)*((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x-
1/2-1/2*5^(1/2))*5^(1/2))^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1
/2)*EllipticF(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),1/5*5^(1/2)*((1
/2*5^(1/2)-1/2)*5^(1/2))^(1/2))+1/60*5^(3/4)*sum(_alpha*(5^(1/2)-1)*((2*x-1
+5^(1/2))/(5^(1/2)-1))^(1/2)*(-2*x+1+5^(1/2))^(1/2)*(-x/(5^(1/2)-1))^(1/2)/
(x*(x^2-x-1))^(1/2)*(3*_alpha^3-_alpha^2+2*_alpha+1+5^(1/2)*(_alpha^3-_alph
a^2-1))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),-1/6*5^(1/
2)*_alpha^3-1/6*_alpha^3+1/3*_alpha^2+1/6*_alpha-1/6*5^(1/2)+1/2-1/6*_alpha
```

```
*5^(1/2),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)),_alpha=RootOf(_Z^4+
1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\left(x^4+1\right)\sqrt{x^3-x^2-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^3-x^2-x)^(1/2)/(x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 1)/((x^4 + 1)*sqrt(x^3 - x^2 - x)), x)
```

mupad [B] time = 0.05, size = 682, normalized size = 5.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/((x^4 + 1)*(x^3 - x^2 - x)^(1/2)),x)
```

```
[Out] (2*(5^(1/2)/2 + 1/2)*(x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/2 - 1/2)/(5^
(1/2)/2 - 1/2))^(1/2)*ellipticF(asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2
)/2 + 1/2)/(5^(1/2)/2 - 1/2))*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 + 1/2))^(1/
2))/(x^3 - x^2 - x*(5^(1/2)/2 - 1/2)*(5^(1/2)/2 + 1/2))^(1/2) - ((5^(1/2)/2
 + 1/2)*(x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/2 - 1/2)/(5^(1/2)/2 - 1/2
))^(1/2)*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 + 1/2))^(1/2)*ellipticPi(2^(1/2)
*(5^(1/2)/2 + 1/2)*(- 1/2 + 1i/2), asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(
1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2)))/(x^3 - x^2 - x*(5^(1/2)/2 - 1/2)*(5^(1/2)
/2 + 1/2))^(1/2) - ((5^(1/2)/2 + 1/2)*(x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(
1/2)/2 - 1/2)/(5^(1/2)/2 - 1/2))^(1/2)*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 +
1/2))^(1/2)*ellipticPi(2^(1/2)*(5^(1/2)/2 + 1/2)*(1/2 - 1i/2), asin((x/(5^(
1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2)))/(x^3 - x^2 -
x*(5^(1/2)/2 - 1/2)*(5^(1/2)/2 + 1/2))^(1/2) - ((5^(1/2)/2 + 1/2)*(x/(5^(1/
2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/2 - 1/2)/(5^(1/2)/2 - 1/2))^(1/2)*((5^(1/2
)/2 - x + 1/2)/(5^(1/2)/2 + 1/2))^(1/2)*ellipticPi(2^(1/2)*(5^(1/2)/2 + 1/2
)*(1/2 + 1i/2), asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/2 + 1/2)/(5^(1
/2)/2 - 1/2)))/(x^3 - x^2 - x*(5^(1/2)/2 - 1/2)*(5^(1/2)/2 + 1/2))^(1/2) -
((5^(1/2)/2 + 1/2)*(x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/2 - 1/2)/(5^(1
/2)/2 - 1/2))^(1/2)*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 + 1/2))^(1/2)*ellipti
cPi(2^(1/2)*(5^(1/2)/2 + 1/2)*(- 1/2 - 1i/2), asin((x/(5^(1/2)/2 + 1/2))^(1
/2)), -(5^(1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2)))/(x^3 - x^2 - x*(5^(1/2)/2 - 1/
2)*(5^(1/2)/2 + 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2+1\right)}{\sqrt{x\left(x^2-x-1\right)}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/(x**3-x**2-x)**(1/2)/(x**4+1),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)/(sqrt(x*(x**2 - x - 1))*(x**4 + 1)), x)
```

3.1573 $\int \sqrt{-1-11x-36x^2-27x^3+16x^4+9x^5+x^6}\,dx$

Optimal. Leaf size=129

$$-\frac{325}{128}\log\left(x^2+5x+1\right)+\frac{\sqrt{x^6+9x^5+16x^4-27x^3-36x^2-11x-1}\,\left(16x^3+104x^2-6x-185\right)}{64\left(x^2+5x+1\right)}+\frac{325}{128}\log\left(-2\right.$$

Rubi [A] time = 0.08, antiderivative size = 209, normalized size of antiderivative = 1.62, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {6688, 6719, 1661, 640, 612, 621, 206}

$$-\frac{65\sqrt{-\left(\left(-x^2+x+1\right)\left(x^2+5x+1\right)^2\right)}(1-2x)}{64\left(x^2+5x+1\right)}-\frac{x\left(-x^2+x+1\right)\sqrt{-\left(\left(-x^2+x+1\right)\left(x^2+5x+1\right)^2\right)}}{4\left(x^2+5x+1\right)}-\frac{15\left(-x^2+x+1\right)\sqrt{-\left(\left(-x^2+x+1\right)\left(x^2+5x+1\right)^2\right)}}{8\left(x^2+5x+1\right)}+\frac{325\sqrt{-\left(\left(-x^2+x+1\right)\left(x^2+5x+1\right)^2\right)}\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)}{128\sqrt{x^2-x-1}\left(x^2+5x+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - 11*x - 36*x^2 - 27*x^3 + 16*x^4 + 9*x^5 + x^6],x]
```

```
[Out] (-65*(1 - 2*x)*Sqrt[-((1 + x - x^2)*(1 + 5*x + x^2)^2)])/(64*(1 + 5*x + x^2
)) - (15*(1 + x - x^2)*Sqrt[-((1 + x - x^2)*(1 + 5*x + x^2)^2)])/(8*(1 + 5*
x + x^2)) - (x*(1 + x - x^2)*Sqrt[-((1 + x - x^2)*(1 + 5*x + x^2)^2)])/(4*(
1 + 5*x + x^2)) + (325*Sqrt[-((1 + x - x^2)*(1 + 5*x + x^2)^2)]*ArcTanh[(1
- 2*x)/(2*Sqrt[-1 - x + x^2])])/(128*Sqrt[-1 - x + x^2]*(1 + 5*x + x^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \sqrt{-1-11x-36x^2-27x^3+16x^4+9x^5+x^6}\,dx = \int \sqrt{\left(-1-x+x^2\right)\left(1+5x+x^2\right)^2}\,dx$$

$$= \frac{\sqrt{\left(-1-x+x^2\right)\left(1+5x+x^2\right)^2}\int \sqrt{-1-x+x^2}\,\left(1+5x+x\right.}{\sqrt{-1-x+x^2}\,\left(1+5x+x^2\right)}$$

$$= -\frac{x\left(1+x-x^2\right)\sqrt{-\left(\left(1+x-x^2\right)\left(1+5x+x^2\right)^2\right)}}{4\left(1+5x+x^2\right)} + \frac{\sqrt{(-1}}{}$$

$$= -\frac{15\left(1+x-x^2\right)\sqrt{-\left(\left(1+x-x^2\right)\left(1+5x+x^2\right)^2\right)}}{8\left(1+5x+x^2\right)} - \frac{x\left(1+\right.}{}$$

$$= -\frac{65(1-2x)\sqrt{-\left(\left(1+x-x^2\right)\left(1+5x+x^2\right)^2\right)}}{64\left(1+5x+x^2\right)} - \frac{15\left(1+x-\right.}{}$$

$$= -\frac{65(1-2x)\sqrt{-\left(\left(1+x-x^2\right)\left(1+5x+x^2\right)^2\right)}}{64\left(1+5x+x^2\right)} - \frac{15\left(1+x-\right.}{}$$

$$= -\frac{65(1-2x)\sqrt{-\left(\left(1+x-x^2\right)\left(1+5x+x^2\right)^2\right)}}{64\left(1+5x+x^2\right)} - \frac{15\left(1+x-\right.}{}$$

Mathematica [A] time = 0.09, size = 101, normalized size = 0.78

$$\frac{\sqrt{x^2-x-1}\,\left(x^2+5x+1\right)\left(325\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)+2\sqrt{x^2-x-1}\,\left(16x^3+104x^2-6x-185\right)\right)}{128\sqrt{\left(x^2-x-1\right)\left(x^2+5x+1\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - 11*x - 36*x^2 - 27*x^3 + 16*x^4 + 9*x^5 + x^6],x]
```

```
[Out] (Sqrt[-1 - x + x^2]*(1 + 5*x + x^2)*(2*Sqrt[-1 - x + x^2]*(-185 - 6*x + 104
*x^2 + 16*x^3) + 325*ArcTanh[(1 - 2*x)/(2*Sqrt[-1 - x + x^2])]))/(128*Sqrt[
(-1 - x + x^2)*(1 + 5*x + x^2)^2])
```

IntegrateAlgebraic [A] time = 0.26, size = 129, normalized size = 1.00

$$-\frac{325}{128}\log\left(x^2+5x+1\right)+\frac{\sqrt{x^6+9x^5+16x^4-27x^3-36x^2-11x-1}\,\left(16x^3+104x^2-6x-185\right)}{64\left(x^2+5x+1\right)}+\frac{325}{128}\log\left(-2x^3-9x^2+2\sqrt{x^6+9x^5+16x^4-27x^3-36x^2-11x-1}+3x+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 - 11*x - 36*x^2 - 27*x^3 + 16*x^4 + 9*x^5 + x^6]
,x]
```

```
[Out] ((-185 - 6*x + 104*x^2 + 16*x^3)*Sqrt[-1 - 11*x - 36*x^2 - 27*x^3 + 16*x^4
+ 9*x^5 + x^6])/(64*(1 + 5*x + x^2)) - (325*Log[1 + 5*x + x^2])/128 + (325*
Log[1 + 3*x - 9*x^2 - 2*x^3 + 2*Sqrt[-1 - 11*x - 36*x^2 - 27*x^3 + 16*x^4 +
 9*x^5 + x^6]])/128
```

fricas [A] time = 0.39, size = 139, normalized size = 1.08

$$\frac{569\,x^{2}+2600\,\left(x^{2}+5\,x+1\right)\log\left(-\frac{2\,x^{3}+9\,x^{2}-3\,x-2\,\sqrt{x^{6}+9\,x^{5}+16\,x^{4}-27\,x^{3}-36\,x^{2}-11\,x-1}-1}{x^{2}+5\,x+1}\right)+16\,\sqrt{x^{6}+9\,x^{5}+16\,x^{4}-27\,x^{3}-36\,x^{2}-11\,x-1}\left(16\,x^{3}+104\,x^{2}-6\,x-185\right)+2845\,x+569}{1024\,\left(x^{2}+5\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+9*x^5+16*x^4-27*x^3-36*x^2-11*x-1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/1024*(569*x^2 + 2600*(x^2 + 5*x + 1)*log(-(2*x^3 + 9*x^2 - 3*x - 2*sqrt(x
^6 + 9*x^5 + 16*x^4 - 27*x^3 - 36*x^2 - 11*x - 1) - 1)/(x^2 + 5*x + 1)) + 1
6*sqrt(x^6 + 9*x^5 + 16*x^4 - 27*x^3 - 36*x^2 - 11*x - 1)*(16*x^3 + 104*x^2
 - 6*x - 185) + 2845*x + 569)/(x^2 + 5*x + 1)
```

giac [A] time = 0.30, size = 97, normalized size = 0.75

$$\frac{325}{128}\,\log\left(\left|-2\,x+2\,\sqrt{x^{2}-x-1}+1\right|\right)\mathrm{sgn}\left(x^{2}+5\,x+1\right)+\frac{1}{64}\left(2\left(4\left(2\,x\mathrm{sgn}\left(x^{2}+5\,x+1\right)+13\,\mathrm{sgn}\left(x^{2}+5\,x+1\right)\right)x-3\,\mathrm{sgn}\left(x^{2}+5\,x+1\right)\right)x-185\,\mathrm{sgn}\left(x^{2}+5\,x+1\right)\right)\sqrt{x^{2}-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+9*x^5+16*x^4-27*x^3-36*x^2-11*x-1)^(1/2),x, algorithm="giac"
)
```

```
[Out] 325/128*log(abs(-2*x + 2*sqrt(x^2 - x - 1) + 1))*sgn(x^2 + 5*x + 1) + 1/64*
(2*(4*(2*x*sgn(x^2 + 5*x + 1) + 13*sgn(x^2 + 5*x + 1))*x - 3*sgn(x^2 + 5*x
+ 1))*x - 185*sgn(x^2 + 5*x + 1))*sqrt(x^2 - x - 1)
```

maple [A] time = 0.01, size = 120, normalized size = 0.93

$$-\frac{\sqrt{x^{6}+9x^{5}+16x^{4}-27x^{3}-36x^{2}-11x-1}\,\left(-32x\left(x^{2}-x-1\right)^{\frac{3}{2}}-240\left(x^{2}-x-1\right)^{\frac{3}{2}}-260x\sqrt{x^{2}-x-1}+325\ln\left(x-\frac{1}{2}+\sqrt{x^{2}-x-1}\right)+130\sqrt{x^{2}-x-1}\right)}{128\left(x^{2}+5x+1\right)\sqrt{x^{2}-x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+9*x^5+16*x^4-27*x^3-36*x^2-11*x-1)^(1/2),x)
```

```
[Out] -1/128*(x^6+9*x^5+16*x^4-27*x^3-36*x^2-11*x-1)^(1/2)*(-32*x*(x^2-x-1)^(3/2)
-240*(x^2-x-1)^(3/2)-260*x*(x^2-x-1)^(1/2)+325*ln(x-1/2+(x^2-x-1)^(1/2))+13
0*(x^2-x-1)^(1/2))/(x^2+5*x+1)/(x^2-x-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{6}+9\,x^{5}+16\,x^{4}-27\,x^{3}-36\,x^{2}-11\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+9*x^5+16*x^4-27*x^3-36*x^2-11*x-1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(x^6 + 9*x^5 + 16*x^4 - 27*x^3 - 36*x^2 - 11*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^6+9\,x^5+16\,x^4-27\,x^3-36\,x^2-11\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((16*x^4 - 36*x^2 - 27*x^3 - 11*x + 9*x^5 + x^6 - 1)^(1/2),x)
[Out] int((16*x^4 - 36*x^2 - 27*x^3 - 11*x + 9*x^5 + x^6 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^6+9x^5+16x^4-27x^3-36x^2-11x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+9*x**5+16*x**4-27*x**3-36*x**2-11*x-1)**(1/2),x)
[Out] Integral(sqrt(x**6 + 9*x**5 + 16*x**4 - 27*x**3 - 36*x**2 - 11*x - 1), x)
```

3.1574 $\int \frac{(-q+2px^3)(aq+bx+apx^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^4}\,dx$

Optimal. Leaf size=129

$$\frac{\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}\,(2ap^2x^6+4apqx^3-4apqx^2+2aq^2+3bpx^4+3bqx)}{6x^3}-bpq\log\left(\sqrt{p^2x^6+2pqx^3-\right.$$

Rubi [F] time = 1.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-q+2px^3)(aq+bx+apx^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-q + 2*p*x^3)*(a*q + b*x + a*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3
+ p^2*x^6])/x^4,x]
```

```
[Out] 2*b*p*Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6], x] - a*q^2*De
fer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x^4, x] - b*q*Defer[In
t][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x^3, x] + a*p*q*Defer[Int][S
qrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x, x] + 2*a*p^2*Defer[Int][x^2*S
qrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6], x]
```

Rubi steps

$$\int \frac{(-q+2px^3)(aq+bx+apx^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^4}\,dx = \int \left(2bp\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}-\right.$$

$$= (2bp)\int \sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\,dx$$

Mathematica [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(-q+2px^3)(aq+bx+apx^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-q + 2*p*x^3)*(a*q + b*x + a*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*
q*x^3 + p^2*x^6])/x^4,x]
```

```
[Out] Integrate[((-q + 2*p*x^3)*(a*q + b*x + a*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*
q*x^3 + p^2*x^6])/x^4, x]
```

IntegrateAlgebraic [A] time = 0.48, size = 129, normalized size = 1.00

$$\frac{\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}\,(2ap^2x^6+4apqx^3-4apqx^2+2aq^2+3bpx^4+3bqx)}{6x^3}-bpq\log\left(\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+px^3+q\right)+bpq\log(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-q + 2*p*x^3)*(a*q + b*x + a*p*x^3)*Sqrt[q^2 - 2*p*q*x
^2 + 2*p*q*x^3 + p^2*x^6])/x^4,x]
```

```
[Out] (Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(2*a*q^2 + 3*b*q*x - 4*a*p*q*x
^2 + 4*a*p*q*x^3 + 3*b*p*x^4 + 2*a*p^2*x^6))/(6*x^3) + b*p*q*Log[x] - b*p*q
*Log[q + p*x^3 + Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(a*p*x^3+a*q+b*x)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(
1/2)/x^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6 + 2\,pqx^3 - 2\,pqx^2 + q^2}\,\left(apx^3 + aq + bx\right)\left(2\,px^3 - q\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(a*p*x^3+a*q+b*x)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(
1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)*(a*p*x^3 + a*q + b*x)
*(2*p*x^3 - q)/x^4, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\left(2p\,x^3 - q\right)\left(ap\,x^3 + aq + bx\right)\sqrt{p^2x^6 + 2pq\,x^3 - 2pq\,x^2 + q^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*p*x^3-q)*(a*p*x^3+a*q+b*x)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)/x
^4,x)
```

```
[Out] int((2*p*x^3-q)*(a*p*x^3+a*q+b*x)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)/x
^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6 + 2\,pqx^3 - 2\,pqx^2 + q^2}\,\left(apx^3 + aq + bx\right)\left(2\,px^3 - q\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(a*p*x^3+a*q+b*x)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(
1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)*(a*p*x^3 + a*q + b*x)
*(2*p*x^3 - q)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(q - 2\,p\,x^3\right)\,\left(a\,p\,x^3 + b\,x + a\,q\right)\,\sqrt{p^2\,x^6 + 2\,p\,q\,x^3 - 2\,p\,q\,x^2 + q^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q - 2*p*x^3)*(a*q + b*x + a*p*x^3)*(p^2*x^6 + q^2 - 2*p*q*x^2 + 2*p*
q*x^3)^(1/2))/x^4,x)
```

```
[Out] -int(((q - 2*p*x^3)*(a*q + b*x + a*p*x^3)*(p^2*x^6 + q^2 - 2*p*q*x^2 + 2*p*
q*x^3)^(1/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2px^3-q\right)\left(apx^3+aq+bx\right)\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x**3-q)*(a*p*x**3+a*q+b*x)*(p**2*x**6+2*p*q*x**3-2*p*q*x**2+
q**2)**(1/2)/x**4,x)
```

```
[Out] Integral((2*p*x**3 - q)*(a*p*x**3 + a*q + b*x)*sqrt(p**2*x**6 + 2*p*q*x**3
- 2*p*q*x**2 + q**2)/x**4, x)
```

3.1575 $\int \frac{-2-x^4+2x^8}{\sqrt[4]{-1+x^4}\left(-2-x^4+x^8\right)}\,dx$

Optimal. Leaf size=129

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{3}\sqrt[4]{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4-1}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-1}}\right)}{6\sqrt[4]{2}}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{3}\sqrt[4]{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4-1}}\right)-$$

Rubi [A] time = 0.35, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6728, 240, 212, 206, 203, 377}

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{3}\sqrt[4]{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4-1}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-1}}\right)}{6\sqrt[4]{2}}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{3}\sqrt[4]{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4-1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-1}}\right)}{6\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 - x^4 + 2*x^8)/((-1 + x^4)^(1/4)*(-2 - x^4 + x^8)),x]
```

```
[Out] ArcTan[x/(-1 + x^4)^(1/4)] - (2^(1/4)*ArcTan[x/(2^(1/4)*(-1 + x^4)^(1/4))])
/3 - ArcTan[(2^(1/4)*x)/(-1 + x^4)^(1/4)]/(6*2^(1/4)) + ArcTanh[x/(-1 + x^4
)^(1/4)] - (2^(1/4)*ArcTanh[x/(2^(1/4)*(-1 + x^4)^(1/4))])/3 - ArcTanh[(2^(
1/4)*x)/(-1 + x^4)^(1/4)]/(6*2^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2-x^4+2x^8}{\sqrt[4]{-1+x^4}\left(-2-x^4+x^8\right)}\,dx &= \int \left(\frac{2}{\sqrt[4]{-1+x^4}} + \frac{2+x^4}{\sqrt[4]{-1+x^4}\left(-2-x^4+x^8\right)}\right)dx \\
&= 2\int \frac{1}{\sqrt[4]{-1+x^4}}\,dx + \int \frac{2+x^4}{\sqrt[4]{-1+x^4}\left(-2-x^4+x^8\right)}\,dx \\
&= 2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right) + \int \left(\frac{8}{3\sqrt[4]{-1+x^4}\left(-4+2x^4\right)} - \frac{}{3\sqrt[4]{-1}}\right. \\
&= -\left(\frac{2}{3}\int \frac{1}{\sqrt[4]{-1+x^4}\left(2+2x^4\right)}\,dx\right) + \frac{8}{3}\int \frac{1}{\sqrt[4]{-1+x^4}\left(-4+2x^4\right)}\,dx + \text{Subst}\Bigg(\\
&= \tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right) + \tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right) - \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{2-4x^4}\,dx, x, \frac{}{\sqrt[4]{-1}}\right. \\
&= \tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right) + \tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right) - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x^2}\,dx, x, \frac{}{\sqrt[4]{}}\right. \\
&= \tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right) - \frac{1}{3}\sqrt[4]{2}\,\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-1+x^4}}\right) - \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{-1+x^4}}\right)}{6\sqrt[4]{2}} + \tanh
\end{aligned}$$

Mathematica [A] time = 0.11, size = 129, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right) - \frac{1}{3}\sqrt[4]{2}\,\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4-1}}\right) - \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-1}}\right)}{6\sqrt[4]{2}} + \tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right) - \frac{1}{3}\sqrt[4]{2}\,\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4-1}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-1}}\right)}{6\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 - x^4 + 2*x^8)/((-1 + x^4)^(1/4)*(-2 - x^4 + x^8)),x]
```

```
[Out] ArcTan[x/(-1 + x^4)^(1/4)] - (2^(1/4)*ArcTan[x/(2^(1/4)*(-1 + x^4)^(1/4))])
/3 - ArcTan[(2^(1/4)*x)/(-1 + x^4)^(1/4)]/(6*2^(1/4)) + ArcTanh[x/(-1 + x^4
)^(1/4)] - (2^(1/4)*ArcTanh[x/(2^(1/4)*(-1 + x^4)^(1/4))])/3 - ArcTanh[(2^(
1/4)*x)/(-1 + x^4)^(1/4)]/(6*2^(1/4))
```

IntegrateAlgebraic [A] time = 0.50, size = 129, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right) - \frac{1}{3}\sqrt[4]{2}\,\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4-1}}\right) - \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-1}}\right)}{6\sqrt[4]{2}} + \tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right) - \frac{1}{3}\sqrt[4]{2}\,\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4-1}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-1}}\right)}{6\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 - x^4 + 2*x^8)/((-1 + x^4)^(1/4)*(-2 - x^4 + x^8)),x
]
```

```
[Out] ArcTan[x/(-1 + x^4)^(1/4)] - (2^(1/4)*ArcTan[x/(2^(1/4)*(-1 + x^4)^(1/4))])
/3 - ArcTan[(2^(1/4)*x)/(-1 + x^4)^(1/4)]/(6*2^(1/4)) + ArcTanh[x/(-1 + x^4
)^(1/4)] - (2^(1/4)*ArcTanh[x/(2^(1/4)*(-1 + x^4)^(1/4))])/3 - ArcTanh[(2^(
1/4)*x)/(-1 + x^4)^(1/4)]/(6*2^(1/4))
```

fricas [B] time = 0.44, size = 255, normalized size = 1.98

$$-\frac{1}{6}\cdot 8^{\frac{3}{4}}\arctan\left(\frac{8^{\frac{3}{4}}\sqrt{2}\,x\sqrt{\frac{\sqrt{2}\,x^2+2\,\sqrt{x^4-1}}{x^2}}-2\cdot 8^{\frac{3}{4}}\left(x^4-1\right)^{\frac{1}{4}}}{8\,x}\right)-\frac{1}{6}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{3}{4}}x\sqrt{\frac{\sqrt{2}\,x^2+\sqrt{x^4-1}}{x^2}}-2^{\frac{3}{4}}\left(x^4-1\right)^{\frac{1}{4}}}{2\,x}\right)-\frac{1}{24}\cdot 8^{\frac{3}{4}}\log\left(\frac{8^{\frac{1}{4}}x+2\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{24}\cdot 8^{\frac{3}{4}}\log\left(-\frac{8^{\frac{1}{4}}x-2\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{24}\cdot 2^{\frac{3}{4}}\log\left(\frac{2^{\frac{1}{4}}x+\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{24}\cdot 2^{\frac{3}{4}}\log\left(-\frac{2^{\frac{1}{4}}x-\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)-\arctan\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\log\left(\frac{x+\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{2}\log\left(-\frac{x-\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-x^4-2)/(x^4-1)^(1/4)/(x^8-x^4-2),x, algorithm="fricas")
```

```
[Out] -1/6*8^(3/4)*arctan(1/8*(8^(3/4)*sqrt(2)*x*sqrt((sqrt(2)*x^2 + 2*sqrt(x^4 -
 1))/x^2) - 2*8^(3/4)*(x^4 - 1)^(1/4))/x) - 1/6*2^(3/4)*arctan(1/2*(2^(3/4)
*x*sqrt((sqrt(2)*x^2 + sqrt(x^4 - 1))/x^2) - 2^(3/4)*(x^4 - 1)^(1/4))/x) -
1/24*8^(3/4)*log((8^(1/4)*x + 2*(x^4 - 1)^(1/4))/x) + 1/24*8^(3/4)*log(-(8^
(1/4)*x - 2*(x^4 - 1)^(1/4))/x) - 1/24*2^(3/4)*log((2^(1/4)*x + (x^4 - 1)^(
1/4))/x) + 1/24*2^(3/4)*log(-(2^(1/4)*x - (x^4 - 1)^(1/4))/x) - arctan((x^4
 - 1)^(1/4)/x) + 1/2*log((x + (x^4 - 1)^(1/4))/x) - 1/2*log(-(x - (x^4 - 1)
^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^{8}-x^{4}-2}{\left(x^{8}-x^{4}-2\right)\left(x^{4}-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-x^4-2)/(x^4-1)^(1/4)/(x^8-x^4-2),x, algorithm="giac")
```

```
[Out] integrate((2*x^8 - x^4 - 2)/((x^8 - x^4 - 2)*(x^4 - 1)^(1/4)), x)
```

maple [F] time = 1.25, size = 0, normalized size = 0.00

$$\int \frac{2x^{8}-x^{4}-2}{\left(x^{4}-1\right)^{\frac{1}{4}}\left(x^{8}-x^{4}-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8-x^4-2)/(x^4-1)^(1/4)/(x^8-x^4-2),x)
```

```
[Out] int((2*x^8-x^4-2)/(x^4-1)^(1/4)/(x^8-x^4-2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^{8}-x^{4}-2}{\left(x^{8}-x^{4}-2\right)\left(x^{4}-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-x^4-2)/(x^4-1)^(1/4)/(x^8-x^4-2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^8 - x^4 - 2)/((x^8 - x^4 - 2)*(x^4 - 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{-2\,x^8+x^4+2}{{\left(x^4-1\right)}^{1/4}\,\left(-x^8+x^4+2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 2*x^8 + 2)/((x^4 - 1)^(1/4)*(x^4 - x^8 + 2)),x)
```

```
[Out] int((x^4 - 2*x^8 + 2)/((x^4 - 1)^(1/4)*(x^4 - x^8 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^8 - x^4 - 2}{\sqrt[4]{(x-1)(x+1)\left(x^2+1\right)}\left(x^4-2\right)\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8-x**4-2)/(x**4-1)**(1/4)/(x**8-x**4-2),x)
```

```
[Out] Integral((2*x**8 - x**4 - 2)/(((x - 1)*(x + 1)*(x**2 + 1))**(1/4)*(x**4 - 2
)*(x**4 + 1)), x)
```

3.1576 $\int \frac{(b+ax^2)\sqrt{bx+ax^3}}{x^2(-b+ax^2)}\,dx$

Optimal. Leaf size=130

$$\frac{2\sqrt{ax^3+bx}}{x}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)$$

Rubi [A] time = 0.60, antiderivative size = 176, normalized size of antiderivative = 1.35, number of steps used = 13, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2056, 466, 474, 12, 490, 1211, 220, 1699, 205, 208}

$$\frac{2\sqrt{ax^3+bx}}{x}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2+b}}\right)}{\sqrt{x}\sqrt{ax^2+b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2+b}}\right)}{\sqrt{x}\sqrt{ax^2+b}}$$

Antiderivative was successfully verified.

```
[In] Int[((b + a*x^2)*Sqrt[b*x + a*x^3])/(x^2*(-b + a*x^2)),x]
```

```
[Out] (2*Sqrt[b*x + a*x^3])/x + (Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3]*ArcTan
[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/Sqrt[b + a*x^2]])/(Sqrt[x]*Sqrt[b + a*x^
2]) - (Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3]*ArcTanh[(Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x])/Sqrt[b + a*x^2]])/(Sqrt[x]*Sqrt[b + a*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
```

```
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\left(b+ax^2\right)\sqrt{bx+ax^3}}{x^2\left(-b+ax^2\right)}\,dx = \frac{\sqrt{bx+ax^3}\int \frac{\left(b+ax^2\right)^{3/2}}{x^{3/2}\left(-b+ax^2\right)}\,dx}{\sqrt{x}\sqrt{b+ax^2}}$$

$$= \frac{\left(2\sqrt{bx+ax^3}\right)\text{Subst}\left(\int \frac{\left(b+ax^4\right)^{3/2}}{x^2\left(-b+ax^4\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{b+ax^2}}$$

$$= \frac{2\sqrt{bx+ax^3}}{x} - \frac{\left(2\sqrt{bx+ax^3}\right)\text{Subst}\left(\int -\frac{4ab^2x^2}{\left(-b+ax^4\right)\sqrt{b+ax^4}}\,dx, x, \sqrt{x}\right)}{b\sqrt{x}\sqrt{b+ax^2}}$$

$$= \frac{2\sqrt{bx+ax^3}}{x} + \frac{\left(8ab\sqrt{bx+ax^3}\right)\text{Subst}\left(\int \frac{x^2}{\left(-b+ax^4\right)\sqrt{b+ax^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{b+ax^2}}$$

$$= \frac{2\sqrt{bx+ax^3}}{x} - \frac{\left(4\sqrt{a}\,b\sqrt{bx+ax^3}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{b}-\sqrt{a}\,x^2\right)\sqrt{b+ax^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{b+ax^2}} + \frac{\left(4\sqrt{a}\,b\right.}{}$$

$$= \frac{2\sqrt{bx+ax^3}}{x} + \frac{\left(2\sqrt{a}\sqrt{b}\sqrt{bx+ax^3}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{a}\,x^2}{\left(\sqrt{b}+\sqrt{a}\,x^2\right)\sqrt{b+ax^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{b+ax^2}} - \frac{\left(2\sqrt{a}\right.}{}$$

$$= \frac{2\sqrt{bx+ax^3}}{x} - \frac{\left(2\sqrt{a}\,b\sqrt{bx+ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-2\sqrt{a}\,bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b+ax^2}}\right)}{\sqrt{x}\sqrt{b+ax^2}} + \frac{\left(2\sqrt{a}\,b\sqrt{b\right.}{}$$

$$= \frac{2\sqrt{bx+ax^3}}{x} + \frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{b+ax^2}}\right)}{\sqrt{x}\sqrt{b+ax^2}} - \frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}}{\sqrt{x}\sqrt{b}}$$

Mathematica [C] time = 0.07, size = 75, normalized size = 0.58

$$\frac{6\left(ax^2+b\right)-8ax^2\sqrt{\frac{ax^2}{b}+1}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{ax^2}{b},\frac{ax^2}{b}\right)}{3\sqrt{x\left(ax^2+b\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((b + a*x^2)*Sqrt[b*x + a*x^3])/(x^2*(-b + a*x^2)),x]
```

```
[Out] (6*(b + a*x^2) - 8*a*x^2*Sqrt[1 + (a*x^2)/b]*AppellF1[3/4, 1/2, 1, 7/4, -((
a*x^2)/b), (a*x^2)/b])/(3*Sqrt[x*(b + a*x^2)])
```

IntegrateAlgebraic [A] time = 0.41, size = 130, normalized size = 1.00

$$\frac{2\sqrt{ax^3+bx}}{x}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b + a*x^2)*Sqrt[b*x + a*x^3])/(x^2*(-b + a*x^2)),x]
```

```
[Out] (2*Sqrt[b*x + a*x^3])/x + Sqrt[2]*a^(1/4)*b^(1/4)*ArcTan[(Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[b*x + a*x^3])/(b + a*x^2)] - Sqrt[2]*a^(1/4)*b^(1/4)*ArcTanh[(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3])/(b + a*x^2)]
```

fricas [B] time = 74.88, size = 1139, normalized size = 8.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)*(a*x^3+b*x)^(1/2)/x^2/(a*x^2-b),x, algorithm="fricas")
```

```
[Out] -1/4*(4*4^(1/4)*(a*b)^(1/4)*x*arctan(-1/2*(sqrt(2)*sqrt(20*a^4*b + 44*a^3*b
^2 + 8*a^2*b^3 + (4*a^4 + 41*a^3*b + 26*a^2*b^2 + a*b^3)*sqrt(a*b))*(4^(3/4
)*((a^3 + 2*a^2*b)*x^4 - 2*(5*a^2*b + a*b^2)*x^3 + a*b^2 + 2*b^3 + 6*(a^2*b
 + 2*a*b^2)*x^2 - 2*(5*a*b^2 + b^3)*x)*(a*b)^(3/4) - 4^(1/4)*((5*a^3*b + a^
2*b^2)*x^4 + 5*a*b^3 + b^4 - 8*(a^3*b + 2*a^2*b^2)*x^3 + 6*(5*a^2*b^2 + a*b
^3)*x^2 - 8*(a^2*b^2 + 2*a*b^3)*x)*(a*b)^(1/4)) + 4*sqrt(a*x^3 + b*x)*(4^(3
/4)*(4*a^4*b - 9*a^3*b^2 + 6*a^2*b^3 - a*b^4)*(a*b)^(3/4)*x + 4^(1/4)*(4*a^
4*b^2 - 9*a^3*b^3 + 6*a^2*b^4 - a*b^5 + (4*a^5*b - 9*a^4*b^2 + 6*a^3*b^3 -
a^2*b^4)*x^2)*(a*b)^(1/4)))/(4*a^4*b^3 - 9*a^3*b^4 + 6*a^2*b^5 - a*b^6 + (4
*a^6*b - 9*a^5*b^2 + 6*a^4*b^3 - a^3*b^4)*x^4 - 2*(4*a^5*b^2 - 9*a^4*b^3 +
6*a^3*b^4 - a^2*b^5)*x^2)) + 4^(1/4)*(a*b)^(1/4)*x*log(-(4^(3/4)*((5*a^3 +
a^2*b)*x^4 - 8*(a^3 + 2*a^2*b)*x^3 + 5*a*b^2 + b^3 + 6*(5*a^2*b + a*b^2)*x^
2 - 8*(a^2*b + 2*a*b^2)*x)*(a*b)^(3/4) + 8*(5*a^2*b^2 + a*b^3 + (5*a^3*b +
a^2*b^2)*x^2 - 4*(a^3*b + 2*a^2*b^2)*x - 2*(a^2*b + 2*a*b^2 + (a^3 + 2*a^2*
b)*x^2 - (5*a^2*b + a*b^2)*x)*sqrt(a*b))*sqrt(a*x^3 + b*x) - 4*4^(1/4)*((a^
4 + 2*a^3*b)*x^4 + a^2*b^2 + 2*a*b^3 - 2*(5*a^3*b + a^2*b^2)*x^3 + 6*(a^3*b
 + 2*a^2*b^2)*x^2 - 2*(5*a^2*b^2 + a*b^3)*x)*(a*b)^(1/4))/(a^2*x^4 - 2*a*b*
x^2 + b^2)) - 4^(1/4)*(a*b)^(1/4)*x*log((4^(3/4)*((5*a^3 + a^2*b)*x^4 - 8*(
a^3 + 2*a^2*b)*x^3 + 5*a*b^2 + b^3 + 6*(5*a^2*b + a*b^2)*x^2 - 8*(a^2*b + 2
*a*b^2)*x)*(a*b)^(3/4) - 8*(5*a^2*b^2 + a*b^3 + (5*a^3*b + a^2*b^2)*x^2 - 4
*(a^3*b + 2*a^2*b^2)*x - 2*(a^2*b + 2*a*b^2 + (a^3 + 2*a^2*b)*x^2 - (5*a^2*
b + a*b^2)*x)*sqrt(a*b))*sqrt(a*x^3 + b*x) - 4*4^(1/4)*((a^4 + 2*a^3*b)*x^4
 + a^2*b^2 + 2*a*b^3 - 2*(5*a^3*b + a^2*b^2)*x^3 + 6*(a^3*b + 2*a^2*b^2)*x^
2 - 2*(5*a^2*b^2 + a*b^3)*x)*(a*b)^(1/4))/(a^2*x^4 - 2*a*b*x^2 + b^2)) - 8*
sqrt(a*x^3 + b*x))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a x^3+b x}\left(a x^2+b\right)}{\left(a x^2-b\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)*(a*x^3+b*x)^(1/2)/x^2/(a*x^2-b),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^3 + b*x)*(a*x^2 + b)/((a*x^2 - b)*x^2), x)
```

maple [C] time = 0.04, size = 643, normalized size = 4.95

$$2a\left(\frac{2b\sqrt{\frac{xa}{\sqrt{-ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a x^3+bx}}-\frac{b\sqrt{\frac{xa}{\sqrt{-ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a x^3+bx}}+\frac{b\sqrt{-ab}\,\sqrt{\frac{xa}{\sqrt{-ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{a^2\sqrt{a x^3+bx}\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)}+\frac{b\sqrt{-ab}\,\sqrt{\frac{xa}{\sqrt{-ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{a^2\sqrt{a x^3+bx}\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)}\right)+\frac{2a x^2+2b}{\sqrt{x\left(a x^2+b\right)}}-\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a}+\frac{\sqrt{-ab}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a}\right)}{\sqrt{a x^3+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)*(a*x^3+b*x)^(1/2)/x^2/(a*x^2-b),x)
```

```
[Out] 2*a*(2/a*b*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x*a/(
-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)*EllipticE(((x+1/a*(-a*b)^(1/2))*a/(-a*
b)^(1/2))^(1/2),1/2*2^(1/2))-1/a*b*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b
)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)*EllipticF(((x+
1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+b/a^2*(-a*b)^(1/2)*(x*
a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(
1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2))*EllipticPi(((x+1
/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2)/(-1/a*(-a*b)^(1/2)
-1/a*(a*b)^(1/2)),1/2*2^(1/2))+b/a^2*(-a*b)^(1/2)*(x*a/(-a*b)^(1/2)+1)^(1/2
```

```
)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)
/(-1/a*(-a*b)^(1/2)+1/a*(a*b)^(1/2))*EllipticPi(((x+1/a*(-a*b)^(1/2))*a/(-a
*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2)/(-1/a*(-a*b)^(1/2)+1/a*(a*b)^(1/2)),1/2*
2^(1/2)))+2*(a*x^2+b)/(x*(a*x^2+b))^(1/2)-2*(-a*b)^(1/2)*((x+1/a*(-a*b)^(1/
2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-
x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x+
1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*Ellip
ticF(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3+bx}\left(ax^2+b\right)}{\left(ax^2-b\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)*(a*x^3+b*x)^(1/2)/x^2/(a*x^2-b),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^3 + b*x)*(a*x^2 + b)/((a*x^2 - b)*x^2), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b*x + a*x^3)^(1/2)*(b + a*x^2))/(x^2*(b - a*x^2)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(ax^2+b\right)}\left(ax^2+b\right)}{x^2\left(ax^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b)*(a*x**3+b*x)**(1/2)/x**2/(a*x**2-b),x)
[Out] Integral(sqrt(x*(a*x**2 + b))*(a*x**2 + b)/(x**2*(a*x**2 - b)), x)
```

3.1577 $$\int \frac{(-2ab+(3a-b)x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{x(-b+x)\sqrt[4]{x^2(-a+x)(-b+x)}\left(a^3-3a^2x+(3a-bd)x^2+(-1+d)x^3\right)}\,dx$$

Optimal. Leaf size=130

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^3(-a-b)+abx^2+x^4}}{a-x}\right)-2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^3(-a-b)+abx^2+x^4}}{a-x}\right)-\frac{4\left(x^3(-a-b)+abx\right.}{x^2(x-b)}$$

Rubi [F] time = 28.98, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2ab+(3a-b)x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{x(-b+x)\sqrt[4]{x^2(-a+x)(-b+x)}\left(a^3-3a^2x+(3a-bd)x^2+(-1+d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*a*b + (3*a - b)*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/(x*(-b + x)*(
x^2*(-a + x)*(-b + x))^(1/4)*(a^3 - 3*a^2*x + (3*a - b*d)*x^2 + (-1 + d)*x^
3)),x]
```

```
[Out] (4*(a - x)*(1 - x/b)^(1/4)*AppellF1[-1/2, -11/4, 5/4, 1/2, x/a, x/b])/(((a
- x)*(b - x)*x^2)^(1/4)*(1 - x/a)^(3/4)) + (2*(3*a - 7*b)*Sqrt[x]*(-a + x)^
(1/4)*(-b + x)^(1/4)*Defer[Subst][Defer[Int][(-a + x^2)^(11/4)/((-b + x^2)^
(5/4)*(a^3 - 3*a^2*x^2 + 3*a*(1 - (b*d)/(3*a))*x^4 - (1 - d)*x^6)), x], x,
Sqrt[x]])/((a - x)*(b - x)*x^2)^(1/4) + (4*b*(3*a - b*d)*Sqrt[x]*(-a + x)^(
1/4)*(-b + x)^(1/4)*Defer[Subst][Defer[Int][(x^2*(-a + x^2)^(11/4))/((-b +
x^2)^(5/4)*(a^3 - 3*a^2*x^2 + 3*a*(1 - (b*d)/(3*a))*x^4 - (1 - d)*x^6)), x]
, x, Sqrt[x]])/(a^2*((a - x)*(b - x)*x^2)^(1/4)) - (4*b*(1 - d)*Sqrt[x]*(-a
 + x)^(1/4)*(-b + x)^(1/4)*Defer[Subst][Defer[Int][(x^4*(-a + x^2)^(11/4))/
((-b + x^2)^(5/4)*(a^3 - 3*a^2*x^2 + 3*a*(1 - (b*d)/(3*a))*x^4 - (1 - d)*x^
6)), x], x, Sqrt[x]])/(a^2*((a - x)*(b - x)*x^2)^(1/4))
```

Rubi steps

$$\int \frac{(-2ab+(3a-b)x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{x(-b+x)\sqrt[4]{x^2(-a+x)(-b+x)}\left(a^3-3a^2x+(3a-bd)x^2+(-1+d)x^3\right)}dx = \frac{\left(\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\int \frac{}{x^{3/2}\sqrt[4]{-a}}}{\sqrt[4]{x^2(-a}}$$

$$= \frac{\left(\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\int \frac{(-a+}{x^{3/2}(-b+}}{\sqrt[4]{x^2(-a+x}}$$

$$= \frac{\left(\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\int \frac{}{x^{3/2}(-b+}}{\sqrt[4]{x^2(-a+x}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\text{Subst}\Big(}{\sqrt[4]{x}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\text{Subst}\Big(}{}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\text{Subst}\Big(}{a^2}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right)\text{Subst}\Big(}{}$$

$$= \frac{\left(2(3a-7b)\sqrt{x}\sqrt[4]{-a+x}\sqrt[4]{-b+x}\right.}{}$$

$$= \frac{4(a-x)\sqrt[4]{1-\frac{x}{b}}F_1\left(-\frac{1}{2};-\frac{11}{4},\frac{5}{4};\frac{1}{2};\right.}{\sqrt[4]{(a-x)(b-x)x^2}\left(1-\frac{x}{a}\right)^{3/4}}$$

Mathematica [F] time = 7.86, size = 0, normalized size = 0.00

$$\int \frac{(-2ab+(3a-b)x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{x(-b+x)\sqrt[4]{x^2(-a+x)(-b+x)}\left(a^3-3a^2x+(3a-bd)x^2+(-1+d)x^3\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2*a*b + (3*a - b)*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/(x*(-b
+ x)*(x^2*(-a + x)*(-b + x))^(1/4)*(a^3 - 3*a^2*x + (3*a - b*d)*x^2 + (-1 +
 d)*x^3)),x]
```

```
[Out] Integrate[((-2*a*b + (3*a - b)*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/(x*(-b
+ x)*(x^2*(-a + x)*(-b + x))^(1/4)*(a^3 - 3*a^2*x + (3*a - b*d)*x^2 + (-1 +
 d)*x^3)), x]
```

IntegrateAlgebraic [A] time = 3.80, size = 130, normalized size = 1.00

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^3(-a-b)+abx^2+x^4}}{a-x}\right)-2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^3(-a-b)+abx^2+x^4}}{a-x}\right)-\frac{4\left(x^3(-a-b)+abx^2+x^4\right)^{3/4}}{x^2(x-b)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*a*b + (3*a - b)*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3)
)/(x*(-b + x)*(x^2*(-a + x)*(-b + x))^(1/4)*(a^3 - 3*a^2*x + (3*a - b*d)*x^
2 + (-1 + d)*x^3)),x]
```

```
[Out] (-4*(a*b*x^2 + (-a - b)*x^3 + x^4)^(3/4))/(x^2*(-b + x)) + 2*d^(1/4)*ArcTan
[(d^(1/4)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/4))/(a - x)] - 2*d^(1/4)*ArcTan
h[(d^(1/4)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/4))/(a - x)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b+(3*a-b)*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/x/(-b+x)/(x^2*(-a+x
)*(-b+x))^(1/4)/(a^3-3*a^2*x+(-b*d+3*a)*x^2+(-1+d)*x^3),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^3-3\,a^2x+3\,ax^2-x^3\right)(2\,ab-(3\,a-b)x)}{\left((a-x)(b-x)x^2\right)^{\frac{1}{4}}\left((d-1)x^3+a^3-3\,a^2x-(bd-3\,a)x^2\right)(b-x)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b+(3*a-b)*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/x/(-b+x)/(x^2*(-a+x
)*(-b+x))^(1/4)/(a^3-3*a^2*x+(-b*d+3*a)*x^2+(-1+d)*x^3),x, algorithm="giac"
)
```

```
[Out] integrate(-(a^3 - 3*a^2*x + 3*a*x^2 - x^3)*(2*a*b - (3*a - b)*x)/(((a - x)*
(b - x)*x^2)^(1/4)*((d - 1)*x^3 + a^3 - 3*a^2*x - (b*d - 3*a)*x^2)*(b - x)*
x), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{(-2ab+(3a-b)\,x)\left(-a^3+3a^2x-3a\,x^2+x^3\right)}{x\,(-b+x)\left(x^2\,(-a+x)\,(-b+x)\right)^{\frac{1}{4}}\left(a^3-3a^2x+(-bd+3a)\,x^2+(-1+d)\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a*b+(3*a-b)*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/x/(-b+x)/(x^2*(-a+x)*(-b+
x))^(1/4)/(a^3-3*a^2*x+(-b*d+3*a)*x^2+(-1+d)*x^3),x)
```

```
[Out] int((-2*a*b+(3*a-b)*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/x/(-b+x)/(x^2*(-a+x)*(-b+
x))^(1/4)/(a^3-3*a^2*x+(-b*d+3*a)*x^2+(-1+d)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^3-3\,a^2x+3\,ax^2-x^3\right)(2\,ab-(3\,a-b)x)}{\left((a-x)(b-x)x^2\right)^{\frac{1}{4}}\left((d-1)x^3+a^3-3\,a^2x-(bd-3\,a)x^2\right)(b-x)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b+(3*a-b)*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/x/(-b+x)/(x^2*(-a+x
)*(-b+x))^(1/4)/(a^3-3*a^2*x+(-b*d+3*a)*x^2+(-1+d)*x^3),x, algorithm="maxim
a")
```

```
[Out] -integrate((a^3 - 3*a^2*x + 3*a*x^2 - x^3)*(2*a*b - (3*a - b)*x)/(((a - x)*
(b - x)*x^2)^(1/4)*((d - 1)*x^3 + a^3 - 3*a^2*x - (b*d - 3*a)*x^2)*(b - x)*
x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{(2\,a\,b-x\,(3\,a-b))\,\left(a^3-3\,a^2\,x+3\,a\,x^2-x^3\right)}{x\,(b-x)\,\left(x^2\,(a-x)\,(b-x)\right)^{1/4}\,\left(x^2\,(3\,a-b\,d)-3\,a^2\,x+a^3+x^3\,(d-1)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*a*b - x*(3*a - b))*(3*a*x^2 - 3*a^2*x + a^3 - x^3))/(x*(b - x)*(x^
2*(a - x)*(b - x))^(1/4)*(x^2*(3*a - b*d) - 3*a^2*x + a^3 + x^3*(d - 1))),x
)
```

```
[Out] -int(((2*a*b - x*(3*a - b))*(3*a*x^2 - 3*a^2*x + a^3 - x^3))/(x*(b - x)*(x^
2*(a - x)*(b - x))^(1/4)*(x^2*(3*a - b*d) - 3*a^2*x + a^3 + x^3*(d - 1))),
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b+(3*a-b)*x)*(-a**3+3*a**2*x-3*a*x**2+x**3)/x/(-b+x)/(x**2*
(-a+x)*(-b+x))**(1/4)/(a**3-3*a**2*x+(-b*d+3*a)*x**2+(-1+d)*x**3),x)
```

```
[Out] Timed out
```

3.1578 $\int \frac{2b+ax^2}{\sqrt[4]{b+ax^2}\,(bn+anx^2+2x^4)}\,dx$

Optimal. Leaf size=130

$$\frac{\tan^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}\,\sqrt[4]{n}}-\frac{\sqrt[4]{n}\,\sqrt{ax^2+b}}{2^{3/4}}}{x\sqrt[4]{ax^2+b}}\right)}{2^{3/4}n^{3/4}}+\frac{\tanh^{-1}\left(\frac{\frac{\sqrt[4]{n}\,\sqrt{ax^2+b}}{2^{3/4}}+\frac{x^2}{\sqrt[4]{2}\,\sqrt[4]{n}}}{x\sqrt[4]{ax^2+b}}\right)}{2^{3/4}n^{3/4}}$$

Rubi [C] time = 0.84, antiderivative size = 571, normalized size of antiderivative = 4.39, number of steps used = 10, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1692, 399, 490, 1218}

$$\frac{\sqrt[4]{b}\left(\frac{\sqrt{a^2n-8b}}{\sqrt{n}}+a\right)\sqrt{-\frac{ax^2}{b}}\,\Pi\left(-\frac{2\sqrt{b}}{\sqrt{-na^2-\sqrt{n}\,\sqrt{a^2n-8b}\,a+4b}};\sin^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)\middle|-1\right)}{2x\sqrt{-a\sqrt{n}\,\sqrt{a^2n-8b}+a^2(-n)+4b}}-\frac{\sqrt[4]{b}\left(\frac{\sqrt{a^2n-8b}}{\sqrt{n}}+a\right)\sqrt{-\frac{ax^2}{b}}\,\Pi\left(\frac{2\sqrt{b}}{\sqrt{-na^2-\sqrt{n}\,\sqrt{a^2n-8b}\,a+4b}};\sin^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)\middle|-1\right)}{2x\sqrt{-a\sqrt{n}\,\sqrt{a^2n-8b}+a^2(-n)+4b}}+\frac{\sqrt[4]{b}\left(a-\frac{\sqrt{a^2n-8b}}{\sqrt{n}}\right)\sqrt{-\frac{ax^2}{b}}\,\Pi\left(-\frac{2\sqrt{b}}{\sqrt{-na^2+\sqrt{n}\,\sqrt{a^2n-8b}\,a+4b}};\sin^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)\middle|-1\right)}{2x\sqrt{a\sqrt{n}\,\sqrt{a^2n-8b}+a^2(-n)+4b}}-\frac{\sqrt[4]{b}\left(a-\frac{\sqrt{a^2n-8b}}{\sqrt{n}}\right)\sqrt{-\frac{ax^2}{b}}\,\Pi\left(\frac{2\sqrt{b}}{\sqrt{-na^2+\sqrt{n}\,\sqrt{a^2n-8b}\,a+4b}};\sin^{-1}\left(\frac{\sqrt[4]{ax^2+b}}{\sqrt[4]{b}}\right)\middle|-1\right)}{2x\sqrt{a\sqrt{n}\,\sqrt{a^2n-8b}+a^2(-n)+4b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*b + a*x^2)/((b + a*x^2)^(1/4)*(b*n + a*n*x^2 + 2*x^4)),x]
```

```
[Out] (b^(1/4)*(a + Sqrt[-8*b + a^2*n]/Sqrt[n])*Sqrt[-((a*x^2)/b)]*EllipticPi[(-2
*Sqrt[b])/Sqrt[4*b - a^2*n - a*Sqrt[n]*Sqrt[-8*b + a^2*n]], ArcSin[(b + a*x
^2)^(1/4)/b^(1/4)], -1])/(2*Sqrt[4*b - a^2*n - a*Sqrt[n]*Sqrt[-8*b + a^2*n]
]*x) - (b^(1/4)*(a + Sqrt[-8*b + a^2*n]/Sqrt[n])*Sqrt[-((a*x^2)/b)]*Ellipti
cPi[(2*Sqrt[b])/Sqrt[4*b - a^2*n - a*Sqrt[n]*Sqrt[-8*b + a^2*n]], ArcSin[(b
 + a*x^2)^(1/4)/b^(1/4)], -1])/(2*Sqrt[4*b - a^2*n - a*Sqrt[n]*Sqrt[-8*b +
a^2*n]]*x) + (b^(1/4)*(a - Sqrt[-8*b + a^2*n]/Sqrt[n])*Sqrt[-((a*x^2)/b)]*E
llipticPi[(-2*Sqrt[b])/Sqrt[4*b - a^2*n + a*Sqrt[n]*Sqrt[-8*b + a^2*n]], Ar
cSin[(b + a*x^2)^(1/4)/b^(1/4)], -1])/(2*Sqrt[4*b - a^2*n + a*Sqrt[n]*Sqrt[
-8*b + a^2*n]]*x) - (b^(1/4)*(a - Sqrt[-8*b + a^2*n]/Sqrt[n])*Sqrt[-((a*x^2
)/b)]*EllipticPi[(2*Sqrt[b])/Sqrt[4*b - a^2*n + a*Sqrt[n]*Sqrt[-8*b + a^2*n
]], ArcSin[(b + a*x^2)^(1/4)/b^(1/4)], -1])/(2*Sqrt[4*b - a^2*n + a*Sqrt[n]
*Sqrt[-8*b + a^2*n]]*x)
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
```

4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rubi steps

$$\int \frac{2b+ax^2}{\sqrt[4]{b+ax^2}\left(bn+anx^2+2x^4\right)}dx = \int\left(\frac{a-\frac{\sqrt{-8b+a^2n}}{\sqrt{n}}}{\left(an-\sqrt{n}\sqrt{-8b+a^2n}+4x^2\right)\sqrt[4]{b+ax^2}}+\frac{a+\frac{\sqrt{-8b+a^2n}}{\sqrt{n}}}{\left(an+\sqrt{n}\sqrt{-8b+a^2n}+4x\right.}\right.$$

$$=\left(a-\frac{\sqrt{-8b+a^2n}}{\sqrt{n}}\right)\int\frac{1}{\left(an-\sqrt{n}\sqrt{-8b+a^2n}+4x^2\right)\sqrt[4]{b+ax^2}}dx+\left(a+\right.$$

$$=\frac{\left(2\left(a-\frac{\sqrt{-8b+a^2n}}{\sqrt{n}}\right)\sqrt{-\frac{ax^2}{b}}\right)\text{Subst}\left(\int\frac{x^2}{\left(-4b+a\left(an-\sqrt{n}\sqrt{-8b+a^2n}\right)+4x^4\right)\sqrt{1-\frac{x^4}{b}}}dx,x\right.}{x}$$

$$=-\frac{\left(\left(a-\frac{\sqrt{-8b+a^2n}}{\sqrt{n}}\right)\sqrt{-\frac{ax^2}{b}}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{4b-a^2n+a\sqrt{n}\sqrt{-8b+a^2n}}-2x^2\right)\sqrt{1-\frac{x^4}{b}}}dx,x\right.}{2x}$$

$$=\frac{\sqrt[4]{b}\left(a+\frac{\sqrt{-8b+a^2n}}{\sqrt{n}}\right)\sqrt{-\frac{ax^2}{b}}\,\Pi\left(-\frac{2\sqrt{b}}{\sqrt{4b-a^2n-a\sqrt{n}\sqrt{-8b+a^2n}}};\sin^{-1}\left(\frac{\sqrt[4]{b+ax^2}}{\sqrt[4]{b}}\right)\middle|-1\right)}{2\sqrt{4b-a^2n-a\sqrt{n}\sqrt{-8b+a^2n}}\,x}$$

Mathematica [F] time = 0.30, size = 0, normalized size = 0.00

$$\int\frac{2b+ax^2}{\sqrt[4]{b+ax^2}\left(bn+anx^2+2x^4\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2*b + a*x^2)/((b + a*x^2)^(1/4)*(b*n + a*n*x^2 + 2*x^4)),x]
[Out] Integrate[(2*b + a*x^2)/((b + a*x^2)^(1/4)*(b*n + a*n*x^2 + 2*x^4)), x]
```

IntegrateAlgebraic [A] time = 0.46, size = 130, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}\sqrt[4]{n}}-\frac{\sqrt[4]{n}\sqrt{ax^2+b}}{2^{3/4}}}{x\sqrt[4]{ax^2+b}}\right)}{2^{3/4}n^{3/4}}+\frac{\tanh^{-1}\left(\frac{\frac{\sqrt[4]{n}\sqrt{ax^2+b}}{2^{3/4}}+\frac{x^2}{\sqrt[4]{2}\sqrt[4]{n}}}{x\sqrt[4]{ax^2+b}}\right)}{2^{3/4}n^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*b + a*x^2)/((b + a*x^2)^(1/4)*(b*n + a*n*x^2 + 2*x^4)
),x]
[Out] ArcTan[(x^2/(2^(1/4)*n^(1/4)) - (n^(1/4)*Sqrt[b + a*x^2])/2^(3/4))/(x*(b +
a*x^2)^(1/4))]/(2^(3/4)*n^(3/4)) + ArcTanh[(x^2/(2^(1/4)*n^(1/4)) + (n^(1/4
)*Sqrt[b + a*x^2])/2^(3/4))/(x*(b + a*x^2)^(1/4))]/(2^(3/4)*n^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b)/(a*x^2+b)^(1/4)/(2*x^4+1881*a*x^2+1881*b),x, algorith
m="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + 2b}{\left(2x^4 + 1881ax^2 + 1881b\right)\left(ax^2 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b)/(a*x^2+b)^(1/4)/(2*x^4+1881*a*x^2+1881*b),x, algorith
m="giac")
```

```
[Out] integrate((a*x^2 + 2*b)/((2*x^4 + 1881*a*x^2 + 1881*b)*(a*x^2 + b)^(1/4)),
x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2 + 2b}{\left(a\,x^2 + b\right)^{\frac{1}{4}}\left(2x^4 + 1881a\,x^2 + 1881b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+2*b)/(a*x^2+b)^(1/4)/(2*x^4+1881*a*x^2+1881*b),x)
```

```
[Out] int((a*x^2+2*b)/(a*x^2+b)^(1/4)/(2*x^4+1881*a*x^2+1881*b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + 2b}{\left(2x^4 + 1881ax^2 + 1881b\right)\left(ax^2 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b)/(a*x^2+b)^(1/4)/(2*x^4+1881*a*x^2+1881*b),x, algorith
m="maxima")
```

```
[Out] integrate((a*x^2 + 2*b)/((2*x^4 + 1881*a*x^2 + 1881*b)*(a*x^2 + b)^(1/4)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x^2 + 2\,b}{\left(a\,x^2 + b\right)^{1/4}\,\left(2\,x^4 + 1881\,a\,x^2 + 1881\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b + a*x^2)/((b + a*x^2)^(1/4)*(1881*b + 1881*a*x^2 + 2*x^4)),x)
```

```
[Out] int((2*b + a*x^2)/((b + a*x^2)^(1/4)*(1881*b + 1881*a*x^2 + 2*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + 2b}{\sqrt[4]{ax^2 + b}\,\left(1881ax^2 + 1881b + 2x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+2*b)/(a*x**2+b)**(1/4)/(2*x**4+1881*a*x**2+1881*b),x)
```

```
[Out] Integral((a*x**2 + 2*b)/((a*x**2 + b)**(1/4)*(1881*a*x**2 + 1881*b + 2*x**4
)), x)
```

3.1579 $\int \frac{x\sqrt{x+x^2}}{\sqrt{x^2+x\sqrt{x+x^2}}}\,dx$

Optimal. Leaf size=130

$$\frac{\sqrt{x^2+x}\sqrt{x\left(\sqrt{x^2+x}+x\right)}\left(-384x^2-136x+255\right)}{960x}+\sqrt{x\left(\sqrt{x^2+x}+x\right)}\left(\frac{1}{960}\left(384x^2+568x-85\right)-\frac{17\sqrt{\sqrt{x^2-}}}{}\right.$$

Rubi [F] time = 1.63, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\sqrt{x+x^2}}{\sqrt{x^2+x\sqrt{x+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*Sqrt[x + x^2])/Sqrt[x^2 + x*Sqrt[x + x^2]],x]
```

```
[Out] (2*Sqrt[x + x^2]*Defer[Subst][Defer[Int][(x^4*Sqrt[1 + x^2])/Sqrt[x^4 + x^2
*Sqrt[x^2 + x^4]], x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + x])
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{x+x^2}}{\sqrt{x^2+x\sqrt{x+x^2}}}\,dx &= \frac{\sqrt{x+x^2}\int \frac{x^{3/2}\sqrt{1+x}}{\sqrt{x^2+x\sqrt{x+x^2}}}\,dx}{\sqrt{x}\sqrt{1+x}}\\ &= \frac{\left(2\sqrt{x+x^2}\right)\text{Subst}\left(\int \frac{x^4\sqrt{1+x^2}}{\sqrt{x^4+x^2\sqrt{x^2+x^4}}}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1+x}}\end{aligned}$$

Mathematica [C] time = 0.43, size = 137, normalized size = 1.05

$$\frac{\left(x+\sqrt{x(x+1)}\right)^2\sqrt{x\left(x+\sqrt{x(x+1)}\right)}\left(x+\sqrt{x(x+1)}+1\right)\left(17\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1+\frac{1}{2\left(x+\sqrt{x(x+1)}\right)}\right)+10\left(8x^2+\left(8\sqrt{x(x+1)}+11\right)x+7\sqrt{x(x+1)}\right)\right)}{80\sqrt{x(x+1)}\left(2x+2\sqrt{x(x+1)}+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[x + x^2])/Sqrt[x^2 + x*Sqrt[x + x^2]],x]
```

```
[Out] ((x + Sqrt[x*(1 + x)])^2*Sqrt[x*(x + Sqrt[x*(1 + x)])]*(1 + x + Sqrt[x*(1 +
 x)])*(10*(8*x^2 + 7*Sqrt[x*(1 + x)] + x*(11 + 8*Sqrt[x*(1 + x)])) + 17*Hyp
ergeometric2F1[-5/2, 1, -3/2, 1 + 1/(2*(x + Sqrt[x*(1 + x)]))]))/(80*Sqrt[x
*(1 + x)]*(1 + 2*x + 2*Sqrt[x*(1 + x)])^3)
```

IntegrateAlgebraic [A] time = 4.64, size = 130, normalized size = 1.00

$$\frac{\sqrt{x^2+x}\sqrt{x\left(\sqrt{x^2+x}+x\right)}\left(-384x^2-136x+255\right)}{960x}+\sqrt{x\left(\sqrt{x^2+x}+x\right)}\left(\frac{1}{960}\left(384x^2+568x-85\right)-\frac{17\sqrt{\sqrt{x^2+x}-x}\tanh^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+x}-x}\right)}{64\sqrt{2}x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[x + x^2])/Sqrt[x^2 + x*Sqrt[x + x^2]],x]
```

```
[Out] ((255 - 136*x - 384*x^2)*Sqrt[x + x^2]*Sqrt[x*(x + Sqrt[x + x^2])])/(960*x)
 + Sqrt[x*(x + Sqrt[x + x^2])]*((-85 + 568*x + 384*x^2)/960 - (17*Sqrt[-x +
 Sqrt[x + x^2]]*ArcTanh[Sqrt[2]*Sqrt[-x + Sqrt[x + x^2]]])/(64*Sqrt[2]*x))
```

fricas [A] time = 0.42, size = 118, normalized size = 0.91

$$\frac{255\sqrt{2}x\log\left(\frac{4x^2-2\sqrt{x^2+\sqrt{x^2+x}x}\left(\sqrt{2}x+\sqrt{2}\sqrt{x^2+x}\right)+4\sqrt{x^2+x}x+x}{x}\right)+4\left(384x^3+568x^2-\left(384x^2+136x-255\right)\sqrt{x^2+x}-85x\right)\sqrt{x^2+\sqrt{x^2+x}x}}{3840x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/3840*(255*sqrt(2)*x*log((4*x^2 - 2*sqrt(x^2 + sqrt(x^2 + x)*x)*(sqrt(2)*x
 + sqrt(2)*sqrt(x^2 + x)) + 4*sqrt(x^2 + x)*x + x)/x) + 4*(384*x^3 + 568*x^
2 - (384*x^2 + 136*x - 255)*sqrt(x^2 + x) - 85*x)*sqrt(x^2 + sqrt(x^2 + x)*
x))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+x}\,x}{\sqrt{x^2+\sqrt{x^2+x}\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + x)*x/sqrt(x^2 + sqrt(x^2 + x)*x), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{x^2+x}}{\sqrt{x^2+x\sqrt{x^2+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x)
```

```
[Out] int(x*(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+x}\,x}{\sqrt{x^2+\sqrt{x^2+x}\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(x^2 + x)*x/sqrt(x^2 + sqrt(x^2 + x)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{x^2+x}}{\sqrt{x^2+x\,\sqrt{x^2+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + x^2)^(1/2))/(x^2 + x*(x + x^2)^(1/2))^(1/2),x)
```

```
[Out] int((x*(x + x^2)^(1/2))/(x^2 + x*(x + x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{x(x+1)}}{\sqrt{x\left(x+\sqrt{x^2+x}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+x)**(1/2)/(x**2+x*(x**2+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(x*(x + 1))/sqrt(x*(x + sqrt(x**2 + x))), x)
```

3.1580 $\int \frac{\sqrt{1+x}}{1+\sqrt{x+\sqrt{1+x}}}\,dx$

Optimal. Leaf size=130

$$\sqrt{x+1}\left(\sqrt{x+\sqrt{x+1}}-2\right)-\frac{3}{2}\sqrt{x+\sqrt{x+1}}-\frac{23}{4}\log\left(2\sqrt{x+1}-2\sqrt{x+\sqrt{x+1}}+1\right)-\frac{4}{3}\log\left(\sqrt{x+1}-\sqrt{x+\right.$$

Rubi [A] time = 0.34, antiderivative size = 182, normalized size of antiderivative = 1.40, number of steps used = 18, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6742, 612, 621, 206, 734, 843, 724}

$$\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)-2\sqrt{x+1}-2\sqrt{x+\sqrt{x+1}}-\frac{2}{3}\log\left(1-\sqrt{x+1}\right)+\frac{8}{3}\log\left(\sqrt{x+1}+2\right)+\frac{2}{3}\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)+\frac{15}{4}\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)-\frac{8}{3}\tanh^{-1}\left(\frac{3\sqrt{x+1}+4}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 + Sqrt[x + Sqrt[1 + x]]),x]
```

```
[Out] -2*Sqrt[1 + x] - 2*Sqrt[x + Sqrt[1 + x]] + (Sqrt[x + Sqrt[1 + x]]*(1 + 2*Sq
rt[1 + x]))/2 + (2*ArcTanh[(1 - 3*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/
3 + (15*ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/4 - (8*ArcT
anh[(4 + 3*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/3 - (2*Log[1 - Sqrt[1 +
 x]])/3 + (8*Log[2 + Sqrt[1 + x]])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{1+x}}{1+\sqrt{x+\sqrt{1+x}}}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{1+\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)$$

$$= 2\,\text{Subst}\left(\int \left(-1-\frac{1}{3(-1+x)}+\frac{4}{3(2+x)}+\sqrt{-1+x+x^2}+\frac{\sqrt{-1+x+x^2}}{3(-1+x)}-\frac{4\sqrt{-1+ \ldots}}{3(2+ \ldots}\right.\right.$$

$$= -2\sqrt{1+x}-\frac{2}{3}\log\left(1-\sqrt{1+x}\right)+\frac{8}{3}\log\left(2+\sqrt{1+x}\right)+\frac{2}{3}\,\text{Subst}\left(\int \frac{\sqrt{-1+x+x^2}}{-1+x} \ldots\right.$$

$$= -2\sqrt{1+x}-2\sqrt{x+\sqrt{1+x}}+\frac{1}{2}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right)-\frac{2}{3}\log\left(1-\sqrt{1+x}\right)+ \ldots$$

$$= -2\sqrt{1+x}-2\sqrt{x+\sqrt{1+x}}+\frac{1}{2}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right)-\frac{2}{3}\log\left(1-\sqrt{1+x}\right)+ \ldots$$

$$= -2\sqrt{1+x}-2\sqrt{x+\sqrt{1+x}}+\frac{1}{2}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right)-\frac{5}{4}\tanh^{-1}\left(\frac{1+2\sqrt{1 \ldots}}{2\sqrt{x+\sqrt{1 \ldots}}}\right. \ldots$$

$$= -2\sqrt{1+x}-2\sqrt{x+\sqrt{1+x}}+\frac{1}{2}\sqrt{x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right)+\frac{2}{3}\tanh^{-1}\left(\frac{1-3\sqrt{1 \ldots}}{2\sqrt{x+\sqrt{1 \ldots}}}\right. \ldots$$

Mathematica [A] time = 0.13, size = 170, normalized size = 1.31

$$\frac{1}{12}\left(-24\sqrt{x+1}+12\sqrt{x+1}\sqrt{x+\sqrt{x+1}}-18\sqrt{x+\sqrt{x+1}}-8\log\left(1-\sqrt{x+1}\right)+32\log\left(\sqrt{x+1}+2\right)+8\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)+45\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)-32\tanh^{-1}\left(\frac{3\sqrt{x+1}+4}{2\sqrt{x+\sqrt{x+1}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 + Sqrt[x + Sqrt[1 + x]]),x]
```

```
[Out] (-24*Sqrt[1 + x] - 18*Sqrt[x + Sqrt[1 + x]] + 12*Sqrt[1 + x]*Sqrt[x + Sqrt[
1 + x]] + 8*ArcTanh[(1 - 3*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] + 45*Arc
Tanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] - 32*ArcTanh[(4 + 3*Sqr
t[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] - 8*Log[1 - Sqrt[1 + x]] + 32*Log[2 +
Sqrt[1 + x]])/12
```

IntegrateAlgebraic [A] time = 0.35, size = 127, normalized size = 0.98

$$\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}-3\right)-2\sqrt{x+1}+\frac{16}{3}\log\left(-\sqrt{x+1}+\sqrt{x+\sqrt{x+1}}-1\right)-\frac{4}{3}\log\left(-\sqrt{x+1}+\sqrt{x+\sqrt{x+1}}+2\right)-\frac{23}{4}\log\left(-2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]/(1 + Sqrt[x + Sqrt[1 + x]]),x]
```

```
[Out] -2*Sqrt[1 + x] + (Sqrt[x + Sqrt[1 + x]]*(-3 + 2*Sqrt[1 + x]))/2 + (16*Log[-
1 - Sqrt[1 + x] + Sqrt[x + Sqrt[1 + x]]])/3 - (4*Log[2 - Sqrt[1 + x] + Sqrt
[x + Sqrt[1 + x]]])/3 - (23*Log[-1 - 2*Sqrt[1 + x] + 2*Sqrt[x + Sqrt[1 + x]
]])/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1+(x+(1+x)^(1/2))^(1/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.73, size = 157, normalized size = 1.21

$$\frac{1}{2}\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}-3\right)-2\sqrt{x+1}-\frac{8}{3}\log\left(-\sqrt{x+\sqrt{x+1}}+\sqrt{x+1}+3\right)+\frac{8}{3}\log\left(-\sqrt{x+\sqrt{x+1}}+\sqrt{x+1}+1\right)-\frac{15}{4}\log\left(-2\sqrt{x+\sqrt{x+1}}+2\sqrt{x+1}+1\right)+\frac{8}{3}\log\left(\sqrt{x+1}+2\right)-\frac{2}{3}\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}+2\right|\right)+\frac{2}{3}\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}\right|\right)-\frac{2}{3}\log\left(\left|\sqrt{x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1+(x+(1+x)^(1/2))^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) - 3) - 2*sqrt(x + 1) - 8/3*log(-sq
rt(x + sqrt(x + 1)) + sqrt(x + 1) + 3) + 8/3*log(-sqrt(x + sqrt(x + 1)) + s
qrt(x + 1) + 1) - 15/4*log(-2*sqrt(x + sqrt(x + 1)) + 2*sqrt(x + 1) + 1) +
8/3*log(sqrt(x + 1) + 2) - 2/3*log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1)
+ 2)) + 2/3*log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1))) - 2/3*log(abs(sqr
t(x + 1) - 1))
```

maple [B] time = 0.02, size = 238, normalized size = 1.83

$$\frac{\left(1+2\sqrt{1+x}\right)\sqrt{x+\sqrt{1+x}}}{2}-\frac{5\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}\right)}{4}+\frac{2\sqrt{\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2}}{3}+\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2}\right)-\frac{2\,\mathrm{arctanh}\left(\frac{-1+3\sqrt{1+x}}{2\sqrt{\left(-1+\sqrt{1+x}\right)^{2}+3\sqrt{1+x}-2}}\right)}{3}-\frac{8\sqrt{\left(2+\sqrt{1+x}\right)^{2}-5-3\sqrt{1+x}}}{3}+4\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{\left(2+\sqrt{1+x}\right)^{2}-5-3\sqrt{1+x}}\right)+\frac{8\,\mathrm{arctanh}\left(\frac{-3\sqrt{1+x}-4}{2\sqrt{\left(2+\sqrt{1+x}\right)^{2}-5-3\sqrt{1+x}}}\right)}{3}-2\sqrt{1+x}-\frac{2\ln\left(-1+\sqrt{1+x}\right)}{3}+\frac{8\ln\left(2+\sqrt{1+x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1+(x+(1+x)^(1/2))^(1/2)),x)
```

```
[Out] 1/2*(1+2*(1+x)^(1/2))*(x+(1+x)^(1/2))^(1/2)-5/4*ln(1/2+(1+x)^(1/2)+(x+(1+x)
^(1/2))^(1/2))+2/3*((-1+(1+x)^(1/2))^2+3*(1+x)^(1/2)-2)^(1/2)+ln(1/2+(1+x)^
(1/2)+((-1+(1+x)^(1/2))^2+3*(1+x)^(1/2)-2)^(1/2))-2/3*arctanh(1/2*(-1+3*(1+
x)^(1/2))/((-1+(1+x)^(1/2))^2+3*(1+x)^(1/2)-2)^(1/2))-8/3*((2+(1+x)^(1/2))^
2-5-3*(1+x)^(1/2))^(1/2)+4*ln(1/2+(1+x)^(1/2)+((2+(1+x)^(1/2))^2-5-3*(1+x)^
(1/2))^(1/2))+8/3*arctanh(1/2*(-3*(1+x)^(1/2)-4)/((2+(1+x)^(1/2))^2-5-3*(1+
x)^(1/2))^(1/2))-2*(1+x)^(1/2)-2/3*ln(-1+(1+x)^(1/2))+8/3*ln(2+(1+x)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1+(x+(1+x)^(1/2))^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + 1)/(sqrt(x + sqrt(x + 1)) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/((x + (x + 1)^(1/2))^(1/2) + 1),x)
```

```
[Out] int((x + 1)^(1/2)/((x + (x + 1)^(1/2))^(1/2) + 1), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1+(x+(1+x)**(1/2))**(1/2)),x)
```

```
[Out] Integral(sqrt(x + 1)/(sqrt(x + sqrt(x + 1)) + 1), x)
```

3.1581 $\int \frac{1+x}{(-1+x)(1+2x)\sqrt[3]{-1+3x^2}}\,dx$

Optimal. Leaf size=131

$$\frac{\log\left(\sqrt[3]{3x^2-1}-\sqrt[3]{2}\,x\right)}{3\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2}\,\sqrt[3]{3x^2-1}\,x+\left(3x^2-1\right)^{2/3}\right)}{6\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{3x^2-1}}{\sqrt[3]{3x^2-1}+2\sqrt[3]{2}\,x}\right)}{\sqrt[3]{2}\,\sqrt{3}}$$

Rubi [C] time = 0.61, antiderivative size = 265, normalized size of antiderivative = 2.02, number of steps used = 20, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {6742, 757, 430, 429, 444, 55, 617, 204, 31, 56}

$$-\frac{2x\sqrt[3]{1-3x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,x^2\right)}{3\sqrt[3]{3x^2-1}}-\frac{x\sqrt[3]{1-3x^2}\,F_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};4x^2,3x^2\right)}{3\sqrt[3]{3x^2-1}}-\frac{\log\left(1-4x^2\right)}{12\sqrt[3]{2}}-\frac{\log\left(1-x^2\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{3x^2-1}\right)}{2\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{3x^2-1}+1\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-2\,2^{2/3}\sqrt[3]{3x^2-1}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{3x^2-1}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x)/((-1 + x)*(1 + 2*x)*(-1 + 3*x^2)^(1/3)),x]
```

```
[Out] (-2*x*(1 - 3*x^2)^(1/3)*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2])/(3*(-1 + 3*
x^2)^(1/3)) - (x*(1 - 3*x^2)^(1/3)*AppellF1[1/2, 1, 1/3, 3/2, 4*x^2, 3*x^2]
)/(3*(-1 + 3*x^2)^(1/3)) + ArcTan[(1 - 2*2^(2/3)*(-1 + 3*x^2)^(1/3))/Sqrt[3
]]/(2*2^(1/3)*Sqrt[3]) + ArcTan[(1 + 2^(2/3)*(-1 + 3*x^2)^(1/3))/Sqrt[3]]/(
2^(1/3)*Sqrt[3]) - Log[1 - 4*x^2]/(12*2^(1/3)) - Log[1 - x^2]/(6*2^(1/3)) +
 Log[2^(1/3) - (-1 + 3*x^2)^(1/3)]/(2*2^(1/3)) + Log[1 + 2^(2/3)*(-1 + 3*x^
2)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1+x}{(-1+x)(1+2x)\sqrt[3]{-1+3x^2}}\,dx = \int \left(\frac{2}{3(-1+x)\sqrt[3]{-1+3x^2}} - \frac{1}{3(1+2x)\sqrt[3]{-1+3x^2}}\right)dx$$
$$= -\left(\frac{1}{3}\int \frac{1}{(1+2x)\sqrt[3]{-1+3x^2}}\,dx\right) + \frac{2}{3}\int \frac{1}{(-1+x)\sqrt[3]{-1+3x^2}}\,dx$$
$$= -\left(\frac{1}{3}\int \left(\frac{1}{\left(1-4x^2\right)\sqrt[3]{-1+3x^2}} + \frac{2x}{\sqrt[3]{-1+3x^2}\left(-1+4x^2\right)}\right)dx\right) + \frac{2}{3}\int \Bigg(\frac{}{(-}$$
$$= -\left(\frac{1}{3}\int \frac{1}{\left(1-4x^2\right)\sqrt[3]{-1+3x^2}}\,dx\right) + \frac{2}{3}\int \frac{1}{\left(-1+x^2\right)\sqrt[3]{-1+3x^2}}\,dx + \frac{2}{3}\int$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{-1+3x}}\,dx, x, x^2\right) - \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{-1+3x}\,(-1+}\right.$$
$$= -\frac{2x\sqrt[3]{1-3x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,x^2\right)}{3\sqrt[3]{-1+3x^2}} - \frac{x\sqrt[3]{1-3x^2}\,F_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};4x^2,3x^2\right)}{3\sqrt[3]{-1+3x^2}} -$$
$$= -\frac{2x\sqrt[3]{1-3x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,x^2\right)}{3\sqrt[3]{-1+3x^2}} - \frac{x\sqrt[3]{1-3x^2}\,F_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};4x^2,3x^2\right)}{3\sqrt[3]{-1+3x^2}} -$$
$$= -\frac{2x\sqrt[3]{1-3x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,x^2\right)}{3\sqrt[3]{-1+3x^2}} - \frac{x\sqrt[3]{1-3x^2}\,F_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};4x^2,3x^2\right)}{3\sqrt[3]{-1+3x^2}} -$$

Mathematica [F] time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{1+x}{(-1+x)(1+2x)\sqrt[3]{-1+3x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x)/((-1 + x)*(1 + 2*x)*(-1 + 3*x^2)^(1/3)),x]
[Out] Integrate[(1 + x)/((-1 + x)*(1 + 2*x)*(-1 + 3*x^2)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.21, size = 131, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{3x^2-1}-\sqrt[3]{2}\,x\right)}{3\sqrt[3]{2}} - \frac{\log\left(2^{2/3}x^2+\sqrt[3]{2}\,\sqrt[3]{3x^2-1}\,x+\left(3x^2-1\right)^{2/3}\right)}{6\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{3x^2-1}}{\sqrt[3]{3x^2-1}+2\sqrt[3]{2}\,x}\right)}{\sqrt[3]{2}\,\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((-1 + x)*(1 + 2*x)*(-1 + 3*x^2)^(1/3)),x]
[Out] ArcTan[(Sqrt[3]*(-1 + 3*x^2)^(1/3))/(2*2^(1/3)*x + (-1 + 3*x^2)^(1/3))]/(2^
(1/3)*Sqrt[3]) + Log[-(2^(1/3)*x) + (-1 + 3*x^2)^(1/3)]/(3*2^(1/3)) - Log[2
^(2/3)*x^2 + 2^(1/3)*x*(-1 + 3*x^2)^(1/3) + (-1 + 3*x^2)^(2/3)]/(6*2^(1/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(1+2*x)/(3*x^2-1)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(3\,x^2-1\right)^{\frac{1}{3}}(2\,x+1)(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(1+2*x)/(3*x^2-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/((3*x^2 - 1)^(1/3)*(2*x + 1)*(x - 1)), x)
```

maple [C] time = 28.24, size = 581, normalized size = 4.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-1+x)/(1+2*x)/(3*x^2-1)^(1/3),x)
```

```
[Out] RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln((3*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-3*RootOf(Root
Of(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*(3*x^2-1)^(2/3)*
x+RootOf(_Z^3-4)^2*(3*x^2-1)^(1/3)*x^2+12*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3
-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*(3*x^2-1)^(1/3)*x^2-3*RootOf(_Z^3-4)*x^2
-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^2+2*(3*x^2-1)^(2
/3)*x+RootOf(_Z^3-4)+6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
)/(1+2*x)/(-1+x)^2)+1/6*RootOf(_Z^3-4)*ln(-(3*RootOf(RootOf(_Z^3-4)^2+6*_Z*
RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+9*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-9*RootOf(_Z^3-4)*RootOf(Roo
tOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*(3*x^2-1)^(1/3)*x^2+2*RootOf(_Z^
3-4)*x^3+6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+3*RootO
f(_Z^3-4)*x^2+9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^2-3*
(3*x^2-1)^(2/3)*x-RootOf(_Z^3-4)-3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3
-4)+36*_Z^2))/(1+2*x)/(-1+x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(3\,x^2-1\right)^{\frac{1}{3}}(2\,x+1)(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(1+2*x)/(3*x^2-1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/((3*x^2 - 1)^(1/3)*(2*x + 1)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+1}{(2\,x+1)\,\left(3\,x^2-1\right)^{1/3}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((2*x + 1)*(3*x^2 - 1)^(1/3)*(x - 1)),x)
```

```
[Out] int((x + 1)/((2*x + 1)*(3*x^2 - 1)^(1/3)*(x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{(x-1)(2x+1)\sqrt[3]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(1+2*x)/(3*x**2-1)**(1/3),x)
[Out] Integral((x + 1)/((x - 1)*(2*x + 1)*(3*x**2 - 1)**(1/3)), x)
```

3.1582 $\int \frac{-b+ax^2}{x^2\sqrt[3]{-x+x^3}}\,dx$

Optimal. Leaf size=131

$$\frac{1}{4}\left(a-i\sqrt{3}\,a\right)\log\left(-2i\sqrt[3]{x^3-x}+\sqrt{3}\,x-ix\right)+\frac{1}{4}\left(a+i\sqrt{3}\,a\right)\log\left(2i\sqrt[3]{x^3-x}+\sqrt{3}\,x+ix\right)-\frac{1}{2}a\log\left(\sqrt[3]{x^3-x}-x\right)-$$

Rubi [A] time = 0.12, antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2038, 2011, 329, 275, 239}

$$-\frac{3a\sqrt[3]{x}\sqrt[3]{x^2-1}\log\left(x^{2/3}-\sqrt[3]{x^2-1}\right)}{4\sqrt[3]{x^3-x}}+\frac{\sqrt{3}\,a\sqrt[3]{x}\sqrt[3]{x^2-1}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{x^3-x}}-\frac{3b\left(x^3-x\right)^{2/3}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^2)/(x^2*(-x + x^3)^(1/3)),x]
```

```
[Out] (-3*b*(-x + x^3)^(2/3))/(4*x^2) + (Sqrt[3]*a*x^(1/3)*(-1 + x^2)^(1/3)*ArcTa
n[(1 + (2*x^(2/3))/(-1 + x^2)^(1/3))/Sqrt[3]])/(2*(-x + x^3)^(1/3)) - (3*a*
x^(1/3)*(-1 + x^2)^(1/3)*Log[x^(2/3) - (-1 + x^2)^(1/3)])/(4*(-x + x^3)^(1/
3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
```

```
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-b+ax^2}{x^2\sqrt[3]{-x+x^3}}\,dx &= -\frac{3b\left(-x+x^3\right)^{2/3}}{4x^2}+a\int \frac{1}{\sqrt[3]{-x+x^3}}\,dx\\ &= -\frac{3b\left(-x+x^3\right)^{2/3}}{4x^2}+\frac{\left(a\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\right)\int \frac{1}{\sqrt[3]{x}\,\sqrt[3]{-1+x^2}}\,dx}{\sqrt[3]{-x+x^3}}\\ &= -\frac{3b\left(-x+x^3\right)^{2/3}}{4x^2}+\frac{\left(3a\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{-1+x^6}}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x+x^3}}\\ &= -\frac{3b\left(-x+x^3\right)^{2/3}}{4x^2}+\frac{\left(3a\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x^3}}\,dx,x,x^{2/3}\right)}{2\sqrt[3]{-x+x^3}}\\ &= -\frac{3b\left(-x+x^3\right)^{2/3}}{4x^2}+\frac{\sqrt{3}\,a\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{-x+x^3}}-\frac{3a\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\,\log\left(x^{2/3}-\sqrt[3]{\right.}}{4\sqrt[3]{-x+x^3}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 160, normalized size = 1.22

$$\frac{-2a\sqrt[3]{x^2-1}\,x^{4/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2-1}}\right)+a\sqrt[3]{x^2-1}\,x^{4/3}\log\left(\frac{x^{4/3}}{\left(x^2-1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2-1}}+1\right)+2\sqrt{3}\,a\sqrt[3]{x^2-1}\,x^{4/3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1}{\sqrt{3}}\right)-3bx^2+3b}{4x\sqrt[3]{x\left(x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^2)/(x^2*(-x + x^3)^(1/3)),x]
```

```
[Out] (3*b - 3*b*x^2 + 2*Sqrt[3]*a*x^(4/3)*(-1 + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3
))/(-1 + x^2)^(1/3))/Sqrt[3]] - 2*a*x^(4/3)*(-1 + x^2)^(1/3)*Log[1 - x^(2/3
)/(-1 + x^2)^(1/3)] + a*x^(4/3)*(-1 + x^2)^(1/3)*Log[1 + x^(4/3)/(-1 + x^2)
^(2/3) + x^(2/3)/(-1 + x^2)^(1/3)])/(4*x*(x*(-1 + x^2))^(1/3))
```

IntegrateAlgebraic [A] time = 0.27, size = 110, normalized size = 0.84

$$-\frac{1}{2}a\log\left(\sqrt[3]{x^3-x}-x\right)+\frac{1}{2}\sqrt{3}\,a\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-x}+x}\right)+\frac{1}{4}a\log\left(\sqrt[3]{x^3-x}\,x+\left(x^3-x\right)^{2/3}+x^2\right)-\frac{3b\left(x^3-x\right)^{2/3}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^2)/(x^2*(-x + x^3)^(1/3)),x]
```

```
[Out] (-3*b*(-x + x^3)^(2/3))/(4*x^2) + (Sqrt[3]*a*ArcTan[(Sqrt[3]*x)/(x + 2*(-x
+ x^3)^(1/3))])/2 - (a*Log[-x + (-x + x^3)^(1/3)])/2 + (a*Log[x^2 + x*(-x +
 x^3)^(1/3) + (-x + x^3)^(2/3)])/4
```

fricas [A] time = 1.22, size = 112, normalized size = 0.85

$$\frac{2\sqrt{3}\,ax^2\arctan\left(-\frac{44032959556\sqrt{3}\left(x^3-x\right)^{\frac{1}{3}}x+\sqrt{3}\left(16754327161\,x^2-2707204793\right)-10524305234\sqrt{3}\left(x^3-x\right)^{\frac{2}{3}}}{81835897185\,x^2-1102302937}\right)-ax^2\log\left(-3\left(x^3-x\right)^{\frac{1}{3}}x+3\left(x^3-x\right)^{\frac{2}{3}}+1\right)-3\left(x^3-x\right)^{\frac{2}{3}}b}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/x^2/(x^3-x)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*a*x^2*arctan(-(44032959556*sqrt(3)*(x^3 - x)^(1/3)*x + sqrt(
3)*(16754327161*x^2 - 2707204793) - 10524305234*sqrt(3)*(x^3 - x)^(2/3))/(8
1835897185*x^2 - 1102302937)) - a*x^2*log(-3*(x^3 - x)^(1/3)*x + 3*(x^3 - x
)^(2/3) + 1) - 3*(x^3 - x)^(2/3)*b)/x^2
```

giac [A] time = 0.24, size = 78, normalized size = 0.60

$$-\frac{1}{2}\sqrt{3}a\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{4}a\log\left(\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{2}a\log\left(\left|\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)-\frac{3}{4}b\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/x^2/(x^3-x)^(1/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*(-1/x^2 + 1)^(1/3) + 1)) + 1/4*a*log((
-1/x^2 + 1)^(2/3) + (-1/x^2 + 1)^(1/3) + 1) - 1/2*a*log(abs((-1/x^2 + 1)^(1
/3) - 1)) - 3/4*b*(-1/x^2 + 1)^(2/3)
```

maple [C] time = 0.32, size = 55, normalized size = 0.42

$$-\frac{3b\left(x^2-1\right)}{4x\left(x\left(x^2-1\right)\right)^{\frac{1}{3}}}+\frac{3a\left(-\mathrm{signum}\left(x^2-1\right)\right)^{\frac{1}{3}}x^{\frac{2}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^2\right)}{2\mathrm{signum}\left(x^2-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)/x^2/(x^3-x)^(1/3),x)
```

```
[Out] -3/4*b*(x^2-1)/x/(x*(x^2-1))^(1/3)+3/2*a/signum(x^2-1)^(1/3)*(-signum(x^2-1
))^(1/3)*x^(2/3)*hypergeom([1/3,1/3],[4/3],x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax^2-b}{\left(x^3-x\right)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/x^2/(x^3-x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 - b)/((x^3 - x)^(1/3)*x^2), x)
```

mupad [B] time = 1.25, size = 46, normalized size = 0.35

$$\frac{3\,a\,x\,{\left(1-x^2\right)}^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ x^2\right)}{2\,{\left(x^3-x\right)}^{1/3}}-\frac{3\,b\,{\left(x^3-x\right)}^{2/3}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^2)/(x^2*(x^3 - x)^(1/3)),x)
```

```
[Out] (3*a*x*(1 - x^2)^(1/3)*hypergeom([1/3, 1/3], 4/3, x^2))/(2*(x^3 - x)^(1/3))
 - (3*b*(x^3 - x)^(2/3))/(4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax^2-b}{x^2\sqrt[3]{x\left(x-1\right)\left(x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)/x**2/(x**3-x)**(1/3),x)
```

```
[Out] Integral((a*x**2 - b)/(x**2*(x*(x - 1)*(x + 1))**(1/3)), x)
```

3.1583 $\int \frac{1+x^4}{\sqrt{-x-x^2+x^3}\left(-1+x^4\right)} \, dx$

Optimal. Leaf size=131

$$-\tan^{-1}\left(\frac{\sqrt{x^3-x^2-x}}{x^2-x-1}\right)-\frac{1}{2}\sqrt{\frac{1}{5}+\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1-2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)-\frac{1}{2}\sqrt{\frac{1}{5}-\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1+2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)$$

Rubi [C] time = 1.91, antiderivative size = 611, normalized size of antiderivative = 4.66, number of steps used = 27, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2056, 6725, 716, 1098, 934, 168, 538, 537}

$$\frac{\sqrt{x}\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}\sqrt{\frac{\left(1+\sqrt{5}\right)x+2}{\left(1-\sqrt{5}\right)x+2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-\left(\left(1-\sqrt{5}\right)x\right)-2}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{\left(1-\sqrt{5}\right)x+2}}\sqrt{x^3-x^2-x}}-\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(\frac{1}{2}\left(-1-\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{2\sqrt{x^3-x^2-x}}-\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(-\frac{1}{2}i\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{2\sqrt{x^3-x^2-x}}-\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(\frac{1}{2}i\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{2\sqrt{x^3-x^2-x}}-\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\Pi\left(\frac{1}{2}\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{2\sqrt{x^3-x^2-x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^4)/(Sqrt[-x - x^2 + x^3]*(-1 + x^4)),x]
```

```
[Out] (Sqrt[x]*Sqrt[-2 - (1 - Sqrt[5])*x]*Sqrt[(2 + (1 + Sqrt[5])*x)/(2 + (1 - Sq
rt[5])*x)]*EllipticF[ArcSin[(Sqrt[2]*5^(1/4)*Sqrt[x])/Sqrt[-2 - (1 - Sqrt[5
])*x]], (5 - Sqrt[5])/10])/(5^(1/4)*Sqrt[(2 + (1 - Sqrt[5])*x)^(-1)]*Sqrt[-
x - x^2 + x^3]) - (Sqrt[3 + Sqrt[5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[
1 - (2*x)/(1 + Sqrt[5])]*EllipticPi[(-1 - Sqrt[5])/2, ArcSin[Sqrt[2/(1 + Sq
rt[5])]*Sqrt[x]], (-3 - Sqrt[5])/2])/(2*Sqrt[-x - x^2 + x^3]) - (Sqrt[3 + S
qrt[5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1 + Sqrt[5])]*Elli
pticPi[(-1/2*I)*(1 + Sqrt[5]), ArcSin[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]], (-3 -
 Sqrt[5])/2])/(2*Sqrt[-x - x^2 + x^3]) - (Sqrt[3 + Sqrt[5]]*Sqrt[x]*Sqrt[-1
 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1 + Sqrt[5])]*EllipticPi[(I/2)*(1 + Sqrt[
5]), ArcSin[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]], (-3 - Sqrt[5])/2])/(2*Sqrt[-x -
 x^2 + x^3]) - (Sqrt[3 + Sqrt[5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 -
 (2*x)/(1 + Sqrt[5])]*EllipticPi[(1 + Sqrt[5])/2, ArcSin[Sqrt[2/(1 + Sqrt[5
])]*Sqrt[x]], (-3 - Sqrt[5])/2])/(2*Sqrt[-x - x^2 + x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 716

```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2,
Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; FreeQ
```

```
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^4}{\sqrt{-x-x^2+x^3}\left(-1+x^4\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{1+x^4}{\sqrt{x}\sqrt{-1-x+x^2}\left(-1+x^4\right)}\,dx}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{-1-x+x^2}}+\frac{2}{\sqrt{x}\sqrt{-1-x+x^2}\left(-1+x^4\right)}\right)dx}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-1-x+x^2}}\,dx}{\sqrt{-x-x^2+x^3}} + \frac{\left(2\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-1-x+x^2}\left(-1\cdots\right.}}{\sqrt{-x-x^2+x^3}}\cdots$$

$$= \frac{\left(2\sqrt{x}\sqrt{-1-x+x^2}\right)\int \left(-\frac{1}{2\sqrt{x}\left(1-x^2\right)\sqrt{-1-x+x^2}}-\frac{1}{2\sqrt{x}\left(1+x^2\right)\sqrt{-1-x+x^2}}\right)dx}{\sqrt{-x-x^2+x^3}} + \cdots$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - \cdots$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - \cdots$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - \cdots$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - \cdots$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} + \cdots$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} + \cdots$$

$$= \frac{\sqrt{x}\sqrt{-2-\left(1-\sqrt{5}\right)x}\sqrt{\frac{2+\left(1+\sqrt{5}\right)x}{2+\left(1-\sqrt{5}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{5}\sqrt{x}}{\sqrt{-2-\left(1-\sqrt{5}\right)x}}\right)|\frac{1}{10}\left(5-\sqrt{5}\right)\right)}{\sqrt[4]{5}\sqrt{\frac{1}{2+\left(1-\sqrt{5}\right)x}}\sqrt{-x-x^2+x^3}} - \cdots$$

Mathematica [C] time = 8.75, size = 1848, normalized size = 14.11

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^4)/(Sqrt[-x - x^2 + x^3]*(-1 + x^4)),x]
```

```
[Out] -(((-1 + x^(-4))*x^(3/2)*(1 + x^4)*(((-4*I)*Sqrt[1 - x^(-2) - x^(-1)]*Ellip
ticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], -3/2 - Sqrt[5]/2])/Sqrt[-1 +
 Sqrt[5]] + ((2*I)*Sqrt[1 - x^(-2) - x^(-1)]*EllipticPi[(-1/2*I)*(1 + Sqrt[
5]), I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], (-3 - Sqrt[5])/2])/Sqrt[-1 +
 Sqrt[5]] + ((2*I)*Sqrt[1 - x^(-2) - x^(-1)]*EllipticPi[(I/2)*(1 + Sqrt[5])
, I*ArcSinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], (-3 - Sqrt[5])/2])/Sqrt[-1 + Sq
rt[5]] + ((2*I)*Sqrt[1 - x^(-2) - x^(-1)]*EllipticPi[(1 + Sqrt[5])/2, I*Arc
Sinh[Sqrt[2/(1 + Sqrt[5])]/Sqrt[x]], (-3 - Sqrt[5])/2])/Sqrt[-1 + Sqrt[5]]
+ ((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5])] - 2/S
qrt[x])^2*Sqrt[(I*Sqrt[2*(1 + Sqrt[5])] + 2/Sqrt[x])/((1 + 2*I)*Sqrt[2] - S
qrt[10] + (2*Sqrt[-1 + Sqrt[5]])/Sqrt[x] - ((2*I)*Sqrt[1 + Sqrt[5]])/Sqrt[x
])]*Sqrt[(I*Sqrt[2*(1 + Sqrt[5])] - 2/Sqrt[x])/(Sqrt[2]*((-1 + 2*I) + Sqrt[
5]) - (2*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]]))/Sqrt[x])]*Sqrt[(Sqrt[2
]*((-1 - 2*I) + Sqrt[5]) + (2*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]]))/S
qrt[x])/(Sqrt[2]*((-1 + 2*I) + Sqrt[5]) - (2*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1
 + Sqrt[5]]))/Sqrt[x])]*((2 + Sqrt[2*(-1 + Sqrt[5])])*EllipticF[ArcSin[Sqrt
[((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5])] + 2/Sq
rt[x]))/((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5])]
 - 2/Sqrt[x]))]], -3/5 - (4*I)/5] - 2*Sqrt[2*(-1 + Sqrt[5])]*EllipticPi[((-
2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]]))/((2
 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])), Arc
Sin[Sqrt[((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5])
] + 2/Sqrt[x]))/((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + S
qrt[5])] - 2/Sqrt[x]))]], -3/5 - (4*I)/5]))/((-2 + Sqrt[2*(-1 + Sqrt[5])])*
(2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])) -
((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5])] - 2/Sqr
t[x])^2*Sqrt[(I*Sqrt[2*(1 + Sqrt[5])] + 2/Sqrt[x])/((1 + 2*I)*Sqrt[2] - Sqr
t[10] + (2*Sqrt[-1 + Sqrt[5]])/Sqrt[x] - ((2*I)*Sqrt[1 + Sqrt[5]])/Sqrt[x])
]*Sqrt[(I*Sqrt[2*(1 + Sqrt[5])] - 2/Sqrt[x])/(Sqrt[2]*((-1 + 2*I) + Sqrt[5]
) - (2*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]]))/Sqrt[x])]*Sqrt[(Sqrt[2]*
((-1 - 2*I) + Sqrt[5]) + (2*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]]))/Sqr
t[x])/(Sqrt[2]*((-1 + 2*I) + Sqrt[5]) - (2*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 +
 Sqrt[5]]))/Sqrt[x])]*((-2 + Sqrt[2*(-1 + Sqrt[5])])*EllipticF[ArcSin[Sqrt[
((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5])] + 2/Sqr
t[x]))/((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5])]
- 2/Sqrt[x]))]], -3/5 - (4*I)/5] - 2*Sqrt[2*(-1 + Sqrt[5])]*EllipticPi[((2
+ Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]]))/((-2
+ Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])), ArcS
in[Sqrt[((Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sqrt[5])]
 + 2/Sqrt[x]))/((Sqrt[-1 + Sqrt[5]] + I*Sqrt[1 + Sqrt[5]])*(Sqrt[2*(-1 + Sq
rt[5])] - 2/Sqrt[x]))]], -3/5 - (4*I)/5]))/((-2 + Sqrt[2*(-1 + Sqrt[5])])*(
2 + Sqrt[2*(-1 + Sqrt[5])])*(Sqrt[-1 + Sqrt[5]] - I*Sqrt[1 + Sqrt[5]]))))/(
Sqrt[2]*Sqrt[x*(-1 - x + x^2)]*(-x^(-4) + x^4)))
```

IntegrateAlgebraic [A] time = 0.32, size = 131, normalized size = 1.00

$$-\tan^{-1}\left(\frac{\sqrt{x^3-x^2-x}}{x^2-x-1}\right)-\frac{1}{2}\sqrt{\frac{1}{5}+\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1-2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)-\frac{1}{2}\sqrt{\frac{1}{5}-\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1+2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)/(Sqrt[-x - x^2 + x^3]*(-1 + x^4)),x]
```

```
[Out] -ArcTan[Sqrt[-x - x^2 + x^3]/(-1 - x + x^2)] - (Sqrt[1/5 + (2*I)/5]*ArcTan[
(Sqrt[1 - 2*I]*Sqrt[-x - x^2 + x^3])/(-1 - x + x^2)])/2 - (Sqrt[1/5 - (2*I)
/5]*ArcTan[(Sqrt[1 + 2*I]*Sqrt[-x - x^2 + x^3])/(-1 - x + x^2)])/2
```

fricas [B] time = 0.91, size = 2485, normalized size = 18.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^3-x^2-x)^(1/2)/(x^4-1),x, algorithm="fricas")
```

```
[Out] 1/160*5^(1/4)*(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 5)*log(5*(5*x^4 -
20*x^3 + 5^(1/4)*sqrt(x^3 - x^2 - x)*(sqrt(5)*sqrt(2)*(x^2 - 6*x - 1) - 5*s
qrt(2)*(x^2 - 2*x - 1))*sqrt(sqrt(5) + 5) + 30*x^2 + 20*sqrt(5)*(x^3 - x^2
- x) + 20*x + 5)/(x^4 + 2*x^2 + 1)) - 1/160*5^(1/4)*(sqrt(5)*sqrt(2) - sqrt
(2))*sqrt(sqrt(5) + 5)*log(5*(5*x^4 - 20*x^3 - 5^(1/4)*sqrt(x^3 - x^2 - x)*
(sqrt(5)*sqrt(2)*(x^2 - 6*x - 1) - 5*sqrt(2)*(x^2 - 2*x - 1))*sqrt(sqrt(5)
+ 5) + 30*x^2 + 20*sqrt(5)*(x^3 - x^2 - x) + 20*x + 5)/(x^4 + 2*x^2 + 1)) -
 1/20*5^(1/4)*sqrt(2)*sqrt(sqrt(5) + 5)*arctan(-1/200*(100*x^11 + 1300*x^10
 - 6700*x^9 - 4400*x^8 + 28400*x^7 + 1400*x^6 - 28400*x^5 - 4400*x^4 + 6700
*x^3 + 1300*x^2 + 5*sqrt(x^3 - x^2 - x)*(5^(3/4)*(sqrt(5)*sqrt(2)*(x^10 - 4
*x^9 - 17*x^8 + 56*x^7 + 78*x^6 - 136*x^5 - 78*x^4 + 56*x^3 + 17*x^2 - 4*x
- 1) - sqrt(2)*(x^10 + 8*x^9 - 57*x^8 - 24*x^7 + 294*x^6 + 64*x^5 - 294*x^4
 - 24*x^3 + 57*x^2 + 8*x - 1)) + 4*5^(1/4)*(sqrt(5)*sqrt(2)*(3*x^9 + 7*x^8
+ 14*x^7 - 81*x^6 - 10*x^5 + 81*x^4 + 14*x^3 - 7*x^2 + 3*x) - 5*sqrt(2)*(x^
9 + 9*x^8 - 18*x^7 - 15*x^6 + 26*x^5 + 15*x^4 - 18*x^3 - 9*x^2 + x)))*sqrt(
sqrt(5) + 5) - sqrt(5)*(240*x^10 + 160*x^9 - 1680*x^8 - 480*x^7 + 3200*x^6
+ 480*x^5 - 1680*x^4 - 160*x^3 + 240*x^2 + sqrt(x^3 - x^2 - x)*(5^(3/4)*(sq
rt(5)*sqrt(2)*(x^10 - 6*x^9 - 25*x^8 + 120*x^7 - 58*x^6 - 196*x^5 + 58*x^4
+ 120*x^3 + 25*x^2 - 6*x - 1) - sqrt(2)*(x^10 - 2*x^9 - 17*x^8 + 216*x^7 -
306*x^6 - 396*x^5 + 306*x^4 + 216*x^3 + 17*x^2 - 2*x - 1)) + 4*5^(1/4)*(sqr
t(5)*sqrt(2)*(3*x^9 - 3*x^8 - 56*x^7 + 69*x^6 + 90*x^5 - 69*x^4 - 56*x^3 +
3*x^2 + 3*x) - 5*sqrt(2)*(x^9 + 3*x^8 - 28*x^7 + 11*x^6 + 54*x^5 - 11*x^4 -
 28*x^3 - 3*x^2 + x)))*sqrt(sqrt(5) + 5) + 4*sqrt(5)*(5*x^11 - 25*x^10 - 10
5*x^9 + 440*x^8 + 50*x^7 - 830*x^6 - 50*x^5 + 440*x^4 + 105*x^3 - 25*x^2 -
sqrt(5)*(x^11 + 3*x^10 - 37*x^9 + 96*x^8 - 6*x^7 - 166*x^6 + 6*x^5 + 96*x^4
 + 37*x^3 + 3*x^2 - x) - 5*x) - 80*sqrt(5)*(x^10 + 6*x^9 - 23*x^8 - 2*x^7 +
 40*x^6 + 2*x^5 - 23*x^4 - 6*x^3 + x^2))*sqrt((5*x^4 - 20*x^3 + 5^(1/4)*sqr
t(x^3 - x^2 - x)*(sqrt(5)*sqrt(2)*(x^2 - 6*x - 1) - 5*sqrt(2)*(x^2 - 2*x -
1))*sqrt(sqrt(5) + 5) + 30*x^2 + 20*sqrt(5)*(x^3 - x^2 - x) + 20*x + 5)/(x^
4 + 2*x^2 + 1)) + 20*sqrt(5)*(5*x^11 - 15*x^10 - 15*x^9 + 20*x^8 - 20*x^7 +
 70*x^6 + 20*x^5 + 20*x^4 + 15*x^3 - 15*x^2 - sqrt(5)*(x^11 + 13*x^10 - 67*
x^9 - 44*x^8 + 284*x^7 + 14*x^6 - 284*x^5 - 44*x^4 + 67*x^3 + 13*x^2 - x) -
 5*x) - 100*sqrt(5)*(x^11 - 3*x^10 - 3*x^9 + 4*x^8 - 4*x^7 + 14*x^6 + 4*x^5
 + 4*x^4 + 3*x^3 - 3*x^2 - x) - 100*x)/(x^11 - 9*x^10 - 45*x^9 + 180*x^8 +
18*x^7 - 326*x^6 - 18*x^5 + 180*x^4 + 45*x^3 - 9*x^2 - x)) - 1/20*5^(1/4)*s
qrt(2)*sqrt(sqrt(5) + 5)*arctan(1/200*(100*x^11 + 1300*x^10 - 6700*x^9 - 44
00*x^8 + 28400*x^7 + 1400*x^6 - 28400*x^5 - 4400*x^4 + 6700*x^3 + 1300*x^2
- 5*sqrt(x^3 - x^2 - x)*(5^(3/4)*(sqrt(5)*sqrt(2)*(x^10 - 4*x^9 - 17*x^8 +
56*x^7 + 78*x^6 - 136*x^5 - 78*x^4 + 56*x^3 + 17*x^2 - 4*x - 1) - sqrt(2)*(
x^10 + 8*x^9 - 57*x^8 - 24*x^7 + 294*x^6 + 64*x^5 - 294*x^4 - 24*x^3 + 57*x
^2 + 8*x - 1)) + 4*5^(1/4)*(sqrt(5)*sqrt(2)*(3*x^9 + 7*x^8 + 14*x^7 - 81*x^
6 - 10*x^5 + 81*x^4 + 14*x^3 - 7*x^2 + 3*x) - 5*sqrt(2)*(x^9 + 9*x^8 - 18*x
^7 - 15*x^6 + 26*x^5 + 15*x^4 - 18*x^3 - 9*x^2 + x)))*sqrt(sqrt(5) + 5) - s
qrt(5)*(240*x^10 + 160*x^9 - 1680*x^8 - 480*x^7 + 3200*x^6 + 480*x^5 - 1680
*x^4 - 160*x^3 + 240*x^2 - sqrt(x^3 - x^2 - x)*(5^(3/4)*(sqrt(5)*sqrt(2)*(x
^10 - 6*x^9 - 25*x^8 + 120*x^7 - 58*x^6 - 196*x^5 + 58*x^4 + 120*x^3 + 25*x
^2 - 6*x - 1) - sqrt(2)*(x^10 - 2*x^9 - 17*x^8 + 216*x^7 - 306*x^6 - 396*x^
5 + 306*x^4 + 216*x^3 + 17*x^2 - 2*x - 1)) + 4*5^(1/4)*(sqrt(5)*sqrt(2)*(3*
x^9 - 3*x^8 - 56*x^7 + 69*x^6 + 90*x^5 - 69*x^4 - 56*x^3 + 3*x^2 + 3*x) - 5
*sqrt(2)*(x^9 + 3*x^8 - 28*x^7 + 11*x^6 + 54*x^5 - 11*x^4 - 28*x^3 - 3*x^2
+ x)))*sqrt(sqrt(5) + 5) + 4*sqrt(5)*(5*x^11 - 25*x^10 - 105*x^9 + 440*x^8
+ 50*x^7 - 830*x^6 - 50*x^5 + 440*x^4 + 105*x^3 - 25*x^2 - sqrt(5)*(x^11 +
3*x^10 - 37*x^9 + 96*x^8 - 6*x^7 - 166*x^6 + 6*x^5 + 96*x^4 + 37*x^3 + 3*x^
2 - x) - 5*x) - 80*sqrt(5)*(x^10 + 6*x^9 - 23*x^8 - 2*x^7 + 40*x^6 + 2*x^5
```

```
- 23*x^4 - 6*x^3 + x^2))*sqrt((5*x^4 - 20*x^3 - 5^(1/4)*sqrt(x^3 - x^2 - x)
*(sqrt(5)*sqrt(2)*(x^2 - 6*x - 1) - 5*sqrt(2)*(x^2 - 2*x - 1))*sqrt(sqrt(5)
 + 5) + 30*x^2 + 20*sqrt(5)*(x^3 - x^2 - x) + 20*x + 5)/(x^4 + 2*x^2 + 1))
+ 20*sqrt(5)*(5*x^11 - 15*x^10 - 15*x^9 + 20*x^8 - 20*x^7 + 70*x^6 + 20*x^5
 + 20*x^4 + 15*x^3 - 15*x^2 - sqrt(5)*(x^11 + 13*x^10 - 67*x^9 - 44*x^8 + 2
84*x^7 + 14*x^6 - 284*x^5 - 44*x^4 + 67*x^3 + 13*x^2 - x) - 5*x) - 100*sqrt
(5)*(x^11 - 3*x^10 - 3*x^9 + 4*x^8 - 4*x^7 + 14*x^6 + 4*x^5 + 4*x^4 + 3*x^3
 - 3*x^2 - x) - 100*x)/(x^11 - 9*x^10 - 45*x^9 + 180*x^8 + 18*x^7 - 326*x^6
 - 18*x^5 + 180*x^4 + 45*x^3 - 9*x^2 - x)) + 1/2*arctan(1/2*(x^2 - 2*x - 1)
/sqrt(x^3 - x^2 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^4-1\right)\sqrt{x^3-x^2-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^3-x^2-x)^(1/2)/(x^4-1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/((x^4 - 1)*sqrt(x^3 - x^2 - x)), x)
```

maple [C] time = 0.03, size = 1009, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^3-x^2-x)^(1/2)/(x^4-1),x)
```

```
[Out] 2/5*(1/2*5^(1/2)-1/2)*((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x-
1/2-1/2*5^(1/2))*5^(1/2))^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1
/2)*EllipticF(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),1/5*5^(1/2)*((1
/2*5^(1/2)-1/2)*5^(1/2))^(1/2))+1/5*(1/2*5^(1/2)-1/2)*((x-1/2+1/2*5^(1/2))/
(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x-1/2-1/2*5^(1/2))*5^(1/2))^(1/2)*(-x/(1/2*5^
(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(-1/2-1/2*5^(1/2))*EllipticPi(((x-1/2+1
/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(-1/2-1/2*5^(1/2)),1
/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))-1/5*(1/2*5^(1/2)-1/2)*((x-1/2
+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x-1/2-1/2*5^(1/2))*5^(1/2))^(1/
2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(3/2-1/2*5^(1/2))*Ellipti
cPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(3/2-1/
2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))+1/10*I*(x/(1/2*5^
(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x
*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(
1/2)/(1/2-I-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))
^(1/2),(1/2-1/2*5^(1/2))/(1/2-I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)
*5^(1/2))^(1/2))*5^(1/2)-1/10*I*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+
1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*
(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(1/2-I-1/2*5^(1/2))*Elliptic
Pi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2-I-1
/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))-1/10*I*(x/(1/2*5
^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*
x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^
(1/2)/(1/2+I-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2)
)^(1/2),(1/2-1/2*5^(1/2))/(1/2+I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2
)*5^(1/2))^(1/2))*5^(1/2)+1/10*I*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)
+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)
*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(1/2+I-1/2*5^(1/2))*Ellipti
cPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2+I-
1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^4-1\right)\sqrt{x^3-x^2-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^3-x^2-x)^(1/2)/(x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/((x^4 - 1)*sqrt(x^3 - x^2 - x)), x)
```

mupad [B] time = 0.85, size = 658, normalized size = 5.02

$$\frac{2\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}}{\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}-\frac{\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\Pi\left(-\frac{\sqrt{5}}{2}-\frac{1}{2};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)}{\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}-\frac{\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\Pi\left(\frac{\sqrt{5}}{2}+\frac{1}{2};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)}{\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}-\frac{\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\Pi\left(-\frac{\sqrt{5}\,1\mathrm{i}}{2}-\frac{1}{2}\mathrm{i};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)}{\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}-\frac{\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\Pi\left(\frac{\sqrt{5}\,1\mathrm{i}}{2}+\frac{1}{2}\mathrm{i};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)}{\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/((x^4 - 1)*(x^3 - x^2 - x)^(1/2)),x)
```

```
[Out] (2*(5^(1/2)/2 + 1/2)*(x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/2 - 1/2)/(5^
(1/2)/2 - 1/2))^(1/2)*ellipticF(asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2
)/2 + 1/2)/(5^(1/2)/2 - 1/2))*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 + 1/2))^(1/
2))/(x^3 - x^2 - x*(5^(1/2)/2 - 1/2)*(5^(1/2)/2 + 1/2))^(1/2) - ((5^(1/2)/2
 + 1/2)*(x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/2 - 1/2)/(5^(1/2)/2 - 1/2
))^(1/2)*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 + 1/2))^(1/2)*ellipticPi(- 5^(1/
2)/2 - 1/2, asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/2 + 1/2)/(5^(1/2)/
2 - 1/2)))/(x^3 - x^2 - x*(5^(1/2)/2 - 1/2)*(5^(1/2)/2 + 1/2))^(1/2) - ((5^
(1/2)/2 + 1/2)*(x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/2 - 1/2)/(5^(1/2)/
2 - 1/2))^(1/2)*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 + 1/2))^(1/2)*ellipticPi(
5^(1/2)/2 + 1/2, asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/2 + 1/2)/(5^(
1/2)/2 - 1/2)))/(x^3 - x^2 - x*(5^(1/2)/2 - 1/2)*(5^(1/2)/2 + 1/2))^(1/2) -
 ((5^(1/2)/2 + 1/2)*(x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/2 - 1/2)/(5^(
1/2)/2 - 1/2))^(1/2)*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 + 1/2))^(1/2)*ellipt
icPi(- (5^(1/2)*1i)/2 - 1i/2, asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/
2 + 1/2)/(5^(1/2)/2 - 1/2)))/(x^3 - x^2 - x*(5^(1/2)/2 - 1/2)*(5^(1/2)/2 +
1/2))^(1/2) - ((5^(1/2)/2 + 1/2)*(x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/
2 - 1/2)/(5^(1/2)/2 - 1/2))^(1/2)*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 + 1/2))
^(1/2)*ellipticPi((5^(1/2)*1i)/2 + 1i/2, asin((x/(5^(1/2)/2 + 1/2))^(1/2)),
 -(5^(1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2)))/(x^3 - x^2 - x*(5^(1/2)/2 - 1/2)*(5
^(1/2)/2 + 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\sqrt{x\left(x^2-x-1\right)}\,(x-1)\,(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**3-x**2-x)**(1/2)/(x**4-1),x)
```

```
[Out] Integral((x**4 + 1)/(sqrt(x*(x**2 - x - 1))*(x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.1584 $\int \frac{(-1+x^3)\sqrt[3]{1+x^6}}{x^2(1+x^3)}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt[3]{x^6+1}}{x}-\frac{1}{3}\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^6+1}+2x\right)+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2^{2/3}\sqrt[3]{x^6+1}-x}\right)}{\sqrt{3}}+\frac{\log\left(2^{2/3}\sqrt[3]{x^6+1}\,x-\sqrt[3]{2}\left(x^6+1\right)^{2/3}-2x^2\right)}{3\;2^{2/3}}$$

Rubi [F] time = 0.59, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^3)\sqrt[3]{1+x^6}}{x^2(1+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^3)*(1 + x^6)^(1/3))/(x^2*(1 + x^3)),x]
```

```
[Out] Hypergeometric2F1[-1/3, -1/6, 5/6, -x^6]/x - (2*Defer[Int][(1 + x^6)^(1/3)/
(1 + x), x])/3 + (2*(1 - I*Sqrt[3])*Defer[Int][(1 + x^6)^(1/3)/(-1 - I*Sqrt
[3] + 2*x), x])/3 + (2*(1 + I*Sqrt[3])*Defer[Int][(1 + x^6)^(1/3)/(-1 + I*S
qrt[3] + 2*x), x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^3)\sqrt[3]{1+x^6}}{x^2(1+x^3)}\,dx &= \int \left(-\frac{\sqrt[3]{1+x^6}}{x^2}-\frac{2\sqrt[3]{1+x^6}}{3(1+x)}+\frac{2(1+x)\sqrt[3]{1+x^6}}{3\left(1-x+x^2\right)}\right)dx\\
&= -\left(\frac{2}{3}\int \frac{\sqrt[3]{1+x^6}}{1+x}\,dx\right)+\frac{2}{3}\int \frac{(1+x)\sqrt[3]{1+x^6}}{1-x+x^2}\,dx-\int \frac{\sqrt[3]{1+x^6}}{x^2}\,dx\\
&= \frac{{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-x^6\right)}{x}-\frac{2}{3}\int \frac{\sqrt[3]{1+x^6}}{1+x}\,dx+\frac{2}{3}\int \left(\frac{\left(1-i\sqrt{3}\right)\sqrt[3]{1+x^6}}{-1-i\sqrt{3}+2x}+\frac{\left(1+i\sqrt{3}\right)}{-1+i\sqrt{}}\right.\\
&= \frac{{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-x^6\right)}{x}-\frac{2}{3}\int \frac{\sqrt[3]{1+x^6}}{1+x}\,dx+\frac{1}{3}\left(2\left(1-i\sqrt{3}\right)\right)\int \frac{\sqrt[3]{1+x^6}}{-1-i\sqrt{3}+2x}\,dx
\end{aligned}$$

Mathematica [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^3)\sqrt[3]{1+x^6}}{x^2(1+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^3)*(1 + x^6)^(1/3))/(x^2*(1 + x^3)),x]
```

```
[Out] Integrate[((-1 + x^3)*(1 + x^6)^(1/3))/(x^2*(1 + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.72, size = 131, normalized size = 1.00

$$\frac{\sqrt[3]{x^6+1}}{x}-\frac{1}{3}\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^6+1}+2x\right)+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2^{2/3}\sqrt[3]{x^6+1}-x}\right)}{\sqrt{3}}+\frac{\log\left(2^{2/3}\sqrt[3]{x^6+1}\,x-\sqrt[3]{2}\left(x^6+1\right)^{2/3}-2x^2\right)}{3\;2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)*(1 + x^6)^(1/3))/(x^2*(1 + x^3)),x]
```

```
[Out] (1 + x^6)^(1/3)/x + (2^(1/3)*ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 + x^6)^(1/
3))])/Sqrt[3] - (2^(1/3)*Log[2*x + 2^(2/3)*(1 + x^6)^(1/3)])/3 + Log[-2*x^2
 + 2^(2/3)*x*(1 + x^6)^(1/3) - 2^(1/3)*(1 + x^6)^(2/3)]/(3*2^(2/3))
```

fricas [B] time = 31.70, size = 327, normalized size = 2.50

$$\frac{2\sqrt{3}\,(-2)^{\frac{1}{3}}\,x\arctan\left(\frac{6\sqrt{3}\,(-2)^{\frac{2}{3}}\left(x^{14}-14x^{11}+6x^{8}-14x^{5}+x^{2}\right)\left(x^{6}+1\right)^{\frac{1}{3}}+6\sqrt{3}\,(-2)^{\frac{1}{3}}\left(x^{13}-2x^{10}-6x^{7}-2x^{4}+x\right)\left(x^{6}+1\right)^{\frac{2}{3}}+\sqrt{3}\left(x^{18}-30x^{15}+51x^{12}-52x^{9}+51x^{6}-30x^{3}+1\right)}{3\left(x^{18}+6x^{15}-93x^{12}+20x^{9}-93x^{6}+6x^{3}+1\right)}\right)+2\,(-2)^{\frac{1}{3}}\,x\log\left(-\frac{6\,(-2)^{\frac{1}{3}}\left(x^{6}+1\right)^{\frac{1}{3}}x^{2}-(-2)^{\frac{2}{3}}\left(x^{6}+2x^{3}+1\right)-6\left(x^{6}+1\right)^{\frac{2}{3}}x}{x^{6}+2x^{3}+1}\right)-(-2)^{\frac{1}{3}}\,x\log\left(-\frac{3\,(-2)^{\frac{2}{3}}\left(x^{7}-4x^{4}+x\right)\left(x^{6}+1\right)^{\frac{2}{3}}+(-2)^{\frac{1}{3}}\left(x^{12}-14x^{9}+6x^{6}-14x^{3}+1\right)-12\left(x^{8}-x^{5}+x^{2}\right)\left(x^{6}+1\right)^{\frac{1}{3}}}{x^{12}+4x^{9}+6x^{6}+4x^{3}+1}\right)+18\left(x^{6}+1\right)^{\frac{1}{3}}}{18\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6+1)^(1/3)/x^2/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/18*(2*sqrt(3)*(-2)^(1/3)*x*arctan(1/3*(6*sqrt(3)*(-2)^(2/3)*(x^14 - 14*x^
11 + 6*x^8 - 14*x^5 + x^2)*(x^6 + 1)^(1/3) + 6*sqrt(3)*(-2)^(1/3)*(x^13 - 2
*x^10 - 6*x^7 - 2*x^4 + x)*(x^6 + 1)^(2/3) + sqrt(3)*(x^18 - 30*x^15 + 51*x
^12 - 52*x^9 + 51*x^6 - 30*x^3 + 1))/(x^18 + 6*x^15 - 93*x^12 + 20*x^9 - 93
*x^6 + 6*x^3 + 1)) + 2*(-2)^(1/3)*x*log(-(6*(-2)^(1/3)*(x^6 + 1)^(1/3)*x^2
 - (-2)^(2/3)*(x^6 + 2*x^3 + 1) - 6*(x^6 + 1)^(2/3)*x)/(x^6 + 2*x^3 + 1)) -
(-2)^(1/3)*x*log(-(3*(-2)^(2/3)*(x^7 - 4*x^4 + x)*(x^6 + 1)^(2/3) + (-2)^(1
/3)*(x^12 - 14*x^9 + 6*x^6 - 14*x^3 + 1) - 12*(x^8 - x^5 + x^2)*(x^6 + 1)^(
1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1)) + 18*(x^6 + 1)^(1/3))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}+1\right)^{\frac{1}{3}}\left(x^{3}-1\right)}{\left(x^{3}+1\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6+1)^(1/3)/x^2/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 1)^(1/3)*(x^3 - 1)/((x^3 + 1)*x^2), x)
```

maple [F] time = 2.42, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}-1\right)\left(x^{6}+1\right)^{\frac{1}{3}}}{x^{2}\left(x^{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)*(x^6+1)^(1/3)/x^2/(x^3+1),x)
```

```
[Out] int((x^3-1)*(x^6+1)^(1/3)/x^2/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}+1\right)^{\frac{1}{3}}\left(x^{3}-1\right)}{\left(x^{3}+1\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6+1)^(1/3)/x^2/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + 1)^(1/3)*(x^3 - 1)/((x^3 + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3-1\right)\,\left(x^6+1\right)^{1/3}}{x^2\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)*(x^6 + 1)^(1/3))/(x^2*(x^3 + 1)),x)
```

```
[Out] int(((x^3 - 1)*(x^6 + 1)^(1/3))/(x^2*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\left(x^2+1\right)\left(x^4-x^2+1\right)}\,(x-1)\left(x^2+x+1\right)}{x^2\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)*(x**6+1)**(1/3)/x**2/(x**3+1),x)
```

```
[Out] Integral(((x**2 + 1)*(x**4 - x**2 + 1))**(1/3)*(x - 1)*(x**2 + x + 1)/(x**2
*(x + 1)*(x**2 - x + 1)), x)
```

3.1585 $\int \frac{(-2+x^6)\sqrt[3]{2+x^6}}{x^2(2+2x^3+x^6)}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt[3]{x^6+2}}{x}-\frac{1}{3}\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^6+2}+2x\right)+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^6+2}-x}\right)}{\sqrt{3}}+\frac{\log\left(2^{2/3}\sqrt[3]{x^6+2}\,x-\sqrt[3]{2}\left(x^6+2\right)^{2/3}-2x^2\right)}{3\ 2^{2/3}}$$

Rubi [C] time = 1.18, antiderivative size = 195, normalized size of antiderivative = 1.49, number of steps used = 29, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6728, 364, 1562, 465, 429, 510}

$$\frac{ix^5F_1\left(\frac{5}{6};1,-\frac{1}{3};\frac{11}{6};-\frac{ix^6}{2},-\frac{x^6}{2}\right)}{5\ 2^{2/3}}-\frac{ix^5F_1\left(\frac{5}{6};1,-\frac{1}{3};\frac{11}{6};\frac{ix^6}{2},-\frac{x^6}{2}\right)}{5\ 2^{2/3}}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};-\frac{ix^6}{2},-\frac{x^6}{2}\right)}{2^{2/3}}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)x^2F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};\frac{ix^6}{2},-\frac{x^6}{2}\right)}{2^{2/3}}+\frac{\sqrt[3]{2}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{x^6}{2}\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-2 + x^6)*(2 + x^6)^(1/3))/(x^2*(2 + 2*x^3 + x^6)),x]
```

```
[Out] ((1/2 - I/2)*x^2*AppellF1[1/3, 1, -1/3, 4/3, (-1/2*I)*x^6, -1/2*x^6])/2^(2/
3) + ((1/2 + I/2)*x^2*AppellF1[1/3, 1, -1/3, 4/3, (I/2)*x^6, -1/2*x^6])/2^(
2/3) + ((I/5)*x^5*AppellF1[5/6, 1, -1/3, 11/6, (-1/2*I)*x^6, -1/2*x^6])/2^(
2/3) - ((I/5)*x^5*AppellF1[5/6, 1, -1/3, 11/6, (I/2)*x^6, -1/2*x^6])/2^(2/3
) + (2^(1/3)*Hypergeometric2F1[-1/3, -1/6, 5/6, -1/2*x^6])/x
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1562

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(
2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)/(d^2 - e^2*x^(2*n)))^(-q), x], x]
, x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !IntegerQ[p]
```

```
&& ILtQ[q, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-2+x^6\right)\sqrt[3]{2+x^6}}{x^2\left(2+2x^3+x^6\right)}\,dx &= \int \left(-\frac{\sqrt[3]{2+x^6}}{x^2}+\frac{2x\left(1+x^3\right)\sqrt[3]{2+x^6}}{2+2x^3+x^6}\right)dx \\
&= 2\int \frac{x\left(1+x^3\right)\sqrt[3]{2+x^6}}{2+2x^3+x^6}\,dx-\int \frac{\sqrt[3]{2+x^6}}{x^2}\,dx \\
&= \frac{\sqrt[3]{2}\;{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{x^6}{2}\right)}{x}+2\int \left(\frac{x\sqrt[3]{2+x^6}}{2+2x^3+x^6}+\frac{x^4\sqrt[3]{2+x^6}}{2+2x^3+x^6}\right)dx \\
&= \frac{\sqrt[3]{2}\;{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{x^6}{2}\right)}{x}+2\int \frac{x\sqrt[3]{2+x^6}}{2+2x^3+x^6}\,dx+2\int \frac{x^4\sqrt[3]{2+x^6}}{2+2x^3+x^6}\,dx \\
&= \frac{\sqrt[3]{2}\;{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{x^6}{2}\right)}{x}+2\int \left(\frac{ix\sqrt[3]{2+x^6}}{(-2+2i)-2x^3}+\frac{ix\sqrt[3]{2+x^6}}{(2+2i)+2x^3}\right)dx+2\int \Bigg(-\frac{(}{(}
\\
&= \frac{\sqrt[3]{2}\;{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{x^6}{2}\right)}{x}+(-2-2i)\int \frac{x\sqrt[3]{2+x^6}}{(-2+2i)-2x^3}\,dx+2i\int \frac{x\sqrt[3]{2+x^6}}{(-2+2i)-2x^3}
\\
&= \frac{\sqrt[3]{2}\;{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{x^6}{2}\right)}{x}+(-2-2i)\int \left(\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x\sqrt[3]{2+x^6}}{2i+x^6}-\frac{x^4\sqrt[3]{2+x^6}}{2\left(2i+x^6\right)}\right)dx+2
\\
&= \frac{\sqrt[3]{2}\;{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};-\frac{x^6}{2}\right)}{x}+i\int \frac{x^4\sqrt[3]{2+x^6}}{-2i+x^6}\,dx-i\int \frac{x^4\sqrt[3]{2+x^6}}{2i+x^6}\,dx+(1-i)\int \frac{x\sqrt[3]{}}{-}
\\
&= \frac{ix^5F_1\left(\frac{5}{6};1,-\frac{1}{3};\frac{11}{6};-\frac{ix^6}{2},-\frac{x^6}{2}\right)}{5\;2^{2/3}}-\frac{ix^5F_1\left(\frac{5}{6};1,-\frac{1}{3};\frac{11}{6};\frac{ix^6}{2},-\frac{x^6}{2}\right)}{5\;2^{2/3}}+\frac{\sqrt[3]{2}\;{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};\right.}{x}
\\
&= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};-\frac{ix^6}{2},-\frac{x^6}{2}\right)}{2^{2/3}}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)x^2F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};\frac{ix^6}{2},-\frac{x^6}{2}\right)}{2^{2/3}}+\frac{ix^5F_1}{}
\end{aligned}
$$

Mathematica [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(-2+x^6\right)\sqrt[3]{2+x^6}}{x^2\left(2+2x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + x^6)*(2 + x^6)^(1/3))/(x^2*(2 + 2*x^3 + x^6)),x]
[Out] Integrate[((-2 + x^6)*(2 + x^6)^(1/3))/(x^2*(2 + 2*x^3 + x^6)), x]
```

IntegrateAlgebraic [A] time = 1.18, size = 131, normalized size = 1.00

$$\frac{\sqrt[3]{x^6+2}}{x}-\frac{1}{3}\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^6+2}+2x\right)+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^6+2}-x}\right)}{\sqrt{3}}+\frac{\log\left(2^{2/3}\sqrt[3]{x^6+2}\,x-\sqrt[3]{2}\left(x^6+2\right)^{2/3}-2x^2\right)}{3\;2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^6)*(2 + x^6)^(1/3))/(x^2*(2 + 2*x^3 + x^6)),x]
```

```
[Out] (2 + x^6)^(1/3)/x + (2^(1/3)*ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(2 + x^6)^(1/
3))])/Sqrt[3] - (2^(1/3)*Log[2*x + 2^(2/3)*(2 + x^6)^(1/3)])/3 + Log[-2*x^2
 + 2^(2/3)*x*(2 + x^6)^(1/3) - 2^(1/3)*(2 + x^6)^(2/3)]/(3*2^(2/3))
```

fricas [B] time = 174.84, size = 334, normalized size = 2.55

$$\frac{2\sqrt{3}\left(-2\right)^{\frac{1}{3}} x \arctan\left(\frac{6\sqrt{3}\left(-2\right)^{\frac{2}{3}}\left(x^{14}-14x^{11}+8x^{8}-28x^{5}+4x^{2}\right)\left(x^{6}+2\right)^{\frac{1}{3}}+6\sqrt{3}\left(-2\right)^{\frac{1}{3}}\left(x^{13}-2x^{10}-4x^{7}-4x^{4}+4x\right)\left(x^{6}+2\right)^{\frac{2}{3}}+\sqrt{3}\left(x^{18}-30x^{15}+54x^{12}-112x^{9}+108x^{6}-120x^{3}+8\right)}{3\left(x^{18}+6x^{15}-90x^{12}+32x^{9}-180x^{6}+24x^{3}+8\right)}\right)+2\left(-2\right)^{\frac{1}{3}} x \log\left(\frac{6\left(-2\right)^{\frac{1}{3}}\left(x^{6}+2\right)^{\frac{1}{3}}x^{2}-\left(-2\right)^{\frac{2}{3}}\left(x^{6}+2x^{3}+2\right)-6\left(x^{6}+2\right)^{\frac{2}{3}}x}{x^{6}+2x^{3}+2}\right)-\left(-2\right)^{\frac{1}{3}} x \log\left(-\frac{3\left(-2\right)^{\frac{2}{3}}\left(x^{7}-4x^{4}+2x\right)\left(x^{6}+2\right)^{\frac{2}{3}}+\left(-2\right)^{\frac{1}{3}}\left(x^{12}-14x^{9}+8x^{6}-28x^{3}+4\right)-12\left(x^{8}-x^{5}+2x^{2}\right)\left(x^{6}+2\right)^{\frac{1}{3}}}{x^{12}+4x^{9}+8x^{6}+8x^{3}+4}\right)+18\left(x^{6}+2\right)^{\frac{1}{3}}}{18x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6+2)^(1/3)/x^2/(x^6+2*x^3+2),x, algorithm="fricas")
```

```
[Out] 1/18*(2*sqrt(3)*(-2)^(1/3)*x*arctan(1/3*(6*sqrt(3)*(-2)^(2/3)*(x^14 - 14*x^
11 + 8*x^8 - 28*x^5 + 4*x^2)*(x^6 + 2)^(1/3) + 6*sqrt(3)*(-2)^(1/3)*(x^13 -
 2*x^10 - 4*x^7 - 4*x^4 + 4*x)*(x^6 + 2)^(2/3) + sqrt(3)*(x^18 - 30*x^15 +
54*x^12 - 112*x^9 + 108*x^6 - 120*x^3 + 8))/(x^18 + 6*x^15 - 90*x^12 + 32*x
^9 - 180*x^6 + 24*x^3 + 8)) + 2*(-2)^(1/3)*x*log((6*(-2)^(1/3)*(x^6 + 2)^(1
/3)*x^2 - (-2)^(2/3)*(x^6 + 2*x^3 + 2) - 6*(x^6 + 2)^(2/3)*x)/(x^6 + 2*x^3
+ 2)) - (-2)^(1/3)*x*log(-(3*(-2)^(2/3)*(x^7 - 4*x^4 + 2*x)*(x^6 + 2)^(2/3)
 + (-2)^(1/3)*(x^12 - 14*x^9 + 8*x^6 - 28*x^3 + 4) - 12*(x^8 - x^5 + 2*x^2)
*(x^6 + 2)^(1/3))/(x^12 + 4*x^9 + 8*x^6 + 8*x^3 + 4)) + 18*(x^6 + 2)^(1/3))
/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}+2\right)^{\frac{1}{3}}\left(x^{6}-2\right)}{\left(x^{6}+2x^{3}+2\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6+2)^(1/3)/x^2/(x^6+2*x^3+2),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 2)^(1/3)*(x^6 - 2)/((x^6 + 2*x^3 + 2)*x^2), x)
```

maple [C] time = 4.87, size = 1600, normalized size = 12.21

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-2)*(x^6+2)^(1/3)/x^2/(x^6+2*x^3+2),x)
```

```
[Out] (x^6+2)^(1/3)/x+(-1/3*ln((3*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_
Z^2)*RootOf(_Z^3+2)^3*x^9+18*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*
_Z^2)^2*RootOf(_Z^3+2)^2*x^9+RootOf(_Z^3+2)*x^12+6*RootOf(RootOf(_Z^3+2)^2+
3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^12+9*(x^12+4*x^6+4)^(1/3)*RootOf(RootOf(_Z^3+
2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)*x^7-2*RootOf(_Z^3+2)*x^9-12
*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^9+9*(x^12+4*x^6+4)^(
2/3)*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)^2*x
^2+6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)^3*x
^3+36*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)^2*RootOf(_Z^3+2)^
2*x^3+4*RootOf(_Z^3+2)*x^6+24*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9
*_Z^2)*x^6+18*(x^12+4*x^6+4)^(1/3)*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3
+2)+9*_Z^2)*RootOf(_Z^3+2)*x-4*RootOf(_Z^3+2)*x^3-24*RootOf(RootOf(_Z^3+2)^
2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^3+4*RootOf(_Z^3+2)+24*RootOf(RootOf(_Z^3+2)
^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2))/(x^6+2*x^3+2)/(x^6+2))*RootOf(_Z^3+2)-ln((3
*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)^3*x^9+1
```

```
8*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)^2*RootOf(_Z^3+2)^2*x^
9+RootOf(_Z^3+2)*x^12+6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)
*x^12+9*(x^12+4*x^6+4)^(1/3)*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*
_Z^2)*RootOf(_Z^3+2)*x^7-2*RootOf(_Z^3+2)*x^9-12*RootOf(RootOf(_Z^3+2)^2+3*
_Z*RootOf(_Z^3+2)+9*_Z^2)*x^9+9*(x^12+4*x^6+4)^(2/3)*RootOf(RootOf(_Z^3+2)^
2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)^2*x^2+6*RootOf(RootOf(_Z^3+2)^
2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)^3*x^3+36*RootOf(RootOf(_Z^3+2)
^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)^2*RootOf(_Z^3+2)^2*x^3+4*RootOf(_Z^3+2)*x^6+
24*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^6+18*(x^12+4*x^6+4
)^(1/3)*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)*
x-4*RootOf(_Z^3+2)*x^3-24*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^
2)*x^3+4*RootOf(_Z^3+2)+24*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z
^2))/(x^6+2*x^3+2)/(x^6+2))*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_
Z^2)+RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*ln(-(-3*RootOf(Roo
tOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)^3*x^9-18*RootOf(Ro
otOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)^2*RootOf(_Z^3+2)^2*x^9+RootOf(_Z
^3+2)*x^12+6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^12+3*(x^
12+4*x^6+4)^(1/3)*RootOf(_Z^3+2)^2*x^7+9*(x^12+4*x^6+4)^(1/3)*RootOf(RootOf
(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)*x^7+9*(x^12+4*x^6+4)^
(2/3)*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)^2*
x^2-6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)^3*
x^3-36*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)^2*RootOf(_Z^3+2)
^2*x^3+4*RootOf(_Z^3+2)*x^6+24*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+
9*_Z^2)*x^6-6*(x^12+4*x^6+4)^(2/3)*x^2+6*(x^12+4*x^6+4)^(1/3)*RootOf(_Z^3+2
)^2*x+18*(x^12+4*x^6+4)^(1/3)*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9
*_Z^2)*RootOf(_Z^3+2)*x+4*RootOf(_Z^3+2)+24*RootOf(RootOf(_Z^3+2)^2+3*_Z*Ro
otOf(_Z^3+2)+9*_Z^2))/(x^6+2*x^3+2)/(x^6+2)))/(x^6+2)^(2/3)*((x^6+2)^2)^(1/
3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+2\right)^{\frac{1}{3}}\left(x^6-2\right)}{\left(x^6+2\,x^3+2\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6+2)^(1/3)/x^2/(x^6+2*x^3+2),x, algorithm="maxima")
[Out] integrate((x^6 + 2)^(1/3)*(x^6 - 2)/((x^6 + 2*x^3 + 2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6-2\right)\,\left(x^6+2\right)^{1/3}}{x^2\,\left(x^6+2\,x^3+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 2)*(x^6 + 2)^(1/3))/(x^2*(2*x^3 + x^6 + 2)),x)
[Out] int(((x^6 - 2)*(x^6 + 2)^(1/3))/(x^2*(2*x^3 + x^6 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6-2\right)\sqrt[3]{x^6+2}}{x^2\left(x^6+2x^3+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-2)*(x**6+2)**(1/3)/x**2/(x**6+2*x**3+2),x)
[Out] Integral((x**6 - 2)*(x**6 + 2)**(1/3)/(x**2*(x**6 + 2*x**3 + 2)), x)
```

3.1586 $\int \frac{\left(1+x^3\right)^{2/3}\left(-1-2x^3+2x^6\right)}{x^6\left(-1+x^3+2x^6\right)}\,dx$

Optimal. Leaf size=131

$$-\frac{2\log\left(3^{2/3}\sqrt[3]{x^3+1}-3x\right)}{\sqrt[3]{3}}+2\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{x^3+1}+\sqrt[3]{3}x}\right)+\frac{\left(x^3+1\right)^{2/3}\left(-17x^3-2\right)}{10x^5}+\frac{\log\left(3^{2/3}\sqrt[3]{x^3+1}\,x+\sqrt[3]{3}\right.}{\sqrt[3]{3}}$$

Rubi [A] time = 0.53, antiderivative size = 134, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {1586, 6725, 271, 264, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{2\log\left(1-\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+1}}\right)}{\sqrt[3]{3}}+2\sqrt[6]{3}\tan^{-1}\left(\frac{2x}{\sqrt[6]{3}\sqrt[3]{x^3+1}}+\frac{1}{\sqrt{3}}\right)-\frac{\left(x^3+1\right)^{2/3}}{5x^5}-\frac{17\left(x^3+1\right)^{2/3}}{10x^2}+\frac{\log\left(\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+1}}+\frac{3^{2/3}x^2}{\left(x^3+1\right)^{2/3}}+1\right)}{\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^3)^(2/3)*(-1 - 2*x^3 + 2*x^6))/(x^6*(-1 + x^3 + 2*x^6)),x]
```

```
[Out] -1/5*(1 + x^3)^(2/3)/x^5 - (17*(1 + x^3)^(2/3))/(10*x^2) + 2*3^(1/6)*ArcTan
[1/Sqrt[3] + (2*x)/(3^(1/6)*(1 + x^3)^(1/3))] - (2*Log[1 - (3^(1/3)*x)/(1 +
 x^3)^(1/3)])/3^(1/3) + Log[1 + (3^(2/3)*x^2)/(1 + x^3)^(2/3) + (3^(1/3)*x)
/(1 + x^3)^(1/3)]/3^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(1+x^3\right)^{2/3}\left(-1-2x^3+2x^6\right)}{x^6\left(-1+x^3+2x^6\right)}\,dx = \int \frac{-1-2x^3+2x^6}{x^6\sqrt[3]{1+x^3}\left(-1+2x^3\right)}\,dx$$

$$= \int \left(\frac{1}{x^6\sqrt[3]{1+x^3}}+\frac{4}{x^3\sqrt[3]{1+x^3}}-\frac{6}{\sqrt[3]{1+x^3}\left(-1+2x^3\right)}\right)dx$$

$$= 4\int \frac{1}{x^3\sqrt[3]{1+x^3}}\,dx - 6\int \frac{1}{\sqrt[3]{1+x^3}\left(-1+2x^3\right)}\,dx + \int \frac{1}{x^6\sqrt[3]{1+x^3}}\,dx$$

$$= -\frac{\left(1+x^3\right)^{2/3}}{5x^5} - \frac{2\left(1+x^3\right)^{2/3}}{x^2} - \frac{3}{5}\int \frac{1}{x^3\sqrt[3]{1+x^3}}\,dx - 6\,\mathrm{Subst}\left(\int \frac{1}{-1+3x^3}\,d\right.$$

$$= -\frac{\left(1+x^3\right)^{2/3}}{5x^5} - \frac{17\left(1+x^3\right)^{2/3}}{10x^2} - 2\,\mathrm{Subst}\left(\int \frac{1}{-1+\sqrt[3]{3}\,x}\,dx, x, \frac{x}{\sqrt[3]{1+x^3}}\right) - 2$$

$$= -\frac{\left(1+x^3\right)^{2/3}}{5x^5} - \frac{17\left(1+x^3\right)^{2/3}}{10x^2} - \frac{2\log\left(1-\frac{\sqrt[3]{3}\,x}{\sqrt[3]{1+x^3}}\right)}{\sqrt[3]{3}} + 3\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt[3]{3}\,x+}\right.$$

$$= -\frac{\left(1+x^3\right)^{2/3}}{5x^5} - \frac{17\left(1+x^3\right)^{2/3}}{10x^2} - \frac{2\log\left(1-\frac{\sqrt[3]{3}\,x}{\sqrt[3]{1+x^3}}\right)}{\sqrt[3]{3}} + \frac{\log\left(1+\frac{3^{2/3}x^2}{\left(1+x^3\right)^{2/3}}+\frac{\sqrt[3]{}}{\sqrt[3]{1}}\right.}{\sqrt[3]{3}}$$

$$= -\frac{\left(1+x^3\right)^{2/3}}{5x^5} - \frac{17\left(1+x^3\right)^{2/3}}{10x^2} + 2\sqrt[6]{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}\,x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right) - \frac{2\log\left(1-\frac{\sqrt[3]{3}\,x}{\sqrt[3]{1+x^3}}\right.}{\sqrt[3]{3}}$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(1+x^3\right)^{2/3}\left(-1-2x^3+2x^6\right)}{x^6\left(-1+x^3+2x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^3)^(2/3)*(-1 - 2*x^3 + 2*x^6))/(x^6*(-1 + x^3 + 2*x^6)),x
]
```

```
[Out] Integrate[((1 + x^3)^(2/3)*(-1 - 2*x^3 + 2*x^6))/(x^6*(-1 + x^3 + 2*x^6)),
x]
```

IntegrateAlgebraic [A] time = 0.35, size = 131, normalized size = 1.00

$$-\frac{2\log\left(3^{2/3}\sqrt[3]{x^3+1}-3x\right)}{\sqrt[3]{3}} + 2\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{x^3+1}+\sqrt[3]{3}\,x}\right) + \frac{\left(x^3+1\right)^{2/3}\left(-17x^3-2\right)}{10x^5} + \frac{\log\left(3^{2/3}\sqrt[3]{x^3+1}\,x+\sqrt[3]{3}\left(x^3+1\right)^{2/3}+3x^2\right)}{\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)^(2/3)*(-1 - 2*x^3 + 2*x^6))/(x^6*(-1 + x^3 + 
2*x^6)),x]
```

```
[Out] ((-2 - 17*x^3)*(1 + x^3)^(2/3))/(10*x^5) + 2*3^(1/6)*ArcTan[(3^(5/6)*x)/(3^
(1/3)*x + 2*(1 + x^3)^(1/3))] - (2*Log[-3*x + 3^(2/3)*(1 + x^3)^(1/3)])/3^(
1/3) + Log[3*x^2 + 3^(2/3)*x*(1 + x^3)^(1/3) + 3^(1/3)*(1 + x^3)^(2/3)]/3^(
1/3)
```

fricas [B] time = 2.76, size = 292, normalized size = 2.23

$$\frac{20\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^5\log\left(\frac{9\cdot 3^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^3+1\right)^{\frac{1}{3}}x^2+3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(2x^3-1\right)-9\left(x^3+1\right)^{\frac{2}{3}}x}{2x^3-1}\right)-10\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^5\log\left(-\frac{3\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(7x^4+x\right)\left(x^3+1\right)^{\frac{2}{3}}-3^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(31x^6+23x^3+1\right)-9\left(5x^5+2x^2\right)\left(x^3+1\right)^{\frac{1}{3}}}{4x^6-4x^3+1}\right)-60\cdot 3^{\frac{1}{6}}(-1)^{\frac{1}{3}}x^5\arctan\left(\frac{3^{\frac{1}{6}}\left(6\cdot 3^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(14x^7-5x^4-x\right)\left(x^3+1\right)^{\frac{2}{3}}+18(-1)^{\frac{1}{3}}\left(31x^8+23x^5+x^2\right)\left(x^3+1\right)^{\frac{1}{3}}-3^{\frac{1}{3}}\left(127x^9+201x^6+48x^3+1\right)\right)}{3\left(251x^9+231x^6+6x^3-1\right)}\right)-9\left(17x^3+2\right)\left(x^3+1\right)^{\frac{2}{3}}}{90x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6-2*x^3-1)/x^6/(2*x^6+x^3-1),x, algorithm="fri
cas")
```

```
[Out] 1/90*(20*3^(2/3)*(-1)^(1/3)*x^5*log((9*3^(1/3)*(-1)^(2/3)*(x^3 + 1)^(1/3)*x
^2 + 3^(2/3)*(-1)^(1/3)*(2*x^3 - 1) - 9*(x^3 + 1)^(2/3)*x)/(2*x^3 - 1)) - 1
0*3^(2/3)*(-1)^(1/3)*x^5*log(-(3*3^(2/3)*(-1)^(1/3)*(7*x^4 + x)*(x^3 + 1)^(
2/3) - 3^(1/3)*(-1)^(2/3)*(31*x^6 + 23*x^3 + 1) - 9*(5*x^5 + 2*x^2)*(x^3 +
1)^(1/3))/(4*x^6 - 4*x^3 + 1)) - 60*3^(1/6)*(-1)^(1/3)*x^5*arctan(1/3*3^(1/
6)*(6*3^(2/3)*(-1)^(2/3)*(14*x^7 - 5*x^4 - x)*(x^3 + 1)^(2/3) + 18*(-1)^(1/
3)*(31*x^8 + 23*x^5 + x^2)*(x^3 + 1)^(1/3) - 3^(1/3)*(127*x^9 + 201*x^6 + 4
8*x^3 + 1))/(251*x^9 + 231*x^6 + 6*x^3 - 1)) - 9*(17*x^3 + 2)*(x^3 + 1)^(2/
3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^6-2\,x^3-1\right)\left(x^3+1\right)^{\frac{2}{3}}}{\left(2\,x^6+x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6-2*x^3-1)/x^6/(2*x^6+x^3-1),x, algorithm="gia
c")
```

```
[Out] integrate((2*x^6 - 2*x^3 - 1)*(x^3 + 1)^(2/3)/((2*x^6 + x^3 - 1)*x^6), x)
```

maple [C] time = 2.80, size = 613, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)*(2*x^6-2*x^3-1)/x^6/(2*x^6+x^3-1),x)
```

```
[Out] -1/10*(17*x^6+19*x^3+2)/x^5/(x^3+1)^(1/3)+6*RootOf(RootOf(_Z^3+9)^2+9*_Z*Ro
otOf(_Z^3+9)+81*_Z^2)*ln(-(81*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+8
1*_Z^2)^2*RootOf(_Z^3+9)^2*x^3+12*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+
9)+81*_Z^2)*RootOf(_Z^3+9)^3*x^3+15*(x^3+1)^(2/3)*RootOf(RootOf(_Z^3+9)^2+9
*_Z*RootOf(_Z^3+9)+81*_Z^2)*RootOf(_Z^3+9)^2*x+63*(x^3+1)^(1/3)*RootOf(Root
Of(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*_Z^2)*RootOf(_Z^3+9)*x^2+5*RootOf(_Z^3+
9)^2*(x^3+1)^(1/3)*x^2+27*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*_Z
^2)*x^3+4*RootOf(_Z^3+9)*x^3+6*x*(x^3+1)^(2/3)+27*RootOf(RootOf(_Z^3+9)^2+9
*_Z*RootOf(_Z^3+9)+81*_Z^2)+4*RootOf(_Z^3+9))/(2*x^3-1))+2/3*RootOf(_Z^3+9)
*ln((108*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*_Z^2)^2*RootOf(_Z^3
+9)^2*x^3+9*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*_Z^2)*RootOf(_Z^
3+9)^3*x^3+15*(x^3+1)^(2/3)*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*
_Z^2)*RootOf(_Z^3+9)^2*x-18*(x^3+1)^(1/3)*RootOf(RootOf(_Z^3+9)^2+9*_Z*Root
Of(_Z^3+9)+81*_Z^2)*RootOf(_Z^3+9)*x^2+5*RootOf(_Z^3+9)^2*(x^3+1)^(1/3)*x^2
-144*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*_Z^2)*x^3-12*RootOf(_Z^
3+9)*x^3-21*x*(x^3+1)^(2/3)-36*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+
81*_Z^2)-3*RootOf(_Z^3+9))/(2*x^3-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^6-2\,x^3-1\right)\left(x^3+1\right)^{\frac{2}{3}}}{\left(2\,x^6+x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6-2*x^3-1)/x^6/(2*x^6+x^3-1),x, algorithm="max
ima")
```

```
[Out] integrate((2*x^6 - 2*x^3 - 1)*(x^3 + 1)^(2/3)/((2*x^6 + x^3 - 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^3+1\right)^{2/3}\,\left(-2\,x^6+2\,x^3+1\right)}{x^6\,\left(2\,x^6+x^3-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3 + 1)^(2/3)*(2*x^3 - 2*x^6 + 1))/(x^6*(x^3 + 2*x^6 - 1)),x)
[Out] int(-((x^3 + 1)^(2/3)*(2*x^3 - 2*x^6 + 1))/(x^6*(x^3 + 2*x^6 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)*(2*x**6-2*x**3-1)/x**6/(2*x**6+x**3-1),x)
[Out] Timed out
```

3.1587 $\int \frac{(1+x^3)\sqrt{x-x^4}}{2+4x^3+3x^6}\,dx$

Optimal. Leaf size=131

$$-\frac{1}{18}\sqrt{8+7i\sqrt{2}}\ \tan^{-1}\left(\frac{\sqrt{2-\frac{i}{\sqrt{2}}}\,x\sqrt{x-x^4}}{x^3-1}\right)-\frac{1}{18}\sqrt{8-7i\sqrt{2}}\ \tan^{-1}\left(\frac{\sqrt{2+\frac{i}{\sqrt{2}}}\,x\sqrt{x-x^4}}{x^3-1}\right)-\frac{2}{9}\tan^{-1}\left(\frac{x^2}{\sqrt{x-x^4}}\right)$$

Rubi [B] time = 0.86, antiderivative size = 323, normalized size of antiderivative = 2.47, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2056, 6715, 1692, 402, 216, 377, 205}

$$-\frac{\left(2+i\sqrt{2}\right)\sqrt{x-x^4}\sin^{-1}\left(x^{3/2}\right)}{18\sqrt{x}\sqrt{1-x^3}}-\frac{\left(2-i\sqrt{2}\right)\sqrt{x-x^4}\sin^{-1}\left(x^{3/2}\right)}{18\sqrt{x}\sqrt{1-x^3}}+\frac{\left(\sqrt{2}+i\right)\sqrt{x-x^4}\tan^{-1}\left(\frac{x^{3/2}}{\sqrt{\frac{-\sqrt{2}+2i}{-\sqrt{2}+5i}}\sqrt{1-x^3}}\right)}{9\sqrt{2}\sqrt{\frac{-\sqrt{2}+2i}{-\sqrt{2}+5i}}\sqrt{x}\sqrt{1-x^3}}+\frac{\left(7+4i\sqrt{2}\right)\sqrt{x-x^4}\tan^{-1}\left(\frac{x^{3/2}}{\sqrt{\frac{\sqrt{2}+2i}{\sqrt{2}+5i}}\sqrt{1-x^3}}\right)}{9\sqrt{2}\sqrt{-8+7i\sqrt{2}}\sqrt{x}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((1 + x^3)*Sqrt[x - x^4])/(2 + 4*x^3 + 3*x^6),x]
```

```
[Out] -1/18*((2 - I*Sqrt[2])*Sqrt[x - x^4]*ArcSin[x^(3/2)])/(Sqrt[x]*Sqrt[1 - x^3
]) - ((2 + I*Sqrt[2])*Sqrt[x - x^4]*ArcSin[x^(3/2)])/(18*Sqrt[x]*Sqrt[1 - x
^3]) + ((I + Sqrt[2])*Sqrt[x - x^4]*ArcTan[x^(3/2)/(Sqrt[(2*I - Sqrt[2])/(5
*I - Sqrt[2])]*Sqrt[1 - x^3])])/(9*Sqrt[2]*Sqrt[(2*I - Sqrt[2])/(5*I - Sqrt
[2])]*Sqrt[x]*Sqrt[1 - x^3]) + ((7 + (4*I)*Sqrt[2])*Sqrt[x - x^4]*ArcTan[x^
(3/2)/(Sqrt[(2*I + Sqrt[2])/(5*I + Sqrt[2])]*Sqrt[1 - x^3])])/(9*Sqrt[2]*Sq
rt[-8 + (7*I)*Sqrt[2]]*Sqrt[x]*Sqrt[1 - x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{(1+x^3)\sqrt{x-x^4}}{2+4x^3+3x^6}\,dx = \frac{\sqrt{x-x^4}\int \frac{\sqrt{x}\sqrt{1-x^3}(1+x^3)}{2+4x^3+3x^6}\,dx}{\sqrt{x}\sqrt{1-x^3}}$$

$$= \frac{\left(2\sqrt{x-x^4}\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}(1+x^2)}{2+4x^2+3x^4}\,dx, x, x^{3/2}\right)}{3\sqrt{x}\sqrt{1-x^3}}$$

$$= \frac{\left(2\sqrt{x-x^4}\right)\text{Subst}\left(\int \left(\frac{\left(1-\frac{i}{\sqrt{2}}\right)\sqrt{1-x^2}}{4-2i\sqrt{2}+6x^2} + \frac{\left(1+\frac{i}{\sqrt{2}}\right)\sqrt{1-x^2}}{4+2i\sqrt{2}+6x^2}\right)dx, x, x^{3/2}\right)}{3\sqrt{x}\sqrt{1-x^3}}$$

$$= \frac{\left(\left(2-i\sqrt{2}\right)\sqrt{x-x^4}\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{4-2i\sqrt{2}+6x^2}\,dx, x, x^{3/2}\right)}{3\sqrt{x}\sqrt{1-x^3}} + \frac{\left(\left(2+i\sqrt{2}\right)\sqrt{x-x^4}\right)\text{Subst}\left(\int \right.}{3\sqrt{x}\sqrt{1-}}$$

$$= -\frac{\left(\left(2-i\sqrt{2}\right)\sqrt{x-x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, x^{3/2}\right)}{18\sqrt{x}\sqrt{1-x^3}} - \frac{\left(\left(2-i\sqrt{2}\right)\left(-5+i\sqrt{2}\right)\sqrt{x-x^4}\right)}{9\sqrt{x}}$$

$$= -\frac{\left(2-i\sqrt{2}\right)\sqrt{x-x^4}\sin^{-1}\left(x^{3/2}\right)}{18\sqrt{x}\sqrt{1-x^3}} - \frac{\left(2+i\sqrt{2}\right)\sqrt{x-x^4}\sin^{-1}\left(x^{3/2}\right)}{18\sqrt{x}\sqrt{1-x^3}} - \frac{\left(\left(2-i\sqrt{2}\right)\left(-5\right.\right.}{}$$

$$= -\frac{\left(2-i\sqrt{2}\right)\sqrt{x-x^4}\sin^{-1}\left(x^{3/2}\right)}{18\sqrt{x}\sqrt{1-x^3}} - \frac{\left(2+i\sqrt{2}\right)\sqrt{x-x^4}\sin^{-1}\left(x^{3/2}\right)}{18\sqrt{x}\sqrt{1-x^3}} + \frac{\sqrt{\frac{2i-\sqrt{2}}{5i-\sqrt{2}}}\left(5+i\sqrt{\right.}{}$$

Mathematica [B] time = 3.32, size = 1192, normalized size = 9.10

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + x^3)*Sqrt[x - x^4])/(2 + 4*x^3 + 3*x^6),x]
```

```
[Out] (Sqrt[x - x^4]*(-4*Sqrt[2*(-5 + I*Sqrt[2])]*Sqrt[-2 + I*Sqrt[2]]*Log[-Sqrt[
-2/3 - (I/3)*Sqrt[2]] + x^(3/2)] + 4*Sqrt[2*(-5 + I*Sqrt[2])]*Sqrt[-2 + I*S
qrt[2]]*Log[Sqrt[-2/3 - (I/3)*Sqrt[2]] + x^(3/2)] - (7*I)*Sqrt[-5 + I*Sqrt[
2]]*Sqrt[-2 + I*Sqrt[2]]*Log[-Sqrt[-6 - (3*I)*Sqrt[2]] + 3*x^(3/2)] + (7*I)
*Sqrt[-5 + I*Sqrt[2]]*Sqrt[-2 + I*Sqrt[2]]*Log[Sqrt[-6 - (3*I)*Sqrt[2]] + 3
```

```
*x^(3/2)] + (7*I)*Sqrt[-5 - I*Sqrt[2]]*Sqrt[-2 - I*Sqrt[2]]*Log[-Sqrt[-6 +
(3*I)*Sqrt[2]] + 3*x^(3/2)] - 4*Sqrt[-2 - I*Sqrt[2]]*Sqrt[-10 - (2*I)*Sqrt[
2]]*Log[-Sqrt[-6 + (3*I)*Sqrt[2]] + 3*x^(3/2)] - (7*I)*Sqrt[-5 - I*Sqrt[2]]
*Sqrt[-2 - I*Sqrt[2]]*Log[Sqrt[-6 + (3*I)*Sqrt[2]] + 3*x^(3/2)] + 4*Sqrt[-2
 - I*Sqrt[2]]*Sqrt[-10 - (2*I)*Sqrt[2]]*Log[Sqrt[-6 + (3*I)*Sqrt[2]] + 3*x^
(3/2)] + 72*Log[x^(3/2) + Sqrt[-1 + x^3]] - (7*I)*Sqrt[-5 + I*Sqrt[2]]*Sqrt
[-2 + I*Sqrt[2]]*Log[-3 - Sqrt[-6 - (3*I)*Sqrt[2]]*x^(3/2) + Sqrt[-15 - (3*
I)*Sqrt[2]]*Sqrt[-1 + x^3]] - 4*Sqrt[2*(-5 + I*Sqrt[2])]*Sqrt[-2 + I*Sqrt[2
]]*Log[-3 - Sqrt[-6 - (3*I)*Sqrt[2]]*x^(3/2) + Sqrt[-15 - (3*I)*Sqrt[2]]*Sq
rt[-1 + x^3]] + (7*I)*Sqrt[-5 + I*Sqrt[2]]*Sqrt[-2 + I*Sqrt[2]]*Log[-3 + Sq
rt[-6 - (3*I)*Sqrt[2]]*x^(3/2) + Sqrt[-15 - (3*I)*Sqrt[2]]*Sqrt[-1 + x^3]]
+ 4*Sqrt[2*(-5 + I*Sqrt[2])]*Sqrt[-2 + I*Sqrt[2]]*Log[-3 + Sqrt[-6 - (3*I)*
Sqrt[2]]*x^(3/2) + Sqrt[-15 - (3*I)*Sqrt[2]]*Sqrt[-1 + x^3]] + (7*I)*Sqrt[-
5 - I*Sqrt[2]]*Sqrt[-2 - I*Sqrt[2]]*Log[-3 - Sqrt[-6 + (3*I)*Sqrt[2]]*x^(3/
2) + Sqrt[(3*I)*(5*I + Sqrt[2])]*Sqrt[-1 + x^3]] - 4*Sqrt[-2 - I*Sqrt[2]]*S
qrt[-10 - (2*I)*Sqrt[2]]*Log[-3 - Sqrt[-6 + (3*I)*Sqrt[2]]*x^(3/2) + Sqrt[(
3*I)*(5*I + Sqrt[2])]*Sqrt[-1 + x^3]] - (7*I)*Sqrt[-5 - I*Sqrt[2]]*Sqrt[-2
- I*Sqrt[2]]*Log[-3 + Sqrt[-6 + (3*I)*Sqrt[2]]*x^(3/2) + Sqrt[(3*I)*(5*I +
Sqrt[2])]*Sqrt[-1 + x^3]] + 4*Sqrt[-2 - I*Sqrt[2]]*Sqrt[-10 - (2*I)*Sqrt[2]
]*Log[-3 + Sqrt[-6 + (3*I)*Sqrt[2]]*x^(3/2) + Sqrt[(3*I)*(5*I + Sqrt[2])]*S
qrt[-1 + x^3]]))/(324*Sqrt[x]*Sqrt[-1 + x^3])
```

IntegrateAlgebraic [A] time = 1.25, size = 121, normalized size = 0.92

$$-\frac{2}{9}\tan^{-1}\left(\frac{x^2}{\sqrt{x-x^4}}\right)+\frac{1}{18}\sqrt{8+7i\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\frac{i}{\sqrt{2}}}\,x^2}{\sqrt{x-x^4}}\right)+\frac{1}{18}\sqrt{8-7i\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\frac{i}{\sqrt{2}}}\,x^2}{\sqrt{x-x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)*Sqrt[x - x^4])/(2 + 4*x^3 + 3*x^6),x]
```

```
[Out] (-2*ArcTan[x^2/Sqrt[x - x^4]])/9 + (Sqrt[8 + (7*I)*Sqrt[2]]*ArcTan[(Sqrt[2
- I/Sqrt[2]]*x^2)/Sqrt[x - x^4]])/18 + (Sqrt[8 - (7*I)*Sqrt[2]]*ArcTan[(Sqr
t[2 + I/Sqrt[2]]*x^2)/Sqrt[x - x^4]])/18
```

fricas [B] time = 12.10, size = 2741, normalized size = 20.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(-x^4+x)^(1/2)/(3*x^6+4*x^3+2),x, algorithm="fricas")
```

```
[Out] -1/1008*2^(3/4)*sqrt(4*sqrt(2) + 9)*(4*sqrt(2) - 9)*log(-6360849/7*(168*x^6
 - 168*x^3 + 2*2^(3/4)*(28*x^4 - 3*sqrt(2)*(7*x^4 - 2*x) - 10*x)*sqrt(-x^4
+ x)*sqrt(4*sqrt(2) + 9) - 7*sqrt(2)*(19*x^6 - 12*x^3 + 2))/(3*x^6 + 4*x^3
+ 2)) + 1/1008*2^(3/4)*sqrt(4*sqrt(2) + 9)*(4*sqrt(2) - 9)*log(-6360849/7*(
168*x^6 - 168*x^3 - 2*2^(3/4)*(28*x^4 - 3*sqrt(2)*(7*x^4 - 2*x) - 10*x)*sqr
t(-x^4 + x)*sqrt(4*sqrt(2) + 9) - 7*sqrt(2)*(19*x^6 - 12*x^3 + 2))/(3*x^6 +
 4*x^3 + 2)) - 1/36*2^(3/4)*sqrt(4*sqrt(2) + 9)*arctan(-1/14*(8632050444092
280152834837119864926*x^36 + 91615215589322098617786305925997 92*x^33 - 1726
093558958158956618684547000192 8*x^30 - 22417539871410114792789642783506784*
x^27 + 86322513700512212295212545730757 84*x^24 + 179412084973962182067995 99
939395968*x^21 + 11168886727388974655327562902 5344*x^18 - 54935026225746501
72977792469639936*x^15 + 30905337002436620764838266192416*x^12 + 7906518639
719023682366715187514 88*x^9 - 13803150598393579452945086291366 4*x^6 + 12890
476859438719685214517708 8*x^3 + 12721698*sqrt(2)*sqrt(-x^4 + x)*(2^(3/4)*(1
550682228692016032742176 52*x^34 + 252837956784573083685349366*x^31 - 299319
523809873331574781920*x^28 - 586004622273400652833815356*x^25 + 12199718377
7565423757231456*x^22 + 419025032959227012317601072*x^19 + 2807605866947102
```

7380499328*x^16 - 90013541853774632322919520*x^13 - 49183740894388759197559
68*x^10 + 5625954326986205601627616*x^7 + 48339376368627709608960*x^4 - 3*s
qrt(2)*(16825352941418359482113253*x^34 + 59987578296866209972620350*x^31 -
 6580181442381755563592634*x^28 - 133348333567725981519681428*x^25 - 432024
77060524375944383896*x^22 + 90505619247446912851428400*x^19 + 3824689815010
5540704718192*x^16 - 19238762657512258793540000*x^13 - 56490395563655905046
31024*x^10 + 1892796862144660136321888*x^7 - 11363374832743038268448*x^4 +
145841485650323828672*x) - 619706421311517572800*x) - 16*2^(1/4)*(340616517
42756490436056644*x^34 + 95703917494121772333264282*x^31 + 2258857934151042
1818747546*x^28 - 144065524109011381824443844*x^25 - 1112611315731164515683
61236*x^22 + 45474265224424729086049080*x^19 + 64882637947323391431307656*x
^16 + 5306970009315400116931728*x^13 - 10505618506466998937300016*x^10 - 23
92945547527543470258048*x^7 + 207197976670170578006208*x^4 + sqrt(2)*(12030
880734706239561839937*x^34 + 44933301219178895918527241*x^31 + 469447063961
73846244715276*x^28 - 35613080256963383211529986*x^25 - 9706855294632781490
4168272*x^22 - 32620008949099351619636380*x^19 + 38469908845526136232962336
*x^16 + 24972611089770871113434152*x^13 - 504271448925687639314320*x^10 - 1
692005295003728616548448*x^7 + 146510610963976919718464*x^4)))*sqrt(4*sqrt(
2) + 9) + 99*sqrt(649/7)*(sqrt(2)*(2^(3/4)*(3121596530260239669336820157913
*x^36 + 5834255341413061687138179876792*x^33 - 6888167414217170094657282052
608*x^30 - 13995366187142943155635818597216*x^27 + 452321879686174477334898
1882116*x^24 + 10950989276996393893976849951808*x^21 - 10935492134993033672
70942631488*x^18 - 3077112817453062541515435758592*x^15 + 38146043227532731
9018147934640*x^12 + 300850897180239679913838976896*x^9 - 58555914471835493
587554868992*x^6 - 5108884480838567481905664*x^3 - 2*sqrt(2)*(1086133862743
827732006103951791*x^36 + 2078536628748868754952479046197*x^33 - 2374524441
070525146385161658610*x^30 - 4992736270966904035107639005666*x^27 + 1517995
330234711341166019538136*x^24 + 3915463113897739790410043247464*x^21 - 3380
72542128739686425119214608*x^18 - 1104683428514778558567052777424*x^15 + 12
4434881558629873274651070128*x^12 + 108391044654310593595674374672*x^9 - 20
827411787368918663045438624*x^6 + 1406128425854828903266144*x^3 - 135071238
92806437370880) + 46404763143266731467456) - 16*2^(1/4)*(248250202572925434
417171709641*x^36 + 392158279593892776753649524707*x^33 - 76436551095212858
3802234715988*x^30 - 1158625043830399121658197199928*x^27 + 744974044625885
970803079278080*x^24 + 1186605786206243645868681816096*x^21 - 2106127429938
96435791727033808*x^18 - 457525832401022893299363583856*x^15 - 200297823085
24211147631862800*x^12 + 38604316354119658596668068688*x^9 + 58044133521231
1981979699072*x^6 - 14158202308552722075699904*x^3 - 18*sqrt(2)*(8564251745
904023585195931603*x^36 + 12750457151213196303514367536*x^33 - 285341546460
70291083552637573*x^30 - 38161333576607079640202027588*x^27 + 3188378469461
4085184693919794*x^24 + 40679760423015746022246526032*x^21 - 12754053545003
855227347594068*x^18 - 17014472745812489197655856688*x^15 + 815653447800013
477852821480*x^12 + 1792322302234552578825256576*x^9 - 22771207005398409855
974432*x^6 + 555955717496406285267328*x^3)))*sqrt(4*sqrt(2) + 9) - 28*(5242
36500266665825730685749760*x^34 + 1332404845015756903230965089536*x^31 - 13
0480878679520904884096757504*x^28 - 2107192014832089761023279214592*x^25 -
612492456088768126642309490688*x^22 + 988722784766968088259898358784*x^19 +
 228997944867525635371563924480*x^16 - 235754311387964830721719050240*x^13
- 8539169464378485377627922432*x^10 + 23301104080816172474328858624*x^7 - 3
204348545010516418409545728*x^4 - sqrt(2)*(328841013685710897201295863120*x
^34 + 712423623928727497352013174712*x^31 - 507875961967949035452802264160*
x^28 - 1634077001729216627778034549952*x^25 - 30119552846381515879483912064
*x^22 + 1159152878738410570103304835584*x^19 + 2523903873210454265494853324
80*x^16 - 254589968502812873449866789248*x^13 - 430690856539146879483690032
64*x^10 + 19342332894031279621629314176*x^7 - 43928868748816117016977920*x^
4 - 6360849*sqrt(2)*(33227621146716099962831*x^34 + 75316178531714961613054
*x^31 - 46850423411517051236214*x^28 - 168426333235143900655124*x^25 - 1055
4911710188156761256*x^22 + 114645820898039007484144*x^19 + 2763443711778416
3769424*x^16 - 23483842011297496036128*x^13 - 3713017786705821859664*x^10 +

1934426021997052439648*x^7 + 492184585689176067 2*x^4 - 66678885297238592*x
) + 5972513105046841150837 76*x) - 16*sqrt(2)*(102474016362147716966746323 03
*x^34 + 3275263563747727892387456203*x^31 - 8096225442382114936789735337 6*x
^28 - 8751472050731700302970842847 0*x^25 + 8017623632658754805790099058 4*x^
22 + 1326428712108859789300848594 04*x^19 + 8584531730908213324425520080*x^1
6 - 4730794725927535364230964077 6*x^13 - 1825305533887870011870081627 2*x^10
 - 1029941776235616195421930656*x^7 + 1416148371835824525647109 76*x^4))*sqr
t(-x^4 + x))*sqrt(-(168*x^6 - 168*x^3 + 2*2^(3/4)*(28*x^4 - 3*sqrt(2)*(7*x^
4 - 2*x) - 10*x)*sqrt(-x^4 + x)*sqrt(4*sqrt(2) + 9) - 7*sqrt(2)*(19*x^6 - 1
2*x^3 + 2))/(3*x^6 + 4*x^3 + 2)) + 56*sqrt(2)*(2182418482886443893898488792
58650*x^36 + 251359916402604912653099443236315*x^33 - 653777886516007933475
373096546014*x^30 - 750700155453942593740040211672026*x^27 + 68686297097696
6659426192517349092*x^24 + 795085121396131507309277160898104*x^21 - 2896327
26736470239458313325939952*x^18 - 341211879567795800265863199007312*x^15 +
407683031555959101622376518999 68*x^12 + 45519347077752749837422618635632*x^
9 - 2637210339419949781840773265504*x^6 + 161689148264887783439633993 6*x^3
 - 14541470867755473408047552) - 712415088*sqrt(2)*(183796416602620291374850
17*x^36 + 2223165894705251197775568 3*x^33 - 5466217163608129353325317 6*x^30
 - 6842526718534935820089018 0*x^27 + 5221394974334514499212957 6*x^24 + 7131
476166868839974340492 0*x^21 - 1648247557527246217211577 6*x^18 - 28047105217
963056593939424*x^15 + 7272734837387065125899 04*x^12 + 29435813709713822165
83536*x^9 - 1982277045598782561742 08*x^6 + 4380445167874176424128*x^3 + sqr
t(2)*(48817688806532198011423 20*x^36 + 3033693318603343422286517*x^33 - 128
50910120393402178568823*x^30 - 1050947654605577383881213 4*x^27 + 1010649165
786860204483312 8*x^24 + 1080460061543325160387408 8*x^21 - 25314197193122561
90585608*x^18 - 3921202481378418716194160*x^15 + 5524089652659637311593 28*x
^12 + 5711300437144308221194 40*x^9 - 1401821623186625583871 84*x^6 + 3097547
919702057133088*x^3)) - 1156571977971226039247832192)/(37624270817611600887
70877747039271*x^36 + 1117645559198961908509726860134 6040*x^33 - 4052067875
89120595364629876997268*x^30 - 2644172742138643686024591451779 3136*x^27 - 1
56230668603551454695830712789669 08*x^24 + 1837821549178104921850567643631 68
64*x^21 + 175017611259170808101087053587595 84*x^18 - 211638299290701493270 8
830590815104*x^15 - 534636994493997946751646000909067 2*x^12 - 9954848624110
516056736194835831 04*x^9 + 1093151270114057899444501852842 88*x^6 + 10832968
4388704307699361100 8*x^3 - 7632135417534638943928384)) - 1/36*2^(3/4)*sqrt(
4*sqrt(2) + 9)*arctan(1/14*(863205044409228015283483711986492 6*x^36 + 91615
21558932209861778630592599792*x^33 - 17260935589581589566186845470001928*x^
30 - 22417539871410114792789642783506784*x^27 + 863225137005122122952125457
3075784*x^24 + 17941208497396218206799599939395968*x^21 + 1116888672738897 4
6553275629025344*x^18 - 5493502622574650172977792469639936*x^15 + 309053370
0243662076483826619241 6*x^12 + 7906518639719023682366715187514 88*x^9 - 1380
3150598393579452945086291366 4*x^6 + 128904768594387196852145177088*x^3 - 12
721698*sqrt(2)*sqrt(-x^4 + x)*(2^(3/4)*(1550682228692016032742176 52*x^34 +
2528379567845730836853493 66*x^31 - 2993195238098733315747819 20*x^28 - 58600
4622273400652833815356*x^25 + 1219971837775654237572314 56*x^22 + 4190250329
59227012317601072*x^19 + 2807605866947102738049932 8*x^16 - 9001354185377463
2322919520*x^13 - 4918374089438875919755968*x^10 + 562595432698620560162761
6*x^7 + 4833937636862770960896 0*x^4 - 3*sqrt(2)*(168253529414183594821132 53
*x^34 + 5998757829686620997262035 0*x^31 - 6580181442381755563592634*x^28 -
1333483335677259815196814 28*x^25 - 4320247706052437594438389 6*x^22 + 905056
19247446912851428400*x^19 + 3824689815010554070471819 2*x^16 - 1923876265751
2258793540000*x^13 - 5649039556365590504631024*x^10 + 189279686214466013632
1888*x^7 - 1136337483274303826844 8*x^4 + 145841485650323828672*x) - 6197064
21311517572800*x) - 16*2^(1/4)*(3406165174275649043605664 4*x^34 + 957039174
94121772333264282*x^31 + 2258857934151042181874754 6*x^28 - 1440655241090113
81824443844*x^25 - 1112611315731164515683612 36*x^22 + 454742652244247290860
49080*x^19 + 6488263794732339143130765 6*x^16 + 5306970009315400116931728*x^
13 - 1050561850646699893730001 6*x^10 - 2392945547527543470258048*x^7 + 2071
97976670170578006208*x^4 + sqrt(2)*(120308807347062395618399 37*x^34 + 44933

3012191788959185272241*x^31 + 46944706396173846244715276*x^28 - 356130802569
63383211529986*x^25 - 97068552946327814904168272*x^22 - 3262000894909935161
9636380*x^19 + 38469908845526136232962336*x^16 + 24972611089770871113434152
*x^13 - 504271448925687639314320*x^10 - 1692005295003728616548448*x^7 + 146
510610963976919718464*x^4)))*sqrt(4*sqrt(2) + 9) - 99*sqrt(649/7)*(sqrt(2)*
(2^(3/4)*(3121596530260239669336820157913*x^36 + 58342553414130616871381798
76792*x^33 - 6888167414217170094657282052608*x^30 - 13995366187142943155635
818597216*x^27 + 4523218796861744773348981882116*x^24 + 1095098927699639389
3976849951808*x^21 - 1093549213499303367270942631488*x^18 - 307711281745306
2541515435758592*x^15 + 381460432275327319018147934640*x^12 + 3008508971802
39679913838976896*x^9 - 58555914471835493587554868992*x^6 - 510888448083856
7481905664*x^3 - 2*sqrt(2)*(1086133862743827732006103951791*x^36 + 20785366
28748868754952479046197*x^33 - 2374524441070525146385161658610*x^30 - 49927
36270966904035107639005666*x^27 + 1517995330234711341166019538136*x^24 + 39
15463113897739790410043247464*x^21 - 338072542128739686425119214608*x^18 -
1104683428514778558567052777424*x^15 + 124434881558629873274651070128*x^12
+ 108391044654310593595674374672*x^9 - 20827411787368918663045438624*x^6 +
1406128425854828903266144*x^3 - 13507123892806437370880) + 4640476314326673
1467456) - 16*2^(1/4)*(248250202572925434417171709641*x^36 + 39215827959389
2776753649524707*x^33 - 764365510952128583802234715988*x^30 - 1158625043830
399121658197199928*x^27 + 744974044625885970803079278080*x^24 + 11866057862
06243645868681816096*x^21 - 210612742993896435791727033808*x^18 - 457525832
401022893299363583856*x^15 - 20029782308524211147631862800*x^12 + 386043163
54119658596668068688*x^9 + 580441335212311981979699072*x^6 - 14158202308552
722075699904*x^3 - 18*sqrt(2)*(8564251745904023585195931603*x^36 + 12750457
151213196303514367536*x^33 - 28534154646070291083552637573*x^30 - 381613335
76607079640202027588*x^27 + 31883784694614085184693919794*x^24 + 4067976042
3015746022246526032*x^21 - 12754053545003855227347594068*x^18 - 17014472745
812489197655856688*x^15 + 815653447800013477852821480*x^12 + 17923223022345
52578825256576*x^9 - 22771207005398409855974432*x^6 + 555955717496406285267
328*x^3)))*sqrt(4*sqrt(2) + 9) + 28*(524236500266665825730685749760*x^34 +
1332404845015756903230965089536*x^31 - 130480878679520904884096757504*x^28
- 2107192014832089761023279214592*x^25 - 612492456088768126642309490688*x^2
2 + 988722784766968088259898358784*x^19 + 228997944867525635371563924480*x^
16 - 235754311387964830721719050240*x^13 - 8539169464378485377627922432*x^1
0 + 23301104080816172474328858624*x^7 - 3204348545010516418409545728*x^4 -
sqrt(2)*(328841013685710897201295863120*x^34 + 7124236239287274973520131747
12*x^31 - 507875961967949035452802264160*x^28 - 163407700172921662777803454
9952*x^25 - 30119552846381515879483912064*x^22 + 11591528787384105701033048
35584*x^19 + 252390387321045426549485332480*x^16 - 254589968502812873449866
789248*x^13 - 43069085653914687948369003264*x^10 + 193423328940312796216293
14176*x^7 - 43928868748816117016977920*x^4 - 6360849*sqrt(2)*(3322762114671
6099962831*x^34 + 75316178531714961613054*x^31 - 46850423411517051236214*x^
28 - 168426333235143900655124*x^25 - 10554911710188156761256*x^22 + 1146458
20898039007484144*x^19 + 27634437117784163769424*x^16 - 2348384201129749603
6128*x^13 - 3713017786705821859664*x^10 + 1934426021997052439648*x^7 + 4921
845856891760672*x^4 - 66678885297238592*x) + 597251310504684115083776*x) -
16*sqrt(2)*(10247401636214771696674632303*x^34 + 32752635637477278923874562
03*x^31 - 80962254423821149367897353376*x^28 - 8751472050731700302970842847
0*x^25 + 80176236326587548057900990584*x^22 + 13264287121088597893008485940
4*x^19 + 8584531730908213324425520080*x^16 - 47307947259275353642309640776*
x^13 - 18253055338878700118700816272*x^10 - 1029941776235616195421930656*x^
7 + 141614837183582452564710976*x^4))*sqrt(-x^4 + x))*sqrt(-(168*x^6 - 168*
x^3 - 2*2^(3/4)*(28*x^4 - 3*sqrt(2)*(7*x^4 - 2*x) - 10*x)*sqrt(-x^4 + x)*sq
rt(4*sqrt(2) + 9) - 7*sqrt(2)*(19*x^6 - 12*x^3 + 2))/(3*x^6 + 4*x^3 + 2)) +
56*sqrt(2)*(218241848288644389389848879258650*x^36 + 251359916402604912653
099443236315*x^33 - 653777886516007933475373096546014*x^30 - 75070015545394
2593740040211672026*x^27 + 686862970976966659426192517349092*x^24 + 7950851
21396131507309277160898104*x^21 - 289632726736470239458313325939952*x^18 -

```
341211879567795800265863199007312*x^15 + 4076830315559591016223765189968*x
^12 + 4551934707775274983742261863563 2*x^9 - 2637210339419949781840773265504*x^6 + 1616891482648877834396339936*x^3 - 1454147086775547340804755 2) - 71
2415088*sqrt(2)*(18379641660262029137485017*x^36 + 222316589470525119777556
83*x^33 - 546621716360812935332531 76*x^30 - 684252671853493582008901 80*x^27
 + 5221394974334514499212957 6*x^24 + 7131476166868839974340492 0*x^21 - 1648
2475575272462172115776*x^18 - 28047105217963056593939424*x^15 + 72727348373
870651258990 4*x^12 + 29435813709713822165835 36*x^9 - 1982277045598782561742
08*x^6 + 4380445167874176424128*x^3 + sqrt(2)*(48817688806532198011423 20*x^
36 + 30336933186033434222865 17*x^33 - 12850910120393402178568823*x^30 - 105
0947654605577383881213 4*x^27 + 1010649165786860204483312 8*x^24 + 1080460061
5433251603874088*x^21 - 25314197193122561905856 08*x^18 - 39212024813784187 1
6194160*x^15 + 55240896526596373115932 8*x^12 + 5711300437144308221194 40*x^9
 - 1401821623186625583871 84*x^6 + 3097547919702057133088*x^3)) - 1156571977
971226039247832192)/(37624270817611600887708777470392 71*x^36 + 111764555919
8961908509726860134604 0*x^33 - 4052067875891205953646298769972 68*x^30 - 264
4172742138643686024591451779313 6*x^27 - 15623066860355145469583071278966908
*x^24 + 18378215491781049218505676436316864*x^21 + 175017611259170808101087
05358759584*x^18 - 21163829929070149327088305908151 04*x^15 - 53463699449399
794675164600090906 72*x^12 - 99548486241105160567361948358310 4*x^9 + 1093151
270114057899444501852842 88*x^6 + 1083296843887043076993611008*x^3 - 7632135
417534638943928384)) + 1/9*arctan(2*sqrt(-x^4 + x)*x/(2*x^3 - 1))
```

giac [B] time = 0.35, size = 207, normalized size = 1.58

$$-\frac{1}{36}\sqrt{18\sqrt{2}+16}\arctan\left(\frac{2\left(\frac{9}{2}\right)^{\frac{3}{4}}\left(\left(\frac{9}{2}\right)^{\frac{1}{4}}\left(\sqrt{6}-2\sqrt{3}\right)+6\sqrt{\frac{1}{x^3}-1}\right)}{9\left(\sqrt{6}+2\sqrt{3}\right)}\right)+\frac{1}{36}\sqrt{18\sqrt{2}+16}\arctan\left(\frac{2\left(\frac{9}{2}\right)^{\frac{3}{4}}\left(\left(\frac{9}{2}\right)^{\frac{1}{4}}\left(\sqrt{6}-2\sqrt{3}\right)-6\sqrt{\frac{1}{x^3}-1}\right)}{9\left(\sqrt{6}+2\sqrt{3}\right)}\right)-\frac{1}{72}\sqrt{18\sqrt{2}-16}\log\left(\frac{1}{3}\left(\sqrt{6}\left(\frac{9}{2}\right)^{\frac{1}{4}}-2\left(\frac{9}{2}\right)^{\frac{1}{4}}\sqrt{3}\right)\sqrt{\frac{1}{x^3}-1}+3\sqrt{\frac{1}{2}}+\frac{1}{x^3}-1\right)+\frac{1}{72}\sqrt{18\sqrt{2}-16}\log\left(-\frac{1}{3}\left(\sqrt{6}\left(\frac{9}{2}\right)^{\frac{1}{4}}-2\left(\frac{9}{2}\right)^{\frac{1}{4}}\sqrt{3}\right)\sqrt{\frac{1}{x^3}-1}+3\sqrt{\frac{1}{2}}+\frac{1}{x^3}-1\right)+\frac{2}{9}\arctan\left(\sqrt{\frac{1}{x^3}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(-x^4+x)^(1/2)/(3*x^6+4*x^3+2),x, algorithm="giac")
```

```
[Out] -1/36*sqrt(18*sqrt(2) + 16)*arctan(2/9*(9/2)^(3/4)*((9/2)^(1/4)*(sqrt(6) -
2*sqrt(3)) + 6*sqrt(1/x^3 - 1))/(sqrt(6) + 2*sqrt(3))) + 1/36*sqrt(18*sqrt(
2) + 16)*arctan(2/9*(9/2)^(3/4)*((9/2)^(1/4)*(sqrt(6) - 2*sqrt(3)) - 6*sqrt
(1/x^3 - 1))/(sqrt(6) + 2*sqrt(3))) - 1/72*sqrt(18*sqrt(2) - 16)*log(1/3*(s
qrt(6)*(9/2)^(1/4) - 2*(9/2)^(1/4)*sqrt(3))*sqrt(1/x^3 - 1) + 3*sqrt(1/2) +
 1/x^3 - 1) + 1/72*sqrt(18*sqrt(2) - 16)*log(-1/3*(sqrt(6)*(9/2)^(1/4) - 2*
(9/2)^(1/4)*sqrt(3))*sqrt(1/x^3 - 1) + 3*sqrt(1/2) + 1/x^3 - 1) + 2/9*arcta
n(sqrt(1/x^3 - 1))
```

maple [C] time = 0.46, size = 667, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)*(-x^4+x)^(1/2)/(3*x^6+4*x^3+2),x)
```

```
[Out] -2/3*(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x
))^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2))/(-1+x))^(1/2)
*((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2)/(-3/2+1/2*I*3^(1
/2))/(-x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/2)))^(1/2)*(Ellipti
cF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3
^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
-EllipticPi(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),(-1/
2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/
2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+1/108*4^(1/2)*sum(_alp
ha^2*(_alpha^3-1)*(-1+x)^2*(3*_alpha^5+3*_alpha^4+3*_alpha^3+7*_alpha^2+7*_
alpha+7)*(1-I*3^(1/2))*(x/(-1+x)*(-3+I*3^(1/2))/(I*3^(1/2)-1))^(1/2)*(1/(-1
+x)*(I*3^(1/2)+2*x+1)/(-1-I*3^(1/2)))^(1/2)*(1/(-1+x)*(1+2*x-I*3^(1/2))/(I*
3^(1/2)-1))^(1/2)/(-3+I*3^(1/2))/(-x*(-1+x)*(I*3^(1/2)+2*x+1)*(1+2*x-I*3^(1
/2)))^(1/2)*(2*EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x
```

```
))^(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-
1/2*I*3^(1/2)))^(1/2))-(-3*_alpha^5-4*_alpha^2)*EllipticPi(((-3/2+1/2*I*3^(
1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),-1/4*I*3^(1/2)*_alpha^5+3/4*_alp
ha^5-1/3*I*3^(1/2)*_alpha^2+_alpha^2-1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))
*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))),_alph
a=RootOf(3*_Z^6+4*_Z^3+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+x}\left(x^3+1\right)}{3x^6+4x^3+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(-x^4+x)^(1/2)/(3*x^6+4*x^3+2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + x)*(x^3 + 1)/(3*x^6 + 4*x^3 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x-x^4}\,\left(x^3+1\right)}{3x^6+4x^3+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - x^4)^(1/2)*(x^3 + 1))/(4*x^3 + 3*x^6 + 2),x)
[Out] int(((x - x^4)^(1/2)*(x^3 + 1))/(4*x^3 + 3*x^6 + 2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)*(-x**4+x)**(1/2)/(3*x**6+4*x**3+2),x)
[Out] Timed out
```

3.1588 $\int \frac{-1+x}{(1+x)\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=131

$$\frac{2\sqrt{x^2+1}\,(3x-6)+2\left(3x^2-6x+1\right)}{3\left(\sqrt{x^2+1}+x\right)^{3/2}}-4\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{1+\sqrt{2}}}\right)+4\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{\sqrt{2}-1}}\right)$$

Rubi [A] time = 0.37, antiderivative size = 134, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6742, 2117, 14, 2119, 1628, 828, 826, 1166, 207, 203}

$$\sqrt{\sqrt{x^2+1}+x}-\frac{4}{\sqrt{\sqrt{x^2+1}+x}}-\frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}}-\frac{4\tan^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{1+\sqrt{2}}}+\frac{4\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{\sqrt{2}-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x)/((1 + x)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/3*1/(x + Sqrt[1 + x^2])^(3/2) - 4/Sqrt[x + Sqrt[1 + x^2]] + Sqrt[x + Sqr
t[1 + x^2]] - (4*ArcTan[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]])/Sqrt[1
 + Sqrt[2]] + (4*ArcTanh[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]])/Sqrt[-
1 + Sqrt[2]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-1+x}{(1+x)\sqrt{x+\sqrt{1+x^2}}}\,dx = \int \left(\frac{1}{\sqrt{x+\sqrt{1+x^2}}} - \frac{2}{(1+x)\sqrt{x+\sqrt{1+x^2}}}\right)dx$$

$$= -\left(2\int \frac{1}{(1+x)\sqrt{x+\sqrt{1+x^2}}}\,dx\right) + \int \frac{1}{\sqrt{x+\sqrt{1+x^2}}}\,dx$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x^2}{x^{5/2}}\,dx, x, x+\sqrt{1+x^2}\right) - 2\operatorname{Subst}\left(\int \frac{1+x^2}{x^{3/2}\left(-1+2x+x^2\right)}\,dx, x\right.$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{x^{5/2}}+\frac{1}{\sqrt{x}}\right)dx, x, x+\sqrt{1+x^2}\right) - 2\operatorname{Subst}\left(\int \left(\frac{1}{x^{3/2}}+\frac{2(1}{x^{3/2}\left(-1+\right.}\right.\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \frac{4}{\sqrt{x+\sqrt{1+x^2}}} + \sqrt{x+\sqrt{1+x^2}} - 4\operatorname{Subst}\left(\int \frac{}{x^{3/2}\left(-\right.}\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} - \frac{4}{\sqrt{x+\sqrt{1+x^2}}} + \sqrt{x+\sqrt{1+x^2}} + 4\operatorname{Subst}\left(\int \frac{}{\sqrt{x}\left(-\right.}\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} - \frac{4}{\sqrt{x+\sqrt{1+x^2}}} + \sqrt{x+\sqrt{1+x^2}} + 8\operatorname{Subst}\left(\int \frac{-1}{-1+2}\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} - \frac{4}{\sqrt{x+\sqrt{1+x^2}}} + \sqrt{x+\sqrt{1+x^2}} - 4\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}}\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} - \frac{4}{\sqrt{x+\sqrt{1+x^2}}} + \sqrt{x+\sqrt{1+x^2}} - \frac{4\tan^{-1}\left(\frac{\sqrt{x+\sqrt{1+x^2}}}{\sqrt{1+\sqrt{2}}}\right)}{\sqrt{1+\sqrt{2}}}$$

Mathematica [A] time = 0.36, size = 148, normalized size = 1.13

$$\sqrt{\sqrt{x^2+1}+x} - \frac{4}{\sqrt{\sqrt{x^2+1}+x}} - \frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}} + 4\left(\sqrt{2}-1\right)\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}}\right) + 4\sqrt{\sqrt{2}-1}\left(1+\sqrt{2}\right)\tanh^{-1}\left(\frac{1}{\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/((1 + x)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/3*1/(x + Sqrt[1 + x^2])^(3/2) - 4/Sqrt[x + Sqrt[1 + x^2]] + Sqrt[x + Sqr
t[1 + x^2]] + 4*(-1 + Sqrt[2])*Sqrt[1 + Sqrt[2]]*ArcTan[1/(Sqrt[-1 + Sqrt[2
]]*Sqrt[x + Sqrt[1 + x^2]])] + 4*Sqrt[-1 + Sqrt[2]]*(1 + Sqrt[2])*ArcTanh[1
/(Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]])]
```

IntegrateAlgebraic [A] time = 0.24, size = 131, normalized size = 1.00

$$\frac{2\sqrt{x^2+1}\,(3x-6)+2\left(3x^2-6x+1\right)}{3\left(\sqrt{x^2+1}+x\right)^{3/2}} - 4\sqrt{\sqrt{2}-1}\tan^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right) + 4\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/((1 + x)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] (2*(-6 + 3*x)*Sqrt[1 + x^2] + 2*(1 - 6*x + 3*x^2))/(3*(x + Sqrt[1 + x^2])^(
3/2)) - 4*Sqrt[-1 + Sqrt[2]]*ArcTan[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^
```

```
2]]] + 4*Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]
]]
```

fricas [A] time = 0.43, size = 164, normalized size = 1.25

$$-\frac{2}{3}\left(x^2-\sqrt{x^2+1}\,(x-6)-6\,x-1\right)\sqrt{x+\sqrt{x^2+1}}+8\,\sqrt{\sqrt{2}-1}\,\arctan\left(\sqrt{x+\sqrt{2}+\sqrt{x^2+1}+1}\,\sqrt{\sqrt{2}-1}-\sqrt{x+\sqrt{x^2+1}}\,\sqrt{\sqrt{2}-1}\right)+2\,\sqrt{\sqrt{2}+1}\,\log\left(4\,\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+4\,\sqrt{x+\sqrt{x^2+1}}\right)-2\,\sqrt{\sqrt{2}+1}\,\log\left(-4\,\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+4\,\sqrt{x+\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(x^2 - sqrt(x^2 + 1)*(x - 6) - 6*x - 1)*sqrt(x + sqrt(x^2 + 1)) + 8*sq
rt(sqrt(2) - 1)*arctan(sqrt(x + sqrt(2) + sqrt(x^2 + 1) + 1)*sqrt(sqrt(2) -
 1) - sqrt(x + sqrt(x^2 + 1))*sqrt(sqrt(2) - 1)) + 2*sqrt(sqrt(2) + 1)*log(
4*sqrt(sqrt(2) + 1)*(sqrt(2) - 1) + 4*sqrt(x + sqrt(x^2 + 1))) - 2*sqrt(sqr
t(2) + 1)*log(-4*sqrt(sqrt(2) + 1)*(sqrt(2) - 1) + 4*sqrt(x + sqrt(x^2 + 1)
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x+\sqrt{x^2+1}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/(sqrt(x + sqrt(x^2 + 1))*(x + 1)), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{-1+x}{(1+x)\,\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(1+x)/(x+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int((-1+x)/(1+x)/(x+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x+\sqrt{x^2+1}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/(sqrt(x + sqrt(x^2 + 1))*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x-1}{\sqrt{x+\sqrt{x^2+1}}\,\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x + (x^2 + 1)^(1/2))^(1/2)*(x + 1)),x)
```

```
[Out] int((x - 1)/((x + (x^2 + 1)^(1/2))^(1/2)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{(x+1)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral((x - 1)/((x + 1)*sqrt(x + sqrt(x**2 + 1))), x)
```

3.1589 $\int \frac{1+x}{(-1+x)\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=131

$$\frac{2\sqrt{x^2+1}\,(3x+6)+2\left(3x^2+6x+1\right)}{3\left(\sqrt{x^2+1}+x\right)^{3/2}}+4\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{\sqrt{2}-1}}\right)-4\sqrt{\sqrt{2}-1}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{1+\sqrt{2}}}\right)$$

Rubi [A] time = 0.31, antiderivative size = 134, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6742, 2117, 14, 2119, 1628, 828, 826, 1166, 207, 203}

$$\sqrt{\sqrt{x^2+1}+x}+\frac{4}{\sqrt{\sqrt{x^2+1}+x}}-\frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}}+\frac{4\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{\sqrt{2}-1}}-\frac{4\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{1+\sqrt{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x)/((-1 + x)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/3*1/(x + Sqrt[1 + x^2])^(3/2) + 4/Sqrt[x + Sqrt[1 + x^2]] + Sqrt[x + Sqr
t[1 + x^2]] + (4*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]])/Sqrt[-1
 + Sqrt[2]] - (4*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]])/Sqrt[
1 + Sqrt[2]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1+x}{(-1+x)\sqrt{x+\sqrt{1+x^2}}}\,dx = \int \left(\frac{1}{\sqrt{x+\sqrt{1+x^2}}} + \frac{2}{(-1+x)\sqrt{x+\sqrt{1+x^2}}}\right)dx$$

$$= 2\int \frac{1}{(-1+x)\sqrt{x+\sqrt{1+x^2}}}\,dx + \int \frac{1}{\sqrt{x+\sqrt{1+x^2}}}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{x^{5/2}}\,dx, x, x+\sqrt{1+x^2}\right) + 2\,\text{Subst}\left(\int \frac{1+x^2}{x^{3/2}\left(-1-2x+x^2\right)}\,dx, x, x\right.$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^{5/2}} + \frac{1}{\sqrt{x}}\right)dx, x, x+\sqrt{1+x^2}\right) + 2\,\text{Subst}\left(\int \left(\frac{1}{x^{3/2}} + \frac{2(1+}{x^{3/2}\left(-1-\right.}\right.\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} - \frac{4}{\sqrt{x+\sqrt{1+x^2}}} + \sqrt{x+\sqrt{1+x^2}} + 4\,\text{Subst}\left(\int \frac{1}{x^{3/2}\left(-1\right.}\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \frac{4}{\sqrt{x+\sqrt{1+x^2}}} + \sqrt{x+\sqrt{1+x^2}} - 4\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1\right.}\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \frac{4}{\sqrt{x+\sqrt{1+x^2}}} + \sqrt{x+\sqrt{1+x^2}} - 8\,\text{Subst}\left(\int \frac{1-x}{-1-2x^2}\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \frac{4}{\sqrt{x+\sqrt{1+x^2}}} + \sqrt{x+\sqrt{1+x^2}} + 4\,\text{Subst}\left(\int \frac{1}{-1-\sqrt{2}}\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \frac{4}{\sqrt{x+\sqrt{1+x^2}}} + \sqrt{x+\sqrt{1+x^2}} + \frac{4\tan^{-1}\left(\frac{\sqrt{x+\sqrt{1+x^2}}}{\sqrt{-1+\sqrt{2}}}\right)}{\sqrt{-1+\sqrt{2}}}$$

Mathematica [A] time = 0.36, size = 148, normalized size = 1.13

$$\sqrt{\sqrt{x^2+1}+x} + \frac{4}{\sqrt{\sqrt{x^2+1}+x}} - \frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}} - 4\sqrt{\sqrt{2}-1}\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{1}{\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}}\right) - 4\left(\sqrt{2}-1\right)\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\frac{1}{\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-1 + x)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/3*1/(x + Sqrt[1 + x^2])^(3/2) + 4/Sqrt[x + Sqrt[1 + x^2]] + Sqrt[x + Sqr
t[1 + x^2]] - 4*Sqrt[-1 + Sqrt[2]]*(1 + Sqrt[2])*ArcTan[1/(Sqrt[1 + Sqrt[2]
]*Sqrt[x + Sqrt[1 + x^2]])] - 4*(-1 + Sqrt[2])*Sqrt[1 + Sqrt[2]]*ArcTanh[1/
(Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]])]
```

IntegrateAlgebraic [A] time = 0.23, size = 131, normalized size = 1.00

$$\frac{2\sqrt{x^2+1}\,(3x+6)+2\left(3x^2+6x+1\right)}{3\left(\sqrt{x^2+1}+x\right)^{3/2}} + 4\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right) - 4\sqrt{\sqrt{2}-1}\,\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((-1 + x)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] (2*(6 + 3*x)*Sqrt[1 + x^2] + 2*(1 + 6*x + 3*x^2))/(3*(x + Sqrt[1 + x^2])^(3
/2)) + 4*Sqrt[1 + Sqrt[2]]*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]
```

```
] - 4*Sqrt[-1 + Sqrt[2]]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]
]
```

fricas [A] time = 0.44, size = 164, normalized size = 1.25

$$-\frac{2}{3}\left(x^2-\sqrt{x^2+1}\,(x+6)+6\,x-1\right)\sqrt{x+\sqrt{x^2+1}}-8\sqrt{\sqrt{2}+1}\arctan\left(\sqrt{x+\sqrt{2}+\sqrt{x^2+1}-1}\sqrt{\sqrt{2}+1}-\sqrt{x+\sqrt{x^2+1}}\sqrt{\sqrt{2}+1}\right)-2\sqrt{\sqrt{2}-1}\log\left(4\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+4\sqrt{x+\sqrt{x^2+1}}\right)+2\sqrt{\sqrt{2}-1}\log\left(-4\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+4\sqrt{x+\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(x^2 - sqrt(x^2 + 1)*(x + 6) + 6*x - 1)*sqrt(x + sqrt(x^2 + 1)) - 8*sq
rt(sqrt(2) + 1)*arctan(sqrt(x + sqrt(2) + sqrt(x^2 + 1) - 1)*sqrt(sqrt(2) +
 1) - sqrt(x + sqrt(x^2 + 1))*sqrt(sqrt(2) + 1)) - 2*sqrt(sqrt(2) - 1)*log(
4*(sqrt(2) + 1)*sqrt(sqrt(2) - 1) + 4*sqrt(x + sqrt(x^2 + 1))) + 2*sqrt(sqr
t(2) - 1)*log(-4*(sqrt(2) + 1)*sqrt(sqrt(2) - 1) + 4*sqrt(x + sqrt(x^2 + 1)
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x+\sqrt{x^2+1}}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x + sqrt(x^2 + 1))*(x - 1)), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1+x}{(-1+x)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-1+x)/(x+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int((1+x)/(-1+x)/(x+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x+\sqrt{x^2+1}}\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x + sqrt(x^2 + 1))*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+1}{\sqrt{x+\sqrt{x^2+1}}\,\left(x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x + (x^2 + 1)^(1/2))^(1/2)*(x - 1)),x)
```

```
[Out] int((x + 1)/((x + (x^2 + 1)^(1/2))^(1/2)*(x - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{(x-1)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/(x+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral((x + 1)/((x - 1)*sqrt(x + sqrt(x**2 + 1))), x)
```

3.1590 $\int \frac{\sqrt{-x+x^2}}{\sqrt{x^2-x\sqrt{-x+x^2}}}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{x^2-x}\sqrt{-x\left(\sqrt{x^2-x}-x\right)}(8x-9)}{12x}+\sqrt{x\left(x-\sqrt{x^2-x}\right)}\left(\frac{3\sqrt{\sqrt{x^2-x}+x}\tanh^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2-x}+x}\right)}{4\sqrt{2}x}+\frac{1}{12}\right.$$

Rubi [F] time = 1.89, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-x+x^2}}{\sqrt{x^2-x\sqrt{-x+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[-x + x^2]/Sqrt[x^2 - x*Sqrt[-x + x^2]],x]
```

```
[Out] (2*Sqrt[-x + x^2]*Defer[Subst][Defer[Int][(x^2*Sqrt[-1 + x^2])/Sqrt[x^4 - x
^2*Sqrt[-x^2 + x^4]], x], x, Sqrt[x]])/(Sqrt[-1 + x]*Sqrt[x])
```

Rubi steps

$$\int \frac{\sqrt{-x+x^2}}{\sqrt{x^2-x\sqrt{-x+x^2}}}\,dx=\frac{\sqrt{-x+x^2}\int \frac{\sqrt{-1+x}\sqrt{x}}{\sqrt{x^2-x\sqrt{-x+x^2}}}\,dx}{\sqrt{-1+x}\sqrt{x}}$$
$$=\frac{\left(2\sqrt{-x+x^2}\right)\text{Subst}\left(\int \frac{x^2\sqrt{-1+x^2}}{\sqrt{x^4-x^2\sqrt{-x^2+x^4}}}\,dx,x,\sqrt{x}\right)}{\sqrt{-1+x}\sqrt{x}}$$

Mathematica [C] time = 0.39, size = 160, normalized size = 1.22

$$\frac{\sqrt{(x-1)x}\left(\sqrt{(x-1)x}-x\right)\left(9\left(-2x+2\sqrt{(x-1)x}+1\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{1}{2\left(\sqrt{(x-1)x}-x\right)}\right)-24x^2+24\sqrt{(x-1)x}\,x+22x-10\sqrt{(x-1)x}\right)}{12\sqrt{x\left(x-\sqrt{(x-1)x}\right)}\left(4x^2-\left(4\sqrt{(x-1)x}+5\right)x+3\sqrt{(x-1)x}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x + x^2]/Sqrt[x^2 - x*Sqrt[-x + x^2]],x]
```

```
[Out] (Sqrt[(-1 + x)*x]*(-x + Sqrt[(-1 + x)*x])*(22*x - 24*x^2 - 10*Sqrt[(-1 + x)
*x] + 24*x*Sqrt[(-1 + x)*x] + 9*(1 - 2*x + 2*Sqrt[(-1 + x)*x])*Hypergeometr
ic2F1[-1/2, 1, 1/2, 1 + 1/(2*(-x + Sqrt[(-1 + x)*x]))]))/(12*Sqrt[x*(x - Sq
rt[(-1 + x)*x])]*(1 + 4*x^2 + 3*Sqrt[(-1 + x)*x] - x*(5 + 4*Sqrt[(-1 + x)*x
])))
```

IntegrateAlgebraic [A] time = 4.55, size = 131, normalized size = 1.00

$$\frac{\sqrt{x^2-x}\sqrt{-x\left(\sqrt{x^2-x}-x\right)}(8x-9)}{12x}+\sqrt{x\left(x-\sqrt{x^2-x}\right)}\left(\frac{3\sqrt{\sqrt{x^2-x}+x}\tanh^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2-x}+x}\right)}{4\sqrt{2}x}+\frac{1}{12}(8x-19)\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-x + x^2]/Sqrt[x^2 - x*Sqrt[-x + x^2]],x]
```

```
[Out] ((-9 + 8*x)*Sqrt[-x + x^2]*Sqrt[-(x*(-x + Sqrt[-x + x^2]))])/(12*x) + Sqrt[
x*(x - Sqrt[-x + x^2])]*((-19 + 8*x)/12 + (3*Sqrt[x + Sqrt[-x + x^2]]*ArcTa
nh[Sqrt[2]*Sqrt[x + Sqrt[-x + x^2]]])/(4*Sqrt[2]*x))
```

fricas [A] time = 0.43, size = 123, normalized size = 0.94

$$\frac{9\sqrt{2}\,x\log\left(-\frac{4\,x^2+2\,\sqrt{x^2-\sqrt{x^2-x}\,x}\left(\sqrt{2}\,x-\sqrt{2}\,\sqrt{x^2-x}\right)-4\,\sqrt{x^2-x}\,x-x}{x}\right)+4\left(8\,x^2+\sqrt{x^2-x}\,(8\,x-9)-19\,x\right)\sqrt{x^2-\sqrt{x^2-x}\,x}}{48\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)^(1/2)/(x^2-x*(x^2-x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(9*sqrt(2)*x*log(-(4*x^2 + 2*sqrt(x^2 - sqrt(x^2 - x)*x)*(sqrt(2)*x -
sqrt(2)*sqrt(x^2 - x)) - 4*sqrt(x^2 - x)*x - x)/x) + 4*(8*x^2 + sqrt(x^2 -
x)*(8*x - 9) - 19*x)*sqrt(x^2 - sqrt(x^2 - x)*x))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-x}}{\sqrt{x^2-\sqrt{x^2-x}\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)^(1/2)/(x^2-x*(x^2-x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x)/sqrt(x^2 - sqrt(x^2 - x)*x), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-x}}{\sqrt{x^2-x\sqrt{x^2-x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x)^(1/2)/(x^2-x*(x^2-x)^(1/2))^(1/2),x)
```

```
[Out] int((x^2-x)^(1/2)/(x^2-x*(x^2-x)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-x}}{\sqrt{x^2-\sqrt{x^2-x}\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)^(1/2)/(x^2-x*(x^2-x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x)/sqrt(x^2 - sqrt(x^2 - x)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^2-x}}{\sqrt{x^2-x\,\sqrt{x^2-x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x)^(1/2)/(x^2 - x*(x^2 - x)^(1/2))^(1/2),x)
[Out] int((x^2 - x)^(1/2)/(x^2 - x*(x^2 - x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x(x-1)}}{\sqrt{x\left(x-\sqrt{x^2-x}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x)**(1/2)/(x**2-x*(x**2-x)**(1/2))**(1/2),x)
[Out] Integral(sqrt(x*(x - 1))/sqrt(x*(x - sqrt(x**2 - x))), x)
```

3.1591 $\int \frac{6+2x+x^2}{(2+x)\left(2+x^2\right)\sqrt[3]{2+x+x^2}}\,dx$

Optimal. Leaf size=132

$$\frac{\log\left(2\sqrt[3]{x^2+x+2}+2^{2/3}x\right)}{2^{2/3}}-\frac{\log\left(\sqrt[3]{2}x^2-2^{2/3}\sqrt[3]{x^2+x+2}\,x+2\left(x^2+x+2\right)^{2/3}\right)}{2\,2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2+x+2}}{\sqrt{3}}-\frac{2^{2/3}x}{\sqrt{3}}}{\sqrt[3]{x^2+x+2}}\right)}{2^{2/3}}$$

Rubi [F] time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{6+2x+x^2}{(2+x)\left(2+x^2\right)\sqrt[3]{2+x+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(6 + 2*x + x^2)/((2 + x)*(2 + x^2)*(2 + x + x^2)^(1/3)),x]
```

```
[Out] (-3*((1 - I*Sqrt[7] + 2*x)/(2 + x))^(1/3)*((1 + I*Sqrt[7] + 2*x)/(2 + x))^(
1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (3 - I*Sqrt[7])/(2*(2 + x)), (3 + I*Sqrt[
7])/(2*(2 + x))])/(2*2^(2/3)*(2 + x + x^2)^(1/3)) + 2*Defer[Int][1/((2 + x^
2)*(2 + x + x^2)^(1/3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{6+2x+x^2}{(2+x)\left(2+x^2\right)\sqrt[3]{2+x+x^2}}\,dx &= \int\left(\frac{1}{(2+x)\sqrt[3]{2+x+x^2}}+\frac{2}{\left(2+x^2\right)\sqrt[3]{2+x+x^2}}\right)dx\\
&= 2\int\frac{1}{\left(2+x^2\right)\sqrt[3]{2+x+x^2}}\,dx+\int\frac{1}{(2+x)\sqrt[3]{2+x+x^2}}\,dx\\
&= 2\int\frac{1}{\left(2+x^2\right)\sqrt[3]{2+x+x^2}}\,dx-\frac{\left(\sqrt[3]{\frac{1-i\sqrt{7}+2x}{2+x}}\sqrt[3]{\frac{1+i\sqrt{7}+2x}{2+x}}\right)\text{Subst}\left(\int\frac{}{\sqrt[3]{x}\sqrt[3]{1-}}\right.}{2^{2/3}\left(\frac{1}{2+x}\right)^{2/3}\sqrt[3]{2-}}\\
&= -\frac{3\sqrt[3]{\frac{1-i\sqrt{7}+2x}{2+x}}\sqrt[3]{\frac{1+i\sqrt{7}+2x}{2+x}}\,F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{3-i\sqrt{7}}{2(2+x)},\frac{3+i\sqrt{7}}{2(2+x)}\right)}{2\,2^{2/3}\sqrt[3]{2+x+x^2}}+2\int\frac{1}{\left(2+x^2\right)\sqrt[3]{2-}}
\end{aligned}$$

Mathematica [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{6+2x+x^2}{(2+x)\left(2+x^2\right)\sqrt[3]{2+x+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(6 + 2*x + x^2)/((2 + x)*(2 + x^2)*(2 + x + x^2)^(1/3)),x]
```

```
[Out] Integrate[(6 + 2*x + x^2)/((2 + x)*(2 + x^2)*(2 + x + x^2)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.19, size = 132, normalized size = 1.00

$$\frac{\log\left(2\sqrt[3]{x^2+x+2}+2^{2/3}x\right)}{2^{2/3}}-\frac{\log\left(\sqrt[3]{2}x^2-2^{2/3}\sqrt[3]{x^2+x+2}\,x+2\left(x^2+x+2\right)^{2/3}\right)}{2\,2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2+x+2}}{\sqrt{3}}-\frac{2^{2/3}x}{\sqrt{3}}}{\sqrt[3]{x^2+x+2}}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(6 + 2*x + x^2)/((2 + x)*(2 + x^2)*(2 + x + x^2)^(1/3)),
x]
```

```
[Out] -((Sqrt[3]*ArcTan[(-((2^(2/3)*x)/Sqrt[3]) + (2 + x + x^2)^(1/3)/Sqrt[3])/(2
 + x + x^2)^(1/3)])/2^(2/3)) + Log[2^(2/3)*x + 2*(2 + x + x^2)^(1/3)]/2^(2/
3) - Log[2^(1/3)*x^2 - 2^(2/3)*x*(2 + x + x^2)^(1/3) + 2*(2 + x + x^2)^(2/3
)]/(2*2^(2/3))
```

fricas [B] time = 10.73, size = 395, normalized size = 2.99

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(12\cdot 4^{\frac{2}{3}}\left(x^{7}+x^{6}-x^{5}-2x^{4}-10x^{3}-8x^{2}-8x\right)\left(x^{2}+x+2\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(x^{9}+24x^{8}-36x^{7}-64x^{6}-276x^{5}-168x^{4}-136x^{3}+144x^{2}+96x+64\right)+12\left(x^{8}-14x^{7}-10x^{6}-20x^{5}+20x^{4}+16x^{3}+16x^{2}\right)\left(x^{2}+x+2\right)^{\frac{1}{3}}\right)}{6\left(x^{9}-48x^{8}-36x^{7}-64x^{6}+84x^{5}+120x^{4}+152x^{3}+144x^{2}+96x+64\right)}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(-\frac{6\cdot 4^{\frac{1}{3}}\left(x^{2}+x+2\right)^{\frac{1}{3}}x^{2}+4^{\frac{2}{3}}\left(x^{3}+2x^{2}+2x+4\right)+12\left(x^{2}+x+2\right)^{\frac{2}{3}}x}{x^{3}+2x^{2}+2x+4}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(\frac{6\cdot 4^{\frac{2}{3}}\left(x^{4}-x^{3}-x^{2}-2x\right)\left(x^{2}+x+2\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(x^{6}-14x^{5}-10x^{4}-20x^{3}+20x^{2}+16x+16\right)-6\left(x^{5}-4x^{4}-4x^{3}-8x^{2}\right)\left(x^{2}+x+2\right)^{\frac{1}{3}}}{x^{6}+4x^{5}+8x^{4}+16x^{3}+20x^{2}+16x+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+6)/(2+x)/(x^2+2)/(x^2+x+2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^(2/3)*(x^7 + x^6 - x^
5 - 2*x^4 - 10*x^3 - 8*x^2 - 8*x)*(x^2 + x + 2)^(2/3) + 4^(1/3)*(x^9 + 24*x
^8 - 36*x^7 - 64*x^6 - 276*x^5 - 168*x^4 - 136*x^3 + 144*x^2 + 96*x + 64) +
 12*(x^8 - 14*x^7 - 10*x^6 - 20*x^5 + 20*x^4 + 16*x^3 + 16*x^2)*(x^2 + x +
2)^(1/3))/(x^9 - 48*x^8 - 36*x^7 - 64*x^6 + 84*x^5 + 120*x^4 + 152*x^3 + 14
4*x^2 + 96*x + 64)) + 1/12*4^(2/3)*log(-(6*4^(1/3)*(x^2 + x + 2)^(1/3)*x^2
+ 4^(2/3)*(x^3 + 2*x^2 + 2*x + 4) + 12*(x^2 + x + 2)^(2/3)*x)/(x^3 + 2*x^2
+ 2*x + 4)) - 1/24*4^(2/3)*log((6*4^(2/3)*(x^4 - x^3 - x^2 - 2*x)*(x^2 + x
+ 2)^(2/3) + 4^(1/3)*(x^6 - 14*x^5 - 10*x^4 - 20*x^3 + 20*x^2 + 16*x + 16)
- 6*(x^5 - 4*x^4 - 4*x^3 - 8*x^2)*(x^2 + x + 2)^(1/3))/(x^6 + 4*x^5 + 8*x^4
 + 16*x^3 + 20*x^2 + 16*x + 16))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+2x+6}{\left(x^2+x+2\right)^{\frac{1}{3}}\left(x^2+2\right)(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+6)/(2+x)/(x^2+2)/(x^2+x+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 2*x + 6)/((x^2 + x + 2)^(1/3)*(x^2 + 2)*(x + 2)), x)
```

maple [C] time = 5.44, size = 662, normalized size = 5.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x+6)/(2+x)/(x^2+2)/(x^2+x+2)^(1/3),x)
```

```
[Out] -1/2*ln(-(RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2
)^3*x^3+2*(x^2+x+2)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+2*_Z*Roo
tOf(_Z^3-2)+4*_Z^2)*x+(x^2+x+2)^(1/3)*RootOf(_Z^3-2)^2*x^2-2*(x^2+x+2)^(1/3
)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2-2*
(x^2+x+2)^(2/3)*x+2*RootOf(_Z^3-2)*x^2+2*RootOf(_Z^3-2)*x+4*RootOf(_Z^3-2))
/(2+x)/(x^2+2))*RootOf(_Z^3-2)-ln(-(RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^
3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3+2*(x^2+x+2)^(2/3)*RootOf(_Z^3-2)^2*RootOf
(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+(x^2+x+2)^(1/3)*RootOf(_Z^3
-2)^2*x^2-2*(x^2+x+2)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_
Z^2)*RootOf(_Z^3-2)*x^2-2*(x^2+x+2)^(2/3)*x+2*RootOf(_Z^3-2)*x^2+2*RootOf(_
Z^3-2)*x+4*RootOf(_Z^3-2))/(2+x)/(x^2+2))*RootOf(RootOf(_Z^3-2)^2+2*_Z*Root
Of(_Z^3-2)+4*_Z^2)+RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*ln((
RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3+2*
(x^2+x+2)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2
```

```
)+4*_Z^2)*x-2*(x^2+x+2)^(1/3)*RootOf(_Z^3-2)^2*x^2-2*(x^2+x+2)^(1/3)*RootOf
(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2+RootOf(_Z^
3-2)*x^3+4*(x^2+x+2)^(2/3)*x-2*RootOf(_Z^3-2)*x^2-2*RootOf(_Z^3-2)*x-4*Root
Of(_Z^3-2))/(2+x)/(x^2+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 2x + 6}{\left(x^2 + x + 2\right)^{\frac{1}{3}}\left(x^2 + 2\right)(x + 2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+6)/(2+x)/(x^2+2)/(x^2+x+2)^(1/3),x, algorithm="maxima")
[Out] integrate((x^2 + 2*x + 6)/((x^2 + x + 2)^(1/3)*(x^2 + 2)*(x + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 + 2x + 6}{\left(x^2 + 2\right)\,(x + 2)\,\left(x^2 + x + 2\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 + 6)/((x^2 + 2)*(x + 2)*(x + x^2 + 2)^(1/3)),x)
[Out] int((2*x + x^2 + 6)/((x^2 + 2)*(x + 2)*(x + x^2 + 2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 2x + 6}{(x + 2)\left(x^2 + 2\right)\sqrt[3]{x^2 + x + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x+6)/(2+x)/(x**2+2)/(x**2+x+2)**(1/3),x)
[Out] Integral((x**2 + 2*x + 6)/((x + 2)*(x**2 + 2)*(x**2 + x + 2)**(1/3)), x)
```

3.1592 $\int \frac{-1+x}{x\sqrt[3]{1+2x+2x^2+x^3}}\,dx$

Optimal. Leaf size=132

$$-\log\left(\sqrt[3]{x^3+2x^2+2x+1}-x-1\right)+\frac{1}{2}\log\left(x^2+\left(x^3+2x^2+2x+1\right)^{2/3}+(x+1)\sqrt[3]{x^3+2x^2+2x+1}+2x+1\right.$$

Rubi [F] time = 0.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+x}{x\sqrt[3]{1+2x+2x^2+x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x)/(x*(1 + 2*x + 2*x^2 + x^3)^(1/3)),x]
```

```
[Out] (3*(1 + x)*(1 - (2*(1 + x))/(1 - I*Sqrt[3]))^(1/3)*(1 - (2*(1 + x))/(1 + I*
Sqrt[3]))^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (2*(1 + x))/(1 - I*Sqrt[3]), (
2*(1 + x))/(1 + I*Sqrt[3])])/(2*(1 + 2*x + 2*x^2 + x^3)^(1/3)) - ((1 + x)^(
1/3)*(1 + x + x^2)^(1/3)*Defer[Subst][Defer[Int][1/((-2/3 + x)*(1/3 + x)^(1
/3)*(7/9 - x/3 + x^2)^(1/3)), x], x, 2/3 + x])/(1 + 2*x + 2*x^2 + x^3)^(1/3
)
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x}{x\sqrt[3]{1+2x+2x^2+x^3}}\,dx &= \int \left(\frac{1}{\sqrt[3]{1+2x+2x^2+x^3}}-\frac{1}{x\sqrt[3]{1+2x+2x^2+x^3}}\right)dx\\
&= \int \frac{1}{\sqrt[3]{1+2x+2x^2+x^3}}\,dx-\int \frac{1}{x\sqrt[3]{1+2x+2x^2+x^3}}\,dx\\
&= \text{Subst}\left(\int \frac{1}{\sqrt[3]{\frac{7}{27}+\frac{2x}{3}+x^3}}\,dx,x,\frac{2}{3}+x\right)-\text{Subst}\left(\int \frac{1}{\left(-\frac{2}{3}+x\right)\sqrt[3]{\frac{7}{27}+\frac{2x}{3}+x^3}}\,d\right.\\
&= \frac{\left(\sqrt[3]{1+x}\sqrt[3]{1+x+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{\frac{1}{3}+x}\sqrt[3]{\frac{7}{9}-\frac{x}{3}+x^2}}\,dx,x,\frac{2}{3}+x\right)}{\sqrt[3]{1+2x+2x^2+x^3}}-\frac{\left(\sqrt[3]{1+x}\sqrt[3]{1+x\right.}}{}\\
&= \frac{\left(\sqrt[3]{\frac{-i+\sqrt{3}-2ix}{i+\sqrt{3}}}\sqrt[3]{\frac{i+\sqrt{3}+2ix}{-i+\sqrt{3}}}\sqrt[3]{1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\sqrt[3]{1-\frac{2x}{1-i\sqrt{3}}}\sqrt[3]{1-\frac{2x}{1+i\sqrt{3}}}}\,dx,x,1+x\right)}{\sqrt[3]{1+2x+2x^2+x^3}}\\
&= \frac{3\sqrt[3]{-\frac{i-\sqrt{3}+2ix}{i+\sqrt{3}}}\sqrt[3]{-\frac{i+\sqrt{3}+2ix}{i-\sqrt{3}}}(1+x)F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{2(1+x)}{1-i\sqrt{3}},\frac{2(1+x)}{1+i\sqrt{3}}\right)}{2\sqrt[3]{1+2x+2x^2+x^3}}-\frac{\left(\sqrt[3]{1+x}\sqrt[3]{1+\right.}}{}
\end{aligned}$$

Mathematica [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{-1+x}{x\sqrt[3]{1+2x+2x^2+x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x)/(x*(1 + 2*x + 2*x^2 + x^3)^(1/3)),x]
```

```
[Out] Integrate[(-1 + x)/(x*(1 + 2*x + 2*x^2 + x^3)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.28, size = 132, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^3+2x^2+2x+1}-x-1\right)+\frac{1}{2}\log\left(x^2+\left(x^3+2x^2+2x+1\right)^{2/3}+(x+1)\sqrt[3]{x^3+2x^2+2x+1}+2x+1\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^3+2x^2+2x+1}}{\sqrt[3]{x^3+2x^2+2x+1}+2x+2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/(x*(1 + 2*x + 2*x^2 + x^3)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*(1 + 2*x + 2*x^2 + x^3)^(1/3))/(2 + 2*x + (1 + 2*
x + 2*x^2 + x^3)^(1/3))]) - Log[-1 - x + (1 + 2*x + 2*x^2 + x^3)^(1/3)] + L
og[1 + 2*x + x^2 + (1 + x)*(1 + 2*x + 2*x^2 + x^3)^(1/3) + (1 + 2*x + 2*x^2
 + x^3)^(2/3)]/2
```

fricas [A] time = 0.68, size = 123, normalized size = 0.93

$$\sqrt{3}\arctan\left(-\frac{4\sqrt{3}\left(x^3+2x^2+2x+1\right)^{\frac{1}{3}}(x+1)+\sqrt{3}\left(x^2+x+1\right)-2\sqrt{3}\left(x^3+2x^2+2x+1\right)^{\frac{2}{3}}}{9x^2+17x+9}\right)-\frac{1}{2}\log\left(-\frac{3\left(x^3+2x^2+2x+1\right)^{\frac{1}{3}}(x+1)-x-3\left(x^3+2x^2+2x+1\right)^{\frac{2}{3}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x/(x^3+2*x^2+2*x+1)^(1/3),x, algorithm="fricas")
```

```
[Out] sqrt(3)*arctan(-(4*sqrt(3)*(x^3 + 2*x^2 + 2*x + 1)^(1/3)*(x + 1) + sqrt(3)*
(x^2 + x + 1) - 2*sqrt(3)*(x^3 + 2*x^2 + 2*x + 1)^(2/3))/(9*x^2 + 17*x + 9)
) - 1/2*log(-(3*(x^3 + 2*x^2 + 2*x + 1)^(1/3)*(x + 1) - x - 3*(x^3 + 2*x^2
+ 2*x + 1)^(2/3))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^3+2x^2+2x+1\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x/(x^3+2*x^2+2*x+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/((x^3 + 2*x^2 + 2*x + 1)^(1/3)*x), x)
```

maple [C] time = 1.19, size = 455, normalized size = 3.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/x/(x^3+2*x^2+2*x+1)^(1/3),x)
```

```
[Out] RootOf(_Z^2-_Z+1)*ln((-RootOf(_Z^2-_Z+1)^2*x^2+2*RootOf(_Z^2-_Z+1)^2*x+Root
Of(_Z^2-_Z+1)*x^2+111*(x^3+2*x^2+2*x+1)^(2/3)+111*x*(x^3+2*x^2+2*x+1)^(1/3)
-RootOf(_Z^2-_Z+1)^2-39*RootOf(_Z^2-_Z+1)*x+110*x^2+111*(x^3+2*x^2+2*x+1)^(
1/3)+RootOf(_Z^2-_Z+1)+187*x+110)/x)-ln((-RootOf(_Z^2-_Z+1)^2*x^2+2*RootOf(
_Z^2-_Z+1)^2*x+RootOf(_Z^2-_Z+1)*x^2+111*(x^3+2*x^2+2*x+1)^(2/3)+111*x*(x^3
+2*x^2+2*x+1)^(1/3)-RootOf(_Z^2-_Z+1)^2+35*RootOf(_Z^2-_Z+1)*x+110*x^2+111*
(x^3+2*x^2+2*x+1)^(1/3)+RootOf(_Z^2-_Z+1)+150*x+110)/x)*RootOf(_Z^2-_Z+1)+l
n((-RootOf(_Z^2-_Z+1)^2*x^2+2*RootOf(_Z^2-_Z+1)^2*x+RootOf(_Z^2-_Z+1)*x^2+1
11*(x^3+2*x^2+2*x+1)^(2/3)+111*x*(x^3+2*x^2+2*x+1)^(1/3)-RootOf(_Z^2-_Z+1)^
2+35*RootOf(_Z^2-_Z+1)*x+110*x^2+111*(x^3+2*x^2+2*x+1)^(1/3)+RootOf(_Z^2-_Z
+1)+150*x+110)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^3+2\,x^2+2\,x+1\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x/(x^3+2*x^2+2*x+1)^(1/3),x, algorithm="maxima")
[Out] integrate((x - 1)/((x^3 + 2*x^2 + 2*x + 1)^(1/3)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x-1}{x\,\left(x^3+2\,x^2+2\,x+1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(x*(2*x + 2*x^2 + x^3 + 1)^(1/3)),x)
[Out] int((x - 1)/(x*(2*x + 2*x^2 + x^3 + 1)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{x\sqrt[3]{(x+1)\left(x^2+x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x/(x**3+2*x**2+2*x+1)**(1/3),x)
[Out] Integral((x - 1)/(x*((x + 1)*(x**2 + x + 1))**(1/3)), x)
```

3.1593 $$\int \frac{(-2q+px^3)(aq+bx^2+apx^3)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^7}\,dx$$

Optimal. Leaf size=132

$$\frac{\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}\,(2ap^2x^6-4apqx^4+4apqx^3+2aq^2+3bpx^5+3bqx^2)}{6x^6}-bpq\log\left(\sqrt{p^2x^6-2pqx^4+2\right.$$

Rubi [F] time = 1.37, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2q+px^3)(aq+bx^2+apx^3)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*q + p*x^3)*(a*q + b*x^2 + a*p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^
4 + p^2*x^6])/x^7,x]
```

```
[Out] -2*a*q^2*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^7, x] - 2
*b*q*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^5, x] - a*p*q
*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^4, x] + b*p*Defer
[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^2, x] + a*p^2*Defer[Int
][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x, x]
```

Rubi steps

$$\int \frac{(-2q+px^3)(aq+bx^2+apx^3)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^7}\,dx = \int \left(-\frac{2aq^2\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^7}-\right.$$

$$= (bp)\int \frac{\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^2}\,dx + ($$

Mathematica [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{(-2q+px^3)(aq+bx^2+apx^3)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2*q + p*x^3)*(a*q + b*x^2 + a*p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*
p*q*x^4 + p^2*x^6])/x^7,x]
```

```
[Out] Integrate[((-2*q + p*x^3)*(a*q + b*x^2 + a*p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*
p*q*x^4 + p^2*x^6])/x^7, x]
```

IntegrateAlgebraic [A] time = 0.53, size = 132, normalized size = 1.00

$$\frac{\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}\,(2ap^2x^6-4apqx^4+4apqx^3+2aq^2+3bpx^5+3bqx^2)}{6x^6}-bpq\log\left(\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}+px^3+q\right)+2bpq\log(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*q + p*x^3)*(a*q + b*x^2 + a*p*x^3)*Sqrt[q^2 + 2*p*q
*x^3 - 2*p*q*x^4 + p^2*x^6])/x^7,x]
```

```
[Out] (Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(2*a*q^2 + 3*b*q*x^2 + 4*a*p*q
*x^3 - 4*a*p*q*x^4 + 3*b*p*x^5 + 2*a*p^2*x^6))/(6*x^6) + 2*b*p*q*Log[x] - b
*p*q*Log[q + p*x^3 + Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(a*p*x^3+b*x^2+a*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)
^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6 - 2\,pqx^4 + 2\,pqx^3 + q^2}\left(apx^3 + bx^2 + aq\right)\left(px^3 - 2\,q\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(a*p*x^3+b*x^2+a*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)
^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(p^2*x^6 - 2*p*q*x^4 + 2*p*q*x^3 + q^2)*(a*p*x^3 + b*x^2 + a*
q)*(p*x^3 - 2*q)/x^7, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(p\,x^{3}-2q\right)\left(ap\,x^{3}+b\,x^{2}+aq\right)\sqrt{p^{2}x^{6}-2pq\,x^{4}+2pq\,x^{3}+q^{2}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^3-2*q)*(a*p*x^3+b*x^2+a*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)
/x^7,x)
```

```
[Out] int((p*x^3-2*q)*(a*p*x^3+b*x^2+a*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)
/x^7,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6 - 2\,pqx^4 + 2\,pqx^3 + q^2}\left(apx^3 + bx^2 + aq\right)\left(px^3 - 2\,q\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(a*p*x^3+b*x^2+a*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)
^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(p^2*x^6 - 2*p*q*x^4 + 2*p*q*x^3 + q^2)*(a*p*x^3 + b*x^2 + a*
q)*(p*x^3 - 2*q)/x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(2\,q-p\,x^3\right)\,\left(a\,p\,x^3+b\,x^2+a\,q\right)\,\sqrt{p^2\,x^6-2\,p\,q\,x^4+2\,p\,q\,x^3+q^2}}{x^7} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*q - p*x^3)*(a*q + b*x^2 + a*p*x^3)*(p^2*x^6 + q^2 + 2*p*q*x^3 - 2*
p*q*x^4)^(1/2))/x^7,x)
```

```
[Out] int(-((2*q - p*x^3)*(a*q + b*x^2 + a*p*x^3)*(p^2*x^6 + q^2 + 2*p*q*x^3 - 2*
p*q*x^4)^(1/2))/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(px^3-2q\right)\left(apx^3+aq+bx^2\right)\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**3-2*q)*(a*p*x**3+b*x**2+a*q)*(p**2*x**6-2*p*q*x**4+2*p*q*x*
*3+q**2)**(1/2)/x**7,x)
```

```
[Out] Integral((p*x**3 - 2*q)*(a*p*x**3 + a*q + b*x**2)*sqrt(p**2*x**6 - 2*p*q*x*
*4 + 2*p*q*x**3 + q**2)/x**7, x)
```

3.1594 $\int \frac{1}{x\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=132

$$\frac{4\left(256a^5x^2+32a^3bx+245ab^2\right)\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{1155b^4x^3}-\frac{4\sqrt{a^2x^2-bx}\left(-256a^4x^2-160a^2bx+105b^2\right)\sqrt{x\left(\sqrt{\right.}}}{1155b^4x^4}$$

Rubi [F] time = 4.78, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2)),x
]
```

```
[Out] (2*Sqrt[x]*Sqrt[-b + a^2*x]*Defer[Subst][Defer[Int][1/(x^2*Sqrt[-b + a^2*x^
2]*(a*x^4 + x^2*Sqrt[-(b*x^2) + a^2*x^4])^(3/2)), x], x, Sqrt[x]])/Sqrt[-(b
*x) + a^2*x^2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx=\frac{\left(\sqrt{x}\sqrt{-b+a^2x}\right)\int \frac{1}{x^{3/2}\sqrt{-b+a^2x}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx}{\sqrt{-bx+a^2x^2}}$$

$$=\frac{\left(2\sqrt{x}\sqrt{-b+a^2x}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{-b+a^2x^2}\left(ax^4+x^2\sqrt{-bx^2+a^2x^4}\right)^{3/2}}\,d\right.}{\sqrt{-bx+a^2x^2}}$$

Mathematica [F] time = 1.62, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3
/2)),x]
```

```
[Out] Integrate[1/(x*Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3
/2)), x]
```

IntegrateAlgebraic [A] time = 5.45, size = 132, normalized size = 1.00

$$\frac{4\left(256a^5x^2+32a^3bx+245ab^2\right)\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{1155b^4x^3}-\frac{4\sqrt{a^2x^2-bx}\left(-256a^4x^2-160a^2bx+105b^2\right)\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{1155b^4x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2
*x^2])^(3/2)),x]
```

```
[Out] (-4*Sqrt[-(b*x) + a^2*x^2]*(105*b^2 - 160*a^2*b*x - 256*a^4*x^2)*Sqrt[x*(a*
x + Sqrt[-(b*x) + a^2*x^2])])/(1155*b^4*x^4) + (4*(245*a*b^2 + 32*a^3*b*x +
 256*a^5*x^2)*Sqrt[x*(a*x + Sqrt[-(b*x) + a^2*x^2])])/(1155*b^4*x^3)
```

fricas [A] time = 0.41, size = 93, normalized size = 0.70

$$\frac{4\left(256\,a^5x^3+32\,a^3bx^2+245\,ab^2x+\left(256\,a^4x^2+160\,a^2bx-105\,b^2\right)\sqrt{a^2x^2-bx}\right)\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}}{1155\,b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, al
gorithm="fricas")
```

```
[Out] 4/1155*(256*a^5*x^3 + 32*a^3*b*x^2 + 245*a*b^2*x + (256*a^4*x^2 + 160*a^2*b
*x - 105*b^2)*sqrt(a^2*x^2 - b*x))*sqrt(a*x^2 + sqrt(a^2*x^2 - b*x)*x)/(b^4
*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2-bx}\left(ax^2+\sqrt{a^2x^2-bx}\,x\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, al
gorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 - b*x)*(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2)*x),
x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a^2x^2-bx}\,\left(a\,x^2+x\sqrt{a^2x^2-bx}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

```
[Out] int(1/x/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2-bx}\left(ax^2+\sqrt{a^2x^2-bx}\,x\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, al
gorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 - b*x)*(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2)*x),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\sqrt{a^2 x^2 - b x}\,\left(a x^2 + x\sqrt{a^2 x^2 - b x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2*x^2 - b*x)^(1/2)*(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x*(a^2*x^2 - b*x)^(1/2)*(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(x\left(ax + \sqrt{a^2x^2 - bx}\right)\right)^{\frac{3}{2}}\sqrt{x\left(a^2x - b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2*x**2-b*x)**(1/2)/(a*x**2+x*(a**2*x**2-b*x)**(1/2))**(3/
2),x)
```

```
[Out] Integral(1/(x*(x*(a*x + sqrt(a**2*x**2 - b*x)))**(3/2)*sqrt(x*(a**2*x - b))
), x)
```

3.1595 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(-1+x^2)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=132

$$\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{2}-1\right)}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)$$

Rubi [C] time = 0.72, antiderivative size = 161, normalized size of antiderivative = 1.22, number of steps used = 12, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6725, 2133, 725, 206}

$$-\frac{1}{4}\sqrt{1-i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)+\frac{1}{4}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)+\frac{1}{4}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)-\frac{1}{4}\sqrt{1+i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((-1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] -1/4*(Sqrt[1 - I]*ArcTanh[(1 - I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])]) + (Sqrt
[1 - I]*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/4 + (Sqrt[1 + I]*
ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/4 - (Sqrt[1 + I]*ArcTanh[
(1 + I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(-1+x^2\right)\sqrt{1+x^4}}\,dx = \int \left(-\frac{\sqrt{x^2+\sqrt{1+x^4}}}{2(1-x)\sqrt{1+x^4}} - \frac{\sqrt{x^2+\sqrt{1+x^4}}}{2(1+x)\sqrt{1+x^4}}\right)dx$$

$$= -\left(\frac{1}{2}\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1-x)\sqrt{1+x^4}}\,dx\right) - \frac{1}{2}\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$

$$= -\left(\left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{1}{(1-x)\sqrt{1-ix^2}}\,dx\right) - \left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{1}{(1+x)\sqrt{1-ix^2}}\,dx - \left(\frac{1}{4}+\frac{i}{4}\right)\int$$

$$= -\left(\left(-\frac{1}{4}-\frac{i}{4}\right)\text{Subst}\left(\int \frac{1}{(1+i)-x^2}\,dx, x, \frac{-1-ix}{\sqrt{1+ix^2}}\right)\right) - \left(-\frac{1}{4}-\frac{i}{4}\right)\text{Subst}\left(\int \frac{}{(1+}\right.$$

$$= -\frac{1}{4}\sqrt{1-i}\,\tanh^{-1}\left(\frac{1-ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) + \frac{1}{4}\sqrt{1-i}\,\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) + \frac{1}{4}\sqrt{1}$$

Mathematica [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(-1+x^2\right)\sqrt{1+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((-1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((-1 + x^2)*Sqrt[1 + x^4]), x]
```

IntegrateAlgebraic [A] time = 0.62, size = 190, normalized size = 1.44

$$\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right) - \sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2 + Sqrt[1 + x^4]]/((-1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] Sqrt[(-1 + Sqrt[2])/2]*ArcTan[(-Sqrt[-1/2 + 1/Sqrt[2]] + Sqrt[-1/2 + 1/Sqrt
[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 + x^4
]])] - Sqrt[(1 + Sqrt[2])/2]*ArcTanh[(-Sqrt[1/2 + 1/Sqrt[2]] + Sqrt[1/2 + 1
/Sqrt[2]]*x^2 + Sqrt[1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 +
 x^4]])]
```

fricas [B] time = 5.30, size = 289, normalized size = 2.19

$$-\frac{1}{2}\sqrt{2}\sqrt{\sqrt{2}-1}\arctan\left(-\frac{\left(\sqrt{2}x^2+x^2-\sqrt{x^4+1}\left(\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+3}+\sqrt{2}+1\right)+\left(x^2+\sqrt{2}\left(x^2-2\right)-3\right)\sqrt{-2\sqrt{2}+3}-1\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}-1}}{2x}\right) + \frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}+1}\log\left(-\frac{\sqrt{2}x^2+2x^2+\left(x^3-\sqrt{2}x-\sqrt{x^4+1}x-x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}+1}+\sqrt{x^4+1}\left(\sqrt{2}+1\right)}{x^2-1}\right) - \frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}+1}\log\left(-\frac{\sqrt{2}x^2+2x^2-\left(x^3-\sqrt{2}x-\sqrt{x^4+1}x-x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}+1}+\sqrt{x^4+1}\left(\sqrt{2}+1\right)}{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^2-1)/(x^4+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/2*sqrt(2)*sqrt(sqrt(2) - 1)*arctan(-1/2*(sqrt(2)*x^2 + x^2 - sqrt(x^4 +
1)*((sqrt(2) + 1)*sqrt(-2*sqrt(2) + 3) + sqrt(2) + 1) + (x^2 + sqrt(2)*(x^2
 - 2) - 3)*sqrt(-2*sqrt(2) + 3) - 1)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(sqrt(2)
 - 1)/x) + 1/8*sqrt(2)*sqrt(sqrt(2) + 1)*log(-(sqrt(2)*x^2 + 2*x^2 + (x^3 -
 sqrt(2)*x - sqrt(x^4 + 1)*x - x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(sqrt(2) +
1) + sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 - 1)) - 1/8*sqrt(2)*sqrt(sqrt(2) + 1
```

```
)*log(-(sqrt(2)*x^2 + 2*x^2 - (x^3 - sqrt(2)*x - sqrt(x^4 + 1)*x - x)*sqrt(
x^2 + sqrt(x^4 + 1))*sqrt(sqrt(2) + 1) + sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2
- 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^2-1)/(x^4+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\left(x^2-1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^2-1)/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^2-1)/(x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^2-1)/(x^4+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\left(x^2-1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^2 - 1)*(x^4 + 1)^(1/2)),x)
```

```
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^2 - 1)*(x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x-1)(x+1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**2-1)/(x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x - 1)*(x + 1)*sqrt(x**4 + 1)), x)
```

3.1596 $\int \frac{1}{(1+2x)\sqrt[3]{-1+4x+4x^2}}\,dx$

Optimal. Leaf size=133

$$-\frac{\log\left(2^{2/3}\sqrt[3]{4x^2+4x-1}+2\right)}{4\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}\left(4x^2+4x-1\right)^{2/3}+2^{2/3}\sqrt[3]{4x^2+4x-1}-2\right)}{8\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\sqrt[3]{}}{}\right.}{4\sqrt[3]{2}}$$

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 0.66, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {694, 266, 56, 617, 204, 31}

$$\frac{\log(2x+1)}{4\sqrt[3]{2}}-\frac{3\log\left(\sqrt[3]{(2x+1)^2-2}+\sqrt[3]{2}\right)}{8\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2^{2/3}\sqrt[3]{(2x+1)^2-2}}{\sqrt{3}}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + 2*x)*(-1 + 4*x + 4*x^2)^(1/3)),x]
```

```
[Out] -1/4*(Sqrt[3]*ArcTan[(1 - 2^(2/3)*(-2 + (1 + 2*x)^2)^(1/3))/Sqrt[3]])/2^(1/
3) + Log[1 + 2*x]/(4*2^(1/3)) - (3*Log[2^(1/3) + (-2 + (1 + 2*x)^2)^(1/3)])
/(8*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
```

```
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+2x)\sqrt[3]{-1+4x+4x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{-2+x^2}}\,dx, x, 1+2x\right)\\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{-2+x}\,x}\,dx, x, (1+2x)^2\right)\\
&= \frac{\log(1+2x)}{4\sqrt[3]{2}} + \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{-2+(1+2x)^2}\right) - \frac{3\,\text{Subst}}{}\\
&= \frac{\log(1+2x)}{4\sqrt[3]{2}} - \frac{3\log\left(\sqrt[3]{2}+\sqrt[3]{-2+(1+2x)^2}\right)}{8\sqrt[3]{2}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-2^{2/3}\right.}{4\sqrt[3]{2}}\\
&= -\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1-2^{2/3}\sqrt[3]{-2+(1+2x)^2}}{\sqrt{3}}\right)}{4\sqrt[3]{2}} + \frac{\log(1+2x)}{4\sqrt[3]{2}} - \frac{3\log\left(\sqrt[3]{2}+\sqrt[3]{-2+(1+2x)^2}\right)}{8\sqrt[3]{2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.30

$$\frac{3}{16}\left((2x+1)^2-2\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{1}{2}\left(2-(2x+1)^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + 2*x)*(-1 + 4*x + 4*x^2)^(1/3)),x]
```

```
[Out] (3*(-2 + (1 + 2*x)^2)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (2 - (1 + 2*x)^2
)/2])/16
```

IntegrateAlgebraic [A] time = 0.24, size = 133, normalized size = 1.00

$$-\frac{\log\left(2^{2/3}\sqrt[3]{4x^2+4x-1}+2\right)}{4\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}\left(4x^2+4x-1\right)^{2/3}+2^{2/3}\sqrt[3]{4x^2+4x-1}-2\right)}{8\sqrt[3]{2}} - \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\sqrt[3]{4x^2+4x-1}}{\sqrt{3}}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + 2*x)*(-1 + 4*x + 4*x^2)^(1/3)),x]
```

```
[Out] -1/4*(Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(-1 + 4*x + 4*x^2)^(1/3))/Sqrt[3]
])/2^(1/3) - Log[2 + 2^(2/3)*(-1 + 4*x + 4*x^2)^(1/3)]/(4*2^(1/3)) + Log[-2
 + 2^(2/3)*(-1 + 4*x + 4*x^2)^(1/3) - 2^(1/3)*(-1 + 4*x + 4*x^2)^(2/3)]/(8*
2^(1/3))
```

fricas [A] time = 0.40, size = 124, normalized size = 0.93

$$\frac{1}{8}\sqrt{3}\,2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{1}{6}}\left(2\sqrt{2}\,(-1)^{\frac{1}{3}}\left(4x^2+4x-1\right)^{\frac{1}{3}}+2^{\frac{5}{6}}\right)\right) - \frac{1}{16}\cdot 2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}\,(-1)^{\frac{2}{3}}\left(4x^2+4x-1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}+\left(4x^2+4x-1\right)^{\frac{2}{3}}\right) + \frac{1}{8}\cdot 2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}\,(-1)^{\frac{2}{3}}+\left(4x^2+4x-1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(4*x^2+4*x-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2*sqrt(2)*(-1)^(
1/3)*(4*x^2 + 4*x - 1)^(1/3) + 2^(5/6))) - 1/16*2^(2/3)*(-1)^(1/3)*log(-2^(
1/3)*(-1)^(2/3)*(4*x^2 + 4*x - 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (4*x^2 + 4*x
```

```
- 1)^(2/3)) + 1/8*2^(2/3)*(-1)^(1/3)*log(2^(1/3)*(-1)^(2/3) + (4*x^2 + 4*x
- 1)^(1/3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(4\,x^2+4\,x-1\right)^{\frac{1}{3}}(2\,x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(4*x^2+4*x-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((4*x^2 + 4*x - 1)^(1/3)*(2*x + 1)), x)
```

maple [C] time = 8.08, size = 1244, normalized size = 9.35

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x)/(4*x^2+4*x-1)^(1/3),x)
```

```
[Out] 1/2*RootOf(RootOf(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4)+16*_Z^2)*ln((8*RootOf(RootO
f(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4)+16*_Z^2)*RootOf(_Z^3+4)^3*x^2+160*RootOf(Ro
otOf(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4)+16*_Z^2)^2*RootOf(_Z^3+4)^2*x^2+8*RootOf
(RootOf(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4)+16*_Z^2)*RootOf(_Z^3+4)^3*x-48*(4*x^2
+4*x-1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4)+16*_Z^2)*RootOf(_
Z^3+4)^2+160*RootOf(RootOf(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4)+16*_Z^2)^2*RootOf(
_Z^3+4)^2*x-9*(4*x^2+4*x-1)^(1/3)*RootOf(_Z^3+4)^2-96*(4*x^2+4*x-1)^(1/3)*R
ootOf(RootOf(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4)+16*_Z^2)*RootOf(_Z^3+4)+4*RootOf
(_Z^3+4)*x^2+80*RootOf(RootOf(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4)+16*_Z^2)*x^2+4*
RootOf(_Z^3+4)*x+18*(4*x^2+4*x-1)^(2/3)+80*RootOf(RootOf(_Z^3+4)^2+4*_Z*Roo
tOf(_Z^3+4)+16*_Z^2)*x-7*RootOf(_Z^3+4)-140*RootOf(RootOf(_Z^3+4)^2+4*_Z*Ro
otOf(_Z^3+4)+16*_Z^2))/(1+2*x)^2)-1/8*ln((32*RootOf(RootOf(_Z^3+4)^2+4*_Z*R
ootOf(_Z^3+4)+16*_Z^2)*RootOf(_Z^3+4)^3*x^2+160*RootOf(RootOf(_Z^3+4)^2+4*_
Z*RootOf(_Z^3+4)+16*_Z^2)^2*RootOf(_Z^3+4)^2*x^2+32*RootOf(RootOf(_Z^3+4)^2
+4*_Z*RootOf(_Z^3+4)+16*_Z^2)*RootOf(_Z^3+4)^3*x+48*(4*x^2+4*x-1)^(2/3)*Roo
tOf(RootOf(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4)+16*_Z^2)*RootOf(_Z^3+4)^2+160*Root
Of(RootOf(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4)+16*_Z^2)^2*RootOf(_Z^3+4)^2*x+15*(4
*x^2+4*x-1)^(1/3)*RootOf(_Z^3+4)^2+96*(4*x^2+4*x-1)^(1/3)*RootOf(RootOf(_Z^
3+4)^2+4*_Z*RootOf(_Z^3+4)+16*_Z^2)*RootOf(_Z^3+4)-48*RootOf(_Z^3+4)*x^2-24
0*RootOf(RootOf(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4)+16*_Z^2)*x^2-48*RootOf(_Z^3+4
)*x-30*(4*x^2+4*x-1)^(2/3)-240*RootOf(RootOf(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4)+
16*_Z^2)*x+28*RootOf(_Z^3+4)+140*RootOf(RootOf(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4
)+16*_Z^2))/(1+2*x)^2)*RootOf(_Z^3+4)-1/2*ln((32*RootOf(RootOf(_Z^3+4)^2+4*
_Z*RootOf(_Z^3+4)+16*_Z^2)*RootOf(_Z^3+4)^3*x^2+160*RootOf(RootOf(_Z^3+4)^2
+4*_Z*RootOf(_Z^3+4)+16*_Z^2)^2*RootOf(_Z^3+4)^2*x^2+32*RootOf(RootOf(_Z^3+
4)^2+4*_Z*RootOf(_Z^3+4)+16*_Z^2)*RootOf(_Z^3+4)^3*x+48*(4*x^2+4*x-1)^(2/3)
*RootOf(RootOf(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4)+16*_Z^2)*RootOf(_Z^3+4)^2+160*
RootOf(RootOf(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4)+16*_Z^2)^2*RootOf(_Z^3+4)^2*x+1
5*(4*x^2+4*x-1)^(1/3)*RootOf(_Z^3+4)^2+96*(4*x^2+4*x-1)^(1/3)*RootOf(RootOf
(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4)+16*_Z^2)*RootOf(_Z^3+4)-48*RootOf(_Z^3+4)*x^
2-240*RootOf(RootOf(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4)+16*_Z^2)*x^2-48*RootOf(_Z
^3+4)*x-30*(4*x^2+4*x-1)^(2/3)-240*RootOf(RootOf(_Z^3+4)^2+4*_Z*RootOf(_Z^3
+4)+16*_Z^2)*x+28*RootOf(_Z^3+4)+140*RootOf(RootOf(_Z^3+4)^2+4*_Z*RootOf(_Z
^3+4)+16*_Z^2))/(1+2*x)^2)*RootOf(RootOf(_Z^3+4)^2+4*_Z*RootOf(_Z^3+4)+16*_
Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(4\,x^2+4\,x-1\right)^{\frac{1}{3}}(2\,x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(4*x^2+4*x-1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((4*x^2 + 4*x - 1)^(1/3)*(2*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(2\,x+1)\,\left(4\,x^2+4\,x-1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + 1)*(4*x + 4*x^2 - 1)^(1/3)),x)
```

```
[Out] int(1/((2*x + 1)*(4*x + 4*x^2 - 1)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x+1)\sqrt[3]{4x^2+4x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(4*x**2+4*x-1)**(1/3),x)
```

```
[Out] Integral(1/((2*x + 1)*(4*x**2 + 4*x - 1)**(1/3)), x)
```

3.1597 $$\int \frac{x^3(-b+x)(-2ab+(3a-b)x)}{(-a+x)\left(x^2(-a+x)(-b+x)\right)^{3/4}\left(-a^3d+3a^2dx+(b-3ad)x^2+(-1+d)x^3\right)}\,dx$$

Optimal. Leaf size=133

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x^3(-a-b)+abx^2+x^4\right)^{3/4}}{x^2(x-b)}\right)-2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x^3(-a-b)+abx^2+x^4\right)^{3/4}}{x^2(x-b)}\right)-\frac{4\sqrt[4]{x^3(-a-b)}}{a-}$$

Rubi [F] time = 26.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3(-b+x)(-2ab+(3a-b)x)}{(-a+x)\left(x^2(-a+x)(-b+x)\right)^{3/4}\left(-a^3d+3a^2dx+(b-3ad)x^2+(-1+d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*(-b + x)*(-2*a*b + (3*a - b)*x))/((-a + x)*(x^2*(-a + x)*(-b + x))
^(3/4)*(-(a^3*d) + 3*a^2*d*x + (b - 3*a*d)*x^2 + (-1 + d)*x^3)),x]
```

```
[Out] (-4*(3*a - b)*(b - x)*x^2)/(3*a*(1 - d)*((a - x)*(b - x)*x^2)^(3/4)) - (2*(
3*a - b)*((b*(a - x))/(a*(b - x)))^(3/4)*(b - x)*x^2*Hypergeometric2F1[1/2,
 3/4, 3/2, -(((a - b)*x)/(a*(b - x)))])/(3*a*(1 - d)*((a - x)*(b - x)*x^2)^
(3/4)) + (2*a^3*(3*a - b)*d*x^(3/2)*(-a + x)^(3/4)*(-b + x)^(3/4)*Defer[Sub
st][Defer[Int][(-b + x^2)^(1/4)/((-a + x^2)^(7/4)*(a^3*d - 3*a^2*d*x^2 - b*
(1 - (3*a*d)/b)*x^4 + (1 - d)*x^6)), x], x, Sqrt[x]])/((1 - d)*((a - x)*(b
- x)*x^2)^(3/4)) - (6*a^2*(3*a - b)*d*x^(3/2)*(-a + x)^(3/4)*(-b + x)^(3/4)
*Defer[Subst][Defer[Int][(x^2*(-b + x^2)^(1/4))/((-a + x^2)^(7/4)*(a^3*d -
3*a^2*d*x^2 - b*(1 - (3*a*d)/b)*x^4 + (1 - d)*x^6)), x], x, Sqrt[x]])/((1 -
 d)*((a - x)*(b - x)*x^2)^(3/4)) + (2*(b^2 + 9*a^2*d - a*(b + 5*b*d))*x^(3/
2)*(-a + x)^(3/4)*(-b + x)^(3/4)*Defer[Subst][Defer[Int][(x^4*(-b + x^2)^(1
/4))/((-a + x^2)^(7/4)*(a^3*d - 3*a^2*d*x^2 - b*(1 - (3*a*d)/b)*x^4 + (1 -
d)*x^6)), x], x, Sqrt[x]])/((1 - d)*((a - x)*(b - x)*x^2)^(3/4))
```

Rubi steps

$$\int \frac{x^3(-b+x)(-2ab+(3a-b)x)}{(-a+x)\left(x^2(-a+x)(-b+x)\right)^{3/4}\left(-a^3d+3a^2dx+(b-3ad)x^2+(-1+d)x^3\right)}\,dx = \frac{\left(x^{3/2}(-a+x)^{3/4}(-b+x)^{3/4}\right)}{\left(x^2(-\right.}$$

$$= \frac{\left(2x^{3/2}(-a+x)^{3/4}(-b+x)^{3/4}\right.}{}$$

$$= \frac{\left(2x^{3/2}(-a+x)^{3/4}(-b+x)^{3/4}\right.}{}$$

$$= -\frac{\left(2(3a-b)x^{3/2}(-a+x)^{3/4}(\right.}{(1-d)\left(\right.}$$

$$= -\frac{4(3a-b)(b-x)x^2}{3a(1-d)\left((a-x)(b-x)x^2\right.}$$

$$= -\frac{4(3a-b)(b-x)x^2}{3a(1-d)\left((a-x)(b-x)x^2\right.}$$

Mathematica [F] time = 5.34, size = 0, normalized size = 0.00

$$\int \frac{x^3(-b+x)(-2ab+(3a-b)x)}{(-a+x)\left(x^2(-a+x)(-b+x)\right)^{3/4}\left(-a^3d+3a^2dx+(b-3ad)x^2+(-1+d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(-b + x)*(-2*a*b + (3*a - b)*x))/((-a + x)*(x^2*(-a + x)*(-b
 + x))^(3/4)*(-(a^3*d) + 3*a^2*d*x + (b - 3*a*d)*x^2 + (-1 + d)*x^3)),x]
```

```
[Out] Integrate[(x^3*(-b + x)*(-2*a*b + (3*a - b)*x))/((-a + x)*(x^2*(-a + x)*(-b
 + x))^(3/4)*(-(a^3*d) + 3*a^2*d*x + (b - 3*a*d)*x^2 + (-1 + d)*x^3)), x]
```

IntegrateAlgebraic [A] time = 6.79, size = 133, normalized size = 1.00

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x^3(-a-b)+abx^2+x^4\right)^{3/4}}{x^2(x-b)}\right)-2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x^3(-a-b)+abx^2+x^4\right)^{3/4}}{x^2(x-b)}\right)-\frac{4\sqrt[4]{x^3(-a-b)+abx^2+x^4}}{a-x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(-b + x)*(-2*a*b + (3*a - b)*x))/((-a + x)*(x^2*(-a
 + x)*(-b + x))^(3/4)*(-(a^3*d) + 3*a^2*d*x + (b - 3*a*d)*x^2 + (-1 + d)*x^
3)),x]
```

```
[Out] (-4*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/4))/(a - x) + 2*d^(1/4)*ArcTan[(d^(1/
4)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(3/4))/(x^2*(-b + x))] - 2*d^(1/4)*ArcTan
h[(d^(1/4)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(3/4))/(x^2*(-b + x))]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-b+x)*(-2*a*b+(3*a-b)*x)/(-a+x)/(x^2*(-a+x)*(-b+x))^(3/4)/(-
a^3*d+3*a^2*d*x+(-3*a*d+b)*x^2+(-1+d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2\,ab-(3\,a-b)x)(b-x)x^3}{\left(a^3d-3\,a^2dx-(d-1)x^3+(3\,ad-b)x^2\right)\left((a-x)(b-x)x^2\right)^{\frac{3}{4}}(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-b+x)*(-2*a*b+(3*a-b)*x)/(-a+x)/(x^2*(-a+x)*(-b+x))^(3/4)/(-
a^3*d+3*a^2*d*x+(-3*a*d+b)*x^2+(-1+d)*x^3),x, algorithm="giac")
```

```
[Out] integrate((2*a*b - (3*a - b)*x)*(b - x)*x^3/((a^3*d - 3*a^2*d*x - (d - 1)*x
^3 + (3*a*d - b)*x^2)*((a - x)*(b - x)*x^2)^(3/4)*(a - x)), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x^3(-b+x)(-2ab+(3a-b)x)}{(-a+x)\left(x^2(-a+x)(-b+x)\right)^{\frac{3}{4}}\left(-a^3d+3a^2dx+(-3ad+b)x^2+(-1+d)x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-b+x)*(-2*a*b+(3*a-b)*x)/(-a+x)/(x^2*(-a+x)*(-b+x))^(3/4)/(-a^3*d+
3*a^2*d*x+(-3*a*d+b)*x^2+(-1+d)*x^3),x)
```

```
[Out] int(x^3*(-b+x)*(-2*a*b+(3*a-b)*x)/(-a+x)/(x^2*(-a+x)*(-b+x))^(3/4)/(-a^3*d+
3*a^2*d*x+(-3*a*d+b)*x^2+(-1+d)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2\,ab-(3\,a-b)x)(b-x)x^3}{\left(a^3d-3\,a^2dx-(d-1)x^3+(3\,ad-b)x^2\right)\left((a-x)(b-x)x^2\right)^{\frac{3}{4}}(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-b+x)*(-2*a*b+(3*a-b)*x)/(-a+x)/(x^2*(-a+x)*(-b+x))^(3/4)/(-
a^3*d+3*a^2*d*x+(-3*a*d+b)*x^2+(-1+d)*x^3),x, algorithm="maxima")
```

```
[Out] integrate((2*a*b - (3*a - b)*x)*(b - x)*x^3/((a^3*d - 3*a^2*d*x - (d - 1)*x
^3 + (3*a*d - b)*x^2)*((a - x)*(b - x)*x^2)^(3/4)*(a - x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^3\,\left(2\,a\,b-x\,\left(3\,a-b\right)\right)\,\left(b-x\right)}{\left(a-x\right)\,{\left(x^2\,\left(a-x\right)\,\left(b-x\right)\right)}^{3/4}\,\left(x^2\,\left(b-3\,a\,d\right)-a^3\,d+x^3\,\left(d-1\right)+3\,a^2\,d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(2*a*b - x*(3*a - b))*(b - x))/((a - x)*(x^2*(a - x)*(b - x))^(3/
4)*(x^2*(b - 3*a*d) - a^3*d + x^3*(d - 1) + 3*a^2*d*x)),x)
```

```
[Out] -int((x^3*(2*a*b - x*(3*a - b))*(b - x))/((a - x)*(x^2*(a - x)*(b - x))^(3/
4)*(x^2*(b - 3*a*d) - a^3*d + x^3*(d - 1) + 3*a^2*d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-b+x)*(-2*a*b+(3*a-b)*x)/(-a+x)/(x**2*(-a+x)*(-b+x))**(3/4)
/(-a**3*d+3*a**2*d*x+(-3*a*d+b)*x**2+(-1+d)*x**3),x)
```

```
[Out] Timed out
```

3.1598 $\int \frac{x^2}{\sqrt{-x-x^2+x^3}\left(-1+x^4\right)}\,dx$

Optimal. Leaf size=133

$$-\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x^3-x^2-x}}{x^2-x-1}\right)+\frac{1}{4}\sqrt{\frac{1}{5}+\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1-2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)+\frac{1}{4}\sqrt{\frac{1}{5}-\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1+2i}\sqrt{x^3-x^2-}}{x^2-x-1}\right.$$

Rubi [C] time = 3.08, antiderivative size = 467, normalized size of antiderivative = 3.51, number of steps used = 55, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2056, 6725, 957, 716, 1098, 934, 168, 538, 537, 1134, 1184}

$$-\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\,\Pi\left(\frac{1}{2}\left(-1-\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{4\sqrt{x^3-x^2-x}}+\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\,\Pi\left(-\frac{1}{2}i\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{4\sqrt{x^3-x^2-x}}+\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\,\Pi\left(\frac{1}{2}i\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{4\sqrt{x^3-x^2-x}}-\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{2x+\sqrt{5}-1}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\,\Pi\left(\frac{1}{2}\left(1+\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)|\frac{1}{2}\left(-3-\sqrt{5}\right)\right)}{4\sqrt{x^3-x^2-x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[-x - x^2 + x^3]*(-1 + x^4)),x]
```

```
[Out] -1/4*(Sqrt[3 + Sqrt[5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1
+ Sqrt[5])]*EllipticPi[(-1 - Sqrt[5])/2, ArcSin[Sqrt[2/(1 + Sqrt[5])]*Sqrt[
x]], (-3 - Sqrt[5])/2])/Sqrt[-x - x^2 + x^3] + (Sqrt[3 + Sqrt[5]]*Sqrt[x]*S
qrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1 + Sqrt[5])]*EllipticPi[(-1/2*I)*(
1 + Sqrt[5]), ArcSin[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]], (-3 - Sqrt[5])/2])/(4*
Sqrt[-x - x^2 + x^3]) + (Sqrt[3 + Sqrt[5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]
*Sqrt[1 - (2*x)/(1 + Sqrt[5])]*EllipticPi[(I/2)*(1 + Sqrt[5]), ArcSin[Sqrt[
2/(1 + Sqrt[5])]*Sqrt[x]], (-3 - Sqrt[5])/2])/(4*Sqrt[-x - x^2 + x^3]) - (S
qrt[3 + Sqrt[5]]*Sqrt[x]*Sqrt[-1 + Sqrt[5] + 2*x]*Sqrt[1 - (2*x)/(1 + Sqrt[
5])]*EllipticPi[(1 + Sqrt[5])/2, ArcSin[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]], (-3
 - Sqrt[5])/2])/(4*Sqrt[-x - x^2 + x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 716

```
Int[(x_)^(m_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2,
Subst[Int[x^(2*m + 1)/Sqrt[a + b*x^2 + c*x^4], x], x, Sqrt[x]], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m^2, 1/4]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 957

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1098

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*Sqrt[(
2*a + (b + q)*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]],
(b + q)/(2*q)])/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)]), x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1134

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, -Dist[(b - q)/(2*c), Int[1/Sqrt[a + b*x^2 + c*x^4], x
], x] + Dist[1/(2*c), Int[(b - q + 2*c*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e*x*(b + q + 2*c*x^2))/(2*c*Sqrt
[a + b*x^2 + c*x^4]), x] - Simp[(e*q*Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q
)*x^2)]*Sqrt[(2*a + (b + q)*x^2)/q]*EllipticE[ArcSin[x/Sqrt[(2*a + (b + q)*
x^2)/(2*q)]], (b + q)/(2*q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b
 + q)*x^2)]), x] /; EqQ[2*c*d - e*(b - q), 0]] /; FreeQ[{a, b, c, d, e}, x]
 && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{-x-x^2+x^3}\left(-1+x^4\right)}dx = \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{x^{3/2}}{\sqrt{-1-x+x^2}\left(-1+x^4\right)}dx}{\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \left(-\frac{x^{3/2}}{2\left(1-x^2\right)\sqrt{-1-x+x^2}} - \frac{x^{3/2}}{2\left(1+x^2\right)\sqrt{-1-x+x^2}}\right)dx}{\sqrt{-x-x^2+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{x^{3/2}}{\left(1-x^2\right)\sqrt{-1-x+x^2}}dx}{2\sqrt{-x-x^2+x^3}} - \frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{x^{3/2}}{\left(1+x^2\right)\sqrt{-1}}}{2\sqrt{-x-x^2+x^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-1-x+x^2}\right)\int \left(\frac{ix^{3/2}}{2(i-x)\sqrt{-1-x+x^2}} + \frac{ix^{3/2}}{2(i+x)\sqrt{-1-x+x^2}}\right)dx}{2\sqrt{-x-x^2+x^3}} - \frac{\left(\sqrt{x}\sqrt{-1-\right.}{}$$

$$= -\frac{\left(i\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{x^{3/2}}{(i-x)\sqrt{-1-x+x^2}}dx}{4\sqrt{-x-x^2+x^3}} - \frac{\left(i\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{x^{3/2}}{(i+x)\sqrt{-1-}}}{4\sqrt{-x-x^2+x^3}}$$

$$= -\frac{\left(i\sqrt{x}\sqrt{-1-x+x^2}\right)\int \left(-\frac{i}{\sqrt{x}\sqrt{-1-x+x^2}} + \frac{1}{(-i-x)\sqrt{x}\sqrt{-1-x+x^2}} + \frac{\sqrt{x}}{\sqrt{-1-x+x^2}}\right)dx}{4\sqrt{-x-x^2+x^3}}$$

$$= -\frac{\left(i\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{1}{(-i-x)\sqrt{x}\sqrt{-1-x+x^2}}dx}{4\sqrt{-x-x^2+x^3}} - \frac{\left(i\sqrt{x}\sqrt{-1-x+x^2}\right)\int \frac{}{\sqrt{x}(-i}}{4\sqrt{-x-x^2+x}}$$

$$= -\frac{\left(i\sqrt{x}\sqrt{-1-\sqrt{5}+2x}\sqrt{-1+\sqrt{5}+2x}\right)\int \frac{1}{(-i-x)\sqrt{x}\sqrt{-1-\sqrt{5}+2x}\sqrt{-1+\sqrt{5}+2x}}d}{4\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(i\sqrt{x}\sqrt{-1-\sqrt{5}+2x}\sqrt{-1+\sqrt{5}+2x}\right)\mathrm{Subst}\left(\int \frac{1}{\left(i-x^2\right)\sqrt{-1-\sqrt{5}+2x^2}\sqrt{-1+\sqrt{5}+}}\right.}{2\sqrt{-x-x^2+x^3}}$$

$$= \frac{\left(i\sqrt{x}\sqrt{-1+\sqrt{5}+2x}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\right)\mathrm{Subst}\left(\int \frac{1}{\left(i-x^2\right)\sqrt{-1+\sqrt{5}+2x^2}\sqrt{1+\frac{2x^2}{-1-\sqrt{5}}}}dx\right.}{2\sqrt{-x-x^2+x^3}}$$

$$= -\frac{\sqrt{3+\sqrt{5}}\sqrt{x}\sqrt{-1+\sqrt{5}+2x}\sqrt{1-\frac{2x}{1+\sqrt{5}}}\,\Pi\left(\frac{1}{2}\left(-1-\sqrt{5}\right);\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{}}}\right.\right.}{4\sqrt{-x-x^2+x^3}}$$

Mathematica [C] time = 0.92, size = 430, normalized size = 3.23

$$\frac{(1+i)\sqrt{\frac{x}{2-2\sqrt{5}}}\sqrt{-x^2+x+1}\left((-1+i)\sqrt{5}\,\Pi\left(\frac{1}{2}\left(-5-3\sqrt{5}\right);\sin^{-1}\left(\frac{\sqrt{2x+\sqrt{5}-1}}{\sqrt{2}\sqrt[4]{5}}\right)|\frac{1}{2}\left(5+\sqrt{5}\right)\right)+(1-i)\left(2\sqrt{5}-5\right)\Pi\left(\frac{1}{2}\left(5-\sqrt{5}\right);\sin^{-1}\left(\frac{\sqrt{2x+\sqrt{5}-1}}{\sqrt{2}\sqrt[4]{5}}\right)|\frac{1}{2}\left(5+\sqrt{5}\right)\right)+\sqrt{5}\,\Pi\left(\frac{2\sqrt{5}}{(-1-2i)+\sqrt{5}};\sin^{-1}\left(\frac{\sqrt{2x+\sqrt{5}-1}}{\sqrt{2}\sqrt[4]{5}}\right)|\frac{1}{2}\left(5+\sqrt{5}\right)\right)-(2+i)\Pi\left(\frac{2\sqrt{5}}{(-1-2i)+\sqrt{5}};\sin^{-1}\left(\frac{\sqrt{2x+\sqrt{5}-1}}{\sqrt{2}\sqrt[4]{5}}\right)|\frac{1}{2}\left(5+\sqrt{5}\right)\right)-i\sqrt{5}\,\Pi\left(\frac{2\sqrt{5}}{(-1+2i)+\sqrt{5}};\sin^{-1}\left(\frac{\sqrt{2x+\sqrt{5}-1}}{\sqrt{2}\sqrt[4]{5}}\right)|\frac{1}{2}\left(5+\sqrt{5}\right)\right)+(1+2i)\Pi\left(\frac{2\sqrt{5}}{(-1+2i)+\sqrt{5}};\sin^{-1}\left(\frac{\sqrt{2x+\sqrt{5}-1}}{\sqrt{2}\sqrt[4]{5}}\right)|\frac{1}{2}\left(5+\sqrt{5}\right)\right)\right)}{\left(3\sqrt{5}-5\right)\sqrt{x\left(x^2-x-1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(Sqrt[-x - x^2 + x^3]*(-1 + x^4)),x]
```

```
[Out] ((1 + I)*Sqrt[x/(2 - 2*Sqrt[5])]*Sqrt[1 + x - x^2]*((-1 + I)*Sqrt[5]*Ellipt
icPi[(-5 - 3*Sqrt[5])/2, ArcSin[Sqrt[-1 + Sqrt[5] + 2*x]/(Sqrt[2]*5^(1/4))]
, (5 + Sqrt[5])/2] + (1 - I)*(-5 + 2*Sqrt[5])*EllipticPi[(5 - Sqrt[5])/2, A
rcSin[Sqrt[-1 + Sqrt[5] + 2*x]/(Sqrt[2]*5^(1/4))], (5 + Sqrt[5])/2] - (2 +
I)*EllipticPi[(2*Sqrt[5])/((-1 - 2*I) + Sqrt[5]), ArcSin[Sqrt[-1 + Sqrt[5]
+ 2*x]/(Sqrt[2]*5^(1/4))], (5 + Sqrt[5])/2] + Sqrt[5]*EllipticPi[(2*Sqrt[5]
)/((-1 - 2*I) + Sqrt[5]), ArcSin[Sqrt[-1 + Sqrt[5] + 2*x]/(Sqrt[2]*5^(1/4))
], (5 + Sqrt[5])/2] + (1 + 2*I)*EllipticPi[(2*Sqrt[5])/((-1 + 2*I) + Sqrt[5
```

```
]), ArcSin[Sqrt[-1 + Sqrt[5] + 2*x]/(Sqrt[2]*5^(1/4))], (5 + Sqrt[5])/2] -
I*Sqrt[5]*EllipticPi[(2*Sqrt[5])/((-1 + 2*I) + Sqrt[5]), ArcSin[Sqrt[-1 + S
qrt[5] + 2*x]/(Sqrt[2]*5^(1/4))], (5 + Sqrt[5])/2]))/((-5 + 3*Sqrt[5])*Sqrt
[x*(-1 - x + x^2)])
```

IntegrateAlgebraic [A] time = 0.32, size = 133, normalized size = 1.00

$$-\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x^3-x^2-x}}{x^2-x-1}\right)+\frac{1}{4}\sqrt{\frac{1}{5}+\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1-2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)+\frac{1}{4}\sqrt{\frac{1}{5}-\frac{2i}{5}}\tan^{-1}\left(\frac{\sqrt{1+2i}\sqrt{x^3-x^2-x}}{x^2-x-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[-x - x^2 + x^3]*(-1 + x^4)),x]
```

```
[Out] -1/2*ArcTan[Sqrt[-x - x^2 + x^3]/(-1 - x + x^2)] + (Sqrt[1/5 + (2*I)/5]*Arc
Tan[(Sqrt[1 - 2*I]*Sqrt[-x - x^2 + x^3])/(-1 - x + x^2)])/4 + (Sqrt[1/5 - (
2*I)/5]*ArcTan[(Sqrt[1 + 2*I]*Sqrt[-x - x^2 + x^3])/(-1 - x + x^2)])/4
```

fricas [B] time = 0.93, size = 2485, normalized size = 18.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3-x^2-x)^(1/2)/(x^4-1),x, algorithm="fricas")
```

```
[Out] -1/320*5^(1/4)*(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 5)*log(5*(5*x^4 -
 20*x^3 + 5^(1/4)*sqrt(x^3 - x^2 - x)*(sqrt(5)*sqrt(2)*(x^2 - 6*x - 1) - 5*
sqrt(2)*(x^2 - 2*x - 1))*sqrt(sqrt(5) + 5) + 30*x^2 + 20*sqrt(5)*(x^3 - x^2
 - x) + 20*x + 5)/(x^4 + 2*x^2 + 1)) + 1/320*5^(1/4)*(sqrt(5)*sqrt(2) - sqr
t(2))*sqrt(sqrt(5) + 5)*log(5*(5*x^4 - 20*x^3 - 5^(1/4)*sqrt(x^3 - x^2 - x)
*(sqrt(5)*sqrt(2)*(x^2 - 6*x - 1) - 5*sqrt(2)*(x^2 - 2*x - 1))*sqrt(sqrt(5)
 + 5) + 30*x^2 + 20*sqrt(5)*(x^3 - x^2 - x) + 20*x + 5)/(x^4 + 2*x^2 + 1))
+ 1/40*5^(1/4)*sqrt(2)*sqrt(sqrt(5) + 5)*arctan(-1/200*(100*x^11 + 1300*x^1
0 - 6700*x^9 - 4400*x^8 + 28400*x^7 + 1400*x^6 - 28400*x^5 - 4400*x^4 + 670
0*x^3 + 1300*x^2 + 5*sqrt(x^3 - x^2 - x)*(5^(3/4)*(sqrt(5)*sqrt(2)*(x^10 -
4*x^9 - 17*x^8 + 56*x^7 + 78*x^6 - 136*x^5 - 78*x^4 + 56*x^3 + 17*x^2 - 4*x
 - 1) - sqrt(2)*(x^10 + 8*x^9 - 57*x^8 - 24*x^7 + 294*x^6 + 64*x^5 - 294*x^
4 - 24*x^3 + 57*x^2 + 8*x - 1)) + 4*5^(1/4)*(sqrt(5)*sqrt(2)*(3*x^9 + 7*x^8
 + 14*x^7 - 81*x^6 - 10*x^5 + 81*x^4 + 14*x^3 - 7*x^2 + 3*x) - 5*sqrt(2)*(x
^9 + 9*x^8 - 18*x^7 - 15*x^6 + 26*x^5 + 15*x^4 - 18*x^3 - 9*x^2 + x)))*sqrt
(sqrt(5) + 5) - sqrt(5)*(240*x^10 + 160*x^9 - 1680*x^8 - 480*x^7 + 3200*x^6
 + 480*x^5 - 1680*x^4 - 160*x^3 + 240*x^2 + sqrt(x^3 - x^2 - x)*(5^(3/4)*(s
qrt(5)*sqrt(2)*(x^10 - 6*x^9 - 25*x^8 + 120*x^7 - 58*x^6 - 196*x^5 + 58*x^4
 + 120*x^3 + 25*x^2 - 6*x - 1) - sqrt(2)*(x^10 - 2*x^9 - 17*x^8 + 216*x^7 -
 306*x^6 - 396*x^5 + 306*x^4 + 216*x^3 + 17*x^2 - 2*x - 1)) + 4*5^(1/4)*(sq
rt(5)*sqrt(2)*(3*x^9 - 3*x^8 - 56*x^7 + 69*x^6 + 90*x^5 - 69*x^4 - 56*x^3 +
 3*x^2 + 3*x) - 5*sqrt(2)*(x^9 + 3*x^8 - 28*x^7 + 11*x^6 + 54*x^5 - 11*x^4
- 28*x^3 - 3*x^2 + x)))*sqrt(sqrt(5) + 5) + 4*sqrt(5)*(5*x^11 - 25*x^10 - 1
05*x^9 + 440*x^8 + 50*x^7 - 830*x^6 - 50*x^5 + 440*x^4 + 105*x^3 - 25*x^2 -
 sqrt(5)*(x^11 + 3*x^10 - 37*x^9 + 96*x^8 - 6*x^7 - 166*x^6 + 6*x^5 + 96*x^
4 + 37*x^3 + 3*x^2 - x) - 5*x) - 80*sqrt(5)*(x^10 + 6*x^9 - 23*x^8 - 2*x^7
+ 40*x^6 + 2*x^5 - 23*x^4 - 6*x^3 + x^2))*sqrt((5*x^4 - 20*x^3 + 5^(1/4)*sq
rt(x^3 - x^2 - x)*(sqrt(5)*sqrt(2)*(x^2 - 6*x - 1) - 5*sqrt(2)*(x^2 - 2*x -
 1))*sqrt(sqrt(5) + 5) + 30*x^2 + 20*sqrt(5)*(x^3 - x^2 - x) + 20*x + 5)/(x
^4 + 2*x^2 + 1)) + 20*sqrt(5)*(5*x^11 - 15*x^10 - 15*x^9 + 20*x^8 - 20*x^7
+ 70*x^6 + 20*x^5 + 20*x^4 + 15*x^3 - 15*x^2 - sqrt(5)*(x^11 + 13*x^10 - 67
*x^9 - 44*x^8 + 284*x^7 + 14*x^6 - 284*x^5 - 44*x^4 + 67*x^3 + 13*x^2 - x)
- 5*x) - 100*sqrt(5)*(x^11 - 3*x^10 - 3*x^9 + 4*x^8 - 4*x^7 + 14*x^6 + 4*x^
5 + 4*x^4 + 3*x^3 - 3*x^2 - x) - 100*x)/(x^11 - 9*x^10 - 45*x^9 + 180*x^8 +
 18*x^7 - 326*x^6 - 18*x^5 + 180*x^4 + 45*x^3 - 9*x^2 - x)) + 1/40*5^(1/4)*
sqrt(2)*sqrt(sqrt(5) + 5)*arctan(1/200*(100*x^11 + 1300*x^10 - 6700*x^9 - 4
```

```
400*x^8 + 28400*x^7 + 1400*x^6 - 28400*x^5 - 4400*x^4 + 6700*x^3 + 1300*x^2
 - 5*sqrt(x^3 - x^2 - x)*(5^(3/4)*(sqrt(5)*sqrt(2)*(x^10 - 4*x^9 - 17*x^8 +
 56*x^7 + 78*x^6 - 136*x^5 - 78*x^4 + 56*x^3 + 17*x^2 - 4*x - 1) - sqrt(2)*
(x^10 + 8*x^9 - 57*x^8 - 24*x^7 + 294*x^6 + 64*x^5 - 294*x^4 - 24*x^3 + 57*
x^2 + 8*x - 1)) + 4*5^(1/4)*(sqrt(5)*sqrt(2)*(3*x^9 + 7*x^8 + 14*x^7 - 81*x
^6 - 10*x^5 + 81*x^4 + 14*x^3 - 7*x^2 + 3*x) - 5*sqrt(2)*(x^9 + 9*x^8 - 18*
x^7 - 15*x^6 + 26*x^5 + 15*x^4 - 18*x^3 - 9*x^2 + x)))*sqrt(sqrt(5) + 5) -
sqrt(5)*(240*x^10 + 160*x^9 - 1680*x^8 - 480*x^7 + 3200*x^6 + 480*x^5 - 168
0*x^4 - 160*x^3 + 240*x^2 - sqrt(x^3 - x^2 - x)*(5^(3/4)*(sqrt(5)*sqrt(2)*(
x^10 - 6*x^9 - 25*x^8 + 120*x^7 - 58*x^6 - 196*x^5 + 58*x^4 + 120*x^3 + 25*
x^2 - 6*x - 1) - sqrt(2)*(x^10 - 2*x^9 - 17*x^8 + 216*x^7 - 306*x^6 - 396*x
^5 + 306*x^4 + 216*x^3 + 17*x^2 - 2*x - 1)) + 4*5^(1/4)*(sqrt(5)*sqrt(2)*(3
*x^9 - 3*x^8 - 56*x^7 + 69*x^6 + 90*x^5 - 69*x^4 - 56*x^3 + 3*x^2 + 3*x) -
5*sqrt(2)*(x^9 + 3*x^8 - 28*x^7 + 11*x^6 + 54*x^5 - 11*x^4 - 28*x^3 - 3*x^2
 + x)))*sqrt(sqrt(5) + 5) + 4*sqrt(5)*(5*x^11 - 25*x^10 - 105*x^9 + 440*x^8
 + 50*x^7 - 830*x^6 - 50*x^5 + 440*x^4 + 105*x^3 - 25*x^2 - sqrt(5)*(x^11 +
 3*x^10 - 37*x^9 + 96*x^8 - 6*x^7 - 166*x^6 + 6*x^5 + 96*x^4 + 37*x^3 + 3*x
^2 - x) - 5*x) - 80*sqrt(5)*(x^10 + 6*x^9 - 23*x^8 - 2*x^7 + 40*x^6 + 2*x^5
 - 23*x^4 - 6*x^3 + x^2))*sqrt((5*x^4 - 20*x^3 - 5^(1/4)*sqrt(x^3 - x^2 - x
)*(sqrt(5)*sqrt(2)*(x^2 - 6*x - 1) - 5*sqrt(2)*(x^2 - 2*x - 1))*sqrt(sqrt(5
) + 5) + 30*x^2 + 20*sqrt(5)*(x^3 - x^2 - x) + 20*x + 5)/(x^4 + 2*x^2 + 1))
 + 20*sqrt(5)*(5*x^11 - 15*x^10 - 15*x^9 + 20*x^8 - 20*x^7 + 70*x^6 + 20*x^
5 + 20*x^4 + 15*x^3 - 15*x^2 - sqrt(5)*(x^11 + 13*x^10 - 67*x^9 - 44*x^8 +
284*x^7 + 14*x^6 - 284*x^5 - 44*x^4 + 67*x^3 + 13*x^2 - x) - 5*x) - 100*sqr
t(5)*(x^11 - 3*x^10 - 3*x^9 + 4*x^8 - 4*x^7 + 14*x^6 + 4*x^5 + 4*x^4 + 3*x^
3 - 3*x^2 - x) - 100*x)/(x^11 - 9*x^10 - 45*x^9 + 180*x^8 + 18*x^7 - 326*x^
6 - 18*x^5 + 180*x^4 + 45*x^3 - 9*x^2 - x)) + 1/4*arctan(1/2*(x^2 - 2*x - 1
)/sqrt(x^3 - x^2 - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^4-1\right)\sqrt{x^3-x^2-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3-x^2-x)^(1/2)/(x^4-1),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^4 - 1)*sqrt(x^3 - x^2 - x)), x)
```

maple [C] time = 0.03, size = 898, normalized size = 6.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3-x^2-x)^(1/2)/(x^4-1),x)
```

```
[Out] 1/10*(1/2*5^(1/2)-1/2)*((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2)*(-5*(x
-1/2-1/2*5^(1/2))*5^(1/2))^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(
1/2)/(-1/2-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^
(1/2),(1/2-1/2*5^(1/2))/(-1/2-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5
^(1/2))^(1/2))-1/10*(1/2*5^(1/2)-1/2)*((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2
))^(1/2)*(-5*(x-1/2-1/2*5^(1/2))*5^(1/2))^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2
)/(x^3-x^2-x)^(1/2)/(3/2-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*
5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(3/2-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5
^(1/2)-1/2)*5^(1/2))^(1/2))-1/20*I*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/
2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/
2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(1/2-I-1/2*5^(1/2))*Ellip
ticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2-
I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))*5^(1/2)+1/20*
I*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))
```

```
^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(
x^3-x^2-x)^(1/2)/(1/2-I-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*5
^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2-I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*
5^(1/2)-1/2)*5^(1/2))^(1/2))+1/20*I*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1
/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2))^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1
/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/(x^3-x^2-x)^(1/2)/(1/2+I-1/2*5^(1/2))*Elli
pticPi(((x-1/2+1/2*5^(1/2))/(1/2*5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2
+I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2*5^(1/2)-1/2)*5^(1/2))^(1/2))*5^(1/2)-1/20
*I*(x/(1/2*5^(1/2)-1/2)-1/2/(1/2*5^(1/2)-1/2)+1/2/(1/2*5^(1/2)-1/2)*5^(1/2)
)^(1/2)*(-5*x*5^(1/2)+5/2*5^(1/2)+25/2)^(1/2)*(-x/(1/2*5^(1/2)-1/2))^(1/2)/
(x^3-x^2-x)^(1/2)/(1/2+I-1/2*5^(1/2))*EllipticPi(((x-1/2+1/2*5^(1/2))/(1/2*
5^(1/2)-1/2))^(1/2),(1/2-1/2*5^(1/2))/(1/2+I-1/2*5^(1/2)),1/5*5^(1/2)*((1/2
*5^(1/2)-1/2)*5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^4-1\right)\sqrt{x^3-x^2-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3-x^2-x)^(1/2)/(x^4-1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^4 - 1)*sqrt(x^3 - x^2 - x)), x)
```

mupad [B] time = 0.06, size = 533, normalized size = 4.01

$$-\frac{\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\Pi\left(-\frac{\sqrt{5}}{2}-\frac{1}{2};\operatorname{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)}{2\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}-\frac{\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\Pi\left(\frac{\sqrt{5}}{2}+\frac{1}{2};\operatorname{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)}{2\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}+\frac{\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\Pi\left(-\frac{\sqrt{5}\,1\mathrm{i}}{2}-\frac{1}{2}\mathrm{i};\operatorname{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)}{2\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}+\frac{\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\sqrt{\frac{x+\frac{\sqrt{5}}{2}-\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\sqrt{\frac{\frac{\sqrt{5}}{2}-x+\frac{1}{2}}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\,\Pi\left(\frac{\sqrt{5}\,1\mathrm{i}}{2}+\frac{1}{2}\mathrm{i};\operatorname{asin}\left(\sqrt{\frac{x}{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\middle|-\frac{\frac{\sqrt{5}}{2}+\frac{1}{2}}{\frac{\sqrt{5}}{2}-\frac{1}{2}}\right)}{2\sqrt{x^3-x^2-\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^4 - 1)*(x^3 - x^2 - x)^(1/2)),x)
```

```
[Out] ((5^(1/2)/2 + 1/2)*(x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/2)/2 - 1/2)/(5^(1
/2)/2 - 1/2))^(1/2)*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 + 1/2))^(1/2)*ellipti
cPi(- (5^(1/2)*1i)/2 - 1i/2, asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/2
 + 1/2)/(5^(1/2)/2 - 1/2)))/(2*(x^3 - x^2 - x*(5^(1/2)/2 - 1/2)*(5^(1/2)/2
+ 1/2))^(1/2)) - ((5^(1/2)/2 + 1/2)*(x/(5^(1/2)/2 + 1/2))^(1/2)*((x + 5^(1/
2)/2 - 1/2)/(5^(1/2)/2 - 1/2))^(1/2)*((5^(1/2)/2 - x + 1/2)/(5^(1/2)/2 + 1/
2))^(1/2)*ellipticPi(5^(1/2)/2 + 1/2, asin((x/(5^(1/2)/2 + 1/2))^(1/2)), -(
5^(1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2)))/(2*(x^3 - x^2 - x*(5^(1/2)/2 - 1/2)*(5
^(1/2)/2 + 1/2))^(1/2)) - ((5^(1/2)/2 + 1/2)*(x/(5^(1/2)/2 + 1/2))^(1/2)*((
x + 5^(1/2)/2 - 1/2)/(5^(1/2)/2 - 1/2))^(1/2)*((5^(1/2)/2 - x + 1/2)/(5^(1/
2)/2 + 1/2))^(1/2)*ellipticPi(- 5^(1/2)/2 - 1/2, asin((x/(5^(1/2)/2 + 1/2))
^(1/2)), -(5^(1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2)))/(2*(x^3 - x^2 - x*(5^(1/2)/
2 - 1/2)*(5^(1/2)/2 + 1/2))^(1/2)) + ((5^(1/2)/2 + 1/2)*(x/(5^(1/2)/2 + 1/2
))^(1/2)*((x + 5^(1/2)/2 - 1/2)/(5^(1/2)/2 - 1/2))^(1/2)*((5^(1/2)/2 - x +
1/2)/(5^(1/2)/2 + 1/2))^(1/2)*ellipticPi((5^(1/2)*1i)/2 + 1i/2, asin((x/(5^
(1/2)/2 + 1/2))^(1/2)), -(5^(1/2)/2 + 1/2)/(5^(1/2)/2 - 1/2)))/(2*(x^3 - x^
2 - x*(5^(1/2)/2 - 1/2)*(5^(1/2)/2 + 1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x\left(x^2-x-1\right)}\,(x-1)(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**3-x**2-x)**(1/2)/(x**4-1),x)
```

```
[Out] Integral(x**2/(sqrt(x*(x**2 - x - 1))*(x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.1599 $\int \frac{(-1+x^2)\sqrt{1+x^2+x^4}}{(1+x^2)(1+x+x^2+x^3+x^4)}\,dx$

Optimal. Leaf size=133

$$-2\tan^{-1}\left(\frac{\sqrt{x^4+x^2+1}}{x^2-x+1}\right)+\sqrt{\frac{1}{5}\left(2+2\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{x^4+x^2+1}}{\sqrt{2+\sqrt{5}}\,(x^2-x+1)}\right)+\sqrt{\frac{1}{5}\left(2\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{x^4}}{\sqrt{\sqrt{5}-}}\right.$$

Rubi [F] time = 1.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^2)\sqrt{1+x^2+x^4}}{(1+x^2)(1+x+x^2+x^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^2)*Sqrt[1 + x^2 + x^4])/((1 + x^2)*(1 + x + x^2 + x^3 + x^4)),
x]
```

```
[Out] (-2*x*Sqrt[1 + x^2 + x^4])/(1 + x^2) - ArcTan[x/Sqrt[1 + x^2 + x^4]] + (2*(
1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqr
t[1 + x^2 + x^4] - (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF
[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4]) + Defer[Int][Sqrt[1 + x^2 + x^4
]/(1 + x + x^2 + x^3 + x^4), x] + 2*Defer[Int][(x*Sqrt[1 + x^2 + x^4])/(1 +
 x + x^2 + x^3 + x^4), x] + 2*Defer[Int][(x^2*Sqrt[1 + x^2 + x^4])/(1 + x +
 x^2 + x^3 + x^4), x]
```

Rubi steps

$$\int \frac{(-1+x^2)\sqrt{1+x^2+x^4}}{(1+x^2)(1+x+x^2+x^3+x^4)}\,dx=\int\left(-\frac{2\sqrt{1+x^2+x^4}}{1+x^2}+\frac{(1+2x+2x^2)\sqrt{1+x^2+x^4}}{1+x+x^2+x^3+x^4}\right)dx$$

$$=-\left(2\int\frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx\right)+\int\frac{(1+2x+2x^2)\sqrt{1+x^2+x^4}}{1+x+x^2+x^3+x^4}\,dx$$

$$=-\left(2\int\frac{x^2}{\sqrt{1+x^2+x^4}}\,dx\right)-2\int\frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx+\int\left(\frac{\sqrt{}}{1+}\right.$$

$$=-\left(2\int\frac{1}{\sqrt{1+x^2+x^4}}\,dx\right)+2\int\frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx+2\int\frac{x\sqrt{1+x^2}}{1+x+x^2+}$$

$$=-\frac{2x\sqrt{1+x^2+x^4}}{1+x^2}+\frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}-\frac{3(1+x^2)}{}$$

$$=-\frac{2x\sqrt{1+x^2+x^4}}{1+x^2}-\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)+\frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E(}{\sqrt{1+x^2+x^}}$$

Mathematica [C] time = 139.37, size = 4461, normalized size = 33.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^2)*Sqrt[1 + x^2 + x^4])/((1 + x^2)*(1 + x + x^2 + x^3 +
x^4)),x]
```

```
[Out] ((1 + x^2)*Sqrt[1 + x^2 + x^4]*(1 + x + x^2 + x^3 + x^4)*((-2*Sqrt[1 + x^2
+ x^4])/(1 + x^2) + ((1 + 2*x + 2*x^2)*Sqrt[1 + x^2 + x^4])/(1 + x + x^2 +
x^3 + x^4))*(((-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*
EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/Sqrt[1 + x^2 + x^4] + ((-1)
^(1/15)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(
2/15), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/((1 - (-1)^(1/5))*(-(-1)^(1/5)
 - (-1)^(2/5))*(-(-1)^(1/5) + (-1)^(3/5))*(-(-1)^(1/5) - (-1)^(4/5))*Sqrt[1
 + x^2 + x^4]) - ((-1)^(4/15)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*
x^2]*EllipticPi[(-1)^(2/15), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/((1 - (-
1)^(1/5))*(-(-1)^(1/5) - (-1)^(2/5))*(-(-1)^(1/5) + (-1)^(3/5))*(-(-1)^(1/5
) - (-1)^(4/5))*Sqrt[1 + x^2 + x^4]) + ((-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]
*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(2/15), I*ArcSinh[(-1)^(5/6)*x],
(-1)^(2/3)])/((1 - (-1)^(1/5))*(-(-1)^(1/5) - (-1)^(2/5))*(-(-1)^(1/5) + (-
1)^(3/5))*(-(-1)^(1/5) - (-1)^(4/5))*Sqrt[1 + x^2 + x^4]) - ((-1)^(13/15)*S
qrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(2/15), I*
ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/((1 - (-1)^(1/5))*(-(-1)^(1/5) - (-1)^(
2/5))*(-(-1)^(1/5) + (-1)^(3/5))*(-(-1)^(1/5) - (-1)^(4/5))*Sqrt[1 + x^2 +
x^4]) - (2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*Ell
ipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/((-1 + (-1)^(1/5)
)*(-1 - (-1)^(2/5))*(-1 + (-1)^(3/5))*(-1 - (-1)^(4/5))*Sqrt[1 + x^2 + x^4]
) + ((-1)^(1/15)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*Elliptic
Pi[(-1)^(8/15), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/((1 + (-1)^(4/5))*((-
1)^(1/5) + (-1)^(4/5))*(-(-1)^(2/5) + (-1)^(4/5))*((-1)^(3/5) + (-1)^(4/5))
*Sqrt[1 + x^2 + x^4]) - ((-1)^(4/15)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)
^(2/3)*x^2]*EllipticPi[(-1)^(8/15), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(
(1 + (-1)^(4/5))*((-1)^(1/5) + (-1)^(4/5))*(-(-1)^(2/5) + (-1)^(4/5))*((-1)
^(3/5) + (-1)^(4/5))*Sqrt[1 + x^2 + x^4]) - ((-1)^(7/15)*Sqrt[1 + (-1)^(1/3
)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(8/15), I*ArcSinh[(-1)^(5/6
)*x], (-1)^(2/3)])/((1 + (-1)^(4/5))*((-1)^(1/5) + (-1)^(4/5))*(-(-1)^(2/5)
 + (-1)^(4/5))*((-1)^(3/5) + (-1)^(4/5))*Sqrt[1 + x^2 + x^4]) + ((-1)^(2/3)
*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(8/15),
I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/((1 + (-1)^(4/5))*((-1)^(1/5) + (-1)^
(4/5))*(-(-1)^(2/5) + (-1)^(4/5))*((-1)^(3/5) + (-1)^(4/5))*Sqrt[1 + x^2 +
x^4]) + ((-1)^(4/15)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*Elli
pticPi[-(-1)^(11/15), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/((1 - (-1)^(3/5
))*((-1)^(1/5) - (-1)^(3/5))*(-(-1)^(2/5) - (-1)^(3/5))*(-(-1)^(3/5) - (-1)
^(4/5))*Sqrt[1 + x^2 + x^4]) + ((-1)^(7/15)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1
 - (-1)^(2/3)*x^2]*EllipticPi[-(-1)^(11/15), I*ArcSinh[(-1)^(5/6)*x], (-1)^
(2/3)])/((1 - (-1)^(3/5))*((-1)^(1/5) - (-1)^(3/5))*(-(-1)^(2/5) - (-1)^(3/
5))*(-(-1)^(3/5) - (-1)^(4/5))*Sqrt[1 + x^2 + x^4]) + ((-1)^(2/3)*Sqrt[1 +
(-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[-(-1)^(11/15), I*ArcSin
h[(-1)^(5/6)*x], (-1)^(2/3)])/((1 - (-1)^(3/5))*((-1)^(1/5) - (-1)^(3/5))*(
-(-1)^(2/5) - (-1)^(3/5))*(-(-1)^(3/5) - (-1)^(4/5))*Sqrt[1 + x^2 + x^4]) +
 ((-1)^(13/15)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi
[-(-1)^(11/15), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/((1 - (-1)^(3/5))*((-
1)^(1/5) - (-1)^(3/5))*(-(-1)^(2/5) - (-1)^(3/5))*(-(-1)^(3/5) - (-1)^(4/5)
)*Sqrt[1 + x^2 + x^4]) - ((-1)^(1/15)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1
)^(2/3)*x^2]*EllipticPi[(-1)^(14/15), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])
/((1 + (-1)^(2/5))*((-1)^(1/5) + (-1)^(2/5))*((-1)^(2/5) + (-1)^(3/5))*((-1
)^(2/5) - (-1)^(4/5))*Sqrt[1 + x^2 + x^4]) + ((-1)^(7/15)*Sqrt[1 + (-1)^(1/
3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(14/15), I*ArcSinh[(-1)^(5
/6)*x], (-1)^(2/3)])/((1 + (-1)^(2/5))*((-1)^(1/5) + (-1)^(2/5))*((-1)^(2/5
) + (-1)^(3/5))*((-1)^(2/5) - (-1)^(4/5))*Sqrt[1 + x^2 + x^4]) + ((-1)^(2/3
)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(14/15)
, I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/((1 + (-1)^(2/5))*((-1)^(1/5) + (-1
)^(2/5))*((-1)^(2/5) + (-1)^(3/5))*((-1)^(2/5) - (-1)^(4/5))*Sqrt[1 + x^2 +
 x^4]) + ((-1)^(13/15)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*El
```

```
lipticPi[(-1)^(14/15), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/((1 + (-1)^(2/
5))*((-1)^(1/5) + (-1)^(2/5))*((-1)^(2/5) + (-1)^(3/5))*((-1)^(2/5) - (-1)^
(4/5))*Sqrt[1 + x^2 + x^4]) - ((-1)^(1/5)*((-3 + (-1)^(2/5) - 3*(-1)^(4/5))
*Sqrt[6 - 5*(-1)^(1/5) + 5*(-1)^(2/5) - 6*(-1)^(3/5) + 5*(-1)^(4/5)]*Log[(-
1)^(1/5) + x^2] + (4 - 5*(-1)^(1/5) + 8*(-1)^(2/5) - 8*(-1)^(3/5) + 5*(-1)^
(4/5))*Sqrt[6 - 6*(-1)^(1/5) + 5*(-1)^(2/5) - 5*(-1)^(3/5) + 5*(-1)^(4/5)]*
Log[-(-1)^(2/5) + x^2] + 2*Sqrt[6 - 5*(-1)^(1/5) + 5*(-1)^(2/5) - 5*(-1)^(3
/5) + 6*(-1)^(4/5)]*Log[(-1)^(3/5) + x^2] - 2*(-1)^(1/5)*Sqrt[6 - 5*(-1)^(1
/5) + 5*(-1)^(2/5) - 5*(-1)^(3/5) + 6*(-1)^(4/5)]*Log[(-1)^(3/5) + x^2] - (
-1)^(2/5)*Sqrt[6 - 5*(-1)^(1/5) + 5*(-1)^(2/5) - 5*(-1)^(3/5) + 6*(-1)^(4/5
)]*Log[(-1)^(3/5) + x^2] + 2*(-1)^(3/5)*Sqrt[6 - 5*(-1)^(1/5) + 5*(-1)^(2/5
) - 5*(-1)^(3/5) + 6*(-1)^(4/5)]*Log[(-1)^(3/5) + x^2] - (-1)^(4/5)*Sqrt[6
- 5*(-1)^(1/5) + 5*(-1)^(2/5) - 5*(-1)^(3/5) + 6*(-1)^(4/5)]*Log[(-1)^(3/5)
 + x^2] + 3*Sqrt[6 - 5*(-1)^(1/5) + 6*(-1)^(2/5) - 5*(-1)^(3/5) + 5*(-1)^(4
/5)]*Log[-(-1)^(4/5) + x^2] - 4*(-1)^(1/5)*Sqrt[6 - 5*(-1)^(1/5) + 6*(-1)^(
2/5) - 5*(-1)^(3/5) + 5*(-1)^(4/5)]*Log[-(-1)^(4/5) + x^2] + 3*(-1)^(2/5)*S
qrt[6 - 5*(-1)^(1/5) + 6*(-1)^(2/5) - 5*(-1)^(3/5) + 5*(-1)^(4/5)]*Log[-(-1
)^(4/5) + x^2] + 3*Sqrt[6 - 5*(-1)^(1/5) + 5*(-1)^(2/5) - 6*(-1)^(3/5) + 5*
(-1)^(4/5)]*Log[2 - (-1)^(1/5) + x^2 - 2*(-1)^(1/5)*x^2 + 2*Sqrt[1 - (-1)^(
1/5) + (-1)^(2/5)]*Sqrt[1 + x^2 + x^4]] - (-1)^(2/5)*Sqrt[6 - 5*(-1)^(1/5)
+ 5*(-1)^(2/5) - 6*(-1)^(3/5) + 5*(-1)^(4/5)]*Log[2 - (-1)^(1/5) + x^2 - 2*
(-1)^(1/5)*x^2 + 2*Sqrt[1 - (-1)^(1/5) + (-1)^(2/5)]*Sqrt[1 + x^2 + x^4]] +
 3*(-1)^(4/5)*Sqrt[6 - 5*(-1)^(1/5) + 5*(-1)^(2/5) - 6*(-1)^(3/5) + 5*(-1)^
(4/5)]*Log[2 - (-1)^(1/5) + x^2 - 2*(-1)^(1/5)*x^2 + 2*Sqrt[1 - (-1)^(1/5)
+ (-1)^(2/5)]*Sqrt[1 + x^2 + x^4]] - 2*Sqrt[6 - 5*(-1)^(1/5) + 5*(-1)^(2/5)
 - 5*(-1)^(3/5) + 6*(-1)^(4/5)]*Log[2 - (-1)^(3/5) + x^2 - 2*(-1)^(3/5)*x^2
 + 2*Sqrt[1 - (-1)^(1/5) - (-1)^(3/5)]*Sqrt[1 + x^2 + x^4]] + 2*(-1)^(1/5)*
Sqrt[6 - 5*(-1)^(1/5) + 5*(-1)^(2/5) - 5*(-1)^(3/5) + 6*(-1)^(4/5)]*Log[2 -
 (-1)^(3/5) + x^2 - 2*(-1)^(3/5)*x^2 + 2*Sqrt[1 - (-1)^(1/5) - (-1)^(3/5)]*
Sqrt[1 + x^2 + x^4]] + (-1)^(2/5)*Sqrt[6 - 5*(-1)^(1/5) + 5*(-1)^(2/5) - 5*
(-1)^(3/5) + 6*(-1)^(4/5)]*Log[2 - (-1)^(3/5) + x^2 - 2*(-1)^(3/5)*x^2 + 2*
Sqrt[1 - (-1)^(1/5) - (-1)^(3/5)]*Sqrt[1 + x^2 + x^4]] - 2*(-1)^(3/5)*Sqrt[
6 - 5*(-1)^(1/5) + 5*(-1)^(2/5) - 5*(-1)^(3/5) + 6*(-1)^(4/5)]*Log[2 - (-1)
^(3/5) + x^2 - 2*(-1)^(3/5)*x^2 + 2*Sqrt[1 - (-1)^(1/5) - (-1)^(3/5)]*Sqrt[
1 + x^2 + x^4]] + (-1)^(4/5)*Sqrt[6 - 5*(-1)^(1/5) + 5*(-1)^(2/5) - 5*(-1)^
(3/5) + 6*(-1)^(4/5)]*Log[2 - (-1)^(3/5) + x^2 - 2*(-1)^(3/5)*x^2 + 2*Sqrt[
1 - (-1)^(1/5) - (-1)^(3/5)]*Sqrt[1 + x^2 + x^4]] - 4*Sqrt[6 - 6*(-1)^(1/5)
 + 5*(-1)^(2/5) - 5*(-1)^(3/5) + 5*(-1)^(4/5)]*Log[2 + (-1)^(2/5) + x^2 + 2
*(-1)^(2/5)*x^2 + 2*Sqrt[1 + (-1)^(2/5) + (-1)^(4/5)]*Sqrt[1 + x^2 + x^4]]
+ 5*(-1)^(1/5)*Sqrt[6 - 6*(-1)^(1/5) + 5*(-1)^(2/5) - 5*(-1)^(3/5) + 5*(-1)
^(4/5)]*Log[2 + (-1)^(2/5) + x^2 + 2*(-1)^(2/5)*x^2 + 2*Sqrt[1 + (-1)^(2/5)
 + (-1)^(4/5)]*Sqrt[1 + x^2 + x^4]] - 8*(-1)^(2/5)*Sqrt[6 - 6*(-1)^(1/5) +
5*(-1)^(2/5) - 5*(-1)^(3/5) + 5*(-1)^(4/5)]*Log[2 + (-1)^(2/5) + x^2 + 2*(-
1)^(2/5)*x^2 + 2*Sqrt[1 + (-1)^(2/5) + (-1)^(4/5)]*Sqrt[1 + x^2 + x^4]] + 8
*(-1)^(3/5)*Sqrt[6 - 6*(-1)^(1/5) + 5*(-1)^(2/5) - 5*(-1)^(3/5) + 5*(-1)^(4
/5)]*Log[2 + (-1)^(2/5) + x^2 + 2*(-1)^(2/5)*x^2 + 2*Sqrt[1 + (-1)^(2/5) +
(-1)^(4/5)]*Sqrt[1 + x^2 + x^4]] - 5*(-1)^(4/5)*Sqrt[6 - 6*(-1)^(1/5) + 5*(
-1)^(2/5) - 5*(-1)^(3/5) + 5*(-1)^(4/5)]*Log[2 + (-1)^(2/5) + x^2 + 2*(-1)^
(2/5)*x^2 + 2*Sqrt[1 + (-1)^(2/5) + (-1)^(4/5)]*Sqrt[1 + x^2 + x^4]] - 3*Sq
rt[6 - 5*(-1)^(1/5) + 6*(-1)^(2/5) - 5*(-1)^(3/5) + 5*(-1)^(4/5)]*Log[2 + (
-1)^(4/5) + x^2 + 2*(-1)^(4/5)*x^2 + 2*Sqrt[1 - (-1)^(3/5) + (-1)^(4/5)]*Sq
rt[1 + x^2 + x^4]] + 4*(-1)^(1/5)*Sqrt[6 - 5*(-1)^(1/5) + 6*(-1)^(2/5) - 5*
(-1)^(3/5) + 5*(-1)^(4/5)]*Log[2 + (-1)^(4/5) + x^2 + 2*(-1)^(4/5)*x^2 + 2*
Sqrt[1 - (-1)^(3/5) + (-1)^(4/5)]*Sqrt[1 + x^2 + x^4]] - 3*(-1)^(2/5)*Sqrt[
6 - 5*(-1)^(1/5) + 6*(-1)^(2/5) - 5*(-1)^(3/5) + 5*(-1)^(4/5)]*Log[2 + (-1)
^(4/5) + x^2 + 2*(-1)^(4/5)*x^2 + 2*Sqrt[1 - (-1)^(3/5) + (-1)^(4/5)]*Sqrt[
1 + x^2 + x^4]]))/(2*(-1 + (-1)^(1/5))^2*(1 + (-1)^(1/5))^4*(1 - (-1)^(1/5)
 + (-1)^(2/5))^(5/2)*Sqrt[6 - 5*(-1)^(1/5) + 5*(-1)^(2/5) - 6*(-1)^(3/5) +
5*(-1)^(4/5)]))))/((-1 + x)*(1 + x)*(1 - x + x^2)*(1 + x + x^2))
```

IntegrateAlgebraic [A] time = 1.14, size = 133, normalized size = 1.00

$$-2\tan^{-1}\left(\frac{\sqrt{x^4+x^2+1}}{x^2-x+1}\right)+\sqrt{\frac{1}{5}\left(2+2\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\sqrt{5}-2}\sqrt{x^4+x^2+1}}{x^2-x+1}\right)+\sqrt{\frac{1}{5}\left(2\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{5}}\sqrt{x^4+x^2+1}}{x^2-x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*Sqrt[1 + x^2 + x^4])/((1 + x^2)*(1 + x + x^2
 + x^3 + x^4)),x]
```

```
[Out] -2*ArcTan[Sqrt[1 + x^2 + x^4]/(1 - x + x^2)] + Sqrt[(2 + 2*Sqrt[5])/5]*ArcT
an[(Sqrt[-2 + Sqrt[5]]*Sqrt[1 + x^2 + x^4])/(1 - x + x^2)] + Sqrt[(-2 + 2*S
qrt[5])/5]*ArcTanh[(Sqrt[2 + Sqrt[5]]*Sqrt[1 + x^2 + x^4])/(1 - x + x^2)]
```

fricas [B] time = 0.64, size = 306, normalized size = 2.30

$$\frac{1}{5}\sqrt{5}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{\sqrt{2}\left(2x^4+\sqrt{5}x^2+x^2+2\right)\sqrt{2\sqrt{5}+2}\sqrt{\sqrt{5}+1}+2\sqrt{x^4+x^2+1}\left(2x^2+\sqrt{5}x-x+2\right)\sqrt{2\sqrt{5}+2}}{8\left(x^4-x^3+x^2-x+1\right)}\right)+\frac{1}{20}\sqrt{5}\sqrt{2\sqrt{5}-2}\log\left(-\frac{2\sqrt{x^4+x^2+1}\left(2x^2-\sqrt{5}x+x+2\right)+\left(x^4+3x^2+\sqrt{5}\left(x^4+x^2+1\right)+1\right)\sqrt{2\sqrt{5}-2}}{x^4+x^3+x^2+x+1}\right)-\frac{1}{20}\sqrt{5}\sqrt{2\sqrt{5}-2}\log\left(-\frac{2\sqrt{x^4+x^2+1}\left(2x^2-\sqrt{5}x+x+2\right)-\left(x^4+3x^2+\sqrt{5}\left(x^4+x^2+1\right)+1\right)\sqrt{2\sqrt{5}-2}}{x^4+x^3+x^2+x+1}\right)-\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+x^2+1)^(1/2)/(x^2+1)/(x^4+x^3+x^2+x+1),x, algorithm=
"fricas")
```

```
[Out] 1/5*sqrt(5)*sqrt(2*sqrt(5) + 2)*arctan(1/8*(sqrt(2)*(2*x^4 + sqrt(5)*x^2 +
x^2 + 2)*sqrt(2*sqrt(5) + 2)*sqrt(sqrt(5) + 1) + 2*sqrt(x^4 + x^2 + 1)*(2*x
^2 + sqrt(5)*x - x + 2)*sqrt(2*sqrt(5) + 2))/(x^4 - x^3 + x^2 - x + 1)) + 1
/20*sqrt(5)*sqrt(2*sqrt(5) - 2)*log(-(2*sqrt(x^4 + x^2 + 1)*(2*x^2 - sqrt(5
)*x + x + 2) + (x^4 + 3*x^2 + sqrt(5)*(x^4 + x^2 + 1) + 1)*sqrt(2*sqrt(5) -
 2))/(x^4 + x^3 + x^2 + x + 1)) - 1/20*sqrt(5)*sqrt(2*sqrt(5) - 2)*log(-(2*
sqrt(x^4 + x^2 + 1)*(2*x^2 - sqrt(5)*x + x + 2) - (x^4 + 3*x^2 + sqrt(5)*(x
^4 + x^2 + 1) + 1)*sqrt(2*sqrt(5) - 2))/(x^4 + x^3 + x^2 + x + 1)) - arctan
(x/sqrt(x^4 + x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+x^2+1}\left(x^2-1\right)}{\left(x^4+x^3+x^2+x+1\right)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+x^2+1)^(1/2)/(x^2+1)/(x^4+x^3+x^2+x+1),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 - 1)/((x^4 + x^3 + x^2 + x + 1)*(x^2 + 1
)), x)
```

maple [C] time = 0.10, size = 712, normalized size = 5.35

$$\frac{8\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}\left(1+i\sqrt{3}\right)}-\frac{8\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\mathrm{EllipticE}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}\left(1+i\sqrt{3}\right)}-\frac{2\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x,-\frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}}-\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}\left(1+i\sqrt{3}\right)}+\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4+\_Z^3+\_Z^2+\_Z+1\right)}\left(-2\_\alpha^2-\_\alpha-2\right)\left(-\frac{\operatorname{arctanh}\left(\frac{\left(2\_\alpha^2+1\right)\left(6\_\alpha^3-3\_\alpha^2+11x^2-6\_\alpha+4\right)}{22\sqrt{-\_\alpha^3-\_\alpha}\sqrt{x^4+x^2+1}}\right)}{\sqrt{-\_\alpha^3-\_\alpha}}-\frac{\sqrt{2}\left(-\_\alpha^3-\_\alpha^2-\_\alpha-1\right)\sqrt{x^2+2-ix^2\sqrt{3}}\sqrt{x^2+2+ix^2\sqrt{3}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x,-\frac{i\_\alpha^3\sqrt{3}}{2}-\frac{\_\alpha^3}{2},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{i\sqrt{3}-1}\sqrt{x^4+x^2+1}}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(x^4+x^2+1)^(1/2)/(x^2+1)/(x^4+x^3+x^2+x+1),x)
```

```
[Out] 8/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2
*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticF(1/2*x*(-2+2
*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-8/(-2+2*I*3^(1/2))^(1/2)*(1+1
/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^
2+1)^(1/2)/(1+I*3^(1/2))*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I
*3^(1/2))^(1/2))-2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))
^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1
```

```
/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2
)/(-1/2+1/2*I*3^(1/2))^(1/2))+2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/
2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*Ellipti
cF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-8/(-2+2*I*3^(1/
2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(
1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2
),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-
2+2*I*3^(1/2))^(1/2)))+1/10*sum((-2*_alpha^2-_alpha-2)*(-1/(-_alpha^3-_alph
a)^(1/2)*arctanh(1/22*(2*_alpha^2+1)*(6*_alpha^3-3*_alpha^2+11*x^2-6*_alpha
+4)/(-_alpha^3-_alpha)^(1/2)/(x^4+x^2+1)^(1/2))-2^(1/2)*(-_alpha^3-_alpha^2
-_alpha-1)/(I*3^(1/2)-1)^(1/2)*(x^2+2-I*3^(1/2)*x^2)^(1/2)*(x^2+2+I*3^(1/2)
*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/2*
I*_alpha^3*3^(1/2)-1/2*_alpha^3,(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1
/2))^(1/2))),_alpha=RootOf(_Z^4+_Z^3+_Z^2+_Z+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+x^2+1}\left(x^2-1\right)}{\left(x^4+x^3+x^2+x+1\right)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^4+x^2+1)^(1/2)/(x^2+1)/(x^4+x^3+x^2+x+1),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 - 1)/((x^4 + x^3 + x^2 + x + 1)*(x^2 + 1
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2-1\right)\sqrt{x^4+x^2+1}}{\left(x^2+1\right)\left(x^4+x^3+x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*(x^2 + x^4 + 1)^(1/2))/((x^2 + 1)*(x + x^2 + x^3 + x^4 + 1))
,x)
```

```
[Out] int(((x^2 - 1)*(x^2 + x^4 + 1)^(1/2))/((x^2 + 1)*(x + x^2 + x^3 + x^4 + 1))
, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}\,(x-1)\,(x+1)}{\left(x^2+1\right)\left(x^4+x^3+x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**4+x**2+1)**(1/2)/(x**2+1)/(x**4+x**3+x**2+x+1),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x - 1)*(x + 1)/((x**2 + 1)*(x
**4 + x**3 + x**2 + x + 1)), x)
```

3.1600 $\int \frac{x^6}{(-b+ax^4)(b+ax^4)^{3/4}}\,dx$

Optimal. Leaf size=133

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2a^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}a^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2a^{7/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}a^{7/4}}$$

Rubi [A] time = 0.11, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {494, 481, 298, 203, 206}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2a^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}a^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2a^{7/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((-b + a*x^4)*(b + a*x^4)^(3/4)),x]
```

```
[Out] -1/2*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)]/a^(7/4) + ArcTan[(2^(1/4)*a^(1/4
)*x)/(b + a*x^4)^(1/4)]/(2*2^(3/4)*a^(7/4)) + ArcTanh[(a^(1/4)*x)/(b + a*x^
4)^(1/4)]/(2*a^(7/4)) - ArcTanh[(2^(1/4)*a^(1/4)*x)/(b + a*x^4)^(1/4)]/(2*2
^(3/4)*a^(7/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
```

```
tQ[-1, p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\left(-b+ax^4\right)\left(b+ax^4\right)^{3/4}}\,dx &= b\,\text{Subst}\left(\int \frac{x^6}{\left(1-ax^4\right)\left(-b+2abx^4\right)}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)\\ &= \frac{\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{a} + \frac{b\,\text{Subst}\left(\int \frac{x^2}{-b+2abx^4}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{a}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{2a^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{2a^{3/2}} - \frac{\text{Subst}\left(\int\right.}{}\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{2a^{7/4}} + \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{2\,2^{3/4}a^{7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{2a^{7/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{2\,2^{3/4}a^{7/4}}\end{aligned}$$

Mathematica [C] time = 0.06, size = 63, normalized size = 0.47

$$-\frac{x^7\left(\frac{ax^4}{b}+1\right)^{3/4} F_1\left(\frac{7}{4};\frac{3}{4},1;\frac{11}{4};-\frac{ax^4}{b},\frac{ax^4}{b}\right)}{7b\left(ax^4+b\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((-b + a*x^4)*(b + a*x^4)^(3/4)),x]
```

```
[Out] -1/7*(x^7*(1 + (a*x^4)/b)^(3/4)*AppellF1[7/4, 3/4, 1, 11/4, -((a*x^4)/b), (
a*x^4)/b])/(b*(b + a*x^4)^(3/4))
```

IntegrateAlgebraic [A] time = 0.65, size = 133, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2a^{7/4}} + \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}a^{7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2a^{7/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/((-b + a*x^4)*(b + a*x^4)^(3/4)),x]
```

```
[Out] -1/2*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)]/a^(7/4) + ArcTan[(2^(1/4)*a^(1/4
)*x)/(b + a*x^4)^(1/4)]/(2*2^(3/4)*a^(7/4)) + ArcTanh[(a^(1/4)*x)/(b + a*x^
4)^(1/4)]/(2*a^(7/4)) - ArcTanh[(2^(1/4)*a^(1/4)*x)/(b + a*x^4)^(1/4)]/(2*2
^(3/4)*a^(7/4))
```

fricas [B] time = 0.44, size = 302, normalized size = 2.27

$$\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{a^7}^{\frac{1}{4}}\arctan\left(\frac{4\left(\left(\frac{1}{8}\right)^{\frac{3}{4}}a^5\sqrt{\frac{2\sqrt{\frac{1}{2}}a^4\sqrt{\frac{1}{a^7}}x^2+\sqrt{ax^4+b}}{x^2}}\frac{1}{a^7}^{\frac{3}{4}}x-\left(\frac{1}{8}\right)^{\frac{3}{4}}\left(ax^4+b\right)^{\frac{1}{4}}a^5\frac{1}{a^7}^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{4}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{a^7}^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{8}\right)^{\frac{1}{4}}a^2\frac{1}{a^7}^{\frac{1}{4}}x+\left(ax^4+b\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{4}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{a^7}^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{8}\right)^{\frac{1}{4}}a^2\frac{1}{a^7}^{\frac{1}{4}}x-\left(ax^4+b\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{a^7}^{\frac{1}{4}}\arctan\left(\frac{a^5\frac{1}{a^7}^{\frac{3}{4}}x\sqrt{\frac{a^4\sqrt{\frac{1}{a^7}}x^2+\sqrt{ax^4+b}}{x^2}}-\left(ax^4+b\right)^{\frac{1}{4}}a^5\frac{1}{a^7}^{\frac{3}{4}}}{x}\right)+\frac{1}{4}\frac{1}{a^7}^{\frac{1}{4}}\log\left(\frac{a^2\frac{1}{a^7}^{\frac{1}{4}}x+\left(ax^4+b\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{4}\frac{1}{a^7}^{\frac{1}{4}}\log\left(-\frac{a^2\frac{1}{a^7}^{\frac{1}{4}}x-\left(ax^4+b\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a*x^4-b)/(a*x^4+b)^(3/4),x, algorithm="fricas")
```

```
[Out] (1/8)^(1/4)*(a^(-7))^(1/4)*arctan(4*((1/8)^(3/4)*a^5*sqrt((2*sqrt(1/2)*a^4*
sqrt(a^(-7))*x^2 + sqrt(a*x^4 + b))/x^2)*(a^(-7))^(3/4)*x - (1/8)^(3/4)*(a*
x^4 + b)^(1/4)*a^5*(a^(-7))^(3/4))/x) - 1/4*(1/8)^(1/4)*(a^(-7))^(1/4)*log(
```

```
(2*(1/8)^(1/4)*a^2*(a^(-7))^(1/4)*x + (a*x^4 + b)^(1/4))/x) + 1/4*(1/8)^(1/
4)*(a^(-7))^(1/4)*log(-(2*(1/8)^(1/4)*a^2*(a^(-7))^(1/4)*x - (a*x^4 + b)^(1
/4))/x) - (a^(-7))^(1/4)*arctan((a^5*(a^(-7))^(3/4)*x*sqrt((a^4*sqrt(a^(-7)
)*x^2 + sqrt(a*x^4 + b))/x^2) - (a*x^4 + b)^(1/4)*a^5*(a^(-7))^(3/4))/x) +
1/4*(a^(-7))^(1/4)*log((a^2*(a^(-7))^(1/4)*x + (a*x^4 + b)^(1/4))/x) - 1/4*
(a^(-7))^(1/4)*log(-(a^2*(a^(-7))^(1/4)*x - (a*x^4 + b)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(ax^4+b\right)^{\frac{3}{4}}\left(ax^4-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a*x^4-b)/(a*x^4+b)^(3/4),x, algorithm="giac")
[Out] integrate(x^6/((a*x^4 + b)^(3/4)*(a*x^4 - b)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a\,x^4-b\right)\left(a\,x^4+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^4-b)/(a*x^4+b)^(3/4),x)
[Out] int(x^6/(a*x^4-b)/(a*x^4+b)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(ax^4+b\right)^{\frac{3}{4}}\left(ax^4-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a*x^4-b)/(a*x^4+b)^(3/4),x, algorithm="maxima")
[Out] integrate(x^6/((a*x^4 + b)^(3/4)*(a*x^4 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^6}{\left(a\,x^4+b\right)^{3/4}\,\left(b-a\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^6/((b + a*x^4)^(3/4)*(b - a*x^4)),x)
[Out] -int(x^6/((b + a*x^4)^(3/4)*(b - a*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(ax^4-b\right)\left(ax^4+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(a*x**4-b)/(a*x**4+b)**(3/4),x)
[Out] Integral(x**6/((a*x**4 - b)*(a*x**4 + b)**(3/4)), x)
```

3.1601 $\int \frac{-b+ax^3}{x^3(b+ax^3)\sqrt[4]{-bx+ax^4}}\,dx$

Optimal. Leaf size=133

$$\frac{2\,2^{3/4}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3b}+\frac{2\,2^{3/4}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3b}-\frac{4\left(ax^4-bx\right)^{3/4}}{9bx^3}$$

Rubi [C] time = 0.41, antiderivative size = 56, normalized size of antiderivative = 0.42, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2056, 466, 465, 511, 510}

$$\frac{4\left(b-ax^3\right)\,_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{2ax^3}{b-ax^3}\right)}{9bx^2\sqrt[4]{ax^4-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^3)/(x^3*(b + a*x^3)*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (4*(b - a*x^3)*Hypergeometric2F1[-3/4, 1, 1/4, (-2*a*x^3)/(b - a*x^3)])/(9*
b*x^2*(-(b*x) + a*x^4)^(1/4))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
```

```
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{-b+ax^3}{x^3\left(b+ax^3\right)\sqrt[4]{-bx+ax^4}}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\int \frac{\left(-b+ax^3\right)^{3/4}}{x^{13/4}\left(b+ax^3\right)}\,dx}{\sqrt[4]{-bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\operatorname{Subst}\left(\int \frac{\left(-b+ax^{12}\right)^{3/4}}{x^{10}\left(b+ax^{12}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\operatorname{Subst}\left(\int \frac{\left(-b+ax^4\right)^{3/4}}{x^4\left(b+ax^4\right)}\,dx,x,x^{3/4}\right)}{3\sqrt[4]{-bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\left(-b+ax^3\right)\right)\operatorname{Subst}\left(\int \frac{\left(1-\frac{ax^4}{b}\right)^{3/4}}{x^4\left(b+ax^4\right)}\,dx,x,x^{3/4}\right)}{3\left(1-\frac{ax^3}{b}\right)^{3/4}\sqrt[4]{-bx+ax^4}}$$

$$= \frac{4\left(b-ax^3\right){}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{2ax^3}{b-ax^3}\right)}{9bx^2\sqrt[4]{-bx+ax^4}}$$

Mathematica [C] time = 0.05, size = 78, normalized size = 0.59

$$-\frac{4\left(\frac{ax^3}{b}+1\right)^{3/4}\left(ax^4-bx\right)^{3/4}{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};\frac{2ax^3}{ax^3+b}\right)}{9bx^3\left(1-\frac{ax^3}{b}\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-b + a*x^3)/(x^3*(b + a*x^3)*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(1 + (a*x^3)/b)^(3/4)*(-(b*x) + a*x^4)^(3/4)*Hypergeometric2F1[-3/4, -3
/4, 1/4, (2*a*x^3)/(b + a*x^3)])/(9*b*x^3*(1 - (a*x^3)/b)^(3/4))
```

IntegrateAlgebraic [A] time = 0.48, size = 133, normalized size = 1.00

$$\frac{2\,2^{3/4}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3b}+\frac{2\,2^{3/4}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3b}-\frac{4\left(ax^4-bx\right)^{3/4}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^3)/(x^3*(b + a*x^3)*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(-(b*x) + a*x^4)^(3/4))/(9*b*x^3) + (2*2^(3/4)*a^(3/4)*ArcTan[(2^(1/4)*
a^(1/4)*(-(b*x) + a*x^4)^(3/4))/(-b + a*x^3)])/(3*b) + (2*2^(3/4)*a^(3/4)*A
rcTanh[(2^(1/4)*a^(1/4)*(-(b*x) + a*x^4)^(3/4))/(-b + a*x^3)])/(3*b)
```

fricas [B] time = 90.95, size = 488, normalized size = 3.67

$$-\frac{12\cdot 8^{\frac{1}{4}}bx^3\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}\arctan\left(\frac{16\cdot 8^{\frac{1}{4}}\left(ax^4-bx\right)^{\frac{1}{4}}a^4bx^2\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}+4\cdot 8^{\frac{3}{4}}\left(ax^4-bx\right)^{\frac{3}{4}}a^2b^3\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}+\sqrt{2}\left(8\cdot 8^{\frac{1}{4}}\sqrt{ax^4-bx}\,a^2bx\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}+8^{\frac{3}{4}}\left(3ab^3x^3-b^4\right)\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}\right)\sqrt{\sqrt{2}a^2b^2\sqrt{\frac{a^3}{b^4}}}}{8\left(a^5x^3+a^4b\right)}\right)-3\cdot 8^{\frac{1}{4}}bx^3\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}\log\left(\frac{4\sqrt{2}\left(ax^4-bx\right)^{\frac{1}{4}}ab^2x^2\sqrt{\frac{a^3}{b^4}}+8^{\frac{3}{4}}\sqrt{ax^4-bx}\,b^3x\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}+4\left(ax^4-bx\right)^{\frac{3}{4}}a^2+8^{\frac{1}{4}}\left(3a^2bx^3-ab^2\right)\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}}{ax^3+b}\right)+3\cdot 8^{\frac{1}{4}}bx^3\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}\log\left(\frac{4\sqrt{2}\left(ax^4-bx\right)^{\frac{1}{4}}ab^2x^2\sqrt{\frac{a^3}{b^4}}-8^{\frac{3}{4}}\sqrt{ax^4-bx}\,b^3x\left(\frac{a^3}{b^4}\right)^{\frac{3}{4}}+4\left(ax^4-bx\right)^{\frac{3}{4}}a^2-8^{\frac{1}{4}}\left(3a^2bx^3-ab^2\right)\left(\frac{a^3}{b^4}\right)^{\frac{1}{4}}}{ax^3+b}\right)+8\left(ax^4-bx\right)^{\frac{3}{4}}}{18bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)/x^3/(a*x^3+b)/(a*x^4-b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/18*(12*8^(1/4)*b*x^3*(a^3/b^4)^(1/4)*arctan(1/8*(16*8^(1/4)*(a*x^4 - b*x
)^(1/4)*a^4*b*x^2*(a^3/b^4)^(1/4) + 4*8^(3/4)*(a*x^4 - b*x)^(3/4)*a^2*b^3*(
a^3/b^4)^(3/4) + sqrt(2)*(8*8^(1/4)*sqrt(a*x^4 - b*x)*a^2*b*x*(a^3/b^4)^(1/
4) + 8^(3/4)*(3*a*b^3*x^3 - b^4)*(a^3/b^4)^(3/4))*sqrt(sqrt(2)*a^2*b^2*sqrt
(a^3/b^4)))/(a^5*x^3 + a^4*b)) - 3*8^(1/4)*b*x^3*(a^3/b^4)^(1/4)*log((4*sqr
t(2)*(a*x^4 - b*x)^(1/4)*a*b^2*x^2*sqrt(a^3/b^4) + 8^(3/4)*sqrt(a*x^4 - b*x
)*b^3*x*(a^3/b^4)^(3/4) + 4*(a*x^4 - b*x)^(3/4)*a^2 + 8^(1/4)*(3*a^2*b*x^3
- a*b^2)*(a^3/b^4)^(1/4))/(a*x^3 + b)) + 3*8^(1/4)*b*x^3*(a^3/b^4)^(1/4)*lo
g((4*sqrt(2)*(a*x^4 - b*x)^(1/4)*a*b^2*x^2*sqrt(a^3/b^4) - 8^(3/4)*sqrt(a*x
^4 - b*x)*b^3*x*(a^3/b^4)^(3/4) + 4*(a*x^4 - b*x)^(3/4)*a^2 - 8^(1/4)*(3*a^
2*b*x^3 - a*b^2)*(a^3/b^4)^(1/4))/(a*x^3 + b)) + 8*(a*x^4 - b*x)^(3/4))/(b*
x^3)
```

giac [B] time = 0.23, size = 215, normalized size = 1.62

$$\frac{2 \cdot 2^{\frac{1}{4}} (-a)^{\frac{3}{4}} \arctan\left(\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{3b} + \frac{2 \cdot 2^{\frac{1}{4}} (-a)^{\frac{3}{4}} \arctan\left(-\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{3b} - \frac{2^{\frac{1}{4}} (-a)^{\frac{3}{4}} \log\left(2^{\frac{3}{4}} (-a)^{\frac{1}{4}} \left(a-\frac{b}{x^3}\right)^{\frac{1}{4}} + \sqrt{2}\sqrt{-a} + \sqrt{a-\frac{b}{x^3}}\right)}{3b} + \frac{2^{\frac{1}{4}} (-a)^{\frac{3}{4}} \log\left(-2^{\frac{3}{4}} (-a)^{\frac{1}{4}} \left(a-\frac{b}{x^3}\right)^{\frac{1}{4}} + \sqrt{2}\sqrt{-a} + \sqrt{a-\frac{b}{x^3}}\right)}{3b} - \frac{4\left(a-\frac{b}{x^3}\right)^{\frac{3}{4}}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)/x^3/(a*x^3+b)/(a*x^4-b*x)^(1/4),x, algorithm="giac")
```

```
[Out] 2/3*2^(1/4)*(-a)^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4)*(-a)^(1/4) + 2*(a - b/x^
3)^(1/4))/(-a)^(1/4))/b + 2/3*2^(1/4)*(-a)^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/
4)*(-a)^(1/4) - 2*(a - b/x^3)^(1/4))/(-a)^(1/4))/b - 1/3*2^(1/4)*(-a)^(3/4)
*log(2^(3/4)*(-a)^(1/4)*(a - b/x^3)^(1/4) + sqrt(2)*sqrt(-a) + sqrt(a - b/x
^3))/b + 1/3*2^(1/4)*(-a)^(3/4)*log(-2^(3/4)*(-a)^(1/4)*(a - b/x^3)^(1/4) +
 sqrt(2)*sqrt(-a) + sqrt(a - b/x^3))/b - 4/9*(a - b/x^3)^(3/4)/b
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{a\,x^3-b}{x^3\left(a\,x^3+b\right)\left(a\,x^4-bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b)/x^3/(a*x^3+b)/(a*x^4-b*x)^(1/4),x)
```

```
[Out] int((a*x^3-b)/x^3/(a*x^3+b)/(a*x^4-b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3-b}{\left(ax^4-bx\right)^{\frac{1}{4}}\left(ax^3+b\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)/x^3/(a*x^3+b)/(a*x^4-b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^3 - b)/((a*x^4 - b*x)^(1/4)*(a*x^3 + b)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{b-a\,x^3}{x^3\,{\left(a\,x^4-b\,x\right)}^{1/4}\,\left(a\,x^3+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^3)/(x^3*(a*x^4 - b*x)^(1/4)*(b + a*x^3)),x)
[Out] int(-(b - a*x^3)/(x^3*(a*x^4 - b*x)^(1/4)*(b + a*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3 - b}{x^3 \sqrt[4]{x\left(ax^3 - b\right)}\left(ax^3 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b)/x**3/(a*x**3+b)/(a*x**4-b*x)**(1/4),x)
[Out] Integral((a*x**3 - b)/(x**3*(x*(a*x**3 - b))**(1/4)*(a*x**3 + b)), x)
```

3.1602 $\int \frac{-b+ax^4}{\sqrt[4]{b+ax^4}\left(-b+3ax^4\right)}\,dx$

Optimal. Leaf size=133

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{6\sqrt[4]{a}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{3\sqrt{2}\,\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{6\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{3\sqrt{2}\,\sqrt[4]{a}}$$

Rubi [A] time = 0.07, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {530, 240, 212, 206, 203, 377}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{6\sqrt[4]{a}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{3\sqrt{2}\,\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{6\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{3\sqrt{2}\,\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^4)/((b + a*x^4)^(1/4)*(-b + 3*a*x^4)),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)]/(6*a^(1/4)) + ArcTan[(Sqrt[2]*a^(1/4)
*x)/(b + a*x^4)^(1/4)]/(3*Sqrt[2]*a^(1/4)) + ArcTanh[(a^(1/4)*x)/(b + a*x^4
)^(1/4)]/(6*a^(1/4)) + ArcTanh[(Sqrt[2]*a^(1/4)*x)/(b + a*x^4)^(1/4)]/(3*Sq
rt[2]*a^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-b+ax^4}{\sqrt[4]{b+ax^4}\left(-b+3ax^4\right)}\,dx &= \frac{1}{3}\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx - \frac{1}{3}(2b)\int \frac{1}{\sqrt[4]{b+ax^4}\left(-b+3ax^4\right)}\,dx \\
&= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right) - \frac{1}{3}(2b)\,\text{Subst}\left(\int \frac{1}{-b+4abx^4}\,dx, x, \frac{x}{\sqrt[4]{b+}}\right. \\
&= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right) + \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right. \\
&= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{6\sqrt[4]{a}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{3\sqrt{2}\,\sqrt[4]{a}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{6\sqrt[4]{a}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{3\sqrt{2}\,\sqrt[4]{a}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 108, normalized size = 0.81

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)+\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{6\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^4)/((b + a*x^4)^(1/4)*(-b + 3*a*x^4)),x]
```

```
[Out] (ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)] + Sqrt[2]*ArcTan[(Sqrt[2]*a^(1/4)*x)
/(b + a*x^4)^(1/4)] + ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)] + Sqrt[2]*ArcT
anh[(Sqrt[2]*a^(1/4)*x)/(b + a*x^4)^(1/4)])/(6*a^(1/4))
```

IntegrateAlgebraic [A] time = 0.45, size = 133, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{6\sqrt[4]{a}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{3\sqrt{2}\,\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{6\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{3\sqrt{2}\,\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^4)/((b + a*x^4)^(1/4)*(-b + 3*a*x^4)),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)]/(6*a^(1/4)) + ArcTan[(Sqrt[2]*a^(1/4)
*x)/(b + a*x^4)^(1/4)]/(3*Sqrt[2]*a^(1/4)) + ArcTanh[(a^(1/4)*x)/(b + a*x^4
)^(1/4)]/(6*a^(1/4)) + ArcTanh[(Sqrt[2]*a^(1/4)*x)/(b + a*x^4)^(1/4)]/(3*Sq
rt[2]*a^(1/4))
```

fricas [B] time = 0.43, size = 237, normalized size = 1.78

$$\frac{2\left(\frac{1}{4}\right)^{\frac{1}{4}}\arctan\left(\frac{\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}x\sqrt{\frac{2\sqrt{a}\,x^2+\sqrt{ax^4+b}}{x^2}}}{a^{\frac{1}{4}}}-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(ax^4+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}}{x}\right)}{3a^{\frac{1}{4}}}+\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}a^{\frac{1}{4}}x+\left(ax^4+b\right)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}}-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}a^{\frac{1}{4}}x-\left(ax^4+b\right)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}}+\frac{\arctan\left(\frac{\frac{x\sqrt{\frac{\sqrt{a}\,x^2+\sqrt{ax^4+b}}{x^2}}}{a^{\frac{1}{4}}}-\frac{\left(ax^4+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}}{x}\right)}{3a^{\frac{1}{4}}}+\frac{\log\left(\frac{a^{\frac{1}{4}}x+\left(ax^4+b\right)^{\frac{1}{4}}}{x}\right)}{12a^{\frac{1}{4}}}-\frac{\log\left(-\frac{a^{\frac{1}{4}}x-\left(ax^4+b\right)^{\frac{1}{4}}}{x}\right)}{12a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)/(a*x^4+b)^(1/4)/(3*a*x^4-b),x, algorithm="fricas")
```

```
[Out] 2/3*(1/4)^(1/4)*arctan(((1/4)^(1/4)*x*sqrt((2*sqrt(a)*x^2 + sqrt(a*x^4 + b)
)/x^2)/a^(1/4) - (1/4)^(1/4)*(a*x^4 + b)^(1/4)/a^(1/4))/x)/a^(1/4) + 1/6*(1
/4)^(1/4)*log((4*(1/4)^(3/4)*a^(1/4)*x + (a*x^4 + b)^(1/4))/x)/a^(1/4) - 1/
6*(1/4)^(1/4)*log(-(4*(1/4)^(3/4)*a^(1/4)*x - (a*x^4 + b)^(1/4))/x)/a^(1/4)
 + 1/3*arctan((x*sqrt((sqrt(a)*x^2 + sqrt(a*x^4 + b))/x^2)/a^(1/4) - (a*x^4
 + b)^(1/4)/a^(1/4))/x)/a^(1/4) + 1/12*log((a^(1/4)*x + (a*x^4 + b)^(1/4))/
x)/a^(1/4) - 1/12*log(-(a^(1/4)*x - (a*x^4 + b)^(1/4))/x)/a^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4 - b}{\left(3\,ax^4 - b\right)\left(ax^4 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)/(a*x^4+b)^(1/4)/(3*a*x^4-b),x, algorithm="giac")
[Out] integrate((a*x^4 - b)/((3*a*x^4 - b)*(a*x^4 + b)^(1/4)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{a\,x^4 - b}{\left(a\,x^4 + b\right)^{\frac{1}{4}}\left(3a\,x^4 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b)/(a*x^4+b)^(1/4)/(3*a*x^4-b),x)
[Out] int((a*x^4-b)/(a*x^4+b)^(1/4)/(3*a*x^4-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4 - b}{\left(3\,ax^4 - b\right)\left(ax^4 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)/(a*x^4+b)^(1/4)/(3*a*x^4-b),x, algorithm="maxima")
[Out] integrate((a*x^4 - b)/((3*a*x^4 - b)*(a*x^4 + b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b - a\,x^4}{\left(a\,x^4 + b\right)^{1/4}\,\left(b - 3\,a\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a*x^4)/((b + a*x^4)^(1/4)*(b - 3*a*x^4)),x)
[Out] int((b - a*x^4)/((b + a*x^4)^(1/4)*(b - 3*a*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4 - b}{\sqrt[4]{ax^4 + b}\,\left(3ax^4 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b)/(a*x**4+b)**(1/4)/(3*a*x**4-b),x)
[Out] Integral((a*x**4 - b)/((a*x**4 + b)**(1/4)*(3*a*x**4 - b)), x)
```

3.1603 $\int \frac{-2b+ax^2}{\sqrt[4]{-b+ax^2}\left(-b+ax^2+cx^4\right)}\,dx$

Optimal. Leaf size=133

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}+\sqrt{c}\,x^2}\right)}{\sqrt{2}\sqrt[4]{c}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}\sqrt[4]{c}}$$

Rubi [C] time = 15.78, antiderivative size = 2670, normalized size of antiderivative = 20.08, number of steps used = 18, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1692, 399, 490, 1217, 220, 1707}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(-2*b + a*x^2)/((-b + a*x^2)^(1/4)*(-b + a*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(Sqrt[b]*Sqrt[-a - Sqrt[a^2 + 4*b*c]]*Sqrt[(a*x^2)/b]*ArcTan[(Sqrt[a]*
Sqrt[-a - Sqrt[a^2 + 4*b*c]]*(-b + a*x^2)^(1/4))/(2^(1/4)*Sqrt[b]*c^(1/4)*(
-a^2 - 2*b*c - a*Sqrt[a^2 + 4*b*c])^(1/4)*Sqrt[(a*x^2)/b])])/(2^(1/4)*Sqrt[
a]*c^(1/4)*(-a^2 - 2*b*c - a*Sqrt[a^2 + 4*b*c])^(1/4)*x) - (Sqrt[b]*Sqrt[a
+ Sqrt[a^2 + 4*b*c]]*Sqrt[(a*x^2)/b]*ArcTan[(Sqrt[a]*Sqrt[a + Sqrt[a^2 + 4*
b*c]]*(-b + a*x^2)^(1/4))/(2^(1/4)*Sqrt[b]*c^(1/4)*(-a^2 - 2*b*c - a*Sqrt[a
^2 + 4*b*c])^(1/4)*Sqrt[(a*x^2)/b])])/(2*2^(1/4)*Sqrt[a]*c^(1/4)*(-a^2 - 2*
b*c - a*Sqrt[a^2 + 4*b*c])^(1/4)*x) - (Sqrt[b]*Sqrt[a - Sqrt[a^2 + 4*b*c]]*
Sqrt[(a*x^2)/b]*ArcTan[(Sqrt[a]*Sqrt[a - Sqrt[a^2 + 4*b*c]]*(-b + a*x^2)^(1
/4))/(2^(1/4)*Sqrt[b]*c^(1/4)*(-a^2 - 2*b*c + a*Sqrt[a^2 + 4*b*c])^(1/4)*Sq
rt[(a*x^2)/b])])/(2*2^(1/4)*Sqrt[a]*c^(1/4)*(-a^2 - 2*b*c + a*Sqrt[a^2 + 4*
b*c])^(1/4)*x) - (Sqrt[b]*Sqrt[-a + Sqrt[a^2 + 4*b*c]]*Sqrt[(a*x^2)/b]*ArcT
an[(Sqrt[a]*Sqrt[-a + Sqrt[a^2 + 4*b*c]]*(-b + a*x^2)^(1/4))/(2^(1/4)*Sqrt[
b]*c^(1/4)*(-a^2 - 2*b*c + a*Sqrt[a^2 + 4*b*c])^(1/4)*Sqrt[(a*x^2)/b])])/(2
*2^(1/4)*Sqrt[a]*c^(1/4)*(-a^2 - 2*b*c + a*Sqrt[a^2 + 4*b*c])^(1/4)*x) - ((
a + Sqrt[a^2 + 4*b*c])*(2*Sqrt[b] - (Sqrt[2]*Sqrt[-a^2 - 2*b*c - a*Sqrt[a^2
 + 4*b*c]])/Sqrt[c])*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[-b + a*x^2])^2]*(Sqrt[b]
+ Sqrt[-b + a*x^2])*EllipticF[2*ArcTan[(-b + a*x^2)^(1/4)/b^(1/4)], 1/2])/(
4*b^(1/4)*(a^2 + 4*b*c + a*Sqrt[a^2 + 4*b*c])*x) - ((a + Sqrt[a^2 + 4*b*c])
*(2*Sqrt[b] + (Sqrt[2]*Sqrt[-a^2 - 2*b*c - a*Sqrt[a^2 + 4*b*c]])/Sqrt[c])*S
qrt[(a*x^2)/(Sqrt[b] + Sqrt[-b + a*x^2])^2]*(Sqrt[b] + Sqrt[-b + a*x^2])*El
lipticF[2*ArcTan[(-b + a*x^2)^(1/4)/b^(1/4)], 1/2])/(4*b^(1/4)*(a^2 + 4*b*c
 + a*Sqrt[a^2 + 4*b*c])*x) - ((a - Sqrt[a^2 + 4*b*c])*(2*Sqrt[b]*Sqrt[c] -
Sqrt[-2*a^2 - 4*b*c + 2*a*Sqrt[a^2 + 4*b*c]])*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[
-b + a*x^2])^2]*(Sqrt[b] + Sqrt[-b + a*x^2])*EllipticF[2*ArcTan[(-b + a*x^2
)^(1/4)/b^(1/4)], 1/2])/(4*b^(1/4)*Sqrt[c]*(a^2 + 4*b*c - a*Sqrt[a^2 + 4*b*
c])*x) - ((a - Sqrt[a^2 + 4*b*c])*(2*Sqrt[b]*Sqrt[c] + Sqrt[-2*a^2 - 4*b*c
+ 2*a*Sqrt[a^2 + 4*b*c]])*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[-b + a*x^2])^2]*(Sqr
t[b] + Sqrt[-b + a*x^2])*EllipticF[2*ArcTan[(-b + a*x^2)^(1/4)/b^(1/4)], 1/
2])/(4*b^(1/4)*Sqrt[c]*(a^2 + 4*b*c - a*Sqrt[a^2 + 4*b*c])*x) + ((a + Sqrt[
a^2 + 4*b*c])*(Sqrt[2]*Sqrt[b]*Sqrt[c] + Sqrt[-a^2 - 2*b*c - a*Sqrt[a^2 + 4
*b*c]])^2*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[-b + a*x^2])^2]*(Sqrt[b] + Sqrt[-b +
 a*x^2])*EllipticPi[-1/4*(Sqrt[2]*Sqrt[b]*Sqrt[c] - Sqrt[-a^2 - 2*b*c - a*S
qrt[a^2 + 4*b*c]])^2/(Sqrt[2]*Sqrt[b]*Sqrt[c]*Sqrt[-a^2 - 2*b*c - a*Sqrt[a^
2 + 4*b*c]]), 2*ArcTan[(-b + a*x^2)^(1/4)/b^(1/4)], 1/2])/(4*Sqrt[2]*b^(1/4
)*Sqrt[c]*Sqrt[-a^2 - 2*b*c - a*Sqrt[a^2 + 4*b*c]]*(a^2 + 4*b*c + a*Sqrt[a^
2 + 4*b*c])*x) - ((a + Sqrt[a^2 + 4*b*c])*(Sqrt[2]*Sqrt[b]*Sqrt[c] - Sqrt[-
a^2 - 2*b*c - a*Sqrt[a^2 + 4*b*c]])^2*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[-b + a*x
^2])^2]*(Sqrt[b] + Sqrt[-b + a*x^2])*EllipticPi[(Sqrt[2]*Sqrt[b]*Sqrt[c] +
Sqrt[-a^2 - 2*b*c - a*Sqrt[a^2 + 4*b*c]])^2/(4*Sqrt[2]*Sqrt[b]*Sqrt[c]*Sqrt
```

```
[-a^2 - 2*b*c - a*Sqrt[a^2 + 4*b*c]]), 2*ArcTan[(-b + a*x^2)^(1/4)/b^(1/4)]
, 1/2])/(4*Sqrt[2]*b^(1/4)*Sqrt[c]*Sqrt[-a^2 - 2*b*c - a*Sqrt[a^2 + 4*b*c]]
*(a^2 + 4*b*c + a*Sqrt[a^2 + 4*b*c])*x) + ((a - Sqrt[a^2 + 4*b*c])*(Sqrt[2]
*Sqrt[b]*Sqrt[c] + Sqrt[-a^2 - 2*b*c + a*Sqrt[a^2 + 4*b*c]])^2*Sqrt[(a*x^2)
/(Sqrt[b] + Sqrt[-b + a*x^2])^2]*(Sqrt[b] + Sqrt[-b + a*x^2])*EllipticPi[-1
/4*(Sqrt[2]*Sqrt[b]*Sqrt[c] - Sqrt[-a^2 - 2*b*c + a*Sqrt[a^2 + 4*b*c]])^2/(
Sqrt[2]*Sqrt[b]*Sqrt[c]*Sqrt[-a^2 - 2*b*c + a*Sqrt[a^2 + 4*b*c]]), 2*ArcTan
[(-b + a*x^2)^(1/4)/b^(1/4)], 1/2])/(4*Sqrt[2]*b^(1/4)*Sqrt[c]*(a^2 + 4*b*c
 - a*Sqrt[a^2 + 4*b*c])*Sqrt[-a^2 - 2*b*c + a*Sqrt[a^2 + 4*b*c]]*x) - ((a -
 Sqrt[a^2 + 4*b*c])*(Sqrt[2]*Sqrt[b]*Sqrt[c] - Sqrt[-a^2 - 2*b*c + a*Sqrt[a
^2 + 4*b*c]])^2*Sqrt[(a*x^2)/(Sqrt[b] + Sqrt[-b + a*x^2])^2]*(Sqrt[b] + Sqr
t[-b + a*x^2])*EllipticPi[(Sqrt[2]*Sqrt[b]*Sqrt[c] + Sqrt[-a^2 - 2*b*c + a*
Sqrt[a^2 + 4*b*c]])^2/(4*Sqrt[2]*Sqrt[b]*Sqrt[c]*Sqrt[-a^2 - 2*b*c + a*Sqrt
[a^2 + 4*b*c]]), 2*ArcTan[(-b + a*x^2)^(1/4)/b^(1/4)], 1/2])/(4*Sqrt[2]*b^(
1/4)*Sqrt[c]*(a^2 + 4*b*c - a*Sqrt[a^2 + 4*b*c])*Sqrt[-a^2 - 2*b*c + a*Sqrt
[a^2 + 4*b*c]]*x)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
```

```
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{-2b+ax^2}{\sqrt[4]{-b+ax^2}\left(-b+ax^2+cx^4\right)}\,dx = \int \left(\frac{a-\sqrt{a^2+4bc}}{\sqrt[4]{-b+ax^2}\left(a-\sqrt{a^2+4bc}+2cx^2\right)} + \frac{a+\sqrt{a^2+4bc}}{\sqrt[4]{-b+ax^2}\left(a+\sqrt{a^2+4bc}+2\ldots\right.}\right.\ldots$$

$$= \left(a-\sqrt{a^2+4bc}\right)\int \frac{1}{\sqrt[4]{-b+ax^2}\left(a-\sqrt{a^2+4bc}+2cx^2\right)}\,dx + \left(a+\sqrt{a^2+4\ldots\right.$$

$$= \frac{\left(2\left(a-\sqrt{a^2+4bc}\right)\sqrt{\frac{ax^2}{b}}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{b}}\left(2bc+a\left(a-\sqrt{a^2+4bc}\right)+2cx^4\right)}\,dx, x, \sqrt[4]{-\ldots}\right.}{x}\ldots$$

$$= -\frac{\left(\left(a-\sqrt{a^2+4bc}\right)\sqrt{\frac{ax^2}{b}}\right)\operatorname{Subst}\left(\int \frac{1}{\left(\sqrt{-a^2-2bc+a\sqrt{a^2+4bc}}-\sqrt{2}\sqrt{c}x^2\right)\sqrt{1+\frac{x^4}{b}}}\,dx,\ldots\right.}{\sqrt{2}\sqrt{c}\,x}\ldots$$

$$= -\frac{\left(\left(a+\sqrt{a^2+4bc}\right)\left(\sqrt{2}\sqrt{b}\sqrt{c}-\sqrt{-a^2-2bc-a\sqrt{a^2+4bc}}\right)\sqrt{\frac{ax^2}{b}}\right)\operatorname{Subs}\ldots}{\sqrt{2}\sqrt{c}\left(a^2+4bc+a\sqrt{a^2+4bc}\right)x}\ldots$$

$$= \frac{\sqrt{b}\left(a^3+4abc+\left(a^2+2bc\right)\sqrt{a^2+4bc}\right)\sqrt{\frac{ax^2}{b}}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{-a-\sqrt{a^2+4bc}}\,\sqrt[4]{\ldots}}{\sqrt[4]{2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{-a^2-2bc-a\sqrt{a\ldots}}}\right.\ldots}{\sqrt[4]{2}\sqrt{a}\sqrt[4]{c}\sqrt{-a-\sqrt{a^2+4bc}}\sqrt[4]{-a^2-2bc-a\sqrt{a^2+4bc}}\left(a^2+4bc+a\sqrt{a^2\ldots}\right.}\ldots$$

Mathematica [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{-2b+ax^2}{\sqrt[4]{-b+ax^2}\left(-b+ax^2+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*b + a*x^2)/((-b + a*x^2)^(1/4)*(-b + a*x^2 + c*x^4)),x]
[Out] Integrate[(-2*b + a*x^2)/((-b + a*x^2)^(1/4)*(-b + a*x^2 + c*x^4)), x]
```

IntegrateAlgebraic [A] time = 0.38, size = 133, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}+\sqrt{c}\,x^2}\right)}{\sqrt{2}\sqrt[4]{c}} - \frac{\tan^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*b + a*x^2)/((-b + a*x^2)^(1/4)*(-b + a*x^2 + c*x^4))
,x]
```

```
[Out] -(ArcTan[(-((c^(1/4)*x^2)/Sqrt[2]) + Sqrt[-b + a*x^2]/(Sqrt[2]*c^(1/4)))/(x
*(-b + a*x^2)^(1/4))]/(Sqrt[2]*c^(1/4))) + ArcTanh[(Sqrt[2]*c^(1/4)*x*(-b +
 a*x^2)^(1/4))/(Sqrt[c]*x^2 + Sqrt[-b + a*x^2])]/(Sqrt[2]*c^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-2*b)/(a*x^2-b)^(1/4)/(c*x^4+a*x^2-b),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - 2b}{\left(cx^4 + ax^2 - b\right)\left(ax^2 - b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-2*b)/(a*x^2-b)^(1/4)/(c*x^4+a*x^2-b),x, algorithm="giac")
```

```
[Out] integrate((a*x^2 - 2*b)/((c*x^4 + a*x^2 - b)*(a*x^2 - b)^(1/4)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2 - 2b}{\left(a\,x^2 - b\right)^{\frac{1}{4}} \left(c\,x^4 + a\,x^2 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-2*b)/(a*x^2-b)^(1/4)/(c*x^4+a*x^2-b),x)
```

```
[Out] int((a*x^2-2*b)/(a*x^2-b)^(1/4)/(c*x^4+a*x^2-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - 2b}{\left(cx^4 + ax^2 - b\right)\left(ax^2 - b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-2*b)/(a*x^2-b)^(1/4)/(c*x^4+a*x^2-b),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^2 - 2*b)/((c*x^4 + a*x^2 - b)*(a*x^2 - b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{2\,b - a\,x^2}{\left(a\,x^2 - b\right)^{1/4}\,\left(c\,x^4 + a\,x^2 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*b - a*x^2)/((a*x^2 - b)^(1/4)*(a*x^2 - b + c*x^4)),x)
```

```
[Out] int(-(2*b - a*x^2)/((a*x^2 - b)^(1/4)*(a*x^2 - b + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - 2b}{\sqrt[4]{ax^2 - b}\left(ax^2 - b + cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-2*b)/(a*x**2-b)**(1/4)/(c*x**4+a*x**2-b),x)
[Out] Integral((a*x**2 - 2*b)/((a*x**2 - b)**(1/4)*(a*x**2 - b + c*x**4)), x)
```

3.1604 $\displaystyle\int \frac{(-q+px^2)\sqrt{q^2+p^2x^4}}{x^2(aq+bx+apx^2)}\,dx$

Optimal. Leaf size=133

$$\frac{2\sqrt{2a^2pq-b^2}\tan^{-1}\left(\frac{x\sqrt{2a^2pq-b^2}}{a\sqrt{p^2x^4+q^2}+apx^2+aq+bx}\right)}{a^2}-\frac{b\log\left(\sqrt{p^2x^4+q^2}+px^2+q\right)}{a^2}+\frac{b\log(x)}{a^2}+\frac{\sqrt{p^2x^4+q^2}}{ax}$$

Rubi [C] time = 7.11, antiderivative size = 1209, normalized size of antiderivative = 9.09, number of steps used = 42, number of rules used = 20, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.465, Rules used = {6728, 277, 305, 220, 1196, 266, 50, 63, 208, 1729, 1209, 1198, 1217, 1707, 1248, 735, 844, 217, 206, 725}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[((-q + p*x^2)*Sqrt[q^2 + p^2*x^4])/(x^2*(a*q + b*x + a*p*x^2)),x]
```

```
[Out] (b*Sqrt[q^2 + p^2*x^4])/(2*a^2*q) - ((b - Sqrt[b^2 - 4*a^2*p*q])*Sqrt[q^2 +
 p^2*x^4])/(4*a^2*q) - ((b + Sqrt[b^2 - 4*a^2*p*q])*Sqrt[q^2 + p^2*x^4])/(4
*a^2*q) + Sqrt[q^2 + p^2*x^4]/(a*x) + (Sqrt[-b^2 + 2*a^2*p*q]*ArcTan[(Sqrt[
-b^2 + 2*a^2*p*q]*x)/(a*Sqrt[q^2 + p^2*x^4])])/a^2 - ((b - Sqrt[b^2 - 4*a^2
*p*q])*ArcTanh[(p*x^2)/Sqrt[q^2 + p^2*x^4]])/(4*a^2) - ((b + Sqrt[b^2 - 4*a
^2*p*q])*ArcTanh[(p*x^2)/Sqrt[q^2 + p^2*x^4]])/(4*a^2) + (Sqrt[b^2 - 2*a^2*
p*q]*(b + Sqrt[b^2 - 4*a^2*p*q])*Sqrt[b^2 - 2*a^2*p*q - b*Sqrt[b^2 - 4*a^2*
p*q]]*ArcTanh[(p*(4*a^2*q^2 + (b - Sqrt[b^2 - 4*a^2*p*q])^2*x^2))/(2*Sqrt[2
]*Sqrt[b^2 - 2*a^2*p*q]*Sqrt[b^2 - 2*a^2*p*q - b*Sqrt[b^2 - 4*a^2*p*q]]*Sqr
t[q^2 + p^2*x^4])])/(4*Sqrt[2]*a^4*p*q) + (Sqrt[b^2 - 2*a^2*p*q]*(b - Sqrt[
b^2 - 4*a^2*p*q])*Sqrt[b^2 - 2*a^2*p*q + b*Sqrt[b^2 - 4*a^2*p*q]]*ArcTanh[(
p*(4*a^2*q^2 + (b + Sqrt[b^2 - 4*a^2*p*q])^2*x^2))/(2*Sqrt[2]*Sqrt[b^2 - 2*
a^2*p*q]*Sqrt[b^2 - 2*a^2*p*q + b*Sqrt[b^2 - 4*a^2*p*q]]*Sqrt[q^2 + p^2*x^4
])])/(4*Sqrt[2]*a^4*p*q) - (b*ArcTanh[Sqrt[q^2 + p^2*x^4]/q])/(2*a^2) - (Sq
rt[p]*Sqrt[q]*(q + p*x^2)*Sqrt[(q^2 + p^2*x^4)/(q + p*x^2)^2]*EllipticF[2*A
rcTan[(Sqrt[p]*x)/Sqrt[q]], 1/2])/(a*Sqrt[q^2 + p^2*x^4]) + (b*(b - Sqrt[b^
2 - 4*a^2*p*q])*(q + p*x^2)*Sqrt[(q^2 + p^2*x^4)/(q + p*x^2)^2]*EllipticF[2
*ArcTan[(Sqrt[p]*x)/Sqrt[q]], 1/2])/(4*a^3*Sqrt[p]*Sqrt[q]*Sqrt[q^2 + p^2*x
^4]) - ((b^2 - 2*a^2*p*q)*(b - Sqrt[b^2 - 4*a^2*p*q])*(q + p*x^2)*Sqrt[(q^2
 + p^2*x^4)/(q + p*x^2)^2]*EllipticF[2*ArcTan[(Sqrt[p]*x)/Sqrt[q]], 1/2])/(
4*a^3*b*Sqrt[p]*Sqrt[q]*Sqrt[q^2 + p^2*x^4]) + (b*(b + Sqrt[b^2 - 4*a^2*p*q
])*(q + p*x^2)*Sqrt[(q^2 + p^2*x^4)/(q + p*x^2)^2]*EllipticF[2*ArcTan[(Sqrt
[p]*x)/Sqrt[q]], 1/2])/(4*a^3*Sqrt[p]*Sqrt[q]*Sqrt[q^2 + p^2*x^4]) - ((b^2
- 2*a^2*p*q)*(b + Sqrt[b^2 - 4*a^2*p*q])*(q + p*x^2)*Sqrt[(q^2 + p^2*x^4)/(
q + p*x^2)^2]*EllipticF[2*ArcTan[(Sqrt[p]*x)/Sqrt[q]], 1/2])/(4*a^3*b*Sqrt[
p]*Sqrt[q]*Sqrt[q^2 + p^2*x^4])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
```

```
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Dist[d, I
nt[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, Int[(x*(a + c*x^4)^p)/(d
```

```
^2 - e^2*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(-q+px^2\right)\sqrt{q^2+p^2x^4}}{x^2\left(aq+bx+apx^2\right)}\,dx = \int\left(-\frac{\sqrt{q^2+p^2x^4}}{ax^2}+\frac{b\sqrt{q^2+p^2x^4}}{a^2qx}+\frac{\left(-b^2+2a^2pq-abpx\right)\sqrt{q^2+p^2x^4}}{a^2q\left(aq+bx+apx^2\right)}\right)dx$$

$$= -\frac{\int\frac{\sqrt{q^2+p^2x^4}}{x^2}\,dx}{a}+\frac{\int\frac{\left(-b^2+2a^2pq-abpx\right)\sqrt{q^2+p^2x^4}}{aq+bx+apx^2}\,dx}{a^2q}+\frac{b\int\frac{\sqrt{q^2+p^2x^4}}{x}\,dx}{a^2q}$$

$$= \frac{\sqrt{q^2+p^2x^4}}{ax}-\frac{\left(2p^2\right)\int\frac{x^2}{\sqrt{q^2+p^2x^4}}\,dx}{a}+\frac{\int\left(\frac{\left(-abp-ap\sqrt{b^2-4a^2pq}\right)\sqrt{q^2+p^2x^4}}{b-\sqrt{b^2-4a^2pq}+2apx}+\frac{\left(-abp+ap\sqrt{b^2\cdots}\right.}{b+\sqrt{b^2\cdots}}\right.}{a^2q}$$

$$= \frac{b\sqrt{q^2+p^2x^4}}{2a^2q}+\frac{\sqrt{q^2+p^2x^4}}{ax}+\frac{(bq)\,\text{Subst}\left(\int\frac{1}{x\sqrt{q^2+p^2x}}\,dx,x,x^4\right)}{4a^2}-\frac{(2pq)\int\frac{1}{\sqrt{q^2+p\cdots}}}{a}$$

$$= \frac{b\sqrt{q^2+p^2x^4}}{2a^2q}+\frac{\sqrt{q^2+p^2x^4}}{ax}-\frac{2px\sqrt{q^2+p^2x^4}}{a\left(q+px^2\right)}+\frac{2\sqrt{p}\sqrt{q}\left(q+px^2\right)\sqrt{\frac{q^2+p^2x^4}{\left(q+px^2\right)^2}}\,E\cdots}{a\sqrt{q^2+p^2x^4}}$$

$$= \frac{b\sqrt{q^2+p^2x^4}}{2a^2q}+\frac{\sqrt{q^2+p^2x^4}}{ax}-\frac{2px\sqrt{q^2+p^2x^4}}{a\left(q+px^2\right)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{q^2+p^2x^4}}{q}\right)}{2a^2}+\frac{2\sqrt{p}\sqrt{\cdots}}{\cdots}$$

$$= \frac{b\sqrt{q^2+p^2x^4}}{2a^2q}-\frac{\left(b-\sqrt{b^2-4a^2pq}\right)\sqrt{q^2+p^2x^4}}{4a^2q}-\frac{\left(b+\sqrt{b^2-4a^2pq}\right)\sqrt{q^2+p^2x^{\cdots}}}{4a^2q}$$

$$= \frac{b\sqrt{q^2+p^2x^4}}{2a^2q}-\frac{\left(b-\sqrt{b^2-4a^2pq}\right)\sqrt{q^2+p^2x^4}}{4a^2q}-\frac{\left(b+\sqrt{b^2-4a^2pq}\right)\sqrt{q^2+p^2x^{\cdots}}}{4a^2q}$$

$$= \frac{b\sqrt{q^2+p^2x^4}}{2a^2q}-\frac{\left(b-\sqrt{b^2-4a^2pq}\right)\sqrt{q^2+p^2x^4}}{4a^2q}-\frac{\left(b+\sqrt{b^2-4a^2pq}\right)\sqrt{q^2+p^2x^{\cdots}}}{4a^2q}$$

$$= \frac{b\sqrt{q^2+p^2x^4}}{2a^2q}-\frac{\left(b-\sqrt{b^2-4a^2pq}\right)\sqrt{q^2+p^2x^4}}{4a^2q}-\frac{\left(b+\sqrt{b^2-4a^2pq}\right)\sqrt{q^2+p^2x^{\cdots}}}{4a^2q}$$

Mathematica [C] time = 8.74, size = 3835, normalized size = 28.83

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((-q + p*x^2)*Sqrt[q^2 + p^2*x^4])/(x^2*(a*q + b*x + a*p*x^2)),x]
```

```
[Out] Sqrt[q^2 + p^2*x^4]/(a*x) - ((b*(2*ArcTanh[(p*x^2)/Sqrt[q^2 + p^2*x^4]] + 2
*ArcTanh[Sqrt[q^2 + p^2*x^4]/q] + (Sqrt[2]*Sqrt[b^2 - 2*a^2*p*q]*(b^2 - 4*a
^2*p*q - b*Sqrt[b^2 - 4*a^2*p*q])*ArcTanh[(p*(b*(b - Sqrt[b^2 - 4*a^2*p*q])
*x^2 + 2*a^2*q*(q - p*x^2)))/(Sqrt[2]*Sqrt[b^2 - 2*a^2*p*q]*Sqrt[b^2 - 2*a^
2*p*q - b*Sqrt[b^2 - 4*a^2*p*q]]*Sqrt[q^2 + p^2*x^4])])/(b*Sqrt[b^2 - 4*a^2
*p*q]*Sqrt[b^2 - 2*a^2*p*q - b*Sqrt[b^2 - 4*a^2*p*q]]) - (Sqrt[2]*Sqrt[b^2
- 2*a^2*p*q]*(b^2 - 4*a^2*p*q + b*Sqrt[b^2 - 4*a^2*p*q])*ArcTanh[(p*(b*(b +
 Sqrt[b^2 - 4*a^2*p*q])*x^2 + 2*a^2*q*(q - p*x^2)))/(Sqrt[2]*Sqrt[b^2 - 2*a
^2*p*q]*Sqrt[b^2 - 2*a^2*p*q + b*Sqrt[b^2 - 4*a^2*p*q]]*Sqrt[q^2 + p^2*x^4]
)])/(b*Sqrt[b^2 - 4*a^2*p*q]*Sqrt[b^2 - 2*a^2*p*q + b*Sqrt[b^2 - 4*a^2*p*q]
])))/(4*a) + (I*b^2*Sqrt[1 - (I*p*x^2)/q]*Sqrt[1 + (I*p*x^2)/q]*EllipticF[I
*ArcSinh[Sqrt[(I*p)/q]*x], -1])/(a^2*Sqrt[(I*p)/q]*Sqrt[q^2 + p^2*x^4]) - (
(2*I)*p*q*Sqrt[1 - (I*p*x^2)/q]*Sqrt[1 + (I*p*x^2)/q]*EllipticF[I*ArcSinh[S
qrt[(I*p)/q]*x], -1])/(Sqrt[(I*p)/q]*Sqrt[q^2 + p^2*x^4]) + (I*b^6*Sqrt[1 -
 (I*p*x^2)/q]*Sqrt[1 + (I*p*x^2)/q]*EllipticPi[((-2*I)*a^2*p*q)/(-b^2 + 2*a
^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q]), I*ArcSinh[Sqrt[(I*p)/q]*x], -1])/(a^4*
p^2*Sqrt[(I*p)/q]*(-b^2 + 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q])*(-1/2*(b^2
 - 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q])/(a^2*p^2) + (b^2 - 2*a^2*p*q + Sq
rt[b^4 - 4*a^2*b^2*p*q])/(2*a^2*p^2))*Sqrt[q^2 + p^2*x^4]) - ((6*I)*b^4*q*S
qrt[1 - (I*p*x^2)/q]*Sqrt[1 + (I*p*x^2)/q]*EllipticPi[((-2*I)*a^2*p*q)/(-b^
2 + 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q]), I*ArcSinh[Sqrt[(I*p)/q]*x], -1]
)/(a^2*p*Sqrt[(I*p)/q]*(-b^2 + 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q])*(-1/2
*(b^2 - 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q])/(a^2*p^2) + (b^2 - 2*a^2*p*q
 + Sqrt[b^4 - 4*a^2*b^2*p*q])/(2*a^2*p^2))*Sqrt[q^2 + p^2*x^4]) + ((8*I)*b^
2*q^2*Sqrt[1 - (I*p*x^2)/q]*Sqrt[1 + (I*p*x^2)/q]*EllipticPi[((-2*I)*a^2*p*
q)/(-b^2 + 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q]), I*ArcSinh[Sqrt[(I*p)/q]*
x], -1])/(Sqrt[(I*p)/q]*(-b^2 + 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q])*(-1/
2*(b^2 - 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q])/(a^2*p^2) + (b^2 - 2*a^2*p*
q + Sqrt[b^4 - 4*a^2*b^2*p*q])/(2*a^2*p^2))*Sqrt[q^2 + p^2*x^4]) + (I*b^4*S
qrt[b^4 - 4*a^2*b^2*p*q]*Sqrt[1 - (I*p*x^2)/q]*Sqrt[1 + (I*p*x^2)/q]*Ellipt
icPi[((-2*I)*a^2*p*q)/(-b^2 + 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q]), I*Arc
Sinh[Sqrt[(I*p)/q]*x], -1])/(a^4*p^2*Sqrt[(I*p)/q]*(-b^2 + 2*a^2*p*q - Sqrt
[b^4 - 4*a^2*b^2*p*q])*(-1/2*(b^2 - 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q])/
(a^2*p^2) + (b^2 - 2*a^2*p*q + Sqrt[b^4 - 4*a^2*b^2*p*q])/(2*a^2*p^2))*Sqrt
[q^2 + p^2*x^4]) - ((4*I)*b^2*q*Sqrt[b^4 - 4*a^2*b^2*p*q]*Sqrt[1 - (I*p*x^2
)/q]*Sqrt[1 + (I*p*x^2)/q]*EllipticPi[((-2*I)*a^2*p*q)/(-b^2 + 2*a^2*p*q -
Sqrt[b^4 - 4*a^2*b^2*p*q]), I*ArcSinh[Sqrt[(I*p)/q]*x], -1])/(a^2*p*Sqrt[(I
*p)/q]*(-b^2 + 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q])*(-1/2*(b^2 - 2*a^2*p*
q - Sqrt[b^4 - 4*a^2*b^2*p*q])/(a^2*p^2) + (b^2 - 2*a^2*p*q + Sqrt[b^4 - 4*
a^2*b^2*p*q])/(2*a^2*p^2))*Sqrt[q^2 + p^2*x^4]) + ((4*I)*q^2*Sqrt[b^4 - 4*a
^2*b^2*p*q]*Sqrt[1 - (I*p*x^2)/q]*Sqrt[1 + (I*p*x^2)/q]*EllipticPi[((-2*I)*
a^2*p*q)/(-b^2 + 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q]), I*ArcSinh[Sqrt[(I*
p)/q]*x], -1])/(Sqrt[(I*p)/q]*(-b^2 + 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q]
)*(-1/2*(b^2 - 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q])/(a^2*p^2) + (b^2 - 2*
a^2*p*q + Sqrt[b^4 - 4*a^2*b^2*p*q])/(2*a^2*p^2))*Sqrt[q^2 + p^2*x^4]) + (I
*b^6*Sqrt[1 - (I*p*x^2)/q]*Sqrt[1 + (I*p*x^2)/q]*EllipticPi[((-2*I)*a^2*p*q
)/(-b^2 + 2*a^2*p*q + Sqrt[b^4 - 4*a^2*b^2*p*q]), I*ArcSinh[Sqrt[(I*p)/q]*x
], -1])/(a^4*p^2*Sqrt[(I*p)/q]*(-b^2 + 2*a^2*p*q + Sqrt[b^4 - 4*a^2*b^2*p*q
])*((b^2 - 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q])/(2*a^2*p^2) - (b^2 - 2*a^
2*p*q + Sqrt[b^4 - 4*a^2*b^2*p*q])/(2*a^2*p^2))*Sqrt[q^2 + p^2*x^4]) - ((6*
I)*b^4*q*Sqrt[1 - (I*p*x^2)/q]*Sqrt[1 + (I*p*x^2)/q]*EllipticPi[((-2*I)*a^2
*p*q)/(-b^2 + 2*a^2*p*q + Sqrt[b^4 - 4*a^2*b^2*p*q]), I*ArcSinh[Sqrt[(I*p)/
```

```
q]*x], -1])/(a^2*p*Sqrt[(I*p)/q]*(-b^2 + 2*a^2*p*q + Sqrt[b^4 - 4*a^2*b^2*p
*q])*((b^2 - 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q])/(2*a^2*p^2) - (b^2 - 2*
a^2*p*q + Sqrt[b^4 - 4*a^2*b^2*p*q])/(2*a^2*p^2))*Sqrt[q^2 + p^2*x^4]) + ((
8*I)*b^2*q^2*Sqrt[1 - (I*p*x^2)/q]*Sqrt[1 + (I*p*x^2)/q]*EllipticPi[((-2*I)
*a^2*p*q)/(-b^2 + 2*a^2*p*q + Sqrt[b^4 - 4*a^2*b^2*p*q]), I*ArcSinh[Sqrt[(I
*p)/q]*x], -1])/(Sqrt[(I*p)/q]*(-b^2 + 2*a^2*p*q + Sqrt[b^4 - 4*a^2*b^2*p*q
])*((b^2 - 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q])/(2*a^2*p^2) - (b^2 - 2*a^
2*p*q + Sqrt[b^4 - 4*a^2*b^2*p*q])/(2*a^2*p^2))*Sqrt[q^2 + p^2*x^4]) - (I*b
^4*Sqrt[b^4 - 4*a^2*b^2*p*q]*Sqrt[1 - (I*p*x^2)/q]*Sqrt[1 + (I*p*x^2)/q]*El
lipticPi[((-2*I)*a^2*p*q)/(-b^2 + 2*a^2*p*q + Sqrt[b^4 - 4*a^2*b^2*p*q]), I
*ArcSinh[Sqrt[(I*p)/q]*x], -1])/(a^4*p^2*Sqrt[(I*p)/q]*(-b^2 + 2*a^2*p*q +
Sqrt[b^4 - 4*a^2*b^2*p*q])*((b^2 - 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q])/(
2*a^2*p^2) - (b^2 - 2*a^2*p*q + Sqrt[b^4 - 4*a^2*b^2*p*q])/(2*a^2*p^2))*Sqr
t[q^2 + p^2*x^4]) + ((4*I)*b^2*q*Sqrt[b^4 - 4*a^2*b^2*p*q]*Sqrt[1 - (I*p*x^
2)/q]*Sqrt[1 + (I*p*x^2)/q]*EllipticPi[((-2*I)*a^2*p*q)/(-b^2 + 2*a^2*p*q +
 Sqrt[b^4 - 4*a^2*b^2*p*q]), I*ArcSinh[Sqrt[(I*p)/q]*x], -1])/(a^2*p*Sqrt[(
I*p)/q]*(-b^2 + 2*a^2*p*q + Sqrt[b^4 - 4*a^2*b^2*p*q])*((b^2 - 2*a^2*p*q -
Sqrt[b^4 - 4*a^2*b^2*p*q])/(2*a^2*p^2) - (b^2 - 2*a^2*p*q + Sqrt[b^4 - 4*a^
2*b^2*p*q])/(2*a^2*p^2))*Sqrt[q^2 + p^2*x^4]) - ((4*I)*q^2*Sqrt[b^4 - 4*a^2
*b^2*p*q]*Sqrt[1 - (I*p*x^2)/q]*Sqrt[1 + (I*p*x^2)/q]*EllipticPi[((-2*I)*a^
2*p*q)/(-b^2 + 2*a^2*p*q + Sqrt[b^4 - 4*a^2*b^2*p*q]), I*ArcSinh[Sqrt[(I*p)
/q]*x], -1])/(Sqrt[(I*p)/q]*(-b^2 + 2*a^2*p*q + Sqrt[b^4 - 4*a^2*b^2*p*q])*
((b^2 - 2*a^2*p*q - Sqrt[b^4 - 4*a^2*b^2*p*q])/(2*a^2*p^2) - (b^2 - 2*a^2*p
*q + Sqrt[b^4 - 4*a^2*b^2*p*q])/(2*a^2*p^2))*Sqrt[q^2 + p^2*x^4]))/a
```

IntegrateAlgebraic [A] time = 1.63, size = 133, normalized size = 1.00

$$\frac{2\sqrt{2a^2pq-b^2}\,\tan^{-1}\left(\frac{x\sqrt{2a^2pq-b^2}}{a\sqrt{p^2x^4+q^2}+apx^2+aq+bx}\right)}{a^2}-\frac{b\log\left(\sqrt{p^2x^4+q^2}+px^2+q\right)}{a^2}+\frac{b\log(x)}{a^2}+\frac{\sqrt{p^2x^4+q^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-q + p*x^2)*Sqrt[q^2 + p^2*x^4])/(x^2*(a*q + b*x + a*p
*x^2)),x]
```

```
[Out] Sqrt[q^2 + p^2*x^4]/(a*x) + (2*Sqrt[-b^2 + 2*a^2*p*q]*ArcTan[(Sqrt[-b^2 + 2
*a^2*p*q]*x)/(a*q + b*x + a*p*x^2 + a*Sqrt[q^2 + p^2*x^4])])/a^2 + (b*Log[x
])/a^2 - (b*Log[q + p*x^2 + Sqrt[q^2 + p^2*x^4]])/a^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^2-q)*(p^2*x^4+q^2)^(1/2)/x^2/(a*p*x^2+a*q+b*x),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^4+q^2}\left(px^2-q\right)}{\left(apx^2+aq+bx\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^2-q)*(p^2*x^4+q^2)^(1/2)/x^2/(a*p*x^2+a*q+b*x),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(p^2*x^4 + q^2)*(p*x^2 - q)/((a*p*x^2 + a*q + b*x)*x^2), x)
```

maple [C] time = 0.10, size = 6694, normalized size = 50.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^2-q)*(p^2*x^4+q^2)^(1/2)/x^2/(a*p*x^2+a*q+b*x),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^4 + q^2}\left(px^2 - q\right)}{\left(apx^2 + aq + bx\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^2-q)*(p^2*x^4+q^2)^(1/2)/x^2/(a*p*x^2+a*q+b*x),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(p^2*x^4 + q^2)*(p*x^2 - q)/((a*p*x^2 + a*q + b*x)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{p^2\,x^4+q^2}\,\left(q-p\,x^2\right)}{x^2\,\left(a\,p\,x^2+b\,x+a\,q\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((p^2*x^4 + q^2)^(1/2)*(q - p*x^2))/(x^2*(a*q + b*x + a*p*x^2)),x)
```

```
[Out] int(-((p^2*x^4 + q^2)^(1/2)*(q - p*x^2))/(x^2*(a*q + b*x + a*p*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(px^2 - q\right)\sqrt{p^2x^4 + q^2}}{x^2\left(apx^2 + aq + bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**2-q)*(p**2*x**4+q**2)**(1/2)/x**2/(a*p*x**2+a*q+b*x),x)
```

```
[Out] Integral((p*x**2 - q)*sqrt(p**2*x**4 + q**2)/(x**2*(a*p*x**2 + a*q + b*x)),
 x)
```

3.1605 $\int \frac{\sqrt[3]{1+x^5}\left(-3+2x^5\right)}{x^2\left(2-x^3+2x^5\right)}\,dx$

Optimal. Leaf size=133

$$\frac{3\sqrt[3]{x^5+1}}{2x}+\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^5+1}-x\right)}{2\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^5+1}+x}\right)}{2\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^5+1}x+2^{2/3}\left(x^5+1\right)^{2/3}+x^2\right)}{4\sqrt[3]{2}}$$

Rubi [F] time = 0.85, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{1+x^5}\left(-3+2x^5\right)}{x^2\left(2-x^3+2x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^5)^(1/3)*(-3 + 2*x^5))/(x^2*(2 - x^3 + 2*x^5)),x]
```

```
[Out] (3*Hypergeometric2F1[-1/3, -1/5, 4/5, -x^5])/(2*x) - (3*Defer[Int][(x*(1 + 
x^5)^(1/3))/(2 - x^3 + 2*x^5), x])/2 + 5*Defer[Int][(x^3*(1 + x^5)^(1/3))/(
2 - x^3 + 2*x^5), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{1+x^5}\left(-3+2x^5\right)}{x^2\left(2-x^3+2x^5\right)}\,dx &= \int \left(-\frac{3\sqrt[3]{1+x^5}}{2x^2}+\frac{x\left(-3+10x^2\right)\sqrt[3]{1+x^5}}{2\left(2-x^3+2x^5\right)}\right)dx\\
&= \frac{1}{2}\int \frac{x\left(-3+10x^2\right)\sqrt[3]{1+x^5}}{2-x^3+2x^5}\,dx-\frac{3}{2}\int \frac{\sqrt[3]{1+x^5}}{x^2}\,dx\\
&= \frac{3\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{5};\frac{4}{5};-x^5\right)}{2x}+\frac{1}{2}\int \left(-\frac{3x\sqrt[3]{1+x^5}}{2-x^3+2x^5}+\frac{10x^3\sqrt[3]{1+x^5}}{2-x^3+2x^5}\right)dx\\
&= \frac{3\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{5};\frac{4}{5};-x^5\right)}{2x}-\frac{3}{2}\int \frac{x\sqrt[3]{1+x^5}}{2-x^3+2x^5}\,dx+5\int \frac{x^3\sqrt[3]{1+x^5}}{2-x^3+2x^5}\,dx
\end{aligned}$$

Mathematica [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{1+x^5}\left(-3+2x^5\right)}{x^2\left(2-x^3+2x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^5)^(1/3)*(-3 + 2*x^5))/(x^2*(2 - x^3 + 2*x^5)),x]
```

```
[Out] Integrate[((1 + x^5)^(1/3)*(-3 + 2*x^5))/(x^2*(2 - x^3 + 2*x^5)), x]
```

IntegrateAlgebraic [A] time = 2.68, size = 133, normalized size = 1.00

$$\frac{3\sqrt[3]{x^5+1}}{2x}+\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^5+1}-x\right)}{2\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^5+1}+x}\right)}{2\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^5+1}x+2^{2/3}\left(x^5+1\right)^{2/3}+x^2\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^5)^(1/3)*(-3 + 2*x^5))/(x^2*(2 - x^3 + 2*x^5)),x
]
```

```
[Out] (3*(1 + x^5)^(1/3))/(2*x) + (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*2^(1/3)*(1 +
 x^5)^(1/3))])/(2*2^(1/3)) + Log[-x + 2^(1/3)*(1 + x^5)^(1/3)]/(2*2^(1/3))
- Log[x^2 + 2^(1/3)*x*(1 + x^5)^(1/3) + 2^(2/3)*(1 + x^5)^(2/3)]/(4*2^(1/3)
)
```

fricas [B] time = 100.24, size = 384, normalized size = 2.89

$$\frac{2\sqrt{3}\,2^{\frac{2}{3}}x\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(24\sqrt{2}\left(2x^{11}+x^9-x^7+4x^6+x^4+2x\right)\left(x^5+1\right)^{\frac{2}{3}}+2^{\frac{5}{6}}\left(8x^{15}+60x^{13}+24x^{11}+24x^{10}-x^9+120x^8+24x^6+24x^5+60x^3+8\right)+12\cdot 2^{\frac{1}{6}}\left(4x^{12}+14x^{10}+x^8+8x^7+14x^5+4x^2\right)\left(x^5+1\right)^{\frac{1}{3}}\right)}{6\left(8x^{15}-12x^{13}-48x^{11}+24x^{10}-x^9-24x^8-48x^6+24x^5-12x^3+8\right)}\right)+2\cdot 2^{\frac{2}{3}}x\log\left(\frac{3\cdot 2^{\frac{2}{3}}\left(x^5+1\right)^{\frac{1}{3}}x^2-6\left(x^5+1\right)^{\frac{2}{3}}x+2^{\frac{1}{3}}\left(2x^5-x^3+2\right)}{2x^5-x^3+2}\right)-2^{\frac{2}{3}}x\log\left(\frac{12\cdot 2^{\frac{1}{3}}\left(x^6+x^4+x\right)\left(x^5+1\right)^{\frac{2}{3}}+2^{\frac{2}{3}}\left(4x^{10}+14x^8+x^6+8x^5+14x^3+4\right)+6\left(4x^7+x^5+4x^2\right)\left(x^5+1\right)^{\frac{1}{3}}}{4x^{10}-4x^8+x^6+8x^5-4x^3+4}\right)+36\left(x^5+1\right)^{\frac{1}{3}}}{24x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(1/3)*(2*x^5-3)/x^2/(2*x^5-x^3+2),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(3)*2^(2/3)*x*arctan(1/6*sqrt(3)*2^(1/6)*(24*sqrt(2)*(2*x^11 +
x^9 - x^7 + 4*x^6 + x^4 + 2*x)*(x^5 + 1)^(2/3) + 2^(5/6)*(8*x^15 + 60*x^13
+ 24*x^11 + 24*x^10 - x^9 + 120*x^8 + 24*x^6 + 24*x^5 + 60*x^3 + 8) + 12*2^
(1/6)*(4*x^12 + 14*x^10 + x^8 + 8*x^7 + 14*x^5 + 4*x^2)*(x^5 + 1)^(1/3))/(8
*x^15 - 12*x^13 - 48*x^11 + 24*x^10 - x^9 - 24*x^8 - 48*x^6 + 24*x^5 - 12*x
^3 + 8)) + 2*2^(2/3)*x*log((3*2^(2/3)*(x^5 + 1)^(1/3)*x^2 - 6*(x^5 + 1)^(2/
3)*x + 2^(1/3)*(2*x^5 - x^3 + 2))/(2*x^5 - x^3 + 2)) - 2^(2/3)*x*log((12*2^
(1/3)*(x^6 + x^4 + x)*(x^5 + 1)^(2/3) + 2^(2/3)*(4*x^10 + 14*x^8 + x^6 + 8*
x^5 + 14*x^3 + 4) + 6*(4*x^7 + x^5 + 4*x^2)*(x^5 + 1)^(1/3))/(4*x^10 - 4*x^
8 + x^6 + 8*x^5 - 4*x^3 + 4)) + 36*(x^5 + 1)^(1/3))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^5-3\right)\left(x^5+1\right)^{\frac{1}{3}}}{\left(2x^5-x^3+2\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(1/3)*(2*x^5-3)/x^2/(2*x^5-x^3+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^5 - 3)*(x^5 + 1)^(1/3)/((2*x^5 - x^3 + 2)*x^2), x)
```

maple [C] time = 8.29, size = 1656, normalized size = 12.45

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)^(1/3)*(2*x^5-3)/x^2/(2*x^5-x^3+2),x)
```

```
[Out] 3/2*(x^5+1)^(1/3)/x+(-1/4*ln(-(RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+
4*_Z^2)*RootOf(_Z^3-4)^3*x^8+4*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+
4*_Z^2)^2*RootOf(_Z^3-4)^2*x^8+4*RootOf(_Z^3-4)*x^10+16*RootOf(RootOf(_Z^3-
4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^10+12*(x^10+2*x^5+1)^(1/3)*RootOf(RootOf
(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x^6+2*RootOf(_Z^3-4)*
x^8+8*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^8+6*(x^10+2*x^5
+1)^(2/3)*RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z
^2)*x^2+RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^
3*x^3+4*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4
)^2*x^3+8*RootOf(_Z^3-4)*x^5+32*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)
+4*_Z^2)*x^5+12*(x^10+2*x^5+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z
^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x+2*RootOf(_Z^3-4)*x^3+8*RootOf(RootOf(_Z^3-4)
^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^3+4*RootOf(_Z^3-4)+16*RootOf(RootOf(_Z^3-4
)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(2*x^5-x^3+2)/(1+x)/(x^4-x^3+x^2-x+1))*Roo
tOf(_Z^3-4)-1/2*ln(-(RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*Ro
```

```
otOf(_Z^3-4)^3*x^8+4*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*
RootOf(_Z^3-4)^2*x^8+4*RootOf(_Z^3-4)*x^10+16*RootOf(RootOf(_Z^3-4)^2+2*_Z*
RootOf(_Z^3-4)+4*_Z^2)*x^10+12*(x^10+2*x^5+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2
+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x^6+2*RootOf(_Z^3-4)*x^8+8*Root
Of(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^8+6*(x^10+2*x^5+1)^(2/3)*
RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2+Ro
otOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x^3+4*Ro
otOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+8*
RootOf(_Z^3-4)*x^5+32*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x
^5+12*(x^10+2*x^5+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z
^2)*RootOf(_Z^3-4)*x+2*RootOf(_Z^3-4)*x^3+8*RootOf(RootOf(_Z^3-4)^2+2*_Z*Ro
otOf(_Z^3-4)+4*_Z^2)*x^3+4*RootOf(_Z^3-4)+16*RootOf(RootOf(_Z^3-4)^2+2*_Z*R
ootOf(_Z^3-4)+4*_Z^2))/(2*x^5-x^3+2)/(1+x)/(x^4-x^3+x^2-x+1))*RootOf(RootOf
(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+1/2*RootOf(RootOf(_Z^3-4)^2+2*_Z*Roo
tOf(_Z^3-4)+4*_Z^2)*ln((-RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2
)*RootOf(_Z^3-4)^3*x^8-4*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2
)^2*RootOf(_Z^3-4)^2*x^8+4*RootOf(_Z^3-4)*x^10+16*RootOf(RootOf(_Z^3-4)^2+2
*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^10+6*RootOf(_Z^3-4)^2*(x^10+2*x^5+1)^(1/3)*x^6
+12*(x^10+2*x^5+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2
)*RootOf(_Z^3-4)*x^6+6*(x^10+2*x^5+1)^(2/3)*RootOf(_Z^3-4)^2*RootOf(RootOf(
_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-RootOf(RootOf(_Z^3-4)^2+2*_Z*Root
Of(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x^3-4*RootOf(RootOf(_Z^3-4)^2+2*_Z*Root
Of(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+8*RootOf(_Z^3-4)*x^5+32*RootOf(Ro
otOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^5+12*(x^10+2*x^5+1)^(2/3)*x^2+
6*RootOf(_Z^3-4)^2*(x^10+2*x^5+1)^(1/3)*x+12*(x^10+2*x^5+1)^(1/3)*RootOf(Ro
otOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x+4*RootOf(_Z^3-4
)+16*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(2*x^5-x^3+2)/(1+
x)/(x^4-x^3+x^2-x+1)))/(x^5+1)^(2/3)*((x^5+1)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 x^5 - 3\right)\left(x^5 + 1\right)^{\frac{1}{3}}}{\left(2 x^5 - x^3 + 2\right)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(1/3)*(2*x^5-3)/x^2/(2*x^5-x^3+2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^5 - 3)*(x^5 + 1)^(1/3)/((2*x^5 - x^3 + 2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^5 + 1\right)^{1/3}\,\left(2\,x^5 - 3\right)}{x^2\,\left(2\,x^5 - x^3 + 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 + 1)^(1/3)*(2*x^5 - 3))/(x^2*(2*x^5 - x^3 + 2)),x)
```

```
[Out] int(((x^5 + 1)^(1/3)*(2*x^5 - 3))/(x^2*(2*x^5 - x^3 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{(x+1)\left(x^4 - x^3 + x^2 - x + 1\right)}\left(2x^5 - 3\right)}{x^2\left(2x^5 - x^3 + 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+1)**(1/3)*(2*x**5-3)/x**2/(2*x**5-x**3+2),x)
```

```
[Out] Integral(((x + 1)*(x**4 - x**3 + x**2 - x + 1))**(1/3)*(2*x**5 - 3)/(x**2*(
2*x**5 - x**3 + 2)), x)
```

3.1606 $\int \frac{x^2(4b+ax^5)}{(-b+ax^5)^{3/4}(-b+cx^4+ax^5)}\,dx$

Optimal. Leaf size=133

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5-b}}{\sqrt{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5-b}}\right)}{c^{3/4}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x\sqrt[4]{ax^5-b}}{\sqrt{ax^5-b}-\sqrt{c}\,x^2}\right)}{c^{3/4}}$$

Rubi [F] time = 2.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2(4b+ax^5)}{(-b+ax^5)^{3/4}(-b+cx^4+ax^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(4*b + a*x^5))/((-b + a*x^5)^(3/4)*(-b + c*x^4 + a*x^5)),x]
```

```
[Out] (c^2*x*(1 - (a*x^5)/b)^(3/4)*Hypergeometric2F1[1/5, 3/4, 6/5, (a*x^5)/b])/(
a^2*(-b + a*x^5)^(3/4)) - (c*x^2*(1 - (a*x^5)/b)^(3/4)*Hypergeometric2F1[2/
5, 3/4, 7/5, (a*x^5)/b])/(2*a*(-b + a*x^5)^(3/4)) + (x^3*(1 - (a*x^5)/b)^(3
/4)*Hypergeometric2F1[3/5, 3/4, 8/5, (a*x^5)/b])/(3*(-b + a*x^5)^(3/4)) - (
b*c^2*Defer[Int][1/((b - c*x^4 - a*x^5)*(-b + a*x^5)^(3/4)), x])/a^2 - (b*c
*Defer[Int][x/((-b + a*x^5)^(3/4)*(-b + c*x^4 + a*x^5)), x])/a + 5*b*Defer[
Int][x^2/((-b + a*x^5)^(3/4)*(-b + c*x^4 + a*x^5)), x] - (c^3*Defer[Int][x^
4/((-b + a*x^5)^(3/4)*(-b + c*x^4 + a*x^5)), x])/a^2
```

Rubi steps

$$\int \frac{x^2(4b+ax^5)}{(-b+ax^5)^{3/4}(-b+cx^4+ax^5)}\,dx=\int\left(\frac{c^2}{a^2(-b+ax^5)^{3/4}}-\frac{cx}{a(-b+ax^5)^{3/4}}+\frac{x^2}{(-b+ax^5)^{3/4}}+\frac{bc^2-abcx}{a^2(-b+ax^5)^{3/4}}\right.$$

$$=\frac{\int\frac{bc^2-abcx+5a^2bx^2-c^3x^4}{(-b+ax^5)^{3/4}(-b+cx^4+ax^5)}\,dx}{a^2}-\frac{c\int\frac{x}{(-b+ax^5)^{3/4}}\,dx}{a}+\frac{c^2\int\frac{1}{(-b+ax^5)^{3/4}}\,dx}{a^2}+\int$$

$$=\frac{\int\left(-\frac{bc^2}{(b-cx^4-ax^5)(-b+ax^5)^{3/4}}-\frac{abcx}{(-b+ax^5)^{3/4}(-b+cx^4+ax^5)}+\frac{5a^2bx^2}{(-b+ax^5)^{3/4}(-b+cx^4+ax^5)}-\right.}{a^2}$$

$$=\frac{c^2x\left(1-\frac{ax^5}{b}\right)^{3/4}{}_2F_1\left(\frac{1}{5},\frac{3}{4};\frac{6}{5};\frac{ax^5}{b}\right)}{a^2(-b+ax^5)^{3/4}}-\frac{cx^2\left(1-\frac{ax^5}{b}\right)^{3/4}{}_2F_1\left(\frac{2}{5},\frac{3}{4};\frac{7}{5};\frac{ax^5}{b}\right)}{2a(-b+ax^5)^{3/4}}+$$

Mathematica [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{x^2(4b+ax^5)}{(-b+ax^5)^{3/4}(-b+cx^4+ax^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(4*b + a*x^5))/((-b + a*x^5)^(3/4)*(-b + c*x^4 + a*x^5)),x]
```

```
[Out] Integrate[(x^2*(4*b + a*x^5))/((-b + a*x^5)^(3/4)*(-b + c*x^4 + a*x^5)), x]
```

IntegrateAlgebraic [A] time = 13.37, size = 133, normalized size = 1.00

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5-b}}{\sqrt{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5-b}}\right)}{c^{3/4}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x\sqrt[4]{ax^5-b}}{\sqrt{ax^5-b}-\sqrt{c}x^2}\right)}{c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(4*b + a*x^5))/((-b + a*x^5)^(3/4)*(-b + c*x^4 + a*
x^5)),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[2]*c^(1/4)*x*(-b + a*x^5)^(1/4))/(-(Sqrt[c]*x^2) +
Sqrt[-b + a*x^5])])/c^(3/4)) + (Sqrt[2]*ArcTanh[((c^(1/4)*x^2)/Sqrt[2] + Sq
rt[-b + a*x^5]/(Sqrt[2]*c^(1/4)))/(x*(-b + a*x^5)^(1/4))])/c^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^5+4*b)/(a*x^5-b)^(3/4)/(a*x^5+c*x^4-b),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^5+4\,b\right)x^2}{\left(ax^5+cx^4-b\right)\left(ax^5-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^5+4*b)/(a*x^5-b)^(3/4)/(a*x^5+c*x^4-b),x, algorithm="gia
c")
```

```
[Out] integrate((a*x^5 + 4*b)*x^2/((a*x^5 + c*x^4 - b)*(a*x^5 - b)^(3/4)), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a\,x^5+4b\right)}{\left(a\,x^5-b\right)^{\frac{3}{4}}\left(a\,x^5+c\,x^4-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^5+4*b)/(a*x^5-b)^(3/4)/(a*x^5+c*x^4-b),x)
```

```
[Out] int(x^2*(a*x^5+4*b)/(a*x^5-b)^(3/4)/(a*x^5+c*x^4-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^5+4\,b\right)x^2}{\left(ax^5+cx^4-b\right)\left(ax^5-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^5+4*b)/(a*x^5-b)^(3/4)/(a*x^5+c*x^4-b),x, algorithm="max
ima")
```

```
[Out] integrate((a*x^5 + 4*b)*x^2/((a*x^5 + c*x^4 - b)*(a*x^5 - b)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\left(a\,x^5+4\,b\right)}{{\left(a\,x^5-b\right)}^{3/4}\,\left(a\,x^5+c\,x^4-b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(4*b + a*x^5))/((a*x^5 - b)^(3/4)*(a*x^5 - b + c*x^4)),x)
```

```
[Out] int((x^2*(4*b + a*x^5))/((a*x^5 - b)^(3/4)*(a*x^5 - b + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a x^{5} + 4 b\right)}{\left(a x^{5} - b\right)^{\frac{3}{4}} \left(a x^{5} - b + c x^{4}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x**5+4*b)/(a*x**5-b)**(3/4)/(a*x**5+c*x**4-b),x)
```

```
[Out] Integral(x**2*(a*x**5 + 4*b)/((a*x**5 - b)**(3/4)*(a*x**5 - b + c*x**4)), x
)
```

3.1607 $\int \frac{(-1+x^6)(1+x^6)^{2/3}}{x^3(2-x^3+2x^6)}\,dx$

Optimal. Leaf size=133

$$\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^6+1}-x\right)}{6\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^6+1}+x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{(x^6+1)^{2/3}}{4x^2}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^6+1}\,x+2^{2/3}(x^6+1)^{2/3}+x^2\right)}{12\ 2^{2/3}}$$

Rubi [C] time = 0.89, antiderivative size = 241, normalized size of antiderivative = 1.81, number of steps used = 16, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {6728, 275, 364, 1438, 429, 465, 510}

$$\frac{(-\sqrt{15}+i)\,xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};-\frac{8x^6}{7-i\sqrt{15}},-x^6\right)}{\sqrt{15}+7i}+\frac{(\sqrt{15}+i)\,xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};-\frac{8x^6}{7+i\sqrt{15}},-x^6\right)}{-\sqrt{15}+7i}+\frac{x^4F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-x^6,-\frac{8x^6}{7-i\sqrt{15}}\right)}{7-i\sqrt{15}}+\frac{x^4F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-x^6,-\frac{8x^6}{7+i\sqrt{15}}\right)}{7+i\sqrt{15}}+\frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};-x^6\right)}{4x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^6)*(1 + x^6)^(2/3))/(x^3*(2 - x^3 + 2*x^6)),x]
```

```
[Out] ((I - Sqrt[15])*x*AppellF1[1/6, 1, -2/3, 7/6, (-8*x^6)/(7 - I*Sqrt[15]), -x
^6])/(7*I + Sqrt[15]) + ((I + Sqrt[15])*x*AppellF1[1/6, 1, -2/3, 7/6, (-8*x
^6)/(7 + I*Sqrt[15]), -x^6])/(7*I - Sqrt[15]) + (x^4*AppellF1[2/3, -2/3, 1,
 5/3, -x^6, (-8*x^6)/(7 - I*Sqrt[15])])/(7 - I*Sqrt[15]) + (x^4*AppellF1[2/
3, -2/3, 1, 5/3, -x^6, (-8*x^6)/(7 + I*Sqrt[15])])/(7 + I*Sqrt[15]) + Hyper
geometric2F1[-2/3, -1/3, 2/3, -x^6]/(4*x^2)
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(-1+x^6\right)\left(1+x^6\right)^{2/3}}{x^3\left(2-x^3+2x^6\right)}\,dx = \int \left(-\frac{\left(1+x^6\right)^{2/3}}{2x^3}+\frac{\left(-1+4x^3\right)\left(1+x^6\right)^{2/3}}{2\left(2-x^3+2x^6\right)}\right)dx$$

$$= -\left(\frac{1}{2}\int \frac{\left(1+x^6\right)^{2/3}}{x^3}\,dx\right)+\frac{1}{2}\int \frac{\left(-1+4x^3\right)\left(1+x^6\right)^{2/3}}{2-x^3+2x^6}\,dx$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{\left(1+x^3\right)^{2/3}}{x^2}\,dx,x,x^2\right)\right)+\frac{1}{2}\int \left(\frac{4\left(1+x^6\right)^{2/3}}{-1-i\sqrt{15}+4x^3}+\frac{4\left(1+x^6\right)^{2/3}}{-1+i\sqrt{15}+4x^3}\right.$$

$$= \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};-x^6\right)}{4x^2}+2\int \frac{\left(1+x^6\right)^{2/3}}{-1-i\sqrt{15}+4x^3}\,dx+2\int \frac{\left(1+x^6\right)^{2/3}}{-1+i\sqrt{15}+4x^3}\,dx$$

$$= \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};-x^6\right)}{4x^2}+2\int \left(\frac{\left(i-\sqrt{15}\right)\left(1+x^6\right)^{2/3}}{2\left(7i+\sqrt{15}+8ix^6\right)}+\frac{2x^3\left(1+x^6\right)^{2/3}}{7-i\sqrt{15}+8x^6}\right)dx+2\int \left(\right.$$

$$= \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};-x^6\right)}{4x^2}+4\int \frac{x^3\left(1+x^6\right)^{2/3}}{7-i\sqrt{15}+8x^6}\,dx+4\int \frac{x^3\left(1+x^6\right)^{2/3}}{7+i\sqrt{15}+8x^6}\,dx+\left(-i-\right.$$

$$= \frac{\left(i-\sqrt{15}\right)xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};-\frac{8x^6}{7-i\sqrt{15}},-x^6\right)}{7i+\sqrt{15}}+\frac{\left(i+\sqrt{15}\right)xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};-\frac{8x^6}{7+i\sqrt{15}},-x\right.}{7i-\sqrt{15}}$$

$$= \frac{\left(i-\sqrt{15}\right)xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};-\frac{8x^6}{7-i\sqrt{15}},-x^6\right)}{7i+\sqrt{15}}+\frac{\left(i+\sqrt{15}\right)xF_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};-\frac{8x^6}{7+i\sqrt{15}},-x\right.}{7i-\sqrt{15}}$$

Mathematica [F] time = 1.40, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^6\right)\left(1+x^6\right)^{2/3}}{x^3\left(2-x^3+2x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^6)*(1 + x^6)^(2/3))/(x^3*(2 - x^3 + 2*x^6)),x]
[Out] Integrate[((-1 + x^6)*(1 + x^6)^(2/3))/(x^3*(2 - x^3 + 2*x^6)), x]
```

IntegrateAlgebraic [A] time = 1.18, size = 133, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^6+1}-x\right)}{6\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^6+1}+x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\left(x^6+1\right)^{2/3}}{4x^2}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^6+1}\,x+2^{2/3}\left(x^6+1\right)^{2/3}+x^2\right)}{12\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^6)*(1 + x^6)^(2/3))/(x^3*(2 - x^3 + 2*x^6)),x]
```

```
[Out] (1 + x^6)^(2/3)/(4*x^2) - ArcTan[(Sqrt[3]*x)/(x + 2*2^(1/3)*(1 + x^6)^(1/3)
)]/(2*2^(2/3)*Sqrt[3]) + Log[-x + 2^(1/3)*(1 + x^6)^(1/3)]/(6*2^(2/3)) - Lo
g[x^2 + 2^(1/3)*x*(1 + x^6)^(1/3) + 2^(2/3)*(1 + x^6)^(2/3)]/(12*2^(2/3))
```

fricas [B] time = 146.00, size = 349, normalized size = 2.62

$$-\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}x^2\arctan\left(\frac{4^{\frac{1}{6}}\left(12\cdot 4^{\frac{2}{3}}\sqrt{3}\left(2x^{13}+x^{10}+3x^7+x^4+2x\right)\left(x^6+1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\sqrt{3}\left(8x^{18}+60x^{15}+48x^{12}+119x^9+48x^6+60x^3+8\right)+12\sqrt{3}\left(4x^{14}+14x^{11}+9x^8+14x^5+4x^2\right)\left(x^6+1\right)^{\frac{1}{3}}\right)}{6\left(8x^{18}-12x^{15}-24x^{12}-25x^9-24x^6-12x^3+8\right)}\right)-2\cdot 4^{\frac{2}{3}}x^2\log\left(-\frac{6\cdot 4^{\frac{1}{3}}\left(x^6+1\right)^{\frac{1}{3}}x^2+4^{\frac{2}{3}}\left(2x^6-x^3+2\right)-12\left(x^6+1\right)^{\frac{2}{3}}x}{2x^6-x^3+2}\right)+4^{\frac{2}{3}}x^2\log\left(\frac{6\cdot 4^{\frac{2}{3}}\left(x^7+x^4+x\right)\left(x^6+1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(4x^{12}+14x^9+9x^6+14x^3+4\right)+6\left(4x^8+x^5+4x^2\right)\left(x^6+1\right)^{\frac{1}{3}}}{4x^{12}-4x^9+9x^6-4x^3+4}\right)-36\left(x^6+1\right)^{\frac{2}{3}}}{144x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)*(x^6+1)^(2/3)/x^3/(2*x^6-x^3+2),x, algorithm="fricas")
```

```
[Out] -1/144*(4*4^(1/6)*sqrt(3)*x^2*arctan(1/6*4^(1/6)*(12*4^(2/3)*sqrt(3)*(2*x^1
3 + x^10 + 3*x^7 + x^4 + 2*x)*(x^6 + 1)^(2/3) + 4^(1/3)*sqrt(3)*(8*x^18 + 6
0*x^15 + 48*x^12 + 119*x^9 + 48*x^6 + 60*x^3 + 8) + 12*sqrt(3)*(4*x^14 + 14
*x^11 + 9*x^8 + 14*x^5 + 4*x^2)*(x^6 + 1)^(1/3))/(8*x^18 - 12*x^15 - 24*x^1
2 - 25*x^9 - 24*x^6 - 12*x^3 + 8)) - 2*4^(2/3)*x^2*log(-(6*4^(1/3)*(x^6 + 1
)^(1/3)*x^2 + 4^(2/3)*(2*x^6 - x^3 + 2) - 12*(x^6 + 1)^(2/3)*x)/(2*x^6 - x^
3 + 2)) + 4^(2/3)*x^2*log((6*4^(2/3)*(x^7 + x^4 + x)*(x^6 + 1)^(2/3) + 4^(1
/3)*(4*x^12 + 14*x^9 + 9*x^6 + 14*x^3 + 4) + 6*(4*x^8 + x^5 + 4*x^2)*(x^6 +
 1)^(1/3))/(4*x^12 - 4*x^9 + 9*x^6 - 4*x^3 + 4)) - 36*(x^6 + 1)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+1\right)^{\frac{2}{3}}\left(x^6-1\right)}{\left(2x^6-x^3+2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)*(x^6+1)^(2/3)/x^3/(2*x^6-x^3+2),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 1)^(2/3)*(x^6 - 1)/((2*x^6 - x^3 + 2)*x^3), x)
```

maple [F] time = 4.82, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6-1\right)\left(x^6+1\right)^{\frac{2}{3}}}{x^3\left(2x^6-x^3+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)*(x^6+1)^(2/3)/x^3/(2*x^6-x^3+2),x)
```

```
[Out] int((x^6-1)*(x^6+1)^(2/3)/x^3/(2*x^6-x^3+2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+1\right)^{\frac{2}{3}}\left(x^6-1\right)}{\left(2x^6-x^3+2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)*(x^6+1)^(2/3)/x^3/(2*x^6-x^3+2),x, algorithm="maxima")
[Out] integrate((x^6 + 1)^(2/3)*(x^6 - 1)/((2*x^6 - x^3 + 2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6-1\right)\,\left(x^6+1\right)^{2/3}}{x^3\,\left(2\,x^6-x^3+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 1)*(x^6 + 1)^(2/3))/(x^3*(2*x^6 - x^3 + 2)),x)
[Out] int(((x^6 - 1)*(x^6 + 1)^(2/3))/(x^3*(2*x^6 - x^3 + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)*(x**6+1)**(2/3)/x**3/(2*x**6-x**3+2),x)
[Out] Timed out
```

3.1608 $\int \frac{x^2(2b+ax^6)}{(-b+ax^6)^{3/4}(-b+cx^4+ax^6)}\,dx$

Optimal. Leaf size=133

$$\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6-b}}{\sqrt{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}x^2}{\sqrt{2}}}{x\sqrt[4]{ax^6-b}}\right)}{\sqrt{2}c^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x\sqrt[4]{ax^6-b}}{\sqrt{ax^6-b}-\sqrt{c}x^2}\right)}{\sqrt{2}c^{3/4}}$$

Rubi [F] time = 2.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2(2b+ax^6)}{(-b+ax^6)^{3/4}(-b+cx^4+ax^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(2*b + a*x^6))/((-b + a*x^6)^(3/4)*(-b + c*x^4 + a*x^6)),x]
```

```
[Out] (Sqrt[(a*x^6)/(Sqrt[b] + Sqrt[-b + a*x^6])^2]*(Sqrt[b] + Sqrt[-b + a*x^6])*
EllipticF[2*ArcTan[(-b + a*x^6)^(1/4)/b^(1/4)], 1/2])/(3*a*b^(1/4)*x^3) - (
c*x*(1 - (a*x^6)/b)^(3/4)*Hypergeometric2F1[1/6, 3/4, 7/6, (a*x^6)/b])/(a*(
-b + a*x^6)^(3/4)) + (b*c*Defer[Int][1/((b - c*x^4 - a*x^6)*(-b + a*x^6)^(3
/4)), x])/a + 3*b*Defer[Int][x^2/((-b + a*x^6)^(3/4)*(-b + c*x^4 + a*x^6)),
 x] + (c^2*Defer[Int][x^4/((-b + a*x^6)^(3/4)*(-b + c*x^4 + a*x^6)), x])/a
```

Rubi steps

$$\int \frac{x^2(2b+ax^6)}{(-b+ax^6)^{3/4}(-b+cx^4+ax^6)}\,dx = \int\left(-\frac{c}{a(-b+ax^6)^{3/4}}+\frac{x^2}{(-b+ax^6)^{3/4}}-\frac{bc-3abx^2-c^2x^4}{a(-b+ax^6)^{3/4}(-b+cx^4+a\cdots}\right.$$

$$=-\frac{\int\frac{bc-3abx^2-c^2x^4}{(-b+ax^6)^{3/4}(-b+cx^4+ax^6)}\,dx}{a}-\frac{c\int\frac{1}{(-b+ax^6)^{3/4}}\,dx}{a}+\int\frac{x^2}{(-b+ax^6)^{3/4}}\,dx$$

$$=\frac{1}{3}\text{Subst}\left(\int\frac{1}{(-b+ax^2)^{3/4}}\,dx,x,x^3\right)-\frac{\int\left(-\frac{bc}{(b-cx^4-ax^6)(-b+ax^6)^{3/4}}-\frac{\cdots}{(-b+\cdots}\right.}{\cdots}$$

$$=-\frac{cx\left(1-\frac{ax^6}{b}\right)^{3/4}{}_2F_1\left(\frac{1}{6},\frac{3}{4};\frac{7}{6};\frac{ax^6}{b}\right)}{a(-b+ax^6)^{3/4}}+(3b)\int\frac{x^2}{(-b+ax^6)^{3/4}(-b+cx^4+\cdots}$$

$$=\frac{\sqrt{\frac{ax^6}{\left(\sqrt{b}+\sqrt{-b+ax^6}\right)^2}}\left(\sqrt{b}+\sqrt{-b+ax^6}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-b+ax^6}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{3a\sqrt[4]{b}x^3}-\frac{cx(1\cdots}{\cdots}$$

Mathematica [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x^2(2b+ax^6)}{(-b+ax^6)^{3/4}(-b+cx^4+ax^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(2*b + a*x^6))/((-b + a*x^6)^(3/4)*(-b + c*x^4 + a*x^6)),x]
```

```
[Out] Integrate[(x^2*(2*b + a*x^6))/((-b + a*x^6)^(3/4)*(-b + c*x^4 + a*x^6)), x]
```

IntegrateAlgebraic [A] time = 15.82, size = 133, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6-b}}{\sqrt{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}x^2}{\sqrt{2}}}{x\sqrt[4]{ax^6-b}}\right)}{\sqrt{2}c^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x\sqrt[4]{ax^6-b}}{\sqrt{ax^6-b}-\sqrt{c}x^2}\right)}{\sqrt{2}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(2*b + a*x^6))/((-b + a*x^6)^(3/4)*(-b + c*x^4 + a*
x^6)),x]
```

```
[Out] -(ArcTan[(Sqrt[2]*c^(1/4)*x*(-b + a*x^6)^(1/4))/(-(Sqrt[c]*x^2) + Sqrt[-b +
 a*x^6])]/(Sqrt[2]*c^(3/4))) + ArcTanh[((c^(1/4)*x^2)/Sqrt[2] + Sqrt[-b + a
*x^6]/(Sqrt[2]*c^(1/4)))/(x*(-b + a*x^6)^(1/4))]/(Sqrt[2]*c^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^6+2*b)/(a*x^6-b)^(3/4)/(a*x^6+c*x^4-b),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^6+2b)x^2}{(ax^6+cx^4-b)(ax^6-b)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^6+2*b)/(a*x^6-b)^(3/4)/(a*x^6+c*x^4-b),x, algorithm="gia
c")
```

```
[Out] integrate((a*x^6 + 2*b)*x^2/((a*x^6 + c*x^4 - b)*(a*x^6 - b)^(3/4)), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x^2(ax^6+2b)}{(ax^6-b)^{\frac{3}{4}}(ax^6+cx^4-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^6+2*b)/(a*x^6-b)^(3/4)/(a*x^6+c*x^4-b),x)
```

```
[Out] int(x^2*(a*x^6+2*b)/(a*x^6-b)^(3/4)/(a*x^6+c*x^4-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^6+2b)x^2}{(ax^6+cx^4-b)(ax^6-b)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^6+2*b)/(a*x^6-b)^(3/4)/(a*x^6+c*x^4-b),x, algorithm="max
ima")
```

```
[Out] integrate((a*x^6 + 2*b)*x^2/((a*x^6 + c*x^4 - b)*(a*x^6 - b)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a\,x^6+2\,b\right)}{{\left(a\,x^6-b\right)}^{3/4}\,\left(a\,x^6+c\,x^4-b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(2*b + a*x^6))/((a*x^6 - b)^(3/4)*(a*x^6 - b + c*x^4)),x)
```

```
[Out] int((x^2*(2*b + a*x^6))/((a*x^6 - b)^(3/4)*(a*x^6 - b + c*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x**6+2*b)/(a*x**6-b)**(3/4)/(a*x**6+c*x**4-b),x)
```

```
[Out] Timed out
```

3.1609 $\int x^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=133

$$\frac{x\left(16x^8+36x^4+9\right)\sqrt{\sqrt{x^4+1}+x^2}+x\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}\left(16x^6+28x^2\right)}{48\left(2x^4+1\right)+96\sqrt{x^4+1}\,x^2}-\frac{3\tan^{-1}\left(\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}\right)}{16\sqrt{2}}$$

Rubi [F] time = 0.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

```
[Out] Defer[Int][x^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]], x]
```

Rubi steps

$$\int x^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int x^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}\,dx$$

Mathematica [A] time = 0.23, size = 189, normalized size = 1.42

$$\frac{\sqrt{x^4+1}\sqrt{x^2\left(\sqrt{x^4+1}+x^2\right)}\left(\sqrt{2}\sqrt{x^2\left(\sqrt{x^4+1}+x^2\right)}\left(16x^8+36x^4+16\sqrt{x^4+1}\,x^6+28\sqrt{x^4+1}\,x^2+9\right)-9\left(2x^4+2\sqrt{x^4+1}\,x^2+1\right)\tan^{-1}\left(\sqrt{\left(\sqrt{x^4+1}+x^2\right)^2-1}\right)\right)}{48\sqrt{2}\left(\sqrt{x^4+1}+x^2\right)^{3/2}\left(x^5+\sqrt{x^4+1}\,x^3+x\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

```
[Out] (Sqrt[1 + x^4]*Sqrt[x^2*(x^2 + Sqrt[1 + x^4])]*(Sqrt[2]*Sqrt[x^2*(x^2 + Sqr
t[1 + x^4])]*(9 + 36*x^4 + 16*x^8 + 28*x^2*Sqrt[1 + x^4] + 16*x^6*Sqrt[1 +
x^4]) - 9*(1 + 2*x^4 + 2*x^2*Sqrt[1 + x^4])*ArcTan[Sqrt[-1 + (x^2 + Sqrt[1
+ x^4])^2]]))/(48*Sqrt[2]*(x^2 + Sqrt[1 + x^4])^(3/2)*(x + x^5 + x^3*Sqrt[1
 + x^4]))
```

IntegrateAlgebraic [A] time = 0.20, size = 133, normalized size = 1.00

$$\frac{x\left(16x^8+36x^4+9\right)\sqrt{\sqrt{x^4+1}+x^2}+x\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}\left(16x^6+28x^2\right)}{48\left(2x^4+1\right)+96\sqrt{x^4+1}\,x^2}-\frac{3\tan^{-1}\left(\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

```
[Out] (x*Sqrt[1 + x^4]*(28*x^2 + 16*x^6)*Sqrt[x^2 + Sqrt[1 + x^4]] + x*(9 + 36*x^
4 + 16*x^8)*Sqrt[x^2 + Sqrt[1 + x^4]])/(96*x^2*Sqrt[1 + x^4] + 48*(1 + 2*x^
4)) - (3*ArcTan[Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]]])/(16*Sqrt[2])
```

fricas [A] time = 0.89, size = 81, normalized size = 0.61

$$-\frac{1}{48}\left(2x^5-10\sqrt{x^4+1}\,x^3-9x\right)\sqrt{x^2+\sqrt{x^4+1}}+\frac{3}{32}\sqrt{2}\arctan\left(-\frac{\left(\sqrt{2}\,x^2-\sqrt{2}\,\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/48*(2*x^5 - 10*sqrt(x^4 + 1)*x^3 - 9*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 3/32
*sqrt(2)*arctan(-1/2*(sqrt(2)*x^2 - sqrt(2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(
x^4 + 1))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^2, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int x^2\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int(x^2*(x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

```
[Out] int(x^2*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{x^2+\sqrt{x^4+1}}\,\sqrt{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**4+1)**(1/2)*(x**2+(x**4+1)**(1/2))**(1/2),x)
[Out] Integral(x**2*sqrt(x**2 + sqrt(x**4 + 1))*sqrt(x**4 + 1), x)
```

3.1610 $\int \sqrt{1+\sqrt{1+\sqrt{1+x}}}\,dx$

Optimal. Leaf size=133

$$\frac{8}{315}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\,(35x+27)+\sqrt{x+1}\left(\frac{8}{63}\sqrt{\sqrt{x+1}+1}\sqrt{\sqrt{\sqrt{x+1}+1}+1}+\frac{8}{315}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right.$$

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 0.53, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {371, 1398, 772}

$$\frac{8}{9}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{9/2}-\frac{24}{7}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{7/2}+\frac{16}{5}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]],x]
```

```
[Out] (16*(1 + Sqrt[1 + Sqrt[1 + x]])^(5/2))/5 - (24*(1 + Sqrt[1 + Sqrt[1 + x]])^
(7/2))/7 + (8*(1 + Sqrt[1 + Sqrt[1 + x]])^(9/2))/9
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1+\sqrt{1+\sqrt{1+x}}}\,dx &= 2\,\mathrm{Subst}\left(\int x\sqrt{1+\sqrt{1+x}}\,dx,x,\sqrt{1+x}\right)\\
&= 2\,\mathrm{Subst}\left(\int \sqrt{1+\sqrt{x}}\,(-1+x)\,dx,x,1+\sqrt{1+x}\right)\\
&= 4\,\mathrm{Subst}\left(\int x\sqrt{1+x}\,\left(-1+x^2\right)\,dx,x,\sqrt{1+\sqrt{1+x}}\right)\\
&= 4\,\mathrm{Subst}\left(\int \left(2(1+x)^{3/2}-3(1+x)^{5/2}+(1+x)^{7/2}\right)dx,x,\sqrt{1+\sqrt{1+x}}\right)\\
&= \frac{16}{5}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{5/2}-\frac{24}{7}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{7/2}+\frac{8}{9}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{9/2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.37

$$\frac{8}{315}\left(35\sqrt{x+1}-65\sqrt{\sqrt{x+1}+1}+61\right)\left(\sqrt{\sqrt{x+1}+1}+1\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]],x]
```

```
[Out] (8*(61 + 35*Sqrt[1 + x] - 65*Sqrt[1 + Sqrt[1 + x]])*(1 + Sqrt[1 + Sqrt[1 +
x]])^(5/2))/315
```

IntegrateAlgebraic [A] time = 0.07, size = 85, normalized size = 0.64

$$\frac{8}{315}\sqrt{\sqrt{x+1}+1}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\left(5\sqrt{x+1}-8\right)+\frac{8}{315}\left(35(x+1)+\sqrt{x+1}-8\right)\sqrt{\sqrt{\sqrt{x+1}+1}+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]],x]
```

```
[Out] (8*Sqrt[1 + Sqrt[1 + x]]*(-8 + 5*Sqrt[1 + x])*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]
]])/315 + (8*(-8 + Sqrt[1 + x] + 35*(1 + x))*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]
])/315
```

fricas [A] time = 0.40, size = 44, normalized size = 0.33

$$\frac{8}{315}\left(\left(5\sqrt{x+1}-8\right)\sqrt{\sqrt{x+1}+1}+35x+\sqrt{x+1}+27\right)\sqrt{\sqrt{\sqrt{x+1}+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/315*((5*sqrt(x + 1) - 8)*sqrt(sqrt(x + 1) + 1) + 35*x + sqrt(x + 1) + 27)
*sqrt(sqrt(sqrt(x + 1) + 1) + 1)
```

giac [C] time = 1.76, size = 312, normalized size = 2.35

$$\frac{8}{315}\left(\left(35\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{9}{2}}-180\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{7}{2}}+378\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{5}{2}}-420\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{3}{2}}+315\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)\operatorname{sgn}\left(4\left(\sqrt{x+1}+1\right)^{2}-8\sqrt{x+1}-7\right)+9\left(5\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{7}{2}}-21\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{5}{2}}+35\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{3}{2}}-35\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)\operatorname{sgn}\left(4\left(\sqrt{x+1}+1\right)^{2}-8\sqrt{x+1}-7\right)-21\left(3\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{5}{2}}-10\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{3}{2}}+15\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)\operatorname{sgn}\left(4\left(\sqrt{x+1}+1\right)^{2}-8\sqrt{x+1}-7\right)-105\left(\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{3}{2}}-3\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)\operatorname{sgn}\left(4\left(\sqrt{x+1}+1\right)^{2}-8\sqrt{x+1}-7\right)\right)\operatorname{sgn}(4x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/315*((35*(sqrt(sqrt(x + 1) + 1) + 1)^(9/2) - 180*(sqrt(sqrt(x + 1) + 1) +
 1)^(7/2) + 378*(sqrt(sqrt(x + 1) + 1) + 1)^(5/2) - 420*(sqrt(sqrt(x + 1) +
 1) + 1)^(3/2) + 315*sqrt(sqrt(sqrt(x + 1) + 1) + 1))*sgn(4*(sqrt(x + 1) +
1)^2 - 8*sqrt(x + 1) - 7) + 9*(5*(sqrt(sqrt(x + 1) + 1) + 1)^(7/2) - 21*(sq
rt(sqrt(x + 1) + 1) + 1)^(5/2) + 35*(sqrt(sqrt(x + 1) + 1) + 1)^(3/2) - 35*
sqrt(sqrt(sqrt(x + 1) + 1) + 1))*sgn(4*(sqrt(x + 1) + 1)^2 - 8*sqrt(x + 1)
- 7) - 21*(3*(sqrt(sqrt(x + 1) + 1) + 1)^(5/2) - 10*(sqrt(sqrt(x + 1) + 1)
+ 1)^(3/2) + 15*sqrt(sqrt(sqrt(x + 1) + 1) + 1))*sgn(4*(sqrt(x + 1) + 1)^2
- 8*sqrt(x + 1) - 7) - 105*((sqrt(sqrt(x + 1) + 1) + 1)^(3/2) - 3*sqrt(sqrt
(sqrt(x + 1) + 1) + 1))*sgn(4*(sqrt(x + 1) + 1)^2 - 8*sqrt(x + 1) - 7))*sgn
(4*x + 1)
```

maple [A] time = 0.01, size = 47, normalized size = 0.35

$$\frac{8\left(1+\sqrt{1+\sqrt{1+x}}\right)^{\frac{9}{2}}}{9}-\frac{24\left(1+\sqrt{1+\sqrt{1+x}}\right)^{\frac{7}{2}}}{7}+\frac{16\left(1+\sqrt{1+\sqrt{1+x}}\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(1+(1+x)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 8/9*(1+(1+(1+x)^(1/2))^(1/2))^(9/2)-24/7*(1+(1+(1+x)^(1/2))^(1/2))^(7/2)+16
/5*(1+(1+(1+x)^(1/2))^(1/2))^(5/2)
```

maxima [C] time = 0.31, size = 46, normalized size = 0.35

$$\frac{8}{9}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{9}{2}}-\frac{24}{7}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{7}{2}}+\frac{16}{5}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*(sqrt(sqrt(x + 1) + 1) + 1)^(9/2) - 24/7*(sqrt(sqrt(x + 1) + 1) + 1)^(7
/2) + 16/5*(sqrt(sqrt(x + 1) + 1) + 1)^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sqrt{\sqrt{x+1}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((x + 1)^(1/2) + 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int((((x + 1)^(1/2) + 1)^(1/2) + 1)^(1/2), x)
```

sympy [A] time = 2.27, size = 230, normalized size = 1.73

$$-\frac{\sqrt{2}\sqrt{x+1}\sqrt{\sqrt{x+1}+1}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{63\pi}-\frac{\sqrt{2}\sqrt{x+1}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi}-\frac{\sqrt{2}\left(x+1\right)\sqrt{\sqrt{\sqrt{x+1}+1}+1}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{9\pi}+\frac{8\sqrt{2}\sqrt{\sqrt{x+1}+1}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi}+\frac{8\sqrt{2}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] -sqrt(2)*sqrt(x + 1)*sqrt(sqrt(x + 1) + 1)*sqrt(sqrt(sqrt(x + 1) + 1) + 1)*
gamma(-1/4)*gamma(1/4)/(63*pi) - sqrt(2)*sqrt(x + 1)*sqrt(sqrt(sqrt(x + 1)
+ 1) + 1)*gamma(-1/4)*gamma(1/4)/(315*pi) - sqrt(2)*(x + 1)*sqrt(sqrt(sqrt(
x + 1) + 1) + 1)*gamma(-1/4)*gamma(1/4)/(9*pi) + 8*sqrt(2)*sqrt(sqrt(x + 1)
 + 1)*sqrt(sqrt(sqrt(x + 1) + 1) + 1)*gamma(-1/4)*gamma(1/4)/(315*pi) + 8*s
qrt(2)*sqrt(sqrt(sqrt(x + 1) + 1) + 1)*gamma(-1/4)*gamma(1/4)/(315*pi)
```

3.1611 $\int \frac{(-2+x^3)(1+x^3)^{2/3}}{x^6(-1+2x^3)}\,dx$

Optimal. Leaf size=134

$$-3^{2/3}\log\left(3^{2/3}\sqrt[3]{x^3+1}-3x\right)+3\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{x^3+1}+\sqrt[3]{3}x}\right)+\frac{\left(x^3+1\right)^{2/3}\left(-19x^3-4\right)}{10x^5}+\frac{1}{2}3^{2/3}\log\left(3^{2/3}\sqrt[3]{x^3+1}x\right.$$

Rubi [A] time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {580, 583, 12, 377, 200, 31, 634, 617, 204, 628}

$$-3^{2/3}\log\left(1-\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+1}}\right)+3\sqrt[6]{3}\tan^{-1}\left(\frac{2x}{\sqrt[6]{3}\sqrt[3]{x^3+1}}+\frac{1}{\sqrt{3}}\right)-\frac{2\left(x^3+1\right)^{2/3}}{5x^5}-\frac{19\left(x^3+1\right)^{2/3}}{10x^2}+\frac{1}{2}3^{2/3}\log\left(\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+1}}+\frac{3^{2/3}x^2}{\left(x^3+1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-2 + x^3)*(1 + x^3)^(2/3))/(x^6*(-1 + 2*x^3)),x]
```

```
[Out] (-2*(1 + x^3)^(2/3))/(5*x^5) - (19*(1 + x^3)^(2/3))/(10*x^2) + 3*3^(1/6)*Ar
cTan[1/Sqrt[3] + (2*x)/(3^(1/6)*(1 + x^3)^(1/3))] - 3^(2/3)*Log[1 - (3^(1/3
)*x)/(1 + x^3)^(1/3)] + (3^(2/3)*Log[1 + (3^(2/3)*x^2)/(1 + x^3)^(2/3) + (3
^(1/3)*x)/(1 + x^3)^(1/3)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
```

```
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-2+x^3\right)\left(1+x^3\right)^{2/3}}{x^6\left(-1+2x^3\right)}\,dx &= -\frac{2\left(1+x^3\right)^{2/3}}{5x^5}-\frac{1}{5}\int \frac{19+7x^3}{x^3\sqrt[3]{1+x^3}\left(-1+2x^3\right)}\,dx \\
&= -\frac{2\left(1+x^3\right)^{2/3}}{5x^5}-\frac{19\left(1+x^3\right)^{2/3}}{10x^2}-\frac{1}{10}\int \frac{90}{\sqrt[3]{1+x^3}\left(-1+2x^3\right)}\,dx \\
&= -\frac{2\left(1+x^3\right)^{2/3}}{5x^5}-\frac{19\left(1+x^3\right)^{2/3}}{10x^2}-9\int \frac{1}{\sqrt[3]{1+x^3}\left(-1+2x^3\right)}\,dx \\
&= -\frac{2\left(1+x^3\right)^{2/3}}{5x^5}-\frac{19\left(1+x^3\right)^{2/3}}{10x^2}-9\,\text{Subst}\left(\int \frac{1}{-1+3x^3}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right) \\
&= -\frac{2\left(1+x^3\right)^{2/3}}{5x^5}-\frac{19\left(1+x^3\right)^{2/3}}{10x^2}-3\,\text{Subst}\left(\int \frac{1}{-1+\sqrt[3]{3}\,x}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right)-3\,\text{Subst} \\
&= -\frac{2\left(1+x^3\right)^{2/3}}{5x^5}-\frac{19\left(1+x^3\right)^{2/3}}{10x^2}-3^{2/3}\log\left(1-\frac{\sqrt[3]{3}\,x}{\sqrt[3]{1+x^3}}\right)+\frac{9}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{3}\,x+}\right. \\
&= -\frac{2\left(1+x^3\right)^{2/3}}{5x^5}-\frac{19\left(1+x^3\right)^{2/3}}{10x^2}-3^{2/3}\log\left(1-\frac{\sqrt[3]{3}\,x}{\sqrt[3]{1+x^3}}\right)+\frac{1}{2}3^{2/3}\log\left(1+\frac{3^{2/3}x^2}{\left(1+x^3\right)^{2}}\right. \\
&= -\frac{2\left(1+x^3\right)^{2/3}}{5x^5}-\frac{19\left(1+x^3\right)^{2/3}}{10x^2}+3\sqrt[6]{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}\,x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)-3^{2/3}\log\left(1-\frac{\sqrt[3]{3}\,x}{\sqrt[3]{1+x^3}}\right.
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 124, normalized size = 0.93

$$3\sqrt[6]{3}\,\tan^{-1}\left(\frac{2x}{\sqrt[6]{3}\,\sqrt[3]{x^3+1}}+\frac{1}{\sqrt{3}}\right)-\frac{\left(x^3+1\right)^{2/3}\left(19x^3+4\right)}{10x^5}+\frac{1}{2}3^{2/3}\left(\log\left(\frac{\sqrt[3]{3}\,x}{\sqrt[3]{x^3+1}}+\frac{3^{2/3}x^2}{\left(x^3+1\right)^{2/3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{3}\,x}{\sqrt[3]{x^3+1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-2 + x^3)*(1 + x^3)^(2/3))/(x^6*(-1 + 2*x^3)),x]
```

```
[Out] -1/10*((1 + x^3)^(2/3)*(4 + 19*x^3))/x^5 + 3*3^(1/6)*ArcTan[1/Sqrt[3] + (2*
x)/(3^(1/6)*(1 + x^3)^(1/3))] + (3^(2/3)*(-2*Log[1 - (3^(1/3)*x)/(1 + x^3)^
(1/3)] + Log[1 + (3^(2/3)*x^2)/(1 + x^3)^(2/3) + (3^(1/3)*x)/(1 + x^3)^(1/3
)]))/2
```

IntegrateAlgebraic [A] time = 0.33, size = 134, normalized size = 1.00

$$-3^{2/3}\log\left(3^{2/3}\sqrt[3]{x^3+1}-3x\right)+3\sqrt[6]{3}\,\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{x^3+1}+\sqrt[3]{3}\,x}\right)+\frac{\left(x^3+1\right)^{2/3}\left(-19x^3-4\right)}{10x^5}+\frac{1}{2}3^{2/3}\log\left(3^{2/3}\sqrt[3]{x^3+1}\,x+\sqrt[3]{3}\left(x^3+1\right)^{2/3}+3x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^3)*(1 + x^3)^(2/3))/(x^6*(-1 + 2*x^3)),x]
```

```
[Out] ((-4 - 19*x^3)*(1 + x^3)^(2/3))/(10*x^5) + 3*3^(1/6)*ArcTan[(3^(5/6)*x)/(3^
(1/3)*x + 2*(1 + x^3)^(1/3))] - 3^(2/3)*Log[-3*x + 3^(2/3)*(1 + x^3)^(1/3)]
 + (3^(2/3)*Log[3*x^2 + 3^(2/3)*x*(1 + x^3)^(1/3) + 3^(1/3)*(1 + x^3)^(2/3)
])/2
```

fricas [B] time = 2.27, size = 274, normalized size = 2.04

$$-\frac{10\sqrt{3}\,(-9)^{\frac{1}{3}}x^5\arctan\left(\frac{2\sqrt{3}\,(-9)^{\frac{2}{3}}\left(14x^7-5x^4-x\right)\left(x^3+1\right)^{\frac{2}{3}}+6\sqrt{3}\,(-9)^{\frac{1}{3}}\left(31x^8+23x^5+x^2\right)\left(x^3+1\right)^{\frac{1}{3}}-\sqrt{3}\left(127x^9+201x^6+48x^3+1\right)}{3\left(251x^9+231x^6+6x^3-1\right)}\right)-10\,(-9)^{\frac{1}{3}}x^5\log\left(\frac{3\,(-9)^{\frac{2}{3}}\left(x^3+1\right)^{\frac{1}{3}}x^2-9\left(x^3+1\right)^{\frac{2}{3}}x+(-9)^{\frac{1}{3}}\left(2x^3-1\right)}{2x^3-1}\right)+5\,(-9)^{\frac{1}{3}}x^5\log\left(-\frac{9\,(-9)^{\frac{1}{3}}\left(7x^4+x\right)\left(x^3+1\right)^{\frac{2}{3}}-(-9)^{\frac{2}{3}}\left(31x^6+23x^3+1\right)-27\left(5x^5+2x^2\right)\left(x^3+1\right)^{\frac{1}{3}}}{4x^6-4x^3+1}\right)+3\left(19x^3+4\right)\left(x^3+1\right)^{\frac{2}{3}}}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3+1)^(2/3)/x^6/(2*x^3-1),x, algorithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*(-9)^(1/3)*x^5*arctan(1/3*(2*sqrt(3)*(-9)^(2/3)*(14*x^7 -
 5*x^4 - x)*(x^3 + 1)^(2/3) + 6*sqrt(3)*(-9)^(1/3)*(31*x^8 + 23*x^5 + x^2)*
(x^3 + 1)^(1/3) - sqrt(3)*(127*x^9 + 201*x^6 + 48*x^3 + 1))/(251*x^9 + 231*
x^6 + 6*x^3 - 1)) - 10*(-9)^(1/3)*x^5*log((3*(-9)^(2/3)*(x^3 + 1)^(1/3)*x^2
 - 9*(x^3 + 1)^(2/3)*x + (-9)^(1/3)*(2*x^3 - 1))/(2*x^3 - 1)) + 5*(-9)^(1/3
)*x^5*log(-(9*(-9)^(1/3)*(7*x^4 + x)*(x^3 + 1)^(2/3) - (-9)^(2/3)*(31*x^6 +
 23*x^3 + 1) - 27*(5*x^5 + 2*x^2)*(x^3 + 1)^(1/3))/(4*x^6 - 4*x^3 + 1)) + 3
*(19*x^3 + 4)*(x^3 + 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+1\right)^{\frac{2}{3}}\left(x^3-2\right)}{\left(2\,x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3+1)^(2/3)/x^6/(2*x^3-1),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)^(2/3)*(x^3 - 2)/((2*x^3 - 1)*x^6), x)
```

maple [C] time = 2.25, size = 612, normalized size = 4.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2)*(x^3+1)^(2/3)/x^6/(2*x^3-1),x)
```

```
[Out] -1/10*(19*x^6+23*x^3+4)/x^5/(x^3+1)^(1/3)+RootOf(_Z^3+9)*ln((108*RootOf(Roo
tOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*_Z^2)^2*RootOf(_Z^3+9)^2*x^3+9*RootOf(
RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*_Z^2)*RootOf(_Z^3+9)^3*x^3+15*(x^3+
1)^(2/3)*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*_Z^2)*RootOf(_Z^3+9
)^2*x-18*(x^3+1)^(1/3)*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*_Z^2)
*RootOf(_Z^3+9)*x^2+5*RootOf(_Z^3+9)^2*(x^3+1)^(1/3)*x^2-144*RootOf(RootOf(
_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*_Z^2)*x^3-12*RootOf(_Z^3+9)*x^3-21*x*(x^3+
1)^(2/3)-36*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*_Z^2)-3*RootOf(_
Z^3+9))/(2*x^3-1))+9*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*_Z^2)*l
n(-(81*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*_Z^2)^2*RootOf(_Z^3+9
)^2*x^3+12*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*_Z^2)*RootOf(_Z^3
+9)^3*x^3+15*(x^3+1)^(2/3)*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*_
Z^2)*RootOf(_Z^3+9)^2*x+63*(x^3+1)^(1/3)*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootO
f(_Z^3+9)+81*_Z^2)*RootOf(_Z^3+9)*x^2+5*RootOf(_Z^3+9)^2*(x^3+1)^(1/3)*x^2+
27*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*_Z^2)*x^3+4*RootOf(_Z^3+9
)*x^3+6*x*(x^3+1)^(2/3)+27*RootOf(RootOf(_Z^3+9)^2+9*_Z*RootOf(_Z^3+9)+81*_
Z^2)+4*RootOf(_Z^3+9))/(2*x^3-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+1\right)^{\frac{2}{3}}\left(x^3-2\right)}{\left(2\,x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3+1)^(2/3)/x^6/(2*x^3-1),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 1)^(2/3)*(x^3 - 2)/((2*x^3 - 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3+1\right)^{2/3}\left(x^3-2\right)}{x^6\left(2x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)^(2/3)*(x^3 - 2))/(x^6*(2*x^3 - 1)),x)
[Out] int(((x^3 + 1)^(2/3)*(x^3 - 2))/(x^6*(2*x^3 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x+1\right)\left(x^2-x+1\right)\right)^{\frac{2}{3}}\left(x^3-2\right)}{x^6\left(2x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2)*(x**3+1)**(2/3)/x**6/(2*x**3-1),x)
[Out] Integral(((x + 1)*(x**2 - x + 1))**(2/3)*(x**3 - 2)/(x**6*(2*x**3 - 1)), x)
```

3.1612 $\int \frac{(-1+x+x^4)\sqrt[4]{-x^3+x^4}}{1+x}\,dx$

Optimal. Leaf size=134

$$-\frac{9869\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)}{4096}+2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^3}}\right)+\frac{9869\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)}{4096}-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^3}}\right)+\frac{\sqrt[4]{x^4-x^3}}{}$$

Rubi [C] time = 0.78, antiderivative size = 223, normalized size of antiderivative = 1.66, number of steps used = 35, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2056, 6733, 6725, 279, 331, 298, 203, 206, 321, 511, 510}

$$\frac{4\sqrt[4]{x^4-x^3}F_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};x,-x\right)}{3\sqrt[4]{1-x}}+\frac{1}{5}\sqrt[4]{x^4-x^3}x^4-\frac{21}{80}\sqrt[4]{x^4-x^3}x^3-\frac{53\sqrt[4]{x^4-x^3}x}{1536}-\frac{6515\sqrt[4]{x^4-x^3}}{6144}-\frac{1677\sqrt[4]{x^4-x^3}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{4096\sqrt[4]{x-1}x^{3/4}}+\frac{1677\sqrt[4]{x^4-x^3}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{4096\sqrt[4]{x-1}x^{3/4}}+\frac{65}{192}\sqrt[4]{x^4-x^3}x^2$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x + x^4)*(-x^3 + x^4)^(1/4))/(1 + x),x]
```

```
[Out] (-6515*(-x^3 + x^4)^(1/4))/6144 - (53*x*(-x^3 + x^4)^(1/4))/1536 + (65*x^2*
(-x^3 + x^4)^(1/4))/192 - (21*x^3*(-x^3 + x^4)^(1/4))/80 + (x^4*(-x^3 + x^4
)^(1/4))/5 + (4*(-x^3 + x^4)^(1/4)*AppellF1[3/4, -1/4, 1, 7/4, x, -x])/(3*(
1 - x)^(1/4)) - (1677*(-x^3 + x^4)^(1/4)*ArcTan[x^(1/4)/(-1 + x)^(1/4)])/(4
096*(-1 + x)^(1/4)*x^(3/4)) + (1677*(-x^3 + x^4)^(1/4)*ArcTanh[x^(1/4)/(-1
+ x)^(1/4)])/(4096*(-1 + x)^(1/4)*x^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{(-1+x+x^4)\sqrt[4]{-x^3+x^4}}{1+x}\,dx = \frac{\sqrt[4]{-x^3+x^4}\int \frac{\sqrt[4]{-1+x}\,x^{3/4}(-1+x+x^4)}{1+x}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{\left(4\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\int \frac{x^6\sqrt[4]{-1+x^4}(-1+x^4+x^{16})}{1+x^4}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{\left(4\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\int \left(-x^2\sqrt[4]{-1+x^4} + x^{10}\sqrt[4]{-1+x^4} - x^{14}\sqrt[4]{-1+x^4} + x\right.\right.}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= -\frac{\left(4\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\int x^2\sqrt[4]{-1+x^4}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-1+x}\,x^{3/4}} + \frac{\left(4\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\right.}{\sqrt[4]{-1}}$$

$$= -\sqrt[4]{-x^3+x^4} + \frac{1}{3}x^2\sqrt[4]{-x^3+x^4} - \frac{1}{4}x^3\sqrt[4]{-x^3+x^4} + \frac{1}{5}x^4\sqrt[4]{-x^3+x^4} + \frac{\left(4\sqrt[4]{-\right.}{}$$

$$= -\sqrt[4]{-x^3+x^4} - \frac{1}{24}x\sqrt[4]{-x^3+x^4} + \frac{17}{48}x^2\sqrt[4]{-x^3+x^4} - \frac{21}{80}x^3\sqrt[4]{-x^3+x^4} + \frac{1}{5}x^4$$

$$= -\frac{103}{96}\sqrt[4]{-x^3+x^4} - \frac{5}{384}x\sqrt[4]{-x^3+x^4} + \frac{65}{192}x^2\sqrt[4]{-x^3+x^4} - \frac{21}{80}x^3\sqrt[4]{-x^3+x^4}$$

$$= -\frac{1571\sqrt[4]{-x^3+x^4}}{1536} - \frac{53x\sqrt[4]{-x^3+x^4}}{1536} + \frac{65}{192}x^2\sqrt[4]{-x^3+x^4} - \frac{21}{80}x^3\sqrt[4]{-x^3+x^4}$$

$$= -\frac{6515\sqrt[4]{-x^3+x^4}}{6144} - \frac{53x\sqrt[4]{-x^3+x^4}}{1536} + \frac{65}{192}x^2\sqrt[4]{-x^3+x^4} - \frac{21}{80}x^3\sqrt[4]{-x^3+x^4}$$

$$= -\frac{6515\sqrt[4]{-x^3+x^4}}{6144} - \frac{53x\sqrt[4]{-x^3+x^4}}{1536} + \frac{65}{192}x^2\sqrt[4]{-x^3+x^4} - \frac{21}{80}x^3\sqrt[4]{-x^3+x^4}$$

$$= -\frac{6515\sqrt[4]{-x^3+x^4}}{6144} - \frac{53x\sqrt[4]{-x^3+x^4}}{1536} + \frac{65}{192}x^2\sqrt[4]{-x^3+x^4} - \frac{21}{80}x^3\sqrt[4]{-x^3+x^4}$$

$$= -\frac{6515\sqrt[4]{-x^3+x^4}}{6144} - \frac{53x\sqrt[4]{-x^3+x^4}}{1536} + \frac{65}{192}x^2\sqrt[4]{-x^3+x^4} - \frac{21}{80}x^3\sqrt[4]{-x^3+x^4}$$

Mathematica [C] time = 0.20, size = 118, normalized size = 0.88

$$\frac{x^3\left(148035x(1-x^2)^{3/4}F_1\left(\frac{7}{4};\frac{3}{4},1;\frac{11}{4};x,-x\right) - 228025(1-x)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{2x}{x+1}\right) + 7(x+1)^{3/4}\left(6144x^5-14208x^4+18464x^3-11460x^2-31515x+32575\right)\right)}{215040\left((x-1)x^3\right)^{3/4}(x+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x + x^4)*(-x^3 + x^4)^(1/4))/(1 + x),x]
```

```
[Out] (x^3*(7*(1 + x)^(3/4)*(32575 - 31515*x - 11460*x^2 + 18464*x^3 - 14208*x^4
+ 6144*x^5) + 148035*x*(1 - x^2)^(3/4)*AppellF1[7/4, 3/4, 1, 11/4, x, -x] -
 228025*(1 - x)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (2*x)/(1 + x)]))/(21
5040*((-1 + x)*x^3)^(3/4)*(1 + x)^(3/4))
```

IntegrateAlgebraic [A] time = 0.89, size = 134, normalized size = 1.00

$$-\frac{9869\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)}{4096}+2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^3}}\right)+\frac{9869\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)}{4096}-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4-x^3}}\right)+\frac{\sqrt[4]{x^4-x^3}\left(6144x^4-8064x^3+10400x^2-1060x-32575\right)}{30720}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x + x^4)*(-x^3 + x^4)^(1/4))/(1 + x),x]
```

```
[Out] ((-x^3 + x^4)^(1/4)*(-32575 - 1060*x + 10400*x^2 - 8064*x^3 + 6144*x^4))/30
720 - (9869*ArcTan[x/(-x^3 + x^4)^(1/4)])/4096 + 2*2^(1/4)*ArcTan[(2^(1/4)*
x)/(-x^3 + x^4)^(1/4)] + (9869*ArcTanh[x/(-x^3 + x^4)^(1/4)])/4096 - 2*2^(1
/4)*ArcTanh[(2^(1/4)*x)/(-x^3 + x^4)^(1/4)]
```

fricas [A] time = 0.41, size = 209, normalized size = 1.56

$$\frac{1}{30720}\left(6144x^4-8064x^3+10400x^2-1060x-32575\right)\left(x^4-x^3\right)^{\frac14}+4\cdot 2^{\frac14}\arctan\left(\frac{2^{\frac34}x\sqrt{\frac{\sqrt{2}x^2+\sqrt{x^4-x^3}}{x^2}}-2^{\frac34}\left(x^4-x^3\right)^{\frac14}}{2x}\right)-2^{\frac14}\log\left(\frac{2^{\frac14}x+\left(x^4-x^3\right)^{\frac14}}{x}\right)+2^{\frac14}\log\left(-\frac{2^{\frac14}x-\left(x^4-x^3\right)^{\frac14}}{x}\right)+\frac{9869}{4096}\arctan\left(\frac{\left(x^4-x^3\right)^{\frac14}}{x}\right)+\frac{9869}{8192}\log\left(\frac{x+\left(x^4-x^3\right)^{\frac14}}{x}\right)-\frac{9869}{8192}\log\left(-\frac{x-\left(x^4-x^3\right)^{\frac14}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x-1)*(x^4-x^3)^(1/4)/(1+x),x, algorithm="fricas")
```

```
[Out] 1/30720*(6144*x^4 - 8064*x^3 + 10400*x^2 - 1060*x - 32575)*(x^4 - x^3)^(1/4
) + 4*2^(1/4)*arctan(1/2*(2^(3/4)*x*sqrt((sqrt(2)*x^2 + sqrt(x^4 - x^3))/x^
2) - 2^(3/4)*(x^4 - x^3)^(1/4))/x) - 2^(1/4)*log((2^(1/4)*x + (x^4 - x^3)^(
1/4))/x) + 2^(1/4)*log(-(2^(1/4)*x - (x^4 - x^3)^(1/4))/x) + 9869/4096*arct
an((x^4 - x^3)^(1/4)/x) + 9869/8192*log((x + (x^4 - x^3)^(1/4))/x) - 9869/8
192*log(-(x - (x^4 - x^3)^(1/4))/x)
```

giac [A] time = 0.22, size = 184, normalized size = 1.37

$$\frac{1}{30720}\left(32575\left(\frac{1}{x}-1\right)^4\left(-\frac{1}{x}+1\right)^{\frac14}+131360\left(\frac{1}{x}-1\right)^3\left(-\frac{1}{x}+1\right)^{\frac14}+188230\left(\frac{1}{x}-1\right)^2\left(-\frac{1}{x}+1\right)^{\frac14}-120744\left(-\frac{1}{x}+1\right)^{\frac54}+25155\left(-\frac{1}{x}+1\right)^{\frac14}\right)x^5+2\cdot 2^{\frac14}\arctan\left(\frac{1}{2}\cdot 2^{\frac34}\left(-\frac{1}{x}+1\right)^{\frac14}\right)+2^{\frac14}\log\left(2^{\frac14}+\left(-\frac{1}{x}+1\right)^{\frac14}\right)-2^{\frac14}\log\left(\left|-2^{\frac14}+\left(-\frac{1}{x}+1\right)^{\frac14}\right|\right)-\frac{9869}{4096}\arctan\left(\left(-\frac{1}{x}+1\right)^{\frac14}\right)-\frac{9869}{8192}\log\left(\left(-\frac{1}{x}+1\right)^{\frac14}+1\right)+\frac{9869}{8192}\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac14}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x-1)*(x^4-x^3)^(1/4)/(1+x),x, algorithm="giac")
```

```
[Out] 1/30720*(32575*(1/x - 1)^4*(-1/x + 1)^(1/4) + 131360*(1/x - 1)^3*(-1/x + 1)
^(1/4) + 188230*(1/x - 1)^2*(-1/x + 1)^(1/4) - 120744*(-1/x + 1)^(5/4) + 25
155*(-1/x + 1)^(1/4))*x^5 + 2*2^(1/4)*arctan(1/2*2^(3/4)*(-1/x + 1)^(1/4))
+ 2^(1/4)*log(2^(1/4) + (-1/x + 1)^(1/4)) - 2^(1/4)*log(abs(-2^(1/4) + (-1/
x + 1)^(1/4))) - 9869/4096*arctan((-1/x + 1)^(1/4)) - 9869/8192*log((-1/x +
 1)^(1/4) + 1) + 9869/8192*log(abs((-1/x + 1)^(1/4) - 1))
```

maple [C] time = 1.79, size = 931, normalized size = 6.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x-1)*(x^4-x^3)^(1/4)/(1+x),x)
```

```
[Out] 1/30720*(6144*x^4-8064*x^3+10400*x^2-1060*x-32575)*(x^3*(-1+x))^(1/4)+(-986
9/8192*ln((2*(x^4-3*x^3+3*x^2-x)^(3/4)-2*(x^4-3*x^3+3*x^2-x)^(1/2)*x+2*(x^4
-3*x^3+3*x^2-x)^(1/4)*x^2-2*x^3+2*(x^4-3*x^3+3*x^2-x)^(1/2)-4*(x^4-3*x^3+3*
x^2-x)^(1/4)*x+5*x^2+2*(x^4-3*x^3+3*x^2-x)^(1/4)-4*x+1)/(-1+x)^2)-9869/8192
*RootOf(_Z^2+1)*ln((2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+1)*x-2*RootOf(_
Z^2+1)*x^3+2*(x^4-3*x^3+3*x^2-x)^(3/4)-2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_
Z^2+1)-2*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+5*RootOf(_Z^2+1)*x^2+4*(x^4-3*x^3+3*
x^2-x)^(1/4)*x-4*RootOf(_Z^2+1)*x-2*(x^4-3*x^3+3*x^2-x)^(1/4)+RootOf(_Z^2+1
))/(-1+x)^2)+RootOf(_Z^4-2)*ln((-2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4-2)
^3*x+2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4-2)^3+2*(x^4-3*x^3+3*x^2-x)^(1/
4)*RootOf(_Z^4-2)^2*x^2-4*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4-2)^2*x-3*Ro
```

```
otOf(_Z^4-2)*x^3+4*(x^4-3*x^3+3*x^2-x)^(3/4)+2*(x^4-3*x^3+3*x^2-x)^(1/4)*Ro
otOf(_Z^4-2)^2+7*RootOf(_Z^4-2)*x^2-5*RootOf(_Z^4-2)*x+RootOf(_Z^4-2))/(-1+
x)^2/(1+x))-RootOf(_Z^2+RootOf(_Z^4-2)^2)*ln(-(2*(x^4-3*x^3+3*x^2-x)^(1/2)*
RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2*x-2*(x^4-3*x^3+3*x^2-x)^(1/2
)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2+2*(x^4-3*x^3+3*x^2-x)^(1/4
)*RootOf(_Z^4-2)^2*x^2-3*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^3-4*(x^4-3*x^3+3*x
^2-x)^(1/4)*RootOf(_Z^4-2)^2*x+7*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^2-4*(x^4-3
*x^3+3*x^2-x)^(3/4)+2*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4-2)^2-5*RootOf(_
Z^2+RootOf(_Z^4-2)^2)*x+RootOf(_Z^2+RootOf(_Z^4-2)^2))/(-1+x)^2/(1+x)))*(x^
3*(-1+x))^(1/4)/(-1+x)/x*(x*(-1+x)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^3\right)^{\frac{1}{4}}\left(x^4+x-1\right)}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x-1)*(x^4-x^3)^(1/4)/(1+x),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^3)^(1/4)*(x^4 + x - 1)/(x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-x^3\right)^{1/4}\,\left(x^4+x-1\right)}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - x^3)^(1/4)*(x + x^4 - 1))/(x + 1),x)
```

```
[Out] int(((x^4 - x^3)^(1/4)*(x + x^4 - 1))/(x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\,(x-1)}\,\left(x^4+x-1\right)}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x-1)*(x**4-x**3)**(1/4)/(1+x),x)
```

```
[Out] Integral((x**3*(x - 1))**(1/4)*(x**4 + x - 1)/(x + 1), x)
```

3.1613 $\int \frac{(-3+x^4)(1+x^4)^{2/3}}{x^3(2+x^3+2x^4)}\,dx$

Optimal. Leaf size=134

$$\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^4+1}+x\right)}{2\,2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^4+1}-x}\right)}{2\,2^{2/3}}+\frac{3\left(x^4+1\right)^{2/3}}{4x^2}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{x^4+1}\,x+2^{2/3}\left(x^4+1\right)^{2/3}+x^2\right)}{4\,2^{2/3}}$$

Rubi [F] time = 1.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}}{x^3\left(2+x^3+2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + x^4)*(1 + x^4)^(2/3))/(x^3*(2 + x^3 + 2*x^4)),x]
```

```
[Out] (3*(1 + x^4)^(2/3))/(4*x^2) + (3*x^2)/(1 - Sqrt[3] - (1 + x^4)^(1/3)) - (3*
3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 + x^4)^(1/3))*Sqrt[(1 + (1 + x^4)^(1/3) +
 (1 + x^4)^(2/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2]*EllipticE[ArcSin[(1 +
Sqrt[3] - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))], -7 + 4*Sqrt[3]
])/(2*x^2*Sqrt[-((1 - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2)])
 + (Sqrt[2]*3^(3/4)*(1 - (1 + x^4)^(1/3))*Sqrt[(1 + (1 + x^4)^(1/3) + (1 +
x^4)^(2/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3
] - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))], -7 + 4*Sqrt[3]])/(x^
2*Sqrt[-((1 - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2)]) + (3*De
fer[Int][(1 + x^4)^(2/3)/(2 + x^3 + 2*x^4), x])/2 + 4*Defer[Int][(x*(1 + x^
4)^(2/3))/(2 + x^3 + 2*x^4), x]
```

Rubi steps

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}}{x^3\left(2+x^3+2x^4\right)}\,dx=\int \left(-\frac{3\left(1+x^4\right)^{2/3}}{2x^3}+\frac{(3+8x)\left(1+x^4\right)^{2/3}}{2\left(2+x^3+2x^4\right)}\right)dx$$

$$=\frac{1}{2}\int \frac{(3+8x)\left(1+x^4\right)^{2/3}}{2+x^3+2x^4}\,dx-\frac{3}{2}\int \frac{\left(1+x^4\right)^{2/3}}{x^3}\,dx$$

$$=\frac{1}{2}\int \left(\frac{3\left(1+x^4\right)^{2/3}}{2+x^3+2x^4}+\frac{8x\left(1+x^4\right)^{2/3}}{2+x^3+2x^4}\right)dx-\frac{3}{4}\text{Subst}\left(\int \frac{\left(1+x^2\right)^{2/3}}{x^2}\,dx,x,x^2\right)$$

$$=\frac{3\left(1+x^4\right)^{2/3}}{4x^2}+\frac{3}{2}\int \frac{\left(1+x^4\right)^{2/3}}{2+x^3+2x^4}\,dx+4\int \frac{x\left(1+x^4\right)^{2/3}}{2+x^3+2x^4}\,dx-\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^2}}\right.$$

$$=\frac{3\left(1+x^4\right)^{2/3}}{4x^2}+\frac{3}{2}\int \frac{\left(1+x^4\right)^{2/3}}{2+x^3+2x^4}\,dx+4\int \frac{x\left(1+x^4\right)^{2/3}}{2+x^3+2x^4}\,dx-\frac{\left(3\sqrt{x^4}\right)\text{Subst}\left(\int\right.}{2}$$

$$=\frac{3\left(1+x^4\right)^{2/3}}{4x^2}+\frac{3}{2}\int \frac{\left(1+x^4\right)^{2/3}}{2+x^3+2x^4}\,dx+4\int \frac{x\left(1+x^4\right)^{2/3}}{2+x^3+2x^4}\,dx+\frac{\left(3\sqrt{x^4}\right)\text{Subst}\left(\int\right.}{2}$$

$$=\frac{3\left(1+x^4\right)^{2/3}}{4x^2}+\frac{3x^2}{1-\sqrt{3}-\sqrt[3]{1+x^4}}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1+x^4}\right)\sqrt{\frac{1+\sqrt[3]{1+x^4}+\ldots}{\left(1-\sqrt{3}-\sqrt[3]{1+x}\ldots\right.}}}{2x^2\sqrt{-\frac{1-\ldots}{\left(1-\sqrt{3}\ldots\right.}}}$$

Mathematica [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}}{x^3\left(2+x^3+2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + x^4)*(1 + x^4)^(2/3))/(x^3*(2 + x^3 + 2*x^4)),x]
```

```
[Out] Integrate[((-3 + x^4)*(1 + x^4)^(2/3))/(x^3*(2 + x^3 + 2*x^4)), x]
```

IntegrateAlgebraic [A] time = 1.22, size = 134, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^4+1}+x\right)}{2\,2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^4+1}-x}\right)}{2\,2^{2/3}}+\frac{3\left(x^4+1\right)^{2/3}}{4x^2}-\frac{\log\left(-\sqrt[3]{2}\sqrt[3]{x^4+1}\,x+2^{2/3}\left(x^4+1\right)^{2/3}+x^2\right)}{4\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 + x^4)^(2/3))/(x^3*(2 + x^3 + 2*x^4)),x]
```

```
[Out] (3*(1 + x^4)^(2/3))/(4*x^2) + (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*2^(1/3)*(
1 + x^4)^(1/3))])/(2*2^(2/3)) + Log[x + 2^(1/3)*(1 + x^4)^(1/3)]/(2*2^(2/3)
) - Log[x^2 - 2^(1/3)*x*(1 + x^4)^(1/3) + 2^(2/3)*(1 + x^4)^(2/3)]/(4*2^(2/
3))
```

fricas [B] time = 96.45, size = 391, normalized size = 2.92

$$\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}\,x^2\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(12\cdot 4^{\frac{2}{3}}\left(2x^9-x^8-x^7+4x^5-x^4+2x\right)\left(x^4+1\right)^{\frac{2}{3}}-4^{\frac{1}{3}}\left(8x^{12}-60x^{11}+24x^{10}+x^9+24x^8-120x^7+24x^6+24x^4-60x^3+8\right)-12\left(4x^{10}-14x^9+x^8+8x^6-14x^5+4x^2\right)\left(x^4+1\right)^{\frac{1}{3}}\right)}{6\left(8x^{12}+12x^{11}-48x^{10}+x^9+24x^8+24x^7-48x^6+24x^4+12x^3+8\right)}\right)+2\cdot 4^{\frac{2}{3}}x^2\log\left(-\frac{6\cdot 4^{\frac{1}{3}}\left(x^4+1\right)^{\frac{1}{3}}x^2+4^{\frac{2}{3}}\left(2x^4+x^3+2\right)+12\left(x^4+1\right)^{\frac{2}{3}}x}{2x^4+x^3+2}\right)-4^{\frac{2}{3}}x^2\log\left(-\frac{6\cdot 4^{\frac{2}{3}}\left(x^5-x^4+x\right)\left(x^4+1\right)^{\frac{2}{3}}-4^{\frac{1}{3}}\left(4x^8-14x^7+x^6+8x^4-14x^3+4\right)-6\left(4x^6-x^5+4x^2\right)\left(x^4+1\right)^{\frac{1}{3}}}{4x^8+4x^7+x^6+8x^4+4x^3+4}\right)+36\left(x^4+1\right)^{\frac{2}{3}}}{48x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)/x^3/(2*x^4+x^3+2),x, algorithm="fricas")
```

```
[Out] 1/48*(4*4^(1/6)*sqrt(3)*x^2*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^(2/3)*(2*x^9 -
 x^8 - x^7 + 4*x^5 - x^4 + 2*x)*(x^4 + 1)^(2/3) - 4^(1/3)*(8*x^12 - 60*x^11
 + 24*x^10 + x^9 + 24*x^8 - 120*x^7 + 24*x^6 + 24*x^4 - 60*x^3 + 8) - 12*(4
*x^10 - 14*x^9 + x^8 + 8*x^6 - 14*x^5 + 4*x^2)*(x^4 + 1)^(1/3))/(8*x^12 + 1
2*x^11 - 48*x^10 + x^9 + 24*x^8 + 24*x^7 - 48*x^6 + 24*x^4 + 12*x^3 + 8)) +
 2*4^(2/3)*x^2*log(-(6*4^(1/3)*(x^4 + 1)^(1/3)*x^2 + 4^(2/3)*(2*x^4 + x^3 +
 2) + 12*(x^4 + 1)^(2/3)*x)/(2*x^4 + x^3 + 2)) - 4^(2/3)*x^2*log(-(6*4^(2/3
)*(x^5 - x^4 + x)*(x^4 + 1)^(2/3) - 4^(1/3)*(4*x^8 - 14*x^7 + x^6 + 8*x^4 -
 14*x^3 + 4) - 6*(4*x^6 - x^5 + 4*x^2)*(x^4 + 1)^(1/3))/(4*x^8 + 4*x^7 + x^
6 + 8*x^4 + 4*x^3 + 4)) + 36*(x^4 + 1)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)^{\frac{2}{3}}\left(x^4-3\right)}{\left(2x^4+x^3+2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)/x^3/(2*x^4+x^3+2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)^(2/3)*(x^4 - 3)/((2*x^4 + x^3 + 2)*x^3), x)
```

maple [C] time = 68.46, size = 644, normalized size = 4.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4+1)^(2/3)/x^3/(2*x^4+x^3+2),x)
```

```
[Out] 3/4*(x^4+1)^(2/3)/x^2+1/2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^
2)*ln(-(RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^
3*x^3+2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2
)^2*x^3-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2
)^2*(x^4+1)^(2/3)*x-RootOf(_Z^3-2)^2*(x^4+1)^(1/3)*x^2-4*RootOf(_Z^3-2)*(x^
4+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2+2*RootOf
(_Z^3-2)*x^4+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^4+2*(x
^4+1)^(2/3)*x+2*RootOf(_Z^3-2)+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2
)+4*_Z^2))/(2*x^4+x^3+2))+1/4*RootOf(_Z^3-2)*ln((RootOf(RootOf(_Z^3-2)^2+2*
_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3+2*RootOf(RootOf(_Z^3-2)^2+2*
_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-2*RootOf(RootOf(_Z^3-2)^2+
2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(x^4+1)^(2/3)*x-RootOf(_Z^3-2)
^2*(x^4+1)^(1/3)*x^2+2*RootOf(_Z^3-2)*(x^4+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2
+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2-2*RootOf(_Z^3-2)*x^4-4*RootOf(RootOf(_Z^3-
2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^4+RootOf(_Z^3-2)*x^3+2*RootOf(RootOf(_Z^
3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^3-4*(x^4+1)^(2/3)*x-2*RootOf(_Z^3-2)-4
*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2))/(2*x^4+x^3+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)^{\frac{2}{3}}\left(x^4-3\right)}{\left(2\,x^4+x^3+2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)/x^3/(2*x^4+x^3+2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)^(2/3)*(x^4 - 3)/((2*x^4 + x^3 + 2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(x^4+1\right)}^{2/3}\,\left(x^4-3\right)}{x^3\,\left(2\,x^4+x^3+2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(2/3)*(x^4 - 3))/(x^3*(x^3 + 2*x^4 + 2)),x)
```

```
[Out] int(((x^4 + 1)^(2/3)*(x^4 - 3))/(x^3*(x^3 + 2*x^4 + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4+1)**(2/3)/x**3/(2*x**4+x**3+2),x)
```

```
[Out] Timed out
```

3.1614 $\int \frac{(-4b+ax^4)(b+ax^4)^{3/4}}{x^8(4b+ax^4)}\,dx$

Optimal. Leaf size=134

$$\frac{3^{3/4}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt{2}\sqrt[4]{ax^4+b}}\right)}{8\sqrt{2}b}+\frac{3^{3/4}a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt{2}\sqrt[4]{ax^4+b}}\right)}{8\sqrt{2}b}+\frac{(6b-ax^4)(ax^4+b)^{3/4}}{42bx^7}$$

Rubi [A] time = 0.19, antiderivative size = 143, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {580, 583, 12, 377, 212, 206, 203}

$$\frac{3^{3/4}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt{2}\sqrt[4]{ax^4+b}}\right)}{8\sqrt{2}b}+\frac{3^{3/4}a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt{2}\sqrt[4]{ax^4+b}}\right)}{8\sqrt{2}b}+\frac{(ax^4+b)^{3/4}}{7x^7}-\frac{a(ax^4+b)^{3/4}}{42bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-4*b + a*x^4)*(b + a*x^4)^(3/4))/(x^8*(4*b + a*x^4)),x]
```

```
[Out] (b + a*x^4)^(3/4)/(7*x^7) - (a*(b + a*x^4)^(3/4))/(42*b*x^3) + (3^(3/4)*a^(
7/4)*ArcTan[(3^(1/4)*a^(1/4)*x)/(Sqrt[2]*(b + a*x^4)^(1/4))])/(8*Sqrt[2]*b)
 + (3^(3/4)*a^(7/4)*ArcTanh[(3^(1/4)*a^(1/4)*x)/(Sqrt[2]*(b + a*x^4)^(1/4))
])/(8*Sqrt[2]*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(-4b+ax^4\right)\left(b+ax^4\right)^{3/4}}{x^8\left(4b+ax^4\right)}\,dx = \frac{\left(b+ax^4\right)^{3/4}}{7x^7}+\frac{\int \frac{8ab^2+44a^2bx^4}{x^4\sqrt[4]{b+ax^4}\left(4b+ax^4\right)}\,dx}{28b}$$

$$= \frac{\left(b+ax^4\right)^{3/4}}{7x^7}-\frac{a\left(b+ax^4\right)^{3/4}}{42bx^3}-\frac{\int -\frac{504a^2b^3}{\sqrt[4]{b+ax^4}\left(4b+ax^4\right)}\,dx}{336b^3}$$

$$= \frac{\left(b+ax^4\right)^{3/4}}{7x^7}-\frac{a\left(b+ax^4\right)^{3/4}}{42bx^3}+\frac{1}{2}\left(3a^2\right)\int \frac{1}{\sqrt[4]{b+ax^4}\left(4b+ax^4\right)}\,dx$$

$$= \frac{\left(b+ax^4\right)^{3/4}}{7x^7}-\frac{a\left(b+ax^4\right)^{3/4}}{42bx^3}+\frac{1}{2}\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{4b-3abx^4}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)$$

$$= \frac{\left(b+ax^4\right)^{3/4}}{7x^7}-\frac{a\left(b+ax^4\right)^{3/4}}{42bx^3}+\frac{\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{2-\sqrt{3}\sqrt{a}x^2}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)}{8b}+\frac{\left(3a^2 \right.}{}$$

$$= \frac{\left(b+ax^4\right)^{3/4}}{7x^7}-\frac{a\left(b+ax^4\right)^{3/4}}{42bx^3}+\frac{3^{3/4}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt{2}\sqrt[4]{b+ax^4}}\right)}{8\sqrt{2}b}+\frac{3^{3/4}a^{7/4}\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{8\sqrt{2}b}$$

Mathematica [A] time = 0.26, size = 151, normalized size = 1.13

$$\frac{3^{3/4}a^{7/4}\left(-\log\left(2-\frac{\sqrt{2}\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)+\log\left(\frac{\sqrt{2}\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}+2\right)+2\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt{2}\sqrt[4]{a+bx^4}}\right)\right)}{16\sqrt{2}b}+\left(\frac{1}{7x^7}-\frac{a}{42bx^3}\right)\left(ax^4+b\right)^{3/4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-4*b + a*x^4)*(b + a*x^4)^(3/4))/(x^8*(4*b + a*x^4)),x]
```

```
[Out] (1/(7*x^7) - a/(42*b*x^3))*(b + a*x^4)^(3/4) + (3^(3/4)*a^(7/4)*(2*ArcTan[(
3^(1/4)*a^(1/4)*x)/(Sqrt[2]*(a + b*x^4)^(1/4))] - Log[2 - (Sqrt[2]*3^(1/4)*
a^(1/4)*x)/(a + b*x^4)^(1/4)] + Log[2 + (Sqrt[2]*3^(1/4)*a^(1/4)*x)/(a + b*
x^4)^(1/4)]))/(16*Sqrt[2]*b)
```

IntegrateAlgebraic [A] time = 0.66, size = 134, normalized size = 1.00

$$\frac{3^{3/4}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt{2}\sqrt[4]{ax^4+b}}\right)}{8\sqrt{2}b}+\frac{3^{3/4}a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt{2}\sqrt[4]{ax^4+b}}\right)}{8\sqrt{2}b}+\frac{\left(6b-ax^4\right)\left(ax^4+b\right)^{3/4}}{42bx^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4*b + a*x^4)*(b + a*x^4)^(3/4))/(x^8*(4*b + a*x^4)),x
]
```

```
[Out] ((6*b - a*x^4)*(b + a*x^4)^(3/4))/(42*b*x^7) + (3^(3/4)*a^(7/4)*ArcTan[(3^(
1/4)*a^(1/4)*x)/(Sqrt[2]*(b + a*x^4)^(1/4))])/(8*Sqrt[2]*b) + (3^(3/4)*a^(7
/4)*ArcTanh[(3^(1/4)*a^(1/4)*x)/(Sqrt[2]*(b + a*x^4)^(1/4))])/(8*Sqrt[2]*b)
```

fricas [B] time = 85.32, size = 494, normalized size = 3.69

$$-\frac{84 \left(\frac{27}{4}\right)^{\frac{1}{4}} \left(\frac{a^{7}}{b^{4}}\right)^{\frac{1}{4}} b x^{7} \arctan\left(-\frac{4 \left(27 \left(\frac{27}{4}\right)^{\frac{1}{4}} \left(a x^{4}+b\right)^{\frac{1}{4}} \left(\frac{a^{7}}{b^{4}}\right)^{\frac{1}{4}} a^{9} b x^{3}+12 \left(\frac{27}{4}\right)^{\frac{3}{4}} \left(a x^{4}+b\right)^{\frac{3}{4}} \left(\frac{a^{7}}{b^{4}}\right)^{\frac{3}{4}} a^{5} b^{3} x-\sqrt{\frac{3}{2}} \sqrt{\sqrt{3} \sqrt{\frac{a^{7}}{b^{4}}} a^{6} b^{2}} \left(18 \left(\frac{27}{4}\right)^{\frac{1}{4}} \sqrt{a x^{4}+b} \left(\frac{a^{7}}{b^{4}}\right)^{\frac{1}{4}} a^{4} b x^{2}+\left(\frac{27}{4}\right)^{\frac{3}{4}} \left(7 a b^{3} x^{4}+4 b^{4}\right) \left(\frac{a^{7}}{b^{4}}\right)^{\frac{3}{4}}\right)\right)}{27 \left(a^{11} x^{4}+4 a^{10} b\right)}\right) - 21 \left(\frac{27}{4}\right)^{\frac{1}{4}} \left(\frac{a^{7}}{b^{4}}\right)^{\frac{1}{4}} b x^{7} \log\left(\frac{18 \sqrt{3} \left(a x^{4}+b\right)^{\frac{1}{4}} \sqrt{\frac{a^{7}}{b^{4}}} a^{2} b^{2} x^{3}+8 \left(\frac{27}{4}\right)^{\frac{3}{4}} \sqrt{a x^{4}+b} \left(\frac{a^{7}}{b^{4}}\right)^{\frac{3}{4}} b^{3} x^{2}+36 \left(a x^{4}+b\right)^{\frac{3}{4}} a^{5} x+3 \left(\frac{27}{4}\right)^{\frac{1}{4}} \left(7 a^{4} b x^{4}+4 a^{3} b^{2}\right) \left(\frac{a^{7}}{b^{4}}\right)^{\frac{1}{4}}}{4 \left(a x^{4}+4 b\right)}\right) + 21 \left(\frac{27}{4}\right)^{\frac{1}{4}} \left(\frac{a^{7}}{b^{4}}\right)^{\frac{1}{4}} b x^{7} \log\left(\frac{18 \sqrt{3} \left(a x^{4}+b\right)^{\frac{1}{4}} \sqrt{\frac{a^{7}}{b^{4}}} a^{2} b^{2} x^{3}-8 \left(\frac{27}{4}\right)^{\frac{3}{4}} \sqrt{a x^{4}+b} \left(\frac{a^{7}}{b^{4}}\right)^{\frac{3}{4}} b^{3} x^{2}+36 \left(a x^{4}+b\right)^{\frac{3}{4}} a^{5} x-3 \left(\frac{27}{4}\right)^{\frac{1}{4}} \left(7 a^{4} b x^{4}+4 a^{3} b^{2}\right) \left(\frac{a^{7}}{b^{4}}\right)^{\frac{1}{4}}}{4 \left(a x^{4}+4 b\right)}\right) + 16 \left(a x^{4}+b\right)^{\frac{3}{4}} \left(a x^{4}-6 b\right)}{672 b x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-4*b)*(a*x^4+b)^(3/4)/x^8/(a*x^4+4*b),x, algorithm="fricas"
)
```

```
[Out] -1/672*(84*(27/4)^(1/4)*(a^7/b^4)^(1/4)*b*x^7*arctan(-4/27*(27*(27/4)^(1/4)
*(a*x^4 + b)^(1/4)*(a^7/b^4)^(1/4)*a^9*b*x^3 + 12*(27/4)^(3/4)*(a*x^4 + b)^
(3/4)*(a^7/b^4)^(3/4)*a^5*b^3*x - sqrt(3/2)*sqrt(sqrt(3)*sqrt(a^7/b^4)*a^6*
b^2)*(18*(27/4)^(1/4)*sqrt(a*x^4 + b)*(a^7/b^4)^(1/4)*a^4*b*x^2 + (27/4)^(3
/4)*(7*a*b^3*x^4 + 4*b^4)*(a^7/b^4)^(3/4)))/(a^11*x^4 + 4*a^10*b)) - 21*(27
/4)^(1/4)*(a^7/b^4)^(1/4)*b*x^7*log(1/4*(18*sqrt(3)*(a*x^4 + b)^(1/4)*sqrt(
a^7/b^4)*a^2*b^2*x^3 + 8*(27/4)^(3/4)*sqrt(a*x^4 + b)*(a^7/b^4)^(3/4)*b^3*x
^2 + 36*(a*x^4 + b)^(3/4)*a^5*x + 3*(27/4)^(1/4)*(7*a^4*b*x^4 + 4*a^3*b^2)*
(a^7/b^4)^(1/4))/(a*x^4 + 4*b)) + 21*(27/4)^(1/4)*(a^7/b^4)^(1/4)*b*x^7*log
(1/4*(18*sqrt(3)*(a*x^4 + b)^(1/4)*sqrt(a^7/b^4)*a^2*b^2*x^3 - 8*(27/4)^(3/
4)*sqrt(a*x^4 + b)*(a^7/b^4)^(3/4)*b^3*x^2 + 36*(a*x^4 + b)^(3/4)*a^5*x - 3
*(27/4)^(1/4)*(7*a^4*b*x^4 + 4*a^3*b^2)*(a^7/b^4)^(1/4))/(a*x^4 + 4*b)) + 1
6*(a*x^4 + b)^(3/4)*(a*x^4 - 6*b))/(b*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a x^{4}+b\right)^{\frac{3}{4}} \left(a x^{4}-4 \, b\right)}{\left(a x^{4}+4 \, b\right) x^{8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-4*b)*(a*x^4+b)^(3/4)/x^8/(a*x^4+4*b),x, algorithm="giac")
```

```
[Out] integrate((a*x^4 + b)^(3/4)*(a*x^4 - 4*b)/((a*x^4 + 4*b)*x^8), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(a \,x^{4}-4 b \right) \left(a \,x^{4}+b \right)^{\frac{3}{4}}}{x^{8} \left(a \,x^{4}+4 b \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-4*b)*(a*x^4+b)^(3/4)/x^8/(a*x^4+4*b),x)
```

```
[Out] int((a*x^4-4*b)*(a*x^4+b)^(3/4)/x^8/(a*x^4+4*b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a x^{4}+b\right)^{\frac{3}{4}} \left(a x^{4}-4 \, b\right)}{\left(a x^{4}+4 \, b\right) x^{8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-4*b)*(a*x^4+b)^(3/4)/x^8/(a*x^4+4*b),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^4 + b)^(3/4)*(a*x^4 - 4*b)/((a*x^4 + 4*b)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(a\,x^4+b\right)^{3/4}\,\left(4\,b-a\,x^4\right)}{x^8\,\left(a\,x^4+4\,b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b + a*x^4)^(3/4)*(4*b - a*x^4))/(x^8*(4*b + a*x^4)),x)
```

```
[Out] int(-((b + a*x^4)^(3/4)*(4*b - a*x^4))/(x^8*(4*b + a*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{4} - 4b\right) \left(ax^{4} + b\right)^{\frac{3}{4}}}{x^{8} \left(ax^{4} + 4b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-4*b)*(a*x**4+b)**(3/4)/x**8/(a*x**4+4*b),x)
```

```
[Out] Integral((a*x**4 - 4*b)*(a*x**4 + b)**(3/4)/(x**8*(a*x**4 + 4*b)), x)
```

3.1615 $\int \frac{(b+ax^4)^{3/4}(2b+ax^4)}{x^8(4b+ax^4)}\,dx$

Optimal. Leaf size=134

$$\frac{3^{3/4}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt{2}\sqrt[4]{ax^4+b}}\right)}{32\sqrt{2}b}+\frac{3^{3/4}a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt{2}\sqrt[4]{ax^4+b}}\right)}{32\sqrt{2}b}+\frac{(-19ax^4-12b)(ax^4+b)^{3/4}}{168bx^7}$$

Rubi [A] time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {580, 583, 12, 377, 212, 206, 203}

$$\frac{3^{3/4}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt{2}\sqrt[4]{ax^4+b}}\right)}{32\sqrt{2}b}+\frac{3^{3/4}a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt{2}\sqrt[4]{ax^4+b}}\right)}{32\sqrt{2}b}-\frac{(ax^4+b)^{3/4}}{14x^7}-\frac{19a(ax^4+b)^{3/4}}{168bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[((b + a*x^4)^(3/4)*(2*b + a*x^4))/(x^8*(4*b + a*x^4)),x]
```

```
[Out] -1/14*(b + a*x^4)^(3/4)/x^7 - (19*a*(b + a*x^4)^(3/4))/(168*b*x^3) + (3^(3/
4)*a^(7/4)*ArcTan[(3^(1/4)*a^(1/4)*x)/(Sqrt[2]*(b + a*x^4)^(1/4))])/(32*Sqr
t[2]*b) + (3^(3/4)*a^(7/4)*ArcTanh[(3^(1/4)*a^(1/4)*x)/(Sqrt[2]*(b + a*x^4)
^(1/4))])/(32*Sqrt[2]*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(b+ax^4\right)^{3/4}\left(2b+ax^4\right)}{x^8\left(4b+ax^4\right)}\,dx = -\frac{\left(b+ax^4\right)^{3/4}}{14x^7}+\frac{\int \frac{38ab^2+20a^2bx^4}{x^4\sqrt[4]{b+ax^4}\left(4b+ax^4\right)}\,dx}{28b}$$

$$= -\frac{\left(b+ax^4\right)^{3/4}}{14x^7}-\frac{19a\left(b+ax^4\right)^{3/4}}{168bx^3}-\frac{\int -\frac{126a^2b^3}{\sqrt[4]{b+ax^4}\left(4b+ax^4\right)}\,dx}{336b^3}$$

$$= -\frac{\left(b+ax^4\right)^{3/4}}{14x^7}-\frac{19a\left(b+ax^4\right)^{3/4}}{168bx^3}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt[4]{b+ax^4}\left(4b+ax^4\right)}\,dx$$

$$= -\frac{\left(b+ax^4\right)^{3/4}}{14x^7}-\frac{19a\left(b+ax^4\right)^{3/4}}{168bx^3}+\frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{4b-3abx^4}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)$$

$$= -\frac{\left(b+ax^4\right)^{3/4}}{14x^7}-\frac{19a\left(b+ax^4\right)^{3/4}}{168bx^3}+\frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{2-\sqrt{3}\sqrt{a}x^2}\,dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)}{32b}+\frac{(3\ldots}{\ldots}$$

$$= -\frac{\left(b+ax^4\right)^{3/4}}{14x^7}-\frac{19a\left(b+ax^4\right)^{3/4}}{168bx^3}+\frac{3^{3/4}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt{2}\sqrt[4]{b+ax^4}}\right)}{32\sqrt{2}b}+\frac{3^{3/4}a^{7/4}\tanh^{-1}\left(\ldots\right)}{32\sqrt{2}\ldots}$$

Mathematica [A] time = 0.16, size = 151, normalized size = 1.13

$$\frac{3^{3/4}a^{7/4}\left(-\log\left(2-\frac{\sqrt{2}\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)+\log\left(\frac{\sqrt{2}\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}+2\right)+2\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt{2}\sqrt[4]{a+bx^4}}\right)\right)}{64\sqrt{2}b}+\left(-\frac{19a}{168bx^3}-\frac{1}{14x^7}\right)\left(ax^4+b\right)^{3/4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((b + a*x^4)^(3/4)*(2*b + a*x^4))/(x^8*(4*b + a*x^4)),x]
```

```
[Out] (-1/14*1/x^7 - (19*a)/(168*b*x^3))*(b + a*x^4)^(3/4) + (3^(3/4)*a^(7/4)*(2*
ArcTan[(3^(1/4)*a^(1/4)*x)/(Sqrt[2]*(a + b*x^4)^(1/4))] - Log[2 - (Sqrt[2]*
3^(1/4)*a^(1/4)*x)/(a + b*x^4)^(1/4)] + Log[2 + (Sqrt[2]*3^(1/4)*a^(1/4)*x)
/(a + b*x^4)^(1/4)]))/(64*Sqrt[2]*b)
```

IntegrateAlgebraic [A] time = 0.61, size = 134, normalized size = 1.00

$$\frac{3^{3/4}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt{2}\sqrt[4]{ax^4+b}}\right)}{32\sqrt{2}b}+\frac{3^{3/4}a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[4]{a}x}{\sqrt{2}\sqrt[4]{ax^4+b}}\right)}{32\sqrt{2}b}+\frac{\left(-19ax^4-12b\right)\left(ax^4+b\right)^{3/4}}{168bx^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b + a*x^4)^(3/4)*(2*b + a*x^4))/(x^8*(4*b + a*x^4)),x]
```

```
[Out] ((-12*b - 19*a*x^4)*(b + a*x^4)^(3/4))/(168*b*x^7) + (3^(3/4)*a^(7/4)*ArcTa
n[(3^(1/4)*a^(1/4)*x)/(Sqrt[2]*(b + a*x^4)^(1/4))])/(32*Sqrt[2]*b) + (3^(3/
4)*a^(7/4)*ArcTanh[(3^(1/4)*a^(1/4)*x)/(Sqrt[2]*(b + a*x^4)^(1/4))])/(32*Sq
rt[2]*b)
```

fricas [B] time = 91.96, size = 495, normalized size = 3.69

$$-\frac{84 \left(\frac{27}{4}\right)^{\frac{1}{4}} \left(\frac{a^7}{b^4}\right)^{\frac{1}{4}} b x^7 \arctan\left(-\frac{4 \left(27 \left(\frac{27}{4}\right)^{\frac{1}{4}} (a x^4+b)^{\frac{1}{4}} \left(\frac{a^7}{b^4}\right)^{\frac{1}{4}} a^9 b x^3 + 12 \left(\frac{27}{4}\right)^{\frac{3}{4}} (a x^4+b)^{\frac{3}{4}} \left(\frac{a^7}{b^4}\right)^{\frac{3}{4}} a^5 b^3 x - \sqrt{\frac{3}{2}} \sqrt{\sqrt{3} \sqrt{\frac{a^7}{b^4}} a^6 b^2} \left(18 \left(\frac{27}{4}\right)^{\frac{1}{4}} \sqrt{a x^4+b} \left(\frac{a^7}{b^4}\right)^{\frac{1}{4}} a^4 b x^2 + \left(\frac{27}{4}\right)^{\frac{3}{4}} (7 a b^3 x^4 + 4 b^4) \left(\frac{a^7}{b^4}\right)^{\frac{3}{4}}\right)\right)}{27 (a^{11} x^4 + 4 a^{10} b)}\right) - 21 \left(\frac{27}{4}\right)^{\frac{1}{4}} \left(\frac{a^7}{b^4}\right)^{\frac{1}{4}} b x^7 \log\left(\frac{18 \sqrt{3} (a x^4+b)^{\frac{1}{4}} \sqrt{\frac{a^7}{b^4}} a^2 b^2 x^3 + 8 \left(\frac{27}{4}\right)^{\frac{3}{4}} \sqrt{a x^4+b} \left(\frac{a^7}{b^4}\right)^{\frac{3}{4}} b^3 x^2 + 36 (a x^4+b)^{\frac{3}{4}} a^5 x + 3 \left(\frac{27}{4}\right)^{\frac{1}{4}} (7 a^4 b x^4 + 4 a^3 b^2) \left(\frac{a^7}{b^4}\right)^{\frac{1}{4}}}{4 (a x^4 + 4 b)}\right) + 21 \left(\frac{27}{4}\right)^{\frac{1}{4}} \left(\frac{a^7}{b^4}\right)^{\frac{1}{4}} b x^7 \log\left(\frac{18 \sqrt{3} (a x^4+b)^{\frac{1}{4}} \sqrt{\frac{a^7}{b^4}} a^2 b^2 x^3 - 8 \left(\frac{27}{4}\right)^{\frac{3}{4}} \sqrt{a x^4+b} \left(\frac{a^7}{b^4}\right)^{\frac{3}{4}} b^3 x^2 + 36 (a x^4+b)^{\frac{3}{4}} a^5 x - 3 \left(\frac{27}{4}\right)^{\frac{1}{4}} (7 a^4 b x^4 + 4 a^3 b^2) \left(\frac{a^7}{b^4}\right)^{\frac{1}{4}}}{4 (a x^4 + 4 b)}\right) + 16 (19 a x^4 + 12 b) (a x^4 + b)^{\frac{3}{4}}}{2688 b x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4)*(a*x^4+2*b)/x^8/(a*x^4+4*b),x, algorithm="fricas"
)
```

```
[Out] -1/2688*(84*(27/4)^(1/4)*(a^7/b^4)^(1/4)*b*x^7*arctan(-4/27*(27*(27/4)^(1/4
)*(a*x^4 + b)^(1/4)*(a^7/b^4)^(1/4)*a^9*b*x^3 + 12*(27/4)^(3/4)*(a*x^4 + b)
^(3/4)*(a^7/b^4)^(3/4)*a^5*b^3*x - sqrt(3/2)*sqrt(sqrt(3)*sqrt(a^7/b^4)*a^6
*b^2)*(18*(27/4)^(1/4)*sqrt(a*x^4 + b)*(a^7/b^4)^(1/4)*a^4*b*x^2 + (27/4)^(
3/4)*(7*a*b^3*x^4 + 4*b^4)*(a^7/b^4)^(3/4)))/(a^11*x^4 + 4*a^10*b)) - 21*(2
7/4)^(1/4)*(a^7/b^4)^(1/4)*b*x^7*log(1/4*(18*sqrt(3)*(a*x^4 + b)^(1/4)*sqrt
(a^7/b^4)*a^2*b^2*x^3 + 8*(27/4)^(3/4)*sqrt(a*x^4 + b)*(a^7/b^4)^(3/4)*b^3*
x^2 + 36*(a*x^4 + b)^(3/4)*a^5*x + 3*(27/4)^(1/4)*(7*a^4*b*x^4 + 4*a^3*b^2)
*(a^7/b^4)^(1/4))/(a*x^4 + 4*b)) + 21*(27/4)^(1/4)*(a^7/b^4)^(1/4)*b*x^7*lo
g(1/4*(18*sqrt(3)*(a*x^4 + b)^(1/4)*sqrt(a^7/b^4)*a^2*b^2*x^3 - 8*(27/4)^(3
/4)*sqrt(a*x^4 + b)*(a^7/b^4)^(3/4)*b^3*x^2 + 36*(a*x^4 + b)^(3/4)*a^5*x -
3*(27/4)^(1/4)*(7*a^4*b*x^4 + 4*a^3*b^2)*(a^7/b^4)^(1/4))/(a*x^4 + 4*b)) +
16*(19*a*x^4 + 12*b)*(a*x^4 + b)^(3/4))/(b*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a x^4 + 2 b\right) \left(a x^4 + b\right)^{\frac{3}{4}}}{\left(a x^4 + 4 b\right) x^8} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4)*(a*x^4+2*b)/x^8/(a*x^4+4*b),x, algorithm="giac")
```

```
[Out] integrate((a*x^4 + 2*b)*(a*x^4 + b)^(3/4)/((a*x^4 + 4*b)*x^8), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(a \,x^{4}+b \right)^{\frac{3}{4}} \left(a \,x^{4}+2 b \right)}{x^{8} \left(a \,x^{4}+4 b \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b)^(3/4)*(a*x^4+2*b)/x^8/(a*x^4+4*b),x)
```

```
[Out] int((a*x^4+b)^(3/4)*(a*x^4+2*b)/x^8/(a*x^4+4*b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a x^4 + 2 b\right) \left(a x^4 + b\right)^{\frac{3}{4}}}{\left(a x^4 + 4 b\right) x^8} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4)*(a*x^4+2*b)/x^8/(a*x^4+4*b),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^4 + 2*b)*(a*x^4 + b)^(3/4)/((a*x^4 + 4*b)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^4+b\right)^{3/4}\,\left(a\,x^4+2\,b\right)}{x^8\,\left(a\,x^4+4\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a*x^4)^(3/4)*(2*b + a*x^4))/(x^8*(4*b + a*x^4)),x)
```

```
[Out] int(((b + a*x^4)^(3/4)*(2*b + a*x^4))/(x^8*(4*b + a*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{4}+b\right)^{\frac{3}{4}}\left(ax^{4}+2b\right)}{x^{8}\left(ax^{4}+4b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b)**(3/4)*(a*x**4+2*b)/x**8/(a*x**4+4*b),x)
```

```
[Out] Integral((a*x**4 + b)**(3/4)*(a*x**4 + 2*b)/(x**8*(a*x**4 + 4*b)), x)
```

3.1616 $\int \frac{x^4 \sqrt[4]{bx^3+ax^4}}{b+ax}\,dx$

Optimal. Leaf size=134

$$\frac{4389b^5\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{4096a^{23/4}}-\frac{4389b^5\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{4096a^{23/4}}+\frac{\sqrt[4]{ax^4+bx^3}\,\left(2048a^4x^4-2432a^3bx^3+3040a^2b^2x^2-4\right.}{10240a^5}$$

Rubi [A] time = 0.32, antiderivative size = 255, normalized size of antiderivative = 1.90, number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2056, 50, 63, 331, 298, 203, 206}

$$\frac{4389b^5\sqrt[4]{ax^4+bx^3}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{4096a^{23/4}x^{3/4}\sqrt[4]{ax+b}}-\frac{4389b^5\sqrt[4]{ax^4+bx^3}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{4096a^{23/4}x^{3/4}\sqrt[4]{ax+b}}+\frac{1463b^4\sqrt[4]{ax^4+bx^3}}{2048a^5}-\frac{209b^3x\sqrt[4]{ax^4+bx^3}}{512a^4}+\frac{19b^2x^2\sqrt[4]{ax^4+bx^3}}{64a^3}-\frac{19bx^3\sqrt[4]{ax^4+bx^3}}{80a^2}+\frac{x^4\sqrt[4]{ax^4+bx^3}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(b*x^3 + a*x^4)^(1/4))/(b + a*x),x]
```

```
[Out] (1463*b^4*(b*x^3 + a*x^4)^(1/4))/(2048*a^5) - (209*b^3*x*(b*x^3 + a*x^4)^(1
/4))/(512*a^4) + (19*b^2*x^2*(b*x^3 + a*x^4)^(1/4))/(64*a^3) - (19*b*x^3*(b
*x^3 + a*x^4)^(1/4))/(80*a^2) + (x^4*(b*x^3 + a*x^4)^(1/4))/(5*a) + (4389*b
^5*(b*x^3 + a*x^4)^(1/4)*ArcTan[(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(4096*a
^(23/4)*x^(3/4)*(b + a*x)^(1/4)) - (4389*b^5*(b*x^3 + a*x^4)^(1/4)*ArcTanh[
(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(4096*a^(23/4)*x^(3/4)*(b + a*x)^(1/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{x^4\sqrt[4]{bx^3+ax^4}}{b+ax}\,dx = \frac{\sqrt[4]{bx^3+ax^4}\int \frac{x^{19/4}}{(b+ax)^{3/4}}\,dx}{x^{3/4}\sqrt[4]{b+ax}}$$

$$= \frac{x^4\sqrt[4]{bx^3+ax^4}}{5a} - \frac{\left(19b\sqrt[4]{bx^3+ax^4}\right)\int \frac{x^{15/4}}{(b+ax)^{3/4}}\,dx}{20ax^{3/4}\sqrt[4]{b+ax}}$$

$$= -\frac{19bx^3\sqrt[4]{bx^3+ax^4}}{80a^2} + \frac{x^4\sqrt[4]{bx^3+ax^4}}{5a} + \frac{\left(57b^2\sqrt[4]{bx^3+ax^4}\right)\int \frac{x^{11/4}}{(b+ax)^{3/4}}\,dx}{64a^2x^{3/4}\sqrt[4]{b+ax}}$$

$$= \frac{19b^2x^2\sqrt[4]{bx^3+ax^4}}{64a^3} - \frac{19bx^3\sqrt[4]{bx^3+ax^4}}{80a^2} + \frac{x^4\sqrt[4]{bx^3+ax^4}}{5a} - \frac{\left(209b^3\sqrt[4]{bx^3+ax^4}\right)\int \frac{x^{7/4}}{(b+ax)^{3}}}{256a^3x^{3/4}\sqrt[4]{b+ax}}$$

$$= -\frac{209b^3x\sqrt[4]{bx^3+ax^4}}{512a^4} + \frac{19b^2x^2\sqrt[4]{bx^3+ax^4}}{64a^3} - \frac{19bx^3\sqrt[4]{bx^3+ax^4}}{80a^2} + \frac{x^4\sqrt[4]{bx^3+ax^4}}{5a} + \frac{(146}{}$$

$$= \frac{1463b^4\sqrt[4]{bx^3+ax^4}}{2048a^5} - \frac{209b^3x\sqrt[4]{bx^3+ax^4}}{512a^4} + \frac{19b^2x^2\sqrt[4]{bx^3+ax^4}}{64a^3} - \frac{19bx^3\sqrt[4]{bx^3+ax^4}}{80a^2} + \frac{x}{}$$

$$= \frac{1463b^4\sqrt[4]{bx^3+ax^4}}{2048a^5} - \frac{209b^3x\sqrt[4]{bx^3+ax^4}}{512a^4} + \frac{19b^2x^2\sqrt[4]{bx^3+ax^4}}{64a^3} - \frac{19bx^3\sqrt[4]{bx^3+ax^4}}{80a^2} + \frac{x}{}$$

$$= \frac{1463b^4\sqrt[4]{bx^3+ax^4}}{2048a^5} - \frac{209b^3x\sqrt[4]{bx^3+ax^4}}{512a^4} + \frac{19b^2x^2\sqrt[4]{bx^3+ax^4}}{64a^3} - \frac{19bx^3\sqrt[4]{bx^3+ax^4}}{80a^2} + \frac{x}{}$$

$$= \frac{1463b^4\sqrt[4]{bx^3+ax^4}}{2048a^5} - \frac{209b^3x\sqrt[4]{bx^3+ax^4}}{512a^4} + \frac{19b^2x^2\sqrt[4]{bx^3+ax^4}}{64a^3} - \frac{19bx^3\sqrt[4]{bx^3+ax^4}}{80a^2} + \frac{x}{}$$

$$= \frac{1463b^4\sqrt[4]{bx^3+ax^4}}{2048a^5} - \frac{209b^3x\sqrt[4]{bx^3+ax^4}}{512a^4} + \frac{19b^2x^2\sqrt[4]{bx^3+ax^4}}{64a^3} - \frac{19bx^3\sqrt[4]{bx^3+ax^4}}{80a^2} + \frac{x}{}$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.37

$$\frac{4x^8\left(\frac{ax}{b}+1\right)^{3/4} {}_2F_1\left(\frac{3}{4},\frac{23}{4};\frac{27}{4};-\frac{ax}{b}\right)}{23\left(x^3(ax+b)\right)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(b*x^3 + a*x^4)^(1/4))/(b + a*x),x]

[Out] (4*x^8*(1 + (a*x)/b)^(3/4)*Hypergeometric2F1[3/4, 23/4, 27/4, -((a*x)/b)])/(23*(x^3*(b + a*x))^(3/4))

IntegrateAlgebraic [A] time = 0.55, size = 134, normalized size = 1.00

$$\frac{4389b^5\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+bx^3}}\right)}{4096a^{23/4}}-\frac{4389b^5\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+bx^3}}\right)}{4096a^{23/4}}+\frac{\sqrt[4]{ax^4+bx^3}\left(2048a^4x^4-2432a^3bx^3+3040a^2b^2x^2-4180ab^3x+7315b^4\right)}{10240a^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^4*(b*x^3 + a*x^4)^(1/4))/(b + a*x),x]

[Out] ((b*x^3 + a*x^4)^(1/4)*(7315*b^4 - 4180*a*b^3*x + 3040*a^2*b^2*x^2 - 2432*a^3*b*x^3 + 2048*a^4*x^4))/(10240*a^5) + (4389*b^5*ArcTan[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(4096*a^(23/4)) - (4389*b^5*ArcTanh[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(4096*a^(23/4))

fricas [B] time = 0.44, size = 275, normalized size = 2.05

$$\frac{87780\,a^5\left(\frac{b^{20}}{a^{23}}\right)^{\frac14}\arctan\left(-\frac{(ax^4+bx^3)^{\frac14}a^{17}b^5\left(\frac{b^{20}}{a^{23}}\right)^{\frac34}-a^{17}x\sqrt{\frac{a^{12}x^2\sqrt{\frac{b^{20}}{a^{23}}}+\sqrt{ax^4+bx^3}\,b^{10}}{x^2}}\left(\frac{b^{20}}{a^{23}}\right)^{\frac34}}{b^{20}x}\right)-21945\,a^5\left(\frac{b^{20}}{a^{23}}\right)^{\frac14}\log\left(\frac{4389\left(a^6x\left(\frac{b^{20}}{a^{23}}\right)^{\frac14}+(ax^4+bx^3)^{\frac14}b^5\right)}{x}\right)+21945\,a^5\left(\frac{b^{20}}{a^{23}}\right)^{\frac14}\log\left(-\frac{4389\left(a^6x\left(\frac{b^{20}}{a^{23}}\right)^{\frac14}-(ax^4+bx^3)^{\frac14}b^5\right)}{x}\right)+4\left(2048\,a^4x^4-2432\,a^3bx^3+3040\,a^2b^2x^2-4180\,ab^3x+7315\,b^4\right)(ax^4+bx^3)^{\frac14}}{40960\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(a*x^4+b*x^3)^(1/4)/(a*x+b),x, algorithm="fricas")

[Out] 1/40960*(87780*a^5*(b^20/a^23)^(1/4)*arctan(-((a*x^4 + b*x^3)^(1/4)*a^17*b^5*(b^20/a^23)^(3/4) - a^17*x*sqrt((a^12*x^2*sqrt(b^20/a^23) + sqrt(a*x^4 + b*x^3)*b^10)/x^2)*(b^20/a^23)^(3/4))/(b^20*x)) - 21945*a^5*(b^20/a^23)^(1/4)*log(4389*(a^6*x*(b^20/a^23)^(1/4) + (a*x^4 + b*x^3)^(1/4)*b^5)/x) + 21945*a^5*(b^20/a^23)^(1/4)*log(-4389*(a^6*x*(b^20/a^23)^(1/4) - (a*x^4 + b*x^3)^(1/4)*b^5)/x) + 4*(2048*a^4*x^4 - 2432*a^3*b*x^3 + 3040*a^2*b^2*x^2 - 4180*a*b^3*x + 7315*b^4)*(a*x^4 + b*x^3)^(1/4))/a^5

giac [B] time = 0.74, size = 295, normalized size = 2.20

$$-\frac{\frac{43890\sqrt{2}(-a)^{\frac14}b^6\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac14}+2\left(a+\frac{b}{x}\right)^{\frac14}\right)}{2(-a)^{\frac14}}\right)}{a^6}+\frac{43890\sqrt{2}(-a)^{\frac14}b^6\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac14}-2\left(a+\frac{b}{x}\right)^{\frac14}\right)}{2(-a)^{\frac14}}\right)}{a^6}+\frac{21945\sqrt{2}(-a)^{\frac14}b^6\log\left(\sqrt{2}(-a)^{\frac14}\left(a+\frac{b}{x}\right)^{\frac14}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{a^6}+\frac{21945\sqrt{2}b^6\log\left(-\sqrt{2}(-a)^{\frac14}\left(a+\frac{b}{x}\right)^{\frac14}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{(-a)^{\frac34}a^5}-\frac{8\left(7315\left(a+\frac{b}{x}\right)^{\frac{17}{4}}b^6-33440\left(a+\frac{b}{x}\right)^{\frac{13}{4}}ab^6+59470\left(a+\frac{b}{x}\right)^{\frac94}a^2b^6-50312\left(a+\frac{b}{x}\right)^{\frac54}a^3b^6+19015\left(a+\frac{b}{x}\right)^{\frac14}a^4b^6\right)x^5}{a^5b^5}}{81920\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(a*x^4+b*x^3)^(1/4)/(a*x+b),x, algorithm="giac")

[Out] -1/81920*(43890*sqrt(2)*(-a)^(1/4)*b^6*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x)^(1/4))/(-a)^(1/4))/a^6 + 43890*sqrt(2)*(-a)^(1/4)*b^6*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(a + b/x)^(1/4))/(-a)^(1/4))/a^6 + 21945*sqrt(2)*(-a)^(1/4)*b^6*log(sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a + b/x))/a^6 + 21945*sqrt(2)*b^6*log(-sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a + b/x))/((-a)^(3/4)*a^5) - 8*(7315*(a + b/x)^(17/4)*b^6 - 33440*(a + b/x)^(13/4)*a*b^6 + 59470*(a + b/x)^(9/4)*a^2*b^6 - 50312*(a + b/x)^(5/4)*a^3*b^6 + 19015*(a + b/x)^(1/4)*a^4*b^6)*x^5/(a^5*b^5))/b

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a\,x^4+b\,x^3\right)^{\frac14}}{ax+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a*x^4+b*x^3)^(1/4)/(a*x+b),x)
```

```
[Out] int(x^4*(a*x^4+b*x^3)^(1/4)/(a*x+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^4 + bx^3)^{\frac{1}{4}} x^4}{ax + b} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^4+b*x^3)^(1/4)/(a*x+b),x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + b*x^3)^(1/4)*x^4/(a*x + b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4 \left(a\,x^4 + b\,x^3\right)^{1/4}}{b + a\,x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a*x^4 + b*x^3)^(1/4))/(b + a*x),x)
```

```
[Out] int((x^4*(a*x^4 + b*x^3)^(1/4))/(b + a*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sqrt[4]{x^3 (ax + b)}}{ax + b} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a*x**4+b*x**3)**(1/4)/(a*x+b),x)
```

```
[Out] Integral(x**4*(x**3*(a*x + b))**(1/4)/(a*x + b), x)
```

3.1617 $\int \frac{b+ax^2}{(-b+ax^2)\sqrt{b^2+a^2x^4}}\,dx$

Optimal. Leaf size=134

$$\frac{\tanh^{-1}\left(\frac{\sqrt{6-4\sqrt{2}}\sqrt{a}\sqrt{b}\,x}{\sqrt{a^2x^4+b^2}+ax^2+b}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{6+4\sqrt{2}}\sqrt{a}\sqrt{b}\,x}{\sqrt{a^2x^4+b^2}+ax^2+b}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}$$

Rubi [A] time = 0.10, antiderivative size = 50, normalized size of antiderivative = 0.37, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1699, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\,x}{\sqrt{a^2x^4+b^2}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^2)/((-b + a*x^2)*Sqrt[b^2 + a^2*x^4]),x]
```

```
[Out] -(ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[b]*x)/Sqrt[b^2 + a^2*x^4]]/(Sqrt[2]*Sqrt[a]
*Sqrt[b]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{b+ax^2}{(-b+ax^2)\sqrt{b^2+a^2x^4}}\,dx &= b\,\text{Subst}\left(\int \frac{1}{-b+2ab^2x^2}\,dx, x, \frac{x}{\sqrt{b^2+a^2x^4}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\,x}{\sqrt{b^2+a^2x^4}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}\end{aligned}$$

Mathematica [C] time = 0.22, size = 95, normalized size = 0.71

$$-\frac{i\sqrt{\frac{a^2x^4}{b^2}+1}\left(F\left(i\sinh^{-1}\left(\sqrt{\frac{ia}{b}}\,x\right)\middle|-1\right)-2\Pi\left(i;i\sinh^{-1}\left(\sqrt{\frac{ia}{b}}\,x\right)\middle|-1\right)\right)}{\sqrt{\frac{ia}{b}}\sqrt{a^2x^4+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^2)/((-b + a*x^2)*Sqrt[b^2 + a^2*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (a^2*x^4)/b^2]*(EllipticF[I*ArcSinh[Sqrt[(I*a)/b]*x], -1] -
2*EllipticPi[I, I*ArcSinh[Sqrt[(I*a)/b]*x], -1]))/(Sqrt[(I*a)/b]*Sqrt[b^2 +
 a^2*x^4])
```

IntegrateAlgebraic [A] time = 0.51, size = 50, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\,x}{\sqrt{a^2x^4+b^2}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^2)/((-b + a*x^2)*Sqrt[b^2 + a^2*x^4]),x]
```

```
[Out] -(ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[b]*x)/Sqrt[b^2 + a^2*x^4]]/(Sqrt[2]*Sqrt[a]
*Sqrt[b]))
```

fricas [A] time = 0.47, size = 132, normalized size = 0.99

$$\left[\frac{1}{4}\sqrt{2}\sqrt{\frac{1}{ab}}\log\left(\frac{a^2x^4-2\sqrt{2}\sqrt{a^2x^4+b^2}\,abx\sqrt{\frac{1}{ab}}+2\,abx^2+b^2}{a^2x^4-2\,abx^2+b^2}\right),\ \frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{ab}}\arctan\left(\frac{\sqrt{2}\sqrt{a^2x^4+b^2}\sqrt{-\frac{1}{ab}}}{2\,x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a*x^2-b)/(a^2*x^4+b^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*sqrt(1/(a*b))*log((a^2*x^4 - 2*sqrt(2)*sqrt(a^2*x^4 + b^2)*a*b
*x*sqrt(1/(a*b)) + 2*a*b*x^2 + b^2)/(a^2*x^4 - 2*a*b*x^2 + b^2)), 1/2*sqrt(
2)*sqrt(-1/(a*b))*arctan(1/2*sqrt(2)*sqrt(a^2*x^4 + b^2)*sqrt(-1/(a*b))/x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2+b}{\sqrt{a^2x^4+b^2}\left(ax^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a*x^2-b)/(a^2*x^4+b^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^2 + b)/(sqrt(a^2*x^4 + b^2)*(a*x^2 - b)), x)
```

maple [C] time = 0.04, size = 152, normalized size = 1.13

$$\frac{\sqrt{1-\frac{ia\,x^2}{b}}\sqrt{1+\frac{ia\,x^2}{b}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{ia}{b}},i\right)}{\sqrt{\frac{ia}{b}}\sqrt{a^2x^4+b^2}}-\frac{2\sqrt{1-\frac{ia\,x^2}{b}}\sqrt{1+\frac{ia\,x^2}{b}}\ \mathrm{EllipticPi}\left(x\sqrt{\frac{ia}{b}},-i,\frac{\sqrt{-\frac{ia}{b}}}{\sqrt{\frac{ia}{b}}}\right)}{\sqrt{\frac{ia}{b}}\sqrt{a^2x^4+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)/(a*x^2-b)/(a^2*x^4+b^2)^(1/2),x)
```

```
[Out] 1/(I*a/b)^(1/2)*(1-I*a/b*x^2)^(1/2)*(1+I*a/b*x^2)^(1/2)/(a^2*x^4+b^2)^(1/2)
*EllipticF(x*(I*a/b)^(1/2),I)-2/(I*a/b)^(1/2)*(1-I*a/b*x^2)^(1/2)*(1+I*a/b*
x^2)^(1/2)/(a^2*x^4+b^2)^(1/2)*EllipticPi(x*(I*a/b)^(1/2),-I,(-I*a/b)^(1/2)
/(I*a/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2+b}{\sqrt{a^2x^4+b^2}\left(ax^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a*x^2-b)/(a^2*x^4+b^2)^(1/2),x, algorithm="maxima")
[Out] integrate((a*x^2 + b)/(sqrt(a^2*x^4 + b^2)*(a*x^2 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a\,x^2+b}{\sqrt{a^2\,x^4+b^2}\,\left(b-a\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x^2)/((b^2 + a^2*x^4)^(1/2)*(b - a*x^2)),x)
[Out] int(-(b + a*x^2)/((b^2 + a^2*x^4)^(1/2)*(b - a*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2+b}{\left(ax^2-b\right)\sqrt{a^2x^4+b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b)/(a*x**2-b)/(a**2*x**4+b**2)**(1/2),x)
[Out] Integral((a*x**2 + b)/((a*x**2 - b)*sqrt(a**2*x**4 + b**2)), x)
```

3.1618 $\int \frac{(1+x^2)\sqrt{1+\sqrt{1+x^2}}}{-1+x^2}\,dx$

Optimal. Leaf size=134

$$\frac{2\sqrt{x^2+1}\,x}{3\sqrt{\sqrt{x^2+1}+1}}+\frac{4x}{3\sqrt{\sqrt{x^2+1}+1}}+2\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}\,\sqrt{\sqrt{x^2+1}+1}}\right)-2\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{}{\sqrt{\sqrt{2}-1}}\right.$$

Rubi [F] time = 0.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+x^2)\sqrt{1+\sqrt{1+x^2}}}{-1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^2)*Sqrt[1 + Sqrt[1 + x^2]])/(-1 + x^2),x]
```

```
[Out] (2*x^3)/(3*(1 + Sqrt[1 + x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 + x^2]] - Def
er[Int][Sqrt[1 + Sqrt[1 + x^2]]/(1 - x), x] - Defer[Int][Sqrt[1 + Sqrt[1 +
x^2]]/(1 + x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^2)\sqrt{1+\sqrt{1+x^2}}}{-1+x^2}\,dx &= \int \left(\sqrt{1+\sqrt{1+x^2}}+\frac{2\sqrt{1+\sqrt{1+x^2}}}{-1+x^2}\right)dx\\
&= 2\int \frac{\sqrt{1+\sqrt{1+x^2}}}{-1+x^2}\,dx+\int \sqrt{1+\sqrt{1+x^2}}\,dx\\
&= \frac{2x^3}{3\left(1+\sqrt{1+x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1+x^2}}}+2\int \left(-\frac{\sqrt{1+\sqrt{1+x^2}}}{2(1-x)}-\frac{\sqrt{1+\sqrt{1+x^2}}}{2(1+x)}\right.\\
&= \frac{2x^3}{3\left(1+\sqrt{1+x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1+x^2}}}-\int \frac{\sqrt{1+\sqrt{1+x^2}}}{1-x}\,dx-\int \frac{\sqrt{1+\sqrt{1+x}}}{1+x}
\end{aligned}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(1+x^2)\sqrt{1+\sqrt{1+x^2}}}{-1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^2)*Sqrt[1 + Sqrt[1 + x^2]])/(-1 + x^2),x]
```

```
[Out] Integrate[((1 + x^2)*Sqrt[1 + Sqrt[1 + x^2]])/(-1 + x^2), x]
```

IntegrateAlgebraic [A] time = 0.40, size = 134, normalized size = 1.00

$$\frac{2\sqrt{x^2+1}\,x}{3\sqrt{\sqrt{x^2+1}+1}}+\frac{4x}{3\sqrt{\sqrt{x^2+1}+1}}+2\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{2}-1}\,x}{\sqrt{\sqrt{x^2+1}+1}}\right)-2\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{2}}\,x}{\sqrt{\sqrt{x^2+1}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^2)*Sqrt[1 + Sqrt[1 + x^2]])/(-1 + x^2),x]
```

```
[Out] (4*x)/(3*Sqrt[1 + Sqrt[1 + x^2]]) + (2*x*Sqrt[1 + x^2])/(3*Sqrt[1 + Sqrt[1
+ x^2]]) + 2*Sqrt[-1 + Sqrt[2]]*ArcTan[(Sqrt[-1 + Sqrt[2]]*x)/Sqrt[1 + Sqrt
[1 + x^2]]] - 2*Sqrt[1 + Sqrt[2]]*ArcTanh[(Sqrt[1 + Sqrt[2]]*x)/Sqrt[1 + Sq
rt[1 + x^2]]]
```

fricas [B] time = 4.12, size = 444, normalized size = 3.31

$$-\frac{12\,x\sqrt{\sqrt{2}-1}\arctan\left(\frac{\left(51\,x^{5}-222\,x^{3}-2\,\sqrt{2}\left(5\,x^{5}-163\,x^{3}-46\,x\right)+2\left(31\,x^{3}+\sqrt{2}\left(41\,x^{3}-81\,x\right)+173\,x\right)\sqrt{x^{2}+1}+11\,x\right)\sqrt{3821\,\sqrt{2}+4841}\,\sqrt{\sqrt{2}-1}+4802\left(x^{4}+\sqrt{2}\left(x^{4}-3\,x^{2}-2\right)+\left(3\,x^{2}+2\,\sqrt{2}\left(x^{2}+1\right)+1\right)\sqrt{x^{2}+1}-1\right)\sqrt{\sqrt{2}-1}\,\sqrt{\sqrt{x^{2}+1}+1}}{2401\left(x^{5}-10\,x^{3}-7\,x\right)}\right)+3\,x\sqrt{\sqrt{2}+1}\log\left(-\frac{2\left(\left(71\,x^{3}-\sqrt{2}\left(61\,x^{3}+325\,x\right)+2\,\sqrt{x^{2}+1}\left(132\,\sqrt{2}\,x-193\,x\right)+457\,x\right)\sqrt{\sqrt{2}+1}+2\left(71\,x^{2}-\sqrt{2}\left(61\,x^{2}+132\right)+\sqrt{x^{2}+1}\left(132\,\sqrt{2}-193\right)+193\right)\sqrt{\sqrt{x^{2}+1}+1}\right)}{x^{3}-x}\right)-3\,x\sqrt{\sqrt{2}+1}\log\left(\frac{2\left(\left(71\,x^{3}-\sqrt{2}\left(61\,x^{3}+325\,x\right)+2\,\sqrt{x^{2}+1}\left(132\,\sqrt{2}\,x-193\,x\right)+457\,x\right)\sqrt{\sqrt{2}+1}-2\left(71\,x^{2}-\sqrt{2}\left(61\,x^{2}+132\right)+\sqrt{x^{2}+1}\left(132\,\sqrt{2}-193\right)+193\right)\sqrt{\sqrt{x^{2}+1}+1}\right)}{x^{3}-x}\right)-4\left(x^{2}+\sqrt{x^{2}+1}-1\right)\sqrt{\sqrt{x^{2}+1}+1}}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(1+(x^2+1)^(1/2))^(1/2)/(x^2-1),x, algorithm="fricas")
```

```
[Out] -1/6*(12*x*sqrt(sqrt(2) - 1)*arctan(1/2401*((51*x^5 - 222*x^3 - 2*sqrt(2)*(
5*x^5 - 163*x^3 - 46*x) + 2*(31*x^3 + sqrt(2)*(41*x^3 - 81*x) + 173*x)*sqrt
(x^2 + 1) + 11*x)*sqrt(3821*sqrt(2) + 4841)*sqrt(sqrt(2) - 1) + 4802*(x^4 +
 sqrt(2)*(x^4 - 3*x^2 - 2) + (3*x^2 + 2*sqrt(2)*(x^2 + 1) + 1)*sqrt(x^2 + 1
) - 1)*sqrt(sqrt(2) - 1)*sqrt(sqrt(x^2 + 1) + 1))/(x^5 - 10*x^3 - 7*x)) + 3
*x*sqrt(sqrt(2) + 1)*log(-2*((71*x^3 - sqrt(2)*(61*x^3 + 325*x) + 2*sqrt(x^
2 + 1)*(132*sqrt(2)*x - 193*x) + 457*x)*sqrt(sqrt(2) + 1) + 2*(71*x^2 - sqr
t(2)*(61*x^2 + 132) + sqrt(x^2 + 1)*(132*sqrt(2) - 193) + 193)*sqrt(sqrt(x^
2 + 1) + 1))/(x^3 - x)) - 3*x*sqrt(sqrt(2) + 1)*log(2*((71*x^3 - sqrt(2)*(6
1*x^3 + 325*x) + 2*sqrt(x^2 + 1)*(132*sqrt(2)*x - 193*x) + 457*x)*sqrt(sqrt
(2) + 1) - 2*(71*x^2 - sqrt(2)*(61*x^2 + 132) + sqrt(x^2 + 1)*(132*sqrt(2)
- 193) + 193)*sqrt(sqrt(x^2 + 1) + 1))/(x^3 - x)) - 4*(x^2 + sqrt(x^2 + 1)
- 1)*sqrt(sqrt(x^2 + 1) + 1))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}+1\right)\sqrt{\sqrt{x^{2}+1}+1}}{x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(1+(x^2+1)^(1/2))^(1/2)/(x^2-1),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)*sqrt(sqrt(x^2 + 1) + 1)/(x^2 - 1), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}+1\right)\sqrt{1+\sqrt{x^{2}+1}}}{x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(1+(x^2+1)^(1/2))^(1/2)/(x^2-1),x)
```

```
[Out] int((x^2+1)*(1+(x^2+1)^(1/2))^(1/2)/(x^2-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}+1\right)\sqrt{\sqrt{x^{2}+1}+1}}{x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(1+(x^2+1)^(1/2))^(1/2)/(x^2-1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)*sqrt(sqrt(x^2 + 1) + 1)/(x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^2+1)\sqrt{\sqrt{x^2+1}+1}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*((x^2 + 1)^(1/2) + 1)^(1/2))/(x^2 - 1),x)
[Out] int(((x^2 + 1)*((x^2 + 1)^(1/2) + 1)^(1/2))/(x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2+1)\sqrt{\sqrt{x^2+1}+1}}{(x-1)(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(1+(x**2+1)**(1/2))**(1/2)/(x**2-1),x)
[Out] Integral((x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1)/((x - 1)*(x + 1)), x)
```

3.1619 $\int \frac{1}{\sqrt[3]{-b+ax^3}}\,dx$

Optimal. Leaf size=135

$$\frac{\log\left(a^{2/3}x^2+\sqrt[3]{a}\,x\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}\right)}{6\sqrt[3]{a}}-\frac{\log\left(\sqrt[3]{ax^3-b}-\sqrt[3]{a}\,x\right)}{3\sqrt[3]{a}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a}\,x}{2\sqrt[3]{ax^3-b}+\sqrt[3]{a}\,x}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Rubi [A] time = 0.01, antiderivative size = 74, normalized size of antiderivative = 0.55, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {239}

$$\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{ax^3-b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{\log\left(\sqrt[3]{ax^3-b}-\sqrt[3]{a}\,x\right)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^3)^(-1/3),x]
```

```
[Out] ArcTan[(1 + (2*a^(1/3)*x)/(-b + a*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*a^(1/3)) -
Log[-(a^(1/3)*x) + (-b + a*x^3)^(1/3)]/(2*a^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-b+ax^3}}\,dx=\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{-b+ax^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{\log\left(-\sqrt[3]{a}\,x+\sqrt[3]{-b+ax^3}\right)}{2\sqrt[3]{a}}$$

Mathematica [A] time = 0.07, size = 118, normalized size = 0.87

$$\frac{\log\left(\frac{a^{2/3}x^2}{\left(ax^3-b\right)^{2/3}}+\frac{\sqrt[3]{a}\,x}{\sqrt[3]{ax^3-b}}+1\right)-2\log\left(1-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{ax^3-b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{ax^3-b}}+1}{\sqrt{3}}\right)}{6\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^3)^(-1/3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*a^(1/3)*x)/(-b + a*x^3)^(1/3))/Sqrt[3]] - 2*Log[1
 - (a^(1/3)*x)/(-b + a*x^3)^(1/3)] + Log[1 + (a^(2/3)*x^2)/(-b + a*x^3)^(2/
3) + (a^(1/3)*x)/(-b + a*x^3)^(1/3)])/(6*a^(1/3))
```

IntegrateAlgebraic [A] time = 0.25, size = 135, normalized size = 1.00

$$\frac{\log\left(a^{2/3}x^2+\sqrt[3]{a}\,x\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}\right)}{6\sqrt[3]{a}}-\frac{\log\left(\sqrt[3]{ax^3-b}-\sqrt[3]{a}\,x\right)}{3\sqrt[3]{a}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a}\,x}{2\sqrt[3]{ax^3-b}+\sqrt[3]{a}\,x}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^3)^(-1/3),x]
```

```
[Out] ArcTan[(Sqrt[3]*a^(1/3)*x)/(a^(1/3)*x + 2*(-b + a*x^3)^(1/3))]/(Sqrt[3]*a^(
1/3)) - Log[-(a^(1/3)*x) + (-b + a*x^3)^(1/3)]/(3*a^(1/3)) + Log[a^(2/3)*x^
2 + a^(1/3)*x*(-b + a*x^3)^(1/3) + (-b + a*x^3)^(2/3)]/(6*a^(1/3))
```

fricas [A] time = 0.42, size = 350, normalized size = 2.59

$$\left[\frac{3\sqrt{\frac{1}{3}}a\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(-3ax^3+3\left(ax^3-b\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}x^2+3\sqrt{\frac{1}{3}}\left((-a)^{\frac{1}{3}}ax^3-\left(ax^3-b\right)^{\frac{1}{3}}ax^2+2\left(ax^3-b\right)^{\frac{2}{3}}(-a)^{\frac{2}{3}}x\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}+2b\right)-2(-a)^{\frac{2}{3}}\log\left(\frac{(-a)^{\frac{1}{3}}x+\left(ax^3-b\right)^{\frac{1}{3}}}{x}\right)+(-a)^{\frac{2}{3}}\log\left(\frac{(-a)^{\frac{2}{3}}x^2-\left(ax^3-b\right)^{\frac{1}{3}}(-a)^{\frac{1}{3}}x+\left(ax^3-b\right)^{\frac{2}{3}}}{x^2}\right)}{6a}, -\frac{6\sqrt{\frac{1}{3}}a\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-a)^{\frac{1}{3}}x-2\left(ax^3-b\right)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}}{x}\right)+2(-a)^{\frac{2}{3}}\log\left(\frac{(-a)^{\frac{1}{3}}x+\left(ax^3-b\right)^{\frac{1}{3}}}{x}\right)-(-a)^{\frac{2}{3}}\log\left(\frac{(-a)^{\frac{2}{3}}x^2-\left(ax^3-b\right)^{\frac{1}{3}}(-a)^{\frac{1}{3}}x+\left(ax^3-b\right)^{\frac{2}{3}}}{x^2}\right)}{6a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3-b)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*sqrt((-a)^(1/3)/a)*log(-3*a*x^3 + 3*(a*x^3 - b)^(1/3)*(
-a)^(2/3)*x^2 + 3*sqrt(1/3)*((-a)^(1/3)*a*x^3 - (a*x^3 - b)^(1/3)*a*x^2 + 2
*(a*x^3 - b)^(2/3)*(-a)^(2/3)*x)*sqrt((-a)^(1/3)/a) + 2*b) - 2*(-a)^(2/3)*l
og(((-a)^(1/3)*x + (a*x^3 - b)^(1/3))/x) + (-a)^(2/3)*log(((-a)^(2/3)*x^2 -
 (a*x^3 - b)^(1/3)*(-a)^(1/3)*x + (a*x^3 - b)^(2/3))/x^2))/a, -1/6*(6*sqrt(
1/3)*a*sqrt(-(-a)^(1/3)/a)*arctan(-sqrt(1/3)*((-a)^(1/3)*x - 2*(a*x^3 - b)^
(1/3))*sqrt(-(-a)^(1/3)/a)/x) + 2*(-a)^(2/3)*log(((-a)^(1/3)*x + (a*x^3 - b
)^(1/3))/x) - (-a)^(2/3)*log(((-a)^(2/3)*x^2 - (a*x^3 - b)^(1/3)*(-a)^(1/3)
*x + (a*x^3 - b)^(2/3))/x^2))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^3-b\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3-b)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*x^3 - b)^(-1/3), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,x^3-b\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3-b)^(1/3),x)
```

```
[Out] int(1/(a*x^3-b)^(1/3),x)
```

maxima [A] time = 0.43, size = 108, normalized size = 0.80

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a^{\frac{1}{3}}+\frac{2\left(ax^3-b\right)^{\frac{1}{3}}}{x}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{1}{3}}}+\frac{\log\left(a^{\frac{2}{3}}+\frac{\left(ax^3-b\right)^{\frac{1}{3}}a^{\frac{1}{3}}}{x}+\frac{\left(ax^3-b\right)^{\frac{2}{3}}}{x^2}\right)}{6a^{\frac{1}{3}}}-\frac{\log\left(-a^{\frac{1}{3}}+\frac{\left(ax^3-b\right)^{\frac{1}{3}}}{x}\right)}{3a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3-b)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(a^(1/3) + 2*(a*x^3 - b)^(1/3)/x)/a^(1/3))/
a^(1/3) + 1/6*log(a^(2/3) + (a*x^3 - b)^(1/3)*a^(1/3)/x + (a*x^3 - b)^(2/3)
/x^2)/a^(1/3) - 1/3*log(-a^(1/3) + (a*x^3 - b)^(1/3)/x)/a^(1/3)
```

mupad [B] time = 0.97, size = 39, normalized size = 0.29

$$\frac{x\left(1-\frac{a\,x^3}{b}\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ \frac{a\,x^3}{b}\right)}{\left(a\,x^3-b\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3 - b)^(1/3),x)
```

```
[Out] (x*(1 - (a*x^3)/b)^(1/3)*hypergeom([1/3, 1/3], 4/3, (a*x^3)/b))/(a*x^3 - b)
^(1/3)
```

sympy [C] time = 0.86, size = 37, normalized size = 0.27

$$\frac{xe^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{ax^3}{b}\right)}{3\sqrt[3]{b}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**3-b)**(1/3),x)
```

```
[Out] x*exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), a*x**3/b)/(3*b**(1/3)*g
amma(4/3))
```

3.1620 $\int \frac{(a^2-2ax+x^2)(-ab-ac+3bc+2(a-b-c)x+x^2)}{((-a+x)(-b+x)(-c+x))^{3/4}(-bc-a^3d+(b+c+3a^2d)x-(1+3ad)x^2+dx^3)}\,dx$

Optimal. Leaf size=135

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3\right)^{3/4}}{(b-x)(x-c)}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3\right)^{3/4}}{(b-x)(x-c)}\right)}{d^{3/4}}$$

Rubi [F] time = 67.84, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a^2-2ax+x^2)(-ab-ac+3bc+2(a-b-c)x+x^2)}{((-a+x)(-b+x)(-c+x))^{3/4}\left(-bc-a^3d+(b+c+3a^2d)x-(1+3ad)x^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a^2 - 2*a*x + x^2)*(-(a*b) - a*c + 3*b*c + 2*(a - b - c)*x + x^2))/((
(-a + x)*(-b + x)*(-c + x))^(3/4)*(-(b*c) - a^3*d + (b + c + 3*a^2*d)*x - (
1 + 3*a*d)*x^2 + d*x^3)),x]
```

```
[Out] (-8*(a - b - c)*(a - x)*(-((b - x)/(a - b)))^(3/4)*(-((c - x)/(a - c)))^(3/
4)*AppellF1[1/4, 3/4, 3/4, 5/4, (a - x)/(a - b), (a - x)/(a - c)])/(d*(-((a
 - x)*(b - x)*(c - x)))^(3/4)) - (4*(1 + 2*a*d)*(a - x)*(-((b - x)/(a - b))
)^(3/4)*(-((c - x)/(a - c)))^(3/4)*AppellF1[1/4, 3/4, 3/4, 5/4, (a - x)/(a
- b), (a - x)/(a - c)])/(d^2*(-((a - x)*(b - x)*(c - x)))^(3/4)) + (4*(a -
x)^2*(-((b - x)/(a - b)))^(3/4)*(-((c - x)/(a - c)))^(3/4)*AppellF1[5/4, 3/
4, 3/4, 9/4, (a - x)/(a - b), (a - x)/(a - c)])/(5*d*(-((a - x)*(b - x)*(c
- x)))^(3/4)) - (8*(a - b)*(a - c)*(a - b - c)*(-a + x)^(3/4)*(-b + x)^(3/4
)*(-c + x)^(3/4)*Defer[Subst][Defer[Int][1/((a - b + x^4)^(3/4)*(a - c + x^
4)^(3/4)*(a^2*(1 + (b*c - a*(b + c))/a^2) + 2*a*(1 - (b + c)/(2*a))*x^4 + x
^8 - d*x^12)), x], x, (-a + x)^(1/4)])/(d*(-((a - x)*(b - x)*(c - x)))^(3/4
)) - (4*(a - b)*(a - c)*(1 + 2*a*d)*(-a + x)^(3/4)*(-b + x)^(3/4)*(-c + x)^
(3/4)*Defer[Subst][Defer[Int][1/((a - b + x^4)^(3/4)*(a - c + x^4)^(3/4)*(a
^2*(1 + (b*c - a*(b + c))/a^2) + 2*a*(1 - (b + c)/(2*a))*x^4 + x^8 - d*x^12
)), x], x, (-a + x)^(1/4)])/(d^2*(-((a - x)*(b - x)*(c - x)))^(3/4)) - (8*(
a - b - c)*(2*a - b - c)*(-a + x)^(3/4)*(-b + x)^(3/4)*(-c + x)^(3/4)*Defer
[Subst][Defer[Int][x^4/((a - b + x^4)^(3/4)*(a - c + x^4)^(3/4)*(a^2*(1 + (
b*c - a*(b + c))/a^2) + 2*a*(1 - (b + c)/(2*a))*x^4 + x^8 - d*x^12)), x], x
, (-a + x)^(1/4)])/(d*(-((a - x)*(b - x)*(c - x)))^(3/4)) + (4*(b + c - 5*a
^2*d - b*c*d - a*(2 - 3*b*d - 3*c*d))*(-a + x)^(3/4)*(-b + x)^(3/4)*(-c + x
)^(3/4)*Defer[Subst][Defer[Int][x^4/((a - b + x^4)^(3/4)*(a - c + x^4)^(3/4
)*(a^2*(1 + (b*c - a*(b + c))/a^2) + 2*a*(1 - (b + c)/(2*a))*x^4 + x^8 - d*
x^12)), x], x, (-a + x)^(1/4)])/(d^2*(-((a - x)*(b - x)*(c - x)))^(3/4)) -
(8*(a - b - c)*(1 + a*d)*(-a + x)^(3/4)*(-b + x)^(3/4)*(-c + x)^(3/4)*Defer
[Subst][Defer[Int][x^8/((a - b + x^4)^(3/4)*(a - c + x^4)^(3/4)*(a^2*(1 + (
b*c - a*(b + c))/a^2) + 2*a*(1 - (b + c)/(2*a))*x^4 + x^8 - d*x^12)), x], x
, (-a + x)^(1/4)])/(d*(-((a - x)*(b - x)*(c - x)))^(3/4)) - (4*(1 + 4*a*d -
 b*d - c*d + a^2*d^2)*(-a + x)^(3/4)*(-b + x)^(3/4)*(-c + x)^(3/4)*Defer[Su
bst][Defer[Int][x^8/((a - b + x^4)^(3/4)*(a - c + x^4)^(3/4)*(a^2*(1 + (b*c
 - a*(b + c))/a^2) + 2*a*(1 - (b + c)/(2*a))*x^4 + x^8 - d*x^12)), x], x, (
-a + x)^(1/4)])/(d^2*(-((a - x)*(b - x)*(c - x)))^(3/4)) + (4*(3*b*c - a*(b
 + c))*(-a + x)^(3/4)*(-b + x)^(3/4)*(-c + x)^(3/4)*Defer[Subst][Defer[Int]
[x^8/((a - b + x^4)^(3/4)*(a - c + x^4)^(3/4)*(-a^2 + a*(b + c - 2*x^4) + b
*(-c + x^4) + x^4*(c - x^4 + d*x^8))), x], x, (-a + x)^(1/4)])/(-((a - x)*(
b - x)*(c - x)))^(3/4)
```

Rubi steps

$$\int \frac{\left(a^2-2ax+x^2\right)\left(-ab-ac+3bc+2(a-b-c)x+x^2\right)}{((-a+x)(-b+x)(-c+x))^{3/4}\left(-bc-a^3d+\left(b+c+3a^2d\right)x-(1+3ad)x^2+dx^3\right)}\,dx = \int \frac{(}{((-a+x)(-b+x}$$

$$= \frac{\left((-a+x)^{3/4}(-b+x\right.}{}$$

$$= \frac{\left((-a+x)^{3/4}(-b+x\right.}{}$$

$$= \frac{\left((-a+x)^{3/4}(-b+x\right.}{}$$

$$= \frac{\left((-a+x)^{3/4}(-b+x\right.}{}$$

$$= \frac{\left(4(-a+x)^{3/4}(-b+\right.}{}$$

$$= \frac{\left(4(-a+x)^{3/4}(-b+\right.}{}$$

$$= -\frac{\left(4(-3bc+a(b+c\right.}{}$$

$$= -\frac{\left(4(-3bc+a(b+c\right.}{}$$

$$= -\frac{\left(4(-3bc+a(b+c\right.}{}$$

$$= -\frac{8(a-b-c)(a-x)}{d(}$$

Mathematica [F] time = 1.39, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2-2ax+x^2\right)\left(-ab-ac+3bc+2(a-b-c)x+x^2\right)}{((-a+x)(-b+x)(-c+x))^{3/4}\left(-bc-a^3d+\left(b+c+3a^2d\right)x-(1+3ad)x^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a^2 - 2*a*x + x^2)*(-(a*b) - a*c + 3*b*c + 2*(a - b - c)*x + x^
2))/(((-a + x)*(-b + x)*(-c + x))^(3/4)*(-(b*c) - a^3*d + (b + c + 3*a^2*d)
*x - (1 + 3*a*d)*x^2 + d*x^3)),x]
```

```
[Out] Integrate[((a^2 - 2*a*x + x^2)*(-(a*b) - a*c + 3*b*c + 2*(a - b - c)*x + x^
2))/(((-a + x)*(-b + x)*(-c + x))^(3/4)*(-(b*c) - a^3*d + (b + c + 3*a^2*d)
*x - (1 + 3*a*d)*x^2 + d*x^3)), x]
```

IntegrateAlgebraic [A] time = 4.16, size = 135, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3\right)^{3/4}}{(b-x)(x-c)}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3\right)^{3/4}}{(b-x)(x-c)}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a^2 - 2*a*x + x^2)*(-(a*b) - a*c + 3*b*c + 2*(a - b -
c)*x + x^2))/(((-a + x)*(-b + x)*(-c + x))^(3/4)*(-(b*c) - a^3*d + (b + c +
 3*a^2*d)*x - (1 + 3*a*d)*x^2 + d*x^3)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(-(a*b*c) + (a*b + a*c + b*c)*x + (-a - b - c)*x^2 + x^
3)^(3/4))/((b - x)*(-c + x))])/d^(3/4) + (2*ArcTanh[(d^(1/4)*(-(a*b*c) + (a
*b + a*c + b*c)*x + (-a - b - c)*x^2 + x^3)^(3/4))/((b - x)*(-c + x))])/d^(
3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-2*a*x+x^2)*(-a*b-a*c+3*b*c+2*(a-b-c)*x+x^2)/((-a+x)*(-b+x)*(
-c+x))^(3/4)/(-b*c-a^3*d+(3*a^2*d+b+c)*x-(3*a*d+1)*x^2+d*x^3),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2-2\,ax+x^2\right)\left(ab+ac-3\,bc-2\,(a-b-c)x-x^2\right)}{\left(a^3d-dx^3+(3\,ad+1)x^2+bc-\left(3\,a^2d+b+c\right)x\right)\left(-(a-x)(b-x)(c-x)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-2*a*x+x^2)*(-a*b-a*c+3*b*c+2*(a-b-c)*x+x^2)/((-a+x)*(-b+x)*(
-c+x))^(3/4)/(-b*c-a^3*d+(3*a^2*d+b+c)*x-(3*a*d+1)*x^2+d*x^3),x, algorithm=
"giac")
```

```
[Out] integrate((a^2 - 2*a*x + x^2)*(a*b + a*c - 3*b*c - 2*(a - b - c)*x - x^2)/(
(a^3*d - d*x^3 + (3*a*d + 1)*x^2 + b*c - (3*a^2*d + b + c)*x)*(-(a - x)*(b
- x)*(c - x))^(3/4)), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2-2ax+x^2\right)\left(-ab-ac+3bc+2\left(a-b-c\right)x+x^2\right)}{\left(\left(-a+x\right)\left(-b+x\right)\left(-c+x\right)\right)^{\frac{3}{4}}\left(-bc-a^3d+\left(3a^2d+b+c\right)x-\left(3ad+1\right)x^2+d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-2*a*x+x^2)*(-a*b-a*c+3*b*c+2*(a-b-c)*x+x^2)/((-a+x)*(-b+x)*(-c+x))
^(3/4)/(-b*c-a^3*d+(3*a^2*d+b+c)*x-(3*a*d+1)*x^2+d*x^3),x)
```

```
[Out] int((a^2-2*a*x+x^2)*(-a*b-a*c+3*b*c+2*(a-b-c)*x+x^2)/((-a+x)*(-b+x)*(-c+x))
^(3/4)/(-b*c-a^3*d+(3*a^2*d+b+c)*x-(3*a*d+1)*x^2+d*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2-2\,ax+x^2\right)\left(ab+ac-3\,bc-2\,(a-b-c)x-x^2\right)}{\left(a^3d-dx^3+(3\,ad+1)x^2+bc-\left(3\,a^2d+b+c\right)x\right)\left(-(a-x)(b-x)(c-x)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-2*a*x+x^2)*(-a*b-a*c+3*b*c+2*(a-b-c)*x+x^2)/((-a+x)*(-b+x)*(
-c+x))^(3/4)/(-b*c-a^3*d+(3*a^2*d+b+c)*x-(3*a*d+1)*x^2+d*x^3),x, algorithm=
"maxima")
```

```
[Out] integrate((a^2 - 2*a*x + x^2)*(a*b + a*c - 3*b*c - 2*(a - b - c)*x - x^2)/(
(a^3*d - d*x^3 + (3*a*d + 1)*x^2 + b*c - (3*a^2*d + b + c)*x)*(-(a - x)*(b
- x)*(c - x))^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2-2\,a\,x+x^2\right)\,\left(a\,b+a\,c-3\,b\,c+2\,x\,\left(b-a+c\right)-x^2\right)}{{\left(-\left(a-x\right)\,\left(b-x\right)\,\left(c-x\right)\right)}^{3/4}\,\left(b\,c-x\,\left(3\,d\,a^2+b+c\right)+a^3\,d-d\,x^3+x^2\,\left(3\,a\,d+1\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a^2 - 2*a*x + x^2)*(a*b + a*c - 3*b*c + 2*x*(b - a + c) - x^2))/((-(a
 - x)*(b - x)*(c - x))^(3/4)*(b*c - x*(b + c + 3*a^2*d) + a^3*d - d*x^3 + x
^2*(3*a*d + 1))),x)
```

```
[Out] int(((a^2 - 2*a*x + x^2)*(a*b + a*c - 3*b*c + 2*x*(b - a + c) - x^2))/((-(a
 - x)*(b - x)*(c - x))^(3/4)*(b*c - x*(b + c + 3*a^2*d) + a^3*d - d*x^3 + x
^2*(3*a*d + 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-2*a*x+x**2)*(-a*b-a*c+3*b*c+2*(a-b-c)*x+x**2)/((-a+x)*(-b+x
)*(-c+x))**(3/4)/(-b*c-a**3*d+(3*a**2*d+b+c)*x-(3*a*d+1)*x**2+d*x**3),x)
```

```
[Out] Timed out
```

3.1621 $\int \frac{(-4+x^5)\sqrt[4]{1-2x^4+x^5}}{x^2(1+x^5)}\,dx$

Optimal. Leaf size=135

$$\frac{4\sqrt[4]{x^5-2x^4+1}}{x}-2^{3/4}\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^5-2x^4+1}}{\sqrt{2}\,x^2-\sqrt{x^5-2x^4+1}}\right)-2^{3/4}\tanh^{-1}\left(\frac{2\sqrt[4]{2}\,x\sqrt[4]{x^5-2x^4+1}}{2x^2+\sqrt{2}\sqrt{x^5-2x^4+1}}\right)$$

Rubi [F] time = 1.36, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-4+x^5)\sqrt[4]{1-2x^4+x^5}}{x^2(1+x^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-4 + x^5)*(1 - 2*x^4 + x^5)^(1/4))/(x^2*(1 + x^5)),x]
```

```
[Out] Defer[Int][(1 - 2*x^4 + x^5)^(1/4)/(-1 - x), x] - 4*Defer[Int][(1 - 2*x^4 +
 x^5)^(1/4)/x^2, x] + Defer[Int][(1 - 2*x^4 + x^5)^(1/4)/(1 - x + x^2 - x^3
 + x^4), x] - 2*Defer[Int][(x*(1 - 2*x^4 + x^5)^(1/4))/(1 - x + x^2 - x^3 +
 x^4), x] + 3*Defer[Int][(x^2*(1 - 2*x^4 + x^5)^(1/4))/(1 - x + x^2 - x^3 +
 x^4), x] + Defer[Int][(x^3*(1 - 2*x^4 + x^5)^(1/4))/(1 - x + x^2 - x^3 + x
^4), x]
```

Rubi steps

$$\int \frac{(-4+x^5)\sqrt[4]{1-2x^4+x^5}}{x^2(1+x^5)}\,dx = \int \left(\frac{\sqrt[4]{1-2x^4+x^5}}{-1-x}-\frac{4\sqrt[4]{1-2x^4+x^5}}{x^2}+\frac{(1-2x+3x^2+x^3)\sqrt[4]{1-2x^4+x^5}}{1-x+x^2-x^3+x^4}\right)d$$

$$=-\left(4\int \frac{\sqrt[4]{1-2x^4+x^5}}{x^2}\,dx\right)+\int \frac{\sqrt[4]{1-2x^4+x^5}}{-1-x}\,dx+\int \frac{(1-2x+3x^2+x^3)\sqrt[4]{}}{1-x+x^2-x^3}$$

$$=-\left(4\int \frac{\sqrt[4]{1-2x^4+x^5}}{x^2}\,dx\right)+\int \frac{\sqrt[4]{1-2x^4+x^5}}{-1-x}\,dx+\int \left(\frac{\sqrt[4]{1-2x^4+x^5}}{1-x+x^2-x^3+x^4}\right.$$

$$=-\left(2\int \frac{x\sqrt[4]{1-2x^4+x^5}}{1-x+x^2-x^3+x^4}\,dx\right)+3\int \frac{x^2\sqrt[4]{1-2x^4+x^5}}{1-x+x^2-x^3+x^4}\,dx-4\int \frac{\sqrt[4]{1-2x}}{x^2}$$

Mathematica [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{(-4+x^5)\sqrt[4]{1-2x^4+x^5}}{x^2(1+x^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-4 + x^5)*(1 - 2*x^4 + x^5)^(1/4))/(x^2*(1 + x^5)),x]
```

```
[Out] Integrate[((-4 + x^5)*(1 - 2*x^4 + x^5)^(1/4))/(x^2*(1 + x^5)), x]
```

IntegrateAlgebraic [A] time = 1.96, size = 135, normalized size = 1.00

$$\frac{4\sqrt[4]{x^5-2x^4+1}}{x}-2^{3/4}\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^5-2x^4+1}}{\sqrt{2}\,x^2-\sqrt{x^5-2x^4+1}}\right)-2^{3/4}\tanh^{-1}\left(\frac{2\sqrt[4]{2}\,x\sqrt[4]{x^5-2x^4+1}}{2x^2+\sqrt{2}\sqrt{x^5-2x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + x^5)*(1 - 2*x^4 + x^5)^(1/4))/(x^2*(1 + x^5)),x]
```

```
[Out] (4*(1 - 2*x^4 + x^5)^(1/4))/x - 2^(3/4)*ArcTan[(2^(3/4)*x*(1 - 2*x^4 + x^5)
^(1/4))/(Sqrt[2]*x^2 - Sqrt[1 - 2*x^4 + x^5])] - 2^(3/4)*ArcTanh[(2*2^(1/4)
*x*(1 - 2*x^4 + x^5)^(1/4))/(2*x^2 + Sqrt[2]*Sqrt[1 - 2*x^4 + x^5])]
```

fricas [B] time = 93.57, size = 809, normalized size = 5.99

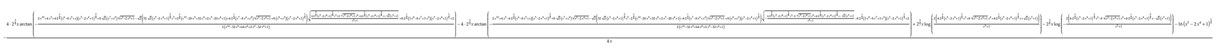

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-4)*(x^5-2*x^4+1)^(1/4)/x^2/(x^5+1),x, algorithm="fricas")
```

```
[Out] -1/4*(4*2^(3/4)*x*arctan(-1/2*(2*x^10 + 4*x^5 + 4*2^(3/4)*(x^6 - 8*x^5 + x)
*(x^5 - 2*x^4 + 1)^(3/4) + 8*sqrt(2)*(x^7 + x^2)*sqrt(x^5 - 2*x^4 + 1) - sq
rt(2)*(32*sqrt(2)*(x^5 - 2*x^4 + 1)^(3/4)*x^5 + 2^(3/4)*(x^10 - 20*x^9 + 32
*x^8 + 2*x^5 - 20*x^4 + 1) + 4*2^(1/4)*(x^7 - 8*x^6 + x^2)*sqrt(x^5 - 2*x^4
 + 1) + 8*(x^8 + x^3)*(x^5 - 2*x^4 + 1)^(1/4))*sqrt((4*2^(3/4)*(x^5 - 2*x^4
 + 1)^(1/4)*x^3 + 8*sqrt(x^5 - 2*x^4 + 1)*x^2 + 4*2^(1/4)*(x^5 - 2*x^4 + 1)
^(3/4)*x + sqrt(2)*(x^5 + 1))/(x^5 + 1)) + 8*2^(1/4)*(3*x^8 - 8*x^7 + 3*x^3
)*(x^5 - 2*x^4 + 1)^(1/4) + 2)/(x^10 - 32*x^9 + 64*x^8 + 2*x^5 - 32*x^4 + 1
)) - 4*2^(3/4)*x*arctan(-1/2*(2*x^10 + 4*x^5 - 4*2^(3/4)*(x^6 - 8*x^5 + x)*
(x^5 - 2*x^4 + 1)^(3/4) + 8*sqrt(2)*(x^7 + x^2)*sqrt(x^5 - 2*x^4 + 1) - sqr
t(2)*(32*sqrt(2)*(x^5 - 2*x^4 + 1)^(3/4)*x^5 - 2^(3/4)*(x^10 - 20*x^9 + 32*
x^8 + 2*x^5 - 20*x^4 + 1) - 4*2^(1/4)*(x^7 - 8*x^6 + x^2)*sqrt(x^5 - 2*x^4
+ 1) + 8*(x^8 + x^3)*(x^5 - 2*x^4 + 1)^(1/4))*sqrt(-(4*2^(3/4)*(x^5 - 2*x^4
 + 1)^(1/4)*x^3 - 8*sqrt(x^5 - 2*x^4 + 1)*x^2 + 4*2^(1/4)*(x^5 - 2*x^4 + 1)
^(3/4)*x - sqrt(2)*(x^5 + 1))/(x^5 + 1)) - 8*2^(1/4)*(3*x^8 - 8*x^7 + 3*x^3
)*(x^5 - 2*x^4 + 1)^(1/4) + 2)/(x^10 - 32*x^9 + 64*x^8 + 2*x^5 - 32*x^4 + 1
)) + 2^(3/4)*x*log(2*(4*2^(3/4)*(x^5 - 2*x^4 + 1)^(1/4)*x^3 + 8*sqrt(x^5 -
2*x^4 + 1)*x^2 + 4*2^(1/4)*(x^5 - 2*x^4 + 1)^(3/4)*x + sqrt(2)*(x^5 + 1))/(
x^5 + 1)) - 2^(3/4)*x*log(-2*(4*2^(3/4)*(x^5 - 2*x^4 + 1)^(1/4)*x^3 - 8*sqr
t(x^5 - 2*x^4 + 1)*x^2 + 4*2^(1/4)*(x^5 - 2*x^4 + 1)^(3/4)*x - sqrt(2)*(x^5
 + 1))/(x^5 + 1)) - 16*(x^5 - 2*x^4 + 1)^(1/4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5-2\,x^4+1\right)^{\frac{1}{4}}\left(x^5-4\right)}{\left(x^5+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-4)*(x^5-2*x^4+1)^(1/4)/x^2/(x^5+1),x, algorithm="giac")
```

```
[Out] integrate((x^5 - 2*x^4 + 1)^(1/4)*(x^5 - 4)/((x^5 + 1)*x^2), x)
```

maple [C] time = 26.98, size = 1652, normalized size = 12.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-4)*(x^5-2*x^4+1)^(1/4)/x^2/(x^5+1),x)
```

```
[Out] 4*(x^5-2*x^4+1)^(1/4)/x+(-RootOf(_Z^4+2)*ln((-x^15*RootOf(_Z^4+2)^2+8*RootO
f(_Z^4+2)^2*x^14-2*RootOf(_Z^4+2)^3*(x^15-6*x^14+12*x^13-8*x^12+3*x^10-12*x
^9+12*x^8+3*x^5-6*x^4+1)^(1/4)*x^11-20*RootOf(_Z^4+2)^2*x^13+8*RootOf(_Z^4+
2)^3*(x^15-6*x^14+12*x^13-8*x^12+3*x^10-12*x^9+12*x^8+3*x^5-6*x^4+1)^(1/4)*
x^10+16*RootOf(_Z^4+2)^2*x^12-8*RootOf(_Z^4+2)^3*(x^15-6*x^14+12*x^13-8*x^1
2+3*x^10-12*x^9+12*x^8+3*x^5-6*x^4+1)^(1/4)*x^9-3*RootOf(_Z^4+2)^2*x^10+16*
```

```
RootOf(_Z^4+2)^2*x^9-4*RootOf(_Z^4+2)^3*(x^15-6*x^14+12*x^13-8*x^12+3*x^10-
12*x^9+12*x^8+3*x^5-6*x^4+1)^(1/4)*x^6-20*RootOf(_Z^4+2)^2*x^8+4*(x^15-6*x^
14+12*x^13-8*x^12+3*x^10-12*x^9+12*x^8+3*x^5-6*x^4+1)^(1/2)*x^7+8*RootOf(_Z
^4+2)^3*(x^15-6*x^14+12*x^13-8*x^12+3*x^10-12*x^9+12*x^8+3*x^5-6*x^4+1)^(1/
4)*x^5-8*(x^15-6*x^14+12*x^13-8*x^12+3*x^10-12*x^9+12*x^8+3*x^5-6*x^4+1)^(1
/2)*x^6+4*(x^15-6*x^14+12*x^13-8*x^12+3*x^10-12*x^9+12*x^8+3*x^5-6*x^4+1)^(
3/4)*RootOf(_Z^4+2)*x^3-3*RootOf(_Z^4+2)^2*x^5+8*RootOf(_Z^4+2)^2*x^4-2*Roo
tOf(_Z^4+2)^3*(x^15-6*x^14+12*x^13-8*x^12+3*x^10-12*x^9+12*x^8+3*x^5-6*x^4+
1)^(1/4)*x+4*(x^15-6*x^14+12*x^13-8*x^12+3*x^10-12*x^9+12*x^8+3*x^5-6*x^4+1
)^(1/2)*x^2-RootOf(_Z^4+2)^2)/(-1+x)^2/(x^4-x^3-x^2-x-1)^2/(1+x)/(x^4-x^3+x
^2-x+1))-RootOf(_Z^2+RootOf(_Z^4+2)^2)*ln((x^15*RootOf(_Z^4+2)^2-8*RootOf(_
Z^4+2)^2*x^14+2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^4+2)^2*(x^15-6*x^14
+12*x^13-8*x^12+3*x^10-12*x^9+12*x^8+3*x^5-6*x^4+1)^(1/4)*x^11+20*RootOf(_Z
^4+2)^2*x^13-8*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^4+2)^2*(x^15-6*x^14+
12*x^13-8*x^12+3*x^10-12*x^9+12*x^8+3*x^5-6*x^4+1)^(1/4)*x^10-16*RootOf(_Z^
4+2)^2*x^12+8*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^4+2)^2*(x^15-6*x^14+1
2*x^13-8*x^12+3*x^10-12*x^9+12*x^8+3*x^5-6*x^4+1)^(1/4)*x^9+3*RootOf(_Z^4+2
)^2*x^10-16*RootOf(_Z^4+2)^2*x^9+4*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^
4+2)^2*(x^15-6*x^14+12*x^13-8*x^12+3*x^10-12*x^9+12*x^8+3*x^5-6*x^4+1)^(1/4
)*x^6+20*RootOf(_Z^4+2)^2*x^8-8*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^4+2
)^2*(x^15-6*x^14+12*x^13-8*x^12+3*x^10-12*x^9+12*x^8+3*x^5-6*x^4+1)^(1/4)*x
^5+4*(x^15-6*x^14+12*x^13-8*x^12+3*x^10-12*x^9+12*x^8+3*x^5-6*x^4+1)^(1/2)*
x^7-8*(x^15-6*x^14+12*x^13-8*x^12+3*x^10-12*x^9+12*x^8+3*x^5-6*x^4+1)^(1/2)
*x^6+4*RootOf(_Z^2+RootOf(_Z^4+2)^2)*(x^15-6*x^14+12*x^13-8*x^12+3*x^10-12*
x^9+12*x^8+3*x^5-6*x^4+1)^(3/4)*x^3+3*RootOf(_Z^4+2)^2*x^5-8*RootOf(_Z^4+2)
^2*x^4+2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^4+2)^2*(x^15-6*x^14+12*x^1
3-8*x^12+3*x^10-12*x^9+12*x^8+3*x^5-6*x^4+1)^(1/4)*x+4*(x^15-6*x^14+12*x^13
-8*x^12+3*x^10-12*x^9+12*x^8+3*x^5-6*x^4+1)^(1/2)*x^2+RootOf(_Z^4+2)^2)/(-1
+x)^2/(x^4-x^3-x^2-x-1)^2/(1+x)/(x^4-x^3+x^2-x+1)))/(x^5-2*x^4+1)^(3/4)*((x
^5-2*x^4+1)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5-2\,x^4+1\right)^{\frac{1}{4}}\left(x^5-4\right)}{\left(x^5+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-4)*(x^5-2*x^4+1)^(1/4)/x^2/(x^5+1),x, algorithm="maxima")
[Out] integrate((x^5 - 2*x^4 + 1)^(1/4)*(x^5 - 4)/((x^5 + 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^5-4\right)\,\left(x^5-2\,x^4+1\right)^{1/4}}{x^2\,\left(x^5+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 - 4)*(x^5 - 2*x^4 + 1)^(1/4))/(x^2*(x^5 + 1)),x)
[Out] int(((x^5 - 4)*(x^5 - 2*x^4 + 1)^(1/4))/(x^2*(x^5 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{(x-1)\left(x^4-x^3-x^2-x-1\right)}\,\left(x^5-4\right)}{x^2\,(x+1)\left(x^4-x^3+x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-4)*(x**5-2*x**4+1)**(1/4)/x**2/(x**5+1),x)
```

```
[Out] Integral(((x - 1)*(x**4 - x**3 - x**2 - x - 1))**(1/4)*(x**5 - 4)/(x**2*(x
+ 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

3.1622 $\int \frac{b+ax^6}{x^6 \sqrt[3]{x+x^3}}\,dx$

Optimal. Leaf size=135

$$\frac{1}{4}\left(a-i\sqrt{3}\,a\right)\log\left(-2i\sqrt[3]{x^3+x}+\sqrt{3}\,x-ix\right)+\frac{1}{4}\left(a+i\sqrt{3}\,a\right)\log\left(2i\sqrt[3]{x^3+x}+\sqrt{3}\,x+ix\right)-\frac{1}{2}a\log\left(\sqrt[3]{x^3+x}-x\right)-$$

Rubi [A] time = 0.18, antiderivative size = 156, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2052, 2011, 329, 275, 239, 2016, 2014}

$$-\frac{3a\sqrt[3]{x}\sqrt[3]{x^2+1}\log\left(x^{2/3}-\sqrt[3]{x^2+1}\right)}{4\sqrt[3]{x^3+x}}+\frac{\sqrt{3}\,a\sqrt[3]{x}\sqrt[3]{x^2+1}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{x^3+x}}-\frac{3b\left(x^3+x\right)^{2/3}}{16x^6}+\frac{9b\left(x^3+x\right)^{2/3}}{40x^4}-\frac{27b\left(x^3+x\right)^{2/3}}{80x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^6)/(x^6*(x + x^3)^(1/3)),x]
```

```
[Out] (-3*b*(x + x^3)^(2/3))/(16*x^6) + (9*b*(x + x^3)^(2/3))/(40*x^4) - (27*b*(x
 + x^3)^(2/3))/(80*x^2) + (Sqrt[3]*a*x^(1/3)*(1 + x^2)^(1/3)*ArcTan[(1 + (2
*x^(2/3))/(1 + x^2)^(1/3))/Sqrt[3]])/(2*(x + x^3)^(1/3)) - (3*a*x^(1/3)*(1
+ x^2)^(1/3)*Log[x^(2/3) - (1 + x^2)^(1/3)])/(4*(x + x^3)^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{b+ax^6}{x^6\sqrt[3]{x+x^3}}\,dx &= \int \left(\frac{a}{\sqrt[3]{x+x^3}}+\frac{b}{x^6\sqrt[3]{x+x^3}}\right)dx\\
&= a\int \frac{1}{\sqrt[3]{x+x^3}}\,dx + b\int \frac{1}{x^6\sqrt[3]{x+x^3}}\,dx\\
&= -\frac{3b\left(x+x^3\right)^{2/3}}{16x^6} - \frac{1}{4}(3b)\int \frac{1}{x^4\sqrt[3]{x+x^3}}\,dx + \frac{\left(a\sqrt[3]{x}\,\sqrt[3]{1+x^2}\right)\int \frac{1}{\sqrt[3]{x}\,\sqrt[3]{1+x^2}}\,dx}{\sqrt[3]{x+x^3}}\\
&= -\frac{3b\left(x+x^3\right)^{2/3}}{16x^6} + \frac{9b\left(x+x^3\right)^{2/3}}{40x^4} + \frac{1}{20}(9b)\int \frac{1}{x^2\sqrt[3]{x+x^3}}\,dx + \frac{\left(3a\sqrt[3]{x}\,\sqrt[3]{1+x^2}\right)\mathrm{Subst}\left(\int\right.}{\sqrt[3]{x+x^3}}\\
&= -\frac{3b\left(x+x^3\right)^{2/3}}{16x^6} + \frac{9b\left(x+x^3\right)^{2/3}}{40x^4} - \frac{27b\left(x+x^3\right)^{2/3}}{80x^2} + \frac{\left(3a\sqrt[3]{x}\,\sqrt[3]{1+x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^3}}\,dx\right.}{2\sqrt[3]{x+x^3}}\\
&= -\frac{3b\left(x+x^3\right)^{2/3}}{16x^6} + \frac{9b\left(x+x^3\right)^{2/3}}{40x^4} - \frac{27b\left(x+x^3\right)^{2/3}}{80x^2} + \frac{\sqrt{3}\,a\sqrt[3]{x}\,\sqrt[3]{1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{x+x^3}} -
\end{aligned}$$

Mathematica [A] time = 0.19, size = 149, normalized size = 1.10

$$\frac{3\sqrt[3]{x}\,\sqrt[3]{x^2+1}\left(\frac{1}{12}a\left(-2\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2+1}}\right)+\log\left(\frac{x^{4/3}}{\left(x^2+1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2+1}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)\right)-\frac{b\left(x^2+1\right)^{2/3}\left(9x^4-6x^2+5\right)}{80x^{16/3}}\right)}{\sqrt[3]{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^6)/(x^6*(x + x^3)^(1/3)),x]
```

```
[Out] (3*x^(1/3)*(1 + x^2)^(1/3)*(-1/80*(b*(1 + x^2)^(2/3)*(5 - 6*x^2 + 9*x^4))/x
^(16/3) + (a*(2*Sqrt[3]*ArcTan[(1 + (2*x^(2/3))/(1 + x^2)^(1/3))/Sqrt[3]] -
 2*Log[1 - x^(2/3)/(1 + x^2)^(1/3)] + Log[1 + x^(4/3)/(1 + x^2)^(2/3) + x^(
2/3)/(1 + x^2)^(1/3)]))/12))/(x + x^3)^(1/3)
```

IntegrateAlgebraic [A] time = 0.30, size = 112, normalized size = 0.83

$$-\frac{1}{2}a\log\left(\sqrt[3]{x^3+x}-x\right)+\frac{1}{2}\sqrt{3}\,a\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3+x}+x}\right)+\frac{1}{4}a\log\left(\sqrt[3]{x^3+x}\,x+\left(x^3+x\right)^{2/3}+x^2\right)-\frac{3b\left(x^3+x\right)^{2/3}\left(9x^4-6x^2+5\right)}{80x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^6)/(x^6*(x + x^3)^(1/3)),x]
```

```
[Out] (-3*b*(x + x^3)^(2/3)*(5 - 6*x^2 + 9*x^4))/(80*x^6) + (Sqrt[3]*a*ArcTan[(Sq
rt[3]*x)/(x + 2*(x + x^3)^(1/3))])/2 - (a*Log[-x + (x + x^3)^(1/3)])/2 + (a
*Log[x^2 + x*(x + x^3)^(1/3) + (x + x^3)^(2/3)])/4
```

fricas [A] time = 1.25, size = 118, normalized size = 0.87

$$\frac{40 \sqrt{3} a x^{6} \arctan\left(-\frac{196 \sqrt{3} \left(x^{3}+x\right)^{\frac{1}{3}} x-\sqrt{3} \left(539 x^{2}+507\right)-1274 \sqrt{3} \left(x^{3}+x\right)^{\frac{2}{3}}}{2205 x^{2}+2197}\right) - 20 a x^{6} \log\left(3 \left(x^{3}+x\right)^{\frac{1}{3}} x - 3 \left(x^{3}+x\right)^{\frac{2}{3}} + 1\right) - 3 \left(9 b x^{4} - 6 b x^{2} + 5 b\right) \left(x^{3}+x\right)^{\frac{2}{3}}}{80 x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^6/(x^3+x)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/80*(40*sqrt(3)*a*x^6*arctan(-(196*sqrt(3)*(x^3 + x)^(1/3)*x - sqrt(3)*(53
9*x^2 + 507) - 1274*sqrt(3)*(x^3 + x)^(2/3))/(2205*x^2 + 2197)) - 20*a*x^6*
log(3*(x^3 + x)^(1/3)*x - 3*(x^3 + x)^(2/3) + 1) - 3*(9*b*x^4 - 6*b*x^2 + 5
*b)*(x^3 + x)^(2/3))/x^6
```

giac [A] time = 0.22, size = 88, normalized size = 0.65

$$-\frac{3}{16} b \left(\frac{1}{x^{2}}+1\right)^{\frac{8}{3}} + \frac{3}{5} b \left(\frac{1}{x^{2}}+1\right)^{\frac{5}{3}} - \frac{1}{2} \sqrt{3} a \arctan\left(\frac{1}{3} \sqrt{3} \left(2 \left(\frac{1}{x^{2}}+1\right)^{\frac{1}{3}}+1\right)\right) + \frac{1}{4} a \log\left(\left(\frac{1}{x^{2}}+1\right)^{\frac{2}{3}} + \left(\frac{1}{x^{2}}+1\right)^{\frac{1}{3}} + 1\right) - \frac{1}{2} a \log\left(\left|\left(\frac{1}{x^{2}}+1\right)^{\frac{1}{3}} - 1\right|\right) - \frac{3}{4} b \left(\frac{1}{x^{2}}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^6/(x^3+x)^(1/3),x, algorithm="giac")
```

```
[Out] -3/16*b*(1/x^2 + 1)^(8/3) + 3/5*b*(1/x^2 + 1)^(5/3) - 1/2*sqrt(3)*a*arctan(
1/3*sqrt(3)*(2*(1/x^2 + 1)^(1/3) + 1)) + 1/4*a*log((1/x^2 + 1)^(2/3) + (1/x
^2 + 1)^(1/3) + 1) - 1/2*a*log(abs((1/x^2 + 1)^(1/3) - 1)) - 3/4*b*(1/x^2 +
 1)^(2/3)
```

maple [C] time = 0.31, size = 51, normalized size = 0.38

$$-\frac{3 b \left(x^{2}+1\right) \left(9 x^{4}-6 x^{2}+5\right)}{80 x^{5} \left(x \left(x^{2}+1\right)\right)^{\frac{1}{3}}} + \frac{3 a x^{\frac{2}{3}} \operatorname{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{3}\right], \left[\frac{4}{3}\right], -x^{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6+b)/x^6/(x^3+x)^(1/3),x)
```

```
[Out] -3/80*b*(x^2+1)*(9*x^4-6*x^2+5)/x^5/(x*(x^2+1))^(1/3)+3/2*a*x^(2/3)*hyperge
om([1/3,1/3],[4/3],-x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a x^{6} + b}{\left(x^{3} + x\right)^{\frac{1}{3}} x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^6/(x^3+x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*x^6 + b)/((x^3 + x)^(1/3)*x^6), x)
```

mupad [B] time = 1.30, size = 54, normalized size = 0.40

$$\frac{3\, a\, x \left(x^{2}+1\right)^{1/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -x^{2}\right)}{2 \left(x^{3}+x\right)^{1/3}} - \frac{3\, b \left(x^{3}+x\right)^{2/3} \left(9 x^{4}-6 x^{2}+5\right)}{80 x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^6)/(x^6*(x + x^3)^(1/3)),x)
```

```
[Out] (3*a*x*(x^2 + 1)^(1/3)*hypergeom([1/3, 1/3], 4/3, -x^2))/(2*(x + x^3)^(1/3)
) - (3*b*(x + x^3)^(2/3)*(9*x^4 - 6*x^2 + 5))/(80*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^6 + b}{x^6 \sqrt[3]{x(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6+b)/x**6/(x**3+x)**(1/3),x)
```

```
[Out] Integral((a*x**6 + b)/(x**6*(x*(x**2 + 1))**(1/3)), x)
```

3.1623 $\int \frac{b^2+a^2x^2}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$

Optimal. Leaf size=135

$$\frac{2\sqrt{a^2x^2+b^2}\,\left(4a^4x^5+19a^2b^2x^3+7b^4x\right)}{5\left(\sqrt{a^2x^2+b^2}+ax\right)^{7/2}}+\frac{2\left(28a^6x^6+147a^4b^2x^4+112a^2b^4x^2+9b^6\right)}{35a\left(\sqrt{a^2x^2+b^2}+ax\right)^{7/2}}$$

Rubi [A] time = 0.09, antiderivative size = 130, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2122, 270}

$$\frac{3b^2\sqrt{\sqrt{a^2x^2+b^2}+ax}}{4a}+\frac{\left(\sqrt{a^2x^2+b^2}+ax\right)^{5/2}}{20a}-\frac{b^6}{28a\left(\sqrt{a^2x^2+b^2}+ax\right)^{7/2}}-\frac{b^4}{4a\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2 + a^2*x^2)/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] -1/28*b^6/(a*(a*x + Sqrt[b^2 + a^2*x^2])^(7/2)) - b^4/(4*a*(a*x + Sqrt[b^2
+ a^2*x^2])^(3/2)) + (3*b^2*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/(4*a) + (a*x +
 Sqrt[b^2 + a^2*x^2])^(5/2)/(20*a)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{b^2+a^2x^2}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(b^2+x^2\right)^3}{x^{9/2}}\,dx,x,ax+\sqrt{b^2+a^2x^2}\right)}{8a}\\
&= \frac{\text{Subst}\left(\int \left(\frac{b^6}{x^{9/2}}+\frac{3b^4}{x^{5/2}}+\frac{3b^2}{\sqrt{x}}+x^{3/2}\right)dx,x,ax+\sqrt{b^2+a^2x^2}\right)}{8a}\\
&= -\frac{b^6}{28a\left(ax+\sqrt{b^2+a^2x^2}\right)^{7/2}}-\frac{b^4}{4a\left(ax+\sqrt{b^2+a^2x^2}\right)^{3/2}}+\frac{3b^2\sqrt{ax+\sqrt{b^2+a^2x^2}}}{4a}+
\end{aligned}$$

Mathematica [A] time = 0.13, size = 111, normalized size = 0.82

$$\frac{105b^2\left(\sqrt{a^2x^2+b^2}+ax\right)^4+7\left(\sqrt{a^2x^2+b^2}+ax\right)^6-35b^4\left(\sqrt{a^2x^2+b^2}+ax\right)^2-5b^6}{140a\left(\sqrt{a^2x^2+b^2}+ax\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2 + a^2*x^2)/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] (-5*b^6 - 35*b^4*(a*x + Sqrt[b^2 + a^2*x^2])^2 + 105*b^2*(a*x + Sqrt[b^2 +
a^2*x^2])^4 + 7*(a*x + Sqrt[b^2 + a^2*x^2])^6)/(140*a*(a*x + Sqrt[b^2 + a^2
*x^2])^(7/2))
```

IntegrateAlgebraic [A] time = 0.18, size = 135, normalized size = 1.00

$$\frac{2\sqrt{a^2x^2+b^2}\left(4a^4x^5+19a^2b^2x^3+7b^4x\right)}{5\left(\sqrt{a^2x^2+b^2}+ax\right)^{7/2}}+\frac{2\left(28a^6x^6+147a^4b^2x^4+112a^2b^4x^2+9b^6\right)}{35a\left(\sqrt{a^2x^2+b^2}+ax\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^2 + a^2*x^2)/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] (2*Sqrt[b^2 + a^2*x^2]*(7*b^4*x + 19*a^2*b^2*x^3 + 4*a^4*x^5))/(5*(a*x + Sq
rt[b^2 + a^2*x^2])^(7/2)) + (2*(9*b^6 + 112*a^2*b^4*x^2 + 147*a^4*b^2*x^4 +
 28*a^6*x^6))/(35*a*(a*x + Sqrt[b^2 + a^2*x^2])^(7/2))
```

fricas [A] time = 0.41, size = 83, normalized size = 0.61

$$-\frac{2\left(5\,a^4x^4+12\,a^2b^2x^2-9\,b^4-\left(5\,a^3x^3+13\,ab^2x\right)\sqrt{a^2x^2+b^2}\right)\sqrt{ax+\sqrt{a^2x^2+b^2}}}{35\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b^2)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] -2/35*(5*a^4*x^4 + 12*a^2*b^2*x^2 - 9*b^4 - (5*a^3*x^3 + 13*a*b^2*x)*sqrt(a
^2*x^2 + b^2))*sqrt(a*x + sqrt(a^2*x^2 + b^2))/(a*b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a^2x^2+b^2}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b^2)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((a^2*x^2 + b^2)/sqrt(a*x + sqrt(a^2*x^2 + b^2)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int\frac{a^2x^2+b^2}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2+b^2)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a^2*x^2+b^2)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2x^2+b^2}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b^2)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a^2*x^2 + b^2)/sqrt(a*x + sqrt(a^2*x^2 + b^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a^2\,x^2+b^2}{\sqrt{a\,x+\sqrt{a^2\,x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2 + a^2*x^2)/(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int((b^2 + a^2*x^2)/(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2x^2+b^2}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2+b**2)/(a*x+(a**2*x**2+b**2)**(1/2))**(1/2),x)
```

```
[Out] Integral((a**2*x**2 + b**2)/sqrt(a*x + sqrt(a**2*x**2 + b**2)), x)
```

3.1624 $\int \frac{x^2}{(b+ax^2)^{3/4}(2b+ax^2)}\,dx$

Optimal. Leaf size=136

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}\sqrt{ax^2+b}}{\sqrt{a}}-\frac{\sqrt{a}\,x^2}{2\sqrt[4]{b}}}{x\sqrt[4]{ax^2+b}}\right)}{2a^{3/2}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{2\sqrt{a}\sqrt[4]{b}\,x\sqrt[4]{ax^2+b}}{2\sqrt{b}\sqrt{ax^2+b}+ax^2}\right)}{2a^{3/2}\sqrt[4]{b}}$$

Rubi [A] time = 0.04, antiderivative size = 115, normalized size of antiderivative = 0.85, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {441}

$$\frac{\tanh^{-1}\left(\frac{b^{3/4}\left(1-\frac{\sqrt{ax^2+b}}{\sqrt{b}}\right)}{\sqrt{a}\,x\sqrt[4]{ax^2+b}}\right)}{a^{3/2}\sqrt[4]{b}}-\frac{\tan^{-1}\left(\frac{b^{3/4}\left(\frac{\sqrt{ax^2+b}}{\sqrt{b}}+1\right)}{\sqrt{a}\,x\sqrt[4]{ax^2+b}}\right)}{a^{3/2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((b + a*x^2)^(3/4)*(2*b + a*x^2)),x]
```

```
[Out] -(ArcTan[(b^(3/4)*(1 + Sqrt[b + a*x^2]/Sqrt[b]))/(Sqrt[a]*x*(b + a*x^2)^(1/
4))]/(a^(3/2)*b^(1/4))) + ArcTanh[(b^(3/4)*(1 - Sqrt[b + a*x^2]/Sqrt[b]))/(
Sqrt[a]*x*(b + a*x^2)^(1/4))]/(a^(3/2)*b^(1/4))
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(b+ax^2\right)^{3/4}\left(2b+ax^2\right)}\,dx=-\frac{\tan^{-1}\left(\frac{b^{3/4}\left(1+\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)}{\sqrt{a}\,x\sqrt[4]{b+ax^2}}\right)}{a^{3/2}\sqrt[4]{b}}+\frac{\tanh^{-1}\left(\frac{b^{3/4}\left(1-\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)}{\sqrt{a}\,x\sqrt[4]{b+ax^2}}\right)}{a^{3/2}\sqrt[4]{b}}$$

Mathematica [C] time = 0.06, size = 67, normalized size = 0.49

$$\frac{x^3\left(\frac{ax^2+b}{b}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{ax^2}{b},-\frac{ax^2}{2b}\right)}{6b\left(ax^2+b\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((b + a*x^2)^(3/4)*(2*b + a*x^2)),x]
```

```
[Out] (x^3*((b + a*x^2)/b)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -((a*x^2)/b), -1/2*(a
*x^2)/b])/(6*b*(b + a*x^2)^(3/4))
```

IntegrateAlgebraic [A] time = 2.42, size = 136, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}\sqrt{ax^2+b}}{\sqrt{a}}-\frac{\sqrt{a}\,x^2}{2\sqrt[4]{b}}}{x\sqrt[4]{ax^2+b}}\right)}{2a^{3/2}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{2\sqrt{a}\,\sqrt[4]{b}\,x\sqrt[4]{ax^2+b}}{2\sqrt{b}\,\sqrt{ax^2+b}+ax^2}\right)}{2a^{3/2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((b + a*x^2)^(3/4)*(2*b + a*x^2)),x]
```

```
[Out] -1/2*ArcTan[(-1/2*(Sqrt[a]*x^2)/b^(1/4) + (b^(1/4)*Sqrt[b + a*x^2])/Sqrt[a]
)/(x*(b + a*x^2)^(1/4))]/(a^(3/2)*b^(1/4)) - ArcTanh[(2*Sqrt[a]*b^(1/4)*x*(
b + a*x^2)^(1/4))/(a*x^2 + 2*Sqrt[b]*Sqrt[b + a*x^2])]/(2*a^(3/2)*b^(1/4))
```

fricas [B] time = 0.43, size = 207, normalized size = 1.52

$$-2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^6b}\right)^{\frac{1}{4}}\arctan\left(\frac{4\left(\sqrt{\frac{1}{2}}\left(\frac{1}{4}\right)^{\frac{3}{4}}a^4bx\sqrt{\frac{a^4x^2\sqrt{-\frac{1}{a^6b}}+2\sqrt{ax^2+b}}{x^2}}\left(-\frac{1}{a^6b}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(ax^2+b\right)^{\frac{1}{4}}a^4b\left(-\frac{1}{a^6b}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^6b}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}a^2x\left(-\frac{1}{a^6b}\right)^{\frac{1}{4}}+\left(ax^2+b\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^6b}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}a^2x\left(-\frac{1}{a^6b}\right)^{\frac{1}{4}}-\left(ax^2+b\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2+b)^(3/4)/(a*x^2+2*b),x, algorithm="fricas")
```

```
[Out] -2*(1/4)^(1/4)*(-1/(a^6*b))^(1/4)*arctan(4*(sqrt(1/2)*(1/4)^(3/4)*a^4*b*x*s
qrt((a^4*x^2*sqrt(-1/(a^6*b)) + 2*sqrt(a*x^2 + b))/x^2)*(-1/(a^6*b))^(3/4)
- (1/4)^(3/4)*(a*x^2 + b)^(1/4)*a^4*b*(-1/(a^6*b))^(3/4))/x) - 1/2*(1/4)^(1
/4)*(-1/(a^6*b))^(1/4)*log(((1/4)^(1/4)*a^2*x*(-1/(a^6*b))^(1/4) + (a*x^2 +
 b)^(1/4))/x) + 1/2*(1/4)^(1/4)*(-1/(a^6*b))^(1/4)*log(-((1/4)^(1/4)*a^2*x*
(-1/(a^6*b))^(1/4) - (a*x^2 + b)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax^2+2\,b\right)\left(ax^2+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2+b)^(3/4)/(a*x^2+2*b),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a*x^2 + 2*b)*(a*x^2 + b)^(3/4)), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a\,x^2+b\right)^{\frac{3}{4}}\left(a\,x^2+2b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2+b)^(3/4)/(a*x^2+2*b),x)
```

```
[Out] int(x^2/(a*x^2+b)^(3/4)/(a*x^2+2*b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax^2+2\,b\right)\left(ax^2+b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2+b)^(3/4)/(a*x^2+2*b),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a*x^2 + 2*b)*(a*x^2 + b)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(a\,x^2+b\right)^{3/4}\,\left(a\,x^2+2\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b + a*x^2)^(3/4)*(2*b + a*x^2)),x)
```

```
[Out] int(x^2/((b + a*x^2)^(3/4)*(2*b + a*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax^2+b\right)^{\frac{3}{4}}\left(ax^2+2b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**2+b)**(3/4)/(a*x**2+2*b),x)
```

```
[Out] Integral(x**2/((a*x**2 + b)**(3/4)*(a*x**2 + 2*b)), x)
```

3.1625 $\int \frac{x}{\left(-b+ax^3\right)^{2/3}}\,dx$

Optimal. Leaf size=136

$$-\frac{\log\left(\sqrt[3]{ax^3-b}-\sqrt[3]{a}\,x\right)}{3a^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a}\,x}{2\sqrt[3]{ax^3-b}+\sqrt[3]{a}\,x}\right)}{\sqrt{3}\,a^{2/3}}+\frac{\log\left(a^{2/3}x^2+\sqrt[3]{a}\,x\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}\right)}{6a^{2/3}}$$

Rubi [A] time = 0.07, antiderivative size = 130, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {331, 292, 31, 634, 617, 204, 628}

$$-\frac{\log\left(1-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{ax^3-b}}\right)}{3a^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{ax^3-b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}}+\frac{\log\left(\frac{a^{2/3}x^2}{\left(ax^3-b\right)^{2/3}}+\frac{\sqrt[3]{a}\,x}{\sqrt[3]{ax^3-b}}+1\right)}{6a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(-b + a*x^3)^(2/3),x]
```

```
[Out] -(ArcTan[(1 + (2*a^(1/3)*x)/(-b + a*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*a^(2/3)))
 - Log[1 - (a^(1/3)*x)/(-b + a*x^3)^(1/3)]/(3*a^(2/3)) + Log[1 + (a^(2/3)*x
^2)/(-b + a*x^3)^(2/3) + (a^(1/3)*x)/(-b + a*x^3)^(1/3)]/(6*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\left(-b+ax^3\right)^{2/3}}\,dx = \text{Subst}\left(\int \frac{x}{1-ax^3}\,dx, x, \frac{x}{\sqrt[3]{-b+ax^3}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{a}x}\,dx, x, \frac{x}{\sqrt[3]{-b+ax^3}}\right)}{3\sqrt[3]{a}} - \frac{\text{Subst}\left(\int \frac{1-\sqrt[3]{a}x}{1+\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{-b+ax^3}}\right)}{3\sqrt[3]{a}}$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{a}x}{\sqrt[3]{-b+ax^3}}\right)}{3a^{2/3}} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}+2a^{2/3}x}{1+\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{-b+ax^3}}\right)}{6a^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{a}x+a^{2/3}x^2}\,d\ldots\right.}{2\sqrt[3]{a}}$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{a}x}{\sqrt[3]{-b+ax^3}}\right)}{3a^{2/3}} + \frac{\log\left(1+\frac{a^{2/3}x^2}{\left(-b+ax^3\right)^{2/3}}+\frac{\sqrt[3]{a}x}{\sqrt[3]{-b+ax^3}}\right)}{6a^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a}\ldots}{\sqrt[3]{-b+\ldots}}\right.}{a^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a}x}{\sqrt[3]{-b+ax^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}} - \frac{\log\left(1-\frac{\sqrt[3]{a}x}{\sqrt[3]{-b+ax^3}}\right)}{3a^{2/3}} + \frac{\log\left(1+\frac{a^{2/3}x^2}{\left(-b+ax^3\right)^{2/3}}+\frac{\sqrt[3]{a}x}{\sqrt[3]{-b+ax^3}}\right)}{6a^{2/3}}$$

Mathematica [C] time = 0.01, size = 44, normalized size = 0.32

$$\frac{x^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{ax^3}{ax^3-b}\right)}{2\left(ax^3-b\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-b + a*x^3)^(2/3),x]
```

```
[Out] (x^2*Hypergeometric2F1[2/3, 1, 5/3, (a*x^3)/(-b + a*x^3)])/(2*(-b + a*x^3)^
(2/3))
```

IntegrateAlgebraic [A] time = 0.23, size = 136, normalized size = 1.00

$$-\frac{\log\left(\sqrt[3]{ax^3-b}-\sqrt[3]{a}x\right)}{3a^{2/3}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a}x}{2\sqrt[3]{ax^3-b}+\sqrt[3]{a}x}\right)}{\sqrt{3}\,a^{2/3}} + \frac{\log\left(a^{2/3}x^2+\sqrt[3]{a}x\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}\right)}{6a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(-b + a*x^3)^(2/3),x]
```

```
[Out] -(ArcTan[(Sqrt[3]*a^(1/3)*x)/(a^(1/3)*x + 2*(-b + a*x^3)^(1/3))]/(Sqrt[3]*a
^(2/3))) - Log[-(a^(1/3)*x) + (-b + a*x^3)^(1/3)]/(3*a^(2/3)) + Log[a^(2/3)
*x^2 + a^(1/3)*x*(-b + a*x^3)^(1/3) + (-b + a*x^3)^(2/3)]/(6*a^(2/3))
```

fricas [A] time = 0.40, size = 181, normalized size = 1.33

$$\frac{2\sqrt{3}\,a\sqrt{-\left(-a^{2}\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-a^{2}\right)^{\frac{1}{3}}ax-2\sqrt{3}\left(ax^{3}-b\right)^{\frac{1}{3}}\left(-a^{2}\right)^{\frac{2}{3}}\right)\sqrt{-\left(-a^{2}\right)^{\frac{1}{3}}}}{3\,a^{2}x}\right)-2\left(-a^{2}\right)^{\frac{2}{3}}\log\left(-\frac{\left(-a^{2}\right)^{\frac{2}{3}}x-\left(ax^{3}-b\right)^{\frac{1}{3}}a}{x}\right)+\left(-a^{2}\right)^{\frac{2}{3}}\log\left(-\frac{\left(-a^{2}\right)^{\frac{1}{3}}ax^{2}-\left(ax^{3}-b\right)^{\frac{1}{3}}\left(-a^{2}\right)^{\frac{2}{3}}x-\left(ax^{3}-b\right)^{\frac{2}{3}}a}{x^{2}}\right)}{6\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^3-b)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*a*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(1/3)*a*x
 - 2*sqrt(3)*(a*x^3 - b)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/(a^2*x)) -
2*(-a^2)^(2/3)*log(-((-a^2)^(2/3)*x - (a*x^3 - b)^(1/3)*a)/x) + (-a^2)^(2/3
)*log(-((-a^2)^(1/3)*a*x^2 - (a*x^3 - b)^(1/3)*(-a^2)^(2/3)*x - (a*x^3 - b)
^(2/3)*a)/x^2))/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax^{3}-b\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^3-b)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x/(a*x^3 - b)^(2/3), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a\,x^{3}-b\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^3-b)^(2/3),x)
```

```
[Out] int(x/(a*x^3-b)^(2/3),x)
```

maxima [A] time = 0.43, size = 108, normalized size = 0.79

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a^{\frac{1}{3}}+\frac{2\left(ax^{3}-b\right)^{\frac{1}{3}}}{x}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}}+\frac{\log\left(a^{\frac{2}{3}}+\frac{\left(ax^{3}-b\right)^{\frac{1}{3}}a^{\frac{1}{3}}}{x}+\frac{\left(ax^{3}-b\right)^{\frac{2}{3}}}{x^{2}}\right)}{6\,a^{\frac{2}{3}}}-\frac{\log\left(-a^{\frac{1}{3}}+\frac{\left(ax^{3}-b\right)^{\frac{1}{3}}}{x}\right)}{3\,a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^3-b)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(a^(1/3) + 2*(a*x^3 - b)^(1/3)/x)/a^(1/3))/a
^(2/3) + 1/6*log(a^(2/3) + (a*x^3 - b)^(1/3)*a^(1/3)/x + (a*x^3 - b)^(2/3)/
x^2)/a^(2/3) - 1/3*log(-a^(1/3) + (a*x^3 - b)^(1/3)/x)/a^(2/3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(a\,x^3-b\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^3 - b)^(2/3),x)
```

```
[Out] int(x/(a*x^3 - b)^(2/3), x)
```

sympy [C] time = 0.94, size = 41, normalized size = 0.30

$$\frac{x^2 e^{-\frac{2i\pi}{3}} \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix} \middle| \frac{ax^3}{b}\right)}{3b^{\frac{2}{3}} \Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x**3-b)**(2/3),x)
```

```
[Out] x**2*exp(-2*I*pi/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), a*x**3/b)/(3*b**(2
/3)*gamma(5/3))
```

3.1626 $\int \frac{x \sqrt[3]{x^2+x^4}}{1+2x^2} \, dx$

Optimal. Leaf size=136

$$\frac{3}{8}\sqrt[3]{x^4+x^2}-\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}+1\right)}{8\ 2^{2/3}}+\frac{\log\left(-2\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}+2^{2/3}\sqrt[3]{x^4+x^2}-1\right)}{16\ 2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\ 2^{2/3}\sqrt[3]{x^4+x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}}$$

Rubi [A] time = 0.21, antiderivative size = 109, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2034, 694, 266, 50, 58, 617, 204, 31}

$$\frac{3\sqrt[3]{\left(2x^2+1\right)^2-1}}{8\ 2^{2/3}}+\frac{\log\left(2x^2+1\right)}{8\ 2^{2/3}}-\frac{3\log\left(\sqrt[3]{\left(2x^2+1\right)^2-1}+1\right)}{16\ 2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\left(2x^2+1\right)^2-1}}{\sqrt{3}}\right)}{8\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(x^2 + x^4)^(1/3))/(1 + 2*x^2),x]
```

```
[Out] (3*(-1 + (1 + 2*x^2)^2)^(1/3))/(8*2^(2/3)) + (Sqrt[3]*ArcTan[(1 - 2*(-1 + (
1 + 2*x^2)^2)^(1/3))/Sqrt[3]])/(8*2^(2/3)) + Log[1 + 2*x^2]/(8*2^(2/3)) - (
3*Log[1 + (-1 + (1 + 2*x^2)^2)^(1/3)])/(16*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x\sqrt[3]{x^2+x^4}}{1+2x^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt[3]{x+x^2}}{1+2x}\,dx, x, x^2\right)$$
$$= \frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt[3]{-\frac{1}{4}+\frac{x^2}{4}}}{x}\,dx, x, 1+2x^2\right)$$
$$= \frac{1}{8}\,\text{Subst}\left(\int \frac{\sqrt[3]{-\frac{1}{4}+\frac{x}{4}}}{x}\,dx, x, \left(1+2x^2\right)^2\right)$$
$$= \frac{3\sqrt[3]{-1+\left(1+2x^2\right)^2}}{8\ 2^{2/3}} - \frac{1}{32}\,\text{Subst}\left(\int \frac{1}{\left(-\frac{1}{4}+\frac{x}{4}\right)^{2/3}x}\,dx, x, \left(1+2x^2\right)^2\right)$$
$$= \frac{3\sqrt[3]{-1+\left(1+2x^2\right)^2}}{8\ 2^{2/3}} + \frac{\log\left(1+2x^2\right)}{8\ 2^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{1}{2^{2/3}}+x}\,dx, x, \sqrt[3]{x^2+x^4}\right)}{16\ 2^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{}{}\right.}{}$$
$$= \frac{3\sqrt[3]{-1+\left(1+2x^2\right)^2}}{8\ 2^{2/3}} + \frac{\log\left(1+2x^2\right)}{8\ 2^{2/3}} - \frac{3\log\left(1+2^{2/3}\sqrt[3]{x^2+x^4}\right)}{16\ 2^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x\right.}{8\ 2}$$
$$= \frac{3\sqrt[3]{-1+\left(1+2x^2\right)^2}}{8\ 2^{2/3}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\ 2^{2/3}\sqrt[3]{x^2+x^4}}{\sqrt{3}}\right)}{8\ 2^{2/3}} + \frac{\log\left(1+2x^2\right)}{8\ 2^{2/3}} - \frac{3\log\left(1+2^{2/3}\sqrt[3]{x^2+\right.}{16\ 2^{2/3}}$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.42

$$\frac{3\sqrt[3]{x^4+x^2}\left(\sqrt[3]{x^2+1}-F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-x^2,-2x^2\right)\right)}{8\sqrt[3]{x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(x^2 + x^4)^(1/3))/(1 + 2*x^2),x]
```

```
[Out] (3*(x^2 + x^4)^(1/3)*((1 + x^2)^(1/3) - AppellF1[1/3, 2/3, 1, 4/3, -x^2, -2
*x^2]))/(8*(1 + x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.27, size = 136, normalized size = 1.00

$$\frac{3}{8}\sqrt[3]{x^4+x^2}-\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}+1\right)}{8\ 2^{2/3}}+\frac{\log\left(-2\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}+2^{2/3}\sqrt[3]{x^4+x^2}-1\right)}{16\ 2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\ 2^{2/3}\sqrt[3]{x^4+x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(x^2 + x^4)^(1/3))/(1 + 2*x^2),x]
```

```
[Out] (3*(x^2 + x^4)^(1/3))/8 + (Sqrt[3]*ArcTan[1/Sqrt[3] - (2*2^(2/3)*(x^2 + x^4
)^(1/3))/Sqrt[3]])/(8*2^(2/3)) - Log[1 + 2^(2/3)*(x^2 + x^4)^(1/3)]/(8*2^(2
/3)) + Log[-1 + 2^(2/3)*(x^2 + x^4)^(1/3) - 2*2^(1/3)*(x^2 + x^4)^(2/3)]/(1
6*2^(2/3))
```

fricas [A] time = 0.41, size = 128, normalized size = 0.94

$$\frac{1}{16}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(-1\right)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(2\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\right)\right)-\frac{1}{64}\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}+4\left(x^4+x^2\right)^{\frac{2}{3}}\right)+\frac{1}{32}\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}+4\left(x^4+x^2\right)^{\frac{1}{3}}\right)+\frac{3}{8}\left(x^4+x^2\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4+x^2)^(1/3)/(2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/16*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*sqrt(3)*(2*4^(2/3)*(-1)^
(2/3)*(x^4 + x^2)^(1/3) - 4^(1/3))) - 1/64*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(
-1)^(1/3)*(x^4 + x^2)^(1/3) + 4^(1/3)*(-1)^(2/3) + 4*(x^4 + x^2)^(2/3)) + 1
/32*4^(2/3)*(-1)^(1/3)*log(-4^(2/3)*(-1)^(1/3) + 4*(x^4 + x^2)^(1/3)) + 3/8
*(x^4 + x^2)^(1/3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^2\right)^{\frac{1}{3}} x}{2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4+x^2)^(1/3)/(2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + x^2)^(1/3)*x/(2*x^2 + 1), x)
```

maple [C] time = 19.82, size = 1333, normalized size = 9.80

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^4+x^2)^(1/3)/(2*x^2+1),x)
```

```
[Out] 3/8*(x^2*(x^2+1))^(1/3)+(1/16*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^
2)*ln((-1698*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)*RootOf(_Z^3+2)
^3*x^6+524*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)^2*RootOf(_Z^3+2)
^2*x^6-3396*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)*RootOf(_Z^3+2)^
3*x^4+1048*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)^2*RootOf(_Z^3+2)
^2*x^4+27168*RootOf(_Z^3+2)*x^6-8384*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3
+2)+_Z^2)*x^6-18048*(x^5+2*x^3+x)^(1/3)*RootOf(_Z^3+2)^2*x^3-30186*(x^5+2*x
^3+x)^(1/3)*RootOf(_Z^3+2)*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)*
x^3+1698*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)*RootOf(_Z^3+2)^3*x
^2-524*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)^2*RootOf(_Z^3+2)^2*x
```

```
^2+9024*(x^5+2*x^3+x)^(2/3)*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)
*RootOf(_Z^3+2)^2+54336*RootOf(_Z^3+2)*x^4-16768*RootOf(RootOf(_Z^3+2)^2+_Z
*RootOf(_Z^3+2)+_Z^2)*x^4-18048*(x^5+2*x^3+x)^(1/3)*RootOf(_Z^3+2)^2*x-3018
6*(x^5+2*x^3+x)^(1/3)*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)*RootO
f(_Z^3+2)*x+3396*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)*RootOf(_Z^
3+2)^3-1048*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)^2*RootOf(_Z^3+2
)^2+21225*RootOf(_Z^3+2)*x^2-6550*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)
+_Z^2)*x^2+12138*(x^5+2*x^3+x)^(2/3)-5943*RootOf(_Z^3+2)+1834*RootOf(RootOf
(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2))/(2*x^2+1)^2/(x^2+1))+1/16*RootOf(_Z^3+2
)*ln(-(524*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)*RootOf(_Z^3+2)^3
*x^6-1698*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)^2*RootOf(_Z^3+2)^
2*x^6+1048*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)*RootOf(_Z^3+2)^3
*x^4-3396*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)^2*RootOf(_Z^3+2)^
2*x^4+7336*RootOf(_Z^3+2)*x^6-23772*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+
2)+_Z^2)*x^6-18048*(x^5+2*x^3+x)^(1/3)*RootOf(_Z^3+2)^2*x^3+12138*(x^5+2*x^
3+x)^(1/3)*RootOf(_Z^3+2)*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)*x
^3-524*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)*RootOf(_Z^3+2)^3*x^2
+1698*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)^2*RootOf(_Z^3+2)^2*x^
2+9024*(x^5+2*x^3+x)^(2/3)*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)*
RootOf(_Z^3+2)^2+14672*RootOf(_Z^3+2)*x^4-47544*RootOf(RootOf(_Z^3+2)^2+_Z*
RootOf(_Z^3+2)+_Z^2)*x^4-18048*(x^5+2*x^3+x)^(1/3)*RootOf(_Z^3+2)^2*x+12138
*(x^5+2*x^3+x)^(1/3)*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)*RootOf
(_Z^3+2)*x-1048*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)*RootOf(_Z^3
+2)^3+3396*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2)^2*RootOf(_Z^3+2)
^2+7598*RootOf(_Z^3+2)*x^2-24621*RootOf(RootOf(_Z^3+2)^2+_Z*RootOf(_Z^3+2)+
_Z^2)*x^2-30186*(x^5+2*x^3+x)^(2/3)+262*RootOf(_Z^3+2)-849*RootOf(RootOf(_Z
^3+2)^2+_Z*RootOf(_Z^3+2)+_Z^2))/(2*x^2+1)^2/(x^2+1)))/x*(x^2*(x^2+1))^(1/3
)*(x*(x^2+1)^2)^(1/3)/(x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^2\right)^{\frac{1}{3}}x}{2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4+x^2)^(1/3)/(2*x^2+1),x, algorithm="maxima")
[Out] integrate((x^4 + x^2)^(1/3)*x/(2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(x^4+x^2\right)^{1/3}}{2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^2 + x^4)^(1/3))/(2*x^2 + 1),x)
[Out] int((x*(x^2 + x^4)^(1/3))/(2*x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{x^2\left(x^2+1\right)}}{2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**4+x**2)**(1/3)/(2*x**2+1),x)
[Out] Integral(x*(x**2*(x**2 + 1))**(1/3)/(2*x**2 + 1), x)
```

3.1627 $\int \frac{b+ax^3}{x^6\left(-b+ax^3\right)\sqrt[4]{bx+ax^4}}\,dx$

Optimal. Leaf size=136

$$-\frac{2\,2^{3/4}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3b^2}-\frac{2\,2^{3/4}a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3b^2}+\frac{4\left(10ax^3+3b\right)\left(ax^4+bx\right)^{3/4}}{63b^2x^6}$$

Rubi [C] time = 4.75, antiderivative size = 132, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2056, 466, 465, 511, 510}

$$-\frac{\Gamma\left(-\frac{3}{4}\right)\left(ax^3+b\right)^2\left(\left(4a^2x^6-7abx^3+3b^2\right)\,_2F_1\left(1,1;\frac{1}{4};-\frac{2ax^3}{b-ax^3}\right)+32ax^3\left(ax^3+b\right)\,_2F_1\left(2,2;\frac{5}{4};-\frac{2ax^3}{b-ax^3}\right)\right)}{21b^2x^5\Gamma\left(\frac{1}{4}\right)\left(b-ax^3\right)^2\sqrt[4]{ax^4+bx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(b + a*x^3)/(x^6*(-b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] -1/21*((b + a*x^3)^2*Gamma[-3/4]*((3*b^2 - 7*a*b*x^3 + 4*a^2*x^6)*Hypergeom
etric2F1[1, 1, 1/4, (-2*a*x^3)/(b - a*x^3)] + 32*a*x^3*(b + a*x^3)*Hypergeo
metric2F1[2, 2, 5/4, (-2*a*x^3)/(b - a*x^3)]))/(b^2*x^5*(b - a*x^3)^2*(b*x
+ a*x^4)^(1/4)*Gamma[1/4])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{b+ax^3}{x^6\left(-b+ax^3\right)\sqrt[4]{bx+ax^4}}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\int \frac{\left(b+ax^3\right)^{3/4}}{x^{25/4}\left(-b+ax^3\right)}\,dx}{\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{\left(b+ax^{12}\right)^{3/4}}{x^{22}\left(-b+ax^{12}\right)}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{\left(b+ax^4\right)^{3/4}}{x^8\left(-b+ax^4\right)}\,dx, x, x^{3/4}\right)}{3\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\left(b+ax^3\right)\right)\text{Subst}\left(\int \frac{\left(1+\frac{ax^4}{b}\right)^{3/4}}{x^8\left(-b+ax^4\right)}\,dx, x, x^{3/4}\right)}{3\left(1+\frac{ax^3}{b}\right)^{3/4}\sqrt[4]{bx+ax^4}}$$

$$= -\frac{\left(b+ax^3\right)^2\Gamma\left(-\frac{3}{4}\right)\left(\left(3b^2-7abx^3+4a^2x^6\right){}_2F_1\left(1,1;\frac{1}{4};-\frac{2ax^3}{b-ax^3}\right)+32ax^3\left(b\right.\right.}{21b^2x^5\left(b-ax^3\right)^2\sqrt[4]{bx+ax^4}\,\Gamma\left(\frac{1}{4}\right)}$$

Mathematica [C] time = 4.89, size = 123, normalized size = 0.90

$$-\frac{\Gamma\left(-\frac{3}{4}\right)\left(x\left(ax^3+b\right)\right)^{7/4}\left(\left(4a^2x^6-7abx^3+3b^2\right){}_2F_1\left(1,1;\frac{1}{4};-\frac{2ax^3}{b-ax^3}\right)+32ax^3\left(ax^3+b\right){}_2F_1\left(2,2;\frac{5}{4};-\frac{2ax^3}{b-ax^3}\right)\right)}{21b^2x^7\Gamma\left(\frac{1}{4}\right)\left(b-ax^3\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b + a*x^3)/(x^6*(-b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] -1/21*((x*(b + a*x^3))^(7/4)*Gamma[-3/4]*((3*b^2 - 7*a*b*x^3 + 4*a^2*x^6)*H
ypergeometric2F1[1, 1, 1/4, (-2*a*x^3)/(b - a*x^3)] + 32*a*x^3*(b + a*x^3)*
Hypergeometric2F1[2, 2, 5/4, (-2*a*x^3)/(b - a*x^3)]))/(b^2*x^7*(b - a*x^3)
^2*Gamma[1/4])
```

IntegrateAlgebraic [A] time = 0.51, size = 136, normalized size = 1.00

$$-\frac{2\,2^{3/4}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3b^2}-\frac{2\,2^{3/4}a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3b^2}+\frac{4\left(10ax^3+3b\right)\left(ax^4+bx\right)^{3/4}}{63b^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^3)/(x^6*(-b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*(3*b + 10*a*x^3)*(b*x + a*x^4)^(3/4))/(63*b^2*x^6) - (2*2^(3/4)*a^(7/4)*
ArcTan[(2^(1/4)*a^(1/4)*(b*x + a*x^4)^(3/4))/(b + a*x^3)])/(3*b^2) - (2*2^(
3/4)*a^(7/4)*ArcTanh[(2^(1/4)*a^(1/4)*(b*x + a*x^4)^(3/4))/(b + a*x^3)])/(3
*b^2)
```

fricas [B] time = 152.07, size = 511, normalized size = 3.76

$$\frac{84\cdot 8^{\frac{1}{4}}b^{2}x^{6}\left(\frac{a^{7}}{b^{8}}\right)^{\frac{1}{4}}\arctan\left(\frac{16\cdot 8^{\frac{1}{4}}\left(ax^{4}+bx\right)^{\frac{1}{4}}a^{9}b^{2}x^{2}\left(\frac{a^{7}}{b^{8}}\right)^{\frac{1}{4}}+4\cdot 8^{\frac{3}{4}}\left(ax^{4}+bx\right)^{\frac{3}{4}}a^{5}b^{6}\left(\frac{a^{7}}{b^{8}}\right)^{\frac{3}{4}}+\sqrt{2}\sqrt{\sqrt{2}a^{6}b^{4}\sqrt{\frac{a^{7}}{b^{8}}}}\left(8\cdot 8^{\frac{1}{4}}\sqrt{ax^{4}+bx}\,a^{4}b^{2}x\left(\frac{a^{7}}{b^{8}}\right)^{\frac{1}{4}}+8^{\frac{3}{4}}\left(3ab^{6}x^{3}+b^{7}\right)\left(\frac{a^{7}}{b^{8}}\right)^{\frac{3}{4}}\right)}{8\left(a^{11}x^{3}-a^{10}b\right)}\right)-21\cdot 8^{\frac{1}{4}}b^{2}x^{6}\left(\frac{a^{7}}{b^{8}}\right)^{\frac{1}{4}}\log\left(\frac{4\sqrt{2}\left(ax^{4}+bx\right)^{\frac{1}{4}}a^{2}b^{4}x^{2}\sqrt{\frac{a^{7}}{b^{8}}}+8^{\frac{3}{4}}\sqrt{ax^{4}+bx}\,b^{6}x\left(\frac{a^{7}}{b^{8}}\right)^{\frac{3}{4}}+4\left(ax^{4}+bx\right)^{\frac{3}{4}}a^{5}+8^{\frac{1}{4}}\left(3a^{4}b^{2}x^{3}+a^{3}b^{3}\right)\left(\frac{a^{7}}{b^{8}}\right)^{\frac{1}{4}}}{ax^{3}-b}\right)+21\cdot 8^{\frac{1}{4}}b^{2}x^{6}\left(\frac{a^{7}}{b^{8}}\right)^{\frac{1}{4}}\log\left(\frac{4\sqrt{2}\left(ax^{4}+bx\right)^{\frac{1}{4}}a^{2}b^{4}x^{2}\sqrt{\frac{a^{7}}{b^{8}}}-8^{\frac{3}{4}}\sqrt{ax^{4}+bx}\,b^{6}x\left(\frac{a^{7}}{b^{8}}\right)^{\frac{3}{4}}+4\left(ax^{4}+bx\right)^{\frac{3}{4}}a^{5}-8^{\frac{1}{4}}\left(3a^{4}b^{2}x^{3}+a^{3}b^{3}\right)\left(\frac{a^{7}}{b^{8}}\right)^{\frac{1}{4}}}{ax^{3}-b}\right)+8\left(ax^{4}+bx\right)^{\frac{3}{4}}\left(10ax^{3}+3b\right)}{126b^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)/x^6/(a*x^3-b)/(a*x^4+b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/126*(84*8^(1/4)*b^2*x^6*(a^7/b^8)^(1/4)*arctan(1/8*(16*8^(1/4)*(a*x^4 + b
*x)^(1/4)*a^9*b^2*x^2*(a^7/b^8)^(1/4) + 4*8^(3/4)*(a*x^4 + b*x)^(3/4)*a^5*b
^6*(a^7/b^8)^(3/4) + sqrt(2)*sqrt(sqrt(2)*a^6*b^4*sqrt(a^7/b^8))*(8*8^(1/4)
*sqrt(a*x^4 + b*x)*a^4*b^2*x*(a^7/b^8)^(1/4) + 8^(3/4)*(3*a*b^6*x^3 + b^7)*
(a^7/b^8)^(3/4)))/(a^11*x^3 - a^10*b)) - 21*8^(1/4)*b^2*x^6*(a^7/b^8)^(1/4)
*log((4*sqrt(2)*(a*x^4 + b*x)^(1/4)*a^2*b^4*x^2*sqrt(a^7/b^8) + 8^(3/4)*sqr
t(a*x^4 + b*x)*b^6*x*(a^7/b^8)^(3/4) + 4*(a*x^4 + b*x)^(3/4)*a^5 + 8^(1/4)*
(3*a^4*b^2*x^3 + a^3*b^3)*(a^7/b^8)^(1/4))/(a*x^3 - b)) + 21*8^(1/4)*b^2*x^
6*(a^7/b^8)^(1/4)*log((4*sqrt(2)*(a*x^4 + b*x)^(1/4)*a^2*b^4*x^2*sqrt(a^7/b
^8) - 8^(3/4)*sqrt(a*x^4 + b*x)*b^6*x*(a^7/b^8)^(3/4) + 4*(a*x^4 + b*x)^(3/
4)*a^5 - 8^(1/4)*(3*a^4*b^2*x^3 + a^3*b^3)*(a^7/b^8)^(1/4))/(a*x^3 - b)) +
8*(a*x^4 + b*x)^(3/4)*(10*a*x^3 + 3*b))/(b^2*x^6)
```

giac [B] time = 0.20, size = 233, normalized size = 1.71

$$-\frac{2\cdot 2^{\frac{1}{4}}\left(-a\right)^{\frac{3}{4}}a\arctan\left(\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}\left(-a\right)^{\frac{1}{4}}+2\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)}{3b^{2}}-\frac{2\cdot 2^{\frac{1}{4}}\left(-a\right)^{\frac{3}{4}}a\arctan\left(-\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}\left(-a\right)^{\frac{1}{4}}-2\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)}{3b^{2}}+\frac{2^{\frac{1}{4}}\left(-a\right)^{\frac{3}{4}}a\log\left(2^{\frac{3}{4}}\left(-a\right)^{\frac{1}{4}}\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a+\frac{b}{x^{3}}}\right)}{3b^{2}}-\frac{2^{\frac{1}{4}}\left(-a\right)^{\frac{3}{4}}a\log\left(-2^{\frac{3}{4}}\left(-a\right)^{\frac{1}{4}}\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a+\frac{b}{x^{3}}}\right)}{3b^{2}}+\frac{4\left(3\left(a+\frac{b}{x^{3}}\right)^{\frac{7}{4}}b^{12}+7\left(a+\frac{b}{x^{3}}\right)^{\frac{3}{4}}ab^{12}\right)}{63b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)/x^6/(a*x^3-b)/(a*x^4+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] -2/3*2^(1/4)*(-a)^(3/4)*a*arctan(1/2*2^(1/4)*(2^(3/4)*(-a)^(1/4) + 2*(a + b
/x^3)^(1/4))/(-a)^(1/4))/b^2 - 2/3*2^(1/4)*(-a)^(3/4)*a*arctan(-1/2*2^(1/4)
*(2^(3/4)*(-a)^(1/4) - 2*(a + b/x^3)^(1/4))/(-a)^(1/4))/b^2 + 1/3*2^(1/4)*(
-a)^(3/4)*a*log(2^(3/4)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(2)*sqrt(-a) + s
qrt(a + b/x^3))/b^2 - 1/3*2^(1/4)*(-a)^(3/4)*a*log(-2^(3/4)*(-a)^(1/4)*(a +
 b/x^3)^(1/4) + sqrt(2)*sqrt(-a) + sqrt(a + b/x^3))/b^2 + 4/63*(3*(a + b/x^
3)^(7/4)*b^12 + 7*(a + b/x^3)^(3/4)*a*b^12)/b^14
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{a\,x^{3}+b}{x^{6}\left(a\,x^{3}-b\right)\left(a\,x^{4}+bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b)/x^6/(a*x^3-b)/(a*x^4+b*x)^(1/4),x)
```

```
[Out] int((a*x^3+b)/x^6/(a*x^3-b)/(a*x^4+b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{3}+b}{\left(ax^{4}+bx\right)^{\frac{1}{4}}\left(ax^{3}-b\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b)/x^6/(a*x^3-b)/(a*x^4+b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^3 + b)/((a*x^4 + b*x)^(1/4)*(a*x^3 - b)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a\,x^3+b}{x^6\,{\left(a\,x^4+b\,x\right)}^{1/4}\,\left(b-a\,x^3\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x^3)/(x^6*(b*x + a*x^4)^(1/4)*(b - a*x^3)),x)
[Out] -int((b + a*x^3)/(x^6*(b*x + a*x^4)^(1/4)*(b - a*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^3+b}{x^6\sqrt[4]{x\left(ax^3+b\right)}\left(ax^3-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b)/x**6/(a*x**3-b)/(a*x**4+b*x)**(1/4),x)
[Out] Integral((a*x**3 + b)/(x**6*(x*(a*x**3 + b))**(1/4)*(a*x**3 - b)), x)
```

3.1628 $\int \frac{(-1+x^4)\sqrt[4]{x^2+x^6}}{1+x^4+x^8}\,dx$

Optimal. Leaf size=136

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^6+x^2}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6+x^2}}{\sqrt{x^6+x^2}-x^2}\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^6+x^2}}\right)+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt{2}}+\frac{\sqrt{x^6+x^2}}{\sqrt{2}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\sqrt{2}}$$

Rubi [C] time = 0.47, antiderivative size = 163, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2056, 6728, 466, 510}

$$\frac{2\left(-\sqrt{3}+i\right)x\sqrt[4]{x^6+x^2}\,F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};-x^4,-\frac{2x^4}{1-i\sqrt{3}}\right)}{3\left(\sqrt{3}+i\right)\sqrt[4]{x^4+1}}+\frac{2\left(\sqrt{3}+i\right)x\sqrt[4]{x^6+x^2}\,F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};-x^4,-\frac{2x^4}{1+i\sqrt{3}}\right)}{3\left(-\sqrt{3}+i\right)\sqrt[4]{x^4+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^4 + x^8),x]
```

```
[Out] (2*(I - Sqrt[3])*x*(x^2 + x^6)^(1/4)*AppellF1[3/8, -1/4, 1, 11/8, -x^4, (-2
*x^4)/(1 - I*Sqrt[3])])/(3*(I + Sqrt[3])*(1 + x^4)^(1/4)) + (2*(I + Sqrt[3]
)*x*(x^2 + x^6)^(1/4)*AppellF1[3/8, -1/4, 1, 11/8, -x^4, (-2*x^4)/(1 + I*Sq
rt[3])])/(3*(I - Sqrt[3])*(1 + x^4)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{1+x^4+x^8}\,dx = \frac{\sqrt[4]{x^2+x^6}\int \frac{\sqrt{x}\left(-1+x^4\right)\sqrt[4]{1+x^4}}{1+x^4+x^8}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\sqrt[4]{x^2+x^6}\int \left(\frac{\left(1+i\sqrt{3}\right)\sqrt{x}\sqrt[4]{1+x^4}}{1-i\sqrt{3}+2x^4}+\frac{\left(1-i\sqrt{3}\right)\sqrt{x}\sqrt[4]{1+x^4}}{1+i\sqrt{3}+2x^4}\right)dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\left(\left(1-i\sqrt{3}\right)\sqrt[4]{x^2+x^6}\right)\int \frac{\sqrt{x}\sqrt[4]{1+x^4}}{1+i\sqrt{3}+2x^4}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(\left(1+i\sqrt{3}\right)\sqrt[4]{x^2+x^6}\right)\int \frac{\sqrt{x}\sqrt[4]{1+x^4}}{1-i\sqrt{3}+2x^4}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\left(2\left(1-i\sqrt{3}\right)\sqrt[4]{x^2+x^6}\right)\text{Subst}\left(\int \frac{x^2\sqrt[4]{1+x^8}}{1+i\sqrt{3}+2x^8}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(2\left(1+i\sqrt{3}\right)\sqrt[4]{x^2+x^6}\right.}{}$$

$$= \frac{2\left(i-\sqrt{3}\right)x\sqrt[4]{x^2+x^6}\,F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};-x^4,-\frac{2x^4}{1-i\sqrt{3}}\right)}{3\left(i+\sqrt{3}\right)\sqrt[4]{1+x^4}} + \frac{2\left(i+\sqrt{3}\right)x\sqrt[4]{x^2+x^6}\,F_1\left(\right.}{3\left(i-\sqrt{3}\right)}$$

Mathematica [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{1+x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^4 + x^8),x]
```

```
[Out] Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^4 + x^8), x]
```

IntegrateAlgebraic [A] time = 0.49, size = 136, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^6+x^2}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6+x^2}}{\sqrt{x^6+x^2}-x^2}\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^6+x^2}}\right)+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt{2}}+\frac{\sqrt{x^6+x^2}}{\sqrt{2}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[x/(x^2 + x^6)^(1/4)]/2 - ArcTan[(Sqrt[2]*x*(x^2 + x^6)^(1/4))/(-x^2
+ Sqrt[x^2 + x^6])]/(2*Sqrt[2]) - ArcTanh[x/(x^2 + x^6)^(1/4)]/2 + ArcTanh[
(x^2/Sqrt[2] + Sqrt[x^2 + x^6]/Sqrt[2])/(x*(x^2 + x^6)^(1/4))]/(2*Sqrt[2])
```

fricas [B] time = 12.14, size = 780, normalized size = 5.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan((x^9 + 2*x^7 + 3*x^5 + 2*x^3 + 2*sqrt(2)*(x^6 + x^2)^(3
/4)*(x^4 - 3*x^2 + 1) + 2*sqrt(2)*(3*x^6 - x^4 + 3*x^2)*(x^6 + x^2)^(1/4) +
 4*sqrt(x^6 + x^2)*(x^5 + x^3 + x) + (16*(x^6 + x^2)^(3/4)*x^2 + 2*sqrt(2)*
sqrt(x^6 + x^2)*(x^5 - 3*x^3 + x) + sqrt(2)*(x^9 - 8*x^7 + x^5 - 8*x^3 + x)
 + 4*(x^6 + x^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt((x^5 + x^3 + 2*sqrt(2)*(x^6
```

```
+ x^2)^(1/4)*x^2 + 4*sqrt(x^6 + x^2)*x + 2*sqrt(2)*(x^6 + x^2)^(3/4) + x)/(
x^5 + x^3 + x)) + x)/(x^9 - 14*x^7 + 3*x^5 - 14*x^3 + x)) + 1/4*sqrt(2)*arc
tan((x^9 + 2*x^7 + 3*x^5 + 2*x^3 - 2*sqrt(2)*(x^6 + x^2)^(3/4)*(x^4 - 3*x^2
 + 1) - 2*sqrt(2)*(3*x^6 - x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 4*sqrt(x^6 + x^
2)*(x^5 + x^3 + x) + (16*(x^6 + x^2)^(3/4)*x^2 - 2*sqrt(2)*sqrt(x^6 + x^2)*
(x^5 - 3*x^3 + x) - sqrt(2)*(x^9 - 8*x^7 + x^5 - 8*x^3 + x) + 4*(x^6 + x^4
+ x^2)*(x^6 + x^2)^(1/4))*sqrt((x^5 + x^3 - 2*sqrt(2)*(x^6 + x^2)^(1/4)*x^2
 + 4*sqrt(x^6 + x^2)*x - 2*sqrt(2)*(x^6 + x^2)^(3/4) + x)/(x^5 + x^3 + x))
+ x)/(x^9 - 14*x^7 + 3*x^5 - 14*x^3 + x)) + 1/16*sqrt(2)*log(4*(x^5 + x^3 +
 2*sqrt(2)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(x^6 + x^2)*x + 2*sqrt(2)*(x^6 + x
^2)^(3/4) + x)/(x^5 + x^3 + x)) - 1/16*sqrt(2)*log(4*(x^5 + x^3 - 2*sqrt(2)
*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(x^6 + x^2)*x - 2*sqrt(2)*(x^6 + x^2)^(3/4)
+ x)/(x^5 + x^3 + x)) + 1/4*arctan(2*((x^6 + x^2)^(1/4)*x^2 + (x^6 + x^2)^(
3/4))/(x^5 - x^3 + x)) + 1/4*log(-(x^5 + x^3 - 2*(x^6 + x^2)^(1/4)*x^2 + 2*
sqrt(x^6 + x^2)*x + x - 2*(x^6 + x^2)^(3/4))/(x^5 - x^3 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}+x^{2}\right)^{\frac{1}{4}} \left(x^{4}-1\right)}{x^{8}+x^{4}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^2)^(1/4)*(x^4 - 1)/(x^8 + x^4 + 1), x)
```

maple [C] time = 7.29, size = 420, normalized size = 3.09

$$\frac{\ln\left(-\frac{-x^{5}+2\left(x^{6}+x^{2}\right)^{\frac{3}{4}}-2\sqrt{x^{6}+x^{2}}\, x+2\left(x^{6}+x^{2}\right)^{\frac{1}{4}} x^{2}-x^{3}-x}{\left(x^{4}-x^{2}+1\right) x}\right)}{4}+\frac{\operatorname{RootOf}\left(\_Z^{4}+1\right) \ln\left(\frac{\operatorname{RootOf}\left(\_Z^{4}+1\right)^{3} x^{5}-\operatorname{RootOf}\left(\_Z^{4}+1\right)^{3} x^{3}-2 \operatorname{RootOf}\left(\_Z^{4}+1\right)^{2}\left(x^{6}+x^{2}\right)^{\frac{1}{4}} x^{2}+\operatorname{RootOf}\left(\_Z^{4}+1\right)^{3} x-2\sqrt{x^{6}+x^{2}}\, \operatorname{RootOf}\left(\_Z^{4}+1\right) x-2\left(x^{6}+x^{2}\right)^{\frac{3}{4}}}{x\left(x^{2}+x+1\right)\left(x^{2}-x+1\right)}\right)}{4}+\frac{\operatorname{RootOf}\left(\_Z^{4}+1\right)^{3} \ln\left(-\frac{2\sqrt{x^{6}+x^{2}}\, \operatorname{RootOf}\left(\_Z^{4}+1\right)^{3} x-\operatorname{RootOf}\left(\_Z^{4}+1\right) x^{5}-2 \operatorname{RootOf}\left(\_Z^{4}+1\right)^{2}\left(x^{6}+x^{2}\right)^{\frac{1}{4}} x^{2}+\operatorname{RootOf}\left(\_Z^{4}+1\right) x^{3}+2\left(x^{6}+x^{2}\right)^{\frac{3}{4}}-\operatorname{RootOf}\left(\_Z^{4}+1\right) x}{x\left(x^{2}+x+1\right)\left(x^{2}-x+1\right)}\right)}{4}-\frac{\operatorname{RootOf}\left(\_Z^{4}+1\right)^{2} \ln\left(-\frac{-x^{5} \operatorname{RootOf}\left(\_Z^{4}+1\right)^{2}+2 \operatorname{RootOf}\left(\_Z^{4}+1\right)^{2}\sqrt{x^{6}+x^{2}}\, x-\operatorname{RootOf}\left(\_Z^{4}+1\right)^{2} x^{3}-x \operatorname{RootOf}\left(\_Z^{4}+1\right)^{2}+2\left(x^{6}+x^{2}\right)^{\frac{3}{4}}-2\left(x^{6}+x^{2}\right)^{\frac{1}{4}} x^{2}}{\left(x^{4}-x^{2}+1\right) x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^6+x^2)^(1/4)/(x^8+x^4+1),x)
```

```
[Out] 1/4*ln(-(-x^5+2*(x^6+x^2)^(3/4)-2*(x^6+x^2)^(1/2)*x+2*(x^6+x^2)^(1/4)*x^2-x
^3-x)/(x^4-x^2+1)/x)+1/4*RootOf(_Z^4+1)*ln((RootOf(_Z^4+1)^3*x^5-RootOf(_Z^
4+1)^3*x^3-2*RootOf(_Z^4+1)^2*(x^6+x^2)^(1/4)*x^2+RootOf(_Z^4+1)^3*x-2*(x^6
+x^2)^(1/2)*RootOf(_Z^4+1)*x-2*(x^6+x^2)^(3/4))/x/(x^2+x+1)/(x^2-x+1))+1/4*
RootOf(_Z^4+1)^3*ln(-(2*(x^6+x^2)^(1/2)*RootOf(_Z^4+1)^3*x-RootOf(_Z^4+1)*x
^5-2*RootOf(_Z^4+1)^2*(x^6+x^2)^(1/4)*x^2+RootOf(_Z^4+1)*x^3+2*(x^6+x^2)^(3
/4)-RootOf(_Z^4+1)*x)/x/(x^2+x+1)/(x^2-x+1))-1/4*RootOf(_Z^4+1)^2*ln(-(-x^5
*RootOf(_Z^4+1)^2+2*RootOf(_Z^4+1)^2*(x^6+x^2)^(1/2)*x-RootOf(_Z^4+1)^2*x^3
-x*RootOf(_Z^4+1)^2+2*(x^6+x^2)^(3/4)-2*(x^6+x^2)^(1/4)*x^2)/(x^4-x^2+1)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}+x^{2}\right)^{\frac{1}{4}} \left(x^{4}-1\right)}{x^{8}+x^{4}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + x^2)^(1/4)*(x^4 - 1)/(x^8 + x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(x^6+x^2\right)}^{1/4}\,\left(x^4-1\right)}{x^8+x^4+1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + x^6)^(1/4)*(x^4 - 1))/(x^4 + x^8 + 1),x)
```

```
[Out] int(((x^2 + x^6)^(1/4)*(x^4 - 1))/(x^4 + x^8 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^{2}\left(x^{4}+1\right)}\,(x-1)\,(x+1)\left(x^{2}+1\right)}{\left(x^{2}-x+1\right)\left(x^{2}+x+1\right)\left(x^{4}-x^{2}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**6+x**2)**(1/4)/(x**8+x**4+1),x)
```

```
[Out] Integral((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)/((x**2 - x + 1
)*(x**2 + x + 1)*(x**4 - x**2 + 1)), x)
```

3.1629 $\int \frac{(-1+x^4)\sqrt[4]{x^2+x^6}}{1+x^4+x^8}\,dx$

Optimal. Leaf size=136

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^6+x^2}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2}x\sqrt[4]{x^6+x^2}}{\sqrt{x^6+x^2}-x^2}\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^6+x^2}}\right)+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt{2}}+\frac{\sqrt{x^6+x^2}}{\sqrt{2}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\sqrt{2}}$$

Rubi [C] time = 0.43, antiderivative size = 163, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2056, 6728, 466, 510}

$$\frac{2\left(-\sqrt{3}+i\right)x\sqrt[4]{x^6+x^2}\,F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};-x^4,-\frac{2x^4}{1-i\sqrt{3}}\right)}{3\left(\sqrt{3}+i\right)\sqrt[4]{x^4+1}}+\frac{2\left(\sqrt{3}+i\right)x\sqrt[4]{x^6+x^2}\,F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};-x^4,-\frac{2x^4}{1+i\sqrt{3}}\right)}{3\left(-\sqrt{3}+i\right)\sqrt[4]{x^4+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^4 + x^8),x]
```

```
[Out] (2*(I - Sqrt[3])*x*(x^2 + x^6)^(1/4)*AppellF1[3/8, -1/4, 1, 11/8, -x^4, (-2
*x^4)/(1 - I*Sqrt[3])])/(3*(I + Sqrt[3])*(1 + x^4)^(1/4)) + (2*(I + Sqrt[3]
)*x*(x^2 + x^6)^(1/4)*AppellF1[3/8, -1/4, 1, 11/8, -x^4, (-2*x^4)/(1 + I*Sq
rt[3])])/(3*(I - Sqrt[3])*(1 + x^4)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{1+x^4+x^8}\,dx = \frac{\sqrt[4]{x^2+x^6}\int \frac{\sqrt{x}\left(-1+x^4\right)\sqrt[4]{1+x^4}}{1+x^4+x^8}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\sqrt[4]{x^2+x^6}\int \left(\frac{\left(1+i\sqrt{3}\right)\sqrt{x}\sqrt[4]{1+x^4}}{1-i\sqrt{3}+2x^4}+\frac{\left(1-i\sqrt{3}\right)\sqrt{x}\sqrt[4]{1+x^4}}{1+i\sqrt{3}+2x^4}\right)dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\left(\left(1-i\sqrt{3}\right)\sqrt[4]{x^2+x^6}\right)\int \frac{\sqrt{x}\sqrt[4]{1+x^4}}{1+i\sqrt{3}+2x^4}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(\left(1+i\sqrt{3}\right)\sqrt[4]{x^2+x^6}\right)\int \frac{\sqrt{x}\sqrt[4]{1+x^4}}{1-i\sqrt{3}+2x^4}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\left(2\left(1-i\sqrt{3}\right)\sqrt[4]{x^2+x^6}\right)\text{Subst}\left(\int \frac{x^2\sqrt[4]{1+x^8}}{1+i\sqrt{3}+2x^8}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(2\left(1+i\sqrt{3}\right)\sqrt[4]{x^2+x^6}\right.}{\sqrt{}}$$

$$= \frac{2\left(i-\sqrt{3}\right)x\sqrt[4]{x^2+x^6}\,F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};-x^4,-\frac{2x^4}{1-i\sqrt{3}}\right)}{3\left(i+\sqrt{3}\right)\sqrt[4]{1+x^4}} + \frac{2\left(i+\sqrt{3}\right)x\sqrt[4]{x^2+x^6}\,F_1\left(\right.}{3\left(i-\sqrt{3}\right)}$$

Mathematica [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{1+x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^4 + x^8),x]
```

```
[Out] Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^4 + x^8), x]
```

IntegrateAlgebraic [A] time = 0.00, size = 136, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^6+x^2}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^6+x^2}}{\sqrt{x^6+x^2}-x^2}\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^6+x^2}}\right)+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt{2}}+\frac{\sqrt{x^6+x^2}}{\sqrt{2}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[x/(x^2 + x^6)^(1/4)]/2 - ArcTan[(Sqrt[2]*x*(x^2 + x^6)^(1/4))/(-x^2
+ Sqrt[x^2 + x^6])]/(2*Sqrt[2]) - ArcTanh[x/(x^2 + x^6)^(1/4)]/2 + ArcTanh[
(x^2/Sqrt[2] + Sqrt[x^2 + x^6]/Sqrt[2])/(x*(x^2 + x^6)^(1/4))]/(2*Sqrt[2])
```

fricas [B] time = 12.43, size = 780, normalized size = 5.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan((x^9 + 2*x^7 + 3*x^5 + 2*x^3 + 2*sqrt(2)*(x^6 + x^2)^(3
/4)*(x^4 - 3*x^2 + 1) + 2*sqrt(2)*(3*x^6 - x^4 + 3*x^2)*(x^6 + x^2)^(1/4) +
 4*sqrt(x^6 + x^2)*(x^5 + x^3 + x) + (16*(x^6 + x^2)^(3/4)*x^2 + 2*sqrt(2)*
sqrt(x^6 + x^2)*(x^5 - 3*x^3 + x) + sqrt(2)*(x^9 - 8*x^7 + x^5 - 8*x^3 + x)
 + 4*(x^6 + x^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt((x^5 + x^3 + 2*sqrt(2)*(x^6
```

```
+ x^2)^(1/4)*x^2 + 4*sqrt(x^6 + x^2)*x + 2*sqrt(2)*(x^6 + x^2)^(3/4) + x)/(
x^5 + x^3 + x)) + x)/(x^9 - 14*x^7 + 3*x^5 - 14*x^3 + x)) + 1/4*sqrt(2)*arc
tan((x^9 + 2*x^7 + 3*x^5 + 2*x^3 - 2*sqrt(2)*(x^6 + x^2)^(3/4)*(x^4 - 3*x^2
 + 1) - 2*sqrt(2)*(3*x^6 - x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 4*sqrt(x^6 + x^
2)*(x^5 + x^3 + x) + (16*(x^6 + x^2)^(3/4)*x^2 - 2*sqrt(2)*sqrt(x^6 + x^2)*
(x^5 - 3*x^3 + x) - sqrt(2)*(x^9 - 8*x^7 + x^5 - 8*x^3 + x) + 4*(x^6 + x^4
+ x^2)*(x^6 + x^2)^(1/4))*sqrt((x^5 + x^3 - 2*sqrt(2)*(x^6 + x^2)^(1/4)*x^2
 + 4*sqrt(x^6 + x^2)*x - 2*sqrt(2)*(x^6 + x^2)^(3/4) + x)/(x^5 + x^3 + x))
+ x)/(x^9 - 14*x^7 + 3*x^5 - 14*x^3 + x)) + 1/16*sqrt(2)*log(4*(x^5 + x^3 +
 2*sqrt(2)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(x^6 + x^2)*x + 2*sqrt(2)*(x^6 + x
^2)^(3/4) + x)/(x^5 + x^3 + x)) - 1/16*sqrt(2)*log(4*(x^5 + x^3 - 2*sqrt(2)
*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(x^6 + x^2)*x - 2*sqrt(2)*(x^6 + x^2)^(3/4)
+ x)/(x^5 + x^3 + x)) + 1/4*arctan(2*((x^6 + x^2)^(1/4)*x^2 + (x^6 + x^2)^(
3/4))/(x^5 - x^3 + x)) + 1/4*log(-(x^5 + x^3 - 2*(x^6 + x^2)^(1/4)*x^2 + 2*
sqrt(x^6 + x^2)*x + x - 2*(x^6 + x^2)^(3/4))/(x^5 - x^3 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}{x^8+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^2)^(1/4)*(x^4 - 1)/(x^8 + x^4 + 1), x)
```

maple [C] time = 10.49, size = 488, normalized size = 3.59

$$-\frac{\ln\left(-\frac{x^5+2\left(x^6+x^2\right)^{\frac{3}{4}}+2\sqrt{x^6+x^2}\,x+2\left(x^6+x^2\right)^{\frac{1}{4}}x^2+x^3+x}{\left(x^4-x^2+1\right)x}\right)}{4}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)x^5-2\,\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{x^6+x^2}\,x+\mathrm{RootOf}\left(\_Z^2+1\right)x^3+2\left(x^6+x^2\right)^{\frac{3}{4}}-2\left(x^6+x^2\right)^{\frac{1}{4}}x^2+\mathrm{RootOf}\left(\_Z^2+1\right)x}{\left(x^4-x^2+1\right)x}\right)}{4}-\frac{\mathrm{RootOf}\left(\_Z^2-\mathrm{RootOf}\left(\_Z^2+1\right)\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2-\mathrm{RootOf}\left(\_Z^2+1\right)\right)\mathrm{RootOf}\left(\_Z^2+1\right)x^5-\mathrm{RootOf}\left(\_Z^2-\mathrm{RootOf}\left(\_Z^2+1\right)\right)\mathrm{RootOf}\left(\_Z^2+1\right)x^3-2\,\mathrm{RootOf}\left(\_Z^2-\mathrm{RootOf}\left(\_Z^2+1\right)\right)\sqrt{x^6+x^2}\,x+2\left(x^6+x^2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)x^2+2\left(x^6+x^2\right)^{\frac{3}{4}}+\mathrm{RootOf}\left(\_Z^2-\mathrm{RootOf}\left(\_Z^2+1\right)\right)\mathrm{RootOf}\left(\_Z^2+1\right)x}{x\left(x^2-x+1\right)\left(x^2+x+1\right)}\right)}{4}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\mathrm{RootOf}\left(\_Z^2-\mathrm{RootOf}\left(\_Z^2+1\right)\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^2-\mathrm{RootOf}\left(\_Z^2+1\right)\right)x^5+2\,\mathrm{RootOf}\left(\_Z^2-\mathrm{RootOf}\left(\_Z^2+1\right)\right)\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{x^6+x^2}\,x-2\left(x^6+x^2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)x^2+\mathrm{RootOf}\left(\_Z^2-\mathrm{RootOf}\left(\_Z^2+1\right)\right)x^3+2\left(x^6+x^2\right)^{\frac{3}{4}}-\mathrm{RootOf}\left(\_Z^2-\mathrm{RootOf}\left(\_Z^2+1\right)\right)x}{x\left(x^2-x+1\right)\left(x^2+x+1\right)}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^6+x^2)^(1/4)/(x^8+x^4+1),x)
```

```
[Out] -1/4*ln(-(x^5+2*(x^6+x^2)^(3/4)+2*(x^6+x^2)^(1/2)*x+2*(x^6+x^2)^(1/4)*x^2+x
^3+x)/(x^4-x^2+1)/x)+1/4*RootOf(_Z^2+1)*ln(-(RootOf(_Z^2+1)*x^5-2*RootOf(_Z
^2+1)*(x^6+x^2)^(1/2)*x+RootOf(_Z^2+1)*x^3+2*(x^6+x^2)^(3/4)-2*(x^6+x^2)^(1
/4)*x^2+RootOf(_Z^2+1)*x)/(x^4-x^2+1)/x)-1/4*RootOf(_Z^2-RootOf(_Z^2+1))*ln
(-(RootOf(_Z^2-RootOf(_Z^2+1))*RootOf(_Z^2+1)*x^5-RootOf(_Z^2-RootOf(_Z^2+1
))*RootOf(_Z^2+1)*x^3-2*RootOf(_Z^2-RootOf(_Z^2+1))*(x^6+x^2)^(1/2)*x+2*(x^
6+x^2)^(1/4)*RootOf(_Z^2+1)*x^2+2*(x^6+x^2)^(3/4)+RootOf(_Z^2-RootOf(_Z^2+1
))*RootOf(_Z^2+1)*x)/x/(x^2-x+1)/(x^2+x+1))+1/4*RootOf(_Z^2+1)*RootOf(_Z^2-
RootOf(_Z^2+1))*ln(-(-RootOf(_Z^2-RootOf(_Z^2+1))*x^5+2*RootOf(_Z^2-RootOf(
_Z^2+1))*RootOf(_Z^2+1)*(x^6+x^2)^(1/2)*x-2*(x^6+x^2)^(1/4)*RootOf(_Z^2+1)*
x^2+RootOf(_Z^2-RootOf(_Z^2+1))*x^3+2*(x^6+x^2)^(3/4)-RootOf(_Z^2-RootOf(_Z
^2+1))*x)/x/(x^2-x+1)/(x^2+x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}{x^8+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + x^2)^(1/4)*(x^4 - 1)/(x^8 + x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(x^6+x^2\right)}^{1/4}\,\left(x^4-1\right)}{x^8+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + x^6)^(1/4)*(x^4 - 1))/(x^4 + x^8 + 1),x)
```

```
[Out] int(((x^2 + x^6)^(1/4)*(x^4 - 1))/(x^4 + x^8 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\left(x^4+1\right)}\,(x-1)\,(x+1)\left(x^2+1\right)}{\left(x^2-x+1\right)\left(x^2+x+1\right)\left(x^4-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**6+x**2)**(1/4)/(x**8+x**4+1),x)
```

```
[Out] Integral((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)/((x**2 - x + 1
)*(x**2 + x + 1)*(x**4 - x**2 + 1)), x)
```

3.1630 $\int \frac{x\sqrt{ax+\sqrt{-b+ax}}}{\sqrt{-b+ax}}\,dx$

Optimal. Leaf size=136

$$\frac{(-48b^2-8b+5)\log\left(2\sqrt{ax-b}-2\sqrt{\sqrt{ax-b}+ax}+1\right)}{64a^2}+\frac{\sqrt{\sqrt{ax-b}+ax}\,(8ax-12b+15)}{96a^2}+\frac{\sqrt{ax-b}\,(24ax+3$$

Rubi [A] time = 0.54, antiderivative size = 166, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1661, 640, 612, 621, 206}

$$\frac{\sqrt{ax-b}\left(\sqrt{ax-b}+ax\right)^{3/2}}{2a^2}-\frac{5\left(\sqrt{ax-b}+ax\right)^{3/2}}{12a^2}+\frac{(12b+5)\left(2\sqrt{ax-b}+1\right)\sqrt{\sqrt{ax-b}+ax}}{32a^2}-\frac{(1-4b)(12b+5)\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{2\sqrt{\sqrt{ax-b}+ax}}\right)}{64a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a*x + Sqrt[-b + a*x]])/Sqrt[-b + a*x],x]
```

```
[Out] (-5*(a*x + Sqrt[-b + a*x])^(3/2))/(12*a^2) + (Sqrt[-b + a*x]*(a*x + Sqrt[-b
 + a*x])^(3/2))/(2*a^2) + ((5 + 12*b)*Sqrt[a*x + Sqrt[-b + a*x]]*(1 + 2*Sqr
t[-b + a*x]))/(32*a^2) - ((1 - 4*b)*(5 + 12*b)*ArcTanh[(1 + 2*Sqrt[-b + a*x
])/(2*Sqrt[a*x + Sqrt[-b + a*x]])])/(64*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x\sqrt{ax+\sqrt{-b+ax}}}{\sqrt{-b+ax}}\,dx = \frac{2\,\text{Subst}\left(\int \left(b+x^2\right)\sqrt{b+x+x^2}\,dx, x, \sqrt{-b+ax}\right)}{a^2}$$

$$= \frac{\sqrt{-b+ax}\left(ax+\sqrt{-b+ax}\right)^{3/2}}{2a^2} + \frac{\text{Subst}\left(\int \left(3b-\frac{5x}{2}\right)\sqrt{b+x+x^2}\,dx, x, \sqrt{-b+a}\right.}{2a^2}$$

$$= -\frac{5\left(ax+\sqrt{-b+ax}\right)^{3/2}}{12a^2} + \frac{\sqrt{-b+ax}\left(ax+\sqrt{-b+ax}\right)^{3/2}}{2a^2} + \frac{(5+12b)\,\text{Subst}\left(\int \sqrt{\right.}{}$$

$$= -\frac{5\left(ax+\sqrt{-b+ax}\right)^{3/2}}{12a^2} + \frac{\sqrt{-b+ax}\left(ax+\sqrt{-b+ax}\right)^{3/2}}{2a^2} + \frac{(5+12b)\sqrt{ax+\sqrt{-b}}}{3}$$

$$= -\frac{5\left(ax+\sqrt{-b+ax}\right)^{3/2}}{12a^2} + \frac{\sqrt{-b+ax}\left(ax+\sqrt{-b+ax}\right)^{3/2}}{2a^2} + \frac{(5+12b)\sqrt{ax+\sqrt{-b}}}{3}$$

$$= -\frac{5\left(ax+\sqrt{-b+ax}\right)^{3/2}}{12a^2} + \frac{\sqrt{-b+ax}\left(ax+\sqrt{-b+ax}\right)^{3/2}}{2a^2} + \frac{(5+12b)\sqrt{ax+\sqrt{-b}}}{3}$$

Mathematica [A] time = 0.21, size = 132, normalized size = 0.97

$$\frac{3\left(48b^2+8b-5\right)\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{2\sqrt{\sqrt{ax-b}+ax}}\right)+2\sqrt{\sqrt{ax-b}+ax}\left(12b\left(6\sqrt{ax-b}-1\right)+8a\left(6x\sqrt{ax-b}+x\right)-10\sqrt{ax-b}+15\right)}{192a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a*x + Sqrt[-b + a*x]])/Sqrt[-b + a*x],x]
```

```
[Out] (2*Sqrt[a*x + Sqrt[-b + a*x]]*(15 - 10*Sqrt[-b + a*x] + 12*b*(-1 + 6*Sqrt[-
b + a*x]) + 8*a*(x + 6*x*Sqrt[-b + a*x])) + 3*(-5 + 8*b + 48*b^2)*ArcTanh[(
1 + 2*Sqrt[-b + a*x])/(2*Sqrt[a*x + Sqrt[-b + a*x]])])/(192*a^2)
```

IntegrateAlgebraic [A] time = 0.32, size = 135, normalized size = 0.99

$$\frac{\left(-48b^2-8b+5\right)\log\left(-2\sqrt{ax-b}+2\sqrt{\sqrt{ax-b}+ax}-1\right)}{64a^2}+\frac{\sqrt{\sqrt{ax-b}+ax}\left(48(ax-b)^{3/2}+8(ax-b)+120b\sqrt{ax-b}-10\sqrt{ax-b}-4b+15\right)}{96a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[a*x + Sqrt[-b + a*x]])/Sqrt[-b + a*x],x]
```

```
[Out] (Sqrt[a*x + Sqrt[-b + a*x]]*(15 - 4*b - 10*Sqrt[-b + a*x] + 120*b*Sqrt[-b +
 a*x] + 8*(-b + a*x) + 48*(-b + a*x)^(3/2)))/(96*a^2) + ((5 - 8*b - 48*b^2)
*Log[-1 - 2*Sqrt[-b + a*x] + 2*Sqrt[a*x + Sqrt[-b + a*x]]])/(64*a^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+(a*x-b)^(1/2))^(1/2)/(a*x-b)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.75, size = 112, normalized size = 0.82

$$-\frac{3\left(48\,b^{2}+8\,b-5\right)\log\left(\left|-2\,\sqrt{ax-b}+2\,\sqrt{ax+\sqrt{ax-b}}-1\right|\right)-2\,\sqrt{ax+\sqrt{ax-b}}\left(2\,\sqrt{ax-b}\left(4\,\sqrt{ax-b}\left(6\,\sqrt{ax-b}+1\right)+60\,b-5\right)-4\,b+15\right)}{192\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+(a*x-b)^(1/2))^(1/2)/(a*x-b)^(1/2),x, algorithm="giac")
```

```
[Out] -1/192*(3*(48*b^2 + 8*b - 5)*log(abs(-2*sqrt(a*x - b) + 2*sqrt(a*x + sqrt(a
*x - b)) - 1)) - 2*sqrt(a*x + sqrt(a*x - b))*(2*sqrt(a*x - b)*(4*sqrt(a*x -
 b)*(6*sqrt(a*x - b) + 1) + 60*b - 5) - 4*b + 15))/a^2
```

maple [B] time = 0.01, size = 251, normalized size = 1.85

$$\frac{\sqrt{ax-b}\,\left(ax+\sqrt{ax-b}\right)^{\frac{3}{2}}}{2a^{2}}-\frac{5\left(ax+\sqrt{ax-b}\right)^{\frac{3}{2}}}{12a^{2}}+\frac{5\sqrt{ax+\sqrt{ax-b}}\,\sqrt{ax-b}}{16a^{2}}+\frac{5\sqrt{ax+\sqrt{ax-b}}}{32a^{2}}+\frac{\ln\left(\frac{1}{2}+\sqrt{ax-b}+\sqrt{ax+\sqrt{ax-b}}\right)b}{8a^{2}}-\frac{5\ln\left(\frac{1}{2}+\sqrt{ax-b}+\sqrt{ax+\sqrt{ax-b}}\right)}{64a^{2}}+\frac{3b\sqrt{ax+\sqrt{ax-b}}\,\sqrt{ax-b}}{4a^{2}}+\frac{3b\sqrt{ax+\sqrt{ax-b}}}{8a^{2}}+\frac{3\ln\left(\frac{1}{2}+\sqrt{ax-b}+\sqrt{ax+\sqrt{ax-b}}\right)b^{2}}{4a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x+(a*x-b)^(1/2))^(1/2)/(a*x-b)^(1/2),x)
```

```
[Out] 1/2/a^2*(a*x-b)^(1/2)*(a*x+(a*x-b)^(1/2))^(3/2)-5/12/a^2*(a*x+(a*x-b)^(1/2)
)^(3/2)+5/16/a^2*(a*x+(a*x-b)^(1/2))^(1/2)*(a*x-b)^(1/2)+5/32/a^2*(a*x+(a*x
-b)^(1/2))^(1/2)+1/8/a^2*ln(1/2+(a*x-b)^(1/2)+(a*x+(a*x-b)^(1/2))^(1/2))*b-
5/64/a^2*ln(1/2+(a*x-b)^(1/2)+(a*x+(a*x-b)^(1/2))^(1/2))+3/4/a^2*b*(a*x+(a*
x-b)^(1/2))^(1/2)*(a*x-b)^(1/2)+3/8/a^2*b*(a*x+(a*x-b)^(1/2))^(1/2)+3/4/a^2
*ln(1/2+(a*x-b)^(1/2)+(a*x+(a*x-b)^(1/2))^(1/2))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+\sqrt{ax-b}}\,x}{\sqrt{ax-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+(a*x-b)^(1/2))^(1/2)/(a*x-b)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + sqrt(a*x - b))*x/sqrt(a*x - b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{a\,x+\sqrt{a\,x-b}}}{\sqrt{a\,x-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + (a*x - b)^(1/2))^(1/2))/(a*x - b)^(1/2),x)
```

```
[Out] int((x*(a*x + (a*x - b)^(1/2))^(1/2))/(a*x - b)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{ax+\sqrt{ax-b}}}{\sqrt{ax-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+(a*x-b)**(1/2))**(1/2)/(a*x-b)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*x + sqrt(a*x - b))/sqrt(a*x - b), x)
```

3.1631 $\int \frac{(-1+ax)\sqrt{ax+\sqrt{-b+ax}}}{\sqrt{-b+ax}}\,dx$

Optimal. Leaf size=136

$$\frac{(-48b^2+56b-11)\log\left(2\sqrt{ax-b}-2\sqrt{\sqrt{ax-b}+ax}+1\right)}{64a}+\frac{\sqrt{\sqrt{ax-b}+ax}\,(8ax-12b-33)}{96a}+\frac{\sqrt{ax-b}\,(24a\ldots}{}$$

Rubi [A] time = 0.46, antiderivative size = 166, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {1661, 640, 612, 621, 206}

$$\frac{\sqrt{ax-b}\left(\sqrt{ax-b}+ax\right)^{3/2}}{2a}-\frac{5\left(\sqrt{ax-b}+ax\right)^{3/2}}{12a}-\frac{(11-12b)\left(2\sqrt{ax-b}+1\right)\sqrt{\sqrt{ax-b}+ax}}{32a}+\frac{(11-12b)(1-4b)\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{2\sqrt{\sqrt{ax-b}+ax}}\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + a*x)*Sqrt[a*x + Sqrt[-b + a*x]])/Sqrt[-b + a*x],x]
```

```
[Out] (-5*(a*x + Sqrt[-b + a*x])^(3/2))/(12*a) + (Sqrt[-b + a*x]*(a*x + Sqrt[-b +
 a*x])^(3/2))/(2*a) - ((11 - 12*b)*Sqrt[a*x + Sqrt[-b + a*x]]*(1 + 2*Sqrt[-
b + a*x]))/(32*a) + ((11 - 12*b)*(1 - 4*b)*ArcTanh[(1 + 2*Sqrt[-b + a*x])/(
2*Sqrt[a*x + Sqrt[-b + a*x]])])/(64*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(-1+ax)\sqrt{ax+\sqrt{-b+ax}}}{\sqrt{-b+ax}}\,dx = \frac{2\,\text{Subst}\left(\int \left(-1+b+x^2\right)\sqrt{b+x+x^2}\,dx, x, \sqrt{-b+ax}\right)}{a}$$

$$= \frac{\sqrt{-b+ax}\left(ax+\sqrt{-b+ax}\right)^{3/2}}{2a} + \frac{\text{Subst}\left(\int \left(-4+3b-\frac{5x}{2}\right)\sqrt{b+x+x^2}\,dx, x\right.}{2a}$$

$$= -\frac{5\left(ax+\sqrt{-b+ax}\right)^{3/2}}{12a} + \frac{\sqrt{-b+ax}\left(ax+\sqrt{-b+ax}\right)^{3/2}}{2a} - \frac{(11-12b)\,\text{Subs}}{}$$

$$= -\frac{5\left(ax+\sqrt{-b+ax}\right)^{3/2}}{12a} + \frac{\sqrt{-b+ax}\left(ax+\sqrt{-b+ax}\right)^{3/2}}{2a} - \frac{(11-12b)\sqrt{ax}}{}$$

$$= -\frac{5\left(ax+\sqrt{-b+ax}\right)^{3/2}}{12a} + \frac{\sqrt{-b+ax}\left(ax+\sqrt{-b+ax}\right)^{3/2}}{2a} - \frac{(11-12b)\sqrt{ax}}{}$$

$$= -\frac{5\left(ax+\sqrt{-b+ax}\right)^{3/2}}{12a} + \frac{\sqrt{-b+ax}\left(ax+\sqrt{-b+ax}\right)^{3/2}}{2a} - \frac{(11-12b)\sqrt{ax}}{}$$

Mathematica [A] time = 0.17, size = 132, normalized size = 0.97

$$\frac{3\left(48b^2-56b+11\right)\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{2\sqrt{\sqrt{ax-b}+ax}}\right)+2\sqrt{\sqrt{ax-b}+ax}\left(12b\left(6\sqrt{ax-b}-1\right)+8a\left(6x\sqrt{ax-b}+x\right)-106\sqrt{ax-b}-33\right)}{192a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + a*x)*Sqrt[a*x + Sqrt[-b + a*x]])/Sqrt[-b + a*x],x]
```

```
[Out] (2*Sqrt[a*x + Sqrt[-b + a*x]]*(-33 - 106*Sqrt[-b + a*x] + 12*b*(-1 + 6*Sqrt
[-b + a*x]) + 8*a*(x + 6*x*Sqrt[-b + a*x])) + 3*(11 - 56*b + 48*b^2)*ArcTan
h[(1 + 2*Sqrt[-b + a*x])/(2*Sqrt[a*x + Sqrt[-b + a*x]])])/(192*a)
```

IntegrateAlgebraic [A] time = 0.42, size = 139, normalized size = 1.02

$$\frac{\left(-48b^2+56b-11\right)\log\left(a\left(-2\sqrt{ax-b}-1\right)+2a\sqrt{\sqrt{ax-b}+ax}\right)}{64a}+\frac{\sqrt{\sqrt{ax-b}+ax}\left(48(ax-b)^{3/2}+8(ax-b)+120b\sqrt{ax-b}-106\sqrt{ax-b}-4b-33\right)}{96a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + a*x)*Sqrt[a*x + Sqrt[-b + a*x]])/Sqrt[-b + a*x],x
]
```

```
[Out] (Sqrt[a*x + Sqrt[-b + a*x]]*(-33 - 4*b - 106*Sqrt[-b + a*x] + 120*b*Sqrt[-b
 + a*x] + 8*(-b + a*x) + 48*(-b + a*x)^(3/2)))/(96*a) + ((-11 + 56*b - 48*b
^2)*Log[a*(-1 - 2*Sqrt[-b + a*x]) + 2*a*Sqrt[a*x + Sqrt[-b + a*x]]])/(64*a)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)*(a*x+(a*x-b)^(1/2))^(1/2)/(a*x-b)^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [A] time = 1.28, size = 181, normalized size = 1.33

$$-\frac{3\left(48b^{2}+8b-5\right)\log\left(\left|-2\sqrt{ax-b}+2\sqrt{ax+\sqrt{ax-b}}-1\right|\right)-48\left(4b-1\right)\log\left(\left|-2\sqrt{ax-b}+2\sqrt{ax+\sqrt{ax-b}}-1\right|\right)-2\sqrt{ax+\sqrt{ax-b}}\left(2\sqrt{ax-b}\left(4\sqrt{ax-b}\left(6\sqrt{ax-b}+1\right)+60b-5\right)-4b+15\right)+96\sqrt{ax+\sqrt{ax-b}}\left(2\sqrt{ax-b}+1\right)}{192a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)*(a*x+(a*x-b)^(1/2))^(1/2)/(a*x-b)^(1/2),x, algorithm="gia
c")
```

```
[Out] -1/192*(3*(48*b^2 + 8*b - 5)*log(abs(-2*sqrt(a*x - b) + 2*sqrt(a*x + sqrt(a
*x - b)) - 1)) - 48*(4*b - 1)*log(abs(-2*sqrt(a*x - b) + 2*sqrt(a*x + sqrt(
a*x - b)) - 1)) - 2*sqrt(a*x + sqrt(a*x - b))*(2*sqrt(a*x - b)*(4*sqrt(a*x
- b)*(6*sqrt(a*x - b) + 1) + 60*b - 5) - 4*b + 15) + 96*sqrt(a*x + sqrt(a*x
 - b))*(2*sqrt(a*x - b) + 1))/a
```

maple [B] time = 0.01, size = 251, normalized size = 1.85

$$\frac{\sqrt{ax-b}\,\left(ax+\sqrt{ax-b}\right)^{\frac{3}{2}}}{2a}-\frac{5\left(ax+\sqrt{ax-b}\right)^{\frac{3}{2}}}{12a}-\frac{11\sqrt{ax-b}\,\sqrt{ax+\sqrt{ax-b}}}{16a}-\frac{11\sqrt{ax+\sqrt{ax-b}}}{32a}-\frac{7\ln\left(\frac{1}{2}+\sqrt{ax-b}+\sqrt{ax+\sqrt{ax-b}}\right)b}{8a}+\frac{11\ln\left(\frac{1}{2}+\sqrt{ax-b}+\sqrt{ax+\sqrt{ax-b}}\right)}{64a}+\frac{3b\sqrt{ax+\sqrt{ax-b}}\,\sqrt{ax-b}}{4a}+\frac{3b\sqrt{ax+\sqrt{ax-b}}}{8a}+\frac{3\ln\left(\frac{1}{2}+\sqrt{ax-b}+\sqrt{ax+\sqrt{ax-b}}\right)b^{2}}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)*(a*x+(a*x-b)^(1/2))^(1/2)/(a*x-b)^(1/2),x)
```

```
[Out] 1/2/a*(a*x-b)^(1/2)*(a*x+(a*x-b)^(1/2))^(3/2)-5/12/a*(a*x+(a*x-b)^(1/2))^(3
/2)-11/16*(a*x-b)^(1/2)*(a*x+(a*x-b)^(1/2))^(1/2)/a-11/32*(a*x+(a*x-b)^(1/2
))^(1/2)/a-7/8/a*ln(1/2+(a*x-b)^(1/2)+(a*x+(a*x-b)^(1/2))^(1/2))*b+11/64/a*
ln(1/2+(a*x-b)^(1/2)+(a*x+(a*x-b)^(1/2))^(1/2))+3/4/a*b*(a*x+(a*x-b)^(1/2))
^(1/2)*(a*x-b)^(1/2)+3/8/a*b*(a*x+(a*x-b)^(1/2))^(1/2)+3/4/a*ln(1/2+(a*x-b)
^(1/2)+(a*x+(a*x-b)^(1/2))^(1/2))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+\sqrt{ax-b}}\,(ax-1)}{\sqrt{ax-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)*(a*x+(a*x-b)^(1/2))^(1/2)/(a*x-b)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(a*x + sqrt(a*x - b))*(a*x - 1)/sqrt(a*x - b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x+\sqrt{a\,x-b}}\,\left(a\,x-1\right)}{\sqrt{a\,x-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (a*x - b)^(1/2))^(1/2)*(a*x - 1))/(a*x - b)^(1/2),x)
```

```
[Out] int(((a*x + (a*x - b)^(1/2))^(1/2)*(a*x - 1))/(a*x - b)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-1)\sqrt{ax+\sqrt{ax-b}}}{\sqrt{ax-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)*(a*x+(a*x-b)**(1/2))**(1/2)/(a*x-b)**(1/2),x)
```

```
[Out] Integral((a*x - 1)*sqrt(a*x + sqrt(a*x - b))/sqrt(a*x - b), x)
```

3.1632 $\int \frac{-2+x^4}{\sqrt[4]{1+x^4}\left(-1-x^4+2x^8\right)}\,dx$

Optimal. Leaf size=137

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{6\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{6\sqrt[4]{2}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}-x^2}\right)}{6\sqrt{2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}+x^2}\right)}{6\sqrt{2}}$$

Rubi [A] time = 0.27, antiderivative size = 193, normalized size of antiderivative = 1.41, number of steps used = 16, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {6728, 377, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{6\sqrt[4]{2}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+1}}\right)}{6\sqrt{2}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+1}}+1\right)}{6\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{6\sqrt[4]{2}}-\frac{5\log\left(-\frac{\sqrt{2}x}{\sqrt[4]{x^4+1}}+\frac{x^2}{\sqrt{x^4+1}}+1\right)}{12\sqrt{2}}+\frac{5\log\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+1}}+\frac{x^2}{\sqrt{x^4+1}}+1\right)}{12\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^4)/((1 + x^4)^(1/4)*(-1 - x^4 + 2*x^8)),x]
```

```
[Out] ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(6*2^(1/4)) - (5*ArcTan[1 - (Sqrt[2]*x)
/(1 + x^4)^(1/4)])/(6*Sqrt[2]) + (5*ArcTan[1 + (Sqrt[2]*x)/(1 + x^4)^(1/4)]
)/(6*Sqrt[2]) + ArcTanh[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(6*2^(1/4)) - (5*Log[1
 + x^2/Sqrt[1 + x^4] - (Sqrt[2]*x)/(1 + x^4)^(1/4)])/(12*Sqrt[2]) + (5*Log[
1 + x^2/Sqrt[1 + x^4] + (Sqrt[2]*x)/(1 + x^4)^(1/4)])/(12*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-2+x^4}{\sqrt[4]{1+x^4}\left(-1-x^4+2x^8\right)}\,dx = \int \left(-\frac{4}{3\sqrt[4]{1+x^4}\left(-4+4x^4\right)}+\frac{10}{3\sqrt[4]{1+x^4}\left(2+4x^4\right)}\right)dx$$

$$= -\left(\frac{4}{3}\int \frac{1}{\sqrt[4]{1+x^4}\left(-4+4x^4\right)}\,dx\right)+\frac{10}{3}\int \frac{1}{\sqrt[4]{1+x^4}\left(2+4x^4\right)}\,dx$$

$$= -\left(\frac{4}{3}\,\text{Subst}\left(\int \frac{1}{-4+8x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)\right)+\frac{10}{3}\,\text{Subst}\left(\int \frac{1}{2+2x^4}\,dx,x,\frac{x}{\sqrt[4]{1+\cdots}}\right.$$

$$= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right.$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{6\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{6\sqrt[4]{2}}+\frac{5}{12}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right.$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{6\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{6\sqrt[4]{2}}-\frac{5\log\left(1+\frac{x^2}{\sqrt{1+x^4}}-\frac{\sqrt{2}x}{\sqrt[4]{1+x^4}}\right)}{12\sqrt{2}}+\frac{5\log\left(1+\cdots\right.}{1\cdots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right)}{6\sqrt[4]{2}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{1+x^4}}\right)}{6\sqrt{2}}+\frac{5\tan^{-1}\left(1+\frac{\sqrt{2}x}{\sqrt[4]{1+x^4}}\right)}{6\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{1+x^4}}\right.}{6\sqrt[4]{2}}$$

Mathematica [A] time = 0.12, size = 171, normalized size = 1.25

$$\frac{2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)+2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)-5\left(2\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+1}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+1}}+1\right)+\log\left(-\frac{\sqrt{2}x}{\sqrt[4]{x^4+1}}+\frac{x^2}{\sqrt{x^4+1}}+1\right)-\log\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+1}}+\frac{x^2}{\sqrt{x^4+1}}+1\right)\right)}{12\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^4)/((1 + x^4)^(1/4)*(-1 - x^4 + 2*x^8)),x]
```

```
[Out] (2*2^(1/4)*ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)] + 2*2^(1/4)*ArcTanh[(2^(1/4)
*x)/(1 + x^4)^(1/4)] - 5*(2*ArcTan[1 - (Sqrt[2]*x)/(1 + x^4)^(1/4)] - 2*Arc
Tan[1 + (Sqrt[2]*x)/(1 + x^4)^(1/4)] + Log[1 + x^2/Sqrt[1 + x^4] - (Sqrt[2]
*x)/(1 + x^4)^(1/4)] - Log[1 + x^2/Sqrt[1 + x^4] + (Sqrt[2]*x)/(1 + x^4)^(1
/4)]))/(12*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.44, size = 137, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{6\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^4+1}}\right)}{6\sqrt[4]{2}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}-x^2}\right)}{6\sqrt{2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}+x^2}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x^4)/((1 + x^4)^(1/4)*(-1 - x^4 + 2*x^8)),x]
```

```
[Out] ArcTan[(2^(1/4)*x)/(1 + x^4)^(1/4)]/(6*2^(1/4)) + (5*ArcTan[(Sqrt[2]*x*(1 +
 x^4)^(1/4))/(-x^2 + Sqrt[1 + x^4])])/(6*Sqrt[2]) + ArcTanh[(2^(1/4)*x)/(1
+ x^4)^(1/4)]/(6*2^(1/4)) + (5*ArcTanh[(Sqrt[2]*x*(1 + x^4)^(1/4))/(x^2 + S
qrt[1 + x^4])])/(6*Sqrt[2])
```

fricas [B] time = 9.97, size = 610, normalized size = 4.45

$$-\frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{4\cdot 2^{\frac{3}{4}}\left(x^4+1\right)^{\frac{1}{4}}x^3+4\cdot 2^{\frac{1}{4}}\left(x^4+1\right)^{\frac{3}{4}}x+2^{\frac{3}{4}}\left(2\cdot 2^{\frac{3}{4}}\sqrt{x^4+1}\,x^2+2^{\frac{1}{4}}\left(3x^4+1\right)\right)}{2\left(x^4-1\right)}\right)+\frac{1}{48}\cdot 2^{\frac{3}{4}}\log\left(\frac{4\sqrt{2}\left(x^4+1\right)^{\frac{1}{4}}x^3+4\cdot 2^{\frac{1}{4}}\sqrt{x^4+1}\,x^2+2^{\frac{3}{4}}\left(3x^4+1\right)+4\left(x^4+1\right)^{\frac{3}{4}}x}{x^4-1}\right)-\frac{1}{48}\cdot 2^{\frac{3}{4}}\log\left(\frac{4\sqrt{2}\left(x^4+1\right)^{\frac{1}{4}}x^3-4\cdot 2^{\frac{1}{4}}\sqrt{x^4+1}\,x^2-2^{\frac{3}{4}}\left(3x^4+1\right)+4\left(x^4+1\right)^{\frac{3}{4}}x}{x^4-1}\right)+\frac{5}{12}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^4+1\right)^{\frac{3}{4}}x^2-\sqrt{2}\left(x^4+1\right)^{\frac{5}{4}}-\left(2x^5-\sqrt{2}\left(x^4+1\right)^{\frac{3}{4}}x^2-\sqrt{2}\left(x^4+1\right)^{\frac{5}{4}}+2x\right)\sqrt{\frac{2x^4+2\sqrt{2}\left(x^4+1\right)^{\frac{1}{4}}x^3+4\sqrt{x^4+1}\,x^2+2\sqrt{2}\left(x^4+1\right)^{\frac{3}{4}}x+1}{2x^4+1}}}{2\left(x^5+x\right)}\right)+\frac{5}{12}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^4+1\right)^{\frac{3}{4}}x^2-\sqrt{2}\left(x^4+1\right)^{\frac{5}{4}}+\left(2x^5+\sqrt{2}\left(x^4+1\right)^{\frac{3}{4}}x^2+\sqrt{2}\left(x^4+1\right)^{\frac{5}{4}}+2x\right)\sqrt{\frac{2x^4-2\sqrt{2}\left(x^4+1\right)^{\frac{1}{4}}x^3+4\sqrt{x^4+1}\,x^2-2\sqrt{2}\left(x^4+1\right)^{\frac{3}{4}}x+1}{2x^4+1}}}{2\left(x^5+x\right)}\right)+\frac{5}{48}\sqrt{2}\log\left(\frac{2x^4+2\sqrt{2}\left(x^4+1\right)^{\frac{1}{4}}x^3+4\sqrt{x^4+1}\,x^2+2\sqrt{2}\left(x^4+1\right)^{\frac{3}{4}}x+1}{2x^4+1}\right)-\frac{5}{48}\sqrt{2}\log\left(\frac{2x^4-2\sqrt{2}\left(x^4+1\right)^{\frac{1}{4}}x^3+4\sqrt{x^4+1}\,x^2-2\sqrt{2}\left(x^4+1\right)^{\frac{3}{4}}x+1}{2x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2)/(x^4+1)^(1/4)/(2*x^8-x^4-1),x, algorithm="fricas")
```

```
[Out] -1/12*2^(3/4)*arctan(1/2*(4*2^(3/4)*(x^4 + 1)^(1/4)*x^3 + 4*2^(1/4)*(x^4 +
1)^(3/4)*x + 2^(3/4)*(2*2^(3/4)*sqrt(x^4 + 1)*x^2 + 2^(1/4)*(3*x^4 + 1)))/(
x^4 - 1)) + 1/48*2^(3/4)*log((4*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 4*2^(1/4)*sqr
t(x^4 + 1)*x^2 + 2^(3/4)*(3*x^4 + 1) + 4*(x^4 + 1)^(3/4)*x)/(x^4 - 1)) - 1/
48*2^(3/4)*log((4*sqrt(2)*(x^4 + 1)^(1/4)*x^3 - 4*2^(1/4)*sqrt(x^4 + 1)*x^2
 - 2^(3/4)*(3*x^4 + 1) + 4*(x^4 + 1)^(3/4)*x)/(x^4 - 1)) + 5/12*sqrt(2)*arc
tan(1/2*(sqrt(2)*(x^4 + 1)^(3/4)*x^2 - sqrt(2)*(x^4 + 1)^(5/4) - (2*x^5 - s
qrt(2)*(x^4 + 1)^(3/4)*x^2 - sqrt(2)*(x^4 + 1)^(5/4) + 2*x)*sqrt((2*x^4 + 2
*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 4*sqrt(x^4 + 1)*x^2 + 2*sqrt(2)*(x^4 + 1)^(3
/4)*x + 1)/(2*x^4 + 1)))/(x^5 + x)) + 5/12*sqrt(2)*arctan(1/2*(sqrt(2)*(x^4
 + 1)^(3/4)*x^2 - sqrt(2)*(x^4 + 1)^(5/4) + (2*x^5 + sqrt(2)*(x^4 + 1)^(3/4
)*x^2 + sqrt(2)*(x^4 + 1)^(5/4) + 2*x)*sqrt((2*x^4 - 2*sqrt(2)*(x^4 + 1)^(1
/4)*x^3 + 4*sqrt(x^4 + 1)*x^2 - 2*sqrt(2)*(x^4 + 1)^(3/4)*x + 1)/(2*x^4 + 1
)))/(x^5 + x)) + 5/48*sqrt(2)*log((2*x^4 + 2*sqrt(2)*(x^4 + 1)^(1/4)*x^3 +
4*sqrt(x^4 + 1)*x^2 + 2*sqrt(2)*(x^4 + 1)^(3/4)*x + 1)/(2*x^4 + 1)) - 5/48*
sqrt(2)*log((2*x^4 - 2*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 4*sqrt(x^4 + 1)*x^2 -
2*sqrt(2)*(x^4 + 1)^(3/4)*x + 1)/(2*x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-2}{\left(2\,x^8-x^4-1\right)\left(x^4+1\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2)/(x^4+1)^(1/4)/(2*x^8-x^4-1),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 2)/((2*x^8 - x^4 - 1)*(x^4 + 1)^(1/4)), x)
```

maple [C] time = 3.90, size = 569, normalized size = 4.15

$$\frac{\operatorname{RootOf}\left(\_Z^4-8\right)\ln\left(-\frac{\sqrt{x^4+1}\,\operatorname{RootOf}\left(\_Z^4-8\right)^3 x^2+2\left(x^4+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4-8\right)^2 x^3+3\operatorname{RootOf}\left(\_Z^4-8\right)x^4+4\left(x^4+1\right)^{\frac{3}{4}}x+\operatorname{RootOf}\left(\_Z^4-8\right)}{(-1+x)(1+x)\left(x^2+1\right)}\right)}{24}-\frac{\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)\ln\left(-\frac{\sqrt{x^4+1}\,\operatorname{RootOf}\left(\_Z^4-8\right)^2\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)x^2-2\left(x^4+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4-8\right)^2 x^3-3\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)x^4+4\left(x^4+1\right)^{\frac{3}{4}}x-\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)}{(-1+x)(1+x)\left(x^2+1\right)}\right)}{24}+\frac{5\ln\left(-\frac{4\operatorname{RootOf}\left(\_Z^4-8\right)^2\sqrt{x^4+1}\,x^2+2\operatorname{RootOf}\left(\_Z^4-8\right)^2x^4+8\left(x^4+1\right)^{\frac{3}{4}}x+8x^3\left(x^4+1\right)^{\frac{1}{4}}+\operatorname{RootOf}\left(\_Z^4-8\right)^2}{2x^4+1}\right)\operatorname{RootOf}\left(\_Z^4-8\right)^2}{48}-\frac{5\ln\left(-\frac{4\operatorname{RootOf}\left(\_Z^4-8\right)^2\sqrt{x^4+1}\,x^2+2\operatorname{RootOf}\left(\_Z^4-8\right)^2x^4+8\left(x^4+1\right)^{\frac{3}{4}}x+8x^3\left(x^4+1\right)^{\frac{1}{4}}+\operatorname{RootOf}\left(\_Z^4-8\right)^2}{2x^4+1}\right)\operatorname{RootOf}\left(\_Z^4-8\right)\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)}{48}+\frac{5\operatorname{RootOf}\left(\_Z^4-8\right)\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)\operatorname{RootOf}\left(\_Z^4-8\right)^3\left(x^4+1\right)^{\frac{1}{4}}x^3-2\operatorname{RootOf}\left(\_Z^4-8\right)^2\sqrt{x^4+1}\,x^2+2\operatorname{RootOf}\left(\_Z^4-8\right)\sqrt{x^4+1}\,\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)x^2-8\left(x^4+1\right)^{\frac{3}{4}}x-\operatorname{RootOf}\left(\_Z^4-8\right)^2-\operatorname{RootOf}\left(\_Z^4-8\right)\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-8\right)^2\right)}{2x^4+1}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2)/(x^4+1)^(1/4)/(2*x^8-x^4-1),x)
```

```
[Out] 1/24*RootOf(_Z^4-8)*ln(-((x^4+1)^(1/2)*RootOf(_Z^4-8)^3*x^2+2*(x^4+1)^(1/4)
*RootOf(_Z^4-8)^2*x^3+3*RootOf(_Z^4-8)*x^4+4*(x^4+1)^(3/4)*x+RootOf(_Z^4-8)
)/(-1+x)/(1+x)/(x^2+1))-1/24*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln(-((x^4+1)^(1/
2)*RootOf(_Z^4-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^2-2*(x^4+1)^(1/4)*RootO
f(_Z^4-8)^2*x^3-3*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^4+4*(x^4+1)^(3/4)*x-RootO
f(_Z^2+RootOf(_Z^4-8)^2))/(-1+x)/(1+x)/(x^2+1))+5/48*ln(-(4*RootOf(_Z^4-8)^
2*(x^4+1)^(1/2)*x^2+2*RootOf(_Z^4-8)^2*x^4+8*(x^4+1)^(3/4)*x+8*x^3*(x^4+1)^
(1/4)+RootOf(_Z^4-8)^2)/(2*x^4+1))*RootOf(_Z^4-8)^2-5/48*ln(-(4*RootOf(_Z^4
-8)^2*(x^4+1)^(1/2)*x^2+2*RootOf(_Z^4-8)^2*x^4+8*(x^4+1)^(3/4)*x+8*x^3*(x^4
+1)^(1/4)+RootOf(_Z^4-8)^2)/(2*x^4+1))*RootOf(_Z^4-8)*RootOf(_Z^2+RootOf(_Z
^4-8)^2)+5/24*RootOf(_Z^4-8)*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln(-(RootOf(_Z^2
+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^3*(x^4+1)^(1/4)*x^3-2*RootOf(_Z^4-8)^2*(x
^4+1)^(1/2)*x^2+2*RootOf(_Z^4-8)*(x^4+1)^(1/2)*RootOf(_Z^2+RootOf(_Z^4-8)^2
)*x^2-8*(x^4+1)^(3/4)*x-RootOf(_Z^4-8)^2-RootOf(_Z^4-8)*RootOf(_Z^2+RootOf(
_Z^4-8)^2))/(2*x^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-2}{\left(2\,x^8-x^4-1\right)\left(x^4+1\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2)/(x^4+1)^(1/4)/(2*x^8-x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 2)/((2*x^8 - x^4 - 1)*(x^4 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^4-2}{\left(x^4+1\right)^{1/4}\,\left(-2\,x^8+x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 2)/((x^4 + 1)^(1/4)*(x^4 - 2*x^8 + 1)),x)
```

```
[Out] -int((x^4 - 2)/((x^4 + 1)^(1/4)*(x^4 - 2*x^8 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-2}{(x-1)\,(x+1)\left(x^2+1\right)\sqrt[4]{x^4+1}\,\left(2x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2)/(x**4+1)**(1/4)/(2*x**8-x**4-1),x)
```

```
[Out] Integral((x**4 - 2)/((x - 1)*(x + 1)*(x**2 + 1)*(x**4 + 1)**(1/4)*(2*x**4 +
 1)), x)
```

3.1633 $\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{5x}{12b^2\left(ax^2+b^2\right)\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{5\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{8\sqrt{a}\,b^{7/2}}+\frac{x\left(15ax^2+23b^2\right)}{24b^3\left(ax^2+b^2\right)^{3/2}\sqrt{\sqrt{ax^2+b^2}+b}}$$

Rubi [F] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^(5/2),x]
[Out] Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^{5/2}}\,dx=\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^{5/2}}\,dx$$

Mathematica [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^(5/2),x]
[Out] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^(5/2), x]
```

IntegrateAlgebraic [A] time = 0.29, size = 137, normalized size = 1.00

$$\frac{5x}{12b^2\left(ax^2+b^2\right)\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{5\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{8\sqrt{a}\,b^{7/2}}+\frac{x\left(15ax^2+23b^2\right)}{24b^3\left(ax^2+b^2\right)^{3/2}\sqrt{\sqrt{ax^2+b^2}+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^(5/2),x]
[Out] (5*x)/(12*b^2*(b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]]) + (x*(23*b^2 + 15*
a*x^2))/(24*b^3*(b^2 + a*x^2)^(3/2)*Sqrt[b + Sqrt[b^2 + a*x^2]]) + (5*ArcTa
n[(Sqrt[a]*x)/(Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/(8*Sqrt[a]*b^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\left(ax^2+b^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 + b^2)^(5/2), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{\left(a\,x^2+b^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^(5/2),x)
```

```
[Out] int((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\left(ax^2+b^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 + b^2)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b+\sqrt{b^2+a\,x^2}}}{{\left(b^2+a\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(a*x^2 + b^2)^(5/2),x)
```

```
[Out] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(a*x^2 + b^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b + \sqrt{a x^2 + b^2}}}{\left(a x^2 + b^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x**2+b**2)**(1/2))**(1/2)/(a*x**2+b**2)**(5/2),x)
[Out] Integral(sqrt(b + sqrt(a*x**2 + b**2))/(a*x**2 + b**2)**(5/2), x)
```

3.1634 $\int \frac{\sqrt[4]{-b+ax^2}}{x} \, dx$

Optimal. Leaf size=138

$$2\sqrt[4]{ax^2-b}+\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}-\sqrt{b}}\right)}{\sqrt{2}}-\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.21, antiderivative size = 211, normalized size of antiderivative = 1.53, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 50, 63, 211, 1165, 628, 1162, 617, 204}

$$2\sqrt[4]{ax^2-b}+\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)}{2\sqrt{2}}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)}{2\sqrt{2}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}\right)}{\sqrt{2}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^2)^(1/4)/x,x]
```

```
[Out] 2*(-b + a*x^2)^(1/4) + (b^(1/4)*ArcTan[1 - (Sqrt[2]*(-b + a*x^2)^(1/4))/b^(
1/4)])/Sqrt[2] - (b^(1/4)*ArcTan[1 + (Sqrt[2]*(-b + a*x^2)^(1/4))/b^(1/4)])
/Sqrt[2] + (b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4) + Sqrt
[-b + a*x^2]])/(2*Sqrt[2]) - (b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a
*x^2)^(1/4) + Sqrt[-b + a*x^2]])/(2*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{-b+ax^2}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt[4]{-b+ax}}{x}\,dx, x, x^2\right)\\
&= 2\sqrt[4]{-b+ax^2} - \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{x(-b+ax)^{3/4}}\,dx, x, x^2\right)\\
&= 2\sqrt[4]{-b+ax^2} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^2}\right)}{a}\\
&= 2\sqrt[4]{-b+ax^2} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^2}\right)}{a} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+\cdots}\right.}{a}\\
&= 2\sqrt[4]{-b+ax^2} + \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^2}\right)}{2\sqrt{2}} + \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}-\cdots}{-\sqrt{b}+\sqrt{2}\sqrt[4]{\cdots}}\right.}{2\cdots}\\
&= 2\sqrt[4]{-b+ax^2} + \frac{\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^2}+\sqrt{-b+ax^2}\right)}{2\sqrt{2}} - \frac{\sqrt[4]{b}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\cdots\right.}{2\cdots}\\
&= 2\sqrt[4]{-b+ax^2} + \frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)}{\sqrt{2}} - \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)}{\sqrt{2}} + \frac{\sqrt[4]{b}\log\left(\sqrt{b}-\cdots\right.}{\cdots}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 211, normalized size = 1.53

$$2\sqrt[4]{ax^2-b}+\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)}{2\sqrt{2}}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)}{2\sqrt{2}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}\right)}{\sqrt{2}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^2)^(1/4)/x,x]
```

```
[Out] 2*(-b + a*x^2)^(1/4) + (b^(1/4)*ArcTan[1 - (Sqrt[2]*(-b + a*x^2)^(1/4))/b^(
1/4)])/Sqrt[2] - (b^(1/4)*ArcTan[1 + (Sqrt[2]*(-b + a*x^2)^(1/4))/b^(1/4)])
/Sqrt[2] + (b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4) + Sqrt
[-b + a*x^2]])/(2*Sqrt[2]) - (b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a
*x^2)^(1/4) + Sqrt[-b + a*x^2]])/(2*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.22, size = 138, normalized size = 1.00

$$2\sqrt[4]{ax^2-b}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}}-\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}+\sqrt{b}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^2)^(1/4)/x,x]
```

```
[Out] 2*(-b + a*x^2)^(1/4) - (b^(1/4)*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a*x^
2]/(Sqrt[2]*b^(1/4)))/(-b + a*x^2)^(1/4)])/Sqrt[2] - (b^(1/4)*ArcTanh[(Sqrt
[2]*b^(1/4)*(-b + a*x^2)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^2])])/Sqrt[2]
```

fricas [A] time = 0.43, size = 122, normalized size = 0.88

$$-2(-b)^{\frac{1}{4}}\arctan\left(\frac{(-b)^{\frac{3}{4}}\sqrt{\sqrt{ax^2-b}+\sqrt{-b}}-\left(ax^2-b\right)^{\frac{1}{4}}(-b)^{\frac{3}{4}}}{b}\right)-\frac{1}{2}(-b)^{\frac{1}{4}}\log\left(\left(ax^2-b\right)^{\frac{1}{4}}+(-b)^{\frac{1}{4}}\right)+\frac{1}{2}(-b)^{\frac{1}{4}}\log\left(\left(ax^2-b\right)^{\frac{1}{4}}-(-b)^{\frac{1}{4}}\right)+2\left(ax^2-b\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)^(1/4)/x,x, algorithm="fricas")
```

```
[Out] -2*(-b)^(1/4)*arctan(((-b)^(3/4)*sqrt(sqrt(a*x^2 - b) + sqrt(-b)) - (a*x^2
- b)^(1/4)*(-b)^(3/4))/b) - 1/2*(-b)^(1/4)*log((a*x^2 - b)^(1/4) + (-b)^(1/
4)) + 1/2*(-b)^(1/4)*log((a*x^2 - b)^(1/4) - (-b)^(1/4)) + 2*(a*x^2 - b)^(1
/4)
```

giac [A] time = 0.27, size = 175, normalized size = 1.27

$$-\frac{1}{2}\sqrt{2}b^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)-\frac{1}{2}\sqrt{2}b^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)-\frac{1}{4}\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)+\frac{1}{4}\sqrt{2}b^{\frac{1}{4}}\log\left(-\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)+2\left(ax^2-b\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)^(1/4)/x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*b^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^2 - b)^(1
/4))/b^(1/4)) - 1/2*sqrt(2)*b^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
2*(a*x^2 - b)^(1/4))/b^(1/4)) - 1/4*sqrt(2)*b^(1/4)*log(sqrt(2)*(a*x^2 - b)
^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b)) + 1/4*sqrt(2)*b^(1/4)*log(-sqrt
(2)*(a*x^2 - b)^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b)) + 2*(a*x^2 - b)^
(1/4)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{2}-b\right)^{\frac{1}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)^(1/4)/x,x)
```

```
[Out] int((a*x^2-b)^(1/4)/x,x)
```

maxima [A] time = 0.42, size = 175, normalized size = 1.27

$$-\frac{1}{2}\sqrt{2}\,b^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)-\frac{1}{2}\sqrt{2}\,b^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)-\frac{1}{4}\sqrt{2}\,b^{\frac{1}{4}}\log\left(\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)+\frac{1}{4}\sqrt{2}\,b^{\frac{1}{4}}\log\left(-\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)+2\left(ax^2-b\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)^(1/4)/x,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*b^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^2 - b)^(1
/4))/b^(1/4)) - 1/2*sqrt(2)*b^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
2*(a*x^2 - b)^(1/4))/b^(1/4)) - 1/4*sqrt(2)*b^(1/4)*log(sqrt(2)*(a*x^2 - b)
^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b)) + 1/4*sqrt(2)*b^(1/4)*log(-sqrt
(2)*(a*x^2 - b)^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b)) + 2*(a*x^2 - b)^
(1/4)
```

mupad [B] time = 1.03, size = 64, normalized size = 0.46

$$2\left(a\,x^2-b\right)^{1/4}-(-b)^{1/4}\,\mathrm{atan}\left(\frac{\left(a\,x^2-b\right)^{1/4}}{(-b)^{1/4}}\right)-(-b)^{1/4}\,\mathrm{atanh}\left(\frac{\left(a\,x^2-b\right)^{1/4}}{(-b)^{1/4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 - b)^(1/4)/x,x)
```

```
[Out] 2*(a*x^2 - b)^(1/4) - (-b)^(1/4)*atan((a*x^2 - b)^(1/4)/(-b)^(1/4)) - (-b)^
(1/4)*atanh((a*x^2 - b)^(1/4)/(-b)^(1/4))
```

sympy [C] time = 0.96, size = 48, normalized size = 0.35

$$-\frac{\sqrt[4]{a}\,\sqrt{x}\,\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^2}\right)}{2\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)**(1/4)/x,x)
```

```
[Out] -a**(1/4)*sqrt(x)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*exp_polar(2*I*p
i)/(a*x**2))/(2*gamma(3/4))
```

3.1635 $\int \frac{(b+ax^2)\sqrt[3]{-x+x^3}}{x^2}\,dx$

Optimal. Leaf size=138

$$\frac{1}{6}(a-3b)\log\left(\sqrt[3]{x^3-x}-x\right)+\frac{1}{6}\left(\sqrt{3}\,a-3\sqrt{3}\,b\right)\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-x}+x}\right)+\frac{\sqrt[3]{x^3-x}\left(ax^2-3b\right)}{2x}+\frac{1}{12}(3b-a)\log\left(\sqrt[3]{x^3-\right.$$

Rubi [A] time = 0.22, antiderivative size = 225, normalized size of antiderivative = 1.63, number of steps used = 12, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2038, 2004, 2032, 329, 275, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{2}x\sqrt[3]{x^3-x}\,(a-3b)+\frac{x^{2/3}\left(x^2-1\right)^{2/3}(a-3b)\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2-1}}\right)}{6\left(x^3-x\right)^{2/3}}-\frac{x^{2/3}\left(x^2-1\right)^{2/3}(a-3b)\log\left(\frac{x^{4/3}}{\left(x^2-1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2-1}}+1\right)}{12\left(x^3-x\right)^{2/3}}+\frac{x^{2/3}\left(x^2-1\right)^{2/3}(a-3b)\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1}{\sqrt{3}}\right)}{2\sqrt{3}\left(x^3-x\right)^{2/3}}+\frac{3b\left(x^3-x\right)^{4/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((b + a*x^2)*(-x + x^3)^(1/3))/x^2,x]
```

```
[Out] ((a - 3*b)*x*(-x + x^3)^(1/3))/2 + (3*b*(-x + x^3)^(4/3))/(2*x^2) + ((a - 3
*b)*x^(2/3)*(-1 + x^2)^(2/3)*ArcTan[(1 + (2*x^(2/3))/(-1 + x^2)^(1/3))/Sqrt
[3]])/(2*Sqrt[3]*(-x + x^3)^(2/3)) + ((a - 3*b)*x^(2/3)*(-1 + x^2)^(2/3)*Lo
g[1 - x^(2/3)/(-1 + x^2)^(1/3)])/(6*(-x + x^3)^(2/3)) - ((a - 3*b)*x^(2/3)*
(-1 + x^2)^(2/3)*Log[1 + x^(4/3)/(-1 + x^2)^(2/3) + x^(2/3)/(-1 + x^2)^(1/3
)])/(12*(-x + x^3)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, -(n*p) - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\int \frac{\left(b+ax^2\right)\sqrt[3]{-x+x^3}}{x^2}\,dx = \frac{3b\left(-x+x^3\right)^{4/3}}{2x^2} + (a-3b)\int \sqrt[3]{-x+x^3}\,dx$$

$$= \frac{1}{2}(a-3b)x\sqrt[3]{-x+x^3} + \frac{3b\left(-x+x^3\right)^{4/3}}{2x^2} + \frac{1}{3}(-a+3b)\int \frac{x}{\left(-x+x^3\right)^{2/3}}\,dx$$

$$= \frac{1}{2}(a-3b)x\sqrt[3]{-x+x^3} + \frac{3b\left(-x+x^3\right)^{4/3}}{2x^2} + \frac{\left((-a+3b)x^{2/3}\left(-1+x^2\right)^{2/3}\right)\int \frac{\sqrt[3]{x}}{\left(-1+x^2\right)^{2/3}}\,d}{3\left(-x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}(a-3b)x\sqrt[3]{-x+x^3} + \frac{3b\left(-x+x^3\right)^{4/3}}{2x^2} + \frac{\left((-a+3b)x^{2/3}\left(-1+x^2\right)^{2/3}\right)\mathrm{Subst}\left(\int \frac{}{(-1}\right.}{\left(-x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}(a-3b)x\sqrt[3]{-x+x^3} + \frac{3b\left(-x+x^3\right)^{4/3}}{2x^2} + \frac{\left((-a+3b)x^{2/3}\left(-1+x^2\right)^{2/3}\right)\mathrm{Subst}\left(\int \frac{}{(-1}\right.}{2\left(-x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}(a-3b)x\sqrt[3]{-x+x^3} + \frac{3b\left(-x+x^3\right)^{4/3}}{2x^2} + \frac{\left((-a+3b)x^{2/3}\left(-1+x^2\right)^{2/3}\right)\mathrm{Subst}\left(\int \frac{x}{1-x}\right.}{2\left(-x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}(a-3b)x\sqrt[3]{-x+x^3} + \frac{3b\left(-x+x^3\right)^{4/3}}{2x^2} + \frac{\left((-a+3b)x^{2/3}\left(-1+x^2\right)^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{1-x}\right.}{6\left(-x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}(a-3b)x\sqrt[3]{-x+x^3} + \frac{3b\left(-x+x^3\right)^{4/3}}{2x^2} + \frac{(a-3b)x^{2/3}\left(-1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{6\left(-x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}(a-3b)x\sqrt[3]{-x+x^3} + \frac{3b\left(-x+x^3\right)^{4/3}}{2x^2} + \frac{(a-3b)x^{2/3}\left(-1+x^2\right)^{2/3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{6\left(-x+x^3\right)^{2/3}}$$

$$= \frac{1}{2}(a-3b)x\sqrt[3]{-x+x^3} + \frac{3b\left(-x+x^3\right)^{4/3}}{2x^2} + \frac{(a-3b)x^{2/3}\left(-1+x^2\right)^{2/3}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\left(-x+x^3\right)^{2/3}}$$

Mathematica [C] time = 0.03, size = 66, normalized size = 0.48

$$\frac{3\sqrt[3]{x\left(x^2-1\right)}\left(x^2(a-3b)\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^2\right)-2b\left(1-x^2\right)^{4/3}\right)}{4x\sqrt[3]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b + a*x^2)*(-x + x^3)^(1/3))/x^2,x]
```

```
[Out] (3*(x*(-1 + x^2))^(1/3)*(-2*b*(1 - x^2)^(4/3) + (a - 3*b)*x^2*Hypergeometri
c2F1[-1/3, 2/3, 5/3, x^2]))/(4*x*(1 - x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.36, size = 138, normalized size = 1.00

$$\frac{1}{6}(a-3b)\log\left(\sqrt[3]{x^3-x}-x\right)+\frac{1}{6}\left(\sqrt{3}a-3\sqrt{3}b\right)\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x}+x}\right)+\frac{\sqrt[3]{x^3-x}\left(ax^2-3b\right)}{2x}+\frac{1}{12}(3b-a)\log\left(\sqrt[3]{x^3-x}\,x+\left(x^3-x\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b + a*x^2)*(-x + x^3)^(1/3))/x^2,x]
```

```
[Out] ((-3*b + a*x^2)*(-x + x^3)^(1/3))/(2*x) + ((Sqrt[3]*a - 3*Sqrt[3]*b)*ArcTan
[(Sqrt[3]*x)/(x + 2*(-x + x^3)^(1/3))])/6 + ((a - 3*b)*Log[-x + (-x + x^3)^
(1/3)])/6 + ((-a + 3*b)*Log[x^2 + x*(-x + x^3)^(1/3) + (-x + x^3)^(2/3)])/1
2
```

fricas [A] time = 77.59, size = 123, normalized size = 0.89

$$\frac{2\sqrt{3}(a-3b)x\arctan\left(-\frac{44032959556\sqrt{3}(x^3-x)^{\frac{1}{3}}x+\sqrt{3}(16754327161x^2-2707204793)-10524305234\sqrt{3}(x^3-x)^{\frac{2}{3}}}{81835897185x^2-1102302937}\right)+(a-3b)x\log\left(-3\left(x^3-x\right)^{\frac{1}{3}}x+3\left(x^3-x\right)^{\frac{2}{3}}+1\right)+6\left(ax^2-3b\right)\left(x^3-x\right)^{\frac{1}{3}}}{12x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)*(x^3-x)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*sqrt(3)*(a - 3*b)*x*arctan(-(44032959556*sqrt(3)*(x^3 - x)^(1/3)*x
+ sqrt(3)*(16754327161*x^2 - 2707204793) - 10524305234*sqrt(3)*(x^3 - x)^(2
/3))/(81835897185*x^2 - 1102302937)) + (a - 3*b)*x*log(-3*(x^3 - x)^(1/3)*x
 + 3*(x^3 - x)^(2/3) + 1) + 6*(a*x^2 - 3*b)*(x^3 - x)^(1/3))/x
```

giac [A] time = 0.86, size = 105, normalized size = 0.76

$$\frac{1}{2}ax^2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}-\frac{1}{6}\sqrt{3}(a-3b)\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{12}(a-3b)\log\left(\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}(a-3b)\log\left(\left|\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)-\frac{3}{2}b\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)*(x^3-x)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*a*x^2*(-1/x^2 + 1)^(1/3) - 1/6*sqrt(3)*(a - 3*b)*arctan(1/3*sqrt(3)*(2*
(-1/x^2 + 1)^(1/3) + 1)) - 1/12*(a - 3*b)*log((-1/x^2 + 1)^(2/3) + (-1/x^2
+ 1)^(1/3) + 1) + 1/6*(a - 3*b)*log(abs((-1/x^2 + 1)^(1/3) - 1)) - 3/2*b*(-
1/x^2 + 1)^(1/3)
```

maple [C] time = 1.60, size = 800, normalized size = 5.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)*(x^3-x)^(1/3)/x^2,x)
```

```
[Out] 1/2*(a*x^2-3*b)*(x*(x^2-1))^(1/3)/x+1/36*(a-3*b)*(RootOf(_Z^2+6*_Z+36)*ln(-
(47*RootOf(_Z^2+6*_Z+36)^2*x^4+3207*RootOf(_Z^2+6*_Z+36)*x^4+2925*(x^6-2*x^
4+x^2)^(1/3)*RootOf(_Z^2+6*_Z+36)*x^2-235*RootOf(_Z^2+6*_Z+36)^2*x^2+6930*x
^4+2925*RootOf(_Z^2+6*_Z+36)*(x^6-2*x^4+x^2)^(2/3)+5238*(x^6-2*x^4+x^2)^(1/
3)*x^2-5601*RootOf(_Z^2+6*_Z+36)*x^2+5238*(x^6-2*x^4+x^2)^(2/3)-2925*RootOf
(_Z^2+6*_Z+36)*(x^6-2*x^4+x^2)^(1/3)+188*RootOf(_Z^2+6*_Z+36)^2-11340*x^2-5
238*(x^6-2*x^4+x^2)^(1/3)+2394*RootOf(_Z^2+6*_Z+36)+4410)/(-1+x)/(1+x))-ln(
(-47*RootOf(_Z^2+6*_Z+36)^2*x^4+2643*RootOf(_Z^2+6*_Z+36)*x^4+2925*(x^6-2*x
^4+x^2)^(1/3)*RootOf(_Z^2+6*_Z+36)*x^2+235*RootOf(_Z^2+6*_Z+36)^2*x^2+10620
*x^4+2925*RootOf(_Z^2+6*_Z+36)*(x^6-2*x^4+x^2)^(2/3)+12312*(x^6-2*x^4+x^2)^
(1/3)*x^2-2781*RootOf(_Z^2+6*_Z+36)*x^2+12312*(x^6-2*x^4+x^2)^(2/3)-2925*Ro
otOf(_Z^2+6*_Z+36)*(x^6-2*x^4+x^2)^(1/3)-188*RootOf(_Z^2+6*_Z+36)^2-13806*x
^2-12312*(x^6-2*x^4+x^2)^(1/3)+138*RootOf(_Z^2+6*_Z+36)+3186)/(-1+x)/(1+x))
*RootOf(_Z^2+6*_Z+36)-6*ln((-47*RootOf(_Z^2+6*_Z+36)^2*x^4+2643*RootOf(_Z^2
+6*_Z+36)*x^4+2925*(x^6-2*x^4+x^2)^(1/3)*RootOf(_Z^2+6*_Z+36)*x^2+235*RootO
f(_Z^2+6*_Z+36)^2*x^2+10620*x^4+2925*RootOf(_Z^2+6*_Z+36)*(x^6-2*x^4+x^2)^(
2/3)+12312*(x^6-2*x^4+x^2)^(1/3)*x^2-2781*RootOf(_Z^2+6*_Z+36)*x^2+12312*(x
^6-2*x^4+x^2)^(2/3)-2925*RootOf(_Z^2+6*_Z+36)*(x^6-2*x^4+x^2)^(1/3)-188*Roo
tOf(_Z^2+6*_Z+36)^2-13806*x^2-12312*(x^6-2*x^4+x^2)^(1/3)+138*RootOf(_Z^2+6
```

```
*_Z+36)+3186)/(-1+x)/(1+x)))*(x*(x^2-1))^(1/3)/x*(x^2*(x^2-1)^2)^(1/3)/(x^2
-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^2+b)(x^3-x)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)*(x^3-x)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 + b)*(x^3 - x)^(1/3)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^3-x)^{1/3}\,(a\,x^2+b)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - x)^(1/3)*(b + a*x^2))/x^2,x)
```

```
[Out] int(((x^3 - x)^(1/3)*(b + a*x^2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x\,(x-1)\,(x+1)}\,(ax^2+b)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b)*(x**3-x)**(1/3)/x**2,x)
```

```
[Out] Integral((x*(x - 1)*(x + 1))**(1/3)*(a*x**2 + b)/x**2, x)
```

3.1636 $\int \frac{-b+ax}{x\sqrt[3]{b^3+a^3x^3}}\,dx$

Optimal. Leaf size=138

$$-\log\left(\sqrt[3]{a^3x^3+b^3}-ax-b\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a^3x^3+b^3}}{\sqrt[3]{a^3x^3+b^3}+2ax+2b}\right)+\frac{1}{2}\log\left(\left(a^3x^3+b^3\right)^{2/3}+(ax+b)\sqrt[3]{a^3x^3+b}\right.$$

Rubi [A] time = 0.15, antiderivative size = 126, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1844, 239, 266, 55, 617, 204, 31}

$$-\frac{1}{2}\log\left(b-\sqrt[3]{a^3x^3+b^3}\right)-\frac{1}{2}\log\left(\sqrt[3]{a^3x^3+b^3}-ax\right)+\frac{\tan^{-1}\left(\frac{\frac{2ax}{\sqrt[3]{a^3x^3+b^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a^3x^3+b^3}+b}{\sqrt{3}b}\right)}{\sqrt{3}}+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x)/(x*(b^3 + a^3*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*a*x)/(b^3 + a^3*x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - ArcTan[(b + 2
*(b^3 + a^3*x^3)^(1/3))/(Sqrt[3]*b)]/Sqrt[3] + Log[x]/2 - Log[b - (b^3 + a^
3*x^3)^(1/3)]/2 - Log[-(a*x) + (b^3 + a^3*x^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-b+ax}{x\sqrt[3]{b^3+a^3x^3}}\,dx &= \int \left(\frac{a}{\sqrt[3]{b^3+a^3x^3}} - \frac{b}{x\sqrt[3]{b^3+a^3x^3}}\right) dx \\
&= a\int \frac{1}{\sqrt[3]{b^3+a^3x^3}}\,dx - b\int \frac{1}{x\sqrt[3]{b^3+a^3x^3}}\,dx \\
&= \frac{\tan^{-1}\left(\frac{1+\frac{2ax}{\sqrt[3]{b^3+a^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{2}\log\left(-ax+\sqrt[3]{b^3+a^3x^3}\right) - \frac{1}{3}b\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{b^3+a^3x}}\,dx, x, x^3\right) \\
&= \frac{\tan^{-1}\left(\frac{1+\frac{2ax}{\sqrt[3]{b^3+a^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log(x)}{2} - \frac{1}{2}\log\left(-ax+\sqrt[3]{b^3+a^3x^3}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{b-x}\,dx, x, \sqrt[3]{b^3 -}\right. \\
&= \frac{\tan^{-1}\left(\frac{1+\frac{2ax}{\sqrt[3]{b^3+a^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log(x)}{2} - \frac{1}{2}\log\left(b-\sqrt[3]{b^3+a^3x^3}\right) - \frac{1}{2}\log\left(-ax+\sqrt[3]{b^3+a^3x^3}\right) + \text{Sul} \\
&= \frac{\tan^{-1}\left(\frac{1+\frac{2ax}{\sqrt[3]{b^3+a^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b^3+a^3x^3}}{b}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log(x)}{2} - \frac{1}{2}\log\left(b-\sqrt[3]{b^3+a^3x^3}\right) - \frac{1}{2}\log\left(-a\right.
\end{aligned}$$

Mathematica [A] time = 0.13, size = 169, normalized size = 1.22

$$\frac{1}{6}\left(-2\log\left(1-\frac{ax}{\sqrt[3]{a^3x^3+b^3}}\right) - 3\log\left(b-\sqrt[3]{a^3x^3+b^3}\right) + 2\sqrt{3}\tan^{-1}\left(\frac{\frac{2ax}{\sqrt[3]{a^3x^3+b^3}}+1}{\sqrt{3}}\right) - 2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3x^3+b^3}+b}{\sqrt{3}b}\right) + \log\left(\frac{ax}{\sqrt[3]{a^3x^3+b^3}} + \frac{a^2x^2}{\left(a^3x^3+b^3\right)^{2/3}} + 1\right) + 3\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x)/(x*(b^3 + a^3*x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*a*x)/(b^3 + a^3*x^3)^(1/3))/Sqrt[3]] - 2*Sqrt[3]*
ArcTan[(b + 2*(b^3 + a^3*x^3)^(1/3))/(Sqrt[3]*b)] + 3*Log[x] - 2*Log[1 - (a
*x)/(b^3 + a^3*x^3)^(1/3)] + Log[1 + (a^2*x^2)/(b^3 + a^3*x^3)^(2/3) + (a*x
)/(b^3 + a^3*x^3)^(1/3)] - 3*Log[b - (b^3 + a^3*x^3)^(1/3)])/6
```

IntegrateAlgebraic [A] time = 0.62, size = 138, normalized size = 1.00

$$-\log\left(\sqrt[3]{a^3x^3+b^3}-ax-b\right) - \sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a^3x^3+b^3}}{\sqrt[3]{a^3x^3+b^3}+2ax+2b}\right) + \frac{1}{2}\log\left(\left(a^3x^3+b^3\right)^{2/3} + (ax+b)\sqrt[3]{a^3x^3+b^3} + a^2x^2 + 2abx + b^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x)/(x*(b^3 + a^3*x^3)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*(b^3 + a^3*x^3)^(1/3))/(2*b + 2*a*x + (b^3 + a^3*
x^3)^(1/3))]) - Log[-b - a*x + (b^3 + a^3*x^3)^(1/3)] + Log[b^2 + 2*a*b*x +
 a^2*x^2 + (b + a*x)*(b^3 + a^3*x^3)^(1/3) + (b^3 + a^3*x^3)^(2/3)]/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/x/(a^3*x^3+b^3)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax - b}{\left(a^3x^3 + b^3\right)^{\frac{1}{3}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/x/(a^3*x^3+b^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*x - b)/((a^3*x^3 + b^3)^(1/3)*x), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{ax - b}{x\left(a^3x^3 + b^3\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-b)/x/(a^3*x^3+b^3)^(1/3),x)
```

```
[Out] int((a*x-b)/x/(a^3*x^3+b^3)^(1/3),x)
```

maxima [A] time = 0.41, size = 213, normalized size = 1.54

$$-\frac{1}{6}a\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+\frac{2\left(a^3x^3+b^3\right)^{\frac{1}{3}}}{x}\right)}{3a}\right)}{a} - \frac{\log\left(a^2+\frac{\left(a^3x^3+b^3\right)^{\frac{1}{3}}a}{x}+\frac{\left(a^3x^3+b^3\right)^{\frac{2}{3}}}{x^2}\right)}{a} + \frac{2\log\left(-a+\frac{\left(a^3x^3+b^3\right)^{\frac{1}{3}}}{x}\right)}{a}\right) - \frac{1}{6}b\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b+2\left(a^3x^3+b^3\right)^{\frac{1}{3}}\right)}{3b}\right)}{b} - \frac{\log\left(b^2+\left(a^3x^3+b^3\right)^{\frac{1}{3}}b+\left(a^3x^3+b^3\right)^{\frac{2}{3}}\right)}{b} + \frac{2\log\left(-b+\left(a^3x^3+b^3\right)^{\frac{1}{3}}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/x/(a^3*x^3+b^3)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(a^3*x^3 + b^3)^(1/3)/x)/a)/a -
 log(a^2 + (a^3*x^3 + b^3)^(1/3)*a/x + (a^3*x^3 + b^3)^(2/3)/x^2)/a + 2*log
(-a + (a^3*x^3 + b^3)^(1/3)/x)/a) - 1/6*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b
+ 2*(a^3*x^3 + b^3)^(1/3))/b)/b - log(b^2 + (a^3*x^3 + b^3)^(1/3)*b + (a^3*
x^3 + b^3)^(2/3))/b + 2*log(-b + (a^3*x^3 + b^3)^(1/3))/b)
```

mupad [B] time = 1.35, size = 160, normalized size = 1.16

$$\frac{a\,x\,\left(\frac{a^3\,x^3}{b^3}+1\right)^{1/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ -\frac{a^3\,x^3}{b^3}\right)}{\left(a^3\,x^3+b^3\right)^{1/3}} - \ln\left(b^2\,\left(a^3\,x^3+b^3\right)^{1/3}-9\,b^3\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)^2\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right) + \ln\left(b^2\,\left(a^3\,x^3+b^3\right)^{1/3}-9\,b^3\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)^2\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right) - \frac{\ln\left(b^2\,\left(a^3\,x^3+b^3\right)^{1/3}-b^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x)/(x*(b^3 + a^3*x^3)^(1/3)),x)
```

```
[Out] log(b^2*(b^3 + a^3*x^3)^(1/3) - 9*b^3*((3^(1/2)*1i)/6 + 1/6)^2)*((3^(1/2)*1
i)/6 + 1/6) - log(b^2*(b^3 + a^3*x^3)^(1/3) - 9*b^3*((3^(1/2)*1i)/6 - 1/6)^
2)*((3^(1/2)*1i)/6 - 1/6) - log(b^2*(b^3 + a^3*x^3)^(1/3) - b^3)/3 + (a*x*(
```

```
(a^3*x^3)/b^3 + 1)^(1/3)*hypergeom([1/3, 1/3], 4/3, -(a^3*x^3)/b^3))/(b^3 +
 a^3*x^3)^(1/3)
```

sympy [C] time = 4.31, size = 76, normalized size = 0.55

$$\frac{ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{a^3x^3e^{i\pi}}{b^3}\right)}{3b\Gamma\left(\frac{4}{3}\right)}+\frac{b\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{b^3e^{i\pi}}{a^3x^3}\right)}{3ax\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/x/(a**3*x**3+b**3)**(1/3),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), a**3*x**3*exp_polar(I*pi)/b**3)/(3
*b*gamma(4/3)) + b*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b**3*exp_polar(I*pi
)/(a**3*x**3))/(3*a*x*gamma(4/3))
```

3.1637 $\int \frac{x^2 \sqrt[4]{-x^3+x^4}}{2+x} \, dx$

Optimal. Leaf size=138

$$\frac{1135}{64}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)-8\,2^{3/4}\sqrt[4]{3}\,\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\,x}{\sqrt[4]{x^4-x^3}}\right)-\frac{1135}{64}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)+8\,2^{3/4}\sqrt[4]{3}\,\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\,x}{\sqrt[4]{x^4-x^3}}\right)$$

Rubi [A] time = 0.23, antiderivative size = 258, normalized size of antiderivative = 1.87, number of steps used = 27, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2042, 101, 157, 50, 63, 240, 212, 206, 203, 105, 93, 298}

$$-\frac{25}{24}\sqrt[4]{x^4-x^3}\,x+\frac{401}{96}\sqrt[4]{x^4-x^3}-\frac{1135\sqrt[4]{x^4-x^3}\,\tan^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{64\sqrt[4]{x-1}\,x^{3/4}}-\frac{8\,2^{3/4}\sqrt[4]{3}\,\sqrt[4]{x^4-x^3}\,\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{\sqrt[4]{x-1}\,x^{3/4}}-\frac{1135\sqrt[4]{x^4-x^3}\,\tanh^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{64\sqrt[4]{x-1}\,x^{3/4}}+\frac{8\,2^{3/4}\sqrt[4]{3}\,\sqrt[4]{x^4-x^3}\,\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{\sqrt[4]{x-1}\,x^{3/4}}+\frac{1}{3}\sqrt[4]{x^4-x^3}\,x^2$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(-x^3 + x^4)^(1/4))/(2 + x),x]
```

```
[Out] (401*(-x^3 + x^4)^(1/4))/96 - (25*x*(-x^3 + x^4)^(1/4))/24 + (x^2*(-x^3 + x
^4)^(1/4))/3 - (1135*(-x^3 + x^4)^(1/4)*ArcTan[(-1 + x)^(1/4)/x^(1/4)])/(64
*(-1 + x)^(1/4)*x^(3/4)) - (8*2^(3/4)*3^(1/4)*(-x^3 + x^4)^(1/4)*ArcTan[((3
/2)^(1/4)*x^(1/4))/(-1 + x)^(1/4)])/((-1 + x)^(1/4)*x^(3/4)) - (1135*(-x^3
+ x^4)^(1/4)*ArcTanh[(-1 + x)^(1/4)/x^(1/4)])/(64*(-1 + x)^(1/4)*x^(3/4)) +
 (8*2^(3/4)*3^(1/4)*(-x^3 + x^4)^(1/4)*ArcTanh[((3/2)^(1/4)*x^(1/4))/(-1 +
x)^(1/4)])/((-1 + x)^(1/4)*x^(3/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
```

```
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\int \frac{x^2 \sqrt[4]{-x^3+x^4}}{2+x}\,dx = \frac{\sqrt[4]{-x^3+x^4}\int \frac{\sqrt[4]{-1+x}\,x^{11/4}}{2+x}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{1}{3}x^2\sqrt[4]{-x^3+x^4} - \frac{\sqrt[4]{-x^3+x^4}\int \frac{x^{7/4}\left(-\frac{11}{2}+\frac{25x}{4}\right)}{(-1+x)^{3/4}(2+x)}\,dx}{3\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{1}{3}x^2\sqrt[4]{-x^3+x^4} - \frac{\left(25\sqrt[4]{-x^3+x^4}\right)\int \frac{x^{7/4}}{(-1+x)^{3/4}}\,dx}{12\sqrt[4]{-1+x}\,x^{3/4}} + \frac{\left(6\sqrt[4]{-x^3+x^4}\right)\int \frac{x^{7/4}}{(-1+x)^{3/4}(2+x)}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= -\frac{25}{24}x\sqrt[4]{-x^3+x^4} + \frac{1}{3}x^2\sqrt[4]{-x^3+x^4} - \frac{\left(175\sqrt[4]{-x^3+x^4}\right)\int \frac{x^{3/4}}{(-1+x)^{3/4}}\,dx}{96\sqrt[4]{-1+x}\,x^{3/4}} + \frac{\left(6\sqrt[4]{-x^3+x^4}\right)\int}{\sqrt[4]{-1+x}}$$

$$= \frac{401}{96}\sqrt[4]{-x^3+x^4} - \frac{25}{24}x\sqrt[4]{-x^3+x^4} + \frac{1}{3}x^2\sqrt[4]{-x^3+x^4} - \frac{\left(175\sqrt[4]{-x^3+x^4}\right)\int \frac{1}{(-1+x)^{3/4}\sqrt[4]{x}}\,dx}{128\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{401}{96}\sqrt[4]{-x^3+x^4} - \frac{25}{24}x\sqrt[4]{-x^3+x^4} + \frac{1}{3}x^2\sqrt[4]{-x^3+x^4} - \frac{\left(175\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^4}}\right.}{32\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{401}{96}\sqrt[4]{-x^3+x^4} - \frac{25}{24}x\sqrt[4]{-x^3+x^4} + \frac{1}{3}x^2\sqrt[4]{-x^3+x^4} - \frac{\left(175\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\int \frac{1}{1-x^4}\,d\right.}{32\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{401}{96}\sqrt[4]{-x^3+x^4} - \frac{25}{24}x\sqrt[4]{-x^3+x^4} + \frac{1}{3}x^2\sqrt[4]{-x^3+x^4} - \frac{8\,2^{3/4}\sqrt[4]{3}\sqrt[4]{-x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\,\sqrt[4]{}}{\sqrt[4]{-1+}}\right.}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{401}{96}\sqrt[4]{-x^3+x^4} - \frac{25}{24}x\sqrt[4]{-x^3+x^4} + \frac{1}{3}x^2\sqrt[4]{-x^3+x^4} - \frac{1135\sqrt[4]{-x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{64\sqrt[4]{-1+x}\,x^{3/4}} -$$

Mathematica [C] time = 0.04, size = 124, normalized size = 0.90

$$\frac{4\sqrt[4]{(x-1)x^3}\left(\sqrt[4]{x}\,{}_2F_1\left(-\frac{11}{4},\frac{1}{4};\frac{5}{4};1-x\right)-3\sqrt[4]{x}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{4};\frac{5}{4};1-x\right)+6\sqrt[4]{x}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};1-x\right)-12\sqrt[4]{x}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};1-x\right)+8\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{2(x-1)}{3x}\right)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(-x^3 + x^4)^(1/4))/(2 + x),x]
```

```
[Out] (4*((-1 + x)*x^3)^(1/4)*(x^(1/4)*Hypergeometric2F1[-11/4, 1/4, 5/4, 1 - x]
- 3*x^(1/4)*Hypergeometric2F1[-7/4, 1/4, 5/4, 1 - x] + 6*x^(1/4)*Hypergeome
tric2F1[-3/4, 1/4, 5/4, 1 - x] - 12*x^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4
, 1 - x] + 8*Hypergeometric2F1[1/4, 1, 5/4, (2*(-1 + x))/(3*x)]))/x
```

IntegrateAlgebraic [A] time = 0.65, size = 138, normalized size = 1.00

$$\frac{1135}{64}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)-8\,2^{3/4}\sqrt[4]{3}\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\,x}{\sqrt[4]{x^4-x^3}}\right)-\frac{1135}{64}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)+8\,2^{3/4}\sqrt[4]{3}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\,x}{\sqrt[4]{x^4-x^3}}\right)+\frac{1}{96}\sqrt[4]{x^4-x^3}\left(32x^2-100x+401\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(-x^3 + x^4)^(1/4))/(2 + x),x]
```

```
[Out] ((401 - 100*x + 32*x^2)*(-x^3 + x^4)^(1/4))/96 + (1135*ArcTan[x/(-x^3 + x^4
)^(1/4)])/64 - 8*2^(3/4)*3^(1/4)*ArcTan[((3/2)^(1/4)*x)/(-x^3 + x^4)^(1/4)]
```

```
 - (1135*ArcTanh[x/(-x^3 + x^4)^(1/4)])/64 + 8*2^(3/4)*3^(1/4)*ArcTanh[((3/
2)^(1/4)*x)/(-x^3 + x^4)^(1/4)]
```

fricas [A] time = 0.43, size = 207, normalized size = 1.50

$$\frac{1}{96}(x^4-x^3)^{\frac{1}{4}}(32x^2-100x+401) - 16\cdot 24^{\frac{1}{4}}\arctan\left(\frac{24^{\frac{3}{4}}\sqrt{2}x\sqrt{\frac{\sqrt{6}x^2+2\sqrt{x^4-x^3}}{x^2}} - 2\cdot 24^{\frac{3}{4}}(x^4-x^3)^{\frac{1}{4}}}{24x}\right) + 4\cdot 24^{\frac{1}{4}}\log\left(\frac{24^{\frac{1}{4}}x+2(x^4-x^3)^{\frac{1}{4}}}{x}\right) - 4\cdot 24^{\frac{1}{4}}\log\left(-\frac{24^{\frac{1}{4}}x-2(x^4-x^3)^{\frac{1}{4}}}{x}\right) - \frac{1135}{64}\arctan\left(\frac{(x^4-x^3)^{\frac{1}{4}}}{x}\right) - \frac{1135}{128}\log\left(\frac{x+(x^4-x^3)^{\frac{1}{4}}}{x}\right) + \frac{1135}{128}\log\left(-\frac{x-(x^4-x^3)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4-x^3)^(1/4)/(2+x),x, algorithm="fricas")
```

```
[Out] 1/96*(x^4 - x^3)^(1/4)*(32*x^2 - 100*x + 401) - 16*24^(1/4)*arctan(1/24*(24
^(3/4)*sqrt(2)*x*sqrt((sqrt(6)*x^2 + 2*sqrt(x^4 - x^3))/x^2) - 2*24^(3/4)*(
x^4 - x^3)^(1/4))/x) + 4*24^(1/4)*log((24^(1/4)*x + 2*(x^4 - x^3)^(1/4))/x)
 - 4*24^(1/4)*log(-(24^(1/4)*x - 2*(x^4 - x^3)^(1/4))/x) - 1135/64*arctan((
x^4 - x^3)^(1/4)/x) - 1135/128*log((x + (x^4 - x^3)^(1/4))/x) + 1135/128*lo
g(-(x - (x^4 - x^3)^(1/4))/x)
```

giac [A] time = 0.63, size = 149, normalized size = 1.08

$$-\frac{1}{96}\left(401\left(\frac{1}{x}-1\right)^2\left(-\frac{1}{x}+1\right)^{\frac{1}{4}} - 702\left(-\frac{1}{x}+1\right)^{\frac{5}{4}} + 333\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)x^3 - 8\cdot 24^{\frac{1}{4}}\arctan\left(\frac{2}{3}\left(\frac{3}{2}\right)^{\frac{3}{4}}\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right) - 4\cdot 24^{\frac{1}{4}}\log\left(\left(\frac{3}{2}\right)^{\frac{1}{4}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right) + 4\cdot 24^{\frac{1}{4}}\log\left(\left|-\left(\frac{3}{2}\right)^{\frac{1}{4}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right|\right) + \frac{1135}{64}\arctan\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right) + \frac{1135}{128}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right) - \frac{1135}{128}\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4-x^3)^(1/4)/(2+x),x, algorithm="giac")
```

```
[Out] -1/96*(401*(1/x - 1)^2*(-1/x + 1)^(1/4) - 702*(-1/x + 1)^(5/4) + 333*(-1/x
+ 1)^(1/4))*x^3 - 8*24^(1/4)*arctan(2/3*(3/2)^(3/4)*(-1/x + 1)^(1/4)) - 4*2
4^(1/4)*log((3/2)^(1/4) + (-1/x + 1)^(1/4)) + 4*24^(1/4)*log(abs(-(3/2)^(1/
4) + (-1/x + 1)^(1/4))) + 1135/64*arctan((-1/x + 1)^(1/4)) + 1135/128*log((
-1/x + 1)^(1/4) + 1) - 1135/128*log(abs((-1/x + 1)^(1/4) - 1))
```

maple [C] time = 2.45, size = 1030, normalized size = 7.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4-x^3)^(1/4)/(2+x),x)
```

```
[Out] 1/96*(32*x^2-100*x+401)*(x^3*(-1+x))^(1/4)+(-1135/128*ln((2*(x^4-3*x^3+3*x^
2-x)^(3/4)+2*(x^4-3*x^3+3*x^2-x)^(1/2)*x+2*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+2*
x^3-2*(x^4-3*x^3+3*x^2-x)^(1/2)-4*(x^4-3*x^3+3*x^2-x)^(1/4)*x-5*x^2+2*(x^4-
3*x^3+3*x^2-x)^(1/4)+4*x-1)/(-1+x)^2)-4*RootOf(_Z^2+RootOf(_Z^4-24)^2)*ln((
(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4-24)^2*RootOf(_Z^2+RootOf(_Z^4-24)^2)*
x-(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4-24)^2*RootOf(_Z^2+RootOf(_Z^4-24)^2
)-2*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4-24)^2*x^2+4*(x^4-3*x^3+3*x^2-x)^(
1/4)*RootOf(_Z^4-24)^2*x-5*RootOf(_Z^2+RootOf(_Z^4-24)^2)*x^3+12*(x^4-3*x^3
+3*x^2-x)^(3/4)-2*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4-24)^2+12*RootOf(_Z^
2+RootOf(_Z^4-24)^2)*x^2-9*RootOf(_Z^2+RootOf(_Z^4-24)^2)*x+2*RootOf(_Z^2+R
ootOf(_Z^4-24)^2))/(-1+x)^2/(2+x))+4*RootOf(_Z^4-24)*ln((RootOf(_Z^4-24)^3*
(x^4-3*x^3+3*x^2-x)^(1/2)*x-RootOf(_Z^4-24)^3*(x^4-3*x^3+3*x^2-x)^(1/2)+2*(
x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4-24)^2*x^2-4*(x^4-3*x^3+3*x^2-x)^(1/4)*
RootOf(_Z^4-24)^2*x+5*RootOf(_Z^4-24)*x^3+12*(x^4-3*x^3+3*x^2-x)^(3/4)+2*(x
^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4-24)^2-12*RootOf(_Z^4-24)*x^2+9*RootOf(_
Z^4-24)*x-2*RootOf(_Z^4-24))/(-1+x)^2/(2+x))-1135/3072*RootOf(_Z^4-24)^3*Ro
otOf(_Z^2+RootOf(_Z^4-24)^2)*ln((-2*RootOf(_Z^4-24)^3*(x^4-3*x^3+3*x^2-x)^(
1/2)*RootOf(_Z^2+RootOf(_Z^4-24)^2)*x+2*RootOf(_Z^2+RootOf(_Z^4-24)^2)*Root
Of(_Z^4-24)^3*x^3+2*RootOf(_Z^4-24)^3*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2
+RootOf(_Z^4-24)^2)-5*RootOf(_Z^2+RootOf(_Z^4-24)^2)*RootOf(_Z^4-24)^3*x^2+
4*RootOf(_Z^2+RootOf(_Z^4-24)^2)*RootOf(_Z^4-24)^3*x-RootOf(_Z^4-24)^3*Root
Of(_Z^2+RootOf(_Z^4-24)^2)+48*(x^4-3*x^3+3*x^2-x)^(3/4)-48*(x^4-3*x^3+3*x^2
```

```
-x)^(1/4)*x^2+96*(x^4-3*x^3+3*x^2-x)^(1/4)*x-48*(x^4-3*x^3+3*x^2-x)^(1/4))/
(-1+x)^2))*(x^3*(-1+x))^(1/4)/(-1+x)/x*(x*(-1+x)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^3\right)^{\frac{1}{4}}x^2}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^4-x^3)^(1/4)/(2+x),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^3)^(1/4)*x^2/(x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\left(x^4-x^3\right)^{1/4}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^4 - x^3)^(1/4))/(x + 2),x)
```

```
[Out] int((x^2*(x^4 - x^3)^(1/4))/(x + 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt[4]{x^3\left(x-1\right)}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**4-x**3)**(1/4)/(2+x),x)
```

```
[Out] Integral(x**2*(x**3*(x - 1))**(1/4)/(x + 2), x)
```

3.1638 $\int \frac{(-1+x)(-3+8x-8x^2+12x^4)}{x\left(\frac{1-2x^2}{1+2x^2}\right)^{2/3}(1+2x^2)(3-7x+7x^2-6x^3+2x^4)}\,dx$

Optimal. Leaf size=138

$$\log\left(\sqrt[3]{\frac{1-2x^2}{2x^2+1}}+x-1\right)-\frac{1}{2}\log\left(x^2+\left(\frac{1-2x^2}{2x^2+1}\right)^{2/3}+(1-x)\sqrt[3]{\frac{1-2x^2}{2x^2+1}}-2x+1\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x-\sqrt{3}}{-2\sqrt[3]{\frac{1-2x^2}{2x^2+1}}+x-1}\right.$$

Rubi [F] time = 10.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x)\left(-3+8x-8x^2+12x^4\right)}{x\left(\frac{1-2x^2}{1+2x^2}\right)^{2/3}\left(1+2x^2\right)\left(3-7x+7x^2-6x^3+2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x)*(-3 + 8*x - 8*x^2 + 12*x^4))/(x*((1 - 2*x^2)/(1 + 2*x^2))^(2/
3)*(1 + 2*x^2)*(3 - 7*x + 7*x^2 - 6*x^3 + 2*x^4)),x]
```

```
[Out] (6*x*(1 - 2*x^2)^(2/3)*AppellF1[1/2, 2/3, 1/3, 3/2, 2*x^2, -2*x^2])/(((1 -
2*x^2)/(1 + 2*x^2))^(2/3)*(1 + 2*x^2)^(2/3)) + (Sqrt[3]*(1 - 2*x^2)^(2/3)*A
rcTan[1/Sqrt[3] + (2*(1 + 2*x^2)^(1/3))/(Sqrt[3]*(1 - 2*x^2)^(1/3))])/(2*((
1 - 2*x^2)/(1 + 2*x^2))^(2/3)*(1 + 2*x^2)^(2/3)) - ((1 - 2*x^2)^(2/3)*Log[x
])/(2*((1 - 2*x^2)/(1 + 2*x^2))^(2/3)*(1 + 2*x^2)^(2/3)) + (3*(1 - 2*x^2)^(
2/3)*Log[(1 - 2*x^2)^(1/3) - (1 + 2*x^2)^(1/3)])/(4*((1 - 2*x^2)/(1 + 2*x^2
))^(2/3)*(1 + 2*x^2)^(2/3)) - (22*(1 - 2*x^2)^(2/3)*Defer[Int][1/((1 - 2*x^
2)^(2/3)*(1 + 2*x^2)^(1/3)*(3 - 7*x + 7*x^2 - 6*x^3 + 2*x^4)), x])/(((1 - 2
*x^2)/(1 + 2*x^2))^(2/3)*(1 + 2*x^2)^(2/3)) + (51*(1 - 2*x^2)^(2/3)*Defer[I
nt][x/((1 - 2*x^2)^(2/3)*(1 + 2*x^2)^(1/3)*(3 - 7*x + 7*x^2 - 6*x^3 + 2*x^4
)), x])/(((1 - 2*x^2)/(1 + 2*x^2))^(2/3)*(1 + 2*x^2)^(2/3)) - (44*(1 - 2*x^
2)^(2/3)*Defer[Int][x^2/((1 - 2*x^2)^(2/3)*(1 + 2*x^2)^(1/3)*(3 - 7*x + 7*x
^2 - 6*x^3 + 2*x^4)), x])/(((1 - 2*x^2)/(1 + 2*x^2))^(2/3)*(1 + 2*x^2)^(2/3
)) + (22*(1 - 2*x^2)^(2/3)*Defer[Int][x^3/((1 - 2*x^2)^(2/3)*(1 + 2*x^2)^(1
/3)*(3 - 7*x + 7*x^2 - 6*x^3 + 2*x^4)), x])/(((1 - 2*x^2)/(1 + 2*x^2))^(2/3
)*(1 + 2*x^2)^(2/3))
```

Rubi steps

$$\int \frac{(-1+x)\left(-3+8x-8x^2+12x^4\right)}{x\left(\frac{1-2x^2}{1+2x^2}\right)^{2/3}\left(1+2x^2\right)\left(3-7x+7x^2-6x^3+2x^4\right)}dx = \frac{\left(1-2x^2\right)^{2/3}\int \frac{(-1+x)\left(-3+8x-8x^2+12x^4\right)}{x\left(1-2x^2\right)^{2/3}\sqrt[3]{1+2x^2}\left(3-7x+7x^2-6x^3+2x^4\right)}dx}{\left(\frac{1-2x^2}{1+2x^2}\right)^{2/3}\left(1+2x^2\right)^{2/3}}$$

$$= \frac{\left(1-2x^2\right)^{2/3}\int\left(\frac{6}{\left(1-2x^2\right)^{2/3}\sqrt[3]{1+2x^2}} + \frac{1}{x\left(1-2x^2\right)^{2/3}\sqrt[3]{1+2x^2}} + \cdots\right.}{\left(\frac{1-2x^2}{1+2x^2}\right)^{2/3}\left(1+2x\cdots\right.}$$

$$= \frac{\left(1-2x^2\right)^{2/3}\int \frac{1}{x\left(1-2x^2\right)^{2/3}\sqrt[3]{1+2x^2}}dx}{\left(\frac{1-2x^2}{1+2x^2}\right)^{2/3}\left(1+2x^2\right)^{2/3}} + \frac{\left(1-2x^2\right)^{2/3}\int \cdots}{\cdots}$$

$$= \frac{6x\left(1-2x^2\right)^{2/3}F_1\left(\frac{1}{2};\frac{2}{3},\frac{1}{3};\frac{3}{2};2x^2,-2x^2\right)}{\left(\frac{1-2x^2}{1+2x^2}\right)^{2/3}\left(1+2x^2\right)^{2/3}} + \frac{\left(1-2x^2\right)\cdots}{\cdots}$$

$$= \frac{6x\left(1-2x^2\right)^{2/3}F_1\left(\frac{1}{2};\frac{2}{3},\frac{1}{3};\frac{3}{2};2x^2,-2x^2\right)}{\left(\frac{1-2x^2}{1+2x^2}\right)^{2/3}\left(1+2x^2\right)^{2/3}} + \frac{\sqrt{3}\left(1-\cdots\right.}{2\cdots}$$

Mathematica [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(-1+x)\left(-3+8x-8x^2+12x^4\right)}{x\left(\frac{1-2x^2}{1+2x^2}\right)^{2/3}\left(1+2x^2\right)\left(3-7x+7x^2-6x^3+2x^4\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x)*(-3 + 8*x - 8*x^2 + 12*x^4))/(x*((1 - 2*x^2)/(1 + 2*x^2
))^(2/3)*(1 + 2*x^2)*(3 - 7*x + 7*x^2 - 6*x^3 + 2*x^4)),x]
```

```
[Out] Integrate[((-1 + x)*(-3 + 8*x - 8*x^2 + 12*x^4))/(x*((1 - 2*x^2)/(1 + 2*x^2
))^(2/3)*(1 + 2*x^2)*(3 - 7*x + 7*x^2 - 6*x^3 + 2*x^4)), x]
```

IntegrateAlgebraic [A] time = 3.17, size = 138, normalized size = 1.00

$$\log\left(\sqrt[3]{\frac{1-2x^2}{2x^2+1}}+x-1\right)-\frac{1}{2}\log\left(x^2+\left(\frac{1-2x^2}{2x^2+1}\right)^{2/3}+(1-x)\sqrt[3]{\frac{1-2x^2}{2x^2+1}}-2x+1\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x-\sqrt{3}}{-2\sqrt[3]{\frac{1-2x^2}{2x^2+1}}+x-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)*(-3 + 8*x - 8*x^2 + 12*x^4))/(x*((1 - 2*x^2)/(
1 + 2*x^2))^(2/3)*(1 + 2*x^2)*(3 - 7*x + 7*x^2 - 6*x^3 + 2*x^4)),x]
```

```
[Out] Sqrt[3]*ArcTan[(-Sqrt[3] + Sqrt[3]*x)/(-1 + x - 2*((1 - 2*x^2)/(1 + 2*x^2))
^(1/3))] + Log[-1 + x + ((1 - 2*x^2)/(1 + 2*x^2))^(1/3)] - Log[1 - 2*x + x^
2 + (1 - x)*((1 - 2*x^2)/(1 + 2*x^2))^(1/3) + ((1 - 2*x^2)/(1 + 2*x^2))^(2/
3)]/2
```

fricas [B] time = 3.36, size = 278, normalized size = 2.01

$$\sqrt{3}\arctan\left(\frac{434\sqrt{3}\left(2x^3-2x^2+x-1\right)\left(-\frac{2x^2-1}{2x^2+1}\right)^{\frac{2}{3}}+682\sqrt{3}\left(2x^4-4x^3+3x^2-2x+1\right)\left(-\frac{2x^2-1}{2x^2+1}\right)^{\frac{1}{3}}+\sqrt{3}\left(242x^5-726x^4+847x^3-1095x^2+363x+124\right)}{2662x^5-7986x^4+9317x^3-5969x^2+3993x-1674}\right)+\frac{1}{2}\log\left(\frac{2x^5-6x^4+7x^3-7x^2+3\left(2x^3-2x^2+x-1\right)\left(-\frac{2x^2-1}{2x^2+1}\right)^{\frac{2}{3}}+3\left(2x^4-4x^3+3x^2-2x+1\right)\left(-\frac{2x^2-1}{2x^2+1}\right)^{\frac{1}{3}}+3x}{2x^5-6x^4+7x^3-7x^2+3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(12*x^4-8*x^2+8*x-3)/x/((-2*x^2+1)/(2*x^2+1))^(2/3)/(2*x^2
+1)/(2*x^4-6*x^3+7*x^2-7*x+3),x, algorithm="fricas")
```

```
[Out] sqrt(3)*arctan((434*sqrt(3)*(2*x^3 - 2*x^2 + x - 1)*(-(2*x^2 - 1)/(2*x^2 +
1))^(2/3) + 682*sqrt(3)*(2*x^4 - 4*x^3 + 3*x^2 - 2*x + 1)*(-(2*x^2 - 1)/(2*
x^2 + 1))^(1/3) + sqrt(3)*(242*x^5 - 726*x^4 + 847*x^3 - 1095*x^2 + 363*x +
 124))/(2662*x^5 - 7986*x^4 + 9317*x^3 - 5969*x^2 + 3993*x - 1674)) + 1/2*l
og((2*x^5 - 6*x^4 + 7*x^3 - 7*x^2 + 3*(2*x^3 - 2*x^2 + x - 1)*(-(2*x^2 - 1)
/(2*x^2 + 1))^(2/3) + 3*(2*x^4 - 4*x^3 + 3*x^2 - 2*x + 1)*(-(2*x^2 - 1)/(2*
x^2 + 1))^(1/3) + 3*x)/(2*x^5 - 6*x^4 + 7*x^3 - 7*x^2 + 3*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(12\,x^4-8\,x^2+8\,x-3\right)(x-1)}{\left(2\,x^4-6\,x^3+7\,x^2-7\,x+3\right)\left(2\,x^2+1\right)x\left(-\frac{2\,x^2-1}{2\,x^2+1}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(12*x^4-8*x^2+8*x-3)/x/((-2*x^2+1)/(2*x^2+1))^(2/3)/(2*x^2
+1)/(2*x^4-6*x^3+7*x^2-7*x+3),x, algorithm="giac")
```

```
[Out] integrate((12*x^4 - 8*x^2 + 8*x - 3)*(x - 1)/((2*x^4 - 6*x^3 + 7*x^2 - 7*x
+ 3)*(2*x^2 + 1)*x*(-(2*x^2 - 1)/(2*x^2 + 1))^(2/3)), x)
```

maple [C] time = 4.91, size = 2147, normalized size = 15.56

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)*(12*x^4-8*x^2+8*x-3)/x/((-2*x^2+1)/(2*x^2+1))^(2/3)/(2*x^2+1)/(2
*x^4-6*x^3+7*x^2-7*x+3),x)
```

```
[Out] -2*ln(-(-11+33*x-114*RootOf(4*_Z^2+2*_Z+1)*x^2+28*RootOf(4*_Z^2+2*_Z+1)*x^5
-84*RootOf(4*_Z^2+2*_Z+1)*x^4+42*RootOf(4*_Z^2+2*_Z+1)*x+36*x*(-(2*x^2-1)/(
2*x^2+1))^(1/3)+22*x^5-55*x^2+77*x^3-66*x^4-18*(-(2*x^2-1)/(2*x^2+1))^(2/3)
-18*(-(2*x^2-1)/(2*x^2+1))^(1/3)-18*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^
2+1))^(1/3)*x^2+12*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x+12*
RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(2/3)*x^3-12*RootOf(4*_Z^2+2*_
Z+1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^4-12*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/
(2*x^2+1))^(2/3)*x^2+24*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)*
x^3+6*x*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(2/3)+98*x^3*RootOf(4*
_Z^2+2*_Z+1)+28*RootOf(4*_Z^2+2*_Z+1)^2+8*RootOf(4*_Z^2+2*_Z+1)-56*RootOf(4
*_Z^2+2*_Z+1)^2*x^2-36*(-(2*x^2-1)/(2*x^2+1))^(2/3)*x^2-54*(-(2*x^2-1)/(2*x
^2+1))^(1/3)*x^2-36*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^4+72*(-(2*x^2-1)/(2*x^2+
1))^(1/3)*x^3+18*(-(2*x^2-1)/(2*x^2+1))^(2/3)*x-6*RootOf(4*_Z^2+2*_Z+1)*(-(
2*x^2-1)/(2*x^2+1))^(2/3)-6*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(1
/3)+36*x^3*(-(2*x^2-1)/(2*x^2+1))^(2/3))/(2*x^4-6*x^3+7*x^2-7*x+3)/x)*RootO
f(4*_Z^2+2*_Z+1)+2*RootOf(4*_Z^2+2*_Z+1)*ln((8-12*x-58*RootOf(4*_Z^2+2*_Z+1
)*x^2+28*RootOf(4*_Z^2+2*_Z+1)*x^5-84*RootOf(4*_Z^2+2*_Z+1)*x^4+42*RootOf(4
*_Z^2+2*_Z+1)*x-30*x*(-(2*x^2-1)/(2*x^2+1))^(1/3)-8*x^5+12*x^2-28*x^3+24*x^
4+15*(-(2*x^2-1)/(2*x^2+1))^(2/3)+15*(-(2*x^2-1)/(2*x^2+1))^(1/3)-18*RootOf
(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^2+12*RootOf(4*_Z^2+2*_Z+1)*(
-(2*x^2-1)/(2*x^2+1))^(1/3)*x+12*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1
))^(2/3)*x^3-12*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^4-12*R
ootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(2/3)*x^2+24*RootOf(4*_Z^2+2*_Z
+1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^3+6*x*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/
(2*x^2+1))^(2/3)+98*x^3*RootOf(4*_Z^2+2*_Z+1)-28*RootOf(4*_Z^2+2*_Z+1)^2-20
```

```
*RootOf(4*_Z^2+2*_Z+1)+56*RootOf(4*_Z^2+2*_Z+1)^2*x^2+30*(-(2*x^2-1)/(2*x^2
+1))^(2/3)*x^2+45*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^2+30*(-(2*x^2-1)/(2*x^2+1)
)^(1/3)*x^4-60*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^3-15*(-(2*x^2-1)/(2*x^2+1))^(
2/3)*x-6*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(2/3)-6*RootOf(4*_Z^2
+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)-30*x^3*(-(2*x^2-1)/(2*x^2+1))^(2/3))/
(2*x^4-6*x^3+7*x^2-7*x+3)/x)-ln(-(-11+33*x-114*RootOf(4*_Z^2+2*_Z+1)*x^2+28
*RootOf(4*_Z^2+2*_Z+1)*x^5-84*RootOf(4*_Z^2+2*_Z+1)*x^4+42*RootOf(4*_Z^2+2*
_Z+1)*x+36*x*(-(2*x^2-1)/(2*x^2+1))^(1/3)+22*x^5-55*x^2+77*x^3-66*x^4-18*(-
(2*x^2-1)/(2*x^2+1))^(2/3)-18*(-(2*x^2-1)/(2*x^2+1))^(1/3)-18*RootOf(4*_Z^2
+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^2+12*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2
-1)/(2*x^2+1))^(1/3)*x+12*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(2/3
)*x^3-12*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^4-12*RootOf(4
*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(2/3)*x^2+24*RootOf(4*_Z^2+2*_Z+1)*(-(
2*x^2-1)/(2*x^2+1))^(1/3)*x^3+6*x*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+
1))^(2/3)+98*x^3*RootOf(4*_Z^2+2*_Z+1)+28*RootOf(4*_Z^2+2*_Z+1)^2+8*RootOf(
4*_Z^2+2*_Z+1)-56*RootOf(4*_Z^2+2*_Z+1)^2*x^2-36*(-(2*x^2-1)/(2*x^2+1))^(2/
3)*x^2-54*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^2-36*(-(2*x^2-1)/(2*x^2+1))^(1/3)*
x^4+72*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^3+18*(-(2*x^2-1)/(2*x^2+1))^(2/3)*x-6
*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(2/3)-6*RootOf(4*_Z^2+2*_Z+1)
*(-(2*x^2-1)/(2*x^2+1))^(1/3)+36*x^3*(-(2*x^2-1)/(2*x^2+1))^(2/3))/(2*x^4-6
*x^3+7*x^2-7*x+3)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(12\,x^4-8\,x^2+8\,x-3\right)(x-1)}{\left(2\,x^4-6\,x^3+7\,x^2-7\,x+3\right)\left(2\,x^2+1\right)x\left(-\frac{2\,x^2-1}{2\,x^2+1}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(12*x^4-8*x^2+8*x-3)/x/((-2*x^2+1)/(2*x^2+1))^(2/3)/(2*x^2
+1)/(2*x^4-6*x^3+7*x^2-7*x+3),x, algorithm="maxima")
```

```
[Out] integrate((12*x^4 - 8*x^2 + 8*x - 3)*(x - 1)/((2*x^4 - 6*x^3 + 7*x^2 - 7*x
+ 3)*(2*x^2 + 1)*x*(-(2*x^2 - 1)/(2*x^2 + 1))^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x-1)\,\left(12\,x^4-8\,x^2+8\,x-3\right)}{x\,\left(2\,x^2+1\right)\,{\left(-\frac{2\,x^2-1}{2\,x^2+1}\right)}^{2/3}\,\left(2\,x^4-6\,x^3+7\,x^2-7\,x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)*(8*x - 8*x^2 + 12*x^4 - 3))/(x*(2*x^2 + 1)*(-(2*x^2 - 1)/(2*x^
2 + 1))^(2/3)*(7*x^2 - 7*x - 6*x^3 + 2*x^4 + 3)),x)
```

```
[Out] int(((x - 1)*(8*x - 8*x^2 + 12*x^4 - 3))/(x*(2*x^2 + 1)*(-(2*x^2 - 1)/(2*x^
2 + 1))^(2/3)*(7*x^2 - 7*x - 6*x^3 + 2*x^4 + 3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(12*x**4-8*x**2+8*x-3)/x/((-2*x**2+1)/(2*x**2+1))**(2/3)/(
2*x**2+1)/(2*x**4-6*x**3+7*x**2-7*x+3),x)
```

```
[Out] Timed out
```

3.1639 $\int \frac{\left(-1+x^3\right)^{2/3}\left(1+x^3+x^6\right)}{x^6\left(-1+x^6\right)}\,dx$

Optimal. Leaf size=138

$$-\frac{\log\left(2^{2/3}\sqrt[3]{x^3-1}-2x\right)}{3\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3-1}+x}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\left(x^3-1\right)^{2/3}\left(3x^3+2\right)}{10x^5}+\frac{\log\left(2^{2/3}\sqrt[3]{x^3-1}\,x+\sqrt[3]{2}\left(x^3-1\right)^{2/3}+2x^2\right.}{6\sqrt[3]{2}}$$

Rubi [C] time = 0.78, antiderivative size = 402, normalized size of antiderivative = 2.91, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1586, 6725, 271, 264, 2148, 6728}

$$-\frac{\log\left(2^{2/3}\sqrt[3]{x^3-1}-x+1\right)}{4\sqrt[3]{2}}-\frac{\log\left(2\,2^{2/3}\sqrt[3]{x^3-1}-2x-i\sqrt{3}-1\right)}{4\sqrt[3]{2}}-\frac{\log\left(2\,2^{2/3}\sqrt[3]{x^3-1}-2x+i\sqrt{3}-1\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{x^3-1}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2+\frac{\sqrt[3]{2}\left(2x-i\sqrt{3}+1\right)}{\sqrt[3]{x^3-1}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2+\frac{\sqrt[3]{2}\left(2x+i\sqrt{3}+1\right)}{\sqrt[3]{x^3-1}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\left(x^3-1\right)^{2/3}}{5x^5}+\frac{3\left(x^3-1\right)^{2/3}}{10x^2}+\frac{\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}}+\frac{\log\left(-\left(-2x-i\sqrt{3}+1\right)^2\left(2x-i\sqrt{3}+1\right)\right)}{12\sqrt[3]{2}}+\frac{\log\left(-\left(-2x+i\sqrt{3}+1\right)^2\left(2x+i\sqrt{3}+1\right)\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(2/3)*(1 + x^3 + x^6))/(x^6*(-1 + x^6)),x]
```

```
[Out] (-1 + x^3)^(2/3)/(5*x^5) + (3*(-1 + x^3)^(2/3))/(10*x^2) + ArcTan[(1 - (2^(
1/3)*(1 - x))/(-1 + x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + ArcTan[(2 +
(2^(1/3)*(1 - I*Sqrt[3] + 2*x))/(-1 + x^3)^(1/3))/(2*Sqrt[3])]/(2*2^(1/3)*S
qrt[3]) + ArcTan[(2 + (2^(1/3)*(1 + I*Sqrt[3] + 2*x))/(-1 + x^3)^(1/3))/(2*
Sqrt[3])]/(2*2^(1/3)*Sqrt[3]) + Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) + Log[-
((1 - I*Sqrt[3] - 2*x)^2*(1 - I*Sqrt[3] + 2*x))]/(12*2^(1/3)) + Log[-((1 +
I*Sqrt[3] - 2*x)^2*(1 + I*Sqrt[3] + 2*x))]/(12*2^(1/3)) - Log[1 - x + 2^(2/
3)*(-1 + x^3)^(1/3)]/(4*2^(1/3)) - Log[-1 - I*Sqrt[3] - 2*x + 2*2^(2/3)*(-1
 + x^3)^(1/3)]/(4*2^(1/3)) - Log[-1 + I*Sqrt[3] - 2*x + 2*2^(2/3)*(-1 + x^3
)^(1/3)]/(4*2^(1/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(1+x^3+x^6\right)}{x^6\left(-1+x^6\right)}\,dx = \int \frac{1+x^3+x^6}{x^6\sqrt[3]{-1+x^3}\left(1+x^3\right)}\,dx$$

$$= \int \left(\frac{1}{x^6\sqrt[3]{-1+x^3}} + \frac{1}{3(1+x)\sqrt[3]{-1+x^3}} + \frac{2-x}{3\left(1-x+x^2\right)\sqrt[3]{-1+x^3}}\right)dx$$

$$= \frac{1}{3}\int \frac{1}{(1+x)\sqrt[3]{-1+x^3}}\,dx + \frac{1}{3}\int \frac{2-x}{\left(1-x+x^2\right)\sqrt[3]{-1+x^3}}\,dx + \int \frac{1}{x^6\sqrt[3]{-1+\cdots}}$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{5x^5} + \frac{\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{\log\left(1-x+2^{2/\cdots}\right.}{4\sqrt[3]{2}\cdots}$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{5x^5} + \frac{3\left(-1+x^3\right)^{2/3}}{10x^2} + \frac{\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \cdots$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{5x^5} + \frac{3\left(-1+x^3\right)^{2/3}}{10x^2} + \frac{\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2+\frac{\sqrt[3]{2}\left(1-i\sqrt{3}+2x\right)}{\sqrt[3]{-1+x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} \cdots$$

Mathematica [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(1+x^3+x^6\right)}{x^6\left(-1+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^3)^(2/3)*(1 + x^3 + x^6))/(x^6*(-1 + x^6)),x]
[Out] Integrate[((-1 + x^3)^(2/3)*(1 + x^3 + x^6))/(x^6*(-1 + x^6)), x]
```

IntegrateAlgebraic [A] time = 0.37, size = 138, normalized size = 1.00

$$-\frac{\log\left(2^{2/3}\sqrt[3]{x^3-1}-2x\right)}{3\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3-1}+x}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\left(x^3-1\right)^{2/3}\left(3x^3+2\right)}{10x^5} + \frac{\log\left(2^{2/3}\sqrt[3]{x^3-1}\,x+\sqrt[3]{2}\left(x^3-1\right)^{2/3}+2x^2\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(2/3)*(1 + x^3 + x^6))/(x^6*(-1 + x^6)),x]
```

```
[Out] ((-1 + x^3)^(2/3)*(2 + 3*x^3))/(10*x^5) + ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(
-1 + x^3)^(1/3))]/(2^(1/3)*Sqrt[3]) - Log[-2*x + 2^(2/3)*(-1 + x^3)^(1/3)]/
(3*2^(1/3)) + Log[2*x^2 + 2^(2/3)*x*(-1 + x^3)^(1/3) + 2^(1/3)*(-1 + x^3)^(
2/3)]/(6*2^(1/3))
```

fricas [B] time = 2.45, size = 301, normalized size = 2.18

$$-\frac{10\sqrt{6}\,2^{\frac{1}{6}}\,(-1)^{\frac{1}{3}}\,x^5\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(x^3-1\right)^{\frac{2}{3}}+12\sqrt{6}\,(-1)^{\frac{1}{3}}\left(19x^8-16x^5+x^2\right)\left(x^3-1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right)\right)}{6\left(109x^9-105x^6+3x^3+1\right)}\right)-10\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^5\log\left(-\frac{6\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^3-1\right)^{\frac{1}{3}}x^2+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3+1\right)-6\left(x^3-1\right)^{\frac{2}{3}}x}{x^3+1}\right)+5\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^5\log\left(-\frac{3\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(5x^4-x\right)\left(x^3-1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(19x^6-16x^3+1\right)-12\left(2x^5-x^2\right)\left(x^3-1\right)^{\frac{1}{3}}}{x^6+2x^3+1}\right)-18\left(3x^3+2\right)\left(x^3-1\right)^{\frac{2}{3}}}{180x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+x^3+1)/x^6/(x^6-1),x, algorithm="fricas")
```

```
[Out] -1/180*(10*sqrt(6)*2^(1/6)*(-1)^(1/3)*x^5*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(
2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(x^3 - 1)^(2/3) + 12*sqrt(6)*(-1)^(1/3)
*(19*x^8 - 16*x^5 + x^2)*(x^3 - 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^
6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*(-1)^(1/3)*x
^5*log(-(6*2^(1/3)*(-1)^(2/3)*(x^3 - 1)^(1/3)*x^2 + 2^(2/3)*(-1)^(1/3)*(x^3
 + 1) - 6*(x^3 - 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(2/3)*(-1)^(1/3)*x^5*log(-(3*
2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(x^3 - 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(19*x^6
 - 16*x^3 + 1) - 12*(2*x^5 - x^2)*(x^3 - 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 18*
(3*x^3 + 2)*(x^3 - 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^3+1\right)\left(x^3-1\right)^{\frac{2}{3}}}{\left(x^6-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+x^3+1)/x^6/(x^6-1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^3 + 1)*(x^3 - 1)^(2/3)/((x^6 - 1)*x^6), x)
```

maple [C] time = 2.68, size = 929, normalized size = 6.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)*(x^6+x^3+1)/x^6/(x^6-1),x)
```

```
[Out] 1/10*(3*x^6-x^3-2)/x^5/(x^3-1)^(1/3)-1/6*ln((18*RootOf(RootOf(_Z^3+4)^2+6*_
Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-6*RootOf(RootOf(_Z^3+4)^2+
6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+12*(x^3-1)^(2/3)*RootOf(R
ootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x-24*(x^3-1)^
(1/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)*x
^2+(x^3-1)^(1/3)*RootOf(_Z^3+4)^2*x^2+6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf
(_Z^3+4)+36*_Z^2)*x^3-2*RootOf(_Z^3+4)*x^3+2*x*(x^3-1)^(2/3)-6*RootOf(RootO
f(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)+2*RootOf(_Z^3+4))/(1+x)/(x^2-x+1))
*RootOf(_Z^3+4)-ln((18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)
^2*RootOf(_Z^3+4)^2*x^3-6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z
^2)*RootOf(_Z^3+4)^3*x^3+12*(x^3-1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*Root
Of(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x-24*(x^3-1)^(1/3)*RootOf(RootOf(_Z^3+
4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)*x^2+(x^3-1)^(1/3)*RootOf(_
Z^3+4)^2*x^2+6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3-2*R
ootOf(_Z^3+4)*x^3+2*x*(x^3-1)^(2/3)-6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_
Z^3+4)+36*_Z^2)+2*RootOf(_Z^3+4))/(1+x)/(x^2-x+1))*RootOf(RootOf(_Z^3+4)^2+
6*_Z*RootOf(_Z^3+4)+36*_Z^2)+RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36
*_Z^2)*ln((18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf
(_Z^3+4)^2*x^3+9*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootO
```

```
f(_Z^3+4)^3*x^3-12*(x^3-1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4
)+36*_Z^2)*RootOf(_Z^3+4)^2*x+24*(x^3-1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z
*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)*x^2+5*(x^3-1)^(1/3)*RootOf(_Z^3+4)^
2*x^2-18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3-9*RootOf(
_Z^3+4)*x^3+10*x*(x^3-1)^(2/3)+6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4
)+36*_Z^2)+3*RootOf(_Z^3+4))/(1+x)/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^3+1\right)\left(x^3-1\right)^{\frac{2}{3}}}{\left(x^6-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+x^3+1)/x^6/(x^6-1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + x^3 + 1)*(x^3 - 1)^(2/3)/((x^6 - 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3-1\right)^{2/3}\,\left(x^6+x^3+1\right)}{x^6\,\left(x^6-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(x^3 + x^6 + 1))/(x^6*(x^6 - 1)),x)
```

```
[Out] int(((x^3 - 1)^(2/3)*(x^3 + x^6 + 1))/(x^6*(x^6 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x^6+x^3+1\right)}{x^6\left(x-1\right)\left(x+1\right)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)*(x**6+x**3+1)/x**6/(x**6-1),x)
```

```
[Out] Integral(((x - 1)*(x**2 + x + 1))**(2/3)*(x**6 + x**3 + 1)/(x**6*(x - 1)*(x
 + 1)*(x**2 - x + 1)*(x**2 + x + 1)), x)
```

3.1640 $\int \frac{x}{\left(x^2(-a+x)\right)^{2/3}(-ad+(-1+d)x)}\,dx$

Optimal. Leaf size=139

$$-\frac{\log\left(d^{2/3}\left(x^3-ax^2\right)^{2/3}+\sqrt[3]{d}\,x\sqrt[3]{x^3-ax^2}+x^2\right)}{2a\sqrt[3]{d}}+\frac{\log\left(x-\sqrt[3]{d}\,\sqrt[3]{x^3-ax^2}\right)}{a\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{d}\sqrt[3]{x^3-ax^2}+x}\right)}{a\sqrt[3]{d}}$$

Rubi [A] time = 0.36, antiderivative size = 194, normalized size of antiderivative = 1.40, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {6719, 91}

$$-\frac{x^{4/3}(x-a)^{2/3}\log(-ad-(1-d)x)}{2a\sqrt[3]{d}\left(-\left(x^2(a-x)\right)\right)^{2/3}}+\frac{3x^{4/3}(x-a)^{2/3}\log\left(\frac{\sqrt[3]{x}}{\sqrt[3]{d}}-\sqrt[3]{x-a}\right)}{2a\sqrt[3]{d}\left(-\left(x^2(a-x)\right)\right)^{2/3}}+\frac{\sqrt{3}x^{4/3}(x-a)^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{x-a}}+\frac{1}{\sqrt{3}}\right)}{a\sqrt[3]{d}\left(-\left(x^2(a-x)\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((x^2*(-a + x))^(2/3)*(-(a*d) + (-1 + d)*x)),x]
```

```
[Out] (Sqrt[3]*x^(4/3)*(-a + x)^(2/3)*ArcTan[1/Sqrt[3] + (2*x^(1/3))/(Sqrt[3]*d^(
1/3)*(-a + x)^(1/3))])/(a*d^(1/3)*(-((a - x)*x^2))^(2/3)) - (x^(4/3)*(-a +
x)^(2/3)*Log[-(a*d) - (1 - d)*x])/(2*a*d^(1/3)*(-((a - x)*x^2))^(2/3)) + (3
*x^(4/3)*(-a + x)^(2/3)*Log[x^(1/3)/d^(1/3) - (-a + x)^(1/3)])/(2*a*d^(1/3)
*(-((a - x)*x^2))^(2/3))
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(x^2(-a+x)\right)^{2/3}(-ad+(-1+d)x)}\,dx &= \frac{\left(x^{4/3}(-a+x)^{2/3}\right)\int \frac{1}{\sqrt[3]{x}(-a+x)^{2/3}(-ad+(-1+d)x)}\,dx}{\left(x^2(-a+x)\right)^{2/3}}\\ &= \frac{\sqrt{3}x^{4/3}(-a+x)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{-a+x}}\right)}{a\sqrt[3]{d}\left(-\left((a-x)x^2\right)\right)^{2/3}}-\frac{x^{4/3}(-a+x)^{2/3}\log(-a}{2a\sqrt[3]{d}\left(-\left((a-x\right.\right.}\end{aligned}$$

Mathematica [C] time = 0.03, size = 46, normalized size = 0.33

$$-\frac{3x^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x}{d(x-a)}\right)}{2ad\left(x^2(x-a)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((x^2*(-a + x))^(2/3)*(-(a*d) + (-1 + d)*x)),x]
```

```
[Out] (-3*x^2*Hypergeometric2F1[2/3, 1, 5/3, x/(d*(-a + x))])/(2*a*d*(x^2*(-a + x
))^(2/3))
```

IntegrateAlgebraic [A] time = 0.41, size = 139, normalized size = 1.00

$$-\frac{\log\left(d^{2/3}\left(x^3-ax^2\right)^{2/3}+\sqrt[3]{d}\,x\sqrt[3]{x^3-ax^2}+x^2\right)}{2a\sqrt[3]{d}}+\frac{\log\left(x-\sqrt[3]{d}\sqrt[3]{x^3-ax^2}\right)}{a\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{d}\sqrt[3]{x^3-ax^2}+x}\right)}{a\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((x^2*(-a + x))^(2/3)*(-(a*d) + (-1 + d)*x)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*d^(1/3)*(-(a*x^2) + x^3)^(1/3))])/(a*d^(
1/3)) + Log[x - d^(1/3)*(-(a*x^2) + x^3)^(1/3)]/(a*d^(1/3)) - Log[x^2 + d^(
1/3)*x*(-(a*x^2) + x^3)^(1/3) + d^(2/3)*(-(a*x^2) + x^3)^(2/3)]/(2*a*d^(1/3
))
```

fricas [A] time = 0.42, size = 338, normalized size = 2.43

$$\left[\frac{\sqrt{3}d\sqrt{-\frac{1}{d^{\frac{2}{3}}}}\log\left(\frac{2adx-(2d+1)x^2-\sqrt{3}\left(d^{\frac{1}{3}}x^2+\left(-ax^2+x^3\right)^{\frac{1}{3}}d^{\frac{2}{3}}x-2\left(-ax^2+x^3\right)^{\frac{2}{3}}d\right)\sqrt{-\frac{1}{d^{\frac{2}{3}}}}+3\left(-ax^2+x^3\right)^{\frac{1}{3}}d^{\frac{1}{3}}x}{adx-(d-1)x^2}\right)+2d^{\frac{2}{3}}\log\left(-\frac{d^{\frac{2}{3}}x-\left(-ax^2+x^3\right)^{\frac{1}{3}}d}{x}\right)-d^{\frac{2}{3}}\log\left(\frac{d^{\frac{1}{3}}x^2+\left(-ax^2+x^3\right)^{\frac{1}{3}}d^{\frac{2}{3}}x+\left(-ax^2+x^3\right)^{\frac{2}{3}}d}{x^2}\right)}{2ad}, -\frac{2\sqrt{3}d^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}x+2\left(-ax^2+x^3\right)^{\frac{1}{3}}d^{\frac{2}{3}}\right)}{3d^{\frac{1}{3}}x}\right)-2d^{\frac{2}{3}}\log\left(-\frac{d^{\frac{2}{3}}x-\left(-ax^2+x^3\right)^{\frac{1}{3}}d}{x}\right)+d^{\frac{2}{3}}\log\left(\frac{d^{\frac{1}{3}}x^2+\left(-ax^2+x^3\right)^{\frac{1}{3}}d^{\frac{2}{3}}x+\left(-ax^2+x^3\right)^{\frac{2}{3}}d}{x^2}\right)}{2ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2*(-a+x))^(2/3)/(-a*d+(-1+d)*x),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*d*sqrt(-1/d^(2/3))*log((2*a*d*x - (2*d + 1)*x^2 - sqrt(3)*(d^
(1/3)*x^2 + (-a*x^2 + x^3)^(1/3)*d^(2/3)*x - 2*(-a*x^2 + x^3)^(2/3)*d)*sqrt
(-1/d^(2/3)) + 3*(-a*x^2 + x^3)^(1/3)*d^(1/3)*x)/(a*d*x - (d - 1)*x^2)) + 2
*d^(2/3)*log(-(d^(2/3)*x - (-a*x^2 + x^3)^(1/3)*d)/x) - d^(2/3)*log((d^(1/3
)*x^2 + (-a*x^2 + x^3)^(1/3)*d^(2/3)*x + (-a*x^2 + x^3)^(2/3)*d)/x^2))/(a*d
), -1/2*(2*sqrt(3)*d^(2/3)*arctan(1/3*sqrt(3)*(d^(1/3)*x + 2*(-a*x^2 + x^3)
^(1/3)*d^(2/3))/(d^(1/3)*x)) - 2*d^(2/3)*log(-(d^(2/3)*x - (-a*x^2 + x^3)^(
1/3)*d)/x) + d^(2/3)*log((d^(1/3)*x^2 + (-a*x^2 + x^3)^(1/3)*d^(2/3)*x + (-
a*x^2 + x^3)^(2/3)*d)/x^2))/(a*d)]
```

giac [A] time = 0.22, size = 107, normalized size = 0.77

$$-\frac{\sqrt{3}|d|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}d^{\frac{1}{3}}\left(2\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}+\frac{1}{d^{\frac{1}{3}}}\right)\right)}{ad}-\frac{|d|^{\frac{2}{3}}\log\left(\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}+\frac{\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}}{d^{\frac{1}{3}}}+\frac{1}{d^{\frac{2}{3}}}\right)}{2ad}+\frac{\log\left(\left|\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}-\frac{1}{d^{\frac{1}{3}}}\right|\right)}{ad^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2*(-a+x))^(2/3)/(-a*d+(-1+d)*x),x, algorithm="giac")
```

```
[Out] -sqrt(3)*abs(d)^(2/3)*arctan(1/3*sqrt(3)*d^(1/3)*(2*(-a/x + 1)^(1/3) + 1/d^
(1/3)))/(a*d) - 1/2*abs(d)^(2/3)*log((-a/x + 1)^(2/3) + (-a/x + 1)^(1/3)/d^
(1/3) + 1/d^(2/3))/(a*d) + log(abs((-a/x + 1)^(1/3) - 1/d^(1/3)))/(a*d^(1/3
))
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^2\left(-a+x\right)\right)^{\frac{2}{3}}\left(-ad+\left(-1+d\right)x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2*(-a+x))^(2/3)/(-a*d+(-1+d)*x),x)
```

```
[Out] int(x/(x^2*(-a+x))^(2/3)/(-a*d+(-1+d)*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\left(-(a-x)x^2\right)^{\frac{2}{3}}(ad-(d-1)x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2*(-a+x))^(2/3)/(-a*d+(-1+d)*x),x, algorithm="maxima")
```

```
[Out] -integrate(x/((-(a - x)*x^2)^(2/3)*(a*d - (d - 1)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x}{(a\,d-x\,(d-1))\,\left(-x^2\,(a-x)\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((a*d - x*(d - 1))*(-x^2*(a - x))^(2/3)),x)
```

```
[Out] int(-x/((a*d - x*(d - 1))*(-x^2*(a - x))^(2/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^2\,(-a+x)\right)^{\frac{2}{3}}(-ad+dx-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2*(-a+x))**(2/3)/(-a*d+(-1+d)*x),x)
```

```
[Out] Integral(x/((x**2*(-a + x))**(2/3)*(-a*d + d*x - x)), x)
```

3.1641 $\int \frac{(-b+ax^2)^{3/4}}{x}\,dx$

Optimal. Leaf size=139

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}-\sqrt{b}}\right)}{\sqrt{2}}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}}+\frac{2}{3}\left(ax^2-b\right)^{3/4}$$

Rubi [A] time = 0.24, antiderivative size = 213, normalized size of antiderivative = 1.53, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 50, 63, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)}{2\sqrt{2}}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)}{2\sqrt{2}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}\right)}{\sqrt{2}}-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}}+\frac{2}{3}\left(ax^2-b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^2)^(3/4)/x,x]
```

```
[Out] (2*(-b + a*x^2)^(3/4))/3 + (b^(3/4)*ArcTan[1 - (Sqrt[2]*(-b + a*x^2)^(1/4))
/b^(1/4)])/Sqrt[2] - (b^(3/4)*ArcTan[1 + (Sqrt[2]*(-b + a*x^2)^(1/4))/b^(1/
4)])/Sqrt[2] - (b^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4) +
Sqrt[-b + a*x^2]])/(2*Sqrt[2]) + (b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b
 + a*x^2)^(1/4) + Sqrt[-b + a*x^2]])/(2*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(-b+ax^2\right)^{3/4}}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(-b+ax)^{3/4}}{x}\,dx, x, x^2\right)$$

$$= \frac{2}{3}\left(-b+ax^2\right)^{3/4} - \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{-b+ax}}\,dx, x, x^2\right)$$

$$= \frac{2}{3}\left(-b+ax^2\right)^{3/4} - \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^2}\right)}{a}$$

$$= \frac{2}{3}\left(-b+ax^2\right)^{3/4} + \frac{b\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^2}\right)}{a} - \frac{b\,\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^2}\ldots\right.}{a}$$

$$= \frac{2}{3}\left(-b+ax^2\right)^{3/4} - \frac{b^{3/4}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^2}\right)}{2\sqrt{2}} - \frac{b^{3/4}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}\ldots}{-\sqrt{b}+\sqrt{2}\ldots}\right.}{2\ldots}$$

$$= \frac{2}{3}\left(-b+ax^2\right)^{3/4} - \frac{b^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^2}+\sqrt{-b+ax^2}\right)}{2\sqrt{2}} + \frac{b^{3/4}\log\left(\sqrt{b}+\sqrt{2}\ldots\right.}{\ldots}$$

$$= \frac{2}{3}\left(-b+ax^2\right)^{3/4} + \frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)}{\sqrt{2}} - \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)}{\sqrt{2}} - \frac{b^{3/4}\log\left(\sqrt{b}\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.03, size = 79, normalized size = 0.57

$$\frac{2}{3}\left(ax^2-b\right)^{3/4}+(-b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^2-b}}{\sqrt[4]{-b}}\right)-(-b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^2-b}}{\sqrt[4]{-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^2)^(3/4)/x,x]
```

```
[Out] (2*(-b + a*x^2)^(3/4))/3 + (-b)^(3/4)*ArcTan[(-b + a*x^2)^(1/4)/(-b)^(1/4)]
 - (-b)^(3/4)*ArcTanh[(-b + a*x^2)^(1/4)/(-b)^(1/4)]
```

IntegrateAlgebraic [A] time = 0.21, size = 139, normalized size = 1.00

$$-\frac{b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}+\sqrt{b}}\right)}{\sqrt{2}}+\frac{2}{3}\left(ax^2-b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^2)^(3/4)/x,x]
```

```
[Out] (2*(-b + a*x^2)^(3/4))/3 - (b^(3/4)*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b +
a*x^2]/(Sqrt[2]*b^(1/4)))/(-b + a*x^2)^(1/4)])/Sqrt[2] + (b^(3/4)*ArcTanh[(
Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^2])])/Sqrt[2]
```

fricas [A] time = 0.40, size = 159, normalized size = 1.14

$$2\left(-b^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-b^3\right)^{\frac{1}{4}}\left(ax^2-b\right)^{\frac{1}{4}}b^2-\sqrt{\sqrt{ax^2-b}\,b^4-\sqrt{-b^3}\,b^3}\left(-b^3\right)^{\frac{1}{4}}}{b^3}\right)-\frac{1}{2}\left(-b^3\right)^{\frac{1}{4}}\log\left(\left(ax^2-b\right)^{\frac{1}{4}}b^2+\left(-b^3\right)^{\frac{3}{4}}\right)+\frac{1}{2}\left(-b^3\right)^{\frac{1}{4}}\log\left(\left(ax^2-b\right)^{\frac{1}{4}}b^2-\left(-b^3\right)^{\frac{3}{4}}\right)+\frac{2}{3}\left(ax^2-b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)^(3/4)/x,x, algorithm="fricas")
```

```
[Out] 2*(-b^3)^(1/4)*arctan(-((-b^3)^(1/4)*(a*x^2 - b)^(1/4)*b^2 - sqrt(sqrt(a*x^
2 - b)*b^4 - sqrt(-b^3)*b^3)*(-b^3)^(1/4))/b^3) - 1/2*(-b^3)^(1/4)*log((a*x
^2 - b)^(1/4)*b^2 + (-b^3)^(3/4)) + 1/2*(-b^3)^(1/4)*log((a*x^2 - b)^(1/4)*
b^2 - (-b^3)^(3/4)) + 2/3*(a*x^2 - b)^(3/4)
```

giac [A] time = 0.13, size = 175, normalized size = 1.26

$$-\frac{1}{2}\sqrt{2}b^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)-\frac{1}{2}\sqrt{2}b^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)+\frac{1}{4}\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)-\frac{1}{4}\sqrt{2}b^{\frac{3}{4}}\log\left(-\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)+\frac{2}{3}\left(ax^2-b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)^(3/4)/x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*b^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^2 - b)^(1
/4))/b^(1/4)) - 1/2*sqrt(2)*b^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
2*(a*x^2 - b)^(1/4))/b^(1/4)) + 1/4*sqrt(2)*b^(3/4)*log(sqrt(2)*(a*x^2 - b)
^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b)) - 1/4*sqrt(2)*b^(3/4)*log(-sqrt
(2)*(a*x^2 - b)^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b)) + 2/3*(a*x^2 - b
)^(3/4)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^2-b\right)^{\frac{3}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)^(3/4)/x,x)
```

```
[Out] int((a*x^2-b)^(3/4)/x,x)
```

maxima [A] time = 0.42, size = 178, normalized size = 1.28

$$-\frac{1}{4}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}\right)b+\frac{2}{3}\left(ax^2-b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)^(3/4)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^2 - b)^(1/4))/
b^(1/4))/b^(1/4) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^
2 - b)^(1/4))/b^(1/4))/b^(1/4) - sqrt(2)*log(sqrt(2)*(a*x^2 - b)^(1/4)*b^(1
/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(1/4) + sqrt(2)*log(-sqrt(2)*(a*x^2 - b)
^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(1/4))*b + 2/3*(a*x^2 - b)^(3
/4)
```

mupad [B] time = 1.06, size = 63, normalized size = 0.45

$$\frac{2\left(a\,x^2-b\right)^{3/4}}{3}+{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(a\,x^2-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)-{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(a\,x^2-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 - b)^(3/4)/x,x)
```

```
[Out] (2*(a*x^2 - b)^(3/4))/3 + (-b)^(3/4)*atan((a*x^2 - b)^(1/4)/(-b)^(1/4)) - (
-b)^(3/4)*atanh((a*x^2 - b)^(1/4)/(-b)^(1/4))
```

sympy [C] time = 1.07, size = 48, normalized size = 0.35

$$-\frac{a^{\frac{3}{4}}x^{\frac{3}{2}}\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^2}\right)}{2\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)**(3/4)/x,x)
```

```
[Out] -a**(3/4)*x**(3/2)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), b*exp_polar(2*I*
pi)/(a*x**2))/(2*gamma(1/4))
```

3.1642 $\int \frac{-2x+x^2}{\left(1-x+x^2\right)\sqrt[4]{1+x^4}}\,dx$

Optimal. Leaf size=139

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{\tan^{-1}\left(\frac{\left(\sqrt{2}x-\sqrt{2}\right)\sqrt[4]{x^4+1}}{\sqrt{x^4+1}-x^2+2x-1}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt{2}x-\sqrt{2}\right)\sqrt[4]{x^4+1}}{\sqrt{x^4+1}+x^2-2x+1}\right)}{\sqrt{2}}$$

Rubi [C] time = 1.85, antiderivative size = 851, normalized size of antiderivative = 6.12, number of steps used = 53, number of rules used = 21, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {1593, 6728, 240, 212, 206, 203, 2153, 1240, 377, 208, 205, 510, 1248, 746, 399, 490, 1213, 537, 444, 63, 298}

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Int[(-2*x + x^2)/((1 - x + x^2)*(1 + x^4)^(1/4)),x]
```

```
[Out] -1/6*((1 + I*Sqrt[3])*x^3*AppellF1[3/4, 1/4, 1, 7/4, -x^4, (-2*x^4)/(1 - I*
Sqrt[3])]) - ((1 - I*Sqrt[3])*x^3*AppellF1[3/4, 1/4, 1, 7/4, -x^4, (-2*x^4)
/(1 + I*Sqrt[3])])/6 + ArcTan[x/(1 + x^4)^(1/4)]/2 - ((1 + I*Sqrt[3])*(-((I
 - Sqrt[3])/(I + Sqrt[3])))^(1/4)*ArcTan[x/((-((I - Sqrt[3])/(I + Sqrt[3]))
)^(1/4)*(1 + x^4)^(1/4))])/4 - ((-((I - Sqrt[3])/(I + Sqrt[3])))^(3/4)*ArcT
an[((-((I - Sqrt[3])/(I + Sqrt[3])))^(1/4)*x)/(1 + x^4)^(1/4)])/2 - (((1 -
I*Sqrt[3])/2)^(3/4)*ArcTan[(2^(1/4)*(1 + x^4)^(1/4))/(1 - I*Sqrt[3])^(1/4)]
)/2 - (((1 + I*Sqrt[3])/2)^(3/4)*ArcTan[(2^(1/4)*(1 + x^4)^(1/4))/(1 + I*Sq
rt[3])^(1/4)])/2 + ArcTanh[x/(1 + x^4)^(1/4)]/2 - ((1 + I*Sqrt[3])*(-((I -
Sqrt[3])/(I + Sqrt[3])))^(1/4)*ArcTanh[x/((-((I - Sqrt[3])/(I + Sqrt[3])))^
(1/4)*(1 + x^4)^(1/4))])/4 - ((-((I - Sqrt[3])/(I + Sqrt[3])))^(3/4)*ArcTan
h[((-((I - Sqrt[3])/(I + Sqrt[3])))^(1/4)*x)/(1 + x^4)^(1/4)])/2 + (((1 - I
*Sqrt[3])/2)^(3/4)*ArcTanh[(2^(1/4)*(1 + x^4)^(1/4))/(1 - I*Sqrt[3])^(1/4)]
)/2 + (((1 + I*Sqrt[3])/2)^(3/4)*ArcTanh[(2^(1/4)*(1 + x^4)^(1/4))/(1 + I*S
qrt[3])^(1/4)])/2 - ((I/2)*Sqrt[-x^4]*EllipticPi[(-I - Sqrt[3])/2, ArcSin[(
1 + x^4)^(1/4)], -1])/x^2 + ((I/2)*Sqrt[-x^4]*EllipticPi[(I - Sqrt[3])/2, A
rcSin[(1 + x^4)^(1/4)], -1])/x^2 + ((I/2)*Sqrt[-x^4]*EllipticPi[1/Sqrt[(1 -
 I*Sqrt[3])/2], ArcSin[(1 + x^4)^(1/4)], -1])/x^2 - ((I/2)*Sqrt[-x^4]*Ellip
ticPi[1/Sqrt[(1 + I*Sqrt[3])/2], ArcSin[(1 + x^4)^(1/4)], -1])/x^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2153

```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^nn)^p, (c/(c^2 - d^2*x^(2*n)) - (d*x^n)/
(c^2 - d^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !
IntegerQ[p] && ILtQ[q, 0] && IGtQ[Log[2, nn/n], 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-2x+x^2}{\left(1-x+x^2\right)\sqrt[4]{1+x^4}}\,dx = \int \frac{(-2+x)x}{\left(1-x+x^2\right)\sqrt[4]{1+x^4}}\,dx$$
$$= \int \left(\frac{1}{\sqrt[4]{1+x^4}} - \frac{1+x}{\left(1-x+x^2\right)\sqrt[4]{1+x^4}}\right) dx$$
$$= \int \frac{1}{\sqrt[4]{1+x^4}}\,dx - \int \frac{1+x}{\left(1-x+x^2\right)\sqrt[4]{1+x^4}}\,dx$$
$$= -\int \left(\frac{1-i\sqrt{3}}{\left(-1-i\sqrt{3}+2x\right)\sqrt[4]{1+x^4}} + \frac{1+i\sqrt{3}}{\left(-1+i\sqrt{3}+2x\right)\sqrt[4]{1+x^4}}\right) dx + \text{Subst}\left(\int \frac{1}{1-}\right.$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right) - \left(1-i\right.$$
$$= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) + \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) - \left(1-i\sqrt{3}\right)\int \left(\frac{i-\sqrt{3}}{2\left(i+\sqrt{3}+2ix^2\right)\sqrt[4]{1-}}\right.$$
$$= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) + \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) + 2i\int \frac{1}{\left(-i+\sqrt{3}-2ix^2\right)\sqrt[4]{1+x^4}}\,dx -$$
$$= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) + \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) + 2i\int \left(\frac{1+i\sqrt{3}}{2\left(i+\sqrt{3}+2ix^4\right)\sqrt[4]{1+x^4}} + \right.$$
$$= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) + \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) - 2\int \frac{x^2}{\sqrt[4]{1+x^4}\left(1-i\sqrt{3}+2x^4\right)}\,dx - 2$$
$$= -\frac{2x^3F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1-i\sqrt{3}}\right)}{3\left(1-i\sqrt{3}\right)} - \frac{2x^3F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1+i\sqrt{3}}\right)}{3\left(1+i\sqrt{3}\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+}}\right.$$
$$= -\frac{2x^3F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1-i\sqrt{3}}\right)}{3\left(1-i\sqrt{3}\right)} - \frac{2x^3F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1+i\sqrt{3}}\right)}{3\left(1+i\sqrt{3}\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+}}\right.$$
$$= -\frac{2x^3F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1-i\sqrt{3}}\right)}{3\left(1-i\sqrt{3}\right)} - \frac{2x^3F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1+i\sqrt{3}}\right)}{3\left(1+i\sqrt{3}\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+}}\right.$$
$$= -\frac{2x^3F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1-i\sqrt{3}}\right)}{3\left(1-i\sqrt{3}\right)} - \frac{2x^3F_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};-x^4,-\frac{2x^4}{1+i\sqrt{3}}\right)}{3\left(1+i\sqrt{3}\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{1+}}\right.$$

Mathematica [**F**] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{-2x+x^2}{\left(1-x+x^2\right)\sqrt[4]{1+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*x + x^2)/((1 - x + x^2)*(1 + x^4)^(1/4)),x]
```

```
[Out] Integrate[(-2*x + x^2)/((1 - x + x^2)*(1 + x^4)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 12.17, size = 139, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{\tan^{-1}\left(\frac{\left(\sqrt{2}x-\sqrt{2}\right)\sqrt[4]{x^4+1}}{\sqrt{x^4+1}-x^2+2x-1}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt{2}x-\sqrt{2}\right)\sqrt[4]{x^4+1}}{\sqrt{x^4+1}+x^2-2x+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*x + x^2)/((1 - x + x^2)*(1 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)]/2 - ArcTan[((-Sqrt[2] + Sqrt[2]*x)*(1 + x^4)^(1/4
))/(-1 + 2*x - x^2 + Sqrt[1 + x^4])]/Sqrt[2] + ArcTanh[x/(1 + x^4)^(1/4)]/2
 - ArcTanh[((-Sqrt[2] + Sqrt[2]*x)*(1 + x^4)^(1/4))/(1 - 2*x + x^2 + Sqrt[1
 + x^4])]/Sqrt[2]
```

fricas [B] time = 12.15, size = 1172, normalized size = 8.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x)/(x^2-x+1)/(x^4+1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(-(x^8 - 4*x^7 + 10*x^6 - 16*x^5 + 19*x^4 - 16*x^3 + sqrt
(2)*(x^5 - 7*x^4 + 15*x^3 - 15*x^2 + 7*x - 1)*(x^4 + 1)^(3/4) + 10*x^2 - sq
rt(2)*(x^7 - x^6 - 6*x^5 + 16*x^4 - 16*x^3 + 6*x^2 + x - 1)*(x^4 + 1)^(1/4)
 + 2*(x^6 - 4*x^5 + 8*x^4 - 10*x^3 + 8*x^2 - 4*x + 1)*sqrt(x^4 + 1) - (sqrt
(2)*(x^6 - 8*x^5 + 22*x^4 - 30*x^3 + 22*x^2 - 8*x + 1)*sqrt(x^4 + 1) + 4*(x
^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1)*(x^4 + 1)^(3/4) + sqrt(2)*(2*x^8 -
10*x^7 + 19*x^6 - 22*x^5 + 21*x^4 - 22*x^3 + 19*x^2 - 10*x + 2) + 2*(x^7 -
5*x^6 + 12*x^5 - 18*x^4 + 18*x^3 - 12*x^2 + 5*x - 1)*(x^4 + 1)^(1/4))*sqrt(
(x^4 - 2*x^3 - sqrt(2)*(x^4 + 1)^(3/4)*(x - 1) + 3*x^2 - sqrt(2)*(x^4 + 1)^
(1/4)*(x^3 - 3*x^2 + 3*x - 1) + 2*sqrt(x^4 + 1)*(x^2 - 2*x + 1) - 2*x + 1)/
(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) - 4*x + 1)/(3*x^8 - 12*x^7 + 14*x^6 - 11*x
^4 + 14*x^2 - 12*x + 3)) - 1/2*sqrt(2)*arctan(-(x^8 - 4*x^7 + 10*x^6 - 16*x
^5 + 19*x^4 - 16*x^3 - sqrt(2)*(x^5 - 7*x^4 + 15*x^3 - 15*x^2 + 7*x - 1)*(x
^4 + 1)^(3/4) + 10*x^2 + sqrt(2)*(x^7 - x^6 - 6*x^5 + 16*x^4 - 16*x^3 + 6*x
^2 + x - 1)*(x^4 + 1)^(1/4) + 2*(x^6 - 4*x^5 + 8*x^4 - 10*x^3 + 8*x^2 - 4*x
 + 1)*sqrt(x^4 + 1) + (sqrt(2)*(x^6 - 8*x^5 + 22*x^4 - 30*x^3 + 22*x^2 - 8*
x + 1)*sqrt(x^4 + 1) - 4*(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1)*(x^4 + 1
)^(3/4) + sqrt(2)*(2*x^8 - 10*x^7 + 19*x^6 - 22*x^5 + 21*x^4 - 22*x^3 + 19*
x^2 - 10*x + 2) - 2*(x^7 - 5*x^6 + 12*x^5 - 18*x^4 + 18*x^3 - 12*x^2 + 5*x
- 1)*(x^4 + 1)^(1/4))*sqrt((x^4 - 2*x^3 + sqrt(2)*(x^4 + 1)^(3/4)*(x - 1) +
 3*x^2 + sqrt(2)*(x^4 + 1)^(1/4)*(x^3 - 3*x^2 + 3*x - 1) + 2*sqrt(x^4 + 1)*
(x^2 - 2*x + 1) - 2*x + 1)/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) - 4*x + 1)/(3*x
^8 - 12*x^7 + 14*x^6 - 11*x^4 + 14*x^2 - 12*x + 3)) - 1/8*sqrt(2)*log(4*(x^
4 - 2*x^3 + sqrt(2)*(x^4 + 1)^(3/4)*(x - 1) + 3*x^2 + sqrt(2)*(x^4 + 1)^(1/
4)*(x^3 - 3*x^2 + 3*x - 1) + 2*sqrt(x^4 + 1)*(x^2 - 2*x + 1) - 2*x + 1)/(x^
4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/8*sqrt(2)*log(4*(x^4 - 2*x^3 - sqrt(2)*(x
^4 + 1)^(3/4)*(x - 1) + 3*x^2 - sqrt(2)*(x^4 + 1)^(1/4)*(x^3 - 3*x^2 + 3*x
- 1) + 2*sqrt(x^4 + 1)*(x^2 - 2*x + 1) - 2*x + 1)/(x^4 - 2*x^3 + 3*x^2 - 2*
x + 1)) + 1/4*arctan(2*(x^4 + 1)^(1/4)*x^3 + 2*(x^4 + 1)^(3/4)*x) + 1/4*log
(2*x^4 + 2*(x^4 + 1)^(1/4)*x^3 + 2*sqrt(x^4 + 1)*x^2 + 2*(x^4 + 1)^(3/4)*x
+ 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-2\,x}{\left(x^4+1\right)^{\frac{1}{4}}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x)/(x^2-x+1)/(x^4+1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 2*x)/((x^4 + 1)^(1/4)*(x^2 - x + 1)), x)
```

maple [C] time = 8.95, size = 591, normalized size = 4.25

$$\frac{\ln\left(2\left(x^4+1\right)^{\frac{3}{4}}x+2x^2\sqrt{x^4+1}+2x^3\left(x^4+1\right)^{\frac{1}{4}}+2x^4+1\right)}{4}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-2\,\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{x^4+1}\,x^2+2\,\mathrm{RootOf}\left(\_Z^2+1\right)x^4+2\left(x^4+1\right)^{\frac{3}{4}}x-2x^3\left(x^4+1\right)^{\frac{1}{4}}+\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{4} \text{[illegible]}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x)/(x^2-x+1)/(x^4+1)^(1/4),x)
```

```
[Out] 1/4*ln(2*(x^4+1)^(3/4)*x+2*x^2*(x^4+1)^(1/2)+2*x^3*(x^4+1)^(1/4)+2*x^4+1)+1
/4*RootOf(_Z^2+1)*ln(-2*RootOf(_Z^2+1)*(x^4+1)^(1/2)*x^2+2*RootOf(_Z^2+1)*x
^4+2*(x^4+1)^(3/4)*x-2*x^3*(x^4+1)^(1/4)+RootOf(_Z^2+1))-1/2*RootOf(_Z^2+Ro
otOf(_Z^2+1))*ln(((x^4+1)^(1/2)*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2+1)*
x^2-2*(x^4+1)^(1/2)*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2+1)*x+RootOf(_Z^
2+1)*(x^4+1)^(1/4)*x^3+(x^4+1)^(3/4)*x+(x^4+1)^(1/2)*RootOf(_Z^2+RootOf(_Z^
2+1))*RootOf(_Z^2+1)-3*RootOf(_Z^2+1)*(x^4+1)^(1/4)*x^2+2*RootOf(_Z^2+RootO
f(_Z^2+1))*x^3-(x^4+1)^(3/4)+3*RootOf(_Z^2+1)*(x^4+1)^(1/4)*x-3*RootOf(_Z^2
+RootOf(_Z^2+1))*x^2-RootOf(_Z^2+1)*(x^4+1)^(1/4)+2*RootOf(_Z^2+RootOf(_Z^2
+1))*x)/(x^2-x+1)^2)-1/2*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))*ln(((x^
4+1)^(1/2)*RootOf(_Z^2+RootOf(_Z^2+1))*x^2-RootOf(_Z^2+1)*(x^4+1)^(1/4)*x^3
+2*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2+1)*x^3+(x^4+1)^(3/4)*x-2*(x^4+1)
^(1/2)*RootOf(_Z^2+RootOf(_Z^2+1))*x+3*RootOf(_Z^2+1)*(x^4+1)^(1/4)*x^2-3*R
ootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2+1)*x^2-(x^4+1)^(3/4)+(x^4+1)^(1/2)*R
ootOf(_Z^2+RootOf(_Z^2+1))-3*RootOf(_Z^2+1)*(x^4+1)^(1/4)*x+2*RootOf(_Z^2+R
ootOf(_Z^2+1))*RootOf(_Z^2+1)*x+RootOf(_Z^2+1)*(x^4+1)^(1/4))/(x^2-x+1)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - 2\,x}{\left(x^4+1\right)^{\frac{1}{4}}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x)/(x^2-x+1)/(x^4+1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 2*x)/((x^4 + 1)^(1/4)*(x^2 - x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{2\,x - x^2}{\left(x^4+1\right)^{1/4}\,\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - x^2)/((x^4 + 1)^(1/4)*(x^2 - x + 1)),x)
```

```
[Out] int(-(2*x - x^2)/((x^4 + 1)^(1/4)*(x^2 - x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(x-2)}{\sqrt[4]{x^4+1}\,\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x)/(x**2-x+1)/(x**4+1)**(1/4),x)
```

```
[Out] Integral(x*(x - 2)/((x**4 + 1)**(1/4)*(x**2 - x + 1)), x)
```

3.1643 $\int \frac{1+k^4x^4}{\sqrt{(1-x)x(1-k^2x)}\left(-1+k^4x^4\right)}\,dx$

Optimal. Leaf size=139

$$-\frac{\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^3+(-k^2-1)x^2+x}}\right)}{2(k-1)}-\frac{\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^3+(-k^2-1)x^2+x}}\right)}{2(k+1)}-\frac{\tan^{-1}\left(\frac{\sqrt{k^2+1}\sqrt{k^2x^3+(-k^2-1)x^2+x}}{(x-1)(k^2x-1)}\right)}{\sqrt{k^2+1}}$$

Rubi [C] time = 4.73, antiderivative size = 450, normalized size of antiderivative = 3.24, number of steps used = 27, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6718, 6725, 115, 6688, 934, 12, 168, 537}

$$\frac{(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\,\Pi\left(-\frac{1}{k};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{\sqrt{-k^2}\sqrt{x-x^2}\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\,\Pi\left(\frac{1}{k};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{\sqrt{-k^2}\sqrt{x-x^2}\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\,\Pi\left(-\frac{1}{\sqrt{-k^2}};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{\sqrt{-k^2}\sqrt{x-x^2}\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\,\Pi\left(\frac{1}{\sqrt{-k^2}};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{\sqrt{-k^2}\sqrt{x-x^2}\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + k^4*x^4)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^4*x^4)),x]
```

```
[Out] (2*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticF[ArcSin[Sqrt[x]], k^2])/Sqr
t[(1 - x)*x*(1 - k^2*x)] + ((1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^2*x]*Ellipt
icPi[-k^(-1), ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/(Sqrt[-k^2]*Sqrt[(1 - x
)*x*(1 - k^2*x)]*Sqrt[x - x^2]) + ((1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^2*x]
*EllipticPi[k^(-1), ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/(Sqrt[-k^2]*Sqrt[
(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2]) + ((1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 -
k^2*x]*EllipticPi[-(1/Sqrt[-k^2]), ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/(S
qrt[-k^2]*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2]) + ((1 - x)*Sqrt[-x]*Sq
rt[x]*Sqrt[1 - k^2*x]*EllipticPi[1/Sqrt[-k^2], ArcSin[Sqrt[-k^2]*Sqrt[-x]],
 k^(-2)])/(Sqrt[-k^2]*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+k^4x^4}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(-1+k^4x^4\right)}\,dx = \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{1+k^4x^4}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\left(-1+k^4x^4\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \left(\frac{1}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}}+\frac{2}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\left(-1+k^4x^4\right)}\right) \cdots}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x} \cdots\right.}{\sqrt{(1-\cdots}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\cdots}{\sqrt{(1-x)x\cdots}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\cdots}{\cdots}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \cdots}{\sqrt{(1-x)x\left(\cdots\right.}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \cdots}{\cdots}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \cdots}{2\sqrt{(1-x)x\left(\cdots\right.}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{2-2x}\,\sqrt{1-x}\,\sqrt{-x}\,\sqrt{x}\cdots\right.}{\sqrt{2}\,\sqrt{(1\cdots}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{2-2x}\,\sqrt{1-x}\,\sqrt{-x}\,\sqrt{x}\cdots\right.}{2\sqrt{(1-\cdots}}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(\sqrt{2-2x}\,\sqrt{1-x}\,\sqrt{-x}\,\sqrt{x}\cdots\right.}{\cdots}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-k^2x}\cdots}{\sqrt{-k^2}\,\sqrt{(1-x)\cdots}}$$

Mathematica [C] time = 4.06, size = 247, normalized size = 1.78

$$\frac{i\sqrt{\frac{1}{x-1}+1}\,(x-1)^{3/2}\sqrt{\frac{1-\frac{1}{k^2}}{x-1}+1}\,\left(-(k+1)\left(\left(k^2+1\right)\Pi\left(\frac{k-1}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+i(k-1)\left((k+i)\Pi\left(\frac{k-i}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)-(k-i)\Pi\left(\frac{k+i}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)\right)+2\left(k^4+1\right)F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+\left(k^3-k^2+k-1\right)\Pi\left(1+\frac{1}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)}{\left(k^4-1\right)\sqrt{(x-1)x\left(k^2x-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + k^4*x^4)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^4*x^4)),x]
```

```
[Out] (I*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (1 - k^(-2))/(-1 + x)]*(-1 + x)^(3/2)*(
2*(1 + k^4)*EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] + (-1 + k - k^
2 + k^3)*EllipticPi[1 + k^(-1), I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] - (1
 + k)*((1 + k^2)*EllipticPi[(-1 + k)/k, I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-
2)] + I*(-1 + k)*((I + k)*EllipticPi[(-I + k)/k, I*ArcSinh[1/Sqrt[-1 + x]],
 1 - k^(-2)] - (-I + k)*EllipticPi[(I + k)/k, I*ArcSinh[1/Sqrt[-1 + x]], 1
- k^(-2)]))))/((-1 + k^4)*Sqrt[(-1 + x)*x*(-1 + k^2*x)])
```

IntegrateAlgebraic [A] time = 0.31, size = 139, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^3+(-k^2-1)x^2+x}}\right)}{2(k-1)}-\frac{\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^3+(-k^2-1)x^2+x}}\right)}{2(k+1)}-\frac{\tan^{-1}\left(\frac{\sqrt{k^2+1}\sqrt{k^2x^3+(-k^2-1)x^2+x}}{(x-1)(k^2x-1)}\right)}{\sqrt{k^2+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + k^4*x^4)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^4*x^4
)),x]
```

```
[Out] -1/2*ArcTan[((-1 + k)*x)/Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3]]/(-1 + k) - Arc
Tan[((1 + k)*x)/Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3]]/(2*(1 + k)) - ArcTan[(S
qrt[1 + k^2]*Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3])/((-1 + x)*(-1 + k^2*x))]/S
qrt[1 + k^2]
```

fricas [B] time = 0.52, size = 286, normalized size = 2.06

$$\frac{2\sqrt{k^2+1}\left(k^2-1\right)\arctan\left(\frac{\sqrt{k^2x^3-(k^2+1)x^2+x}\left(k^2x^2-2\left(k^2+1\right)x+1\right)\sqrt{k^2+1}}{2\left(\left(k^4+k^2\right)x^3-\left(k^4+2k^2+1\right)x^2+\left(k^2+1\right)x\right)}\right)+\left(k^3-k^2+k-1\right)\arctan\left(\frac{\sqrt{k^2x^3-(k^2+1)x^2+x}\left(k^2x^2-2\left(k^2+k+1\right)x+1\right)}{2\left(\left(k^3+k^2\right)x^3-\left(k^3+k^2+k+1\right)x^2+(k+1)x\right)}\right)+\left(k^3+k^2+k+1\right)\arctan\left(\frac{\sqrt{k^2x^3-(k^2+1)x^2+x}\left(k^2x^2-2\left(k^2-k+1\right)x+1\right)}{2\left(\left(k^3-k^2\right)x^3-\left(k^3-k^2+k-1\right)x^2+(k-1)x\right)}\right)}{4\left(k^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^4*x^4+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^4*x^4-1),x, algorithm="
fricas")
```

```
[Out] 1/4*(2*sqrt(k^2 + 1)*(k^2 - 1)*arctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)
*(k^2*x^2 - 2*(k^2 + 1)*x + 1)*sqrt(k^2 + 1)/((k^4 + k^2)*x^3 - (k^4 + 2*k^
2 + 1)*x^2 + (k^2 + 1)*x)) + (k^3 - k^2 + k - 1)*arctan(1/2*sqrt(k^2*x^3 -
(k^2 + 1)*x^2 + x)*(k^2*x^2 - 2*(k^2 + k + 1)*x + 1)/((k^3 + k^2)*x^3 - (k^
3 + k^2 + k + 1)*x^2 + (k + 1)*x)) + (k^3 + k^2 + k + 1)*arctan(1/2*sqrt(k^
2*x^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2 - 2*(k^2 - k + 1)*x + 1)/((k^3 - k^2)*x
^3 - (k^3 - k^2 + k - 1)*x^2 + (k - 1)*x)))/(k^4 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^4x^4+1}{\left(k^4x^4-1\right)\sqrt{\left(k^2x-1\right)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^4*x^4+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^4*x^4-1),x, algorithm="
giac")
```

```
[Out] integrate((k^4*x^4 + 1)/((k^4*x^4 - 1)*sqrt((k^2*x - 1)*(x - 1)*x)), x)
```

maple [C] time = 0.04, size = 570, normalized size = 4.10

$$-\frac{2\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\mathrm{EllipticF}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^2\sqrt{k^2x^3-k^2x^2-x^2+x}}-\frac{\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\mathrm{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}-\frac{1}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^3\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}-\frac{1}{k}\right)}+\frac{\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\mathrm{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}+\frac{1}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^3\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}+\frac{1}{k}\right)}-\frac{i\sqrt{-k^2x+1}\,\sqrt{\frac{x}{\frac{1}{k^2}-1}-\frac{1}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\mathrm{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}-\frac{i}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^3\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}-\frac{i}{k}\right)}+\frac{i\sqrt{-k^2x+1}\,\sqrt{\frac{x}{\frac{1}{k^2}-1}-\frac{1}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\mathrm{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}+\frac{i}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^3\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}+\frac{i}{k}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^4*x^4+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^4*x^4-1),x)
```

```
[Out] -2/k^2*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x
^3-k^2*x^2-x^2+x)^(1/2)*EllipticF((-(x-1/k^2)*k^2)^(1/2),(1/k^2/(1/k^2-1))^
(1/2))-1/k^3*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/
(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2-1/k)*EllipticPi((-(x-1/k^2)*k^2)^(1/2)
,1/k^2/(1/k^2-1/k),(1/k^2/(1/k^2-1))^(1/2))+1/k^3*(-(x-1/k^2)*k^2)^(1/2)*((
-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1
/k)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+1/k),(1/k^2/(1/k^2-1))^(
1/2))-I/k^3*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2
)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2-I/k)*EllipticPi((-(x-1/k^2)*k^2)^(1/
2),1/k^2/(1/k^2-I/k),(1/k^2/(1/k^2-1))^(1/2))+I/k^3*(-k^2*x+1)^(1/2)*(1/(1/
k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/
k^2+I/k)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+I/k),(1/k^2/(1/k^2-
1))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^4x^4+1}{\left(k^4x^4-1\right)\sqrt{\left(k^2x-1\right)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^4*x^4+1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^4*x^4-1),x, algorithm="
maxima")
```

```
[Out] integrate((k^4*x^4 + 1)/((k^4*x^4 - 1)*sqrt((k^2*x - 1)*(x - 1)*x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^4*x^4 + 1)/((k^4*x^4 - 1)*(x*(k^2*x - 1)*(x - 1))^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^4x^4+1}{\sqrt{x\,(x-1)\left(k^2x-1\right)}\,(kx-1)\,(kx+1)\left(k^2x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k**4*x**4+1)/((1-x)*x*(-k**2*x+1))**(1/2)/(k**4*x**4-1),x)
```

```
[Out] Integral((k**4*x**4 + 1)/(sqrt(x*(x - 1)*(k**2*x - 1))*(k*x - 1)*(k*x + 1)*
(k**2*x**2 + 1)), x)
```

3.1644 $\int \frac{(-1+x^8)(1+x^8)}{\sqrt[4]{-1-x^4+x^8}\,(1-3x^8+x^{16})}\,dx$

Optimal. Leaf size=139

$$-\frac{x}{2\sqrt[4]{x^8-x^4-1}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^8-x^4-1}}{\sqrt{2}x^2-\sqrt{x^8-x^4-1}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}x\sqrt[4]{x^8-x^4-1}}{2x^2+\sqrt{2}\sqrt{x^8-x^4-1}}\right)}{4\,2^{3/4}}$$

Rubi [F] time = 1.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^8)(1+x^8)}{\sqrt[4]{-1-x^4+x^8}\,(1-3x^8+x^{16})}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^8)*(1 + x^8))/((-1 - x^4 + x^8)^(1/4)*(1 - 3*x^8 + x^16)),x]
```

```
[Out] (x*(1 - (2*x^4)/(1 - Sqrt[5]))^(1/4)*(1 - (2*x^4)/(1 + Sqrt[5]))^(1/4)*Appe
llF1[1/4, 1/4, 1/4, 5/4, (2*x^4)/(1 + Sqrt[5]), (2*x^4)/(1 - Sqrt[5])])/(-1
 - x^4 + x^8)^(1/4) - (x*(Sqrt[3 + Sqrt[5]] - Sqrt[2]*x^4)^(1/4)*(1 + Sqrt[
(3 + Sqrt[5])/2]*x^4)^(1/4)*AppellF1[1/4, 5/4, 1/4, 5/4, Sqrt[2/(3 + Sqrt[5
])]*x^4, -(Sqrt[(3 + Sqrt[5])/2]*x^4)])/(2*(3 + Sqrt[5])^(1/8)*(-1 - x^4 +
x^8)^(1/4)) - (x*(Sqrt[3 - Sqrt[5]] + Sqrt[2]*x^4)^(1/4)*(1 - Sqrt[(3 - Sqr
t[5])/2]*x^4)^(1/4)*AppellF1[1/4, 5/4, 1/4, 5/4, -(Sqrt[(3 + Sqrt[5])/2]*x^
4), Sqrt[(3 - Sqrt[5])/2]*x^4])/(2*(3 - Sqrt[5])^(1/8)*(-1 - x^4 + x^8)^(1/
4)) - (Sqrt[3 - Sqrt[5]]*Defer[Int][1/((Sqrt[3 - Sqrt[5]] - Sqrt[2]*x^4)*(-
1 - x^4 + x^8)^(1/4)), x])/2 - (Sqrt[3 + Sqrt[5]]*Defer[Int][1/((Sqrt[3 + S
qrt[5]] + Sqrt[2]*x^4)*(-1 - x^4 + x^8)^(1/4)), x])/2
```

Rubi steps

$$\int \frac{(-1+x^8)(1+x^8)}{\sqrt[4]{-1-x^4+x^8}\,(1-3x^8+x^{16})}\,dx = \int \frac{-1+x^{16}}{\sqrt[4]{-1-x^4+x^8}\,(1-3x^8+x^{16})}\,dx$$

$$= \int \left(\frac{1}{\sqrt[4]{-1-x^4+x^8}} - \frac{2-3x^8}{\sqrt[4]{-1-x^4+x^8}\,(1-3x^8+x^{16})}\right)dx$$

$$= \int \frac{1}{\sqrt[4]{-1-x^4+x^8}}\,dx - \int \frac{2-3x^8}{\sqrt[4]{-1-x^4+x^8}\,(1-3x^8+x^{16})}\,dx$$

$$= \frac{\left(\sqrt[4]{1+\frac{2x^4}{-1-\sqrt{5}}}\sqrt[4]{1+\frac{2x^4}{-1+\sqrt{5}}}\right)\int \frac{1}{\sqrt[4]{1+\frac{2x^4}{-1-\sqrt{5}}}\sqrt[4]{1+\frac{2x^4}{-1+\sqrt{5}}}}\,dx}{\sqrt[4]{-1-x^4+x^8}} - \int\Bigg(\frac{}{\sqrt[4]{-1-x\cdots}}$$

$$= \frac{x\sqrt[4]{1-\frac{2x^4}{1-\sqrt{5}}}\sqrt[4]{1-\frac{2x^4}{1+\sqrt{5}}}\,F_1\left(\frac{1}{4};\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{2x^4}{1+\sqrt{5}},\frac{2x^4}{1-\sqrt{5}}\right)}{\sqrt[4]{-1-x^4+x^8}} - \left(-3-\sqrt{5}\right)\int \cdots$$

$$= \frac{x\sqrt[4]{1-\frac{2x^4}{1-\sqrt{5}}}\sqrt[4]{1-\frac{2x^4}{1+\sqrt{5}}}\,F_1\left(\frac{1}{4};\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{2x^4}{1+\sqrt{5}},\frac{2x^4}{1-\sqrt{5}}\right)}{\sqrt[4]{-1-x^4+x^8}} - \left(-3-\sqrt{5}\right)\int\Bigg(\cdots$$

$$= \frac{x\sqrt[4]{1-\frac{2x^4}{1-\sqrt{5}}}\sqrt[4]{1-\frac{2x^4}{1+\sqrt{5}}}\,F_1\left(\frac{1}{4};\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{2x^4}{1+\sqrt{5}},\frac{2x^4}{1-\sqrt{5}}\right)}{\sqrt[4]{-1-x^4+x^8}} - \frac{1}{2}\sqrt{3-\sqrt{5}}\int \cdots$$

$$= \frac{x\sqrt[4]{1-\frac{2x^4}{1-\sqrt{5}}}\sqrt[4]{1-\frac{2x^4}{1+\sqrt{5}}}\,F_1\left(\frac{1}{4};\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{2x^4}{1+\sqrt{5}},\frac{2x^4}{1-\sqrt{5}}\right)}{\sqrt[4]{-1-x^4+x^8}} - \frac{1}{2}\sqrt{3-\sqrt{5}}\int \cdots$$

$$= \frac{x\sqrt[4]{1-\frac{2x^4}{1-\sqrt{5}}}\sqrt[4]{1-\frac{2x^4}{1+\sqrt{5}}}\,F_1\left(\frac{1}{4};\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{2x^4}{1+\sqrt{5}},\frac{2x^4}{1-\sqrt{5}}\right)}{\sqrt[4]{-1-x^4+x^8}} - \frac{1}{2}\sqrt{3-\sqrt{5}}\int \cdots$$

$$= \frac{x\sqrt[4]{1-\frac{2x^4}{1-\sqrt{5}}}\sqrt[4]{1-\frac{2x^4}{1+\sqrt{5}}}\,F_1\left(\frac{1}{4};\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{2x^4}{1+\sqrt{5}},\frac{2x^4}{1-\sqrt{5}}\right)}{\sqrt[4]{-1-x^4+x^8}} - \frac{x\sqrt[4]{\sqrt{3+\sqrt{5}}-\cdots}}{\cdots}$$

Mathematica [F] time = 2.24, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^8)(1+x^8)}{\sqrt[4]{-1-x^4+x^8}\,(1-3x^8+x^{16})}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^8)*(1 + x^8))/((-1 - x^4 + x^8)^(1/4)*(1 - 3*x^8 + x^16)
),x]
```

```
[Out] Integrate[((-1 + x^8)*(1 + x^8))/((-1 - x^4 + x^8)^(1/4)*(1 - 3*x^8 + x^16)
), x]
```

IntegrateAlgebraic [A] time = 2.43, size = 139, normalized size = 1.00

$$-\frac{x}{2\sqrt[4]{x^8-x^4-1}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^8-x^4-1}}{\sqrt{2}\,x^2-\sqrt{x^8-x^4-1}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}\,x\sqrt[4]{x^8-x^4-1}}{2x^2+\sqrt{2}\,\sqrt{x^8-x^4-1}}\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^8)*(1 + x^8))/((-1 - x^4 + x^8)^(1/4)*(1 - 3*x^
8 + x^16)),x]
```

```
[Out] -1/2*x/(-1 - x^4 + x^8)^(1/4) + ArcTan[(2^(3/4)*x*(-1 - x^4 + x^8)^(1/4))/(
Sqrt[2]*x^2 - Sqrt[-1 - x^4 + x^8])]/(4*2^(3/4)) - ArcTanh[(2*2^(1/4)*x*(-1
 - x^4 + x^8)^(1/4))/(2*x^2 + Sqrt[2]*Sqrt[-1 - x^4 + x^8])]/(4*2^(3/4))
```

fricas [B] time = 26.73, size = 658, normalized size = 4.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)*(x^8+1)/(x^8-x^4-1)^(1/4)/(x^16-3*x^8+1),x, algorithm="fr
icas")
```

```
[Out] -1/32*(4*2^(1/4)*(x^8 - x^4 - 1)*arctan(1/2*(4*2^(1/4)*(x^8 - x^4 - 1)^(1/4
)*x^3 + 2*2^(3/4)*(x^8 - x^4 - 1)^(3/4)*x + sqrt(2)*(2*2^(3/4)*(x^8 - x^4 -
 1)^(1/4)*x^3 - 4*sqrt(x^8 - x^4 - 1)*x^2 + 2*2^(1/4)*(x^8 - x^4 - 1)^(3/4)
*x - sqrt(2)*(x^8 + x^4 - 1))*sqrt((x^8 + x^4 + 4*2^(1/4)*(x^8 - x^4 - 1)^(
1/4)*x^3 + 4*sqrt(2)*sqrt(x^8 - x^4 - 1)*x^2 + 2*2^(3/4)*(x^8 - x^4 - 1)^(3
/4)*x - 1)/(x^8 + x^4 - 1)))/(x^8 - 3*x^4 - 1)) + 4*2^(1/4)*(x^8 - x^4 - 1)
*arctan(1/2*(4*2^(1/4)*(x^8 - x^4 - 1)^(1/4)*x^3 + 2*2^(3/4)*(x^8 - x^4 - 1
)^(3/4)*x + sqrt(2)*(2*2^(3/4)*(x^8 - x^4 - 1)^(1/4)*x^3 + 4*sqrt(x^8 - x^4
 - 1)*x^2 + 2*2^(1/4)*(x^8 - x^4 - 1)^(3/4)*x + sqrt(2)*(x^8 + x^4 - 1))*sq
rt((x^8 + x^4 - 4*2^(1/4)*(x^8 - x^4 - 1)^(1/4)*x^3 + 4*sqrt(2)*sqrt(x^8 -
x^4 - 1)*x^2 - 2*2^(3/4)*(x^8 - x^4 - 1)^(3/4)*x - 1)/(x^8 + x^4 - 1)))/(x^
8 - 3*x^4 - 1)) + 2^(1/4)*(x^8 - x^4 - 1)*log(2*(x^8 + x^4 + 4*2^(1/4)*(x^8
 - x^4 - 1)^(1/4)*x^3 + 4*sqrt(2)*sqrt(x^8 - x^4 - 1)*x^2 + 2*2^(3/4)*(x^8
- x^4 - 1)^(3/4)*x - 1)/(x^8 + x^4 - 1)) - 2^(1/4)*(x^8 - x^4 - 1)*log(2*(x
^8 + x^4 - 4*2^(1/4)*(x^8 - x^4 - 1)^(1/4)*x^3 + 4*sqrt(2)*sqrt(x^8 - x^4 -
 1)*x^2 - 2*2^(3/4)*(x^8 - x^4 - 1)^(3/4)*x - 1)/(x^8 + x^4 - 1)) + 16*(x^8
 - x^4 - 1)^(3/4)*x)/(x^8 - x^4 - 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)*(x^8+1)/(x^8-x^4-1)^(1/4)/(x^16-3*x^8+1),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [C] time = 3.66, size = 286, normalized size = 2.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-1)*(x^8+1)/(x^8-x^4-1)^(1/4)/(x^16-3*x^8+1),x)
```

```
[Out] -1/2*x/(x^8-x^4-1)^(1/4)-1/16*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln(-(x^8*RootOf
(_Z^2+RootOf(_Z^4+8)^2)+RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*(x^8
-x^4-1)^(1/2)*x^2+2*(x^8-x^4-1)^(1/4)*RootOf(_Z^4+8)^2*x^3-3*RootOf(_Z^2+Ro
otOf(_Z^4+8)^2)*x^4+4*(x^8-x^4-1)^(3/4)*x-RootOf(_Z^2+RootOf(_Z^4+8)^2))/(x
^8+x^4-1))-1/16*RootOf(_Z^4+8)*ln((-RootOf(_Z^4+8)*x^8+RootOf(_Z^4+8)^3*(x^
8-x^4-1)^(1/2)*x^2+2*(x^8-x^4-1)^(1/4)*RootOf(_Z^4+8)^2*x^3+3*RootOf(_Z^4+8
)*x^4-4*(x^8-x^4-1)^(3/4)*x+RootOf(_Z^4+8))/(x^8+x^4-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^8+1)(x^8-1)}{(x^{16}-3x^8+1)(x^8-x^4-1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)*(x^8+1)/(x^8-x^4-1)^(1/4)/(x^16-3*x^8+1),x, algorithm="ma
xima")
```

```
[Out] integrate((x^8 + 1)*(x^8 - 1)/((x^16 - 3*x^8 + 1)*(x^8 - x^4 - 1)^(1/4)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^8-1)\,(x^8+1)}{(x^8-x^4-1)^{1/4}\,(x^{16}-3x^8+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^8 - 1)*(x^8 + 1))/((x^8 - x^4 - 1)^(1/4)*(x^16 - 3*x^8 + 1)),x)
```

```
[Out] int(((x^8 - 1)*(x^8 + 1))/((x^8 - x^4 - 1)^(1/4)*(x^16 - 3*x^8 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-1)*(x**8+1)/(x**8-x**4-1)**(1/4)/(x**16-3*x**8+1),x)
```

```
[Out] Timed out
```

3.1645 $\int \frac{1}{\sqrt[3]{x^2(-a+x)}\,(-ad+(-1+d)x)}\,dx$

Optimal. Leaf size=140

$$\frac{\log\left(x-\sqrt[3]{d}\,\sqrt[3]{x^3-ax^2}\right)}{ad^{2/3}}-\frac{\log\left(d^{2/3}\left(x^3-ax^2\right)^{2/3}+\sqrt[3]{d}\,x\sqrt[3]{x^3-ax^2}+x^2\right)}{2ad^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{d}\,\sqrt[3]{x^3-ax^2}+x}\right)}{ad^{2/3}}$$

Rubi [A] time = 0.38, antiderivative size = 248, normalized size of antiderivative = 1.77, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2081, 2077, 91}

$$-\frac{\left(-a^2x\right)^{2/3}\sqrt[3]{a^2(x-a)}\log((d-1)x-ad)}{2a^3d^{2/3}\sqrt[3]{x^2(x-a)}}+\frac{3\left(-a^2x\right)^{2/3}\sqrt[3]{a^2(x-a)}\log\left(-\sqrt[3]{\frac{2}{3}}\sqrt[3]{d}\sqrt[3]{a^2(x-a)}-\sqrt[3]{\frac{2}{3}}\sqrt[3]{-a^2x}\right)}{2a^3d^{2/3}\sqrt[3]{x^2(x-a)}}+\frac{\sqrt{3}\left(-a^2x\right)^{2/3}\sqrt[3]{a^2(x-a)}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a^2(x-a)}}{\sqrt{3}\sqrt[3]{-a^2x}}\right)}{a^3d^{2/3}\sqrt[3]{x^2(x-a)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((x^2*(-a + x))^(1/3)*(-(a*d) + (-1 + d)*x)),x]
```

```
[Out] (Sqrt[3]*(-(a^2*x))^(2/3)*(a^2*(-a + x))^(1/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3
)*(a^2*(-a + x))^(1/3))/(Sqrt[3]*(-(a^2*x))^(1/3))])/(a^3*d^(2/3)*(x^2*(-a
+ x))^(1/3)) - ((-(a^2*x))^(2/3)*(a^2*(-a + x))^(1/3)*Log[-(a*d) + (-1 + d)
*x])/(2*a^3*d^(2/3)*(x^2*(-a + x))^(1/3)) + (3*(-(a^2*x))^(2/3)*(a^2*(-a +
x))^(1/3)*Log[-((2/3)^(1/3)*(-(a^2*x))^(1/3)) - (2/3)^(1/3)*d^(1/3)*(a^2*(-
a + x))^(1/3)])/(2*a^3*d^(2/3)*(x^2*(-a + x))^(1/3))
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 2077

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Dist[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int
[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d, e
, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 2081

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x^2(-a+x)}\,(-ad+(-1+d)x)}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{3}(a(-1+d)-3ad)+(-1+d)x\right)\sqrt[3]{-\frac{2a^3}{27}-\frac{a^2x}{3}+x^3}}\,dx, x, -$$

$$= \frac{\left(2^{2/3}\left(-a^2x\right)^{2/3}\sqrt[3]{a^2(-a+x)}\right)\text{Subst}\left(\int \frac{1}{\left(-\frac{2a^3}{9}-\frac{2a^2x}{3}\right)^{2/3}\sqrt[3]{-\frac{2a^3}{9}+\frac{a^2x}{3}}\left(\frac{1}{3}(a(-1+\right.}\right.}{3\sqrt[3]{-ax^2+x^3}}$$

$$= \frac{\sqrt{3}\sqrt[3]{-a^2(a-x)}\left(-a^2x\right)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{-a^2(a-x)}}{\sqrt{3}\sqrt[3]{-a^2x}}\right)}{a^3d^{2/3}\sqrt[3]{-ax^2+x^3}} - \frac{\sqrt[3]{-a^2(a-x)}}{2}$$

Mathematica [C] time = 0.03, size = 42, normalized size = 0.30

$$-\frac{3x\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{x}{d(x-a)}\right)}{ad\sqrt[3]{x^2(x-a)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((x^2*(-a + x))^(1/3)*(-(a*d) + (-1 + d)*x)),x]
```

```
[Out] (-3*x*Hypergeometric2F1[1/3, 1, 4/3, x/(d*(-a + x))])/(a*d*(x^2*(-a + x))^(
1/3))
```

IntegrateAlgebraic [A] time = 0.35, size = 140, normalized size = 1.00

$$\frac{\log\left(x-\sqrt[3]{d}\sqrt[3]{x^3-ax^2}\right)}{ad^{2/3}} - \frac{\log\left(d^{2/3}\left(x^3-ax^2\right)^{2/3}+\sqrt[3]{d}\,x\sqrt[3]{x^3-ax^2}+x^2\right)}{2ad^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{d}\sqrt[3]{x^3-ax^2}+x}\right)}{ad^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((x^2*(-a + x))^(1/3)*(-(a*d) + (-1 + d)*x)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*d^(1/3)*(-(a*x^2) + x^3)^(1/3))])/(a*d
^(2/3))) + Log[x - d^(1/3)*(-(a*x^2) + x^3)^(1/3)]/(a*d^(2/3)) - Log[x^2 +
d^(1/3)*x*(-(a*x^2) + x^3)^(1/3) + d^(2/3)*(-(a*x^2) + x^3)^(2/3)]/(2*a*d^(
2/3))
```

fricas [A] time = 0.40, size = 153, normalized size = 1.09

$$\frac{2\sqrt{3}\left(d^2\right)^{\frac{1}{6}}d\arctan\left(\frac{\sqrt{3}\left(d^2\right)^{\frac{1}{6}}\left(2\left(-ax^2+x^3\right)^{\frac{1}{3}}d+\left(d^2\right)^{\frac{1}{3}}x\right)}{3dx}\right)+2\left(d^2\right)^{\frac{2}{3}}\log\left(\frac{\left(-ax^2+x^3\right)^{\frac{1}{3}}d-\left(d^2\right)^{\frac{1}{3}}x}{x}\right)-\left(d^2\right)^{\frac{2}{3}}\log\left(\frac{\left(-ax^2+x^3\right)^{\frac{2}{3}}d^2+\left(-ax^2+x^3\right)^{\frac{1}{3}}\left(d^2\right)^{\frac{1}{3}}dx+\left(d^2\right)^{\frac{2}{3}}x^2}{x^2}\right)}{2ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-a+x))^(1/3)/(-a*d+(-1+d)*x),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(1/3*sqrt(3)*(d^2)^(1/6)*(2*(-a*x^2 + x^
3)^(1/3)*d + (d^2)^(1/3)*x)/(d*x)) + 2*(d^2)^(2/3)*log(((-a*x^2 + x^3)^(1/3
)*d - (d^2)^(1/3)*x)/x) - (d^2)^(2/3)*log(((-a*x^2 + x^3)^(2/3)*d^2 + (-a*x
^2 + x^3)^(1/3)*(d^2)^(1/3)*d*x + (d^2)^(2/3)*x^2)/x^2))/(a*d^2)
```

giac [A] time = 0.44, size = 103, normalized size = 0.74

$$\frac{\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}d^{\frac{1}{3}}\left(2\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}+\frac{1}{d^{\frac{1}{3}}}\right)\right)}{a|d|^{\frac{2}{3}}}-\frac{|d|^{\frac{4}{3}}\log\left(\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}+\frac{\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}}{d^{\frac{1}{3}}}+\frac{1}{d^{\frac{2}{3}}}\right)}{2\,ad^{2}}+\frac{\log\left(\left|\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}-\frac{1}{d^{\frac{1}{3}}}\right|\right)}{ad^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-a+x))^(1/3)/(-a*d+(-1+d)*x),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*d^(1/3)*(2*(-a/x + 1)^(1/3) + 1/d^(1/3)))/(a*abs
(d)^(2/3)) - 1/2*abs(d)^(4/3)*log((-a/x + 1)^(2/3) + (-a/x + 1)^(1/3)/d^(1/
3) + 1/d^(2/3))/(a*d^2) + log(abs((-a/x + 1)^(1/3) - 1/d^(1/3)))/(a*d^(2/3)
)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{2}\left(-a+x\right)\right)^{\frac{1}{3}}\left(-ad+\left(-1+d\right)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(-a+x))^(1/3)/(-a*d+(-1+d)*x),x)
```

```
[Out] int(1/(x^2*(-a+x))^(1/3)/(-a*d+(-1+d)*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(-(a-x)x^{2}\right)^{\frac{1}{3}}\left(ad-(d-1)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-a+x))^(1/3)/(-a*d+(-1+d)*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/((-(a - x)*x^2)^(1/3)*(a*d - (d - 1)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{1}{\left(a\,d-x\,\left(d-1\right)\right)\,{\left(-x^2\,\left(a-x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a*d - x*(d - 1))*(-x^2*(a - x))^(1/3)),x)
```

```
[Out] int(-1/((a*d - x*(d - 1))*(-x^2*(a - x))^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x^{2}\left(-a+x\right)}\left(-ad+dx-x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(-a+x))**(1/3)/(-a*d+(-1+d)*x),x)
```

```
[Out] Integral(1/((x**2*(-a + x))**(1/3)*(-a*d + d*x - x)), x)
```

3.1646 $\int \frac{\sqrt[3]{6+2x+x^2}}{1+x}\,dx$

Optimal. Leaf size=140

$$\frac{3}{2}\sqrt[3]{x^2+2x+6}+\frac{1}{2}\sqrt[3]{5}\log\left(5^{2/3}\sqrt[3]{x^2+2x+6}-5\right)-\frac{1}{4}\sqrt[3]{5}\log\left(\sqrt[3]{5}\left(x^2+2x+6\right)^{2/3}+5^{2/3}\sqrt[3]{x^2+2x+6}+5\right)-$$

Rubi [A] time = 0.09, antiderivative size = 103, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {694, 266, 50, 57, 617, 204, 31}

$$\frac{3}{2}\sqrt[3]{(x+1)^2+5}-\frac{1}{2}\sqrt[3]{5}\log(x+1)+\frac{3}{4}\sqrt[3]{5}\log\left(\sqrt[3]{5}-\sqrt[3]{(x+1)^2+5}\right)-\frac{1}{2}\sqrt{3}\sqrt[3]{5}\tan^{-1}\left(\frac{2\sqrt[3]{(x+1)^2+5}+\sqrt[3]{5}}{\sqrt{3}\sqrt[3]{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(6 + 2*x + x^2)^(1/3)/(1 + x),x]
```

```
[Out] (3*(5 + (1 + x)^2)^(1/3))/2 - (Sqrt[3]*5^(1/3)*ArcTan[(5^(1/3) + 2*(5 + (1
+ x)^2)^(1/3))/(Sqrt[3]*5^(1/3))])/2 - (5^(1/3)*Log[1 + x])/2 + (3*5^(1/3)*
Log[5^(1/3) - (5 + (1 + x)^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{6+2x+x^2}}{1+x}\,dx &= \text{Subst}\left(\int \frac{\sqrt[3]{5+x^2}}{x}\,dx, x, 1+x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt[3]{5+x}}{x}\,dx, x, (1+x)^2\right)\\ &= \frac{3}{2}\sqrt[3]{5+(1+x)^2} + \frac{5}{2}\text{Subst}\left(\int \frac{1}{x(5+x)^{2/3}}\,dx, x, (1+x)^2\right)\\ &= \frac{3}{2}\sqrt[3]{5+(1+x)^2} - \frac{1}{2}\sqrt[3]{5}\log(1+x) - \frac{1}{4}\left(3\sqrt[3]{5}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{5}-x}\,dx, x, \sqrt[3]{5+(1+x)^2}\right) -\\ &= \frac{3}{2}\sqrt[3]{5+(1+x)^2} - \frac{1}{2}\sqrt[3]{5}\log(1+x) + \frac{3}{4}\sqrt[3]{5}\log\left(\sqrt[3]{5}-\sqrt[3]{5+(1+x)^2}\right) + \frac{1}{2}\left(3\sqrt[3]{5}\right)\text{Subst}\Big(\\ &= \frac{3}{2}\sqrt[3]{5+(1+x)^2} - \frac{1}{2}\sqrt{3}\sqrt[3]{5}\tan^{-1}\left(\frac{5+2\,5^{2/3}\sqrt[3]{5+(1+x)^2}}{5\sqrt{3}}\right) - \frac{1}{2}\sqrt[3]{5}\log(1+x) + \frac{3}{4}\sqrt[3]{5}\,\text{lo}\end{aligned}$$

Mathematica [A] time = 0.06, size = 131, normalized size = 0.94

$$\frac{1}{4}\left(6\sqrt[3]{x^2+2x+6} + 2\sqrt[3]{5}\log\left(\sqrt[3]{5}-\sqrt[3]{(x+1)^2+5}\right) - \sqrt[3]{5}\log\left(\left((x+1)^2+5\right)^{2/3} + \sqrt[3]{5}\sqrt[3]{(x+1)^2+5} + 5^{2/3}\right) - 2\sqrt{3}\sqrt[3]{5}\tan^{-1}\left(\frac{2\sqrt[3]{(x+1)^2+5}+\sqrt[3]{5}}{\sqrt{3}\sqrt[3]{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(6 + 2*x + x^2)^(1/3)/(1 + x),x]
```

```
[Out] (6*(6 + 2*x + x^2)^(1/3) - 2*Sqrt[3]*5^(1/3)*ArcTan[(5^(1/3) + 2*(5 + (1 +
x)^2)^(1/3))/(Sqrt[3]*5^(1/3))] + 2*5^(1/3)*Log[5^(1/3) - (5 + (1 + x)^2)^(
1/3)] - 5^(1/3)*Log[5^(2/3) + 5^(1/3)*(5 + (1 + x)^2)^(1/3) + (5 + (1 + x)^
2)^(2/3)])/4
```

IntegrateAlgebraic [A] time = 0.23, size = 140, normalized size = 1.00

$$\frac{3}{2}\sqrt[3]{x^2+2x+6} + \frac{1}{2}\sqrt[3]{5}\log\left(5^{2/3}\sqrt[3]{x^2+2x+6}-5\right) - \frac{1}{4}\sqrt[3]{5}\log\left(\sqrt[3]{5}\left(x^2+2x+6\right)^{2/3} + 5^{2/3}\sqrt[3]{x^2+2x+6}+5\right) - \frac{1}{2}\sqrt{3}\sqrt[3]{5}\tan^{-1}\left(\frac{2\sqrt[3]{x^2+2x+6}}{\sqrt{3}\sqrt[3]{5}} + \frac{1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(6 + 2*x + x^2)^(1/3)/(1 + x),x]
```

```
[Out] (3*(6 + 2*x + x^2)^(1/3))/2 - (Sqrt[3]*5^(1/3)*ArcTan[1/Sqrt[3] + (2*(6 + 2
*x + x^2)^(1/3))/(Sqrt[3]*5^(1/3))])/2 + (5^(1/3)*Log[-5 + 5^(2/3)*(6 + 2*x
 + x^2)^(1/3)])/2 - (5^(1/3)*Log[5 + 5^(2/3)*(6 + 2*x + x^2)^(1/3) + 5^(1/3
)*(6 + 2*x + x^2)^(2/3)])/4
```

fricas [A] time = 0.40, size = 102, normalized size = 0.73

$$-\frac{1}{2}\cdot 5^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{2}{15}\cdot 5^{\frac{2}{3}}\sqrt{3}\left(x^2+2x+6\right)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right) - \frac{1}{4}\cdot 5^{\frac{1}{3}}\log\left(5^{\frac{2}{3}} + 5^{\frac{1}{3}}\left(x^2+2x+6\right)^{\frac{1}{3}} + \left(x^2+2x+6\right)^{\frac{2}{3}}\right) + \frac{1}{2}\cdot 5^{\frac{1}{3}}\log\left(-5^{\frac{1}{3}} + \left(x^2+2x+6\right)^{\frac{1}{3}}\right) + \frac{3}{2}\left(x^2+2x+6\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+6)^(1/3)/(1+x),x, algorithm="fricas")
```

```
[Out] -1/2*5^(1/3)*sqrt(3)*arctan(2/15*5^(2/3)*sqrt(3)*(x^2 + 2*x + 6)^(1/3) + 1/
3*sqrt(3)) - 1/4*5^(1/3)*log(5^(2/3) + 5^(1/3)*(x^2 + 2*x + 6)^(1/3) + (x^2
 + 2*x + 6)^(2/3)) + 1/2*5^(1/3)*log(-5^(1/3) + (x^2 + 2*x + 6)^(1/3)) + 3/
2*(x^2 + 2*x + 6)^(1/3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+2\,x+6\right)^{\frac{1}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+6)^(1/3)/(1+x),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 2*x + 6)^(1/3)/(x + 1), x)
```

maple [C] time = 8.35, size = 1820, normalized size = 13.00

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x+6)^(1/3)/(1+x),x)
```

```
[Out] 3/2*(x^2+2*x+6)^(1/3)+(5/12*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)
+25*_Z^2)*ln(-(31104*RootOf(_Z^3-5)^2*(x^4+4*x^3+16*x^2+24*x+36)^(1/3)-9828
*RootOf(_Z^3-5)-2718*RootOf(_Z^3-5)*x^2-4572*RootOf(_Z^3-5)*x-2000*RootOf(3
6*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)^2*RootOf(_Z^3-5)^2*x^2+150
*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*RootOf(_Z^3-5)^3*
x^2-2400*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)^2*RootOf(
_Z^3-5)^2*x+180*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*Ro
otOf(_Z^3-5)^3*x+495*(x^4+4*x^3+16*x^2+24*x+36)^(1/3)*RootOf(36*RootOf(_Z^3
-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*RootOf(_Z^3-5)*x^2+131040*RootOf(36*Roo
tOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)-108*RootOf(_Z^3-5)*x^4-432*Root
Of(_Z^3-5)*x^3+36240*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^
2)*x^2+60960*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*x+144
0*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*x^4+5760*RootOf(
36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*x^3-22950*(x^4+4*x^3+16*x
^2+24*x+36)^(2/3)-200*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z
^2)^2*RootOf(_Z^3-5)^2*x^4-800*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3
-5)+25*_Z^2)^2*RootOf(_Z^3-5)^2*x^3+990*(x^4+4*x^3+16*x^2+24*x+36)^(1/3)*Ro
otOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*RootOf(_Z^3-5)*x+15*
RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*RootOf(_Z^3-5)^3*x
^4+60*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*RootOf(_Z^3-
5)^3*x^3-4320*(x^4+4*x^3+16*x^2+24*x+36)^(2/3)*RootOf(36*RootOf(_Z^3-5)^2+3
0*_Z*RootOf(_Z^3-5)+25*_Z^2)*RootOf(_Z^3-5)^2+5184*RootOf(_Z^3-5)^2*(x^4+4*
x^3+16*x^2+24*x+36)^(1/3)*x^2+10368*RootOf(_Z^3-5)^2*(x^4+4*x^3+16*x^2+24*x
+36)^(1/3)*x+2970*(x^4+4*x^3+16*x^2+24*x+36)^(1/3)*RootOf(36*RootOf(_Z^3-5)
^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*RootOf(_Z^3-5))/(x^2+2*x+6)/(1+x)^2)+1/2*R
ootOf(_Z^3-5)*ln(-(-62208*RootOf(_Z^3-5)^2*(x^4+4*x^3+16*x^2+24*x+36)^(1/3)
+314496*RootOf(_Z^3-5)+115776*RootOf(_Z^3-5)*x^2+180864*RootOf(_Z^3-5)*x-25
0*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)^2*RootOf(_Z^3-5)
^2*x^2+4800*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*RootOf
(_Z^3-5)^3*x^2-300*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)
^2*RootOf(_Z^3-5)^2*x+5760*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+
25*_Z^2)*RootOf(_Z^3-5)^3*x-7650*(x^4+4*x^3+16*x^2+24*x+36)^(1/3)*RootOf(36
*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*RootOf(_Z^3-5)*x^2-16380*Ro
```

```
otOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)+6336*RootOf(_Z^3-5)*
x^4+25344*RootOf(_Z^3-5)*x^3-6030*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_
Z^3-5)+25*_Z^2)*x^2-9420*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25
*_Z^2)*x-330*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*x^4-1
320*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*x^3+5940*(x^4+
4*x^3+16*x^2+24*x+36)^(2/3)-25*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3
-5)+25*_Z^2)^2*RootOf(_Z^3-5)^2*x^4-100*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*Ro
otOf(_Z^3-5)+25*_Z^2)^2*RootOf(_Z^3-5)^2*x^3-15300*(x^4+4*x^3+16*x^2+24*x+3
6)^(1/3)*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*RootOf(_Z
^3-5)*x+480*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*RootOf
(_Z^3-5)^3*x^4+1920*RootOf(36*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2
)*RootOf(_Z^3-5)^3*x^3+8640*(x^4+4*x^3+16*x^2+24*x+36)^(2/3)*RootOf(36*Root
Of(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*RootOf(_Z^3-5)^2-10368*RootOf(_Z
^3-5)^2*(x^4+4*x^3+16*x^2+24*x+36)^(1/3)*x^2-20736*RootOf(_Z^3-5)^2*(x^4+4*
x^3+16*x^2+24*x+36)^(1/3)*x-45900*(x^4+4*x^3+16*x^2+24*x+36)^(1/3)*RootOf(3
6*RootOf(_Z^3-5)^2+30*_Z*RootOf(_Z^3-5)+25*_Z^2)*RootOf(_Z^3-5))/(x^2+2*x+6
)/(1+x)^2))/(x^2+2*x+6)^(2/3)*((x^2+2*x+6)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+2\,x+6\right)^{\frac{1}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+6)^(1/3)/(1+x),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 2*x + 6)^(1/3)/(x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+2\,x+6\right)^{1/3}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 + 6)^(1/3)/(x + 1),x)
```

```
[Out] int((2*x + x^2 + 6)^(1/3)/(x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x^2+2x+6}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x+6)**(1/3)/(1+x),x)
```

```
[Out] Integral((x**2 + 2*x + 6)**(1/3)/(x + 1), x)
```

3.1647 $\int \frac{-1+x}{x\sqrt[3]{-1+x^3}}\,dx$

Optimal. Leaf size=140

$$-\frac{1}{6}\log\left(\left(x^3-1\right)^{2/3}-\sqrt[3]{x^3-1}+1\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}\left(\frac{x+1}{-2\sqrt[3]{x^3-1}+x-1}\right)$$

Rubi [A] time = 0.07, antiderivative size = 94, normalized size of antiderivative = 0.67, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1844, 239, 266, 56, 618, 204, 31}

$$\frac{1}{2}\log\left(\sqrt[3]{x^3-1}+1\right)-\frac{1}{2}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(x*(-1 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + ArcTan[(1 - 2*(-1 +
x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 + (-1 + x^3)^(1/3)]/2 - Log
[-x + (-1 + x^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x}{x\sqrt[3]{-1+x^3}}\,dx &= \int \left(\frac{1}{\sqrt[3]{-1+x^3}} - \frac{1}{x\sqrt[3]{-1+x^3}}\right) dx \\
&= \int \frac{1}{\sqrt[3]{-1+x^3}}\,dx - \int \frac{1}{x\sqrt[3]{-1+x^3}}\,dx \\
&= \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{2}\log\left(-x+\sqrt[3]{-1+x^3}\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x}\,x}\,dx, x, x^3\right) \\
&= \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} - \frac{1}{2}\log\left(-x+\sqrt[3]{-1+x^3}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[3]{-1+x^3}\right) \\
&= \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1+\sqrt[3]{-1+x^3}\right) - \frac{1}{2}\log\left(-x+\sqrt[3]{-1+x^3}\right) + \text{Subst}\left(\int \right. \\
&= \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{-1+2\sqrt[3]{-1+x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1+\sqrt[3]{-1+x^3}\right) - \frac{1}{2}\log\left(-x+\sqrt[3]{\right.
\end{aligned}
$$

Mathematica [C] time = 0.06, size = 103, normalized size = 0.74

$$
\frac{1}{6}\left(-3\left(x^3-1\right)^{2/3}{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-x^3\right)-2\log\left(1-\frac{x}{\sqrt[3]{x^3-1}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)+\log\left(\frac{x}{\sqrt[3]{x^3-1}}+\frac{x^2}{\left(x^3-1\right)^{2/3}}+1\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(x*(-1 + x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]] - 3*(-1 + x^3)^(2/3
)*Hypergeometric2F1[2/3, 1, 5/3, 1 - x^3] - 2*Log[1 - x/(-1 + x^3)^(1/3)] +
 Log[1 + x^2/(-1 + x^3)^(2/3) + x/(-1 + x^3)^(1/3)])/6
```

IntegrateAlgebraic [A] time = 3.58, size = 140, normalized size = 1.00

$$
-\frac{1}{6}\log\left(\left(x^3-1\right)^{2/3}-\sqrt[3]{x^3-1}+1\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^3-1}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}\left(\frac{x+1}{-2\sqrt[3]{x^3-1}+x-1}\right)+\frac{1}{6}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/(x*(-1 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x^3)^(1/3))]/Sqrt[3] + ArcTan[1/Sqrt[3] - (
2*(-1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - (2*ArcTanh[(1 + x)/(-1 + x - 2*(-1 +
```

```
x^3)^(1/3))])/3 - Log[1 - (-1 + x^3)^(1/3) + (-1 + x^3)^(2/3)]/6 + Log[x^2
 + x*(-1 + x^3)^(1/3) + (-1 + x^3)^(2/3)]/6
```

fricas [A] time = 0.78, size = 196, normalized size = 1.40

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(x^{4}+2x^{3}+x^{2}-x-1\right)\left(x^{3}-1\right)^{\frac{2}{3}}-2\sqrt{3}\left(x^{5}+x^{4}-x^{3}-2x^{2}-x\right)\left(x^{3}-1\right)^{\frac{1}{3}}+\sqrt{3}\left(x^{5}+2x^{4}+2x^{3}-x^{2}-2x-1\right)}{3\left(2x^{6}+3x^{5}-4x^{3}-3x^{2}+1\right)}\right)+\frac{1}{3}\log\left(-x^{3}-x^{2}-\left(x^{3}-1\right)^{\frac{2}{3}}(x+1)-\left(x^{3}-1\right)^{\frac{1}{3}}\left(x^{2}+x\right)+1\right)-\frac{1}{6}\log\left(-x^{3}+\left(x^{3}-1\right)^{\frac{2}{3}}(x+1)-\left(x^{3}-1\right)^{\frac{1}{3}}(x+1)+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x/(x^3-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*(2*sqrt(3)*(x^4 + 2*x^3 + x^2 - x - 1)*(x^3 - 1)^(2/
3) - 2*sqrt(3)*(x^5 + x^4 - x^3 - 2*x^2 - x)*(x^3 - 1)^(1/3) + sqrt(3)*(x^5
 + 2*x^4 + 2*x^3 - x^2 - 2*x - 1))/(2*x^6 + 3*x^5 - 4*x^3 - 3*x^2 + 1)) + 1
/3*log(-x^3 - x^2 - (x^3 - 1)^(2/3)*(x + 1) - (x^3 - 1)^(1/3)*(x^2 + x) + 1
) - 1/6*log(-x^3 + (x^3 - 1)^(2/3)*(x + 1) - (x^3 - 1)^(1/3)*(x + 1) + x +
1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^{3}-1\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x/(x^3-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/((x^3 - 1)^(1/3)*x), x)
```

maple [C] time = 0.34, size = 113, normalized size = 0.81

$$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\mathrm{signum}\left(x^{3}-1\right)\right)^{\frac{1}{3}}\left(\frac{2\pi\sqrt{3}\,x^{3}\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^{3}\right)}{9\Gamma\left(\frac{2}{3}\right)}+\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{6\pi\,\mathrm{signum}\left(x^{3}-1\right)^{\frac{1}{3}}}+\frac{\left(-\mathrm{signum}\left(x^{3}-1\right)\right)^{\frac{1}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^{3}\right)}{\mathrm{signum}\left(x^{3}-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/x/(x^3-1)^(1/3),x)
```

```
[Out] -1/6/Pi*3^(1/2)*GAMMA(2/3)/signum(x^3-1)^(1/3)*(-signum(x^3-1))^(1/3)*(2/9*
Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1,1,4/3],[2,2],x^3)+2/3*(-1/6*Pi*3^(1/
2)-3/2*ln(3)+3*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(2/3))+1/signum(x^3-1)^(1/3)*(-s
ignum(x^3-1))^(1/3)*x*hypergeom([1/3,1/3],[4/3],x^3)
```

maxima [A] time = 0.41, size = 124, normalized size = 0.89

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^{3}-1\right)^{\frac{1}{3}}-1\right)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^{3}-1\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{1}{6}\log\left(\left(x^{3}-1\right)^{\frac{2}{3}}-\left(x^{3}-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(x^{3}-1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(\frac{\left(x^{3}-1\right)^{\frac{1}{3}}}{x}+\frac{\left(x^{3}-1\right)^{\frac{2}{3}}}{x^{2}}+1\right)-\frac{1}{3}\log\left(\frac{\left(x^{3}-1\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x/(x^3-1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3) - 1)) - 1/3*sqrt(3)*arct
an(1/3*sqrt(3)*(2*(x^3 - 1)^(1/3)/x + 1)) - 1/6*log((x^3 - 1)^(2/3) - (x^3
- 1)^(1/3) + 1) + 1/3*log((x^3 - 1)^(1/3) + 1) + 1/6*log((x^3 - 1)^(1/3)/x
+ (x^3 - 1)^(2/3)/x^2 + 1) - 1/3*log((x^3 - 1)^(1/3)/x - 1)
```

mupad [B] time = 1.24, size = 100, normalized size = 0.71

$$\frac{\ln\left(\left(x^{3}-1\right)^{1/3}+1\right)}{3}+\ln\left(9\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^{2}+\left(x^{3}-1\right)^{1/3}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(9\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^{2}+\left(x^{3}-1\right)^{1/3}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{x\left(1-x^{3}\right)^{1/3}{}_{2}F_{1}\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ x^{3}\right)}{\left(x^{3}-1\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(x*(x^3 - 1)^(1/3)),x)
```

```
[Out] log((x^3 - 1)^(1/3) + 1)/3 + log(9*((3^(1/2)*1i)/6 - 1/6)^2 + (x^3 - 1)^(1/
3))*((3^(1/2)*1i)/6 - 1/6) - log(9*((3^(1/2)*1i)/6 + 1/6)^2 + (x^3 - 1)^(1/
3))*((3^(1/2)*1i)/6 + 1/6) + (x*(1 - x^3)^(1/3)*hypergeom([1/3, 1/3], 4/3,
x^3))/(x^3 - 1)^(1/3)
```

sympy [C] time = 2.42, size = 60, normalized size = 0.43

$$\frac{xe^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{e^{2i\pi}}{x^3}\right)}{3x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/x/(x**3-1)**(1/3),x)
```

```
[Out] x*exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3)/(3*gamma(4/3)) +
gamma(1/3)*hyper((1/3, 1/3), (4/3,), exp_polar(2*I*pi)/x**3)/(3*x*gamma(4/3
))
```

3.1648 $\int \frac{\left(-1+x^3\right)^{2/3}\left(1+x^3\right)}{x^6\left(-2+x^3\right)}\,dx$

Optimal. Leaf size=140

$$\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}-x\right)}{4\,2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}+x}\right)}{4\,2^{2/3}}+\frac{\left(x^3-1\right)^{2/3}\left(11x^3+4\right)}{40x^5}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}\,x+2^{2/3}\left(x^3-1\right)^{2/3}\right.}{8\,2^{2/3}}$$

Rubi [A] time = 0.14, antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {580, 583, 12, 377, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(\sqrt[3]{2}-\frac{x}{\sqrt[3]{x^3-1}}\right)}{4\,2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2^{2/3}x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{4\,2^{2/3}}+\frac{\left(x^3-1\right)^{2/3}}{10x^5}+\frac{11\left(x^3-1\right)^{2/3}}{40x^2}-\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+\frac{x^2}{\left(x^3-1\right)^{2/3}}+2^{2/3}\right)}{8\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(2/3)*(1 + x^3))/(x^6*(-2 + x^3)),x]
```

```
[Out] (-1 + x^3)^(2/3)/(10*x^5) + (11*(-1 + x^3)^(2/3))/(40*x^2) - (Sqrt[3]*ArcTa
n[(1 + (2^(2/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]])/(4*2^(2/3)) + Log[2^(1/3) -
x/(-1 + x^3)^(1/3)]/(4*2^(2/3)) - Log[2^(2/3) + x^2/(-1 + x^3)^(2/3) + (2^(
1/3)*x)/(-1 + x^3)^(1/3)]/(8*2^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(1+x^3\right)}{x^6\left(-2+x^3\right)}\,dx = \frac{\left(-1+x^3\right)^{2/3}}{10x^5} - \frac{1}{10}\int \frac{11-13x^3}{x^3\left(-2+x^3\right)\sqrt[3]{-1+x^3}}\,dx$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{10x^5} + \frac{11\left(-1+x^3\right)^{2/3}}{40x^2} + \frac{1}{40}\int \frac{30}{\left(-2+x^3\right)\sqrt[3]{-1+x^3}}\,dx$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{10x^5} + \frac{11\left(-1+x^3\right)^{2/3}}{40x^2} + \frac{3}{4}\int \frac{1}{\left(-2+x^3\right)\sqrt[3]{-1+x^3}}\,dx$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{10x^5} + \frac{11\left(-1+x^3\right)^{2/3}}{40x^2} + \frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{-2+x^3}\,dx, x, \frac{x}{\sqrt[3]{-1+x^3}}\right)$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{10x^5} + \frac{11\left(-1+x^3\right)^{2/3}}{40x^2} + \frac{\operatorname{Subst}\left(\int \frac{1}{-\sqrt[3]{2}+x}\,dx, x, \frac{x}{\sqrt[3]{-1+x^3}}\right)}{4\ 2^{2/3}} + \frac{\operatorname{Subst}\left(\int \frac{\ }{2^{2/}}\right.}{}$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{10x^5} + \frac{11\left(-1+x^3\right)^{2/3}}{40x^2} + \frac{\log\left(\sqrt[3]{2} - \frac{x}{\sqrt[3]{-1+x^3}}\right)}{4\ 2^{2/3}} - \frac{\operatorname{Subst}\left(\int \frac{\sqrt[3]{2}+2x}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx\right.}{8\ 2^{2/3}}$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{10x^5} + \frac{11\left(-1+x^3\right)^{2/3}}{40x^2} + \frac{\log\left(\sqrt[3]{2} - \frac{x}{\sqrt[3]{-1+x^3}}\right)}{4\ 2^{2/3}} - \frac{\log\left(2^{2/3} + \frac{x^2}{\left(-1+x^3\right)^{2/3}} + \right.}{8\ 2^{2/3}}$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{10x^5} + \frac{11\left(-1+x^3\right)^{2/3}}{40x^2} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{4\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2} - \frac{x}{\sqrt[3]{-1+x^3}}\right)}{4\ 2^{2/3}} -$$

Mathematica [A] time = 0.16, size = 135, normalized size = 0.96

$$\left(\frac{1}{10x^5} + \frac{11}{40x^2}\right)\left(x^3-1\right)^{2/3} - \frac{-2\log\left(2 - \frac{2^{2/3}x}{\sqrt[3]{1-x^3}}\right) + 2\sqrt{3}\tan^{-1}\left(\frac{\frac{2^{2/3}x}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right) + \log\left(\frac{2^{2/3}x}{\sqrt[3]{1-x^3}} + \frac{\sqrt[3]{2}x^2}{\left(1-x^3\right)^{2/3}} + 2\right)}{8\ 2^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^3)^(2/3)*(1 + x^3))/(x^6*(-2 + x^3)),x]
```

```
[Out] (1/(10*x^5) + 11/(40*x^2))*(-1 + x^3)^(2/3) - (2*Sqrt[3]*ArcTan[(1 + (2^(2/
3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - 2*Log[2 - (2^(2/3)*x)/(1 - x^3)^(1/3)] +
Log[2 + (2^(1/3)*x^2)/(1 - x^3)^(2/3) + (2^(2/3)*x)/(1 - x^3)^(1/3)])/(8*2^
(2/3))
```

IntegrateAlgebraic [A] time = 0.33, size = 140, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1} - x\right)}{4\ 2^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}+x}\right)}{4\ 2^{2/3}} + \frac{\left(x^3-1\right)^{2/3}\left(11x^3+4\right)}{40x^5} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}\,x + 2^{2/3}\left(x^3-1\right)^{2/3} + x^2\right)}{8\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(2/3)*(1 + x^3))/(x^6*(-2 + x^3)),x]
```

```
[Out] ((-1 + x^3)^(2/3)*(4 + 11*x^3))/(40*x^5) - (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x +
 2*2^(1/3)*(-1 + x^3)^(1/3))])/(4*2^(2/3)) + Log[-x + 2^(1/3)*(-1 + x^3)^(1
/3)]/(4*2^(2/3)) - Log[x^2 + 2^(1/3)*x*(-1 + x^3)^(1/3) + 2^(2/3)*(-1 + x^3
)^(2/3)]/(8*2^(2/3))
```

fricas [B] time = 2.85, size = 266, normalized size = 1.90

$$\frac{20 \cdot 4^{\frac{1}{6}} \sqrt{3} x^{5} \arctan\left(\frac{4^{\frac{1}{6}} \sqrt{3} \left(12 \cdot 4^{\frac{2}{3}} \left(2 x^{7}-5 x^{4}+2 x\right) \left(x^{3}-1\right)^{\frac{2}{3}}+4^{\frac{1}{3}} \left(91 x^{9}-168 x^{6}+84 x^{3}-8\right)+12 \left(19 x^{8}-22 x^{5}+4 x^{2}\right) \left(x^{3}-1\right)^{\frac{1}{3}}\right)}{6 \left(53 x^{9}-48 x^{6}-12 x^{3}+8\right)}\right)+10 \cdot 4^{\frac{2}{3}} x^{5} \log\left(\frac{6 \cdot 4^{\frac{1}{3}} \left(x^{3}-1\right)^{\frac{1}{3}} x^{2}+4^{\frac{2}{3}} \left(x^{3}-2\right)-12 \left(x^{3}-1\right)^{\frac{2}{3}} x}{x^{3}-2}\right)-5 \cdot 4^{\frac{2}{3}} x^{5} \log\left(\frac{6 \cdot 4^{\frac{2}{3}} \left(2 x^{4}-x\right) \left(x^{3}-1\right)^{\frac{2}{3}}+4^{\frac{1}{3}} \left(19 x^{6}-22 x^{3}+4\right)+6 \left(5 x^{5}-4 x^{2}\right) \left(x^{3}-1\right)^{\frac{1}{3}}}{x^{6}-4 x^{3}+4}\right)+12 \left(11 x^{3}+4\right) \left(x^{3}-1\right)^{\frac{2}{3}}}{480 x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^3+1)/x^6/(x^3-2),x, algorithm="fricas")
```

```
[Out] 1/480*(20*4^(1/6)*sqrt(3)*x^5*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^(2/3)*(2*x^7
 - 5*x^4 + 2*x)*(x^3 - 1)^(2/3) + 4^(1/3)*(91*x^9 - 168*x^6 + 84*x^3 - 8) +
 12*(19*x^8 - 22*x^5 + 4*x^2)*(x^3 - 1)^(1/3))/(53*x^9 - 48*x^6 - 12*x^3 +
8)) + 10*4^(2/3)*x^5*log((6*4^(1/3)*(x^3 - 1)^(1/3)*x^2 + 4^(2/3)*(x^3 - 2)
 - 12*(x^3 - 1)^(2/3)*x)/(x^3 - 2)) - 5*4^(2/3)*x^5*log((6*4^(2/3)*(2*x^4 -
 x)*(x^3 - 1)^(2/3) + 4^(1/3)*(19*x^6 - 22*x^3 + 4) + 6*(5*x^5 - 4*x^2)*(x^
3 - 1)^(1/3))/(x^6 - 4*x^3 + 4)) + 12*(11*x^3 + 4)*(x^3 - 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}+1\right) \left(x^{3}-1\right)^{\frac{2}{3}}}{\left(x^{3}-2\right) x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^3+1)/x^6/(x^3-2),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)*(x^3 - 1)^(2/3)/((x^3 - 2)*x^6), x)
```

maple [C] time = 2.70, size = 889, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)*(x^3+1)/x^6/(x^3-2),x)
```

```
[Out] 1/40*(11*x^6-7*x^3-4)/x^5/(x^3-1)^(1/3)+1/8*RootOf(_Z^3-2)*ln((3*RootOf(Roo
tOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^3+27*RootOf(R
ootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^3+15*(x^3
-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-
2)^2*x+2*RootOf(_Z^3-2)^2*(x^3-1)^(1/3)*x^2-3*(x^3-1)^(1/3)*RootOf(RootOf(_
Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^2-2*RootOf(_Z^3-2)*x
^3-18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+x*(x^3-1)^(2
/3)+2*RootOf(_Z^3-2)+18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2
))/(x^3-2))-1/8*ln((6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*
RootOf(_Z^3-2)^3*x^3-18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2
)^2*RootOf(_Z^3-2)^2*x^3-30*(x^3-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*Root
Of(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x-RootOf(_Z^3-2)^2*(x^3-1)^(1/3)*x^2+2
4*(x^3-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf
(_Z^3-2)*x^2+6*RootOf(_Z^3-2)*x^3-18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z
^3-2)+36*_Z^2)*x^3-8*x*(x^3-1)^(2/3)-4*RootOf(_Z^3-2)+12*RootOf(RootOf(_Z^3
-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2))/(x^3-2))*RootOf(_Z^3-2)-3/4*ln((6*RootO
f(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^3-18*Roo
tOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-30
*(x^3-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(
_Z^3-2)^2*x-RootOf(_Z^3-2)^2*(x^3-1)^(1/3)*x^2+24*(x^3-1)^(1/3)*RootOf(Root
Of(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^2+6*RootOf(_Z^3-
2)*x^3-18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3-8*x*(x^3
-1)^(2/3)-4*RootOf(_Z^3-2)+12*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+3
6*_Z^2))/(x^3-2))*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+1\right)\left(x^3-1\right)^{\frac{2}{3}}}{\left(x^3-2\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^3+1)/x^6/(x^3-2),x, algorithm="maxima")
[Out] integrate((x^3 + 1)*(x^3 - 1)^(2/3)/((x^3 - 2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3-1\right)^{2/3}\left(x^3+1\right)}{x^6\left(x^3-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(x^3 + 1))/(x^6*(x^3 - 2)),x)
[Out] int(((x^3 - 1)^(2/3)*(x^3 + 1))/(x^6*(x^3 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}{x^6\left(x^3-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)*(x**3+1)/x**6/(x**3-2),x)
[Out] Integral(((x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)/(x**6*(x**3
 - 2)), x)
```

3.1649 $\int \frac{(-4+x^2)\sqrt[3]{x+x^3}}{x^4(2+x^2)}\,dx$

Optimal. Leaf size=140

$$-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{x^3+x}-x\right)}{4\sqrt[3]{2}}-\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3+x}+x}\right)}{4\sqrt[3]{2}}-\frac{3\sqrt[3]{x^3+x}\left(2x^2-1\right)}{4x^3}+\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{x^3+x}\,x+2^{2/3}\left(x^3+x\right)^{2}\right.}{8\sqrt[3]{2}}$$

Rubi [A] time = 0.38, antiderivative size = 228, normalized size of antiderivative = 1.63, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {2056, 580, 583, 12, 466, 465, 494, 292, 31, 634, 617, 204, 628}

$$-\frac{3\sqrt[3]{x^3+x}}{2x}+\frac{3\sqrt[3]{x^3+x}}{4x^3}-\frac{3\sqrt[3]{x^3+x}\log\left(\sqrt[3]{2}-\frac{x^{2/3}}{\sqrt[3]{x^2+1}}\right)}{4\sqrt[3]{2}\sqrt[3]{x}\sqrt[3]{x^2+1}}+\frac{3\sqrt[3]{x^3+x}\log\left(\frac{x^{4/3}}{(x^2+1)^{2/3}}+\frac{\sqrt[3]{2}x^{2/3}}{\sqrt[3]{x^2+1}}+2^{2/3}\right)}{8\sqrt[3]{2}\sqrt[3]{x}\sqrt[3]{x^2+1}}-\frac{3\sqrt{3}\sqrt[3]{x^3+x}\tan^{-1}\left(\frac{\frac{2^{2/3}x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt[3]{x}\sqrt[3]{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((-4 + x^2)*(x + x^3)^(1/3))/(x^4*(2 + x^2)),x]
```

```
[Out] (3*(x + x^3)^(1/3))/(4*x^3) - (3*(x + x^3)^(1/3))/(2*x) - (3*Sqrt[3]*(x + x
^3)^(1/3)*ArcTan[(1 + (2^(2/3)*x^(2/3))/(1 + x^2)^(1/3))/Sqrt[3]])/(4*2^(1/
3)*x^(1/3)*(1 + x^2)^(1/3)) - (3*(x + x^3)^(1/3)*Log[2^(1/3) - x^(2/3)/(1 +
 x^2)^(1/3)])/(4*2^(1/3)*x^(1/3)*(1 + x^2)^(1/3)) + (3*(x + x^3)^(1/3)*Log[
2^(2/3) + x^(4/3)/(1 + x^2)^(2/3) + (2^(1/3)*x^(2/3))/(1 + x^2)^(1/3)])/(8*
2^(1/3)*x^(1/3)*(1 + x^2)^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\left(-4+x^2\right)\sqrt[3]{x+x^3}}{x^4\left(2+x^2\right)}\,dx = \frac{\sqrt[3]{x+x^3}\int \frac{\left(-4+x^2\right)\sqrt[3]{1+x^2}}{x^{11/3}\left(2+x^2\right)}\,dx}{\sqrt[3]{x}\,\sqrt[3]{1+x^2}}$$

$$= \frac{3\sqrt[3]{x+x^3}}{4x^3} + \frac{\left(3\sqrt[3]{x+x^3}\right)\int \frac{\frac{32}{3}+\frac{40x^2}{3}}{x^{5/3}\left(1+x^2\right)^{2/3}\left(2+x^2\right)}\,dx}{16\sqrt[3]{x}\,\sqrt[3]{1+x^2}}$$

$$= \frac{3\sqrt[3]{x+x^3}}{4x^3} - \frac{3\sqrt[3]{x+x^3}}{2x} - \frac{\left(9\sqrt[3]{x+x^3}\right)\int -\frac{32\sqrt[3]{x}}{3\left(1+x^2\right)^{2/3}\left(2+x^2\right)}\,dx}{64\sqrt[3]{x}\,\sqrt[3]{1+x^2}}$$

$$= \frac{3\sqrt[3]{x+x^3}}{4x^3} - \frac{3\sqrt[3]{x+x^3}}{2x} + \frac{\left(3\sqrt[3]{x+x^3}\right)\int \frac{\sqrt[3]{x}}{\left(1+x^2\right)^{2/3}\left(2+x^2\right)}\,dx}{2\sqrt[3]{x}\,\sqrt[3]{1+x^2}}$$

$$= \frac{3\sqrt[3]{x+x^3}}{4x^3} - \frac{3\sqrt[3]{x+x^3}}{2x} + \frac{\left(9\sqrt[3]{x+x^3}\right)\text{Subst}\left(\int \frac{x^3}{\left(1+x^6\right)^{2/3}\left(2+x^6\right)}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{x}\,\sqrt[3]{1+x^2}}$$

$$= \frac{3\sqrt[3]{x+x^3}}{4x^3} - \frac{3\sqrt[3]{x+x^3}}{2x} + \frac{\left(9\sqrt[3]{x+x^3}\right)\text{Subst}\left(\int \frac{x}{\left(1+x^3\right)^{2/3}\left(2+x^3\right)}\,dx, x, x^{2/3}\right)}{4\sqrt[3]{x}\,\sqrt[3]{1+x^2}}$$

$$= \frac{3\sqrt[3]{x+x^3}}{4x^3} - \frac{3\sqrt[3]{x+x^3}}{2x} + \frac{\left(9\sqrt[3]{x+x^3}\right)\text{Subst}\left(\int \frac{x}{2-x^3}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{4\sqrt[3]{x}\,\sqrt[3]{1+x^2}}$$

$$= \frac{3\sqrt[3]{x+x^3}}{4x^3} - \frac{3\sqrt[3]{x+x^3}}{2x} + \frac{\left(3\sqrt[3]{x+x^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{4\sqrt[3]{2}\,\sqrt[3]{x}\,\sqrt[3]{1+x^2}} - \frac{\left(3\sqrt[3]{x+x^3}\right)\text{S}\cdots}{\cdots}$$

$$= \frac{3\sqrt[3]{x+x^3}}{4x^3} - \frac{3\sqrt[3]{x+x^3}}{2x} - \frac{3\sqrt[3]{x+x^3}\,\log\left(\sqrt[3]{2}-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{4\sqrt[3]{2}\,\sqrt[3]{x}\,\sqrt[3]{1+x^2}} - \frac{\left(9\sqrt[3]{x+x^3}\right)\text{Subst}\left(\int \frac{\cdots}{2^{2/3}+\cdots}\right.}{8\sqrt[3]{x}\,\sqrt[3]{1+\cdots}}$$

$$= \frac{3\sqrt[3]{x+x^3}}{4x^3} - \frac{3\sqrt[3]{x+x^3}}{2x} - \frac{3\sqrt[3]{x+x^3}\,\log\left(\sqrt[3]{2}-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{4\sqrt[3]{2}\,\sqrt[3]{x}\,\sqrt[3]{1+x^2}} + \frac{3\sqrt[3]{x+x^3}\,\log\left(2^{2/3}+\frac{x^{4/\cdots}}{\left(1+x^2\right)\cdots}\right.}{8\sqrt[3]{2}\,\sqrt[3]{x}\,\sqrt[3]{1+\cdots}}$$

$$= \frac{3\sqrt[3]{x+x^3}}{4x^3} - \frac{3\sqrt[3]{x+x^3}}{2x} - \frac{3\sqrt{3}\,\sqrt[3]{x+x^3}\,\tan^{-1}\left(\frac{1+\frac{2^{2/3}x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\,\sqrt[3]{x}\,\sqrt[3]{1+x^2}} - \frac{3\sqrt[3]{x+x^3}\,\log\left(\sqrt[3]{2}-\frac{x^{2\cdots}}{\sqrt[3]{1\cdots}}\right.}{4\sqrt[3]{2}\,\sqrt[3]{x}\,\sqrt[3]{1+x^2}}$$

Mathematica [C] time = 0.09, size = 102, normalized size = 0.73

$$\frac{3\sqrt[3]{x^3+x}\left(9\left(x^2+2\right)x^2\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{x^2}{2x^2+2}\right)+\left(9x^4-6x^2\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x^2}{2x^2+2}\right)-4\left(11x^4+7x^2-4\right)\right)}{64x^3\left(x^2+1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-4 + x^2)*(x + x^3)^(1/3))/(x^4*(2 + x^2)),x]
```

```
[Out] (3*(x + x^3)^(1/3)*(-4*(-4 + 7*x^2 + 11*x^4) + (-6*x^2 + 9*x^4)*Hypergeomet
ric2F1[2/3, 1, 5/3, x^2/(2 + 2*x^2)] + 9*x^2*(2 + x^2)*Hypergeometric2F1[2/
3, 2, 5/3, x^2/(2 + 2*x^2)]))/(64*x^3*(1 + x^2))
```

IntegrateAlgebraic [A] time = 0.41, size = 140, normalized size = 1.00

$$-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{x^3+x}-x\right)}{4\sqrt[3]{2}}-\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3+x}+x}\right)}{4\sqrt[3]{2}}-\frac{3\sqrt[3]{x^3+x}\left(2x^2-1\right)}{4x^3}+\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{x^3+x}\,x+2^{2/3}\left(x^3+x\right)^{2/3}+x^2\right)}{8\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + x^2)*(x + x^3)^(1/3))/(x^4*(2 + x^2)),x]
```

```
[Out] (-3*(-1 + 2*x^2)*(x + x^3)^(1/3))/(4*x^3) - (3*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(
x + 2*2^(1/3)*(x + x^3)^(1/3))])/(4*2^(1/3)) - (3*Log[-x + 2^(1/3)*(x + x^3
)^(1/3)])/(4*2^(1/3)) + (3*Log[x^2 + 2^(1/3)*x*(x + x^3)^(1/3) + 2^(2/3)*(x
 + x^3)^(2/3)])/(8*2^(1/3))
```

fricas [B] time = 2.11, size = 288, normalized size = 2.06

$$\frac{2\sqrt{3}\,2^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^3\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(24\sqrt{2}(-1)^{\frac{1}{3}}\left(2x^4+5x^2+2\right)\left(x^3+x\right)^{\frac{2}{3}}-12\cdot 2^{\frac{1}{6}}(-1)^{\frac{2}{3}}\left(19x^5+22x^3+4x\right)\left(x^3+x\right)^{\frac{1}{3}}-2^{\frac{5}{6}}\left(91x^6+168x^4+84x^2+8\right)\right)}{6\left(53x^6+48x^4-12x^2-8\right)}\right)-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^3\log\left(\frac{12\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^3+x\right)^{\frac{2}{3}}\left(2x^2+1\right)-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(19x^4+22x^2+4\right)+6\left(5x^3+4x\right)\left(x^3+x\right)^{\frac{1}{3}}}{x^4+4x^2+4}\right)+2\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^3\log\left(\frac{3\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3+x\right)^{\frac{1}{3}}x-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^2+2\right)+6\left(x^3+x\right)^{\frac{2}{3}}}{x^2+2}\right)-12\left(x^3+x\right)^{\frac{1}{3}}\left(2x^2-1\right)}{16x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)*(x^3+x)^(1/3)/x^4/(x^2+2),x, algorithm="fricas")
```

```
[Out] 1/16*(2*sqrt(3)*2^(2/3)*(-1)^(1/3)*x^3*arctan(1/6*sqrt(3)*2^(1/6)*(24*sqrt(
2)*(-1)^(1/3)*(2*x^4 + 5*x^2 + 2)*(x^3 + x)^(2/3) - 12*2^(1/6)*(-1)^(2/3)*(
19*x^5 + 22*x^3 + 4*x)*(x^3 + x)^(1/3) - 2^(5/6)*(91*x^6 + 168*x^4 + 84*x^2
 + 8))/(53*x^6 + 48*x^4 - 12*x^2 - 8)) - 2^(2/3)*(-1)^(1/3)*x^3*log((12*2^(
1/3)*(-1)^(2/3)*(x^3 + x)^(2/3)*(2*x^2 + 1) - 2^(2/3)*(-1)^(1/3)*(19*x^4 +
22*x^2 + 4) + 6*(5*x^3 + 4*x)*(x^3 + x)^(1/3))/(x^4 + 4*x^2 + 4)) + 2*2^(2/
3)*(-1)^(1/3)*x^3*log((3*2^(2/3)*(-1)^(1/3)*(x^3 + x)^(1/3)*x - 2^(1/3)*(-1
)^(2/3)*(x^2 + 2) + 6*(x^3 + x)^(2/3))/(x^2 + 2)) - 12*(x^3 + x)^(1/3)*(2*x
^2 - 1))/x^3
```

giac [A] time = 0.18, size = 97, normalized size = 0.69

$$\frac{3}{4}\sqrt{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{1}{2}\right)^{\frac{2}{3}}\left(\left(\frac{1}{2}\right)^{\frac{1}{3}}+2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right)\right)+\frac{3}{4}\left(\frac{1}{x^2}+1\right)^{\frac{4}{3}}+\frac{3}{16}\cdot 4^{\frac{1}{3}}\log\left(\left(\frac{1}{2}\right)^{\frac{2}{3}}+\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}\right)-\frac{3}{4}\left(\frac{1}{2}\right)^{\frac{1}{3}}\log\left(\left|-\left(\frac{1}{2}\right)^{\frac{1}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right|\right)-\frac{9}{4}\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)*(x^3+x)^(1/3)/x^4/(x^2+2),x, algorithm="giac")
```

```
[Out] 3/4*sqrt(3)*(1/2)^(1/3)*arctan(2/3*sqrt(3)*(1/2)^(2/3)*((1/2)^(1/3) + 2*(1/
x^2 + 1)^(1/3))) + 3/4*(1/x^2 + 1)^(4/3) + 3/16*4^(1/3)*log((1/2)^(2/3) + (
1/2)^(1/3)*(1/x^2 + 1)^(1/3) + (1/x^2 + 1)^(2/3)) - 3/4*(1/2)^(1/3)*log(abs
(-(1/2)^(1/3) + (1/x^2 + 1)^(1/3))) - 9/4*(1/x^2 + 1)^(1/3)
```

maple [C] time = 8.74, size = 1587, normalized size = 11.34

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-4)*(x^3+x)^(1/3)/x^4/(x^2+2),x)
```

```
[Out] -3/4*(2*x^4+x^2-1)/x^3*(x*(x^2+1))^(1/3)/(x^2+1)+(3/4*RootOf(RootOf(_Z^3+4)
^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*ln(-(3*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_
Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x^4+4*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_
Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x^4+9*(x^6+2*x^4+x^2)^(2/3)*RootOf(_Z^3+4
)^2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)-24*RootOf(_Z^3+4)^2
*(x^6+2*x^4+x^2)^(1/3)*x^2-18*RootOf(_Z^3+4)*RootOf(RootOf(_Z^3+4)^2+2*_Z*R
ootOf(_Z^3+4)+4*_Z^2)*(x^6+2*x^4+x^2)^(1/3)*x^2+33*RootOf(_Z^3+4)*x^4+44*Ro
otOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^4-3*RootOf(RootOf(_Z^3+
4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3-4*RootOf(RootOf(_Z^3+4)^2
+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2-24*RootOf(_Z^3+4)^2*(x^6+2*
x^4+x^2)^(1/3)-18*(x^6+2*x^4+x^2)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf
(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)+63*RootOf(_Z^3+4)*x^2+84*RootOf(RootOf(_Z^3
+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2-48*(x^6+2*x^4+x^2)^(2/3)+30*RootOf(_Z
^3+4)+40*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(x^2+1)/(x^2+
2))-3/8*ln((RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3
+4)^3*x^4-4*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z
^3+4)^2*x^4+9*(x^6+2*x^4+x^2)^(2/3)*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^
2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)+15*RootOf(_Z^3+4)^2*(x^6+2*x^4+x^2)^(1/3)*x^2
-18*RootOf(_Z^3+4)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*(x^6
+2*x^4+x^2)^(1/3)*x^2-13*RootOf(_Z^3+4)*x^4+52*RootOf(RootOf(_Z^3+4)^2+2*_Z
*RootOf(_Z^3+4)+4*_Z^2)*x^4-RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_
Z^2)*RootOf(_Z^3+4)^3+4*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)
^2*RootOf(_Z^3+4)^2+15*RootOf(_Z^3+4)^2*(x^6+2*x^4+x^2)^(1/3)-18*(x^6+2*x^4
+x^2)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3
+4)-21*RootOf(_Z^3+4)*x^2+84*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*
_Z^2)*x^2+30*(x^6+2*x^4+x^2)^(2/3)-8*RootOf(_Z^3+4)+32*RootOf(RootOf(_Z^3+4
)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(x^2+1)/(x^2+2))*RootOf(_Z^3+4)-3/4*ln((Ro
otOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x^4-4*Ro
otOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x^4+9*
(x^6+2*x^4+x^2)^(2/3)*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(
_Z^3+4)+4*_Z^2)+15*RootOf(_Z^3+4)^2*(x^6+2*x^4+x^2)^(1/3)*x^2-18*RootOf(_Z^
3+4)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*(x^6+2*x^4+x^2)^(1
/3)*x^2-13*RootOf(_Z^3+4)*x^4+52*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4
)+4*_Z^2)*x^4-RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z
^3+4)^3+4*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3
+4)^2+15*RootOf(_Z^3+4)^2*(x^6+2*x^4+x^2)^(1/3)-18*(x^6+2*x^4+x^2)^(1/3)*Ro
otOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)-21*RootOf(
_Z^3+4)*x^2+84*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2+30*(
x^6+2*x^4+x^2)^(2/3)-8*RootOf(_Z^3+4)+32*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootO
f(_Z^3+4)+4*_Z^2))/(x^2+1)/(x^2+2))*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^
3+4)+4*_Z^2))*(x*(x^2+1))^(1/3)/x*(x^2*(x^2+1)^2)^(1/3)/(x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3\left(18\,x^{5}+7\left(x^{3}+x\right)x^{2}+8\,x^{3}-10\,x\right)\left(x^{2}+1\right)^{\frac{1}{3}}}{56\left(x^{\frac{17}{3}}+2\,x^{\frac{11}{3}}\right)}+\int\frac{9\left(3\,x^{4}-x^{2}-4\right)\left(x^{2}+1\right)^{\frac{1}{3}}}{7\left(x^{\frac{23}{3}}+4\,x^{\frac{17}{3}}+4\,x^{\frac{11}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)*(x^3+x)^(1/3)/x^4/(x^2+2),x, algorithm="maxima")
```

```
[Out] -3/56*(18*x^5 + 7*(x^3 + x)*x^2 + 8*x^3 - 10*x)*(x^2 + 1)^(1/3)/(x^(17/3) +
 2*x^(11/3)) + integrate(9/7*(3*x^4 - x^2 - 4)*(x^2 + 1)^(1/3)/(x^(23/3) +
4*x^(17/3) + 4*x^(11/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(x^{2}-4\right)\,{\left(x^{3}+x\right)}^{1/3}}{x^{4}\,\left(x^{2}+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 4)*(x + x^3)^(1/3))/(x^4*(x^2 + 2)),x)
```

```
[Out] int(((x^2 - 4)*(x + x^3)^(1/3))/(x^4*(x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x\left(x^2+1\right)}\,(x-2)\,(x+2)}{x^4\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4)*(x**3+x)**(1/3)/x**4/(x**2+2),x)
```

```
[Out] Integral((x*(x**2 + 1))**(1/3)*(x - 2)*(x + 2)/(x**4*(x**2 + 2)), x)
```

3.1650 $\int \frac{x}{\left(b+ax^2\right)\sqrt{-bx+ax^3}}\,dx$

Optimal. Leaf size=140

$$\frac{\tanh^{-1}\left(\frac{\frac{a^{3/4}x^2}{2\sqrt[4]{b}}-\frac{b^{3/4}}{2\sqrt[4]{a}}+\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{ax^3-bx}}\right)}{4a^{3/4}b^{3/4}}-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3-bx}}{-2\sqrt{a}\sqrt{b}x+ax^2-b}\right)}{4a^{3/4}b^{3/4}}$$

Rubi [A] time = 0.56, antiderivative size = 181, normalized size of antiderivative = 1.29, number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2042, 466, 490, 1211, 224, 221, 1699, 208, 205}

$$\frac{\sqrt{x}\sqrt{ax^2-b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2-b}}\right)}{2\sqrt{2}(-a)^{3/4}b^{3/4}\sqrt{ax^3-bx}}-\frac{\sqrt{x}\sqrt{ax^2-b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2-b}}\right)}{2\sqrt{2}(-a)^{3/4}b^{3/4}\sqrt{ax^3-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((b + a*x^2)*Sqrt[-(b*x) + a*x^3]),x]
```

```
[Out] (Sqrt[x]*Sqrt[-b + a*x^2]*ArcTan[(Sqrt[2]*(-a)^(1/4)*b^(1/4)*Sqrt[x])/Sqrt[
-b + a*x^2]])/(2*Sqrt[2]*(-a)^(3/4)*b^(3/4)*Sqrt[-(b*x) + a*x^3]) - (Sqrt[x
]*Sqrt[-b + a*x^2]*ArcTanh[(Sqrt[2]*(-a)^(1/4)*b^(1/4)*Sqrt[x])/Sqrt[-b + a
*x^2]])/(2*Sqrt[2]*(-a)^(3/4)*b^(3/4)*Sqrt[-(b*x) + a*x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
```

```
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(b+ax^2\right)\sqrt{-bx+ax^3}}\,dx &= \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\int \frac{\sqrt{x}}{\sqrt{-b+ax^2}\left(b+ax^2\right)}\,dx}{\sqrt{-bx+ax^3}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-b+ax^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{-b+ax^4}\left(b+ax^4\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{-bx+ax^3}}\\
&= \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(\sqrt{b}-\sqrt{-a}\,x^2\right)\sqrt{-b+ax^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{-a}\sqrt{-bx+ax^3}} - \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\mathrm{S}\cdots}{\cdots}\\
&= -\frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{-a}\,x^2}{\left(\sqrt{b}+\sqrt{-a}\,x^2\right)\sqrt{-b+ax^4}}\,dx,x,\sqrt{x}\right)}{2\sqrt{-a}\sqrt{b}\sqrt{-bx+ax^3}} + \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\cdots}{\cdots}\\
&= -\frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b}-2\sqrt{-a}\,bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{-b+ax^2}}\right)}{2\sqrt{-a}\sqrt{-bx+ax^3}} + \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\mathrm{Su}\cdots}{2\cdots}\\
&= \frac{\sqrt{x}\sqrt{-b+ax^2}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a}\sqrt[4]{b}\sqrt{x}}{\sqrt{-b+ax^2}}\right)}{2\sqrt{2}\,(-a)^{3/4}b^{3/4}\sqrt{-bx+ax^3}} - \frac{\sqrt{x}\sqrt{-b+ax^2}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a}\sqrt[4]{b}\sqrt{x}}{\sqrt{-b+ax^2}}\right)}{2\sqrt{2}\,(-a)^{3/4}b^{3/4}\sqrt{-bx+ax^3}}
\end{aligned}
$$

Mathematica [C] time = 0.05, size = 68, normalized size = 0.49

$$
\frac{2x^2\sqrt{\frac{b-ax^2}{b}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{ax^2}{b},-\frac{ax^2}{b}\right)}{3b\sqrt{ax^3-bx}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((b + a*x^2)*Sqrt[-(b*x) + a*x^3]),x]
```

```
[Out] (2*x^2*Sqrt[(b - a*x^2)/b]*AppellF1[3/4, 1/2, 1, 7/4, (a*x^2)/b, -((a*x^2)/
b)])/(3*b*Sqrt[-(b*x) + a*x^3])
```

IntegrateAlgebraic [A] time = 0.45, size = 150, normalized size = 1.07

$$\frac{\tan^{-1}\left(\frac{\frac{a^{3/4}x^2}{2\sqrt[4]{b}}-\frac{b^{3/4}}{2\sqrt[4]{a}}-\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{ax^3-bx}}\right)}{4a^{3/4}b^{3/4}}+\frac{\tanh^{-1}\left(\frac{\frac{a^{3/4}x^2}{2\sqrt[4]{b}}-\frac{b^{3/4}}{2\sqrt[4]{a}}+\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{ax^3-bx}}\right)}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((b + a*x^2)*Sqrt[-(b*x) + a*x^3]),x]
```

```
[Out] ArcTan[(-1/2*b^(3/4)/a^(1/4) - a^(1/4)*b^(1/4)*x + (a^(3/4)*x^2)/(2*b^(1/4)
))/Sqrt[-(b*x) + a*x^3]]/(4*a^(3/4)*b^(3/4)) + ArcTanh[(-1/2*b^(3/4)/a^(1/4
) + a^(1/4)*b^(1/4)*x + (a^(3/4)*x^2)/(2*b^(1/4)))/Sqrt[-(b*x) + a*x^3]]/(4
*a^(3/4)*b^(3/4))
```

fricas [B] time = 0.53, size = 363, normalized size = 2.59

$$-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{ax^3-bx}\,ab\left(-\frac{1}{a^3b^3}\right)^{\frac{1}{4}}}{ax^2-b}\right)+\frac{1}{8}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\log\left(\frac{a^2x^4-6\,abx^2+b^2+4\left(4\left(\frac{1}{4}\right)^{\frac{3}{4}}a^3b^3x\left(-\frac{1}{a^3b^3}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(a^2bx^2-ab^2\right)\left(-\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\right)\sqrt{ax^3-bx}+4\left(a^3b^2x^3-a^2b^3x\right)\sqrt{-\frac{1}{a^3b^3}}}{a^2x^4+2\,abx^2+b^2}\right)-\frac{1}{8}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\log\left(\frac{a^2x^4-6\,abx^2+b^2-4\left(4\left(\frac{1}{4}\right)^{\frac{3}{4}}a^3b^3x\left(-\frac{1}{a^3b^3}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(a^2bx^2-ab^2\right)\left(-\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\right)\sqrt{ax^3-bx}+4\left(a^3b^2x^3-a^2b^3x\right)\sqrt{-\frac{1}{a^3b^3}}}{a^2x^4+2\,abx^2+b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^2+b)/(a*x^3-b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(1/4)^(1/4)*(-1/(a^3*b^3))^(1/4)*arctan(2*(1/4)^(1/4)*sqrt(a*x^3 - b*x
)*a*b*(-1/(a^3*b^3))^(1/4)/(a*x^2 - b)) + 1/8*(1/4)^(1/4)*(-1/(a^3*b^3))^(1
/4)*log((a^2*x^4 - 6*a*b*x^2 + b^2 + 4*(4*(1/4)^(3/4)*a^3*b^3*x*(-1/(a^3*b^
3))^(3/4) + (1/4)^(1/4)*(a^2*b*x^2 - a*b^2)*(-1/(a^3*b^3))^(1/4))*sqrt(a*x^
3 - b*x) + 4*(a^3*b^2*x^3 - a^2*b^3*x)*sqrt(-1/(a^3*b^3)))/(a^2*x^4 + 2*a*b
*x^2 + b^2)) - 1/8*(1/4)^(1/4)*(-1/(a^3*b^3))^(1/4)*log((a^2*x^4 - 6*a*b*x^
2 + b^2 - 4*(4*(1/4)^(3/4)*a^3*b^3*x*(-1/(a^3*b^3))^(3/4) + (1/4)^(1/4)*(a^
2*b*x^2 - a*b^2)*(-1/(a^3*b^3))^(1/4))*sqrt(a*x^3 - b*x) + 4*(a^3*b^2*x^3 -
 a^2*b^3*x)*sqrt(-1/(a^3*b^3)))/(a^2*x^4 + 2*a*b*x^2 + b^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{ax^3-bx}\left(ax^2+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^2+b)/(a*x^3-b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(a*x^3 - b*x)*(a*x^2 + b)), x)
```

maple [C] time = 0.05, size = 284, normalized size = 2.03

$$\frac{\sqrt{ab}\,\sqrt{\frac{xa}{\sqrt{ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},-\frac{\sqrt{ab}}{a\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{2a^2\sqrt{a\,x^3-bx}\,\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)}+\frac{\sqrt{ab}\,\sqrt{\frac{xa}{\sqrt{ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},-\frac{\sqrt{ab}}{a\left(-\frac{\sqrt{ab}}{a}+\frac{\sqrt{-ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{2a^2\sqrt{a\,x^3-bx}\,\left(-\frac{\sqrt{ab}}{a}+\frac{\sqrt{-ab}}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2+b)/(a*x^3-b*x)^(1/2),x)
```

```
[Out] 1/2/a^2*(a*b)^(1/2)*(x*a/(a*b)^(1/2)+1)^(1/2)*(-2*x*a/(a*b)^(1/2)+2)^(1/2)*
(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/
2))*EllipticPi(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2),-1/a*(a*b)^(1/2)/(
-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2)),1/2*2^(1/2))+1/2/a^2*(a*b)^(1/2)*(x*a/(a
*b)^(1/2)+1)^(1/2)*(-2*x*a/(a*b)^(1/2)+2)^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a
*x^3-b*x)^(1/2)/(-1/a*(a*b)^(1/2)+1/a*(-a*b)^(1/2))*EllipticPi(((x+1/a*(a*b
)^(1/2))*a/(a*b)^(1/2))^(1/2),-1/a*(a*b)^(1/2)/(-1/a*(a*b)^(1/2)+1/a*(-a*b)
^(1/2)),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{ax^3 - bx}\left(ax^2 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^2+b)/(a*x^3-b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(a*x^3 - b*x)*(a*x^2 + b)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x^3 - b*x)^(1/2)*(b + a*x^2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x\left(ax^2 - b\right)}\left(ax^2 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x**2+b)/(a*x**3-b*x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(x*(a*x**2 - b))*(a*x**2 + b)), x)
```

3.1651 $\int \frac{-b+ax^2}{(b+ax^2)\sqrt{-bx+ax^3}}\,dx$

Optimal. Leaf size=140

$$-\frac{\tanh^{-1}\left(\frac{\frac{a^{3/4}x^2}{2\sqrt[4]{b}}-\frac{b^{3/4}}{2\sqrt[4]{a}}+\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{ax^3-bx}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}}-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3-bx}}{-2\sqrt{a}\sqrt{b}\,x+ax^2-b}\right)}{2\sqrt[4]{a}\sqrt[4]{b}}$$

Rubi [A] time = 0.22, antiderivative size = 176, normalized size of antiderivative = 1.26, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2056, 466, 405}

$$-\frac{\sqrt{x}\sqrt{ax^2-b}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{b}-\sqrt{a}x\right)}{\sqrt[4]{b}\sqrt{ax^2-b}}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3-bx}}-\frac{\sqrt{x}\sqrt{ax^2-b}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}x+\sqrt{b}\right)}{\sqrt[4]{b}\sqrt{ax^2-b}}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^2)/((b + a*x^2)*Sqrt[-(b*x) + a*x^3]),x]
```

```
[Out] -((Sqrt[x]*Sqrt[-b + a*x^2]*ArcTan[(a^(1/4)*Sqrt[x]*(Sqrt[b] - Sqrt[a]*x))/
(b^(1/4)*Sqrt[-b + a*x^2])])/(a^(1/4)*b^(1/4)*Sqrt[-(b*x) + a*x^3])) - (Sqr
t[x]*Sqrt[-b + a*x^2]*ArcTanh[(a^(1/4)*Sqrt[x]*(Sqrt[b] + Sqrt[a]*x))/(b^(1
/4)*Sqrt[-b + a*x^2])])/(a^(1/4)*b^(1/4)*Sqrt[-(b*x) + a*x^3])
```

Rule 405

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[-(a*b), 4]}, Simp[(a*ArcTan[(q*x*(a + q^2*x^2))/(a*Sqrt[a + b*x^4])])/(
2*c*q), x] + Simp[(a*ArcTanh[(q*x*(a - q^2*x^2))/(a*Sqrt[a + b*x^4])])/(2*c
*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && NegQ[a*b]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{-b+ax^2}{\left(b+ax^2\right)\sqrt{-bx+ax^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\int \frac{\sqrt{-b+ax^2}}{\sqrt{x}\left(b+ax^2\right)}\,dx}{\sqrt{-bx+ax^3}}$$
$$= \frac{\left(2\sqrt{x}\sqrt{-b+ax^2}\right)\text{Subst}\left(\int \frac{\sqrt{-b+ax^4}}{b+ax^4}\,dx, x, \sqrt{x}\right)}{\sqrt{-bx+ax^3}}$$
$$= -\frac{\sqrt{x}\sqrt{-b+ax^2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{b}-\sqrt{a}\,x\right)}{\sqrt[4]{b}\sqrt{-b+ax^2}}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{-bx+ax^3}} - \frac{\sqrt{x}\sqrt{-b+ax^2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{b}+\sqrt{\ldots}\right.}{\sqrt[4]{b}\sqrt{-b+ax\ldots}}\right.}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{-bx+ax^3}}$$

Mathematica [C] time = 0.05, size = 62, normalized size = 0.44

$$\frac{2\sqrt{ax^3-bx}\,F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{ax^2}{b},-\frac{ax^2}{b}\right)}{b\sqrt{1-\frac{ax^2}{b}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-b + a*x^2)/((b + a*x^2)*Sqrt[-(b*x) + a*x^3]),x]
```

```
[Out] (2*Sqrt[-(b*x) + a*x^3]*AppellF1[1/4, -1/2, 1, 5/4, (a*x^2)/b, -((a*x^2)/b)
])/(b*Sqrt[1 - (a*x^2)/b])
```

IntegrateAlgebraic [A] time = 0.42, size = 150, normalized size = 1.07

$$\frac{\tan^{-1}\left(\frac{\frac{a^{3/4}x^2}{2\sqrt[4]{b}}-\frac{b^{3/4}}{2\sqrt[4]{a}}-\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{ax^3-bx}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}} - \frac{\tanh^{-1}\left(\frac{\frac{a^{3/4}x^2}{2\sqrt[4]{b}}-\frac{b^{3/4}}{2\sqrt[4]{a}}+\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{ax^3-bx}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^2)/((b + a*x^2)*Sqrt[-(b*x) + a*x^3]),x]
```

```
[Out] ArcTan[(-1/2*b^(3/4)/a^(1/4) - a^(1/4)*b^(1/4)*x + (a^(3/4)*x^2)/(2*b^(1/4)
))/Sqrt[-(b*x) + a*x^3]]/(2*a^(1/4)*b^(1/4)) - ArcTanh[(-1/2*b^(3/4)/a^(1/4
) + a^(1/4)*b^(1/4)*x + (a^(3/4)*x^2)/(2*b^(1/4)))/Sqrt[-(b*x) + a*x^3]]/(2
*a^(1/4)*b^(1/4))
```

fricas [B] time = 0.53, size = 344, normalized size = 2.46

$$\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab}\right)^{\frac{1}{4}}\arctan\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{ax^3-bx}\,ab\left(-\frac{1}{ab}\right)^{\frac{3}{4}}}{ax^2-b}\right)+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab}\right)^{\frac{1}{4}}\log\left(\frac{a^2x^4-6abx^2+b^2+8\sqrt{ax^3-bx}\left(\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{ab}\right)^{\frac{1}{4}}+\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(a^2bx^2-ab^2\right)\left(-\frac{1}{ab}\right)^{\frac{3}{4}}\right)-4\left(a^2bx^3-ab^2x\right)\sqrt{-\frac{1}{ab}}}{a^2x^4+2abx^2+b^2}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab}\right)^{\frac{1}{4}}\log\left(\frac{a^2x^4-6abx^2+b^2-8\sqrt{ax^3-bx}\left(\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{ab}\right)^{\frac{1}{4}}+\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(a^2bx^2-ab^2\right)\left(-\frac{1}{ab}\right)^{\frac{3}{4}}\right)-4\left(a^2bx^3-ab^2x\right)\sqrt{-\frac{1}{ab}}}{a^2x^4+2abx^2+b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(a*x^2+b)/(a*x^3-b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] (1/4)^(1/4)*(-1/(a*b))^(1/4)*arctan(4*(1/4)^(3/4)*sqrt(a*x^3 - b*x)*a*b*(-1
/(a*b))^(3/4)/(a*x^2 - b)) + 1/4*(1/4)^(1/4)*(-1/(a*b))^(1/4)*log((a^2*x^4
- 6*a*b*x^2 + b^2 + 8*sqrt(a*x^3 - b*x)*((1/4)^(1/4)*a*b*x*(-1/(a*b))^(1/4)
 + (1/4)^(3/4)*(a^2*b*x^2 - a*b^2)*(-1/(a*b))^(3/4)) - 4*(a^2*b*x^3 - a*b^2
*x)*sqrt(-1/(a*b)))/(a^2*x^4 + 2*a*b*x^2 + b^2)) - 1/4*(1/4)^(1/4)*(-1/(a*b
))^(1/4)*log((a^2*x^4 - 6*a*b*x^2 + b^2 - 8*sqrt(a*x^3 - b*x)*((1/4)^(1/4)*
a*b*x*(-1/(a*b))^(1/4) + (1/4)^(3/4)*(a^2*b*x^2 - a*b^2)*(-1/(a*b))^(3/4))
- 4*(a^2*b*x^3 - a*b^2*x)*sqrt(-1/(a*b)))/(a^2*x^4 + 2*a*b*x^2 + b^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2-b}{\sqrt{ax^3-bx}\left(ax^2+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(a*x^2+b)/(a*x^3-b*x)^(1/2),x, algorithm="giac")
[Out] integrate((a*x^2 - b)/(sqrt(a*x^3 - b*x)*(a*x^2 + b)), x)
```

maple [C] time = 0.04, size = 400, normalized size = 2.86

$$\frac{\sqrt{ab}\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}}\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a x^3-bx}}-2b\left(\frac{\sqrt{ab}\sqrt{\frac{xa}{\sqrt{ab}}+1}\sqrt{-\frac{2xa}{\sqrt{ab}}+2}\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},-\frac{\sqrt{ab}}{a\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{2\sqrt{-ab}\,a\sqrt{a x^3-bx}\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)}-\frac{\sqrt{ab}\sqrt{\frac{xa}{\sqrt{ab}}+1}\sqrt{-\frac{2xa}{\sqrt{ab}}+2}\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},-\frac{\sqrt{ab}}{a\left(-\frac{\sqrt{ab}}{a}+\frac{\sqrt{-ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{2\sqrt{-ab}\,a\sqrt{a x^3-bx}\left(-\frac{\sqrt{ab}}{a}+\frac{\sqrt{-ab}}{a}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)/(a*x^2+b)/(a*x^3-b*x)^(1/2),x)
[Out] 1/a*(a*b)^(1/2)*((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2)*(-2*(x-1/a*(a*b)^
(1/2))*a/(a*b)^(1/2))^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)*Elli
pticF(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2),1/2*2^(1/2))-2*b*(1/2/(-a*b
)^(1/2)/a*(a*b)^(1/2)*(x*a/(a*b)^(1/2)+1)^(1/2)*(-2*x*a/(a*b)^(1/2)+2)^(1/2
)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(
1/2))*EllipticPi(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2),-1/a*(a*b)^(1/2)
/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2)),1/2*2^(1/2))-1/2/(-a*b)^(1/2)/a*(a*b)^
(1/2)*(x*a/(a*b)^(1/2)+1)^(1/2)*(-2*x*a/(a*b)^(1/2)+2)^(1/2)*(-x*a/(a*b)^(1
/2))^(1/2)/(a*x^3-b*x)^(1/2)/(-1/a*(a*b)^(1/2)+1/a*(-a*b)^(1/2))*EllipticPi
(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2),-1/a*(a*b)^(1/2)/(-1/a*(a*b)^(1/
2)+1/a*(-a*b)^(1/2)),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2-b}{\sqrt{ax^3-bx}\left(ax^2+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(a*x^2+b)/(a*x^3-b*x)^(1/2),x, algorithm="maxima")
[Out] integrate((a*x^2 - b)/(sqrt(a*x^3 - b*x)*(a*x^2 + b)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^2)/((a*x^3 - b*x)^(1/2)*(b + a*x^2)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2-b}{\sqrt{x\left(ax^2-b\right)}\left(ax^2+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)/(a*x**2+b)/(a*x**3-b*x)**(1/2),x)
[Out] Integral((a*x**2 - b)/(sqrt(x*(a*x**2 - b))*(a*x**2 + b)), x)
```

3.1652 $\int \frac{\sqrt{-bx+ax^3}}{-b^2+a^2x^4}\,dx$

Optimal. Leaf size=140

$$\frac{\tanh^{-1}\left(\frac{\frac{a^{3/4}x^2}{2\sqrt[4]{b}}-\frac{b^{3/4}}{2\sqrt[4]{a}}+\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{ax^3-bx}}\right)}{4a^{3/4}b^{3/4}}-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3-bx}}{-2\sqrt{a}\sqrt{b}\,x+ax^2-b}\right)}{4a^{3/4}b^{3/4}}$$

Rubi [A] time = 0.44, antiderivative size = 181, normalized size of antiderivative = 1.29, number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2056, 1254, 466, 490, 1211, 224, 221, 1699, 208, 205}

$$\frac{\sqrt{ax^3-bx}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2-b}}\right)}{2\sqrt{2}\,(-a)^{3/4}b^{3/4}\sqrt{x}\sqrt{ax^2-b}}-\frac{\sqrt{ax^3-bx}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2-b}}\right)}{2\sqrt{2}\,(-a)^{3/4}b^{3/4}\sqrt{x}\sqrt{ax^2-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-(b*x) + a*x^3]/(-b^2 + a^2*x^4),x]
```

```
[Out] (Sqrt[-(b*x) + a*x^3]*ArcTan[(Sqrt[2]*(-a)^(1/4)*b^(1/4)*Sqrt[x])/Sqrt[-b +
 a*x^2]])/(2*Sqrt[2]*(-a)^(3/4)*b^(3/4)*Sqrt[x]*Sqrt[-b + a*x^2]) - (Sqrt[-
(b*x) + a*x^3]*ArcTanh[(Sqrt[2]*(-a)^(1/4)*b^(1/4)*Sqrt[x])/Sqrt[-b + a*x^2
]])/(2*Sqrt[2]*(-a)^(3/4)*b^(3/4)*Sqrt[x]*Sqrt[-b + a*x^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
```

```
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1254

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e)^p, x] /
; FreeQ[{a, c, d, e, f, q, m, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p
]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\sqrt{-bx+ax^3}}{-b^2+a^2x^4}\,dx = \frac{\sqrt{-bx+ax^3}\int \frac{\sqrt{x}\sqrt{-b+ax^2}}{-b^2+a^2x^4}\,dx}{\sqrt{x}\sqrt{-b+ax^2}}$$

$$= \frac{\sqrt{-bx+ax^3}\int \frac{\sqrt{x}}{\sqrt{-b+ax^2}\left(b+ax^2\right)}\,dx}{\sqrt{x}\sqrt{-b+ax^2}}$$

$$= \frac{\left(2\sqrt{-bx+ax^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-b+ax^4}\left(b+ax^4\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{-b+ax^2}}$$

$$= \frac{\sqrt{-bx+ax^3}\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{b}-\sqrt{-a}\,x^2\right)\sqrt{-b+ax^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a}\sqrt{x}\sqrt{-b+ax^2}} - \frac{\sqrt{-bx+ax^3}\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{b}+\sqrt{-a}\,x^2\right)\ldots}\right.}{\sqrt{-a}\sqrt{x}\sqrt{-b+\ldots}}$$

$$= -\frac{\sqrt{-bx+ax^3}\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{-a}\,x^2}{\left(\sqrt{b}+\sqrt{-a}\,x^2\right)\sqrt{-b+ax^4}}\,dx, x, \sqrt{x}\right)}{2\sqrt{-a}\sqrt{b}\sqrt{x}\sqrt{-b+ax^2}} + \frac{\sqrt{-bx+ax^3}\,\text{Subst}\left(\int \frac{\sqrt{b}+\ldots}{\left(\sqrt{b}-\sqrt{-a}\ldots\right.}\right.}{2\sqrt{-a}\sqrt{b}\sqrt{x}\sqrt{-\ldots}}$$

$$= -\frac{\sqrt{-bx+ax^3}\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-2\sqrt{-a}\,bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{-b+ax^2}}\right)}{2\sqrt{-a}\sqrt{x}\sqrt{-b+ax^2}} + \frac{\sqrt{-bx+ax^3}\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+2\sqrt{-a}\,bx^2}\ldots\right.}{2\sqrt{-a}\sqrt{x}\sqrt{-b+ax\ldots}}$$

$$= \frac{\sqrt{-bx+ax^3}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a}\sqrt[4]{b}\sqrt{x}}{\sqrt{-b+ax^2}}\right)}{2\sqrt{2}\,(-a)^{3/4}b^{3/4}\sqrt{x}\sqrt{-b+ax^2}} - \frac{\sqrt{-bx+ax^3}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a}\sqrt[4]{b}\sqrt{x}}{\sqrt{-b+ax^2}}\right)}{2\sqrt{2}\,(-a)^{3/4}b^{3/4}\sqrt{x}\sqrt{-b+ax^2}}$$

Mathematica [C] time = 1.03, size = 121, normalized size = 0.86

$$\frac{\sqrt{-\frac{\sqrt{a}\,x}{\sqrt{b}}}\sqrt{1-\frac{ax^2}{b}}\sqrt{ax^3-bx}\left(\Pi\left(i; i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{a}\,x}{\sqrt{b}}}\right)\middle|-1\right)-\Pi\left(-i; i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{a}\,x}{\sqrt{b}}}\right)\middle|-1\right)\right)}{ax\left(ax^2-b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(b*x) + a*x^3]/(-b^2 + a^2*x^4),x]
```

```
[Out] (Sqrt[-((Sqrt[a]*x)/Sqrt[b])]*Sqrt[1 - (a*x^2)/b]*Sqrt[-(b*x) + a*x^3]*(-El
lipticPi[-I, I*ArcSinh[Sqrt[-((Sqrt[a]*x)/Sqrt[b])]], -1] + EllipticPi[I, I
*ArcSinh[Sqrt[-((Sqrt[a]*x)/Sqrt[b])]], -1]))/(a*x*(-b + a*x^2))
```

IntegrateAlgebraic [A] time = 0.42, size = 150, normalized size = 1.07

$$\frac{\tan^{-1}\left(\frac{\frac{a^{3/4}x^2}{2\sqrt[4]{b}}-\frac{b^{3/4}}{2\sqrt[4]{a}}-\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{ax^3-bx}}\right)}{4a^{3/4}b^{3/4}} + \frac{\tanh^{-1}\left(\frac{\frac{a^{3/4}x^2}{2\sqrt[4]{b}}-\frac{b^{3/4}}{2\sqrt[4]{a}}+\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{ax^3-bx}}\right)}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-(b*x) + a*x^3]/(-b^2 + a^2*x^4),x]
```

```
[Out] ArcTan[(-1/2*b^(3/4)/a^(1/4) - a^(1/4)*b^(1/4)*x + (a^(3/4)*x^2)/(2*b^(1/4)
))/Sqrt[-(b*x) + a*x^3]]/(4*a^(3/4)*b^(3/4)) + ArcTanh[(-1/2*b^(3/4)/a^(1/4
) + a^(1/4)*b^(1/4)*x + (a^(3/4)*x^2)/(2*b^(1/4)))/Sqrt[-(b*x) + a*x^3]]/(4
*a^(3/4)*b^(3/4))
```

fricas [B] time = 0.53, size = 363, normalized size = 2.59

$$-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{ax^3-bx}\,ab\left(-\frac{1}{a^3b^3}\right)^{\frac{1}{4}}}{ax^2-b}\right)+\frac{1}{8}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\log\left(\frac{a^2x^4-6abx^2+b^2+4\left(4\left(\frac{1}{4}\right)^{\frac{3}{4}}a^3b^3x\left(-\frac{1}{a^3b^3}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(a^2bx^2-ab^2\right)\left(-\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\right)\sqrt{ax^3-bx}+4\left(a^3b^2x^3-a^2b^3x\right)\sqrt{-\frac{1}{a^3b^3}}}{a^2x^4+2abx^2+b^2}\right)-\frac{1}{8}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\log\left(\frac{a^2x^4-6abx^2+b^2-4\left(4\left(\frac{1}{4}\right)^{\frac{3}{4}}a^3b^3x\left(-\frac{1}{a^3b^3}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(a^2bx^2-ab^2\right)\left(-\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\right)\sqrt{ax^3-bx}+4\left(a^3b^2x^3-a^2b^3x\right)\sqrt{-\frac{1}{a^3b^3}}}{a^2x^4+2abx^2+b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b*x)^(1/2)/(a^2*x^4-b^2),x, algorithm="fricas")
```

```
[Out] -1/2*(1/4)^(1/4)*(-1/(a^3*b^3))^(1/4)*arctan(2*(1/4)^(1/4)*sqrt(a*x^3 - b*x
)*a*b*(-1/(a^3*b^3))^(1/4)/(a*x^2 - b)) + 1/8*(1/4)^(1/4)*(-1/(a^3*b^3))^(1
/4)*log((a^2*x^4 - 6*a*b*x^2 + b^2 + 4*(4*(1/4)^(3/4)*a^3*b^3*x*(-1/(a^3*b^
3))^(3/4) + (1/4)^(1/4)*(a^2*b*x^2 - a*b^2)*(-1/(a^3*b^3))^(1/4))*sqrt(a*x^
3 - b*x) + 4*(a^3*b^2*x^3 - a^2*b^3*x)*sqrt(-1/(a^3*b^3)))/(a^2*x^4 + 2*a*b
*x^2 + b^2)) - 1/8*(1/4)^(1/4)*(-1/(a^3*b^3))^(1/4)*log((a^2*x^4 - 6*a*b*x^
2 + b^2 - 4*(4*(1/4)^(3/4)*a^3*b^3*x*(-1/(a^3*b^3))^(3/4) + (1/4)^(1/4)*(a^
2*b*x^2 - a*b^2)*(-1/(a^3*b^3))^(1/4))*sqrt(a*x^3 - b*x) + 4*(a^3*b^2*x^3 -
 a^2*b^3*x)*sqrt(-1/(a^3*b^3)))/(a^2*x^4 + 2*a*b*x^2 + b^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3 - bx}}{a^2x^4 - b^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b*x)^(1/2)/(a^2*x^4-b^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^3 - b*x)/(a^2*x^4 - b^2), x)
```

maple [C] time = 0.05, size = 601, normalized size = 4.29

$$\frac{\sqrt{ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}}\,\sqrt{-\frac{xa}{\sqrt{ab}}}\left(-\frac{2\sqrt{ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)}{a}+\frac{\sqrt{ab}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)}{a}\right)}{2ba\sqrt{a x^3-bx}}-\frac{-\frac{2b\sqrt{\frac{xa}{\sqrt{ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a x^3-bx}}+\frac{b\sqrt{\frac{xa}{\sqrt{ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a x^3-bx}}-\frac{b\sqrt{ab}\,\sqrt{\frac{xa}{\sqrt{ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},-\frac{\sqrt{ab}}{a\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{a^2\sqrt{a x^3-bx}\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)}-\frac{b\sqrt{ab}\,\sqrt{\frac{xa}{\sqrt{ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},-\frac{\sqrt{ab}}{a\left(-\frac{\sqrt{ab}}{a}+\frac{\sqrt{-ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{a^2\sqrt{a x^3-bx}\left(-\frac{\sqrt{ab}}{a}+\frac{\sqrt{-ab}}{a}\right)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b*x)^(1/2)/(a^2*x^4-b^2),x)
```

```
[Out] 1/2/b/a*(a*b)^(1/2)*((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2)*(-2*(x-1/a*(a
*b)^(1/2))*a/(a*b)^(1/2))^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)*
(-2/a*(a*b)^(1/2)*EllipticE(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2),1/2*2
^(1/2))+1/a*(a*b)^(1/2)*EllipticF(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2)
,1/2*2^(1/2)))-1/2/b*(-2/a*b*(x*a/(a*b)^(1/2)+1)^(1/2)*(-2*x*a/(a*b)^(1/2)+
2)^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)*EllipticE(((x+1/a*(a*b)
^(1/2))*a/(a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*b*(x*a/(a*b)^(1/2)+1)^(1/2)*(
-2*x*a/(a*b)^(1/2)+2)^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)*Elli
pticF(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2),1/2*2^(1/2))-b/a^2*(a*b)^(1
/2)*(x*a/(a*b)^(1/2)+1)^(1/2)*(-2*x*a/(a*b)^(1/2)+2)^(1/2)*(-x*a/(a*b)^(1/2
))^(1/2)/(a*x^3-b*x)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2))*EllipticPi((
(x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2),-1/a*(a*b)^(1/2)/(-1/a*(-a*b)^(1/2
)-1/a*(a*b)^(1/2)),1/2*2^(1/2))-b/a^2*(a*b)^(1/2)*(x*a/(a*b)^(1/2)+1)^(1/2)
*(-2*x*a/(a*b)^(1/2)+2)^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)/(-
1/a*(a*b)^(1/2)+1/a*(-a*b)^(1/2))*EllipticPi(((x+1/a*(a*b)^(1/2))*a/(a*b)^(
1/2))^(1/2),-1/a*(a*b)^(1/2)/(-1/a*(a*b)^(1/2)+1/a*(-a*b)^(1/2)),1/2*2^(1/2
)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3 - bx}}{a^2x^4 - b^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b*x)^(1/2)/(a^2*x^4-b^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^3 - b*x)/(a^2*x^4 - b^2), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x^3 - b*x)^(1/2)/(b^2 - a^2*x^4),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(ax^2-b\right)}}{\left(ax^2-b\right)\left(ax^2+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b*x)**(1/2)/(a**2*x**4-b**2),x)
[Out] Integral(sqrt(x*(a*x**2 - b))/((a*x**2 - b)*(a*x**2 + b)), x)
```

3.1653 $\int \frac{(1+x^5)^{2/3}(-3+2x^5)(2+x^3+2x^5)}{x^6(2-x^3+2x^5)}\,dx$

Optimal. Leaf size=140

$$\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^5+1}-x\right)}{2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^5+1}+x}\right)}{2^{2/3}}+\frac{3\left(x^5+1\right)^{2/3}\left(2x^5+5x^3+2\right)}{10x^5}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^5+1}x+2^{2/3}\left(x^5+\right.\right.}{2\ 2^{2/3}}$$

Rubi [F] time = 1.29, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+x^5)^{2/3}(-3+2x^5)(2+x^3+2x^5)}{x^6(2-x^3+2x^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^5)^(2/3)*(-3 + 2*x^5)*(2 + x^3 + 2*x^5))/(x^6*(2 - x^3 + 2*x^5)
),x]
```

```
[Out] (3*(1 + x^5)^(2/3))/5 + (3*(1 + x^5)^(2/3))/(5*x^5) + (3*Hypergeometric2F1[
-2/3, -2/5, 3/5, -x^5])/(2*x^2) - 3*Defer[Int][(1 + x^5)^(2/3)/(2 - x^3 + 2
*x^5), x] + 10*Defer[Int][(x^2*(1 + x^5)^(2/3))/(2 - x^3 + 2*x^5), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^5)^{2/3}(-3+2x^5)(2+x^3+2x^5)}{x^6(2-x^3+2x^5)}\,dx &= \int \left(-\frac{3(1+x^5)^{2/3}}{x^6}-\frac{3(1+x^5)^{2/3}}{x^3}+\frac{2(1+x^5)^{2/3}}{x}+\frac{(-3+10x^2)(1}{2-x^3+}\right.\\
&= 2\int \frac{(1+x^5)^{2/3}}{x}\,dx-3\int \frac{(1+x^5)^{2/3}}{x^6}\,dx-3\int \frac{(1+x^5)^{2/3}}{x^3}\,dx+\int\\
&= \frac{3\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{5};\frac{3}{5};-x^5\right)}{2x^2}+\frac{2}{5}\text{Subst}\left(\int \frac{(1+x)^{2/3}}{x}\,dx,x,x^5\right)-\frac{3}{5}\text{Subs}\\
&= \frac{3}{5}(1+x^5)^{2/3}+\frac{3(1+x^5)^{2/3}}{5x^5}+\frac{3\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{5};\frac{3}{5};-x^5\right)}{2x^2}-3\int \frac{(1+}{2-x^3}
\end{aligned}$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(1+x^5)^{2/3}(-3+2x^5)(2+x^3+2x^5)}{x^6(2-x^3+2x^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^5)^(2/3)*(-3 + 2*x^5)*(2 + x^3 + 2*x^5))/(x^6*(2 - x^3 +
2*x^5)),x]
```

```
[Out] Integrate[((1 + x^5)^(2/3)*(-3 + 2*x^5)*(2 + x^3 + 2*x^5))/(x^6*(2 - x^3 +
2*x^5)), x]
```

IntegrateAlgebraic [A] time = 3.57, size = 140, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^5+1}-x\right)}{2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^5+1}+x}\right)}{2^{2/3}}+\frac{3\left(x^5+1\right)^{2/3}\left(2x^5+5x^3+2\right)}{10x^5}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^5+1}x+2^{2/3}\left(x^5+1\right)^{2/3}+x^2\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^5)^(2/3)*(-3 + 2*x^5)*(2 + x^3 + 2*x^5))/(x^6*(2
 - x^3 + 2*x^5)),x]
```

```
[Out] (3*(1 + x^5)^(2/3)*(2 + 5*x^3 + 2*x^5))/(10*x^5) - (Sqrt[3]*ArcTan[(Sqrt[3]
*x)/(x + 2*2^(1/3)*(1 + x^5)^(1/3))])/2^(2/3) + Log[-x + 2^(1/3)*(1 + x^5)^
(1/3)]/2^(2/3) - Log[x^2 + 2^(1/3)*x*(1 + x^5)^(1/3) + 2^(2/3)*(1 + x^5)^(2
/3)]/(2*2^(2/3))
```

fricas [B] time = 174.90, size = 399, normalized size = 2.85

$$-\frac{20 \cdot 4^{\frac{1}{6}} \sqrt{3} x^5 \arctan\left(\frac{4^{\frac{1}{6}} \sqrt{3}\left(12 \cdot 4^{\frac{2}{3}}\left(2 x^{11}+x^9-x^7+4 x^6+x^4+2 x\right)\left(x^5+1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(8 x^{15}+60 x^{13}+24 x^{11}+24 x^{10}-x^9+120 x^8+24 x^6+24 x^5+60 x^3+8\right)+12\left(4 x^{12}+14 x^{10}+x^8+8 x^7+14 x^5+4 x^2\right)\left(x^5+1\right)^{\frac{1}{3}}\right)}{6\left(8 x^{15}-12 x^{13}-48 x^{11}+24 x^{10}-x^9-24 x^8-48 x^6+24 x^5-12 x^3+8\right)}\right) - 10 \cdot 4^{\frac{2}{3}} x^5 \log\left(\frac{6 \cdot 4^{\frac{1}{3}}\left(x^5+1\right)^{\frac{1}{3}} x^2+4^{\frac{2}{3}}\left(2 x^5-x^3+2\right)-12\left(x^5+1\right)^{\frac{2}{3}} x}{2 x^5-x^3+2}\right) + 5 \cdot 4^{\frac{2}{3}} x^5 \log\left(\frac{6 \cdot 4^{\frac{2}{3}}\left(x^6+x^4+x\right)\left(x^5+1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(4 x^{10}+14 x^8+x^6+8 x^5+14 x^3+4\right)+6\left(4 x^7+x^5+4 x^2\right)\left(x^5+1\right)^{\frac{1}{3}}}{4 x^{10}-4 x^8+x^6+8 x^5-4 x^3+4}\right) - 36\left(2 x^5+5 x^3+2\right)\left(x^5+1\right)^{\frac{2}{3}}}{120 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)*(2*x^5-3)*(2*x^5+x^3+2)/x^6/(2*x^5-x^3+2),x, algori
thm="fricas")
```

```
[Out] -1/120*(20*4^(1/6)*sqrt(3)*x^5*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^(2/3)*(2*x^
11 + x^9 - x^7 + 4*x^6 + x^4 + 2*x)*(x^5 + 1)^(2/3) + 4^(1/3)*(8*x^15 + 60*
x^13 + 24*x^11 + 24*x^10 - x^9 + 120*x^8 + 24*x^6 + 24*x^5 + 60*x^3 + 8) +
12*(4*x^12 + 14*x^10 + x^8 + 8*x^7 + 14*x^5 + 4*x^2)*(x^5 + 1)^(1/3))/(8*x^
15 - 12*x^13 - 48*x^11 + 24*x^10 - x^9 - 24*x^8 - 48*x^6 + 24*x^5 - 12*x^3
+ 8)) - 10*4^(2/3)*x^5*log((6*4^(1/3)*(x^5 + 1)^(1/3)*x^2 + 4^(2/3)*(2*x^5
- x^3 + 2) - 12*(x^5 + 1)^(2/3)*x)/(2*x^5 - x^3 + 2)) + 5*4^(2/3)*x^5*log((
6*4^(2/3)*(x^6 + x^4 + x)*(x^5 + 1)^(2/3) + 4^(1/3)*(4*x^10 + 14*x^8 + x^6
+ 8*x^5 + 14*x^3 + 4) + 6*(4*x^7 + x^5 + 4*x^2)*(x^5 + 1)^(1/3))/(4*x^10 -
4*x^8 + x^6 + 8*x^5 - 4*x^3 + 4)) - 36*(2*x^5 + 5*x^3 + 2)*(x^5 + 1)^(2/3))
/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 x^5+x^3+2\right)\left(2 x^5-3\right)\left(x^5+1\right)^{\frac{2}{3}}}{\left(2 x^5-x^3+2\right)x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)*(2*x^5-3)*(2*x^5+x^3+2)/x^6/(2*x^5-x^3+2),x, algori
thm="giac")
```

```
[Out] integrate((2*x^5 + x^3 + 2)*(2*x^5 - 3)*(x^5 + 1)^(2/3)/((2*x^5 - x^3 + 2)*
x^6), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5+1\right)^{\frac{2}{3}}\left(2x^5-3\right)\left(2x^5+x^3+2\right)}{x^6\left(2x^5-x^3+2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)^(2/3)*(2*x^5-3)*(2*x^5+x^3+2)/x^6/(2*x^5-x^3+2),x)
```

```
[Out] int((x^5+1)^(2/3)*(2*x^5-3)*(2*x^5+x^3+2)/x^6/(2*x^5-x^3+2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 x^5+x^3+2\right)\left(2 x^5-3\right)\left(x^5+1\right)^{\frac{2}{3}}}{\left(2 x^5-x^3+2\right)x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(2/3)*(2*x^5-3)*(2*x^5+x^3+2)/x^6/(2*x^5-x^3+2),x, algori
thm="maxima")
```

```
[Out] integrate((2*x^5 + x^3 + 2)*(2*x^5 - 3)*(x^5 + 1)^(2/3)/((2*x^5 - x^3 + 2)*
x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^5+1\right)^{2/3}\,\left(2\,x^5-3\right)\,\left(2\,x^5+x^3+2\right)}{x^6\,\left(2\,x^5-x^3+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 + 1)^(2/3)*(2*x^5 - 3)*(x^3 + 2*x^5 + 2))/(x^6*(2*x^5 - x^3 + 2))
,x)
```

```
[Out] int(((x^5 + 1)^(2/3)*(2*x^5 - 3)*(x^3 + 2*x^5 + 2))/(x^6*(2*x^5 - x^3 + 2))
, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((x+1)\left(x^4-x^3+x^2-x+1\right)\right)^{\frac{2}{3}}\left(2x^5-3\right)\left(2x^5+x^3+2\right)}{x^6\left(2x^5-x^3+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+1)**(2/3)*(2*x**5-3)*(2*x**5+x**3+2)/x**6/(2*x**5-x**3+2),x
)
```

```
[Out] Integral(((x + 1)*(x**4 - x**3 + x**2 - x + 1))**(2/3)*(2*x**5 - 3)*(2*x**5
 + x**3 + 2)/(x**6*(2*x**5 - x**3 + 2)), x)
```

3.1654 $\int \frac{b+ax^6}{\left(-b+ax^6\right)\sqrt[3]{-b+a^3x^3+ax^6}}\,dx$

Optimal. Leaf size=140

$$\frac{\log\left(\sqrt[3]{a^3x^3+ax^6-b}-ax\right)}{3a}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,ax}{2\sqrt[3]{a^3x^3+ax^6-b}+ax}\right)}{\sqrt{3}\,a}-\frac{\log\left(ax\sqrt[3]{a^3x^3+ax^6-b}+\left(a^3x^3+ax^6-b\right)^{2/3}+a^2x^2\right)}{6a}$$

Rubi [F] time = 0.79, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{b+ax^6}{\left(-b+ax^6\right)\sqrt[3]{-b+a^3x^3+ax^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(b + a*x^6)/((-b + a*x^6)*(-b + a^3*x^3 + a*x^6)^(1/3)),x]
```

```
[Out] (x*(1 + (2*Sqrt[a]*x^3)/(a^(5/2) - Sqrt[a^5 + 4*b]))^(1/3)*(1 + (2*Sqrt[a]*
x^3)/(a^(5/2) + Sqrt[a^5 + 4*b]))^(1/3)*AppellF1[1/3, 1/3, 1/3, 4/3, (-2*a*
x^3)/(a^3 - Sqrt[a^6 + 4*a*b]), (-2*a*x^3)/(a^3 + Sqrt[a^6 + 4*a*b])])/(-b
+ a^3*x^3 + a*x^6)^(1/3) - Sqrt[b]*Defer[Int][1/((Sqrt[b] - Sqrt[a]*x^3)*(-
b + a^3*x^3 + a*x^6)^(1/3)), x] - Sqrt[b]*Defer[Int][1/((Sqrt[b] + Sqrt[a]*
x^3)*(-b + a^3*x^3 + a*x^6)^(1/3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{b+ax^6}{\left(-b+ax^6\right)\sqrt[3]{-b+a^3x^3+ax^6}}\,dx &= \int \left(\frac{1}{\sqrt[3]{-b+a^3x^3+ax^6}}+\frac{2b}{\left(-b+ax^6\right)\sqrt[3]{-b+a^3x^3+ax^6}}\right)dx\\
&= (2b)\int \frac{1}{\left(-b+ax^6\right)\sqrt[3]{-b+a^3x^3+ax^6}}\,dx+\int \frac{1}{\sqrt[3]{-b+a^3x^3+ax^6}}\,dx\\
&= (2b)\int \left(-\frac{1}{2\sqrt{b}\left(\sqrt{b}-\sqrt{a}\,x^3\right)\sqrt[3]{-b+a^3x^3+ax^6}}-\frac{1}{2\sqrt{b}\left(\sqrt{b}+\sqrt{a}\,x^3\right)}\right.\\
&= \frac{x\sqrt[3]{1+\frac{2\sqrt{a}\,x^3}{a^{5/2}-\sqrt{a^5+4b}}}\sqrt[3]{1+\frac{2\sqrt{a}\,x^3}{a^{5/2}+\sqrt{a^5+4b}}}\,F_1\left(\frac{1}{3};\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{2ax^3}{a^3-\sqrt{a^6+4ab}},-\frac{2a}{a^3+\sqrt{a}}\right.}{\sqrt[3]{-b+a^3x^3+ax^6}}
\end{aligned}$$

Mathematica [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{b+ax^6}{\left(-b+ax^6\right)\sqrt[3]{-b+a^3x^3+ax^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(b + a*x^6)/((-b + a*x^6)*(-b + a^3*x^3 + a*x^6)^(1/3)),x]
```

```
[Out] Integrate[(b + a*x^6)/((-b + a*x^6)*(-b + a^3*x^3 + a*x^6)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 2.79, size = 144, normalized size = 1.03

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,ax}{2\sqrt[3]{a^3x^3+ax^6-b}+ax}\right)}{\sqrt{3}\,a}+\frac{\log\left(a^2x-a\sqrt[3]{a^3x^3+ax^6-b}\right)}{3a}-\frac{\log\left(ax\sqrt[3]{a^3x^3+ax^6-b}+\left(a^3x^3+ax^6-b\right)^{2/3}+a^2x^2\right)}{6a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^6)/((-b + a*x^6)*(-b + a^3*x^3 + a*x^6)^(1/3)),
x]
```

```
[Out] -(ArcTan[(Sqrt[3]*a*x)/(a*x + 2*(-b + a^3*x^3 + a*x^6)^(1/3))]/(Sqrt[3]*a))
 + Log[a^2*x - a*(-b + a^3*x^3 + a*x^6)^(1/3)]/(3*a) - Log[a^2*x^2 + a*x*(-
b + a^3*x^3 + a*x^6)^(1/3) + (-b + a^3*x^3 + a*x^6)^(2/3)]/(6*a)
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/(a*x^6-b)/(a*x^6+a^3*x^3-b)^(1/3),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^6 + b}{\left(ax^6 + a^3x^3 - b\right)^{\frac{1}{3}}\left(ax^6 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/(a*x^6-b)/(a*x^6+a^3*x^3-b)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*x^6 + b)/((a*x^6 + a^3*x^3 - b)^(1/3)*(a*x^6 - b)), x)
```

maple [**F**] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{a\,x^6 + b}{\left(a\,x^6 - b\right)\left(a\,x^6 + a^3x^3 - b\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6+b)/(a*x^6-b)/(a*x^6+a^3*x^3-b)^(1/3),x)
```

```
[Out] int((a*x^6+b)/(a*x^6-b)/(a*x^6+a^3*x^3-b)^(1/3),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^6 + b}{\left(ax^6 + a^3x^3 - b\right)^{\frac{1}{3}}\left(ax^6 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/(a*x^6-b)/(a*x^6+a^3*x^3-b)^(1/3),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^6 + b)/((a*x^6 + a^3*x^3 - b)^(1/3)*(a*x^6 - b)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a\,x^6 + b}{\left(b - a\,x^6\right)\,\left(a^3\,x^3 + a\,x^6 - b\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x^6)/((b - a*x^6)*(a*x^6 - b + a^3*x^3)^(1/3)),x)
[Out] int(-(b + a*x^6)/((b - a*x^6)*(a*x^6 - b + a^3*x^3)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^6 + b}{(ax^6 - b)\sqrt[3]{a^3x^3 + ax^6 - b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6+b)/(a*x**6-b)/(a*x**6+a**3*x**3-b)**(1/3),x)
[Out] Integral((a*x**6 + b)/((a*x**6 - b)*(a**3*x**3 + a*x**6 - b)**(1/3)), x)
```

3.1655 $\int \frac{(1+x^2)(1+x^8)\sqrt{1+x^2+x^4+x^6+x^8}}{x^7(-1+x^2)}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{x^8+x^6+x^4+x^2+1}\,\left(8x^8+26x^6+65x^4+26x^2+8\right)}{48x^6}-\frac{65}{32}\log\left(-2x^4-x^2+2\sqrt{x^8+x^6+x^4+x^2+1}-2\right)+2\sqrt{}$$

Rubi [F] time = 1.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+x^2\right)\left(1+x^8\right)\sqrt{1+x^2+x^4+x^6+x^8}}{x^7\left(-1+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^2)*(1 + x^8)*Sqrt[1 + x^2 + x^4 + x^6 + x^8])/(x^7*(-1 + x^2)),
x]
```

```
[Out] -Defer[Int][Sqrt[1 + x^2 + x^4 + x^6 + x^8]/x^7, x] - 2*Defer[Int][Sqrt[1 +
 x^2 + x^4 + x^6 + x^8]/x^5, x] - 2*Defer[Int][Sqrt[1 + x^2 + x^4 + x^6 + x
^8]/x^3, x] + Defer[Subst][Defer[Int][Sqrt[1 + x + x^2 + x^3 + x^4], x], x,
 x^2]/2 + 2*Defer[Subst][Defer[Int][Sqrt[1 + x + x^2 + x^3 + x^4]/(-1 + x),
 x], x, x^2] - Defer[Subst][Defer[Int][Sqrt[1 + x + x^2 + x^3 + x^4]/x, x],
 x, x^2]
```

Rubi steps

$$\int \frac{\left(1+x^2\right)\left(1+x^8\right)\sqrt{1+x^2+x^4+x^6+x^8}}{x^7\left(-1+x^2\right)}\,dx = \int\left(-\frac{\sqrt{1+x^2+x^4+x^6+x^8}}{x^7}-\frac{2\sqrt{1+x^2+x^4+x^6+x^8}}{x^5}-2\sqrt{}\right.$$

$$= -\left(2\int\frac{\sqrt{1+x^2+x^4+x^6+x^8}}{x^5}\,dx\right)-2\int\frac{\sqrt{1+x^2+x^4+x^6+}}{x^3}$$

$$= \frac{1}{2}\text{Subst}\left(\int\sqrt{1+x+x^2+x^3+x^4}\,dx,x,x^2\right)-2\int\frac{\sqrt{1+x^2+}}{}$$

Mathematica [C] time = 3.16, size = 687, normalized size = 4.91

$$\frac{\frac{15(-1)^{3/5}\sqrt{-\frac{\sqrt[5]{-1}\left(\sqrt[5]{-1}-1\right)\left((-1)^{4/5}-x^2\right)\left(x^2+\sqrt[5]{-1}\right)}{\left(1-\sqrt[5]{-1}+(-1)^{2/5}\right)^2\left((-1)^{2/5}-x^2\right)^2}}\sqrt{\frac{x^2+(-1)^{4/5}+(-1)^{2/5}-\sqrt[5]{-1}+1}{\left(\sqrt[5]{-1}-1\right)\left((-1)^{2/5}-x^2\right)}}\left((-1)^{2/5}-x^2\right)^2\left(\left(45-32\sqrt[5]{-1}+38(-1)^{2/5}-32(-1)^{3/5}+45(-1)^{4/5}\right)F\left(\sin^{-1}\left(\sqrt{-\frac{\sqrt[5]{-1}\left(-1+\sqrt[5]{-1}\right)\left(x^2+\sqrt[5]{-1}\right)}{\left(1-\sqrt[5]{-1}+(-1)^{2/5}\right)\left((-1)^{2/5}-x^2\right)}}\right)|\frac{1-\sqrt[5]{-1}+(-1)^{2/5}}{\left(-1+\sqrt[5]{-1}\right)^2}\right)+13\left(1-\sqrt[5]{-1}+2(-1)^{3/5}\right)\Pi\left(\frac{-1+\sqrt[5]{-1}-(-1)^{2/5}}{-1+\sqrt[5]{-1}};\sin^{-1}\left(\sqrt{-\frac{\sqrt[5]{-1}\left(-1+\sqrt[5]{-1}\right)\left(x^2+\sqrt[5]{-1}\right)}{\left(1-\sqrt[5]{-1}+(-1)^{2/5}\right)\left((-1)^{2/5}-x^2\right)}}\right)|\frac{1-\sqrt[5]{-1}+(-1)^{2/5}}{\left(-1+\sqrt[5]{-1}\right)^2}\right)+\sqrt[5]{-1}\left(13\left(2-(-1)^{2/5}+(-1)^{3/5}\right)\Pi\left(\frac{-1+\sqrt[5]{-1}-(-1)^{4/5}}{-1+\sqrt[5]{-1}};\sin^{-1}\left(\sqrt{-\frac{\sqrt[5]{-1}\left(-1+\sqrt[5]{-1}\right)\left(x^2+\sqrt[5]{-1}\right)}{\left(1-\sqrt[5]{-1}+(-1)^{2/5}\right)\left((-1)^{2/5}-x^2\right)}}\right)|\frac{1-\sqrt[5]{-1}+(-1)^{2/5}}{\left(-1+\sqrt[5]{-1}\right)^2}\right)-64\left(1-\sqrt[5]{-1}+(-1)^{2/5}\right)\Pi\left(1-\sqrt[5]{-1}+(-1)^{4/5};\sin^{-1}\left(\sqrt{-\frac{\sqrt[5]{-1}\left(-1+\sqrt[5]{-1}\right)\left(x^2+\sqrt[5]{-1}\right)}{\left(1-\sqrt[5]{-1}+(-1)^{2/5}\right)\left((-1)^{2/5}-x^2\right)}}\right)|\frac{1-\sqrt[5]{-1}+(-1)^{2/5}}{\left(-1+\sqrt[5]{-1}\right)^2}\right)\right)\right)}{\left((-1)^{2/5}-1\right)^2}+\frac{\left(x^8+x^6+x^4+x^2+1\right)\left(8x^8+26x^6+65x^4+26x^2+8\right)}{x^6}}{48\sqrt{x^8+x^6+x^4+x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + x^2)*(1 + x^8)*Sqrt[1 + x^2 + x^4 + x^6 + x^8])/(x^7*(-1 +
x^2)),x]
```

```
[Out] (((1 + x^2 + x^4 + x^6 + x^8)*(8 + 26*x^2 + 65*x^4 + 26*x^6 + 8*x^8))/x^6 +
 (15*(-1)^(3/5)*((-1)^(2/5) - x^2)^2*Sqrt[-(((-1)^(1/5)*(-1 + (-1)^(1/5))*(
(-1)^(4/5) - x^2)*((-1)^(1/5) + x^2))/((1 - (-1)^(1/5) + (-1)^(2/5))^2*((-1
)^(2/5) - x^2)^2))]*Sqrt[(1 - (-1)^(1/5) + (-1)^(2/5) + (-1)^(4/5) + x^2)/(
(-1 + (-1)^(1/5))*((-1)^(2/5) - x^2))]*((45 - 32*(-1)^(1/5) + 38*(-1)^(2/5)
 - 32*(-1)^(3/5) + 45*(-1)^(4/5))*EllipticF[ArcSin[Sqrt[-(((-1)^(1/5)*(-1 +
 (-1)^(1/5))*((-1)^(1/5) + x^2))/((1 - (-1)^(1/5) + (-1)^(2/5))*((-1)^(2/5)
 - x^2)))]], (1 - (-1)^(1/5) + (-1)^(2/5))/(-1 + (-1)^(1/5))^2] + 13*(1 - (
-1)^(1/5) + 2*(-1)^(3/5))*EllipticPi[(-1 + (-1)^(1/5) - (-1)^(2/5))/(-1 + (
```

```
-1)^(1/5)), ArcSin[Sqrt[-(((-1)^(1/5)*(-1 + (-1)^(1/5))*((-1)^(1/5) + x^2))
/((1 - (-1)^(1/5) + (-1)^(2/5))*((-1)^(2/5) - x^2)))]], (1 - (-1)^(1/5) + (
-1)^(2/5))/(-1 + (-1)^(1/5))^2] + (-1)^(1/5)*(13*(2 - (-1)^(2/5) + (-1)^(3/
5))*EllipticPi[(-1 + (-1)^(1/5) - (-1)^(4/5))/(-1 + (-1)^(1/5)), ArcSin[Sqr
t[-(((-1)^(1/5)*(-1 + (-1)^(1/5))*((-1)^(1/5) + x^2))/((1 - (-1)^(1/5) + (-
1)^(2/5))*((-1)^(2/5) - x^2)))]], (1 - (-1)^(1/5) + (-1)^(2/5))/(-1 + (-1)^
(1/5))^2] - 64*(1 - (-1)^(1/5) + (-1)^(2/5))*EllipticPi[1 - (-1)^(1/5) + (-
1)^(4/5), ArcSin[Sqrt[-(((-1)^(1/5)*(-1 + (-1)^(1/5))*((-1)^(1/5) + x^2))/(
(1 - (-1)^(1/5) + (-1)^(2/5))*((-1)^(2/5) - x^2)))]], (1 - (-1)^(1/5) + (-1
)^(2/5))/(-1 + (-1)^(1/5))^2])))/(-1 + (-1)^(2/5))^2)/(48*Sqrt[1 + x^2 + x^
4 + x^6 + x^8])
```

IntegrateAlgebraic [A] time = 0.50, size = 140, normalized size = 1.00

$$\frac{\sqrt{x^8+x^6+x^4+x^2+1}\,(8x^8+26x^6+65x^4+26x^2+8)}{48x^6}-\frac{65}{32}\log\left(-2x^4-x^2+2\sqrt{x^8+x^6+x^4+x^2+1}-2\right)+2\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5}\,x^2}{x^4-2x^2-\sqrt{x^8+x^6+x^4+x^2+1}+1}\right)+\frac{65\log(x)}{16}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^2)*(1 + x^8)*Sqrt[1 + x^2 + x^4 + x^6 + x^8])/(x
^7*(-1 + x^2)),x]
```

```
[Out] (Sqrt[1 + x^2 + x^4 + x^6 + x^8]*(8 + 26*x^2 + 65*x^4 + 26*x^6 + 8*x^8))/(4
8*x^6) + 2*Sqrt[5]*ArcTanh[(Sqrt[5]*x^2)/(1 - 2*x^2 + x^4 - Sqrt[1 + x^2 +
x^4 + x^6 + x^8])] + (65*Log[x])/16 - (65*Log[-2 - x^2 - 2*x^4 + 2*Sqrt[1 +
 x^2 + x^4 + x^6 + x^8]])/32
```

fricas [A] time = 0.49, size = 165, normalized size = 1.18

$$\frac{48\sqrt{5}\,x^6\log\left(-\frac{9\,x^8+4\,x^6+14\,x^4-4\,\sqrt{5}\,\sqrt{x^8+x^6+x^4+x^2+1}\,(x^4+1)+4\,x^2+9}{x^8-4\,x^6+6\,x^4-4\,x^2+1}\right)+195\,x^6\log\left(\frac{2\,x^4+x^2+2\,\sqrt{x^8+x^6+x^4+x^2+1}+2}{x^2}\right)+2\,(8\,x^8+26\,x^6+65\,x^4+26\,x^2+8)\sqrt{x^8+x^6+x^4+x^2+1}}{96\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^8+1)*(x^8+x^6+x^4+x^2+1)^(1/2)/x^7/(x^2-1),x, algorith
m="fricas")
```

```
[Out] 1/96*(48*sqrt(5)*x^6*log(-(9*x^8 + 4*x^6 + 14*x^4 - 4*sqrt(5)*sqrt(x^8 + x^
6 + x^4 + x^2 + 1)*(x^4 + 1) + 4*x^2 + 9)/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
) + 195*x^6*log((2*x^4 + x^2 + 2*sqrt(x^8 + x^6 + x^4 + x^2 + 1) + 2)/x^2)
+ 2*(8*x^8 + 26*x^6 + 65*x^4 + 26*x^2 + 8)*sqrt(x^8 + x^6 + x^4 + x^2 + 1))
/x^6
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^8+x^6+x^4+x^2+1}\,(x^8+1)(x^2+1)}{(x^2-1)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^8+1)*(x^8+x^6+x^4+x^2+1)^(1/2)/x^7/(x^2-1),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(x^8 + x^6 + x^4 + x^2 + 1)*(x^8 + 1)*(x^2 + 1)/((x^2 - 1)*x^
7), x)
```

maple [C] time = 0.98, size = 172, normalized size = 1.23

$$\frac{65x^{12}+91x^{10}+99x^8+99x^6+99x^4+34x^2+8}{48x^6\sqrt{x^8+x^6+x^4+x^2+1}}+\frac{(8x^2+26)\sqrt{x^8+x^6+x^4+x^2+1}}{48}-\frac{65\ln\left(\frac{-2-x^2-2x^4+2\sqrt{x^8+x^6+x^4+x^2+1}}{x^2}\right)}{32}-\mathrm{RootOf}(\_Z^2-5)\ln\left(\frac{\mathrm{RootOf}(\_Z^2-5)x^4+2\sqrt{x^8+x^6+x^4+x^2+1}+\mathrm{RootOf}(\_Z^2-5)}{(1+x)^2(-1+x)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^8+1)*(x^8+x^6+x^4+x^2+1)^(1/2)/x^7/(x^2-1),x)
```

```
[Out] 1/48*(65*x^12+91*x^10+99*x^8+99*x^6+99*x^4+34*x^2+8)/x^6/(x^8+x^6+x^4+x^2+1
)^(1/2)+1/48*(8*x^2+26)*(x^8+x^6+x^4+x^2+1)^(1/2)-65/32*ln((-2-x^2-2*x^4+2*
(x^8+x^6+x^4+x^2+1)^(1/2))/x^2)-RootOf(_Z^2-5)*ln((RootOf(_Z^2-5)*x^4+2*(x^
8+x^6+x^4+x^2+1)^(1/2)+RootOf(_Z^2-5))/(1+x)^2/(-1+x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^8+x^6+x^4+x^2+1}\left(x^8+1\right)\left(x^2+1\right)}{\left(x^2-1\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^8+1)*(x^8+x^6+x^4+x^2+1)^(1/2)/x^7/(x^2-1),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(x^8 + x^6 + x^4 + x^2 + 1)*(x^8 + 1)*(x^2 + 1)/((x^2 - 1)*x^
7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+1\right)\,\left(x^8+1\right)\,\sqrt{x^8+x^6+x^4+x^2+1}}{x^7\,\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*(x^8 + 1)*(x^2 + x^4 + x^6 + x^8 + 1)^(1/2))/(x^7*(x^2 - 1))
,x)
```

```
[Out] int(((x^2 + 1)*(x^8 + 1)*(x^2 + x^4 + x^6 + x^8 + 1)^(1/2))/(x^7*(x^2 - 1))
, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(x^4-x^3+x^2-x+1\right)\left(x^4+x^3+x^2+x+1\right)}\,\left(x^2+1\right)\left(x^8+1\right)}{x^7\,(x-1)\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**8+1)*(x**8+x**6+x**4+x**2+1)**(1/2)/x**7/(x**2-1),x)
```

```
[Out] Integral(sqrt((x**4 - x**3 + x**2 - x + 1)*(x**4 + x**3 + x**2 + x + 1))*(x
**2 + 1)*(x**8 + 1)/(x**7*(x - 1)*(x + 1)), x)
```

3.1656 $\int \frac{x^2\sqrt{x+x^2}}{\sqrt{x^2+x\sqrt{x+x^2}}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{x^2+x}\sqrt{x\left(\sqrt{x^2+x}+x\right)}\left(-3072x^3-640x^2+840x-1575\right)}{10752x}+\sqrt{x\left(\sqrt{x^2+x}+x\right)}\left(\frac{75\sqrt{\sqrt{x^2+x}-x}\,\tanh^{-}}{512\sqrt{}}\right.$$

Rubi [F] time = 1.73, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\sqrt{x+x^2}}{\sqrt{x^2+x\sqrt{x+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*Sqrt[x + x^2])/Sqrt[x^2 + x*Sqrt[x + x^2]],x]
```

```
[Out] (2*Sqrt[x + x^2]*Defer[Subst][Defer[Int][(x^6*Sqrt[1 + x^2])/Sqrt[x^4 + x^2
*Sqrt[x^2 + x^4]], x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + x])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{x+x^2}}{\sqrt{x^2+x\sqrt{x+x^2}}}\,dx &= \frac{\sqrt{x+x^2}\int \frac{x^{5/2}\sqrt{1+x}}{\sqrt{x^2+x\sqrt{x+x^2}}}\,dx}{\sqrt{x}\sqrt{1+x}}\\
&= \frac{\left(2\sqrt{x+x^2}\right)\text{Subst}\left(\int \frac{x^6\sqrt{1+x^2}}{\sqrt{x^4+x^2\sqrt{x^2+x^4}}}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1+x}}
\end{aligned}$$

Mathematica [C] time = 0.50, size = 139, normalized size = 0.99

$$\frac{\left(x+\sqrt{x(x+1)}\right)^3\sqrt{x\left(x+\sqrt{x(x+1)}\right)}\left(x+\sqrt{x(x+1)}+1\right)\left(28x\left(32x^2+\left(32\sqrt{x(x+1)}+46\right)x+30\sqrt{x(x+1)}+11\right)-25\,{}_2F_1\left(-\frac{7}{2},2;-\frac{5}{2};1+\frac{1}{2\left(x+\sqrt{x(x+1)}\right)}\right)\right)}{672\sqrt{x(x+1)}\left(2x+2\sqrt{x(x+1)}+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[x + x^2])/Sqrt[x^2 + x*Sqrt[x + x^2]],x]
```

```
[Out] ((x + Sqrt[x*(1 + x)])^3*Sqrt[x*(x + Sqrt[x*(1 + x)])]*(1 + x + Sqrt[x*(1 +
 x)])*(28*x*(11 + 32*x^2 + 30*Sqrt[x*(1 + x)] + x*(46 + 32*Sqrt[x*(1 + x)])
) - 25*Hypergeometric2F1[-7/2, 2, -5/2, 1 + 1/(2*(x + Sqrt[x*(1 + x)]))]))/
(672*Sqrt[x*(1 + x)]*(1 + 2*x + 2*Sqrt[x*(1 + x)])^4)
```

IntegrateAlgebraic [A] time = 4.92, size = 140, normalized size = 1.00

$$\frac{\sqrt{x^2+x}\sqrt{x\left(\sqrt{x^2+x}+x\right)}\left(-3072x^3-640x^2+840x-1575\right)}{10752x}+\sqrt{x\left(\sqrt{x^2+x}+x\right)}\left(\frac{75\sqrt{\sqrt{x^2+x}-x}\,\tanh^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+x}-x}\right)}{512\sqrt{2}\,x}+\frac{3072x^3+3968x^2-120x+525}{10752}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[x + x^2])/Sqrt[x^2 + x*Sqrt[x + x^2]],x]
```

```
[Out] (Sqrt[x + x^2]*(-1575 + 840*x - 640*x^2 - 3072*x^3)*Sqrt[x*(x + Sqrt[x + x^
2])])/(10752*x) + Sqrt[x*(x + Sqrt[x + x^2])]*((525 - 120*x + 3968*x^2 + 30
72*x^3)/10752 + (75*Sqrt[-x + Sqrt[x + x^2]]*ArcTanh[Sqrt[2]*Sqrt[-x + Sqrt
[x + x^2]]])/(512*Sqrt[2]*x))
```

fricas [A] time = 0.40, size = 128, normalized size = 0.91

$$\frac{1575 \sqrt{2} x \log\left(\frac{4 x^2+2 \sqrt{x^2+\sqrt{x^2+x} x} \left(\sqrt{2} x+\sqrt{2} \sqrt{x^2+x}\right)+4 \sqrt{x^2+x} x+x}{x}\right) + 4 \left(3072 x^4 + 3968 x^3 - 120 x^2 - \left(3072 x^3 + 640 x^2 - 840 x + 1575\right) \sqrt{x^2+x} + 525 x\right) \sqrt{x^2 + \sqrt{x^2+x} x}}{43008 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/43008*(1575*sqrt(2)*x*log((4*x^2 + 2*sqrt(x^2 + sqrt(x^2 + x)*x)*(sqrt(2)
*x + sqrt(2)*sqrt(x^2 + x)) + 4*sqrt(x^2 + x)*x + x)/x) + 4*(3072*x^4 + 396
8*x^3 - 120*x^2 - (3072*x^3 + 640*x^2 - 840*x + 1575)*sqrt(x^2 + x) + 525*x
)*sqrt(x^2 + sqrt(x^2 + x)*x))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+x} x^2}{\sqrt{x^2+\sqrt{x^2+x} x}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(x^2 + x)*x^2/sqrt(x^2 + sqrt(x^2 + x)*x), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{x^2+x}}{\sqrt{x^2+x\sqrt{x^2+x}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x)
```

```
[Out] int(x^2*(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+x} x^2}{\sqrt{x^2+\sqrt{x^2+x} x}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(x^2 + x)*x^2/sqrt(x^2 + sqrt(x^2 + x)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \sqrt{x^2+x}}{\sqrt{x^2+x \sqrt{x^2+x}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x + x^2)^(1/2))/(x^2 + x*(x + x^2)^(1/2))^(1/2),x)
```

```
[Out] int((x^2*(x + x^2)^(1/2))/(x^2 + x*(x + x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\sqrt{x\left(x+1\right)}}{\sqrt{x\left(x+\sqrt{x^{2}+x}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+x)**(1/2)/(x**2+x*(x**2+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x*(x + 1))/sqrt(x*(x + sqrt(x**2 + x))), x)
```

3.1657 $\int \frac{\sqrt[3]{-6-2x+x^2}}{-1+x}\,dx$

Optimal. Leaf size=141

$$\frac{3}{2}\sqrt[3]{x^2-2x-6}-\frac{1}{2}\sqrt[3]{7}\log\left(7^{2/3}\sqrt[3]{x^2-2x-6}+7\right)+\frac{1}{4}\sqrt[3]{7}\log\left(-\sqrt[3]{7}\left(x^2-2x-6\right)^{2/3}+7^{2/3}\sqrt[3]{x^2-2x-6}-7\right)+\frac{1}{2}$$

Rubi [A] time = 0.09, antiderivative size = 103, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {694, 266, 50, 58, 617, 204, 31}

$$\frac{3}{2}\sqrt[3]{(x-1)^2-7}-\frac{3}{4}\sqrt[3]{7}\log\left(\sqrt[3]{(x-1)^2-7}+\sqrt[3]{7}\right)+\frac{1}{2}\sqrt[3]{7}\log(1-x)+\frac{1}{2}\sqrt{3}\sqrt[3]{7}\tan^{-1}\left(\frac{\sqrt[3]{7}-2\sqrt[3]{(x-1)^2-7}}{\sqrt{3}\sqrt[3]{7}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-6 - 2*x + x^2)^(1/3)/(-1 + x),x]
```

```
[Out] (3*(-7 + (-1 + x)^2)^(1/3))/2 + (Sqrt[3]*7^(1/3)*ArcTan[(7^(1/3) - 2*(-7 +
(-1 + x)^2)^(1/3))/(Sqrt[3]*7^(1/3))])/2 - (3*7^(1/3)*Log[7^(1/3) + (-7 + (
-1 + x)^2)^(1/3)])/4 + (7^(1/3)*Log[1 - x])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{-6-2x+x^2}}{-1+x}\,dx &= \text{Subst}\left(\int \frac{\sqrt[3]{-7+x^2}}{x}\,dx, x, -1+x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt[3]{-7+x}}{x}\,dx, x, (-1+x)^2\right)\\
&= \frac{3}{2}\sqrt[3]{-7+(-1+x)^2} - \frac{7}{2}\text{Subst}\left(\int \frac{1}{(-7+x)^{2/3}x}\,dx, x, (-1+x)^2\right)\\
&= \frac{3}{2}\sqrt[3]{-7+(-1+x)^2} + \frac{1}{2}\sqrt[3]{7}\log(1-x) - \frac{1}{4}\left(3\sqrt[3]{7}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{7}+x}\,dx, x, \sqrt[3]{-7+(-1
\\
&= \frac{3}{2}\sqrt[3]{-7+(-1+x)^2} - \frac{3}{4}\sqrt[3]{7}\log\left(\sqrt[3]{7}+\sqrt[3]{-7+(-1+x)^2}\right) + \frac{1}{2}\sqrt[3]{7}\log(1-x) - \frac{1}{2}\left(3\sqrt[3]{7}
\\
&= \frac{3}{2}\sqrt[3]{-7+(-1+x)^2} + \frac{1}{2}\sqrt{3}\sqrt[3]{7}\tan^{-1}\left(\frac{7-2\,7^{2/3}\sqrt[3]{-7+(-1+x)^2}}{7\sqrt{3}}\right) - \frac{3}{4}\sqrt[3]{7}\log\left(\sqrt[3]{7}+
\end{aligned}$$

Mathematica [A] time = 0.07, size = 129, normalized size = 0.91

$$\frac{1}{4}\left(6\sqrt[3]{x^2-2x-6}+2\sqrt{3}\sqrt[3]{7}\tan^{-1}\left(\frac{7-2\,7^{2/3}\sqrt[3]{x^2-2x-6}}{7\sqrt{3}}\right)-2\sqrt[3]{7}\log\left(\sqrt[3]{(x-1)^2-7}+\sqrt[3]{7}\right)+\sqrt[3]{7}\log\left(\left((x-1)^2-7\right)^{2/3}-\sqrt[3]{7}\sqrt[3]{(x-1)^2-7}+7^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-6 - 2*x + x^2)^(1/3)/(-1 + x),x]
```

```
[Out] (6*(-6 - 2*x + x^2)^(1/3) + 2*Sqrt[3]*7^(1/3)*ArcTan[(7 - 2*7^(2/3)*(-6 - 2
*x + x^2)^(1/3))/(7*Sqrt[3])] - 2*7^(1/3)*Log[7^(1/3) + (-7 + (-1 + x)^2)^(
1/3)] + 7^(1/3)*Log[7^(2/3) - 7^(1/3)*(-7 + (-1 + x)^2)^(1/3) + (-7 + (-1 +
 x)^2)^(2/3)])/4
```

IntegrateAlgebraic [A] time = 0.31, size = 141, normalized size = 1.00

$$\frac{3}{2}\sqrt[3]{x^2-2x-6}-\frac{1}{2}\sqrt[3]{7}\log\left(7^{2/3}\sqrt[3]{x^2-2x-6}+7\right)+\frac{1}{4}\sqrt[3]{7}\log\left(-\sqrt[3]{7}\left(x^2-2x-6\right)^{2/3}+7^{2/3}\sqrt[3]{x^2-2x-6}-7\right)+\frac{1}{2}\sqrt{3}\sqrt[3]{7}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^2-2x-6}}{\sqrt{3}\sqrt[3]{7}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-6 - 2*x + x^2)^(1/3)/(-1 + x),x]
```

```
[Out] (3*(-6 - 2*x + x^2)^(1/3))/2 + (Sqrt[3]*7^(1/3)*ArcTan[1/Sqrt[3] - (2*(-6 -
 2*x + x^2)^(1/3))/(Sqrt[3]*7^(1/3))])/2 - (7^(1/3)*Log[7 + 7^(2/3)*(-6 - 2
*x + x^2)^(1/3)])/2 + (7^(1/3)*Log[-7 + 7^(2/3)*(-6 - 2*x + x^2)^(1/3) - 7^
(1/3)*(-6 - 2*x + x^2)^(2/3)])/4
```

fricas [A] time = 0.39, size = 102, normalized size = 0.72

$$\frac{1}{2}\sqrt{3}\,(-7)^{\frac{1}{3}}\arctan\left(\frac{2}{21}\sqrt{3}\,(-7)^{\frac{2}{3}}\left(x^2-2x-6\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-\frac{1}{4}(-7)^{\frac{1}{3}}\log\left((-7)^{\frac{2}{3}}+(-7)^{\frac{1}{3}}\left(x^2-2x-6\right)^{\frac{1}{3}}+\left(x^2-2x-6\right)^{\frac{2}{3}}\right)+\frac{1}{2}(-7)^{\frac{1}{3}}\log\left(-(-7)^{\frac{1}{3}}+\left(x^2-2x-6\right)^{\frac{1}{3}}\right)+\frac{3}{2}\left(x^2-2x-6\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-6)^(1/3)/(-1+x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*(-7)^(1/3)*arctan(2/21*sqrt(3)*(-7)^(2/3)*(x^2 - 2*x - 6)^(1/3)
 - 1/3*sqrt(3)) - 1/4*(-7)^(1/3)*log((-7)^(2/3) + (-7)^(1/3)*(x^2 - 2*x - 6
)^(1/3) + (x^2 - 2*x - 6)^(2/3)) + 1/2*(-7)^(1/3)*log(-(-7)^(1/3) + (x^2 -
2*x - 6)^(1/3)) + 3/2*(x^2 - 2*x - 6)^(1/3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2 - 2x - 6\right)^{\frac{1}{3}}}{x - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-6)^(1/3)/(-1+x),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 2*x - 6)^(1/3)/(x - 1), x)
```

maple [C] time = 8.15, size = 2698, normalized size = 19.13

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x-6)^(1/3)/(-1+x),x)
```

```
[Out] 3/2*(x^2-2*x-6)^(1/3)+(1/2*RootOf(_Z^3+7)*ln((-8424*RootOf(_Z^3+7)^2*(x^4-4
*x^3-8*x^2+24*x+36)^(1/3)+16758*(x^4-4*x^3-8*x^2+24*x+36)^(1/3)*RootOf(36*R
ootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)*RootOf(_Z^3+7)*x^2+100548*(x^
4-4*x^3-8*x^2+24*x+36)^(2/3)-252*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z
^3+7)+49*_Z^2)*x^4+1008*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*
_Z^2)*x^3-32004*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)+52
1208*RootOf(_Z^3+7)+5838*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49
*_Z^2)*x^2-13692*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)*x
+4104*RootOf(_Z^3+7)*x^4-16416*RootOf(_Z^3+7)*x^3-95076*RootOf(_Z^3+7)*x^2+
222984*RootOf(_Z^3+7)*x+798*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)
+49*_Z^2)*RootOf(_Z^3+7)^3*x^4-3192*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf
(_Z^3+7)+49*_Z^2)*RootOf(_Z^3+7)^3*x^3-33516*(x^4-4*x^3-8*x^2+24*x+36)^(1/3
)*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)*RootOf(_Z^3+7)*x
-49*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)^2*RootOf(_Z^3+
7)^2*x^4+196*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)^2*Roo
tOf(_Z^3+7)^2*x^3+98*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^
2)^2*RootOf(_Z^3+7)^2*x^2-1596*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3
+7)+49*_Z^2)*RootOf(_Z^3+7)^3*x^2-588*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*Root
Of(_Z^3+7)+49*_Z^2)^2*RootOf(_Z^3+7)^2*x+9576*RootOf(36*RootOf(_Z^3+7)^2+42
*_Z*RootOf(_Z^3+7)+49*_Z^2)*RootOf(_Z^3+7)^3*x-15120*(x^4-4*x^3-8*x^2+24*x+
36)^(2/3)*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)*RootOf(_
Z^3+7)^2+1404*RootOf(_Z^3+7)^2*(x^4-4*x^3-8*x^2+24*x+36)^(1/3)*x^2-2808*Roo
tOf(_Z^3+7)^2*(x^4-4*x^3-8*x^2+24*x+36)^(1/3)*x-100548*(x^4-4*x^3-8*x^2+24*
x+36)^(1/3)*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)*RootOf
(_Z^3+7))/(x^2-2*x-6)/(-1+x)^2)-1/2*ln((-43092*RootOf(_Z^3+7)^2*(x^4-4*x^3-
8*x^2+24*x+36)^(1/3)+819*(x^4-4*x^3-8*x^2+24*x+36)^(1/3)*RootOf(36*RootOf(_
Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)*RootOf(_Z^3+7)*x^2+4914*(x^4-4*x^3-8
*x^2+24*x+36)^(2/3)-5460*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49
*_Z^2)*x^4+21840*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)*x
^3-320040*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)-260604*R
ootOf(_Z^3+7)+64260*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2
)*x^2-172200*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)*x-444
6*RootOf(_Z^3+7)*x^4+17784*RootOf(_Z^3+7)*x^3+52326*RootOf(_Z^3+7)*x^2-1402
```

```
20*RootOf(_Z^3+7)*x+399*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*
_Z^2)*RootOf(_Z^3+7)^3*x^4-1596*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^
3+7)+49*_Z^2)*RootOf(_Z^3+7)^3*x^3-1638*(x^4-4*x^3-8*x^2+24*x+36)^(1/3)*Roo
tOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)*RootOf(_Z^3+7)*x+490*
RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)^2*RootOf(_Z^3+7)^2
*x^4-1960*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)^2*RootOf
(_Z^3+7)^2*x^3-980*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)
^2*RootOf(_Z^3+7)^2*x^2-798*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)
+49*_Z^2)*RootOf(_Z^3+7)^3*x^2+5880*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf
(_Z^3+7)+49*_Z^2)^2*RootOf(_Z^3+7)^2*x+4788*RootOf(36*RootOf(_Z^3+7)^2+42*_
Z*RootOf(_Z^3+7)+49*_Z^2)*RootOf(_Z^3+7)^3*x+7560*(x^4-4*x^3-8*x^2+24*x+36)
^(2/3)*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)*RootOf(_Z^3
+7)^2+7182*RootOf(_Z^3+7)^2*(x^4-4*x^3-8*x^2+24*x+36)^(1/3)*x^2-14364*RootO
f(_Z^3+7)^2*(x^4-4*x^3-8*x^2+24*x+36)^(1/3)*x-4914*(x^4-4*x^3-8*x^2+24*x+36
)^(1/3)*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)*RootOf(_Z^
3+7))/(x^2-2*x-6)/(-1+x)^2)*RootOf(_Z^3+7)-7/12*ln((-43092*RootOf(_Z^3+7)^2
*(x^4-4*x^3-8*x^2+24*x+36)^(1/3)+819*(x^4-4*x^3-8*x^2+24*x+36)^(1/3)*RootOf
(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)*RootOf(_Z^3+7)*x^2+4914*
(x^4-4*x^3-8*x^2+24*x+36)^(2/3)-5460*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootO
f(_Z^3+7)+49*_Z^2)*x^4+21840*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7
)+49*_Z^2)*x^3-320040*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z
^2)-260604*RootOf(_Z^3+7)+64260*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^
3+7)+49*_Z^2)*x^2-172200*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49
*_Z^2)*x-4446*RootOf(_Z^3+7)*x^4+17784*RootOf(_Z^3+7)*x^3+52326*RootOf(_Z^3
+7)*x^2-140220*RootOf(_Z^3+7)*x+399*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf
(_Z^3+7)+49*_Z^2)*RootOf(_Z^3+7)^3*x^4-1596*RootOf(36*RootOf(_Z^3+7)^2+42*_
Z*RootOf(_Z^3+7)+49*_Z^2)*RootOf(_Z^3+7)^3*x^3-1638*(x^4-4*x^3-8*x^2+24*x+3
6)^(1/3)*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)*RootOf(_Z
^3+7)*x+490*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)^2*Root
Of(_Z^3+7)^2*x^4-1960*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z
^2)^2*RootOf(_Z^3+7)^2*x^3-980*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3
+7)+49*_Z^2)^2*RootOf(_Z^3+7)^2*x^2-798*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*Ro
otOf(_Z^3+7)+49*_Z^2)*RootOf(_Z^3+7)^3*x^2+5880*RootOf(36*RootOf(_Z^3+7)^2+
42*_Z*RootOf(_Z^3+7)+49*_Z^2)^2*RootOf(_Z^3+7)^2*x+4788*RootOf(36*RootOf(_Z
^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)*RootOf(_Z^3+7)^3*x+7560*(x^4-4*x^3-8*
x^2+24*x+36)^(2/3)*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2)
*RootOf(_Z^3+7)^2+7182*RootOf(_Z^3+7)^2*(x^4-4*x^3-8*x^2+24*x+36)^(1/3)*x^2
-14364*RootOf(_Z^3+7)^2*(x^4-4*x^3-8*x^2+24*x+36)^(1/3)*x-4914*(x^4-4*x^3-8
*x^2+24*x+36)^(1/3)*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*RootOf(_Z^3+7)+49*_Z^2
)*RootOf(_Z^3+7))/(x^2-2*x-6)/(-1+x)^2)*RootOf(36*RootOf(_Z^3+7)^2+42*_Z*Ro
otOf(_Z^3+7)+49*_Z^2))/(x^2-2*x-6)^(2/3)*((x^2-2*x-6)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-2\,x-6\right)^{\frac{1}{3}}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-6)^(1/3)/(-1+x),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 2*x - 6)^(1/3)/(x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2-2\,x-6\right)^{1/3}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 2*x - 6)^(1/3)/(x - 1),x)
```

```
[Out] int((x^2 - 2*x - 6)^(1/3)/(x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x^2 - 2x - 6}}{x - 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x-6)**(1/3)/(-1+x),x)
```

```
[Out] Integral((x**2 - 2*x - 6)**(1/3)/(x - 1), x)
```

3.1658 $\int \frac{-1+x}{\sqrt[3]{1-x-x^2+x^3}} \, dx$

Optimal. Leaf size=141

$$\frac{(x-1)^{2/3}\sqrt[3]{x+1}\left(\sqrt[3]{x-1}\,(x+1)^{2/3}+\frac{2}{3}\log\left(\sqrt[3]{x-1}-\sqrt[3]{x+1}\right)-\frac{1}{3}\log\left((x-1)^{2/3}+\sqrt[3]{x+1}\,\sqrt[3]{x-1}+(x+1)^{2/}\right.\right.}{\sqrt[3]{(x-1)^2(x+1)}}$$

Rubi [A] time = 0.20, antiderivative size = 214, normalized size of antiderivative = 1.52, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2081, 2077, 21, 50, 60}

$$-\frac{(1-x)(x+1)}{\sqrt[3]{x^3-x^2-x+1}}+\frac{(3-3x)^{2/3}\sqrt[3]{x+1}\log\left(-\frac{8}{3}(x-1)\right)}{3\,3^{2/3}\sqrt[3]{x^3-x^2-x+1}}+\frac{(3-3x)^{2/3}\sqrt[3]{x+1}\log\left(\frac{\sqrt[3]{3}\sqrt[3]{x+1}}{\sqrt[3]{3-3x}}+1\right)}{3^{2/3}\sqrt[3]{x^3-x^2-x+1}}+\frac{2(3-3x)^{2/3}\sqrt[3]{x+1}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x+1}}{\sqrt[6]{3}\sqrt[3]{3-3x}}\right)}{3\sqrt[6]{3}\sqrt[3]{x^3-x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(1 - x - x^2 + x^3)^(1/3),x]
```

```
[Out] -(((1 - x)*(1 + x))/(1 - x - x^2 + x^3)^(1/3)) + (2*(3 - 3*x)^(2/3)*(1 + x)
^(1/3)*ArcTan[1/Sqrt[3] - (2*(1 + x)^(1/3))/(3^(1/6)*(3 - 3*x)^(1/3))])/(3*
3^(1/6)*(1 - x - x^2 + x^3)^(1/3)) + ((3 - 3*x)^(2/3)*(1 + x)^(1/3)*Log[(-8
*(-1 + x))/3])/(3*3^(2/3)*(1 - x - x^2 + x^3)^(1/3)) + ((3 - 3*x)^(2/3)*(1
+ x)^(1/3)*Log[1 + (3^(1/3)*(1 + x)^(1/3))/(3 - 3*x)^(1/3)])/(3^(2/3)*(1 -
x - x^2 + x^3)^(1/3))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 2077

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Dist[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int
[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d, e
, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 2081

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{-1+x}{\sqrt[3]{1-x-x^2+x^3}}\,dx = \text{Subst}\left(\int \frac{-\frac{2}{3}+x}{\sqrt[3]{\frac{16}{27}-\frac{4x}{3}+x^3}}\,dx, x, -\frac{1}{3}+x\right)$$

$$= \frac{\left(4\,2^{2/3}(1-x)^{2/3}\sqrt[3]{1+x}\right)\text{Subst}\left(\int \frac{-\frac{2}{3}+x}{\left(\frac{16}{9}-\frac{8x}{3}\right)^{2/3}\sqrt[3]{\frac{16}{9}+\frac{4x}{3}}}\,dx, x, -\frac{1}{3}+x\right)}{3\sqrt[3]{1-x-x^2+x^3}}$$

$$= -\frac{\left((1-x)^{2/3}\sqrt[3]{1+x}\right)\text{Subst}\left(\int \frac{\sqrt[3]{\frac{16}{9}-\frac{8x}{3}}}{\sqrt[3]{\frac{16}{9}+\frac{4x}{3}}}\,dx, x, -\frac{1}{3}+x\right)}{\sqrt[3]{2}\sqrt[3]{1-x-x^2+x^3}}$$

$$= -\frac{(1-x)(1+x)}{\sqrt[3]{1-x-x^2+x^3}} - \frac{\left(8\,2^{2/3}(1-x)^{2/3}\sqrt[3]{1+x}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{16}{9}-\frac{8x}{3}\right)^{2/3}\sqrt[3]{\frac{16}{9}+\frac{4x}{3}}}\,dx, x, -\frac{1}{3}+x\right.}{9\sqrt[3]{1-x-x^2+x^3}}$$

$$= -\frac{(1-x)(1+x)}{\sqrt[3]{1-x-x^2+x^3}} + \frac{2(1-x)^{2/3}\sqrt[3]{1+x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1+x}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{\sqrt{3}\sqrt[3]{1-x-x^2+x^3}} + \frac{(1-x)^{2/3}\sqrt[3]{1+x}\,\log}{3\sqrt[3]{1-x-x^2+}}$$

Mathematica [C] time = 0.02, size = 48, normalized size = 0.34

$$\frac{3\left((x-1)^2(x+1)\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{4}{3};\frac{7}{3};\frac{1-x}{2}\right)}{4\sqrt[3]{2}\,(x+1)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(1 - x - x^2 + x^3)^(1/3),x]
```

```
[Out] (3*((-1 + x)^2*(1 + x))^(2/3)*Hypergeometric2F1[1/3, 4/3, 7/3, (1 - x)/2])/
(4*2^(1/3)*(1 + x)^(2/3))
```

IntegrateAlgebraic [A] time = 5.10, size = 141, normalized size = 1.00

$$\frac{(x-1)^{2/3}\sqrt[3]{x+1}\left(\sqrt[3]{x-1}\,(x+1)^{2/3}+\frac{2}{3}\log\left(\sqrt[3]{x-1}-\sqrt[3]{x+1}\right)-\frac{1}{3}\log\left((x-1)^{2/3}+\sqrt[3]{x+1}\,\sqrt[3]{x-1}+(x+1)^{2/3}\right)+\frac{2\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{x+1}}{2\sqrt[3]{x-1}+\sqrt[3]{x+1}}\right)}{\sqrt{3}}\right)}{\sqrt[3]{(x-1)^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/(1 - x - x^2 + x^3)^(1/3),x]
```

```
[Out] ((-1 + x)^(2/3)*(1 + x)^(1/3)*((-1 + x)^(1/3)*(1 + x)^(2/3) + (2*ArcTan[(Sq
rt[3]*(1 + x)^(1/3))/(2*(-1 + x)^(1/3) + (1 + x)^(1/3))])/Sqrt[3] + (2*Log[
(-1 + x)^(1/3) - (1 + x)^(1/3)])/3 - Log[(-1 + x)^(2/3) + (-1 + x)^(1/3)*(1
 + x)^(1/3) + (1 + x)^(2/3)]/3))/((-1 + x)^2*(1 + x))^(1/3)
```

fricas [A] time = 0.40, size = 161, normalized size = 1.14

$$\frac{2\sqrt{3}(x-1)\arctan\left(\frac{\sqrt{3}(x-1)+2\sqrt{3}(x^3-x^2-x+1)^{\frac{1}{3}}}{3(x-1)}\right)-(x-1)\log\left(\frac{x^2+(x^3-x^2-x+1)^{\frac{1}{3}}(x-1)-2x+(x^3-x^2-x+1)^{\frac{2}{3}}+1}{x^2-2x+1}\right)+2(x-1)\log\left(-\frac{x-(x^3-x^2-x+1)^{\frac{1}{3}}-1}{x-1}\right)+3(x^3-x^2-x+1)^{\frac{2}{3}}}{3(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^3-x^2-x+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*(x - 1)*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt(3)*(x^3 - x^2 -
 x + 1)^(1/3))/(x - 1)) - (x - 1)*log((x^2 + (x^3 - x^2 - x + 1)^(1/3)*(x -
 1) - 2*x + (x^3 - x^2 - x + 1)^(2/3) + 1)/(x^2 - 2*x + 1)) + 2*(x - 1)*log
(-(x - (x^3 - x^2 - x + 1)^(1/3) - 1)/(x - 1)) + 3*(x^3 - x^2 - x + 1)^(2/3
))/(x - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^3-x^2-x+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^3-x^2-x+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/(x^3 - x^2 - x + 1)^(1/3), x)
```

maple [C] time = 0.43, size = 361, normalized size = 2.56

$$\frac{(-1+x)(1+x)}{\left((-1+x)^2(1+x)\right)^{\frac{1}{3}}}+\frac{2\ln\left(-\frac{-4\,\mathrm{RootOf}(\_Z^2+\_Z+1)^2x^2+3\,\mathrm{RootOf}(\_Z^2+\_Z+1)(x^3-x^2-x+1)^{\frac{2}{3}}-3\,\mathrm{RootOf}(\_Z^2+\_Z+1)(x^3-x^2-x+1)^{\frac{1}{3}}x+4\,\mathrm{RootOf}(\_Z^2+\_Z+1)^2x-4\,\mathrm{RootOf}(\_Z^2+\_Z+1)x^2+3\,\mathrm{RootOf}(\_Z^2+\_Z+1)(x^3-x^2-x+1)^{\frac{1}{3}}-3(x^3-x^2-x+1)^{\frac{1}{3}}x+2\,\mathrm{RootOf}(\_Z^2+\_Z+1)x-x^2+3(x^3-x^2-x+1)^{\frac{1}{3}}+2\,\mathrm{RootOf}(\_Z^2+\_Z+1)+1}{-1+x}\right)}{3}+\frac{2\,\mathrm{RootOf}(\_Z^2+\_Z+1)\ln\left(\frac{-2\,\mathrm{RootOf}(\_Z^2+\_Z+1)^2x^2+3\,\mathrm{RootOf}(\_Z^2+\_Z+1)(x^3-x^2-x+1)^{\frac{2}{3}}+2\,\mathrm{RootOf}(\_Z^2+\_Z+1)^2x-5\,\mathrm{RootOf}(\_Z^2+\_Z+1)x^2+3(x^3-x^2-x+1)^{\frac{2}{3}}-3(x^3-x^2-x+1)^{\frac{1}{3}}x+6\,\mathrm{RootOf}(\_Z^2+\_Z+1)x-2x^2+3(x^3-x^2-x+1)^{\frac{1}{3}}-\mathrm{RootOf}(\_Z^2+\_Z+1)+4x-2}{-1+x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(x^3-x^2-x+1)^(1/3),x)
```

```
[Out] (-1+x)*(1+x)/((-1+x)^2*(1+x))^(1/3)+2/3*ln(-(-4*RootOf(_Z^2+_Z+1)^2*x^2+3*R
ootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(2/3)-3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(1/3
)*x+4*RootOf(_Z^2+_Z+1)^2*x-4*RootOf(_Z^2+_Z+1)*x^2+3*RootOf(_Z^2+_Z+1)*(x^
3-x^2-x+1)^(1/3)-3*(x^3-x^2-x+1)^(1/3)*x+2*RootOf(_Z^2+_Z+1)*x-x^2+3*(x^3-x
^2-x+1)^(1/3)+2*RootOf(_Z^2+_Z+1)+1)/(-1+x))+2/3*RootOf(_Z^2+_Z+1)*ln((-2*R
ootOf(_Z^2+_Z+1)^2*x^2+3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(2/3)+2*RootOf(_Z^
2+_Z+1)^2*x-5*RootOf(_Z^2+_Z+1)*x^2+3*(x^3-x^2-x+1)^(2/3)-3*(x^3-x^2-x+1)^(
1/3)*x+6*RootOf(_Z^2+_Z+1)*x-2*x^2+3*(x^3-x^2-x+1)^(1/3)-RootOf(_Z^2+_Z+1)+
4*x-2)/(-1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^3-x^2-x+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^3-x^2-x+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/(x^3 - x^2 - x + 1)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x-1}{{\left(x^3-x^2-x+1\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(x^3 - x^2 - x + 1)^(1/3),x)
```

```
[Out] int((x - 1)/(x^3 - x^2 - x + 1)^(1/3), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt[3]{(x-1)^2(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**3-x**2-x+1)**(1/3),x)
```

```
[Out] Integral((x - 1)/((x - 1)**2*(x + 1))**(1/3), x)
```

3.1659 $\int \frac{x}{\sqrt[3]{-1-x+x^2+x^3}}\,dx$

Optimal. Leaf size=141

$$\frac{\sqrt[3]{x-1}\,(x+1)^{2/3}\left((x-1)^{2/3}\sqrt[3]{x+1}+\frac{1}{3}\log\left(\sqrt[3]{x-1}-\sqrt[3]{x+1}\right)-\frac{1}{6}\log\left((x-1)^{2/3}+\sqrt[3]{x+1}\,\sqrt[3]{x-1}+(x+1)^{2/}\right.\right.}{\sqrt[3]{(x-1)(x+1)^2}}$$

Rubi [A] time = 0.20, antiderivative size = 191, normalized size of antiderivative = 1.35, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2081, 2077, 80, 60}

$$-\frac{(1-x)(x+1)}{\sqrt[3]{x^3+x^2-x-1}}+\frac{(-x-1)^{2/3}\sqrt[3]{x-1}\log\left(\frac{\sqrt[3]{x-1}}{\sqrt[3]{-x-1}}+1\right)}{2\sqrt[3]{x^3+x^2-x-1}}+\frac{(-x-1)^{2/3}\sqrt[3]{x-1}\log\left(-\frac{8}{3}(x+1)\right)}{6\sqrt[3]{x^3+x^2-x-1}}+\frac{(-x-1)^{2/3}\sqrt[3]{x-1}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x-1}}{\sqrt{3}\sqrt[3]{-x-1}}\right)}{\sqrt{3}\sqrt[3]{x^3+x^2-x-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 - x + x^2 + x^3)^(1/3),x]
```

```
[Out] -(((1 - x)*(1 + x))/(-1 - x + x^2 + x^3)^(1/3)) + ((-1 - x)^(2/3)*(-1 + x)^
(1/3)*ArcTan[1/Sqrt[3] - (2*(-1 + x)^(1/3))/(Sqrt[3]*(-1 - x)^(1/3))])/(Sqr
t[3]*(-1 - x + x^2 + x^3)^(1/3)) + ((-1 - x)^(2/3)*(-1 + x)^(1/3)*Log[1 + (
-1 + x)^(1/3)/(-1 - x)^(1/3)])/(2*(-1 - x + x^2 + x^3)^(1/3)) + ((-1 - x)^(
2/3)*(-1 + x)^(1/3)*Log[(-8*(1 + x))/3])/(6*(-1 - x + x^2 + x^3)^(1/3))
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 2077

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Dist[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int
[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d, e
, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 2081

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{-1-x+x^2+x^3}}\,dx = \text{Subst}\left(\int \frac{-\frac{1}{3}+x}{\sqrt[3]{-\frac{16}{27}-\frac{4x}{3}+x^3}}\,dx, x, \frac{1}{3}+x\right)$$

$$= \frac{\left(4\,2^{2/3}(-1-x)^{2/3}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{-\frac{1}{3}+x}{\left(-\frac{16}{9}-\frac{8x}{3}\right)^{2/3}\sqrt[3]{-\frac{16}{9}+\frac{4x}{3}}}\,dx, x, \frac{1}{3}+x\right)}{3\sqrt[3]{-1-x+x^2+x^3}}$$

$$= -\frac{(1-x)(1+x)}{\sqrt[3]{-1-x+x^2+x^3}} - \frac{\left(4\,2^{2/3}(-1-x)^{2/3}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\left(-\frac{16}{9}-\frac{8x}{3}\right)^{2/3}\sqrt[3]{-\frac{16}{9}+\frac{4x}{3}}}\,dx, x\right.}{9\sqrt[3]{-1-x+x^2+x^3}}$$

$$= -\frac{(1-x)(1+x)}{\sqrt[3]{-1-x+x^2+x^3}} + \frac{(-1-x)^{2/3}\sqrt[3]{-1+x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{-1+x}}{\sqrt{3}\sqrt[3]{-1-x}}\right)}{\sqrt{3}\sqrt[3]{-1-x+x^2+x^3}} + \frac{(-1-x)^{2/3}\sqrt[3]{-}}{2\sqrt[3]{-1}}$$

Mathematica [C] time = 0.02, size = 58, normalized size = 0.41

$$\frac{(x-1)\left(-\sqrt[3]{2}\,(x+1)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{1-x}{2}\right)+4x+4\right)}{4\sqrt[3]{(x-1)(x+1)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 - x + x^2 + x^3)^(1/3),x]
```

```
[Out] ((-1 + x)*(4 + 4*x - 2^(1/3)*(1 + x)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3,
 (1 - x)/2]))/(4*((-1 + x)*(1 + x)^2)^(1/3))
```

IntegrateAlgebraic [A] time = 5.28, size = 152, normalized size = 1.08

$$\frac{\sqrt[3]{x-1}\,(x+1)^{2/3}\left(\frac{2\sqrt[3]{x+1}}{\sqrt[3]{x-1}\left(\frac{x+1}{x-1}-1\right)}+\frac{1}{3}\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-1}}-1\right)-\frac{1}{6}\log\left(\frac{(x+1)^{2/3}}{(x-1)^{2/3}}+\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-1}}+1\right)-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{x+1}}{\sqrt{3}\sqrt[3]{x-1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}\right)}{\sqrt[3]{(x-1)(x+1)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(-1 - x + x^2 + x^3)^(1/3),x]
```

```
[Out] ((-1 + x)^(1/3)*(1 + x)^(2/3)*((2*(1 + x)^(1/3))/((-1 + x)^(1/3)*(-1 + (1 +
 x)/(-1 + x))) - ArcTan[1/Sqrt[3] + (2*(1 + x)^(1/3))/(Sqrt[3]*(-1 + x)^(1/
3))]/Sqrt[3] + Log[-1 + (1 + x)^(1/3)/(-1 + x)^(1/3)]/3 - Log[1 + (1 + x)^(
1/3)/(-1 + x)^(1/3) + (1 + x)^(2/3)/(-1 + x)^(2/3)]/6))/((-1 + x)*(1 + x)^2
)^(1/3)
```

fricas [A] time = 0.41, size = 151, normalized size = 1.07

$$\frac{2\sqrt{3}\,(x+1)\arctan\left(\frac{\sqrt{3}\,(x+1)+2\sqrt{3}\left(x^3+x^2-x-1\right)^{\frac{1}{3}}}{3\,(x+1)}\right)-(x+1)\log\left(\frac{x^2+\left(x^3+x^2-x-1\right)^{\frac{1}{3}}(x+1)+2\,x+\left(x^3+x^2-x-1\right)^{\frac{2}{3}}+1}{x^2+2\,x+1}\right)+2\,(x+1)\log\left(-\frac{x-\left(x^3+x^2-x-1\right)^{\frac{1}{3}}+1}{x+1}\right)+6\left(x^3+x^2-x-1\right)^{\frac{2}{3}}}{6\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+x^2-x-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(x + 1)*arctan(1/3*(sqrt(3)*(x + 1) + 2*sqrt(3)*(x^3 + x^2 -
 x - 1)^(1/3))/(x + 1)) - (x + 1)*log((x^2 + (x^3 + x^2 - x - 1)^(1/3)*(x +
```

```
 1) + 2*x + (x^3 + x^2 - x - 1)^(2/3) + 1)/(x^2 + 2*x + 1)) + 2*(x + 1)*log
(-(x - (x^3 + x^2 - x - 1)^(1/3) + 1)/(x + 1)) + 6*(x^3 + x^2 - x - 1)^(2/3
))/(x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+x^2-x-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+x^2-x-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x/(x^3 + x^2 - x - 1)^(1/3), x)
```

maple [C] time = 0.42, size = 343, normalized size = 2.43

$$\frac{(-1+x)(1+x)}{\left((-1+x)(1+x)^2\right)^{\frac{1}{3}}}+\frac{\ln\left(\frac{-4\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x^2+3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^3+x^2-x-1\right)^{\frac{2}{3}}-3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^3+x^2-x-1\right)^{\frac{1}{3}}x-4\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x-4\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2-3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^3+x^2-x-1\right)^{\frac{1}{3}}-3x\left(x^3+x^2-x-1\right)^{\frac{1}{3}}-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x-x^2-3\left(x^3+x^2-x-1\right)^{\frac{1}{3}}+2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)+1}{1+x}\right)}{3}+\frac{\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(-\frac{-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x^2+3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(x^3+x^2-x-1\right)^{\frac{2}{3}}-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x-5\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2+3\left(x^3+x^2-x-1\right)^{\frac{2}{3}}-3x\left(x^3+x^2-x-1\right)^{\frac{1}{3}}-6\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x-2x^2-3\left(x^3+x^2-x-1\right)^{\frac{1}{3}}-\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-4x-2}{1+x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3+x^2-x-1)^(1/3),x)
```

```
[Out] (-1+x)*(1+x)/((-1+x)*(1+x)^2)^(1/3)+1/3*ln((-4*RootOf(_Z^2+_Z+1)^2*x^2+3*Ro
otOf(_Z^2+_Z+1)*(x^3+x^2-x-1)^(2/3)-3*RootOf(_Z^2+_Z+1)*(x^3+x^2-x-1)^(1/3)
*x-4*RootOf(_Z^2+_Z+1)^2*x-4*RootOf(_Z^2+_Z+1)*x^2-3*RootOf(_Z^2+_Z+1)*(x^3
+x^2-x-1)^(1/3)-3*x*(x^3+x^2-x-1)^(1/3)-2*RootOf(_Z^2+_Z+1)*x-x^2-3*(x^3+x^
2-x-1)^(1/3)+2*RootOf(_Z^2+_Z+1)+1)/(1+x))+1/3*RootOf(_Z^2+_Z+1)*ln(-(-2*Ro
otOf(_Z^2+_Z+1)^2*x^2+3*RootOf(_Z^2+_Z+1)*(x^3+x^2-x-1)^(2/3)-2*RootOf(_Z^2
+_Z+1)^2*x-5*RootOf(_Z^2+_Z+1)*x^2+3*(x^3+x^2-x-1)^(2/3)-3*x*(x^3+x^2-x-1)^
(1/3)-6*RootOf(_Z^2+_Z+1)*x-2*x^2-3*(x^3+x^2-x-1)^(1/3)-RootOf(_Z^2+_Z+1)-4
*x-2)/(1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+x^2-x-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+x^2-x-1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^3 + x^2 - x - 1)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(x^3+x^2-x-1\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 - x + x^3 - 1)^(1/3),x)
```

```
[Out] int(x/(x^2 - x + x^3 - 1)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{\left(x-1\right)\left(x+1\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3+x**2-x-1)**(1/3),x)
```

```
[Out] Integral(x/((x - 1)*(x + 1)**2)**(1/3), x)
```

3.1660 $\int \frac{(3+2x)\left(1+x+3x^3\right)^{2/3}}{x^3\left(1+x+x^3\right)}\,dx$

Optimal. Leaf size=141

$$-2^{2/3}\log\left(2^{2/3}\sqrt[3]{3x^3+x+1}-2x\right)+2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{3x^3+x+1}+x}\right)-\frac{3\left(3x^3+x+1\right)^{2/3}}{2x^2}+\frac{\log\left(2^{2/3}\sqrt[3]{3x^3+x\right.}{}$$

Rubi [F] time = 1.76, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(3+2x)\left(1+x+3x^3\right)^{2/3}}{x^3\left(1+x+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((3 + 2*x)*(1 + x + 3*x^3)^(2/3))/(x^3*(1 + x + x^3)),x]
```

```
[Out] (3*(1 + x + 3*x^3)^(2/3)*Defer[Int][(((2/(-9 + Sqrt[85]))^(1/3) - ((-9 + Sq
rt[85])/2)^(1/3) + 3*x)^(2/3)*(1 + (2/(-9 + Sqrt[85]))^(2/3) + ((-9 + Sqrt[
85])/2)^(2/3) - 3*((2/(-9 + Sqrt[85]))^(1/3) - ((-9 + Sqrt[85])/2)^(1/3))*x
 + 9*x^2)^(2/3))/x^3, x])/(((2/(-9 + Sqrt[85]))^(1/3) - ((-9 + Sqrt[85])/2)
^(1/3) + 3*x)^(2/3)*(1 + (2/(-9 + Sqrt[85]))^(2/3) + ((-9 + Sqrt[85])/2)^(2
/3) - 3*((2/(-9 + Sqrt[85]))^(1/3) - ((-9 + Sqrt[85])/2)^(1/3))*x + 9*x^2)^
(2/3)) - ((1 + x + 3*x^3)^(2/3)*Defer[Int][(((2/(-9 + Sqrt[85]))^(1/3) - ((
-9 + Sqrt[85])/2)^(1/3) + 3*x)^(2/3)*(1 + (2/(-9 + Sqrt[85]))^(2/3) + ((-9
+ Sqrt[85])/2)^(2/3) - 3*((2/(-9 + Sqrt[85]))^(1/3) - ((-9 + Sqrt[85])/2)^(
1/3))*x + 9*x^2)^(2/3))/x^2, x])/(((2/(-9 + Sqrt[85]))^(1/3) - ((-9 + Sqrt[
85])/2)^(1/3) + 3*x)^(2/3)*(1 + (2/(-9 + Sqrt[85]))^(2/3) + ((-9 + Sqrt[85]
)/2)^(2/3) - 3*((2/(-9 + Sqrt[85]))^(1/3) - ((-9 + Sqrt[85])/2)^(1/3))*x +
9*x^2)^(2/3)) + ((1 + x + 3*x^3)^(2/3)*Defer[Int][(((2/(-9 + Sqrt[85]))^(1/
3) - ((-9 + Sqrt[85])/2)^(1/3) + 3*x)^(2/3)*(1 + (2/(-9 + Sqrt[85]))^(2/3)
+ ((-9 + Sqrt[85])/2)^(2/3) - 3*((2/(-9 + Sqrt[85]))^(1/3) - ((-9 + Sqrt[85
])/2)^(1/3))*x + 9*x^2)^(2/3))/x, x])/(((2/(-9 + Sqrt[85]))^(1/3) - ((-9 +
Sqrt[85])/2)^(1/3) + 3*x)^(2/3)*(1 + (2/(-9 + Sqrt[85]))^(2/3) + ((-9 + Sqr
t[85])/2)^(2/3) - 3*((2/(-9 + Sqrt[85]))^(1/3) - ((-9 + Sqrt[85])/2)^(1/3))
*x + 9*x^2)^(2/3)) - 4*Defer[Int][(1 + x + 3*x^3)^(2/3)/(1 + x + x^3), x] +
 Defer[Int][(x*(1 + x + 3*x^3)^(2/3))/(1 + x + x^3), x] - Defer[Int][(x^2*(
1 + x + 3*x^3)^(2/3))/(1 + x + x^3), x]
```

Rubi steps

$$\int \frac{(3+2x)\left(1+x+3x^3\right)^{2/3}}{x^3\left(1+x+x^3\right)}\,dx = \int\left(\frac{3\left(1+x+3x^3\right)^{2/3}}{x^3} - \frac{\left(1+x+3x^3\right)^{2/3}}{x^2} + \frac{\left(1+x+3x^3\right)^{2/3}}{x} + \frac{\left(-4+x-x^2 \cdots\right.}{1+\cdots}\right.$$

$$= 3\int \frac{\left(1+x+3x^3\right)^{2/3}}{x^3}\,dx - \int \frac{\left(1+x+3x^3\right)^{2/3}}{x^2}\,dx + \int \frac{\left(1+x+3x^3\right)^{2/3}}{x}\,dx + \cdots$$

$$= -\frac{\left(1+x+3x^3\right)^{2/3}\int \frac{\left(\sqrt[3]{\frac{2}{-9+\sqrt{85}}}-\sqrt[3]{\frac{1}{2}\left(-9+\sqrt{85}\right)}+3x\right)^{2/3}\left(1+\left(\frac{2}{-9+\sqrt{85}}\right)^{2/3}+\left(\frac{1}{2}\left(-9\cdots\right.\right.\right.}{x^2}}{\left(\sqrt[3]{\frac{2}{-9+\sqrt{85}}}-\sqrt[3]{\frac{1}{2}\left(-9+\sqrt{85}\right)}+3x\right)^{2/3}\left(1+\left(\frac{2}{-9+\sqrt{85}}\right)^{2/3}+\left(\frac{1}{2}\left(-9+\sqrt{85}\cdots\right.\right.\right.}$$

$$= -\left(4\int \frac{\left(1+x+3x^3\right)^{2/3}}{1+x+x^3}\,dx\right) - \frac{\left(1+x+3x^3\right)^{2/3}\int \frac{\left(\sqrt[3]{\frac{2}{-9+\sqrt{85}}}-\sqrt[3]{\frac{1}{2}\left(-9+\cdots\right.}\right.}{\cdots}}{\left(\sqrt[3]{\frac{2}{-9+\sqrt{85}}}-\sqrt[3]{\frac{1}{2}\left(-9+\sqrt{85}\right)}+3x\right)^{2/3}\left(1+\cdots\right.}$$

Mathematica [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(3+2x)\left(1+x+3x^3\right)^{2/3}}{x^3\left(1+x+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((3 + 2*x)*(1 + x + 3*x^3)^(2/3))/(x^3*(1 + x + x^3)),x]
```

```
[Out] Integrate[((3 + 2*x)*(1 + x + 3*x^3)^(2/3))/(x^3*(1 + x + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.50, size = 141, normalized size = 1.00

$$-2^{2/3}\log\left(2^{2/3}\sqrt[3]{3x^3+x+1}-2x\right)+2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{3x^3+x+1}+x}\right)-\frac{3\left(3x^3+x+1\right)^{2/3}}{2x^2}+\frac{\log\left(2^{2/3}\sqrt[3]{3x^3+x+1}\,x+\sqrt[3]{2}\left(3x^3+x+1\right)^{2/3}+2x^2\right)}{\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3 + 2*x)*(1 + x + 3*x^3)^(2/3))/(x^3*(1 + x + x^3)),x]
```

```
[Out] (-3*(1 + x + 3*x^3)^(2/3))/(2*x^2) + 2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x
+ 2^(2/3)*(1 + x + 3*x^3)^(1/3))] - 2^(2/3)*Log[-2*x + 2^(2/3)*(1 + x + 3*x
^3)^(1/3)] + Log[2*x^2 + 2^(2/3)*x*(1 + x + 3*x^3)^(1/3) + 2^(1/3)*(1 + x +
 3*x^3)^(2/3)]/2^(1/3)
```

fricas [B] time = 10.65, size = 380, normalized size = 2.70

$$\frac{2\sqrt{3}(-4)^{\frac{1}{3}}x^2\arctan\left(\frac{3\sqrt{3}(-4)^{\frac{2}{3}}\left(7x^7+8x^5+8x^4+x^3+2x^2+x\right)\left(3x^3+x+1\right)^{\frac{2}{3}}-6\sqrt{3}(-4)^{\frac{1}{3}}\left(55x^8+20x^6+20x^5+x^4+2x^3+x^2\right)\left(3x^3+x+1\right)^{\frac{1}{3}}+\sqrt{3}\left(433x^9+255x^7+255x^6+39x^5+78x^4+40x^3+3x^2+3x+1\right)}{3\left(323x^9+105x^7+105x^6-3x^5-6x^4-4x^3-3x^2-3x-1\right)}\right)+2(-4)^{\frac{1}{3}}x^2\log\left(\frac{3(-4)^{\frac{2}{3}}\left(3x^3+x+1\right)^{\frac{1}{3}}x^2-6\left(3x^3+x+1\right)^{\frac{2}{3}}x-(-4)^{\frac{1}{3}}\left(x^3+x+1\right)}{x^3+x+1}\right)-(-4)^{\frac{1}{3}}x^2\log\left(-\frac{6(-4)^{\frac{1}{3}}\left(7x^4+x^2+x\right)\left(3x^3+x+1\right)^{\frac{2}{3}}-(-4)^{\frac{2}{3}}\left(55x^6+20x^4+20x^3+x^2+2x+1\right)-24\left(4x^5+x^3+x^2\right)\left(3x^3+x+1\right)^{\frac{1}{3}}}{x^6+2x^4+2x^3+x^2+2x+1}\right)-9\left(3x^3+x+1\right)^{\frac{2}{3}}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)*(3*x^3+x+1)^(2/3)/x^3/(x^3+x+1),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(-4)^(1/3)*x^2*arctan(1/3*(3*sqrt(3)*(-4)^(2/3)*(7*x^7 + 8*x
^5 + 8*x^4 + x^3 + 2*x^2 + x)*(3*x^3 + x + 1)^(2/3) - 6*sqrt(3)*(-4)^(1/3)*
(55*x^8 + 20*x^6 + 20*x^5 + x^4 + 2*x^3 + x^2)*(3*x^3 + x + 1)^(1/3) + sqrt
(3)*(433*x^9 + 255*x^7 + 255*x^6 + 39*x^5 + 78*x^4 + 40*x^3 + 3*x^2 + 3*x +
 1))/(323*x^9 + 105*x^7 + 105*x^6 - 3*x^5 - 6*x^4 - 4*x^3 - 3*x^2 - 3*x - 1
)) + 2*(-4)^(1/3)*x^2*log((3*(-4)^(2/3)*(3*x^3 + x + 1)^(1/3)*x^2 - 6*(3*x^
```

```
3 + x + 1)^(2/3)*x - (-4)^(1/3)*(x^3 + x + 1))/(x^3 + x + 1)) - (-4)^(1/3)*
x^2*log(-(6*(-4)^(1/3)*(7*x^4 + x^2 + x)*(3*x^3 + x + 1)^(2/3) - (-4)^(2/3)
*(55*x^6 + 20*x^4 + 20*x^3 + x^2 + 2*x + 1) - 24*(4*x^5 + x^3 + x^2)*(3*x^3
 + x + 1)^(1/3))/(x^6 + 2*x^4 + 2*x^3 + x^2 + 2*x + 1)) - 9*(3*x^3 + x + 1)
^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^3+x+1\right)^{\frac{2}{3}}(2\,x+3)}{\left(x^3+x+1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)*(3*x^3+x+1)^(2/3)/x^3/(x^3+x+1),x, algorithm="giac")
```

```
[Out] integrate((3*x^3 + x + 1)^(2/3)*(2*x + 3)/((x^3 + x + 1)*x^3), x)
```

maple [C] time = 4.49, size = 695, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)*(3*x^3+x+1)^(2/3)/x^3/(x^3+x+1),x)
```

```
[Out] -3/2*(3*x^3+x+1)^(2/3)/x^2+RootOf(_Z^3+4)*ln(-(RootOf(RootOf(_Z^3+4)^2+2*_Z
*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x^3+5*RootOf(RootOf(_Z^3+4)^2+2*_Z
*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+4*(3*x^3+x+1)^(2/3)*RootOf(R
ootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*x+4*RootOf(_Z^
3+4)^2*(3*x^3+x+1)^(1/3)*x^2+(3*x^3+x+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z
*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*x^2-5*RootOf(_Z^3+4)*x^3-25*RootOf(R
ootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^3-7*(3*x^3+x+1)^(2/3)*x-RootO
f(_Z^3+4)*x-5*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x-RootOf(
_Z^3+4)-5*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(x^3+x+1))+2
*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*ln((5*RootOf(RootOf(_Z
^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x^3+4*RootOf(RootOf(_Z
^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+8*(3*x^3+x+1)^(2
/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*x+
8*RootOf(_Z^3+4)^2*(3*x^3+x+1)^(1/3)*x^2+14*(3*x^3+x+1)^(1/3)*RootOf(RootOf
(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*x^2+15*RootOf(_Z^3+4)
*x^3+12*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^3-2*(3*x^3+x+
1)^(2/3)*x+5*RootOf(_Z^3+4)*x+4*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)
+4*_Z^2)*x+5*RootOf(_Z^3+4)+4*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4
*_Z^2))/(x^3+x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^3+x+1\right)^{\frac{2}{3}}(2\,x+3)}{\left(x^3+x+1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)*(3*x^3+x+1)^(2/3)/x^3/(x^3+x+1),x, algorithm="maxima")
```

```
[Out] integrate((3*x^3 + x + 1)^(2/3)*(2*x + 3)/((x^3 + x + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x+3)\,\left(3\,x^3+x+1\right)^{2/3}}{x^3\,\left(x^3+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 3)*(x + 3*x^3 + 1)^(2/3))/(x^3*(x + x^3 + 1)),x)
[Out] int(((2*x + 3)*(x + 3*x^3 + 1)^(2/3))/(x^3*(x + x^3 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)*(3*x**3+x+1)**(2/3)/x**3/(x**3+x+1),x)
[Out] Timed out
```

3.1661 $\displaystyle\int \frac{x^2\left(-3ab^3+2b^2(3a+b)x-3b(a+b)x^2+x^4\right)}{\left(x(-a+x)(-b+x)^3\right)^{3/4}\left(ab^3-b^2(3a+b)x+3b(a+b)x^2-(a+3b+d)x^3+x^4\right)}\,dx$

Optimal. Leaf size=141

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{ab^3x+x^3\left(3ab+3b^2\right)+x^2\left(-3ab^2-b^3\right)+x^4(-a-3b)+x^5}}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{ab^3x+x^3\left(3ab+3b^2\right)+x^2\left(-3ab^2-b^3\right)+x^4(-a-3b)+x^5}}\right)}{d^{3/4}}$$

Rubi [F] time = 29.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(-3ab^3+2b^2(3a+b)x-3b(a+b)x^2+x^4\right)}{\left(x(-a+x)(-b+x)^3\right)^{3/4}\left(ab^3-b^2(3a+b)x+3b(a+b)x^2-(a+3b+d)x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(-3*a*b^3 + 2*b^2*(3*a + b)*x - 3*b*(a + b)*x^2 + x^4))/((x*(-a +
x)*(-b + x)^3)^(3/4)*(a*b^3 - b^2*(3*a + b)*x + 3*b*(a + b)*x^2 - (a + 3*b
+ d)*x^3 + x^4)),x]
```

```
[Out] (4*(b - x)^2*x*(1 - x/a)^(3/4)*(1 - x/b)^(1/4)*AppellF1[1/4, 3/4, 1/4, 5/4,
 x/a, x/b])/((a - x)*(b - x)^3*x)^(3/4) - (4*a*b^3*x^(3/4)*(-a + x)^(3/4)*(
-b + x)^(9/4)*Defer[Subst][Defer[Int][1/((-a + x^4)^(3/4)*(-b + x^4)^(1/4)*
(a*b^3 - 3*a*b^2*(1 + b/(3*a))*x^4 + 3*a*b*(1 + b/a)*x^8 - a*(1 + (3*b + d)
/a)*x^12 + x^16)), x], x, x^(1/4)])/((a - x)*(b - x)^3*x)^(3/4) + (4*b^2*(3
*a + b)*x^(3/4)*(-a + x)^(3/4)*(-b + x)^(9/4)*Defer[Subst][Defer[Int][x^4/(
(-a + x^4)^(3/4)*(-b + x^4)^(1/4)*(a*b^3 - 3*a*b^2*(1 + b/(3*a))*x^4 + 3*a*
b*(1 + b/a)*x^8 - a*(1 + (3*b + d)/a)*x^12 + x^16)), x], x, x^(1/4)])/((a -
 x)*(b - x)^3*x)^(3/4) - (12*b*(2*a + b)*x^(3/4)*(-a + x)^(3/4)*(-b + x)^(9
/4)*Defer[Subst][Defer[Int][x^8/((-a + x^4)^(3/4)*(-b + x^4)^(1/4)*(a*b^3 -
 3*a*b^2*(1 + b/(3*a))*x^4 + 3*a*b*(1 + b/a)*x^8 - a*(1 + (3*b + d)/a)*x^12
 + x^16)), x], x, x^(1/4)])/((a - x)*(b - x)^3*x)^(3/4) + (4*(a + 5*b + d)*
x^(3/4)*(-a + x)^(3/4)*(-b + x)^(9/4)*Defer[Subst][Defer[Int][x^12/((-a + x
^4)^(3/4)*(-b + x^4)^(1/4)*(a*b^3 - 3*a*b^2*(1 + b/(3*a))*x^4 + 3*a*b*(1 +
b/a)*x^8 - a*(1 + (3*b + d)/a)*x^12 + x^16)), x], x, x^(1/4)])/((a - x)*(b
- x)^3*x)^(3/4)
```

Rubi steps

$$\int \frac{x^2\left(-3ab^3+2b^2(3a+b)x-3b(a+b)x^2+x^4\right)}{\left(x(-a+x)(-b+x)^3\right)^{3/4}\left(ab^3-b^2(3a+b)x+3b(a+b)x^2-(a+3b+d)x^3+x^4\right)}dx = \frac{\left(x^{3/4}(-a+x)^{3/4}(-b+\right.}{}$$

$$= \frac{\left(x^{3/4}(-a+x)^{3/4}(-b+\right.}{}$$

$$= \frac{\left(x^{3/4}(-a+x)^{3/4}(-b+\right.}{}$$

$$= \frac{\left(4x^{3/4}(-a+x)^{3/4}(-b\right.}{}$$

$$= \frac{\left(4x^{3/4}(-a+x)^{3/4}(-b\right.}{}$$

$$= \frac{\left(4x^{3/4}(-a+x)^{3/4}(-b\right.}{}$$

$$= -\frac{\left(4x^{3/4}(-a+x)^{3/4}(-\right.}{}$$

$$= -\frac{\left(4ab^3x^{3/4}(-a+x)^{3/\right.}{}$$

$$= \frac{4(b-x)^2x\left(1-\frac{x}{a}\right)^{3/4}}{\left((a-x\right.}$$

Mathematica [F] time = 4.26, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-3ab^3+2b^2(3a+b)x-3b(a+b)x^2+x^4\right)}{\left(x(-a+x)(-b+x)^3\right)^{3/4}\left(ab^3-b^2(3a+b)x+3b(a+b)x^2-(a+3b+d)x^3+x^4\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(-3*a*b^3 + 2*b^2*(3*a + b)*x - 3*b*(a + b)*x^2 + x^4))/((x*
(-a + x)*(-b + x)^3)^(3/4)*(a*b^3 - b^2*(3*a + b)*x + 3*b*(a + b)*x^2 - (a
+ 3*b + d)*x^3 + x^4)),x]
```

```
[Out] Integrate[(x^2*(-3*a*b^3 + 2*b^2*(3*a + b)*x - 3*b*(a + b)*x^2 + x^4))/((x*
(-a + x)*(-b + x)^3)^(3/4)*(a*b^3 - b^2*(3*a + b)*x + 3*b*(a + b)*x^2 - (a
+ 3*b + d)*x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 3.98, size = 141, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{ab^3x+x^3(3ab+3b^2)+x^2(-3ab^2-b^3)+x^4(-a-3b)+x^5}}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{ab^3x+x^3(3ab+3b^2)+x^2(-3ab^2-b^3)+x^4(-a-3b)+x^5}}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(-3*a*b^3 + 2*b^2*(3*a + b)*x - 3*b*(a + b)*x^2 + x
^4))/((x*(-a + x)*(-b + x)^3)^(3/4)*(a*b^3 - b^2*(3*a + b)*x + 3*b*(a + b)*
x^2 - (a + 3*b + d)*x^3 + x^4)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*x)/(a*b^3*x + (-3*a*b^2 - b^3)*x^2 + (3*a*b + 3*b^2)*x^3
 + (-a - 3*b)*x^4 + x^5)^(1/4)]])/d^(3/4) - (2*ArcTanh[(d^(1/4)*x)/(a*b^3*x
+ (-3*a*b^2 - b^3)*x^2 + (3*a*b + 3*b^2)*x^3 + (-a - 3*b)*x^4 + x^5)^(1/4)]
)/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-3*a*b^3+2*b^2*(3*a+b)*x-3*b*(a+b)*x^2+x^4)/(x*(-a+x)*(-b+x)
^3)^(3/4)/(a*b^3-b^2*(3*a+b)*x+3*b*(a+b)*x^2-(a+3*b+d)*x^3+x^4),x, algorith
m="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(3\,ab^3-2\,(3\,a+b)b^2x+3\,(a+b)bx^2-x^4\right)x^2}{\left((a-x)(b-x)^3x\right)^{\frac{3}{4}}\left(ab^3-(3\,a+b)b^2x+3\,(a+b)bx^2-(a+3\,b+d)x^3+x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-3*a*b^3+2*b^2*(3*a+b)*x-3*b*(a+b)*x^2+x^4)/(x*(-a+x)*(-b+x)
^3)^(3/4)/(a*b^3-b^2*(3*a+b)*x+3*b*(a+b)*x^2-(a+3*b+d)*x^3+x^4),x, algorith
m="giac")
```

```
[Out] integrate(-(3*a*b^3 - 2*(3*a + b)*b^2*x + 3*(a + b)*b*x^2 - x^4)*x^2/(((a -
 x)*(b - x)^3*x)^(3/4)*(a*b^3 - (3*a + b)*b^2*x + 3*(a + b)*b*x^2 - (a + 3*
b + d)*x^3 + x^4)), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-3a\,b^3+2b^2\,(3a+b)\,x-3b\,(a+b)\,x^2+x^4\right)}{\left(x\,(-a+x)\,(-b+x)^3\right)^{\frac{3}{4}}\left(a\,b^3-b^2\,(3a+b)\,x+3b\,(a+b)\,x^2-(a+3b+d)\,x^3+x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-3*a*b^3+2*b^2*(3*a+b)*x-3*b*(a+b)*x^2+x^4)/(x*(-a+x)*(-b+x)^3)^(3
/4)/(a*b^3-b^2*(3*a+b)*x+3*b*(a+b)*x^2-(a+3*b+d)*x^3+x^4),x)
```

```
[Out] int(x^2*(-3*a*b^3+2*b^2*(3*a+b)*x-3*b*(a+b)*x^2+x^4)/(x*(-a+x)*(-b+x)^3)^(3
/4)/(a*b^3-b^2*(3*a+b)*x+3*b*(a+b)*x^2-(a+3*b+d)*x^3+x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(3\,ab^3-2\,(3\,a+b)b^2x+3\,(a+b)bx^2-x^4\right)x^2}{\left((a-x)(b-x)^3x\right)^{\frac{3}{4}}\left(ab^3-(3\,a+b)b^2x+3\,(a+b)bx^2-(a+3\,b+d)x^3+x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-3*a*b^3+2*b^2*(3*a+b)*x-3*b*(a+b)*x^2+x^4)/(x*(-a+x)*(-b+x)
^3)^(3/4)/(a*b^3-b^2*(3*a+b)*x+3*b*(a+b)*x^2-(a+3*b+d)*x^3+x^4),x, algorith
m="maxima")
```

```
[Out] -integrate((3*a*b^3 - 2*(3*a + b)*b^2*x + 3*(a + b)*b*x^2 - x^4)*x^2/(((a -
 x)*(b - x)^3*x)^(3/4)*(a*b^3 - (3*a + b)*b^2*x + 3*(a + b)*b*x^2 - (a + 3*
b + d)*x^3 + x^4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^2\,\left(3\,a\,b^3-x^4-2\,b^2\,x\,\left(3\,a+b\right)+3\,b\,x^2\,\left(a+b\right)\right)}{{\left(x\,\left(a-x\right)\,{\left(b-x\right)}^3\right)}^{3/4}\,\left(a\,b^3-x^3\,\left(a+3\,b+d\right)+x^4-b^2\,x\,\left(3\,a+b\right)+3\,b\,x^2\,\left(a+b\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(3*a*b^3 - x^4 - 2*b^2*x*(3*a + b) + 3*b*x^2*(a + b)))/((x*(a - x
)*(b - x)^3)^(3/4)*(a*b^3 - x^3*(a + 3*b + d) + x^4 - b^2*x*(3*a + b) + 3*b
*x^2*(a + b))),x)
```

```
[Out] int(-(x^2*(3*a*b^3 - x^4 - 2*b^2*x*(3*a + b) + 3*b*x^2*(a + b)))/((x*(a - x
)*(b - x)^3)^(3/4)*(a*b^3 - x^3*(a + 3*b + d) + x^4 - b^2*x*(3*a + b) + 3*b
*x^2*(a + b))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-3*a*b**3+2*b**2*(3*a+b)*x-3*b*(a+b)*x**2+x**4)/(x*(-a+x)*(
-b+x)**3)**(3/4)/(a*b**3-b**2*(3*a+b)*x+3*b*(a+b)*x**2-(a+3*b+d)*x**3+x**4)
,x)
```

```
[Out] Timed out
```

3.1662 $\int \frac{x^6}{(-b+ax^4)^{3/4}(b+ax^4)}\,dx$

Optimal. Leaf size=141

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2a^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2\,2^{3/4}a^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2a^{7/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2\,2^{3/4}a^{7/4}}$$

Rubi [A] time = 0.13, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {494, 481, 298, 203, 206}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2a^{7/4}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2\,2^{3/4}a^{7/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2a^{7/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2\,2^{3/4}a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((-b + a*x^4)^(3/4)*(b + a*x^4)),x]
```

```
[Out] -1/2*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)]/a^(7/4) + ArcTan[(2^(1/4)*a^(1/
4)*x)/(-b + a*x^4)^(1/4)]/(2*2^(3/4)*a^(7/4)) + ArcTanh[(a^(1/4)*x)/(-b + a
*x^4)^(1/4)]/(2*a^(7/4)) - ArcTanh[(2^(1/4)*a^(1/4)*x)/(-b + a*x^4)^(1/4)]/
(2*2^(3/4)*a^(7/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
```

```
tQ[-1, p, 0]
```

Rubi steps

$$\int \frac{x^6}{\left(-b+ax^4\right)^{3/4}\left(b+ax^4\right)}\,dx = -\left(b\,\text{Subst}\left(\int \frac{x^6}{\left(1-ax^4\right)\left(b-2abx^4\right)}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{a} - \frac{b\,\text{Subst}\left(\int \frac{x^2}{b-2abx^4}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{2a^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{2a^{3/2}} - \frac{\text{Subst}}{}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{2a^{7/4}} + \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{2\,2^{3/4}a^{7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{2a^{7/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{2\,2^{3/4}a^{7/4}}$$

Mathematica [C] time = 0.06, size = 67, normalized size = 0.48

$$\frac{x^7\left(\frac{b-ax^4}{b}\right)^{3/4}F_1\left(\frac{7}{4};\frac{3}{4},1;\frac{11}{4};\frac{ax^4}{b},-\frac{ax^4}{b}\right)}{7b\left(ax^4-b\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((-b + a*x^4)^(3/4)*(b + a*x^4)),x]
```

```
[Out] (x^7*((b - a*x^4)/b)^(3/4)*AppellF1[7/4, 3/4, 1, 11/4, (a*x^4)/b, -((a*x^4)
/b)])/(7*b*(-b + a*x^4)^(3/4))
```

IntegrateAlgebraic [A] time = 0.81, size = 141, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2a^{7/4}} + \frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2\,2^{3/4}a^{7/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2a^{7/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2\,2^{3/4}a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/((-b + a*x^4)^(3/4)*(b + a*x^4)),x]
```

```
[Out] -1/2*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)]/a^(7/4) + ArcTan[(2^(1/4)*a^(1/
4)*x)/(-b + a*x^4)^(1/4)]/(2*2^(3/4)*a^(7/4)) + ArcTanh[(a^(1/4)*x)/(-b + a
*x^4)^(1/4)]/(2*a^(7/4)) - ArcTanh[(2^(1/4)*a^(1/4)*x)/(-b + a*x^4)^(1/4)]/
(2*2^(3/4)*a^(7/4))
```

fricas [B] time = 0.44, size = 318, normalized size = 2.26

$$\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{a^7}^{\frac{1}{4}}\arctan\left(\frac{4\left(\left(\frac{1}{8}\right)^{\frac{3}{4}}a^5\sqrt{\frac{2\sqrt{\frac{1}{2}}a^4\sqrt{\frac{1}{a^7}}x^2+\sqrt{ax^4-b}}{x^2}}\frac{1}{a^7}^{\frac{3}{4}}x-\left(\frac{1}{8}\right)^{\frac{3}{4}}\left(ax^4-b\right)^{\frac{1}{4}}a^5\frac{1}{a^7}^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{4}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{a^7}^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{8}\right)^{\frac{1}{4}}a^2\frac{1}{a^7}^{\frac{1}{4}}x+\left(ax^4-b\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{4}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{a^7}^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{8}\right)^{\frac{1}{4}}a^2\frac{1}{a^7}^{\frac{1}{4}}x-\left(ax^4-b\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{a^7}^{\frac{1}{4}}\arctan\left(\frac{a^5\frac{1}{a^7}^{\frac{3}{4}}x\sqrt{\frac{a^4\sqrt{\frac{1}{a^7}}x^2+\sqrt{ax^4-b}}{x^2}}-\left(ax^4-b\right)^{\frac{1}{4}}a^5\frac{1}{a^7}^{\frac{3}{4}}}{x}\right)+\frac{1}{4}\frac{1}{a^7}^{\frac{1}{4}}\log\left(\frac{a^2\frac{1}{a^7}^{\frac{1}{4}}x+\left(ax^4-b\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{4}\frac{1}{a^7}^{\frac{1}{4}}\log\left(-\frac{a^2\frac{1}{a^7}^{\frac{1}{4}}x-\left(ax^4-b\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a*x^4-b)^(3/4)/(a*x^4+b),x, algorithm="fricas")
```

```
[Out] (1/8)^(1/4)*(a^(-7))^(1/4)*arctan(4*((1/8)^(3/4)*a^5*sqrt((2*sqrt(1/2)*a^4*
sqrt(a^(-7))*x^2 + sqrt(a*x^4 - b))/x^2)*(a^(-7))^(3/4)*x - (1/8)^(3/4)*(a*
x^4 - b)^(1/4)*a^5*(a^(-7))^(3/4))/x) - 1/4*(1/8)^(1/4)*(a^(-7))^(1/4)*log(
```

```
(2*(1/8)^(1/4)*a^2*(a^(-7))^(1/4)*x + (a*x^4 - b)^(1/4))/x) + 1/4*(1/8)^(1/
4)*(a^(-7))^(1/4)*log(-(2*(1/8)^(1/4)*a^2*(a^(-7))^(1/4)*x - (a*x^4 - b)^(1
/4))/x) - (a^(-7))^(1/4)*arctan((a^5*(a^(-7))^(3/4)*x*sqrt((a^4*sqrt(a^(-7)
)*x^2 + sqrt(a*x^4 - b))/x^2) - (a*x^4 - b)^(1/4)*a^5*(a^(-7))^(3/4))/x) +
1/4*(a^(-7))^(1/4)*log((a^2*(a^(-7))^(1/4)*x + (a*x^4 - b)^(1/4))/x) - 1/4*
(a^(-7))^(1/4)*log(-(a^2*(a^(-7))^(1/4)*x - (a*x^4 - b)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(ax^4+b\right)\left(ax^4-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a*x^4-b)^(3/4)/(a*x^4+b),x, algorithm="giac")
```

```
[Out] integrate(x^6/((a*x^4 + b)*(a*x^4 - b)^(3/4)), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a\,x^4-b\right)^{\frac{3}{4}}\left(a\,x^4+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^4-b)^(3/4)/(a*x^4+b),x)
```

```
[Out] int(x^6/(a*x^4-b)^(3/4)/(a*x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(ax^4+b\right)\left(ax^4-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a*x^4-b)^(3/4)/(a*x^4+b),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((a*x^4 + b)*(a*x^4 - b)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(a\,x^4+b\right)\,{\left(a\,x^4-b\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((b + a*x^4)*(a*x^4 - b)^(3/4)),x)
```

```
[Out] int(x^6/((b + a*x^4)*(a*x^4 - b)^(3/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(ax^4-b\right)^{\frac{3}{4}}\left(ax^4+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(a*x**4-b)**(3/4)/(a*x**4+b),x)
```

```
[Out] Integral(x**6/((a*x**4 - b)**(3/4)*(a*x**4 + b)), x)
```

3.1663 $\int \frac{-b+2ax^2}{(-b+ax^2)\sqrt[4]{bx^2+ax^4}}\,dx$

Optimal. Leaf size=141

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{\sqrt[4]{a}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{\sqrt[4]{2}\,\sqrt[4]{a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{\sqrt[4]{2}\,\sqrt[4]{a}}$$

Rubi [A] time = 0.35, antiderivative size = 265, normalized size of antiderivative = 1.88, number of steps used = 13, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {2056, 584, 329, 240, 212, 206, 203, 466, 377}

$$\frac{2\sqrt{x}\,\sqrt[4]{ax^2+b}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{\sqrt[4]{a}\,\sqrt[4]{ax^4+bx^2}}-\frac{\sqrt{x}\,\sqrt[4]{ax^2+b}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{\sqrt[4]{2}\,\sqrt[4]{a}\,\sqrt[4]{ax^4+bx^2}}+\frac{2\sqrt{x}\,\sqrt[4]{ax^2+b}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{\sqrt[4]{a}\,\sqrt[4]{ax^4+bx^2}}-\frac{\sqrt{x}\,\sqrt[4]{ax^2+b}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{\sqrt[4]{2}\,\sqrt[4]{a}\,\sqrt[4]{ax^4+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + 2*a*x^2)/((-b + a*x^2)*(b*x^2 + a*x^4)^(1/4)),x]
```

```
[Out] (2*Sqrt[x]*(b + a*x^2)^(1/4)*ArcTan[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(
a^(1/4)*(b*x^2 + a*x^4)^(1/4)) - (Sqrt[x]*(b + a*x^2)^(1/4)*ArcTan[(2^(1/4)
*a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(2^(1/4)*a^(1/4)*(b*x^2 + a*x^4)^(1/4
)) + (2*Sqrt[x]*(b + a*x^2)^(1/4)*ArcTanh[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/
4)])/(a^(1/4)*(b*x^2 + a*x^4)^(1/4)) - (Sqrt[x]*(b + a*x^2)^(1/4)*ArcTanh[(
2^(1/4)*a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(2^(1/4)*a^(1/4)*(b*x^2 + a*x^
4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{-b+2ax^2}{\left(-b+ax^2\right)\sqrt[4]{bx^2+ax^4}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{b+ax^2}\right)\int \frac{-b+2ax^2}{\sqrt{x}\left(-b+ax^2\right)\sqrt[4]{b+ax^2}}\,dx}{\sqrt[4]{bx^2+ax^4}}$$

$$= \frac{\left(\sqrt{x}\sqrt[4]{b+ax^2}\right)\int \left(\frac{2}{\sqrt{x}\sqrt[4]{b+ax^2}}+\frac{b}{\sqrt{x}\left(-b+ax^2\right)\sqrt[4]{b+ax^2}}\right)dx}{\sqrt[4]{bx^2+ax^4}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{b+ax^2}\right)\int \frac{1}{\sqrt{x}\sqrt[4]{b+ax^2}}\,dx}{\sqrt[4]{bx^2+ax^4}} + \frac{\left(b\sqrt{x}\sqrt[4]{b+ax^2}\right)\int \frac{1}{\sqrt{x}\left(-b+ax^2\right)\sqrt[4]{b+ax^2}}\,dx}{\sqrt[4]{bx^2+ax^4}}$$

$$= \frac{\left(4\sqrt{x}\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{bx^2+ax^4}} + \frac{\left(2b\sqrt{x}\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{}{(-b+}\right.}{\sqrt[4]{bx^2+ax^4}}$$

$$= \frac{\left(4\sqrt{x}\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{\sqrt[4]{bx^2+ax^4}} + \frac{\left(2b\sqrt{x}\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{}{-b}\right.}{\sqrt[4]{bx^2+ax^4}}$$

$$= -\frac{\left(\sqrt{x}\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\sqrt{a}x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{\sqrt[4]{bx^2+ax^4}} - \frac{\left(\sqrt{x}\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int\right.}{\sqrt[4]{bx^2+}}$$

$$= \frac{2\sqrt{x}\sqrt[4]{b+ax^2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{\sqrt[4]{a}\sqrt[4]{bx^2+ax^4}} - \frac{\sqrt{x}\sqrt[4]{b+ax^2}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{\sqrt[4]{2}\sqrt[4]{a}\sqrt[4]{bx^2+ax^4}} + \frac{2\sqrt{x}\sqrt[4]{b+a}}{\sqrt[4]{a}}$$

Mathematica [A] time = 0.18, size = 161, normalized size = 1.14

$$\frac{\sqrt{x}\sqrt[4]{ax^2+b}\left(4\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)-2^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)+4\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)-2^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{x^2(ax^2+b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + 2*a*x^2)/((-b + a*x^2)*(b*x^2 + a*x^4)^(1/4)),x]
```

```
[Out] (Sqrt[x]*(b + a*x^2)^(1/4)*(4*ArcTan[(a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)] -
 2^(3/4)*ArcTan[(2^(1/4)*a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)] + 4*ArcTanh[(a
^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)] - 2^(3/4)*ArcTanh[(2^(1/4)*a^(1/4)*Sqrt[
x])/(b + a*x^2)^(1/4)]))/(2*a^(1/4)*(x^2*(b + a*x^2))^(1/4))
```

IntegrateAlgebraic [A] time = 0.43, size = 141, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+bx^2}}\right)}{\sqrt[4]{a}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4+bx^2}}\right)}{\sqrt[4]{2}\sqrt[4]{a}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+bx^2}}\right)}{\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4+bx^2}}\right)}{\sqrt[4]{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + 2*a*x^2)/((-b + a*x^2)*(b*x^2 + a*x^4)^(1/4)),x]
```

```
[Out] (2*ArcTan[(a^(1/4)*x)/(b*x^2 + a*x^4)^(1/4)])/a^(1/4) - ArcTan[(2^(1/4)*a^(
1/4)*x)/(b*x^2 + a*x^4)^(1/4)]/(2^(1/4)*a^(1/4)) + (2*ArcTanh[(a^(1/4)*x)/(
b*x^2 + a*x^4)^(1/4)])/a^(1/4) - ArcTanh[(2^(1/4)*a^(1/4)*x)/(b*x^2 + a*x^4
)^(1/4)]/(2^(1/4)*a^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2-b)/(a*x^2-b)/(a*x^4+b*x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.23, size = 377, normalized size = 2.67

$$\frac{\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a}+\frac{\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a}-\frac{\sqrt{2}(-a)^{\frac{3}{4}}\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)}{2a}+\frac{\sqrt{2}(-a)^{\frac{3}{4}}\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)}{2a}-\frac{2^{\frac{1}{4}}(-a)^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{2a}-\frac{2^{\frac{1}{4}}(-a)^{\frac{3}{4}}\arctan\left(-\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{2a}+\frac{2^{\frac{1}{4}}(-a)^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)}{4a}-\frac{2^{\frac{1}{4}}(-a)^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a+\frac{b}{x^2}}\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2-b)/(a*x^2-b)/(a*x^4+b*x^2)^(1/4),x, algorithm="giac")
```

```
[Out] sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^2)^(
1/4))/(-a)^(1/4))/a + sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^
(1/4) - 2*(a + b/x^2)^(1/4))/(-a)^(1/4))/a - 1/2*sqrt(2)*(-a)^(3/4)*log(sqr
t(2)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sqrt(a + b/x^2))/a + 1/2*sqr
t(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sqrt
(a + b/x^2))/a - 1/2*2^(1/4)*(-a)^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4)*(-a)^(1
/4) + 2*(a + b/x^2)^(1/4))/(-a)^(1/4))/a - 1/2*2^(1/4)*(-a)^(3/4)*arctan(-1
/2*2^(1/4)*(2^(3/4)*(-a)^(1/4) - 2*(a + b/x^2)^(1/4))/(-a)^(1/4))/a + 1/4*2
^(1/4)*(-a)^(3/4)*log(2^(3/4)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(2)*sqrt(-
a) + sqrt(a + b/x^2))/a - 1/4*2^(1/4)*(-a)^(3/4)*log(-2^(3/4)*(-a)^(1/4)*(a
 + b/x^2)^(1/4) + sqrt(2)*sqrt(-a) + sqrt(a + b/x^2))/a
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{2a\,x^{2}-b}{\left(a\,x^{2}-b\right)\left(a\,x^{4}+b\,x^{2}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x^2-b)/(a*x^2-b)/(a*x^4+b*x^2)^(1/4),x)
[Out] int((2*a*x^2-b)/(a*x^2-b)/(a*x^4+b*x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax^{2}-b}{\left(ax^{4}+bx^{2}\right)^{\frac{1}{4}}\left(ax^{2}-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2-b)/(a*x^2-b)/(a*x^4+b*x^2)^(1/4),x, algorithm="maxima")
[Out] integrate((2*a*x^2 - b)/((a*x^4 + b*x^2)^(1/4)*(a*x^2 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b-2\,a\,x^{2}}{\left(b-a\,x^{2}\right)\,\left(a\,x^{4}+b\,x^{2}\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - 2*a*x^2)/((b - a*x^2)*(a*x^4 + b*x^2)^(1/4)),x)
[Out] int((b - 2*a*x^2)/((b - a*x^2)*(a*x^4 + b*x^2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2ax^{2}-b}{\sqrt[4]{x^{2}\left(ax^{2}+b\right)}\,\left(ax^{2}-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x**2-b)/(a*x**2-b)/(a*x**4+b*x**2)**(1/4),x)
[Out] Integral((2*a*x**2 - b)/((x**2*(a*x**2 + b))**(1/4)*(a*x**2 - b)), x)
```

3.1664 $\int \frac{-3b+2ax^4}{\left(-2b+ax^4\right)\sqrt[4]{b+ax^4}}\,dx$

Optimal. Leaf size=141

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{\sqrt[4]{a}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}\sqrt[4]{3}\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}\sqrt[4]{3}\sqrt[4]{a}}$$

Rubi [A] time = 0.11, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {530, 240, 212, 206, 203, 377}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{\sqrt[4]{a}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}\sqrt[4]{3}\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}\sqrt[4]{3}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-3*b + 2*a*x^4)/((-2*b + a*x^4)*(b + a*x^4)^(1/4)),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)]/a^(1/4) - ArcTan[((3/2)^(1/4)*a^(1/4)
*x)/(b + a*x^4)^(1/4)]/(2*2^(3/4)*3^(1/4)*a^(1/4)) + ArcTanh[(a^(1/4)*x)/(b
 + a*x^4)^(1/4)]/a^(1/4) - ArcTanh[((3/2)^(1/4)*a^(1/4)*x)/(b + a*x^4)^(1/4
)]/(2*2^(3/4)*3^(1/4)*a^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\int \frac{-3b+2ax^4}{\left(-2b+ax^4\right)\sqrt[4]{b+ax^4}}\,dx = 2\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx + b\int \frac{1}{\left(-2b+ax^4\right)\sqrt[4]{b+ax^4}}\,dx$$

$$= 2\,\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right) + b\,\text{Subst}\left(\int \frac{1}{-2b+3abx^4}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right.$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{3}\sqrt{a}x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}+\sqrt{3}\sqrt{a}x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{2\sqrt{2}} + \text{Su}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b+ax^4}}\right)}{\sqrt[4]{a}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\sqrt[4]{a}x}{\sqrt[4]{b+ax^4}}\right)}{2\,2^{3/4}\sqrt[4]{3}\sqrt[4]{a}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b+ax^4}}\right)}{\sqrt[4]{a}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\sqrt[4]{a}x}{\sqrt[4]{b+ax^4}}\right)}{2\,2^{3/4}\sqrt[4]{3}\sqrt[4]{a}}$$

Mathematica [A] time = 0.12, size = 128, normalized size = 0.91

$$\frac{12\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right) - \sqrt[4]{2}\,3^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right) + 12\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right) - \sqrt[4]{2}\,3^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{12\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3*b + 2*a*x^4)/((-2*b + a*x^4)*(b + a*x^4)^(1/4)),x]
```

```
[Out] (12*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)] - 2^(1/4)*3^(3/4)*ArcTan[((3/2)^(
1/4)*a^(1/4)*x)/(b + a*x^4)^(1/4)] + 12*ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/
4)] - 2^(1/4)*3^(3/4)*ArcTanh[((3/2)^(1/4)*a^(1/4)*x)/(b + a*x^4)^(1/4)])/(
12*a^(1/4))
```

IntegrateAlgebraic [A] time = 0.59, size = 141, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{\sqrt[4]{a}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}\sqrt[4]{3}\sqrt[4]{a}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{\sqrt[4]{a}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{2\,2^{3/4}\sqrt[4]{3}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*b + 2*a*x^4)/((-2*b + a*x^4)*(b + a*x^4)^(1/4)),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)]/a^(1/4) - ArcTan[((3/2)^(1/4)*a^(1/4)
*x)/(b + a*x^4)^(1/4)]/(2*2^(3/4)*3^(1/4)*a^(1/4)) + ArcTanh[(a^(1/4)*x)/(b
 + a*x^4)^(1/4)]/a^(1/4) - ArcTanh[((3/2)^(1/4)*a^(1/4)*x)/(b + a*x^4)^(1/4
)]/(2*2^(3/4)*3^(1/4)*a^(1/4))
```

fricas [B] time = 0.44, size = 241, normalized size = 1.71

$$-\frac{\left(\frac{1}{24}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\frac{\left(\frac{1}{24}\right)^{\frac{1}{4}}x\sqrt{\frac{3\sqrt{\frac{1}{6}}\sqrt{a}x^2+\sqrt{ax^4+b}}{x^2}}}{a^{\frac{1}{4}}} - \frac{\left(\frac{1}{24}\right)^{\frac{1}{4}}\left(ax^4+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{x}\right)}{a^{\frac{1}{4}}} - \frac{\left(\frac{1}{24}\right)^{\frac{1}{4}}\log\left(\frac{12\left(\frac{1}{24}\right)^{\frac{3}{4}}a^{\frac{1}{4}}x+\left(ax^4+b\right)^{\frac{1}{4}}}{x}\right)}{4a^{\frac{1}{4}}} + \frac{\left(\frac{1}{24}\right)^{\frac{1}{4}}\log\left(-\frac{12\left(\frac{1}{24}\right)^{\frac{3}{4}}a^{\frac{1}{4}}x-\left(ax^4+b\right)^{\frac{1}{4}}}{x}\right)}{4a^{\frac{1}{4}}} + \frac{2\arctan\left(\frac{\frac{x\sqrt{\frac{\sqrt{a}x^2+\sqrt{ax^4+b}}{x^2}}}{a^{\frac{1}{4}}} - \frac{\left(ax^4+b\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}}{x}\right)}{a^{\frac{1}{4}}} + \frac{\log\left(\frac{a^{\frac{1}{4}}x+\left(ax^4+b\right)^{\frac{1}{4}}}{x}\right)}{2a^{\frac{1}{4}}} - \frac{\log\left(-\frac{a^{\frac{1}{4}}x-\left(ax^4+b\right)^{\frac{1}{4}}}{x}\right)}{2a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^4-3*b)/(a*x^4-2*b)/(a*x^4+b)^(1/4),x, algorithm="fricas")
```

```
[Out] -(1/24)^(1/4)*arctan(2*((1/24)^(1/4)*x*sqrt((3*sqrt(1/6)*sqrt(a)*x^2 + sqrt
(a*x^4 + b))/x^2)/a^(1/4) - (1/24)^(1/4)*(a*x^4 + b)^(1/4)/a^(1/4))/x)/a^(1
/4) - 1/4*(1/24)^(1/4)*log((12*(1/24)^(3/4)*a^(1/4)*x + (a*x^4 + b)^(1/4))/
x)/a^(1/4) + 1/4*(1/24)^(1/4)*log(-(12*(1/24)^(3/4)*a^(1/4)*x - (a*x^4 + b)
^(1/4))/x)/a^(1/4) + 2*arctan((x*sqrt((sqrt(a)*x^2 + sqrt(a*x^4 + b))/x^2)/
a^(1/4) - (a*x^4 + b)^(1/4)/a^(1/4))/x)/a^(1/4) + 1/2*log((a^(1/4)*x + (a*x
^4 + b)^(1/4))/x)/a^(1/4) - 1/2*log(-(a^(1/4)*x - (a*x^4 + b)^(1/4))/x)/a^(
1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax^4 - 3\,b}{\left(ax^4 + b\right)^{\frac{1}{4}}\left(ax^4 - 2\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^4-3*b)/(a*x^4-2*b)/(a*x^4+b)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((2*a*x^4 - 3*b)/((a*x^4 + b)^(1/4)*(a*x^4 - 2*b)), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{2a\,x^4 - 3b}{\left(a\,x^4 - 2b\right)\left(a\,x^4 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x^4-3*b)/(a*x^4-2*b)/(a*x^4+b)^(1/4),x)
```

```
[Out] int((2*a*x^4-3*b)/(a*x^4-2*b)/(a*x^4+b)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax^4 - 3\,b}{\left(ax^4 + b\right)^{\frac{1}{4}}\left(ax^4 - 2\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^4-3*b)/(a*x^4-2*b)/(a*x^4+b)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((2*a*x^4 - 3*b)/((a*x^4 + b)^(1/4)*(a*x^4 - 2*b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,b - 2\,a\,x^4}{\left(a\,x^4 + b\right)^{1/4}\,\left(2\,b - a\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*b - 2*a*x^4)/((b + a*x^4)^(1/4)*(2*b - a*x^4)),x)
```

```
[Out] int((3*b - 2*a*x^4)/((b + a*x^4)^(1/4)*(2*b - a*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2ax^4 - 3b}{\left(ax^4 - 2b\right)\sqrt[4]{ax^4 + b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x**4-3*b)/(a*x**4-2*b)/(a*x**4+b)**(1/4),x)
```

```
[Out] Integral((2*a*x**4 - 3*b)/((a*x**4 - 2*b)*(a*x**4 + b)**(1/4)), x)
```

3.1665 $\int \frac{b+ax^4}{\sqrt[4]{-b+ax^4}\left(b+3ax^4\right)}\,dx$

Optimal. Leaf size=141

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{6\sqrt[4]{a}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{3\sqrt{2}\,\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{6\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{3\sqrt{2}\,\sqrt[4]{a}}$$

Rubi [A] time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {530, 240, 212, 206, 203, 377}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{6\sqrt[4]{a}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{3\sqrt{2}\,\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{6\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{3\sqrt{2}\,\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^4)/((-b + a*x^4)^(1/4)*(b + 3*a*x^4)),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)]/(6*a^(1/4)) + ArcTan[(Sqrt[2]*a^(1/4
)*x)/(-b + a*x^4)^(1/4)]/(3*Sqrt[2]*a^(1/4)) + ArcTanh[(a^(1/4)*x)/(-b + a*
x^4)^(1/4)]/(6*a^(1/4)) + ArcTanh[(Sqrt[2]*a^(1/4)*x)/(-b + a*x^4)^(1/4)]/(
3*Sqrt[2]*a^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{b+ax^4}{\sqrt[4]{-b+ax^4}\,(b+3ax^4)}\,dx &= \frac{1}{3}\int \frac{1}{\sqrt[4]{-b+ax^4}}\,dx+\frac{1}{3}(2b)\int \frac{1}{\sqrt[4]{-b+ax^4}\,(b+3ax^4)}\,dx\\ &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{1-ax^4}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)+\frac{1}{3}(2b)\,\mathrm{Subst}\left(\int \frac{1}{b-4abx^4}\,dx,x,\frac{x}{\sqrt[4]{-b\ldots}}\right.\\ &= \frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)+\frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{a}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{-b+\ldots}}\right.\\ &= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{6\sqrt[4]{a}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{3\sqrt{2}\,\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{6\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{3\sqrt{2}\,\sqrt[4]{a}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 116, normalized size = 0.82

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{6\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^4)/((-b + a*x^4)^(1/4)*(b + 3*a*x^4)),x]
```

```
[Out] (ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)] + Sqrt[2]*ArcTan[(Sqrt[2]*a^(1/4)*x
)/(-b + a*x^4)^(1/4)] + ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)] + Sqrt[2]*A
rcTanh[(Sqrt[2]*a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(6*a^(1/4))
```

IntegrateAlgebraic [A] time = 0.45, size = 141, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{6\sqrt[4]{a}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{3\sqrt{2}\,\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{6\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{3\sqrt{2}\,\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^4)/((-b + a*x^4)^(1/4)*(b + 3*a*x^4)),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)]/(6*a^(1/4)) + ArcTan[(Sqrt[2]*a^(1/4
)*x)/(-b + a*x^4)^(1/4)]/(3*Sqrt[2]*a^(1/4)) + ArcTanh[(a^(1/4)*x)/(-b + a*
x^4)^(1/4)]/(6*a^(1/4)) + ArcTanh[(Sqrt[2]*a^(1/4)*x)/(-b + a*x^4)^(1/4)]/(
3*Sqrt[2]*a^(1/4))
```

fricas [B] time = 0.43, size = 253, normalized size = 1.79

$$\frac{2\left(\frac{1}{4}\right)^{\frac{1}{4}}\arctan\left(\frac{\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}x\sqrt{\frac{2\sqrt{a}x^2+\sqrt{ax^4-b}}{x^2}}}{a^{\frac{1}{4}}}-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}(ax^4-b)^{\frac{1}{4}}}{a^{\frac{1}{4}}}}{x}\right)}{3a^{\frac{1}{4}}}+\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}a^{\frac{1}{4}}x+(ax^4-b)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}}-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}a^{\frac{1}{4}}x-(ax^4-b)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}}+\frac{\arctan\left(\frac{\frac{x\sqrt{\frac{\sqrt{a}x^2+\sqrt{ax^4-b}}{x^2}}}{a^{\frac{1}{4}}}-\frac{(ax^4-b)^{\frac{1}{4}}}{a^{\frac{1}{4}}}}{x}\right)}{3a^{\frac{1}{4}}}+\frac{\log\left(\frac{a^{\frac{1}{4}}x+(ax^4-b)^{\frac{1}{4}}}{x}\right)}{12a^{\frac{1}{4}}}-\frac{\log\left(-\frac{a^{\frac{1}{4}}x-(ax^4-b)^{\frac{1}{4}}}{x}\right)}{12a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)/(a*x^4-b)^(1/4)/(3*a*x^4+b),x, algorithm="fricas")
```

```
[Out] 2/3*(1/4)^(1/4)*arctan(((1/4)^(1/4)*x*sqrt((2*sqrt(a)*x^2 + sqrt(a*x^4 - b)
)/x^2)/a^(1/4) - (1/4)^(1/4)*(a*x^4 - b)^(1/4)/a^(1/4))/x)/a^(1/4) + 1/6*(1
/4)^(1/4)*log((4*(1/4)^(3/4)*a^(1/4)*x + (a*x^4 - b)^(1/4))/x)/a^(1/4) - 1/
6*(1/4)^(1/4)*log(-(4*(1/4)^(3/4)*a^(1/4)*x - (a*x^4 - b)^(1/4))/x)/a^(1/4)
 + 1/3*arctan((x*sqrt((sqrt(a)*x^2 + sqrt(a*x^4 - b))/x^2)/a^(1/4) - (a*x^4
 - b)^(1/4)/a^(1/4))/x)/a^(1/4) + 1/12*log((a^(1/4)*x + (a*x^4 - b)^(1/4))/
x)/a^(1/4) - 1/12*log(-(a^(1/4)*x - (a*x^4 - b)^(1/4))/x)/a^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4+b}{\left(3\,ax^4+b\right)\left(ax^4-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)/(a*x^4-b)^(1/4)/(3*a*x^4+b),x, algorithm="giac")
[Out] integrate((a*x^4 + b)/((3*a*x^4 + b)*(a*x^4 - b)^(1/4)), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{a\,x^4+b}{\left(a\,x^4-b\right)^{\frac{1}{4}}\left(3a\,x^4+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b)/(a*x^4-b)^(1/4)/(3*a*x^4+b),x)
[Out] int((a*x^4+b)/(a*x^4-b)^(1/4)/(3*a*x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4+b}{\left(3\,ax^4+b\right)\left(ax^4-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)/(a*x^4-b)^(1/4)/(3*a*x^4+b),x, algorithm="maxima")
[Out] integrate((a*x^4 + b)/((3*a*x^4 + b)*(a*x^4 - b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x^4+b}{{\left(a\,x^4-b\right)}^{1/4}\,\left(3\,a\,x^4+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^4)/((a*x^4 - b)^(1/4)*(b + 3*a*x^4)),x)
[Out] int((b + a*x^4)/((a*x^4 - b)^(1/4)*(b + 3*a*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4+b}{\sqrt[4]{ax^4-b}\,\left(3ax^4+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b)/(a*x**4-b)**(1/4)/(3*a*x**4+b),x)
[Out] Integral((a*x**4 + b)/((a*x**4 - b)**(1/4)*(3*a*x**4 + b)), x)
```

3.1666 $\int \frac{(2+x^6)(-1-x^4+x^6)}{\sqrt[4]{1-x^4-x^6}\,(-1+x^6)^2}\,dx$

Optimal. Leaf size=141

$$-\frac{\left(-x^6-x^4+1\right)^{3/4}x}{2\left(x^6-1\right)}-\frac{5\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{-x^6-x^4+1}}{\sqrt{-x^6-x^4+1}-x^2}\right)}{4\sqrt{2}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{-x^6-x^4+1}}{x^2+\sqrt{-x^6-x^4+1}}\right)}{4\sqrt{2}}$$

Rubi [F] time = 2.74, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(2+x^6\right)\left(-1-x^4+x^6\right)}{\sqrt[4]{1-x^4-x^6}\left(-1+x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((2 + x^6)*(-1 - x^4 + x^6))/((1 - x^4 - x^6)^(1/4)*(-1 + x^6)^2),x]
```

```
[Out] Defer[Int][(1 - x^4 - x^6)^(-1/4), x] - Defer[Int][1/((-1 + I*Sqrt[3] - 2*x
)^2*(1 - x^4 - x^6)^(1/4)), x]/3 + ((I/3)*Defer[Int][1/((-1 + I*Sqrt[3] - 2
*x)*(1 - x^4 - x^6)^(1/4)), x])/Sqrt[3] - Defer[Int][1/((1 + I*Sqrt[3] - 2*
x)^2*(1 - x^4 - x^6)^(1/4)), x]/3 + ((I/3)*Defer[Int][1/((1 + I*Sqrt[3] - 2
*x)*(1 - x^4 - x^6)^(1/4)), x])/Sqrt[3] - Defer[Int][1/((-1 + x)^2*(1 - x^4
 - x^6)^(1/4)), x]/12 + (5*Defer[Int][1/((-1 + x)*(1 - x^4 - x^6)^(1/4)), x
])/12 - Defer[Int][1/((1 + x)^2*(1 - x^4 - x^6)^(1/4)), x]/12 - (5*Defer[In
t][1/((1 + x)*(1 - x^4 - x^6)^(1/4)), x])/12 + (5*(3 + (5*I)*Sqrt[3])*Defer
[Int][1/((-1 - I*Sqrt[3] + 2*x)*(1 - x^4 - x^6)^(1/4)), x])/36 - (5*(3 - (5
*I)*Sqrt[3])*Defer[Int][1/((1 - I*Sqrt[3] + 2*x)*(1 - x^4 - x^6)^(1/4)), x]
)/36 - Defer[Int][1/((-1 + I*Sqrt[3] + 2*x)^2*(1 - x^4 - x^6)^(1/4)), x]/3
+ ((I/3)*Defer[Int][1/((-1 + I*Sqrt[3] + 2*x)*(1 - x^4 - x^6)^(1/4)), x])/S
qrt[3] + (5*(3 - (5*I)*Sqrt[3])*Defer[Int][1/((-1 + I*Sqrt[3] + 2*x)*(1 - x
^4 - x^6)^(1/4)), x])/36 - Defer[Int][1/((1 + I*Sqrt[3] + 2*x)^2*(1 - x^4 -
 x^6)^(1/4)), x]/3 + ((I/3)*Defer[Int][1/((1 + I*Sqrt[3] + 2*x)*(1 - x^4 -
x^6)^(1/4)), x])/Sqrt[3] - (5*(3 + (5*I)*Sqrt[3])*Defer[Int][1/((1 + I*Sqrt
[3] + 2*x)*(1 - x^4 - x^6)^(1/4)), x])/36
```

Rubi steps

$$\int \frac{\left(2+x^6\right)\left(-1-x^4+x^6\right)}{\sqrt[4]{1-x^4-x^6}\left(-1+x^6\right)^2}\,dx = \int \left(\frac{1}{\sqrt[4]{1-x^4-x^6}}-\frac{1}{12(-1+x)^2\sqrt[4]{1-x^4-x^6}}-\frac{1}{12(1+x)^2\sqrt[4]{1-x^4-x^6}}+\frac{}{6(-}\right.$$

$$=-\left(\frac{1}{12}\int \frac{1}{(-1+x)^2\sqrt[4]{1-x^4-x^6}}\,dx\right)-\frac{1}{12}\int \frac{1}{(1+x)^2\sqrt[4]{1-x^4-x^6}}\,dx+\frac{1}{4}\int \frac{}{(1}$$

$$=-\left(\frac{1}{12}\int \frac{1}{(-1+x)^2\sqrt[4]{1-x^4-x^6}}\,dx\right)-\frac{1}{12}\int \frac{1}{(1+x)^2\sqrt[4]{1-x^4-x^6}}\,dx+\frac{1}{4}\int \Bigl(-$$

$$=-\left(\frac{1}{12}\int \frac{1}{(-1+x)^2\sqrt[4]{1-x^4-x^6}}\,dx\right)-\frac{1}{12}\int \frac{1}{(1+x)^2\sqrt[4]{1-x^4-x^6}}\,dx-\frac{1}{3}\int \frac{}{(-}$$

Mathematica [F] time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{(2+x^6)(-1-x^4+x^6)}{\sqrt[4]{1-x^4-x^6}\,(-1+x^6)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((2 + x^6)*(-1 - x^4 + x^6))/((1 - x^4 - x^6)^(1/4)*(-1 + x^6)^2)
,x]
```

```
[Out] Integrate[((2 + x^6)*(-1 - x^4 + x^6))/((1 - x^4 - x^6)^(1/4)*(-1 + x^6)^2)
, x]
```

IntegrateAlgebraic [A] time = 2.88, size = 141, normalized size = 1.00

$$-\frac{(-x^6-x^4+1)^{3/4}\,x}{2(x^6-1)}-\frac{5\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{-x^6-x^4+1}}{\sqrt{-x^6-x^4+1}-x^2}\right)}{4\sqrt{2}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{-x^6-x^4+1}}{x^2+\sqrt{-x^6-x^4+1}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^6)*(-1 - x^4 + x^6))/((1 - x^4 - x^6)^(1/4)*(-1
+ x^6)^2),x]
```

```
[Out] -1/2*(x*(1 - x^4 - x^6)^(3/4))/(-1 + x^6) - (5*ArcTan[(Sqrt[2]*x*(1 - x^4 -
 x^6)^(1/4))/(-x^2 + Sqrt[1 - x^4 - x^6])])/(4*Sqrt[2]) - (5*ArcTanh[(Sqrt[
2]*x*(1 - x^4 - x^6)^(1/4))/(x^2 + Sqrt[1 - x^4 - x^6])])/(4*Sqrt[2])
```

fricas [B] time = 10.15, size = 852, normalized size = 6.04

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)*(x^6-x^4-1)/(-x^6-x^4+1)^(1/4)/(x^6-1)^2,x, algorithm="fr
icas")
```

```
[Out] -1/32*(20*sqrt(2)*(x^6 - 1)*arctan(-(x^12 - 2*x^6 + 2*sqrt(2)*(x^7 + 4*x^5
- x)*(-x^6 - x^4 + 1)^(3/4) + 2*sqrt(2)*(3*x^9 + 4*x^7 - 3*x^3)*(-x^6 - x^4
 + 1)^(1/4) - 4*(x^8 - x^2)*sqrt(-x^6 - x^4 + 1) - (16*(-x^6 - x^4 + 1)^(3/
4)*x^5 + 2*sqrt(2)*(x^8 + 4*x^6 - x^2)*sqrt(-x^6 - x^4 + 1) - sqrt(2)*(x^12
 + 10*x^10 + 8*x^8 - 2*x^6 - 10*x^4 + 1) - 4*(x^9 - x^3)*(-x^6 - x^4 + 1)^(
1/4))*sqrt((x^6 + 2*sqrt(2)*(-x^6 - x^4 + 1)^(1/4)*x^3 - 4*sqrt(-x^6 - x^4
+ 1)*x^2 + 2*sqrt(2)*(-x^6 - x^4 + 1)^(3/4)*x - 1)/(x^6 - 1)) + 1)/(x^12 +
16*x^10 + 16*x^8 - 2*x^6 - 16*x^4 + 1)) - 20*sqrt(2)*(x^6 - 1)*arctan(-(x^1
2 - 2*x^6 - 2*sqrt(2)*(x^7 + 4*x^5 - x)*(-x^6 - x^4 + 1)^(3/4) - 2*sqrt(2)*
(3*x^9 + 4*x^7 - 3*x^3)*(-x^6 - x^4 + 1)^(1/4) - 4*(x^8 - x^2)*sqrt(-x^6 -
x^4 + 1) - (16*(-x^6 - x^4 + 1)^(3/4)*x^5 - 2*sqrt(2)*(x^8 + 4*x^6 - x^2)*s
qrt(-x^6 - x^4 + 1) + sqrt(2)*(x^12 + 10*x^10 + 8*x^8 - 2*x^6 - 10*x^4 + 1)
 - 4*(x^9 - x^3)*(-x^6 - x^4 + 1)^(1/4))*sqrt((x^6 - 2*sqrt(2)*(-x^6 - x^4
+ 1)^(1/4)*x^3 - 4*sqrt(-x^6 - x^4 + 1)*x^2 - 2*sqrt(2)*(-x^6 - x^4 + 1)^(3
/4)*x - 1)/(x^6 - 1)) + 1)/(x^12 + 16*x^10 + 16*x^8 - 2*x^6 - 16*x^4 + 1))
- 5*sqrt(2)*(x^6 - 1)*log(4*(x^6 + 2*sqrt(2)*(-x^6 - x^4 + 1)^(1/4)*x^3 - 4
*sqrt(-x^6 - x^4 + 1)*x^2 + 2*sqrt(2)*(-x^6 - x^4 + 1)^(3/4)*x - 1)/(x^6 -
1)) + 5*sqrt(2)*(x^6 - 1)*log(4*(x^6 - 2*sqrt(2)*(-x^6 - x^4 + 1)^(1/4)*x^3
 - 4*sqrt(-x^6 - x^4 + 1)*x^2 - 2*sqrt(2)*(-x^6 - x^4 + 1)^(3/4)*x - 1)/(x^
6 - 1)) + 16*(-x^6 - x^4 + 1)^(3/4)*x)/(x^6 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^6-x^4-1)(x^6+2)}{(x^6-1)^2(-x^6-x^4+1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)*(x^6-x^4-1)/(-x^6-x^4+1)^(1/4)/(x^6-1)^2,x, algorithm="gi
ac")
```

```
[Out] integrate((x^6 - x^4 - 1)*(x^6 + 2)/((x^6 - 1)^2*(-x^6 - x^4 + 1)^(1/4)), x
)
```

maple [C] time = 9.94, size = 321, normalized size = 2.28

$$\frac{x\left(x^{6}+x^{4}-1\right)}{2\left(x^{6}-1\right)\left(-x^{6}-x^{4}+1\right)^{\frac{1}{4}}}-\frac{5\,\mathrm{RootOf}\left(\_Z^{4}+1\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}x^{6}-2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}\left(-x^{6}-x^{4}+1\right)^{\frac{1}{4}}x^{3}-2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}x^{4}+2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)\left(-x^{6}-x^{4}+1\right)^{\frac{3}{4}}x+2x^{2}\sqrt{-x^{6}-x^{4}+1}+\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}}{(-1+x)(1+x)\left(x^{2}+x+1\right)\left(x^{2}-x+1\right)}\right)}{8}-\frac{5\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}x^{6}-2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}x^{4}+2\left(-x^{6}-x^{4}+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}x^{3}-2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)\sqrt{-x^{6}-x^{4}+1}\,x^{2}+2\left(-x^{6}-x^{4}+1\right)^{\frac{3}{4}}x+\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}}{(-1+x)(1+x)\left(x^{2}+x+1\right)\left(x^{2}-x+1\right)}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+2)*(x^6-x^4-1)/(-x^6-x^4+1)^(1/4)/(x^6-1)^2,x)
```

```
[Out] 1/2*x*(x^6+x^4-1)/(x^6-1)/(-x^6-x^4+1)^(1/4)-5/8*RootOf(_Z^4+1)*ln(-(-RootO
f(_Z^4+1)^2*x^6-2*RootOf(_Z^4+1)^3*(-x^6-x^4+1)^(1/4)*x^3-2*RootOf(_Z^4+1)^
2*x^4+2*RootOf(_Z^4+1)*(-x^6-x^4+1)^(3/4)*x+2*x^2*(-x^6-x^4+1)^(1/2)+RootOf
(_Z^4+1)^2)/(-1+x)/(1+x)/(x^2+x+1)/(x^2-x+1))-5/8*RootOf(_Z^4+1)^3*ln((-Roo
tOf(_Z^4+1)^3*x^6-2*RootOf(_Z^4+1)^3*x^4+2*(-x^6-x^4+1)^(1/4)*RootOf(_Z^4+1
)^2*x^3-2*RootOf(_Z^4+1)*(-x^6-x^4+1)^(1/2)*x^2+2*(-x^6-x^4+1)^(3/4)*x+Root
Of(_Z^4+1)^3)/(-1+x)/(1+x)/(x^2+x+1)/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}-x^{4}-1\right)\left(x^{6}+2\right)}{\left(x^{6}-1\right)^{2}\left(-x^{6}-x^{4}+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)*(x^6-x^4-1)/(-x^6-x^4+1)^(1/4)/(x^6-1)^2,x, algorithm="ma
xima")
```

```
[Out] integrate((x^6 - x^4 - 1)*(x^6 + 2)/((x^6 - 1)^2*(-x^6 - x^4 + 1)^(1/4)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^{6}+2\right)\,\left(-x^{6}+x^{4}+1\right)}{{\left(x^{6}-1\right)}^{2}\,{\left(-x^{6}-x^{4}+1\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^6 + 2)*(x^4 - x^6 + 1))/((x^6 - 1)^2*(1 - x^6 - x^4)^(1/4)),x)
```

```
[Out] int(-((x^6 + 2)*(x^4 - x^6 + 1))/((x^6 - 1)^2*(1 - x^6 - x^4)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6} + 2\right) \left(x^{6} - x^{4} - 1\right)}{\left(x - 1\right)^{2} \left(x + 1\right)^{2} \left(x^{2} - x + 1\right)^{2} \left(x^{2} + x + 1\right)^{2} \sqrt[4]{- x^{6} - x^{4} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+2)*(x**6-x**4-1)/(-x**6-x**4+1)**(1/4)/(x**6-1)**2,x)
```

```
[Out] Integral((x**6 + 2)*(x**6 - x**4 - 1)/((x - 1)**2*(x + 1)**2*(x**2 - x + 1)
**2*(x**2 + x + 1)**2*(-x**6 - x**4 + 1)**(1/4)), x)
```

3.1667 $\int \frac{(-2b+ax^6)(b-cx^4+ax^6)}{x^2(b+ax^6)^{3/4}(b+cx^4+ax^6)}\,dx$

Optimal. Leaf size=141

$$\sqrt{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x\sqrt[4]{ax^6+b}}{\sqrt{ax^6+b}-\sqrt{c}\,x^2}\right)-\sqrt{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6+b}}{\sqrt{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^6+b}}\right)+\frac{2\sqrt[4]{ax^6+b}}{x}$$

Rubi [F] time = 3.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2b+ax^6)(b-cx^4+ax^6)}{x^2(b+ax^6)^{3/4}(b+cx^4+ax^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*b + a*x^6)*(b - c*x^4 + a*x^6))/(x^2*(b + a*x^6)^(3/4)*(b + c*x^4
+ a*x^6)),x]
```

```
[Out] (-4*Sqrt[b]*c*(1 + (a*x^6)/b)^(3/4)*EllipticF[ArcTan[(Sqrt[a]*x^3)/Sqrt[b]]
/2, 2])/(3*Sqrt[a]*(b + a*x^6)^(3/4)) + (2*b*(1 + (a*x^6)/b)^(3/4)*Hypergeo
metric2F1[-1/6, 3/4, 5/6, -((a*x^6)/b)])/(x*(b + a*x^6)^(3/4)) + (2*c^2*x*(
1 + (a*x^6)/b)^(3/4)*Hypergeometric2F1[1/6, 3/4, 7/6, -((a*x^6)/b)])/(a*(b
+ a*x^6)^(3/4)) + (a*x^5*(1 + (a*x^6)/b)^(3/4)*Hypergeometric2F1[3/4, 5/6,
11/6, -((a*x^6)/b)])/(5*(b + a*x^6)^(3/4)) - (2*b*c^2*Defer[Int][1/((b + a*
x^6)^(3/4)*(b + c*x^4 + a*x^6)), x])/a + 6*b*c*Defer[Int][x^2/((b + a*x^6)^
(3/4)*(b + c*x^4 + a*x^6)), x] - (2*c^3*Defer[Int][x^4/((b + a*x^6)^(3/4)*(
b + c*x^4 + a*x^6)), x])/a
```

Rubi steps

$$\int \frac{(-2b+ax^6)(b-cx^4+ax^6)}{x^2(b+ax^6)^{3/4}(b+cx^4+ax^6)}\,dx=\int\left(\frac{2c^2}{a(b+ax^6)^{3/4}}-\frac{2b}{x^2(b+ax^6)^{3/4}}-\frac{2cx^2}{(b+ax^6)^{3/4}}+\frac{ax^4}{(b+ax^6)^{3/4}}+\frac{-\ldots}{a}\right.$$

$$=a\int\frac{x^4}{(b+ax^6)^{3/4}}\,dx-(2b)\int\frac{1}{x^2(b+ax^6)^{3/4}}\,dx-(2c)\int\frac{x^2}{(b+ax^6)^{3/4}}$$

$$=-\left(\frac{1}{3}(2c)\,\text{Subst}\left(\int\frac{1}{(b+ax^2)^{3/4}}\,dx,x,x^3\right)\right)+\frac{(2c)\int\left(-\frac{bc}{(b+ax^6)^{3/4}(b+cx^4+a}\right.}{}$$

$$=\frac{2b\left(1+\frac{ax^6}{b}\right)^{3/4}{}_2F_1\left(-\frac{1}{6},\frac{3}{4};\frac{5}{6};-\frac{ax^6}{b}\right)}{x(b+ax^6)^{3/4}}+\frac{2c^2x\left(1+\frac{ax^6}{b}\right)^{3/4}{}_2F_1\left(\frac{1}{6},\frac{3}{4};\frac{7}{6};-\ldots\right.}{a(b+ax^6)^{3/4}}$$

$$=-\frac{4\sqrt{b}\,c\left(1+\frac{ax^6}{b}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{a}\,x^3}{\sqrt{b}}\right)\middle|2\right)}{3\sqrt{a}\,(b+ax^6)^{3/4}}+\frac{2b\left(1+\frac{ax^6}{b}\right)^{3/4}{}_2F_1\left(-\frac{1}{6},\frac{3}{4}\ldots\right.}{x(b+ax^6)^{3/4}}$$

Mathematica [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{(-2b+ax^6)(b-cx^4+ax^6)}{x^2(b+ax^6)^{3/4}(b+cx^4+ax^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2*b + a*x^6)*(b - c*x^4 + a*x^6))/(x^2*(b + a*x^6)^(3/4)*(b +
c*x^4 + a*x^6)),x]
```

```
[Out] Integrate[((-2*b + a*x^6)*(b - c*x^4 + a*x^6))/(x^2*(b + a*x^6)^(3/4)*(b +
c*x^4 + a*x^6)), x]
```

IntegrateAlgebraic [A] time = 15.65, size = 141, normalized size = 1.00

$$\sqrt{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x\sqrt[4]{ax^6+b}}{\sqrt{ax^6+b}-\sqrt{c}\,x^2}\right)-\sqrt{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6+b}}{\sqrt{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^6+b}}\right)+\frac{2\sqrt[4]{ax^6+b}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*b + a*x^6)*(b - c*x^4 + a*x^6))/(x^2*(b + a*x^6)^(3
/4)*(b + c*x^4 + a*x^6)),x]
```

```
[Out] (2*(b + a*x^6)^(1/4))/x + Sqrt[2]*c^(1/4)*ArcTan[(Sqrt[2]*c^(1/4)*x*(b + a*
x^6)^(1/4))/(-(Sqrt[c]*x^2) + Sqrt[b + a*x^6])] - Sqrt[2]*c^(1/4)*ArcTanh[(
(c^(1/4)*x^2)/Sqrt[2] + Sqrt[b + a*x^6]/(Sqrt[2]*c^(1/4)))/(x*(b + a*x^6)^(
1/4))]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-2*b)*(a*x^6-c*x^4+b)/x^2/(a*x^6+b)^(3/4)/(a*x^6+c*x^4+b),x
, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^6-cx^4+b)(ax^6-2\,b)}{(ax^6+cx^4+b)(ax^6+b)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-2*b)*(a*x^6-c*x^4+b)/x^2/(a*x^6+b)^(3/4)/(a*x^6+c*x^4+b),x
, algorithm="giac")
```

```
[Out] integrate((a*x^6 - c*x^4 + b)*(a*x^6 - 2*b)/((a*x^6 + c*x^4 + b)*(a*x^6 + b
)^(3/4)*x^2), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{(a\,x^6-2b)(a\,x^6-c\,x^4+b)}{x^2(a\,x^6+b)^{\frac{3}{4}}(a\,x^6+c\,x^4+b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6-2*b)*(a*x^6-c*x^4+b)/x^2/(a*x^6+b)^(3/4)/(a*x^6+c*x^4+b),x)
```

```
[Out] int((a*x^6-2*b)*(a*x^6-c*x^4+b)/x^2/(a*x^6+b)^(3/4)/(a*x^6+c*x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^6 - cx^4 + b)(ax^6 - 2b)}{(ax^6 + cx^4 + b)(ax^6 + b)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-2*b)*(a*x^6-c*x^4+b)/x^2/(a*x^6+b)^(3/4)/(a*x^6+c*x^4+b),x
, algorithm="maxima")
```

```
[Out] integrate((a*x^6 - c*x^4 + b)*(a*x^6 - 2*b)/((a*x^6 + c*x^4 + b)*(a*x^6 + b
)^(3/4)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(2\,b - a\,x^6)\,(a\,x^6 - c\,x^4 + b)}{x^2\,(a\,x^6 + b)^{3/4}\,(a\,x^6 + c\,x^4 + b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*b - a*x^6)*(b + a*x^6 - c*x^4))/(x^2*(b + a*x^6)^(3/4)*(b + a*x^6
+ c*x^4)),x)
```

```
[Out] int(-((2*b - a*x^6)*(b + a*x^6 - c*x^4))/(x^2*(b + a*x^6)^(3/4)*(b + a*x^6
+ c*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6-2*b)*(a*x**6-c*x**4+b)/x**2/(a*x**6+b)**(3/4)/(a*x**6+c*x
**4+b),x)
```

```
[Out] Timed out
```

3.1668 $\int \frac{\sqrt[4]{1+2x^4}\left(-1-x^4+2x^8\right)}{x^6\left(2+x^4\right)}\,dx$

Optimal. Leaf size=141

$$\frac{9}{8}\sqrt[4]{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}x}{\sqrt[4]{2x^4+1}}\right)-\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{2x^4+1}}\right)-\frac{9}{8}\sqrt[4]{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}x}{\sqrt[4]{2x^4+1}}\right)+\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{2x^4+1}}\right)+\frac{\sqrt[4]{2x^4+\ldots}}{\ldots}$$

Rubi [C] time = 0.37, antiderivative size = 123, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6725, 264, 277, 331, 298, 203, 206, 510}

$$\frac{3}{8}x^3F_1\left(\frac{3}{4};1,-\frac{1}{4};\frac{7}{4};-\frac{x^4}{2},-2x^4\right)+\frac{\sqrt[4]{2x^4+1}}{4x}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{2x^4+1}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{2x^4+1}}\right)}{4\,2^{3/4}}+\frac{\left(2x^4+1\right)^{5/4}}{10x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((1 + 2*x^4)^(1/4)*(-1 - x^4 + 2*x^8))/(x^6*(2 + x^4)),x]
```

```
[Out] (1 + 2*x^4)^(1/4)/(4*x) + (1 + 2*x^4)^(5/4)/(10*x^5) + (3*x^3*AppellF1[3/4,
 1, -1/4, 7/4, -1/2*x^4, -2*x^4])/8 + ArcTan[(2^(1/4)*x)/(1 + 2*x^4)^(1/4)]
/(4*2^(3/4)) - ArcTanh[(2^(1/4)*x)/(1 + 2*x^4)^(1/4)]/(4*2^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{1+2x^4}\left(-1-x^4+2x^8\right)}{x^6\left(2+x^4\right)}\,dx &= \int \left(-\frac{\sqrt[4]{1+2x^4}}{2x^6}-\frac{\sqrt[4]{1+2x^4}}{4x^2}+\frac{9x^2\sqrt[4]{1+2x^4}}{4\left(2+x^4\right)}\right)dx\\
&= -\left(\frac{1}{4}\int \frac{\sqrt[4]{1+2x^4}}{x^2}\,dx\right)-\frac{1}{2}\int \frac{\sqrt[4]{1+2x^4}}{x^6}\,dx+\frac{9}{4}\int \frac{x^2\sqrt[4]{1+2x^4}}{2+x^4}\,dx\\
&= \frac{\sqrt[4]{1+2x^4}}{4x}+\frac{\left(1+2x^4\right)^{5/4}}{10x^5}+\frac{3}{8}x^3F_1\left(\frac{3}{4};1,-\frac{1}{4};\frac{7}{4};-\frac{x^4}{2},-2x^4\right)-\frac{1}{2}\int \frac{x^2}{\left(1+2x\right.}\\
&= \frac{\sqrt[4]{1+2x^4}}{4x}+\frac{\left(1+2x^4\right)^{5/4}}{10x^5}+\frac{3}{8}x^3F_1\left(\frac{3}{4};1,-\frac{1}{4};\frac{7}{4};-\frac{x^4}{2},-2x^4\right)-\frac{1}{2}\text{Subst}\left(\int\right.\\
&= \frac{\sqrt[4]{1+2x^4}}{4x}+\frac{\left(1+2x^4\right)^{5/4}}{10x^5}+\frac{3}{8}x^3F_1\left(\frac{3}{4};1,-\frac{1}{4};\frac{7}{4};-\frac{x^4}{2},-2x^4\right)-\frac{\text{Subst}\left(\int \frac{}{1-}\right.}{}\\
&= \frac{\sqrt[4]{1+2x^4}}{4x}+\frac{\left(1+2x^4\right)^{5/4}}{10x^5}+\frac{3}{8}x^3F_1\left(\frac{3}{4};1,-\frac{1}{4};\frac{7}{4};-\frac{x^4}{2},-2x^4\right)+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}}{\sqrt[4]{1+2}}\right.}{4\,2^{3/4}}\end{aligned}$$

Mathematica [C] time = 0.14, size = 99, normalized size = 0.70

$$\frac{2}{7}x^7F_1\left(\frac{7}{4};\frac{3}{4},1;\frac{11}{4};-2x^4,-\frac{x^4}{2}\right)+\frac{5x^3\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{3x^4}{x^4+2}\right)}{12\sqrt[4]{2}\left(x^4+2\right)^{3/4}}+\frac{\sqrt[4]{2x^4+1}\left(9x^4+2\right)}{20x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + 2*x^4)^(1/4)*(-1 - x^4 + 2*x^8))/(x^6*(2 + x^4)),x]
```

```
[Out] ((1 + 2*x^4)^(1/4)*(2 + 9*x^4))/(20*x^5) + (2*x^7*AppellF1[7/4, 3/4, 1, 11/
4, -2*x^4, -1/2*x^4])/7 + (5*x^3*Hypergeometric2F1[3/4, 3/4, 7/4, (-3*x^4)/
(2 + x^4)])/(12*2^(1/4)*(2 + x^4)^(3/4))
```

IntegrateAlgebraic [A] time = 0.50, size = 141, normalized size = 1.00

$$\frac{9}{8}\sqrt[4]{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\,x}{\sqrt[4]{2x^4+1}}\right)-\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{2x^4+1}}\right)-\frac{9}{8}\sqrt[4]{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}\,x}{\sqrt[4]{2x^4+1}}\right)+\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{2x^4+1}}\right)+\frac{\sqrt[4]{2x^4+1}\,(9x^4+2)}{20x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + 2*x^4)^(1/4)*(-1 - x^4 + 2*x^8))/(x^6*(2 + x^4)),x
]
```

```
[Out] ((1 + 2*x^4)^(1/4)*(2 + 9*x^4))/(20*x^5) + (9*(3/2)^(1/4)*ArcTan[((3/2)^(1/
4)*x)/(1 + 2*x^4)^(1/4)])/8 - 2^(1/4)*ArcTan[(2^(1/4)*x)/(1 + 2*x^4)^(1/4)]
 - (9*(3/2)^(1/4)*ArcTanh[((3/2)^(1/4)*x)/(1 + 2*x^4)^(1/4)])/8 + 2^(1/4)*A
rcTanh[(2^(1/4)*x)/(1 + 2*x^4)^(1/4)]
```

fricas [B] time = 32.41, size = 519, normalized size = 3.68

$$-\frac{180\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}x^5\arctan\left(-\frac{12\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}(2x^4+1)^{\frac{1}{4}}x^3+12\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}(2x^4+1)^{\frac{3}{4}}x-\sqrt{3}\left(4\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\sqrt{2x^4+1}\,x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}(7x^4+2)\right)\sqrt{\sqrt{3}\sqrt{2}}}{6(x^4+2)}\right)+45\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}x^5\log\left(\frac{6\sqrt{3}\sqrt{2}(2x^4+1)^{\frac{1}{4}}x^3+6\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\sqrt{2x^4+1}\,x^2+3^{\frac{3}{4}}2^{\frac{1}{4}}(7x^4+2)+12(2x^4+1)^{\frac{3}{4}}x}{x^4+2}\right)-45\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}x^5\log\left(\frac{6\sqrt{3}\sqrt{2}(2x^4+1)^{\frac{1}{4}}x^3-6\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\sqrt{2x^4+1}\,x^2-3^{\frac{3}{4}}2^{\frac{1}{4}}(7x^4+2)+12(2x^4+1)^{\frac{3}{4}}x}{x^4+2}\right)-320\cdot 2^{\frac{1}{4}}x^5\arctan\left(-2\cdot 2^{\frac{3}{4}}(2x^4+1)^{\frac{1}{4}}x^3-2\cdot 2^{\frac{1}{4}}(2x^4+1)^{\frac{3}{4}}x+\frac{1}{2}2^{\frac{3}{4}}\left(2\cdot 2^{\frac{3}{4}}\sqrt{2x^4+1}\,x^2+2^{\frac{1}{4}}(4x^4+1)\right)\right)-80\cdot 2^{\frac{1}{4}}x^5\log\left(4\sqrt{2}(2x^4+1)^{\frac{1}{4}}x^3+4\cdot 2^{\frac{1}{4}}\sqrt{2x^4+1}\,x^2+2^{\frac{3}{4}}(4x^4+1)+4(2x^4+1)^{\frac{3}{4}}x\right)+80\cdot 2^{\frac{1}{4}}x^5\log\left(4\sqrt{2}(2x^4+1)^{\frac{1}{4}}x^3-4\cdot 2^{\frac{1}{4}}\sqrt{2x^4+1}\,x^2-2^{\frac{3}{4}}(4x^4+1)+4(2x^4+1)^{\frac{3}{4}}x\right)-16(9x^4+2)(2x^4+1)^{\frac{1}{4}}}{320x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+1)^(1/4)*(2*x^8-x^4-1)/x^6/(x^4+2),x, algorithm="fricas")
```

```
[Out] -1/320*(180*3^(1/4)*2^(3/4)*x^5*arctan(-1/6*(12*3^(3/4)*2^(1/4)*(2*x^4 + 1)
^(1/4)*x^3 + 12*3^(1/4)*2^(3/4)*(2*x^4 + 1)^(3/4)*x - sqrt(3)*(4*3^(3/4)*2^
(1/4)*sqrt(2*x^4 + 1)*x^2 + 3^(1/4)*2^(3/4)*(7*x^4 + 2))*sqrt(sqrt(3)*sqrt(
2)))/(x^4 + 2)) + 45*3^(1/4)*2^(3/4)*x^5*log((6*sqrt(3)*sqrt(2)*(2*x^4 + 1)
^(1/4)*x^3 + 6*3^(1/4)*2^(3/4)*sqrt(2*x^4 + 1)*x^2 + 3^(3/4)*2^(1/4)*(7*x^4
 + 2) + 12*(2*x^4 + 1)^(3/4)*x)/(x^4 + 2)) - 45*3^(1/4)*2^(3/4)*x^5*log((6*
sqrt(3)*sqrt(2)*(2*x^4 + 1)^(1/4)*x^3 - 6*3^(1/4)*2^(3/4)*sqrt(2*x^4 + 1)*x
^2 - 3^(3/4)*2^(1/4)*(7*x^4 + 2) + 12*(2*x^4 + 1)^(3/4)*x)/(x^4 + 2)) - 320
*2^(1/4)*x^5*arctan(-2*2^(3/4)*(2*x^4 + 1)^(1/4)*x^3 - 2*2^(1/4)*(2*x^4 + 1
)^(3/4)*x + 1/2*2^(3/4)*(2*2^(3/4)*sqrt(2*x^4 + 1)*x^2 + 2^(1/4)*(4*x^4 + 1
))) - 80*2^(1/4)*x^5*log(4*sqrt(2)*(2*x^4 + 1)^(1/4)*x^3 + 4*2^(1/4)*sqrt(2
*x^4 + 1)*x^2 + 2^(3/4)*(4*x^4 + 1) + 4*(2*x^4 + 1)^(3/4)*x) + 80*2^(1/4)*x
^5*log(4*sqrt(2)*(2*x^4 + 1)^(1/4)*x^3 - 4*2^(1/4)*sqrt(2*x^4 + 1)*x^2 - 2^
(3/4)*(4*x^4 + 1) + 4*(2*x^4 + 1)^(3/4)*x) - 16*(9*x^4 + 2)*(2*x^4 + 1)^(1/
4))/x^5
```

giac [A] time = 0.20, size = 177, normalized size = 1.26

$$-\frac{1}{16}\cdot 54^{\frac{3}{4}}\arctan\left(\frac{24^{\frac{3}{4}}(2x^4+1)^{\frac{1}{4}}}{12x}\right)-\frac{1}{32}\cdot 54^{\frac{3}{4}}\log\left(\frac{1}{2}\cdot 24^{\frac{1}{4}}+\frac{(2x^4+1)^{\frac{1}{4}}}{x}\right)+\frac{1}{32}\cdot 54^{\frac{3}{4}}\log\left(-\frac{1}{2}\cdot 24^{\frac{1}{4}}+\frac{(2x^4+1)^{\frac{1}{4}}}{x}\right)+2^{\frac{1}{4}}\arctan\left(\frac{2^{\frac{3}{4}}(2x^4+1)^{\frac{1}{4}}}{2x}\right)+\frac{1}{2}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+\frac{(2x^4+1)^{\frac{1}{4}}}{x}\right)-\frac{1}{2}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{1}{4}}+\frac{(2x^4+1)^{\frac{1}{4}}}{x}\right)+\frac{(2x^4+1)^{\frac{1}{4}}\left(\frac{1}{x^4}+2\right)}{10x}+\frac{(2x^4+1)^{\frac{1}{4}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+1)^(1/4)*(2*x^8-x^4-1)/x^6/(x^4+2),x, algorithm="giac")
```

```
[Out] -1/16*54^(3/4)*arctan(1/12*24^(3/4)*(2*x^4 + 1)^(1/4)/x) - 1/32*54^(3/4)*lo
g(1/2*24^(1/4) + (2*x^4 + 1)^(1/4)/x) + 1/32*54^(3/4)*log(-1/2*24^(1/4) + (
2*x^4 + 1)^(1/4)/x) + 2^(1/4)*arctan(1/2*2^(3/4)*(2*x^4 + 1)^(1/4)/x) + 1/2
*2^(1/4)*log(2^(1/4) + (2*x^4 + 1)^(1/4)/x) - 1/2*2^(1/4)*log(-2^(1/4) + (2
*x^4 + 1)^(1/4)/x) + 1/10*(2*x^4 + 1)^(1/4)*(1/x^4 + 2)/x + 1/4*(2*x^4 + 1)
^(1/4)/x
```

maple [F] time = 2.14, size = 0, normalized size = 0.00

$$\int \frac{(2x^4+1)^{\frac{1}{4}}\left(2x^8-x^4-1\right)}{x^6\left(x^4+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+1)^(1/4)*(2*x^8-x^4-1)/x^6/(x^4+2),x)
[Out] int((2*x^4+1)^(1/4)*(2*x^8-x^4-1)/x^6/(x^4+2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^8 - x^4 - 1\right)\left(2\,x^4 + 1\right)^{\frac{1}{4}}}{\left(x^4 + 2\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+1)^(1/4)*(2*x^8-x^4-1)/x^6/(x^4+2),x, algorithm="maxima")
[Out] integrate((2*x^8 - x^4 - 1)*(2*x^4 + 1)^(1/4)/((x^4 + 2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(2\,x^4 + 1\right)^{1/4}\,\left(-2\,x^8 + x^4 + 1\right)}{x^6\,\left(x^4 + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*x^4 + 1)^(1/4)*(x^4 - 2*x^8 + 1))/(x^6*(x^4 + 2)),x)
[Out] int(-((2*x^4 + 1)^(1/4)*(x^4 - 2*x^8 + 1))/(x^6*(x^4 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)\,\left(x^2+1\right)\left(2x^4+1\right)^{\frac{5}{4}}}{x^6\left(x^4+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+1)**(1/4)*(2*x**8-x**4-1)/x**6/(x**4+2),x)
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)*(2*x**4 + 1)**(5/4)/(x**6*(x**4 + 2)),
x)
```

3.1669 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=141

$$\sqrt{\sqrt{x^2+1}+x}-\frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}}-2\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{x^2+1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2+1}+x}}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{x^2+1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2+1}+x}}\right)$$

Rubi [A] time = 0.41, antiderivative size = 180, normalized size of antiderivative = 1.28, number of steps used = 16, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {6725, 2117, 14, 2122, 329, 211, 1165, 628, 1162, 617, 204}

$$\sqrt{\sqrt{x^2+1}+x}-\frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}}+\sqrt{2}\log\left(\sqrt{x^2+1}-\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right)-\sqrt{2}\log\left(\sqrt{x^2+1}+\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right)+2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\sqrt{x^2+1}+x}\right)-2\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/3*1/(x + Sqrt[1 + x^2])^(3/2) + Sqrt[x + Sqrt[1 + x^2]] + 2*Sqrt[2]*ArcT
an[1 - Sqrt[2]*Sqrt[x + Sqrt[1 + x^2]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt
[x + Sqrt[1 + x^2]]] + Sqrt[2]*Log[1 + x + Sqrt[1 + x^2] - Sqrt[2]*Sqrt[x +
 Sqrt[1 + x^2]]] - Sqrt[2]*Log[1 + x + Sqrt[1 + x^2] + Sqrt[2]*Sqrt[x + Sqr
t[1 + x^2]]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+x^2}{\left(1+x^2\right)\sqrt{x+\sqrt{1+x^2}}}\,dx = \int \left(\frac{1}{\sqrt{x+\sqrt{1+x^2}}} - \frac{2}{\left(1+x^2\right)\sqrt{x+\sqrt{1+x^2}}}\right)dx$$

$$= -\left(2\int \frac{1}{\left(1+x^2\right)\sqrt{x+\sqrt{1+x^2}}}\,dx\right) + \int \frac{1}{\sqrt{x+\sqrt{1+x^2}}}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1+x^2}{x^{5/2}}\,dx, x, x+\sqrt{1+x^2}\right) - 4\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx, x, x+\sqrt{1}\right.$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{x^{5/2}} + \frac{1}{\sqrt{x}}\right)dx, x, x+\sqrt{1+x^2}\right) - 8\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x+}\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} - 4\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x+\sqrt{1+x^2}}\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} - 2\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x+}\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} + \sqrt{2}\,\log\left(1+x+\sqrt{1+x^2} - \sqrt{2}\sqrt{x+}\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} + 2\sqrt{2}\,\tan^{-1}\left(1-\sqrt{2}\sqrt{x+\sqrt{1+x^2}}\right) -$$

Mathematica [A] time = 0.12, size = 180, normalized size = 1.28

$$\sqrt{\sqrt{x^2+1}+x} - \frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}} + \sqrt{2}\log\left(\sqrt{x^2+1} - \sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right) - \sqrt{2}\log\left(\sqrt{x^2+1} + \sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right) + 2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\sqrt{x^2+1}+x}\right) - 2\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/3*1/(x + Sqrt[1 + x^2])^(3/2) + Sqrt[x + Sqrt[1 + x^2]] + 2*Sqrt[2]*ArcT
an[1 - Sqrt[2]*Sqrt[x + Sqrt[1 + x^2]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt
[x + Sqrt[1 + x^2]]] + Sqrt[2]*Log[1 + x + Sqrt[1 + x^2] - Sqrt[2]*Sqrt[x +
 Sqrt[1 + x^2]]] - Sqrt[2]*Log[1 + x + Sqrt[1 + x^2] + Sqrt[2]*Sqrt[x + Sqr
t[1 + x^2]]]
```

IntegrateAlgebraic [A] time = 0.18, size = 141, normalized size = 1.00

$$\sqrt{\sqrt{x^2+1}+x} - \frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}} - 2\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{x^2+1}}{\sqrt{2}} + \frac{x}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2+1}+x}}\right) - 2\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{x^2+1}}{\sqrt{2}} + \frac{x}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2+1}+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/3*1/(x + Sqrt[1 + x^2])^(3/2) + Sqrt[x + Sqrt[1 + x^2]] - 2*Sqrt[2]*ArcT
an[(-(1/Sqrt[2]) + x/Sqrt[2] + Sqrt[1 + x^2]/Sqrt[2])/Sqrt[x + Sqrt[1 + x^2
]]] - 2*Sqrt[2]*ArcTanh[(1/Sqrt[2] + x/Sqrt[2] + Sqrt[1 + x^2]/Sqrt[2])/Sqr
t[x + Sqrt[1 + x^2]]]
```

fricas [B] time = 0.42, size = 216, normalized size = 1.53

$$-\frac{2}{3}\left(x^2-\sqrt{x^2+1}\,x-1\right)\sqrt{x+\sqrt{x^2+1}}+4\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+x+\sqrt{x^2+1}+1}-\sqrt{2}\sqrt{x+\sqrt{x^2+1}}-1\right)+4\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+4x+4\sqrt{x^2+1}+4}-\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+1\right)-\sqrt{2}\log\left(4\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+4x+4\sqrt{x^2+1}+4\right)+\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+4x+4\sqrt{x^2+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(x^2 - sqrt(x^2 + 1)*x - 1)*sqrt(x + sqrt(x^2 + 1)) + 4*sqrt(2)*arctan
(sqrt(2)*sqrt(sqrt(2)*sqrt(x + sqrt(x^2 + 1)) + x + sqrt(x^2 + 1) + 1) - sq
rt(2)*sqrt(x + sqrt(x^2 + 1)) - 1) + 4*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*s
qrt(2)*sqrt(x + sqrt(x^2 + 1)) + 4*x + 4*sqrt(x^2 + 1) + 4) - sqrt(2)*sqrt(
x + sqrt(x^2 + 1)) + 1) - sqrt(2)*log(4*sqrt(2)*sqrt(x + sqrt(x^2 + 1)) + 4
*x + 4*sqrt(x^2 + 1) + 4) + sqrt(2)*log(-4*sqrt(2)*sqrt(x + sqrt(x^2 + 1))
+ 4*x + 4*sqrt(x^2 + 1) + 4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^2+1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/((x^2 + 1)*sqrt(x + sqrt(x^2 + 1))), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^2+1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int((x^2-1)/(x^2+1)/(x+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^2+1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)/((x^2 + 1)*sqrt(x + sqrt(x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2-1}{\left(x^2+1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + 1)*(x + (x^2 + 1)^(1/2))^(1/2)),x)
```

```
[Out] int((x^2 - 1)/((x^2 + 1)*(x + (x^2 + 1)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{\sqrt{x+\sqrt{x^2+1}}\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(sqrt(x + sqrt(x**2 + 1))*(x**2 + 1)), x)
```

3.1670 $\int \frac{(1+x^2)\sqrt[3]{x+2x^3}}{x^4(-1+x^2)}\,dx$

Optimal. Leaf size=142

$$\sqrt[3]{3}\log\left(3^{2/3}\sqrt[3]{2x^3+x}-3x\right)+3^{5/6}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2x^3+x}+\sqrt[3]{3}x}\right)+\frac{3\sqrt[3]{2x^3+x}\,(10x^2+1)}{8x^3}-\frac{1}{2}\sqrt[3]{3}\log\left(3^{2/3}\sqrt[3]{2x^3+\right.$$

Rubi [A] time = 0.42, antiderivative size = 242, normalized size of antiderivative = 1.70, number of steps used = 13, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {2056, 580, 583, 12, 466, 465, 494, 292, 31, 634, 617, 204, 628}

$$\frac{15\sqrt[3]{2x^3+x}}{4x}+\frac{3\sqrt[3]{2x^3+x}}{8x^3}+\frac{\sqrt[3]{3}\sqrt[3]{2x^3+x}\log\left(1-\frac{\sqrt[3]{3}x^{2/3}}{\sqrt[3]{2x^2+1}}\right)}{\sqrt[3]{x}\sqrt[3]{2x^2+1}}-\frac{\sqrt[3]{3}\sqrt[3]{2x^3+x}\log\left(\frac{3^{2/3}x^{4/3}}{(2x^2+1)^{2/3}}+\frac{\sqrt[3]{3}x^{2/3}}{\sqrt[3]{2x^2+1}}+1\right)}{2\sqrt[3]{x}\sqrt[3]{2x^2+1}}+\frac{3^{5/6}\sqrt[3]{2x^3+x}\tan^{-1}\left(\frac{2x^{2/3}}{\sqrt[6]{3}\sqrt[3]{2x^2+1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{x}\sqrt[3]{2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(x + 2*x^3)^(1/3))/(x^4*(-1 + x^2)),x]
```

```
[Out] (3*(x + 2*x^3)^(1/3))/(8*x^3) + (15*(x + 2*x^3)^(1/3))/(4*x) + (3^(5/6)*(x
+ 2*x^3)^(1/3)*ArcTan[1/Sqrt[3] + (2*x^(2/3))/(3^(1/6)*(1 + 2*x^2)^(1/3))])
/(x^(1/3)*(1 + 2*x^2)^(1/3)) + (3^(1/3)*(x + 2*x^3)^(1/3)*Log[1 - (3^(1/3)*
x^(2/3))/(1 + 2*x^2)^(1/3)])/(x^(1/3)*(1 + 2*x^2)^(1/3)) - (3^(1/3)*(x + 2*
x^3)^(1/3)*Log[1 + (3^(2/3)*x^(4/3))/(1 + 2*x^2)^(2/3) + (3^(1/3)*x^(2/3))/
(1 + 2*x^2)^(1/3)])/(2*x^(1/3)*(1 + 2*x^2)^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
```

```
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{(1+x^2)\sqrt[3]{x+2x^3}}{x^4(-1+x^2)}\,dx = \frac{\sqrt[3]{x+2x^3}\int \frac{(1+x^2)\sqrt[3]{1+2x^2}}{x^{11/3}(-1+x^2)}\,dx}{\sqrt[3]{x}\sqrt[3]{1+2x^2}}$$

$$= \frac{3\sqrt[3]{x+2x^3}}{8x^3} - \frac{\left(3\sqrt[3]{x+2x^3}\right)\int \frac{-\frac{20}{3}-\frac{28x^2}{3}}{x^{5/3}(-1+x^2)(1+2x^2)^{2/3}}\,dx}{8\sqrt[3]{x}\sqrt[3]{1+2x^2}}$$

$$= \frac{3\sqrt[3]{x+2x^3}}{8x^3} + \frac{15\sqrt[3]{x+2x^3}}{4x} - \frac{\left(9\sqrt[3]{x+2x^3}\right)\int -\frac{32\sqrt[3]{x}}{3(-1+x^2)(1+2x^2)^{2/3}}\,dx}{16\sqrt[3]{x}\sqrt[3]{1+2x^2}}$$

$$= \frac{3\sqrt[3]{x+2x^3}}{8x^3} + \frac{15\sqrt[3]{x+2x^3}}{4x} + \frac{\left(6\sqrt[3]{x+2x^3}\right)\int \frac{\sqrt[3]{x}}{(-1+x^2)(1+2x^2)^{2/3}}\,dx}{\sqrt[3]{x}\sqrt[3]{1+2x^2}}$$

$$= \frac{3\sqrt[3]{x+2x^3}}{8x^3} + \frac{15\sqrt[3]{x+2x^3}}{4x} + \frac{\left(18\sqrt[3]{x+2x^3}\right)\text{Subst}\left(\int \frac{x^3}{(-1+x^6)(1+2x^6)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1+2x^2}}$$

$$= \frac{3\sqrt[3]{x+2x^3}}{8x^3} + \frac{15\sqrt[3]{x+2x^3}}{4x} + \frac{\left(9\sqrt[3]{x+2x^3}\right)\text{Subst}\left(\int \frac{x}{(-1+x^3)(1+2x^3)^{2/3}}\,dx, x, x^{2/3}\right)}{\sqrt[3]{x}\sqrt[3]{1+2x^2}}$$

$$= \frac{3\sqrt[3]{x+2x^3}}{8x^3} + \frac{15\sqrt[3]{x+2x^3}}{4x} + \frac{\left(9\sqrt[3]{x+2x^3}\right)\text{Subst}\left(\int \frac{x}{-1+3x^3}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{1+2x^2}}\right)}{\sqrt[3]{x}\sqrt[3]{1+2x^2}}$$

$$= \frac{3\sqrt[3]{x+2x^3}}{8x^3} + \frac{15\sqrt[3]{x+2x^3}}{4x} + \frac{\left(3^{2/3}\sqrt[3]{x+2x^3}\right)\text{Subst}\left(\int \frac{1}{-1+\sqrt[3]{3}x}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{1+2x^2}}\right)}{\sqrt[3]{x}\sqrt[3]{1+2x^2}} - \ldots$$

$$= \frac{3\sqrt[3]{x+2x^3}}{8x^3} + \frac{15\sqrt[3]{x+2x^3}}{4x} + \frac{\sqrt[3]{3}\sqrt[3]{x+2x^3}\log\left(1-\frac{\sqrt[3]{3}x^{2/3}}{\sqrt[3]{1+2x^2}}\right)}{\sqrt[3]{x}\sqrt[3]{1+2x^2}} - \frac{\left(\sqrt[3]{3}\sqrt[3]{x+2x^3}\right)\text{Su}\ldots}{\ldots}$$

$$= \frac{3\sqrt[3]{x+2x^3}}{8x^3} + \frac{15\sqrt[3]{x+2x^3}}{4x} + \frac{\sqrt[3]{3}\sqrt[3]{x+2x^3}\log\left(1-\frac{\sqrt[3]{3}x^{2/3}}{\sqrt[3]{1+2x^2}}\right)}{\sqrt[3]{x}\sqrt[3]{1+2x^2}} - \frac{\sqrt[3]{3}\sqrt[3]{x+2x^3}\log\ldots}{2\sqrt[3]{x}\ldots}$$

$$= \frac{3\sqrt[3]{x+2x^3}}{8x^3} + \frac{15\sqrt[3]{x+2x^3}}{4x} + \frac{3^{5/6}\sqrt[3]{x+2x^3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}x^{2/3}}{\sqrt[3]{1+2x^2}}}{\sqrt{3}}\right)}{\sqrt[3]{x}\sqrt[3]{1+2x^2}} + \frac{\sqrt[3]{3}\sqrt[3]{x+2x^3}\log\ldots}{\sqrt[3]{x}\sqrt[3]{1+\ldots}}$$

Mathematica [C] time = 4.23, size = 103, normalized size = 0.73

$$\frac{3(2x^2+1)\sqrt[3]{2x^3+x}\left(2(6x^4-7x^2+1)\,{}_2F_1\left(1,1;\frac{2}{3};\frac{3x^2}{x^2-1}\right)+27(2x^4+x^2)\,{}_2F_1\left(2,2;\frac{5}{3};\frac{3x^2}{x^2-1}\right)-(x^2-1)^2\right)}{8x^3(x^2-1)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + x^2)*(x + 2*x^3)^(1/3))/(x^4*(-1 + x^2)),x]
```

```
[Out] (3*(1 + 2*x^2)*(x + 2*x^3)^(1/3)*(-(-1 + x^2)^2 + 2*(1 - 7*x^2 + 6*x^4)*Hyp
ergeometric2F1[1, 1, 2/3, (3*x^2)/(-1 + x^2)] + 27*(x^2 + 2*x^4)*Hypergeome
tric2F1[2, 2, 5/3, (3*x^2)/(-1 + x^2)]))/(8*x^3*(-1 + x^2)^2)
```

IntegrateAlgebraic [A] time = 0.38, size = 142, normalized size = 1.00

$$\sqrt[3]{3}\log\left(3^{2/3}\sqrt[3]{2x^3+x}-3x\right)+3^{5/6}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2x^3+x}+\sqrt[3]{3}x}\right)+\frac{3\sqrt[3]{2x^3+x}\left(10x^2+1\right)}{8x^3}-\frac{1}{2}\sqrt[3]{3}\log\left(3^{2/3}\sqrt[3]{2x^3+x}x+\sqrt[3]{3}\left(2x^3+x\right)^{2/3}+3x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^2)*(x + 2*x^3)^(1/3))/(x^4*(-1 + x^2)),x]
```

```
[Out] (3*(1 + 10*x^2)*(x + 2*x^3)^(1/3))/(8*x^3) + 3^(5/6)*ArcTan[(3^(5/6)*x)/(3^
(1/3)*x + 2*(x + 2*x^3)^(1/3))] + 3^(1/3)*Log[-3*x + 3^(2/3)*(x + 2*x^3)^(1
/3)] - (3^(1/3)*Log[3*x^2 + 3^(2/3)*x*(x + 2*x^3)^(1/3) + 3^(1/3)*(x + 2*x^
3)^(2/3)])/2
```

fricas [B] time = 1.63, size = 263, normalized size = 1.85

$$\frac{8\cdot 3^{\frac{5}{6}}x^3\arctan\left(\frac{6\cdot 3^{\frac{5}{6}}\left(8x^4-7x^2-1\right)\left(2x^3+x\right)^{\frac{2}{3}}-\sqrt{3}\left(377x^6+300x^4+51x^2+1\right)-18\cdot 3^{\frac{1}{6}}\left(55x^5+25x^3+x\right)\left(2x^3+x\right)^{\frac{1}{3}}}{3\left(487x^6+240x^4+3x^2-1\right)}\right)-4\cdot 3^{\frac{1}{3}}x^3\log\left(\frac{3\cdot 3^{\frac{2}{3}}\left(2x^3+x\right)^{\frac{2}{3}}\left(8x^2+1\right)+3^{\frac{1}{3}}\left(55x^4+25x^2+1\right)+9\left(7x^3+2x\right)\left(2x^3+x\right)^{\frac{1}{3}}}{x^4-2x^2+1}\right)+8\cdot 3^{\frac{1}{3}}x^3\log\left(-\frac{3^{\frac{2}{3}}\left(x^2-1\right)-9\cdot 3^{\frac{1}{3}}\left(2x^3+x\right)^{\frac{1}{3}}x+9\left(2x^3+x\right)^{\frac{2}{3}}}{x^2-1}\right)+9\left(2x^3+x\right)^{\frac{1}{3}}\left(10x^2+1\right)}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(2*x^3+x)^(1/3)/x^4/(x^2-1),x, algorithm="fricas")
```

```
[Out] 1/24*(8*3^(5/6)*x^3*arctan(1/3*(6*3^(5/6)*(8*x^4 - 7*x^2 - 1)*(2*x^3 + x)^(
2/3) - sqrt(3)*(377*x^6 + 300*x^4 + 51*x^2 + 1) - 18*3^(1/6)*(55*x^5 + 25*x
^3 + x)*(2*x^3 + x)^(1/3))/(487*x^6 + 240*x^4 + 3*x^2 - 1)) - 4*3^(1/3)*x^3
*log((3*3^(2/3)*(2*x^3 + x)^(2/3)*(8*x^2 + 1) + 3^(1/3)*(55*x^4 + 25*x^2 +
1) + 9*(7*x^3 + 2*x)*(2*x^3 + x)^(1/3))/(x^4 - 2*x^2 + 1)) + 8*3^(1/3)*x^3*
log(-(3^(2/3)*(x^2 - 1) - 9*3^(1/3)*(2*x^3 + x)^(1/3)*x + 9*(2*x^3 + x)^(2/
3))/(x^2 - 1)) + 9*(2*x^3 + x)^(1/3)*(10*x^2 + 1))/x^3
```

giac [A] time = 0.23, size = 90, normalized size = 0.63

$$-3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(3^{\frac{1}{3}}+2\left(\frac{1}{x^2}+2\right)^{\frac{1}{3}}\right)\right)+\frac{3}{8}\left(\frac{1}{x^2}+2\right)^{\frac{4}{3}}-\frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(3^{\frac{2}{3}}+3^{\frac{1}{3}}\left(\frac{1}{x^2}+2\right)^{\frac{1}{3}}+\left(\frac{1}{x^2}+2\right)^{\frac{2}{3}}\right)+3^{\frac{1}{3}}\log\left(\left|-3^{\frac{1}{3}}+\left(\frac{1}{x^2}+2\right)^{\frac{1}{3}}\right|\right)+3\left(\frac{1}{x^2}+2\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(2*x^3+x)^(1/3)/x^4/(x^2-1),x, algorithm="giac")
```

```
[Out] -3^(5/6)*arctan(1/3*3^(1/6)*(3^(1/3) + 2*(1/x^2 + 2)^(1/3))) + 3/8*(1/x^2 +
 2)^(4/3) - 1/2*3^(1/3)*log(3^(2/3) + 3^(1/3)*(1/x^2 + 2)^(1/3) + (1/x^2 +
2)^(2/3)) + 3^(1/3)*log(abs(-3^(1/3) + (1/x^2 + 2)^(1/3))) + 3*(1/x^2 + 2)^
(1/3)
```

maple [C] time = 9.39, size = 1861, normalized size = 13.11

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(2*x^3+x)^(1/3)/x^4/(x^2-1),x)
```

```
[Out] 3/8*(20*x^4+12*x^2+1)/x^3*(x*(2*x^2+1))^(1/3)/(2*x^2+1)+(-1/3*ln((178*RootO
f(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)^2*RootOf(_Z^3-3)^2*x^4+1212*
RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)*RootOf(_Z^3-3)^3*x^4-62
3*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)^2*RootOf(_Z^3-3)^2*x^
2-4242*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)*RootOf(_Z^3-3)^3
```

```
*x^2+19197*(4*x^6+4*x^4+x^2)^(2/3)*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z
^3-3)+_Z^2)*RootOf(_Z^3-3)^2-18738*(4*x^6+4*x^4+x^2)^(1/3)*RootOf(9*RootOf(
_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)*RootOf(_Z^3-3)*x^2+58968*(4*x^6+4*x^4+x
^2)^(1/3)*RootOf(_Z^3-3)^2*x^2-41652*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(
_Z^3-3)+_Z^2)*x^4-283608*RootOf(_Z^3-3)*x^4-356*RootOf(9*RootOf(_Z^3-3)^2+3
*_Z*RootOf(_Z^3-3)+_Z^2)^2*RootOf(_Z^3-3)^2-2424*RootOf(9*RootOf(_Z^3-3)^2+
3*_Z*RootOf(_Z^3-3)+_Z^2)*RootOf(_Z^3-3)^3-9369*(4*x^6+4*x^4+x^2)^(1/3)*Roo
tOf(_Z^3-3)*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)+29484*(4*x^
6+4*x^4+x^2)^(1/3)*RootOf(_Z^3-3)^2-32040*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*Ro
otOf(_Z^3-3)+_Z^2)*x^2-218160*RootOf(_Z^3-3)*x^2+84321*(4*x^6+4*x^4+x^2)^(2
/3)-5607*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)-38178*RootOf(_
Z^3-3))/(2*x^2+1)/(-1+x)/(1+x))*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-
3)+_Z^2)-ln((178*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)^2*Root
Of(_Z^3-3)^2*x^4+1212*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)*R
ootOf(_Z^3-3)^3*x^4-623*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)
^2*RootOf(_Z^3-3)^2*x^2-4242*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+
_Z^2)*RootOf(_Z^3-3)^3*x^2+19197*(4*x^6+4*x^4+x^2)^(2/3)*RootOf(9*RootOf(_Z
^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)*RootOf(_Z^3-3)^2-18738*(4*x^6+4*x^4+x^2)^
(1/3)*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)*RootOf(_Z^3-3)*x^
2+58968*(4*x^6+4*x^4+x^2)^(1/3)*RootOf(_Z^3-3)^2*x^2-41652*RootOf(9*RootOf(
_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)*x^4-283608*RootOf(_Z^3-3)*x^4-356*RootO
f(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)^2*RootOf(_Z^3-3)^2-2424*Root
Of(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)*RootOf(_Z^3-3)^3-9369*(4*x^
6+4*x^4+x^2)^(1/3)*RootOf(_Z^3-3)*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^
3-3)+_Z^2)+29484*(4*x^6+4*x^4+x^2)^(1/3)*RootOf(_Z^3-3)^2-32040*RootOf(9*Ro
otOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)*x^2-218160*RootOf(_Z^3-3)*x^2+8432
1*(4*x^6+4*x^4+x^2)^(2/3)-5607*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3
)+_Z^2)-38178*RootOf(_Z^3-3))/(2*x^2+1)/(-1+x)/(1+x))*RootOf(_Z^3-3)+RootOf
(_Z^3-3)*ln(-(-226*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)^2*Ro
otOf(_Z^3-3)^2*x^4-1212*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)
*RootOf(_Z^3-3)^3*x^4+791*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^
2)^2*RootOf(_Z^3-3)^2*x^2+4242*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3
)+_Z^2)*RootOf(_Z^3-3)^3*x^2+19197*(4*x^6+4*x^4+x^2)^(2/3)*RootOf(9*RootOf(
_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)*RootOf(_Z^3-3)^2-19656*(4*x^6+4*x^4+x^2
)^(1/3)*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)*RootOf(_Z^3-3)*
x^2+56214*(4*x^6+4*x^4+x^2)^(1/3)*RootOf(_Z^3-3)^2*x^2-54918*RootOf(9*RootO
f(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)*x^4-294516*RootOf(_Z^3-3)*x^4+452*Roo
tOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)^2*RootOf(_Z^3-3)^2+2424*Ro
otOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)*RootOf(_Z^3-3)^3-9828*(4*
x^6+4*x^4+x^2)^(1/3)*RootOf(_Z^3-3)*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_
Z^3-3)+_Z^2)+28107*(4*x^6+4*x^4+x^2)^(1/3)*RootOf(_Z^3-3)^2-33561*RootOf(9*
RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3-3)+_Z^2)*x^2-179982*RootOf(_Z^3-3)*x^2+88
452*(4*x^6+4*x^4+x^2)^(2/3)-3051*RootOf(9*RootOf(_Z^3-3)^2+3*_Z*RootOf(_Z^3
-3)+_Z^2)-16362*RootOf(_Z^3-3))/(2*x^2+1)/(-1+x)/(1+x)))*(x*(2*x^2+1))^(1/3
)/x*(x^2*(2*x^2+1)^2)^(1/3)/(2*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^3+x\right)^{\frac{1}{3}}\left(x^2+1\right)}{\left(x^2-1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(2*x^3+x)^(1/3)/x^4/(x^2-1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^3 + x)^(1/3)*(x^2 + 1)/((x^2 - 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^3+x\right)^{1/3}\,\left(x^2+1\right)}{x^4\,\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 2*x^3)^(1/3)*(x^2 + 1))/(x^4*(x^2 - 1)),x)
[Out] int(((x + 2*x^3)^(1/3)*(x^2 + 1))/(x^4*(x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x\left(2x^2+1\right)}\left(x^2+1\right)}{x^4\left(x-1\right)\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(2*x**3+x)**(1/3)/x**4/(x**2-1),x)
[Out] Integral((x*(2*x**2 + 1))**(1/3)*(x**2 + 1)/(x**4*(x - 1)*(x + 1)), x)
```

3.1671 $\int \frac{(-b+a^3x^3)\sqrt[3]{b+a^3x^3}}{x^5}\,dx$

Optimal. Leaf size=142

$$\frac{(b-3a^3x^3)\sqrt[3]{a^3x^3+b}}{4x^4}-\frac{1}{3}a^4\log\left(\sqrt[3]{a^3x^3+b}-ax\right)-\frac{a^4\tan^{-1}\left(\frac{\sqrt{3}\,ax}{2\sqrt[3]{a^3x^3+b}+ax}\right)}{\sqrt{3}}+\frac{1}{6}a^4\log\left(ax\sqrt[3]{a^3x^3+b}+(a^3x^3+\right.$$

Rubi [A] time = 0.10, antiderivative size = 151, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {451, 277, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{a^3\sqrt[3]{a^3x^3+b}}{x}+\frac{(a^3x^3+b)^{4/3}}{4x^4}-\frac{1}{3}a^4\log\left(1-\frac{ax}{\sqrt[3]{a^3x^3+b}}\right)-\frac{a^4\tan^{-1}\left(\frac{\frac{2ax}{\sqrt[3]{a^3x^3+b}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}a^4\log\left(\frac{ax}{\sqrt[3]{a^3x^3+b}}+\frac{a^2x^2}{(a^3x^3+b)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a^3*x^3)*(b + a^3*x^3)^(1/3))/x^5,x]
```

```
[Out] -((a^3*(b + a^3*x^3)^(1/3))/x) + (b + a^3*x^3)^(4/3)/(4*x^4) - (a^4*ArcTan[
(1 + (2*a*x)/(b + a^3*x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - (a^4*Log[1 - (a*x)/(b
 + a^3*x^3)^(1/3)])/3 + (a^4*Log[1 + (a^2*x^2)/(b + a^3*x^3)^(2/3) + (a*x)/
(b + a^3*x^3)^(1/3)])/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(-b+a^3x^3\right)\sqrt[3]{b+a^3x^3}}{x^5}\,dx = \frac{\left(b+a^3x^3\right)^{4/3}}{4x^4}+a^3\int \frac{\sqrt[3]{b+a^3x^3}}{x^2}\,dx$$

$$= -\frac{a^3\sqrt[3]{b+a^3x^3}}{x}+\frac{\left(b+a^3x^3\right)^{4/3}}{4x^4}+a^6\int \frac{x}{\left(b+a^3x^3\right)^{2/3}}\,dx$$

$$= -\frac{a^3\sqrt[3]{b+a^3x^3}}{x}+\frac{\left(b+a^3x^3\right)^{4/3}}{4x^4}+a^6\,\text{Subst}\left(\int \frac{x}{1-a^3x^3}\,dx,x,\frac{x}{\sqrt[3]{b+a^3x^3}}\right)$$

$$= -\frac{a^3\sqrt[3]{b+a^3x^3}}{x}+\frac{\left(b+a^3x^3\right)^{4/3}}{4x^4}+\frac{1}{3}a^5\,\text{Subst}\left(\int \frac{1}{1-ax}\,dx,x,\frac{x}{\sqrt[3]{b+a^3x^3}}\right)-\frac{1}{3}a^5$$

$$= -\frac{a^3\sqrt[3]{b+a^3x^3}}{x}+\frac{\left(b+a^3x^3\right)^{4/3}}{4x^4}-\frac{1}{3}a^4\log\left(1-\frac{ax}{\sqrt[3]{b+a^3x^3}}\right)+\frac{1}{6}a^4\,\text{Subst}\left(\int \frac{a}{1+}\right.$$

$$= -\frac{a^3\sqrt[3]{b+a^3x^3}}{x}+\frac{\left(b+a^3x^3\right)^{4/3}}{4x^4}-\frac{1}{3}a^4\log\left(1-\frac{ax}{\sqrt[3]{b+a^3x^3}}\right)+\frac{1}{6}a^4\log\left(1+\frac{a}{\left(b+\right.}\right.$$

$$= -\frac{a^3\sqrt[3]{b+a^3x^3}}{x}+\frac{\left(b+a^3x^3\right)^{4/3}}{4x^4}-\frac{a^4\tan^{-1}\left(\frac{1+\frac{2ax}{\sqrt[3]{b+a^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}a^4\log\left(1-\frac{ax}{\sqrt[3]{b+a^3x^3}}\right.$$

Mathematica [C] time = 0.06, size = 74, normalized size = 0.52

$$\frac{\sqrt[3]{a^3x^3+b}\left(-\frac{4a^3x^3\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{a^3x^3}{b}\right)}{\sqrt[3]{\frac{a^3x^3}{b}+1}}+a^3x^3+b\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + a^3*x^3)*(b + a^3*x^3)^(1/3))/x^5,x]
```

```
[Out] ((b + a^3*x^3)^(1/3)*(b + a^3*x^3 - (4*a^3*x^3*Hypergeometric2F1[-1/3, -1/3
, 2/3, -((a^3*x^3)/b)])/(1 + (a^3*x^3)/b)^(1/3)))/(4*x^4)
```

IntegrateAlgebraic [A] time = 0.25, size = 142, normalized size = 1.00

$$\frac{(b-3a^3x^3)\sqrt[3]{a^3x^3+b}}{4x^4}-\frac{1}{3}a^4\log\left(\sqrt[3]{a^3x^3+b}-ax\right)-\frac{a^4\tan^{-1}\left(\frac{\sqrt{3}ax}{2\sqrt[3]{a^3x^3+b}+ax}\right)}{\sqrt{3}}+\frac{1}{6}a^4\log\left(ax\sqrt[3]{a^3x^3+b}+\left(a^3x^3+b\right)^{2/3}+a^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a^3*x^3)*(b + a^3*x^3)^(1/3))/x^5,x]
```

```
[Out] ((b - 3*a^3*x^3)*(b + a^3*x^3)^(1/3))/(4*x^4) - (a^4*ArcTan[(Sqrt[3]*a*x)/(
a*x + 2*(b + a^3*x^3)^(1/3))])/Sqrt[3] - (a^4*Log[-(a*x) + (b + a^3*x^3)^(1
/3)])/3 + (a^4*Log[a^2*x^2 + a*x*(b + a^3*x^3)^(1/3) + (b + a^3*x^3)^(2/3)]
)/6
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3-b)*(a^3*x^3+b)^(1/3)/x^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a^3x^3+b\right)^{\frac{1}{3}}\left(a^3x^3-b\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3-b)*(a^3*x^3+b)^(1/3)/x^5,x, algorithm="giac")
```

```
[Out] integrate((a^3*x^3 + b)^(1/3)*(a^3*x^3 - b)/x^5, x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int\frac{\left(a^3x^3-b\right)\left(a^3x^3+b\right)^{\frac{1}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^3*x^3-b)*(a^3*x^3+b)^(1/3)/x^5,x)
```

```
[Out] int((a^3*x^3-b)*(a^3*x^3+b)^(1/3)/x^5,x)
```

maxima [A] time = 0.42, size = 133, normalized size = 0.94

$$\frac{1}{6}\left(2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(a+\frac{2\left(a^3x^3+b\right)^{\frac{1}{3}}}{x}\right)}{3\,a}\right)+a\log\left(a^2+\frac{\left(a^3x^3+b\right)^{\frac{1}{3}}a}{x}+\frac{\left(a^3x^3+b\right)^{\frac{2}{3}}}{x^2}\right)-2\,a\log\left(-a+\frac{\left(a^3x^3+b\right)^{\frac{1}{3}}}{x}\right)-\frac{6\left(a^3x^3+b\right)^{\frac{1}{3}}}{x}\right)a^3+\frac{\left(a^3x^3+b\right)^{\frac{4}{3}}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3-b)*(a^3*x^3+b)^(1/3)/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(a + 2*(a^3*x^3 + b)^(1/3)/x)/a) + a*lo
g(a^2 + (a^3*x^3 + b)^(1/3)*a/x + (a^3*x^3 + b)^(2/3)/x^2) - 2*a*log(-a + (
a^3*x^3 + b)^(1/3)/x) - 6*(a^3*x^3 + b)^(1/3)/x)*a^3 + 1/4*(a^3*x^3 + b)^(4
/3)/x^4
```

mupad [B] time = 1.63, size = 66, normalized size = 0.46

$$\frac{\left(a^3\,x^3+b\right)^{4/3}}{4\,x^4}-\frac{a^3\,\left(a^3\,x^3+b\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},-\frac{1}{3};\ \frac{2}{3};\ -\frac{a^3\,x^3}{b}\right)}{x\,\left(\frac{a^3\,x^3}{b}+1\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b + a^3*x^3)^(1/3)*(b - a^3*x^3))/x^5,x)
```

```
[Out] (b + a^3*x^3)^(4/3)/(4*x^4) - (a^3*(b + a^3*x^3)^(1/3)*hypergeom([-1/3, -1/
3], 2/3, -(a^3*x^3)/b))/(x*((a^3*x^3)/b + 1)^(1/3))
```

sympy [C] time = 2.63, size = 114, normalized size = 0.80

$$-\frac{a^4\sqrt[3]{1+\frac{b}{a^3x^3}}\,\Gamma\left(-\frac{4}{3}\right)}{3\Gamma\left(-\frac{1}{3}\right)}+\frac{a^3\sqrt[3]{b}\,\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{a^3x^3e^{i\pi}}{b}\right)}{3x\Gamma\left(\frac{2}{3}\right)}-\frac{ab\sqrt[3]{1+\frac{b}{a^3x^3}}\,\Gamma\left(-\frac{4}{3}\right)}{3x^3\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**3*x**3-b)*(a**3*x**3+b)**(1/3)/x**5,x)
```

```
[Out] -a**4*(1 + b/(a**3*x**3))**(1/3)*gamma(-4/3)/(3*gamma(-1/3)) + a**3*b**(1/3
)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), a**3*x**3*exp_polar(I*pi)/b)/(3*x
*gamma(2/3)) - a*b*(1 + b/(a**3*x**3))**(1/3)*gamma(-4/3)/(3*x**3*gamma(-1/
3))
```

3.1672 $\int \frac{-3b+ax^2}{\sqrt[4]{3b-2ax^2}\left(3b-2ax^2+3x^4\right)}\,dx$

Optimal. Leaf size=142

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{3}x^2}{\sqrt{2}}-\frac{\sqrt{3b-2ax^2}}{\sqrt{2}\sqrt[4]{3}}}{x\sqrt[4]{3b-2ax^2}}\right)}{2\sqrt{2}\sqrt[4]{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}x\sqrt[4]{3b-2ax^2}}{\sqrt{3b-2ax^2}+\sqrt{3}x^2}\right)}{2\sqrt{2}\sqrt[4]{3}}$$

Rubi [C] time = 0.84, antiderivative size = 539, normalized size of antiderivative = 3.80, number of steps used = 10, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {1692, 399, 490, 1218}

$$\frac{\sqrt[4]{b}\left(\sqrt{a^2-9b}+a\right)\sqrt{\frac{ax^2}{b}}\,\Pi\left(-\frac{3\sqrt{b}}{\sqrt{-2a^2-2\sqrt{a^2-9b}\,a+9b}};\sin^{-1}\left(\frac{\sqrt[4]{3b-2ax^2}}{\sqrt[4]{3}\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt{2}\,3^{3/4}x\sqrt{-2a\sqrt{a^2-9b}-2a^2+9b}}-\frac{\sqrt[4]{b}\left(\sqrt{a^2-9b}+a\right)\sqrt{\frac{ax^2}{b}}\,\Pi\left(\frac{3\sqrt{b}}{\sqrt{-2a^2-2\sqrt{a^2-9b}\,a+9b}};\sin^{-1}\left(\frac{\sqrt[4]{3b-2ax^2}}{\sqrt[4]{3}\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt{2}\,3^{3/4}x\sqrt{-2a\sqrt{a^2-9b}-2a^2+9b}}+\frac{\sqrt[4]{b}\left(a-\sqrt{a^2-9b}\right)\sqrt{\frac{ax^2}{b}}\,\Pi\left(-\frac{3\sqrt{b}}{\sqrt{-2a^2+2\sqrt{a^2-9b}\,a+9b}};\sin^{-1}\left(\frac{\sqrt[4]{3b-2ax^2}}{\sqrt[4]{3}\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt{2}\,3^{3/4}x\sqrt{2a\sqrt{a^2-9b}-2a^2+9b}}-\frac{\sqrt[4]{b}\left(a-\sqrt{a^2-9b}\right)\sqrt{\frac{ax^2}{b}}\,\Pi\left(\frac{3\sqrt{b}}{\sqrt{-2a^2+2\sqrt{a^2-9b}\,a+9b}};\sin^{-1}\left(\frac{\sqrt[4]{3b-2ax^2}}{\sqrt[4]{3}\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt{2}\,3^{3/4}x\sqrt{2a\sqrt{a^2-9b}-2a^2+9b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-3*b + a*x^2)/((3*b - 2*a*x^2)^(1/4)*(3*b - 2*a*x^2 + 3*x^4)),x]
```

```
[Out] ((a + Sqrt[a^2 - 9*b])*b^(1/4)*Sqrt[(a*x^2)/b]*EllipticPi[(-3*Sqrt[b])/Sqrt
[-2*a^2 - 2*a*Sqrt[a^2 - 9*b] + 9*b], ArcSin[(3*b - 2*a*x^2)^(1/4)/(3^(1/4)
*b^(1/4))], -1])/(Sqrt[2]*3^(3/4)*Sqrt[-2*a^2 - 2*a*Sqrt[a^2 - 9*b] + 9*b]*
x) - ((a + Sqrt[a^2 - 9*b])*b^(1/4)*Sqrt[(a*x^2)/b]*EllipticPi[(3*Sqrt[b])/
Sqrt[-2*a^2 - 2*a*Sqrt[a^2 - 9*b] + 9*b], ArcSin[(3*b - 2*a*x^2)^(1/4)/(3^(
1/4)*b^(1/4))], -1])/(Sqrt[2]*3^(3/4)*Sqrt[-2*a^2 - 2*a*Sqrt[a^2 - 9*b] + 9
*b]*x) + ((a - Sqrt[a^2 - 9*b])*b^(1/4)*Sqrt[(a*x^2)/b]*EllipticPi[(-3*Sqrt
[b])/Sqrt[-2*a^2 + 2*a*Sqrt[a^2 - 9*b] + 9*b], ArcSin[(3*b - 2*a*x^2)^(1/4)
/(3^(1/4)*b^(1/4))], -1])/(Sqrt[2]*3^(3/4)*Sqrt[-2*a^2 + 2*a*Sqrt[a^2 - 9*b
] + 9*b]*x) - ((a - Sqrt[a^2 - 9*b])*b^(1/4)*Sqrt[(a*x^2)/b]*EllipticPi[(3*
Sqrt[b])/Sqrt[-2*a^2 + 2*a*Sqrt[a^2 - 9*b] + 9*b], ArcSin[(3*b - 2*a*x^2)^(
1/4)/(3^(1/4)*b^(1/4))], -1])/(Sqrt[2]*3^(3/4)*Sqrt[-2*a^2 + 2*a*Sqrt[a^2 -
 9*b] + 9*b]*x)
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
```

```
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{-3b+ax^2}{\sqrt[4]{3b-2ax^2}\left(3b-2ax^2+3x^4\right)}dx = \int \left(\frac{a+\sqrt{a^2-9b}}{\left(-2a-2\sqrt{a^2-9b}+6x^2\right)\sqrt[4]{3b-2ax^2}} + \frac{a-\sqrt{a^2-9b}}{\left(-2a+2\sqrt{a^2-9b}+6x^2\right)\sqrt[4]{\ }}\right.$$

$$= \left(a-\sqrt{a^2-9b}\right)\int \frac{1}{\left(-2a+2\sqrt{a^2-9b}+6x^2\right)\sqrt[4]{3b-2ax^2}}dx + \left(a+\sqrt{a^2}\right.$$

$$= \frac{\left(2\sqrt{\frac{2}{3}}\left(a-\sqrt{a^2-9b}\right)\sqrt{\frac{ax^2}{b}}\right)\text{Subst}\left(\int \frac{x^2}{\left(-2a\left(-2a+2\sqrt{a^2-9b}\right)-18b+6x^4\right)\sqrt{1-\frac{x^4}{3b}}}d\right.}{x}$$

$$= -\frac{\left(\left(a-\sqrt{a^2-9b}\right)\sqrt{\frac{ax^2}{b}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-2a^2+2a\sqrt{a^2-9b}+9b}-\sqrt{3}x^2\right)\sqrt{1-\frac{x^4}{3b}}}dx,x,\right.}{3\sqrt{2}x}$$

$$= \frac{\left(a+\sqrt{a^2-9b}\right)\sqrt[4]{b}\sqrt{\frac{ax^2}{b}}\,\Pi\left(-\frac{3\sqrt{b}}{\sqrt{-2a^2-2a\sqrt{a^2-9b}+9b}};\sin^{-1}\left(\frac{\sqrt[4]{3b-2ax^2}}{\sqrt[4]{3}\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt{2}\,3^{3/4}\sqrt{-2a^2-2a\sqrt{a^2-9b}+9b}\,x}$$

Mathematica [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{-3b+ax^2}{\sqrt[4]{3b-2ax^2}\left(3b-2ax^2+3x^4\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3*b + a*x^2)/((3*b - 2*a*x^2)^(1/4)*(3*b - 2*a*x^2 + 3*x^4)),x]
[Out] Integrate[(-3*b + a*x^2)/((3*b - 2*a*x^2)^(1/4)*(3*b - 2*a*x^2 + 3*x^4)), x
]
```

IntegrateAlgebraic [A] time = 0.47, size = 142, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{3}x^2}{\sqrt{2}}-\frac{\sqrt{3b-2ax^2}}{\sqrt{2}\sqrt[4]{3}}}{x\sqrt[4]{3b-2ax^2}}\right)}{2\sqrt{2}\sqrt[4]{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3}x\sqrt[4]{3b-2ax^2}}{\sqrt{3b-2ax^2}+\sqrt{3}x^2}\right)}{2\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*b + a*x^2)/((3*b - 2*a*x^2)^(1/4)*(3*b - 2*a*x^2 + 3
*x^4)),x]
[Out] -1/2*ArcTan[((3^(1/4)*x^2)/Sqrt[2] - Sqrt[3*b - 2*a*x^2]/(Sqrt[2]*3^(1/4)))
/(x*(3*b - 2*a*x^2)^(1/4))]/(Sqrt[2]*3^(1/4)) - ArcTanh[(Sqrt[2]*3^(1/4)*x*
(3*b - 2*a*x^2)^(1/4))/(Sqrt[3]*x^2 + Sqrt[3*b - 2*a*x^2])]/(2*Sqrt[2]*3^(1
/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-3*b)/(-2*a*x^2+3*b)^(1/4)/(3*x^4-2*a*x^2+3*b),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - 3\,b}{\left(3\,x^4 - 2\,ax^2 + 3\,b\right)\left(-2\,ax^2 + 3\,b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-3*b)/(-2*a*x^2+3*b)^(1/4)/(3*x^4-2*a*x^2+3*b),x, algorithm
="giac")
```

```
[Out] integrate((a*x^2 - 3*b)/((3*x^4 - 2*a*x^2 + 3*b)*(-2*a*x^2 + 3*b)^(1/4)), x
)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2 - 3b}{\left(-2a\,x^2 + 3b\right)^{\frac{1}{4}}\left(3x^4 - 2a\,x^2 + 3b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-3*b)/(-2*a*x^2+3*b)^(1/4)/(3*x^4-2*a*x^2+3*b),x)
```

```
[Out] int((a*x^2-3*b)/(-2*a*x^2+3*b)^(1/4)/(3*x^4-2*a*x^2+3*b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - 3\,b}{\left(3\,x^4 - 2\,ax^2 + 3\,b\right)\left(-2\,ax^2 + 3\,b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-3*b)/(-2*a*x^2+3*b)^(1/4)/(3*x^4-2*a*x^2+3*b),x, algorithm
="maxima")
```

```
[Out] integrate((a*x^2 - 3*b)/((3*x^4 - 2*a*x^2 + 3*b)*(-2*a*x^2 + 3*b)^(1/4)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{3\,b - a\,x^2}{\left(3\,b - 2\,a\,x^2\right)^{1/4}\,\left(3\,x^4 - 2\,a\,x^2 + 3\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*b - a*x^2)/((3*b - 2*a*x^2)^(1/4)*(3*b - 2*a*x^2 + 3*x^4)),x)
```

```
[Out] int(-(3*b - a*x^2)/((3*b - 2*a*x^2)^(1/4)*(3*b - 2*a*x^2 + 3*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - 3b}{\sqrt[4]{-2ax^2 + 3b}\,\left(-2ax^2 + 3b + 3x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-3*b)/(-2*a*x**2+3*b)**(1/4)/(3*x**4-2*a*x**2+3*b),x)
```

```
[Out] Integral((a*x**2 - 3*b)/((-2*a*x**2 + 3*b)**(1/4)*(-2*a*x**2 + 3*b + 3*x**4
)), x)
```

3.1673 $\int \frac{(-4b+ax^5)(b-cx^4+ax^5)}{x^2(b+ax^5)^{3/4}(b+cx^4+ax^5)}\,dx$

Optimal. Leaf size=142

$$2\sqrt{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x\sqrt[4]{ax^5+b}}{\sqrt{ax^5+b}-\sqrt{c}\,x^2}\right)-2\sqrt{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5+b}}{\sqrt{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5+b}}\right)+\frac{4\sqrt[4]{ax^5+b}}{x}$$

Rubi [F] time = 3.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-4b+ax^5)(b-cx^4+ax^5)}{x^2(b+ax^5)^{3/4}(b+cx^4+ax^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-4*b + a*x^5)*(b - c*x^4 + a*x^5))/(x^2*(b + a*x^5)^(3/4)*(b + c*x^4
 + a*x^5)),x]
```

```
[Out] (4*b*(1 + (a*x^5)/b)^(3/4)*Hypergeometric2F1[-1/5, 3/4, 4/5, -((a*x^5)/b)])
/(x*(b + a*x^5)^(3/4)) - (2*c^3*x*(1 + (a*x^5)/b)^(3/4)*Hypergeometric2F1[1
/5, 3/4, 6/5, -((a*x^5)/b)])/(a^2*(b + a*x^5)^(3/4)) + (c^2*x^2*(1 + (a*x^5
)/b)^(3/4)*Hypergeometric2F1[2/5, 3/4, 7/5, -((a*x^5)/b)])/(a*(b + a*x^5)^(
3/4)) - (2*c*x^3*(1 + (a*x^5)/b)^(3/4)*Hypergeometric2F1[3/5, 3/4, 8/5, -((
a*x^5)/b)])/(3*(b + a*x^5)^(3/4)) + (a*x^4*(1 + (a*x^5)/b)^(3/4)*Hypergeome
tric2F1[3/4, 4/5, 9/5, -((a*x^5)/b)])/(4*(b + a*x^5)^(3/4)) + (2*b*c^3*Defe
r[Int][1/((b + a*x^5)^(3/4)*(b + c*x^4 + a*x^5)), x])/a^2 - (2*b*c^2*Defer[
Int][x/((b + a*x^5)^(3/4)*(b + c*x^4 + a*x^5)), x])/a + 10*b*c*Defer[Int][x
^2/((b + a*x^5)^(3/4)*(b + c*x^4 + a*x^5)), x] + (2*c^4*Defer[Int][x^4/((b
 + a*x^5)^(3/4)*(b + c*x^4 + a*x^5)), x])/a^2
```

Rubi steps

$$\begin{aligned}\int \frac{(-4b+ax^5)(b-cx^4+ax^5)}{x^2(b+ax^5)^{3/4}(b+cx^4+ax^5)}\,dx &= \int \left(-\frac{2c^3}{a^2(b+ax^5)^{3/4}}-\frac{4b}{x^2(b+ax^5)^{3/4}}+\frac{2c^2x}{a(b+ax^5)^{3/4}}-\frac{2cx^2}{(b+ax^5)^{3/4}}\right.\\ &= \frac{2\int \frac{bc^3-abc^2x+5a^2bcx^2+c^4x^4}{(b+ax^5)^{3/4}(b+cx^4+ax^5)}\,dx}{a^2}+a\int \frac{x^3}{(b+ax^5)^{3/4}}\,dx-(4b)\int \frac{1}{x^2(b+ax}\\ &= \frac{2\int \left(\frac{bc^3}{(b+ax^5)^{3/4}(b+cx^4+ax^5)}-\frac{abc^2x}{(b+ax^5)^{3/4}(b+cx^4+ax^5)}+\frac{5a^2bcx^2}{(b+ax^5)^{3/4}(b+cx^4+ax^5)}+\right.}{a^2}\\ &= \frac{4b\left(1+\frac{ax^5}{b}\right)^{3/4}{}_2F_1\left(-\frac{1}{5},\frac{3}{4};\frac{4}{5};-\frac{ax^5}{b}\right)}{x(b+ax^5)^{3/4}}-\frac{2c^3x\left(1+\frac{ax^5}{b}\right)^{3/4}{}_2F_1\left(\frac{1}{5},\frac{3}{4};\frac{6}{5};-\right.}{a^2(b+ax^5)^{3/4}}\end{aligned}$$

Mathematica [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{(-4b+ax^5)(b-cx^4+ax^5)}{x^2(b+ax^5)^{3/4}(b+cx^4+ax^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-4*b + a*x^5)*(b - c*x^4 + a*x^5))/(x^2*(b + a*x^5)^(3/4)*(b +
c*x^4 + a*x^5)),x]
```

```
[Out] Integrate[((-4*b + a*x^5)*(b - c*x^4 + a*x^5))/(x^2*(b + a*x^5)^(3/4)*(b +
c*x^4 + a*x^5)), x]
```

IntegrateAlgebraic [A] time = 13.18, size = 142, normalized size = 1.00

$$2\sqrt{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x\sqrt[4]{ax^5+b}}{\sqrt{ax^5+b}-\sqrt{c}\,x^2}\right)-2\sqrt{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5+b}}{\sqrt{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5+b}}\right)+\frac{4\sqrt[4]{ax^5+b}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4*b + a*x^5)*(b - c*x^4 + a*x^5))/(x^2*(b + a*x^5)^(3
/4)*(b + c*x^4 + a*x^5)),x]
```

```
[Out] (4*(b + a*x^5)^(1/4))/x + 2*Sqrt[2]*c^(1/4)*ArcTan[(Sqrt[2]*c^(1/4)*x*(b +
a*x^5)^(1/4))/(-(Sqrt[c]*x^2) + Sqrt[b + a*x^5])] - 2*Sqrt[2]*c^(1/4)*ArcTa
nh[((c^(1/4)*x^2)/Sqrt[2] + Sqrt[b + a*x^5]/(Sqrt[2]*c^(1/4)))/(x*(b + a*x^
5)^(1/4))]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5-4*b)*(a*x^5-c*x^4+b)/x^2/(a*x^5+b)^(3/4)/(a*x^5+c*x^4+b),x
, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^5-cx^4+b\right)\left(ax^5-4\,b\right)}{\left(ax^5+cx^4+b\right)\left(ax^5+b\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5-4*b)*(a*x^5-c*x^4+b)/x^2/(a*x^5+b)^(3/4)/(a*x^5+c*x^4+b),x
, algorithm="giac")
```

```
[Out] integrate((a*x^5 - c*x^4 + b)*(a*x^5 - 4*b)/((a*x^5 + c*x^4 + b)*(a*x^5 + b
)^(3/4)*x^2), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^5-4b\right)\left(a\,x^5-c\,x^4+b\right)}{x^2\left(a\,x^5+b\right)^{\frac{3}{4}}\left(a\,x^5+c\,x^4+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^5-4*b)*(a*x^5-c*x^4+b)/x^2/(a*x^5+b)^(3/4)/(a*x^5+c*x^4+b),x)
```

```
[Out] int((a*x^5-4*b)*(a*x^5-c*x^4+b)/x^2/(a*x^5+b)^(3/4)/(a*x^5+c*x^4+b),x)
```

maximA [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^5 - cx^4 + b\right)\left(ax^5 - 4\,b\right)}{\left(ax^5 + cx^4 + b\right)\left(ax^5 + b\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5-4*b)*(a*x^5-c*x^4+b)/x^2/(a*x^5+b)^(3/4)/(a*x^5+c*x^4+b),x
, algorithm="maxima")
```

```
[Out] integrate((a*x^5 - c*x^4 + b)*(a*x^5 - 4*b)/((a*x^5 + c*x^4 + b)*(a*x^5 + b
)^(3/4)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(4\,b - a\,x^5\right)\,\left(a\,x^5 - c\,x^4 + b\right)}{x^2\,{\left(a\,x^5 + b\right)}^{3/4}\,\left(a\,x^5 + c\,x^4 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((4*b - a*x^5)*(b + a*x^5 - c*x^4))/(x^2*(b + a*x^5)^(3/4)*(b + a*x^5
+ c*x^4)),x)
```

```
[Out] int(-((4*b - a*x^5)*(b + a*x^5 - c*x^4))/(x^2*(b + a*x^5)^(3/4)*(b + a*x^5
+ c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^5 - 4b\right)\left(ax^5 + b - cx^4\right)}{x^2\left(ax^5 + b\right)^{\frac{3}{4}}\left(ax^5 + b + cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**5-4*b)*(a*x**5-c*x**4+b)/x**2/(a*x**5+b)**(3/4)/(a*x**5+c*x
**4+b),x)
```

```
[Out] Integral((a*x**5 - 4*b)*(a*x**5 + b - c*x**4)/(x**2*(a*x**5 + b)**(3/4)*(a*
x**5 + b + c*x**4)), x)
```

3.1674 $\int \frac{(-1+x^3)(1+x^6)^{2/3}(1-x^3+x^6)}{x^6(1+x^3)}\,dx$

Optimal. Leaf size=142

$$-2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^6+1}+2x\right)-2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2^{2/3}\sqrt[3]{x^6+1}-x}\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^6+1}\,x-\sqrt[3]{2}\left(x^6+1\right)^{2/3}-2x^2\right)}{\sqrt[3]{2}}+\frac{(x^6}{}$$

Rubi [F] time = 0.93, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^3)(1+x^6)^{2/3}(1-x^3+x^6)}{x^6(1+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^3)*(1 + x^6)^(2/3)*(1 - x^3 + x^6))/(x^6*(1 + x^3)),x]
```

```
[Out] Hypergeometric2F1[-5/6, -2/3, 1/6, -x^6]/(5*x^5) - (3*Hypergeometric2F1[-2/
3, -1/3, 2/3, -x^6])/(2*x^2) + x*Hypergeometric2F1[-2/3, 1/6, 7/6, -x^6] -
2*Defer[Int][(1 + x^6)^(2/3)/(1 + x), x] + 2*(1 + I*Sqrt[3])*Defer[Int][(1
+ x^6)^(2/3)/(-1 - I*Sqrt[3] + 2*x), x] + 2*(1 - I*Sqrt[3])*Defer[Int][(1 +
 x^6)^(2/3)/(-1 + I*Sqrt[3] + 2*x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^3)(1+x^6)^{2/3}(1-x^3+x^6)}{x^6(1+x^3)}\,dx &= \int \left((1+x^6)^{2/3}-\frac{(1+x^6)^{2/3}}{x^6}+\frac{3(1+x^6)^{2/3}}{x^3}-\frac{2(1+x^6)^{2/3}}{1+x}+\frac{2(-2+}{1}\right.\\
&= -\left(2\int \frac{(1+x^6)^{2/3}}{1+x}\,dx\right)+2\int \frac{(-2+x)(1+x^6)^{2/3}}{1-x+x^2}\,dx+3\int \frac{(1+x^6)^{2}}{x^3}\\
&= \frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};-x^6\right)}{5x^5}+x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};-x^6\right)+\frac{3}{2}\text{Subst}\left(\int \frac{(1+x^3)^{2}}{x^2}\right.\\
&= \frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};-x^6\right)}{5x^5}-\frac{3\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};-x^6\right)}{2x^2}+x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};-x^6\right)
\end{aligned}$$

Mathematica [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^3)(1+x^6)^{2/3}(1-x^3+x^6)}{x^6(1+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^3)*(1 + x^6)^(2/3)*(1 - x^3 + x^6))/(x^6*(1 + x^3)),x]
```

```
[Out] Integrate[((-1 + x^3)*(1 + x^6)^(2/3)*(1 - x^3 + x^6))/(x^6*(1 + x^3)), x]
```

IntegrateAlgebraic [A] time = 1.49, size = 142, normalized size = 1.00

$$-2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^6+1}+2x\right)-2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2^{2/3}\sqrt[3]{x^6+1}-x}\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^6+1}\,x-\sqrt[3]{2}\left(x^6+1\right)^{2/3}-2x^2\right)}{\sqrt[3]{2}}+\frac{\left(x^6+1\right)^{2/3}\left(2x^6-15x^3+2\right)}{10x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)*(1 + x^6)^(2/3)*(1 - x^3 + x^6))/(x^6*(1 + x
^3)),x]
```

```
[Out] ((1 + x^6)^(2/3)*(2 - 15*x^3 + 2*x^6))/(10*x^5) - 2^(2/3)*Sqrt[3]*ArcTan[(S
qrt[3]*x)/(-x + 2^(2/3)*(1 + x^6)^(1/3))] - 2^(2/3)*Log[2*x + 2^(2/3)*(1 +
x^6)^(1/3)] + Log[-2*x^2 + 2^(2/3)*x*(1 + x^6)^(1/3) - 2^(1/3)*(1 + x^6)^(2
/3)]/2^(1/3)
```

fricas [B] time = 51.28, size = 344, normalized size = 2.42

$$\frac{10\sqrt{3}\,(-4)^{\frac{1}{3}}x^5\arctan\left(\frac{3\sqrt{3}(-4)^{\frac{2}{3}}\left(x^{13}-2x^{10}-6x^7-2x^4+x\right)\left(x^6+1\right)^{\frac{2}{3}}+6\sqrt{3}(-4)^{\frac{1}{3}}\left(x^{14}-14x^{11}+6x^8-14x^5+x^2\right)\left(x^6+1\right)^{\frac{1}{3}}-\sqrt{3}\left(x^{18}-30x^{15}+51x^{12}-52x^9+51x^6-30x^3+1\right)}{3\left(x^{18}+6x^{15}-93x^{12}+20x^9-93x^6+6x^3+1\right)}\right)+10(-4)^{\frac{1}{3}}x^5\log\left(\frac{3(-4)^{\frac{2}{3}}\left(x^6+1\right)^{\frac{1}{3}}x^2+6\left(x^6+1\right)^{\frac{2}{3}}x-(-4)^{\frac{1}{3}}\left(x^6+2x^3+1\right)}{x^6+2x^3+1}\right)-5(-4)^{\frac{1}{3}}x^5\log\left(\frac{6(-4)^{\frac{1}{3}}\left(x^7-4x^4+x\right)\left(x^6+1\right)^{\frac{2}{3}}+(-4)^{\frac{2}{3}}\left(x^{12}-14x^9+6x^6-14x^3+1\right)+24\left(x^8-x^5+x^2\right)\left(x^6+1\right)^{\frac{1}{3}}}{x^{12}+4x^9+6x^6+4x^3+1}\right)+3\left(2x^6-15x^3+2\right)\left(x^6+1\right)^{\frac{2}{3}}}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6+1)^(2/3)*(x^6-x^3+1)/x^6/(x^3+1),x, algorithm="frica
s")
```

```
[Out] 1/30*(10*sqrt(3)*(-4)^(1/3)*x^5*arctan(1/3*(3*sqrt(3)*(-4)^(2/3)*(x^13 - 2*
x^10 - 6*x^7 - 2*x^4 + x)*(x^6 + 1)^(2/3) + 6*sqrt(3)*(-4)^(1/3)*(x^14 - 14
*x^11 + 6*x^8 - 14*x^5 + x^2)*(x^6 + 1)^(1/3) - sqrt(3)*(x^18 - 30*x^15 + 5
1*x^12 - 52*x^9 + 51*x^6 - 30*x^3 + 1))/(x^18 + 6*x^15 - 93*x^12 + 20*x^9 -
 93*x^6 + 6*x^3 + 1)) + 10*(-4)^(1/3)*x^5*log((3*(-4)^(2/3)*(x^6 + 1)^(1/3)
*x^2 + 6*(x^6 + 1)^(2/3)*x - (-4)^(1/3)*(x^6 + 2*x^3 + 1))/(x^6 + 2*x^3 + 1
)) - 5*(-4)^(1/3)*x^5*log((6*(-4)^(1/3)*(x^7 - 4*x^4 + x)*(x^6 + 1)^(2/3) +
 (-4)^(2/3)*(x^12 - 14*x^9 + 6*x^6 - 14*x^3 + 1) + 24*(x^8 - x^5 + x^2)*(x^
6 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1)) + 3*(2*x^6 - 15*x^3 + 2)*
(x^6 + 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6-x^3+1\right)\left(x^6+1\right)^{\frac{2}{3}}\left(x^3-1\right)}{\left(x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6+1)^(2/3)*(x^6-x^3+1)/x^6/(x^3+1),x, algorithm="giac"
)
```

```
[Out] integrate((x^6 - x^3 + 1)*(x^6 + 1)^(2/3)*(x^3 - 1)/((x^3 + 1)*x^6), x)
```

maple [F] time = 2.21, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-1\right)\left(x^6+1\right)^{\frac{2}{3}}\left(x^6-x^3+1\right)}{x^6\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)*(x^6+1)^(2/3)*(x^6-x^3+1)/x^6/(x^3+1),x)
```

```
[Out] int((x^3-1)*(x^6+1)^(2/3)*(x^6-x^3+1)/x^6/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6-x^3+1\right)\left(x^6+1\right)^{\frac{2}{3}}\left(x^3-1\right)}{\left(x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(x^6+1)^(2/3)*(x^6-x^3+1)/x^6/(x^3+1),x, algorithm="maxim
a")
```

```
[Out] integrate((x^6 - x^3 + 1)*(x^6 + 1)^(2/3)*(x^3 - 1)/((x^3 + 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3-1\right)\left(x^6+1\right)^{2/3}\left(x^6-x^3+1\right)}{x^6\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)*(x^6 + 1)^(2/3)*(x^6 - x^3 + 1))/(x^6*(x^3 + 1)),x)
```

```
[Out] int(((x^3 - 1)*(x^6 + 1)^(2/3)*(x^6 - x^3 + 1))/(x^6*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x^2+1\right)\left(x^4-x^2+1\right)\right)^{\frac{2}{3}}(x-1)\left(x^2+x+1\right)\left(x^6-x^3+1\right)}{x^6(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)*(x**6+1)**(2/3)*(x**6-x**3+1)/x**6/(x**3+1),x)
```

```
[Out] Integral(((x**2 + 1)*(x**4 - x**2 + 1))**(2/3)*(x - 1)*(x**2 + x + 1)*(x**6
 - x**3 + 1)/(x**6*(x + 1)*(x**2 - x + 1)), x)
```

3.1675 $\int \frac{(-b^2+ax^2)^2 \sqrt{b+\sqrt{b^2+ax^2}}}{(b^2+ax^2)^2}\,dx$

Optimal. Leaf size=142

$$\frac{x(2ax^2+11b^2)}{3\sqrt{ax^2+b^2}\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}+\frac{2x(2abx^2+5b^3)}{3(ax^2+b^2)\sqrt{\sqrt{ax^2+b^2}+b}}$$

Rubi [F] time = 2.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-b^2+ax^2)^2\sqrt{b+\sqrt{b^2+ax^2}}}{(b^2+ax^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]])/(b^2 + a*x^2)^2,x]
```

```
[Out] (2*a*x^3)/(3*(b + Sqrt[b^2 + a*x^2])^(3/2)) + (2*b*x)/Sqrt[b + Sqrt[b^2 + a
*x^2]] - b*Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b - Sqrt[-a]*x), x] - b*
Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b + Sqrt[-a]*x), x] - a*b^2*Defer[I
nt][Sqrt[b + Sqrt[b^2 + a*x^2]]/(Sqrt[-a]*b - a*x)^2, x] - a*b^2*Defer[Int]
[Sqrt[b + Sqrt[b^2 + a*x^2]]/(Sqrt[-a]*b + a*x)^2, x]
```

Rubi steps

$$\int \frac{(-b^2+ax^2)^2\sqrt{b+\sqrt{b^2+ax^2}}}{(b^2+ax^2)^2}\,dx = \int \left(\sqrt{b+\sqrt{b^2+ax^2}}+\frac{4b^4\sqrt{b+\sqrt{b^2+ax^2}}}{(b^2+ax^2)^2}-\frac{4b^2\sqrt{b+\sqrt{b^2+ax^2}}}{b^2+ax^2}\right)$$

$$= -\left((4b^2)\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b^2+ax^2}\,dx\right)+(4b^4)\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{(b^2+ax^2)^2}\,dx+\int$$

$$= \frac{2ax^3}{3\left(b+\sqrt{b^2+ax^2}\right)^{3/2}}+\frac{2bx}{\sqrt{b+\sqrt{b^2+ax^2}}}-(4b^2)\int\left(\frac{\sqrt{b+\sqrt{b^2+ax}}}{2b\left(b-\sqrt{-a}\,x\right.}\right.$$

$$= \frac{2ax^3}{3\left(b+\sqrt{b^2+ax^2}\right)^{3/2}}+\frac{2bx}{\sqrt{b+\sqrt{b^2+ax^2}}}-(2b)\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b-\sqrt{-a}\,x}$$

$$= \frac{2ax^3}{3\left(b+\sqrt{b^2+ax^2}\right)^{3/2}}+\frac{2bx}{\sqrt{b+\sqrt{b^2+ax^2}}}-(2b)\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b-\sqrt{-a}\,x}$$

$$= \frac{2ax^3}{3\left(b+\sqrt{b^2+ax^2}\right)^{3/2}}+\frac{2bx}{\sqrt{b+\sqrt{b^2+ax^2}}}+b\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b-\sqrt{-a}\,x}\,dx$$

Mathematica [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{(-b^2+ax^2)^2\sqrt{b+\sqrt{b^2+ax^2}}}{(b^2+ax^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]])/(b^2 + a*x^2)^2,x]
[Out] Integrate[((-b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]])/(b^2 + a*x^2)^2, x
]
```

IntegrateAlgebraic [A] time = 0.36, size = 142, normalized size = 1.00

$$\frac{x(2ax^2+11b^2)}{3\sqrt{ax^2+b^2}\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}+\frac{2x(2abx^2+5b^3)}{3(ax^2+b^2)\sqrt{\sqrt{ax^2+b^2}+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]])/(b^2 + a*
x^2)^2,x]
[Out] (x*(11*b^2 + 2*a*x^2))/(3*Sqrt[b^2 + a*x^2]*Sqrt[b + Sqrt[b^2 + a*x^2]]) +
(2*x*(5*b^3 + 2*a*b*x^2))/(3*(b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]]) - (
5*b^(3/2)*ArcTan[(Sqrt[a]*x)/(Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/Sqrt[a
]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b^2)^2*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^2,x, algori
thm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^2-b^2)^2\sqrt{b+\sqrt{ax^2+b^2}}}{(ax^2+b^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b^2)^2*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^2,x, algori
thm="giac")
[Out] integrate((a*x^2 - b^2)^2*sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 + b^2)^2, x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(a\,x^2-b^2)^2\sqrt{b+\sqrt{a\,x^2+b^2}}}{(a\,x^2+b^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b^2)^2*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^2,x)
```

```
[Out] int((a*x^2-b^2)^2*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^2-b^2)^2\sqrt{b+\sqrt{ax^2+b^2}}}{(ax^2+b^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b^2)^2*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^2,x, algori
thm="maxima")
```

```
[Out] integrate((a*x^2 - b^2)^2*sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 + b^2)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a\,x^2-b^2)^2\,\sqrt{b+\sqrt{b^2+a\,x^2}}}{(b^2+a\,x^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^2 - b^2)^2*(b + (a*x^2 + b^2)^(1/2))^(1/2))/(a*x^2 + b^2)^2,x)
```

```
[Out] int(((a*x^2 - b^2)^2*(b + (a*x^2 + b^2)^(1/2))^(1/2))/(a*x^2 + b^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}\,(ax^2-b^2)^2}{(ax^2+b^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b**2)**2*(b+(a*x**2+b**2)**(1/2))**(1/2)/(a*x**2+b**2)**2
,x)
```

```
[Out] Integral(sqrt(b + sqrt(a*x**2 + b**2))*(a*x**2 - b**2)**2/(a*x**2 + b**2)**
2, x)
```

3.1676 $\int \frac{(2+x^3)(1+2x^3)^{2/3}}{x^6(-1+x^3)}\,dx$

Optimal. Leaf size=143

$$3^{2/3}\log\left(3^{2/3}\sqrt[3]{2x^3+1}-3x\right)-3\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2x^3+1}+\sqrt[3]{3}x}\right)+\frac{(2x^3+1)^{2/3}(23x^3+4)}{10x^5}-\frac{1}{2}3^{2/3}\log\left(3^{2/3}\sqrt[3]{2x^3+1}\right.$$

Rubi [A] time = 0.17, antiderivative size = 148, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {580, 583, 12, 377, 200, 31, 634, 617, 204, 628}

$$3^{2/3}\log\left(1-\frac{\sqrt[3]{3}x}{\sqrt[3]{2x^3+1}}\right)-3\sqrt[6]{3}\tan^{-1}\left(\frac{2x}{\sqrt[6]{3}\sqrt[3]{2x^3+1}}+\frac{1}{\sqrt{3}}\right)+\frac{2(2x^3+1)^{2/3}}{5x^5}+\frac{23(2x^3+1)^{2/3}}{10x^2}-\frac{1}{2}3^{2/3}\log\left(\frac{\sqrt[3]{3}x}{\sqrt[3]{2x^3+1}}+\frac{3^{2/3}x^2}{(2x^3+1)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x^3)*(1 + 2*x^3)^(2/3))/(x^6*(-1 + x^3)),x]
```

```
[Out] (2*(1 + 2*x^3)^(2/3))/(5*x^5) + (23*(1 + 2*x^3)^(2/3))/(10*x^2) - 3*3^(1/6)
*ArcTan[1/Sqrt[3] + (2*x)/(3^(1/6)*(1 + 2*x^3)^(1/3))] + 3^(2/3)*Log[1 - (3
^(1/3)*x)/(1 + 2*x^3)^(1/3)] - (3^(2/3)*Log[1 + (3^(2/3)*x^2)/(1 + 2*x^3)^(
2/3) + (3^(1/3)*x)/(1 + 2*x^3)^(1/3)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
```

```
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(2+x^3\right)\left(1+2x^3\right)^{2/3}}{x^6\left(-1+x^3\right)}\,dx = \frac{2\left(1+2x^3\right)^{2/3}}{5x^5} - \frac{1}{5}\int \frac{-23-22x^3}{x^3\left(-1+x^3\right)\sqrt[3]{1+2x^3}}\,dx$$

$$= \frac{2\left(1+2x^3\right)^{2/3}}{5x^5} + \frac{23\left(1+2x^3\right)^{2/3}}{10x^2} - \frac{1}{10}\int -\frac{90}{\left(-1+x^3\right)\sqrt[3]{1+2x^3}}\,dx$$

$$= \frac{2\left(1+2x^3\right)^{2/3}}{5x^5} + \frac{23\left(1+2x^3\right)^{2/3}}{10x^2} + 9\int \frac{1}{\left(-1+x^3\right)\sqrt[3]{1+2x^3}}\,dx$$

$$= \frac{2\left(1+2x^3\right)^{2/3}}{5x^5} + \frac{23\left(1+2x^3\right)^{2/3}}{10x^2} + 9\,\text{Subst}\left(\int \frac{1}{-1+3x^3}\,dx, x, \frac{x}{\sqrt[3]{1+2x^3}}\right)$$

$$= \frac{2\left(1+2x^3\right)^{2/3}}{5x^5} + \frac{23\left(1+2x^3\right)^{2/3}}{10x^2} + 3\,\text{Subst}\left(\int \frac{1}{-1+\sqrt[3]{3}\,x}\,dx, x, \frac{x}{\sqrt[3]{1+2x^3}}\right) + 3\,\text{Sul}$$

$$= \frac{2\left(1+2x^3\right)^{2/3}}{5x^5} + \frac{23\left(1+2x^3\right)^{2/3}}{10x^2} + 3^{2/3}\log\left(1-\frac{\sqrt[3]{3}\,x}{\sqrt[3]{1+2x^3}}\right) - \frac{9}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{3}\,x}\right.$$

$$= \frac{2\left(1+2x^3\right)^{2/3}}{5x^5} + \frac{23\left(1+2x^3\right)^{2/3}}{10x^2} + 3^{2/3}\log\left(1-\frac{\sqrt[3]{3}\,x}{\sqrt[3]{1+2x^3}}\right) - \frac{1}{2}3^{2/3}\log\left(1+\frac{3^{2/3}}{(1+2\right.$$

$$= \frac{2\left(1+2x^3\right)^{2/3}}{5x^5} + \frac{23\left(1+2x^3\right)^{2/3}}{10x^2} - 3\sqrt[6]{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}\,x}{\sqrt[3]{1+2x^3}}}{\sqrt{3}}\right) + 3^{2/3}\log\left(1-\frac{\sqrt[3]{3}\,x}{\sqrt[3]{1+2}}\right.$$

Mathematica [A] time = 0.28, size = 128, normalized size = 0.90

$$-3\sqrt[6]{3}\tan^{-1}\left(\frac{2x}{\sqrt[6]{3}\sqrt[3]{x^3+2}}+\frac{1}{\sqrt{3}}\right)+\frac{\left(2x^3+1\right)^{2/3}\left(23x^3+4\right)}{10x^5}+\frac{1}{2}3^{2/3}\left(2\log\left(1-\frac{\sqrt[3]{3}\,x}{\sqrt[3]{x^3+2}}\right)-\log\left(\frac{\sqrt[3]{3}\,x}{\sqrt[3]{x^3+2}}+\frac{3^{2/3}x^2}{\left(x^3+2\right)^{2/3}}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + x^3)*(1 + 2*x^3)^(2/3))/(x^6*(-1 + x^3)),x]
```

```
[Out] ((1 + 2*x^3)^(2/3)*(4 + 23*x^3))/(10*x^5) - 3*3^(1/6)*ArcTan[1/Sqrt[3] + (2
*x)/(3^(1/6)*(2 + x^3)^(1/3))] + (3^(2/3)*(2*Log[1 - (3^(1/3)*x)/(2 + x^3)^
(1/3)] - Log[1 + (3^(2/3)*x^2)/(2 + x^3)^(2/3) + (3^(1/3)*x)/(2 + x^3)^(1/3
)]))/2
```

IntegrateAlgebraic [A] time = 0.34, size = 143, normalized size = 1.00

$$3^{2/3}\log\left(3^{2/3}\sqrt[3]{2x^3+1}-3x\right)-3\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2x^3+1}+\sqrt[3]{3}\,x}\right)+\frac{\left(2x^3+1\right)^{2/3}\left(23x^3+4\right)}{10x^5}-\frac{1}{2}3^{2/3}\log\left(3^{2/3}\sqrt[3]{2x^3+1}\,x+\sqrt[3]{3}\left(2x^3+1\right)^{2/3}+3x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x^3)*(1 + 2*x^3)^(2/3))/(x^6*(-1 + x^3)),x]
```

```
[Out] ((1 + 2*x^3)^(2/3)*(4 + 23*x^3))/(10*x^5) - 3*3^(1/6)*ArcTan[(3^(5/6)*x)/(3
^(1/3)*x + 2*(1 + 2*x^3)^(1/3))] + 3^(2/3)*Log[-3*x + 3^(2/3)*(1 + 2*x^3)^(
1/3)] - (3^(2/3)*Log[3*x^2 + 3^(2/3)*x*(1 + 2*x^3)^(1/3) + 3^(1/3)*(1 + 2*x
^3)^(2/3)])/2
```

fricas [B] time = 2.38, size = 281, normalized size = 1.97

$$-\frac{10\cdot 9^{\frac{1}{3}}\sqrt{3}\,x^5\arctan\left(\frac{2\cdot 9^{\frac{2}{3}}\sqrt{3}\left(8x^7-7x^4-x\right)\left(2x^3+1\right)^{\frac{2}{3}}-6\cdot 9^{\frac{1}{3}}\sqrt{3}\left(55x^8+25x^5+x^2\right)\left(2x^3+1\right)^{\frac{1}{3}}-\sqrt{3}\left(377x^9+300x^6+51x^3+1\right)}{3\left(487x^9+240x^6+3x^3-1\right)}\right)-10\cdot 9^{\frac{1}{3}}x^5\log\left(\frac{3\cdot 9^{\frac{2}{3}}\left(2x^3+1\right)^{\frac{1}{3}}x^2-9\left(2x^3+1\right)^{\frac{2}{3}}x-9^{\frac{1}{3}}\left(x^3-1\right)}{x^3-1}\right)+5\cdot 9^{\frac{1}{3}}x^5\log\left(\frac{9\cdot 9^{\frac{1}{3}}\left(8x^4+x\right)\left(2x^3+1\right)^{\frac{2}{3}}+9^{\frac{2}{3}}\left(55x^6+25x^3+1\right)+27\left(7x^5+2x^2\right)\left(2x^3+1\right)^{\frac{1}{3}}}{x^6-2x^3+1}\right)-3\left(23x^3+4\right)\left(2x^3+1\right)^{\frac{2}{3}}}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(2*x^3+1)^(2/3)/x^6/(x^3-1),x, algorithm="fricas")
```

```
[Out] -1/30*(10*9^(1/3)*sqrt(3)*x^5*arctan(1/3*(2*9^(2/3)*sqrt(3)*(8*x^7 - 7*x^4
- x)*(2*x^3 + 1)^(2/3) - 6*9^(1/3)*sqrt(3)*(55*x^8 + 25*x^5 + x^2)*(2*x^3 +
 1)^(1/3) - sqrt(3)*(377*x^9 + 300*x^6 + 51*x^3 + 1))/(487*x^9 + 240*x^6 +
3*x^3 - 1)) - 10*9^(1/3)*x^5*log((3*9^(2/3)*(2*x^3 + 1)^(1/3)*x^2 - 9*(2*x^
3 + 1)^(2/3)*x - 9^(1/3)*(x^3 - 1))/(x^3 - 1)) + 5*9^(1/3)*x^5*log((9*9^(1/
3)*(8*x^4 + x)*(2*x^3 + 1)^(2/3) + 9^(2/3)*(55*x^6 + 25*x^3 + 1) + 27*(7*x^
5 + 2*x^2)*(2*x^3 + 1)^(1/3))/(x^6 - 2*x^3 + 1)) - 3*(23*x^3 + 4)*(2*x^3 +
1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^3+1\right)^{\frac{2}{3}}\left(x^3+2\right)}{\left(x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(2*x^3+1)^(2/3)/x^6/(x^3-1),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 1)^(2/3)*(x^3 + 2)/((x^3 - 1)*x^6), x)
```

maple [C] time = 2.84, size = 843, normalized size = 5.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2)*(2*x^3+1)^(2/3)/x^6/(x^3-1),x)
```

```
[Out] 1/10*(46*x^6+31*x^3+4)/x^5/(2*x^3+1)^(1/3)+RootOf(_Z^3-9)*ln((6*RootOf(Root
Of(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3+135*RootOf(R
ootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+21*(2*x
^3+1)^(2/3)*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81
*_Z^2)*x+8*(2*x^3+1)^(1/3)*RootOf(_Z^3-9)^2*x^2+9*(2*x^3+1)^(1/3)*RootOf(Ro
otOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*x^2-4*RootOf(_Z^
3-9)*x^3-90*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3-3*(2*x
^3+1)^(2/3)*x-2*RootOf(_Z^3-9)-45*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-
9)+81*_Z^2))/(-1+x)/(x^2+x+1))-ln(-(3*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_
Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3+54*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf
(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3-3*(2*x^3+1)^(1/3)*RootOf(_Z^3-9)^2
*x^2-27*(2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2
)*RootOf(_Z^3-9)*x^2+5*RootOf(_Z^3-9)*x^3+90*RootOf(RootOf(_Z^3-9)^2+9*_Z*R
ootOf(_Z^3-9)+81*_Z^2)*x^3+9*(2*x^3+1)^(2/3)*x+RootOf(_Z^3-9)+18*RootOf(Roo
tOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2))/(-1+x)/(x^2+x+1))*RootOf(_Z^3-9
)-9*ln(-(3*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3
-9)^3*x^3+54*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(
_Z^3-9)^2*x^3-3*(2*x^3+1)^(1/3)*RootOf(_Z^3-9)^2*x^2-27*(2*x^3+1)^(1/3)*Roo
tOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*x^2+5*Root
Of(_Z^3-9)*x^3+90*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3+
9*(2*x^3+1)^(2/3)*x+RootOf(_Z^3-9)+18*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_
Z^3-9)+81*_Z^2))/(-1+x)/(x^2+x+1))*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3
-9)+81*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^3+1\right)^{\frac{2}{3}}\left(x^3+2\right)}{\left(x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)*(2*x^3+1)^(2/3)/x^6/(x^3-1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^3 + 1)^(2/3)*(x^3 + 2)/((x^3 - 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3+2\right)\,{\left(2\,x^3+1\right)}^{2/3}}{x^6\,\left(x^3-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 2)*(2*x^3 + 1)^(2/3))/(x^6*(x^3 - 1)),x)
```

```
[Out] int(((x^3 + 2)*(2*x^3 + 1)^(2/3))/(x^6*(x^3 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3} + 2\right) \left(2 x^{3} + 1\right)^{\frac{2}{3}}}{x^{6} \left(x - 1\right) \left(x^{2} + x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2)*(2*x**3+1)**(2/3)/x**6/(x**3-1),x)
```

```
[Out] Integral((x**3 + 2)*(2*x**3 + 1)**(2/3)/(x**6*(x - 1)*(x**2 + x + 1)), x)
```

3.1677 $\int \frac{(-4+x^2)\sqrt[4]{2-x^2-2x^4}}{x^2(-2+x^2)}\,dx$

Optimal. Leaf size=143

$$-\frac{2\sqrt[4]{-2x^4-x^2+2}}{x}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{-2x^4-x^2+2}}{\sqrt{2}x^2-\sqrt{-2x^4-x^2+2}}\right)}{\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}x\sqrt[4]{-2x^4-x^2+2}}{2x^2+\sqrt{2}\sqrt{-2x^4-x^2+2}}\right)}{\sqrt[4]{2}}$$

Rubi [F] time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-4+x^2)\sqrt[4]{2-x^2-2x^4}}{x^2(-2+x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-4 + x^2)*(2 - x^2 - 2*x^4)^(1/4))/(x^2*(-2 + x^2)),x]
```

```
[Out] (-2*(2 - x^2 - 2*x^4)^(1/4)*AppellF1[-1/2, -1/4, -1/4, 1/2, (-4*x^2)/(1 + S
qrt[17]), (-4*x^2)/(1 - Sqrt[17])])/(x*(1 + (4*x^2)/(1 - Sqrt[17]))^(1/4)*(
1 + (4*x^2)/(1 + Sqrt[17]))^(1/4)) + Defer[Int][(2 - x^2 - 2*x^4)^(1/4)/(2
- x^2), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-4+x^2)\sqrt[4]{2-x^2-2x^4}}{x^2(-2+x^2)}\,dx &= \int \left(\frac{2\sqrt[4]{2-x^2-2x^4}}{x^2}+\frac{\sqrt[4]{2-x^2-2x^4}}{2-x^2}\right)dx\\
&= 2\int \frac{\sqrt[4]{2-x^2-2x^4}}{x^2}\,dx+\int \frac{\sqrt[4]{2-x^2-2x^4}}{2-x^2}\,dx\\
&= \frac{\left(2\sqrt[4]{2-x^2-2x^4}\right)\int \frac{\sqrt[4]{1-\frac{4x^2}{-1-\sqrt{17}}}\sqrt[4]{1-\frac{4x^2}{-1+\sqrt{17}}}}{x^2}\,dx}{\sqrt[4]{1-\frac{4x^2}{-1-\sqrt{17}}}\sqrt[4]{1-\frac{4x^2}{-1+\sqrt{17}}}}+\int \frac{\sqrt[4]{2-x^2-2x^4}}{2-x^2}\,dx\\
&= -\frac{2\sqrt[4]{2-x^2-2x^4}F_1\left(-\frac{1}{2};-\frac{1}{4},-\frac{1}{4};\frac{1}{2};-\frac{4x^2}{1+\sqrt{17}},-\frac{4x^2}{1-\sqrt{17}}\right)}{x\sqrt[4]{1+\frac{4x^2}{1-\sqrt{17}}}\sqrt[4]{1+\frac{4x^2}{1+\sqrt{17}}}}+\int \frac{\sqrt[4]{2-x^2-2x^4}}{2-x^2}\,dx
\end{aligned}$$

Mathematica [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(-4+x^2)\sqrt[4]{2-x^2-2x^4}}{x^2(-2+x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-4 + x^2)*(2 - x^2 - 2*x^4)^(1/4))/(x^2*(-2 + x^2)),x]
```

```
[Out] Integrate[((-4 + x^2)*(2 - x^2 - 2*x^4)^(1/4))/(x^2*(-2 + x^2)), x]
```

IntegrateAlgebraic [A] time = 0.33, size = 143, normalized size = 1.00

$$-\frac{2\sqrt[4]{-2x^4-x^2+2}}{x}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{-2x^4-x^2+2}}{\sqrt{2}x^2-\sqrt{-2x^4-x^2+2}}\right)}{\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}x\sqrt[4]{-2x^4-x^2+2}}{2x^2+\sqrt{2}\sqrt{-2x^4-x^2+2}}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + x^2)*(2 - x^2 - 2*x^4)^(1/4))/(x^2*(-2 + x^2)),x]
```

```
[Out] (-2*(2 - x^2 - 2*x^4)^(1/4))/x + ArcTan[(2^(3/4)*x*(2 - x^2 - 2*x^4)^(1/4))
/(Sqrt[2]*x^2 - Sqrt[2 - x^2 - 2*x^4])]/2^(1/4) + ArcTanh[(2*2^(1/4)*x*(2 -
 x^2 - 2*x^4)^(1/4))/(2*x^2 + Sqrt[2]*Sqrt[2 - x^2 - 2*x^4])]/2^(1/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)*(-2*x^4-x^2+2)^(1/4)/x^2/(x^2-2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-2\,x^4-x^2+2\right)^{\frac{1}{4}}\left(x^2-4\right)}{\left(x^2-2\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)*(-2*x^4-x^2+2)^(1/4)/x^2/(x^2-2),x, algorithm="giac")
```

```
[Out] integrate((-2*x^4 - x^2 + 2)^(1/4)*(x^2 - 4)/((x^2 - 2)*x^2), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-4\right)\left(-2x^4-x^2+2\right)^{\frac{1}{4}}}{x^2\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-4)*(-2*x^4-x^2+2)^(1/4)/x^2/(x^2-2),x)
```

```
[Out] int((x^2-4)*(-2*x^4-x^2+2)^(1/4)/x^2/(x^2-2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-2\,x^4-x^2+2\right)^{\frac{1}{4}}\left(x^2-4\right)}{\left(x^2-2\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)*(-2*x^4-x^2+2)^(1/4)/x^2/(x^2-2),x, algorithm="maxima")
```

```
[Out] integrate((-2*x^4 - x^2 + 2)^(1/4)*(x^2 - 4)/((x^2 - 2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2-4\right)\,\left(-2\,x^4-x^2+2\right)^{1/4}}{x^2\,\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 4)*(2 - 2*x^4 - x^2)^(1/4))/(x^2*(x^2 - 2)),x)
```

```
[Out] int(((x^2 - 4)*(2 - 2*x^4 - x^2)^(1/4))/(x^2*(x^2 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-2)(x+2)\sqrt[4]{-2x^4-x^2+2}}{x^2\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4)*(-2*x**4-x**2+2)**(1/4)/x**2/(x**2-2),x)
[Out] Integral((x - 2)*(x + 2)*(-2*x**4 - x**2 + 2)**(1/4)/(x**2*(x**2 - 2)), x)
```

3.1678 $\int \frac{1}{(1+x)\left(-2+2x+x^2-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{i\sqrt{-x^4+x^2+2x-2}\left(-\frac{1}{4}i\tanh^{-1}\left(-\sqrt{x^2+2x+2}+x+1\right)-\frac{7i\tanh^{-1}\left(\frac{\sqrt{x^2+2x+2}}{\sqrt{5}}-\frac{x}{\sqrt{5}}+\frac{1}{\sqrt{5}}\right)}{20\sqrt{5}}-\frac{i(x^3-2x^2-2x+4)}{20(x-1)^2\sqrt{x^2+2x+2}}\right)}{(x-1)\sqrt{x^2+2x+2}}$$

Rubi [F] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(1+x)\left(-2+2x+x^2-x^4\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 + x)*(-2 + 2*x + x^2 - x^4)^(3/2)),x]
```

```
[Out] Defer[Int][1/((1 + x)*(-2 + 2*x + x^2 - x^4)^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{(1+x)\left(-2+2x+x^2-x^4\right)^{3/2}}\,dx=\int \frac{1}{(1+x)\left(-2+2x+x^2-x^4\right)^{3/2}}\,dx$$

Mathematica [A] time = 0.14, size = 121, normalized size = 0.85

$$\frac{7\sqrt{5}\sqrt{x^2+2x+2}(x-1)^2\tanh^{-1}\left(\frac{2x+3}{\sqrt{5}\sqrt{x^2+2x+2}}\right)-25\sqrt{x^2+2x+2}(x-1)^2\tanh^{-1}\left(\sqrt{(x+1)^2+1}\right)+10\left(x^3-2x^2-2x+4\right)}{200(x-1)\sqrt{-x^4+x^2+2x-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(-2 + 2*x + x^2 - x^4)^(3/2)),x]
```

```
[Out] (10*(4 - 2*x - 2*x^2 + x^3) + 7*Sqrt[5]*(-1 + x)^2*Sqrt[2 + 2*x + x^2]*ArcT
anh[(3 + 2*x)/(Sqrt[5]*Sqrt[2 + 2*x + x^2])] - 25*(-1 + x)^2*Sqrt[2 + 2*x +
 x^2]*ArcTanh[Sqrt[1 + (1 + x)^2]])/(200*(-1 + x)*Sqrt[-2 + 2*x + x^2 - x^4
])
```

IntegrateAlgebraic [A] time = 9.27, size = 143, normalized size = 1.00

$$-\frac{i\sqrt{-x^4+x^2+2x-2}\left(-\frac{1}{4}i\tanh^{-1}\left(-\sqrt{x^2+2x+2}+x+1\right)-\frac{7i\tanh^{-1}\left(\frac{\sqrt{x^2+2x+2}}{\sqrt{5}}-\frac{x}{\sqrt{5}}+\frac{1}{\sqrt{5}}\right)}{20\sqrt{5}}-\frac{i(x^3-2x^2-2x+4)}{20(x-1)^2\sqrt{x^2+2x+2}}\right)}{(x-1)\sqrt{x^2+2x+2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x)*(-2 + 2*x + x^2 - x^4)^(3/2)),x]
```

```
[Out] ((-I)*Sqrt[-2 + 2*x + x^2 - x^4]*(((-1/20*I)*(4 - 2*x - 2*x^2 + x^3))/((-1
+ x)^2*Sqrt[2 + 2*x + x^2]) - (I/4)*ArcTanh[1 + x - Sqrt[2 + 2*x + x^2]] -
(((7*I)/20)*ArcTanh[1/Sqrt[5] - x/Sqrt[5] + Sqrt[2 + 2*x + x^2]/Sqrt[5]])/S
qrt[5]))/((-1 + x)*Sqrt[2 + 2*x + x^2])
```

fricas [A] time = 0.40, size = 174, normalized size = 1.22

$$-\frac{7\sqrt{5}\left(x^5-x^4-x^3-x^2+4x-2\right)\arctan\left(\frac{\sqrt{5}\sqrt{-x^4+x^2+2x-2}(2x+3)}{5(x^3+x^2-2)}\right)-25\left(x^5-x^4-x^3-x^2+4x-2\right)\arctan\left(\frac{\sqrt{-x^4+x^2+2x-2}}{x^3+x^2-2}\right)+10\sqrt{-x^4+x^2+2x-2}\left(x^3-2x^2-2x+4\right)}{200\left(x^5-x^4-x^3-x^2+4x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^4+x^2+2*x-2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/200*(7*sqrt(5)*(x^5 - x^4 - x^3 - x^2 + 4*x - 2)*arctan(1/5*sqrt(5)*sqrt
(-x^4 + x^2 + 2*x - 2)*(2*x + 3)/(x^3 + x^2 - 2)) - 25*(x^5 - x^4 - x^3 - x
^2 + 4*x - 2)*arctan(sqrt(-x^4 + x^2 + 2*x - 2)/(x^3 + x^2 - 2)) + 10*sqrt(
-x^4 + x^2 + 2*x - 2)*(x^3 - 2*x^2 - 2*x + 4))/(x^5 - x^4 - x^3 - x^2 + 4*x
 - 2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^4+x^2+2*x-2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.03, size = 253, normalized size = 1.77

$$-\frac{\left(7\arctan\left(\frac{(3+2x)\sqrt{5}}{5\sqrt{-x^2-2x-2}}\right)\sqrt{5}\,\sqrt{-x^2-2x-2}\,x^2-14\arctan\left(\frac{(3+2x)\sqrt{5}}{5\sqrt{-x^2-2x-2}}\right)\sqrt{5}\,\sqrt{-x^2-2x-2}\,x-25\arctan\left(\frac{1}{\sqrt{-x^2-2x-2}}\right)\sqrt{-x^2-2x-2}\,x^2+7\sqrt{5}\,\arctan\left(\frac{(3+2x)\sqrt{5}}{5\sqrt{-x^2-2x-2}}\right)\sqrt{-x^2-2x-2}+50\arctan\left(\frac{1}{\sqrt{-x^2-2x-2}}\right)\sqrt{-x^2-2x-2}\,x+10x^3-25\arctan\left(\frac{1}{\sqrt{-x^2-2x-2}}\right)\sqrt{-x^2-2x-2}-20x^2-20x+40\right)(-1+x)\left(x^2+2x+2\right)}{200\left(-x^4+x^2+2x-2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(-x^4+x^2+2*x-2)^(3/2),x)
```

```
[Out] -1/200*(7*arctan(1/5*(3+2*x)*5^(1/2)/(-x^2-2*x-2)^(1/2))*5^(1/2)*(-x^2-2*x-
2)^(1/2)*x^2-14*arctan(1/5*(3+2*x)*5^(1/2)/(-x^2-2*x-2)^(1/2))*5^(1/2)*(-x^
2-2*x-2)^(1/2)*x-25*arctan(1/(-x^2-2*x-2)^(1/2))*(-x^2-2*x-2)^(1/2)*x^2+7*5
^(1/2)*arctan(1/5*(3+2*x)*5^(1/2)/(-x^2-2*x-2)^(1/2))*(-x^2-2*x-2)^(1/2)+50
*arctan(1/(-x^2-2*x-2)^(1/2))*(-x^2-2*x-2)^(1/2)*x+10*x^3-25*arctan(1/(-x^2
-2*x-2)^(1/2))*(-x^2-2*x-2)^(1/2)-20*x^2-20*x+40)*(-1+x)*(x^2+2*x+2)/(-x^4+
x^2+2*x-2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+x^2+2x-2\right)^{\frac{3}{2}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^4+x^2+2*x-2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2*x - 2)^(3/2)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x+1)\,\left(-x^4+x^2+2x-2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)*(2*x + x^2 - x^4 - 2)^(3/2)),x)
```

```
[Out] int(1/((x + 1)*(2*x + x^2 - x^4 - 2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(x-1\right)^{2}\left(x^{2}+2x+2\right)\right)^{\frac{3}{2}}\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x**4+x**2+2*x-2)**(3/2),x)
```

```
[Out] Integral(1/((-(x - 1)**2*(x**2 + 2*x + 2))**(3/2)*(x + 1)), x)
```

3.1679 $\int \frac{(-3+2x^4)(1+2x^4)^{2/3}}{x^3(2-x^3+4x^4)}\,dx$

Optimal. Leaf size=143

$$\frac{\log\left(\sqrt[3]{2}\sqrt[3]{2x^4+1}-x\right)}{2\,2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{2x^4+1}+x}\right)}{2\,2^{2/3}}+\frac{3\left(2x^4+1\right)^{2/3}}{4x^2}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{2x^4+1}\,x+2^{2/3}\left(2x^4+1\right)^{2/3}+\right.}{4\,2^{2/3}}$$

Rubi [F] time = 1.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-3+2x^4\right)\left(1+2x^4\right)^{2/3}}{x^3\left(2-x^3+4x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + 2*x^4)*(1 + 2*x^4)^(2/3))/(x^3*(2 - x^3 + 4*x^4)),x]
```

```
[Out] (3*(1 + 2*x^4)^(2/3))/(4*x^2) + (6*x^2)/(1 - Sqrt[3] - (1 + 2*x^4)^(1/3)) -
 (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 + 2*x^4)^(1/3))*Sqrt[(1 + (1 + 2*x^4)
^(1/3) + (1 + 2*x^4)^(2/3))/(1 - Sqrt[3] - (1 + 2*x^4)^(1/3))^2]*EllipticE[
ArcSin[(1 + Sqrt[3] - (1 + 2*x^4)^(1/3))/(1 - Sqrt[3] - (1 + 2*x^4)^(1/3))]
, -7 + 4*Sqrt[3]])/(2*x^2*Sqrt[-((1 - (1 + 2*x^4)^(1/3))/(1 - Sqrt[3] - (1
+ 2*x^4)^(1/3))^2)]) + (Sqrt[2]*3^(3/4)*(1 - (1 + 2*x^4)^(1/3))*Sqrt[(1 + (
1 + 2*x^4)^(1/3) + (1 + 2*x^4)^(2/3))/(1 - Sqrt[3] - (1 + 2*x^4)^(1/3))^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - (1 + 2*x^4)^(1/3))/(1 - Sqrt[3] - (1 + 2*x^
4)^(1/3))], -7 + 4*Sqrt[3]])/(x^2*Sqrt[-((1 - (1 + 2*x^4)^(1/3))/(1 - Sqrt[
3] - (1 + 2*x^4)^(1/3))^2)]) - (3*Defer[Int][(1 + 2*x^4)^(2/3)/(2 - x^3 + 4
*x^4), x])/2 + 8*Defer[Int][(x*(1 + 2*x^4)^(2/3))/(2 - x^3 + 4*x^4), x]
```

Rubi steps

$$\int \frac{(-3+2x^4)(1+2x^4)^{2/3}}{x^3(2-x^3+4x^4)}\,dx = \int \left(-\frac{3(1+2x^4)^{2/3}}{2x^3} + \frac{(-3+16x)(1+2x^4)^{2/3}}{2(2-x^3+4x^4)}\right) dx$$

$$= \frac{1}{2}\int \frac{(-3+16x)(1+2x^4)^{2/3}}{2-x^3+4x^4}\,dx - \frac{3}{2}\int \frac{(1+2x^4)^{2/3}}{x^3}\,dx$$

$$= \frac{1}{2}\int \left(-\frac{3(1+2x^4)^{2/3}}{2-x^3+4x^4} + \frac{16x(1+2x^4)^{2/3}}{2-x^3+4x^4}\right) dx - \frac{3}{4}\text{Subst}\left(\int \frac{(1+2x^2)^{2/3}}{x^2}\,dx, x, x$$

$$= \frac{3(1+2x^4)^{2/3}}{4x^2} - \frac{3}{2}\int \frac{(1+2x^4)^{2/3}}{2-x^3+4x^4}\,dx - 2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+2x^2}}\,dx, x, x^2\right) + 8\int$$

$$= \frac{3(1+2x^4)^{2/3}}{4x^2} - \frac{3}{2}\int \frac{(1+2x^4)^{2/3}}{2-x^3+4x^4}\,dx + 8\int \frac{x(1+2x^4)^{2/3}}{2-x^3+4x^4}\,dx - \frac{\left(3\sqrt{x^4}\right)\text{Subst}(}{}$$

$$= \frac{3(1+2x^4)^{2/3}}{4x^2} - \frac{3}{2}\int \frac{(1+2x^4)^{2/3}}{2-x^3+4x^4}\,dx + 8\int \frac{x(1+2x^4)^{2/3}}{2-x^3+4x^4}\,dx + \frac{\left(3\sqrt{x^4}\right)\text{Subst}(}{}$$

$$= \frac{3(1+2x^4)^{2/3}}{4x^2} + \frac{6x^2}{1-\sqrt{3}-\sqrt[3]{1+2x^4}} - \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1+2x^4}\right)\sqrt{\frac{1+\sqrt[3]{1+2}}{(1-\sqrt{}}}}{2x^2\sqrt{-}}$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(-3+2x^4)(1+2x^4)^{2/3}}{x^3(2-x^3+4x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + 2*x^4)*(1 + 2*x^4)^(2/3))/(x^3*(2 - x^3 + 4*x^4)),x]
[Out] Integrate[((-3 + 2*x^4)*(1 + 2*x^4)^(2/3))/(x^3*(2 - x^3 + 4*x^4)), x]
```

IntegrateAlgebraic [A] time = 1.18, size = 143, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{2}\sqrt[3]{2x^4+1}-x\right)}{2\,2^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{2x^4+1}+x}\right)}{2\,2^{2/3}} + \frac{3(2x^4+1)^{2/3}}{4x^2} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{2x^4+1}\,x + 2^{2/3}(2x^4+1)^{2/3} + x^2\right)}{4\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + 2*x^4)*(1 + 2*x^4)^(2/3))/(x^3*(2 - x^3 + 4*x^4))
,x]
[Out] (3*(1 + 2*x^4)^(2/3))/(4*x^2) - (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*2^(1/3)*
(1 + 2*x^4)^(1/3))])/(2*2^(2/3)) + Log[-x + 2^(1/3)*(1 + 2*x^4)^(1/3)]/(2*2
^(2/3)) - Log[x^2 + 2^(1/3)*x*(1 + 2*x^4)^(1/3) + 2^(2/3)*(1 + 2*x^4)^(2/3)
]/(4*2^(2/3))
```

fricas [B] time = 104.81, size = 404, normalized size = 2.83

$$-\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}x^2\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(12\cdot 4^{\frac{2}{3}}(8x^9+2x^8-x^7+8x^5+x^4+2x)(2x^4+1)^{\frac{2}{3}}+4^{\frac{1}{3}}(64x^{12}+240x^{11}+48x^{10}-x^9+96x^8+240x^7+24x^6+48x^4+60x^3+8)+12(16x^{10}+28x^9+x^8+16x^6+14x^5+4x^2)(2x^4+1)^{\frac{1}{3}}\right)}{6(64x^{12}-48x^{11}-96x^{10}-x^9+96x^8-48x^7-48x^6+48x^4-12x^3+8)}\right) - 2\cdot 4^{\frac{2}{3}}x^2\log\left(\frac{6\cdot 4^{\frac{1}{3}}(2x^4+1)^{\frac{1}{3}}x^2+4^{\frac{2}{3}}(4x^4-x^3+2)-12(2x^4+1)^{\frac{2}{3}}x}{4x^4-x^3+2}\right) + 4^{\frac{2}{3}}x^2\log\left(\frac{6\cdot 4^{\frac{2}{3}}(2x^5+x^4+x)(2x^4+1)^{\frac{2}{3}}+4^{\frac{1}{3}}(16x^8+28x^7+x^6+16x^4+14x^3+4)+6(8x^6+x^5+4x^2)(2x^4+1)^{\frac{1}{3}}}{16x^8-8x^7+x^6+16x^4-4x^3+4}\right) - 36(2x^4+1)^{\frac{2}{3}}}{48x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-3)*(2*x^4+1)^(2/3)/x^3/(4*x^4-x^3+2),x, algorithm="fricas"
)
```

```
[Out] -1/48*(4*4^(1/6)*sqrt(3)*x^2*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^(2/3)*(8*x^9
+ 2*x^8 - x^7 + 8*x^5 + x^4 + 2*x)*(2*x^4 + 1)^(2/3) + 4^(1/3)*(64*x^12 + 2
40*x^11 + 48*x^10 - x^9 + 96*x^8 + 240*x^7 + 24*x^6 + 48*x^4 + 60*x^3 + 8)
+ 12*(16*x^10 + 28*x^9 + x^8 + 16*x^6 + 14*x^5 + 4*x^2)*(2*x^4 + 1)^(1/3))/
(64*x^12 - 48*x^11 - 96*x^10 - x^9 + 96*x^8 - 48*x^7 - 48*x^6 + 48*x^4 - 12
*x^3 + 8)) - 2*4^(2/3)*x^2*log((6*4^(1/3)*(2*x^4 + 1)^(1/3)*x^2 + 4^(2/3)*(
4*x^4 - x^3 + 2) - 12*(2*x^4 + 1)^(2/3)*x)/(4*x^4 - x^3 + 2)) + 4^(2/3)*x^2
*log((6*4^(2/3)*(2*x^5 + x^4 + x)*(2*x^4 + 1)^(2/3) + 4^(1/3)*(16*x^8 + 28*
x^7 + x^6 + 16*x^4 + 14*x^3 + 4) + 6*(8*x^6 + x^5 + 4*x^2)*(2*x^4 + 1)^(1/3
))/(16*x^8 - 8*x^7 + x^6 + 16*x^4 - 4*x^3 + 4)) - 36*(2*x^4 + 1)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 x^4+1\right)^{\frac{2}{3}} \left(2 x^4-3\right)}{\left(4 x^4-x^3+2\right) x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-3)*(2*x^4+1)^(2/3)/x^3/(4*x^4-x^3+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^4 + 1)^(2/3)*(2*x^4 - 3)/((4*x^4 - x^3 + 2)*x^3), x)
```

maple [C] time = 85.36, size = 669, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-3)*(2*x^4+1)^(2/3)/x^3/(4*x^4-x^3+2),x)
```

```
[Out] 3/4*(2*x^4+1)^(2/3)/x^2+1/2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_
Z^2)*ln(-(RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2
)^3*x^3+2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2
)^2*(2*x^4+1)^(2/3)*x+2*RootOf(_Z^3-2)*(2*x^4+1)^(1/3)*RootOf(RootOf(_Z^3-2
)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2+2*RootOf(_Z^3-2)^2*(2*x^4+1)^(1/3)*x^2+
4*RootOf(_Z^3-2)*x^4+RootOf(_Z^3-2)*x^3+4*(2*x^4+1)^(2/3)*x+2*RootOf(_Z^3-2
))/(4*x^4-x^3+2))-1/2*ln((RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^
2)*RootOf(_Z^3-2)^3*x^3+2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^
2)*RootOf(_Z^3-2)^2*(2*x^4+1)^(2/3)*x+2*RootOf(_Z^3-2)*(2*x^4+1)^(1/3)*Root
Of(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2-RootOf(_Z^3-2)^2*(2*x^4
+1)^(1/3)*x^2-4*RootOf(_Z^3-2)*x^4-2*(2*x^4+1)^(2/3)*x-2*RootOf(_Z^3-2))/(4
*x^4-x^3+2))*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-1/4*ln((Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3+2*Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(2*x^4+1
)^(2/3)*x+2*RootOf(_Z^3-2)*(2*x^4+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*Roo
tOf(_Z^3-2)+4*_Z^2)*x^2-RootOf(_Z^3-2)^2*(2*x^4+1)^(1/3)*x^2-4*RootOf(_Z^3-
2)*x^4-2*(2*x^4+1)^(2/3)*x-2*RootOf(_Z^3-2))/(4*x^4-x^3+2))*RootOf(_Z^3-2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 x^4+1\right)^{\frac{2}{3}} \left(2 x^4-3\right)}{\left(4 x^4-x^3+2\right) x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-3)*(2*x^4+1)^(2/3)/x^3/(4*x^4-x^3+2),x, algorithm="maxima"
)
```

```
[Out] integrate((2*x^4 + 1)^(2/3)*(2*x^4 - 3)/((4*x^4 - x^3 + 2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^4+1\right)^{2/3}\,\left(2\,x^4-3\right)}{x^3\,\left(4\,x^4-x^3+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^4 + 1)^(2/3)*(2*x^4 - 3))/(x^3*(4*x^4 - x^3 + 2)),x)
[Out] int(((2*x^4 + 1)^(2/3)*(2*x^4 - 3))/(x^3*(4*x^4 - x^3 + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-3)*(2*x**4+1)**(2/3)/x**3/(4*x**4-x**3+2),x)
[Out] Timed out
```

3.1680 $\int \frac{-b+ax^3}{x^6(b+ax^3)\sqrt[4]{-bx+ax^4}}\,dx$

Optimal. Leaf size=143

$$-\frac{2\,2^{3/4}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,(ax^4-bx)^{3/4}}{ax^3-b}\right)}{3b^2}-\frac{2\,2^{3/4}a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,(ax^4-bx)^{3/4}}{ax^3-b}\right)}{3b^2}-\frac{4\left(3b-10ax^3\right)\left(ax^4-bx\right)^{3/4}}{63b^2x^6}$$

Rubi [C] time = 0.57, antiderivative size = 132, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2056, 466, 465, 511, 510}

$$-\frac{\Gamma\left(-\frac{3}{4}\right)\left(b-ax^3\right)^2\left(\left(4a^2x^6+7abx^3+3b^2\right)\,{}_2F_1\left(1,1;\frac{1}{4};\frac{2ax^3}{ax^3+b}\right)-32ax^3\left(b-ax^3\right)\,{}_2F_1\left(2,2;\frac{5}{4};\frac{2ax^3}{ax^3+b}\right)\right)}{21b^2x^5\Gamma\left(\frac{1}{4}\right)\left(ax^3+b\right)^2\sqrt[4]{ax^4-bx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-b + a*x^3)/(x^6*(b + a*x^3)*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] -1/21*((b - a*x^3)^2*Gamma[-3/4]*((3*b^2 + 7*a*b*x^3 + 4*a^2*x^6)*Hypergeom
etric2F1[1, 1, 1/4, (2*a*x^3)/(b + a*x^3)] - 32*a*x^3*(b - a*x^3)*Hypergeom
etric2F1[2, 2, 5/4, (2*a*x^3)/(b + a*x^3)]))/(b^2*x^5*(b + a*x^3)^2*(-(b*x)
 + a*x^4)^(1/4)*Gamma[1/4])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{-b+ax^3}{x^6\left(b+ax^3\right)\sqrt[4]{-bx+ax^4}}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\int \frac{\left(-b+ax^3\right)^{3/4}}{x^{25/4}\left(b+ax^3\right)}\,dx}{\sqrt[4]{-bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{\left(-b+ax^{12}\right)^{3/4}}{x^{22}\left(b+ax^{12}\right)}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\int \frac{\left(-b+ax^4\right)^{3/4}}{x^8\left(b+ax^4\right)}\,dx, x, x^{3/4}\right)}{3\sqrt[4]{-bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\left(-b+ax^3\right)\right)\text{Subst}\left(\int \frac{\left(1-\frac{ax^4}{b}\right)^{3/4}}{x^8\left(b+ax^4\right)}\,dx, x, x^{3/4}\right)}{3\left(1-\frac{ax^3}{b}\right)^{3/4}\sqrt[4]{-bx+ax^4}}$$

$$= -\frac{\left(b-ax^3\right)^2\Gamma\left(-\frac{3}{4}\right)\left(\left(3b^2+7abx^3+4a^2x^6\right){}_2F_1\left(1,1;\frac{1}{4};\frac{2ax^3}{b+ax^3}\right)-32ax^3\left(b-ax^3\right.\right.}{21b^2x^5\left(b+ax^3\right)^2\sqrt[4]{-bx+ax^4}\,\Gamma\left(\frac{1}{4}\right)}$$

Mathematica [C] time = 0.36, size = 132, normalized size = 0.92

$$-\frac{4\left(ax^4-bx\right)^{3/4}\left(4a^2x^6-6ax^3\left(4ax^3-3b\right){}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{2ax^3}{b-ax^3}\right)-24ax^3\left(ax^3+b\right){}_2F_1\left(\frac{1}{4},2;\frac{5}{4};-\frac{2ax^3}{b-ax^3}\right)-7abx^3+3b^2\right)}{63b^2x^6\left(b-ax^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-b + a*x^3)/(x^6*(b + a*x^3)*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(-(b*x) + a*x^4)^(3/4)*(3*b^2 - 7*a*b*x^3 + 4*a^2*x^6 - 6*a*x^3*(-3*b +
 4*a*x^3)*Hypergeometric2F1[1/4, 1, 5/4, (-2*a*x^3)/(b - a*x^3)] - 24*a*x^3
*(b + a*x^3)*Hypergeometric2F1[1/4, 2, 5/4, (-2*a*x^3)/(b - a*x^3)]))/(63*b
^2*x^6*(b - a*x^3))
```

IntegrateAlgebraic [A] time = 0.53, size = 143, normalized size = 1.00

$$-\frac{2\,2^{3/4}a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3b^2}-\frac{2\,2^{3/4}a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3b^2}-\frac{4\left(3b-10ax^3\right)\left(ax^4-bx\right)^{3/4}}{63b^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^3)/(x^6*(b + a*x^3)*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(3*b - 10*a*x^3)*(-(b*x) + a*x^4)^(3/4))/(63*b^2*x^6) - (2*2^(3/4)*a^(7
/4)*ArcTan[(2^(1/4)*a^(1/4)*(-(b*x) + a*x^4)^(3/4))/(-b + a*x^3)])/(3*b^2)
- (2*2^(3/4)*a^(7/4)*ArcTanh[(2^(1/4)*a^(1/4)*(-(b*x) + a*x^4)^(3/4))/(-b +
 a*x^3)])/(3*b^2)
```

fricas [B] time = 152.90, size = 520, normalized size = 3.64

$$\frac{84\cdot 8^{\frac{1}{4}} b^{2} x^{6} \left(\frac{a^{7}}{b^{8}}\right)^{\frac{1}{4}} \arctan\left(\frac{16\cdot 8^{\frac{1}{4}} (a x^{4}-b x)^{\frac{1}{4}} a^{9} b^{2} x^{2} \left(\frac{a^{7}}{b^{8}}\right)^{\frac{1}{4}}+4\cdot 8^{\frac{3}{4}} (a x^{4}-b x)^{\frac{3}{4}} a^{5} b^{6} \left(\frac{a^{7}}{b^{8}}\right)^{\frac{3}{4}}+\sqrt{2}\sqrt{\sqrt{2} a^{6} b^{4} \sqrt{\frac{a^{7}}{b^{8}}}}\left(8\cdot 8^{\frac{1}{4}} \sqrt{a x^{4}-b x}\, a^{4} b^{2} x \left(\frac{a^{7}}{b^{8}}\right)^{\frac{1}{4}}+8^{\frac{3}{4}} (3 a b^{6} x^{3}-b^{7}) \left(\frac{a^{7}}{b^{8}}\right)^{\frac{3}{4}}\right)}{8\,(a^{11} x^{3}+a^{10} b)}\right) - 21\cdot 8^{\frac{1}{4}} b^{2} x^{6} \left(\frac{a^{7}}{b^{8}}\right)^{\frac{1}{4}} \log\left(\frac{4\sqrt{2} (a x^{4}-b x)^{\frac{1}{4}} a^{2} b^{4} x^{2} \sqrt{\frac{a^{7}}{b^{8}}}+8^{\frac{3}{4}} \sqrt{a x^{4}-b x}\, b^{6} x \left(\frac{a^{7}}{b^{8}}\right)^{\frac{3}{4}}+4 (a x^{4}-b x)^{\frac{3}{4}} a^{5}+8^{\frac{1}{4}} (3 a^{4} b^{2} x^{3}-a^{3} b^{3}) \left(\frac{a^{7}}{b^{8}}\right)^{\frac{1}{4}}}{a x^{3}+b}\right) + 21\cdot 8^{\frac{1}{4}} b^{2} x^{6} \left(\frac{a^{7}}{b^{8}}\right)^{\frac{1}{4}} \log\left(\frac{4\sqrt{2} (a x^{4}-b x)^{\frac{1}{4}} a^{2} b^{4} x^{2} \sqrt{\frac{a^{7}}{b^{8}}}-8^{\frac{3}{4}} \sqrt{a x^{4}-b x}\, b^{6} x \left(\frac{a^{7}}{b^{8}}\right)^{\frac{3}{4}}+4 (a x^{4}-b x)^{\frac{3}{4}} a^{5}-8^{\frac{1}{4}} (3 a^{4} b^{2} x^{3}-a^{3} b^{3}) \left(\frac{a^{7}}{b^{8}}\right)^{\frac{1}{4}}}{a x^{3}+b}\right) + 8 (a x^{4}-b x)^{\frac{3}{4}} (10 a x^{3}-3 b)}{126\, b^{2} x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)/x^6/(a*x^3+b)/(a*x^4-b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/126*(84*8^(1/4)*b^2*x^6*(a^7/b^8)^(1/4)*arctan(1/8*(16*8^(1/4)*(a*x^4 - b
*x)^(1/4)*a^9*b^2*x^2*(a^7/b^8)^(1/4) + 4*8^(3/4)*(a*x^4 - b*x)^(3/4)*a^5*b
^6*(a^7/b^8)^(3/4) + sqrt(2)*sqrt(sqrt(2)*a^6*b^4*sqrt(a^7/b^8))*(8*8^(1/4)
*sqrt(a*x^4 - b*x)*a^4*b^2*x*(a^7/b^8)^(1/4) + 8^(3/4)*(3*a*b^6*x^3 - b^7)*
(a^7/b^8)^(3/4)))/(a^11*x^3 + a^10*b)) - 21*8^(1/4)*b^2*x^6*(a^7/b^8)^(1/4)
*log((4*sqrt(2)*(a*x^4 - b*x)^(1/4)*a^2*b^4*x^2*sqrt(a^7/b^8) + 8^(3/4)*sqr
t(a*x^4 - b*x)*b^6*x*(a^7/b^8)^(3/4) + 4*(a*x^4 - b*x)^(3/4)*a^5 + 8^(1/4)*
(3*a^4*b^2*x^3 - a^3*b^3)*(a^7/b^8)^(1/4))/(a*x^3 + b)) + 21*8^(1/4)*b^2*x^
6*(a^7/b^8)^(1/4)*log((4*sqrt(2)*(a*x^4 - b*x)^(1/4)*a^2*b^4*x^2*sqrt(a^7/b
^8) - 8^(3/4)*sqrt(a*x^4 - b*x)*b^6*x*(a^7/b^8)^(3/4) + 4*(a*x^4 - b*x)^(3/
4)*a^5 - 8^(1/4)*(3*a^4*b^2*x^3 - a^3*b^3)*(a^7/b^8)^(1/4))/(a*x^3 + b)) +
8*(a*x^4 - b*x)^(3/4)*(10*a*x^3 - 3*b))/(b^2*x^6)
```

giac [B] time = 0.22, size = 241, normalized size = 1.69

$$-\frac{2\cdot 2^{\frac{1}{4}} (-a)^{\frac{3}{4}} a \arctan\left(\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}} (-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x^{3}}\right)^{\frac{1}{4}}\right)}{2 (-a)^{\frac{1}{4}}}\right)}{3 b^{2}} - \frac{2\cdot 2^{\frac{1}{4}} (-a)^{\frac{3}{4}} a \arctan\left(-\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}} (-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x^{3}}\right)^{\frac{1}{4}}\right)}{2 (-a)^{\frac{1}{4}}}\right)}{3 b^{2}} + \frac{2^{\frac{1}{4}} (-a)^{\frac{3}{4}} a \log\left(2^{\frac{3}{4}} (-a)^{\frac{1}{4}}\left(a-\frac{b}{x^{3}}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a-\frac{b}{x^{3}}}\right)}{3 b^{2}} - \frac{2^{\frac{1}{4}} (-a)^{\frac{3}{4}} a \log\left(-2^{\frac{3}{4}} (-a)^{\frac{1}{4}}\left(a-\frac{b}{x^{3}}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a-\frac{b}{x^{3}}}\right)}{3 b^{2}} + \frac{4\left(3\left(a-\frac{b}{x^{3}}\right)^{\frac{7}{4}} b^{12}+7\left(a-\frac{b}{x^{3}}\right)^{\frac{3}{4}} a b^{12}\right)}{63 b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)/x^6/(a*x^3+b)/(a*x^4-b*x)^(1/4),x, algorithm="giac")
```

```
[Out] -2/3*2^(1/4)*(-a)^(3/4)*a*arctan(1/2*2^(1/4)*(2^(3/4)*(-a)^(1/4) + 2*(a - b
/x^3)^(1/4))/(-a)^(1/4))/b^2 - 2/3*2^(1/4)*(-a)^(3/4)*a*arctan(-1/2*2^(1/4)
*(2^(3/4)*(-a)^(1/4) - 2*(a - b/x^3)^(1/4))/(-a)^(1/4))/b^2 + 1/3*2^(1/4)*(
-a)^(3/4)*a*log(2^(3/4)*(-a)^(1/4)*(a - b/x^3)^(1/4) + sqrt(2)*sqrt(-a) + s
qrt(a - b/x^3))/b^2 - 1/3*2^(1/4)*(-a)^(3/4)*a*log(-2^(3/4)*(-a)^(1/4)*(a -
 b/x^3)^(1/4) + sqrt(2)*sqrt(-a) + sqrt(a - b/x^3))/b^2 + 4/63*(3*(a - b/x^
3)^(7/4)*b^12 + 7*(a - b/x^3)^(3/4)*a*b^12)/b^14
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{a\,x^{3}-b}{x^{6} \left(a\,x^{3}+b\right) \left(a\,x^{4}-b x\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b)/x^6/(a*x^3+b)/(a*x^4-b*x)^(1/4),x)
```

```
[Out] int((a*x^3-b)/x^6/(a*x^3+b)/(a*x^4-b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a x^{3} - b}{{\left(a x^{4} - b x\right)}^{\frac{1}{4}} {\left(a x^{3} + b\right)} x^{6}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)/x^6/(a*x^3+b)/(a*x^4-b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^3 - b)/((a*x^4 - b*x)^(1/4)*(a*x^3 + b)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{b-a\,x^3}{x^6\,{\left(a\,x^4-b\,x\right)}^{1/4}\,\left(a\,x^3+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^3)/(x^6*(a*x^4 - b*x)^(1/4)*(b + a*x^3)),x)
[Out] int(-(b - a*x^3)/(x^6*(a*x^4 - b*x)^(1/4)*(b + a*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{3} - b}{x^{6} \sqrt[4]{x \left(ax^{3} - b\right)} \left(ax^{3} + b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b)/x**6/(a*x**3+b)/(a*x**4-b*x)**(1/4),x)
[Out] Integral((a*x**3 - b)/(x**6*(x*(a*x**3 - b))**(1/4)*(a*x**3 + b)), x)
```

3.1681 $\int \frac{\left(-1+x^3\right)^{2/3}\left(4+4x^3+x^6\right)}{x^9\left(1+x^3\right)}\,dx$

Optimal. Leaf size=143

$$-\frac{1}{3}2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^3-1}-2x\right)+\frac{2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2^{2/3}\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}+\frac{\log\left(2^{2/3}\sqrt[3]{x^3-1}\,x+\sqrt[3]{2}\left(x^3-1\right)^{2/3}+2x^2\right)}{3\sqrt[3]{2}}+\frac{\left(x^3-1\right.}{}$$

Rubi [A] time = 0.42, antiderivative size = 277, normalized size of antiderivative = 1.94, number of steps used = 23, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {28, 586, 580, 583, 12, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{2}{3}2^{2/3}\log\left(1-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{x^3-1}}\right)+\frac{\log\left(1-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{x^3-1}}\right)}{12\sqrt[3]{2}}+\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{\left(x^3-1\right)^{2/3}}{2x^8}+\frac{11\left(x^3-1\right)^{2/3}}{20x^5}-\frac{79\left(x^3-1\right)^{2/3}}{80x^2}+\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}\,x}{\sqrt[3]{x^3-1}}+\frac{2^{2/3}x^2}{\left(x^3-1\right)^{2/3}}+1\right)-\frac{\log\left(\frac{\sqrt[3]{2}\,x}{\sqrt[3]{x^3-1}}+\frac{2^{2/3}x^2}{\left(x^3-1\right)^{2/3}}+1\right)}{24\sqrt[3]{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^3)^(2/3)*(4 + 4*x^3 + x^6))/(x^9*(1 + x^3)),x]
```

```
[Out] -1/2*(-1 + x^3)^(2/3)/x^8 + (11*(-1 + x^3)^(2/3))/(20*x^5) - (79*(-1 + x^3)
^(2/3))/(80*x^2) - ArcTan[(1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]]/(4*
2^(1/3)*Sqrt[3]) + (2*2^(2/3)*ArcTan[(1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/S
qrt[3]])/Sqrt[3] + Log[1 - (2^(1/3)*x)/(-1 + x^3)^(1/3)]/(12*2^(1/3)) - (2*
2^(2/3)*Log[1 - (2^(1/3)*x)/(-1 + x^3)^(1/3)])/3 - Log[1 + (2^(2/3)*x^2)/(-
1 + x^3)^(2/3) + (2^(1/3)*x)/(-1 + x^3)^(1/3)]/(24*2^(1/3)) + (2^(2/3)*Log[
1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) + (2^(1/3)*x)/(-1 + x^3)^(1/3)])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 586

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[e, Int[(g*x)^m*
(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^(r - 1), x], x] + Dist[f/e^n, Int[(
g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^(r - 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g, m, p, q}, x] && IGtQ[n, 0] && IGtQ[r, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(4+4x^3+x^6\right)}{x^9\left(1+x^3\right)}\,dx = \int \frac{\left(-1+x^3\right)^{2/3}\left(2+x^3\right)^2}{x^9\left(1+x^3\right)}\,dx$$

$$= \frac{1}{8}\int \frac{\left(-1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(1+x^3\right)}\,dx + 2\int \frac{\left(-1+x^3\right)^{2/3}\left(2+x^3\right)}{x^9\left(1+x^3\right)}\,dx$$

$$= -\frac{\left(-1+x^3\right)^{2/3}}{2x^8} - \frac{\left(-1+x^3\right)^{2/3}}{20x^5} + \frac{1}{40}\int \frac{9-x^3}{x^3\sqrt[3]{-1+x^3}\left(1+x^3\right)}\,dx + \frac{1}{4}\int \frac{}{x^6}$$

$$= -\frac{\left(-1+x^3\right)^{2/3}}{2x^8} + \frac{11\left(-1+x^3\right)^{2/3}}{20x^5} + \frac{9\left(-1+x^3\right)^{2/3}}{80x^2} + \frac{1}{80}\int -\frac{20}{\sqrt[3]{-1+x^3}\,(1}$$

$$= -\frac{\left(-1+x^3\right)^{2/3}}{2x^8} + \frac{11\left(-1+x^3\right)^{2/3}}{20x^5} - \frac{79\left(-1+x^3\right)^{2/3}}{80x^2} + \frac{1}{40}\int \frac{160}{\sqrt[3]{-1+x^3}\,(1}$$

$$= -\frac{\left(-1+x^3\right)^{2/3}}{2x^8} + \frac{11\left(-1+x^3\right)^{2/3}}{20x^5} - \frac{79\left(-1+x^3\right)^{2/3}}{80x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-2x^3}\right.$$

$$= -\frac{\left(-1+x^3\right)^{2/3}}{2x^8} + \frac{11\left(-1+x^3\right)^{2/3}}{20x^5} - \frac{79\left(-1+x^3\right)^{2/3}}{80x^2} - \frac{1}{12}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}}\right.$$

$$= -\frac{\left(-1+x^3\right)^{2/3}}{2x^8} + \frac{11\left(-1+x^3\right)^{2/3}}{20x^5} - \frac{79\left(-1+x^3\right)^{2/3}}{80x^2} + \frac{\log\left(1-\frac{\sqrt[3]{2}x}{\sqrt[3]{-1+x^3}}\right)}{12\sqrt[3]{2}} -$$

$$= -\frac{\left(-1+x^3\right)^{2/3}}{2x^8} + \frac{11\left(-1+x^3\right)^{2/3}}{20x^5} - \frac{79\left(-1+x^3\right)^{2/3}}{80x^2} + \frac{\log\left(1-\frac{\sqrt[3]{2}x}{\sqrt[3]{-1+x^3}}\right)}{12\sqrt[3]{2}} -$$

$$= -\frac{\left(-1+x^3\right)^{2/3}}{2x^8} + \frac{11\left(-1+x^3\right)^{2/3}}{20x^5} - \frac{79\left(-1+x^3\right)^{2/3}}{80x^2} - \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} +$$

$$= -\frac{\left(-1+x^3\right)^{2/3}}{2x^8} + \frac{11\left(-1+x^3\right)^{2/3}}{20x^5} - \frac{79\left(-1+x^3\right)^{2/3}}{80x^2} - \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} +$$

Mathematica [A] time = 0.27, size = 139, normalized size = 0.97

$$\frac{1}{30}\left(5\,2^{2/3}\left(-2\log\left(1-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)+\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}+1\right)\right)-\frac{3\left(x^3-1\right)^{2/3}\left(2x^6-2x^3+5\right)}{x^8}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^3)^(2/3)*(4 + 4*x^3 + x^6))/(x^9*(1 + x^3)),x]
```

```
[Out] ((-3*(-1 + x^3)^(2/3)*(5 - 2*x^3 + 2*x^6))/x^8 + 5*2^(2/3)*(2*Sqrt[3]*ArcTa
n[(1 + (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (2^(1/3)*x)/(1 -
 x^3)^(1/3)] + Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) + (2^(1/3)*x)/(1 - x^3
)^(1/3)]))/30
```

IntegrateAlgebraic [A] time = 0.37, size = 143, normalized size = 1.00

$$-\frac{1}{3}2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^3-1}-2x\right)+\frac{2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}+\frac{\log\left(2^{2/3}\sqrt[3]{x^3-1}\,x+\sqrt[3]{2}\left(x^3-1\right)^{2/3}+2x^2\right)}{3\sqrt[3]{2}}+\frac{\left(x^3-1\right)^{2/3}\left(-2x^6+2x^3-5\right)}{10x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(2/3)*(4 + 4*x^3 + x^6))/(x^9*(1 + x^3)),x]
```

```
[Out] ((-1 + x^3)^(2/3)*(-5 + 2*x^3 - 2*x^6))/(10*x^8) + (2^(2/3)*ArcTan[(Sqrt[3]
*x)/(x + 2^(2/3)*(-1 + x^3)^(1/3))])/Sqrt[3] - (2^(2/3)*Log[-2*x + 2^(2/3)*
(-1 + x^3)^(1/3)])/3 + Log[2*x^2 + 2^(2/3)*x*(-1 + x^3)^(1/3) + 2^(1/3)*(-1
 + x^3)^(2/3)]/(3*2^(1/3))
```

fricas [B] time = 2.55, size = 276, normalized size = 1.93

$$-\frac{10\sqrt{3}\,(-4)^{\frac{1}{3}}x^8\arctan\left(\frac{3\sqrt{3}(-4)^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(x^3-1\right)^{\frac{2}{3}}+6\sqrt{3}(-4)^{\frac{1}{3}}\left(19x^8-16x^5+x^2\right)\left(x^3-1\right)^{\frac{1}{3}}-\sqrt{3}\left(71x^9-111x^6+33x^3-1\right)}{3\left(109x^9-105x^6+3x^3+1\right)}\right)-10\,(-4)^{\frac{1}{3}}x^8\log\left(-\frac{3(-4)^{\frac{2}{3}}\left(x^3-1\right)^{\frac{1}{3}}x^2-6\left(x^3-1\right)^{\frac{2}{3}}x+(-4)^{\frac{1}{3}}\left(x^3+1\right)}{x^3+1}\right)+5\,(-4)^{\frac{1}{3}}x^8\log\left(-\frac{6(-4)^{\frac{1}{3}}\left(5x^4-x\right)\left(x^3-1\right)^{\frac{2}{3}}-(-4)^{\frac{2}{3}}\left(19x^6-16x^3+1\right)-24\left(2x^5-x^2\right)\left(x^3-1\right)^{\frac{1}{3}}}{x^6+2x^3+1}\right)+9\left(2x^6-2x^3+5\right)\left(x^3-1\right)^{\frac{2}{3}}}{90x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+4*x^3+4)/x^9/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/90*(10*sqrt(3)*(-4)^(1/3)*x^8*arctan(1/3*(3*sqrt(3)*(-4)^(2/3)*(5*x^7 +
4*x^4 - x)*(x^3 - 1)^(2/3) + 6*sqrt(3)*(-4)^(1/3)*(19*x^8 - 16*x^5 + x^2)*(
x^3 - 1)^(1/3) - sqrt(3)*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^
6 + 3*x^3 + 1)) - 10*(-4)^(1/3)*x^8*log(-(3*(-4)^(2/3)*(x^3 - 1)^(1/3)*x^2
- 6*(x^3 - 1)^(2/3)*x + (-4)^(1/3)*(x^3 + 1))/(x^3 + 1)) + 5*(-4)^(1/3)*x^8
*log(-(6*(-4)^(1/3)*(5*x^4 - x)*(x^3 - 1)^(2/3) - (-4)^(2/3)*(19*x^6 - 16*x
^3 + 1) - 24*(2*x^5 - x^2)*(x^3 - 1)^(1/3))/(x^6 + 2*x^3 + 1)) + 9*(2*x^6 -
 2*x^3 + 5)*(x^3 - 1)^(2/3))/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+4x^3+4\right)\left(x^3-1\right)^{\frac{2}{3}}}{\left(x^3+1\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+4*x^3+4)/x^9/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 4*x^3 + 4)*(x^3 - 1)^(2/3)/((x^3 + 1)*x^9), x)
```

maple [C] time = 2.62, size = 628, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)*(x^6+4*x^3+4)/x^9/(x^3+1),x)
```

```
[Out] -1/10*(2*x^9-4*x^6+7*x^3-5)/x^8/(x^3-1)^(1/3)+2*RootOf(RootOf(_Z^3+4)^2+6*_
Z*RootOf(_Z^3+4)+36*_Z^2)*ln((9*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)
+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+36*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+
4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+12*(x^3-1)^(2/3)*RootOf(RootOf(_Z^3+4)^2
+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x+4*(x^3-1)^(1/3)*RootOf(_Z^
3+4)^2*x^2+30*(x^3-1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*
_Z^2)*RootOf(_Z^3+4)*x^2+3*RootOf(_Z^3+4)*x^3+12*RootOf(RootOf(_Z^3+4)^2+6*
_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3+2*x*(x^3-1)^(2/3)-3*RootOf(_Z^3+4)-12*RootOf
(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2))/(1+x)/(x^2-x+1))+1/3*RootOf
(_Z^3+4)*ln(-(3*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf
(_Z^3+4)^3*x^3+27*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*Ro
otOf(_Z^3+4)^2*x^3+6*(x^3-1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3
```

```
+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x+2*(x^3-1)^(1/3)*RootOf(_Z^3+4)^2*x^2-3*(x^3
-1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+
4)*x^2-3*RootOf(_Z^3+4)*x^3-27*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+
36*_Z^2)*x^3-5*x*(x^3-1)^(2/3)+RootOf(_Z^3+4)+9*RootOf(RootOf(_Z^3+4)^2+6*_
Z*RootOf(_Z^3+4)+36*_Z^2))/(1+x)/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}+4\,x^{3}+4\right)\left(x^{3}-1\right)^{\frac{2}{3}}}{\left(x^{3}+1\right)x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+4*x^3+4)/x^9/(x^3+1),x, algorithm="maxima")
[Out] integrate((x^6 + 4*x^3 + 4)*(x^3 - 1)^(2/3)/((x^3 + 1)*x^9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3-1\right)^{2/3}\,\left(x^6+4\,x^3+4\right)}{x^9\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(4*x^3 + x^6 + 4))/(x^9*(x^3 + 1)),x)
[Out] int(((x^3 - 1)^(2/3)*(4*x^3 + x^6 + 4))/(x^9*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}\left(x^{3}+2\right)^{2}}{x^{9}\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)*(x**6+4*x**3+4)/x**9/(x**3+1),x)
[Out] Integral(((x - 1)*(x**2 + x + 1))**(2/3)*(x**3 + 2)**2/(x**9*(x + 1)*(x**2
- x + 1)), x)
```

3.1682 $\int \frac{(1+x^3)^{2/3}(-1-2x^3+2x^6)}{x^9(-1+x^3)}\,dx$

Optimal. Leaf size=143

$$-\frac{1}{3}2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^3+1}-2x\right)+\frac{2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}+\frac{\log\left(2^{2/3}\sqrt[3]{x^3+1}\,x+\sqrt[3]{2}\left(x^3+1\right)^{2/3}+2x^2\right)}{3\sqrt[3]{2}}+\frac{\left(x^3+1\right)^{2}}{}$$

Rubi [F] time = 0.71, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+x^3\right)^{2/3}\left(-1-2x^3+2x^6\right)}{x^9\left(-1+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^3)^(2/3)*(-1 - 2*x^3 + 2*x^6))/(x^9*(-1 + x^3)),x]
```

```
[Out] -1/2*(1 + x^3)^(2/3)/x^2 - (1 + x^3)^(5/3)/(8*x^8) - (21*(1 + x^3)^(5/3))/(
40*x^5) + ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[-x + (1
 + x^3)^(1/3)]/2 - Defer[Int][(1 + x^3)^(2/3)/(-1 + x), x]/3 + ((1 - I*Sqrt
[3])*Defer[Int][(1 + x^3)^(2/3)/(1 - I*Sqrt[3] + 2*x), x])/3 + ((1 + I*Sqrt
[3])*Defer[Int][(1 + x^3)^(2/3)/(1 + I*Sqrt[3] + 2*x), x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+x^3\right)^{2/3}\left(-1-2x^3+2x^6\right)}{x^9\left(-1+x^3\right)}\,dx &= \int \left(-\frac{\left(1+x^3\right)^{2/3}}{3(-1+x)}+\frac{\left(1+x^3\right)^{2/3}}{x^9}+\frac{3\left(1+x^3\right)^{2/3}}{x^6}+\frac{\left(1+x^3\right)^{2/3}}{x^3}+\frac{(2+x)\left(1+\right.}{3\left(1+x+\right.}\right.\\
&= -\left(\frac{1}{3}\int \frac{\left(1+x^3\right)^{2/3}}{-1+x}\,dx\right)+\frac{1}{3}\int \frac{(2+x)\left(1+x^3\right)^{2/3}}{1+x+x^2}\,dx+3\int \frac{\left(1+x^3\right)^{2/3}}{x^6}\,dx+\\
&= -\frac{\left(1+x^3\right)^{2/3}}{2x^2}-\frac{\left(1+x^3\right)^{5/3}}{8x^8}-\frac{3\left(1+x^3\right)^{5/3}}{5x^5}-\frac{1}{3}\int \frac{\left(1+x^3\right)^{2/3}}{-1+x}\,dx+\frac{1}{3}\int \left(\left(1\right.\right.\\
&= -\frac{\left(1+x^3\right)^{2/3}}{2x^2}-\frac{\left(1+x^3\right)^{5/3}}{8x^8}-\frac{21\left(1+x^3\right)^{5/3}}{40x^5}+\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(-x\right.
\end{aligned}$$

Mathematica [A] time = 0.24, size = 131, normalized size = 0.92

$$\frac{-2\log\left(1-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3+1}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)+\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3+1}}+\frac{2^{2/3}x^2}{\left(x^3+1\right)^{2/3}}+1\right)}{3\sqrt[3]{2}}-\frac{\left(x^3+1\right)^{2/3}\left(41x^6+26x^3+5\right)}{40x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)^(2/3)*(-1 - 2*x^3 + 2*x^6))/(x^9*(-1 + x^3)),x]
```

```
[Out] -1/40*((1 + x^3)^(2/3)*(5 + 26*x^3 + 41*x^6))/x^8 + (2*Sqrt[3]*ArcTan[(1 +
(2*2^(1/3)*x)/(1 + x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (2^(1/3)*x)/(1 + x^3)^(
```

```
1/3)] + Log[1 + (2^(2/3)*x^2)/(1 + x^3)^(2/3) + (2^(1/3)*x)/(1 + x^3)^(1/3)
])/(3*2^(1/3))
```

IntegrateAlgebraic [A] time = 0.37, size = 143, normalized size = 1.00

$$-\frac{1}{3}2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^3+1}-2x\right)+\frac{2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}+\frac{\log\left(2^{2/3}\sqrt[3]{x^3+1}\,x+\sqrt[3]{2}\left(x^3+1\right)^{2/3}+2x^2\right)}{3\sqrt[3]{2}}+\frac{\left(x^3+1\right)^{2/3}\left(-41x^6-26x^3-5\right)}{40x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)^(2/3)*(-1 - 2*x^3 + 2*x^6))/(x^9*(-1 + x^3)),
x]
```

```
[Out] ((1 + x^3)^(2/3)*(-5 - 26*x^3 - 41*x^6))/(40*x^8) + (2^(2/3)*ArcTan[(Sqrt[3
]*x)/(x + 2^(2/3)*(1 + x^3)^(1/3))])/Sqrt[3] - (2^(2/3)*Log[-2*x + 2^(2/3)*
(1 + x^3)^(1/3)])/3 + Log[2*x^2 + 2^(2/3)*x*(1 + x^3)^(1/3) + 2^(1/3)*(1 +
x^3)^(2/3)]/(3*2^(1/3))
```

fricas [B] time = 2.53, size = 271, normalized size = 1.90

$$-\frac{40\sqrt{3}\left(-4\right)^{\frac{1}{3}}x^8\arctan\left(\frac{3\sqrt{3}\left(-4\right)^{\frac{2}{3}}\left(5x^7-4x^4-x\right)\left(x^3+1\right)^{\frac{2}{3}}+6\sqrt{3}\left(-4\right)^{\frac{1}{3}}\left(19x^8+16x^5+x^2\right)\left(x^3+1\right)^{\frac{1}{3}}-\sqrt{3}\left(71x^9+111x^6+33x^3+1\right)}{3\left(109x^9+105x^6+3x^3-1\right)}\right)-40\left(-4\right)^{\frac{1}{3}}x^8\log\left(\frac{3\left(-4\right)^{\frac{2}{3}}\left(x^3+1\right)^{\frac{1}{3}}x^2-6\left(x^3+1\right)^{\frac{2}{3}}x+\left(-4\right)^{\frac{1}{3}}\left(x^3-1\right)}{x^3-1}\right)+20\left(-4\right)^{\frac{1}{3}}x^8\log\left(-\frac{6\left(-4\right)^{\frac{1}{3}}\left(5x^4+x\right)\left(x^3+1\right)^{\frac{2}{3}}-\left(-4\right)^{\frac{2}{3}}\left(19x^6+16x^3+1\right)-24\left(2x^5+x^2\right)\left(x^3+1\right)^{\frac{1}{3}}}{x^6-2x^3+1}\right)+9\left(41x^6+26x^3+5\right)\left(x^3+1\right)^{\frac{2}{3}}}{360x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6-2*x^3-1)/x^9/(x^3-1),x, algorithm="fricas")
```

```
[Out] -1/360*(40*sqrt(3)*(-4)^(1/3)*x^8*arctan(1/3*(3*sqrt(3)*(-4)^(2/3)*(5*x^7 -
 4*x^4 - x)*(x^3 + 1)^(2/3) + 6*sqrt(3)*(-4)^(1/3)*(19*x^8 + 16*x^5 + x^2)*
(x^3 + 1)^(1/3) - sqrt(3)*(71*x^9 + 111*x^6 + 33*x^3 + 1))/(109*x^9 + 105*x
^6 + 3*x^3 - 1)) - 40*(-4)^(1/3)*x^8*log((3*(-4)^(2/3)*(x^3 + 1)^(1/3)*x^2
- 6*(x^3 + 1)^(2/3)*x + (-4)^(1/3)*(x^3 - 1))/(x^3 - 1)) + 20*(-4)^(1/3)*x^
8*log(-(6*(-4)^(1/3)*(5*x^4 + x)*(x^3 + 1)^(2/3) - (-4)^(2/3)*(19*x^6 + 16*
x^3 + 1) - 24*(2*x^5 + x^2)*(x^3 + 1)^(1/3))/(x^6 - 2*x^3 + 1)) + 9*(41*x^6
 + 26*x^3 + 5)*(x^3 + 1)^(2/3))/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^6-2x^3-1\right)\left(x^3+1\right)^{\frac{2}{3}}}{\left(x^3-1\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6-2*x^3-1)/x^9/(x^3-1),x, algorithm="giac")
```

```
[Out] integrate((2*x^6 - 2*x^3 - 1)*(x^3 + 1)^(2/3)/((x^3 - 1)*x^9), x)
```

maple [C] time = 2.45, size = 931, normalized size = 6.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)*(2*x^6-2*x^3-1)/x^9/(x^3-1),x)
```

```
[Out] -1/40*(41*x^9+67*x^6+31*x^3+5)/x^8/(x^3+1)^(1/3)+2*RootOf(RootOf(_Z^3+4)^2+
6*_Z*RootOf(_Z^3+4)+36*_Z^2)*ln((-18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z
^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-9*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf
(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+12*(x^3+1)^(2/3)*RootOf(RootOf(_Z^3+
4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x-24*(x^3+1)^(1/3)*RootO
f(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)*x^2-5*(x^3+1
)^(1/3)*RootOf(_Z^3+4)^2*x^2+18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)
+36*_Z^2)*x^3+9*RootOf(_Z^3+4)*x^3-10*x*(x^3+1)^(2/3)+6*RootOf(RootOf(_Z^3+
```

```
4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)+3*RootOf(_Z^3+4))/(-1+x)/(x^2+x+1))-1/3*l
n(-(18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4
)^2*x^3-6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+
4)^3*x^3+12*(x^3+1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z
^2)*RootOf(_Z^3+4)^2*x-24*(x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf
(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)*x^2+(x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x^2+6*R
ootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3-2*RootOf(_Z^3+4)*x^
3+2*x*(x^3+1)^(2/3)+6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)-
2*RootOf(_Z^3+4))/(-1+x)/(x^2+x+1))*RootOf(_Z^3+4)-2*ln(-(18*RootOf(RootOf(
_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-6*RootOf(Root
Of(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+12*(x^3+1)^(
2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*
x-24*(x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*Roo
tOf(_Z^3+4)*x^2+(x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x^2+6*RootOf(RootOf(_Z^3+4)^
2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3-2*RootOf(_Z^3+4)*x^3+2*x*(x^3+1)^(2/3)+6
*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)-2*RootOf(_Z^3+4))/(-1
+x)/(x^2+x+1))*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{6}-2\,x^{3}-1\right)\left(x^{3}+1\right)^{\frac{2}{3}}}{\left(x^{3}-1\right)x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6-2*x^3-1)/x^9/(x^3-1),x, algorithm="maxima")
[Out] integrate((2*x^6 - 2*x^3 - 1)*(x^3 + 1)^(2/3)/((x^3 - 1)*x^9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(x^{3}+1\right)^{2/3}\,\left(-2\,x^{6}+2\,x^{3}+1\right)}{x^{9}\,\left(x^{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3 + 1)^(2/3)*(2*x^3 - 2*x^6 + 1))/(x^9*(x^3 - 1)),x)
[Out] -int(((x^3 + 1)^(2/3)*(2*x^3 - 2*x^6 + 1))/(x^9*(x^3 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x+1\right)\left(x^{2}-x+1\right)\right)^{\frac{2}{3}}\left(2x^{6}-2x^{3}-1\right)}{x^{9}\left(x-1\right)\left(x^{2}+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)*(2*x**6-2*x**3-1)/x**9/(x**3-1),x)
[Out] Integral(((x + 1)*(x**2 - x + 1))**(2/3)*(2*x**6 - 2*x**3 - 1)/(x**9*(x - 1
)*(x**2 + x + 1)), x)
```

3.1683 $\int \frac{x-\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=143

$$\frac{2x}{3\sqrt{\sqrt{x^2+1}+1}}+\sqrt{x^2+1}\left(\frac{2}{3}\sqrt{\sqrt{x^2+1}+1}-\frac{2x}{3\sqrt{\sqrt{x^2+1}+1}}\right)-\frac{4}{3}\sqrt{\sqrt{x^2+1}+1}-2\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\sqrt{x^2+}}}\right.$$

Rubi [F] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x-\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x - Sqrt[1 + x^2])/Sqrt[1 + Sqrt[1 + x^2]],x]
```

```
[Out] -2*Sqrt[1 + Sqrt[1 + x^2]] + (2*(1 + Sqrt[1 + x^2])^(3/2))/3 - Defer[Int][S
qrt[1 + x^2]/Sqrt[1 + Sqrt[1 + x^2]], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x-\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\,dx &= \int \left(\frac{x}{\sqrt{1+\sqrt{1+x^2}}}-\frac{\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\right)dx\\
&= \int \frac{x}{\sqrt{1+\sqrt{1+x^2}}}\,dx-\int \frac{\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+\sqrt{x}}}\,dx,x,1+x^2\right)-\int \frac{\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\,dx\\
&= -\int \frac{\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\,dx+\text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx,x,\sqrt{1+x^2}\right)\\
&= -\int \frac{\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\,dx+\text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right)dx,x,\sqrt{1+x^2}\right)\\
&= -2\sqrt{1+\sqrt{1+x^2}}+\frac{2}{3}\left(1+\sqrt{1+x^2}\right)^{3/2}-\int \frac{\sqrt{1+x^2}}{\sqrt{1+\sqrt{1+x^2}}}\,dx
\end{aligned}$$

Mathematica [C] time = 0.38, size = 126, normalized size = 0.88

$$\frac{\sqrt{\sqrt{x^2+1}+1}\left(-6\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}-\frac{\sqrt{x^2+1}}{2}\right)-4x^2+4\sqrt{x^2+1}\,x+8\sqrt{x^2+1}-3\sqrt{2}\sqrt{\sqrt{x^2+1}-1}\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}-1}}{\sqrt{2}}\right)-8x-2\right)}{6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 + x^2])/Sqrt[1 + Sqrt[1 + x^2]],x]
```

```
[Out] (Sqrt[1 + Sqrt[1 + x^2]]*(-2 - 8*x - 4*x^2 + 8*Sqrt[1 + x^2] + 4*x*Sqrt[1 +
 x^2] - 3*Sqrt[2]*Sqrt[-1 + Sqrt[1 + x^2]]*ArcTan[Sqrt[-1 + Sqrt[1 + x^2]]/
Sqrt[2]] - 6*Hypergeometric2F1[-1/2, 1, 1/2, 1/2 - Sqrt[1 + x^2]/2]))/(6*x)
```

IntegrateAlgebraic [A] time = 0.43, size = 143, normalized size = 1.00

$$\frac{2x}{3\sqrt{\sqrt{x^2+1}+1}}+\sqrt{x^2+1}\left(\frac{2}{3}\sqrt{\sqrt{x^2+1}+1}-\frac{2x}{3\sqrt{\sqrt{x^2+1}+1}}\right)-\frac{4}{3}\sqrt{\sqrt{x^2+1}+1}-2\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}-\frac{\sqrt{\sqrt{x^2+1}+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x - Sqrt[1 + x^2])/Sqrt[1 + Sqrt[1 + x^2]],x]
```

```
[Out] (2*x)/(3*Sqrt[1 + Sqrt[1 + x^2]]) - (4*Sqrt[1 + Sqrt[1 + x^2]])/3 + Sqrt[1
+ x^2]*((-2*x)/(3*Sqrt[1 + Sqrt[1 + x^2]]) + (2*Sqrt[1 + Sqrt[1 + x^2]])/3)
 - 2*Sqrt[2]*ArcTan[x/(Sqrt[2]*Sqrt[1 + Sqrt[1 + x^2]]) - Sqrt[1 + Sqrt[1 +
 x^2]]/Sqrt[2]]
```

fricas [A] time = 1.16, size = 64, normalized size = 0.45

$$\frac{3\sqrt{2}x\arctan\left(\frac{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}{x}\right)-2\left(x^2-\sqrt{x^2+1}(x+2)+2x+2\right)\sqrt{\sqrt{x^2+1}+1}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+1)^(1/2))/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*sqrt(2)*x*arctan(sqrt(2)*sqrt(sqrt(x^2 + 1) + 1)/x) - 2*(x^2 - sqrt(
x^2 + 1)*(x + 2) + 2*x + 2)*sqrt(sqrt(x^2 + 1) + 1))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x-\sqrt{x^2+1}}{\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+1)^(1/2))/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + 1))/sqrt(sqrt(x^2 + 1) + 1), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int\frac{x-\sqrt{x^2+1}}{\sqrt{1+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+1)^(1/2))/(1+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int((x-(x^2+1)^(1/2))/(1+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{3}\left(\sqrt{x^2+1}+1\right)^{\frac{3}{2}}-2\sqrt{\sqrt{x^2+1}+1}-\int\frac{\sqrt{x^2+1}}{\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+1)^(1/2))/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(sqrt(x^2 + 1) + 1)^(3/2) - 2*sqrt(sqrt(x^2 + 1) + 1) - integrate(sqrt(
x^2 + 1)/sqrt(sqrt(x^2 + 1) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x-\sqrt{x^2+1}}{\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (x^2 + 1)^(1/2))/((x^2 + 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int((x - (x^2 + 1)^(1/2))/((x^2 + 1)^(1/2) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{x^2+1}}{\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+1)**(1/2))/(1+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral((x - sqrt(x**2 + 1))/sqrt(sqrt(x**2 + 1) + 1), x)
```

3.1684 $\int \frac{(-4+x^3)(-2+x^3)(-1+x^3)^{2/3}}{x^9(-2+3x^3)}\,dx$

Optimal. Leaf size=144

$$\frac{5\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}+x\right)}{3\,2^{2/3}}+\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}-x}\right)}{2^{2/3}\sqrt{3}}-\frac{5\log\left(-\sqrt[3]{2}\sqrt[3]{x^3-1}\,x+2^{2/3}\left(x^3-1\right)^{2/3}+x^2\right)}{6\,2^{2/3}}+\frac{\left(x^3-1\right)^{2/3}\left(16x^6\right.}{10x^8}$$

Rubi [C] time = 0.61, antiderivative size = 141, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {6725, 271, 264, 277, 239, 430, 429}

$$-\frac{15x\left(x^3-1\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,\frac{3x^3}{2}\right)}{2\left(1-x^3\right)^{2/3}}+\frac{5}{2}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{5\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\left(x^3-1\right)^{5/3}}{2x^8}-\frac{9\left(x^3-1\right)^{5/3}}{10x^5}+\frac{5\left(x^3-1\right)^{2/3}}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-4 + x^3)*(-2 + x^3)*(-1 + x^3)^(2/3))/(x^9*(-2 + 3*x^3)),x]
```

```
[Out] (5*(-1 + x^3)^(2/3))/(2*x^2) - (-1 + x^3)^(5/3)/(2*x^8) - (9*(-1 + x^3)^(5/
3))/(10*x^5) - (15*x*(-1 + x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, x^3, (3*x
^3)/2])/(2*(1 - x^3)^(2/3)) - (5*ArcTan[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3
]])/Sqrt[3] + (5*Log[-x + (-1 + x^3)^(1/3)])/2
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(-4+x^3)(-2+x^3)(-1+x^3)^{2/3}}{x^9(-2+3x^3)}\,dx = \int \left(-\frac{4(-1+x^3)^{2/3}}{x^9} - \frac{3(-1+x^3)^{2/3}}{x^6} - \frac{5(-1+x^3)^{2/3}}{x^3} + \frac{15(-1+x^3}{-2+3x^3}\right.$$

$$= -\left(3\int \frac{(-1+x^3)^{2/3}}{x^6}\,dx\right) - 4\int \frac{(-1+x^3)^{2/3}}{x^9}\,dx - 5\int \frac{(-1+x^3)^{2/3}}{x^3}\,d$$

$$= \frac{5(-1+x^3)^{2/3}}{2x^2} - \frac{(-1+x^3)^{5/3}}{2x^8} - \frac{3(-1+x^3)^{5/3}}{5x^5} - \frac{3}{2}\int \frac{(-1+x^3)^{2/3}}{x^6}\,d$$

$$= \frac{5(-1+x^3)^{2/3}}{2x^2} - \frac{(-1+x^3)^{5/3}}{2x^8} - \frac{9(-1+x^3)^{5/3}}{10x^5} - \frac{15x(-1+x^3)^{2/3}F_1}{2(1-}$$

Mathematica [A] time = 0.19, size = 140, normalized size = 0.97

$$\frac{5\left(2\log\left(\frac{2^{2/3}x}{\sqrt[3]{1-x^3}}+2\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2^{2/3}x}{\sqrt[3]{1-x^3}}-1}{\sqrt{3}}\right)-\log\left(-\frac{2^{2/3}x}{\sqrt[3]{1-x^3}}+\frac{\sqrt[3]{2}x^2}{(1-x^3)^{2/3}}+2\right)\right)}{6\,2^{2/3}}+\frac{(x^3-1)^{2/3}(16x^6+4x^3+5)}{10x^8}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-4 + x^3)*(-2 + x^3)*(-1 + x^3)^(2/3))/(x^9*(-2 + 3*x^3)),x]
```

```
[Out] ((-1 + x^3)^(2/3)*(5 + 4*x^3 + 16*x^6))/(10*x^8) + (5*(2*Sqrt[3]*ArcTan[(-1
 + (2^(2/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - Log[2 + (2^(1/3)*x^2)/(1 - x^3)^
(2/3) - (2^(2/3)*x)/(1 - x^3)^(1/3)] + 2*Log[2 + (2^(2/3)*x)/(1 - x^3)^(1/3
)]))/(6*2^(2/3))
```

IntegrateAlgebraic [A] time = 0.36, size = 144, normalized size = 1.00

$$\frac{5\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}+x\right)}{3\,2^{2/3}}+\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}-x}\right)}{2^{2/3}\sqrt{3}}-\frac{5\log\left(-\sqrt[3]{2}\sqrt[3]{x^3-1}\,x+2^{2/3}(x^3-1)^{2/3}+x^2\right)}{6\,2^{2/3}}+\frac{(x^3-1)^{2/3}(16x^6+4x^3+5)}{10x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + x^3)*(-2 + x^3)*(-1 + x^3)^(2/3))/(x^9*(-2 + 3*x^
3)),x]
```

```
[Out] ((-1 + x^3)^(2/3)*(5 + 4*x^3 + 16*x^6))/(10*x^8) + (5*ArcTan[(Sqrt[3]*x)/(-
x + 2*2^(1/3)*(-1 + x^3)^(1/3))])/(2^(2/3)*Sqrt[3]) + (5*Log[x + 2^(1/3)*(-
```

```
1 + x^3)^(1/3)])/(3*2^(2/3)) - (5*Log[x^2 - 2^(1/3)*x*(-1 + x^3)^(1/3) + 2^
(2/3)*(-1 + x^3)^(2/3)])/(6*2^(2/3))
```

fricas [B] time = 2.80, size = 267, normalized size = 1.85

$$\frac{100 \cdot 4^{\frac{1}{6}} \sqrt{3} x^{8} \arctan\left(\frac{4^{\frac{1}{6}} \left(12 \cdot 4^{\frac{2}{3}} \sqrt{3} \left(3 x^{4} - 2 x\right) \left(x^{3} - 1\right)^{\frac{2}{3}} - 4^{\frac{1}{3}} \sqrt{3} \left(27 x^{9} - 72 x^{6} + 36 x^{3} + 8\right) - 12 \sqrt{3} \left(9 x^{8} - 6 x^{5} - 4 x^{2}\right) \left(x^{3} - 1\right)^{\frac{1}{3}}\right)}{6 \left(27 x^{9} - 36 x^{3} + 8\right)}\right) + 50 \cdot 4^{\frac{2}{3}} x^{8} \log\left(\frac{6 \cdot 4^{\frac{1}{3}} \left(x^{3} - 1\right)^{\frac{1}{3}} x^{2} + 4^{\frac{2}{3}} \left(3 x^{3} - 2\right) + 12 \left(x^{3} - 1\right)^{\frac{2}{3}} x}{3 x^{3} - 2}\right) - 25 \cdot 4^{\frac{2}{3}} x^{8} \log\left(\frac{6 \cdot 4^{\frac{2}{3}} \left(x^{3} - 1\right)^{\frac{2}{3}} x - 4^{\frac{1}{3}} \left(9 x^{6} - 6 x^{3} - 4\right) + 6 \left(3 x^{5} - 4 x^{2}\right) \left(x^{3} - 1\right)^{\frac{1}{3}}}{9 x^{6} - 12 x^{3} + 4}\right) + 36 \left(16 x^{6} + 4 x^{3} + 5\right) \left(x^{3} - 1\right)^{\frac{2}{3}}}{360 x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^3-2)*(x^3-1)^(2/3)/x^9/(3*x^3-2),x, algorithm="fricas"
)
```

```
[Out] 1/360*(100*4^(1/6)*sqrt(3)*x^8*arctan(1/6*4^(1/6)*(12*4^(2/3)*sqrt(3)*(3*x^
4 - 2*x)*(x^3 - 1)^(2/3) - 4^(1/3)*sqrt(3)*(27*x^9 - 72*x^6 + 36*x^3 + 8) -
 12*sqrt(3)*(9*x^8 - 6*x^5 - 4*x^2)*(x^3 - 1)^(1/3))/(27*x^9 - 36*x^3 + 8))
 + 50*4^(2/3)*x^8*log((6*4^(1/3)*(x^3 - 1)^(1/3)*x^2 + 4^(2/3)*(3*x^3 - 2)
+ 12*(x^3 - 1)^(2/3)*x)/(3*x^3 - 2)) - 25*4^(2/3)*x^8*log((6*4^(2/3)*(x^3 -
 1)^(2/3)*x - 4^(1/3)*(9*x^6 - 6*x^3 - 4) + 6*(3*x^5 - 4*x^2)*(x^3 - 1)^(1/
3))/(9*x^6 - 12*x^3 + 4)) + 36*(16*x^6 + 4*x^3 + 5)*(x^3 - 1)^(2/3))/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3} - 1\right)^{\frac{2}{3}} \left(x^{3} - 2\right) \left(x^{3} - 4\right)}{\left(3 x^{3} - 2\right) x^{9}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^3-2)*(x^3-1)^(2/3)/x^9/(3*x^3-2),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 1)^(2/3)*(x^3 - 2)*(x^3 - 4)/((3*x^3 - 2)*x^9), x)
```

maple [C] time = 2.50, size = 564, normalized size = 3.92

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-4)*(x^3-2)*(x^3-1)^(2/3)/x^9/(3*x^3-2),x)
```

```
[Out] 1/10*(16*x^9-12*x^6+x^3-5)/x^8/(x^3-1)^(1/3)+5/6*RootOf(_Z^3-2)*ln(-(18*Roo
tOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-3*
RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^3-1
8*(x^3-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf
(_Z^3-2)^2*x-18*(x^3-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+3
6*_Z^2)*RootOf(_Z^3-2)*x^2-3*RootOf(_Z^3-2)^2*(x^3-1)^(1/3)*x^2-6*RootOf(Ro
otOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+RootOf(_Z^3-2)*x^3+12*RootO
f(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-2*RootOf(_Z^3-2))/(3*x^3-2)
)+5*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*ln(-(18*RootOf(Roo
tOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-3*RootOf(
RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^3+18*(x^3-
1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2
)^2*x+3*RootOf(_Z^3-2)^2*(x^3-1)^(1/3)*x^2+12*RootOf(RootOf(_Z^3-2)^2+6*_Z*
RootOf(_Z^3-2)+36*_Z^2)*x^3-2*RootOf(_Z^3-2)*x^3+6*x*(x^3-1)^(2/3)-12*RootO
f(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)+2*RootOf(_Z^3-2))/(3*x^3-2)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3} - 1\right)^{\frac{2}{3}} \left(x^{3} - 2\right) \left(x^{3} - 4\right)}{\left(3 x^{3} - 2\right) x^{9}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^3-2)*(x^3-1)^(2/3)/x^9/(3*x^3-2),x, algorithm="maxima"
)
```

```
[Out] integrate((x^3 - 1)^(2/3)*(x^3 - 2)*(x^3 - 4)/((3*x^3 - 2)*x^9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3-1\right)^{2/3}\,\left(x^3-2\right)\,\left(x^3-4\right)}{x^9\,\left(3\,x^3-2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(x^3 - 2)*(x^3 - 4))/(x^9*(3*x^3 - 2)),x)
```

```
[Out] int(((x^3 - 1)^(2/3)*(x^3 - 2)*(x^3 - 4))/(x^9*(3*x^3 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x - 1\right) \left(x^{2} + x + 1\right)\right)^{\frac{2}{3}} \left(x^{3} - 4\right) \left(x^{3} - 2\right)}{x^{9} \left(3 x^{3} - 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-4)*(x**3-2)*(x**3-1)**(2/3)/x**9/(3*x**3-2),x)
```

```
[Out] Integral(((x - 1)*(x**2 + x + 1))**(2/3)*(x**3 - 4)*(x**3 - 2)/(x**9*(3*x**
3 - 2)), x)
```

3.1685 $\int \frac{1}{(-1+x)\left(-2x^2-3x^3+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=144

$$\frac{\sqrt{x^4-3x^3-2x^2}\,\left(-453x^3+1555x^2+238x-136\right)}{544x^3\left(x^2-3x-2\right)}+\frac{1}{4}\tan^{-1}\left(\frac{\frac{x^2}{2}-\frac{1}{2}\sqrt{x^4-3x^3-2x^2}-\frac{x}{2}}{x}\right)-\frac{119\tan^{-1}\left(\frac{\frac{x^2}{\sqrt{2}}-\frac{\sqrt{x^4-3x}}{\sqrt{}}}{x}\right)}{32\sqrt{2}}$$

Rubi [B] time = 0.23, antiderivative size = 303, normalized size of antiderivative = 2.10, number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2056, 960, 740, 12, 724, 204, 834, 806}

$$\frac{x(13-3x)}{17\sqrt{x^4-3x^3-2x^2}}+\frac{13-3x}{17x\sqrt{x^4-3x^3-2x^2}}+\frac{13-3x}{17\sqrt{x^4-3x^3-2x^2}}-\frac{(10-x)x}{34\sqrt{x^4-3x^3-2x^2}}-\frac{69\left(-x^2+3x+2\right)}{136x\sqrt{x^4-3x^3-2x^2}}+\frac{373\left(-x^2+3x+2\right)}{544\sqrt{x^4-3x^3-2x^2}}+\frac{x\sqrt{x^2-3x-2}\tan^{-1}\left(\frac{x+7}{4\sqrt{x^2-3x-2}}\right)}{8\sqrt{x^4-3x^3-2x^2}}-\frac{119x\sqrt{x^2-3x-2}\tan^{-1}\left(\frac{3x+4}{2\sqrt{2}\sqrt{x^2-3x-2}}\right)}{64\sqrt{2}\sqrt{x^4-3x^3-2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)*(-2*x^2 - 3*x^3 + x^4)^(3/2)),x]
```

```
[Out] (13 - 3*x)/(17*Sqrt[-2*x^2 - 3*x^3 + x^4]) + (13 - 3*x)/(17*x*Sqrt[-2*x^2 -
 3*x^3 + x^4]) + ((13 - 3*x)*x)/(17*Sqrt[-2*x^2 - 3*x^3 + x^4]) - ((10 - x)
*x)/(34*Sqrt[-2*x^2 - 3*x^3 + x^4]) + (373*(2 + 3*x - x^2))/(544*Sqrt[-2*x^
2 - 3*x^3 + x^4]) - (69*(2 + 3*x - x^2))/(136*x*Sqrt[-2*x^2 - 3*x^3 + x^4])
 + (x*Sqrt[-2 - 3*x + x^2]*ArcTan[(7 + x)/(4*Sqrt[-2 - 3*x + x^2])])/(8*Sqr
t[-2*x^2 - 3*x^3 + x^4]) - (119*x*Sqrt[-2 - 3*x + x^2]*ArcTan[(4 + 3*x)/(2*
Sqrt[2]*Sqrt[-2 - 3*x + x^2])])/(64*Sqrt[2]*Sqrt[-2*x^2 - 3*x^3 + x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{(-1+x)\left(-2x^2-3x^3+x^4\right)^{3/2}}\,dx = \frac{\left(x\sqrt{-2-3x+x^2}\right)\int \frac{1}{(-1+x)x^3\left(-2-3x+x^2\right)^{3/2}}\,dx}{\sqrt{-2x^2-3x^3+x^4}}$$

$$= \frac{\left(x\sqrt{-2-3x+x^2}\right)\int \left(\frac{1}{(-1+x)\left(-2-3x+x^2\right)^{3/2}} - \frac{1}{x^3\left(-2-3x+x^2\right)^{3/2}} - \frac{1}{x^2\left(-2-3x+x^2\right)^{3/2}} - \right.}{\sqrt{-2x^2-3x^3+x^4}}$$

$$= \frac{\left(x\sqrt{-2-3x+x^2}\right)\int \frac{1}{(-1+x)\left(-2-3x+x^2\right)^{3/2}}\,dx}{\sqrt{-2x^2-3x^3+x^4}} - \frac{\left(x\sqrt{-2-3x+x^2}\right)\int \frac{1}{x^3(-2-3x+}}{\sqrt{-2x^2-3x^3+x^4}}$$

$$= \frac{13-3x}{17\sqrt{-2x^2-3x^3+x^4}} + \frac{13-3x}{17x\sqrt{-2x^2-3x^3+x^4}} + \frac{(13-3x)x}{17\sqrt{-2x^2-3x^3+x^4}} -$$

$$= \frac{13-3x}{17\sqrt{-2x^2-3x^3+x^4}} + \frac{13-3x}{17x\sqrt{-2x^2-3x^3+x^4}} + \frac{(13-3x)x}{17\sqrt{-2x^2-3x^3+x^4}} -$$

$$= \frac{13-3x}{17\sqrt{-2x^2-3x^3+x^4}} + \frac{13-3x}{17x\sqrt{-2x^2-3x^3+x^4}} + \frac{(13-3x)x}{17\sqrt{-2x^2-3x^3+x^4}} -$$

$$= \frac{13-3x}{17\sqrt{-2x^2-3x^3+x^4}} + \frac{13-3x}{17x\sqrt{-2x^2-3x^3+x^4}} + \frac{(13-3x)x}{17\sqrt{-2x^2-3x^3+x^4}} -$$

$$= \frac{13-3x}{17\sqrt{-2x^2-3x^3+x^4}} + \frac{13-3x}{17x\sqrt{-2x^2-3x^3+x^4}} + \frac{(13-3x)x}{17\sqrt{-2x^2-3x^3+x^4}} -$$

Mathematica [A] time = 0.07, size = 126, normalized size = 0.88

$$\frac{-1812x^3+6220x^2+2023\sqrt{2}\sqrt{x^2-3x-2}\,x^2\tan^{-1}\left(\frac{-3x-4}{2\sqrt{2}\sqrt{x^2-3x-2}}\right)-272\sqrt{x^2-3x-2}\,x^2\tan^{-1}\left(\frac{-x-7}{4\sqrt{x^2-3x-2}}\right)+952x-544}{2176x\sqrt{x^2\left(x^2-3x-2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)*(-2*x^2 - 3*x^3 + x^4)^(3/2)),x]
```

```
[Out] (-544 + 952*x + 6220*x^2 - 1812*x^3 + 2023*Sqrt[2]*x^2*Sqrt[-2 - 3*x + x^2]
*ArcTan[(-4 - 3*x)/(2*Sqrt[2]*Sqrt[-2 - 3*x + x^2])] - 272*x^2*Sqrt[-2 - 3*
x + x^2]*ArcTan[(-7 - x)/(4*Sqrt[-2 - 3*x + x^2])])/(2176*x*Sqrt[x^2*(-2 -
3*x + x^2)])
```

IntegrateAlgebraic [A] time = 1.36, size = 144, normalized size = 1.00

$$\frac{\sqrt{x^4-3x^3-2x^2}\left(-453x^3+1555x^2+238x-136\right)}{544x^3\left(x^2-3x-2\right)}+\frac{1}{4}\tan^{-1}\left(\frac{\frac{x^2}{2}-\frac{1}{2}\sqrt{x^4-3x^3-2x^2}-\frac{x}{2}}{x}\right)-\frac{119\tan^{-1}\left(\frac{\frac{x^2}{\sqrt{2}}-\frac{\sqrt{x^4-3x^3-2x^2}}{\sqrt{2}}}{x}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x)*(-2*x^2 - 3*x^3 + x^4)^(3/2)),x]
```

```
[Out] ((-136 + 238*x + 1555*x^2 - 453*x^3)*Sqrt[-2*x^2 - 3*x^3 + x^4])/(544*x^3*(
-2 - 3*x + x^2)) + ArcTan[(-1/2*x + x^2/2 - Sqrt[-2*x^2 - 3*x^3 + x^4]/2)/x
]/4 - (119*ArcTan[(x^2/Sqrt[2] - Sqrt[-2*x^2 - 3*x^3 + x^4]/Sqrt[2])/x])/(3
2*Sqrt[2])
```

fricas [A] time = 0.41, size = 168, normalized size = 1.17

$$-\frac{906\,x^{5}-2718\,x^{4}-1812\,x^{3}-2023\,\sqrt{2}\left(x^{5}-3\,x^{4}-2\,x^{3}\right)\arctan\left(-\frac{\sqrt{2}\,x^{2}-\sqrt{2}\,\sqrt{x^{4}-3\,x^{3}-2\,x^{2}}}{2\,x}\right)+272\left(x^{5}-3\,x^{4}-2\,x^{3}\right)\arctan\left(-\frac{x^{2}-x-\sqrt{x^{4}-3\,x^{3}-2\,x^{2}}}{2\,x}\right)+2\,\sqrt{x^{4}-3\,x^{3}-2\,x^{2}}\left(453\,x^{3}-1555\,x^{2}-238\,x+136\right)}{1088\left(x^{5}-3\,x^{4}-2\,x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^4-3*x^3-2*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1088*(906*x^5 - 2718*x^4 - 1812*x^3 - 2023*sqrt(2)*(x^5 - 3*x^4 - 2*x^3)
*arctan(-1/2*(sqrt(2)*x^2 - sqrt(2)*sqrt(x^4 - 3*x^3 - 2*x^2))/x) + 272*(x^
5 - 3*x^4 - 2*x^3)*arctan(-1/2*(x^2 - x - sqrt(x^4 - 3*x^3 - 2*x^2))/x) + 2
*sqrt(x^4 - 3*x^3 - 2*x^2)*(453*x^3 - 1555*x^2 - 238*x + 136))/(x^5 - 3*x^4
 - 2*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^4-3*x^3-2*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 113, normalized size = 0.78

$$-\frac{x\left(x^{2}-3x-2\right)\left(2023\sqrt{2}\,\arctan\left(\frac{(4+3x)\sqrt{2}}{4\sqrt{x^{2}-3x-2}}\right)x^{2}\sqrt{x^{2}-3x-2}\,-272\arctan\left(\frac{7+x}{4\sqrt{x^{2}-3x-2}}\right)x^{2}\sqrt{x^{2}-3x-2}\,+1812x^{3}-6220x^{2}-952x+544\right)}{2176\left(x^{4}-3x^{3}-2x^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)/(x^4-3*x^3-2*x^2)^(3/2),x)
```

```
[Out] -1/2176*x*(x^2-3*x-2)*(2023*2^(1/2)*arctan(1/4*(4+3*x)*2^(1/2)/(x^2-3*x-2)^
(1/2))*x^2*(x^2-3*x-2)^(1/2)-272*arctan(1/4*(7+x)/(x^2-3*x-2)^(1/2))*x^2*(x
^2-3*x-2)^(1/2)+1812*x^3-6220*x^2-952*x+544)/(x^4-3*x^3-2*x^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{4}-3\,x^{3}-2\,x^{2}\right)^{\frac{3}{2}}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x^4-3*x^3-2*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 - 3*x^3 - 2*x^2)^(3/2)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x-1)\,\left(x^{4}-3\,x^{3}-2\,x^{2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)*(x^4 - 3*x^3 - 2*x^2)^(3/2)),x)
```

```
[Out] int(1/((x - 1)*(x^4 - 3*x^3 - 2*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2 \left(x^2 - 3x - 2\right)\right)^{\frac{3}{2}} (x - 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(x**4-3*x**3-2*x**2)**(3/2),x)
```

```
[Out] Integral(1/((x**2*(x**2 - 3*x - 2))**(3/2)*(x - 1)), x)
```

3.1686 $\int \frac{1}{(b+2ax^3)\sqrt[4]{bx+ax^4}}\,dx$

Optimal. Leaf size=144

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x\sqrt[4]{ax^4+bx}}{\sqrt{ax^4+bx}-\sqrt{a}\,x^2}\right)}{3\sqrt[4]{a}\,b}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^4+bx}}{\sqrt{2}\sqrt[4]{a}}+\frac{\sqrt[4]{a}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^4+bx}}\right)}{3\sqrt[4]{a}\,b}$$

Rubi [B] time = 0.40, antiderivative size = 351, normalized size of antiderivative = 2.44, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2056, 466, 465, 377, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{x}\sqrt[4]{ax^3+b}\log\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3+b}}+1\right)}{3\sqrt{2}\sqrt[4]{a}\,b\sqrt[4]{ax^4+bx}}+\frac{\sqrt[4]{x}\sqrt[4]{ax^3+b}\log\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3+b}}+1\right)}{3\sqrt{2}\sqrt[4]{a}\,b\sqrt[4]{ax^4+bx}}-\frac{\sqrt{2}\sqrt[4]{x}\sqrt[4]{ax^3+b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{a}\,b\sqrt[4]{ax^4+bx}}+\frac{\sqrt{2}\sqrt[4]{x}\sqrt[4]{ax^3+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3+b}}+1\right)}{3\sqrt[4]{a}\,b\sqrt[4]{ax^4+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b + 2*a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] -1/3*(Sqrt[2]*x^(1/4)*(b + a*x^3)^(1/4)*ArcTan[1 - (Sqrt[2]*a^(1/4)*x^(3/4)
)/(b + a*x^3)^(1/4)])/(a^(1/4)*b*(b*x + a*x^4)^(1/4)) + (Sqrt[2]*x^(1/4)*(b
 + a*x^3)^(1/4)*ArcTan[1 + (Sqrt[2]*a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)])/(3
*a^(1/4)*b*(b*x + a*x^4)^(1/4)) - (x^(1/4)*(b + a*x^3)^(1/4)*Log[1 + (Sqrt[
a]*x^(3/2))/Sqrt[b + a*x^3] - (Sqrt[2]*a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)])
/(3*Sqrt[2]*a^(1/4)*b*(b*x + a*x^4)^(1/4)) + (x^(1/4)*(b + a*x^3)^(1/4)*Log
[1 + (Sqrt[a]*x^(3/2))/Sqrt[b + a*x^3] + (Sqrt[2]*a^(1/4)*x^(3/4))/(b + a*x
^3)^(1/4)])/(3*Sqrt[2]*a^(1/4)*b*(b*x + a*x^4)^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
```

```
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{(b+2ax^3)\sqrt[4]{bx+ax^4}}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\int \frac{1}{\sqrt[4]{x}\sqrt[4]{b+ax^3}(b+2ax^3)}\,dx}{\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{b+ax^{12}}(b+2ax^{12})}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{b+ax^4}(b+2ax^4)}\,dx, x, x^{3/4}\right)}{3\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{b+abx^4}\,dx, x, \frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(2\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1-\sqrt{a}\,x^2}{b+abx^4}\,dx, x, \frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}} + \frac{\left(2\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int\right.}{3\sqrt[4]{bx+a}}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{a}}-\frac{\sqrt{2}\,x}{\sqrt[4]{a}}+x^2}\,dx, x, \frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt{a}\,b\sqrt[4]{bx+ax^4}} + \frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\right.}{3\sqrt{a}\,b}$$

$$= -\frac{\sqrt[4]{x}\sqrt[4]{b+ax^3}\log\left(1+\frac{\sqrt{a}\,x^{3/2}}{\sqrt{b+ax^3}}-\frac{\sqrt{2}\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt{2}\sqrt[4]{a}\,b\sqrt[4]{bx+ax^4}} + \frac{\sqrt[4]{x}\sqrt[4]{b+ax^3}\log\left(1+\frac{\sqrt{a}\,x^{3/2}}{\sqrt{b+ax^3}}+\right.}{3\sqrt{2}\sqrt[4]{a}\,b\sqrt[4]{bx+ax^4}}$$

$$= -\frac{\sqrt{2}\sqrt[4]{x}\sqrt[4]{b+ax^3}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{a}\,b\sqrt[4]{bx+ax^4}} + \frac{\sqrt{2}\sqrt[4]{x}\sqrt[4]{b+ax^3}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b+ax}}\right.}{3\sqrt[4]{a}\,b\sqrt[4]{bx+ax^4}}$$

Mathematica [C] time = 0.04, size = 75, normalized size = 0.52

$$\frac{4x\sqrt[4]{\frac{ax^3}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{ax^3}{2ax^3+b}\right)}{3b\sqrt[4]{x(ax^3+b)}\sqrt[4]{\frac{2ax^3}{b}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((b + 2*a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*x*(1 + (a*x^3)/b)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (a*x^3)/(b + 2*
a*x^3)])/(3*b*(x*(b + a*x^3))^(1/4)*(1 + (2*a*x^3)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.48, size = 144, normalized size = 1.00

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x\sqrt[4]{ax^4+bx}}{\sqrt{ax^4+bx}-\sqrt{a}\,x^2}\right)}{3\sqrt[4]{a}\,b} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^4+bx}}{\sqrt{2}\sqrt[4]{a}}+\frac{\sqrt[4]{a}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^4+bx}}\right)}{3\sqrt[4]{a}\,b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((b + 2*a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[2]*a^(1/4)*x*(b*x + a*x^4)^(1/4))/(-(Sqrt[a]*x^2) + S
qrt[b*x + a*x^4])])/(3*a^(1/4)*b) + (Sqrt[2]*ArcTanh[((a^(1/4)*x^2)/Sqrt[2]
```

```
 + Sqrt[b*x + a*x^4]/(Sqrt[2]*a^(1/4)))/(x*(b*x + a*x^4)^(1/4))])/(3*a^(1/4
)*b)
```

fricas [B] time = 125.46, size = 355, normalized size = 2.47

$$-\frac{2}{3}\left(-\frac{1}{ab^4}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(ab^4\sqrt{\frac{1}{b^2}}x\left(-\frac{1}{ab^4}\right)^{\frac{3}{4}}+ab^3x\left(-\frac{1}{ab^4}\right)^{\frac{3}{4}}\right)\left(ax^4+bx\right)^{\frac{3}{4}}-\left(ax^4+bx\right)^{\frac{1}{4}}\left(\left(ab^2x^3+b^3\right)\sqrt{\frac{1}{b^2}}\left(-\frac{1}{ab^4}\right)^{\frac{1}{4}}-\left(abx^3+b^2\right)\left(-\frac{1}{ab^4}\right)^{\frac{1}{4}}\right)}{2\left(ax^4+bx\right)}\right)+\frac{1}{6}\left(-\frac{1}{ab^4}\right)^{\frac{1}{4}}\log\left(-\frac{2\left(ax^4+bx\right)^{\frac{3}{4}}ab^2\left(-\frac{1}{ab^4}\right)^{\frac{3}{4}}+2\sqrt{ax^4+bx}\,abx\sqrt{-\frac{1}{ab^4}}+2\left(ax^4+bx\right)^{\frac{1}{4}}ax^2\left(-\frac{1}{ab^4}\right)^{\frac{1}{4}}-1}{2ax^3+b}\right)-\frac{1}{6}\left(-\frac{1}{ab^4}\right)^{\frac{1}{4}}\log\left(\frac{2\left(ax^4+bx\right)^{\frac{3}{4}}ab^2\left(-\frac{1}{ab^4}\right)^{\frac{3}{4}}-2\sqrt{ax^4+bx}\,abx\sqrt{-\frac{1}{ab^4}}+2\left(ax^4+bx\right)^{\frac{1}{4}}ax^2\left(-\frac{1}{ab^4}\right)^{\frac{1}{4}}+1}{2ax^3+b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a*x^3+b)/(a*x^4+b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] -2/3*(-1/(a*b^4))^(1/4)*arctan(1/2*((a*b^4*sqrt(b^(-2))*x*(-1/(a*b^4))^(3/4
) + a*b^3*x*(-1/(a*b^4))^(3/4))*(a*x^4 + b*x)^(3/4) - (a*x^4 + b*x)^(1/4)*(
(a*b^2*x^3 + b^3)*sqrt(b^(-2))*(-1/(a*b^4))^(1/4) - (a*b*x^3 + b^2)*(-1/(a*
b^4))^(1/4)))/(a*x^4 + b*x)) + 1/6*(-1/(a*b^4))^(1/4)*log(-(2*(a*x^4 + b*x)
^(3/4)*a*b^2*(-1/(a*b^4))^(3/4) + 2*sqrt(a*x^4 + b*x)*a*b*x*sqrt(-1/(a*b^4)
) + 2*(a*x^4 + b*x)^(1/4)*a*x^2*(-1/(a*b^4))^(1/4) - 1)/(2*a*x^3 + b)) - 1/
6*(-1/(a*b^4))^(1/4)*log((2*(a*x^4 + b*x)^(3/4)*a*b^2*(-1/(a*b^4))^(3/4) -
2*sqrt(a*x^4 + b*x)*a*b*x*sqrt(-1/(a*b^4)) + 2*(a*x^4 + b*x)^(1/4)*a*x^2*(-
1/(a*b^4))^(1/4) + 1)/(2*a*x^3 + b))
```

giac [A] time = 0.21, size = 162, normalized size = 1.12

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}+2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2a^{\frac{1}{4}}}\right)}{3a^{\frac{1}{4}}b}-\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}-2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2a^{\frac{1}{4}}}\right)}{3a^{\frac{1}{4}}b}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}a^{\frac{1}{4}}+\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)}{6a^{\frac{1}{4}}b}-\frac{\sqrt{2}\log\left(-\sqrt{2}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}a^{\frac{1}{4}}+\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)}{6a^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a*x^3+b)/(a*x^4+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4) + 2*(a + b/x^3)^(1/4))/a^(
1/4))/(a^(1/4)*b) - 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4) - 2*(a
 + b/x^3)^(1/4))/a^(1/4))/(a^(1/4)*b) + 1/6*sqrt(2)*log(sqrt(2)*(a + b/x^3)
^(1/4)*a^(1/4) + sqrt(a + b/x^3) + sqrt(a))/(a^(1/4)*b) - 1/6*sqrt(2)*log(-
sqrt(2)*(a + b/x^3)^(1/4)*a^(1/4) + sqrt(a + b/x^3) + sqrt(a))/(a^(1/4)*b)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2a\,x^3+b\right)\left(a\,x^4+bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a*x^3+b)/(a*x^4+b*x)^(1/4),x)
```

```
[Out] int(1/(2*a*x^3+b)/(a*x^4+b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^4+bx\right)^{\frac{1}{4}}\left(2\,ax^3+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a*x^3+b)/(a*x^4+b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x^4 + b*x)^(1/4)*(2*a*x^3 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\,x^4+b\,x\right)^{1/4}\,\left(2\,a\,x^3+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + a*x^4)^(1/4)*(b + 2*a*x^3)),x)
[Out] int(1/((b*x + a*x^4)^(1/4)*(b + 2*a*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x\left(ax^3+b\right)}\,\left(2ax^3+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a*x**3+b)/(a*x**4+b*x)**(1/4),x)
[Out] Integral(1/((x*(a*x**3 + b))**(1/4)*(2*a*x**3 + b)), x)
```

3.1687 $\int \frac{\sqrt[3]{1-x^7}\left(-2+x^3+2x^7\right)\left(3+4x^7\right)}{x^2\left(-1+x^7\right)\left(-4+x^3+4x^7\right)}\,dx$

Optimal. Leaf size=144

$$\frac{3\sqrt[3]{1-x^7}}{2x}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^7}-x\right)}{2\,2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\,2^{2/3}\sqrt[3]{1-x^7}+x}\right)}{2\,2^{2/3}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^7}\,x+2\sqrt[3]{2}\left(1-x^7\right)^{2/3}+x^2\right)}{4\,2^{2/3}}$$

Rubi [F] time = 1.60, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{1-x^7}\left(-2+x^3+2x^7\right)\left(3+4x^7\right)}{x^2\left(-1+x^7\right)\left(-4+x^3+4x^7\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 - x^7)^(1/3)*(-2 + x^3 + 2*x^7)*(3 + 4*x^7))/(x^2*(-1 + x^7)*(-4 +
x^3 + 4*x^7)),x]
```

```
[Out] (3*Hypergeometric2F1[-1/7, 2/3, 6/7, x^7])/(2*x) - (x^2*Hypergeometric2F1[2
/7, 2/3, 9/7, x^7])/4 - (x^6*Hypergeometric2F1[2/3, 6/7, 13/7, x^7])/3 - (7
*Defer[Int][x/((1 - x^7)^(2/3)*(-4 + x^3 + 4*x^7)), x])/2 + Defer[Int][x^4/
((1 - x^7)^(2/3)*(-4 + x^3 + 4*x^7)), x]/2
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{1-x^7}\left(-2+x^3+2x^7\right)\left(3+4x^7\right)}{x^2\left(-1+x^7\right)\left(-4+x^3+4x^7\right)}\,dx &= -\int \frac{\left(-2+x^3+2x^7\right)\left(3+4x^7\right)}{x^2\left(1-x^7\right)^{2/3}\left(-4+x^3+4x^7\right)}\,dx\\
&= -\int \left(\frac{3}{2x^2\left(1-x^7\right)^{2/3}}+\frac{x}{2\left(1-x^7\right)^{2/3}}+\frac{2x^5}{\left(1-x^7\right)^{2/3}}-\frac{x\left(-7+\right.}{2\left(1-x^7\right)^{2/3}\left(-4\right.}\right.\\
&= -\left(\frac{1}{2}\int \frac{x}{\left(1-x^7\right)^{2/3}}\,dx\right)+\frac{1}{2}\int \frac{x\left(-7+x^3\right)}{\left(1-x^7\right)^{2/3}\left(-4+x^3+4x^7\right)}\,dx-\frac{3}{2}\int\\
&= \frac{3\,{}_2F_1\left(-\frac{1}{7},\frac{2}{3};\frac{6}{7};x^7\right)}{2x}-\frac{1}{4}x^2\,{}_2F_1\left(\frac{2}{7},\frac{2}{3};\frac{9}{7};x^7\right)-\frac{1}{3}x^6\,{}_2F_1\left(\frac{2}{3},\frac{6}{7};\frac{13}{7};x^7\right)\\
&= \frac{3\,{}_2F_1\left(-\frac{1}{7},\frac{2}{3};\frac{6}{7};x^7\right)}{2x}-\frac{1}{4}x^2\,{}_2F_1\left(\frac{2}{7},\frac{2}{3};\frac{9}{7};x^7\right)-\frac{1}{3}x^6\,{}_2F_1\left(\frac{2}{3},\frac{6}{7};\frac{13}{7};x^7\right)
\end{aligned}$$

Mathematica [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{1-x^7}\left(-2+x^3+2x^7\right)\left(3+4x^7\right)}{x^2\left(-1+x^7\right)\left(-4+x^3+4x^7\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 - x^7)^(1/3)*(-2 + x^3 + 2*x^7)*(3 + 4*x^7))/(x^2*(-1 + x^7)*
(-4 + x^3 + 4*x^7)),x]
```

```
[Out] Integrate[((1 - x^7)^(1/3)*(-2 + x^3 + 2*x^7)*(3 + 4*x^7))/(x^2*(-1 + x^7)*
(-4 + x^3 + 4*x^7)), x]
```

IntegrateAlgebraic [A] time = 18.13, size = 144, normalized size = 1.00

$$\frac{3\sqrt[3]{1-x^7}}{2x}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^7}-x\right)}{2\ 2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\ 2^{2/3}\sqrt[3]{1-x^7}+x}\right)}{2\ 2^{2/3}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^7}\,x+2\sqrt[3]{2}\left(1-x^7\right)^{2/3}+x^2\right)}{4\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 - x^7)^(1/3)*(-2 + x^3 + 2*x^7)*(3 + 4*x^7))/(x^2*(-
1 + x^7)*(-4 + x^3 + 4*x^7)),x]
```

```
[Out] (3*(1 - x^7)^(1/3))/(2*x) - (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*2^(2/3)*(1 -
 x^7)^(1/3))])/(2*2^(2/3)) - Log[-x + 2^(2/3)*(1 - x^7)^(1/3)]/(2*2^(2/3))
+ Log[x^2 + 2^(2/3)*x*(1 - x^7)^(1/3) + 2*2^(1/3)*(1 - x^7)^(2/3)]/(4*2^(2/
3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^7+1)^(1/3)*(2*x^7+x^3-2)*(4*x^7+3)/x^2/(x^7-1)/(4*x^7+x^3-4),
x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4x^7+3\right)\left(2x^7+x^3-2\right)\left(-x^7+1\right)^{\frac{1}{3}}}{\left(4x^7+x^3-4\right)\left(x^7-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^7+1)^(1/3)*(2*x^7+x^3-2)*(4*x^7+3)/x^2/(x^7-1)/(4*x^7+x^3-4),
x, algorithm="giac")
```

```
[Out] integrate((4*x^7 + 3)*(2*x^7 + x^3 - 2)*(-x^7 + 1)^(1/3)/((4*x^7 + x^3 - 4)
*(x^7 - 1)*x^2), x)
```

maple [C] time = 60.16, size = 1664, normalized size = 11.56

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^7+1)^(1/3)*(2*x^7+x^3-2)*(4*x^7+3)/x^2/(x^7-1)/(4*x^7+x^3-4),x)
```

```
[Out] -3/2*(x^7-1)/x/(-x^7+1)^(2/3)+(1/2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3
+2)+4*_Z^2)*ln((4*RootOf(_Z^3+2)*x^14+16*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootO
f(_Z^3+2)+4*_Z^2)*x^14-RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*
RootOf(_Z^3+2)^3*x^10-4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)
^2*RootOf(_Z^3+2)^2*x^10+6*(x^14-2*x^7+1)^(1/3)*RootOf(_Z^3+2)^2*x^8+12*(x^
14-2*x^7+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootO
f(_Z^3+2)*x^8-8*RootOf(_Z^3+2)*x^7-32*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_
Z^3+2)+4*_Z^2)*x^7+6*(x^14-2*x^7+1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*Root
Of(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x^2+RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf
(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^3+4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf
(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^3-6*(x^14-2*x^7+1)^(2/3)*x^2-6*(x^14-
```

```
2*x^7+1)^(1/3)*RootOf(_Z^3+2)^2*x-12*(x^14-2*x^7+1)^(1/3)*RootOf(RootOf(_Z^
3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x+4*RootOf(_Z^3+2)+16*Roo
tOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2))/(4*x^7+x^3-4)/(x^6+x^5+x^
4+x^3+x^2+x+1)/(-1+x))-1/4*ln(-(4*RootOf(_Z^3+2)*x^14+16*RootOf(RootOf(_Z^3
+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^14+RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(
_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^10+4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf
(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^10+12*(x^14-2*x^7+1)^(1/3)*RootOf(Roo
tOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x^8-RootOf(_Z^3+2)
*x^10-4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^10-8*RootOf(_
Z^3+2)*x^7-32*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^7+6*(x^
14-2*x^7+1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootO
f(_Z^3+2)^2*x^2-RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(
_Z^3+2)^3*x^3-4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootO
f(_Z^3+2)^2*x^3-12*(x^14-2*x^7+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf
(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x+RootOf(_Z^3+2)*x^3+4*RootOf(RootOf(_Z^3+2
)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^3+4*RootOf(_Z^3+2)+16*RootOf(RootOf(_Z^3+
2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2))/(4*x^7+x^3-4)/(x^6+x^5+x^4+x^3+x^2+x+1)/(
-1+x))*RootOf(_Z^3+2)-1/2*ln(-(4*RootOf(_Z^3+2)*x^14+16*RootOf(RootOf(_Z^3+
2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^14+RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_
Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^10+4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(
_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^10+12*(x^14-2*x^7+1)^(1/3)*RootOf(Root
Of(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x^8-RootOf(_Z^3+2)*
x^10-4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^10-8*RootOf(_Z
^3+2)*x^7-32*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^7+6*(x^1
4-2*x^7+1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf
(_Z^3+2)^2*x^2-RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_
Z^3+2)^3*x^3-4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf
(_Z^3+2)^2*x^3-12*(x^14-2*x^7+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(
_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x+RootOf(_Z^3+2)*x^3+4*RootOf(RootOf(_Z^3+2)
^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^3+4*RootOf(_Z^3+2)+16*RootOf(RootOf(_Z^3+2
)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2))/(4*x^7+x^3-4)/(x^6+x^5+x^4+x^3+x^2+x+1)/(-
1+x))*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2))/(-x^7+1)^(2/3)*(
(x^7-1)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^7+3\right)\left(2\,x^7+x^3-2\right)\left(-x^7+1\right)^{\frac{1}{3}}}{\left(4\,x^7+x^3-4\right)\left(x^7-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^7+1)^(1/3)*(2*x^7+x^3-2)*(4*x^7+3)/x^2/(x^7-1)/(4*x^7+x^3-4),
x, algorithm="maxima")
```

```
[Out] integrate((4*x^7 + 3)*(2*x^7 + x^3 - 2)*(-x^7 + 1)^(1/3)/((4*x^7 + x^3 - 4)
*(x^7 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(4\,x^7+3\right)\,\left(2\,x^7+x^3-2\right)}{x^2\,{\left(1-x^7\right)}^{2/3}\,\left(4\,x^7+x^3-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((4*x^7 + 3)*(x^3 + 2*x^7 - 2))/(x^2*(1 - x^7)^(2/3)*(x^3 + 4*x^7 - 4)
),x)
```

```
[Out] int(-((4*x^7 + 3)*(x^3 + 2*x^7 - 2))/(x^2*(1 - x^7)^(2/3)*(x^3 + 4*x^7 - 4)
), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**7+1)**(1/3)*(2*x**7+x**3-2)*(4*x**7+3)/x**2/(x**7-1)/(4*x**7
+x**3-4),x)
```

```
[Out] Timed out
```

3.1688 $\int \frac{1}{x^2\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$

Optimal. Leaf size=144

$$-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{4bx}-\frac{1}{2x\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{\sqrt{2}\sqrt{b}}\right)}{2\sqrt{2}\,b^{3/2}}$$

Rubi [F] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

```
[Out] Defer[Int][1/(x^2*Sqrt[b + Sqrt[b^2 + a*x^2]]), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{b+\sqrt{b^2+ax^2}}}\,dx=\int \frac{1}{x^2\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Mathematica [C] time = 0.40, size = 143, normalized size = 0.99

$$-\frac{\sqrt{\sqrt{ax^2+b^2}+b}\left(9ax^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b-\sqrt{b^2+ax^2}}{2b}\right)+2b\left(\sqrt{ax^2+b^2}+b\right){}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b-\sqrt{b^2+ax^2}}{2b}\right)+2b\left(5\sqrt{ax^2+b^2}-7b\right)\right)}{24abx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

```
[Out] -1/24*(Sqrt[b + Sqrt[b^2 + a*x^2]]*(2*b*(-7*b + 5*Sqrt[b^2 + a*x^2]) + 2*b*
(b + Sqrt[b^2 + a*x^2])*Hypergeometric2F1[-3/2, 1, -1/2, (b - Sqrt[b^2 + a*
x^2])/(2*b)] + 9*a*x^2*Hypergeometric2F1[-1/2, 1, 1/2, (b - Sqrt[b^2 + a*x^
2])/(2*b)]))/(a*b*x^3)
```

IntegrateAlgebraic [A] time = 0.27, size = 112, normalized size = 0.78

$$-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{4bx}-\frac{1}{2x\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{4\sqrt{2}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

```
[Out] -1/2*1/(x*Sqrt[b + Sqrt[b^2 + a*x^2]]) - Sqrt[b + Sqrt[b^2 + a*x^2]]/(4*b*x
) - (Sqrt[a]*ArcTan[(Sqrt[a]*x)/(Sqrt[2]*Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]
])])/(4*Sqrt[2]*b^(3/2))
```

fricas [A] time = 44.20, size = 282, normalized size = 1.96

$$\left[\frac{\sqrt{\frac{1}{2}}\, a x^{3} \sqrt{-\frac{a}{b}} \log\left(-\frac{a^{2} x^{3}+4 a b^{2} x-4 \sqrt{a x^{2}+b^{2}}\, a b x+4 \left(2 \sqrt{\frac{1}{2}} \sqrt{a x^{2}+b^{2}}\, b^{2} \sqrt{-\frac{a}{b}}-\sqrt{\frac{1}{2}} \left(a b x^{2}+2 b^{3}\right) \sqrt{-\frac{a}{b}}\right) \sqrt{b+\sqrt{a x^{2}+b^{2}}}}{x^{3}}\right)-2 \left(a x^{2}-2 b^{2}+2 \sqrt{a x^{2}+b^{2}}\, b\right) \sqrt{b+\sqrt{a x^{2}+b^{2}}}}{8 a b x^{3}}, \frac{\sqrt{\frac{1}{2}}\, a x^{3} \sqrt{\frac{a}{b}} \arctan\left(\frac{2 \sqrt{\frac{1}{2}} \sqrt{b+\sqrt{a x^{2}+b^{2}}}\, b \sqrt{\frac{a}{b}}}{a x}\right)-\left(a x^{2}-2 b^{2}+2 \sqrt{a x^{2}+b^{2}}\, b\right) \sqrt{b+\sqrt{a x^{2}+b^{2}}}}{4 a b x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(1/2)*a*x^3*sqrt(-a/b)*log(-(a^2*x^3 + 4*a*b^2*x - 4*sqrt(a*x^2 +
 b^2)*a*b*x + 4*(2*sqrt(1/2)*sqrt(a*x^2 + b^2)*b^2*sqrt(-a/b) - sqrt(1/2)*(
a*b*x^2 + 2*b^3)*sqrt(-a/b))*sqrt(b + sqrt(a*x^2 + b^2)))/x^3) - 2*(a*x^2 -
 2*b^2 + 2*sqrt(a*x^2 + b^2)*b)*sqrt(b + sqrt(a*x^2 + b^2)))/(a*b*x^3), 1/4
*(sqrt(1/2)*a*x^3*sqrt(a/b)*arctan(2*sqrt(1/2)*sqrt(b + sqrt(a*x^2 + b^2))*
b*sqrt(a/b)/(a*x)) - (a*x^2 - 2*b^2 + 2*sqrt(a*x^2 + b^2)*b)*sqrt(b + sqrt(
a*x^2 + b^2)))/(a*b*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b+\sqrt{a x^{2}+b^{2}}}\, x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b + sqrt(a*x^2 + b^2))*x^2), x)
```

maple [C] time = 0.04, size = 31, normalized size = 0.22

$$-\frac{\sqrt{2}\ \mathrm{hypergeom}\left(\left[-\frac{1}{2}, \frac{1}{4}, \frac{3}{4}\right], \left[\frac{1}{2}, \frac{3}{2}\right], -\frac{x^{2} a}{b^{2}}\right)}{2 \left(b^{2}\right)^{\frac{1}{4}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] -1/2/(b^2)^(1/4)*2^(1/2)/x*hypergeom([-1/2,1/4,3/4],[1/2,3/2],-x^2*a/b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b+\sqrt{a x^{2}+b^{2}}}\, x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b + sqrt(a*x^2 + b^2))*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2} \sqrt{b+\sqrt{b^{2}+a\, x^{2}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b + (a*x^2 + b^2)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^2*(b + (a*x^2 + b^2)^(1/2))^(1/2)), x)
```

sympy [C] time = 0.99, size = 46, normalized size = 0.32

$$-\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_{3}F_{2}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, \frac{3}{2} \end{matrix}\middle| \frac{ax^{2}e^{i\pi}}{b^{2}}\right)}{2\pi\sqrt{b}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b+(a*x**2+b**2)**(1/2))**(1/2),x)
```

```
[Out] -gamma(1/4)*gamma(3/4)*hyper((-1/2, 1/4, 3/4), (1/2, 3/2), a*x**2*exp_polar
(I*pi)/b**2)/(2*pi*sqrt(b)*x)
```

3.1689 $\int \frac{\sqrt[4]{-b+ax^3}}{x} \, dx$

Optimal. Leaf size=145

$$\frac{4}{3}\sqrt[4]{ax^3-b}+\frac{1}{3}\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}-\sqrt{b}}\right)-\frac{1}{3}\sqrt{2}\sqrt[4]{b}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)$$

Rubi [A] time = 0.23, antiderivative size = 218, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 50, 63, 211, 1165, 628, 1162, 617, 204}

$$\frac{4}{3}\sqrt[4]{ax^3-b}+\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{3\sqrt{2}}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{3\sqrt{2}}+\frac{1}{3}\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}\right)-\frac{1}{3}\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^3)^(1/4)/x,x]
```

```
[Out] (4*(-b + a*x^3)^(1/4))/3 + (Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*(-b + a*x^3
)^(1/4))/b^(1/4)])/3 - (Sqrt[2]*b^(1/4)*ArcTan[1 + (Sqrt[2]*(-b + a*x^3)^(1
/4))/b^(1/4)])/3 + (b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4
) + Sqrt[-b + a*x^3]])/(3*Sqrt[2]) - (b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)
*(-b + a*x^3)^(1/4) + Sqrt[-b + a*x^3]])/(3*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{-b+ax^3}}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[4]{-b+ax}}{x}\,dx, x, x^3\right)\\
&= \frac{4}{3}\sqrt[4]{-b+ax^3} - \frac{1}{3}b\,\text{Subst}\left(\int \frac{1}{x(-b+ax)^{3/4}}\,dx, x, x^3\right)\\
&= \frac{4}{3}\sqrt[4]{-b+ax^3} - \frac{(4b)\,\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{3a}\\
&= \frac{4}{3}\sqrt[4]{-b+ax^3} - \frac{\left(2\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{3a} - \frac{\left(2\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-\right.}{3a}\\
&= \frac{4}{3}\sqrt[4]{-b+ax^3} + \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{3\sqrt{2}} + \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}-2x}{-\sqrt{b}+\sqrt{2}\sqrt[4]{b}x-\right.}{3\sqrt{2}}\\
&= \frac{4}{3}\sqrt[4]{-b+ax^3} + \frac{\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^3}+\sqrt{-b+ax^3}\right)}{3\sqrt{2}} - \frac{\sqrt[4]{b}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{-\right.}{3\sqrt{}}\\
&= \frac{4}{3}\sqrt[4]{-b+ax^3} + \frac{1}{3}\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right) - \frac{1}{3}\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)
\end{aligned}$$

Mathematica [A] time = 0.08, size = 211, normalized size = 1.46

$$\frac{1}{6}\left(8\sqrt[4]{ax^3-b}+\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)-\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)+2\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}\right)-2\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^3)^(1/4)/x,x]
```

```
[Out] (8*(-b + a*x^3)^(1/4) + 2*Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*(-b + a*x^3)^
(1/4))/b^(1/4)] - 2*Sqrt[2]*b^(1/4)*ArcTan[1 + (Sqrt[2]*(-b + a*x^3)^(1/4))
/b^(1/4)] + Sqrt[2]*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4
) + Sqrt[-b + a*x^3]] - Sqrt[2]*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b +
 a*x^3)^(1/4) + Sqrt[-b + a*x^3]])/6
```

IntegrateAlgebraic [A] time = 0.25, size = 144, normalized size = 0.99

$$\frac{4}{3}\sqrt[4]{ax^3-b}-\frac{1}{3}\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)-\frac{1}{3}\sqrt{2}\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}+\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^3)^(1/4)/x,x]
```

```
[Out] (4*(-b + a*x^3)^(1/4))/3 - (Sqrt[2]*b^(1/4)*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sq
rt[-b + a*x^3]/(Sqrt[2]*b^(1/4)))/(-b + a*x^3)^(1/4)])/3 - (Sqrt[2]*b^(1/4)
*ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^3])]
)/3
```

fricas [A] time = 0.40, size = 122, normalized size = 0.84

$$-\frac{4}{3}\left(-b\right)^{\frac{1}{4}}\arctan\left(\frac{\left(-b\right)^{\frac{3}{4}}\sqrt{\sqrt{ax^3-b}+\sqrt{-b}}-\left(ax^3-b\right)^{\frac{1}{4}}\left(-b\right)^{\frac{3}{4}}}{b}\right)-\frac{1}{3}\left(-b\right)^{\frac{1}{4}}\log\left(\left(ax^3-b\right)^{\frac{1}{4}}+\left(-b\right)^{\frac{1}{4}}\right)+\frac{1}{3}\left(-b\right)^{\frac{1}{4}}\log\left(\left(ax^3-b\right)^{\frac{1}{4}}-\left(-b\right)^{\frac{1}{4}}\right)+\frac{4}{3}\left(ax^3-b\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(1/4)/x,x, algorithm="fricas")
```

```
[Out] -4/3*(-b)^(1/4)*arctan(((-b)^(3/4)*sqrt(sqrt(a*x^3 - b) + sqrt(-b)) - (a*x^
3 - b)^(1/4)*(-b)^(3/4))/b) - 1/3*(-b)^(1/4)*log((a*x^3 - b)^(1/4) + (-b)^(
1/4)) + 1/3*(-b)^(1/4)*log((a*x^3 - b)^(1/4) - (-b)^(1/4)) + 4/3*(a*x^3 - b
)^(1/4)
```

giac [A] time = 0.48, size = 175, normalized size = 1.21

$$-\frac{1}{3}\sqrt{2}b^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)-\frac{1}{3}\sqrt{2}b^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)-\frac{1}{6}\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)+\frac{1}{6}\sqrt{2}b^{\frac{1}{4}}\log\left(-\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)+\frac{4}{3}\left(ax^3-b\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(1/4)/x,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*b^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1
/4))/b^(1/4)) - 1/3*sqrt(2)*b^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
2*(a*x^3 - b)^(1/4))/b^(1/4)) - 1/6*sqrt(2)*b^(1/4)*log(sqrt(2)*(a*x^3 - b)
^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b)) + 1/6*sqrt(2)*b^(1/4)*log(-sqrt
(2)*(a*x^3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b)) + 4/3*(a*x^3 - b
)^(1/4)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{3}-b\right)^{\frac{1}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b)^(1/4)/x,x)
```

```
[Out] int((a*x^3-b)^(1/4)/x,x)
```

maxima [A] time = 0.43, size = 175, normalized size = 1.21

$$-\frac{1}{3}\sqrt{2}b^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)-\frac{1}{3}\sqrt{2}b^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)-\frac{1}{6}\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)+\frac{1}{6}\sqrt{2}b^{\frac{1}{4}}\log\left(-\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)+\frac{4}{3}\left(ax^3-b\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(1/4)/x,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(2)*b^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1
/4))/b^(1/4)) - 1/3*sqrt(2)*b^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
2*(a*x^3 - b)^(1/4))/b^(1/4)) - 1/6*sqrt(2)*b^(1/4)*log(sqrt(2)*(a*x^3 - b)
^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b)) + 1/6*sqrt(2)*b^(1/4)*log(-sqrt
(2)*(a*x^3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b)) + 4/3*(a*x^3 - b
)^(1/4)
```

mupad [B] time = 1.02, size = 64, normalized size = 0.44

$$\frac{4\left(a\,x^3-b\right)^{1/4}}{3}-\frac{2\,{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(a\,x^3-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{3}-\frac{2\,{\left(-b\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(a\,x^3-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 - b)^(1/4)/x,x)
```

```
[Out] (4*(a*x^3 - b)^(1/4))/3 - (2*(-b)^(1/4)*atan((a*x^3 - b)^(1/4)/(-b)^(1/4)))
/3 - (2*(-b)^(1/4)*atanh((a*x^3 - b)^(1/4)/(-b)^(1/4)))/3
```

sympy [C] time = 0.97, size = 48, normalized size = 0.33

$$-\frac{\sqrt[4]{a}\,x^{\frac{3}{4}}\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\,\frac{be^{2i\pi}}{ax^3}\right)}{3\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b)**(1/4)/x,x)
```

```
[Out] -a**(1/4)*x**(3/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*exp_polar(2*I*
pi)/(a*x**3))/(3*gamma(3/4))
```

3.1690 $\int \frac{(-b+ax^3)^{3/4}}{x}\,dx$

Optimal. Leaf size=145

$$\frac{1}{3}\sqrt{2}\,b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}-\sqrt{b}}\right)+\frac{1}{3}\sqrt{2}\,b^{3/4}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\,\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)+\frac{4}{9}\left(ax^3-b\right)^{3/4}$$

Rubi [A] time = 0.20, antiderivative size = 218, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 50, 63, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{3\sqrt{2}}+\frac{b^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{3\sqrt{2}}+\frac{1}{3}\sqrt{2}\,b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}\right)-\frac{1}{3}\sqrt{2}\,b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}+1\right)+\frac{4}{9}\left(ax^3-b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^3)^(3/4)/x,x]
```

```
[Out] (4*(-b + a*x^3)^(3/4))/9 + (Sqrt[2]*b^(3/4)*ArcTan[1 - (Sqrt[2]*(-b + a*x^3
)^(1/4))/b^(1/4)])/3 - (Sqrt[2]*b^(3/4)*ArcTan[1 + (Sqrt[2]*(-b + a*x^3)^(1
/4))/b^(1/4)])/3 - (b^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4
) + Sqrt[-b + a*x^3]])/(3*Sqrt[2]) + (b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)
*(-b + a*x^3)^(1/4) + Sqrt[-b + a*x^3]])/(3*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(-b+ax^3\right)^{3/4}}{x}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(-b+ax)^{3/4}}{x}\,dx,x,x^3\right)$$
$$= \frac{4}{9}\left(-b+ax^3\right)^{3/4} - \frac{1}{3}b\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{-b+ax}}\,dx,x,x^3\right)$$
$$= \frac{4}{9}\left(-b+ax^3\right)^{3/4} - \frac{(4b)\,\text{Subst}\left(\int \frac{x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{-b+ax^3}\right)}{3a}$$
$$= \frac{4}{9}\left(-b+ax^3\right)^{3/4} + \frac{(2b)\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{-b+ax^3}\right)}{3a} - \frac{(2b)\,\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{-\right.}}{3a}$$
$$= \frac{4}{9}\left(-b+ax^3\right)^{3/4} - \frac{b^{3/4}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx,x,\sqrt[4]{-b+ax^3}\right)}{3\sqrt{2}} - \frac{b^{3/4}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}}{-\sqrt{b}+\sqrt{2}}\right.}{3}$$
$$= \frac{4}{9}\left(-b+ax^3\right)^{3/4} - \frac{b^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^3}+\sqrt{-b+ax^3}\right)}{3\sqrt{2}} + \frac{b^{3/4}\log\left(\sqrt{b}+\sqrt{2}\right.}{}$$
$$= \frac{4}{9}\left(-b+ax^3\right)^{3/4} + \frac{1}{3}\sqrt{2}\,b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right) - \frac{1}{3}\sqrt{2}\,b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+a}}{\sqrt[4]{b}}\right.$$

Mathematica [A] time = 0.03, size = 84, normalized size = 0.58

$$\frac{4}{9}\left(ax^3-b\right)^{3/4}+\frac{2}{3}(-b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^3-b}}{\sqrt[4]{-b}}\right)-\frac{2}{3}(-b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^3-b}}{\sqrt[4]{-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^3)^(3/4)/x,x]
```

```
[Out] (4*(-b + a*x^3)^(3/4))/9 + (2*(-b)^(3/4)*ArcTan[(-b + a*x^3)^(1/4)/(-b)^(1/
4)])/3 - (2*(-b)^(3/4)*ArcTanh[(-b + a*x^3)^(1/4)/(-b)^(1/4)])/3
```

IntegrateAlgebraic [A] time = 0.20, size = 144, normalized size = 0.99

$$-\frac{1}{3}\sqrt{2}\,b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)+\frac{1}{3}\sqrt{2}\,b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}+\sqrt{b}}\right)+\frac{4}{9}\left(ax^3-b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^3)^(3/4)/x,x]
```

```
[Out] (4*(-b + a*x^3)^(3/4))/9 - (Sqrt[2]*b^(3/4)*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sq
rt[-b + a*x^3]/(Sqrt[2]*b^(1/4)))/(-b + a*x^3)^(1/4)])/3 + (Sqrt[2]*b^(3/4)
*ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^3])]
)/3
```

fricas [A] time = 0.40, size = 159, normalized size = 1.10

$$\frac{4}{3}\left(-b^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^3-b\right)^{\frac{1}{4}}\left(-b^3\right)^{\frac{1}{4}}b^2-\sqrt{\sqrt{ax^3-b}\,b^4-\sqrt{-b^3}\,b^3}\left(-b^3\right)^{\frac{1}{4}}}{b^3}\right)-\frac{1}{3}\left(-b^3\right)^{\frac{1}{4}}\log\left(\left(ax^3-b\right)^{\frac{1}{4}}b^2+\left(-b^3\right)^{\frac{3}{4}}\right)+\frac{1}{3}\left(-b^3\right)^{\frac{1}{4}}\log\left(\left(ax^3-b\right)^{\frac{1}{4}}b^2-\left(-b^3\right)^{\frac{3}{4}}\right)+\frac{4}{9}\left(ax^3-b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(3/4)/x,x, algorithm="fricas")
```

```
[Out] 4/3*(-b^3)^(1/4)*arctan(-((a*x^3 - b)^(1/4)*(-b^3)^(1/4)*b^2 - sqrt(sqrt(a*
x^3 - b)*b^4 - sqrt(-b^3)*b^3)*(-b^3)^(1/4))/b^3) - 1/3*(-b^3)^(1/4)*log((a
*x^3 - b)^(1/4)*b^2 + (-b^3)^(3/4)) + 1/3*(-b^3)^(1/4)*log((a*x^3 - b)^(1/4
)*b^2 - (-b^3)^(3/4)) + 4/9*(a*x^3 - b)^(3/4)
```

giac [A] time = 0.31, size = 175, normalized size = 1.21

$$-\frac{1}{3}\sqrt{2}\,b^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)-\frac{1}{3}\sqrt{2}\,b^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)+\frac{1}{6}\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)-\frac{1}{6}\sqrt{2}\,b^{\frac{3}{4}}\log\left(-\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)+\frac{4}{9}\left(ax^3-b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(3/4)/x,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*b^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1
/4))/b^(1/4)) - 1/3*sqrt(2)*b^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
2*(a*x^3 - b)^(1/4))/b^(1/4)) + 1/6*sqrt(2)*b^(3/4)*log(sqrt(2)*(a*x^3 - b)
^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b)) - 1/6*sqrt(2)*b^(3/4)*log(-sqrt
(2)*(a*x^3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b)) + 4/9*(a*x^3 - b
)^(3/4)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^3-b\right)^{\frac{3}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b)^(3/4)/x,x)
```

```
[Out] int((a*x^3-b)^(3/4)/x,x)
```

maxima [A] time = 0.51, size = 178, normalized size = 1.23

$$-\frac{1}{6}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}\right)b+\frac{4}{9}\left(ax^3-b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(3/4)/x,x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1/4))/
b^(1/4))/b^(1/4) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^
3 - b)^(1/4))/b^(1/4))/b^(1/4) - sqrt(2)*log(sqrt(2)*(a*x^3 - b)^(1/4)*b^(1
/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(1/4) + sqrt(2)*log(-sqrt(2)*(a*x^3 - b)
^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(1/4))*b + 4/9*(a*x^3 - b)^(3
/4)
```

mupad [B] time = 0.99, size = 64, normalized size = 0.44

$$\frac{4\left(a\,x^3-b\right)^{3/4}}{9}+\frac{2\,{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(a\,x^3-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{3}-\frac{2\,{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(a\,x^3-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 - b)^(3/4)/x,x)
```

```
[Out] (4*(a*x^3 - b)^(3/4))/9 + (2*(-b)^(3/4)*atan((a*x^3 - b)^(1/4)/(-b)^(1/4)))
/3 - (2*(-b)^(3/4)*atanh((a*x^3 - b)^(1/4)/(-b)^(1/4)))/3
```

sympy [C] time = 1.09, size = 48, normalized size = 0.33

$$-\frac{a^{\frac{3}{4}}x^{\frac{9}{4}}\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^3}\right)}{3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b)**(3/4)/x,x)
```

```
[Out] -a**(3/4)*x**(9/4)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), b*exp_polar(2*I*
pi)/(a*x**3))/(3*gamma(1/4))
```

3.1691 $\int \frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)\left(-1-x^3+x^4\right)}{x^6\left(-2-x^3+2x^4\right)}\,dx$

Optimal. Leaf size=145

$$-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^4-1}-x\right)}{4\,2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^4-1}+x}\right)}{4\,2^{2/3}}+\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^4-1}\,x+2^{2/3}\left(x^4-1\right)^{2/3}+x^2\right)}{8\,2^{2/3}}+\frac{3\left(x^4-1\right)^{2/3}(}{40}$$

Rubi [F] time = 1.41, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)\left(-1-x^3+x^4\right)}{x^6\left(-2-x^3+2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^4)^(2/3)*(3 + x^4)*(-1 - x^3 + x^4))/(x^6*(-2 - x^3 + 2*x^4)),
x]
```

```
[Out] (-3*(-1 + x^4)^(2/3))/(8*x^2) + (3*x^2)/(2*(1 + Sqrt[3] + (-1 + x^4)^(1/3))
) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + (-1 + x^4)^(1/3))*Sqrt[(1 - (-1 + x^4
)^(1/3) + (-1 + x^4)^(2/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))^2]*EllipticE[A
rcSin[(1 - Sqrt[3] + (-1 + x^4)^(1/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))], -
7 - 4*Sqrt[3]])/(4*x^2*Sqrt[(1 + (-1 + x^4)^(1/3))/(1 + Sqrt[3] + (-1 + x^4
)^(1/3))^2]) + (3^(3/4)*(1 + (-1 + x^4)^(1/3))*Sqrt[(1 - (-1 + x^4)^(1/3) +
 (-1 + x^4)^(2/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))^2]*EllipticF[ArcSin[(1
- Sqrt[3] + (-1 + x^4)^(1/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))], -7 - 4*Sqr
t[3]])/(Sqrt[2]*x^2*Sqrt[(1 + (-1 + x^4)^(1/3))/(1 + Sqrt[3] + (-1 + x^4)^(
1/3))^2]) - (3*(-1 + x^4)^(2/3)*Hypergeometric2F1[-5/4, -2/3, -1/4, x^4])/(
10*x^5*(1 - x^4)^(2/3)) - ((-1 + x^4)^(2/3)*Hypergeometric2F1[-2/3, -1/4, 3
/4, x^4])/(2*x*(1 - x^4)^(2/3)) + (3*Defer[Int][(-1 + x^4)^(2/3)/(-2 - x^3
+ 2*x^4), x])/4 - 2*Defer[Int][(x*(-1 + x^4)^(2/3))/(-2 - x^3 + 2*x^4), x]
```

Rubi steps

$$\int \frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)\left(-1-x^3+x^4\right)}{x^6\left(-2-x^3+2x^4\right)}\,dx = \int \left(\frac{3\left(-1+x^4\right)^{2/3}}{2x^6}+\frac{3\left(-1+x^4\right)^{2/3}}{4x^3}+\frac{\left(-1+x^4\right)^{2/3}}{2x^2}+\frac{(3-8x)\left(-1+\right.}{4\left(-2-x^3+\right.}\right.$$

$$= \frac{1}{4}\int \frac{(3-8x)\left(-1+x^4\right)^{2/3}}{-2-x^3+2x^4}\,dx+\frac{1}{2}\int \frac{\left(-1+x^4\right)^{2/3}}{x^2}\,dx+\frac{3}{4}\int \frac{\left(-1+x\right.}{x^3}$$

$$= \frac{1}{4}\int \left(\frac{3\left(-1+x^4\right)^{2/3}}{-2-x^3+2x^4}-\frac{8x\left(-1+x^4\right)^{2/3}}{-2-x^3+2x^4}\right)dx+\frac{3}{8}\,\mathrm{Subst}\left(\int \frac{\left(-1+x^2\right.}{x^2}\right.$$

$$= -\frac{3\left(-1+x^4\right)^{2/3}}{8x^2}-\frac{3\left(-1+x^4\right)^{2/3}\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};x^4\right)}{10x^5\left(1-x^4\right)^{2/3}}-\frac{\left(-1+x^4\right)^{2/}}{2x}$$

$$= -\frac{3\left(-1+x^4\right)^{2/3}}{8x^2}-\frac{3\left(-1+x^4\right)^{2/3}\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};x^4\right)}{10x^5\left(1-x^4\right)^{2/3}}-\frac{\left(-1+x^4\right)^{2/}}{2x}$$

$$= -\frac{3\left(-1+x^4\right)^{2/3}}{8x^2}-\frac{3\left(-1+x^4\right)^{2/3}\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};x^4\right)}{10x^5\left(1-x^4\right)^{2/3}}-\frac{\left(-1+x^4\right)^{2/}}{2x}$$

$$= -\frac{3\left(-1+x^4\right)^{2/3}}{8x^2}+\frac{3x^2}{2\left(1+\sqrt{3}+\sqrt[3]{-1+x^4}\right)}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1+\sqrt[3]{\right.}}{}$$

Mathematica [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)\left(-1-x^3+x^4\right)}{x^6\left(-2-x^3+2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^4)^(2/3)*(3 + x^4)*(-1 - x^3 + x^4))/(x^6*(-2 - x^3 + 2*
x^4)),x]
```

```
[Out] Integrate[((-1 + x^4)^(2/3)*(3 + x^4)*(-1 - x^3 + x^4))/(x^6*(-2 - x^3 + 2*
x^4)), x]
```

IntegrateAlgebraic [A] time = 3.53, size = 145, normalized size = 1.00

$$-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^4-1}-x\right)}{4\,2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^4-1}+x}\right)}{4\,2^{2/3}}+\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^4-1}\,x+2^{2/3}\left(x^4-1\right)^{2/3}+x^2\right)}{8\,2^{2/3}}+\frac{3\left(x^4-1\right)^{2/3}\left(4x^4-5x^3-4\right)}{40x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)^(2/3)*(3 + x^4)*(-1 - x^3 + x^4))/(x^6*(-2 -
 x^3 + 2*x^4)),x]
```

```
[Out] (3*(-1 + x^4)^(2/3)*(-4 - 5*x^3 + 4*x^4))/(40*x^5) + (Sqrt[3]*ArcTan[(Sqrt[
3]*x)/(x + 2*2^(1/3)*(-1 + x^4)^(1/3))])/(4*2^(2/3)) - Log[-x + 2^(1/3)*(-1
 + x^4)^(1/3)]/(4*2^(2/3)) + Log[x^2 + 2^(1/3)*x*(-1 + x^4)^(1/3) + 2^(2/3)
*(-1 + x^4)^(2/3)]/(8*2^(2/3))
```

fricas [B] time = 126.15, size = 433, normalized size = 2.99

$$-\frac{20\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}x^5\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(12\cdot 4^{\frac{2}{3}}(-1)^{\frac{2}{3}}(2x^9+x^8-x^7-4x^5-x^4+2x)(x^4-1)^{\frac{2}{3}}-12(-1)^{\frac{1}{3}}(4x^{10}+14x^9+x^8-8x^6-14x^5+4x^2)(x^4-1)^{\frac{1}{3}}+4^{\frac{1}{3}}(8x^{12}+60x^{11}+24x^{10}-x^9-24x^8-120x^7-24x^6+24x^4+60x^3-8)\right)}{6(8x^{12}-12x^{11}-48x^{10}-x^9-24x^8+24x^7+48x^6+24x^4-12x^3-8)}\right)-10\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^5\log\left(\frac{6\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}(x^4-1)^{\frac{1}{3}}x^2-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}(2x^4-x^3-2)-12(x^4-1)^{\frac{2}{3}}x}{2x^4-x^3-2}\right)+5\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^5\log\left(-\frac{6\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^5+x^4-x)(x^4-1)^{\frac{2}{3}}-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}(4x^8+14x^7+x^6-8x^4-14x^3+4)-6(4x^6+x^5-4x^2)(x^4-1)^{\frac{1}{3}}}{4x^8-4x^7+x^6-8x^4+4x^3+4}\right)-36(4x^4-5x^3-4)(x^4-1)^{\frac{2}{3}}}{480x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)*(x^4-x^3-1)/x^6/(2*x^4-x^3-2),x, algorithm=
"fricas")
```

```
[Out] -1/480*(20*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^5*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^
(2/3)*(-1)^(2/3)*(2*x^9 + x^8 - x^7 - 4*x^5 - x^4 + 2*x)*(x^4 - 1)^(2/3) -
12*(-1)^(1/3)*(4*x^10 + 14*x^9 + x^8 - 8*x^6 - 14*x^5 + 4*x^2)*(x^4 - 1)^(1
/3) + 4^(1/3)*(8*x^12 + 60*x^11 + 24*x^10 - x^9 - 24*x^8 - 120*x^7 - 24*x^6
 + 24*x^4 + 60*x^3 - 8))/(8*x^12 - 12*x^11 - 48*x^10 - x^9 - 24*x^8 + 24*x^
7 + 48*x^6 + 24*x^4 - 12*x^3 - 8)) - 10*4^(2/3)*(-1)^(1/3)*x^5*log((6*4^(1/
3)*(-1)^(2/3)*(x^4 - 1)^(1/3)*x^2 - 4^(2/3)*(-1)^(1/3)*(2*x^4 - x^3 - 2) -
12*(x^4 - 1)^(2/3)*x)/(2*x^4 - x^3 - 2)) + 5*4^(2/3)*(-1)^(1/3)*x^5*log(-(6
*4^(2/3)*(-1)^(1/3)*(x^5 + x^4 - x)*(x^4 - 1)^(2/3) - 4^(1/3)*(-1)^(2/3)*(4
*x^8 + 14*x^7 + x^6 - 8*x^4 - 14*x^3 + 4) - 6*(4*x^6 + x^5 - 4*x^2)*(x^4 -
1)^(1/3))/(4*x^8 - 4*x^7 + x^6 - 8*x^4 + 4*x^3 + 4)) - 36*(4*x^4 - 5*x^3 -
4)*(x^4 - 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4-x^3-1)(x^4+3)(x^4-1)^{\frac{2}{3}}}{(2x^4-x^3-2)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)*(x^4-x^3-1)/x^6/(2*x^4-x^3-2),x, algorithm=
"giac")
```

```
[Out] integrate((x^4 - x^3 - 1)*(x^4 + 3)*(x^4 - 1)^(2/3)/((2*x^4 - x^3 - 2)*x^6)
, x)
```

maple [C] time = 68.53, size = 809, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(2/3)*(x^4+3)*(x^4-x^3-1)/x^6/(2*x^4-x^3-2),x)
```

```
[Out] 3/40*(4*x^8-5*x^7-8*x^4+5*x^3+4)/x^5/(x^4-1)^(1/3)-1/8*ln((2*RootOf(RootOf(
_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^3-2*RootOf(_Z^3+
2)^2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*(x^4-1)^(2/3)*x-4*
RootOf(_Z^3+2)*(x^4-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*
_Z^2)*x^2-4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^4-RootOf(
_Z^3+2)^2*(x^4-1)^(1/3)*x^2-2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4
*_Z^2)*x^3+4*(x^4-1)^(2/3)*x+4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+
4*_Z^2))/(2*x^4-x^3-2))*RootOf(_Z^3+2)-1/4*ln((2*RootOf(RootOf(_Z^3+2)^2+2*
_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^3-2*RootOf(_Z^3+2)^2*RootOf(
RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*(x^4-1)^(2/3)*x-4*RootOf(_Z^3+
2)*(x^4-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^2-4*
RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^4-RootOf(_Z^3+2)^2*(x
^4-1)^(1/3)*x^2-2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^3+4
*(x^4-1)^(2/3)*x+4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2))/(2*
x^4-x^3-2))*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)+1/8*RootOf(
_Z^3+2)*ln(-(RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_
Z^3+2)^2*x^3-RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4
*_Z^2)*(x^4-1)^(2/3)*x+RootOf(_Z^3+2)*(x^4-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2
```

```
+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^2+2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3
+2)+4*_Z^2)*x^4+RootOf(_Z^3+2)^2*(x^4-1)^(1/3)*x^2-(x^4-1)^(2/3)*x-2*RootOf
(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2))/(2*x^4-x^3-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4 - x^3 - 1)(x^4 + 3)(x^4 - 1)^{\frac{2}{3}}}{(2\,x^4 - x^3 - 2)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)*(x^4-x^3-1)/x^6/(2*x^4-x^3-2),x, algorithm=
"maxima")
```

```
[Out] integrate((x^4 - x^3 - 1)*(x^4 + 3)*(x^4 - 1)^(2/3)/((2*x^4 - x^3 - 2)*x^6)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^4 - 1)^{2/3}\,(x^4 + 3)\,(-x^4 + x^3 + 1)}{x^6\,(-2\,x^4 + x^3 + 2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)^(2/3)*(x^4 + 3)*(x^3 - x^4 + 1))/(x^6*(x^3 - 2*x^4 + 2)),x)
```

```
[Out] int(((x^4 - 1)^(2/3)*(x^4 + 3)*(x^3 - x^4 + 1))/(x^6*(x^3 - 2*x^4 + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(2/3)*(x**4+3)*(x**4-x**3-1)/x**6/(2*x**4-x**3-2),x)
```

```
[Out] Timed out
```

3.1692 $\int \frac{\left(-b+ax^4\right)^{3/4}}{x}\,dx$

Optimal. Leaf size=145

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{ax^4-b}}{\sqrt{ax^4-b}-\sqrt{b}}\right)}{2\sqrt{2}}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^4-b}}{\sqrt{2}\,\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^4-b}}\right)}{2\sqrt{2}}+\frac{1}{3}\left(ax^4-b\right)^{3/4}$$

Rubi [A] time = 0.21, antiderivative size = 218, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 50, 63, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{ax^4-b}+\sqrt{ax^4-b}+\sqrt{b}\right)}{4\sqrt{2}}+\frac{b^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{ax^4-b}+\sqrt{ax^4-b}+\sqrt{b}\right)}{4\sqrt{2}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{ax^4-b}}{\sqrt[4]{b}}\right)}{2\sqrt{2}}-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{ax^4-b}}{\sqrt[4]{b}}+1\right)}{2\sqrt{2}}+\frac{1}{3}\left(ax^4-b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^4)^(3/4)/x,x]
```

```
[Out] (-b + a*x^4)^(3/4)/3 + (b^(3/4)*ArcTan[1 - (Sqrt[2]*(-b + a*x^4)^(1/4))/b^(
1/4)])/(2*Sqrt[2]) - (b^(3/4)*ArcTan[1 + (Sqrt[2]*(-b + a*x^4)^(1/4))/b^(1/
4)])/(2*Sqrt[2]) - (b^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^4)^(1/4
) + Sqrt[-b + a*x^4]])/(4*Sqrt[2]) + (b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)
*(-b + a*x^4)^(1/4) + Sqrt[-b + a*x^4]])/(4*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(-b+ax^4\right)^{3/4}}{x}\,dx = \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{(-b+ax)^{3/4}}{x}\,dx, x, x^4\right)$$

$$= \frac{1}{3}\left(-b+ax^4\right)^{3/4} - \frac{1}{4} b\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt[4]{-b+ax}}\,dx, x, x^4\right)$$

$$= \frac{1}{3}\left(-b+ax^4\right)^{3/4} - \frac{b\,\mathrm{Subst}\left(\int \frac{x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^4}\right)}{a}$$

$$= \frac{1}{3}\left(-b+ax^4\right)^{3/4} + \frac{b\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^4}\right)}{2a} - \frac{b\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^4}\right)}{2a}$$

$$= \frac{1}{3}\left(-b+ax^4\right)^{3/4} - \frac{b^{3/4}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^4}\right)}{4\sqrt{2}} - \frac{b^{3/4}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}}{-\sqrt{b}+\sqrt{2}}\right.}{4}$$

$$= \frac{1}{3}\left(-b+ax^4\right)^{3/4} - \frac{b^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^4}+\sqrt{-b+ax^4}\right)}{4\sqrt{2}} + \frac{b^{3/4}\log\left(\sqrt{b}+\sqrt{2}\right.}{}$$

$$= \frac{1}{3}\left(-b+ax^4\right)^{3/4} + \frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^4}}{\sqrt[4]{b}}\right)}{2\sqrt{2}} - \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^4}}{\sqrt[4]{b}}\right)}{2\sqrt{2}} - \frac{b^{3/4}\log\left(\sqrt{b}\right.}{}$$

Mathematica [A] time = 0.03, size = 84, normalized size = 0.58

$$\frac{1}{3}\left(ax^4-b\right)^{3/4}+\frac{1}{2}(-b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^4-b}}{\sqrt[4]{-b}}\right)-\frac{1}{2}(-b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^4-b}}{\sqrt[4]{-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^4)^(3/4)/x,x]
```

```
[Out] (-b + a*x^4)^(3/4)/3 + ((-b)^(3/4)*ArcTan[(-b + a*x^4)^(1/4)/(-b)^(1/4)])/2
 - ((-b)^(3/4)*ArcTanh[(-b + a*x^4)^(1/4)/(-b)^(1/4)])/2
```

IntegrateAlgebraic [A] time = 0.19, size = 144, normalized size = 0.99

$$-\frac{b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt{ax^4-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^4-b}}\right)}{2\sqrt{2}}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^4-b}}{\sqrt{ax^4-b}+\sqrt{b}}\right)}{2\sqrt{2}}+\frac{1}{3}\left(ax^4-b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^4)^(3/4)/x,x]
```

```
[Out] (-b + a*x^4)^(3/4)/3 - (b^(3/4)*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a*x^
4]/(Sqrt[2]*b^(1/4)))/(-b + a*x^4)^(1/4)])/(2*Sqrt[2]) + (b^(3/4)*ArcTanh[(
Sqrt[2]*b^(1/4)*(-b + a*x^4)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^4])])/(2*Sqrt[
2])
```

fricas [A] time = 0.41, size = 158, normalized size = 1.09

$$\left(-b^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^4-b\right)^{\frac{1}{4}}\left(-b^3\right)^{\frac{1}{4}}b^2-\sqrt{\sqrt{ax^4-b}\,b^4-\sqrt{-b^3}\,b^3}\left(-b^3\right)^{\frac{1}{4}}}{b^3}\right)-\frac{1}{4}\left(-b^3\right)^{\frac{1}{4}}\log\left(\left(ax^4-b\right)^{\frac{1}{4}}b^2+\left(-b^3\right)^{\frac{3}{4}}\right)+\frac{1}{4}\left(-b^3\right)^{\frac{1}{4}}\log\left(\left(ax^4-b\right)^{\frac{1}{4}}b^2-\left(-b^3\right)^{\frac{3}{4}}\right)+\frac{1}{3}\left(ax^4-b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4)/x,x, algorithm="fricas")
```

```
[Out] (-b^3)^(1/4)*arctan(-((a*x^4 - b)^(1/4)*(-b^3)^(1/4)*b^2 - sqrt(sqrt(a*x^4
- b)*b^4 - sqrt(-b^3)*b^3)*(-b^3)^(1/4))/b^3) - 1/4*(-b^3)^(1/4)*log((a*x^4
 - b)^(1/4)*b^2 + (-b^3)^(3/4)) + 1/4*(-b^3)^(1/4)*log((a*x^4 - b)^(1/4)*b^
2 - (-b^3)^(3/4)) + 1/3*(a*x^4 - b)^(3/4)
```

giac [A] time = 0.30, size = 175, normalized size = 1.21

$$-\frac{1}{4}\sqrt{2}b^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)-\frac{1}{4}\sqrt{2}b^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)+\frac{1}{8}\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)-\frac{1}{8}\sqrt{2}b^{\frac{3}{4}}\log\left(-\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)+\frac{1}{3}\left(ax^4-b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4)/x,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*b^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^4 - b)^(1
/4))/b^(1/4)) - 1/4*sqrt(2)*b^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
2*(a*x^4 - b)^(1/4))/b^(1/4)) + 1/8*sqrt(2)*b^(3/4)*log(sqrt(2)*(a*x^4 - b)
^(1/4)*b^(1/4) + sqrt(a*x^4 - b) + sqrt(b)) - 1/8*sqrt(2)*b^(3/4)*log(-sqrt
(2)*(a*x^4 - b)^(1/4)*b^(1/4) + sqrt(a*x^4 - b) + sqrt(b)) + 1/3*(a*x^4 - b
)^(3/4)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^4-b\right)^{\frac{3}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b)^(3/4)/x,x)
```

```
[Out] int((a*x^4-b)^(3/4)/x,x)
```

maxima [A] time = 0.41, size = 178, normalized size = 1.23

$$-\frac{1}{8}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}\right)b+\frac{1}{3}\left(ax^4-b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4)/x,x, algorithm="maxima")
```

```
[Out] -1/8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^4 - b)^(1/4))/
b^(1/4))/b^(1/4) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^
4 - b)^(1/4))/b^(1/4))/b^(1/4) - sqrt(2)*log(sqrt(2)*(a*x^4 - b)^(1/4)*b^(1
/4) + sqrt(a*x^4 - b) + sqrt(b))/b^(1/4) + sqrt(2)*log(-sqrt(2)*(a*x^4 - b)
^(1/4)*b^(1/4) + sqrt(a*x^4 - b) + sqrt(b))/b^(1/4))*b + 1/3*(a*x^4 - b)^(3
/4)
```

mupad [B] time = 1.02, size = 64, normalized size = 0.44

$$\frac{\left(a\,x^4-b\right)^{3/4}}{3}+\frac{{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(a\,x^4-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{2}-\frac{{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(a\,x^4-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4 - b)^(3/4)/x,x)
```

```
[Out] (a*x^4 - b)^(3/4)/3 + ((-b)^(3/4)*atan((a*x^4 - b)^(1/4)/(-b)^(1/4)))/2 - (
(-b)^(3/4)*atanh((a*x^4 - b)^(1/4)/(-b)^(1/4)))/2
```

sympy [C] time = 1.06, size = 46, normalized size = 0.32

$$-\frac{a^{\frac{3}{4}}x^3\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^4}\right)}{4\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b)**(3/4)/x,x)
```

```
[Out] -a**(3/4)*x**3*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), b*exp_polar(2*I*pi)/
(a*x**4))/(4*gamma(1/4))
```

3.1693

$$\int \frac{-3ab^3+2b^2(3a+b)x-3b(a+b)x^2+x^4}{\sqrt[4]{x(-a+x)(-b+x)^3}\left(ab^3d-b^2(3a+b)dx+3b(a+b)dx^2-(1+ad+3bd)x^3+dx^4\right)}\,dx$$

Optimal. Leaf size=145

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{ab^3x+x^3(3ab+3b^2)+x^2(-3ab^2-b^3)+x^4(-a-3b)+x^5}}{x}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{ab^3x+x^3(3ab+3b^2)+x^2(-3ab^2-b^3)+x^4(-a-3b)+x^5}}{x}\right)}{d^{3/4}}$$

Rubi [F] time = 37.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3ab^3+2b^2(3a+b)x-3b(a+b)x^2+x^4}{\sqrt[4]{x(-a+x)(-b+x)^3}\left(ab^3d-b^2(3a+b)dx+3b(a+b)dx^2-(1+ad+3bd)x^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3*a*b^3 + 2*b^2*(3*a + b)*x - 3*b*(a + b)*x^2 + x^4)/((x*(-a + x)*(-b
 + x)^3)^(1/4)*(a*b^3*d - b^2*(3*a + b)*d*x + 3*b*(a + b)*d*x^2 - (1 + a*d
+ 3*b*d)*x^3 + d*x^4)),x]
```

```
[Out] (12*a*b*x^(1/4)*(-a + x)^(1/4)*(-b + x)^(3/4)*Defer[Subst][Defer[Int][(x^2*
(-b + x^4)^(5/4))/((-a + x^4)^(1/4)*(-(a*b^3*d) + 3*a*b^2*(1 + b/(3*a))*d*x
^4 - 3*a*b*(1 + b/a)*d*x^8 + (1 + (a + 3*b)*d)*x^12 - d*x^16)), x], x, x^(1
/4)])/((a - x)*(b - x)^3*x)^(1/4) + (8*b*x^(1/4)*(-a + x)^(1/4)*(-b + x)^(3
/4)*Defer[Subst][Defer[Int][(x^6*(-b + x^4)^(5/4))/((-a + x^4)^(1/4)*(a*b^3
*d - 3*a*b^2*(1 + b/(3*a))*d*x^4 + 3*a*b*(1 + b/a)*d*x^8 - (1 + (a + 3*b)*d
)*x^12 + d*x^16)), x], x, x^(1/4)])/((a - x)*(b - x)^3*x)^(1/4) + (4*x^(1/4
)*(-a + x)^(1/4)*(-b + x)^(3/4)*Defer[Subst][Defer[Int][(x^10*(-b + x^4)^(5
/4))/((-a + x^4)^(1/4)*(a*b^3*d - 3*a*b^2*(1 + b/(3*a))*d*x^4 + 3*a*b*(1 +
b/a)*d*x^8 - (1 + (a + 3*b)*d)*x^12 + d*x^16)), x], x, x^(1/4)])/((a - x)*(
b - x)^3*x)^(1/4)
```

Rubi steps

$$\int \frac{-3ab^3+2b^2(3a+b)x-3b(a+b)x^2+x^4}{\sqrt[4]{x(-a+x)(-b+x)^3}\left(ab^3d-b^2(3a+b)dx+3b(a+b)dx^2-(1+ad+3bd)x^3+dx^4\right)}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{-a+x}\,(-b\right.}{}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt[4]{-a+x}\,(-b\right.}{}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt[4]{-a+x}\,(-b\right.}{}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-a+x}\,(-b\right.}{}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-a+x}\,(-b\right.}{}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-a+x}\,(-b\right.}{}$$

Mathematica [F] time = 2.59, size = 0, normalized size = 0.00

$$\int \frac{-3ab^3+2b^2(3a+b)x-3b(a+b)x^2+x^4}{\sqrt[4]{x(-a+x)(-b+x)^3}\left(ab^3d-b^2(3a+b)dx+3b(a+b)dx^2-(1+ad+3bd)x^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3*a*b^3 + 2*b^2*(3*a + b)*x - 3*b*(a + b)*x^2 + x^4)/((x*(-a +
x)*(-b + x)^3)^(1/4)*(a*b^3*d - b^2*(3*a + b)*d*x + 3*b*(a + b)*d*x^2 - (1
+ a*d + 3*b*d)*x^3 + d*x^4)),x]
```

```
[Out] Integrate[(-3*a*b^3 + 2*b^2*(3*a + b)*x - 3*b*(a + b)*x^2 + x^4)/((x*(-a +
x)*(-b + x)^3)^(1/4)*(a*b^3*d - b^2*(3*a + b)*d*x + 3*b*(a + b)*d*x^2 - (1
+ a*d + 3*b*d)*x^3 + d*x^4)), x]
```

IntegrateAlgebraic [A] time = 0.51, size = 145, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{ab^3x+x^3\left(3ab+3b^2\right)+x^2\left(-3ab^2-b^3\right)+x^4(-a-3b)+x^5}}{x}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{ab^3x+x^3\left(3ab+3b^2\right)+x^2\left(-3ab^2-b^3\right)+x^4(-a-3b)+x^5}}{x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*a*b^3 + 2*b^2*(3*a + b)*x - 3*b*(a + b)*x^2 + x^4)/(
(x*(-a + x)*(-b + x)^3)^(1/4)*(a*b^3*d - b^2*(3*a + b)*d*x + 3*b*(a + b)*d*
x^2 - (1 + a*d + 3*b*d)*x^3 + d*x^4)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(a*b^3*x + (-3*a*b^2 - b^3)*x^2 + (3*a*b + 3*b^2)*x^3 +
(-a - 3*b)*x^4 + x^5)^(1/4))/x]])/d^(3/4) - (2*ArcTanh[(d^(1/4)*(a*b^3*x + (
-3*a*b^2 - b^3)*x^2 + (3*a*b + 3*b^2)*x^3 + (-a - 3*b)*x^4 + x^5)^(1/4))/x]
)/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*a*b^3+2*b^2*(3*a+b)*x-3*b*(a+b)*x^2+x^4)/(x*(-a+x)*(-b+x)^3)^
(1/4)/(a*b^3*d-b^2*(3*a+b)*d*x+3*b*(a+b)*d*x^2-(a*d+3*b*d+1)*x^3+d*x^4),x,
algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{3\,ab^3 - 2\,(3\,a + b)b^2x + 3\,(a + b)bx^2 - x^4}{\left(ab^3d - (3\,a + b)b^2dx + 3\,(a + b)bdx^2 + dx^4 - (ad + 3\,bd + 1)x^3\right)\left((a - x)(b - x)^3x\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*a*b^3+2*b^2*(3*a+b)*x-3*b*(a+b)*x^2+x^4)/(x*(-a+x)*(-b+x)^3)^
(1/4)/(a*b^3*d-b^2*(3*a+b)*d*x+3*b*(a+b)*d*x^2-(a*d+3*b*d+1)*x^3+d*x^4),x,
algorithm="giac")
```

```
[Out] integrate(-(3*a*b^3 - 2*(3*a + b)*b^2*x + 3*(a + b)*b*x^2 - x^4)/((a*b^3*d
- (3*a + b)*b^2*d*x + 3*(a + b)*b*d*x^2 + d*x^4 - (a*d + 3*b*d + 1)*x^3)*((
a - x)*(b - x)^3*x)^(1/4)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{-3a\,b^3 + 2b^2\,(3a + b)\,x - 3b\,(a + b)\,x^2 + x^4}{\left(x\,(-a + x)\,(-b + x)^3\right)^{\frac{1}{4}}\left(a\,b^3d - b^2\,(3a + b)\,dx + 3b\,(a + b)\,d\,x^2 - (ad + 3bd + 1)\,x^3 + d\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*a*b^3+2*b^2*(3*a+b)*x-3*b*(a+b)*x^2+x^4)/(x*(-a+x)*(-b+x)^3)^(1/4)/
(a*b^3*d-b^2*(3*a+b)*d*x+3*b*(a+b)*d*x^2-(a*d+3*b*d+1)*x^3+d*x^4),x)
```

```
[Out] int((-3*a*b^3+2*b^2*(3*a+b)*x-3*b*(a+b)*x^2+x^4)/(x*(-a+x)*(-b+x)^3)^(1/4)/
(a*b^3*d-b^2*(3*a+b)*d*x+3*b*(a+b)*d*x^2-(a*d+3*b*d+1)*x^3+d*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3\,ab^3 - 2\,(3\,a + b)b^2x + 3\,(a + b)bx^2 - x^4}{\left(ab^3d - (3\,a + b)b^2dx + 3\,(a + b)bdx^2 + dx^4 - (ad + 3\,bd + 1)x^3\right)\left((a - x)(b - x)^3x\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*a*b^3+2*b^2*(3*a+b)*x-3*b*(a+b)*x^2+x^4)/(x*(-a+x)*(-b+x)^3)^
(1/4)/(a*b^3*d-b^2*(3*a+b)*d*x+3*b*(a+b)*d*x^2-(a*d+3*b*d+1)*x^3+d*x^4),x,
algorithm="maxima")
```

```
[Out] -integrate((3*a*b^3 - 2*(3*a + b)*b^2*x + 3*(a + b)*b*x^2 - x^4)/((a*b^3*d
- (3*a + b)*b^2*d*x + 3*(a + b)*b*d*x^2 + d*x^4 - (a*d + 3*b*d + 1)*x^3)*((
a - x)*(b - x)^3*x)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{3\,a\,b^3 - x^4 - 2\,b^2\,x\,\left(3\,a + b\right) + 3\,b\,x^2\,\left(a + b\right)}{\left(x\,\left(a - x\right)\,{\left(b - x\right)}^3\right)^{1/4}\,\left(d\,x^4 - x^3\,\left(a\,d + 3\,b\,d + 1\right) + a\,b^3\,d + 3\,b\,d\,x^2\,\left(a + b\right) - b^2\,d\,x\,\left(3\,a + b\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*a*b^3 - x^4 - 2*b^2*x*(3*a + b) + 3*b*x^2*(a + b))/((x*(a - x)*(b -
 x)^3)^(1/4)*(d*x^4 - x^3*(a*d + 3*b*d + 1) + a*b^3*d + 3*b*d*x^2*(a + b) -
 b^2*d*x*(3*a + b))),x)
```

```
[Out] int(-(3*a*b^3 - x^4 - 2*b^2*x*(3*a + b) + 3*b*x^2*(a + b))/((x*(a - x)*(b -
 x)^3)^(1/4)*(d*x^4 - x^3*(a*d + 3*b*d + 1) + a*b^3*d + 3*b*d*x^2*(a + b) -
 b^2*d*x*(3*a + b))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*a*b**3+2*b**2*(3*a+b)*x-3*b*(a+b)*x**2+x**4)/(x*(-a+x)*(-b+x)
**3)**(1/4)/(a*b**3*d-b**2*(3*a+b)*d*x+3*b*(a+b)*d*x**2-(a*d+3*b*d+1)*x**3+
d*x**4),x)
```

```
[Out] Timed out
```

3.1694 $\int \frac{(-b+ax^5)^{3/4}}{x}\,dx$

Optimal. Leaf size=145

$$\frac{1}{5}\sqrt{2}\,b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{ax^5-b}}{\sqrt{ax^5-b}-\sqrt{b}}\right)+\frac{1}{5}\sqrt{2}\,b^{3/4}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5-b}}{\sqrt{2}\,\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^5-b}}\right)+\frac{4}{15}\left(ax^5-b\right)^{3/4}$$

Rubi [A] time = 0.21, antiderivative size = 218, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 50, 63, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{ax^5-b}+\sqrt{ax^5-b}+\sqrt{b}\right)}{5\sqrt{2}}+\frac{b^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{ax^5-b}+\sqrt{ax^5-b}+\sqrt{b}\right)}{5\sqrt{2}}+\frac{1}{5}\sqrt{2}\,b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{ax^5-b}}{\sqrt[4]{b}}\right)-\frac{1}{5}\sqrt{2}\,b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{ax^5-b}}{\sqrt[4]{b}}+1\right)+\frac{4}{15}\left(ax^5-b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^5)^(3/4)/x,x]
```

```
[Out] (4*(-b + a*x^5)^(3/4))/15 + (Sqrt[2]*b^(3/4)*ArcTan[1 - (Sqrt[2]*(-b + a*x^
5)^(1/4))/b^(1/4)])/5 - (Sqrt[2]*b^(3/4)*ArcTan[1 + (Sqrt[2]*(-b + a*x^5)^(
1/4))/b^(1/4)])/5 - (b^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^5)^(1/
4) + Sqrt[-b + a*x^5]])/(5*Sqrt[2]) + (b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4
)*(-b + a*x^5)^(1/4) + Sqrt[-b + a*x^5]])/(5*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(-b+ax^5\right)^{3/4}}{x}\,dx = \frac{1}{5}\,\text{Subst}\left(\int \frac{(-b+ax)^{3/4}}{x}\,dx, x, x^5\right)$$

$$= \frac{4}{15}\left(-b+ax^5\right)^{3/4} - \frac{1}{5}b\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{-b+ax}}\,dx, x, x^5\right)$$

$$= \frac{4}{15}\left(-b+ax^5\right)^{3/4} - \frac{(4b)\,\text{Subst}\left(\int \frac{x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^5}\right)}{5a}$$

$$= \frac{4}{15}\left(-b+ax^5\right)^{3/4} + \frac{(2b)\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^5}\right)}{5a} - \frac{(2b)\,\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{\cdots}\right.}{5a}$$

$$= \frac{4}{15}\left(-b+ax^5\right)^{3/4} - \frac{b^{3/4}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^5}\right)}{5\sqrt{2}} - \frac{b^{3/4}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{\cdots}}{-\sqrt{b}+\sqrt{\cdots}}\right.}{\cdots}$$

$$= \frac{4}{15}\left(-b+ax^5\right)^{3/4} - \frac{b^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^5}+\sqrt{-b+ax^5}\right)}{5\sqrt{2}} + \frac{b^{3/4}\log\left(\sqrt{b}+\sqrt{2}\cdots\right.}{\cdots}$$

$$= \frac{4}{15}\left(-b+ax^5\right)^{3/4} + \frac{1}{5}\sqrt{2}\,b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^5}}{\sqrt[4]{b}}\right) - \frac{1}{5}\sqrt{2}\,b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+\cdots}}{\sqrt[4]{b}}\right.$$

Mathematica [A] time = 0.03, size = 84, normalized size = 0.58

$$\frac{4}{15}\left(ax^5-b\right)^{3/4}+\frac{2}{5}(-b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^5-b}}{\sqrt[4]{-b}}\right)-\frac{2}{5}(-b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^5-b}}{\sqrt[4]{-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^5)^(3/4)/x,x]
```

```
[Out] (4*(-b + a*x^5)^(3/4))/15 + (2*(-b)^(3/4)*ArcTan[(-b + a*x^5)^(1/4)/(-b)^(1
/4)])/5 - (2*(-b)^(3/4)*ArcTanh[(-b + a*x^5)^(1/4)/(-b)^(1/4)])/5
```

IntegrateAlgebraic [A] time = 0.19, size = 144, normalized size = 0.99

$$-\frac{1}{5}\sqrt{2}\,b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt{ax^5-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^5-b}}\right)+\frac{1}{5}\sqrt{2}\,b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^5-b}}{\sqrt{ax^5-b}+\sqrt{b}}\right)+\frac{4}{15}\left(ax^5-b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^5)^(3/4)/x,x]
```

```
[Out] (4*(-b + a*x^5)^(3/4))/15 - (Sqrt[2]*b^(3/4)*ArcTan[(-(b^(1/4)/Sqrt[2]) + S
qrt[-b + a*x^5]/(Sqrt[2]*b^(1/4)))/(-b + a*x^5)^(1/4)])/5 + (Sqrt[2]*b^(3/4
)*ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^5)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^5])
])/5
```

fricas [A] time = 0.41, size = 159, normalized size = 1.10

$$\frac{4}{5}\left(-b^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^5-b\right)^{\frac{1}{4}}\left(-b^3\right)^{\frac{1}{4}}b^2-\sqrt{\sqrt{ax^5-b}\,b^4-\sqrt{-b^3}\,b^3}\,\left(-b^3\right)^{\frac{1}{4}}}{b^3}\right)-\frac{1}{5}\left(-b^3\right)^{\frac{1}{4}}\log\left(\left(ax^5-b\right)^{\frac{1}{4}}b^2+\left(-b^3\right)^{\frac{3}{4}}\right)+\frac{1}{5}\left(-b^3\right)^{\frac{1}{4}}\log\left(\left(ax^5-b\right)^{\frac{1}{4}}b^2-\left(-b^3\right)^{\frac{3}{4}}\right)+\frac{4}{15}\left(ax^5-b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5-b)^(3/4)/x,x, algorithm="fricas")
```

```
[Out] 4/5*(-b^3)^(1/4)*arctan(-((a*x^5 - b)^(1/4)*(-b^3)^(1/4)*b^2 - sqrt(sqrt(a*
x^5 - b)*b^4 - sqrt(-b^3)*b^3)*(-b^3)^(1/4))/b^3) - 1/5*(-b^3)^(1/4)*log((a
*x^5 - b)^(1/4)*b^2 + (-b^3)^(3/4)) + 1/5*(-b^3)^(1/4)*log((a*x^5 - b)^(1/4
)*b^2 - (-b^3)^(3/4)) + 4/15*(a*x^5 - b)^(3/4)
```

giac [A] time = 0.14, size = 175, normalized size = 1.21

$$-\frac{1}{5}\sqrt{2}\,b^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+2\left(ax^5-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)-\frac{1}{5}\sqrt{2}\,b^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-2\left(ax^5-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)+\frac{1}{10}\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{2}\left(ax^5-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^5-b}+\sqrt{b}\right)-\frac{1}{10}\sqrt{2}\,b^{\frac{3}{4}}\log\left(-\sqrt{2}\left(ax^5-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^5-b}+\sqrt{b}\right)+\frac{4}{15}\left(ax^5-b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5-b)^(3/4)/x,x, algorithm="giac")
```

```
[Out] -1/5*sqrt(2)*b^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^5 - b)^(1
/4))/b^(1/4)) - 1/5*sqrt(2)*b^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
2*(a*x^5 - b)^(1/4))/b^(1/4)) + 1/10*sqrt(2)*b^(3/4)*log(sqrt(2)*(a*x^5 - b
)^(1/4)*b^(1/4) + sqrt(a*x^5 - b) + sqrt(b)) - 1/10*sqrt(2)*b^(3/4)*log(-sq
rt(2)*(a*x^5 - b)^(1/4)*b^(1/4) + sqrt(a*x^5 - b) + sqrt(b)) + 4/15*(a*x^5
- b)^(3/4)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^5-b\right)^{\frac{3}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^5-b)^(3/4)/x,x)
```

```
[Out] int((a*x^5-b)^(3/4)/x,x)
```

maxima [A] time = 0.42, size = 178, normalized size = 1.23

$$-\frac{1}{10}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^5-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^5-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^5-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^5-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^5-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^5-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}\right)b+\frac{4}{15}\left(ax^5-b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5-b)^(3/4)/x,x, algorithm="maxima")
```

```
[Out] -1/10*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^5 - b)^(1/4))
/b^(1/4))/b^(1/4) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x
^5 - b)^(1/4))/b^(1/4))/b^(1/4) - sqrt(2)*log(sqrt(2)*(a*x^5 - b)^(1/4)*b^(
1/4) + sqrt(a*x^5 - b) + sqrt(b))/b^(1/4) + sqrt(2)*log(-sqrt(2)*(a*x^5 - b
)^(1/4)*b^(1/4) + sqrt(a*x^5 - b) + sqrt(b))/b^(1/4))*b + 4/15*(a*x^5 - b)^
(3/4)
```

mupad [B] time = 1.23, size = 64, normalized size = 0.44

$$\frac{4\left(a\,x^5-b\right)^{3/4}}{15}+\frac{2\,{(-b)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(a\,x^5-b\right)}^{1/4}}{{(-b)}^{1/4}}\right)}{5}-\frac{2\,{(-b)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(a\,x^5-b\right)}^{1/4}}{{(-b)}^{1/4}}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^5 - b)^(3/4)/x,x)
```

```
[Out] (4*(a*x^5 - b)^(3/4))/15 + (2*(-b)^(3/4)*atan((a*x^5 - b)^(1/4)/(-b)^(1/4))
)/5 - (2*(-b)^(3/4)*atanh((a*x^5 - b)^(1/4)/(-b)^(1/4)))/5
```

sympy [C] time = 1.08, size = 48, normalized size = 0.33

$$-\frac{a^{\frac{3}{4}}x^{\frac{15}{4}}\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^5}\right)}{5\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**5-b)**(3/4)/x,x)
```

```
[Out] -a**(3/4)*x**(15/4)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), b*exp_polar(2*I
*pi)/(a*x**5))/(5*gamma(1/4))
```

3.1695 $\int \frac{(-1+x^3)^{2/3}(4+x^3+x^6)}{x^9(-2+x^3)}\,dx$

Optimal. Leaf size=145

$$\frac{5\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}-x\right)}{12\ 2^{2/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}+x}\right)}{4\ 2^{2/3}\sqrt{3}}-\frac{5\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}\,x+2^{2/3}\left(x^3-1\right)^{2/3}+x^2\right)}{24\ 2^{2/3}}+\frac{\left(x^3-1\right)^{2/3}(7x}{40x}$$

Rubi [C] time = 0.35, antiderivative size = 143, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6725, 271, 264, 277, 239, 430, 429}

$$-\frac{5x\left(x^3-1\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,\frac{x^3}{2}\right)}{8\left(1-x^3\right)^{2/3}}+\frac{5}{8}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{5\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{\left(x^3-1\right)^{5/3}}{4x^8}-\frac{9\left(x^3-1\right)^{5/3}}{20x^5}+\frac{5\left(x^3-1\right)^{2/3}}{8x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^3)^(2/3)*(4 + x^3 + x^6))/(x^9*(-2 + x^3)),x]
```

```
[Out] (5*(-1 + x^3)^(2/3))/(8*x^2) - (-1 + x^3)^(5/3)/(4*x^8) - (9*(-1 + x^3)^(5/
3))/(20*x^5) - (5*x*(-1 + x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, x^3, x^3/2
])/(8*(1 - x^3)^(2/3)) - (5*ArcTan[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]])/(
4*Sqrt[3]) + (5*Log[-x + (-1 + x^3)^(1/3)])/8
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(4+x^3+x^6\right)}{x^9\left(-2+x^3\right)}\,dx = \int\left(-\frac{2\left(-1+x^3\right)^{2/3}}{x^9}-\frac{3\left(-1+x^3\right)^{2/3}}{2x^6}-\frac{5\left(-1+x^3\right)^{2/3}}{4x^3}+\frac{5\left(-1+x^3\right)^{2/3}}{4\left(-2+x^3\right)}\right)dx$$

$$= -\left(\frac{5}{4}\int\frac{\left(-1+x^3\right)^{2/3}}{x^3}\,dx\right)+\frac{5}{4}\int\frac{\left(-1+x^3\right)^{2/3}}{-2+x^3}\,dx-\frac{3}{2}\int\frac{\left(-1+x^3\right)^{2/3}}{x^6}\,dx-2\int$$

$$= \frac{5\left(-1+x^3\right)^{2/3}}{8x^2}-\frac{\left(-1+x^3\right)^{5/3}}{4x^8}-\frac{3\left(-1+x^3\right)^{5/3}}{10x^5}-\frac{3}{4}\int\frac{\left(-1+x^3\right)^{2/3}}{x^6}\,dx-\frac{5}{4}\int$$

$$= \frac{5\left(-1+x^3\right)^{2/3}}{8x^2}-\frac{\left(-1+x^3\right)^{5/3}}{4x^8}-\frac{9\left(-1+x^3\right)^{5/3}}{20x^5}-\frac{5x\left(-1+x^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\right.}{8\left(1-x^3\right)^{2/3}}$$

Mathematica [A] time = 0.18, size = 138, normalized size = 0.95

$$\frac{\left(x^3-1\right)^{2/3}\left(7x^6+8x^3+10\right)}{40x^8}-\frac{5\left(-2\log\left(2-\frac{2^{2/3}x}{\sqrt[3]{1-x^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2^{2/3}x}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)+\log\left(\frac{2^{2/3}x}{\sqrt[3]{1-x^3}}+\frac{\sqrt[3]{2}x^2}{\left(1-x^3\right)^{2/3}}+2\right)\right)}{24\,2^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^3)^(2/3)*(4 + x^3 + x^6))/(x^9*(-2 + x^3)),x]
```

```
[Out] ((-1 + x^3)^(2/3)*(10 + 8*x^3 + 7*x^6))/(40*x^8) - (5*(2*Sqrt[3]*ArcTan[(1
+ (2^(2/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - 2*Log[2 - (2^(2/3)*x)/(1 - x^3)^(
1/3)] + Log[2 + (2^(1/3)*x^2)/(1 - x^3)^(2/3) + (2^(2/3)*x)/(1 - x^3)^(1/3)
]))/(24*2^(2/3))
```

IntegrateAlgebraic [A] time = 0.35, size = 145, normalized size = 1.00

$$\frac{5\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}-x\right)}{12\,2^{2/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}+x}\right)}{4\,2^{2/3}\sqrt{3}}-\frac{5\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}\,x+2^{2/3}\left(x^3-1\right)^{2/3}+x^2\right)}{24\,2^{2/3}}+\frac{\left(x^3-1\right)^{2/3}\left(7x^6+8x^3+10\right)}{40x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(2/3)*(4 + x^3 + x^6))/(x^9*(-2 + x^3)),x]
```

```
[Out] ((-1 + x^3)^(2/3)*(10 + 8*x^3 + 7*x^6))/(40*x^8) - (5*ArcTan[(Sqrt[3]*x)/(x
 + 2*2^(1/3)*(-1 + x^3)^(1/3))])/(4*2^(2/3)*Sqrt[3]) + (5*Log[-x + 2^(1/3)*
```

```
(-1 + x^3)^(1/3)])/(12*2^(2/3)) - (5*Log[x^2 + 2^(1/3)*x*(-1 + x^3)^(1/3) +
 2^(2/3)*(-1 + x^3)^(2/3)])/(24*2^(2/3))
```

fricas [B] time = 2.96, size = 277, normalized size = 1.91

$$\frac{100 \cdot 4^{\frac{1}{6}} \sqrt{3} x^8 \arctan\left(\frac{4^{\frac{1}{6}}\left(12 \cdot 4^{\frac{2}{3}} \sqrt{3}\left(2 x^7-5 x^4+2 x\right)\left(x^3-1\right)^{\frac{2}{3}}+4^{\frac{1}{3}} \sqrt{3}\left(91 x^9-168 x^6+84 x^3-8\right)+12 \sqrt{3}\left(19 x^8-22 x^5+4 x^2\right)\left(x^3-1\right)^{\frac{1}{3}}\right)}{6\left(53 x^9-48 x^6-12 x^3+8\right)}\right)+50 \cdot 4^{\frac{2}{3}} x^8 \log\left(\frac{6 \cdot 4^{\frac{1}{3}}\left(x^3-1\right)^{\frac{1}{3}} x^2+4^{\frac{2}{3}}\left(x^3-2\right)-12\left(x^3-1\right)^{\frac{2}{3}} x}{x^3-2}\right)-25 \cdot 4^{\frac{2}{3}} x^8 \log\left(\frac{6 \cdot 4^{\frac{2}{3}}\left(2 x^4-x\right)\left(x^3-1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(19 x^6-22 x^3+4\right)+6\left(5 x^5-4 x^2\right)\left(x^3-1\right)^{\frac{1}{3}}}{x^6-4 x^3+4}\right)+36\left(7 x^6+8 x^3+10\right)\left(x^3-1\right)^{\frac{2}{3}}}{1440 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+x^3+4)/x^9/(x^3-2),x, algorithm="fricas")
```

```
[Out] 1/1440*(100*4^(1/6)*sqrt(3)*x^8*arctan(1/6*4^(1/6)*(12*4^(2/3)*sqrt(3)*(2*x
^7 - 5*x^4 + 2*x)*(x^3 - 1)^(2/3) + 4^(1/3)*sqrt(3)*(91*x^9 - 168*x^6 + 84*
x^3 - 8) + 12*sqrt(3)*(19*x^8 - 22*x^5 + 4*x^2)*(x^3 - 1)^(1/3))/(53*x^9 -
48*x^6 - 12*x^3 + 8)) + 50*4^(2/3)*x^8*log((6*4^(1/3)*(x^3 - 1)^(1/3)*x^2 +
 4^(2/3)*(x^3 - 2) - 12*(x^3 - 1)^(2/3)*x)/(x^3 - 2)) - 25*4^(2/3)*x^8*log(
(6*4^(2/3)*(2*x^4 - x)*(x^3 - 1)^(2/3) + 4^(1/3)*(19*x^6 - 22*x^3 + 4) + 6*
(5*x^5 - 4*x^2)*(x^3 - 1)^(1/3))/(x^6 - 4*x^3 + 4)) + 36*(7*x^6 + 8*x^3 + 1
0)*(x^3 - 1)^(2/3))/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^3+4\right)\left(x^3-1\right)^{\frac{2}{3}}}{\left(x^3-2\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+x^3+4)/x^9/(x^3-2),x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^3 + 4)*(x^3 - 1)^(2/3)/((x^3 - 2)*x^9), x)
```

maple [C] time = 2.60, size = 826, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)*(x^6+x^3+4)/x^9/(x^3-2),x)
```

```
[Out] 1/40*(7*x^9+x^6+2*x^3-10)/x^8/(x^3-1)^(1/3)-5/24*ln((-36*RootOf(RootOf(_Z^3
-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^3+3*RootOf(RootOf(_
Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^3+30*(x^3-1)^(2/3)
*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x+30
*(x^3-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(
_Z^3-2)*x^2+RootOf(_Z^3-2)^2*(x^3-1)^(1/3)*x^2+24*RootOf(RootOf(_Z^3-2)^2+6
*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3-2*RootOf(_Z^3-2)*x^3+2*x*(x^3-1)^(2/3)-24*R
ootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)+2*RootOf(_Z^3-2))/(x^3-
2))*RootOf(_Z^3-2)-5/4*ln((-36*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+
36*_Z^2)^2*RootOf(_Z^3-2)^2*x^3+3*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-
2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^3+30*(x^3-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+6
*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x+30*(x^3-1)^(1/3)*RootOf(Root
Of(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^2+RootOf(_Z^3-2)
^2*(x^3-1)^(1/3)*x^2+24*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2
)*x^3-2*RootOf(_Z^3-2)*x^3+2*x*(x^3-1)^(2/3)-24*RootOf(RootOf(_Z^3-2)^2+6*_
Z*RootOf(_Z^3-2)+36*_Z^2)+2*RootOf(_Z^3-2))/(x^3-2))*RootOf(RootOf(_Z^3-2)^
2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)+5/4*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3
-2)+36*_Z^2)*ln(-(-18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^
2*RootOf(_Z^3-2)^2*x^3+3*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^
2)*RootOf(_Z^3-2)^3*x^3+3*RootOf(_Z^3-2)^2*(x^3-1)^(1/3)*x^2-18*RootOf(Root
Of(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+3*RootOf(_Z^3-2)*x^3+6*x*(x^3
-1)^(2/3)+12*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-2*RootOf(
_Z^3-2))/(x^3-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^3+4\right)\left(x^3-1\right)^{\frac{2}{3}}}{\left(x^3-2\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+x^3+4)/x^9/(x^3-2),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + x^3 + 4)*(x^3 - 1)^(2/3)/((x^3 - 2)*x^9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3-1\right)^{2/3}\,\left(x^6+x^3+4\right)}{x^9\,\left(x^3-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(x^3 + x^6 + 4))/(x^9*(x^3 - 2)),x)
```

```
[Out] int(((x^3 - 1)^(2/3)*(x^3 + x^6 + 4))/(x^9*(x^3 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x^6+x^3+4\right)}{x^9\left(x^3-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)*(x**6+x**3+4)/x**9/(x**3-2),x)
```

```
[Out] Integral(((x - 1)*(x**2 + x + 1))**(2/3)*(x**6 + x**3 + 4)/(x**9*(x**3 - 2)
), x)
```

3.1696 $\int \frac{-3-4x+3x^6}{(1+2x+x^6)\sqrt[3]{1+2x+2x^3+x^6}}\,dx$

Optimal. Leaf size=145

$$\frac{\log\left(2^{2/3}\sqrt[3]{x^6+2x^3+2x+1}-2x\right)}{\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^6+2x^3+2x+1}+x}\right)}{\sqrt[3]{2}}-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^6+2x^3+2x+1}\,x+\sqrt[3]{\cdots}\right.}{2\sqrt[3]{2}}$$

Rubi [F] time = 1.94, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3-4x+3x^6}{(1+2x+x^6)\sqrt[3]{1+2x+2x^3+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3 - 4*x + 3*x^6)/((1 + 2*x + x^6)*(1 + 2*x + 2*x^3 + x^6)^(1/3)),x]
```

```
[Out] 3*Defer[Int][(1 + 2*x + 2*x^3 + x^6)^(-1/3), x] - Defer[Int][1/((1 + x)*(1
+ 2*x + 2*x^3 + x^6)^(1/3)), x] - 5*Defer[Int][1/((1 + x - x^2 + x^3 - x^4
+ x^5)*(1 + 2*x + 2*x^3 + x^6)^(1/3)), x] - 4*Defer[Int][x/((1 + x - x^2 +
x^3 - x^4 + x^5)*(1 + 2*x + 2*x^3 + x^6)^(1/3)), x] + 3*Defer[Int][x^2/((1
+ x - x^2 + x^3 - x^4 + x^5)*(1 + 2*x + 2*x^3 + x^6)^(1/3)), x] - 2*Defer[I
nt][x^3/((1 + x - x^2 + x^3 - x^4 + x^5)*(1 + 2*x + 2*x^3 + x^6)^(1/3)), x]
 + Defer[Int][x^4/((1 + x - x^2 + x^3 - x^4 + x^5)*(1 + 2*x + 2*x^3 + x^6)^
(1/3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3-4x+3x^6}{(1+2x+x^6)\sqrt[3]{1+2x+2x^3+x^6}}\,dx &= \int \left(\frac{3}{\sqrt[3]{1+2x+2x^3+x^6}}-\frac{2(3+5x)}{(1+2x+x^6)\sqrt[3]{1+2x+2x^3+x^6}}\right)dx\\
&= -\left(2\int \frac{3+5x}{(1+2x+x^6)\sqrt[3]{1+2x+2x^3+x^6}}\,dx\right)+3\int \frac{1}{\sqrt[3]{1+2x+2x^3 \cdots}}\\
&= -\left(2\int\left(\frac{1}{2(1+x)\sqrt[3]{1+2x+2x^3+x^6}}+\frac{5+4x-3x^2 \cdots}{2\left(1+x-x^2+x^3-x^4+x \cdots\right.}\right.\right.\\
&= 3\int \frac{1}{\sqrt[3]{1+2x+2x^3+x^6}}\,dx-\int \frac{1}{(1+x)\sqrt[3]{1+2x+2x^3+x^6}}\,dx- \cdots\\
&= 3\int \frac{1}{\sqrt[3]{1+2x+2x^3+x^6}}\,dx-\int \frac{1}{(1+x)\sqrt[3]{1+2x+2x^3+x^6}}\,dx- \cdots\\
&= -\left(2\int \frac{x^3}{\left(1+x-x^2+x^3-x^4+x^5\right)\sqrt[3]{1+2x+2x^3+x^6}}\,dx\right)+3\int \cdots
\end{aligned}$$

Mathematica [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{-3-4x+3x^6}{(1+2x+x^6)\sqrt[3]{1+2x+2x^3+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3 - 4*x + 3*x^6)/((1 + 2*x + x^6)*(1 + 2*x + 2*x^3 + x^6)^(1/3)
),x]
```

```
[Out] Integrate[(-3 - 4*x + 3*x^6)/((1 + 2*x + x^6)*(1 + 2*x + 2*x^3 + x^6)^(1/3)
), x]
```

IntegrateAlgebraic [A] time = 0.55, size = 145, normalized size = 1.00

$$\frac{\log \left(2^{2/3} \sqrt[3]{x^6+2 x^3+2 x+1}-2 x\right)}{\sqrt[3]{2}}-\frac{\sqrt{3} \tan ^{-1}\left(\frac{\sqrt{3} x}{2^{2/3} \sqrt[3]{x^6+2 x^3+2 x+1}+x}\right)}{\sqrt[3]{2}}-\frac{\log \left(2 x^2+2^{2/3} \sqrt[3]{x^6+2 x^3+2 x+1} x+\sqrt[3]{2} \left(x^6+2 x^3+2 x+1\right)^{2/3}\right)}{2 \sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 - 4*x + 3*x^6)/((1 + 2*x + x^6)*(1 + 2*x + 2*x^3 + x
^6)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(1 + 2*x + 2*x^3 + x^6)^(1/3))])
/2^(1/3)) + Log[-2*x + 2^(2/3)*(1 + 2*x + 2*x^3 + x^6)^(1/3)]/2^(1/3) - Log
[2*x^2 + 2^(2/3)*x*(1 + 2*x + 2*x^3 + x^6)^(1/3) + 2^(1/3)*(1 + 2*x + 2*x^3
 + x^6)^(2/3)]/(2*2^(1/3))
```

fricas [B] time = 43.55, size = 478, normalized size = 3.30

$$-\frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3} 2^{\frac{1}{6}} \left(2^{\frac{5}{6}} \left(x^{18}+36 x^{15}+6 x^{13}+183 x^{12}+144 x^{10}+288 x^{9}+12 x^{8}+372 x^{7}+183 x^{6}+144 x^{5}+144 x^{4}+44 x^{3}+12 x^{2}+6 x+1\right)+12 \sqrt{2} \left(x^{14}+18 x^{11}+4 x^{9}+38 x^{8}+36 x^{6}+18 x^{5}+4 x^{4}+4 x^{3}+x^{2}\right) \left(x^{6}+2 x^{3}+2 x+1\right)^{\frac{1}{3}}+12 \cdot 2^{\frac{1}{6}} \left(x^{13}+6 x^{10}+4 x^{8}+2 x^{7}+12 x^{5}+6 x^{4}+4 x^{3}+4 x^{2}+x\right) \left(x^{6}+2 x^{3}+2 x+1\right)^{\frac{2}{3}}\right)}{6 \left(x^{18}+6 x^{13}-105 x^{12}-216 x^{9}+12 x^{8}-204 x^{7}-105 x^{6}+8 x^{3}+12 x^{2}+6 x+1\right)}\right)+\frac{1}{6} \cdot 2^{\frac{2}{3}} \log\left(\frac{6 \cdot 2^{\frac{1}{3}} \left(x^{6}+2 x^{3}+2 x+1\right)^{\frac{1}{3}} x^{2}+2^{\frac{2}{3}} \left(x^{6}+2 x+1\right)-6 \left(x^{6}+2 x^{3}+2 x+1\right)^{\frac{2}{3}} x}{x^{6}+2 x+1}\right)-\frac{1}{12} \cdot 2^{\frac{2}{3}} \log\left(\frac{3 \cdot 2^{\frac{2}{3}} \left(x^{7}+6 x^{4}+2 x^{2}+x\right) \left(x^{6}+2 x^{3}+2 x+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}} \left(x^{12}+18 x^{9}+4 x^{7}+38 x^{6}+36 x^{4}+18 x^{3}+4 x^{2}+4 x+1\right)+12 \left(x^{8}+3 x^{5}+2 x^{3}+x^{2}\right) \left(x^{6}+2 x^{3}+2 x+1\right)^{\frac{1}{3}}}{x^{12}+4 x^{7}+2 x^{6}+4 x^{2}+4 x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-4*x-3)/(x^6+2*x+1)/(x^6+2*x^3+2*x+1)^(1/3),x, algorithm="f
ricas")
```

```
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^18 + 36*x^15 +
6*x^13 + 183*x^12 + 144*x^10 + 288*x^9 + 12*x^8 + 372*x^7 + 183*x^6 + 144*x
^5 + 144*x^4 + 44*x^3 + 12*x^2 + 6*x + 1) + 12*sqrt(2)*(x^14 + 18*x^11 + 4*
x^9 + 38*x^8 + 36*x^6 + 18*x^5 + 4*x^4 + 4*x^3 + x^2)*(x^6 + 2*x^3 + 2*x +
1)^(1/3) + 12*2^(1/6)*(x^13 + 6*x^10 + 4*x^8 + 2*x^7 + 12*x^5 + 6*x^4 + 4*x
^3 + 4*x^2 + x)*(x^6 + 2*x^3 + 2*x + 1)^(2/3))/(x^18 + 6*x^13 - 105*x^12 -
216*x^9 + 12*x^8 - 204*x^7 - 105*x^6 + 8*x^3 + 12*x^2 + 6*x + 1)) + 1/6*2^(
2/3)*log((6*2^(1/3)*(x^6 + 2*x^3 + 2*x + 1)^(1/3)*x^2 + 2^(2/3)*(x^6 + 2*x
+ 1) - 6*(x^6 + 2*x^3 + 2*x + 1)^(2/3)*x)/(x^6 + 2*x + 1)) - 1/12*2^(2/3)*l
og((3*2^(2/3)*(x^7 + 6*x^4 + 2*x^2 + x)*(x^6 + 2*x^3 + 2*x + 1)^(2/3) + 2^(
1/3)*(x^12 + 18*x^9 + 4*x^7 + 38*x^6 + 36*x^4 + 18*x^3 + 4*x^2 + 4*x + 1) +
 12*(x^8 + 3*x^5 + 2*x^3 + x^2)*(x^6 + 2*x^3 + 2*x + 1)^(1/3))/(x^12 + 4*x^
7 + 2*x^6 + 4*x^2 + 4*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3 \, x^{6} - 4 \, x - 3}{{\left(x^{6} + 2 \, x^{3} + 2 \, x + 1\right)}^{\frac{1}{3}} {\left(x^{6} + 2 \, x + 1\right)}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-4*x-3)/(x^6+2*x+1)/(x^6+2*x^3+2*x+1)^(1/3),x, algorithm="g
iac")
```

```
[Out] integrate((3*x^6 - 4*x - 3)/((x^6 + 2*x^3 + 2*x + 1)^(1/3)*(x^6 + 2*x + 1))
, x)
```

maple [C] time = 31.12, size = 1188, normalized size = 8.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^6-4*x-3)/(x^6+2*x+1)/(x^6+2*x^3+2*x+1)^(1/3),x)
```

```
[Out] -1/2*ln(-(RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4
)^3*x^3+2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3
-4)^2*x^3-RootOf(_Z^3-4)*x^6-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+
4*_Z^2)*x^6-(x^6+2*x^3+2*x+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^
3-4)+4*_Z^2)*RootOf(_Z^3-4)^2*x-2*(x^6+2*x^3+2*x+1)^(1/3)*RootOf(_Z^3-4)^2*
x^2-2*(x^6+2*x^3+2*x+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4
*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3-4*RootOf(RootOf(_Z^3-4)^2+2*
_Z*RootOf(_Z^3-4)+4*_Z^2)*x^3-4*(x^6+2*x^3+2*x+1)^(2/3)*x-2*RootOf(_Z^3-4)*
x-4*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-RootOf(_Z^3-4)-2*
RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(1+x)/(x^5-x^4+x^3-x^2
+x+1))*RootOf(_Z^3-4)-ln(-(RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z
^2)*RootOf(_Z^3-4)^3*x^3+2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z
^2)^2*RootOf(_Z^3-4)^2*x^3-RootOf(_Z^3-4)*x^6-2*RootOf(RootOf(_Z^3-4)^2+2*_
Z*RootOf(_Z^3-4)+4*_Z^2)*x^6-(x^6+2*x^3+2*x+1)^(2/3)*RootOf(RootOf(_Z^3-4)^
2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2*x-2*(x^6+2*x^3+2*x+1)^(1/3)*
RootOf(_Z^3-4)^2*x^2-2*(x^6+2*x^3+2*x+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z
*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3-4*RootOf(Ro
otOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^3-4*(x^6+2*x^3+2*x+1)^(2/3)*x-
2*RootOf(_Z^3-4)*x-4*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-
RootOf(_Z^3-4)-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(1+x)
/(x^5-x^4+x^3-x^2+x+1))*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)
+RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*ln(-(2*RootOf(RootOf(_
Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+2*RootOf(RootOf
(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^6+(x^6+2*x^3+2*x+1)^(2/3)*RootOf(R
ootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2*x-(x^6+2*x^3+2
*x+1)^(1/3)*RootOf(_Z^3-4)^2*x^2+2*(x^6+2*x^3+2*x+1)^(1/3)*RootOf(RootOf(_Z
^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x^2+8*RootOf(RootOf(_Z^3
-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^3-2*(x^6+2*x^3+2*x+1)^(2/3)*x+4*RootOf(
RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+2*RootOf(RootOf(_Z^3-4)^2+2*
_Z*RootOf(_Z^3-4)+4*_Z^2))/(1+x)/(x^5-x^4+x^3-x^2+x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^6 - 4\,x - 3}{\left(x^6 + 2\,x^3 + 2\,x + 1\right)^{\frac{1}{3}}\left(x^6 + 2\,x + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-4*x-3)/(x^6+2*x+1)/(x^6+2*x^3+2*x+1)^(1/3),x, algorithm="m
axima")
```

```
[Out] integrate((3*x^6 - 4*x - 3)/((x^6 + 2*x^3 + 2*x + 1)^(1/3)*(x^6 + 2*x + 1))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{-3\,x^6 + 4\,x + 3}{\left(x^6 + 2\,x + 1\right)\,\left(x^6 + 2\,x^3 + 2\,x + 1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x - 3*x^6 + 3)/((2*x + x^6 + 1)*(2*x + 2*x^3 + x^6 + 1)^(1/3)),x)
```

```
[Out] int(-(4*x - 3*x^6 + 3)/((2*x + x^6 + 1)*(2*x + 2*x^3 + x^6 + 1)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^6 - 4x - 3}{\sqrt[3]{\left(x^2 + 1\right)\left(x^4 - x^2 + 2x + 1\right)}\,(x + 1)\left(x^5 - x^4 + x^3 - x^2 + x + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**6-4*x-3)/(x**6+2*x+1)/(x**6+2*x**3+2*x+1)**(1/3),x)
```

```
[Out] Integral((3*x**6 - 4*x - 3)/(((x**2 + 1)*(x**4 - x**2 + 2*x + 1))**(1/3)*(x
 + 1)*(x**5 - x**4 + x**3 - x**2 + x + 1)), x)
```

3.1697 $\int \frac{(-b+ax^6)^{3/4}}{x}\,dx$

Optimal. Leaf size=145

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}}{\sqrt{ax^6-b}-\sqrt{b}}\right)}{3\sqrt{2}}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^6-b}}\right)}{3\sqrt{2}}+\frac{2}{9}\left(ax^6-b\right)^{3/4}$$

Rubi [A] time = 0.21, antiderivative size = 218, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 50, 63, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}+\sqrt{ax^6-b}+\sqrt{b}\right)}{6\sqrt{2}}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}+\sqrt{ax^6-b}+\sqrt{b}\right)}{6\sqrt{2}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^6-b}}{\sqrt[4]{b}}\right)}{3\sqrt{2}}-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^6-b}}{\sqrt[4]{b}}+1\right)}{3\sqrt{2}}+\frac{2}{9}\left(ax^6-b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^6)^(3/4)/x,x]
```

```
[Out] (2*(-b + a*x^6)^(3/4))/9 + (b^(3/4)*ArcTan[1 - (Sqrt[2]*(-b + a*x^6)^(1/4))
/b^(1/4)])/(3*Sqrt[2]) - (b^(3/4)*ArcTan[1 + (Sqrt[2]*(-b + a*x^6)^(1/4))/b
^(1/4)])/(3*Sqrt[2]) - (b^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^6)^
(1/4) + Sqrt[-b + a*x^6]])/(6*Sqrt[2]) + (b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(
1/4)*(-b + a*x^6)^(1/4) + Sqrt[-b + a*x^6]])/(6*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(-b+ax^6\right)^{3/4}}{x}\,dx = \frac{1}{6}\text{Subst}\left(\int \frac{(-b+ax)^{3/4}}{x}\,dx,x,x^6\right)$$

$$= \frac{2}{9}\left(-b+ax^6\right)^{3/4} - \frac{1}{6}b\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{-b+ax}}\,dx,x,x^6\right)$$

$$= \frac{2}{9}\left(-b+ax^6\right)^{3/4} - \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{-b+ax^6}\right)}{3a}$$

$$= \frac{2}{9}\left(-b+ax^6\right)^{3/4} + \frac{b\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{-b+ax^6}\right)}{3a} - \frac{b\,\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{-b+a\ldots}\right.}{3a}$$

$$= \frac{2}{9}\left(-b+ax^6\right)^{3/4} - \frac{b^{3/4}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx,x,\sqrt[4]{-b+ax^6}\right)}{6\sqrt{2}} - \frac{b^{3/4}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{\ldots}}{-\sqrt{b}+\sqrt{2}\ldots}\right.}{6\ldots}$$

$$= \frac{2}{9}\left(-b+ax^6\right)^{3/4} - \frac{b^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^6}+\sqrt{-b+ax^6}\right)}{6\sqrt{2}} + \frac{b^{3/4}\log\left(\sqrt{b}+\sqrt{2}\ldots\right.}{\ldots}$$

$$= \frac{2}{9}\left(-b+ax^6\right)^{3/4} + \frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^6}}{\sqrt[4]{b}}\right)}{3\sqrt{2}} - \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^6}}{\sqrt[4]{b}}\right)}{3\sqrt{2}} - \frac{b^{3/4}\log\left(\sqrt{\ldots}\right.}{\ldots}$$

Mathematica [A] time = 0.03, size = 84, normalized size = 0.58

$$\frac{2}{9}\left(ax^6-b\right)^{3/4}+\frac{1}{3}(-b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^6-b}}{\sqrt[4]{-b}}\right)-\frac{1}{3}(-b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^6-b}}{\sqrt[4]{-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^6)^(3/4)/x,x]
```

```
[Out] (2*(-b + a*x^6)^(3/4))/9 + ((-b)^(3/4)*ArcTan[(-b + a*x^6)^(1/4)/(-b)^(1/4)
])/3 - ((-b)^(3/4)*ArcTanh[(-b + a*x^6)^(1/4)/(-b)^(1/4)])/3
```

IntegrateAlgebraic [A] time = 0.19, size = 144, normalized size = 0.99

$$-\frac{b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt{ax^6-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^6-b}}\right)}{3\sqrt{2}}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}}{\sqrt{ax^6-b}+\sqrt{b}}\right)}{3\sqrt{2}}+\frac{2}{9}\left(ax^6-b\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^6)^(3/4)/x,x]
```

```
[Out] (2*(-b + a*x^6)^(3/4))/9 - (b^(3/4)*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b +
a*x^6]/(Sqrt[2]*b^(1/4)))/(-b + a*x^6)^(1/4)])/(3*Sqrt[2]) + (b^(3/4)*ArcTa
nh[(Sqrt[2]*b^(1/4)*(-b + a*x^6)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^6])])/(3*S
qrt[2])
```

fricas [A] time = 0.42, size = 159, normalized size = 1.10

$$\frac{2}{3}\left(-b^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^6-b\right)^{\frac{1}{4}}\left(-b^3\right)^{\frac{1}{4}}b^2-\sqrt{\sqrt{ax^6-b}\,b^4-\sqrt{-b^3}\,b^3}\,\left(-b^3\right)^{\frac{1}{4}}}{b^3}\right)-\frac{1}{6}\left(-b^3\right)^{\frac{1}{4}}\log\left(\left(ax^6-b\right)^{\frac{1}{4}}b^2+\left(-b^3\right)^{\frac{3}{4}}\right)+\frac{1}{6}\left(-b^3\right)^{\frac{1}{4}}\log\left(\left(ax^6-b\right)^{\frac{1}{4}}b^2-\left(-b^3\right)^{\frac{3}{4}}\right)+\frac{2}{9}\left(ax^6-b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-b)^(3/4)/x,x, algorithm="fricas")
```

```
[Out] 2/3*(-b^3)^(1/4)*arctan(-((a*x^6 - b)^(1/4)*(-b^3)^(1/4)*b^2 - sqrt(sqrt(a*
x^6 - b)*b^4 - sqrt(-b^3)*b^3)*(-b^3)^(1/4))/b^3) - 1/6*(-b^3)^(1/4)*log((a
*x^6 - b)^(1/4)*b^2 + (-b^3)^(3/4)) + 1/6*(-b^3)^(1/4)*log((a*x^6 - b)^(1/4
)*b^2 - (-b^3)^(3/4)) + 2/9*(a*x^6 - b)^(3/4)
```

giac [A] time = 0.25, size = 175, normalized size = 1.21

$$-\frac{1}{6}\sqrt{2}b^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^6-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)-\frac{1}{6}\sqrt{2}b^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^6-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)+\frac{1}{12}\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{2}\left(ax^6-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^6-b}+\sqrt{b}\right)-\frac{1}{12}\sqrt{2}b^{\frac{3}{4}}\log\left(-\sqrt{2}\left(ax^6-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^6-b}+\sqrt{b}\right)+\frac{2}{9}\left(ax^6-b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-b)^(3/4)/x,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*b^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^6 - b)^(1
/4))/b^(1/4)) - 1/6*sqrt(2)*b^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
2*(a*x^6 - b)^(1/4))/b^(1/4)) + 1/12*sqrt(2)*b^(3/4)*log(sqrt(2)*(a*x^6 - b
)^(1/4)*b^(1/4) + sqrt(a*x^6 - b) + sqrt(b)) - 1/12*sqrt(2)*b^(3/4)*log(-sq
rt(2)*(a*x^6 - b)^(1/4)*b^(1/4) + sqrt(a*x^6 - b) + sqrt(b)) + 2/9*(a*x^6 -
 b)^(3/4)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{6}-b\right)^{\frac{3}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6-b)^(3/4)/x,x)
```

```
[Out] int((a*x^6-b)^(3/4)/x,x)
```

maxima [A] time = 0.42, size = 178, normalized size = 1.23

$$-\frac{1}{12}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^6-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^6-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^6-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^6-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^6-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^6-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}\right)b+\frac{2}{9}\left(ax^6-b\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-b)^(3/4)/x,x, algorithm="maxima")
```

```
[Out] -1/12*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^6 - b)^(1/4))
/b^(1/4))/b^(1/4) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x
^6 - b)^(1/4))/b^(1/4))/b^(1/4) - sqrt(2)*log(sqrt(2)*(a*x^6 - b)^(1/4)*b^(
1/4) + sqrt(a*x^6 - b) + sqrt(b))/b^(1/4) + sqrt(2)*log(-sqrt(2)*(a*x^6 - b
)^(1/4)*b^(1/4) + sqrt(a*x^6 - b) + sqrt(b))/b^(1/4))*b + 2/9*(a*x^6 - b)^(
3/4)
```

mupad [B] time = 1.02, size = 64, normalized size = 0.44

$$\frac{2\left(a\,x^6-b\right)^{3/4}}{9}+\frac{{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(a\,x^6-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{3}-\frac{{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(a\,x^6-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6 - b)^(3/4)/x,x)
```

```
[Out] (2*(a*x^6 - b)^(3/4))/9 + ((-b)^(3/4)*atan((a*x^6 - b)^(1/4)/(-b)^(1/4)))/3
 - ((-b)^(3/4)*atanh((a*x^6 - b)^(1/4)/(-b)^(1/4)))/3
```

sympy [C] time = 1.08, size = 48, normalized size = 0.33

$$-\frac{a^{\frac{3}{4}}x^{\frac{9}{2}}\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^6}\right)}{6\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6-b)**(3/4)/x,x)
```

```
[Out] -a**(3/4)*x**(9/2)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), b*exp_polar(2*I*
pi)/(a*x**6))/(6*gamma(1/4))
```

3.1698 $\int \frac{b+ax^6}{\sqrt[3]{-x+x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{1}{162}(-14a-81b)\log\left(\sqrt[3]{x^3-x}-x\right)+\frac{1}{162}\left(14\sqrt{3}\,a+81\sqrt{3}\,b\right)\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-x}+x}\right)+\frac{1}{324}(14a+81b)\log\left(\sqrt[3]{x^3-\right.$$

Rubi [A] time = 0.23, antiderivative size = 271, normalized size of antiderivative = 1.87, number of steps used = 13, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2053, 2011, 329, 275, 239, 2024}

$$\frac{7}{27}a\left(x^3-x\right)^{2/3}+\frac{1}{6}a\left(x^3-x\right)^{2/3}x^4+\frac{7}{36}a\left(x^3-x\right)^{2/3}x^2-\frac{7a\sqrt[3]{x^2-1}\,\sqrt[3]{x}\,\log\left(x^{2/3}-\sqrt[3]{x^2-1}\right)}{54\sqrt[3]{x^3-x}}+\frac{7a\sqrt[3]{x^2-1}\,\sqrt[3]{x}\,\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1}{\sqrt{3}}\right)}{27\sqrt{3}\,\sqrt[3]{x^3-x}}-\frac{3b\sqrt[3]{x^2-1}\,\sqrt[3]{x}\,\log\left(x^{2/3}-\sqrt[3]{x^2-1}\right)}{4\sqrt[3]{x^3-x}}+\frac{\sqrt{3}\,b\sqrt[3]{x^2-1}\,\sqrt[3]{x}\,\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{x^3-x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^6)/(-x + x^3)^(1/3),x]
```

```
[Out] (7*a*(-x + x^3)^(2/3))/27 + (7*a*x^2*(-x + x^3)^(2/3))/36 + (a*x^4*(-x + x^
3)^(2/3))/6 + (7*a*x^(1/3)*(-1 + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3))/(-1 + x
^2)^(1/3))/Sqrt[3]])/(27*Sqrt[3]*(-x + x^3)^(1/3)) + (Sqrt[3]*b*x^(1/3)*(-1
 + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3))/(-1 + x^2)^(1/3))/Sqrt[3]])/(2*(-x +
x^3)^(1/3)) - (7*a*x^(1/3)*(-1 + x^2)^(1/3)*Log[x^(2/3) - (-1 + x^2)^(1/3)]
)/(54*(-x + x^3)^(1/3)) - (3*b*x^(1/3)*(-1 + x^2)^(1/3)*Log[x^(2/3) - (-1 +
 x^2)^(1/3)])/(4*(-x + x^3)^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
```

```
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2053

```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Expan
dIntegrand[Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && (Po
lyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IntegerQ[p] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{b+ax^6}{\sqrt[3]{-x+x^3}}\,dx &= \int \left(\frac{b}{\sqrt[3]{-x+x^3}}+\frac{ax^6}{\sqrt[3]{-x+x^3}}\right)dx \\
&= a\int \frac{x^6}{\sqrt[3]{-x+x^3}}\,dx+b\int \frac{1}{\sqrt[3]{-x+x^3}}\,dx \\
&= \frac{1}{6}ax^4\left(-x+x^3\right)^{2/3}+\frac{1}{9}(7a)\int \frac{x^4}{\sqrt[3]{-x+x^3}}\,dx+\frac{\left(b\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\right)\int \frac{1}{\sqrt[3]{x}\,\sqrt[3]{-1+x^2}}\,dx}{\sqrt[3]{-x+x^3}} \\
&= \frac{7}{36}ax^2\left(-x+x^3\right)^{2/3}+\frac{1}{6}ax^4\left(-x+x^3\right)^{2/3}+\frac{1}{27}(14a)\int \frac{x^2}{\sqrt[3]{-x+x^3}}\,dx+\frac{\left(3b\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\right)\mathrm{Sub}\ldots}{\sqrt[3]{-x\ldots}} \\
&= \frac{7}{27}a\left(-x+x^3\right)^{2/3}+\frac{7}{36}ax^2\left(-x+x^3\right)^{2/3}+\frac{1}{6}ax^4\left(-x+x^3\right)^{2/3}+\frac{1}{81}(14a)\int \frac{1}{\sqrt[3]{-x+x^3}}\,dx+\frac{\left(3b\ldots\right.}{\ldots} \\
&= \frac{7}{27}a\left(-x+x^3\right)^{2/3}+\frac{7}{36}ax^2\left(-x+x^3\right)^{2/3}+\frac{1}{6}ax^4\left(-x+x^3\right)^{2/3}+\frac{\sqrt{3}\,b\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x\ldots}{\sqrt[3]{-\ldots}}}{\sqrt{3}}\right.\ldots}{2\sqrt[3]{-x+x^3}} \\
&= \frac{7}{27}a\left(-x+x^3\right)^{2/3}+\frac{7}{36}ax^2\left(-x+x^3\right)^{2/3}+\frac{1}{6}ax^4\left(-x+x^3\right)^{2/3}+\frac{\sqrt{3}\,b\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x\ldots}{\sqrt[3]{-\ldots}}}{\sqrt{3}}\right.\ldots}{2\sqrt[3]{-x+x^3}} \\
&= \frac{7}{27}a\left(-x+x^3\right)^{2/3}+\frac{7}{36}ax^2\left(-x+x^3\right)^{2/3}+\frac{1}{6}ax^4\left(-x+x^3\right)^{2/3}+\frac{\sqrt{3}\,b\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x\ldots}{\sqrt[3]{-\ldots}}}{\sqrt{3}}\right.\ldots}{2\sqrt[3]{-x+x^3}} \\
&= \frac{7}{27}a\left(-x+x^3\right)^{2/3}+\frac{7}{36}ax^2\left(-x+x^3\right)^{2/3}+\frac{1}{6}ax^4\left(-x+x^3\right)^{2/3}+\frac{7a\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-1+x\ldots}}}{\sqrt{3}}\right.\ldots}{27\sqrt{3}\,\sqrt[3]{-x+x^3}}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 220, normalized size = 1.52

$$\frac{\sqrt[3]{x}\,\sqrt[3]{x^2-1}\left(-2(14a+81b)\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2-1}}\right)+2\sqrt{3}(14a+81b)\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1}{\sqrt{3}}\right)+54a\left(x^2-1\right)^{2/3}x^{14/3}+63a\left(x^2-1\right)^{2/3}x^{8/3}+84a\left(x^2-1\right)^{2/3}x^{2/3}+14a\log\left(\frac{x^{4/3}}{\left(x^2-1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2-1}}+1\right)+81b\log\left(\frac{x^{4/3}}{\left(x^2-1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2-1}}+1\right)\right)}{324\sqrt[3]{x\left(x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^6)/(-x + x^3)^(1/3),x]
```

```
[Out] (x^(1/3)*(-1 + x^2)^(1/3)*(84*a*x^(2/3)*(-1 + x^2)^(2/3) + 63*a*x^(8/3)*(-1
 + x^2)^(2/3) + 54*a*x^(14/3)*(-1 + x^2)^(2/3) + 2*Sqrt[3]*(14*a + 81*b)*Ar
cTan[(1 + (2*x^(2/3))/(-1 + x^2)^(1/3))/Sqrt[3]] - 2*(14*a + 81*b)*Log[1 -
x^(2/3)/(-1 + x^2)^(1/3)] + 14*a*Log[1 + x^(4/3)/(-1 + x^2)^(2/3) + x^(2/3)
```

```
/(-1 + x^2)^(1/3)] + 81*b*Log[1 + x^(4/3)/(-1 + x^2)^(2/3) + x^(2/3)/(-1 +
x^2)^(1/3)]))/(324*(x*(-1 + x^2))^(1/3))
```

IntegrateAlgebraic [A] time = 0.48, size = 145, normalized size = 1.00

$$\frac{1}{162}(-14a-81b)\log\left(\sqrt[3]{x^3-x}-x\right)+\frac{1}{162}\left(14\sqrt{3}a+81\sqrt{3}b\right)\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x}+x}\right)+\frac{1}{324}(14a+81b)\log\left(\sqrt[3]{x^3-x}\,x+\left(x^3-x\right)^{2/3}+x^2\right)+\frac{1}{108}\left(x^3-x\right)^{2/3}\left(18ax^4+21ax^2+28a\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^6)/(-x + x^3)^(1/3),x]
```

```
[Out] ((-x + x^3)^(2/3)*(28*a + 21*a*x^2 + 18*a*x^4))/108 + ((14*Sqrt[3]*a + 81*S
qrt[3]*b)*ArcTan[(Sqrt[3]*x)/(x + 2*(-x + x^3)^(1/3))])/162 + ((-14*a - 81*
b)*Log[-x + (-x + x^3)^(1/3)])/162 + ((14*a + 81*b)*Log[x^2 + x*(-x + x^3)^
(1/3) + (-x + x^3)^(2/3)])/324
```

fricas [A] time = 155.52, size = 128, normalized size = 0.88

$$\frac{1}{162}\sqrt{3}(14a+81b)\arctan\left(-\frac{44032959556\sqrt{3}\left(x^3-x\right)^{\frac{1}{3}}x+\sqrt{3}\left(16754327161x^2-2707204793\right)-10524305234\sqrt{3}\left(x^3-x\right)^{\frac{2}{3}}}{81835897185x^2-1102302937}\right)-\frac{1}{324}(14a+81b)\log\left(-3\left(x^3-x\right)^{\frac{1}{3}}x+3\left(x^3-x\right)^{\frac{2}{3}}+1\right)+\frac{1}{108}\left(18ax^4+21ax^2+28a\right)\left(x^3-x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/(x^3-x)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/162*sqrt(3)*(14*a + 81*b)*arctan(-(44032959556*sqrt(3)*(x^3 - x)^(1/3)*x
+ sqrt(3)*(16754327161*x^2 - 2707204793) - 10524305234*sqrt(3)*(x^3 - x)^(2
/3))/(81835897185*x^2 - 1102302937)) - 1/324*(14*a + 81*b)*log(-3*(x^3 - x)
^(1/3)*x + 3*(x^3 - x)^(2/3) + 1) + 1/108*(18*a*x^4 + 21*a*x^2 + 28*a)*(x^3
 - x)^(2/3)
```

giac [A] time = 0.25, size = 133, normalized size = 0.92

$$\frac{1}{108}\left(28a\left(\frac{1}{x^2}-1\right)^2\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}-77a\left(-\frac{1}{x^2}+1\right)^{\frac{5}{3}}+67a\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}\right)x^6-\frac{1}{162}\sqrt{3}(14a+81b)\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{324}(14a+81b)\log\left(\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{162}(14a+81b)\log\left(\left|\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/(x^3-x)^(1/3),x, algorithm="giac")
```

```
[Out] 1/108*(28*a*(1/x^2 - 1)^2*(-1/x^2 + 1)^(2/3) - 77*a*(-1/x^2 + 1)^(5/3) + 67
*a*(-1/x^2 + 1)^(2/3))*x^6 - 1/162*sqrt(3)*(14*a + 81*b)*arctan(1/3*sqrt(3)
*(2*(-1/x^2 + 1)^(1/3) + 1)) + 1/324*(14*a + 81*b)*log((-1/x^2 + 1)^(2/3) +
 (-1/x^2 + 1)^(1/3) + 1) - 1/162*(14*a + 81*b)*log(abs((-1/x^2 + 1)^(1/3) -
 1))
```

maple [C] time = 0.38, size = 98, normalized size = 0.68

$$\frac{a\left(18x^4+21x^2+28\right)x\left(x^2-1\right)}{108\left(x\left(x^2-1\right)\right)^{\frac{1}{3}}}+\frac{3b\left(-\mathrm{signum}\left(x^2-1\right)\right)^{\frac{1}{3}}x^{\frac{2}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^2\right)}{2\mathrm{signum}\left(x^2-1\right)^{\frac{1}{3}}}+\frac{7a\left(-\mathrm{signum}\left(x^2-1\right)\right)^{\frac{1}{3}}x^{\frac{2}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^2\right)}{27\mathrm{signum}\left(x^2-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6+b)/(x^3-x)^(1/3),x)
```

```
[Out] 1/108*a*(18*x^4+21*x^2+28)*x*(x^2-1)/(x*(x^2-1))^(1/3)+3/2*b/signum(x^2-1)^
(1/3)*(-signum(x^2-1))^(1/3)*x^(2/3)*hypergeom([1/3,1/3],[4/3],x^2)+7/27*a/
signum(x^2-1)^(1/3)*(-signum(x^2-1))^(1/3)*x^(2/3)*hypergeom([1/3,1/3],[4/3
],x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax^6+b}{\left(x^3-x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/(x^3-x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*x^6 + b)/(x^3 - x)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x^6+b}{{\left(x^3-x\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^6)/(x^3 - x)^(1/3),x)
```

```
[Out] int((b + a*x^6)/(x^3 - x)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{6} + b}{\sqrt[3]{x \left(x - 1\right) \left(x + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6+b)/(x**3-x)**(1/3),x)
```

```
[Out] Integral((a*x**6 + b)/(x*(x - 1)*(x + 1))**(1/3), x)
```

3.1699 $\int \frac{\sqrt[4]{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=145

$$\frac{2b\sqrt[4]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{a}-\frac{b\tan^{-1}\left(\sqrt[4]{2}\sqrt[4]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}\right)}{\sqrt[4]{2}\,a}-\frac{b\tanh^{-1}\left(\sqrt[4]{2}\sqrt[4]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}\right)}{\sqrt[4]{2}\,a}$$

Rubi [F] time = 0.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[4]{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])^(1/4)/Sqrt[-(a/b^2) + (a^2
*x^2)/b^2],x]
```

```
[Out] Defer[Int][(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])^(1/4)/Sqrt[-(a/b^2)
 + (a^2*x^2)/b^2], x]
```

Rubi steps

$$\int \frac{\sqrt[4]{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt[4]{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Mathematica [A] time = 1.13, size = 231, normalized size = 1.59

$$-\frac{ax\sqrt[4]{x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(bx\sqrt{\frac{a(ax^2-1)}{b^2}}+ax^2-1\right)\left(-2\sqrt[4]{2}\sqrt[4]{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}+\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)^2+a}}{\sqrt[4]{a}}\right)+\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)^2+a}}{\sqrt[4]{a}}\right)\right)}{\sqrt[4]{2}\sqrt{\frac{a(ax^2-1)}{b^2}}\left(ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])^(1/4)/Sqrt[-(a/b^2)
+ (a^2*x^2)/b^2],x]
```

```
[Out] -((a*x*(x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2]))^(1/4)*(-1 + a*x^2 + b*x*Sqr
t[(a*(-1 + a*x^2))/b^2])*(-2*2^(1/4)*(a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^
2]))^(1/4) + a^(1/4)*ArcTan[(a + (a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])^2)^(1
/4)/a^(1/4)] + a^(1/4)*ArcTanh[(a + (a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])^2)
^(1/4)/a^(1/4)]))/(2^(1/4)*Sqrt[(a*(-1 + a*x^2))/b^2]*(a*x*(a*x + b*Sqrt[(a
*(-1 + a*x^2))/b^2]))^(5/4)))
```

IntegrateAlgebraic [A] time = 3.76, size = 206, normalized size = 1.42

$$\frac{2b\sqrt[4]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{a}+\frac{b\log\left(\sqrt[4]{2}\sqrt[4]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}-1\right)}{2\sqrt[4]{2}\,a}-\frac{b\log\left(\sqrt[4]{2}\,a\sqrt[4]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}+a\right)}{2\sqrt[4]{2}\,a}-\frac{b\tan^{-1}\left(\sqrt[4]{2}\sqrt[4]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}\right)}{\sqrt[4]{2}\,a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])^(1/4)/Sqrt[
-(a/b^2) + (a^2*x^2)/b^2],x]
```

```
[Out] (2*b*(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])^(1/4))/a - (b*ArcTan[2^(1
/4)*(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])^(1/4)])/(2^(1/4)*a) + (b*L
og[-1 + 2^(1/4)*(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])^(1/4)])/(2*2^(
1/4)*a) - (b*Log[a + 2^(1/4)*a*(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])
^(1/4)])/(2*2^(1/4)*a)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/4)/(-a/b^2+a^2*x^2/b^2)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\, bx\right)^{\frac{1}{4}}}{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/4)/(-a/b^2+a^2*x^2/b^2)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)^(1/4)/sqrt(a^2*x^2/b^2 -
a/b^2), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\left(a\, x^2 + bx\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}\right)^{\frac{1}{4}}}{\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/4)/(-a/b^2+a^2*x^2/b^2)^(1/2)
,x)
```

```
[Out] int((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/4)/(-a/b^2+a^2*x^2/b^2)^(1/2)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\, bx\right)^{\frac{1}{4}}}{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/4)/(-a/b^2+a^2*x^2/b^2)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)^(1/4)/sqrt(a^2*x^2/b^2 -
a/b^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^2+b\,x\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}\right)^{1/4}}{\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/4)/((a^2*x^2)/b^2 - a/b^
2)^(1/2),x)
```

```
[Out] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/4)/((a^2*x^2)/b^2 - a/b^
2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x\left(ax+b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)}}{\sqrt{\frac{a\left(ax^2-1\right)}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x*(-a/b**2+a**2*x**2/b**2)**(1/2))**(1/4)/(-a/b**2+a**2
*x**2/b**2)**(1/2),x)
```

```
[Out] Integral((x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))**(1/4)/sqrt(a*(a*x**2
- 1)/b**2), x)
```

3.1700 $\int \frac{1}{\sqrt[3]{x^2(-a+x)}\,(a+(-1+d)x)}\,dx$

Optimal. Leaf size=146

$$\frac{\log\left(\sqrt[3]{d}\,x\sqrt[3]{x^3-ax^2}+\left(x^3-ax^2\right)^{2/3}+d^{2/3}x^2\right)}{2a\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{x^3-ax^2}-\sqrt[3]{d}\,x\right)}{a\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\,x}{2\sqrt[3]{x^3-ax^2}+\sqrt[3]{d}\,x}\right)}{a\sqrt[3]{d}}$$

Rubi [A] time = 0.34, antiderivative size = 246, normalized size of antiderivative = 1.68, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2081, 2077, 91}

$$\frac{\left(-a^2x\right)^{2/3}\sqrt[3]{a^2(x-a)}\log(a+(d-1)x)}{2a^3\sqrt[3]{d}\sqrt[3]{x^2(x-a)}}-\frac{3\left(-a^2x\right)^{2/3}\sqrt[3]{a^2(x-a)}\log\left(-\frac{\sqrt[3]{\frac{2}{3}}\sqrt[3]{a^2(x-a)}}{\sqrt[3]{d}}-\sqrt[3]{\frac{2}{3}}\sqrt[3]{-a^2x}\right)}{2a^3\sqrt[3]{d}\sqrt[3]{x^2(x-a)}}-\frac{\sqrt{3}\left(-a^2x\right)^{2/3}\sqrt[3]{a^2(x-a)}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a^2(x-a)}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{-a^2x}}\right)}{a^3\sqrt[3]{d}\sqrt[3]{x^2(x-a)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((x^2*(-a + x))^(1/3)*(a + (-1 + d)*x)),x]
```

```
[Out] -((Sqrt[3]*(-(a^2*x))^(2/3)*(a^2*(-a + x))^(1/3)*ArcTan[1/Sqrt[3] - (2*(a^2
*(-a + x))^(1/3))/(Sqrt[3]*d^(1/3)*(-(a^2*x))^(1/3))])/(a^3*d^(1/3)*(x^2*(-
a + x))^(1/3))) + ((-(a^2*x))^(2/3)*(a^2*(-a + x))^(1/3)*Log[a + (-1 + d)*x
])/(2*a^3*d^(1/3)*(x^2*(-a + x))^(1/3)) - (3*(-(a^2*x))^(2/3)*(a^2*(-a + x)
)^(1/3)*Log[-((2/3)^(1/3)*(-(a^2*x))^(1/3)) - ((2/3)^(1/3)*(a^2*(-a + x))^(
1/3))/d^(1/3)])/(2*a^3*d^(1/3)*(x^2*(-a + x))^(1/3))
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 2077

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Dist[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int
[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d, e
, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 2081

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x^2(-a+x)}\,(a+(-1+d)x)}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{3}(3a+a(-1+d))+(-1+d)x\right)\sqrt[3]{-\frac{2a^3}{27}-\frac{a^2x}{3}+x^3}}\,dx, x, -\frac{a}{3}\right.$$

$$= \frac{\left(2^{2/3}\left(-a^2x\right)^{2/3}\sqrt[3]{a^2(-a+x)}\right)\text{Subst}\left(\int \frac{1}{\left(-\frac{2a^3}{9}-\frac{2a^2x}{3}\right)^{2/3}\sqrt[3]{-\frac{2a^3}{9}+\frac{a^2x}{3}}\left(\frac{1}{3}(3a+a(-1+\right.}\right.}{3\sqrt[3]{-ax^2+x^3}}$$

$$= -\frac{\sqrt{3}\sqrt[3]{-a^2(a-x)}\left(-a^2x\right)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{-a^2(a-x)}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{-a^2x}}\right)}{a^3\sqrt[3]{d}\sqrt[3]{-ax^2+x^3}} + \frac{\sqrt[3]{-a^2(a-x)}\left(\right.}{2a^3\sqrt[3]{}}$$

Mathematica [C] time = 0.02, size = 37, normalized size = 0.25

$$\frac{3x\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{dx}{x-a}\right)}{a\sqrt[3]{x^2(x-a)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((x^2*(-a + x))^(1/3)*(a + (-1 + d)*x)),x]
```

```
[Out] (3*x*Hypergeometric2F1[1/3, 1, 4/3, (d*x)/(-a + x)])/(a*(x^2*(-a + x))^(1/3
))
```

IntegrateAlgebraic [A] time = 0.38, size = 146, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{d}\,x\sqrt[3]{x^3-ax^2}+\left(x^3-ax^2\right)^{2/3}+d^{2/3}x^2\right)}{2a\sqrt[3]{d}} - \frac{\log\left(\sqrt[3]{x^3-ax^2}-\sqrt[3]{d}\,x\right)}{a\sqrt[3]{d}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\,x}{2\sqrt[3]{x^3-ax^2}+\sqrt[3]{d}\,x}\right)}{a\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((x^2*(-a + x))^(1/3)*(a + (-1 + d)*x)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*x)/(d^(1/3)*x + 2*(-(a*x^2) + x^3)^(1/3))]
)/(a*d^(1/3)) - Log[-(d^(1/3)*x) + (-(a*x^2) + x^3)^(1/3)]/(a*d^(1/3)) + Lo
g[d^(2/3)*x^2 + d^(1/3)*x*(-(a*x^2) + x^3)^(1/3) + (-(a*x^2) + x^3)^(2/3)]/
(2*a*d^(1/3))
```

fricas [A] time = 0.44, size = 377, normalized size = 2.58

$$\left[\frac{\sqrt{3}d\sqrt{\frac{(-d)^{\frac{1}{3}}}{d}}\log\left(-\frac{(d+2)x^2-2ax-3\left(-ax^2+x^3\right)^{\frac{1}{3}}(-d)^{\frac{2}{3}}x-\sqrt{3}\left((-d)^{\frac{1}{3}}dx^2-\left(-ax^2+x^3\right)^{\frac{1}{3}}dx+2\left(-ax^2+x^3\right)^{\frac{2}{3}}(-d)^{\frac{2}{3}}\right)\sqrt{\frac{(-d)^{\frac{1}{3}}}{d}}}{(d-1)x^2+ax}\right)-2(-d)^{\frac{2}{3}}\log\left(\frac{(-d)^{\frac{1}{3}}x+\left(-ax^2+x^3\right)^{\frac{1}{3}}}{x}\right)+(-d)^{\frac{2}{3}}\log\left(\frac{(-d)^{\frac{2}{3}}x^2-\left(-ax^2+x^3\right)^{\frac{1}{3}}(-d)^{\frac{1}{3}}x+\left(-ax^2+x^3\right)^{\frac{2}{3}}}{x^2}\right)}{2ad}, -\frac{2\sqrt{3}d\sqrt{-\frac{(-d)^{\frac{1}{3}}}{d}}\arctan\left(-\frac{\sqrt{3}\left((-d)^{\frac{1}{3}}x-2\left(-ax^2+x^3\right)^{\frac{1}{3}}\right)\sqrt{-\frac{(-d)^{\frac{1}{3}}}{d}}}{3x}\right)+2(-d)^{\frac{2}{3}}\log\left(\frac{(-d)^{\frac{1}{3}}x+\left(-ax^2+x^3\right)^{\frac{1}{3}}}{x}\right)-(-d)^{\frac{2}{3}}\log\left(\frac{(-d)^{\frac{2}{3}}x^2-\left(-ax^2+x^3\right)^{\frac{1}{3}}(-d)^{\frac{1}{3}}x+\left(-ax^2+x^3\right)^{\frac{2}{3}}}{x^2}\right)}{2ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-a+x))^(1/3)/(a+(-1+d)*x),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*d*sqrt((-d)^(1/3)/d)*log(-((d + 2)*x^2 - 2*a*x - 3*(-a*x^2 +
x^3)^(1/3)*(-d)^(2/3)*x - sqrt(3)*((-d)^(1/3)*d*x^2 - (-a*x^2 + x^3)^(1/3)*
d*x + 2*(-a*x^2 + x^3)^(2/3)*(-d)^(2/3))*sqrt((-d)^(1/3)/d))/((d - 1)*x^2 +
 a*x)) - 2*(-d)^(2/3)*log(((-d)^(1/3)*x + (-a*x^2 + x^3)^(1/3))/x) + (-d)^(
2/3)*log(((-d)^(2/3)*x^2 - (-a*x^2 + x^3)^(1/3)*(-d)^(1/3)*x + (-a*x^2 + x^
3)^(2/3))/x^2))/(a*d), -1/2*(2*sqrt(3)*d*sqrt(-(-d)^(1/3)/d)*arctan(-1/3*sq
rt(3)*((-d)^(1/3)*x - 2*(-a*x^2 + x^3)^(1/3))*sqrt(-(-d)^(1/3)/d)/x) + 2*(-
```

```
d)^(2/3)*log(((-d)^(1/3)*x + (-a*x^2 + x^3)^(1/3))/x) - (-d)^(2/3)*log(((-d
)^(2/3)*x^2 - (-a*x^2 + x^3)^(1/3)*(-d)^(1/3)*x + (-a*x^2 + x^3)^(2/3))/x^2
))/(a*d)]
```

giac [A] time = 0.22, size = 100, normalized size = 0.68

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}+2\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}\right)}{3d^{\frac{1}{3}}}\right)}{ad^{\frac{1}{3}}}+\frac{\log\left(d^{\frac{2}{3}}+d^{\frac{1}{3}}\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}+\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}\right)}{2\,ad^{\frac{1}{3}}}-\frac{\log\left(\left|-d^{\frac{1}{3}}+\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}\right|\right)}{ad^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-a+x))^(1/3)/(a+(-1+d)*x),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(d^(1/3) + 2*(-a/x + 1)^(1/3))/d^(1/3))/(a*d^(1
/3)) + 1/2*log(d^(2/3) + d^(1/3)*(-a/x + 1)^(1/3) + (-a/x + 1)^(2/3))/(a*d^
(1/3)) - log(abs(-d^(1/3) + (-a/x + 1)^(1/3)))/(a*d^(1/3))
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{2}\left(-a+x\right)\right)^{\frac{1}{3}}\left(a+\left(-1+d\right)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(-a+x))^(1/3)/(a+(-1+d)*x),x)
```

```
[Out] int(1/(x^2*(-a+x))^(1/3)/(a+(-1+d)*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-(a-x)x^{2}\right)^{\frac{1}{3}}\left((d-1)x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-a+x))^(1/3)/(a+(-1+d)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((-(a - x)*x^2)^(1/3)*((d - 1)*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+x\,\left(d-1\right)\right)\,{\left(-x^2\,\left(a-x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + x*(d - 1))*(-x^2*(a - x))^(1/3)),x)
```

```
[Out] int(1/((a + x*(d - 1))*(-x^2*(a - x))^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x^{2}\left(-a+x\right)}\left(a+dx-x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(-a+x))**(1/3)/(a+(-1+d)*x),x)
```

```
[Out] Integral(1/((x**2*(-a + x))**(1/3)*(a + d*x - x)), x)
```

3.1701 $\int \frac{(2+x-x^3-x^4)^{2/3}(6+2x+x^4)(-2-x+x^3+x^4)}{x^6(-2-x+2x^3+x^4)}\,dx$

Optimal. Leaf size=146

$$-\log\left(\sqrt[3]{-x^4-x^3+x+2}-x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{-x^4-x^3+x+2}+x}\right)+\frac{3\left(-x^4-x^3+x+2\right)^{2/3}\left(2x^4-3x^3-2x\right.}{10x^5}$$

Rubi [F] time = 2.68, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(2+x-x^3-x^4)^{2/3}(6+2x+x^4)(-2-x+x^3+x^4)}{x^6(-2-x+2x^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((2 + x - x^3 - x^4)^(2/3)*(6 + 2*x + x^4)*(-2 - x + x^3 + x^4))/(x^6*(
-2 - x + 2*x^3 + x^4)),x]
```

```
[Out] Defer[Int][(2 + x - x^3 - x^4)^(2/3)/(1 - x), x] + 6*Defer[Int][(2 + x - x^
3 - x^4)^(2/3)/x^6, x] + 2*Defer[Int][(2 + x - x^3 - x^4)^(2/3)/x^5, x] + 3
*Defer[Int][(2 + x - x^3 - x^4)^(2/3)/x^3, x] + Defer[Int][(2 + x - x^3 - x
^4)^(2/3)/x^2, x]/2 + Defer[Int][(2 + x - x^3 - x^4)^(2/3)/x, x]/4 - Defer[
Int][(2 + x - x^3 - x^4)^(2/3)/(2 + x), x]/4 + (1 - I*Sqrt[3])*Defer[Int][(
2 + x - x^3 - x^4)^(2/3)/(1 - I*Sqrt[3] + 2*x), x] + (1 + I*Sqrt[3])*Defer[
Int][(2 + x - x^3 - x^4)^(2/3)/(1 + I*Sqrt[3] + 2*x), x]
```

Rubi steps

$$\int \frac{(2+x-x^3-x^4)^{2/3}(6+2x+x^4)(-2-x+x^3+x^4)}{x^6(-2-x+2x^3+x^4)}\,dx = \int \left(\frac{(2+x-x^3-x^4)^{2/3}}{1-x}+\frac{6(2+x-x^3-x^4)^{2/3}}{x^6}\right.$$
$$=\frac{1}{4}\int \frac{(2+x-x^3-x^4)^{2/3}}{x}\,dx-\frac{1}{4}\int \frac{(2+x-x^3}{2+x}$$
$$=\frac{1}{4}\int \frac{(2+x-x^3-x^4)^{2/3}}{x}\,dx-\frac{1}{4}\int \frac{(2+x-x^3}{2+x}$$
$$=\frac{1}{4}\int \frac{(2+x-x^3-x^4)^{2/3}}{x}\,dx-\frac{1}{4}\int \frac{(2+x-x^3}{2+x}$$

Mathematica [F] time = 1.25, size = 0, normalized size = 0.00

$$\int \frac{(2+x-x^3-x^4)^{2/3}(6+2x+x^4)(-2-x+x^3+x^4)}{x^6(-2-x+2x^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((2 + x - x^3 - x^4)^(2/3)*(6 + 2*x + x^4)*(-2 - x + x^3 + x^4))/
(x^6*(-2 - x + 2*x^3 + x^4)),x]
```

```
[Out] Integrate[((2 + x - x^3 - x^4)^(2/3)*(6 + 2*x + x^4)*(-2 - x + x^3 + x^4))/
(x^6*(-2 - x + 2*x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.23, size = 146, normalized size = 1.00

$$-\log\left(\sqrt[3]{-x^4-x^3+x+2}-x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{-x^4-x^3+x+2}+x}\right)+\frac{3\left(-x^4-x^3+x+2\right)^{2/3}\left(2x^4-3x^3-2x-4\right)}{10x^5}+\frac{1}{2}\log\left(x^2+\sqrt[3]{-x^4-x^3+x+2}\,x+\left(-x^4-x^3+x+2\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x - x^3 - x^4)^(2/3)*(6 + 2*x + x^4)*(-2 - x + x^3
 + x^4))/(x^6*(-2 - x + 2*x^3 + x^4)),x]
```

```
[Out] (3*(2 + x - x^3 - x^4)^(2/3)*(-4 - 2*x - 3*x^3 + 2*x^4))/(10*x^5) + Sqrt[3]
*ArcTan[(Sqrt[3]*x)/(x + 2*(2 + x - x^3 - x^4)^(1/3))] - Log[-x + (2 + x -
x^3 - x^4)^(1/3)] + Log[x^2 + x*(2 + x - x^3 - x^4)^(1/3) + (2 + x - x^3 -
x^4)^(2/3)]/2
```

fricas [A] time = 3.23, size = 204, normalized size = 1.40

$$-\frac{10\sqrt{3}x^5\arctan\left(-\frac{49772\sqrt{3}\left(-x^4-x^3+x+2\right)^{\frac{1}{3}}x^2-31378\sqrt{3}\left(-x^4-x^3+x+2\right)^{\frac{2}{3}}x-\sqrt{3}\left(17661x^4+26125x^3-17661x-35322\right)}{24389x^4-72947x^3-24389x-48778}\right)+5x^5\log\left(\frac{x^4+2x^3-3\left(-x^4-x^3+x+2\right)^{\frac{1}{3}}x^2+3\left(-x^4-x^3+x+2\right)^{\frac{2}{3}}x-x-2}{x^4+2x^3-x-2}\right)-3\left(2x^4-3x^3-2x-4\right)\left(-x^4-x^3+x+2\right)^{\frac{2}{3}}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-x^3+x+2)^(2/3)*(x^4+2*x+6)*(x^4+x^3-x-2)/x^6/(x^4+2*x^3-x-2
),x, algorithm="fricas")
```

```
[Out] -1/10*(10*sqrt(3)*x^5*arctan(-(49772*sqrt(3)*(-x^4 - x^3 + x + 2)^(1/3)*x^2
 - 31378*sqrt(3)*(-x^4 - x^3 + x + 2)^(2/3)*x - sqrt(3)*(17661*x^4 + 26125*
x^3 - 17661*x - 35322))/(24389*x^4 - 72947*x^3 - 24389*x - 48778)) + 5*x^5*
log((x^4 + 2*x^3 - 3*(-x^4 - x^3 + x + 2)^(1/3)*x^2 + 3*(-x^4 - x^3 + x + 2
)^(2/3)*x - x - 2)/(x^4 + 2*x^3 - x - 2)) - 3*(2*x^4 - 3*x^3 - 2*x - 4)*(-x
^4 - x^3 + x + 2)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3-x-2\right)\left(x^4+2x+6\right)\left(-x^4-x^3+x+2\right)^{\frac{2}{3}}}{\left(x^4+2x^3-x-2\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-x^3+x+2)^(2/3)*(x^4+2*x+6)*(x^4+x^3-x-2)/x^6/(x^4+2*x^3-x-2
),x, algorithm="giac")
```

```
[Out] integrate((x^4 + x^3 - x - 2)*(x^4 + 2*x + 6)*(-x^4 - x^3 + x + 2)^(2/3)/((
x^4 + 2*x^3 - x - 2)*x^6), x)
```

maple [C] time = 3.80, size = 698, normalized size = 4.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4-x^3+x+2)^(2/3)*(x^4+2*x+6)*(x^4+x^3-x-2)/x^6/(x^4+2*x^3-x-2),x)
```

```
[Out] -3/10*(2*x^8-x^7-3*x^6-4*x^5-7*x^4+2*x^3+2*x^2+8*x+8)/x^5/(-x^4-x^3+x+2)^(1
/3)+RootOf(_Z^2-_Z+1)*ln(-(-RootOf(_Z^2-_Z+1)^2*x^3-RootOf(_Z^2-_Z+1)*x^4+R
ootOf(_Z^2-_Z+1)*(-x^4-x^3+x+2)^(2/3)*x+RootOf(_Z^2-_Z+1)*(-x^4-x^3+x+2)^(1
/3)*x^2+(-x^4-x^3+x+2)^(2/3)*x+(-x^4-x^3+x+2)^(1/3)*x^2+RootOf(_Z^2-_Z+1)*x
+2*RootOf(_Z^2-_Z+1))/(2+x)/(RootOf(_Z^2-_Z+1)*x+1)/(-1-x+RootOf(_Z^2-_Z+1)
*x)/(-1+x))-ln((RootOf(_Z^2-_Z+1)^2*x^3-RootOf(_Z^2-_Z+1)*x^4+RootOf(_Z^2-_
Z+1)*(-x^4-x^3+x+2)^(2/3)*x+RootOf(_Z^2-_Z+1)*(-x^4-x^3+x+2)^(1/3)*x^2-2*Ro
otOf(_Z^2-_Z+1)*x^3+x^4-2*(-x^4-x^3+x+2)^(2/3)*x-2*(-x^4-x^3+x+2)^(1/3)*x^2
+x^3+RootOf(_Z^2-_Z+1)*x+2*RootOf(_Z^2-_Z+1)-x-2)/(2+x)/(RootOf(_Z^2-_Z+1)*
```

```
x+1)/(-1-x+RootOf(_Z^2-_Z+1)*x)/(-1+x))*RootOf(_Z^2-_Z+1)+ln((RootOf(_Z^2-_
Z+1)^2*x^3-RootOf(_Z^2-_Z+1)*x^4+RootOf(_Z^2-_Z+1)*(-x^4-x^3+x+2)^(2/3)*x+R
ootOf(_Z^2-_Z+1)*(-x^4-x^3+x+2)^(1/3)*x^2-2*RootOf(_Z^2-_Z+1)*x^3+x^4-2*(-x
^4-x^3+x+2)^(2/3)*x-2*(-x^4-x^3+x+2)^(1/3)*x^2+x^3+RootOf(_Z^2-_Z+1)*x+2*Ro
otOf(_Z^2-_Z+1)-x-2)/(2+x)/(RootOf(_Z^2-_Z+1)*x+1)/(-1-x+RootOf(_Z^2-_Z+1)*
x)/(-1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3-x-2\right)\left(x^4+2\,x+6\right)\left(-x^4-x^3+x+2\right)^{\frac{2}{3}}}{\left(x^4+2\,x^3-x-2\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-x^3+x+2)^(2/3)*(x^4+2*x+6)*(x^4+x^3-x-2)/x^6/(x^4+2*x^3-x-2
),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + x^3 - x - 2)*(x^4 + 2*x + 6)*(-x^4 - x^3 + x + 2)^(2/3)/((
x^4 + 2*x^3 - x - 2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4+2\,x+6\right)\,{\left(-x^4-x^3+x+2\right)}^{5/3}}{x^6\,\left(-x^4-2\,x^3+x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + x^4 + 6)*(x - x^3 - x^4 + 2)^(5/3))/(x^6*(x - 2*x^3 - x^4 + 2))
,x)
```

```
[Out] int(((2*x + x^4 + 6)*(x - x^3 - x^4 + 2)^(5/3))/(x^6*(x - 2*x^3 - x^4 + 2))
, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+2x+6\right)\left(-x^4-x^3+x+2\right)^{\frac{2}{3}}\left(x^4+x^3-x-2\right)}{x^6\left(x-1\right)\left(x+2\right)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4-x**3+x+2)**(2/3)*(x**4+2*x+6)*(x**4+x**3-x-2)/x**6/(x**4+2
*x**3-x-2),x)
```

```
[Out] Integral((x**4 + 2*x + 6)*(-x**4 - x**3 + x + 2)**(2/3)*(x**4 + x**3 - x -
2)/(x**6*(x - 1)*(x + 2)*(x**2 + x + 1)), x)
```

3.1702 $\int \frac{(-3+x^4)(1+x^4)^{2/3}(2+x^3+2x^4)}{x^6(4-x^3+4x^4)}\,dx$

Optimal. Leaf size=146

$$\frac{3\log\left(2^{2/3}\sqrt[3]{x^4+1}-x\right)}{16\sqrt[3]{2}}-\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\,2^{2/3}\sqrt[3]{x^4+1}+x}\right)}{16\sqrt[3]{2}}-\frac{3\log\left(2^{2/3}\sqrt[3]{x^4+1}\,x+2\sqrt[3]{2}\left(x^4+1\right)^{2/3}+x^2\right)}{32\sqrt[3]{2}}+\frac{3\left(x^4+1\right)^{2/3}}{}$$

Rubi [F] time = 1.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-3+x^4)\left(1+x^4\right)^{2/3}(2+x^3+2x^4)}{x^6(4-x^3+4x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + x^4)*(1 + x^4)^(2/3)*(2 + x^3 + 2*x^4))/(x^6*(4 - x^3 + 4*x^4)),
x]
```

```
[Out] (9*(1 + x^4)^(2/3))/(16*x^2) + (9*x^2)/(4*(1 - Sqrt[3] - (1 + x^4)^(1/3)))
- (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 + x^4)^(1/3))*Sqrt[(1 + (1 + x^4)^(1
/3) + (1 + x^4)^(2/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2]*EllipticE[ArcSin[
(1 + Sqrt[3] - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))], -7 + 4*Sq
rt[3]])/(8*x^2*Sqrt[-((1 - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))
^2)]) + (3*3^(3/4)*(1 - (1 + x^4)^(1/3))*Sqrt[(1 + (1 + x^4)^(1/3) + (1 + x
^4)^(2/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3]
 - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))], -7 + 4*Sqrt[3]])/(2*S
qrt[2]*x^2*Sqrt[-((1 - (1 + x^4)^(1/3))/(1 - Sqrt[3] - (1 + x^4)^(1/3))^2)]
) + (3*Hypergeometric2F1[-5/4, -2/3, -1/4, -x^4])/(10*x^5) - Hypergeometric
2F1[-2/3, -1/4, 3/4, -x^4]/(2*x) - (9*Defer[Int][(1 + x^4)^(2/3)/(4 - x^3 +
 4*x^4), x])/8 + 6*Defer[Int][(x*(1 + x^4)^(2/3))/(4 - x^3 + 4*x^4), x]
```

Rubi steps

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}\left(2+x^3+2x^4\right)}{x^6\left(4-x^3+4x^4\right)}\,dx = \int \left(-\frac{3\left(1+x^4\right)^{2/3}}{2x^6} - \frac{9\left(1+x^4\right)^{2/3}}{8x^3} + \frac{\left(1+x^4\right)^{2/3}}{2x^2} + \frac{3(-3+16x)\left(1\right.}{8\left(4-x^3+\right.}\right.$$

$$= \frac{3}{8}\int \frac{(-3+16x)\left(1+x^4\right)^{2/3}}{4-x^3+4x^4}\,dx + \frac{1}{2}\int \frac{\left(1+x^4\right)^{2/3}}{x^2}\,dx - \frac{9}{8}\int \frac{\left(1+\right.}{}$$

$$= \frac{3\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};-x^4\right)}{10x^5} - \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{4};\frac{3}{4};-x^4\right)}{2x} + \frac{3}{8}\int \left(-\frac{3\left(1+\right.}{4-x^3}\right.$$

$$= \frac{9\left(1+x^4\right)^{2/3}}{16x^2} + \frac{3\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};-x^4\right)}{10x^5} - \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{4};\frac{3}{4};-x^4\right)}{2x} -$$

$$= \frac{9\left(1+x^4\right)^{2/3}}{16x^2} + \frac{3\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};-x^4\right)}{10x^5} - \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{4};\frac{3}{4};-x^4\right)}{2x} -$$

$$= \frac{9\left(1+x^4\right)^{2/3}}{16x^2} + \frac{3\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};-x^4\right)}{10x^5} - \frac{{}_2F_1\left(-\frac{2}{3},-\frac{1}{4};\frac{3}{4};-x^4\right)}{2x} -$$

$$= \frac{9\left(1+x^4\right)^{2/3}}{16x^2} + \frac{9x^2}{4\left(1-\sqrt{3}-\sqrt[3]{1+x^4}\right)} - \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1}\right.}{}$$

Mathematica [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(-3+x^4\right)\left(1+x^4\right)^{2/3}\left(2+x^3+2x^4\right)}{x^6\left(4-x^3+4x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + x^4)*(1 + x^4)^(2/3)*(2 + x^3 + 2*x^4))/(x^6*(4 - x^3 + 4*
x^4)),x]
```

```
[Out] Integrate[((-3 + x^4)*(1 + x^4)^(2/3)*(2 + x^3 + 2*x^4))/(x^6*(4 - x^3 + 4*
x^4)), x]
```

IntegrateAlgebraic [A] time = 3.59, size = 146, normalized size = 1.00

$$\frac{3\log\left(2^{2/3}\sqrt[3]{x^4+1}-x\right)}{16\sqrt[3]{2}} - \frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\,2^{2/3}\sqrt[3]{x^4+1}+x}\right)}{16\sqrt[3]{2}} - \frac{3\log\left(2^{2/3}\sqrt[3]{x^4+1}\,x+2\sqrt[3]{2}\left(x^4+1\right)^{2/3}+x^2\right)}{32\sqrt[3]{2}} + \frac{3\left(x^4+1\right)^{2/3}\left(8x^4+15x^3+8\right)}{80x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 + x^4)^(2/3)*(2 + x^3 + 2*x^4))/(x^6*(4 -
 x^3 + 4*x^4)),x]
```

```
[Out] (3*(1 + x^4)^(2/3)*(8 + 15*x^3 + 8*x^4))/(80*x^5) - (3*Sqrt[3]*ArcTan[(Sqrt
[3]*x)/(x + 2*2^(2/3)*(1 + x^4)^(1/3))])/(16*2^(1/3)) + (3*Log[-x + 2^(2/3)
*(1 + x^4)^(1/3)])/(16*2^(1/3)) - (3*Log[x^2 + 2^(2/3)*x*(1 + x^4)^(1/3) +
2*2^(1/3)*(1 + x^4)^(2/3)])/(32*2^(1/3))
```

fricas [B] time = 108.41, size = 410, normalized size = 2.81

$$-\frac{10\sqrt{3}\,2^{\frac{2}{3}}x^5\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}\left(64x^{12}+240x^{11}+48x^{10}-x^9+192x^8+480x^7+48x^6+192x^4+240x^3+64\right)+12\sqrt{2}\left(16x^{10}+28x^9+x^8+32x^6+28x^5+16x^2\right)\left(x^4+1\right)^{\frac{1}{3}}+48\cdot 2^{\frac{1}{6}}\left(8x^9+2x^8-x^7+16x^5+2x^4+8x\right)\left(x^4+1\right)^{\frac{2}{3}}\right)}{6\left(64x^{12}-48x^{11}-96x^{10}-x^9+192x^8-96x^7-96x^6+192x^4-48x^3+64\right)}\right)-10\cdot 2^{\frac{2}{3}}x^5\log\left(\frac{6\cdot 2^{\frac{1}{3}}\left(x^4+1\right)^{\frac{1}{3}}x^2+2^{\frac{2}{3}}\left(4x^4-x^3+4\right)-12\left(x^4+1\right)^{\frac{2}{3}}x}{4x^4-x^3+4}\right)+5\cdot 2^{\frac{2}{3}}x^5\log\left(\frac{12\cdot 2^{\frac{2}{3}}\left(2x^5+x^4+2x\right)\left(x^4+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(16x^8+28x^7+x^6+32x^4+28x^3+16\right)+6\left(8x^6+x^5+8x^2\right)\left(x^4+1\right)^{\frac{1}{3}}}{16x^8-8x^7+x^6+32x^4-8x^3+16}\right)-12\left(8x^4+15x^3+8\right)\left(x^4+1\right)^{\frac{2}{3}}}{320x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)*(2*x^4+x^3+2)/x^6/(4*x^4-x^3+4),x, algorith
m="fricas")
```

```
[Out] -1/320*(10*sqrt(3)*2^(2/3)*x^5*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(64*x^12
 + 240*x^11 + 48*x^10 - x^9 + 192*x^8 + 480*x^7 + 48*x^6 + 192*x^4 + 240*x^
3 + 64) + 12*sqrt(2)*(16*x^10 + 28*x^9 + x^8 + 32*x^6 + 28*x^5 + 16*x^2)*(x
^4 + 1)^(1/3) + 48*2^(1/6)*(8*x^9 + 2*x^8 - x^7 + 16*x^5 + 2*x^4 + 8*x)*(x^
4 + 1)^(2/3))/(64*x^12 - 48*x^11 - 96*x^10 - x^9 + 192*x^8 - 96*x^7 - 96*x^
6 + 192*x^4 - 48*x^3 + 64)) - 10*2^(2/3)*x^5*log((6*2^(1/3)*(x^4 + 1)^(1/3)
*x^2 + 2^(2/3)*(4*x^4 - x^3 + 4) - 12*(x^4 + 1)^(2/3)*x)/(4*x^4 - x^3 + 4))
 + 5*2^(2/3)*x^5*log((12*2^(2/3)*(2*x^5 + x^4 + 2*x)*(x^4 + 1)^(2/3) + 2^(1
/3)*(16*x^8 + 28*x^7 + x^6 + 32*x^4 + 28*x^3 + 16) + 6*(8*x^6 + x^5 + 8*x^2
)*(x^4 + 1)^(1/3))/(16*x^8 - 8*x^7 + x^6 + 32*x^4 - 8*x^3 + 16)) - 12*(8*x^
4 + 15*x^3 + 8)*(x^4 + 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^4+x^3+2\right)\left(x^4+1\right)^{\frac{2}{3}}\left(x^4-3\right)}{\left(4\,x^4-x^3+4\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)*(2*x^4+x^3+2)/x^6/(4*x^4-x^3+4),x, algorith
m="giac")
```

```
[Out] integrate((2*x^4 + x^3 + 2)*(x^4 + 1)^(2/3)*(x^4 - 3)/((4*x^4 - x^3 + 4)*x^
6), x)
```

maple [C] time = 103.81, size = 668, normalized size = 4.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4+1)^(2/3)*(2*x^4+x^3+2)/x^6/(4*x^4-x^3+4),x)
```

```
[Out] 3/80*(8*x^8+15*x^7+16*x^4+15*x^3+8)/x^5/(x^4+1)^(1/3)+3/8*RootOf(RootOf(_Z^
3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)*ln(-(4*RootOf(RootOf(_Z^3-4)^2+4*_Z*Roo
tOf(_Z^3-4)+16*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+RootOf(RootOf(_Z^3-4)^2+4*_Z*Ro
otOf(_Z^3-4)+16*_Z^2)*RootOf(_Z^3-4)^3*x^3-4*(x^4+1)^(2/3)*RootOf(RootOf(_Z
^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)*RootOf(_Z^3-4)^2*x+8*RootOf(_Z^3-4)*(x
^4+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)*x^2-16*Roo
tOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)*x^4+RootOf(_Z^3-4)^2*(x^4
+1)^(1/3)*x^2-4*RootOf(_Z^3-4)*x^4+4*(x^4+1)^(2/3)*x-16*RootOf(RootOf(_Z^3-
4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)-4*RootOf(_Z^3-4))/(4*x^4-x^3+4))+3/32*Roo
tOf(_Z^3-4)*ln((4*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)^2*Ro
otOf(_Z^3-4)^2*x^3+RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)*Roo
tOf(_Z^3-4)^3*x^3-4*(x^4+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-
4)+16*_Z^2)*RootOf(_Z^3-4)^2*x-4*RootOf(_Z^3-4)*(x^4+1)^(1/3)*RootOf(RootOf
(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)*x^2+16*RootOf(RootOf(_Z^3-4)^2+4*_Z
*RootOf(_Z^3-4)+16*_Z^2)*x^4+RootOf(_Z^3-4)^2*(x^4+1)^(1/3)*x^2+4*RootOf(_Z
^3-4)*x^4+4*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-4)+16*_Z^2)*x^3+RootOf
(_Z^3-4)*x^3-8*(x^4+1)^(2/3)*x+16*RootOf(RootOf(_Z^3-4)^2+4*_Z*RootOf(_Z^3-
4)+16*_Z^2)+4*RootOf(_Z^3-4))/(4*x^4-x^3+4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^4+x^3+2\right)\left(x^4+1\right)^{\frac{2}{3}}\left(x^4-3\right)}{\left(4\,x^4-x^3+4\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4+1)^(2/3)*(2*x^4+x^3+2)/x^6/(4*x^4-x^3+4),x, algorith
m="maxima")
```

```
[Out] integrate((2*x^4 + x^3 + 2)*(x^4 + 1)^(2/3)*(x^4 - 3)/((4*x^4 - x^3 + 4)*x^
6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4+1\right)^{2/3}\,\left(x^4-3\right)\,\left(2\,x^4+x^3+2\right)}{x^6\,\left(4\,x^4-x^3+4\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(2/3)*(x^4 - 3)*(x^3 + 2*x^4 + 2))/(x^6*(4*x^4 - x^3 + 4)),x
)
```

```
[Out] int(((x^4 + 1)^(2/3)*(x^4 - 3)*(x^3 + 2*x^4 + 2))/(x^6*(4*x^4 - x^3 + 4)),
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4+1)**(2/3)*(2*x**4+x**3+2)/x**6/(4*x**4-x**3+4),x)
```

```
[Out] Timed out
```

3.1703 $\int \frac{\left(-1-x^4+2x^6\right)\sqrt[3]{x-x^5+x^7}}{\left(1+x^2-x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=146

$$\frac{1}{6}\log\left(\sqrt[3]{x^7-x^5+x}+x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^7-x^5+x}}{\sqrt[3]{x^7-x^5+x}-2x}\right)}{2\sqrt{3}}-\frac{1}{12}\log\left(x^2-\sqrt[3]{x^7-x^5+x}\,x+\left(x^7-x^5+x\right)^{2/3}\right)-\frac{\sqrt[3]{x^7-x^5+}}{2\left(x^6-x^4+x^{}\right.}$$

Rubi [F] time = 2.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1-x^4+2x^6\right)\sqrt[3]{x-x^5+x^7}}{\left(1+x^2-x^4+x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 - x^4 + 2*x^6)*(x - x^5 + x^7)^(1/3))/(1 + x^2 - x^4 + x^6)^2,x]
```

```
[Out] (-9*(x - x^5 + x^7)^(1/3)*Defer[Subst][Defer[Int][(x*(1 - x^6 + x^9)^(1/3))
/(1 + x^3 - x^6 + x^9)^2, x], x, x^(2/3)])/(2*x^(1/3)*(1 - x^4 + x^6)^(1/3)
) - (3*(x - x^5 + x^7)^(1/3)*Defer[Subst][Defer[Int][(x^4*(1 - x^6 + x^9)^(
1/3))/(1 + x^3 - x^6 + x^9)^2, x], x, x^(2/3)])/(x^(1/3)*(1 - x^4 + x^6)^(1
/3)) + (3*(x - x^5 + x^7)^(1/3)*Defer[Subst][Defer[Int][(x^7*(1 - x^6 + x^9
)^(1/3))/(1 + x^3 - x^6 + x^9)^2, x], x, x^(2/3)])/(2*x^(1/3)*(1 - x^4 + x^
6)^(1/3)) + (3*(x - x^5 + x^7)^(1/3)*Defer[Subst][Defer[Int][(x*(1 - x^6 +
x^9)^(1/3))/(1 + x^3 - x^6 + x^9), x], x, x^(2/3)])/(x^(1/3)*(1 - x^4 + x^6
)^(1/3))
```

Rubi steps

$$\int \frac{\left(-1-x^4+2x^6\right)\sqrt[3]{x-x^5+x^7}}{\left(1+x^2-x^4+x^6\right)^2}\,dx = \frac{\sqrt[3]{x-x^5+x^7}\int \frac{\sqrt[3]{x}\sqrt[3]{1-x^4+x^6}\left(-1-x^4+2x^6\right)}{\left(1+x^2-x^4+x^6\right)^2}dx}{\sqrt[3]{x}\sqrt[3]{1-x^4+x^6}}$$

$$= \frac{\left(3\sqrt[3]{x-x^5+x^7}\right)\text{Subst}\left(\int \frac{x^3\sqrt[3]{1-x^{12}+x^{18}}\left(-1-x^{12}+2x^{18}\right)}{\left(1+x^6-x^{12}+x^{18}\right)^2}dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1-x^4+x^6}}$$

$$= \frac{\left(3\sqrt[3]{x-x^5+x^7}\right)\text{Subst}\left(\int \frac{x\sqrt[3]{1-x^6+x^9}\left(-1-x^6+2x^9\right)}{\left(1+x^3-x^6+x^9\right)^2}dx,x,x^{2/3}\right)}{2\sqrt[3]{x}\sqrt[3]{1-x^4+x^6}}$$

$$= \frac{\left(3\sqrt[3]{x-x^5+x^7}\right)\text{Subst}\left(\int \left(\frac{x\left(-3-2x^3+x^6\right)\sqrt[3]{1-x^6+x^9}}{\left(1+x^3-x^6+x^9\right)^2}+\frac{2x\sqrt[3]{1-x^6+x^9}}{1+x^3-x^6+x^9}\right)dx,x,x\right.}{2\sqrt[3]{x}\sqrt[3]{1-x^4+x^6}}$$

$$= \frac{\left(3\sqrt[3]{x-x^5+x^7}\right)\text{Subst}\left(\int \frac{x\left(-3-2x^3+x^6\right)\sqrt[3]{1-x^6+x^9}}{\left(1+x^3-x^6+x^9\right)^2}dx,x,x^{2/3}\right)}{2\sqrt[3]{x}\sqrt[3]{1-x^4+x^6}} + \frac{\left(3\sqrt[3]{x-x\right.}{}$$

$$= \frac{\left(3\sqrt[3]{x-x^5+x^7}\right)\text{Subst}\left(\int \left(-\frac{3x\sqrt[3]{1-x^6+x^9}}{\left(1+x^3-x^6+x^9\right)^2}-\frac{2x^4\sqrt[3]{1-x^6+x^9}}{\left(1+x^3-x^6+x^9\right)^2}+\frac{x^7\sqrt[3]{1-x^6+x^9}}{\left(1+x^3-x^6+x^9\right.}\right.\right.}{2\sqrt[3]{x}\sqrt[3]{1-x^4+x^6}}$$

$$= \frac{\left(3\sqrt[3]{x-x^5+x^7}\right)\text{Subst}\left(\int \frac{x^7\sqrt[3]{1-x^6+x^9}}{\left(1+x^3-x^6+x^9\right)^2}dx,x,x^{2/3}\right)}{2\sqrt[3]{x}\sqrt[3]{1-x^4+x^6}} - \frac{\left(3\sqrt[3]{x-x^5+x^7}\right)}{}$$

Mathematica [F] time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{\left(-1-x^4+2x^6\right)\sqrt[3]{x-x^5+x^7}}{\left(1+x^2-x^4+x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 - x^4 + 2*x^6)*(x - x^5 + x^7)^(1/3))/(1 + x^2 - x^4 + x^6)^
2,x]
```

```
[Out] Integrate[((-1 - x^4 + 2*x^6)*(x - x^5 + x^7)^(1/3))/(1 + x^2 - x^4 + x^6)^
2, x]
```

IntegrateAlgebraic [A] time = 4.92, size = 146, normalized size = 1.00

$$\frac{1}{6}\log\left(\sqrt[3]{x^7-x^5+x}+x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^7-x^5+x}}{\sqrt[3]{x^7-x^5+x}-2x}\right)}{2\sqrt{3}}-\frac{1}{12}\log\left(x^2-\sqrt[3]{x^7-x^5+x}\,x+\left(x^7-x^5+x\right)^{2/3}\right)-\frac{\sqrt[3]{x^7-x^5+x}\,x}{2\left(x^6-x^4+x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 - x^4 + 2*x^6)*(x - x^5 + x^7)^(1/3))/(1 + x^2 - x^
4 + x^6)^2,x]
```

```
[Out] -1/2*(x*(x - x^5 + x^7)^(1/3))/(1 + x^2 - x^4 + x^6) - ArcTan[(Sqrt[3]*(x -
 x^5 + x^7)^(1/3))/(-2*x + (x - x^5 + x^7)^(1/3))]/(2*Sqrt[3]) + Log[x + (x
 - x^5 + x^7)^(1/3)]/6 - Log[x^2 - x*(x - x^5 + x^7)^(1/3) + (x - x^5 + x^7
)^(2/3)]/12
```

fricas [A] time = 3.05, size = 198, normalized size = 1.36

$$-\frac{2\sqrt{3}\left(x^{6}-x^{4}+x^{2}+1\right)\arctan\left(-\frac{2\sqrt{3}\left(x^{7}-x^{5}+x\right)^{\frac{1}{3}}x+\sqrt{3}\left(x^{6}-x^{4}-x^{2}+1\right)-2\sqrt{3}\left(x^{7}-x^{5}+x\right)^{\frac{2}{3}}}{x^{6}-x^{4}+x^{2}+1}\right)-\left(x^{6}-x^{4}+x^{2}+1\right)\log\left(\frac{x^{6}-x^{4}+x^{2}+3\left(x^{7}-x^{5}+x\right)^{\frac{1}{3}}x+3\left(x^{7}-x^{5}+x\right)^{\frac{2}{3}}+1}{x^{6}-x^{4}+x^{2}+1}\right)+6\left(x^{7}-x^{5}+x\right)^{\frac{1}{3}}x}{12\left(x^{6}-x^{4}+x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6-x^4-1)*(x^7-x^5+x)^(1/3)/(x^6-x^4+x^2+1)^2,x, algorithm="f
ricas")
```

```
[Out] -1/12*(2*sqrt(3)*(x^6 - x^4 + x^2 + 1)*arctan(-(2*sqrt(3)*(x^7 - x^5 + x)^(
1/3)*x + sqrt(3)*(x^6 - x^4 - x^2 + 1) - 2*sqrt(3)*(x^7 - x^5 + x)^(2/3))/(
x^6 - x^4 + x^2 + 1)) - (x^6 - x^4 + x^2 + 1)*log((x^6 - x^4 + x^2 + 3*(x^7
 - x^5 + x)^(1/3)*x + 3*(x^7 - x^5 + x)^(2/3) + 1)/(x^6 - x^4 + x^2 + 1)) +
 6*(x^7 - x^5 + x)^(1/3)*x)/(x^6 - x^4 + x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{7}-x^{5}+x\right)^{\frac{1}{3}}\left(2\,x^{6}-x^{4}-1\right)}{\left(x^{6}-x^{4}+x^{2}+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6-x^4-1)*(x^7-x^5+x)^(1/3)/(x^6-x^4+x^2+1)^2,x, algorithm="g
iac")
```

```
[Out] integrate((x^7 - x^5 + x)^(1/3)*(2*x^6 - x^4 - 1)/(x^6 - x^4 + x^2 + 1)^2,
x)
```

maple [C] time = 9.22, size = 1306, normalized size = 8.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6-x^4-1)*(x^7-x^5+x)^(1/3)/(x^6-x^4+x^2+1)^2,x)
```

```
[Out] -1/2*x/(x^6-x^4+x^2+1)*(x*(x^6-x^4+1))^(1/3)+(1/6*ln(-(8*RootOf(4*_Z^2+2*_Z
+1)^2*x^12+6*RootOf(4*_Z^2+2*_Z+1)*x^12-16*RootOf(4*_Z^2+2*_Z+1)^2*x^10+x^1
2-12*RootOf(4*_Z^2+2*_Z+1)*x^10-2*x^10+4*RootOf(4*_Z^2+2*_Z+1)*x^8+6*(x^14-
2*x^12+x^10+2*x^8-2*x^6+x^2)^(1/3)*RootOf(4*_Z^2+2*_Z+1)*x^6+24*RootOf(4*_Z
^2+2*_Z+1)^2*x^6+x^8+14*RootOf(4*_Z^2+2*_Z+1)*x^6-6*(x^14-2*x^12+x^10+2*x^8
-2*x^6+x^2)^(1/3)*RootOf(4*_Z^2+2*_Z+1)*x^4-16*RootOf(4*_Z^2+2*_Z+1)^2*x^4+
2*x^6-12*RootOf(4*_Z^2+2*_Z+1)*x^4-8*RootOf(4*_Z^2+2*_Z+1)^2*x^2-2*x^4+6*Ro
otOf(4*_Z^2+2*_Z+1)*(x^14-2*x^12+x^10+2*x^8-2*x^6+x^2)^(2/3)-2*RootOf(4*_Z^
2+2*_Z+1)*x^2+3*(x^14-2*x^12+x^10+2*x^8-2*x^6+x^2)^(2/3)+6*RootOf(4*_Z^2+2*
_Z+1)*(x^14-2*x^12+x^10+2*x^8-2*x^6+x^2)^(1/3)+8*RootOf(4*_Z^2+2*_Z+1)^2+6*
RootOf(4*_Z^2+2*_Z+1)+1)/(x^6-x^4+1)/(x^6-x^4+x^2+1))-1/6*ln((4*RootOf(4*_Z
^2+2*_Z+1)^2*x^12-2*RootOf(4*_Z^2+2*_Z+1)*x^12-8*RootOf(4*_Z^2+2*_Z+1)^2*x^
10+4*RootOf(4*_Z^2+2*_Z+1)*x^10-2*RootOf(4*_Z^2+2*_Z+1)*x^8+12*RootOf(4*_Z^
2+2*_Z+1)^2*x^6+x^8-3*(x^14-2*x^12+x^10+2*x^8-2*x^6+x^2)^(1/3)*x^6-4*RootOf
(4*_Z^2+2*_Z+1)*x^6-8*RootOf(4*_Z^2+2*_Z+1)^2*x^4-x^6+3*(x^14-2*x^12+x^10+2
*x^8-2*x^6+x^2)^(1/3)*x^4+4*RootOf(4*_Z^2+2*_Z+1)*x^4-4*RootOf(4*_Z^2+2*_Z+
1)^2*x^2+6*RootOf(4*_Z^2+2*_Z+1)*(x^14-2*x^12+x^10+2*x^8-2*x^6+x^2)^(2/3)+4
*RootOf(4*_Z^2+2*_Z+1)^2+x^2-3*(x^14-2*x^12+x^10+2*x^8-2*x^6+x^2)^(1/3)-2*R
ootOf(4*_Z^2+2*_Z+1))/(x^6-x^4+1)/(x^6-x^4+x^2+1))-1/3*ln((4*RootOf(4*_Z^2+
2*_Z+1)^2*x^12-2*RootOf(4*_Z^2+2*_Z+1)*x^12-8*RootOf(4*_Z^2+2*_Z+1)^2*x^10+
4*RootOf(4*_Z^2+2*_Z+1)*x^10-2*RootOf(4*_Z^2+2*_Z+1)*x^8+12*RootOf(4*_Z^2+2
*_Z+1)^2*x^6+x^8-3*(x^14-2*x^12+x^10+2*x^8-2*x^6+x^2)^(1/3)*x^6-4*RootOf(4*
_Z^2+2*_Z+1)*x^6-8*RootOf(4*_Z^2+2*_Z+1)^2*x^4-x^6+3*(x^14-2*x^12+x^10+2*x^
8-2*x^6+x^2)^(1/3)*x^4+4*RootOf(4*_Z^2+2*_Z+1)*x^4-4*RootOf(4*_Z^2+2*_Z+1)^
```

```
2*x^2+6*RootOf(4*_Z^2+2*_Z+1)*(x^14-2*x^12+x^10+2*x^8-2*x^6+x^2)^(2/3)+4*Ro
otOf(4*_Z^2+2*_Z+1)^2+x^2-3*(x^14-2*x^12+x^10+2*x^8-2*x^6+x^2)^(1/3)-2*Root
Of(4*_Z^2+2*_Z+1))/(x^6-x^4+1)/(x^6-x^4+x^2+1))*RootOf(4*_Z^2+2*_Z+1))*(x*(
x^6-x^4+1))^(1/3)*(x^2*(x^6-x^4+1)^2)^(1/3)/x/(x^6-x^4+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^7 - x^5 + x\right)^{\frac{1}{3}} \left(2\,x^6 - x^4 - 1\right)}{\left(x^6 - x^4 + x^2 + 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6-x^4-1)*(x^7-x^5+x)^(1/3)/(x^6-x^4+x^2+1)^2,x, algorithm="m
axima")
```

```
[Out] integrate((x^7 - x^5 + x)^(1/3)*(2*x^6 - x^4 - 1)/(x^6 - x^4 + x^2 + 1)^2,
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(-2\,x^6 + x^4 + 1\right)\,\left(x^7 - x^5 + x\right)^{1/3}}{\left(x^6 - x^4 + x^2 + 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^4 - 2*x^6 + 1)*(x - x^5 + x^7)^(1/3))/(x^2 - x^4 + x^6 + 1)^2,x)
```

```
[Out] int(-((x^4 - 2*x^6 + 1)*(x - x^5 + x^7)^(1/3))/(x^2 - x^4 + x^6 + 1)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**6-x**4-1)*(x**7-x**5+x)**(1/3)/(x**6-x**4+x**2+1)**2,x)
```

```
[Out] Timed out
```

3.1704 $\int \frac{x^4(-4b+ax^3)}{\sqrt[4]{-b+ax^3}\,(-b^2+2abx^3-a^2x^6+x^8)}\,dx$

Optimal. Leaf size=146

$$\tan^{-1}\left(\frac{\sqrt[4]{ax^3-b}}{x}\right)+\tanh^{-1}\left(\frac{x\left(ax^3-b\right)^{3/4}}{b-ax^3}\right)-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3-b}}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}+x^2}\right)}{\sqrt{2}}$$

Rubi [F] time = 2.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4\left(-4b+ax^3\right)}{\sqrt[4]{-b+ax^3}\left(-b^2+2abx^3-a^2x^6+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(-4*b + a*x^3))/((-b + a*x^3)^(1/4)*(-b^2 + 2*a*b*x^3 - a^2*x^6 +
x^8)),x]
```

```
[Out] 2*b*Defer[Int][1/((-b + a*x^3)^(1/4)*(b - a*x^3 - x^4)), x] - (a*Defer[Int]
[x^3/((-b + a*x^3)^(1/4)*(-b + a*x^3 - x^4)), x])/2 - 2*b*Defer[Int][1/((-b
 + a*x^3)^(1/4)*(b - a*x^3 + x^4)), x] + (a*Defer[Int][x^3/((-b + a*x^3)^(1
/4)*(-b + a*x^3 + x^4)), x])/2
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(-4b+ax^3\right)}{\sqrt[4]{-b+ax^3}\left(-b^2+2abx^3-a^2x^6+x^8\right)}\,dx &= \int \left(\frac{4b-ax^3}{2\sqrt[4]{-b+ax^3}\left(b-ax^3-x^4\right)}+\frac{-4b+ax^3}{2\sqrt[4]{-b+ax^3}\left(b-ax^3+x^4\right)}\right) d\\
&= \frac{1}{2}\int \frac{4b-ax^3}{\sqrt[4]{-b+ax^3}\left(b-ax^3-x^4\right)}\,dx+\frac{1}{2}\int \frac{-4b+ax^3}{\sqrt[4]{-b+ax^3}\left(b-ax^3+\right.}\\
&= \frac{1}{2}\int \left(-\frac{ax^3}{\sqrt[4]{-b+ax^3}\left(-b+ax^3-x^4\right)}-\frac{4b}{\sqrt[4]{-b+ax^3}\left(b-ax^3+x^4\right)}\right.\\
&= -\left(\frac{1}{2}a\int \frac{x^3}{\sqrt[4]{-b+ax^3}\left(-b+ax^3-x^4\right)}\,dx\right)+\frac{1}{2}a\int \frac{x^3}{\sqrt[4]{-b+ax^3}\left(-\right.}
\end{aligned}$$

Mathematica [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(-4b+ax^3\right)}{\sqrt[4]{-b+ax^3}\left(-b^2+2abx^3-a^2x^6+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(-4*b + a*x^3))/((-b + a*x^3)^(1/4)*(-b^2 + 2*a*b*x^3 - a^2*
x^6 + x^8)),x]
```

```
[Out] Integrate[(x^4*(-4*b + a*x^3))/((-b + a*x^3)^(1/4)*(-b^2 + 2*a*b*x^3 - a^2*
x^6 + x^8)), x]
```

IntegrateAlgebraic [A] time = 1.82, size = 146, normalized size = 1.00

$$\tan^{-1}\left(\frac{\sqrt[4]{ax^3-b}}{x}\right)+\tanh^{-1}\left(\frac{x\left(ax^3-b\right)^{3/4}}{b-ax^3}\right)-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}}{x\sqrt[4]{ax^3-b}}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}+x^2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(-4*b + a*x^3))/((-b + a*x^3)^(1/4)*(-b^2 + 2*a*b*x
^3 - a^2*x^6 + x^8)),x]
```

```
[Out] ArcTan[(-b + a*x^3)^(1/4)/x] - ArcTan[(-(x^2/Sqrt[2]) + Sqrt[-b + a*x^3]/Sq
rt[2])/(x*(-b + a*x^3)^(1/4))]/Sqrt[2] + ArcTanh[(x*(-b + a*x^3)^(3/4))/(b
- a*x^3)] + ArcTanh[(Sqrt[2]*x*(-b + a*x^3)^(1/4))/(x^2 + Sqrt[-b + a*x^3])
]/Sqrt[2]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^3-4*b)/(a*x^3-b)^(1/4)/(-a^2*x^6+x^8+2*a*b*x^3-b^2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(ax^3-4\,b\right)x^4}{\left(a^2x^6-x^8-2\,abx^3+b^2\right)\left(ax^3-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^3-4*b)/(a*x^3-b)^(1/4)/(-a^2*x^6+x^8+2*a*b*x^3-b^2),x, a
lgorithm="giac")
```

```
[Out] integrate(-(a*x^3 - 4*b)*x^4/((a^2*x^6 - x^8 - 2*a*b*x^3 + b^2)*(a*x^3 - b)
^(1/4)), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a\,x^3-4b\right)}{\left(a\,x^3-b\right)^{\frac{1}{4}}\left(-a^2x^6+x^8+2ab\,x^3-b^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a*x^3-4*b)/(a*x^3-b)^(1/4)/(-a^2*x^6+x^8+2*a*b*x^3-b^2),x)
```

```
[Out] int(x^4*(a*x^3-4*b)/(a*x^3-b)^(1/4)/(-a^2*x^6+x^8+2*a*b*x^3-b^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(ax^3-4\,b\right)x^4}{\left(a^2x^6-x^8-2\,abx^3+b^2\right)\left(ax^3-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a*x^3-4*b)/(a*x^3-b)^(1/4)/(-a^2*x^6+x^8+2*a*b*x^3-b^2),x, a
lgorithm="maxima")
```

```
[Out] -integrate((a*x^3 - 4*b)*x^4/((a^2*x^6 - x^8 - 2*a*b*x^3 + b^2)*(a*x^3 - b)
^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int -\frac{x^4\,\left(4\,b-a\,x^3\right)}{{\left(a\,x^3-b\right)}^{1/4}\,\left(a^2\,x^6-2\,a\,b\,x^3+b^2-x^8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(4*b - a*x^3))/((a*x^3 - b)^(1/4)*(b^2 - x^8 + a^2*x^6 - 2*a*b*x^3
)),x)
```

```
[Out] -int(-(x^4*(4*b - a*x^3))/((a*x^3 - b)^(1/4)*(b^2 - x^8 + a^2*x^6 - 2*a*b*x
^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a*x**3-4*b)/(a*x**3-b)**(1/4)/(-a**2*x**6+x**8+2*a*b*x**3-b
**2),x)
```

```
[Out] Timed out
```

3.1705 $\int \frac{\sqrt{-1+x^2}\,\sqrt{x^2+x\sqrt{-1+x^2}}}{1+x^2}\,dx$

Optimal. Leaf size=146

$$\sqrt{x^2+\sqrt{x^2-1}\,x}-\sqrt{2\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{x^2+\sqrt{x^2-1}\,x}}{\sqrt{1+\sqrt{2}}}\right)-\frac{\tanh^{-1}\left(\sqrt{2}\sqrt{x^2+\sqrt{x^2-1}\,x}\right)}{\sqrt{2}}+\sqrt{2\left(\sqrt{2}-1\right)}\ \text{ta}$$

Rubi [F] time = 1.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-1+x^2}\,\sqrt{x^2+x\sqrt{-1+x^2}}}{1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-1 + x^2]*Sqrt[x^2 + x*Sqrt[-1 + x^2]])/(1 + x^2),x]
```

```
[Out] (I/2)*Defer[Int][(Sqrt[-1 + x^2]*Sqrt[x^2 + x*Sqrt[-1 + x^2]])/(I - x), x]
+ (I/2)*Defer[Int][(Sqrt[-1 + x^2]*Sqrt[x^2 + x*Sqrt[-1 + x^2]])/(I + x), x
]
```

Rubi steps

$$\int \frac{\sqrt{-1+x^2}\,\sqrt{x^2+x\sqrt{-1+x^2}}}{1+x^2}\,dx=\int \left(\frac{i\sqrt{-1+x^2}\,\sqrt{x^2+x\sqrt{-1+x^2}}}{2(i-x)}+\frac{i\sqrt{-1+x^2}\,\sqrt{x^2+x\sqrt{-1+x^2}}}{2(i+x)}\right)dx$$

$$=\frac{1}{2}i\int \frac{\sqrt{-1+x^2}\,\sqrt{x^2+x\sqrt{-1+x^2}}}{i-x}\,dx+\frac{1}{2}i\int \frac{\sqrt{-1+x^2}\,\sqrt{x^2+x\sqrt{-1+}}}{i+x}$$

Mathematica [C] time = 2.72, size = 808, normalized size = 5.53

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[-1 + x^2]*Sqrt[x^2 + x*Sqrt[-1 + x^2]])/(1 + x^2),x]
```

```
[Out] (Sqrt[-1 + x^2]*(x + Sqrt[-1 + x^2])*(2*Sqrt[2]*Sqrt[x*(x + Sqrt[-1 + x^2])
] + 4*Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[1 + Sqrt[2]]*(4 + 3*Sqrt[2] + Sqrt[2]*
(x + Sqrt[-1 + x^2])^2))/(Sqrt[1 + (x + Sqrt[-1 + x^2])^2]*(3 + 2*Sqrt[2] +
 (x + Sqrt[-1 + x^2])^2))] + 2*Log[x + Sqrt[-1 + x^2]] - 2*Sqrt[-1 + Sqrt[2
]]*Log[17 - 12*Sqrt[2] + (6 - 4*Sqrt[2])*(x + Sqrt[-1 + x^2])^2 + (x + Sqrt
[-1 + x^2])^4] - (2*I)*Sqrt[1 + Sqrt[2]]*Log[17 + 12*Sqrt[2] + (6 + 4*Sqrt[
2])*(x + Sqrt[-1 + x^2])^2 + (x + Sqrt[-1 + x^2])^4] + I*Sqrt[1 + Sqrt[2]]*
Log[17 + 12*Sqrt[2] + (34 + 24*Sqrt[2])*(x + Sqrt[-1 + x^2])^2 + (5 + 4*Sqr
t[2])*(x + Sqrt[-1 + x^2])^4 - 2*Sqrt[2*(1 + Sqrt[2])]*(2 + Sqrt[2])*(x + S
qrt[-1 + x^2])^3*Sqrt[x*(x + Sqrt[-1 + x^2])] - (2*Sqrt[2*(1 + Sqrt[2])]*(1
0 + 7*Sqrt[2])*(x*(x + Sqrt[-1 + x^2]))^(3/2))/x] + I*Sqrt[1 + Sqrt[2]]*Log
[17 + 12*Sqrt[2] + (34 + 24*Sqrt[2])*(x + Sqrt[-1 + x^2])^2 + (5 + 4*Sqrt[2
])*(x + Sqrt[-1 + x^2])^4 + 2*Sqrt[2*(1 + Sqrt[2])]*(2 + Sqrt[2])*(x + Sqrt
[-1 + x^2])^3*Sqrt[x*(x + Sqrt[-1 + x^2])] + (2*Sqrt[2*(1 + Sqrt[2])]*(10 +
 7*Sqrt[2])*(x*(x + Sqrt[-1 + x^2]))^(3/2))/x] - 2*Log[1 + Sqrt[1 + (x + Sq
```

```
rt[-1 + x^2])^2]] + 2*Sqrt[-1 + Sqrt[2]]*Log[11 - 8*Sqrt[2] - (x + Sqrt[-1
+ x^2])^4 - 12*Sqrt[-1 + Sqrt[2]]*Sqrt[x*(x + Sqrt[-1 + x^2])] + 8*Sqrt[2*(
-1 + Sqrt[2])]*Sqrt[x*(x + Sqrt[-1 + x^2])] - 2*(x + Sqrt[-1 + x^2])^2*(1 +
 2*Sqrt[-1 + Sqrt[2]]*Sqrt[x*(x + Sqrt[-1 + x^2])])]))/(2*Sqrt[2]*(-1 + x^2
 + x*Sqrt[-1 + x^2]))
```

IntegrateAlgebraic [A] time = 2.10, size = 146, normalized size = 1.00

$$\sqrt{x^2+\sqrt{x^2-1}\,x}-\sqrt{2\left(1+\sqrt{2}\right)}\tan^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{x^2+\sqrt{x^2-1}\,x}\right)-\frac{\tanh^{-1}\left(\sqrt{2}\sqrt{x^2+\sqrt{x^2-1}\,x}\right)}{\sqrt{2}}+\sqrt{2\left(\sqrt{2}-1\right)}\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{x^2+\sqrt{x^2-1}\,x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^2]*Sqrt[x^2 + x*Sqrt[-1 + x^2]])/(1 + x^2),
x]
```

```
[Out] Sqrt[x^2 + x*Sqrt[-1 + x^2]] - Sqrt[2*(1 + Sqrt[2])]*ArcTan[Sqrt[-1 + Sqrt[
2]]*Sqrt[x^2 + x*Sqrt[-1 + x^2]]] - ArcTanh[Sqrt[2]*Sqrt[x^2 + x*Sqrt[-1 +
x^2]]]/Sqrt[2] + Sqrt[2*(-1 + Sqrt[2])]*ArcTanh[Sqrt[1 + Sqrt[2]]*Sqrt[x^2
+ x*Sqrt[-1 + x^2]]]
```

fricas [B] time = 22.97, size = 516, normalized size = 3.53

$$-\sqrt{2\sqrt{2}+2}\arctan\left(\frac{45602\left(4x^4-6x^2-\sqrt{2}\left(x^4-1\right)-\left(4x^3-\sqrt{2}\left(x^3-3x\right)-4x\right)\sqrt{x^2-1}-2\right)\sqrt{x^2+\sqrt{x^2-1}\,x}\sqrt{2\sqrt{2}+2}+\left(1902x^4-3056x^2-\sqrt{2}\left(1403x^4-2494x^2+343\right)+2\left(904x^3-\sqrt{2}\left(499x^3-873x\right)-1216x\right)\sqrt{x^2-1}+530\right)\sqrt{76309\sqrt{2}+105481}\sqrt{2\sqrt{2}+2}}{45602\left(7x^4-10x^2-1\right)}\right)+\frac{1}{4}\sqrt{2}\log\left(4x^2+2\left(2\sqrt{2}\sqrt{x^2-1}\,x-\sqrt{2}\left(2x^2-1\right)\right)\sqrt{x^2+\sqrt{x^2-1}\,x}-4\sqrt{x^2-1}\,x-1\right)+\frac{1}{4}\sqrt{2\sqrt{2}-2}\log\left(-\frac{4\left(31x^2+\sqrt{2}\left(109x^2+78\right)-\sqrt{x^2-1}\left(109\sqrt{2}x+31x\right)-187\right)\sqrt{x^2+\sqrt{x^2-1}\,x}+\left(280x^2+\sqrt{2}\left(249x^2-187\right)-2\sqrt{x^2-1}\left(31\sqrt{2}x+218x\right)+156\right)\sqrt{2\sqrt{2}-2}}{x^2+1}\right)-\frac{1}{4}\sqrt{2\sqrt{2}-2}\log\left(-\frac{4\left(31x^2+\sqrt{2}\left(109x^2+78\right)-\sqrt{x^2-1}\left(109\sqrt{2}x+31x\right)-187\right)\sqrt{x^2+\sqrt{x^2-1}\,x}-\left(280x^2+\sqrt{2}\left(249x^2-187\right)-2\sqrt{x^2-1}\left(31\sqrt{2}x+218x\right)+156\right)\sqrt{2\sqrt{2}-2}}{x^2+1}\right)+\sqrt{x^2+\sqrt{x^2-1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)*(x^2+x*(x^2-1)^(1/2))^(1/2)/(x^2+1),x, algorithm="f
ricas")
```

```
[Out] -sqrt(2*sqrt(2) + 2)*arctan(1/45602*(45602*(4*x^4 - 6*x^2 - sqrt(2)*(x^4 -
1) - (4*x^3 - sqrt(2)*(x^3 - 3*x) - 4*x)*sqrt(x^2 - 1) - 2)*sqrt(x^2 + sqrt
(x^2 - 1)*x)*sqrt(2*sqrt(2) + 2) + (1902*x^4 - 3056*x^2 - sqrt(2)*(1403*x^4
 - 2494*x^2 + 343) + 2*(904*x^3 - sqrt(2)*(499*x^3 - 873*x) - 1216*x)*sqrt(
x^2 - 1) + 530)*sqrt(76309*sqrt(2) + 105481)*sqrt(2*sqrt(2) + 2))/(7*x^4 -
10*x^2 - 1)) + 1/4*sqrt(2)*log(4*x^2 + 2*(2*sqrt(2)*sqrt(x^2 - 1)*x - sqrt(
2)*(2*x^2 - 1))*sqrt(x^2 + sqrt(x^2 - 1)*x) - 4*sqrt(x^2 - 1)*x - 1) + 1/4*
sqrt(2*sqrt(2) - 2)*log(-(4*(31*x^2 + sqrt(2)*(109*x^2 + 78) - sqrt(x^2 - 1
)*(109*sqrt(2)*x + 31*x) - 187)*sqrt(x^2 + sqrt(x^2 - 1)*x) + (280*x^2 + sq
rt(2)*(249*x^2 - 187) - 2*sqrt(x^2 - 1)*(31*sqrt(2)*x + 218*x) + 156)*sqrt(
2*sqrt(2) - 2))/(x^2 + 1)) - 1/4*sqrt(2*sqrt(2) - 2)*log(-(4*(31*x^2 + sqrt
(2)*(109*x^2 + 78) - sqrt(x^2 - 1)*(109*sqrt(2)*x + 31*x) - 187)*sqrt(x^2 +
 sqrt(x^2 - 1)*x) - (280*x^2 + sqrt(2)*(249*x^2 - 187) - 2*sqrt(x^2 - 1)*(3
1*sqrt(2)*x + 218*x) + 156)*sqrt(2*sqrt(2) - 2))/(x^2 + 1)) + sqrt(x^2 + sq
rt(x^2 - 1)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^2-1}\,x}\,\sqrt{x^2-1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)*(x^2+x*(x^2-1)^(1/2))^(1/2)/(x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^2 - 1)*x)*sqrt(x^2 - 1)/(x^2 + 1), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-1}\,\sqrt{x^2+x\sqrt{x^2-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(1/2)*(x^2+x*(x^2-1)^(1/2))^(1/2)/(x^2+1),x)
```

```
[Out] int((x^2-1)^(1/2)*(x^2+x*(x^2-1)^(1/2))^(1/2)/(x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^2 - 1}\, x}\, \sqrt{x^2 - 1}}{x^2 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)*(x^2+x*(x^2-1)^(1/2))^(1/2)/(x^2+1),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^2 - 1)*x)*sqrt(x^2 - 1)/(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^2 - 1}\, \sqrt{x\, \sqrt{x^2 - 1} + x^2}}{x^2 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)^(1/2)*(x*(x^2 - 1)^(1/2) + x^2)^(1/2))/(x^2 + 1),x)
```

```
[Out] int(((x^2 - 1)^(1/2)*(x*(x^2 - 1)^(1/2) + x^2)^(1/2))/(x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(x + \sqrt{x^2 - 1}\right)}\, \sqrt{(x - 1)(x + 1)}}{x^2 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(1/2)*(x**2+x*(x**2-1)**(1/2))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt(x*(x + sqrt(x**2 - 1)))*sqrt((x - 1)*(x + 1))/(x**2 + 1), x)
```

3.1706 $\int \frac{x}{\left(x^2(-a+x)\right)^{2/3}(a+(-1+d)x)}\,dx$

Optimal. Leaf size=147

$$-\frac{\log\left(\sqrt[3]{x^3-ax^2}-\sqrt[3]{d}\,x\right)}{ad^{2/3}}+\frac{\log\left(\sqrt[3]{d}\,x\sqrt[3]{x^3-ax^2}+\left(x^3-ax^2\right)^{2/3}+d^{2/3}x^2\right)}{2ad^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\,x}{2\sqrt[3]{x^3-ax^2}+\sqrt[3]{d}\,x}\right)}{ad^{2/3}}$$

Rubi [A] time = 0.36, antiderivative size = 192, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6719, 91}

$$\frac{x^{4/3}(x-a)^{2/3}\log(a-(1-d)x)}{2ad^{2/3}\left(-\left(x^2(a-x)\right)\right)^{2/3}}-\frac{3x^{4/3}(x-a)^{2/3}\log\left(\sqrt[3]{d}\sqrt[3]{x}-\sqrt[3]{x-a}\right)}{2ad^{2/3}\left(-\left(x^2(a-x)\right)\right)^{2/3}}-\frac{\sqrt{3}x^{4/3}(x-a)^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{x-a}}+\frac{1}{\sqrt{3}}\right)}{ad^{2/3}\left(-\left(x^2(a-x)\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((x^2*(-a + x))^(2/3)*(a + (-1 + d)*x)),x]
```

```
[Out] -((Sqrt[3]*x^(4/3)*(-a + x)^(2/3)*ArcTan[1/Sqrt[3] + (2*d^(1/3)*x^(1/3))/(S
qrt[3]*(-a + x)^(1/3))])/(a*d^(2/3)*(-((a - x)*x^2))^(2/3))) + (x^(4/3)*(-a
 + x)^(2/3)*Log[a - (1 - d)*x])/(2*a*d^(2/3)*(-((a - x)*x^2))^(2/3)) - (3*x
^(4/3)*(-a + x)^(2/3)*Log[d^(1/3)*x^(1/3) - (-a + x)^(1/3)])/(2*a*d^(2/3)*(
-((a - x)*x^2))^(2/3))
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{x}{\left(x^2(-a+x)\right)^{2/3}(a+(-1+d)x)}\,dx=\frac{\left(x^{4/3}(-a+x)^{2/3}\right)\int \frac{1}{\sqrt[3]{x}(-a+x)^{2/3}(a+(-1+d)x)}\,dx}{\left(x^2(-a+x)\right)^{2/3}}$$

$$=-\frac{\sqrt{3}x^{4/3}(-a+x)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{-a+x}}\right)}{ad^{2/3}\left(-\left((a-x)x^2\right)\right)^{2/3}}+\frac{x^{4/3}(-a+x)^{2/3}\log(a-(1 \ldots}{2ad^{2/3}\left(-\left((a-x)x^2\right)\right)^{2 \ldots}}$$

Mathematica [C] time = 0.02, size = 41, normalized size = 0.28

$$\frac{3x^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{dx}{x-a}\right)}{2a\left(x^2(x-a)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((x^2*(-a + x))^(2/3)*(a + (-1 + d)*x)),x]
```

```
[Out] (3*x^2*Hypergeometric2F1[2/3, 1, 5/3, (d*x)/(-a + x)])/(2*a*(x^2*(-a + x))^
(2/3))
```

IntegrateAlgebraic [A] time = 0.40, size = 147, normalized size = 1.00

$$-\frac{\log\left(\sqrt[3]{x^3-ax^2}-\sqrt[3]{d}\,x\right)}{ad^{2/3}}+\frac{\log\left(\sqrt[3]{d}\,x\sqrt[3]{x^3-ax^2}+\left(x^3-ax^2\right)^{2/3}+d^{2/3}x^2\right)}{2ad^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\,x}{2\sqrt[3]{x^3-ax^2}+\sqrt[3]{d}\,x}\right)}{ad^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((x^2*(-a + x))^(2/3)*(a + (-1 + d)*x)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*x)/(d^(1/3)*x + 2*(-(a*x^2) + x^3)^(1/3)
)]])/(a*d^(2/3))) - Log[-(d^(1/3)*x) + (-(a*x^2) + x^3)^(1/3)]/(a*d^(2/3)) +
 Log[d^(2/3)*x^2 + d^(1/3)*x*(-(a*x^2) + x^3)^(1/3) + (-(a*x^2) + x^3)^(2/3
)]/(2*a*d^(2/3))
```

fricas [A] time = 0.40, size = 185, normalized size = 1.26

$$\frac{2\sqrt{3}\,d\sqrt{-\left(-d^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{3}\left(\left(-d^2\right)^{\frac{1}{3}}dx-2\left(-ax^2+x^3\right)^{\frac{1}{3}}\left(-d^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-d^2\right)^{\frac{1}{3}}}}{3\,d^2x}\right)-2\left(-d^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(-d^2\right)^{\frac{2}{3}}x-\left(-ax^2+x^3\right)^{\frac{1}{3}}d}{x}\right)+\left(-d^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(-d^2\right)^{\frac{1}{3}}dx^2-\left(-ax^2+x^3\right)^{\frac{1}{3}}\left(-d^2\right)^{\frac{2}{3}}x-\left(-ax^2+x^3\right)^{\frac{2}{3}}d}{x^2}\right)}{2\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2*(-a+x))^(2/3)/(a+(-1+d)*x),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*d*sqrt(-(-d^2)^(1/3))*arctan(-1/3*sqrt(3)*((-d^2)^(1/3)*d*x
- 2*(-a*x^2 + x^3)^(1/3)*(-d^2)^(2/3))*sqrt(-(-d^2)^(1/3))/(d^2*x)) - 2*(-d
^2)^(2/3)*log(-((-d^2)^(2/3)*x - (-a*x^2 + x^3)^(1/3)*d)/x) + (-d^2)^(2/3)*
log(-((-d^2)^(1/3)*d*x^2 - (-a*x^2 + x^3)^(1/3)*(-d^2)^(2/3)*x - (-a*x^2 +
x^3)^(2/3)*d)/x^2))/(a*d^2)
```

giac [A] time = 0.22, size = 99, normalized size = 0.67

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}+2\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)}{ad^{\frac{2}{3}}}+\frac{\log\left(d^{\frac{2}{3}}+d^{\frac{1}{3}}\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}+\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}\right)}{2\,ad^{\frac{2}{3}}}-\frac{\log\left(\left|-d^{\frac{1}{3}}+\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}\right|\right)}{ad^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2*(-a+x))^(2/3)/(a+(-1+d)*x),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(d^(1/3) + 2*(-a/x + 1)^(1/3))/d^(1/3))/(a*d^(2/
3)) + 1/2*log(d^(2/3) + d^(1/3)*(-a/x + 1)^(1/3) + (-a/x + 1)^(2/3))/(a*d^(
2/3)) - log(abs(-d^(1/3) + (-a/x + 1)^(1/3)))/(a*d^(2/3))
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^2\left(-a+x\right)\right)^{\frac{2}{3}}\left(a+\left(-1+d\right)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2*(-a+x))^(2/3)/(a+(-1+d)*x),x)
```

```
[Out] int(x/(x^2*(-a+x))^(2/3)/(a+(-1+d)*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-(a-x)x^2\right)^{\frac{2}{3}}((d-1)x+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2*(-a+x))^(2/3)/(a+(-1+d)*x),x, algorithm="maxima")
[Out] integrate(x/((-(a - x)*x^2)^(2/3)*((d - 1)*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a+x\,(d-1))\,\left(-x^2\,(a-x)\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + x*(d - 1))*(-x^2*(a - x))^(2/3)),x)
[Out] int(x/((a + x*(d - 1))*(-x^2*(a - x))^(2/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^2\,(-a+x)\right)^{\frac{2}{3}}(a+dx-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2*(-a+x))**(2/3)/(a+(-1+d)*x),x)
[Out] Integral(x/((x**2*(-a + x))**(2/3)*(a + d*x - x)), x)
```

3.1707 $\int \frac{b-cx+ax^2}{(-b+ax^2)\sqrt{bx+ax^3}}\,dx$

Optimal. Leaf size=147

$$-\frac{\left(2\sqrt{a}\sqrt{b}+c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(2\sqrt{a}\sqrt{b}-c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}$$

Rubi [C] time = 1.71, antiderivative size = 284, normalized size of antiderivative = 1.93, number of steps used = 13, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2056, 6725, 329, 220, 933, 168, 537}

$$-\frac{\sqrt{x}\left(\frac{c}{\sqrt{a}}+2\sqrt{b}\right)\sqrt{\frac{ax^2}{b}+1}\,\Pi\left(\frac{\sqrt{-a}}{\sqrt{a}};\sin^{-1}\left(\frac{\sqrt[4]{-a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{ax^3+bx}}-\frac{\sqrt{x}\left(2\sqrt{b}-\frac{c}{\sqrt{a}}\right)\sqrt{\frac{ax^2}{b}+1}\,\Pi\left(\frac{\sqrt{a}}{\sqrt{-a}};\sin^{-1}\left(\frac{\sqrt[4]{-a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{ax^3+bx}}+\frac{\sqrt{x}\left(\sqrt{a}x+\sqrt{b}\right)\sqrt{\frac{ax^2+b}{\left(\sqrt{a}x+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(b - c*x + a*x^2)/((-b + a*x^2)*Sqrt[b*x + a*x^3]),x]
```

```
[Out] (Sqrt[x]*(Sqrt[b] + Sqrt[a]*x)*Sqrt[(b + a*x^2)/(Sqrt[b] + Sqrt[a]*x)^2]*El
lipticF[2*ArcTan[(a^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(a^(1/4)*b^(1/4)*Sqrt[b*
x + a*x^3]) - ((2*Sqrt[b] + c/Sqrt[a])*Sqrt[x]*Sqrt[1 + (a*x^2)/b]*Elliptic
Pi[Sqrt[-a]/Sqrt[a], ArcSin[((-a)^(1/4)*Sqrt[x])/b^(1/4)], -1])/((-a)^(1/4)
*b^(1/4)*Sqrt[b*x + a*x^3]) - ((2*Sqrt[b] - c/Sqrt[a])*Sqrt[x]*Sqrt[1 + (a*
x^2)/b]*EllipticPi[Sqrt[a]/Sqrt[-a], ArcSin[((-a)^(1/4)*Sqrt[x])/b^(1/4)],
-1])/((-a)^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)]])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
```

```
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{b-cx+ax^2}{\left(-b+ax^2\right)\sqrt{bx+ax^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \frac{b-cx+ax^2}{\sqrt{x}\left(-b+ax^2\right)\sqrt{b+ax^2}}\,dx}{\sqrt{bx+ax^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{b+ax^2}}+\frac{2b-cx}{\sqrt{x}\left(-b+ax^2\right)\sqrt{b+ax^2}}\right)dx}{\sqrt{bx+ax^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx}{\sqrt{bx+ax^3}} + \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \frac{2b-cx}{\sqrt{x}\left(-b+ax^2\right)\sqrt{b+ax^2}}\,dx}{\sqrt{bx+ax^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \left(-\frac{2b^{3/2}-\frac{bc}{\sqrt{a}}}{2b\sqrt{x}\left(\sqrt{b}-\sqrt{a}\,x\right)\sqrt{b+ax^2}}-\frac{2b^{3/2}+\frac{bc}{\sqrt{a}}}{2b\sqrt{x}\left(\sqrt{b}+\sqrt{a}\,x\right)\sqrt{b+ax^2}}\right)dx}{\sqrt{bx+ax^3}} + \frac{\left(2\sqrt{x}\sqrt{b}\right.}{}$$

$$= \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}\,x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} - \frac{\left(\left(2\sqrt{b}-\frac{c}{\sqrt{a}}\right)\sqrt{x}\sqrt{b+ax}\right.}{2\sqrt{bx}}$$

$$= \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}\,x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} - \frac{\left(\left(2\sqrt{b}-\frac{c}{\sqrt{a}}\right)\sqrt{x}\sqrt{1+\frac{ax}{b}}\right.}{}$$

$$= \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}\,x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} + \frac{\left(\left(2\sqrt{b}-\frac{c}{\sqrt{a}}\right)\sqrt{x}\sqrt{1+\frac{ax}{b}}\right.}{}$$

$$= \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}\,x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} - \frac{\left(2\sqrt{b}+\frac{c}{\sqrt{a}}\right)\sqrt{x}\sqrt{1+\frac{ax^2}{b}}}{\sqrt[4]{-a}\sqrt[4]{}}$$

Mathematica [C] time = 0.51, size = 133, normalized size = 0.90

$$-\frac{2x\sqrt{\frac{ax^2}{b}+1}\left(x\left(3axF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{ax^2}{b},\frac{ax^2}{b}\right)-5cF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{ax^2}{b},\frac{ax^2}{b}\right)\right)+15bF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{ax^2}{b},\frac{ax^2}{b}\right)\right)}{15b\sqrt{x\left(ax^2+b\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b - c*x + a*x^2)/((-b + a*x^2)*Sqrt[b*x + a*x^3]),x]
```

```
[Out] (-2*x*Sqrt[1 + (a*x^2)/b]*(15*b*AppellF1[1/4, 1/2, 1, 5/4, -((a*x^2)/b), (a
*x^2)/b] + x*(-5*c*AppellF1[3/4, 1/2, 1, 7/4, -((a*x^2)/b), (a*x^2)/b] + 3*
a*x*AppellF1[5/4, 1/2, 1, 9/4, -((a*x^2)/b), (a*x^2)/b])))/(15*b*Sqrt[x*(b
+ a*x^2)])
```

IntegrateAlgebraic [A] time = 0.44, size = 147, normalized size = 1.00

$$-\frac{\left(2\sqrt{a}\sqrt{b}+c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(2\sqrt{a}\sqrt{b}-c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b - c*x + a*x^2)/((-b + a*x^2)*Sqrt[b*x + a*x^3]),x]
```

```
[Out] -1/2*((2*Sqrt[a]*Sqrt[b] + c)*ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[b*x + a*
x^3])/(b + a*x^2)])/(Sqrt[2]*a^(3/4)*b^(3/4)) - ((2*Sqrt[a]*Sqrt[b] - c)*Ar
cTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3])/(b + a*x^2)])/(2*Sqrt[2]*
a^(3/4)*b^(3/4))
```

fricas [B] time = 0.82, size = 1553, normalized size = 10.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-c*x+b)/(a*x^2-b)/(a*x^3+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(1/2)*sqrt(-(a*b*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)) +
4*c)/(a*b))*log(-(16*a^2*b^4 - b^2*c^4 + (16*a^4*b^2 - a^2*c^4)*x^4 + 6*(16
*a^3*b^3 - a*b*c^4)*x^2 + 4*sqrt(1/2)*(4*a^2*b^3*c + a*b^2*c^3 + (4*a^3*b^2
*c + a^2*b*c^3)*x^2 + 4*(4*a^3*b^3 + a^2*b^2*c^2)*x - 2*(a^4*b^3*x^2 + a^3*
b^3*c*x + a^3*b^4)*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)))*sqrt(a*x
^3 + b*x)*sqrt(-(a*b*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)) + 4*c)/
(a*b)) - 4*((4*a^4*b^3 - a^3*b^2*c^2)*x^3 + (4*a^3*b^4 - a^2*b^3*c^2)*x)*sq
rt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)))/(a^2*x^4 - 2*a*b*x^2 + b^2))
+ 1/8*sqrt(1/2)*sqrt(-(a*b*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)) +
 4*c)/(a*b))*log(-(16*a^2*b^4 - b^2*c^4 + (16*a^4*b^2 - a^2*c^4)*x^4 + 6*(1
6*a^3*b^3 - a*b*c^4)*x^2 - 4*sqrt(1/2)*(4*a^2*b^3*c + a*b^2*c^3 + (4*a^3*b^
2*c + a^2*b*c^3)*x^2 + 4*(4*a^3*b^3 + a^2*b^2*c^2)*x - 2*(a^4*b^3*x^2 + a^3
*b^3*c*x + a^3*b^4)*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)))*sqrt(a*
x^3 + b*x)*sqrt(-(a*b*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)) + 4*c)
/(a*b)) - 4*((4*a^4*b^3 - a^3*b^2*c^2)*x^3 + (4*a^3*b^4 - a^2*b^3*c^2)*x)*s
qrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)))/(a^2*x^4 - 2*a*b*x^2 + b^2))
 - 1/8*sqrt(1/2)*sqrt((a*b*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)) -
 4*c)/(a*b))*log(-(16*a^2*b^4 - b^2*c^4 + (16*a^4*b^2 - a^2*c^4)*x^4 + 6*(1
6*a^3*b^3 - a*b*c^4)*x^2 + 4*sqrt(1/2)*(4*a^2*b^3*c + a*b^2*c^3 + (4*a^3*b^
2*c + a^2*b*c^3)*x^2 + 4*(4*a^3*b^3 + a^2*b^2*c^2)*x + 2*(a^4*b^3*x^2 + a^3
*b^3*c*x + a^3*b^4)*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)))*sqrt(a*
x^3 + b*x)*sqrt((a*b*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)) - 4*c)/
(a*b)) + 4*((4*a^4*b^3 - a^3*b^2*c^2)*x^3 + (4*a^3*b^4 - a^2*b^3*c^2)*x)*sq
rt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)))/(a^2*x^4 - 2*a*b*x^2 + b^2))
```

```
+ 1/8*sqrt(1/2)*sqrt((a*b*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)) -
4*c)/(a*b))*log(-(16*a^2*b^4 - b^2*c^4 + (16*a^4*b^2 - a^2*c^4)*x^4 + 6*(16
*a^3*b^3 - a*b*c^4)*x^2 - 4*sqrt(1/2)*(4*a^2*b^3*c + a*b^2*c^3 + (4*a^3*b^2
*c + a^2*b*c^3)*x^2 + 4*(4*a^3*b^3 + a^2*b^2*c^2)*x + 2*(a^4*b^3*x^2 + a^3*
b^3*c*x + a^3*b^4)*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)))*sqrt(a*x
^3 + b*x)*sqrt((a*b*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)) - 4*c)/(
a*b)) + 4*((4*a^4*b^3 - a^3*b^2*c^2)*x^3 + (4*a^3*b^4 - a^2*b^3*c^2)*x)*sqr
t((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)))/(a^2*x^4 - 2*a*b*x^2 + b^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - cx + b}{\sqrt{ax^3 + bx}\left(ax^2 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-c*x+b)/(a*x^2-b)/(a*x^3+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^2 - c*x + b)/(sqrt(a*x^3 + b*x)*(a*x^2 - b)), x)
```

maple [C] time = 0.05, size = 710, normalized size = 4.83

$$\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a x^3+bx}}-\frac{\sqrt{-ab}\sqrt{\frac{xa}{\sqrt{-ab}}+1}\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)c}{2a^2\sqrt{a x^3+bx}\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)}+\frac{\sqrt{-ab}\sqrt{\frac{xa}{\sqrt{-ab}}+1}\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)b}{\sqrt{ab}\,a\sqrt{a x^3+bx}\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)}-\frac{\sqrt{-ab}\sqrt{\frac{xa}{\sqrt{-ab}}+1}\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)c}{2a^2\sqrt{a x^3+bx}\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)}-\frac{\sqrt{-ab}\sqrt{\frac{xa}{\sqrt{-ab}}+1}\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)b}{\sqrt{ab}\,a\sqrt{a x^3+bx}\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-c*x+b)/(a*x^2-b)/(a*x^3+b*x)^(1/2),x)
```

```
[Out] 1/a*(-a*b)^(1/2)*((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x-1/a*(-a
*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2
)*EllipticF(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-1/2/a^
2*(-a*b)^(1/2)*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x
*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2)
)*EllipticPi(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2)/
(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2)),1/2*2^(1/2))*c+1/(a*b)^(1/2)/a*(-a*b)^(
1/2)*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^
(1/2))^(1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2))*Elliptic
Pi(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2)/(-1/a*(-a*
b)^(1/2)-1/a*(a*b)^(1/2)),1/2*2^(1/2))*b-1/2/a^2*(-a*b)^(1/2)*(x*a/(-a*b)^(
1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^
3+b*x)^(1/2)/(-1/a*(-a*b)^(1/2)+1/a*(a*b)^(1/2))*EllipticPi(((x+1/a*(-a*b)^
(1/2))*a/(-a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2)/(-1/a*(-a*b)^(1/2)+1/a*(a*b)
^(1/2)),1/2*2^(1/2))*c-1/(a*b)^(1/2)/a*(-a*b)^(1/2)*(x*a/(-a*b)^(1/2)+1)^(1
/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/
2)/(-1/a*(-a*b)^(1/2)+1/a*(a*b)^(1/2))*EllipticPi(((x+1/a*(-a*b)^(1/2))*a/(
-a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2)/(-1/a*(-a*b)^(1/2)+1/a*(a*b)^(1/2)),1/
2*2^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - cx + b}{\sqrt{ax^3 + bx}\left(ax^2 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-c*x+b)/(a*x^2-b)/(a*x^3+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 - c*x + b)/(sqrt(a*x^3 + b*x)*(a*x^2 - b)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - c*x + a*x^2)/((b*x + a*x^3)^(1/2)*(b - a*x^2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + b - cx}{\sqrt{x\left(ax^2 + b\right)}\left(ax^2 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-c*x+b)/(a*x**2-b)/(a*x**3+b*x)**(1/2),x)
```

```
[Out] Integral((a*x**2 + b - c*x)/(sqrt(x*(a*x**2 + b))*(a*x**2 - b)), x)
```

3.1708 $\int \frac{b+cx+ax^2}{\left(-b+ax^2\right)\sqrt{bx+ax^3}}\,dx$

Optimal. Leaf size=147

$$-\frac{\left(2\sqrt{a}\,\sqrt{b}-c\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{ax^3+bx}}{ax^2+b}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(2\sqrt{a}\,\sqrt{b}+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{ax^3+bx}}{ax^2+b}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}$$

Rubi [C] time = 1.50, antiderivative size = 284, normalized size of antiderivative = 1.93, number of steps used = 13, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2056, 6725, 329, 220, 933, 168, 537}

$$-\frac{\sqrt{x}\left(2\sqrt{b}-\frac{c}{\sqrt{a}}\right)\sqrt{\frac{ax^2}{b}+1}\,\Pi\left(\frac{\sqrt{-a}}{\sqrt{a}};\sin^{-1}\left(\frac{\sqrt[4]{-a}\,\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{ax^3+bx}}-\frac{\sqrt{x}\left(\frac{c}{\sqrt{a}}+2\sqrt{b}\right)\sqrt{\frac{ax^2}{b}+1}\,\Pi\left(\frac{\sqrt{a}}{\sqrt{-a}};\sin^{-1}\left(\frac{\sqrt[4]{-a}\,\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{ax^3+bx}}+\frac{\sqrt{x}\left(\sqrt{a}\,x+\sqrt{b}\right)\sqrt{\frac{ax^2+b}{\left(\sqrt{a}\,x+\sqrt{b}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{ax^3+bx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(b + c*x + a*x^2)/((-b + a*x^2)*Sqrt[b*x + a*x^3]),x]
```

```
[Out] (Sqrt[x]*(Sqrt[b] + Sqrt[a]*x)*Sqrt[(b + a*x^2)/(Sqrt[b] + Sqrt[a]*x)^2]*El
lipticF[2*ArcTan[(a^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(a^(1/4)*b^(1/4)*Sqrt[b*
x + a*x^3]) - ((2*Sqrt[b] - c/Sqrt[a])*Sqrt[x]*Sqrt[1 + (a*x^2)/b]*Elliptic
Pi[Sqrt[-a]/Sqrt[a], ArcSin[((-a)^(1/4)*Sqrt[x])/b^(1/4)], -1])/((-a)^(1/4)
*b^(1/4)*Sqrt[b*x + a*x^3]) - ((2*Sqrt[b] + c/Sqrt[a])*Sqrt[x]*Sqrt[1 + (a*
x^2)/b]*EllipticPi[Sqrt[a]/Sqrt[-a], ArcSin[((-a)^(1/4)*Sqrt[x])/b^(1/4)],
-1])/((-a)^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
```

```
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{b+cx+ax^2}{\left(-b+ax^2\right)\sqrt{bx+ax^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \frac{b+cx+ax^2}{\sqrt{x}\left(-b+ax^2\right)\sqrt{b+ax^2}}\,dx}{\sqrt{bx+ax^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{b+ax^2}}+\frac{2b+cx}{\sqrt{x}\left(-b+ax^2\right)\sqrt{b+ax^2}}\right)dx}{\sqrt{bx+ax^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx}{\sqrt{bx+ax^3}} + \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \frac{2b+cx}{\sqrt{x}\left(-b+ax^2\right)\sqrt{b+ax^2}}\,dx}{\sqrt{bx+ax^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \left(-\frac{2b^{3/2}+\frac{bc}{\sqrt{a}}}{2b\sqrt{x}\left(\sqrt{b}-\sqrt{a}\,x\right)\sqrt{b+ax^2}}-\frac{2b^{3/2}-\frac{bc}{\sqrt{a}}}{2b\sqrt{x}\left(\sqrt{b}+\sqrt{a}\,x\right)\sqrt{b+ax^2}}\right)dx}{\sqrt{bx+ax^3}} + \frac{\left(2\sqrt{x}\right.\cdots}{\cdots}$$

$$= \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}\,x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} - \frac{\left(\left(2\sqrt{b}-\frac{c}{\sqrt{a}}\right)\sqrt{x}\sqrt{b+\cdots}\right.}{2\cdots}$$

$$= \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}\,x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} - \frac{\left(\left(2\sqrt{b}-\frac{c}{\sqrt{a}}\right)\sqrt{x}\sqrt{1-\cdots}\right.}{\cdots}$$

$$= \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}\,x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} + \frac{\left(\left(2\sqrt{b}-\frac{c}{\sqrt{a}}\right)\sqrt{x}\sqrt{1-\cdots}\right.}{\cdots}$$

$$= \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}\,x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} - \frac{\left(2\sqrt{b}-\frac{c}{\sqrt{a}}\right)\sqrt{x}\sqrt{1+\cdots}}{\sqrt[4]{-a}\cdots}$$

Mathematica [C] time = 0.43, size = 133, normalized size = 0.90

$$-\frac{2x\sqrt{\frac{ax^2}{b}+1}\left(x\left(5cF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{ax^2}{b},\frac{ax^2}{b}\right)+3axF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{ax^2}{b},\frac{ax^2}{b}\right)\right)+15bF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{ax^2}{b},\frac{ax^2}{b}\right)\right)}{15b\sqrt{x\left(ax^2+b\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b + c*x + a*x^2)/((-b + a*x^2)*Sqrt[b*x + a*x^3]),x]
```

```
[Out] (-2*x*Sqrt[1 + (a*x^2)/b]*(15*b*AppellF1[1/4, 1/2, 1, 5/4, -((a*x^2)/b), (a
*x^2)/b] + x*(5*c*AppellF1[3/4, 1/2, 1, 7/4, -((a*x^2)/b), (a*x^2)/b] + 3*a
*x*AppellF1[5/4, 1/2, 1, 9/4, -((a*x^2)/b), (a*x^2)/b])))/(15*b*Sqrt[x*(b +
 a*x^2)])
```

IntegrateAlgebraic [A] time = 0.43, size = 147, normalized size = 1.00

$$-\frac{\left(2\sqrt{a}\sqrt{b}-c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(2\sqrt{a}\sqrt{b}+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + c*x + a*x^2)/((-b + a*x^2)*Sqrt[b*x + a*x^3]),x]
```

```
[Out] -1/2*((2*Sqrt[a]*Sqrt[b] - c)*ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[b*x + a*
x^3])/(b + a*x^2)])/(Sqrt[2]*a^(3/4)*b^(3/4)) - ((2*Sqrt[a]*Sqrt[b] + c)*Ar
cTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3])/(b + a*x^2)])/(2*Sqrt[2]*
a^(3/4)*b^(3/4))
```

fricas [B] time = 0.82, size = 1557, normalized size = 10.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+c*x+b)/(a*x^2-b)/(a*x^3+b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(1/2)*sqrt((a*b*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)) + 4*
c)/(a*b))*log(-(16*a^2*b^4 - b^2*c^4 + (16*a^4*b^2 - a^2*c^4)*x^4 + 6*(16*a
^3*b^3 - a*b*c^4)*x^2 + 4*sqrt(1/2)*(4*a^2*b^3*c + a*b^2*c^3 + (4*a^3*b^2*c
 + a^2*b*c^3)*x^2 - 4*(4*a^3*b^3 + a^2*b^2*c^2)*x - 2*(a^4*b^3*x^2 - a^3*b^
3*c*x + a^3*b^4)*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)))*sqrt(a*x^3
 + b*x)*sqrt((a*b*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)) + 4*c)/(a*
b)) + 4*((4*a^4*b^3 - a^3*b^2*c^2)*x^3 + (4*a^3*b^4 - a^2*b^3*c^2)*x)*sqrt(
(16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)))/(a^2*x^4 - 2*a*b*x^2 + b^2)) - 1
/8*sqrt(1/2)*sqrt((a*b*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)) + 4*c
)/(a*b))*log(-(16*a^2*b^4 - b^2*c^4 + (16*a^4*b^2 - a^2*c^4)*x^4 + 6*(16*a^
3*b^3 - a*b*c^4)*x^2 - 4*sqrt(1/2)*(4*a^2*b^3*c + a*b^2*c^3 + (4*a^3*b^2*c
+ a^2*b*c^3)*x^2 - 4*(4*a^3*b^3 + a^2*b^2*c^2)*x - 2*(a^4*b^3*x^2 - a^3*b^3
*c*x + a^3*b^4)*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)))*sqrt(a*x^3
+ b*x)*sqrt((a*b*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)) + 4*c)/(a*b
)) + 4*((4*a^4*b^3 - a^3*b^2*c^2)*x^3 + (4*a^3*b^4 - a^2*b^3*c^2)*x)*sqrt((
16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)))/(a^2*x^4 - 2*a*b*x^2 + b^2)) + 1/
8*sqrt(1/2)*sqrt(-(a*b*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)) - 4*c
)/(a*b))*log(-(16*a^2*b^4 - b^2*c^4 + (16*a^4*b^2 - a^2*c^4)*x^4 + 6*(16*a^
3*b^3 - a*b*c^4)*x^2 + 4*sqrt(1/2)*(4*a^2*b^3*c + a*b^2*c^3 + (4*a^3*b^2*c
+ a^2*b*c^3)*x^2 - 4*(4*a^3*b^3 + a^2*b^2*c^2)*x + 2*(a^4*b^3*x^2 - a^3*b^3
*c*x + a^3*b^4)*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)))*sqrt(a*x^3
+ b*x)*sqrt(-(a*b*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)) - 4*c)/(a*
b)) - 4*((4*a^4*b^3 - a^3*b^2*c^2)*x^3 + (4*a^3*b^4 - a^2*b^3*c^2)*x)*sqrt(
(16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)))/(a^2*x^4 - 2*a*b*x^2 + b^2)) - 1
```

```
/8*sqrt(1/2)*sqrt(-(a*b*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)) - 4*
c)/(a*b))*log(-(16*a^2*b^4 - b^2*c^4 + (16*a^4*b^2 - a^2*c^4)*x^4 + 6*(16*a
^3*b^3 - a*b*c^4)*x^2 - 4*sqrt(1/2)*(4*a^2*b^3*c + a*b^2*c^3 + (4*a^3*b^2*c
 + a^2*b*c^3)*x^2 - 4*(4*a^3*b^3 + a^2*b^2*c^2)*x + 2*(a^4*b^3*x^2 - a^3*b^
3*c*x + a^3*b^4)*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)))*sqrt(a*x^3
 + b*x)*sqrt(-(a*b*sqrt((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)) - 4*c)/(a
*b)) - 4*((4*a^4*b^3 - a^3*b^2*c^2)*x^3 + (4*a^3*b^4 - a^2*b^3*c^2)*x)*sqrt
((16*a^2*b^2 + 8*a*b*c^2 + c^4)/(a^3*b^3)))/(a^2*x^4 - 2*a*b*x^2 + b^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2+cx+b}{\sqrt{ax^3+bx}\left(ax^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+c*x+b)/(a*x^2-b)/(a*x^3+b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^2 + c*x + b)/(sqrt(a*x^3 + b*x)*(a*x^2 - b)), x)
```

maple [C] time = 0.05, size = 710, normalized size = 4.83

$$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{ax^3+bx}}+\frac{\sqrt{-ab}\,\sqrt{\frac{xa}{\sqrt{-ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)c}{2a^2\sqrt{ax^3+bx}\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)}+\frac{\sqrt{-ab}\,\sqrt{\frac{xa}{\sqrt{-ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)b}{\sqrt{ab}\,a\sqrt{ax^3+bx}\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)}+\frac{\sqrt{-ab}\,\sqrt{\frac{xa}{\sqrt{-ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)c}{2a^2\sqrt{ax^3+bx}\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)}-\frac{\sqrt{-ab}\,\sqrt{\frac{xa}{\sqrt{-ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)b}{\sqrt{ab}\,a\sqrt{ax^3+bx}\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+c*x+b)/(a*x^2-b)/(a*x^3+b*x)^(1/2),x)
```

```
[Out] 1/a*(-a*b)^(1/2)*((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x-1/a*(-a
*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2
)*EllipticF(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/2/a^
2*(-a*b)^(1/2)*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x
*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2)
)*EllipticPi(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2)/
(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2)),1/2*2^(1/2))*c+1/(a*b)^(1/2)/a*(-a*b)^(
1/2)*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^
(1/2))^(1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2))*Elliptic
Pi(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2)/(-1/a*(-a*
b)^(1/2)-1/a*(a*b)^(1/2)),1/2*2^(1/2))*b+1/2/a^2*(-a*b)^(1/2)*(x*a/(-a*b)^(
1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^
3+b*x)^(1/2)/(-1/a*(-a*b)^(1/2)+1/a*(a*b)^(1/2))*EllipticPi(((x+1/a*(-a*b)^
(1/2))*a/(-a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2)/(-1/a*(-a*b)^(1/2)+1/a*(a*b)
^(1/2)),1/2*2^(1/2))*c-1/(a*b)^(1/2)/a*(-a*b)^(1/2)*(x*a/(-a*b)^(1/2)+1)^(1
/2)*(-2*x*a/(-a*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/
2)/(-1/a*(-a*b)^(1/2)+1/a*(a*b)^(1/2))*EllipticPi(((x+1/a*(-a*b)^(1/2))*a/(
-a*b)^(1/2))^(1/2),-1/a*(-a*b)^(1/2)/(-1/a*(-a*b)^(1/2)+1/a*(a*b)^(1/2)),1/
2*2^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2+cx+b}{\sqrt{ax^3+bx}\left(ax^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+c*x+b)/(a*x^2-b)/(a*x^3+b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 + c*x + b)/(sqrt(a*x^3 + b*x)*(a*x^2 - b)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + c*x + a*x^2)/((b*x + a*x^3)^(1/2)*(b - a*x^2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + b + cx}{\sqrt{x\left(ax^2 + b\right)}\left(ax^2 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+c*x+b)/(a*x**2-b)/(a*x**3+b*x)**(1/2),x)
```

```
[Out] Integral((a*x**2 + b + c*x)/(sqrt(x*(a*x**2 + b))*(a*x**2 - b)), x)
```

3.1709 $\int \frac{\sqrt[3]{b-ax^6}\left(b+ax^6\right)}{x^2\left(-b+cx^3+ax^6\right)}\,dx$

Optimal. Leaf size=147

$$-\frac{1}{6}\sqrt[3]{c}\log\left(\sqrt[3]{c}\,x\sqrt[3]{b-ax^6}+\left(b-ax^6\right)^{2/3}+c^{2/3}x^2\right)+\frac{1}{3}\sqrt[3]{c}\log\left(\sqrt[3]{b-ax^6}-\sqrt[3]{c}\,x\right)+\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{c}\,x}{2\sqrt[3]{b-ax^6}+\sqrt[3]{c}\,x}\right)}{\sqrt{3}}+\cdots$$

Rubi [C] time = 3.84, antiderivative size = 1003, normalized size of antiderivative = 6.82, number of steps used = 38, number of rules used = 9, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6728, 365, 364, 1562, 465, 430, 429, 511, 510}

$$-\frac{2a^2c\sqrt[3]{b-ax^6}\,F_1\left(\frac{5}{6};1,-\frac{1}{3};\frac{11}{6};\frac{2a^2x^6}{c^2-\sqrt{c^2+4ab}\,c+2ab},\frac{ax^6}{b}\right)x^5}{5\sqrt{c^2+4ab}\left(c^2-\sqrt{c^2+4ab}\,c+2ab\right)\sqrt[3]{1-\frac{ax^6}{b}}}-\frac{2a^2\left(1-\frac{c}{\sqrt{c^2+4ab}}\right)\sqrt[3]{b-ax^6}\,F_1\left(\frac{5}{6};1,-\frac{1}{3};\frac{11}{6};\frac{2a^2x^6}{c^2-\sqrt{c^2+4ab}\,c+2ab},\frac{ax^6}{b}\right)x^5}{5\left(2ab+c\left(c-\sqrt{c^2+4ab}\right)\right)\sqrt[3]{1-\frac{ax^6}{b}}}-\frac{2a^2\left(\frac{c}{\sqrt{c^2+4ab}}+1\right)\sqrt[3]{b-ax^6}\,F_1\left(\frac{5}{6};1,-\frac{1}{3};\frac{11}{6};\frac{2a^2x^6}{2ab+c\left(c+\sqrt{c^2+4ab}\right)},\frac{ax^6}{b}\right)x^5}{5\left(2ab+c\left(c+\sqrt{c^2+4ab}\right)\right)\sqrt[3]{1-\frac{ax^6}{b}}}+\frac{2a^2c\sqrt[3]{b-ax^6}\,F_1\left(\frac{5}{6};1,-\frac{1}{3};\frac{11}{6};\frac{2a^2x^6}{2ab+c\left(c+\sqrt{c^2+4ab}\right)},\frac{ax^6}{b}\right)x^5}{5\sqrt{c^2+4ab}\left(2ab+c\left(c+\sqrt{c^2+4ab}\right)\right)\sqrt[3]{1-\frac{ax^6}{b}}}-\frac{a\sqrt[3]{b-ax^6}\,F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};\frac{2a^2x^6}{c^2-\sqrt{c^2+4ab}\,c+2ab},\frac{ax^6}{b}\right)x^2}{\sqrt{c^2+4ab}\sqrt[3]{1-\frac{ax^6}{b}}}-\frac{ac\left(1-\frac{c}{\sqrt{c^2+4ab}}\right)\sqrt[3]{b-ax^6}\,F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};\frac{2a^2x^6}{c^2-\sqrt{c^2+4ab}\,c+2ab},\frac{ax^6}{b}\right)x^2}{2\left(2ab+c\left(c-\sqrt{c^2+4ab}\right)\right)\sqrt[3]{1-\frac{ax^6}{b}}}+\frac{a\sqrt[3]{b-ax^6}\,F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};\frac{2a^2x^6}{2ab+c\left(c+\sqrt{c^2+4ab}\right)},\frac{ax^6}{b}\right)x^2}{\sqrt{c^2+4ab}\sqrt[3]{1-\frac{ax^6}{b}}}-\frac{ac\left(\frac{c}{\sqrt{c^2+4ab}}+1\right)\sqrt[3]{b-ax^6}\,F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};\frac{2a^2x^6}{2ab+c\left(c+\sqrt{c^2+4ab}\right)},\frac{ax^6}{b}\right)x^2}{2\left(2ab+c\left(c+\sqrt{c^2+4ab}\right)\right)\sqrt[3]{1-\frac{ax^6}{b}}}+\frac{\sqrt[3]{b-ax^6}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};\frac{ax^6}{b}\right)}{\sqrt[3]{1-\frac{ax^6}{b}}\,x}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((b - a*x^6)^(1/3)*(b + a*x^6))/(x^2*(-b + c*x^3 + a*x^6)),x]
```

```
[Out] -((a*x^2*(b - a*x^6)^(1/3)*AppellF1[1/3, 1, -1/3, 4/3, (2*a^2*x^6)/(2*a*b +
 c^2 - c*Sqrt[4*a*b + c^2]), (a*x^6)/b])/(Sqrt[4*a*b + c^2]*(1 - (a*x^6)/b)
^(1/3))) - (a*c*(1 - c/Sqrt[4*a*b + c^2])*x^2*(b - a*x^6)^(1/3)*AppellF1[1/
3, 1, -1/3, 4/3, (2*a^2*x^6)/(2*a*b + c^2 - c*Sqrt[4*a*b + c^2]), (a*x^6)/b
])/(2*(2*a*b + c*(c - Sqrt[4*a*b + c^2]))*(1 - (a*x^6)/b)^(1/3)) + (a*x^2*(
b - a*x^6)^(1/3)*AppellF1[1/3, 1, -1/3, 4/3, (2*a^2*x^6)/(2*a*b + c*(c + Sq
rt[4*a*b + c^2])), (a*x^6)/b])/(Sqrt[4*a*b + c^2]*(1 - (a*x^6)/b)^(1/3)) -
(a*c*(1 + c/Sqrt[4*a*b + c^2])*x^2*(b - a*x^6)^(1/3)*AppellF1[1/3, 1, -1/3,
 4/3, (2*a^2*x^6)/(2*a*b + c*(c + Sqrt[4*a*b + c^2])), (a*x^6)/b])/(2*(2*a*
b + c*(c + Sqrt[4*a*b + c^2]))*(1 - (a*x^6)/b)^(1/3)) - (2*a^2*c*x^5*(b - a
*x^6)^(1/3)*AppellF1[5/6, 1, -1/3, 11/6, (2*a^2*x^6)/(2*a*b + c^2 - c*Sqrt[
4*a*b + c^2]), (a*x^6)/b])/(5*Sqrt[4*a*b + c^2]*(2*a*b + c^2 - c*Sqrt[4*a*b
 + c^2])*(1 - (a*x^6)/b)^(1/3)) - (2*a^2*(1 - c/Sqrt[4*a*b + c^2])*x^5*(b -
 a*x^6)^(1/3)*AppellF1[5/6, 1, -1/3, 11/6, (2*a^2*x^6)/(2*a*b + c^2 - c*Sqr
t[4*a*b + c^2]), (a*x^6)/b])/(5*(2*a*b + c*(c - Sqrt[4*a*b + c^2]))*(1 - (a
*x^6)/b)^(1/3)) + (2*a^2*c*x^5*(b - a*x^6)^(1/3)*AppellF1[5/6, 1, -1/3, 11/
6, (2*a^2*x^6)/(2*a*b + c*(c + Sqrt[4*a*b + c^2])), (a*x^6)/b])/(5*Sqrt[4*a
*b + c^2]*(2*a*b + c*(c + Sqrt[4*a*b + c^2]))*(1 - (a*x^6)/b)^(1/3)) - (2*a
^2*(1 + c/Sqrt[4*a*b + c^2])*x^5*(b - a*x^6)^(1/3)*AppellF1[5/6, 1, -1/3, 1
1/6, (2*a^2*x^6)/(2*a*b + c*(c + Sqrt[4*a*b + c^2])), (a*x^6)/b])/(5*(2*a*b
 + c*(c + Sqrt[4*a*b + c^2]))*(1 - (a*x^6)/b)^(1/3)) + ((b - a*x^6)^(1/3)*H
ypergeometric2F1[-1/3, -1/6, 5/6, (a*x^6)/b])/(x*(1 - (a*x^6)/b)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1562

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(
2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)/(d^2 - e^2*x^(2*n)))^(-q), x], x]
, x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !IntegerQ[p]
 && ILtQ[q, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{b-ax^6}\left(b+ax^6\right)}{x^2\left(-b+cx^3+ax^6\right)}dx = \int\left(-\frac{\sqrt[3]{b-ax^6}}{x^2}+\frac{x\left(c+2ax^3\right)\sqrt[3]{b-ax^6}}{-b+cx^3+ax^6}\right)dx$$

$$= -\int \frac{\sqrt[3]{b-ax^6}}{x^2}dx + \int \frac{x\left(c+2ax^3\right)\sqrt[3]{b-ax^6}}{-b+cx^3+ax^6}dx$$

$$= -\frac{\sqrt[3]{b-ax^6}\int \frac{\sqrt[3]{1-\frac{ax^6}{b}}}{x^2}dx}{\sqrt[3]{1-\frac{ax^6}{b}}} + \int\left(\frac{cx\sqrt[3]{b-ax^6}}{-b+cx^3+ax^6}+\frac{2ax^4\sqrt[3]{b-ax^6}}{-b+cx^3+ax^6}\right)dx$$

$$= \frac{\sqrt[3]{b-ax^6}\ {}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};\frac{ax^6}{b}\right)}{x\sqrt[3]{1-\frac{ax^6}{b}}} + (2a)\int \frac{x^4\sqrt[3]{b-ax^6}}{-b+cx^3+ax^6}dx + c\int \frac{x\sqrt[3]{b-ax^6}}{-b+cx^3+ax^6}$$

$$= \frac{\sqrt[3]{b-ax^6}\ {}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};\frac{ax^6}{b}\right)}{x\sqrt[3]{1-\frac{ax^6}{b}}} + (2a)\int\left(\frac{\left(-c+\sqrt{4ab+c^2}\right)x\sqrt[3]{b-ax^6}}{\sqrt{4ab+c^2}\left(c-\sqrt{4ab+c^2}+2ax^3\right)}+\right.$$

$$= \frac{\sqrt[3]{b-ax^6}\ {}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};\frac{ax^6}{b}\right)}{x\sqrt[3]{1-\frac{ax^6}{b}}} + \frac{(2ac)\int \frac{x\sqrt[3]{b-ax^6}}{c-\sqrt{4ab+c^2}+2ax^3}dx}{\sqrt{4ab+c^2}} - \frac{(2ac)\int \frac{x\sqrt[3]{b-ax^6}}{c+\sqrt{4ab+c^2}+2a}}{\sqrt{4ab+c^2}}$$

$$= \frac{\sqrt[3]{b-ax^6}\ {}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};\frac{ax^6}{b}\right)}{x\sqrt[3]{1-\frac{ax^6}{b}}} - \frac{(2ac)\int\left(\frac{\left(c+\sqrt{4ab+c^2}\right)x\sqrt[3]{b-ax^6}}{2\left(2ab+c^2+c\sqrt{4ab+c^2}-2a^2x^6\right)}+\frac{ax^4\sqrt[3]{b-a}}{-2ab-c^2-c\sqrt{4ab+}}\right.}{\sqrt{4ab+c^2}}$$

$$= \frac{\sqrt[3]{b-ax^6}\ {}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};\frac{ax^6}{b}\right)}{x\sqrt[3]{1-\frac{ax^6}{b}}} - \frac{\left(2a^2c\right)\int \frac{x^4\sqrt[3]{b-ax^6}}{-2ab-c^2-c\sqrt{4ab+c^2}+2a^2x^6}dx}{\sqrt{4ab+c^2}} + \frac{\left(2a^2c\right)\int -2}{}$$

$$= \frac{\sqrt[3]{b-ax^6}\ {}_2F_1\left(-\frac{1}{3},-\frac{1}{6};\frac{5}{6};\frac{ax^6}{b}\right)}{x\sqrt[3]{1-\frac{ax^6}{b}}} + \frac{1}{2}\left(ac\left(1-\frac{c}{\sqrt{4ab+c^2}}\right)\right)\text{Subst}\left(\int \frac{}{-2ab-c^2+c}\right.$$

$$= -\frac{2a^2cx^5\sqrt[3]{b-ax^6}\,F_1\left(\frac{5}{6};1,-\frac{1}{3};\frac{11}{6};\frac{2a^2x^6}{2ab+c^2-c\sqrt{4ab+c^2}},\frac{ax^6}{b}\right)}{5\sqrt{4ab+c^2}\left(2ab+c^2-c\sqrt{4ab+c^2}\right)\sqrt[3]{1-\frac{ax^6}{b}}} - \frac{2a^2\left(1-\frac{c}{\sqrt{4ab+c^2}}\right)x^5\sqrt[3]{b}}{5\left(2ab+c\right.}$$

$$= -\frac{ax^2\sqrt[3]{b-ax^6}\,F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};\frac{2a^2x^6}{2ab+c^2-c\sqrt{4ab+c^2}},\frac{ax^6}{b}\right)}{\sqrt{4ab+c^2}\sqrt[3]{1-\frac{ax^6}{b}}} - \frac{ac\left(1-\frac{c}{\sqrt{4ab+c^2}}\right)x^2\sqrt[3]{b-ax^6}}{2\left(2ab+c\left(c-\right.\right.}$$

Mathematica [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{b-ax^6}\left(b+ax^6\right)}{x^2\left(-b+cx^3+ax^6\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[((b - a*x^6)^(1/3)*(b + a*x^6))/(x^2*(-b + c*x^3 + a*x^6)),x]
[Out] Integrate[((b - a*x^6)^(1/3)*(b + a*x^6))/(x^2*(-b + c*x^3 + a*x^6)), x]
```

IntegrateAlgebraic [A] time = 2.21, size = 147, normalized size = 1.00

$$-\frac{1}{6}\sqrt[3]{c}\log\left(\sqrt[3]{c}\,x\sqrt[3]{b-ax^6}+\left(b-ax^6\right)^{2/3}+c^{2/3}x^2\right)+\frac{1}{3}\sqrt[3]{c}\log\left(\sqrt[3]{b-ax^6}-\sqrt[3]{c}\,x\right)+\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{c}\,x}{2\sqrt[3]{b-ax^6}+\sqrt[3]{c}\,x}\right)}{\sqrt{3}}+\frac{\sqrt[3]{b-ax^6}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b - a*x^6)^(1/3)*(b + a*x^6))/(x^2*(-b + c*x^3 + a*x^6
)),x]
```

```
[Out] (b - a*x^6)^(1/3)/x + (c^(1/3)*ArcTan[(Sqrt[3]*c^(1/3)*x)/(c^(1/3)*x + 2*(b
 - a*x^6)^(1/3))])/Sqrt[3] + (c^(1/3)*Log[-(c^(1/3)*x) + (b - a*x^6)^(1/3)]
)/3 - (c^(1/3)*Log[c^(2/3)*x^2 + c^(1/3)*x*(b - a*x^6)^(1/3) + (b - a*x^6)^
(2/3)])/6
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^6+b)^(1/3)*(a*x^6+b)/x^2/(a*x^6+c*x^3-b),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^6+b\right)\left(-ax^6+b\right)^{\frac{1}{3}}}{\left(ax^6+cx^3-b\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^6+b)^(1/3)*(a*x^6+b)/x^2/(a*x^6+c*x^3-b),x, algorithm="giac
")
```

```
[Out] integrate((a*x^6 + b)*(-a*x^6 + b)^(1/3)/((a*x^6 + c*x^3 - b)*x^2), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(-a\,x^6+b\right)^{\frac{1}{3}}\left(a\,x^6+b\right)}{x^2\left(a\,x^6+c\,x^3-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x^6+b)^(1/3)*(a*x^6+b)/x^2/(a*x^6+c*x^3-b),x)
```

```
[Out] int((-a*x^6+b)^(1/3)*(a*x^6+b)/x^2/(a*x^6+c*x^3-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^6+b\right)\left(-ax^6+b\right)^{\frac{1}{3}}}{\left(ax^6+cx^3-b\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^6+b)^(1/3)*(a*x^6+b)/x^2/(a*x^6+c*x^3-b),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x^6 + b)*(-a*x^6 + b)^(1/3)/((a*x^6 + c*x^3 - b)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^6+b\right)\,{\left(b-a\,x^6\right)}^{1/3}}{x^2\,\left(a\,x^6+c\,x^3-b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a*x^6)*(b - a*x^6)^(1/3))/(x^2*(a*x^6 - b + c*x^3)),x)
[Out] int(((b + a*x^6)*(b - a*x^6)^(1/3))/(x^2*(a*x^6 - b + c*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x**6+b)**(1/3)*(a*x**6+b)/x**2/(a*x**6+c*x**3-b),x)
[Out] Timed out
```

3.1710 $\int \frac{x^2(4+7x^3)}{\sqrt[3]{x+x^4}\,(-1+x^4+x^7)}\,dx$

Optimal. Leaf size=147

$$2\tanh^{-1}\left(1-2x\sqrt[3]{x^4+x}\right)-\sqrt{3}\tan^{-1}\left(\frac{3\sqrt{3}x\sqrt[3]{x^4+x}-3x^2\sqrt[3]{x^4+x}}{-3\sqrt[3]{x^4+x}\,x+\sqrt{3}\sqrt[3]{x^4+x}\,x^2+2\sqrt{3}x-6}\right)-\tanh^{-1}\left(\frac{\sqrt[3]{x^4+x}\,x\cdots}{\sqrt[3]{x^4+x}\,x+2\left(x^4+\cdots\right.}\right.$$

Rubi [F] time = 1.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(4+7x^3\right)}{\sqrt[3]{x+x^4}\left(-1+x^4+x^7\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(4 + 7*x^3))/((x + x^4)^(1/3)*(-1 + x^4 + x^7)),x]
```

```
[Out] (12*x^(1/3)*(1 + x^3)^(1/3)*Defer[Subst][Defer[Int][x^7/((1 + x^9)^(1/3)*(-
1 + x^12 + x^21)), x], x, x^(1/3)])/(x + x^4)^(1/3) + (21*x^(1/3)*(1 + x^3)
^(1/3)*Defer[Subst][Defer[Int][x^16/((1 + x^9)^(1/3)*(-1 + x^12 + x^21)), x
], x, x^(1/3)])/(x + x^4)^(1/3)
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(4+7x^3\right)}{\sqrt[3]{x+x^4}\left(-1+x^4+x^7\right)}\,dx &= \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^3}\right)\int \frac{x^{5/3}\left(4+7x^3\right)}{\sqrt[3]{1+x^3}\left(-1+x^4+x^7\right)}\,dx}{\sqrt[3]{x+x^4}}\\ &= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^3}\right)\text{Subst}\left(\int \frac{x^7\left(4+7x^9\right)}{\sqrt[3]{1+x^9}\left(-1+x^{12}+x^{21}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x+x^4}}\\ &= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^3}\right)\text{Subst}\left(\int \left(\frac{4x^7}{\sqrt[3]{1+x^9}\left(-1+x^{12}+x^{21}\right)}+\frac{7x^{16}}{\sqrt[3]{1+x^9}\left(-1+x^{12}+x^{21}\right)}\right)dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x+x^4}}\\ &= \frac{\left(12\sqrt[3]{x}\sqrt[3]{1+x^3}\right)\text{Subst}\left(\int \frac{x^7}{\sqrt[3]{1+x^9}\left(-1+x^{12}+x^{21}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x+x^4}}+\frac{\left(21\sqrt[3]{x}\sqrt[3]{1+x^3}\right)\text{Sub}\cdots}{\cdots}\end{aligned}$$

Mathematica [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(4+7x^3\right)}{\sqrt[3]{x+x^4}\left(-1+x^4+x^7\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(4 + 7*x^3))/((x + x^4)^(1/3)*(-1 + x^4 + x^7)),x]
```

```
[Out] Integrate[(x^2*(4 + 7*x^3))/((x + x^4)^(1/3)*(-1 + x^4 + x^7)), x]
```

IntegrateAlgebraic [A] time = 10.23, size = 147, normalized size = 1.00

$$2\tanh^{-1}\left(1-2x\sqrt[3]{x^4+x}\right)-\sqrt{3}\tan^{-1}\left(\frac{3\sqrt{3}x\sqrt[3]{x^4+x}-3x^2\sqrt[3]{x^4+x}}{-3\sqrt[3]{x^4+x}\,x+\sqrt{3}\sqrt[3]{x^4+x}\,x^2+2\sqrt{3}x-6}\right)-\tanh^{-1}\left(\frac{\sqrt[3]{x^4+x}\,x+1}{\sqrt[3]{x^4+x}\,x+2\left(x^4+x\right)^{2/3}x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(4 + 7*x^3))/((x + x^4)^(1/3)*(-1 + x^4 + x^7)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(3*Sqrt[3]*x*(x + x^4)^(1/3) - 3*x^2*(x + x^4)^(1/3))/(-6
+ 2*Sqrt[3]*x - 3*x*(x + x^4)^(1/3) + Sqrt[3]*x^2*(x + x^4)^(1/3))]) + 2*Ar
cTanh[1 - 2*x*(x + x^4)^(1/3)] - ArcTanh[(1 + x*(x + x^4)^(1/3))/(1 + x*(x
+ x^4)^(1/3) + 2*x^2*(x + x^4)^(2/3))]
```

fricas [A] time = 3.21, size = 103, normalized size = 0.70

$$-\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(x^4+x\right)^{\frac{2}{3}}x^2-4\sqrt{3}\left(x^4+x\right)^{\frac{1}{3}}x-\sqrt{3}\left(x^7+x^4\right)}{x^7+x^4+8}\right)+\frac{1}{2}\log\left(\frac{x^7+x^4-3\left(x^4+x\right)^{\frac{2}{3}}x^2+3\left(x^4+x\right)^{\frac{1}{3}}x-1}{x^7+x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(7*x^3+4)/(x^4+x)^(1/3)/(x^7+x^4-1),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan((2*sqrt(3)*(x^4 + x)^(2/3)*x^2 - 4*sqrt(3)*(x^4 + x)^(1/3)*
x - sqrt(3)*(x^7 + x^4))/(x^7 + x^4 + 8)) + 1/2*log((x^7 + x^4 - 3*(x^4 + x
)^(2/3)*x^2 + 3*(x^4 + x)^(1/3)*x - 1)/(x^7 + x^4 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(7x^3+4\right)x^2}{\left(x^7+x^4-1\right)\left(x^4+x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(7*x^3+4)/(x^4+x)^(1/3)/(x^7+x^4-1),x, algorithm="giac")
```

```
[Out] integrate((7*x^3 + 4)*x^2/((x^7 + x^4 - 1)*(x^4 + x)^(1/3)), x)
```

maple [C] time = 10.55, size = 319, normalized size = 2.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(7*x^3+4)/(x^4+x)^(1/3)/(x^7+x^4-1),x)
```

```
[Out] ln((67419187*RootOf(_Z^2+_Z+1)^2*x^7-92588403*x^7*RootOf(_Z^2+_Z+1)-8449994
2*x^7+67419187*RootOf(_Z^2+_Z+1)^2*x^4+168096051*RootOf(_Z^2+_Z+1)*(x^4+x)^
(2/3)*x^2-92588403*RootOf(_Z^2+_Z+1)*x^4+311926719*x^2*(x^4+x)^(2/3)-844999
42*x^4+143830668*RootOf(_Z^2+_Z+1)*(x^4+x)^(1/3)*x-134838374*RootOf(_Z^2+_Z
+1)^2-168096051*x*(x^4+x)^(1/3)-286757503*RootOf(_Z^2+_Z+1)-126749913)/(x^7
+x^4-1))+RootOf(_Z^2+_Z+1)*ln(-(42249971*RootOf(_Z^2+_Z+1)^2*x^7+194169100*
x^7*RootOf(_Z^2+_Z+1)+202257561*x^7+42249971*RootOf(_Z^2+_Z+1)^2*x^4+168096
051*RootOf(_Z^2+_Z+1)*(x^4+x)^(2/3)*x^2+194169100*RootOf(_Z^2+_Z+1)*x^4-143
830668*x^2*(x^4+x)^(2/3)+202257561*x^4-311926719*RootOf(_Z^2+_Z+1)*(x^4+x)^
(1/3)*x-84499942*RootOf(_Z^2+_Z+1)^2-168096051*x*(x^4+x)^(1/3)-92588403*Roo
tOf(_Z^2+_Z+1)+67419187)/(x^7+x^4-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(7x^3+4\right)x^2}{\left(x^7+x^4-1\right)\left(x^4+x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(7*x^3+4)/(x^4+x)^(1/3)/(x^7+x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((7*x^3 + 4)*x^2/((x^7 + x^4 - 1)*(x^4 + x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(7\,x^3+4\right)}{{\left(x^4+x\right)}^{1/3}\,\left(x^7+x^4-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(7*x^3 + 4))/((x + x^4)^(1/3)*(x^4 + x^7 - 1)),x)
```

```
[Out] int((x^2*(7*x^3 + 4))/((x + x^4)^(1/3)*(x^4 + x^7 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(7 x^{3} + 4\right)}{\sqrt[3]{x \left(x + 1\right) \left(x^{2} - x + 1\right)} \left(x^{7} + x^{4} - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(7*x**3+4)/(x**4+x)**(1/3)/(x**7+x**4-1),x)
```

```
[Out] Integral(x**2*(7*x**3 + 4)/((x*(x + 1)*(x**2 - x + 1))**(1/3)*(x**7 + x**4
- 1)), x)
```

3.1711 $\int \frac{\sqrt{x-\sqrt{-1+x^2}}}{x^2} dx$

Optimal. Leaf size=147

$$\sqrt{x-\sqrt{x^2-1}}\left(\sqrt{\sqrt{x^2-1}+x}\left(-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{x^2-1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2-1}+x}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{x^2-1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2-1}+x}}\right)}{\sqrt{2}}\right)-\frac{1}{x}\right)$$

Rubi [A] time = 0.14, antiderivative size = 200, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2119, 457, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2\left(x-\sqrt{x^2-1}\right)^{3/2}}{\left(x-\sqrt{x^2-1}\right)^2+1}+\frac{\log\left(-\sqrt{x^2-1}-\sqrt{2}\sqrt{x-\sqrt{x^2-1}}+x+1\right)}{2\sqrt{2}}-\frac{\log\left(-\sqrt{x^2-1}+\sqrt{2}\sqrt{x-\sqrt{x^2-1}}+x+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x-\sqrt{x^2-1}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{x-\sqrt{x^2-1}}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x - Sqrt[-1 + x^2]]/x^2,x]
```

```
[Out] (-2*(x - Sqrt[-1 + x^2])^(3/2))/(1 + (x - Sqrt[-1 + x^2])^2) - ArcTan[1 - S
qrt[2]*Sqrt[x - Sqrt[-1 + x^2]]]/Sqrt[2] + ArcTan[1 + Sqrt[2]*Sqrt[x - Sqrt
[-1 + x^2]]]/Sqrt[2] + Log[1 + x - Sqrt[-1 + x^2] - Sqrt[2]*Sqrt[x - Sqrt[-
1 + x^2]]]/(2*Sqrt[2]) - Log[1 + x - Sqrt[-1 + x^2] + Sqrt[2]*Sqrt[x - Sqrt
[-1 + x^2]]]/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{x-\sqrt{-1+x^2}}}{x^2}\,dx = 2\,\text{Subst}\left(\int \frac{\sqrt{x}\,\left(-1+x^2\right)}{\left(1+x^2\right)^2}\,dx, x, x-\sqrt{-1+x^2}\right)$$

$$= -\frac{2\left(x-\sqrt{-1+x^2}\right)^{3/2}}{1+\left(x-\sqrt{-1+x^2}\right)^2} + \text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, x-\sqrt{-1+x^2}\right)$$

$$= -\frac{2\left(x-\sqrt{-1+x^2}\right)^{3/2}}{1+\left(x-\sqrt{-1+x^2}\right)^2} + 2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{x-\sqrt{-1+x^2}}\right)$$

$$= -\frac{2\left(x-\sqrt{-1+x^2}\right)^{3/2}}{1+\left(x-\sqrt{-1+x^2}\right)^2} - \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x-\sqrt{-1+x^2}}\right) + \text{Subst}\left(\int \frac{1+}{1+}\right.$$

$$= -\frac{2\left(x-\sqrt{-1+x^2}\right)^{3/2}}{1+\left(x-\sqrt{-1+x^2}\right)^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x-\sqrt{-1+x^2}}\right) + \frac{1}{2}\text{Su}$$

$$= -\frac{2\left(x-\sqrt{-1+x^2}\right)^{3/2}}{1+\left(x-\sqrt{-1+x^2}\right)^2} + \frac{\log\left(1+x-\sqrt{-1+x^2}-\sqrt{2}\sqrt{x-\sqrt{-1+x^2}}\right)}{2\sqrt{2}} - \frac{\log\left(1+\right.}{}$$

$$= -\frac{2\left(x-\sqrt{-1+x^2}\right)^{3/2}}{1+\left(x-\sqrt{-1+x^2}\right)^2} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{x-\sqrt{-1+x^2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{x-\sqrt{-}}\right.}{\sqrt{2}}$$

Mathematica [C] time = 0.23, size = 102, normalized size = 0.69

$$\frac{4\sqrt{x^2-1}\left(x-\sqrt{x^2-1}\right)^{5/2}\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\left(x-\sqrt{x^2-1}\right)^2\right)-2\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\left(x-\sqrt{x^2-1}\right)^2\right)\right)}{-3x^2+3\sqrt{x^2-1}\,x+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x - Sqrt[-1 + x^2]]/x^2,x]
```

```
[Out] (4*Sqrt[-1 + x^2]*(x - Sqrt[-1 + x^2])^(5/2)*(Hypergeometric2F1[3/4, 1, 7/4
, -(x - Sqrt[-1 + x^2])^2] - 2*Hypergeometric2F1[3/4, 2, 7/4, -(x - Sqrt[-1
 + x^2])^2]))/(3 - 3*x^2 + 3*x*Sqrt[-1 + x^2])
```

IntegrateAlgebraic [A] time = 0.29, size = 135, normalized size = 0.92

$$-\frac{\sqrt{x-\sqrt{x^2-1}}}{x} + \frac{\tan^{-1}\left(\frac{-\frac{\sqrt{x^2-1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x-\sqrt{x^2-1}}}\right)}{\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{\frac{\sqrt{x^2-1}}{\sqrt{2}}-\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{x-\sqrt{x^2-1}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x - Sqrt[-1 + x^2]]/x^2,x]
```

```
[Out] -(Sqrt[x - Sqrt[-1 + x^2]]/x) + ArcTan[(-(1/Sqrt[2]) + x/Sqrt[2] - Sqrt[-1
+ x^2]/Sqrt[2])/Sqrt[x - Sqrt[-1 + x^2]]]/Sqrt[2] + ArcTanh[(-(1/Sqrt[2]) -
 x/Sqrt[2] + Sqrt[-1 + x^2]/Sqrt[2])/Sqrt[x - Sqrt[-1 + x^2]]]/Sqrt[2]
```

fricas [B] time = 0.42, size = 226, normalized size = 1.54

$$-\frac{4\sqrt{2}x\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x-\sqrt{x^2-1}}+x-\sqrt{x^2-1}+1}-\sqrt{2}\sqrt{x-\sqrt{x^2-1}}-1\right)+4\sqrt{2}x\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x-\sqrt{x^2-1}}+4x-4\sqrt{x^2-1}+4}-\sqrt{2}\sqrt{x-\sqrt{x^2-1}}+1\right)+\sqrt{2}x\log\left(4\sqrt{2}\sqrt{x-\sqrt{x^2-1}}+4x-4\sqrt{x^2-1}+4\right)-\sqrt{2}x\log\left(-4\sqrt{2}\sqrt{x-\sqrt{x^2-1}}+4x-4\sqrt{x^2-1}+4\right)+4\sqrt{x-\sqrt{x^2-1}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2-1)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*x*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x - sqrt(x^2 - 1)) + x -
 sqrt(x^2 - 1) + 1) - sqrt(2)*sqrt(x - sqrt(x^2 - 1)) - 1) + 4*sqrt(2)*x*ar
ctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x - sqrt(x^2 - 1)) + 4*x - 4*sqrt(x^2
 - 1) + 4) - sqrt(2)*sqrt(x - sqrt(x^2 - 1)) + 1) + sqrt(2)*x*log(4*sqrt(2)
*sqrt(x - sqrt(x^2 - 1)) + 4*x - 4*sqrt(x^2 - 1) + 4) - sqrt(2)*x*log(-4*sq
rt(2)*sqrt(x - sqrt(x^2 - 1)) + 4*x - 4*sqrt(x^2 - 1) + 4) + 4*sqrt(x - sqr
t(x^2 - 1)))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x-\sqrt{x^2-1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2-1)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x - sqrt(x^2 - 1))/x^2, x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x-\sqrt{x^2-1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2-1)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int((x-(x^2-1)^(1/2))^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x-\sqrt{x^2-1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2-1)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x - sqrt(x^2 - 1))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x-\sqrt{x^2-1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (x^2 - 1)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int((x - (x^2 - 1)^(1/2))^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x-\sqrt{x^2-1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2-1)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x - sqrt(x**2 - 1))/x**2, x)
```

3.1712 $\int \frac{\left(ax+\sqrt{-bx+a^2x^2}\right)^{3/4}}{\sqrt{-bx+a^2x^2}}\,dx$

Optimal. Leaf size=147

$$\frac{4\left(\sqrt{a^2x^2-bx}+ax\right)^{3/4}}{3a}+\frac{\sqrt[4]{2}\,b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\sqrt[4]{\sqrt{a^2x^2-bx}+ax}}{\sqrt[4]{b}}\right)}{a^{7/4}}-\frac{\sqrt[4]{2}\,b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\sqrt[4]{\sqrt{a^2x^2-bx}+ax}}{\sqrt[4]{b}}\right)}{a^{7/4}}$$

Rubi [A] time = 0.23, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {2121, 50, 63, 298, 205, 208}

$$\frac{4\left(\sqrt{a^2x^2-bx}+ax\right)^{3/4}}{3a}+\frac{\sqrt[4]{2}\,b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\sqrt[4]{\sqrt{a^2x^2-bx}+ax}}{\sqrt[4]{b}}\right)}{a^{7/4}}-\frac{\sqrt[4]{2}\,b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\sqrt[4]{\sqrt{a^2x^2-bx}+ax}}{\sqrt[4]{b}}\right)}{a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + Sqrt[-(b*x) + a^2*x^2])^(3/4)/Sqrt[-(b*x) + a^2*x^2],x]
```

```
[Out] (4*(a*x + Sqrt[-(b*x) + a^2*x^2])^(3/4))/(3*a) + (2^(1/4)*b^(3/4)*ArcTan[(2
^(1/4)*a^(1/4)*(a*x + Sqrt[-(b*x) + a^2*x^2])^(1/4))/b^(1/4)])/a^(7/4) - (2
^(1/4)*b^(3/4)*ArcTanh[(2^(1/4)*a^(1/4)*(a*x + Sqrt[-(b*x) + a^2*x^2])^(1/4
))/b^(1/4)])/a^(7/4)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
```

```
tQ[a/b, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2*(i/c)^m
)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(ax+\sqrt{-bx+a^2x^2}\right)^{3/4}}{\sqrt{-bx+a^2x^2}}\,dx = 2\,\text{Subst}\left(\int \frac{x^{3/4}}{-b+2ax}\,dx, x, ax+\sqrt{-bx+a^2x^2}\right)$$
$$= \frac{4\left(ax+\sqrt{-bx+a^2x^2}\right)^{3/4}}{3a} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{x}\,(-b+2ax)}\,dx, x, ax+\sqrt{-bx+a^2x^2}\right)}{a}$$
$$= \frac{4\left(ax+\sqrt{-bx+a^2x^2}\right)^{3/4}}{3a} + \frac{(4b)\,\text{Subst}\left(\int \frac{x^2}{-b+2ax^4}\,dx, x, \sqrt[4]{ax+\sqrt{-bx+a^2x^2}}\right.}{a}$$
$$= \frac{4\left(ax+\sqrt{-bx+a^2x^2}\right)^{3/4}}{3a} - \frac{\left(\sqrt{2}\,b\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{2}\,\sqrt{a}\,x^2}\,dx, x, \sqrt[4]{ax+\sqrt{-bx\cdots}}\right.}{a^{3/2}}$$
$$= \frac{4\left(ax+\sqrt{-bx+a^2x^2}\right)^{3/4}}{3a} + \frac{\sqrt[4]{2}\,b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,\sqrt[4]{ax+\sqrt{-bx+a^2x^2}}}{\sqrt[4]{b}}\right)}{a^{7/4}} - \frac{\sqrt[4]{2}\,b^{3/4}\,\text{ta}\cdots}{\cdots}$$

Mathematica [F] time = 1.52, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+\sqrt{-bx+a^2x^2}\right)^{3/4}}{\sqrt{-bx+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*x + Sqrt[-(b*x) + a^2*x^2])^(3/4)/Sqrt[-(b*x) + a^2*x^2],x]
[Out] Integrate[(a*x + Sqrt[-(b*x) + a^2*x^2])^(3/4)/Sqrt[-(b*x) + a^2*x^2], x]
```

IntegrateAlgebraic [F] time = 16.30, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+\sqrt{-bx+a^2x^2}\right)^{3/4}}{\sqrt{-bx+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x + Sqrt[-(b*x) + a^2*x^2])^(3/4)/Sqrt[-(b*x) + a^2*x
^2],x]
[Out] Defer[IntegrateAlgebraic][(a*x + Sqrt[-(b*x) + a^2*x^2])^(3/4)/Sqrt[-(b*x)
+ a^2*x^2], x]
```

fricas [B] time = 0.43, size = 268, normalized size = 1.82

$$-\frac{12 \left(\frac{1}{8}\right)^{\frac{1}{4}} a \left(\frac{b^3}{a^7}\right)^{\frac{1}{4}} \arctan\left(-\frac{2 \left(\left(\frac{1}{8}\right)^{\frac{1}{4}} \left(ax+\sqrt{a^2x^2-bx}\right)^{\frac{1}{4}} a^2 b^2 \left(\frac{b^3}{a^7}\right)^{\frac{1}{4}} - \left(\frac{1}{8}\right)^{\frac{1}{4}} \sqrt{\sqrt{\frac{1}{2}} a^3 b^3 \sqrt{\frac{b^3}{a^7}} + \sqrt{ax+\sqrt{a^2x^2-bx}}\, b^4}\, a^2 \left(\frac{b^3}{a^7}\right)^{\frac{1}{4}}\right)}{b^3}\right) + 3 \left(\frac{1}{8}\right)^{\frac{1}{4}} a \left(\frac{b^3}{a^7}\right)^{\frac{1}{4}} \log\left(4 \left(\frac{1}{8}\right)^{\frac{3}{4}} a^5 \left(\frac{b^3}{a^7}\right)^{\frac{3}{4}} + \left(ax + \sqrt{a^2x^2 - bx}\right)^{\frac{1}{4}} b^2\right) - 3 \left(\frac{1}{8}\right)^{\frac{1}{4}} a \left(\frac{b^3}{a^7}\right)^{\frac{1}{4}} \log\left(-4 \left(\frac{1}{8}\right)^{\frac{3}{4}} a^5 \left(\frac{b^3}{a^7}\right)^{\frac{3}{4}} + \left(ax + \sqrt{a^2x^2 - bx}\right)^{\frac{1}{4}} b^2\right) - 4 \left(ax + \sqrt{a^2x^2 - bx}\right)^{\frac{3}{4}}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2-b*x)^(1/2))^(3/4)/(a^2*x^2-b*x)^(1/2),x, algorithm=
"fricas")
```

```
[Out] -1/3*(12*(1/8)^(1/4)*a*(b^3/a^7)^(1/4)*arctan(-2*((1/8)^(1/4)*(a*x + sqrt(a
^2*x^2 - b*x))^(1/4)*a^2*b^2*(b^3/a^7)^(1/4) - (1/8)^(1/4)*sqrt(sqrt(1/2)*a
^3*b^3*sqrt(b^3/a^7) + sqrt(a*x + sqrt(a^2*x^2 - b*x))*b^4)*a^2*(b^3/a^7)^(
1/4))/b^3) + 3*(1/8)^(1/4)*a*(b^3/a^7)^(1/4)*log(4*(1/8)^(3/4)*a^5*(b^3/a^7
)^(3/4) + (a*x + sqrt(a^2*x^2 - b*x))^(1/4)*b^2) - 3*(1/8)^(1/4)*a*(b^3/a^7
)^(1/4)*log(-4*(1/8)^(3/4)*a^5*(b^3/a^7)^(3/4) + (a*x + sqrt(a^2*x^2 - b*x)
)^(1/4)*b^2) - 4*(a*x + sqrt(a^2*x^2 - b*x))^(3/4))/a
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2-b*x)^(1/2))^(3/4)/(a^2*x^2-b*x)^(1/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + \sqrt{a^2x^2 - bx}\right)^{\frac{3}{4}}}{\sqrt{a^2x^2 - bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+(a^2*x^2-b*x)^(1/2))^(3/4)/(a^2*x^2-b*x)^(1/2),x)
```

```
[Out] int((a*x+(a^2*x^2-b*x)^(1/2))^(3/4)/(a^2*x^2-b*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + \sqrt{a^2x^2 - bx}\right)^{\frac{3}{4}}}{\sqrt{a^2x^2 - bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2-b*x)^(1/2))^(3/4)/(a^2*x^2-b*x)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + sqrt(a^2*x^2 - b*x))^(3/4)/sqrt(a^2*x^2 - b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a\,x+\sqrt{a^2\,x^2-b\,x}\right)}^{3/4}}{\sqrt{a^2\,x^2-b\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + (a^2*x^2 - b*x)^(1/2))^(3/4)/(a^2*x^2 - b*x)^(1/2),x)
[Out] int((a*x + (a^2*x^2 - b*x)^(1/2))^(3/4)/(a^2*x^2 - b*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + \sqrt{a^2x^2 - bx}\right)^{\frac{3}{4}}}{\sqrt{x\left(a^2x - b\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a**2*x**2-b*x)**(1/2))**(3/4)/(a**2*x**2-b*x)**(1/2),x)
[Out] Integral((a*x + sqrt(a**2*x**2 - b*x))**(3/4)/sqrt(x*(a**2*x - b)), x)
```

3.1713 $\int \frac{\sqrt[4]{-bx^3+ax^4}\left(-d+cx^4\right)}{x^4}\,dx$

Optimal. Leaf size=148

$$\frac{3b^2c\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^3}}\right)}{16a^{7/4}}-\frac{3b^2c\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^3}}\right)}{16a^{7/4}}+\frac{\sqrt[4]{ax^4-bx^3}\left(-128a^3dx^2-32a^2bdx+180ab^2cx^4+160ab^2d-45\right.}{360ab^2x^3}$$

Rubi [A] time = 0.44, antiderivative size = 239, normalized size of antiderivative = 1.61, number of steps used = 12, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2052, 2004, 2024, 2032, 63, 331, 298, 203, 206, 2016, 2014}

$$\frac{3b^2cx^{9/4}(ax-b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax-b}}\right)}{16a^{7/4}\left(ax^4-bx^3\right)^{3/4}}-\frac{3b^2cx^{9/4}(ax-b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax-b}}\right)}{16a^{7/4}\left(ax^4-bx^3\right)^{3/4}}-\frac{16ad\left(ax^4-bx^3\right)^{5/4}}{45b^2x^5}+\frac{1}{2}cx\sqrt[4]{ax^4-bx^3}-\frac{bc\sqrt[4]{ax^4-bx^3}}{8a}-\frac{4d\left(ax^4-bx^3\right)^{5/4}}{9bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-(b*x^3) + a*x^4)^(1/4)*(-d + c*x^4))/x^4,x]
```

```
[Out] -1/8*(b*c*(-(b*x^3) + a*x^4)^(1/4))/a + (c*x*(-(b*x^3) + a*x^4)^(1/4))/2 -
(4*d*(-(b*x^3) + a*x^4)^(5/4))/(9*b*x^6) - (16*a*d*(-(b*x^3) + a*x^4)^(5/4)
)/(45*b^2*x^5) + (3*b^2*c*x^(9/4)*(-b + a*x)^(3/4)*ArcTan[(a^(1/4)*x^(1/4))
/(-b + a*x)^(1/4)])/(16*a^(7/4)*(-(b*x^3) + a*x^4)^(3/4)) - (3*b^2*c*x^(9/4
)*(-b + a*x)^(3/4)*ArcTanh[(a^(1/4)*x^(1/4))/(-b + a*x)^(1/4)])/(16*a^(7/4)
*(-(b*x^3) + a*x^4)^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\sqrt[4]{-bx^3+ax^4}\left(-d+cx^4\right)}{x^4}\,dx = \int \left(c\sqrt[4]{-bx^3+ax^4} - \frac{d\sqrt[4]{-bx^3+ax^4}}{x^4}\right)dx$$

$$= c\int \sqrt[4]{-bx^3+ax^4}\,dx - d\int \frac{\sqrt[4]{-bx^3+ax^4}}{x^4}\,dx$$

$$= \frac{1}{2}cx\sqrt[4]{-bx^3+ax^4} - \frac{4d\left(-bx^3+ax^4\right)^{5/4}}{9bx^6} - \frac{1}{8}(bc)\int \frac{x^3}{\left(-bx^3+ax^4\right)^{3/4}}\,dx - \frac{(4ad)}{}$$

$$= -\frac{bc\sqrt[4]{-bx^3+ax^4}}{8a} + \frac{1}{2}cx\sqrt[4]{-bx^3+ax^4} - \frac{4d\left(-bx^3+ax^4\right)^{5/4}}{9bx^6} - \frac{16ad\left(-bx^3+ax\right.}{45b^2x^5}$$

$$= -\frac{bc\sqrt[4]{-bx^3+ax^4}}{8a} + \frac{1}{2}cx\sqrt[4]{-bx^3+ax^4} - \frac{4d\left(-bx^3+ax^4\right)^{5/4}}{9bx^6} - \frac{16ad\left(-bx^3+ax\right.}{45b^2x^5}$$

$$= -\frac{bc\sqrt[4]{-bx^3+ax^4}}{8a} + \frac{1}{2}cx\sqrt[4]{-bx^3+ax^4} - \frac{4d\left(-bx^3+ax^4\right)^{5/4}}{9bx^6} - \frac{16ad\left(-bx^3+ax\right.}{45b^2x^5}$$

$$= -\frac{bc\sqrt[4]{-bx^3+ax^4}}{8a} + \frac{1}{2}cx\sqrt[4]{-bx^3+ax^4} - \frac{4d\left(-bx^3+ax^4\right)^{5/4}}{9bx^6} - \frac{16ad\left(-bx^3+ax\right.}{45b^2x^5}$$

$$= -\frac{bc\sqrt[4]{-bx^3+ax^4}}{8a} + \frac{1}{2}cx\sqrt[4]{-bx^3+ax^4} - \frac{4d\left(-bx^3+ax^4\right)^{5/4}}{9bx^6} - \frac{16ad\left(-bx^3+ax\right.}{45b^2x^5}$$

$$= -\frac{bc\sqrt[4]{-bx^3+ax^4}}{8a} + \frac{1}{2}cx\sqrt[4]{-bx^3+ax^4} - \frac{4d\left(-bx^3+ax^4\right)^{5/4}}{9bx^6} - \frac{16ad\left(-bx^3+ax\right.}{45b^2x^5}$$

Mathematica [C] time = 0.28, size = 237, normalized size = 1.60

$$-\frac{4\sqrt[4]{x^3(ax-b)}\left(4a^6dx^2\sqrt[4]{1-\frac{ax}{b}}+a^5bdx\sqrt[4]{1-\frac{ax}{b}}-5a^4b^2d\sqrt[4]{1-\frac{ax}{b}}-24a^2b^4cx^2\sqrt[4]{1-\frac{ax}{b}}+5b^6c\,{}_2F_1\left(-\frac{17}{4},-\frac{9}{4};-\frac{5}{4};\frac{ax}{b}\right)-20b^6c\,{}_2F_1\left(-\frac{13}{4},-\frac{9}{4};-\frac{5}{4};\frac{ax}{b}\right)+30b^6c\,{}_2F_1\left(-\frac{9}{4},-\frac{9}{4};-\frac{5}{4};\frac{ax}{b}\right)-15b^6c\sqrt[4]{1-\frac{ax}{b}}+39ab^5cx\sqrt[4]{1-\frac{ax}{b}}\right)}{45a^4b^2x^3\sqrt[4]{1-\frac{ax}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-(b*x^3) + a*x^4)^(1/4)*(-d + c*x^4))/x^4,x]
```

```
[Out] (-4*(x^3*(-b + a*x))^(1/4)*(-15*b^6*c*(1 - (a*x)/b)^(1/4) - 5*a^4*b^2*d*(1
- (a*x)/b)^(1/4) + 39*a*b^5*c*x*(1 - (a*x)/b)^(1/4) + a^5*b*d*x*(1 - (a*x)/
b)^(1/4) - 24*a^2*b^4*c*x^2*(1 - (a*x)/b)^(1/4) + 4*a^6*d*x^2*(1 - (a*x)/b)
^(1/4) + 5*b^6*c*Hypergeometric2F1[-17/4, -9/4, -5/4, (a*x)/b] - 20*b^6*c*H
ypergeometric2F1[-13/4, -9/4, -5/4, (a*x)/b] + 30*b^6*c*Hypergeometric2F1[-
9/4, -9/4, -5/4, (a*x)/b]))/(45*a^4*b^2*x^3*(1 - (a*x)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.82, size = 148, normalized size = 1.00

$$\frac{3b^2c\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^3}}\right)}{16a^{7/4}} - \frac{3b^2c\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^3}}\right)}{16a^{7/4}} + \frac{\sqrt[4]{ax^4-bx^3}\left(-128a^3dx^2-32a^2bdx+180ab^2cx^4+160ab^2d-45b^3cx^3\right)}{360ab^2x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-(b*x^3) + a*x^4)^(1/4)*(-d + c*x^4))/x^4,x]
```

```
[Out] ((-(b*x^3) + a*x^4)^(1/4)*(160*a*b^2*d - 32*a^2*b*d*x - 128*a^3*d*x^2 - 45*
b^3*c*x^3 + 180*a*b^2*c*x^4))/(360*a*b^2*x^3) + (3*b^2*c*ArcTan[(a^(1/4)*x)
```

```
/(-(b*x^3) + a*x^4)^(1/4)])/(16*a^(7/4)) - (3*b^2*c*ArcTanh[(a^(1/4)*x)/(-(
b*x^3) + a*x^4)^(1/4)])/(16*a^(7/4))
```

fricas [B] time = 0.43, size = 334, normalized size = 2.26

$$\frac{540 \left(\frac{b^8c^4}{a^7}\right)^{\frac{1}{4}} ab^2x^3 \arctan\left(-\frac{\left(\frac{b^8c^4}{a^7}\right)^{\frac{3}{4}} \left(ax^4-bx^3\right)^{\frac{1}{4}} a^5b^2c-\left(\frac{b^8c^4}{a^7}\right)^{\frac{3}{4}} a^5x\sqrt{\frac{\sqrt{ax^4-bx^3}\, b^4c^2+\sqrt{\frac{b^8c^4}{a^7}}\, a^4x^2}{x^2}}}{b^8c^4x}\right) - 135 \left(\frac{b^8c^4}{a^7}\right)^{\frac{1}{4}} ab^2x^3 \log\left(\frac{3\left(\left(ax^4-bx^3\right)^{\frac{1}{4}} b^2c+\left(\frac{b^8c^4}{a^7}\right)^{\frac{1}{4}} a^2x\right)}{x}\right) + 135 \left(\frac{b^8c^4}{a^7}\right)^{\frac{1}{4}} ab^2x^3 \log\left(\frac{3\left(\left(ax^4-bx^3\right)^{\frac{1}{4}} b^2c-\left(\frac{b^8c^4}{a^7}\right)^{\frac{1}{4}} a^2x\right)}{x}\right) + 4\left(180\, ab^2cx^4 - 45\, b^3cx^3 - 128\, a^3dx^2 - 32\, a^2bdx + 160\, ab^2d\right)\left(ax^4-bx^3\right)^{\frac{1}{4}}}{1440\, ab^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x^3)^(1/4)*(c*x^4-d)/x^4,x, algorithm="fricas")
```

```
[Out] 1/1440*(540*(b^8*c^4/a^7)^(1/4)*a*b^2*x^3*arctan(-((b^8*c^4/a^7)^(3/4)*(a*x
^4 - b*x^3)^(1/4)*a^5*b^2*c - (b^8*c^4/a^7)^(3/4)*a^5*x*sqrt((sqrt(a*x^4 -
b*x^3)*b^4*c^2 + sqrt(b^8*c^4/a^7)*a^4*x^2)/x^2))/(b^8*c^4*x)) - 135*(b^8*c
^4/a^7)^(1/4)*a*b^2*x^3*log(3*((a*x^4 - b*x^3)^(1/4)*b^2*c + (b^8*c^4/a^7)^
(1/4)*a^2*x)/x) + 135*(b^8*c^4/a^7)^(1/4)*a*b^2*x^3*log(3*((a*x^4 - b*x^3)^
(1/4)*b^2*c - (b^8*c^4/a^7)^(1/4)*a^2*x)/x) + 4*(180*a*b^2*c*x^4 - 45*b^3*c
*x^3 - 128*a^3*d*x^2 - 32*a^2*b*d*x + 160*a*b^2*d)*(a*x^4 - b*x^3)^(1/4))/(
a*b^2*x^3)
```

giac [B] time = 1.26, size = 296, normalized size = 2.00

$$\frac{\frac{270 \sqrt{2}\, b^3c \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}\, a} + \frac{270 \sqrt{2}\, b^3c \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}\, a} + \frac{135 \sqrt{2}\, b^3c \log\left(\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a-\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x}}\right)}{(-a)^{\frac{3}{4}}\, a} + \frac{135 \sqrt{2}\,(-a)^{\frac{1}{4}} b^3c \log\left(-\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a-\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x}}\right)}{a^2} + \frac{360\left(\left(a-\frac{b}{x}\right)^{\frac{5}{4}} b^3c+3\left(a-\frac{b}{x}\right)^{\frac{1}{4}} ab^3c\right)x^2}{ab^2} + \frac{256\left(5\left(a-\frac{b}{x}\right)^{\frac{9}{4}} b^8d-9\left(a-\frac{b}{x}\right)^{\frac{5}{4}} ab^8d\right)}{b^9}}{2880\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x^3)^(1/4)*(c*x^4-d)/x^4,x, algorithm="giac")
```

```
[Out] 1/2880*(270*sqrt(2)*b^3*c*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a - b
/x)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a) + 270*sqrt(2)*b^3*c*arctan(-1/2*sqrt(
2)*(sqrt(2)*(-a)^(1/4) - 2*(a - b/x)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a) + 13
5*sqrt(2)*b^3*c*log(sqrt(2)*(-a)^(1/4)*(a - b/x)^(1/4) + sqrt(-a) + sqrt(a
- b/x))/((-a)^(3/4)*a) + 135*sqrt(2)*(-a)^(1/4)*b^3*c*log(-sqrt(2)*(-a)^(1/
4)*(a - b/x)^(1/4) + sqrt(-a) + sqrt(a - b/x))/a^2 + 360*((a - b/x)^(5/4)*b
^3*c + 3*(a - b/x)^(1/4)*a*b^3*c)*x^2/(a*b^2) + 256*(5*(a - b/x)^(9/4)*b^8*
d - 9*(a - b/x)^(5/4)*a*b^8*d)/b^9)/b
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^4-b\,x^3\right)^{\frac{1}{4}} \left(c\,x^4-d\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b*x^3)^(1/4)*(c*x^4-d)/x^4,x)
```

```
[Out] int((a*x^4-b*x^3)^(1/4)*(c*x^4-d)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4-bx^3\right)^{\frac{1}{4}}\left(cx^4-d\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x^3)^(1/4)*(c*x^4-d)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 - b*x^3)^(1/4)*(c*x^4 - d)/x^4, x)
```

mupad [B] time = 1.85, size = 111, normalized size = 0.75

$$\frac{4\,d\,{\left(a\,x^4-b\,x^3\right)}^{1/4}}{9\,x^3}-\frac{16\,a^2\,d\,{\left(a\,x^4-b\,x^3\right)}^{1/4}}{45\,b^2\,x}+\frac{4\,c\,x\,{\left(a\,x^4-b\,x^3\right)}^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{7}{4};\ \frac{11}{4};\ \frac{a\,x}{b}\right)}{7\,{\left(1-\frac{a\,x}{b}\right)}^{1/4}}-\frac{4\,a\,d\,{\left(a\,x^4-b\,x^3\right)}^{1/4}}{45\,b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((d - c*x^4)*(a*x^4 - b*x^3)^(1/4))/x^4,x)
```

```
[Out] (4*d*(a*x^4 - b*x^3)^(1/4))/(9*x^3) - (16*a^2*d*(a*x^4 - b*x^3)^(1/4))/(45*
b^2*x) + (4*c*x*(a*x^4 - b*x^3)^(1/4)*hypergeom([-1/4, 7/4], 11/4, (a*x)/b)
)/(7*(1 - (a*x)/b)^(1/4)) - (4*a*d*(a*x^4 - b*x^3)^(1/4))/(45*b*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\,(ax-b)}\,\left(cx^4-d\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b*x**3)**(1/4)*(c*x**4-d)/x**4,x)
```

```
[Out] Integral((x**3*(a*x - b))**(1/4)*(c*x**4 - d)/x**4, x)
```

3.1714 $\int \frac{(1+x^3)^{2/3}(2+x^3)}{x^6(4+x^3)}\,dx$

Optimal. Leaf size=149

$$-\frac{\log\left(6^{2/3}\sqrt[3]{x^3+1}-3x\right)}{16\sqrt[3]{6}}+\frac{\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\,2^{2/3}\sqrt[3]{x^3+1}+\sqrt[3]{3}x}\right)}{16\sqrt[3]{2}}+\frac{\left(x^3+1\right)^{2/3}\left(-13x^3-8\right)}{80x^5}+\frac{\log\left(6^{2/3}\sqrt[3]{x^3+1}\,x+2\sqrt[3]{6}\right.}{32\sqrt[3]{6}}$$

Rubi [A] time = 0.20, antiderivative size = 162, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {580, 583, 12, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+1}}\right)}{16\sqrt[3]{6}}+\frac{\sqrt[6]{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{6}x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{16\sqrt[3]{2}}-\frac{\left(x^3+1\right)^{2/3}}{10x^5}-\frac{13\left(x^3+1\right)^{2/3}}{80x^2}+\frac{\log\left(\frac{2^{2/3}\sqrt[3]{3}x}{\sqrt[3]{x^3+1}}+\frac{3^{2/3}x^2}{\left(x^3+1\right)^{2/3}}+2\sqrt[3]{2}\right)}{32\sqrt[3]{6}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^3)^(2/3)*(2 + x^3))/(x^6*(4 + x^3)),x]
```

```
[Out] -1/10*(1 + x^3)^(2/3)/x^5 - (13*(1 + x^3)^(2/3))/(80*x^2) + (3^(1/6)*ArcTan
[(1 + (6^(1/3)*x)/(1 + x^3)^(1/3))/Sqrt[3]])/(16*2^(1/3)) - Log[2^(2/3) - (
3^(1/3)*x)/(1 + x^3)^(1/3)]/(16*6^(1/3)) + Log[2*2^(1/3) + (3^(2/3)*x^2)/(1
 + x^3)^(2/3) + (2^(2/3)*3^(1/3)*x)/(1 + x^3)^(1/3)]/(32*6^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(4+x^3\right)}\,dx &= -\frac{\left(1+x^3\right)^{2/3}}{10x^5}+\frac{1}{20}\int \frac{26+14x^3}{x^3\sqrt[3]{1+x^3}\left(4+x^3\right)}\,dx \\
&= -\frac{\left(1+x^3\right)^{2/3}}{10x^5}-\frac{13\left(1+x^3\right)^{2/3}}{80x^2}-\frac{1}{160}\int -\frac{60}{\sqrt[3]{1+x^3}\left(4+x^3\right)}\,dx \\
&= -\frac{\left(1+x^3\right)^{2/3}}{10x^5}-\frac{13\left(1+x^3\right)^{2/3}}{80x^2}+\frac{3}{8}\int \frac{1}{\sqrt[3]{1+x^3}\left(4+x^3\right)}\,dx \\
&= -\frac{\left(1+x^3\right)^{2/3}}{10x^5}-\frac{13\left(1+x^3\right)^{2/3}}{80x^2}+\frac{3}{8}\text{Subst}\left(\int \frac{1}{4-3x^3}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right) \\
&= -\frac{\left(1+x^3\right)^{2/3}}{10x^5}-\frac{13\left(1+x^3\right)^{2/3}}{80x^2}+\frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{3}x}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right)}{16\sqrt[3]{2}}+\frac{\text{Subst}\left(\int \frac{\cdots}{2\sqrt[3]{2}-\cdots}\right.}{\cdots} \\
&= -\frac{\left(1+x^3\right)^{2/3}}{10x^5}-\frac{13\left(1+x^3\right)^{2/3}}{80x^2}-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{3}x}{\sqrt[3]{1+x^3}}\right)}{16\sqrt[3]{6}}+\frac{3\,\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{3}x+3^{2/3}\cdots}\right.}{16\ 2^{2/3}} \\
&= -\frac{\left(1+x^3\right)^{2/3}}{10x^5}-\frac{13\left(1+x^3\right)^{2/3}}{80x^2}-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{3}x}{\sqrt[3]{1+x^3}}\right)}{16\sqrt[3]{6}}+\frac{\log\left(2\sqrt[3]{2}+\frac{3^{2/3}x^2}{\left(1+x^3\right)^{2/3}}+\frac{2^{2/3}\cdots}{\sqrt[3]{1+\cdots}}\right.}{32\sqrt[3]{6}} \\
&= -\frac{\left(1+x^3\right)^{2/3}}{10x^5}-\frac{13\left(1+x^3\right)^{2/3}}{80x^2}+\frac{\sqrt[6]{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{6}x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{16\sqrt[3]{2}}-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{3}x}{\sqrt[3]{1+x^3}}\right)}{16\sqrt[3]{6}}+\frac{\log\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.27, size = 135, normalized size = 0.91

$$
\frac{1}{192}\left(6\ 2^{2/3}\sqrt[6]{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{6}x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)+6^{2/3}\left(\log\left(\frac{2\sqrt[3]{6}x}{\sqrt[3]{x^3+1}}+\frac{6^{2/3}x^2}{\left(x^3+1\right)^{2/3}}+4\right)-2\log\left(2-\frac{\sqrt[3]{6}x}{\sqrt[3]{x^3+1}}\right)\right)\right)+\left(x^3+1\right)^{2/3}\left(-\frac{1}{10x^5}-\frac{13}{80x^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)^(2/3)*(2 + x^3))/(x^6*(4 + x^3)),x]
```

```
[Out] (-1/10*1/x^5 - 13/(80*x^2))*(1 + x^3)^(2/3) + (6*2^(2/3)*3^(1/6)*ArcTan[(1
+ (6^(1/3)*x)/(1 + x^3)^(1/3))/Sqrt[3]] + 6^(2/3)*(-2*Log[2 - (6^(1/3)*x)/(
1 + x^3)^(1/3)] + Log[4 + (6^(2/3)*x^2)/(1 + x^3)^(2/3) + (2*6^(1/3)*x)/(1
+ x^3)^(1/3)]))/192
```

IntegrateAlgebraic [A] time = 0.38, size = 149, normalized size = 1.00

$$
-\frac{\log\left(6^{2/3}\sqrt[3]{x^3+1}-3x\right)}{16\sqrt[3]{6}}+\frac{\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\ 2^{2/3}\sqrt[3]{x^3+1}+\sqrt[3]{3}x}\right)}{16\sqrt[3]{2}}+\frac{\left(x^3+1\right)^{2/3}\left(-13x^3-8\right)}{80x^5}+\frac{\log\left(6^{2/3}\sqrt[3]{x^3+1}\,x+2\sqrt[3]{6}\left(x^3+1\right)^{2/3}+3x^2\right)}{32\sqrt[3]{6}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)^(2/3)*(2 + x^3))/(x^6*(4 + x^3)),x]
```

```
[Out] ((-8 - 13*x^3)*(1 + x^3)^(2/3))/(80*x^5) + (3^(1/6)*ArcTan[(3^(5/6)*x)/(3^(
1/3)*x + 2*2^(2/3)*(1 + x^3)^(1/3))])/(16*2^(1/3)) - Log[-3*x + 6^(2/3)*(1
+ x^3)^(1/3)]/(16*6^(1/3)) + Log[3*x^2 + 6^(2/3)*x*(1 + x^3)^(1/3) + 2*6^(1
/3)*(1 + x^3)^(2/3)]/(32*6^(1/3))
```

fricas [B] time = 3.11, size = 303, normalized size = 2.03

$$\frac{30 \cdot 6^{\frac{1}{6}} \sqrt{2} \left(-1\right)^{\frac{1}{3}} x^{5} \arctan\left(\frac{6^{\frac{1}{6}} \left(24 \cdot 6^{\frac{2}{3}} \sqrt{2} \left(-1\right)^{\frac{2}{3}} \left(5 x^{7}+22 x^{4}+8 x\right) \left(x^{3}+1\right)^{\frac{2}{3}}-36 \sqrt{2} \left(-1\right)^{\frac{1}{3}} \left(109 x^{8}+116 x^{5}+16 x^{2}\right) \left(x^{3}+1\right)^{\frac{1}{3}}+6^{\frac{1}{3}} \sqrt{2} \left(1189 x^{9}+2064 x^{6}+912 x^{3}+64\right)\right)}{6 \left(971 x^{9}+960 x^{6}-48 x^{3}-64\right)}\right)+10 \cdot 6^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} x^{5} \log\left(-\frac{18 \cdot 6^{\frac{1}{3}} \left(-1\right)^{\frac{2}{3}} \left(x^{3}+1\right)^{\frac{1}{3}} x^{2}-6^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} \left(x^{3}+4\right)-36 \left(x^{3}+1\right)^{\frac{2}{3}} x}{x^{3}+4}\right)-5 \cdot 6^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} x^{5} \log\left(-\frac{12 \cdot 6^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} \left(5 x^{4}+2 x\right) \left(x^{3}+1\right)^{\frac{2}{3}}-6^{\frac{1}{3}} \left(-1\right)^{\frac{2}{3}} \left(109 x^{6}+116 x^{3}+16\right)-18 \left(11 x^{5}+8 x^{2}\right) \left(x^{3}+1\right)^{\frac{1}{3}}}{x^{6}+8 x^{3}+16}\right)-36 \left(13 x^{3}+8\right) \left(x^{3}+1\right)^{\frac{2}{3}}}{2880 x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^3+2)/x^6/(x^3+4),x, algorithm="fricas")
```

```
[Out] 1/2880*(30*6^(1/6)*sqrt(2)*(-1)^(1/3)*x^5*arctan(1/6*6^(1/6)*(24*6^(2/3)*sq
rt(2)*(-1)^(2/3)*(5*x^7 + 22*x^4 + 8*x)*(x^3 + 1)^(2/3) - 36*sqrt(2)*(-1)^(
1/3)*(109*x^8 + 116*x^5 + 16*x^2)*(x^3 + 1)^(1/3) + 6^(1/3)*sqrt(2)*(1189*x
^9 + 2064*x^6 + 912*x^3 + 64))/(971*x^9 + 960*x^6 - 48*x^3 - 64)) + 10*6^(2
/3)*(-1)^(1/3)*x^5*log(-(18*6^(1/3)*(-1)^(2/3)*(x^3 + 1)^(1/3)*x^2 - 6^(2/3
)*(-1)^(1/3)*(x^3 + 4) - 36*(x^3 + 1)^(2/3)*x)/(x^3 + 4)) - 5*6^(2/3)*(-1)^
(1/3)*x^5*log(-(12*6^(2/3)*(-1)^(1/3)*(5*x^4 + 2*x)*(x^3 + 1)^(2/3) - 6^(1/
3)*(-1)^(2/3)*(109*x^6 + 116*x^3 + 16) - 18*(11*x^5 + 8*x^2)*(x^3 + 1)^(1/3
))/(x^6 + 8*x^3 + 16)) - 36*(13*x^3 + 8)*(x^3 + 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}+2\right) \left(x^{3}+1\right)^{\frac{2}{3}}}{\left(x^{3}+4\right) x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^3+2)/x^6/(x^3+4),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 2)*(x^3 + 1)^(2/3)/((x^3 + 4)*x^6), x)
```

maple [C] time = 2.63, size = 485, normalized size = 3.26

$$-\frac{13 x^{6}+21 x^{3}+8}{80 x^{5} \left(x^{3}+1\right)^{\frac{1}{3}}}+\frac{3 \mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^{3}+36\right)^{2}+36 \_Z \mathrm{RootOf}\left(\_Z^{3}+36\right)+1296 \_Z^{2}\right) \ln\left(\frac{216 \mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^{3}+36\right)^{2}+36 \_Z \mathrm{RootOf}\left(\_Z^{3}+36\right)+1296 \_Z^{2}\right)^{2} \mathrm{RootOf}\left(\_Z^{3}+36\right)^{2} x^{3}+3 \mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^{3}+36\right)^{2}+36 \_Z \mathrm{RootOf}\left(\_Z^{3}+36\right)+1296 \_Z^{2}\right) \mathrm{RootOf}\left(\_Z^{3}+36\right)^{3} x^{3}+108 \left(x^{3}+1\right)^{\frac{1}{3}} \mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^{3}+36\right)^{2}+36 \_Z \mathrm{RootOf}\left(\_Z^{3}+36\right)+1296 \_Z^{2}\right) \mathrm{RootOf}\left(\_Z^{3}+36\right) x^{2}+288 \mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^{3}+36\right)^{2}+36 \_Z \mathrm{RootOf}\left(\_Z^{3}+36\right)+1296 \_Z^{2}\right) x^{3}+4 \mathrm{RootOf}\left(\_Z^{3}+36\right) x^{3}+36 x \left(x^{3}+1\right)^{\frac{2}{3}}+288 \mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^{3}+36\right)^{2}+36 \_Z \mathrm{RootOf}\left(\_Z^{3}+36\right)+1296 \_Z^{2}\right)+4 \mathrm{RootOf}\left(\_Z^{3}+36\right)}{x^{3}+4}\right)}{8}+\frac{\mathrm{RootOf}\left(\_Z^{3}+36\right) \ln\left(-\frac{54 \mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^{3}+36\right)^{2}+36 \_Z \mathrm{RootOf}\left(\_Z^{3}+36\right)+1296 \_Z^{2}\right)^{2} \mathrm{RootOf}\left(\_Z^{3}+36\right)^{2} x^{3}+3 \mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^{3}+36\right)^{2}+36 \_Z \mathrm{RootOf}\left(\_Z^{3}+36\right)+1296 \_Z^{2}\right) \mathrm{RootOf}\left(\_Z^{3}+36\right)^{3} x^{3}-54 \left(x^{3}+1\right)^{\frac{1}{3}} \mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^{3}+36\right)^{2}+36 \_Z \mathrm{RootOf}\left(\_Z^{3}+36\right)+1296 \_Z^{2}\right) \mathrm{RootOf}\left(\_Z^{3}+36\right) x^{2}-126 \mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^{3}+36\right)^{2}+36 \_Z \mathrm{RootOf}\left(\_Z^{3}+36\right)+1296 \_Z^{2}\right) x^{3}-7 \mathrm{RootOf}\left(\_Z^{3}+36\right) x^{3}-18 x \left(x^{3}+1\right)^{\frac{2}{3}}-72 \mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^{3}+36\right)^{2}+36 \_Z \mathrm{RootOf}\left(\_Z^{3}+36\right)+1296 \_Z^{2}\right)-4 \mathrm{RootOf}\left(\_Z^{3}+36\right)}{x^{3}+4}\right)}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)*(x^3+2)/x^6/(x^3+4),x)
```

```
[Out] -1/80*(13*x^6+21*x^3+8)/x^5/(x^3+1)^(1/3)+3/8*RootOf(RootOf(_Z^3+36)^2+36*_
Z*RootOf(_Z^3+36)+1296*_Z^2)*ln((216*RootOf(RootOf(_Z^3+36)^2+36*_Z*RootOf(
_Z^3+36)+1296*_Z^2)^2*RootOf(_Z^3+36)^2*x^3+3*RootOf(RootOf(_Z^3+36)^2+36*_
Z*RootOf(_Z^3+36)+1296*_Z^2)*RootOf(_Z^3+36)^3*x^3+108*(x^3+1)^(1/3)*RootOf
(RootOf(_Z^3+36)^2+36*_Z*RootOf(_Z^3+36)+1296*_Z^2)*RootOf(_Z^3+36)*x^2+288
*RootOf(RootOf(_Z^3+36)^2+36*_Z*RootOf(_Z^3+36)+1296*_Z^2)*x^3+4*RootOf(_Z^
3+36)*x^3+36*x*(x^3+1)^(2/3)+288*RootOf(RootOf(_Z^3+36)^2+36*_Z*RootOf(_Z^3
+36)+1296*_Z^2)+4*RootOf(_Z^3+36))/(x^3+4))+1/96*RootOf(_Z^3+36)*ln(-(54*Ro
otOf(RootOf(_Z^3+36)^2+36*_Z*RootOf(_Z^3+36)+1296*_Z^2)^2*RootOf(_Z^3+36)^2
*x^3+3*RootOf(RootOf(_Z^3+36)^2+36*_Z*RootOf(_Z^3+36)+1296*_Z^2)*RootOf(_Z^
3+36)^3*x^3-54*(x^3+1)^(1/3)*RootOf(RootOf(_Z^3+36)^2+36*_Z*RootOf(_Z^3+36)
+1296*_Z^2)*RootOf(_Z^3+36)*x^2-126*RootOf(RootOf(_Z^3+36)^2+36*_Z*RootOf(_
Z^3+36)+1296*_Z^2)*x^3-7*RootOf(_Z^3+36)*x^3-18*x*(x^3+1)^(2/3)-72*RootOf(R
ootOf(_Z^3+36)^2+36*_Z*RootOf(_Z^3+36)+1296*_Z^2)-4*RootOf(_Z^3+36))/(x^3+4
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}+2\right) \left(x^{3}+1\right)^{\frac{2}{3}}}{\left(x^{3}+4\right) x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^3+2)/x^6/(x^3+4),x, algorithm="maxima")
[Out] integrate((x^3 + 2)*(x^3 + 1)^(2/3)/((x^3 + 4)*x^6), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3+1\right)^{2/3}\,\left(x^3+2\right)}{x^6\,\left(x^3+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)^(2/3)*(x^3 + 2))/(x^6*(x^3 + 4)),x)
[Out] int(((x^3 + 1)^(2/3)*(x^3 + 2))/(x^6*(x^3 + 4)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x+1\right)\left(x^2-x+1\right)\right)^{\frac{2}{3}}\left(x^3+2\right)}{x^6\left(x^3+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)*(x**3+2)/x**6/(x**3+4),x)
[Out] Integral(((x + 1)*(x**2 - x + 1))**(2/3)*(x**3 + 2)/(x**6*(x**3 + 4)), x)
```

3.1715 $\int \frac{(-1+x^3)(1+3x^3)^{2/3}}{x^6(1+x^3)}\,dx$

Optimal. Leaf size=149

$$-\frac{2}{3}2^{2/3}\log\left(2^{2/3}\sqrt[3]{3x^3+1}-2x\right)+\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{3x^3+1}+x}\right)}{\sqrt{3}}+\frac{(3x^3+1)^{2/3}(1-2x^3)}{5x^5}+\frac{1}{3}2^{2/3}\log\left(2^{2/3}\sqrt[3]{3x^3+1}\,x+\right.$$

Rubi [A] time = 0.18, antiderivative size = 158, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {580, 583, 12, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{2}{3}2^{2/3}\log\left(1-\frac{\sqrt[3]{2}x}{\sqrt[3]{3x^3+1}}\right)+\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{3x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{(3x^3+1)^{2/3}}{5x^5}-\frac{2(3x^3+1)^{2/3}}{5x^2}+\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{3x^3+1}}+\frac{2^{2/3}x^2}{(3x^3+1)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)*(1 + 3*x^3)^(2/3))/(x^6*(1 + x^3)),x]
```

```
[Out] (1 + 3*x^3)^(2/3)/(5*x^5) - (2*(1 + 3*x^3)^(2/3))/(5*x^2) + (2*2^(2/3)*ArcT
an[(1 + (2*2^(1/3)*x)/(1 + 3*x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - (2*2^(2/3)*Log
[1 - (2^(1/3)*x)/(1 + 3*x^3)^(1/3)])/3 + (2^(2/3)*Log[1 + (2^(2/3)*x^2)/(1
+ 3*x^3)^(2/3) + (2^(1/3)*x)/(1 + 3*x^3)^(1/3)])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(-1+x^3\right)\left(1+3x^3\right)^{2/3}}{x^6\left(1+x^3\right)}\,dx = \frac{\left(1+3x^3\right)^{2/3}}{5x^5} + \frac{1}{5}\int \frac{4+24x^3}{x^3\left(1+x^3\right)\sqrt[3]{1+3x^3}}\,dx$$
$$= \frac{\left(1+3x^3\right)^{2/3}}{5x^5} - \frac{2\left(1+3x^3\right)^{2/3}}{5x^2} - \frac{1}{10}\int -\frac{40}{\left(1+x^3\right)\sqrt[3]{1+3x^3}}\,dx$$
$$= \frac{\left(1+3x^3\right)^{2/3}}{5x^5} - \frac{2\left(1+3x^3\right)^{2/3}}{5x^2} + 4\int \frac{1}{\left(1+x^3\right)\sqrt[3]{1+3x^3}}\,dx$$
$$= \frac{\left(1+3x^3\right)^{2/3}}{5x^5} - \frac{2\left(1+3x^3\right)^{2/3}}{5x^2} + 4\,\text{Subst}\left(\int \frac{1}{1-2x^3}\,dx, x, \frac{x}{\sqrt[3]{1+3x^3}}\right)$$
$$= \frac{\left(1+3x^3\right)^{2/3}}{5x^5} - \frac{2\left(1+3x^3\right)^{2/3}}{5x^2} + \frac{4}{3}\,\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1+3x^3}}\right) + \frac{4}{3}\,\text{Subst}$$
$$= \frac{\left(1+3x^3\right)^{2/3}}{5x^5} - \frac{2\left(1+3x^3\right)^{2/3}}{5x^2} - \frac{2}{3}2^{2/3}\log\left(1-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1+3x^3}}\right) + 2\,\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\right.$$
$$= \frac{\left(1+3x^3\right)^{2/3}}{5x^5} - \frac{2\left(1+3x^3\right)^{2/3}}{5x^2} - \frac{2}{3}2^{2/3}\log\left(1-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1+3x^3}}\right) + \frac{1}{3}2^{2/3}\log\left(1+\frac{2^{2/3}}{\left(1+3\right.}\right.$$
$$= \frac{\left(1+3x^3\right)^{2/3}}{5x^5} - \frac{2\left(1+3x^3\right)^{2/3}}{5x^2} + \frac{2\,2^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1+3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}2^{2/3}\log\left(1-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1+3x^3}}\right.$$

Mathematica [A] time = 0.27, size = 130, normalized size = 0.87

$$\frac{1}{3}2^{2/3}\left(-2\log\left(1-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{x^3+3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{x^3+3}}+1}{\sqrt{3}}\right)+\log\left(\frac{\sqrt[3]{2}\,x}{\sqrt[3]{x^3+3}}+\frac{2^{2/3}x^2}{\left(x^3+3\right)^{2/3}}+1\right)\right)+\left(3x^3+1\right)^{2/3}\left(\frac{1}{5x^5}-\frac{2}{5x^2}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^3)*(1 + 3*x^3)^(2/3))/(x^6*(1 + x^3)),x]
```

```
[Out] (1/(5*x^5) - 2/(5*x^2))*(1 + 3*x^3)^(2/3) + (2^(2/3)*(2*Sqrt[3]*ArcTan[(1 +
 (2*2^(1/3)*x)/(3 + x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (2^(1/3)*x)/(3 + x^3)^
(1/3)] + Log[1 + (2^(2/3)*x^2)/(3 + x^3)^(2/3) + (2^(1/3)*x)/(3 + x^3)^(1/3
)]))/3
```

IntegrateAlgebraic [A] time = 0.36, size = 149, normalized size = 1.00

$$-\frac{2}{3}2^{2/3}\log\left(2^{2/3}\sqrt[3]{3x^3+1}-2x\right)+\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2^{2/3}\sqrt[3]{3x^3+1}+x}\right)}{\sqrt{3}}+\frac{\left(3x^3+1\right)^{2/3}\left(1-2x^3\right)}{5x^5}+\frac{1}{3}2^{2/3}\log\left(2^{2/3}\sqrt[3]{3x^3+1}\,x+\sqrt[3]{2}\left(3x^3+1\right)^{2/3}+2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)*(1 + 3*x^3)^(2/3))/(x^6*(1 + x^3)),x]
```

```
[Out] ((1 - 2*x^3)*(1 + 3*x^3)^(2/3))/(5*x^5) + (2*2^(2/3)*ArcTan[(Sqrt[3]*x)/(x
+ 2^(2/3)*(1 + 3*x^3)^(1/3))])/Sqrt[3] - (2*2^(2/3)*Log[-2*x + 2^(2/3)*(1 +
 3*x^3)^(1/3)])/3 + (2^(2/3)*Log[2*x^2 + 2^(2/3)*x*(1 + 3*x^3)^(1/3) + 2^(1
/3)*(1 + 3*x^3)^(2/3)])/3
```

fricas [B] time = 2.74, size = 279, normalized size = 1.87

$$\frac{10\sqrt{3}\,(-4)^{\frac{1}{3}}x^5\arctan\left(\frac{3\sqrt{3}(-4)^{\frac{2}{3}}(7x^7+8x^4+x)(3x^3+1)^{\frac{2}{3}}-6\sqrt{3}(-4)^{\frac{1}{3}}(55x^8+20x^5+x^2)(3x^3+1)^{\frac{1}{3}}+\sqrt{3}(433x^9+255x^6+39x^3+1)}{3(323x^9+105x^6-3x^3-1)}\right)+10\,(-4)^{\frac{1}{3}}x^5\log\left(-\frac{3(-4)^{\frac{2}{3}}(3x^3+1)^{\frac{1}{3}}x^2-6(3x^3+1)^{\frac{2}{3}}x-(-4)^{\frac{1}{3}}(x^3+1)}{x^3+1}\right)-5\,(-4)^{\frac{1}{3}}x^5\log\left(-\frac{6(-4)^{\frac{1}{3}}(7x^4+x)(3x^3+1)^{\frac{2}{3}}-(-4)^{\frac{2}{3}}(55x^6+20x^3+1)-24(4x^5+x^2)(3x^3+1)^{\frac{1}{3}}}{x^6+2x^3+1}\right)-9\left(3x^3+1\right)^{\frac{2}{3}}\left(2x^3-1\right)}{45x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(3*x^3+1)^(2/3)/x^6/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/45*(10*sqrt(3)*(-4)^(1/3)*x^5*arctan(1/3*(3*sqrt(3)*(-4)^(2/3)*(7*x^7 + 8
*x^4 + x)*(3*x^3 + 1)^(2/3) - 6*sqrt(3)*(-4)^(1/3)*(55*x^8 + 20*x^5 + x^2)*
(3*x^3 + 1)^(1/3) + sqrt(3)*(433*x^9 + 255*x^6 + 39*x^3 + 1))/(323*x^9 + 10
5*x^6 - 3*x^3 - 1)) + 10*(-4)^(1/3)*x^5*log(-(3*(-4)^(2/3)*(3*x^3 + 1)^(1/3
)*x^2 - 6*(3*x^3 + 1)^(2/3)*x - (-4)^(1/3)*(x^3 + 1))/(x^3 + 1)) - 5*(-4)^(
1/3)*x^5*log(-(6*(-4)^(1/3)*(7*x^4 + x)*(3*x^3 + 1)^(2/3) - (-4)^(2/3)*(55*
x^6 + 20*x^3 + 1) - 24*(4*x^5 + x^2)*(3*x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
- 9*(3*x^3 + 1)^(2/3)*(2*x^3 - 1))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^3+1\right)^{\frac{2}{3}}\left(x^3-1\right)}{\left(x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(3*x^3+1)^(2/3)/x^6/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((3*x^3 + 1)^(2/3)*(x^3 - 1)/((x^3 + 1)*x^6), x)
```

maple [C] time = 2.74, size = 958, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)*(3*x^3+1)^(2/3)/x^6/(x^3+1),x)
```

```
[Out] -1/5*(6*x^6-x^3-1)/x^5/(3*x^3+1)^(1/3)-2/3*ln(-(6*RootOf(RootOf(_Z^3+4)^2+6
*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3-54*RootOf(RootOf(_Z^3+4)^2
+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-24*(3*x^3+1)^(2/3)*Roo
tOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x+RootOf
(_Z^3+4)^2*(3*x^3+1)^(1/3)*x^2+48*(3*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+6
*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)*x^2+6*RootOf(_Z^3+4)*x^3-54*Root
Of(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3+2*(3*x^3+1)^(2/3)*x+2*
RootOf(_Z^3+4)-18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2))/(1+
x)/(x^2-x+1))*RootOf(_Z^3+4)-4*ln(-(6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_
Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3-54*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf
(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-24*(3*x^3+1)^(2/3)*RootOf(RootOf(_
Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x+RootOf(_Z^3+4)^2*(
3*x^3+1)^(1/3)*x^2+48*(3*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_
Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)*x^2+6*RootOf(_Z^3+4)*x^3-54*RootOf(RootOf(_Z
^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3+2*(3*x^3+1)^(2/3)*x+2*RootOf(_Z^3+
4)-18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2))/(1+x)/(x^2-x+1)
)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)+4*RootOf(RootOf(_Z^3
+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*ln((15*RootOf(RootOf(_Z^3+4)^2+6*_Z*Root
Of(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+54*RootOf(RootOf(_Z^3+4)^2+6*_Z*Ro
otOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-24*(3*x^3+1)^(2/3)*RootOf(Root
Of(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x+7*RootOf(_Z^3+
4)^2*(3*x^3+1)^(1/3)*x^2+48*(3*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*Ro
otOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)*x^2-25*RootOf(_Z^3+4)*x^3-90*RootOf(Ro
otOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3+14*(3*x^3+1)^(2/3)*x-5*Root
Of(_Z^3+4)-18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2))/(1+x)/(
x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^3+1\right)^{\frac{2}{3}}\left(x^3-1\right)}{\left(x^3+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)*(3*x^3+1)^(2/3)/x^6/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((3*x^3 + 1)^(2/3)*(x^3 - 1)/((x^3 + 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3-1\right)\,\left(3\,x^3+1\right)^{2/3}}{x^6\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)*(3*x^3 + 1)^(2/3))/(x^6*(x^3 + 1)),x)
```

```
[Out] int(((x^3 - 1)*(3*x^3 + 1)^(2/3))/(x^6*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\left(3x^3+1\right)^{\frac{2}{3}}\left(x^2+x+1\right)}{x^6\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)*(3*x**3+1)**(2/3)/x**6/(x**3+1),x)
```

```
[Out] Integral((x - 1)*(3*x**3 + 1)**(2/3)*(x**2 + x + 1)/(x**6*(x + 1)*(x**2 - x
 + 1)), x)
```

3.1716 $\int \frac{3-8x+8x^2-12x^4}{x\sqrt[3]{\frac{1-2x^2}{1+2x^2}}(1+2x^2)(3-7x+7x^2-6x^3+2x^4)}\,dx$

Optimal. Leaf size=149

$$\log\left(\sqrt[3]{\frac{1-2x^2}{2x^2+1}}+x-1\right)-\frac{1}{2}\log\left(x^2+\left(\frac{1-2x^2}{2x^2+1}\right)^{2/3}+(1-x)\sqrt[3]{\frac{1-2x^2}{2x^2+1}}-2x+1\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\frac{1-2x^2}{2x^2+1}}}{\sqrt[3]{\frac{1-2x^2}{2x^2+1}}-2x+\right.$$

Rubi [F] time = 7.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3-8x+8x^2-12x^4}{x\sqrt[3]{\frac{1-2x^2}{1+2x^2}}\,(1+2x^2)\,(3-7x+7x^2-6x^3+2x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3 - 8*x + 8*x^2 - 12*x^4)/(x*((1 - 2*x^2)/(1 + 2*x^2))^(1/3)*(1 + 2*x^
2)*(3 - 7*x + 7*x^2 - 6*x^3 + 2*x^4)),x]
```

```
[Out] (Sqrt[3]*(1 - 2*x^2)^(1/3)*ArcTan[1/Sqrt[3] + (2*(1 - 2*x^2)^(1/3))/(Sqrt[3
]*(1 + 2*x^2)^(1/3))])/(2*((1 - 2*x^2)/(1 + 2*x^2))^(1/3)*(1 + 2*x^2)^(1/3)
) - ((1 - 2*x^2)^(1/3)*Log[x])/(2*((1 - 2*x^2)/(1 + 2*x^2))^(1/3)*(1 + 2*x^
2)^(1/3)) + (3*(1 - 2*x^2)^(1/3)*Log[(1 - 2*x^2)^(1/3) - (1 + 2*x^2)^(1/3)]
)/(4*((1 - 2*x^2)/(1 + 2*x^2))^(1/3)*(1 + 2*x^2)^(1/3)) + ((1 - 2*x^2)^(1/3
)*Defer[Int][1/((1 - 2*x^2)^(1/3)*(1 + 2*x^2)^(2/3)*(-3 + 7*x - 7*x^2 + 6*x
^3 - 2*x^4)), x])/(((1 - 2*x^2)/(1 + 2*x^2))^(1/3)*(1 + 2*x^2)^(1/3)) + ((1
 - 2*x^2)^(1/3)*Defer[Int][x/((1 - 2*x^2)^(1/3)*(1 + 2*x^2)^(2/3)*(3 - 7*x
+ 7*x^2 - 6*x^3 + 2*x^4)), x])/(((1 - 2*x^2)/(1 + 2*x^2))^(1/3)*(1 + 2*x^2)
^(1/3)) + (6*(1 - 2*x^2)^(1/3)*Defer[Int][x^2/((1 - 2*x^2)^(1/3)*(1 + 2*x^2
)^(2/3)*(3 - 7*x + 7*x^2 - 6*x^3 + 2*x^4)), x])/(((1 - 2*x^2)/(1 + 2*x^2))^
(1/3)*(1 + 2*x^2)^(1/3)) - (14*(1 - 2*x^2)^(1/3)*Defer[Int][x^3/((1 - 2*x^2
)^(1/3)*(1 + 2*x^2)^(2/3)*(3 - 7*x + 7*x^2 - 6*x^3 + 2*x^4)), x])/(((1 - 2*
x^2)/(1 + 2*x^2))^(1/3)*(1 + 2*x^2)^(1/3))
```

Rubi steps

$$\int \frac{3-8x+8x^2-12x^4}{x\sqrt[3]{\frac{1-2x^2}{1+2x^2}}\left(1+2x^2\right)\left(3-7x+7x^2-6x^3+2x^4\right)}\,dx = \frac{\sqrt[3]{1-2x^2}\int \frac{3-8x+8x^2-12x^4}{x\sqrt[3]{1-2x^2}\left(1+2x^2\right)^{2/3}\left(3-7x+7x^2-6x^3+2x^4\right)}\,dx}{\sqrt[3]{\frac{1-2x^2}{1+2x^2}}\sqrt[3]{1+2x^2}}$$

$$= \frac{\sqrt[3]{1-2x^2}\int \left(\frac{1}{x\sqrt[3]{1-2x^2}\left(1+2x^2\right)^{2/3}} + \frac{-1+x+6x^2-14x^3}{\sqrt[3]{1-2x^2}\left(1+2x^2\right)^{2/3}\left(3-7x+7x^2-\right.}\right.}{\sqrt[3]{\frac{1-2x^2}{1+2x^2}}\sqrt[3]{1+2x^2}}$$

$$= \frac{\sqrt[3]{1-2x^2}\int \frac{1}{x\sqrt[3]{1-2x^2}\left(1+2x^2\right)^{2/3}}\,dx}{\sqrt[3]{\frac{1-2x^2}{1+2x^2}}\sqrt[3]{1+2x^2}} + \frac{\sqrt[3]{1-2x^2}\int \frac{}{\sqrt[3]{1-2x^2}\left(1+\right.}}{\sqrt[3]{\frac{1-2x^2}{1+2x^2}}}$$

$$= \frac{\sqrt[3]{1-2x^2}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-2x}\,x(1+2x)^{2/3}}\,dx, x, x^2\right)}{2\sqrt[3]{\frac{1-2x^2}{1+2x^2}}\sqrt[3]{1+2x^2}} + \frac{\sqrt[3]{1-2x^2}}{}$$

$$= \frac{\sqrt{3}\sqrt[3]{1-2x^2}\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{1-2x^2}}{\sqrt{3}\sqrt[3]{1+2x^2}}\right)}{2\sqrt[3]{\frac{1-2x^2}{1+2x^2}}\sqrt[3]{1+2x^2}} - \frac{\sqrt[3]{1-2x^2}\log(}{2\sqrt[3]{\frac{1-2x^2}{1+2x^2}}\sqrt[3]{1+2}}$$

Mathematica [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{3-8x+8x^2-12x^4}{x\sqrt[3]{\frac{1-2x^2}{1+2x^2}}\left(1+2x^2\right)\left(3-7x+7x^2-6x^3+2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3 - 8*x + 8*x^2 - 12*x^4)/(x*((1 - 2*x^2)/(1 + 2*x^2))^(1/3)*(1
+ 2*x^2)*(3 - 7*x + 7*x^2 - 6*x^3 + 2*x^4)),x]
```

```
[Out] Integrate[(3 - 8*x + 8*x^2 - 12*x^4)/(x*((1 - 2*x^2)/(1 + 2*x^2))^(1/3)*(1
+ 2*x^2)*(3 - 7*x + 7*x^2 - 6*x^3 + 2*x^4)), x]
```

IntegrateAlgebraic [A] time = 0.38, size = 149, normalized size = 1.00

$$\log\left(\sqrt[3]{\frac{1-2x^2}{2x^2+1}}+x-1\right)-\frac{1}{2}\log\left(x^2+\left(\frac{1-2x^2}{2x^2+1}\right)^{2/3}+(1-x)\sqrt[3]{\frac{1-2x^2}{2x^2+1}}-2x+1\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\frac{1-2x^2}{2x^2+1}}}{\sqrt[3]{\frac{1-2x^2}{2x^2+1}}-2x+2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - 8*x + 8*x^2 - 12*x^4)/(x*((1 - 2*x^2)/(1 + 2*x^2))^
(1/3)*(1 + 2*x^2)*(3 - 7*x + 7*x^2 - 6*x^3 + 2*x^4)),x]
```

```
[Out] Sqrt[3]*ArcTan[(Sqrt[3]*((1 - 2*x^2)/(1 + 2*x^2))^(1/3))/(2 - 2*x + ((1 - 2
*x^2)/(1 + 2*x^2))^(1/3))] + Log[-1 + x + ((1 - 2*x^2)/(1 + 2*x^2))^(1/3)]
- Log[1 - 2*x + x^2 + (1 - x)*((1 - 2*x^2)/(1 + 2*x^2))^(1/3) + ((1 - 2*x^2
)/(1 + 2*x^2))^(2/3)]/2
```

fricas [A] time = 3.95, size = 279, normalized size = 1.87

$$-\sqrt{3}\arctan\left(\frac{434\sqrt{3}\left(2x^3-2x^2+x-1\right)\left(-\frac{2x^2-1}{2x^2+1}\right)^{\frac{2}{3}}+682\sqrt{3}\left(2x^4-4x^3+3x^2-2x+1\right)\left(-\frac{2x^2-1}{2x^2+1}\right)^{\frac{1}{3}}+\sqrt{3}\left(242x^5-726x^4+847x^3-1095x^2+363x+124\right)}{2662x^5-7986x^4+9317x^3-5969x^2+3993x-1674}\right)+\frac{1}{2}\log\left(\frac{2x^5-6x^4+7x^3-7x^2+3\left(2x^3-2x^2+x-1\right)\left(-\frac{2x^2-1}{2x^2+1}\right)^{\frac{2}{3}}+3\left(2x^4-4x^3+3x^2-2x+1\right)\left(-\frac{2x^2-1}{2x^2+1}\right)^{\frac{1}{3}}+3x}{2x^5-6x^4+7x^3-7x^2+3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-12*x^4+8*x^2-8*x+3)/x/((-2*x^2+1)/(2*x^2+1))^(1/3)/(2*x^2+1)/(2
*x^4-6*x^3+7*x^2-7*x+3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan((434*sqrt(3)*(2*x^3 - 2*x^2 + x - 1)*(-(2*x^2 - 1)/(2*x^2 +
 1))^(2/3) + 682*sqrt(3)*(2*x^4 - 4*x^3 + 3*x^2 - 2*x + 1)*(-(2*x^2 - 1)/(2
*x^2 + 1))^(1/3) + sqrt(3)*(242*x^5 - 726*x^4 + 847*x^3 - 1095*x^2 + 363*x
+ 124))/(2662*x^5 - 7986*x^4 + 9317*x^3 - 5969*x^2 + 3993*x - 1674)) + 1/2*
log((2*x^5 - 6*x^4 + 7*x^3 - 7*x^2 + 3*(2*x^3 - 2*x^2 + x - 1)*(-(2*x^2 - 1
)/(2*x^2 + 1))^(2/3) + 3*(2*x^4 - 4*x^3 + 3*x^2 - 2*x + 1)*(-(2*x^2 - 1)/(2
*x^2 + 1))^(1/3) + 3*x)/(2*x^5 - 6*x^4 + 7*x^3 - 7*x^2 + 3*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{12\,x^4 - 8\,x^2 + 8\,x - 3}{\left(2\,x^4 - 6\,x^3 + 7\,x^2 - 7\,x + 3\right)\left(2\,x^2 + 1\right)x\left(-\frac{2\,x^2-1}{2\,x^2+1}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-12*x^4+8*x^2-8*x+3)/x/((-2*x^2+1)/(2*x^2+1))^(1/3)/(2*x^2+1)/(2
*x^4-6*x^3+7*x^2-7*x+3),x, algorithm="giac")
```

```
[Out] integrate(-(12*x^4 - 8*x^2 + 8*x - 3)/((2*x^4 - 6*x^3 + 7*x^2 - 7*x + 3)*(2
*x^2 + 1)*x*(-(2*x^2 - 1)/(2*x^2 + 1))^(1/3)), x)
```

maple [C] time = 5.12, size = 2146, normalized size = 14.40

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-12*x^4+8*x^2-8*x+3)/x/((-2*x^2+1)/(2*x^2+1))^(1/3)/(2*x^2+1)/(2*x^4-6
*x^3+7*x^2-7*x+3),x)
```

```
[Out] 2*ln(-(-11+33*x-114*RootOf(4*_Z^2+2*_Z+1)*x^2+28*RootOf(4*_Z^2+2*_Z+1)*x^5-
84*RootOf(4*_Z^2+2*_Z+1)*x^4+42*RootOf(4*_Z^2+2*_Z+1)*x+36*x*(-(2*x^2-1)/(2
*x^2+1))^(1/3)+22*x^5-55*x^2+77*x^3-66*x^4-18*(-(2*x^2-1)/(2*x^2+1))^(2/3)-
18*(-(2*x^2-1)/(2*x^2+1))^(1/3)-18*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2
+1))^(1/3)*x^2+12*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x+12*R
ootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(2/3)*x^3-12*RootOf(4*_Z^2+2*_Z
+1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^4-12*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(
2*x^2+1))^(2/3)*x^2+24*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x
^3+6*x*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(2/3)+98*x^3*RootOf(4*_
Z^2+2*_Z+1)+28*RootOf(4*_Z^2+2*_Z+1)^2+8*RootOf(4*_Z^2+2*_Z+1)-56*RootOf(4*
_Z^2+2*_Z+1)^2*x^2-36*(-(2*x^2-1)/(2*x^2+1))^(2/3)*x^2-54*(-(2*x^2-1)/(2*x^
2+1))^(1/3)*x^2-36*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^4+72*(-(2*x^2-1)/(2*x^2+1
))^(1/3)*x^3+18*(-(2*x^2-1)/(2*x^2+1))^(2/3)*x-6*RootOf(4*_Z^2+2*_Z+1)*(-(2
*x^2-1)/(2*x^2+1))^(2/3)-6*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(1/
3)+36*x^3*(-(2*x^2-1)/(2*x^2+1))^(2/3))/(2*x^4-6*x^3+7*x^2-7*x+3)/x)*RootOf
(4*_Z^2+2*_Z+1)-2*RootOf(4*_Z^2+2*_Z+1)*ln((8-12*x-58*RootOf(4*_Z^2+2*_Z+1)
*x^2+28*RootOf(4*_Z^2+2*_Z+1)*x^5-84*RootOf(4*_Z^2+2*_Z+1)*x^4+42*RootOf(4*
_Z^2+2*_Z+1)*x-30*x*(-(2*x^2-1)/(2*x^2+1))^(1/3)-8*x^5+12*x^2-28*x^3+24*x^4
+15*(-(2*x^2-1)/(2*x^2+1))^(2/3)+15*(-(2*x^2-1)/(2*x^2+1))^(1/3)-18*RootOf(
4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^2+12*RootOf(4*_Z^2+2*_Z+1)*(-
(2*x^2-1)/(2*x^2+1))^(1/3)*x+12*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1)
)^(2/3)*x^3-12*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^4-12*Ro
otOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(2/3)*x^2+24*RootOf(4*_Z^2+2*_Z+
1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^3+6*x*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(
2*x^2+1))^(2/3)+98*x^3*RootOf(4*_Z^2+2*_Z+1)-28*RootOf(4*_Z^2+2*_Z+1)^2-20*
RootOf(4*_Z^2+2*_Z+1)+56*RootOf(4*_Z^2+2*_Z+1)^2*x^2+30*(-(2*x^2-1)/(2*x^2+
1))^(2/3)*x^2+45*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^2+30*(-(2*x^2-1)/(2*x^2+1))
```

```
^(1/3)*x^4-60*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^3-15*(-(2*x^2-1)/(2*x^2+1))^(2
/3)*x-6*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(2/3)-6*RootOf(4*_Z^2+
2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)-30*x^3*(-(2*x^2-1)/(2*x^2+1))^(2/3))/(
2*x^4-6*x^3+7*x^2-7*x+3)/x)-ln((8-12*x-58*RootOf(4*_Z^2+2*_Z+1)*x^2+28*Root
Of(4*_Z^2+2*_Z+1)*x^5-84*RootOf(4*_Z^2+2*_Z+1)*x^4+42*RootOf(4*_Z^2+2*_Z+1)
*x-30*x*(-(2*x^2-1)/(2*x^2+1))^(1/3)-8*x^5+12*x^2-28*x^3+24*x^4+15*(-(2*x^2
-1)/(2*x^2+1))^(2/3)+15*(-(2*x^2-1)/(2*x^2+1))^(1/3)-18*RootOf(4*_Z^2+2*_Z+
1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^2+12*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2
*x^2+1))^(1/3)*x+12*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(2/3)*x^3-
12*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^4-12*RootOf(4*_Z^2+
2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(2/3)*x^2+24*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-
1)/(2*x^2+1))^(1/3)*x^3+6*x*RootOf(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(2
/3)+98*x^3*RootOf(4*_Z^2+2*_Z+1)-28*RootOf(4*_Z^2+2*_Z+1)^2-20*RootOf(4*_Z^
2+2*_Z+1)+56*RootOf(4*_Z^2+2*_Z+1)^2*x^2+30*(-(2*x^2-1)/(2*x^2+1))^(2/3)*x^
2+45*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^2+30*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^4-6
0*(-(2*x^2-1)/(2*x^2+1))^(1/3)*x^3-15*(-(2*x^2-1)/(2*x^2+1))^(2/3)*x-6*Root
Of(4*_Z^2+2*_Z+1)*(-(2*x^2-1)/(2*x^2+1))^(2/3)-6*RootOf(4*_Z^2+2*_Z+1)*(-(2
*x^2-1)/(2*x^2+1))^(1/3)-30*x^3*(-(2*x^2-1)/(2*x^2+1))^(2/3))/(2*x^4-6*x^3+
7*x^2-7*x+3)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{12\,x^4-8\,x^2+8\,x-3}{\left(2\,x^4-6\,x^3+7\,x^2-7\,x+3\right)\left(2\,x^2+1\right)x\left(-\frac{2\,x^2-1}{2\,x^2+1}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-12*x^4+8*x^2-8*x+3)/x/((-2*x^2+1)/(2*x^2+1))^(1/3)/(2*x^2+1)/(2
*x^4-6*x^3+7*x^2-7*x+3),x, algorithm="maxima")
```

```
[Out] -integrate((12*x^4 - 8*x^2 + 8*x - 3)/((2*x^4 - 6*x^3 + 7*x^2 - 7*x + 3)*(2
*x^2 + 1)*x*(-(2*x^2 - 1)/(2*x^2 + 1))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{12\,x^4-8\,x^2+8\,x-3}{x\,\left(2\,x^2+1\right)\,{\left(-\frac{2\,x^2-1}{2\,x^2+1}\right)}^{1/3}\,\left(2\,x^4-6\,x^3+7\,x^2-7\,x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(8*x - 8*x^2 + 12*x^4 - 3)/(x*(2*x^2 + 1)*(-(2*x^2 - 1)/(2*x^2 + 1))^(
1/3)*(7*x^2 - 7*x - 6*x^3 + 2*x^4 + 3)),x)
```

```
[Out] -int((8*x - 8*x^2 + 12*x^4 - 3)/(x*(2*x^2 + 1)*(-(2*x^2 - 1)/(2*x^2 + 1))^(
1/3)*(7*x^2 - 7*x - 6*x^3 + 2*x^4 + 3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-12*x**4+8*x**2-8*x+3)/x/((-2*x**2+1)/(2*x**2+1))**(1/3)/(2*x**2
+1)/(2*x**4-6*x**3+7*x**2-7*x+3),x)
```

```
[Out] Timed out
```

3.1717 $\int \frac{x^2}{\sqrt{bx+ax^3}\left(-b^2+a^2x^4\right)}\,dx$

Optimal. Leaf size=149

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{4\sqrt{2}\,a^{5/4}b^{5/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{4\sqrt{2}\,a^{5/4}b^{5/4}}+\frac{\sqrt{ax^3+bx}}{2ab\left(ax^2+b\right)}$$

Rubi [A] time = 0.34, antiderivative size = 187, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2056, 1254, 466, 471, 21, 404, 212, 206, 203}

$$-\frac{\sqrt{x}\sqrt{ax^2+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2+b}}\right)}{4\sqrt{2}\,a^{5/4}b^{5/4}\sqrt{ax^3+bx}}-\frac{\sqrt{x}\sqrt{ax^2+b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2+b}}\right)}{4\sqrt{2}\,a^{5/4}b^{5/4}\sqrt{ax^3+bx}}+\frac{x}{2ab\sqrt{ax^3+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[b*x + a*x^3]*(-b^2 + a^2*x^4)),x]
```

```
[Out] x/(2*a*b*Sqrt[b*x + a*x^3]) - (Sqrt[x]*Sqrt[b + a*x^2]*ArcTan[(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x])/Sqrt[b + a*x^2]])/(4*Sqrt[2]*a^(5/4)*b^(5/4)*Sqrt[b*x
+ a*x^3]) - (Sqrt[x]*Sqrt[b + a*x^2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x])/Sqrt[b + a*x^2]])/(4*Sqrt[2]*a^(5/4)*b^(5/4)*Sqrt[b*x + a*x^3])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 404

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[a/c,
 Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1254

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e)^p, x] /
; FreeQ[{a, c, d, e, f, q, m, q}, x] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p
]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{bx+ax^3}\left(-b^2+a^2x^4\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \frac{x^{3/2}}{\sqrt{b+ax^2}\left(-b^2+a^2x^4\right)}\,dx}{\sqrt{bx+ax^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \frac{x^{3/2}}{\left(-b+ax^2\right)\left(b+ax^2\right)^{3/2}}\,dx}{\sqrt{bx+ax^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b+ax^2}\right)\text{Subst}\left(\int \frac{x^4}{\left(-b+ax^4\right)\left(b+ax^4\right)^{3/2}}\,dx, x, \sqrt{x}\right)}{\sqrt{bx+ax^3}}$$

$$= \frac{x}{2ab\sqrt{bx+ax^3}} - \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\text{Subst}\left(\int \frac{-b-ax^4}{\left(-b+ax^4\right)\sqrt{b+ax^4}}\,dx, x, \sqrt{x}\right)}{2ab\sqrt{bx+ax^3}}$$

$$= \frac{x}{2ab\sqrt{bx+ax^3}} + \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\text{Subst}\left(\int \frac{\sqrt{b+ax^4}}{-b+ax^4}\,dx, x, \sqrt{x}\right)}{2ab\sqrt{bx+ax^3}}$$

$$= \frac{x}{2ab\sqrt{bx+ax^3}} - \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\text{Subst}\left(\int \frac{1}{1-4abx^4}\,dx, x, \frac{\sqrt{x}}{\sqrt{b+ax^2}}\right)}{2ab\sqrt{bx+ax^3}}$$

$$= \frac{x}{2ab\sqrt{bx+ax^3}} - \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\text{Subst}\left(\int \frac{1}{1-2\sqrt{a}\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b+ax^2}}\right)}{4ab\sqrt{bx+ax^3}} - \frac{\left(\sqrt{x}\right.}{}$$

$$= \frac{x}{2ab\sqrt{bx+ax^3}} - \frac{\sqrt{x}\sqrt{b+ax^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{b+ax^2}}\right)}{4\sqrt{2}\,a^{5/4}b^{5/4}\sqrt{bx+ax^3}} - \frac{\sqrt{x}\sqrt{b+ax^2}\tanh^{-1}\left(\right.}{4\sqrt{2}\,a^{5/4}b^{5/4}\sqrt{b}}$$

Mathematica [C] time = 0.69, size = 93, normalized size = 0.62

$$\frac{bx\sqrt{\frac{ax^2}{b}+1}-x\left(ax^2+b\right)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{ax^2}{b},\frac{ax^2}{b}\right)}{2ab^2\sqrt{x\left(ax^2+b\right)}\sqrt{\frac{ax^2}{b}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(Sqrt[b*x + a*x^3]*(-b^2 + a^2*x^4)),x]
```

```
[Out] (b*x*Sqrt[1 + (a*x^2)/b] - x*(b + a*x^2)*AppellF1[1/4, -1/2, 1, 5/4, -((a*x
^2)/b), (a*x^2)/b])/(2*a*b^2*Sqrt[x*(b + a*x^2)]*Sqrt[1 + (a*x^2)/b])
```

IntegrateAlgebraic [A] time = 0.43, size = 149, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{4\sqrt{2}\,a^{5/4}b^{5/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{4\sqrt{2}\,a^{5/4}b^{5/4}}+\frac{\sqrt{ax^3+bx}}{2ab\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[b*x + a*x^3]*(-b^2 + a^2*x^4)),x]
```

```
[Out] Sqrt[b*x + a*x^3]/(2*a*b*(b + a*x^2)) - ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[b*x + a*x^3])/(b + a*x^2)]/(4*Sqrt[2]*a^(5/4)*b^(5/4)) - ArcTanh[(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3])/(b + a*x^2)]/(4*Sqrt[2]*a^(5/4)*b^(5/4)
)
```

fricas [B] time = 0.53, size = 426, normalized size = 2.86

$$-\frac{4\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(a^{2}bx^{2}+ab^{2}\right)\left(\frac{1}{a^{5}b^{5}}\right)^{\frac{1}{4}}\arctan\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{ax^{3}+bx}\,a^{4}b^{4}\left(\frac{1}{a^{5}b^{5}}\right)^{\frac{3}{4}}}{ax^{2}+b}\right)+\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(a^{2}bx^{2}+ab^{2}\right)\left(\frac{1}{a^{5}b^{5}}\right)^{\frac{1}{4}}\log\left(\frac{a^{2}x^{4}+6abx^{2}+b^{2}+8\left(\left(\frac{1}{4}\right)^{\frac{1}{4}}a^{2}b^{2}x\left(\frac{1}{a^{5}b^{5}}\right)^{\frac{1}{4}}+\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(a^{5}b^{4}x^{2}+a^{4}b^{5}\right)\left(\frac{1}{a^{5}b^{5}}\right)^{\frac{3}{4}}\right)\sqrt{ax^{3}+bx}+4\left(a^{4}b^{3}x^{3}+a^{3}b^{4}x\right)\sqrt{\frac{1}{a^{5}b^{5}}}}{a^{2}x^{4}-2abx^{2}+b^{2}}\right)-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(a^{2}bx^{2}+ab^{2}\right)\left(\frac{1}{a^{5}b^{5}}\right)^{\frac{1}{4}}\log\left(\frac{a^{2}x^{4}+6abx^{2}+b^{2}-8\left(\left(\frac{1}{4}\right)^{\frac{1}{4}}a^{2}b^{2}x\left(\frac{1}{a^{5}b^{5}}\right)^{\frac{1}{4}}+\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(a^{5}b^{4}x^{2}+a^{4}b^{5}\right)\left(\frac{1}{a^{5}b^{5}}\right)^{\frac{3}{4}}\right)\sqrt{ax^{3}+bx}+4\left(a^{4}b^{3}x^{3}+a^{3}b^{4}x\right)\sqrt{\frac{1}{a^{5}b^{5}}}}{a^{2}x^{4}-2abx^{2}+b^{2}}\right)-8\sqrt{ax^{3}+bx}}{16\left(a^{2}bx^{2}+ab^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^3+b*x)^(1/2)/(a^2*x^4-b^2),x, algorithm="fricas")
```

```
[Out] -1/16*(4*(1/4)^(1/4)*(a^2*b*x^2 + a*b^2)*(1/(a^5*b^5))^(1/4)*arctan(4*(1/4)
^(3/4)*sqrt(a*x^3 + b*x)*a^4*b^4*(1/(a^5*b^5))^(3/4)/(a*x^2 + b)) + (1/4)^(
1/4)*(a^2*b*x^2 + a*b^2)*(1/(a^5*b^5))^(1/4)*log((a^2*x^4 + 6*a*b*x^2 + b^2
 + 8*((1/4)^(1/4)*a^2*b^2*x*(1/(a^5*b^5))^(1/4) + (1/4)^(3/4)*(a^5*b^4*x^2
+ a^4*b^5)*(1/(a^5*b^5))^(3/4))*sqrt(a*x^3 + b*x) + 4*(a^4*b^3*x^3 + a^3*b^
4*x)*sqrt(1/(a^5*b^5)))/(a^2*x^4 - 2*a*b*x^2 + b^2)) - (1/4)^(1/4)*(a^2*b*x
^2 + a*b^2)*(1/(a^5*b^5))^(1/4)*log((a^2*x^4 + 6*a*b*x^2 + b^2 - 8*((1/4)^(
1/4)*a^2*b^2*x*(1/(a^5*b^5))^(1/4) + (1/4)^(3/4)*(a^5*b^4*x^2 + a^4*b^5)*(1
/(a^5*b^5))^(3/4))*sqrt(a*x^3 + b*x) + 4*(a^4*b^3*x^3 + a^3*b^4*x)*sqrt(1/(
a^5*b^5)))/(a^2*x^4 - 2*a*b*x^2 + b^2)) - 8*sqrt(a*x^3 + b*x))/(a^2*b*x^2 +
 a*b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(a^{2}x^{4}-b^{2}\right)\sqrt{ax^{3}+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^3+b*x)^(1/2)/(a^2*x^4-b^2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*x^4 - b^2)*sqrt(a*x^3 + b*x)), x)
```

maple [C] time = 0.05, size = 448, normalized size = 3.01

$$\frac{\frac{\sqrt{-ab}\sqrt{\frac{xa}{\sqrt{-ab}}+1}\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{2\sqrt{ab}\,a\sqrt{ax^{3}+bx}\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)}-\frac{\sqrt{-ab}\sqrt{\frac{xa}{\sqrt{-ab}}+1}\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{2\sqrt{ab}\,a\sqrt{ax^{3}+bx}\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)}}{2a}+\frac{\frac{x}{b\sqrt{\left(x^{2}+\frac{b}{a}\right)xa}}+\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{2ba\sqrt{ax^{3}+bx}}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^3+b*x)^(1/2)/(a^2*x^4-b^2),x)
```

```
[Out] 1/2/a*(1/2/(a*b)^(1/2)/a*(-a*b)^(1/2)*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-
a*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(-a*b
)^(1/2)-1/a*(a*b)^(1/2))*EllipticPi(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(
1/2),-1/a*(-a*b)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2)),1/2*2^(1/2))-1/2
/(a*b)^(1/2)/a*(-a*b)^(1/2)*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(1/2)
+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(-a*b)^(1/2)+1/
a*(a*b)^(1/2))*EllipticPi(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),-1/a*
(-a*b)^(1/2)/(-1/a*(-a*b)^(1/2)+1/a*(a*b)^(1/2)),1/2*2^(1/2)))+1/2/a*(x/b/(
(x^2+b/a)*x*a)^(1/2)+1/2/b/a*(-a*b)^(1/2)*((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1
/2))^(1/2)*(-2*(x-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x*a/(-a*b)^(1/2
))^(1/2)/(a*x^3+b*x)^(1/2)*EllipticF(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^
(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(a^{2}x^{4}-b^{2}\right)\sqrt{ax^{3}+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^3+b*x)^(1/2)/(a^2*x^4-b^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*x^4 - b^2)*sqrt(a*x^3 + b*x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((b^2 - a^2*x^4)*(b*x + a*x^3)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x\left(ax^2+b\right)}\left(ax^2-b\right)\left(ax^2+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**3+b*x)**(1/2)/(a**2*x**4-b**2),x)
```

```
[Out] Integral(x**2/(sqrt(x*(a*x**2 + b))*(a*x**2 - b)*(a*x**2 + b)), x)
```

3.1718 $\int \frac{b+ax^6}{x^6\left(-b+ax^3\right)\sqrt[4]{bx+ax^4}}\,dx$

Optimal. Leaf size=149

$$-\frac{2^{3/4}\left(a^{3/4}b+a^{7/4}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3b^2}-\frac{2^{3/4}\left(a^{3/4}b+a^{7/4}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3b^2}+\frac{4\left(ax^4+bx\right)^{3/4}\left(ax^3+b\right.}{21b^2x^6}$$

Rubi [A] time = 1.16, antiderivative size = 253, normalized size of antiderivative = 1.70, number of steps used = 14, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {2056, 6725, 271, 264, 466, 465, 494, 461, 212, 206, 203}

$$-\frac{2^{3/4}a^{3/4}\sqrt[4]{x}(a+b)\sqrt[4]{ax^3+b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3b^2\sqrt[4]{ax^4+bx}}-\frac{2^{3/4}a^{3/4}\sqrt[4]{x}(a+b)\sqrt[4]{ax^3+b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3b^2\sqrt[4]{ax^4+bx}}+\frac{4(a+b)\left(ax^3+b\right)^2}{21ab^2x^5\sqrt[4]{ax^4+bx}}-\frac{4\left(ax^3+b\right)}{21ax^5\sqrt[4]{ax^4+bx}}-\frac{4\left(ax^3+b\right)}{21bx^2\sqrt[4]{ax^4+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^6)/(x^6*(-b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(b + a*x^3))/(21*a*x^5*(b*x + a*x^4)^(1/4)) - (4*(b + a*x^3))/(21*b*x^2
*(b*x + a*x^4)^(1/4)) + (4*(a + b)*(b + a*x^3)^2)/(21*a*b^2*x^5*(b*x + a*x^
4)^(1/4)) - (2^(3/4)*a^(3/4)*(a + b)*x^(1/4)*(b + a*x^3)^(1/4)*ArcTan[(2^(1
/4)*a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)])/(3*b^2*(b*x + a*x^4)^(1/4)) - (2^(
3/4)*a^(3/4)*(a + b)*x^(1/4)*(b + a*x^3)^(1/4)*ArcTanh[(2^(1/4)*a^(1/4)*x^(
3/4))/(b + a*x^3)^(1/4)])/(3*b^2*(b*x + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{b+ax^6}{x^6\left(-b+ax^3\right)\sqrt[4]{bx+ax^4}}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\int \frac{b+ax^6}{x^{25/4}\left(-b+ax^3\right)\sqrt[4]{b+ax^3}}\,dx}{\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\int \left(\frac{b}{ax^{25/4}\sqrt[4]{b+ax^3}}+\frac{1}{x^{13/4}\sqrt[4]{b+ax^3}}+\frac{ab+b^2}{ax^{25/4}\left(-b+ax^3\right)\sqrt[4]{b+ax^3}}\right)dx}{\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\int \frac{1}{x^{13/4}\sqrt[4]{b+ax^3}}\,dx}{\sqrt[4]{bx+ax^4}} + \frac{\left(b\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\int \frac{1}{x^{25/4}\sqrt[4]{b+ax^3}}\,dx}{a\sqrt[4]{bx+ax^4}} + \frac{\left(b(a+\right.}{}$$

$$= -\frac{4\left(b+ax^3\right)}{21ax^5\sqrt[4]{bx+ax^4}} - \frac{4\left(b+ax^3\right)}{9bx^2\sqrt[4]{bx+ax^4}} - \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\int \frac{1}{x^{13/4}\sqrt[4]{b+ax^3}}\,dx}{7\sqrt[4]{bx+ax^4}} + -$$

$$= -\frac{4\left(b+ax^3\right)}{21ax^5\sqrt[4]{bx+ax^4}} - \frac{4\left(b+ax^3\right)}{21bx^2\sqrt[4]{bx+ax^4}} + \frac{\left(4b(a+b)\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{}{x^8}\right.}{3a\sqrt[4]{bx+ax^4}}$$

$$= -\frac{4\left(b+ax^3\right)}{21ax^5\sqrt[4]{bx+ax^4}} - \frac{4\left(b+ax^3\right)}{21bx^2\sqrt[4]{bx+ax^4}} + \frac{\left(4(a+b)\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{(}{x^8(-}\right.}{3ab\sqrt[4]{bx+ax^4}}$$

$$= -\frac{4\left(b+ax^3\right)}{21ax^5\sqrt[4]{bx+ax^4}} - \frac{4\left(b+ax^3\right)}{21bx^2\sqrt[4]{bx+ax^4}} + \frac{\left(4(a+b)\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int\left(-\right.\right.}{3ab\sqrt[4]{bx+}}$$

$$= -\frac{4\left(b+ax^3\right)}{21ax^5\sqrt[4]{bx+ax^4}} - \frac{4\left(b+ax^3\right)}{21bx^2\sqrt[4]{bx+ax^4}} + \frac{4(a+b)\left(b+ax^3\right)^2}{21ab^2x^5\sqrt[4]{bx+ax^4}} + \frac{\left(4a(a+b)\sqrt[4]{x}\right.}{}$$

$$= -\frac{4\left(b+ax^3\right)}{21ax^5\sqrt[4]{bx+ax^4}} - \frac{4\left(b+ax^3\right)}{21bx^2\sqrt[4]{bx+ax^4}} + \frac{4(a+b)\left(b+ax^3\right)^2}{21ab^2x^5\sqrt[4]{bx+ax^4}} - \frac{\left(2a(a+b)\sqrt[4]{x}\right.}{}$$

$$= -\frac{4\left(b+ax^3\right)}{21ax^5\sqrt[4]{bx+ax^4}} - \frac{4\left(b+ax^3\right)}{21bx^2\sqrt[4]{bx+ax^4}} + \frac{4(a+b)\left(b+ax^3\right)^2}{21ab^2x^5\sqrt[4]{bx+ax^4}} - \frac{2^{3/4}a^{3/4}(a+b)}{}$$

Mathematica [C] time = 5.22, size = 97, normalized size = 0.65

$$\frac{4\left(\left(ax^3+b\right)^2-\frac{7ax^6(a+b)\sqrt[4]{\frac{ax^3}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{2ax^3}{b-ax^3}\right)}{\sqrt[4]{1-\frac{ax^3}{b}}}\right)}{21b^2x^5\sqrt[4]{x\left(ax^3+b\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b + a*x^6)/(x^6*(-b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*((b + a*x^3)^2 - (7*a*(a + b)*x^6*(1 + (a*x^3)/b)^(1/4)*Hypergeometric2F
1[1/4, 1/4, 5/4, (-2*a*x^3)/(b - a*x^3)])/(1 - (a*x^3)/b)^(1/4)))/(21*b^2*x
^5*(x*(b + a*x^3))^(1/4))
```

IntegrateAlgebraic [A] time = 0.90, size = 149, normalized size = 1.00

$$-\frac{2^{3/4}\left(a^{3/4}b+a^{7/4}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3b^2}-\frac{2^{3/4}\left(a^{3/4}b+a^{7/4}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3b^2}+\frac{4\left(ax^4+bx\right)^{3/4}\left(ax^3+b\right)}{21b^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^6)/(x^6*(-b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*(b + a*x^3)*(b*x + a*x^4)^(3/4))/(21*b^2*x^6) - (2^(3/4)*(a^(7/4) + a^(3
/4)*b)*ArcTan[(2^(1/4)*a^(1/4)*(b*x + a*x^4)^(3/4))/(b + a*x^3)])/(3*b^2) -
 (2^(3/4)*(a^(7/4) + a^(3/4)*b)*ArcTanh[(2^(1/4)*a^(1/4)*(b*x + a*x^4)^(3/4
))/(b + a*x^3)])/(3*b^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^6/(a*x^3-b)/(a*x^4+b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.30, size = 272, normalized size = 1.83

$$-\frac{\sqrt{2}\left(2^{\frac{3}{4}}(-a)^{\frac{3}{4}}a+2^{\frac{3}{4}}(-a)^{\frac{3}{4}}b\right)\arctan\left(\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{6b^2}-\frac{\sqrt{2}\left(2^{\frac{3}{4}}(-a)^{\frac{3}{4}}a+2^{\frac{3}{4}}(-a)^{\frac{3}{4}}b\right)\arctan\left(-\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{6b^2}+\frac{\sqrt{2}\left(2^{\frac{3}{4}}(-a)^{\frac{3}{4}}a+2^{\frac{3}{4}}(-a)^{\frac{3}{4}}b\right)\log\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)}{12b^2}-\frac{\sqrt{2}\left(2^{\frac{3}{4}}(-a)^{\frac{3}{4}}a+2^{\frac{3}{4}}(-a)^{\frac{3}{4}}b\right)\log\left(-2^{\frac{3}{4}}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a+\frac{b}{x^3}}\right)}{12b^2}+\frac{4\left(a+\frac{b}{x^3}\right)^{\frac{7}{4}}}{21b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^6/(a*x^3-b)/(a*x^4+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*(2^(3/4)*(-a)^(3/4)*a + 2^(3/4)*(-a)^(3/4)*b)*arctan(1/2*2^(1/
4)*(2^(3/4)*(-a)^(1/4) + 2*(a + b/x^3)^(1/4))/(-a)^(1/4))/b^2 - 1/6*sqrt(2)
*(2^(3/4)*(-a)^(3/4)*a + 2^(3/4)*(-a)^(3/4)*b)*arctan(-1/2*2^(1/4)*(2^(3/4)
*(-a)^(1/4) - 2*(a + b/x^3)^(1/4))/(-a)^(1/4))/b^2 + 1/12*sqrt(2)*(2^(3/4)*
(-a)^(3/4)*a + 2^(3/4)*(-a)^(3/4)*b)*log(2^(3/4)*(-a)^(1/4)*(a + b/x^3)^(1/
4) + sqrt(2)*sqrt(-a) + sqrt(a + b/x^3))/b^2 - 1/12*sqrt(2)*(2^(3/4)*(-a)^(
3/4)*a + 2^(3/4)*(-a)^(3/4)*b)*log(-2^(3/4)*(-a)^(1/4)*(a + b/x^3)^(1/4) +
sqrt(2)*sqrt(-a) + sqrt(a + b/x^3))/b^2 + 4/21*(a + b/x^3)^(7/4)/b^2
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{a\,x^{6}+b}{x^{6}\left(a\,x^{3}-b\right)\left(a\,x^{4}+bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6+b)/x^6/(a*x^3-b)/(a*x^4+b*x)^(1/4),x)
```

```
[Out] int((a*x^6+b)/x^6/(a*x^3-b)/(a*x^4+b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{6}+b}{\left(ax^{4}+bx\right)^{\frac{1}{4}}\left(ax^{3}-b\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^6/(a*x^3-b)/(a*x^4+b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^6 + b)/((a*x^4 + b*x)^(1/4)*(a*x^3 - b)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{a\,x^{6}+b}{x^{6}\,{\left(a\,x^{4}+b\,x\right)}^{1/4}\,\left(b-a\,x^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x^6)/(x^6*(b*x + a*x^4)^(1/4)*(b - a*x^3)),x)
[Out] -int((b + a*x^6)/(x^6*(b*x + a*x^4)^(1/4)*(b - a*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^6 + b}{x^6 \sqrt[4]{x\left(ax^3 + b\right)}\left(ax^3 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6+b)/x**6/(a*x**3-b)/(a*x**4+b*x)**(1/4),x)
[Out] Integral((a*x**6 + b)/(x**6*(x*(a*x**3 + b))**(1/4)*(a*x**3 - b)), x)
```

3.1719 $\int \frac{\sqrt[3]{-1+2x^3+x^8}\,(3+5x^8)}{x^2(-1+x^8)}\,dx$

Optimal. Leaf size=149

$$\frac{3\sqrt[3]{x^8+2x^3-1}}{x}+\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^8+2x^3-1}-2x\right)+\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2^{2/3}\sqrt[3]{x^8+2x^3-1}+x}\right)-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{\ }\right.}{}$$

Rubi [F] time = 1.59, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{-1+2x^3+x^8}\,(3+5x^8)}{x^2(-1+x^8)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + 2*x^3 + x^8)^(1/3)*(3 + 5*x^8))/(x^2*(-1 + x^8)),x]
```

```
[Out] Defer[Int][(-1 + 2*x^3 + x^8)^(1/3)/(-1 - x), x] + I*Defer[Int][(-1 + 2*x^3
 + x^8)^(1/3)/(I - x), x] + (-1)^(3/4)*Defer[Int][(-1 + 2*x^3 + x^8)^(1/3)/
((-1)^(1/4) - x), x] - (-1)^(1/4)*Defer[Int][(-1 + 2*x^3 + x^8)^(1/3)/(-(-1
)^(3/4) - x), x] + Defer[Int][(-1 + 2*x^3 + x^8)^(1/3)/(-1 + x), x] - 3*Def
er[Int][(-1 + 2*x^3 + x^8)^(1/3)/x^2, x] + I*Defer[Int][(-1 + 2*x^3 + x^8)^
(1/3)/(I + x), x] + (-1)^(3/4)*Defer[Int][(-1 + 2*x^3 + x^8)^(1/3)/((-1)^(1
/4) + x), x] - (-1)^(1/4)*Defer[Int][(-1 + 2*x^3 + x^8)^(1/3)/(-(-1)^(3/4)
+ x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{-1+2x^3+x^8}\,(3+5x^8)}{x^2(-1+x^8)}\,dx &= \int \left(\frac{\sqrt[3]{-1+2x^3+x^8}}{-1-x}+\frac{\sqrt[3]{-1+2x^3+x^8}}{-1+x}-\frac{3\sqrt[3]{-1+2x^3+x^8}}{x^2}+\frac{2\sqrt[3]{-1+2}}{1+}\right.\\
&= 2\int \frac{\sqrt[3]{-1+2x^3+x^8}}{1+x^2}\,dx-3\int \frac{\sqrt[3]{-1+2x^3+x^8}}{x^2}\,dx+4\int \frac{x^2\sqrt[3]{-1+2x^3+}}{1+x^4}\\
&= 2\int \left(\frac{i\sqrt[3]{-1+2x^3+x^8}}{2(i-x)}+\frac{i\sqrt[3]{-1+2x^3+x^8}}{2(i+x)}\right)dx-3\int \frac{\sqrt[3]{-1+2x^3+x^8}}{x^2}\,dx\\
&= i\int \frac{\sqrt[3]{-1+2x^3+x^8}}{i-x}\,dx+i\int \frac{\sqrt[3]{-1+2x^3+x^8}}{i+x}\,dx-2\int \frac{\sqrt[3]{-1+2x^3+x^8}}{i-x^2}\\
&= i\int \frac{\sqrt[3]{-1+2x^3+x^8}}{i-x}\,dx+i\int \frac{\sqrt[3]{-1+2x^3+x^8}}{i+x}\,dx-2\int \left(-\frac{(-1)^{3/4}\sqrt[3]{-1+}}{2(\sqrt[4]{-1}}\right.\\
&= i\int \frac{\sqrt[3]{-1+2x^3+x^8}}{i-x}\,dx+i\int \frac{\sqrt[3]{-1+2x^3+x^8}}{i+x}\,dx-3\int \frac{\sqrt[3]{-1+2x^3+x^8}}{x^2}
\end{aligned}$$

Mathematica [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-1+2x^3+x^8}\,(3+5x^8)}{x^2(-1+x^8)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + 2*x^3 + x^8)^(1/3)*(3 + 5*x^8))/(x^2*(-1 + x^8)),x]
```

[Out] Integrate[((-1 + 2*x^3 + x^8)^(1/3)*(3 + 5*x^8))/(x^2*(-1 + x^8)), x]

IntegrateAlgebraic [A] time = 2.83, size = 149, normalized size = 1.00

$$\frac{3\sqrt[3]{x^8+2x^3-1}}{x}+\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^8+2x^3-1}-2x\right)+\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^8+2x^3-1}+x}\right)-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^8+2x^3-1}x+\sqrt[3]{2}\left(x^8+2x^3-1\right)^{2/3}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((-1 + 2*x^3 + x^8)^(1/3)*(3 + 5*x^8))/(x^2*(-1 + x^8)), x]

[Out] (3*(-1 + 2*x^3 + x^8)^(1/3))/x + 2^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(-1 + 2*x^3 + x^8)^(1/3))] + 2^(1/3)*Log[-2*x + 2^(2/3)*(-1 + 2*x^3 + x^8)^(1/3)] - Log[2*x^2 + 2^(2/3)*x*(-1 + 2*x^3 + x^8)^(1/3) + 2^(1/3)*(-1 + 2*x^3 + x^8)^(2/3)]/2^(2/3)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^8+2*x^3-1)^(1/3)*(5*x^8+3)/x^2/(x^8-1),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(5x^8+3\right)\left(x^8+2x^3-1\right)^{\frac{1}{3}}}{\left(x^8-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^8+2*x^3-1)^(1/3)*(5*x^8+3)/x^2/(x^8-1),x, algorithm="giac")

[Out] integrate((5*x^8 + 3)*(x^8 + 2*x^3 - 1)^(1/3)/((x^8 - 1)*x^2), x)

maple [C] time = 9.68, size = 2335, normalized size = 15.67

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8+2*x^3-1)^(1/3)*(5*x^8+3)/x^2/(x^8-1),x)

[Out] 3*(x^8+2*x^3-1)^(1/3)/x+(RootOf(_Z^3-2)*ln((-2*x^16*RootOf(_Z^3-2)-2*x^16*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^11-4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^11+3*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x^9-12*RootOf(_Z^3-2)*x^11-12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^11-8*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^6-8*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^6+4*RootOf(_Z^3-2)*x^8+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^8+6*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x^2+6*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x^4+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-16*RootOf(_Z^3-2)*x^6-16*x^6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-3*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x+12*RootOf(_Z^3-2)*x^3+12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*

```
_Z^2)*x^3-2*RootOf(_Z^3-2)-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*
_Z^2))/(x^8+2*x^3-1)/(-1+x)/(1+x)/(x^2+1)/(x^4+1))-ln(-(-x^16*RootOf(_Z^3-2
)-x^16*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+2*RootOf(RootOf(
_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^11+2*RootOf(RootOf
(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^11-3*(x^16+4*x^
11-2*x^8+4*x^6-4*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4
*_Z^2)*RootOf(_Z^3-2)*x^9-4*RootOf(_Z^3-2)*x^11-4*RootOf(RootOf(_Z^3-2)^2+2
*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^11+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-
2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^6+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-
2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^6+2*RootOf(_Z^3-2)*x^8+2*RootOf(RootOf(_Z^3
-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^8+3*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(
2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x
^2-6*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*R
ootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^4-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*Roo
tOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*Roo
tOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-4*RootOf(_Z^3-2)*x^6-4*x^6*RootO
f(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+3*(x^16+4*x^11-2*x^8+4*x^6-4
*x^3+1)^(2/3)*x^2+3*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(1/3)*RootOf(RootOf(_
Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x+4*RootOf(_Z^3-2)*x^3+
4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^3-RootOf(_Z^3-2)-Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2))/(x^8+2*x^3-1)/(-1+x)/(1+
x)/(x^2+1)/(x^4+1))*RootOf(_Z^3-2)-2*ln(-(-x^16*RootOf(_Z^3-2)-x^16*RootOf(
RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+2*RootOf(RootOf(_Z^3-2)^2+2*_Z
*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^11+2*RootOf(RootOf(_Z^3-2)^2+2*_
Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^11-3*(x^16+4*x^11-2*x^8+4*x^6
-4*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(
_Z^3-2)*x^9-4*RootOf(_Z^3-2)*x^11-4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^
3-2)+4*_Z^2)*x^11+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*Roo
tOf(_Z^3-2)^3*x^6+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*R
ootOf(_Z^3-2)^2*x^6+2*RootOf(_Z^3-2)*x^8+2*RootOf(RootOf(_Z^3-2)^2+2*_Z*Roo
tOf(_Z^3-2)+4*_Z^2)*x^8+3*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(2/3)*RootOf(Ro
otOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x^2-6*(x^16+4*x
^11-2*x^8+4*x^6-4*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+
4*_Z^2)*RootOf(_Z^3-2)*x^4-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*
_Z^2)*RootOf(_Z^3-2)^3*x^3-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*
_Z^2)^2*RootOf(_Z^3-2)^2*x^3-4*RootOf(_Z^3-2)*x^6-4*x^6*RootOf(RootOf(_Z^3-
2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+3*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(2/3)*
x^2+3*(x^16+4*x^11-2*x^8+4*x^6-4*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*
RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x+4*RootOf(_Z^3-2)*x^3+4*RootOf(RootO
f(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^3-RootOf(_Z^3-2)-RootOf(RootOf(_Z
^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2))/(x^8+2*x^3-1)/(-1+x)/(1+x)/(x^2+1)/(x^
4+1))*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2))/(x^8+2*x^3-1)^(2
/3)*((x^8+2*x^3-1)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^{8}+3\right)\left(x^{8}+2\,x^{3}-1\right)^{\frac{1}{3}}}{\left(x^{8}-1\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+2*x^3-1)^(1/3)*(5*x^8+3)/x^2/(x^8-1),x, algorithm="maxima")
[Out] integrate((5*x^8 + 3)*(x^8 + 2*x^3 - 1)^(1/3)/((x^8 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^8+3\right)\,{\left(x^8+2\,x^3-1\right)}^{1/3}}{x^2\,\left(x^8-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x^8 + 3)*(2*x^3 + x^8 - 1)^(1/3))/(x^2*(x^8 - 1)),x)
```

```
[Out] int(((5*x^8 + 3)*(2*x^3 + x^8 - 1)^(1/3))/(x^2*(x^8 - 1)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^8+3)\sqrt[3]{x^8+2x^3-1}}{x^2(x-1)(x+1)(x^2+1)(x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8+2*x**3-1)**(1/3)*(5*x**8+3)/x**2/(x**8-1),x)
```

```
[Out] Integral((5*x**8 + 3)*(x**8 + 2*x**3 - 1)**(1/3)/(x**2*(x - 1)*(x + 1)*(x**
2 + 1)*(x**4 + 1)), x)
```

3.1720 $\int \frac{\sqrt{b^2+ax^2}}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$

Optimal. Leaf size=149

$$-\frac{2bx}{3\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{2x\sqrt{ax^2+b^2}}{3\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{2\sqrt{2}\,b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{a}}$$

Rubi [F] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{b^2+ax^2}}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b^2 + a*x^2]/Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] Defer[Int][Sqrt[b^2 + a*x^2]/Sqrt[b + Sqrt[b^2 + a*x^2]], x]
```

Rubi steps

$$\int \frac{\sqrt{b^2+ax^2}}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx=\int \frac{\sqrt{b^2+ax^2}}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Mathematica [C] time = 0.43, size = 212, normalized size = 1.42

$$\frac{6b^2\left(\sqrt{ax^2+b^2}+b\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b-\sqrt{b^2+ax^2}}{2b}\right)-6b^2\sqrt{ax^2+b^2}+4ax^2\sqrt{ax^2+b^2}+3\sqrt{2}\,b^{3/2}\sqrt{\sqrt{ax^2+b^2}-b}\left(\sqrt{ax^2+b^2}+b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{ax^2+b^2}-b}}{\sqrt{2}\sqrt{b}}\right)-4abx^2-6b^3}{6ax\sqrt{\sqrt{ax^2+b^2}+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b^2 + a*x^2]/Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] (-6*b^3 - 4*a*b*x^2 - 6*b^2*Sqrt[b^2 + a*x^2] + 4*a*x^2*Sqrt[b^2 + a*x^2] +
 3*Sqrt[2]*b^(3/2)*Sqrt[-b + Sqrt[b^2 + a*x^2]]*(b + Sqrt[b^2 + a*x^2])*Arc
Tan[Sqrt[-b + Sqrt[b^2 + a*x^2]]/(Sqrt[2]*Sqrt[b])] + 6*b^2*(b + Sqrt[b^2 +
 a*x^2])*Hypergeometric2F1[-1/2, 1, 1/2, (b - Sqrt[b^2 + a*x^2])/(2*b)])/(6
*a*x*Sqrt[b + Sqrt[b^2 + a*x^2]])
```

IntegrateAlgebraic [A] time = 0.22, size = 81, normalized size = 0.54

$$\frac{2ax^3}{3\left(\sqrt{ax^2+b^2}+b\right)^{3/2}}+\frac{\sqrt{2}\,b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b^2 + a*x^2]/Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] (2*a*x^3)/(3*(b + Sqrt[b^2 + a*x^2])^(3/2)) + (Sqrt[2]*b^(3/2)*ArcTan[(Sqrt
[a]*x)/(Sqrt[2]*Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/Sqrt[a]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^(1/2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2 + b^2}}{\sqrt{b + \sqrt{ax^2 + b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^(1/2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(a*x^2 + b^2)/sqrt(b + sqrt(a*x^2 + b^2)), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\,x^2 + b^2}}{\sqrt{b + \sqrt{a\,x^2 + b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b^2)^(1/2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x^2+b^2)^(1/2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2 + b^2}}{\sqrt{b + \sqrt{ax^2 + b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^(1/2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(a*x^2 + b^2)/sqrt(b + sqrt(a*x^2 + b^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b^2 + a\,x^2}}{\sqrt{b + \sqrt{b^2 + a\,x^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b^2)^(1/2)/(b + (a*x^2 + b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x^2 + b^2)^(1/2)/(b + (a*x^2 + b^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2+b^2}}{\sqrt{b+\sqrt{ax^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b**2)**(1/2)/(b+(a*x**2+b**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**2 + b**2)/sqrt(b + sqrt(a*x**2 + b**2)), x)
```

3.1721 $\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^3}\,dx$

Optimal. Leaf size=149

$$\frac{35\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{64\sqrt{a}\,b^{9/2}}+\frac{7x\left(15ax^2+23b^2\right)}{192b^4\left(ax^2+b^2\right)^{3/2}\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{x\left(35ax^2+59b^2\right)}{96b^3\left(ax^2+b^2\right)^2\sqrt{\sqrt{ax^2+b^2}+b}}$$

Rubi [F] time = 1.58, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^3,x]
```

```
[Out] (3*Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b - Sqrt[-a]*x), x])/(16*b^5) +
(3*Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b + Sqrt[-a]*x), x])/(16*b^5) +
((-a)^(3/2)*Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(Sqrt[-a]*b - a*x)^3, x]
)/(8*b^3) - (3*a*Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(Sqrt[-a]*b - a*x)^
2, x])/(16*b^4) + ((-a)^(3/2)*Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(Sqrt[
-a]*b + a*x)^3, x])/(8*b^3) - (3*a*Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(
Sqrt[-a]*b + a*x)^2, x])/(16*b^4)
```

Rubi steps

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^3}\,dx=\int\left(-\frac{a^3\sqrt{b+\sqrt{b^2+ax^2}}}{8(-a)^{3/2}b^3\left(\sqrt{-a}\,b-ax\right)^3}-\frac{3a\sqrt{b+\sqrt{b^2+ax^2}}}{16b^4\left(\sqrt{-a}\,b-ax\right)^2}-\frac{a^3\sqrt{b+\sqrt{b^2+ax^2}}}{8(-a)^{3/2}b^3\left(\sqrt{-a}\,b+ax\right)^3}-\frac{3a}{16\cdots}\right.$$

$$=-\frac{(3a)\int\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(\sqrt{-a}\,b-ax\right)^2}dx}{16b^4}-\frac{(3a)\int\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(\sqrt{-a}\,b+ax\right)^2}dx}{16b^4}-\frac{(3a)\int\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{-ab^2-a^2x^2}dx}{8b^4}+\frac{(-a)^{3/2}\int\cdots}{8\cdots}$$

$$=-\frac{(3a)\int\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(\sqrt{-a}\,b-ax\right)^2}dx}{16b^4}-\frac{(3a)\int\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(\sqrt{-a}\,b+ax\right)^2}dx}{16b^4}-\frac{(3a)\int\left(-\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{2ab\left(b-\sqrt{-a}\,x\right)}-\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{2ab\left(b+\sqrt{-a}\,x\right)}\cdots\right.}{8b^4}$$

$$=\frac{3\int\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b-\sqrt{-a}\,x}dx}{16b^5}+\frac{3\int\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b+\sqrt{-a}\,x}dx}{16b^5}-\frac{(3a)\int\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(\sqrt{-a}\,b-ax\right)^2}dx}{16b^4}-\frac{(3a)\int\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(\sqrt{-a}\,b+ax\right)^2}\cdots}{16b^4}$$

Mathematica [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^3,x]
[Out] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^3, x]
```

IntegrateAlgebraic [A] time = 0.33, size = 149, normalized size = 1.00

$$\frac{35\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{64\sqrt{a}\,b^{9/2}}+\frac{7x\left(15ax^2+23b^2\right)}{192b^4\left(ax^2+b^2\right)^{3/2}\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{x\left(35ax^2+59b^2\right)}{96b^3\left(ax^2+b^2\right)^2\sqrt{\sqrt{ax^2+b^2}+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^3,x]
[Out] (7*x*(23*b^2 + 15*a*x^2))/(192*b^4*(b^2 + a*x^2)^(3/2)*Sqrt[b + Sqrt[b^2 +
a*x^2]]) + (x*(59*b^2 + 35*a*x^2))/(96*b^3*(b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^
2 + a*x^2]]) + (35*ArcTan[(Sqrt[a]*x)/(Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])
])/(64*Sqrt[a]*b^(9/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^3,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\left(ax^2+b^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^3,x, algorithm="giac")
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 + b^2)^3, x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{\left(a\,x^2+b^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^3,x)
[Out] int((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\left(ax^2+b^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 + b^2)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b+\sqrt{b^2+a\,x^2}}}{\left(b^2+a\,x^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(a*x^2 + b^2)^3,x)
[Out] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(a*x^2 + b^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\left(ax^2+b^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x**2+b**2)**(1/2))**(1/2)/(a*x**2+b**2)**3,x)
[Out] Integral(sqrt(b + sqrt(a*x**2 + b**2))/(a*x**2 + b**2)**3, x)
```

3.1722 $\int \frac{\left(-b+ax^2\right)^{3/4}}{x^3}\,dx$

Optimal. Leaf size=150

$$-\frac{\left(ax^2-b\right)^{3/4}}{2x^2}-\frac{3a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}-\sqrt{b}}\right)}{4\sqrt{2}\sqrt[4]{b}}-\frac{3a\tanh^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^2-b}}\right)}{4\sqrt{2}\sqrt[4]{b}}$$

Rubi [A] time = 0.23, antiderivative size = 225, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 47, 63, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\left(ax^2-b\right)^{3/4}}{2x^2}+\frac{3a\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)}{8\sqrt{2}\sqrt[4]{b}}-\frac{3a\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)}{8\sqrt{2}\sqrt[4]{b}}-\frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}}+\frac{3a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^2)^(3/4)/x^3,x]
```

```
[Out] -1/2*(-b + a*x^2)^(3/4)/x^2 - (3*a*ArcTan[1 - (Sqrt[2]*(-b + a*x^2)^(1/4))/
b^(1/4)])/(4*Sqrt[2]*b^(1/4)) + (3*a*ArcTan[1 + (Sqrt[2]*(-b + a*x^2)^(1/4)
)/b^(1/4)])/(4*Sqrt[2]*b^(1/4)) + (3*a*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b +
a*x^2)^(1/4) + Sqrt[-b + a*x^2]])/(8*Sqrt[2]*b^(1/4)) - (3*a*Log[Sqrt[b] +
Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4) + Sqrt[-b + a*x^2]])/(8*Sqrt[2]*b^(1/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(-b+ax^2\right)^{3/4}}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(-b+ax)^{3/4}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{\left(-b+ax^2\right)^{3/4}}{2x^2} + \frac{1}{8}(3a)\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{-b+ax}}\,dx, x, x^2\right)$$

$$= -\frac{\left(-b+ax^2\right)^{3/4}}{2x^2} + \frac{3}{2}\,\text{Subst}\left(\int \frac{x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^2}\right)$$

$$= -\frac{\left(-b+ax^2\right)^{3/4}}{2x^2} - \frac{3}{4}\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^2}\right) + \frac{3}{4}\,\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{\ldots}\right.$$

$$= -\frac{\left(-b+ax^2\right)^{3/4}}{2x^2} + \frac{1}{8}(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{2}\sqrt[4]{b}\,x+x^2}\,dx, x, \sqrt[4]{-b+ax^2}\right) + \frac{1}{8}(3a)\,\text{Subst}\left(\ldots\right.$$

$$= -\frac{\left(-b+ax^2\right)^{3/4}}{2x^2} + \frac{3a\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^2}+\sqrt{-b+ax^2}\right)}{8\sqrt{2}\sqrt[4]{b}} - \frac{3a\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\ldots\right.}{8\ldots}$$

$$= -\frac{\left(-b+ax^2\right)^{3/4}}{2x^2} - \frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}} + \frac{3a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}} + \frac{3a\log\left(\sqrt{b}-\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.27

$$\frac{2a\left(ax^2-b\right)^{7/4}\,{}_2F_1\left(\frac{7}{4},2;\frac{11}{4};1-\frac{ax^2}{b}\right)}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^2)^(3/4)/x^3,x]
```

```
[Out] (2*a*(-b + a*x^2)^(7/4)*Hypergeometric2F1[7/4, 2, 11/4, 1 - (a*x^2)/b])/(7*
b^2)
```

IntegrateAlgebraic [A] time = 0.36, size = 149, normalized size = 0.99

$$-\frac{\left(ax^2-b\right)^{3/4}}{2x^2}+\frac{3a\tan^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^2-b}}\right)}{4\sqrt{2}\sqrt[4]{b}}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}+\sqrt{b}}\right)}{4\sqrt{2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^2)^(3/4)/x^3,x]
```

```
[Out] -1/2*(-b + a*x^2)^(3/4)/x^2 + (3*a*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a
*x^2]/(Sqrt[2]*b^(1/4)))/(-b + a*x^2)^(1/4)])/(4*Sqrt[2]*b^(1/4)) - (3*a*Ar
cTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^2])])/(
4*Sqrt[2]*b^(1/4))
```

fricas [A] time = 0.42, size = 204, normalized size = 1.36

$$-\frac{12\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}x^2\arctan\left(-\frac{\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}\left(ax^2-b\right)^{\frac{1}{4}}a^3-\sqrt{\sqrt{ax^2-b}\,a^6-\sqrt{-\frac{a^4}{b}}\,a^4b}\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}}{a^4}\right)-3\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}x^2\log\left(27\left(ax^2-b\right)^{\frac{1}{4}}a^3+27\left(-\frac{a^4}{b}\right)^{\frac{3}{4}}b\right)+3\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}x^2\log\left(27\left(ax^2-b\right)^{\frac{1}{4}}a^3-27\left(-\frac{a^4}{b}\right)^{\frac{3}{4}}b\right)+4\left(ax^2-b\right)^{\frac{3}{4}}}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)^(3/4)/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(12*(-a^4/b)^(1/4)*x^2*arctan(-((-a^4/b)^(1/4)*(a*x^2 - b)^(1/4)*a^3 -
 sqrt(sqrt(a*x^2 - b)*a^6 - sqrt(-a^4/b)*a^4*b)*(-a^4/b)^(1/4))/a^4) - 3*(-
a^4/b)^(1/4)*x^2*log(27*(a*x^2 - b)^(1/4)*a^3 + 27*(-a^4/b)^(3/4)*b) + 3*(-
a^4/b)^(1/4)*x^2*log(27*(a*x^2 - b)^(1/4)*a^3 - 27*(-a^4/b)^(3/4)*b) + 4*(a
*x^2 - b)^(3/4))/x^2
```

giac [A] time = 0.17, size = 196, normalized size = 1.31

$$\frac{\frac{6\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{6\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{3\sqrt{2}a^2\log\left(\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}+\frac{3\sqrt{2}a^2\log\left(-\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}-\frac{8\left(ax^2-b\right)^{\frac{3}{4}}a}{x^2}}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)^(3/4)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(6*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^2 - b)^(1/
4))/b^(1/4))/b^(1/4) + 6*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
 2*(a*x^2 - b)^(1/4))/b^(1/4))/b^(1/4) - 3*sqrt(2)*a^2*log(sqrt(2)*(a*x^2 -
 b)^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(1/4) + 3*sqrt(2)*a^2*log(
-sqrt(2)*(a*x^2 - b)^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(1/4) - 8
*(a*x^2 - b)^(3/4)*a/x^2)/a
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{2}-b\right)^{\frac{3}{4}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)^(3/4)/x^3,x)
```

```
[Out] int((a*x^2-b)^(3/4)/x^3,x)
```

maxima [A] time = 0.41, size = 181, normalized size = 1.21

$$\frac{3}{16}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^{2}-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^{2}-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^{2}-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^{2}-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^{2}-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^{2}-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}\right)a-\frac{\left(ax^{2}-b\right)^{\frac{3}{4}}}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)^(3/4)/x^3,x, algorithm="maxima")
```

```
[Out] 3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^2 - b)^(1/4))/
b^(1/4))/b^(1/4) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^
2 - b)^(1/4))/b^(1/4))/b^(1/4) - sqrt(2)*log(sqrt(2)*(a*x^2 - b)^(1/4)*b^(1
/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(1/4) + sqrt(2)*log(-sqrt(2)*(a*x^2 - b)
^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(1/4))*a - 1/2*(a*x^2 - b)^(3
/4)/x^2
```

mupad [B] time = 1.17, size = 69, normalized size = 0.46

$$\frac{3\,a\,\mathrm{atan}\left(\frac{\left(a\,x^2-b\right)^{1/4}}{(-b)^{1/4}}\right)}{4\,(-b)^{1/4}}-\frac{\left(a\,x^2-b\right)^{3/4}}{2\,x^2}-\frac{3\,a\,\mathrm{atanh}\left(\frac{\left(a\,x^2-b\right)^{1/4}}{(-b)^{1/4}}\right)}{4\,(-b)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 - b)^(3/4)/x^3,x)
```

```
[Out] (3*a*atan((a*x^2 - b)^(1/4)/(-b)^(1/4)))/(4*(-b)^(1/4)) - (a*x^2 - b)^(3/4)
/(2*x^2) - (3*a*atanh((a*x^2 - b)^(1/4)/(-b)^(1/4)))/(4*(-b)^(1/4))
```

sympy [C] time = 1.18, size = 44, normalized size = 0.29

$$-\frac{a^{\frac{3}{4}}\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^{2}}\right)}{2\sqrt{x}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)**(3/4)/x**3,x)
```

```
[Out] -a**(3/4)*gamma(1/4)*hyper((-3/4, 1/4), (5/4,), b*exp_polar(2*I*pi)/(a*x**2
))/(2*sqrt(x)*gamma(5/4))
```

3.1723 $\int \frac{(1+x^3)(-1+2x^3)^{2/3}}{x^6(1+2x^3)}\,dx$

Optimal. Leaf size=150

$$\frac{2}{3}\sqrt[3]{2}\log\left(\sqrt[3]{2}\sqrt[3]{2x^3-1}-2x\right)-\frac{2\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt[3]{2}\sqrt[3]{2x^3-1}+x}\right)}{\sqrt{3}}+\frac{\left(2x^3-1\right)^{2/3}\left(9x^3-2\right)}{10x^5}-\frac{1}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}\sqrt[3]{2x^3-1}\,x\right.$$

Rubi [A] time = 0.18, antiderivative size = 159, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {580, 583, 12, 377, 200, 31, 634, 617, 204, 628}

$$\frac{2}{3}\sqrt[3]{2}\log\left(1-\frac{2^{2/3}x}{\sqrt[3]{2x^3-1}}\right)-\frac{2\sqrt[3]{2}\tan^{-1}\left(\frac{\frac{2\,2^{2/3}x}{\sqrt[3]{2x^3-1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\left(2x^3-1\right)^{2/3}}{5x^5}+\frac{9\left(2x^3-1\right)^{2/3}}{10x^2}-\frac{1}{3}\sqrt[3]{2}\log\left(\frac{2^{2/3}x}{\sqrt[3]{2x^3-1}}+\frac{2\sqrt[3]{2}x^2}{\left(2x^3-1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^3)*(-1 + 2*x^3)^(2/3))/(x^6*(1 + 2*x^3)),x]
```

```
[Out] -1/5*(-1 + 2*x^3)^(2/3)/x^5 + (9*(-1 + 2*x^3)^(2/3))/(10*x^2) - (2*2^(1/3)*
ArcTan[(1 + (2*2^(2/3)*x)/(-1 + 2*x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + (2*2^(1/3
)*Log[1 - (2^(2/3)*x)/(-1 + 2*x^3)^(1/3)])/3 - (2^(1/3)*Log[1 + (2*2^(1/3)*
x^2)/(-1 + 2*x^3)^(2/3) + (2^(2/3)*x)/(-1 + 2*x^3)^(1/3)])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(1+x^3\right)\left(-1+2x^3\right)^{2/3}}{x^6\left(1+2x^3\right)}\,dx = -\frac{\left(-1+2x^3\right)^{2/3}}{5x^5}+\frac{1}{5}\int \frac{9-2x^3}{x^3\sqrt[3]{-1+2x^3}\left(1+2x^3\right)}\,dx$$
$$= -\frac{\left(-1+2x^3\right)^{2/3}}{5x^5}+\frac{9\left(-1+2x^3\right)^{2/3}}{10x^2}+\frac{1}{10}\int -\frac{40}{\sqrt[3]{-1+2x^3}\left(1+2x^3\right)}\,dx$$
$$= -\frac{\left(-1+2x^3\right)^{2/3}}{5x^5}+\frac{9\left(-1+2x^3\right)^{2/3}}{10x^2}-4\int \frac{1}{\sqrt[3]{-1+2x^3}\left(1+2x^3\right)}\,dx$$
$$= -\frac{\left(-1+2x^3\right)^{2/3}}{5x^5}+\frac{9\left(-1+2x^3\right)^{2/3}}{10x^2}-4\,\mathrm{Subst}\left(\int \frac{1}{1-4x^3}\,dx,x,\frac{x}{\sqrt[3]{-1+2x^3}}\right)$$
$$= -\frac{\left(-1+2x^3\right)^{2/3}}{5x^5}+\frac{9\left(-1+2x^3\right)^{2/3}}{10x^2}-\frac{4}{3}\mathrm{Subst}\left(\int \frac{1}{1-2^{2/3}x}\,dx,x,\frac{x}{\sqrt[3]{-1+2x^3}}\right)-$$
$$= -\frac{\left(-1+2x^3\right)^{2/3}}{5x^5}+\frac{9\left(-1+2x^3\right)^{2/3}}{10x^2}+\frac{2}{3}\sqrt[3]{2}\log\left(1-\frac{2^{2/3}x}{\sqrt[3]{-1+2x^3}}\right)-2\,\mathrm{Subst}\left(\int\right.$$
$$= -\frac{\left(-1+2x^3\right)^{2/3}}{5x^5}+\frac{9\left(-1+2x^3\right)^{2/3}}{10x^2}+\frac{2}{3}\sqrt[3]{2}\log\left(1-\frac{2^{2/3}x}{\sqrt[3]{-1+2x^3}}\right)-\frac{1}{3}\sqrt[3]{2}\log\left(1\right.$$
$$= -\frac{\left(-1+2x^3\right)^{2/3}}{5x^5}+\frac{9\left(-1+2x^3\right)^{2/3}}{10x^2}-\frac{2\sqrt[3]{2}\tan^{-1}\left(\frac{1+\frac{2\,2^{2/3}x}{\sqrt[3]{-1+2x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2}{3}\sqrt[3]{2}\log\left(1-\right.$$

Mathematica [A] time = 0.17, size = 139, normalized size = 0.93

$$\left(\frac{9}{10x^2}-\frac{1}{5x^5}\right)\left(2x^3-1\right)^{2/3}-\frac{1}{3}\sqrt[3]{2}\left(-2\log\left(1-\frac{2^{2/3}x}{\sqrt[3]{2-x^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\,2^{2/3}x}{\sqrt[3]{2-x^3}}+1}{\sqrt{3}}\right)+\log\left(\frac{2^{2/3}x}{\sqrt[3]{2-x^3}}+\frac{2\sqrt[3]{2}x^2}{\left(2-x^3\right)^{2/3}}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + x^3)*(-1 + 2*x^3)^(2/3))/(x^6*(1 + 2*x^3)),x]
```

```
[Out] (-1/5*1/x^5 + 9/(10*x^2))*(-1 + 2*x^3)^(2/3) - (2^(1/3)*(2*Sqrt[3]*ArcTan[(
1 + (2*2^(2/3)*x)/(2 - x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (2^(2/3)*x)/(2 - x^
3)^(1/3)] + Log[1 + (2*2^(1/3)*x^2)/(2 - x^3)^(2/3) + (2^(2/3)*x)/(2 - x^3)
^(1/3)]))/3
```

IntegrateAlgebraic [A] time = 0.38, size = 150, normalized size = 1.00

$$\frac{2}{3}\sqrt[3]{2}\log\left(\sqrt[3]{2}\sqrt[3]{2x^3-1}-2x\right)-\frac{2\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt[3]{2}\sqrt[3]{2x^3-1}+x}\right)}{\sqrt{3}}+\frac{\left(2x^3-1\right)^{2/3}\left(9x^3-2\right)}{10x^5}-\frac{1}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}\sqrt[3]{2x^3-1}x+2^{2/3}\left(2x^3-1\right)^{2/3}+4x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)*(-1 + 2*x^3)^(2/3))/(x^6*(1 + 2*x^3)),x]
```

```
[Out] ((-1 + 2*x^3)^(2/3)*(-2 + 9*x^3))/(10*x^5) - (2*2^(1/3)*ArcTan[(Sqrt[3]*x)/
(x + 2^(1/3)*(-1 + 2*x^3)^(1/3))])/Sqrt[3] + (2*2^(1/3)*Log[-2*x + 2^(1/3)*
(-1 + 2*x^3)^(1/3)])/3 - (2^(1/3)*Log[4*x^2 + 2*2^(1/3)*x*(-1 + 2*x^3)^(1/3
) + 2^(2/3)*(-1 + 2*x^3)^(2/3)])/3
```

fricas [B] time = 2.60, size = 290, normalized size = 1.93

$$-\frac{20\,\sqrt{3}\,2^{\frac{1}{3}}x^{5}\arctan\left(\frac{6\,\sqrt{3}\,2^{\frac{2}{3}}\left(20\,x^{7}+8\,x^{4}-x\right)\left(2\,x^{3}-1\right)^{\frac{2}{3}}-12\,\sqrt{3}\,2^{\frac{1}{3}}\left(76\,x^{8}-32\,x^{5}+x^{2}\right)\left(2\,x^{3}-1\right)^{\frac{1}{3}}-\sqrt{3}\left(568\,x^{9}-444\,x^{6}+66\,x^{3}-1\right)}{3\left(872\,x^{9}-420\,x^{6}+6\,x^{3}+1\right)}\right)-20\cdot 2^{\frac{1}{3}}x^{5}\log\left(-\frac{6\cdot 2^{\frac{2}{3}}\left(2\,x^{3}-1\right)^{\frac{1}{3}}x^{2}-6\left(2\,x^{3}-1\right)^{\frac{2}{3}}x-2^{\frac{1}{3}}\left(2\,x^{3}+1\right)}{2\,x^{3}+1}\right)+10\cdot 2^{\frac{1}{3}}x^{5}\log\left(\frac{6\cdot 2^{\frac{1}{3}}\left(10\,x^{4}-x\right)\left(2\,x^{3}-1\right)^{\frac{2}{3}}+2^{\frac{2}{3}}\left(76\,x^{6}-32\,x^{3}+1\right)+24\left(4\,x^{5}-x^{2}\right)\left(2\,x^{3}-1\right)^{\frac{1}{3}}}{4\,x^{6}+4\,x^{3}+1}\right)-9\left(9\,x^{3}-2\right)\left(2\,x^{3}-1\right)^{\frac{2}{3}}}{90\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(2*x^3-1)^(2/3)/x^6/(2*x^3+1),x, algorithm="fricas")
```

```
[Out] -1/90*(20*sqrt(3)*2^(1/3)*x^5*arctan(1/3*(6*sqrt(3)*2^(2/3)*(20*x^7 + 8*x^4
 - x)*(2*x^3 - 1)^(2/3) - 12*sqrt(3)*2^(1/3)*(76*x^8 - 32*x^5 + x^2)*(2*x^3
 - 1)^(1/3) - sqrt(3)*(568*x^9 - 444*x^6 + 66*x^3 - 1))/(872*x^9 - 420*x^6
+ 6*x^3 + 1)) - 20*2^(1/3)*x^5*log(-(6*2^(2/3)*(2*x^3 - 1)^(1/3)*x^2 - 6*(2
*x^3 - 1)^(2/3)*x - 2^(1/3)*(2*x^3 + 1))/(2*x^3 + 1)) + 10*2^(1/3)*x^5*log(
(6*2^(1/3)*(10*x^4 - x)*(2*x^3 - 1)^(2/3) + 2^(2/3)*(76*x^6 - 32*x^3 + 1) +
 24*(4*x^5 - x^2)*(2*x^3 - 1)^(1/3))/(4*x^6 + 4*x^3 + 1)) - 9*(9*x^3 - 2)*(
2*x^3 - 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{3}-1\right)^{\frac{2}{3}}\left(x^{3}+1\right)}{\left(2\,x^{3}+1\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(2*x^3-1)^(2/3)/x^6/(2*x^3+1),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 - 1)^(2/3)*(x^3 + 1)/((2*x^3 + 1)*x^6), x)
```

maple [C] time = 2.84, size = 631, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)*(2*x^3-1)^(2/3)/x^6/(2*x^3+1),x)
```

```
[Out] 1/10*(18*x^6-13*x^3+2)/x^5/(2*x^3-1)^(1/3)+4*RootOf(RootOf(_Z^3-2)^2+6*_Z*R
ootOf(_Z^3-2)+36*_Z^2)*ln((144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+
36*_Z^2)^2*RootOf(_Z^3-2)^2*x^3+36*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3
-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^3-24*(2*x^3-1)^(2/3)*RootOf(RootOf(_Z^3-2)^
2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x+60*(2*x^3-1)^(1/3)*RootOf
(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^2+8*RootOf(
_Z^3-2)^2*(2*x^3-1)^(1/3)*x^2-24*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2
)+36*_Z^2)*x^3-6*RootOf(_Z^3-2)*x^3+2*(2*x^3-1)^(2/3)*x+12*RootOf(RootOf(_Z
^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)+3*RootOf(_Z^3-2))/(2*x^3+1))+2/3*RootO
f(_Z^3-2)*ln(-(108*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*R
ootOf(_Z^3-2)^2*x^3+12*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)
*RootOf(_Z^3-2)^3*x^3-12*(2*x^3-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootO
f(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x-6*(2*x^3-1)^(1/3)*RootOf(RootOf(_Z^3-
2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^2+4*RootOf(_Z^3-2)^2*(2*
x^3-1)^(1/3)*x^2+54*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^
3+6*RootOf(_Z^3-2)*x^3-5*(2*x^3-1)^(2/3)*x-9*RootOf(RootOf(_Z^3-2)^2+6*_Z*R
ootOf(_Z^3-2)+36*_Z^2)-RootOf(_Z^3-2))/(2*x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{3}-1\right)^{\frac{2}{3}}\left(x^{3}+1\right)}{\left(2\,x^{3}+1\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(2*x^3-1)^(2/3)/x^6/(2*x^3+1),x, algorithm="maxima")
[Out] integrate((2*x^3 - 1)^(2/3)*(x^3 + 1)/((2*x^3 + 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3+1\right)\,\left(2\,x^3-1\right)^{2/3}}{x^6\,\left(2\,x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)*(2*x^3 - 1)^(2/3))/(x^6*(2*x^3 + 1)),x)
[Out] int(((x^3 + 1)*(2*x^3 - 1)^(2/3))/(x^6*(2*x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)\left(2x^3-1\right)^{\frac{2}{3}}\left(x^2-x+1\right)}{x^6\left(2x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)*(2*x**3-1)**(2/3)/x**6/(2*x**3+1),x)
[Out] Integral((x + 1)*(2*x**3 - 1)**(2/3)*(x**2 - x + 1)/(x**6*(2*x**3 + 1)), x)
```

3.1724 $\int \frac{(1-x^3)^{2/3}(-1+4x^3)}{x^6(-2+3x^3)}\,dx$

Optimal. Leaf size=150

$$\frac{5\log\left(\sqrt[3]{2}\sqrt[3]{1-x^3}-x\right)}{12\ 2^{2/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{1-x^3}+x}\right)}{4\ 2^{2/3}\sqrt{3}}+\frac{(1-x^3)^{2/3}(29x^3-4)}{40x^5}-\frac{5\log\left(\sqrt[3]{2}\sqrt[3]{1-x^3}x+2^{2/3}(1-x^3)^{2/3}+\right.}{24\ 2^{2/3}}$$

Rubi [A] time = 0.16, antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {580, 583, 12, 377, 200, 31, 634, 617, 204, 628}

$$\frac{5\log\left(\sqrt[3]{2}-\frac{x}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}}-\frac{5\tan^{-1}\left(\frac{\frac{2^{2/3}x}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{4\ 2^{2/3}\sqrt{3}}-\frac{(1-x^3)^{2/3}}{10x^5}+\frac{29(1-x^3)^{2/3}}{40x^2}-\frac{5\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+\frac{x^2}{(1-x^3)^{2/3}}+2^{2/3}\right)}{24\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x^3)^(2/3)*(-1 + 4*x^3))/(x^6*(-2 + 3*x^3)),x]
```

```
[Out] -1/10*(1 - x^3)^(2/3)/x^5 + (29*(1 - x^3)^(2/3))/(40*x^2) - (5*ArcTan[(1 +
(2^(2/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(4*2^(2/3)*Sqrt[3]) + (5*Log[2^(1/3)
 - x/(1 - x^3)^(1/3)])/(12*2^(2/3)) - (5*Log[2^(2/3) + x^2/(1 - x^3)^(2/3)
+ (2^(1/3)*x)/(1 - x^3)^(1/3)])/(24*2^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}(-1+4x^3)}{x^6(-2+3x^3)}\,dx = -\frac{(1-x^3)^{2/3}}{10x^5} - \frac{1}{10}\int \frac{-29+31x^3}{x^3\sqrt[3]{1-x^3}\,(-2+3x^3)}\,dx$$

$$= -\frac{(1-x^3)^{2/3}}{10x^5} + \frac{29(1-x^3)^{2/3}}{40x^2} - \frac{1}{40}\int -\frac{50}{\sqrt[3]{1-x^3}\,(-2+3x^3)}\,dx$$

$$= -\frac{(1-x^3)^{2/3}}{10x^5} + \frac{29(1-x^3)^{2/3}}{40x^2} + \frac{5}{4}\int \frac{1}{\sqrt[3]{1-x^3}\,(-2+3x^3)}\,dx$$

$$= -\frac{(1-x^3)^{2/3}}{10x^5} + \frac{29(1-x^3)^{2/3}}{40x^2} + \frac{5}{4}\,\text{Subst}\left(\int \frac{1}{-2+x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{(1-x^3)^{2/3}}{10x^5} + \frac{29(1-x^3)^{2/3}}{40x^2} + \frac{5\,\text{Subst}\left(\int \frac{1}{-\sqrt[3]{2}+x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}} + \frac{5\,\text{Subst}\left(\int \frac{-}{2^{2/3}}\right.}{}$$

$$= -\frac{(1-x^3)^{2/3}}{10x^5} + \frac{29(1-x^3)^{2/3}}{40x^2} + \frac{5\log\left(\sqrt[3]{2} - \frac{x}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}} - \frac{5\,\text{Subst}\left(\int \frac{\sqrt[3]{2}+2x}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx,\right.}{24\ 2^{2/3}}$$

$$= -\frac{(1-x^3)^{2/3}}{10x^5} + \frac{29(1-x^3)^{2/3}}{40x^2} + \frac{5\log\left(\sqrt[3]{2} - \frac{x}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}} - \frac{5\log\left(2^{2/3} + \frac{x^2}{(1-x^3)^{2/3}} + \frac{\sqrt[3]{}}{\sqrt[3]{1}}\right.}{24\ 2^{2/3}}$$

$$= -\frac{(1-x^3)^{2/3}}{10x^5} + \frac{29(1-x^3)^{2/3}}{40x^2} - \frac{5\tan^{-1}\left(\frac{1+\frac{2^{2/3}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\ 2^{2/3}\sqrt{3}} + \frac{5\log\left(\sqrt[3]{2} - \frac{x}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}} - \frac{5\log}{}$$

Mathematica [A] time = 0.18, size = 129, normalized size = 0.86

$$\left(\frac{29}{40x^2} - \frac{1}{10x^5}\right)(1-x^3)^{2/3} - \frac{5\left(-2\log\left(2 - \frac{2^{2/3}x}{\sqrt[3]{x^3-1}}\right) + 2\sqrt{3}\tan^{-1}\left(\frac{\frac{2^{2/3}x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right) + \log\left(\frac{2^{2/3}x}{\sqrt[3]{x^3-1}} + \frac{\sqrt[3]{2}x^2}{(x^3-1)^{2/3}} + 2\right)\right)}{24\ 2^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - x^3)^(2/3)*(-1 + 4*x^3))/(x^6*(-2 + 3*x^3)),x]
```

```
[Out] (-1/10*1/x^5 + 29/(40*x^2))*(1 - x^3)^(2/3) - (5*(2*Sqrt[3]*ArcTan[(1 + (2^
(2/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] - 2*Log[2 - (2^(2/3)*x)/(-1 + x^3)^(1/3
)] + Log[2 + (2^(1/3)*x^2)/(-1 + x^3)^(2/3) + (2^(2/3)*x)/(-1 + x^3)^(1/3)]
))/(24*2^(2/3))
```

IntegrateAlgebraic [A] time = 0.34, size = 150, normalized size = 1.00

$$\frac{5\log\left(\sqrt[3]{2}\sqrt[3]{1-x^3} - x\right)}{12\ 2^{2/3}} - \frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{1-x^3}+x}\right)}{4\ 2^{2/3}\sqrt{3}} + \frac{(1-x^3)^{2/3}(29x^3-4)}{40x^5} - \frac{5\log\left(\sqrt[3]{2}\sqrt[3]{1-x^3}\,x + 2^{2/3}(1-x^3)^{2/3} + x^2\right)}{24\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 - x^3)^(2/3)*(-1 + 4*x^3))/(x^6*(-2 + 3*x^3)),x]
```

```
[Out] ((1 - x^3)^(2/3)*(-4 + 29*x^3))/(40*x^5) - (5*ArcTan[(Sqrt[3]*x)/(x + 2*2^(
1/3)*(1 - x^3)^(1/3))])/(4*2^(2/3)*Sqrt[3]) + (5*Log[-x + 2^(1/3)*(1 - x^3)
^(1/3)])/(12*2^(2/3)) - (5*Log[x^2 + 2^(1/3)*x*(1 - x^3)^(1/3) + 2^(2/3)*(1
 - x^3)^(2/3)])/(24*2^(2/3))
```

fricas [B] time = 2.92, size = 278, normalized size = 1.85

$$\frac{100 \cdot 4^{\frac{1}{6}} \sqrt{3}\, x^5 \arctan\left(\frac{4^{\frac{1}{6}}\left(12 \cdot 4^{\frac{2}{3}} \sqrt{3}\left(3 x^4-2 x\right)\left(-x^3+1\right)^{\frac{2}{3}}-4^{\frac{1}{3}} \sqrt{3}\left(27 x^9-72 x^6+36 x^3+8\right)+12 \sqrt{3}\left(9 x^8-6 x^5-4 x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6\left(27 x^9-36 x^3+8\right)}\right)+50 \cdot 4^{\frac{2}{3}} x^5 \log\left(-\frac{6 \cdot 4^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}} x^2-4^{\frac{2}{3}}\left(3 x^3-2\right)-12\left(-x^3+1\right)^{\frac{2}{3}} x}{3 x^3-2}\right)-25 \cdot 4^{\frac{2}{3}} x^5 \log\left(\frac{6 \cdot 4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{2}{3}} x-4^{\frac{1}{3}}\left(9 x^6-6 x^3-4\right)-6\left(3 x^5-4 x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}}{9 x^6-12 x^3+4}\right)+36\left(29 x^3-4\right)\left(-x^3+1\right)^{\frac{2}{3}}}{1440 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)*(4*x^3-1)/x^6/(3*x^3-2),x, algorithm="fricas")
```

```
[Out] 1/1440*(100*4^(1/6)*sqrt(3)*x^5*arctan(1/6*4^(1/6)*(12*4^(2/3)*sqrt(3)*(3*x
^4 - 2*x)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(27*x^9 - 72*x^6 + 36*x^3 + 8)
 + 12*sqrt(3)*(9*x^8 - 6*x^5 - 4*x^2)*(-x^3 + 1)^(1/3))/(27*x^9 - 36*x^3 +
8)) + 50*4^(2/3)*x^5*log(-(6*4^(1/3)*(-x^3 + 1)^(1/3)*x^2 - 4^(2/3)*(3*x^3
- 2) - 12*(-x^3 + 1)^(2/3)*x)/(3*x^3 - 2)) - 25*4^(2/3)*x^5*log((6*4^(2/3)*
(-x^3 + 1)^(2/3)*x - 4^(1/3)*(9*x^6 - 6*x^3 - 4) - 6*(3*x^5 - 4*x^2)*(-x^3
+ 1)^(1/3))/(9*x^6 - 12*x^3 + 4)) + 36*(29*x^3 - 4)*(-x^3 + 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4 x^3-1\right)\left(-x^3+1\right)^{\frac{2}{3}}}{\left(3 x^3-2\right) x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)*(4*x^3-1)/x^6/(3*x^3-2),x, algorithm="giac")
```

```
[Out] integrate((4*x^3 - 1)*(-x^3 + 1)^(2/3)/((3*x^3 - 2)*x^6), x)
```

maple [C] time = 2.67, size = 906, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)*(4*x^3-1)/x^6/(3*x^3-2),x)
```

```
[Out] -1/40*(29*x^6-33*x^3+4)/x^5/(-x^3+1)^(1/3)-5/24*ln((3*RootOf(RootOf(_Z^3-2)
^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^3+36*RootOf(RootOf(_Z^3-
2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-18*(-x^3+1)^(2/3)*
RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x-3*R
ootOf(_Z^3-2)^2*(-x^3+1)^(1/3)*x^2-18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^
2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^2+2*RootOf(_Z^3-2)*x^3+24*R
ootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3-6*(-x^3+1)^(2/3)*x-
2*RootOf(_Z^3-2)-24*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2))/(
3*x^3-2))*RootOf(_Z^3-2)-5/4*ln((3*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3
-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^3+36*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z
^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-18*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-
2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x-3*RootOf(_Z^3-2)^2*(-x
^3+1)^(1/3)*x^2-18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-
2)+36*_Z^2)*RootOf(_Z^3-2)*x^2+2*RootOf(_Z^3-2)*x^3+24*RootOf(RootOf(_Z^3-2
)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3-6*(-x^3+1)^(2/3)*x-2*RootOf(_Z^3-2)-24
*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2))/(3*x^3-2))*RootOf(Ro
otOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)+5/4*RootOf(RootOf(_Z^3-2)^2+6*_
Z*RootOf(_Z^3-2)+36*_Z^2)*ln((3*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)
+36*_Z^2)*RootOf(_Z^3-2)^3*x^3+36*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-
2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^3+18*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^
2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x+18*(-x^3+1)^(1/3)*RootOf(
RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^2-RootOf(_Z^
3-2)*x^3-12*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+2*Root
Of(_Z^3-2)+24*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2))/(3*x^3-
2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^3-1\right)\left(-x^3+1\right)^{\frac{2}{3}}}{\left(3\,x^3-2\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)*(4*x^3-1)/x^6/(3*x^3-2),x, algorithm="maxima")
[Out] integrate((4*x^3 - 1)*(-x^3 + 1)^(2/3)/((3*x^3 - 2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-x^3\right)^{2/3}\,\left(4\,x^3-1\right)}{x^6\,\left(3\,x^3-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - x^3)^(2/3)*(4*x^3 - 1))/(x^6*(3*x^3 - 2)),x)
[Out] int(((1 - x^3)^(2/3)*(4*x^3 - 1))/(x^6*(3*x^3 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(4x^3-1\right)}{x^6\left(3x^3-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)*(4*x**3-1)/x**6/(3*x**3-2),x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)*(4*x**3 - 1)/(x**6*(3*x**3 - 2)),
 x)
```

3.1725 $\int \frac{\sqrt[4]{-b+ax^3}}{x^4} dx$

Optimal. Leaf size=150

$$-\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}-\sqrt{b}}\right)}{6\sqrt{2}b^{3/4}}+\frac{a\tanh^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{6\sqrt{2}b^{3/4}}-\frac{\sqrt[4]{ax^3-b}}{3x^3}$$

Rubi [A] time = 0.22, antiderivative size = 225, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 47, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{12\sqrt{2}b^{3/4}}+\frac{a\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{12\sqrt{2}b^{3/4}}-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}\right)}{6\sqrt{2}b^{3/4}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}+1\right)}{6\sqrt{2}b^{3/4}}-\frac{\sqrt[4]{ax^3-b}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^3)^(1/4)/x^4,x]
```

```
[Out] -1/3*(-b + a*x^3)^(1/4)/x^3 - (a*ArcTan[1 - (Sqrt[2]*(-b + a*x^3)^(1/4))/b^
(1/4)])/(6*Sqrt[2]*b^(3/4)) + (a*ArcTan[1 + (Sqrt[2]*(-b + a*x^3)^(1/4))/b^
(1/4)])/(6*Sqrt[2]*b^(3/4)) - (a*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^3)
^(1/4) + Sqrt[-b + a*x^3]])/(12*Sqrt[2]*b^(3/4)) + (a*Log[Sqrt[b] + Sqrt[2]
*b^(1/4)*(-b + a*x^3)^(1/4) + Sqrt[-b + a*x^3]])/(12*Sqrt[2]*b^(3/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt[4]{-b+ax^3}}{x^4}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[4]{-b+ax}}{x^2}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt[4]{-b+ax^3}}{3x^3} + \frac{1}{12}a\,\text{Subst}\left(\int \frac{1}{x(-b+ax)^{3/4}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt[4]{-b+ax^3}}{3x^3} + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)$$

$$= -\frac{\sqrt[4]{-b+ax^3}}{3x^3} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{6\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{6\sqrt{b}}$$

$$= -\frac{\sqrt[4]{-b+ax^3}}{3x^3} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{12\sqrt{2}\,b^{3/4}} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}-2x}{-\sqrt{b}+\sqrt{2}\sqrt[4]{b}x-x^2}\,dx \ldots\right.}{12\sqrt{2}\,b^{3/4}}$$

$$= -\frac{\sqrt[4]{-b+ax^3}}{3x^3} - \frac{a\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^3}+\sqrt{-b+ax^3}\right)}{12\sqrt{2}\,b^{3/4}} + \frac{a\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+\ldots}\right.}{12\sqrt{2}\,b^{3/\ldots}}$$

$$= -\frac{\sqrt[4]{-b+ax^3}}{3x^3} - \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)}{6\sqrt{2}\,b^{3/4}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)}{6\sqrt{2}\,b^{3/4}} - \frac{a\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\ldots\right.}{12\ldots}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.27

$$\frac{4a\left(ax^3-b\right)^{5/4}\,{}_2F_1\left(\frac{5}{4},2;\frac{9}{4};1-\frac{ax^3}{b}\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^3)^(1/4)/x^4,x]
```

```
[Out] (4*a*(-b + a*x^3)^(5/4)*Hypergeometric2F1[5/4, 2, 9/4, 1 - (a*x^3)/b])/(15*
b^2)
```

IntegrateAlgebraic [A] time = 0.33, size = 149, normalized size = 0.99

$$\frac{a\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{6\sqrt{2}\,b^{3/4}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}+\sqrt{b}}\right)}{6\sqrt{2}\,b^{3/4}}-\frac{\sqrt[4]{ax^3-b}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^3)^(1/4)/x^4,x]
```

```
[Out] -1/3*(-b + a*x^3)^(1/4)/x^3 + (a*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a*x
^3]/(Sqrt[2]*b^(1/4)))/(-b + a*x^3)^(1/4)])/(6*Sqrt[2]*b^(3/4)) + (a*ArcTan
h[(Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^3])])/(6*Sq
rt[2]*b^(3/4))
```

fricas [A] time = 0.41, size = 198, normalized size = 1.32

$$\frac{4\left(-\frac{a^4}{b^3}\right)^{\frac{1}{4}}x^3\arctan\left(-\frac{\left(ax^3-b\right)^{\frac{1}{4}}a\left(-\frac{a^4}{b^3}\right)^{\frac{3}{4}}b^2-\sqrt{\sqrt{ax^3-b}\,a^2+\sqrt{-\frac{a^4}{b^3}}\,b^2}\left(-\frac{a^4}{b^3}\right)^{\frac{3}{4}}b^2}{a^4}\right)+\left(-\frac{a^4}{b^3}\right)^{\frac{1}{4}}x^3\log\left(\left(ax^3-b\right)^{\frac{1}{4}}a+\left(-\frac{a^4}{b^3}\right)^{\frac{1}{4}}b\right)-\left(-\frac{a^4}{b^3}\right)^{\frac{1}{4}}x^3\log\left(\left(ax^3-b\right)^{\frac{1}{4}}a-\left(-\frac{a^4}{b^3}\right)^{\frac{1}{4}}b\right)-4\left(ax^3-b\right)^{\frac{1}{4}}}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(1/4)/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(4*(-a^4/b^3)^(1/4)*x^3*arctan(-((a*x^3 - b)^(1/4)*a*(-a^4/b^3)^(3/4)*
b^2 - sqrt(sqrt(a*x^3 - b)*a^2 + sqrt(-a^4/b^3)*b^2)*(-a^4/b^3)^(3/4)*b^2)/
a^4) + (-a^4/b^3)^(1/4)*x^3*log((a*x^3 - b)^(1/4)*a + (-a^4/b^3)^(1/4)*b) -
 (-a^4/b^3)^(1/4)*x^3*log((a*x^3 - b)^(1/4)*a - (-a^4/b^3)^(1/4)*b) - 4*(a*
x^3 - b)^(1/4))/x^3
```

giac [A] time = 0.18, size = 195, normalized size = 1.30

$$\frac{\frac{2\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{2\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{\sqrt{2}a^2\log\left(\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}a^2\log\left(-\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{8\left(ax^3-b\right)^{\frac{1}{4}}a}{x^3}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(1/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(2*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1/
4))/b^(1/4))/b^(3/4) + 2*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
 2*(a*x^3 - b)^(1/4))/b^(1/4))/b^(3/4) + sqrt(2)*a^2*log(sqrt(2)*(a*x^3 - b
)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(3/4) - sqrt(2)*a^2*log(-sqr
t(2)*(a*x^3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(3/4) - 8*(a*
x^3 - b)^(1/4)*a/x^3)/a
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^3-b\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b)^(1/4)/x^4,x)
```

```
[Out] int((a*x^3-b)^(1/4)/x^4,x)
```

maxima [A] time = 0.41, size = 182, normalized size = 1.21

$$\frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+2\,(ax^3-b)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{12\,b^{\frac{3}{4}}}+\frac{\sqrt{2}\,a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-2\,(ax^3-b)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{12\,b^{\frac{3}{4}}}+\frac{\sqrt{2}\,a\log\left(\sqrt{2}\,(ax^3-b)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{24\,b^{\frac{3}{4}}}-\frac{\sqrt{2}\,a\log\left(-\sqrt{2}\,(ax^3-b)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{24\,b^{\frac{3}{4}}}-\frac{(ax^3-b)^{\frac{1}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(1/4)/x^4,x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1/4))/b
^(1/4))/b^(3/4) + 1/12*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(
a*x^3 - b)^(1/4))/b^(1/4))/b^(3/4) + 1/24*sqrt(2)*a*log(sqrt(2)*(a*x^3 - b)
^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(3/4) - 1/24*sqrt(2)*a*log(-s
qrt(2)*(a*x^3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(3/4) - 1/3
*(a*x^3 - b)^(1/4)/x^3
```

mupad [B] time = 1.18, size = 69, normalized size = 0.46

$$-\frac{\left(a\,x^3-b\right)^{1/4}}{3\,x^3}-\frac{a\,\mathrm{atan}\left(\frac{\left(a\,x^3-b\right)^{1/4}}{(-b)^{1/4}}\right)}{6\,(-b)^{3/4}}-\frac{a\,\mathrm{atanh}\left(\frac{\left(a\,x^3-b\right)^{1/4}}{(-b)^{1/4}}\right)}{6\,(-b)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 - b)^(1/4)/x^4,x)
```

```
[Out] - (a*x^3 - b)^(1/4)/(3*x^3) - (a*atan((a*x^3 - b)^(1/4)/(-b)^(1/4)))/(6*(-b
)^(3/4)) - (a*atanh((a*x^3 - b)^(1/4)/(-b)^(1/4)))/(6*(-b)^(3/4))
```

sympy [C] time = 1.13, size = 44, normalized size = 0.29

$$-\frac{\sqrt[4]{a}\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{be^{2i\pi}}{ax^3}\right)}{3x^{\frac{9}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b)**(1/4)/x**4,x)
```

```
[Out] -a**(1/4)*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), b*exp_polar(2*I*pi)/(a*x**3
))/(3*x**(9/4)*gamma(7/4))
```

3.1726 $\int \frac{(-b+ax^3)^{3/4}}{x^4}\,dx$

Optimal. Leaf size=150

$$-\frac{(ax^3-b)^{3/4}}{3x^3}-\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}-\sqrt{b}}\right)}{2\sqrt{2}\sqrt[4]{b}}-\frac{a\tanh^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{2\sqrt{2}\sqrt[4]{b}}$$

Rubi [A] time = 0.21, antiderivative size = 225, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 47, 63, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(ax^3-b)^{3/4}}{3x^3}+\frac{a\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{4\sqrt{2}\sqrt[4]{b}}-\frac{a\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{4\sqrt{2}\sqrt[4]{b}}-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}\right)}{2\sqrt{2}\sqrt[4]{b}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^3)^(3/4)/x^4,x]
```

```
[Out] -1/3*(-b + a*x^3)^(3/4)/x^3 - (a*ArcTan[1 - (Sqrt[2]*(-b + a*x^3)^(1/4))/b^
(1/4)])/(2*Sqrt[2]*b^(1/4)) + (a*ArcTan[1 + (Sqrt[2]*(-b + a*x^3)^(1/4))/b^
(1/4)])/(2*Sqrt[2]*b^(1/4)) + (a*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^3)
^(1/4) + Sqrt[-b + a*x^3]])/(4*Sqrt[2]*b^(1/4)) - (a*Log[Sqrt[b] + Sqrt[2]*
b^(1/4)*(-b + a*x^3)^(1/4) + Sqrt[-b + a*x^3]])/(4*Sqrt[2]*b^(1/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(-b+ax^3\right)^{3/4}}{x^4}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(-b+ax)^{3/4}}{x^2}\,dx, x, x^3\right)$$

$$= -\frac{\left(-b+ax^3\right)^{3/4}}{3x^3} + \frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{-b+ax}}\,dx, x, x^3\right)$$

$$= -\frac{\left(-b+ax^3\right)^{3/4}}{3x^3} + \text{Subst}\left(\int \frac{x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)$$

$$= -\frac{\left(-b+ax^3\right)^{3/4}}{3x^3} - \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{\right.$$

$$= -\frac{\left(-b+ax^3\right)^{3/4}}{3x^3} + \frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{2}\sqrt[4]{b}x+x^2}\,dx, x, \sqrt[4]{-b+ax^3}\right) + \frac{1}{4}a\,\text{Subst}\left(\int \frac{}{\sqrt{b}}\right.$$

$$= -\frac{\left(-b+ax^3\right)^{3/4}}{3x^3} + \frac{a\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^3}+\sqrt{-b+ax^3}\right)}{4\sqrt{2}\sqrt[4]{b}} - \frac{a\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{\right.}{4\sqrt{}}$$

$$= -\frac{\left(-b+ax^3\right)^{3/4}}{3x^3} - \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)}{2\sqrt{2}\sqrt[4]{b}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)}{2\sqrt{2}\sqrt[4]{b}} + \frac{a\log\left(\sqrt{b}-\sqrt{2}\right.}{}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.27

$$\frac{4a\left(ax^3-b\right)^{7/4}\,{}_2F_1\left(\frac{7}{4},2;\frac{11}{4};1-\frac{ax^3}{b}\right)}{21b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^3)^(3/4)/x^4,x]
```

```
[Out] (4*a*(-b + a*x^3)^(7/4)*Hypergeometric2F1[7/4, 2, 11/4, 1 - (a*x^3)/b])/(21
*b^2)
```

IntegrateAlgebraic [A] time = 0.23, size = 149, normalized size = 0.99

$$-\frac{\left(ax^3-b\right)^{3/4}}{3x^3}+\frac{a\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{2\sqrt{2}\sqrt[4]{b}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}+\sqrt{b}}\right)}{2\sqrt{2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^3)^(3/4)/x^4,x]
```

```
[Out] -1/3*(-b + a*x^3)^(3/4)/x^3 + (a*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a*x
^3]/(Sqrt[2]*b^(1/4)))/(-b + a*x^3)^(1/4)])/(2*Sqrt[2]*b^(1/4)) - (a*ArcTan
h[(Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^3])])/(2*Sq
rt[2]*b^(1/4))
```

fricas [A] time = 0.42, size = 201, normalized size = 1.34

$$-\frac{12\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}x^3\arctan\left(-\frac{\left(ax^3-b\right)^{\frac{1}{4}}\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}a^3-\sqrt{\sqrt{ax^3-b}\,a^6-\sqrt{-\frac{a^4}{b}}\,a^4b}\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}}{a^4}\right)-3\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}x^3\log\left(\left(ax^3-b\right)^{\frac{1}{4}}a^3+\left(-\frac{a^4}{b}\right)^{\frac{3}{4}}b\right)+3\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}x^3\log\left(\left(ax^3-b\right)^{\frac{1}{4}}a^3-\left(-\frac{a^4}{b}\right)^{\frac{3}{4}}b\right)+4\left(ax^3-b\right)^{\frac{3}{4}}}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(3/4)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(12*(-a^4/b)^(1/4)*x^3*arctan(-((a*x^3 - b)^(1/4)*(-a^4/b)^(1/4)*a^3
 - sqrt(sqrt(a*x^3 - b)*a^6 - sqrt(-a^4/b)*a^4*b)*(-a^4/b)^(1/4))/a^4) - 3*(
-a^4/b)^(1/4)*x^3*log((a*x^3 - b)^(1/4)*a^3 + (-a^4/b)^(3/4)*b) + 3*(-a^4/b
)^(1/4)*x^3*log((a*x^3 - b)^(1/4)*a^3 - (-a^4/b)^(3/4)*b) + 4*(a*x^3 - b)^(
3/4))/x^3
```

giac [A] time = 0.32, size = 196, normalized size = 1.31

$$\frac{\frac{6\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{6\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{3\sqrt{2}a^2\log\left(\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}+\frac{3\sqrt{2}a^2\log\left(-\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}-\frac{8\left(ax^3-b\right)^{\frac{3}{4}}a}{x^3}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(6*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1/
4))/b^(1/4))/b^(1/4) + 6*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
 2*(a*x^3 - b)^(1/4))/b^(1/4))/b^(1/4) - 3*sqrt(2)*a^2*log(sqrt(2)*(a*x^3 -
 b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(1/4) + 3*sqrt(2)*a^2*log(
-sqrt(2)*(a*x^3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(1/4) - 8
*(a*x^3 - b)^(3/4)*a/x^3)/a
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{3}-b\right)^{\frac{3}{4}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b)^(3/4)/x^4,x)
```

```
[Out] int((a*x^3-b)^(3/4)/x^4,x)
```

maxima [A] time = 0.41, size = 181, normalized size = 1.21

$$\frac{1}{8}\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+2\left(ax^{3}-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-2\left(ax^{3}-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\left(ax^{3}-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^{3}-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\left(ax^{3}-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^{3}-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}\right)a-\frac{\left(ax^{3}-b\right)^{\frac{3}{4}}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(3/4)/x^4,x, algorithm="maxima")
```

```
[Out] 1/8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1/4))/b
^(1/4))/b^(1/4) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^3
 - b)^(1/4))/b^(1/4))/b^(1/4) - sqrt(2)*log(sqrt(2)*(a*x^3 - b)^(1/4)*b^(1/
4) + sqrt(a*x^3 - b) + sqrt(b))/b^(1/4) + sqrt(2)*log(-sqrt(2)*(a*x^3 - b)^
(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(1/4))*a - 1/3*(a*x^3 - b)^(3/
4)/x^3
```

mupad [B] time = 1.11, size = 69, normalized size = 0.46

$$\frac{a\,\mathrm{atan}\left(\frac{{\left(a\,x^3-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{2\,{\left(-b\right)}^{1/4}}-\frac{{\left(a\,x^3-b\right)}^{3/4}}{3\,x^3}-\frac{a\,\mathrm{atanh}\left(\frac{{\left(a\,x^3-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{2\,{\left(-b\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 - b)^(3/4)/x^4,x)
```

```
[Out] (a*atan((a*x^3 - b)^(1/4)/(-b)^(1/4)))/(2*(-b)^(1/4)) - (a*x^3 - b)^(3/4)/(
3*x^3) - (a*atanh((a*x^3 - b)^(1/4)/(-b)^(1/4)))/(2*(-b)^(1/4))
```

sympy [C] time = 1.23, size = 44, normalized size = 0.29

$$-\frac{a^{\frac{3}{4}}\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^{3}}\right)}{3x^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b)**(3/4)/x**4,x)
```

```
[Out] -a**(3/4)*gamma(1/4)*hyper((-3/4, 1/4), (5/4,), b*exp_polar(2*I*pi)/(a*x**3
))/(3*x**(3/4)*gamma(5/4))
```

3.1727 $\int \frac{(-a+x)(-3ab+(a+2b)x)\left(-b^3+3b^2x-3bx^2+x^3\right)}{x\left(x(-a+x)(-b+x)^2\right)^{3/4}\left(ab^2-b(2a+b)x+(a+2b)x^2+(-1+d)x^3\right)}\,dx$

Optimal. Leaf size=150

$$-2\sqrt[4]{d}\,\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}}\right)+2\sqrt[4]{d}\,\tanh^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b}}\right.$$

Rubi [F] time = 36.93, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-a+x)(-3ab+(a+2b)x)\left(-b^3+3b^2x-3bx^2+x^3\right)}{x\left(x(-a+x)(-b+x)^2\right)^{3/4}\left(ab^2-b(2a+b)x+(a+2b)x^2+(-1+d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-a + x)*(-3*a*b + (a + 2*b)*x)*(-b^3 + 3*b^2*x - 3*b*x^2 + x^3))/(x*(
x*(-a + x)*(-b + x)^2)^(3/4)*(a*b^2 - b*(2*a + b)*x + (a + 2*b)*x^2 + (-1 +
 d)*x^3)),x]
```

```
[Out] (4*(a - x)*(b - x)^2*AppellF1[-3/4, -1/4, -3/2, 1/4, x/a, x/b])/((-((a - x)
*(b - x)^2*x))^(3/4)*(1 - x/a)^(1/4)*Sqrt[1 - x/b]) - (4*(5*a + b)*x^(3/4)*
(-a + x)^(3/4)*(-b + x)^(3/2)*Defer[Subst][Defer[Int][((-a + x^4)^(1/4)*(-b
 + x^4)^(3/2))/(-(b^2*x^4) + 2*b*x^8 + (-1 + d)*x^12 + a*(b - x^4)^2), x],
x, x^(1/4)])/(-((a - x)*(b - x)^2*x))^(3/4) + (12*(a + 2*b)*x^(3/4)*(-a + x
)^(3/4)*(-b + x)^(3/2)*Defer[Subst][Defer[Int][(x^4*(-a + x^4)^(1/4)*(-b +
x^4)^(3/2))/(-(b^2*x^4) + 2*b*x^8 + (-1 + d)*x^12 + a*(b - x^4)^2), x], x,
x^(1/4)])/(b*(-((a - x)*(b - x)^2*x))^(3/4)) - (12*(1 - d)*x^(3/4)*(-a + x)
^(3/4)*(-b + x)^(3/2)*Defer[Subst][Defer[Int][(x^8*(-a + x^4)^(1/4)*(-b + x
^4)^(3/2))/(-(b^2*x^4) + 2*b*x^8 + (-1 + d)*x^12 + a*(b - x^4)^2), x], x, x
^(1/4)])/(b*(-((a - x)*(b - x)^2*x))^(3/4))
```

Rubi steps

$$\int \frac{(-a+x)(-3ab+(a+2b)x)\left(-b^3+3b^2x-3bx^2+x^3\right)}{x\left(x(-a+x)(-b+x)^2\right)^{3/4}\left(ab^2-b(2a+b)x+(a+2b)x^2+(-1+d)x^3\right)}dx = \frac{\left(x^{3/4}(-a+x)^{3/4}(-b+x)^{3/2}\right)\int \frac{}{x^{7/4}}}{\left(x(-a+\right.}$$

$$= \frac{\left(x^{3/4}(-a+x)^{3/4}(-b+x)^{3/2}\right)\int \frac{}{x^{7/4}}}{\left(x(-a+x\right.}$$

$$= \frac{\left(x^{3/4}(-a+x)^{3/4}(-b+x)^{3/2}\right)\int \frac{}{x^{7/4}}}{\left(x(-a+x)(-\right.}$$

$$= \frac{\left(4x^{3/4}(-a+x)^{3/4}(-b+x)^{3/2}\right)\mathrm{Sub}}{\left(x(-\right.}$$

$$= \frac{\left(4x^{3/4}(-a+x)^{3/4}(-b+x)^{3/2}\right)\mathrm{Sub}}{}$$

$$= \frac{\left(4x^{3/4}(-a+x)^{3/4}(-b+x)^{3/2}\right)\mathrm{Sub}}{}$$

$$= \frac{\left(4x^{3/4}(-a+x)^{3/4}(-b+x)^{3/2}\right)\mathrm{Sub}}{}$$

$$= -\frac{\left(4(5a+b)x^{3/4}(-a+x)^{3/4}(-b+\right.}{}$$

$$= \frac{4(a-x)(b-x)^2F_1\left(-\frac{3}{4};-\frac{1}{4},-\frac{3}{2};\right.}{\left(-\left((a-x)(b-x)^2x\right)\right)^{3/4}\sqrt[4]{1-\frac{x}{a}}}$$

Mathematica [F] time = 7.11, size = 0, normalized size = 0.00

$$\int \frac{(-a+x)(-3ab+(a+2b)x)\left(-b^3+3b^2x-3bx^2+x^3\right)}{x\left(x(-a+x)(-b+x)^2\right)^{3/4}\left(ab^2-b(2a+b)x+(a+2b)x^2+(-1+d)x^3\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-a + x)*(-3*a*b + (a + 2*b)*x)*(-b^3 + 3*b^2*x - 3*b*x^2 + x^3)
)/(x*(x*(-a + x)*(-b + x)^2)^(3/4)*(a*b^2 - b*(2*a + b)*x + (a + 2*b)*x^2 +
 (-1 + d)*x^3)),x]
```

```
[Out] Integrate[((-a + x)*(-3*a*b + (a + 2*b)*x)*(-b^3 + 3*b^2*x - 3*b*x^2 + x^3)
)/(x*(x*(-a + x)*(-b + x)^2)^(3/4)*(a*b^2 - b*(2*a + b)*x + (a + 2*b)*x^2 +
 (-1 + d)*x^3)), x]
```

IntegrateAlgebraic [A] time = 4.72, size = 150, normalized size = 1.00

$$-2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}}\right)+2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}}\right)-\frac{4\sqrt[4]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-a + x)*(-3*a*b + (a + 2*b)*x)*(-b^3 + 3*b^2*x - 3*b*x
^2 + x^3))/(x*(x*(-a + x)*(-b + x)^2)^(3/4)*(a*b^2 - b*(2*a + b)*x + (a + 2
*b)*x^2 + (-1 + d)*x^3)),x]
```

```
[Out] (-4*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/4))/x - 2*d^
(1/4)*ArcTan[(d^(1/4)*x)/(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 +
 x^4)^(1/4)] + 2*d^(1/4)*ArcTanh[(d^(1/4)*x)/(-(a*b^2*x) + (2*a*b + b^2)*x^
2 + (-a - 2*b)*x^3 + x^4)^(1/4)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-3*a*b+(a+2*b)*x)*(-b^3+3*b^2*x-3*b*x^2+x^3)/x/(x*(-a+x)*
(-b+x)^2)^(3/4)/(a*b^2-b*(2*a+b)*x+(a+2*b)*x^2+(-1+d)*x^3),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(b^3-3\,b^2x+3\,bx^2-x^3\right)(3\,ab-(a+2\,b)x)(a-x)}{\left(-(a-x)(b-x)^2x\right)^{\frac{3}{4}}\left((d-1)x^3+ab^2-(2\,a+b)bx+(a+2\,b)x^2\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-3*a*b+(a+2*b)*x)*(-b^3+3*b^2*x-3*b*x^2+x^3)/x/(x*(-a+x)*
(-b+x)^2)^(3/4)/(a*b^2-b*(2*a+b)*x+(a+2*b)*x^2+(-1+d)*x^3),x, algorithm="gi
ac")
```

```
[Out] integrate(-(b^3 - 3*b^2*x + 3*b*x^2 - x^3)*(3*a*b - (a + 2*b)*x)*(a - x)/((
-(a - x)*(b - x)^2*x)^(3/4)*((d - 1)*x^3 + a*b^2 - (2*a + b)*b*x + (a + 2*b
)*x^2)*x), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(-a+x)\,(-3ab+(a+2b)\,x)\left(-b^3+3b^2x-3b\,x^2+x^3\right)}{x\left(x\,(-a+x)\,(-b+x)^2\right)^{\frac{3}{4}}\left(a\,b^2-b\,(2a+b)\,x+(a+2b)\,x^2+(-1+d)\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x)*(-3*a*b+(a+2*b)*x)*(-b^3+3*b^2*x-3*b*x^2+x^3)/x/(x*(-a+x)*(-b+x)
^2)^(3/4)/(a*b^2-b*(2*a+b)*x+(a+2*b)*x^2+(-1+d)*x^3),x)
```

```
[Out] int((-a+x)*(-3*a*b+(a+2*b)*x)*(-b^3+3*b^2*x-3*b*x^2+x^3)/x/(x*(-a+x)*(-b+x)
^2)^(3/4)/(a*b^2-b*(2*a+b)*x+(a+2*b)*x^2+(-1+d)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(b^3-3\,b^2x+3\,bx^2-x^3\right)(3\,ab-(a+2\,b)x)(a-x)}{\left(-(a-x)(b-x)^2x\right)^{\frac{3}{4}}\left((d-1)x^3+ab^2-(2\,a+b)bx+(a+2\,b)x^2\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-3*a*b+(a+2*b)*x)*(-b^3+3*b^2*x-3*b*x^2+x^3)/x/(x*(-a+x)*
(-b+x)^2)^(3/4)/(a*b^2-b*(2*a+b)*x+(a+2*b)*x^2+(-1+d)*x^3),x, algorithm="ma
xima")
```

```
[Out] -integrate((b^3 - 3*b^2*x + 3*b*x^2 - x^3)*(3*a*b - (a + 2*b)*x)*(a - x)/((
-(a - x)*(b - x)^2*x)^(3/4)*((d - 1)*x^3 + a*b^2 - (2*a + b)*b*x + (a + 2*b
)*x^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(a-x)\,(3\,a\,b-x\,(a+2\,b))\,\left(b^3-3\,b^2\,x+3\,b\,x^2-x^3\right)}{x\,\left(-x\,(a-x)\,(b-x)^2\right)^{3/4}\,\left(a\,b^2+x^2\,(a+2\,b)+x^3\,(d-1)-b\,x\,(2\,a+b)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a - x)*(3*a*b - x*(a + 2*b))*(3*b*x^2 - 3*b^2*x + b^3 - x^3))/(x*(-x
*(a - x)*(b - x)^2)^(3/4)*(a*b^2 + x^2*(a + 2*b) + x^3*(d - 1) - b*x*(2*a +
 b))),x)
```

```
[Out] int(-((a - x)*(3*a*b - x*(a + 2*b))*(3*b*x^2 - 3*b^2*x + b^3 - x^3))/(x*(-x
*(a - x)*(b - x)^2)^(3/4)*(a*b^2 + x^2*(a + 2*b) + x^3*(d - 1) - b*x*(2*a +
 b))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-3*a*b+(a+2*b)*x)*(-b**3+3*b**2*x-3*b*x**2+x**3)/x/(x*(-a
+x)*(-b+x)**2)**(3/4)/(a*b**2-b*(2*a+b)*x+(a+2*b)*x**2+(-1+d)*x**3),x)
```

```
[Out] Timed out
```

3.1728 $\int \frac{\left(-b+ax^5\right)^{3/4}}{x^6}\,dx$

Optimal. Leaf size=150

$$-\frac{\left(ax^5-b\right)^{3/4}}{5x^5}-\frac{3a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^5-b}}{\sqrt{ax^5-b}-\sqrt{b}}\right)}{10\sqrt{2}\sqrt[4]{b}}-\frac{3a\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^5-b}}\right)}{10\sqrt{2}\sqrt[4]{b}}$$

Rubi [A] time = 0.21, antiderivative size = 225, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 47, 63, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\left(ax^5-b\right)^{3/4}}{5x^5}+\frac{3a\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^5-b}+\sqrt{ax^5-b}+\sqrt{b}\right)}{20\sqrt{2}\sqrt[4]{b}}-\frac{3a\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^5-b}+\sqrt{ax^5-b}+\sqrt{b}\right)}{20\sqrt{2}\sqrt[4]{b}}-\frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^5-b}}{\sqrt[4]{b}}\right)}{10\sqrt{2}\sqrt[4]{b}}+\frac{3a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^5-b}}{\sqrt[4]{b}}+1\right)}{10\sqrt{2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^5)^(3/4)/x^6,x]
```

```
[Out] -1/5*(-b + a*x^5)^(3/4)/x^5 - (3*a*ArcTan[1 - (Sqrt[2]*(-b + a*x^5)^(1/4))/
b^(1/4)])/(10*Sqrt[2]*b^(1/4)) + (3*a*ArcTan[1 + (Sqrt[2]*(-b + a*x^5)^(1/4
))/b^(1/4)])/(10*Sqrt[2]*b^(1/4)) + (3*a*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b
+ a*x^5)^(1/4) + Sqrt[-b + a*x^5]])/(20*Sqrt[2]*b^(1/4)) - (3*a*Log[Sqrt[b]
 + Sqrt[2]*b^(1/4)*(-b + a*x^5)^(1/4) + Sqrt[-b + a*x^5]])/(20*Sqrt[2]*b^(1
/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(-b+ax^5\right)^{3/4}}{x^6}\,dx = \frac{1}{5}\text{Subst}\left(\int \frac{(-b+ax)^{3/4}}{x^2}\,dx, x, x^5\right)$$

$$= -\frac{\left(-b+ax^5\right)^{3/4}}{5x^5} + \frac{1}{20}(3a)\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{-b+ax}}\,dx, x, x^5\right)$$

$$= -\frac{\left(-b+ax^5\right)^{3/4}}{5x^5} + \frac{3}{5}\,\text{Subst}\left(\int \frac{x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^5}\right)$$

$$= -\frac{\left(-b+ax^5\right)^{3/4}}{5x^5} - \frac{3}{10}\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^5}\right) + \frac{3}{10}\,\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x,\right.$$

$$= -\frac{\left(-b+ax^5\right)^{3/4}}{5x^5} + \frac{1}{20}(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{2}\sqrt[4]{b}\,x+x^2}\,dx, x, \sqrt[4]{-b+ax^5}\right) + \frac{1}{20}(3a)\,\text{Subs}$$

$$= -\frac{\left(-b+ax^5\right)^{3/4}}{5x^5} + \frac{3a\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^5}+\sqrt{-b+ax^5}\right)}{20\sqrt{2}\sqrt[4]{b}} - \frac{3a\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\right.}{20}$$

$$= -\frac{\left(-b+ax^5\right)^{3/4}}{5x^5} - \frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^5}}{\sqrt[4]{b}}\right)}{10\sqrt{2}\sqrt[4]{b}} + \frac{3a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^5}}{\sqrt[4]{b}}\right)}{10\sqrt{2}\sqrt[4]{b}} + \frac{3a\log\left(\sqrt{b}-\right.}{}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.27

$$\frac{4a\left(ax^5-b\right)^{7/4}\,{}_2F_1\left(\frac{7}{4},2;\frac{11}{4};1-\frac{ax^5}{b}\right)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^5)^(3/4)/x^6,x]
```

```
[Out] (4*a*(-b + a*x^5)^(7/4)*Hypergeometric2F1[7/4, 2, 11/4, 1 - (a*x^5)/b])/(35
*b^2)
```

IntegrateAlgebraic [A] time = 0.22, size = 149, normalized size = 0.99

$$-\frac{\left(ax^5-b\right)^{3/4}}{5x^5}+\frac{3a\tan^{-1}\left(\frac{\frac{\sqrt{ax^5-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^5-b}}\right)}{10\sqrt{2}\sqrt[4]{b}}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^5-b}}{\sqrt{ax^5-b}+\sqrt{b}}\right)}{10\sqrt{2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^5)^(3/4)/x^6,x]
```

```
[Out] -1/5*(-b + a*x^5)^(3/4)/x^5 + (3*a*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a
*x^5]/(Sqrt[2]*b^(1/4)))/(-b + a*x^5)^(1/4)])/(10*Sqrt[2]*b^(1/4)) - (3*a*A
rcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^5)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^5])])/
(10*Sqrt[2]*b^(1/4))
```

fricas [A] time = 0.41, size = 204, normalized size = 1.36

$$-\frac{12\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}x^5\arctan\left(-\frac{\left(ax^5-b\right)^{\frac{1}{4}}\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}a^3-\sqrt{\sqrt{ax^5-b}\,a^6-\sqrt{-\frac{a^4}{b}}\,a^4b}\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}}{a^4}\right)-3\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}x^5\log\left(27\left(ax^5-b\right)^{\frac{1}{4}}a^3+27\left(-\frac{a^4}{b}\right)^{\frac{3}{4}}b\right)+3\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}x^5\log\left(27\left(ax^5-b\right)^{\frac{1}{4}}a^3-27\left(-\frac{a^4}{b}\right)^{\frac{3}{4}}b\right)+4\left(ax^5-b\right)^{\frac{3}{4}}}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5-b)^(3/4)/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(12*(-a^4/b)^(1/4)*x^5*arctan(-((a*x^5 - b)^(1/4)*(-a^4/b)^(1/4)*a^3
- sqrt(sqrt(a*x^5 - b)*a^6 - sqrt(-a^4/b)*a^4*b)*(-a^4/b)^(1/4))/a^4) - 3*(
-a^4/b)^(1/4)*x^5*log(27*(a*x^5 - b)^(1/4)*a^3 + 27*(-a^4/b)^(3/4)*b) + 3*(
-a^4/b)^(1/4)*x^5*log(27*(a*x^5 - b)^(1/4)*a^3 - 27*(-a^4/b)^(3/4)*b) + 4*(
a*x^5 - b)^(3/4))/x^5
```

giac [A] time = 0.17, size = 196, normalized size = 1.31

$$\frac{\frac{6\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^5-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{6\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^5-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{3\sqrt{2}a^2\log\left(\sqrt{2}\left(ax^5-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^5-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}+\frac{3\sqrt{2}a^2\log\left(-\sqrt{2}\left(ax^5-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^5-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}-\frac{8\left(ax^5-b\right)^{\frac{3}{4}}a}{x^5}}{40a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5-b)^(3/4)/x^6,x, algorithm="giac")
```

```
[Out] 1/40*(6*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^5 - b)^(1/
4))/b^(1/4))/b^(1/4) + 6*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
 2*(a*x^5 - b)^(1/4))/b^(1/4))/b^(1/4) - 3*sqrt(2)*a^2*log(sqrt(2)*(a*x^5 -
 b)^(1/4)*b^(1/4) + sqrt(a*x^5 - b) + sqrt(b))/b^(1/4) + 3*sqrt(2)*a^2*log(
-sqrt(2)*(a*x^5 - b)^(1/4)*b^(1/4) + sqrt(a*x^5 - b) + sqrt(b))/b^(1/4) - 8
*(a*x^5 - b)^(3/4)*a/x^5)/a
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{5}-b\right)^{\frac{3}{4}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^5-b)^(3/4)/x^6,x)
```

```
[Out] int((a*x^5-b)^(3/4)/x^6,x)
```

maxima [A] time = 0.41, size = 181, normalized size = 1.21

$$\frac{3}{40}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^5-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^5-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^5-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^5-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^5-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^5-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}\right)a-\frac{\left(ax^5-b\right)^{\frac{3}{4}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5-b)^(3/4)/x^6,x, algorithm="maxima")
```

```
[Out] 3/40*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^5 - b)^(1/4))/
b^(1/4))/b^(1/4) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^
5 - b)^(1/4))/b^(1/4))/b^(1/4) - sqrt(2)*log(sqrt(2)*(a*x^5 - b)^(1/4)*b^(1
/4) + sqrt(a*x^5 - b) + sqrt(b))/b^(1/4) + sqrt(2)*log(-sqrt(2)*(a*x^5 - b)
^(1/4)*b^(1/4) + sqrt(a*x^5 - b) + sqrt(b))/b^(1/4))*a - 1/5*(a*x^5 - b)^(3
/4)/x^5
```

mupad [B] time = 1.20, size = 69, normalized size = 0.46

$$\frac{3\,a\,\mathrm{atan}\left(\frac{{\left(a\,x^5-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{10\,{\left(-b\right)}^{1/4}}-\frac{{\left(a\,x^5-b\right)}^{3/4}}{5\,x^5}-\frac{3\,a\,\mathrm{atanh}\left(\frac{{\left(a\,x^5-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{10\,{\left(-b\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^5 - b)^(3/4)/x^6,x)
```

```
[Out] (3*a*atan((a*x^5 - b)^(1/4)/(-b)^(1/4)))/(10*(-b)^(1/4)) - (a*x^5 - b)^(3/4
)/(5*x^5) - (3*a*atanh((a*x^5 - b)^(1/4)/(-b)^(1/4)))/(10*(-b)^(1/4))
```

sympy [C] time = 1.35, size = 44, normalized size = 0.29

$$-\frac{a^{\frac{3}{4}}\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{3}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{be^{2i\pi}}{ax^5}\right)}{5x^{\frac{5}{4}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**5-b)**(3/4)/x**6,x)
```

```
[Out] -a**(3/4)*gamma(1/4)*hyper((-3/4, 1/4), (5/4,), b*exp_polar(2*I*pi)/(a*x**5
))/(5*x**(5/4)*gamma(5/4))
```

3.1729 $\int \frac{(-b+ax^6)^{3/4}}{x^7}\,dx$

Optimal. Leaf size=150

$$-\frac{(ax^6-b)^{3/4}}{6x^6}-\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}}{\sqrt{ax^6-b}-\sqrt{b}}\right)}{4\sqrt{2}\sqrt[4]{b}}-\frac{a\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^6-b}}\right)}{4\sqrt{2}\sqrt[4]{b}}$$

Rubi [A] time = 0.20, antiderivative size = 225, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 47, 63, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(ax^6-b)^{3/4}}{6x^6}+\frac{a\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}+\sqrt{ax^6-b}+\sqrt{b}\right)}{8\sqrt{2}\sqrt[4]{b}}-\frac{a\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}+\sqrt{ax^6-b}+\sqrt{b}\right)}{8\sqrt{2}\sqrt[4]{b}}-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^6-b}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^6-b}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^6)^(3/4)/x^7,x]
```

```
[Out] -1/6*(-b + a*x^6)^(3/4)/x^6 - (a*ArcTan[1 - (Sqrt[2]*(-b + a*x^6)^(1/4))/b^
(1/4)])/(4*Sqrt[2]*b^(1/4)) + (a*ArcTan[1 + (Sqrt[2]*(-b + a*x^6)^(1/4))/b^
(1/4)])/(4*Sqrt[2]*b^(1/4)) + (a*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^6)
^(1/4) + Sqrt[-b + a*x^6]])/(8*Sqrt[2]*b^(1/4)) - (a*Log[Sqrt[b] + Sqrt[2]*
b^(1/4)*(-b + a*x^6)^(1/4) + Sqrt[-b + a*x^6]])/(8*Sqrt[2]*b^(1/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(-b+ax^6\right)^{3/4}}{x^7}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{(-b+ax)^{3/4}}{x^2}\,dx, x, x^6\right)$$

$$= -\frac{\left(-b+ax^6\right)^{3/4}}{6x^6} + \frac{1}{8}a\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{-b+ax}}\,dx, x, x^6\right)$$

$$= -\frac{\left(-b+ax^6\right)^{3/4}}{6x^6} + \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^6}\right)$$

$$= -\frac{\left(-b+ax^6\right)^{3/4}}{6x^6} - \frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^6}\right) + \frac{1}{4}\,\text{Subst}\Bigg(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{\ }$$

$$= -\frac{\left(-b+ax^6\right)^{3/4}}{6x^6} + \frac{1}{8}a\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{2}\sqrt[4]{b}\,x+x^2}\,dx, x, \sqrt[4]{-b+ax^6}\right) + \frac{1}{8}a\,\text{Subst}\Bigg(\int$$

$$= -\frac{\left(-b+ax^6\right)^{3/4}}{6x^6} + \frac{a\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^6}+\sqrt{-b+ax^6}\right)}{8\sqrt{2}\sqrt[4]{b}} - \frac{a\log\Big(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\,\sqrt[4]{-}}{8\sqrt{2}}$$

$$= -\frac{\left(-b+ax^6\right)^{3/4}}{6x^6} - \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^6}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^6}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}} + \frac{a\log\Big(\sqrt{b}-\sqrt{2}}{}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.27

$$\frac{2a\left(ax^6-b\right)^{7/4}{}_2F_1\left(\frac{7}{4},2;\frac{11}{4};1-\frac{ax^6}{b}\right)}{21b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^6)^(3/4)/x^7,x]
```

```
[Out] (2*a*(-b + a*x^6)^(7/4)*Hypergeometric2F1[7/4, 2, 11/4, 1 - (a*x^6)/b])/(21
*b^2)
```

IntegrateAlgebraic [A] time = 0.20, size = 149, normalized size = 0.99

$$-\frac{\left(ax^6-b\right)^{3/4}}{6x^6}+\frac{a\tan^{-1}\left(\frac{\frac{\sqrt{ax^6-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^6-b}}\right)}{4\sqrt{2}\sqrt[4]{b}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}}{\sqrt{ax^6-b}+\sqrt{b}}\right)}{4\sqrt{2}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^6)^(3/4)/x^7,x]
```

```
[Out] -1/6*(-b + a*x^6)^(3/4)/x^6 + (a*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a*x
^6]/(Sqrt[2]*b^(1/4)))/(-b + a*x^6)^(1/4)])/(4*Sqrt[2]*b^(1/4)) - (a*ArcTan
h[(Sqrt[2]*b^(1/4)*(-b + a*x^6)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^6])])/(4*Sq
rt[2]*b^(1/4))
```

fricas [A] time = 0.42, size = 201, normalized size = 1.34

$$-\frac{12\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}x^6\arctan\left(-\frac{\left(ax^6-b\right)^{\frac{1}{4}}\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}a^3-\sqrt{\sqrt{ax^6-b}\,a^6-\sqrt{-\frac{a^4}{b}}\,a^4b}\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}}{a^4}\right)-3\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}x^6\log\left(\left(ax^6-b\right)^{\frac{1}{4}}a^3+\left(-\frac{a^4}{b}\right)^{\frac{3}{4}}b\right)+3\left(-\frac{a^4}{b}\right)^{\frac{1}{4}}x^6\log\left(\left(ax^6-b\right)^{\frac{1}{4}}a^3-\left(-\frac{a^4}{b}\right)^{\frac{3}{4}}b\right)+4\left(ax^6-b\right)^{\frac{3}{4}}}{24x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-b)^(3/4)/x^7,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(-a^4/b)^(1/4)*x^6*arctan(-((a*x^6 - b)^(1/4)*(-a^4/b)^(1/4)*a^3
- sqrt(sqrt(a*x^6 - b)*a^6 - sqrt(-a^4/b)*a^4*b)*(-a^4/b)^(1/4))/a^4) - 3*(
-a^4/b)^(1/4)*x^6*log((a*x^6 - b)^(1/4)*a^3 + (-a^4/b)^(3/4)*b) + 3*(-a^4/b
)^(1/4)*x^6*log((a*x^6 - b)^(1/4)*a^3 - (-a^4/b)^(3/4)*b) + 4*(a*x^6 - b)^(
3/4))/x^6
```

giac [A] time = 0.29, size = 196, normalized size = 1.31

$$\frac{\frac{6\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^6-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{6\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^6-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{3\sqrt{2}a^2\log\left(\sqrt{2}\left(ax^6-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^6-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}+\frac{3\sqrt{2}a^2\log\left(-\sqrt{2}\left(ax^6-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^6-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}-\frac{8\left(ax^6-b\right)^{\frac{3}{4}}a}{x^6}}{48a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-b)^(3/4)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(6*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^6 - b)^(1/
4))/b^(1/4))/b^(1/4) + 6*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
 2*(a*x^6 - b)^(1/4))/b^(1/4))/b^(1/4) - 3*sqrt(2)*a^2*log(sqrt(2)*(a*x^6 -
 b)^(1/4)*b^(1/4) + sqrt(a*x^6 - b) + sqrt(b))/b^(1/4) + 3*sqrt(2)*a^2*log(
-sqrt(2)*(a*x^6 - b)^(1/4)*b^(1/4) + sqrt(a*x^6 - b) + sqrt(b))/b^(1/4) - 8
*(a*x^6 - b)^(3/4)*a/x^6)/a
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{6}-b\right)^{\frac{3}{4}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6-b)^(3/4)/x^7,x)
```

```
[Out] int((a*x^6-b)^(3/4)/x^7,x)
```

maxima [A] time = 0.51, size = 181, normalized size = 1.21

$$\frac{1}{16}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^{6}-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^{6}-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^{6}-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^{6}-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^{6}-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^{6}-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}\right)a-\frac{\left(ax^{6}-b\right)^{\frac{3}{4}}}{6x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6-b)^(3/4)/x^7,x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^6 - b)^(1/4))/
b^(1/4))/b^(1/4) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^
6 - b)^(1/4))/b^(1/4))/b^(1/4) - sqrt(2)*log(sqrt(2)*(a*x^6 - b)^(1/4)*b^(1
/4) + sqrt(a*x^6 - b) + sqrt(b))/b^(1/4) + sqrt(2)*log(-sqrt(2)*(a*x^6 - b)
^(1/4)*b^(1/4) + sqrt(a*x^6 - b) + sqrt(b))/b^(1/4))*a - 1/6*(a*x^6 - b)^(3
/4)/x^6
```

mupad [B] time = 1.24, size = 69, normalized size = 0.46

$$\frac{a\,\mathrm{atan}\left(\frac{\left(a\,x^6-b\right)^{1/4}}{(-b)^{1/4}}\right)}{4\,(-b)^{1/4}}-\frac{\left(a\,x^6-b\right)^{3/4}}{6\,x^6}-\frac{a\,\mathrm{atanh}\left(\frac{\left(a\,x^6-b\right)^{1/4}}{(-b)^{1/4}}\right)}{4\,(-b)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6 - b)^(3/4)/x^7,x)
```

```
[Out] (a*atan((a*x^6 - b)^(1/4)/(-b)^(1/4)))/(4*(-b)^(1/4)) - (a*x^6 - b)^(3/4)/(
6*x^6) - (a*atanh((a*x^6 - b)^(1/4)/(-b)^(1/4)))/(4*(-b)^(1/4))
```

sympy [C] time = 1.43, size = 44, normalized size = 0.29

$$-\frac{a^{\frac{3}{4}}\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^{6}}\right)}{6x^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6-b)**(3/4)/x**7,x)
```

```
[Out] -a**(3/4)*gamma(1/4)*hyper((-3/4, 1/4), (5/4,), b*exp_polar(2*I*pi)/(a*x**6
))/(6*x**(3/2)*gamma(5/4))
```

3.1730 $\int \frac{(-4+5x^7)\sqrt[3]{-2x+2x^3-x^8}}{(2+x^7)(2-2x^2+x^7)}\,dx$

Optimal. Leaf size=150

$$-\frac{\log\left(2^{2/3}\sqrt[3]{-x^8+2x^3-2x}-2x\right)}{2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{-x^8+2x^3-2x}+x}\right)}{2^{2/3}}+\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{-x^8+2x^3-2x}\,x+\sqrt[3]{2}\left(-\right.\right.}{2\,2^{2/3}}$$

Rubi [F] time = 14.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-4+5x^7)\sqrt[3]{-2x+2x^3-x^8}}{(2+x^7)(2-2x^2+x^7)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-4 + 5*x^7)*(-2*x + 2*x^3 - x^8)^(1/3))/((2 + x^7)*(2 - 2*x^2 + x^7))
,x]

[Out] (-3*(-2)^(1/7)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer[Int][(-2 + 2*x
^6 - x^21)^(-2/3), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/3)) + (3
*2^(1/7)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer[Int][(-2 + 2*x^6 - x
^21)^(-2/3), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/3)) + (3*(-1)^
(2/7)*2^(1/7)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer[Int][(-2 + 2*x^
6 - x^21)^(-2/3), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/3)) - (3*
(-1)^(3/7)*2^(1/7)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer[Int][(-2 +
 2*x^6 - x^21)^(-2/3), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/3))
+ (3*(-1)^(4/7)*2^(1/7)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer[Int][
(-2 + 2*x^6 - x^21)^(-2/3), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1
/3)) - (3*(-1)^(5/7)*2^(1/7)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer[
Int][(-2 + 2*x^6 - x^21)^(-2/3), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^
7)^(1/3)) + (3*(-1)^(6/7)*2^(1/7)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][D
efer[Int][(-2 + 2*x^6 - x^21)^(-2/3), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2
 - x^7)^(1/3)) + ((-2)^(4/21)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer
[Int][1/(((-2)^(1/21) - x)*(-2 + 2*x^6 - x^21)^(2/3)), x], x, x^(1/3)])/(x^
(1/3)*(-2 + 2*x^2 - x^7)^(1/3)) + (2^(4/21)*(-2*x + 2*x^3 - x^8)^(1/3)*Defe
r[Subst][Defer[Int][1/((-2^(1/21) - x)*(-2 + 2*x^6 - x^21)^(2/3)), x], x, x
^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/3)) + ((-1)^(8/21)*2^(4/21)*(-2*x +
 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer[Int][1/((-((-1)^(2/21)*2^(1/21)) - x
)*(-2 + 2*x^6 - x^21)^(2/3)), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^
(1/3)) + ((-1)^(4/7)*2^(4/21)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer
[Int][1/(((-1)^(1/7)*2^(1/21) - x)*(-2 + 2*x^6 - x^21)^(2/3)), x], x, x^(1/
3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/3)) + ((-1)^(16/21)*2^(4/21)*(-2*x + 2*
x^3 - x^8)^(1/3)*Defer[Subst][Defer[Int][1/((-((-1)^(4/21)*2^(1/21)) - x)*(
-2 + 2*x^6 - x^21)^(2/3)), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/
3)) + ((-1)^(20/21)*2^(4/21)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer[
Int][1/(((-1)^(5/21)*2^(1/21) - x)*(-2 + 2*x^6 - x^21)^(2/3)), x], x, x^(1/
3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/3)) - ((-1)^(1/7)*2^(4/21)*(-2*x + 2*x^
3 - x^8)^(1/3)*Defer[Subst][Defer[Int][1/((-((-1)^(2/7)*2^(1/21)) - x)*(-2
+ 2*x^6 - x^21)^(2/3)), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/3))
 - (15*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer[Int][x^3/(-2 + 2*x^6 -
 x^21)^(2/3), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/3)) + ((-2)^(
4/21)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer[Int][1/(((-2)^(1/21) +
(-1)^(1/3)*x)*(-2 + 2*x^6 - x^21)^(2/3)), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2
*x^2 - x^7)^(1/3)) + (2^(4/21)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defe
r[Int][1/((-2^(1/21) + (-1)^(1/3)*x)*(-2 + 2*x^6 - x^21)^(2/3)), x], x, x^(
1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/3)) + ((-1)^(8/21)*2^(4/21)*(-2*x + 2
```

```
*x^3 - x^8)^(1/3)*Defer[Subst][Defer[Int][1/((-((-1)^(2/21)*2^(1/21)) + (-1
)^(1/3)*x)*(-2 + 2*x^6 - x^21)^(2/3)), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^
2 - x^7)^(1/3)) + ((-1)^(4/7)*2^(4/21)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Sub
st][Defer[Int][1/(((-1)^(1/7)*2^(1/21) + (-1)^(1/3)*x)*(-2 + 2*x^6 - x^21)^
(2/3)), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/3)) + ((-1)^(16/21)
*2^(4/21)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer[Int][1/((-((-1)^(4/
21)*2^(1/21)) + (-1)^(1/3)*x)*(-2 + 2*x^6 - x^21)^(2/3)), x], x, x^(1/3)])/
(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/3)) + ((-1)^(20/21)*2^(4/21)*(-2*x + 2*x^3 -
 x^8)^(1/3)*Defer[Subst][Defer[Int][1/(((-1)^(5/21)*2^(1/21) + (-1)^(1/3)*x
)*(-2 + 2*x^6 - x^21)^(2/3)), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^
(1/3)) - ((-1)^(1/7)*2^(4/21)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer
[Int][1/((-((-1)^(2/7)*2^(1/21)) + (-1)^(1/3)*x)*(-2 + 2*x^6 - x^21)^(2/3))
, x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/3)) + ((-2)^(4/21)*(-2*x
+ 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer[Int][1/(((-2)^(1/21) - (-1)^(2/3)*x
)*(-2 + 2*x^6 - x^21)^(2/3)), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^
(1/3)) + (2^(4/21)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer[Int][1/((-
2^(1/21) - (-1)^(2/3)*x)*(-2 + 2*x^6 - x^21)^(2/3)), x], x, x^(1/3)])/(x^(1
/3)*(-2 + 2*x^2 - x^7)^(1/3)) + ((-1)^(8/21)*2^(4/21)*(-2*x + 2*x^3 - x^8)^
(1/3)*Defer[Subst][Defer[Int][1/((-((-1)^(2/21)*2^(1/21)) - (-1)^(2/3)*x)*(
-2 + 2*x^6 - x^21)^(2/3)), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/
3)) + ((-1)^(4/7)*2^(4/21)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer[In
t][1/(((-1)^(1/7)*2^(1/21) - (-1)^(2/3)*x)*(-2 + 2*x^6 - x^21)^(2/3)), x],
x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/3)) + ((-1)^(16/21)*2^(4/21)*(-
2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer[Int][1/((-((-1)^(4/21)*2^(1/21)
) - (-1)^(2/3)*x)*(-2 + 2*x^6 - x^21)^(2/3)), x], x, x^(1/3)])/(x^(1/3)*(-2
 + 2*x^2 - x^7)^(1/3)) + ((-1)^(20/21)*2^(4/21)*(-2*x + 2*x^3 - x^8)^(1/3)*
Defer[Subst][Defer[Int][1/(((-1)^(5/21)*2^(1/21) - (-1)^(2/3)*x)*(-2 + 2*x^
6 - x^21)^(2/3)), x], x, x^(1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/3)) - ((-
1)^(1/7)*2^(4/21)*(-2*x + 2*x^3 - x^8)^(1/3)*Defer[Subst][Defer[Int][1/((-(
(-1)^(2/7)*2^(1/21)) - (-1)^(2/3)*x)*(-2 + 2*x^6 - x^21)^(2/3)), x], x, x^(
1/3)])/(x^(1/3)*(-2 + 2*x^2 - x^7)^(1/3))
```

Rubi steps

$$\int \frac{\left(-4+5x^7\right)\sqrt[3]{-2x+2x^3-x^8}}{\left(2+x^7\right)\left(2-2x^2+x^7\right)}\,dx = \frac{\sqrt[3]{-2x+2x^3-x^8}\int \frac{\sqrt[3]{x}\sqrt[3]{-2+2x^2-x^7}\left(-4+5x^7\right)}{\left(2+x^7\right)\left(2-2x^2+x^7\right)}\,dx}{\sqrt[3]{x}\sqrt[3]{-2+2x^2-x^7}}$$

$$= -\frac{\sqrt[3]{-2x+2x^3-x^8}\int \frac{\sqrt[3]{x}\left(-4+5x^7\right)}{\left(-2+2x^2-x^7\right)^{2/3}\left(2+x^7\right)}\,dx}{\sqrt[3]{x}\sqrt[3]{-2+2x^2-x^7}}$$

$$= -\frac{\sqrt[3]{-2x+2x^3-x^8}\int \left(\frac{5\sqrt[3]{x}}{\left(-2+2x^2-x^7\right)^{2/3}} - \frac{14\sqrt[3]{x}}{\left(-2+2x^2-x^7\right)^{2/3}\left(2+x^7\right)}\right)dx}{\sqrt[3]{x}\sqrt[3]{-2+2x^2-x^7}}$$

$$= -\frac{\left(5\sqrt[3]{-2x+2x^3-x^8}\right)\int \frac{\sqrt[3]{x}}{\left(-2+2x^2-x^7\right)^{2/3}}\,dx}{\sqrt[3]{x}\sqrt[3]{-2+2x^2-x^7}} + \frac{\left(14\sqrt[3]{-2x+2x^3-x^8}\right)\int \frac{}{(-2}}{\sqrt[3]{x}\sqrt[3]{-2+2x^2}}$$

$$= \frac{\left(14\sqrt[3]{-2x+2x^3-x^8}\right)\int\left(-\frac{\sqrt[3]{x}}{7\,2^{6/7}\left(-\sqrt[7]{2}-x\right)\left(-2+2x^2-x^7\right)^{2/3}} - \frac{\sqrt[3]{x}}{7\,2^{6/7}\left(-\sqrt[7]{2}+\sqrt[7]{-1}\,x\right)(}\right.}{}$$

$$= -\frac{\left(15\sqrt[3]{-2x+2x^3-x^8}\right)\mathrm{Subst}\left(\int \frac{x^3}{\left(-2+2x^6-x^{21}\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{-2+2x^2-x^7}} - \frac{\left(\sqrt[7]{2}\sqrt[3]{-2x}\right.}{}$$

$$= -\frac{\left(15\sqrt[3]{-2x+2x^3-x^8}\right)\mathrm{Subst}\left(\int \frac{x^3}{\left(-2+2x^6-x^{21}\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{-2+2x^2-x^7}} - \frac{\left(3\sqrt[7]{2}\sqrt[3]{-2}\right.}{}$$

$$= -\frac{\left(15\sqrt[3]{-2x+2x^3-x^8}\right)\mathrm{Subst}\left(\int \frac{x^3}{\left(-2+2x^6-x^{21}\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{-2+2x^2-x^7}} - \frac{\left(3\sqrt[7]{2}\sqrt[3]{-2}\right.}{}$$

$$= -\frac{\left(15\sqrt[3]{-2x+2x^3-x^8}\right)\mathrm{Subst}\left(\int \frac{x^3}{\left(-2+2x^6-x^{21}\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{-2+2x^2-x^7}} - \frac{\left(3\sqrt[7]{-2}\sqrt[3]{-}\right.}{}$$

$$= -\frac{\left(15\sqrt[3]{-2x+2x^3-x^8}\right)\mathrm{Subst}\left(\int \frac{x^3}{\left(-2+2x^6-x^{21}\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{-2+2x^2-x^7}} - \frac{\left(3\sqrt[7]{-2}\sqrt[3]{-}\right.}{}$$

$$= -\frac{\left(15\sqrt[3]{-2x+2x^3-x^8}\right)\mathrm{Subst}\left(\int \frac{x^3}{\left(-2+2x^6-x^{21}\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{-2+2x^2-x^7}} - \frac{\left(3\sqrt[7]{-2}\sqrt[3]{-}\right.}{}$$

Mathematica [F] time = 3.15, size = 0, normalized size = 0.00

$$\int \frac{\left(-4+5x^7\right)\sqrt[3]{-2x+2x^3-x^8}}{\left(2+x^7\right)\left(2-2x^2+x^7\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-4 + 5*x^7)*(-2*x + 2*x^3 - x^8)^(1/3))/((2 + x^7)*(2 - 2*x^2 + 
 x^7)),x]
```

```
[Out] Integrate[((-4 + 5*x^7)*(-2*x + 2*x^3 - x^8)^(1/3))/((2 + x^7)*(2 - 2*x^2 +
 x^7)), x]
```

IntegrateAlgebraic [A] time = 0.41, size = 150, normalized size = 1.00

$$-\frac{\log\left(2^{2/3}\sqrt[3]{-x^8+2x^3-2x}-2x\right)}{2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{-x^8+2x^3-2x}+x}\right)}{2^{2/3}}+\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{-x^8+2x^3-2x}\,x+\sqrt[3]{2}\left(-x^8+2x^3-2x\right)^{2/3}\right)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + 5*x^7)*(-2*x + 2*x^3 - x^8)^(1/3))/((2 + x^7)*(2
- 2*x^2 + x^7)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(-2*x + 2*x^3 - x^8)^(1/3))])/2^
(2/3)) - Log[-2*x + 2^(2/3)*(-2*x + 2*x^3 - x^8)^(1/3)]/2^(2/3) + Log[2*x^2
 + 2^(2/3)*x*(-2*x + 2*x^3 - x^8)^(1/3) + 2^(1/3)*(-2*x + 2*x^3 - x^8)^(2/3
)]/(2*2^(2/3))
```

fricas [B] time = 7.94, size = 390, normalized size = 2.60

$$\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(-\frac{4^{\frac{1}{6}}\sqrt{3}\left(6\cdot 4^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(x^{15}-18x^{10}+4x^8+36x^5-36x^3+4x\right)\left(-x^8+2x^3-2x\right)^{\frac{1}{3}}-12(-1)^{\frac{1}{3}}\left(x^{14}-6x^9+4x^7-12x^2+4\right)\left(-x^8+2x^3-2x\right)^{\frac{2}{3}}-4^{\frac{1}{3}}\left(x^{21}-36x^{16}+6x^{14}+180x^{11}-144x^9+12x^7-216x^6+360x^4-144x^2+8\right)\right)}{6\left(x^{21}+6x^{14}-108x^{11}+12x^7+216x^6-216x^4+8\right)}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{6\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^8+2x^3-2x\right)^{\frac{2}{3}}\left(x^7-6x^2+2\right)+4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{14}-18x^9+4x^7+36x^4-36x^2+4\right)+24\left(x^8-3x^3+2x\right)\left(-x^8+2x^3-2x\right)^{\frac{1}{3}}}{x^{14}+4x^7+4}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{3\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^8+2x^3-2x\right)^{\frac{1}{3}}x+4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^7+2\right)+6\left(-x^8+2x^3-2x\right)^{\frac{2}{3}}}{x^7+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^7-4)*(-x^8+2*x^3-2*x)^(1/3)/(x^7+2)/(x^7-2*x^2+2),x, algorit
hm="fricas")
```

```
[Out] 1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(-1/6*4^(1/6)*sqrt(3)*(6*4^(2/3)*(-1)^
(2/3)*(x^15 - 18*x^10 + 4*x^8 + 36*x^5 - 36*x^3 + 4*x)*(-x^8 + 2*x^3 - 2*x)
^(1/3) - 12*(-1)^(1/3)*(x^14 - 6*x^9 + 4*x^7 - 12*x^2 + 4)*(-x^8 + 2*x^3 -
2*x)^(2/3) - 4^(1/3)*(x^21 - 36*x^16 + 6*x^14 + 180*x^11 - 144*x^9 + 12*x^7
 - 216*x^6 + 360*x^4 - 144*x^2 + 8))/(x^21 + 6*x^14 - 108*x^11 + 12*x^7 + 2
16*x^6 - 216*x^4 + 8)) - 1/24*4^(2/3)*(-1)^(1/3)*log(-(6*4^(1/3)*(-1)^(2/3)
*(-x^8 + 2*x^3 - 2*x)^(2/3)*(x^7 - 6*x^2 + 2) + 4^(2/3)*(-1)^(1/3)*(x^14 -
18*x^9 + 4*x^7 + 36*x^4 - 36*x^2 + 4) + 24*(x^8 - 3*x^3 + 2*x)*(-x^8 + 2*x^
3 - 2*x)^(1/3))/(x^14 + 4*x^7 + 4)) + 1/12*4^(2/3)*(-1)^(1/3)*log(-(3*4^(2/
3)*(-1)^(1/3)*(-x^8 + 2*x^3 - 2*x)^(1/3)*x + 4^(1/3)*(-1)^(2/3)*(x^7 + 2) +
 6*(-x^8 + 2*x^3 - 2*x)^(2/3))/(x^7 + 2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^8+2x^3-2x\right)^{\frac{1}{3}}\left(5x^7-4\right)}{\left(x^7-2x^2+2\right)\left(x^7+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^7-4)*(-x^8+2*x^3-2*x)^(1/3)/(x^7+2)/(x^7-2*x^2+2),x, algorit
hm="giac")
```

```
[Out] integrate((-x^8 + 2*x^3 - 2*x)^(1/3)*(5*x^7 - 4)/((x^7 - 2*x^2 + 2)*(x^7 +
2)), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^7-4\right)\left(-x^8+2x^3-2x\right)^{\frac{1}{3}}}{\left(x^7+2\right)\left(x^7-2x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^7-4)*(-x^8+2*x^3-2*x)^(1/3)/(x^7+2)/(x^7-2*x^2+2),x)
```

```
[Out] int((5*x^7-4)*(-x^8+2*x^3-2*x)^(1/3)/(x^7+2)/(x^7-2*x^2+2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^8+2\,x^3-2\,x\right)^{\frac{1}{3}}\left(5\,x^7-4\right)}{\left(x^7-2\,x^2+2\right)\left(x^7+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^7-4)*(-x^8+2*x^3-2*x)^(1/3)/(x^7+2)/(x^7-2*x^2+2),x, algorit
hm="maxima")
```

```
[Out] integrate((-x^8 + 2*x^3 - 2*x)^(1/3)*(5*x^7 - 4)/((x^7 - 2*x^2 + 2)*(x^7 +
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^7-4\right)\,{\left(-x^8+2\,x^3-2\,x\right)}^{1/3}}{\left(x^7+2\right)\,\left(x^7-2\,x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x^7 - 4)*(2*x^3 - 2*x - x^8)^(1/3))/((x^7 + 2)*(x^7 - 2*x^2 + 2)),x
)
```

```
[Out] int(((5*x^7 - 4)*(2*x^3 - 2*x - x^8)^(1/3))/((x^7 + 2)*(x^7 - 2*x^2 + 2)),
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**7-4)*(-x**8+2*x**3-2*x)**(1/3)/(x**7+2)/(x**7-2*x**2+2),x)
```

```
[Out] Timed out
```

3.1731 $\int \frac{\sqrt{-b^4+a^4x^4}\left(b^4+a^4x^4\right)}{b^8+a^8x^8}\,dx$

Optimal. Leaf size=150

$$-\frac{\tan^{-1}\left(\frac{-\frac{a^3x^4}{2^{3/4}b}+\frac{b^3}{2^{3/4}a}+\frac{abx^2}{\sqrt[4]{2}}}{x\sqrt{a^4x^4-b^4}}\right)}{2\,2^{3/4}ab}-\frac{\tanh^{-1}\left(\frac{2^{3/4}abx\sqrt{a^4x^4-b^4}}{a^4x^4+\sqrt{2}\,a^2b^2x^2-b^4}\right)}{2\,2^{3/4}ab}$$

Rubi [C] time = 0.73, antiderivative size = 400, normalized size of antiderivative = 2.67, number of steps used = 18, number of rules used = 7, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6725, 406, 224, 221, 409, 1219, 1218}

$$-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,\Pi\left(\frac{a^6}{\left(-a^8\right)^{3/4}};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}+\frac{\left(a^4-\sqrt{-a^8}\right)b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a^5\sqrt{a^4x^4-b^4}}+\frac{\left(\sqrt{-a^8}+a^4\right)b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a^5\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,\Pi\left(\frac{\sqrt[4]{-a^8}}{a^2};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,\Pi\left(-\frac{\sqrt{-\sqrt{-a^8}}}{a^2};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,\Pi\left(\frac{\sqrt{-\sqrt{-a^8}}}{a^2};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-b^4 + a^4*x^4]*(b^4 + a^4*x^4))/(b^8 + a^8*x^8),x]
```

```
[Out] ((a^4 - Sqrt[-a^8])*b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticF[ArcSin[(a*x)/b], -1
])/(2*a^5*Sqrt[-b^4 + a^4*x^4]) + ((a^4 + Sqrt[-a^8])*b*Sqrt[1 - (a^4*x^4)/
b^4]*EllipticF[ArcSin[(a*x)/b], -1])/(2*a^5*Sqrt[-b^4 + a^4*x^4]) - (b*Sqrt
[1 - (a^4*x^4)/b^4]*EllipticPi[a^6/(-a^8)^(3/4), ArcSin[(a*x)/b], -1])/(2*a
*Sqrt[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[(-a^8)^(1/4)
/a^2, ArcSin[(a*x)/b], -1])/(2*a*Sqrt[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x
^4)/b^4]*EllipticPi[-(Sqrt[-Sqrt[-a^8]]/a^2), ArcSin[(a*x)/b], -1])/(2*a*Sq
rt[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[Sqrt[-Sqrt[-a^8
]]/a^2, ArcSin[(a*x)/b], -1])/(2*a*Sqrt[-b^4 + a^4*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{-b^4+a^4x^4}\,\left(b^4+a^4x^4\right)}{b^8+a^8x^8}\,dx = \int \left(-\frac{\sqrt{-a^8}\left(a^4b^4+\sqrt{-a^8}\,b^4\right)\sqrt{-b^4+a^4x^4}}{2a^8b^4\left(b^4-\sqrt{-a^8}\,x^4\right)}+\frac{\sqrt{-a^8}\left(a^4b^4-\sqrt{-a^8}\,b^4\right)\sqrt{-b\cdots}}{2a^8b^4\left(b^4+\sqrt{-a^8}\,x^4\right.\cdots}\right.$$

$$=\frac{\left(a^4+\sqrt{-a^8}\right)\int\frac{\sqrt{-b^4+a^4x^4}}{b^4+\sqrt{-a^8}\,x^4}\,dx}{2a^4}-\frac{\left(\sqrt{-a^8}\left(a^4b^4+\sqrt{-a^8}\,b^4\right)\right)\int\frac{\sqrt{-b^4+a^4x^4}}{b^4-\sqrt{-a^8}\,x^4}\,dx}{2a^8b^4}$$

$$=\frac{1}{2}\left(1+\frac{a^4}{\sqrt{-a^8}}\right)\int\frac{1}{\sqrt{-b^4+a^4x^4}}\,dx+\frac{\left(a^4+\sqrt{-a^8}\right)\int\frac{1}{\sqrt{-b^4+a^4x^4}}\,dx}{2a^4}-b^4\int\cdots$$

$$=-\left(\frac{1}{2}\int\frac{1}{\left(1-\frac{\sqrt[4]{-a^8}\,x^2}{b^2}\right)\sqrt{-b^4+a^4x^4}}\,dx\right)-\frac{1}{2}\int\frac{1}{\left(1+\frac{\sqrt[4]{-a^8}\,x^2}{b^2}\right)\sqrt{-b^4+a^4x^4}}\,d\cdots$$

$$=\frac{\left(1+\frac{a^4}{\sqrt{-a^8}}\right)b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{-b^4+a^4x^4}}+\frac{\left(a^4+\sqrt{-a^8}\right)b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin\cdots\right.}{2a^5\sqrt{-b^4+a^4x^4}}$$

$$=\frac{\left(1+\frac{a^4}{\sqrt{-a^8}}\right)b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{-b^4+a^4x^4}}+\frac{\left(a^4+\sqrt{-a^8}\right)b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin\cdots\right.}{2a^5\sqrt{-b^4+a^4x^4}}$$

Mathematica [C] time = 0.87, size = 192, normalized size = 1.28

$$-\frac{i\sqrt{1-\frac{a^4x^4}{b^4}}\left(2F\left(i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}\,x\right)\middle|-1\right)-\Pi\left(-\sqrt[4]{-1};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}\,x\right)\middle|-1\right)-\Pi\left(\sqrt[4]{-1};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}\,x\right)\middle|-1\right)-\Pi\left(-(-1)^{3/4};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}\,x\right)\middle|-1\right)-\Pi\left((-1)^{3/4};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}\,x\right)\middle|-1\right)\right)}{2\sqrt{-\frac{a^2}{b^2}}\sqrt{a^4x^4-b^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-b^4 + a^4*x^4]*(b^4 + a^4*x^4))/(b^8 + a^8*x^8),x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (a^4*x^4)/b^4]*(2*EllipticF[I*ArcSinh[Sqrt[-(a^2/b^2)]*x
], -1] - EllipticPi[-(-1)^(1/4), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] - Ellip
ticPi[(-1)^(1/4), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] - EllipticPi[-(-1)^(3/
4), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] - EllipticPi[(-1)^(3/4), I*ArcSinh[S
qrt[-(a^2/b^2)]*x], -1]))/(Sqrt[-(a^2/b^2)]*Sqrt[-b^4 + a^4*x^4])
```

IntegrateAlgebraic [A] time = 0.63, size = 156, normalized size = 1.04

$$\frac{\tanh^{-1}\left(\frac{-\frac{a^3x^4}{2^{3/4}b}+\frac{b^3}{2^{3/4}a}-\frac{abx^2}{\sqrt[4]{2}}}{x\sqrt{a^4x^4-b^4}}\right)}{2\,2^{3/4}ab}-\frac{\tan^{-1}\left(\frac{-\frac{a^3x^4}{2^{3/4}b}+\frac{b^3}{2^{3/4}a}+\frac{abx^2}{\sqrt[4]{2}}}{x\sqrt{a^4x^4-b^4}}\right)}{2\,2^{3/4}ab}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-b^4 + a^4*x^4]*(b^4 + a^4*x^4))/(b^8 + a^8*x^8),x
]
```

```
[Out] -1/2*ArcTan[(b^3/(2^(3/4)*a) + (a*b*x^2)/2^(1/4) - (a^3*x^4)/(2^(3/4)*b))/(
x*Sqrt[-b^4 + a^4*x^4])]/(2^(3/4)*a*b) + ArcTanh[(b^3/(2^(3/4)*a) - (a*b*x^
2)/2^(1/4) - (a^3*x^4)/(2^(3/4)*b))/(x*Sqrt[-b^4 + a^4*x^4])]/(2*2^(3/4)*a*
b)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^4)^(1/2)*(a^4*x^4+b^4)/(a^8*x^8+b^8),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^4x^4+b^4\right)\sqrt{a^4x^4-b^4}}{a^8x^8+b^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^4)^(1/2)*(a^4*x^4+b^4)/(a^8*x^8+b^8),x, algorithm="gia
c")
```

```
[Out] integrate((a^4*x^4 + b^4)*sqrt(a^4*x^4 - b^4)/(a^8*x^8 + b^8), x)
```

maple [B] time = 0.05, size = 289, normalized size = 1.93

$$\frac{\sqrt{2}\,\ln\left(\frac{\frac{a^4x^4-b^4}{2x^2}-\frac{\sqrt{\sqrt{2}\,\sqrt{a^4b^4}}\,\sqrt{a^4x^4-b^4}\,\sqrt{2}}{2x}+\frac{\sqrt{2}\,\sqrt{a^4b^4}}{2}}{\frac{a^4x^4-b^4}{2x^2}+\frac{\sqrt{\sqrt{2}\,\sqrt{a^4b^4}}\,\sqrt{a^4x^4-b^4}\,\sqrt{2}}{2x}+\frac{\sqrt{2}\,\sqrt{a^4b^4}}{2}}\right)}{8\sqrt{\sqrt{2}\,\sqrt{a^4b^4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{a^4x^4-b^4}\,\sqrt{2}}{\sqrt{\sqrt{2}\,\sqrt{a^4b^4}}\,x}+1\right)}{4\sqrt{\sqrt{2}\,\sqrt{a^4b^4}}}-\frac{\sqrt{2}\,\arctan\left(-\frac{\sqrt{a^4x^4-b^4}\,\sqrt{2}}{\sqrt{\sqrt{2}\,\sqrt{a^4b^4}}\,x}+1\right)}{4\sqrt{\sqrt{2}\,\sqrt{a^4b^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^4*x^4-b^4)^(1/2)*(a^4*x^4+b^4)/(a^8*x^8+b^8),x)
```

```
[Out] 1/8*2^(1/2)/(2^(1/2)*(a^4*b^4)^(1/2))^(1/2)*ln((1/2*(a^4*x^4-b^4)/x^2-1/2*(
2^(1/2)*(a^4*b^4)^(1/2))^(1/2)*(a^4*x^4-b^4)^(1/2)*2^(1/2)/x+1/2*2^(1/2)*(a
^4*b^4)^(1/2))/(1/2*(a^4*x^4-b^4)/x^2+1/2*(2^(1/2)*(a^4*b^4)^(1/2))^(1/2)*(
a^4*x^4-b^4)^(1/2)*2^(1/2)/x+1/2*2^(1/2)*(a^4*b^4)^(1/2)))+1/4*2^(1/2)/(2^(
1/2)*(a^4*b^4)^(1/2))^(1/2)*arctan(1/(2^(1/2)*(a^4*b^4)^(1/2))^(1/2)*(a^4*x
^4-b^4)^(1/2)*2^(1/2)/x+1)-1/4*2^(1/2)/(2^(1/2)*(a^4*b^4)^(1/2))^(1/2)*arct
an(-1/(2^(1/2)*(a^4*b^4)^(1/2))^(1/2)*(a^4*x^4-b^4)^(1/2)*2^(1/2)/x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^4x^4+b^4\right)\sqrt{a^4x^4-b^4}}{a^8x^8+b^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^4)^(1/2)*(a^4*x^4+b^4)/(a^8*x^8+b^8),x, algorithm="max
ima")
```

```
[Out] integrate((a^4*x^4 + b^4)*sqrt(a^4*x^4 - b^4)/(a^8*x^8 + b^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^4\,x^4+b^4\right)\,\sqrt{a^4\,x^4-b^4}}{a^8\,x^8+b^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b^4 + a^4*x^4)*(a^4*x^4 - b^4)^(1/2))/(b^8 + a^8*x^8),x)
```

```
[Out] int(((b^4 + a^4*x^4)*(a^4*x^4 - b^4)^(1/2))/(b^8 + a^8*x^8), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(ax-b)(ax+b)\left(a^2x^2+b^2\right)}\left(a^4x^4+b^4\right)}{a^8x^8+b^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**4*x**4-b**4)**(1/2)*(a**4*x**4+b**4)/(a**8*x**8+b**8),x)
```

```
[Out] Integral(sqrt((a*x - b)*(a*x + b)*(a**2*x**2 + b**2))*(a**4*x**4 + b**4)/(a
**8*x**8 + b**8), x)
```

3.1732 $\int \frac{-b^8+a^8x^8}{\sqrt{-b^4+a^4x^4}\left(b^8+a^8x^8\right)}\,dx$

Optimal. Leaf size=150

$$-\frac{\tan^{-1}\left(\frac{-\frac{a^3x^4}{2^{3/4}b}+\frac{b^3}{2^{3/4}a}+\frac{abx^2}{\sqrt[4]{2}}}{x\sqrt{a^4x^4-b^4}}\right)}{2\,2^{3/4}ab}-\frac{\tanh^{-1}\left(\frac{2^{3/4}abx\sqrt{a^4x^4-b^4}}{a^4x^4+\sqrt{2}\,a^2b^2x^2-b^4}\right)}{2\,2^{3/4}ab}$$

Rubi [C] time = 0.65, antiderivative size = 400, normalized size of antiderivative = 2.67, number of steps used = 19, number of rules used = 8, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1586, 6725, 406, 224, 221, 409, 1219, 1218}

$$-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,\Pi\left(\frac{a^6}{\left(-a^8\right)^{3/4}};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}+\frac{\left(a^4-\sqrt{-a^8}\right)b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a^5\sqrt{a^4x^4-b^4}}+\frac{\left(\sqrt{-a^8}+a^4\right)b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a^5\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,\Pi\left(\frac{\sqrt[4]{-a^8}}{a^2};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,\Pi\left(-\frac{\sqrt{-\sqrt{-a^8}}}{a^2};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,\Pi\left(\frac{\sqrt{-\sqrt{-a^8}}}{a^2};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b^8 + a^8*x^8)/(Sqrt[-b^4 + a^4*x^4]*(b^8 + a^8*x^8)),x]
```

```
[Out] ((a^4 - Sqrt[-a^8])*b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticF[ArcSin[(a*x)/b], -1
])/(2*a^5*Sqrt[-b^4 + a^4*x^4]) + ((a^4 + Sqrt[-a^8])*b*Sqrt[1 - (a^4*x^4)/
b^4]*EllipticF[ArcSin[(a*x)/b], -1])/(2*a^5*Sqrt[-b^4 + a^4*x^4]) - (b*Sqrt
[1 - (a^4*x^4)/b^4]*EllipticPi[a^6/(-a^8)^(3/4), ArcSin[(a*x)/b], -1])/(2*a
*Sqrt[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[(-a^8)^(1/4)
/a^2, ArcSin[(a*x)/b], -1])/(2*a*Sqrt[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x
^4)/b^4]*EllipticPi[-(Sqrt[-Sqrt[-a^8]]/a^2), ArcSin[(a*x)/b], -1])/(2*a*Sq
rt[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[Sqrt[-Sqrt[-a^8
]]/a^2, ArcSin[(a*x)/b], -1])/(2*a*Sqrt[-b^4 + a^4*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
```

```
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-b^8+a^8x^8}{\sqrt{-b^4+a^4x^4}\left(b^8+a^8x^8\right)}\,dx = \int \frac{\sqrt{-b^4+a^4x^4}\left(b^4+a^4x^4\right)}{b^8+a^8x^8}\,dx$$

$$= \int \left(-\frac{\sqrt{-a^8}\left(a^4b^4+\sqrt{-a^8}\,b^4\right)\sqrt{-b^4+a^4x^4}}{2a^8b^4\left(b^4-\sqrt{-a^8}\,x^4\right)} + \frac{\sqrt{-a^8}\left(a^4b^4-\sqrt{-a^8}\,b^4\right)\sqrt{-b \ldots}}{2a^8b^4\left(b^4+\sqrt{-a^8}\,x^4\right) \ldots}\right.$$

$$= \frac{\left(a^4+\sqrt{-a^8}\right)\int \frac{\sqrt{-b^4+a^4x^4}}{b^4+\sqrt{-a^8}\,x^4}\,dx}{2a^4} - \frac{\left(\sqrt{-a^8}\left(a^4b^4+\sqrt{-a^8}\,b^4\right)\right)\int \frac{\sqrt{-b^4+a^4x^4}}{b^4-\sqrt{-a^8}\,x^4}\,dx}{2a^8b^4}$$

$$= \frac{1}{2}\left(1+\frac{a^4}{\sqrt{-a^8}}\right)\int \frac{1}{\sqrt{-b^4+a^4x^4}}\,dx + \frac{\left(a^4+\sqrt{-a^8}\right)\int \frac{1}{\sqrt{-b^4+a^4x^4}}\,dx}{2a^4} - b^4\int \ldots$$

$$= -\left(\frac{1}{2}\int \frac{1}{\left(1-\frac{\sqrt[4]{-a^8}\,x^2}{b^2}\right)\sqrt{-b^4+a^4x^4}}\,dx\right) - \frac{1}{2}\int \frac{1}{\left(1+\frac{\sqrt[4]{-a^8}\,x^2}{b^2}\right)\sqrt{-b^4+a^4x^4}}\,d\ldots$$

$$= \frac{\left(1+\frac{a^4}{\sqrt{-a^8}}\right)b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{-b^4+a^4x^4}} + \frac{\left(a^4+\sqrt{-a^8}\right)b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin \ldots\right.}{2a^5\sqrt{-b^4+a^4x^4}}$$

$$= \frac{\left(1+\frac{a^4}{\sqrt{-a^8}}\right)b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{-b^4+a^4x^4}} + \frac{\left(a^4+\sqrt{-a^8}\right)b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin \ldots\right.}{2a^5\sqrt{-b^4+a^4x^4}}$$

Mathematica [C] time = 0.60, size = 192, normalized size = 1.28

$$-\frac{i\sqrt{1-\frac{a^4x^4}{b^4}}\left(2F\left(i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}\,x\right)\middle|-1\right)-\Pi\left(-\sqrt[4]{-1};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}\,x\right)\middle|-1\right)-\Pi\left(\sqrt[4]{-1};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}\,x\right)\middle|-1\right)-\Pi\left(-(-1)^{3/4};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}\,x\right)\middle|-1\right)-\Pi\left((-1)^{3/4};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}\,x\right)\middle|-1\right)\right)}{2\sqrt{-\frac{a^2}{b^2}}\sqrt{a^4x^4-b^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b^8 + a^8*x^8)/(Sqrt[-b^4 + a^4*x^4]*(b^8 + a^8*x^8)),x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (a^4*x^4)/b^4]*(2*EllipticF[I*ArcSinh[Sqrt[-(a^2/b^2)]*x
], -1] - EllipticPi[-(-1)^(1/4), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] - Ellip
ticPi[(-1)^(1/4), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] - EllipticPi[-(-1)^(3/
4), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] - EllipticPi[(-1)^(3/4), I*ArcSinh[S
qrt[-(a^2/b^2)]*x], -1]))/(Sqrt[-(a^2/b^2)]*Sqrt[-b^4 + a^4*x^4])
```

IntegrateAlgebraic [A] time = 0.71, size = 156, normalized size = 1.04

$$\frac{\tanh^{-1}\left(\frac{-\frac{a^3x^4}{2^{3/4}b}+\frac{b^3}{2^{3/4}a}-\frac{abx^2}{\sqrt[4]{2}}}{x\sqrt{a^4x^4-b^4}}\right)}{2\,2^{3/4}ab}-\frac{\tan^{-1}\left(\frac{-\frac{a^3x^4}{2^{3/4}b}+\frac{b^3}{2^{3/4}a}+\frac{abx^2}{\sqrt[4]{2}}}{x\sqrt{a^4x^4-b^4}}\right)}{2\,2^{3/4}ab}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b^8 + a^8*x^8)/(Sqrt[-b^4 + a^4*x^4]*(b^8 + a^8*x^8)),
x]
```

```
[Out] -1/2*ArcTan[(b^3/(2^(3/4)*a) + (a*b*x^2)/2^(1/4) - (a^3*x^4)/(2^(3/4)*b))/(
x*Sqrt[-b^4 + a^4*x^4])]/(2^(3/4)*a*b) + ArcTanh[(b^3/(2^(3/4)*a) - (a*b*x^
2)/2^(1/4) - (a^3*x^4)/(2^(3/4)*b))/(x*Sqrt[-b^4 + a^4*x^4])]/(2*2^(3/4)*a*
b)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^8*x^8-b^8)/(a^4*x^4-b^4)^(1/2)/(a^8*x^8+b^8),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^8x^8-b^8}{\left(a^8x^8+b^8\right)\sqrt{a^4x^4-b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^8*x^8-b^8)/(a^4*x^4-b^4)^(1/2)/(a^8*x^8+b^8),x, algorithm="gia
c")
```

```
[Out] integrate((a^8*x^8 - b^8)/((a^8*x^8 + b^8)*sqrt(a^4*x^4 - b^4)), x)
```

maple [C] time = 0.06, size = 281, normalized size = 1.87

$$\frac{\sqrt{\frac{a^2x^2}{b^2}+1}\,\sqrt{1-\frac{a^2x^2}{b^2}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{a^2}{b^2}},i\right)}{\sqrt{-\frac{a^2}{b^2}}\,\sqrt{a^4x^4-b^4}}-\frac{b^8\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^8a^8+b^8\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{\_\alpha^2\left(a^4\_\alpha^6+b^4x^2\right)a^4}{b^4\sqrt{a^4\_\alpha^4-b^4}\,\sqrt{a^4x^4-b^4}}\right)}{\sqrt{a^4\_\alpha^4-b^4}}+\frac{2\_\alpha^7a^8\sqrt{\frac{a^2x^2}{b^2}+1}\,\sqrt{1-\frac{a^2x^2}{b^2}}\,\mathrm{EllipticPi}\left(x\sqrt{-\frac{a^2}{b^2}},\frac{\_\alpha^6a^6}{b^6},\frac{\sqrt{\frac{a^2}{b^2}}}{\sqrt{-\frac{a^2}{b^2}}}\right)}{\sqrt{-\frac{a^2}{b^2}}\,b^8\sqrt{a^4x^4-b^4}}}{\_\alpha^7}\right)}{8a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^8*x^8-b^8)/(a^4*x^4-b^4)^(1/2)/(a^8*x^8+b^8),x)
```

```
[Out] 1/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4
)^(1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)-1/8*b^8/a^8*sum(1/_alpha^7*(-1/(_al
pha^4*a^4-b^4)^(1/2)*arctanh(_alpha^2/b^4*(_alpha^6*a^4+b^4*x^2)*a^4/(_alph
a^4*a^4-b^4)^(1/2)/(a^4*x^4-b^4)^(1/2))+2/(-a^2/b^2)^(1/2)*_alpha^7*a^8/b^8
*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticPi
(x*(-a^2/b^2)^(1/2),_alpha^6*a^6/b^6,(a^2/b^2)^(1/2)/(-a^2/b^2)^(1/2))),_al
pha=RootOf(_Z^8*a^8+b^8))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^8x^8 - b^8}{\left(a^8x^8 + b^8\right)\sqrt{a^4x^4 - b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^8*x^8-b^8)/(a^4*x^4-b^4)^(1/2)/(a^8*x^8+b^8),x, algorithm="max
ima")
```

```
[Out] integrate((a^8*x^8 - b^8)/((a^8*x^8 + b^8)*sqrt(a^4*x^4 - b^4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{b^8 - a^8\,x^8}{\sqrt{a^4\,x^4 - b^4}\,\left(a^8\,x^8 + b^8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^8 - a^8*x^8)/((a^4*x^4 - b^4)^(1/2)*(b^8 + a^8*x^8)),x)
```

```
[Out] int(-(b^8 - a^8*x^8)/((a^4*x^4 - b^4)^(1/2)*(b^8 + a^8*x^8)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax - b)(ax + b)\left(a^2x^2 + b^2\right)\left(a^4x^4 + b^4\right)}{\sqrt{(ax - b)(ax + b)\left(a^2x^2 + b^2\right)}\left(a^8x^8 + b^8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**8*x**8-b**8)/(a**4*x**4-b**4)**(1/2)/(a**8*x**8+b**8),x)
```

```
[Out] Integral((a*x - b)*(a*x + b)*(a**2*x**2 + b**2)*(a**4*x**4 + b**4)/(sqrt((a
*x - b)*(a*x + b)*(a**2*x**2 + b**2))*(a**8*x**8 + b**8)), x)
```

3.1733 $\quad \int \frac{x^3\sqrt{x+x^2}}{\sqrt{x^2+x\sqrt{x+x^2}}}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{x^2+x}\sqrt{x\left(\sqrt{x^2+x}+x\right)}\left(-1146880x^4-168960x^3+201344x^2-264264x+495495\right)}{5160960x}+\sqrt{x\left(\sqrt{x^2+x}+x\right)}\Bigg(1$$

Rubi [F] time = 1.76, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3\sqrt{x+x^2}}{\sqrt{x^2+x\sqrt{x+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*Sqrt[x + x^2])/Sqrt[x^2 + x*Sqrt[x + x^2]],x]
```

```
[Out] (2*Sqrt[x + x^2]*Defer[Subst][Defer[Int][(x^8*Sqrt[1 + x^2])/Sqrt[x^4 + x^2
*Sqrt[x^2 + x^4]], x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[1 + x])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\sqrt{x+x^2}}{\sqrt{x^2+x\sqrt{x+x^2}}}\,dx &= \frac{\sqrt{x+x^2}\int \frac{x^{7/2}\sqrt{1+x}}{\sqrt{x^2+x\sqrt{x+x^2}}}\,dx}{\sqrt{x}\sqrt{1+x}}\\ &= \frac{\left(2\sqrt{x+x^2}\right)\text{Subst}\left(\int \frac{x^8\sqrt{1+x^2}}{\sqrt{x^4+x^2\sqrt{x^2+x^4}}}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{1+x}}\end{aligned}$$

Mathematica [C] time = 0.57, size = 156, normalized size = 1.04

$$\frac{\left(x+\sqrt{x(x+1)}\right)^4\sqrt{x\left(x+\sqrt{x(x+1)}\right)}\left(x+\sqrt{x(x+1)}+1\right)\left(11\,{}_2F_1\left(-\frac{9}{2},3;-\frac{7}{2};1+\frac{1}{2\left(x+\sqrt{x(x+1)}\right)}\right)+72x\left(32x^3+4\left(8\sqrt{x(x+1)}+13\right)x^2+\left(36\sqrt{x(x+1)}+19\right)x+5\sqrt{x(x+1)}\right)\right)}{1152\sqrt{x(x+1)}\left(2x+2\sqrt{x(x+1)}+1\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[x + x^2])/Sqrt[x^2 + x*Sqrt[x + x^2]],x]
```

```
[Out] ((x + Sqrt[x*(1 + x)])^4*Sqrt[x*(x + Sqrt[x*(1 + x)])]*(1 + x + Sqrt[x*(1 +
 x)])*(72*x*(32*x^3 + 5*Sqrt[x*(1 + x)] + 4*x^2*(13 + 8*Sqrt[x*(1 + x)]) +
x*(19 + 36*Sqrt[x*(1 + x)])) + 11*Hypergeometric2F1[-9/2, 3, -7/2, 1 + 1/(2
*(x + Sqrt[x*(1 + x)]))]))/(1152*Sqrt[x*(1 + x)]*(1 + 2*x + 2*Sqrt[x*(1 + x
)])^5)
```

IntegrateAlgebraic [A] time = 5.12, size = 150, normalized size = 1.00

$$\frac{\sqrt{x^2+x}\sqrt{x\left(\sqrt{x^2+x}+x\right)}\left(-1146880x^4-168960x^3+201344x^2-264264x+495495\right)}{5160960x}+\sqrt{x\left(\sqrt{x^2+x}+x\right)}\left(\frac{1146880x^4+1387520x^3-18304x^2+37752x-165165}{5160960}-\frac{1573\sqrt{\sqrt{x^2+x}-x}\tanh^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+x}-x}\right)}{16384\sqrt{2}\,x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[x + x^2])/Sqrt[x^2 + x*Sqrt[x + x^2]],x]
```

```
[Out] (Sqrt[x + x^2]*(495495 - 264264*x + 201344*x^2 - 168960*x^3 - 1146880*x^4)*
Sqrt[x*(x + Sqrt[x + x^2])])/(5160960*x) + Sqrt[x*(x + Sqrt[x + x^2])]*((-1
65165 + 37752*x - 18304*x^2 + 1387520*x^3 + 1146880*x^4)/5160960 - (1573*Sq
rt[-x + Sqrt[x + x^2]]*ArcTanh[Sqrt[2]*Sqrt[-x + Sqrt[x + x^2]]])/(16384*Sq
rt[2]*x))
```

fricas [A] time = 0.58, size = 138, normalized size = 0.92

$$\frac{495495 \sqrt{2} x \log\left(\frac{4 x^2-2 \sqrt{x^2+\sqrt{x^2+x} x}\left(\sqrt{2} x+\sqrt{2} \sqrt{x^2+x}\right)+4 \sqrt{x^2+x} x+x}{x}\right)+4 \left(1146880 x^5+1387520 x^4-18304 x^3+37752 x^2-\left(1146880 x^4+168960 x^3-201344 x^2+264264 x-495495\right) \sqrt{x^2+x}-165165 x\right) \sqrt{x^2+\sqrt{x^2+x} x}}{20643840 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/20643840*(495495*sqrt(2)*x*log((4*x^2 - 2*sqrt(x^2 + sqrt(x^2 + x)*x)*(sq
rt(2)*x + sqrt(2)*sqrt(x^2 + x)) + 4*sqrt(x^2 + x)*x + x)/x) + 4*(1146880*x
^5 + 1387520*x^4 - 18304*x^3 + 37752*x^2 - (1146880*x^4 + 168960*x^3 - 2013
44*x^2 + 264264*x - 495495)*sqrt(x^2 + x) - 165165*x)*sqrt(x^2 + sqrt(x^2 +
 x)*x))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+x}\, x^3}{\sqrt{x^2+\sqrt{x^2+x}\, x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(x^2 + x)*x^3/sqrt(x^2 + sqrt(x^2 + x)*x), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt{x^2+x}}{\sqrt{x^2+x\sqrt{x^2+x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x)
```

```
[Out] int(x^3*(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+x}\, x^3}{\sqrt{x^2+\sqrt{x^2+x}\, x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x)^(1/2)/(x^2+x*(x^2+x)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(x^2 + x)*x^3/sqrt(x^2 + sqrt(x^2 + x)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{x^2+x}}{\sqrt{x^2+x\,\sqrt{x^2+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x + x^2)^(1/2))/(x^2 + x*(x + x^2)^(1/2))^(1/2),x)
[Out] int((x^3*(x + x^2)^(1/2))/(x^2 + x*(x + x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{x\,(x+1)}}{\sqrt{x\left(x+\sqrt{x^2+x}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+x)**(1/2)/(x**2+x*(x**2+x)**(1/2))**(1/2),x)
[Out] Integral(x**3*sqrt(x*(x + 1))/sqrt(x*(x + sqrt(x**2 + x))), x)
```

3.1734 $\int \frac{(-b+ax^2)^{3/4}(3b+2ax^2)}{x}\,dx$

Optimal. Leaf size=151

$$\frac{3b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}-\sqrt{b}}\right)}{\sqrt{2}}+\frac{3b^{7/4}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}}+\frac{2}{7}\left(ax^2-b\right)^{3/4}\left(2ax^2+5b\right)$$

Rubi [A] time = 0.24, antiderivative size = 230, normalized size of antiderivative = 1.52, number of steps used = 13, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {446, 80, 50, 63, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)}{2\sqrt{2}}+\frac{3b^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)}{2\sqrt{2}}+\frac{3b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}\right)}{\sqrt{2}}-\frac{3b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}}+2b\left(ax^2-b\right)^{3/4}+\frac{4}{7}\left(ax^2-b\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a*x^2)^(3/4)*(3*b + 2*a*x^2))/x,x]
```

```
[Out] 2*b*(-b + a*x^2)^(3/4) + (4*(-b + a*x^2)^(7/4))/7 + (3*b^(7/4)*ArcTan[1 - (
Sqrt[2]*(-b + a*x^2)^(1/4))/b^(1/4)])/Sqrt[2] - (3*b^(7/4)*ArcTan[1 + (Sqrt
[2]*(-b + a*x^2)^(1/4))/b^(1/4)])/Sqrt[2] - (3*b^(7/4)*Log[Sqrt[b] - Sqrt[2
]*b^(1/4)*(-b + a*x^2)^(1/4) + Sqrt[-b + a*x^2]])/(2*Sqrt[2]) + (3*b^(7/4)*
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4) + Sqrt[-b + a*x^2]])/(2*Sq
rt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-b+ax^2\right)^{3/4}\left(3b+2ax^2\right)}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(-b+ax)^{3/4}(3b+2ax)}{x}\,dx,x,x^2\right)\\
&= \frac{4}{7}\left(-b+ax^2\right)^{7/4}+\frac{1}{2}(3b)\operatorname{Subst}\left(\int \frac{(-b+ax)^{3/4}}{x}\,dx,x,x^2\right)\\
&= 2b\left(-b+ax^2\right)^{3/4}+\frac{4}{7}\left(-b+ax^2\right)^{7/4}-\frac{1}{2}\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{-b+ax}}\,dx,x,x\right.\\
&= 2b\left(-b+ax^2\right)^{3/4}+\frac{4}{7}\left(-b+ax^2\right)^{7/4}-\frac{\left(6b^2\right)\operatorname{Subst}\left(\int \frac{x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{-b+ax^2}\right.}{a}\\
&= 2b\left(-b+ax^2\right)^{3/4}+\frac{4}{7}\left(-b+ax^2\right)^{7/4}+\frac{\left(3b^2\right)\operatorname{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{-b+ax}\right.}{a}\\
&= 2b\left(-b+ax^2\right)^{3/4}+\frac{4}{7}\left(-b+ax^2\right)^{7/4}-\frac{\left(3b^{7/4}\right)\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx,x,\right.}{2\sqrt{2}}\\
&= 2b\left(-b+ax^2\right)^{3/4}+\frac{4}{7}\left(-b+ax^2\right)^{7/4}-\frac{3b^{7/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^2}+\right.}{2\sqrt{2}}\\
&= 2b\left(-b+ax^2\right)^{3/4}+\frac{4}{7}\left(-b+ax^2\right)^{7/4}+\frac{3b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)}{\sqrt{2}}-\frac{3b^{7/4}\,\mathrm{t}}{}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 91, normalized size = 0.60

$$\frac{2}{7}\left(ax^2-b\right)^{3/4}\left(2ax^2+5b\right)+3b(-b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{ax^2-b}}{\sqrt[4]{-b}}\right)+3(-b)^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{ax^2-b}}{\sqrt[4]{-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + a*x^2)^(3/4)*(3*b + 2*a*x^2))/x,x]
```

```
[Out] (2*(-b + a*x^2)^(3/4)*(5*b + 2*a*x^2))/7 + 3*(-b)^(3/4)*b*ArcTan[(-b + a*x^
2)^(1/4)/(-b)^(1/4)] + 3*(-b)^(7/4)*ArcTanh[(-b + a*x^2)^(1/4)/(-b)^(1/4)]
```

IntegrateAlgebraic [A] time = 0.24, size = 150, normalized size = 0.99

$$-\frac{3b^{7/4}\tan^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^2-b}}\right)}{\sqrt{2}}+\frac{3b^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}+\sqrt{b}}\right)}{\sqrt{2}}+\frac{2}{7}\left(ax^2-b\right)^{3/4}\left(2ax^2+5b\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^2)^(3/4)*(3*b + 2*a*x^2))/x,x]
```

```
[Out] (2*(-b + a*x^2)^(3/4)*(5*b + 2*a*x^2))/7 - (3*b^(7/4)*ArcTan[(-(b^(1/4)/Sqr
t[2]) + Sqrt[-b + a*x^2]/(Sqrt[2]*b^(1/4)))/(-b + a*x^2)^(1/4)])/Sqrt[2] +
(3*b^(7/4)*ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4))/(Sqrt[b] + Sqrt[-b
+ a*x^2])])/Sqrt[2]
```

fricas [A] time = 0.76, size = 173, normalized size = 1.15

$$\frac{2}{7}\left(2\,ax^2+5\,b\right)\left(ax^2-b\right)^{\frac{3}{4}}+6\left(-b^7\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-b^7\right)^{\frac{1}{4}}\left(ax^2-b\right)^{\frac{1}{4}}b^5-\sqrt{\sqrt{ax^2-b}\,b^{10}-\sqrt{-b^7}\,b^7}\left(-b^7\right)^{\frac{1}{4}}}{b^7}\right)-\frac{3}{2}\left(-b^7\right)^{\frac{1}{4}}\log\left(27\left(ax^2-b\right)^{\frac{1}{4}}b^5+27\left(-b^7\right)^{\frac{3}{4}}\right)+\frac{3}{2}\left(-b^7\right)^{\frac{1}{4}}\log\left(27\left(ax^2-b\right)^{\frac{1}{4}}b^5-27\left(-b^7\right)^{\frac{3}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)^(3/4)*(2*a*x^2+3*b)/x,x, algorithm="fricas")
```

```
[Out] 2/7*(2*a*x^2 + 5*b)*(a*x^2 - b)^(3/4) + 6*(-b^7)^(1/4)*arctan(-((-b^7)^(1/4
)*(a*x^2 - b)^(1/4)*b^5 - sqrt(sqrt(a*x^2 - b)*b^10 - sqrt(-b^7)*b^7)*(-b^7
)^(1/4))/b^7) - 3/2*(-b^7)^(1/4)*log(27*(a*x^2 - b)^(1/4)*b^5 + 27*(-b^7)^(
3/4)) + 3/2*(-b^7)^(1/4)*log(27*(a*x^2 - b)^(1/4)*b^5 - 27*(-b^7)^(3/4))
```

giac [A] time = 0.26, size = 189, normalized size = 1.25

$$-\frac{3}{2}\sqrt{2}b^{\frac{7}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)-\frac{3}{2}\sqrt{2}b^{\frac{7}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)+\frac{3}{4}\sqrt{2}b^{\frac{7}{4}}\log\left(\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)-\frac{3}{4}\sqrt{2}b^{\frac{7}{4}}\log\left(-\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)+\frac{4}{7}\left(ax^2-b\right)^{\frac{7}{4}}+2\left(ax^2-b\right)^{\frac{3}{4}}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)^(3/4)*(2*a*x^2+3*b)/x,x, algorithm="giac")
```

```
[Out] -3/2*sqrt(2)*b^(7/4)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^2 - b)^(1
/4))/b^(1/4)) - 3/2*sqrt(2)*b^(7/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
2*(a*x^2 - b)^(1/4))/b^(1/4)) + 3/4*sqrt(2)*b^(7/4)*log(sqrt(2)*(a*x^2 - b)
^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b)) - 3/4*sqrt(2)*b^(7/4)*log(-sqrt
(2)*(a*x^2 - b)^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b)) + 4/7*(a*x^2 - b
)^(7/4) + 2*(a*x^2 - b)^(3/4)*b
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{2}-b\right)^{\frac{3}{4}}\left(2a\,x^{2}+3b\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)^(3/4)*(2*a*x^2+3*b)/x,x)
```

```
[Out] int((a*x^2-b)^(3/4)*(2*a*x^2+3*b)/x,x)
```

maxima [A] time = 0.41, size = 195, normalized size = 1.29

$$-\frac{1}{4}\left(3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^2-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^2-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}\right)b-8\left(ax^2-b\right)^{\frac{3}{4}}\right)b+\frac{4}{7}\left(ax^2-b\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)^(3/4)*(2*a*x^2+3*b)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^2 - b)^(1/4
))/b^(1/4))/b^(1/4) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a
*x^2 - b)^(1/4))/b^(1/4))/b^(1/4) - sqrt(2)*log(sqrt(2)*(a*x^2 - b)^(1/4)*b
^(1/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(1/4) + sqrt(2)*log(-sqrt(2)*(a*x^2 -
 b)^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(1/4))*b - 8*(a*x^2 - b)^(
3/4))*b + 4/7*(a*x^2 - b)^(7/4)
```

mupad [B] time = 1.24, size = 78, normalized size = 0.52

$$\frac{4\left(a\,x^2-b\right)^{7/4}}{7}-3\,{\left(-b\right)}^{7/4}\,\mathrm{atan}\left(\frac{{\left(a\,x^2-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)+3\,{\left(-b\right)}^{7/4}\,\mathrm{atanh}\left(\frac{{\left(a\,x^2-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)+2\,b\,{\left(a\,x^2-b\right)}^{3/4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^2 - b)^(3/4)*(3*b + 2*a*x^2))/x,x)
```

```
[Out] (4*(a*x^2 - b)^(7/4))/7 - 3*(-b)^(7/4)*atan((a*x^2 - b)^(1/4)/(-b)^(1/4)) +
 3*(-b)^(7/4)*atanh((a*x^2 - b)^(1/4)/(-b)^(1/4)) + 2*b*(a*x^2 - b)^(3/4)
```

sympy [A] time = 10.02, size = 80, normalized size = 0.53

$$-\frac{3a^{\frac{3}{4}}bx^{\frac{3}{2}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^2}\right)}{2\Gamma\left(\frac{1}{4}\right)}+2a\left(\begin{cases}\frac{x^2(-b)^{\frac{3}{4}}}{2} & \text{for } a=0\\ \frac{2\left(ax^2-b\right)^{\frac{7}{4}}}{7a} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)**(3/4)*(2*a*x**2+3*b)/x,x)
```

```
[Out] -3*a**(3/4)*b*x**(3/2)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), b*exp_polar(
2*I*pi)/(a*x**2))/(2*gamma(1/4)) + 2*a*Piecewise((x**2*(-b)**(3/4)/2, Eq(a,
 0)), (2*(a*x**2 - b)**(7/4)/(7*a), True))
```

3.1735 $\int \frac{(-6+x^2)(2-x^2+x^3)^{2/3}}{x^3(-2+x^2+x^3)} \, dx$

Optimal. Leaf size=151

$$-\frac{3\left(x^3-x^2+2\right)^{2/3}}{2x^2}-2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^3-x^2+2}-2x\right)+\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^3-x^2+2}\,x+\sqrt[3]{2}\left(x^3-x^2+2\right)^{2/3}\right)}{\sqrt[3]{2}}+2^{2/}$$

Rubi [F] time = 2.88, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-6+x^2\right)\left(2-x^2+x^3\right)^{2/3}}{x^3\left(-2+x^2+x^3\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[((-6 + x^2)*(2 - x^2 + x^3)^(2/3))/(x^3*(-2 + x^2 + x^3)),x]
```

```
[Out] ((9*I)*(2 - x^2 + x^3)^(2/3)*Defer[Subst][Defer[Int][(((1 + (26 - 15*Sqrt[3
])^(2/3))/(3*(26 - 15*Sqrt[3])^(1/3)) + x)^(2/3)*((-1 + (26 - 15*Sqrt[3])^(
-2/3) + (26 - 15*Sqrt[3])^(2/3))/9 - ((1 + (26 - 15*Sqrt[3])^(2/3))*x)/(3*(
26 - 15*Sqrt[3])^(1/3)) + x^2)^(2/3))/((-8/3 + 2*I) - 2*x), x], x, -1/3 + x
])/((-1 + (26 - 15*Sqrt[3])^(-1/3) + (26 - 15*Sqrt[3])^(1/3) + 3*x)^(2/3)*(
-1 + (26 - 15*Sqrt[3])^(-2/3) + (26 - 15*Sqrt[3])^(2/3) + ((1 + (26 - 15*Sq
rt[3])^(2/3))*(1 - 3*x))/(26 - 15*Sqrt[3])^(1/3) + (-1 + 3*x)^2)^(2/3)) + (
9*(2 - x^2 + x^3)^(2/3)*Defer[Subst][Defer[Int][(((1 + (26 - 15*Sqrt[3])^(2
/3))/(3*(26 - 15*Sqrt[3])^(1/3)) + x)^(2/3)*((-1 + (26 - 15*Sqrt[3])^(-2/3)
 + (26 - 15*Sqrt[3])^(2/3))/9 - ((1 + (26 - 15*Sqrt[3])^(2/3))*x)/(3*(26 -
15*Sqrt[3])^(1/3)) + x^2)^(2/3))/(2/3 - x), x], x, -1/3 + x])/((-1 + (26 -
15*Sqrt[3])^(-1/3) + (26 - 15*Sqrt[3])^(1/3) + 3*x)^(2/3)*(-1 + (26 - 15*Sq
rt[3])^(-2/3) + (26 - 15*Sqrt[3])^(2/3) + ((1 + (26 - 15*Sqrt[3])^(2/3))*(1
 - 3*x))/(26 - 15*Sqrt[3])^(1/3) + (-1 + 3*x)^2)^(2/3)) + (27*(2 - x^2 + x^
3)^(2/3)*Defer[Subst][Defer[Int][(((1 + (26 - 15*Sqrt[3])^(2/3))/(3*(26 - 1
5*Sqrt[3])^(1/3)) + x)^(2/3)*((-1 + (26 - 15*Sqrt[3])^(-2/3) + (26 - 15*Sqr
t[3])^(2/3))/9 - ((1 + (26 - 15*Sqrt[3])^(2/3))*x)/(3*(26 - 15*Sqrt[3])^(1/
3)) + x^2)^(2/3))/(1/3 + x)^3, x], x, -1/3 + x])/((-1 + (26 - 15*Sqrt[3])^(
-1/3) + (26 - 15*Sqrt[3])^(1/3) + 3*x)^(2/3)*(-1 + (26 - 15*Sqrt[3])^(-2/3)
 + (26 - 15*Sqrt[3])^(2/3) + ((1 + (26 - 15*Sqrt[3])^(2/3))*(1 - 3*x))/(26
- 15*Sqrt[3])^(1/3) + (-1 + 3*x)^2)^(2/3)) + (9*(2 - x^2 + x^3)^(2/3)*Defer
[Subst][Defer[Int][(((1 + (26 - 15*Sqrt[3])^(2/3))/(3*(26 - 15*Sqrt[3])^(1/
3)) + x)^(2/3)*((-1 + (26 - 15*Sqrt[3])^(-2/3) + (26 - 15*Sqrt[3])^(2/3))/9
 - ((1 + (26 - 15*Sqrt[3])^(2/3))*x)/(3*(26 - 15*Sqrt[3])^(1/3)) + x^2)^(2/
3))/(1/3 + x), x], x, -1/3 + x])/((-1 + (26 - 15*Sqrt[3])^(-1/3) + (26 - 15
*Sqrt[3])^(1/3) + 3*x)^(2/3)*(-1 + (26 - 15*Sqrt[3])^(-2/3) + (26 - 15*Sqrt
[3])^(2/3) + ((1 + (26 - 15*Sqrt[3])^(2/3))*(1 - 3*x))/(26 - 15*Sqrt[3])^(1
/3) + (-1 + 3*x)^2)^(2/3)) + ((9*I)*(2 - x^2 + x^3)^(2/3)*Defer[Subst][Defe
r[Int][(((1 + (26 - 15*Sqrt[3])^(2/3))/(3*(26 - 15*Sqrt[3])^(1/3)) + x)^(2/
3)*((-1 + (26 - 15*Sqrt[3])^(-2/3) + (26 - 15*Sqrt[3])^(2/3))/9 - ((1 + (26
 - 15*Sqrt[3])^(2/3))*x)/(3*(26 - 15*Sqrt[3])^(1/3)) + x^2)^(2/3))/((8/3 +
2*I) + 2*x), x], x, -1/3 + x])/((-1 + (26 - 15*Sqrt[3])^(-1/3) + (26 - 15*S
qrt[3])^(1/3) + 3*x)^(2/3)*(-1 + (26 - 15*Sqrt[3])^(-2/3) + (26 - 15*Sqrt[3
])^(2/3) + ((1 + (26 - 15*Sqrt[3])^(2/3))*(1 - 3*x))/(26 - 15*Sqrt[3])^(1/3
) + (-1 + 3*x)^2)^(2/3))
```

Rubi steps

$$\int \frac{(-6+x^2)(2-x^2+x^3)^{2/3}}{x^3(-2+x^2+x^3)}\,dx = \int \left(\frac{(2-x^2+x^3)^{2/3}}{1-x} + \frac{3(2-x^2+x^3)^{2/3}}{x^3} + \frac{(2-x^2+x^3)^{2/3}}{x} + \frac{(2-x^2+x^3}{2+2x+x}\right.$$

$$= 3\int \frac{(2-x^2+x^3)^{2/3}}{x^3}\,dx + \int \frac{(2-x^2+x^3)^{2/3}}{1-x}\,dx + \int \frac{(2-x^2+x^3)^{2/3}}{x}\,dx +$$

$$= 3\,\text{Subst}\left(\int \frac{\left(\frac{52}{27}-\frac{x}{3}+x^3\right)^{2/3}}{\left(\frac{1}{3}+x\right)^3}\,dx, x, -\frac{1}{3}+x\right) + \int \left(\frac{i(2-x^2+x^3)^{2/3}}{(-2+2i)-2x} + \frac{i(2}{(2}\right.$$

$$= i\int \frac{(2-x^2+x^3)^{2/3}}{(-2+2i)-2x}\,dx + i\int \frac{(2-x^2+x^3)^{2/3}}{(2+2i)+2x}\,dx + \frac{\left(3\sqrt[3]{3}\,(2-x^2+x^3\right.}{\left(\frac{1}{3}\left(-1+\frac{1}{\sqrt[3]{26-15\sqrt{3}}}+\right.\right.}$$

$$= i\,\text{Subst}\left(\int \frac{\left(\frac{52}{27}-\frac{x}{3}+x^3\right)^{2/3}}{\left(-\frac{8}{3}+2i\right)-2x}\,dx, x, -\frac{1}{3}+x\right) + i\,\text{Subst}\left(\int \frac{\left(\frac{52}{27}-\frac{x}{3}+x^3\right)^{2/3}}{\left(\frac{8}{3}+2i\right)+2x}\right.$$

$$= \frac{\left(3i\sqrt[3]{3}\,(2-x^2+x^3)^{2/3}\right)\text{Subst}\left(\int \frac{\left(\frac{1+(26-15\sqrt{3})^{2/3}}{3\sqrt[3]{26-15\sqrt{3}}}+x\right)^{2/3}\left(\frac{1}{9}\left(-1+\frac{1}{(26-15\sqrt{3})^{2/3}}+(26\right.\right.}{\left(-\frac{8}{3}+2i\right)-}\right.}{\left(\frac{1}{3}\left(-1+\frac{1}{\sqrt[3]{26-15\sqrt{3}}}+\sqrt[3]{26-15\sqrt{3}}\right)+x\right)^{2/3}\left(-1+\frac{1}{(26-15\sqrt{3})^{2/3}}+(26-15\right.}$$

Mathematica [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{(-6+x^2)(2-x^2+x^3)^{2/3}}{x^3(-2+x^2+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-6 + x^2)*(2 - x^2 + x^3)^(2/3))/(x^3*(-2 + x^2 + x^3)),x]
[Out] Integrate[((-6 + x^2)*(2 - x^2 + x^3)^(2/3))/(x^3*(-2 + x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.48, size = 151, normalized size = 1.00

$$-\frac{3(x^3-x^2+2)^{2/3}}{2x^2} - 2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^3-x^2+2}-2x\right) + \frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^3-x^2+2}\,x+\sqrt[3]{2}\,(x^3-x^2+2)^{2/3}\right)}{\sqrt[3]{2}} + 2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2^{2/3}\sqrt[3]{x^3-x^2+2}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-6 + x^2)*(2 - x^2 + x^3)^(2/3))/(x^3*(-2 + x^2 + x^3)
),x]
```

```
[Out] (-3*(2 - x^2 + x^3)^(2/3))/(2*x^2) + 2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x
 + 2^(2/3)*(2 - x^2 + x^3)^(1/3))] - 2^(2/3)*Log[-2*x + 2^(2/3)*(2 - x^2 + x
^3)^(1/3)] + Log[2*x^2 + 2^(2/3)*x*(2 - x^2 + x^3)^(1/3) + 2^(1/3)*(2 - x^2
 + x^3)^(2/3)]/2^(1/3)
```

fricas [B] time = 15.18, size = 420, normalized size = 2.78

$$-\frac{2\sqrt{3}\,(-4)^{\frac{1}{3}}x^2\arctan\left(\frac{3\sqrt{3}(-4)^{\frac{2}{3}}\left(5x^7+4x^6-x^5-8x^4+4x^3-4x\right)\left(x^3-x^2+2\right)^{\frac{2}{3}}+6\sqrt{3}(-4)^{\frac{1}{3}}\left(19x^8-16x^7+x^6+32x^5-4x^4+4x^2\right)\left(x^3-x^2+2\right)^{\frac{1}{3}}-\sqrt{3}\left(71x^9-111x^8+33x^7+221x^6-132x^5+6x^4+132x^3-12x^2+8\right)}{3\left(109x^9-105x^8+3x^7+211x^6-12x^5-6x^4+12x^3+12x^2-8\right)}\right)-2(-4)^{\frac{1}{3}}x^2\log\left(-\frac{3(-4)^{\frac{2}{3}}\left(x^3-x^2+2\right)^{\frac{1}{3}}x^2-6\left(x^3-x^2+2\right)^{\frac{2}{3}}x+(-4)^{\frac{1}{3}}\left(x^3+x^2-2\right)}{x^3+x^2-2}\right)+(-4)^{\frac{1}{3}}x^2\log\left(-\frac{6(-4)^{\frac{1}{3}}\left(5x^4-x^3+2x\right)\left(x^3-x^2+2\right)^{\frac{2}{3}}-(-4)^{\frac{2}{3}}\left(19x^6-16x^5+x^4+32x^3-4x^2+4\right)-24\left(2x^5-x^4+2x^2\right)\left(x^3-x^2+2\right)^{\frac{1}{3}}}{x^6+2x^5+x^4-4x^3-4x^2+4}\right)+9\left(x^3-x^2+2\right)^{\frac{2}{3}}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-6)*(x^3-x^2+2)^(2/3)/x^3/(x^3+x^2-2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-4)^(1/3)*x^2*arctan(1/3*(3*sqrt(3)*(-4)^(2/3)*(5*x^7 + 4*
x^6 - x^5 - 8*x^4 + 4*x^3 - 4*x)*(x^3 - x^2 + 2)^(2/3) + 6*sqrt(3)*(-4)^(1/
3)*(19*x^8 - 16*x^7 + x^6 + 32*x^5 - 4*x^4 + 4*x^2)*(x^3 - x^2 + 2)^(1/3) -
 sqrt(3)*(71*x^9 - 111*x^8 + 33*x^7 + 221*x^6 - 132*x^5 + 6*x^4 + 132*x^3 -
 12*x^2 + 8))/(109*x^9 - 105*x^8 + 3*x^7 + 211*x^6 - 12*x^5 - 6*x^4 + 12*x^
3 + 12*x^2 - 8)) - 2*(-4)^(1/3)*x^2*log(-(3*(-4)^(2/3)*(x^3 - x^2 + 2)^(1/3
)*x^2 - 6*(x^3 - x^2 + 2)^(2/3)*x + (-4)^(1/3)*(x^3 + x^2 - 2))/(x^3 + x^2
- 2)) + (-4)^(1/3)*x^2*log(-(6*(-4)^(1/3)*(5*x^4 - x^3 + 2*x)*(x^3 - x^2 +
2)^(2/3) - (-4)^(2/3)*(19*x^6 - 16*x^5 + x^4 + 32*x^3 - 4*x^2 + 4) - 24*(2*
x^5 - x^4 + 2*x^2)*(x^3 - x^2 + 2)^(1/3))/(x^6 + 2*x^5 + x^4 - 4*x^3 - 4*x^
2 + 4)) + 9*(x^3 - x^2 + 2)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-x^2+2\right)^{\frac{2}{3}}\left(x^2-6\right)}{\left(x^3+x^2-2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-6)*(x^3-x^2+2)^(2/3)/x^3/(x^3+x^2-2),x, algorithm="giac")
```

```
[Out] integrate((x^3 - x^2 + 2)^(2/3)*(x^2 - 6)/((x^3 + x^2 - 2)*x^3), x)
```

maple [C] time = 4.93, size = 661, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-6)*(x^3-x^2+2)^(2/3)/x^3/(x^3+x^2-2),x)
```

```
[Out] -3/2*(x^3-x^2+2)^(2/3)/x^2+RootOf(_Z^3+4)*ln((RootOf(RootOf(_Z^3+4)^2+2*_Z*
RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x^3+RootOf(RootOf(_Z^3+4)^2+2*_Z*Ro
otOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-3*(x^3-x^2+2)^(1/3)*RootOf(Root
Of(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*x^2-3*RootOf(_Z^3+4
)*x^3-3*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^3+RootOf(_Z^3
+4)*x^2+RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2-3*(x^3-x^2+
2)^(2/3)*x-2*RootOf(_Z^3+4)-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4
*_Z^2))/(x^2+2*x+2)/(-1+x))+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4
*_Z^2)*ln(-(3*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z
^3+4)^3*x^3+4*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(
_Z^3+4)^2*x^3+4*(x^3-x^2+2)^(2/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+
4)+4*_Z^2)*RootOf(_Z^3+4)^2*x+4*(x^3-x^2+2)^(1/3)*RootOf(_Z^3+4)^2*x^2+10*(
x^3-x^2+2)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf
(_Z^3+4)*x^2+3*RootOf(_Z^3+4)*x^3+4*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^
3+4)+4*_Z^2)*x^3-3*RootOf(_Z^3+4)*x^2-4*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf
```

```
(_Z^3+4)+4*_Z^2)*x^2+2*(x^3-x^2+2)^(2/3)*x+6*RootOf(_Z^3+4)+8*RootOf(RootOf
(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(x^2+2*x+2)/(-1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3 - x^2 + 2\right)^{\frac{2}{3}}\left(x^2 - 6\right)}{\left(x^3 + x^2 - 2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-6)*(x^3-x^2+2)^(2/3)/x^3/(x^3+x^2-2),x, algorithm="maxima")
[Out] integrate((x^3 - x^2 + 2)^(2/3)*(x^2 - 6)/((x^3 + x^2 - 2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2 - 6\right)\,\left(x^3 - x^2 + 2\right)^{2/3}}{x^3\,\left(x^3 + x^2 - 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 6)*(x^3 - x^2 + 2)^(2/3))/(x^3*(x^2 + x^3 - 2)),x)
[Out] int(((x^2 - 6)*(x^3 - x^2 + 2)^(2/3))/(x^3*(x^2 + x^3 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((x+1)\left(x^2 - 2x + 2\right)\right)^{\frac{2}{3}}\left(x^2 - 6\right)}{x^3\,(x-1)\left(x^2 + 2x + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-6)*(x**3-x**2+2)**(2/3)/x**3/(x**3+x**2-2),x)
[Out] Integral(((x + 1)*(x**2 - 2*x + 2))**(2/3)*(x**2 - 6)/(x**3*(x - 1)*(x**2 +
 2*x + 2)), x)
```

3.1736 $\int \frac{x^3(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}\left(ad-dx+x^4\right)}\,dx$

Optimal. Leaf size=151

$$-\frac{\log\left(a^2d^{2/3}\left(x^3-ax^2\right)^{2/3}+a^2\sqrt[3]{d}\,x^2\sqrt[3]{x^3-ax^2}+a^2x^4\right)}{2\sqrt[3]{d}}+\frac{\log\left(ax^2-a\sqrt[3]{d}\,\sqrt[3]{x^3-ax^2}\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{d}\,\sqrt[3]{x^3-ax^2}+x^2}\right.}{\sqrt[3]{d}}$$

Rubi [F] time = 2.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}\left(ad-dx+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*(-4*a + 3*x))/((x^2*(-a + x))^(2/3)*(a*d - d*x + x^4)),x]
```

```
[Out] (-12*a*x^(4/3)*(-a + x)^(2/3)*Defer[Subst][Defer[Int][x^7/((-a + x^3)^(2/3)
*(a*d - d*x^3 + x^12)), x], x, x^(1/3)])/(-((a - x)*x^2))^(2/3) + (9*x^(4/3
)*(-a + x)^(2/3)*Defer[Subst][Defer[Int][x^10/((-a + x^3)^(2/3)*(a*d - d*x^
3 + x^12)), x], x, x^(1/3)])/(-((a - x)*x^2))^(2/3)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}\left(ad-dx+x^4\right)}\,dx &= \frac{\left(x^{4/3}(-a+x)^{2/3}\right)\int \frac{x^{5/3}(-4a+3x)}{(-a+x)^{2/3}\left(ad-dx+x^4\right)}\,dx}{\left(x^2(-a+x)\right)^{2/3}}\\
&= \frac{\left(3x^{4/3}(-a+x)^{2/3}\right)\text{Subst}\left(\int \frac{x^7\left(-4a+3x^3\right)}{\left(-a+x^3\right)^{2/3}\left(ad-dx^3+x^{12}\right)}\,dx,x,\sqrt[3]{x}\right)}{\left(x^2(-a+x)\right)^{2/3}}\\
&= \frac{\left(3x^{4/3}(-a+x)^{2/3}\right)\text{Subst}\left(\int\left(-\frac{4ax^7}{\left(-a+x^3\right)^{2/3}\left(ad-dx^3+x^{12}\right)}+\frac{3x^{10}}{\left(-a+x^3\right)^{2/3}\left(ad-dx^3+x^{12}\right)}\right)\right.}{\left(x^2(-a+x)\right)^{2/3}}\\
&= \frac{\left(9x^{4/3}(-a+x)^{2/3}\right)\text{Subst}\left(\int \frac{x^{10}}{\left(-a+x^3\right)^{2/3}\left(ad-dx^3+x^{12}\right)}\,dx,x,\sqrt[3]{x}\right)}{\left(x^2(-a+x)\right)^{2/3}}-\frac{\left(12ax^{4/3}(-a\right.}{}
\end{aligned}$$

Mathematica [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{x^3(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}\left(ad-dx+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(-4*a + 3*x))/((x^2*(-a + x))^(2/3)*(a*d - d*x + x^4)),x]
```

```
[Out] Integrate[(x^3*(-4*a + 3*x))/((x^2*(-a + x))^(2/3)*(a*d - d*x + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.52, size = 151, normalized size = 1.00

$$-\frac{\log\left(a^2d^{2/3}\left(x^3-ax^2\right)^{2/3}+a^2\sqrt[3]{d}\,x^2\sqrt[3]{x^3-ax^2}+a^2x^4\right)}{2\sqrt[3]{d}}+\frac{\log\left(ax^2-a\sqrt[3]{d}\,\sqrt[3]{x^3-ax^2}\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{d}\,\sqrt[3]{x^3-ax^2}+x^2}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(-4*a + 3*x))/((x^2*(-a + x))^(2/3)*(a*d - d*x + x^
4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*d^(1/3)*(-(a*x^2) + x^3)^(1/3))])/d^
(1/3) + Log[a*x^2 - a*d^(1/3)*(-(a*x^2) + x^3)^(1/3)]/d^(1/3) - Log[a^2*x^4
 + a^2*d^(1/3)*x^2*(-(a*x^2) + x^3)^(1/3) + a^2*d^(2/3)*(-(a*x^2) + x^3)^(2
/3)]/(2*d^(1/3))
```

fricas [A] time = 0.41, size = 338, normalized size = 2.24

$$\left[\frac{\sqrt{3}d\sqrt{-\frac{1}{d^{\frac{2}{3}}}}\log\left(-\frac{x^4-3\left(-ax^2+x^3\right)^{\frac{1}{3}}d^{\frac{1}{3}}x^2-2ad+2dx+\sqrt{3}\left(d^{\frac{1}{3}}x^4+\left(-ax^2+x^3\right)^{\frac{1}{3}}d^{\frac{2}{3}}x^2-2\left(-ax^2+x^3\right)^{\frac{2}{3}}d\right)\sqrt{-\frac{1}{d^{\frac{2}{3}}}}}{x^4+ad-dx}\right)+2d^{\frac{2}{3}}\log\left(\frac{d^{\frac{2}{3}}x^2-\left(-ax^2+x^3\right)^{\frac{1}{3}}d}{x^2}\right)-d^{\frac{2}{3}}\log\left(\frac{d^{\frac{1}{3}}x^4+\left(-ax^2+x^3\right)^{\frac{1}{3}}d^{\frac{2}{3}}x^2+\left(-ax^2+x^3\right)^{\frac{2}{3}}d}{x^4}\right)}{2d},-\frac{2\sqrt{3}d^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}x^2+2\left(-ax^2+x^3\right)^{\frac{1}{3}}d^{\frac{2}{3}}\right)}{3d^{\frac{1}{3}}x^2}\right)-2d^{\frac{2}{3}}\log\left(\frac{d^{\frac{2}{3}}x^2-\left(-ax^2+x^3\right)^{\frac{1}{3}}d}{x^2}\right)+d^{\frac{2}{3}}\log\left(\frac{d^{\frac{1}{3}}x^4+\left(-ax^2+x^3\right)^{\frac{1}{3}}d^{\frac{2}{3}}x^2+\left(-ax^2+x^3\right)^{\frac{2}{3}}d}{x^4}\right)}{2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*a+3*x)/(x^2*(-a+x))^(2/3)/(x^4+a*d-d*x),x, algorithm="fri
cas")
```

```
[Out] [1/2*(sqrt(3)*d*sqrt(-1/d^(2/3))*log(-(x^4 - 3*(-a*x^2 + x^3)^(1/3)*d^(1/3)
*x^2 - 2*a*d + 2*d*x + sqrt(3)*(d^(1/3)*x^4 + (-a*x^2 + x^3)^(1/3)*d^(2/3)*
x^2 - 2*(-a*x^2 + x^3)^(2/3)*d)*sqrt(-1/d^(2/3)))/(x^4 + a*d - d*x)) + 2*d^
(2/3)*log((d^(2/3)*x^2 - (-a*x^2 + x^3)^(1/3)*d)/x^2) - d^(2/3)*log((d^(1/3
)*x^4 + (-a*x^2 + x^3)^(1/3)*d^(2/3)*x^2 + (-a*x^2 + x^3)^(2/3)*d)/x^4))/d,
 -1/2*(2*sqrt(3)*d^(2/3)*arctan(1/3*sqrt(3)*(d^(1/3)*x^2 + 2*(-a*x^2 + x^3)
^(1/3)*d^(2/3))/(d^(1/3)*x^2)) - 2*d^(2/3)*log((d^(2/3)*x^2 - (-a*x^2 + x^3
)^(1/3)*d)/x^2) + d^(2/3)*log((d^(1/3)*x^4 + (-a*x^2 + x^3)^(1/3)*d^(2/3)*x
^2 + (-a*x^2 + x^3)^(2/3)*d)/x^4))/d]
```

giac [A] time = 0.27, size = 235, normalized size = 1.56

$$-\sqrt{3}\left(-\frac{1}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}d\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}+\sqrt{3}a|d|^{\frac{2}{3}}-\sqrt{3}d\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}}{d\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}+a|d|^{\frac{2}{3}}-d\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}}\right)-\frac{1}{2}\left(-\frac{1}{d}\right)^{\frac{1}{3}}\log\left(\frac{3}{4}a^2d^{\frac{4}{3}}+\frac{1}{4}\left(2d\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}-ad^{\frac{2}{3}}-2d\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}\right)^2\right)+\frac{1}{2}\left(-\frac{1}{d}\right)^{\frac{1}{3}}\log\left(\left(\sqrt{3}d\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}+\sqrt{3}a|d|^{\frac{2}{3}}-\sqrt{3}d\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}\right)^2+\left(d\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}+a|d|^{\frac{2}{3}}-d\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*a+3*x)/(x^2*(-a+x))^(2/3)/(x^4+a*d-d*x),x, algorithm="gia
c")
```

```
[Out] -sqrt(3)*(-1/d)^(1/3)*arctan(-(sqrt(3)*d*(-a/x + 1)^(4/3) + sqrt(3)*a*abs(d
)^(2/3) - sqrt(3)*d*(-a/x + 1)^(1/3))/(d*(-a/x + 1)^(4/3) + a*abs(d)^(2/3)
- d*(-a/x + 1)^(1/3))) - 1/2*(-1/d)^(1/3)*log(3/4*a^2*d^(4/3) + 1/4*(2*d*(-
a/x + 1)^(4/3) - a*d^(2/3) - 2*d*(-a/x + 1)^(1/3))^2) + 1/2*(-1/d)^(1/3)*lo
g((sqrt(3)*d*(-a/x + 1)^(4/3) + sqrt(3)*a*abs(d)^(2/3) - sqrt(3)*d*(-a/x +
1)^(1/3))^2 + (d*(-a/x + 1)^(4/3) + a*abs(d)^(2/3) - d*(-a/x + 1)^(1/3))^2)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(-4a+3x\right)}{\left(x^2\left(-a+x\right)\right)^{\frac{2}{3}}\left(x^4+ad-dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-4*a+3*x)/(x^2*(-a+x))^(2/3)/(x^4+a*d-d*x),x)
```

```
[Out] int(x^3*(-4*a+3*x)/(x^2*(-a+x))^(2/3)/(x^4+a*d-d*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(4a-3x)x^3}{(x^4+ad-dx)\left(-(a-x)x^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*a+3*x)/(x^2*(-a+x))^(2/3)/(x^4+a*d-d*x),x, algorithm="max
ima")
```

```
[Out] -integrate((4*a - 3*x)*x^3/((x^4 + a*d - d*x)*(-(a - x)*x^2)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^3\,(4a-3x)}{\left(-x^2\,(a-x)\right)^{2/3}\,\left(x^4-d\,x+a\,d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(4*a - 3*x))/((-x^2*(a - x))^(2/3)*(a*d - d*x + x^4)),x)
```

```
[Out] int(-(x^3*(4*a - 3*x))/((-x^2*(a - x))^(2/3)*(a*d - d*x + x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-4*a+3*x)/(x**2*(-a+x))**(2/3)/(x**4+a*d-d*x),x)
```

```
[Out] Timed out
```

3.1737 $\int \frac{-1+k^4x^4}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(1+k^4x^4\right)}\,dx$

Optimal. Leaf size=151

$$-\frac{\tan^{-1}\left(\frac{\sqrt{k^2-\sqrt{2}k+1}\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{(x-1)\left(k^2x-1\right)}\right)}{\sqrt{k^2-\sqrt{2}k+1}}-\frac{\tan^{-1}\left(\frac{\sqrt{k^2+\sqrt{2}k+1}\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{(x-1)\left(k^2x-1\right)}\right)}{\sqrt{k^2+\sqrt{2}k+1}}$$

Rubi [C] time = 3.84, antiderivative size = 487, normalized size of antiderivative = 3.23, number of steps used = 28, number of rules used = 9, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {6718, 6688, 6725, 714, 115, 934, 12, 168, 537}

$$\frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\Pi\left(\frac{k^2}{\left(-k^4\right)^{3/4}};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{\sqrt{-k^2}\sqrt{x-x^2}\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\Pi\left(\frac{\sqrt[4]{-k^4}}{k^2};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{\sqrt{-k^2}\sqrt{x-x^2}\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\Pi\left(-\frac{\sqrt{-\sqrt{-k^4}}}{k^2};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{\sqrt{-k^2}\sqrt{x-x^2}\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\Pi\left(\frac{\sqrt{-\sqrt{-k^4}}}{k^2};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{\sqrt{-k^2}\sqrt{x-x^2}\sqrt{(1-x)x\left(1-k^2x\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + k^4*x^4)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(1 + k^4*x^4)),x]
```

```
[Out] (2*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticF[ArcSin[Sqrt[x]], k^2])/Sqr
t[(1 - x)*x*(1 - k^2*x)] + ((1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^2*x]*Ellipt
icPi[k^2/(-k^4)^(3/4), ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/(Sqrt[-k^2]*Sq
rt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2]) + ((1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1
 - k^2*x]*EllipticPi[(-k^4)^(1/4)/k^2, ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)]
)/(Sqrt[-k^2]*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2]) + ((1 - x)*Sqrt[-x
]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticPi[-(Sqrt[-Sqrt[-k^4]]/k^2), ArcSin[Sqrt[
-k^2]*Sqrt[-x]], k^(-2)])/(Sqrt[-k^2]*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x -
x^2]) + ((1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticPi[Sqrt[-Sqrt[-k^
4]]/k^2, ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/(Sqrt[-k^2]*Sqrt[(1 - x)*x*(
1 - k^2*x)]*Sqrt[x - x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
```

```
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 714

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c*x)/b]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0]
 && RationalQ[b]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !FreeQ[w, x] &&  !Fr
eeQ[z, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+k^4x^4}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(1+k^4x^4\right)}\,dx = \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{-1+k^4x^4}{\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\left(1+k^4x^4\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{-1+k^4x^4}{\sqrt{1-k^2x}\sqrt{x-x^2}\left(1+k^4x^4\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \left(\frac{1}{\sqrt{1-k^2x}\sqrt{x-x^2}} - \frac{2}{\sqrt{1-k^2x}\sqrt{x-x^2}\left(1+k^4x^4\right)}\right)dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-k^2x}\sqrt{x-x^2}}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \cdots}{\sqrt{(1-x)x(\cdots}}$$

$$= \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\cdots}{\cdots}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \cdots}{\sqrt{(1-x)x(\cdots}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int\left(\cdots\right.}{\cdots}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \cdots}{2\sqrt{(1-x)x(\cdots}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{2-2x}\sqrt{1-x}\sqrt{-x}\sqrt{x}\cdots\right.}{\sqrt{2}\sqrt{(1\cdots}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{2-2x}\sqrt{1-x}\sqrt{-x}\sqrt{x}\cdots\right.}{2\sqrt{(1-\cdots}}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(\sqrt{2-2x}\sqrt{1-x}\sqrt{-x}\sqrt{x}\cdots\right.}{\cdots}$$

$$= \frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\,\Pi\cdots}{\sqrt{-k^2}\sqrt{(1-x\cdots}}$$

Mathematica [C] time = 14.60, size = 10871, normalized size = 71.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + k^4*x^4)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(1 + k^4*x^4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.35, size = 151, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{k^2-\sqrt{2}k+1}\sqrt{k^2x^3+(-k^2-1)x^2+x}}{(x-1)(k^2x-1)}\right)}{\sqrt{k^2-\sqrt{2}k+1}}-\frac{\tan^{-1}\left(\frac{\sqrt{k^2+\sqrt{2}k+1}\sqrt{k^2x^3+(-k^2-1)x^2+x}}{(x-1)(k^2x-1)}\right)}{\sqrt{k^2+\sqrt{2}k+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + k^4*x^4)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(1 + k^4*x^4
)),x]
```

```
[Out] -(ArcTan[(Sqrt[1 - Sqrt[2]*k + k^2]*Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3])/((-
1 + x)*(-1 + k^2*x))]/Sqrt[1 - Sqrt[2]*k + k^2]) - ArcTan[(Sqrt[1 + Sqrt[2]
*k + k^2]*Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3])/((-1 + x)*(-1 + k^2*x))]/Sqrt
[1 + Sqrt[2]*k + k^2]
```

fricas [B] time = 0.55, size = 1197, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^4*x^4-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^4*x^4+1),x, algorithm="
fricas")
```

```
[Out] -1/4*sqrt(-(k^2 + 2*sqrt(1/2)*(k^4 + 1)*sqrt(k^2/(k^8 + 2*k^4 + 1)) + 1)/(k
^4 + 1))*log((k^4*x^4 + 4*k^2*x^2 - 2*(k^4 + k^2)*x^3 + 4*sqrt(1/2)*((k^6 +
 k^2)*x^3 - (k^6 + k^4 + k^2 + 1)*x^2 + (k^4 + 1)*x)*sqrt(k^2/(k^8 + 2*k^4
+ 1)) - 2*(k^2 + 1)*x + 2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(2*k^2*x - (k^4
 + k^2)*x^2 - k^2 + 2*sqrt(1/2)*(k^4 + (k^6 + k^2)*x^2 - (k^6 + k^4 + k^2 +
 1)*x + 1)*sqrt(k^2/(k^8 + 2*k^4 + 1)) - 1)*sqrt(-(k^2 + 2*sqrt(1/2)*(k^4 +
 1)*sqrt(k^2/(k^8 + 2*k^4 + 1)) + 1)/(k^4 + 1)) + 1)/(k^4*x^4 + 1)) + 1/4*s
qrt(-(k^2 + 2*sqrt(1/2)*(k^4 + 1)*sqrt(k^2/(k^8 + 2*k^4 + 1)) + 1)/(k^4 + 1
))*log((k^4*x^4 + 4*k^2*x^2 - 2*(k^4 + k^2)*x^3 + 4*sqrt(1/2)*((k^6 + k^2)*
x^3 - (k^6 + k^4 + k^2 + 1)*x^2 + (k^4 + 1)*x)*sqrt(k^2/(k^8 + 2*k^4 + 1))
- 2*(k^2 + 1)*x - 2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(2*k^2*x - (k^4 + k^2
)*x^2 - k^2 + 2*sqrt(1/2)*(k^4 + (k^6 + k^2)*x^2 - (k^6 + k^4 + k^2 + 1)*x
+ 1)*sqrt(k^2/(k^8 + 2*k^4 + 1)) - 1)*sqrt(-(k^2 + 2*sqrt(1/2)*(k^4 + 1)*sq
rt(k^2/(k^8 + 2*k^4 + 1)) + 1)/(k^4 + 1)) + 1)/(k^4*x^4 + 1)) - 1/4*sqrt(-(
k^2 - 2*sqrt(1/2)*(k^4 + 1)*sqrt(k^2/(k^8 + 2*k^4 + 1)) + 1)/(k^4 + 1))*log
((k^4*x^4 + 4*k^2*x^2 - 2*(k^4 + k^2)*x^3 - 4*sqrt(1/2)*((k^6 + k^2)*x^3 -
(k^6 + k^4 + k^2 + 1)*x^2 + (k^4 + 1)*x)*sqrt(k^2/(k^8 + 2*k^4 + 1)) - 2*(k
^2 + 1)*x + 2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(2*k^2*x - (k^4 + k^2)*x^2
- k^2 - 2*sqrt(1/2)*(k^4 + (k^6 + k^2)*x^2 - (k^6 + k^4 + k^2 + 1)*x + 1)*s
qrt(k^2/(k^8 + 2*k^4 + 1)) - 1)*sqrt(-(k^2 - 2*sqrt(1/2)*(k^4 + 1)*sqrt(k^2
/(k^8 + 2*k^4 + 1)) + 1)/(k^4 + 1)) + 1)/(k^4*x^4 + 1)) + 1/4*sqrt(-(k^2 -
2*sqrt(1/2)*(k^4 + 1)*sqrt(k^2/(k^8 + 2*k^4 + 1)) + 1)/(k^4 + 1))*log((k^4*
x^4 + 4*k^2*x^2 - 2*(k^4 + k^2)*x^3 - 4*sqrt(1/2)*((k^6 + k^2)*x^3 - (k^6 +
 k^4 + k^2 + 1)*x^2 + (k^4 + 1)*x)*sqrt(k^2/(k^8 + 2*k^4 + 1)) - 2*(k^2 + 1
)*x - 2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(2*k^2*x - (k^4 + k^2)*x^2 - k^2
- 2*sqrt(1/2)*(k^4 + (k^6 + k^2)*x^2 - (k^6 + k^4 + k^2 + 1)*x + 1)*sqrt(k^
```

```
2/(k^8 + 2*k^4 + 1)) - 1)*sqrt(-(k^2 - 2*sqrt(1/2)*(k^4 + 1)*sqrt(k^2/(k^8
+ 2*k^4 + 1)) + 1)/(k^4 + 1)) + 1)/(k^4*x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^4x^4-1}{\left(k^4x^4+1\right)\sqrt{\left(k^2x-1\right)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^4*x^4-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^4*x^4+1),x, algorithm="
giac")
```

```
[Out] integrate((k^4*x^4 - 1)/((k^4*x^4 + 1)*sqrt((k^2*x - 1)*(x - 1)*x)), x)
```

maple [C] time = 0.07, size = 255, normalized size = 1.69

$$-\frac{2\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\operatorname{EllipticF}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^2\sqrt{k^2x^3-k^2x^2-x^2+x}}+\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(k^4\_Z^4+1\right)}\frac{\left(k^6\_\alpha^3+k^4\_\alpha^2+\_\alpha k^2+1\right)\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\operatorname{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{k^6\_\alpha^3+k^4\_\alpha^2+\_\alpha k^2+1}{k^4+1},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{\_\alpha^3\left(k^4+1\right)\sqrt{x\left(k^2x^2-k^2x-x+1\right)}}}{k^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^4*x^4-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^4*x^4+1),x)
```

```
[Out] -2/k^2*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x
^3-k^2*x^2-x^2+x)^(1/2)*EllipticF((-(x-1/k^2)*k^2)^(1/2),(1/k^2/(1/k^2-1))^
(1/2))+1/k^4*sum(1/_alpha^3*(_alpha^3*k^6+_alpha^2*k^4+_alpha*k^2+1)/(k^4+1
)*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(x*(k^2*x^2
-k^2*x-x+1))^(1/2)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),(_alpha^3*k^6+_alpha^2
*k^4+_alpha*k^2+1)/(k^4+1),(1/k^2/(1/k^2-1))^(1/2)),_alpha=RootOf(_Z^4*k^4+
1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^4x^4-1}{\left(k^4x^4+1\right)\sqrt{\left(k^2x-1\right)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^4*x^4-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^4*x^4+1),x, algorithm="
maxima")
```

```
[Out] integrate((k^4*x^4 - 1)/((k^4*x^4 + 1)*sqrt((k^2*x - 1)*(x - 1)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{k^4\,x^4-1}{\left(k^4\,x^4+1\right)\,\sqrt{x\,\left(k^2\,x-1\right)\,\left(x-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^4*x^4 - 1)/((k^4*x^4 + 1)*(x*(k^2*x - 1)*(x - 1))^(1/2)),x)
```

```
[Out] int((k^4*x^4 - 1)/((k^4*x^4 + 1)*(x*(k^2*x - 1)*(x - 1))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k**4*x**4-1)/((1-x)*x*(-k**2*x+1))**(1/2)/(k**4*x**4+1),x)
```

```
[Out] Timed out
```

3.1738 $\int \frac{1-x^4}{\left(1+x^4\right)\sqrt[4]{x^3+x^5}}\,dx$

Optimal. Leaf size=151

$$\frac{\tan^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{\sqrt[8]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{\sqrt[8]{2}}-\frac{\tan^{-1}\left(\frac{2^{5/8}x\sqrt[4]{x^5+x^3}}{\sqrt[4]{2}x^2-\sqrt{x^5+x^3}}\right)}{2^{5/8}}+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{2^{3/8}}+\frac{\sqrt{x^5+x^3}}{2^{5/8}}}{x\sqrt[4]{x^5+x^3}}\right)}{2^{5/8}}$$

Rubi [C] time = 0.66, antiderivative size = 97, normalized size of antiderivative = 0.64, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2056, 1586, 6715, 6725, 429}

$$\frac{(2-2i)x\sqrt[4]{x^2+1}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};-ix^2,-x^2\right)}{\sqrt[4]{x^5+x^3}}+\frac{(2+2i)x\sqrt[4]{x^2+1}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};ix^2,-x^2\right)}{\sqrt[4]{x^5+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x^4)/((1 + x^4)*(x^3 + x^5)^(1/4)),x]
```

```
[Out] ((2 - 2*I)*x*(1 + x^2)^(1/4)*AppellF1[1/8, 1, -3/4, 9/8, (-I)*x^2, -x^2])/(
x^3 + x^5)^(1/4) + ((2 + 2*I)*x*(1 + x^2)^(1/4)*AppellF1[1/8, 1, -3/4, 9/8,
 I*x^2, -x^2])/(x^3 + x^5)^(1/4)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1-x^4}{\left(1+x^4\right)\sqrt[4]{x^3+x^5}}\,dx = \frac{\left(x^{3/4}\sqrt[4]{1+x^2}\right)\int \frac{1-x^4}{x^{3/4}\sqrt[4]{1+x^2}\left(1+x^4\right)}\,dx}{\sqrt[4]{x^3+x^5}}$$
$$= \frac{\left(x^{3/4}\sqrt[4]{1+x^2}\right)\int \frac{\left(1-x^2\right)\left(1+x^2\right)^{3/4}}{x^{3/4}\left(1+x^4\right)}\,dx}{\sqrt[4]{x^3+x^5}}$$
$$= \frac{\left(4x^{3/4}\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{\left(1-x^8\right)\left(1+x^8\right)^{3/4}}{1+x^{16}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{x^3+x^5}}$$
$$= \frac{\left(4x^{3/4}\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+x^8\right)^{3/4}}{i-x^8} - \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(1+x^8\right)^{3/4}}{i+x^8}\right)dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{x^3+x^5}}$$
$$= -\frac{\left((2-2i)x^{3/4}\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{\left(1+x^8\right)^{3/4}}{i+x^8}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{x^3+x^5}} + \frac{\left((2+2i)x^{3/4}\sqrt[4]{1+x^2}\right)\text{Sub}}{\sqrt[4]{x^3+}}$$
$$= \frac{(2-2i)x\sqrt[4]{1+x^2}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};-ix^2,-x^2\right)}{\sqrt[4]{x^3+x^5}} + \frac{(2+2i)x\sqrt[4]{1+x^2}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};ix^2,\right.}{\sqrt[4]{x^3+x^5}}$$

Mathematica [F] time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{\left(1+x^4\right)\sqrt[4]{x^3+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^4)/((1 + x^4)*(x^3 + x^5)^(1/4)),x]
```

```
[Out] Integrate[(1 - x^4)/((1 + x^4)*(x^3 + x^5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.55, size = 151, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{x^5+x^3}}\right)}{\sqrt[8]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{x^5+x^3}}\right)}{\sqrt[8]{2}} - \frac{\tan^{-1}\left(\frac{2^{5/8}x\sqrt[4]{x^5+x^3}}{\sqrt[4]{2}\,x^2-\sqrt{x^5+x^3}}\right)}{2^{5/8}} + \frac{\tanh^{-1}\left(\frac{\frac{x^2}{2^{3/8}}+\frac{\sqrt{x^5+x^3}}{2^{5/8}}}{x\sqrt[4]{x^5+x^3}}\right)}{2^{5/8}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^4)/((1 + x^4)*(x^3 + x^5)^(1/4)),x]
```

```
[Out] ArcTan[(2^(1/8)*x)/(x^3 + x^5)^(1/4)]/2^(1/8) - ArcTan[(2^(5/8)*x*(x^3 + x^
5)^(1/4))/(2^(1/4)*x^2 - Sqrt[x^3 + x^5])]/2^(5/8) + ArcTanh[(2^(1/8)*x)/(x
^3 + x^5)^(1/4)]/2^(1/8) + ArcTanh[(x^2/2^(3/8) + Sqrt[x^3 + x^5]/2^(5/8))/
(x*(x^3 + x^5)^(1/4))]/2^(5/8)
```

fricas [B] time = 23.57, size = 2372, normalized size = 15.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^4+1)/(x^5+x^3)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/2*2^(7/8)*arctan(-1/2*(4*(x^5 + x^3)^(3/4)*(2^(5/8)*x + 2^(1/8)*(x^2 + 1
)) - (2^(5/8)*(x^6 + 4*x^5 + 4*x^4 + 4*x^3 + x^2) + 2*sqrt(x^5 + x^3)*(2^(7
/8)*(x^3 + 2*x^2 + x) + 2*2^(3/8)*(x^3 + x^2 + x)) + 2*2^(1/8)*(x^6 + 2*x^5
 + 4*x^4 + 2*x^3 + x^2))*sqrt(3*2^(3/4) - 4*2^(1/4)) + 4*(x^5 + x^3)^(1/4)*
(2^(7/8)*x^3 + 2^(3/8)*(x^4 + x^2)))/(x^6 + x^2)) + 1/8*2^(7/8)*log((2^(7/8
)*(x^6 - 2*x^5 + 4*x^4 - 2*x^3 + x^2) + 2*(x^5 + x^3)^(3/4)*(2*x^2 - sqrt(2
)*(x^2 - 2*x + 1) - 2*x + 2) - 2*sqrt(x^5 + x^3)*(2^(5/8)*(x^3 - 2*x^2 + x)
 - 2*2^(1/8)*(x^3 - x^2 + x)) - 2^(3/8)*(x^6 - 4*x^5 + 4*x^4 - 4*x^3 + x^2)
 - 2*(x^5 + x^3)^(1/4)*(2^(3/4)*(x^4 - 2*x^3 + x^2) - 2*2^(1/4)*(x^4 - x^3
+ x^2)))/(x^6 + x^2)) - 1/8*2^(7/8)*log(-(2^(7/8)*(x^6 - 2*x^5 + 4*x^4 - 2*
x^3 + x^2) - 2*(x^5 + x^3)^(3/4)*(2*x^2 - sqrt(2)*(x^2 - 2*x + 1) - 2*x + 2
) - 2*sqrt(x^5 + x^3)*(2^(5/8)*(x^3 - 2*x^2 + x) - 2*2^(1/8)*(x^3 - x^2 + x
)) - 2^(3/8)*(x^6 - 4*x^5 + 4*x^4 - 4*x^3 + x^2) + 2*(x^5 + x^3)^(1/4)*(2^(
3/4)*(x^4 - 2*x^3 + x^2) - 2*2^(1/4)*(x^4 - x^3 + x^2)))/(x^6 + x^2)) - 1/2
*2^(3/8)*arctan(-(x^10 + 64*x^8 + 130*x^6 + 64*x^4 + x^2 + 2*(x^5 + x^3)^(3
/4)*(2^(5/8)*(x^6 - 79*x^4 - 79*x^2 + 1) + 2*2^(1/8)*(11*x^5 + 16*x^3 + 11*
x)) + 16*sqrt(2)*(x^9 + 5*x^7 + 5*x^5 + x^3) + 4*sqrt(x^5 + x^3)*(2^(3/4)*(
15*x^6 + 32*x^4 + 15*x^2) + 2^(1/4)*(x^7 + 33*x^5 + 33*x^3 + x)) - (16*(x^5
 + x^3)^(3/4)*(2^(3/4)*(x^5 - 14*x^4 + 4*x^3 - 14*x^2 + x) - 2^(1/4)*(x^5 -
 28*x^4 + 4*x^3 - 28*x^2 + x)) + 2^(5/8)*(x^10 - 6*x^9 - 220*x^8 + 26*x^7 -
 446*x^6 + 26*x^5 - 220*x^4 - 6*x^3 + x^2) + 2*sqrt(x^5 + x^3)*(2^(7/8)*(x^
7 - 11*x^6 - 79*x^5 - 16*x^4 - 79*x^3 - 11*x^2 + x) - 2^(3/8)*(x^7 - 22*x^6
 - 79*x^5 - 32*x^4 - 79*x^3 - 22*x^2 + x)) - 4*(x^8 - 30*x^7 + 33*x^6 - 64*
x^5 + 33*x^4 - 30*x^3 + x^2 - sqrt(2)*(x^8 - 15*x^7 + 33*x^6 - 32*x^5 + 33*
x^4 - 15*x^3 + x^2))*(x^5 + x^3)^(1/4) - 2^(1/8)*(x^10 - 12*x^9 - 220*x^8 +
 52*x^7 - 446*x^6 + 52*x^5 - 220*x^4 - 12*x^3 + x^2))*sqrt((3*2^(3/4)*(x^6
+ x^2) + 4*(x^5 + x^3)^(3/4)*(2^(7/8)*(2*x^2 - 3*x + 2) + 2^(3/8)*(3*x^2 -
4*x + 3)) + 8*sqrt(x^5 + x^3)*(3*x^3 - 4*x^2 + sqrt(2)*(2*x^3 - 3*x^2 + 2*x
) + 3*x) + 4*2^(1/4)*(x^6 + x^2) + 4*(x^5 + x^3)^(1/4)*(2^(5/8)*(3*x^4 - 4*
x^3 + 3*x^2) + 2*2^(1/8)*(2*x^4 - 3*x^3 + 2*x^2)))/(x^6 + x^2)) + 2*(x^5 +
x^3)^(1/4)*(2^(7/8)*(3*x^8 - 13*x^6 - 13*x^4 + 3*x^2) + 2*2^(3/8)*(41*x^7 +
 80*x^5 + 41*x^3)))/(x^10 - 384*x^8 - 766*x^6 - 384*x^4 + x^2)) + 1/2*2^(3/
8)*arctan(-(x^10 + 64*x^8 + 130*x^6 + 64*x^4 + x^2 - 2*(x^5 + x^3)^(3/4)*(2
^(5/8)*(x^6 - 79*x^4 - 79*x^2 + 1) + 2*2^(1/8)*(11*x^5 + 16*x^3 + 11*x)) +
16*sqrt(2)*(x^9 + 5*x^7 + 5*x^5 + x^3) + 4*sqrt(x^5 + x^3)*(2^(3/4)*(15*x^6
 + 32*x^4 + 15*x^2) + 2^(1/4)*(x^7 + 33*x^5 + 33*x^3 + x)) - (16*(x^5 + x^3
)^(3/4)*(2^(3/4)*(x^5 - 14*x^4 + 4*x^3 - 14*x^2 + x) - 2^(1/4)*(x^5 - 28*x^
4 + 4*x^3 - 28*x^2 + x)) - 2^(5/8)*(x^10 - 6*x^9 - 220*x^8 + 26*x^7 - 446*x
^6 + 26*x^5 - 220*x^4 - 6*x^3 + x^2) - 2*sqrt(x^5 + x^3)*(2^(7/8)*(x^7 - 11
*x^6 - 79*x^5 - 16*x^4 - 79*x^3 - 11*x^2 + x) - 2^(3/8)*(x^7 - 22*x^6 - 79*
x^5 - 32*x^4 - 79*x^3 - 22*x^2 + x)) - 4*(x^8 - 30*x^7 + 33*x^6 - 64*x^5 +
33*x^4 - 30*x^3 + x^2 - sqrt(2)*(x^8 - 15*x^7 + 33*x^6 - 32*x^5 + 33*x^4 -
15*x^3 + x^2))*(x^5 + x^3)^(1/4) + 2^(1/8)*(x^10 - 12*x^9 - 220*x^8 + 52*x^
7 - 446*x^6 + 52*x^5 - 220*x^4 - 12*x^3 + x^2))*sqrt((3*2^(3/4)*(x^6 + x^2)
 - 4*(x^5 + x^3)^(3/4)*(2^(7/8)*(2*x^2 - 3*x + 2) + 2^(3/8)*(3*x^2 - 4*x +
3)) + 8*sqrt(x^5 + x^3)*(3*x^3 - 4*x^2 + sqrt(2)*(2*x^3 - 3*x^2 + 2*x) + 3*
x) + 4*2^(1/4)*(x^6 + x^2) - 4*(x^5 + x^3)^(1/4)*(2^(5/8)*(3*x^4 - 4*x^3 +
3*x^2) + 2*2^(1/8)*(2*x^4 - 3*x^3 + 2*x^2)))/(x^6 + x^2)) - 2*(x^5 + x^3)^(
1/4)*(2^(7/8)*(3*x^8 - 13*x^6 - 13*x^4 + 3*x^2) + 2*2^(3/8)*(41*x^7 + 80*x^
5 + 41*x^3)))/(x^10 - 384*x^8 - 766*x^6 - 384*x^4 + x^2)) + 1/8*2^(3/8)*log
(4*(3*2^(3/4)*(x^6 + x^2) + 4*(x^5 + x^3)^(3/4)*(2^(7/8)*(2*x^2 - 3*x + 2)
+ 2^(3/8)*(3*x^2 - 4*x + 3)) + 8*sqrt(x^5 + x^3)*(3*x^3 - 4*x^2 + sqrt(2)*(
2*x^3 - 3*x^2 + 2*x) + 3*x) + 4*2^(1/4)*(x^6 + x^2) + 4*(x^5 + x^3)^(1/4)*(
2^(5/8)*(3*x^4 - 4*x^3 + 3*x^2) + 2*2^(1/8)*(2*x^4 - 3*x^3 + 2*x^2)))/(x^6
+ x^2)) - 1/8*2^(3/8)*log(4*(3*2^(3/4)*(x^6 + x^2) - 4*(x^5 + x^3)^(3/4)*(2
^(7/8)*(2*x^2 - 3*x + 2) + 2^(3/8)*(3*x^2 - 4*x + 3)) + 8*sqrt(x^5 + x^3)*(
3*x^3 - 4*x^2 + sqrt(2)*(2*x^3 - 3*x^2 + 2*x) + 3*x) + 4*2^(1/4)*(x^6 + x^2
) - 4*(x^5 + x^3)^(1/4)*(2^(5/8)*(3*x^4 - 4*x^3 + 3*x^2) + 2*2^(1/8)*(2*x^4
 - 3*x^3 + 2*x^2)))/(x^6 + x^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^4-1}{\left(x^5+x^3\right)^{\frac{1}{4}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^4+1)/(x^5+x^3)^(1/4),x, algorithm="giac")
[Out] integrate(-(x^4 - 1)/((x^5 + x^3)^(1/4)*(x^4 + 1)), x)
```

maple [C] time = 53.88, size = 2918, normalized size = 19.32

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^4+1)/(x^5+x^3)^(1/4),x)
[Out] 1/32*ln((x^4*RootOf(_Z^8-128)^9+2*RootOf(_Z^8-128)^8*RootOf(-RootOf(_Z^8-12
8)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^4-2*x^3*RootOf(_Z^8-128)^9-4*RootOf
(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*RootOf(_Z^8-128)^8*x^
3+x^2*RootOf(_Z^8-128)^9+2*RootOf(_Z^8-128)^8*RootOf(-RootOf(_Z^8-128)^5*_Z
+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^2+16*(x^5+x^3)^(1/2)*RootOf(_Z^8-128)^6*Ro
otOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x-8*(x^5+x^3)^(1/
4)*RootOf(_Z^8-128)^6*x^2+8*RootOf(_Z^8-128)^5*x^4+32*RootOf(_Z^8-128)^4*Ro
otOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^3+8*RootOf(_Z^8
-128)^5*x^2+512*(x^5+x^3)^(1/4)*RootOf(_Z^8-128)*RootOf(-RootOf(_Z^8-128)^5
*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^2-128*RootOf(_Z^8-128)*x^4-512*RootOf(-
RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^4+128*RootOf(_Z^8-128
)*x^3+512*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^3+2
56*(x^5+x^3)^(3/4)-128*RootOf(_Z^8-128)*x^2-512*RootOf(-RootOf(_Z^8-128)^5*
_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^2)/(x^2*RootOf(_Z^8-128)^4-2*x*RootOf(_Z
^8-128)^4+RootOf(_Z^8-128)^4-16*x^2+16*x-16)/x^2)*RootOf(_Z^8-128)^5-ln((x^
4*RootOf(_Z^8-128)^9+2*RootOf(_Z^8-128)^8*RootOf(-RootOf(_Z^8-128)^5*_Z+2*R
ootOf(_Z^8-128)^2+32*_Z^2)*x^4-2*x^3*RootOf(_Z^8-128)^9-4*RootOf(-RootOf(_Z
^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*RootOf(_Z^8-128)^8*x^3+x^2*RootO
f(_Z^8-128)^9+2*RootOf(_Z^8-128)^8*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_
Z^8-128)^2+32*_Z^2)*x^2+16*(x^5+x^3)^(1/2)*RootOf(_Z^8-128)^6*RootOf(-RootO
f(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x-8*(x^5+x^3)^(1/4)*RootOf(_
Z^8-128)^6*x^2+8*RootOf(_Z^8-128)^5*x^4+32*RootOf(_Z^8-128)^4*RootOf(-RootO
f(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^3+8*RootOf(_Z^8-128)^5*x^2
+512*(x^5+x^3)^(1/4)*RootOf(_Z^8-128)*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootO
f(_Z^8-128)^2+32*_Z^2)*x^2-128*RootOf(_Z^8-128)*x^4-512*RootOf(-RootOf(_Z^8
-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^4+128*RootOf(_Z^8-128)*x^3+512*R
ootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^3+256*(x^5+x^3
)^(3/4)-128*RootOf(_Z^8-128)*x^2-512*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf
(_Z^8-128)^2+32*_Z^2)*x^2)/(x^2*RootOf(_Z^8-128)^4-2*x*RootOf(_Z^8-128)^4+R
ootOf(_Z^8-128)^4-16*x^2+16*x-16)/x^2)*RootOf(-RootOf(_Z^8-128)^5*_Z+2*Root
Of(_Z^8-128)^2+32*_Z^2)-1/4*RootOf(_Z^8-128)*ln((x^4*RootOf(_Z^8-128)^9-2*x
^3*RootOf(_Z^8-128)^9+x^2*RootOf(_Z^8-128)^9-8*(x^5+x^3)^(1/2)*RootOf(_Z^8-
128)^7*x+16*(x^5+x^3)^(1/4)*RootOf(_Z^8-128)^6*x^2-16*RootOf(_Z^8-128)^5*x^
3-256*RootOf(_Z^8-128)*x^4+256*RootOf(_Z^8-128)*x^3+512*(x^5+x^3)^(3/4)-256
*RootOf(_Z^8-128)*x^2)/(x^2*RootOf(_Z^8-128)^4-2*x*RootOf(_Z^8-128)^4+RootO
f(_Z^8-128)^4+16*x^2-16*x+16)/x^2)+1/8*ln((x^4*RootOf(_Z^8-128)^9-8*RootOf(
_Z^8-128)^8*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^4
-2*x^3*RootOf(_Z^8-128)^9+16*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-12
8)^2+32*_Z^2)*RootOf(_Z^8-128)^8*x^3+x^2*RootOf(_Z^8-128)^9-8*RootOf(_Z^8-1
28)^8*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^2+8*(x^
5+x^3)^(1/2)*RootOf(_Z^8-128)^7*x-16*(x^5+x^3)^(1/4)*RootOf(_Z^8-128)^6*x^2
```

```
+64*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*RootOf(_Z^8
-128)^4*x^4-16*RootOf(_Z^8-128)^5*x^3+64*RootOf(_Z^8-128)^4*RootOf(-RootOf(
_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^2-512*(x^5+x^3)^(1/2)*RootOf
(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*RootOf(_Z^8-128)^2*x-
256*RootOf(_Z^8-128)*x^4+1024*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-1
28)^2+32*_Z^2)*x^4+256*RootOf(_Z^8-128)*x^3-1024*RootOf(-RootOf(_Z^8-128)^5
*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^3+512*(x^5+x^3)^(3/4)-256*RootOf(_Z^8-1
28)*x^2+1024*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^
2)/(x^2*RootOf(_Z^8-128)^4-2*x*RootOf(_Z^8-128)^4+RootOf(_Z^8-128)^4+16*x^2
-16*x+16)/x^2)*RootOf(_Z^8-128)^4*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z
^8-128)^2+32*_Z^2)-1/4*ln((x^4*RootOf(_Z^8-128)^9-8*RootOf(_Z^8-128)^8*Root
Of(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^4-2*x^3*RootOf(_Z
^8-128)^9+16*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*Ro
otOf(_Z^8-128)^8*x^3+x^2*RootOf(_Z^8-128)^9-8*RootOf(_Z^8-128)^8*RootOf(-Ro
otOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^2+8*(x^5+x^3)^(1/2)*Roo
tOf(_Z^8-128)^7*x-16*(x^5+x^3)^(1/4)*RootOf(_Z^8-128)^6*x^2+64*RootOf(-Root
Of(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*RootOf(_Z^8-128)^4*x^4-16*R
ootOf(_Z^8-128)^5*x^3+64*RootOf(_Z^8-128)^4*RootOf(-RootOf(_Z^8-128)^5*_Z+2
*RootOf(_Z^8-128)^2+32*_Z^2)*x^2-512*(x^5+x^3)^(1/2)*RootOf(-RootOf(_Z^8-12
8)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*RootOf(_Z^8-128)^2*x-256*RootOf(_Z^8-
128)*x^4+1024*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x
^4+256*RootOf(_Z^8-128)*x^3-1024*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^
8-128)^2+32*_Z^2)*x^3+512*(x^5+x^3)^(3/4)-256*RootOf(_Z^8-128)*x^2+1024*Roo
tOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x^2)/(x^2*RootOf(_
Z^8-128)^4-2*x*RootOf(_Z^8-128)^4+RootOf(_Z^8-128)^4+16*x^2-16*x+16)/x^2)*R
ootOf(_Z^8-128)-RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)
*ln((RootOf(_Z^8-128)^8*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+
32*_Z^2)*x^4-2*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*
RootOf(_Z^8-128)^8*x^3+RootOf(_Z^8-128)^8*RootOf(-RootOf(_Z^8-128)^5*_Z+2*R
ootOf(_Z^8-128)^2+32*_Z^2)*x^2+8*(x^5+x^3)^(1/2)*RootOf(_Z^8-128)^6*RootOf(
-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_Z^2)*x+4*(x^5+x^3)^(1/4)*Ro
otOf(_Z^8-128)^6*x^2+16*RootOf(_Z^8-128)^4*RootOf(-RootOf(_Z^8-128)^5*_Z+2*
RootOf(_Z^8-128)^2+32*_Z^2)*x^3-32*(x^5+x^3)^(1/2)*RootOf(_Z^8-128)^3*x-256
*(x^5+x^3)^(1/4)*RootOf(_Z^8-128)*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z
^8-128)^2+32*_Z^2)*x^2-256*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)
^2+32*_Z^2)*x^4+256*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^8-128)^2+32*_
Z^2)*x^3+128*(x^5+x^3)^(3/4)-256*RootOf(-RootOf(_Z^8-128)^5*_Z+2*RootOf(_Z^
8-128)^2+32*_Z^2)*x^2)/(x^2*RootOf(_Z^8-128)^4-2*x*RootOf(_Z^8-128)^4+RootO
f(_Z^8-128)^4-16*x^2+16*x-16)/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4-1}{\left(x^5+x^3\right)^{\frac{1}{4}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^4+1)/(x^5+x^3)^(1/4),x, algorithm="maxima")
[Out] -integrate((x^4 - 1)/((x^5 + x^3)^(1/4)*(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^4-1}{\left(x^5+x^3\right)^{1/4}\,\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/((x^3 + x^5)^(1/4)*(x^4 + 1)),x)
```

```
[Out] -int((x^4 - 1)/((x^3 + x^5)^(1/4)*(x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{x^4\sqrt[4]{x^5+x^3}+\sqrt[4]{x^5+x^3}}\,dx-\int \left(-\frac{1}{x^4\sqrt[4]{x^5+x^3}+\sqrt[4]{x^5+x^3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**4+1)/(x**5+x**3)**(1/4),x)
```

```
[Out] -Integral(x**4/(x**4*(x**5 + x**3)**(1/4) + (x**5 + x**3)**(1/4)), x) - Int
egral(-1/(x**4*(x**5 + x**3)**(1/4) + (x**5 + x**3)**(1/4)), x)
```

3.1739 $\int \frac{(1+x^3)^{2/3}(2+x^6)}{x^6(-1+x^3)^2} \, dx$

Optimal. Leaf size=151

$$-\frac{5}{3}2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^3+1}-2x\right)+\frac{5\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}+\frac{5\log\left(2^{2/3}\sqrt[3]{x^3+1}\,x+\sqrt[3]{2}\left(x^3+1\right)^{2/3}+2x^2\right)}{3\sqrt[3]{2}}+\frac{(x^3+$$

Rubi [F] time = 1.86, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+x^3\right)^{2/3}\left(2+x^6\right)}{x^6\left(-1+x^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^3)^(2/3)*(2 + x^6))/(x^6*(-1 + x^3)^2),x]
```

```
[Out] (-2*(1 + x^3)^(2/3))/x^2 - (2*(1 + x^3)^(5/3))/(5*x^5) + (4*ArcTan[(1 + (2*
x)/(1 + x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - 2*Log[-x + (1 + x^3)^(1/3)] - (4*De
fer[Int][(1 + x^3)^(2/3)/(-1 + I*Sqrt[3] - 2*x)^2, x])/3 + (2*(1 - I*Sqrt[3
])*Defer[Int][(1 + x^3)^(2/3)/(-1 + I*Sqrt[3] - 2*x)^2, x])/3 + (((2*I)/3)*
Defer[Int][(1 + x^3)^(2/3)/(-1 + I*Sqrt[3] - 2*x), x])/Sqrt[3] + Defer[Int]
[(1 + x^3)^(2/3)/(-1 + x)^2, x]/3 - 2*Defer[Int][(1 + x^3)^(2/3)/(-1 + x),
x] + (2*(9 - (8*I)*Sqrt[3])*Defer[Int][(1 + x^3)^(2/3)/(1 - I*Sqrt[3] + 2*x
), x])/9 - (4*Defer[Int][(1 + x^3)^(2/3)/(1 + I*Sqrt[3] + 2*x)^2, x])/3 + (
2*(1 + I*Sqrt[3])*Defer[Int][(1 + x^3)^(2/3)/(1 + I*Sqrt[3] + 2*x)^2, x])/3
 + (((2*I)/3)*Defer[Int][(1 + x^3)^(2/3)/(1 + I*Sqrt[3] + 2*x), x])/Sqrt[3]
 + (2*(9 + (8*I)*Sqrt[3])*Defer[Int][(1 + x^3)^(2/3)/(1 + I*Sqrt[3] + 2*x),
 x])/9
```

Rubi steps

$$\int \frac{\left(1+x^3\right)^{2/3}\left(2+x^6\right)}{x^6\left(-1+x^3\right)^2}\,dx = \int \left(\frac{\left(1+x^3\right)^{2/3}}{3(-1+x)^2} - \frac{2\left(1+x^3\right)^{2/3}}{-1+x} + \frac{2\left(1+x^3\right)^{2/3}}{x^6} + \frac{4\left(1+x^3\right)^{2/3}}{x^3} + \frac{(1+x)\left(1+x^3\right)^{2/}}{\left(1+x+x^2\right)^2}\right.$$

$$= \frac{1}{3}\int \frac{\left(1+x^3\right)^{2/3}}{(-1+x)^2}\,dx + \frac{1}{3}\int \frac{(11+6x)\left(1+x^3\right)^{2/3}}{1+x+x^2}\,dx - 2\int \frac{\left(1+x^3\right)^{2/3}}{-1+x}\,dx + 2\int \frac{(1}{}$$

$$= -\frac{2\left(1+x^3\right)^{2/3}}{x^2} - \frac{2\left(1+x^3\right)^{5/3}}{5x^5} + \frac{1}{3}\int \frac{\left(1+x^3\right)^{2/3}}{(-1+x)^2}\,dx + \frac{1}{3}\int \left(\frac{\left(6-\frac{16i}{\sqrt{3}}\right)\left(1+x^3\right)^{2/3}}{1-i\sqrt{3}+2x}\right.$$

$$= -\frac{2\left(1+x^3\right)^{2/3}}{x^2} - \frac{2\left(1+x^3\right)^{5/3}}{5x^5} + \frac{4\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - 2\log\left(-x+\sqrt[3]{1+x^3}\right) + \frac{1}{3}\int$$

$$= -\frac{2\left(1+x^3\right)^{2/3}}{x^2} - \frac{2\left(1+x^3\right)^{5/3}}{5x^5} + \frac{4\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - 2\log\left(-x+\sqrt[3]{1+x^3}\right) + \frac{1}{3}\int$$

$$= -\frac{2\left(1+x^3\right)^{2/3}}{x^2} - \frac{2\left(1+x^3\right)^{5/3}}{5x^5} + \frac{4\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - 2\log\left(-x+\sqrt[3]{1+x^3}\right) + \frac{1}{3}\int$$

Mathematica [A] time = 0.50, size = 138, normalized size = 0.91

$$\frac{5\left(-2\log\left(1-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3+1}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)+\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3+1}}+\frac{2^{2/3}x^2}{\left(x^3+1\right)^{2/3}}+1\right)\right)}{3\sqrt[3]{2}}+\left(x^3+1\right)^{2/3}\left(-\frac{2}{5x^5}-\frac{x}{x^3-1}-\frac{12}{5x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)^(2/3)*(2 + x^6))/(x^6*(-1 + x^3)^2),x]
```

```
[Out] (1 + x^3)^(2/3)*(-2/(5*x^5) - 12/(5*x^2) - x/(-1 + x^3)) + (5*(2*Sqrt[3]*Ar
cTan[(1 + (2*2^(1/3)*x)/(1 + x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (2^(1/3)*x)/(
1 + x^3)^(1/3)] + Log[1 + (2^(2/3)*x^2)/(1 + x^3)^(2/3) + (2^(1/3)*x)/(1 +
x^3)^(1/3)]))/(3*2^(1/3))
```

IntegrateAlgebraic [A] time = 0.46, size = 151, normalized size = 1.00

$$-\frac{5}{3}2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^3+1}-2x\right)+\frac{5\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}+\frac{5\log\left(2^{2/3}\sqrt[3]{x^3+1}\,x+\sqrt[3]{2}\left(x^3+1\right)^{2/3}+2x^2\right)}{3\sqrt[3]{2}}+\frac{\left(x^3+1\right)^{2/3}\left(-17x^6+10x^3+2\right)}{5x^5\left(x^3-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)^(2/3)*(2 + x^6))/(x^6*(-1 + x^3)^2),x]
```

```
[Out] ((1 + x^3)^(2/3)*(2 + 10*x^3 - 17*x^6))/(5*x^5*(-1 + x^3)) + (5*2^(2/3)*Arc
Tan[(Sqrt[3]*x)/(x + 2^(2/3)*(1 + x^3)^(1/3))])/Sqrt[3] - (5*2^(2/3)*Log[-2
*x + 2^(2/3)*(1 + x^3)^(1/3)])/3 + (5*Log[2*x^2 + 2^(2/3)*x*(1 + x^3)^(1/3)
 + 2^(1/3)*(1 + x^3)^(2/3)])/(3*2^(1/3))
```

fricas [B] time = 2.83, size = 297, normalized size = 1.97

$$-\frac{50\sqrt{3}\left(-4\right)^{\frac{1}{3}}\left(x^8-x^5\right)\arctan\left(\frac{3\sqrt{3}\left(-4\right)^{\frac{2}{3}}\left(5x^7-4x^4-x\right)\left(x^3+1\right)^{\frac{2}{3}}+6\sqrt{3}\left(-4\right)^{\frac{1}{3}}\left(19x^8+16x^5+x^2\right)\left(x^3+1\right)^{\frac{1}{3}}-\sqrt{3}\left(71x^9+111x^6+33x^3+1\right)}{3\left(109x^9+105x^6+3x^3-1\right)}\right)-50\left(-4\right)^{\frac{1}{3}}\left(x^8-x^5\right)\log\left(\frac{3\left(-4\right)^{\frac{2}{3}}\left(x^3+1\right)^{\frac{1}{3}}x^2-6\left(x^3+1\right)^{\frac{2}{3}}x+\left(-4\right)^{\frac{1}{3}}\left(x^3-1\right)}{x^3-1}\right)+25\left(-4\right)^{\frac{1}{3}}\left(x^8-x^5\right)\log\left(-\frac{6\left(-4\right)^{\frac{1}{3}}\left(5x^4+x\right)\left(x^3+1\right)^{\frac{2}{3}}-\left(-4\right)^{\frac{2}{3}}\left(19x^6+16x^3+1\right)-24\left(2x^5+x^2\right)\left(x^3+1\right)^{\frac{1}{3}}}{x^6-2x^3+1}\right)+18\left(17x^6-10x^3-2\right)\left(x^3+1\right)^{\frac{2}{3}}}{90\left(x^8-x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^6+2)/x^6/(x^3-1)^2,x, algorithm="fricas")
```

```
[Out] -1/90*(50*sqrt(3)*(-4)^(1/3)*(x^8 - x^5)*arctan(1/3*(3*sqrt(3)*(-4)^(2/3)*(
5*x^7 - 4*x^4 - x)*(x^3 + 1)^(2/3) + 6*sqrt(3)*(-4)^(1/3)*(19*x^8 + 16*x^5
+ x^2)*(x^3 + 1)^(1/3) - sqrt(3)*(71*x^9 + 111*x^6 + 33*x^3 + 1))/(109*x^9
+ 105*x^6 + 3*x^3 - 1)) - 50*(-4)^(1/3)*(x^8 - x^5)*log((3*(-4)^(2/3)*(x^3
+ 1)^(1/3)*x^2 - 6*(x^3 + 1)^(2/3)*x + (-4)^(1/3)*(x^3 - 1))/(x^3 - 1)) + 2
5*(-4)^(1/3)*(x^8 - x^5)*log(-(6*(-4)^(1/3)*(5*x^4 + x)*(x^3 + 1)^(2/3) - (
-4)^(2/3)*(19*x^6 + 16*x^3 + 1) - 24*(2*x^5 + x^2)*(x^3 + 1)^(1/3))/(x^6 -
2*x^3 + 1)) + 18*(17*x^6 - 10*x^3 - 2)*(x^3 + 1)^(2/3))/(x^8 - x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+2\right)\left(x^3+1\right)^{\frac{2}{3}}}{\left(x^3-1\right)^2 x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^6+2)/x^6/(x^3-1)^2,x, algorithm="giac")
```

```
[Out] integrate((x^6 + 2)*(x^3 + 1)^(2/3)/((x^3 - 1)^2*x^6), x)
```

maple [C] time = 2.38, size = 939, normalized size = 6.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)*(x^6+2)/x^6/(x^3-1)^2,x)
```

```
[Out] -1/5*(17*x^9+7*x^6-12*x^3-2)/(x^3-1)/(x^3+1)^(1/3)/x^5-5/3*ln(-(18*RootOf(R
ootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-6*RootO
f(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+12*(x^
3+1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3
+4)^2*x-24*(x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^
2)*RootOf(_Z^3+4)*x^2+(x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x^2+6*RootOf(RootOf(_Z
^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3-2*RootOf(_Z^3+4)*x^3+2*x*(x^3+1)^(
2/3)+6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)-2*RootOf(_Z^3+4
))/(-1+x)/(x^2+x+1))*RootOf(_Z^3+4)-10*ln(-(18*RootOf(RootOf(_Z^3+4)^2+6*_Z
*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-6*RootOf(RootOf(_Z^3+4)^2+6
*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+12*(x^3+1)^(2/3)*RootOf(Ro
otOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x-24*(x^3+1)^(
1/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)*x^
2+(x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x^2+6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(
_Z^3+4)+36*_Z^2)*x^3-2*RootOf(_Z^3+4)*x^3+2*x*(x^3+1)^(2/3)+6*RootOf(RootOf
(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)-2*RootOf(_Z^3+4))/(-1+x)/(x^2+x+1))
*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)+10*RootOf(RootOf(_Z^3
+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*ln(-(18*RootOf(RootOf(_Z^3+4)^2+6*_Z*Roo
tOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+9*RootOf(RootOf(_Z^3+4)^2+6*_Z*
RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3-12*(x^3+1)^(2/3)*RootOf(RootOf
(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x+24*(x^3+1)^(1/3)
*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)*x^2+5*
(x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x^2-18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_
Z^3+4)+36*_Z^2)*x^3-9*RootOf(_Z^3+4)*x^3+10*x*(x^3+1)^(2/3)-6*RootOf(RootOf
(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)-3*RootOf(_Z^3+4))/(-1+x)/(x^2+x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+2\right)\left(x^3+1\right)^{\frac{2}{3}}}{\left(x^3-1\right)^2 x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^6+2)/x^6/(x^3-1)^2,x, algorithm="maxima")
[Out] integrate((x^6 + 2)*(x^3 + 1)^(2/3)/((x^3 - 1)^2*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3+1\right)^{2/3}\,\left(x^6+2\right)}{x^6\,\left(x^3-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)^(2/3)*(x^6 + 2))/(x^6*(x^3 - 1)^2),x)
[Out] int(((x^3 + 1)^(2/3)*(x^6 + 2))/(x^6*(x^3 - 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((x+1)\left(x^2-x+1\right)\right)^{\frac{2}{3}}\left(x^6+2\right)}{x^6\,(x-1)^2\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)*(x**6+2)/x**6/(x**3-1)**2,x)
[Out] Integral(((x + 1)*(x**2 - x + 1))**(2/3)*(x**6 + 2)/(x**6*(x - 1)**2*(x**2
+ x + 1)**2), x)
```

3.1740 $\int \frac{\left(-1+x^3\right)^{2/3}\left(-2+2x^3+x^6\right)}{x^6\left(1+x^3\right)^2}\,dx$

Optimal. Leaf size=151

$$-\frac{7}{3}2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^3-1}-2x\right)+\frac{7\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}+\frac{7\log\left(2^{2/3}\sqrt[3]{x^3-1}x+\sqrt[3]{2}\left(x^3-1\right)^{2/3}+2x^2\right)}{3\sqrt[3]{2}}+\frac{\left(x^3-\right.}{}$$

Rubi [F] time = 1.88, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(-2+2x^3+x^6\right)}{x^6\left(1+x^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^3)^(2/3)*(-2 + 2*x^3 + x^6))/(x^6*(1 + x^3)^2),x]
```

```
[Out] (-3*(-1 + x^3)^(2/3))/x^2 - (2*(-1 + x^3)^(5/3))/(5*x^5) + 2*Sqrt[3]*ArcTan
[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]] - 3*Log[-x + (-1 + x^3)^(1/3)] + (4*
Defer[Int][(-1 + x^3)^(2/3)/(1 + I*Sqrt[3] - 2*x)^2, x])/3 - (2*(1 + I*Sqrt
[3])*Defer[Int][(-1 + x^3)^(2/3)/(1 + I*Sqrt[3] - 2*x)^2, x])/3 - (((2*I)/3
)*Defer[Int][(-1 + x^3)^(2/3)/(1 + I*Sqrt[3] - 2*x), x])/Sqrt[3] - Defer[In
t][(-1 + x^3)^(2/3)/(1 + x)^2, x]/3 - (8*Defer[Int][(-1 + x^3)^(2/3)/(1 + x
), x])/3 + (2*(12 + (11*I)*Sqrt[3])*Defer[Int][(-1 + x^3)^(2/3)/(-1 - I*Sqr
t[3] + 2*x), x])/9 + (4*Defer[Int][(-1 + x^3)^(2/3)/(-1 + I*Sqrt[3] + 2*x)^
2, x])/3 - (2*(1 - I*Sqrt[3])*Defer[Int][(-1 + x^3)^(2/3)/(-1 + I*Sqrt[3] +
 2*x)^2, x])/3 - (((2*I)/3)*Defer[Int][(-1 + x^3)^(2/3)/(-1 + I*Sqrt[3] + 2
*x), x])/Sqrt[3] + (2*(12 - (11*I)*Sqrt[3])*Defer[Int][(-1 + x^3)^(2/3)/(-1
 + I*Sqrt[3] + 2*x), x])/9
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^3\right)^{2/3}\left(-2+2x^3+x^6\right)}{x^6\left(1+x^3\right)^2}\,dx &= \int \left(-\frac{2\left(-1+x^3\right)^{2/3}}{x^6}+\frac{6\left(-1+x^3\right)^{2/3}}{x^3}-\frac{\left(-1+x^3\right)^{2/3}}{3(1+x)^2}-\frac{8\left(-1+x^3\right)^{2/3}}{3(1+x)}+\frac{(-1}{}\right.\\
&= -\left(\frac{1}{3}\int \frac{\left(-1+x^3\right)^{2/3}}{(1+x)^2}\,dx\right)+\frac{1}{3}\int \frac{(-15+8x)\left(-1+x^3\right)^{2/3}}{1-x+x^2}\,dx-2\int \frac{\left(-1+x^3\right.}{x^6}\\
&= -\frac{3\left(-1+x^3\right)^{2/3}}{x^2}-\frac{2\left(-1+x^3\right)^{5/3}}{5x^5}-\frac{1}{3}\int \frac{\left(-1+x^3\right)^{2/3}}{(1+x)^2}\,dx+\frac{1}{3}\int \left(\frac{\left(8+\frac{22i}{\sqrt{3}}\right)}{-1-i}\right.\\
&= -\frac{3\left(-1+x^3\right)^{2/3}}{x^2}-\frac{2\left(-1+x^3\right)^{5/3}}{5x^5}+2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)-3\log\left(-x+\right.\\
&= -\frac{3\left(-1+x^3\right)^{2/3}}{x^2}-\frac{2\left(-1+x^3\right)^{5/3}}{5x^5}+2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)-3\log\left(-x+\right.\\
&= -\frac{3\left(-1+x^3\right)^{2/3}}{x^2}-\frac{2\left(-1+x^3\right)^{5/3}}{5x^5}+2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)-3\log\left(-x+\right.
\end{aligned}$$

Mathematica [A] time = 0.59, size = 146, normalized size = 0.97

$$\frac{7\left(-2\log\left(1-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)+\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}+1\right)\right)}{3\sqrt[3]{2}}+\left(x^3-1\right)^{2/3}\left(\frac{2}{5x^5}-\frac{x}{x^3+1}-\frac{17}{5x^2}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^3)^(2/3)*(-2 + 2*x^3 + x^6))/(x^6*(1 + x^3)^2),x]
```

```
[Out] (-1 + x^3)^(2/3)*(2/(5*x^5) - 17/(5*x^2) - x/(1 + x^3)) + (7*(2*Sqrt[3]*Arc
Tan[(1 + (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (2^(1/3)*x)/(1
 - x^3)^(1/3)] + Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) + (2^(1/3)*x)/(1 - x
^3)^(1/3)]))/(3*2^(1/3))
```

IntegrateAlgebraic [A] time = 0.46, size = 151, normalized size = 1.00

$$-\frac{7}{3}2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^3-1}-2x\right)+\frac{7\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}+\frac{7\log\left(2^{2/3}\sqrt[3]{x^3-1}\,x+\sqrt[3]{2}\left(x^3-1\right)^{2/3}+2x^2\right)}{3\sqrt[3]{2}}+\frac{\left(x^3-1\right)^{2/3}\left(-22x^6-15x^3+2\right)}{5x^5\left(x^3+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(2/3)*(-2 + 2*x^3 + x^6))/(x^6*(1 + x^3)^2),
x]
```

```
[Out] ((-1 + x^3)^(2/3)*(2 - 15*x^3 - 22*x^6))/(5*x^5*(1 + x^3)) + (7*2^(2/3)*Arc
Tan[(Sqrt[3]*x)/(x + 2^(2/3)*(-1 + x^3)^(1/3))])/Sqrt[3] - (7*2^(2/3)*Log[-
2*x + 2^(2/3)*(-1 + x^3)^(1/3)])/3 + (7*Log[2*x^2 + 2^(2/3)*x*(-1 + x^3)^(1
/3) + 2^(1/3)*(-1 + x^3)^(2/3)])/(3*2^(1/3))
```

fricas [B] time = 2.92, size = 294, normalized size = 1.95

$$-\frac{70\sqrt{3}(-4)^{\frac{1}{3}}\left(x^8+x^5\right)\arctan\left(\frac{3\sqrt{3}(-4)^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(x^3-1\right)^{\frac{2}{3}}+6\sqrt{3}(-4)^{\frac{1}{3}}\left(19x^8-16x^5+x^2\right)\left(x^3-1\right)^{\frac{1}{3}}-\sqrt{3}\left(71x^9-111x^6+33x^3-1\right)}{3\left(109x^9-105x^6+3x^3+1\right)}\right)-70(-4)^{\frac{1}{3}}\left(x^8+x^5\right)\log\left(-\frac{3(-4)^{\frac{2}{3}}\left(x^3-1\right)^{\frac{1}{3}}x^2-6\left(x^3-1\right)^{\frac{2}{3}}x+(-4)^{\frac{1}{3}}\left(x^3+1\right)}{x^3+1}\right)+35(-4)^{\frac{1}{3}}\left(x^8+x^5\right)\log\left(-\frac{6(-4)^{\frac{1}{3}}\left(5x^4-x\right)\left(x^3-1\right)^{\frac{2}{3}}-(-4)^{\frac{2}{3}}\left(19x^6-16x^3+1\right)-24\left(2x^5-x^2\right)\left(x^3-1\right)^{\frac{1}{3}}}{x^6+2x^3+1}\right)+18\left(22x^6+15x^3-2\right)\left(x^3-1\right)^{\frac{2}{3}}}{90\left(x^8+x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+2*x^3-2)/x^6/(x^3+1)^2,x, algorithm="fricas")
```

```
[Out] -1/90*(70*sqrt(3)*(-4)^(1/3)*(x^8 + x^5)*arctan(1/3*(3*sqrt(3)*(-4)^(2/3)*(
5*x^7 + 4*x^4 - x)*(x^3 - 1)^(2/3) + 6*sqrt(3)*(-4)^(1/3)*(19*x^8 - 16*x^5
+ x^2)*(x^3 - 1)^(1/3) - sqrt(3)*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9
- 105*x^6 + 3*x^3 + 1)) - 70*(-4)^(1/3)*(x^8 + x^5)*log(-(3*(-4)^(2/3)*(x^3
 - 1)^(1/3)*x^2 - 6*(x^3 - 1)^(2/3)*x + (-4)^(1/3)*(x^3 + 1))/(x^3 + 1)) +
35*(-4)^(1/3)*(x^8 + x^5)*log(-(6*(-4)^(1/3)*(5*x^4 - x)*(x^3 - 1)^(2/3) -
(-4)^(2/3)*(19*x^6 - 16*x^3 + 1) - 24*(2*x^5 - x^2)*(x^3 - 1)^(1/3))/(x^6 +
 2*x^3 + 1)) + 18*(22*x^6 + 15*x^3 - 2)*(x^3 - 1)^(2/3))/(x^8 + x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+2x^3-2\right)\left(x^3-1\right)^{\frac{2}{3}}}{\left(x^3+1\right)^2x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+2*x^3-2)/x^6/(x^3+1)^2,x, algorithm="giac")
```

```
[Out] integrate((x^6 + 2*x^3 - 2)*(x^3 - 1)^(2/3)/((x^3 + 1)^2*x^6), x)
```

maple [C] time = 2.45, size = 572, normalized size = 3.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)*(x^6+2*x^3-2)/x^6/(x^3+1)^2,x)
```

```
[Out] -1/5*(22*x^9-7*x^6-17*x^3+2)/(x^3+1)/(x^3-1)^(1/3)/x^5+14*RootOf(RootOf(_Z^
3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*ln((36*RootOf(RootOf(_Z^3+4)^2+6*_Z*Roo
tOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+3*RootOf(RootOf(_Z^3+4)^2+6*_Z*
RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+18*(x^3-1)^(1/3)*RootOf(RootOf
(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)*x^2+12*RootOf(RootOf
(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3+RootOf(_Z^3+4)*x^3+6*x*(x^3-1)^
(2/3)-12*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)-RootOf(_Z^3+4
))/(1+x)/(x^2-x+1))+7/3*RootOf(_Z^3+4)*ln(-(3*RootOf(RootOf(_Z^3+4)^2+6*_Z*
RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+27*RootOf(RootOf(_Z^3+4)^2+6*_
Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+6*(x^3-1)^(2/3)*RootOf(Roo
tOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x+2*(x^3-1)^(1/
3)*RootOf(_Z^3+4)^2*x^2-3*(x^3-1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf
(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)*x^2-3*RootOf(_Z^3+4)*x^3-27*RootOf(RootOf(
_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3-5*x*(x^3-1)^(2/3)+RootOf(_Z^3+4)
+9*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2))/(1+x)/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6 + 2\,x^3 - 2\right)\left(x^3 - 1\right)^{\frac{2}{3}}}{\left(x^3 + 1\right)^2 x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+2*x^3-2)/x^6/(x^3+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((x^6 + 2*x^3 - 2)*(x^3 - 1)^(2/3)/((x^3 + 1)^2*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3 - 1\right)^{2/3}\,\left(x^6 + 2\,x^3 - 2\right)}{x^6\,\left(x^3 + 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(2*x^3 + x^6 - 2))/(x^6*(x^3 + 1)^2),x)
```

```
[Out] int(((x^3 - 1)^(2/3)*(2*x^3 + x^6 - 2))/(x^6*(x^3 + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x - 1\right)\left(x^2 + x + 1\right)\right)^{\frac{2}{3}}\left(x^6 + 2x^3 - 2\right)}{x^6\left(x + 1\right)^2\left(x^2 - x + 1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)*(x**6+2*x**3-2)/x**6/(x**3+1)**2,x)
```

```
[Out] Integral(((x - 1)*(x**2 + x + 1))**(2/3)*(x**6 + 2*x**3 - 2)/(x**6*(x + 1)*
*2*(x**2 - x + 1)**2), x)
```

3.1741 $\int \frac{(2b+ax^6)(-b-cx^4+ax^6)}{x^2(-b+ax^6)^{3/4}(-b+cx^4+ax^6)}\,dx$

Optimal. Leaf size=151

$$\sqrt{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x\sqrt[4]{ax^6-b}}{\sqrt{ax^6-b}-\sqrt{c}\,x^2}\right)-\sqrt{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6-b}}{\sqrt{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^6-b}}\right)+\frac{2\sqrt[4]{ax^6-b}}{x}$$

Rubi [F] time = 4.37, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(2b+ax^6)(-b-cx^4+ax^6)}{x^2(-b+ax^6)^{3/4}(-b+cx^4+ax^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((2*b + a*x^6)*(-b - c*x^4 + a*x^6))/(x^2*(-b + a*x^6)^(3/4)*(-b + c*x^
4 + a*x^6)),x]
```

```
[Out] (-2*c*Sqrt[(a*x^6)/(Sqrt[b] + Sqrt[-b + a*x^6])^2]*(Sqrt[b] + Sqrt[-b + a*x
^6])*EllipticF[2*ArcTan[(-b + a*x^6)^(1/4)/b^(1/4)], 1/2])/(3*a*b^(1/4)*x^3
) - (2*b*(1 - (a*x^6)/b)^(3/4)*Hypergeometric2F1[-1/6, 3/4, 5/6, (a*x^6)/b]
)/(x*(-b + a*x^6)^(3/4)) + (2*c^2*x*(1 - (a*x^6)/b)^(3/4)*Hypergeometric2F1
[1/6, 3/4, 7/6, (a*x^6)/b])/(a*(-b + a*x^6)^(3/4)) + (a*x^5*(1 - (a*x^6)/b)
^(3/4)*Hypergeometric2F1[3/4, 5/6, 11/6, (a*x^6)/b])/(5*(-b + a*x^6)^(3/4))
 - (2*b*c^2*Defer[Int][1/((b - c*x^4 - a*x^6)*(-b + a*x^6)^(3/4)), x])/a -
6*b*c*Defer[Int][x^2/((-b + a*x^6)^(3/4)*(-b + c*x^4 + a*x^6)), x] - (2*c^3
*Defer[Int][x^4/((-b + a*x^6)^(3/4)*(-b + c*x^4 + a*x^6)), x])/a
```

Rubi steps

$$\int \frac{(2b+ax^6)(-b-cx^4+ax^6)}{x^2(-b+ax^6)^{3/4}(-b+cx^4+ax^6)}\,dx=\int\left(\frac{2c^2}{a(-b+ax^6)^{3/4}}+\frac{2b}{x^2(-b+ax^6)^{3/4}}-\frac{2cx^2}{(-b+ax^6)^{3/4}}+\frac{ax^4}{(-b+ax\ldots}\right.$$

$$=a\int\frac{x^4}{(-b+ax^6)^{3/4}}\,dx+(2b)\int\frac{1}{x^2(-b+ax^6)^{3/4}}\,dx-(2c)\int\frac{x\ldots}{(-b+\ldots}$$

$$=-\left(\frac{1}{3}(2c)\,\text{Subst}\left(\int\frac{1}{(-b+ax^2)^{3/4}}\,dx,x,x^3\right)\right)-\frac{(2c)\int\left(\frac{bc}{(b-cx^4-ax^6)(-b\ldots}\right.}{\ldots}$$

$$=-\frac{2b\left(1-\frac{ax^6}{b}\right)^{3/4}{}_2F_1\left(-\frac{1}{6},\frac{3}{4};\frac{5}{6};\frac{ax^6}{b}\right)}{x(-b+ax^6)^{3/4}}+\frac{2c^2x\left(1-\frac{ax^6}{b}\right)^{3/4}{}_2F_1\left(\frac{1}{6},\frac{3}{4};\frac{7}{6};\ldots\right.}{a(-b+ax^6)^{3/4}}$$

$$=-\frac{2c\sqrt{\frac{ax^6}{\left(\sqrt{b}+\sqrt{-b+ax^6}\right)^2}}\left(\sqrt{b}+\sqrt{-b+ax^6}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-b+ax^6}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3a\sqrt[4]{b}\,x^3}\ldots$$

Mathematica [F] time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{(2b+ax^6)(-b-cx^4+ax^6)}{x^2(-b+ax^6)^{3/4}(-b+cx^4+ax^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((2*b + a*x^6)*(-b - c*x^4 + a*x^6))/(x^2*(-b + a*x^6)^(3/4)*(-b
+ c*x^4 + a*x^6)),x]
```

```
[Out] Integrate[((2*b + a*x^6)*(-b - c*x^4 + a*x^6))/(x^2*(-b + a*x^6)^(3/4)*(-b
+ c*x^4 + a*x^6)), x]
```

IntegrateAlgebraic [A] time = 15.65, size = 151, normalized size = 1.00

$$\sqrt{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x\sqrt[4]{ax^6-b}}{\sqrt{ax^6-b}-\sqrt{c}\,x^2}\right)-\sqrt{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6-b}}{\sqrt{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^6-b}}\right)+\frac{2\sqrt[4]{ax^6-b}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2*b + a*x^6)*(-b - c*x^4 + a*x^6))/(x^2*(-b + a*x^6)^(
3/4)*(-b + c*x^4 + a*x^6)),x]
```

```
[Out] (2*(-b + a*x^6)^(1/4))/x + Sqrt[2]*c^(1/4)*ArcTan[(Sqrt[2]*c^(1/4)*x*(-b +
a*x^6)^(1/4))/(-(Sqrt[c]*x^2) + Sqrt[-b + a*x^6])] - Sqrt[2]*c^(1/4)*ArcTan
h[((c^(1/4)*x^2)/Sqrt[2] + Sqrt[-b + a*x^6]/(Sqrt[2]*c^(1/4)))/(x*(-b + a*x
^6)^(1/4))]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+2*b)*(a*x^6-c*x^4-b)/x^2/(a*x^6-b)^(3/4)/(a*x^6+c*x^4-b),x
, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^6-cx^4-b)(ax^6+2\,b)}{(ax^6+cx^4-b)(ax^6-b)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+2*b)*(a*x^6-c*x^4-b)/x^2/(a*x^6-b)^(3/4)/(a*x^6+c*x^4-b),x
, algorithm="giac")
```

```
[Out] integrate((a*x^6 - c*x^4 - b)*(a*x^6 + 2*b)/((a*x^6 + c*x^4 - b)*(a*x^6 - b
)^(3/4)*x^2), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(a\,x^6+2b)(a\,x^6-c\,x^4-b)}{x^2(a\,x^6-b)^{\frac{3}{4}}(a\,x^6+c\,x^4-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6+2*b)*(a*x^6-c*x^4-b)/x^2/(a*x^6-b)^(3/4)/(a*x^6+c*x^4-b),x)
```

```
[Out] int((a*x^6+2*b)*(a*x^6-c*x^4-b)/x^2/(a*x^6-b)^(3/4)/(a*x^6+c*x^4-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^6 - cx^4 - b\right)\left(ax^6 + 2\,b\right)}{\left(ax^6 + cx^4 - b\right)\left(ax^6 - b\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+2*b)*(a*x^6-c*x^4-b)/x^2/(a*x^6-b)^(3/4)/(a*x^6+c*x^4-b),x
, algorithm="maxima")
```

```
[Out] integrate((a*x^6 - c*x^4 - b)*(a*x^6 + 2*b)/((a*x^6 + c*x^4 - b)*(a*x^6 - b
)^(3/4)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(a\,x^6 + 2\,b\right)\,\left(-a\,x^6 + c\,x^4 + b\right)}{x^2\,\left(a\,x^6 - b\right)^{3/4}\,\left(a\,x^6 + c\,x^4 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*b + a*x^6)*(b - a*x^6 + c*x^4))/(x^2*(a*x^6 - b)^(3/4)*(a*x^6 - b
+ c*x^4)),x)
```

```
[Out] int(-((2*b + a*x^6)*(b - a*x^6 + c*x^4))/(x^2*(a*x^6 - b)^(3/4)*(a*x^6 - b
+ c*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6+2*b)*(a*x**6-c*x**4-b)/x**2/(a*x**6-b)**(3/4)/(a*x**6+c*x
**4-b),x)
```

```
[Out] Timed out
```

3.1742 $\int \frac{\sqrt{c+d\sqrt{b+ax^2}}}{x}\,dx$

Optimal. Leaf size=151

$$2\sqrt{d\sqrt{ax^2+b}+c}+\sqrt{\sqrt{b}\,d-c}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{b}\,d-c}\,\sqrt{d\sqrt{ax^2+b}+c}}{c-\sqrt{b}\,d}\right)+\sqrt{-\sqrt{b}\,d-c}\,\tan^{-1}\left(\frac{\sqrt{-\sqrt{b}\,d-c}\,\sqrt{d\sqrt{ax^2}}}{\sqrt{b}\,d+c}\right.$$

Rubi [A] time = 0.39, antiderivative size = 122, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {371, 1398, 825, 827, 1166, 207}

$$2\sqrt{d\sqrt{ax^2+b}+c}-\sqrt{c-\sqrt{b}\,d}\,\tanh^{-1}\left(\frac{\sqrt{d\sqrt{ax^2+b}+c}}{\sqrt{c-\sqrt{b}\,d}}\right)-\sqrt{\sqrt{b}\,d+c}\,\tanh^{-1}\left(\frac{\sqrt{d\sqrt{ax^2+b}+c}}{\sqrt{\sqrt{b}\,d+c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Sqrt[b + a*x^2]]/x,x]
```

```
[Out] 2*Sqrt[c + d*Sqrt[b + a*x^2]] - Sqrt[c - Sqrt[b]*d]*ArcTanh[Sqrt[c + d*Sqrt
[b + a*x^2]]/Sqrt[c - Sqrt[b]*d]] - Sqrt[c + Sqrt[b]*d]*ArcTanh[Sqrt[c + d*
Sqrt[b + a*x^2]]/Sqrt[c + Sqrt[b]*d]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+d\sqrt{b+ax^2}}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{c+d\sqrt{b+ax}}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{c+d\sqrt{x}}}{-b+x}\,dx, x, b+ax^2\right)\\
&= \text{Subst}\left(\int \frac{x\sqrt{c+dx}}{-b+x^2}\,dx, x, \sqrt{b+ax^2}\right)\\
&= 2\sqrt{c+d\sqrt{b+ax^2}} + \text{Subst}\left(\int \frac{bd+cx}{\sqrt{c+dx}\,(-b+x^2)}\,dx, x, \sqrt{b+ax^2}\right)\\
&= 2\sqrt{c+d\sqrt{b+ax^2}} + 2\,\text{Subst}\left(\int \frac{-c^2+bd^2+cx^2}{c^2-bd^2-2cx^2+x^4}\,dx, x, \sqrt{c+d\sqrt{b+ax^2}}\right)\\
&= 2\sqrt{c+d\sqrt{b+ax^2}} + \left(c-\sqrt{b}\,d\right)\text{Subst}\left(\int \frac{1}{-c+\sqrt{b}\,d+x^2}\,dx, x, \sqrt{c+d\sqrt{b+ax^2}}\right)\\
&= 2\sqrt{c+d\sqrt{b+ax^2}} - \sqrt{c-\sqrt{b}\,d}\,\tanh^{-1}\left(\frac{\sqrt{c+d\sqrt{b+ax^2}}}{\sqrt{c-\sqrt{b}\,d}}\right) - \sqrt{c+\sqrt{b}\,d}\,\tanh^{-1}\left(
\end{aligned}$$

Mathematica [A] time = 0.22, size = 122, normalized size = 0.81

$$2\sqrt{d\sqrt{ax^2+b}+c} - \sqrt{c-\sqrt{b}\,d}\,\tanh^{-1}\left(\frac{\sqrt{d\sqrt{ax^2+b}+c}}{\sqrt{c-\sqrt{b}\,d}}\right) - \sqrt{\sqrt{b}\,d+c}\,\tanh^{-1}\left(\frac{\sqrt{d\sqrt{ax^2+b}+c}}{\sqrt{\sqrt{b}\,d+c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Sqrt[b + a*x^2]]/x,x]
```

```
[Out] 2*Sqrt[c + d*Sqrt[b + a*x^2]] - Sqrt[c - Sqrt[b]*d]*ArcTanh[Sqrt[c + d*Sqrt
[b + a*x^2]]/Sqrt[c - Sqrt[b]*d]] - Sqrt[c + Sqrt[b]*d]*ArcTanh[Sqrt[c + d*
Sqrt[b + a*x^2]]/Sqrt[c + Sqrt[b]*d]]
```

IntegrateAlgebraic [A] time = 0.37, size = 151, normalized size = 1.00

$$2\sqrt{d\sqrt{ax^2+b}+c} + \sqrt{\sqrt{b}\,d-c}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{b}\,d-c}\sqrt{d\sqrt{ax^2+b}+c}}{c-\sqrt{b}\,d}\right) + \sqrt{-\sqrt{b}\,d-c}\,\tan^{-1}\left(\frac{\sqrt{-\sqrt{b}\,d-c}\sqrt{d\sqrt{ax^2+b}+c}}{\sqrt{b}\,d+c}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*Sqrt[b + a*x^2]]/x,x]
```

[Out] 2*Sqrt[c + d*Sqrt[b + a*x^2]] + Sqrt[-c + Sqrt[b]*d]*ArcTan[(Sqrt[-c + Sqrt[b]*d]*Sqrt[c + d*Sqrt[b + a*x^2]])/(c - Sqrt[b]*d)] + Sqrt[-c - Sqrt[b]*d]*ArcTan[(Sqrt[-c - Sqrt[b]*d]*Sqrt[c + d*Sqrt[b + a*x^2]])/(c + Sqrt[b]*d)]

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*(a*x^2+b)^(1/2))^(1/2)/x,x, algorithm="fricas")

[Out] Timed out

giac [**B**] time = 0.20, size = 288, normalized size = 1.91

$$\frac{2\sqrt{\sqrt{ax^2+b}\,d+c}\,d+\frac{\left(\sqrt{b}cd^3\operatorname{sgn}\left(\left(\sqrt{ax^2+b}\,d+c\right)d-cd\right)-bd^3|d|+c^2d|d|\operatorname{sgn}\left(\left(\sqrt{ax^2+b}\,d+c\right)d-cd\right)-\sqrt{b}cd^3\right)\arctan\left(\frac{\sqrt{\sqrt{ax^2+b}\,d+c}}{\sqrt{-c+\sqrt{bd^2}}}\right)}{\left(\sqrt{b}\,d+c\right)\sqrt{\sqrt{b}\,d-c}\,|d|}+\frac{\left(\sqrt{b}cd^3\operatorname{sgn}\left(\left(\sqrt{ax^2+b}\,d+c\right)d-cd\right)+bd^3|d|-c^2d|d|\operatorname{sgn}\left(\left(\sqrt{ax^2+b}\,d+c\right)d-cd\right)-\sqrt{b}cd^3\right)\arctan\left(\frac{\sqrt{\sqrt{ax^2+b}\,d+c}}{\sqrt{-c-\sqrt{bd^2}}}\right)}{\left(\sqrt{b}\,d-c\right)\sqrt{-\sqrt{b}\,d-c}\,|d|}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*(a*x^2+b)^(1/2))^(1/2)/x,x, algorithm="giac")

[Out] (2*sqrt(sqrt(a*x^2 + b)*d + c)*d + (sqrt(b)*c*d^3*sgn((sqrt(a*x^2 + b)*d + c)*d - c*d) - b*d^3*abs(d) + c^2*d*abs(d)*sgn((sqrt(a*x^2 + b)*d + c)*d - c*d) - sqrt(b)*c*d^3)*arctan(sqrt(sqrt(a*x^2 + b)*d + c)/sqrt(-c + sqrt(b*d^2)))/((sqrt(b)*d + c)*sqrt(sqrt(b)*d - c)*abs(d)) + (sqrt(b)*c*d^3*sgn((sqrt(a*x^2 + b)*d + c)*d - c*d) + b*d^3*abs(d) - c^2*d*abs(d)*sgn((sqrt(a*x^2 + b)*d + c)*d - c*d) - sqrt(b)*c*d^3)*arctan(sqrt(sqrt(a*x^2 + b)*d + c)/sqrt(-c - sqrt(b*d^2)))/((sqrt(b)*d - c)*sqrt(-sqrt(b)*d - c)*abs(d)))/d

maple [**F**] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\sqrt{a\,x^2+b}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*(a*x^2+b)^(1/2))^(1/2)/x,x)

[Out] int((c+d*(a*x^2+b)^(1/2))^(1/2)/x,x)

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{ax^2+b}\,d+c}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*(a*x^2+b)^(1/2))^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(sqrt(a*x^2 + b)*d + c)/x, x)

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+d\,\sqrt{a\,x^2+b}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*(b + a*x^2)^(1/2))^(1/2)/x,x)

```
[Out] int((c + d*(b + a*x^2)^(1/2))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d\sqrt{ax^2 + b}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(a*x**2+b)**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c + d*sqrt(a*x**2 + b))/x, x)
```

3.1743 $\int \frac{\left(-b+a^2x^4\right)\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx$

Optimal. Leaf size=151

$$\frac{2a^{5/2}x^6+2a^{3/2}x^4\sqrt{a^2x^4+b}+3\sqrt{a}\,bx^2}{8\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}}-\frac{11b\log\left(\sqrt{a^2x^4+b}+\sqrt{2}\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}+ax^2\right)}{8\sqrt{2}\sqrt{a}}$$

Rubi [F] time = 1.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-b+a^2x^4\right)\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-b + a^2*x^4)*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]])/Sqrt[b + a^2*x^4],x]
```

```
[Out] -((b*ArcTanh[(Sqrt[2]*Sqrt[a]*x)/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]])/(Sqrt[2]
*Sqrt[a])) + a^2*Defer[Int][(x^4*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]])/Sqrt[b +
a^2*x^4], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-b+a^2x^4\right)\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx &= \int \left(-\frac{b\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}+\frac{a^2x^4\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\right)dx\\
&= a^2\int \frac{x^4\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx-b\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx\\
&= a^2\int \frac{x^4\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx-b\,\text{Subst}\left(\int \frac{1}{1-2ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+\sqrt{b}}}\right.\\
&= -\frac{b\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\right)}{\sqrt{2}\sqrt{a}}+a^2\int \frac{x^4\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx
\end{aligned}$$

Mathematica [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(-b+a^2x^4\right)\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-b + a^2*x^4)*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]])/Sqrt[b + a^2*x^4
],x]
```

```
[Out] Integrate[((-b + a^2*x^4)*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]])/Sqrt[b + a^2*x^4
], x]
```

IntegrateAlgebraic [A] time = 0.83, size = 151, normalized size = 1.00

$$\frac{2a^{5/2}x^6+2a^{3/2}x^4\sqrt{a^2x^4+b}+3\sqrt{a}\,bx^2}{8\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}}-\frac{11b\log\left(\sqrt{a^2x^4+b}+\sqrt{2}\,\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}+ax^2\right)}{8\sqrt{2}\,\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a^2*x^4)*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]])/Sqrt[b
+ a^2*x^4],x]
```

```
[Out] (3*Sqrt[a]*b*x^2 + 2*a^(5/2)*x^6 + 2*a^(3/2)*x^4*Sqrt[b + a^2*x^4])/(8*Sqrt
[a]*x*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]) - (11*b*Log[a*x^2 + Sqrt[b + a^2*x^4
] + Sqrt[2]*Sqrt[a]*x*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]])/(8*Sqrt[2]*Sqrt[a])
```

fricas [A] time = 2.44, size = 236, normalized size = 1.56

$$\left[\frac{11\sqrt{2}\sqrt{a}\,b\log\left(4a^2x^4+4\sqrt{a^2x^4+b}\,ax^2-2\left(\sqrt{2}a^{\frac{3}{2}}x^3+\sqrt{2}\sqrt{a^2x^4+b}\,\sqrt{a}\,x\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}+b\right)-4\left(a^2x^3-3\sqrt{a^2x^4+b}\,ax\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{32a},\ \frac{11\sqrt{2}\sqrt{-a}\,b\arctan\left(\frac{\sqrt{2}\sqrt{ax^2+\sqrt{a^2x^4+b}}\,\sqrt{-a}}{2ax}\right)-2\left(a^2x^3-3\sqrt{a^2x^4+b}\,ax\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{16a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^4-b)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/(a^2*x^4+b)^(1/2),x,
algorithm="fricas")
```

```
[Out] [1/32*(11*sqrt(2)*sqrt(a)*b*log(4*a^2*x^4 + 4*sqrt(a^2*x^4 + b)*a*x^2 - 2*(
sqrt(2)*a^(3/2)*x^3 + sqrt(2)*sqrt(a^2*x^4 + b)*sqrt(a)*x)*sqrt(a*x^2 + sqr
t(a^2*x^4 + b)) + b) - 4*(a^2*x^3 - 3*sqrt(a^2*x^4 + b)*a*x)*sqrt(a*x^2 + s
qrt(a^2*x^4 + b)))/a, 1/16*(11*sqrt(2)*sqrt(-a)*b*arctan(1/2*sqrt(2)*sqrt(a
*x^2 + sqrt(a^2*x^4 + b))*sqrt(-a)/(a*x)) - 2*(a^2*x^3 - 3*sqrt(a^2*x^4 + b
)*a*x)*sqrt(a*x^2 + sqrt(a^2*x^4 + b)))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a^2x^4-b\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{\sqrt{a^2x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^4-b)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/(a^2*x^4+b)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate((a^2*x^4 - b)*sqrt(a*x^2 + sqrt(a^2*x^4 + b))/sqrt(a^2*x^4 + b),
x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a^2x^4-b\right)\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}{\sqrt{a^2x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^4-b)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/(a^2*x^4+b)^(1/2),x)
```

```
[Out] int((a^2*x^4-b)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/(a^2*x^4+b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a^2x^4-b\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{\sqrt{a^2x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^4-b)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/(a^2*x^4+b)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate((a^2*x^4 - b)*sqrt(a*x^2 + sqrt(a^2*x^4 + b))/sqrt(a^2*x^4 + b),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{\sqrt{a^2\,x^4+b}+a\,x^2}\,\left(b-a^2\,x^4\right)}{\sqrt{a^2\,x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(((b + a^2*x^4)^(1/2) + a*x^2)^(1/2)*(b - a^2*x^4))/(b + a^2*x^4)^(1/2
),x)
```

```
[Out] int(-(((b + a^2*x^4)^(1/2) + a*x^2)^(1/2)*(b - a^2*x^4))/(b + a^2*x^4)^(1/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2+\sqrt{a^2x^4+b}}\,\left(a^2x^4-b\right)}{\sqrt{a^2x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**4-b)*(a*x**2+(a**2*x**4+b)**(1/2))**(1/2)/(a**2*x**4+b)*
*(1/2),x)
```

```
[Out] Integral(sqrt(a*x**2 + sqrt(a**2*x**4 + b))*(a**2*x**4 - b)/sqrt(a**2*x**4
+ b), x)
```

3.1744 $\int \sqrt{b+a^2x^4}\,\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$

Optimal. Leaf size=151

$$\frac{5b\log\left(\sqrt{a^2x^4+b}+\sqrt{2}\,\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}+ax^2\right)}{8\sqrt{2}\,\sqrt{a}}+\frac{2a^{5/2}x^6+2a^{3/2}x^4\sqrt{a^2x^4+b}+3\sqrt{a}\,bx^2}{8\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}}$$

Rubi [F] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{b+a^2x^4}\,\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] Defer[Int][Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]], x]
```

Rubi steps

$$\int \sqrt{b+a^2x^4}\,\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx=\int \sqrt{b+a^2x^4}\,\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Mathematica [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \sqrt{b+a^2x^4}\,\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] Integrate[Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]], x]
```

IntegrateAlgebraic [A] time = 0.74, size = 151, normalized size = 1.00

$$\frac{5b\log\left(\sqrt{a^2x^4+b}+\sqrt{2}\,\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}+ax^2\right)}{8\sqrt{2}\,\sqrt{a}}+\frac{2a^{5/2}x^6+2a^{3/2}x^4\sqrt{a^2x^4+b}+3\sqrt{a}\,bx^2}{8\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] (3*Sqrt[a]*b*x^2 + 2*a^(5/2)*x^6 + 2*a^(3/2)*x^4*Sqrt[b + a^2*x^4])/(8*Sqrt
[a]*x*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]) + (5*b*Log[a*x^2 + Sqrt[b + a^2*x^4]
 + Sqrt[2]*Sqrt[a]*x*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]])/(8*Sqrt[2]*Sqrt[a])
```

fricas [A] time = 2.21, size = 236, normalized size = 1.56

$$\left[\frac{5\sqrt{2}\sqrt{a}\,b\log\left(4a^2x^4+4\sqrt{a^2x^4+b}\,ax^2+2\left(\sqrt{2}a^{\frac{3}{2}}x^3+\sqrt{2}\sqrt{a^2x^4+b}\,\sqrt{a}\,x\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}+b\right)-4\left(a^2x^3-3\sqrt{a^2x^4+b}\,ax\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{32a},-\frac{5\sqrt{2}\sqrt{-a}\,b\arctan\left(\frac{\sqrt{2}\sqrt{ax^2+\sqrt{a^2x^4+b}}\,\sqrt{-a}}{2ax}\right)+2\left(a^2x^3-3\sqrt{a^2x^4+b}\,ax\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{16a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^4+b)^(1/2)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/32*(5*sqrt(2)*sqrt(a)*b*log(4*a^2*x^4 + 4*sqrt(a^2*x^4 + b)*a*x^2 + 2*(s
qrt(2)*a^(3/2)*x^3 + sqrt(2)*sqrt(a^2*x^4 + b)*sqrt(a)*x)*sqrt(a*x^2 + sqrt
(a^2*x^4 + b)) + b) - 4*(a^2*x^3 - 3*sqrt(a^2*x^4 + b)*a*x)*sqrt(a*x^2 + sq
rt(a^2*x^4 + b)))/a, -1/16*(5*sqrt(2)*sqrt(-a)*b*arctan(1/2*sqrt(2)*sqrt(a*
x^2 + sqrt(a^2*x^4 + b))*sqrt(-a)/(a*x)) + 2*(a^2*x^3 - 3*sqrt(a^2*x^4 + b)
*a*x)*sqrt(a*x^2 + sqrt(a^2*x^4 + b)))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^4+b)^(1/2)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, need to choose a branch for the root of a polyno
mial with parameters. This might be wrong.The choice was done assuming [a,b
,x]=[-53,73,-58]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b,x
]=[-62,-47,-7]Warning, need to choose a branch for the root of a polynomial
 with parameters. This might be wrong.The choice was done assuming [a,b,t_n
ostep]=[-60,56,-86]Warning, need to choose a branch for the root of a polyn
omial with parameters. This might be wrong.The choice was done assuming [a,
b,t_nostep]=[-35,-31,25]schur row 3 6.14083e-09Warning, need to choose a br
anch for the root of a polynomial with parameters. This might be wrong.The
choice was done assuming [a,b,t_nostep]=[37,16,-50]schur row 3 6.81038e-09W
arning, need to choose a branch for the root of a polynomial with parameter
s. This might be wrong.The choice was done assuming [a,b,t_nostep]=[-89,-87
,-6]schur row 3 -3.32574e-09Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b,t_nostep]=[-15,-47,-99]schur row 3 6.63154e-12Warning, need to c
hoose a branch for the root of a polynomial with parameters. This might be
wrong.The choice was done assuming [a,b,t_nostep]=[4,51,52]Warning, need to
 choose a branch for the root of a polynomial with parameters. This might b
e wrong.The choice was done assuming [a,b,t_nostep]=[95,47,-75]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [a,b,t_nostep]=[-81,99,6]Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.The choice was done assuming [a,b,t_nostep]=[60,98,14]schur
row 3 -1.41002e-07Warning, need to choose a branch for the root of a polyno
mial with parameters. This might be wrong.The choice was done assuming [a,b
,t_nostep]=[-61,-5,-81]schur row 3 -3.62988e-11Warning, need to choose a br
anch for the root of a polynomial with parameters. This might be wrong.The
choice was done assuming [a,b,t_nostep]=[69,4,92]Warning, need to choose a
branch for the root of a polynomial with parameters. This might be wrong.Th
e choice was done assuming [a,b,t_nostep]=[-86,29,62]Warning, need to choos
e a branch for the root of a polynomial with parameters. This might be wron
g.The choice was done assuming [a,b,t_nostep]=[97,39,62]Warning, need to ch
oose a branch for the root of a polynomial with parameters. This might be w
rong.The choice was done assuming [a,b,t_nostep]=[12,-89,27]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.The choice was done assuming [a,b,t_nostep]=[75,-77,-41]Warning, n
eed to choose a branch for the root of a polynomial with parameters. This m
ight be wrong.The choice was done assuming [a,b,t_nostep]=[37,-64,-25]Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b,t_nostep]=[-84,-28,-58
```

```
]schur row 3 1.98116e-10Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.The choice was done assumin
g [a,b,t_nostep]=[92,64,-69]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b,t_nostep]=[-71,-97,65]schur row 3 -8.62952e-11Warning, need to c
hoose a branch for the root of a polynomial with parameters. This might be
wrong.The choice was done assuming [a,b,t_nostep]=[38,22,35]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.The choice was done assuming [a,b,t_nostep]=[5,-43,43]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [a,b,t_nostep]=[51,28,-73]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b,t_nostep]=[-20,28,31]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming [a,b,t_nostep]=[23,-43,-65
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [a,b,t_nostep]=[29,-3
1,-73]Warning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming [a,b,t_nostep]=[
-47,29,-38]Warning, need to choose a branch for the root of a polynomial wi
th parameters. This might be wrong.The choice was done assuming [a,b,t_nost
ep]=[-61,93,-37]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b,t
_nostep]=[-32,44,87]Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming [a
,b,t_nostep]=[85,86,-73]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.The choice was done assumin
g [a,b,t_nostep]=[-76,72,56]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b,t_nostep]=[-88,76,-73]Warning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming [a,b,t_nostep]=[-35,-51,70]schur row 3 -2.55512e-09Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [a,b,t_nostep]=[-28,-18,-32]schur r
ow 3 -1.27129e-09Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [a,b,
t_nostep]=[-8,45,5]Warning, need to choose a branch for the root of a polyn
omial with parameters. This might be wrong.The choice was done assuming [a,
b,t_nostep]=[-94,53,-97]schur row 3 -2.84124e-11Warning, need to choose a b
ranch for the root of a polynomial with parameters. This might be wrong.The
 choice was done assuming [a,b,t_nostep]=[-90,15,-80]schur row 3 -4.4424e-1
1Unable to divide, perhaps due to rounding error%%%{%%%{%%{[-2,%%%{4,[1]%%%
}]:[1,0,%%%{1,[1]%%%}]%%},[0]%%%}/%%%{%%{[2,1]:[1,0,%%%{1,[1]%%%}]%%},[0]%%
%},[1]%%%}+%%%{%%%{1,[1]%%%},[0]%%%} / %%%{%%%{1/%%%{%%{[2,1]:[1,0,%%%{1,[1
]%%%}]%%},[0]%%%},[0]%%%},[0]%%%} Error: Bad Argument ValueUnable to divide
, perhaps due to rounding error%%%{%%%{%%{[2,%%%{4,[1]%%%}]:[1,0,%%%{1,[1]%
%%}]%%},[0]%%%}/%%%{%%{[-2,1]:[1,0,%%%{1,[1]%%%}]%%},[0]%%%},[1]%%%}+%%%{%%
%{1,[1]%%%},[0]%%%} / %%%{%%%{1/%%%{%%{[-2,1]:[1,0,%%%{1,[1]%%%}]%%},[0]%%%
},[0]%%%},[0]%%%} Error: Bad Argument ValueEvaluation time: 10.56integrate(
(4*a^2*x^4*sqrt(sqrt(a^2*x^4+b)+a*x^2)*sqrt(a^2*x^4+b)-2*a^2*x^4-4*a*x^2*sq
rt(sqrt(a^2*x^4+b)+a*x^2)*sqrt(a^2*x^4+b)+4*b*sqrt(sqrt(a^2*x^4+b)+a*x^2)*s
qrt(a^2*x^4+b)-2*b+sqrt(sqrt(a^2*x^4+b)+a*x^2)*sqrt(a^2*x^4+b)+2*(a^2*x^4+b
))/(4*a^2*x^4-4*a*x^2+4*b+1),x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2x^4+b}\,\sqrt{a\,x^2+\sqrt{a^2x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^4+b)^(1/2)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x)
```

```
[Out] int((a^2*x^4+b)^(1/2)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2x^4+b}\sqrt{ax^2+\sqrt{a^2x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^4+b)^(1/2)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(a^2*x^4 + b)*sqrt(a*x^2 + sqrt(a^2*x^4 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sqrt{a^2\,x^4+b}+a\,x^2}\,\sqrt{a^2\,x^4+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a^2*x^4)^(1/2) + a*x^2)^(1/2)*(b + a^2*x^4)^(1/2),x)
```

```
[Out] int(((b + a^2*x^4)^(1/2) + a*x^2)^(1/2)*(b + a^2*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^2+\sqrt{a^2x^4+b}}\,\sqrt{a^2x^4+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**4+b)**(1/2)*(a*x**2+(a**2*x**4+b)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**2 + sqrt(a**2*x**4 + b))*sqrt(a**2*x**4 + b), x)
```

3.1745 $\int \frac{1}{(2b+ax)\sqrt[4]{bx^2+ax^3}}\,dx$

Optimal. Leaf size=152

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{a}\,\sqrt[4]{b}\,x\,\sqrt[4]{ax^3+bx^2}}{2\sqrt{b}\,\sqrt{ax^3+bx^2}+ax^2}\right)}{2\sqrt{a}\,b^{3/4}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}\,\sqrt{ax^3+bx^2}}{\sqrt{a}}-\frac{\sqrt{a}\,x^2}{2\sqrt[4]{b}}}{x\,\sqrt[4]{ax^3+bx^2}}\right)}{2\sqrt{a}\,b^{3/4}}$$

Rubi [A] time = 0.37, antiderivative size = 172, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2056, 107, 106, 490, 1211, 221, 1699, 203, 206}

$$\frac{\sqrt{-\frac{ax}{b}}\,\sqrt[4]{ax+b}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{ax+b}}{\sqrt[4]{-b}\,\sqrt{-\frac{ax}{b}}}\right)}{\sqrt{2}\,a\sqrt[4]{-b}\,\sqrt[4]{ax^3+bx^2}}-\frac{\sqrt{-\frac{ax}{b}}\,\sqrt[4]{ax+b}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{ax+b}}{\sqrt[4]{-b}\,\sqrt{-\frac{ax}{b}}}\right)}{\sqrt{2}\,a\sqrt[4]{-b}\,\sqrt[4]{ax^3+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2*b + a*x)*(b*x^2 + a*x^3)^(1/4)),x]
```

```
[Out] (Sqrt[-((a*x)/b)]*(b + a*x)^(1/4)*ArcTan[(Sqrt[2]*(b + a*x)^(1/4))/((-b)^(1
/4)*Sqrt[-((a*x)/b)])])/(Sqrt[2]*a*(-b)^(1/4)*(b*x^2 + a*x^3)^(1/4)) - (Sqr
t[-((a*x)/b)]*(b + a*x)^(1/4)*ArcTanh[(Sqrt[2]*(b + a*x)^(1/4))/((-b)^(1/4)
*Sqrt[-((a*x)/b)])])/(Sqrt[2]*a*(-b)^(1/4)*(b*x^2 + a*x^3)^(1/4))
```

Rule 106

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
1/4)), x_Symbol] :> Dist[-4, Subst[Int[x^2/((b*e - a*f - b*x^4)*Sqrt[c - (d
*e)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f
}, x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 107

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
1/4)), x_Symbol] :> Dist[Sqrt[-((f*(c + d*x))/(d*e - c*f))]/Sqrt[c + d*x],
Int[1/((a + b*x)*Sqrt[-((c*f)/(d*e - c*f)) - (d*f*x)/(d*e - c*f)]*(e + f*x)
^(1/4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[-(f/(d*e - c*f)),
0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{(2b+ax)\sqrt[4]{bx^2+ax^3}}\,dx = \frac{\left(\sqrt{x}\,\sqrt[4]{b+ax}\right)\int \frac{1}{\sqrt{x}\,\sqrt[4]{b+ax}\,(2b+ax)}\,dx}{\sqrt[4]{bx^2+ax^3}}$$

$$= \frac{\left(\sqrt{-\frac{ax}{b}}\,\sqrt[4]{b+ax}\right)\int \frac{1}{\sqrt{-\frac{ax}{b}}\,\sqrt[4]{b+ax}\,(2b+ax)}\,dx}{\sqrt[4]{bx^2+ax^3}}$$

$$= -\frac{\left(4\sqrt{-\frac{ax}{b}}\,\sqrt[4]{b+ax}\right)\mathrm{Subst}\left(\int \frac{x^2}{\left(-ab-ax^4\right)\sqrt{1-\frac{x^4}{b}}}\,dx, x, \sqrt[4]{b+ax}\right)}{\sqrt[4]{bx^2+ax^3}}$$

$$= -\frac{\left(2\sqrt{-\frac{ax}{b}}\,\sqrt[4]{b+ax}\right)\mathrm{Subst}\left(\int \frac{1}{\left(\sqrt{-b}-x^2\right)\sqrt{1-\frac{x^4}{b}}}\,dx, x, \sqrt[4]{b+ax}\right)}{a\sqrt[4]{bx^2+ax^3}} + \frac{\left(2\sqrt{-\frac{ax}{b}}\,\sqrt[4]{b+\right.}{}$$

$$= \frac{\left(\sqrt{-\frac{ax}{b}}\,\sqrt[4]{b+ax}\right)\mathrm{Subst}\left(\int \frac{\sqrt{-b}-x^2}{\left(\sqrt{-b}+x^2\right)\sqrt{1-\frac{x^4}{b}}}\,dx, x, \sqrt[4]{b+ax}\right)}{a\sqrt{-b}\,\sqrt[4]{bx^2+ax^3}} - \frac{\left(\sqrt{-\frac{ax}{b}}\,\sqrt[4]{b+ax}\right)}{}$$

$$= -\frac{\left(\sqrt{-\frac{ax}{b}}\,\sqrt[4]{b+ax}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-b}-2x^2}\,dx, x, \frac{\sqrt[4]{b+ax}}{\sqrt{-\frac{ax}{b}}}\right)}{a\sqrt[4]{bx^2+ax^3}} + \frac{\left(\sqrt{-\frac{ax}{b}}\,\sqrt[4]{b+ax}\right)\mathrm{Subst}\left(\right.}{a\sqrt[4]{bx^2}}$$

$$= \frac{\sqrt{-\frac{ax}{b}}\,\sqrt[4]{b+ax}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b+ax}}{\sqrt[4]{-b}\,\sqrt{-\frac{ax}{b}}}\right)}{\sqrt{2}\,a\sqrt[4]{-b}\,\sqrt[4]{bx^2+ax^3}} - \frac{\sqrt{-\frac{ax}{b}}\,\sqrt[4]{b+ax}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b+ax}}{\sqrt[4]{-b}\,\sqrt{-\frac{ax}{b}}}\right)}{\sqrt{2}\,a\sqrt[4]{-b}\,\sqrt[4]{bx^2+ax^3}}$$

Mathematica [C] time = 0.05, size = 58, normalized size = 0.38

$$\frac{x\sqrt[4]{\frac{ax+b}{b}}\,F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{ax}{b},-\frac{ax}{2b}\right)}{b\sqrt[4]{x^2(ax+b)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2*b + a*x)*(b*x^2 + a*x^3)^(1/4)),x]
```

```
[Out] (x*((b + a*x)/b)^(1/4)*AppellF1[1/2, 1/4, 1, 3/2, -((a*x)/b), -1/2*(a*x)/b]
)/(b*(x^2*(b + a*x))^(1/4))
```

IntegrateAlgebraic [A] time = 0.43, size = 152, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{a}\,\sqrt[4]{b}\,x\,\sqrt[4]{ax^3+bx^2}}{2\sqrt{b}\,\sqrt{ax^3+bx^2}+ax^2}\right)}{2\sqrt{a}\,b^{3/4}} - \frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{b}\,\sqrt{ax^3+bx^2}}{\sqrt{a}}-\frac{\sqrt{a}\,x^2}{2\sqrt[4]{b}}}{x\sqrt[4]{ax^3+bx^2}}\right)}{2\sqrt{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((2*b + a*x)*(b*x^2 + a*x^3)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(-1/2*(Sqrt[a]*x^2)/b^(1/4) + (b^(1/4)*Sqrt[b*x^2 + a*x^3])/Sqr
t[a])/(x*(b*x^2 + a*x^3)^(1/4))]/(Sqrt[a]*b^(3/4)) + ArcTanh[(2*Sqrt[a]*b^(
```

```
1/4)*x*(b*x^2 + a*x^3)^(1/4))/(a*x^2 + 2*Sqrt[b]*Sqrt[b*x^2 + a*x^3])]/(2*S
qrt[a]*b^(3/4))
```

fricas [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+2*b)/(a*x^3+b*x^2)^(1/4),x, algorithm="fricas")
[Out] Timed out
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^3 + bx^2)^{\frac{1}{4}}(ax + 2\,b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+2*b)/(a*x^3+b*x^2)^(1/4),x, algorithm="giac")
[Out] integrate(1/((a*x^3 + b*x^2)^(1/4)*(a*x + 2*b)), x)
```

maple [F]   time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax + 2b)\left(a\,x^3 + b\,x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+2*b)/(a*x^3+b*x^2)^(1/4),x)
[Out] int(1/(a*x+2*b)/(a*x^3+b*x^2)^(1/4),x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^3 + bx^2)^{\frac{1}{4}}(ax + 2\,b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+2*b)/(a*x^3+b*x^2)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((a*x^3 + b*x^2)^(1/4)*(a*x + 2*b)), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(2\,b + a\,x)\,\left(a\,x^3 + b\,x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*b + a*x)*(a*x^3 + b*x^2)^(1/4)),x)
[Out] int(1/((2*b + a*x)*(a*x^3 + b*x^2)^(1/4)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^2\,(ax + b)}\,(ax + 2b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+2*b)/(a*x**3+b*x**2)**(1/4),x)
[Out] Integral(1/((x**2*(a*x + b))**(1/4)*(a*x + 2*b)), x)
```

3.1746 $\int \frac{x(-4a+3x)}{\sqrt[3]{x^2(-a+x)}\left(ad-dx+x^4\right)}\,dx$

Optimal. Leaf size=152

$$-\frac{\log\left(a^2d^{2/3}\left(x^3-ax^2\right)^{2/3}+a^2\sqrt[3]{d}\,x^2\sqrt[3]{x^3-ax^2}+a^2x^4\right)}{2d^{2/3}}+\frac{\log\left(ax^2-a\sqrt[3]{d}\,\sqrt[3]{x^3-ax^2}\right)}{d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x^2}{2\sqrt[3]{d}\,\sqrt[3]{x^3-ax^2}+x^2}\right.}{d^{2/3}}$$

Rubi [F] time = 1.90, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(-4a+3x)}{\sqrt[3]{x^2(-a+x)}\left(ad-dx+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-4*a + 3*x))/((x^2*(-a + x))^(1/3)*(a*d - d*x + x^4)),x]
```

```
[Out] (-12*a*x^(2/3)*(-a + x)^(1/3)*Defer[Subst][Defer[Int][x^3/((-a + x^3)^(1/3)
*(a*d - d*x^3 + x^12)), x], x, x^(1/3)])/(-((a - x)*x^2))^(1/3) + (9*x^(2/3
)*(-a + x)^(1/3)*Defer[Subst][Defer[Int][x^6/((-a + x^3)^(1/3)*(a*d - d*x^3
 + x^12)), x], x, x^(1/3)])/(-((a - x)*x^2))^(1/3)
```

Rubi steps

$$\begin{aligned}
\int \frac{x(-4a+3x)}{\sqrt[3]{x^2(-a+x)}\left(ad-dx+x^4\right)}\,dx &= \frac{\left(x^{2/3}\sqrt[3]{-a+x}\right)\int \frac{\sqrt[3]{x}\,(-4a+3x)}{\sqrt[3]{-a+x}\left(ad-dx+x^4\right)}\,dx}{\sqrt[3]{x^2(-a+x)}}\\
&= \frac{\left(3x^{2/3}\sqrt[3]{-a+x}\right)\text{Subst}\left(\int \frac{x^3\left(-4a+3x^3\right)}{\sqrt[3]{-a+x^3}\left(ad-dx^3+x^{12}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x^2(-a+x)}}\\
&= \frac{\left(3x^{2/3}\sqrt[3]{-a+x}\right)\text{Subst}\left(\int \left(-\frac{4ax^3}{\sqrt[3]{-a+x^3}\left(ad-dx^3+x^{12}\right)}+\frac{3x^6}{\sqrt[3]{-a+x^3}\left(ad-dx^3+x^{12}\right)}\right)dx,x,\right.}{\sqrt[3]{x^2(-a+x)}}\\
&= \frac{\left(9x^{2/3}\sqrt[3]{-a+x}\right)\text{Subst}\left(\int \frac{x^6}{\sqrt[3]{-a+x^3}\left(ad-dx^3+x^{12}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x^2(-a+x)}}-\frac{\left(12ax^{2/3}\sqrt[3]{-a+x}\right.}{}
\end{aligned}$$

Mathematica [C] time = 1.43, size = 646, normalized size = 4.25

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-4*a + 3*x))/((x^2*(-a + x))^(1/3)*(a*d - d*x + x^4)),x]
```

```
[Out] (a*x*(4*RootSum[-d + 3*d*#1 - 3*d*#1^2 + d*#1^3 - a^3*#1^4 & , (6*(x/(-a +
x))^(1/3) - 2*Sqrt[3]*ArcTan[(1 + (2*(x/(-a + x))^(1/3))/#1^(1/3))/Sqrt[3]]
*#1^(1/3) + 2*Log[-(x/(-a + x))^(1/3) + #1^(1/3)]*#1^(1/3) - Log[(x/(-a + x
))^(2/3) + (x/(-a + x))^(1/3)*#1^(1/3) + #1^(2/3)]*#1^(1/3))/(-3*d + 6*d*#1
 - 3*d*#1^2 + 4*a^3*#1^3) & ] + 5*RootSum[-d + 3*d*#1 - 3*d*#1^2 + d*#1^3 -
 a^3*#1^4 & , (-6*(x/(-a + x))^(1/3)*#1 + 2*Sqrt[3]*ArcTan[(1 + (2*(x/(-a +
```

```
x))^(1/3))/#1^(1/3))/Sqrt[3]]*#1^(4/3) - 2*Log[-(x/(-a + x))^(1/3) + #1^(1
/3)]*#1^(4/3) + Log[(x/(-a + x))^(2/3) + (x/(-a + x))^(1/3)*#1^(1/3) + #1^(
2/3)]*#1^(4/3))/(-3*d + 6*d*#1 - 3*d*#1^2 + 4*a^3*#1^3) & ] - RootSum[-d +
3*d*#1 - 3*d*#1^2 + d*#1^3 - a^3*#1^4 & , (-6*(x/(-a + x))^(1/3)*#1^2 + 2*S
qrt[3]*ArcTan[(1 + (2*(x/(-a + x))^(1/3))/#1^(1/3))/Sqrt[3]]*#1^(7/3) - 2*L
og[-(x/(-a + x))^(1/3) + #1^(1/3)]*#1^(7/3) + Log[(x/(-a + x))^(2/3) + (x/(
-a + x))^(1/3)*#1^(1/3) + #1^(2/3)]*#1^(7/3))/(-3*d + 6*d*#1 - 3*d*#1^2 + 4
*a^3*#1^3) & ]))/(2*(x/(-a + x))^(1/3)*(x^2*(-a + x))^(1/3))
```

IntegrateAlgebraic [C] time = 0.59, size = 78, normalized size = 0.51

$$-\frac{a\text{RootSum}\left[\#1^{12}d-3\#1^{9}d+3\#1^{6}d-\#1^{3}d+a^{3}\&,\frac{\log\left(\sqrt[3]{x^3-ax^2}-\#1x\right)-\log(x)}{\#1^4-\#1}\&\right]}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-4*a + 3*x))/((x^2*(-a + x))^(1/3)*(a*d - d*x + x^4)
),x]
```

```
[Out] -((a*RootSum[a^3 - d*#1^3 + 3*d*#1^6 - 3*d*#1^9 + d*#1^12 & , (-Log[x] + Lo
g[(-(a*x^2) + x^3)^(1/3) - x*#1])/(-#1 + #1^4) & ])/d)
```

fricas [A] time = 0.44, size = 156, normalized size = 1.03

$$\frac{2\sqrt{3}\left(d^{2}\right)^{\frac{1}{6}}d\arctan\left(\frac{\sqrt{3}\left(\left(d^{2}\right)^{\frac{1}{3}}x^{2}+2\left(-ax^{2}+x^{3}\right)^{\frac{1}{3}}d\right)\left(d^{2}\right)^{\frac{1}{6}}}{3\,dx^{2}}\right)+2\left(d^{2}\right)^{\frac{2}{3}}\log\left(\frac{\left(d^{2}\right)^{\frac{1}{3}}x^{2}-\left(-ax^{2}+x^{3}\right)^{\frac{1}{3}}d}{x^{2}}\right)-\left(d^{2}\right)^{\frac{2}{3}}\log\left(\frac{\left(d^{2}\right)^{\frac{2}{3}}x^{4}+\left(-ax^{2}+x^{3}\right)^{\frac{1}{3}}\left(d^{2}\right)^{\frac{1}{3}}dx^{2}+\left(-ax^{2}+x^{3}\right)^{\frac{2}{3}}d^{2}}{x^{4}}\right)}{2\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*a+3*x)/(x^2*(-a+x))^(1/3)/(x^4+a*d-d*x),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(1/3*sqrt(3)*((d^2)^(1/3)*x^2 + 2*(-a*x^
2 + x^3)^(1/3)*d)*(d^2)^(1/6)/(d*x^2)) + 2*(d^2)^(2/3)*log(((d^2)^(1/3)*x^2
 - (-a*x^2 + x^3)^(1/3)*d)/x^2) - (d^2)^(2/3)*log(((d^2)^(2/3)*x^4 + (-a*x^
2 + x^3)^(1/3)*(d^2)^(1/3)*d*x^2 + (-a*x^2 + x^3)^(2/3)*d^2)/x^4))/d^2
```

giac [A] time = 0.26, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*a+3*x)/(x^2*(-a+x))^(1/3)/(x^4+a*d-d*x),x, algorithm="giac"
)
```

```
[Out] 0
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{x\left(-4a+3x\right)}{\left(x^{2}\left(-a+x\right)\right)^{\frac{1}{3}}\left(x^{4}+ad-dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-4*a+3*x)/(x^2*(-a+x))^(1/3)/(x^4+a*d-d*x),x)
```

```
[Out] int(x*(-4*a+3*x)/(x^2*(-a+x))^(1/3)/(x^4+a*d-d*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(4\,a-3\,x)x}{\left(x^4+ad-dx\right)\left(-(a-x)x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*a+3*x)/(x^2*(-a+x))^(1/3)/(x^4+a*d-d*x),x, algorithm="maxim
a")
```

```
[Out] -integrate((4*a - 3*x)*x/((x^4 + a*d - d*x)*(-(a - x)*x^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x\,(4\,a-3\,x)}{\left(-x^2\,(a-x)\right)^{1/3}\,\left(x^4-d\,x+a\,d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(4*a - 3*x))/((-x^2*(a - x))^(1/3)*(a*d - d*x + x^4)),x)
```

```
[Out] int(-(x*(4*a - 3*x))/((-x^2*(a - x))^(1/3)*(a*d - d*x + x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*a+3*x)/(x**2*(-a+x))**(1/3)/(x**4+a*d-d*x),x)
```

```
[Out] Timed out
```

3.1747 $\int \frac{\left(-b+ax^5\right)^{3/4}\left(4b+ax^5\right)}{x^4\left(-b+cx^4+ax^5\right)}\,dx$

Optimal. Leaf size=152

$$\sqrt{2}\,c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x\sqrt[4]{ax^5-b}}{\sqrt{ax^5-b}-\sqrt{c}\,x^2}\right)+\sqrt{2}\,c^{3/4}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5-b}}{\sqrt{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5-b}}\right)+\frac{4\left(ax^5-b\right)^{3/4}}{3x^3}$$

Rubi [F] time = 1.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-b+ax^5\right)^{3/4}\left(4b+ax^5\right)}{x^4\left(-b+cx^4+ax^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-b + a*x^5)^(3/4)*(4*b + a*x^5))/(x^4*(-b + c*x^4 + a*x^5)),x]
```

```
[Out] (4*(-b + a*x^5)^(3/4)*Hypergeometric2F1[-3/4, -3/5, 2/5, (a*x^5)/b])/(3*x^3
*(1 - (a*x^5)/b)^(3/4)) + 4*c*Defer[Int][(-b + a*x^5)^(3/4)/(-b + c*x^4 + a
*x^5), x] + 5*a*Defer[Int][(x*(-b + a*x^5)^(3/4))/(-b + c*x^4 + a*x^5), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-b+ax^5\right)^{3/4}\left(4b+ax^5\right)}{x^4\left(-b+cx^4+ax^5\right)}\,dx &= \int \left(-\frac{4\left(-b+ax^5\right)^{3/4}}{x^4}+\frac{(4c+5ax)\left(-b+ax^5\right)^{3/4}}{-b+cx^4+ax^5}\right)dx\\
&= -\left(4\int \frac{\left(-b+ax^5\right)^{3/4}}{x^4}\,dx\right)+\int \frac{(4c+5ax)\left(-b+ax^5\right)^{3/4}}{-b+cx^4+ax^5}\,dx\\
&= -\frac{\left(4\left(-b+ax^5\right)^{3/4}\right)\int \frac{\left(1-\frac{ax^5}{b}\right)^{3/4}}{x^4}\,dx}{\left(1-\frac{ax^5}{b}\right)^{3/4}}+\int \left(\frac{4c\left(-b+ax^5\right)^{3/4}}{-b+cx^4+ax^5}+\frac{5ax\left(-b+ax^5\right)^{3/}}{-b+cx^4+ax^5}\right.\\
&= \frac{4\left(-b+ax^5\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{5};\frac{2}{5};\frac{ax^5}{b}\right)}{3x^3\left(1-\frac{ax^5}{b}\right)^{3/4}}+(5a)\int \frac{x\left(-b+ax^5\right)^{3/4}}{-b+cx^4+ax^5}\,dx+(4c)\int \frac{(}{-}
\end{aligned}$$

Mathematica [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{\left(-b+ax^5\right)^{3/4}\left(4b+ax^5\right)}{x^4\left(-b+cx^4+ax^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-b + a*x^5)^(3/4)*(4*b + a*x^5))/(x^4*(-b + c*x^4 + a*x^5)),x]
```

```
[Out] Integrate[((-b + a*x^5)^(3/4)*(4*b + a*x^5))/(x^4*(-b + c*x^4 + a*x^5)), x]
```

IntegrateAlgebraic [A] time = 1.16, size = 152, normalized size = 1.00

$$\sqrt{2}\,c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x\sqrt[4]{ax^5-b}}{\sqrt{ax^5-b}-\sqrt{c}\,x^2}\right)+\sqrt{2}\,c^{3/4}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5-b}}{\sqrt{2}\,\sqrt[4]{c}}+\frac{\sqrt[4]{c}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5-b}}\right)+\frac{4\left(ax^5-b\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^5)^(3/4)*(4*b + a*x^5))/(x^4*(-b + c*x^4 + a*
x^5)),x]
```

```
[Out] (4*(-b + a*x^5)^(3/4))/(3*x^3) + Sqrt[2]*c^(3/4)*ArcTan[(Sqrt[2]*c^(1/4)*x*
(-b + a*x^5)^(1/4))/(-(Sqrt[c]*x^2) + Sqrt[-b + a*x^5])] + Sqrt[2]*c^(3/4)*
ArcTanh[((c^(1/4)*x^2)/Sqrt[2] + Sqrt[-b + a*x^5]/(Sqrt[2]*c^(1/4)))/(x*(-b
 + a*x^5)^(1/4))]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5-b)^(3/4)*(a*x^5+4*b)/x^4/(a*x^5+c*x^4-b),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^5+4\,b\right)\left(ax^5-b\right)^{\frac{3}{4}}}{\left(ax^5+cx^4-b\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5-b)^(3/4)*(a*x^5+4*b)/x^4/(a*x^5+c*x^4-b),x, algorithm="gia
c")
```

```
[Out] integrate((a*x^5 + 4*b)*(a*x^5 - b)^(3/4)/((a*x^5 + c*x^4 - b)*x^4), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^5-b\right)^{\frac{3}{4}}\left(a\,x^5+4b\right)}{x^4\left(a\,x^5+c\,x^4-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^5-b)^(3/4)*(a*x^5+4*b)/x^4/(a*x^5+c*x^4-b),x)
```

```
[Out] int((a*x^5-b)^(3/4)*(a*x^5+4*b)/x^4/(a*x^5+c*x^4-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^5+4\,b\right)\left(ax^5-b\right)^{\frac{3}{4}}}{\left(ax^5+cx^4-b\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5-b)^(3/4)*(a*x^5+4*b)/x^4/(a*x^5+c*x^4-b),x, algorithm="max
ima")
```

```
[Out] integrate((a*x^5 + 4*b)*(a*x^5 - b)^(3/4)/((a*x^5 + c*x^4 - b)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^5-b\right)^{3/4}\,\left(a\,x^5+4\,b\right)}{x^4\,\left(a\,x^5+c\,x^4-b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^5 - b)^(3/4)*(4*b + a*x^5))/(x^4*(a*x^5 - b + c*x^4)),x)
[Out] int(((a*x^5 - b)^(3/4)*(4*b + a*x^5))/(x^4*(a*x^5 - b + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{5} - b\right)^{\frac{3}{4}} \left(ax^{5} + 4b\right)}{x^{4} \left(ax^{5} - b + cx^{4}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**5-b)**(3/4)*(a*x**5+4*b)/x**4/(a*x**5+c*x**4-b),x)
[Out] Integral((a*x**5 - b)**(3/4)*(a*x**5 + 4*b)/(x**4*(a*x**5 - b + c*x**4)), x
)
```

3.1748 $\int \frac{(1+x^8)^{3/4}}{-1+x^8} \, dx$

Optimal. Leaf size=152

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^8+1}}\right)}{4\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^8+1}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^8+1}}{\sqrt{2}x^2-\sqrt{x^8+1}}\right)}{4\ 2^{3/4}}-\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}x\sqrt[4]{x^8+1}}{\sqrt{2}\sqrt{x^8+1}+2x^2}\right)}{4\ 2^{3/4}}$$

Rubi [C] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 0.14, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {429}

$$-xF_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};x^8,-x^8\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^8)^(3/4)/(-1 + x^8),x]
```

```
[Out] -(x*AppellF1[1/8, 1, -3/4, 9/8, x^8, -x^8])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(1+x^8)^{3/4}}{-1+x^8} \, dx = -xF_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};x^8,-x^8\right)$$

Mathematica [C] time = 0.12, size = 110, normalized size = 0.72

$$\frac{9x\left(x^8+1\right)^{3/4}F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};-x^8,x^8\right)}{\left(x^8-1\right)\left(2x^8\left(4F_1\left(\frac{9}{8};-\frac{3}{4},2;\frac{17}{8};-x^8,x^8\right)+3F_1\left(\frac{9}{8};\frac{1}{4},1;\frac{17}{8};-x^8,x^8\right)\right)+9F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};-x^8,x^8\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^8)^(3/4)/(-1 + x^8),x]
```

```
[Out] (9*x*(1 + x^8)^(3/4)*AppellF1[1/8, -3/4, 1, 9/8, -x^8, x^8])/((-1 + x^8)*(9
*AppellF1[1/8, -3/4, 1, 9/8, -x^8, x^8] + 2*x^8*(4*AppellF1[9/8, -3/4, 2, 1
7/8, -x^8, x^8] + 3*AppellF1[9/8, 1/4, 1, 17/8, -x^8, x^8])))
```

IntegrateAlgebraic [A] time = 0.52, size = 152, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^8+1}}\right)}{4\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^8+1}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^8+1}}{\sqrt{2}x^2-\sqrt{x^8+1}}\right)}{4\ 2^{3/4}}-\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}x\sqrt[4]{x^8+1}}{\sqrt{2}\sqrt{x^8+1}+2x^2}\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^8)^(3/4)/(-1 + x^8),x]
```

```
[Out] -1/4*ArcTan[(2^(1/4)*x)/(1 + x^8)^(1/4)]/2^(1/4) + ArcTan[(2^(3/4)*x*(1 + x
^8)^(1/4))/(Sqrt[2]*x^2 - Sqrt[1 + x^8])]/(4*2^(3/4)) - ArcTanh[(2^(1/4)*x)
/(1 + x^8)^(1/4)]/(4*2^(1/4)) - ArcTanh[(2*2^(1/4)*x*(1 + x^8)^(1/4))/(2*x^
2 + Sqrt[2]*Sqrt[1 + x^8])]/(4*2^(3/4))
```

fricas [B] time = 16.98, size = 677, normalized size = 4.45

$$-\frac{1}{8}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{2^{\frac{1}{4}}\left(x^8+1\right)^{\frac{3}{4}}x^2}{x^9+x}\right)-\frac{1}{32}\cdot 2^{\frac{3}{4}}\log\left(-\frac{4\cdot 2^{\frac{1}{4}}\left(x^8+1\right)^{\frac{1}{4}}x^3+2\cdot 2^{\frac{3}{4}}\left(x^8+1\right)^{\frac{3}{4}}x+4\sqrt{x^8+1}\,x^2+\sqrt{2}\left(x^8+2x^4+1\right)}{x^8-2x^4+1}\right)+\frac{1}{32}\cdot 2^{\frac{3}{4}}\log\left(\frac{4\cdot 2^{\frac{1}{4}}\left(x^8+1\right)^{\frac{1}{4}}x^3+2\cdot 2^{\frac{3}{4}}\left(x^8+1\right)^{\frac{3}{4}}x-4\sqrt{x^8+1}\,x^2-\sqrt{2}\left(x^8+2x^4+1\right)}{x^8-2x^4+1}\right)-\frac{1}{8}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{\left(4\cdot 2^{\frac{1}{4}}\left(x^8+1\right)^{\frac{1}{4}}x^3+2\cdot 2^{\frac{3}{4}}\left(x^8+1\right)^{\frac{3}{4}}x+\sqrt{2}\left(2\cdot 2^{\frac{3}{4}}\left(x^8+1\right)^{\frac{1}{4}}x^3-4\sqrt{x^8+1}\,x^2+2\cdot 2^{\frac{1}{4}}\left(x^8+1\right)^{\frac{3}{4}}x-\sqrt{2}\left(x^8+2x^4+1\right)\right)\right)\sqrt{\frac{x^8+2x^4+4\cdot 2^{\frac{1}{4}}\left(x^8+1\right)^{\frac{1}{4}}x^3+4\sqrt{2}\sqrt{x^8+1}\,x^2+2\cdot 2^{\frac{3}{4}}\left(x^8+1\right)^{\frac{3}{4}}x+1}{x^8+2x^4+1}}}{2\left(x^8-2x^4+1\right)}\right)-\frac{1}{8}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{\left(4\cdot 2^{\frac{1}{4}}\left(x^8+1\right)^{\frac{1}{4}}x^3+2\cdot 2^{\frac{3}{4}}\left(x^8+1\right)^{\frac{3}{4}}x+\sqrt{2}\left(2\cdot 2^{\frac{3}{4}}\left(x^8+1\right)^{\frac{1}{4}}x^3+4\sqrt{x^8+1}\,x^2+2\cdot 2^{\frac{1}{4}}\left(x^8+1\right)^{\frac{3}{4}}x+\sqrt{2}\left(x^8+2x^4+1\right)\right)\right)\sqrt{\frac{x^8+2x^4-4\cdot 2^{\frac{1}{4}}\left(x^8+1\right)^{\frac{1}{4}}x^3+4\sqrt{2}\sqrt{x^8+1}\,x^2-2\cdot 2^{\frac{3}{4}}\left(x^8+1\right)^{\frac{3}{4}}x+1}{x^8+2x^4+1}}}{2\left(x^8-2x^4+1\right)}\right)-\frac{1}{32}\cdot 2^{\frac{1}{4}}\log\left(\frac{2\left(x^8+2x^4+4\cdot 2^{\frac{1}{4}}\left(x^8+1\right)^{\frac{1}{4}}x^3+4\sqrt{2}\sqrt{x^8+1}\,x^2+2\cdot 2^{\frac{3}{4}}\left(x^8+1\right)^{\frac{3}{4}}x+1\right)}{x^8+2x^4+1}\right)+\frac{1}{32}\cdot 2^{\frac{1}{4}}\log\left(\frac{2\left(x^8+2x^4-4\cdot 2^{\frac{1}{4}}\left(x^8+1\right)^{\frac{1}{4}}x^3+4\sqrt{2}\sqrt{x^8+1}\,x^2-2\cdot 2^{\frac{3}{4}}\left(x^8+1\right)^{\frac{3}{4}}x+1\right)}{x^8+2x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)^(3/4)/(x^8-1),x, algorithm="fricas")
```

```
[Out] -1/8*2^(3/4)*arctan(2^(1/4)*(x^8 + 1)^(3/4)*x^2/(x^9 + x)) - 1/32*2^(3/4)*l
og(-(4*2^(1/4)*(x^8 + 1)^(1/4)*x^3 + 2*2^(3/4)*(x^8 + 1)^(3/4)*x + 4*sqrt(x
^8 + 1)*x^2 + sqrt(2)*(x^8 + 2*x^4 + 1))/(x^8 - 2*x^4 + 1)) + 1/32*2^(3/4)*
log((4*2^(1/4)*(x^8 + 1)^(1/4)*x^3 + 2*2^(3/4)*(x^8 + 1)^(3/4)*x - 4*sqrt(x
^8 + 1)*x^2 - sqrt(2)*(x^8 + 2*x^4 + 1))/(x^8 - 2*x^4 + 1)) - 1/8*2^(1/4)*a
rctan(1/2*(4*2^(1/4)*(x^8 + 1)^(1/4)*x^3 + 2*2^(3/4)*(x^8 + 1)^(3/4)*x + sq
rt(2)*(2*2^(3/4)*(x^8 + 1)^(1/4)*x^3 - 4*sqrt(x^8 + 1)*x^2 + 2*2^(1/4)*(x^8
 + 1)^(3/4)*x - sqrt(2)*(x^8 + 2*x^4 + 1))*sqrt((x^8 + 2*x^4 + 4*2^(1/4)*(x
^8 + 1)^(1/4)*x^3 + 4*sqrt(2)*sqrt(x^8 + 1)*x^2 + 2*2^(3/4)*(x^8 + 1)^(3/4)
*x + 1)/(x^8 + 2*x^4 + 1)))/(x^8 - 2*x^4 + 1)) - 1/8*2^(1/4)*arctan(1/2*(4*
2^(1/4)*(x^8 + 1)^(1/4)*x^3 + 2*2^(3/4)*(x^8 + 1)^(3/4)*x + sqrt(2)*(2*2^(3
/4)*(x^8 + 1)^(1/4)*x^3 + 4*sqrt(x^8 + 1)*x^2 + 2*2^(1/4)*(x^8 + 1)^(3/4)*x
 + sqrt(2)*(x^8 + 2*x^4 + 1))*sqrt((x^8 + 2*x^4 - 4*2^(1/4)*(x^8 + 1)^(1/4)
*x^3 + 4*sqrt(2)*sqrt(x^8 + 1)*x^2 - 2*2^(3/4)*(x^8 + 1)^(3/4)*x + 1)/(x^8
+ 2*x^4 + 1)))/(x^8 - 2*x^4 + 1)) - 1/32*2^(1/4)*log(2*(x^8 + 2*x^4 + 4*2^(
1/4)*(x^8 + 1)^(1/4)*x^3 + 4*sqrt(2)*sqrt(x^8 + 1)*x^2 + 2*2^(3/4)*(x^8 + 1
)^(3/4)*x + 1)/(x^8 + 2*x^4 + 1)) + 1/32*2^(1/4)*log(2*(x^8 + 2*x^4 - 4*2^(
1/4)*(x^8 + 1)^(1/4)*x^3 + 4*sqrt(2)*sqrt(x^8 + 1)*x^2 - 2*2^(3/4)*(x^8 + 1
)^(3/4)*x + 1)/(x^8 + 2*x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+1\right)^{\frac{3}{4}}}{x^8-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)^(3/4)/(x^8-1),x, algorithm="giac")
```

```
[Out] integrate((x^8 + 1)^(3/4)/(x^8 - 1), x)
```

maple [F] time = 16.95, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+1\right)^{\frac{3}{4}}}{x^8-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8+1)^(3/4)/(x^8-1),x)
```

```
[Out] int((x^8+1)^(3/4)/(x^8-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+1\right)^{\frac{3}{4}}}{x^8-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)^(3/4)/(x^8-1),x, algorithm="maxima")
```

```
[Out] integrate((x^8 + 1)^(3/4)/(x^8 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^8+1\right)^{3/4}}{x^8-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 + 1)^(3/4)/(x^8 - 1),x)
```

```
[Out] int((x^8 + 1)^(3/4)/(x^8 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+1\right)^{\frac{3}{4}}}{(x-1)\,(x+1)\left(x^2+1\right)\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8+1)**(3/4)/(x**8-1),x)
```

```
[Out] Integral((x**8 + 1)**(3/4)/((x - 1)*(x + 1)*(x**2 + 1)*(x**4 + 1)), x)
```

3.1749 $\int \frac{x^4}{\sqrt[4]{-1+x^4}\left(-1+2x^4+x^8\right)}\,dx$

Optimal. Leaf size=152

$$\frac{\tan^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{x^4-1}}\right)}{4\,2^{5/8}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{x^4-1}}\right)}{4\,2^{5/8}}+\frac{\tan^{-1}\left(\frac{2^{5/8}x\sqrt[4]{x^4-1}}{\sqrt[4]{2}\,x^2-\sqrt{x^4-1}}\right)}{8\sqrt[8]{2}}-\frac{\tanh^{-1}\left(\frac{2\,2^{3/8}x\sqrt[4]{x^4-1}}{2^{3/4}\sqrt{x^4-1}+2x^2}\right)}{8\sqrt[8]{2}}$$

Rubi [A] time = 0.26, antiderivative size = 282, normalized size of antiderivative = 1.86, number of steps used = 16, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1528, 377, 211, 1165, 628, 1162, 617, 204, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{x^4-1}}\right)}{4\,2^{5/8}}-\frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(1-\frac{2^{5/8}x}{\sqrt[4]{x^4-1}}\right)}{4\,2^{5/8}\left(2-\sqrt{2}\right)}+\frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{2^{5/8}x}{\sqrt[4]{x^4-1}}+1\right)}{4\,2^{5/8}\left(2-\sqrt{2}\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{x^4-1}}\right)}{4\,2^{5/8}}-\frac{\left(1-\sqrt{2}\right)\log\left(-\frac{2x}{\sqrt[4]{x^4-1}}+\frac{2^{5/8}x^2}{\sqrt{x^4-1}}+2^{3/8}\right)}{8\,2^{5/8}\left(2-\sqrt{2}\right)}+\frac{\left(1-\sqrt{2}\right)\log\left(\frac{2^{5/8}x}{\sqrt[4]{x^4-1}}+\frac{\sqrt[4]{2}\,x^2}{\sqrt{x^4-1}}+1\right)}{8\,2^{5/8}\left(2-\sqrt{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((-1 + x^4)^(1/4)*(-1 + 2*x^4 + x^8)),x]
```

```
[Out] ArcTan[(2^(1/8)*x)/(-1 + x^4)^(1/4)]/(4*2^(5/8)) - ((1 - Sqrt[2])*ArcTan[1
- (2^(5/8)*x)/(-1 + x^4)^(1/4)])/(4*2^(5/8)*(2 - Sqrt[2])) + ((1 - Sqrt[2])
*ArcTan[1 + (2^(5/8)*x)/(-1 + x^4)^(1/4)])/(4*2^(5/8)*(2 - Sqrt[2])) + ArcT
anh[(2^(1/8)*x)/(-1 + x^4)^(1/4)]/(4*2^(5/8)) - ((1 - Sqrt[2])*Log[2^(3/8)
+ (2^(5/8)*x^2)/Sqrt[-1 + x^4] - (2*x)/(-1 + x^4)^(1/4)])/(8*2^(5/8)*(2 - S
qrt[2])) + ((1 - Sqrt[2])*Log[1 + (2^(1/4)*x^2)/Sqrt[-1 + x^4] + (2^(5/8)*x
)/(-1 + x^4)^(1/4)])/(8*2^(5/8)*(2 - Sqrt[2]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1528

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q, (
f*x)^m/(a + b*x^n + c*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, e, f, q, n}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&  !IntegerQ[q] &&
IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{-1+x^4}\left(-1+2x^4+x^8\right)}\,dx = \int\left(\frac{1-\frac{1}{\sqrt{2}}}{\sqrt[4]{-1+x^4}\left(2-2\sqrt{2}+2x^4\right)}+\frac{1+\frac{1}{\sqrt{2}}}{\sqrt[4]{-1+x^4}\left(2+2\sqrt{2}+2x^4\right)}\right)dx$$

$$= \frac{1}{2}\left(2-\sqrt{2}\right)\int\frac{1}{\sqrt[4]{-1+x^4}\left(2-2\sqrt{2}+2x^4\right)}\,dx+\frac{1}{2}\left(2+\sqrt{2}\right)\int\frac{}{\sqrt[4]{-1+x^4}}$$

$$= \frac{1}{2}\left(2-\sqrt{2}\right)\text{Subst}\left(\int\frac{1}{2-2\sqrt{2}-\left(4-2\sqrt{2}\right)x^4}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)+\frac{1}{2}\left(2-\right.$$

$$= \frac{\text{Subst}\left(\int\frac{1}{1-\sqrt[4]{2}x^2}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)}{4\sqrt{2}}+\frac{\text{Subst}\left(\int\frac{1}{1+\sqrt[4]{2}x^2}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)}{4\sqrt{2}}+\frac{1}{4}\left(2\right.$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{-1+x^4}}\right)}{4\,2^{5/8}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{-1+x^4}}\right)}{4\,2^{5/8}}+\frac{\text{Subst}\left(\int\frac{2^{3/8}+2x}{-\frac{1}{\sqrt[4]{2}}-2^{3/8}x-x^2}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right.}{16\sqrt[8]{2}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{-1+x^4}}\right)}{4\,2^{5/8}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{-1+x^4}}\right)}{4\,2^{5/8}}+\frac{\log\left(2^{3/8}+\frac{2^{5/8}x^2}{\sqrt{-1+x^4}}-\frac{2x}{\sqrt[4]{-1+x^4}}\right)}{16\sqrt[8]{2}}-\frac{\text{lo}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{-1+x^4}}\right)}{4\,2^{5/8}}-\frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(1-\frac{2^{5/8}x}{\sqrt[4]{-1+x^4}}\right)}{4\,2^{5/8}\left(2-\sqrt{2}\right)}+\frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(1+\right.}{4\,2^{5/8}\left(2-\sqrt{2}\right)}$$

Mathematica [A] time = 0.25, size = 175, normalized size = 1.15

$$\frac{4\tan^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^4-1}}\right)+4\tanh^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^4-1}}\right)+\sqrt{2}\left(2\tan^{-1}\left(1-\frac{2^{5/8}x}{\sqrt[4]{x^4-1}}\right)-2\tan^{-1}\left(\frac{2^{5/8}x}{\sqrt[4]{x^4-1}}+1\right)+\log\left(-\frac{2^{5/8}x}{\sqrt[4]{x^4-1}}+\frac{\sqrt[4]{2}x^2}{\sqrt{x^4-1}}+1\right)-\log\left(\frac{2^{5/8}x}{\sqrt[4]{x^4-1}}+\frac{\sqrt[4]{2}x^2}{\sqrt{x^4-1}}+1\right)\right)}{16\,2^{5/8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((-1 + x^4)^(1/4)*(-1 + 2*x^4 + x^8)),x]
```

```
[Out] (4*ArcTan[(2^(1/8)*x)/(-1 + x^4)^(1/4)] + 4*ArcTanh[(2^(1/8)*x)/(-1 + x^4)^
(1/4)] + Sqrt[2]*(2*ArcTan[1 - (2^(5/8)*x)/(-1 + x^4)^(1/4)] - 2*ArcTan[1 +
 (2^(5/8)*x)/(-1 + x^4)^(1/4)] + Log[1 + (2^(1/4)*x^2)/Sqrt[-1 + x^4] - (2^
(5/8)*x)/(-1 + x^4)^(1/4)] - Log[1 + (2^(1/4)*x^2)/Sqrt[-1 + x^4] + (2^(5/8
)*x)/(-1 + x^4)^(1/4)]))/(16*2^(5/8))
```

IntegrateAlgebraic [A] time = 0.50, size = 152, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^4-1}}\right)}{4\,2^{5/8}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^4-1}}\right)}{4\,2^{5/8}}+\frac{\tan^{-1}\left(\frac{2^{5/8}x\sqrt[4]{x^4-1}}{\sqrt[4]{2}x^2-\sqrt{x^4-1}}\right)}{8\sqrt[8]{2}}-\frac{\tanh^{-1}\left(\frac{2\,2^{3/8}x\sqrt[4]{x^4-1}}{2^{3/4}\sqrt{x^4-1}+2x^2}\right)}{8\sqrt[8]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((-1 + x^4)^(1/4)*(-1 + 2*x^4 + x^8)),x]
```

```
[Out] ArcTan[(2^(1/8)*x)/(-1 + x^4)^(1/4)]/(4*2^(5/8)) + ArcTan[(2^(5/8)*x*(-1 +
x^4)^(1/4))/(2^(1/4)*x^2 - Sqrt[-1 + x^4])]/(8*2^(1/8)) + ArcTanh[(2^(1/8)*
x)/(-1 + x^4)^(1/4)]/(4*2^(5/8)) - ArcTanh[(2*2^(3/8)*x*(-1 + x^4)^(1/4))/(
2*x^2 + 2^(3/4)*Sqrt[-1 + x^4])]/(8*2^(1/8))
```

fricas [B] time = 13.02, size = 2047, normalized size = 13.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1)^(1/4)/(x^8+2*x^4-1),x, algorithm="fricas")
```

```
[Out] -1/256*32^(7/8)*sqrt(2)*arctan(-1/32*(2080*x^16 - 3968*x^12 + 2112*x^8 - 12
8*x^4 - sqrt(2)*(32^(5/8)*sqrt(2)*(151*x^16 - 392*x^12 + 254*x^8 - 8*x^4 -
1) + 128*(x^4 - 1)^(3/4)*(2^(3/4)*(25*x^13 - 26*x^9 - x^5) - 2*2^(1/4)*(11*
x^13 - 12*x^9 - x^5)) - 8*32^(1/8)*sqrt(2)*(189*x^16 - 418*x^12 + 236*x^8 -
 2*x^4 - 1) + sqrt(x^4 - 1)*(32^(7/8)*sqrt(2)*(91*x^14 - 123*x^10 + 19*x^6
+ x^2) - 8*32^(3/8)*sqrt(2)*(86*x^14 - 101*x^10 + 8*x^6 + x^2)) + 32*(28*x^
15 - 6*x^11 - 24*x^7 - 2*x^3 + sqrt(2)*(3*x^15 - 27*x^11 + 27*x^7 + x^3))*(
x^4 - 1)^(1/4))*sqrt((12*2^(3/4)*(x^8 + 2*x^4 - 1) + (x^4 - 1)^(3/4)*(32^(7
/8)*sqrt(2)*(x^5 + 2*x) + 4*32^(3/8)*sqrt(2)*(x^5 + 3*x)) + 32*(x^6 + 3*x^2
 + sqrt(2)*(x^6 + 2*x^2))*sqrt(x^4 - 1) + 16*2^(1/4)*(x^8 + 2*x^4 - 1) + 2*
(x^4 - 1)^(1/4)*(32^(5/8)*sqrt(2)*(x^7 + 3*x^3) + 8*32^(1/8)*sqrt(2)*(x^7 +
 2*x^3)))/(x^8 + 2*x^4 - 1)) - 8*(x^4 - 1)^(3/4)*(32^(5/8)*sqrt(2)*(81*x^13
 - 79*x^9 - 3*x^5 + x) - 8*32^(1/8)*sqrt(2)*(5*x^13 - 22*x^9 + 11*x^5)) + 5
12*sqrt(2)*(3*x^16 - 5*x^12 + 3*x^8 - x^4) + 128*sqrt(x^4 - 1)*(2^(3/4)*(17
*x^14 - 30*x^10 + 15*x^6) + 2^(1/4)*(31*x^14 - 33*x^10 + 3*x^6 - x^2)) - 4*
(x^4 - 1)^(1/4)*(32^(7/8)*sqrt(2)*(19*x^15 - 13*x^11 - 9*x^7 + 3*x^3) - 8*3
2^(3/8)*sqrt(2)*(39*x^15 - 82*x^11 + 41*x^7)) + 32)/(383*x^16 - 772*x^12 +
382*x^8 + 4*x^4 - 1)) + 1/256*32^(7/8)*sqrt(2)*arctan(-1/32*(2080*x^16 - 39
68*x^12 + 2112*x^8 - 128*x^4 + sqrt(2)*(32^(5/8)*sqrt(2)*(151*x^16 - 392*x^
12 + 254*x^8 - 8*x^4 - 1) - 128*(x^4 - 1)^(3/4)*(2^(3/4)*(25*x^13 - 26*x^9
 - x^5) - 2*2^(1/4)*(11*x^13 - 12*x^9 - x^5)) - 8*32^(1/8)*sqrt(2)*(189*x^16
 - 418*x^12 + 236*x^8 - 2*x^4 - 1) + sqrt(x^4 - 1)*(32^(7/8)*sqrt(2)*(91*x^
14 - 123*x^10 + 19*x^6 + x^2) - 8*32^(3/8)*sqrt(2)*(86*x^14 - 101*x^10 + 8*
x^6 + x^2)) - 32*(28*x^15 - 6*x^11 - 24*x^7 - 2*x^3 + sqrt(2)*(3*x^15 - 27*
x^11 + 27*x^7 + x^3))*(x^4 - 1)^(1/4))*sqrt((12*2^(3/4)*(x^8 + 2*x^4 - 1) -
 (x^4 - 1)^(3/4)*(32^(7/8)*sqrt(2)*(x^5 + 2*x) + 4*32^(3/8)*sqrt(2)*(x^5 +
3*x)) + 32*(x^6 + 3*x^2 + sqrt(2)*(x^6 + 2*x^2))*sqrt(x^4 - 1) + 16*2^(1/4)
*(x^8 + 2*x^4 - 1) - 2*(x^4 - 1)^(1/4)*(32^(5/8)*sqrt(2)*(x^7 + 3*x^3) + 8*
32^(1/8)*sqrt(2)*(x^7 + 2*x^3)))/(x^8 + 2*x^4 - 1)) + 8*(x^4 - 1)^(3/4)*(32
^(5/8)*sqrt(2)*(81*x^13 - 79*x^9 - 3*x^5 + x) - 8*32^(1/8)*sqrt(2)*(5*x^13
 - 22*x^9 + 11*x^5)) + 512*sqrt(2)*(3*x^16 - 5*x^12 + 3*x^8 - x^4) + 128*sqr
t(x^4 - 1)*(2^(3/4)*(17*x^14 - 30*x^10 + 15*x^6) + 2^(1/4)*(31*x^14 - 33*x^
10 + 3*x^6 - x^2)) + 4*(x^4 - 1)^(1/4)*(32^(7/8)*sqrt(2)*(19*x^15 - 13*x^11
 - 9*x^7 + 3*x^3) - 8*32^(3/8)*sqrt(2)*(39*x^15 - 82*x^11 + 41*x^7)) + 32)/
(383*x^16 - 772*x^12 + 382*x^8 + 4*x^4 - 1)) - 1/1024*32^(7/8)*sqrt(2)*log(
128*(12*2^(3/4)*(x^8 + 2*x^4 - 1) + (x^4 - 1)^(3/4)*(32^(7/8)*sqrt(2)*(x^5
+ 2*x) + 4*32^(3/8)*sqrt(2)*(x^5 + 3*x)) + 32*(x^6 + 3*x^2 + sqrt(2)*(x^6 +
 2*x^2))*sqrt(x^4 - 1) + 16*2^(1/4)*(x^8 + 2*x^4 - 1) + 2*(x^4 - 1)^(1/4)*(
32^(5/8)*sqrt(2)*(x^7 + 3*x^3) + 8*32^(1/8)*sqrt(2)*(x^7 + 2*x^3)))/(x^8 +
2*x^4 - 1)) + 1/1024*32^(7/8)*sqrt(2)*log(128*(12*2^(3/4)*(x^8 + 2*x^4 - 1)
 - (x^4 - 1)^(3/4)*(32^(7/8)*sqrt(2)*(x^5 + 2*x) + 4*32^(3/8)*sqrt(2)*(x^5
+ 3*x)) + 32*(x^6 + 3*x^2 + sqrt(2)*(x^6 + 2*x^2))*sqrt(x^4 - 1) + 16*2^(1/
4)*(x^8 + 2*x^4 - 1) - 2*(x^4 - 1)^(1/4)*(32^(5/8)*sqrt(2)*(x^7 + 3*x^3) +
8*32^(1/8)*sqrt(2)*(x^7 + 2*x^3)))/(x^8 + 2*x^4 - 1)) - 1/128*32^(7/8)*arct
an(1/16*(sqrt(2)*(32^(5/8)*(7*x^8 - 6*x^4 + 1) + sqrt(x^4 - 1)*(32^(7/8)*(3
*x^6 - x^2) + 8*32^(3/8)*(2*x^6 - x^2)) + 8*32^(1/8)*(5*x^8 - 4*x^4 + 1))*s
qrt(3*2^(3/4) - 4*2^(1/4)) + 4*(8*32^(1/8)*x^5 + 32^(5/8)*(x^5 - x))*(x^4 -
 1)^(3/4) + 2*(8*32^(3/8)*x^7 + 32^(7/8)*(x^7 - x^3))*(x^4 - 1)^(1/4))/(x^8
 + 2*x^4 - 1)) + 1/512*32^(7/8)*log((32^(7/8)*(x^8 + 1) + 32*(x^5 - sqrt(2)
*x + x)*(x^4 - 1)^(3/4) - 4*sqrt(x^4 - 1)*(8*32^(1/8)*x^2 - 32^(5/8)*(x^6 +
 x^2)) + 4*32^(3/8)*(x^8 - 2*x^4 - 1) - 32*(x^4 - 1)^(1/4)*(2^(3/4)*x^3 - 2
^(1/4)*(x^7 + x^3)))/(x^8 + 2*x^4 - 1)) - 1/512*32^(7/8)*log(-(32^(7/8)*(x^
8 + 1) - 32*(x^5 - sqrt(2)*x + x)*(x^4 - 1)^(3/4) - 4*sqrt(x^4 - 1)*(8*32^(
1/8)*x^2 - 32^(5/8)*(x^6 + x^2)) + 4*32^(3/8)*(x^8 - 2*x^4 - 1) + 32*(x^4 -
 1)^(1/4)*(2^(3/4)*x^3 - 2^(1/4)*(x^7 + x^3)))/(x^8 + 2*x^4 - 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1)^(1/4)/(x^8+2*x^4-1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [C] time = 17.88, size = 1593, normalized size = 10.48

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4-1)^(1/4)/(x^8+2*x^4-1),x)
```

```
[Out] 1/2*ln(-(-RootOf(_Z^8-8)^9*x^4+64*(x^4-1)^(1/2)*RootOf(_Z^8-8)^6*RootOf(8*R
ootOf(_Z^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*_Z^2)*x^2+4*(x^4-1)^(1/4)*RootOf(_Z
^8-8)^6*x^3-64*RootOf(8*RootOf(_Z^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*_Z^2)*Root
Of(_Z^8-8)^4*x^4-2*x^4*RootOf(_Z^8-8)^5+8*(x^4-1)^(1/2)*RootOf(_Z^8-8)^3*x^
2-128*RootOf(8*RootOf(_Z^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*_Z^2)*x^4+2*RootOf(
_Z^8-8)^5-16*(x^4-1)^(3/4)*x+128*RootOf(8*RootOf(_Z^8-8)^5*_Z+RootOf(_Z^8-8
)^2+256*_Z^2))/(RootOf(_Z^8-8)^4*x^4-2*x^4+2))*RootOf(_Z^8-8)^4*RootOf(8*Ro
otOf(_Z^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*_Z^2)+1/16*ln(-(-RootOf(_Z^8-8)^9*x^
4+64*(x^4-1)^(1/2)*RootOf(_Z^8-8)^6*RootOf(8*RootOf(_Z^8-8)^5*_Z+RootOf(_Z^
8-8)^2+256*_Z^2)*x^2+4*(x^4-1)^(1/4)*RootOf(_Z^8-8)^6*x^3-64*RootOf(8*RootO
f(_Z^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*_Z^2)*RootOf(_Z^8-8)^4*x^4-2*x^4*RootOf
(_Z^8-8)^5+8*(x^4-1)^(1/2)*RootOf(_Z^8-8)^3*x^2-128*RootOf(8*RootOf(_Z^8-8)
^5*_Z+RootOf(_Z^8-8)^2+256*_Z^2)*x^4+2*RootOf(_Z^8-8)^5-16*(x^4-1)^(3/4)*x+
128*RootOf(8*RootOf(_Z^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*_Z^2))/(RootOf(_Z^8-8
)^4*x^4-2*x^4+2))*RootOf(_Z^8-8)-1/16*RootOf(_Z^8-8)*ln((-RootOf(_Z^8-8)^9*
x^4+4*(x^4-1)^(1/4)*RootOf(_Z^8-8)^6*x^3-2*x^4*RootOf(_Z^8-8)^5-8*(x^4-1)^(
1/2)*RootOf(_Z^8-8)^3*x^2+2*RootOf(_Z^8-8)^5+16*(x^4-1)^(3/4)*x)/(RootOf(_Z
^8-8)^4*x^4-2*x^4+2))-1/32*ln(-(4*RootOf(8*RootOf(_Z^8-8)^5*_Z+RootOf(_Z^8-
8)^2+256*_Z^2)*RootOf(_Z^8-8)^8*x^4+(x^4-1)^(1/4)*RootOf(_Z^8-8)^6*x^3-8*Ro
otOf(8*RootOf(_Z^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*_Z^2)*RootOf(_Z^8-8)^4*x^4+
x^4*RootOf(_Z^8-8)^5+32*(x^4-1)^(1/2)*RootOf(8*RootOf(_Z^8-8)^5*_Z+RootOf(_
Z^8-8)^2+256*_Z^2)*RootOf(_Z^8-8)^2*x^2+64*(x^4-1)^(1/4)*RootOf(8*RootOf(_Z
^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*_Z^2)*RootOf(_Z^8-8)*x^3+8*RootOf(_Z^8-8)^4
*RootOf(8*RootOf(_Z^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*_Z^2)-2*RootOf(_Z^8-8)*x
^4-4*(x^4-1)^(3/4)*x+2*RootOf(_Z^8-8))/(RootOf(_Z^8-8)^4*x^4+2*x^4-2))*Root
Of(_Z^8-8)^5-ln(-(4*RootOf(8*RootOf(_Z^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*_Z^2)
*RootOf(_Z^8-8)^8*x^4+(x^4-1)^(1/4)*RootOf(_Z^8-8)^6*x^3-8*RootOf(8*RootOf(
_Z^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*_Z^2)*RootOf(_Z^8-8)^4*x^4+x^4*RootOf(_Z^
8-8)^5+32*(x^4-1)^(1/2)*RootOf(8*RootOf(_Z^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*_
Z^2)*RootOf(_Z^8-8)^2*x^2+64*(x^4-1)^(1/4)*RootOf(8*RootOf(_Z^8-8)^5*_Z+Roo
tOf(_Z^8-8)^2+256*_Z^2)*RootOf(_Z^8-8)*x^3+8*RootOf(_Z^8-8)^4*RootOf(8*Root
Of(_Z^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*_Z^2)-2*RootOf(_Z^8-8)*x^4-4*(x^4-1)^(
3/4)*x+2*RootOf(_Z^8-8))/(RootOf(_Z^8-8)^4*x^4+2*x^4-2))*RootOf(8*RootOf(_Z
^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*_Z^2)+RootOf(8*RootOf(_Z^8-8)^5*_Z+RootOf(_
Z^8-8)^2+256*_Z^2)*ln((4*RootOf(8*RootOf(_Z^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*
_Z^2)*RootOf(_Z^8-8)^8*x^4+(x^4-1)^(1/2)*RootOf(_Z^8-8)^7*x^2+(x^4-1)^(1/4)
*RootOf(_Z^8-8)^6*x^3-8*RootOf(8*RootOf(_Z^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*_
Z^2)*RootOf(_Z^8-8)^4*x^4+32*(x^4-1)^(1/2)*RootOf(8*RootOf(_Z^8-8)^5*_Z+Roo
tOf(_Z^8-8)^2+256*_Z^2)*RootOf(_Z^8-8)^2*x^2+64*(x^4-1)^(1/4)*RootOf(8*Root
Of(_Z^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*_Z^2)*RootOf(_Z^8-8)*x^3+8*RootOf(_Z^8
```

```
-8)^4*RootOf(8*RootOf(_Z^8-8)^5*_Z+RootOf(_Z^8-8)^2+256*_Z^2)+4*(x^4-1)^(3/
4)*x)/(RootOf(_Z^8-8)^4*x^4+2*x^4-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^8+2\,x^4-1\right)\left(x^4-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1)^(1/4)/(x^8+2*x^4-1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((x^8 + 2*x^4 - 1)*(x^4 - 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(x^4-1\right)^{1/4}\,\left(x^8+2\,x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((x^4 - 1)^(1/4)*(2*x^4 + x^8 - 1)),x)
```

```
[Out] int(x^4/((x^4 - 1)^(1/4)*(2*x^4 + x^8 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt[4]{(x-1)\,(x+1)\,\left(x^2+1\right)}\,\left(x^8+2x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4-1)**(1/4)/(x**8+2*x**4-1),x)
```

```
[Out] Integral(x**4/(((x - 1)*(x + 1)*(x**2 + 1))**(1/4)*(x**8 + 2*x**4 - 1)), x)
```

3.1750 $\int \frac{\left(-1+x^4\right)^{3/4}}{-1+2x^4+x^8}\,dx$

Optimal. Leaf size=152

$$\frac{\tan^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt[8]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt[8]{2}}-\frac{\tan^{-1}\left(\frac{2^{5/8}x\sqrt[4]{x^4-1}}{\sqrt[4]{2}x^2-\sqrt{x^4-1}}\right)}{4\,2^{5/8}}+\frac{\tanh^{-1}\left(\frac{2\,2^{3/8}x\sqrt[4]{x^4-1}}{2^{3/4}\sqrt{x^4-1}+2x^2}\right)}{4\,2^{5/8}}$$

Rubi [A] time = 0.22, antiderivative size = 282, normalized size of antiderivative = 1.86, number of steps used = 25, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {1428, 408, 240, 212, 206, 203, 377, 211, 1165, 628, 1162, 617, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt[8]{2}}+\frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(1-\frac{2^{5/8}x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt[8]{2}\left(2-\sqrt{2}\right)}-\frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{2^{5/8}x}{\sqrt[4]{x^4-1}}+1\right)}{4\sqrt[8]{2}\left(2-\sqrt{2}\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt[8]{2}}+\frac{\left(1-\sqrt{2}\right)\log\left(-\frac{2x}{\sqrt[4]{x^4-1}}+\frac{2^{5/8}x^2}{\sqrt{x^4-1}}+2^{3/8}\right)}{8\sqrt[8]{2}\left(2-\sqrt{2}\right)}-\frac{\left(1-\sqrt{2}\right)\log\left(\frac{2^{5/8}x}{\sqrt[4]{x^4-1}}+\frac{\sqrt[4]{2}x^2}{\sqrt{x^4-1}}+1\right)}{8\sqrt[8]{2}\left(2-\sqrt{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)^(3/4)/(-1 + 2*x^4 + x^8),x]
```

```
[Out] ArcTan[(2^(1/8)*x)/(-1 + x^4)^(1/4)]/(4*2^(1/8)) + ((1 - Sqrt[2])*ArcTan[1
- (2^(5/8)*x)/(-1 + x^4)^(1/4)])/(4*2^(1/8)*(2 - Sqrt[2])) - ((1 - Sqrt[2])
*ArcTan[1 + (2^(5/8)*x)/(-1 + x^4)^(1/4)])/(4*2^(1/8)*(2 - Sqrt[2])) + ArcT
anh[(2^(1/8)*x)/(-1 + x^4)^(1/4)]/(4*2^(1/8)) + ((1 - Sqrt[2])*Log[2^(3/8)
+ (2^(5/8)*x^2)/Sqrt[-1 + x^4] - (2*x)/(-1 + x^4)^(1/4)])/(8*2^(1/8)*(2 - S
qrt[2])) - ((1 - Sqrt[2])*Log[1 + (2^(1/4)*x^2)/Sqrt[-1 + x^4] + (2^(5/8)*x
)/(-1 + x^4)^(1/4)])/(8*2^(1/8)*(2 - Sqrt[2]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 408

```
Int[((a_) + (b_.)*(x_)^4)^(p_)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d
, Int[(a + b*x^4)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(p
- 1)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[p, 3/4] || EqQ[p, 5/4])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1428

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^
n)^q/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[(d + e*x^n)^q/(b + r + 2
*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{\left(-1+x^4\right)^{3/4}}{-1+2x^4+x^8}\,dx = \frac{\int \frac{\left(-1+x^4\right)^{3/4}}{2-2\sqrt{2}+2x^4}\,dx}{\sqrt{2}} - \frac{\int \frac{\left(-1+x^4\right)^{3/4}}{2+2\sqrt{2}+2x^4}\,dx}{\sqrt{2}}$$

$$= -\left(\left(-1+\sqrt{2}\right)\int \frac{1}{\sqrt[4]{-1+x^4}\left(2-2\sqrt{2}+2x^4\right)}\,dx\right) + \left(1+\sqrt{2}\right)\int \frac{1}{\sqrt[4]{-1+x^4}\left(2+2\sqrt{2}\right.}$$

$$= -\left(\left(-1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{2-2\sqrt{2}-\left(4-2\sqrt{2}\right)x^4}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)\right) + \left(1+\sqrt{2}\right)\text{Subs}$$

$$= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt[4]{2}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt[4]{2}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right) -$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{-1+x^4}}\right)}{4\sqrt[8]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{-1+x^4}}\right)}{4\sqrt[8]{2}} - \frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt[4]{2}}-2^{3/8}x+x^2}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)}{4\,2^{3/4}\left(2-\sqrt{2}\right)} -$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{-1+x^4}}\right)}{4\sqrt[8]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{-1+x^4}}\right)}{4\sqrt[8]{2}} + \frac{\left(1-\sqrt{2}\right)\log\left(2^{3/8}+\frac{2^{5/8}x^2}{\sqrt{-1+x^4}}-\frac{2x}{\sqrt[4]{-1+x^4}}\right)}{8\sqrt[8]{2}\left(2-\sqrt{2}\right)} - \frac{\left(1-\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{-1+x^4}}\right)}{4\sqrt[8]{2}} + \frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(1-\frac{2^{5/8}x}{\sqrt[4]{-1+x^4}}\right)}{4\sqrt[8]{2}\left(2-\sqrt{2}\right)} - \frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(1+\frac{2^{5/8}x}{\sqrt[4]{-1+x^4}}\right)}{4\sqrt[8]{2}\left(2-\sqrt{2}\right)} + \frac{\tan}{}$$

Mathematica [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^4\right)^{3/4}}{-1+2x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^4)^(3/4)/(-1 + 2*x^4 + x^8),x]
```

```
[Out] Integrate[(-1 + x^4)^(3/4)/(-1 + 2*x^4 + x^8), x]
```

IntegrateAlgebraic [A] time = 0.45, size = 152, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt[8]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt[8]{2}} - \frac{\tan^{-1}\left(\frac{2^{5/8}x\sqrt[4]{x^4-1}}{\sqrt[4]{2}\,x^2-\sqrt{x^4-1}}\right)}{4\,2^{5/8}} + \frac{\tanh^{-1}\left(\frac{2\,2^{3/8}x\sqrt[4]{x^4-1}}{2^{3/4}\sqrt{x^4-1}+2x^2}\right)}{4\,2^{5/8}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)^(3/4)/(-1 + 2*x^4 + x^8),x]
```

```
[Out] ArcTan[(2^(1/8)*x)/(-1 + x^4)^(1/4)]/(4*2^(1/8)) - ArcTan[(2^(5/8)*x*(-1 +
x^4)^(1/4))/(2^(1/4)*x^2 - Sqrt[-1 + x^4])]/(4*2^(5/8)) + ArcTanh[(2^(1/8)*
x)/(-1 + x^4)^(1/4)]/(4*2^(1/8)) + ArcTanh[(2*2^(3/8)*x*(-1 + x^4)^(1/4))/(
2*x^2 + 2^(3/4)*Sqrt[-1 + x^4])]/(4*2^(5/8))
```

fricas [B] time = 11.99, size = 1933, normalized size = 12.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(3/4)/(x^8+2*x^4-1),x, algorithm="fricas")
```

```
[Out] -1/8*2^(7/8)*arctan(1/2*(4*(2^(5/8)*x^5 + 2^(1/8)*(x^5 - x))*(x^4 - 1)^(3/4
) + (2^(5/8)*(5*x^8 - 4*x^4 + 1) + 2*sqrt(x^4 - 1)*(2^(7/8)*(2*x^6 - x^2) +
 2^(3/8)*(3*x^6 - x^2)) + 2^(1/8)*(7*x^8 - 6*x^4 + 1))*sqrt(6*2^(3/4) - 8*2
^(1/4)) + 4*(2^(7/8)*x^7 + 2^(3/8)*(x^7 - x^3))*(x^4 - 1)^(1/4))/(x^8 + 2*x
^4 - 1)) + 1/32*2^(7/8)*log((2^(7/8)*(x^8 - 2*x^4 - 1) + 4*(x^5 - sqrt(2)*x
 + x)*(x^4 - 1)^(3/4) - 4*sqrt(x^4 - 1)*(2^(5/8)*x^2 - 2^(1/8)*(x^6 + x^2))
 + 2*2^(3/8)*(x^8 + 1) - 4*(x^4 - 1)^(1/4)*(2^(3/4)*x^3 - 2^(1/4)*(x^7 + x^
3)))/(x^8 + 2*x^4 - 1)) - 1/32*2^(7/8)*log(-(2^(7/8)*(x^8 - 2*x^4 - 1) - 4*
(x^5 - sqrt(2)*x + x)*(x^4 - 1)^(3/4) - 4*sqrt(x^4 - 1)*(2^(5/8)*x^2 - 2^(1
/8)*(x^6 + x^2)) + 2*2^(3/8)*(x^8 + 1) + 4*(x^4 - 1)^(1/4)*(2^(3/4)*x^3 - 2
^(1/4)*(x^7 + x^3)))/(x^8 + 2*x^4 - 1)) + 1/8*2^(3/8)*arctan(-1/2*(130*x^16
 - 248*x^12 + 132*x^8 - 8*x^4 - sqrt(2)*(16*(x^4 - 1)^(3/4)*(2^(3/4)*(25*x^
13 - 26*x^9 - x^5) - 2*2^(1/4)*(11*x^13 - 12*x^9 - x^5)) + 2^(5/8)*(151*x^1
6 - 392*x^12 + 254*x^8 - 8*x^4 - 1) + 2*sqrt(x^4 - 1)*(2^(7/8)*(91*x^14 - 1
23*x^10 + 19*x^6 + x^2) - 2*2^(3/8)*(86*x^14 - 101*x^10 + 8*x^6 + x^2)) + 4
*(28*x^15 - 6*x^11 - 24*x^7 - 2*x^3 + sqrt(2)*(3*x^15 - 27*x^11 + 27*x^7 +
x^3))*(x^4 - 1)^(1/4) - 2*2^(1/8)*(189*x^16 - 418*x^12 + 236*x^8 - 2*x^4 -
1))*sqrt((3*2^(3/4)*(x^8 + 2*x^4 - 1) + 4*(x^4 - 1)^(3/4)*(2^(7/8)*(x^5 + 2
*x) + 2^(3/8)*(x^5 + 3*x)) + 8*(x^6 + 3*x^2 + sqrt(2)*(x^6 + 2*x^2))*sqrt(x
^4 - 1) + 4*2^(1/4)*(x^8 + 2*x^4 - 1) + 4*(x^4 - 1)^(1/4)*(2^(5/8)*(x^7 + 3
*x^3) + 2*2^(1/8)*(x^7 + 2*x^3)))/(x^8 + 2*x^4 - 1)) - 4*(x^4 - 1)^(3/4)*(2
^(5/8)*(81*x^13 - 79*x^9 - 3*x^5 + x) - 2*2^(1/8)*(5*x^13 - 22*x^9 + 11*x^5
)) + 32*sqrt(2)*(3*x^16 - 5*x^12 + 3*x^8 - x^4) + 8*sqrt(x^4 - 1)*(2^(3/4)*
(17*x^14 - 30*x^10 + 15*x^6) + 2^(1/4)*(31*x^14 - 33*x^10 + 3*x^6 - x^2)) -
 4*(x^4 - 1)^(1/4)*(2^(7/8)*(19*x^15 - 13*x^11 - 9*x^7 + 3*x^3) - 2*2^(3/8)
*(39*x^15 - 82*x^11 + 41*x^7)) + 2)/(383*x^16 - 772*x^12 + 382*x^8 + 4*x^4
- 1)) - 1/8*2^(3/8)*arctan(-1/2*(130*x^16 - 248*x^12 + 132*x^8 - 8*x^4 - sq
rt(2)*(16*(x^4 - 1)^(3/4)*(2^(3/4)*(25*x^13 - 26*x^9 - x^5) - 2*2^(1/4)*(11
*x^13 - 12*x^9 - x^5)) - 2^(5/8)*(151*x^16 - 392*x^12 + 254*x^8 - 8*x^4 - 1
) - 2*sqrt(x^4 - 1)*(2^(7/8)*(91*x^14 - 123*x^10 + 19*x^6 + x^2) - 2*2^(3/8
)*(86*x^14 - 101*x^10 + 8*x^6 + x^2)) + 4*(28*x^15 - 6*x^11 - 24*x^7 - 2*x^
3 + sqrt(2)*(3*x^15 - 27*x^11 + 27*x^7 + x^3))*(x^4 - 1)^(1/4) + 2*2^(1/8)*
(189*x^16 - 418*x^12 + 236*x^8 - 2*x^4 - 1))*sqrt((3*2^(3/4)*(x^8 + 2*x^4 -
 1) - 4*(x^4 - 1)^(3/4)*(2^(7/8)*(x^5 + 2*x) + 2^(3/8)*(x^5 + 3*x)) + 8*(x^
6 + 3*x^2 + sqrt(2)*(x^6 + 2*x^2))*sqrt(x^4 - 1) + 4*2^(1/4)*(x^8 + 2*x^4 -
 1) - 4*(x^4 - 1)^(1/4)*(2^(5/8)*(x^7 + 3*x^3) + 2*2^(1/8)*(x^7 + 2*x^3)))/
(x^8 + 2*x^4 - 1)) + 4*(x^4 - 1)^(3/4)*(2^(5/8)*(81*x^13 - 79*x^9 - 3*x^5 +
 x) - 2*2^(1/8)*(5*x^13 - 22*x^9 + 11*x^5)) + 32*sqrt(2)*(3*x^16 - 5*x^12 +
 3*x^8 - x^4) + 8*sqrt(x^4 - 1)*(2^(3/4)*(17*x^14 - 30*x^10 + 15*x^6) + 2^(
1/4)*(31*x^14 - 33*x^10 + 3*x^6 - x^2)) + 4*(x^4 - 1)^(1/4)*(2^(7/8)*(19*x^
15 - 13*x^11 - 9*x^7 + 3*x^3) - 2*2^(3/8)*(39*x^15 - 82*x^11 + 41*x^7)) + 2
)/(383*x^16 - 772*x^12 + 382*x^8 + 4*x^4 - 1)) + 1/32*2^(3/8)*log(8*(3*2^(3
/4)*(x^8 + 2*x^4 - 1) + 4*(x^4 - 1)^(3/4)*(2^(7/8)*(x^5 + 2*x) + 2^(3/8)*(x
^5 + 3*x)) + 8*(x^6 + 3*x^2 + sqrt(2)*(x^6 + 2*x^2))*sqrt(x^4 - 1) + 4*2^(1
/4)*(x^8 + 2*x^4 - 1) + 4*(x^4 - 1)^(1/4)*(2^(5/8)*(x^7 + 3*x^3) + 2*2^(1/8
)*(x^7 + 2*x^3)))/(x^8 + 2*x^4 - 1)) - 1/32*2^(3/8)*log(8*(3*2^(3/4)*(x^8 +
 2*x^4 - 1) - 4*(x^4 - 1)^(3/4)*(2^(7/8)*(x^5 + 2*x) + 2^(3/8)*(x^5 + 3*x))
 + 8*(x^6 + 3*x^2 + sqrt(2)*(x^6 + 2*x^2))*sqrt(x^4 - 1) + 4*2^(1/4)*(x^8 +
 2*x^4 - 1) - 4*(x^4 - 1)^(1/4)*(2^(5/8)*(x^7 + 3*x^3) + 2*2^(1/8)*(x^7 + 2
*x^3)))/(x^8 + 2*x^4 - 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(3/4)/(x^8+2*x^4-1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
```

```
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [C] time = 12.35, size = 1163, normalized size = 7.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(3/4)/(x^8+2*x^4-1),x)
```

```
[Out] -1/16*RootOf(_Z^8-128)*ln(-(x^4*RootOf(_Z^8-128)^9+8*RootOf(_Z^8-128)^7*(x^
4-1)^(1/2)*x^2-16*RootOf(_Z^8-128)^6*(x^4-1)^(1/4)*x^3+24*RootOf(_Z^8-128)^
5*x^4-32*RootOf(_Z^8-128)^4*(x^4-1)^(3/4)*x+64*RootOf(_Z^8-128)^3*(x^4-1)^(
1/2)*x^2-128*RootOf(_Z^8-128)^2*(x^4-1)^(1/4)*x^3-8*RootOf(_Z^8-128)^5+128*
RootOf(_Z^8-128)*x^4-512*(x^4-1)^(3/4)*x-128*RootOf(_Z^8-128))/(x^4*RootOf(
_Z^8-128)^4-8*x^4+8))-1/256*ln((39*RootOf(_Z^8-128)^8*RootOf(_Z^2+RootOf(_Z
^8-128)^2)*x^4-376*(x^4-1)^(1/2)*RootOf(_Z^8-128)^6*RootOf(_Z^2+RootOf(_Z^8
-128)^2)*x^2-624*RootOf(_Z^8-128)^6*(x^4-1)^(1/4)*x^3+688*RootOf(_Z^2+RootO
f(_Z^8-128)^2)*RootOf(_Z^8-128)^4*x^4+1504*RootOf(_Z^8-128)^4*(x^4-1)^(3/4)
*x-4992*(x^4-1)^(1/2)*RootOf(_Z^8-128)^2*RootOf(_Z^2+RootOf(_Z^8-128)^2)*x^
2-6016*RootOf(_Z^8-128)^2*(x^4-1)^(1/4)*x^3-312*RootOf(_Z^8-128)^4*RootOf(_
Z^2+RootOf(_Z^8-128)^2)+3008*RootOf(_Z^2+RootOf(_Z^8-128)^2)*x^4+19968*(x^4
-1)^(3/4)*x-3008*RootOf(_Z^2+RootOf(_Z^8-128)^2))/(x^4*RootOf(_Z^8-128)^4+8
*x^4-8))*RootOf(_Z^8-128)^5+1/256*ln((39*RootOf(_Z^8-128)^8*RootOf(_Z^2+Roo
tOf(_Z^8-128)^2)*x^4-376*(x^4-1)^(1/2)*RootOf(_Z^8-128)^6*RootOf(_Z^2+RootO
f(_Z^8-128)^2)*x^2-624*RootOf(_Z^8-128)^6*(x^4-1)^(1/4)*x^3+688*RootOf(_Z^2
+RootOf(_Z^8-128)^2)*RootOf(_Z^8-128)^4*x^4+1504*RootOf(_Z^8-128)^4*(x^4-1)
^(3/4)*x-4992*(x^4-1)^(1/2)*RootOf(_Z^8-128)^2*RootOf(_Z^2+RootOf(_Z^8-128)
^2)*x^2-6016*RootOf(_Z^8-128)^2*(x^4-1)^(1/4)*x^3-312*RootOf(_Z^8-128)^4*Ro
otOf(_Z^2+RootOf(_Z^8-128)^2)+3008*RootOf(_Z^2+RootOf(_Z^8-128)^2)*x^4+1996
8*(x^4-1)^(3/4)*x-3008*RootOf(_Z^2+RootOf(_Z^8-128)^2))/(x^4*RootOf(_Z^8-12
8)^4+8*x^4-8))*RootOf(_Z^8-128)^4*RootOf(_Z^2+RootOf(_Z^8-128)^2)+1/128*Roo
tOf(_Z^8-128)^5*ln(-(x^4*RootOf(_Z^8-128)^9+RootOf(_Z^8-128)^8*RootOf(_Z^2+
RootOf(_Z^8-128)^2)*x^4+16*RootOf(_Z^8-128)^5*(x^4-1)^(1/4)*RootOf(_Z^2+Roo
tOf(_Z^8-128)^2)*x^3-8*RootOf(_Z^8-128)^5*x^4-8*RootOf(_Z^2+RootOf(_Z^8-128
)^2)*RootOf(_Z^8-128)^4*x^4-64*RootOf(_Z^8-128)^3*(x^4-1)^(1/2)*x^2+64*(x^4
-1)^(1/2)*RootOf(_Z^8-128)^2*RootOf(_Z^2+RootOf(_Z^8-128)^2)*x^2+8*RootOf(_
Z^8-128)^5+8*RootOf(_Z^8-128)^4*RootOf(_Z^2+RootOf(_Z^8-128)^2)-512*(x^4-1)
^(3/4)*x)/(x^4*RootOf(_Z^8-128)^4+8*x^4-8))+1/16*RootOf(_Z^2+RootOf(_Z^8-12
8)^2)*ln(-((x^4-1)^(1/2)*RootOf(_Z^8-128)^6*RootOf(_Z^2+RootOf(_Z^8-128)^2)
*x^2+2*RootOf(_Z^8-128)^6*(x^4-1)^(1/4)*x^3-2*RootOf(_Z^2+RootOf(_Z^8-128)^
2)*RootOf(_Z^8-128)^4*x^4-16*RootOf(_Z^2+RootOf(_Z^8-128)^2)*x^4-64*(x^4-1)
^(3/4)*x+16*RootOf(_Z^2+RootOf(_Z^8-128)^2))/(x^4*RootOf(_Z^8-128)^4-8*x^4+
8))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}-1\right)^{\frac{3}{4}}}{x^{8}+2\,x^{4}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(3/4)/(x^8+2*x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 1)^(3/4)/(x^8 + 2*x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-1\right)^{3/4}}{x^8+2\,x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)^(3/4)/(2*x^4 + x^8 - 1),x)
```

```
[Out] int((x^4 - 1)^(3/4)/(2*x^4 + x^8 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((x-1)(x+1)\left(x^2+1\right)\right)^{\frac{3}{4}}}{x^8+2x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(3/4)/(x**8+2*x**4-1),x)
```

```
[Out] Integral(((x - 1)*(x + 1)*(x**2 + 1))**(3/4)/(x**8 + 2*x**4 - 1), x)
```

3.1751 $\int \frac{1+x^4}{\left(-1+x^4\right)\sqrt{1+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=152

$$\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}-2\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^2+1}+1}}\right)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}\right)-\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2}}}\right.$$

Rubi [F] time = 0.88, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x^4}{\left(-1+x^4\right)\sqrt{1+\sqrt{1+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^4)/((-1 + x^4)*Sqrt[1 + Sqrt[1 + x^2]]),x]
```

```
[Out] Defer[Int][1/Sqrt[1 + Sqrt[1 + x^2]], x] - (I/2)*Defer[Int][1/((I - x)*Sqrt
[1 + Sqrt[1 + x^2]]), x] - Defer[Int][1/((1 - x)*Sqrt[1 + Sqrt[1 + x^2]]),
x]/2 - (I/2)*Defer[Int][1/((I + x)*Sqrt[1 + Sqrt[1 + x^2]]), x] - Defer[Int
][1/((1 + x)*Sqrt[1 + Sqrt[1 + x^2]]), x]/2
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{\left(-1+x^4\right)\sqrt{1+\sqrt{1+x^2}}}\,dx &= \int \left(\frac{1}{\sqrt{1+\sqrt{1+x^2}}}+\frac{2}{\left(-1+x^4\right)\sqrt{1+\sqrt{1+x^2}}}\right)dx\\
&= 2\int \frac{1}{\left(-1+x^4\right)\sqrt{1+\sqrt{1+x^2}}}\,dx+\int \frac{1}{\sqrt{1+\sqrt{1+x^2}}}\,dx\\
&= 2\int \left(-\frac{1}{2\left(1-x^2\right)\sqrt{1+\sqrt{1+x^2}}}-\frac{1}{2\left(1+x^2\right)\sqrt{1+\sqrt{1+x^2}}}\right)dx+\int \frac{}{\sqrt{1}}\\
&= \int \frac{1}{\sqrt{1+\sqrt{1+x^2}}}\,dx-\int \frac{1}{\left(1-x^2\right)\sqrt{1+\sqrt{1+x^2}}}\,dx-\int \frac{1}{\left(1+x^2\right)\sqrt{1+\sqrt{}}}\\
&= \int \frac{1}{\sqrt{1+\sqrt{1+x^2}}}\,dx-\int \left(\frac{i}{2(i-x)\sqrt{1+\sqrt{1+x^2}}}+\frac{i}{2(i+x)\sqrt{1+\sqrt{1+x^2}}}\right.\\
&= -\left(\frac{1}{2}i\int \frac{1}{(i-x)\sqrt{1+\sqrt{1+x^2}}}\,dx\right)-\frac{1}{2}i\int \frac{1}{(i+x)\sqrt{1+\sqrt{1+x^2}}}\,dx-\frac{1}{2}\int
\end{aligned}$$

Mathematica [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{\left(-1+x^4\right)\sqrt{1+\sqrt{1+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^4)/((-1 + x^4)*Sqrt[1 + Sqrt[1 + x^2]]),x]
```

```
[Out] Integrate[(1 + x^4)/((-1 + x^4)*Sqrt[1 + Sqrt[1 + x^2]]), x]
```

IntegrateAlgebraic [A] time = 0.49, size = 152, normalized size = 1.00

$$\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}-2\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^2+1}+1}}\right)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}\right)-\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{\sqrt{2}-1}\,x}{\sqrt{\sqrt{x^2+1}+1}}\right)-\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{2}}\,x}{\sqrt{\sqrt{x^2+1}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)/((-1 + x^4)*Sqrt[1 + Sqrt[1 + x^2]]),x]
```

```
[Out] (2*x)/Sqrt[1 + Sqrt[1 + x^2]] - 2*ArcTan[x/Sqrt[1 + Sqrt[1 + x^2]]] + Sqrt[
2]*ArcTan[x/(Sqrt[2]*Sqrt[1 + Sqrt[1 + x^2]])] - Sqrt[1 + Sqrt[2]]*ArcTan[(
Sqrt[-1 + Sqrt[2]]*x)/Sqrt[1 + Sqrt[1 + x^2]]] - Sqrt[-1 + Sqrt[2]]*ArcTanh
[(Sqrt[1 + Sqrt[2]]*x)/Sqrt[1 + Sqrt[1 + x^2]]]
```

fricas [B] time = 20.62, size = 520, normalized size = 3.42

$$\frac{4x\sqrt{\sqrt{2}+1}\arctan\left(-\frac{\left(71x^5-874x^3-\sqrt{2}\left(61x^5-548x^3-81x\right)-2\left(51x^3-2\sqrt{2}\left(5x^3-127x\right)-335x\right)\sqrt{x^2+1}-173x\right)\sqrt{3821\sqrt{2}+4841}\sqrt{\sqrt{2}+1}-4802\left(x^4-6x^2+\sqrt{2}\left(3x^2+1\right)+\left(x^2+\sqrt{2}\left(x^2-1\right)+3\right)\sqrt{x^2+1}-3\right)\sqrt{\sqrt{2}+1}\sqrt{\sqrt{x^2+1}+1}}{2401\left(x^5-10x^3-7x\right)}\right)-4\sqrt{2}x\arctan\left(\frac{\sqrt{2}\sqrt{\sqrt{x^2+1}+1}}{x}\right)+x\sqrt{\sqrt{2}-1}\log\left(-\frac{\left(51x^3-2\sqrt{2}\left(5x^3+66x\right)+2\sqrt{x^2+1}\left(61\sqrt{2}x-71x\right)+193x\right)\sqrt{\sqrt{2}-1}+2\left(71x^2-\sqrt{2}\left(61x^2+132\right)+\sqrt{x^2+1}\left(132\sqrt{2}-193\right)+193\right)\sqrt{\sqrt{x^2+1}+1}}{x^3-x}\right)-x\sqrt{\sqrt{2}-1}\log\left(\frac{\left(51x^3-2\sqrt{2}\left(5x^3+66x\right)+2\sqrt{x^2+1}\left(61\sqrt{2}x-71x\right)+193x\right)\sqrt{\sqrt{2}-1}-2\left(71x^2-\sqrt{2}\left(61x^2+132\right)+\sqrt{x^2+1}\left(132\sqrt{2}-193\right)+193\right)\sqrt{\sqrt{x^2+1}+1}}{x^3-x}\right)+2x\arctan\left(\frac{4\left(x^4-12x^2+\left(5x^2-3\right)\sqrt{x^2+1}+3\right)\sqrt{\sqrt{x^2+1}+1}}{x^5-46x^3+17x}\right)+8\sqrt{\sqrt{x^2+1}+1}\left(\sqrt{x^2+1}-1\right)}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*x*sqrt(sqrt(2) + 1)*arctan(-1/2401*((71*x^5 - 874*x^3 - sqrt(2)*(61*
x^5 - 548*x^3 - 81*x) - 2*(51*x^3 - 2*sqrt(2)*(5*x^3 - 127*x) - 335*x)*sqrt
(x^2 + 1) - 173*x)*sqrt(3821*sqrt(2) + 4841)*sqrt(sqrt(2) + 1) - 4802*(x^4
- 6*x^2 + sqrt(2)*(3*x^2 + 1) + (x^2 + sqrt(2)*(x^2 - 1) + 3)*sqrt(x^2 + 1)
 - 3)*sqrt(sqrt(2) + 1)*sqrt(sqrt(x^2 + 1) + 1))/(x^5 - 10*x^3 - 7*x)) - 4*
sqrt(2)*x*arctan(sqrt(2)*sqrt(sqrt(x^2 + 1) + 1)/x) + x*sqrt(sqrt(2) - 1)*l
og(-((51*x^3 - 2*sqrt(2)*(5*x^3 + 66*x) + 2*sqrt(x^2 + 1)*(61*sqrt(2)*x - 7
1*x) + 193*x)*sqrt(sqrt(2) - 1) + 2*(71*x^2 - sqrt(2)*(61*x^2 + 132) + sqrt
(x^2 + 1)*(132*sqrt(2) - 193) + 193)*sqrt(sqrt(x^2 + 1) + 1))/(x^3 - x)) -
x*sqrt(sqrt(2) - 1)*log(((51*x^3 - 2*sqrt(2)*(5*x^3 + 66*x) + 2*sqrt(x^2 +
1)*(61*sqrt(2)*x - 71*x) + 193*x)*sqrt(sqrt(2) - 1) - 2*(71*x^2 - sqrt(2)*(
61*x^2 + 132) + sqrt(x^2 + 1)*(132*sqrt(2) - 193) + 193)*sqrt(sqrt(x^2 + 1)
 + 1))/(x^3 - x)) + 2*x*arctan(4*(x^4 - 12*x^2 + (5*x^2 - 3)*sqrt(x^2 + 1)
+ 3)*sqrt(sqrt(x^2 + 1) + 1)/(x^5 - 46*x^3 + 17*x)) + 8*sqrt(sqrt(x^2 + 1)
+ 1)*(sqrt(x^2 + 1) - 1))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^4-1\right)\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/((x^4 - 1)*sqrt(sqrt(x^2 + 1) + 1)), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^4-1\right)\sqrt{1+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^4-1)/(1+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int((x^4+1)/(x^4-1)/(1+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^4-1\right)\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate((x^4 + 1)/((x^4 - 1)*sqrt(sqrt(x^2 + 1) + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4+1}{\left(x^4-1\right)\,\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/((x^4 - 1)*((x^2 + 1)^(1/2) + 1)^(1/2)),x)
[Out] int((x^4 + 1)/((x^4 - 1)*((x^2 + 1)^(1/2) + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{(x-1)\,(x+1)\left(x^2+1\right)\sqrt{\sqrt{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**4-1)/(1+(x**2+1)**(1/2))**(1/2),x)
[Out] Integral((x**4 + 1)/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1)),
x)
```

3.1752 $\int \frac{1}{x^3\left(-b+ax^2\right)^{3/4}}\,dx$

Optimal. Leaf size=153

$$-\frac{3a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}-\sqrt{b}}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{3a\tanh^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^2-b}}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{ax^2-b}}{2bx^2}$$

Rubi [A] time = 0.22, antiderivative size = 228, normalized size of antiderivative = 1.49, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 51, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3a\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)}{8\sqrt{2}\,b^{7/4}}+\frac{3a\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}+\sqrt{ax^2-b}+\sqrt{b}\right)}{8\sqrt{2}\,b^{7/4}}-\frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{3a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^2-b}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{ax^2-b}}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-b + a*x^2)^(3/4)),x]
```

```
[Out] (-b + a*x^2)^(1/4)/(2*b*x^2) - (3*a*ArcTan[1 - (Sqrt[2]*(-b + a*x^2)^(1/4))
/b^(1/4)])/(4*Sqrt[2]*b^(7/4)) + (3*a*ArcTan[1 + (Sqrt[2]*(-b + a*x^2)^(1/4
))/b^(1/4)])/(4*Sqrt[2]*b^(7/4)) - (3*a*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b +
 a*x^2)^(1/4) + Sqrt[-b + a*x^2]])/(8*Sqrt[2]*b^(7/4)) + (3*a*Log[Sqrt[b] +
 Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4) + Sqrt[-b + a*x^2]])/(8*Sqrt[2]*b^(7/4)
)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^3\left(-b+ax^2\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(-b+ax)^{3/4}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt[4]{-b+ax^2}}{2bx^2} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{x(-b+ax)^{3/4}}\,dx, x, x^2\right)}{8b}$$

$$= \frac{\sqrt[4]{-b+ax^2}}{2bx^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^2}\right)}{2b}$$

$$= \frac{\sqrt[4]{-b+ax^2}}{2bx^2} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^2}\right)}{4b^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+\right.}{4b^{3/2}}$$

$$= \frac{\sqrt[4]{-b+ax^2}}{2bx^2} - \frac{(3a)\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^2}\right)}{8\sqrt{2}\,b^{7/4}} - \frac{(3a)\,\text{Subst}\left(\int \frac{\sqrt{2}}{-\sqrt{b}+}\right.}{}$$

$$= \frac{\sqrt[4]{-b+ax^2}}{2bx^2} - \frac{3a\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^2}+\sqrt{-b+ax^2}\right)}{8\sqrt{2}\,b^{7/4}} + \frac{3a\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{\right.}{}$$

$$= \frac{\sqrt[4]{-b+ax^2}}{2bx^2} - \frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}} + \frac{3a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^2}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}} - \frac{3a\log\left(\sqrt{b}\right.}{}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.25

$$\frac{2a\sqrt[4]{ax^2-b}\ {}_2F_1\left(\frac{1}{4},2;\frac{5}{4};1-\frac{ax^2}{b}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-b + a*x^2)^(3/4)),x]
```

```
[Out] (2*a*(-b + a*x^2)^(1/4)*Hypergeometric2F1[1/4, 2, 5/4, 1 - (a*x^2)/b])/b^2
```

IntegrateAlgebraic [A] time = 0.36, size = 152, normalized size = 0.99

$$\frac{3a\tan^{-1}\left(\frac{\frac{\sqrt{ax^2-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^2-b}}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^2-b}}{\sqrt{ax^2-b}+\sqrt{b}}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{ax^2-b}}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(-b + a*x^2)^(3/4)),x]
```

```
[Out] (-b + a*x^2)^(1/4)/(2*b*x^2) + (3*a*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b +
a*x^2]/(Sqrt[2]*b^(1/4)))/(-b + a*x^2)^(1/4)])/(4*Sqrt[2]*b^(7/4)) + (3*a*A
rcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^2)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^2])])/
(4*Sqrt[2]*b^(7/4))
```

fricas [A] time = 0.41, size = 212, normalized size = 1.39

$$\frac{12\,bx^2\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{(ax^2-b)^{\frac{1}{4}}ab^5\left(-\frac{a^4}{b^7}\right)^{\frac{3}{4}}-\sqrt{b^4\sqrt{-\frac{a^4}{b^7}}+\sqrt{ax^2-b}\,a^2}\,b^5\left(-\frac{a^4}{b^7}\right)^{\frac{3}{4}}}{a^4}\right)+3\,bx^2\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(3\,b^2\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}+3\,(ax^2-b)^{\frac{1}{4}}a\right)-3\,bx^2\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(-3\,b^2\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}+3\,(ax^2-b)^{\frac{1}{4}}a\right)+4\,(ax^2-b)^{\frac{1}{4}}}{8\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^2-b)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/8*(12*b*x^2*(-a^4/b^7)^(1/4)*arctan(-((a*x^2 - b)^(1/4)*a*b^5*(-a^4/b^7)^
(3/4) - sqrt(b^4*sqrt(-a^4/b^7) + sqrt(a*x^2 - b)*a^2)*b^5*(-a^4/b^7)^(3/4)
)/a^4) + 3*b*x^2*(-a^4/b^7)^(1/4)*log(3*b^2*(-a^4/b^7)^(1/4) + 3*(a*x^2 - b
)^(1/4)*a) - 3*b*x^2*(-a^4/b^7)^(1/4)*log(-3*b^2*(-a^4/b^7)^(1/4) + 3*(a*x^
2 - b)^(1/4)*a) + 4*(a*x^2 - b)^(1/4))/(b*x^2)
```

giac [A] time = 0.34, size = 199, normalized size = 1.30

$$\frac{\frac{6\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\,(ax^2-b)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{7}{4}}}+\frac{6\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\,(ax^2-b)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{7}{4}}}+\frac{3\sqrt{2}a^2\log\left(\sqrt{2}\,(ax^2-b)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)}{b^{\frac{7}{4}}}-\frac{3\sqrt{2}a^2\log\left(-\sqrt{2}\,(ax^2-b)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^2-b}+\sqrt{b}\right)}{b^{\frac{7}{4}}}+\frac{8\,(ax^2-b)^{\frac{1}{4}}a}{bx^2}}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^2-b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/16*(6*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^2 - b)^(1/
4))/b^(1/4))/b^(7/4) + 6*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
 2*(a*x^2 - b)^(1/4))/b^(1/4))/b^(7/4) + 3*sqrt(2)*a^2*log(sqrt(2)*(a*x^2 -
 b)^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(7/4) - 3*sqrt(2)*a^2*log(
-sqrt(2)*(a*x^2 - b)^(1/4)*b^(1/4) + sqrt(a*x^2 - b) + sqrt(b))/b^(7/4) + 8
*(a*x^2 - b)^(1/4)*a/(b*x^2))/a
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(a \,x^{2}-b \right)^{\frac{3}{4}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x^2-b)^(3/4),x)
```

```
[Out] int(1/x^3/(a*x^2-b)^(3/4),x)
```

maxima [A] time = 0.51, size = 202, normalized size = 1.32

$$\frac{\left(ax^{2}-b\right)^{\frac{1}{4}}a}{2\left(\left(ax^{2}-b\right)b+b^{2}\right)}+\frac{3\left(\frac{2\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+2\left(ax^{2}-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{2\sqrt{2}\,a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-2\left(ax^{2}-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{\sqrt{2}\,a\log\left(\sqrt{2}\left(ax^{2}-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^{2}-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}\,a\log\left(-\sqrt{2}\left(ax^{2}-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^{2}-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^2-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/2*(a*x^2 - b)^(1/4)*a/((a*x^2 - b)*b + b^2) + 3/16*(2*sqrt(2)*a*arctan(1/
2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^2 - b)^(1/4))/b^(1/4))/b^(3/4) + 2*sqrt
(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^2 - b)^(1/4))/b^(1/4))/
b^(3/4) + sqrt(2)*a*log(sqrt(2)*(a*x^2 - b)^(1/4)*b^(1/4) + sqrt(a*x^2 - b)
 + sqrt(b))/b^(3/4) - sqrt(2)*a*log(-sqrt(2)*(a*x^2 - b)^(1/4)*b^(1/4) + sq
rt(a*x^2 - b) + sqrt(b))/b^(3/4))/b
```

mupad [B] time = 1.26, size = 72, normalized size = 0.47

$$\frac{{\left(a\,x^2-b\right)}^{1/4}}{2\,b\,x^2}+\frac{3\,a\,\mathrm{atan}\left(\frac{{\left(a\,x^2-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{7/4}}+\frac{3\,a\,\mathrm{atanh}\left(\frac{{\left(a\,x^2-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x^2 - b)^(3/4)),x)
```

```
[Out] (a*x^2 - b)^(1/4)/(2*b*x^2) + (3*a*atan((a*x^2 - b)^(1/4)/(-b)^(1/4)))/(4*(
-b)^(7/4)) + (3*a*atanh((a*x^2 - b)^(1/4)/(-b)^(1/4)))/(4*(-b)^(7/4))
```

sympy [C] time = 1.15, size = 42, normalized size = 0.27

$$-\frac{\Gamma\left(\frac{7}{4}\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| {\frac{b e^{2 i \pi}}{a x^{2}}} \right)}}{2 a^{\frac{3}{4}} x^{\frac{7}{2}} \Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*x**2-b)**(3/4),x)
```

```
[Out] -gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*exp_polar(2*I*pi)/(a*x**2))/(2*a**
(3/4)*x**(7/2)*gamma(11/4))
```

3.1753 $\int \frac{ab-2bx+x^2}{\sqrt[4]{x(-a+x)(-b+x)^3}\left(b-(1+ad)x+dx^2\right)} dx$

Optimal. Leaf size=153

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{ab^3x+x^3(3ab+3b^2)+x^2(-3ab^2-b^3)+x^4(-a-3b)+x^5}}{b-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{ab^3x+x^3(3ab+3b^2)+x^2(-3ab^2-b^3)+x^4(-a-3b)+x^5}}{b-x}\right)}{d^{3/4}}$$

Rubi [F] time = 7.49, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ab-2bx+x^2}{\sqrt[4]{x(-a+x)(-b+x)^3}\left(b-(1+ad)x+dx^2\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(a*b - 2*b*x + x^2)/((x*(-a + x)*(-b + x)^3)^(1/4)*(b - (1 + a*d)*x + d
*x^2)),x]
```

```
[Out] (4*x*(1 - x/a)^(1/4)*(1 - x/b)^(3/4)*AppellF1[3/4, 1/4, 3/4, 7/4, x/a, x/b]
)/(3*d*((a - x)*(b - x)^3*x)^(1/4)) + ((1 + a*d - 2*b*d + Sqrt[1 + 2*a*d -
4*b*d + a^2*d^2])*x^(1/4)*(-a + x)^(1/4)*(-b + x)^(3/4)*Defer[Int][1/(x^(1/
4)*(-a + x)^(1/4)*(-b + x)^(3/4)*(-1 - a*d - Sqrt[1 + 2*a*d - 4*b*d + a^2*d
^2] + 2*d*x)), x])/(d*((a - x)*(b - x)^3*x)^(1/4)) + ((1 + a*d - 2*b*d - Sq
rt[1 + 2*a*d - 4*b*d + a^2*d^2])*x^(1/4)*(-a + x)^(1/4)*(-b + x)^(3/4)*Defe
r[Int][1/(x^(1/4)*(-a + x)^(1/4)*(-b + x)^(3/4)*(-1 - a*d + Sqrt[1 + 2*a*d
- 4*b*d + a^2*d^2] + 2*d*x)), x])/(d*((a - x)*(b - x)^3*x)^(1/4))
```

Rubi steps

$$\int \frac{ab-2bx+x^2}{\sqrt[4]{x(-a+x)(-b+x)^3}\left(b-(1+ad)x+dx^2\right)} dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{-a+x}(-b+x)^{3/4}\right)\int \frac{ab-2bx+x^2}{\sqrt[4]{x}\sqrt[4]{-a+x}(-b+x)^{3/4}\left(b-(1+ad)x+dx^2\right)} d\ldots}{\sqrt[4]{x(-a+x)(-b+x)^3}}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt[4]{-a+x}(-b+x)^{3/4}\right)\int\left(\frac{1}{d\sqrt[4]{x}\sqrt[4]{-a+x}(-b+x)^{3/4}}-\frac{b-\ldots}{d\sqrt[4]{x}\sqrt[4]{-a+x}(\ldots}\right.}{\sqrt[4]{x(-a+x)(-b+x)^3}}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt[4]{-a+x}(-b+x)^{3/4}\right)\int \frac{1}{\sqrt[4]{x}\sqrt[4]{-a+x}(-b+x)^{3/4}} dx}{d\sqrt[4]{x(-a+x)(-b+x)^3}} - \frac{\left(\sqrt[4]{x}\sqrt[4]{-a\ldots}\right.}{\ldots}$$

$$= -\frac{\left(\sqrt[4]{x}\sqrt[4]{-a+x}(-b+x)^{3/4}\right)\int\left(\frac{-1-ad+2bd-\sqrt{1+2ad-4bd\ldots}}{\sqrt[4]{x}\sqrt[4]{-a+x}(-b+x)^{3/4}\left(-1-ad-\sqrt{1+2ad\ldots}\right.}\right.}{d\sqrt[4]{x(-a\ldots}}$$

$$= -\frac{\left(\left(-1-ad+2bd-\sqrt{1+2ad-4bd+a^2d^2}\right)\sqrt[4]{x}\sqrt[4]{-a+x}(\ldots\right.}{d\sqrt[4]{x(-a+\ldots}}$$

$$= \frac{4x\sqrt[4]{1-\frac{x}{a}}\left(1-\frac{x}{b}\right)^{3/4}F_1\left(\frac{3}{4};\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{x}{a},\frac{x}{b}\right)}{3d\sqrt[4]{(a-x)(b-x)^3x}} - \frac{\left(\left(-1-ad+2bd-\ldots\right.\right.}{\ldots}$$

Mathematica [F] time = 6.22, size = 0, normalized size = 0.00

$$\int \frac{ab-2bx+x^2}{\sqrt[4]{x(-a+x)(-b+x)^3}\,\left(b-(1+ad)x+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*b - 2*b*x + x^2)/((x*(-a + x)*(-b + x)^3)^(1/4)*(b - (1 + a*d)
*x + d*x^2)),x]
```

```
[Out] Integrate[(a*b - 2*b*x + x^2)/((x*(-a + x)*(-b + x)^3)^(1/4)*(b - (1 + a*d)
*x + d*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.39, size = 153, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{ab^3x+x^3(3ab+3b^2)+x^2(-3ab^2-b^3)+x^4(-a-3b)+x^5}}{b-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{ab^3x+x^3(3ab+3b^2)+x^2(-3ab^2-b^3)+x^4(-a-3b)+x^5}}{b-x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b - 2*b*x + x^2)/((x*(-a + x)*(-b + x)^3)^(1/4)*(b -
(1 + a*d)*x + d*x^2)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(a*b^3*x + (-3*a*b^2 - b^3)*x^2 + (3*a*b + 3*b^2)*x^3 +
 (-a - 3*b)*x^4 + x^5)^(1/4))/(b - x)])/d^(3/4) + (2*ArcTanh[(d^(1/4)*(a*b^
3*x + (-3*a*b^2 - b^3)*x^2 + (3*a*b + 3*b^2)*x^3 + (-a - 3*b)*x^4 + x^5)^(1
/4))/(b - x)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^3)^(1/4)/(b-(a*d+1)*x+d*x^2),x,
algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab-2\,bx+x^2}{\left((a-x)(b-x)^3x\right)^{\frac{1}{4}}\left(dx^2-(ad+1)x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^3)^(1/4)/(b-(a*d+1)*x+d*x^2),x,
algorithm="giac")
```

```
[Out] integrate((a*b - 2*b*x + x^2)/(((a - x)*(b - x)^3*x)^(1/4)*(d*x^2 - (a*d +
1)*x + b)), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{ab-2bx+x^2}{\left(x\,(-a+x)\,(-b+x)^3\right)^{\frac{1}{4}}\left(b-(ad+1)\,x+d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^3)^(1/4)/(b-(a*d+1)*x+d*x^2),x)
```

```
[Out] int((a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^3)^(1/4)/(b-(a*d+1)*x+d*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab-2\,bx+x^2}{\left((a-x)(b-x)^3x\right)^{\frac{1}{4}}\left(dx^2-(ad+1)x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^3)^(1/4)/(b-(a*d+1)*x+d*x^2),x,
algorithm="maxima")
```

```
[Out] integrate((a*b - 2*b*x + x^2)/(((a - x)*(b - x)^3*x)^(1/4)*(d*x^2 - (a*d +
1)*x + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2-2\,b\,x+a\,b}{{\left(x\,\left(a-x\right)\,{\left(b-x\right)}^3\right)}^{1/4}\,\left(d\,x^2+\left(-a\,d-1\right)\,x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b - 2*b*x + x^2)/((x*(a - x)*(b - x)^3)^(1/4)*(b - x*(a*d + 1) + d*x
^2)),x)
```

```
[Out] int((a*b - 2*b*x + x^2)/((x*(a - x)*(b - x)^3)^(1/4)*(b - x*(a*d + 1) + d*x
^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*b*x+x**2)/(x*(-a+x)*(-b+x)**3)**(1/4)/(b-(a*d+1)*x+d*x**2)
,x)
```

```
[Out] Timed out
```

3.1754 $\int \frac{1}{x^4(-b+ax^3)^{3/4}}\,dx$

Optimal. Leaf size=153

$$-\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}-\sqrt{b}}\right)}{2\sqrt{2}\,b^{7/4}}+\frac{a\tanh^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{2\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{ax^3-b}}{3bx^3}$$

Rubi [A] time = 0.23, antiderivative size = 228, normalized size of antiderivative = 1.49, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 51, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{a\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{4\sqrt{2}\,b^{7/4}}-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}\right)}{2\sqrt{2}\,b^{7/4}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}+1\right)}{2\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{ax^3-b}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-b + a*x^3)^(3/4)),x]
```

```
[Out] (-b + a*x^3)^(1/4)/(3*b*x^3) - (a*ArcTan[1 - (Sqrt[2]*(-b + a*x^3)^(1/4))/b
^(1/4)])/(2*Sqrt[2]*b^(7/4)) + (a*ArcTan[1 + (Sqrt[2]*(-b + a*x^3)^(1/4))/b
^(1/4)])/(2*Sqrt[2]*b^(7/4)) - (a*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^3
)^(1/4) + Sqrt[-b + a*x^3]])/(4*Sqrt[2]*b^(7/4)) + (a*Log[Sqrt[b] + Sqrt[2]
*b^(1/4)*(-b + a*x^3)^(1/4) + Sqrt[-b + a*x^3]])/(4*Sqrt[2]*b^(7/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^4\left(-b+ax^3\right)^{3/4}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(-b+ax)^{3/4}}\,dx, x, x^3\right)$$

$$= \frac{\sqrt[4]{-b+ax^3}}{3bx^3} + \frac{a\,\text{Subst}\left(\int \frac{1}{x(-b+ax)^{3/4}}\,dx, x, x^3\right)}{4b}$$

$$= \frac{\sqrt[4]{-b+ax^3}}{3bx^3} + \frac{\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{b}$$

$$= \frac{\sqrt[4]{-b+ax^3}}{3bx^3} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{2b^{3/2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{2b^{3/2}}$$

$$= \frac{\sqrt[4]{-b+ax^3}}{3bx^3} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{4\sqrt{2}\,b^{7/4}} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}-2x}{-\sqrt{b}+\sqrt{2}\sqrt[4]{b}x-x^2 \cdots}\right)}{4\sqrt{2}\,b^{7 \cdots}}$$

$$= \frac{\sqrt[4]{-b+ax^3}}{3bx^3} - \frac{a\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^3}+\sqrt{-b+ax^3}\right)}{4\sqrt{2}\,b^{7/4}} + \frac{a\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{- \cdots}\right)}{4\sqrt{2} \cdots}$$

$$= \frac{\sqrt[4]{-b+ax^3}}{3bx^3} - \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)}{2\sqrt{2}\,b^{7/4}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)}{2\sqrt{2}\,b^{7/4}} - \frac{a\log\left(\sqrt{b}-\sqrt{2} \cdots\right.}{\cdots}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.26

$$\frac{4a\sqrt[4]{ax^3-b}\;{}_2F_1\left(\frac{1}{4},2;\frac{5}{4};1-\frac{ax^3}{b}\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-b + a*x^3)^(3/4)),x]
```

```
[Out] (4*a*(-b + a*x^3)^(1/4)*Hypergeometric2F1[1/4, 2, 5/4, 1 - (a*x^3)/b])/(3*b
^2)
```

IntegrateAlgebraic [A] time = 0.19, size = 152, normalized size = 0.99

$$\frac{a\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{2\sqrt{2}\,b^{7/4}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}+\sqrt{b}}\right)}{2\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{ax^3-b}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(-b + a*x^3)^(3/4)),x]
```

```
[Out] (-b + a*x^3)^(1/4)/(3*b*x^3) + (a*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a*
x^3]/(Sqrt[2]*b^(1/4)))/(-b + a*x^3)^(1/4)])/(2*Sqrt[2]*b^(7/4)) + (a*ArcTa
nh[(Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^3])])/(2*S
qrt[2]*b^(7/4))
```

fricas [A] time = 0.45, size = 209, normalized size = 1.37

$$\frac{12\,bx^3\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^3-b\right)^{\frac{1}{4}}ab^5\left(-\frac{a^4}{b^7}\right)^{\frac{3}{4}}-\sqrt{b^4\sqrt{-\frac{a^4}{b^7}}+\sqrt{ax^3-b}\,a^2}\,b^5\left(-\frac{a^4}{b^7}\right)^{\frac{3}{4}}}{a^4}\right)+3\,bx^3\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}+\left(ax^3-b\right)^{\frac{1}{4}}a\right)-3\,bx^3\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(-b^2\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}+\left(ax^3-b\right)^{\frac{1}{4}}a\right)+4\left(ax^3-b\right)^{\frac{1}{4}}}{12\,bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*x^3-b)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/12*(12*b*x^3*(-a^4/b^7)^(1/4)*arctan(-((a*x^3 - b)^(1/4)*a*b^5*(-a^4/b^7)
^(3/4) - sqrt(b^4*sqrt(-a^4/b^7) + sqrt(a*x^3 - b)*a^2)*b^5*(-a^4/b^7)^(3/4
))/a^4) + 3*b*x^3*(-a^4/b^7)^(1/4)*log(b^2*(-a^4/b^7)^(1/4) + (a*x^3 - b)^(
1/4)*a) - 3*b*x^3*(-a^4/b^7)^(1/4)*log(-b^2*(-a^4/b^7)^(1/4) + (a*x^3 - b)^
(1/4)*a) + 4*(a*x^3 - b)^(1/4))/(b*x^3)
```

giac [A] time = 0.18, size = 199, normalized size = 1.30

$$\frac{\frac{6\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{7}{4}}}+\frac{6\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{7}{4}}}+\frac{3\sqrt{2}a^2\log\left(\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{b^{\frac{7}{4}}}-\frac{3\sqrt{2}a^2\log\left(-\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{b^{\frac{7}{4}}}+\frac{8\left(ax^3-b\right)^{\frac{1}{4}}a}{bx^3}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*x^3-b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/24*(6*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1/
4))/b^(1/4))/b^(7/4) + 6*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
 2*(a*x^3 - b)^(1/4))/b^(1/4))/b^(7/4) + 3*sqrt(2)*a^2*log(sqrt(2)*(a*x^3 -
 b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(7/4) - 3*sqrt(2)*a^2*log(
-sqrt(2)*(a*x^3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(7/4) + 8
*(a*x^3 - b)^(1/4)*a/(b*x^3))/a
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a\,x^3-b\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a*x^3-b)^(3/4),x)
```

```
[Out] int(1/x^4/(a*x^3-b)^(3/4),x)
```

maxima [A] time = 0.40, size = 202, normalized size = 1.32

$$\frac{\left(ax^3-b\right)^{\frac{1}{4}}a}{3\left(\left(ax^3-b\right)b+b^2\right)}+\frac{\frac{2\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{2\sqrt{2}\,a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{\sqrt{2}\,a\log\left(\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}\,a\log\left(-\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*x^3-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/3*(a*x^3 - b)^(1/4)*a/((a*x^3 - b)*b + b^2) + 1/8*(2*sqrt(2)*a*arctan(1/2
*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1/4))/b^(1/4))/b^(3/4) + 2*sqrt(
2)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^3 - b)^(1/4))/b^(1/4))/b
^(3/4) + sqrt(2)*a*log(sqrt(2)*(a*x^3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b)
+ sqrt(b))/b^(3/4) - sqrt(2)*a*log(-sqrt(2)*(a*x^3 - b)^(1/4)*b^(1/4) + sqr
t(a*x^3 - b) + sqrt(b))/b^(3/4))/b
```

mupad [B] time = 1.35, size = 72, normalized size = 0.47

$$\frac{\left(a\,x^3-b\right)^{1/4}}{3\,b\,x^3}+\frac{a\,\mathrm{atan}\left(\frac{\left(a\,x^3-b\right)^{1/4}}{(-b)^{1/4}}\right)}{2\,(-b)^{7/4}}+\frac{a\,\mathrm{atanh}\left(\frac{\left(a\,x^3-b\right)^{1/4}}{(-b)^{1/4}}\right)}{2\,(-b)^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x^3 - b)^(3/4)),x)
```

```
[Out] (a*x^3 - b)^(1/4)/(3*b*x^3) + (a*atan((a*x^3 - b)^(1/4)/(-b)^(1/4)))/(2*(-b
)^(7/4)) + (a*atanh((a*x^3 - b)^(1/4)/(-b)^(1/4)))/(2*(-b)^(7/4))
```

sympy [C] time = 1.21, size = 42, normalized size = 0.27

$$-\frac{\Gamma\left(\frac{7}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| {\frac{be^{2i\pi}}{ax^3}} \right)}{3a^{\frac{3}{4}} x^{\frac{21}{4}} \Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a*x**3-b)**(3/4),x)
```

```
[Out] -gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*exp_polar(2*I*pi)/(a*x**3))/(3*a**
(3/4)*x**(21/4)*gamma(11/4))
```

3.1755 $\int \frac{1}{x^4 \sqrt[4]{-b+ax^3}}\,dx$

Optimal. Leaf size=153

$$-\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}-\sqrt{b}}\right)}{6\sqrt{2}\,b^{5/4}}-\frac{a\tanh^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{6\sqrt{2}\,b^{5/4}}+\frac{\left(ax^3-b\right)^{3/4}}{3bx^3}$$

Rubi [A] time = 0.21, antiderivative size = 228, normalized size of antiderivative = 1.49, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 51, 63, 297, 1162, 617, 204, 1165, 628}

$$\frac{a\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{12\sqrt{2}\,b^{5/4}}-\frac{a\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{12\sqrt{2}\,b^{5/4}}-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}\right)}{6\sqrt{2}\,b^{5/4}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}+1\right)}{6\sqrt{2}\,b^{5/4}}+\frac{\left(ax^3-b\right)^{3/4}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-b + a*x^3)^(1/4)),x]
```

```
[Out] (-b + a*x^3)^(3/4)/(3*b*x^3) - (a*ArcTan[1 - (Sqrt[2]*(-b + a*x^3)^(1/4))/b
^(1/4)])/(6*Sqrt[2]*b^(5/4)) + (a*ArcTan[1 + (Sqrt[2]*(-b + a*x^3)^(1/4))/b
^(1/4)])/(6*Sqrt[2]*b^(5/4)) + (a*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^3
)^(1/4) + Sqrt[-b + a*x^3]])/(12*Sqrt[2]*b^(5/4)) - (a*Log[Sqrt[b] + Sqrt[2
]*b^(1/4)*(-b + a*x^3)^(1/4) + Sqrt[-b + a*x^3]])/(12*Sqrt[2]*b^(5/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[4]{-b+ax^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt[4]{-b+ax}}\,dx, x, x^3\right)$$

$$= \frac{\left(-b+ax^3\right)^{3/4}}{3bx^3} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{-b+ax}}\,dx, x, x^3\right)}{12b}$$

$$= \frac{\left(-b+ax^3\right)^{3/4}}{3bx^3} + \frac{\text{Subst}\left(\int \frac{x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{3b}$$

$$= \frac{\left(-b+ax^3\right)^{3/4}}{3bx^3} - \frac{\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{6b} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{6b}$$

$$= \frac{\left(-b+ax^3\right)^{3/4}}{3bx^3} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{12\sqrt{2}\,b^{5/4}} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}-2x}{-\sqrt{b}+\sqrt{2}\sqrt[4]{b}x-x^2}\right.}{12\sqrt{2}\,b^{5}}$$

$$= \frac{\left(-b+ax^3\right)^{3/4}}{3bx^3} + \frac{a\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^3}+\sqrt{-b+ax^3}\right)}{12\sqrt{2}\,b^{5/4}} - \frac{a\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b}\right.}{12\sqrt{2}}$$

$$= \frac{\left(-b+ax^3\right)^{3/4}}{3bx^3} - \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)}{6\sqrt{2}\,b^{5/4}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)}{6\sqrt{2}\,b^{5/4}} + \frac{a\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{}\right.}{}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.26

$$\frac{4a\left(ax^3-b\right)^{3/4}\ {}_2F_1\left(\frac{3}{4},2;\frac{7}{4};1-\frac{ax^3}{b}\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-b + a*x^3)^(1/4)),x]
```

```
[Out] (4*a*(-b + a*x^3)^(3/4)*Hypergeometric2F1[3/4, 2, 7/4, 1 - (a*x^3)/b])/(9*b
^2)
```

IntegrateAlgebraic [A] time = 0.34, size = 152, normalized size = 0.99

$$\frac{a\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{6\sqrt{2}\,b^{5/4}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}+\sqrt{b}}\right)}{6\sqrt{2}\,b^{5/4}}+\frac{\left(ax^3-b\right)^{3/4}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(-b + a*x^3)^(1/4)),x]
```

```
[Out] (-b + a*x^3)^(3/4)/(3*b*x^3) + (a*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a*
x^3]/(Sqrt[2]*b^(1/4)))/(-b + a*x^3)^(1/4)])/(6*Sqrt[2]*b^(5/4)) - (a*ArcTa
nh[(Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^3])])/(6*S
qrt[2]*b^(5/4))
```

fricas [A] time = 0.42, size = 214, normalized size = 1.40

$$-\frac{4\,bx^3\left(-\frac{a^4}{b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^3-b\right)^{\frac{1}{4}}a^3b\left(-\frac{a^4}{b^5}\right)^{\frac{1}{4}}-\sqrt{-a^4b^3\sqrt{-\frac{a^4}{b^5}}+\sqrt{ax^3-b}\,a^6}\,b\left(-\frac{a^4}{b^5}\right)^{\frac{1}{4}}}{a^4}\right)-bx^3\left(-\frac{a^4}{b^5}\right)^{\frac{1}{4}}\log\left(b^4\left(-\frac{a^4}{b^5}\right)^{\frac{3}{4}}+\left(ax^3-b\right)^{\frac{1}{4}}a^3\right)+bx^3\left(-\frac{a^4}{b^5}\right)^{\frac{1}{4}}\log\left(-b^4\left(-\frac{a^4}{b^5}\right)^{\frac{3}{4}}+\left(ax^3-b\right)^{\frac{1}{4}}a^3\right)-4\left(ax^3-b\right)^{\frac{3}{4}}}{12\,bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*x^3-b)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*x^3*(-a^4/b^5)^(1/4)*arctan(-((a*x^3 - b)^(1/4)*a^3*b*(-a^4/b^5)
^(1/4) - sqrt(-a^4*b^3*sqrt(-a^4/b^5) + sqrt(a*x^3 - b)*a^6)*b*(-a^4/b^5)^(
1/4))/a^4) - b*x^3*(-a^4/b^5)^(1/4)*log(b^4*(-a^4/b^5)^(3/4) + (a*x^3 - b)^
(1/4)*a^3) + b*x^3*(-a^4/b^5)^(1/4)*log(-b^4*(-a^4/b^5)^(3/4) + (a*x^3 - b)
^(1/4)*a^3) - 4*(a*x^3 - b)^(3/4))/(b*x^3)
```

giac [A] time = 0.30, size = 198, normalized size = 1.29

$$\frac{\frac{2\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{5}{4}}}+\frac{2\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^3-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{5}{4}}}-\frac{\sqrt{2}a^2\log\left(\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{b^{\frac{5}{4}}}+\frac{\sqrt{2}a^2\log\left(-\sqrt{2}\left(ax^3-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{b^{\frac{5}{4}}}+\frac{8\left(ax^3-b\right)^{\frac{3}{4}}a}{bx^3}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*x^3-b)^(1/4),x, algorithm="giac")
```

```
[Out] 1/24*(2*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1/
4))/b^(1/4))/b^(5/4) + 2*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
 2*(a*x^3 - b)^(1/4))/b^(1/4))/b^(5/4) - sqrt(2)*a^2*log(sqrt(2)*(a*x^3 - b
)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(5/4) + sqrt(2)*a^2*log(-sqr
t(2)*(a*x^3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(5/4) + 8*(a*
x^3 - b)^(3/4)*a/(b*x^3))/a
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(a\,x^{3}-b\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a*x^3-b)^(1/4),x)
```

```
[Out] int(1/x^4/(a*x^3-b)^(1/4),x)
```

maxima [A] time = 0.41, size = 199, normalized size = 1.30

$$\frac{\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+2\left(ax^{3}-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-2\left(ax^{3}-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(ax^{3}-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^{3}-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(ax^{3}-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^{3}-b}+\sqrt{b}\right)}{b^{\frac{1}{4}}}\right)a}{24\,b}+\frac{\left(ax^{3}-b\right)^{\frac{3}{4}}a}{3\left(\left(ax^{3}-b\right)b+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*x^3-b)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/24*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1/4))/
b^(1/4))/b^(1/4) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^
3 - b)^(1/4))/b^(1/4))/b^(1/4) - sqrt(2)*log(sqrt(2)*(a*x^3 - b)^(1/4)*b^(1
/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(1/4) + sqrt(2)*log(-sqrt(2)*(a*x^3 - b)
^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(1/4))*a/b + 1/3*(a*x^3 - b)^
(3/4)*a/((a*x^3 - b)*b + b^2)
```

mupad [B] time = 1.25, size = 72, normalized size = 0.47

$$\frac{\left(a\,x^3-b\right)^{3/4}}{3\,b\,x^3}-\frac{a\,\mathrm{atan}\left(\frac{{\left(a\,x^3-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{6\,{\left(-b\right)}^{5/4}}+\frac{a\,\mathrm{atanh}\left(\frac{{\left(a\,x^3-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{6\,{\left(-b\right)}^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x^3 - b)^(1/4)),x)
```

```
[Out] (a*x^3 - b)^(3/4)/(3*b*x^3) - (a*atan((a*x^3 - b)^(1/4)/(-b)^(1/4)))/(6*(-b
)^(5/4)) + (a*atanh((a*x^3 - b)^(1/4)/(-b)^(1/4)))/(6*(-b)^(5/4))
```

sympy [C] time = 1.14, size = 42, normalized size = 0.27

$$-\frac{\Gamma\left(\frac{5}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| {\frac{b e^{2 i \pi}}{a x^{3}}} \right)}{3 \sqrt[4]{a} x^{\frac{15}{4}} \Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a*x**3-b)**(1/4),x)
```

```
[Out] -gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*exp_polar(2*I*pi)/(a*x**3))/(3*a**(
1/4)*x**(15/4)*gamma(9/4))
```

3.1756 $\int \frac{1}{x^5\left(-b+ax^4\right)^{3/4}}\,dx$

Optimal. Leaf size=153

$$-\frac{3a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^4-b}}{\sqrt{ax^4-b}-\sqrt{b}}\right)}{8\sqrt{2}\,b^{7/4}}+\frac{3a\tanh^{-1}\left(\frac{\frac{\sqrt{ax^4-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^4-b}}\right)}{8\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{ax^4-b}}{4bx^4}$$

Rubi [A] time = 0.22, antiderivative size = 228, normalized size of antiderivative = 1.49, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 51, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3a\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^4-b}+\sqrt{ax^4-b}+\sqrt{b}\right)}{16\sqrt{2}\,b^{7/4}}+\frac{3a\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^4-b}+\sqrt{ax^4-b}+\sqrt{b}\right)}{16\sqrt{2}\,b^{7/4}}-\frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^4-b}}{\sqrt[4]{b}}\right)}{8\sqrt{2}\,b^{7/4}}+\frac{3a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^4-b}}{\sqrt[4]{b}}+1\right)}{8\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{ax^4-b}}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(-b + a*x^4)^(3/4)),x]
```

```
[Out] (-b + a*x^4)^(1/4)/(4*b*x^4) - (3*a*ArcTan[1 - (Sqrt[2]*(-b + a*x^4)^(1/4))
/b^(1/4)])/(8*Sqrt[2]*b^(7/4)) + (3*a*ArcTan[1 + (Sqrt[2]*(-b + a*x^4)^(1/4
))/b^(1/4)])/(8*Sqrt[2]*b^(7/4)) - (3*a*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b +
 a*x^4)^(1/4) + Sqrt[-b + a*x^4]])/(16*Sqrt[2]*b^(7/4)) + (3*a*Log[Sqrt[b]
+ Sqrt[2]*b^(1/4)*(-b + a*x^4)^(1/4) + Sqrt[-b + a*x^4]])/(16*Sqrt[2]*b^(7/
4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^5\left(-b+ax^4\right)^{3/4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2(-b+ax)^{3/4}}\,dx, x, x^4\right)$$

$$= \frac{\sqrt[4]{-b+ax^4}}{4bx^4} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{x(-b+ax)^{3/4}}\,dx, x, x^4\right)}{16b}$$

$$= \frac{\sqrt[4]{-b+ax^4}}{4bx^4} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^4}\right)}{4b}$$

$$= \frac{\sqrt[4]{-b+ax^4}}{4bx^4} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^4}\right)}{8b^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax}\right.}{8b^{3/2}}$$

$$= \frac{\sqrt[4]{-b+ax^4}}{4bx^4} - \frac{(3a)\,\text{Subst}\left(\int \frac{\sqrt{2}\,\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,x-x^2}\,dx, x, \sqrt[4]{-b+ax^4}\right)}{16\sqrt{2}\,b^{7/4}} - \frac{(3a)\,\text{Subst}\left(\int \frac{\sqrt{2}\,\sqrt[4]{b}}{-\sqrt{b}+\sqrt{2}}\right.}{16}$$

$$= \frac{\sqrt[4]{-b+ax^4}}{4bx^4} - \frac{3a\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{-b+ax^4}+\sqrt{-b+ax^4}\right)}{16\sqrt{2}\,b^{7/4}} + \frac{3a\log\left(\sqrt{b}+\sqrt{2}\,\sqrt[4]{b}\right.}{16}$$

$$= \frac{\sqrt[4]{-b+ax^4}}{4bx^4} - \frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{-b+ax^4}}{\sqrt[4]{b}}\right)}{8\sqrt{2}\,b^{7/4}} + \frac{3a\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{-b+ax^4}}{\sqrt[4]{b}}\right)}{8\sqrt{2}\,b^{7/4}} - \frac{3a\log\left(\sqrt{b}-\right.}{}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.24

$$\frac{a\sqrt[4]{ax^4-b}\ {}_2F_1\left(\frac{1}{4},2;\frac{5}{4};1-\frac{ax^4}{b}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(-b + a*x^4)^(3/4)),x]
```

```
[Out] (a*(-b + a*x^4)^(1/4)*Hypergeometric2F1[1/4, 2, 5/4, 1 - (a*x^4)/b])/b^2
```

IntegrateAlgebraic [A] time = 0.18, size = 152, normalized size = 0.99

$$\frac{3a\tan^{-1}\left(\frac{\frac{\sqrt{ax^4-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^4-b}}\right)}{8\sqrt{2}\,b^{7/4}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^4-b}}{\sqrt{ax^4-b}+\sqrt{b}}\right)}{8\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{ax^4-b}}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(-b + a*x^4)^(3/4)),x]
```

```
[Out] (-b + a*x^4)^(1/4)/(4*b*x^4) + (3*a*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b +
a*x^4]/(Sqrt[2]*b^(1/4)))/(-b + a*x^4)^(1/4)])/(8*Sqrt[2]*b^(7/4)) + (3*a*A
rcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^4)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^4])])/
(8*Sqrt[2]*b^(7/4))
```

fricas [A] time = 0.42, size = 212, normalized size = 1.39

$$\frac{12\,bx^4\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^4-b\right)^{\frac{1}{4}}ab^5\left(-\frac{a^4}{b^7}\right)^{\frac{3}{4}}-\sqrt{b^4\sqrt{-\frac{a^4}{b^7}}+\sqrt{ax^4-b}\,a^2}\,b^5\left(-\frac{a^4}{b^7}\right)^{\frac{3}{4}}}{a^4}\right)+3\,bx^4\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(3\,b^2\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}+3\left(ax^4-b\right)^{\frac{1}{4}}a\right)-3\,bx^4\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(-3\,b^2\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}+3\left(ax^4-b\right)^{\frac{1}{4}}a\right)+4\left(ax^4-b\right)^{\frac{1}{4}}}{16\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a*x^4-b)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/16*(12*b*x^4*(-a^4/b^7)^(1/4)*arctan(-((a*x^4 - b)^(1/4)*a*b^5*(-a^4/b^7)
^(3/4) - sqrt(b^4*sqrt(-a^4/b^7) + sqrt(a*x^4 - b)*a^2)*b^5*(-a^4/b^7)^(3/4
))/a^4) + 3*b*x^4*(-a^4/b^7)^(1/4)*log(3*b^2*(-a^4/b^7)^(1/4) + 3*(a*x^4 -
b)^(1/4)*a) - 3*b*x^4*(-a^4/b^7)^(1/4)*log(-3*b^2*(-a^4/b^7)^(1/4) + 3*(a*x
^4 - b)^(1/4)*a) + 4*(a*x^4 - b)^(1/4))/(b*x^4)
```

giac [A] time = 0.18, size = 199, normalized size = 1.30

$$\frac{\frac{6\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{7}{4}}}+\frac{6\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{7}{4}}}+\frac{3\sqrt{2}a^2\log\left(\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{b^{\frac{7}{4}}}-\frac{3\sqrt{2}a^2\log\left(-\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{b^{\frac{7}{4}}}+\frac{8\left(ax^4-b\right)^{\frac{1}{4}}a}{bx^4}}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a*x^4-b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/32*(6*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^4 - b)^(1/
4))/b^(1/4))/b^(7/4) + 6*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
 2*(a*x^4 - b)^(1/4))/b^(1/4))/b^(7/4) + 3*sqrt(2)*a^2*log(sqrt(2)*(a*x^4 -
 b)^(1/4)*b^(1/4) + sqrt(a*x^4 - b) + sqrt(b))/b^(7/4) - 3*sqrt(2)*a^2*log(
-sqrt(2)*(a*x^4 - b)^(1/4)*b^(1/4) + sqrt(a*x^4 - b) + sqrt(b))/b^(7/4) + 8
*(a*x^4 - b)^(1/4)*a/(b*x^4))/a
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(a\,x^4-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(a*x^4-b)^(3/4),x)
```

```
[Out] int(1/x^5/(a*x^4-b)^(3/4),x)
```

maxima [A] time = 0.64, size = 202, normalized size = 1.32

$$\frac{\left(ax^4-b\right)^{\frac{1}{4}} a}{4\left(\left(ax^4-b\right)b+b^2\right)}+\frac{3\left(\frac{2\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{2\sqrt{2}\,a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{\sqrt{2}\,a\log\left(\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}\,a\log\left(-\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a*x^4-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/4*(a*x^4 - b)^(1/4)*a/((a*x^4 - b)*b + b^2) + 3/32*(2*sqrt(2)*a*arctan(1/
2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^4 - b)^(1/4))/b^(1/4))/b^(3/4) + 2*sqrt
(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^4 - b)^(1/4))/b^(1/4))/
b^(3/4) + sqrt(2)*a*log(sqrt(2)*(a*x^4 - b)^(1/4)*b^(1/4) + sqrt(a*x^4 - b)
 + sqrt(b))/b^(3/4) - sqrt(2)*a*log(-sqrt(2)*(a*x^4 - b)^(1/4)*b^(1/4) + sq
rt(a*x^4 - b) + sqrt(b))/b^(3/4))/b
```

mupad [B] time = 1.30, size = 72, normalized size = 0.47

$$\frac{\left(a\,x^4-b\right)^{1/4}}{4\,b\,x^4}+\frac{3\,a\,\mathrm{atan}\left(\frac{\left(a\,x^4-b\right)^{1/4}}{(-b)^{1/4}}\right)}{8\,(-b)^{7/4}}+\frac{3\,a\,\mathrm{atanh}\left(\frac{\left(a\,x^4-b\right)^{1/4}}{(-b)^{1/4}}\right)}{8\,(-b)^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a*x^4 - b)^(3/4)),x)
```

```
[Out] (a*x^4 - b)^(1/4)/(4*b*x^4) + (3*a*atan((a*x^4 - b)^(1/4)/(-b)^(1/4)))/(8*(
-b)^(7/4)) + (3*a*atanh((a*x^4 - b)^(1/4)/(-b)^(1/4)))/(8*(-b)^(7/4))
```

sympy [C] time = 1.24, size = 41, normalized size = 0.27

$$-\frac{\Gamma\left(\frac{7}{4}\right) {}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{be^{2i\pi}}{ax^4}\right)}{4a^{\frac{3}{4}}x^7\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(a*x**4-b)**(3/4),x)
```

```
[Out] -gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*exp_polar(2*I*pi)/(a*x**4))/(4*a**
(3/4)*x**7*gamma(11/4))
```

3.1757 $\int \frac{ab^3-2(3a-b)b^2x+3(3a-b)bx^2-4ax^3+x^4}{\sqrt[4]{x(-a+x)(-b+x)^3}\left(a^3-\left(3a^2+b^3d\right)x+3\left(a+b^2d\right)x^2-(1+3bd)x^3+dx^4\right)}dx$

Optimal. Leaf size=153

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{ab^3x+x^3\left(3ab+3b^2\right)+x^2\left(-3ab^2-b^3\right)+x^4(-a-3b)+x^5}}{a-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{ab^3x+x^3\left(3ab+3b^2\right)+x^2\left(-3ab^2-b^3\right)+x^4(-a-3b)+x^5}}{a-x}\right)}{d^{3/4}}$$

Rubi [F] time = 19.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ab^3-2(3a-b)b^2x+3(3a-b)bx^2-4ax^3+x^4}{\sqrt[4]{x(-a+x)(-b+x)^3}\left(a^3-\left(3a^2+b^3d\right)x+3\left(a+b^2d\right)x^2-(1+3bd)x^3+dx^4\right)}dx$$

Verification is not applicable to the result.

```
[In] Int[(a*b^3 - 2*(3*a - b)*b^2*x + 3*(3*a - b)*b*x^2 - 4*a*x^3 + x^4)/((x*(-a
 + x)*(-b + x)^3)^(1/4)*(a^3 - (3*a^2 + b^3*d)*x + 3*(a + b^2*d)*x^2 - (1 +
 3*b*d)*x^3 + d*x^4)),x]
```

```
[Out] (4*a*b*(a - x)^(1/4)*(b - x)^(3/4)*x^(1/4)*Defer[Subst][Defer[Int][(x^2*(b
- x^4)^(5/4))/((a - x^4)^(1/4)*(a^3 - 3*a^2*(1 + (b^3*d)/(3*a^2))*x^4 + 3*a
*(1 + (b^2*d)/a)*x^8 - (1 + 3*b*d)*x^12 + d*x^16)), x], x, x^(1/4)])/((a -
x)*(b - x)^3*x)^(1/4) - (8*(2*a - b)*(a - x)^(1/4)*(b - x)^(3/4)*x^(1/4)*De
fer[Subst][Defer[Int][(x^6*(b - x^4)^(5/4))/((a - x^4)^(1/4)*(a^3 - 3*a^2*(
1 + (b^3*d)/(3*a^2))*x^4 + 3*a*(1 + (b^2*d)/a)*x^8 - (1 + 3*b*d)*x^12 + d*x
^16)), x], x, x^(1/4)])/((a - x)*(b - x)^3*x)^(1/4) + (4*(a - x)^(1/4)*(b -
 x)^(3/4)*x^(1/4)*Defer[Subst][Defer[Int][(x^10*(b - x^4)^(5/4))/((a - x^4)
^(1/4)*(a^3 - 3*a^2*(1 + (b^3*d)/(3*a^2))*x^4 + 3*a*(1 + (b^2*d)/a)*x^8 - (
1 + 3*b*d)*x^12 + d*x^16)), x], x, x^(1/4)])/((a - x)*(b - x)^3*x)^(1/4)
```

Rubi steps

$$\int \frac{ab^3-2(3a-b)b^2x+3(3a-b)bx^2-4ax^3+x^4}{\sqrt[4]{x(-a+x)(-b+x)^3}\left(a^3-\left(3a^2+b^3d\right)x+3\left(a+b^2d\right)x^2-(1+3bd)x^3+dx^4\right)}dx = \int \frac{}{\sqrt[4]{(a-x)(b-x)^3x}}$$

$$= \frac{\left(\sqrt[4]{a-x}(b-x)^{3/4}\sqrt[4]{x}\right.}{}$$

$$= \frac{\left(4\sqrt[4]{a-x}(b-x)^{3/4}\sqrt[4]{x}\right.}{}$$

$$= \frac{\left(4\sqrt[4]{a-x}(b-x)^{3/4}\sqrt[4]{x}\right.}{}$$

$$= \frac{\left(4\sqrt[4]{a-x}(b-x)^{3/4}\sqrt[4]{x}\right.}{}$$

Mathematica [F] time = 2.61, size = 0, normalized size = 0.00

$$\int \frac{ab^3-2(3a-b)b^2x+3(3a-b)bx^2-4ax^3+x^4}{\sqrt[4]{x(-a+x)(-b+x)^3}\left(a^3-\left(3a^2+b^3d\right)x+3\left(a+b^2d\right)x^2-(1+3bd)x^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*b^3 - 2*(3*a - b)*b^2*x + 3*(3*a - b)*b*x^2 - 4*a*x^3 + x^4)/(
(x*(-a + x)*(-b + x)^3)^(1/4)*(a^3 - (3*a^2 + b^3*d)*x + 3*(a + b^2*d)*x^2
- (1 + 3*b*d)*x^3 + d*x^4)),x]
```

```
[Out] Integrate[(a*b^3 - 2*(3*a - b)*b^2*x + 3*(3*a - b)*b*x^2 - 4*a*x^3 + x^4)/(
(x*(-a + x)*(-b + x)^3)^(1/4)*(a^3 - (3*a^2 + b^3*d)*x + 3*(a + b^2*d)*x^2
- (1 + 3*b*d)*x^3 + d*x^4)), x]
```

IntegrateAlgebraic [A] time = 0.48, size = 153, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{ab^3x+x^3\left(3ab+3b^2\right)+x^2\left(-3ab^2-b^3\right)+x^4(-a-3b)+x^5}}{a-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{ab^3x+x^3\left(3ab+3b^2\right)+x^2\left(-3ab^2-b^3\right)+x^4(-a-3b)+x^5}}{a-x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b^3 - 2*(3*a - b)*b^2*x + 3*(3*a - b)*b*x^2 - 4*a*x^3
 + x^4)/((x*(-a + x)*(-b + x)^3)^(1/4)*(a^3 - (3*a^2 + b^3*d)*x + 3*(a + b^
2*d)*x^2 - (1 + 3*b*d)*x^3 + d*x^4)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(a*b^3*x + (-3*a*b^2 - b^3)*x^2 + (3*a*b + 3*b^2)*x^3 +
 (-a - 3*b)*x^4 + x^5)^(1/4))/(a - x)])/d^(3/4) + (2*ArcTanh[(d^(1/4)*(a*b^
3*x + (-3*a*b^2 - b^3)*x^2 + (3*a*b + 3*b^2)*x^3 + (-a - 3*b)*x^4 + x^5)^(1
/4))/(a - x)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b^3-2*(3*a-b)*b^2*x+3*(3*a-b)*b*x^2-4*a*x^3+x^4)/(x*(-a+x)*(-b
+x)^3)^(1/4)/(a^3-(b^3*d+3*a^2)*x+3*(b^2*d+a)*x^2-(3*b*d+1)*x^3+d*x^4),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab^3-2\,(3\,a-b)b^2x+3\,(3\,a-b)bx^2-4\,ax^3+x^4}{\left((a-x)(b-x)^3x\right)^{\frac{1}{4}}\left(dx^4-(3\,bd+1)x^3+a^3+3\left(b^2d+a\right)x^2-\left(b^3d+3\,a^2\right)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b^3-2*(3*a-b)*b^2*x+3*(3*a-b)*b*x^2-4*a*x^3+x^4)/(x*(-a+x)*(-b
+x)^3)^(1/4)/(a^3-(b^3*d+3*a^2)*x+3*(b^2*d+a)*x^2-(3*b*d+1)*x^3+d*x^4),x, a
lgorithm="giac")
```

```
[Out] integrate((a*b^3 - 2*(3*a - b)*b^2*x + 3*(3*a - b)*b*x^2 - 4*a*x^3 + x^4)/(
((a - x)*(b - x)^3*x)^(1/4)*(d*x^4 - (3*b*d + 1)*x^3 + a^3 + 3*(b^2*d + a)*
x^2 - (b^3*d + 3*a^2)*x)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{a\,b^3-2\,(3a-b)\,b^2x+3\,(3a-b)\,b\,x^2-4a\,x^3+x^4}{\left(x\,(-a+x)\,(-b+x)^3\right)^{\frac{1}{4}}\left(a^3-\left(b^3d+3a^2\right)x+3\left(b^2d+a\right)x^2-(3bd+1)\,x^3+d\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b^3-2*(3*a-b)*b^2*x+3*(3*a-b)*b*x^2-4*a*x^3+x^4)/(x*(-a+x)*(-b+x)^3)
^(1/4)/(a^3-(b^3*d+3*a^2)*x+3*(b^2*d+a)*x^2-(3*b*d+1)*x^3+d*x^4),x)
```

```
[Out] int((a*b^3-2*(3*a-b)*b^2*x+3*(3*a-b)*b*x^2-4*a*x^3+x^4)/(x*(-a+x)*(-b+x)^3)
^(1/4)/(a^3-(b^3*d+3*a^2)*x+3*(b^2*d+a)*x^2-(3*b*d+1)*x^3+d*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab^3 - 2\,(3\,a-b)b^2x + 3\,(3\,a-b)bx^2 - 4\,ax^3 + x^4}{\left((a-x)(b-x)^3x\right)^{\frac{1}{4}}\left(dx^4 - (3\,bd+1)x^3 + a^3 + 3\left(b^2d+a\right)x^2 - \left(b^3d + 3\,a^2\right)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b^3-2*(3*a-b)*b^2*x+3*(3*a-b)*b*x^2-4*a*x^3+x^4)/(x*(-a+x)*(-b
+x)^3)^(1/4)/(a^3-(b^3*d+3*a^2)*x+3*(b^2*d+a)*x^2-(3*b*d+1)*x^3+d*x^4),x, a
lgorithm="maxima")
```

```
[Out] integrate((a*b^3 - 2*(3*a - b)*b^2*x + 3*(3*a - b)*b*x^2 - 4*a*x^3 + x^4)/(
((a - x)*(b - x)^3*x)^(1/4)*(d*x^4 - (3*b*d + 1)*x^3 + a^3 + 3*(b^2*d + a)*
x^2 - (b^3*d + 3*a^2)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,b^3 - 4\,a\,x^3 + x^4 + 3\,b\,x^2\,\left(3\,a-b\right) - 2\,b^2\,x\,\left(3\,a-b\right)}{{\left(x\,\left(a-x\right)\,{\left(b-x\right)}^3\right)}^{1/4}\,\left(3\,x^2\,\left(d\,b^2+a\right) + d\,x^4 - x^3\,\left(3\,b\,d+1\right) + a^3 - x\,\left(3\,a^2+d\,b^3\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b^3 - 4*a*x^3 + x^4 + 3*b*x^2*(3*a - b) - 2*b^2*x*(3*a - b))/((x*(a
- x)*(b - x)^3)^(1/4)*(3*x^2*(a + b^2*d) + d*x^4 - x^3*(3*b*d + 1) + a^3 -
x*(b^3*d + 3*a^2))),x)
```

```
[Out] int((a*b^3 - 4*a*x^3 + x^4 + 3*b*x^2*(3*a - b) - 2*b^2*x*(3*a - b))/((x*(a
- x)*(b - x)^3)^(1/4)*(3*x^2*(a + b^2*d) + d*x^4 - x^3*(3*b*d + 1) + a^3 -
x*(b^3*d + 3*a^2))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b**3-2*(3*a-b)*b**2*x+3*(3*a-b)*b*x**2-4*a*x**3+x**4)/(x*(-a+x
)*(-b+x)**3)**(1/4)/(a**3-(b**3*d+3*a**2)*x+3*(b**2*d+a)*x**2-(3*b*d+1)*x**
3+d*x**4),x)
```

```
[Out] Timed out
```

3.1758 $\int \frac{1}{x^6\left(-b+ax^5\right)^{3/4}}\,dx$

Optimal. Leaf size=153

$$-\frac{3a\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{ax^5-b}}{\sqrt{ax^5-b}-\sqrt{b}}\right)}{10\sqrt{2}\,b^{7/4}}+\frac{3a\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5-b}}{\sqrt{2}\,\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^5-b}}\right)}{10\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{ax^5-b}}{5bx^5}$$

Rubi [A] time = 0.22, antiderivative size = 228, normalized size of antiderivative = 1.49, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 51, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3a\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{ax^5-b}+\sqrt{ax^5-b}+\sqrt{b}\right)}{20\sqrt{2}\,b^{7/4}}+\frac{3a\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{ax^5-b}+\sqrt{ax^5-b}+\sqrt{b}\right)}{20\sqrt{2}\,b^{7/4}}-\frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{ax^5-b}}{\sqrt[4]{b}}\right)}{10\sqrt{2}\,b^{7/4}}+\frac{3a\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{ax^5-b}}{\sqrt[4]{b}}+1\right)}{10\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{ax^5-b}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(-b + a*x^5)^(3/4)),x]
```

```
[Out] (-b + a*x^5)^(1/4)/(5*b*x^5) - (3*a*ArcTan[1 - (Sqrt[2]*(-b + a*x^5)^(1/4))
/b^(1/4)])/(10*Sqrt[2]*b^(7/4)) + (3*a*ArcTan[1 + (Sqrt[2]*(-b + a*x^5)^(1/
4))/b^(1/4)])/(10*Sqrt[2]*b^(7/4)) - (3*a*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b
 + a*x^5)^(1/4) + Sqrt[-b + a*x^5]])/(20*Sqrt[2]*b^(7/4)) + (3*a*Log[Sqrt[b
] + Sqrt[2]*b^(1/4)*(-b + a*x^5)^(1/4) + Sqrt[-b + a*x^5]])/(20*Sqrt[2]*b^(
7/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^6\left(-b+ax^5\right)^{3/4}}\,dx = \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x^2(-b+ax)^{3/4}}\,dx, x, x^5\right)$$

$$= \frac{\sqrt[4]{-b+ax^5}}{5bx^5} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{x(-b+ax)^{3/4}}\,dx, x, x^5\right)}{20b}$$

$$= \frac{\sqrt[4]{-b+ax^5}}{5bx^5} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^5}\right)}{5b}$$

$$= \frac{\sqrt[4]{-b+ax^5}}{5bx^5} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^5}\right)}{10b^{3/2}} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+\right.}{10b^{3/2}}$$

$$= \frac{\sqrt[4]{-b+ax^5}}{5bx^5} - \frac{(3a)\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^5}\right)}{20\sqrt{2}\,b^{7/4}} - \frac{(3a)\,\text{Subst}\left(\int \frac{\sqrt{2}}{-\sqrt{b}+}\right.}{2}$$

$$= \frac{\sqrt[4]{-b+ax^5}}{5bx^5} - \frac{3a\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^5}+\sqrt{-b+ax^5}\right)}{20\sqrt{2}\,b^{7/4}} + \frac{3a\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{\right.}{2}$$

$$= \frac{\sqrt[4]{-b+ax^5}}{5bx^5} - \frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^5}}{\sqrt[4]{b}}\right)}{10\sqrt{2}\,b^{7/4}} + \frac{3a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^5}}{\sqrt[4]{b}}\right)}{10\sqrt{2}\,b^{7/4}} - \frac{3a\log\left(\sqrt{b}\right.}{}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.26

$$\frac{4a\sqrt[4]{ax^5-b}\,{}_2F_1\left(\frac{1}{4},2;\frac{5}{4};1-\frac{ax^5}{b}\right)}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(-b + a*x^5)^(3/4)),x]
```

```
[Out] (4*a*(-b + a*x^5)^(1/4)*Hypergeometric2F1[1/4, 2, 5/4, 1 - (a*x^5)/b])/(5*b
^2)
```

IntegrateAlgebraic [A] time = 0.18, size = 152, normalized size = 0.99

$$\frac{3a\tan^{-1}\left(\frac{\frac{\sqrt{ax^5-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^5-b}}\right)}{10\sqrt{2}\,b^{7/4}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^5-b}}{\sqrt{ax^5-b}+\sqrt{b}}\right)}{10\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{ax^5-b}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^6*(-b + a*x^5)^(3/4)),x]
```

```
[Out] (-b + a*x^5)^(1/4)/(5*b*x^5) + (3*a*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b +
a*x^5]/(Sqrt[2]*b^(1/4)))/(-b + a*x^5)^(1/4)])/(10*Sqrt[2]*b^(7/4)) + (3*a*
ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^5)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^5])])
/(10*Sqrt[2]*b^(7/4))
```

fricas [A] time = 0.46, size = 212, normalized size = 1.39

$$\frac{12\,bx^5\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^5-b\right)^{\frac{1}{4}}ab^5\left(-\frac{a^4}{b^7}\right)^{\frac{3}{4}}-\sqrt{b^4\sqrt{-\frac{a^4}{b^7}}+\sqrt{ax^5-b}\,a^2}\,b^5\left(-\frac{a^4}{b^7}\right)^{\frac{3}{4}}}{a^4}\right)+3\,bx^5\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(3\,b^2\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}+3\left(ax^5-b\right)^{\frac{1}{4}}a\right)-3\,bx^5\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(-3\,b^2\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}+3\left(ax^5-b\right)^{\frac{1}{4}}a\right)+4\left(ax^5-b\right)^{\frac{1}{4}}}{20\,bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(a*x^5-b)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/20*(12*b*x^5*(-a^4/b^7)^(1/4)*arctan(-((a*x^5 - b)^(1/4)*a*b^5*(-a^4/b^7)
^(3/4) - sqrt(b^4*sqrt(-a^4/b^7) + sqrt(a*x^5 - b)*a^2)*b^5*(-a^4/b^7)^(3/4
))/a^4) + 3*b*x^5*(-a^4/b^7)^(1/4)*log(3*b^2*(-a^4/b^7)^(1/4) + 3*(a*x^5 -
b)^(1/4)*a) - 3*b*x^5*(-a^4/b^7)^(1/4)*log(-3*b^2*(-a^4/b^7)^(1/4) + 3*(a*x
^5 - b)^(1/4)*a) + 4*(a*x^5 - b)^(1/4))/(b*x^5)
```

giac [A] time = 0.32, size = 199, normalized size = 1.30

$$\frac{\frac{6\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^5-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{7}{4}}}+\frac{6\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^5-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{7}{4}}}+\frac{3\sqrt{2}a^2\log\left(\sqrt{2}\left(ax^5-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^5-b}+\sqrt{b}\right)}{b^{\frac{7}{4}}}-\frac{3\sqrt{2}a^2\log\left(-\sqrt{2}\left(ax^5-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^5-b}+\sqrt{b}\right)}{b^{\frac{7}{4}}}+\frac{8\left(ax^5-b\right)^{\frac{1}{4}}a}{bx^5}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(a*x^5-b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/40*(6*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^5 - b)^(1/
4))/b^(1/4))/b^(7/4) + 6*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
 2*(a*x^5 - b)^(1/4))/b^(1/4))/b^(7/4) + 3*sqrt(2)*a^2*log(sqrt(2)*(a*x^5 -
 b)^(1/4)*b^(1/4) + sqrt(a*x^5 - b) + sqrt(b))/b^(7/4) - 3*sqrt(2)*a^2*log(
-sqrt(2)*(a*x^5 - b)^(1/4)*b^(1/4) + sqrt(a*x^5 - b) + sqrt(b))/b^(7/4) + 8
*(a*x^5 - b)^(1/4)*a/(b*x^5))/a
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{6} \left(a\,x^{5}-b\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(a*x^5-b)^(3/4),x)
```

```
[Out] int(1/x^6/(a*x^5-b)^(3/4),x)
```

maxima [A] time = 0.58, size = 202, normalized size = 1.32

$$\frac{\left(ax^{5}-b\right)^{\frac{1}{4}}a}{5\left(\left(ax^{5}-b\right)b+b^{2}\right)}+\frac{3\left(\frac{2\sqrt{2}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^{5}-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{2\sqrt{2}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^{5}-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{\sqrt{2}a\log\left(\sqrt{2}\left(ax^{5}-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^{5}-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}a\log\left(-\sqrt{2}\left(ax^{5}-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^{5}-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}\right)}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(a*x^5-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/5*(a*x^5 - b)^(1/4)*a/((a*x^5 - b)*b + b^2) + 3/40*(2*sqrt(2)*a*arctan(1/
2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^5 - b)^(1/4))/b^(1/4))/b^(3/4) + 2*sqrt
(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^5 - b)^(1/4))/b^(1/4))/
b^(3/4) + sqrt(2)*a*log(sqrt(2)*(a*x^5 - b)^(1/4)*b^(1/4) + sqrt(a*x^5 - b)
 + sqrt(b))/b^(3/4) - sqrt(2)*a*log(-sqrt(2)*(a*x^5 - b)^(1/4)*b^(1/4) + sq
rt(a*x^5 - b) + sqrt(b))/b^(3/4))/b
```

mupad [B] time = 1.31, size = 72, normalized size = 0.47

$$\frac{{\left(a\,x^5-b\right)}^{1/4}}{5\,b\,x^5}+\frac{3\,a\,\mathrm{atan}\left(\frac{{\left(a\,x^5-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{10\,{\left(-b\right)}^{7/4}}+\frac{3\,a\,\mathrm{atanh}\left(\frac{{\left(a\,x^5-b\right)}^{1/4}}{{\left(-b\right)}^{1/4}}\right)}{10\,{\left(-b\right)}^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a*x^5 - b)^(3/4)),x)
```

```
[Out] (a*x^5 - b)^(1/4)/(5*b*x^5) + (3*a*atan((a*x^5 - b)^(1/4)/(-b)^(1/4)))/(10*
(-b)^(7/4)) + (3*a*atanh((a*x^5 - b)^(1/4)/(-b)^(1/4)))/(10*(-b)^(7/4))
```

sympy [C] time = 1.34, size = 42, normalized size = 0.27

$$-\frac{\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^{5}}\right)}{5a^{\frac{3}{4}}x^{\frac{35}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(a*x**5-b)**(3/4),x)
```

```
[Out] -gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*exp_polar(2*I*pi)/(a*x**5))/(5*a**
(3/4)*x**(35/4)*gamma(11/4))
```

3.1759 $\int \frac{1}{x^7\left(-b+ax^6\right)^{3/4}}\,dx$

Optimal. Leaf size=153

$$-\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}}{\sqrt{ax^6-b}-\sqrt{b}}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{a\tanh^{-1}\left(\frac{\frac{\sqrt{ax^6-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^6-b}}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{ax^6-b}}{6bx^6}$$

Rubi [A] time = 0.21, antiderivative size = 228, normalized size of antiderivative = 1.49, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 51, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}+\sqrt{ax^6-b}+\sqrt{b}\right)}{8\sqrt{2}\,b^{7/4}}+\frac{a\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}+\sqrt{ax^6-b}+\sqrt{b}\right)}{8\sqrt{2}\,b^{7/4}}-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^6-b}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^6-b}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{ax^6-b}}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(-b + a*x^6)^(3/4)),x]
```

```
[Out] (-b + a*x^6)^(1/4)/(6*b*x^6) - (a*ArcTan[1 - (Sqrt[2]*(-b + a*x^6)^(1/4))/b
^(1/4)])/(4*Sqrt[2]*b^(7/4)) + (a*ArcTan[1 + (Sqrt[2]*(-b + a*x^6)^(1/4))/b
^(1/4)])/(4*Sqrt[2]*b^(7/4)) - (a*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^6
)^(1/4) + Sqrt[-b + a*x^6]])/(8*Sqrt[2]*b^(7/4)) + (a*Log[Sqrt[b] + Sqrt[2]
*b^(1/4)*(-b + a*x^6)^(1/4) + Sqrt[-b + a*x^6]])/(8*Sqrt[2]*b^(7/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(-b+ax^6\right)^{3/4}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x^2(-b+ax)^{3/4}}\,dx, x, x^6\right)\\
&= \frac{\sqrt[4]{-b+ax^6}}{6bx^6} + \frac{a\,\text{Subst}\left(\int \frac{1}{x(-b+ax)^{3/4}}\,dx, x, x^6\right)}{8b}\\
&= \frac{\sqrt[4]{-b+ax^6}}{6bx^6} + \frac{\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^6}\right)}{2b}\\
&= \frac{\sqrt[4]{-b+ax^6}}{6bx^6} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^6}\right)}{4b^{3/2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^6}\right)}{4b^{3/2}}\\
&= \frac{\sqrt[4]{-b+ax^6}}{6bx^6} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^6}\right)}{8\sqrt{2}\,b^{7/4}} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}-2}{-\sqrt{b}+\sqrt{2}\sqrt[4]{b}}\right.}{8\sqrt{2}}\\
&= \frac{\sqrt[4]{-b+ax^6}}{6bx^6} - \frac{a\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^6}+\sqrt{-b+ax^6}\right)}{8\sqrt{2}\,b^{7/4}} + \frac{a\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\right.}{8}\\
&= \frac{\sqrt[4]{-b+ax^6}}{6bx^6} - \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^6}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^6}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}} - \frac{a\log\left(\sqrt{b}-\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.26

$$\frac{2a\sqrt[4]{ax^6-b}\ {}_2F_1\left(\frac{1}{4},2;\frac{5}{4};1-\frac{ax^6}{b}\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(-b + a*x^6)^(3/4)),x]
```

```
[Out] (2*a*(-b + a*x^6)^(1/4)*Hypergeometric2F1[1/4, 2, 5/4, 1 - (a*x^6)/b])/(3*b
^2)
```

IntegrateAlgebraic [A] time = 0.18, size = 152, normalized size = 0.99

$$\frac{a\tan^{-1}\left(\frac{\frac{\sqrt{ax^6-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^6-b}}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^6-b}}{\sqrt{ax^6-b}+\sqrt{b}}\right)}{4\sqrt{2}\,b^{7/4}}+\frac{\sqrt[4]{ax^6-b}}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(-b + a*x^6)^(3/4)),x]
```

```
[Out] (-b + a*x^6)^(1/4)/(6*b*x^6) + (a*ArcTan[(-(b^(1/4)/Sqrt[2]) + Sqrt[-b + a*
x^6]/(Sqrt[2]*b^(1/4)))/(-b + a*x^6)^(1/4)])/(4*Sqrt[2]*b^(7/4)) + (a*ArcTa
nh[(Sqrt[2]*b^(1/4)*(-b + a*x^6)^(1/4))/(Sqrt[b] + Sqrt[-b + a*x^6])])/(4*S
qrt[2]*b^(7/4))
```

fricas [A] time = 0.44, size = 209, normalized size = 1.37

$$\frac{12\,bx^6\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^6-b\right)^{\frac{1}{4}}ab^5\left(-\frac{a^4}{b^7}\right)^{\frac{3}{4}}-\sqrt{b^4\sqrt{-\frac{a^4}{b^7}}+\sqrt{ax^6-b}\,a^2}\,b^5\left(-\frac{a^4}{b^7}\right)^{\frac{3}{4}}}{a^4}\right)+3\,bx^6\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}+\left(ax^6-b\right)^{\frac{1}{4}}a\right)-3\,bx^6\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(-b^2\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}+\left(ax^6-b\right)^{\frac{1}{4}}a\right)+4\left(ax^6-b\right)^{\frac{1}{4}}}{24\,bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a*x^6-b)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/24*(12*b*x^6*(-a^4/b^7)^(1/4)*arctan(-((a*x^6 - b)^(1/4)*a*b^5*(-a^4/b^7)
^(3/4) - sqrt(b^4*sqrt(-a^4/b^7) + sqrt(a*x^6 - b)*a^2)*b^5*(-a^4/b^7)^(3/4
))/a^4) + 3*b*x^6*(-a^4/b^7)^(1/4)*log(b^2*(-a^4/b^7)^(1/4) + (a*x^6 - b)^(
1/4)*a) - 3*b*x^6*(-a^4/b^7)^(1/4)*log(-b^2*(-a^4/b^7)^(1/4) + (a*x^6 - b)^
(1/4)*a) + 4*(a*x^6 - b)^(1/4))/(b*x^6)
```

giac [A] time = 0.18, size = 199, normalized size = 1.30

$$\frac{\frac{6\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^6-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{7}{4}}}+\frac{6\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^6-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{7}{4}}}+\frac{3\sqrt{2}a^2\log\left(\sqrt{2}\left(ax^6-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^6-b}+\sqrt{b}\right)}{b^{\frac{7}{4}}}-\frac{3\sqrt{2}a^2\log\left(-\sqrt{2}\left(ax^6-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^6-b}+\sqrt{b}\right)}{b^{\frac{7}{4}}}+\frac{8\left(ax^6-b\right)^{\frac{1}{4}}a}{bx^6}}{48\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a*x^6-b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/48*(6*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^6 - b)^(1/
4))/b^(1/4))/b^(7/4) + 6*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
 2*(a*x^6 - b)^(1/4))/b^(1/4))/b^(7/4) + 3*sqrt(2)*a^2*log(sqrt(2)*(a*x^6 -
 b)^(1/4)*b^(1/4) + sqrt(a*x^6 - b) + sqrt(b))/b^(7/4) - 3*sqrt(2)*a^2*log(
-sqrt(2)*(a*x^6 - b)^(1/4)*b^(1/4) + sqrt(a*x^6 - b) + sqrt(b))/b^(7/4) + 8
*(a*x^6 - b)^(1/4)*a/(b*x^6))/a
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{7} \left(a\,x^{6}-b\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(a*x^6-b)^(3/4),x)
```

```
[Out] int(1/x^7/(a*x^6-b)^(3/4),x)
```

maxima [A] time = 0.43, size = 202, normalized size = 1.32

$$\frac{\left(ax^{6}-b\right)^{\frac{1}{4}} a}{6\left(\left(ax^{6}-b\right)b+b^{2}\right)}+\frac{\frac{2\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+2\left(ax^{6}-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{2\sqrt{2}\,a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-2\left(ax^{6}-b\right)^{\frac{1}{4}}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{\sqrt{2}\,a\log\left(\sqrt{2}\left(ax^{6}-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^{6}-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}\,a\log\left(-\sqrt{2}\left(ax^{6}-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^{6}-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a*x^6-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/6*(a*x^6 - b)^(1/4)*a/((a*x^6 - b)*b + b^2) + 1/16*(2*sqrt(2)*a*arctan(1/
2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^6 - b)^(1/4))/b^(1/4))/b^(3/4) + 2*sqrt
(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^6 - b)^(1/4))/b^(1/4))/
b^(3/4) + sqrt(2)*a*log(sqrt(2)*(a*x^6 - b)^(1/4)*b^(1/4) + sqrt(a*x^6 - b)
 + sqrt(b))/b^(3/4) - sqrt(2)*a*log(-sqrt(2)*(a*x^6 - b)^(1/4)*b^(1/4) + sq
rt(a*x^6 - b) + sqrt(b))/b^(3/4))/b
```

mupad [B] time = 1.32, size = 72, normalized size = 0.47

$$\frac{\left(a\,x^6-b\right)^{1/4}}{6\,b\,x^6}+\frac{a\,\mathrm{atan}\left(\frac{\left(a\,x^6-b\right)^{1/4}}{(-b)^{1/4}}\right)}{4\,(-b)^{7/4}}+\frac{a\,\mathrm{atanh}\left(\frac{\left(a\,x^6-b\right)^{1/4}}{(-b)^{1/4}}\right)}{4\,(-b)^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a*x^6 - b)^(3/4)),x)
```

```
[Out] (a*x^6 - b)^(1/4)/(6*b*x^6) + (a*atan((a*x^6 - b)^(1/4)/(-b)^(1/4)))/(4*(-b
)^(7/4)) + (a*atanh((a*x^6 - b)^(1/4)/(-b)^(1/4)))/(4*(-b)^(7/4))
```

sympy [C] time = 1.41, size = 42, normalized size = 0.27

$$-\frac{\Gamma\left(\frac{7}{4}\right) {}_{2}F_{1}\left(\begin{matrix}\frac{3}{4}, \frac{7}{4} \\ \frac{11}{4}\end{matrix}\middle| {\frac{be^{2i\pi}}{ax^{6}}} \right)}{6a^{\frac{3}{4}}x^{\frac{21}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(a*x**6-b)**(3/4),x)
```

```
[Out] -gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*exp_polar(2*I*pi)/(a*x**6))/(6*a**
(3/4)*x**(21/2)*gamma(11/4))
```

3.1760 $\int \frac{b^6+a^6x^6}{\sqrt{b^4+a^4x^4}\left(-b^6+a^6x^6\right)}\,dx$

Optimal. Leaf size=153

$$-\frac{2\tan^{-1}\left(\frac{abx}{\sqrt{a^4x^4+b^4}}\right)}{3ab}+\frac{\tanh^{-1}\left(\frac{\sqrt{6-4\sqrt{2}}\,abx}{\sqrt{a^4x^4+b^4}+a^2x^2+b^2}\right)}{3\sqrt{2}\,ab}-\frac{\tanh^{-1}\left(\frac{\sqrt{6+4\sqrt{2}}\,abx}{\sqrt{a^4x^4+b^4}+a^2x^2+b^2}\right)}{3\sqrt{2}\,ab}$$

Rubi [C] time = 3.93, antiderivative size = 340, normalized size of antiderivative = 2.22, number of steps used = 41, number of rules used = 13, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {6725, 220, 2073, 1211, 1699, 208, 6728, 1725, 1217, 1707, 1248, 725, 206}

$$-\frac{2\tan^{-1}\left(\frac{abx}{\sqrt{a^4x^4+b^4}}\right)}{3ab}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{a^4x^4+b^4}}\right)}{3\sqrt{2}\,ab}-\frac{\left(a-\sqrt{3}\sqrt{-a^2}\right)\left(a^2x^2+b^2\right)\sqrt{\frac{a^4x^4+b^4}{\left(a^2x^2+b^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{6a^2b\sqrt{a^4x^4+b^4}}-\frac{\left(\sqrt{3}\sqrt{-a^2}+a\right)\left(a^2x^2+b^2\right)\sqrt{\frac{a^4x^4+b^4}{\left(a^2x^2+b^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{6a^2b\sqrt{a^4x^4+b^4}}+\frac{\left(a^2x^2+b^2\right)\sqrt{\frac{a^4x^4+b^4}{\left(a^2x^2+b^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{3ab\sqrt{a^4x^4+b^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b^6 + a^6*x^6)/(Sqrt[b^4 + a^4*x^4]*(-b^6 + a^6*x^6)),x]
```

```
[Out] (-2*ArcTan[(a*b*x)/Sqrt[b^4 + a^4*x^4]])/(3*a*b) - ArcTanh[(Sqrt[2]*a*b*x)/
Sqrt[b^4 + a^4*x^4]]/(3*Sqrt[2]*a*b) + ((b^2 + a^2*x^2)*Sqrt[(b^4 + a^4*x^4
)/(b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2])/(3*a*b*Sqrt[b^4 + a
^4*x^4]) - ((a - Sqrt[3]*Sqrt[-a^2])*(b^2 + a^2*x^2)*Sqrt[(b^4 + a^4*x^4)/(
b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2])/(6*a^2*b*Sqrt[b^4 + a^
4*x^4]) - ((a + Sqrt[3]*Sqrt[-a^2])*(b^2 + a^2*x^2)*Sqrt[(b^4 + a^4*x^4)/(b
^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2])/(6*a^2*b*Sqrt[b^4 + a^4
*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[d,
 Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Dist[e, Int[x/((d^2 - e^
2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{b^6+a^6x^6}{\sqrt{b^4+a^4x^4}\left(-b^6+a^6x^6\right)}\,dx = \int \left(\frac{1}{\sqrt{b^4+a^4x^4}}+\frac{2b^6}{\sqrt{b^4+a^4x^4}\left(-b^6+a^6x^6\right)}\right)dx$$

$$= \left(2b^6\right)\int \frac{1}{\sqrt{b^4+a^4x^4}\left(-b^6+a^6x^6\right)}\,dx+\int \frac{1}{\sqrt{b^4+a^4x^4}}\,dx$$

$$= \frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{2ab\sqrt{b^4+a^4x^4}}+\left(2b^6\right)\int \left(-\frac{1}{3b^4\left(b^2-a^2x^2\right)\sqrt{b^4+}}\right.$$

$$= \frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{2ab\sqrt{b^4+a^4x^4}}+\frac{1}{3}b\int \frac{-2b+ax}{\left(b^2-abx+a^2x^2\right)\sqrt{b^4+a^4}}$$

$$= \frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{2ab\sqrt{b^4+a^4x^4}}-\frac{1}{3}\int \frac{1}{\sqrt{b^4+a^4x^4}}\,dx-\frac{1}{3}\int \frac{}{\left(b^2-\right.}$$

$$= \frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{3ab\sqrt{b^4+a^4x^4}}-\frac{1}{3}\left(\left(a-\sqrt{3}\sqrt{-a^2}\right)b\right)\int \frac{}{\left(ab-\sqrt{\right.}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}+\frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{3ab\sqrt{b^4+a^4x^4}}+\frac{1}{3}\left(2a^2\left(a\right.\right.$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}+\frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{3ab\sqrt{b^4+a^4x^4}}-2\frac{\left(a-\sqrt{3}\right.}{}$$

$$= -\frac{2\tan^{-1}\left(\frac{abx}{\sqrt{b^4+a^4x^4}}\right)}{3ab}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}+\frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\right.}{3ab\sqrt{b^4+a^4x^4}}$$

$$= -\frac{2\tan^{-1}\left(\frac{abx}{\sqrt{b^4+a^4x^4}}\right)}{3ab}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}+\frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\right.}{3ab\sqrt{b^4+a^4x^4}}$$

Mathematica [C] time = 0.71, size = 268, normalized size = 1.75

$$-\frac{i\sqrt{\frac{a^4x^4}{b^4}+1}\left(3\sqrt[4]{a^4}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}\,x\right)\middle|-1\right)-2\sqrt[4]{a^4}\,\Pi\left(-\frac{i}{2}-\frac{\sqrt{3}}{2};i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}\,x\right)\middle|-1\right)-2\sqrt[4]{a^4}\,\Pi\left(\frac{1}{2}\left(-i+\sqrt{3}\right);i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}\,x\right)\middle|-1\right)-(1-i)\sqrt{2}\sqrt[4]{b^4}\sqrt{\frac{ia^2}{b^2}}\,\Pi\left(\frac{i\sqrt{a^4}\sqrt{b^4}}{a^2b^2};i\sinh^{-1}\left(\frac{(1+i)\sqrt[4]{a^4}\,x}{\sqrt{2}\sqrt[4]{b^4}}\right)\middle|-1\right)\right)}{3\sqrt[4]{a^4}\sqrt{\frac{ia^2}{b^2}}\sqrt{a^4x^4+b^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^6 + a^6*x^6)/(Sqrt[b^4 + a^4*x^4]*(-b^6 + a^6*x^6)),x]
```

```
[Out] ((-1/3*I)*Sqrt[1 + (a^4*x^4)/b^4]*(3*(a^4)^(1/4)*EllipticF[I*ArcSinh[Sqrt[(
I*a^2)/b^2]*x], -1] - 2*(a^4)^(1/4)*EllipticPi[-1/2*I - Sqrt[3]/2, I*ArcSin
h[Sqrt[(I*a^2)/b^2]*x], -1] - 2*(a^4)^(1/4)*EllipticPi[(-I + Sqrt[3])/2, I*
```

```
ArcSinh[Sqrt[(I*a^2)/b^2]*x], -1] - (1 - I)*Sqrt[2]*Sqrt[(I*a^2)/b^2]*(b^4)
^(1/4)*EllipticPi[(I*Sqrt[a^4]*Sqrt[b^4])/(a^2*b^2), I*ArcSinh[((1 + I)*(a^
4)^(1/4)*x)/(Sqrt[2]*(b^4)^(1/4))], -1]))/((a^4)^(1/4)*Sqrt[(I*a^2)/b^2]*Sq
rt[b^4 + a^4*x^4])
```

IntegrateAlgebraic [A] time = 1.40, size = 71, normalized size = 0.46

$$-\frac{2\tan^{-1}\left(\frac{abx}{\sqrt{a^4x^4+b^4}}\right)}{3ab}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{a^4x^4+b^4}}\right)}{3\sqrt{2}\,ab}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^6 + a^6*x^6)/(Sqrt[b^4 + a^4*x^4]*(-b^6 + a^6*x^6)),x
]
```

```
[Out] (-2*ArcTan[(a*b*x)/Sqrt[b^4 + a^4*x^4]])/(3*a*b) - ArcTanh[(Sqrt[2]*a*b*x)/
Sqrt[b^4 + a^4*x^4]]/(3*Sqrt[2]*a*b)
```

fricas [A] time = 0.55, size = 127, normalized size = 0.83

$$\frac{\sqrt{2}\log\left(\frac{a^4x^4+2\,a^2b^2x^2+b^4-2\,\sqrt{2}\,\sqrt{a^4x^4+b^4}\,abx}{a^4x^4-2\,a^2b^2x^2+b^4}\right)-4\,\arctan\left(\frac{2\,\sqrt{a^4x^4+b^4}\,abx}{a^4x^4-a^2b^2x^2+b^4}\right)}{12\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^6*x^6+b^6)/(a^4*x^4+b^4)^(1/2)/(a^6*x^6-b^6),x, algorithm="fri
cas")
```

```
[Out] 1/12*(sqrt(2)*log((a^4*x^4 + 2*a^2*b^2*x^2 + b^4 - 2*sqrt(2)*sqrt(a^4*x^4 +
 b^4)*a*b*x)/(a^4*x^4 - 2*a^2*b^2*x^2 + b^4)) - 4*arctan(2*sqrt(a^4*x^4 + b
^4)*a*b*x/(a^4*x^4 - a^2*b^2*x^2 + b^4)))/(a*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a^6x^6+b^6}{\left(a^6x^6-b^6\right)\sqrt{a^4x^4+b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^6*x^6+b^6)/(a^4*x^4+b^4)^(1/2)/(a^6*x^6-b^6),x, algorithm="gia
c")
```

```
[Out] integrate((a^6*x^6 + b^6)/((a^6*x^6 - b^6)*sqrt(a^4*x^4 + b^4)), x)
```

maple [C] time = 0.05, size = 825, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^6*x^6+b^6)/(a^4*x^4+b^4)^(1/2)/(a^6*x^6-b^6),x)
```

```
[Out] 1/(I*a^2/b^2)^(1/2)*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/b^2*x^2)^(1/2)/(a^4*x^
4+b^4)^(1/2)*EllipticF(x*(I*a^2/b^2)^(1/2),I)+1/6*b/a*sum((-_alpha*a-2*b)/(
2*_alpha*a+b)*(1/(b^3*(_alpha*a+b))^(1/2)*arctanh((_alpha*a+b)*a*b*(a*x^2+_
alpha*b)/(b^3*(_alpha*a+b))^(1/2)/(a^4*x^4+b^4)^(1/2))+2/(I*a^2/b^2)^(1/2)*
a*(_alpha*a+b)/b^2*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/b^2*x^2)^(1/2)/(a^4*x^4
+b^4)^(1/2)*EllipticPi(x*(I*a^2/b^2)^(1/2),-I*_alpha*a/b,(-I*a^2/b^2)^(1/2)
/(I*a^2/b^2)^(1/2))),_alpha=RootOf(_Z^2*a^2+_Z*a*b+b^2))+1/3*b/a*(-1/4*2^(1
/2)/(b^4)^(1/2)*arctanh(1/4*(2*a^2*b^2*x^2+2*b^4)*2^(1/2)/(b^4)^(1/2)/(a^4*
x^4+b^4)^(1/2))-1/(I*a^2/b^2)^(1/2)/b*a*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/b^
```

```
2*x^2)^(1/2)/(a^4*x^4+b^4)^(1/2)*EllipticPi(x*(I*a^2/b^2)^(1/2),-I,(-I*a^2/
b^2)^(1/2)/(I*a^2/b^2)^(1/2)))-1/6*b/a*sum((-_alpha*a+2*b)/(2*_alpha*a-b)*(
-1/(-b^3*(_alpha*a-b))^(1/2)*arctanh((_alpha*a-b)*a*b*(a*x^2-_alpha*b)/(-b^
3*(_alpha*a-b))^(1/2)/(a^4*x^4+b^4)^(1/2))+2/(I*a^2/b^2)^(1/2)*a*(_alpha*a-
b)/b^2*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/b^2*x^2)^(1/2)/(a^4*x^4+b^4)^(1/2)*
EllipticPi(x*(I*a^2/b^2)^(1/2),I*_alpha*a/b,(-I*a^2/b^2)^(1/2)/(I*a^2/b^2)^
(1/2))),_alpha=RootOf(_Z^2*a^2-_Z*a*b+b^2))-1/3*b/a*(-1/4*2^(1/2)/(b^4)^(1/
2)*arctanh(1/4*(2*a^2*b^2*x^2+2*b^4)*2^(1/2)/(b^4)^(1/2)/(a^4*x^4+b^4)^(1/2
))+1/(I*a^2/b^2)^(1/2)/b*a*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/b^2*x^2)^(1/2)/
(a^4*x^4+b^4)^(1/2)*EllipticPi(x*(I*a^2/b^2)^(1/2),-I,(-I*a^2/b^2)^(1/2)/(I
*a^2/b^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^6x^6+b^6}{\left(a^6x^6-b^6\right)\sqrt{a^4x^4+b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^6*x^6+b^6)/(a^4*x^4+b^4)^(1/2)/(a^6*x^6-b^6),x, algorithm="max
ima")
```

```
[Out] integrate((a^6*x^6 + b^6)/((a^6*x^6 - b^6)*sqrt(a^4*x^4 + b^4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a^6\,x^6+b^6}{\sqrt{a^4\,x^4+b^4}\,\left(b^6-a^6\,x^6\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^6 + a^6*x^6)/((b^4 + a^4*x^4)^(1/2)*(b^6 - a^6*x^6)),x)
```

```
[Out] int(-(b^6 + a^6*x^6)/((b^4 + a^4*x^4)^(1/2)*(b^6 - a^6*x^6)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2+b^2\right)\left(a^4x^4-a^2b^2x^2+b^4\right)}{(ax-b)\,(ax+b)\,\sqrt{a^4x^4+b^4}\,\left(a^2x^2-abx+b^2\right)\left(a^2x^2+abx+b^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**6*x**6+b**6)/(a**4*x**4+b**4)**(1/2)/(a**6*x**6-b**6),x)
```

```
[Out] Integral((a**2*x**2 + b**2)*(a**4*x**4 - a**2*b**2*x**2 + b**4)/((a*x - b)*
(a*x + b)*sqrt(a**4*x**4 + b**4)*(a**2*x**2 - a*b*x + b**2)*(a**2*x**2 + a*
b*x + b**2)), x)
```

3.1761 $\int \frac{\sqrt{-1+x^2}\sqrt{x^2+x\sqrt{-1+x^2}}}{1+x}\,dx$

Optimal. Leaf size=153

$$\frac{1}{4}\sqrt{x^2+\sqrt{x^2-1}\,x}\,(-x-4)+\frac{3}{4}\sqrt{x^2-1}\,\sqrt{x^2+\sqrt{x^2-1}\,x}-\frac{3\log\left(\sqrt{x^2-1}-\sqrt{2}\sqrt{x^2+\sqrt{x^2-1}\,x}+x\right)}{4\sqrt{2}}-\sqrt{2}\tanh$$

Rubi [F] time = 0.37, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-1+x^2}\sqrt{x^2+x\sqrt{-1+x^2}}}{1+x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-1 + x^2]*Sqrt[x^2 + x*Sqrt[-1 + x^2]])/(1 + x),x]
```

```
[Out] Defer[Int][(Sqrt[-1 + x^2]*Sqrt[x^2 + x*Sqrt[-1 + x^2]])/(1 + x), x]
```

Rubi steps

$$\int \frac{\sqrt{-1+x^2}\sqrt{x^2+x\sqrt{-1+x^2}}}{1+x}\,dx=\int \frac{\sqrt{-1+x^2}\sqrt{x^2+x\sqrt{-1+x^2}}}{1+x}\,dx$$

Mathematica [A] time = 0.73, size = 150, normalized size = 0.98

$$\frac{\sqrt{x^2-1}\left(\sqrt{2}\sqrt{x\left(\sqrt{x^2-1}+x\right)}\left(2x^2+2\left(\sqrt{x^2-1}-2\right)x-4\sqrt{x^2-1}-3\right)+3\left(\sqrt{x^2-1}+x\right)\sinh^{-1}\left(\sqrt{x^2-1}+x\right)+4\left(\sqrt{x^2-1}+x\right)\tanh^{-1}\left(\sqrt{\left(\sqrt{x^2-1}+x\right)^2+1}\right)\right)}{4\sqrt{2}\left(x^2+\sqrt{x^2-1}\,x-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^2]*Sqrt[x^2 + x*Sqrt[-1 + x^2]])/(1 + x),x]
```

```
[Out] (Sqrt[-1 + x^2]*(Sqrt[2]*Sqrt[x*(x + Sqrt[-1 + x^2])]*(-3 + 2*x^2 - 4*Sqrt[
-1 + x^2] + 2*x*(-2 + Sqrt[-1 + x^2])) + 3*(x + Sqrt[-1 + x^2])*ArcSinh[x +
 Sqrt[-1 + x^2]] + 4*(x + Sqrt[-1 + x^2])*ArcTanh[Sqrt[1 + (x + Sqrt[-1 + x
^2])^2]]))/(4*Sqrt[2]*(-1 + x^2 + x*Sqrt[-1 + x^2]))
```

IntegrateAlgebraic [A] time = 2.42, size = 153, normalized size = 1.00

$$\frac{1}{4}\sqrt{x^2+\sqrt{x^2-1}\,x}\,(-x-4)+\frac{3}{4}\sqrt{x^2-1}\,\sqrt{x^2+\sqrt{x^2-1}\,x}-\frac{3\log\left(\sqrt{x^2-1}-\sqrt{2}\sqrt{x^2+\sqrt{x^2-1}\,x}+x\right)}{4\sqrt{2}}-\sqrt{2}\tanh^{-1}\left(\sqrt{x^2-1}-\sqrt{2}\sqrt{x^2+\sqrt{x^2-1}\,x}+x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^2]*Sqrt[x^2 + x*Sqrt[-1 + x^2]])/(1 + x),x]
```

```
[Out] ((-4 - x)*Sqrt[x^2 + x*Sqrt[-1 + x^2]])/4 + (3*Sqrt[-1 + x^2]*Sqrt[x^2 + x*
Sqrt[-1 + x^2]])/4 - Sqrt[2]*ArcTanh[x + Sqrt[-1 + x^2] - Sqrt[2]*Sqrt[x^2
+ x*Sqrt[-1 + x^2]]] - (3*Log[x + Sqrt[-1 + x^2] - Sqrt[2]*Sqrt[x^2 + x*Sqr
t[-1 + x^2]]])/(4*Sqrt[2])
```

fricas [A] time = 3.61, size = 153, normalized size = 1.00

$$-\frac{1}{4}\sqrt{x^2+\sqrt{x^2-1}\,x}\left(x-3\sqrt{x^2-1}+4\right)+\frac{1}{4}\sqrt{2}\log\left(4x^2-2\left(2\sqrt{2}\sqrt{x^2-1}\,x-\sqrt{2}\left(2x^2-1\right)\right)\sqrt{x^2+\sqrt{x^2-1}\,x}-4\sqrt{x^2-1}\,x-1\right)+\frac{3}{16}\sqrt{2}\log\left(-4x^2-2\sqrt{x^2+\sqrt{x^2-1}\,x}\left(\sqrt{2}x+\sqrt{2}\sqrt{x^2-1}\right)-4\sqrt{x^2-1}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)*(x^2+x*(x^2-1)^(1/2))^(1/2)/(1+x),x, algorithm="fri
cas")
```

```
[Out] -1/4*sqrt(x^2 + sqrt(x^2 - 1)*x)*(x - 3*sqrt(x^2 - 1) + 4) + 1/4*sqrt(2)*lo
g(4*x^2 - 2*(2*sqrt(2)*sqrt(x^2 - 1)*x - sqrt(2)*(2*x^2 - 1))*sqrt(x^2 + sq
rt(x^2 - 1)*x) - 4*sqrt(x^2 - 1)*x - 1) + 3/16*sqrt(2)*log(-4*x^2 - 2*sqrt(
x^2 + sqrt(x^2 - 1)*x)*(sqrt(2)*x + sqrt(2)*sqrt(x^2 - 1)) - 4*sqrt(x^2 - 1
)*x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^2-1}\,x}\,\sqrt{x^2-1}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)*(x^2+x*(x^2-1)^(1/2))^(1/2)/(1+x),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^2 - 1)*x)*sqrt(x^2 - 1)/(x + 1), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-1}\,\sqrt{x^2+x\sqrt{x^2-1}}}{1+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(1/2)*(x^2+x*(x^2-1)^(1/2))^(1/2)/(1+x),x)
```

```
[Out] int((x^2-1)^(1/2)*(x^2+x*(x^2-1)^(1/2))^(1/2)/(1+x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^2-1}\,x}\,\sqrt{x^2-1}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)*(x^2+x*(x^2-1)^(1/2))^(1/2)/(1+x),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^2 - 1)*x)*sqrt(x^2 - 1)/(x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^2-1}\,\sqrt{x\sqrt{x^2-1}+x^2}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)^(1/2)*(x*(x^2 - 1)^(1/2) + x^2)^(1/2))/(x + 1),x)
```

```
[Out] int(((x^2 - 1)^(1/2)*(x*(x^2 - 1)^(1/2) + x^2)^(1/2))/(x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(x+\sqrt{x^2-1}\right)}\,\sqrt{(x-1)(x+1)}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(1/2)*(x**2+x*(x**2-1)**(1/2))**(1/2)/(1+x),x)
```

```
[Out] Integral(sqrt(x*(x + sqrt(x**2 - 1)))*sqrt((x - 1)*(x + 1))/(x + 1), x)
```

3.1762 $\int \frac{(1+x^4)\sqrt{1+\sqrt{1+x^2}}}{-1+x^4}\,dx$

Optimal. Leaf size=153

$$\frac{2\sqrt{x^2+1}\,x}{3\sqrt{\sqrt{x^2+1}+1}}+\frac{4x}{3\sqrt{\sqrt{x^2+1}+1}}-2\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^2+1}+1}}\right)+\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt{2}}\,\sqrt{\sqrt{x^2+1}+1}}\right)-\sqrt{1}$$

Rubi [F] time = 0.79, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+x^4)\sqrt{1+\sqrt{1+x^2}}}{-1+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^4)*Sqrt[1 + Sqrt[1 + x^2]])/(-1 + x^4),x]
```

```
[Out] (2*x^3)/(3*(1 + Sqrt[1 + x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 + x^2]] - (I/
2)*Defer[Int][Sqrt[1 + Sqrt[1 + x^2]]/(I - x), x] - Defer[Int][Sqrt[1 + Sqr
t[1 + x^2]]/(1 - x), x]/2 - (I/2)*Defer[Int][Sqrt[1 + Sqrt[1 + x^2]]/(I + x
), x] - Defer[Int][Sqrt[1 + Sqrt[1 + x^2]]/(1 + x), x]/2
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^4)\sqrt{1+\sqrt{1+x^2}}}{-1+x^4}\,dx &= \int \left(\sqrt{1+\sqrt{1+x^2}}+\frac{2\sqrt{1+\sqrt{1+x^2}}}{-1+x^4}\right)dx\\
&= 2\int \frac{\sqrt{1+\sqrt{1+x^2}}}{-1+x^4}\,dx+\int \sqrt{1+\sqrt{1+x^2}}\,dx\\
&= \frac{2x^3}{3\left(1+\sqrt{1+x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1+x^2}}}+2\int \left(-\frac{\sqrt{1+\sqrt{1+x^2}}}{2(1-x^2)}-\frac{\sqrt{1+\sqrt{1+x^2}}}{2(1+x^2)}\right.\\
&= \frac{2x^3}{3\left(1+\sqrt{1+x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1+x^2}}}-\int \frac{\sqrt{1+\sqrt{1+x^2}}}{1-x^2}\,dx-\int \frac{\sqrt{1+\sqrt{1+x}}}{1+x^2}\\
&= \frac{2x^3}{3\left(1+\sqrt{1+x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1+x^2}}}-\int \left(\frac{i\sqrt{1+\sqrt{1+x^2}}}{2(i-x)}+\frac{i\sqrt{1+\sqrt{1+x^2}}}{2(i+x)}\right)\\
&= \frac{2x^3}{3\left(1+\sqrt{1+x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1+x^2}}}-\frac{1}{2}i\int \frac{\sqrt{1+\sqrt{1+x^2}}}{i-x}\,dx-\frac{1}{2}i\int \frac{\sqrt{1+\sqrt{}}}{i+}
\end{aligned}$$

Mathematica [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(1+x^4)\sqrt{1+\sqrt{1+x^2}}}{-1+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^4)*Sqrt[1 + Sqrt[1 + x^2]])/(-1 + x^4),x]
```

```
[Out] Integrate[((1 + x^4)*Sqrt[1 + Sqrt[1 + x^2]])/(-1 + x^4), x]
```

IntegrateAlgebraic [A] time = 0.27, size = 153, normalized size = 1.00

$$\frac{2\sqrt{x^2+1}\,x}{3\sqrt{\sqrt{x^2+1}+1}}+\frac{4x}{3\sqrt{\sqrt{x^2+1}+1}}-2\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^2+1}+1}}\right)+\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{2}-1}\,x}{\sqrt{\sqrt{x^2+1}+1}}\right)-\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{2}}\,x}{\sqrt{\sqrt{x^2+1}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^4)*Sqrt[1 + Sqrt[1 + x^2]])/(-1 + x^4),x]
```

```
[Out] (4*x)/(3*Sqrt[1 + Sqrt[1 + x^2]]) + (2*x*Sqrt[1 + x^2])/(3*Sqrt[1 + Sqrt[1
+ x^2]]) - 2*ArcTan[x/Sqrt[1 + Sqrt[1 + x^2]]] + Sqrt[-1 + Sqrt[2]]*ArcTan[
(Sqrt[-1 + Sqrt[2]]*x)/Sqrt[1 + Sqrt[1 + x^2]]] - Sqrt[1 + Sqrt[2]]*ArcTanh
[(Sqrt[1 + Sqrt[2]]*x)/Sqrt[1 + Sqrt[1 + x^2]]]
```

fricas [B] time = 10.95, size = 499, normalized size = 3.26

$$-\frac{12\,x\sqrt{\sqrt{2}-1}\,\arctan\left(\frac{\left(51\,x^{5}-222\,x^{3}-2\,\sqrt{2}\left(5\,x^{5}-163\,x^{3}-46\,x\right)+2\left(31\,x^{3}+\sqrt{2}\left(41\,x^{3}-81\,x\right)+173\,x\right)\sqrt{x^{2}+1}+11\,x\right)\sqrt{3821\,\sqrt{2}+4841}\,\sqrt{\sqrt{2}-1}+4802\left(x^{4}+\sqrt{2}\left(x^{4}-3\,x^{2}-2\right)+\left(3\,x^{2}+2\,\sqrt{2}\left(x^{2}+1\right)+1\right)\sqrt{x^{2}+1}-1\right)\sqrt{\sqrt{2}-1}\,\sqrt{\sqrt{x^{2}+1}+1}}{2401\left(x^{5}-10\,x^{3}-7\,x\right)}\right)+3\,x\sqrt{\sqrt{2}+1}\,\log\left(-\frac{\left(71\,x^{3}-\sqrt{2}\left(61\,x^{3}+325\,x\right)+2\,\sqrt{x^{2}+1}\left(132\,\sqrt{2}\,x-193\,x\right)+457\,x\right)\sqrt{\sqrt{2}+1}+2\left(71\,x^{2}-\sqrt{2}\left(61\,x^{2}+132\right)+\sqrt{x^{2}+1}\left(132\,\sqrt{2}-193\right)+193\right)\sqrt{\sqrt{x^{2}+1}+1}}{x^{3}-x}\right)-3\,x\sqrt{\sqrt{2}+1}\,\log\left(\frac{\left(71\,x^{3}-\sqrt{2}\left(61\,x^{3}+325\,x\right)+2\,\sqrt{x^{2}+1}\left(132\,\sqrt{2}\,x-193\,x\right)+457\,x\right)\sqrt{\sqrt{2}+1}-2\left(71\,x^{2}-\sqrt{2}\left(61\,x^{2}+132\right)+\sqrt{x^{2}+1}\left(132\,\sqrt{2}-193\right)+193\right)\sqrt{\sqrt{x^{2}+1}+1}}{x^{3}-x}\right)-6\,x\arctan\left(\frac{4\left(x^{4}-12\,x^{2}+\left(5\,x^{2}-3\right)\sqrt{x^{2}+1}+3\right)\sqrt{\sqrt{x^{2}+1}+1}}{x^{5}-46\,x^{3}+17\,x}\right)-8\left(x^{2}+\sqrt{x^{2}+1}-1\right)\sqrt{\sqrt{x^{2}+1}+1}}{12\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(1+(x^2+1)^(1/2))^(1/2)/(x^4-1),x, algorithm="fricas")
```

```
[Out] -1/12*(12*x*sqrt(sqrt(2) - 1)*arctan(1/2401*((51*x^5 - 222*x^3 - 2*sqrt(2)*
(5*x^5 - 163*x^3 - 46*x) + 2*(31*x^3 + sqrt(2)*(41*x^3 - 81*x) + 173*x)*sqr
t(x^2 + 1) + 11*x)*sqrt(3821*sqrt(2) + 4841)*sqrt(sqrt(2) - 1) + 4802*(x^4
+ sqrt(2)*(x^4 - 3*x^2 - 2) + (3*x^2 + 2*sqrt(2)*(x^2 + 1) + 1)*sqrt(x^2 +
1) - 1)*sqrt(sqrt(2) - 1)*sqrt(sqrt(x^2 + 1) + 1))/(x^5 - 10*x^3 - 7*x)) +
3*x*sqrt(sqrt(2) + 1)*log(-((71*x^3 - sqrt(2)*(61*x^3 + 325*x) + 2*sqrt(x^2
 + 1)*(132*sqrt(2)*x - 193*x) + 457*x)*sqrt(sqrt(2) + 1) + 2*(71*x^2 - sqrt
(2)*(61*x^2 + 132) + sqrt(x^2 + 1)*(132*sqrt(2) - 193) + 193)*sqrt(sqrt(x^2
 + 1) + 1))/(x^3 - x)) - 3*x*sqrt(sqrt(2) + 1)*log(((71*x^3 - sqrt(2)*(61*x
^3 + 325*x) + 2*sqrt(x^2 + 1)*(132*sqrt(2)*x - 193*x) + 457*x)*sqrt(sqrt(2)
 + 1) - 2*(71*x^2 - sqrt(2)*(61*x^2 + 132) + sqrt(x^2 + 1)*(132*sqrt(2) - 1
93) + 193)*sqrt(sqrt(x^2 + 1) + 1))/(x^3 - x)) - 6*x*arctan(4*(x^4 - 12*x^2
 + (5*x^2 - 3)*sqrt(x^2 + 1) + 3)*sqrt(sqrt(x^2 + 1) + 1)/(x^5 - 46*x^3 + 1
7*x)) - 8*(x^2 + sqrt(x^2 + 1) - 1)*sqrt(sqrt(x^2 + 1) + 1))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)\sqrt{\sqrt{x^2+1}+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(1+(x^2+1)^(1/2))^(1/2)/(x^4-1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)*sqrt(sqrt(x^2 + 1) + 1)/(x^4 - 1), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)\sqrt{1+\sqrt{x^2+1}}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)*(1+(x^2+1)^(1/2))^(1/2)/(x^4-1),x)
```

```
[Out] int((x^4+1)*(1+(x^2+1)^(1/2))^(1/2)/(x^4-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4+1)\sqrt{\sqrt{x^2+1}+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(1+(x^2+1)^(1/2))^(1/2)/(x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)*sqrt(sqrt(x^2 + 1) + 1)/(x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^4+1)\,\sqrt{\sqrt{x^2+1}+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)*((x^2 + 1)^(1/2) + 1)^(1/2))/(x^4 - 1),x)
```

```
[Out] int(((x^4 + 1)*((x^2 + 1)^(1/2) + 1)^(1/2))/(x^4 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4+1)\,\sqrt{\sqrt{x^2+1}+1}}{(x-1)\,(x+1)\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)*(1+(x**2+1)**(1/2))**(1/2)/(x**4-1),x)
```

```
[Out] Integral((x**4 + 1)*sqrt(sqrt(x**2 + 1) + 1)/((x - 1)*(x + 1)*(x**2 + 1)),
x)
```

3.1763 $\int \frac{-b^2+ax^2}{(b^2+ax^2)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$

Optimal. Leaf size=153

$$\frac{2x}{\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{4\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}+\frac{2\sqrt{2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{a}}$$

Rubi [F] time = 1.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-b^2+ax^2}{(b^2+ax^2)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-b^2 + a*x^2)/((b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

```
[Out] Defer[Int][1/Sqrt[b + Sqrt[b^2 + a*x^2]], x] - b*Defer[Int][1/((b - Sqrt[-a
]*x)*Sqrt[b + Sqrt[b^2 + a*x^2]]), x] - b*Defer[Int][1/((b + Sqrt[-a]*x)*Sq
rt[b + Sqrt[b^2 + a*x^2]]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-b^2+ax^2}{(b^2+ax^2)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx &= \int \left(\frac{1}{\sqrt{b+\sqrt{b^2+ax^2}}}-\frac{2b^2}{(b^2+ax^2)\sqrt{b+\sqrt{b^2+ax^2}}}\right)dx\\
&= -\left((2b^2)\int \frac{1}{(b^2+ax^2)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx\right)+\int \frac{1}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx\\
&= -\left((2b^2)\int \left(\frac{1}{2b\left(b-\sqrt{-a}\,x\right)\sqrt{b+\sqrt{b^2+ax^2}}}+\frac{1}{2b\left(b+\sqrt{-a}\,x\right)\sqrt{b+\sqrt{}}}\right.\right.\\
&= -\left(b\int \frac{1}{\left(b-\sqrt{-a}\,x\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx\right)-b\int \frac{1}{\left(b+\sqrt{-a}\,x\right)\sqrt{b+\sqrt{b^2}}}
\end{aligned}$$

Mathematica [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{-b^2+ax^2}{(b^2+ax^2)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-b^2 + a*x^2)/((b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

```
[Out] Integrate[(-b^2 + a*x^2)/((b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]]), x]
```

IntegrateAlgebraic [A] time = 0.25, size = 120, normalized size = 0.78

$$\frac{2x}{\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{4\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b^2 + a*x^2)/((b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]
]),x]
```

```
[Out] (2*x)/Sqrt[b + Sqrt[b^2 + a*x^2]] - (4*Sqrt[b]*ArcTan[(Sqrt[a]*x)/(Sqrt[b]*
Sqrt[b + Sqrt[b^2 + a*x^2]])])/Sqrt[a] + (Sqrt[2]*Sqrt[b]*ArcTan[(Sqrt[a]*x
)/(Sqrt[2]*Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/Sqrt[a]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b^2)/(a*x^2+b^2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2-b^2}{\left(ax^2+b^2\right)\sqrt{b+\sqrt{ax^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b^2)/(a*x^2+b^2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x^2 - b^2)/((a*x^2 + b^2)*sqrt(b + sqrt(a*x^2 + b^2))), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2-b^2}{\left(a\,x^2+b^2\right)\sqrt{b+\sqrt{a\,x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b^2)/(a*x^2+b^2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x^2-b^2)/(a*x^2+b^2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2-b^2}{\left(ax^2+b^2\right)\sqrt{b+\sqrt{ax^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b^2)/(a*x^2+b^2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x^2 - b^2)/((a*x^2 + b^2)*sqrt(b + sqrt(a*x^2 + b^2))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x^2-b^2}{\left(b^2+a\,x^2\right)\,\sqrt{b+\sqrt{b^2+a\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 - b^2)/((a*x^2 + b^2)*(b + (a*x^2 + b^2)^(1/2))^(1/2)),x)
[Out] int((a*x^2 - b^2)/((a*x^2 + b^2)*(b + (a*x^2 + b^2)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2-b^2}{\sqrt{b+\sqrt{ax^2+b^2}}\,\left(ax^2+b^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b**2)/(a*x**2+b**2)/(b+(a*x**2+b**2)**(1/2))**(1/2),x)
[Out] Integral((a*x**2 - b**2)/(sqrt(b + sqrt(a*x**2 + b**2))*(a*x**2 + b**2)), x
)
```

3.1764 $\int \frac{x}{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=153

$$\frac{-12x^3-3x^2+\sqrt{x^2+1}\left(-12x^2+\left(8x^2+12x-4\right)\sqrt{\sqrt{x^2+1}+x}-3x\right)+\left(8x^3+12x^2+4\right)\sqrt{\sqrt{x^2+1}+x}-6x}{24\sqrt{x^2+1}\,x+12\left(2x^2+1\right)}$$

Rubi [A] time = 0.22, antiderivative size = 109, normalized size of antiderivative = 0.71, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6742, 195, 215, 2117, 14, 2119, 448, 2122, 270}

$$\frac{x^2}{4}-\frac{1}{4}\sqrt{x^2+1}\,x+\frac{1}{6}\left(\sqrt{x^2+1}+x\right)^{3/2}+\frac{1}{2}\sqrt{\sqrt{x^2+1}+x}-\frac{1}{2\sqrt{\sqrt{x^2+1}+x}}-\frac{1}{6\left(\sqrt{x^2+1}+x\right)^{3/2}}-\frac{x}{2}-\frac{1}{4}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/2*x + x^2/4 - (x*Sqrt[1 + x^2])/4 - 1/(6*(x + Sqrt[1 + x^2])^(3/2)) - 1/
(2*Sqrt[x + Sqrt[1 + x^2]]) + Sqrt[x + Sqrt[1 + x^2]]/2 + (x + Sqrt[1 + x^2
])^(3/2)/6 - ArcSinh[x]/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
```

```
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x}{1+\sqrt{x+\sqrt{1+x^2}}}\,dx = \int \left(-\frac{1}{2}+\frac{x}{2}-\frac{\sqrt{1+x^2}}{2}+\frac{1}{2}\sqrt{x+\sqrt{1+x^2}}-\frac{1}{2}x\sqrt{x+\sqrt{1+x^2}}+\frac{1}{2}\sqrt{1+x^2}\sqrt{x+\right.$$

$$= -\frac{x}{2}+\frac{x^2}{4}-\frac{1}{2}\int \sqrt{1+x^2}\,dx+\frac{1}{2}\int \sqrt{x+\sqrt{1+x^2}}\,dx-\frac{1}{2}\int x\sqrt{x+\sqrt{1+x^2}}\,dx$$

$$= -\frac{x}{2}+\frac{x^2}{4}-\frac{1}{4}x\sqrt{1+x^2}-\frac{1}{8}\text{Subst}\left(\int \frac{(-1+x^2)(1+x^2)}{x^{5/2}}\,dx,x,x+\sqrt{1+x^2}\right)+\frac{1}{8}$$

$$= -\frac{x}{2}+\frac{x^2}{4}-\frac{1}{4}x\sqrt{1+x^2}-\frac{1}{4}\sinh^{-1}(x)-\frac{1}{8}\text{Subst}\left(\int \left(-\frac{1}{x^{5/2}}+x^{3/2}\right)dx,x,x+\sqrt{1\right.$$

$$= -\frac{x}{2}+\frac{x^2}{4}-\frac{1}{4}x\sqrt{1+x^2}-\frac{1}{6\left(x+\sqrt{1+x^2}\right)^{3/2}}-\frac{1}{2\sqrt{x+\sqrt{1+x^2}}}+\frac{1}{2}\sqrt{x+\sqrt{1+x}}$$

Mathematica [A] time = 1.06, size = 217, normalized size = 1.42

$$\frac{1}{60}\left(15x^2-15\sqrt{x^2+1}\,x-20\left(\sqrt{x^2+1}-2x\right)\sqrt{\sqrt{x^2+1}+x}-\frac{4\sqrt{x^2+1}\left(6x^4+6x^2+3\sqrt{x^2+1}\,x+6\sqrt{x^2+1}\,x^3+2\right)}{\sqrt{\sqrt{x^2+1}+x}\left(x^2+\sqrt{x^2+1}\,x+1\right)}+\frac{4\sqrt{\sqrt{x^2+1}+x}\left(6x^4+21x^2+18\sqrt{x^2+1}\,x+6\sqrt{x^2+1}\,x^3+7\right)}{2x^2+2\sqrt{x^2+1}\,x+1}-30x-15\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] (-30*x + 15*x^2 - 15*x*Sqrt[1 + x^2] - 20*(-2*x + Sqrt[1 + x^2])*Sqrt[x + S
qrt[1 + x^2]] - (4*Sqrt[1 + x^2]*(2 + 6*x^2 + 6*x^4 + 3*x*Sqrt[1 + x^2] + 6
*x^3*Sqrt[1 + x^2]))/(Sqrt[x + Sqrt[1 + x^2]]*(1 + x^2 + x*Sqrt[1 + x^2]))
 + (4*Sqrt[x + Sqrt[1 + x^2]]*(7 + 21*x^2 + 6*x^4 + 18*x*Sqrt[1 + x^2] + 6*x
^3*Sqrt[1 + x^2]))/(1 + 2*x^2 + 2*x*Sqrt[1 + x^2]) - 15*ArcSinh[x])/60
```

IntegrateAlgebraic [A] time = 0.27, size = 153, normalized size = 1.00

$$\frac{-12x^3-3x^2+\sqrt{x^2+1}\left(-12x^2+\left(8x^2+12x-4\right)\sqrt{\sqrt{x^2+1}+x}-3x\right)+\left(8x^3+12x^2+4\right)\sqrt{\sqrt{x^2+1}+x}-6x}{24\sqrt{x^2+1}\,x+12\left(2x^2+1\right)}-\tanh^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}-1}{\sqrt{\sqrt{x^2+1}+x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(1 + Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] (-6*x - 3*x^2 - 12*x^3 + (4 + 12*x^2 + 8*x^3)*Sqrt[x + Sqrt[1 + x^2]] + Sqr
t[1 + x^2]*(-3*x - 12*x^2 + (-4 + 12*x + 8*x^2)*Sqrt[x + Sqrt[1 + x^2]]))/(
24*x*Sqrt[1 + x^2] + 12*(1 + 2*x^2)) - ArcTanh[(-1 + Sqrt[x + Sqrt[1 + x^2]
])/(1 + Sqrt[x + Sqrt[1 + x^2]])]
```

fricas [A] time = 0.42, size = 66, normalized size = 0.43

$$\frac{1}{4}x^2-\frac{1}{3}\left(x^2-\sqrt{x^2+1}\,(x-1)-2\,x-1\right)\sqrt{x+\sqrt{x^2+1}}-\frac{1}{4}\sqrt{x^2+1}\,x-\frac{1}{2}\,x-\frac{1}{2}\log\left(\sqrt{x+\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*x^2 - 1/3*(x^2 - sqrt(x^2 + 1)*(x - 1) - 2*x - 1)*sqrt(x + sqrt(x^2 + 1
)) - 1/4*sqrt(x^2 + 1)*x - 1/2*x - 1/2*log(sqrt(x + sqrt(x^2 + 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x + sqrt(x^2 + 1)) + 1), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x}{1+\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+(x+(x^2+1)^(1/2))^(1/2)),x)
```

```
[Out] int(x/(1+(x+(x^2+1)^(1/2))^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x + sqrt(x^2 + 1)) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + (x^2 + 1)^(1/2))^(1/2) + 1),x)
```

```
[Out] int(x/((x + (x^2 + 1)^(1/2))^(1/2) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+(x+(x**2+1)**(1/2))**(1/2)),x)
```

```
[Out] Integral(x/(sqrt(x + sqrt(x**2 + 1)) + 1), x)
```

3.1765 $\int \frac{-2x+(1+k)x^2}{((1-x)x(1-kx))^{2/3}\left(b-b(1+k)x+(-1+bk)x^2\right)}\,dx$

Optimal. Leaf size=154

$$-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+x^2\right)}{2\sqrt[3]{b}}+\frac{\log\left(x-\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{\sqrt[3]{b}}+$$

Rubi [F] time = 4.80, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2x+(1+k)x^2}{((1-x)x(1-kx))^{2/3}\left(b-b(1+k)x+(-1+bk)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*x + (1 + k)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(b - b*(1 + k)*x + (-
1 + b*k)*x^2)),x]
```

```
[Out] ((1 + Sqrt[4 + b*(1 - k)^2]/Sqrt[b] + k)*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2
/3)*Defer[Int][x^(1/3)/((1 - x)^(2/3)*(1 - k*x)^(2/3)*(-(b*(1 + k)) - Sqrt[
b]*Sqrt[4 + b - 2*b*k + b*k^2] + 2*(-1 + b*k)*x)), x])/((1 - x)*x*(1 - k*x)
)^(2/3) + ((1 - Sqrt[4 + b*(1 - k)^2]/Sqrt[b] + k)*(1 - x)^(2/3)*x^(2/3)*(1
 - k*x)^(2/3)*Defer[Int][x^(1/3)/((1 - x)^(2/3)*(1 - k*x)^(2/3)*(-(b*(1 + k
)) + Sqrt[b]*Sqrt[4 + b - 2*b*k + b*k^2] + 2*(-1 + b*k)*x)), x])/((1 - x)*x
*(1 - k*x))^(2/3)
```

Rubi steps

$$\int \frac{-2x+(1+k)x^2}{((1-x)x(1-kx))^{2/3}\left(b-b(1+k)x+(-1+bk)x^2\right)}\,dx=\int \frac{x(-2+(1+k)x)}{((1-x)x(1-kx))^{2/3}\left(b-b(1+k)x+(-1+bk)x^2\right)}\,d$$
$$=\frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{\sqrt[3]{x}\,(-2+(1+k)x)}{(1-x)^{2/3}(1-kx)^{2/3}\left(b-b(1+k)x+(-1\right.}}{((1-x)x(1-kx))^{2/3}}$$
$$=\frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int\left(\frac{\left(1+k+\frac{\sqrt{4+b-2b}}{\sqrt{b}}\right.}{(1-x)^{2/3}(1-kx)^{2/3}\left(-b(1+k)-\sqrt{b}\right.}\right.}{}$$
$$=\frac{\left(\left(1-\frac{\sqrt{4+b(1-k)^2}}{\sqrt{b}}+k\right)(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{}{(1-x)^{2/}}}{((1-x)x(1-k}$$

Mathematica [F] time = 7.18, size = 0, normalized size = 0.00

$$\int \frac{-2x+(1+k)x^2}{((1-x)x(1-kx))^{2/3}\left(b-b(1+k)x+(-1+bk)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*x + (1 + k)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(b - b*(1 + k)*
x + (-1 + b*k)*x^2)),x]
```

```
[Out] Integrate[(-2*x + (1 + k)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(b - b*(1 + k)*
x + (-1 + b*k)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.39, size = 154, normalized size = 1.00

$$-\frac{\log \left(b^{2/3} \left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{b}\, x \sqrt[3]{kx^3+(-k-1)x^2+x}+x^2\right)}{2 \sqrt[3]{b}}+\frac{\log \left(x-\sqrt[3]{b} \sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{\sqrt[3]{b}}+\frac{\sqrt{3} \tan ^{-1}\left(\frac{\sqrt{3} x}{2 \sqrt[3]{b} \sqrt[3]{kx^3+(-k-1)x^2+x}+x}\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*x + (1 + k)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(b - b
*(1 + k)*x + (-1 + b*k)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3)
)])/b^(1/3) + Log[x - b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3)]/b^(1/3) - L
og[x^2 + b^(1/3)*x*(x + (-1 - k)*x^2 + k*x^3)^(1/3) + b^(2/3)*(x + (-1 - k)
*x^2 + k*x^3)^(2/3)]/(2*b^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x+(1+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-b*(1+k)*x+(b*k-1)*x^
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.58, size = 130, normalized size = 0.84

$$-\frac{\sqrt{3} |b|^{\frac{2}{3}} \arctan \left(\frac{1}{3} \sqrt{3} b^{\frac{1}{3}} \left(2 \left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{1}{3}}+\frac{1}{b^{\frac{1}{3}}}\right)\right)}{b}-\frac{|b|^{\frac{2}{3}} \log \left(\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{2}{3}}+\frac{\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{1}{3}}}{b^{\frac{1}{3}}}+\frac{1}{b^{\frac{2}{3}}}\right)}{2 b}+\frac{\log \left(\left|\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{1}{3}}-\frac{1}{b^{\frac{1}{3}}}\right|\right)}{b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x+(1+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-b*(1+k)*x+(b*k-1)*x^
2),x, algorithm="giac")
```

```
[Out] -sqrt(3)*abs(b)^(2/3)*arctan(1/3*sqrt(3)*b^(1/3)*(2*(k - k/x - 1/x + 1/x^2)
^(1/3) + 1/b^(1/3)))/b - 1/2*abs(b)^(2/3)*log((k - k/x - 1/x + 1/x^2)^(2/3)
 + (k - k/x - 1/x + 1/x^2)^(1/3)/b^(1/3) + 1/b^(2/3))/b + log(abs((k - k/x
- 1/x + 1/x^2)^(1/3) - 1/b^(1/3)))/b^(1/3)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{-2x+(1+k)\, x^{2}}{\left((1-x)\, x\, (-kx+1)\right)^{\frac{2}{3}} \left(b-b\,(1+k)\, x+(bk-1)\, x^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x+(1+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-b*(1+k)*x+(b*k-1)*x^2),x)
```

```
[Out] int((-2*x+(1+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-b*(1+k)*x+(b*k-1)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(k+1)x^2-2\,x}{\left(b(k+1)x-(bk-1)x^2-b\right)\left((kx-1)(x-1)x\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x+(1+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-b*(1+k)*x+(b*k-1)*x^
2),x, algorithm="maxima")
```

```
[Out] -integrate(((k + 1)*x^2 - 2*x)/((b*(k + 1)*x - (b*k - 1)*x^2 - b)*((k*x - 1
)*(x - 1)*x)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{2x-x^2\,(k+1)}{(x\,(k\,x-1)\,(x-1))^{2/3}\,\left((b\,k-1)\,x^2-b\,(k+1)\,x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - x^2*(k + 1))/((x*(k*x - 1)*(x - 1))^(2/3)*(b + x^2*(b*k - 1) -
b*x*(k + 1))),x)
```

```
[Out] int(-(2*x - x^2*(k + 1))/((x*(k*x - 1)*(x - 1))^(2/3)*(b + x^2*(b*k - 1) -
b*x*(k + 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x+(1+k)*x**2)/((1-x)*x*(-k*x+1))**(2/3)/(b-b*(1+k)*x+(b*k-1)*
x**2),x)
```

```
[Out] Timed out
```

3.1766 $\int \frac{(-b+ax^2)\sqrt{-bx+ax^3}}{x^2(b+ax^2)}\,dx$

Optimal. Leaf size=154

$$-\sqrt[4]{a}\sqrt[4]{b}\tanh^{-1}\left(\frac{\frac{a^{3/4}x^2}{2\sqrt[4]{b}}-\frac{b^{3/4}}{2\sqrt[4]{a}}+\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{ax^3-bx}}\right)+\frac{2\sqrt{ax^3-bx}}{x}+\sqrt[4]{a}\sqrt[4]{b}\tan^{-1}\left(\frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3-bx}}{-2\sqrt{a}\sqrt{b}\,x+ax^2-b}\right)$$

Rubi [A] time = 0.75, antiderivative size = 195, normalized size of antiderivative = 1.27, number of steps used = 15, number of rules used = 11, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.306, Rules used = {2056, 466, 474, 12, 490, 1211, 224, 221, 1699, 208, 205}

$$\frac{2\sqrt{ax^3-bx}}{x}+\frac{\sqrt{2}\sqrt[4]{-a}\sqrt[4]{b}\sqrt{ax^3-bx}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2-b}}\right)}{\sqrt{x}\sqrt{ax^2-b}}-\frac{\sqrt{2}\sqrt[4]{-a}\sqrt[4]{b}\sqrt{ax^3-bx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2-b}}\right)}{\sqrt{x}\sqrt{ax^2-b}}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a*x^2)*Sqrt[-(b*x) + a*x^3])/(x^2*(b + a*x^2)),x]
```

```
[Out] (2*Sqrt[-(b*x) + a*x^3])/x + (Sqrt[2]*(-a)^(1/4)*b^(1/4)*Sqrt[-(b*x) + a*x^
3]*ArcTan[(Sqrt[2]*(-a)^(1/4)*b^(1/4)*Sqrt[x])/Sqrt[-b + a*x^2]])/(Sqrt[x]*
Sqrt[-b + a*x^2]) - (Sqrt[2]*(-a)^(1/4)*b^(1/4)*Sqrt[-(b*x) + a*x^3]*ArcTan
h[(Sqrt[2]*(-a)^(1/4)*b^(1/4)*Sqrt[x])/Sqrt[-b + a*x^2]])/(Sqrt[x]*Sqrt[-b
+ a*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\left(-b+ax^2\right)\sqrt{-bx+ax^3}}{x^2\left(b+ax^2\right)}\,dx=\frac{\sqrt{-bx+ax^3}\int \frac{\left(-b+ax^2\right)^{3/2}}{x^{3/2}\left(b+ax^2\right)}\,dx}{\sqrt{x}\sqrt{-b+ax^2}}$$

$$=\frac{\left(2\sqrt{-bx+ax^3}\right)\text{Subst}\left(\int \frac{\left(-b+ax^4\right)^{3/2}}{x^2\left(b+ax^4\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{-b+ax^2}}$$

$$=\frac{2\sqrt{-bx+ax^3}}{x}+\frac{\left(2\sqrt{-bx+ax^3}\right)\text{Subst}\left(\int -\frac{4ab^2x^2}{\sqrt{-b+ax^4}\left(b+ax^4\right)}\,dx,x,\sqrt{x}\right)}{b\sqrt{x}\sqrt{-b+ax^2}}$$

$$=\frac{2\sqrt{-bx+ax^3}}{x}-\frac{\left(8ab\sqrt{-bx+ax^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-b+ax^4}\left(b+ax^4\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{-b+ax^2}}$$

$$=\frac{2\sqrt{-bx+ax^3}}{x}-\frac{\left(4ab\sqrt{-bx+ax^3}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{b}-\sqrt{-a}\,x^2\right)\sqrt{-b+ax^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{-a}\sqrt{x}\sqrt{-b+ax^2}}+$$

$$=\frac{2\sqrt{-bx+ax^3}}{x}+\frac{\left(2a\sqrt{b}\sqrt{-bx+ax^3}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{-a}\,x^2}{\left(\sqrt{b}+\sqrt{-a}\,x^2\right)\sqrt{-b+ax^4}}\,dx,x,\sqrt{x}\right.}{\sqrt{-a}\sqrt{x}\sqrt{-b+ax^2}}$$

$$=\frac{2\sqrt{-bx+ax^3}}{x}+\frac{\left(2ab\sqrt{-bx+ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-2\sqrt{-a}\,bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{-b+ax^2}}\right)}{\sqrt{-a}\sqrt{x}\sqrt{-b+ax^2}}-\Big($$

$$=\frac{2\sqrt{-bx+ax^3}}{x}+\frac{\sqrt{2}\sqrt[4]{-a}\sqrt[4]{b}\sqrt{-bx+ax^3}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a}\sqrt[4]{b}\sqrt{x}}{\sqrt{-b+ax^2}}\right)}{\sqrt{x}\sqrt{-b+ax^2}}-\frac{\sqrt{2}\sqrt[4]{-a}\,\sqrt[4]{}}{}$$

Mathematica [C] time = 0.06, size = 88, normalized size = 0.57

$$-\frac{2\sqrt{ax^3-bx}\left(4ax^2\sqrt{1-\frac{ax^2}{b}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{ax^2}{b},-\frac{ax^2}{b}\right)-3ax^2+3b\right)}{3ax^3-3bx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-b + a*x^2)*Sqrt[-(b*x) + a*x^3])/(x^2*(b + a*x^2)),x]
```

```
[Out] (-2*Sqrt[-(b*x) + a*x^3]*(3*b - 3*a*x^2 + 4*a*x^2*Sqrt[1 - (a*x^2)/b]*Appel
lF1[3/4, 1/2, 1, 7/4, (a*x^2)/b, -((a*x^2)/b)]))/(-3*b*x + 3*a*x^3)
```

IntegrateAlgebraic [A] time = 0.45, size = 165, normalized size = 1.07

$$-\sqrt[4]{a}\sqrt[4]{b}\tan^{-1}\left(\frac{\frac{a^{3/4}x^2}{2\sqrt[4]{b}}-\frac{b^{3/4}}{2\sqrt[4]{a}}-\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{ax^3-bx}}\right)-\sqrt[4]{a}\sqrt[4]{b}\tanh^{-1}\left(\frac{\frac{a^{3/4}x^2}{2\sqrt[4]{b}}-\frac{b^{3/4}}{2\sqrt[4]{a}}+\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{ax^3-bx}}\right)+\frac{2\sqrt{ax^3-bx}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^2)*Sqrt[-(b*x) + a*x^3])/(x^2*(b + a*x^2)),x]
```

```
[Out] (2*Sqrt[-(b*x) + a*x^3])/x - a^(1/4)*b^(1/4)*ArcTan[(-1/2*b^(3/4)/a^(1/4) -
 a^(1/4)*b^(1/4)*x + (a^(3/4)*x^2)/(2*b^(1/4)))/Sqrt[-(b*x) + a*x^3]] - a^(
1/4)*b^(1/4)*ArcTanh[(-1/2*b^(3/4)/a^(1/4) + a^(1/4)*b^(1/4)*x + (a^(3/4)*x
^2)/(2*b^(1/4)))/Sqrt[-(b*x) + a*x^3]]
```

fricas [B] time = 71.91, size = 1180, normalized size = 7.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(a*x^3-b*x)^(1/2)/x^2/(a*x^2+b),x, algorithm="fricas")
```

```
[Out] -1/4*(4*4^(1/4)*(-a*b)^(1/4)*x*arctan(-1/4*(8*sqrt(a*x^3 - b*x)*(4^(3/4)*(4
*a^4*b + 9*a^3*b^2 + 6*a^2*b^3 + a*b^4)*(-a*b)^(3/4)*x - 4^(1/4)*(4*a^4*b^2
 + 9*a^3*b^3 + 6*a^2*b^4 + a*b^5 - (4*a^5*b + 9*a^4*b^2 + 6*a^3*b^3 + a^2*b
^4)*x^2)*(-a*b)^(1/4)) + sqrt(-160*a^4*b + 352*a^3*b^2 - 64*a^2*b^3 + 8*(4*
a^4 - 41*a^3*b + 26*a^2*b^2 - a*b^3)*sqrt(-a*b))*(4^(3/4)*((a^3 - 2*a^2*b)*
x^4 + 2*(5*a^2*b - a*b^2)*x^3 + a*b^2 - 2*b^3 - 6*(a^2*b - 2*a*b^2)*x^2 - 2
*(5*a*b^2 - b^3)*x)*(-a*b)^(3/4) + 4^(1/4)*((5*a^3*b - a^2*b^2)*x^4 + 5*a*b
^3 - b^4 - 8*(a^3*b - 2*a^2*b^2)*x^3 - 6*(5*a^2*b^2 - a*b^3)*x^2 + 8*(a^2*b
^2 - 2*a*b^3)*x)*(-a*b)^(1/4)))/(4*a^4*b^3 + 9*a^3*b^4 + 6*a^2*b^5 + a*b^6
+ (4*a^6*b + 9*a^5*b^2 + 6*a^4*b^3 + a^3*b^4)*x^4 + 2*(4*a^5*b^2 + 9*a^4*b^
3 + 6*a^3*b^4 + a^2*b^5)*x^2)) + 4^(1/4)*(-a*b)^(1/4)*x*log((4^(3/4)*((5*a^
3 - a^2*b)*x^4 - 8*(a^3 - 2*a^2*b)*x^3 + 5*a*b^2 - b^3 - 6*(5*a^2*b - a*b^2
)*x^2 + 8*(a^2*b - 2*a*b^2)*x)*(-a*b)^(3/4) + 8*(5*a^2*b^2 - a*b^3 - (5*a^3
*b - a^2*b^2)*x^2 + 4*(a^3*b - 2*a^2*b^2)*x + 2*(a^2*b - 2*a*b^2 - (a^3 - 2
*a^2*b)*x^2 - (5*a^2*b - a*b^2)*x)*sqrt(-a*b))*sqrt(a*x^3 - b*x) - 4*4^(1/4
)*((a^4 - 2*a^3*b)*x^4 + a^2*b^2 - 2*a*b^3 + 2*(5*a^3*b - a^2*b^2)*x^3 - 6*
(a^3*b - 2*a^2*b^2)*x^2 - 2*(5*a^2*b^2 - a*b^3)*x)*(-a*b)^(1/4))/(a^2*x^4 +
 2*a*b*x^2 + b^2)) - 4^(1/4)*(-a*b)^(1/4)*x*log(-(4^(3/4)*((5*a^3 - a^2*b)*
x^4 - 8*(a^3 - 2*a^2*b)*x^3 + 5*a*b^2 - b^3 - 6*(5*a^2*b - a*b^2)*x^2 + 8*(
a^2*b - 2*a*b^2)*x)*(-a*b)^(3/4) - 8*(5*a^2*b^2 - a*b^3 - (5*a^3*b - a^2*b^
2)*x^2 + 4*(a^3*b - 2*a^2*b^2)*x + 2*(a^2*b - 2*a*b^2 - (a^3 - 2*a^2*b)*x^2
 - (5*a^2*b - a*b^2)*x)*sqrt(-a*b))*sqrt(a*x^3 - b*x) - 4*4^(1/4)*((a^4 - 2
*a^3*b)*x^4 + a^2*b^2 - 2*a*b^3 + 2*(5*a^3*b - a^2*b^2)*x^3 - 6*(a^3*b - 2*
a^2*b^2)*x^2 - 2*(5*a^2*b^2 - a*b^3)*x)*(-a*b)^(1/4))/(a^2*x^4 + 2*a*b*x^2
+ b^2)) - 8*sqrt(a*x^3 - b*x))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a x^3 - b x}\left(a x^2 - b\right)}{\left(a x^2 + b\right) x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(a*x^3-b*x)^(1/2)/x^2/(a*x^2+b),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^3 - b*x)*(a*x^2 - b)/((a*x^2 + b)*x^2), x)
```

maple [C] time = 0.05, size = 617, normalized size = 4.01

$$2a\left(-\frac{2b\sqrt{\frac{xa}{\sqrt{ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a x^3-bx}}+\frac{b\sqrt{\frac{xa}{\sqrt{ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a x^3-bx}}-\frac{b\sqrt{ab}\,\sqrt{\frac{xa}{\sqrt{ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},-\frac{\sqrt{ab}}{a\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{a^2\sqrt{a x^3-bx}\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)}-\frac{b\sqrt{ab}\,\sqrt{\frac{xa}{\sqrt{ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},-\frac{\sqrt{ab}}{a\left(-\frac{\sqrt{ab}}{a}+\frac{\sqrt{-ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{a^2\sqrt{a x^3-bx}\left(-\frac{\sqrt{ab}}{a}+\frac{\sqrt{-ab}}{a}\right)}\right)+\frac{2a x^2-2b}{\sqrt{x\left(a x^2-b\right)}}-\frac{2\sqrt{ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}}\,\sqrt{-\frac{xa}{\sqrt{ab}}}\left(-\frac{2\sqrt{ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)}{a}+\frac{\sqrt{ab}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)}{a}\right)}{\sqrt{a x^3-bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)*(a*x^3-b*x)^(1/2)/x^2/(a*x^2+b),x)
```

```
[Out] 2*a*(-2/a*b*(x*a/(a*b)^(1/2)+1)^(1/2)*(-2*x*a/(a*b)^(1/2)+2)^(1/2)*(-x*a/(a
*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)*EllipticE(((x+1/a*(a*b)^(1/2))*a/(a*b)^(
1/2))^(1/2),1/2*2^(1/2))+1/a*b*(x*a/(a*b)^(1/2)+1)^(1/2)*(-2*x*a/(a*b)^(1/2
)+2)^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)*EllipticF(((x+1/a*(a*
b)^(1/2))*a/(a*b)^(1/2))^(1/2),1/2*2^(1/2))-b/a^2*(a*b)^(1/2)*(x*a/(a*b)^(1
/2)+1)^(1/2)*(-2*x*a/(a*b)^(1/2)+2)^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b
*x)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2))*EllipticPi(((x+1/a*(a*b)^(1/2
))*a/(a*b)^(1/2))^(1/2),-1/a*(a*b)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2)
```

```
),1/2*2^(1/2))-b/a^2*(a*b)^(1/2)*(x*a/(a*b)^(1/2)+1)^(1/2)*(-2*x*a/(a*b)^(1
/2)+2)^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)/(-1/a*(a*b)^(1/2)+1
/a*(-a*b)^(1/2))*EllipticPi(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2),-1/a*
(a*b)^(1/2)/(-1/a*(a*b)^(1/2)+1/a*(-a*b)^(1/2)),1/2*2^(1/2)))+2*(a*x^2-b)/(
x*(a*x^2-b))^(1/2)-2*(a*b)^(1/2)*((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2)*
(-2*(x-1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^
3-b*x)^(1/2)*(-2/a*(a*b)^(1/2)*EllipticE(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2)
)^(1/2),1/2*2^(1/2))+1/a*(a*b)^(1/2)*EllipticF(((x+1/a*(a*b)^(1/2))*a/(a*b)
^(1/2))^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3 - bx}\left(ax^2 - b\right)}{\left(ax^2 + b\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(a*x^3-b*x)^(1/2)/x^2/(a*x^2+b),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^3 - b*x)*(a*x^2 - b)/((a*x^2 + b)*x^2), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*x^3 - b*x)^(1/2)*(b - a*x^2))/(x^2*(b + a*x^2)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(ax^2 - b\right)}\left(ax^2 - b\right)}{x^2\left(ax^2 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)*(a*x**3-b*x)**(1/2)/x**2/(a*x**2+b),x)
[Out] Integral(sqrt(x*(a*x**2 - b))*(a*x**2 - b)/(x**2*(a*x**2 + b)), x)
```

3.1767 $\int \frac{(-3+x^4)(1-x^3+x^4)^{2/3}}{x^3(1+x^3+x^4)}\,dx$

Optimal. Leaf size=154

$$2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^4-x^3+1}+2x\right)+2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2^{2/3}\sqrt[3]{x^4-x^3+1}-x}\right)+\frac{3\left(x^4-x^3+1\right)^{2/3}}{2x^2}-\frac{\log\left(-2x^2+2^{2/3}\sqrt[3]{x^4\right.}{}$$

Rubi [F] time = 0.82, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-3+x^4)(1-x^3+x^4)^{2/3}}{x^3(1+x^3+x^4)}\,dx$$

Verification is not applicable to the result.

[In] Int[((-3 + x^4)*(1 - x^3 + x^4)^(2/3))/(x^3*(1 + x^3 + x^4)),x]

[Out] -3*Defer[Int][(1 - x^3 + x^4)^(2/3)/x^3, x] + 3*Defer[Int][(1 - x^3 + x^4)^(2/3)/(1 + x^3 + x^4), x] + 4*Defer[Int][(x*(1 - x^3 + x^4)^(2/3))/(1 + x^3 + x^4), x]

Rubi steps

$$\begin{aligned}
\int \frac{(-3+x^4)(1-x^3+x^4)^{2/3}}{x^3(1+x^3+x^4)}\,dx &= \int \left(-\frac{3\left(1-x^3+x^4\right)^{2/3}}{x^3}+\frac{(3+4x)\left(1-x^3+x^4\right)^{2/3}}{1+x^3+x^4}\right)dx\\
&= -\left(3\int \frac{\left(1-x^3+x^4\right)^{2/3}}{x^3}\,dx\right)+\int \frac{(3+4x)\left(1-x^3+x^4\right)^{2/3}}{1+x^3+x^4}\,dx\\
&= -\left(3\int \frac{\left(1-x^3+x^4\right)^{2/3}}{x^3}\,dx\right)+\int \left(\frac{3\left(1-x^3+x^4\right)^{2/3}}{1+x^3+x^4}+\frac{4x\left(1-x^3+x^4\right)^{2/3}}{1+x^3+x^4}\right)dx\\
&= -\left(3\int \frac{\left(1-x^3+x^4\right)^{2/3}}{x^3}\,dx\right)+3\int \frac{\left(1-x^3+x^4\right)^{2/3}}{1+x^3+x^4}\,dx+4\int \frac{x\left(1-x^3+x^4\right)^{2/3}}{1+x^3+x^4}
\end{aligned}$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(-3+x^4)(1-x^3+x^4)^{2/3}}{x^3(1+x^3+x^4)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[((-3 + x^4)*(1 - x^3 + x^4)^(2/3))/(x^3*(1 + x^3 + x^4)),x]

[Out] Integrate[((-3 + x^4)*(1 - x^3 + x^4)^(2/3))/(x^3*(1 + x^3 + x^4)), x]

IntegrateAlgebraic [A] time = 0.78, size = 154, normalized size = 1.00

$$2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^4-x^3+1}+2x\right)+2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2^{2/3}\sqrt[3]{x^4-x^3+1}-x}\right)+\frac{3\left(x^4-x^3+1\right)^{2/3}}{2x^2}-\frac{\log\left(-2x^2+2^{2/3}\sqrt[3]{x^4-x^3+1}\,x-\sqrt[3]{2}\left(x^4-x^3+1\right)^{2/3}\right)}{\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^4)*(1 - x^3 + x^4)^(2/3))/(x^3*(1 + x^3 + x^4))
,x]
```

```
[Out] (3*(1 - x^3 + x^4)^(2/3))/(2*x^2) + 2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x
+ 2^(2/3)*(1 - x^3 + x^4)^(1/3))] + 2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3 + x^
4)^(1/3)] - Log[-2*x^2 + 2^(2/3)*x*(1 - x^3 + x^4)^(1/3) - 2^(1/3)*(1 - x^3
 + x^4)^(2/3)]/2^(1/3)
```

fricas [B] time = 71.81, size = 413, normalized size = 2.68

$$\frac{2 \cdot 4^{\frac{1}{3}} \sqrt{3}\, x^2 \arctan\left(\frac{3 \cdot 4^{\frac{2}{3}} \sqrt{3} \left(x^9-4 x^8-5 x^7+2 x^5-4 x^4+x\right) \left(x^4-x^3+1\right)^{\frac{2}{3}}-6 \cdot 4^{\frac{1}{3}} \sqrt{3} \left(x^{10}-16 x^9+19 x^8+2 x^6-16 x^5+x^2\right) \left(x^4-x^3+1\right)^{\frac{1}{3}}-\sqrt{3} \left(x^{12}-33 x^{11}+111 x^{10}-71 x^9+3 x^8-66 x^7+111 x^6+3 x^4-33 x^3+1\right)}{3 \left(x^{12}+3 x^{11}-105 x^{10}+109 x^9+3 x^8+6 x^7-105 x^6+3 x^4+3 x^3+1\right)}\right)+2 \cdot 4^{\frac{1}{3}} x^2 \log\left(-\frac{3 \cdot 4^{\frac{2}{3}} \left(x^4-x^3+1\right)^{\frac{1}{3}} x^2+6 \left(x^4-x^3+1\right)^{\frac{2}{3}} x+4^{\frac{1}{3}} \left(x^4+x^3+1\right)}{x^4+x^3+1}\right)-4^{\frac{1}{3}} x^2 \log\left(-\frac{6 \cdot 4^{\frac{1}{3}} \left(x^5-5 x^4+x\right) \left(x^4-x^3+1\right)^{\frac{2}{3}}-4^{\frac{2}{3}} \left(x^8-16 x^7+19 x^6+2 x^4-16 x^3+1\right)-24 \left(x^6-2 x^5+x^2\right) \left(x^4-x^3+1\right)^{\frac{1}{3}}}{x^8+2 x^7+x^6+2 x^4+2 x^3+1}\right)+9 \left(x^4-x^3+1\right)^{\frac{2}{3}}}{6 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4-x^3+1)^(2/3)/x^3/(x^4+x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*(2*4^(1/3)*sqrt(3)*x^2*arctan(1/3*(3*4^(2/3)*sqrt(3)*(x^9 - 4*x^8 - 5*x
^7 + 2*x^5 - 4*x^4 + x)*(x^4 - x^3 + 1)^(2/3) - 6*4^(1/3)*sqrt(3)*(x^10 - 1
6*x^9 + 19*x^8 + 2*x^6 - 16*x^5 + x^2)*(x^4 - x^3 + 1)^(1/3) - sqrt(3)*(x^1
2 - 33*x^11 + 111*x^10 - 71*x^9 + 3*x^8 - 66*x^7 + 111*x^6 + 3*x^4 - 33*x^3
 + 1))/(x^12 + 3*x^11 - 105*x^10 + 109*x^9 + 3*x^8 + 6*x^7 - 105*x^6 + 3*x^
4 + 3*x^3 + 1)) + 2*4^(1/3)*x^2*log(-(3*4^(2/3)*(x^4 - x^3 + 1)^(1/3)*x^2 +
 6*(x^4 - x^3 + 1)^(2/3)*x + 4^(1/3)*(x^4 + x^3 + 1))/(x^4 + x^3 + 1)) - 4^
(1/3)*x^2*log(-(6*4^(1/3)*(x^5 - 5*x^4 + x)*(x^4 - x^3 + 1)^(2/3) - 4^(2/3)
*(x^8 - 16*x^7 + 19*x^6 + 2*x^4 - 16*x^3 + 1) - 24*(x^6 - 2*x^5 + x^2)*(x^4
 - x^3 + 1)^(1/3))/(x^8 + 2*x^7 + x^6 + 2*x^4 + 2*x^3 + 1)) + 9*(x^4 - x^3
+ 1)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4 - x^3 + 1\right)^{\frac{2}{3}} \left(x^4 - 3\right)}{\left(x^4 + x^3 + 1\right) x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4-x^3+1)^(2/3)/x^3/(x^4+x^3+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 - x^3 + 1)^(2/3)*(x^4 - 3)/((x^4 + x^3 + 1)*x^3), x)
```

maple [C] time = 51.40, size = 725, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3)*(x^4-x^3+1)^(2/3)/x^3/(x^4+x^3+1),x)
```

```
[Out] 3/2*(x^4-x^3+1)^(2/3)/x^2+2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_
Z^2)*ln(-(3*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3
-4)^3*x^3+4*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z
^3-4)^2*x^3-4*(x^4-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)
+4*_Z^2)*RootOf(_Z^3-4)^2*x-4*(x^4-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x^2-10*(x^
4-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_
Z^3-4)*x^2+3*RootOf(_Z^3-4)*x^4+4*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-
4)+4*_Z^2)*x^4-3*RootOf(_Z^3-4)*x^3-4*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_
Z^3-4)+4*_Z^2)*x^3+2*(x^4-x^3+1)^(2/3)*x+3*RootOf(_Z^3-4)+4*RootOf(RootOf(_
Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(x^4+x^3+1))+RootOf(_Z^3-4)*ln((RootO
f(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x^3+3*RootO
f(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-2*(x^
4-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_
```

```
Z^3-4)^2*x-2*(x^4-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x^2+(x^4-x^3+1)^(1/3)*RootO
f(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x^2-RootOf(_Z
^3-4)*x^4-3*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^4+3*RootO
f(_Z^3-4)*x^3+9*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^3-5*(
x^4-x^3+1)^(2/3)*x-RootOf(_Z^3-4)-3*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^
3-4)+4*_Z^2))/(x^4+x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4 - x^3 + 1\right)^{\frac{2}{3}} \left(x^4 - 3\right)}{\left(x^4 + x^3 + 1\right) x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3)*(x^4-x^3+1)^(2/3)/x^3/(x^4+x^3+1),x, algorithm="maxima")
[Out] integrate((x^4 - x^3 + 1)^(2/3)*(x^4 - 3)/((x^4 + x^3 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4 - 3\right) \left(x^4 - x^3 + 1\right)^{2/3}}{x^3 \left(x^4 + x^3 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 3)*(x^4 - x^3 + 1)^(2/3))/(x^3*(x^3 + x^4 + 1)),x)
[Out] int(((x^4 - 3)*(x^4 - x^3 + 1)^(2/3))/(x^3*(x^3 + x^4 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3)*(x**4-x**3+1)**(2/3)/x**3/(x**4+x**3+1),x)
[Out] Timed out
```

3.1768 $\int \frac{(-4b+ax^5)(b+ax^5)^{3/4}}{x^4(2b+cx^4+2ax^5)}\,dx$

Optimal. Leaf size=154

$$\frac{c^{3/4}\tan^{-1}\left(\frac{2^{3/4}\sqrt[4]{c}\,x\sqrt[4]{ax^5+b}}{\sqrt{2}\sqrt{ax^5+b}-\sqrt{c}\,x^2}\right)}{2\sqrt[4]{2}}+\frac{c^{3/4}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5+b}}{\sqrt[4]{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}\,x^2}{2^{3/4}}}{x\sqrt[4]{ax^5+b}}\right)}{2\sqrt[4]{2}}+\frac{2\left(ax^5+b\right)^{3/4}}{3x^3}$$

Rubi [F] time = 1.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-4b+ax^5\right)\left(b+ax^5\right)^{3/4}}{x^4\left(2b+cx^4+2ax^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-4*b + a*x^5)*(b + a*x^5)^(3/4))/(x^4*(2*b + c*x^4 + 2*a*x^5)),x]
```

```
[Out] (2*(b + a*x^5)^(3/4)*Hypergeometric2F1[-3/4, -3/5, 2/5, -((a*x^5)/b)])/(3*x
^3*(1 + (a*x^5)/b)^(3/4)) + 2*c*Defer[Int][(b + a*x^5)^(3/4)/(2*b + c*x^4 +
 2*a*x^5), x] + 5*a*Defer[Int][(x*(b + a*x^5)^(3/4))/(2*b + c*x^4 + 2*a*x^5
), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-4b+ax^5\right)\left(b+ax^5\right)^{3/4}}{x^4\left(2b+cx^4+2ax^5\right)}\,dx &= \int \left(-\frac{2\left(b+ax^5\right)^{3/4}}{x^4}+\frac{(2c+5ax)\left(b+ax^5\right)^{3/4}}{2b+cx^4+2ax^5}\right)dx\\
&= -\left(2\int \frac{\left(b+ax^5\right)^{3/4}}{x^4}\,dx\right)+\int \frac{(2c+5ax)\left(b+ax^5\right)^{3/4}}{2b+cx^4+2ax^5}\,dx\\
&= -\frac{\left(2\left(b+ax^5\right)^{3/4}\right)\int \frac{\left(1+\frac{ax^5}{b}\right)^{3/4}}{x^4}\,dx}{\left(1+\frac{ax^5}{b}\right)^{3/4}}+\int \left(\frac{2c\left(b+ax^5\right)^{3/4}}{2b+cx^4+2ax^5}+\frac{5ax\left(b+ax^5\right)^{3/4}}{2b+cx^4+2ax^5}\right)\\
&= \frac{2\left(b+ax^5\right)^{3/4}{}_2F_1\left(-\frac{3}{4},-\frac{3}{5};\frac{2}{5};-\frac{ax^5}{b}\right)}{3x^3\left(1+\frac{ax^5}{b}\right)^{3/4}}+(5a)\int \frac{x\left(b+ax^5\right)^{3/4}}{2b+cx^4+2ax^5}\,dx+(2c)\int \frac{}{2}
\end{aligned}$$

Mathematica [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\left(-4b+ax^5\right)\left(b+ax^5\right)^{3/4}}{x^4\left(2b+cx^4+2ax^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-4*b + a*x^5)*(b + a*x^5)^(3/4))/(x^4*(2*b + c*x^4 + 2*a*x^5)),
x]
```

```
[Out] Integrate[((-4*b + a*x^5)*(b + a*x^5)^(3/4))/(x^4*(2*b + c*x^4 + 2*a*x^5)),
 x]
```

IntegrateAlgebraic [A] time = 1.37, size = 154, normalized size = 1.00

$$-\frac{c^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt{ax^5+b}}{\sqrt[4]{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}x^2}{2^{3/4}}}{x\sqrt[4]{ax^5+b}}\right)}{2\sqrt[4]{2}}+\frac{c^{3/4}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5+b}}{\sqrt[4]{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}x^2}{2^{3/4}}}{x\sqrt[4]{ax^5+b}}\right)}{2\sqrt[4]{2}}+\frac{2\left(ax^5+b\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4*b + a*x^5)*(b + a*x^5)^(3/4))/(x^4*(2*b + c*x^4 + 2
*a*x^5)),x]
```

```
[Out] (2*(b + a*x^5)^(3/4))/(3*x^3) - (c^(3/4)*ArcTan[(-((c^(1/4)*x^2)/2^(3/4)) +
 Sqrt[b + a*x^5]/(2^(1/4)*c^(1/4)))/(x*(b + a*x^5)^(1/4))])/(2*2^(1/4)) + (
c^(3/4)*ArcTanh[((c^(1/4)*x^2)/2^(3/4) + Sqrt[b + a*x^5]/(2^(1/4)*c^(1/4)))
/(x*(b + a*x^5)^(1/4))])/(2*2^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5-4*b)*(a*x^5+b)^(3/4)/x^4/(2*a*x^5+c*x^4+2*b),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^5+b\right)^{\frac{3}{4}}\left(ax^5-4\,b\right)}{\left(2\,ax^5+cx^4+2\,b\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5-4*b)*(a*x^5+b)^(3/4)/x^4/(2*a*x^5+c*x^4+2*b),x, algorithm=
"giac")
```

```
[Out] integrate((a*x^5 + b)^(3/4)*(a*x^5 - 4*b)/((2*a*x^5 + c*x^4 + 2*b)*x^4), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^5-4b\right)\left(a\,x^5+b\right)^{\frac{3}{4}}}{x^4\left(2a\,x^5+c\,x^4+2b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^5-4*b)*(a*x^5+b)^(3/4)/x^4/(2*a*x^5+c*x^4+2*b),x)
```

```
[Out] int((a*x^5-4*b)*(a*x^5+b)^(3/4)/x^4/(2*a*x^5+c*x^4+2*b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^5+b\right)^{\frac{3}{4}}\left(ax^5-4\,b\right)}{\left(2\,ax^5+cx^4+2\,b\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^5-4*b)*(a*x^5+b)^(3/4)/x^4/(2*a*x^5+c*x^4+2*b),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x^5 + b)^(3/4)*(a*x^5 - 4*b)/((2*a*x^5 + c*x^4 + 2*b)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(a\,x^5+b\right)^{3/4}\,\left(4\,b-a\,x^5\right)}{x^4\,\left(2\,a\,x^5+c\,x^4+2\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b + a*x^5)^(3/4)*(4*b - a*x^5))/(x^4*(2*b + 2*a*x^5 + c*x^4)),x)
```

```
[Out] int(-((b + a*x^5)^(3/4)*(4*b - a*x^5))/(x^4*(2*b + 2*a*x^5 + c*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^5-4b\right)\left(ax^5+b\right)^{\frac{3}{4}}}{x^4\left(2ax^5+2b+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**5-4*b)*(a*x**5+b)**(3/4)/x**4/(2*a*x**5+c*x**4+2*b),x)
```

```
[Out] Integral((a*x**5 - 4*b)*(a*x**5 + b)**(3/4)/(x**4*(2*a*x**5 + 2*b + c*x**4)
), x)
```

3.1769 $\int \frac{-1+x^6}{\sqrt[3]{x^2+x^4}\left(1+x^6\right)}\,dx$

Optimal. Leaf size=154

$$-\frac{\left(x^4+x^2\right)^{2/3}}{x\left(x^2+1\right)}+\frac{\tan^{-1}\left(\frac{3^{2/3}x\sqrt[3]{x^4+x^2}}{\sqrt[3]{3}x^2-\left(x^4+x^2\right)^{2/3}}\right)}{3^{2/3}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[6]{3}x}{\sqrt[3]{x^4+x^2}}\right)}{3\sqrt[6]{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{3}x^2+\frac{\left(x^4+x^2\right)^{2/3}}{\sqrt[6]{3}}}{x\sqrt[3]{x^4+x^2}}\right)}{3\sqrt[6]{3}}$$

Rubi [F] time = 2.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+x^6}{\sqrt[3]{x^2+x^4}\left(1+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x^6)/((x^2 + x^4)^(1/3)*(1 + x^6)),x]
```

```
[Out] (-2*x*(1 + x^2)^(1/3)*AppellF1[1/6, 1/3, 1, 7/6, -x^2, (2*x^2)/(1 - I*Sqrt[
3])])/(x^2 + x^4)^(1/3) - (2*x*(1 + x^2)^(1/3)*AppellF1[1/6, 1/3, 1, 7/6, -
x^2, (2*x^2)/(1 + I*Sqrt[3])])/(x^2 + x^4)^(1/3) + (3*x*(1 + x^2)^(1/3)*Hyp
ergeometric2F1[1/6, 1/3, 7/6, -x^2])/(x^2 + x^4)^(1/3) - ((I/3)*x^(2/3)*(1
 + x^2)^(1/3)*Defer[Subst][Defer[Int][1/((I - x)*(1 + x^6)^(1/3)), x], x, x^
(1/3)])/(x^2 + x^4)^(1/3) - ((I/3)*x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Def
er[Int][1/((I + x)*(1 + x^6)^(1/3)), x], x, x^(1/3)])/(x^2 + x^4)^(1/3) - (
Sqrt[1 - I*Sqrt[3]]*x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][1/((Sqr
t[1 - I*Sqrt[3]] - Sqrt[2]*x)*(1 + x^6)^(1/3)), x], x, x^(1/3)])/(3*(x^2 +
x^4)^(1/3)) - (Sqrt[1 + I*Sqrt[3]]*x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Def
er[Int][1/((Sqrt[1 + I*Sqrt[3]] - Sqrt[2]*x)*(1 + x^6)^(1/3)), x], x, x^(1/
3)])/(3*(x^2 + x^4)^(1/3)) - (Sqrt[1 - I*Sqrt[3]]*x^(2/3)*(1 + x^2)^(1/3)*D
efer[Subst][Defer[Int][1/((Sqrt[1 - I*Sqrt[3]] + Sqrt[2]*x)*(1 + x^6)^(1/3)
), x], x, x^(1/3)])/(3*(x^2 + x^4)^(1/3)) - (Sqrt[1 + I*Sqrt[3]]*x^(2/3)*(1
 + x^2)^(1/3)*Defer[Subst][Defer[Int][1/((Sqrt[1 + I*Sqrt[3]] + Sqrt[2]*x)*
(1 + x^6)^(1/3)), x], x, x^(1/3)])/(3*(x^2 + x^4)^(1/3))
```

Rubi steps

$$\int \frac{-1+x^6}{\sqrt[3]{x^2+x^4}\left(1+x^6\right)}\,dx = \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{-1+x^6}{x^{2/3}\sqrt[3]{1+x^2}\left(1+x^6\right)}\,dx}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{-1+x^{18}}{\sqrt[3]{1+x^6}\left(1+x^{18}\right)}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1+x^6}} - \frac{2}{\sqrt[3]{1+x^6}\left(1+x^{18}\right)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(6x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^6}\left(1+x^{1\cdots}\right.}\right.}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\ {}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(6x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int\left(\frac{1}{9\left(1+x^2\right)\sqrt[3]{1+x^6}} + \frac{2-\cdots}{9\left(1-x^2+x^4\cdots\right.}\right.\right.}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\ {}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(2x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{3\sqrt[3]{x^2+x^4}} - \cdots$$

$$= \frac{3x\sqrt[3]{1+x^2}\ {}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(2x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int\left(\frac{i}{2(i-x)\sqrt[3]{1+x^6}} + \frac{i}{2(i+x)\sqrt[3]{1+x^{\cdots}}}\right.\right.}{3\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\ {}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(ix^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{(i-x)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{3\sqrt[3]{x^2+x^4}} - \cdots$$

$$= -\frac{2x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};\frac{1}{3},1;\frac{7}{6};-x^2,\frac{2x^2}{1-i\sqrt{3}}\right)}{\sqrt[3]{x^2+x^4}} - \frac{2x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};\frac{1}{3},1;\frac{7}{6};-x^2,\frac{2x^2}{1+i\sqrt{3}}\right)}{\sqrt[3]{x^2+x^4}} + \frac{3x\cdots}{\cdots}$$

$$= -\frac{2x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};\frac{1}{3},1;\frac{7}{6};-x^2,\frac{2x^2}{1-i\sqrt{3}}\right)}{\sqrt[3]{x^2+x^4}} - \frac{2x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};\frac{1}{3},1;\frac{7}{6};-x^2,\frac{2x^2}{1+i\sqrt{3}}\right)}{\sqrt[3]{x^2+x^4}} + \frac{3x\cdots}{\cdots}$$

Mathematica [F] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{-1+x^6}{\sqrt[3]{x^2+x^4}\left(1+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^6)/((x^2 + x^4)^(1/3)*(1 + x^6)),x]
[Out] Integrate[(-1 + x^6)/((x^2 + x^4)^(1/3)*(1 + x^6)), x]
```

IntegrateAlgebraic [A] time = 0.55, size = 154, normalized size = 1.00

$$-\frac{\left(x^4+x^2\right)^{2/3}}{x\left(x^2+1\right)}+\frac{\tan^{-1}\left(\frac{3^{2/3}x\sqrt[3]{x^4+x^2}}{\sqrt[3]{3}x^2-\left(x^4+x^2\right)^{2/3}}\right)}{3^{2/3}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[6]{3}x}{\sqrt[3]{x^4+x^2}}\right)}{3\sqrt[6]{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{3}x^2+\frac{\left(x^4+x^2\right)^{2/3}}{\sqrt[6]{3}}}{x\sqrt[3]{x^4+x^2}}\right)}{3\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^6)/((x^2 + x^4)^(1/3)*(1 + x^6)),x]
```

```
[Out] -((x^2 + x^4)^(2/3)/(x*(1 + x^2))) + ArcTan[(3^(2/3)*x*(x^2 + x^4)^(1/3))/(
3^(1/3)*x^2 - (x^2 + x^4)^(2/3))]/3^(2/3) - (2*ArcTanh[(3^(1/6)*x)/(x^2 + x
^4)^(1/3)])/(3*3^(1/6)) - ArcTanh[(3^(1/6)*x^2 + (x^2 + x^4)^(2/3)/3^(1/6))
/(x*(x^2 + x^4)^(1/3))]/(3*3^(1/6))
```

fricas [B] time = 3.07, size = 1749, normalized size = 11.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^4+x^2)^(1/3)/(x^6+1),x, algorithm="fricas")
```

```
[Out] 1/36*(2*3^(5/6)*(x^3 + x)*log((2*3^(5/6)*(13*x^5 - 45*x^4 + 65*x^3 - 45*x^2
 + 13*x) - 6*(x^4 + x^2)^(2/3)*(26*x^2 + sqrt(3)*(15*x^2 - 26*x + 15) - 45*
x + 26) + 3*3^(1/3)*(15*x^5 - 52*x^4 + 75*x^3 - 52*x^2 + 15*x) + 6*(x^4 + x
^2)^(1/3)*(3^(2/3)*(15*x^3 - 26*x^2 + 15*x) + 3^(1/6)*(26*x^3 - 45*x^2 + 26
*x)))/(x^5 - x^3 + x)) - 2*3^(5/6)*(x^3 + x)*log(-(2*3^(5/6)*(13*x^5 - 45*x
^4 + 65*x^3 - 45*x^2 + 13*x) + 6*(x^4 + x^2)^(2/3)*(26*x^2 - sqrt(3)*(15*x^
2 - 26*x + 15) - 45*x + 26) - 3*3^(1/3)*(15*x^5 - 52*x^4 + 75*x^3 - 52*x^2
+ 15*x) - 6*(x^4 + x^2)^(1/3)*(3^(2/3)*(15*x^3 - 26*x^2 + 15*x) - 3^(1/6)*(
26*x^3 - 45*x^2 + 26*x)))/(x^5 - x^3 + x)) - 3^(5/6)*(x^3 + x)*log(12*(1351
*3^(2/3)*(x^5 - x^3 + x) + 2*(x^4 + x^2)^(2/3)*(3^(5/6)*(1351*x^2 - 2340*x
+ 1351) + 3*3^(1/3)*(780*x^2 - 1351*x + 780)) + 6*(x^4 + x^2)^(1/3)*(1351*x
^3 - 2340*x^2 + sqrt(3)*(780*x^3 - 1351*x^2 + 780*x) + 1351*x) + 2340*3^(1/
6)*(x^5 - x^3 + x))/(x^5 - x^3 + x)) + 3^(5/6)*(x^3 + x)*log(12*(1351*3^(2/
3)*(x^5 - x^3 + x) - 2*(x^4 + x^2)^(2/3)*(3^(5/6)*(1351*x^2 - 2340*x + 1351
) - 3*3^(1/3)*(780*x^2 - 1351*x + 780)) + 6*(x^4 + x^2)^(1/3)*(1351*x^3 - 2
340*x^2 - sqrt(3)*(780*x^3 - 1351*x^2 + 780*x) + 1351*x) - 2340*3^(1/6)*(x^
5 - x^3 + x))/(x^5 - x^3 + x)) - 12*3^(1/3)*(x^3 + x)*arctan(1/9*(72*x^8 +
72*x^2 + 2*sqrt(3)*(2*3^(2/3)*(13*x^9 - 45*x^8 - 260*x^7 - 540*x^6 - 663*x^
5 - 540*x^4 - 260*x^3 - 45*x^2 + 13*x) - 12*(x^4 + x^2)^(2/3)*(3^(5/6)*(15*
x^5 - 104*x^4 - 15*x^3 - 104*x^2 + 15*x) - 2*3^(1/3)*(13*x^5 - 90*x^4 - 13*
x^3 - 90*x^2 + 13*x)) + 6*(26*x^7 - 135*x^6 - 312*x^5 - 405*x^4 - 312*x^3 -
 135*x^2 - 3*sqrt(3)*(5*x^7 - 26*x^6 - 60*x^5 - 78*x^4 - 60*x^3 - 26*x^2 +
5*x) + 26*x)*(x^4 + x^2)^(1/3) - 3*3^(1/6)*(15*x^9 - 52*x^8 - 300*x^7 - 624
*x^6 - 765*x^5 - 624*x^4 - 300*x^3 - 52*x^2 + 15*x))*sqrt((1351*3^(2/3)*(x^
5 - x^3 + x) + 2*(x^4 + x^2)^(2/3)*(3^(5/6)*(1351*x^2 - 2340*x + 1351) + 3*
3^(1/3)*(780*x^2 - 1351*x + 780)) + 6*(x^4 + x^2)^(1/3)*(1351*x^3 - 2340*x^
2 + sqrt(3)*(780*x^3 - 1351*x^2 + 780*x) + 1351*x) + 2340*3^(1/6)*(x^5 - x^
3 + x))/(x^5 - x^3 + x)) + 12*(x^4 + x^2)^(2/3)*(3^(2/3)*(x^6 - 12*x^4 - 12
*x^2 + 1) + 9*3^(1/6)*(x^5 + 3*x^3 + x)) + 3*sqrt(3)*(x^9 + 46*x^7 + 99*x^5
 + 46*x^3 + x) + 12*(x^4 + x^2)^(1/3)*(3^(5/6)*(x^7 + 12*x^5 + 12*x^3 + x)
+ 3*3^(1/3)*(5*x^6 + 7*x^4 + 5*x^2)))/(x^9 - 50*x^7 - 93*x^5 - 50*x^3 + x))
 - 12*3^(1/3)*(x^3 + x)*arctan(1/9*(72*x^8 + 72*x^2 + 2*sqrt(3)*(2*3^(2/3)*
(13*x^9 - 45*x^8 - 260*x^7 - 540*x^6 - 663*x^5 - 540*x^4 - 260*x^3 - 45*x^2
 + 13*x) + 12*(x^4 + x^2)^(2/3)*(3^(5/6)*(15*x^5 - 104*x^4 - 15*x^3 - 104*x
^2 + 15*x) + 2*3^(1/3)*(13*x^5 - 90*x^4 - 13*x^3 - 90*x^2 + 13*x)) + 6*(26*
x^7 - 135*x^6 - 312*x^5 - 405*x^4 - 312*x^3 - 135*x^2 + 3*sqrt(3)*(5*x^7 -
26*x^6 - 60*x^5 - 78*x^4 - 60*x^3 - 26*x^2 + 5*x) + 26*x)*(x^4 + x^2)^(1/3)
```

```
 + 3*3^(1/6)*(15*x^9 - 52*x^8 - 300*x^7 - 624*x^6 - 765*x^5 - 624*x^4 - 300
*x^3 - 52*x^2 + 15*x))*sqrt((1351*3^(2/3)*(x^5 - x^3 + x) - 2*(x^4 + x^2)^(
2/3)*(3^(5/6)*(1351*x^2 - 2340*x + 1351) - 3*3^(1/3)*(780*x^2 - 1351*x + 78
0)) + 6*(x^4 + x^2)^(1/3)*(1351*x^3 - 2340*x^2 - sqrt(3)*(780*x^3 - 1351*x^
2 + 780*x) + 1351*x) - 2340*3^(1/6)*(x^5 - x^3 + x))/(x^5 - x^3 + x)) + 12*
(x^4 + x^2)^(2/3)*(3^(2/3)*(x^6 - 12*x^4 - 12*x^2 + 1) - 9*3^(1/6)*(x^5 + 3
*x^3 + x)) - 3*sqrt(3)*(x^9 + 46*x^7 + 99*x^5 + 46*x^3 + x) - 12*(x^4 + x^2
)^(1/3)*(3^(5/6)*(x^7 + 12*x^5 + 12*x^3 + x) - 3*3^(1/3)*(5*x^6 + 7*x^4 + 5
*x^2)))/(x^9 - 50*x^7 - 93*x^5 - 50*x^3 + x)) - 36*(x^4 + x^2)^(2/3))/(x^3
+ x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 - 1}{\left(x^6 + 1\right)\left(x^4 + x^2\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^4+x^2)^(1/3)/(x^6+1),x, algorithm="giac")
[Out] integrate((x^6 - 1)/((x^6 + 1)*(x^4 + x^2)^(1/3)), x)
```

maple [C] time = 33.33, size = 7488, normalized size = 48.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)/(x^4+x^2)^(1/3)/(x^6+1),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 - 1}{\left(x^6 + 1\right)\left(x^4 + x^2\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^4+x^2)^(1/3)/(x^6+1),x, algorithm="maxima")
[Out] integrate((x^6 - 1)/((x^6 + 1)*(x^4 + x^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6 - 1}{\left(x^4 + x^2\right)^{1/3}\,\left(x^6 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)/((x^2 + x^4)^(1/3)*(x^6 + 1)),x)
[Out] int((x^6 - 1)/((x^2 + x^4)^(1/3)*(x^6 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\,(x+1)\left(x^2 - x + 1\right)\left(x^2 + x + 1\right)}{\sqrt[3]{x^2\left(x^2 + 1\right)}\,\left(x^2 + 1\right)\left(x^4 - x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)/(x**4+x**2)**(1/3)/(x**6+1),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1)/((x**2*(x**2 + 1))**
(1/3)*(x**2 + 1)*(x**4 - x**2 + 1)), x)
```

3.1770 $\int \frac{\sqrt[4]{2+3x^4}\,(4+6x^4+x^8)}{x^6(1+x^4)(1+2x^4)}\,dx$

Optimal. Leaf size=154

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{3x^4+2}}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{3x^4+2}}\right)+\frac{4\sqrt[4]{3x^4+2}\,(6x^4-1)}{5x^5}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{3x^4+2}}{\sqrt{3x^4+2}-x^2}\right)}{2\sqrt{2}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{3x}}{\sqrt{3x^4+2}}\right.}{2\sqrt{2}}$$

Rubi [C] time = 0.97, antiderivative size = 157, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {6725, 264, 277, 331, 298, 203, 206, 510}

$$\frac{10}{3}\sqrt[4]{2}\,x^3F_1\left(\frac{3}{4};1,-\frac{1}{4};\frac{7}{4};-2x^4,-\frac{3x^4}{2}\right)+\frac{1}{3}\sqrt[4]{2}\,x^3F_1\left(\frac{3}{4};1,-\frac{1}{4};\frac{7}{4};-x^4,-\frac{3x^4}{2}\right)+\frac{6\sqrt[4]{3x^4+2}}{x}+3\sqrt[4]{3}\tan^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{3x^4+2}}\right)-3\sqrt[4]{3}\tanh^{-1}\left(\frac{\sqrt[4]{3}\,x}{\sqrt[4]{3x^4+2}}\right)-\frac{2\left(3x^4+2\right)^{5/4}}{5x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((2 + 3*x^4)^(1/4)*(4 + 6*x^4 + x^8))/(x^6*(1 + x^4)*(1 + 2*x^4)),x]
```

```
[Out] (6*(2 + 3*x^4)^(1/4))/x - (2*(2 + 3*x^4)^(5/4))/(5*x^5) + (10*2^(1/4)*x^3*A
ppellF1[3/4, 1, -1/4, 7/4, -2*x^4, (-3*x^4)/2])/3 + (2^(1/4)*x^3*AppellF1[3
/4, 1, -1/4, 7/4, -x^4, (-3*x^4)/2])/3 + 3*3^(1/4)*ArcTan[(3^(1/4)*x)/(2 +
3*x^4)^(1/4)] - 3*3^(1/4)*ArcTanh[(3^(1/4)*x)/(2 + 3*x^4)^(1/4)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{2+3x^4}\left(4+6x^4+x^8\right)}{x^6\left(1+x^4\right)\left(1+2x^4\right)}\,dx=\int\left(\frac{4\sqrt[4]{2+3x^4}}{x^6}-\frac{6\sqrt[4]{2+3x^4}}{x^2}+\frac{x^2\sqrt[4]{2+3x^4}}{1+x^4}+\frac{10x^2\sqrt[4]{2+3x^4}}{1+2x^4}\right)dx$$

$$=4\int\frac{\sqrt[4]{2+3x^4}}{x^6}\,dx-6\int\frac{\sqrt[4]{2+3x^4}}{x^2}\,dx+10\int\frac{x^2\sqrt[4]{2+3x^4}}{1+2x^4}\,dx+\int\frac{x^2\sqrt[4]{2+3x}}{1+x^4}$$

$$=\frac{6\sqrt[4]{2+3x^4}}{x}-\frac{2\left(2+3x^4\right)^{5/4}}{5x^5}+\frac{10}{3}\sqrt[4]{2}\,x^3F_1\left(\frac{3}{4};1,-\frac{1}{4};\frac{7}{4};-2x^4,-\frac{3x^4}{2}\right)+\frac{1}{3}\sqrt[4]{2}\,x^3$$

$$=\frac{6\sqrt[4]{2+3x^4}}{x}-\frac{2\left(2+3x^4\right)^{5/4}}{5x^5}+\frac{10}{3}\sqrt[4]{2}\,x^3F_1\left(\frac{3}{4};1,-\frac{1}{4};\frac{7}{4};-2x^4,-\frac{3x^4}{2}\right)+\frac{1}{3}\sqrt[4]{2}\,x^3$$

$$=\frac{6\sqrt[4]{2+3x^4}}{x}-\frac{2\left(2+3x^4\right)^{5/4}}{5x^5}+\frac{10}{3}\sqrt[4]{2}\,x^3F_1\left(\frac{3}{4};1,-\frac{1}{4};\frac{7}{4};-2x^4,-\frac{3x^4}{2}\right)+\frac{1}{3}\sqrt[4]{2}\,x^3$$

$$=\frac{6\sqrt[4]{2+3x^4}}{x}-\frac{2\left(2+3x^4\right)^{5/4}}{5x^5}+\frac{10}{3}\sqrt[4]{2}\,x^3F_1\left(\frac{3}{4};1,-\frac{1}{4};\frac{7}{4};-2x^4,-\frac{3x^4}{2}\right)+\frac{1}{3}\sqrt[4]{2}\,x^3$$

Mathematica [C] time = 0.21, size = 94, normalized size = 0.61

$$\frac{x^3\left(5\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{x^4}{3x^4+2}\right)-{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{x^4}{3x^4+2}\right)\right)}{3\left(3x^4+2\right)^{3/4}}+\left(\frac{24}{5x}-\frac{4}{5x^5}\right)\sqrt[4]{3x^4+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^4)^(1/4)*(4 + 6*x^4 + x^8))/(x^6*(1 + x^4)*(1 + 2*x^4))
,x]
```

```
[Out] (-4/(5*x^5) + 24/(5*x))*(2 + 3*x^4)^(1/4) + (x^3*(5*Hypergeometric2F1[3/4,
1, 7/4, -(x^4/(2 + 3*x^4))] - Hypergeometric2F1[3/4, 1, 7/4, x^4/(2 + 3*x^4
)]))/(3*(2 + 3*x^4)^(3/4))
```

IntegrateAlgebraic [A] time = 0.42, size = 154, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt[4]{3x^4+2}}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt[4]{3x^4+2}}\right)+\frac{4\sqrt[4]{3x^4+2}\left(6x^4-1\right)}{5x^5}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{3x^4+2}}{\sqrt{3x^4+2}-x^2}\right)}{2\sqrt{2}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{3x^4+2}}{\sqrt{3x^4+2}+x^2}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 3*x^4)^(1/4)*(4 + 6*x^4 + x^8))/(x^6*(1 + x^4)*(1
+ 2*x^4)),x]
```

```
[Out] (4*(2 + 3*x^4)^(1/4)*(-1 + 6*x^4))/(5*x^5) + ArcTan[x/(2 + 3*x^4)^(1/4)]/2
+ (5*ArcTan[(Sqrt[2]*x*(2 + 3*x^4)^(1/4))/(-x^2 + Sqrt[2 + 3*x^4])])/(2*Sqr
t[2]) - ArcTanh[x/(2 + 3*x^4)^(1/4)]/2 - (5*ArcTanh[(Sqrt[2]*x*(2 + 3*x^4)^
(1/4))/(x^2 + Sqrt[2 + 3*x^4])])/(2*Sqrt[2])
```

fricas [B] time = 5.78, size = 724, normalized size = 4.70

$$\frac{100\sqrt{2}x^5\arctan\left(-\frac{4x^8+4x^4+\sqrt{2}(3x^4+2)^{\frac{3}{4}}x+\sqrt{2}(4x^7+3x^3)(3x^4+2)^{\frac{1}{4}}+2(2x^6+x^2)\sqrt{3x^4+2}-\left(4(3x^4+2)^{\frac{3}{4}}x^5+\sqrt{2}\sqrt{3x^4+2}x^2-\sqrt{2}(4x^8+x^4-1)+2(2x^7+x^3)(3x^4+2)^{\frac{1}{4}}\right)\sqrt{\frac{2x^4+\sqrt{2}(3x^4+2)^{\frac{1}{4}}x^3+2\sqrt{3x^4+2}x^2+\sqrt{2}(3x^4+2)^{\frac{3}{4}}x+1}{2x^4+1}}+1}{8x^8+4x^4-1}\right)-100\sqrt{2}x^5\arctan\left(-\frac{4x^8+4x^4-\sqrt{2}(3x^4+2)^{\frac{3}{4}}x-\sqrt{2}(4x^7+3x^3)(3x^4+2)^{\frac{1}{4}}+2(2x^6+x^2)\sqrt{3x^4+2}-\left(4(3x^4+2)^{\frac{3}{4}}x^5-\sqrt{2}\sqrt{3x^4+2}x^2+\sqrt{2}(4x^8+x^4-1)+2(2x^7+x^3)(3x^4+2)^{\frac{1}{4}}\right)\sqrt{\frac{2x^4-\sqrt{2}(3x^4+2)^{\frac{1}{4}}x^3+2\sqrt{3x^4+2}x^2-\sqrt{2}(3x^4+2)^{\frac{3}{4}}x+1}{2x^4+1}}+1}{8x^8+4x^4-1}\right)-25\sqrt{2}x^5\log\left(\frac{4\left(2x^4+\sqrt{2}(3x^4+2)^{\frac{1}{4}}x^3+2\sqrt{3x^4+2}x^2+\sqrt{2}(3x^4+2)^{\frac{3}{4}}x+1\right)}{2x^4+1}\right)+25\sqrt{2}x^5\log\left(\frac{4\left(2x^4-\sqrt{2}(3x^4+2)^{\frac{1}{4}}x^3+2\sqrt{3x^4+2}x^2-\sqrt{2}(3x^4+2)^{\frac{3}{4}}x+1\right)}{2x^4+1}\right)+20x^5\arctan\left(\frac{(3x^4+2)^{\frac{1}{4}}x^3+(3x^4+2)^{\frac{3}{4}}x}{x^4+1}\right)+20x^5\log\left(-\frac{2x^4-(3x^4+2)^{\frac{1}{4}}x^3+\sqrt{3x^4+2}x^2-(3x^4+2)^{\frac{3}{4}}x+1}{x^4+1}\right)+64(6x^4-1)(3x^4+2)^{\frac{1}{4}}}{80x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+2)^(1/4)*(x^8+6*x^4+4)/x^6/(x^4+1)/(2*x^4+1),x, algorithm=
"fricas")
```

```
[Out] 1/80*(100*sqrt(2)*x^5*arctan(-(4*x^8 + 4*x^4 + sqrt(2)*(3*x^4 + 2)^(3/4)*x
+ sqrt(2)*(4*x^7 + 3*x^3)*(3*x^4 + 2)^(1/4) + 2*(2*x^6 + x^2)*sqrt(3*x^4 +
2) - (4*(3*x^4 + 2)^(3/4)*x^5 + sqrt(2)*sqrt(3*x^4 + 2)*x^2 - sqrt(2)*(4*x^
8 + x^4 - 1) + 2*(2*x^7 + x^3)*(3*x^4 + 2)^(1/4))*sqrt((2*x^4 + sqrt(2)*(3*
x^4 + 2)^(1/4)*x^3 + 2*sqrt(3*x^4 + 2)*x^2 + sqrt(2)*(3*x^4 + 2)^(3/4)*x +
1)/(2*x^4 + 1)) + 1)/(8*x^8 + 4*x^4 - 1)) - 100*sqrt(2)*x^5*arctan(-(4*x^8
+ 4*x^4 - sqrt(2)*(3*x^4 + 2)^(3/4)*x - sqrt(2)*(4*x^7 + 3*x^3)*(3*x^4 + 2)
^(1/4) + 2*(2*x^6 + x^2)*sqrt(3*x^4 + 2) - (4*(3*x^4 + 2)^(3/4)*x^5 - sqrt(
2)*sqrt(3*x^4 + 2)*x^2 + sqrt(2)*(4*x^8 + x^4 - 1) + 2*(2*x^7 + x^3)*(3*x^4
 + 2)^(1/4))*sqrt((2*x^4 - sqrt(2)*(3*x^4 + 2)^(1/4)*x^3 + 2*sqrt(3*x^4 + 2
)*x^2 - sqrt(2)*(3*x^4 + 2)^(3/4)*x + 1)/(2*x^4 + 1)) + 1)/(8*x^8 + 4*x^4 -
 1)) - 25*sqrt(2)*x^5*log(4*(2*x^4 + sqrt(2)*(3*x^4 + 2)^(1/4)*x^3 + 2*sqrt
(3*x^4 + 2)*x^2 + sqrt(2)*(3*x^4 + 2)^(3/4)*x + 1)/(2*x^4 + 1)) + 25*sqrt(2
)*x^5*log(4*(2*x^4 - sqrt(2)*(3*x^4 + 2)^(1/4)*x^3 + 2*sqrt(3*x^4 + 2)*x^2
- sqrt(2)*(3*x^4 + 2)^(3/4)*x + 1)/(2*x^4 + 1)) + 20*x^5*arctan(((3*x^4 + 2
)^(1/4)*x^3 + (3*x^4 + 2)^(3/4)*x)/(x^4 + 1)) + 20*x^5*log(-(2*x^4 - (3*x^4
 + 2)^(1/4)*x^3 + sqrt(3*x^4 + 2)*x^2 - (3*x^4 + 2)^(3/4)*x + 1)/(x^4 + 1))
 + 64*(6*x^4 - 1)*(3*x^4 + 2)^(1/4))/x^5
```

giac [A] time = 0.17, size = 221, normalized size = 1.44

$$-\frac{5}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2(3x^4+2)^{\frac{1}{4}}}{x}\right)\right)-\frac{5}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2(3x^4+2)^{\frac{1}{4}}}{x}\right)\right)-\frac{5}{8}\sqrt{2}\log\left(\frac{\sqrt{2}(3x^4+2)^{\frac{1}{4}}}{x}+\frac{\sqrt{3x^4+2}}{x^2}+1\right)+\frac{5}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}(3x^4+2)^{\frac{1}{4}}}{x}+\frac{\sqrt{3x^4+2}}{x^2}+1\right)-\frac{2(3x^4+2)^{\frac{1}{4}}\left(\frac{2}{x^4}+3\right)}{5x}+\frac{6(3x^4+2)^{\frac{1}{4}}}{x}-\frac{1}{2}\arctan\left(\frac{(3x^4+2)^{\frac{1}{4}}}{x}\right)-\frac{1}{4}\log\left(\frac{(3x^4+2)^{\frac{1}{4}}}{x}+1\right)+\frac{1}{4}\log\left(\frac{(3x^4+2)^{\frac{1}{4}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+2)^(1/4)*(x^8+6*x^4+4)/x^6/(x^4+1)/(2*x^4+1),x, algorithm=
"giac")
```

```
[Out] -5/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^4 + 2)^(1/4)/x)) - 5/4*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^4 + 2)^(1/4)/x)) - 5/8*sqrt(2)*
log(sqrt(2)*(3*x^4 + 2)^(1/4)/x + sqrt(3*x^4 + 2)/x^2 + 1) + 5/8*sqrt(2)*lo
g(-sqrt(2)*(3*x^4 + 2)^(1/4)/x + sqrt(3*x^4 + 2)/x^2 + 1) - 2/5*(3*x^4 + 2)
^(1/4)*(2/x^4 + 3)/x + 6*(3*x^4 + 2)^(1/4)/x - 1/2*arctan((3*x^4 + 2)^(1/4)
/x) - 1/4*log((3*x^4 + 2)^(1/4)/x + 1) + 1/4*log((3*x^4 + 2)^(1/4)/x - 1)
```

maple [C] time = 3.55, size = 987, normalized size = 6.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4+2)^(1/4)*(x^8+6*x^4+4)/x^6/(x^4+1)/(2*x^4+1),x)
```

```
[Out] 4/5*(18*x^8+9*x^4-2)/x^5/(3*x^4+2)^(3/4)+(1/4*ln((-18*x^12+9*(27*x^12+54*x^
8+36*x^4+8)^(1/4)*x^9-3*(27*x^12+54*x^8+36*x^4+8)^(1/2)*x^6-33*x^8+(27*x^12
```

```
+54*x^8+36*x^4+8)^(3/4)*x^3+12*(27*x^12+54*x^8+36*x^4+8)^(1/4)*x^5-2*(27*x^
12+54*x^8+36*x^4+8)^(1/2)*x^2-20*x^4+4*(27*x^12+54*x^8+36*x^4+8)^(1/4)*x-4)
/(x^4+1)/(3*x^4+2)^2)+1/4*RootOf(_Z^2+1)*ln((18*x^12-9*RootOf(_Z^2+1)*(27*x
^12+54*x^8+36*x^4+8)^(1/4)*x^9-3*(27*x^12+54*x^8+36*x^4+8)^(1/2)*x^6+33*x^8
+RootOf(_Z^2+1)*(27*x^12+54*x^8+36*x^4+8)^(3/4)*x^3-12*RootOf(_Z^2+1)*(27*x
^12+54*x^8+36*x^4+8)^(1/4)*x^5-2*(27*x^12+54*x^8+36*x^4+8)^(1/2)*x^2+20*x^4
-4*RootOf(_Z^2+1)*(27*x^12+54*x^8+36*x^4+8)^(1/4)*x+4)/(x^4+1)/(3*x^4+2)^2)
+5/4*RootOf(_Z^2+RootOf(_Z^2+1))*ln(-(-9*RootOf(_Z^2+1)*x^12+9*RootOf(_Z^2+
RootOf(_Z^2+1))*RootOf(_Z^2+1)*(27*x^12+54*x^8+36*x^4+8)^(1/4)*x^9-21*RootO
f(_Z^2+1)*x^8-3*(27*x^12+54*x^8+36*x^4+8)^(1/2)*x^6+12*RootOf(_Z^2+RootOf(_
Z^2+1))*RootOf(_Z^2+1)*(27*x^12+54*x^8+36*x^4+8)^(1/4)*x^5+RootOf(_Z^2+Root
Of(_Z^2+1))*(27*x^12+54*x^8+36*x^4+8)^(3/4)*x^3-16*RootOf(_Z^2+1)*x^4-2*(27
*x^12+54*x^8+36*x^4+8)^(1/2)*x^2+4*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2+
1)*(27*x^12+54*x^8+36*x^4+8)^(1/4)*x-4*RootOf(_Z^2+1))/(2*x^4+1)/(3*x^4+2)^
2)-5/4*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))*ln((-9*RootOf(_Z^2+1)*x^1
2+9*RootOf(_Z^2+RootOf(_Z^2+1))*(27*x^12+54*x^8+36*x^4+8)^(1/4)*x^9-21*Root
Of(_Z^2+1)*x^8+RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2+1)*(27*x^12+54*x^8+3
6*x^4+8)^(3/4)*x^3+3*(27*x^12+54*x^8+36*x^4+8)^(1/2)*x^6+12*RootOf(_Z^2+Roo
tOf(_Z^2+1))*(27*x^12+54*x^8+36*x^4+8)^(1/4)*x^5-16*RootOf(_Z^2+1)*x^4+2*(2
7*x^12+54*x^8+36*x^4+8)^(1/2)*x^2+4*RootOf(_Z^2+RootOf(_Z^2+1))*(27*x^12+54
*x^8+36*x^4+8)^(1/4)*x-4*RootOf(_Z^2+1))/(2*x^4+1)/(3*x^4+2)^2))/(3*x^4+2)^
(3/4)*((3*x^4+2)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^8+6x^4+4)(3x^4+2)^{\frac{1}{4}}}{(2x^4+1)(x^4+1)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+2)^(1/4)*(x^8+6*x^4+4)/x^6/(x^4+1)/(2*x^4+1),x, algorithm=
"maxima")
```

```
[Out] integrate((x^8 + 6*x^4 + 4)*(3*x^4 + 2)^(1/4)/((2*x^4 + 1)*(x^4 + 1)*x^6),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3x^4+2)^{1/4}\,(x^8+6x^4+4)}{x^6\,(x^4+1)\,(2x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^4 + 2)^(1/4)*(6*x^4 + x^8 + 4))/(x^6*(x^4 + 1)*(2*x^4 + 1)),x)
```

```
[Out] int(((3*x^4 + 2)^(1/4)*(6*x^4 + x^8 + 4))/(x^6*(x^4 + 1)*(2*x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{3x^4+2}\,(x^8+6x^4+4)}{x^6\,(x^4+1)\,(2x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4+2)**(1/4)*(x**8+6*x**4+4)/x**6/(x**4+1)/(2*x**4+1),x)
```

```
[Out] Integral((3*x**4 + 2)**(1/4)*(x**8 + 6*x**4 + 4)/(x**6*(x**4 + 1)*(2*x**4 +
 1)), x)
```

3.1771 $\int \frac{\sqrt[8]{256-256x^2+96x^4-16x^6+x^8}}{-1+x^3}\,dx$

Optimal. Leaf size=154

$$\frac{\sqrt[8]{\left(x^2-4\right)^4}\left(\frac{2\tan^{-1}\left(\frac{\sqrt{x^2-4}}{\sqrt{3}(x-2)}\right)}{\sqrt{3}}-\frac{1}{3}\sqrt{2\left(-3-5i\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{1-\frac{2i}{\sqrt{3}}}\sqrt{x^2-4}}{x-2}\right)-\frac{1}{3}\sqrt{2\left(-3+5i\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{1+\frac{2i}{\sqrt{3}}}\sqrt{x^2}}{x-2}\right.\right.}{\sqrt{x^2-4}}$$

Rubi [A] time = 0.84, antiderivative size = 212, normalized size of antiderivative = 1.38, number of steps used = 19, number of rules used = 14, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2074, 6720, 735, 844, 217, 206, 725, 204, 1020, 1080, 1037, 1031, 207, 203}

$$\frac{\sqrt[8]{\left(x^2-4\right)^4}\tan^{-1}\left(\frac{4-x}{\sqrt{3}\sqrt{x^2-4}}\right)}{\sqrt{3}\sqrt{x^2-4}}-\frac{\sqrt{2\sqrt{21}-3}\sqrt[8]{\left(x^2-4\right)^4}\tan^{-1}\left(\frac{\left(21-4\sqrt{21}\right)x+\sqrt{21}+21}{\sqrt{21\left(2\sqrt{21}-3\right)}\sqrt{x^2-4}}\right)}{3\sqrt{x^2-4}}+\frac{\sqrt{3+2\sqrt{21}}\sqrt[8]{\left(x^2-4\right)^4}\tanh^{-1}\left(\frac{\left(21+4\sqrt{21}\right)x-\sqrt{21}+21}{\sqrt{21\left(3+2\sqrt{21}\right)}\sqrt{x^2-4}}\right)}{3\sqrt{x^2-4}}$$

Antiderivative was successfully verified.

```
[In] Int[(256 - 256*x^2 + 96*x^4 - 16*x^6 + x^8)^(1/8)/(-1 + x^3),x]
```

```
[Out] (((-4 + x^2)^4)^(1/8)*ArcTan[(4 - x)/(Sqrt[3]*Sqrt[-4 + x^2])])/(Sqrt[3]*Sq
rt[-4 + x^2]) - (Sqrt[-3 + 2*Sqrt[21]]*((-4 + x^2)^4)^(1/8)*ArcTan[(21 + Sq
rt[21] + (21 - 4*Sqrt[21])*x)/(Sqrt[21*(-3 + 2*Sqrt[21])]*Sqrt[-4 + x^2])])
/(3*Sqrt[-4 + x^2]) + (Sqrt[3 + 2*Sqrt[21]]*((-4 + x^2)^4)^(1/8)*ArcTanh[(2
1 - Sqrt[21] + (21 + 4*Sqrt[21])*x)/(Sqrt[21*(3 + 2*Sqrt[21])]*Sqrt[-4 + x^
2])])/(3*Sqrt[-4 + x^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1031

```
Int[((g_) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b
 - 2*a*h)*(b^2 - 4*a*c) - b*d*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g
*c)*x, x]/Sqrt[d + f*x^2]], x] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^
2 - 4*a*c, 0] && EqQ[b*h^2*d - 2*g*h*(c*d - a*f) - g^2*b*f, 0]
```

Rule 1037

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + b^2*d*f, 2]}, Dist
[1/(2*q), Int[Simp[h*b*d - g*(c*d - a*f - q) + (h*(c*d - a*f + q) + g*b*f)*
x, x]/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[
h*b*d - g*(c*d - a*f + q) + (h*(c*d - a*f - q) + g*b*f)*x, x]/((a + b*x + c
*x^2)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b
^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
```

```
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\sqrt[8]{256-256x^2+96x^4-16x^6+x^8}}{-1+x^3}\,dx = \int \left(\frac{\sqrt[8]{\left(-4+x^2\right)^4}}{3(-1+x)}+\frac{(-2-x)\sqrt[8]{\left(-4+x^2\right)^4}}{3\left(1+x+x^2\right)}\right)dx$$

$$= \frac{1}{3}\int \frac{\sqrt[8]{\left(-4+x^2\right)^4}}{-1+x}\,dx+\frac{1}{3}\int \frac{(-2-x)\sqrt[8]{\left(-4+x^2\right)^4}}{1+x+x^2}\,dx$$

$$= \frac{\sqrt[8]{\left(-4+x^2\right)^4}\int \frac{\sqrt{-4+x^2}}{-1+x}\,dx}{3\sqrt{-4+x^2}}+\frac{\sqrt[8]{\left(-4+x^2\right)^4}\int \frac{(-2-x)\sqrt{-4+x^2}}{1+x+x^2}\,dx}{3\sqrt{-4+x^2}}$$

$$= \frac{\sqrt[8]{\left(-4+x^2\right)^4}\int \frac{-4+x}{(-1+x)\sqrt{-4+x^2}}\,dx}{3\sqrt{-4+x^2}}+\frac{\sqrt[8]{\left(-4+x^2\right)^4}\int \frac{8+5x-x^2}{\sqrt{-4+x^2}\left(1+x+x^2\right)}}{3\sqrt{-4+x^2}}$$

$$= \frac{\sqrt[8]{\left(-4+x^2\right)^4}\int \frac{9+6x}{\sqrt{-4+x^2}\left(1+x+x^2\right)}\,dx}{3\sqrt{-4+x^2}}-\frac{\sqrt[8]{\left(-4+x^2\right)^4}\int \frac{1}{(-1+x)\sqrt{-4+x^2}}}{\sqrt{-4+x^2}}$$

$$= \frac{\sqrt[8]{\left(-4+x^2\right)^4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,\frac{-4+x}{\sqrt{-4+x^2}}\right)}{\sqrt{-4+x^2}}+\frac{\sqrt[8]{\left(-4+x^2\right)^4}\int \frac{3(7}{6\sqrt{21}}$$

$$= \frac{\sqrt[8]{\left(-4+x^2\right)^4}\tan^{-1}\left(\frac{4-x}{\sqrt{3}\sqrt{-4+x^2}}\right)}{\sqrt{3}\sqrt{-4+x^2}}+\frac{\left(4\sqrt{21}\left(3-2\sqrt{21}\right)\sqrt[8]{\left(-4+x^2\right.}\right.}{}$$

$$= \frac{\sqrt[8]{\left(-4+x^2\right)^4}\tan^{-1}\left(\frac{4-x}{\sqrt{3}\sqrt{-4+x^2}}\right)}{\sqrt{3}\sqrt{-4+x^2}}-\frac{\sqrt{-3+2\sqrt{21}}\sqrt[8]{\left(-4+x^2\right)^4}\tan}{3\sqrt{-4+}}$$

Mathematica [A] time = 0.30, size = 157, normalized size = 1.02

$$\frac{\sqrt[8]{\left(x^2-4\right)^4}\left(-\sqrt{3}\tan^{-1}\left(\frac{x-4}{\sqrt{3}\sqrt{x^2-4}}\right)+(-1)^{2/3}\sqrt{1+4\sqrt[3]{-1}}\tanh^{-1}\left(\frac{4(-1)^{2/3}-x}{\sqrt{1+4\sqrt[3]{-1}}\sqrt{x^2-4}}\right)+\sqrt[3]{-1}\sqrt{1-4(-1)^{2/3}}\tanh^{-1}\left(\frac{x+4\sqrt[3]{-1}}{\sqrt{1-4(-1)^{2/3}}\sqrt{x^2-4}}\right)\right)}{3\sqrt{x^2-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(256 - 256*x^2 + 96*x^4 - 16*x^6 + x^8)^(1/8)/(-1 + x^3),x]
```

```
[Out] (((-4 + x^2)^4)^(1/8)*(-(Sqrt[3]*ArcTan[(-4 + x)/(Sqrt[3]*Sqrt[-4 + x^2])])
 + (-1)^(2/3)*Sqrt[1 + 4*(-1)^(1/3)]*ArcTanh[(4*(-1)^(2/3) - x)/(Sqrt[1 + 4
*(-1)^(1/3)]*Sqrt[-4 + x^2])] + (-1)^(1/3)*Sqrt[1 - 4*(-1)^(2/3)]*ArcTanh[(
4*(-1)^(1/3) + x)/(Sqrt[1 - 4*(-1)^(2/3)]*Sqrt[-4 + x^2])]))/(3*Sqrt[-4 + x
^2])
```

IntegrateAlgebraic [A] time = 20.25, size = 154, normalized size = 1.00

$$\frac{\sqrt[8]{\left(x^2-4\right)^4}\left(\frac{2\tan^{-1}\left(\frac{\sqrt{x^2-4}}{\sqrt{3}(x-2)}\right)}{\sqrt{3}}-\frac{1}{3}\sqrt{2\left(-3-5i\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{1-\frac{2i}{\sqrt{3}}}\sqrt{x^2-4}}{x-2}\right)-\frac{1}{3}\sqrt{2\left(-3+5i\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{1+\frac{2i}{\sqrt{3}}}\sqrt{x^2-4}}{x-2}\right)\right)}{\sqrt{x^2-4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(256 - 256*x^2 + 96*x^4 - 16*x^6 + x^8)^(1/8)/(-1 + x^3)
,x]
```

```
[Out] (((-4 + x^2)^4)^(1/8)*((2*ArcTan[Sqrt[-4 + x^2]/(Sqrt[3]*(-2 + x))])/Sqrt[3
] - (Sqrt[2*(-3 - (5*I)*Sqrt[3])]*ArcTan[(Sqrt[1 - (2*I)/Sqrt[3]]*Sqrt[-4 +
 x^2])/(-2 + x)])/3 - (Sqrt[2*(-3 + (5*I)*Sqrt[3])]*ArcTan[(Sqrt[1 + (2*I)/
Sqrt[3]]*Sqrt[-4 + x^2])/(-2 + x)])/3))/Sqrt[-4 + x^2]
```

fricas [B] time = 0.50, size = 1214, normalized size = 7.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-16*x^6+96*x^4-256*x^2+256)^(1/8)/(x^3-1),x, algorithm="frica
s")
```

```
[Out] -1/63*21^(3/4)*sqrt(3)*sqrt(-4*sqrt(21) + 56)*arctan(1/1764000*sqrt(2)*sqrt
(70*x^2 - 21^(1/4)*(sqrt(21)*(2*x + 3) - 7*x + 7)*sqrt(-4*sqrt(21) + 56) +
(x^8 - 16*x^6 + 96*x^4 - 256*x^2 + 256)^(1/8)*(21^(1/4)*(2*sqrt(21) - 7)*sq
rt(-4*sqrt(21) + 56) - 140*x - 70) + 70*x + 70*sqrt(21) + 70*(x^8 - 16*x^6
+ 96*x^4 - 256*x^2 + 256)^(1/4) + 70)*(sqrt(35)*(21^(3/4)*(13*sqrt(21)*sqrt
(3) + 57*sqrt(3)) + 3*21^(1/4)*(11*sqrt(21)*sqrt(3) - 21*sqrt(3)))*sqrt(-4*
sqrt(21) + 56) + 30*sqrt(35)*(sqrt(21)*(3*sqrt(21)*sqrt(3) + 7*sqrt(3)) + 6
3*sqrt(21)*sqrt(3) - 273*sqrt(3))) + 1/120*sqrt(21)*sqrt(3)*(9*x + 4) + 1/8
40*sqrt(21)*(sqrt(21)*sqrt(3)*(3*x + 8) + 7*sqrt(3)*(x - 4)) - 1/40*sqrt(3)
*(13*x + 8) + 1/25200*(21^(3/4)*(sqrt(21)*sqrt(3)*(13*x + 8) + 3*sqrt(3)*(1
9*x + 4)) + 3*21^(1/4)*(sqrt(21)*sqrt(3)*(11*x + 76) - 21*sqrt(3)*(x + 16))
)*sqrt(-4*sqrt(21) + 56) - 1/25200*(x^8 - 16*x^6 + 96*x^4 - 256*x^2 + 256)^
(1/8)*(30*sqrt(21)*(3*sqrt(21)*sqrt(3) + 7*sqrt(3)) + (21^(3/4)*(13*sqrt(21
)*sqrt(3) + 57*sqrt(3)) + 3*21^(1/4)*(11*sqrt(21)*sqrt(3) - 21*sqrt(3)))*sq
rt(-4*sqrt(21) + 56) + 1890*sqrt(21)*sqrt(3) - 8190*sqrt(3))) - 1/63*21^(3/
4)*sqrt(3)*sqrt(-4*sqrt(21) + 56)*arctan(1/1764000*sqrt(2)*sqrt(70*x^2 + 21
^(1/4)*(sqrt(21)*(2*x + 3) - 7*x + 7)*sqrt(-4*sqrt(21) + 56) - (x^8 - 16*x^
6 + 96*x^4 - 256*x^2 + 256)^(1/8)*(21^(1/4)*(2*sqrt(21) - 7)*sqrt(-4*sqrt(2
1) + 56) + 140*x + 70) + 70*x + 70*sqrt(21) + 70*(x^8 - 16*x^6 + 96*x^4 - 2
56*x^2 + 256)^(1/4) + 70)*(sqrt(35)*(21^(3/4)*(13*sqrt(21)*sqrt(3) + 57*sqr
t(3)) + 3*21^(1/4)*(11*sqrt(21)*sqrt(3) - 21*sqrt(3)))*sqrt(-4*sqrt(21) + 5
6) - 30*sqrt(35)*(sqrt(21)*(3*sqrt(21)*sqrt(3) + 7*sqrt(3)) + 63*sqrt(21)*s
qrt(3) - 273*sqrt(3))) - 1/120*sqrt(21)*sqrt(3)*(9*x + 4) - 1/840*sqrt(21)*
(sqrt(21)*sqrt(3)*(3*x + 8) + 7*sqrt(3)*(x - 4)) + 1/40*sqrt(3)*(13*x + 8)
+ 1/25200*(21^(3/4)*(sqrt(21)*sqrt(3)*(13*x + 8) + 3*sqrt(3)*(19*x + 4)) +
3*21^(1/4)*(sqrt(21)*sqrt(3)*(11*x + 76) - 21*sqrt(3)*(x + 16)))*sqrt(-4*sq
rt(21) + 56) + 1/25200*(x^8 - 16*x^6 + 96*x^4 - 256*x^2 + 256)^(1/8)*(30*sq
rt(21)*(3*sqrt(21)*sqrt(3) + 7*sqrt(3)) - (21^(3/4)*(13*sqrt(21)*sqrt(3) +
57*sqrt(3)) + 3*21^(1/4)*(11*sqrt(21)*sqrt(3) - 21*sqrt(3)))*sqrt(-4*sqrt(2
1) + 56) + 1890*sqrt(21)*sqrt(3) - 8190*sqrt(3))) + 1/420*21^(1/4)*(sqrt(21
) + 14)*sqrt(-4*sqrt(21) + 56)*log(16*x^2 + 8/35*21^(1/4)*(sqrt(21)*(2*x +
3) - 7*x + 7)*sqrt(-4*sqrt(21) + 56) - 8/35*(x^8 - 16*x^6 + 96*x^4 - 256*x^
2 + 256)^(1/8)*(21^(1/4)*(2*sqrt(21) - 7)*sqrt(-4*sqrt(21) + 56) + 140*x +
70) + 16*x + 16*sqrt(21) + 16*(x^8 - 16*x^6 + 96*x^4 - 256*x^2 + 256)^(1/4)
 + 16) - 1/420*21^(1/4)*(sqrt(21) + 14)*sqrt(-4*sqrt(21) + 56)*log(16*x^2 -
 8/35*21^(1/4)*(sqrt(21)*(2*x + 3) - 7*x + 7)*sqrt(-4*sqrt(21) + 56) + 8/35
```

```
*(x^8 - 16*x^6 + 96*x^4 - 256*x^2 + 256)^(1/8)*(21^(1/4)*(2*sqrt(21) - 7)*s
qrt(-4*sqrt(21) + 56) - 140*x - 70) + 16*x + 16*sqrt(21) + 16*(x^8 - 16*x^6
 + 96*x^4 - 256*x^2 + 256)^(1/4) + 16) - 2/3*sqrt(3)*arctan(-1/3*sqrt(3)*(x
 - 1) + 1/3*sqrt(3)*(x^8 - 16*x^6 + 96*x^4 - 256*x^2 + 256)^(1/8))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{8}-16\,x^{6}+96\,x^{4}-256\,x^{2}+256\right)^{\frac{1}{8}}}{x^{3}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-16*x^6+96*x^4-256*x^2+256)^(1/8)/(x^3-1),x, algorithm="giac"
)
```

```
[Out] integrate((x^8 - 16*x^6 + 96*x^4 - 256*x^2 + 256)^(1/8)/(x^3 - 1), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{8}-16x^{6}+96x^{4}-256x^{2}+256\right)^{\frac{1}{8}}}{x^{3}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-16*x^6+96*x^4-256*x^2+256)^(1/8)/(x^3-1),x)
```

```
[Out] int((x^8-16*x^6+96*x^4-256*x^2+256)^(1/8)/(x^3-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{8}-16\,x^{6}+96\,x^{4}-256\,x^{2}+256\right)^{\frac{1}{8}}}{x^{3}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-16*x^6+96*x^4-256*x^2+256)^(1/8)/(x^3-1),x, algorithm="maxim
a")
```

```
[Out] integrate((x^8 - 16*x^6 + 96*x^4 - 256*x^2 + 256)^(1/8)/(x^3 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(x^8-16\,x^6+96\,x^4-256\,x^2+256\right)}^{1/8}}{x^3-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((96*x^4 - 256*x^2 - 16*x^6 + x^8 + 256)^(1/8)/(x^3 - 1),x)
```

```
[Out] int((96*x^4 - 256*x^2 - 16*x^6 + x^8 + 256)^(1/8)/(x^3 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[8]{\left(x-2\right)^{4}\left(x+2\right)^{4}}}{\left(x-1\right)\left(x^{2}+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-16*x**6+96*x**4-256*x**2+256)**(1/8)/(x**3-1),x)
```

```
[Out] Integral(((x - 2)**4*(x + 2)**4)**(1/8)/((x - 1)*(x**2 + x + 1)), x)
```

3.1772

$$\int \frac{-36-6x^2+6x^3+x^6}{x\left(-6+x^3\right)\sqrt[6]{\frac{6+x^3}{-6+x^3}}\left(36-90x+122x^2-96x^3+51x^4-26x^5+15x^6-6x^7+x^8\right)}\,dx$$

Optimal. Leaf size=154

$$-\frac{1}{3}\tan^{-1}\left(\frac{\sqrt[6]{\frac{x^3+6}{x^3-6}}}{x-1}\right)-\frac{1}{6}\tan^{-1}\left(\frac{\sqrt[3]{\frac{x^3+6}{x^3-6}}-x^2+2x-1}{(x-1)\sqrt[6]{\frac{x^3+6}{x^3-6}}}\right)+\frac{\tanh^{-1}\left(\frac{\left(\sqrt{3}x-\sqrt{3}\right)\sqrt[6]{\frac{x^3+6}{x^3-6}}}{\sqrt[3]{\frac{x^3+6}{x^3-6}}+x^2-2x+1}\right)}{2\sqrt{3}}$$

Rubi [F] time = 10.60, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-36-6x^2+6x^3+x^6}{x\left(-6+x^3\right)\sqrt[6]{\frac{6+x^3}{-6+x^3}}\left(36-90x+122x^2-96x^3+51x^4-26x^5+15x^6-6x^7+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-36 - 6*x^2 + 6*x^3 + x^6)/(x*(-6 + x^3)*((6 + x^3)/(-6 + x^3))^(1/6)*
(36 - 90*x + 122*x^2 - 96*x^3 + 51*x^4 - 26*x^5 + 15*x^6 - 6*x^7 + x^8)),x]
```

```
[Out] ((6 + x^3)^(1/6)*ArcTan[(6 + x^3)^(1/6)/(-6 + x^3)^(1/6)])/(9*(-6 + x^3)^(1
/6)*(-((6 + x^3)/(6 - x^3)))^(1/6)) - ((6 + x^3)^(1/6)*ArcTan[Sqrt[3] - (2*
(6 + x^3)^(1/6))/(-6 + x^3)^(1/6)])/(18*(-6 + x^3)^(1/6)*(-((6 + x^3)/(6 -
x^3)))^(1/6)) + ((6 + x^3)^(1/6)*ArcTan[Sqrt[3] + (2*(6 + x^3)^(1/6))/(-6 +
 x^3)^(1/6)])/(18*(-6 + x^3)^(1/6)*(-((6 + x^3)/(6 - x^3)))^(1/6)) + ((6 +
x^3)^(1/6)*Log[1 - (Sqrt[3]*(6 + x^3)^(1/6))/(-6 + x^3)^(1/6) + (6 + x^3)^(
1/3)/(-6 + x^3)^(1/3)])/(12*Sqrt[3]*(-6 + x^3)^(1/6)*(-((6 + x^3)/(6 - x^3)
))^(1/6)) - ((6 + x^3)^(1/6)*Log[1 + (Sqrt[3]*(6 + x^3)^(1/6))/(-6 + x^3)^(
1/6) + (6 + x^3)^(1/3)/(-6 + x^3)^(1/3)])/(12*Sqrt[3]*(-6 + x^3)^(1/6)*(-((
6 + x^3)/(6 - x^3)))^(1/6)) - (90*(6 + x^3)^(1/6)*Defer[Int][1/((-6 + x^3)^
(5/6)*(6 + x^3)^(1/6)*(36 - 90*x + 122*x^2 - 96*x^3 + 51*x^4 - 26*x^5 + 15*
x^6 - 6*x^7 + x^8)), x])/((-6 + x^3)^(1/6)*(-((6 + x^3)/(6 - x^3)))^(1/6))
+ (116*(6 + x^3)^(1/6)*Defer[Int][x/((-6 + x^3)^(5/6)*(6 + x^3)^(1/6)*(36 -
 90*x + 122*x^2 - 96*x^3 + 51*x^4 - 26*x^5 + 15*x^6 - 6*x^7 + x^8)), x])/((
-6 + x^3)^(1/6)*(-((6 + x^3)/(6 - x^3)))^(1/6)) - (90*(6 + x^3)^(1/6)*Defer
[Int][x^2/((-6 + x^3)^(5/6)*(6 + x^3)^(1/6)*(36 - 90*x + 122*x^2 - 96*x^3 +
 51*x^4 - 26*x^5 + 15*x^6 - 6*x^7 + x^8)), x])/((-6 + x^3)^(1/6)*(-((6 + x^
3)/(6 - x^3)))^(1/6)) + (51*(6 + x^3)^(1/6)*Defer[Int][x^3/((-6 + x^3)^(5/6
)*(6 + x^3)^(1/6)*(36 - 90*x + 122*x^2 - 96*x^3 + 51*x^4 - 26*x^5 + 15*x^6
- 6*x^7 + x^8)), x])/((-6 + x^3)^(1/6)*(-((6 + x^3)/(6 - x^3)))^(1/6)) - (2
6*(6 + x^3)^(1/6)*Defer[Int][x^4/((-6 + x^3)^(5/6)*(6 + x^3)^(1/6)*(36 - 90
*x + 122*x^2 - 96*x^3 + 51*x^4 - 26*x^5 + 15*x^6 - 6*x^7 + x^8)), x])/((-6
+ x^3)^(1/6)*(-((6 + x^3)/(6 - x^3)))^(1/6)) + (16*(6 + x^3)^(1/6)*Defer[In
t][x^5/((-6 + x^3)^(5/6)*(6 + x^3)^(1/6)*(36 - 90*x + 122*x^2 - 96*x^3 + 51
*x^4 - 26*x^5 + 15*x^6 - 6*x^7 + x^8)), x])/((-6 + x^3)^(1/6)*(-((6 + x^3)/
(6 - x^3)))^(1/6)) - (6*(6 + x^3)^(1/6)*Defer[Int][x^6/((-6 + x^3)^(5/6)*(6
 + x^3)^(1/6)*(36 - 90*x + 122*x^2 - 96*x^3 + 51*x^4 - 26*x^5 + 15*x^6 - 6*
x^7 + x^8)), x])/((-6 + x^3)^(1/6)*(-((6 + x^3)/(6 - x^3)))^(1/6)) + ((6 +
x^3)^(1/6)*Defer[Int][x^7/((-6 + x^3)^(5/6)*(6 + x^3)^(1/6)*(36 - 90*x + 12
2*x^2 - 96*x^3 + 51*x^4 - 26*x^5 + 15*x^6 - 6*x^7 + x^8)), x])/((-6 + x^3)^
(1/6)*(-((6 + x^3)/(6 - x^3)))^(1/6))
```

Rubi steps

$$\int \frac{-36-6x^2+6x^3+x^6}{x\left(-6+x^3\right)\sqrt[6]{\frac{6+x^3}{-6+x^3}}\left(36-90x+122x^2-96x^3+51x^4-26x^5+15x^6-6x^7+x^8\right)}\,dx = \frac{\sqrt[6]{6+x^3}\int \frac{\cdots}{x\left(-6+x^3\right)^{5\cdots}}}{\cdots}$$

$$= \frac{\sqrt[6]{6+x^3}\int\left(-\frac{\cdots}{x\left(-6+x\cdots\right.}\right.}{\cdots}$$

$$= -\frac{\sqrt[6]{6+x^3}\int\frac{\cdots}{x\left(-6+x^3\cdots\right.}}{\sqrt[6]{-6+x^3}\sqrt[6]{\cdots}}$$

$$= -\frac{\sqrt[6]{6+x^3}\,\mathrm{Subst}\left(\int\cdots\right.}{3\sqrt[6]{-6\cdots}}$$

$$= \frac{\sqrt[6]{6+x^3}\int\frac{\cdots}{\left(-6+x^3\right)^{5/6}\cdots}}{\cdots}$$

$$= \frac{\sqrt[6]{6+x^3}\,\mathrm{Subst}\left(\int\frac{\cdots}{1\cdots}\right.}{9\sqrt[6]{-6+x^3}\cdots}$$

$$= \frac{\sqrt[6]{6+x^3}\tan^{-1}\left(\frac{\sqrt[6]{6\cdots}}{\sqrt[6]{-6\cdots}}\right.}{9\sqrt[6]{-6+x^3}\sqrt[6]{-\frac{6+\cdots}{6-\cdots}}}$$

$$= \frac{\sqrt[6]{6+x^3}\tan^{-1}\left(\frac{\sqrt[6]{6\cdots}}{\sqrt[6]{-6\cdots}}\right.}{9\sqrt[6]{-6+x^3}\sqrt[6]{-\frac{6+\cdots}{6-\cdots}}}$$

$$= \frac{\sqrt[6]{6+x^3}\tan^{-1}\left(\frac{\sqrt[6]{6\cdots}}{\sqrt[6]{-6\cdots}}\right.}{9\sqrt[6]{-6+x^3}\sqrt[6]{-\frac{6+\cdots}{6-\cdots}}}$$

Mathematica [F] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{-36-6x^2+6x^3+x^6}{x\left(-6+x^3\right)\sqrt[6]{\frac{6+x^3}{-6+x^3}}\left(36-90x+122x^2-96x^3+51x^4-26x^5+15x^6-6x^7+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-36 - 6*x^2 + 6*x^3 + x^6)/(x*(-6 + x^3)*((6 + x^3)/(-6 + x^3))^
(1/6)*(36 - 90*x + 122*x^2 - 96*x^3 + 51*x^4 - 26*x^5 + 15*x^6 - 6*x^7 + x^
8)),x]
```

```
[Out] Integrate[(-36 - 6*x^2 + 6*x^3 + x^6)/(x*(-6 + x^3)*((6 + x^3)/(-6 + x^3))^
(1/6)*(36 - 90*x + 122*x^2 - 96*x^3 + 51*x^4 - 26*x^5 + 15*x^6 - 6*x^7 + x^
8)), x]
```

IntegrateAlgebraic [A] time = 3.02, size = 154, normalized size = 1.00

$$-\frac{1}{3}\tan^{-1}\left(\frac{\sqrt[6]{\frac{x^3+6}{x^3-6}}}{x-1}\right)-\frac{1}{6}\tan^{-1}\left(\frac{\sqrt[3]{\frac{x^3+6}{x^3-6}}-x^2+2x-1}{(x-1)\sqrt[6]{\frac{x^3+6}{x^3-6}}}\right)+\frac{\tanh^{-1}\left(\frac{\left(\sqrt{3}\,x-\sqrt{3}\right)\sqrt[6]{\frac{x^3+6}{x^3-6}}}{\sqrt[3]{\frac{x^3+6}{x^3-6}}+x^2-2x+1}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-36 - 6*x^2 + 6*x^3 + x^6)/(x*(-6 + x^3)*((6 + x^3)/(-6
 + x^3))^(1/6)*(36 - 90*x + 122*x^2 - 96*x^3 + 51*x^4 - 26*x^5 + 15*x^6 - 6
*x^7 + x^8)),x]
```

```
[Out] -1/3*ArcTan[((6 + x^3)/(-6 + x^3))^(1/6)/(-1 + x)] - ArcTan[(-1 + 2*x - x^2
 + ((6 + x^3)/(-6 + x^3))^(1/3))/((-1 + x)*((6 + x^3)/(-6 + x^3))^(1/6))]/6
 + ArcTanh[((-Sqrt[3] + Sqrt[3]*x)*((6 + x^3)/(-6 + x^3))^(1/6))/(1 - 2*x +
 x^2 + ((6 + x^3)/(-6 + x^3))^(1/3))]/(2*Sqrt[3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+6*x^3-6*x^2-36)/x/(x^3-6)/((x^3+6)/(x^3-6))^(1/6)/(x^8-6*x^7
+15*x^6-26*x^5+51*x^4-96*x^3+122*x^2-90*x+36),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+6\,x^3-6\,x^2-36}{\left(x^8-6\,x^7+15\,x^6-26\,x^5+51\,x^4-96\,x^3+122\,x^2-90\,x+36\right)\left(x^3-6\right)x\left(\frac{x^3+6}{x^3-6}\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+6*x^3-6*x^2-36)/x/(x^3-6)/((x^3+6)/(x^3-6))^(1/6)/(x^8-6*x^7
+15*x^6-26*x^5+51*x^4-96*x^3+122*x^2-90*x+36),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 6*x^3 - 6*x^2 - 36)/((x^8 - 6*x^7 + 15*x^6 - 26*x^5 + 51*x
^4 - 96*x^3 + 122*x^2 - 90*x + 36)*(x^3 - 6)*x*((x^3 + 6)/(x^3 - 6))^(1/6))
, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+6x^3-6x^2-36}{x\left(x^3-6\right)\left(\frac{x^3+6}{x^3-6}\right)^{\frac{1}{6}}\left(x^8-6x^7+15x^6-26x^5+51x^4-96x^3+122x^2-90x+36\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+6*x^3-6*x^2-36)/x/(x^3-6)/((x^3+6)/(x^3-6))^(1/6)/(x^8-6*x^7+15*x^
6-26*x^5+51*x^4-96*x^3+122*x^2-90*x+36),x)
```

```
[Out] int((x^6+6*x^3-6*x^2-36)/x/(x^3-6)/((x^3+6)/(x^3-6))^(1/6)/(x^8-6*x^7+15*x^
6-26*x^5+51*x^4-96*x^3+122*x^2-90*x+36),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+6\,x^3-6\,x^2-36}{\left(x^8-6\,x^7+15\,x^6-26\,x^5+51\,x^4-96\,x^3+122\,x^2-90\,x+36\right)\left(x^3-6\right)x\left(\frac{x^3+6}{x^3-6}\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+6*x^3-6*x^2-36)/x/(x^3-6)/((x^3+6)/(x^3-6))^(1/6)/(x^8-6*x^7
+15*x^6-26*x^5+51*x^4-96*x^3+122*x^2-90*x+36),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + 6*x^3 - 6*x^2 - 36)/((x^8 - 6*x^7 + 15*x^6 - 26*x^5 + 51*x
^4 - 96*x^3 + 122*x^2 - 90*x + 36)*(x^3 - 6)*x*((x^3 + 6)/(x^3 - 6))^(1/6))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{-x^6-6\,x^3+6\,x^2+36}{x\,{\left(\frac{x^3+6}{x^3-6}\right)}^{1/6}\,\left(x^3-6\right)\,\left(x^8-6\,x^7+15\,x^6-26\,x^5+51\,x^4-96\,x^3+122\,x^2-90\,x+36\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(6*x^2 - 6*x^3 - x^6 + 36)/(x*((x^3 + 6)/(x^3 - 6))^(1/6)*(x^3 - 6)*(1
22*x^2 - 90*x - 96*x^3 + 51*x^4 - 26*x^5 + 15*x^6 - 6*x^7 + x^8 + 36)),x)
```

```
[Out] -int((6*x^2 - 6*x^3 - x^6 + 36)/(x*((x^3 + 6)/(x^3 - 6))^(1/6)*(x^3 - 6)*(1
22*x^2 - 90*x - 96*x^3 + 51*x^4 - 26*x^5 + 15*x^6 - 6*x^7 + x^8 + 36)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+6*x**3-6*x**2-36)/x/(x**3-6)/((x**3+6)/(x**3-6))**(1/6)/(x*
*8-6*x**7+15*x**6-26*x**5+51*x**4-96*x**3+122*x**2-90*x+36),x)
```

```
[Out] Timed out
```

3.1773 $\int \frac{-b+ax^8}{(b+ax^8)\sqrt[4]{b-cx^4+ax^8}}\,dx$

Optimal. Leaf size=154

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x\,\sqrt[4]{ax^8+b-cx^4}}{\sqrt{ax^8+b-cx^4}-\sqrt{c}\,x^2}\right)}{2\sqrt{2}\,\sqrt[4]{c}}-\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^8+b-cx^4}}{\sqrt{2}\,\sqrt[4]{c}}+\frac{\sqrt[4]{c}\,x^2}{\sqrt{2}}}{x\,\sqrt[4]{ax^8+b-cx^4}}\right)}{2\sqrt{2}\,\sqrt[4]{c}}$$

Rubi [F] time = 0.73, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-b+ax^8}{(b+ax^8)\sqrt[4]{b-cx^4+ax^8}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-b + a*x^8)/((b + a*x^8)*(b - c*x^4 + a*x^8)^(1/4)),x]
```

```
[Out] (x*(1 - (2*a*x^4)/(c - Sqrt[-4*a*b + c^2]))^(1/4)*(1 - (2*a*x^4)/(c + Sqrt[
-4*a*b + c^2]))^(1/4)*AppellF1[1/4, 1/4, 1/4, 5/4, (2*a*x^4)/(c + Sqrt[-4*a
*b + c^2]), (2*a*x^4)/(c - Sqrt[-4*a*b + c^2])])/(b - c*x^4 + a*x^8)^(1/4)
- Sqrt[b]*Defer[Int][1/((Sqrt[b] - Sqrt[-a]*x^4)*(b - c*x^4 + a*x^8)^(1/4))
, x] - Sqrt[b]*Defer[Int][1/((Sqrt[b] + Sqrt[-a]*x^4)*(b - c*x^4 + a*x^8)^(
1/4)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-b+ax^8}{(b+ax^8)\sqrt[4]{b-cx^4+ax^8}}\,dx &= \int \left(\frac{1}{\sqrt[4]{b-cx^4+ax^8}}-\frac{2b}{(b+ax^8)\sqrt[4]{b-cx^4+ax^8}}\right)dx\\
&= -\left((2b)\int \frac{1}{(b+ax^8)\sqrt[4]{b-cx^4+ax^8}}\,dx\right)+\int \frac{1}{\sqrt[4]{b-cx^4+ax^8}}\,dx\\
&= -\left((2b)\int \left(\frac{1}{2\sqrt{b}\left(\sqrt{b}-\sqrt{-a}\,x^4\right)\sqrt[4]{b-cx^4+ax^8}}+\frac{1}{2\sqrt{b}\left(\sqrt{b}+\sqrt{-a}\,x^4\right)\sqrt[4]{b-\right.}}\right.\right.\\
&= \frac{x\sqrt[4]{1-\frac{2ax^4}{c-\sqrt{-4ab+c^2}}}\sqrt[4]{1-\frac{2ax^4}{c+\sqrt{-4ab+c^2}}}\,F_1\left(\frac{1}{4};\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{2ax^4}{c+\sqrt{-4ab+c^2}},\frac{2ax^4}{c-\sqrt{-4ab+c^2}}\right)}{\sqrt[4]{b-cx^4+ax^8}}-\sqrt{}
\end{aligned}$$

Mathematica [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{-b+ax^8}{(b+ax^8)\sqrt[4]{b-cx^4+ax^8}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-b + a*x^8)/((b + a*x^8)*(b - c*x^4 + a*x^8)^(1/4)),x]
```

```
[Out] Integrate[(-b + a*x^8)/((b + a*x^8)*(b - c*x^4 + a*x^8)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 3.06, size = 154, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x\sqrt[4]{ax^8+b-cx^4}}{\sqrt{ax^8+b-cx^4}-\sqrt{c}\,x^2}\right)}{2\sqrt{2}\sqrt[4]{c}}-\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^8+b-cx^4}}{\sqrt{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^8+b-cx^4}}\right)}{2\sqrt{2}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^8)/((b + a*x^8)*(b - c*x^4 + a*x^8)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2]*c^(1/4)*x*(b - c*x^4 + a*x^8)^(1/4))/(-(Sqrt[c]*x^2) +
 Sqrt[b - c*x^4 + a*x^8])]/(Sqrt[2]*c^(1/4)) - ArcTanh[((c^(1/4)*x^2)/Sqrt[
2] + Sqrt[b - c*x^4 + a*x^8]/(Sqrt[2]*c^(1/4)))/(x*(b - c*x^4 + a*x^8)^(1/4
))]/(2*Sqrt[2]*c^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-b)/(a*x^8+b)/(a*x^8-c*x^4+b)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^8-b}{\left(ax^8-cx^4+b\right)^{\frac{1}{4}}\left(ax^8+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-b)/(a*x^8+b)/(a*x^8-c*x^4+b)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((a*x^8 - b)/((a*x^8 - c*x^4 + b)^(1/4)*(a*x^8 + b)), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{a\,x^8-b}{\left(a\,x^8+b\right)\left(a\,x^8-c\,x^4+b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^8-b)/(a*x^8+b)/(a*x^8-c*x^4+b)^(1/4),x)
```

```
[Out] int((a*x^8-b)/(a*x^8+b)/(a*x^8-c*x^4+b)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^8-b}{\left(ax^8-cx^4+b\right)^{\frac{1}{4}}\left(ax^8+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-b)/(a*x^8+b)/(a*x^8-c*x^4+b)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^8 - b)/((a*x^8 - c*x^4 + b)^(1/4)*(a*x^8 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{b-a\,x^8}{\left(a\,x^8+b\right)\,{\left(a\,x^8-c\,x^4+b\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^8)/((b + a*x^8)*(b + a*x^8 - c*x^4)^(1/4)),x)
[Out] int(-(b - a*x^8)/((b + a*x^8)*(b + a*x^8 - c*x^4)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{8} - b}{\left(ax^{8} + b\right) \sqrt[4]{ax^{8} + b - cx^{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**8-b)/(a*x**8+b)/(a*x**8-c*x**4+b)**(1/4),x)
[Out] Integral((a*x**8 - b)/((a*x**8 + b)*(a*x**8 + b - c*x**4)**(1/4)), x)
```

3.1774 $\int \frac{1}{x^2\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=154

$$\frac{4\left(8192a^7x^3+1024a^5bx^2+448a^3b^2x+6699ab^3\right)\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{45045b^5x^4}-\frac{4\sqrt{a^2x^2-bx}\left(-8192a^6x^3-5120a\right.}{}$$

Rubi [F] time = 4.77, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2))
,x]
```

```
[Out] (2*Sqrt[x]*Sqrt[-b + a^2*x]*Defer[Subst][Defer[Int][1/(x^4*Sqrt[-b + a^2*x^
2]*(a*x^4 + x^2*Sqrt[-(b*x^2) + a^2*x^4])^(3/2)), x], x, Sqrt[x]])/Sqrt[-(b
*x) + a^2*x^2]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx = \frac{\left(\sqrt{x}\sqrt{-b+a^2x}\right)\int \frac{1}{x^{5/2}\sqrt{-b+a^2x}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx}{\sqrt{-bx+a^2x^2}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-b+a^2x}\right)\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{-b+a^2x^2}\left(ax^4+x^2\sqrt{-bx^2+a^2x^4}\right)^{3/2}}\right.}{\sqrt{-bx+a^2x^2}}$$

Mathematica [F] time = 1.77, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{-bx+a^2x^2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^
(3/2)),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^
(3/2)), x]
```

IntegrateAlgebraic [A] time = 5.57, size = 154, normalized size = 1.00

$$\frac{4\left(8192a^7x^3+1024a^5bx^2+448a^3b^2x+6699ab^3\right)\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{45045b^5x^4}-\frac{4\sqrt{a^2x^2-bx}\left(-8192a^6x^3-5120a^4bx^2-4032a^2b^2x+3003b^3\right)\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}}{45045b^5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[-(b*x) + a^2*x^2]*(a*x^2 + x*Sqrt[-(b*x) + a
^2*x^2])^(3/2)),x]
```

```
[Out] (-4*Sqrt[-(b*x) + a^2*x^2]*(3003*b^3 - 4032*a^2*b^2*x - 5120*a^4*b*x^2 - 81
92*a^6*x^3)*Sqrt[x*(a*x + Sqrt[-(b*x) + a^2*x^2])])/(45045*b^5*x^5) + (4*(6
699*a*b^3 + 448*a^3*b^2*x + 1024*a^5*b*x^2 + 8192*a^7*x^3)*Sqrt[x*(a*x + Sq
rt[-(b*x) + a^2*x^2])])/(45045*b^5*x^4)
```

fricas [A] time = 0.40, size = 115, normalized size = 0.75

$$\frac{4\left(8192\,a^{7}x^{4}+1024\,a^{5}bx^{3}+448\,a^{3}b^{2}x^{2}+6699\,ab^{3}x+\left(8192\,a^{6}x^{3}+5120\,a^{4}bx^{2}+4032\,a^{2}b^{2}x-3003\,b^{3}\right)\sqrt{a^{2}x^{2}-bx}\right)\sqrt{ax^{2}+\sqrt{a^{2}x^{2}-bx}\,x}}{45045\,b^{5}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x,
algorithm="fricas")
```

```
[Out] 4/45045*(8192*a^7*x^4 + 1024*a^5*b*x^3 + 448*a^3*b^2*x^2 + 6699*a*b^3*x + (
8192*a^6*x^3 + 5120*a^4*b*x^2 + 4032*a^2*b^2*x - 3003*b^3)*sqrt(a^2*x^2 - b
*x))*sqrt(a*x^2 + sqrt(a^2*x^2 - b*x)*x)/(b^5*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^{2}x^{2}-bx}\left(ax^{2}+\sqrt{a^{2}x^{2}-bx}\,x\right)^{\frac{3}{2}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x,
algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 - b*x)*(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2)*x^2)
, x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\sqrt{a^{2}x^{2}-bx}\,\left(a\,x^{2}+x\sqrt{a^{2}x^{2}-bx}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

```
[Out] int(1/x^2/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^{2}x^{2}-bx}\left(ax^{2}+\sqrt{a^{2}x^{2}-bx}\,x\right)^{\frac{3}{2}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x,
algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 - b*x)*(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2)*x^2)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \sqrt{a^2 x^2 - b x} \left(a x^2 + x \sqrt{a^2 x^2 - b x}\right)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2*x^2 - b*x)^(1/2)*(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2)),x
)
```

```
[Out] int(1/(x^2*(a^2*x^2 - b*x)^(1/2)*(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(x \left(ax + \sqrt{a^2x^2 - bx}\right)\right)^{\frac{3}{2}} \sqrt{x \left(a^2x - b\right)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2*x**2-b*x)**(1/2)/(a*x**2+x*(a**2*x**2-b*x)**(1/2))**
(3/2),x)
```

```
[Out] Integral(1/(x**2*(x*(a*x + sqrt(a**2*x**2 - b*x)))**(3/2)*sqrt(x*(a**2*x -
b))), x)
```

3.1775 $\int \frac{1+\sqrt{1+x^2}}{1+\sqrt{x+\sqrt{1+x^2}}} \, dx$

Optimal. Leaf size=154

$$\frac{5}{4}\log\left(\sqrt{x^2+1}+x\right)-4\log\left(\sqrt{\sqrt{x^2+1}+x}+1\right)+\frac{-24x^3+214x^2+\sqrt{x^2+1}\left(-24x^2+\left(16x^2+72x+40\right)\sqrt{\sqrt{x^2}\ldots}\right.}{48\sqrt{\ldots}}$$

Rubi [A] time = 0.72, antiderivative size = 208, normalized size of antiderivative = 1.35, number of steps used = 32, number of rules used = 22, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.710, Rules used = {6742, 2117, 1821, 1620, 195, 215, 266, 50, 63, 207, 14, 2122, 270, 2120, 462, 459, 329, 298, 203, 206, 466, 461}

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{x^2+1}\,x+\frac{1}{6}\left(\sqrt{x^2+1}+x\right)^{3/2}-\frac{\sqrt{x^2+1}}{2}+\frac{3}{2}\sqrt{\sqrt{x^2+1}+x}+\frac{3}{2\sqrt{\sqrt{x^2+1}+x}}-\frac{1}{2\left(\sqrt{x^2+1}+x\right)}+\frac{1}{6\left(\sqrt{x^2+1}+x\right)^{3/2}}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-2\log\left(\sqrt{\sqrt{x^2+1}+x}+1\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt{x^2+1}\right)-2\tanh^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)-\frac{\log(x)}{2}+\frac{1}{4}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[1 + x^2])/(1 + Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/4*x^2 - Sqrt[1 + x^2]/2 + (x*Sqrt[1 + x^2])/4 + 1/(6*(x + Sqrt[1 + x^2])
^(3/2)) - 1/(2*(x + Sqrt[1 + x^2])) + 3/(2*Sqrt[x + Sqrt[1 + x^2]]) + (3*Sq
rt[x + Sqrt[1 + x^2]])/2 + (x + Sqrt[1 + x^2])^(3/2)/6 + ArcSinh[x]/4 + Arc
Tanh[Sqrt[1 + x^2]]/2 - 2*ArcTanh[Sqrt[x + Sqrt[1 + x^2]]] - Log[x]/2 + Log
[x + Sqrt[1 + x^2]]/2 - 2*Log[1 + Sqrt[x + Sqrt[1 + x^2]]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
```

```
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+\sqrt{1+x^2}}{1+\sqrt{x+\sqrt{1+x^2}}}\,dx &= \int \left(\frac{1}{1+\sqrt{x+\sqrt{1+x^2}}}+\frac{\sqrt{1+x^2}}{1+\sqrt{x+\sqrt{1+x^2}}}\right)dx \\
&= \int \frac{1}{1+\sqrt{x+\sqrt{1+x^2}}}\,dx+\int \frac{\sqrt{1+x^2}}{1+\sqrt{x+\sqrt{1+x^2}}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{\left(1+\sqrt{x}\right)x^2}\,dx,x,x+\sqrt{1+x^2}\right)+\int \left(\frac{\sqrt{1+x^2}}{2}-\frac{\sqrt{1+x^2}}{2x}-\frac{1+x^2}{2x} \right. \\
&= \frac{1}{2}\int \sqrt{1+x^2}\,dx-\frac{1}{2}\int \frac{\sqrt{1+x^2}}{x}\,dx-\frac{1}{2}\int \frac{1+x^2}{x}\,dx-\frac{1}{2}\int \sqrt{1+x^2}\,\sqrt{x+\sqrt{1+} \\
&= \frac{1}{4}x\sqrt{1+x^2}-\frac{1}{8}\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^{5/2}}\,dx,x,x+\sqrt{1+x^2}\right)+\frac{1}{8}\text{Subst}\left(\int \frac{\left(1+x^2\right.}{x^{5/2}\left(-1+\right.} \right. \\
&= -\frac{x^2}{4}-\frac{\sqrt{1+x^2}}{2}+\frac{1}{4}x\sqrt{1+x^2}-\frac{1}{2\left(x+\sqrt{1+x^2}\right)}+\frac{3}{2\sqrt{x+\sqrt{1+x^2}}}+\sqrt{x+\sqrt{1}} \\
&= -\frac{x^2}{4}-\frac{\sqrt{1+x^2}}{2}+\frac{1}{4}x\sqrt{1+x^2}+\frac{1}{12\left(x+\sqrt{1+x^2}\right)^{3/2}}-\frac{1}{2\left(x+\sqrt{1+x^2}\right)}+\frac{}{2\sqrt{x}} \\
&= -\frac{x^2}{4}-\frac{\sqrt{1+x^2}}{2}+\frac{1}{4}x\sqrt{1+x^2}+\frac{1}{6\left(x+\sqrt{1+x^2}\right)^{3/2}}-\frac{1}{2\left(x+\sqrt{1+x^2}\right)}+\frac{}{2\sqrt{x+}} \\
&= -\frac{x^2}{4}-\frac{\sqrt{1+x^2}}{2}+\frac{1}{4}x\sqrt{1+x^2}+\frac{1}{6\left(x+\sqrt{1+x^2}\right)^{3/2}}-\frac{1}{2\left(x+\sqrt{1+x^2}\right)}+\frac{}{2\sqrt{x+}} \\
&= -\frac{x^2}{4}-\frac{\sqrt{1+x^2}}{2}+\frac{1}{4}x\sqrt{1+x^2}+\frac{1}{6\left(x+\sqrt{1+x^2}\right)^{3/2}}-\frac{1}{2\left(x+\sqrt{1+x^2}\right)}+\frac{}{2\sqrt{x+}}
\end{aligned}
$$

Mathematica [C] time = 2.08, size = 482, normalized size = 3.13

$$
-\frac{2\sqrt{x^2+1}\left(\sqrt{x^2+1}+x\right)^{9/2}\left(\left(4x^2+4\sqrt{x^2+1}\,x+2\right){}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\left(x+\sqrt{x^2+1}\right)^2\right)-x^2-\sqrt{x^2+1}\,x-2\right)}{48x^6+84x^4+39x^2+15\sqrt{x^2+1}\,x+48\sqrt{x^2+1}\,x^5+60\sqrt{x^2+1}\,x^3+3}+\frac{1}{3}\left(\sqrt{x^2+1}-2x\right)\sqrt{\sqrt{x^2+1}+x}-\frac{x^2}{4}+\frac{1}{4}(x-4)\sqrt{x^2+1}+\log\left(\sqrt{x^2+1}+1\right)+\frac{2\sqrt{x^2+1}\left(\sqrt{x^2+1}+x\right)\left(\sqrt{\sqrt{x^2+1}+x}-\tan^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)-\tanh^{-1}\left(\sqrt{\sqrt{x^2+1}+x}\right)\right)}{x^2+\sqrt{x^2+1}\,x+1}-\frac{\left(6x^4+21x^2+18\sqrt{x^2+1}\,x+6\sqrt{x^2+1}\,x^3+7\right)\sqrt{\sqrt{x^2+1}+x}}{15\left(2x^2+2\sqrt{x^2+1}\,x+1\right)}+\frac{\sqrt{x^2+1}\left(6x^4+6x^2+3\sqrt{x^2+1}\,x+6\sqrt{x^2+1}\,x^3+2\right)}{15\left(x^2+\sqrt{x^2+1}\,x+1\right)\sqrt{\sqrt{x^2+1}+x}}+\frac{x}{2}-2\log(x)+\frac{1}{4}\sinh^{-1}(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[1 + x^2])/(1 + Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] x/2 - x^2/4 + ((-4 + x)*Sqrt[1 + x^2])/4 + ((-2*x + Sqrt[1 + x^2])*Sqrt[x +
 Sqrt[1 + x^2]])/3 + (Sqrt[1 + x^2]*(2 + 6*x^2 + 6*x^4 + 3*x*Sqrt[1 + x^2]
+ 6*x^3*Sqrt[1 + x^2]))/(15*Sqrt[x + Sqrt[1 + x^2]]*(1 + x^2 + x*Sqrt[1 + x
^2])) - (Sqrt[x + Sqrt[1 + x^2]]*(7 + 21*x^2 + 6*x^4 + 18*x*Sqrt[1 + x^2] +
 6*x^3*Sqrt[1 + x^2]))/(15*(1 + 2*x^2 + 2*x*Sqrt[1 + x^2])) + ArcSinh[x]/4
+ (2*Sqrt[1 + x^2]*(x + Sqrt[1 + x^2])*(Sqrt[x + Sqrt[1 + x^2]] - ArcTan[Sq
```

```
rt[x + Sqrt[1 + x^2]]] - ArcTanh[Sqrt[x + Sqrt[1 + x^2]]]))/(1 + x^2 + x*Sq
rt[1 + x^2]) - (2*Sqrt[1 + x^2]*(x + Sqrt[1 + x^2])^(9/2)*(-2 - x^2 - x*Sqr
t[1 + x^2] + (2 + 4*x^2 + 4*x*Sqrt[1 + x^2])*Hypergeometric2F1[3/4, 1, 7/4,
 (x + Sqrt[1 + x^2])^2]))/(3 + 39*x^2 + 84*x^4 + 48*x^6 + 15*x*Sqrt[1 + x^2
] + 60*x^3*Sqrt[1 + x^2] + 48*x^5*Sqrt[1 + x^2]) - 2*Log[x] + Log[1 + Sqrt[
1 + x^2]]
```

IntegrateAlgebraic [A] time = 0.24, size = 154, normalized size = 1.00

$$\frac{5}{4}\log\left(\sqrt{x^2+1}+x\right)-4\log\left(\sqrt{\sqrt{x^2+1}+x}+1\right)+\frac{-24x^3+214x^2+\sqrt{x^2+1}\left(-24x^2+\left(16x^2+72x+40\right)\sqrt{\sqrt{x^2+1}+x}+214x-24\right)+\left(16x^3+72x^2+48x+40\right)\sqrt{\sqrt{x^2+1}+x}-36x+104}{48\sqrt{x^2+1}\,x+24\left(2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[1 + x^2])/(1 + Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] (104 - 36*x + 214*x^2 - 24*x^3 + (40 + 48*x + 72*x^2 + 16*x^3)*Sqrt[x + Sqr
t[1 + x^2]] + Sqrt[1 + x^2]*(-24 + 214*x - 24*x^2 + (40 + 72*x + 16*x^2)*Sq
rt[x + Sqrt[1 + x^2]]))/(48*x*Sqrt[1 + x^2] + 24*(1 + 2*x^2)) + (5*Log[x +
Sqrt[1 + x^2]])/4 - 4*Log[1 + Sqrt[x + Sqrt[1 + x^2]]]
```

fricas [A] time = 0.40, size = 84, normalized size = 0.55

$$-\frac{1}{4}x^2+\frac{1}{3}\left(x^2-\sqrt{x^2+1}\,(x-5)-4\,x+5\right)\sqrt{x+\sqrt{x^2+1}}+\frac{1}{4}\sqrt{x^2+1}\,(x-4)+\frac{1}{2}x-4\,\log\left(\sqrt{x+\sqrt{x^2+1}}+1\right)+\frac{5}{2}\log\left(\sqrt{x+\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))/(1+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="frica
s")
```

```
[Out] -1/4*x^2 + 1/3*(x^2 - sqrt(x^2 + 1)*(x - 5) - 4*x + 5)*sqrt(x + sqrt(x^2 +
1)) + 1/4*sqrt(x^2 + 1)*(x - 4) + 1/2*x - 4*log(sqrt(x + sqrt(x^2 + 1)) + 1
) + 5/2*log(sqrt(x + sqrt(x^2 + 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}+1}{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))/(1+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="giac"
)
```

```
[Out] integrate((sqrt(x^2 + 1) + 1)/(sqrt(x + sqrt(x^2 + 1)) + 1), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1+\sqrt{x^2+1}}{1+\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x^2+1)^(1/2))/(1+(x+(x^2+1)^(1/2))^(1/2)),x)
```

```
[Out] int((1+(x^2+1)^(1/2))/(1+(x+(x^2+1)^(1/2))^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x+\frac{1}{2}\int \sqrt{x^2+1}\,dx-\int \frac{x^2+\sqrt{x^2+1}\,x+x}{2\left(x+\sqrt{x^2+1}+2\sqrt{x+\sqrt{x^2+1}}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))/(1+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="maxim
a")
```

```
[Out] 1/2*x + 1/2*integrate(sqrt(x^2 + 1), x) - integrate(1/2*(x^2 + sqrt(x^2 + 1
)*x + x)/(x + sqrt(x^2 + 1) + 2*sqrt(x + sqrt(x^2 + 1)) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^2+1}+1}{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)^(1/2) + 1)/((x + (x^2 + 1)^(1/2))^(1/2) + 1),x)
```

```
[Out] int(((x^2 + 1)^(1/2) + 1)/((x + (x^2 + 1)^(1/2))^(1/2) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}+1}{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x**2+1)**(1/2))/(1+(x+(x**2+1)**(1/2))**(1/2)),x)
```

```
[Out] Integral((sqrt(x**2 + 1) + 1)/(sqrt(x + sqrt(x**2 + 1)) + 1), x)
```

3.1776 $\int \frac{-2+(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(b-b(1+k)x+(-1+bk)x^2)}\,dx$

Optimal. Leaf size=155

$$\frac{\log\left(x-\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{b^{2/3}}-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+x^2\right)}{2b^{2/3}}-\cdots$$

Rubi [F] time = 4.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2+(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,\left(b-b(1+k)x+(-1+bk)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2 + (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(b - b*(1 + k)*x + (-1 +
b*k)*x^2)),x]
```

```
[Out] ((1 + Sqrt[4 + b*(1 - k)^2]/Sqrt[b] + k)*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1
/3)*Defer[Int][1/((1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*(-(b*(1 + k)) - Sqr
t[b]*Sqrt[4 + b - 2*b*k + b*k^2] + 2*(-1 + b*k)*x)), x])/((1 - x)*x*(1 - k*
x))^(1/3) + ((1 - Sqrt[4 + b*(1 - k)^2]/Sqrt[b] + k)*(1 - x)^(1/3)*x^(1/3)*
(1 - k*x)^(1/3)*Defer[Int][1/((1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*(-(b*(1
 + k)) + Sqrt[b]*Sqrt[4 + b - 2*b*k + b*k^2] + 2*(-1 + b*k)*x)), x])/((1 -
x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\int \frac{-2+(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,\left(b-b(1+k)x+(-1+bk)x^2\right)}\,dx = \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{-2+(1+k)x}{\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\,\left(b-b(1+k)x+(-1+bk)x^2\right.}\cdots}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$=\frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int\left(\frac{1+k+\frac{\sqrt{4+b-2bk+bk^2}}{\sqrt{b}}}{\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\left(-b(1+k)-\sqrt{b}\,\sqrt{4+b-2}\cdots\right.}\cdots\right.}{\cdots}$$

$$=\frac{\left(\left(1-\frac{\sqrt{4+b(1-k)^2}}{\sqrt{b}}+k\right)\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{\cdots}{\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-}\cdots}}{\sqrt[3]{(1-x)x(1-kx)}\cdots}$$

Mathematica [F] time = 4.45, size = 0, normalized size = 0.00

$$\int \frac{-2+(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,\left(b-b(1+k)x+(-1+bk)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2 + (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(b - b*(1 + k)*x +
(-1 + b*k)*x^2)),x]
```

```
[Out] Integrate[(-2 + (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(b - b*(1 + k)*x +
(-1 + b*k)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.29, size = 155, normalized size = 1.00

$$\frac{\log\left(x-\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{b^{2/3}}-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+x^2\right)}{2b^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+x}\right)}{b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(b - b*(1
+ k)*x + (-1 + b*k)*x^2)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/
3))])/b^(2/3)) + Log[x - b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3)]/b^(2/3)
- Log[x^2 + b^(1/3)*x*(x + (-1 - k)*x^2 + k*x^3)^(1/3) + b^(2/3)*(x + (-1 -
 k)*x^2 + k*x^3)^(2/3)]/(2*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-b*(1+k)*x+(b*k-1)*x^2),x
, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.59, size = 129, normalized size = 0.83

$$\frac{\sqrt{3}\,|b|^{\frac{4}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\,b^{\frac{1}{3}}\left(2\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{1}{3}}+\frac{1}{b^{\frac{1}{3}}}\right)\right)}{b^2}-\frac{|b|^{\frac{4}{3}}\log\left(\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{2}{3}}+\frac{\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{1}{3}}}{b^{\frac{1}{3}}}+\frac{1}{b^{\frac{2}{3}}}\right)}{2\,b^2}+\frac{\log\left(\left|\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{1}{3}}-\frac{1}{b^{\frac{1}{3}}}\right|\right)}{b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-b*(1+k)*x+(b*k-1)*x^2),x
, algorithm="giac")
```

```
[Out] sqrt(3)*abs(b)^(4/3)*arctan(1/3*sqrt(3)*b^(1/3)*(2*(k - k/x - 1/x + 1/x^2)^
(1/3) + 1/b^(1/3)))/b^2 - 1/2*abs(b)^(4/3)*log((k - k/x - 1/x + 1/x^2)^(2/3
) + (k - k/x - 1/x + 1/x^2)^(1/3)/b^(1/3) + 1/b^(2/3))/b^2 + log(abs((k - k
/x - 1/x + 1/x^2)^(1/3) - 1/b^(1/3)))/b^(2/3)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{-2+(1+k)\,x}{\left((1-x)\,x\,(-kx+1)\right)^{\frac{1}{3}}\left(b-b\,(1+k)\,x+(bk-1)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-b*(1+k)*x+(b*k-1)*x^2),x)
```

```
[Out] int((-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-b*(1+k)*x+(b*k-1)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(k+1)x-2}{\left(b(k+1)x-(bk-1)x^2-b\right)\left((kx-1)(x-1)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-b*(1+k)*x+(b*k-1)*x^2),x
, algorithm="maxima")
```

```
[Out] -integrate(((k + 1)*x - 2)/((b*(k + 1)*x - (b*k - 1)*x^2 - b)*((k*x - 1)*(x
 - 1)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(k+1\right)-2}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/3}\,\left(\left(b\,k-1\right)\,x^2-b\,\left(k+1\right)\,x+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(k + 1) - 2)/((x*(k*x - 1)*(x - 1))^(1/3)*(b + x^2*(b*k - 1) - b*x*(
k + 1))),x)
```

```
[Out] int((x*(k + 1) - 2)/((x*(k*x - 1)*(x - 1))^(1/3)*(b + x^2*(b*k - 1) - b*x*(
k + 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(b-b*(1+k)*x+(b*k-1)*x**2)
,x)
```

```
[Out] Timed out
```

3.1777 $\int \frac{\sqrt[4]{-b+ax^4}\left(-8b+ax^8\right)}{x^{10}\left(b+ax^4\right)}\,dx$

Optimal. Leaf size=155

$$\frac{\left(8a^{9/4}-a^{5/4}b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{2^{3/4}b^2}-\frac{\left(8a^{9/4}-a^{5/4}b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{2^{3/4}b^2}+\frac{\sqrt[4]{ax^4-b}\left(80a^2x^8-9abx^8-16abx^4+8b^2\right)}{9b^2x^9}$$

Rubi [C] time = 0.70, antiderivative size = 232, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6725, 271, 264, 277, 331, 298, 203, 206, 511, 510}

$$\frac{a^{5/4}(8a-b)\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{2b^2}-\frac{a^{5/4}(8a-b)\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{2b^2}+\frac{a^2x^3(8a-b)\sqrt[4]{ax^4-b}\,F_1\left(\frac{3}{4};1,-\frac{1}{4};\frac{7}{4};-\frac{ax^4}{b},\frac{ax^4}{b}\right)}{3b^3\sqrt[4]{1-\frac{ax^4}{b}}}+\frac{a(8a-b)\sqrt[4]{ax^4-b}}{b^2x}+\frac{8a\left(ax^4-b\right)^{5/4}}{9b^2x^5}-\frac{8\left(ax^4-b\right)^{5/4}}{9bx^9}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-b + a*x^4)^(1/4)*(-8*b + a*x^8))/(x^10*(b + a*x^4)),x]
```

```
[Out] (a*(8*a - b)*(-b + a*x^4)^(1/4))/(b^2*x) - (8*(-b + a*x^4)^(5/4))/(9*b*x^9)
 + (8*a*(-b + a*x^4)^(5/4))/(9*b^2*x^5) + (a^2*(8*a - b)*x^3*(-b + a*x^4)^(
1/4)*AppellF1[3/4, 1, -1/4, 7/4, -((a*x^4)/b), (a*x^4)/b])/(3*b^3*(1 - (a*x
^4)/b)^(1/4)) + (a^(5/4)*(8*a - b)*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/
(2*b^2) - (a^(5/4)*(8*a - b)*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(2*b^
2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{-b+ax^4}\left(-8b+ax^8\right)}{x^{10}\left(b+ax^4\right)}dx = \int \left(-\frac{8\sqrt[4]{-b+ax^4}}{x^{10}} + \frac{8a\sqrt[4]{-b+ax^4}}{bx^6} - \frac{a(8a-b)\sqrt[4]{-b+ax^4}}{b^2x^2} + \frac{a^2(8a-b)x^2\sqrt[4]{\cdot}}{b^2\left(b+a\cdot\right.}\right.$$

$$= -\left(8\int \frac{\sqrt[4]{-b+ax^4}}{x^{10}}dx\right) - \frac{(a(8a-b))\int \frac{\sqrt[4]{-b+ax^4}}{x^2}dx}{b^2} + \frac{\left(a^2(8a-b)\right)\int \frac{x^2\sqrt[4]{-b+ax}}{b+ax^4}}{b^2}$$

$$= \frac{a(8a-b)\sqrt[4]{-b+ax^4}}{b^2x} - \frac{8\left(-b+ax^4\right)^{5/4}}{9bx^9} + \frac{8a\left(-b+ax^4\right)^{5/4}}{5b^2x^5} - \frac{\left(a^2(8a-b)\right)\int}{b^2}$$

$$= \frac{a(8a-b)\sqrt[4]{-b+ax^4}}{b^2x} - \frac{8\left(-b+ax^4\right)^{5/4}}{9bx^9} + \frac{8a\left(-b+ax^4\right)^{5/4}}{9b^2x^5} + \frac{a^2(8a-b)x^3\sqrt[4]{-}}{}$$

$$= \frac{a(8a-b)\sqrt[4]{-b+ax^4}}{b^2x} - \frac{8\left(-b+ax^4\right)^{5/4}}{9bx^9} + \frac{8a\left(-b+ax^4\right)^{5/4}}{9b^2x^5} + \frac{a^2(8a-b)x^3\sqrt[4]{-}}{}$$

$$= \frac{a(8a-b)\sqrt[4]{-b+ax^4}}{b^2x} - \frac{8\left(-b+ax^4\right)^{5/4}}{9bx^9} + \frac{8a\left(-b+ax^4\right)^{5/4}}{9b^2x^5} + \frac{a^2(8a-b)x^3\sqrt[4]{-}}{}$$

Mathematica [C] time = 0.33, size = 129, normalized size = 0.83

$$\frac{\left(ax^4-b\right)\left(80a^2x^8-ab\left(9x^4+16\right)x^4+8b^2\right)+\frac{6a^2x^{12}(b-8a)\left(1-\frac{ax^4}{b}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{2ax^4}{ax^4+b}\right)}{\left(\frac{ax^4}{b}+1\right)^{3/4}}}{9b^2x^9\left(ax^4-b\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-b + a*x^4)^(1/4)*(-8*b + a*x^8))/(x^10*(b + a*x^4)),x]
```

```
[Out] ((-b + a*x^4)*(8*b^2 + 80*a^2*x^8 - a*b*x^4*(16 + 9*x^4)) + (6*a^2*(-8*a +
b)*x^12*(1 - (a*x^4)/b)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (2*a*x^4)/(b
 + a*x^4)])/(1 + (a*x^4)/b)^(3/4))/(9*b^2*x^9*(-b + a*x^4)^(3/4))
```

IntegrateAlgebraic [A] time = 0.49, size = 155, normalized size = 1.00

$$\frac{\left(8a^{9/4}-a^{5/4}b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{2^{3/4}b^2} - \frac{\left(8a^{9/4}-a^{5/4}b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{2^{3/4}b^2} + \frac{\sqrt[4]{ax^4-b}\left(80a^2x^8-9abx^8-16abx^4+8b^2\right)}{9b^2x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^4)^(1/4)*(-8*b + a*x^8))/(x^10*(b + a*x^4)),x
]
```

```
[Out] ((-b + a*x^4)^(1/4)*(8*b^2 - 16*a*b*x^4 + 80*a^2*x^8 - 9*a*b*x^8))/(9*b^2*x
^9) + ((8*a^(9/4) - a^(5/4)*b)*ArcTan[(2^(1/4)*a^(1/4)*x)/(-b + a*x^4)^(1/4
)])/(2^(3/4)*b^2) - ((8*a^(9/4) - a^(5/4)*b)*ArcTanh[(2^(1/4)*a^(1/4)*x)/(-
b + a*x^4)^(1/4)])/(2^(3/4)*b^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(1/4)*(a*x^8-8*b)/x^10/(a*x^4+b),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{8}-8\,b\right)\left(ax^{4}-b\right)^{\frac{1}{4}}}{\left(ax^{4}+b\right)x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(1/4)*(a*x^8-8*b)/x^10/(a*x^4+b),x, algorithm="giac")
[Out] integrate((a*x^8 - 8*b)*(a*x^4 - b)^(1/4)/((a*x^4 + b)*x^10), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{4}-b\right)^{\frac{1}{4}}\left(a\,x^{8}-8b\right)}{x^{10}\left(a\,x^{4}+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b)^(1/4)*(a*x^8-8*b)/x^10/(a*x^4+b),x)
[Out] int((a*x^4-b)^(1/4)*(a*x^8-8*b)/x^10/(a*x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{8}-8\,b\right)\left(ax^{4}-b\right)^{\frac{1}{4}}}{\left(ax^{4}+b\right)x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(1/4)*(a*x^8-8*b)/x^10/(a*x^4+b),x, algorithm="maxima")
[Out] integrate((a*x^8 - 8*b)*(a*x^4 - b)^(1/4)/((a*x^4 + b)*x^10), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(a\,x^{4}-b\right)^{1/4}\,\left(8\,b-a\,x^{8}\right)}{x^{10}\,\left(a\,x^{4}+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*x^4 - b)^(1/4)*(8*b - a*x^8))/(x^10*(b + a*x^4)),x)
[Out] int(-((a*x^4 - b)^(1/4)*(8*b - a*x^8))/(x^10*(b + a*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{ax^{4}-b}\,\left(ax^{8}-8b\right)}{x^{10}\left(ax^{4}+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b)**(1/4)*(a*x**8-8*b)/x**10/(a*x**4+b),x)
[Out] Integral((a*x**4 - b)**(1/4)*(a*x**8 - 8*b)/(x**10*(a*x**4 + b)), x)
```

3.1778 $\displaystyle\int \frac{(-6+x^2)(-2+x^2)(2-x^2+x^3)\sqrt[3]{-2+x^2+2x^3}}{x^5(-2+x^2+x^3)^2}\,dx$

Optimal. Leaf size=156

$$-\frac{7}{3}\log\left(\sqrt[3]{2x^3+x^2-2}-x\right)+\frac{7}{6}\log\left(x^2+\sqrt[3]{2x^3+x^2-2}\,x+\left(2x^3+x^2-2\right)^{2/3}\right)-\frac{7\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2x^3+x^2-2}+x}\right)}{\sqrt{3}}+\frac{\sqrt[3]{2x}}{}$$

Rubi [F] time = 9.80, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-6+x^2)(-2+x^2)(2-x^2+x^3)\sqrt[3]{-2+x^2+2x^3}}{x^5(-2+x^2+x^3)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-6 + x^2)*(-2 + x^2)*(2 - x^2 + x^3)*(-2 + x^2 + 2*x^3)^(1/3))/(x^5*(
-2 + x^2 + x^3)^2),x]
```

```
[Out] ((48/5 + (24*I)/5)*(-2 + x^2 + 2*x^3)^(1/3)*Defer[Subst][Defer[Int][((-1/3*
(1 + (107 + 6*Sqrt[318])^(2/3))/(107 + 6*Sqrt[318])^(1/3) + 2*x)^(1/3)*((-1
 + (107 + 6*Sqrt[318])^(-2/3) + (107 + 6*Sqrt[318])^(2/3))/9 + (2*(1 + (107
 + 6*Sqrt[318])^(2/3))*x)/(3*(107 + 6*Sqrt[318])^(1/3)) + 4*x^2)^(1/3))/((-
5/3 + 2*I) - 2*x)^2, x], x, 1/6 + x])/((1 - (1 + (107 + 6*Sqrt[318])^(2/3))
/(107 + 6*Sqrt[318])^(1/3) + 6*x)^(1/3)*(-1 + (107 + 6*Sqrt[318])^(-2/3) +
(107 + 6*Sqrt[318])^(2/3) + ((1 + (107 + 6*Sqrt[318])^(2/3))*(1 + 6*x))/(10
7 + 6*Sqrt[318])^(1/3) + (1 + 6*x)^2)^(1/3)) - (((24*I)/5)*(-2 + x^2 + 2*x^
3)^(1/3)*Defer[Subst][Defer[Int][((-1/3*(1 + (107 + 6*Sqrt[318])^(2/3))/(10
7 + 6*Sqrt[318])^(1/3) + 2*x)^(1/3)*((-1 + (107 + 6*Sqrt[318])^(-2/3) + (10
7 + 6*Sqrt[318])^(2/3))/9 + (2*(1 + (107 + 6*Sqrt[318])^(2/3))*x)/(3*(107 +
 6*Sqrt[318])^(1/3)) + 4*x^2)^(1/3))/((-5/3 + 2*I) - 2*x), x], x, 1/6 + x])
/((1 - (1 + (107 + 6*Sqrt[318])^(2/3))/(107 + 6*Sqrt[318])^(1/3) + 6*x)^(1/
3)*(-1 + (107 + 6*Sqrt[318])^(-2/3) + (107 + 6*Sqrt[318])^(2/3) + ((1 + (10
7 + 6*Sqrt[318])^(2/3))*(1 + 6*x))/(107 + 6*Sqrt[318])^(1/3) + (1 + 6*x)^2)
^(1/3)) + (6*(-2 + x^2 + 2*x^3)^(1/3)*Defer[Subst][Defer[Int][((-1/3*(1 + (
107 + 6*Sqrt[318])^(2/3))/(107 + 6*Sqrt[318])^(1/3) + 2*x)^(1/3)*((-1 + (10
7 + 6*Sqrt[318])^(-2/3) + (107 + 6*Sqrt[318])^(2/3))/9 + (2*(1 + (107 + 6*S
qrt[318])^(2/3))*x)/(3*(107 + 6*Sqrt[318])^(1/3)) + 4*x^2)^(1/3))/(-7/6 + x
)^2, x], x, 1/6 + x])/(5*(1 - (1 + (107 + 6*Sqrt[318])^(2/3))/(107 + 6*Sqrt
[318])^(1/3) + 6*x)^(1/3)*(-1 + (107 + 6*Sqrt[318])^(-2/3) + (107 + 6*Sqrt[
318])^(2/3) + ((1 + (107 + 6*Sqrt[318])^(2/3))*(1 + 6*x))/(107 + 6*Sqrt[318
])^(1/3) + (1 + 6*x)^2)^(1/3)) - (51*(-2 + x^2 + 2*x^3)^(1/3)*Defer[Subst][
Defer[Int][((-1/3*(1 + (107 + 6*Sqrt[318])^(2/3))/(107 + 6*Sqrt[318])^(1/3)
 + 2*x)^(1/3)*((-1 + (107 + 6*Sqrt[318])^(-2/3) + (107 + 6*Sqrt[318])^(2/3)
)/9 + (2*(1 + (107 + 6*Sqrt[318])^(2/3))*x)/(3*(107 + 6*Sqrt[318])^(1/3)) +
 4*x^2)^(1/3))/(-7/6 + x), x], x, 1/6 + x])/(5*(1 - (1 + (107 + 6*Sqrt[318]
)^(2/3))/(107 + 6*Sqrt[318])^(1/3) + 6*x)^(1/3)*(-1 + (107 + 6*Sqrt[318])^(
-2/3) + (107 + 6*Sqrt[318])^(2/3) + ((1 + (107 + 6*Sqrt[318])^(2/3))*(1 + 6
*x))/(107 + 6*Sqrt[318])^(1/3) + (1 + 6*x)^2)^(1/3)) + (18*(-2 + x^2 + 2*x^
3)^(1/3)*Defer[Subst][Defer[Int][((-1/3*(1 + (107 + 6*Sqrt[318])^(2/3))/(10
7 + 6*Sqrt[318])^(1/3) + 2*x)^(1/3)*((-1 + (107 + 6*Sqrt[318])^(-2/3) + (10
7 + 6*Sqrt[318])^(2/3))/9 + (2*(1 + (107 + 6*Sqrt[318])^(2/3))*x)/(3*(107 +
 6*Sqrt[318])^(1/3)) + 4*x^2)^(1/3))/(-1/6 + x)^5, x], x, 1/6 + x])/((1 - (
1 + (107 + 6*Sqrt[318])^(2/3))/(107 + 6*Sqrt[318])^(1/3) + 6*x)^(1/3)*(-1 +
 (107 + 6*Sqrt[318])^(-2/3) + (107 + 6*Sqrt[318])^(2/3) + ((1 + (107 + 6*Sq
rt[318])^(2/3))*(1 + 6*x))/(107 + 6*Sqrt[318])^(1/3) + (1 + 6*x)^2)^(1/3))
```

```
- (3*(-2 + x^2 + 2*x^3)^(1/3)*Defer[Subst][Defer[Int][((-1/3*(1 + (107 + 6*
Sqrt[318])^(2/3))/(107 + 6*Sqrt[318])^(1/3) + 2*x)^(1/3)*((-1 + (107 + 6*Sq
rt[318])^(-2/3) + (107 + 6*Sqrt[318])^(2/3))/9 + (2*(1 + (107 + 6*Sqrt[318]
)^(2/3))*x)/(3*(107 + 6*Sqrt[318])^(1/3)) + 4*x^2)^(1/3))/(-1/6 + x)^3, x],
 x, 1/6 + x])/((1 - (1 + (107 + 6*Sqrt[318])^(2/3))/(107 + 6*Sqrt[318])^(1/
3) + 6*x)^(1/3)*(-1 + (107 + 6*Sqrt[318])^(-2/3) + (107 + 6*Sqrt[318])^(2/3
) + ((1 + (107 + 6*Sqrt[318])^(2/3))*(1 + 6*x))/(107 + 6*Sqrt[318])^(1/3) +
 (1 + 6*x)^2)^(1/3)) + (27*(-2 + x^2 + 2*x^3)^(1/3)*Defer[Subst][Defer[Int]
[((-1/3*(1 + (107 + 6*Sqrt[318])^(2/3))/(107 + 6*Sqrt[318])^(1/3) + 2*x)^(1
/3)*((-1 + (107 + 6*Sqrt[318])^(-2/3) + (107 + 6*Sqrt[318])^(2/3))/9 + (2*(
1 + (107 + 6*Sqrt[318])^(2/3))*x)/(3*(107 + 6*Sqrt[318])^(1/3)) + 4*x^2)^(1
/3))/(-1/6 + x)^2, x], x, 1/6 + x])/((1 - (1 + (107 + 6*Sqrt[318])^(2/3))/(
107 + 6*Sqrt[318])^(1/3) + 6*x)^(1/3)*(-1 + (107 + 6*Sqrt[318])^(-2/3) + (1
07 + 6*Sqrt[318])^(2/3) + ((1 + (107 + 6*Sqrt[318])^(2/3))*(1 + 6*x))/(107
+ 6*Sqrt[318])^(1/3) + (1 + 6*x)^2)^(1/3)) + ((51/5 + (39*I)/5)*(-2 + x^2 +
 2*x^3)^(1/3)*Defer[Subst][Defer[Int][((-1/3*(1 + (107 + 6*Sqrt[318])^(2/3)
)/(107 + 6*Sqrt[318])^(1/3) + 2*x)^(1/3)*((-1 + (107 + 6*Sqrt[318])^(-2/3)
+ (107 + 6*Sqrt[318])^(2/3))/9 + (2*(1 + (107 + 6*Sqrt[318])^(2/3))*x)/(3*(
107 + 6*Sqrt[318])^(1/3)) + 4*x^2)^(1/3))/((5/3 - 2*I) + 2*x), x], x, 1/6 +
 x])/((1 - (1 + (107 + 6*Sqrt[318])^(2/3))/(107 + 6*Sqrt[318])^(1/3) + 6*x)
^(1/3)*(-1 + (107 + 6*Sqrt[318])^(-2/3) + (107 + 6*Sqrt[318])^(2/3) + ((1 +
 (107 + 6*Sqrt[318])^(2/3))*(1 + 6*x))/(107 + 6*Sqrt[318])^(1/3) + (1 + 6*x
)^2)^(1/3)) + ((48/5 - (24*I)/5)*(-2 + x^2 + 2*x^3)^(1/3)*Defer[Subst][Defe
r[Int][((-1/3*(1 + (107 + 6*Sqrt[318])^(2/3))/(107 + 6*Sqrt[318])^(1/3) + 2
*x)^(1/3)*((-1 + (107 + 6*Sqrt[318])^(-2/3) + (107 + 6*Sqrt[318])^(2/3))/9
+ (2*(1 + (107 + 6*Sqrt[318])^(2/3))*x)/(3*(107 + 6*Sqrt[318])^(1/3)) + 4*x
^2)^(1/3))/((5/3 + 2*I) + 2*x)^2, x], x, 1/6 + x])/((1 - (1 + (107 + 6*Sqrt
[318])^(2/3))/(107 + 6*Sqrt[318])^(1/3) + 6*x)^(1/3)*(-1 + (107 + 6*Sqrt[31
8])^(-2/3) + (107 + 6*Sqrt[318])^(2/3) + ((1 + (107 + 6*Sqrt[318])^(2/3))*(
1 + 6*x))/(107 + 6*Sqrt[318])^(1/3) + (1 + 6*x)^2)^(1/3)) + ((51/5 - (63*I)
/5)*(-2 + x^2 + 2*x^3)^(1/3)*Defer[Subst][Defer[Int][((-1/3*(1 + (107 + 6*S
qrt[318])^(2/3))/(107 + 6*Sqrt[318])^(1/3) + 2*x)^(1/3)*((-1 + (107 + 6*Sqr
t[318])^(-2/3) + (107 + 6*Sqrt[318])^(2/3))/9 + (2*(1 + (107 + 6*Sqrt[318])
^(2/3))*x)/(3*(107 + 6*Sqrt[318])^(1/3)) + 4*x^2)^(1/3))/((5/3 + 2*I) + 2*x
), x], x, 1/6 + x])/((1 - (1 + (107 + 6*Sqrt[318])^(2/3))/(107 + 6*Sqrt[318
])^(1/3) + 6*x)^(1/3)*(-1 + (107 + 6*Sqrt[318])^(-2/3) + (107 + 6*Sqrt[318]
)^(2/3) + ((1 + (107 + 6*Sqrt[318])^(2/3))*(1 + 6*x))/(107 + 6*Sqrt[318])^(
1/3) + (1 + 6*x)^2)^(1/3))
```

Rubi steps

$$\int \frac{\left(-6+x^2\right)\left(-2+x^2\right)\left(2-x^2+x^3\right)\sqrt[3]{-2+x^2+2x^3}}{x^5\left(-2+x^2+x^3\right)^2}\,dx = \int \left(\frac{2\sqrt[3]{-2+x^2+2x^3}}{5(-1+x)^2}-\frac{17\sqrt[3]{-2+x^2+2x^3}}{5(-1+x)}+\frac{6\sqrt[3]{-2}}{}\right.$$

$$= \frac{1}{5}\int \frac{(4+17x)\sqrt[3]{-2+x^2+2x^3}}{2+2x+x^2}\,dx+\frac{2}{5}\int \frac{\sqrt[3]{-2+x^2}}{(-1+x}$$

$$= \frac{1}{5}\int \left(\frac{(17+13i)\sqrt[3]{-2+x^2+2x^3}}{(2-2i)+2x}+\frac{(17-13i)\sqrt[3]{-2+}}{(2+2i)+}\right.$$

= rest of steps removed due to Latex formating pro

Mathematica [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{\left(-6+x^2\right)\left(-2+x^2\right)\left(2-x^2+x^3\right)\sqrt[3]{-2+x^2+2x^3}}{x^5\left(-2+x^2+x^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-6 + x^2)*(-2 + x^2)*(2 - x^2 + x^3)*(-2 + x^2 + 2*x^3)^(1/3))/
(x^5*(-2 + x^2 + x^3)^2),x]
```

```
[Out] Integrate[((-6 + x^2)*(-2 + x^2)*(2 - x^2 + x^3)*(-2 + x^2 + 2*x^3)^(1/3))/
(x^5*(-2 + x^2 + x^3)^2), x]
```

IntegrateAlgebraic [A] time = 0.95, size = 156, normalized size = 1.00

$$-\frac{7}{3}\log\left(\sqrt[3]{2x^3+x^2-2}-x\right)+\frac{7}{6}\log\left(x^2+\sqrt[3]{2x^3+x^2-2}\,x+\left(2x^3+x^2-2\right)^{2/3}\right)-\frac{7\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2x^3+x^2-2}+x}\right)}{\sqrt{3}}+\frac{\sqrt[3]{2x^3+x^2-2}\left(-38x^6-27x^5+3x^4+54x^3-12x^2+12\right)}{4x^4\left(x^3+x^2-2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-6 + x^2)*(-2 + x^2)*(2 - x^2 + x^3)*(-2 + x^2 + 2*x^3
)^(1/3))/(x^5*(-2 + x^2 + x^3)^2),x]
```

```
[Out] ((-2 + x^2 + 2*x^3)^(1/3)*(12 - 12*x^2 + 54*x^3 + 3*x^4 - 27*x^5 - 38*x^6))
/(4*x^4*(-2 + x^2 + x^3)) - (7*ArcTan[(Sqrt[3]*x)/(x + 2*(-2 + x^2 + 2*x^3)
^(1/3))])/Sqrt[3] - (7*Log[-x + (-2 + x^2 + 2*x^3)^(1/3)])/3 + (7*Log[x^2 +
 x*(-2 + x^2 + 2*x^3)^(1/3) + (-2 + x^2 + 2*x^3)^(2/3)])/6
```

fricas [A] time = 2.76, size = 212, normalized size = 1.36

$$\frac{28\sqrt{3}\left(x^7+x^6-2x^4\right)\arctan\left(\frac{1078\sqrt{3}\left(2x^3+x^2-2\right)^{\frac{1}{3}}x^2+196\sqrt{3}\left(2x^3+x^2-2\right)^{\frac{2}{3}}x+\sqrt{3}\left(669x^3+32x^2-64\right)}{1315x^3-8x^2+16}\right)-14\left(x^7+x^6-2x^4\right)\log\left(\frac{x^3+3\left(2x^3+x^2-2\right)^{\frac{1}{3}}x^2+x^2-3\left(2x^3+x^2-2\right)^{\frac{2}{3}}x-2}{x^3+x^2-2}\right)-3\left(38x^6+27x^5-3x^4-54x^3+12x^2-12\right)\left(2x^3+x^2-2\right)^{\frac{1}{3}}}{12\left(x^7+x^6-2x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-6)*(x^2-2)*(x^3-x^2+2)*(2*x^3+x^2-2)^(1/3)/x^5/(x^3+x^2-2)^2
,x, algorithm="fricas")
```

```
[Out] 1/12*(28*sqrt(3)*(x^7 + x^6 - 2*x^4)*arctan((1078*sqrt(3)*(2*x^3 + x^2 - 2)
^(1/3)*x^2 + 196*sqrt(3)*(2*x^3 + x^2 - 2)^(2/3)*x + sqrt(3)*(669*x^3 + 32*
x^2 - 64))/(1315*x^3 - 8*x^2 + 16)) - 14*(x^7 + x^6 - 2*x^4)*log((x^3 + 3*(
2*x^3 + x^2 - 2)^(1/3)*x^2 + x^2 - 3*(2*x^3 + x^2 - 2)^(2/3)*x - 2)/(x^3 +
x^2 - 2)) - 3*(38*x^6 + 27*x^5 - 3*x^4 - 54*x^3 + 12*x^2 - 12)*(2*x^3 + x^2
 - 2)^(1/3))/(x^7 + x^6 - 2*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^3+x^2-2\right)^{\frac{1}{3}}\left(x^3-x^2+2\right)\left(x^2-2\right)\left(x^2-6\right)}{\left(x^3+x^2-2\right)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-6)*(x^2-2)*(x^3-x^2+2)*(2*x^3+x^2-2)^(1/3)/x^5/(x^3+x^2-2)^2
,x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + x^2 - 2)^(1/3)*(x^3 - x^2 + 2)*(x^2 - 2)*(x^2 - 6)/((x^3
 + x^2 - 2)^2*x^5), x)
```

maple [C] time = 1.62, size = 1291, normalized size = 8.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-6)*(x^2-2)*(x^3-x^2+2)*(2*x^3+x^2-2)^(1/3)/x^5/(x^3+x^2-2)^2,x)
```

```
[Out] -1/4*(76*x^9+92*x^8+21*x^7-187*x^6-84*x^5+18*x^4+84*x^3-36*x^2+24)/x^4/(x^3
+x^2-2)/(2*x^3+x^2-2)^(2/3)+(7/6*RootOf(_Z^2-2*_Z+4)*ln((2*RootOf(_Z^2-2*_Z
```

```
+4)^2*x^6+RootOf(_Z^2-2*_Z+4)^2*x^5+6*RootOf(_Z^2-2*_Z+4)*x^6+6*RootOf(_Z^2
-2*_Z+4)*(4*x^6+4*x^5+x^4-8*x^3-4*x^2+4)^(1/3)*x^4+7*RootOf(_Z^2-2*_Z+4)*x^
5-8*x^6+3*RootOf(_Z^2-2*_Z+4)*(4*x^6+4*x^5+x^4-8*x^3-4*x^2+4)^(2/3)*x^2+3*R
ootOf(_Z^2-2*_Z+4)*(4*x^6+4*x^5+x^4-8*x^3-4*x^2+4)^(1/3)*x^3-12*(4*x^6+4*x^
5+x^4-8*x^3-4*x^2+4)^(1/3)*x^4-2*RootOf(_Z^2-2*_Z+4)^2*x^3+2*RootOf(_Z^2-2*
_Z+4)*x^4-8*x^5-6*(4*x^6+4*x^5+x^4-8*x^3-4*x^2+4)^(2/3)*x^2-6*(4*x^6+4*x^5+
x^4-8*x^3-4*x^2+4)^(1/3)*x^3-14*RootOf(_Z^2-2*_Z+4)*x^3-2*x^4-6*RootOf(_Z^2
-2*_Z+4)*(4*x^6+4*x^5+x^4-8*x^3-4*x^2+4)^(1/3)*x-8*RootOf(_Z^2-2*_Z+4)*x^2+
16*x^3+12*(4*x^6+4*x^5+x^4-8*x^3-4*x^2+4)^(1/3)*x+8*x^2+8*RootOf(_Z^2-2*_Z+
4)-8)/(x^2+2*x+2)/(2*x^3+x^2-2)/(-1+x))-7/6*ln(-(-2*RootOf(_Z^2-2*_Z+4)^2*x
^6-RootOf(_Z^2-2*_Z+4)^2*x^5+14*RootOf(_Z^2-2*_Z+4)*x^6+6*RootOf(_Z^2-2*_Z+
4)*(4*x^6+4*x^5+x^4-8*x^3-4*x^2+4)^(1/3)*x^4+11*RootOf(_Z^2-2*_Z+4)*x^5-12*
x^6+3*RootOf(_Z^2-2*_Z+4)*(4*x^6+4*x^5+x^4-8*x^3-4*x^2+4)^(2/3)*x^2+3*RootO
f(_Z^2-2*_Z+4)*(4*x^6+4*x^5+x^4-8*x^3-4*x^2+4)^(1/3)*x^3+2*RootOf(_Z^2-2*_Z
+4)^2*x^3+2*RootOf(_Z^2-2*_Z+4)*x^4-10*x^5-22*RootOf(_Z^2-2*_Z+4)*x^3-2*x^4
-6*RootOf(_Z^2-2*_Z+4)*(4*x^6+4*x^5+x^4-8*x^3-4*x^2+4)^(1/3)*x-8*RootOf(_Z^
2-2*_Z+4)*x^2+20*x^3+8*x^2+8*RootOf(_Z^2-2*_Z+4)-8)/(x^2+2*x+2)/(2*x^3+x^2-
2)/(-1+x))*RootOf(_Z^2-2*_Z+4)+7/3*ln(-(-2*RootOf(_Z^2-2*_Z+4)^2*x^6-RootOf
(_Z^2-2*_Z+4)^2*x^5+14*RootOf(_Z^2-2*_Z+4)*x^6+6*RootOf(_Z^2-2*_Z+4)*(4*x^6
+4*x^5+x^4-8*x^3-4*x^2+4)^(1/3)*x^4+11*RootOf(_Z^2-2*_Z+4)*x^5-12*x^6+3*Roo
tOf(_Z^2-2*_Z+4)*(4*x^6+4*x^5+x^4-8*x^3-4*x^2+4)^(2/3)*x^2+3*RootOf(_Z^2-2*
_Z+4)*(4*x^6+4*x^5+x^4-8*x^3-4*x^2+4)^(1/3)*x^3+2*RootOf(_Z^2-2*_Z+4)^2*x^3
+2*RootOf(_Z^2-2*_Z+4)*x^4-10*x^5-22*RootOf(_Z^2-2*_Z+4)*x^3-2*x^4-6*RootOf
(_Z^2-2*_Z+4)*(4*x^6+4*x^5+x^4-8*x^3-4*x^2+4)^(1/3)*x-8*RootOf(_Z^2-2*_Z+4)
*x^2+20*x^3+8*x^2+8*RootOf(_Z^2-2*_Z+4)-8)/(x^2+2*x+2)/(2*x^3+x^2-2)/(-1+x)
))/(2*x^3+x^2-2)^(2/3)*((2*x^3+x^2-2)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^3+x^2-2\right)^{\frac{1}{3}}\left(x^3-x^2+2\right)\left(x^2-2\right)\left(x^2-6\right)}{\left(x^3+x^2-2\right)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-6)*(x^2-2)*(x^3-x^2+2)*(2*x^3+x^2-2)^(1/3)/x^5/(x^3+x^2-2)^2
,x, algorithm="maxima")
```

```
[Out] integrate((2*x^3 + x^2 - 2)^(1/3)*(x^3 - x^2 + 2)*(x^2 - 2)*(x^2 - 6)/((x^3
 + x^2 - 2)^2*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2-2\right)\,\left(x^2-6\right)\,\left(x^3-x^2+2\right)\,{\left(2\,x^3+x^2-2\right)}^{1/3}}{x^5\,{\left(x^3+x^2-2\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 2)*(x^2 - 6)*(x^3 - x^2 + 2)*(x^2 + 2*x^3 - 2)^(1/3))/(x^5*(x^2
 + x^3 - 2)^2),x)
```

```
[Out] int(((x^2 - 2)*(x^2 - 6)*(x^3 - x^2 + 2)*(x^2 + 2*x^3 - 2)^(1/3))/(x^5*(x^2
 + x^3 - 2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x + 1\right) \left(x^{2} - 6\right) \left(x^{2} - 2\right) \left(x^{2} - 2 x + 2\right) \sqrt[3]{2 x^{3} + x^{2} - 2}}{x^{5} \left(x - 1\right)^{2} \left(x^{2} + 2 x + 2\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-6)*(x**2-2)*(x**3-x**2+2)*(2*x**3+x**2-2)**(1/3)/x**5/(x**3
+x**2-2)**2,x)
```

```
[Out] Integral((x + 1)*(x**2 - 6)*(x**2 - 2)*(x**2 - 2*x + 2)*(2*x**3 + x**2 - 2)
**(1/3)/(x**5*(x - 1)**2*(x**2 + 2*x + 2)**2), x)
```

3.1779 $\int \frac{(1+x)\sqrt[4]{x^3+x^5}}{x(-1+x^3)}\,dx$

Optimal. Leaf size=156

$$\frac{2}{3}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^5+x^3}}\right)-\frac{2}{3}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^5+x^3}}\right)-\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^5+x^3}}{\sqrt{x^5+x^3}-x^2}\right)+\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt{2}}+\frac{\sqrt{x^5+x^3}}{\sqrt{2}}}{x\sqrt[4]{x^5+x^3}}\right)$$

Rubi [C] time = 0.74, antiderivative size = 320, normalized size of antiderivative = 2.05, number of steps used = 18, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2056, 6725, 959, 466, 510}

$$-\frac{4\left(1-\sqrt[3]{-1}\right)\sqrt[4]{x^5+x^3}\,F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};-x^2,-\sqrt[3]{-1}\,x^2\right)}{9\sqrt[4]{x^2+1}}-\frac{4\left(1+(-1)^{2/3}\right)\sqrt[4]{x^5+x^3}\,F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};-x^2,(-1)^{2/3}x^2\right)}{9\sqrt[4]{x^2+1}}-\frac{8\sqrt[4]{x^5+x^3}\,F_1\left(\frac{3}{8};1,-\frac{1}{4};\frac{11}{8};x^2,-x^2\right)}{9\sqrt[4]{x^2+1}}-\frac{4\left(1+(-1)^{2/3}\right)x\sqrt[4]{x^5+x^3}\,F_1\left(\frac{7}{8};-\frac{1}{4},1;\frac{15}{8};-x^2,-\sqrt[3]{-1}\,x^2\right)}{21\sqrt[4]{x^2+1}}-\frac{4\left(1-\sqrt[3]{-1}\right)x\sqrt[4]{x^5+x^3}\,F_1\left(\frac{7}{8};-\frac{1}{4},1;\frac{15}{8};-x^2,(-1)^{2/3}x^2\right)}{21\sqrt[4]{x^2+1}}-\frac{8x\sqrt[4]{x^5+x^3}\,F_1\left(\frac{7}{8};1,-\frac{1}{4};\frac{15}{8};x^2,-x^2\right)}{21\sqrt[4]{x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((1 + x)*(x^3 + x^5)^(1/4))/(x*(-1 + x^3)),x]
```

```
[Out] (-4*(1 - (-1)^(1/3))*(x^3 + x^5)^(1/4)*AppellF1[3/8, -1/4, 1, 11/8, -x^2, -
((-1)^(1/3)*x^2)])/(9*(1 + x^2)^(1/4)) - (4*(1 + (-1)^(2/3))*(x^3 + x^5)^(1
/4)*AppellF1[3/8, -1/4, 1, 11/8, -x^2, (-1)^(2/3)*x^2])/(9*(1 + x^2)^(1/4))
 - (8*(x^3 + x^5)^(1/4)*AppellF1[3/8, 1, -1/4, 11/8, x^2, -x^2])/(9*(1 + x^
2)^(1/4)) - (4*(1 + (-1)^(2/3))*x*(x^3 + x^5)^(1/4)*AppellF1[7/8, -1/4, 1,
15/8, -x^2, -((-1)^(1/3)*x^2)])/(21*(1 + x^2)^(1/4)) - (4*(1 - (-1)^(1/3))*
x*(x^3 + x^5)^(1/4)*AppellF1[7/8, -1/4, 1, 15/8, -x^2, (-1)^(2/3)*x^2])/(21
*(1 + x^2)^(1/4)) - (8*x*(x^3 + x^5)^(1/4)*AppellF1[7/8, 1, -1/4, 15/8, x^2
, -x^2])/(21*(1 + x^2)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(1+x)\sqrt[4]{x^3+x^5}}{x\left(-1+x^3\right)}\,dx = \frac{\sqrt[4]{x^3+x^5}\int \frac{(1+x)\sqrt[4]{1+x^2}}{\sqrt[4]{x}\left(-1+x^3\right)}\,dx}{x^{3/4}\sqrt[4]{1+x^2}}$$

$$= \frac{\sqrt[4]{x^3+x^5}\int \left(-\frac{2\sqrt[4]{1+x^2}}{3(1-x)\sqrt[4]{x}} - \frac{\left(1+(-1)^{2/3}\right)\sqrt[4]{1+x^2}}{3\sqrt[4]{x}\left(1+\sqrt[3]{-1}\,x\right)} - \frac{\left(1-\sqrt[3]{-1}\right)\sqrt[4]{1+x^2}}{3\sqrt[4]{x}\left(1-(-1)^{2/3}x\right)}\right)dx}{x^{3/4}\sqrt[4]{1+x^2}}$$

$$= -\frac{\left(2\sqrt[4]{x^3+x^5}\right)\int \frac{\sqrt[4]{1+x^2}}{(1-x)\sqrt[4]{x}}\,dx}{3x^{3/4}\sqrt[4]{1+x^2}} + \frac{\left(\left(-1+\sqrt[3]{-1}\right)\sqrt[4]{x^3+x^5}\right)\int \frac{\sqrt[4]{1+x^2}}{\sqrt[4]{x}\left(1-(-1)^{2/3}x\right)}\,dx}{3x^{3/4}\sqrt[4]{1+x^2}} + \frac{\left(\left(-1-\right.\right.}{}$$

$$= -\frac{\left(2\sqrt[4]{x^3+x^5}\right)\int \frac{\sqrt[4]{1+x^2}}{\sqrt[4]{x}\left(1-x^2\right)}\,dx}{3x^{3/4}\sqrt[4]{1+x^2}} - \frac{\left(2\sqrt[4]{x^3+x^5}\right)\int \frac{x^{3/4}\sqrt[4]{1+x^2}}{1-x^2}\,dx}{3x^{3/4}\sqrt[4]{1+x^2}} + \frac{\left(\left(-1+\sqrt[3]{-1}\right)\sqrt[4]{x^3+\right.}{3x^{3/4}\sqrt[4]{}}$$

$$= -\frac{\left(8\sqrt[4]{x^3+x^5}\right)\mathrm{Subst}\left(\int \frac{x^2\sqrt[4]{1+x^8}}{1-x^8}\,dx, x, \sqrt[4]{x}\right)}{3x^{3/4}\sqrt[4]{1+x^2}} - \frac{\left(8\sqrt[4]{x^3+x^5}\right)\mathrm{Subst}\left(\int \frac{x^6\sqrt[4]{1+x^8}}{1-x^8}\,dx, x,\right.}{3x^{3/4}\sqrt[4]{1+x^2}}$$

$$= -\frac{4\left(1-\sqrt[3]{-1}\right)\sqrt[4]{x^3+x^5}\,F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};-x^2,-\sqrt[3]{-1}\,x^2\right)}{9\sqrt[4]{1+x^2}} - \frac{4\left(1+(-1)^{2/3}\right)\sqrt[4]{x^3+x^5}\,F_1}{9\sqrt[4]{}}$$

Mathematica [F] time = 1.44, size = 0, normalized size = 0.00

$$\int \frac{(1+x)\sqrt[4]{x^3+x^5}}{x\left(-1+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x)*(x^3 + x^5)^(1/4))/(x*(-1 + x^3)),x]
[Out] Integrate[((1 + x)*(x^3 + x^5)^(1/4))/(x*(-1 + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.53, size = 156, normalized size = 1.00

$$\frac{2}{3}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^5+x^3}}\right) - \frac{2}{3}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^5+x^3}}\right) - \frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^5+x^3}}{\sqrt{x^5+x^3}-x^2}\right) + \frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt{2}}+\frac{\sqrt{x^5+x^3}}{\sqrt{2}}}{x\sqrt[4]{x^5+x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x)*(x^3 + x^5)^(1/4))/(x*(-1 + x^3)),x]
[Out] (2*2^(1/4)*ArcTan[(2^(1/4)*x)/(x^3 + x^5)^(1/4)])/3 - (Sqrt[2]*ArcTan[(Sqrt
[2]*x*(x^3 + x^5)^(1/4))/(-x^2 + Sqrt[x^3 + x^5])])/3 - (2*2^(1/4)*ArcTanh[
(2^(1/4)*x)/(x^3 + x^5)^(1/4)])/3 + (Sqrt[2]*ArcTanh[(x^2/Sqrt[2] + Sqrt[x^
3 + x^5]/Sqrt[2])/(x*(x^3 + x^5)^(1/4))])/3
```

fricas [B] time = 5.60, size = 953, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(x^5+x^3)^(1/4)/x/(x^3-1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(2)*arctan((x^6 + 2*x^5 + 3*x^4 + 2*x^3 + 2*sqrt(2)*(x^5 + x^3)^(3
/4)*(x^2 - 3*x + 1) + x^2 + 2*sqrt(2)*(x^5 + x^3)^(1/4)*(3*x^4 - x^3 + 3*x^
2) + 4*sqrt(x^5 + x^3)*(x^3 + x^2 + x) + (2*sqrt(2)*sqrt(x^5 + x^3)*(x^3 -
3*x^2 + x) + 16*(x^5 + x^3)^(3/4)*x + sqrt(2)*(x^6 - 8*x^5 + x^4 - 8*x^3 +
x^2) + 4*(x^5 + x^3)^(1/4)*(x^4 + x^3 + x^2))*sqrt((x^4 + x^3 + 2*sqrt(2)*(
x^5 + x^3)^(1/4)*x^2 + x^2 + 4*sqrt(x^5 + x^3)*x + 2*sqrt(2)*(x^5 + x^3)^(3
/4))/(x^4 + x^3 + x^2)))/(x^6 - 14*x^5 + 3*x^4 - 14*x^3 + x^2)) + 1/3*sqrt(
2)*arctan((x^6 + 2*x^5 + 3*x^4 + 2*x^3 - 2*sqrt(2)*(x^5 + x^3)^(3/4)*(x^2 -
 3*x + 1) + x^2 - 2*sqrt(2)*(x^5 + x^3)^(1/4)*(3*x^4 - x^3 + 3*x^2) + 4*sqr
t(x^5 + x^3)*(x^3 + x^2 + x) - (2*sqrt(2)*sqrt(x^5 + x^3)*(x^3 - 3*x^2 + x)
 - 16*(x^5 + x^3)^(3/4)*x + sqrt(2)*(x^6 - 8*x^5 + x^4 - 8*x^3 + x^2) - 4*(
x^5 + x^3)^(1/4)*(x^4 + x^3 + x^2))*sqrt((x^4 + x^3 - 2*sqrt(2)*(x^5 + x^3)
^(1/4)*x^2 + x^2 + 4*sqrt(x^5 + x^3)*x - 2*sqrt(2)*(x^5 + x^3)^(3/4))/(x^4
+ x^3 + x^2)))/(x^6 - 14*x^5 + 3*x^4 - 14*x^3 + x^2)) + 1/12*sqrt(2)*log(4*
(x^4 + x^3 + 2*sqrt(2)*(x^5 + x^3)^(1/4)*x^2 + x^2 + 4*sqrt(x^5 + x^3)*x +
2*sqrt(2)*(x^5 + x^3)^(3/4))/(x^4 + x^3 + x^2)) - 1/12*sqrt(2)*log(4*(x^4 +
 x^3 - 2*sqrt(2)*(x^5 + x^3)^(1/4)*x^2 + x^2 + 4*sqrt(x^5 + x^3)*x - 2*sqrt
(2)*(x^5 + x^3)^(3/4))/(x^4 + x^3 + x^2)) + 2/3*2^(1/4)*arctan(1/2*(4*2^(3/
4)*(x^5 + x^3)^(1/4)*x^2 + 2^(3/4)*(2*2^(3/4)*sqrt(x^5 + x^3)*x + 2^(1/4)*(
x^4 + 2*x^3 + x^2)) + 4*2^(1/4)*(x^5 + x^3)^(3/4))/(x^4 - 2*x^3 + x^2)) - 1
/6*2^(1/4)*log(-(4*sqrt(2)*(x^5 + x^3)^(1/4)*x^2 + 2^(3/4)*(x^4 + 2*x^3 + x
^2) + 4*2^(1/4)*sqrt(x^5 + x^3)*x + 4*(x^5 + x^3)^(3/4))/(x^4 - 2*x^3 + x^2
)) + 1/6*2^(1/4)*log(-(4*sqrt(2)*(x^5 + x^3)^(1/4)*x^2 - 2^(3/4)*(x^4 + 2*x
^3 + x^2) - 4*2^(1/4)*sqrt(x^5 + x^3)*x + 4*(x^5 + x^3)^(3/4))/(x^4 - 2*x^3
 + x^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{5}+x^{3}\right)^{\frac{1}{4}}(x+1)}{\left(x^{3}-1\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(x^5+x^3)^(1/4)/x/(x^3-1),x, algorithm="giac")
```

```
[Out] integrate((x^5 + x^3)^(1/4)*(x + 1)/((x^3 - 1)*x), x)
```

maple [C] time = 13.23, size = 741, normalized size = 4.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)*(x^5+x^3)^(1/4)/x/(x^3-1),x)
```

```
[Out] -1/3*RootOf(_Z^2+RootOf(_Z^4-2)^2)*ln((-RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootO
f(_Z^4-2)^2*x^4-2*RootOf(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^3-RootOf
(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^2-4*(x^5+x^3)^(1/4)*RootOf(_Z^4-
2)^2*x^2+4*RootOf(_Z^2+RootOf(_Z^4-2)^2)*(x^5+x^3)^(1/2)*x+4*(x^5+x^3)^(3/4
))/(-1+x)^2/x^2)+1/3*RootOf(_Z^4-2)*ln((-RootOf(_Z^4-2)^3*x^4-2*RootOf(_Z^4
-2)^3*x^3+4*(x^5+x^3)^(1/4)*RootOf(_Z^4-2)^2*x^2-RootOf(_Z^4-2)^3*x^2-4*(x^
5+x^3)^(1/2)*RootOf(_Z^4-2)*x+4*(x^5+x^3)^(3/4))/(-1+x)^2/x^2)-1/6*ln((-Roo
tOf(_Z^4-2)^2*x^4-4*RootOf(_Z^4-2)^2*(x^5+x^3)^(1/2)*x-RootOf(_Z^4-2)^2*x^3
-RootOf(_Z^4-2)^2*x^2+4*(x^5+x^3)^(3/4)+4*(x^5+x^3)^(1/4)*x^2)/x^2/(x^2+x+1
))*RootOf(_Z^4-2)^2-1/6*ln((-RootOf(_Z^4-2)^2*x^4-4*RootOf(_Z^4-2)^2*(x^5+x
^3)^(1/2)*x-RootOf(_Z^4-2)^2*x^3-RootOf(_Z^4-2)^2*x^2+4*(x^5+x^3)^(3/4)+4*(
```

```
x^5+x^3)^(1/4)*x^2)/x^2/(x^2+x+1))*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^
4-2)+1/3*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)*ln((-2*RootOf(_Z^2+Ro
otOf(_Z^4-2)^2)*(x^5+x^3)^(1/4)*RootOf(_Z^4-2)^3*x^2-RootOf(_Z^2+RootOf(_Z^
4-2)^2)*RootOf(_Z^4-2)*x^4-RootOf(_Z^4-2)^2*x^4+2*RootOf(_Z^4-2)*RootOf(_Z^
2+RootOf(_Z^4-2)^2)*(x^5+x^3)^(1/2)*x+RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(
_Z^4-2)*x^3-2*RootOf(_Z^4-2)^2*(x^5+x^3)^(1/2)*x+RootOf(_Z^4-2)^2*x^3-RootO
f(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)*x^2-RootOf(_Z^4-2)^2*x^2+4*(x^5+x^3
)^(3/4))/x^2/(x^2+x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5+x^3\right)^{\frac{1}{4}}(x+1)}{\left(x^3-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(x^5+x^3)^(1/4)/x/(x^3-1),x, algorithm="maxima")
[Out] integrate((x^5 + x^3)^(1/4)*(x + 1)/((x^3 - 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^5+x^3\right)^{1/4}\,(x+1)}{x\,\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + x^5)^(1/4)*(x + 1))/(x*(x^3 - 1)),x)
[Out] int(((x^3 + x^5)^(1/4)*(x + 1))/(x*(x^3 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\left(x^2+1\right)}\,(x+1)}{x\,(x-1)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*(x**5+x**3)**(1/4)/x/(x**3-1),x)
[Out] Integral((x**3*(x**2 + 1))**(1/4)*(x + 1)/(x*(x - 1)*(x**2 + x + 1)), x)
```

3.1780 $\int \frac{(1+2x^3)^{4/3}(1+3x^3)}{x^8(1+4x^3)}\,dx$

Optimal. Leaf size=157

$$\frac{2}{3}\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{2x^3+1}+2x\right)-\frac{2\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{2x^3+1}-x}\right)}{\sqrt{3}}-\frac{1}{3}\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{2x^3+1}\,x-\sqrt[3]{2}\left(2x^3+1\right)^{2/3}-2x^2\right)+$$

Rubi [A] time = 0.21, antiderivative size = 176, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {580, 583, 12, 494, 292, 31, 634, 617, 204, 628}

$$-\frac{29\sqrt[3]{2x^3+1}}{14x}+\frac{2}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{2x^3+1}}+1\right)+\frac{2\sqrt[3]{2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{2x^3+1}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\left(2x^3+1\right)^{4/3}}{7x^7}-\frac{\sqrt[3]{2x^3+1}}{28x^4}-\frac{1}{3}\sqrt[3]{2}\log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{2x^3+1}}+\frac{2^{2/3}x^2}{\left(2x^3+1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x^3)^(4/3)*(1 + 3*x^3))/(x^8*(1 + 4*x^3)),x]
```

```
[Out] -1/28*(1 + 2*x^3)^(1/3)/x^4 - (29*(1 + 2*x^3)^(1/3))/(14*x) - (1 + 2*x^3)^(
4/3)/(7*x^7) + (2*2^(1/3)*ArcTan[(1 - (2*2^(1/3)*x)/(1 + 2*x^3)^(1/3))/Sqrt
[3]])/Sqrt[3] - (2^(1/3)*Log[1 + (2^(2/3)*x^2)/(1 + 2*x^3)^(2/3) - (2^(1/3)
*x)/(1 + 2*x^3)^(1/3)])/3 + (2*2^(1/3)*Log[1 + (2^(1/3)*x)/(1 + 2*x^3)^(1/3
)])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(1+2x^3)^{4/3}(1+3x^3)}{x^8(1+4x^3)}\,dx = -\frac{(1+2x^3)^{4/3}}{7x^7} + \frac{1}{7}\int \frac{\sqrt[3]{1+2x^3}\,(1+18x^3)}{x^5(1+4x^3)}\,dx$$

$$= -\frac{\sqrt[3]{1+2x^3}}{28x^4} - \frac{(1+2x^3)^{4/3}}{7x^7} + \frac{1}{28}\int \frac{58+120x^3}{x^2(1+2x^3)^{2/3}(1+4x^3)}\,dx$$

$$= -\frac{\sqrt[3]{1+2x^3}}{28x^4} - \frac{29\sqrt[3]{1+2x^3}}{14x} - \frac{(1+2x^3)^{4/3}}{7x^7} - \frac{1}{28}\int \frac{112x}{(1+2x^3)^{2/3}(1+4x^3)}\,dx$$

$$= -\frac{\sqrt[3]{1+2x^3}}{28x^4} - \frac{29\sqrt[3]{1+2x^3}}{14x} - \frac{(1+2x^3)^{4/3}}{7x^7} - 4\int \frac{x}{(1+2x^3)^{2/3}(1+4x^3)}\,dx$$

$$= -\frac{\sqrt[3]{1+2x^3}}{28x^4} - \frac{29\sqrt[3]{1+2x^3}}{14x} - \frac{(1+2x^3)^{4/3}}{7x^7} - 4\,\text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1+2x^3}}\right)$$

$$= -\frac{\sqrt[3]{1+2x^3}}{28x^4} - \frac{29\sqrt[3]{1+2x^3}}{14x} - \frac{(1+2x^3)^{4/3}}{7x^7} + \frac{1}{3}\left(2\,2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, -$$

$$= -\frac{\sqrt[3]{1+2x^3}}{28x^4} - \frac{29\sqrt[3]{1+2x^3}}{14x} - \frac{(1+2x^3)^{4/3}}{7x^7} + \frac{2}{3}\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1+2x^3}}\right) - \frac{1}{3}\sqrt[3]{2}\,\text{Su}$$

$$= -\frac{\sqrt[3]{1+2x^3}}{28x^4} - \frac{29\sqrt[3]{1+2x^3}}{14x} - \frac{(1+2x^3)^{4/3}}{7x^7} - \frac{1}{3}\sqrt[3]{2}\log\left(1+\frac{2^{2/3}x^2}{(1+2x^3)^{2/3}} - \frac{\sqrt[3]{2}\,x}{\sqrt[3]{1+2}}\right.$$

$$= -\frac{\sqrt[3]{1+2x^3}}{28x^4} - \frac{29\sqrt[3]{1+2x^3}}{14x} - \frac{(1+2x^3)^{4/3}}{7x^7} + \frac{2\sqrt[3]{2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1+2x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\sqrt[3]{2}\log\left(1\right.$$

Mathematica [C] time = 0.10, size = 71, normalized size = 0.45

$$-\frac{2x^2\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{2x^3}{4x^3+1}\right)}{(4x^3+1)^{2/3}} - \frac{\sqrt[3]{2x^3+1}\,(58x^6+9x^3+4)}{28x^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + 2*x^3)^(4/3)*(1 + 3*x^3))/(x^8*(1 + 4*x^3)),x]
```

```
[Out] -1/28*((1 + 2*x^3)^(1/3)*(4 + 9*x^3 + 58*x^6))/x^7 - (2*x^2*Hypergeometric2
F1[2/3, 2/3, 5/3, (2*x^3)/(1 + 4*x^3)])/(1 + 4*x^3)^(2/3)
```

IntegrateAlgebraic [A] time = 0.35, size = 157, normalized size = 1.00

$$\frac{2}{3}\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{2x^3+1}+2x\right) - \frac{2\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{2x^3+1}-x}\right)}{\sqrt{3}} - \frac{1}{3}\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{2x^3+1}\,x - \sqrt[3]{2}\,(2x^3+1)^{2/3} - 2x^2\right) + \frac{\sqrt[3]{2x^3+1}\,(-58x^6-9x^3-4)}{28x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + 2*x^3)^(4/3)*(1 + 3*x^3))/(x^8*(1 + 4*x^3)),x]
```

```
[Out] ((1 + 2*x^3)^(1/3)*(-4 - 9*x^3 - 58*x^6))/(28*x^7) - (2*2^(1/3)*ArcTan[(Sqr
t[3]*x)/(-x + 2^(2/3)*(1 + 2*x^3)^(1/3))])/Sqrt[3] + (2*2^(1/3)*Log[2*x + 2
```

```
^(2/3)*(1 + 2*x^3)^(1/3)])/3 - (2^(1/3)*Log[-2*x^2 + 2^(2/3)*x*(1 + 2*x^3)^
(1/3) - 2^(1/3)*(1 + 2*x^3)^(2/3)])/3
```

fricas [B] time = 2.05, size = 291, normalized size = 1.85

$$-\frac{56\,\sqrt{3}\,2^{\frac{1}{3}}x^{7}\arctan\left(-\frac{6\,\sqrt{3}\,2^{\frac{2}{3}}\left(20\,x^{8}+10\,x^{5}-x^{2}\right)\left(2\,x^{3}+1\right)^{\frac{1}{3}}-6\,\sqrt{3}\,2^{\frac{1}{3}}\left(8\,x^{7}-2\,x^{4}-x\right)\left(2\,x^{3}+1\right)^{\frac{2}{3}}+\sqrt{3}\left(8\,x^{9}+60\,x^{6}+24\,x^{3}-1\right)}{3\left(152\,x^{9}+60\,x^{6}-12\,x^{3}-1\right)}\right)-56\cdot 2^{\frac{1}{3}}x^{7}\log\left(\frac{6\cdot 2^{\frac{1}{3}}\left(2\,x^{3}+1\right)^{\frac{1}{3}}x^{2}+2^{\frac{2}{3}}\left(4\,x^{3}+1\right)+6\left(2\,x^{3}+1\right)^{\frac{2}{3}}x}{4\,x^{3}+1}\right)+28\cdot 2^{\frac{1}{3}}x^{7}\log\left(\frac{3\cdot 2^{\frac{2}{3}}\left(2\,x^{4}-x\right)\left(2\,x^{3}+1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(20\,x^{6}+10\,x^{3}-1\right)+12\left(x^{5}+x^{2}\right)\left(2\,x^{3}+1\right)^{\frac{1}{3}}}{16\,x^{6}+8\,x^{3}+1}\right)+9\left(58\,x^{6}+9\,x^{3}+4\right)\left(2\,x^{3}+1\right)^{\frac{1}{3}}}{252\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)^(4/3)*(3*x^3+1)/x^8/(4*x^3+1),x, algorithm="fricas")
```

```
[Out] -1/252*(56*sqrt(3)*2^(1/3)*x^7*arctan(-1/3*(6*sqrt(3)*2^(2/3)*(20*x^8 + 10*
x^5 - x^2)*(2*x^3 + 1)^(1/3) - 6*sqrt(3)*2^(1/3)*(8*x^7 - 2*x^4 - x)*(2*x^3
 + 1)^(2/3) + sqrt(3)*(8*x^9 + 60*x^6 + 24*x^3 - 1))/(152*x^9 + 60*x^6 - 12
*x^3 - 1)) - 56*2^(1/3)*x^7*log((6*2^(1/3)*(2*x^3 + 1)^(1/3)*x^2 + 2^(2/3)*
(4*x^3 + 1) + 6*(2*x^3 + 1)^(2/3)*x)/(4*x^3 + 1)) + 28*2^(1/3)*x^7*log((3*2
^(2/3)*(2*x^4 - x)*(2*x^3 + 1)^(2/3) - 2^(1/3)*(20*x^6 + 10*x^3 - 1) + 12*(
x^5 + x^2)*(2*x^3 + 1)^(1/3))/(16*x^6 + 8*x^3 + 1)) + 9*(58*x^6 + 9*x^3 + 4
)*(2*x^3 + 1)^(1/3))/x^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^{3}+1\right)\left(2\,x^{3}+1\right)^{\frac{4}{3}}}{\left(4\,x^{3}+1\right)x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)^(4/3)*(3*x^3+1)/x^8/(4*x^3+1),x, algorithm="giac")
```

```
[Out] integrate((3*x^3 + 1)*(2*x^3 + 1)^(4/3)/((4*x^3 + 1)*x^8), x)
```

maple [C] time = 2.62, size = 941, normalized size = 5.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+1)^(4/3)*(3*x^3+1)/x^8/(4*x^3+1),x)
```

```
[Out] -1/28*(116*x^9+76*x^6+17*x^3+4)/x^7/(2*x^3+1)^(2/3)+(4*RootOf(RootOf(_Z^3-2
)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*ln((72*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf
(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^6-12*RootOf(RootOf(_Z^3-2)^2+6*_Z*Ro
otOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^6+18*(4*x^6+4*x^3+1)^(2/3)*RootOf(
RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x^2+36*(4*x^
6+4*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootO
f(_Z^3-2)*x^4+6*(4*x^6+4*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x^4+36*RootOf(RootOf
(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-6*RootOf(Roo
tOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^3+18*(4*x^6+4
*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_
Z^3-2)*x+3*(4*x^6+4*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x-12*RootOf(RootOf(_Z^3-2
)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+2*RootOf(_Z^3-2)*x^3-6*RootOf(RootOf(_
Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)+RootOf(_Z^3-2))/(2*x^3+1)/(4*x^3+1))+
2/3*RootOf(_Z^3-2)*ln(-(-72*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*
_Z^2)^2*RootOf(_Z^3-2)^2*x^6+12*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)
+36*_Z^2)*RootOf(_Z^3-2)^3*x^6+18*(4*x^6+4*x^3+1)^(2/3)*RootOf(RootOf(_Z^3-
2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x^2+6*(4*x^6+4*x^3+1)^(1
/3)*RootOf(_Z^3-2)^2*x^4-36*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*
_Z^2)^2*RootOf(_Z^3-2)^2*x^3+6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+
36*_Z^2)*RootOf(_Z^3-2)^3*x^3-24*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2
)+36*_Z^2)*x^6+4*RootOf(_Z^3-2)*x^6+6*(4*x^6+4*x^3+1)^(2/3)*x^2+3*(4*x^6+4*
x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x-24*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3
-2)+36*_Z^2)*x^3+4*RootOf(_Z^3-2)*x^3-6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf
```

```
(_Z^3-2)+36*_Z^2)+RootOf(_Z^3-2))/(2*x^3+1)/(4*x^3+1)))/(2*x^3+1)^(2/3)*((2
*x^3+1)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^{3}+1\right)\left(2\,x^{3}+1\right)^{\frac{4}{3}}}{\left(4\,x^{3}+1\right)x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+1)^(4/3)*(3*x^3+1)/x^8/(4*x^3+1),x, algorithm="maxima")
[Out] integrate((3*x^3 + 1)*(2*x^3 + 1)^(4/3)/((4*x^3 + 1)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(2\,x^3+1\right)}^{4/3}\,\left(3\,x^3+1\right)}{x^8\,\left(4\,x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^3 + 1)^(4/3)*(3*x^3 + 1))/(x^8*(4*x^3 + 1)),x)
[Out] int(((2*x^3 + 1)^(4/3)*(3*x^3 + 1))/(x^8*(4*x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{3}+1\right)^{\frac{4}{3}}\left(3x^{3}+1\right)}{x^{8}\left(4x^{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+1)**(4/3)*(3*x**3+1)/x**8/(4*x**3+1),x)
[Out] Integral((2*x**3 + 1)**(4/3)*(3*x**3 + 1)/(x**8*(4*x**3 + 1)), x)
```

3.1781 $\int \frac{-1+x^2}{(1+x^2)\sqrt[3]{-x^2+x^4}}\,dx$

Optimal. Leaf size=157

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^4-x^2}}\right)}{\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}x\sqrt[3]{x^4-x^2}}{\sqrt[3]{2}\left(x^4-x^2\right)^{2/3}-2x^2}\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tanh^{-1}\left(\frac{\frac{\sqrt[3]{2}x^2}{\sqrt{3}}+\frac{\left(x^4-x^2\right)^{2/3}}{\sqrt[3]{2}\sqrt{3}}}{x\sqrt[3]{x^4-x^2}}\right)}{2\sqrt[3]{2}}$$

Rubi [C] time = 0.11, antiderivative size = 46, normalized size of antiderivative = 0.29, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2056, 466, 430, 429}

$$-\frac{3x\sqrt[3]{1-x^2}\,F_1\left(\frac{1}{6};-\frac{2}{3},1;\frac{7}{6};x^2,-x^2\right)}{\sqrt[3]{x^4-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^2)/((1 + x^2)*(-x^2 + x^4)^(1/3)),x]
```

```
[Out] (-3*x*(1 - x^2)^(1/3)*AppellF1[1/6, -2/3, 1, 7/6, x^2, -x^2])/(-x^2 + x^4)^
(1/3)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{-1+x^2}{(1+x^2)\sqrt[3]{-x^2+x^4}}\,dx = \frac{\left(x^{2/3}\sqrt[3]{-1+x^2}\right)\int \frac{(-1+x^2)^{2/3}}{x^{2/3}(1+x^2)}\,dx}{\sqrt[3]{-x^2+x^4}}$$
$$= \frac{\left(3x^{2/3}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{(-1+x^6)^{2/3}}{1+x^6}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-x^2+x^4}}$$
$$= \frac{\left(3x^{2/3}\left(-1+x^2\right)\right)\text{Subst}\left(\int \frac{(1-x^6)^{2/3}}{1+x^6}\,dx, x, \sqrt[3]{x}\right)}{(1-x^2)^{2/3}\sqrt[3]{-x^2+x^4}}$$
$$= -\frac{3x\sqrt[3]{1-x^2}\,F_1\left(\frac{1}{6};-\frac{2}{3},1;\frac{7}{6};x^2,-x^2\right)}{\sqrt[3]{-x^2+x^4}}$$

Mathematica [C] time = 0.03, size = 48, normalized size = 0.31

$$\frac{3\left(x^2\left(x^2-1\right)\right)^{2/3}F_1\left(\frac{1}{6};-\frac{2}{3},1;\frac{7}{6};x^2,-x^2\right)}{x\left(1-x^2\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*(-x^2 + x^4)^(1/3)),x]
```

```
[Out] (3*(x^2*(-1 + x^2))^(2/3)*AppellF1[1/6, -2/3, 1, 7/6, x^2, -x^2])/(x*(1 - x
^2)^(2/3))
```

IntegrateAlgebraic [A] time = 0.43, size = 157, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^4-x^2}}\right)}{\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}x\sqrt[3]{x^4-x^2}}{\sqrt[3]{2}\left(x^4-x^2\right)^{2/3}-2x^2}\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tanh^{-1}\left(\frac{\frac{\sqrt[3]{2}x^2}{\sqrt{3}}+\frac{\left(x^4-x^2\right)^{2/3}}{\sqrt[3]{2}\sqrt{3}}}{x\sqrt[3]{x^4-x^2}}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*(-x^2 + x^4)^(1/3)),x]
```

```
[Out] -(ArcTan[(2^(1/3)*x)/(-x^2 + x^4)^(1/3)]/2^(1/3)) - ArcTan[(2^(2/3)*x*(-x^2
 + x^4)^(1/3))/(-2*x^2 + 2^(1/3)*(-x^2 + x^4)^(2/3))]/(2*2^(1/3)) - (Sqrt[3
]*ArcTanh[((2^(1/3)*x^2)/Sqrt[3] + (-x^2 + x^4)^(2/3)/(2^(1/3)*Sqrt[3]))/(x
*(-x^2 + x^4)^(1/3))])/(2*2^(1/3))
```

fricas [B] time = 2.86, size = 2094, normalized size = 13.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^4-x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/32*sqrt(3)*2^(2/3)*log(8500000*(8*sqrt(3)*2^(1/3)*(x^4 - x^2) + 2*(x^4 -
 x^2)^(2/3)*(sqrt(3)*2^(2/3)*(x^2 - 1) + 6*2^(2/3)*x) + 2^(1/3)*(x^5 + 2*x^
3 + x) + 4*(x^4 - x^2)^(1/3)*(3*x^3 + 2*sqrt(3)*x^2 - 3*x))/(x^5 + 2*x^3 +
x)) - 1/32*sqrt(3)*2^(2/3)*log(2125000*(8*sqrt(3)*2^(1/3)*(x^4 - x^2) + 2*(
x^4 - x^2)^(2/3)*(sqrt(3)*2^(2/3)*(x^2 - 1) + 6*2^(2/3)*x) + 2^(1/3)*(x^5 +
```

```
 2*x^3 + x) + 4*(x^4 - x^2)^(1/3)*(3*x^3 + 2*sqrt(3)*x^2 - 3*x))/(x^5 + 2*x
^3 + x)) + 1/32*sqrt(3)*2^(2/3)*log(-2125000*(8*sqrt(3)*2^(1/3)*(x^4 - x^2)
 + 2*(x^4 - x^2)^(2/3)*(sqrt(3)*2^(2/3)*(x^2 - 1) - 6*2^(2/3)*x) - 2^(1/3)*
(x^5 + 2*x^3 + x) - 4*(x^4 - x^2)^(1/3)*(3*x^3 - 2*sqrt(3)*x^2 - 3*x))/(x^5
 + 2*x^3 + x)) + 1/32*sqrt(3)*2^(2/3)*log(-8500000*(8*sqrt(3)*2^(1/3)*(x^4
- x^2) + 2*(x^4 - x^2)^(2/3)*(sqrt(3)*2^(2/3)*(x^2 - 1) - 6*2^(2/3)*x) - 2^
(1/3)*(x^5 + 2*x^3 + x) - 4*(x^4 - x^2)^(1/3)*(3*x^3 - 2*sqrt(3)*x^2 - 3*x)
)/(x^5 + 2*x^3 + x)) + 1/4*2^(2/3)*arctan(-(74071498415429632*x^9 + 1645279
755446275808*x^8 - 2346817955632029696*x^7 - 11516958288123930656*x^6 + 573
0636889080074240*x^5 + 11516958288123930656*x^4 - 2346817955632029696*x^3 -
 1645279755446275808*x^2 - 125*sqrt(34)*(4*sqrt(3)*2^(1/3)*(78465570355328*
x^9 - 3301419835659*x^8 + 1100839094578688*x^7 - 595767752585659*x^6 - 3614
058455553280*x^5 + 595767752585659*x^4 + 1100839094578688*x^3 + 33014198356
59*x^2 + 78465570355328*x) + 16*(x^4 - x^2)^(2/3)*(4*sqrt(3)*2^(2/3)*(15136
88563712*x^6 + 57183135266496*x^5 - 26977277846305*x^4 - 167158338888320*x^
3 + 26977277846305*x^2 + 57183135266496*x - 1513688563712) - 2^(2/3)*(79163
286177664*x^6 - 56815411732213*x^5 - 187311276664960*x^4 + 112551186315710*
x^3 + 187311276664960*x^2 - 56815411732213*x - 79163286177664)) - 2^(1/3)*(
36167723835659*x^9 + 4738598437685248*x^8 - 1343569332842636*x^7 - 16069401
562314752*x^6 + 2036119636643410*x^5 + 16069401562314752*x^4 - 134356933284
2636*x^3 - 4738598437685248*x^2 + 36167723835659*x) - 4*(183204669874443*x^
7 + 4116235393055744*x^6 - 2225700627116645*x^5 - 10698715224852480*x^4 + 2
225700627116645*x^3 + 4116235393055744*x^2 - 531250*sqrt(3)*(1009306368*x^7
 - 511421263*x^6 - 4316628224*x^5 + 1207618962*x^4 + 4316628224*x^3 - 51142
1263*x^2 - 1009306368*x) - 183204669874443*x)*(x^4 - x^2)^(1/3))*sqrt((8*sq
rt(3)*2^(1/3)*(x^4 - x^2) + 2*(x^4 - x^2)^(2/3)*(sqrt(3)*2^(2/3)*(x^2 - 1)
+ 6*2^(2/3)*x) + 2^(1/3)*(x^5 + 2*x^3 + x) + 4*(x^4 - x^2)^(1/3)*(3*x^3 + 2
*sqrt(3)*x^2 - 3*x))/(x^5 + 2*x^3 + x)) - 1062500*(x^4 - x^2)^(2/3)*(2*sqrt
(3)*2^(1/3)*(23651383808*x^6 + 470146644789*x^5 - 226386757120*x^4 - 718099
82630*x^3 + 226386757120*x^2 + 470146644789*x - 23651383808) - 2^(1/3)*(618
463173263*x^6 - 733160605696*x^5 - 6989546598945*x^4 + 2615047352320*x^3 +
6989546598945*x^2 - 733160605696*x - 618463173263)) - 265625*sqrt(3)*(61301
2268401*x^9 - 500076281856*x^8 - 1596364015228*x^7 + 3500533972992*x^6 + 11
774899788070*x^5 - 3500533972992*x^4 - 1596364015228*x^3 + 500076281856*x^2
 + 613012268401*x) - 1062500*(x^4 - x^2)^(1/3)*(sqrt(3)*2^(2/3)*(2171208267
37*x^7 + 155432605696*x^6 + 1229224098945*x^5 - 689287352320*x^4 - 12292240
98945*x^3 + 155432605696*x^2 - 217120826737*x) - 2*2^(2/3)*(71795383808*x^7
 + 1283539269789*x^6 - 948546757120*x^5 - 5040931232630*x^4 + 948546757120*
x^3 + 1283539269789*x^2 - 71795383808*x)) + 74071498415429632*x)/(479958568
556831351*x^9 - 1202832749691437056*x^8 - 12744795130528777828*x^7 + 841982
9247840059392*x^6 + 32209010220853194570*x^5 - 8419829247840059392*x^4 - 12
744795130528777828*x^3 + 1202832749691437056*x^2 + 479958568556831351*x)) -
 1/4*2^(2/3)*arctan((74071498415429632*x^9 + 1645279755446275808*x^8 - 2346
817955632029696*x^7 - 11516958288123930656*x^6 + 5730636889080074240*x^5 +
11516958288123930656*x^4 - 2346817955632029696*x^3 - 1645279755446275808*x^
2 + 125*sqrt(34)*(4*sqrt(3)*2^(1/3)*(78465570355328*x^9 - 3301419835659*x^8
 + 1100839094578688*x^7 - 595767752585659*x^6 - 3614058455553280*x^5 + 5957
67752585659*x^4 + 1100839094578688*x^3 + 3301419835659*x^2 + 78465570355328
*x) + 16*(x^4 - x^2)^(2/3)*(4*sqrt(3)*2^(2/3)*(1513688563712*x^6 + 57183135
266496*x^5 - 26977277846305*x^4 - 167158338888320*x^3 + 26977277846305*x^2
+ 57183135266496*x - 1513688563712) + 2^(2/3)*(79163286177664*x^6 - 5681541
1732213*x^5 - 187311276664960*x^4 + 112551186315710*x^3 + 187311276664960*x
^2 - 56815411732213*x - 79163286177664)) + 2^(1/3)*(36167723835659*x^9 + 47
38598437685248*x^8 - 1343569332842636*x^7 - 16069401562314752*x^6 + 2036119
636643410*x^5 + 16069401562314752*x^4 - 1343569332842636*x^3 - 473859843768
5248*x^2 + 36167723835659*x) + 4*(183204669874443*x^7 + 4116235393055744*x^
6 - 2225700627116645*x^5 - 10698715224852480*x^4 + 2225700627116645*x^3 + 4
116235393055744*x^2 + 531250*sqrt(3)*(1009306368*x^7 - 511421263*x^6 - 4316
628224*x^5 + 1207618962*x^4 + 4316628224*x^3 - 511421263*x^2 - 1009306368*x
```

```
) - 183204669874443*x)*(x^4 - x^2)^(1/3))*sqrt(-(8*sqrt(3)*2^(1/3)*(x^4 - x
^2) + 2*(x^4 - x^2)^(2/3)*(sqrt(3)*2^(2/3)*(x^2 - 1) - 6*2^(2/3)*x) - 2^(1/
3)*(x^5 + 2*x^3 + x) - 4*(x^4 - x^2)^(1/3)*(3*x^3 - 2*sqrt(3)*x^2 - 3*x))/(
x^5 + 2*x^3 + x)) + 1062500*(x^4 - x^2)^(2/3)*(2*sqrt(3)*2^(1/3)*(236513838
08*x^6 + 470146644789*x^5 - 226386757120*x^4 - 71809982630*x^3 + 2263867571
20*x^2 + 470146644789*x - 23651383808) + 2^(1/3)*(618463173263*x^6 - 733160
605696*x^5 - 6989546598945*x^4 + 2615047352320*x^3 + 6989546598945*x^2 - 73
3160605696*x - 618463173263)) + 265625*sqrt(3)*(613012268401*x^9 - 50007628
1856*x^8 - 1596364015228*x^7 + 3500533972992*x^6 + 11774899788070*x^5 - 350
0533972992*x^4 - 1596364015228*x^3 + 500076281856*x^2 + 613012268401*x) + 1
062500*(x^4 - x^2)^(1/3)*(sqrt(3)*2^(2/3)*(217120826737*x^7 + 155432605696*
x^6 + 1229224098945*x^5 - 689287352320*x^4 - 1229224098945*x^3 + 1554326056
96*x^2 - 217120826737*x) + 2*2^(2/3)*(71795383808*x^7 + 128353926978 9*x^6 -
 948546757120*x^5 - 5040931232630*x^4 + 948546757120*x^3 + 128353926978 9*x^
2 - 71795383808*x)) + 74071498415429632*x)/(479958568556831351*x^9 - 120283
2749691437056*x^8 - 12744795130528777828*x^7 + 8419829247840059392*x^6 + 32
209010220853194570*x^5 - 8419829247840059392*x^4 - 12744795130528777828*x^3
 + 1202832749691437056*x^2 + 479958568556831351*x)) + 1/2*2^(2/3)*arctan(-1
/2*(3564544*x^5 + 249106968*x^4 - 21387264*x^3 + 2125000*2^(2/3)*(x^4 - x^2
)^(1/3)*(512*x^3 + 59*x^2 - 512*x) + 1062500*2^(1/3)*(x^4 - x^2)^(2/3)*(59*
x^2 - 2048*x - 59) - 249106968*x^2 - 125*sqrt(34)*2^(1/6)*(4*2^(2/3)*(x^4 -
 x^2)^(2/3)*(15104*x^2 + 527769*x - 15104) + 3481*2^(1/3)*(x^5 + 2*x^3 + x)
 + 4*(x^4 - x^2)^(1/3)*(527769*x^3 - 60416*x^2 - 527769*x)) + 3564544*x)/(2
05379*x^5 - 2168870912*x^4 - 1232274*x^3 + 2168870912*x^2 + 205379*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^4-x^2\right)^{\frac{1}{3}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^4-x^2)^(1/3),x, algorithm="giac")
[Out] integrate((x^2 - 1)/((x^4 - x^2)^(1/3)*(x^2 + 1)), x)
```

maple [F] time = 4.64, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^2+1\right)\left(x^4-x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^4-x^2)^(1/3),x)
[Out] int((x^2-1)/(x^2+1)/(x^4-x^2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^4-x^2\right)^{\frac{1}{3}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^4-x^2)^(1/3),x, algorithm="maxima")
[Out] integrate((x^2 - 1)/((x^4 - x^2)^(1/3)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2-1}{\left(x^2+1\right)\left(x^4-x^2\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + 1)*(x^4 - x^2)^(1/3)),x)
```

```
[Out] int((x^2 - 1)/((x^2 + 1)*(x^4 - x^2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{\sqrt[3]{x^2(x-1)(x+1)}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**4-x**2)**(1/3),x)
```

```
[Out] Integral((x - 1)*(x + 1)/((x**2*(x - 1)*(x + 1))**(1/3)*(x**2 + 1)), x)
```

3.1782 $\int \frac{x^3(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}\left(a-x+dx^4\right)}\,dx$

Optimal. Leaf size=157

$$-\frac{\log\left(a^2d^{2/3}x^4+a^2\sqrt[3]{d}\,x^2\sqrt[3]{x^3-ax^2}+a^2\left(x^3-ax^2\right)^{2/3}\right)}{2d^{2/3}}+\frac{\log\left(a\sqrt[3]{x^3-ax^2}-a\sqrt[3]{d}\,x^2\right)}{d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\,x^2}{2\sqrt[3]{x^3-ax^2}+\sqrt[3]{d}\,x^2}\right.}{d^{2/3}}$$

Rubi [F] time = 2.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}\left(a-x+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*(-4*a + 3*x))/((x^2*(-a + x))^(2/3)*(a - x + d*x^4)),x]
```

```
[Out] (-12*a*x^(4/3)*(-a + x)^(2/3)*Defer[Subst][Defer[Int][x^7/((-a + x^3)^(2/3)
*(a - x^3 + d*x^12)), x], x, x^(1/3)])/(-((a - x)*x^2))^(2/3) + (9*x^(4/3)*
(-a + x)^(2/3)*Defer[Subst][Defer[Int][x^10/((-a + x^3)^(2/3)*(a - x^3 + d*
x^12)), x], x, x^(1/3)])/(-((a - x)*x^2))^(2/3)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}\left(a-x+dx^4\right)}\,dx &= \frac{\left(x^{4/3}(-a+x)^{2/3}\right)\int \frac{x^{5/3}(-4a+3x)}{(-a+x)^{2/3}\left(a-x+dx^4\right)}\,dx}{\left(x^2(-a+x)\right)^{2/3}}\\
&= \frac{\left(3x^{4/3}(-a+x)^{2/3}\right)\operatorname{Subst}\left(\int \frac{x^7\left(-4a+3x^3\right)}{\left(-a+x^3\right)^{2/3}\left(a-x^3+dx^{12}\right)}\,dx,x,\sqrt[3]{x}\right)}{\left(x^2(-a+x)\right)^{2/3}}\\
&= \frac{\left(3x^{4/3}(-a+x)^{2/3}\right)\operatorname{Subst}\left(\int \left(-\frac{4ax^7}{\left(-a+x^3\right)^{2/3}\left(a-x^3+dx^{12}\right)}+\frac{3x^{10}}{\left(-a+x^3\right)^{2/3}\left(a-x^3+dx^{12}\right)}\right)dx\right.}{\left(x^2(-a+x)\right)^{2/3}}\\
&= \frac{\left(9x^{4/3}(-a+x)^{2/3}\right)\operatorname{Subst}\left(\int \frac{x^{10}}{\left(-a+x^3\right)^{2/3}\left(a-x^3+dx^{12}\right)}\,dx,x,\sqrt[3]{x}\right)}{\left(x^2(-a+x)\right)^{2/3}}-\frac{\left(12ax^{4/3}(-a+\right.}{}
\end{aligned}$$

Mathematica [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{x^3(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}\left(a-x+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(-4*a + 3*x))/((x^2*(-a + x))^(2/3)*(a - x + d*x^4)),x]
```

```
[Out] Integrate[(x^3*(-4*a + 3*x))/((x^2*(-a + x))^(2/3)*(a - x + d*x^4)), x]
```

IntegrateAlgebraic [A] time = 0.55, size = 157, normalized size = 1.00

$$-\frac{\log\left(a^2d^{2/3}x^4+a^2\sqrt[3]{d}\,x^2\sqrt[3]{x^3-ax^2}+a^2\left(x^3-ax^2\right)^{2/3}\right)}{2d^{2/3}}+\frac{\log\left(a\sqrt[3]{x^3-ax^2}-a\sqrt[3]{d}\,x^2\right)}{d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\,x^2}{2\sqrt[3]{x^3-ax^2}+\sqrt[3]{d}\,x^2}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(-4*a + 3*x))/((x^2*(-a + x))^(2/3)*(a - x + d*x^4)
),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*x^2)/(d^(1/3)*x^2 + 2*(-(a*x^2) + x^3)^(1/
3))])/d^(2/3) + Log[-(a*d^(1/3)*x^2) + a*(-(a*x^2) + x^3)^(1/3)]/d^(2/3) -
Log[a^2*d^(2/3)*x^4 + a^2*d^(1/3)*x^2*(-(a*x^2) + x^3)^(1/3) + a^2*(-(a*x^2
) + x^3)^(2/3)]/(2*d^(2/3))
```

fricas [A] time = 0.40, size = 158, normalized size = 1.01

$$-\frac{2\sqrt{3}\left(d^2\right)^{\frac{1}{6}}d\arctan\left(\frac{\sqrt{3}\left(\left(d^2\right)^{\frac{1}{3}}dx^2+2\left(-ax^2+x^3\right)^{\frac{1}{3}}\left(d^2\right)^{\frac{2}{3}}\right)\left(d^2\right)^{\frac{1}{6}}}{3d^2x^2}\right)-2\left(d^2\right)^{\frac{2}{3}}\log\left(\frac{\left(d^2\right)^{\frac{2}{3}}x^2-\left(-ax^2+x^3\right)^{\frac{1}{3}}d}{x^2}\right)+\left(d^2\right)^{\frac{2}{3}}\log\left(\frac{\left(d^2\right)^{\frac{1}{3}}dx^4+\left(-ax^2+x^3\right)^{\frac{1}{3}}\left(d^2\right)^{\frac{2}{3}}x^2+\left(-ax^2+x^3\right)^{\frac{2}{3}}d}{x^4}\right)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*a+3*x)/(x^2*(-a+x))^(2/3)/(d*x^4+a-x),x, algorithm="frica
s")
```

```
[Out] -1/2*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(1/3*sqrt(3)*((d^2)^(1/3)*d*x^2 + 2*(-a
*x^2 + x^3)^(1/3)*(d^2)^(2/3))*(d^2)^(1/6)/(d^2*x^2)) - 2*(d^2)^(2/3)*log((
(d^2)^(2/3)*x^2 - (-a*x^2 + x^3)^(1/3)*d)/x^2) + (d^2)^(2/3)*log(((d^2)^(1/
3)*d*x^4 + (-a*x^2 + x^3)^(1/3)*(d^2)^(2/3)*x^2 + (-a*x^2 + x^3)^(2/3)*d)/x
^4))/d^2
```

giac [A] time = 0.50, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*a+3*x)/(x^2*(-a+x))^(2/3)/(d*x^4+a-x),x, algorithm="giac"
)
```

```
[Out] 0
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(-4a+3x\right)}{\left(x^2\left(-a+x\right)\right)^{\frac{2}{3}}\left(d\,x^4+a-x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-4*a+3*x)/(x^2*(-a+x))^(2/3)/(d*x^4+a-x),x)
```

```
[Out] int(x^3*(-4*a+3*x)/(x^2*(-a+x))^(2/3)/(d*x^4+a-x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(4\,a-3\,x)x^3}{\left(dx^4+a-x\right)\left(-(a-x)x^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-4*a+3*x)/(x^2*(-a+x))^(2/3)/(d*x^4+a-x),x, algorithm="maxim
a")
```

```
[Out] -integrate((4*a - 3*x)*x^3/((d*x^4 + a - x)*(-(a - x)*x^2)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^3\,\left(4\,a-3\,x\right)}{{\left(-x^2\,\left(a-x\right)\right)}^{2/3}\,\left(d\,x^4-x+a\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(4*a - 3*x))/((-x^2*(a - x))^(2/3)*(a - x + d*x^4)),x)
```

```
[Out] int(-(x^3*(4*a - 3*x))/((-x^2*(a - x))^(2/3)*(a - x + d*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-4*a+3*x)/(x**2*(-a+x))**(2/3)/(d*x**4+a-x),x)
```

```
[Out] Timed out
```

3.1783 $\int \frac{b+ax^6}{x^6\left(b+ax^3\right)\sqrt[4]{-bx+ax^4}}\,dx$

Optimal. Leaf size=157

$$\frac{2^{3/4}\left(a^{3/4}b+a^{7/4}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3b^2}+\frac{2^{3/4}\left(a^{3/4}b+a^{7/4}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3b^2}+\frac{4\left(ax^4-bx\right)^{3/4}\left(b-ax\right.}{21b^2x^6}$$

Rubi [A] time = 1.19, antiderivative size = 269, normalized size of antiderivative = 1.71, number of steps used = 14, number of rules used = 11, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {2056, 6725, 271, 264, 466, 465, 494, 461, 212, 206, 203}

$$\frac{2^{3/4}a^{3/4}\sqrt[4]{x}(a+b)\sqrt[4]{ax^3-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3b^2\sqrt[4]{ax^4-bx}}+\frac{2^{3/4}a^{3/4}\sqrt[4]{x}(a+b)\sqrt[4]{ax^3-b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3b^2\sqrt[4]{ax^4-bx}}-\frac{4(a+b)\left(b-ax^3\right)^2}{21ab^2x^5\sqrt[4]{ax^4-bx}}+\frac{4\left(b-ax^3\right)}{21ax^5\sqrt[4]{ax^4-bx}}-\frac{4\left(b-ax^3\right)}{21bx^2\sqrt[4]{ax^4-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^6)/(x^6*(b + a*x^3)*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (4*(b - a*x^3))/(21*a*x^5*(-(b*x) + a*x^4)^(1/4)) - (4*(b - a*x^3))/(21*b*x
^2*(-(b*x) + a*x^4)^(1/4)) - (4*(a + b)*(b - a*x^3)^2)/(21*a*b^2*x^5*(-(b*x
) + a*x^4)^(1/4)) + (2^(3/4)*a^(3/4)*(a + b)*x^(1/4)*(-b + a*x^3)^(1/4)*Arc
Tan[(2^(1/4)*a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)])/(3*b^2*(-(b*x) + a*x^4)^
(1/4)) + (2^(3/4)*a^(3/4)*(a + b)*x^(1/4)*(-b + a*x^3)^(1/4)*ArcTanh[(2^(1/
4)*a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)])/(3*b^2*(-(b*x) + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{b+ax^6}{x^6\left(b+ax^3\right)\sqrt[4]{-bx+ax^4}}\,dx = \frac{\left(\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\int \frac{b+ax^6}{x^{25/4}\sqrt[4]{-b+ax^3}\left(b+ax^3\right)}\,dx}{\sqrt[4]{-bx+ax^4}}$$

$$= \frac{\left(\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\int \left(-\frac{b}{ax^{25/4}\sqrt[4]{-b+ax^3}}+\frac{1}{x^{13/4}\sqrt[4]{-b+ax^3}}+\frac{ab+b^2}{ax^{25/4}\sqrt[4]{-b+ax^3}\left(b+ax^3\right)}\right)dx \cdots}{\sqrt[4]{-bx+ax^4}}$$

$$= \frac{\left(\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\int \frac{1}{x^{13/4}\sqrt[4]{-b+ax^3}}\,dx}{\sqrt[4]{-bx+ax^4}} - \frac{\left(b\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\int \frac{1}{x^{25/4}\sqrt[4]{-b+ax^3}}\,dx}{a\sqrt[4]{-bx+ax^4}} + \cdots$$

$$= \frac{4\left(b-ax^3\right)}{21ax^5\sqrt[4]{-bx+ax^4}} - \frac{4\left(b-ax^3\right)}{9bx^2\sqrt[4]{-bx+ax^4}} - \frac{\left(4\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\int \frac{1}{x^{13/4}\sqrt[4]{-b+ax^3}} \cdots}{7\sqrt[4]{-bx+ax^4}}$$

$$= \frac{4\left(b-ax^3\right)}{21ax^5\sqrt[4]{-bx+ax^4}} - \frac{4\left(b-ax^3\right)}{21bx^2\sqrt[4]{-bx+ax^4}} + \frac{\left(4b(a+b)\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\text{Subst}\cdots}{3a\sqrt[4]{-bx\cdots}}$$

$$= \frac{4\left(b-ax^3\right)}{21ax^5\sqrt[4]{-bx+ax^4}} - \frac{4\left(b-ax^3\right)}{21bx^2\sqrt[4]{-bx+ax^4}} + \frac{\left(4(a+b)\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\cdots\right.}{3ab\sqrt[4]{-bx\cdots}}$$

$$= \frac{4\left(b-ax^3\right)}{21ax^5\sqrt[4]{-bx+ax^4}} - \frac{4\left(b-ax^3\right)}{21bx^2\sqrt[4]{-bx+ax^4}} + \frac{\left(4(a+b)\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\text{Subst}\left(\cdots\right.}{3ab\sqrt[4]{\cdots}}$$

$$= \frac{4\left(b-ax^3\right)}{21ax^5\sqrt[4]{-bx+ax^4}} - \frac{4\left(b-ax^3\right)}{21bx^2\sqrt[4]{-bx+ax^4}} - \frac{4(a+b)\left(b-ax^3\right)^2}{21ab^2x^5\sqrt[4]{-bx+ax^4}} - \frac{\left(4a(a+\cdots\right.}{\cdots}$$

$$= \frac{4\left(b-ax^3\right)}{21ax^5\sqrt[4]{-bx+ax^4}} - \frac{4\left(b-ax^3\right)}{21bx^2\sqrt[4]{-bx+ax^4}} - \frac{4(a+b)\left(b-ax^3\right)^2}{21ab^2x^5\sqrt[4]{-bx+ax^4}} + \frac{\left(2a(a+\cdots\right.}{\cdots}$$

$$= \frac{4\left(b-ax^3\right)}{21ax^5\sqrt[4]{-bx+ax^4}} - \frac{4\left(b-ax^3\right)}{21bx^2\sqrt[4]{-bx+ax^4}} - \frac{4(a+b)\left(b-ax^3\right)^2}{21ab^2x^5\sqrt[4]{-bx+ax^4}} + \frac{2^{3/4}a^{3/4}(\cdots}{\cdots}$$

Mathematica [C] time = 5.18, size = 114, normalized size = 0.73

$$\frac{28ax^6(a+b)\sqrt[4]{1-\frac{ax^3}{b}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{2ax^3}{ax^3+b}\right)-4\left(b-ax^3\right)^2\sqrt[4]{\frac{ax^3}{b}+1}}{21b^2x^5\sqrt[4]{\frac{ax^3}{b}+1}\,\sqrt[4]{ax^4-bx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b + a*x^6)/(x^6*(b + a*x^3)*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(b - a*x^3)^2*(1 + (a*x^3)/b)^(1/4) + 28*a*(a + b)*x^6*(1 - (a*x^3)/b)^
(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (2*a*x^3)/(b + a*x^3)])/(21*b^2*x^5*
(1 + (a*x^3)/b)^(1/4)*(-(b*x) + a*x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.91, size = 157, normalized size = 1.00

$$\frac{2^{3/4}\left(a^{3/4}b+a^{7/4}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3b^2}+\frac{2^{3/4}\left(a^{3/4}b+a^{7/4}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\left(ax^4-bx\right)^{3/4}}{ax^3-b}\right)}{3b^2}+\frac{4\left(ax^4-bx\right)^{3/4}\left(b-ax^3\right)}{21b^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^6)/(x^6*(b + a*x^3)*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] (4*(b - a*x^3)*(-(b*x) + a*x^4)^(3/4))/(21*b^2*x^6) + (2^(3/4)*(a^(7/4) + a
^(3/4)*b)*ArcTan[(2^(1/4)*a^(1/4)*(-(b*x) + a*x^4)^(3/4))/(-b + a*x^3)])/(3
*b^2) + (2^(3/4)*(a^(7/4) + a^(3/4)*b)*ArcTanh[(2^(1/4)*a^(1/4)*(-(b*x) + a
*x^4)^(3/4))/(-b + a*x^3)])/(3*b^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^6/(a*x^3+b)/(a*x^4-b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.32, size = 279, normalized size = 1.78

$$\frac{\sqrt{2}\left(2^{\frac{3}{4}}(-a)^{\frac{3}{4}}a+2^{\frac{3}{4}}(-a)^{\frac{3}{4}}b\right)\arctan\left(\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{6b^2}+\frac{\sqrt{2}\left(2^{\frac{3}{4}}(-a)^{\frac{3}{4}}a+2^{\frac{3}{4}}(-a)^{\frac{3}{4}}b\right)\arctan\left(-\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{6b^2}-\frac{\sqrt{2}\left(2^{\frac{3}{4}}(-a)^{\frac{3}{4}}a+2^{\frac{3}{4}}(-a)^{\frac{3}{4}}b\right)\log\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a-\frac{b}{x^3}}\right)}{12b^2}+\frac{\sqrt{2}\left(2^{\frac{3}{4}}(-a)^{\frac{3}{4}}a+2^{\frac{3}{4}}(-a)^{\frac{3}{4}}b\right)\log\left(-2^{\frac{3}{4}}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a-\frac{b}{x^3}}\right)}{12b^2}-\frac{4\left(a-\frac{b}{x^3}\right)^{\frac{7}{4}}}{21b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^6/(a*x^3+b)/(a*x^4-b*x)^(1/4),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(2^(3/4)*(-a)^(3/4)*a + 2^(3/4)*(-a)^(3/4)*b)*arctan(1/2*2^(1/4
)*(2^(3/4)*(-a)^(1/4) + 2*(a - b/x^3)^(1/4))/(-a)^(1/4))/b^2 + 1/6*sqrt(2)*
(2^(3/4)*(-a)^(3/4)*a + 2^(3/4)*(-a)^(3/4)*b)*arctan(-1/2*2^(1/4)*(2^(3/4)*
(-a)^(1/4) - 2*(a - b/x^3)^(1/4))/(-a)^(1/4))/b^2 - 1/12*sqrt(2)*(2^(3/4)*(
-a)^(3/4)*a + 2^(3/4)*(-a)^(3/4)*b)*log(2^(3/4)*(-a)^(1/4)*(a - b/x^3)^(1/4
) + sqrt(2)*sqrt(-a) + sqrt(a - b/x^3))/b^2 + 1/12*sqrt(2)*(2^(3/4)*(-a)^(3
/4)*a + 2^(3/4)*(-a)^(3/4)*b)*log(-2^(3/4)*(-a)^(1/4)*(a - b/x^3)^(1/4) + s
qrt(2)*sqrt(-a) + sqrt(a - b/x^3))/b^2 - 4/21*(a - b/x^3)^(7/4)/b^2
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{a\,x^{6}+b}{x^{6}\left(a\,x^{3}+b\right)\left(a\,x^{4}-bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6+b)/x^6/(a*x^3+b)/(a*x^4-b*x)^(1/4),x)
```

```
[Out] int((a*x^6+b)/x^6/(a*x^3+b)/(a*x^4-b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{6}+b}{\left(ax^{4}-bx\right)^{\frac{1}{4}}\left(ax^{3}+b\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^6/(a*x^3+b)/(a*x^4-b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^6 + b)/((a*x^4 - b*x)^(1/4)*(a*x^3 + b)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x^6+b}{x^6\,{\left(a\,x^4-b\,x\right)}^{1/4}\,\left(a\,x^3+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^6)/(x^6*(a*x^4 - b*x)^(1/4)*(b + a*x^3)),x)
```

```
[Out] int((b + a*x^6)/(x^6*(a*x^4 - b*x)^(1/4)*(b + a*x^3)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^6 + b}{x^6 \sqrt[4]{x\left(ax^3 - b\right)}\left(ax^3 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6+b)/x**6/(a*x**3+b)/(a*x**4-b*x)**(1/4),x)
```

```
[Out] Integral((a*x**6 + b)/(x**6*(x*(a*x**3 - b))**(1/4)*(a*x**3 + b)), x)
```

3.1784 $\int x^2\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$

Optimal. Leaf size=157

$$\frac{b\log\left(i\sqrt{a^2x^4+b}+i\sqrt{2}\,\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}+iax^2\right)}{8\sqrt{2}\,a^{3/2}}-\frac{i\left(2iax^4\sqrt{a^2x^4+b}+ix^2\left(2a^2x^4-b\right)\right)}{8ax\sqrt{\sqrt{a^2x^4+b}+ax^2}}$$

Rubi [F] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] Defer[Int][x^2*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]], x]
```

Rubi steps

$$\int x^2\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx=\int x^2\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Mathematica [F] time = 0.10, size = 0, normalized size = 0.00

$$\int x^2\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] Integrate[x^2*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]], x]
```

IntegrateAlgebraic [A] time = 0.55, size = 157, normalized size = 1.00

$$\frac{b\log\left(i\sqrt{a^2x^4+b}+i\sqrt{2}\,\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}+iax^2\right)}{8\sqrt{2}\,a^{3/2}}-\frac{i\left(2iax^4\sqrt{a^2x^4+b}+ix^2\left(2a^2x^4-b\right)\right)}{8ax\sqrt{\sqrt{a^2x^4+b}+ax^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] ((-1/8*I)*((2*I)*a*x^4*Sqrt[b + a^2*x^4] + I*x^2*(-b + 2*a^2*x^4)))/(a*x*Sq
rt[a*x^2 + Sqrt[b + a^2*x^4]]) + (b*Log[I*a*x^2 + I*Sqrt[b + a^2*x^4] + I*S
qrt[2]*Sqrt[a]*x*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]])/(8*Sqrt[2]*a^(3/2))
```

fricas [A] time = 1.76, size = 236, normalized size = 1.50

$$\left[\frac{\sqrt{2}\,\sqrt{a}\,b\log\left(4\,a^2x^4+4\,\sqrt{a^2x^4+b}\,ax^2+2\left(\sqrt{2}\,a^{\frac{3}{2}}x^3+\sqrt{2}\,\sqrt{a^2x^4+b}\,\sqrt{a}\,x\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}+b\right)+4\left(3\,a^2x^3-\sqrt{a^2x^4+b}\,ax\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{32\,a^2},-\frac{\sqrt{2}\,\sqrt{-a}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{ax^2+\sqrt{a^2x^4+b}}\,\sqrt{-a}}{2\,ax}\right)-2\left(3\,a^2x^3-\sqrt{a^2x^4+b}\,ax\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{16\,a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(sqrt(2)*sqrt(a)*b*log(4*a^2*x^4 + 4*sqrt(a^2*x^4 + b)*a*x^2 + 2*(sqr
t(2)*a^(3/2)*x^3 + sqrt(2)*sqrt(a^2*x^4 + b)*sqrt(a)*x)*sqrt(a*x^2 + sqrt(a
^2*x^4 + b)) + b) + 4*(3*a^2*x^3 - sqrt(a^2*x^4 + b)*a*x)*sqrt(a*x^2 + sqrt
(a^2*x^4 + b)))/a^2, -1/16*(sqrt(2)*sqrt(-a)*b*arctan(1/2*sqrt(2)*sqrt(a*x^
2 + sqrt(a^2*x^4 + b))*sqrt(-a)/(a*x)) - 2*(3*a^2*x^3 - sqrt(a^2*x^4 + b)*a
*x)*sqrt(a*x^2 + sqrt(a^2*x^4 + b)))/a^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{%%%{%%
{[-2,%%%{4,[1]%%%}]:[1,0,%%%{1,[1]%%%}]%%},[0]%%%}/%%%{%%{[2,1]:[1,0,%%%{1,
[1]%%%}]%%},[0]%%%},[1]%%%}+%%%{%%%{1,[1]%%%},[0]%%%} / %%%{%%%{1/%%%{%%{[2
,1]:[1,0,%%%{1,[1]%%%}]%%},[0]%%%},[0]%%%},[0]%%%} Error: Bad Argument Valu
eUnable to divide, perhaps due to rounding error%%%{%%%{%%{[2,%%%{4,[1]%%%}
]:[1,0,%%%{1,[1]%%%}]%%},[0]%%%}/%%%{%%{[-2,1]:[1,0,%%%{1,[1]%%%}]%%},[0]%%
%},[1]%%%}+%%%{%%%{1,[1]%%%},[0]%%%} / %%%{%%%{1/%%%{%%{[-2,1]:[1,0,%%%{1,[
1]%%%}]%%},[0]%%%},[0]%%%},[0]%%%} Error: Bad Argument ValueEvaluation time
: 1integrate((4*a^2*x^6*sqrt(sqrt(a^2*x^4+b)+a*x^2)-4*a^2*x^6-4*a*x^4*sqrt(
sqrt(a^2*x^4+b)+a*x^2)+4*b*x^2*sqrt(sqrt(a^2*x^4+b)+a*x^2)-4*b*x^2+x^2*sqrt
(sqrt(a^2*x^4+b)+a*x^2)+4*x^2*(a^2*x^4+b))/(4*a^2*x^4-4*a*x^2+4*b+1),x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^{2} \sqrt{a\,x^{2}+\sqrt{a^{2} x^{4}+b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x)
```

```
[Out] int(x^2*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^{2}+\sqrt{a^{2}x^{4}+b}}\, x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^4 + b))*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2}\,\sqrt{\sqrt{a^{2}\,x^{4}+b}+a\,x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b + a^2*x^4)^(1/2) + a*x^2)^(1/2),x)
```

```
[Out] int(x^2*((b + a^2*x^4)^(1/2) + a*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{ax^2+\sqrt{a^2x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x**2+(a**2*x**4+b)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*x**2 + sqrt(a**2*x**4 + b)), x)
```

3.1785 $\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=157

$$-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[4]{c}}\right)}{2ac^{5/4}}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[4]{c}}\right)}{2ac^{5/4}}-\frac{3\left(\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c\right)^{3/4}}{ac\sqrt[3]{\sqrt{a^2x^2-b}+ax}}$$

Rubi [F] time = 1.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x + Sqrt
[-b + a^2*x^2])^(1/3))^(1/4)),x]
```

```
[Out] Defer[Int][1/(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x
 + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\sqrt[4]{c+}}$$

Mathematica [C] time = 0.26, size = 74, normalized size = 0.47

$$\frac{4\left(\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c\right)^{3/4}{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};\frac{c+\sqrt[3]{ax+\sqrt{a^2x^2-b}}}{c}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x
+ Sqrt[-b + a^2*x^2])^(1/3))^(1/4)),x]
```

```
[Out] (4*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(3/4)*Hypergeometric2F1[3/4, 2, 7
/4, (c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))/c])/(a*c^2)
```

IntegrateAlgebraic [A] time = 0.39, size = 157, normalized size = 1.00

$$-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[4]{c}}\right)}{2ac^{5/4}}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[4]{c}}\right)}{2ac^{5/4}}-\frac{3\left(\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c\right)^{3/4}}{ac\sqrt[3]{\sqrt{a^2x^2-b}+ax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/3)*(
c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)),x]
```

```
[Out] (-3*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(3/4))/(a*c*(a*x + Sqrt[-b + a^2
*x^2])^(1/3)) - (3*ArcTan[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)/c^(1
/4)])/(2*a*c^(5/4)) + (3*ArcTanh[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/
4)/c^(1/4)])/(2*a*c^(5/4))
```

fricas [**B**] time = 0.56, size = 291, normalized size = 1.85

$$\frac{3\left(4\,abc\left(\frac{1}{a^4c^5}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2c^3\sqrt{\frac{1}{a^4c^5}}+\sqrt{c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}}}\,ac\left(\frac{1}{a^4c^5}\right)^{\frac{1}{4}}-a\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}c\left(\frac{1}{a^4c^5}\right)^{\frac{1}{4}}\right)+abc\left(\frac{1}{a^4c^5}\right)^{\frac{1}{4}}\log\left(a^3c^4\left(\frac{1}{a^4c^5}\right)^{\frac{3}{4}}+\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}\right)-abc\left(\frac{1}{a^4c^5}\right)^{\frac{1}{4}}\log\left(-a^3c^4\left(\frac{1}{a^4c^5}\right)^{\frac{3}{4}}+\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}\right)-4\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{2}{3}}\left(ax-\sqrt{a^2x^2-b}\right)\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{3}{4}}\right)}{4\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^
2-b)^(1/2))^(1/3))^(1/4),x, algorithm="fricas")
```

```
[Out] 3/4*(4*a*b*c*(1/(a^4*c^5))^(1/4)*arctan(sqrt(a^2*c^3*sqrt(1/(a^4*c^5)) + sq
rt(c + (a*x + sqrt(a^2*x^2 - b))^(1/3)))*a*c*(1/(a^4*c^5))^(1/4) - a*(c + (
a*x + sqrt(a^2*x^2 - b))^(1/3))^(1/4)*c*(1/(a^4*c^5))^(1/4)) + a*b*c*(1/(a^
4*c^5))^(1/4)*log(a^3*c^4*(1/(a^4*c^5))^(3/4) + (c + (a*x + sqrt(a^2*x^2 -
b))^(1/3))^(1/4)) - a*b*c*(1/(a^4*c^5))^(1/4)*log(-a^3*c^4*(1/(a^4*c^5))^(3
/4) + (c + (a*x + sqrt(a^2*x^2 - b))^(1/3))^(1/4)) - 4*(a*x + sqrt(a^2*x^2
- b))^(2/3)*(a*x - sqrt(a^2*x^2 - b))*(c + (a*x + sqrt(a^2*x^2 - b))^(1/3))
^(3/4))/(a*b*c)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^
2-b)^(1/2))^(1/3))^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2-b}\,\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-b)^(
1/2))^(1/3))^(1/4),x)
```

```
[Out] int(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-b)^(
1/2))^(1/3))^(1/4),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2-b}\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^
2-b)^(1/2))^(1/3))^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 - b)*(a*x + sqrt(a^2*x^2 - b))^(1/3)*(c + (a*x +
sqrt(a^2*x^2 - b))^(1/3))^(1/4)), x)
```

mupad [B] time = 1.98, size = 99, normalized size = 0.63

$$-\frac{12\left(\frac{c}{\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/3}}+1\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{5}{4};\ \frac{9}{4};\ -\frac{c}{\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/3}}\right)}{5\,a\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/3}\left(c+\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/3}\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x + (a^2*x^2 - b)^(1/2))^(1/3)*(c + (a*x + (a^2*x^2 - b)^(1/2))^(
1/3))^(1/4)*(a^2*x^2 - b)^(1/2)),x)
```

```
[Out] -(12*(c/(a*x + (a^2*x^2 - b)^(1/2))^(1/3) + 1)^(1/4)*hypergeom([1/4, 5/4],
9/4, -c/(a*x + (a^2*x^2 - b)^(1/2))^(1/3)))/(5*a*(a*x + (a^2*x^2 - b)^(1/2)
)^(1/3)*(c + (a*x + (a^2*x^2 - b)^(1/2))^(1/3))^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{a^2x^2-b}}}\,\sqrt[3]{ax+\sqrt{a^2x^2-b}}\,\sqrt{a^2x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*x**2-b)**(1/2)/(a*x+(a**2*x**2-b)**(1/2))**(1/3)/(c+(a*x+
(a**2*x**2-b)**(1/2))**(1/3))**(1/4),x)
```

```
[Out] Integral(1/((c + (a*x + sqrt(a**2*x**2 - b))**(1/3))**(1/4)*(a*x + sqrt(a**
2*x**2 - b))**(1/3)*sqrt(a**2*x**2 - b)), x)
```

3.1786 $$\int \frac{(2a-3b+x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{(-b+x)\sqrt[4]{(-a+x)(-b+x)^2}\left(-a^3-b^2d+\left(3a^2+2bd\right)x-(3a+d)x^2+x^3\right)}\,dx$$

Optimal. Leaf size=158

$$-2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}}{a-x}\right)+2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b}}{a-x}\right.$$

Rubi [F] time = 28.42, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(2a-3b+x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{(-b+x)\sqrt[4]{(-a+x)(-b+x)^2}\left(-a^3-b^2d+\left(3a^2+2bd\right)x-(3a+d)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((2*a - 3*b + x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/((-b + x)*((-a + x)*
(-b + x)^2)^(1/4)*(-a^3 - b^2*d + (3*a^2 + 2*b*d)*x - (3*a + d)*x^2 + x^3))
,x]
```

```
[Out] (2*(a - x))/(-((a - x)*(b - x)^2))^(1/4) - (2*(2*a - 3*b)*(a - x))/((a - b)
*(-((a - x)*(b - x)^2))^(1/4)) - (2*(a + d)*(a - x))/((a - b)*(-((a - x)*(b
 - x)^2))^(1/4)) + (2*(2*a - 3*b)*(b - x)*Sqrt[-a + x])/((a - b)^(3/2)*(-((
a - x)*(b - x)^2))^(1/4)*(1 + Sqrt[-a + x]/Sqrt[a - b])) - (6*(b - x)*Sqrt[
-a + x])/(Sqrt[a - b]*(-((a - x)*(b - x)^2))^(1/4)*(1 + Sqrt[-a + x]/Sqrt[a
 - b])) + (2*(a + d)*(b - x)*Sqrt[-a + x])/((a - b)^(3/2)*(-((a - x)*(b - x
)^2))^(1/4)*(1 + Sqrt[-a + x]/Sqrt[a - b])) + (2*(2*a - 3*b)*(-a + x)^(1/4)
*Sqrt[-((b - x)/((a - b)*(1 + Sqrt[-a + x]/Sqrt[a - b])^2))]*(1 + Sqrt[-a +
 x]/Sqrt[a - b])*EllipticE[2*ArcTan[(-a + x)^(1/4)/(a - b)^(1/4)], 1/2])/((
a - b)^(1/4)*(-((a - x)*(b - x)^2))^(1/4)) - (6*(a - b)^(3/4)*(-a + x)^(1/4
)*Sqrt[-((b - x)/((a - b)*(1 + Sqrt[-a + x]/Sqrt[a - b])^2))]*(1 + Sqrt[-a
+ x]/Sqrt[a - b])*EllipticE[2*ArcTan[(-a + x)^(1/4)/(a - b)^(1/4)], 1/2])/(
-((a - x)*(b - x)^2))^(1/4) + (2*(a + d)*(-a + x)^(1/4)*Sqrt[-((b - x)/((a
- b)*(1 + Sqrt[-a + x]/Sqrt[a - b])^2))]*(1 + Sqrt[-a + x]/Sqrt[a - b])*Ell
ipticE[2*ArcTan[(-a + x)^(1/4)/(a - b)^(1/4)], 1/2])/((a - b)^(1/4)*(-((a -
 x)*(b - x)^2))^(1/4)) - ((2*a - 3*b)*(-a + x)^(1/4)*Sqrt[-((b - x)/((a - b
)*(1 + Sqrt[-a + x]/Sqrt[a - b])^2))]*(1 + Sqrt[-a + x]/Sqrt[a - b])*Ellipt
icF[2*ArcTan[(-a + x)^(1/4)/(a - b)^(1/4)], 1/2])/((a - b)^(1/4)*(-((a - x)
*(b - x)^2))^(1/4)) + (3*(a - b)^(3/4)*(-a + x)^(1/4)*Sqrt[-((b - x)/((a -
b)*(1 + Sqrt[-a + x]/Sqrt[a - b])^2))]*(1 + Sqrt[-a + x]/Sqrt[a - b])*Ellip
ticF[2*ArcTan[(-a + x)^(1/4)/(a - b)^(1/4)], 1/2])/(-((a - x)*(b - x)^2))^(
1/4) - ((a + d)*(-a + x)^(1/4)*Sqrt[-((b - x)/((a - b)*(1 + Sqrt[-a + x]/Sq
rt[a - b])^2))]*(1 + Sqrt[-a + x]/Sqrt[a - b])*EllipticF[2*ArcTan[(-a + x)^
(1/4)/(a - b)^(1/4)], 1/2])/((a - b)^(1/4)*(-((a - x)*(b - x)^2))^(1/4)) -
(4*(a - b)^2*d*(a + d)*(-a + x)^(1/4)*Sqrt[-b + x]*Defer[Subst][Defer[Int][
x^2/((a - b + x^4)^(3/2)*(a^2*(1 + (b*(-2*a + b))/a^2)*d + 2*a*(1 - b/a)*d*
x^4 + d*x^8 - x^12)), x], x, (-a + x)^(1/4)])/(-((a - x)*(b - x)^2))^(1/4)
- (4*(a - b)*d*(3*a - b + 2*d)*(-a + x)^(1/4)*Sqrt[-b + x]*Defer[Subst][Def
er[Int][x^6/((a - b + x^4)^(3/2)*(a^2*(1 + (b*(-2*a + b))/a^2)*d + 2*a*(1 -
 b/a)*d*x^4 + d*x^8 - x^12)), x], x, (-a + x)^(1/4)])/(-((a - x)*(b - x)^2)
)^(1/4) - (4*d*(3*a - 2*b + d)*(-a + x)^(1/4)*Sqrt[-b + x]*Defer[Subst][Def
er[Int][x^10/((a - b + x^4)^(3/2)*(a^2*(1 + (b*(-2*a + b))/a^2)*d + 2*a*(1
- b/a)*d*x^4 + d*x^8 - x^12)), x], x, (-a + x)^(1/4)])/(-((a - x)*(b - x)^2
))^(1/4) - (4*(2*a - 3*b)*d*(-a + x)^(1/4)*Sqrt[-b + x]*Defer[Subst][Defer[
Int][(x^2*Sqrt[a - b + x^4])/(a^2*(1 + b^2/a^2)*d - 2*b*d*x^4 + x^8*(d - x^
4) + 2*a*d*(-b + x^4)), x], x, (-a + x)^(1/4)])/(-((a - x)*(b - x)^2))^(1/4
)
```

Rubi steps

$$\int \frac{(2a-3b+x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{(-b+x)\sqrt[4]{(-a+x)(-b+x)^2}\left(-a^3-b^2d+\left(3a^2+2bd\right)x-(3a+d)x^2+x^3\right)}\,dx = \frac{\left(\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\int \cdots}{\cdots}$$

$$= \frac{\left(\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\int \cdots}{\sqrt[4]{(-\cdots}}$$

$$= \frac{\left(\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\int \cdots}{\sqrt[4]{(-\cdots}}$$

$$= \frac{\left(\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\int \left(\cdots\right.}{\cdots}$$

$$= \frac{\left(\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\int \cdots}{\sqrt[4]{(-\cdots}}$$

$$= -\frac{\left(4\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{\left(4\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{\left(4\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\mathrm{Su}\cdots}{\cdots}$$

$$= \frac{\left(4\sqrt[4]{-a+x}\,\sqrt{-b+x}\right)\mathrm{Sub}\cdots}{\sqrt[4]{(-a\cdots}}$$

$$= \frac{2(a-x)}{\sqrt[4]{-\left((a-x)(b-x)^2\right)}} - \cdots$$

$$= \frac{2(a-x)}{\sqrt[4]{-\left((a-x)(b-x)^2\right)}} - \cdots$$

$$= \frac{2(a-x)}{\sqrt[4]{-\left((a-x)(b-x)^2\right)}} - \cdots$$

Mathematica [C] time = 12.48, size = 2491, normalized size = 15.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2*a - 3*b + x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/((-b + x)*((-a
 + x)*(-b + x)^2)^(1/4)*(-a^3 - b^2*d + (3*a^2 + 2*b*d)*x - (3*a + d)*x^2 +
 x^3)),x]
```

```
[Out] (4*(-a + x))/((b - x)^2*(-a + x))^(1/4) + ((2*I)*(a - b)*d*(a - x)^(7/4)*((
b - x)/(a - x))^(3/2)*((-(EllipticPi[-(1/(Sqrt[a - b]*Sqrt[Root[1 + d*#1 +
(-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 2]])), I*ArcSinh
[Sqrt[-Sqrt[a - b]]/(a - x)^(1/4)], -1]*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*
d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1]]*(1 - 4*(a - b)*Root[1 + d*
#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 2] + 3*(a -
 b)^2*Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^
3 & , 2]^2)*(Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2
*d)*#1^3 & , 1] - Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d
+ b^2*d)*#1^3 & , 3])) + EllipticPi[1/(Sqrt[a - b]*Sqrt[Root[1 + d*#1 + (-2
*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 2]]), I*ArcSinh[Sqr
t[-Sqrt[a - b]]/(a - x)^(1/4)], -1]*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#
1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1]]*(1 - 4*(a - b)*Root[1 + d*#1 +
 (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 2] + 3*(a - b)^
2*Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 &
, 2]^2)*(Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*
#1^3 & , 1] - Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^
2*d)*#1^3 & , 3]) + (EllipticPi[-(1/(Sqrt[a - b]*Sqrt[Root[1 + d*#1 + (-2*a
*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1]])), I*ArcSinh[Sqrt
[-Sqrt[a - b]]/(a - x)^(1/4)], -1] - EllipticPi[1/(Sqrt[a - b]*Sqrt[Root[1
+ d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1]]), I
*ArcSinh[Sqrt[-Sqrt[a - b]]/(a - x)^(1/4)], -1])*(1 - 4*(a - b)*Root[1 + d*
#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1] + 3*(a -
 b)^2*Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^
3 & , 1]^2)*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d +
 b^2*d)*#1^3 & , 2]]*(Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*
b*d + b^2*d)*#1^3 & , 2] - Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d -
 2*a*b*d + b^2*d)*#1^3 & , 3])*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2
+ (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 3]] + EllipticPi[-(1/(Sqrt[a - b]*Sqrt
[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & ,
 3]])), I*ArcSinh[Sqrt[-Sqrt[a - b]]/(a - x)^(1/4)], -1]*Sqrt[Root[1 + d*#1
 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1]]*(Root[1 +
 d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1] - Roo
t[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 2])
*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^
3 & , 2]]*(1 - 4*(a - b)*Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2
*a*b*d + b^2*d)*#1^3 & , 3] + 3*(a - b)^2*Root[1 + d*#1 + (-2*a*d + 2*b*d)*
#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 3]^2) - EllipticPi[1/(Sqrt[a - b]
*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^
3 & , 3]]), I*ArcSinh[Sqrt[-Sqrt[a - b]]/(a - x)^(1/4)], -1]*Sqrt[Root[1 +
d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1]]*(Root
[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1] -
 Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & ,
 2])*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)
*#1^3 & , 2]]*(1 - 4*(a - b)*Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d
 - 2*a*b*d + b^2*d)*#1^3 & , 3] + 3*(a - b)^2*Root[1 + d*#1 + (-2*a*d + 2*b
*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 3]^2)))/(Sqrt[-Sqrt[a - b]]*(
(b - x)^2*(-a + x))^(1/4)*(-b + x)*Sqrt[Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1
^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1]]*Sqrt[Root[1 + d*#1 + (-2*a*d +
2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 2]]*Sqrt[Root[1 + d*#1 + (
```

```
-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 3]]*(-3*a + 3*b -
 2*d + 2*(a - b)^3*d*Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b
*d + b^2*d)*#1^3 & , 1]*Root[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*
a*b*d + b^2*d)*#1^3 & , 2]^2 + 2*(a - b)^3*d*Root[1 + d*#1 + (-2*a*d + 2*b*
d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 1]^2*Root[1 + d*#1 + (-2*a*d +
 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 3] + 2*(a - b)^3*d*Root[1
 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 2]*Roo
t[1 + d*#1 + (-2*a*d + 2*b*d)*#1^2 + (a^2*d - 2*a*b*d + b^2*d)*#1^3 & , 3]^
2))
```

IntegrateAlgebraic [A] time = 1.08, size = 158, normalized size = 1.00

$$-2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}}{a-x}\right)+2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}}{a-x}\right)+\frac{4\left(-ab^2+2abx-ax^2+b^2x-2bx^2+x^3\right)^{3/4}}{(b-x)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2*a - 3*b + x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/((-b
+ x)*((-a + x)*(-b + x)^2)^(1/4)*(-a^3 - b^2*d + (3*a^2 + 2*b*d)*x - (3*a +
 d)*x^2 + x^3)),x]
```

```
[Out] (4*(-(a*b^2) + 2*a*b*x + b^2*x - a*x^2 - 2*b*x^2 + x^3)^(3/4))/(b - x)^2 -
2*d^(1/4)*ArcTan[(d^(1/4)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^
3)^(1/4))/(a - x)] + 2*d^(1/4)*ArcTanh[(d^(1/4)*(-(a*b^2) + (2*a*b + b^2)*x
 + (-a - 2*b)*x^2 + x^3)^(1/4))/(a - x)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-3*b+x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/(-b+x)/((-a+x)*(-b+x)^2)^(
1/4)/(-a^3-b^2*d+(3*a^2+2*b*d)*x-(3*a+d)*x^2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^3-3\,a^2x+3\,ax^2-x^3\right)(2\,a-3\,b+x)}{\left(a^3+b^2d+(3\,a+d)x^2-x^3-\left(3\,a^2+2\,bd\right)x\right)\left(-(a-x)(b-x)^2\right)^{\frac{1}{4}}(b-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-3*b+x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/(-b+x)/((-a+x)*(-b+x)^2)^(
1/4)/(-a^3-b^2*d+(3*a^2+2*b*d)*x-(3*a+d)*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate(-(a^3 - 3*a^2*x + 3*a*x^2 - x^3)*(2*a - 3*b + x)/((a^3 + b^2*d +
(3*a + d)*x^2 - x^3 - (3*a^2 + 2*b*d)*x)*(-(a - x)*(b - x)^2)^(1/4)*(b - x)
), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{(2a-3b+x)\left(-a^3+3a^2x-3a\,x^2+x^3\right)}{(-b+x)\left((-a+x)\,(-b+x)^2\right)^{\frac{1}{4}}\left(-a^3-b^2d+\left(3a^2+2bd\right)x-(3a+d)\,x^2+x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a-3*b+x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/(-b+x)/((-a+x)*(-b+x)^2)^(1/4)/(
-a^3-b^2*d+(3*a^2+2*b*d)*x-(3*a+d)*x^2+x^3),x)
```

```
[Out] int((2*a-3*b+x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/(-b+x)/((-a+x)*(-b+x)^2)^(1/4)/(
-a^3-b^2*d+(3*a^2+2*b*d)*x-(3*a+d)*x^2+x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^3 - 3\,a^2x + 3\,ax^2 - x^3\right)(2\,a - 3\,b + x)}{\left(a^3 + b^2d + (3\,a + d)x^2 - x^3 - \left(3\,a^2 + 2\,bd\right)x\right)\left(-(a - x)(b - x)^2\right)^{\frac{1}{4}}(b - x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-3*b+x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/(-b+x)/((-a+x)*(-b+x)^2)^(
1/4)/(-a^3-b^2*d+(3*a^2+2*b*d)*x-(3*a+d)*x^2+x^3),x, algorithm="maxima")
```

```
[Out] -integrate((a^3 - 3*a^2*x + 3*a*x^2 - x^3)*(2*a - 3*b + x)/((a^3 + b^2*d +
(3*a + d)*x^2 - x^3 - (3*a^2 + 2*b*d)*x)*(-(a - x)*(b - x)^2)^(1/4)*(b - x)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{(2\,a - 3\,b + x)\,\left(a^3 - 3\,a^2\,x + 3\,a\,x^2 - x^3\right)}{(b - x)\,\left(-(a - x)\,(b - x)^2\right)^{1/4}\,\left(b^2\,d - x\,\left(3\,a^2 + 2\,b\,d\right) + x^2\,(3\,a + d) + a^3 - x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*a - 3*b + x)*(3*a*x^2 - 3*a^2*x + a^3 - x^3))/((b - x)*(-(a - x)*(
b - x)^2)^(1/4)*(b^2*d - x*(2*b*d + 3*a^2) + x^2*(3*a + d) + a^3 - x^3)),x)
```

```
[Out] -int(((2*a - 3*b + x)*(3*a*x^2 - 3*a^2*x + a^3 - x^3))/((b - x)*(-(a - x)*(
b - x)^2)^(1/4)*(b^2*d - x*(2*b*d + 3*a^2) + x^2*(3*a + d) + a^3 - x^3)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-3*b+x)*(-a**3+3*a**2*x-3*a*x**2+x**3)/(-b+x)/((-a+x)*(-b+x)*
*2)**(1/4)/(-a**3-b**2*d+(3*a**2+2*b*d)*x-(3*a+d)*x**2+x**3),x)
```

```
[Out] Timed out
```

3.1787 $\int \frac{\sqrt{1-3x^2-2x^4}\left(1+2x^4\right)}{\left(-1+x^2+2x^4\right)\left(-1+2x^2+2x^4\right)}\,dx$

Optimal. Leaf size=158

$$\tan^{-1}\left(\frac{x\sqrt{-2x^4-3x^2+1}}{2x^4+3x^2-1}\right)-i\sqrt{2}\tanh^{-1}\left(\frac{2\sqrt{2}\,x^3-2i\sqrt{-2x^4-3x^2+1}\,x-2ix}{2\sqrt{2}\,x^4+3\sqrt{2}\,x^2-2i\sqrt{-2x^4-3x^2+1}\,x^2-\sqrt{2}\sqrt{-2x^4-3x^2+1}-\cdots}\right.$$

Rubi [C] time = 1.47, antiderivative size = 467, normalized size of antiderivative = 2.96, number of steps used = 32, number of rules used = 8, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6728, 1208, 1180, 524, 424, 419, 1212, 537}

$$-\sqrt{\frac{1}{2}\left(9\sqrt{17}-37\right)}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{17}}}\right)|\frac{1}{4}\left(-13+3\sqrt{17}\right)\right)+\frac{1}{2}\sqrt{3\sqrt{17}-5}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{17}}}\right)|\frac{1}{4}\left(-13+3\sqrt{17}\right)\right)+\frac{\left(1+2\sqrt{3}-\sqrt{17}\right)F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{17}}}\right)|\frac{1}{4}\left(-13+3\sqrt{17}\right)\right)}{\sqrt{2\left(3+\sqrt{17}\right)}}+\frac{\left(1-2\sqrt{3}-\sqrt{17}\right)F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{17}}}\right)|\frac{1}{4}\left(-13+3\sqrt{17}\right)\right)}{\sqrt{2\left(3+\sqrt{17}\right)}}+2\sqrt{\frac{2}{3+\sqrt{17}}}\Pi\left(\frac{1}{4}\left(3-\sqrt{17}\right);\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{17}}}\right)|\frac{1}{4}\left(-13+3\sqrt{17}\right)\right)-\sqrt{\frac{2}{3+\sqrt{17}}}\Pi\left(\frac{3-\sqrt{17}}{2\left(1-\sqrt{3}\right)};\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{17}}}\right)|\frac{1}{4}\left(-13+3\sqrt{17}\right)\right)-\sqrt{\frac{2}{3+\sqrt{17}}}\Pi\left(\frac{3-\sqrt{17}}{2\left(1+\sqrt{3}\right)};\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{17}}}\right)|\frac{1}{4}\left(-13+3\sqrt{17}\right)\right)+2\sqrt{\frac{2}{3+\sqrt{17}}}\Pi\left(\frac{1}{2}\left(-3+\sqrt{17}\right);\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{17}}}\right)|\frac{1}{4}\left(-13+3\sqrt{17}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[1 - 3*x^2 - 2*x^4]*(1 + 2*x^4))/((-1 + x^2 + 2*x^4)*(-1 + 2*x^2 +
 2*x^4)),x]
```

```
[Out] ((1 - 2*Sqrt[3] - Sqrt[17])*EllipticF[ArcSin[(2*x)/Sqrt[-3 + Sqrt[17]]], (-
13 + 3*Sqrt[17])/4])/Sqrt[2*(3 + Sqrt[17])] + ((1 + 2*Sqrt[3] - Sqrt[17])*E
llipticF[ArcSin[(2*x)/Sqrt[-3 + Sqrt[17]]], (-13 + 3*Sqrt[17])/4])/Sqrt[2*(
3 + Sqrt[17])] + (Sqrt[-5 + 3*Sqrt[17]]*EllipticF[ArcSin[(2*x)/Sqrt[-3 + Sq
rt[17]]], (-13 + 3*Sqrt[17])/4])/2 - Sqrt[(-37 + 9*Sqrt[17])/2]*EllipticF[A
rcSin[(2*x)/Sqrt[-3 + Sqrt[17]]], (-13 + 3*Sqrt[17])/4] + 2*Sqrt[2/(3 + Sqr
t[17])]*EllipticPi[(3 - Sqrt[17])/4, ArcSin[(2*x)/Sqrt[-3 + Sqrt[17]]], (-1
3 + 3*Sqrt[17])/4] - Sqrt[2/(3 + Sqrt[17])]*EllipticPi[(3 - Sqrt[17])/(2*(1
 - Sqrt[3])), ArcSin[(2*x)/Sqrt[-3 + Sqrt[17]]], (-13 + 3*Sqrt[17])/4] - Sq
rt[2/(3 + Sqrt[17])]*EllipticPi[(3 - Sqrt[17])/(2*(1 + Sqrt[3])), ArcSin[(2
*x)/Sqrt[-3 + Sqrt[17]]], (-13 + 3*Sqrt[17])/4] + 2*Sqrt[2/(3 + Sqrt[17])]*
EllipticPi[(-3 + Sqrt[17])/2, ArcSin[(2*x)/Sqrt[-3 + Sqrt[17]]], (-13 + 3*S
qrt[17])/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1212

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-3x^2-2x^4}\left(1+2x^4\right)}{\left(-1+x^2+2x^4\right)\left(-1+2x^2+2x^4\right)}dx = \int\left(\frac{\sqrt{1-3x^2-2x^4}}{1+x^2}+\frac{2\sqrt{1-3x^2-2x^4}}{-1+2x^2}-\frac{2\left(1+2x^2\right)\sqrt{1-3x^2-2}}{-1+2x^2+2x^4}\right.$$

$$= 2\int\frac{\sqrt{1-3x^2-2x^4}}{-1+2x^2}dx - 2\int\frac{\left(1+2x^2\right)\sqrt{1-3x^2-2x^4}}{-1+2x^2+2x^4}dx + \int$$

$$= -\left(\frac{1}{2}\int\frac{8+4x^2}{\sqrt{1-3x^2-2x^4}}dx\right)+2\int\frac{1}{\left(1+x^2\right)\sqrt{1-3x^2-2x^4}}dx - 2$$

$$= -\left(4\int\frac{\sqrt{1-3x^2-2x^4}}{2-2\sqrt{3}+4x^2}dx\right)-4\int\frac{\sqrt{1-3x^2-2x^4}}{2+2\sqrt{3}+4x^2}dx-\sqrt{2}\int$$

$$= 2\sqrt{\frac{2}{3+\sqrt{17}}}\,\Pi\left(\frac{1}{4}\left(3-\sqrt{17}\right);\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{17}}}\right)|\frac{1}{4}\left(-13+3\sqrt{1}\right.\right.$$

$$= -\sqrt{2\left(3+\sqrt{17}\right)}\,E\left(\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{17}}}\right)|\frac{1}{4}\left(-13+3\sqrt{17}\right)\right)+\frac{1}{2}\sqrt{}$$

$$= -\sqrt{2\left(3+\sqrt{17}\right)}\,E\left(\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{17}}}\right)|\frac{1}{4}\left(-13+3\sqrt{17}\right)\right)+\frac{1}{2}\sqrt{}$$

$$= \frac{\left(1-2\sqrt{3}-\sqrt{17}\right)F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{17}}}\right)|\frac{1}{4}\left(-13+3\sqrt{17}\right)\right)}{\sqrt{2\left(3+\sqrt{17}\right)}}+\frac{\left(1+2\right.}{}$$

Mathematica [C] time = 0.94, size = 251, normalized size = 1.59

$$i\sqrt{\frac{2}{\sqrt{17}-3}}\left(F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{17}}}\right)|-\frac{13}{4}-\frac{3\sqrt{17}}{4}\right)-2\Pi\left(-\frac{3}{2}-\frac{\sqrt{17}}{2};i\sinh^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{17}}}\right)|-\frac{13}{4}-\frac{3\sqrt{17}}{4}\right)-2\Pi\left(\frac{1}{4}\left(3+\sqrt{17}\right);i\sinh^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{17}}}\right)|\frac{1}{4}\left(-13-3\sqrt{17}\right)\right)+\Pi\left(\frac{3+\sqrt{17}}{2-2\sqrt{3}};i\sinh^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{17}}}\right)|-\frac{13}{4}-\frac{3\sqrt{17}}{4}\right)+\Pi\left(\frac{3+\sqrt{17}}{2+2\sqrt{3}};i\sinh^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{17}}}\right)|-\frac{13}{4}-\frac{3\sqrt{17}}{4}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - 3*x^2 - 2*x^4]*(1 + 2*x^4))/((-1 + x^2 + 2*x^4)*(-1 + 2
*x^2 + 2*x^4)),x]
```

```
[Out] I*Sqrt[2/(-3 + Sqrt[17])]*(EllipticF[I*ArcSinh[(2*x)/Sqrt[3 + Sqrt[17]]], -
13/4 - (3*Sqrt[17])/4] - 2*EllipticPi[-3/2 - Sqrt[17]/2, I*ArcSinh[(2*x)/Sq
rt[3 + Sqrt[17]]], -13/4 - (3*Sqrt[17])/4] - 2*EllipticPi[(3 + Sqrt[17])/4,
 I*ArcSinh[(2*x)/Sqrt[3 + Sqrt[17]]], (-13 - 3*Sqrt[17])/4] + EllipticPi[(3
 + Sqrt[17])/(2 - 2*Sqrt[3]), I*ArcSinh[(2*x)/Sqrt[3 + Sqrt[17]]], -13/4 -
(3*Sqrt[17])/4] + EllipticPi[(3 + Sqrt[17])/(2 + 2*Sqrt[3]), I*ArcSinh[(2*x
)/Sqrt[3 + Sqrt[17]]], -13/4 - (3*Sqrt[17])/4])
```

IntegrateAlgebraic [A] time = 0.47, size = 79, normalized size = 0.50

$$\tan^{-1}\left(\frac{x\sqrt{-2x^4-3x^2+1}}{2x^4+3x^2-1}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x\sqrt{-2x^4-3x^2+1}}{2x^4+3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 - 3*x^2 - 2*x^4]*(1 + 2*x^4))/((-1 + x^2 + 2*x^4
)*(-1 + 2*x^2 + 2*x^4)),x]
```

```
[Out] ArcTan[(x*Sqrt[1 - 3*x^2 - 2*x^4])/(-1 + 3*x^2 + 2*x^4)] - Sqrt[2]*ArcTan[(
Sqrt[2]*x*Sqrt[1 - 3*x^2 - 2*x^4])/(-1 + 3*x^2 + 2*x^4)]
```

fricas [A] time = 0.53, size = 75, normalized size = 0.47

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{2\sqrt{2}\sqrt{-2x^4-3x^2+1}\,x}{2x^4+5x^2-1}\right)+\frac{1}{2}\arctan\left(\frac{2\sqrt{-2x^4-3x^2+1}\,x}{2x^4+4x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^4-3*x^2+1)^(1/2)*(2*x^4+1)/(2*x^4+x^2-1)/(2*x^4+2*x^2-1),x,
 algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(2*sqrt(2)*sqrt(-2*x^4 - 3*x^2 + 1)*x/(2*x^4 + 5*x^2 - 1
)) + 1/2*arctan(2*sqrt(-2*x^4 - 3*x^2 + 1)*x/(2*x^4 + 4*x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^4+1)\sqrt{-2x^4-3x^2+1}}{(2x^4+2x^2-1)(2x^4+x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^4-3*x^2+1)^(1/2)*(2*x^4+1)/(2*x^4+x^2-1)/(2*x^4+2*x^2-1),x,
 algorithm="giac")
```

```
[Out] integrate((2*x^4 + 1)*sqrt(-2*x^4 - 3*x^2 + 1)/((2*x^4 + 2*x^2 - 1)*(2*x^4
+ x^2 - 1)), x)
```

maple [C] time = 0.29, size = 862, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^4-3*x^2+1)^(1/2)*(2*x^4+1)/(2*x^4+x^2-1)/(2*x^4+2*x^2-1),x)
```

```
[Out] -10/(6+2*17^(1/2))^(1/2)*(1-3/2*x^2-1/2*x^2*17^(1/2))^(1/2)*(1-3/2*x^2+1/2*
x^2*17^(1/2))^(1/2)/(-2*x^4-3*x^2+1)^(1/2)*EllipticF(1/2*x*(6+2*17^(1/2))^(
1/2),1/4*I*34^(1/2)-3/4*I*2^(1/2))+16/(6+2*17^(1/2))^(1/2)*(1-3/2*x^2-1/2*x
^2*17^(1/2))^(1/2)*(1-3/2*x^2+1/2*x^2*17^(1/2))^(1/2)/(-2*x^4-3*x^2+1)^(1/2
)/(-3+17^(1/2))*EllipticF(1/2*x*(6+2*17^(1/2))^(1/2),1/4*I*34^(1/2)-3/4*I*2
^(1/2))-16/(6+2*17^(1/2))^(1/2)*(1-3/2*x^2-1/2*x^2*17^(1/2))^(1/2)*(1-3/2*x
^2+1/2*x^2*17^(1/2))^(1/2)/(-2*x^4-3*x^2+1)^(1/2)/(-3+17^(1/2))*EllipticE(1
/2*x*(6+2*17^(1/2))^(1/2),1/4*I*34^(1/2)-3/4*I*2^(1/2))+2/(3/2+1/2*17^(1/2)
)^(1/2)*(1-3/2*x^2-1/2*x^2*17^(1/2))^(1/2)*(1-3/2*x^2+1/2*x^2*17^(1/2))^(1/
2)/(-2*x^4-3*x^2+1)^(1/2)*EllipticPi((3/2+1/2*17^(1/2))^(1/2)*x,2/(3/2+1/2*
17^(1/2)),(3/2-1/2*17^(1/2))^(1/2)/(3/2+1/2*17^(1/2))^(1/2))+2/(3/2+1/2*17^
(1/2))^(1/2)*(1-3/2*x^2-1/2*x^2*17^(1/2))^(1/2)*(1-3/2*x^2+1/2*x^2*17^(1/2)
)^(1/2)/(-2*x^4-3*x^2+1)^(1/2)*EllipticPi((3/2+1/2*17^(1/2))^(1/2)*x,-1/(3/
2+1/2*17^(1/2)),(3/2-1/2*17^(1/2))^(1/2)/(3/2+1/2*17^(1/2))^(1/2))+8/(6+2*1
7^(1/2))^(1/2)*(1-(3/2+1/2*17^(1/2))*x^2)^(1/2)*(1-(3/2-1/2*17^(1/2))*x^2)^
(1/2)/(-2*x^4-3*x^2+1)^(1/2)*EllipticF(1/2*x*(6+2*17^(1/2))^(1/2),1/4*I*34^
(1/2)-3/4*I*2^(1/2))-16/(6+2*17^(1/2))^(1/2)*(1-(3/2+1/2*17^(1/2))*x^2)^(1/
2)*(1-(3/2-1/2*17^(1/2))*x^2)^(1/2)/(-2*x^4-3*x^2+1)^(1/2)/(-3+17^(1/2))*(E
llipticF(1/2*x*(6+2*17^(1/2))^(1/2),1/4*I*34^(1/2)-3/4*I*2^(1/2))-EllipticE
(1/2*x*(6+2*17^(1/2))^(1/2),1/4*I*34^(1/2)-3/4*I*2^(1/2)))+1/4*sum(_alpha*(
1/(-_alpha^2)^(1/2)*arctanh(1/22*(4*_alpha^2+3)*(-17*_alpha^2+11*x^2+4)/(-_
alpha^2)^(1/2)/(-2*x^4-3*x^2+1)^(1/2))-2*2^(1/2)*(_alpha^3+_alpha)/(3+17^(1
/2))^(1/2)*(-3*x^2+2-x^2*17^(1/2))^(1/2)*(-3*x^2+2+x^2*17^(1/2))^(1/2)/(-2*
x^4-3*x^2+1)^(1/2)*EllipticPi((3/2+1/2*17^(1/2))^(1/2)*x,1/2*_alpha^2*17^(1
/2)-3/2*_alpha^2+1/2*17^(1/2)-3/2,(3/2-1/2*17^(1/2))^(1/2)/(3/2+1/2*17^(1/2
))^(1/2))),_alpha=RootOf(2*_Z^4+2*_Z^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^4+1)\sqrt{-2x^4-3x^2+1}}{(2x^4+2x^2-1)(2x^4+x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^4-3*x^2+1)^(1/2)*(2*x^4+1)/(2*x^4+x^2-1)/(2*x^4+2*x^2-1),x,
 algorithm="maxima")
```

```
[Out] integrate((2*x^4 + 1)*sqrt(-2*x^4 - 3*x^2 + 1)/((2*x^4 + 2*x^2 - 1)*(2*x^4
+ x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x^4+1)\,\sqrt{-2x^4-3x^2+1}}{(2x^4+x^2-1)\,(2x^4+2x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^4 + 1)*(1 - 2*x^4 - 3*x^2)^(1/2))/((x^2 + 2*x^4 - 1)*(2*x^2 + 2*x
^4 - 1)),x)
```

```
[Out] int(((2*x^4 + 1)*(1 - 2*x^4 - 3*x^2)^(1/2))/((x^2 + 2*x^4 - 1)*(2*x^2 + 2*x
^4 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**4-3*x**2+1)**(1/2)*(2*x**4+1)/(2*x**4+x**2-1)/(2*x**4+2*x*
*2-1),x)
```

```
[Out] Timed out
```

3.1788 $\int \frac{x(-4a+3x)}{\sqrt[3]{x^2(-a+x)}\left(a-x+dx^4\right)} \, dx$

Optimal. Leaf size=158

$$-\frac{\log \left(a^2 d^{2/3} x^4+a^2 \sqrt[3]{d}\, x^2 \sqrt[3]{x^3-a x^2}+a^2 \left(x^3-a x^2\right)^{2/3}\right)}{2 \sqrt[3]{d}}+\frac{\log \left(a \sqrt[3]{x^3-a x^2}-a \sqrt[3]{d}\, x^2\right)}{\sqrt[3]{d}}-\frac{\sqrt{3} \tan ^{-1}\left(\frac{\sqrt{3} \sqrt[3]{d}\, x^2}{2 \sqrt[3]{x^3-a x^2}+\sqrt[3]{d}\, x^2}\right.}{\sqrt[3]{d}}$$

Rubi [F] time = 1.79, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(-4a+3x)}{\sqrt[3]{x^2(-a+x)}\left(a-x+dx^4\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-4*a + 3*x))/((x^2*(-a + x))^(1/3)*(a - x + d*x^4)),x]
```

```
[Out] (-12*a*x^(2/3)*(-a + x)^(1/3)*Defer[Subst][Defer[Int][x^3/((-a + x^3)^(1/3)
*(a - x^3 + d*x^12)), x], x, x^(1/3)])/(-((a - x)*x^2))^(1/3) + (9*x^(2/3)*
(-a + x)^(1/3)*Defer[Subst][Defer[Int][x^6/((-a + x^3)^(1/3)*(a - x^3 + d*x
^12)), x], x, x^(1/3)])/(-((a - x)*x^2))^(1/3)
```

Rubi steps

$$\begin{aligned} \int \frac{x(-4a+3x)}{\sqrt[3]{x^2(-a+x)}\left(a-x+dx^4\right)} \, dx &= \frac{\left(x^{2/3} \sqrt[3]{-a+x}\right) \int \frac{\sqrt[3]{x}\,(-4a+3x)}{\sqrt[3]{-a+x}\left(a-x+dx^4\right)} \, dx}{\sqrt[3]{x^2(-a+x)}} \\ &= \frac{\left(3x^{2/3} \sqrt[3]{-a+x}\right) \text{Subst}\left(\int \frac{x^3\left(-4a+3x^3\right)}{\sqrt[3]{-a+x^3}\left(a-x^3+dx^{12}\right)} \, dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2(-a+x)}} \\ &= \frac{\left(3x^{2/3} \sqrt[3]{-a+x}\right) \text{Subst}\left(\int \left(-\frac{4ax^3}{\sqrt[3]{-a+x^3}\left(a-x^3+dx^{12}\right)}+\frac{3x^6}{\sqrt[3]{-a+x^3}\left(a-x^3+dx^{12}\right)}\right) dx, x, \sqrt[3]{x}\right.}{\sqrt[3]{x^2(-a+x)}} \\ &= \frac{\left(9x^{2/3} \sqrt[3]{-a+x}\right) \text{Subst}\left(\int \frac{x^6}{\sqrt[3]{-a+x^3}\left(a-x^3+dx^{12}\right)} \, dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2(-a+x)}}-\frac{\left(12ax^{2/3} \sqrt[3]{-a+x}\right)}{} \end{aligned}$$

Mathematica [C] time = 1.20, size = 625, normalized size = 3.96

$$\frac{ax\left(4\,\text{RootSum}\left[\#1^4 a^3 d-\#1^3+3\#1^2-3\#1+1\&,\frac{-\sqrt[3]{\#1}\log\left(\#1^{2/3}+\sqrt[3]{\#1}\sqrt[3]{\frac{x}{x-a}}+\left(\frac{x}{x-a}\right)^{2/3}\right)+2\sqrt[3]{\#1}\log\left(\sqrt[3]{\#1}-\sqrt[3]{\frac{x}{x-a}}\right)-2\sqrt{3}\sqrt[3]{\#1}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{\frac{x}{x-a}}}{\sqrt[3]{\#1}}+1}{\sqrt{3}}\right)+6\sqrt[3]{\frac{x}{x-a}}}{4\#1^3a^3d-3\#1^2+6\#1-3}\&\right]+5\,\text{RootSum}\left[\#1^4 a^3 d-\#1^3+3\#1^2-3\#1+1\&,\frac{-2\#1^{4/3}\log\left(\sqrt[3]{\#1}-\sqrt[3]{\frac{x}{x-a}}\right)+\#1^{4/3}\log\left(\#1^{2/3}+\sqrt[3]{\#1}\sqrt[3]{\frac{x}{x-a}}+\left(\frac{x}{x-a}\right)^{2/3}\right)+2\sqrt{3}\#1^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{\frac{x}{x-a}}}{\sqrt[3]{\#1}}+1}{\sqrt{3}}\right)-6\#1\sqrt[3]{\frac{x}{x-a}}}{4\#1^3a^3d-3\#1^2+6\#1-3}\&\right]-\text{RootSum}\left[\#1^4 a^3 d-\#1^3+3\#1^2-3\#1+1\&,\frac{-2\#1^{7/3}\log\left(\sqrt[3]{\#1}-\sqrt[3]{\frac{x}{x-a}}\right)+\#1^{7/3}\log\left(\#1^{2/3}+\sqrt[3]{\#1}\sqrt[3]{\frac{x}{x-a}}+\left(\frac{x}{x-a}\right)^{2/3}\right)+2\sqrt{3}\#1^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{\frac{x}{x-a}}}{\sqrt[3]{\#1}}+1}{\sqrt{3}}\right)-6\#1^2\sqrt[3]{\frac{x}{x-a}}}{4\#1^3a^3d-3\#1^2+6\#1-3}\&\right]\right)}{2\sqrt[3]{\frac{x}{x-a}}\sqrt[3]{x^2(x-a)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-4*a + 3*x))/((x^2*(-a + x))^(1/3)*(a - x + d*x^4)),x]
```

```
[Out] (a*x*(4*RootSum[1 - 3*#1 + 3*#1^2 - #1^3 + a^3*d*#1^4 & , (6*(x/(-a + x))^(
1/3) - 2*Sqrt[3]*ArcTan[(1 + (2*(x/(-a + x))^(1/3))/#1^(1/3))/Sqrt[3]]*#1^(
1/3) + 2*Log[-(x/(-a + x))^(1/3) + #1^(1/3)]*#1^(1/3) - Log[(x/(-a + x))^(2
/3) + (x/(-a + x))^(1/3)*#1^(1/3) + #1^(2/3)]*#1^(1/3))/(-3 + 6*#1 - 3*#1^2
 + 4*a^3*d*#1^3) & ] + 5*RootSum[1 - 3*#1 + 3*#1^2 - #1^3 + a^3*d*#1^4 & ,
```

```
(-6*(x/(-a + x))^(1/3)*#1 + 2*Sqrt[3]*ArcTan[(1 + (2*(x/(-a + x))^(1/3))/#1
^(1/3))/Sqrt[3]]*#1^(4/3) - 2*Log[-(x/(-a + x))^(1/3) + #1^(1/3)]*#1^(4/3)
+ Log[(x/(-a + x))^(2/3) + (x/(-a + x))^(1/3)*#1^(1/3) + #1^(2/3)]*#1^(4/3)
)/(-3 + 6*#1 - 3*#1^2 + 4*a^3*d*#1^3) & ] - RootSum[1 - 3*#1 + 3*#1^2 - #1^
3 + a^3*d*#1^4 & , (-6*(x/(-a + x))^(1/3)*#1^2 + 2*Sqrt[3]*ArcTan[(1 + (2*(
x/(-a + x))^(1/3))/#1^(1/3))/Sqrt[3]]*#1^(7/3) - 2*Log[-(x/(-a + x))^(1/3)
+ #1^(1/3)]*#1^(7/3) + Log[(x/(-a + x))^(2/3) + (x/(-a + x))^(1/3)*#1^(1/3)
 + #1^(2/3)]*#1^(7/3))/(-3 + 6*#1 - 3*#1^2 + 4*a^3*d*#1^3) & ]))/(2*(x/(-a
+ x))^(1/3)*(x^2*(-a + x))^(1/3))
```

IntegrateAlgebraic [C] time = 0.46, size = 72, normalized size = 0.46

$$-a\text{RootSum}\left[\#1^{12}-3\#1^9+3\#1^6-\#1^3+a^3d\&,\frac{\log\left(\sqrt[3]{x^3-ax^2}-\#1x\right)-\log(x)}{\#1^4-\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-4*a + 3*x))/((x^2*(-a + x))^(1/3)*(a - x + d*x^4)),
x]
```

```
[Out] -(a*RootSum[a^3*d - #1^3 + 3*#1^6 - 3*#1^9 + #1^12 & , (-Log[x] + Log[(-(a*
x^2) + x^3)^(1/3) - x*#1])/(-#1 + #1^4) & ])
```

fricas [A] time = 0.46, size = 330, normalized size = 2.09

$$\left[\frac{\sqrt{3}d\sqrt{-\frac{1}{d^{\frac{2}{3}}}}\log\left(-\frac{dx^4-3\left(-ax^2+x^3\right)^{\frac{1}{3}}d^{\frac{2}{3}}x^2-\sqrt{3}\left(d^{\frac{4}{3}}x^4+\left(-ax^2+x^3\right)^{\frac{1}{3}}dx^2-2\left(-ax^2+x^3\right)^{\frac{2}{3}}d^{\frac{2}{3}}\right)\sqrt{-\frac{1}{d^{\frac{2}{3}}}}-2a+2x}{dx^4+a-x}\right)+2d^{\frac{2}{3}}\log\left(\frac{d^{\frac{1}{3}}x^2-\left(-ax^2+x^3\right)^{\frac{1}{3}}}{x^2}\right)-d^{\frac{2}{3}}\log\left(\frac{d^{\frac{2}{3}}x^4+\left(-ax^2+x^3\right)^{\frac{1}{3}}d^{\frac{1}{3}}x^2+\left(-ax^2+x^3\right)^{\frac{2}{3}}}{x^4}\right)}{2d},\frac{2\sqrt{3}d^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}x^2+2\left(-ax^2+x^3\right)^{\frac{1}{3}}\right)}{3d^{\frac{1}{3}}x^2}\right)+2d^{\frac{2}{3}}\log\left(\frac{d^{\frac{1}{3}}x^2-\left(-ax^2+x^3\right)^{\frac{1}{3}}}{x^2}\right)-d^{\frac{2}{3}}\log\left(\frac{d^{\frac{2}{3}}x^4+\left(-ax^2+x^3\right)^{\frac{1}{3}}d^{\frac{1}{3}}x^2+\left(-ax^2+x^3\right)^{\frac{2}{3}}}{x^4}\right)}{2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*a+3*x)/(x^2*(-a+x))^(1/3)/(d*x^4+a-x),x, algorithm="fricas"
)
```

```
[Out] [1/2*(sqrt(3)*d*sqrt(-1/d^(2/3))*log(-(d*x^4 - 3*(-a*x^2 + x^3)^(1/3)*d^(2/
3)*x^2 - sqrt(3)*(d^(4/3)*x^4 + (-a*x^2 + x^3)^(1/3)*d*x^2 - 2*(-a*x^2 + x^
3)^(2/3)*d^(2/3))*sqrt(-1/d^(2/3)) - 2*a + 2*x)/(d*x^4 + a - x)) + 2*d^(2/3
)*log((d^(1/3)*x^2 - (-a*x^2 + x^3)^(1/3))/x^2) - d^(2/3)*log((d^(2/3)*x^4
+ (-a*x^2 + x^3)^(1/3)*d^(1/3)*x^2 + (-a*x^2 + x^3)^(2/3))/x^4))/d, 1/2*(2*
sqrt(3)*d^(2/3)*arctan(1/3*sqrt(3)*(d^(1/3)*x^2 + 2*(-a*x^2 + x^3)^(1/3))/(
d^(1/3)*x^2)) + 2*d^(2/3)*log((d^(1/3)*x^2 - (-a*x^2 + x^3)^(1/3))/x^2) - d
^(2/3)*log((d^(2/3)*x^4 + (-a*x^2 + x^3)^(1/3)*d^(1/3)*x^2 + (-a*x^2 + x^3)
^(2/3))/x^4))/d]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*a+3*x)/(x^2*(-a+x))^(1/3)/(d*x^4+a-x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
-1/d)^(1/3)*ln((sqrt(3)*(abs(d)^(1/3))^2*sqrt(3)/2*(-a/x+1)^(1/3)*(-a/x+1)-
sqrt(3)*(abs(d)^(1/3))^2*sqrt(3)/2*(-a/x+1)^(1/3)+(abs(d)^(1/3))^2/2*(-a/x+
1)^(1/3)*(-a/x+1)-(abs(d)^(1/3))^2/2*(-a/x+1)^(1/3)+2*d*a)*(sqrt(3)*(abs(d)
^(1/3))^2*sqrt(3)/2*(-a/x+1)^(1/3)*(-a/x+1)-sqrt(3)*(abs(d)^(1/3))^2*sqrt(3
)/2*(-a/x+1)^(1/3)+(abs(d)^(1/3))^2/2*(-a/x+1)^(1/3)*(-a/x+1)-(abs(d)^(1/3)
)^2/2*(-a/x+1)^(1/3)+2*d*a)+(-sqrt(3)*(abs(d)^(1/3))^2/2*(-a/x+1)^(1/3)*(-a
/x+1)+sqrt(3)*(abs(d)^(1/3))^2/2*(-a/x+1)^(1/3)+(abs(d)^(1/3))^2*sqrt(3)/2*
```

```
(-a/x+1)^(1/3)*(-a/x+1)-(abs(d)^(1/3))^2*sqrt(3)/2*(-a/x+1)^(1/3))*(-sqrt(3
)*(abs(d)^(1/3))^2/2*(-a/x+1)^(1/3)*(-a/x+1)+sqrt(3)*(abs(d)^(1/3))^2/2*(-a
/x+1)^(1/3)+(abs(d)^(1/3))^2*sqrt(3)/2*(-a/x+1)^(1/3)*(-a/x+1)-(abs(d)^(1/3
))^2*sqrt(3)/2*(-a/x+1)^(1/3)))-sqrt(3)*(-1/d)^(1/3)*atan((sqrt(3)*(abs(d)^
(1/3))^2*sqrt(3)/2*(-a/x+1)^(1/3)*(-a/x+1)-sqrt(3)*(abs(d)^(1/3))^2*sqrt(3)
/2*(-a/x+1)^(1/3)+(abs(d)^(1/3))^2/2*(-a/x+1)^(1/3)*(-a/x+1)-(abs(d)^(1/3))
^2/2*(-a/x+1)^(1/3)+2*d*a)/(-sqrt(3)*(abs(d)^(1/3))^2/2*(-a/x+1)^(1/3)*(-a/
x+1)+sqrt(3)*(abs(d)^(1/3))^2/2*(-a/x+1)^(1/3)+(abs(d)^(1/3))^2*sqrt(3)/2*(
-a/x+1)^(1/3)*(-a/x+1)-(abs(d)^(1/3))^2*sqrt(3)/2*(-a/x+1)^(1/3)))-(-1/d)^(
1/3)/2*ln((-(d^(1/3))^2/2*(-a/x+1)^(1/3)*(-a/x+1)+(d^(1/3))^2/2*(-a/x+1)^(1
/3)+d*a)^2+(-(d^(1/3))^2*sqrt(3)/2*(-a/x+1)^(1/3)*(-a/x+1)+(d^(1/3))^2*sqrt
(3)/2*(-a/x+1)^(1/3))^2)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x \left(-4 a + 3 x\right)}{\left(x^{2} \left(-a + x\right)\right)^{\frac{1}{3}} \left(d \,x^{4}+a - x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-4*a+3*x)/(x^2*(-a+x))^(1/3)/(d*x^4+a-x),x)
```

```
[Out] int(x*(-4*a+3*x)/(x^2*(-a+x))^(1/3)/(d*x^4+a-x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{{\left(4 \, a - 3 \, x\right)} x}{{\left(d x^{4} + a - x\right)} \left(-{\left(a - x\right)} x^{2}\right)^{\frac{1}{3}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*a+3*x)/(x^2*(-a+x))^(1/3)/(d*x^4+a-x),x, algorithm="maxima"
)
```

```
[Out] -integrate((4*a - 3*x)*x/((d*x^4 + a - x)*(-(a - x)*x^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x\,\left(4\,a-3\,x\right)}{{\left(-x^2\,\left(a-x\right)\right)}^{1/3}\,\left(d\,x^4-x+a\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(4*a - 3*x))/((-x^2*(a - x))^(1/3)*(a - x + d*x^4)),x)
```

```
[Out] int(-(x*(4*a - 3*x))/((-x^2*(a - x))^(1/3)*(a - x + d*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-4*a+3*x)/(x**2*(-a+x))**(1/3)/(d*x**4+a-x),x)
```

```
[Out] Timed out
```

3.1789 $\int \frac{1}{\left(-1+x^2\right)\sqrt[3]{-x^2+x^3}}\,dx$

Optimal. Leaf size=159

$$-\frac{3\left(x^3-x^2\right)^{2/3}}{2(x-1)x}+\frac{\log\left(2^{2/3}\sqrt[3]{x^3-x^2}-2x\right)}{2\sqrt[3]{2}}-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^3-x^2}\,x+\sqrt[3]{2}\left(x^3-x^2\right)^{2/3}\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2^{2/3}\sqrt[3]{x^3-x^2}\cdots}\right.}{2\sqrt[3]{2}}$$

Rubi [A] time = 0.08, antiderivative size = 182, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2056, 848, 96, 91}

$$-\frac{3x}{2\sqrt[3]{x^3-x^2}}+\frac{3\sqrt[3]{x-1}\,x^{2/3}\log\left(\frac{\sqrt[3]{x-1}}{\sqrt[3]{2}}-\sqrt[3]{x}\right)}{4\sqrt[3]{2}\sqrt[3]{x^3-x^2}}-\frac{\sqrt[3]{x-1}\,x^{2/3}\log(x+1)}{4\sqrt[3]{2}\sqrt[3]{x^3-x^2}}+\frac{\sqrt{3}\sqrt[3]{x-1}\,x^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{x-1}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt[3]{x^3-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x^2)*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (-3*x)/(2*(-x^2 + x^3)^(1/3)) + (Sqrt[3]*(-1 + x)^(1/3)*x^(2/3)*ArcTan[1/Sq
rt[3] + (2^(2/3)*(-1 + x)^(1/3))/(Sqrt[3]*x^(1/3))])/(2*2^(1/3)*(-x^2 + x^3
)^(1/3)) + (3*(-1 + x)^(1/3)*x^(2/3)*Log[(-1 + x)^(1/3)/2^(1/3) - x^(1/3)])
/(4*2^(1/3)*(-x^2 + x^3)^(1/3)) - ((-1 + x)^(1/3)*x^(2/3)*Log[1 + x])/(4*2^
(1/3)*(-x^2 + x^3)^(1/3))
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{\left(-1+x^2\right)\sqrt[3]{-x^2+x^3}}\,dx = \frac{\left(\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{1}{\sqrt[3]{-1+x}\,x^{2/3}\left(-1+x^2\right)}\,dx}{\sqrt[3]{-x^2+x^3}}$$

$$= \frac{\left(\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{1}{(-1+x)^{4/3}x^{2/3}(1+x)}\,dx}{\sqrt[3]{-x^2+x^3}}$$

$$= -\frac{3x}{2\sqrt[3]{-x^2+x^3}} - \frac{\left(\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{1}{\sqrt[3]{-1+x}\,x^{2/3}(1+x)}\,dx}{2\sqrt[3]{-x^2+x^3}}$$

$$= -\frac{3x}{2\sqrt[3]{-x^2+x^3}} + \frac{\sqrt{3}\,\sqrt[3]{-1+x}\,x^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}\sqrt[3]{-1+x}}{\sqrt{3}\,\sqrt[3]{x}}\right)}{2\sqrt[3]{2}\,\sqrt[3]{-x^2+x^3}} + \frac{3\sqrt[3]{-1+x}\,x^{2/3}\log\left(-\frac{\sqrt[3]{}}{}\right.}{4\sqrt[3]{2}\,\sqrt[3]{-x^2+}}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.26

$$-\frac{3\left((x-1)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x-1}{2x}\right)+4x\right)}{8\sqrt[3]{(x-1)x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x^2)*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (-3*(4*x + (-1 + x)*Hypergeometric2F1[2/3, 1, 5/3, (-1 + x)/(2*x)]))/(8*((-
1 + x)*x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.42, size = 159, normalized size = 1.00

$$-\frac{3\left(x^3-x^2\right)^{2/3}}{2(x-1)x}+\frac{\log\left(2^{2/3}\sqrt[3]{x^3-x^2}-2x\right)}{2\sqrt[3]{2}}-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^3-x^2}\,x+\sqrt[3]{2}\left(x^3-x^2\right)^{2/3}\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3-x^2}+x}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x^2)*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (-3*(-x^2 + x^3)^(2/3))/(2*(-1 + x)*x) - (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2
^(2/3)*(-x^2 + x^3)^(1/3))])/(2*2^(1/3)) + Log[-2*x + 2^(2/3)*(-x^2 + x^3)^
(1/3)]/(2*2^(1/3)) - Log[2*x^2 + 2^(2/3)*x*(-x^2 + x^3)^(1/3) + 2^(1/3)*(-x
^2 + x^3)^(2/3)]/(4*2^(1/3))
```

fricas [A] time = 0.42, size = 163, normalized size = 1.03

$$\frac{2\sqrt{3}\,2^{\frac{2}{3}}\left(x^2-x\right)\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}x+2\sqrt{2}\left(x^3-x^2\right)^{\frac{1}{3}}\right)}{6x}\right)+2\cdot 2^{\frac{2}{3}}\left(x^2-x\right)\log\left(-\frac{2^{\frac{1}{3}}x-\left(x^3-x^2\right)^{\frac{1}{3}}}{x}\right)-2^{\frac{2}{3}}\left(x^2-x\right)\log\left(\frac{2^{\frac{2}{3}}x^2+2^{\frac{1}{3}}\left(x^3-x^2\right)^{\frac{1}{3}}x+\left(x^3-x^2\right)^{\frac{2}{3}}}{x^2}\right)-12\left(x^3-x^2\right)^{\frac{2}{3}}}{8\left(x^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x^3-x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/8*(2*sqrt(3)*2^(2/3)*(x^2 - x)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*x + 2*
sqrt(2)*(x^3 - x^2)^(1/3))/x) + 2*2^(2/3)*(x^2 - x)*log(-(2^(1/3)*x - (x^3
- x^2)^(1/3))/x) - 2^(2/3)*(x^2 - x)*log((2^(2/3)*x^2 + 2^(1/3)*(x^3 - x^2)
^(1/3)*x + (x^3 - x^2)^(2/3))/x^2) - 12*(x^3 - x^2)^(2/3))/(x^2 - x)
```

giac [A] time = 0.20, size = 98, normalized size = 0.62

$$\frac{1}{4}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{8}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}\right)+\frac{1}{4}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}\right|\right)-\frac{3}{2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x^3-x^2)^(1/3),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-1/x + 1)^(1/3
))) - 1/8*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-1/x + 1)^(1/3) + (-1/x + 1)^(2/3)
) + 1/4*2^(2/3)*log(abs(-2^(1/3) + (-1/x + 1)^(1/3))) - 3/2/(-1/x + 1)^(1/3
)
```

maple [C] time = 2.91, size = 714, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)/(x^3-x^2)^(1/3),x)
```

```
[Out] -3/2*x/((-1+x)*x^2)^(1/3)+1/4*RootOf(_Z^3-4)*ln((2*RootOf(RootOf(_Z^3-4)^2+
_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x^2-RootOf(RootOf(_Z^3-4)^2+_Z*Roo
tOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x^2-4*RootOf(RootOf(_Z^3-4)^2+_Z*RootO
f(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x+2*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3
-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x-24*(x^3-x^2)^(2/3)*RootOf(RootOf(_Z^3-4)^2+_
Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2+48*(x^3-x^2)^(1/3)*RootOf(_Z^3-4)^2
*x+18*(x^3-x^2)^(1/3)*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3
-4)+_Z^2)*x-44*RootOf(_Z^3-4)*x^2+22*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3
-4)+_Z^2)*x^2+4*RootOf(_Z^3-4)*x-2*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4
)+_Z^2)*x-60*(x^3-x^2)^(2/3))/x/(1+x))+1/4*RootOf(RootOf(_Z^3-4)^2+_Z*RootO
f(_Z^3-4)+_Z^2)*ln((RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(
_Z^3-4)^3*x^2-2*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z
^3-4)^2*x^2-2*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4
)^3*x+4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*
x+24*(x^3-x^2)^(2/3)*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf
(_Z^3-4)^2-48*(x^3-x^2)^(1/3)*RootOf(_Z^3-4)^2*x-30*(x^3-x^2)^(1/3)*RootOf(
_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x+26*RootOf(_Z^3-4)
*x^2-52*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x^2-10*RootOf(_Z^3-
4)*x+20*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x+36*(x^3-x^2)^(2/3
))/x/(1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3-x^2\right)^{\frac{1}{3}}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x^3-x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 - x^2)^(1/3)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2-1\right)\,{\left(x^3-x^2\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)*(x^3 - x^2)^(1/3)),x)
```

```
[Out] int(1/((x^2 - 1)*(x^3 - x^2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x^2 (x-1)} (x-1) (x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)/(x**3-x**2)**(1/3),x)
```

```
[Out] Integral(1/((x**2*(x - 1))**(1/3)*(x - 1)*(x + 1)), x)
```

3.1790 $\int \frac{-b^2+a^2x^4}{\sqrt{-bx+ax^3}\left(b^2+cx^2+a^2x^4\right)}\,dx$

Optimal. Leaf size=159

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{2ab+c}\sqrt{ax^3-bx}}{x\sqrt{2ab+c}-ax^2+b}\right)}{\sqrt{2}\sqrt[4]{2ab+c}}-\frac{\tanh^{-1}\left(\frac{\frac{ax^2}{\sqrt{2}\sqrt[4]{2ab+c}}+\frac{x\sqrt[4]{2ab+c}}{\sqrt{2}}-\frac{b}{\sqrt{2}\sqrt[4]{2ab+c}}}{\sqrt{ax^3-bx}}\right)}{\sqrt{2}\sqrt[4]{2ab+c}}$$

Rubi [C] time = 3.22, antiderivative size = 880, normalized size of antiderivative = 5.53, number of steps used = 21, number of rules used = 10, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2056, 1586, 6715, 6728, 406, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{b}\left(1-\frac{2ab-c}{\sqrt{c^2-4a^2b^2}}\right)\sqrt{x}\sqrt{1-\frac{ax^2}{b}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{ax^3-bx}}+\frac{\sqrt[4]{b}\left(\frac{2ab-c}{\sqrt{c^2-4a^2b^2}}+1\right)\sqrt{x}\sqrt{1-\frac{ax^2}{b}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{ax^3-bx}}+\frac{\sqrt[4]{b}\left(4a^2b^2-c\left(c+\sqrt{c^2-4a^2b^2}\right)\right)\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\Pi\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{b}}{\sqrt{-c-\sqrt{c^2-4a^2b^2}}};\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{c^2-4a^2b^2}\left(c+\sqrt{c^2-4a^2b^2}\right)\sqrt{ax^3-bx}}+\frac{\sqrt[4]{b}\left(4a^2b^2-c\left(c+\sqrt{c^2-4a^2b^2}\right)\right)\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\Pi\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}}{\sqrt{-c-\sqrt{c^2-4a^2b^2}}};\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{c^2-4a^2b^2}\left(c+\sqrt{c^2-4a^2b^2}\right)\sqrt{ax^3-bx}}-\frac{\sqrt[4]{b}\left(4a^2b^2-c\left(c-\sqrt{c^2-4a^2b^2}\right)\right)\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\Pi\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{b}}{\sqrt{\sqrt{c^2-4a^2b^2}-c}};\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{c^2-4a^2b^2}\left(c-\sqrt{c^2-4a^2b^2}\right)\sqrt{ax^3-bx}}-\frac{\sqrt[4]{b}\left(4a^2b^2-c\left(c-\sqrt{c^2-4a^2b^2}\right)\right)\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\Pi\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}}{\sqrt{\sqrt{c^2-4a^2b^2}-c}};\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{c^2-4a^2b^2}\left(c-\sqrt{c^2-4a^2b^2}\right)\sqrt{ax^3-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b^2 + a^2*x^4)/(Sqrt[-(b*x) + a*x^3]*(b^2 + c*x^2 + a^2*x^4)),x]
```

```
[Out] (b^(1/4)*(1 - (2*a*b - c)/Sqrt[-4*a^2*b^2 + c^2])*Sqrt[x]*Sqrt[1 - (a*x^2)/
b]*EllipticF[ArcSin[(a^(1/4)*Sqrt[x])/b^(1/4)], -1])/(a^(1/4)*Sqrt[-(b*x) +
 a*x^3]) + (b^(1/4)*(1 + (2*a*b - c)/Sqrt[-4*a^2*b^2 + c^2])*Sqrt[x]*Sqrt[1
 - (a*x^2)/b]*EllipticF[ArcSin[(a^(1/4)*Sqrt[x])/b^(1/4)], -1])/(a^(1/4)*Sq
rt[-(b*x) + a*x^3]) + (b^(1/4)*(4*a^2*b^2 - c*(c + Sqrt[-4*a^2*b^2 + c^2]))
*Sqrt[x]*Sqrt[1 - (a*x^2)/b]*EllipticPi[-((Sqrt[2]*Sqrt[a]*Sqrt[b])/Sqrt[-c
 - Sqrt[-4*a^2*b^2 + c^2]]), ArcSin[(a^(1/4)*Sqrt[x])/b^(1/4)], -1])/(a^(1/
4)*Sqrt[-4*a^2*b^2 + c^2]*(c + Sqrt[-4*a^2*b^2 + c^2])*Sqrt[-(b*x) + a*x^3]
) + (b^(1/4)*(4*a^2*b^2 - c*(c + Sqrt[-4*a^2*b^2 + c^2]))*Sqrt[x]*Sqrt[1 -
(a*x^2)/b]*EllipticPi[(Sqrt[2]*Sqrt[a]*Sqrt[b])/Sqrt[-c - Sqrt[-4*a^2*b^2 +
 c^2]], ArcSin[(a^(1/4)*Sqrt[x])/b^(1/4)], -1])/(a^(1/4)*Sqrt[-4*a^2*b^2 +
c^2]*(c + Sqrt[-4*a^2*b^2 + c^2])*Sqrt[-(b*x) + a*x^3]) - (b^(1/4)*(4*a^2*b
^2 - c*(c - Sqrt[-4*a^2*b^2 + c^2]))*Sqrt[x]*Sqrt[1 - (a*x^2)/b]*EllipticPi
[-((Sqrt[2]*Sqrt[a]*Sqrt[b])/Sqrt[-c + Sqrt[-4*a^2*b^2 + c^2]]), ArcSin[(a^
(1/4)*Sqrt[x])/b^(1/4)], -1])/(a^(1/4)*Sqrt[-4*a^2*b^2 + c^2]*(c - Sqrt[-4*
a^2*b^2 + c^2])*Sqrt[-(b*x) + a*x^3]) - (b^(1/4)*(4*a^2*b^2 - c*(c - Sqrt[-
4*a^2*b^2 + c^2]))*Sqrt[x]*Sqrt[1 - (a*x^2)/b]*EllipticPi[(Sqrt[2]*Sqrt[a]*
Sqrt[b])/Sqrt[-c + Sqrt[-4*a^2*b^2 + c^2]], ArcSin[(a^(1/4)*Sqrt[x])/b^(1/4
)], -1])/(a^(1/4)*Sqrt[-4*a^2*b^2 + c^2]*(c - Sqrt[-4*a^2*b^2 + c^2])*Sqrt[
-(b*x) + a*x^3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-b^2+a^2x^4}{\sqrt{-bx+ax^3}\left(b^2+cx^2+a^2x^4\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\int \frac{-b^2+a^2x^4}{\sqrt{x}\sqrt{-b+ax^2}\left(b^2+cx^2+a^2x^4\right)}\,dx}{\sqrt{-bx+ax^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\int \frac{\sqrt{-b+ax^2}\left(b+ax^2\right)}{\sqrt{x}\left(b^2+cx^2+a^2x^4\right)}\,dx}{\sqrt{-bx+ax^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-b+ax^2}\right)\text{Subst}\left(\int \frac{\sqrt{-b+ax^4}\left(b+ax^4\right)}{b^2+cx^4+a^2x^8}\,dx, x, \sqrt{x}\right)}{\sqrt{-bx+ax^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-b+ax^2}\right)\text{Subst}\left(\int\left(\frac{\left(a+\frac{a(2ab-c)}{\sqrt{-4a^2b^2+c^2}}\right)\sqrt{-b+ax^4}}{c-\sqrt{-4a^2b^2+c^2}+2a^2x^4}+\frac{\left(a-\frac{a(2ab-c)}{\sqrt{-4a^2b^2+c^2}}\right)\sqrt{-b-\cdots}}{c+\sqrt{-4a^2b^2+c^2}+2a^2\cdots}\right.\right.}{\sqrt{-bx+ax^3}}$$

$$= \frac{\left(2a\left(1-\frac{2ab-c}{\sqrt{-4a^2b^2+c^2}}\right)\sqrt{x}\sqrt{-b+ax^2}\right)\text{Subst}\left(\int \frac{\sqrt{-b+ax^4}}{c+\sqrt{-4a^2b^2+c^2}+2a^2x^4}\,dx, x\cdots\right.}{\sqrt{-bx+ax^3}}$$

$$= \frac{\left(\left(1-\frac{2ab-c}{\sqrt{-4a^2b^2+c^2}}\right)\sqrt{x}\sqrt{-b+ax^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+ax^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{-bx+ax^3}} + \frac{\left(\left(1\cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(\left(1+\frac{2ab-c}{\sqrt{-4a^2b^2+c^2}}\right)\left(2ab+c-\sqrt{-4a^2b^2+c^2}\right)\sqrt{x}\sqrt{-b+ax^2}\right)\text{Subst}\cdots}{2\left(c-\sqrt{-4a^2b^2+c^2}\right)\sqrt{-bx+a\cdots}}$$

$$= \frac{\sqrt[4]{b}\left(1-\frac{2ab-c}{\sqrt{-4a^2b^2+c^2}}\right)\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{-bx+ax^3}} + \frac{\sqrt[4]{b}\left(1+\cdots\right.}{\cdots}$$

$$= \frac{\sqrt[4]{b}\left(1-\frac{2ab-c}{\sqrt{-4a^2b^2+c^2}}\right)\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{-bx+ax^3}} + \frac{\sqrt[4]{b}\left(1+\cdots\right.}{\cdots}$$

Mathematica [C] time = 1.49, size = 396, normalized size = 2.49

$$-\frac{i\sqrt{1-\frac{b}{ax^2}}\sqrt{ax^3-bx}\left(-\Pi\left(-\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}\sqrt{\frac{\sqrt{c^2-4a^2b^2}-c}{b^2}}};i\sinh^{-1}\left(\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}\sqrt{\frac{\sqrt{c^2-4a^2b^2}-c}{b^2}}};i\sinh^{-1}\left(\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(-\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}\sqrt{-\frac{c+\sqrt{c^2-4a^2b^2}}{b^2}}};i\sinh^{-1}\left(\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}\sqrt{-\frac{c+\sqrt{c^2-4a^2b^2}}{b^2}}};i\sinh^{-1}\left(\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{x}}\right)\middle|-1\right)+2F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{x^{3/2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\left(a-\frac{b}{x^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b^2 + a^2*x^4)/(Sqrt[-(b*x) + a*x^3]*(b^2 + c*x^2 + a^2*x^4)),x
]
```

```
[Out] ((-I)*Sqrt[1 - b/(a*x^2)]*Sqrt[-(b*x) + a*x^3]*(2*EllipticF[I*ArcSinh[Sqrt[
-(Sqrt[b]/Sqrt[a])]/Sqrt[x]], -1] - EllipticPi[-((Sqrt[2]*Sqrt[a])/(Sqrt[b]
*Sqrt[(-c + Sqrt[-4*a^2*b^2 + c^2])/b^2])), I*ArcSinh[Sqrt[-(Sqrt[b]/Sqrt[a
])]/Sqrt[x]], -1] - EllipticPi[(Sqrt[2]*Sqrt[a])/(Sqrt[b]*Sqrt[(-c + Sqrt[-
4*a^2*b^2 + c^2])/b^2]), I*ArcSinh[Sqrt[-(Sqrt[b]/Sqrt[a])]/Sqrt[x]], -1] -
```

```
 EllipticPi[-((Sqrt[2]*Sqrt[a])/(Sqrt[b]*Sqrt[-((c + Sqrt[-4*a^2*b^2 + c^2]
)/b^2)])), I*ArcSinh[Sqrt[-(Sqrt[b]/Sqrt[a])]/Sqrt[x]], -1] - EllipticPi[(S
qrt[2]*Sqrt[a])/(Sqrt[b]*Sqrt[-((c + Sqrt[-4*a^2*b^2 + c^2])/b^2)]), I*ArcS
inh[Sqrt[-(Sqrt[b]/Sqrt[a])]/Sqrt[x]], -1]))/(Sqrt[-(Sqrt[b]/Sqrt[a])]*(a -
 b/x^2)*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.97, size = 159, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{2ab+c}\sqrt{ax^3-bx}}{x\sqrt{2ab+c}-ax^2+b}\right)}{\sqrt{2}\sqrt[4]{2ab+c}}-\frac{\tanh^{-1}\left(\frac{\frac{ax^2}{\sqrt{2}\sqrt[4]{2ab+c}}+\frac{x\sqrt[4]{2ab+c}}{\sqrt{2}}-\frac{b}{\sqrt{2}\sqrt[4]{2ab+c}}}{\sqrt{ax^3-bx}}\right)}{\sqrt{2}\sqrt[4]{2ab+c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b^2 + a^2*x^4)/(Sqrt[-(b*x) + a*x^3]*(b^2 + c*x^2 + a^
2*x^4)),x]
```

```
[Out] ArcTan[(Sqrt[2]*(2*a*b + c)^(1/4)*Sqrt[-(b*x) + a*x^3])/(b + Sqrt[2*a*b + c
]*x - a*x^2)]/(Sqrt[2]*(2*a*b + c)^(1/4)) - ArcTanh[(-(b/(Sqrt[2]*(2*a*b +
c)^(1/4))) + ((2*a*b + c)^(1/4)*x)/Sqrt[2] + (a*x^2)/(Sqrt[2]*(2*a*b + c)^(
1/4)))/Sqrt[-(b*x) + a*x^3]]/(Sqrt[2]*(2*a*b + c)^(1/4))
```

fricas [B] time = 0.68, size = 401, normalized size = 2.52

$$\left(-\frac{1}{2ab+c}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{ax^3-bx}(2ab+c)\left(-\frac{1}{2ab+c}\right)^{\frac{3}{4}}}{ax^2-b}\right)+\frac{1}{4}\left(-\frac{1}{2ab+c}\right)^{\frac{1}{4}}\log\left(\frac{a^2x^4-(4ab+c)x^2+b^2+2\sqrt{ax^3-bx}\left((2ab+c)x\left(-\frac{1}{2ab+c}\right)^{\frac{1}{4}}-\left(2ab^2-\left(2a^2b+ac\right)x^2+bc\right)\left(-\frac{1}{2ab+c}\right)^{\frac{3}{4}}\right)-2\left(\left(2a^2b+ac\right)x^3-\left(2ab^2+bc\right)x\right)\sqrt{-\frac{1}{2ab+c}}}{a^2x^4+cx^2+b^2}\right)-\frac{1}{4}\left(-\frac{1}{2ab+c}\right)^{\frac{1}{4}}\log\left(\frac{a^2x^4-(4ab+c)x^2+b^2-2\sqrt{ax^3-bx}\left((2ab+c)x\left(-\frac{1}{2ab+c}\right)^{\frac{1}{4}}-\left(2ab^2-\left(2a^2b+ac\right)x^2+bc\right)\left(-\frac{1}{2ab+c}\right)^{\frac{3}{4}}\right)-2\left(\left(2a^2b+ac\right)x^3-\left(2ab^2+bc\right)x\right)\sqrt{-\frac{1}{2ab+c}}}{a^2x^4+cx^2+b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^4-b^2)/(a*x^3-b*x)^(1/2)/(a^2*x^4+c*x^2+b^2),x, algorithm=
"fricas")
```

```
[Out] (-1/(2*a*b + c))^(1/4)*arctan(sqrt(a*x^3 - b*x)*(2*a*b + c)*(-1/(2*a*b + c)
)^(3/4)/(a*x^2 - b)) + 1/4*(-1/(2*a*b + c))^(1/4)*log((a^2*x^4 - (4*a*b + c
)*x^2 + b^2 + 2*sqrt(a*x^3 - b*x)*((2*a*b + c)*x*(-1/(2*a*b + c))^(1/4) - (
2*a*b^2 - (2*a^2*b + a*c)*x^2 + b*c)*(-1/(2*a*b + c))^(3/4)) - 2*((2*a^2*b
+ a*c)*x^3 - (2*a*b^2 + b*c)*x)*sqrt(-1/(2*a*b + c)))/(a^2*x^4 + c*x^2 + b^
2)) - 1/4*(-1/(2*a*b + c))^(1/4)*log((a^2*x^4 - (4*a*b + c)*x^2 + b^2 - 2*s
qrt(a*x^3 - b*x)*((2*a*b + c)*x*(-1/(2*a*b + c))^(1/4) - (2*a*b^2 - (2*a^2*
b + a*c)*x^2 + b*c)*(-1/(2*a*b + c))^(3/4)) - 2*((2*a^2*b + a*c)*x^3 - (2*a
*b^2 + b*c)*x)*sqrt(-1/(2*a*b + c)))/(a^2*x^4 + c*x^2 + b^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2x^4-b^2}{\left(a^2x^4+cx^2+b^2\right)\sqrt{ax^3-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^4-b^2)/(a*x^3-b*x)^(1/2)/(a^2*x^4+c*x^2+b^2),x, algorithm=
"giac")
```

```
[Out] integrate((a^2*x^4 - b^2)/((a^2*x^4 + c*x^2 + b^2)*sqrt(a*x^3 - b*x)), x)
```

maple [C] time = 0.09, size = 375, normalized size = 2.36

$$\frac{\sqrt{ab}\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}}\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a x^3-bx}}+\frac{\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(a^2\_Z^4+c\_Z^2+b^2\right)}\frac{\left(-\_\alpha^2c-2b^2\right)\sqrt{ab}\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}}\sqrt{-\frac{\left(x-\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}}\sqrt{-\frac{xa}{\sqrt{ab}}}\left(a\left(a^2\_\alpha^3+ab\_\alpha+c\_\alpha\right)-a^2\sqrt{ab}\,\_\alpha^2-\sqrt{ab}\,ab-\sqrt{ab}\,c\right)\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},\frac{-\sqrt{ab}\,\_\alpha^3a^2+\_\alpha^2a^2b-\sqrt{ab}\,\_\alpha ab-\sqrt{ab}\,\_\alpha c+ab^2+bc}{b(2ab+c)},\frac{\sqrt{2}}{2}\right)}{\_\alpha\left(2\_\alpha^2a^2+c\right)(2ab+c)\sqrt{x\left(a x^2-b\right)}}\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^4-b^2)/(a*x^3-b*x)^(1/2)/(a^2*x^4+c*x^2+b^2),x)
```

```
[Out] 1/a*(a*b)^(1/2)*((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2)*(-2*(x-1/a*(a*b)^
(1/2))*a/(a*b)^(1/2))^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)*Elli
pticF(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/2/a/b*2^(1/2
)*sum((-_alpha^2*c-2*b^2)/_alpha/(2*_alpha^2*a^2+c)/(2*a*b+c)*(a*b)^(1/2)*(
(x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2)*(-(x-1/a*(a*b)^(1/2))*a/(a*b)^(1/2
))^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(x*(a*x^2-b))^(1/2)*(a*(_alpha^3*a^2+_alp
ha*a*b+_alpha*c)-a^2*(a*b)^(1/2)*_alpha^2-(a*b)^(1/2)*a*b-(a*b)^(1/2)*c)*El
lipticPi(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2),(-(a*b)^(1/2)*_alpha^3*a
^2+_alpha^2*a^2*b-(a*b)^(1/2)*_alpha*a*b-(a*b)^(1/2)*_alpha*c+a*b^2+b*c)/b/
(2*a*b+c),1/2*2^(1/2)),_alpha=RootOf(_Z^4*a^2+_Z^2*c+b^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2x^4 - b^2}{\left(a^2x^4 + cx^2 + b^2\right)\sqrt{ax^3 - bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^4-b^2)/(a*x^3-b*x)^(1/2)/(a^2*x^4+c*x^2+b^2),x, algorithm=
"maxima")
```

```
[Out] integrate((a^2*x^4 - b^2)/((a^2*x^4 + c*x^2 + b^2)*sqrt(a*x^3 - b*x)), x)
```

mupad [B] time = 4.81, size = 165, normalized size = 1.04

$$\frac{\ln\left(\frac{b-x\,\sqrt{-c-2\,a\,b}+2\,\sqrt{a\,x^3-b\,x}\,{\left(-c-2\,a\,b\right)}^{1/4}-a\,x^2}{b+x\,\sqrt{-c-2\,a\,b}-a\,x^2}\right)}{2\,{\left(-c-2\,a\,b\right)}^{1/4}}+\frac{\ln\left(\frac{b+x\,\sqrt{-c-2\,a\,b}-a\,x^2-\sqrt{a\,x^3-b\,x}\,{\left(-c-2\,a\,b\right)}^{1/4}\,2{}\mathrm{i}}{x\,\sqrt{-c-2\,a\,b}-b+a\,x^2}\right)\,1{}\mathrm{i}}{2\,{\left(-c-2\,a\,b\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^2 - a^2*x^4)/((a*x^3 - b*x)^(1/2)*(c*x^2 + b^2 + a^2*x^4)),x)
```

```
[Out] log((b - x*(- c - 2*a*b)^(1/2) + 2*(a*x^3 - b*x)^(1/2)*(- c - 2*a*b)^(1/4)
- a*x^2)/(b + x*(- c - 2*a*b)^(1/2) - a*x^2))/(2*(- c - 2*a*b)^(1/4)) + (lo
g((b + x*(- c - 2*a*b)^(1/2) - (a*x^3 - b*x)^(1/2)*(- c - 2*a*b)^(1/4)*2i -
 a*x^2)/(x*(- c - 2*a*b)^(1/2) - b + a*x^2))*1i)/(2*(- c - 2*a*b)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2 - b\right)\left(ax^2 + b\right)}{\sqrt{x\left(ax^2 - b\right)}\,\left(a^2x^4 + b^2 + cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**4-b**2)/(a*x**3-b*x)**(1/2)/(a**2*x**4+c*x**2+b**2),x)
```

```
[Out] Integral((a*x**2 - b)*(a*x**2 + b)/(sqrt(x*(a*x**2 - b))*(a**2*x**4 + b**2
+ c*x**2)), x)
```

3.1791 $\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x\left(bx^2+a(q+px^3)^2\right)}\,dx$

Optimal. Leaf size=159

$$\frac{\sqrt{2apq+b}\tanh^{-1}\left(\frac{\sqrt{b}\,x^2\sqrt{2apq+b}}{\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}\,(apx^3+aq)+ap^2x^6+2apqx^3+aq^2+bx^2}\right)}{a\sqrt{b}}+\frac{\log\left(\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+px^3+\right.}{a}$$

Rubi [F] time = 5.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x\left(bx^2+a(q+px^3)^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6])/(x*(b*x^2
+ a*(q + p*x^3)^2)),x]
```

```
[Out] -(Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x, x]/(a*q)) + (b*
Defer[Int][(x*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6])/(a*q^2 + b*x^2 +
 2*a*p*q*x^3 + a*p^2*x^6), x])/(a*q) + 4*p*Defer[Int][(x^2*Sqrt[q^2 - 2*p*q
*x^2 + 2*p*q*x^3 + p^2*x^6])/(a*q^2 + b*x^2 + 2*a*p*q*x^3 + a*p^2*x^6), x]
+ (p^2*Defer[Int][(x^5*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6])/(a*q^2
+ b*x^2 + 2*a*p*q*x^3 + a*p^2*x^6), x])/q
```

Rubi steps

$$\begin{aligned}\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x\left(bx^2+a(q+px^3)^2\right)}\,dx &= \int\left(-\frac{\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{aqx}+\frac{x\left(b+4apqx+ap^2x^4\right)\sqrt{\ }}{aq\left(aq^2+bx^2\right.}\right.\\ &= -\frac{\int\frac{\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x}\,dx}{aq}+\frac{\int\frac{x\left(b+4apqx+ap^2x^4\right)\sqrt{q^2-2pqx^2+2pqx^3+}}{aq^2+bx^2+2apqx^3+ap^2x^6}}{aq}\\ &= -\frac{\int\frac{\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x}\,dx}{aq}+\frac{\int\left(\frac{bx\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{aq^2+bx^2+2apqx^3+ap^2x^6}+\frac{4apqx}{a}\right.}{}\\ &= (4p)\int\frac{x^2\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{aq^2+bx^2+2apqx^3+ap^2x^6}\,dx-\frac{\int\frac{\sqrt{q^2-2pqx^2+2pqx^3}}{x}}{aq}\end{aligned}$$

Mathematica [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x\left(bx^2+a(q+px^3)^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6])/(x*(
b*x^2 + a*(q + p*x^3)^2)),x]
```

```
[Out] Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6])/(x*(
b*x^2 + a*(q + p*x^3)^2)), x]
```

IntegrateAlgebraic [A] time = 0.74, size = 159, normalized size = 1.00

$$\frac{\sqrt{2apq+b}\tanh^{-1}\left(\frac{\sqrt{b}\,x^2\sqrt{2apq+b}}{\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}\left(apx^3+aq\right)+ap^2x^6+2apqx^3+aq^2+bx^2}\right)}{a\sqrt{b}}+\frac{\log\left(\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+px^3+q\right)}{a}-\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x
^6])/(x*(b*x^2 + a*(q + p*x^3)^2)),x]
```

```
[Out] (Sqrt[b + 2*a*p*q]*ArcTanh[(Sqrt[b]*Sqrt[b + 2*a*p*q]*x^2)/(a*q^2 + b*x^2 +
 2*a*p*q*x^3 + a*p^2*x^6 + (a*q + a*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3
 + p^2*x^6])])/(a*Sqrt[b]) - Log[x]/a + Log[q + p*x^3 + Sqrt[q^2 - 2*p*q*x^
2 + 2*p*q*x^3 + p^2*x^6]]/a
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)/x/(b*x^2+a*(p
*x^3+q)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6+2\,pqx^3-2\,pqx^2+q^2}\left(2\,px^3-q\right)}{\left(\left(px^3+q\right)^2a+bx^2\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)/x/(b*x^2+a*(p
*x^3+q)^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)*(2*p*x^3 - q)/(((p*x^
3 + q)^2*a + b*x^2)*x), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(2p\,x^3-q\right)\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}}{x\left(b\,x^2+a\left(p\,x^3+q\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)/x/(b*x^2+a*(p*x^3+q
)^2),x)
```

```
[Out] int((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)/x/(b*x^2+a*(p*x^3+q
)^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6+2\,pqx^3-2\,pqx^2+q^2}\left(2\,px^3-q\right)}{\left(\left(px^3+q\right)^2a+bx^2\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)/x/(b*x^2+a*(p
*x^3+q)^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)*(2*p*x^3 - q)/(((p*x^
3 + q)^2*a + b*x^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(q-2\,p\,x^3\right)\,\sqrt{p^2\,x^6+2\,p\,q\,x^3-2\,p\,q\,x^2+q^2}}{x\,\left(a\,{\left(p\,x^3+q\right)}^2+b\,x^2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q - 2*p*x^3)*(p^2*x^6 + q^2 - 2*p*q*x^2 + 2*p*q*x^3)^(1/2))/(x*(a*(q
 + p*x^3)^2 + b*x^2)),x)
```

```
[Out] int(-((q - 2*p*x^3)*(p^2*x^6 + q^2 - 2*p*q*x^2 + 2*p*q*x^3)^(1/2))/(x*(a*(q
 + p*x^3)^2 + b*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 p x^{3} - q\right) \sqrt{p^{2} x^{6} + 2 p q x^{3} - 2 p q x^{2} + q^{2}}}{x \left(a p^{2} x^{6} + 2 a p q x^{3} + a q^{2} + b x^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x**3-q)*(p**2*x**6+2*p*q*x**3-2*p*q*x**2+q**2)**(1/2)/x/(b*x
**2+a*(p*x**3+q)**2),x)
```

```
[Out] Integral((2*p*x**3 - q)*sqrt(p**2*x**6 + 2*p*q*x**3 - 2*p*q*x**2 + q**2)/(x
*(a*p**2*x**6 + 2*a*p*q*x**3 + a*q**2 + b*x**2)), x)
```

3.1792 $\int \frac{(-2q+px^3)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x\left(bx^4+a(q+px^3)^2\right)}\,dx$

Optimal. Leaf size=159

$$\frac{\sqrt{2apq+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x^4\sqrt{2apq+b}}{\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}\,(apx^3+aq)+ap^2x^6+2apqx^3+aq^2+bx^4}\right)}{a\sqrt{b}}+\frac{\log\left(\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}\right.+px}{a}$$

Rubi [F] time = 6.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2q+px^3)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x\left(bx^4+a(q+px^3)^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6])/(x*(b*x^4
+ a*(q + p*x^3)^2)),x]
```

```
[Out] (-2*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x, x])/(a*q) + 5
*p*Defer[Int][(x^2*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6])/(a*q^2 + 2*
a*p*q*x^3 + b*x^4 + a*p^2*x^6), x] + (2*b*Defer[Int][(x^3*Sqrt[q^2 + 2*p*q*
x^3 - 2*p*q*x^4 + p^2*x^6])/(a*q^2 + 2*a*p*q*x^3 + b*x^4 + a*p^2*x^6), x])/
(a*q) + (2*p^2*Defer[Int][(x^5*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6])
/(a*q^2 + 2*a*p*q*x^3 + b*x^4 + a*p^2*x^6), x])/q
```

Rubi steps

$$\int \frac{(-2q+px^3)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x\left(bx^4+a(q+px^3)^2\right)}\,dx=\int \left(-\frac{2\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{aqx}+\frac{x^2\left(5apq+2bx+2a\right.}{aq\left(aq^2\right.}\right.$$

$$=\frac{\int \frac{x^2(5apq+2bx+2ap^2x^3)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{aq^2+2apqx^3+bx^4+ap^2x^6}\,dx}{aq}-\frac{2\int \frac{\sqrt{q^2+2pqx^3-}}{}}{a}$$

$$=\frac{\int \left(\frac{5apqx^2\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{aq^2+2apqx^3+bx^4+ap^2x^6}+\frac{2bx^3\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{aq^2+2apqx^3+bx^4+ap^2x^6}+\right.}{aq}$$

$$=(5p)\int \frac{x^2\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{aq^2+2apqx^3+bx^4+ap^2x^6}\,dx-\frac{2\int \frac{\sqrt{q^2+2pqx^3-}}{x}}{a}$$

Mathematica [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{(-2q+px^3)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x\left(bx^4+a(q+px^3)^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6])/(x*(
b*x^4 + a*(q + p*x^3)^2)),x]
```

```
[Out] Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6])/(x*(
b*x^4 + a*(q + p*x^3)^2)), x]
```

IntegrateAlgebraic [A] time = 0.78, size = 159, normalized size = 1.00

$$\frac{\sqrt{2apq+b}\tanh^{-1}\left(\frac{\sqrt{b}\,x^4\sqrt{2apq+b}}{\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}\left(apx^3+aq\right)+ap^2x^6+2apqx^3+aq^2+bx^4}\right)}{a\sqrt{b}}+\frac{\log\left(\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}+px^3+q\right)}{a}-\frac{2\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x
^6])/(x*(b*x^4 + a*(q + p*x^3)^2)),x]
```

```
[Out] (Sqrt[b + 2*a*p*q]*ArcTanh[(Sqrt[b]*Sqrt[b + 2*a*p*q]*x^4)/(a*q^2 + 2*a*p*q
*x^3 + b*x^4 + a*p^2*x^6 + (a*q + a*p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4
 + p^2*x^6])])/(a*Sqrt[b]) - (2*Log[x])/a + Log[q + p*x^3 + Sqrt[q^2 + 2*p*
q*x^3 - 2*p*q*x^4 + p^2*x^6]]/a
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)/x/(b*x^4+a*(p
*x^3+q)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6-2\,pqx^4+2\,pqx^3+q^2}\left(px^3-2\,q\right)}{\left(bx^4+\left(px^3+q\right)^2a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)/x/(b*x^4+a*(p
*x^3+q)^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(p^2*x^6 - 2*p*q*x^4 + 2*p*q*x^3 + q^2)*(p*x^3 - 2*q)/((b*x^4
 + (p*x^3 + q)^2*a)*x), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(p\,x^3-2q\right)\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}}{x\left(b\,x^4+a\left(p\,x^3+q\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)/x/(b*x^4+a*(p*x^3+q
)^2),x)
```

```
[Out] int((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)/x/(b*x^4+a*(p*x^3+q
)^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6-2\,pqx^4+2\,pqx^3+q^2}\left(px^3-2\,q\right)}{\left(bx^4+\left(px^3+q\right)^2a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)/x/(b*x^4+a*(p
*x^3+q)^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(p^2*x^6 - 2*p*q*x^4 + 2*p*q*x^3 + q^2)*(p*x^3 - 2*q)/((b*x^4
 + (p*x^3 + q)^2*a)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(2\,q-p\,x^3\right)\,\sqrt{p^2\,x^6-2\,p\,q\,x^4+2\,p\,q\,x^3+q^2}}{x\,\left(a\,{\left(p\,x^3+q\right)}^2+b\,x^4\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*q - p*x^3)*(p^2*x^6 + q^2 + 2*p*q*x^3 - 2*p*q*x^4)^(1/2))/(x*(a*(q
 + p*x^3)^2 + b*x^4)),x)
```

```
[Out] int(-((2*q - p*x^3)*(p^2*x^6 + q^2 + 2*p*q*x^3 - 2*p*q*x^4)^(1/2))/(x*(a*(q
 + p*x^3)^2 + b*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(px^{3} - 2q\right) \sqrt{p^{2} x^{6} - 2 pq x^{4} + 2 pq x^{3} + q^{2}}}{x \left(ap^{2} x^{6} + 2 apq x^{3} + aq^{2} + bx^{4}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**3-2*q)*(p**2*x**6-2*p*q*x**4+2*p*q*x**3+q**2)**(1/2)/x/(b*x
**4+a*(p*x**3+q)**2),x)
```

```
[Out] Integral((p*x**3 - 2*q)*sqrt(p**2*x**6 - 2*p*q*x**4 + 2*p*q*x**3 + q**2)/(x
*(a*p**2*x**6 + 2*a*p*q*x**3 + a*q**2 + b*x**4)), x)
```

3.1793 $\int \frac{\sqrt{1+\sqrt{1-\sqrt{1+\frac{1}{x^2}}}}}{x}\,dx$

Optimal. Leaf size=160

$$-4\sqrt{\sqrt{1-\sqrt{\frac{1}{x^2}+1}}+1}+\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{1-\sqrt{\frac{x^2+1}{x^2}}}+1}}{\sqrt{\sqrt{2}-1}}\right)+2\tanh^{-1}\left(\sqrt{\sqrt{1-\sqrt{\frac{x^2+1}{x^2}}}+1}\right)+\sqrt{1+\sqrt{2}$$

Rubi [A] time = 1.03, antiderivative size = 148, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6742, 2073, 207, 1166, 203}

$$-4\sqrt{\sqrt{1-\sqrt{\frac{1}{x^2}+1}}+1}+\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{1-\sqrt{\frac{1}{x^2}+1}}+1}}{\sqrt{\sqrt{2}-1}}\right)+2\tanh^{-1}\left(\sqrt{\sqrt{1-\sqrt{\frac{1}{x^2}+1}}+1}\right)+\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{1-\sqrt{\frac{1}{x^2}+1}}+1}}{\sqrt{1+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 - Sqrt[1 + x^(-2)]]]/x,x]
```

```
[Out] -4*Sqrt[1 + Sqrt[1 - Sqrt[1 + x^(-2)]]] + Sqrt[-1 + Sqrt[2]]*ArcTan[Sqrt[1
+ Sqrt[1 - Sqrt[1 + x^(-2)]]]/Sqrt[-1 + Sqrt[2]]] + 2*ArcTanh[Sqrt[1 + Sqrt
[1 - Sqrt[1 + x^(-2)]]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[1 + Sqrt[1 - Sqrt
[1 + x^(-2)]]]/Sqrt[1 + Sqrt[2]]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{1+\sqrt{1-\sqrt{1+\frac{1}{x^2}}}}}{x}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1+\sqrt{1-\sqrt{1+x}}}}{x}\,dx,x,\frac{1}{x^2}\right)\right)$$
$$= -\text{Subst}\left(\int \frac{\sqrt{1+\sqrt{1-x}}\,x}{-1+x^2}\,dx,x,\sqrt{1+\frac{1}{x^2}}\right)$$
$$= -\left(2\,\text{Subst}\left(\int \frac{\sqrt{1+x}\,(-1+x^2)}{x\,(-2+x^2)}\,dx,x,\sqrt{1-\sqrt{1+\frac{1}{x^2}}}\right)\right)$$
$$= -\left(4\,\text{Subst}\left(\int \frac{x^4\,(-2+x^2)}{1+x^2-3x^4+x^6}\,dx,x,\sqrt{1+\sqrt{1-\sqrt{1+\frac{1}{x^2}}}}\right)\right)$$
$$= -\left(4\,\text{Subst}\left(\int \left(1-\frac{1+x^2-x^4}{1+x^2-3x^4+x^6}\right)dx,x,\sqrt{1+\sqrt{1-\sqrt{1+\frac{1}{x^2}}}}\right)\right)$$
$$= -4\sqrt{1+\sqrt{1-\sqrt{1+\frac{1}{x^2}}}} + 4\,\text{Subst}\left(\int \frac{1+x^2-x^4}{1+x^2-3x^4+x^6}\,dx,x,\sqrt{1+\sqrt{1-\sqrt{\ldots}}}\right.$$
$$= -4\sqrt{1+\sqrt{1-\sqrt{1+\frac{1}{x^2}}}} + 4\,\text{Subst}\left(\int \left(-\frac{1}{2\,(-1+x^2)}+\frac{-1-x^2}{2\,(-1-2x^2+x^4)}\right) d\ldots\right.$$
$$= -4\sqrt{1+\sqrt{1-\sqrt{1+\frac{1}{x^2}}}} - 2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+\sqrt{1-\sqrt{1+\frac{1}{x^2}}}}\right)\ldots$$
$$= -4\sqrt{1+\sqrt{1-\sqrt{1+\frac{1}{x^2}}}} + 2\tanh^{-1}\left(\sqrt{1+\sqrt{1-\sqrt{1+\frac{1}{x^2}}}}\right) + \left(-1-\sqrt{2}\right)\text{Su}\ldots$$
$$= -4\sqrt{1+\sqrt{1-\sqrt{1+\frac{1}{x^2}}}} + \sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{1+\sqrt{1-\sqrt{1+\frac{1}{x^2}}}}}{\sqrt{-1+\sqrt{2}}}\right) + 2\tan\ldots$$

Mathematica [A] time = 0.26, size = 148, normalized size = 0.92

$$-4\sqrt{\sqrt{1-\sqrt{\frac{1}{x^2}+1}}+1}+\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{1-\sqrt{\frac{1}{x^2}+1}}+1}}{\sqrt{\sqrt{2}-1}}\right)+2\tanh^{-1}\left(\sqrt{\sqrt{1-\sqrt{\frac{1}{x^2}+1}}+1}\right)+\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{1-\sqrt{\frac{1}{x^2}+1}}+1}}{\sqrt{1+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 - Sqrt[1 + x^(-2)]]]/x,x]
```

```
[Out] -4*Sqrt[1 + Sqrt[1 - Sqrt[1 + x^(-2)]]] + Sqrt[-1 + Sqrt[2]]*ArcTan[Sqrt[1
+ Sqrt[1 - Sqrt[1 + x^(-2)]]]/Sqrt[-1 + Sqrt[2]]] + 2*ArcTanh[Sqrt[1 + Sqrt
[1 - Sqrt[1 + x^(-2)]]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[1 + Sqrt[1 - Sqrt
[1 + x^(-2)]]]/Sqrt[1 + Sqrt[2]]]
```

IntegrateAlgebraic [A] time = 0.89, size = 160, normalized size = 1.00

$$-4\sqrt{\sqrt{1-\sqrt{\frac{1}{x^2}+1}}+1}+\sqrt{\sqrt{2}-1}\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{1-\sqrt{\frac{x^2+1}{x^2}}}+1}\right)+2\tanh^{-1}\left(\sqrt{\sqrt{1-\sqrt{\frac{x^2+1}{x^2}}}+1}\right)+\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{1-\sqrt{\frac{x^2+1}{x^2}}}+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 - Sqrt[1 + x^(-2)]]]/x,x]
```

```
[Out] -4*Sqrt[1 + Sqrt[1 - Sqrt[1 + x^(-2)]]] + Sqrt[-1 + Sqrt[2]]*ArcTan[Sqrt[1
+ Sqrt[2]]*Sqrt[1 + Sqrt[1 - Sqrt[(1 + x^2)/x^2]]]] + 2*ArcTanh[Sqrt[1 + Sq
rt[1 - Sqrt[(1 + x^2)/x^2]]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[-1 + Sqrt[2]
]*Sqrt[1 + Sqrt[1 - Sqrt[(1 + x^2)/x^2]]]]
```

fricas [B] time = 127.13, size = 1028, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1-(1+1/x^2)^(1/2))^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(sqrt(2) - 1)*arctan(1/2098*(2*sqrt(2)*(4432*x^4 - 132*x^2 + sqrt(2)*(3
208*x^4 + 37*x^2) + (6832*x^4 - 52*x^2 + sqrt(2)*(4824*x^4 - 49*x^2))*sqrt(
(x^2 + 1)/x^2))*sqrt(6301*sqrt(2) - 8849)*sqrt(sqrt(2) - 1)*sqrt(-sqrt((x^2
 + 1)/x^2) + 1) - sqrt(2)*(14496*x^4 + 16124*x^2 + sqrt(2)*(10432*x^4 + 117
24*x^2 - 101) + 4*(2008*x^4 - 3*x^2 + sqrt(2)*(1408*x^4 - 23*x^2))*sqrt((x^
2 + 1)/x^2) - 150)*sqrt(6301*sqrt(2) - 8849)*sqrt(sqrt(2) - 1) + 4196*((40*
x^4 + 5*x^2 + 4*sqrt(2)*(6*x^4 - x^2) + (40*sqrt(2)*x^4 + 56*x^4 - x^2)*sqr
t((x^2 + 1)/x^2))*sqrt(sqrt(2) - 1)*sqrt(-sqrt((x^2 + 1)/x^2) + 1) - (32*x^
4 + 18*x^2 + sqrt(2)*(8*x^4 - 13*x^2) + (32*x^4 - 2*x^2 + sqrt(2)*(24*x^4 +
 x^2))*sqrt((x^2 + 1)/x^2))*sqrt(sqrt(2) - 1))*sqrt(sqrt(-sqrt((x^2 + 1)/x^
2) + 1) + 1))/(64*x^4 + 112*x^2 - 1)) - 1/4*sqrt(sqrt(2) + 1)*log(4*(101*sq
rt(2)*x^2 + 150*x^2 - (101*sqrt(2)*x^2 + 150*x^2)*sqrt((x^2 + 1)/x^2))*sqrt
(sqrt(2) + 1)*sqrt(-sqrt((x^2 + 1)/x^2) + 1) + 2*(404*x^2 + sqrt(2)*(300*x^
2 + 49) - 4*(75*sqrt(2)*x^2 + 101*x^2)*sqrt((x^2 + 1)/x^2) + 52)*sqrt(sqrt(
2) + 1) - 4*(150*sqrt(2)*x^2 + 202*x^2 + (101*sqrt(2)*x^2 + 150*x^2 - (101*
sqrt(2)*x^2 + 150*x^2)*sqrt((x^2 + 1)/x^2))*sqrt(-sqrt((x^2 + 1)/x^2) + 1)
- 2*(75*sqrt(2)*x^2 + 101*x^2)*sqrt((x^2 + 1)/x^2))*sqrt(sqrt(-sqrt((x^2 +
1)/x^2) + 1) + 1)) + 1/4*sqrt(sqrt(2) + 1)*log(-4*(101*sqrt(2)*x^2 + 150*x^
2 - (101*sqrt(2)*x^2 + 150*x^2)*sqrt((x^2 + 1)/x^2))*sqrt(sqrt(2) + 1)*sqrt
(-sqrt((x^2 + 1)/x^2) + 1) - 2*(404*x^2 + sqrt(2)*(300*x^2 + 49) - 4*(75*sq
rt(2)*x^2 + 101*x^2)*sqrt((x^2 + 1)/x^2) + 52)*sqrt(sqrt(2) + 1) - 4*(150*s
qrt(2)*x^2 + 202*x^2 + (101*sqrt(2)*x^2 + 150*x^2 - (101*sqrt(2)*x^2 + 150*
x^2)*sqrt((x^2 + 1)/x^2))*sqrt(-sqrt((x^2 + 1)/x^2) + 1) - 2*(75*sqrt(2)*x^
2 + 101*x^2)*sqrt((x^2 + 1)/x^2))*sqrt(sqrt(-sqrt((x^2 + 1)/x^2) + 1) + 1))
 - 4*sqrt(sqrt(-sqrt((x^2 + 1)/x^2) + 1) + 1) + log(2*(x^2*sqrt((x^2 + 1)/x
^2) + x^2)*sqrt(sqrt(-sqrt((x^2 + 1)/x^2) + 1) + 1)*sqrt(-sqrt((x^2 + 1)/x^
2) + 1) + 2*(x^2*sqrt((x^2 + 1)/x^2) + x^2)*sqrt(-sqrt((x^2 + 1)/x^2) + 1)
- 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1-(1+1/x^2)^(1/2))^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error index.cc index_gcd Error: Bad Argument
 ValueError index.cc index_gcd Error: Bad Argument ValueError index.cc inde
```

```
x_gcd Error: Bad Argument ValueError index.cc index_gcd Error: Bad Argument
 ValueEvaluation time: 1.7Done
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\sqrt{1-\sqrt{1+\frac{1}{x^2}}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(1-(1+1/x^2)^(1/2))^(1/2))^(1/2)/x,x)
[Out] int((1+(1-(1+1/x^2)^(1/2))^(1/2))^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{-\sqrt{\frac{1}{x^2}+1}+1}+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1-(1+1/x^2)^(1/2))^(1/2))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(-sqrt(1/x^2 + 1) + 1) + 1)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{1-\sqrt{\frac{1}{x^2}+1}}+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - (1/x^2 + 1)^(1/2))^(1/2) + 1)^(1/2)/x,x)
[Out] int(((1 - (1/x^2 + 1)^(1/2))^(1/2) + 1)^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{1-\sqrt{1+\frac{1}{x^2}}}+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1-(1+1/x**2)**(1/2))**(1/2))**(1/2)/x,x)
[Out] Integral(sqrt(sqrt(1 - sqrt(1 + x**(-2))) + 1)/x, x)
```

3.1794 $\int \frac{-2x+(1+k)x^2}{((1-x)x(1-kx))^{2/3}\left(1-(1+k)x+(-b+k)x^2\right)} dx$

Optimal. Leaf size=160

$$\frac{\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[3]{b}\,x\right)}{b^{2/3}}-\frac{\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2b^{2/3}}+\frac{\sqrt{}}{}$$

Rubi [F] time = 3.88, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2x+(1+k)x^2}{((1-x)x(1-kx))^{2/3}\left(1-(1+k)x+(-b+k)x^2\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*x + (1 + k)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(1 - (1 + k)*x + (-b
+ k)*x^2)),x]
```

```
[Out] ((1 + Sqrt[4*b + (-1 + k)^2] + k)*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Def
er[Int][x^(1/3)/((1 - x)^(2/3)*(1 - k*x)^(2/3)*(-1 - k - Sqrt[1 + 4*b - 2*k
 + k^2] + 2*(-b + k)*x)), x])/((1 - x)*x*(1 - k*x))^(2/3) + ((1 - Sqrt[4*b
+ (-1 + k)^2] + k)*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Int][x^(1/3)
/((1 - x)^(2/3)*(1 - k*x)^(2/3)*(-1 - k + Sqrt[1 + 4*b - 2*k + k^2] + 2*(-b
 + k)*x)), x])/((1 - x)*x*(1 - k*x))^(2/3)
```

Rubi steps

$$\begin{aligned}\int \frac{-2x+(1+k)x^2}{((1-x)x(1-kx))^{2/3}\left(1-(1+k)x+(-b+k)x^2\right)} dx &= \int \frac{x(-2+(1+k)x)}{((1-x)x(1-kx))^{2/3}\left(1-(1+k)x+(-b+k)x^2\right)} dx\\ &= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{\sqrt[3]{x}\,(-2+(1+k)x)}{(1-x)^{2/3}(1-kx)^{2/3}\left(1-(1+k)x+(-b+k)x\right.}}{((1-x)x(1-kx))^{2/3}}\\ &= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int\left(\frac{\left(1+k+\sqrt{1+4b-2k+k^2}\right)}{(1-x)^{2/3}(1-kx)^{2/3}\left(-1-k-\sqrt{1+4b-2k}\right.}\right.}{((1-x)\cdot}\\ &= \frac{\left(\left(1-\sqrt{4b+(-1+k)^2}+k\right)(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int -}{((1-x)x(1-kx))}\end{aligned}$$

Mathematica [F] time = 7.32, size = 0, normalized size = 0.00

$$\int \frac{-2x+(1+k)x^2}{((1-x)x(1-kx))^{2/3}\left(1-(1+k)x+(-b+k)x^2\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*x + (1 + k)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(1 - (1 + k)*x
+ (-b + k)*x^2)),x]
```

```
[Out] Integrate[(-2*x + (1 + k)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(1 - (1 + k)*x
+ (-b + k)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.35, size = 160, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[3]{b}\,x\right)}{b^{2/3}}-\frac{\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2b^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*x + (1 + k)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(1 - (
1 + k)*x + (-b + k)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(x + (-1 - k)*x^2 + k*x^
3)^(1/3))])/b^(2/3) + Log[-(b^(1/3)*x) + (x + (-1 - k)*x^2 + k*x^3)^(1/3)]/
b^(2/3) - Log[b^(2/3)*x^2 + b^(1/3)*x*(x + (-1 - k)*x^2 + k*x^3)^(1/3) + (x
 + (-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x+(1+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-(1+k)*x+(-b+k)*x^2),
x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.53, size = 122, normalized size = 0.76

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+2\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{2}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{1}{3}}+\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{2}{3}}\right)}{2b^{\frac{2}{3}}}+\frac{\log\left(\left|-b^{\frac{1}{3}}+\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{1}{3}}\right|\right)}{b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x+(1+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-(1+k)*x+(-b+k)*x^2),
x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(k - k/x - 1/x + 1/x^2)^(1/3))/b^(
1/3))/b^(2/3) - 1/2*log(b^(2/3) + b^(1/3)*(k - k/x - 1/x + 1/x^2)^(1/3) + (
k - k/x - 1/x + 1/x^2)^(2/3))/b^(2/3) + log(abs(-b^(1/3) + (k - k/x - 1/x +
 1/x^2)^(1/3)))/b^(2/3)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{-2x+(1+k)\,x^2}{\left((1-x)\,x\,(-kx+1)\right)^{\frac{2}{3}}\left(1-(1+k)\,x+(-b+k)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x+(1+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-(1+k)*x+(-b+k)*x^2),x)
```

```
[Out] int((-2*x+(1+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-(1+k)*x+(-b+k)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(k+1)x^2-2\,x}{\left((kx-1)(x-1)x\right)^{\frac{2}{3}}\left((b-k)x^2+(k+1)x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x+(1+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-(1+k)*x+(-b+k)*x^2),
x, algorithm="maxima")
```

```
[Out] -integrate(((k + 1)*x^2 - 2*x)/(((k*x - 1)*(x - 1)*x)^(2/3)*((b - k)*x^2 +
(k + 1)*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x-x^2\,\left(k+1\right)}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{2/3}\,\left(\left(b-k\right)\,x^2+\left(k+1\right)\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2*(k + 1))/((x*(k*x - 1)*(x - 1))^(2/3)*(x*(k + 1) + x^2*(b -
k) - 1)),x)
```

```
[Out] int((2*x - x^2*(k + 1))/((x*(k*x - 1)*(x - 1))^(2/3)*(x*(k + 1) + x^2*(b -
k) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x+(1+k)*x**2)/((1-x)*x*(-k*x+1))**(2/3)/(1-(1+k)*x+(-b+k)*x**
2),x)
```

```
[Out] Timed out
```

3.1795 $\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx$

Optimal. Leaf size=161

$$\frac{\log\left(k^{2/3}x^2+\sqrt[3]{k}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2\sqrt[3]{k}}-\frac{\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[3]{k}\,x\right)}{\sqrt[3]{k}}+$$

Rubi [F] time = 0.63, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
```

```
[Out] (3*(1 - x)^(1/3)*x*(1 - k*x)^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, x, k*x])/(2
*((1 - x)*x*(1 - k*x))^(1/3)) + ((1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defe
r[Int][1/((1 - x)^(1/3)*x^(1/3)*(1 + (-1 - k)*x)*(1 - k*x)^(1/3)), x])/((1
- x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\begin{aligned}\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx &= \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{2-(1+k)x}{\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\,(1-(1+k)x)}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}\\ &= \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{1}{\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}+\frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int}{\sqrt[3]{(1-x)}}\\ &= \frac{3\sqrt[3]{1-x}\,x\sqrt[3]{1-kx}\,F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};x,kx\right)}{2\sqrt[3]{(1-x)x(1-kx)}}+\frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int}{\sqrt[3]{(1-x)x(1}}\end{aligned}$$

Mathematica [F] time = 1.60, size = 0, normalized size = 0.00

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
```

```
[Out] Integrate[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)), x]
```

IntegrateAlgebraic [A] time = 0.25, size = 161, normalized size = 1.00

$$\frac{\log\left(k^{2/3}x^2+\sqrt[3]{k}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2\sqrt[3]{k}}-\frac{\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[3]{k}\,x\right)}{\sqrt[3]{k}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{k}\,x}{2\sqrt[3]{kx^3+(-k-1)x^2+x}+\sqrt[3]{k}\,x}\right)}{\sqrt[3]{k}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k
)*x)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*k^(1/3)*x)/(k^(1/3)*x + 2*(x + (-1 - k)*x^2 + k*x^
3)^(1/3))])/k^(1/3) - Log[-(k^(1/3)*x) + (x + (-1 - k)*x^2 + k*x^3)^(1/3)]/
k^(1/3) + Log[k^(2/3)*x^2 + k^(1/3)*x*(x + (-1 - k)*x^2 + k*x^3)^(1/3) + (x
 + (-1 - k)*x^2 + k*x^3)^(2/3)]/(2*k^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(k+1)x-2}{((kx-1)(x-1)x)^{\frac{1}{3}}\,((k+1)x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="gi
ac")
```

```
[Out] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{2-(1+k)\,x}{((1-x)\,x\,(-kx+1))^{\frac{1}{3}}\,(1-(1+k)\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x)
```

```
[Out] int((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(k+1)x-2}{((kx-1)(x-1)x)^{\frac{1}{3}}\,((k+1)x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="ma
xima")
```

```
[Out] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(k+1)-2}{(x\,(k+1)-1)\,{(x\,(k\,x-1)\,(x-1))}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(k + 1) - 2)/((x*(k + 1) - 1)*(x*(k*x - 1)*(x - 1))^(1/3)),x)
```

```
[Out] int((x*(k + 1) - 2)/((x*(k + 1) - 1)*(x*(k*x - 1)*(x - 1))^(1/3)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx + x - 2}{\sqrt[3]{x(x-1)(kx-1)}\,(kx + x - 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(1-(1+k)*x),x)
[Out] Integral((k*x + x - 2)/((x*(x - 1)*(k*x - 1))**(1/3)*(k*x + x - 1)), x)
```

3.1796 $\int \frac{-2+(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$

Optimal. Leaf size=161

$$-\frac{\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-$$

Rubi [F] time = 3.49, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2+(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2 + (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x + (-b + k)
*x^2)),x]
```

```
[Out] ((1 + Sqrt[4*b + (-1 + k)^2] + k)*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Def
er[Int][1/((1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*(-1 - k - Sqrt[1 + 4*b - 2
*k + k^2] + 2*(-b + k)*x)), x])/((1 - x)*x*(1 - k*x))^(1/3) + ((1 - Sqrt[4*
b + (-1 + k)^2] + k)*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Int][1/((1
 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*(-1 - k + Sqrt[1 + 4*b - 2*k + k^2] + 2
*(-b + k)*x)), x])/((1 - x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\int \frac{-2+(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx=\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{-2+(1+k)x}{\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$=\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int\left(\frac{1+k+\sqrt{1+4b-2k+k^2}}{\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\left(-1-k-\sqrt{1+4b-2k+k^2}+2\right.}\right.}{\sqrt[3]{(1-x)x(}}$$

$$=\frac{\left(\left(1-\sqrt{4b+(-1+k)^2}+k\right)\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{}{\sqrt[3]{1-x}\sqrt[3]{}}}{\sqrt[3]{(1-x)x(1-kx)}}$$

Mathematica [F] time = 6.57, size = 0, normalized size = 0.00

$$\int \frac{-2+(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2 + (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x + (-
b + k)*x^2)),x]
```

```
[Out] Integrate[(-2 + (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x + (-
b + k)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.29, size = 161, normalized size = 1.00

$$-\frac{\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 +
k)*x + (-b + k)*x^2)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(x + (-1 - k)*x^2 + k*
x^3)^(1/3))])/b^(1/3)) + Log[-(b^(1/3)*x) + (x + (-1 - k)*x^2 + k*x^3)^(1/3
)]/b^(1/3) - Log[b^(2/3)*x^2 + b^(1/3)*x*(x + (-1 - k)*x^2 + k*x^3)^(1/3) +
 (x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x+(-b+k)*x^2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.30, size = 121, normalized size = 0.75

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+2\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{1}{3}}+\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{2}{3}}\right)}{2b^{\frac{1}{3}}}+\frac{\log\left(\left|-b^{\frac{1}{3}}+\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{1}{3}}\right|\right)}{b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x+(-b+k)*x^2),x, a
lgorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(k - k/x - 1/x + 1/x^2)^(1/3))/b^(1
/3))/b^(1/3) - 1/2*log(b^(2/3) + b^(1/3)*(k - k/x - 1/x + 1/x^2)^(1/3) + (k
 - k/x - 1/x + 1/x^2)^(2/3))/b^(1/3) + log(abs(-b^(1/3) + (k - k/x - 1/x +
1/x^2)^(1/3)))/b^(1/3)
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{-2+(1+k)\,x}{\left((1-x)\,x\,(-kx+1)\right)^{\frac{1}{3}}\left(1-(1+k)\,x+(-b+k)\,x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x+(-b+k)*x^2),x)
```

```
[Out] int((-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x+(-b+k)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(k+1)x-2}{\left((kx-1)(x-1)x\right)^{\frac{1}{3}}\left((b-k)x^{2}+(k+1)x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x+(-b+k)*x^2),x, a
lgorithm="maxima")
```

```
[Out] -integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((b - k)*x^2 + (k +
 1)*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x\,(k+1)-2}{(x\,(k\,x-1)\,(x-1))^{1/3}\,\left((b-k)\,x^2+(k+1)\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(k + 1) - 2)/((x*(k*x - 1)*(x - 1))^(1/3)*(x*(k + 1) + x^2*(b - k)
- 1)),x)
```

```
[Out] int(-(x*(k + 1) - 2)/((x*(k*x - 1)*(x - 1))^(1/3)*(x*(k + 1) + x^2*(b - k)
- 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(1-(1+k)*x+(-b+k)*x**2),x)
```

```
[Out] Timed out
```

3.1797 $\int \frac{(3+2x^2)(1+2x^2+2x^3)^{2/3}}{x^3(-1-2x^2+x^3)}\,dx$

Optimal. Leaf size=161

$$\frac{3\left(2x^3+2x^2+1\right)^{2/3}}{2x^2}+3^{2/3}\log\left(3^{2/3}\sqrt[3]{2x^3+2x^2+1}-3x\right)-\frac{1}{2}3^{2/3}\log\left(3x^2+3^{2/3}\sqrt[3]{2x^3+2x^2+1}\,x+\sqrt[3]{3}\left(2x^3+\right.\right.$$

Rubi [F] time = 2.59, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(3+2x^2\right)\left(1+2x^2+2x^3\right)^{2/3}}{x^3\left(-1-2x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((3 + 2*x^2)*(1 + 2*x^2 + 2*x^3)^(2/3))/(x^3*(-1 - 2*x^2 + x^3)),x]
```

```
[Out] 3*Defer[Int][(1 + 2*x^2 + 2*x^3)^(2/3)/(-1 - 2*x^2 + x^3), x] + 8*Defer[Int
][(x*(1 + 2*x^2 + 2*x^3)^(2/3))/(-1 - 2*x^2 + x^3), x] - 4*Defer[Int][(x^2*
(1 + 2*x^2 + 2*x^3)^(2/3))/(-1 - 2*x^2 + x^3), x] - (27*2^(2/3)*(1 + 2*x^2
+ 2*x^3)^(2/3)*Defer[Subst][Defer[Int][((((31 + 3*Sqrt[105])^(1/3)*(2*2^(1/
3) + (31 - 3*Sqrt[105])^(2/3)))/6 + 2*x)^(2/3)*((-4 + (62 - 6*Sqrt[105])^(2
/3) + (8*2^(1/3))/(31 - 3*Sqrt[105])^(2/3))/9 - (2*((62 - 6*Sqrt[105])^(1/3
) + (2*2^(2/3))/(31 - 3*Sqrt[105])^(1/3))*x)/3 + 4*x^2)^(2/3))/(-1/3 + x)^3
, x], x, 1/3 + x])/((4 + (31 + 3*Sqrt[105])^(1/3)*(2*2^(1/3) + (31 - 3*Sqrt
[105])^(2/3)) + 12*x)^(2/3)*(-4 + (62 - 6*Sqrt[105])^(2/3) + (8*2^(1/3))/(3
1 - 3*Sqrt[105])^(2/3) - 2*((62 - 6*Sqrt[105])^(1/3) + (2*2^(2/3))/(31 - 3*
Sqrt[105])^(1/3))*(1 + 3*x) + 4*(1 + 3*x)^2)^(2/3)) + (36*2^(2/3)*(1 + 2*x^
2 + 2*x^3)^(2/3)*Defer[Subst][Defer[Int][((((31 + 3*Sqrt[105])^(1/3)*(2*2^(
1/3) + (31 - 3*Sqrt[105])^(2/3)))/6 + 2*x)^(2/3)*((-4 + (62 - 6*Sqrt[105])^
(2/3) + (8*2^(1/3))/(31 - 3*Sqrt[105])^(2/3))/9 - (2*((62 - 6*Sqrt[105])^(1
/3) + (2*2^(2/3))/(31 - 3*Sqrt[105])^(1/3))*x)/3 + 4*x^2)^(2/3))/(-1/3 + x)
, x], x, 1/3 + x])/((4 + (31 + 3*Sqrt[105])^(1/3)*(2*2^(1/3) + (31 - 3*Sqrt
[105])^(2/3)) + 12*x)^(2/3)*(-4 + (62 - 6*Sqrt[105])^(2/3) + (8*2^(1/3))/(3
1 - 3*Sqrt[105])^(2/3) - 2*((62 - 6*Sqrt[105])^(1/3) + (2*2^(2/3))/(31 - 3*
Sqrt[105])^(1/3))*(1 + 3*x) + 4*(1 + 3*x)^2)^(2/3))
```

Rubi steps

$$\int \frac{\left(3+2x^2\right)\left(1+2x^2+2x^3\right)^{2/3}}{x^3\left(-1-2x^2+x^3\right)}\,dx = \int \left(-\frac{3\left(1+2x^2+2x^3\right)^{2/3}}{x^3}+\frac{4\left(1+2x^2+2x^3\right)^{2/3}}{x}+\frac{\left(3+8x-4x^2\right)\left(1+2x^2\right.}{-1-2x^2+x^3}\right.$$

$$= -\left(3\int \frac{\left(1+2x^2+2x^3\right)^{2/3}}{x^3}\,dx\right)+4\int \frac{\left(1+2x^2+2x^3\right)^{2/3}}{x}\,dx+\int \frac{(3+8x-}{-}$$

$$= -\left(3\,\mathrm{Subst}\left(\int \frac{\left(\frac{31}{27}-\frac{2x}{3}+2x^3\right)^{2/3}}{\left(-\frac{1}{3}+x\right)^3}\,dx, x, \frac{1}{3}+x\right)\right)+4\,\mathrm{Subst}\left(\int \frac{\left(\frac{31}{27}-\frac{2x}{3}+2\right.}{-\frac{1}{3}+x}\right.$$

$$= 3\int \frac{\left(1+2x^2+2x^3\right)^{2/3}}{-1-2x^2+x^3}\,dx-4\int \frac{x^2\left(1+2x^2+2x^3\right)^{2/3}}{-1-2x^2+x^3}\,dx+8\int \frac{x\left(1+2x^2\right.}{-1-2x}$$

Mathematica [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\left(3+2x^2\right)\left(1+2x^2+2x^3\right)^{2/3}}{x^3\left(-1-2x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((3 + 2*x^2)*(1 + 2*x^2 + 2*x^3)^(2/3))/(x^3*(-1 - 2*x^2 + x^3)),
x]
```

```
[Out] Integrate[((3 + 2*x^2)*(1 + 2*x^2 + 2*x^3)^(2/3))/(x^3*(-1 - 2*x^2 + x^3)),
 x]
```

IntegrateAlgebraic [A] time = 0.44, size = 161, normalized size = 1.00

$$\frac{3\left(2x^3+2x^2+1\right)^{2/3}}{2x^2}+3^{2/3}\log\left(3^{2/3}\sqrt[3]{2x^3+2x^2+1}-3x\right)-\frac{1}{2}3^{2/3}\log\left(3x^2+3^{2/3}\sqrt[3]{2x^3+2x^2+1}\,x+\sqrt[3]{3}\left(2x^3+2x^2+1\right)^{2/3}\right)-3\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2x^3+2x^2+1}+\sqrt[3]{3}\,x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((3 + 2*x^2)*(1 + 2*x^2 + 2*x^3)^(2/3))/(x^3*(-1 - 2*x^2
 + x^3)),x]
```

```
[Out] (3*(1 + 2*x^2 + 2*x^3)^(2/3))/(2*x^2) - 3*3^(1/6)*ArcTan[(3^(5/6)*x)/(3^(1/
3)*x + 2*(1 + 2*x^2 + 2*x^3)^(1/3))] + 3^(2/3)*Log[-3*x + 3^(2/3)*(1 + 2*x^
2 + 2*x^3)^(1/3)] - (3^(2/3)*Log[3*x^2 + 3^(2/3)*x*(1 + 2*x^2 + 2*x^3)^(1/3
) + 3^(1/3)*(1 + 2*x^2 + 2*x^3)^(2/3)])/2
```

fricas [B] time = 18.75, size = 438, normalized size = 2.72

$$-\frac{2\cdot 9^{\frac{1}{3}}\sqrt{3}x^2\arctan\left(\frac{2\cdot 9^{\frac{2}{3}}\sqrt{3}\left(8x^7-14x^6-4x^5-7x^4-4x^3-x\right)\left(2x^3+2x^2+1\right)^{\frac{2}{3}}-6\cdot 9^{\frac{1}{3}}\sqrt{3}\left(55x^8+50x^7+4x^6+25x^5+4x^4+x^2\right)\left(2x^3+2x^2+1\right)^{\frac{1}{3}}-\sqrt{3}\left(377x^9+600x^8+204x^7+308x^6+204x^5+12x^4+51x^3+6x^2+1\right)}{3\left(487x^9+480x^8+12x^7+232x^6+12x^5-12x^4+3x^3-6x^2-1\right)}\right)-2\cdot 9^{\frac{1}{3}}x^2\log\left(\frac{3\cdot 9^{\frac{2}{3}}\left(2x^3+2x^2+1\right)^{\frac{1}{3}}x^2-9\left(2x^3+2x^2+1\right)^{\frac{2}{3}}x-9^{\frac{1}{3}}\left(x^3-2x^2-1\right)}{x^3-2x^2-1}\right)+9^{\frac{1}{3}}x^2\log\left(\frac{9\cdot 9^{\frac{1}{3}}\left(8x^4+2x^3+x\right)\left(2x^3+2x^2+1\right)^{\frac{2}{3}}+9^{\frac{2}{3}}\left(55x^6+50x^5+4x^4+25x^3+4x^2+1\right)+27\left(7x^5+4x^4+2x^2\right)\left(2x^3+2x^2+1\right)^{\frac{1}{3}}}{x^6-4x^5+4x^4-2x^3+4x^2+1}\right)-9\left(2x^3+2x^2+1\right)^{\frac{2}{3}}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)*(2*x^3+2*x^2+1)^(2/3)/x^3/(x^3-2*x^2-1),x, algorithm="f
ricas")
```

```
[Out] -1/6*(2*9^(1/3)*sqrt(3)*x^2*arctan(1/3*(2*9^(2/3)*sqrt(3)*(8*x^7 - 14*x^6 -
 4*x^5 - 7*x^4 - 4*x^3 - x)*(2*x^3 + 2*x^2 + 1)^(2/3) - 6*9^(1/3)*sqrt(3)*(
```

```
55*x^8 + 50*x^7 + 4*x^6 + 25*x^5 + 4*x^4 + x^2)*(2*x^3 + 2*x^2 + 1)^(1/3) -
 sqrt(3)*(377*x^9 + 600*x^8 + 204*x^7 + 308*x^6 + 204*x^5 + 12*x^4 + 51*x^3
 + 6*x^2 + 1))/(487*x^9 + 480*x^8 + 12*x^7 + 232*x^6 + 12*x^5 - 12*x^4 + 3*
x^3 - 6*x^2 - 1)) - 2*9^(1/3)*x^2*log((3*9^(2/3)*(2*x^3 + 2*x^2 + 1)^(1/3)*
x^2 - 9*(2*x^3 + 2*x^2 + 1)^(2/3)*x - 9^(1/3)*(x^3 - 2*x^2 - 1))/(x^3 - 2*x
^2 - 1)) + 9^(1/3)*x^2*log((9*9^(1/3)*(8*x^4 + 2*x^3 + x)*(2*x^3 + 2*x^2 +
1)^(2/3) + 9^(2/3)*(55*x^6 + 50*x^5 + 4*x^4 + 25*x^3 + 4*x^2 + 1) + 27*(7*x
^5 + 4*x^4 + 2*x^2)*(2*x^3 + 2*x^2 + 1)^(1/3))/(x^6 - 4*x^5 + 4*x^4 - 2*x^3
 + 4*x^2 + 1)) - 9*(2*x^3 + 2*x^2 + 1)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 x^3+2 x^2+1\right)^{\frac{2}{3}}\left(2 x^2+3\right)}{\left(x^3-2 x^2-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)*(2*x^3+2*x^2+1)^(2/3)/x^3/(x^3-2*x^2-1),x, algorithm="g
iac")
```

```
[Out] integrate((2*x^3 + 2*x^2 + 1)^(2/3)*(2*x^2 + 3)/((x^3 - 2*x^2 - 1)*x^3), x)
```

maple [C] time = 4.82, size = 1001, normalized size = 6.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+3)*(2*x^3+2*x^2+1)^(2/3)/x^3/(x^3-2*x^2-1),x)
```

```
[Out] 3/2*(2*x^3+2*x^2+1)^(2/3)/x^2+RootOf(_Z^3-9)*ln((2*RootOf(RootOf(_Z^3-9)^2+
3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^3*x^3+15*RootOf(RootOf(_Z^3-9)^2
+3*_Z*RootOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+7*(2*x^3+2*x^2+1)^(2/3)
*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x+8*(
2*x^3+2*x^2+1)^(1/3)*RootOf(_Z^3-9)^2*x^2+3*(2*x^3+2*x^2+1)^(1/3)*RootOf(_Z
^3-9)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x^2-4*RootOf(_Z^3
-9)*x^3-30*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x^3-4*RootOf
(_Z^3-9)*x^2-30*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x^2-3*(
2*x^3+2*x^2+1)^(2/3)*x-2*RootOf(_Z^3-9)-15*RootOf(RootOf(_Z^3-9)^2+3*_Z*Roo
tOf(_Z^3-9)+9*_Z^2))/(x^3-2*x^2-1))-ln(-(RootOf(RootOf(_Z^3-9)^2+3*_Z*RootO
f(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^3*x^3+6*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootO
f(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2*x^3-3*(2*x^3+2*x^2+1)^(1/3)*RootOf(_Z^
3-9)^2*x^2-9*(2*x^3+2*x^2+1)^(1/3)*RootOf(_Z^3-9)*RootOf(RootOf(_Z^3-9)^2+3
*_Z*RootOf(_Z^3-9)+9*_Z^2)*x^2+5*RootOf(_Z^3-9)*x^3+30*RootOf(RootOf(_Z^3-9
)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x^3+2*RootOf(_Z^3-9)*x^2+12*RootOf(RootOf(_
Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x^2+9*(2*x^3+2*x^2+1)^(2/3)*x+RootOf(_
Z^3-9)+6*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2))/(x^3-2*x^2-1)
)*RootOf(_Z^3-9)-3*ln(-(RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)
*RootOf(_Z^3-9)^3*x^3+6*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)
^2*RootOf(_Z^3-9)^2*x^3-3*(2*x^3+2*x^2+1)^(1/3)*RootOf(_Z^3-9)^2*x^2-9*(2*x
^3+2*x^2+1)^(1/3)*RootOf(_Z^3-9)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9
)+9*_Z^2)*x^2+5*RootOf(_Z^3-9)*x^3+30*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_
Z^3-9)+9*_Z^2)*x^3+2*RootOf(_Z^3-9)*x^2+12*RootOf(RootOf(_Z^3-9)^2+3*_Z*Roo
tOf(_Z^3-9)+9*_Z^2)*x^2+9*(2*x^3+2*x^2+1)^(2/3)*x+RootOf(_Z^3-9)+6*RootOf(R
ootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2))/(x^3-2*x^2-1))*RootOf(RootOf(_
Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 x^3+2 x^2+1\right)^{\frac{2}{3}}\left(2 x^2+3\right)}{\left(x^3-2 x^2-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)*(2*x^3+2*x^2+1)^(2/3)/x^3/(x^3-2*x^2-1),x, algorithm="m
axima")
```

```
[Out] integrate((2*x^3 + 2*x^2 + 1)^(2/3)*(2*x^2 + 3)/((x^3 - 2*x^2 - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(2\,x^2+3\right)\,{\left(2\,x^3+2\,x^2+1\right)}^{2/3}}{x^3\,\left(-x^3+2\,x^2+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*x^2 + 3)*(2*x^2 + 2*x^3 + 1)^(2/3))/(x^3*(2*x^2 - x^3 + 1)),x)
```

```
[Out] int(-((2*x^2 + 3)*(2*x^2 + 2*x^3 + 1)^(2/3))/(x^3*(2*x^2 - x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{2} + 3\right)\left(2x^{3} + 2x^{2} + 1\right)^{\frac{2}{3}}}{x^{3}\left(x^{3} - 2x^{2} - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+3)*(2*x**3+2*x**2+1)**(2/3)/x**3/(x**3-2*x**2-1),x)
```

```
[Out] Integral((2*x**2 + 3)*(2*x**3 + 2*x**2 + 1)**(2/3)/(x**3*(x**3 - 2*x**2 - 1
)), x)
```

3.1798 $\int \frac{-b+cx+ax^2}{\left(b+ax^2\right)\sqrt{-bx+ax^3}}\,dx$

Optimal. Leaf size=161

$$\frac{\left(-2\sqrt{a}\sqrt{b}-c\right)\tan^{-1}\left(\frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3-bx}}{-2\sqrt{a}\sqrt{b}x+ax^2-b}\right)}{4a^{3/4}b^{3/4}}+\frac{\left(c-2\sqrt{a}\sqrt{b}\right)\tanh^{-1}\left(\frac{2\sqrt{a}\sqrt{b}x+ax^2-b}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3-bx}}\right)}{4a^{3/4}b^{3/4}}$$

Rubi [C] time = 1.70, antiderivative size = 257, normalized size of antiderivative = 1.60, number of steps used = 14, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2056, 6725, 329, 224, 221, 933, 168, 537}

$$-\frac{\sqrt{x}\left(2a\sqrt{b}+\sqrt{-a}c\right)\sqrt{1-\frac{ax^2}{b}}\Pi\left(\frac{\sqrt{-a}}{\sqrt{a}};\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{a^{5/4}\sqrt[4]{b}\sqrt{ax^3-bx}}-\frac{\sqrt{x}\left(\frac{c}{\sqrt{-a}}+2\sqrt{b}\right)\sqrt{1-\frac{ax^2}{b}}\Pi\left(\frac{\sqrt{a}}{\sqrt{-a}};\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3-bx}}+\frac{2\sqrt[4]{b}\sqrt{x}\sqrt{1-\frac{ax^2}{b}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{ax^3-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + c*x + a*x^2)/((b + a*x^2)*Sqrt[-(b*x) + a*x^3]),x]
```

```
[Out] (2*b^(1/4)*Sqrt[x]*Sqrt[1 - (a*x^2)/b]*EllipticF[ArcSin[(a^(1/4)*Sqrt[x])/b
^(1/4)], -1])/(a^(1/4)*Sqrt[-(b*x) + a*x^3]) - ((2*a*Sqrt[b] + Sqrt[-a]*c)*
Sqrt[x]*Sqrt[1 - (a*x^2)/b]*EllipticPi[Sqrt[-a]/Sqrt[a], ArcSin[(a^(1/4)*Sq
rt[x])/b^(1/4)], -1])/(a^(5/4)*b^(1/4)*Sqrt[-(b*x) + a*x^3]) - ((2*Sqrt[b]
+ c/Sqrt[-a])*Sqrt[x]*Sqrt[1 - (a*x^2)/b]*EllipticPi[Sqrt[a]/Sqrt[-a], ArcS
in[(a^(1/4)*Sqrt[x])/b^(1/4)], -1])/(a^(1/4)*b^(1/4)*Sqrt[-(b*x) + a*x^3])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
```

```
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[
a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x]
, x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a
*e^2, 0] &&  !GtQ[a, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-b+cx+ax^2}{\left(b+ax^2\right)\sqrt{-bx+ax^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\int \frac{-b+cx+ax^2}{\sqrt{x}\sqrt{-b+ax^2}\left(b+ax^2\right)}\,dx}{\sqrt{-bx+ax^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\int \left(\frac{1}{\sqrt{x}\sqrt{-b+ax^2}} - \frac{2b-cx}{\sqrt{x}\sqrt{-b+ax^2}\left(b+ax^2\right)}\right)dx}{\sqrt{-bx+ax^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\int \frac{1}{\sqrt{x}\sqrt{-b+ax^2}}\,dx}{\sqrt{-bx+ax^3}} - \frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\int \frac{2b-cx}{\sqrt{x}\sqrt{-b+ax^2}\left(b+ax^2\right)}\,dx}{\sqrt{-bx+ax^3}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{-b+ax^2}\right)\int \left(\frac{2b^{3/2}-\frac{bc}{\sqrt{-a}}}{2b\sqrt{x}\left(\sqrt{b}-\sqrt{-a}\,x\right)\sqrt{-b+ax^2}} + \frac{2b^{3/2}+\frac{bc}{\sqrt{-a}}}{2b\sqrt{x}\left(\sqrt{b}+\sqrt{-a}\,x\right)\sqrt{-b+ax^2}}\right)dx}{\sqrt{-bx+ax^3}} + \cdots$$

$$= -\frac{\left(\left(2\sqrt{b}+\frac{c}{\sqrt{-a}}\right)\sqrt{x}\sqrt{-b+ax^2}\right)\int \frac{1}{\sqrt{x}\left(\sqrt{b}+\sqrt{-a}\,x\right)\sqrt{-b+ax^2}}\,dx}{2\sqrt{-bx+ax^3}} - \frac{\left(\left(2b^{3/2}-\frac{bc}{\sqrt{-a}}\right)\cdots\right.}{\cdots}$$

$$= \frac{2\sqrt[4]{b}\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{-bx+ax^3}} - \frac{\left(\left(2\sqrt{b}+\frac{c}{\sqrt{-a}}\right)\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\right)\int \cdots}{2\sqrt{-bx+\cdots}}$$

$$= \frac{2\sqrt[4]{b}\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{-bx+ax^3}} + \frac{\left(\left(2\sqrt{b}+\frac{c}{\sqrt{-a}}\right)\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\right)\text{Subs}\cdots}{\cdots}$$

$$= \frac{2\sqrt[4]{b}\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{-bx+ax^3}} - \frac{\left(2a\sqrt{b}+\sqrt{-a}\,c\right)\sqrt{x}\sqrt{1-\frac{ax^2}{b}}\,\Pi\left(\cdots\right.}{a^{5/4}\sqrt[4]{b}\sqrt{-bx+\cdots}}$$

Mathematica [C] time = 0.60, size = 136, normalized size = 0.84

$$-\frac{2x\sqrt{1-\frac{ax^2}{b}}\left(15bF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{ax^2}{b},-\frac{ax^2}{b}\right)-x\left(5cF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{ax^2}{b},-\frac{ax^2}{b}\right)+3axF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{ax^2}{b},-\frac{ax^2}{b}\right)\right)\right)}{15b\sqrt{ax^3-bx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-b + c*x + a*x^2)/((b + a*x^2)*Sqrt[-(b*x) + a*x^3]),x]
```

```
[Out] (-2*x*Sqrt[1 - (a*x^2)/b]*(15*b*AppellF1[1/4, 1/2, 1, 5/4, (a*x^2)/b, -((a*
x^2)/b)] - x*(5*c*AppellF1[3/4, 1/2, 1, 7/4, (a*x^2)/b, -((a*x^2)/b)] + 3*a
*x*AppellF1[5/4, 1/2, 1, 9/4, (a*x^2)/b, -((a*x^2)/b)])))/(15*b*Sqrt[-(b*x)
 + a*x^3])
```

IntegrateAlgebraic [A] time = 0.52, size = 159, normalized size = 0.99

$$\frac{\left(2\sqrt{a}\sqrt{b}+c\right)\tan^{-1}\left(\frac{-2\sqrt{a}\sqrt{b}\,x+ax^2-b}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3-bx}}\right)}{4a^{3/4}b^{3/4}} + \frac{\left(c-2\sqrt{a}\sqrt{b}\right)\tanh^{-1}\left(\frac{2\sqrt{a}\sqrt{b}\,x+ax^2-b}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3-bx}}\right)}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + c*x + a*x^2)/((b + a*x^2)*Sqrt[-(b*x) + a*x^3]),x]
```

[Out] ((2*Sqrt[a]*Sqrt[b] + c)*ArcTan[(-b - 2*Sqrt[a]*Sqrt[b]*x + a*x^2)/(2*a^(1/4)*b^(1/4)*Sqrt[-(b*x) + a*x^3])])/(4*a^(3/4)*b^(3/4)) + ((-2*Sqrt[a]*Sqrt[b] + c)*ArcTanh[(-b + 2*Sqrt[a]*Sqrt[b]*x + a*x^2)/(2*a^(1/4)*b^(1/4)*Sqrt[-(b*x) + a*x^3])])/(4*a^(3/4)*b^(3/4))

fricas [B] time = 0.86, size = 1593, normalized size = 9.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^2+c*x-b)/(a*x^2+b)/(a*x^3-b*x)^(1/2),x, algorithm="fricas")

[Out] -1/8*sqrt(1/2)*sqrt(-(a*b*sqrt(-(16*a^2*b^2 - 8*a*b*c^2 + c^4)/(a^3*b^3)) + 4*c)/(a*b))*log(-(16*a^2*b^4 - b^2*c^4 + (16*a^4*b^2 - a^2*c^4)*x^4 - 6*(16*a^3*b^3 - a*b*c^4)*x^2 + 4*sqrt(1/2)*(4*a^2*b^3*c - a*b^2*c^3 - (4*a^3*b^2*c - a^2*b*c^3)*x^2 - 4*(4*a^3*b^3 - a^2*b^2*c^2)*x + 2*(a^4*b^3*x^2 - a^3*b^3*c*x - a^3*b^4)*sqrt(-(16*a^2*b^2 - 8*a*b*c^2 + c^4)/(a^3*b^3)))*sqrt(a*x^3 - b*x)*sqrt(-(a*b*sqrt(-(16*a^2*b^2 - 8*a*b*c^2 + c^4)/(a^3*b^3)) + 4*c)/(a*b)) + 4*((4*a^4*b^3 + a^3*b^2*c^2)*x^3 - (4*a^3*b^4 + a^2*b^3*c^2)*x)*sqrt(-(16*a^2*b^2 - 8*a*b*c^2 + c^4)/(a^3*b^3)))/(a^2*x^4 + 2*a*b*x^2 + b^2)) + 1/8*sqrt(1/2)*sqrt(-(a*b*sqrt(-(16*a^2*b^2 - 8*a*b*c^2 + c^4)/(a^3*b^3)) + 4*c)/(a*b))*log(-(16*a^2*b^4 - b^2*c^4 + (16*a^4*b^2 - a^2*c^4)*x^4 - 6*(16*a^3*b^3 - a*b*c^4)*x^2 - 4*sqrt(1/2)*(4*a^2*b^3*c - a*b^2*c^3 - (4*a^3*b^2*c - a^2*b*c^3)*x^2 - 4*(4*a^3*b^3 - a^2*b^2*c^2)*x + 2*(a^4*b^3*x^2 - a^3*b^3*c*x - a^3*b^4)*sqrt(-(16*a^2*b^2 - 8*a*b*c^2 + c^4)/(a^3*b^3)))*sqrt(a*x^3 - b*x)*sqrt(-(a*b*sqrt(-(16*a^2*b^2 - 8*a*b*c^2 + c^4)/(a^3*b^3)) + 4*c)/(a*b)) + 4*((4*a^4*b^3 + a^3*b^2*c^2)*x^3 - (4*a^3*b^4 + a^2*b^3*c^2)*x)*sqrt(-(16*a^2*b^2 - 8*a*b*c^2 + c^4)/(a^3*b^3)))/(a^2*x^4 + 2*a*b*x^2 + b^2)) - 1/8*sqrt(1/2)*sqrt((a*b*sqrt(-(16*a^2*b^2 - 8*a*b*c^2 + c^4)/(a^3*b^3)) - 4*c)/(a*b))*log(-(16*a^2*b^4 - b^2*c^4 + (16*a^4*b^2 - a^2*c^4)*x^4 - 6*(16*a^3*b^3 - a*b*c^4)*x^2 + 4*sqrt(1/2)*(4*a^2*b^3*c - a*b^2*c^3 - (4*a^3*b^2*c - a^2*b*c^3)*x^2 - 4*(4*a^3*b^3 - a^2*b^2*c^2)*x - 2*(a^4*b^3*x^2 - a^3*b^3*c*x - a^3*b^4)*sqrt(-(16*a^2*b^2 - 8*a*b*c^2 + c^4)/(a^3*b^3)))*sqrt(a*x^3 - b*x)*sqrt((a*b*sqrt(-(16*a^2*b^2 - 8*a*b*c^2 + c^4)/(a^3*b^3)) - 4*c)/(a*b)) - 4*((4*a^4*b^3 + a^3*b^2*c^2)*x^3 - (4*a^3*b^4 + a^2*b^3*c^2)*x)*sqrt(-(16*a^2*b^2 - 8*a*b*c^2 + c^4)/(a^3*b^3)))/(a^2*x^4 + 2*a*b*x^2 + b^2)) + 1/8*sqrt(1/2)*sqrt((a*b*sqrt(-(16*a^2*b^2 - 8*a*b*c^2 + c^4)/(a^3*b^3)) - 4*c)/(a*b))*log(-(16*a^2*b^4 - b^2*c^4 + (16*a^4*b^2 - a^2*c^4)*x^4 - 6*(16*a^3*b^3 - a*b*c^4)*x^2 - 4*sqrt(1/2)*(4*a^2*b^3*c - a*b^2*c^3 - (4*a^3*b^2*c - a^2*b*c^3)*x^2 - 4*(4*a^3*b^3 - a^2*b^2*c^2)*x - 2*(a^4*b^3*x^2 - a^3*b^3*c*x - a^3*b^4)*sqrt(-(16*a^2*b^2 - 8*a*b*c^2 + c^4)/(a^3*b^3)))*sqrt(a*x^3 - b*x)*sqrt((a*b*sqrt(-(16*a^2*b^2 - 8*a*b*c^2 + c^4)/(a^3*b^3)) - 4*c)/(a*b)) - 4*((4*a^4*b^3 + a^3*b^2*c^2)*x^3 - (4*a^3*b^4 + a^2*b^3*c^2)*x)*sqrt(-(16*a^2*b^2 - 8*a*b*c^2 + c^4)/(a^3*b^3)))/(a^2*x^4 + 2*a*b*x^2 + b^2))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + cx - b}{\sqrt{ax^3 - bx}\,(ax^2 + b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^2+c*x-b)/(a*x^2+b)/(a*x^3-b*x)^(1/2),x, algorithm="giac")

[Out] integrate((a*x^2 + c*x - b)/(sqrt(a*x^3 - b*x)*(a*x^2 + b)), x)

maple [C] time = 0.06, size = 681, normalized size = 4.23

$$\frac{\sqrt{ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}}\,\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a\,x^3-bx}}+\frac{\sqrt{ab}\,\sqrt{\frac{xa}{\sqrt{ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},-\frac{\sqrt{ab}}{a\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)c}{2a^2\sqrt{a\,x^3-bx}\,\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)}-\frac{\sqrt{ab}\,\sqrt{\frac{xa}{\sqrt{ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},-\frac{\sqrt{ab}}{a\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)b}{\sqrt{-ab}\,a\sqrt{a\,x^3-bx}\,\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)}+\frac{\sqrt{ab}\,\sqrt{\frac{xa}{\sqrt{ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},-\frac{\sqrt{ab}}{a\left(-\frac{\sqrt{ab}}{a}+\frac{\sqrt{-ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)c}{2a^2\sqrt{a\,x^3-bx}\,\left(-\frac{\sqrt{ab}}{a}+\frac{\sqrt{-ab}}{a}\right)}+\frac{\sqrt{ab}\,\sqrt{\frac{xa}{\sqrt{ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{ab}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{a}\right)a}{\sqrt{ab}}},-\frac{\sqrt{ab}}{a\left(-\frac{\sqrt{ab}}{a}+\frac{\sqrt{-ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)b}{\sqrt{-ab}\,a\sqrt{a\,x^3-bx}\,\left(-\frac{\sqrt{ab}}{a}+\frac{\sqrt{-ab}}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+c*x-b)/(a*x^2+b)/(a*x^3-b*x)^(1/2),x)
```

```
[Out] 1/a*(a*b)^(1/2)*((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2)*(-2*(x-1/a*(a*b)^
(1/2))*a/(a*b)^(1/2))^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)*Elli
pticF(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/2/a^2*(a*b)^
(1/2)*(x*a/(a*b)^(1/2)+1)^(1/2)*(-2*x*a/(a*b)^(1/2)+2)^(1/2)*(-x*a/(a*b)^(1
/2))^(1/2)/(a*x^3-b*x)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2))*EllipticPi
(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2),-1/a*(a*b)^(1/2)/(-1/a*(-a*b)^(1
/2)-1/a*(a*b)^(1/2)),1/2*2^(1/2))*c-1/(-a*b)^(1/2)/a*(a*b)^(1/2)*(x*a/(a*b)
^(1/2)+1)^(1/2)*(-2*x*a/(a*b)^(1/2)+2)^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^
3-b*x)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2))*EllipticPi(((x+1/a*(a*b)^(
1/2))*a/(a*b)^(1/2))^(1/2),-1/a*(a*b)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1
/2)),1/2*2^(1/2))*b+1/2/a^2*(a*b)^(1/2)*(x*a/(a*b)^(1/2)+1)^(1/2)*(-2*x*a/(
a*b)^(1/2)+2)^(1/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)/(-1/a*(a*b)^
(1/2)+1/a*(-a*b)^(1/2))*EllipticPi(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2
),-1/a*(a*b)^(1/2)/(-1/a*(a*b)^(1/2)+1/a*(-a*b)^(1/2)),1/2*2^(1/2))*c+1/(-a
*b)^(1/2)/a*(a*b)^(1/2)*(x*a/(a*b)^(1/2)+1)^(1/2)*(-2*x*a/(a*b)^(1/2)+2)^(1
/2)*(-x*a/(a*b)^(1/2))^(1/2)/(a*x^3-b*x)^(1/2)/(-1/a*(a*b)^(1/2)+1/a*(-a*b)
^(1/2))*EllipticPi(((x+1/a*(a*b)^(1/2))*a/(a*b)^(1/2))^(1/2),-1/a*(a*b)^(1/
2)/(-1/a*(a*b)^(1/2)+1/a*(-a*b)^(1/2)),1/2*2^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + cx - b}{\sqrt{ax^3 - bx}\,(ax^2 + b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+c*x-b)/(a*x^2+b)/(a*x^3-b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 + c*x - b)/(sqrt(a*x^3 - b*x)*(a*x^2 + b)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x - b + a*x^2)/((a*x^3 - b*x)^(1/2)*(b + a*x^2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - b + cx}{\sqrt{x\left(ax^2 - b\right)}\,\left(ax^2 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+c*x-b)/(a*x**2+b)/(a*x**3-b*x)**(1/2),x)
```

```
[Out] Integral((a*x**2 - b + c*x)/(sqrt(x*(a*x**2 - b))*(a*x**2 + b)), x)
```

3.1799 $\int \frac{-4-2x+2x^2+x^4}{x(-2+x^2)\sqrt[4]{\frac{2+x^2}{-2+x^2}}(8-10x+4x^2+4x^3-4x^4+x^5)}\,dx$

Optimal. Leaf size=161

$$\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt{2}}+\frac{\sqrt{\frac{x^2+2}{x^2-2}}}{\sqrt{2}}-\sqrt{2}\,x+\frac{1}{\sqrt{2}}}{(x-1)\sqrt[4]{\frac{x^2+2}{x^2-2}}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{-\frac{x^2}{\sqrt{2}}+\frac{\sqrt{\frac{x^2+2}{x^2-2}}}{\sqrt{2}}+\sqrt{2}\,x-\frac{1}{\sqrt{2}}}{(x-1)\sqrt[4]{\frac{x^2+2}{x^2-2}}}\right)}{2\sqrt{2}}$$

Rubi [F] time = 6.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-4-2x+2x^2+x^4}{x\left(-2+x^2\right)\sqrt[4]{\frac{2+x^2}{-2+x^2}}\left(8-10x+4x^2+4x^3-4x^4+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-4 - 2*x + 2*x^2 + x^4)/(x*(-2 + x^2)*((2 + x^2)/(-2 + x^2))^(1/4)*(8
 - 10*x + 4*x^2 + 4*x^3 - 4*x^4 + x^5)),x]
```

```
[Out] -1/4*((2 + x^2)^(1/4)*ArcTan[1 - (Sqrt[2]*(2 + x^2)^(1/4))/(-2 + x^2)^(1/4)
])/(Sqrt[2]*(-2 + x^2)^(1/4)*(-((2 + x^2)/(2 - x^2)))^(1/4)) + ((2 + x^2)^(
1/4)*ArcTan[1 + (Sqrt[2]*(2 + x^2)^(1/4))/(-2 + x^2)^(1/4)])/(4*Sqrt[2]*(-2
 + x^2)^(1/4)*(-((2 + x^2)/(2 - x^2)))^(1/4)) + ((2 + x^2)^(1/4)*Log[1 - (S
qrt[2]*(2 + x^2)^(1/4))/(-2 + x^2)^(1/4) + Sqrt[2 + x^2]/Sqrt[-2 + x^2]])/(
8*Sqrt[2]*(-2 + x^2)^(1/4)*(-((2 + x^2)/(2 - x^2)))^(1/4)) - ((2 + x^2)^(1/
4)*Log[1 + (Sqrt[2]*(2 + x^2)^(1/4))/(-2 + x^2)^(1/4) + Sqrt[2 + x^2]/Sqrt[
-2 + x^2]])/(8*Sqrt[2]*(-2 + x^2)^(1/4)*(-((2 + x^2)/(2 - x^2)))^(1/4)) - (
7*(2 + x^2)^(1/4)*Defer[Int][1/((-2 + x^2)^(3/4)*(2 + x^2)^(1/4)*(8 - 10*x
+ 4*x^2 + 4*x^3 - 4*x^4 + x^5)), x])/((-2 + x^2)^(1/4)*(-((2 + x^2)/(2 - x^
2)))^(1/4)) + (4*(2 + x^2)^(1/4)*Defer[Int][x/((-2 + x^2)^(3/4)*(2 + x^2)^(
1/4)*(8 - 10*x + 4*x^2 + 4*x^3 - 4*x^4 + x^5)), x])/((-2 + x^2)^(1/4)*(-((2
 + x^2)/(2 - x^2)))^(1/4)) + (2*(2 + x^2)^(1/4)*Defer[Int][x^2/((-2 + x^2)^
(3/4)*(2 + x^2)^(1/4)*(8 - 10*x + 4*x^2 + 4*x^3 - 4*x^4 + x^5)), x])/((-2 +
 x^2)^(1/4)*(-((2 + x^2)/(2 - x^2)))^(1/4)) - ((2 + x^2)^(1/4)*Defer[Int][x
^3/((-2 + x^2)^(3/4)*(2 + x^2)^(1/4)*(8 - 10*x + 4*x^2 + 4*x^3 - 4*x^4 + x^
5)), x])/((-2 + x^2)^(1/4)*(-((2 + x^2)/(2 - x^2)))^(1/4)) + ((2 + x^2)^(1/
4)*Defer[Int][x^4/((-2 + x^2)^(3/4)*(2 + x^2)^(1/4)*(8 - 10*x + 4*x^2 + 4*x
^3 - 4*x^4 + x^5)), x])/(2*(-2 + x^2)^(1/4)*(-((2 + x^2)/(2 - x^2)))^(1/4))
```

Rubi steps

$$\int \frac{-4-2x+2x^2+x^4}{x\left(-2+x^2\right)\sqrt[4]{\frac{2+x^2}{-2+x^2}}\left(8-10x+4x^2+4x^3-4x^4+x^5\right)}\,dx = \frac{\sqrt[4]{2+x^2}\int \frac{-4-2x+2x^2+x^4}{x\left(-2+x^2\right)^{3/4}\sqrt[4]{2+x^2}\left(8-10x+4x^2+4x^3-4x^4+x^5\right.}}{\sqrt[4]{-2+x^2}\sqrt[4]{\frac{2+x^2}{-2+x^2}}}$$

$$= \frac{\sqrt[4]{2+x^2}\int\left(-\frac{1}{2x\left(-2+x^2\right)^{3/4}\sqrt[4]{2+x^2}}+\frac{-14}{2\left(-2+x^2\right)^{3/4}\sqrt[4]{2+}}\right.}{\sqrt[4]{-2+x^2}\sqrt[4]{\frac{2+x^2}{-2+x^2}}}$$

$$= -\frac{\sqrt[4]{2+x^2}\int \frac{1}{x\left(-2+x^2\right)^{3/4}\sqrt[4]{2+x^2}}\,dx}{2\sqrt[4]{-2+x^2}\sqrt[4]{\frac{2+x^2}{-2+x^2}}} + \frac{\sqrt[4]{2+x^2}\int}{}$$

$$= -\frac{\sqrt[4]{2+x^2}\,\text{Subst}\left(\int \frac{1}{(-2+x)^{3/4}x\sqrt[4]{2+x}}\,dx, x, x^2\right)}{4\sqrt[4]{-2+x^2}\sqrt[4]{\frac{2+x^2}{-2+x^2}}} +$$

$$= \frac{\sqrt[4]{2+x^2}\int \frac{x^4}{\left(-2+x^2\right)^{3/4}\sqrt[4]{2+x^2}\left(8-10x+4x^2+4x^3-4x^4+x^5\right)}}{2\sqrt[4]{-2+x^2}\sqrt[4]{\frac{2+x^2}{-2+x^2}}}$$

$$= \frac{\sqrt[4]{2+x^2}\int \frac{x^4}{\left(-2+x^2\right)^{3/4}\sqrt[4]{2+x^2}\left(8-10x+4x^2+4x^3-4x^4+x^5\right)}}{2\sqrt[4]{-2+x^2}\sqrt[4]{\frac{2+x^2}{-2+x^2}}}$$

$$= \frac{\sqrt[4]{2+x^2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{2+x^2}}{\sqrt[4]{-2+x^2}}\right)}{8\sqrt[4]{-2+x^2}\sqrt[4]{\frac{2+x^2}{-2+x^2}}} + $$

$$= \frac{\sqrt[4]{2+x^2}\log\left(1-\frac{\sqrt{2}\sqrt[4]{2+x^2}}{\sqrt[4]{-2+x^2}}+\frac{\sqrt{2+x^2}}{\sqrt{-2+x^2}}\right)}{8\sqrt{2}\sqrt[4]{-2+x^2}\sqrt[4]{-\frac{2+x^2}{2-x^2}}} - \frac{\sqrt[4]{2+}}{}$$

$$= -\frac{\sqrt[4]{2+x^2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2+x^2}}{\sqrt[4]{-2+x^2}}\right)}{4\sqrt{2}\sqrt[4]{-2+x^2}\sqrt[4]{-\frac{2+x^2}{2-x^2}}} + \frac{\sqrt[4]{2+x^2}\tan}{4\sqrt{2}\sqrt[4]{-}}$$

Mathematica [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{-4-2x+2x^2+x^4}{x\left(-2+x^2\right)\sqrt[4]{\frac{2+x^2}{-2+x^2}}\left(8-10x+4x^2+4x^3-4x^4+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-4 - 2*x + 2*x^2 + x^4)/(x*(-2 + x^2)*((2 + x^2)/(-2 + x^2))^(1/
4)*(8 - 10*x + 4*x^2 + 4*x^3 - 4*x^4 + x^5)),x]
```

```
[Out] Integrate[(-4 - 2*x + 2*x^2 + x^4)/(x*(-2 + x^2)*((2 + x^2)/(-2 + x^2))^(1/
4)*(8 - 10*x + 4*x^2 + 4*x^3 - 4*x^4 + x^5)), x]
```

IntegrateAlgebraic [A] time = 0.52, size = 161, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt{2}}+\frac{\sqrt{\frac{x^2+2}{x^2-2}}}{\sqrt{2}}-\sqrt{2}\,x+\frac{1}{\sqrt{2}}}{(x-1)\sqrt[4]{\frac{x^2+2}{x^2-2}}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{-\frac{x^2}{\sqrt{2}}+\frac{\sqrt{\frac{x^2+2}{x^2-2}}}{\sqrt{2}}+\sqrt{2}\,x-\frac{1}{\sqrt{2}}}{(x-1)\sqrt[4]{\frac{x^2+2}{x^2-2}}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-4 - 2*x + 2*x^2 + x^4)/(x*(-2 + x^2)*((2 + x^2)/(-2 +
x^2))^(1/4)*(8 - 10*x + 4*x^2 + 4*x^3 - 4*x^4 + x^5)),x]
```

```
[Out] -1/2*ArcTan[(-(1/Sqrt[2]) + Sqrt[2]*x - x^2/Sqrt[2] + Sqrt[(2 + x^2)/(-2 +
x^2)]/Sqrt[2])/((-1 + x)*((2 + x^2)/(-2 + x^2))^(1/4))]/Sqrt[2] + ArcTanh[(
1/Sqrt[2] - Sqrt[2]*x + x^2/Sqrt[2] + Sqrt[(2 + x^2)/(-2 + x^2)]/Sqrt[2])/(
(-1 + x)*((2 + x^2)/(-2 + x^2))^(1/4))]/(2*Sqrt[2])
```

fricas [B] time = 46.39, size = 1858, normalized size = 11.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2-2*x-4)/x/(x^2-2)/((x^2+2)/(x^2-2))^(1/4)/(x^5-4*x^4+4*
x^3+4*x^2-10*x+8),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(-(x^12 - 8*x^11 + 24*x^10 - 24*x^9 - 36*x^8 + 128*x^7 -
128*x^6 - 16*x^5 + 164*x^4 - 160*x^3 + 2*sqrt(2)*(3*x^9 - 15*x^8 + 18*x^7 +
 30*x^6 - 94*x^5 + 58*x^4 + 60*x^3 - 108*x^2 + 64*x - 16)*((x^2 + 2)/(x^2 -
 2))^(3/4) + 64*x^2 + 2*sqrt(2)*(x^11 - 7*x^10 + 17*x^9 - 7*x^8 - 48*x^7 +
100*x^6 - 58*x^5 - 54*x^4 + 124*x^3 - 116*x^2 + 64*x - 16)*((x^2 + 2)/(x^2
- 2))^(1/4) - (2*sqrt(2)*(3*x^10 - 18*x^9 + 33*x^8 + 12*x^7 - 124*x^6 + 152
*x^5 + 2*x^4 - 168*x^3 + 172*x^2 - 80*x + 16)*sqrt((x^2 + 2)/(x^2 - 2)) + 1
6*(x^9 - 5*x^8 + 6*x^7 + 10*x^6 - 31*x^5 + 19*x^4 + 20*x^3 - 36*x^2 + 20*x
- 4)*((x^2 + 2)/(x^2 - 2))^(3/4) + sqrt(2)*(x^12 - 8*x^11 + 24*x^10 - 24*x^
9 - 30*x^8 + 104*x^7 - 92*x^6 - 40*x^5 + 144*x^4 - 64*x^3 - 88*x^2 + 96*x -
 32) + 4*(x^11 - 7*x^10 + 17*x^9 - 7*x^8 - 44*x^7 + 88*x^6 - 46*x^5 - 58*x^
4 + 108*x^3 - 68*x^2 + 16*x)*((x^2 + 2)/(x^2 - 2))^(1/4))*sqrt((x^6 - 4*x^5
 + 4*x^4 + 4*x^3 - 2*sqrt(2)*(x^3 - x^2 - 2*x + 2)*((x^2 + 2)/(x^2 - 2))^(3
/4) - 10*x^2 - 2*sqrt(2)*(x^5 - 3*x^4 + x^3 + 5*x^2 - 6*x + 2)*((x^2 + 2)/(
x^2 - 2))^(1/4) + 4*(x^4 - 2*x^3 - x^2 + 4*x - 2)*sqrt((x^2 + 2)/(x^2 - 2))
 + 8*x)/(x^6 - 4*x^5 + 4*x^4 + 4*x^3 - 10*x^2 + 8*x)) + 4*(x^10 - 6*x^9 + 1
1*x^8 + 4*x^7 - 40*x^6 + 48*x^5 + 2*x^4 - 56*x^3 + 52*x^2 - 16*x)*sqrt((x^2
 + 2)/(x^2 - 2)))/(x^12 - 8*x^11 + 24*x^10 - 24*x^9 - 52*x^8 + 192*x^7 - 22
4*x^6 + 48*x^5 + 212*x^4 - 416*x^3 + 448*x^2 - 256*x + 64)) - 1/4*sqrt(2)*a
rctan(-(x^12 - 8*x^11 + 24*x^10 - 24*x^9 - 36*x^8 + 128*x^7 - 128*x^6 - 16*
x^5 + 164*x^4 - 160*x^3 - 2*sqrt(2)*(3*x^9 - 15*x^8 + 18*x^7 + 30*x^6 - 94*
x^5 + 58*x^4 + 60*x^3 - 108*x^2 + 64*x - 16)*((x^2 + 2)/(x^2 - 2))^(3/4) +
64*x^2 - 2*sqrt(2)*(x^11 - 7*x^10 + 17*x^9 - 7*x^8 - 48*x^7 + 100*x^6 - 58*
x^5 - 54*x^4 + 124*x^3 - 116*x^2 + 64*x - 16)*((x^2 + 2)/(x^2 - 2))^(1/4) +
 (2*sqrt(2)*(3*x^10 - 18*x^9 + 33*x^8 + 12*x^7 - 124*x^6 + 152*x^5 + 2*x^4
- 168*x^3 + 172*x^2 - 80*x + 16)*sqrt((x^2 + 2)/(x^2 - 2)) - 16*(x^9 - 5*x^
8 + 6*x^7 + 10*x^6 - 31*x^5 + 19*x^4 + 20*x^3 - 36*x^2 + 20*x - 4)*((x^2 +
2)/(x^2 - 2))^(3/4) + sqrt(2)*(x^12 - 8*x^11 + 24*x^10 - 24*x^9 - 30*x^8 +
104*x^7 - 92*x^6 - 40*x^5 + 144*x^4 - 64*x^3 - 88*x^2 + 96*x - 32) - 4*(x^1
1 - 7*x^10 + 17*x^9 - 7*x^8 - 44*x^7 + 88*x^6 - 46*x^5 - 58*x^4 + 108*x^3 -
 68*x^2 + 16*x)*((x^2 + 2)/(x^2 - 2))^(1/4))*sqrt((x^6 - 4*x^5 + 4*x^4 + 4*
x^3 + 2*sqrt(2)*(x^3 - x^2 - 2*x + 2)*((x^2 + 2)/(x^2 - 2))^(3/4) - 10*x^2
+ 2*sqrt(2)*(x^5 - 3*x^4 + x^3 + 5*x^2 - 6*x + 2)*((x^2 + 2)/(x^2 - 2))^(1/
4) + 4*(x^4 - 2*x^3 - x^2 + 4*x - 2)*sqrt((x^2 + 2)/(x^2 - 2)) + 8*x)/(x^6
```

```
- 4*x^5 + 4*x^4 + 4*x^3 - 10*x^2 + 8*x)) + 4*(x^10 - 6*x^9 + 11*x^8 + 4*x^7
 - 40*x^6 + 48*x^5 + 2*x^4 - 56*x^3 + 52*x^2 - 16*x)*sqrt((x^2 + 2)/(x^2 -
2)))/(x^12 - 8*x^11 + 24*x^10 - 24*x^9 - 52*x^8 + 192*x^7 - 224*x^6 + 48*x^
5 + 212*x^4 - 416*x^3 + 448*x^2 - 256*x + 64)) + 1/16*sqrt(2)*log(4*(x^6 -
4*x^5 + 4*x^4 + 4*x^3 + 2*sqrt(2)*(x^3 - x^2 - 2*x + 2)*((x^2 + 2)/(x^2 - 2
))^(3/4) - 10*x^2 + 2*sqrt(2)*(x^5 - 3*x^4 + x^3 + 5*x^2 - 6*x + 2)*((x^2 +
 2)/(x^2 - 2))^(1/4) + 4*(x^4 - 2*x^3 - x^2 + 4*x - 2)*sqrt((x^2 + 2)/(x^2
- 2)) + 8*x)/(x^6 - 4*x^5 + 4*x^4 + 4*x^3 - 10*x^2 + 8*x)) - 1/16*sqrt(2)*l
og(4*(x^6 - 4*x^5 + 4*x^4 + 4*x^3 - 2*sqrt(2)*(x^3 - x^2 - 2*x + 2)*((x^2 +
 2)/(x^2 - 2))^(3/4) - 10*x^2 - 2*sqrt(2)*(x^5 - 3*x^4 + x^3 + 5*x^2 - 6*x
+ 2)*((x^2 + 2)/(x^2 - 2))^(1/4) + 4*(x^4 - 2*x^3 - x^2 + 4*x - 2)*sqrt((x^
2 + 2)/(x^2 - 2)) + 8*x)/(x^6 - 4*x^5 + 4*x^4 + 4*x^3 - 10*x^2 + 8*x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+2x^2-2x-4}{\left(x^5-4x^4+4x^3+4x^2-10x+8\right)\left(x^2-2\right)x\left(\frac{x^2+2}{x^2-2}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2-2*x-4)/x/(x^2-2)/((x^2+2)/(x^2-2))^(1/4)/(x^5-4*x^4+4*
x^3+4*x^2-10*x+8),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 2*x^2 - 2*x - 4)/((x^5 - 4*x^4 + 4*x^3 + 4*x^2 - 10*x + 8)
*(x^2 - 2)*x*((x^2 + 2)/(x^2 - 2))^(1/4)), x)
```

maple [C] time = 7.60, size = 1065, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x^2-2*x-4)/x/(x^2-2)/((x^2+2)/(x^2-2))^(1/4)/(x^5-4*x^4+4*x^3+4*
x^2-10*x+8),x)
```

```
[Out] -1/4*RootOf(_Z^4+1)*ln((2*(-(-x^2-2)/(x^2-2))^(1/2)*x^4*RootOf(_Z^4+1)^3-4*
(-(-x^2-2)/(x^2-2))^(1/2)*x^3*RootOf(_Z^4+1)^3-2*(-(-x^2-2)/(x^2-2))^(1/4)*
x^5*RootOf(_Z^4+1)^2-2*(-(-x^2-2)/(x^2-2))^(1/2)*x^2*RootOf(_Z^4+1)^3+6*(-(
-x^2-2)/(x^2-2))^(1/4)*x^4*RootOf(_Z^4+1)^2+RootOf(_Z^4+1)*x^6+2*(-(-x^2-2)
/(x^2-2))^(3/4)*x^3+8*(-(-x^2-2)/(x^2-2))^(1/2)*RootOf(_Z^4+1)^3*x-2*(-(-x^
2-2)/(x^2-2))^(1/4)*x^3*RootOf(_Z^4+1)^2-4*RootOf(_Z^4+1)*x^5-2*(-(-x^2-2)/
(x^2-2))^(3/4)*x^2-4*(-(-x^2-2)/(x^2-2))^(1/2)*RootOf(_Z^4+1)^3-10*(-(-x^2-
2)/(x^2-2))^(1/4)*RootOf(_Z^4+1)^2*x^2+4*RootOf(_Z^4+1)*x^4-4*(-(-x^2-2)/(x
^2-2))^(3/4)*x+12*(-(-x^2-2)/(x^2-2))^(1/4)*x*RootOf(_Z^4+1)^2+4*RootOf(_Z^
4+1)*x^3+4*(-(-x^2-2)/(x^2-2))^(3/4)-4*(-(-x^2-2)/(x^2-2))^(1/4)*RootOf(_Z^
4+1)^2-12*RootOf(_Z^4+1)*x^2+8*RootOf(_Z^4+1)*x-4*RootOf(_Z^4+1))/(x^5-4*x^
4+4*x^3+4*x^2-10*x+8)/x)+1/4*RootOf(_Z^4+1)^3*ln((-RootOf(_Z^4+1)^3*x^6+2*(
-(-x^2-2)/(x^2-2))^(1/4)*x^5*RootOf(_Z^4+1)^2+4*RootOf(_Z^4+1)^3*x^5-2*x^4*
(-(-x^2-2)/(x^2-2))^(1/2)*RootOf(_Z^4+1)-6*(-(-x^2-2)/(x^2-2))^(1/4)*x^4*Ro
otOf(_Z^4+1)^2-4*RootOf(_Z^4+1)^3*x^4+2*(-(-x^2-2)/(x^2-2))^(3/4)*x^3+4*x^3
*(-(-x^2-2)/(x^2-2))^(1/2)*RootOf(_Z^4+1)+2*(-(-x^2-2)/(x^2-2))^(1/4)*x^3*R
ootOf(_Z^4+1)^2-4*RootOf(_Z^4+1)^3*x^3-2*(-(-x^2-2)/(x^2-2))^(3/4)*x^2+2*x^
2*(-(-x^2-2)/(x^2-2))^(1/2)*RootOf(_Z^4+1)+10*(-(-x^2-2)/(x^2-2))^(1/4)*Roo
tOf(_Z^4+1)^2*x^2+12*RootOf(_Z^4+1)^3*x^2-4*(-(-x^2-2)/(x^2-2))^(3/4)*x-8*(
-(-x^2-2)/(x^2-2))^(1/2)*RootOf(_Z^4+1)*x-12*(-(-x^2-2)/(x^2-2))^(1/4)*x*Ro
otOf(_Z^4+1)^2-8*RootOf(_Z^4+1)^3*x+4*(-(-x^2-2)/(x^2-2))^(3/4)+4*(-(-x^2-2
)/(x^2-2))^(1/2)*RootOf(_Z^4+1)+4*(-(-x^2-2)/(x^2-2))^(1/4)*RootOf(_Z^4+1)^
2+4*RootOf(_Z^4+1)^3)/(x^5-4*x^4+4*x^3+4*x^2-10*x+8)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+2\,x^2-2\,x-4}{\left(x^5-4\,x^4+4\,x^3+4\,x^2-10\,x+8\right)\left(x^2-2\right)x\left(\frac{x^2+2}{x^2-2}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x^2-2*x-4)/x/(x^2-2)/((x^2+2)/(x^2-2))^(1/4)/(x^5-4*x^4+4*
x^3+4*x^2-10*x+8),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 2*x^2 - 2*x - 4)/((x^5 - 4*x^4 + 4*x^3 + 4*x^2 - 10*x + 8)
*(x^2 - 2)*x*((x^2 + 2)/(x^2 - 2))^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{-x^4-2\,x^2+2\,x+4}{x\,{\left(\frac{x^2+2}{x^2-2}\right)}^{1/4}\,\left(x^2-2\right)\,\left(x^5-4\,x^4+4\,x^3+4\,x^2-10\,x+8\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 2*x^2 - x^4 + 4)/(x*((x^2 + 2)/(x^2 - 2))^(1/4)*(x^2 - 2)*(4*x^
2 - 10*x + 4*x^3 - 4*x^4 + x^5 + 8)),x)
```

```
[Out] int(-(2*x - 2*x^2 - x^4 + 4)/(x*((x^2 + 2)/(x^2 - 2))^(1/4)*(x^2 - 2)*(4*x^
2 - 10*x + 4*x^3 - 4*x^4 + x^5 + 8)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x**2-2*x-4)/x/(x**2-2)/((x**2+2)/(x**2-2))**(1/4)/(x**5-4
*x**4+4*x**3+4*x**2-10*x+8),x)
```

```
[Out] Timed out
```

3.1800 $\int \frac{-d+cx}{x^4 \sqrt[3]{-b+ax^3}} dx$

Optimal. Leaf size=162

$$\frac{ad\log\left(\sqrt[3]{ax^3-b}+\sqrt[3]{b}\right)}{9b^{4/3}}-\frac{ad\log\left(-\sqrt[3]{b}\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}+b^{2/3}\right)}{18b^{4/3}}+\frac{ad\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}\,b^{4/3}}+\frac{\left(ax^3-b\right)}{6}$$

Rubi [A] time = 0.18, antiderivative size = 142, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1844, 266, 51, 56, 617, 204, 31, 264}

$$\frac{ad\log\left(\sqrt[3]{ax^3-b}+\sqrt[3]{b}\right)}{6b^{4/3}}+\frac{ad\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}\,b^{4/3}}-\frac{ad\log(x)}{6b^{4/3}}+\frac{c\left(ax^3-b\right)^{2/3}}{2bx^2}-\frac{d\left(ax^3-b\right)^{2/3}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-d + c*x)/(x^4*(-b + a*x^3)^(1/3)),x]
```

```
[Out] -1/3*(d*(-b + a*x^3)^(2/3))/(b*x^3) + (c*(-b + a*x^3)^(2/3))/(2*b*x^2) + (a
*d*ArcTan[(b^(1/3) - 2*(-b + a*x^3)^(1/3))/(Sqrt[3]*b^(1/3))])/(3*Sqrt[3]*b
^(4/3)) - (a*d*Log[x])/(6*b^(4/3)) + (a*d*Log[b^(1/3) + (-b + a*x^3)^(1/3)]
)/(6*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-d+cx}{x^4\sqrt[3]{-b+ax^3}}\,dx &= \int \left(-\frac{d}{x^4\sqrt[3]{-b+ax^3}}+\frac{c}{x^3\sqrt[3]{-b+ax^3}}\right)dx \\
&= c\int \frac{1}{x^3\sqrt[3]{-b+ax^3}}\,dx-d\int \frac{1}{x^4\sqrt[3]{-b+ax^3}}\,dx \\
&= \frac{c\left(-b+ax^3\right)^{2/3}}{2bx^2}-\frac{1}{3}d\,\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{-b+ax}}\,dx,x,x^3\right) \\
&= -\frac{d\left(-b+ax^3\right)^{2/3}}{3bx^3}+\frac{c\left(-b+ax^3\right)^{2/3}}{2bx^2}-\frac{(ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{-b+ax}}\,dx,x,x^3\right)}{9b} \\
&= -\frac{d\left(-b+ax^3\right)^{2/3}}{3bx^3}+\frac{c\left(-b+ax^3\right)^{2/3}}{2bx^2}-\frac{ad\log(x)}{6b^{4/3}}+\frac{(ad)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+x}\,dx,x,\sqrt[3]{-b+ax^3}\right)}{6b^{4/3}} - \cdots \\
&= -\frac{d\left(-b+ax^3\right)^{2/3}}{3bx^3}+\frac{c\left(-b+ax^3\right)^{2/3}}{2bx^2}-\frac{ad\log(x)}{6b^{4/3}}+\frac{ad\log\left(\sqrt[3]{b}+\sqrt[3]{-b+ax^3}\right)}{6b^{4/3}}-\frac{(ad)\,\text{Subs}\cdots}{\cdots} \\
&= -\frac{d\left(-b+ax^3\right)^{2/3}}{3bx^3}+\frac{c\left(-b+ax^3\right)^{2/3}}{2bx^2}+\frac{ad\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-b+ax^3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}}-\frac{ad\log(x)}{6b^{4/3}}+\frac{ad\log\left(\sqrt[3]{b}+\cdots\right)}{6b^{4/3}\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.05, size = 53, normalized size = 0.33

$$\frac{\left(ax^3-b\right)^{2/3}\left(bc-adx^2\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};1-\frac{ax^3}{b}\right)\right)}{2b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-d + c*x)/(x^4*(-b + a*x^3)^(1/3)),x]
```

```
[Out] ((-b + a*x^3)^(2/3)*(b*c - a*d*x^2*Hypergeometric2F1[2/3, 2, 5/3, 1 - (a*x^
3)/b]))/(2*b^2*x^2)
```

IntegrateAlgebraic [A] time = 9.10, size = 162, normalized size = 1.00

$$\frac{ad\log\left(\sqrt[3]{ax^3-b}+\sqrt[3]{b}\right)}{9b^{4/3}}-\frac{ad\log\left(-\sqrt[3]{b}\,\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}+b^{2/3}\right)}{18b^{4/3}}+\frac{ad\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{ax^3-b}}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,b^{4/3}}+\frac{\left(ax^3-b\right)^{2/3}(3cx-2d)}{6bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-d + c*x)/(x^4*(-b + a*x^3)^(1/3)),x]
```

```
[Out] ((-2*d + 3*c*x)*(-b + a*x^3)^(2/3))/(6*b*x^3) + (a*d*ArcTan[1/Sqrt[3] - (2*
(-b + a*x^3)^(1/3))/(Sqrt[3]*b^(1/3))])/(3*Sqrt[3]*b^(4/3)) + (a*d*Log[b^(1
/3) + (-b + a*x^3)^(1/3)])/(9*b^(4/3)) - (a*d*Log[b^(2/3) - b^(1/3)*(-b + a
*x^3)^(1/3) + (-b + a*x^3)^(2/3)])/(18*b^(4/3))
```

fricas [A] time = 83.14, size = 352, normalized size = 2.17

$$\left[\frac{3\sqrt{\frac{1}{3}}\,abdx^3\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2ax^3-3\sqrt{\frac{1}{3}}\left(2\left(ax^3-b\right)^{\frac{2}{3}}b^{\frac{2}{3}}+\left(ax^3-b\right)^{\frac{1}{3}}b-b^{\frac{4}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-3\left(ax^3-b\right)^{\frac{1}{3}}b^{\frac{2}{3}}-3b}{x^3}\right)+2ab^{\frac{2}{3}}dx^3\log\left(\frac{\left(ax^3-b\right)^{\frac{1}{3}}+b^{\frac{1}{3}}}{x}\right)-ab^{\frac{2}{3}}dx^3\log\left(\frac{\left(ax^3-b\right)^{\frac{2}{3}}-\left(ax^3-b\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}}{x^2}\right)+3\left(ax^3-b\right)^{\frac{2}{3}}(3bcx-2bd)}{18b^2x^3},-\frac{6\sqrt{\frac{1}{3}}\,ab^{\frac{2}{3}}dx^3\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(ax^3-b\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}}\right)-2ab^{\frac{2}{3}}dx^3\log\left(\frac{\left(ax^3-b\right)^{\frac{1}{3}}+b^{\frac{1}{3}}}{x}\right)+ab^{\frac{2}{3}}dx^3\log\left(\frac{\left(ax^3-b\right)^{\frac{2}{3}}-\left(ax^3-b\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}}{x^2}\right)-3\left(ax^3-b\right)^{\frac{2}{3}}(3bcx-2bd)}{18b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x-d)/x^4/(a*x^3-b)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*a*b*d*x^3*sqrt(-1/b^(2/3))*log((2*a*x^3 - 3*sqrt(1/3)*(2
*(a*x^3 - b)^(2/3)*b^(2/3) + (a*x^3 - b)^(1/3)*b - b^(4/3))*sqrt(-1/b^(2/3)
) - 3*(a*x^3 - b)^(1/3)*b^(2/3) - 3*b)/x^3) + 2*a*b^(2/3)*d*x^3*log(((a*x^3
 - b)^(1/3) + b^(1/3))/x) - a*b^(2/3)*d*x^3*log(((a*x^3 - b)^(2/3) - (a*x^3
 - b)^(1/3)*b^(1/3) + b^(2/3))/x^2) + 3*(a*x^3 - b)^(2/3)*(3*b*c*x - 2*b*d)
)/(b^2*x^3), -1/18*(6*sqrt(1/3)*a*b^(2/3)*d*x^3*arctan(sqrt(1/3)*(2*(a*x^3
- b)^(1/3) - b^(1/3))/b^(1/3)) - 2*a*b^(2/3)*d*x^3*log(((a*x^3 - b)^(1/3) +
 b^(1/3))/x) + a*b^(2/3)*d*x^3*log(((a*x^3 - b)^(2/3) - (a*x^3 - b)^(1/3)*b
^(1/3) + b^(2/3))/x^2) - 3*(a*x^3 - b)^(2/3)*(3*b*c*x - 2*b*d))/(b^2*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{cx-d}{\left(ax^3-b\right)^{\frac{1}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x-d)/x^4/(a*x^3-b)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*x - d)/((a*x^3 - b)^(1/3)*x^4), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int\frac{cx-d}{x^4\left(a\,x^3-b\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x-d)/x^4/(a*x^3-b)^(1/3),x)
```

```
[Out] int((c*x-d)/x^4/(a*x^3-b)^(1/3),x)
```

maxima [A] time = 0.45, size = 152, normalized size = 0.94

$$-\frac{1}{18}\left(\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\left(ax^3-b\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}+\frac{6\left(ax^3-b\right)^{\frac{2}{3}}a}{\left(ax^3-b\right)b+b^2}+\frac{a\log\left(\left(ax^3-b\right)^{\frac{2}{3}}-\left(ax^3-b\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{4}{3}}}-\frac{2a\log\left(\left(ax^3-b\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{b^{\frac{4}{3}}}\right)d+\frac{\left(ax^3-b\right)^{\frac{2}{3}}c}{2bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x-d)/x^4/(a*x^3-b)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/18*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*(a*x^3 - b)^(1/3) - b^(1/3))/b^(1/
3))/b^(4/3) + 6*(a*x^3 - b)^(2/3)*a/((a*x^3 - b)*b + b^2) + a*log((a*x^3 -
b)^(2/3) - (a*x^3 - b)^(1/3)*b^(1/3) + b^(2/3))/b^(4/3) - 2*a*log((a*x^3 -
b)^(1/3) + b^(1/3))/b^(4/3))*d + 1/2*(a*x^3 - b)^(2/3)*c/(b*x^2)
```

mupad [B] time = 1.78, size = 184, normalized size = 1.14

$$\frac{c\left(a\,x^3-b\right)^{2/3}}{2\,b\,x^2}-\frac{\ln\left(\frac{\left(a\,d+\sqrt{3}\,a\,d\,1\mathrm{i}\right)^2}{36\,b^{5/3}}+\frac{a^2\,d^2\,\left(a\,x^3-b\right)^{1/3}}{9\,b^2}\right)\left(a\,d+\sqrt{3}\,a\,d\,1\mathrm{i}\right)}{18\,b^{4/3}}-\frac{\ln\left(\frac{\left(a\,d-\sqrt{3}\,a\,d\,1\mathrm{i}\right)^2}{36\,b^{5/3}}+\frac{a^2\,d^2\,\left(a\,x^3-b\right)^{1/3}}{9\,b^2}\right)\left(a\,d-\sqrt{3}\,a\,d\,1\mathrm{i}\right)}{18\,b^{4/3}}-\frac{d\left(a\,x^3-b\right)^{2/3}}{3\,b\,x^3}+\frac{a\,d\,\ln\left(\left(a\,x^3-b\right)^{1/3}+b^{1/3}\right)}{9\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(d - c*x)/(x^4*(a*x^3 - b)^(1/3)),x)
```

```
[Out] (c*(a*x^3 - b)^(2/3))/(2*b*x^2) - (log((a*d + 3^(1/2)*a*d*1i)^2/(36*b^(5/3)
) + (a^2*d^2*(a*x^3 - b)^(1/3))/(9*b^2))*(a*d + 3^(1/2)*a*d*1i))/(18*b^(4/3
)) - (log((a*d - 3^(1/2)*a*d*1i)^2/(36*b^(5/3)) + (a^2*d^2*(a*x^3 - b)^(1/3
))/(9*b^2))*(a*d - 3^(1/2)*a*d*1i))/(18*b^(4/3)) - (d*(a*x^3 - b)^(2/3))/(3
*b*x^3) + (a*d*log((a*x^3 - b)^(1/3) + b^(1/3)))/(9*b^(4/3))
```

sympy [C] time = 2.70, size = 126, normalized size = 0.78

$$c\left(\begin{cases}-\frac{a^{\frac{2}{3}}\left(-1+\frac{b}{ax^3}\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{2}{3}\right)}{3b\Gamma\left(\frac{1}{3}\right)} & \text{for } \left|\frac{b}{ax^3}\right|>1\\ -\frac{a^{\frac{2}{3}}\left(1-\frac{b}{ax^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{2}{3}\right)}{3b\Gamma\left(\frac{1}{3}\right)} & \text{otherwise}\end{cases}\right)+\frac{d\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^3}\right)}{3\sqrt[3]{a}\,x^4\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x-d)/x**4/(a*x**3-b)**(1/3),x)
```

```
[Out] c*Piecewise((-a**(2/3)*(-1 + b/(a*x**3))**(2/3)*exp(2*I*pi/3)*gamma(-2/3)/(
3*b*gamma(1/3)), Abs(b/(a*x**3)) > 1), (-a**(2/3)*(1 - b/(a*x**3))**(2/3)*g
amma(-2/3)/(3*b*gamma(1/3)), True)) + d*gamma(4/3)*hyper((1/3, 4/3), (7/3,)
, b*exp_polar(2*I*pi)/(a*x**3))/(3*a**(1/3)*x**4*gamma(7/3))
```

3.1801 $\int \frac{\sqrt[4]{-bx^3+ax^4}\left(-d+cx^4\right)}{x^2}\,dx$

Optimal. Leaf size=162

$$\frac{\left(2048a^4d+77b^4c\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^3}}\right)}{1024a^{15/4}}+\frac{\left(-2048a^4d-77b^4c\right)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^3}}\right)}{1024a^{15/4}}+\frac{\sqrt[4]{ax^4-bx^3}\,\left(384a^3cx^4+6144\right.}{}$$

Rubi [B] time = 0.70, antiderivative size = 392, normalized size of antiderivative = 2.42, number of steps used = 19, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2052, 2020, 2032, 63, 331, 298, 203, 206, 2021, 2024}

$$\frac{77b^4cx^{9/4}(ax-b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax-b}}\right)}{1024a^{15/4}\left(ax^4-bx^3\right)^{3/4}}-\frac{77b^4cx^{9/4}(ax-b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax-b}}\right)}{1024a^{15/4}\left(ax^4-bx^3\right)^{3/4}}-\frac{77b^3c\sqrt[4]{ax^4-bx^3}}{1536a^3}-\frac{11b^2cx\sqrt[4]{ax^4-bx^3}}{384a^2}+\frac{1}{4}cx^3\sqrt[4]{ax^4-bx^3}-\frac{bcx^2\sqrt[4]{ax^4-bx^3}}{48a}+\frac{4d\sqrt[4]{ax^4-bx^3}}{x}+\frac{2\sqrt[4]{a}\,dx^{9/4}(ax-b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax-b}}\right)}{\left(ax^4-bx^3\right)^{3/4}}-\frac{2\sqrt[4]{a}\,dx^{9/4}(ax-b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax-b}}\right)}{\left(ax^4-bx^3\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((-(b*x^3) + a*x^4)^(1/4)*(-d + c*x^4))/x^2,x]
```

```
[Out] (-77*b^3*c*(-(b*x^3) + a*x^4)^(1/4))/(1536*a^3) + (4*d*(-(b*x^3) + a*x^4)^(
1/4))/x - (11*b^2*c*x*(-(b*x^3) + a*x^4)^(1/4))/(384*a^2) - (b*c*x^2*(-(b*x
^3) + a*x^4)^(1/4))/(48*a) + (c*x^3*(-(b*x^3) + a*x^4)^(1/4))/4 + (77*b^4*c
*x^(9/4)*(-b + a*x)^(3/4)*ArcTan[(a^(1/4)*x^(1/4))/(-b + a*x)^(1/4)])/(1024
*a^(15/4)*(-(b*x^3) + a*x^4)^(3/4)) + (2*a^(1/4)*d*x^(9/4)*(-b + a*x)^(3/4)
*ArcTan[(a^(1/4)*x^(1/4))/(-b + a*x)^(1/4)])/(-(b*x^3) + a*x^4)^(3/4) - (77
*b^4*c*x^(9/4)*(-b + a*x)^(3/4)*ArcTanh[(a^(1/4)*x^(1/4))/(-b + a*x)^(1/4)]
)/(1024*a^(15/4)*(-(b*x^3) + a*x^4)^(3/4)) - (2*a^(1/4)*d*x^(9/4)*(-b + a*x
)^(3/4)*ArcTanh[(a^(1/4)*x^(1/4))/(-b + a*x)^(1/4)])/(-(b*x^3) + a*x^4)^(3/
4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\sqrt[4]{-bx^3+ax^4}\left(-d+cx^4\right)}{x^2}\,dx = \int \left(-\frac{d\sqrt[4]{-bx^3+ax^4}}{x^2}+cx^2\sqrt[4]{-bx^3+ax^4}\right)dx$$

$$= c\int x^2\sqrt[4]{-bx^3+ax^4}\,dx - d\int \frac{\sqrt[4]{-bx^3+ax^4}}{x^2}\,dx$$

$$= \frac{4d\sqrt[4]{-bx^3+ax^4}}{x} + \frac{1}{4}cx^3\sqrt[4]{-bx^3+ax^4} - \frac{1}{16}(bc)\int \frac{x^5}{\left(-bx^3+ax^4\right)^{3/4}}\,dx - (ad$$

$$= \frac{4d\sqrt[4]{-bx^3+ax^4}}{x} - \frac{bcx^2\sqrt[4]{-bx^3+ax^4}}{48a} + \frac{1}{4}cx^3\sqrt[4]{-bx^3+ax^4} - \frac{\left(11b^2c\right)\int \frac{}{\left(-bx\right.}}{192a}$$

$$= \frac{4d\sqrt[4]{-bx^3+ax^4}}{x} - \frac{11b^2cx\sqrt[4]{-bx^3+ax^4}}{384a^2} - \frac{bcx^2\sqrt[4]{-bx^3+ax^4}}{48a} + \frac{1}{4}cx^3\sqrt[4]{-bx^3}$$

$$= -\frac{77b^3c\sqrt[4]{-bx^3+ax^4}}{1536a^3} + \frac{4d\sqrt[4]{-bx^3+ax^4}}{x} - \frac{11b^2cx\sqrt[4]{-bx^3+ax^4}}{384a^2} - \frac{bcx^2\sqrt[4]{-b}}{48}$$

$$= -\frac{77b^3c\sqrt[4]{-bx^3+ax^4}}{1536a^3} + \frac{4d\sqrt[4]{-bx^3+ax^4}}{x} - \frac{11b^2cx\sqrt[4]{-bx^3+ax^4}}{384a^2} - \frac{bcx^2\sqrt[4]{-b}}{48}$$

$$= -\frac{77b^3c\sqrt[4]{-bx^3+ax^4}}{1536a^3} + \frac{4d\sqrt[4]{-bx^3+ax^4}}{x} - \frac{11b^2cx\sqrt[4]{-bx^3+ax^4}}{384a^2} - \frac{bcx^2\sqrt[4]{-b}}{48}$$

$$= -\frac{77b^3c\sqrt[4]{-bx^3+ax^4}}{1536a^3} + \frac{4d\sqrt[4]{-bx^3+ax^4}}{x} - \frac{11b^2cx\sqrt[4]{-bx^3+ax^4}}{384a^2} - \frac{bcx^2\sqrt[4]{-b}}{48}$$

$$= -\frac{77b^3c\sqrt[4]{-bx^3+ax^4}}{1536a^3} + \frac{4d\sqrt[4]{-bx^3+ax^4}}{x} - \frac{11b^2cx\sqrt[4]{-bx^3+ax^4}}{384a^2} - \frac{bcx^2\sqrt[4]{-b}}{48}$$

$$= -\frac{77b^3c\sqrt[4]{-bx^3+ax^4}}{1536a^3} + \frac{4d\sqrt[4]{-bx^3+ax^4}}{x} - \frac{11b^2cx\sqrt[4]{-bx^3+ax^4}}{384a^2} - \frac{bcx^2\sqrt[4]{-b}}{48}$$

Mathematica [C] time = 0.12, size = 167, normalized size = 1.03

$$-\frac{4\sqrt[4]{x^3(ax-b)}\left(a^4(-d)\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};\frac{ax}{b}\right)+b^4c\,{}_2F_1\left(-\frac{17}{4},-\frac{1}{4};\frac{3}{4};\frac{ax}{b}\right)-4b^4c\,{}_2F_1\left(-\frac{13}{4},-\frac{1}{4};\frac{3}{4};\frac{ax}{b}\right)+6b^4c\,{}_2F_1\left(-\frac{9}{4},-\frac{1}{4};\frac{3}{4};\frac{ax}{b}\right)-4b^4c\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{4};\frac{3}{4};\frac{ax}{b}\right)+b^4c\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};\frac{ax}{b}\right)\right)}{a^4x\sqrt[4]{1-\frac{ax}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-(b*x^3) + a*x^4)^(1/4)*(-d + c*x^4))/x^2,x]
```

```
[Out] (-4*(x^3*(-b + a*x))^(1/4)*(b^4*c*Hypergeometric2F1[-17/4, -1/4, 3/4, (a*x)
/b] - 4*b^4*c*Hypergeometric2F1[-13/4, -1/4, 3/4, (a*x)/b] + 6*b^4*c*Hyperg
eometric2F1[-9/4, -1/4, 3/4, (a*x)/b] - 4*b^4*c*Hypergeometric2F1[-5/4, -1/
4, 3/4, (a*x)/b] + b^4*c*Hypergeometric2F1[-1/4, -1/4, 3/4, (a*x)/b] - a^4*
d*Hypergeometric2F1[-1/4, -1/4, 3/4, (a*x)/b]))/(a^4*x*(1 - (a*x)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.68, size = 162, normalized size = 1.00

$$\frac{\left(2048a^4d+77b^4c\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^3}}\right)}{1024a^{15/4}}+\frac{\left(-2048a^4d-77b^4c\right)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^3}}\right)}{1024a^{15/4}}+\frac{\sqrt[4]{ax^4-bx^3}\left(384a^3cx^4+6144a^3d-32a^2bcx^3-44ab^2cx^2-77b^3cx\right)}{1536a^3x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-(b*x^3) + a*x^4)^(1/4)*(-d + c*x^4))/x^2,x]
```

```
[Out] ((-(b*x^3) + a*x^4)^(1/4)*(6144*a^3*d - 77*b^3*c*x - 44*a*b^2*c*x^2 - 32*a^
2*b*c*x^3 + 384*a^3*c*x^4))/(1536*a^3*x) + ((77*b^4*c + 2048*a^4*d)*ArcTan[
(a^(1/4)*x)/(-(b*x^3) + a*x^4)^(1/4)])/(1024*a^(15/4)) + ((-77*b^4*c - 2048
*a^4*d)*ArcTanh[(a^(1/4)*x)/(-(b*x^3) + a*x^4)^(1/4)])/(1024*a^(15/4))
```

fricas [B] time = 0.45, size = 810, normalized size = 5.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x^3)^(1/4)*(c*x^4-d)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6144*(12*a^3*x*((35153041*b^16*c^4 + 3739918336*a^4*b^12*c^3*d + 14920817
0496*a^8*b^8*c^2*d^2 + 2645699854336*a^12*b^4*c*d^3 + 17592186044416*a^16*d
^4)/a^15)^(1/4)*arctan((a^11*x*sqrt((a^8*x^2*sqrt((35153041*b^16*c^4 + 3739
918336*a^4*b^12*c^3*d + 149208170496*a^8*b^8*c^2*d^2 + 2645699854336*a^12*b
^4*c*d^3 + 17592186044416*a^16*d^4)/a^15) + (5929*b^8*c^2 + 315392*a^4*b^4*
c*d + 4194304*a^8*d^2)*sqrt(a*x^4 - b*x^3))/x^2)*((35153041*b^16*c^4 + 3739
918336*a^4*b^12*c^3*d + 149208170496*a^8*b^8*c^2*d^2 + 2645699854336*a^12*b
^4*c*d^3 + 17592186044416*a^16*d^4)/a^15)^(3/4) - (77*a^11*b^4*c + 2048*a^1
5*d)*(a*x^4 - b*x^3)^(1/4)*((35153041*b^16*c^4 + 3739918336*a^4*b^12*c^3*d
+ 149208170496*a^8*b^8*c^2*d^2 + 2645699854336*a^12*b^4*c*d^3 + 17592186044
416*a^16*d^4)/a^15)^(3/4))/((35153041*b^16*c^4 + 3739918336*a^4*b^12*c^3*d
+ 149208170496*a^8*b^8*c^2*d^2 + 2645699854336*a^12*b^4*c*d^3 + 17592186044
416*a^16*d^4)*x)) - 3*a^3*x*((35153041*b^16*c^4 + 3739918336*a^4*b^12*c^3*d
 + 149208170496*a^8*b^8*c^2*d^2 + 2645699854336*a^12*b^4*c*d^3 + 1759218604
4416*a^16*d^4)/a^15)^(1/4)*log((a^4*x*((35153041*b^16*c^4 + 3739918336*a^4*
b^12*c^3*d + 149208170496*a^8*b^8*c^2*d^2 + 2645699854336*a^12*b^4*c*d^3 +
17592186044416*a^16*d^4)/a^15)^(1/4) + (77*b^4*c + 2048*a^4*d)*(a*x^4 - b*x
^3)^(1/4))/x) + 3*a^3*x*((35153041*b^16*c^4 + 3739918336*a^4*b^12*c^3*d + 1
49208170496*a^8*b^8*c^2*d^2 + 2645699854336*a^12*b^4*c*d^3 + 17592186044416
*a^16*d^4)/a^15)^(1/4)*log(-(a^4*x*((35153041*b^16*c^4 + 3739918336*a^4*b^1
2*c^3*d + 149208170496*a^8*b^8*c^2*d^2 + 2645699854336*a^12*b^4*c*d^3 + 175
92186044416*a^16*d^4)/a^15)^(1/4) - (77*b^4*c + 2048*a^4*d)*(a*x^4 - b*x^3)
^(1/4))/x) + 4*(384*a^3*c*x^4 - 32*a^2*b*c*x^3 - 44*a*b^2*c*x^2 - 77*b^3*c*
x + 6144*a^3*d)*(a*x^4 - b*x^3)^(1/4))/(a^3*x)
```

giac [B] time = 0.50, size = 350, normalized size = 2.16

$$\frac{49152\left(a-\frac{b}{x}\right)^{\frac{1}{4}}bd+\frac{6\sqrt{2}\left(77b^5c+2048a^4bd\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}a^3}+\frac{6\sqrt{2}\left(77b^5c+2048a^4bd\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}a^3}+\frac{3\sqrt{2}\left(77b^5c+2048a^4bd\right)\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x}}\right)}{(-a)^{\frac{3}{4}}a^3}-\frac{3\sqrt{2}\left(77b^5c+2048a^4bd\right)\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x}}\right)}{(-a)^{\frac{3}{4}}a^3}+\frac{8\left(77\left(a-\frac{b}{x}\right)^{\frac{13}{4}}b^5c-275\left(a-\frac{b}{x}\right)^{\frac{9}{4}}ab^5c+351\left(a-\frac{b}{x}\right)^{\frac{5}{4}}a^2b^5c+231\left(a-\frac{b}{x}\right)^{\frac{1}{4}}a^3b^5c\right)x^4}{a^3b^4}}{12288b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x^3)^(1/4)*(c*x^4-d)/x^2,x, algorithm="giac")
```

```
[Out] 1/12288*(49152*(a - b/x)^(1/4)*b*d + 6*sqrt(2)*(77*b^5*c + 2048*a^4*b*d)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a - b/x)^(1/4))/(-a)^(1/4))/((-a)
^(3/4)*a^3) + 6*sqrt(2)*(77*b^5*c + 2048*a^4*b*d)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(-a)^(1/4) - 2*(a - b/x)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a^3) + 3*sqrt(2
)*(77*b^5*c + 2048*a^4*b*d)*log(sqrt(2)*(-a)^(1/4)*(a - b/x)^(1/4) + sqrt(-
a) + sqrt(a - b/x))/((-a)^(3/4)*a^3) - 3*sqrt(2)*(77*b^5*c + 2048*a^4*b*d)*
```

```
log(-sqrt(2)*(-a)^(1/4)*(a - b/x)^(1/4) + sqrt(-a) + sqrt(a - b/x))/((-a)^(
3/4)*a^3) + 8*(77*(a - b/x)^(13/4)*b^5*c - 275*(a - b/x)^(9/4)*a*b^5*c + 35
1*(a - b/x)^(5/4)*a^2*b^5*c + 231*(a - b/x)^(1/4)*a^3*b^5*c)*x^4/(a^3*b^4))
/b
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{4}-b\,x^{3}\right)^{\frac{1}{4}} \left(c\,x^{4}-d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b*x^3)^(1/4)*(c*x^4-d)/x^2,x)
[Out] int((a*x^4-b*x^3)^(1/4)*(c*x^4-d)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{4}-bx^{3}\right)^{\frac{1}{4}} \left(cx^{4}-d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x^3)^(1/4)*(c*x^4-d)/x^2,x, algorithm="maxima")
[Out] integrate((a*x^4 - b*x^3)^(1/4)*(c*x^4 - d)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(d-c\,x^4\right)\,{\left(a\,x^4-b\,x^3\right)}^{1/4}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((d - c*x^4)*(a*x^4 - b*x^3)^(1/4))/x^2,x)
[Out] -int(((d - c*x^4)*(a*x^4 - b*x^3)^(1/4))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^{3} \left(ax - b\right)} \left(cx^{4} - d\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b*x**3)**(1/4)*(c*x**4-d)/x**2,x)
[Out] Integral((x**3*(a*x - b))**(1/4)*(c*x**4 - d)/x**2, x)
```

3.1802 $\int \frac{x^3(5-4(1+k)x+3kx^2)}{((1-x)x(1-kx))^{2/3}(-b+b(1+k)x-bkx^2+x^5)} dx$

Optimal. Leaf size=162

$$-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{b}\,x^2\sqrt[3]{kx^3+(-k-1)x^2+x}+x^4\right)}{2\sqrt[3]{b}}+\frac{\log\left(x^2-\sqrt[3]{b}\,\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{\sqrt[3]{b}}$$

Rubi [F] time = 23.78, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3\left(5-4(1+k)x+3kx^2\right)}{((1-x)x(1-kx))^{2/3}\left(-b+b(1+k)x-bkx^2+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*(5 - 4*(1 + k)*x + 3*k*x^2))/(((1 - x)*x*(1 - k*x))^(2/3)*(-b + b*
(1 + k)*x - b*k*x^2 + x^5)),x]
```

```
[Out] (9*k*x*((1 - x)/(1 - k*x))^(2/3)*(1 - k*x)*Hypergeometric2F1[1/3, 2/3, 4/3,
 ((1 - k)*x)/(1 - k*x)])/((1 - x)*x*(1 - k*x))^(2/3) + (9*b*k*(1 + k)*(1 -
x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst][Defer[Int][x^3/((1 - x^3)^(2/
3)*(1 - k*x^3)^(2/3)*(b - b*(1 + k)*x^3 + b*k*x^6 - x^15)), x], x, x^(1/3)]
)/((1 - x)*x*(1 - k*x))^(2/3) + (12*(1 + k)*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)
^(2/3)*Defer[Subst][Defer[Int][x^12/((1 - x^3)^(2/3)*(1 - k*x^3)^(2/3)*(b -
 b*(1 + k)*x^3 + b*k*x^6 - x^15)), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(
2/3) + (9*b*k*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst][Defer[Int]
[1/((1 - x^3)^(2/3)*(1 - k*x^3)^(2/3)*(-b + b*(1 + k)*x^3 - b*k*x^6 + x^15)
), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(2/3) + (9*b*k^2*(1 - x)^(2/3)*x^
(2/3)*(1 - k*x)^(2/3)*Defer[Subst][Defer[Int][x^6/((1 - x^3)^(2/3)*(1 - k*x
^3)^(2/3)*(-b + b*(1 + k)*x^3 - b*k*x^6 + x^15)), x], x, x^(1/3)])/((1 - x)
*x*(1 - k*x))^(2/3) + (15*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst
][Defer[Int][x^9/((1 - x^3)^(2/3)*(1 - k*x^3)^(2/3)*(-b + b*(1 + k)*x^3 - b
*k*x^6 + x^15)), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(2/3)
```

Rubi steps

$$\int \frac{x^3\left(5-4(1+k)x+3kx^2\right)}{((1-x)x(1-kx))^{2/3}\left(-b+b(1+k)x-bkx^2+x^5\right)}dx = \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{x^{7/3}\left(5-4(1+k)x+3kx^2\right.}{(1-x)^{2/3}(1-kx)^{2/3}\left(-b+b(1+k)x-\right.}}{((1-x)x(1-kx))^{2/3}}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\text{Subst}\left(\int \frac{x^9\left(5-4\right.}{\left(1-x^3\right)^{2/3}\left(1-kx^3\right)^{2/3}}\right.}{((1-x)x(1-kx))^{2/3}}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\text{Subst}\left(\int\left(\frac{3k}{\left(1-x^3\right)^{2/3}\left(1-kx^3\right)^{2}}\right.\right.}{((1-x)x(}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\text{Subst}\left(\int \frac{3bk-3bk(1+k)x^3-}{\left(1-x^3\right)^{2/3}\left(1-kx^3\right)^{2/3}}\right.}{((1-x)x(1-kx))^{2/3}}$$

$$= \frac{9kx\left(\frac{1-x}{1-kx}\right)^{2/3}(1-kx)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{(1-k)x}{1-kx}\right)}{((1-x)x(1-kx))^{2/3}} + \frac{\left(3(1-x\right.}{}$$

$$= \frac{9kx\left(\frac{1-x}{1-kx}\right)^{2/3}(1-kx)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{(1-k)x}{1-kx}\right)}{((1-x)x(1-kx))^{2/3}} + \frac{\left(15(1-\right.}{}$$

Mathematica [F] time = 1.27, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(5-4(1+k)x+3kx^2\right)}{((1-x)x(1-kx))^{2/3}\left(-b+b(1+k)x-bkx^2+x^5\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(5 - 4*(1 + k)*x + 3*k*x^2))/(((1 - x)*x*(1 - k*x))^(2/3)*(-
b + b*(1 + k)*x - b*k*x^2 + x^5)),x]
```

```
[Out] Integrate[(x^3*(5 - 4*(1 + k)*x + 3*k*x^2))/(((1 - x)*x*(1 - k*x))^(2/3)*(-
b + b*(1 + k)*x - b*k*x^2 + x^5)), x]
```

IntegrateAlgebraic [A] time = 6.40, size = 162, normalized size = 1.00

$$-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{b}\,x^2\sqrt[3]{kx^3+(-k-1)x^2+x}+x^4\right)}{2\sqrt[3]{b}}+\frac{\log\left(x^2-\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+x^2}\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(5 - 4*(1 + k)*x + 3*k*x^2))/(((1 - x)*x*(1 - k*x))
^(2/3)*(-b + b*(1 + k)*x - b*k*x^2 + x^5)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(
1/3))])/b^(1/3) + Log[x^2 - b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3)]/b^(1/
3) - Log[x^4 + b^(1/3)*x^2*(x + (-1 - k)*x^2 + k*x^3)^(1/3) + b^(2/3)*(x +
(-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5-4*(1+k)*x+3*k*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-b+b*(1+k)*x-
b*k*x^2+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,kx^2 - 4\,(k+1)x + 5\right)x^3}{\left(x^5 - bkx^2 + b(k+1)x - b\right)\left((kx-1)(x-1)x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5-4*(1+k)*x+3*k*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-b+b*(1+k)*x-
b*k*x^2+x^5),x, algorithm="giac")
```

```
[Out] integrate((3*k*x^2 - 4*(k + 1)*x + 5)*x^3/((x^5 - b*k*x^2 + b*(k + 1)*x - b
)*((k*x - 1)*(x - 1)*x)^(2/3)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(5 - 4\,(1+k)\,x + 3k\,x^2\right)}{\left((1-x)\,x\,(-kx+1)\right)^{\frac{2}{3}}\left(-b + b\,(1+k)\,x - bk\,x^2 + x^5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(5-4*(1+k)*x+3*k*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-b+b*(1+k)*x-b*k*x^
2+x^5),x)
```

```
[Out] int(x^3*(5-4*(1+k)*x+3*k*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-b+b*(1+k)*x-b*k*x^
2+x^5),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,kx^2 - 4\,(k+1)x + 5\right)x^3}{\left(x^5 - bkx^2 + b(k+1)x - b\right)\left((kx-1)(x-1)x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5-4*(1+k)*x+3*k*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-b+b*(1+k)*x-
b*k*x^2+x^5),x, algorithm="maxima")
```

```
[Out] integrate((3*k*x^2 - 4*(k + 1)*x + 5)*x^3/((x^5 - b*k*x^2 + b*(k + 1)*x - b
)*((k*x - 1)*(x - 1)*x)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^3\,\left(3\,k\,x^2 - 4\,x\,\left(k+1\right) + 5\right)}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{2/3}\,\left(-x^5 + b\,k\,x^2 - b\,\left(k+1\right)\,x + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(3*k*x^2 - 4*x*(k + 1) + 5))/((x*(k*x - 1)*(x - 1))^(2/3)*(b - x^
5 - b*x*(k + 1) + b*k*x^2)),x)
```

```
[Out] -int((x^3*(3*k*x^2 - 4*x*(k + 1) + 5))/((x*(k*x - 1)*(x - 1))^(2/3)*(b - x^
5 - b*x*(k + 1) + b*k*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(3kx^2 - 4kx - 4x + 5\right)}{\left(x\,(x-1)\,(kx-1)\right)^{\frac{2}{3}}\left(-bkx^2 + bkx + bx - b + x^5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(5-4*(1+k)*x+3*k*x**2)/((1-x)*x*(-k*x+1))**(2/3)/(-b+b*(1+k)
*x-b*k*x**2+x**5),x)
```

```
[Out] Integral(x**3*(3*k*x**2 - 4*k*x - 4*x + 5)/((x*(x - 1)*(k*x - 1))**(2/3)*(-
b*k*x**2 + b*k*x + b*x - b + x**5)), x)
```

3.1803 $\int \frac{(-8+x^5)(2+x^5)\sqrt[4]{2-3x^4+x^5}}{x^6(4-3x^4+2x^5)}\,dx$

Optimal. Leaf size=162

$$\frac{\sqrt[4]{x^5-3x^4+2}\,(2x^5+9x^4+4)}{5x^5}+\frac{3\sqrt[4]{3}\tan^{-1}\left(\frac{6^{3/4}x\sqrt[4]{x^5-3x^4+2}}{\sqrt{6}\sqrt{x^5-3x^4+2}-3x^2}\right)}{2\,2^{3/4}}-\frac{3\sqrt[4]{3}\tanh^{-1}\left(\frac{6^{3/4}x\sqrt[4]{x^5-3x^4+2}}{3x^2+\sqrt{6}\sqrt{x^5-3x^4+2}}\right)}{2\,2^{3/4}}$$

Rubi [F] time = 1.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-8+x^5)(2+x^5)\sqrt[4]{2-3x^4+x^5}}{x^6(4-3x^4+2x^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-8 + x^5)*(2 + x^5)*(2 - 3*x^4 + x^5)^(1/4))/(x^6*(4 - 3*x^4 + 2*x^5)
),x]
```

```
[Out] -4*Defer[Int][(2 - 3*x^4 + x^5)^(1/4)/x^6, x] - 3*Defer[Int][(2 - 3*x^4 + x
^5)^(1/4)/x^2, x] + Defer[Int][(2 - 3*x^4 + x^5)^(1/4)/x, x]/2 - 9*Defer[In
t][(x^2*(2 - 3*x^4 + x^5)^(1/4))/(4 - 3*x^4 + 2*x^5), x] + (15*Defer[Int][(
x^3*(2 - 3*x^4 + x^5)^(1/4))/(4 - 3*x^4 + 2*x^5), x])/2
```

Rubi steps

$$\begin{aligned}
\int \frac{(-8+x^5)(2+x^5)\sqrt[4]{2-3x^4+x^5}}{x^6(4-3x^4+2x^5)}\,dx &= \int \left(-\frac{4\sqrt[4]{2-3x^4+x^5}}{x^6}-\frac{3\sqrt[4]{2-3x^4+x^5}}{x^2}+\frac{\sqrt[4]{2-3x^4+x^5}}{2x}+\frac{3x^2(-6 \cdots}{2(\cdots}\right.\\
&= \frac{1}{2}\int \frac{\sqrt[4]{2-3x^4+x^5}}{x}\,dx+\frac{3}{2}\int \frac{x^2(-6+5x)\sqrt[4]{2-3x^4+x^5}}{4-3x^4+2x^5}\,dx-3\int \cdots\\
&= \frac{1}{2}\int \frac{\sqrt[4]{2-3x^4+x^5}}{x}\,dx+\frac{3}{2}\int \left(-\frac{6x^2\sqrt[4]{2-3x^4+x^5}}{4-3x^4+2x^5}+\frac{5x^3\sqrt[4]{2-3x^4+\cdots}}{4-3x^4+2x^{\cdots}}\right.\\
&= \frac{1}{2}\int \frac{\sqrt[4]{2-3x^4+x^5}}{x}\,dx-3\int \frac{\sqrt[4]{2-3x^4+x^5}}{x^2}\,dx-4\int \frac{\sqrt[4]{2-3x^4+x^{\cdots}}}{x^6}\cdots
\end{aligned}$$

Mathematica [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{(-8+x^5)(2+x^5)\sqrt[4]{2-3x^4+x^5}}{x^6(4-3x^4+2x^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-8 + x^5)*(2 + x^5)*(2 - 3*x^4 + x^5)^(1/4))/(x^6*(4 - 3*x^4 +
2*x^5)),x]
```

```
[Out] Integrate[((-8 + x^5)*(2 + x^5)*(2 - 3*x^4 + x^5)^(1/4))/(x^6*(4 - 3*x^4 +
2*x^5)), x]
```

IntegrateAlgebraic [A] time = 2.97, size = 167, normalized size = 1.03

$$\frac{\sqrt[4]{x^5-3x^4+2}\,(2x^5+9x^4+4)}{5x^5}-\frac{3\sqrt[4]{3}\tan^{-1}\left(\frac{\frac{\sqrt{x^5-3x^4+2}}{\sqrt[4]{6}}-\frac{\sqrt[4]{3}x^2}{2^{3/4}}}{x\sqrt[4]{x^5-3x^4+2}}\right)}{2\,2^{3/4}}-\frac{3\sqrt[4]{3}\tanh^{-1}\left(\frac{6^{3/4}x\sqrt[4]{x^5-3x^4+2}}{3x^2+\sqrt{6}\sqrt{x^5-3x^4+2}}\right)}{2\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-8 + x^5)*(2 + x^5)*(2 - 3*x^4 + x^5)^(1/4))/(x^6*(4 -
 3*x^4 + 2*x^5)),x]
```

```
[Out] ((2 - 3*x^4 + x^5)^(1/4)*(4 + 9*x^4 + 2*x^5))/(5*x^5) - (3*3^(1/4)*ArcTan[(
-((3^(1/4)*x^2)/2^(3/4)) + Sqrt[2 - 3*x^4 + x^5]/6^(1/4))/(x*(2 - 3*x^4 + x
^5)^(1/4))])/(2*2^(3/4)) - (3*3^(1/4)*ArcTanh[(6^(3/4)*x*(2 - 3*x^4 + x^5)^
(1/4))/(3*x^2 + Sqrt[6]*Sqrt[2 - 3*x^4 + x^5])])/(2*2^(3/4))
```

fricas [B] time = 88.79, size = 1060, normalized size = 6.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-8)*(x^5+2)*(x^5-3*x^4+2)^(1/4)/x^6/(2*x^5-3*x^4+4),x, algori
thm="fricas")
```

```
[Out] -1/80*(60*3^(1/4)*2^(1/4)*x^5*arctan(-1/3*(12*x^10 - 36*x^9 + 27*x^8 + 48*x
^5 - 72*x^4 + 18*3^(3/4)*2^(3/4)*(2*x^8 - 7*x^7 + 4*x^3)*(x^5 - 3*x^4 + 2)^
(1/4) + 12*sqrt(3)*sqrt(2)*(2*x^7 - 3*x^6 + 4*x^2)*sqrt(x^5 - 3*x^4 + 2) +
12*3^(1/4)*2^(1/4)*(2*x^6 - 15*x^5 + 4*x)*(x^5 - 3*x^4 + 2)^(3/4) - sqrt(3)
*(48*sqrt(3)*sqrt(2)*(x^5 - 3*x^4 + 2)^(3/4)*x^5 + 2*3^(3/4)*2^(3/4)*(2*x^7
 - 15*x^6 + 4*x^2)*sqrt(x^5 - 3*x^4 + 2) + 3^(1/4)*2^(1/4)*(4*x^10 - 72*x^9
 + 171*x^8 + 16*x^5 - 144*x^4 + 16) + 12*(2*x^8 - 3*x^7 + 4*x^3)*(x^5 - 3*x
^4 + 2)^(1/4))*sqrt((12*3^(1/4)*2^(1/4)*(x^5 - 3*x^4 + 2)^(1/4)*x^3 + 4*3^(
3/4)*2^(3/4)*(x^5 - 3*x^4 + 2)^(3/4)*x + 24*sqrt(x^5 - 3*x^4 + 2)*x^2 + sqr
t(3)*sqrt(2)*(2*x^5 - 3*x^4 + 4))/(2*x^5 - 3*x^4 + 4)) + 48)/(4*x^10 - 108*
x^9 + 297*x^8 + 16*x^5 - 216*x^4 + 16)) - 60*3^(1/4)*2^(1/4)*x^5*arctan(-1/
3*(12*x^10 - 36*x^9 + 27*x^8 + 48*x^5 - 72*x^4 - 18*3^(3/4)*2^(3/4)*(2*x^8
- 7*x^7 + 4*x^3)*(x^5 - 3*x^4 + 2)^(1/4) + 12*sqrt(3)*sqrt(2)*(2*x^7 - 3*x^
6 + 4*x^2)*sqrt(x^5 - 3*x^4 + 2) - 12*3^(1/4)*2^(1/4)*(2*x^6 - 15*x^5 + 4*x
)*(x^5 - 3*x^4 + 2)^(3/4) - sqrt(3)*(48*sqrt(3)*sqrt(2)*(x^5 - 3*x^4 + 2)^(
3/4)*x^5 - 2*3^(3/4)*2^(3/4)*(2*x^7 - 15*x^6 + 4*x^2)*sqrt(x^5 - 3*x^4 + 2)
 - 3^(1/4)*2^(1/4)*(4*x^10 - 72*x^9 + 171*x^8 + 16*x^5 - 144*x^4 + 16) + 12
*(2*x^8 - 3*x^7 + 4*x^3)*(x^5 - 3*x^4 + 2)^(1/4))*sqrt(-(12*3^(1/4)*2^(1/4)
*(x^5 - 3*x^4 + 2)^(1/4)*x^3 + 4*3^(3/4)*2^(3/4)*(x^5 - 3*x^4 + 2)^(3/4)*x
- 24*sqrt(x^5 - 3*x^4 + 2)*x^2 - sqrt(3)*sqrt(2)*(2*x^5 - 3*x^4 + 4))/(2*x^
5 - 3*x^4 + 4)) + 48)/(4*x^10 - 108*x^9 + 297*x^8 + 16*x^5 - 216*x^4 + 16))
 + 15*3^(1/4)*2^(1/4)*x^5*log(3*(12*3^(1/4)*2^(1/4)*(x^5 - 3*x^4 + 2)^(1/4)
*x^3 + 4*3^(3/4)*2^(3/4)*(x^5 - 3*x^4 + 2)^(3/4)*x + 24*sqrt(x^5 - 3*x^4 +
2)*x^2 + sqrt(3)*sqrt(2)*(2*x^5 - 3*x^4 + 4))/(2*x^5 - 3*x^4 + 4)) - 15*3^(
1/4)*2^(1/4)*x^5*log(-3*(12*3^(1/4)*2^(1/4)*(x^5 - 3*x^4 + 2)^(1/4)*x^3 + 4
*3^(3/4)*2^(3/4)*(x^5 - 3*x^4 + 2)^(3/4)*x - 24*sqrt(x^5 - 3*x^4 + 2)*x^2 -
 sqrt(3)*sqrt(2)*(2*x^5 - 3*x^4 + 4))/(2*x^5 - 3*x^4 + 4)) - 16*(2*x^5 + 9*
x^4 + 4)*(x^5 - 3*x^4 + 2)^(1/4))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5 - 3\,x^4 + 2\right)^{\frac{1}{4}}\left(x^5 + 2\right)\left(x^5 - 8\right)}{\left(2\,x^5 - 3\,x^4 + 4\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-8)*(x^5+2)*(x^5-3*x^4+2)^(1/4)/x^6/(2*x^5-3*x^4+4),x, algori
thm="giac")
```

```
[Out] integrate((x^5 - 3*x^4 + 2)^(1/4)*(x^5 + 2)*(x^5 - 8)/((2*x^5 - 3*x^4 + 4)*
x^6), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(x^5-8)(x^5+2)(x^5-3x^4+2)^{\frac{1}{4}}}{x^6(2x^5-3x^4+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-8)*(x^5+2)*(x^5-3*x^4+2)^(1/4)/x^6/(2*x^5-3*x^4+4),x)
```

```
[Out] int((x^5-8)*(x^5+2)*(x^5-3*x^4+2)^(1/4)/x^6/(2*x^5-3*x^4+4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^5-3\,x^4+2)^{\frac{1}{4}}(x^5+2)(x^5-8)}{(2\,x^5-3\,x^4+4)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-8)*(x^5+2)*(x^5-3*x^4+2)^(1/4)/x^6/(2*x^5-3*x^4+4),x, algori
thm="maxima")
```

```
[Out] integrate((x^5 - 3*x^4 + 2)^(1/4)*(x^5 + 2)*(x^5 - 8)/((2*x^5 - 3*x^4 + 4)*
x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^5+2)\,(x^5-8)\,(x^5-3\,x^4+2)^{1/4}}{x^6\,(2\,x^5-3\,x^4+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 + 2)*(x^5 - 8)*(x^5 - 3*x^4 + 2)^(1/4))/(x^6*(2*x^5 - 3*x^4 + 4))
,x)
```

```
[Out] int(((x^5 + 2)*(x^5 - 8)*(x^5 - 3*x^4 + 2)^(1/4))/(x^6*(2*x^5 - 3*x^4 + 4))
, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{(x-1)(x^4-2x^3-2x^2-2x-2)}\,(x^5-8)(x^5+2)}{x^6(2x^5-3x^4+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-8)*(x**5+2)*(x**5-3*x**4+2)**(1/4)/x**6/(2*x**5-3*x**4+4),x
)
```

```
[Out] Integral(((x - 1)*(x**4 - 2*x**3 - 2*x**2 - 2*x - 2))**(1/4)*(x**5 - 8)*(x*
*5 + 2)/(x**6*(2*x**5 - 3*x**4 + 4)), x)
```

3.1804 $\int \frac{x^2}{(-1+x^4)\sqrt[4]{x^2+x^6}}\,dx$

Optimal. Leaf size=162

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{4\,2^{3/4}}$$

Rubi [C] time = 0.14, antiderivative size = 46, normalized size of antiderivative = 0.28, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2042, 466, 510}

$$-\frac{2x^3\sqrt[4]{x^4+1}F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*x^3*(1 + x^4)^(1/4)*AppellF1[5/8, 1, 1/4, 13/8, x^4, -x^4])/(5*(x^2 + x
^6)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] && !IntegerQ[p]
&& NeQ[b*c - a*d, 0] && !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\int \frac{x^2}{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}\,dx = \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^4}\right)\int \frac{x^{3/2}}{\left(-1+x^4\right)\sqrt[4]{1+x^4}}\,dx}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{x^4}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= -\frac{2x^3\sqrt[4]{1+x^4}\,F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^2+x^6}}$$

Mathematica [C] time = 0.03, size = 44, normalized size = 0.27

$$-\frac{2x\left(x^6+x^2\right)^{3/4}F_1\left(\frac{5}{8};\frac{1}{4},1;\frac{13}{8};-x^4,x^4\right)}{5\left(x^4+1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*x*(x^2 + x^6)^(3/4)*AppellF1[5/8, 1/4, 1, 13/8, -x^4, x^4])/(5*(1 + x^4
)^(3/4))
```

IntegrateAlgebraic [A] time = 0.51, size = 162, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}\,x^2-\sqrt{x^6+x^2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] -1/4*ArcTan[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/2^(1/4) - ArcTan[(2^(3/4)*x*(x^2
 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^2 + x^6])]/(4*2^(3/4)) - ArcTanh[(2^(1
/4)*x)/(x^2 + x^6)^(1/4)]/(4*2^(1/4)) + ArcTanh[(x^2/2^(1/4) + Sqrt[x^2 + x
^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))]/(4*2^(3/4))
```

fricas [B] time = 13.73, size = 712, normalized size = 4.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/8*2^(3/4)*arctan(1/2*2^(3/4)*(x^6 + x^2)^(1/4)*(x^4 + 1)/(x^5 + x)) - 1/3
2*2^(3/4)*log((4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 2*2^(3/4)*(x^6 + x^2)^(3/4
) + sqrt(2)*(x^5 + 2*x^3 + x) + 4*sqrt(x^6 + x^2)*x)/(x^5 - 2*x^3 + x)) + 1
/32*2^(3/4)*log(-(4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 2*2^(3/4)*(x^6 + x^2)^(
3/4) - sqrt(2)*(x^5 + 2*x^3 + x) - 4*sqrt(x^6 + x^2)*x)/(x^5 - 2*x^3 + x))
+ 1/8*2^(1/4)*arctan(1/2*(4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(2*2^(3
/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 + 2*x^3 + x) - 4*sqrt(x^6 + x^2)*x
 + 2*2^(1/4)*(x^6 + x^2)^(3/4))*sqrt((x^5 + 2*x^3 + 4*2^(1/4)*(x^6 + x^2)^(
1/4)*x^2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x + 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(
```

```
x^5 + 2*x^3 + x)) + 2*2^(3/4)*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 1/8*2
^(1/4)*arctan(1/2*(4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(2*2^(3/4)*(x^
6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 4*sqrt(x^6 + x^2)*x + 2*2^
(1/4)*(x^6 + x^2)^(3/4))*sqrt((x^5 + 2*x^3 - 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^
2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x - 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2
*x^3 + x)) + 2*2^(3/4)*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 1/32*2^(1/4)
*log(2*(x^5 + 2*x^3 + 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^6
+ x^2)*x + 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x)) - 1/32*2^(1/
4)*log(2*(x^5 + 2*x^3 - 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^
6 + x^2)*x - 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^6 + x^2)^(1/4)*(x^4 - 1)), x)
```

maple [C] time = 22.36, size = 631, normalized size = 3.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4-1)/(x^6+x^2)^(1/4),x)
```

```
[Out] -1/16*RootOf(_Z^4-8)*ln((RootOf(_Z^4-8)^3*(x^6+x^2)^(1/2)*x+RootOf(_Z^4-8)*
x^5+2*RootOf(_Z^4-8)^2*(x^6+x^2)^(1/4)*x^2+2*RootOf(_Z^4-8)*x^3+4*(x^6+x^2)
^(3/4)+RootOf(_Z^4-8)*x)/(1+x)^2/(-1+x)^2/x)-1/16*RootOf(_Z^2+RootOf(_Z^4-8
)^2)*ln(-(RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*(x^6+x^2)^(1/2)*x-
RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^5+2*RootOf(_Z^4-8)^2*(x^6+x^2)^(1/4)*x^2-2*
RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^3-4*(x^6+x^2)^(3/4)-RootOf(_Z^2+RootOf(_Z^4
-8)^2)*x)/(1+x)^2/(-1+x)^2/x)+1/32*ln((RootOf(_Z^4-8)^2*x^2+2*RootOf(_Z^4-8
)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))/x/(x^2+1))*RootOf(_Z^4-8)^3+1/32*ln(
(RootOf(_Z^4-8)^2*x^2+2*RootOf(_Z^4-8)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))
/x/(x^2+1))*RootOf(_Z^4-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)-1/32*RootOf(_Z^4
-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln((RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootO
f(_Z^4-8)^2*x^5-RootOf(_Z^4-8)^3*x^5-2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf
(_Z^4-8)^2*x^3+2*RootOf(_Z^4-8)^3*x^3-8*RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootO
f(_Z^4-8)*(x^6+x^2)^(1/4)*x^2+RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^
2*x-8*RootOf(_Z^2+RootOf(_Z^4-8)^2)*(x^6+x^2)^(1/2)*x-RootOf(_Z^4-8)^3*x-8*
(x^6+x^2)^(1/2)*RootOf(_Z^4-8)*x-16*(x^6+x^2)^(3/4))/x/(x^2+1)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^6 + x^2)^(1/4)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(x^6+x^2\right)}^{1/4}\,\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + x^6)^(1/4)*(x^4 - 1)),x)
```

```
[Out] int(x^2/((x^2 + x^6)^(1/4)*(x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[4]{x^2 \left(x^4+1\right)} \left(x-1\right) \left(x+1\right) \left(x^2+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4-1)/(x**6+x**2)**(1/4),x)
```

```
[Out] Integral(x**2/((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.1805 $\int \frac{x^2}{(-1+x^4)\sqrt[4]{x^2+x^6}}\,dx$

Optimal. Leaf size=162

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{4\,2^{3/4}}$$

Rubi [C] time = 0.13, antiderivative size = 46, normalized size of antiderivative = 0.28, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2042, 466, 510}

$$-\frac{2x^3\sqrt[4]{x^4+1}\,F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*x^3*(1 + x^4)^(1/4)*AppellF1[5/8, 1, 1/4, 13/8, x^4, -x^4])/(5*(x^2 + x
^6)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] && !IntegerQ[p]
&& NeQ[b*c - a*d, 0] && !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\int \frac{x^2}{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}\,dx = \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^4}\right)\int \frac{x^{3/2}}{\left(-1+x^4\right)\sqrt[4]{1+x^4}}\,dx}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{x^4}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= -\frac{2x^3\sqrt[4]{1+x^4}\,F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^2+x^6}}$$

Mathematica [C] time = 0.02, size = 44, normalized size = 0.27

$$-\frac{2x\left(x^6+x^2\right)^{3/4}F_1\left(\frac{5}{8};\frac{1}{4},1;\frac{13}{8};-x^4,x^4\right)}{5\left(x^4+1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*x*(x^2 + x^6)^(3/4)*AppellF1[5/8, 1/4, 1, 13/8, -x^4, x^4])/(5*(1 + x^4
)^(3/4))
```

IntegrateAlgebraic [A] time = 0.00, size = 162, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}\,x^2-\sqrt{x^6+x^2}}\right)}{4\;2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{4\;2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] -1/4*ArcTan[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/2^(1/4) - ArcTan[(2^(3/4)*x*(x^2
 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^2 + x^6])]/(4*2^(3/4)) - ArcTanh[(2^(1
/4)*x)/(x^2 + x^6)^(1/4)]/(4*2^(1/4)) + ArcTanh[(x^2/2^(1/4) + Sqrt[x^2 + x
^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))]/(4*2^(3/4))
```

fricas [B] time = 13.51, size = 712, normalized size = 4.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/8*2^(3/4)*arctan(1/2*2^(3/4)*(x^6 + x^2)^(1/4)*(x^4 + 1)/(x^5 + x)) - 1/3
2*2^(3/4)*log((4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 2*2^(3/4)*(x^6 + x^2)^(3/4
) + sqrt(2)*(x^5 + 2*x^3 + x) + 4*sqrt(x^6 + x^2)*x)/(x^5 - 2*x^3 + x)) + 1
/32*2^(3/4)*log(-(4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 2*2^(3/4)*(x^6 + x^2)^(
3/4) - sqrt(2)*(x^5 + 2*x^3 + x) - 4*sqrt(x^6 + x^2)*x)/(x^5 - 2*x^3 + x))
+ 1/8*2^(1/4)*arctan(1/2*(4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(2*2^(3
/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 + 2*x^3 + x) - 4*sqrt(x^6 + x^2)*x
 + 2*2^(1/4)*(x^6 + x^2)^(3/4))*sqrt((x^5 + 2*x^3 + 4*2^(1/4)*(x^6 + x^2)^(
1/4)*x^2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x + 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(
```

```
x^5 + 2*x^3 + x)) + 2*2^(3/4)*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 1/8*2
^(1/4)*arctan(1/2*(4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(2*2^(3/4)*(x^
6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 4*sqrt(x^6 + x^2)*x + 2*2^
(1/4)*(x^6 + x^2)^(3/4))*sqrt((x^5 + 2*x^3 - 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^
2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x - 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2
*x^3 + x)) + 2*2^(3/4)*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 1/32*2^(1/4)
*log(2*(x^5 + 2*x^3 + 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^6
+ x^2)*x + 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x)) - 1/32*2^(1/
4)*log(2*(x^5 + 2*x^3 - 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^
6 + x^2)*x - 2*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^6 + x^2)^(1/4)*(x^4 - 1)), x)
```

maple [C] time = 22.58, size = 632, normalized size = 3.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4-1)/(x^6+x^2)^(1/4),x)
```

```
[Out] -1/16*RootOf(_Z^4-8)*ln((RootOf(_Z^4-8)^3*(x^6+x^2)^(1/2)*x+RootOf(_Z^4-8)*
x^5+2*RootOf(_Z^4-8)^2*(x^6+x^2)^(1/4)*x^2+2*RootOf(_Z^4-8)*x^3+4*(x^6+x^2)
^(3/4)+RootOf(_Z^4-8)*x)/(1+x)^2/(-1+x)^2/x)+1/16*RootOf(_Z^2+RootOf(_Z^4-8
)^2)*ln((RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*(x^6+x^2)^(1/2)*x-R
ootOf(_Z^2+RootOf(_Z^4-8)^2)*x^5-2*RootOf(_Z^4-8)^2*(x^6+x^2)^(1/4)*x^2-2*R
ootOf(_Z^2+RootOf(_Z^4-8)^2)*x^3+4*(x^6+x^2)^(3/4)-RootOf(_Z^2+RootOf(_Z^4-
8)^2)*x)/(1+x)^2/(-1+x)^2/x)-1/32*ln(-(RootOf(_Z^4-8)^2*x^2-2*RootOf(_Z^4-8
)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))/x/(x^2+1))*RootOf(_Z^4-8)^3-1/32*ln(
-(RootOf(_Z^4-8)^2*x^2-2*RootOf(_Z^4-8)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2)
)/x/(x^2+1))*RootOf(_Z^4-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)+1/32*RootOf(_Z^
4-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln((RootOf(_Z^4-8)^3*x^5-RootOf(_Z^2+R
ootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x^5-2*RootOf(_Z^4-8)^3*x^3+2*RootOf(_Z^2+
RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x^3-8*RootOf(_Z^2+RootOf(_Z^4-8)^2)*Root
Of(_Z^4-8)*(x^6+x^2)^(1/4)*x^2+RootOf(_Z^4-8)^3*x-RootOf(_Z^2+RootOf(_Z^4-8
)^2)*RootOf(_Z^4-8)^2*x+8*(x^6+x^2)^(1/2)*RootOf(_Z^4-8)*x+8*RootOf(_Z^2+Ro
otOf(_Z^4-8)^2)*(x^6+x^2)^(1/2)*x-16*(x^6+x^2)^(3/4))/x/(x^2+1)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^6 + x^2)^(1/4)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(x^6+x^2\right)}^{1/4}\,\left(x^4-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + x^6)^(1/4)*(x^4 - 1)),x)
[Out] int(x^2/((x^2 + x^6)^(1/4)*(x^4 - 1)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[4]{x^2(x^4+1)}\,(x-1)(x+1)(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4-1)/(x**6+x**2)**(1/4),x)
[Out] Integral(x**2/((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.1806 $\int \frac{1+x^4}{(-1+x^4)\sqrt[4]{x^2+x^6}}\,dx$

Optimal. Leaf size=162

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{2\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\,2^{3/4}}$$

Rubi [C] time = 0.09, antiderivative size = 42, normalized size of antiderivative = 0.26, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2056, 466, 429}

$$-\frac{2x\sqrt[4]{x^4+1}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, -3/4, 9/8, x^4, -x^4])/(x^2 + x^6)^(
1/4)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1+x^4}{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{\left(1+x^4\right)^{3/4}}{\sqrt{x}\left(-1+x^4\right)}\,dx}{\sqrt[4]{x^2+x^6}}$$
$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{\left(1+x^8\right)^{3/4}}{-1+x^8}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$
$$= -\frac{2x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^2+x^6}}$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.26

$$-\frac{2x\sqrt[4]{x^4+1}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};-x^4,x^4\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*x*(1 + x^4)^(1/4)*AppellF1[1/8, -3/4, 1, 9/8, -x^4, x^4])/(x^2 + x^6)^(
1/4)
```

IntegrateAlgebraic [A] time = 0.51, size = 162, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{2\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/2^(1/4) + ArcTan[(2^(3/4)*x*(x^2
 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^2 + x^6])]/(2*2^(3/4)) - ArcTanh[(2^(1
/4)*x)/(x^2 + x^6)^(1/4)]/(2*2^(1/4)) - ArcTanh[(x^2/2^(1/4) + Sqrt[x^2 + x
^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))]/(2*2^(3/4))
```

fricas [B] time = 49.88, size = 1002, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/4*2^(3/4)*arctan(1/2*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + 2^(3/4)*(2*2^(3/
4)*sqrt(x^6 + x^2)*x + 2^(1/4)*(x^5 + 2*x^3 + x)) + 4*2^(1/4)*(x^6 + x^2)^(
3/4))/(x^5 - 2*x^3 + x)) - 1/16*2^(3/4)*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4)*x
^2 + 2^(3/4)*(x^5 + 2*x^3 + x) + 4*2^(1/4)*sqrt(x^6 + x^2)*x + 4*(x^6 + x^2
)^(3/4))/(x^5 - 2*x^3 + x)) + 1/16*2^(3/4)*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4
)*x^2 - 2^(3/4)*(x^5 + 2*x^3 + x) - 4*2^(1/4)*sqrt(x^6 + x^2)*x + 4*(x^6 +
x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 1/4*2^(1/4)*arctan(-1/2*(2*x^9 + 8*x^7 + 1
2*x^5 + 8*x^3 + 4*2^(3/4)*(x^6 + x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 8*sqrt(2)*s
qrt(x^6 + x^2)*(x^5 + 2*x^3 + x) - sqrt(2)*(32*sqrt(2)*(x^6 + x^2)^(3/4)*x^
2 + 2^(3/4)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3 + x) + 4*2^(1/4)*sqrt(x^6 + x^2)
```

```
*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt((4*2^(3/
4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x
+ 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x)) + 8*2^(1/4)*(3*x^6 - 2*x^
4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3 + x)) -
1/4*2^(1/4)*arctan(-1/2*(2*x^9 + 8*x^7 + 12*x^5 + 8*x^3 - 4*2^(3/4)*(x^6 +
x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 8*sqrt(2)*sqrt(x^6 + x^2)*(x^5 + 2*x^3 + x)
- sqrt(2)*(32*sqrt(2)*(x^6 + x^2)^(3/4)*x^2 - 2^(3/4)*(x^9 - 16*x^7 - 2*x^5
 - 16*x^3 + x) - 4*2^(1/4)*sqrt(x^6 + x^2)*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x
^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt(-(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(
2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(
x^5 + 2*x^3 + x)) - 8*2^(1/4)*(3*x^6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2
*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3 + x)) - 1/16*2^(1/4)*log(8*(4*2^(3/4)*(x
^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x + 4*2
^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x)) + 1/16*2^(1/4)*log(-8*(4*2^(3/
4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*x
+ 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/((x^6 + x^2)^(1/4)*(x^4 - 1)), x)
```

maple [C] time = 21.05, size = 636, normalized size = 3.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^4-1)/(x^6+x^2)^(1/4),x)
```

```
[Out] 1/8*RootOf(_Z^4+8)*ln(-(RootOf(_Z^4+8)^3*(x^6+x^2)^(1/2)*x-RootOf(_Z^4+8)*x
^5-2*RootOf(_Z^4+8)^2*(x^6+x^2)^(1/4)*x^2+2*RootOf(_Z^4+8)*x^3+4*(x^6+x^2)^
(3/4)-RootOf(_Z^4+8)*x)/x/(x^2+1)^2)-1/8*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln(-
(RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*(x^6+x^2)^(1/2)*x+RootOf(_Z
^2+RootOf(_Z^4+8)^2)*x^5+2*RootOf(_Z^4+8)^2*(x^6+x^2)^(1/4)*x^2-2*RootOf(_Z
^2+RootOf(_Z^4+8)^2)*x^3+4*(x^6+x^2)^(3/4)+RootOf(_Z^2+RootOf(_Z^4+8)^2)*x)
/x/(x^2+1)^2)+1/16*ln((RootOf(_Z^4+8)^2*x^2+2*RootOf(_Z^4+8)*(x^6+x^2)^(1/4
)*x+2*(x^6+x^2)^(1/2))/(1+x)/x/(-1+x))*RootOf(_Z^4+8)^3+1/16*ln((RootOf(_Z^
4+8)^2*x^2+2*RootOf(_Z^4+8)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))/(1+x)/x/(-
1+x))*RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z^4+8)^2)-1/16*RootOf(_Z^4+8)^2*
RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln(-(RootOf(_Z^4+8)^3*x^5-RootOf(_Z^2+RootOf(
_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^5+2*RootOf(_Z^4+8)^3*x^3-2*RootOf(_Z^2+RootOf
(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^3-8*RootOf(_Z^4+8)*RootOf(_Z^2+RootOf(_Z^4+8
)^2)*(x^6+x^2)^(1/4)*x^2+RootOf(_Z^4+8)^3*x-RootOf(_Z^2+RootOf(_Z^4+8)^2)*R
ootOf(_Z^4+8)^2*x-8*(x^6+x^2)^(1/2)*RootOf(_Z^4+8)*x-8*RootOf(_Z^2+RootOf(_
Z^4+8)^2)*(x^6+x^2)^(1/2)*x-16*(x^6+x^2)^(3/4))/(1+x)^2/(-1+x)^2/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/((x^6 + x^2)^(1/4)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4+1}{\left(x^6+x^2\right)^{1/4}\,\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/((x^2 + x^6)^(1/4)*(x^4 - 1)),x)
```

```
[Out] int((x^4 + 1)/((x^2 + x^6)^(1/4)*(x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\sqrt[4]{x^2\left(x^4+1\right)}\,(x-1)\,(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**4-1)/(x**6+x**2)**(1/4),x)
```

```
[Out] Integral((x**4 + 1)/((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)),
x)
```

3.1807 $\int \frac{1+x^4}{\left(-1+x^4\right) \sqrt[4]{x^2+x^6}} \, dx$

Optimal. Leaf size=162

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{2\ 2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\ 2^{3/4}}$$

Rubi [C] time = 0.10, antiderivative size = 42, normalized size of antiderivative = 0.26, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2056, 466, 429}

$$-\frac{2x\sqrt[4]{x^4+1}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, -3/4, 9/8, x^4, -x^4])/(x^2 + x^6)^(
1/4)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1+x^4}{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{\left(1+x^4\right)^{3/4}}{\sqrt{x}\left(-1+x^4\right)}\,dx}{\sqrt[4]{x^2+x^6}}$$
$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{\left(1+x^8\right)^{3/4}}{-1+x^8}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$
$$= -\frac{2x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^2+x^6}}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.26

$$-\frac{2x\sqrt[4]{x^4+1}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};-x^4,x^4\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*x*(1 + x^4)^(1/4)*AppellF1[1/8, -3/4, 1, 9/8, -x^4, x^4])/(x^2 + x^6)^(
1/4)
```

IntegrateAlgebraic [A] time = 0.00, size = 162, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{2\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/2^(1/4) + ArcTan[(2^(3/4)*x*(x^2
 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^2 + x^6])]/(2*2^(3/4)) - ArcTanh[(2^(1
/4)*x)/(x^2 + x^6)^(1/4)]/(2*2^(1/4)) - ArcTanh[(x^2/2^(1/4) + Sqrt[x^2 + x
^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))]/(2*2^(3/4))
```

fricas [B] time = 50.47, size = 1002, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/4*2^(3/4)*arctan(1/2*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + 2^(3/4)*(2*2^(3/
4)*sqrt(x^6 + x^2)*x + 2^(1/4)*(x^5 + 2*x^3 + x)) + 4*2^(1/4)*(x^6 + x^2)^(
3/4))/(x^5 - 2*x^3 + x)) - 1/16*2^(3/4)*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4)*x
^2 + 2^(3/4)*(x^5 + 2*x^3 + x) + 4*2^(1/4)*sqrt(x^6 + x^2)*x + 4*(x^6 + x^2
)^(3/4))/(x^5 - 2*x^3 + x)) + 1/16*2^(3/4)*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4
)*x^2 - 2^(3/4)*(x^5 + 2*x^3 + x) - 4*2^(1/4)*sqrt(x^6 + x^2)*x + 4*(x^6 +
x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 1/4*2^(1/4)*arctan(-1/2*(2*x^9 + 8*x^7 + 1
2*x^5 + 8*x^3 + 4*2^(3/4)*(x^6 + x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 8*sqrt(2)*s
qrt(x^6 + x^2)*(x^5 + 2*x^3 + x) - sqrt(2)*(32*sqrt(2)*(x^6 + x^2)^(3/4)*x^
2 + 2^(3/4)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3 + x) + 4*2^(1/4)*sqrt(x^6 + x^2)
```

```
*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt((4*2^(3/
4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x
+ 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x)) + 8*2^(1/4)*(3*x^6 - 2*x^
4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3 + x)) -
1/4*2^(1/4)*arctan(-1/2*(2*x^9 + 8*x^7 + 12*x^5 + 8*x^3 - 4*2^(3/4)*(x^6 +
x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 8*sqrt(2)*sqrt(x^6 + x^2)*(x^5 + 2*x^3 + x)
- sqrt(2)*(32*sqrt(2)*(x^6 + x^2)^(3/4)*x^2 - 2^(3/4)*(x^9 - 16*x^7 - 2*x^5
 - 16*x^3 + x) - 4*2^(1/4)*sqrt(x^6 + x^2)*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x
^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt(-(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(
2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(
x^5 + 2*x^3 + x)) - 8*2^(1/4)*(3*x^6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2
*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3 + x)) - 1/16*2^(1/4)*log(8*(4*2^(3/4)*(x
^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x + 4*2
^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x)) + 1/16*2^(1/4)*log(-8*(4*2^(3/
4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*x
+ 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)/((x^6 + x^2)^(1/4)*(x^4 - 1)), x)
```

maple [C] time = 21.34, size = 633, normalized size = 3.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^4-1)/(x^6+x^2)^(1/4),x)
```

```
[Out] -1/8*RootOf(_Z^4-8)*ln((RootOf(_Z^4-8)^3*(x^6+x^2)^(1/2)*x+RootOf(_Z^4-8)*x
^5+2*RootOf(_Z^4-8)^2*(x^6+x^2)^(1/4)*x^2+2*RootOf(_Z^4-8)*x^3+4*(x^6+x^2)^
(3/4)+RootOf(_Z^4-8)*x)/(1+x)^2/(-1+x)^2/x)+1/8*RootOf(_Z^2+RootOf(_Z^4-8)^
2)*ln((RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*(x^6+x^2)^(1/2)*x-Roo
tOf(_Z^2+RootOf(_Z^4-8)^2)*x^5-2*RootOf(_Z^4-8)^2*(x^6+x^2)^(1/4)*x^2-2*Roo
tOf(_Z^2+RootOf(_Z^4-8)^2)*x^3+4*(x^6+x^2)^(3/4)-RootOf(_Z^2+RootOf(_Z^4-8)
^2)*x)/(1+x)^2/(-1+x)^2/x)+1/16*ln(-(RootOf(_Z^4-8)^2*x^2-2*RootOf(_Z^4-8)*
(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))/x/(x^2+1))*RootOf(_Z^4-8)^3+1/16*ln(-(
RootOf(_Z^4-8)^2*x^2-2*RootOf(_Z^4-8)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))/
x/(x^2+1))*RootOf(_Z^4-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)-1/16*RootOf(_Z^4-
8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln(-(RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootO
f(_Z^4-8)^2*x^5-RootOf(_Z^4-8)^3*x^5-2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf
(_Z^4-8)^2*x^3+2*RootOf(_Z^4-8)^3*x^3+8*RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootO
f(_Z^4-8)*(x^6+x^2)^(1/4)*x^2+RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^
2*x-8*RootOf(_Z^2+RootOf(_Z^4-8)^2)*(x^6+x^2)^(1/2)*x-RootOf(_Z^4-8)^3*x-8*
(x^6+x^2)^(1/2)*RootOf(_Z^4-8)*x+16*(x^6+x^2)^(3/4))/x/(x^2+1)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/((x^6 + x^2)^(1/4)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4+1}{\left(x^6+x^2\right)^{1/4}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/((x^2 + x^6)^(1/4)*(x^4 - 1)),x)
```

```
[Out] int((x^4 + 1)/((x^2 + x^6)^(1/4)*(x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\sqrt[4]{x^2\left(x^4+1\right)}\,(x-1)\,(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**4-1)/(x**6+x**2)**(1/4),x)
```

```
[Out] Integral((x**4 + 1)/((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)),
x)
```

3.1808 $\int \frac{1-x^2+x^4}{(-1+x^4)\sqrt[4]{x^2+x^6}}\,dx$

Optimal. Leaf size=162

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}+\frac{3\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}-\frac{3\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{4\,2^{3/4}}$$

Rubi [C] time = 0.77, antiderivative size = 127, normalized size of antiderivative = 0.78, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2056, 6715, 6725, 245, 1438, 429, 510}

$$-\frac{4\sqrt[4]{x^4+1}\,xF_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^6+x^2}}+\frac{2\sqrt[4]{x^4+1}\,x^3F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^6+x^2}}+\frac{2\sqrt[4]{x^4+1}\,x\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x^2 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-4*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, x^4, -x^4])/(x^2 + x^6)^(1
/4) + (2*x^3*(1 + x^4)^(1/4)*AppellF1[5/8, 1, 1/4, 13/8, x^4, -x^4])/(5*(x^
2 + x^6)^(1/4)) + (2*x*(1 + x^4)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -x^
4])/(x^2 + x^6)^(1/4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
```

```
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1-x^2+x^4}{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{1-x^2+x^4}{\sqrt{x}\left(-1+x^4\right)\sqrt[4]{1+x^4}}\,dx}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\mathrm{Subst}\left(\int \frac{1-x^4+x^8}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\mathrm{Subst}\left(\int \left(\frac{1}{\sqrt[4]{1+x^8}}+\frac{2-x^4}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\right)dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\mathrm{Subst}\left(\int \frac{2-x^4}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\mathrm{Subst}\left(\int\left(-\frac{1}{2\left(1-x^4\right)\sqrt[4]{1+x^8}}-\frac{3}{2\left(1+x^4\right)\sqrt[4]{1}}\right.\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-x^4\right)\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} - \frac{(3}{}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\mathrm{Subst}\left(\int\left(\frac{1}{\left(1-x^8\right)\sqrt[4]{1+x^8}}-\frac{x^4}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\right.\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-x^8\right)\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} + \frac{(\sqrt{}}{}$$

$$= -\frac{4x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^2+x^6}} + \frac{2x^3\sqrt[4]{1+x^4}\,F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^2+x^6}} + \frac{2x\sqrt[4]{1+}}{}$$

Mathematica [C] time = 0.28, size = 95, normalized size = 0.59

$$-\frac{2\sqrt[4]{x^4+1}\left(45xF_1\left(\frac{1}{8};\frac{1}{4},1;\frac{9}{8};-x^4,x^4\right)+5x^5F_1\left(\frac{9}{8};\frac{1}{4},1;\frac{17}{8};-x^4,x^4\right)-9x^3F_1\left(\frac{5}{8};\frac{1}{4},1;\frac{13}{8};-x^4,x^4\right)\right)}{45\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^2 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*(1 + x^4)^(1/4)*(45*x*AppellF1[1/8, 1/4, 1, 9/8, -x^4, x^4] - 9*x^3*App
ellF1[5/8, 1/4, 1, 13/8, -x^4, x^4] + 5*x^5*AppellF1[9/8, 1/4, 1, 17/8, -x^
4, x^4]))/(45*(x^2 + x^6)^(1/4))
```

IntegrateAlgebraic [A] time = 0.78, size = 162, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}+\frac{3\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}-\frac{3\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^2 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] -1/4*ArcTan[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/2^(1/4) + (3*ArcTan[(2^(3/4)*x*(
x^2 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^2 + x^6])])/(4*2^(3/4)) - ArcTanh[(
2^(1/4)*x)/(x^2 + x^6)^(1/4)]/(4*2^(1/4)) - (3*ArcTanh[(x^2/2^(1/4) + Sqrt[
x^2 + x^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))])/(4*2^(3/4))
```

fricas [B] time = 99.26, size = 1002, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/8*2^(3/4)*arctan(1/2*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + 2^(3/4)*(2*2^(3/
4)*sqrt(x^6 + x^2)*x + 2^(1/4)*(x^5 + 2*x^3 + x)) + 4*2^(1/4)*(x^6 + x^2)^(
3/4))/(x^5 - 2*x^3 + x)) - 1/32*2^(3/4)*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4)*x
^2 + 2^(3/4)*(x^5 + 2*x^3 + x) + 4*2^(1/4)*sqrt(x^6 + x^2)*x + 4*(x^6 + x^2
)^(3/4))/(x^5 - 2*x^3 + x)) + 1/32*2^(3/4)*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4
)*x^2 - 2^(3/4)*(x^5 + 2*x^3 + x) - 4*2^(1/4)*sqrt(x^6 + x^2)*x + 4*(x^6 +
x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 3/8*2^(1/4)*arctan(-1/2*(2*x^9 + 8*x^7 + 1
2*x^5 + 8*x^3 + 4*2^(3/4)*(x^6 + x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 8*sqrt(2)*s
qrt(x^6 + x^2)*(x^5 + 2*x^3 + x) - sqrt(2)*(32*sqrt(2)*(x^6 + x^2)^(3/4)*x^
2 + 2^(3/4)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3 + x) + 4*2^(1/4)*sqrt(x^6 + x^2)
*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt((4*2^(3/
4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x
+ 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x)) + 8*2^(1/4)*(3*x^6 - 2*x^
4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3 + x)) -
3/8*2^(1/4)*arctan(-1/2*(2*x^9 + 8*x^7 + 12*x^5 + 8*x^3 - 4*2^(3/4)*(x^6 +
x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 8*sqrt(2)*sqrt(x^6 + x^2)*(x^5 + 2*x^3 + x)
- sqrt(2)*(32*sqrt(2)*(x^6 + x^2)^(3/4)*x^2 - 2^(3/4)*(x^9 - 16*x^7 - 2*x^5
 - 16*x^3 + x) - 4*2^(1/4)*sqrt(x^6 + x^2)*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x
^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt(-(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(
2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(
x^5 + 2*x^3 + x)) - 8*2^(1/4)*(3*x^6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2
*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3 + x)) - 3/32*2^(1/4)*log(8*(4*2^(3/4)*(x
^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x + 4*2
^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x)) + 3/32*2^(1/4)*log(-8*(4*2^(3/
4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*x
+ 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 - x^2 + 1}{\left(x^6 + x^2\right)^{\frac{1}{4}}\left(x^4 - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^4 - x^2 + 1)/((x^6 + x^2)^(1/4)*(x^4 - 1)), x)
```

maple [C] time = 23.25, size = 627, normalized size = 3.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^2+1)/(x^4-1)/(x^6+x^2)^(1/4),x)
```

```
[Out] -3/16*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln((-RootOf(_Z^4+8)^2*x^5+2*RootOf(_Z^2
+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*(x^6+x^2)^(1/4)*x^2+2*x^3*RootOf(_Z^4+8
)^2+4*RootOf(_Z^2+RootOf(_Z^4+8)^2)*(x^6+x^2)^(3/4)-RootOf(_Z^4+8)^2*x+8*(x
^6+x^2)^(1/2)*x)/x/(x^2+1)^2)+3/16*RootOf(_Z^4+8)*ln(-(RootOf(_Z^4+8)^3*(x^
6+x^2)^(1/2)*x-RootOf(_Z^4+8)*x^5-2*RootOf(_Z^4+8)^2*(x^6+x^2)^(1/4)*x^2+2*
RootOf(_Z^4+8)*x^3+4*(x^6+x^2)^(3/4)-RootOf(_Z^4+8)*x)/x/(x^2+1)^2)+1/32*ln
((RootOf(_Z^4+8)^2*x^2+2*RootOf(_Z^4+8)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2)
)/(1+x)/x/(-1+x))*RootOf(_Z^4+8)^3+1/32*ln((RootOf(_Z^4+8)^2*x^2+2*RootOf(_
Z^4+8)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))/(1+x)/x/(-1+x))*RootOf(_Z^4+8)^
2*RootOf(_Z^2+RootOf(_Z^4+8)^2)-1/32*RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z
^4+8)^2)*ln((RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^5-RootOf(_Z^4
+8)^3*x^5+2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^3-2*RootOf(_Z^
4+8)^3*x^3+8*RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)*(x^6+x^2)^(1/4)*x
^2+RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x+8*RootOf(_Z^2+RootOf(_Z
^4+8)^2)*(x^6+x^2)^(1/2)*x-RootOf(_Z^4+8)^3*x+8*(x^6+x^2)^(1/2)*RootOf(_Z^4
+8)*x+16*(x^6+x^2)^(3/4))/(1+x)^2/(-1+x)^2/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 - x^2 + 1}{\left(x^6 + x^2\right)^{\frac{1}{4}}\left(x^4 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^2 + 1)/((x^6 + x^2)^(1/4)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4 - x^2 + 1}{\left(x^6 + x^2\right)^{1/4}\,\left(x^4 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x^2 + 1)/((x^2 + x^6)^(1/4)*(x^4 - 1)),x)
```

```
[Out] int((x^4 - x^2 + 1)/((x^2 + x^6)^(1/4)*(x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 - x^2 + 1}{\sqrt[4]{x^2\left(x^4 + 1\right)}\,\left(x - 1\right)\left(x + 1\right)\left(x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**2+1)/(x**4-1)/(x**6+x**2)**(1/4),x)
```

```
[Out] Integral((x**4 - x**2 + 1)/((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2
+ 1)), x)
```

3.1809 $\int \frac{1-x^2+x^4}{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}\,dx$

Optimal. Leaf size=162

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}+\frac{3\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}-\frac{3\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{4\,2^{3/4}}$$

Rubi [C] time = 0.62, antiderivative size = 127, normalized size of antiderivative = 0.78, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2056, 6715, 6725, 245, 1438, 429, 510}

$$-\frac{4\sqrt[4]{x^4+1}\,xF_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^6+x^2}}+\frac{2\sqrt[4]{x^4+1}\,x^3F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^6+x^2}}+\frac{2\sqrt[4]{x^4+1}\,x\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x^2 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-4*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, x^4, -x^4])/(x^2 + x^6)^(1
/4) + (2*x^3*(1 + x^4)^(1/4)*AppellF1[5/8, 1, 1/4, 13/8, x^4, -x^4])/(5*(x^
2 + x^6)^(1/4)) + (2*x*(1 + x^4)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -x^
4])/(x^2 + x^6)^(1/4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
```

```
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1-x^2+x^4}{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}\,dx = \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^4}\right)\int \frac{1-x^2+x^4}{\sqrt{x}\left(-1+x^4\right)\sqrt[4]{1+x^4}}\,dx}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1-x^4+x^8}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[4]{1+x^8}}+\frac{2-x^4}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\right)dx,x,\sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{2-x^4}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int\left(-\frac{1}{2\left(1-x^4\right)\sqrt[4]{1+x^8}}-\frac{3}{2\left(1+x^4\right)\sqrt[4]{\ }}\right.\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(1-x^4\right)\sqrt[4]{1+x^8}}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} - \frac{(3}{}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int\left(\frac{1}{\left(1-x^8\right)\sqrt[4]{1+x^8}}-\frac{x^4}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\right.\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(1-x^8\right)\sqrt[4]{1+x^8}}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} + \frac{(}{}$$

$$= -\frac{4x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^2+x^6}} + \frac{2x^3\sqrt[4]{1+x^4}\,F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^2+x^6}} + \frac{2x\sqrt[4]{1+}}{}$$

Mathematica [C] time = 0.18, size = 95, normalized size = 0.59

$$-\frac{2\sqrt[4]{x^4+1}\left(45xF_1\left(\frac{1}{8};\frac{1}{4},1;\frac{9}{8};-x^4,x^4\right)+5x^5F_1\left(\frac{9}{8};\frac{1}{4},1;\frac{17}{8};-x^4,x^4\right)-9x^3F_1\left(\frac{5}{8};\frac{1}{4},1;\frac{13}{8};-x^4,x^4\right)\right)}{45\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^2 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*(1 + x^4)^(1/4)*(45*x*AppellF1[1/8, 1/4, 1, 9/8, -x^4, x^4] - 9*x^3*App
ellF1[5/8, 1/4, 1, 13/8, -x^4, x^4] + 5*x^5*AppellF1[9/8, 1/4, 1, 17/8, -x^
4, x^4]))/(45*(x^2 + x^6)^(1/4))
```

IntegrateAlgebraic [A] time = 0.00, size = 162, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}+\frac{3\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}-\frac{3\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^2 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] -1/4*ArcTan[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/2^(1/4) + (3*ArcTan[(2^(3/4)*x*(
x^2 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^2 + x^6])])/(4*2^(3/4)) - ArcTanh[(
2^(1/4)*x)/(x^2 + x^6)^(1/4)]/(4*2^(1/4)) - (3*ArcTanh[(x^2/2^(1/4) + Sqrt[
x^2 + x^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))])/(4*2^(3/4))
```

fricas [B] time = 98.76, size = 1002, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/8*2^(3/4)*arctan(1/2*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + 2^(3/4)*(2*2^(3/
4)*sqrt(x^6 + x^2)*x + 2^(1/4)*(x^5 + 2*x^3 + x)) + 4*2^(1/4)*(x^6 + x^2)^(
3/4))/(x^5 - 2*x^3 + x)) - 1/32*2^(3/4)*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4)*x
^2 + 2^(3/4)*(x^5 + 2*x^3 + x) + 4*2^(1/4)*sqrt(x^6 + x^2)*x + 4*(x^6 + x^2
)^(3/4))/(x^5 - 2*x^3 + x)) + 1/32*2^(3/4)*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4
)*x^2 - 2^(3/4)*(x^5 + 2*x^3 + x) - 4*2^(1/4)*sqrt(x^6 + x^2)*x + 4*(x^6 +
x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 3/8*2^(1/4)*arctan(-1/2*(2*x^9 + 8*x^7 + 1
2*x^5 + 8*x^3 + 4*2^(3/4)*(x^6 + x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 8*sqrt(2)*s
qrt(x^6 + x^2)*(x^5 + 2*x^3 + x) - sqrt(2)*(32*sqrt(2)*(x^6 + x^2)^(3/4)*x^
2 + 2^(3/4)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3 + x) + 4*2^(1/4)*sqrt(x^6 + x^2)
*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt((4*2^(3/
4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x
+ 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x)) + 8*2^(1/4)*(3*x^6 - 2*x^
4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3 + x)) -
3/8*2^(1/4)*arctan(-1/2*(2*x^9 + 8*x^7 + 12*x^5 + 8*x^3 - 4*2^(3/4)*(x^6 +
x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 8*sqrt(2)*sqrt(x^6 + x^2)*(x^5 + 2*x^3 + x)
- sqrt(2)*(32*sqrt(2)*(x^6 + x^2)^(3/4)*x^2 - 2^(3/4)*(x^9 - 16*x^7 - 2*x^5
 - 16*x^3 + x) - 4*2^(1/4)*sqrt(x^6 + x^2)*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x
^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt(-(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(
2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(
x^5 + 2*x^3 + x)) - 8*2^(1/4)*(3*x^6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2
*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3 + x)) - 3/32*2^(1/4)*log(8*(4*2^(3/4)*(x
^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x + 4*2
^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x)) + 3/32*2^(1/4)*log(-8*(4*2^(3/
4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*x
+ 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 - x^2 + 1}{\left(x^6 + x^2\right)^{\frac{1}{4}} \left(x^4 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^2+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="giac")

[Out] integrate((x^4 - x^2 + 1)/((x^6 + x^2)^(1/4)*(x^4 - 1)), x)

maple [C] time = 23.58, size = 633, normalized size = 3.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-x^2+1)/(x^4-1)/(x^6+x^2)^(1/4),x)

[Out] 1/16*RootOf(_Z^4-8)*ln(-(RootOf(_Z^4-8)^3*(x^6+x^2)^(1/2)*x+RootOf(_Z^4-8)*x^5-2*RootOf(_Z^4-8)^2*(x^6+x^2)^(1/4)*x^2+2*RootOf(_Z^4-8)*x^3-4*(x^6+x^2)^(3/4)+RootOf(_Z^4-8)*x)/(1+x)^2/(-1+x)^2/x)+1/16*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln((RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*(x^6+x^2)^(1/2)*x-RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^5-2*RootOf(_Z^4-8)^2*(x^6+x^2)^(1/4)*x^2-2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^3+4*(x^6+x^2)^(3/4)-RootOf(_Z^2+RootOf(_Z^4-8)^2)*x)/(1+x)^2/(-1+x)^2/x)+3/32*ln(-(RootOf(_Z^4-8)^2*x^2-2*RootOf(_Z^4-8)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))/x/(x^2+1))*RootOf(_Z^4-8)^3+3/32*ln(-(RootOf(_Z^4-8)^2*x^2-2*RootOf(_Z^4-8)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))/x/(x^2+1))*RootOf(_Z^4-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)-3/32*RootOf(_Z^4-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln((RootOf(_Z^4-8)^3*x^5-RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x^5-2*RootOf(_Z^4-8)^3*x^3+2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x^3-8*RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)*(x^6+x^2)^(1/4)*x^2+RootOf(_Z^4-8)^3*x-RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x+8*(x^6+x^2)^(1/2)*RootOf(_Z^4-8)*x+8*RootOf(_Z^2+RootOf(_Z^4-8)^2)*(x^6+x^2)^(1/2)*x-16*(x^6+x^2)^(3/4))/x/(x^2+1)^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 - x^2 + 1}{\left(x^6 + x^2\right)^{\frac{1}{4}} \left(x^4 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^2+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="maxima")

[Out] integrate((x^4 - x^2 + 1)/((x^6 + x^2)^(1/4)*(x^4 - 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4 - x^2 + 1}{\left(x^6 + x^2\right)^{1/4}\, \left(x^4 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4 - x^2 + 1)/((x^2 + x^6)^(1/4)*(x^4 - 1)),x)

[Out] int((x^4 - x^2 + 1)/((x^2 + x^6)^(1/4)*(x^4 - 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 - x^2 + 1}{\sqrt[4]{x^2 \left(x^4 + 1\right)}\, (x - 1)(x + 1)\left(x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4-x**2+1)/(x**4-1)/(x**6+x**2)**(1/4),x)

[Out] Integral((x**4 - x**2 + 1)/((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)), x)

3.1810 $\int \frac{1+x^2+x^4}{(-1+x^4)\sqrt[4]{x^2+x^6}}\,dx$

Optimal. Leaf size=162

$$-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{4\,2^{3/4}}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{4\,2^{3/4}}$$

Rubi [C] time = 0.64, antiderivative size = 127, normalized size of antiderivative = 0.78, number of steps used = 15, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2056, 6715, 6725, 245, 1438, 429, 510}

$$-\frac{4\sqrt[4]{x^4+1}\,xF_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^6+x^2}}-\frac{2\sqrt[4]{x^4+1}\,x^3F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^6+x^2}}+\frac{2\sqrt[4]{x^4+1}\,x\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^2 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-4*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, x^4, -x^4])/(x^2 + x^6)^(1
/4) - (2*x^3*(1 + x^4)^(1/4)*AppellF1[5/8, 1, 1/4, 13/8, x^4, -x^4])/(5*(x^
2 + x^6)^(1/4)) + (2*x*(1 + x^4)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -x^
4])/(x^2 + x^6)^(1/4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
```

```
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^2+x^4}{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}\,dx = \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^4}\right)\int \frac{1+x^2+x^4}{\sqrt{x}\left(-1+x^4\right)\sqrt[4]{1+x^4}}\,dx}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1+x^4+x^8}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[4]{1+x^8}}+\frac{2+x^4}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\right)dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{2+x^4}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \left(-\frac{3}{2\left(1-x^4\right)\sqrt[4]{1+x^8}}-\frac{1}{2\left(1+x^4\right)\sqrt[4]{}}\right.\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(1+x^4\right)\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} - \frac{(3}{}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\left(1-x^8\right)\sqrt[4]{1+x^8}}+\frac{x^4}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\right.\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(1-x^8\right)\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} - \frac{(\sqrt{}}{}$$

$$= -\frac{4x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{2x^3\sqrt[4]{1+x^4}\,F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^2+x^6}} + \frac{2x\sqrt[4]{1+}}{}$$

Mathematica [C] time = 0.20, size = 95, normalized size = 0.59

$$-\frac{2\sqrt[4]{x^4+1}\left(45xF_1\left(\frac{1}{8};\frac{1}{4},1;\frac{9}{8};-x^4,x^4\right)+5x^5F_1\left(\frac{9}{8};\frac{1}{4},1;\frac{17}{8};-x^4,x^4\right)+9x^3F_1\left(\frac{5}{8};\frac{1}{4},1;\frac{13}{8};-x^4,x^4\right)\right)}{45\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*(1 + x^4)^(1/4)*(45*x*AppellF1[1/8, 1/4, 1, 9/8, -x^4, x^4] + 9*x^3*App
ellF1[5/8, 1/4, 1, 13/8, -x^4, x^4] + 5*x^5*AppellF1[9/8, 1/4, 1, 17/8, -x^
4, x^4]))/(45*(x^2 + x^6)^(1/4))
```

IntegrateAlgebraic [A] time = 0.78, size = 162, normalized size = 1.00

$$-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{4\,2^{3/4}}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-3*ArcTan[(2^(1/4)*x)/(x^2 + x^6)^(1/4)])/(4*2^(1/4)) + ArcTan[(2^(3/4)*x*
(x^2 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^2 + x^6])]/(4*2^(3/4)) - (3*ArcTan
h[(2^(1/4)*x)/(x^2 + x^6)^(1/4)])/(4*2^(1/4)) - ArcTanh[(x^2/2^(1/4) + Sqrt
[x^2 + x^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))]/(4*2^(3/4))
```

fricas [B] time = 97.40, size = 1002, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] -3/8*2^(3/4)*arctan(1/2*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + 2^(3/4)*(2*2^(3/
4)*sqrt(x^6 + x^2)*x + 2^(1/4)*(x^5 + 2*x^3 + x)) + 4*2^(1/4)*(x^6 + x^2)^(
3/4))/(x^5 - 2*x^3 + x)) - 3/32*2^(3/4)*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4)*x
^2 + 2^(3/4)*(x^5 + 2*x^3 + x) + 4*2^(1/4)*sqrt(x^6 + x^2)*x + 4*(x^6 + x^2
)^(3/4))/(x^5 - 2*x^3 + x)) + 3/32*2^(3/4)*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4
)*x^2 - 2^(3/4)*(x^5 + 2*x^3 + x) - 4*2^(1/4)*sqrt(x^6 + x^2)*x + 4*(x^6 +
x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 1/8*2^(1/4)*arctan(-1/2*(2*x^9 + 8*x^7 + 1
2*x^5 + 8*x^3 + 4*2^(3/4)*(x^6 + x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 8*sqrt(2)*s
qrt(x^6 + x^2)*(x^5 + 2*x^3 + x) - sqrt(2)*(32*sqrt(2)*(x^6 + x^2)^(3/4)*x^
2 + 2^(3/4)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3 + x) + 4*2^(1/4)*sqrt(x^6 + x^2)
*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt((4*2^(3/
4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x
+ 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x)) + 8*2^(1/4)*(3*x^6 - 2*x^
4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3 + x)) -
1/8*2^(1/4)*arctan(-1/2*(2*x^9 + 8*x^7 + 12*x^5 + 8*x^3 - 4*2^(3/4)*(x^6 +
x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 8*sqrt(2)*sqrt(x^6 + x^2)*(x^5 + 2*x^3 + x)
- sqrt(2)*(32*sqrt(2)*(x^6 + x^2)^(3/4)*x^2 - 2^(3/4)*(x^9 - 16*x^7 - 2*x^5
 - 16*x^3 + x) - 4*2^(1/4)*sqrt(x^6 + x^2)*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x
^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt(-(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(
2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(
x^5 + 2*x^3 + x)) - 8*2^(1/4)*(3*x^6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2
*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3 + x)) - 1/32*2^(1/4)*log(8*(4*2^(3/4)*(x
^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x + 4*2
^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x)) + 1/32*2^(1/4)*log(-8*(4*2^(3/
4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*x
+ 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+x^2+1}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^4 + x^2 + 1)/((x^6 + x^2)^(1/4)*(x^4 - 1)), x)
```

maple [C] time = 23.74, size = 633, normalized size = 3.91

$$\frac{3\,\mathrm{RootOf}\left(\_Z^{4}-8\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^{4}-8\right)^{3}\sqrt{x^{6}+x^{2}}\,x+\mathrm{RootOf}\left(\_Z^{4}-8\right)x^{5}-2\,\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}\left(x^{6}+x^{2}\right)^{\frac{1}{4}}x^{2}+2\,\mathrm{RootOf}\left(\_Z^{4}-8\right)x^{3}-4\left(x^{6}+x^{2}\right)^{\frac{3}{4}}+\mathrm{RootOf}\left(\_Z^{4}-8\right)x}{\left(1+x\right)^{2}\left(-1+x\right)^{2}x}\right)}{16}-\frac{3\,\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}\right)\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}\sqrt{x^{6}+x^{2}}\,x-\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}\right)x^{5}+2\,\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}\left(x^{6}+x^{2}\right)^{\frac{1}{4}}x^{2}-2\,\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}\right)x^{3}-4\left(x^{6}+x^{2}\right)^{\frac{3}{4}}-\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}\right)x}{\left(1+x\right)^{2}\left(-1+x\right)^{2}x}\right)}{16}-\frac{\ln\left(\frac{\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}x^{2}+2\,\mathrm{RootOf}\left(\_Z^{4}-8\right)\left(x^{6}+x^{2}\right)^{\frac{1}{4}}x+2\sqrt{x^{6}+x^{2}}}{x\left(x^{2}+1\right)}\right)\mathrm{RootOf}\left(\_Z^{4}-8\right)^{3}}{32}-\frac{\ln\left(\frac{\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}x^{2}+2\,\mathrm{RootOf}\left(\_Z^{4}-8\right)\left(x^{6}+x^{2}\right)^{\frac{1}{4}}x+2\sqrt{x^{6}+x^{2}}}{x\left(x^{2}+1\right)}\right)\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}\right)}{32}+\frac{\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^{4}-8\right)^{3}x^{5}-\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}\right)\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}x^{5}-2\,\mathrm{RootOf}\left(\_Z^{4}-8\right)^{3}x^{3}+2\,\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}\right)\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}x^{3}+8\,\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}\right)\mathrm{RootOf}\left(\_Z^{4}-8\right)\left(x^{6}+x^{2}\right)^{\frac{1}{4}}x^{2}+\mathrm{RootOf}\left(\_Z^{4}-8\right)^{3}x-\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}\right)\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}x+8\sqrt{x^{6}+x^{2}}\,\mathrm{RootOf}\left(\_Z^{4}-8\right)x+8\,\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}-8\right)^{2}\right)\sqrt{x^{6}+x^{2}}\,x+16\left(x^{6}+x^{2}\right)^{\frac{3}{4}}}{x\left(x^{2}+1\right)^{2}}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)/(x^4-1)/(x^6+x^2)^(1/4),x)
```

```
[Out] 3/16*RootOf(_Z^4-8)*ln(-(RootOf(_Z^4-8)^3*(x^6+x^2)^(1/2)*x+RootOf(_Z^4-8)*
x^5-2*RootOf(_Z^4-8)^2*(x^6+x^2)^(1/4)*x^2+2*RootOf(_Z^4-8)*x^3-4*(x^6+x^2)
^(3/4)+RootOf(_Z^4-8)*x)/(1+x)^2/(-1+x)^2/x)-3/16*RootOf(_Z^2+RootOf(_Z^4-8
)^2)*ln(-(RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*(x^6+x^2)^(1/2)*x-
RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^5+2*RootOf(_Z^4-8)^2*(x^6+x^2)^(1/4)*x^2-2*
RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^3-4*(x^6+x^2)^(3/4)-RootOf(_Z^2+RootOf(_Z^4
-8)^2)*x)/(1+x)^2/(-1+x)^2/x)-1/32*ln((RootOf(_Z^4-8)^2*x^2+2*RootOf(_Z^4-8
)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))/x/(x^2+1))*RootOf(_Z^4-8)^3-1/32*ln(
(RootOf(_Z^4-8)^2*x^2+2*RootOf(_Z^4-8)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))
/x/(x^2+1))*RootOf(_Z^4-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)+1/32*RootOf(_Z^4
-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln(-(RootOf(_Z^4-8)^3*x^5-RootOf(_Z^2+R
ootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x^5-2*RootOf(_Z^4-8)^3*x^3+2*RootOf(_Z^2+
RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x^3+8*RootOf(_Z^2+RootOf(_Z^4-8)^2)*Root
Of(_Z^4-8)*(x^6+x^2)^(1/4)*x^2+RootOf(_Z^4-8)^3*x-RootOf(_Z^2+RootOf(_Z^4-8
)^2)*RootOf(_Z^4-8)^2*x+8*(x^6+x^2)^(1/2)*RootOf(_Z^4-8)*x+8*RootOf(_Z^2+Ro
otOf(_Z^4-8)^2)*(x^6+x^2)^(1/2)*x+16*(x^6+x^2)^(3/4))/x/(x^2+1)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}+x^{2}+1}{\left(x^{6}+x^{2}\right)^{\frac{1}{4}}\left(x^{4}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + x^2 + 1)/((x^6 + x^2)^(1/4)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4+x^2+1}{{\left(x^6+x^2\right)}^{1/4}\,\left(x^4-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4 + 1)/((x^2 + x^6)^(1/4)*(x^4 - 1)),x)
```

```
[Out] int((x^2 + x^4 + 1)/((x^2 + x^6)^(1/4)*(x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2} - x + 1\right) \left(x^{2} + x + 1\right)}{\sqrt[4]{x^{2} \left(x^{4} + 1\right)} \left(x - 1\right) \left(x + 1\right) \left(x^{2} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)/(x**4-1)/(x**6+x**2)**(1/4),x)
```

```
[Out] Integral((x**2 - x + 1)*(x**2 + x + 1)/((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x
 + 1)*(x**2 + 1)), x)
```

3.1811 $\int \frac{1+x^2+x^4}{(-1+x^4)\sqrt[4]{x^2+x^6}}\,dx$

Optimal. Leaf size=162

$$-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{4\,2^{3/4}}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{4\,2^{3/4}}$$

Rubi [C] time = 0.58, antiderivative size = 127, normalized size of antiderivative = 0.78, number of steps used = 15, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2056, 6715, 6725, 245, 1438, 429, 510}

$$-\frac{4\sqrt[4]{x^4+1}\,xF_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^6+x^2}}-\frac{2\sqrt[4]{x^4+1}\,x^3F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^6+x^2}}+\frac{2\sqrt[4]{x^4+1}\,x\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^2 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-4*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, x^4, -x^4])/(x^2 + x^6)^(1
/4) - (2*x^3*(1 + x^4)^(1/4)*AppellF1[5/8, 1, 1/4, 13/8, x^4, -x^4])/(5*(x^
2 + x^6)^(1/4)) + (2*x*(1 + x^4)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -x^
4])/(x^2 + x^6)^(1/4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)
/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
```

```
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^2+x^4}{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{1+x^2+x^4}{\sqrt{x}\left(-1+x^4\right)\sqrt[4]{1+x^4}}\,dx}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\operatorname{Subst}\left(\int \frac{1+x^4+x^8}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\operatorname{Subst}\left(\int \left(\frac{1}{\sqrt[4]{1+x^8}} + \frac{2+x^4}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\right)dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\operatorname{Subst}\left(\int \frac{2+x^4}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\operatorname{Subst}\left(\int\left(-\frac{3}{2\left(1-x^4\right)\sqrt[4]{1+x^8}} - \frac{1}{2\left(1+x^4\right)\sqrt[4]{}}\right.\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+x^4\right)\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} - \frac{(3}{}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\operatorname{Subst}\left(\int\left(\frac{1}{\left(1-x^8\right)\sqrt[4]{1+x^8}} + \frac{x^4}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\right.\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-x^8\right)\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} - \frac{(}{}$$

$$= -\frac{4x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{2x^3\sqrt[4]{1+x^4}\,F_1\left(\frac{5}{8};1,\frac{1}{4};\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^2+x^6}} + \frac{2x\sqrt[4]{1+}}{}$$

Mathematica [C] time = 0.20, size = 95, normalized size = 0.59

$$-\frac{2\sqrt[4]{x^4+1}\left(45xF_1\left(\frac{1}{8};\frac{1}{4},1;\frac{9}{8};-x^4,x^4\right)+5x^5F_1\left(\frac{9}{8};\frac{1}{4},1;\frac{17}{8};-x^4,x^4\right)+9x^3F_1\left(\frac{5}{8};\frac{1}{4},1;\frac{13}{8};-x^4,x^4\right)\right)}{45\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*(1 + x^4)^(1/4)*(45*x*AppellF1[1/8, 1/4, 1, 9/8, -x^4, x^4] + 9*x^3*App
ellF1[5/8, 1/4, 1, 13/8, -x^4, x^4] + 5*x^5*AppellF1[9/8, 1/4, 1, 17/8, -x^
4, x^4]))/(45*(x^2 + x^6)^(1/4))
```

IntegrateAlgebraic [A] time = 0.00, size = 162, normalized size = 1.00

$$-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{4\,2^{3/4}}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2 + x^4)/((-1 + x^4)*(x^2 + x^6)^(1/4)),x]
```

```
[Out] (-3*ArcTan[(2^(1/4)*x)/(x^2 + x^6)^(1/4)])/(4*2^(1/4)) + ArcTan[(2^(3/4)*x*
(x^2 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^2 + x^6])]/(4*2^(3/4)) - (3*ArcTan
h[(2^(1/4)*x)/(x^2 + x^6)^(1/4)])/(4*2^(1/4)) - ArcTanh[(x^2/2^(1/4) + Sqrt
[x^2 + x^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))]/(4*2^(3/4))
```

fricas [B] time = 97.63, size = 1002, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] -3/8*2^(3/4)*arctan(1/2*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + 2^(3/4)*(2*2^(3/
4)*sqrt(x^6 + x^2)*x + 2^(1/4)*(x^5 + 2*x^3 + x)) + 4*2^(1/4)*(x^6 + x^2)^(
3/4))/(x^5 - 2*x^3 + x)) - 3/32*2^(3/4)*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4)*x
^2 + 2^(3/4)*(x^5 + 2*x^3 + x) + 4*2^(1/4)*sqrt(x^6 + x^2)*x + 4*(x^6 + x^2
)^(3/4))/(x^5 - 2*x^3 + x)) + 3/32*2^(3/4)*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4
)*x^2 - 2^(3/4)*(x^5 + 2*x^3 + x) - 4*2^(1/4)*sqrt(x^6 + x^2)*x + 4*(x^6 +
x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 1/8*2^(1/4)*arctan(-1/2*(2*x^9 + 8*x^7 + 1
2*x^5 + 8*x^3 + 4*2^(3/4)*(x^6 + x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 8*sqrt(2)*s
qrt(x^6 + x^2)*(x^5 + 2*x^3 + x) - sqrt(2)*(32*sqrt(2)*(x^6 + x^2)^(3/4)*x^
2 + 2^(3/4)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3 + x) + 4*2^(1/4)*sqrt(x^6 + x^2)
*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt((4*2^(3/
4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x
+ 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x)) + 8*2^(1/4)*(3*x^6 - 2*x^
4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3 + x)) -
1/8*2^(1/4)*arctan(-1/2*(2*x^9 + 8*x^7 + 12*x^5 + 8*x^3 - 4*2^(3/4)*(x^6 +
x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 8*sqrt(2)*sqrt(x^6 + x^2)*(x^5 + 2*x^3 + x)
- sqrt(2)*(32*sqrt(2)*(x^6 + x^2)^(3/4)*x^2 - 2^(3/4)*(x^9 - 16*x^7 - 2*x^5
 - 16*x^3 + x) - 4*2^(1/4)*sqrt(x^6 + x^2)*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x
^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt(-(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(
2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(
x^5 + 2*x^3 + x)) - 8*2^(1/4)*(3*x^6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2
*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3 + x)) - 1/32*2^(1/4)*log(8*(4*2^(3/4)*(x
^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x + 4*2
^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x)) + 1/32*2^(1/4)*log(-8*(4*2^(3/
4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*x
+ 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+x^2+1}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^4 + x^2 + 1)/((x^6 + x^2)^(1/4)*(x^4 - 1)), x)
```

maple [C] time = 23.08, size = 636, normalized size = 3.93

$$-\frac{\mathrm{RootOf}\left(\_Z^4+8\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+8\right)^3\sqrt{x^6+x^2}\,x-\mathrm{RootOf}\left(\_Z^4+8\right)x^5+2\,\mathrm{RootOf}\left(\_Z^4+8\right)^2\left(x^6+x^2\right)^{\frac{1}{4}}x^2+2\,\mathrm{RootOf}\left(\_Z^4+8\right)x^3-4\left(x^6+x^2\right)^{\frac{3}{4}}-\mathrm{RootOf}\left(\_Z^4+8\right)x}{x\left(x^2+1\right)^2}\right)}{16}-\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)\mathrm{RootOf}\left(\_Z^4+8\right)^2\sqrt{x^6+x^2}\,x+\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)x^5+2\,\mathrm{RootOf}\left(\_Z^4+8\right)^2\left(x^6+x^2\right)^{\frac{1}{4}}x^2-2\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)x^3+4\left(x^6+x^2\right)^{\frac{3}{4}}+\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)x}{x\left(x^2+1\right)^2}\right)}{16}-\frac{3\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^4+8\right)^2x^2-2\,\mathrm{RootOf}\left(\_Z^4+8\right)\left(x^6+x^2\right)^{\frac{1}{4}}x+2\sqrt{x^6+x^2}}{(1+x)x(-1+x)}\right)\mathrm{RootOf}\left(\_Z^4+8\right)^3}{32}-\frac{3\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^4+8\right)^2x^2-2\,\mathrm{RootOf}\left(\_Z^4+8\right)\left(x^6+x^2\right)^{\frac{1}{4}}x+2\sqrt{x^6+x^2}}{(1+x)x(-1+x)}\right)\mathrm{RootOf}\left(\_Z^4+8\right)^2\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)}{32}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)/(x^4-1)/(x^6+x^2)^(1/4),x)
```

```
[Out] -1/16*RootOf(_Z^4+8)*ln((RootOf(_Z^4+8)^3*(x^6+x^2)^(1/2)*x-RootOf(_Z^4+8)*
x^5+2*RootOf(_Z^4+8)^2*(x^6+x^2)^(1/4)*x^2+2*RootOf(_Z^4+8)*x^3-4*(x^6+x^2)
^(3/4)-RootOf(_Z^4+8)*x)/x/(x^2+1)^2)-1/16*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln
(-(RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*(x^6+x^2)^(1/2)*x+RootOf(
_Z^2+RootOf(_Z^4+8)^2)*x^5+2*RootOf(_Z^4+8)^2*(x^6+x^2)^(1/4)*x^2-2*RootOf(
_Z^2+RootOf(_Z^4+8)^2)*x^3+4*(x^6+x^2)^(3/4)+RootOf(_Z^2+RootOf(_Z^4+8)^2)*
x)/x/(x^2+1)^2)-3/32*ln(-(RootOf(_Z^4+8)^2*x^2-2*RootOf(_Z^4+8)*(x^6+x^2)^(
1/4)*x+2*(x^6+x^2)^(1/2))/(1+x)/x/(-1+x))*RootOf(_Z^4+8)^3-3/32*ln(-(RootOf
(_Z^4+8)^2*x^2-2*RootOf(_Z^4+8)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))/(1+x)/
x/(-1+x))*RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z^4+8)^2)+3/32*RootOf(_Z^4+8
)^2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln((RootOf(_Z^4+8)^3*x^5-RootOf(_Z^2+Root
Of(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^5+2*RootOf(_Z^4+8)^3*x^3-2*RootOf(_Z^2+Roo
tOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^3+8*RootOf(_Z^4+8)*(x^6+x^2)^(1/4)*RootOf
(_Z^2+RootOf(_Z^4+8)^2)*x^2+RootOf(_Z^4+8)^3*x-RootOf(_Z^2+RootOf(_Z^4+8)^2
)*RootOf(_Z^4+8)^2*x-8*(x^6+x^2)^(1/2)*RootOf(_Z^4+8)*x-8*RootOf(_Z^2+RootO
f(_Z^4+8)^2)*(x^6+x^2)^(1/2)*x+16*(x^6+x^2)^(3/4))/(1+x)^2/(-1+x)^2/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+x^2+1}{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^4-1)/(x^6+x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + x^2 + 1)/((x^6 + x^2)^(1/4)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4+x^2+1}{{\left(x^6+x^2\right)}^{1/4}\,\left(x^4-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4 + 1)/((x^2 + x^6)^(1/4)*(x^4 - 1)),x)
```

```
[Out] int((x^2 + x^4 + 1)/((x^2 + x^6)^(1/4)*(x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-x+1\right)\left(x^2+x+1\right)}{\sqrt[4]{x^2\left(x^4+1\right)}\,(x-1)\,(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)/(x**4-1)/(x**6+x**2)**(1/4),x)
```

```
[Out] Integral((x**2 - x + 1)*(x**2 + x + 1)/((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x
 + 1)*(x**2 + 1)), x)
```

3.1812 $\int \frac{(-4+x^3)(-2+x^3)(-1+x^3)^{2/3}}{x^6(-2+x^3+x^6)}\,dx$

Optimal. Leaf size=162

$$-\sqrt[3]{\frac{2}{3}}\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3-1}-3x\right)+\sqrt[3]{2}\,\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2}\,\sqrt[3]{x^3-1}+\sqrt[3]{3}\,x}\right)+\frac{(x^3-1)^{2/3}(8-13x^3)}{10x^5}+\frac{\log\left(\sqrt[3]{2}\,3^{2/3}\right.}{}$$

Rubi [A] time = 0.60, antiderivative size = 157, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1586, 6725, 271, 264, 377, 200, 31, 634, 617, 204, 628}

$$-\sqrt[3]{\frac{2}{3}}\log\left(\sqrt[3]{2}-\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3-1}}\right)+\sqrt[3]{2}\sqrt[6]{3}\tan^{-1}\left(\frac{2^{2/3}x}{\sqrt[6]{3}\sqrt[3]{x^3-1}}+\frac{1}{\sqrt{3}}\right)+\frac{4(x^3-1)^{2/3}}{5x^5}-\frac{13(x^3-1)^{2/3}}{10x^2}+\frac{\log\left(\frac{\sqrt[3]{6}x}{\sqrt[3]{x^3-1}}+\frac{3^{2/3}x^2}{(x^3-1)^{2/3}}+2^{2/3}\right)}{2^{2/3}\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((-4 + x^3)*(-2 + x^3)*(-1 + x^3)^(2/3))/(x^6*(-2 + x^3 + x^6)),x]
```

```
[Out] (4*(-1 + x^3)^(2/3))/(5*x^5) - (13*(-1 + x^3)^(2/3))/(10*x^2) + 2^(1/3)*3^(
1/6)*ArcTan[1/Sqrt[3] + (2^(2/3)*x)/(3^(1/6)*(-1 + x^3)^(1/3))] - (2/3)^(1/
3)*Log[2^(1/3) - (3^(1/3)*x)/(-1 + x^3)^(1/3)] + Log[2^(2/3) + (3^(2/3)*x^2
)/(-1 + x^3)^(2/3) + (6^(1/3)*x)/(-1 + x^3)^(1/3)]/(2^(2/3)*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(-4+x^3)(-2+x^3)(-1+x^3)^{2/3}}{x^6(-2+x^3+x^6)}\,dx = \int \frac{(-4+x^3)(-2+x^3)}{x^6\sqrt[3]{-1+x^3}\,(2+x^3)}\,dx$$

$$= \int \left(\frac{4}{x^6\sqrt[3]{-1+x^3}} - \frac{5}{x^3\sqrt[3]{-1+x^3}} + \frac{6}{\sqrt[3]{-1+x^3}\,(2+x^3)}\right) dx$$

$$= 4\int \frac{1}{x^6\sqrt[3]{-1+x^3}}\,dx - 5\int \frac{1}{x^3\sqrt[3]{-1+x^3}}\,dx + 6\int \frac{1}{\sqrt[3]{-1+x^3}\,(2+x^3}$$

$$= \frac{4(-1+x^3)^{2/3}}{5x^5} - \frac{5(-1+x^3)^{2/3}}{2x^2} + \frac{12}{5}\int \frac{1}{x^3\sqrt[3]{-1+x^3}}\,dx + 6\,\text{Subst}\Big($$

$$= \frac{4(-1+x^3)^{2/3}}{5x^5} - \frac{13(-1+x^3)^{2/3}}{10x^2} + \sqrt[3]{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{3}\,x}\,dx, x,\right.$$

$$= \frac{4(-1+x^3)^{2/3}}{5x^5} - \frac{13(-1+x^3)^{2/3}}{10x^2} - \sqrt[3]{\frac{2}{3}}\log\left(\sqrt[3]{2} - \frac{\sqrt[3]{3}\,x}{\sqrt[3]{-1+x^3}}\right) + \frac{3}{}$$

$$= \frac{4(-1+x^3)^{2/3}}{5x^5} - \frac{13(-1+x^3)^{2/3}}{10x^2} - \sqrt[3]{\frac{2}{3}}\log\left(\sqrt[3]{2} - \frac{\sqrt[3]{3}\,x}{\sqrt[3]{-1+x^3}}\right) + \frac{\text{lo}}{}$$

$$= \frac{4(-1+x^3)^{2/3}}{5x^5} - \frac{13(-1+x^3)^{2/3}}{10x^2} + \sqrt[3]{2}\,\sqrt[6]{3}\,\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{3}\,x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right) - \sqrt[3]{}$$

Mathematica [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(-4+x^3)(-2+x^3)(-1+x^3)^{2/3}}{x^6(-2+x^3+x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-4 + x^3)*(-2 + x^3)*(-1 + x^3)^(2/3))/(x^6*(-2 + x^3 + x^6)),x
]
```

```
[Out] Integrate[((-4 + x^3)*(-2 + x^3)*(-1 + x^3)^(2/3))/(x^6*(-2 + x^3 + x^6)),
x]
```

IntegrateAlgebraic [A] time = 0.45, size = 162, normalized size = 1.00

$$-\sqrt[3]{\frac{2}{3}}\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3-1}-3x\right)+\sqrt[3]{2}\,\sqrt[6]{3}\,\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2}\,\sqrt[3]{x^3-1}+\sqrt[3]{3}\,x}\right)+\frac{(x^3-1)^{2/3}(8-13x^3)}{10x^5}+\frac{\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3-1}\,x+2^{2/3}\sqrt[3]{3}\,(x^3-1)^{2/3}+3x^2\right)}{2^{2/3}\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + x^3)*(-2 + x^3)*(-1 + x^3)^(2/3))/(x^6*(-2 + x^3
+ x^6)),x]
```

```
[Out] ((8 - 13*x^3)*(-1 + x^3)^(2/3))/(10*x^5) + 2^(1/3)*3^(1/6)*ArcTan[(3^(5/6)*
x)/(3^(1/3)*x + 2*2^(1/3)*(-1 + x^3)^(1/3))] - (2/3)^(1/3)*Log[-3*x + 2^(1/
3)*3^(2/3)*(-1 + x^3)^(1/3)] + Log[3*x^2 + 2^(1/3)*3^(2/3)*x*(-1 + x^3)^(1/
3) + 2^(2/3)*3^(1/3)*(-1 + x^3)^(2/3)]/(2^(2/3)*3^(1/3))
```

fricas [B] time = 2.70, size = 291, normalized size = 1.80

$$\frac{10\cdot 3^{\frac{2}{3}}(-2)^{\frac{1}{3}}x^5\log\left(-\frac{9\cdot 3^{\frac{1}{3}}(-2)^{\frac{2}{3}}(x^3-1)^{\frac{1}{3}}x^2+3^{\frac{2}{3}}(-2)^{\frac{1}{3}}(x^3+2)-18(x^3-1)^{\frac{2}{3}}x}{x^3+2}\right)-5\cdot 3^{\frac{2}{3}}(-2)^{\frac{1}{3}}x^5\log\left(-\frac{12\cdot 3^{\frac{2}{3}}(-2)^{\frac{1}{3}}(4x^4-x)(x^3-1)^{\frac{2}{3}}-3^{\frac{1}{3}}(-2)^{\frac{2}{3}}(55x^6-50x^3+4)-18(7x^5-4x^2)(x^3-1)^{\frac{1}{3}}}{x^6+4x^3+4}\right)-30\cdot 3^{\frac{1}{6}}(-2)^{\frac{1}{3}}x^5\arctan\left(\frac{3^{\frac{1}{6}}\left(12\cdot 3^{\frac{2}{3}}(-2)^{\frac{2}{3}}(4x^7+7x^4-2x)(x^3-1)^{\frac{2}{3}}+18(-2)^{\frac{1}{3}}(55x^8-50x^5+4x^2)(x^3-1)^{\frac{1}{3}}-3^{\frac{1}{3}}(377x^9-600x^6+204x^3-8)\right)}{3(487x^9-480x^6+12x^3+8)}\right)-9(13x^3-8)(x^3-1)^{\frac{2}{3}}}{90x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^3-2)*(x^3-1)^(2/3)/x^6/(x^6+x^3-2),x, algorithm="frica
s")
```

```
[Out] 1/90*(10*3^(2/3)*(-2)^(1/3)*x^5*log(-(9*3^(1/3)*(-2)^(2/3)*(x^3 - 1)^(1/3)*
x^2 + 3^(2/3)*(-2)^(1/3)*(x^3 + 2) - 18*(x^3 - 1)^(2/3)*x)/(x^3 + 2)) - 5*3
^(2/3)*(-2)^(1/3)*x^5*log(-(12*3^(2/3)*(-2)^(1/3)*(4*x^4 - x)*(x^3 - 1)^(2/
3) - 3^(1/3)*(-2)^(2/3)*(55*x^6 - 50*x^3 + 4) - 18*(7*x^5 - 4*x^2)*(x^3 - 1
)^(1/3))/(x^6 + 4*x^3 + 4)) - 30*3^(1/6)*(-2)^(1/3)*x^5*arctan(1/3*3^(1/6)*
(12*3^(2/3)*(-2)^(2/3)*(4*x^7 + 7*x^4 - 2*x)*(x^3 - 1)^(2/3) + 18*(-2)^(1/3
)*(55*x^8 - 50*x^5 + 4*x^2)*(x^3 - 1)^(1/3) - 3^(1/3)*(377*x^9 - 600*x^6 +
204*x^3 - 8))/(487*x^9 - 480*x^6 + 12*x^3 + 8)) - 9*(13*x^3 - 8)*(x^3 - 1)^
(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-1\right)^{\frac{2}{3}}\left(x^3-2\right)\left(x^3-4\right)}{\left(x^6+x^3-2\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^3-2)*(x^3-1)^(2/3)/x^6/(x^6+x^3-2),x, algorithm="giac"
)
```

```
[Out] integrate((x^3 - 1)^(2/3)*(x^3 - 2)*(x^3 - 4)/((x^6 + x^3 - 2)*x^6), x)
```

maple [C] time = 3.05, size = 910, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-4)*(x^3-2)*(x^3-1)^(2/3)/x^6/(x^6+x^3-2),x)
```

```
[Out] -1/10*(13*x^6-21*x^3+8)/x^5/(x^3-1)^(1/3)+6*RootOf(RootOf(_Z^3+18)^2+18*_Z*
RootOf(_Z^3+18)+324*_Z^2)*ln((15*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3
+18)+324*_Z^2)*RootOf(_Z^3+18)^3*x^3+108*RootOf(RootOf(_Z^3+18)^2+18*_Z*Roo
tOf(_Z^3+18)+324*_Z^2)^2*RootOf(_Z^3+18)^2*x^3-42*(x^3-1)^(2/3)*RootOf(Root
Of(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)*RootOf(_Z^3+18)^2*x+8*RootOf(
_Z^3+18)^2*(x^3-1)^(1/3)*x^2+126*(x^3-1)^(1/3)*RootOf(RootOf(_Z^3+18)^2+18*
_Z*RootOf(_Z^3+18)+324*_Z^2)*RootOf(_Z^3+18)*x^2-25*RootOf(_Z^3+18)*x^3-180
*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)*x^3+48*x*(x^3-1)^
(2/3)+10*RootOf(_Z^3+18)+72*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+
324*_Z^2))/(x^3+2))-1/3*ln(-(9*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+1
8)+324*_Z^2)*RootOf(_Z^3+18)^3*x^3-108*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootO
f(_Z^3+18)+324*_Z^2)^2*RootOf(_Z^3+18)^2*x^3-42*(x^3-1)^(2/3)*RootOf(RootOf
(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)*RootOf(_Z^3+18)^2*x-RootOf(_Z^3
+18)^2*(x^3-1)^(1/3)*x^2+126*(x^3-1)^(1/3)*RootOf(RootOf(_Z^3+18)^2+18*_Z*R
ootOf(_Z^3+18)+324*_Z^2)*RootOf(_Z^3+18)*x^2+6*RootOf(_Z^3+18)*x^3-72*RootO
f(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)*x^3-6*x*(x^3-1)^(2/3)-6
*RootOf(_Z^3+18)+72*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2
))/(x^3+2))*RootOf(_Z^3+18)-6*ln(-(9*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(
_Z^3+18)+324*_Z^2)*RootOf(_Z^3+18)^3*x^3-108*RootOf(RootOf(_Z^3+18)^2+18*_Z
*RootOf(_Z^3+18)+324*_Z^2)^2*RootOf(_Z^3+18)^2*x^3-42*(x^3-1)^(2/3)*RootOf(
RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)*RootOf(_Z^3+18)^2*x-RootO
f(_Z^3+18)^2*(x^3-1)^(1/3)*x^2+126*(x^3-1)^(1/3)*RootOf(RootOf(_Z^3+18)^2+1
8*_Z*RootOf(_Z^3+18)+324*_Z^2)*RootOf(_Z^3+18)*x^2+6*RootOf(_Z^3+18)*x^3-72
*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)*x^3-6*x*(x^3-1)^(
2/3)-6*RootOf(_Z^3+18)+72*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+32
4*_Z^2))/(x^3+2))*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-1\right)^{\frac{2}{3}}\left(x^3-2\right)\left(x^3-4\right)}{\left(x^6+x^3-2\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4)*(x^3-2)*(x^3-1)^(2/3)/x^6/(x^6+x^3-2),x, algorithm="maxim
a")
```

```
[Out] integrate((x^3 - 1)^(2/3)*(x^3 - 2)*(x^3 - 4)/((x^6 + x^3 - 2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3-1\right)^{2/3}\,\left(x^3-2\right)\,\left(x^3-4\right)}{x^6\,\left(x^6+x^3-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(x^3 - 2)*(x^3 - 4))/(x^6*(x^3 + x^6 - 2)),x)
```

```
[Out] int(((x^3 - 1)^(2/3)*(x^3 - 2)*(x^3 - 4))/(x^6*(x^3 + x^6 - 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-4)*(x**3-2)*(x**3-1)**(2/3)/x**6/(x**6+x**3-2),x)
```

```
[Out] Timed out
```

3.1813 $\int \frac{(-1+x^4)\sqrt[4]{x^2+x^6}}{1+x^8}\,dx$

Optimal. Leaf size=162

$$\frac{\tan^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\,2^{3/8}}+\frac{\tan^{-1}\left(\frac{2^{5/8}x\sqrt[4]{x^6+x^2}}{\sqrt[4]{2}x^2-\sqrt{x^6+x^2}}\right)}{2\,2^{7/8}}-\frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\,2^{3/8}}+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{2^{3/8}}+\frac{\sqrt{x^6+x^2}}{2^{5/8}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\,2^{7/8}}$$

Rubi [C] time = 0.38, antiderivative size = 105, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2056, 6725, 466, 510}

$$-\frac{\left(\frac{1}{3}-\frac{i}{3}\right)x\sqrt[4]{x^6+x^2}\,F_1\left(\frac{3}{8};1,-\frac{1}{4};\frac{11}{8};-ix^4,-x^4\right)}{\sqrt[4]{x^4+1}}-\frac{\left(\frac{1}{3}+\frac{i}{3}\right)x\sqrt[4]{x^6+x^2}\,F_1\left(\frac{3}{8};1,-\frac{1}{4};\frac{11}{8};ix^4,-x^4\right)}{\sqrt[4]{x^4+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^8),x]
```

```
[Out] ((-1/3 + I/3)*x*(x^2 + x^6)^(1/4)*AppellF1[3/8, 1, -1/4, 11/8, (-I)*x^4, -x
^4])/(1 + x^4)^(1/4) - ((1/3 + I/3)*x*(x^2 + x^6)^(1/4)*AppellF1[3/8, 1, -1
/4, 11/8, I*x^4, -x^4])/(1 + x^4)^(1/4)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(-1+x^4)\sqrt[4]{x^2+x^6}}{1+x^8}\,dx = \frac{\sqrt[4]{x^2+x^6}\int \frac{\sqrt{x}(-1+x^4)\sqrt[4]{1+x^4}}{1+x^8}dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$
$$= \frac{\sqrt[4]{x^2+x^6}\int\left(-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x}\sqrt[4]{1+x^4}}{i-x^4}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{x}\sqrt[4]{1+x^4}}{i+x^4}\right)dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$
$$= -\frac{\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{x^2+x^6}\right)\int\frac{\sqrt{x}\sqrt[4]{1+x^4}}{i-x^4}dx}{\sqrt{x}\sqrt[4]{1+x^4}}+\frac{\left(\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt[4]{x^2+x^6}\right)\int\frac{\sqrt{x}\sqrt[4]{1+x^4}}{i+x^4}dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$
$$= -\frac{\left((1+i)\sqrt[4]{x^2+x^6}\right)\text{Subst}\left(\int\frac{x^2\sqrt[4]{1+x^8}}{i-x^8}dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt[4]{1+x^4}}+\frac{\left((1-i)\sqrt[4]{x^2+x^6}\right)\text{Subst}\left(\int \right.}{\sqrt{x}\sqrt[4]{1+x^4}}$$
$$= -\frac{\left(\frac{1}{3}-\frac{i}{3}\right)x\sqrt[4]{x^2+x^6}F_1\left(\frac{3}{8};1,-\frac{1}{4};\frac{11}{8};-ix^4,-x^4\right)}{\sqrt[4]{1+x^4}}-\frac{\left(\frac{1}{3}+\frac{i}{3}\right)x\sqrt[4]{x^2+x^6}F_1\left(\frac{3}{8};1,-\frac{1}{4};\right.}{\sqrt[4]{1+x^4}}$$

Mathematica [F] time = 1.48, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^4)\sqrt[4]{x^2+x^6}}{1+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^8),x]
```

```
[Out] Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^8), x]
```

IntegrateAlgebraic [A] time = 0.61, size = 162, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\,2^{3/8}}+\frac{\tan^{-1}\left(\frac{2^{5/8}x\sqrt[4]{x^6+x^2}}{\sqrt[4]{2}x^2-\sqrt{x^6+x^2}}\right)}{2\,2^{7/8}}-\frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\,2^{3/8}}+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{2^{3/8}}+\frac{\sqrt{x^6+x^2}}{2^{5/8}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\,2^{7/8}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^8),x]
```

```
[Out] ArcTan[(2^(1/8)*x)/(x^2 + x^6)^(1/4)]/(2*2^(3/8)) + ArcTan[(2^(5/8)*x*(x^2
+ x^6)^(1/4))/(2^(1/4)*x^2 - Sqrt[x^2 + x^6])]/(2*2^(7/8)) - ArcTanh[(2^(1/
8)*x)/(x^2 + x^6)^(1/4)]/(2*2^(3/8)) + ArcTanh[(x^2/2^(3/8) + Sqrt[x^2 + x^
6]/2^(5/8))/(x*(x^2 + x^6)^(1/4))]/(2*2^(7/8))
```

fricas [B] time = 13.40, size = 1273, normalized size = 7.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/32*8^(7/8)*sqrt(2)*arctan(1/8*(2*(x^6 + x^2)^(3/4)*(8^(7/8)*sqrt(2)*x^2
+ 2*8^(3/8)*sqrt(2)*(x^4 + 1)) + (8^(3/4)*(x^9 + 4*x^5 + x) + 2*(x^6 + x^2)
^(3/4)*(8^(5/8)*sqrt(2)*x^2 + 2*8^(1/8)*sqrt(2)*(x^4 + 1)) + 8*sqrt(x^6 + x
^2)*(2*x^3 + sqrt(2)*(x^5 + x)) + 8*8^(1/4)*(x^7 + x^3) + (x^6 + x^2)^(1/4)
*(4*8^(3/8)*sqrt(2)*x^4 + 8^(7/8)*sqrt(2)*(x^6 + x^2)))*sqrt(((x^6 + x^2)^(
3/4)*(4*8^(3/8)*sqrt(2)*x^2 - 8^(7/8)*sqrt(2)*(x^4 + 1)) + 2*sqrt(2)*(x^9 +
```

```
 x) - 4*sqrt(x^6 + x^2)*(8^(3/4)*x^3 - 2*8^(1/4)*(x^5 + x)) + 2*(x^6 + x^2)
^(1/4)*(4*8^(1/8)*sqrt(2)*x^4 - 8^(5/8)*sqrt(2)*(x^6 + x^2)))/(x^9 + x)) +
4*(x^6 + x^2)^(1/4)*(8^(5/8)*sqrt(2)*x^4 + 2*8^(1/8)*sqrt(2)*(x^6 + x^2)))/
(x^9 + x)) - 1/32*8^(7/8)*sqrt(2)*arctan(1/8*(2*(x^6 + x^2)^(3/4)*(8^(7/8)*
sqrt(2)*x^2 + 2*8^(3/8)*sqrt(2)*(x^4 + 1)) - (8^(3/4)*(x^9 + 4*x^5 + x) - 2
*(x^6 + x^2)^(3/4)*(8^(5/8)*sqrt(2)*x^2 + 2*8^(1/8)*sqrt(2)*(x^4 + 1)) + 8*
sqrt(x^6 + x^2)*(2*x^3 + sqrt(2)*(x^5 + x)) + 8*8^(1/4)*(x^7 + x^3) - (x^6
+ x^2)^(1/4)*(4*8^(3/8)*sqrt(2)*x^4 + 8^(7/8)*sqrt(2)*(x^6 + x^2)))*sqrt(-(
(x^6 + x^2)^(3/4)*(4*8^(3/8)*sqrt(2)*x^2 - 8^(7/8)*sqrt(2)*(x^4 + 1)) - 2*s
qrt(2)*(x^9 + x) + 4*sqrt(x^6 + x^2)*(8^(3/4)*x^3 - 2*8^(1/4)*(x^5 + x)) +
2*(x^6 + x^2)^(1/4)*(4*8^(1/8)*sqrt(2)*x^4 - 8^(5/8)*sqrt(2)*(x^6 + x^2)))/
(x^9 + x)) + 4*(x^6 + x^2)^(1/4)*(8^(5/8)*sqrt(2)*x^4 + 2*8^(1/8)*sqrt(2)*(
x^6 + x^2)))/(x^9 + x)) - 1/128*8^(7/8)*sqrt(2)*log(4*((x^6 + x^2)^(3/4)*(4
*8^(3/8)*sqrt(2)*x^2 - 8^(7/8)*sqrt(2)*(x^4 + 1)) + 2*sqrt(2)*(x^9 + x) - 4
*sqrt(x^6 + x^2)*(8^(3/4)*x^3 - 2*8^(1/4)*(x^5 + x)) + 2*(x^6 + x^2)^(1/4)*
(4*8^(1/8)*sqrt(2)*x^4 - 8^(5/8)*sqrt(2)*(x^6 + x^2)))/(x^9 + x)) + 1/128*8
^(7/8)*sqrt(2)*log(-4*((x^6 + x^2)^(3/4)*(4*8^(3/8)*sqrt(2)*x^2 - 8^(7/8)*s
qrt(2)*(x^4 + 1)) - 2*sqrt(2)*(x^9 + x) + 4*sqrt(x^6 + x^2)*(8^(3/4)*x^3 -
2*8^(1/4)*(x^5 + x)) + 2*(x^6 + x^2)^(1/4)*(4*8^(1/8)*sqrt(2)*x^4 - 8^(5/8)
*sqrt(2)*(x^6 + x^2)))/(x^9 + x)) - 1/16*8^(7/8)*arctan(1/8*(8^(5/8)*(x^6 +
 x^2)^(1/4)*(x^4 + 1) + 2^(3/4)*(8^(3/8)*(x^6 + x^2)^(1/4)*(x^4 + 1) + 2*8^
(1/8)*(x^6 + x^2)^(3/4)) - 2*8^(3/8)*(x^6 + x^2)^(3/4))/(x^5 + x)) - 1/64*8
^(7/8)*log((8^(3/4)*(x^7 + x^3) + (x^6 + x^2)^(3/4)*(4*8^(1/8)*x^2 + 8^(5/8
)*(x^4 + 1)) + 4*sqrt(x^6 + x^2)*(x^5 + sqrt(2)*x^3 + x) + 8^(1/4)*(x^9 + 4
*x^5 + x) + (x^6 + x^2)^(1/4)*(8^(7/8)*x^4 + 2*8^(3/8)*(x^6 + x^2)))/(x^9 +
 x)) + 1/64*8^(7/8)*log((8^(3/4)*(x^7 + x^3) - (x^6 + x^2)^(3/4)*(4*8^(1/8)
*x^2 + 8^(5/8)*(x^4 + 1)) + 4*sqrt(x^6 + x^2)*(x^5 + sqrt(2)*x^3 + x) + 8^(
1/4)*(x^9 + 4*x^5 + x) - (x^6 + x^2)^(1/4)*(8^(7/8)*x^4 + 2*8^(3/8)*(x^6 +
x^2)))/(x^9 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8+1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^2)^(1/4)*(x^4 - 1)/(x^8 + 1), x)
```

maple [C] time = 114.53, size = 2866, normalized size = 17.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^6+x^2)^(1/4)/(x^8+1),x)
```

```
[Out] 1/8*RootOf(_Z^8-32)*ln(-(RootOf(_Z^8-32)^11*x^5-2*RootOf(_Z^8-32)^11*x^3+Ro
otOf(_Z^8-32)^11*x-8*x^3*RootOf(_Z^8-32)^7-32*(x^6+x^2)^(1/2)*RootOf(_Z^8-3
2)^5*x-64*x^5*RootOf(_Z^8-32)^3+64*x^3*RootOf(_Z^8-32)^3+128*(x^6+x^2)^(1/4
)*RootOf(_Z^8-32)^2*x^2-64*x*RootOf(_Z^8-32)^3+256*(x^6+x^2)^(3/4))/x/(Root
Of(_Z^8-32)^4*x^4-2*RootOf(_Z^8-32)^4*x^2+RootOf(_Z^8-32)^4+8*x^4-8*x^2+8))
-1/32*ln(-(-RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf
(_Z^8-32)^10*x^5+2*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)
*RootOf(_Z^8-32)^10*x^3-RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*
_Z^2)*RootOf(_Z^8-32)^10*x-8*RootOf(_Z^8-32)^6*RootOf(2*RootOf(_Z^8-32)^5*_
Z+RootOf(_Z^8-32)^2+64*_Z^2)*x^3+32*(x^6+x^2)^(1/4)*RootOf(_Z^8-32)^5*RootO
f(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*x^2-32*(x^6+x^2)^(1/2)*
RootOf(_Z^8-32)^4*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*
```

x+64*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-3
2)^2*x^5-64*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf
(_Z^8-32)^2*x^3+16*(x^6+x^2)^(1/4)*RootOf(_Z^8-32)^2*x^2-32*(x^6+x^2)^(1/2)
*RootOf(_Z^8-32)*x+64*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z
^2)*RootOf(_Z^8-32)^2*x+32*(x^6+x^2)^(3/4))/(RootOf(_Z^8-32)^4*x^4-2*RootOf
(_Z^8-32)^4*x^2+RootOf(_Z^8-32)^4-8*x^4+8*x^2-8)/x)*RootOf(_Z^8-32)^5-ln(-(
-RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^1
0*x^5+2*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^
8-32)^10*x^3-RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootO
f(_Z^8-32)^10*x-8*RootOf(_Z^8-32)^6*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z
^8-32)^2+64*_Z^2)*x^3+32*(x^6+x^2)^(1/4)*RootOf(_Z^8-32)^5*RootOf(2*RootOf(
_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*x^2-32*(x^6+x^2)^(1/2)*RootOf(_Z^8
-32)^4*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*x+64*RootOf
(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^2*x^5-64
*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^2
x^3+16(x^6+x^2)^(1/4)*RootOf(_Z^8-32)^2*x^2-32*(x^6+x^2)^(1/2)*RootOf(_Z^
8-32)*x+64*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(
_Z^8-32)^2*x+32*(x^6+x^2)^(3/4))/(RootOf(_Z^8-32)^4*x^4-2*RootOf(_Z^8-32)^4
*x^2+RootOf(_Z^8-32)^4-8*x^4+8*x^2-8)/x)*RootOf(2*RootOf(_Z^8-32)^5*_Z+Root
Of(_Z^8-32)^2+64*_Z^2)-RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_
Z^2)*ln(-(RootOf(_Z^8-32)^11*x^5-4*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^
8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^10*x^5-2*RootOf(_Z^8-32)^11*x^3+8*RootOf(2
*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^10*x^3+Roo
tOf(_Z^8-32)^11*x-4*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2
)*RootOf(_Z^8-32)^10*x+4*x^5*RootOf(_Z^8-32)^7-32*RootOf(_Z^8-32)^6*RootOf(
2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*x^3-128*(x^6+x^2)^(1/4)*R
ootOf(_Z^8-32)^5*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*x
^2-128*(x^6+x^2)^(1/2)*RootOf(_Z^8-32)^4*RootOf(2*RootOf(_Z^8-32)^5*_Z+Root
Of(_Z^8-32)^2+64*_Z^2)*x+4*x*RootOf(_Z^8-32)^7-32*x^5*RootOf(_Z^8-32)^3+256
*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^2
*x^5+32*x^3*RootOf(_Z^8-32)^3-256*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8
-32)^2+64*_Z^2)*RootOf(_Z^8-32)^2*x^3-64*(x^6+x^2)^(1/4)*RootOf(_Z^8-32)^2*
x^2-32*x*RootOf(_Z^8-32)^3+256*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32
)^2+64*_Z^2)*RootOf(_Z^8-32)^2*x+128*(x^6+x^2)^(3/4))/(RootOf(_Z^8-32)^4*x^
4-2*RootOf(_Z^8-32)^4*x^2+RootOf(_Z^8-32)^4-8*x^4+8*x^2-8)/x)-1/4*ln(-(Root
Of(_Z^8-32)^11*x^5+16*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z
^2)*RootOf(_Z^8-32)^10*x^5-2*RootOf(_Z^8-32)^11*x^3-32*RootOf(2*RootOf(_Z^8
-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^10*x^3+RootOf(_Z^8-32)
^11*x+16*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z
^8-32)^10*x-64*RootOf(_Z^8-32)^6*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-
32)^2+64*_Z^2)*x^5-8*x^3*RootOf(_Z^8-32)^7+32*(x^6+x^2)^(1/2)*RootOf(_Z^8-3
2)^5*x-64*RootOf(_Z^8-32)^6*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2
+64*_Z^2)*x-64*x^5*RootOf(_Z^8-32)^3-512*RootOf(2*RootOf(_Z^8-32)^5*_Z+Root
Of(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^2*x^5+64*x^3*RootOf(_Z^8-32)^3+512*R
ootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^2*x
^3-128*(x^6+x^2)^(1/4)*RootOf(_Z^8-32)^2*x^2+2048*(x^6+x^2)^(1/2)*RootOf(2*
RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*x-64*x*RootOf(_Z^8-32)^3-51
2*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^
2*x+256*(x^6+x^2)^(3/4))/x/(RootOf(_Z^8-32)^4*x^4-2*RootOf(_Z^8-32)^4*x^2+R
ootOf(_Z^8-32)^4+8*x^4-8*x^2+8))*RootOf(_Z^8-32)^4*RootOf(2*RootOf(_Z^8-32)
^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)-1/8*ln(-(RootOf(_Z^8-32)^11*x^5+16*RootOf(
2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^10*x^5-2*
RootOf(_Z^8-32)^11*x^3-32*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+6
4*_Z^2)*RootOf(_Z^8-32)^10*x^3+RootOf(_Z^8-32)^11*x+16*RootOf(2*RootOf(_Z^8
-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^10*x-64*RootOf(_Z^8-32
)^6*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*x^5-8*x^3*Root
Of(_Z^8-32)^7+32*(x^6+x^2)^(1/2)*RootOf(_Z^8-32)^5*x-64*RootOf(_Z^8-32)^6*R
ootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*x-64*x^5*RootOf(_Z^
8-32)^3-512*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf

```
(_Z^8-32)^2*x^5+64*x^3*RootOf(_Z^8-32)^3+512*RootOf(2*RootOf(_Z^8-32)^5*_Z+
RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^2*x^3-128*(x^6+x^2)^(1/4)*RootOf
(_Z^8-32)^2*x^2+2048*(x^6+x^2)^(1/2)*RootOf(2*RootOf(_Z^8-32)^5*_Z+RootOf(_
Z^8-32)^2+64*_Z^2)*x-64*x*RootOf(_Z^8-32)^3-512*RootOf(2*RootOf(_Z^8-32)^5*
_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^2*x+256*(x^6+x^2)^(3/4))/x/(R
ootOf(_Z^8-32)^4*x^4-2*RootOf(_Z^8-32)^4*x^2+RootOf(_Z^8-32)^4+8*x^4-8*x^2+
8))*RootOf(_Z^8-32)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8+1),x, algorithm="maxima")
[Out] integrate((x^6 + x^2)^(1/4)*(x^4 - 1)/(x^8 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6+x^2\right)^{1/4}\,\left(x^4-1\right)}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + x^6)^(1/4)*(x^4 - 1))/(x^8 + 1),x)
[Out] int(((x^2 + x^6)^(1/4)*(x^4 - 1))/(x^8 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\left(x^4+1\right)}\,(x-1)\,(x+1)\left(x^2+1\right)}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**6+x**2)**(1/4)/(x**8+1),x)
[Out] Integral((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)/(x**8 + 1), x)
```

3.1814 $\int \frac{(-1+x^4)\sqrt[4]{x^2+x^6}}{1+x^8}\,dx$

Optimal. Leaf size=162

$$\frac{\tan^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\,2^{3/8}}+\frac{\tan^{-1}\left(\frac{2^{5/8}x\sqrt[4]{x^6+x^2}}{\sqrt[4]{2}x^2-\sqrt{x^6+x^2}}\right)}{2\,2^{7/8}}-\frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\,2^{3/8}}+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{2^{3/8}}+\frac{\sqrt{x^6+x^2}}{2^{5/8}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\,2^{7/8}}$$

Rubi [C] time = 0.31, antiderivative size = 105, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2056, 6725, 466, 510}

$$-\frac{\left(\frac{1}{3}-\frac{i}{3}\right)x\sqrt[4]{x^6+x^2}\,F_1\left(\frac{3}{8};1,-\frac{1}{4};\frac{11}{8};-ix^4,-x^4\right)}{\sqrt[4]{x^4+1}}-\frac{\left(\frac{1}{3}+\frac{i}{3}\right)x\sqrt[4]{x^6+x^2}\,F_1\left(\frac{3}{8};1,-\frac{1}{4};\frac{11}{8};ix^4,-x^4\right)}{\sqrt[4]{x^4+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^8),x]
```

```
[Out] ((-1/3 + I/3)*x*(x^2 + x^6)^(1/4)*AppellF1[3/8, 1, -1/4, 11/8, (-I)*x^4, -x
^4])/(1 + x^4)^(1/4) - ((1/3 + I/3)*x*(x^2 + x^6)^(1/4)*AppellF1[3/8, 1, -1
/4, 11/8, I*x^4, -x^4])/(1 + x^4)^(1/4)
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(-1+x^4)\sqrt[4]{x^2+x^6}}{1+x^8}\,dx = \frac{\sqrt[4]{x^2+x^6}\int \frac{\sqrt{x}(-1+x^4)\sqrt[4]{1+x^4}}{1+x^8}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$
$$= \frac{\sqrt[4]{x^2+x^6}\int \left(-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x}\sqrt[4]{1+x^4}}{i-x^4}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{x}\sqrt[4]{1+x^4}}{i+x^4}\right)dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$
$$= -\frac{\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{x^2+x^6}\right)\int \frac{\sqrt{x}\sqrt[4]{1+x^4}}{i-x^4}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt[4]{x^2+x^6}\right)\int \frac{\sqrt{x}\sqrt[4]{1+x^4}}{i+x^4}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$
$$= -\frac{\left((1+i)\sqrt[4]{x^2+x^6}\right)\text{Subst}\left(\int \frac{x^2\sqrt[4]{1+x^8}}{i-x^8}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left((1-i)\sqrt[4]{x^2+x^6}\right)\text{Subst}\left(\int \frac{x^2\sqrt[4]{\ }}{i+}\right.}{\sqrt{x}\sqrt[4]{1+x^4}}$$
$$= -\frac{\left(\frac{1}{3}-\frac{i}{3}\right)x\sqrt[4]{x^2+x^6}\,F_1\left(\frac{3}{8};1,-\frac{1}{4};\frac{11}{8};-ix^4,-x^4\right)}{\sqrt[4]{1+x^4}} - \frac{\left(\frac{1}{3}+\frac{i}{3}\right)x\sqrt[4]{x^2+x^6}\,F_1\left(\frac{3}{8};1,-\frac{1}{4};\frac{11}{8}\right.}{\sqrt[4]{1+x^4}}$$

Mathematica [F] time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^4)\sqrt[4]{x^2+x^6}}{1+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^8),x]
[Out] Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^8), x]
```

IntegrateAlgebraic [A] time = 0.00, size = 162, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\,2^{3/8}}+\frac{\tan^{-1}\left(\frac{2^{5/8}x\sqrt[4]{x^6+x^2}}{\sqrt[4]{2}x^2-\sqrt{x^6+x^2}}\right)}{2\,2^{7/8}}-\frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\,2^{3/8}}+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{2^{3/8}}+\frac{\sqrt{x^6+x^2}}{2^{5/8}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\,2^{7/8}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 + x^8),x]
[Out] ArcTan[(2^(1/8)*x)/(x^2 + x^6)^(1/4)]/(2*2^(3/8)) + ArcTan[(2^(5/8)*x*(x^2
+ x^6)^(1/4))/(2^(1/4)*x^2 - Sqrt[x^2 + x^6])]/(2*2^(7/8)) - ArcTanh[(2^(1/
8)*x)/(x^2 + x^6)^(1/4)]/(2*2^(3/8)) + ArcTanh[(x^2/2^(3/8) + Sqrt[x^2 + x^
6]/2^(5/8))/(x*(x^2 + x^6)^(1/4))]/(2*2^(7/8))
```

fricas [B] time = 13.40, size = 1273, normalized size = 7.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8+1),x, algorithm="fricas")
[Out] -1/32*8^(7/8)*sqrt(2)*arctan(1/8*(2*(x^6 + x^2)^(3/4)*(8^(7/8)*sqrt(2)*x^2
+ 2*8^(3/8)*sqrt(2)*(x^4 + 1)) + (8^(3/4)*(x^9 + 4*x^5 + x) + 2*(x^6 + x^2)
^(3/4)*(8^(5/8)*sqrt(2)*x^2 + 2*8^(1/8)*sqrt(2)*(x^4 + 1)) + 8*sqrt(x^6 + x
^2)*(2*x^3 + sqrt(2)*(x^5 + x)) + 8*8^(1/4)*(x^7 + x^3) + (x^6 + x^2)^(1/4)
*(4*8^(3/8)*sqrt(2)*x^4 + 8^(7/8)*sqrt(2)*(x^6 + x^2)))*sqrt(((x^6 + x^2)^(
3/4)*(4*8^(3/8)*sqrt(2)*x^2 - 8^(7/8)*sqrt(2)*(x^4 + 1)) + 2*sqrt(2)*(x^9 +
```

```
x) - 4*sqrt(x^6 + x^2)*(8^(3/4)*x^3 - 2*8^(1/4)*(x^5 + x)) + 2*(x^6 + x^2)
^(1/4)*(4*8^(1/8)*sqrt(2)*x^4 - 8^(5/8)*sqrt(2)*(x^6 + x^2)))/(x^9 + x)) +
4*(x^6 + x^2)^(1/4)*(8^(5/8)*sqrt(2)*x^4 + 2*8^(1/8)*sqrt(2)*(x^6 + x^2)))/
(x^9 + x)) - 1/32*8^(7/8)*sqrt(2)*arctan(1/8*(2*(x^6 + x^2)^(3/4)*(8^(7/8)*
sqrt(2)*x^2 + 2*8^(3/8)*sqrt(2)*(x^4 + 1)) - (8^(3/4)*(x^9 + 4*x^5 + x) - 2
*(x^6 + x^2)^(3/4)*(8^(5/8)*sqrt(2)*x^2 + 2*8^(1/8)*sqrt(2)*(x^4 + 1)) + 8*
sqrt(x^6 + x^2)*(2*x^3 + sqrt(2)*(x^5 + x)) + 8*8^(1/4)*(x^7 + x^3) - (x^6
+ x^2)^(1/4)*(4*8^(3/8)*sqrt(2)*x^4 + 8^(7/8)*sqrt(2)*(x^6 + x^2)))*sqrt(-(
(x^6 + x^2)^(3/4)*(4*8^(3/8)*sqrt(2)*x^2 - 8^(7/8)*sqrt(2)*(x^4 + 1)) - 2*s
qrt(2)*(x^9 + x) + 4*sqrt(x^6 + x^2)*(8^(3/4)*x^3 - 2*8^(1/4)*(x^5 + x)) +
2*(x^6 + x^2)^(1/4)*(4*8^(1/8)*sqrt(2)*x^4 - 8^(5/8)*sqrt(2)*(x^6 + x^2)))/
(x^9 + x)) + 4*(x^6 + x^2)^(1/4)*(8^(5/8)*sqrt(2)*x^4 + 2*8^(1/8)*sqrt(2)*(
x^6 + x^2)))/(x^9 + x)) - 1/128*8^(7/8)*sqrt(2)*log(4*((x^6 + x^2)^(3/4)*(4
*8^(3/8)*sqrt(2)*x^2 - 8^(7/8)*sqrt(2)*(x^4 + 1)) + 2*sqrt(2)*(x^9 + x) - 4
*sqrt(x^6 + x^2)*(8^(3/4)*x^3 - 2*8^(1/4)*(x^5 + x)) + 2*(x^6 + x^2)^(1/4)*
(4*8^(1/8)*sqrt(2)*x^4 - 8^(5/8)*sqrt(2)*(x^6 + x^2)))/(x^9 + x)) + 1/128*8
^(7/8)*sqrt(2)*log(-4*((x^6 + x^2)^(3/4)*(4*8^(3/8)*sqrt(2)*x^2 - 8^(7/8)*s
qrt(2)*(x^4 + 1)) - 2*sqrt(2)*(x^9 + x) + 4*sqrt(x^6 + x^2)*(8^(3/4)*x^3 -
2*8^(1/4)*(x^5 + x)) + 2*(x^6 + x^2)^(1/4)*(4*8^(1/8)*sqrt(2)*x^4 - 8^(5/8)
*sqrt(2)*(x^6 + x^2)))/(x^9 + x)) - 1/16*8^(7/8)*arctan(1/8*(8^(5/8)*(x^6 +
x^2)^(1/4)*(x^4 + 1) + 2^(3/4)*(8^(3/8)*(x^6 + x^2)^(1/4)*(x^4 + 1) + 2*8^
(1/8)*(x^6 + x^2)^(3/4)) - 2*8^(3/8)*(x^6 + x^2)^(3/4))/(x^5 + x)) - 1/64*8
^(7/8)*log((8^(3/4)*(x^7 + x^3) + (x^6 + x^2)^(3/4)*(4*8^(1/8)*x^2 + 8^(5/8
)*(x^4 + 1)) + 4*sqrt(x^6 + x^2)*(x^5 + sqrt(2)*x^3 + x) + 8^(1/4)*(x^9 + 4
*x^5 + x) + (x^6 + x^2)^(1/4)*(8^(7/8)*x^4 + 2*8^(3/8)*(x^6 + x^2)))/(x^9 +
x)) + 1/64*8^(7/8)*log((8^(3/4)*(x^7 + x^3) - (x^6 + x^2)^(3/4)*(4*8^(1/8)
*x^2 + 8^(5/8)*(x^4 + 1)) + 4*sqrt(x^6 + x^2)*(x^5 + sqrt(2)*x^3 + x) + 8^(
1/4)*(x^9 + 4*x^5 + x) - (x^6 + x^2)^(1/4)*(8^(7/8)*x^4 + 2*8^(3/8)*(x^6 +
x^2)))/(x^9 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8+1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^2)^(1/4)*(x^4 - 1)/(x^8 + 1), x)
```

maple [C] time = 111.25, size = 2859, normalized size = 17.65

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^6+x^2)^(1/4)/(x^8+1),x)
```

```
[Out] -1/8*RootOf(_Z^8-32)*ln((RootOf(_Z^8-32)^11*x^5-2*RootOf(_Z^8-32)^11*x^3+Ro
otOf(_Z^8-32)^11*x-8*x^3*RootOf(_Z^8-32)^7-32*(x^6+x^2)^(1/2)*RootOf(_Z^8-3
2)^5*x-64*x^5*RootOf(_Z^8-32)^3+64*x^3*RootOf(_Z^8-32)^3-128*(x^6+x^2)^(1/4
)*RootOf(_Z^8-32)^2*x^2-64*x*RootOf(_Z^8-32)^3-256*(x^6+x^2)^(3/4))/x/(Root
Of(_Z^8-32)^4*x^4-2*RootOf(_Z^8-32)^4*x^2+RootOf(_Z^8-32)^4+8*x^4-8*x^2+8))
-1/32*ln(-(RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf
(_Z^8-32)^10*x^5-2*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2
)*RootOf(_Z^8-32)^10*x^3+RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+6
4*_Z^2)*RootOf(_Z^8-32)^10*x+8*RootOf(_Z^8-32)^6*RootOf(-2*RootOf(_Z^8-32)^
5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*x^3-32*RootOf(_Z^8-32)^5*RootOf(-2*RootOf(_
Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*(x^6+x^2)^(1/4)*x^2+32*(x^6+x^2)^(1
/2)*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-3
```

2)^4*x-64*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(
_Z^8-32)^2*x^5+64*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)
*RootOf(_Z^8-32)^2*x^3+16*(x^6+x^2)^(1/4)*RootOf(_Z^8-32)^2*x^2-64*RootOf(-
2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^2*x-32*(x
^6+x^2)^(1/2)*RootOf(_Z^8-32)*x+32*(x^6+x^2)^(3/4))/(RootOf(_Z^8-32)^4*x^4-
2*RootOf(_Z^8-32)^4*x^2+RootOf(_Z^8-32)^4-8*x^4+8*x^2-8)/x)*RootOf(_Z^8-32)
^5+ln(-(RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z
^8-32)^10*x^5-2*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*R
ootOf(_Z^8-32)^10*x^3+RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_
Z^2)*RootOf(_Z^8-32)^10*x+8*RootOf(_Z^8-32)^6*RootOf(-2*RootOf(_Z^8-32)^5*_
Z+RootOf(_Z^8-32)^2+64*_Z^2)*x^3-32*RootOf(_Z^8-32)^5*RootOf(-2*RootOf(_Z^8
-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*(x^6+x^2)^(1/4)*x^2+32*(x^6+x^2)^(1/2)
*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^
4*x-64*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^
8-32)^2*x^5+64*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*Ro
otOf(_Z^8-32)^2*x^3+16*(x^6+x^2)^(1/4)*RootOf(_Z^8-32)^2*x^2-64*RootOf(-2*R
ootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^2*x-32*(x^6+
x^2)^(1/2)*RootOf(_Z^8-32)*x+32*(x^6+x^2)^(3/4))/(RootOf(_Z^8-32)^4*x^4-2*R
ootOf(_Z^8-32)^4*x^2+RootOf(_Z^8-32)^4-8*x^4+8*x^2-8)/x)*RootOf(-2*RootOf(_
Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)+RootOf(-2*RootOf(_Z^8-32)^5*_Z+Root
Of(_Z^8-32)^2+64*_Z^2)*ln(-(RootOf(_Z^8-32)^11*x^5+4*RootOf(-2*RootOf(_Z^8-
32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^10*x^5-2*RootOf(_Z^8-32
)^11*x^3-8*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf
(_Z^8-32)^10*x^3+RootOf(_Z^8-32)^11*x+4*RootOf(-2*RootOf(_Z^8-32)^5*_Z+Root
Of(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^10*x+4*x^5*RootOf(_Z^8-32)^7+32*Root
Of(_Z^8-32)^6*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*x^3
+128*RootOf(_Z^8-32)^5*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*
_Z^2)*(x^6+x^2)^(1/4)*x^2+4*x*RootOf(_Z^8-32)^7+128*(x^6+x^2)^(1/2)*RootOf(
-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^4*x-32*x
^5*RootOf(_Z^8-32)^3-256*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+6
4*_Z^2)*RootOf(_Z^8-32)^2*x^5+32*x^3*RootOf(_Z^8-32)^3+256*RootOf(-2*RootOf
(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^2*x^3-64*(x^6+x^2
)^(1/4)*RootOf(_Z^8-32)^2*x^2-32*x*RootOf(_Z^8-32)^3-256*RootOf(-2*RootOf(_
Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^2*x+128*(x^6+x^2)^(
3/4))/(RootOf(_Z^8-32)^4*x^4-2*RootOf(_Z^8-32)^4*x^2+RootOf(_Z^8-32)^4-8*x^
4+8*x^2-8)/x)+1/4*ln(-(RootOf(_Z^8-32)^11*x^5-16*RootOf(-2*RootOf(_Z^8-32)^
5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^10*x^5-2*RootOf(_Z^8-32)^11
*x^3+32*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z
^8-32)^10*x^3+RootOf(_Z^8-32)^11*x-16*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf
(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^10*x+64*RootOf(_Z^8-32)^6*RootOf(-2*Ro
otOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*x^5-8*x^3*RootOf(_Z^8-32)^7+6
4*RootOf(_Z^8-32)^6*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^
2)*x+32*(x^6+x^2)^(1/2)*RootOf(_Z^8-32)^5*x-64*x^5*RootOf(_Z^8-32)^3+512*Ro
otOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^2*x
^5+64*x^3*RootOf(_Z^8-32)^3-512*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-
32)^2+64*_Z^2)*RootOf(_Z^8-32)^2*x^3-128*(x^6+x^2)^(1/4)*RootOf(_Z^8-32)^2*
x^2-64*x*RootOf(_Z^8-32)^3+512*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-3
2)^2+64*_Z^2)*RootOf(_Z^8-32)^2*x-2048*(x^6+x^2)^(1/2)*RootOf(-2*RootOf(_Z^
8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*x+256*(x^6+x^2)^(3/4))/x/(RootOf(_Z^8
-32)^4*x^4-2*RootOf(_Z^8-32)^4*x^2+RootOf(_Z^8-32)^4+8*x^4-8*x^2+8))*RootOf
(_Z^8-32)^4*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)-1/8*l
n(-(RootOf(_Z^8-32)^11*x^5-16*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32
)^2+64*_Z^2)*RootOf(_Z^8-32)^10*x^5-2*RootOf(_Z^8-32)^11*x^3+32*RootOf(-2*R
ootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^10*x^3+RootO
f(_Z^8-32)^11*x-16*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2
)*RootOf(_Z^8-32)^10*x+64*RootOf(_Z^8-32)^6*RootOf(-2*RootOf(_Z^8-32)^5*_Z+
RootOf(_Z^8-32)^2+64*_Z^2)*x^5-8*x^3*RootOf(_Z^8-32)^7+64*RootOf(_Z^8-32)^6
*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*x+32*(x^6+x^2)^(
1/2)*RootOf(_Z^8-32)^5*x-64*x^5*RootOf(_Z^8-32)^3+512*RootOf(-2*RootOf(_Z^8

```
-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootOf(_Z^8-32)^2*x^5+64*x^3*RootOf(_Z
^8-32)^3-512*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*Root
Of(_Z^8-32)^2*x^3-128*(x^6+x^2)^(1/4)*RootOf(_Z^8-32)^2*x^2-64*x*RootOf(_Z^
8-32)^3+512*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_Z^8-32)^2+64*_Z^2)*RootO
f(_Z^8-32)^2*x-2048*(x^6+x^2)^(1/2)*RootOf(-2*RootOf(_Z^8-32)^5*_Z+RootOf(_
Z^8-32)^2+64*_Z^2)*x+256*(x^6+x^2)^(3/4))/x/(RootOf(_Z^8-32)^4*x^4-2*RootOf
(_Z^8-32)^4*x^2+RootOf(_Z^8-32)^4+8*x^4-8*x^2+8))*RootOf(_Z^8-32)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8+1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + x^2)^(1/4)*(x^4 - 1)/(x^8 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6+x^2\right)^{1/4}\,\left(x^4-1\right)}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + x^6)^(1/4)*(x^4 - 1))/(x^8 + 1),x)
```

```
[Out] int(((x^2 + x^6)^(1/4)*(x^4 - 1))/(x^8 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\left(x^4+1\right)}\,(x-1)\,(x+1)\left(x^2+1\right)}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**6+x**2)**(1/4)/(x**8+1),x)
```

```
[Out] Integral((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)/(x**8 + 1), x)
```

3.1815 $\int \frac{-1+x^8}{\sqrt[4]{x^2+x^6}\left(1+x^8\right)}\,dx$

Optimal. Leaf size=162

$$-\frac{\tan^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[8]{2}}+\frac{\tan^{-1}\left(\frac{2^{5/8}x\sqrt[4]{x^6+x^2}}{\sqrt[4]{2}x^2-\sqrt{x^6+x^2}}\right)}{2\,2^{5/8}}-\frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[8]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{2^{3/8}}+\frac{\sqrt{x^6+x^2}}{2^{5/8}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\,2^{5/8}}$$

Rubi [C] time = 0.53, antiderivative size = 97, normalized size of antiderivative = 0.60, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2056, 1586, 6715, 6725, 429}

$$-\frac{(1-i)x\sqrt[4]{x^4+1}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};-ix^4,-x^4\right)}{\sqrt[4]{x^6+x^2}}-\frac{(1+i)x\sqrt[4]{x^4+1}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};ix^4,-x^4\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^8)/((x^2 + x^6)^(1/4)*(1 + x^8)),x]
```

```
[Out] ((-1 + I)*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, -3/4, 9/8, (-I)*x^4, -x^4])/(x
^2 + x^6)^(1/4) - ((1 + I)*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, -3/4, 9/8, I*
x^4, -x^4])/(x^2 + x^6)^(1/4)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+x^8}{\sqrt[4]{x^2+x^6}\,(1+x^8)}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{-1+x^8}{\sqrt{x}\sqrt[4]{1+x^4}(1+x^8)}\,dx}{\sqrt[4]{x^2+x^6}}$$
$$= \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{(-1+x^4)(1+x^4)^{3/4}}{\sqrt{x}(1+x^8)}\,dx}{\sqrt[4]{x^2+x^6}}$$
$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{(-1+x^8)(1+x^8)^{3/4}}{1+x^{16}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$
$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \left(-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(1+x^8)^{3/4}}{i-x^8}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)(1+x^8)^{3/4}}{i+x^8}\right)dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$
$$= -\frac{\left((1+i)\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{(1+x^8)^{3/4}}{i-x^8}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left((1-i)\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\right.}{\sqrt[4]{x^2+x}}$$
$$= -\frac{(1-i)x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};-ix^4,-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{(1+i)x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};ix^4,-\right.}{\sqrt[4]{x^2+x^6}}$$

Mathematica [F] time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{-1+x^8}{\sqrt[4]{x^2+x^6}\,(1+x^8)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^8)/((x^2 + x^6)^(1/4)*(1 + x^8)),x]
[Out] Integrate[(-1 + x^8)/((x^2 + x^6)^(1/4)*(1 + x^8)), x]
```

IntegrateAlgebraic [A] time = 0.66, size = 162, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[8]{2}}+\frac{\tan^{-1}\left(\frac{2^{5/8}x\sqrt[4]{x^6+x^2}}{\sqrt[4]{2}x^2-\sqrt{x^6+x^2}}\right)}{2\,2^{5/8}}-\frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[8]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{2^{3/8}}+\frac{\sqrt{x^6+x^2}}{2^{5/8}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\,2^{5/8}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^8)/((x^2 + x^6)^(1/4)*(1 + x^8)),x]
[Out] -1/2*ArcTan[(2^(1/8)*x)/(x^2 + x^6)^(1/4)]/2^(1/8) + ArcTan[(2^(5/8)*x*(x^2
 + x^6)^(1/4))/(2^(1/4)*x^2 - Sqrt[x^2 + x^6])]/(2*2^(5/8)) - ArcTanh[(2^(1
/8)*x)/(x^2 + x^6)^(1/4)]/(2*2^(1/8)) - ArcTanh[(x^2/2^(3/8) + Sqrt[x^2 + x
^6]/2^(5/8))/(x*(x^2 + x^6)^(1/4))]/(2*2^(5/8))
```

fricas [B] time = 115.50, size = 2347, normalized size = 14.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/(x^6+x^2)^(1/4)/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/4*2^(7/8)*arctan(1/2*(4*(x^6 + x^2)^(3/4)*(2^(5/8)*x^2 + 2^(1/8)*(x^4 +
1)) + (2^(5/8)*(x^9 + 4*x^7 + 4*x^5 + 4*x^3 + x) + 2*sqrt(x^6 + x^2)*(2^(7/
8)*(x^5 + 2*x^3 + x) + 2*2^(3/8)*(x^5 + x^3 + x)) + 2*2^(1/8)*(x^9 + 2*x^7
+ 4*x^5 + 2*x^3 + x))*sqrt(3*2^(3/4) - 4*2^(1/4)) + 4*(x^6 + x^2)^(1/4)*(2^
(7/8)*x^4 + 2^(3/8)*(x^6 + x^2)))/(x^9 + x)) - 1/16*2^(7/8)*log(-(2^(7/8)*(
x^9 - 2*x^7 + 4*x^5 - 2*x^3 + x) + 2*(x^6 + x^2)^(3/4)*(2*x^4 - 2*x^2 - sqr
t(2)*(x^4 - 2*x^2 + 1) + 2) - 2*sqrt(x^6 + x^2)*(2^(5/8)*(x^5 - 2*x^3 + x)
- 2*2^(1/8)*(x^5 - x^3 + x)) - 2^(3/8)*(x^9 - 4*x^7 + 4*x^5 - 4*x^3 + x) -
2*(x^6 + x^2)^(1/4)*(2^(3/4)*(x^6 - 2*x^4 + x^2) - 2*2^(1/4)*(x^6 - x^4 + x
^2)))/(x^9 + x)) + 1/16*2^(7/8)*log((2^(7/8)*(x^9 - 2*x^7 + 4*x^5 - 2*x^3 +
 x) - 2*(x^6 + x^2)^(3/4)*(2*x^4 - 2*x^2 - sqrt(2)*(x^4 - 2*x^2 + 1) + 2) -
 2*sqrt(x^6 + x^2)*(2^(5/8)*(x^5 - 2*x^3 + x) - 2*2^(1/8)*(x^5 - x^3 + x))
- 2^(3/8)*(x^9 - 4*x^7 + 4*x^5 - 4*x^3 + x) + 2*(x^6 + x^2)^(1/4)*(2^(3/4)*
(x^6 - 2*x^4 + x^2) - 2*2^(1/4)*(x^6 - x^4 + x^2)))/(x^9 + x)) - 1/4*2^(3/8
)*arctan((x^17 + 64*x^13 + 130*x^9 + 64*x^5 + 2*(x^6 + x^2)^(3/4)*(2^(5/8)*
(x^12 - 79*x^8 - 79*x^4 + 1) + 2*2^(1/8)*(11*x^10 + 16*x^6 + 11*x^2)) + 16*
sqrt(2)*(x^15 + 5*x^11 + 5*x^7 + x^3) + 4*sqrt(x^6 + x^2)*(2^(3/4)*(15*x^11
 + 32*x^7 + 15*x^3) + 2^(1/4)*(x^13 + 33*x^9 + 33*x^5 + x)) + (16*(x^6 + x^
2)^(3/4)*(2^(3/4)*(x^10 - 14*x^8 + 4*x^6 - 14*x^4 + x^2) - 2^(1/4)*(x^10 -
28*x^8 + 4*x^6 - 28*x^4 + x^2)) + 2^(5/8)*(x^17 - 6*x^15 - 220*x^13 + 26*x^
11 - 446*x^9 + 26*x^7 - 220*x^5 - 6*x^3 + x) + 2*sqrt(x^6 + x^2)*(2^(7/8)*(
x^13 - 11*x^11 - 79*x^9 - 16*x^7 - 79*x^5 - 11*x^3 + x) - 2^(3/8)*(x^13 - 2
2*x^11 - 79*x^9 - 32*x^7 - 79*x^5 - 22*x^3 + x)) - 4*(x^14 - 30*x^12 + 33*x
^10 - 64*x^8 + 33*x^6 - 30*x^4 + x^2 - sqrt(2)*(x^14 - 15*x^12 + 33*x^10 -
32*x^8 + 33*x^6 - 15*x^4 + x^2))*(x^6 + x^2)^(1/4) - 2^(1/8)*(x^17 - 12*x^1
5 - 220*x^13 + 52*x^11 - 446*x^9 + 52*x^7 - 220*x^5 - 12*x^3 + x))*sqrt((3*
2^(3/4)*(x^9 + x) + 4*(x^6 + x^2)^(3/4)*(2^(7/8)*(2*x^4 - 3*x^2 + 2) + 2^(3
/8)*(3*x^4 - 4*x^2 + 3)) + 8*sqrt(x^6 + x^2)*(3*x^5 - 4*x^3 + sqrt(2)*(2*x^
5 - 3*x^3 + 2*x) + 3*x) + 4*2^(1/4)*(x^9 + x) + 4*(x^6 + x^2)^(1/4)*(2^(5/8
)*(3*x^6 - 4*x^4 + 3*x^2) + 2*2^(1/8)*(2*x^6 - 3*x^4 + 2*x^2)))/(x^9 + x))
+ 2*(x^6 + x^2)^(1/4)*(2^(7/8)*(3*x^14 - 13*x^10 - 13*x^6 + 3*x^2) + 2*2^(3
/8)*(41*x^12 + 80*x^8 + 41*x^4)) + x)/(x^17 - 384*x^13 - 766*x^9 - 384*x^5
+ x)) + 1/4*2^(3/8)*arctan((x^17 + 64*x^13 + 130*x^9 + 64*x^5 - 2*(x^6 + x^
2)^(3/4)*(2^(5/8)*(x^12 - 79*x^8 - 79*x^4 + 1) + 2*2^(1/8)*(11*x^10 + 16*x^
6 + 11*x^2)) + 16*sqrt(2)*(x^15 + 5*x^11 + 5*x^7 + x^3) + 4*sqrt(x^6 + x^2)
*(2^(3/4)*(15*x^11 + 32*x^7 + 15*x^3) + 2^(1/4)*(x^13 + 33*x^9 + 33*x^5 + x
)) + (16*(x^6 + x^2)^(3/4)*(2^(3/4)*(x^10 - 14*x^8 + 4*x^6 - 14*x^4 + x^2)
- 2^(1/4)*(x^10 - 28*x^8 + 4*x^6 - 28*x^4 + x^2)) - 2^(5/8)*(x^17 - 6*x^15
- 220*x^13 + 26*x^11 - 446*x^9 + 26*x^7 - 220*x^5 - 6*x^3 + x) - 2*sqrt(x^6
 + x^2)*(2^(7/8)*(x^13 - 11*x^11 - 79*x^9 - 16*x^7 - 79*x^5 - 11*x^3 + x) -
 2^(3/8)*(x^13 - 22*x^11 - 79*x^9 - 32*x^7 - 79*x^5 - 22*x^3 + x)) - 4*(x^1
4 - 30*x^12 + 33*x^10 - 64*x^8 + 33*x^6 - 30*x^4 + x^2 - sqrt(2)*(x^14 - 15
*x^12 + 33*x^10 - 32*x^8 + 33*x^6 - 15*x^4 + x^2))*(x^6 + x^2)^(1/4) + 2^(1
/8)*(x^17 - 12*x^15 - 220*x^13 + 52*x^11 - 446*x^9 + 52*x^7 - 220*x^5 - 12*
x^3 + x))*sqrt((3*2^(3/4)*(x^9 + x) - 4*(x^6 + x^2)^(3/4)*(2^(7/8)*(2*x^4 -
 3*x^2 + 2) + 2^(3/8)*(3*x^4 - 4*x^2 + 3)) + 8*sqrt(x^6 + x^2)*(3*x^5 - 4*x
^3 + sqrt(2)*(2*x^5 - 3*x^3 + 2*x) + 3*x) + 4*2^(1/4)*(x^9 + x) - 4*(x^6 +
x^2)^(1/4)*(2^(5/8)*(3*x^6 - 4*x^4 + 3*x^2) + 2*2^(1/8)*(2*x^6 - 3*x^4 + 2*
x^2)))/(x^9 + x)) - 2*(x^6 + x^2)^(1/4)*(2^(7/8)*(3*x^14 - 13*x^10 - 13*x^6
 + 3*x^2) + 2*2^(3/8)*(41*x^12 + 80*x^8 + 41*x^4)) + x)/(x^17 - 384*x^13 -
766*x^9 - 384*x^5 + x)) - 1/16*2^(3/8)*log(4*(3*2^(3/4)*(x^9 + x) + 4*(x^6
+ x^2)^(3/4)*(2^(7/8)*(2*x^4 - 3*x^2 + 2) + 2^(3/8)*(3*x^4 - 4*x^2 + 3)) +
8*sqrt(x^6 + x^2)*(3*x^5 - 4*x^3 + sqrt(2)*(2*x^5 - 3*x^3 + 2*x) + 3*x) + 4
*2^(1/4)*(x^9 + x) + 4*(x^6 + x^2)^(1/4)*(2^(5/8)*(3*x^6 - 4*x^4 + 3*x^2) +
 2*2^(1/8)*(2*x^6 - 3*x^4 + 2*x^2)))/(x^9 + x)) + 1/16*2^(3/8)*log(4*(3*2^(
3/4)*(x^9 + x) - 4*(x^6 + x^2)^(3/4)*(2^(7/8)*(2*x^4 - 3*x^2 + 2) + 2^(3/8)
*(3*x^4 - 4*x^2 + 3)) + 8*sqrt(x^6 + x^2)*(3*x^5 - 4*x^3 + sqrt(2)*(2*x^5 -
 3*x^3 + 2*x) + 3*x) + 4*2^(1/4)*(x^9 + x) - 4*(x^6 + x^2)^(1/4)*(2^(5/8)*(
3*x^6 - 4*x^4 + 3*x^2) + 2*2^(1/8)*(2*x^6 - 3*x^4 + 2*x^2)))/(x^9 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 - 1}{\left(x^8 + 1\right)\left(x^6 + x^2\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/(x^6+x^2)^(1/4)/(x^8+1),x, algorithm="giac")
[Out] integrate((x^8 - 1)/((x^8 + 1)*(x^6 + x^2)^(1/4)), x)
```

maple [C] time = 133.74, size = 2799, normalized size = 17.28

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-1)/(x^6+x^2)^(1/4)/(x^8+1),x)
[Out] -1/8*RootOf(_Z^8-128)*ln((RootOf(_Z^8-128)^9*x^5-2*x^3*RootOf(_Z^8-128)^9+R
ootOf(_Z^8-128)^9*x-8*(x^6+x^2)^(1/2)*RootOf(_Z^8-128)^7*x-16*RootOf(_Z^8-1
28)^6*(x^6+x^2)^(1/4)*x^2-16*RootOf(_Z^8-128)^5*x^3-256*RootOf(_Z^8-128)*x^
5+256*RootOf(_Z^8-128)*x^3-512*(x^6+x^2)^(3/4)-256*RootOf(_Z^8-128)*x)/(x^4
*RootOf(_Z^8-128)^4-2*x^2*RootOf(_Z^8-128)^4+RootOf(_Z^8-128)^4+16*x^4-16*x
^2+16)/x)-RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*ln((Roo
tOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x
^5-2*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-
128)^8*x^3+RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf
(_Z^8-128)^8*x+8*(x^6+x^2)^(1/2)*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-
128)^2+64*_Z^2)*RootOf(_Z^8-128)^6*x-2*RootOf(_Z^8-128)^6*(x^6+x^2)^(1/4)*x
^2+16*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8
-128)^4*x^3-16*(x^6+x^2)^(1/2)*RootOf(_Z^8-128)^3*x-256*RootOf(-RootOf(_Z^8
-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x^5+256*RootOf(_Z^8-128)*RootOf(-Roo
tOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*(x^6+x^2)^(1/4)*x^2+256*Root
Of(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x^3-64*(x^6+x^2)^(3/4
)-256*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x)/(x^4*Roo
tOf(_Z^8-128)^4-2*x^2*RootOf(_Z^8-128)^4+RootOf(_Z^8-128)^4-16*x^4+16*x^2-1
6)/x)+1/8*ln((RootOf(_Z^8-128)^9*x^5-16*RootOf(-RootOf(_Z^8-128)^5*_Z+RootO
f(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x^5-2*x^3*RootOf(_Z^8-128)^9+32*R
ootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8
*x^3+RootOf(_Z^8-128)^9*x-16*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)
^2+64*_Z^2)*RootOf(_Z^8-128)^8*x+8*(x^6+x^2)^(1/2)*RootOf(_Z^8-128)^7*x+128
*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)
^4*x^5+16*RootOf(_Z^8-128)^6*(x^6+x^2)^(1/4)*x^2-16*RootOf(_Z^8-128)^5*x^3+
128*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-1
28)^4*x-1024*RootOf(_Z^8-128)^2*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-1
28)^2+64*_Z^2)*(x^6+x^2)^(1/2)*x-256*RootOf(_Z^8-128)*x^5+2048*RootOf(-Root
Of(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x^5+256*RootOf(_Z^8-128)*x^3-
2048*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x^3-512*(x^6
+x^2)^(3/4)-256*RootOf(_Z^8-128)*x+2048*RootOf(-RootOf(_Z^8-128)^5*_Z+RootO
f(_Z^8-128)^2+64*_Z^2)*x)/(x^4*RootOf(_Z^8-128)^4-2*x^2*RootOf(_Z^8-128)^4+
RootOf(_Z^8-128)^4+16*x^4-16*x^2+16)/x)*RootOf(_Z^8-128)^4*RootOf(-RootOf(_
Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)-1/8*ln((RootOf(_Z^8-128)^9*x^5-16
*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)
^8*x^5-2*x^3*RootOf(_Z^8-128)^9+32*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^
8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x^3+RootOf(_Z^8-128)^9*x-16*RootOf(-Ro
otOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x+8*(x^6
+x^2)^(1/2)*RootOf(_Z^8-128)^7*x+128*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_
Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^4*x^5+16*RootOf(_Z^8-128)^6*(x^6+x^2)^
(1/4)*x^2-16*RootOf(_Z^8-128)^5*x^3+128*RootOf(-RootOf(_Z^8-128)^5*_Z+RootO
```

```
f(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^4*x-1024*RootOf(_Z^8-128)^2*RootOf(
-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*(x^6+x^2)^(1/2)*x-256*Ro
otOf(_Z^8-128)*x^5+2048*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64
*_Z^2)*x^5+256*RootOf(_Z^8-128)*x^3-2048*RootOf(-RootOf(_Z^8-128)^5*_Z+Root
Of(_Z^8-128)^2+64*_Z^2)*x^3-512*(x^6+x^2)^(3/4)-256*RootOf(_Z^8-128)*x+2048
*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x)/(x^4*RootOf(_
Z^8-128)^4-2*x^2*RootOf(_Z^8-128)^4+RootOf(_Z^8-128)^4+16*x^4-16*x^2+16)/x)
*RootOf(_Z^8-128)-1/64*ln(-(RootOf(_Z^8-128)^9*x^5+4*RootOf(-RootOf(_Z^8-12
8)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x^5-2*x^3*RootOf(_Z^
8-128)^9-8*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf
(_Z^8-128)^8*x^3+RootOf(_Z^8-128)^9*x+4*RootOf(-RootOf(_Z^8-128)^5*_Z+RootO
f(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x+32*(x^6+x^2)^(1/2)*RootOf(-Root
Of(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^6*x+8*RootOf
(_Z^8-128)^5*x^5-8*RootOf(_Z^8-128)^6*(x^6+x^2)^(1/4)*x^2+64*RootOf(-RootOf
(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^4*x^3+8*RootOf
(_Z^8-128)^5*x-128*RootOf(_Z^8-128)*x^5-1024*RootOf(-RootOf(_Z^8-128)^5*_Z+
RootOf(_Z^8-128)^2+64*_Z^2)*x^5+1024*RootOf(_Z^8-128)*RootOf(-RootOf(_Z^8-1
28)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*(x^6+x^2)^(1/4)*x^2+128*RootOf(_Z^8-12
8)*x^3+1024*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x^3+2
56*(x^6+x^2)^(3/4)-128*RootOf(_Z^8-128)*x-1024*RootOf(-RootOf(_Z^8-128)^5*_
Z+RootOf(_Z^8-128)^2+64*_Z^2)*x)/(x^4*RootOf(_Z^8-128)^4-2*x^2*RootOf(_Z^8-
128)^4+RootOf(_Z^8-128)^4-16*x^4+16*x^2-16)/x)*RootOf(_Z^8-128)^5+ln(-(Root
Of(_Z^8-128)^9*x^5+4*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z
^2)*RootOf(_Z^8-128)^8*x^5-2*x^3*RootOf(_Z^8-128)^9-8*RootOf(-RootOf(_Z^8-1
28)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x^3+RootOf(_Z^8-128
)^9*x+4*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z
^8-128)^8*x+32*(x^6+x^2)^(1/2)*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-12
8)^2+64*_Z^2)*RootOf(_Z^8-128)^6*x+8*RootOf(_Z^8-128)^5*x^5-8*RootOf(_Z^8-1
28)^6*(x^6+x^2)^(1/4)*x^2+64*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)
^2+64*_Z^2)*RootOf(_Z^8-128)^4*x^3+8*RootOf(_Z^8-128)^5*x-128*RootOf(_Z^8-1
28)*x^5-1024*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x^5+
1024*RootOf(_Z^8-128)*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_
Z^2)*(x^6+x^2)^(1/4)*x^2+128*RootOf(_Z^8-128)*x^3+1024*RootOf(-RootOf(_Z^8-
128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x^3+256*(x^6+x^2)^(3/4)-128*RootOf(_Z
^8-128)*x-1024*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x)
/(x^4*RootOf(_Z^8-128)^4-2*x^2*RootOf(_Z^8-128)^4+RootOf(_Z^8-128)^4-16*x^4
+16*x^2-16)/x)*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8-1}{\left(x^8+1\right)\left(x^6+x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/(x^6+x^2)^(1/4)/(x^8+1),x, algorithm="maxima")
[Out] integrate((x^8 - 1)/((x^8 + 1)*(x^6 + x^2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8-1}{\left(x^6+x^2\right)^{1/4}\,\left(x^8+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 - 1)/((x^2 + x^6)^(1/4)*(x^8 + 1)),x)
[Out] int((x^8 - 1)/((x^2 + x^6)^(1/4)*(x^8 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-1)/(x**6+x**2)**(1/4)/(x**8+1),x)
```

```
[Out] Timed out
```

3.1816 $\int \frac{-1+x^8}{\sqrt[4]{x^2+x^6}\left(1+x^8\right)}\,dx$

Optimal. Leaf size=162

$$-\frac{\tan^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[8]{2}}+\frac{\tan^{-1}\left(\frac{2^{5/8}x\sqrt[4]{x^6+x^2}}{\sqrt[4]{2}x^2-\sqrt{x^6+x^2}}\right)}{2\,2^{5/8}}-\frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[8]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{2^{3/8}}+\frac{\sqrt{x^6+x^2}}{2^{5/8}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\,2^{5/8}}$$

Rubi [C] time = 0.44, antiderivative size = 97, normalized size of antiderivative = 0.60, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2056, 1586, 6715, 6725, 429}

$$-\frac{(1-i)x\sqrt[4]{x^4+1}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};-ix^4,-x^4\right)}{\sqrt[4]{x^6+x^2}}-\frac{(1+i)x\sqrt[4]{x^4+1}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};ix^4,-x^4\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^8)/((x^2 + x^6)^(1/4)*(1 + x^8)),x]
```

```
[Out] ((-1 + I)*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, -3/4, 9/8, (-I)*x^4, -x^4])/(x
^2 + x^6)^(1/4) - ((1 + I)*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, -3/4, 9/8, I*
x^4, -x^4])/(x^2 + x^6)^(1/4)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+x^8}{\sqrt[4]{x^2+x^6}\left(1+x^8\right)}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{-1+x^8}{\sqrt{x}\sqrt[4]{1+x^4}\left(1+x^8\right)}\,dx}{\sqrt[4]{x^2+x^6}}$$
$$= \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{\left(-1+x^4\right)\left(1+x^4\right)^{3/4}}{\sqrt{x}\left(1+x^8\right)}\,dx}{\sqrt[4]{x^2+x^6}}$$
$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{\left(-1+x^8\right)\left(1+x^8\right)^{3/4}}{1+x^{16}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$
$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \left(-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+x^8\right)^{3/4}}{i-x^8}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(1+x^8\right)^{3/4}}{i+x^8}\right)dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$
$$= -\frac{\left((1+i)\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{\left(1+x^8\right)^{3/4}}{i-x^8}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left((1-i)\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\right.}{\sqrt[4]{x^2+x}}$$
$$= -\frac{(1-i)x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};-ix^4,-x^4\right)}{\sqrt[4]{x^2+x^6}} - \frac{(1+i)x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};ix^4,-\right.}{\sqrt[4]{x^2+x^6}}$$

Mathematica [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{-1+x^8}{\sqrt[4]{x^2+x^6}\left(1+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^8)/((x^2 + x^6)^(1/4)*(1 + x^8)),x]
[Out] Integrate[(-1 + x^8)/((x^2 + x^6)^(1/4)*(1 + x^8)), x]
```

IntegrateAlgebraic [A] time = 0.00, size = 162, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[8]{2}} + \frac{\tan^{-1}\left(\frac{2^{5/8}x\sqrt[4]{x^6+x^2}}{\sqrt[4]{2}x^2-\sqrt{x^6+x^2}}\right)}{2\,2^{5/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[8]{2}} - \frac{\tanh^{-1}\left(\frac{\frac{x^2}{2^{3/8}}+\frac{\sqrt{x^6+x^2}}{2^{5/8}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\,2^{5/8}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^8)/((x^2 + x^6)^(1/4)*(1 + x^8)),x]
[Out] -1/2*ArcTan[(2^(1/8)*x)/(x^2 + x^6)^(1/4)]/2^(1/8) + ArcTan[(2^(5/8)*x*(x^2
 + x^6)^(1/4))/(2^(1/4)*x^2 - Sqrt[x^2 + x^6])]/(2*2^(5/8)) - ArcTanh[(2^(1
/8)*x)/(x^2 + x^6)^(1/4)]/(2*2^(1/8)) - ArcTanh[(x^2/2^(3/8) + Sqrt[x^2 + x
^6]/2^(5/8))/(x*(x^2 + x^6)^(1/4))]/(2*2^(5/8))
```

fricas [B] time = 116.03, size = 2347, normalized size = 14.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/(x^6+x^2)^(1/4)/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/4*2^(7/8)*arctan(1/2*(4*(x^6 + x^2)^(3/4)*(2^(5/8)*x^2 + 2^(1/8)*(x^4 +
1)) + (2^(5/8)*(x^9 + 4*x^7 + 4*x^5 + 4*x^3 + x) + 2*sqrt(x^6 + x^2)*(2^(7/
8)*(x^5 + 2*x^3 + x) + 2*2^(3/8)*(x^5 + x^3 + x)) + 2*2^(1/8)*(x^9 + 2*x^7
+ 4*x^5 + 2*x^3 + x))*sqrt(3*2^(3/4) - 4*2^(1/4)) + 4*(x^6 + x^2)^(1/4)*(2^
(7/8)*x^4 + 2^(3/8)*(x^6 + x^2)))/(x^9 + x)) - 1/16*2^(7/8)*log(-(2^(7/8)*(
x^9 - 2*x^7 + 4*x^5 - 2*x^3 + x) + 2*(x^6 + x^2)^(3/4)*(2*x^4 - 2*x^2 - sqr
t(2)*(x^4 - 2*x^2 + 1) + 2) - 2*sqrt(x^6 + x^2)*(2^(5/8)*(x^5 - 2*x^3 + x)
- 2*2^(1/8)*(x^5 - x^3 + x)) - 2^(3/8)*(x^9 - 4*x^7 + 4*x^5 - 4*x^3 + x) -
2*(x^6 + x^2)^(1/4)*(2^(3/4)*(x^6 - 2*x^4 + x^2) - 2*2^(1/4)*(x^6 - x^4 + x
^2)))/(x^9 + x)) + 1/16*2^(7/8)*log((2^(7/8)*(x^9 - 2*x^7 + 4*x^5 - 2*x^3 +
 x) - 2*(x^6 + x^2)^(3/4)*(2*x^4 - 2*x^2 - sqrt(2)*(x^4 - 2*x^2 + 1) + 2) -
 2*sqrt(x^6 + x^2)*(2^(5/8)*(x^5 - 2*x^3 + x) - 2*2^(1/8)*(x^5 - x^3 + x))
- 2^(3/8)*(x^9 - 4*x^7 + 4*x^5 - 4*x^3 + x) + 2*(x^6 + x^2)^(1/4)*(2^(3/4)*
(x^6 - 2*x^4 + x^2) - 2*2^(1/4)*(x^6 - x^4 + x^2)))/(x^9 + x)) - 1/4*2^(3/8
)*arctan((x^17 + 64*x^13 + 130*x^9 + 64*x^5 + 2*(x^6 + x^2)^(3/4)*(2^(5/8)*
(x^12 - 79*x^8 - 79*x^4 + 1) + 2*2^(1/8)*(11*x^10 + 16*x^6 + 11*x^2)) + 16*
sqrt(2)*(x^15 + 5*x^11 + 5*x^7 + x^3) + 4*sqrt(x^6 + x^2)*(2^(3/4)*(15*x^11
 + 32*x^7 + 15*x^3) + 2^(1/4)*(x^13 + 33*x^9 + 33*x^5 + x)) + (16*(x^6 + x^
2)^(3/4)*(2^(3/4)*(x^10 - 14*x^8 + 4*x^6 - 14*x^4 + x^2) - 2^(1/4)*(x^10 -
28*x^8 + 4*x^6 - 28*x^4 + x^2)) + 2^(5/8)*(x^17 - 6*x^15 - 220*x^13 + 26*x^
11 - 446*x^9 + 26*x^7 - 220*x^5 - 6*x^3 + x) + 2*sqrt(x^6 + x^2)*(2^(7/8)*(
x^13 - 11*x^11 - 79*x^9 - 16*x^7 - 79*x^5 - 11*x^3 + x) - 2^(3/8)*(x^13 - 2
2*x^11 - 79*x^9 - 32*x^7 - 79*x^5 - 22*x^3 + x)) - 4*(x^14 - 30*x^12 + 33*x
^10 - 64*x^8 + 33*x^6 - 30*x^4 + x^2 - sqrt(2)*(x^14 - 15*x^12 + 33*x^10 -
32*x^8 + 33*x^6 - 15*x^4 + x^2))*(x^6 + x^2)^(1/4) - 2^(1/8)*(x^17 - 12*x^1
5 - 220*x^13 + 52*x^11 - 446*x^9 + 52*x^7 - 220*x^5 - 12*x^3 + x))*sqrt((3*
2^(3/4)*(x^9 + x) + 4*(x^6 + x^2)^(3/4)*(2^(7/8)*(2*x^4 - 3*x^2 + 2) + 2^(3
/8)*(3*x^4 - 4*x^2 + 3)) + 8*sqrt(x^6 + x^2)*(3*x^5 - 4*x^3 + sqrt(2)*(2*x^
5 - 3*x^3 + 2*x) + 3*x) + 4*2^(1/4)*(x^9 + x) + 4*(x^6 + x^2)^(1/4)*(2^(5/8
)*(3*x^6 - 4*x^4 + 3*x^2) + 2*2^(1/8)*(2*x^6 - 3*x^4 + 2*x^2)))/(x^9 + x))
+ 2*(x^6 + x^2)^(1/4)*(2^(7/8)*(3*x^14 - 13*x^10 - 13*x^6 + 3*x^2) + 2*2^(3
/8)*(41*x^12 + 80*x^8 + 41*x^4)) + x)/(x^17 - 384*x^13 - 766*x^9 - 384*x^5
+ x)) + 1/4*2^(3/8)*arctan((x^17 + 64*x^13 + 130*x^9 + 64*x^5 - 2*(x^6 + x^
2)^(3/4)*(2^(5/8)*(x^12 - 79*x^8 - 79*x^4 + 1) + 2*2^(1/8)*(11*x^10 + 16*x^
6 + 11*x^2)) + 16*sqrt(2)*(x^15 + 5*x^11 + 5*x^7 + x^3) + 4*sqrt(x^6 + x^2)
*(2^(3/4)*(15*x^11 + 32*x^7 + 15*x^3) + 2^(1/4)*(x^13 + 33*x^9 + 33*x^5 + x
)) + (16*(x^6 + x^2)^(3/4)*(2^(3/4)*(x^10 - 14*x^8 + 4*x^6 - 14*x^4 + x^2)
- 2^(1/4)*(x^10 - 28*x^8 + 4*x^6 - 28*x^4 + x^2)) - 2^(5/8)*(x^17 - 6*x^15
- 220*x^13 + 26*x^11 - 446*x^9 + 26*x^7 - 220*x^5 - 6*x^3 + x) - 2*sqrt(x^6
 + x^2)*(2^(7/8)*(x^13 - 11*x^11 - 79*x^9 - 16*x^7 - 79*x^5 - 11*x^3 + x) -
 2^(3/8)*(x^13 - 22*x^11 - 79*x^9 - 32*x^7 - 79*x^5 - 22*x^3 + x)) - 4*(x^1
4 - 30*x^12 + 33*x^10 - 64*x^8 + 33*x^6 - 30*x^4 + x^2 - sqrt(2)*(x^14 - 15
*x^12 + 33*x^10 - 32*x^8 + 33*x^6 - 15*x^4 + x^2))*(x^6 + x^2)^(1/4) + 2^(1
/8)*(x^17 - 12*x^15 - 220*x^13 + 52*x^11 - 446*x^9 + 52*x^7 - 220*x^5 - 12*
x^3 + x))*sqrt((3*2^(3/4)*(x^9 + x) - 4*(x^6 + x^2)^(3/4)*(2^(7/8)*(2*x^4 -
 3*x^2 + 2) + 2^(3/8)*(3*x^4 - 4*x^2 + 3)) + 8*sqrt(x^6 + x^2)*(3*x^5 - 4*x
^3 + sqrt(2)*(2*x^5 - 3*x^3 + 2*x) + 3*x) + 4*2^(1/4)*(x^9 + x) - 4*(x^6 +
x^2)^(1/4)*(2^(5/8)*(3*x^6 - 4*x^4 + 3*x^2) + 2*2^(1/8)*(2*x^6 - 3*x^4 + 2*
x^2)))/(x^9 + x)) - 2*(x^6 + x^2)^(1/4)*(2^(7/8)*(3*x^14 - 13*x^10 - 13*x^6
 + 3*x^2) + 2*2^(3/8)*(41*x^12 + 80*x^8 + 41*x^4)) + x)/(x^17 - 384*x^13 -
766*x^9 - 384*x^5 + x)) - 1/16*2^(3/8)*log(4*(3*2^(3/4)*(x^9 + x) + 4*(x^6
+ x^2)^(3/4)*(2^(7/8)*(2*x^4 - 3*x^2 + 2) + 2^(3/8)*(3*x^4 - 4*x^2 + 3)) +
8*sqrt(x^6 + x^2)*(3*x^5 - 4*x^3 + sqrt(2)*(2*x^5 - 3*x^3 + 2*x) + 3*x) + 4
*2^(1/4)*(x^9 + x) + 4*(x^6 + x^2)^(1/4)*(2^(5/8)*(3*x^6 - 4*x^4 + 3*x^2) +
 2*2^(1/8)*(2*x^6 - 3*x^4 + 2*x^2)))/(x^9 + x)) + 1/16*2^(3/8)*log(4*(3*2^(
3/4)*(x^9 + x) - 4*(x^6 + x^2)^(3/4)*(2^(7/8)*(2*x^4 - 3*x^2 + 2) + 2^(3/8)
*(3*x^4 - 4*x^2 + 3)) + 8*sqrt(x^6 + x^2)*(3*x^5 - 4*x^3 + sqrt(2)*(2*x^5 -
 3*x^3 + 2*x) + 3*x) + 4*2^(1/4)*(x^9 + x) - 4*(x^6 + x^2)^(1/4)*(2^(5/8)*(
3*x^6 - 4*x^4 + 3*x^2) + 2*2^(1/8)*(2*x^6 - 3*x^4 + 2*x^2)))/(x^9 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8-1}{\left(x^8+1\right)\left(x^6+x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^8-1)/(x^6+x^2)^(1/4)/(x^8+1),x, algorithm="giac")

[Out] integrate((x^8 - 1)/((x^8 + 1)*(x^6 + x^2)^(1/4)), x)

maple [C] time = 129.41, size = 2801, normalized size = 17.29

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8-1)/(x^6+x^2)^(1/4)/(x^8+1),x)

[Out] RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*ln(-(RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x^5-2*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x^3+RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x+8*(x^6+x^2)^(1/2)*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^6*x+2*RootOf(_Z^8-128)^6*(x^6+x^2)^(1/4)*x^2+16*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^4*x^3-16*(x^6+x^2)^(1/2)*RootOf(_Z^8-128)^3*x-256*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x^5-256*RootOf(_Z^8-128)*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*(x^6+x^2)^(1/4)*x^2+256*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x^3+64*(x^6+x^2)^(3/4)-256*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x)/(x^4*RootOf(_Z^8-128)^4-2*x^2*RootOf(_Z^8-128)^4+RootOf(_Z^8-128)^4-16*x^4+16*x^2-16)/x)-1/8*ln(-(RootOf(_Z^8-128)^9*x^5-16*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x^5-2*x^3*RootOf(_Z^8-128)^9+32*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x^3+RootOf(_Z^8-128)^9*x-16*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x+8*(x^6+x^2)^(1/2)*RootOf(_Z^8-128)^7*x+128*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^4*x^5-16*RootOf(_Z^8-128)^6*(x^6+x^2)^(1/4)*x^2-16*RootOf(_Z^8-128)^5*x^3+128*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^4*x-1024*RootOf(_Z^8-128)^2*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*(x^6+x^2)^(1/2)*x-256*RootOf(_Z^8-128)*x^5+2048*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x^5+256*RootOf(_Z^8-128)*x^3-2048*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x^3+512*(x^6+x^2)^(3/4)-256*RootOf(_Z^8-128)*x+2048*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x)/(x^4*RootOf(_Z^8-128)^4-2*x^2*RootOf(_Z^8-128)^4+RootOf(_Z^8-128)^4+16*x^4-16*x^2+16)/x)*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^4+1/8*ln(-(RootOf(_Z^8-128)^9*x^5-16*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x^5-2*x^3*RootOf(_Z^8-128)^9+32*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x^3+RootOf(_Z^8-128)^9*x-16*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x+8*(x^6+x^2)^(1/2)*RootOf(_Z^8-128)^7*x+128*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^4*x^5-16*RootOf(_Z^8-128)^6*(x^6+x^2)^(1/4)*x^2-16*RootOf(_Z^8-128)^5*x^3+128*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^4*x-1024*RootOf(_Z^8-128)^2*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*(x^6+x^2)^(1/2)*x-256*RootOf(_Z^8-128)*x^5+2048*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x^5+256*RootOf(_Z^8-128)*x^3-2048*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x^3+512*(x^6+x^2)^(3/4)-256*RootOf(_Z^8-128)*x+2048*RootOf

```
(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x)/(x^4*RootOf(_Z^8-128
)^4-2*x^2*RootOf(_Z^8-128)^4+RootOf(_Z^8-128)^4+16*x^4-16*x^2+16)/x)*RootOf
(_Z^8-128)+1/8*RootOf(_Z^8-128)*ln(-(RootOf(_Z^8-128)^9*x^5-2*x^3*RootOf(_Z
^8-128)^9+RootOf(_Z^8-128)^9*x-8*(x^6+x^2)^(1/2)*RootOf(_Z^8-128)^7*x+16*Ro
otOf(_Z^8-128)^6*(x^6+x^2)^(1/4)*x^2-16*RootOf(_Z^8-128)^5*x^3-256*RootOf(_
Z^8-128)*x^5+256*RootOf(_Z^8-128)*x^3+512*(x^6+x^2)^(3/4)-256*RootOf(_Z^8-1
28)*x)/(x^4*RootOf(_Z^8-128)^4-2*x^2*RootOf(_Z^8-128)^4+RootOf(_Z^8-128)^4+
16*x^4-16*x^2+16)/x)+1/64*ln((RootOf(_Z^8-128)^9*x^5+4*RootOf(-RootOf(_Z^8-
128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x^5-2*x^3*RootOf(_
Z^8-128)^9-8*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*Root
Of(_Z^8-128)^8*x^3+RootOf(_Z^8-128)^9*x+4*RootOf(-RootOf(_Z^8-128)^5*_Z+Roo
tOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x+32*(x^6+x^2)^(1/2)*RootOf(-Ro
otOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^6*x+8*Root
Of(_Z^8-128)^5*x^5+8*RootOf(_Z^8-128)^6*(x^6+x^2)^(1/4)*x^2+64*RootOf(-Root
Of(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^4*x^3+8*Root
Of(_Z^8-128)^5*x-128*RootOf(_Z^8-128)*x^5-1024*RootOf(-RootOf(_Z^8-128)^5*_
Z+RootOf(_Z^8-128)^2+64*_Z^2)*x^5-1024*RootOf(_Z^8-128)*RootOf(-RootOf(_Z^8
-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*(x^6+x^2)^(1/4)*x^2+128*RootOf(_Z^8-
128)*x^3+1024*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x^3
-256*(x^6+x^2)^(3/4)-128*RootOf(_Z^8-128)*x-1024*RootOf(-RootOf(_Z^8-128)^5
*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x)/(x^4*RootOf(_Z^8-128)^4-2*x^2*RootOf(_Z^
8-128)^4+RootOf(_Z^8-128)^4-16*x^4+16*x^2-16)/x)*RootOf(_Z^8-128)^5-ln((Roo
tOf(_Z^8-128)^9*x^5+4*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_
Z^2)*RootOf(_Z^8-128)^8*x^5-2*x^3*RootOf(_Z^8-128)^9-8*RootOf(-RootOf(_Z^8-
128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_Z^8-128)^8*x^3+RootOf(_Z^8-12
8)^9*x+4*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*RootOf(_
Z^8-128)^8*x+32*(x^6+x^2)^(1/2)*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-1
28)^2+64*_Z^2)*RootOf(_Z^8-128)^6*x+8*RootOf(_Z^8-128)^5*x^5+8*RootOf(_Z^8-
128)^6*(x^6+x^2)^(1/4)*x^2+64*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128
)^2+64*_Z^2)*RootOf(_Z^8-128)^4*x^3+8*RootOf(_Z^8-128)^5*x-128*RootOf(_Z^8-
128)*x^5-1024*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x^5
-1024*RootOf(_Z^8-128)*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*
_Z^2)*(x^6+x^2)^(1/4)*x^2+128*RootOf(_Z^8-128)*x^3+1024*RootOf(-RootOf(_Z^8
-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x^3-256*(x^6+x^2)^(3/4)-128*RootOf(_
Z^8-128)*x-1024*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)*x
)/(x^4*RootOf(_Z^8-128)^4-2*x^2*RootOf(_Z^8-128)^4+RootOf(_Z^8-128)^4-16*x^
4+16*x^2-16)/x)*RootOf(-RootOf(_Z^8-128)^5*_Z+RootOf(_Z^8-128)^2+64*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8-1}{\left(x^8+1\right)\left(x^6+x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/(x^6+x^2)^(1/4)/(x^8+1),x, algorithm="maxima")
[Out] integrate((x^8 - 1)/((x^8 + 1)*(x^6 + x^2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8-1}{\left(x^6+x^2\right)^{1/4}\,\left(x^8+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 - 1)/((x^2 + x^6)^(1/4)*(x^8 + 1)),x)
[Out] int((x^8 - 1)/((x^2 + x^6)^(1/4)*(x^8 + 1)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-1)/(x**6+x**2)**(1/4)/(x**8+1),x)
```

```
[Out] Timed out
```

3.1817 $\int \frac{x^5\left(8-7(1+k)x+6kx^2\right)}{((1-x)x(1-kx))^{2/3}\left(-b+b(1+k)x-bkx^2+x^8\right)}\,dx$

Optimal. Leaf size=162

$$-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{b}\,x^3\sqrt[3]{kx^3+(-k-1)x^2+x}+x^6\right)}{2\sqrt[3]{b}}+\frac{\log\left(x^3-\sqrt[3]{b}\,\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{\sqrt[3]{b}}$$

Rubi [F] time = 27.47, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^5\left(8-7(1+k)x+6kx^2\right)}{((1-x)x(1-kx))^{2/3}\left(-b+b(1+k)x-bkx^2+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^5*(8 - 7*(1 + k)*x + 6*k*x^2))/(((1 - x)*x*(1 - k*x))^(2/3)*(-b + b*
(1 + k)*x - b*k*x^2 + x^8)),x]
```

```
[Out] (21*(1 + k)*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst][Defer[Int][x
^18/((1 - x^3)^(2/3)*(1 - k*x^3)^(2/3)*(b - b*(1 + k)*x^3 + b*k*x^6 - x^24)
), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(2/3) + (24*(1 - x)^(2/3)*x^(2/3)
*(1 - k*x)^(2/3)*Defer[Subst][Defer[Int][x^15/((1 - x^3)^(2/3)*(1 - k*x^3)^
(2/3)*(-b + b*(1 + k)*x^3 - b*k*x^6 + x^24)), x], x, x^(1/3)])/((1 - x)*x*(
1 - k*x))^(2/3) + (18*k*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst][
Defer[Int][x^21/((1 - x^3)^(2/3)*(1 - k*x^3)^(2/3)*(-b + b*(1 + k)*x^3 - b*
k*x^6 + x^24)), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(2/3)
```

Rubi steps

$$\int \frac{x^5\left(8-7(1+k)x+6kx^2\right)}{((1-x)x(1-kx))^{2/3}\left(-b+b(1+k)x-bkx^2+x^8\right)}\,dx=\frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{x^{13/3}\left(8-7(1+k)x+6kx^2\right)}{(1-x)^{2/3}(1-kx)^{2/3}\left(-b+b(1+k)x-bkx\right.}}{((1-x)x(1-kx))^{2/3}}$$

$$=\frac{\left(3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\text{Subst}\left(\int \frac{x^{15}\left(8-7(1+\right.}{\left(1-x^3\right)^{2/3}\left(1-kx^3\right)^{2/3}\left(-b\right.}\right.}{((1-x)x(1-kx))^{2/3}}$$

$$=\frac{\left(3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\text{Subst}\left(\int\left(\frac{7(1+}{\left(1-x^3\right)^{2/3}\left(1-kx^3\right)^{2/3}(b\right.}\right.\right.}{}$$

$$=\frac{\left(24(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\text{Subst}\left(\int \frac{}{\left(1-x^3\right)^{2/3}\left(1-kx^3\right)^{2/3}(-\right.}\right.}{((1-x)x(1-kx))^{2/3}}$$

Mathematica [F] time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(8-7(1+k)x+6kx^2\right)}{((1-x)x(1-kx))^{2/3}\left(-b+b(1+k)x-bkx^2+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^5*(8 - 7*(1 + k)*x + 6*k*x^2))/(((1 - x)*x*(1 - k*x))^(2/3)*(-
b + b*(1 + k)*x - b*k*x^2 + x^8)),x]
```

```
[Out] Integrate[(x^5*(8 - 7*(1 + k)*x + 6*k*x^2))/(((1 - x)*x*(1 - k*x))^(2/3)*(-
b + b*(1 + k)*x - b*k*x^2 + x^8)), x]
```

IntegrateAlgebraic [A] time = 8.21, size = 162, normalized size = 1.00

$$-\frac{\log \left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{b}\,x^3\sqrt[3]{kx^3+(-k-1)x^2+x}+x^6\right)}{2\sqrt[3]{b}}+\frac{\log \left(x^3-\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x^3}{2\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+x^3}\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(8 - 7*(1 + k)*x + 6*k*x^2))/(((1 - x)*x*(1 - k*x))
^(2/3)*(-b + b*(1 + k)*x - b*k*x^2 + x^8)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*x^3)/(x^3 + 2*b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(
1/3))])/b^(1/3) + Log[x^3 - b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3)]/b^(1/
3) - Log[x^6 + b^(1/3)*x^3*(x + (-1 - k)*x^2 + k*x^3)^(1/3) + b^(2/3)*(x +
(-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(8-7*(1+k)*x+6*k*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-b+b*(1+k)*x-
b*k*x^2+x^8),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(6\,kx^2-7\,(k+1)x+8\right)x^5}{\left(x^8-bkx^2+b(k+1)x-b\right)\left((kx-1)(x-1)x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(8-7*(1+k)*x+6*k*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-b+b*(1+k)*x-
b*k*x^2+x^8),x, algorithm="giac")
```

```
[Out] integrate((6*k*x^2 - 7*(k + 1)*x + 8)*x^5/((x^8 - b*k*x^2 + b*(k + 1)*x - b
)*((k*x - 1)*(x - 1)*x)^(2/3)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(8-7\,(1+k)\,x+6k\,x^2\right)}{\left((1-x)\,x\,(-kx+1)\right)^{\frac{2}{3}}\left(-b+b\,(1+k)\,x-bk\,x^2+x^8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(8-7*(1+k)*x+6*k*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-b+b*(1+k)*x-b*k*x^
2+x^8),x)
```

```
[Out] int(x^5*(8-7*(1+k)*x+6*k*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-b+b*(1+k)*x-b*k*x^
2+x^8),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(6\,kx^2-7\,(k+1)x+8\right)x^5}{\left(x^8-bkx^2+b(k+1)x-b\right)\left((kx-1)(x-1)x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(8-7*(1+k)*x+6*k*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-b+b*(1+k)*x-
b*k*x^2+x^8),x, algorithm="maxima")
```

```
[Out] integrate((6*k*x^2 - 7*(k + 1)*x + 8)*x^5/((x^8 - b*k*x^2 + b*(k + 1)*x - b
)*((k*x - 1)*(x - 1)*x)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^5\,\left(6\,k\,x^2-7\,x\,\left(k+1\right)+8\right)}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{2/3}\,\left(-x^8+b\,k\,x^2-b\,\left(k+1\right)\,x+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^5*(6*k*x^2 - 7*x*(k + 1) + 8))/((x*(k*x - 1)*(x - 1))^(2/3)*(b - x^
8 - b*x*(k + 1) + b*k*x^2)),x)
```

```
[Out] -int((x^5*(6*k*x^2 - 7*x*(k + 1) + 8))/((x*(k*x - 1)*(x - 1))^(2/3)*(b - x^
8 - b*x*(k + 1) + b*k*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \left(6 k x^{2} - 7 k x - 7 x + 8\right)}{\left(x \left(x - 1\right) \left(k x - 1\right)\right)^{\frac{2}{3}} \left(- b k x^{2} + b k x + b x - b + x^{8}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(8-7*(1+k)*x+6*k*x**2)/((1-x)*x*(-k*x+1))**(2/3)/(-b+b*(1+k)
*x-b*k*x**2+x**8),x)
```

```
[Out] Integral(x**5*(6*k*x**2 - 7*k*x - 7*x + 8)/((x*(x - 1)*(k*x - 1))**(2/3)*(-
b*k*x**2 + b*k*x + b*x - b + x**8)), x)
```

3.1818 $\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^4} dx$

Optimal. Leaf size=162

$$\frac{\sqrt{\sqrt{ax^2+b^2}+b}\left(\sqrt{a}\left(2b\sqrt{ax^2+b^2}-10b^2\right)-3a^{3/2}x^2\right)}{24\sqrt{a}\,b^2x^3}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{\sqrt{2}\sqrt{b}}\right)}{4\sqrt{2}\,b^{5/2}}$$

Rubi [F] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^4} dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b + Sqrt[b^2 + a*x^2]]/x^4,x]
```

```
[Out] Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/x^4, x]
```

Rubi steps

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^4} dx = \int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^4} dx$$

Mathematica [C] time = 0.19, size = 81, normalized size = 0.50

$$\frac{\sqrt{\sqrt{ax^2+b^2}+b}\left(\left(\sqrt{ax^2+b^2}+b\right){}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b-\sqrt{b^2+ax^2}}{2b}\right)-6b\right)}{12bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/x^4,x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 + a*x^2]]*(-6*b + (b + Sqrt[b^2 + a*x^2])*Hypergeometric
2F1[-3/2, 1, -1/2, (b - Sqrt[b^2 + a*x^2])/(2*b)]))/(12*b*x^3)
```

IntegrateAlgebraic [A] time = 0.28, size = 130, normalized size = 0.80

$$\frac{\sqrt{\sqrt{ax^2+b^2}+b}\left(\sqrt{a}\left(2b\sqrt{ax^2+b^2}-10b^2\right)-3a^{3/2}x^2\right)}{24\sqrt{a}\,b^2x^3}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{8\sqrt{2}\,b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + Sqrt[b^2 + a*x^2]]/x^4,x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 + a*x^2]]*(-3*a^(3/2)*x^2 + Sqrt[a]*(-10*b^2 + 2*b*Sqrt[
b^2 + a*x^2])))/(24*Sqrt[a]*b^2*x^3) - (a^(3/2)*ArcTan[(Sqrt[a]*x)/(Sqrt[2]
*Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/(8*Sqrt[2]*b^(5/2))
```

fricas [A] time = 96.58, size = 280, normalized size = 1.73

$$\left[\frac{3\sqrt{\frac{1}{2}}\,ax^3\sqrt{-\frac{a}{b}}\log\left(-\frac{a^2x^3+4\,ab^2x-4\sqrt{ax^2+b^2}\,abx+4\left(2\sqrt{\frac{1}{2}}\sqrt{ax^2+b^2}\,b^2\sqrt{-\frac{a}{b}}-\sqrt{\frac{1}{2}}\left(abx^2+2\,b^3\right)\sqrt{-\frac{a}{b}}\right)\sqrt{b+\sqrt{ax^2+b^2}}}{x^3}\right)-2\left(3\,ax^2+10\,b^2-2\sqrt{ax^2+b^2}\,b\right)\sqrt{b+\sqrt{ax^2+b^2}}}{48\,b^2x^3}, \frac{3\sqrt{\frac{1}{2}}\,ax^3\sqrt{\frac{a}{b}}\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{b+\sqrt{ax^2+b^2}}\,b\sqrt{\frac{a}{b}}}{ax}\right)-\left(3\,ax^2+10\,b^2-2\sqrt{ax^2+b^2}\,b\right)\sqrt{b+\sqrt{ax^2+b^2}}}{24\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(1/2)*a*x^3*sqrt(-a/b)*log(-(a^2*x^3 + 4*a*b^2*x - 4*sqrt(a*x^
2 + b^2)*a*b*x + 4*(2*sqrt(1/2)*sqrt(a*x^2 + b^2)*b^2*sqrt(-a/b) - sqrt(1/2
)*(a*b*x^2 + 2*b^3)*sqrt(-a/b))*sqrt(b + sqrt(a*x^2 + b^2)))/x^3) - 2*(3*a*
x^2 + 10*b^2 - 2*sqrt(a*x^2 + b^2)*b)*sqrt(b + sqrt(a*x^2 + b^2)))/(b^2*x^3
), 1/24*(3*sqrt(1/2)*a*x^3*sqrt(a/b)*arctan(2*sqrt(1/2)*sqrt(b + sqrt(a*x^2
 + b^2))*b*sqrt(a/b)/(a*x)) - (3*a*x^2 + 10*b^2 - 2*sqrt(a*x^2 + b^2)*b)*sq
rt(b + sqrt(a*x^2 + b^2)))/(b^2*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{a x^2+b^2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/x^4, x)
```

maple [C] time = 0.04, size = 31, normalized size = 0.19

$$-\frac{\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\ \mathrm{hypergeom}\left(\left[-\frac{3}{2},-\frac{1}{4},\frac{1}{4}\right],\left[-\frac{1}{2},\frac{1}{2}\right],-\frac{x^2 a}{b^2}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+(a*x^2+b^2)^(1/2))^(1/2)/x^4,x)
```

```
[Out] -1/3*(b^2)^(1/4)*2^(1/2)/x^3*hypergeom([-3/2,-1/4,1/4],[-1/2,1/2],-x^2*a/b^
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{a x^2+b^2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b+\sqrt{b^2+a\,x^2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/x^4,x)
```

```
[Out] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/x^4, x)
```

sympy [C] time = 1.12, size = 51, normalized size = 0.31

$$\frac{\sqrt{b}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_{3}F_{2}\left(\begin{matrix}-\frac{3}{2},-\frac{1}{4},\frac{1}{4}\\-\frac{1}{2},\frac{1}{2}\end{matrix}\middle|\frac{a x^2 e^{i\pi}}{b^2}\right)}{12\pi x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x**2+b**2)**(1/2))**(1/2)/x**4,x)
```

```
[Out] sqrt(b)*gamma(-1/4)*gamma(1/4)*hyper((-3/2, -1/4, 1/4), (-1/2, 1/2), a*x**2
*exp_polar(I*pi)/b**2)/(12*pi*x**3)
```

3.1819 $\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^4\sqrt{b^2+ax^2}}\,dx$

Optimal. Leaf size=162

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{\sqrt{2}\sqrt{b}}\right)}{4\sqrt{2}\,b^{7/2}}+\frac{\sqrt{\sqrt{ax^2+b^2}+b}\,\left(15a^{3/2}x^2+\sqrt{a}\left(2b^2-10b\sqrt{ax^2+b^2}\right)\right)}{24\sqrt{a}\,b^3x^3}$$

Rubi [F] time = 0.64, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^4\sqrt{b^2+ax^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b + Sqrt[b^2 + a*x^2]]/(x^4*Sqrt[b^2 + a*x^2]),x]
[Out] Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(x^4*Sqrt[b^2 + a*x^2]), x]
```

Rubi steps

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^4\sqrt{b^2+ax^2}}\,dx=\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^4\sqrt{b^2+ax^2}}\,dx$$

Mathematica [C] time = 0.20, size = 61, normalized size = 0.38

$$-\frac{\left(\sqrt{ax^2+b^2}+b\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{b-\sqrt{b^2+ax^2}}{2b}\right)}{6b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(x^4*Sqrt[b^2 + a*x^2]),x]
[Out] -1/6*((b + Sqrt[b^2 + a*x^2])^(3/2)*Hypergeometric2F1[-3/2, 2, -1/2, (b - S
qrt[b^2 + a*x^2])/(2*b)])/(b^2*x^3)
```

IntegrateAlgebraic [A] time = 0.29, size = 130, normalized size = 0.80

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{8\sqrt{2}\,b^{7/2}}+\frac{\sqrt{\sqrt{ax^2+b^2}+b}\,\left(15a^{3/2}x^2+\sqrt{a}\left(2b^2-10b\sqrt{ax^2+b^2}\right)\right)}{24\sqrt{a}\,b^3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + Sqrt[b^2 + a*x^2]]/(x^4*Sqrt[b^2 + a*x^2]),x]
[Out] (Sqrt[b + Sqrt[b^2 + a*x^2]]*(15*a^(3/2)*x^2 + Sqrt[a]*(2*b^2 - 10*b*Sqrt[b
^2 + a*x^2])))/(24*Sqrt[a]*b^3*x^3) + (5*a^(3/2)*ArcTan[(Sqrt[a]*x)/(Sqrt[2
]*Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/(8*Sqrt[2]*b^(7/2))
```

fricas [A] time = 138.23, size = 280, normalized size = 1.73

$$\left[\frac{15\sqrt{\frac{1}{2}}\,ax^3\sqrt{-\frac{a}{b}}\,\log\left(-\frac{a^2x^3+4\,ab^2x-4\,\sqrt{ax^2+b^2}\,abx-4\left(2\sqrt{\frac{1}{2}}\,\sqrt{ax^2+b^2}\,b^2\sqrt{-\frac{a}{b}}-\sqrt{\frac{1}{2}}\,(abx^2+2\,b^3)\sqrt{-\frac{a}{b}}\right)\sqrt{b+\sqrt{ax^2+b^2}}}{x^3}\right)+2\left(15\,ax^2+2\,b^2-10\,\sqrt{ax^2+b^2}\,b\right)\sqrt{b+\sqrt{ax^2+b^2}}}{48\,b^3x^3},\ -\frac{15\sqrt{\frac{1}{2}}\,ax^3\sqrt{\frac{a}{b}}\,\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,\sqrt{b+\sqrt{ax^2+b^2}}\,b\sqrt{\frac{a}{b}}}{ax}\right)-\left(15\,ax^2+2\,b^2-10\,\sqrt{ax^2+b^2}\,b\right)\sqrt{b+\sqrt{ax^2+b^2}}}{24\,b^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^4/(a*x^2+b^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/48*(15*sqrt(1/2)*a*x^3*sqrt(-a/b)*log(-(a^2*x^3 + 4*a*b^2*x - 4*sqrt(a*x
^2 + b^2)*a*b*x - 4*(2*sqrt(1/2)*sqrt(a*x^2 + b^2)*b^2*sqrt(-a/b) - sqrt(1/
2)*(a*b*x^2 + 2*b^3)*sqrt(-a/b))*sqrt(b + sqrt(a*x^2 + b^2)))/x^3) + 2*(15*
a*x^2 + 2*b^2 - 10*sqrt(a*x^2 + b^2)*b)*sqrt(b + sqrt(a*x^2 + b^2)))/(b^3*x
^3), -1/24*(15*sqrt(1/2)*a*x^3*sqrt(a/b)*arctan(2*sqrt(1/2)*sqrt(b + sqrt(a
*x^2 + b^2))*b*sqrt(a/b)/(a*x)) - (15*a*x^2 + 2*b^2 - 10*sqrt(a*x^2 + b^2)*
b)*sqrt(b + sqrt(a*x^2 + b^2)))/(b^3*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\sqrt{ax^2+b^2}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^4/(a*x^2+b^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(sqrt(a*x^2 + b^2)*x^4), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{x^4\sqrt{a\,x^2+b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+(a*x^2+b^2)^(1/2))^(1/2)/x^4/(a*x^2+b^2)^(1/2),x)
```

```
[Out] int((b+(a*x^2+b^2)^(1/2))^(1/2)/x^4/(a*x^2+b^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\sqrt{ax^2+b^2}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^4/(a*x^2+b^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(sqrt(a*x^2 + b^2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b+\sqrt{b^2+a\,x^2}}}{x^4\,\sqrt{b^2+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(x^4*(a*x^2 + b^2)^(1/2)),x)
```

```
[Out] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(x^4*(a*x^2 + b^2)^(1/2)), x)
```

sympy [C] time = 1.44, size = 49, normalized size = 0.30

$$-\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_{3}F_{2}\left(\begin{matrix}-\frac{3}{2},\frac{1}{4},\frac{3}{4}\\-\frac{1}{2},\frac{1}{2}\end{matrix}\middle|\frac{ax^{2}e^{i\pi}}{b^{2}}\right)}{3\pi\sqrt{b}\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x**2+b**2)**(1/2))**(1/2)/x**4/(a*x**2+b**2)**(1/2),x)
```

```
[Out] -gamma(1/4)*gamma(3/4)*hyper((-3/2, 1/4, 3/4), (-1/2, 1/2), a*x**2*exp_pola
r(I*pi)/b**2)/(3*pi*sqrt(b)*x**3)
```

3.1820 $\int \frac{(1+x^3)\sqrt{-2-x^3+x^6}}{x^4(-1-2x^3+x^6)}\,dx$

Optimal. Leaf size=163

$$\frac{\sqrt{x^6-x^3-2}}{3x^3}-\frac{\tan^{-1}\left(\frac{\sqrt{x^6-x^3-2}}{\sqrt{2}(x^3+1)}\right)}{\sqrt{2}}+\frac{1}{3}\sqrt{4+3\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{3\sqrt{2}-4}\sqrt{x^6-x^3-2}}{x^3+1}\right)-\frac{1}{3}\sqrt{3\sqrt{2}-4}\,\tanh^{-1}\left(\sqrt{4-\right.$$

Rubi [A] time = 0.96, antiderivative size = 224, normalized size of antiderivative = 1.37, number of steps used = 26, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {6728, 1357, 732, 843, 621, 206, 724, 204, 734, 6715, 1019, 1076, 1032}

$$\frac{\sqrt{x^6-x^3-2}}{3x^3}+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{x^3+4}{2\sqrt{2}\sqrt{x^6-x^3-2}}\right)-\frac{\tan^{-1}\left(\frac{x^3+4}{2\sqrt{2}\sqrt{x^6-x^3-2}}\right)}{6\sqrt{2}}-\frac{(1+\sqrt{2})\tan^{-1}\left(\frac{-((1-2\sqrt{2})x^3)-\sqrt{2}+5}{2\sqrt[4]{2}\sqrt{x^6-x^3-2}}\right)}{3\,2^{3/4}}-\frac{(1-\sqrt{2})\tanh^{-1}\left(\frac{-((1+2\sqrt{2})x^3)+\sqrt{2}+5}{2\sqrt[4]{2}\sqrt{x^6-x^3-2}}\right)}{3\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^3)*Sqrt[-2 - x^3 + x^6])/(x^4*(-1 - 2*x^3 + x^6)),x]
```

```
[Out] Sqrt[-2 - x^3 + x^6]/(3*x^3) - ArcTan[(4 + x^3)/(2*Sqrt[2]*Sqrt[-2 - x^3 +
x^6])]/(6*Sqrt[2]) + (Sqrt[2]*ArcTan[(4 + x^3)/(2*Sqrt[2]*Sqrt[-2 - x^3 + x
^6])])/3 - ((1 + Sqrt[2])*ArcTan[(5 - Sqrt[2] - (1 - 2*Sqrt[2])*x^3)/(2*2^(
1/4)*Sqrt[-2 - x^3 + x^6])])/(3*2^(3/4)) - ((1 - Sqrt[2])*ArcTanh[(5 + Sqrt
[2] - (1 + 2*Sqrt[2])*x^3)/(2*2^(1/4)*Sqrt[-2 - x^3 + x^6])])/(3*2^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
```

```
|| LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
```

```
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(1+x^3\right)\sqrt{-2-x^3+x^6}}{x^4\left(-1-2x^3+x^6\right)}\,dx &= \int \left(-\frac{\sqrt{-2-x^3+x^6}}{x^4}+\frac{\sqrt{-2-x^3+x^6}}{x}-\frac{x^2\left(-3+x^3\right)\sqrt{-2-x^3+x^6}}{-1-2x^3+x^6}\right)dx \\
&= -\int \frac{\sqrt{-2-x^3+x^6}}{x^4}\,dx+\int \frac{\sqrt{-2-x^3+x^6}}{x}\,dx-\int \frac{x^2\left(-3+x^3\right)\sqrt{-2-x^3+\cdots}}{-1-2x^3+x^6} \\
&= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{-2-x+x^2}}{x^2}\,dx,x,x^3\right)\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{-2-x+x^2}}{x}\,dx,x,\cdots\right. \\
&= \frac{\sqrt{-2-x^3+x^6}}{3x^3}-\frac{1}{6}\operatorname{Subst}\left(\int \frac{4+x}{x\sqrt{-2-x+x^2}}\,dx,x,x^3\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{-\cdots}{x\sqrt{-2\cdots}}\right. \\
&= \frac{\sqrt{-2-x^3+x^6}}{3x^3}-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-2-x+x^2}}\,dx,x,x^3\right)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{\cdots}{x\sqrt{-2\cdots}}\right. \\
&= \frac{\sqrt{-2-x^3+x^6}}{3x^3}-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,\frac{-4-x^3}{\sqrt{-2-x^3+x^6}}\right)-\frac{1}{3}\operatorname{Subst}\left(\int\cdots\right. \\
&= \frac{\sqrt{-2-x^3+x^6}}{3x^3}-\frac{\tan^{-1}\left(\frac{4+x^3}{2\sqrt{2}\sqrt{-2-x^3+x^6}}\right)}{6\sqrt{2}}+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{4+x^3}{2\sqrt{2}\sqrt{-2-x^3+x^6}}\right)\cdots \\
&= \frac{\sqrt{-2-x^3+x^6}}{3x^3}-\frac{\tan^{-1}\left(\frac{4+x^3}{2\sqrt{2}\sqrt{-2-x^3+x^6}}\right)}{6\sqrt{2}}+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{4+x^3}{2\sqrt{2}\sqrt{-2-x^3+x^6}}\right)\cdots
\end{aligned}
$$

Mathematica [A] time = 0.34, size = 326, normalized size = 2.00

$$\frac{1}{12}\left(\frac{4\sqrt{x^6-x^3-2}}{x^3}+3\sqrt{2}\tan^{-1}\left(\frac{x^3+4}{2\sqrt{2}\sqrt{x^6-x^3-2}}\right)-2\,2^{3/4}\tan^{-1}\left(\frac{\left(2\sqrt{2}-1\right)x^3-\sqrt{2}+5}{2\sqrt[4]{2}\sqrt{x^6-x^3-2}}\right)-2\sqrt[4]{2}\tan^{-1}\left(\frac{\left(2\sqrt{2}-1\right)x^3-\sqrt{2}+5}{2\sqrt[4]{2}\sqrt{x^6-x^3-2}}\right)-6\tanh^{-1}\left(\frac{1-2x^3}{2\sqrt{x^6-x^3-2}}\right)-6\tanh^{-1}\left(\frac{2x^3-1}{2\sqrt{x^6-x^3-2}}\right)+2\,2^{3/4}\tanh^{-1}\left(\frac{-\left(\left(1+2\sqrt{2}\right)x^3\right)+\sqrt{2}+5}{2\sqrt[4]{2}\sqrt{x^6-x^3-2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{-\left(\left(1+2\sqrt{2}\right)x^3\right)+\sqrt{2}+5}{2\sqrt[4]{2}\sqrt{x^6-x^3-2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)*Sqrt[-2 - x^3 + x^6])/(x^4*(-1 - 2*x^3 + x^6)),x]
```

```
[Out] ((4*Sqrt[-2 - x^3 + x^6])/x^3 + 3*Sqrt[2]*ArcTan[(4 + x^3)/(2*Sqrt[2]*Sqrt[
-2 - x^3 + x^6])] - 2*2^(1/4)*ArcTan[(5 - Sqrt[2] + (-1 + 2*Sqrt[2])*x^3)/(
2*2^(1/4)*Sqrt[-2 - x^3 + x^6])] - 2*2^(3/4)*ArcTan[(5 - Sqrt[2] + (-1 + 2*
Sqrt[2])*x^3)/(2*2^(1/4)*Sqrt[-2 - x^3 + x^6])] - 6*ArcTanh[(1 - 2*x^3)/(2*
Sqrt[-2 - x^3 + x^6])] - 6*ArcTanh[(-1 + 2*x^3)/(2*Sqrt[-2 - x^3 + x^6])] -
 2*2^(1/4)*ArcTanh[(5 + Sqrt[2] - (1 + 2*Sqrt[2])*x^3)/(2*2^(1/4)*Sqrt[-2 -
 x^3 + x^6])] + 2*2^(3/4)*ArcTanh[(5 + Sqrt[2] - (1 + 2*Sqrt[2])*x^3)/(2*2^
(1/4)*Sqrt[-2 - x^3 + x^6])])/12
```

IntegrateAlgebraic [A] time = 0.51, size = 162, normalized size = 0.99

$$\frac{\sqrt{x^6-x^3-2}}{3x^3}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x^6-x^3-2}}{x^3-2}\right)}{\sqrt{2}}-\frac{1}{3}\sqrt{4+3\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\frac{3}{\sqrt{2}}}\sqrt{x^6-x^3-2}}{x^3-2}\right)-\frac{1}{3}\sqrt{3\sqrt{2}-4}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{\sqrt{2}}-2}\sqrt{x^6-x^3-2}}{x^3-2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)*Sqrt[-2 - x^3 + x^6])/(x^4*(-1 - 2*x^3 + x^6)
),x]
```

```
[Out] Sqrt[-2 - x^3 + x^6]/(3*x^3) + ArcTan[(Sqrt[2]*Sqrt[-2 - x^3 + x^6])/(-2 +
x^3)]/Sqrt[2] - (Sqrt[4 + 3*Sqrt[2]]*ArcTan[(Sqrt[2 + 3/Sqrt[2]]*Sqrt[-2 -
x^3 + x^6])/(-2 + x^3)])/3 - (Sqrt[-4 + 3*Sqrt[2]]*ArcTanh[(Sqrt[-2 + 3/Sqr
t[2]]*Sqrt[-2 - x^3 + x^6])/(-2 + x^3)])/3
```

fricas [B] time = 0.42, size = 289, normalized size = 1.77

$$-\frac{3\sqrt{2}x^3\arctan\left(-\frac{1}{2}\sqrt{2}x^3+\frac{1}{2}\sqrt{2}\sqrt{x^6-x^3-2}\right)-4x^3\sqrt{3\sqrt{2}+4}\arctan\left(\frac{1}{2}\sqrt{2x^6-3x^3+\sqrt{2}\left(2x^3-1\right)-2\sqrt{x^6-x^3-2}\left(x^3+\sqrt{2}-1\right)+1}\sqrt{3\sqrt{2}+4}\left(\sqrt{2}-2\right)-\frac{1}{2}\left(2x^3-\sqrt{2}\left(x^3-3\right)+\sqrt{x^6-x^3-2}\left(\sqrt{2}-2\right)-4\right)\sqrt{3\sqrt{2}+4}\right)+x^3\sqrt{3\sqrt{2}-4}\log\left(-x^3+\sqrt{3\sqrt{2}-4}\left(\sqrt{2}+1\right)+\sqrt{2}+\sqrt{x^6-x^3-2}+1\right)-x^3\sqrt{3\sqrt{2}-4}\log\left(-x^3-\sqrt{3\sqrt{2}-4}\left(\sqrt{2}+1\right)+\sqrt{2}+\sqrt{x^6-x^3-2}+1\right)-2x^3-2\sqrt{x^6-x^3-2}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(x^6-x^3-2)^(1/2)/x^4/(x^6-2*x^3-1),x, algorithm="fricas"
)
```

```
[Out] -1/6*(3*sqrt(2)*x^3*arctan(-1/2*sqrt(2)*x^3 + 1/2*sqrt(2)*sqrt(x^6 - x^3 -
2)) - 4*x^3*sqrt(3*sqrt(2) + 4)*arctan(1/2*sqrt(2*x^6 - 3*x^3 + sqrt(2)*(2*
x^3 - 1) - 2*sqrt(x^6 - x^3 - 2)*(x^3 + sqrt(2) - 1) + 1)*sqrt(3*sqrt(2) +
4)*(sqrt(2) - 2) - 1/2*(2*x^3 - sqrt(2)*(x^3 - 3) + sqrt(x^6 - x^3 - 2)*(sq
rt(2) - 2) - 4)*sqrt(3*sqrt(2) + 4)) + x^3*sqrt(3*sqrt(2) - 4)*log(-x^3 + s
qrt(3*sqrt(2) - 4)*(sqrt(2) + 1) + sqrt(2) + sqrt(x^6 - x^3 - 2) + 1) - x^3
*sqrt(3*sqrt(2) - 4)*log(-x^3 - sqrt(3*sqrt(2) - 4)*(sqrt(2) + 1) + sqrt(2)
 + sqrt(x^6 - x^3 - 2) + 1) - 2*x^3 - 2*sqrt(x^6 - x^3 - 2))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(x^6-x^3-2)^(1/2)/x^4/(x^6-2*x^3-1),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError >> type
```

maple [F] time = 1.78, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+1\right)\sqrt{x^6-x^3-2}}{x^4\left(x^6-2x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)*(x^6-x^3-2)^(1/2)/x^4/(x^6-2*x^3-1),x)
```

```
[Out] int((x^3+1)*(x^6-x^3-2)^(1/2)/x^4/(x^6-2*x^3-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^6-x^3-2}\left(x^3+1\right)}{\left(x^6-2\,x^3-1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(x^6-x^3-2)^(1/2)/x^4/(x^6-2*x^3-1),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(x^6 - x^3 - 2)*(x^3 + 1)/((x^6 - 2*x^3 - 1)*x^4), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^3+1\right)\sqrt{x^6-x^3-2}}{x^4\left(-x^6+2x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3 + 1)*(x^6 - x^3 - 2)^(1/2))/(x^4*(2*x^3 - x^6 + 1)),x)
[Out] int(-((x^3 + 1)*(x^6 - x^3 - 2)^(1/2))/(x^4*(2*x^3 - x^6 + 1)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)*(x**6-x**3-2)**(1/2)/x**4/(x**6-2*x**3-1),x)
[Out] Timed out
```

3.1821 $\int \frac{x^2(-4+7x^3)}{\sqrt[3]{-x+x^4}\,(-1-x^4+x^7)}\,dx$

Optimal. Leaf size=163

$$2\tanh^{-1}\left(1-2x\sqrt[3]{x^4-x}\right)-\sqrt{3}\tan^{-1}\left(\frac{3\sqrt{3}\,x\sqrt[3]{x^4-x}-3x^2\sqrt[3]{x^4-x}}{-3\sqrt[3]{x^4-x}\,x+\sqrt{3}\sqrt[3]{x^4-x}\,x^2+2\sqrt{3}\,x-6}\right)-\tanh^{-1}\left(\frac{\sqrt[3]{x^4-x}\,x+}{\sqrt[3]{x^4-x}\,x+2\left(x^4-\right.}\right.$$

Rubi [F] time = 1.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(-4+7x^3\right)}{\sqrt[3]{-x+x^4}\left(-1-x^4+x^7\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(-4 + 7*x^3))/((-x + x^4)^(1/3)*(-1 - x^4 + x^7)),x]
```

```
[Out] (-12*x^(1/3)*(-1 + x^3)^(1/3)*Defer[Subst][Defer[Int][x^7/((-1 + x^9)^(1/3)
*(-1 - x^12 + x^21)), x], x, x^(1/3)])/(-x + x^4)^(1/3) + (21*x^(1/3)*(-1 +
 x^3)^(1/3)*Defer[Subst][Defer[Int][x^16/((-1 + x^9)^(1/3)*(-1 - x^12 + x^2
1)), x], x, x^(1/3)])/(-x + x^4)^(1/3)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(-4+7x^3\right)}{\sqrt[3]{-x+x^4}\left(-1-x^4+x^7\right)}\,dx &= \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^3}\right)\int \frac{x^{5/3}\left(-4+7x^3\right)}{\sqrt[3]{-1+x^3}\left(-1-x^4+x^7\right)}\,dx}{\sqrt[3]{-x+x^4}}\\
&= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^3}\right)\text{Subst}\left(\int \frac{x^7\left(-4+7x^9\right)}{\sqrt[3]{-1+x^9}\left(-1-x^{12}+x^{21}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x+x^4}}\\
&= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^3}\right)\text{Subst}\left(\int \left(-\frac{4x^7}{\sqrt[3]{-1+x^9}\left(-1-x^{12}+x^{21}\right)}+\frac{7x^{16}}{\sqrt[3]{-1+x^9}\left(-1-x^{12}+x^{21}\right)}\right)dx,x,\sqrt[3]{\right.}{\sqrt[3]{-x+x^4}}\\
&= -\frac{\left(12\sqrt[3]{x}\sqrt[3]{-1+x^3}\right)\text{Subst}\left(\int \frac{x^7}{\sqrt[3]{-1+x^9}\left(-1-x^{12}+x^{21}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-x+x^4}}+\frac{\left(21\sqrt[3]{x}\sqrt[3]{-1+x^{\right.}}{}
\end{aligned}$$

Mathematica [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-4+7x^3\right)}{\sqrt[3]{-x+x^4}\left(-1-x^4+x^7\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(-4 + 7*x^3))/((-x + x^4)^(1/3)*(-1 - x^4 + x^7)),x]
```

```
[Out] Integrate[(x^2*(-4 + 7*x^3))/((-x + x^4)^(1/3)*(-1 - x^4 + x^7)), x]
```

IntegrateAlgebraic [A] time = 9.85, size = 163, normalized size = 1.00

$$2\tanh^{-1}\left(1-2x\sqrt[3]{x^4-x}\right)-\sqrt{3}\tan^{-1}\left(\frac{3\sqrt{3}\,x\sqrt[3]{x^4-x}-3x^2\sqrt[3]{x^4-x}}{-3\sqrt[3]{x^4-x}\,x+\sqrt{3}\sqrt[3]{x^4-x}\,x^2+2\sqrt{3}\,x-6}\right)-\tanh^{-1}\left(\frac{\sqrt[3]{x^4-x}\,x+1}{\sqrt[3]{x^4-x}\,x+2\left(x^4-x\right)^{2/3}x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(-4 + 7*x^3))/((-x + x^4)^(1/3)*(-1 - x^4 + x^7)),x
]
```

```
[Out] -(Sqrt[3]*ArcTan[(3*Sqrt[3]*x*(-x + x^4)^(1/3) - 3*x^2*(-x + x^4)^(1/3))/(-
6 + 2*Sqrt[3]*x - 3*x*(-x + x^4)^(1/3) + Sqrt[3]*x^2*(-x + x^4)^(1/3))]) +
2*ArcTanh[1 - 2*x*(-x + x^4)^(1/3)] - ArcTanh[(1 + x*(-x + x^4)^(1/3))/(1 +
 x*(-x + x^4)^(1/3) + 2*x^2*(-x + x^4)^(2/3))]
```

fricas [A] time = 3.18, size = 119, normalized size = 0.73

$$-\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(x^4-x\right)^{\frac{2}{3}}x^2-4\sqrt{3}\left(x^4-x\right)^{\frac{1}{3}}x-\sqrt{3}\left(x^7-x^4\right)}{x^7-x^4+8}\right)+\frac{1}{2}\log\left(\frac{x^7-x^4-3\left(x^4-x\right)^{\frac{2}{3}}x^2+3\left(x^4-x\right)^{\frac{1}{3}}x-1}{x^7-x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(7*x^3-4)/(x^4-x)^(1/3)/(x^7-x^4-1),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan((2*sqrt(3)*(x^4 - x)^(2/3)*x^2 - 4*sqrt(3)*(x^4 - x)^(1/3)*
x - sqrt(3)*(x^7 - x^4))/(x^7 - x^4 + 8)) + 1/2*log((x^7 - x^4 - 3*(x^4 - x
)^(2/3)*x^2 + 3*(x^4 - x)^(1/3)*x - 1)/(x^7 - x^4 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(7\,x^3-4\right)x^2}{\left(x^7-x^4-1\right)\left(x^4-x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(7*x^3-4)/(x^4-x)^(1/3)/(x^7-x^4-1),x, algorithm="giac")
```

```
[Out] integrate((7*x^3 - 4)*x^2/((x^7 - x^4 - 1)*(x^4 - x)^(1/3)), x)
```

maple [C] time = 17.46, size = 516, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(7*x^3-4)/(x^4-x)^(1/3)/(x^7-x^4-1),x)
```

```
[Out] RootOf(_Z^2+_Z+1)*ln(-(2750978024320396805141*RootOf(_Z^2+_Z+1)^2*x^7-39764
0531996478951563644*x^7*RootOf(_Z^2+_Z+1)+10027471246530585051439424*x^7-27
50978024320396805141*RootOf(_Z^2+_Z+1)^2*x^4+9624328758485465306265498*Root
Of(_Z^2+_Z+1)*(x^4-x)^(2/3)*x^2+397640531996478951563644*RootOf(_Z^2+_Z+1)*
x^4+20449832047033328657637351*x^2*(x^4-x)^(2/3)-10027471246530585051439424
*x^4+9624328758485465306265498*RootOf(_Z^2+_Z+1)*(x^4-x)^(1/3)*x-3081095387
23884442175792*RootOf(_Z^2+_Z+1)^2+20449832047033328657637351*x*(x^4-x)^(1/
3)+9716610729782380212458491*RootOf(_Z^2+_Z+1)+10117002239803179560827276)/
(x^7-x^4-1))-ln(-(2750978024320396805141*RootOf(_Z^2+_Z+1)^2*x^7+4031424880
45119745173926*x^7*RootOf(_Z^2+_Z+1)+10427862756551384399808209*x^7-2750978
024320396805141*RootOf(_Z^2+_Z+1)^2*x^4-9624328758485465306265498*RootOf(_Z
^2+_Z+1)*(x^4-x)^(2/3)*x^2-403142488045119745173926*RootOf(_Z^2+_Z+1)*x^4+1
0825503288547863351371853*x^2*(x^4-x)^(2/3)-10427862756551384399808209*x^4-
9624328758485465306265498*RootOf(_Z^2+_Z+1)*(x^4-x)^(1/3)*x-308109538723884
442175792*RootOf(_Z^2+_Z+1)^2+10825503288547863351371853*x*(x^4-x)^(1/3)-10
332829807230149096810075*RootOf(_Z^2+_Z+1)+9228197129691490619293)/(x^7-x^
4-1))*RootOf(_Z^2+_Z+1)-ln(-(2750978024320396805141*RootOf(_Z^2+_Z+1)^2*x^7
+403142488045119745173926*x^7*RootOf(_Z^2+_Z+1)+10427862756551384399808209*
x^7-2750978024320396805141*RootOf(_Z^2+_Z+1)^2*x^4-962432875848546530626549
8*RootOf(_Z^2+_Z+1)*(x^4-x)^(2/3)*x^2-403142488045119745173926*RootOf(_Z^2+
```

```
_Z+1)*x^4+10825503288547863351371853*x^2*(x^4-x)^(2/3)-10427862756551384399
808209*x^4-9624328758485465306265498*RootOf(_Z^2+_Z+1)*(x^4-x)^(1/3)*x-3081
09538723884442175792*RootOf(_Z^2+_Z+1)^2+10825503288547863351371853*x*(x^4-
x)^(1/3)-10332829807230149096810075*RootOf(_Z^2+_Z+1)+922819712969149061929
93)/(x^7-x^4-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(7x^3-4)x^2}{(x^7-x^4-1)(x^4-x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(7*x^3-4)/(x^4-x)^(1/3)/(x^7-x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((7*x^3 - 4)*x^2/((x^7 - x^4 - 1)*(x^4 - x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2\,(7x^3-4)}{(x^4-x)^{1/3}\,(-x^7+x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(7*x^3 - 4))/((x^4 - x)^(1/3)*(x^4 - x^7 + 1)),x)
```

```
[Out] -int((x^2*(7*x^3 - 4))/((x^4 - x)^(1/3)*(x^4 - x^7 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\,(7x^3-4)}{\sqrt[3]{x\,(x-1)\,(x^2+x+1)}\,(x^7-x^4-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(7*x**3-4)/(x**4-x)**(1/3)/(x**7-x**4-1),x)
```

```
[Out] Integral(x**2*(7*x**3 - 4)/((x*(x - 1)*(x**2 + x + 1))**(1/3)*(x**7 - x**4
- 1)), x)
```

3.1822 $\int \frac{\sqrt{1+2x^2-x^4}\left(-1+x^4\right)\left(1+x^4\right)}{\left(-1-x^2+x^4\right)\left(1+3x^2-x^4-3x^6+x^8\right)}\,dx$

Optimal. Leaf size=163

$$-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x\sqrt{-x^4+2x^2+1}}{x^4-2x^2-1}\right)-\tanh^{-1}\left(\frac{x\sqrt{-x^4+2x^2+1}}{x^4-2x^2-1}\right)+\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\right.$$

Rubi [F] time = 3.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+2x^2-x^4}\left(-1+x^4\right)\left(1+x^4\right)}{\left(-1-x^2+x^4\right)\left(1+3x^2-x^4-3x^6+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[1 + 2*x^2 - x^4]*(-1 + x^4)*(1 + x^4))/((-1 - x^2 + x^4)*(1 + 3*x
^2 - x^4 - 3*x^6 + x^8)),x]
```

```
[Out] (2*EllipticE[ArcSin[Sqrt[-1 + Sqrt[2]]*x], -3 - 2*Sqrt[2]])/Sqrt[1 + Sqrt[2
]] - ((1 + 2*Sqrt[2] - Sqrt[5])*EllipticF[ArcSin[Sqrt[-1 + Sqrt[2]]*x], -3
- 2*Sqrt[2]])/(2*Sqrt[-1 + Sqrt[2]]) - ((1 + 2*Sqrt[2] + Sqrt[5])*EllipticF
[ArcSin[Sqrt[-1 + Sqrt[2]]*x], -3 - 2*Sqrt[2]])/(2*Sqrt[-1 + Sqrt[2]]) + El
lipticPi[(2*(1 + Sqrt[2]))/(1 - Sqrt[5]), ArcSin[Sqrt[-1 + Sqrt[2]]*x], -3
- 2*Sqrt[2]]/Sqrt[-1 + Sqrt[2]] + EllipticPi[(2*(1 + Sqrt[2]))/(1 + Sqrt[5]
), ArcSin[Sqrt[-1 + Sqrt[2]]*x], -3 - 2*Sqrt[2]]/Sqrt[-1 + Sqrt[2]] + 2*Def
er[Int][Sqrt[1 + 2*x^2 - x^4]/(1 + 3*x^2 - x^4 - 3*x^6 + x^8), x] - Defer[I
nt][(x^2*Sqrt[1 + 2*x^2 - x^4])/(1 + 3*x^2 - x^4 - 3*x^6 + x^8), x] - 4*Def
er[Int][(x^4*Sqrt[1 + 2*x^2 - x^4])/(1 + 3*x^2 - x^4 - 3*x^6 + x^8), x] + 2
*Defer[Int][(x^6*Sqrt[1 + 2*x^2 - x^4])/(1 + 3*x^2 - x^4 - 3*x^6 + x^8), x]
```

Rubi steps

$$\int \frac{\sqrt{1+2x^2-x^4}\left(-1+x^4\right)\left(1+x^4\right)}{\left(-1-x^2+x^4\right)\left(1+3x^2-x^4-3x^6+x^8\right)}\,dx = \int\left(\frac{\left(1-2x^2\right)\sqrt{1+2x^2-x^4}}{-1-x^2+x^4} + \frac{\sqrt{1+2x^2-x^4}\left(2-x^2-4x^4+\right.}{1+3x^2-x^4-3x^6+x^8}\right.$$

$$= \int \frac{\left(1-2x^2\right)\sqrt{1+2x^2-x^4}}{-1-x^2+x^4}\,dx + \int \frac{\sqrt{1+2x^2-x^4}\left(2-x^2-4x\right.}{1+3x^2-x^4-3x^6+}$$

$$= \int\left(-\frac{2\sqrt{1+2x^2-x^4}}{-1-\sqrt{5}+2x^2} - \frac{2\sqrt{1+2x^2-x^4}}{-1+\sqrt{5}+2x^2}\right)dx + \int\left(\frac{2\sqrt{1+}}{1+3x^2-x}\right.$$

$$= -\left(2\int \frac{\sqrt{1+2x^2-x^4}}{-1-\sqrt{5}+2x^2}\,dx\right) - 2\int \frac{\sqrt{1+2x^2-x^4}}{-1+\sqrt{5}+2x^2}\,dx + 2\int \frac{}{1+}$$

$$= \frac{1}{2}\int \frac{-3-\sqrt{5}+2x^2}{\sqrt{1+2x^2-x^4}}\,dx + \frac{1}{2}\int \frac{-3+\sqrt{5}+2x^2}{\sqrt{1+2x^2-x^4}}\,dx + 2\int \frac{}{1+3}$$

$$= 2\int \frac{\sqrt{1+2x^2-x^4}}{1+3x^2-x^4-3x^6+x^8}\,dx + 2\int \frac{x^6\sqrt{1+2x^2-x^4}}{1+3x^2-x^4-3x^6+x^8}$$

$$= \frac{\Pi\left(\frac{2(1+\sqrt{2})}{1-\sqrt{5}};\sin^{-1}\left(\sqrt{-1+\sqrt{2}}\,x\right)\middle|-3-2\sqrt{2}\right)}{\sqrt{-1+\sqrt{2}}} + \frac{\Pi\left(\frac{2(1+\sqrt{2})}{1+\sqrt{5}};\mathrm{s}\right.}{}$$

$$= \frac{2E\left(\sin^{-1}\left(\sqrt{-1+\sqrt{2}}\,x\right)\middle|-3-2\sqrt{2}\right)}{\sqrt{1+\sqrt{2}}} - \frac{\left(1+2\sqrt{2}-\sqrt{5}\right)F\left(\mathrm{s}\right.}{2}$$

Mathematica [C] time = 7.79, size = 11852, normalized size = 72.71

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 + 2*x^2 - x^4]*(-1 + x^4)*(1 + x^4))/((-1 - x^2 + x^4)*(1
 + 3*x^2 - x^4 - 3*x^6 + x^8)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 1.09, size = 163, normalized size = 1.00

$$-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x\sqrt{-x^4+2x^2+1}}{x^4-2x^2-1}\right) - \tanh^{-1}\left(\frac{x\sqrt{-x^4+2x^2+1}}{x^4-2x^2-1}\right) + \sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x\sqrt{-x^4+2x^2+1}}{x^4-2x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + 2*x^2 - x^4]*(-1 + x^4)*(1 + x^4))/((-1 - x^2
+ x^4)*(1 + 3*x^2 - x^4 - 3*x^6 + x^8)),x]
```

```
[Out] -(Sqrt[(1 + Sqrt[5])/10]*ArcTan[(Sqrt[-1/2 + Sqrt[5]/2]*x*Sqrt[1 + 2*x^2 -
x^4])/(-1 - 2*x^2 + x^4)]) - ArcTanh[(x*Sqrt[1 + 2*x^2 - x^4])/(-1 - 2*x^2
+ x^4)] + Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[(Sqrt[1/2 + Sqrt[5]/2]*x*Sqrt[1 +
 2*x^2 - x^4])/(-1 - 2*x^2 + x^4)]
```

fricas [B] time = 1.05, size = 485, normalized size = 2.98

$$\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+1}\arctan\left(-\frac{2\sqrt{10}\left(5x^5-10x^3-\sqrt{5}\left(x^5-x\right)-5x\right)\sqrt{-x^4+2x^2+1}\sqrt{\sqrt{5}+1}-\sqrt{10}\left(5x^8-25x^6+25x^4+25x^2+\sqrt{5}\left(x^8-9x^6+13x^4+9x^2+1\right)+5\right)\sqrt{\sqrt{5}+1}\sqrt{\sqrt{5}-2}}{20\left(x^8-5x^6+3x^4+5x^2+1\right)}\right) + \frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(\frac{\sqrt{10}\left(5x^8-35x^6+45x^4+35x^2+\sqrt{5}\left(3x^8-17x^6+19x^4+17x^2+3\right)+5\right)\sqrt{\sqrt{5}-1}+20\left(3x^5-7x^3+\sqrt{5}\left(x^5-3x^3-x\right)-3x\right)\sqrt{-x^4+2x^2+1}}{x^8-3x^6-x^4+3x^2+1}\right) - \frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(-\frac{\sqrt{10}\left(5x^8-35x^6+45x^4+35x^2+\sqrt{5}\left(3x^8-17x^6+19x^4+17x^2+3\right)+5\right)\sqrt{\sqrt{5}-1}-20\left(3x^5-7x^3+\sqrt{5}\left(x^5-3x^3-x\right)-3x\right)\sqrt{-x^4+2x^2+1}}{x^8-3x^6-x^4+3x^2+1}\right) + \frac{1}{2}\log\left(-\frac{x^4-3x^2-2\sqrt{-x^4+2x^2+1}\,x-1}{x^4-x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+2*x^2+1)^(1/2)*(x^4-1)*(x^4+1)/(x^4-x^2-1)/(x^8-3*x^6-x^4+3
*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(10)*sqrt(sqrt(5) + 1)*arctan(-1/20*(2*sqrt(10)*(5*x^5 - 10*x^3 -
sqrt(5)*(x^5 - x) - 5*x)*sqrt(-x^4 + 2*x^2 + 1)*sqrt(sqrt(5) + 1) - sqrt(10
)*(5*x^8 - 25*x^6 + 25*x^4 + 25*x^2 + sqrt(5)*(x^8 - 9*x^6 + 13*x^4 + 9*x^2
 + 1) + 5)*sqrt(sqrt(5) + 1)*sqrt(sqrt(5) - 2))/(x^8 - 5*x^6 + 3*x^4 + 5*x^
2 + 1)) + 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log((sqrt(10)*(5*x^8 - 35*x^6 + 4
5*x^4 + 35*x^2 + sqrt(5)*(3*x^8 - 17*x^6 + 19*x^4 + 17*x^2 + 3) + 5)*sqrt(s
qrt(5) - 1) + 20*(3*x^5 - 7*x^3 + sqrt(5)*(x^5 - 3*x^3 - x) - 3*x)*sqrt(-x^
4 + 2*x^2 + 1))/(x^8 - 3*x^6 - x^4 + 3*x^2 + 1)) - 1/40*sqrt(10)*sqrt(sqrt(
5) - 1)*log(-(sqrt(10)*(5*x^8 - 35*x^6 + 45*x^4 + 35*x^2 + sqrt(5)*(3*x^8 -
 17*x^6 + 19*x^4 + 17*x^2 + 3) + 5)*sqrt(sqrt(5) - 1) - 20*(3*x^5 - 7*x^3 +
 sqrt(5)*(x^5 - 3*x^3 - x) - 3*x)*sqrt(-x^4 + 2*x^2 + 1))/(x^8 - 3*x^6 - x^
4 + 3*x^2 + 1)) + 1/2*log(-(x^4 - 3*x^2 - 2*sqrt(-x^4 + 2*x^2 + 1)*x - 1)/(
x^4 - x^2 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)\left(x^4-1\right)\sqrt{-x^4+2x^2+1}}{\left(x^8-3x^6-x^4+3x^2+1\right)\left(x^4-x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+2*x^2+1)^(1/2)*(x^4-1)*(x^4+1)/(x^4-x^2-1)/(x^8-3*x^6-x^4+3
*x^2+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)*(x^4 - 1)*sqrt(-x^4 + 2*x^2 + 1)/((x^8 - 3*x^6 - x^4 +
3*x^2 + 1)*(x^4 - x^2 - 1)), x)
```

maple [C] time = 0.32, size = 488, normalized size = 2.99

$$-\frac{\sqrt{1-\left(\sqrt{2}-1\right)x^2}\,\sqrt{1-\left(-1-\sqrt{2}\right)x^2}\,\mathrm{EllipticF}\left(x\sqrt{\sqrt{2}-1}, i+i\sqrt{2}\right)}{\sqrt{\sqrt{2}-1}\,\sqrt{-x^4+2x^2+1}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^8-3\_Z^6-\_Z^4+3\_Z^2+1\right)}\_\alpha\left(2\_\alpha^6-6\_\alpha^4+3\right)\left(\frac{\mathrm{arctanh}\left(\frac{\left(\_\alpha^2-1\right)\left(2\_\alpha^6-4\_\alpha^4-6\_\alpha^2+x^2-1\right)}{\sqrt{-\_\alpha^4+2\_\alpha^2+1}\,\sqrt{-x^4+2x^2+1}}\right)}{\sqrt{-\_\alpha^4+2\_\alpha^2+1}}-\frac{2\left(-\_\alpha^7+3\_\alpha^5+\_\alpha^3-3\_\alpha\right)\sqrt{-\sqrt{2}\,x^2+x^2+1}\,\sqrt{\sqrt{2}\,x^2+x^2+1}\,\mathrm{EllipticPi}\left(x\sqrt{\sqrt{2}-1}, -\_\alpha^6\sqrt{2}-\_\alpha^6+3\_\alpha^4\sqrt{2}+3\_\alpha^4+\_\alpha^2\sqrt{2}+\_\alpha^2-3\sqrt{2}-3, \frac{\sqrt{-1-\sqrt{2}}}{\sqrt{\sqrt{2}-1}}\right)}{\sqrt{\sqrt{2}-1}\,\sqrt{-x^4+2x^2+1}}\right)}{20}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4-\_Z^2-1\right)}\_\alpha\left(\frac{\mathrm{arctanh}\left(\frac{\left(\_\alpha^2-1\right)\left(-2\_\alpha^2+x^2-1\right)}{\sqrt{\_\alpha^2}\,\sqrt{-x^4+2x^2+1}}\right)}{\sqrt{\_\alpha^2}}-\frac{2\left(\_\alpha^3-\_\alpha\right)\sqrt{-\sqrt{2}\,x^2+x^2+1}\,\sqrt{\sqrt{2}\,x^2+x^2+1}\,\mathrm{EllipticPi}\left(x\sqrt{\sqrt{2}-1}, \_\alpha^2\sqrt{2}+\_\alpha^2-\sqrt{2}-1, \frac{\sqrt{-1-\sqrt{2}}}{\sqrt{\sqrt{2}-1}}\right)}{\sqrt{\sqrt{2}-1}\,\sqrt{-x^4+2x^2+1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+2*x^2+1)^(1/2)*(x^4-1)*(x^4+1)/(x^4-x^2-1)/(x^8-3*x^6-x^4+3*x^2+1
),x)
```

```
[Out] -1/(2^(1/2)-1)^(1/2)*(1-(2^(1/2)-1)*x^2)^(1/2)*(1-(-1-2^(1/2))*x^2)^(1/2)/(
-x^4+2*x^2+1)^(1/2)*EllipticF(x*(2^(1/2)-1)^(1/2),I+I*2^(1/2))-1/20*sum(_al
pha*(2*_alpha^6-6*_alpha^4+3)*(1/(-_alpha^4+2*_alpha^2+1)^(1/2)*arctanh((_a
lpha^2-1)*(2*_alpha^6-4*_alpha^4-6*_alpha^2+x^2-1)/(-_alpha^4+2*_alpha^2+1)
^(1/2)/(-x^4+2*x^2+1)^(1/2))-2*(-_alpha^7+3*_alpha^5+_alpha^3-3*_alpha)/(2^
(1/2)-1)^(1/2)*(-2^(1/2)*x^2+x^2+1)^(1/2)*(2^(1/2)*x^2+x^2+1)^(1/2)/(-x^4+2
*x^2+1)^(1/2)*EllipticPi(x*(2^(1/2)-1)^(1/2),-_alpha^6*2^(1/2)-_alpha^6+3*_
alpha^4*2^(1/2)+3*_alpha^4+_alpha^2*2^(1/2)+_alpha^2-3*2^(1/2)-3,(-1-2^(1/2
))^(1/2)/(2^(1/2)-1)^(1/2))),_alpha=RootOf(_Z^8-3*_Z^6-_Z^4+3*_Z^2+1))-1/4*
sum(_alpha*(1/(_alpha^2)^(1/2)*arctanh((_alpha^2-1)*(-2*_alpha^2+x^2-1)/(_a
lpha^2)^(1/2)/(-x^4+2*x^2+1)^(1/2))-2*(_alpha^3-_alpha)/(2^(1/2)-1)^(1/2)*(
-2^(1/2)*x^2+x^2+1)^(1/2)*(2^(1/2)*x^2+x^2+1)^(1/2)/(-x^4+2*x^2+1)^(1/2)*El
lipticPi(x*(2^(1/2)-1)^(1/2),_alpha^2*2^(1/2)+_alpha^2-2^(1/2)-1,(-1-2^(1/2
))^(1/2)/(2^(1/2)-1)^(1/2))),_alpha=RootOf(_Z^4-_Z^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)\left(x^4-1\right)\sqrt{-x^4+2x^2+1}}{\left(x^8-3x^6-x^4+3x^2+1\right)\left(x^4-x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+2*x^2+1)^(1/2)*(x^4-1)*(x^4+1)/(x^4-x^2-1)/(x^8-3*x^6-x^4+3
*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)*(x^4 - 1)*sqrt(-x^4 + 2*x^2 + 1)/((x^8 - 3*x^6 - x^4 +
3*x^2 + 1)*(x^4 - x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(x^4-1\right)\left(x^4+1\right)\sqrt{-x^4+2x^2+1}}{\left(-x^4+x^2+1\right)\left(x^8-3x^6-x^4+3x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^4 - 1)*(x^4 + 1)*(2*x^2 - x^4 + 1)^(1/2))/((x^2 - x^4 + 1)*(3*x^2
- x^4 - 3*x^6 + x^8 + 1)),x)
```

```
[Out] -int(((x^4 - 1)*(x^4 + 1)*(2*x^2 - x^4 + 1)^(1/2))/((x^2 - x^4 + 1)*(3*x^2
- x^4 - 3*x^6 + x^8 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+2*x**2+1)**(1/2)*(x**4-1)*(x**4+1)/(x**4-x**2-1)/(x**8-3*x
**6-x**4+3*x**2+1),x)
```

```
[Out] Timed out
```

3.1823 $\int \frac{1}{(-1+x^2)\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=163

$$\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{\sqrt{2}-1}}\right)-\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{1+\sqrt{2}}}\right)+\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{\sqrt{2}-1}}\right)-\sqrt{}$$

Rubi [A] time = 0.34, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {6725, 2119, 1628, 828, 826, 1166, 204, 206, 207, 203}

$$-\frac{\tan^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{1+\sqrt{2}}}+\frac{\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{\sqrt{2}-1}}-\frac{\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{1+\sqrt{2}}}+\frac{\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{\sqrt{2}-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x^2)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -(ArcTan[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[1 + Sqrt[2]]) + A
rcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[-1 + Sqrt[2]] - ArcTa
nh[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[1 + Sqrt[2]] + ArcTanh[
Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[-1 + Sqrt[2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{(-1+x^2)\sqrt{x+\sqrt{1+x^2}}}\,dx = \int \left(-\frac{1}{2(1-x)\sqrt{x+\sqrt{1+x^2}}} - \frac{1}{2(1+x)\sqrt{x+\sqrt{1+x^2}}}\right)dx$$

$$= -\left(\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x+\sqrt{1+x^2}}}\,dx\right) - \frac{1}{2}\int \frac{1}{(1+x)\sqrt{x+\sqrt{1+x^2}}}\,dx$$

$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x^2}{x^{3/2}(1+2x-x^2)}\,dx, x, x+\sqrt{1+x^2}\right)\right) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{}{x^{3/2}(-}\right.$$

$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{1}{x^{3/2}} + \frac{2(1+x)}{x^{3/2}(1+2x-x^2)}\right)dx, x, x+\sqrt{1+x^2}\right)\right) - \frac{1}{2}\operatorname{Subs}$$

$$= -\operatorname{Subst}\left(\int \frac{1+x}{x^{3/2}(1+2x-x^2)}\,dx, x, x+\sqrt{1+x^2}\right) - \operatorname{Subst}\left(\int \frac{1-x}{x^{3/2}(-1+2}\right.$$

$$= -\operatorname{Subst}\left(\int \frac{-1+x}{\sqrt{x}(1+2x-x^2)}\,dx, x, x+\sqrt{1+x^2}\right) + \operatorname{Subst}\left(\int \frac{-1-}{\sqrt{x}(-1+2}\right.$$

$$= -\left(2\operatorname{Subst}\left(\int \frac{-1+x^2}{1+2x^2-x^4}\,dx, x, \sqrt{x+\sqrt{1+x^2}}\right)\right) + 2\operatorname{Subst}\left(\int \frac{-1-}{-1+2x^2}\right.$$

$$= -\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}-x^2}\,dx, x, \sqrt{x+\sqrt{1+x^2}}\right) - \operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}-x^2}\,d\right.$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{x+\sqrt{1+x^2}}}{\sqrt{-1+\sqrt{2}}}\right)}{\sqrt{-1+\sqrt{2}}} - \frac{\tan^{-1}\left(\frac{\sqrt{x+\sqrt{1+x^2}}}{\sqrt{1+\sqrt{2}}}\right)}{\sqrt{1+\sqrt{2}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{1+x^2}}}{\sqrt{-1+\sqrt{2}}}\right)}{\sqrt{-1+\sqrt{2}}} - \frac{\tanh^{-1}\left(\right.}{\sqrt{1}}$$

Mathematica [A] time = 0.35, size = 198, normalized size = 1.21

$$-\frac{\frac{(\sqrt{2}-2)\tan^{-1}\left(\frac{1}{\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}}\right)}{\sqrt{\sqrt{2}-1}} + \frac{(2+\sqrt{2})\tan^{-1}\left(\frac{1}{\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}}\right)}{\sqrt{1+\sqrt{2}}} - \frac{(\sqrt{2}-2)\tanh^{-1}\left(\frac{1}{\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}}\right)}{\sqrt{\sqrt{2}-1}} - \frac{(2+\sqrt{2})\tanh^{-1}\left(\frac{1}{\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}}\right)}{\sqrt{1+\sqrt{2}}}}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x^2)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -((((-2 + Sqrt[2])*ArcTan[1/(Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]])])/
Sqrt[-1 + Sqrt[2]] + ((2 + Sqrt[2])*ArcTan[1/(Sqrt[1 + Sqrt[2]]*Sqrt[x + Sq
rt[1 + x^2]])])/Sqrt[1 + Sqrt[2]] - ((-2 + Sqrt[2])*ArcTanh[1/(Sqrt[-1 + Sq
rt[2]]*Sqrt[x + Sqrt[1 + x^2]])])/Sqrt[-1 + Sqrt[2]] - ((2 + Sqrt[2])*ArcTa
nh[1/(Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]])])/Sqrt[1 + Sqrt[2]])/Sqrt[
2])
```

IntegrateAlgebraic [A] time = 0.29, size = 163, normalized size = 1.00

$$-\sqrt{\sqrt{2}-1}\tan^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right) + \sqrt{1+\sqrt{2}}\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right) - \sqrt{\sqrt{2}-1}\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right) + \sqrt{1+\sqrt{2}}\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x^2)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -(Sqrt[-1 + Sqrt[2]]*ArcTan[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]]) +
Sqrt[1 + Sqrt[2]]*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]] - Sqrt[
```

```
-1 + Sqrt[2]]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]] + Sqrt[1
+ Sqrt[2]]*ArcTanh[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]]
```

fricas [B] time = 0.61, size = 251, normalized size = 1.54

$$-2\sqrt{\sqrt{2}+1}\arctan\left(\sqrt{x+\sqrt{2}+\sqrt{x^2+1}-1}\sqrt{\sqrt{2}+1}-\sqrt{x+\sqrt{x^2+1}}\sqrt{\sqrt{2}+1}\right)+2\sqrt{\sqrt{2}-1}\arctan\left(\sqrt{x+\sqrt{2}+\sqrt{x^2+1}+1}\sqrt{\sqrt{2}-1}-\sqrt{x+\sqrt{x^2+1}}\sqrt{\sqrt{2}-1}\right)-\frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+\sqrt{x+\sqrt{x^2+1}}\right)+\frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(-\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+\sqrt{x+\sqrt{x^2+1}}\right)+\frac{1}{2}\sqrt{\sqrt{2}+1}\log\left(\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+\sqrt{x+\sqrt{x^2+1}}\right)-\frac{1}{2}\sqrt{\sqrt{2}+1}\log\left(-\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+\sqrt{x+\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(2) + 1)*arctan(sqrt(x + sqrt(2) + sqrt(x^2 + 1) - 1)*sqrt(sqrt
(2) + 1) - sqrt(x + sqrt(x^2 + 1))*sqrt(sqrt(2) + 1)) + 2*sqrt(sqrt(2) - 1)
*arctan(sqrt(x + sqrt(2) + sqrt(x^2 + 1) + 1)*sqrt(sqrt(2) - 1) - sqrt(x +
sqrt(x^2 + 1))*sqrt(sqrt(2) - 1)) - 1/2*sqrt(sqrt(2) - 1)*log((sqrt(2) + 1)
*sqrt(sqrt(2) - 1) + sqrt(x + sqrt(x^2 + 1))) + 1/2*sqrt(sqrt(2) - 1)*log(-
(sqrt(2) + 1)*sqrt(sqrt(2) - 1) + sqrt(x + sqrt(x^2 + 1))) + 1/2*sqrt(sqrt(
2) + 1)*log(sqrt(sqrt(2) + 1)*(sqrt(2) - 1) + sqrt(x + sqrt(x^2 + 1))) - 1/
2*sqrt(sqrt(2) + 1)*log(-sqrt(sqrt(2) + 1)*(sqrt(2) - 1) + sqrt(x + sqrt(x^
2 + 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - 1)*sqrt(x + sqrt(x^2 + 1))), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)/(x+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(1/(x^2-1)/(x+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - 1)*sqrt(x + sqrt(x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2-1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)*(x + (x^2 + 1)^(1/2))^(1/2)),x)
[Out] int(1/((x^2 - 1)*(x + (x^2 + 1)^(1/2))^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x-1)(x+1)\sqrt{x+\sqrt{x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)/(x+(x**2+1)**(1/2))**(1/2),x)
[Out] Integral(1/((x - 1)*(x + 1)*sqrt(x + sqrt(x**2 + 1))), x)
```

3.1824 $\int \frac{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$

Optimal. Leaf size=163

$$\frac{1}{2}x\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}+\frac{(3-2ax^2)\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{4b}+\frac{5\tanh^{-1}\left(\sqrt{2}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+a\right.}{4\sqrt{2}\,b}$$

Rubi [F] time = 0.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[-(a/b^2) + (a^2*x^2)/b^2]/Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^
2)/b^2]],x]
```

```
[Out] Defer[Int][Sqrt[-(a/b^2) + (a^2*x^2)/b^2]/Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) +
(a^2*x^2)/b^2]], x]
```

Rubi steps

$$\int \frac{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx=\int \frac{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$$

Mathematica [B] time = 21.92, size = 11560, normalized size = 70.92

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(a/b^2) + (a^2*x^2)/b^2]/Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (
a^2*x^2)/b^2]],x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 3.16, size = 163, normalized size = 1.00

$$\frac{1}{2}x\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}+\frac{(3-2ax^2)\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{4b}+\frac{5\tanh^{-1}\left(\sqrt{2}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}\right)}{4\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-(a/b^2) + (a^2*x^2)/b^2]/Sqrt[a*x^2 + b*x*Sqrt[-(a
/b^2) + (a^2*x^2)/b^2]],x]
```

```
[Out] ((3 - 2*a*x^2)*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]])/(4*b) + (x
*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/
b^2]])/2 + (5*ArcTanh[Sqrt[2]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^
2]]])/(4*Sqrt[2]*b)
```

fricas [A] time = 15.92, size = 168, normalized size = 1.03

$$-\frac{4\left(2\,ax^2-2\,bx\sqrt{\frac{a^2x^2-a}{b^2}}-3\right)\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}-5\,\sqrt{2}\,\log\left(4\,ax^2-4\,bx\sqrt{\frac{a^2x^2-a}{b^2}}-2\,\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\left(2\,\sqrt{2}\,bx\sqrt{\frac{a^2x^2-a}{b^2}}-\sqrt{2}\left(2\,ax^2-1\right)\right)-1\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b^2+a^2*x^2/b^2)^(1/2)/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))
^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(4*(2*a*x^2 - 2*b*x*sqrt((a^2*x^2 - a)/b^2) - 3)*sqrt(a*x^2 + b*x*sqr
t((a^2*x^2 - a)/b^2)) - 5*sqrt(2)*log(4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b^
2) - 2*sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*(2*sqrt(2)*b*x*sqrt((a^2*x
^2 - a)/b^2) - sqrt(2)*(2*a*x^2 - 1)) - 1))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}}{\sqrt{ax^2+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b^2+a^2*x^2/b^2)^(1/2)/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))
^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*x^2/b^2 - a/b^2)/sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*
b*x), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a\,x^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a/b^2+a^2*x^2/b^2)^(1/2)/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)
,x)
```

```
[Out] int((-a/b^2+a^2*x^2/b^2)^(1/2)/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}}{\sqrt{ax^2+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b^2+a^2*x^2/b^2)^(1/2)/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2/b^2 - a/b^2)/sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*
b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}}{\sqrt{a\,x^2+b\,x\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a^2*x^2)/b^2 - a/b^2)^(1/2)/(a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2
))^(1/2),x)
```

```
[Out] int(((a^2*x^2)/b^2 - a/b^2)^(1/2)/(a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2
))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a(ax^2-1)}{b^2}}}{\sqrt{x\left(ax+b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b**2+a**2*x**2/b**2)**(1/2)/(a*x**2+b*x*(-a/b**2+a**2*x**2/b*
*2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(a*x**2 - 1)/b**2)/sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/
b**2))), x)
```

3.1825 $\int \sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}} \sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\, dx$

Optimal. Leaf size=163

$$\frac{5}{12}x\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}+\frac{\left(-2ax^2-9\right)\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{24b}+\frac{3\tanh^{-1}\left(\sqrt{2}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\cdots}\right.}{8\sqrt{2}\,b}$$

Rubi [F] time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}} \sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\, dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^
2)/b^2]],x]
```

```
[Out] Defer[Int][Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) +
(a^2*x^2)/b^2]], x]
```

$$\int \sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}} \sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\, dx=\int \sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}} \sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\, dx$$

Mathematica [B] time = 4.60, size = 714, normalized size = 4.38

$$\frac{\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)^6\left(bx\sqrt{\frac{a(ax^2-1)}{b^2}}+ax^2-1\right)\left(2bx\sqrt{\frac{a(ax^2-1)}{b^2}}+2ax^2-1\right)^2\left(12\sqrt{a}\,x\left(4a^2x^3+ax\left(4bx\sqrt{\frac{a(ax^2-1)}{b^2}}-3\right)-b\sqrt{\frac{a(ax^2-1)}{b^2}}\right)\tanh^{-1}\left(\frac{\sqrt{\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)^2+a}}{\sqrt{a}}\right)+\sqrt{2}x\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(32a^3x^5+16a^2x^3\left(2bx\sqrt{\frac{a(ax^2-1)}{b^2}}-5\right)-ax\left(64bx\sqrt{\frac{a(ax^2-1)}{b^2}}-37\right)+9b\sqrt{\frac{a(ax^2-1)}{b^2}}\right)-3\sqrt{x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(2bx\sqrt{\frac{a(ax^2-1)}{b^2}}+2ax^2-1\right)\tanh^{-1}\left(\sqrt{2}\sqrt{x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\right)\right)}{24\sqrt{2}a^2b^2\sqrt{\frac{a(ax^2-1)}{b^2}}\sqrt{x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(4096a^7x^{13}+1024a^6x^{11}\left(4bx\sqrt{\frac{a(ax^2-1)}{b^2}}-13\right)-256a^5x^9\left(44bx\sqrt{\frac{a(ax^2-1)}{b^2}}-65\right)+768a^4x^7\left(15bx\sqrt{\frac{a(ax^2-1)}{b^2}}-13\right)-224a^3x^5\left(24bx\sqrt{\frac{a(ax^2-1)}{b^2}}-13\right)+28a^2x^3\left(40bx\sqrt{\frac{a(ax^2-1)}{b^2}}-13\right)-ax\left(84bx\sqrt{\frac{a(ax^2-1)}{b^2}}-13\right)+b\sqrt{\frac{a(ax^2-1)}{b^2}}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (
a^2*x^2)/b^2]],x]
```

```
[Out] ((a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])^6*(-1 + a*x^2 + b*x*Sqrt[(a*(-1 + a*x
^2))/b^2])*(-1 + 2*a*x^2 + 2*b*x*Sqrt[(a*(-1 + a*x^2))/b^2])^2*(Sqrt[2]*x*S
qrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*(32*a^3*x^5 + 9*b*Sqrt[(a*(-1
 + a*x^2))/b^2] + 16*a^2*x^3*(-5 + 2*b*x*Sqrt[(a*(-1 + a*x^2))/b^2]) - a*x*
(-37 + 64*b*x*Sqrt[(a*(-1 + a*x^2))/b^2])) - 3*Sqrt[x*(a*x + b*Sqrt[(a*(-1
+ a*x^2))/b^2])]*Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*(-1 + 2*a*x
^2 + 2*b*x*Sqrt[(a*(-1 + a*x^2))/b^2])*ArcTanh[Sqrt[2]*Sqrt[x*(a*x + b*Sqrt
[(a*(-1 + a*x^2))/b^2])]] + 12*Sqrt[a]*x*(4*a^2*x^3 - b*Sqrt[(a*(-1 + a*x^2
))/b^2] + a*x*(-3 + 4*b*x*Sqrt[(a*(-1 + a*x^2))/b^2]))*ArcTanh[Sqrt[a + (a*
x + b*Sqrt[(a*(-1 + a*x^2))/b^2])^2]/Sqrt[a]]))/(24*Sqrt[2]*a^2*b^2*Sqrt[(a
*(-1 + a*x^2))/b^2]*Sqrt[x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*Sqrt[a*x*(
a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*(4096*a^7*x^13 + b*Sqrt[(a*(-1 + a*x^2
))/b^2] + 1024*a^6*x^11*(-13 + 4*b*x*Sqrt[(a*(-1 + a*x^2))/b^2]) + 768*a^4*
x^7*(-13 + 15*b*x*Sqrt[(a*(-1 + a*x^2))/b^2]) - 224*a^3*x^5*(-13 + 24*b*x*S
qrt[(a*(-1 + a*x^2))/b^2]) + 28*a^2*x^3*(-13 + 40*b*x*Sqrt[(a*(-1 + a*x^2))
/b^2]) - 256*a^5*x^9*(-65 + 44*b*x*Sqrt[(a*(-1 + a*x^2))/b^2]) - a*x*(-13 +
 84*b*x*Sqrt[(a*(-1 + a*x^2))/b^2])))
```

IntegrateAlgebraic [A] time = 3.34, size = 223, normalized size = 1.37

$$\frac{5}{12}x\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}+\frac{(-2ax^2-9)\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{24b}+\frac{3\log\left(\sqrt{2}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}+1\right)}{16\sqrt{2}b}-\frac{3\log\left(\sqrt{2}b\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}-b\right)}{16\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*Sqrt[a*x^2 + b*x*Sqrt[-(a
/b^2) + (a^2*x^2)/b^2]],x]
```

```
[Out] ((-9 - 2*a*x^2)*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]])/(24*b) +
(5*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x
^2)/b^2]])/12 + (3*Log[1 + Sqrt[2]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^
2)/b^2]]])/(16*Sqrt[2]*b) - (3*Log[-b + Sqrt[2]*b*Sqrt[a*x^2 + b*x*Sqrt[-(a
/b^2) + (a^2*x^2)/b^2]]])/(16*Sqrt[2]*b)
```

fricas [A] time = 20.41, size = 168, normalized size = 1.03

$$-\frac{4\left(2\,ax^2-10\,bx\sqrt{\frac{a^2x^2-a}{b^2}}+9\right)\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}-9\sqrt{2}\log\left(4\,ax^2-4\,bx\sqrt{\frac{a^2x^2-a}{b^2}}-2\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\left(2\sqrt{2}\,bx\sqrt{\frac{a^2x^2-a}{b^2}}-\sqrt{2}\left(2\,ax^2-1\right)\right)-1\right)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b^2+a^2*x^2/b^2)^(1/2)*(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))
^(1/2),x, algorithm="fricas")
```

```
[Out] -1/96*(4*(2*a*x^2 - 10*b*x*sqrt((a^2*x^2 - a)/b^2) + 9)*sqrt(a*x^2 + b*x*sq
rt((a^2*x^2 - a)/b^2)) - 9*sqrt(2)*log(4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b
^2) - 2*sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*(2*sqrt(2)*b*x*sqrt((a^2*
x^2 - a)/b^2) - sqrt(2)*(2*a*x^2 - 1)) - 1))/b
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b^2+a^2*x^2/b^2)^(1/2)*(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))
^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\,\sqrt{a\,x^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a/b^2+a^2*x^2/b^2)^(1/2)*(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)
,x)
```

```
[Out] int((-a/b^2+a^2*x^2/b^2)^(1/2)*(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^2+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,bx}\,\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b^2+a^2*x^2/b^2)^(1/2)*(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)*sqrt(a^2*x^2/b^2 - a/
b^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a\,x^2+b\,x\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}}\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)*((a^2*x^2)/b^2 - a/b^
2)^(1/2),x)
```

```
[Out] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)*((a^2*x^2)/b^2 - a/b^
2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x\left(ax+b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)}\sqrt{\frac{a\left(ax^2-1\right)}{b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b**2+a**2*x**2/b**2)**(1/2)*(a*x**2+b*x*(-a/b**2+a**2*x**2/b*
*2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))*sqrt(a*(a*x**2 - 1
)/b**2), x)
```

3.1826 $\int \frac{(-3+x)(-2+x)\left(2-x+2x^3\right)^{2/3}}{x^6\left(-2+x+2x^3\right)} \, dx$

Optimal. Leaf size=164

$$2\sqrt[3]{2}\log\left(\sqrt[3]{2}\sqrt[3]{2x^3-x+2}-2x\right)-2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt[3]{2}\sqrt[3]{2x^3-x+2}+x}\right)+\frac{3\left(2x^3-x+2\right)^{2/3}\left(7x^3-x+2\right)}{10x^5}-\sqrt[3]{2}$$

Rubi [F] time = 4.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-3+x)(-2+x)\left(2-x+2x^3\right)^{2/3}}{x^6\left(-2+x+2x^3\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + x)*(-2 + x)*(2 - x + 2*x^3)^(2/3))/(x^6*(-2 + x + 2*x^3)),x]
```

```
[Out] (-27*(2 - x + 2*x^3)^(2/3)*Defer[Int][(((6^(2/3) + 6^(1/3)*(18 - Sqrt[318])
^(2/3))/(3*(18 - Sqrt[318])^(1/3)) + 2*x)^(2/3)*((-6 + (108 - 6*Sqrt[318])^
(2/3) + (6*6^(1/3))/(18 - Sqrt[318])^(2/3))/9 - (2*(18 + Sqrt[318])^(1/3)*(
6^(1/3) + (18 - Sqrt[318])^(2/3))*x)/3 + 4*x^2)^(2/3))/x^6, x])/(((108 - 6*
Sqrt[318])^(1/3) + 6^(2/3)/(18 - Sqrt[318])^(1/3) + 6*x)^(2/3)*(-6 + (108 -
 6*Sqrt[318])^(2/3) + (6*6^(1/3))/(18 - Sqrt[318])^(2/3) - 6*(18 + Sqrt[318
])^(1/3)*(6^(1/3) + (18 - Sqrt[318])^(2/3))*x + 36*x^2)^(2/3)) + (9*(2 - x
+ 2*x^3)^(2/3)*Defer[Int][(((6^(2/3) + 6^(1/3)*(18 - Sqrt[318])^(2/3))/(3*(
18 - Sqrt[318])^(1/3)) + 2*x)^(2/3)*((-6 + (108 - 6*Sqrt[318])^(2/3) + (6*6
^(1/3))/(18 - Sqrt[318])^(2/3))/9 - (2*(18 + Sqrt[318])^(1/3)*(6^(1/3) + (1
8 - Sqrt[318])^(2/3))*x)/3 + 4*x^2)^(2/3))/x^5, x])/(((108 - 6*Sqrt[318])^(
1/3) + 6^(2/3)/(18 - Sqrt[318])^(1/3) + 6*x)^(2/3)*(-6 + (108 - 6*Sqrt[318]
)^(2/3) + (6*6^(1/3))/(18 - Sqrt[318])^(2/3) - 6*(18 + Sqrt[318])^(1/3)*(6^
(1/3) + (18 - Sqrt[318])^(2/3))*x + 36*x^2)^(2/3)) - (27*(2 - x + 2*x^3)^(2
/3)*Defer[Int][(((6^(2/3) + 6^(1/3)*(18 - Sqrt[318])^(2/3))/(3*(18 - Sqrt[3
18])^(1/3)) + 2*x)^(2/3)*((-6 + (108 - 6*Sqrt[318])^(2/3) + (6*6^(1/3))/(18
 - Sqrt[318])^(2/3))/9 - (2*(18 + Sqrt[318])^(1/3)*(6^(1/3) + (18 - Sqrt[31
8])^(2/3))*x)/3 + 4*x^2)^(2/3))/x^3, x])/(((108 - 6*Sqrt[318])^(1/3) + 6^(2
/3)/(18 - Sqrt[318])^(1/3) + 6*x)^(2/3)*(-6 + (108 - 6*Sqrt[318])^(2/3) + (
6*6^(1/3))/(18 - Sqrt[318])^(2/3) - 6*(18 + Sqrt[318])^(1/3)*(6^(1/3) + (18
 - Sqrt[318])^(2/3))*x + 36*x^2)^(2/3)) - (9*(2 - x + 2*x^3)^(2/3)*Defer[In
t][(((6^(2/3) + 6^(1/3)*(18 - Sqrt[318])^(2/3))/(3*(18 - Sqrt[318])^(1/3))
+ 2*x)^(2/3)*((-6 + (108 - 6*Sqrt[318])^(2/3) + (6*6^(1/3))/(18 - Sqrt[318]
)^(2/3))/9 - (2*(18 + Sqrt[318])^(1/3)*(6^(1/3) + (18 - Sqrt[318])^(2/3))*x
)/3 + 4*x^2)^(2/3))/x^2, x])/(2*((108 - 6*Sqrt[318])^(1/3) + 6^(2/3)/(18 -
Sqrt[318])^(1/3) + 6*x)^(2/3)*(-6 + (108 - 6*Sqrt[318])^(2/3) + (6*6^(1/3))
/(18 - Sqrt[318])^(2/3) - 6*(18 + Sqrt[318])^(1/3)*(6^(1/3) + (18 - Sqrt[31
8])^(2/3))*x + 36*x^2)^(2/3)) - (9*(2 - x + 2*x^3)^(2/3)*Defer[Int][(((6^(2
/3) + 6^(1/3)*(18 - Sqrt[318])^(2/3))/(3*(18 - Sqrt[318])^(1/3)) + 2*x)^(2/
3)*((-6 + (108 - 6*Sqrt[318])^(2/3) + (6*6^(1/3))/(18 - Sqrt[318])^(2/3))/9
 - (2*(18 + Sqrt[318])^(1/3)*(6^(1/3) + (18 - Sqrt[318])^(2/3))*x)/3 + 4*x^
2)^(2/3))/x, x])/(4*((108 - 6*Sqrt[318])^(1/3) + 6^(2/3)/(18 - Sqrt[318])^(
1/3) + 6*x)^(2/3)*(-6 + (108 - 6*Sqrt[318])^(2/3) + (6*6^(1/3))/(18 - Sqrt[
318])^(2/3) - 6*(18 + Sqrt[318])^(1/3)*(6^(1/3) + (18 - Sqrt[318])^(2/3))*x
 + 36*x^2)^(2/3)) + (25*Defer[Int][(2 - x + 2*x^3)^(2/3)/(-2 + x + 2*x^3),
x])/4 + Defer[Int][(x*(2 - x + 2*x^3)^(2/3))/(-2 + x + 2*x^3), x] + Defer[I
nt][(x^2*(2 - x + 2*x^3)^(2/3))/(-2 + x + 2*x^3), x]/2
```

Rubi steps

$$\int \frac{(-3+x)(-2+x)\left(2-x+2x^3\right)^{2/3}}{x^6\left(-2+x+2x^3\right)}\,dx = \int\left(-\frac{3\left(2-x+2x^3\right)^{2/3}}{x^6}+\frac{\left(2-x+2x^3\right)^{2/3}}{x^5}-\frac{3\left(2-x+2x^3\right)^{2/3}}{x^3}-\frac{(\cdots}{}\right.$$

$$= -\left(\frac{1}{4}\int\frac{\left(2-x+2x^3\right)^{2/3}}{x}\,dx\right)+\frac{1}{4}\int\frac{\left(25+4x+2x^2\right)\left(2-x+2x^3\right)^{2\cdots}}{-2+x+2x^3}$$

$$= \frac{1}{4}\int\left(\frac{25\left(2-x+2x^3\right)^{2/3}}{-2+x+2x^3}+\frac{4x\left(2-x+2x^3\right)^{2/3}}{-2+x+2x^3}+\frac{2x^2\left(2-x+2x^3\right)\cdots}{-2+x+2x^3}\right.$$

$$= \frac{1}{2}\int\frac{x^2\left(2-x+2x^3\right)^{2/3}}{-2+x+2x^3}\,dx+\frac{25}{4}\int\frac{\left(2-x+2x^3\right)^{2/3}}{-2+x+2x^3}\,dx-\frac{(\cdots}{4\left(\frac{6^{2/3}+\sqrt[3]{6}\cdots}{3\sqrt[3]{\cdots}}\right.}$$

Mathematica [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{(-3+x)(-2+x)\left(2-x+2x^3\right)^{2/3}}{x^6\left(-2+x+2x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + x)*(-2 + x)*(2 - x + 2*x^3)^(2/3))/(x^6*(-2 + x + 2*x^3)),
x]
```

```
[Out] Integrate[((-3 + x)*(-2 + x)*(2 - x + 2*x^3)^(2/3))/(x^6*(-2 + x + 2*x^3)),
 x]
```

IntegrateAlgebraic [A] time = 0.57, size = 164, normalized size = 1.00

$$2\sqrt[3]{2}\log\left(\sqrt[3]{2}\sqrt[3]{2x^3-x+2}-2x\right)-2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt[3]{2}\sqrt[3]{2x^3-x+2}+x}\right)+\frac{3\left(2x^3-x+2\right)^{2/3}\left(7x^3-x+2\right)}{10x^5}-\sqrt[3]{2}\log\left(2\sqrt[3]{2}\sqrt[3]{2x^3-x+2}x+2^{2/3}\left(2x^3-x+2\right)^{2/3}+4x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x)*(-2 + x)*(2 - x + 2*x^3)^(2/3))/(x^6*(-2 + x +
 2*x^3)),x]
```

```
[Out] (3*(2 - x + 2*x^3)^(2/3)*(2 - x + 7*x^3))/(10*x^5) - 2*2^(1/3)*Sqrt[3]*ArcT
an[(Sqrt[3]*x)/(x + 2^(1/3)*(2 - x + 2*x^3)^(1/3))] + 2*2^(1/3)*Log[-2*x +
2^(1/3)*(2 - x + 2*x^3)^(1/3)] - 2^(1/3)*Log[4*x^2 + 2*2^(1/3)*x*(2 - x + 2
*x^3)^(1/3) + 2^(2/3)*(2 - x + 2*x^3)^(2/3)]
```

fricas [B] time = 10.31, size = 424, normalized size = 2.59

$$-\frac{20\sqrt{3}\,2^{\frac{1}{3}}x^5\arctan\left(\frac{6\sqrt{3}\,2^{\frac{2}{3}}\left(20x^7+8x^5-16x^4-x^3+4x^2-4x\right)\left(2x^3-x+2\right)^{\frac{2}{3}}-12\sqrt{3}\,2^{\frac{1}{3}}\left(76x^8-32x^6+64x^5+x^4-4x^3+4x^2\right)\left(2x^3-x+2\right)^{\frac{1}{3}}-\sqrt{3}\left(568x^9-444x^7+888x^6+66x^5-264x^4+263x^3+6x^2-12x+8\right)}{3\left(872x^9-420x^7+840x^6+6x^5-24x^4+25x^3-6x^2+12x-8\right)}\right)-20\cdot 2^{\frac{1}{3}}x^5\log\left(-\frac{6\cdot 2^{\frac{2}{3}}\left(2x^3-x+2\right)^{\frac{1}{3}}x^2-6\left(2x^3-x+2\right)^{\frac{2}{3}}x-2^{\frac{1}{3}}\left(2x^3+x-2\right)}{2x^3+x-2}\right)+10\cdot 2^{\frac{1}{3}}x^5\log\left(\frac{6\cdot 2^{\frac{1}{3}}\left(10x^4-x^2+2x\right)\left(2x^3-x+2\right)^{\frac{2}{3}}+2^{\frac{2}{3}}\left(76x^6-32x^4+64x^3+x^2-4x+4\right)+24\left(4x^5-x^3+2x^2\right)\left(2x^3-x+2\right)^{\frac{1}{3}}}{4x^6+4x^4-8x^3+x^2-4x+4}\right)-9\left(7x^3-x+2\right)\left(2x^3-x+2\right)^{\frac{2}{3}}}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(-2+x)*(2*x^3-x+2)^(2/3)/x^6/(2*x^3+x-2),x, algorithm="fri
cas")
```

```
[Out] -1/30*(20*sqrt(3)*2^(1/3)*x^5*arctan(1/3*(6*sqrt(3)*2^(2/3)*(20*x^7 + 8*x^5
 - 16*x^4 - x^3 + 4*x^2 - 4*x)*(2*x^3 - x + 2)^(2/3) - 12*sqrt(3)*2^(1/3)*(
76*x^8 - 32*x^6 + 64*x^5 + x^4 - 4*x^3 + 4*x^2)*(2*x^3 - x + 2)^(1/3) - sqr
t(3)*(568*x^9 - 444*x^7 + 888*x^6 + 66*x^5 - 264*x^4 + 263*x^3 + 6*x^2 - 12
*x + 8))/(872*x^9 - 420*x^7 + 840*x^6 + 6*x^5 - 24*x^4 + 25*x^3 - 6*x^2 + 1
2*x - 8)) - 20*2^(1/3)*x^5*log(-(6*2^(2/3)*(2*x^3 - x + 2)^(1/3)*x^2 - 6*(2
*x^3 - x + 2)^(2/3)*x - 2^(1/3)*(2*x^3 + x - 2))/(2*x^3 + x - 2)) + 10*2^(1
/3)*x^5*log((6*2^(1/3)*(10*x^4 - x^2 + 2*x)*(2*x^3 - x + 2)^(2/3) + 2^(2/3)
*(76*x^6 - 32*x^4 + 64*x^3 + x^2 - 4*x + 4) + 24*(4*x^5 - x^3 + 2*x^2)*(2*x
^3 - x + 2)^(1/3))/(4*x^6 + 4*x^4 - 8*x^3 + x^2 - 4*x + 4)) - 9*(7*x^3 - x
+ 2)*(2*x^3 - x + 2)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^3 - x + 2\right)^{\frac{2}{3}}(x-2)(x-3)}{\left(2\,x^3 + x - 2\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(-2+x)*(2*x^3-x+2)^(2/3)/x^6/(2*x^3+x-2),x, algorithm="gia
c")
```

```
[Out] integrate((2*x^3 - x + 2)^(2/3)*(x - 2)*(x - 3)/((2*x^3 + x - 2)*x^6), x)
```

maple [C] time = 4.76, size = 743, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+x)*(-2+x)*(2*x^3-x+2)^(2/3)/x^6/(2*x^3+x-2),x)
```

```
[Out] 3/10*(14*x^6-9*x^4+18*x^3+x^2-4*x+4)/x^5/(2*x^3-x+2)^(1/3)+8*RootOf(RootOf(
_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*ln(-(24*RootOf(RootOf(_Z^3-2)^2+4*_Z
*RootOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)^3*x^3+64*RootOf(RootOf(_Z^3-2)^2+4*
_Z*RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-16*(2*x^3-x+2)^(2/3)*Root
Of(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)^2*x+8*(2*x^
3-x+2)^(1/3)*RootOf(_Z^3-2)^2*x^2+40*(2*x^3-x+2)^(1/3)*RootOf(RootOf(_Z^3-2
)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)*x^2-6*RootOf(_Z^3-2)*x^3-16
*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^3+2*(2*x^3-x+2)^(2/
3)*x+3*RootOf(_Z^3-2)*x+8*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z
^2)*x-6*RootOf(_Z^3-2)-16*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z
^2))/(2*x^3+x-2))+2*RootOf(_Z^3-2)*ln((8*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootO
f(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)^3*x^3+48*RootOf(RootOf(_Z^3-2)^2+4*_Z*Roo
tOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-8*(2*x^3-x+2)^(2/3)*RootOf(Root
Of(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)^2*x+4*(2*x^3-x+2)^
(1/3)*RootOf(_Z^3-2)^2*x^2-4*(2*x^3-x+2)^(1/3)*RootOf(RootOf(_Z^3-2)^2+4*_Z
*RootOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)*x^2+6*RootOf(_Z^3-2)*x^3+36*RootOf(
RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^3-5*(2*x^3-x+2)^(2/3)*x-Roo
tOf(_Z^3-2)*x-6*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x+2*Ro
otOf(_Z^3-2)+12*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2))/(2*x^
3+x-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^3 - x + 2\right)^{\frac{2}{3}}(x-2)(x-3)}{\left(2\,x^3 + x - 2\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(-2+x)*(2*x^3-x+2)^(2/3)/x^6/(2*x^3+x-2),x, algorithm="max
ima")
```

```
[Out] integrate((2*x^3 - x + 2)^(2/3)*(x - 2)*(x - 3)/((2*x^3 + x - 2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x-2)\,(x-3)\,\left(2\,x^3-x+2\right)^{2/3}}{x^6\,\left(2\,x^3+x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 2)*(x - 3)*(2*x^3 - x + 2)^(2/3))/(x^6*(x + 2*x^3 - 2)),x)
```

```
[Out] int(((x - 2)*(x - 3)*(2*x^3 - x + 2)^(2/3))/(x^6*(x + 2*x^3 - 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(-2+x)*(2*x**3-x+2)**(2/3)/x**6/(2*x**3+x-2),x)
```

```
[Out] Timed out
```

3.1827 $\int \frac{(1+x^3)^{2/3}(2+x^3)}{x^6(-2-x^3+x^6)}\,dx$

Optimal. Leaf size=164

$$\frac{\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3+1}-3x\right)}{3\,2^{2/3}\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2}\sqrt[3]{x^3+1}+\sqrt[3]{3}x}\right)}{2^{2/3}3^{5/6}}+\frac{\left(x^3+1\right)^{5/3}}{5x^5}-\frac{\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3+1}\,x+2^{2/3}\sqrt[3]{3}\,\left(x^3+1\right)^{2/3}+\right.}{6\,2^{2/3}\sqrt[3]{3}}$$

Rubi [A] time = 0.24, antiderivative size = 166, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1586, 583, 12, 377, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(\sqrt[3]{2}-\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+1}}\right)}{3\,2^{2/3}\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}x}{\sqrt[6]{3}\sqrt[3]{x^3+1}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}3^{5/6}}+\frac{\left(x^3+1\right)^{2/3}}{5x^5}+\frac{\left(x^3+1\right)^{2/3}}{5x^2}-\frac{\log\left(\frac{\sqrt[3]{6}x}{\sqrt[3]{x^3+1}}+\frac{3^{2/3}x^2}{\left(x^3+1\right)^{2/3}}+2^{2/3}\right)}{6\,2^{2/3}\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^3)^(2/3)*(2 + x^3))/(x^6*(-2 - x^3 + x^6)),x]
```

```
[Out] (1 + x^3)^(2/3)/(5*x^5) + (1 + x^3)^(2/3)/(5*x^2) - ArcTan[1/Sqrt[3] + (2^(
2/3)*x)/(3^(1/6)*(1 + x^3)^(1/3))]/(2^(2/3)*3^(5/6)) + Log[2^(1/3) - (3^(1/
3)*x)/(1 + x^3)^(1/3)]/(3*2^(2/3)*3^(1/3)) - Log[2^(2/3) + (3^(2/3)*x^2)/(1
 + x^3)^(2/3) + (6^(1/3)*x)/(1 + x^3)^(1/3)]/(6*2^(2/3)*3^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
```

```
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\int \frac{\left(1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(-2-x^3+x^6\right)}\,dx=\int \frac{2+x^3}{x^6\left(-2+x^3\right)\sqrt[3]{1+x^3}}\,dx$$
$$=\frac{\left(1+x^3\right)^{2/3}}{5x^5}+\frac{1}{10}\int \frac{8+6x^3}{x^3\left(-2+x^3\right)\sqrt[3]{1+x^3}}\,dx$$
$$=\frac{\left(1+x^3\right)^{2/3}}{5x^5}+\frac{\left(1+x^3\right)^{2/3}}{5x^2}+\frac{1}{40}\int \frac{40}{\left(-2+x^3\right)\sqrt[3]{1+x^3}}\,dx$$
$$=\frac{\left(1+x^3\right)^{2/3}}{5x^5}+\frac{\left(1+x^3\right)^{2/3}}{5x^2}+\int \frac{1}{\left(-2+x^3\right)\sqrt[3]{1+x^3}}\,dx$$
$$=\frac{\left(1+x^3\right)^{2/3}}{5x^5}+\frac{\left(1+x^3\right)^{2/3}}{5x^2}+\text{Subst}\left(\int \frac{1}{-2+3x^3}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right)$$
$$=\frac{\left(1+x^3\right)^{2/3}}{5x^5}+\frac{\left(1+x^3\right)^{2/3}}{5x^2}+\frac{\text{Subst}\left(\int \frac{1}{-\sqrt[3]{2}+\sqrt[3]{3}x}\,dx,x,\frac{x}{\sqrt[3]{1+x^3}}\right)}{3\,2^{2/3}}+\frac{\text{Subst}\left(\int \frac{-2\sqrt[3]{2}-\sqrt[3]{3}\ldots}{2^{2/3}+\sqrt[3]{6}x+3^{2\ldots}}\ldots\right.}{3\,2^{2\ldots}}$$
$$=\frac{\left(1+x^3\right)^{2/3}}{5x^5}+\frac{\left(1+x^3\right)^{2/3}}{5x^2}+\frac{\log\left(\sqrt[3]{2}-\frac{\sqrt[3]{3}x}{\sqrt[3]{1+x^3}}\right)}{3\,2^{2/3}\sqrt[3]{3}}-\frac{\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{6}x+3^{2/3}x^2}\,dx,x,\frac{x}{\sqrt[3]{1+\ldots}}\right.}{2\sqrt[3]{2}}$$
$$=\frac{\left(1+x^3\right)^{2/3}}{5x^5}+\frac{\left(1+x^3\right)^{2/3}}{5x^2}+\frac{\log\left(\sqrt[3]{2}-\frac{\sqrt[3]{3}x}{\sqrt[3]{1+x^3}}\right)}{3\,2^{2/3}\sqrt[3]{3}}-\frac{\log\left(2^{2/3}+\frac{3^{2/3}x^2}{\left(1+x^3\right)^{2/3}}+\frac{\sqrt[3]{6}x}{\sqrt[3]{1+x^3}}\right)}{6\,2^{2/3}\sqrt[3]{3}}+\frac{\text{Su}\ldots}{\ldots}$$
$$=\frac{\left(1+x^3\right)^{2/3}}{5x^5}+\frac{\left(1+x^3\right)^{2/3}}{5x^2}-\frac{\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{3}x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{2^{2/3}3^{5/6}}+\frac{\log\left(\sqrt[3]{2}-\frac{\sqrt[3]{3}x}{\sqrt[3]{1+x^3}}\right)}{3\,2^{2/3}\sqrt[3]{3}}-\frac{\log\left(2^{2/3}+\frac{3^{\ldots}}{(1+\ldots}\right.}{6\,2^{\ldots}}$$

Mathematica [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\left(1+x^3\right)^{2/3}\left(2+x^3\right)}{x^6\left(-2-x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^3)^(2/3)*(2 + x^3))/(x^6*(-2 - x^3 + x^6)),x]
```

```
[Out] Integrate[((1 + x^3)^(2/3)*(2 + x^3))/(x^6*(-2 - x^3 + x^6)), x]
```

IntegrateAlgebraic [A] time = 0.43, size = 164, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3+1}-3x\right)}{3\,2^{2/3}\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2}\sqrt[3]{x^3+1}+\sqrt[3]{3}x}\right)}{2^{2/3}3^{5/6}}+\frac{\left(x^3+1\right)^{5/3}}{5x^5}-\frac{\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3+1}\,x+2^{2/3}\sqrt[3]{3}\left(x^3+1\right)^{2/3}+3x^2\right)}{6\,2^{2/3}\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)^(2/3)*(2 + x^3))/(x^6*(-2 - x^3 + x^6)),x]
```

```
[Out] (1 + x^3)^(5/3)/(5*x^5) - ArcTan[(3^(5/6)*x)/(3^(1/3)*x + 2*2^(1/3)*(1 + x^
3)^(1/3))]/(2^(2/3)*3^(5/6)) + Log[-3*x + 2^(1/3)*3^(2/3)*(1 + x^3)^(1/3)]/
(3*2^(2/3)*3^(1/3)) - Log[3*x^2 + 2^(1/3)*3^(2/3)*x*(1 + x^3)^(1/3) + 2^(2/
3)*3^(1/3)*(1 + x^3)^(2/3)]/(6*2^(2/3)*3^(1/3))
```

fricas [B] time = 2.64, size = 253, normalized size = 1.54

$$\frac{10 \cdot 12^{\frac{2}{3}} x^5 \log\left(\frac{18 \cdot 12^{\frac{1}{3}} (x^3+1)^{\frac{1}{3}} x^2 - 12^{\frac{2}{3}} (x^3-2) - 36 (x^3+1)^{\frac{2}{3}} x}{x^3-2}\right) - 5 \cdot 12^{\frac{2}{3}} x^5 \log\left(\frac{6 \cdot 12^{\frac{2}{3}} (4x^4+x)(x^3+1)^{\frac{2}{3}} + 12^{\frac{1}{3}} (55x^6+50x^3+4) + 18 (7x^5+4x^2)(x^3+1)^{\frac{1}{3}}}{x^6-4x^3+4}\right) - 60 \cdot 12^{\frac{1}{6}} x^5 \arctan\left(\frac{12^{\frac{1}{6}} \left(12 \cdot 12^{\frac{2}{3}} (4x^7-7x^4-2x)(x^3+1)^{\frac{2}{3}} - 12^{\frac{1}{3}} (377x^9+600x^6+204x^3+8) - 36 (55x^8+50x^5+4x^2)(x^3+1)^{\frac{1}{3}}\right)}{6 (487x^9+480x^6+12x^3-8)}\right) + 216 (x^3+1)^{\frac{5}{3}}}{1080 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^3+2)/x^6/(x^6-x^3-2),x, algorithm="fricas")
```

```
[Out] 1/1080*(10*12^(2/3)*x^5*log((18*12^(1/3)*(x^3 + 1)^(1/3)*x^2 - 12^(2/3)*(x^
3 - 2) - 36*(x^3 + 1)^(2/3)*x)/(x^3 - 2)) - 5*12^(2/3)*x^5*log((6*12^(2/3)*
(4*x^4 + x)*(x^3 + 1)^(2/3) + 12^(1/3)*(55*x^6 + 50*x^3 + 4) + 18*(7*x^5 +
4*x^2)*(x^3 + 1)^(1/3))/(x^6 - 4*x^3 + 4)) - 60*12^(1/6)*x^5*arctan(1/6*12^
(1/6)*(12*12^(2/3)*(4*x^7 - 7*x^4 - 2*x)*(x^3 + 1)^(2/3) - 12^(1/3)*(377*x^
9 + 600*x^6 + 204*x^3 + 8) - 36*(55*x^8 + 50*x^5 + 4*x^2)*(x^3 + 1)^(1/3))/
(487*x^9 + 480*x^6 + 12*x^3 - 8)) + 216*(x^3 + 1)^(5/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+2\right)\left(x^3+1\right)^{\frac{2}{3}}}{\left(x^6-x^3-2\right)x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^3+2)/x^6/(x^6-x^3-2),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 2)*(x^3 + 1)^(2/3)/((x^6 - x^3 - 2)*x^6), x)
```

maple [C] time = 3.10, size = 606, normalized size = 3.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)*(x^3+2)/x^6/(x^6-x^3-2),x)
```

```
[Out] 1/5*(x^6+2*x^3+1)/x^5/(x^3+1)^(1/3)+1/18*RootOf(_Z^3-18)*ln(-(-270*RootOf(R
ootOf(_Z^3-18)^2+18*_Z*RootOf(_Z^3-18)+324*_Z^2)^2*RootOf(_Z^3-18)^2*x^3-9*
RootOf(RootOf(_Z^3-18)^2+18*_Z*RootOf(_Z^3-18)+324*_Z^2)*RootOf(_Z^3-18)^3*
x^3+42*(x^3+1)^(2/3)*RootOf(RootOf(_Z^3-18)^2+18*_Z*RootOf(_Z^3-18)+324*_Z^
2)*RootOf(_Z^3-18)^2*x+18*(x^3+1)^(1/3)*RootOf(RootOf(_Z^3-18)^2+18*_Z*Root
Of(_Z^3-18)+324*_Z^2)*RootOf(_Z^3-18)*x^2-7*(x^3+1)^(1/3)*RootOf(_Z^3-18)^2
*x^2-450*RootOf(RootOf(_Z^3-18)^2+18*_Z*RootOf(_Z^3-18)+324*_Z^2)*x^3-15*Ro
otOf(_Z^3-18)*x^3+48*x*(x^3+1)^(2/3)-180*RootOf(RootOf(_Z^3-18)^2+18*_Z*Roo
tOf(_Z^3-18)+324*_Z^2)-6*RootOf(_Z^3-18))/(x^3-2))+RootOf(RootOf(_Z^3-18)^2
+18*_Z*RootOf(_Z^3-18)+324*_Z^2)*ln((-162*RootOf(RootOf(_Z^3-18)^2+18*_Z*Ro
otOf(_Z^3-18)+324*_Z^2)^2*RootOf(_Z^3-18)^2*x^3-15*RootOf(RootOf(_Z^3-18)^2
+18*_Z*RootOf(_Z^3-18)+324*_Z^2)*RootOf(_Z^3-18)^3*x^3+42*(x^3+1)^(2/3)*Roo
tOf(RootOf(_Z^3-18)^2+18*_Z*RootOf(_Z^3-18)+324*_Z^2)*RootOf(_Z^3-18)^2*x-1
44*(x^3+1)^(1/3)*RootOf(RootOf(_Z^3-18)^2+18*_Z*RootOf(_Z^3-18)+324*_Z^2)*R
ootOf(_Z^3-18)*x^2-7*(x^3+1)^(1/3)*RootOf(_Z^3-18)^2*x^2+108*RootOf(RootOf(
_Z^3-18)^2+18*_Z*RootOf(_Z^3-18)+324*_Z^2)*x^3+10*RootOf(_Z^3-18)*x^3-6*x*(
x^3+1)^(2/3)+108*RootOf(RootOf(_Z^3-18)^2+18*_Z*RootOf(_Z^3-18)+324*_Z^2)+1
0*RootOf(_Z^3-18))/(x^3-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+2\right)\left(x^3+1\right)^{\frac{2}{3}}}{\left(x^6-x^3-2\right)x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^3+2)/x^6/(x^6-x^3-2),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 2)*(x^3 + 1)^(2/3)/((x^6 - x^3 - 2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^3+1\right)^{2/3}\left(x^3+2\right)}{x^6\left(-x^6+x^3+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3 + 1)^(2/3)*(x^3 + 2))/(x^6*(x^3 - x^6 + 2)),x)
```

```
[Out] int(-((x^3 + 1)^(2/3)*(x^3 + 2))/(x^6*(x^3 - x^6 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((x+1)\left(x^2-x+1\right)\right)^{\frac{2}{3}}\left(x^3+2\right)}{x^6(x+1)\left(x^3-2\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)*(x**3+2)/x**6/(x**6-x**3-2),x)
```

```
[Out] Integral(((x + 1)*(x**2 - x + 1))**(2/3)*(x**3 + 2)/(x**6*(x + 1)*(x**3 - 2
)*(x**2 - x + 1)), x)
```

3.1828 $\int \frac{1}{(2+x)\sqrt[3]{1+x+x^2}}\,dx$

Optimal. Leaf size=165

$$\frac{\log\left(3\sqrt[3]{x^2+x+1}+\sqrt[3]{3}x-\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\log\left(3^{2/3}x^2+9\left(x^2+x+1\right)^{2/3}+\left(3\sqrt[3]{3}-3\sqrt[3]{3}x\right)\sqrt[3]{x^2+x+1}-2\,3^{2/3}x+\right.}{6\sqrt[3]{3}}$$

Rubi [A] time = 0.01, antiderivative size = 84, normalized size of antiderivative = 0.51, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {750}

$$\frac{\log\left(-3^{2/3}\sqrt[3]{x^2+x+1}-x+1\right)}{2\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{2(1-x)}{3\sqrt[6]{3}\sqrt[3]{x^2+x+1}}+\frac{1}{\sqrt{3}}\right)}{3^{5/6}}-\frac{\log(x+2)}{2\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + x)*(1 + x + x^2)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2*(1 - x))/(3*3^(1/6)*(1 + x + x^2)^(1/3))]/3^(5/6))
- Log[2 + x]/(2*3^(1/3)) + Log[1 - x - 3^(2/3)*(1 + x + x^2)^(1/3)]/(2*3^(1
/3))
```

Rule 750

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[3*c*e^2*(2*c*d - b*e), 3]}, -Simp[(Sqrt[3]*c*e*ArcTan
[1/Sqrt[3] + (2*(c*d - b*e - c*e*x))/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3))])/
q^2, x] + (-Simp[(3*c*e*Log[d + e*x])/(2*q^2), x] + Simp[(3*c*e*Log[c*d - b
*e - c*e*x - q*(a + b*x + c*x^2)^(1/3)])/(2*q^2), x])] /; FreeQ[{a, b, c, d
, e}, x] && NeQ[2*c*d - b*e, 0] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*c*
e^2, 0] && PosQ[c*e^2*(2*c*d - b*e)]
```

Rubi steps

$$\int \frac{1}{(2+x)\sqrt[3]{1+x+x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2(1-x)}{3\sqrt[6]{3}\sqrt[3]{1+x+x^2}}\right)}{3^{5/6}}-\frac{\log(2+x)}{2\sqrt[3]{3}}+\frac{\log\left(1-x-3^{2/3}\sqrt[3]{1+x+x^2}\right)}{2\sqrt[3]{3}}$$

Mathematica [C] time = 0.06, size = 118, normalized size = 0.72

$$-\frac{3\sqrt[3]{\frac{2x-i\sqrt{3}+1}{x+2}}\sqrt[3]{\frac{2x+i\sqrt{3}+1}{x+2}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{3-i\sqrt{3}}{2x+4},\frac{3+i\sqrt{3}}{2x+4}\right)}{2\,2^{2/3}\sqrt[3]{x^2+x+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 + x)*(1 + x + x^2)^(1/3)),x]
```

```
[Out] (-3*((1 - I*Sqrt[3] + 2*x)/(2 + x))^(1/3)*((1 + I*Sqrt[3] + 2*x)/(2 + x))^(
1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (3 - I*Sqrt[3])/(4 + 2*x), (3 + I*Sqrt[3]
)/(4 + 2*x)])/(2*2^(2/3)*(1 + x + x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.20, size = 165, normalized size = 1.00

$$\frac{\log\left(3\sqrt[3]{x^2+x+1}+\sqrt[3]{3}x-\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\log\left(3^{2/3}x^2+9\left(x^2+x+1\right)^{2/3}+\left(3\sqrt[3]{3}-3\sqrt[3]{3}x\right)\sqrt[3]{x^2+x+1}-2\,3^{2/3}x+3^{2/3}\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2+x+1}}{\sqrt{3}}-\frac{2x}{3\sqrt[6]{3}}+\frac{2}{3\sqrt[6]{3}}}{\sqrt[3]{x^2+x+1}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((2 + x)*(1 + x + x^2)^(1/3)),x]
```

```
[Out] -(ArcTan[(2/(3*3^(1/6)) - (2*x)/(3*3^(1/6)) + (1 + x + x^2)^(1/3)/Sqrt[3])/
(1 + x + x^2)^(1/3)]/3^(5/6)) + Log[-3^(1/3) + 3^(1/3)*x + 3*(1 + x + x^2)^
(1/3)]/(3*3^(1/3)) - Log[3^(2/3) - 2*3^(2/3)*x + 3^(2/3)*x^2 + (3*3^(1/3) -
 3*3^(1/3)*x)*(1 + x + x^2)^(1/3) + 9*(1 + x + x^2)^(2/3)]/(6*3^(1/3))
```

fricas [A] time = 2.21, size = 165, normalized size = 1.00

$$-\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(\frac{3\cdot 3^{\frac{2}{3}}\left(x^2+x+1\right)^{\frac{2}{3}}+3^{\frac{1}{3}}\left(x^2-2x+1\right)-3\left(x^2+x+1\right)^{\frac{1}{3}}(x-1)}{x^2+4x+4}\right)+\frac{1}{9}\cdot 3^{\frac{2}{3}}\log\left(\frac{3^{\frac{1}{3}}(x-1)+3\left(x^2+x+1\right)^{\frac{1}{3}}}{x+2}\right)-\frac{1}{3}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{1}{6}}\left(6\cdot 3^{\frac{2}{3}}\left(x^2+x+1\right)^{\frac{2}{3}}(x-1)+3^{\frac{1}{3}}\left(x^3+6x^2+12x+8\right)+6\left(x^2+x+1\right)^{\frac{1}{3}}\left(x^2-2x+1\right)\right)}{3\left(x^3-12x^2-6x-10\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+x+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/18*3^(2/3)*log((3*3^(2/3)*(x^2 + x + 1)^(2/3) + 3^(1/3)*(x^2 - 2*x + 1)
- 3*(x^2 + x + 1)^(1/3)*(x - 1))/(x^2 + 4*x + 4)) + 1/9*3^(2/3)*log((3^(1/3
)*(x - 1) + 3*(x^2 + x + 1)^(1/3))/(x + 2)) - 1/3*3^(1/6)*arctan(1/3*3^(1/6
)*(6*3^(2/3)*(x^2 + x + 1)^(2/3)*(x - 1) + 3^(1/3)*(x^3 + 6*x^2 + 12*x + 8)
 + 6*(x^2 + x + 1)^(1/3)*(x^2 - 2*x + 1))/(x^3 - 12*x^2 - 6*x - 10))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+x+1\right)^{\frac{1}{3}}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+x+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + x + 1)^(1/3)*(x + 2)), x)
```

maple [C] time = 14.23, size = 1377, normalized size = 8.35

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+x)/(x^2+x+1)^(1/3),x)
```

```
[Out] 1/9*RootOf(_Z^3-9)*ln(-(13110*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9
*_Z^2)*RootOf(_Z^3-9)^3*x^3+43587*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-
9)+9*_Z^2)^2*RootOf(_Z^3-9)^2*x^3-26220*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf
(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^3*x^2-87174*RootOf(RootOf(_Z^3-9)^2+3*_Z*Ro
otOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2*x^2-131841*(x^2+x+1)^(2/3)*RootOf(R
ootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^2*x+52440*RootOf
(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^3*x+174348*Roo
tOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2*x+13184
1*(x^2+x+1)^(2/3)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootO
f(_Z^3-9)^2-43947*(x^2+x+1)^(1/3)*RootOf(_Z^3-9)^2*x^2+132858*(x^2+x+1)^(1/
3)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)*x^2+8
7894*(x^2+x+1)^(1/3)*RootOf(_Z^3-9)^2*x-265716*(x^2+x+1)^(1/3)*RootOf(RootO
```

```
f(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)*x+43700*RootOf(_Z^3-
9)*x^3+145290*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x^3-43947
*(x^2+x+1)^(1/3)*RootOf(_Z^3-9)^2+132858*(x^2+x+1)^(1/3)*RootOf(RootOf(_Z^3
-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)-485070*RootOf(_Z^3-9)*x^2-
1612719*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x^2-794097*(x^2
+x+1)^(2/3)*x-222870*RootOf(_Z^3-9)*x-740979*RootOf(RootOf(_Z^3-9)^2+3*_Z*R
ootOf(_Z^3-9)+9*_Z^2)*x+794097*(x^2+x+1)^(2/3)-397670*RootOf(_Z^3-9)-132213
9*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2))/(2+x)^3)+1/3*RootOf(
RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*ln((14529*RootOf(RootOf(_Z^3-9
)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^3*x^3+39330*RootOf(RootOf(_Z
^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2*x^3-29058*RootOf(Roo
tOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^3*x^2-78660*RootOf
(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2*x^2-131841
*(x^2+x+1)^(2/3)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf
(_Z^3-9)^2*x+58116*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*Root
Of(_Z^3-9)^3*x+157320*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)^2
*RootOf(_Z^3-9)^2*x+131841*(x^2+x+1)^(2/3)*RootOf(RootOf(_Z^3-9)^2+3*_Z*Roo
tOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^2-43947*(x^2+x+1)^(1/3)*RootOf(_Z^3-9)^2
*x^2-264699*(x^2+x+1)^(1/3)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_
Z^2)*RootOf(_Z^3-9)*x^2+87894*(x^2+x+1)^(1/3)*RootOf(_Z^3-9)^2*x+529398*(x^
2+x+1)^(1/3)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^
3-9)*x-4843*RootOf(_Z^3-9)*x^3-13110*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z
^3-9)+9*_Z^2)*x^3-43947*(x^2+x+1)^(1/3)*RootOf(_Z^3-9)^2-264699*(x^2+x+1)^(
1/3)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)+450
399*RootOf(_Z^3-9)*x^2+1219230*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+
9*_Z^2)*x^2+398574*(x^2+x+1)^(2/3)*x+421341*RootOf(_Z^3-9)*x+1140570*RootOf
(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x-398574*(x^2+x+1)^(2/3)+4407
13*RootOf(_Z^3-9)+1193010*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^
2))/(2+x)^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+x+1\right)^{\frac{1}{3}}(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+x+1)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^2 + x + 1)^(1/3)*(x + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x+2)\left(x^2+x+1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 2)*(x + x^2 + 1)^(1/3)),x)
[Out] int(1/((x + 2)*(x + x^2 + 1)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+2)\sqrt[3]{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x**2+x+1)**(1/3),x)
[Out] Integral(1/((x + 2)*(x**2 + x + 1)**(1/3)), x)
```

3.1829 $\int \frac{(-3+2x)\sqrt[3]{-1+x+x^3}}{x^2(2-2x+x^3)}\,dx$

Optimal. Leaf size=165

$$\frac{3\sqrt[3]{x^3+x-1}}{2x}+\frac{1}{2}\sqrt[3]{\frac{3}{2}}\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3+x-1}-3x\right)+\frac{3^{5/6}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2}\sqrt[3]{x^3+x-1}+\sqrt[3]{3}x}\right)}{2\sqrt[3]{2}}-\frac{1}{4}\sqrt[3]{\frac{3}{2}}\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3+\ldots}\right.$$

Rubi [F] time = 1.58, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-3+2x)\sqrt[3]{-1+x+x^3}}{x^2\left(2-2x+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + 2*x)*(-1 + x + x^3)^(1/3))/(x^2*(2 - 2*x + x^3)),x]
```

```
[Out] (-9*(-1 + x + x^3)^(1/3)*Defer[Int][(((2*(3/(9 + Sqrt[93]))^(1/3) - (2*(9 +
 Sqrt[93]))^(1/3))/6^(2/3) + x)^(1/3)*((6 + 6*3^(1/3)*(2/(9 + Sqrt[93]))^(2
/3) + 2^(1/3)*(3*(9 + Sqrt[93]))^(2/3))/18 - (((6/(9 + Sqrt[93]))^(1/3) - (
(9 + Sqrt[93])/2)^(1/3))*x)/3^(2/3) + x^2)^(1/3))/x^2, x])/(2^(1/3)*(6^(1/3
)*(2*(3/(9 + Sqrt[93]))^(1/3) - (2*(9 + Sqrt[93]))^(1/3)) + 6*x)^(1/3)*(6 +
 6*3^(1/3)*(2/(9 + Sqrt[93]))^(2/3) + 2^(1/3)*(3*(9 + Sqrt[93]))^(2/3) - 6*
3^(1/3)*((6/(9 + Sqrt[93]))^(1/3) - ((9 + Sqrt[93])/2)^(1/3))*x + 18*x^2)^(
1/3)) - (3*(-1 + x + x^3)^(1/3)*Defer[Int][(((2*(3/(9 + Sqrt[93]))^(1/3) -
(2*(9 + Sqrt[93]))^(1/3))/6^(2/3) + x)^(1/3)*((6 + 6*3^(1/3)*(2/(9 + Sqrt[9
3]))^(2/3) + 2^(1/3)*(3*(9 + Sqrt[93]))^(2/3))/18 - (((6/(9 + Sqrt[93]))^(1
/3) - ((9 + Sqrt[93])/2)^(1/3))*x)/3^(2/3) + x^2)^(1/3))/x, x])/(2^(1/3)*(6
^(1/3)*(2*(3/(9 + Sqrt[93]))^(1/3) - (2*(9 + Sqrt[93]))^(1/3)) + 6*x)^(1/3)
*(6 + 6*3^(1/3)*(2/(9 + Sqrt[93]))^(2/3) + 2^(1/3)*(3*(9 + Sqrt[93]))^(2/3)
 - 6*3^(1/3)*((6/(9 + Sqrt[93]))^(1/3) - ((9 + Sqrt[93])/2)^(1/3))*x + 18*x
^2)^(1/3)) - Defer[Int][(-1 + x + x^3)^(1/3)/(2 - 2*x + x^3), x] + (3*Defer
[Int][(x*(-1 + x + x^3)^(1/3))/(2 - 2*x + x^3), x])/2 + Defer[Int][(x^2*(-1
 + x + x^3)^(1/3))/(2 - 2*x + x^3), x]/2
```

Rubi steps

$$\int \frac{(-3+2x)\sqrt[3]{-1+x+x^3}}{x^2\left(2-2x+x^3\right)}\,dx = \int \left(-\frac{3\sqrt[3]{-1+x+x^3}}{2x^2} - \frac{\sqrt[3]{-1+x+x^3}}{2x} + \frac{\left(-2+3x+x^2\right)\sqrt[3]{-1+x+x^3}}{2\left(2-2x+x^3\right)}\right) dx$$

$$= -\left(\frac{1}{2}\int \frac{\sqrt[3]{-1+x+x^3}}{x}\,dx\right) + \frac{1}{2}\int \frac{\left(-2+3x+x^2\right)\sqrt[3]{-1+x+x^3}}{2-2x+x^3}\,dx - \frac{3}{2}\int \cdots$$

$$= \frac{1}{2}\int \left(-\frac{2\sqrt[3]{-1+x+x^3}}{2-2x+x^3} + \frac{3x\sqrt[3]{-1+x+x^3}}{2-2x+x^3} + \frac{x^2\sqrt[3]{-1+x+x^3}}{2-2x+x^3}\right) dx - \cdots$$

$$= \frac{1}{2}\int \frac{x^2\sqrt[3]{-1+x+x^3}}{2-2x+x^3}\,dx + \frac{3}{2}\int \frac{x\sqrt[3]{-1+x+x^3}}{2-2x+x^3}\,dx - \frac{\sqrt[3]{-1+x+x^3}\int \cdots}{2\sqrt[3]{\frac{2\sqrt[3]{\frac{3}{9+\sqrt{93}}} - \sqrt[3]{2\left(9+\sqrt{93}\right)} \cdots}{6^{2/3}}}}$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(-3+2x)\sqrt[3]{-1+x+x^3}}{x^2\left(2-2x+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + 2*x)*(-1 + x + x^3)^(1/3))/(x^2*(2 - 2*x + x^3)),x]
```

```
[Out] Integrate[((-3 + 2*x)*(-1 + x + x^3)^(1/3))/(x^2*(2 - 2*x + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.52, size = 165, normalized size = 1.00

$$\frac{3\sqrt[3]{x^3+x-1}}{2x} + \frac{1}{2}\sqrt[3]{\frac{3}{2}}\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3+x-1}-3x\right) + \frac{3^{5/6}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2}\sqrt[3]{x^3+x-1}+\sqrt[3]{3}x}\right)}{2\sqrt[3]{2}} - \frac{1}{4}\sqrt[3]{\frac{3}{2}}\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3+x-1}\,x+2^{2/3}\sqrt[3]{3}\left(x^3+x-1\right)^{2/3}+3x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + 2*x)*(-1 + x + x^3)^(1/3))/(x^2*(2 - 2*x + x^3)),
x]
```

```
[Out] (3*(-1 + x + x^3)^(1/3))/(2*x) + (3^(5/6)*ArcTan[(3^(5/6)*x)/(3^(1/3)*x + 2
*2^(1/3)*(-1 + x + x^3)^(1/3))])/(2*2^(1/3)) + ((3/2)^(1/3)*Log[-3*x + 2^(1
/3)*3^(2/3)*(-1 + x + x^3)^(1/3)])/2 - ((3/2)^(1/3)*Log[3*x^2 + 2^(1/3)*3^(
2/3)*x*(-1 + x + x^3)^(1/3) + 2^(2/3)*3^(1/3)*(-1 + x + x^3)^(2/3)])/4
```

fricas [B] time = 13.49, size = 406, normalized size = 2.46

$$\frac{2\cdot 3^{\frac{5}{6}}2^{\frac{2}{3}}x\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(24\cdot 3^{\frac{1}{3}}\sqrt{2}\left(4x^7-7x^5+7x^4-2x^3+4x^2-2x\right)\left(x^3+x-1\right)^{\frac{2}{3}}-12\cdot 3^{\frac{2}{3}}2^{\frac{1}{6}}\left(55x^8+50x^6-50x^5+4x^4-8x^3+4x^2\right)\left(x^3+x-1\right)^{\frac{1}{3}}-2^{\frac{5}{6}}\left(377x^9+600x^7-600x^6+204x^5-408x^4+212x^3-24x^2+24x-8\right)\right)}{6\left(487x^9+480x^7-480x^6+12x^5-24x^4+4x^3+24x^2-24x+8\right)}\right)+2\cdot 3^{\frac{1}{3}}2^{\frac{2}{3}}x\log\left(-\frac{9\cdot 3^{\frac{1}{3}}2^{\frac{2}{3}}\left(x^3+x-1\right)^{\frac{1}{3}}x^2-3^{\frac{2}{3}}2^{\frac{1}{3}}\left(x^3-2x+2\right)-18\left(x^3+x-1\right)^{\frac{2}{3}}x}{x^3-2x+2}\right)-3^{\frac{1}{3}}2^{\frac{2}{3}}x\log\left(\frac{12\cdot 3^{\frac{2}{3}}2^{\frac{1}{3}}\left(4x^4+x^2-x\right)\left(x^3+x-1\right)^{\frac{2}{3}}+3^{\frac{1}{3}}2^{\frac{2}{3}}\left(55x^6+50x^4-50x^3+4x^2-8x+4\right)+18\left(7x^5+4x^3-4x^2\right)\left(x^3+x-1\right)^{\frac{1}{3}}}{x^6-4x^4+4x^3+4x^2-8x+4}\right)+36\left(x^3+x-1\right)^{\frac{1}{3}}}{24x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^3+x-1)^(1/3)/x^2/(x^3-2*x+2),x, algorithm="fricas")
```

```
[Out] 1/24*(2*3^(5/6)*2^(2/3)*x*arctan(1/6*sqrt(3)*2^(1/6)*(24*3^(1/3)*sqrt(2)*(4
*x^7 - 7*x^5 + 7*x^4 - 2*x^3 + 4*x^2 - 2*x)*(x^3 + x - 1)^(2/3) - 12*3^(2/3
)*2^(1/6)*(55*x^8 + 50*x^6 - 50*x^5 + 4*x^4 - 8*x^3 + 4*x^2)*(x^3 + x - 1)^
```

```
(1/3) - 2^(5/6)*(377*x^9 + 600*x^7 - 600*x^6 + 204*x^5 - 408*x^4 + 212*x^3
- 24*x^2 + 24*x - 8))/(487*x^9 + 480*x^7 - 480*x^6 + 12*x^5 - 24*x^4 + 4*x^
3 + 24*x^2 - 24*x + 8)) + 2*3^(1/3)*2^(2/3)*x*log(-(9*3^(1/3)*2^(2/3)*(x^3
+ x - 1)^(1/3)*x^2 - 3^(2/3)*2^(1/3)*(x^3 - 2*x + 2) - 18*(x^3 + x - 1)^(2/
3)*x)/(x^3 - 2*x + 2)) - 3^(1/3)*2^(2/3)*x*log((12*3^(2/3)*2^(1/3)*(4*x^4 +
 x^2 - x)*(x^3 + x - 1)^(2/3) + 3^(1/3)*2^(2/3)*(55*x^6 + 50*x^4 - 50*x^3 +
 4*x^2 - 8*x + 4) + 18*(7*x^5 + 4*x^3 - 4*x^2)*(x^3 + x - 1)^(1/3))/(x^6 -
4*x^4 + 4*x^3 + 4*x^2 - 8*x + 4)) + 36*(x^3 + x - 1)^(1/3))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+x-1\right)^{\frac{1}{3}}(2x-3)}{\left(x^3-2x+2\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^3+x-1)^(1/3)/x^2/(x^3-2*x+2),x, algorithm="giac")
```

```
[Out] integrate((x^3 + x - 1)^(1/3)*(2*x - 3)/((x^3 - 2*x + 2)*x^2), x)
```

maple [C] time = 4.31, size = 2200, normalized size = 13.33

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*x)*(x^3+x-1)^(1/3)/x^2/(x^3-2*x+2),x)
```

```
[Out] 3/2*(x^3+x-1)^(1/3)/x+(-1/4*ln((9*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3
-12)+36*_Z^2)^2*RootOf(_Z^3-12)^2*x^6+3*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootO
f(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)^3*x^6+9*RootOf(RootOf(_Z^3-12)^2+6*_Z*R
ootOf(_Z^3-12)+36*_Z^2)^2*RootOf(_Z^3-12)^2*x^4+3*RootOf(RootOf(_Z^3-12)^2+
6*_Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)^3*x^4-9*RootOf(RootOf(_Z^3-12
)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)^2*RootOf(_Z^3-12)^2*x^3-3*RootOf(RootOf(_
Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)^3*x^3+9*(x^6+2*x^4-
2*x^3+x^2-2*x+1)^(2/3)*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^
2)*RootOf(_Z^3-12)^2*x^2-18*(x^6+2*x^4-2*x^3+x^2-2*x+1)^(1/3)*RootOf(_Z^3-1
2)*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x^4-12*RootOf(Roo
tOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x^6-4*RootOf(_Z^3-12)*x^6-18*(
x^6+2*x^4-2*x^3+x^2-2*x+1)^(1/3)*RootOf(_Z^3-12)*RootOf(RootOf(_Z^3-12)^2+6
*_Z*RootOf(_Z^3-12)+36*_Z^2)*x^2-24*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z
^3-12)+36*_Z^2)*x^4-8*RootOf(_Z^3-12)*x^4+18*(x^6+2*x^4-2*x^3+x^2-2*x+1)^(1
/3)*RootOf(_Z^3-12)*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*
x+24*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x^3+8*RootOf(_Z
^3-12)*x^3+18*x^2*(x^6+2*x^4-2*x^3+x^2-2*x+1)^(2/3)-12*RootOf(RootOf(_Z^3-1
2)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x^2-4*RootOf(_Z^3-12)*x^2+24*RootOf(Root
Of(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x+8*RootOf(_Z^3-12)*x-12*RootOf
(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)-4*RootOf(_Z^3-12))/(x^3-2*
x+2)/(x^3+x-1))*RootOf(_Z^3-12)-3/2*ln((9*RootOf(RootOf(_Z^3-12)^2+6*_Z*Roo
tOf(_Z^3-12)+36*_Z^2)^2*RootOf(_Z^3-12)^2*x^6+3*RootOf(RootOf(_Z^3-12)^2+6*
_Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)^3*x^6+9*RootOf(RootOf(_Z^3-12)^
2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)^2*RootOf(_Z^3-12)^2*x^4+3*RootOf(RootOf(_Z^
3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)^3*x^4-9*RootOf(RootOf
(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)^2*RootOf(_Z^3-12)^2*x^3-3*RootOf(
RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)^3*x^3+9*(x^
6+2*x^4-2*x^3+x^2-2*x+1)^(2/3)*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12
)+36*_Z^2)*RootOf(_Z^3-12)^2*x^2-18*(x^6+2*x^4-2*x^3+x^2-2*x+1)^(1/3)*RootO
f(_Z^3-12)*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x^4-12*Ro
otOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x^6-4*RootOf(_Z^3-12)*
x^6-18*(x^6+2*x^4-2*x^3+x^2-2*x+1)^(1/3)*RootOf(_Z^3-12)*RootOf(RootOf(_Z^3
```

```
-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x^2-24*RootOf(RootOf(_Z^3-12)^2+6*_Z*R
ootOf(_Z^3-12)+36*_Z^2)*x^4-8*RootOf(_Z^3-12)*x^4+18*(x^6+2*x^4-2*x^3+x^2-2
*x+1)^(1/3)*RootOf(_Z^3-12)*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+3
6*_Z^2)*x+24*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x^3+8*R
ootOf(_Z^3-12)*x^3+18*x^2*(x^6+2*x^4-2*x^3+x^2-2*x+1)^(2/3)-12*RootOf(RootO
f(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x^2-4*RootOf(_Z^3-12)*x^2+24*Roo
tOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x+8*RootOf(_Z^3-12)*x-1
2*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)-4*RootOf(_Z^3-12))
/(x^3-2*x+2)/(x^3+x-1))*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z
^2)+1/4*RootOf(_Z^3-12)*ln((9*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)
+36*_Z^2)^2*RootOf(_Z^3-12)^2*x^6+3*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z
^3-12)+36*_Z^2)*RootOf(_Z^3-12)^3*x^6+9*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootO
f(_Z^3-12)+36*_Z^2)^2*RootOf(_Z^3-12)^2*x^4+3*RootOf(RootOf(_Z^3-12)^2+6*_Z
*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)^3*x^4-9*RootOf(RootOf(_Z^3-12)^2+
6*_Z*RootOf(_Z^3-12)+36*_Z^2)^2*RootOf(_Z^3-12)^2*x^3-3*RootOf(RootOf(_Z^3-
12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)^3*x^3-9*(x^6+2*x^4-2*x^
3+x^2-2*x+1)^(2/3)*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*R
ootOf(_Z^3-12)^2*x^2+30*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z
^2)*x^6-3*(x^6+2*x^4-2*x^3+x^2-2*x+1)^(1/3)*RootOf(_Z^3-12)^2*x^4+10*RootOf
(_Z^3-12)*x^6+42*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x^4
-3*(x^6+2*x^4-2*x^3+x^2-2*x+1)^(1/3)*RootOf(_Z^3-12)^2*x^2+14*RootOf(_Z^3-1
2)*x^4-42*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x^3+3*(x^6
+2*x^4-2*x^3+x^2-2*x+1)^(1/3)*RootOf(_Z^3-12)^2*x-14*RootOf(_Z^3-12)*x^3+12
*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x^2+4*RootOf(_Z^3-1
2)*x^2-24*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x-8*RootOf
(_Z^3-12)*x+12*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)+4*Roo
tOf(_Z^3-12))/(x^3-2*x+2)/(x^3+x-1)))/(x^3+x-1)^(2/3)*((x^3+x-1)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+x-1\right)^{\frac{1}{3}}(2x-3)}{\left(x^3-2x+2\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^3+x-1)^(1/3)/x^2/(x^3-2*x+2),x, algorithm="maxima")
[Out] integrate((x^3 + x - 1)^(1/3)*(2*x - 3)/((x^3 - 2*x + 2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x-3)\left(x^3+x-1\right)^{1/3}}{x^2\left(x^3-2x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x - 3)*(x + x^3 - 1)^(1/3))/(x^2*(x^3 - 2*x + 2)),x)
[Out] int(((2*x - 3)*(x + x^3 - 1)^(1/3))/(x^2*(x^3 - 2*x + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x-3)\sqrt[3]{x^3+x-1}}{x^2\left(x^3-2x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x**3+x-1)**(1/3)/x**2/(x**3-2*x+2),x)
[Out] Integral((2*x - 3)*(x**3 + x - 1)**(1/3)/(x**2*(x**3 - 2*x + 2)), x)
```

3.1830 $\int \frac{-\left((2a-b)b^2\right)+(4a-b)bx-(2a+b)x^2+x^3}{(-a+x)\sqrt[4]{(-a+x)(-b+x)^2}\left(b^2+ad-(2b+d)x+x^2\right)}\,dx$

Optimal. Leaf size=165

$$-2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}}{b-x}\right)+2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b}}{b-x}\right.$$

Rubi [C] time = 3.34, antiderivative size = 325, normalized size of antiderivative = 1.97, number of steps used = 8, number of rules used = 5, integrand size = 81, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {6688, 6719, 6728, 137, 136}

$$\frac{4(a-b)(b-x)\left(\sqrt{-4a+4b+d}+\sqrt{d}\right)F_1\left(-\frac{1}{4};-\frac{3}{2},1;\frac{3}{4};\frac{a-x}{a-b},\frac{2(a-x)}{2a-2b-d-\sqrt{d}\sqrt{-4a+4b+d}}\right)}{\sqrt{d}\left(-\sqrt{d}\sqrt{-4a+4b+d}+2a-2b-d\right)\sqrt{-\frac{b-x}{a-b}}\sqrt[4]{-\left((a-x)(b-x)^2\right)}}+\frac{4(a-b)(b-x)\left(1-\frac{\sqrt{-4a+4b+d}}{\sqrt{d}}\right)F_1\left(-\frac{1}{4};-\frac{3}{2},1;\frac{3}{4};\frac{a-x}{a-b},\frac{2(a-x)}{2a-2b-d+\sqrt{d}\sqrt{-4a+4b+d}}\right)}{\left(\sqrt{d}\sqrt{-4a+4b+d}+2a-2b-d\right)\sqrt{-\frac{b-x}{a-b}}\sqrt[4]{-\left((a-x)(b-x)^2\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-((2*a - b)*b^2) + (4*a - b)*b*x - (2*a + b)*x^2 + x^3)/((-a + x)*((-a
 + x)*(-b + x)^2)^(1/4)*(b^2 + a*d - (2*b + d)*x + x^2)),x]
```

```
[Out] (4*(a - b)*(Sqrt[d] + Sqrt[-4*a + 4*b + d])*(b - x)*AppellF1[-1/4, -3/2, 1,
 3/4, (a - x)/(a - b), (2*(a - x))/(2*a - 2*b - d - Sqrt[d]*Sqrt[-4*a + 4*b
 + d])])/(Sqrt[d]*(2*a - 2*b - d - Sqrt[d]*Sqrt[-4*a + 4*b + d])*Sqrt[-((b
- x)/(a - b))]*(-((a - x)*(b - x)^2))^(1/4)) + (4*(a - b)*(1 - Sqrt[-4*a +
4*b + d]/Sqrt[d])*(b - x)*AppellF1[-1/4, -3/2, 1, 3/4, (a - x)/(a - b), (2*
(a - x))/(2*a - 2*b - d + Sqrt[d]*Sqrt[-4*a + 4*b + d])])/((2*a - 2*b - d +
 Sqrt[d]*Sqrt[-4*a + 4*b + d])*Sqrt[-((b - x)/(a - b))]*(-((a - x)*(b - x)^
2))^(1/4))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-\left((2a-b)b^2\right)+(4a-b)bx-(2a+b)x^2+x^3}{(-a+x)\sqrt[4]{(-a+x)(-b+x)^2}\left(b^2+ad-(2b+d)x+x^2\right)}\,dx = \int \frac{(b-x)^4(-2a+b+x)}{\left(-\left((a-x)(b-x)^2\right)\right)^{5/4}\left(b^2+ad-(2b+d)\right.}$$

$$= -\frac{\left(\sqrt[4]{a-x}\sqrt{b-x}\right)\int \frac{(b-x)^{3/2}(-2a+b+x)}{(a-x)^{5/4}\left(b^2+ad-(2b+d)x+x^2\right)}\,d}{\sqrt[4]{-\left((a-x)(b-x)^2\right)}}$$

$$= -\frac{\left(\sqrt[4]{a-x}\sqrt{b-x}\right)\int\left(\frac{\left(1+\frac{\sqrt{-4a+4b+d}}{\sqrt{d}}\right)(b-x)^{3/}}{(a-x)^{5/4}\left(-2b-d-\sqrt{d}\sqrt{-4a+4b}\right.}\right.}{\sqrt[4]{-\left((a-x\right.}}$$

$$= -\frac{\left(\left(1-\frac{\sqrt{-4a+4b+d}}{\sqrt{d}}\right)\sqrt[4]{a-x}\sqrt{b-x}\right)\int \frac{}{(a-x)^{5/4}(}}{\sqrt[4]{-\left((a-x)(b-x)^2\right.}}$$

$$= \frac{\left((a-b)\left(1-\frac{\sqrt{-4a+4b+d}}{\sqrt{d}}\right)\sqrt[4]{a-x}\,(b-x)\right)\int \frac{}{(a-}}{\sqrt{-\frac{b-x}{a-b}}\sqrt[4]{-\left((a-x)(b\right.}}$$

$$= \frac{4(a-b)\left(\sqrt{d}+\sqrt{-4a+4b+d}\right)(b-x)F_1\left(-\right.}{\sqrt{d}\left(2a-2b-d-\sqrt{d}\sqrt{-4a+4b+d}\right.}$$

Mathematica [C] time = 18.73, size = 736, normalized size = 4.46

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-((2*a - b)*b^2) + (4*a - b)*b*x - (2*a + b)*x^2 + x^3)/((-a + x
)*((-a + x)*(-b + x)^2)^(1/4)*(b^2 + a*d - (2*b + d)*x + x^2)),x]
```

```
[Out] (-4*(b - x))/((b - x)^2*(-a + x))^(1/4) - (I*Sqrt[2]*d*Sqrt[1 - a/(a - x) +
 b/(a - x)]*(a - x)^(11/4)*(b - x)^3*((4*a - 4*b - d + Sqrt[d*(-4*a + 4*b +
 d)])*Sqrt[-((-2*a + 2*b + d + Sqrt[d*(-4*a + 4*b + d)])/(a - b)^2)]*Ellipt
icPi[-(Sqrt[2]/(Sqrt[a - b]*Sqrt[(2*a - 2*b - d + Sqrt[d*(-4*a + 4*b + d)])
/(a - b)^2])), I*ArcSinh[Sqrt[-Sqrt[a - b]]/(a - x)^(1/4)], -1] - (4*a - 4*
b - d + Sqrt[d*(-4*a + 4*b + d)])*Sqrt[-((-2*a + 2*b + d + Sqrt[d*(-4*a + 4
*b + d)])/(a - b)^2)]*EllipticPi[Sqrt[2]/(Sqrt[a - b]*Sqrt[(2*a - 2*b - d +
 Sqrt[d*(-4*a + 4*b + d)])/(a - b)^2]), I*ArcSinh[Sqrt[-Sqrt[a - b]]/(a - x
)^(1/4)], -1] - (4*a - 4*b - d - Sqrt[d*(-4*a + 4*b + d)])*Sqrt[(2*a - 2*b
- d + Sqrt[d*(-4*a + 4*b + d)])/(a - b)^2]*(EllipticPi[-(Sqrt[2]/(Sqrt[a -
b]*Sqrt[-((-2*a + 2*b + d + Sqrt[d*(-4*a + 4*b + d)])/(a - b)^2)])), I*ArcS
inh[Sqrt[-Sqrt[a - b]]/(a - x)^(1/4)], -1] - EllipticPi[Sqrt[2]/(Sqrt[a - b
]*Sqrt[-((-2*a + 2*b + d + Sqrt[d*(-4*a + 4*b + d)])/(a - b)^2)]), I*ArcSin
h[Sqrt[-Sqrt[a - b]]/(a - x)^(1/4)], -1]))/(Sqrt[-Sqrt[a - b]]*(a - b)*Sqr
```

```
t[d*(-4*a + 4*b + d)]*Sqrt[(2*a - 2*b - d + Sqrt[d*(-4*a + 4*b + d)])/(a -
b)^2]*Sqrt[-((-2*a + 2*b + d + Sqrt[d*(-4*a + 4*b + d)])/(a - b)^2)]*(-((a
- x)*(b - x)^2))^(5/4)*(-a + x)*(-b + x))
```

IntegrateAlgebraic [A] time = 0.95, size = 165, normalized size = 1.00

$$-2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}}{b-x}\right)+2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}}{b-x}\right)-\frac{4\left(-ab^2+2abx-ax^2+b^2x-2bx^2+x^3\right)^{3/4}}{(x-a)(b-x)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-((2*a - b)*b^2) + (4*a - b)*b*x - (2*a + b)*x^2 + x^3)
/((-a + x)*((-a + x)*(-b + x)^2)^(1/4)*(b^2 + a*d - (2*b + d)*x + x^2)),x]
```

```
[Out] (-4*(-(a*b^2) + 2*a*b*x + b^2*x - a*x^2 - 2*b*x^2 + x^3)^(3/4))/((b - x)*(-
a + x)) - 2*d^(1/4)*ArcTan[(d^(1/4)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b
)*x^2 + x^3)^(1/4))/(b - x)] + 2*d^(1/4)*ArcTanh[(d^(1/4)*(-(a*b^2) + (2*a*
b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/4))/(b - x)]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2*a-b)*b^2+(4*a-b)*b*x-(2*a+b)*x^2+x^3)/(-a+x)/((-a+x)*(-b+x)^
2)^(1/4)/(b^2+a*d-(2*b+d)*x+x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2\,a-b)b^2-(4\,a-b)bx+(2\,a+b)x^2-x^3}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{4}}\left(b^2+ad-(2\,b+d)x+x^2\right)(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2*a-b)*b^2+(4*a-b)*b*x-(2*a+b)*x^2+x^3)/(-a+x)/((-a+x)*(-b+x)^
2)^(1/4)/(b^2+a*d-(2*b+d)*x+x^2),x, algorithm="giac")
```

```
[Out] integrate(((2*a - b)*b^2 - (4*a - b)*b*x + (2*a + b)*x^2 - x^3)/((-(a - x)*
(b - x)^2)^(1/4)*(b^2 + a*d - (2*b + d)*x + x^2)*(a - x)), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{-\left(2a-b\right)b^2+\left(4a-b\right)bx-\left(2a+b\right)x^2+x^3}{\left(-a+x\right)\left(\left(-a+x\right)\left(-b+x\right)^2\right)^{\frac{1}{4}}\left(b^2+ad-\left(2b+d\right)x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(2*a-b)*b^2+(4*a-b)*b*x-(2*a+b)*x^2+x^3)/(-a+x)/((-a+x)*(-b+x)^2)^(1/
4)/(b^2+a*d-(2*b+d)*x+x^2),x)
```

```
[Out] int((-(2*a-b)*b^2+(4*a-b)*b*x-(2*a+b)*x^2+x^3)/(-a+x)/((-a+x)*(-b+x)^2)^(1/
4)/(b^2+a*d-(2*b+d)*x+x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2\,a-b)b^2-(4\,a-b)bx+(2\,a+b)x^2-x^3}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{4}}\left(b^2+ad-(2\,b+d)x+x^2\right)(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2*a-b)*b^2+(4*a-b)*b*x-(2*a+b)*x^2+x^3)/(-a+x)/((-a+x)*(-b+x)^
2)^(1/4)/(b^2+a*d-(2*b+d)*x+x^2),x, algorithm="maxima")
```

```
[Out] integrate(((2*a - b)*b^2 - (4*a - b)*b*x + (2*a + b)*x^2 - x^3)/((-(a - x)*
(b - x)^2)^(1/4)*(b^2 + a*d - (2*b + d)*x + x^2)*(a - x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int -\frac{b^2\,\left(2\,a-b\right)+x^2\,\left(2\,a+b\right)-x^3-b\,x\,\left(4\,a-b\right)}{\left(a-x\right)\,{\left(-\left(a-x\right)\,{\left(b-x\right)}^2\right)}^{1/4}\,\left(a\,d-x\,\left(2\,b+d\right)+b^2+x^2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*(2*a - b) + x^2*(2*a + b) - x^3 - b*x*(4*a - b))/((a - x)*(-(a - x
)*(b - x)^2)^(1/4)*(a*d - x*(2*b + d) + b^2 + x^2)),x)
```

```
[Out] -int(-(b^2*(2*a - b) + x^2*(2*a + b) - x^3 - b*x*(4*a - b))/((a - x)*(-(a -
 x)*(b - x)^2)^(1/4)*(a*d - x*(2*b + d) + b^2 + x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(2*a-b)*b**2+(4*a-b)*b*x-(2*a+b)*x**2+x**3)/(-a+x)/((-a+x)*(-b+
x)**2)**(1/4)/(b**2+a*d-(2*b+d)*x+x**2),x)
```

```
[Out] Timed out
```

3.1831 $\int \frac{(-b+ax^4)(b+ax^4)^{3/4}}{x^8(b+2ax^4)} \, dx$

Optimal. Leaf size=165

$$-\frac{3a^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x\sqrt[4]{ax^4+b}}{\sqrt{ax^4+b}-\sqrt{a}\,x^2}\right)}{2\sqrt{2}\,b}-\frac{3a^{7/4}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^4+b}}{\sqrt{2}\sqrt[4]{a}}+\frac{\sqrt[4]{a}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^4+b}}\right)}{2\sqrt{2}\,b}+\frac{(b-6ax^4)(ax^4+b)^{3/4}}{7bx^7}$$

Rubi [A] time = 0.29, antiderivative size = 255, normalized size of antiderivative = 1.55, number of steps used = 13, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {580, 583, 12, 377, 211, 1165, 628, 1162, 617, 204}

$$\frac{3a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{2\sqrt{2}\,b}-\frac{3a^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}+1\right)}{2\sqrt{2}\,b}+\frac{3a^{7/4}\log\left(-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}+\frac{\sqrt{a}\,x^2}{\sqrt{ax^4+b}}+1\right)}{4\sqrt{2}\,b}-\frac{3a^{7/4}\log\left(\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}+\frac{\sqrt{a}\,x^2}{\sqrt{ax^4+b}}+1\right)}{4\sqrt{2}\,b}+\frac{(ax^4+b)^{3/4}}{7x^7}-\frac{6a(ax^4+b)^{3/4}}{7bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a*x^4)*(b + a*x^4)^(3/4))/(x^8*(b + 2*a*x^4)),x]
```

```
[Out] (b + a*x^4)^(3/4)/(7*x^7) - (6*a*(b + a*x^4)^(3/4))/(7*b*x^3) + (3*a^(7/4)*
ArcTan[1 - (Sqrt[2]*a^(1/4)*x)/(b + a*x^4)^(1/4)])/(2*Sqrt[2]*b) - (3*a^(7/
4)*ArcTan[1 + (Sqrt[2]*a^(1/4)*x)/(b + a*x^4)^(1/4)])/(2*Sqrt[2]*b) + (3*a^
(7/4)*Log[1 + (Sqrt[a]*x^2)/Sqrt[b + a*x^4] - (Sqrt[2]*a^(1/4)*x)/(b + a*x^
4)^(1/4)])/(4*Sqrt[2]*b) - (3*a^(7/4)*Log[1 + (Sqrt[a]*x^2)/Sqrt[b + a*x^4]
 + (Sqrt[2]*a^(1/4)*x)/(b + a*x^4)^(1/4)])/(4*Sqrt[2]*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
```

```
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(-b+ax^4)(b+ax^4)^{3/4}}{x^8(b+2ax^4)}\,dx = \frac{(b+ax^4)^{3/4}}{7x^7} + \frac{\int \frac{18ab^2+15a^2bx^4}{x^4\sqrt[4]{b+ax^4}(b+2ax^4)}\,dx}{7b}$$

$$= \frac{(b+ax^4)^{3/4}}{7x^7} - \frac{6a(b+ax^4)^{3/4}}{7bx^3} - \frac{\int \frac{63a^2b^3}{\sqrt[4]{b+ax^4}(b+2ax^4)}\,dx}{21b^3}$$

$$= \frac{(b+ax^4)^{3/4}}{7x^7} - \frac{6a(b+ax^4)^{3/4}}{7bx^3} - (3a^2)\int \frac{1}{\sqrt[4]{b+ax^4}\,(b+2ax^4)}\,dx$$

$$= \frac{(b+ax^4)^{3/4}}{7x^7} - \frac{6a(b+ax^4)^{3/4}}{7bx^3} - (3a^2)\,\mathrm{Subst}\left(\int \frac{1}{b+abx^4}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)$$

$$= \frac{(b+ax^4)^{3/4}}{7x^7} - \frac{6a(b+ax^4)^{3/4}}{7bx^3} - \frac{1}{2}(3a^2)\,\mathrm{Subst}\left(\int \frac{1-\sqrt{a}\,x^2}{b+abx^4}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right) -$$

$$= \frac{(b+ax^4)^{3/4}}{7x^7} - \frac{6a(b+ax^4)^{3/4}}{7bx^3} - \frac{(3a^{3/2})\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{a}}-\frac{\sqrt{2}x}{\sqrt[4]{a}}+x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{4b} - \frac{(3}{}$$

$$= \frac{(b+ax^4)^{3/4}}{7x^7} - \frac{6a(b+ax^4)^{3/4}}{7bx^3} + \frac{3a^{7/4}\log\left(1+\frac{\sqrt{a}\,x^2}{\sqrt{b+ax^4}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{4\sqrt{2}\,b} - \frac{3a^{7/4}\log\left(1\right.}{}$$

$$= \frac{(b+ax^4)^{3/4}}{7x^7} - \frac{6a(b+ax^4)^{3/4}}{7bx^3} + \frac{3a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{2\sqrt{2}\,b} - \frac{3a^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}}{\sqrt[4]{b+}}\right.}{2\sqrt{2}\,b}$$

Mathematica [A] time = 0.29, size = 202, normalized size = 1.22

$$\left(\frac{1}{7x^7}-\frac{6a}{7bx^3}\right)(ax^4+b)^{3/4} - \frac{3a^{7/4}\left(-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}+1\right)-\log\left(-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}+\frac{\sqrt{a}\,x^2}{\sqrt{a+bx^4}}+1\right)+\log\left(\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}+\frac{\sqrt{a}\,x^2}{\sqrt{a+bx^4}}+1\right)\right)}{4\sqrt{2}\,b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-b + a*x^4)*(b + a*x^4)^(3/4))/(x^8*(b + 2*a*x^4)),x]
```

```
[Out] (1/(7*x^7) - (6*a)/(7*b*x^3))*(b + a*x^4)^(3/4) - (3*a^(7/4)*(-2*ArcTan[1 -
 (Sqrt[2]*a^(1/4)*x)/(a + b*x^4)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*a^(1/4)*x)/
(a + b*x^4)^(1/4)] - Log[1 + (Sqrt[a]*x^2)/Sqrt[a + b*x^4] - (Sqrt[2]*a^(1/
4)*x)/(a + b*x^4)^(1/4)] + Log[1 + (Sqrt[a]*x^2)/Sqrt[a + b*x^4] + (Sqrt[2]
*a^(1/4)*x)/(a + b*x^4)^(1/4)]))/(4*Sqrt[2]*b)
```

IntegrateAlgebraic [A] time = 0.53, size = 165, normalized size = 1.00

$$-\frac{3a^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x\sqrt[4]{ax^4+b}}{\sqrt{ax^4+b}-\sqrt{a}\,x^2}\right)}{2\sqrt{2}\,b} - \frac{3a^{7/4}\tanh^{-1}\left(\frac{\frac{\sqrt{ax^4+b}}{\sqrt{2}\sqrt[4]{a}}+\frac{\sqrt[4]{a}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^4+b}}\right)}{2\sqrt{2}\,b} + \frac{(b-6ax^4)(ax^4+b)^{3/4}}{7bx^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^4)*(b + a*x^4)^(3/4))/(x^8*(b + 2*a*x^4)),x]
```

```
[Out] ((b - 6*a*x^4)*(b + a*x^4)^(3/4))/(7*b*x^7) - (3*a^(7/4)*ArcTan[(Sqrt[2]*a^
(1/4)*x*(b + a*x^4)^(1/4))/(-(Sqrt[a]*x^2) + Sqrt[b + a*x^4])])/(2*Sqrt[2]*
```

```
b) - (3*a^(7/4)*ArcTanh[((a^(1/4)*x^2)/Sqrt[2] + Sqrt[b + a*x^4]/(Sqrt[2]*a
^(1/4)))/(x*(b + a*x^4)^(1/4))])/(2*Sqrt[2]*b)
```

fricas [B] time = 67.71, size = 429, normalized size = 2.60

$$\frac{84\left(-\frac{a^7}{b^4}\right)^{\frac{1}{4}}bx^7\arctan\left(\frac{\left(\left(-\frac{a^7}{b^4}\right)^{\frac{3}{4}}a^5b^3x^2+\sqrt{\frac{a^{10}}{b^2}}\left(-\frac{a^7}{b^4}\right)^{\frac{3}{4}}b^4x^2\right)(ax^4+b)^{\frac{3}{4}}-(ax^4+b)^{\frac{1}{4}}\left((a^4b^2x^4+a^3b^3)\sqrt{\frac{a^{10}}{b^2}}\left(-\frac{a^7}{b^4}\right)^{\frac{1}{4}}-(a^9bx^4+a^8b^2)\left(-\frac{a^7}{b^4}\right)^{\frac{1}{4}}\right)}{2(a^{11}x^5+a^{10}bx)}\right)-21\left(-\frac{a^7}{b^4}\right)^{\frac{1}{4}}bx^7\log\left(-\frac{27\left(2(ax^4+b)^{\frac{1}{4}}\left(-\frac{a^7}{b^4}\right)^{\frac{1}{4}}a^4x^3+2\sqrt{ax^4+b}\sqrt{-\frac{a^7}{b^4}}a^2bx^2-a^5+2(ax^4+b)^{\frac{3}{4}}\left(-\frac{a^7}{b^4}\right)^{\frac{3}{4}}b^2x\right)}{2ax^4+b}\right)+21\left(-\frac{a^7}{b^4}\right)^{\frac{1}{4}}bx^7\log\left(\frac{27\left(2(ax^4+b)^{\frac{1}{4}}\left(-\frac{a^7}{b^4}\right)^{\frac{1}{4}}a^4x^3-2\sqrt{ax^4+b}\sqrt{-\frac{a^7}{b^4}}a^2bx^2+a^5+2(ax^4+b)^{\frac{3}{4}}\left(-\frac{a^7}{b^4}\right)^{\frac{3}{4}}b^2x\right)}{2ax^4+b}\right)-8(6ax^4-b)(ax^4+b)^{\frac{3}{4}}}{56bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)*(a*x^4+b)^(3/4)/x^8/(2*a*x^4+b),x, algorithm="fricas")
```

```
[Out] 1/56*(84*(-a^7/b^4)^(1/4)*b*x^7*arctan(1/2*(((-a^7/b^4)^(3/4)*a^5*b^3*x^2 +
 sqrt(a^10/b^2)*(-a^7/b^4)^(3/4)*b^4*x^2)*(a*x^4 + b)^(3/4) - (a*x^4 + b)^(
1/4)*((a^4*b^2*x^4 + a^3*b^3)*sqrt(a^10/b^2)*(-a^7/b^4)^(1/4) - (a^9*b*x^4
+ a^8*b^2)*(-a^7/b^4)^(1/4)))/(a^11*x^5 + a^10*b*x)) - 21*(-a^7/b^4)^(1/4)*
b*x^7*log(-27*(2*(a*x^4 + b)^(1/4)*(-a^7/b^4)^(1/4)*a^4*x^3 + 2*sqrt(a*x^4
+ b)*sqrt(-a^7/b^4)*a^2*b*x^2 - a^5 + 2*(a*x^4 + b)^(3/4)*(-a^7/b^4)^(3/4)*
b^2*x)/(2*a*x^4 + b)) + 21*(-a^7/b^4)^(1/4)*b*x^7*log(27*(2*(a*x^4 + b)^(1/
4)*(-a^7/b^4)^(1/4)*a^4*x^3 - 2*sqrt(a*x^4 + b)*sqrt(-a^7/b^4)*a^2*b*x^2 +
a^5 + 2*(a*x^4 + b)^(3/4)*(-a^7/b^4)^(3/4)*b^2*x)/(2*a*x^4 + b)) - 8*(6*a*x
^4 - b)*(a*x^4 + b)^(3/4))/(b*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^4+b)^{\frac{3}{4}}(ax^4-b)}{(2ax^4+b)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)*(a*x^4+b)^(3/4)/x^8/(2*a*x^4+b),x, algorithm="giac")
```

```
[Out] integrate((a*x^4 + b)^(3/4)*(a*x^4 - b)/((2*a*x^4 + b)*x^8), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(a\,x^4-b)(a\,x^4+b)^{\frac{3}{4}}}{x^8(2a\,x^4+b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b)*(a*x^4+b)^(3/4)/x^8/(2*a*x^4+b),x)
```

```
[Out] int((a*x^4-b)*(a*x^4+b)^(3/4)/x^8/(2*a*x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^4+b)^{\frac{3}{4}}(ax^4-b)}{(2ax^4+b)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)*(a*x^4+b)^(3/4)/x^8/(2*a*x^4+b),x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + b)^(3/4)*(a*x^4 - b)/((2*a*x^4 + b)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(a\,x^4+b)^{3/4}\,(b-a\,x^4)}{x^8\,(2\,a\,x^4+b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b + a*x^4)^(3/4)*(b - a*x^4))/(x^8*(b + 2*a*x^4)),x)
```

```
[Out] int(-((b + a*x^4)^(3/4)*(b - a*x^4))/(x^8*(b + 2*a*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4 - b\right)\left(ax^4 + b\right)^{\frac{3}{4}}}{x^8\left(2ax^4 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b)*(a*x**4+b)**(3/4)/x**8/(2*a*x**4+b),x)
```

```
[Out] Integral((a*x**4 - b)*(a*x**4 + b)**(3/4)/(x**8*(2*a*x**4 + b)), x)
```

3.1832 $\int \frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}{x}\,dx$

Optimal. Leaf size=166

$$-8\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}+2\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}}{\sqrt{\sqrt{2}-1}}\right)+4\tanh^{-1}\left(\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\right)+2\sqrt{1+\sqrt{2}}$$

Rubi [A] time = 0.98, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {6742, 2073, 207, 1166, 203}

$$-8\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}+2\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}{\sqrt{\sqrt{2}-1}}\right)+4\tanh^{-1}\left(\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)+2\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}{\sqrt{1+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]]/x,x]
```

```
[Out] -8*Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]] + 2*Sqrt[-1 + Sqrt[2]]*ArcTan[Sqrt[
1 - Sqrt[1 - Sqrt[1 - x^(-1)]]]/Sqrt[-1 + Sqrt[2]]] + 4*ArcTanh[Sqrt[1 - Sq
rt[1 - Sqrt[1 - x^(-1)]]]] + 2*Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[1 - Sqrt[1 -
Sqrt[1 - x^(-1)]]]/Sqrt[1 + Sqrt[2]]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}{x}\,dx = 2\,\text{Subst}\left(\int \frac{\sqrt{1-\sqrt{1-x}}\,x}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x}}\right)$$

$$= -\left(4\,\text{Subst}\left(\int \frac{\sqrt{1-x}\,(-1+x^2)}{x(-2+x^2)}\,dx, x, \sqrt{1-\sqrt{1-\frac{1}{x}}}\right)\right)$$

$$= -\left(8\,\text{Subst}\left(\int \frac{x^4(-2+x^2)}{1+x^2-3x^4+x^6}\,dx, x, \sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)\right)$$

$$= -\left(8\,\text{Subst}\left(\int \left(1-\frac{1+x^2-x^4}{1+x^2-3x^4+x^6}\right)dx, x, \sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)\right)$$

$$= -8\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}} + 8\,\text{Subst}\left(\int \frac{1+x^2-x^4}{1+x^2-3x^4+x^6}\,dx, x, \sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right.$$

$$= -8\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}} + 8\,\text{Subst}\left(\int \left(-\frac{1}{2(-1+x^2)} + \frac{-1-x^2}{2(-1-2x^2+x^4)}\right)dx, x,\right.$$

$$= -8\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}} - 4\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right) + 4\,\text{Su}$$

$$= -8\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}} + 4\tanh^{-1}\left(\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right) - \left(2\left(1-\sqrt{2}\right)\right)\text{Subst}$$

$$= -8\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}} + 2\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}{\sqrt{-1+\sqrt{2}}}\right) + 4\tanh^{-1}$$

Mathematica [A] time = 0.26, size = 166, normalized size = 1.00

$$-8\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}} + 2\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}}{\sqrt{\sqrt{2}-1}}\right) + 4\tanh^{-1}\left(\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\right) + 2\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}}{\sqrt{1+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]]/x,x]
```

```
[Out] -8*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]] + 2*Sqrt[-1 + Sqrt[2]]*ArcTan[Sqrt[
1 - Sqrt[1 - Sqrt[(-1 + x)/x]]]/Sqrt[-1 + Sqrt[2]]] + 4*ArcTanh[Sqrt[1 - Sq
rt[1 - Sqrt[(-1 + x)/x]]]] + 2*Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[1 - Sqrt[1 -
Sqrt[(-1 + x)/x]]]/Sqrt[1 + Sqrt[2]]]
```

IntegrateAlgebraic [A] time = 1.28, size = 166, normalized size = 1.00

$$-8\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}} + 2\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\right) + 4\tanh^{-1}\left(\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\right) + 2\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]]/x,x]
```

```
[Out] -8*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]] + 2*Sqrt[-1 + Sqrt[2]]*ArcTan[Sqrt[
1 + Sqrt[2]]*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]]] + 4*ArcTanh[Sqrt[1 - Sqr
t[1 - Sqrt[(-1 + x)/x]]]] + 2*Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[-1 + Sqrt[2]]*
Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]]]
```

fricas [A] time = 0.48, size = 243, normalized size = 1.46

$$4\sqrt{\sqrt{2}-1}\arctan\left(-\sqrt{\sqrt{2}-\sqrt{-\sqrt{\frac{x-1}{x}}+1}}\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}\right)+\sqrt{\sqrt{2}+1}\log\left(2\sqrt{\sqrt{2}+1}+2\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}\right)-\sqrt{\sqrt{2}+1}\log\left(-2\sqrt{\sqrt{2}+1}+2\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}\right)-8\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}+2\log\left(\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}+1\right)-2\log\left(\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(1-(1-1/x)^(1/2))^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 4*sqrt(sqrt(2) - 1)*arctan(-sqrt(sqrt(2) - sqrt(-sqrt((x - 1)/x) + 1))*(sqr
t(2) + 1)*sqrt(sqrt(2) - 1) + (sqrt(2) + 1)*sqrt(sqrt(2) - 1)*sqrt(-sqrt(-s
qrt((x - 1)/x) + 1) + 1)) + sqrt(sqrt(2) + 1)*log(2*sqrt(sqrt(2) + 1) + 2*s
qrt(-sqrt(-sqrt((x - 1)/x) + 1) + 1)) - sqrt(sqrt(2) + 1)*log(-2*sqrt(sqrt(
2) + 1) + 2*sqrt(-sqrt(-sqrt((x - 1)/x) + 1) + 1)) - 8*sqrt(-sqrt(-sqrt((x
- 1)/x) + 1) + 1) + 2*log(sqrt(-sqrt(-sqrt((x - 1)/x) + 1) + 1) + 1) - 2*lo
g(sqrt(-sqrt(-sqrt((x - 1)/x) + 1) + 1) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\sqrt{-\sqrt{-\frac{1}{x}+1}+1}+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(1-(1-1/x)^(1/2))^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-sqrt(-sqrt(-1/x + 1) + 1) + 1)/x, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(1-(1-1/x)^(1/2))^(1/2))^(1/2)/x,x)
```

```
[Out] int((1-(1-(1-1/x)^(1/2))^(1/2))^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\sqrt{-\sqrt{-\frac{1}{x}+1}+1}+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(1-(1-1/x)^(1/2))^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-sqrt(-sqrt(-1/x + 1) + 1) + 1)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - (1 - (1 - 1/x)^(1/2))^(1/2))^(1/2)/x,x)
[Out] int((1 - (1 - (1 - 1/x)^(1/2))^(1/2))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(1-(1-1/x)**(1/2))**(1/2))**(1/2)/x,x)
[Out] Integral(sqrt(1 - sqrt(1 - sqrt(1 - 1/x)))/x, x)
```

3.1833 $\int \frac{-2abx+(a+b)x^2}{(x(-a+x)(-b+x))^{2/3}\left(abd-(a+b)dx+(-1+d)x^2\right)} dx$

Optimal. Leaf size=166

$$-\frac{\log\left(d^{2/3}\left(x^2(-a-b)+abx+x^3\right)^{2/3}+\sqrt[3]{d}\,x\sqrt[3]{x^2(-a-b)+abx+x^3}+x^2\right)}{2\sqrt[3]{d}}+\frac{\log\left(x-\sqrt[3]{d}\sqrt[3]{x^2(-a-b)+abx+\cdots}\right.}{\sqrt[3]{d}}$$

Rubi [F] time = 6.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2abx+(a+b)x^2}{(x(-a+x)(-b+x))^{2/3}\left(abd-(a+b)dx+(-1+d)x^2\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*a*b*x + (a + b)*x^2)/((x*(-a + x)*(-b + x))^(2/3)*(a*b*d - (a + b)*
d*x + (-1 + d)*x^2)),x]
```

```
[Out] ((a + b + Sqrt[2*a*b*(2 - d) + a^2*d + b^2*d]/Sqrt[d])*x^(2/3)*(-a + x)^(2/
3)*(-b + x)^(2/3)*Defer[Int][x^(1/3)/((-a + x)^(2/3)*(-b + x)^(2/3)*(-((a +
 b)*d) - Sqrt[d]*Sqrt[4*a*b + a^2*d - 2*a*b*d + b^2*d] + 2*(-1 + d)*x)), x]
)/((a - x)*(b - x)*x)^(2/3) + ((a + b - Sqrt[2*a*b*(2 - d) + a^2*d + b^2*d]
/Sqrt[d])*x^(2/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Int][x^(1/3)/((-a + x
)^(2/3)*(-b + x)^(2/3)*(-((a + b)*d) + Sqrt[d]*Sqrt[4*a*b + a^2*d - 2*a*b*d
 + b^2*d] + 2*(-1 + d)*x)), x])/((a - x)*(b - x)*x)^(2/3)
```

Rubi steps

$$\int \frac{-2abx+(a+b)x^2}{(x(-a+x)(-b+x))^{2/3}\left(abd-(a+b)dx+(-1+d)x^2\right)} dx = \int \frac{x(-2ab+(a+b)x)}{(x(-a+x)(-b+x))^{2/3}\left(abd-(a+b)dx+(-\cdots\right.} dx$$

$$= \frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{\sqrt[3]{x}(-2a\cdots}{(-a+x)^{2/3}(-b+x)^{2/3}(a\cdots}}{(x(-a+x)(-b+x))^{2/3}}$$

$$= \frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \left(\frac{\cdots}{(-a+x)^{2/3}(-b+x)^{2/3}(\cdots}\right.}{\cdots}$$

$$= \frac{\left(\left(a+b-\frac{\sqrt{2ab(2-d)+a^2d+b^2d}}{\sqrt{d}}\right)x^{2/3}(-a+x)^{2/3}(-b\cdots\right.}{(x\cdots}$$

Mathematica [F] time = 10.17, size = 0, normalized size = 0.00

$$\int \frac{-2abx+(a+b)x^2}{(x(-a+x)(-b+x))^{2/3}\left(abd-(a+b)dx+(-1+d)x^2\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*a*b*x + (a + b)*x^2)/((x*(-a + x)*(-b + x))^(2/3)*(a*b*d - (a
 + b)*d*x + (-1 + d)*x^2)),x]
```

```
[Out] Integrate[(-2*a*b*x + (a + b)*x^2)/((x*(-a + x)*(-b + x))^(2/3)*(a*b*d - (a
 + b)*d*x + (-1 + d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.85, size = 166, normalized size = 1.00

$$-\frac{\log\left(d^{2/3}\left(x^2(-a-b)+abx+x^3\right)^{2/3}+\sqrt[3]{d}\,x\sqrt[3]{x^2(-a-b)+abx+x^3}+x^2\right)}{2\sqrt[3]{d}}+\frac{\log\left(x-\sqrt[3]{d}\,\sqrt[3]{x^2(-a-b)+abx+x^3}\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{d}\,\sqrt[3]{x^2(-a-b)+abx+x^3}+x}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*a*b*x + (a + b)*x^2)/((x*(-a + x)*(-b + x))^(2/3)*(a
*b*d - (a + b)*d*x + (-1 + d)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*d^(1/3)*(a*b*x + (-a - b)*x^2 + x^3)^(1/
3))])/d^(1/3) + Log[x - d^(1/3)*(a*b*x + (-a - b)*x^2 + x^3)^(1/3)]/d^(1/3)
 - Log[x^2 + d^(1/3)*x*(a*b*x + (-a - b)*x^2 + x^3)^(1/3) + d^(2/3)*(a*b*x
+ (-a - b)*x^2 + x^3)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b*x+(a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a*b*d-(a+b)*d*x+(-1
+d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,abx-(a+b)x^2}{\left(abd-(a+b)dx+(d-1)x^2\right)\left((a-x)(b-x)x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b*x+(a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a*b*d-(a+b)*d*x+(-1
+d)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-(2*a*b*x - (a + b)*x^2)/((a*b*d - (a + b)*d*x + (d - 1)*x^2)*((a
 - x)*(b - x)*x)^(2/3)), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{-2abx+(a+b)\,x^2}{\left(x\,(-a+x)\,(-b+x)\right)^{\frac{2}{3}}\left(abd-(a+b)\,dx+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a*b*x+(a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a*b*d-(a+b)*d*x+(-1+d)*x^
2),x)
```

```
[Out] int((-2*a*b*x+(a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a*b*d-(a+b)*d*x+(-1+d)*x^
2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,abx-(a+b)x^2}{\left(abd-(a+b)dx+(d-1)x^2\right)\left((a-x)(b-x)x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b*x+(a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a*b*d-(a+b)*d*x+(-1
+d)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((2*a*b*x - (a + b)*x^2)/((a*b*d - (a + b)*d*x + (d - 1)*x^2)*((a
 - x)*(b - x)*x)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\right)-2\,a\,b\,x}{{\left(x\,\left(a-x\right)\,\left(b-x\right)\right)}^{2/3}\,\left(\left(d-1\right)\,x^2-d\,\left(a+b\right)\,x+a\,b\,d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b) - 2*a*b*x)/((x*(a - x)*(b - x))^(2/3)*(x^2*(d - 1) - d*x*(
a + b) + a*b*d)),x)
```

```
[Out] int((x^2*(a + b) - 2*a*b*x)/((x*(a - x)*(b - x))^(2/3)*(x^2*(d - 1) - d*x*(
a + b) + a*b*d)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b*x+(a+b)*x**2)/(x*(-a+x)*(-b+x))**(2/3)/(a*b*d-(a+b)*d*x+(
-1+d)*x**2),x)
```

```
[Out] Timed out
```

3.1834 $\int \frac{\sqrt[4]{-b+ax^3}}{x^7}\,dx$

Optimal. Leaf size=166

$$-\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}-\sqrt{b}}\right)}{16\sqrt{2}\,b^{7/4}}+\frac{a^2\tanh^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{16\sqrt{2}\,b^{7/4}}+\frac{\left(ax^3-4b\right)\sqrt[4]{ax^3-b}}{24bx^6}$$

Rubi [A] time = 0.25, antiderivative size = 257, normalized size of antiderivative = 1.55, number of steps used = 13, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {266, 47, 51, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a^2\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{32\sqrt{2}\,b^{7/4}}+\frac{a^2\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{32\sqrt{2}\,b^{7/4}}-\frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}\right)}{16\sqrt{2}\,b^{7/4}}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}+1\right)}{16\sqrt{2}\,b^{7/4}}+\frac{a\sqrt[4]{ax^3-b}}{24bx^3}-\frac{\sqrt[4]{ax^3-b}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^3)^(1/4)/x^7,x]
```

```
[Out] -1/6*(-b + a*x^3)^(1/4)/x^6 + (a*(-b + a*x^3)^(1/4))/(24*b*x^3) - (a^2*ArcT
an[1 - (Sqrt[2]*(-b + a*x^3)^(1/4))/b^(1/4)])/(16*Sqrt[2]*b^(7/4)) + (a^2*A
rcTan[1 + (Sqrt[2]*(-b + a*x^3)^(1/4))/b^(1/4)])/(16*Sqrt[2]*b^(7/4)) - (a^
2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4) + Sqrt[-b + a*x^3]])/(32
*Sqrt[2]*b^(7/4)) + (a^2*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4) +
 Sqrt[-b + a*x^3]])/(32*Sqrt[2]*b^(7/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt[4]{-b+ax^3}}{x^7}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[4]{-b+ax}}{x^3}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt[4]{-b+ax^3}}{6x^6} + \frac{1}{24}a\,\text{Subst}\left(\int \frac{1}{x^2(-b+ax)^{3/4}}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt[4]{-b+ax^3}}{6x^6} + \frac{a\sqrt[4]{-b+ax^3}}{24bx^3} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x(-b+ax)^{3/4}}\,dx, x, x^3\right)}{32b}$$
$$= -\frac{\sqrt[4]{-b+ax^3}}{6x^6} + \frac{a\sqrt[4]{-b+ax^3}}{24bx^3} + \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{8b}$$
$$= -\frac{\sqrt[4]{-b+ax^3}}{6x^6} + \frac{a\sqrt[4]{-b+ax^3}}{24bx^3} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{16b^{3/2}} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{b}+x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx,\right.}{16b^{3/2}}$$
$$= -\frac{\sqrt[4]{-b+ax^3}}{6x^6} + \frac{a\sqrt[4]{-b+ax^3}}{24bx^3} - \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{32\sqrt{2}\,b^{7/4}} - \frac{a^2\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{\sqrt[4]{-b+ax^3}}{6x^6} + \frac{a\sqrt[4]{-b+ax^3}}{24bx^3} - \frac{a^2\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^3}+\sqrt{-b+ax^3}\right)}{32\sqrt{2}\,b^{7/4}} + \frac{a^2\log\left(\sqrt{b}\right.}{}$$
$$= -\frac{\sqrt[4]{-b+ax^3}}{6x^6} + \frac{a\sqrt[4]{-b+ax^3}}{24bx^3} - \frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)}{16\sqrt{2}\,b^{7/4}} + \frac{a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)}{16\sqrt{2}\,b^{7/4}} - \frac{a^2}{}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.25

$$\frac{4a^2\left(ax^3-b\right)^{5/4}\,{}_2F_1\left(\frac{5}{4},3;\frac{9}{4};1-\frac{ax^3}{b}\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^3)^(1/4)/x^7,x]
```

```
[Out] (4*a^2*(-b + a*x^3)^(5/4)*Hypergeometric2F1[5/4, 3, 9/4, 1 - (a*x^3)/b])/(1
5*b^3)
```

IntegrateAlgebraic [A] time = 0.51, size = 165, normalized size = 0.99

$$\frac{a^2\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{16\sqrt{2}\,b^{7/4}} + \frac{a^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}+\sqrt{b}}\right)}{16\sqrt{2}\,b^{7/4}} + \frac{\left(ax^3-4b\right)\sqrt[4]{ax^3-b}}{24bx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^3)^(1/4)/x^7,x]
```

```
[Out] ((-4*b + a*x^3)*(-b + a*x^3)^(1/4))/(24*b*x^6) + (a^2*ArcTan[(-(b^(1/4)/Sqr
t[2]) + Sqrt[-b + a*x^3]/(Sqrt[2]*b^(1/4)))/(-b + a*x^3)^(1/4)])/(16*Sqrt[2
]*b^(7/4)) + (a^2*ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4))/(Sqrt[b] + S
qrt[-b + a*x^3])])/(16*Sqrt[2]*b^(7/4))
```

fricas [A] time = 0.49, size = 224, normalized size = 1.35

$$\frac{12\left(-\frac{a^8}{b^7}\right)^{\frac{1}{4}} bx^6 \arctan\left(-\frac{(ax^3-b)^{\frac{1}{4}}a^2\left(-\frac{a^8}{b^7}\right)^{\frac{3}{4}}b^5-\sqrt{\sqrt{ax^3-b}\,a^4+\sqrt{-\frac{a^8}{b^7}}\,b^4}\left(-\frac{a^8}{b^7}\right)^{\frac{3}{4}}b^5}{a^8}\right)+3\left(-\frac{a^8}{b^7}\right)^{\frac{1}{4}}bx^6\log\left((ax^3-b)^{\frac{1}{4}}a^2+\left(-\frac{a^8}{b^7}\right)^{\frac{1}{4}}b^2\right)-3\left(-\frac{a^8}{b^7}\right)^{\frac{1}{4}}bx^6\log\left((ax^3-b)^{\frac{1}{4}}a^2-\left(-\frac{a^8}{b^7}\right)^{\frac{1}{4}}b^2\right)+4(ax^3-b)^{\frac{1}{4}}(ax^3-4b)}{96\,bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(1/4)/x^7,x, algorithm="fricas")
```

```
[Out] 1/96*(12*(-a^8/b^7)^(1/4)*b*x^6*arctan(-((a*x^3 - b)^(1/4)*a^2*(-a^8/b^7)^(
3/4)*b^5 - sqrt(sqrt(a*x^3 - b)*a^4 + sqrt(-a^8/b^7)*b^4)*(-a^8/b^7)^(3/4)*
b^5)/a^8) + 3*(-a^8/b^7)^(1/4)*b*x^6*log((a*x^3 - b)^(1/4)*a^2 + (-a^8/b^7)
^(1/4)*b^2) - 3*(-a^8/b^7)^(1/4)*b*x^6*log((a*x^3 - b)^(1/4)*a^2 - (-a^8/b^
7)^(1/4)*b^2) + 4*(a*x^3 - b)^(1/4)*(a*x^3 - 4*b))/(b*x^6)
```

giac [A] time = 0.35, size = 223, normalized size = 1.34

$$\frac{\frac{6\sqrt{2}a^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2(ax^3-b)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{7}{4}}}+\frac{6\sqrt{2}a^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2(ax^3-b)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{7}{4}}}+\frac{3\sqrt{2}a^3\log\left(\sqrt{2}(ax^3-b)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{b^{\frac{7}{4}}}-\frac{3\sqrt{2}a^3\log\left(-\sqrt{2}(ax^3-b)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{b^{\frac{7}{4}}}+\frac{8\left((ax^3-b)^{\frac{5}{4}}a^3-3(ax^3-b)^{\frac{1}{4}}a^3b\right)}{a^2bx^6}}{192\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(1/4)/x^7,x, algorithm="giac")
```

```
[Out] 1/192*(6*sqrt(2)*a^3*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1
/4))/b^(1/4))/b^(7/4) + 6*sqrt(2)*a^3*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)
- 2*(a*x^3 - b)^(1/4))/b^(1/4))/b^(7/4) + 3*sqrt(2)*a^3*log(sqrt(2)*(a*x^3
- b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(7/4) - 3*sqrt(2)*a^3*log
(-sqrt(2)*(a*x^3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(7/4) +
8*((a*x^3 - b)^(5/4)*a^3 - 3*(a*x^3 - b)^(1/4)*a^3*b)/(a^2*b*x^6))/a
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^3-b\right)^{\frac{1}{4}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-b)^(1/4)/x^7,x)
```

```
[Out] int((a*x^3-b)^(1/4)/x^7,x)
```

maxima [A] time = 0.45, size = 247, normalized size = 1.49

$$\frac{(ax^3-b)^{\frac{5}{4}}a^2-3(ax^3-b)^{\frac{1}{4}}a^2b}{24\left((ax^3-b)^2b+2(ax^3-b)b^2+b^3\right)}+\frac{\frac{2\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2(ax^3-b)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{2\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2(ax^3-b)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{\sqrt{2}a^2\log\left(\sqrt{2}(ax^3-b)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}a^2\log\left(-\sqrt{2}(ax^3-b)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^3-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-b)^(1/4)/x^7,x, algorithm="maxima")
```

```
[Out] 1/24*((a*x^3 - b)^(5/4)*a^2 - 3*(a*x^3 - b)^(1/4)*a^2*b)/((a*x^3 - b)^2*b +
 2*(a*x^3 - b)*b^2 + b^3) + 1/64*(2*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)
*b^(1/4) + 2*(a*x^3 - b)^(1/4))/b^(1/4))/b^(3/4) + 2*sqrt(2)*a^2*arctan(-1/
2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^3 - b)^(1/4))/b^(1/4))/b^(3/4) + sqrt(2
)*a^2*log(sqrt(2)*(a*x^3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^
(3/4) - sqrt(2)*a^2*log(-sqrt(2)*(a*x^3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b
) + sqrt(b))/b^(3/4))/b
```

mupad [B] time = 1.39, size = 95, normalized size = 0.57

$$\frac{\left(a\,x^3-b\right)^{5/4}}{24\,b\,x^6}-\frac{\left(a\,x^3-b\right)^{1/4}}{8\,x^6}+\frac{a^2\,\mathrm{atan}\left(\frac{\left(a\,x^3-b\right)^{1/4}}{(-b)^{1/4}}\right)}{16\,(-b)^{7/4}}-\frac{a^2\,\mathrm{atan}\left(\frac{\left(a\,x^3-b\right)^{1/4}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,1\mathrm{i}}{16\,(-b)^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 - b)^(1/4)/x^7,x)
```

```
[Out] (a*x^3 - b)^(5/4)/(24*b*x^6) - (a*x^3 - b)^(1/4)/(8*x^6) + (a^2*atan((a*x^3
 - b)^(1/4)/(-b)^(1/4)))/(16*(-b)^(7/4)) - (a^2*atan(((a*x^3 - b)^(1/4)*1i)
/(-b)^(1/4))*1i)/(16*(-b)^(7/4))
```

sympy [C] time = 1.36, size = 44, normalized size = 0.27

$$-\frac{\sqrt[4]{a}\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^3}\right)}{3x^{\frac{21}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-b)**(1/4)/x**7,x)
```

```
[Out] -a**(1/4)*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), b*exp_polar(2*I*pi)/(a*x**
3))/(3*x**(21/4)*gamma(11/4))
```

3.1835 $\int \frac{(b+2ax^2)\sqrt[4]{bx^2+ax^4}}{-b+ax^2}\,dx$

Optimal. Leaf size=166

$$-\frac{7b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{2a^{3/4}}+\frac{3\sqrt[4]{2}\,b\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{a^{3/4}}+\frac{7b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{2a^{3/4}}-\frac{3\sqrt[4]{2}\,b\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{a^{3/4}}+x\sqrt[4]{ax^4+b}$$

Rubi [A] time = 0.42, antiderivative size = 290, normalized size of antiderivative = 1.75, number of steps used = 14, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2056, 581, 584, 329, 331, 298, 203, 206, 466, 494}

$$-\frac{7b\sqrt[4]{ax^4+bx^2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{2a^{3/4}\sqrt{x}\sqrt[4]{ax^2+b}}+\frac{3\sqrt[4]{2}\,b\sqrt[4]{ax^4+bx^2}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{a^{3/4}\sqrt{x}\sqrt[4]{ax^2+b}}+\frac{7b\sqrt[4]{ax^4+bx^2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{2a^{3/4}\sqrt{x}\sqrt[4]{ax^2+b}}-\frac{3\sqrt[4]{2}\,b\sqrt[4]{ax^4+bx^2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+b}}\right)}{a^{3/4}\sqrt{x}\sqrt[4]{ax^2+b}}+x\sqrt[4]{ax^4+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((b + 2*a*x^2)*(b*x^2 + a*x^4)^(1/4))/(-b + a*x^2),x]
```

```
[Out] x*(b*x^2 + a*x^4)^(1/4) - (7*b*(b*x^2 + a*x^4)^(1/4)*ArcTan[(a^(1/4)*Sqrt[x
])/(b + a*x^2)^(1/4)])/(2*a^(3/4)*Sqrt[x]*(b + a*x^2)^(1/4)) + (3*2^(1/4)*b
*(b*x^2 + a*x^4)^(1/4)*ArcTan[(2^(1/4)*a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])
/(a^(3/4)*Sqrt[x]*(b + a*x^2)^(1/4)) + (7*b*(b*x^2 + a*x^4)^(1/4)*ArcTanh[(
a^(1/4)*Sqrt[x])/(b + a*x^2)^(1/4)])/(2*a^(3/4)*Sqrt[x]*(b + a*x^2)^(1/4))
- (3*2^(1/4)*b*(b*x^2 + a*x^4)^(1/4)*ArcTanh[(2^(1/4)*a^(1/4)*Sqrt[x])/(b +
 a*x^2)^(1/4)])/(a^(3/4)*Sqrt[x]*(b + a*x^2)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{(b+2ax^2)\sqrt[4]{bx^2+ax^4}}{-b+ax^2}\,dx = \frac{\sqrt[4]{bx^2+ax^4}\int \frac{\sqrt{x}\sqrt[4]{b+ax^2}(b+2ax^2)}{-b+ax^2}\,dx}{\sqrt{x}\sqrt[4]{b+ax^2}}$$

$$= x\sqrt[4]{bx^2+ax^4} + \frac{\sqrt[4]{bx^2+ax^4}\int \frac{\sqrt{x}(5ab^2+7a^2bx^2)}{(-b+ax^2)(b+ax^2)^{3/4}}\,dx}{2a\sqrt{x}\sqrt[4]{b+ax^2}}$$

$$= x\sqrt[4]{bx^2+ax^4} + \frac{\sqrt[4]{bx^2+ax^4}\int \left(\frac{7ab\sqrt{x}}{(b+ax^2)^{3/4}} + \frac{12ab^2\sqrt{x}}{(-b+ax^2)(b+ax^2)^{3/4}}\right)dx}{2a\sqrt{x}\sqrt[4]{b+ax^2}}$$

$$= x\sqrt[4]{bx^2+ax^4} + \frac{\left(7b\sqrt[4]{bx^2+ax^4}\right)\int \frac{\sqrt{x}}{(b+ax^2)^{3/4}}\,dx}{2\sqrt{x}\sqrt[4]{b+ax^2}} + \frac{\left(6b^2\sqrt[4]{bx^2+ax^4}\right)\int \frac{}{(-b+ax^2)}}{\sqrt{x}\sqrt[4]{b+ax^2}}$$

$$= x\sqrt[4]{bx^2+ax^4} + \frac{\left(7b\sqrt[4]{bx^2+ax^4}\right)\text{Subst}\left(\int \frac{x^2}{(b+ax^4)^{3/4}}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt[4]{b+ax^2}} + \frac{\left(12b^2\sqrt[4]{bx^2}\right.}{}$$

$$= x\sqrt[4]{bx^2+ax^4} + \frac{\left(7b\sqrt[4]{bx^2+ax^4}\right)\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{\sqrt{x}\sqrt[4]{b+ax^2}} + \frac{\left(12b^2\sqrt[4]{bx^2}\right.}{}$$

$$= x\sqrt[4]{bx^2+ax^4} + \frac{\left(7b\sqrt[4]{bx^2+ax^4}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{2\sqrt{a}\sqrt{x}\sqrt[4]{b+ax^2}} - \frac{\left(7b\sqrt[4]{bx^2}\right.}{}$$

$$= x\sqrt[4]{bx^2+ax^4} - \frac{7b\sqrt[4]{bx^2+ax^4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b+ax^2}}\right)}{2a^{3/4}\sqrt{x}\sqrt[4]{b+ax^2}} + \frac{3\sqrt[4]{2}\,b\sqrt[4]{bx^2+ax^4}\tan^{-1}\left(\frac{\sqrt[4]{2}}{\sqrt[4]{}}\right.}{a^{3/4}\sqrt{x}\sqrt[4]{b+ax^2}}$$

Mathematica [C] time = 0.29, size = 156, normalized size = 0.94

$$\frac{x^3\left(-3ax^2\left(1-\frac{a^2x^4}{b^2}\right)^{3/4}F_1\left(\frac{7}{4};\frac{3}{4},1;\frac{11}{4};-\frac{ax^2}{b},\frac{ax^2}{b}\right)-5b\left(\frac{ax^2}{b}+1\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{2ax^2}{b-ax^2}\right)+3\left(ax^2+b\right)\left(1-\frac{ax^2}{b}\right)^{3/4}\right)}{3\left(x^2\left(ax^2+b\right)\right)^{3/4}\left(1-\frac{ax^2}{b}\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((b + 2*a*x^2)*(b*x^2 + a*x^4)^(1/4))/(-b + a*x^2),x]
```

```
[Out] (x^3*(3*(b + a*x^2)*(1 - (a*x^2)/b)^(3/4) - 3*a*x^2*(1 - (a^2*x^4)/b^2)^(3/
4)*AppellF1[7/4, 3/4, 1, 11/4, -((a*x^2)/b), (a*x^2)/b] - 5*b*(1 + (a*x^2)/
b)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (-2*a*x^2)/(b - a*x^2)]))/(3*(x^2
*(b + a*x^2))^(3/4)*(1 - (a*x^2)/b)^(3/4))
```

IntegrateAlgebraic [A] time = 0.59, size = 166, normalized size = 1.00

$$-\frac{7b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{2a^{3/4}}+\frac{3\sqrt[4]{2}\,b\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{a^{3/4}}+\frac{7b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{2a^{3/4}}-\frac{3\sqrt[4]{2}\,b\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^2}}\right)}{a^{3/4}}+x\sqrt[4]{ax^4+bx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b + 2*a*x^2)*(b*x^2 + a*x^4)^(1/4))/(-b + a*x^2),x]
```

```
[Out] x*(b*x^2 + a*x^4)^(1/4) - (7*b*ArcTan[(a^(1/4)*x)/(b*x^2 + a*x^4)^(1/4)])/(
2*a^(3/4)) + (3*2^(1/4)*b*ArcTan[(2^(1/4)*a^(1/4)*x)/(b*x^2 + a*x^4)^(1/4)]
```

```
)/a^(3/4) + (7*b*ArcTanh[(a^(1/4)*x)/(b*x^2 + a*x^4)^(1/4)])/(2*a^(3/4)) -
(3*2^(1/4)*b*ArcTanh[(2^(1/4)*a^(1/4)*x)/(b*x^2 + a*x^4)^(1/4)])/a^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2+b)*(a*x^4+b*x^2)^(1/4)/(a*x^2-b),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.28, size = 394, normalized size = 2.37

$$\left(a + \frac{b}{x^2}\right)^{\frac{1}{4}} x^2 - \frac{3 \cdot 2^{\frac{3}{4}} (-a)^{\frac{1}{4}} b \arctan\left(\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}} + 2\left(a + \frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{2a} - \frac{3 \cdot 2^{\frac{3}{4}} (-a)^{\frac{1}{4}} b \arctan\left(-\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}} - 2\left(a + \frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{2a} + \frac{3 \cdot 2^{\frac{3}{4}} b \log\left(2^{\frac{3}{4}} (-a)^{\frac{1}{4}} \left(a + \frac{b}{x^2}\right)^{\frac{1}{4}} + \sqrt{2}\sqrt{-a} + \sqrt{a + \frac{b}{x^2}}\right)}{4(-a)^{\frac{3}{4}}} + \frac{3 \cdot 2^{\frac{3}{4}} (-a)^{\frac{1}{4}} b \log\left(-2^{\frac{3}{4}} (-a)^{\frac{1}{4}} \left(a + \frac{b}{x^2}\right)^{\frac{1}{4}} + \sqrt{2}\sqrt{-a} + \sqrt{a + \frac{b}{x^2}}\right)}{4a} + \frac{7\sqrt{2} (-a)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}} + 2\left(a + \frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{4a} + \frac{7\sqrt{2} (-a)^{\frac{1}{4}} b \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}} - 2\left(a + \frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{4a} + \frac{7\sqrt{2} (-a)^{\frac{1}{4}} b \log\left(\sqrt{2} (-a)^{\frac{1}{4}} \left(a + \frac{b}{x^2}\right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a + \frac{b}{x^2}}\right)}{8a} + \frac{7\sqrt{2} b \log\left(-\sqrt{2} (-a)^{\frac{1}{4}} \left(a + \frac{b}{x^2}\right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a + \frac{b}{x^2}}\right)}{8(-a)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2+b)*(a*x^4+b*x^2)^(1/4)/(a*x^2-b),x, algorithm="giac")
```

```
[Out] (a + b/x^2)^(1/4)*x^2 - 3/2*2^(3/4)*(-a)^(1/4)*b*arctan(1/2*2^(1/4)*(2^(3/4
)*(-a)^(1/4) + 2*(a + b/x^2)^(1/4))/(-a)^(1/4))/a - 3/2*2^(3/4)*(-a)^(1/4)*
b*arctan(-1/2*2^(1/4)*(2^(3/4)*(-a)^(1/4) - 2*(a + b/x^2)^(1/4))/(-a)^(1/4)
)/a + 3/4*2^(3/4)*b*log(2^(3/4)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(2)*sqrt
(-a) + sqrt(a + b/x^2))/(-a)^(3/4) + 3/4*2^(3/4)*(-a)^(1/4)*b*log(-2^(3/4)*
(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(2)*sqrt(-a) + sqrt(a + b/x^2))/a + 7/4*
sqrt(2)*(-a)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^2)
^(1/4))/(-a)^(1/4))/a + 7/4*sqrt(2)*(-a)^(1/4)*b*arctan(-1/2*sqrt(2)*(sqrt(
2)*(-a)^(1/4) - 2*(a + b/x^2)^(1/4))/(-a)^(1/4))/a + 7/8*sqrt(2)*(-a)^(1/4)
*b*log(sqrt(2)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sqrt(a + b/x^2))/a
 + 7/8*sqrt(2)*b*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^2)^(1/4) + sqrt(-a) + sqr
t(a + b/x^2))/(-a)^(3/4)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\left(2 a \,x^{2}+b \right) \left(a \,x^{4}+b \,x^{2}\right)^{\frac{1}{4}}}{a \,x^{2}-b}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x^2+b)*(a*x^4+b*x^2)^(1/4)/(a*x^2-b),x)
```

```
[Out] int((2*a*x^2+b)*(a*x^4+b*x^2)^(1/4)/(a*x^2-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(a x^{4} + b x^{2}\right)}^{\frac{1}{4}} {\left(2 \, a x^{2} + b\right)}}{a x^{2} - b}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x^2+b)*(a*x^4+b*x^2)^(1/4)/(a*x^2-b),x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + b*x^2)^(1/4)*(2*a*x^2 + b)/(a*x^2 - b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(2\,a\,x^2+b\right)\,{\left(a\,x^4+b\,x^2\right)}^{1/4}}{b-a\,x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b + 2*a*x^2)*(a*x^4 + b*x^2)^(1/4))/(b - a*x^2),x)
[Out] int(-((b + 2*a*x^2)*(a*x^4 + b*x^2)^(1/4))/(b - a*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2 \left(ax^2+b\right)} \left(2ax^2+b\right)}{ax^2-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x**2+b)*(a*x**4+b*x**2)**(1/4)/(a*x**2-b),x)
[Out] Integral((x**2*(a*x**2 + b))**(1/4)*(2*a*x**2 + b)/(a*x**2 - b), x)
```

3.1836 $\int \frac{-b^4+a^4x^4}{\sqrt{-b^2x+a^2x^3}\left(b^4+a^4x^4\right)}\,dx$

Optimal. Leaf size=166

$$\frac{\tan^{-1}\left(\frac{2^{3/4}\sqrt{a}\,\sqrt{b}\,\sqrt{a^2x^3-b^2x}}{-a^2x^2+\sqrt{2}\,abx+b^2}\right)}{2^{3/4}\sqrt{a}\,\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\frac{a^{3/2}x^2}{2^{3/4}\sqrt{b}}-\frac{b^{3/2}}{2^{3/4}\sqrt{a}}+\frac{\sqrt{a}\,\sqrt{b}\,x}{\sqrt[4]{2}}}{\sqrt{a^2x^3-b^2x}}\right)}{2^{3/4}\sqrt{a}\,\sqrt{b}}$$

Rubi [C] time = 1.71, antiderivative size = 518, normalized size of antiderivative = 3.12, number of steps used = 21, number of rules used = 10, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2056, 1586, 6715, 6725, 406, 224, 221, 409, 1219, 1218}

$$-\frac{\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,\Pi\left(\frac{\sqrt[4]{-a^4}}{a};\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{a^2x^3-b^2x}}-\frac{\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,\Pi\left(-\frac{\sqrt{-\sqrt{-a^4}}}{a};\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{a^2x^3-b^2x}}-\frac{\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,\Pi\left(\frac{\sqrt{-\sqrt{-a^4}}}{a};\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{a^2x^3-b^2x}}+\frac{\left(a^2-\sqrt{-a^4}\right)\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{a^{5/2}\sqrt{a^2x^3-b^2x}}+\frac{\left(\sqrt{-a^4}+a^2\right)\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{a^{5/2}\sqrt{a^2x^3-b^2x}}-\frac{\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,\Pi\left(\frac{a^3}{\left(-a^4\right)^{3/4}};\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{a^2x^3-b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b^4 + a^4*x^4)/(Sqrt[-(b^2*x) + a^2*x^3]*(b^4 + a^4*x^4)),x]
```

```
[Out] ((a^2 - Sqrt[-a^4])*Sqrt[b]*Sqrt[x]*Sqrt[1 - (a^2*x^2)/b^2]*EllipticF[ArcSi
n[(Sqrt[a]*Sqrt[x])/Sqrt[b]], -1])/(a^(5/2)*Sqrt[-(b^2*x) + a^2*x^3]) + ((a
^2 + Sqrt[-a^4])*Sqrt[b]*Sqrt[x]*Sqrt[1 - (a^2*x^2)/b^2]*EllipticF[ArcSin[(
Sqrt[a]*Sqrt[x])/Sqrt[b]], -1])/(a^(5/2)*Sqrt[-(b^2*x) + a^2*x^3]) - (Sqrt[
b]*Sqrt[x]*Sqrt[1 - (a^2*x^2)/b^2]*EllipticPi[a^3/(-a^4)^(3/4), ArcSin[(Sqr
t[a]*Sqrt[x])/Sqrt[b]], -1])/(Sqrt[a]*Sqrt[-(b^2*x) + a^2*x^3]) - (Sqrt[b]*
Sqrt[x]*Sqrt[1 - (a^2*x^2)/b^2]*EllipticPi[(-a^4)^(1/4)/a, ArcSin[(Sqrt[a]*
Sqrt[x])/Sqrt[b]], -1])/(Sqrt[a]*Sqrt[-(b^2*x) + a^2*x^3]) - (Sqrt[b]*Sqrt[
x]*Sqrt[1 - (a^2*x^2)/b^2]*EllipticPi[-(Sqrt[-Sqrt[-a^4]]/a), ArcSin[(Sqrt[
a]*Sqrt[x])/Sqrt[b]], -1])/(Sqrt[a]*Sqrt[-(b^2*x) + a^2*x^3]) - (Sqrt[b]*Sq
rt[x]*Sqrt[1 - (a^2*x^2)/b^2]*EllipticPi[Sqrt[-Sqrt[-a^4]]/a, ArcSin[(Sqrt[
a]*Sqrt[x])/Sqrt[b]], -1])/(Sqrt[a]*Sqrt[-(b^2*x) + a^2*x^3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-b^4+a^4x^4}{\sqrt{-b^2x+a^2x^3}\left(b^4+a^4x^4\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\int \frac{-b^4+a^4x^4}{\sqrt{x}\sqrt{-b^2+a^2x^2}\left(b^4+a^4x^4\right)}\,dx}{\sqrt{-b^2x+a^2x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\int \frac{\sqrt{-b^2+a^2x^2}\left(b^2+a^2x^2\right)}{\sqrt{x}\left(b^4+a^4x^4\right)}\,dx}{\sqrt{-b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\mathrm{Subst}\left(\int \frac{\sqrt{-b^2+a^2x^4}\left(b^2+a^2x^4\right)}{b^4+a^4x^8}\,dx, x, \sqrt{x}\right)}{\sqrt{-b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\mathrm{Subst}\left(\int\left(-\frac{\sqrt{-a^4}\left(a^2b^2+\sqrt{-a^4}\,b^2\right)\sqrt{-b^2+a^2x^4}}{2a^4b^2\left(b^2-\sqrt{-a^4}\,x^4\right)}+\frac{\sqrt{-a^4}\left(a^2b^2-\sqrt{-a^4}\right.}{2a^4b^2\left(b^2+\right.}\right.\right.}{\sqrt{-b^2x+a^2x^3}}$$

$$= \frac{\left(\left(a^2+\sqrt{-a^4}\right)\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\mathrm{Subst}\left(\int \frac{\sqrt{-b^2+a^2x^4}}{b^2+\sqrt{-a^4}\,x^4}\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-b^2x+a^2x^3}} - \frac{\left(\sqrt{-a^4}\left(a^2\right.\right.}{}$$

$$= \frac{\left(\left(a^2+\sqrt{-a^4}\right)\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-b^2+a^2x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^4}\sqrt{-b^2x+a^2x^3}} - \frac{\left(\left(a^2+\sqrt{-}\right.\right.}{}$$

$$= -\frac{\left(\left(a^2+\sqrt{-a^4}\right)\left(a^2b^2+\sqrt{-a^4}\,b^2\right)\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{-\sqrt{-a^4}}\,x^2}{b}\right)\sqrt{-}}\right.}{2a^2\sqrt{-a^4}\,b^2\sqrt{-b^2x+a^2x^3}}$$

$$= \frac{\left(a^2+\sqrt{-a^4}\right)\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{a^{5/2}\sqrt{-b^2x+a^2x^3}} + \frac{\left(a^2+\sqrt{-a^4}\right)\sqrt{b}\sqrt{x}}{\sqrt{a}\sqrt{}}$$

$$= \frac{\left(a^2+\sqrt{-a^4}\right)\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{a^{5/2}\sqrt{-b^2x+a^2x^3}} + \frac{\left(a^2+\sqrt{-a^4}\right)\sqrt{b}\sqrt{x}}{\sqrt{a}\sqrt{}}$$

Mathematica [C] time = 1.48, size = 204, normalized size = 1.23

$$-\frac{ix^{3/2}\sqrt{1-\frac{b^2}{a^2x^2}}\left(2F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{b}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(-\sqrt[4]{-1};i\sinh^{-1}\left(\frac{\sqrt{-\frac{b}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(\sqrt[4]{-1};i\sinh^{-1}\left(\frac{\sqrt{-\frac{b}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(-(-1)^{3/4};i\sinh^{-1}\left(\frac{\sqrt{-\frac{b}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left((-1)^{3/4};i\sinh^{-1}\left(\frac{\sqrt{-\frac{b}{a}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{\sqrt{-\frac{b}{a}}\sqrt{a^2x^3-b^2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b^4 + a^4*x^4)/(Sqrt[-(b^2*x) + a^2*x^3]*(b^4 + a^4*x^4)),x]
```

```
[Out] ((-I)*Sqrt[1 - b^2/(a^2*x^2)]*x^(3/2)*(2*EllipticF[I*ArcSinh[Sqrt[-(b/a)]/S
qrt[x]], -1] - EllipticPi[-(-1)^(1/4), I*ArcSinh[Sqrt[-(b/a)]/Sqrt[x]], -1]
 - EllipticPi[(-1)^(1/4), I*ArcSinh[Sqrt[-(b/a)]/Sqrt[x]], -1] - EllipticPi
[-(-1)^(3/4), I*ArcSinh[Sqrt[-(b/a)]/Sqrt[x]], -1] - EllipticPi[(-1)^(3/4),
 I*ArcSinh[Sqrt[-(b/a)]/Sqrt[x]], -1]))/(Sqrt[-(b/a)]*Sqrt[-(b^2*x) + a^2*x
^3])
```

IntegrateAlgebraic [A] time = 0.57, size = 166, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2^{3/4}\sqrt{a}\sqrt{b}\sqrt{a^2x^3-b^2x}}{-a^2x^2+\sqrt{2}abx+b^2}\right)}{2^{3/4}\sqrt{a}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\frac{a^{3/2}x^2}{2^{3/4}\sqrt{b}}-\frac{b^{3/2}}{2^{3/4}\sqrt{a}}+\frac{\sqrt{a}\sqrt{b}x}{\sqrt[4]{2}}}{\sqrt{a^2x^3-b^2x}}\right)}{2^{3/4}\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b^4 + a^4*x^4)/(Sqrt[-(b^2*x) + a^2*x^3]*(b^4 + a^4*x^
4)),x]
```

```
[Out] ArcTan[(2^(3/4)*Sqrt[a]*Sqrt[b]*Sqrt[-(b^2*x) + a^2*x^3])/(b^2 + Sqrt[2]*a*
b*x - a^2*x^2)]/(2^(3/4)*Sqrt[a]*Sqrt[b]) - ArcTanh[(-(b^(3/2)/(2^(3/4)*Sqr
t[a])) + (Sqrt[a]*Sqrt[b]*x)/2^(1/4) + (a^(3/2)*x^2)/(2^(3/4)*Sqrt[b]))/Sqr
t[-(b^2*x) + a^2*x^3]]/(2^(3/4)*Sqrt[a]*Sqrt[b])
```

fricas [B] time = 0.67, size = 979, normalized size = 5.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^4)/(a^2*x^3-b^2*x)^(1/2)/(a^4*x^4+b^4),x, algorithm="f
ricas")
```

```
[Out] -1/2*sqrt(2)*(1/2)^(1/4)*(1/(a^2*b^2))^(1/4)*arctan(1/2*((2*sqrt(2)*(1/2)^(
3/4)*a^2*b^2*x*(1/(a^2*b^2))^(3/4) - sqrt(2)*(1/2)^(1/4)*(a^2*x^2 - b^2)*(1
/(a^2*b^2))^(1/4))*sqrt(a^2*x^3 - b^2*x) - (2*a^2*x^3 - 2*b^2*x - (2*sqrt(2
)*(1/2)^(3/4)*a^2*b^2*x*(1/(a^2*b^2))^(3/4) + sqrt(2)*(1/2)^(1/4)*(a^2*x^2
- b^2)*(1/(a^2*b^2))^(1/4))*sqrt(a^2*x^3 - b^2*x))*sqrt((a^4*x^4 + b^4 + 8*
sqrt(1/2)*(a^4*b^2*x^3 - a^2*b^4*x)*sqrt(1/(a^2*b^2)) + 4*(sqrt(2)*(1/2)^(1
/4)*a^2*b^2*x*(1/(a^2*b^2))^(1/4) + sqrt(2)*(1/2)^(3/4)*(a^4*b^2*x^2 - a^2*
b^4)*(1/(a^2*b^2))^(3/4))*sqrt(a^2*x^3 - b^2*x))/(a^4*x^4 + b^4)))/(a^2*x^3
 - b^2*x)) - 1/2*sqrt(2)*(1/2)^(1/4)*(1/(a^2*b^2))^(1/4)*arctan(1/2*((2*sqr
t(2)*(1/2)^(3/4)*a^2*b^2*x*(1/(a^2*b^2))^(3/4) - sqrt(2)*(1/2)^(1/4)*(a^2*x
^2 - b^2)*(1/(a^2*b^2))^(1/4))*sqrt(a^2*x^3 - b^2*x) + (2*a^2*x^3 - 2*b^2*x
 + (2*sqrt(2)*(1/2)^(3/4)*a^2*b^2*x*(1/(a^2*b^2))^(3/4) + sqrt(2)*(1/2)^(1/
4)*(a^2*x^2 - b^2)*(1/(a^2*b^2))^(1/4))*sqrt(a^2*x^3 - b^2*x))*sqrt((a^4*x^
4 + b^4 + 8*sqrt(1/2)*(a^4*b^2*x^3 - a^2*b^4*x)*sqrt(1/(a^2*b^2)) - 4*(sqrt
(2)*(1/2)^(1/4)*a^2*b^2*x*(1/(a^2*b^2))^(1/4) + sqrt(2)*(1/2)^(3/4)*(a^4*b^
2*x^2 - a^2*b^4)*(1/(a^2*b^2))^(3/4))*sqrt(a^2*x^3 - b^2*x))/(a^4*x^4 + b^4
)))/(a^2*x^3 - b^2*x)) - 1/8*sqrt(2)*(1/2)^(1/4)*(1/(a^2*b^2))^(1/4)*log((a
^4*x^4 + b^4 + 8*sqrt(1/2)*(a^4*b^2*x^3 - a^2*b^4*x)*sqrt(1/(a^2*b^2)) + 4*
(sqrt(2)*(1/2)^(1/4)*a^2*b^2*x*(1/(a^2*b^2))^(1/4) + sqrt(2)*(1/2)^(3/4)*(a
^4*b^2*x^2 - a^2*b^4)*(1/(a^2*b^2))^(3/4))*sqrt(a^2*x^3 - b^2*x))/(a^4*x^4
+ b^4)) + 1/8*sqrt(2)*(1/2)^(1/4)*(1/(a^2*b^2))^(1/4)*log((a^4*x^4 + b^4 +
8*sqrt(1/2)*(a^4*b^2*x^3 - a^2*b^4*x)*sqrt(1/(a^2*b^2)) - 4*(sqrt(2)*(1/2)^
(1/4)*a^2*b^2*x*(1/(a^2*b^2))^(1/4) + sqrt(2)*(1/2)^(3/4)*(a^4*b^2*x^2 - a^
2*b^4)*(1/(a^2*b^2))^(3/4))*sqrt(a^2*x^3 - b^2*x))/(a^4*x^4 + b^4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a^4x^4-b^4}{\left(a^4x^4+b^4\right)\sqrt{a^2x^3-b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^4)/(a^2*x^3-b^2*x)^(1/2)/(a^4*x^4+b^4),x, algorithm="g
iac")
```

```
[Out] integrate((a^4*x^4 - b^4)/((a^4*x^4 + b^4)*sqrt(a^2*x^3 - b^2*x)), x)
```

maple [C] time = 0.07, size = 247, normalized size = 1.49

$$\frac{b\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{2\left(x-\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{ax}{b}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a^2x^3-b^2x}}-\frac{b\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(a^4\_Z^4+b^4\right)}\frac{\left(a^3\_\alpha^3-\_\alpha^2a^2b+\_\alpha a\,b^2-b^3\right)\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{\left(x-\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{ax}{b}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}},-\frac{a^3\_\alpha^3-\_\alpha^2a^2b+\_\alpha a\,b^2-b^3}{2b^3},\frac{\sqrt{2}}{2}\right)}{\_\alpha^3\sqrt{x\left(a^2x^2-b^2\right)}}\right)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^4*x^4-b^4)/(a^2*x^3-b^2*x)^(1/2)/(a^4*x^4+b^4),x)
```

```
[Out] b/a*((x+b/a)/b*a)^(1/2)*(-2*(x-b/a)/b*a)^(1/2)*(-a*x/b)^(1/2)/(a^2*x^3-b^2*
x)^(1/2)*EllipticF(((x+b/a)/b*a)^(1/2),1/2*2^(1/2))-1/4*b/a^4*2^(1/2)*sum(1
/_alpha^3*(_alpha^3*a^3-_alpha^2*a^2*b+_alpha*a*b^2-b^3)*((x+b/a)/b*a)^(1/2
)*(-(x-b/a)/b*a)^(1/2)*(-a*x/b)^(1/2)/(x*(a^2*x^2-b^2))^(1/2)*EllipticPi(((
x+b/a)/b*a)^(1/2),-1/2*(_alpha^3*a^3-_alpha^2*a^2*b+_alpha*a*b^2-b^3)/b^3,1
/2*2^(1/2)),_alpha=RootOf(_Z^4*a^4+b^4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^4x^4-b^4}{\left(a^4x^4+b^4\right)\sqrt{a^2x^3-b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^4)/(a^2*x^3-b^2*x)^(1/2)/(a^4*x^4+b^4),x, algorithm="m
axima")
```

```
[Out] integrate((a^4*x^4 - b^4)/((a^4*x^4 + b^4)*sqrt(a^2*x^3 - b^2*x)), x)
```

mupad [B] time = 7.93, size = 202, normalized size = 1.22

$$\frac{2^{1/4}\sqrt{-\frac{1}{8}\mathrm{i}}\,\ln\left(\frac{(-1)^{1/4}2^{3/4}b^2-(-1)^{1/4}2^{3/4}a^2x^2-2\,(-1)^{3/4}2^{1/4}a\,b\,x+\sqrt{a}\,\sqrt{b}\,\sqrt{a^2x^3-b^2x}\,4\mathrm{i}}{-a^2x^2+1\mathrm{i}\,\sqrt{2}\,a\,b\,x+b^2}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{2^{1/4}\sqrt{\frac{1}{8}\mathrm{i}}\,\ln\left(\frac{(-1)^{3/4}2^{3/4}b^2-(-1)^{3/4}2^{3/4}a^2x^2-2\,(-1)^{1/4}2^{1/4}a\,b\,x+\sqrt{a}\,\sqrt{b}\,\sqrt{a^2x^3-b^2x}\,4\mathrm{i}}{a^2x^2+1\mathrm{i}\,\sqrt{2}\,a\,b\,x-b^2}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^4 - a^4*x^4)/((b^4 + a^4*x^4)*(a^2*x^3 - b^2*x)^(1/2)),x)
```

```
[Out] (2^(1/4)*(-1i/8)^(1/2)*log((a^(1/2)*b^(1/2)*(a^2*x^3 - b^2*x)^(1/2)*4i + (-
1)^(1/4)*2^(3/4)*b^2 - (-1)^(1/4)*2^(3/4)*a^2*x^2 - 2*(-1)^(3/4)*2^(1/4)*a*
b*x)/(b^2 - a^2*x^2 + 2^(1/2)*a*b*x*1i)))/(a^(1/2)*b^(1/2)) + (2^(1/4)*(1i/
8)^(1/2)*log((a^(1/2)*b^(1/2)*(a^2*x^3 - b^2*x)^(1/2)*4i + (-1)^(3/4)*2^(3/
4)*b^2 - (-1)^(3/4)*2^(3/4)*a^2*x^2 - 2*(-1)^(1/4)*2^(1/4)*a*b*x)/(a^2*x^2
- b^2 + 2^(1/2)*a*b*x*1i)))/(a^(1/2)*b^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-b)\,(ax+b)\left(a^2x^2+b^2\right)}{\sqrt{x\,(ax-b)\,(ax+b)}\,\left(a^4x^4+b^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**4*x**4-b**4)/(a**2*x**3-b**2*x)**(1/2)/(a**4*x**4+b**4),x)
```

```
[Out] Integral((a*x - b)*(a*x + b)*(a**2*x**2 + b**2)/(sqrt(x*(a*x - b)*(a*x + b)
)*(a**4*x**4 + b**4)), x)
```

3.1837 $\int \frac{-b^6+a^6x^6}{\sqrt{b^4+a^4x^4}\left(b^6+a^6x^6\right)}\,dx$

Optimal. Leaf size=166

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{a^4x^4+b^4}+a^2x^2+b^2}\right)}{3ab}+\frac{2\tanh^{-1}\left(\frac{\sqrt{4-2\sqrt{3}}\,abx}{\sqrt{a^4x^4+b^4}+a^2x^2+b^2}\right)}{3ab}-\frac{2\tanh^{-1}\left(\frac{\sqrt{4+2\sqrt{3}}\,abx}{\sqrt{a^4x^4+b^4}+a^2x^2+b^2}\right)}{3ab}$$

Rubi [C] time = 2.20, antiderivative size = 405, normalized size of antiderivative = 2.44, number of steps used = 17, number of rules used = 9, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6725, 220, 2073, 1211, 1699, 205, 6728, 1217, 1707}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{a^4x^4+b^4}}\right)}{3\sqrt{2}\,ab}-\frac{2\tan^{-1}\left(\frac{\sqrt{-a^2}\,bx}{\sqrt{a^4x^4+b^4}}\right)}{3\sqrt{-a^2}\,b}+\frac{\left(a^2x^2+b^2\right)\sqrt{\frac{a^4x^4+b^4}{\left(a^2x^2+b^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{3ab\sqrt{a^4x^4+b^4}}-\frac{\left(a^2-\sqrt{3}\sqrt{-a^4}\right)\left(a^2x^2+b^2\right)\sqrt{\frac{a^4x^4+b^4}{\left(a^2x^2+b^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{3a\left(3a^2-\sqrt{3}\sqrt{-a^4}\right)b\sqrt{a^4x^4+b^4}}-\frac{\left(\sqrt{3}\sqrt{-a^4}+a^2\right)\left(a^2x^2+b^2\right)\sqrt{\frac{a^4x^4+b^4}{\left(a^2x^2+b^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{3a\left(\sqrt{3}\sqrt{-a^4}+3a^2\right)b\sqrt{a^4x^4+b^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b^6 + a^6*x^6)/(Sqrt[b^4 + a^4*x^4]*(b^6 + a^6*x^6)),x]
```

```
[Out] -1/3*ArcTan[(Sqrt[2]*a*b*x)/Sqrt[b^4 + a^4*x^4]]/(Sqrt[2]*a*b) - (2*ArcTan[
(Sqrt[-a^2]*b*x)/Sqrt[b^4 + a^4*x^4]])/(3*Sqrt[-a^2]*b) + ((b^2 + a^2*x^2)*
Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2])/
(3*a*b*Sqrt[b^4 + a^4*x^4]) - ((a^2 - Sqrt[3]*Sqrt[-a^4])*(b^2 + a^2*x^2)*S
qrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*EllipticF[2*ArcTan[(a*x)/b], 1/2])/(
3*a*(3*a^2 - Sqrt[3]*Sqrt[-a^4])*b*Sqrt[b^4 + a^4*x^4]) - ((a^2 + Sqrt[3]*S
qrt[-a^4])*(b^2 + a^2*x^2)*Sqrt[(b^4 + a^4*x^4)/(b^2 + a^2*x^2)^2]*Elliptic
F[2*ArcTan[(a*x)/b], 1/2])/(3*a*(3*a^2 + Sqrt[3]*Sqrt[-a^4])*b*Sqrt[b^4 + a
^4*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
```

```
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-b^6+a^6x^6}{\sqrt{b^4+a^4x^4}\left(b^6+a^6x^6\right)}\,dx = \int\left(\frac{1}{\sqrt{b^4+a^4x^4}}-\frac{2b^6}{\sqrt{b^4+a^4x^4}\left(b^6+a^6x^6\right)}\right)dx$$

$$= -\left(\left(2b^6\right)\int \frac{1}{\sqrt{b^4+a^4x^4}\left(b^6+a^6x^6\right)}\,dx\right)+\int \frac{1}{\sqrt{b^4+a^4x^4}}\,dx$$

$$= \frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{2ab\sqrt{b^4+a^4x^4}}-\left(2b^6\right)\int\left(\frac{1}{3b^4\left(b^2+a^2x^2\right)\sqrt{b^4+\cdots}}\right.$$

$$= \frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{2ab\sqrt{b^4+a^4x^4}}-\frac{1}{3}\left(2b^2\right)\int\frac{1}{\left(b^2+a^2x^2\right)\sqrt{b^4+a^4\cdots}}$$

$$= \frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{2ab\sqrt{b^4+a^4x^4}}-\frac{1}{3}\int\frac{1}{\sqrt{b^4+a^4x^4}}\,dx-\frac{1}{3}\int\frac{\cdots}{\left(b^2\cdots\right.}$$

$$= \frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{3ab\sqrt{b^4+a^4x^4}}-\frac{1}{3}b^2\,\text{Subst}\left(\int\frac{1}{b^2+2a^2b^4x^2}\,dx,\cdots\right.$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}+\frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{3ab\sqrt{b^4+a^4x^4}}-\frac{\left(2\left(a^2-\cdots\right.\right.}{3\left(\cdots\right.}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{2}\,ab}-\frac{2\tan^{-1}\left(\frac{\sqrt{-a^2}\,bx}{\sqrt{b^4+a^4x^4}}\right)}{3\sqrt{-a^2}\,b}+\frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}\,F\left(2\tan^{-1}\cdots\right.}{3ab\sqrt{b^4+a^4x^4}}$$

Mathematica [C] time = 0.71, size = 195, normalized size = 1.17

$$-\frac{(-1)^{5/6}\sqrt{\frac{a^4x^4}{b^4}+1}\left(\left(3-3i\sqrt{3}\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}x\right)\middle|-1\right)+4\left((-1)^{2/3}\Pi\left(-i;i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}x\right)\middle|-1\right)+\left(\sqrt[3]{-1}-1\right)\Pi\left(\sqrt[6]{-1};i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}x\right)\middle|-1\right)+(-1)^{2/3}\Pi\left((-1)^{5/6};i\sinh^{-1}\left(\sqrt{\frac{ia^2}{b^2}}x\right)\middle|-1\right)\right)\right)}{6\sqrt{\frac{ia^2}{b^2}}\sqrt{a^4x^4+b^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b^6 + a^6*x^6)/(Sqrt[b^4 + a^4*x^4]*(b^6 + a^6*x^6)),x]
```

```
[Out] -1/6*((-1)^(5/6)*Sqrt[1 + (a^4*x^4)/b^4]*((3 - (3*I)*Sqrt[3])*EllipticF[I*A
rcSinh[Sqrt[(I*a^2)/b^2]*x], -1] + 4*((-1)^(2/3)*EllipticPi[-I, I*ArcSinh[S
qrt[(I*a^2)/b^2]*x], -1] + (-1 + (-1)^(1/3))*EllipticPi[(-1)^(1/6), I*ArcSi
nh[Sqrt[(I*a^2)/b^2]*x], -1] + (-1)^(2/3)*EllipticPi[(-1)^(5/6), I*ArcSinh[
Sqrt[(I*a^2)/b^2]*x], -1])))/(Sqrt[(I*a^2)/b^2]*Sqrt[b^4 + a^4*x^4])
```

IntegrateAlgebraic [A] time = 1.20, size = 84, normalized size = 0.51

$$-\frac{2\tanh^{-1}\left(\frac{abx}{\sqrt{a^4x^4+b^4}}\right)}{3ab}-\frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{a^4x^4+b^4}+a^2x^2+b^2}\right)}{3ab}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-b^6 + a^6*x^6)/(Sqrt[b^4 + a^4*x^4]*(b^6 + a^6*x^6)),x
]

[Out] -1/3*(Sqrt[2]*ArcTan[(Sqrt[2]*a*b*x)/(b^2 + a^2*x^2 + Sqrt[b^4 + a^4*x^4])]
)/(a*b) - (2*ArcTanh[(a*b*x)/Sqrt[b^4 + a^4*x^4]])/(3*a*b)

fricas [A] time = 0.60, size = 101, normalized size = 0.61

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\,abx}{\sqrt{a^4x^4+b^4}}\right)-2\,\log\left(\frac{a^4x^4+a^2b^2x^2+b^4-2\,\sqrt{a^4x^4+b^4}\,abx}{a^4x^4-a^2b^2x^2+b^4}\right)}{6\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^6*x^6-b^6)/(a^4*x^4+b^4)^(1/2)/(a^6*x^6+b^6),x, algorithm="fri
cas")

[Out] -1/6*(sqrt(2)*arctan(sqrt(2)*a*b*x/sqrt(a^4*x^4 + b^4)) - 2*log((a^4*x^4 +
a^2*b^2*x^2 + b^4 - 2*sqrt(a^4*x^4 + b^4)*a*b*x)/(a^4*x^4 - a^2*b^2*x^2 + b
^4)))/(a*b)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^6x^6-b^6}{\left(a^6x^6+b^6\right)\sqrt{a^4x^4+b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^6*x^6-b^6)/(a^4*x^4+b^4)^(1/2)/(a^6*x^6+b^6),x, algorithm="gia
c")

[Out] integrate((a^6*x^6 - b^6)/((a^6*x^6 + b^6)*sqrt(a^4*x^4 + b^4)), x)

maple [C] time = 0.09, size = 441, normalized size = 2.66

$$\frac{\sqrt{1-\frac{ia^2x^2}{b^2}}\,\sqrt{1+\frac{ia^2x^2}{b^2}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{ia^2}{b^2}},i\right)}{\sqrt{\frac{ia^2}{b^2}}\,\sqrt{a^4x^4+b^4}}-\frac{b^2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(a^4\_Z^4-a^2b^2\_Z^2+b^4\right)}\frac{\left(-\_\alpha^2a^2+2b^2\right)\left(-\frac{\operatorname{arctanh}\left(\frac{\_\alpha^2\left(-\_\alpha^2a^2+a^2x^2+b^2\right)a^2}{\sqrt{a^2b^2\_\alpha^2}\,\sqrt{a^4x^4+b^4}}\right)}{\sqrt{a^2b^2\_\alpha^2}}+\frac{2a^2\_\alpha\left(\_\alpha^2a^2-b^2\right)\sqrt{1-\frac{ia^2x^2}{b^2}}\,\sqrt{1+\frac{ia^2x^2}{b^2}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{ia^2}{b^2}},\frac{i\left(\_\alpha^2a^2-b^2\right)}{b^2},\frac{\sqrt{-\frac{ia^2}{b^2}}}{\sqrt{\frac{ia^2}{b^2}}}\right)}{\sqrt{\frac{ia^2}{b^2}}\,b^4\sqrt{a^4x^4+b^4}}\right)}{\_\alpha\left(2\_\alpha^2a^2-b^2\right)}\right)}{6a^2}-\frac{2\sqrt{1-\frac{ia^2x^2}{b^2}}\,\sqrt{1+\frac{ia^2x^2}{b^2}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{ia^2}{b^2}},i,\frac{\sqrt{-\frac{ia^2}{b^2}}}{\sqrt{\frac{ia^2}{b^2}}}\right)}{3\sqrt{\frac{ia^2}{b^2}}\,\sqrt{a^4x^4+b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^6*x^6-b^6)/(a^4*x^4+b^4)^(1/2)/(a^6*x^6+b^6),x)

[Out] 1/(I*a^2/b^2)^(1/2)*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/b^2*x^2)^(1/2)/(a^4*x^
4+b^4)^(1/2)*EllipticF(x*(I*a^2/b^2)^(1/2),I)-1/6*b^2/a^2*sum((-_alpha^2*a^
2+2*b^2)/_alpha/(2*_alpha^2*a^2-b^2)*(-1/(a^2*b^2*_alpha^2)^(1/2)*arctanh(_
alpha^2*(-_alpha^2*a^2+a^2*x^2+b^2)*a^2/(a^2*b^2*_alpha^2)^(1/2)/(a^4*x^4+b
^4)^(1/2))+2/(I*a^2/b^2)^(1/2)*a^2*_alpha*(_alpha^2*a^2-b^2)/b^4*(1-I*a^2/b
^2*x^2)^(1/2)*(1+I*a^2/b^2*x^2)^(1/2)/(a^4*x^4+b^4)^(1/2)*EllipticPi(x*(I*a
^2/b^2)^(1/2),I*(_alpha^2*a^2-b^2)/b^2,(-I*a^2/b^2)^(1/2)/(I*a^2/b^2)^(1/2)
)),_alpha=RootOf(_Z^4*a^4-_Z^2*a^2*b^2+b^4))-2/3/(I*a^2/b^2)^(1/2)*(1-I*a^2
/b^2*x^2)^(1/2)*(1+I*a^2/b^2*x^2)^(1/2)/(a^4*x^4+b^4)^(1/2)*EllipticPi(x*(I
*a^2/b^2)^(1/2),I,(-I*a^2/b^2)^(1/2)/(I*a^2/b^2)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^6x^6-b^6}{\left(a^6x^6+b^6\right)\sqrt{a^4x^4+b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^6*x^6-b^6)/(a^4*x^4+b^4)^(1/2)/(a^6*x^6+b^6),x, algorithm="max
ima")
```

```
[Out] integrate((a^6*x^6 - b^6)/((a^6*x^6 + b^6)*sqrt(a^4*x^4 + b^4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{b^6 - a^6\,x^6}{\sqrt{a^4\,x^4 + b^4}\,\left(a^6\,x^6 + b^6\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^6 - a^6*x^6)/((b^4 + a^4*x^4)^(1/2)*(b^6 + a^6*x^6)),x)
```

```
[Out] int(-(b^6 - a^6*x^6)/((b^4 + a^4*x^4)^(1/2)*(b^6 + a^6*x^6)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax - b)\,(ax + b)\left(a^2x^2 - abx + b^2\right)\left(a^2x^2 + abx + b^2\right)}{\left(a^2x^2 + b^2\right)\sqrt{a^4x^4 + b^4}\,\left(a^4x^4 - a^2b^2x^2 + b^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**6*x**6-b**6)/(a**4*x**4+b**4)**(1/2)/(a**6*x**6+b**6),x)
```

```
[Out] Integral((a*x - b)*(a*x + b)*(a**2*x**2 - a*b*x + b**2)*(a**2*x**2 + a*b*x
+ b**2)/((a**2*x**2 + b**2)*sqrt(a**4*x**4 + b**4)*(a**4*x**4 - a**2*b**2*x
**2 + b**4)), x)
```

3.1838 $\int \frac{-2ab+(a+b)x}{\sqrt[3]{x(-a+x)(-b+x)}\left(abd-(a+b)dx+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=167

$$\frac{\log\left(x-\sqrt[3]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}\right)}{d^{2/3}}-\frac{\log\left(d^{2/3}\left(x^2(-a-b)+abx+x^3\right)^{2/3}+\sqrt[3]{d}\,x\sqrt[3]{x^2(-a-b)+abx+x^3}+x^2\right)}{2d^{2/3}}$$

Rubi [F] time = 5.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2ab+(a+b)x}{\sqrt[3]{x(-a+x)(-b+x)}\left(abd-(a+b)dx+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*a*b + (a + b)*x)/((x*(-a + x)*(-b + x))^(1/3)*(a*b*d - (a + b)*d*x
+ (-1 + d)*x^2)),x]
```

```
[Out] ((a + b + Sqrt[2*a*b*(2 - d) + a^2*d + b^2*d]/Sqrt[d])*x^(1/3)*(-a + x)^(1/
3)*(-b + x)^(1/3)*Defer[Int][1/(x^(1/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*(-((a
 + b)*d) - Sqrt[d]*Sqrt[4*a*b + a^2*d - 2*a*b*d + b^2*d] + 2*(-1 + d)*x)),
x])/((a - x)*(b - x)*x)^(1/3) + ((a + b - Sqrt[2*a*b*(2 - d) + a^2*d + b^2*
d]/Sqrt[d])*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Int][1/(x^(1/3)*(-a
 + x)^(1/3)*(-b + x)^(1/3)*(-((a + b)*d) + Sqrt[d]*Sqrt[4*a*b + a^2*d - 2*a
*b*d + b^2*d] + 2*(-1 + d)*x)), x])/((a - x)*(b - x)*x)^(1/3)
```

Rubi steps

$$\int \frac{-2ab+(a+b)x}{\sqrt[3]{x(-a+x)(-b+x)}\left(abd-(a+b)dx+(-1+d)x^2\right)}\,dx=\frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\int \frac{-2ab+(a+b)x}{\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\left(abd-(a+b)dx\right.}}{\sqrt[3]{x(-a+x)(-b+x)}}$$

$$=\frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\int\left(\frac{a+b+\frac{\sqrt{4a}}{}}{\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\left(-((a+b)d)-\right.}\right.}{}$$

$$=\frac{\left(\left(a+b-\frac{\sqrt{2ab(2-d)+a^2d+b^2d}}{\sqrt{d}}\right)\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)}{\sqrt[3]{x(-a}}$$

Mathematica [F] time = 9.89, size = 0, normalized size = 0.00

$$\int \frac{-2ab+(a+b)x}{\sqrt[3]{x(-a+x)(-b+x)}\left(abd-(a+b)dx+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*a*b + (a + b)*x)/((x*(-a + x)*(-b + x))^(1/3)*(a*b*d - (a + b
)*d*x + (-1 + d)*x^2)),x]
```

```
[Out] Integrate[(-2*a*b + (a + b)*x)/((x*(-a + x)*(-b + x))^(1/3)*(a*b*d - (a + b
)*d*x + (-1 + d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.47, size = 167, normalized size = 1.00

$$\frac{\log\left(x-\sqrt[3]{d}\,\sqrt[3]{x^2(-a-b)+abx+x^3}\right)}{d^{2/3}}-\frac{\log\left(d^{2/3}\left(x^2(-a-b)+abx+x^3\right)^{2/3}+\sqrt[3]{d}\,x\sqrt[3]{x^2(-a-b)+abx+x^3}+x^2\right)}{2d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{d}\,\sqrt[3]{x^2(-a-b)+abx+x^3}+x}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*a*b + (a + b)*x)/((x*(-a + x)*(-b + x))^(1/3)*(a*b*d
 - (a + b)*d*x + (-1 + d)*x^2)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*d^(1/3)*(a*b*x + (-a - b)*x^2 + x^3)^(
1/3))])/d^(2/3)) + Log[x - d^(1/3)*(a*b*x + (-a - b)*x^2 + x^3)^(1/3)]/d^(2
/3) - Log[x^2 + d^(1/3)*x*(a*b*x + (-a - b)*x^2 + x^3)^(1/3) + d^(2/3)*(a*b
*x + (-a - b)*x^2 + x^3)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(a*b*d-(a+b)*d*x+(-1+d)*
x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,ab-(a+b)x}{\left(abd-(a+b)dx+(d-1)x^2\right)\left((a-x)(b-x)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(a*b*d-(a+b)*d*x+(-1+d)*
x^2),x, algorithm="giac")
```

```
[Out] integrate(-(2*a*b - (a + b)*x)/((a*b*d - (a + b)*d*x + (d - 1)*x^2)*((a - x
)*(b - x)*x)^(1/3)), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{-2ab+\left(a+b\right)x}{\left(x\left(-a+x\right)\left(-b+x\right)\right)^{\frac{1}{3}}\left(abd-\left(a+b\right)dx+\left(-1+d\right)x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(a*b*d-(a+b)*d*x+(-1+d)*x^2),x
)
```

```
[Out] int((-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(a*b*d-(a+b)*d*x+(-1+d)*x^2),x
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,ab-(a+b)x}{\left(abd-(a+b)dx+(d-1)x^2\right)\left((a-x)(b-x)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(a*b*d-(a+b)*d*x+(-1+d)*
x^2),x, algorithm="maxima")
```

```
[Out] -integrate((2*a*b - (a + b)*x)/((a*b*d - (a + b)*d*x + (d - 1)*x^2)*((a - x
)*(b - x)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{2\,a\,b-x\,\left(a+b\right)}{{\left(x\,\left(a-x\right)\,\left(b-x\right)\right)}^{1/3}\,\left(\left(d-1\right)\,x^2-d\,\left(a+b\right)\,x+a\,b\,d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*a*b - x*(a + b))/((x*(a - x)*(b - x))^(1/3)*(x^2*(d - 1) - d*x*(a +
 b) + a*b*d)),x)
```

```
[Out] int(-(2*a*b - x*(a + b))/((x*(a - x)*(b - x))^(1/3)*(x^2*(d - 1) - d*x*(a +
 b) + a*b*d)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))**(1/3)/(a*b*d-(a+b)*d*x+(-1+d)
*x**2),x)
```

```
[Out] Timed out
```

3.1839 $\int \frac{1}{x^7\left(-b+ax^3\right)^{3/4}}\,dx$

Optimal. Leaf size=167

$$-\frac{7a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}-\sqrt{b}}\right)}{16\sqrt{2}\,b^{11/4}}+\frac{7a^2\tanh^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{16\sqrt{2}\,b^{11/4}}+\frac{\sqrt[4]{ax^3-b}\left(7ax^3+4b\right)}{24b^2x^6}$$

Rubi [A] time = 0.26, antiderivative size = 260, normalized size of antiderivative = 1.56, number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 51, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{7a^2\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{32\sqrt{2}\,b^{11/4}}+\frac{7a^2\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{32\sqrt{2}\,b^{11/4}}-\frac{7a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}\right)}{16\sqrt{2}\,b^{11/4}}+\frac{7a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}+1\right)}{16\sqrt{2}\,b^{11/4}}+\frac{7a\sqrt[4]{ax^3-b}}{24b^2x^3}+\frac{\sqrt[4]{ax^3-b}}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(-b + a*x^3)^(3/4)),x]
```

```
[Out] (-b + a*x^3)^(1/4)/(6*b*x^6) + (7*a*(-b + a*x^3)^(1/4))/(24*b^2*x^3) - (7*a
^2*ArcTan[1 - (Sqrt[2]*(-b + a*x^3)^(1/4))/b^(1/4)])/(16*Sqrt[2]*b^(11/4))
+ (7*a^2*ArcTan[1 + (Sqrt[2]*(-b + a*x^3)^(1/4))/b^(1/4)])/(16*Sqrt[2]*b^(1
1/4)) - (7*a^2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4) + Sqrt[-b +
 a*x^3]])/(32*Sqrt[2]*b^(11/4)) + (7*a^2*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b
+ a*x^3)^(1/4) + Sqrt[-b + a*x^3]])/(32*Sqrt[2]*b^(11/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^7\left(-b+ax^3\right)^{3/4}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^3(-b+ax)^{3/4}}\,dx, x, x^3\right)$$
$$= \frac{\sqrt[4]{-b+ax^3}}{6bx^6} + \frac{(7a)\,\text{Subst}\left(\int \frac{1}{x^2(-b+ax)^{3/4}}\,dx, x, x^3\right)}{24b}$$
$$= \frac{\sqrt[4]{-b+ax^3}}{6bx^6} + \frac{7a\sqrt[4]{-b+ax^3}}{24b^2x^3} + \frac{\left(7a^2\right)\text{Subst}\left(\int \frac{1}{x(-b+ax)^{3/4}}\,dx, x, x^3\right)}{32b^2}$$
$$= \frac{\sqrt[4]{-b+ax^3}}{6bx^6} + \frac{7a\sqrt[4]{-b+ax^3}}{24b^2x^3} + \frac{(7a)\,\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{8b^2}$$
$$= \frac{\sqrt[4]{-b+ax^3}}{6bx^6} + \frac{7a\sqrt[4]{-b+ax^3}}{24b^2x^3} + \frac{(7a)\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{16b^{5/2}} + \frac{(7a)\,\text{Subst}}{}$$
$$= \frac{\sqrt[4]{-b+ax^3}}{6bx^6} + \frac{7a\sqrt[4]{-b+ax^3}}{24b^2x^3} - \frac{\left(7a^2\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{32\sqrt{2}\,b^{11/4}} - \frac{(}{}$$
$$= \frac{\sqrt[4]{-b+ax^3}}{6bx^6} + \frac{7a\sqrt[4]{-b+ax^3}}{24b^2x^3} - \frac{7a^2\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^3}+\sqrt{-b+ax^3}\right)}{32\sqrt{2}\,b^{11/4}} + \frac{7}{}$$
$$= \frac{\sqrt[4]{-b+ax^3}}{6bx^6} + \frac{7a\sqrt[4]{-b+ax^3}}{24b^2x^3} - \frac{7a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)}{16\sqrt{2}\,b^{11/4}} + \frac{7a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+}}{\sqrt[4]{b}}\right)}{16\sqrt{2}\,b^{11/4}}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.25

$$\frac{4a^2\sqrt[4]{ax^3-b}\;{}_2F_1\left(\frac{1}{4},3;\frac{5}{4};1-\frac{ax^3}{b}\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(-b + a*x^3)^(3/4)),x]
```

```
[Out] (4*a^2*(-b + a*x^3)^(1/4)*Hypergeometric2F1[1/4, 3, 5/4, 1 - (a*x^3)/b])/(3
*b^3)
```

IntegrateAlgebraic [A] time = 0.28, size = 166, normalized size = 0.99

$$\frac{7a^2\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{16\sqrt{2}\,b^{11/4}} + \frac{7a^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}+\sqrt{b}}\right)}{16\sqrt{2}\,b^{11/4}} + \frac{\sqrt[4]{ax^3-b}\left(7ax^3+4b\right)}{24b^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(-b + a*x^3)^(3/4)),x]
```

```
[Out] ((-b + a*x^3)^(1/4)*(4*b + 7*a*x^3))/(24*b^2*x^6) + (7*a^2*ArcTan[(-(b^(1/4
)/Sqrt[2]) + Sqrt[-b + a*x^3]/(Sqrt[2]*b^(1/4)))/(-b + a*x^3)^(1/4)])/(16*S
qrt[2]*b^(11/4)) + (7*a^2*ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4))/(Sqr
t[b] + Sqrt[-b + a*x^3])])/(16*Sqrt[2]*b^(11/4))
```

fricas [A] time = 0.49, size = 234, normalized size = 1.40

$$\frac{84\, b^{2} x^{6} \left(-\frac{a^{8}}{b^{11}}\right)^{\frac{1}{4}} \arctan\left(-\frac{\left(a x^{3}-b\right)^{\frac{1}{4}} a^{2} b^{8} \left(-\frac{a^{8}}{b^{11}}\right)^{\frac{3}{4}}-\sqrt{b^{6} \sqrt{-\frac{a^{8}}{b^{11}}}+\sqrt{a x^{3}-b}\, a^{4}}\, b^{8} \left(-\frac{a^{8}}{b^{11}}\right)^{\frac{3}{4}}}{a^{8}}\right)+21\, b^{2} x^{6} \left(-\frac{a^{8}}{b^{11}}\right)^{\frac{1}{4}} \log\left(7\, b^{3} \left(-\frac{a^{8}}{b^{11}}\right)^{\frac{1}{4}}+7\left(a x^{3}-b\right)^{\frac{1}{4}} a^{2}\right)-21\, b^{2} x^{6} \left(-\frac{a^{8}}{b^{11}}\right)^{\frac{1}{4}} \log\left(-7\, b^{3} \left(-\frac{a^{8}}{b^{11}}\right)^{\frac{1}{4}}+7\left(a x^{3}-b\right)^{\frac{1}{4}} a^{2}\right)+4\left(7\, a x^{3}+4\, b\right)\left(a x^{3}-b\right)^{\frac{1}{4}}}{96\, b^{2} x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a*x^3-b)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/96*(84*b^2*x^6*(-a^8/b^11)^(1/4)*arctan(-((a*x^3 - b)^(1/4)*a^2*b^8*(-a^8
/b^11)^(3/4) - sqrt(b^6*sqrt(-a^8/b^11) + sqrt(a*x^3 - b)*a^4)*b^8*(-a^8/b^
11)^(3/4))/a^8) + 21*b^2*x^6*(-a^8/b^11)^(1/4)*log(7*b^3*(-a^8/b^11)^(1/4)
+ 7*(a*x^3 - b)^(1/4)*a^2) - 21*b^2*x^6*(-a^8/b^11)^(1/4)*log(-7*b^3*(-a^8/
b^11)^(1/4) + 7*(a*x^3 - b)^(1/4)*a^2) + 4*(7*a*x^3 + 4*b)*(a*x^3 - b)^(1/4
))/(b^2*x^6)
```

giac [A] time = 0.19, size = 224, normalized size = 1.34

$$\frac{\frac{42\,\sqrt{2} a^{3} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2} b^{\frac{1}{4}}+2\left(a x^{3}-b\right)^{\frac{1}{4}}\right)}{2\, b^{\frac{1}{4}}}\right)}{b^{\frac{11}{4}}}+\frac{42\,\sqrt{2} a^{3} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2} b^{\frac{1}{4}}-2\left(a x^{3}-b\right)^{\frac{1}{4}}\right)}{2\, b^{\frac{1}{4}}}\right)}{b^{\frac{11}{4}}}+\frac{21\,\sqrt{2} a^{3} \log\left(\sqrt{2}\left(a x^{3}-b\right)^{\frac{1}{4}} b^{\frac{1}{4}}+\sqrt{a x^{3}-b}+\sqrt{b}\right)}{b^{\frac{11}{4}}}-\frac{21\,\sqrt{2} a^{3} \log\left(-\sqrt{2}\left(a x^{3}-b\right)^{\frac{1}{4}} b^{\frac{1}{4}}+\sqrt{a x^{3}-b}+\sqrt{b}\right)}{b^{\frac{11}{4}}}+\frac{8\left(7\left(a x^{3}-b\right)^{\frac{5}{4}} a^{3}+11\left(a x^{3}-b\right)^{\frac{1}{4}} a^{3} b\right)}{a^{2} b^{2} x^{6}}}{192\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a*x^3-b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/192*(42*sqrt(2)*a^3*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(
1/4))/b^(1/4))/b^(11/4) + 42*sqrt(2)*a^3*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/
4) - 2*(a*x^3 - b)^(1/4))/b^(1/4))/b^(11/4) + 21*sqrt(2)*a^3*log(sqrt(2)*(a
*x^3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(11/4) - 21*sqrt(2)*
a^3*log(-sqrt(2)*(a*x^3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(
11/4) + 8*(7*(a*x^3 - b)^(5/4)*a^3 + 11*(a*x^3 - b)^(1/4)*a^3*b)/(a^2*b^2*x
^6))/a
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{7}\left(a\,x^{3}-b\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(a*x^3-b)^(3/4),x)
```

```
[Out] int(1/x^7/(a*x^3-b)^(3/4),x)
```

maxima [A] time = 0.42, size = 250, normalized size = 1.50

$$\frac{7\left(a x^{3}-b\right)^{\frac{5}{4}} a^{2}+11\left(a x^{3}-b\right)^{\frac{1}{4}} a^{2} b}{24\left(\left(a x^{3}-b\right)^{2} b^{2}+2\left(a x^{3}-b\right) b^{3}+b^{4}\right)}+\frac{7\left(\frac{2\,\sqrt{2} a^{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2} b^{\frac{1}{4}}+2\left(a x^{3}-b\right)^{\frac{1}{4}}\right)}{2\, b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{2\,\sqrt{2} a^{2} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2} b^{\frac{1}{4}}-2\left(a x^{3}-b\right)^{\frac{1}{4}}\right)}{2\, b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{\sqrt{2} a^{2} \log\left(\sqrt{2}\left(a x^{3}-b\right)^{\frac{1}{4}} b^{\frac{1}{4}}+\sqrt{a x^{3}-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2} a^{2} \log\left(-\sqrt{2}\left(a x^{3}-b\right)^{\frac{1}{4}} b^{\frac{1}{4}}+\sqrt{a x^{3}-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}\right)}{64\, b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a*x^3-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/24*(7*(a*x^3 - b)^(5/4)*a^2 + 11*(a*x^3 - b)^(1/4)*a^2*b)/((a*x^3 - b)^2*
b^2 + 2*(a*x^3 - b)*b^3 + b^4) + 7/64*(2*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sq
rt(2)*b^(1/4) + 2*(a*x^3 - b)^(1/4))/b^(1/4))/b^(3/4) + 2*sqrt(2)*a^2*arcta
n(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^3 - b)^(1/4))/b^(1/4))/b^(3/4) + s
```

```
qrt(2)*a^2*log(sqrt(2)*(a*x^3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b
))/b^(3/4) - sqrt(2)*a^2*log(-sqrt(2)*(a*x^3 - b)^(1/4)*b^(1/4) + sqrt(a*x^
3 - b) + sqrt(b))/b^(3/4))/b^2
```

mupad [B] time = 1.40, size = 98, normalized size = 0.59

$$\frac{11\left(a\,x^3-b\right)^{1/4}}{24\,b\,x^6}+\frac{7\left(a\,x^3-b\right)^{5/4}}{24\,b^2\,x^6}-\frac{7\,a^2\,\mathrm{atan}\left(\frac{\left(a\,x^3-b\right)^{1/4}}{(-b)^{1/4}}\right)}{16\,(-b)^{11/4}}+\frac{a^2\,\mathrm{atan}\left(\frac{\left(a\,x^3-b\right)^{1/4}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,7\mathrm{i}}{16\,(-b)^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a*x^3 - b)^(3/4)),x)
```

```
[Out] (11*(a*x^3 - b)^(1/4))/(24*b*x^6) + (7*(a*x^3 - b)^(5/4))/(24*b^2*x^6) - (7
*a^2*atan((a*x^3 - b)^(1/4)/(-b)^(1/4)))/(16*(-b)^(11/4)) + (a^2*atan(((a*x
^3 - b)^(1/4)*1i)/(-b)^(1/4))*7i)/(16*(-b)^(11/4))
```

sympy [C] time = 1.69, size = 42, normalized size = 0.25

$$-\frac{\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^3}\right)}{3a^{\frac{3}{4}}x^{\frac{33}{4}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(a*x**3-b)**(3/4),x)
```

```
[Out] -gamma(11/4)*hyper((3/4, 11/4), (15/4,), b*exp_polar(2*I*pi)/(a*x**3))/(3*a
**(3/4)*x**(33/4)*gamma(15/4))
```

3.1840 $\int \frac{1}{x^7 \sqrt[4]{-b+ax^3}} \, dx$

Optimal. Leaf size=167

$$-\frac{5a^2 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}-\sqrt{b}}\right)}{48\sqrt{2}\,b^{9/4}}-\frac{5a^2 \tanh^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{48\sqrt{2}\,b^{9/4}}+\frac{\left(ax^3-b\right)^{3/4}\left(5ax^3+4b\right)}{24b^2x^6}$$

Rubi [A] time = 0.27, antiderivative size = 260, normalized size of antiderivative = 1.56, number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 51, 63, 297, 1162, 617, 204, 1165, 628}

$$\frac{5a^2\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{96\sqrt{2}\,b^{9/4}}-\frac{5a^2\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}+\sqrt{ax^3-b}+\sqrt{b}\right)}{96\sqrt{2}\,b^{9/4}}-\frac{5a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}\right)}{48\sqrt{2}\,b^{9/4}}+\frac{5a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^3-b}}{\sqrt[4]{b}}+1\right)}{48\sqrt{2}\,b^{9/4}}+\frac{5a\left(ax^3-b\right)^{3/4}}{24b^2x^3}+\frac{\left(ax^3-b\right)^{3/4}}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(-b + a*x^3)^(1/4)),x]
```

```
[Out] (-b + a*x^3)^(3/4)/(6*b*x^6) + (5*a*(-b + a*x^3)^(3/4))/(24*b^2*x^3) - (5*a
^2*ArcTan[1 - (Sqrt[2]*(-b + a*x^3)^(1/4))/b^(1/4)])/(48*Sqrt[2]*b^(9/4)) +
 (5*a^2*ArcTan[1 + (Sqrt[2]*(-b + a*x^3)^(1/4))/b^(1/4)])/(48*Sqrt[2]*b^(9/
4)) + (5*a^2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4) + Sqrt[-b + a
*x^3]])/(96*Sqrt[2]*b^(9/4)) - (5*a^2*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*(-b + a
*x^3)^(1/4) + Sqrt[-b + a*x^3]])/(96*Sqrt[2]*b^(9/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^7\sqrt[4]{-b+ax^3}}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{x^3\sqrt[4]{-b+ax}}\,dx, x, x^3\right)$$

$$= \frac{\left(-b+ax^3\right)^{3/4}}{6bx^6} + \frac{(5a)\,\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt[4]{-b+ax}}\,dx, x, x^3\right)}{24b}$$

$$= \frac{\left(-b+ax^3\right)^{3/4}}{6bx^6} + \frac{5a\left(-b+ax^3\right)^{3/4}}{24b^2x^3} + \frac{\left(5a^2\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt[4]{-b+ax}}\,dx, x, x^3\right)}{96b^2}$$

$$= \frac{\left(-b+ax^3\right)^{3/4}}{6bx^6} + \frac{5a\left(-b+ax^3\right)^{3/4}}{24b^2x^3} + \frac{(5a)\,\mathrm{Subst}\left(\int \frac{x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{24b^2}$$

$$= \frac{\left(-b+ax^3\right)^{3/4}}{6bx^6} + \frac{5a\left(-b+ax^3\right)^{3/4}}{24b^2x^3} - \frac{(5a)\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{48b^2} + \frac{(5a)\,\mathrm{Subst}\left(\right.}{}$$

$$= \frac{\left(-b+ax^3\right)^{3/4}}{6bx^6} + \frac{5a\left(-b+ax^3\right)^{3/4}}{24b^2x^3} + \frac{\left(5a^2\right)\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx, x, \sqrt[4]{-b+ax^3}\right)}{96\sqrt{2}\,b^{9/4}} + \frac{(5}{}$$

$$= \frac{\left(-b+ax^3\right)^{3/4}}{6bx^6} + \frac{5a\left(-b+ax^3\right)^{3/4}}{24b^2x^3} + \frac{5a^2\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^3}+\sqrt{-b+ax^3}\right)}{96\sqrt{2}\,b^{9/4}} - \frac{5a}{}$$

$$= \frac{\left(-b+ax^3\right)^{3/4}}{6bx^6} + \frac{5a\left(-b+ax^3\right)^{3/4}}{24b^2x^3} - \frac{5a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^3}}{\sqrt[4]{b}}\right)}{48\sqrt{2}\,b^{9/4}} + \frac{5a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+a}}{\sqrt[4]{b}}\right.}{48\sqrt{2}\,b^{9/4}}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.25

$$\frac{4a^2\left(ax^3-b\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};1-\frac{ax^3}{b}\right)}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(-b + a*x^3)^(1/4)),x]
```

```
[Out] (4*a^2*(-b + a*x^3)^(3/4)*Hypergeometric2F1[3/4, 3, 7/4, 1 - (a*x^3)/b])/(9
*b^3)
```

IntegrateAlgebraic [A] time = 0.29, size = 166, normalized size = 0.99

$$\frac{5a^2\tan^{-1}\left(\frac{\frac{\sqrt{ax^3-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^3-b}}\right)}{48\sqrt{2}\,b^{9/4}} - \frac{5a^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^3-b}}{\sqrt{ax^3-b}+\sqrt{b}}\right)}{48\sqrt{2}\,b^{9/4}} + \frac{\left(ax^3-b\right)^{3/4}\left(5ax^3+4b\right)}{24b^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(-b + a*x^3)^(1/4)),x]
```

```
[Out] ((-b + a*x^3)^(3/4)*(4*b + 5*a*x^3))/(24*b^2*x^6) + (5*a^2*ArcTan[(-(b^(1/4
)/Sqrt[2]) + Sqrt[-b + a*x^3]/(Sqrt[2]*b^(1/4)))/(-b + a*x^3)^(1/4)])/(48*S
qrt[2]*b^(9/4)) - (5*a^2*ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^3)^(1/4))/(Sqrt
[b] + Sqrt[-b + a*x^3])])/(48*Sqrt[2]*b^(9/4))
```

fricas [A] time = 0.48, size = 240, normalized size = 1.44

$$-\frac{20 \, b^{2} x^{6} \left(-\frac{a^{8}}{b^{9}}\right)^{\frac{1}{4}} \arctan\left(-\frac{125 \, {\left(a x^{3} - b\right)}^{\frac{1}{4}} a^{6} b^{2} \left(-\frac{a^{8}}{b^{9}}\right)^{\frac{1}{4}} - \sqrt{-15625 \, a^{8} b^{5} \sqrt{-\frac{a^{8}}{b^{9}}} + 15625 \, \sqrt{a x^{3} - b} a^{12}} b^{2} \left(-\frac{a^{8}}{b^{9}}\right)^{\frac{1}{4}}}{125 \, a^{8}}\right) - 5 \, b^{2} x^{6} \left(-\frac{a^{8}}{b^{9}}\right)^{\frac{1}{4}} \log\left(125 \, b^{7} \left(-\frac{a^{8}}{b^{9}}\right)^{\frac{3}{4}} + 125 \, {\left(a x^{3} - b\right)}^{\frac{1}{4}} a^{6}\right) + 5 \, b^{2} x^{6} \left(-\frac{a^{8}}{b^{9}}\right)^{\frac{1}{4}} \log\left(-125 \, b^{7} \left(-\frac{a^{8}}{b^{9}}\right)^{\frac{3}{4}} + 125 \, {\left(a x^{3} - b\right)}^{\frac{1}{4}} a^{6}\right) - 4 \, {\left(5 \, a x^{3} + 4 \, b\right)} {\left(a x^{3} - b\right)}^{\frac{3}{4}}}{96 \, b^{2} x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a*x^3-b)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/96*(20*b^2*x^6*(-a^8/b^9)^(1/4)*arctan(-1/125*(125*(a*x^3 - b)^(1/4)*a^6
*b^2*(-a^8/b^9)^(1/4) - sqrt(-15625*a^8*b^5*sqrt(-a^8/b^9) + 15625*sqrt(a*x
^3 - b)*a^12)*b^2*(-a^8/b^9)^(1/4))/a^8) - 5*b^2*x^6*(-a^8/b^9)^(1/4)*log(1
25*b^7*(-a^8/b^9)^(3/4) + 125*(a*x^3 - b)^(1/4)*a^6) + 5*b^2*x^6*(-a^8/b^9)
^(1/4)*log(-125*b^7*(-a^8/b^9)^(3/4) + 125*(a*x^3 - b)^(1/4)*a^6) - 4*(5*a*
x^3 + 4*b)*(a*x^3 - b)^(3/4))/(b^2*x^6)
```

giac [A] time = 0.33, size = 224, normalized size = 1.34

$$\frac{\frac{10 \, \sqrt{2} a^{3} \arctan\left(\frac{\sqrt{2} {\left(\sqrt{2} b^{\frac{1}{4}} + 2 \, {\left(a x^{3} - b\right)}^{\frac{1}{4}}\right)}}{2 \, b^{\frac{1}{4}}}\right)}{b^{\frac{9}{4}}} + \frac{10 \, \sqrt{2} a^{3} \arctan\left(-\frac{\sqrt{2} {\left(\sqrt{2} b^{\frac{1}{4}} - 2 \, {\left(a x^{3} - b\right)}^{\frac{1}{4}}\right)}}{2 \, b^{\frac{1}{4}}}\right)}{b^{\frac{9}{4}}} - \frac{5 \, \sqrt{2} a^{3} \log\left(\sqrt{2} {\left(a x^{3} - b\right)}^{\frac{1}{4}} b^{\frac{1}{4}} + \sqrt{a x^{3} - b} + \sqrt{b}\right)}{b^{\frac{9}{4}}} + \frac{5 \, \sqrt{2} a^{3} \log\left(-\sqrt{2} {\left(a x^{3} - b\right)}^{\frac{1}{4}} b^{\frac{1}{4}} + \sqrt{a x^{3} - b} + \sqrt{b}\right)}{b^{\frac{9}{4}}} + \frac{8 \, {\left(5 \, {\left(a x^{3} - b\right)}^{\frac{7}{4}} a^{3} + 9 \, {\left(a x^{3} - b\right)}^{\frac{3}{4}} a^{3} b\right)}}{a^{2} b^{2} x^{6}}}{192 \, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a*x^3-b)^(1/4),x, algorithm="giac")
```

```
[Out] 1/192*(10*sqrt(2)*a^3*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(
1/4))/b^(1/4))/b^(9/4) + 10*sqrt(2)*a^3*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4
) - 2*(a*x^3 - b)^(1/4))/b^(1/4))/b^(9/4) - 5*sqrt(2)*a^3*log(sqrt(2)*(a*x^
3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(9/4) + 5*sqrt(2)*a^3*l
og(-sqrt(2)*(a*x^3 - b)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(9/4)
+ 8*(5*(a*x^3 - b)^(7/4)*a^3 + 9*(a*x^3 - b)^(3/4)*a^3*b)/(a^2*b^2*x^6))/a
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{7} \left(a \,x^{3}-b \right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(a*x^3-b)^(1/4),x)
```

```
[Out] int(1/x^7/(a*x^3-b)^(1/4),x)
```

maxima [A] time = 0.43, size = 241, normalized size = 1.44

$$\frac{5 \, {\left(\frac{2 \, \sqrt{2} \arctan\left(\frac{\sqrt{2} {\left(\sqrt{2} b^{\frac{1}{4}} + 2 \, {\left(a x^{3} - b\right)}^{\frac{1}{4}}\right)}}{2 \, b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}} + \frac{2 \, \sqrt{2} \arctan\left(-\frac{\sqrt{2} {\left(\sqrt{2} b^{\frac{1}{4}} - 2 \, {\left(a x^{3} - b\right)}^{\frac{1}{4}}\right)}}{2 \, b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}} - \frac{\sqrt{2} \log\left(\sqrt{2} {\left(a x^{3} - b\right)}^{\frac{1}{4}} b^{\frac{1}{4}} + \sqrt{a x^{3} - b} + \sqrt{b}\right)}{b^{\frac{1}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2} {\left(a x^{3} - b\right)}^{\frac{1}{4}} b^{\frac{1}{4}} + \sqrt{a x^{3} - b} + \sqrt{b}\right)}{b^{\frac{1}{4}}}\right)} a^{2}}{192 \, b^{2}} + \frac{5 \, {\left(a x^{3} - b\right)}^{\frac{7}{4}} a^{2} + 9 \, {\left(a x^{3} - b\right)}^{\frac{3}{4}} a^{2} b}{24 \, {\left({\left(a x^{3} - b\right)}^{2} b^{2} + 2 \, {\left(a x^{3} - b\right)} b^{3} + b^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a*x^3-b)^(1/4),x, algorithm="maxima")
```

```
[Out] 5/192*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^3 - b)^(1/4))
/b^(1/4))/b^(1/4) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x
^3 - b)^(1/4))/b^(1/4))/b^(1/4) - sqrt(2)*log(sqrt(2)*(a*x^3 - b)^(1/4)*b^(
1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(1/4) + sqrt(2)*log(-sqrt(2)*(a*x^3 - b
)^(1/4)*b^(1/4) + sqrt(a*x^3 - b) + sqrt(b))/b^(1/4))*a^2/b^2 + 1/24*(5*(a*
```

```
x^3 - b)^(7/4)*a^2 + 9*(a*x^3 - b)^(3/4)*a^2*b)/((a*x^3 - b)^2*b^2 + 2*(a*x
^3 - b)*b^3 + b^4)
```

mupad [B] time = 1.32, size = 98, normalized size = 0.59

$$\frac{3\left(a\,x^3-b\right)^{3/4}}{8\,b\,x^6}+\frac{5\left(a\,x^3-b\right)^{7/4}}{24\,b^2\,x^6}+\frac{5\,a^2\,\mathrm{atan}\left(\frac{\left(a\,x^3-b\right)^{1/4}}{(-b)^{1/4}}\right)}{48\,(-b)^{9/4}}+\frac{a^2\,\mathrm{atan}\left(\frac{\left(a\,x^3-b\right)^{1/4}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,5\mathrm{i}}{48\,(-b)^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a*x^3 - b)^(1/4)),x)
```

```
[Out] (3*(a*x^3 - b)^(3/4))/(8*b*x^6) + (5*(a*x^3 - b)^(7/4))/(24*b^2*x^6) + (5*a
^2*atan((a*x^3 - b)^(1/4)/(-b)^(1/4)))/(48*(-b)^(9/4)) + (a^2*atan(((a*x^3
- b)^(1/4)*1i)/(-b)^(1/4))*5i)/(48*(-b)^(9/4))
```

sympy [C] time = 1.44, size = 42, normalized size = 0.25

$$-\frac{\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^3}\right)}{3\sqrt[4]{a}\,x^{\frac{27}{4}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(a*x**3-b)**(1/4),x)
```

```
[Out] -gamma(9/4)*hyper((1/4, 9/4), (13/4,), b*exp_polar(2*I*pi)/(a*x**3))/(3*a**
(1/4)*x**(27/4)*gamma(13/4))
```

3.1841 $\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt[3]{x^2+x^4}}\,dx$

Optimal. Leaf size=167

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}-x}\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}+x}\right)}{2\sqrt[3]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^4+x^2}}\right)}{\sqrt[3]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x^2+\frac{\left(x^4+x^2\right)^{2/3}}{\sqrt[3]{2}}}{x\sqrt[3]{x^4+x^2}}\right)}{2\sqrt[3]{2}}$$

Rubi [C] time = 0.10, antiderivative size = 42, normalized size of antiderivative = 0.25, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2056, 466, 429}

$$-\frac{3x\sqrt[3]{x^2+1}F_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};x^2,-x^2\right)}{\sqrt[3]{x^4+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^2)/((-1 + x^2)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] (-3*x*(1 + x^2)^(1/3)*AppellF1[1/6, 1, -2/3, 7/6, x^2, -x^2])/(x^2 + x^4)^(
1/3)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt[3]{x^2+x^4}}\,dx = \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{\left(1+x^2\right)^{2/3}}{x^{2/3}\left(-1+x^2\right)}\,dx}{\sqrt[3]{x^2+x^4}}$$
$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{\left(1+x^6\right)^{2/3}}{-1+x^6}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}}$$
$$= -\frac{3x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,-\frac{2}{3};\frac{7}{6};x^2,-x^2\right)}{\sqrt[3]{x^2+x^4}}$$

Mathematica [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt[3]{x^2+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^2)/((-1 + x^2)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] Integrate[(1 + x^2)/((-1 + x^2)*(x^2 + x^4)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.52, size = 167, normalized size = 1.00

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}-x}\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}+x}\right)}{2\sqrt[3]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^4+x^2}}\right)}{\sqrt[3]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x^2+\frac{\left(x^4+x^2\right)^{2/3}}{\sqrt[3]{2}}}{x\sqrt[3]{x^4+x^2}}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 + x^2)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(x^2 + x^4)^(1/3))])/2^(1/3)
 - (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(x^2 + x^4)^(1/3))])/(2*2^(1/3)
) - ArcTanh[(2^(1/3)*x)/(x^2 + x^4)^(1/3)]/2^(1/3) - ArcTanh[(2^(1/3)*x^2 +
 (x^2 + x^4)^(2/3)/2^(1/3))/(x*(x^2 + x^4)^(1/3))]/(2*2^(1/3))
```

fricas [A] time = 1.11, size = 232, normalized size = 1.39

$$-\frac{1}{4}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}\left(x^5+8x^4-2x^3+8x^2+x\right)+8\sqrt{2}\left(x^4+x^2\right)^{\frac{1}{3}}\left(x^3+2x^2+x\right)+8\cdot 2^{\frac{1}{6}}\left(x^4+x^2\right)^{\frac{2}{3}}\left(x^2-2x+1\right)\right)}{6\left(x^5-8x^4-2x^3-8x^2+x\right)}\right)+\frac{1}{4}\cdot 2^{\frac{2}{3}}\log\left(-\frac{2^{\frac{2}{3}}\left(x^3-2x^2+x\right)+4\cdot 2^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}x-4\left(x^4+x^2\right)^{\frac{2}{3}}}{x^3+2x^2+x}\right)-\frac{1}{8}\cdot 2^{\frac{2}{3}}\log\left(\frac{2\cdot 2^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(x^3+2x^2+x\right)+4\left(x^4+x^2\right)^{\frac{1}{3}}x}{x^3+2x^2+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^4+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^5 + 8*x^4 - 2*x
^3 + 8*x^2 + x) + 8*sqrt(2)*(x^4 + x^2)^(1/3)*(x^3 + 2*x^2 + x) + 8*2^(1/6)
*(x^4 + x^2)^(2/3)*(x^2 - 2*x + 1))/(x^5 - 8*x^4 - 2*x^3 - 8*x^2 + x)) + 1/
4*2^(2/3)*log(-(2^(2/3)*(x^3 - 2*x^2 + x) + 4*2^(1/3)*(x^4 + x^2)^(1/3)*x -
 4*(x^4 + x^2)^(2/3))/(x^3 + 2*x^2 + x)) - 1/8*2^(2/3)*log((2*2^(2/3)*(x^4
+ x^2)^(2/3) + 2^(1/3)*(x^3 + 2*x^2 + x) + 4*(x^4 + x^2)^(1/3)*x)/(x^3 + 2*
x^2 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^4+x^2\right)^{\frac{1}{3}}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^4+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/((x^4 + x^2)^(1/3)*(x^2 - 1)), x)
```

maple [C] time = 25.52, size = 2591, normalized size = 15.51

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-1)/(x^4+x^2)^(1/3),x)
```

```
[Out] 1/4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*ln((1106640125761716*x^
2*(x^4+x^2)^(2/3)+1721440195629336*(x^4+x^2)^(2/3)*x+1106640125761716*(x^4+
x^2)^(2/3)+41772375754200*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2
*RootOf(_Z^3-4)^2*x^2-36525817072836*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3
-4)+_Z^2)*RootOf(_Z^3-4)^3*x^2+122536959111840*RootOf(_Z^3-4)^4*RootOf(Root
Of(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*(x^4+x^2)^(2/3)*x^2-2144315288715600
*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x^4+206541191229100*RootOf
(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x^3-103270595614550*RootOf(RootOf
(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x^5-2144315288715600*RootOf(RootOf(_Z^3-
4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x^2-103270595614550*RootOf(RootOf(_Z^3-4)^2+_Z
*RootOf(_Z^3-4)+_Z^2)*x+1874991943072248*RootOf(_Z^3-4)*x^4-180599873304578
*RootOf(_Z^3-4)*x^3+1874991943072248*RootOf(_Z^3-4)*x^2+90299936652289*Root
Of(_Z^3-4)*x-46413750838000*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)
^2*RootOf(_Z^3-4)^2*x+40584241192040*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3
-4)+_Z^2)*RootOf(_Z^3-4)^3*x+90299936652289*RootOf(_Z^3-4)*x^5+928275016760
00*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x^3-8
1168482384080*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4
)^3*x^3-46413750838000*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*Ro
otOf(_Z^3-4)^2*x^5+40584241192040*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)
+_Z^2)*RootOf(_Z^3-4)^3*x^5+41772375754200*RootOf(RootOf(_Z^3-4)^2+_Z*RootO
f(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x^4-36525817072836*RootOf(RootOf(_Z^3-4)
^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x^4+122536959111840*RootOf(_Z^3
-4)^4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*(x^4+x^2)^(2/3)-208
5103979818662*RootOf(_Z^3-4)^2*(x^4+x^2)^(1/3)*x^3+797336343950328*RootOf(R
ootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2*(x^4+x^2)^(2/3)+21
9484629454596*RootOf(_Z^3-4)^2*(x^4+x^2)^(1/3)*x^2-2085103979818662*RootOf(
_Z^3-4)^2*(x^4+x^2)^(1/3)*x-306342397779600*RootOf(_Z^3-4)^4*RootOf(RootOf(
_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*(x^4+x^2)^(2/3)*x-219484629454596*RootO
f(_Z^3-4)^4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*(x^4+x^2)^(1/3)
*x^3-107343584948232*RootOf(_Z^3-4)^3*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^
3-4)+_Z^2)^2*(x^4+x^2)^(1/3)*x^3+548711573636490*RootOf(_Z^3-4)^4*RootOf(Ro
otOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*(x^4+x^2)^(1/3)*x^2+268358962370580*
RootOf(_Z^3-4)^3*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*(x^4+x^2
)^(1/3)*x^2-219484629454596*RootOf(_Z^3-4)^4*RootOf(RootOf(_Z^3-4)^2+_Z*Roo
tOf(_Z^3-4)+_Z^2)*(x^4+x^2)^(1/3)*x-107343584948232*RootOf(_Z^3-4)^3*RootOf
(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*(x^4+x^2)^(1/3)*x+7973363439503
28*RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*(x^4+x^
2)^(2/3)*x^2+242958643915260*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2
)*RootOf(_Z^3-4)^2*(x^4+x^2)^(2/3)*x-1019764057008204*RootOf(_Z^3-4)*RootOf
(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*(x^4+x^2)^(1/3)*x^3+1073435849482
32*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*(x^4+x^2)
^(1/3)*x^2-1019764057008204*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)
*RootOf(_Z^3-4)*(x^4+x^2)^(1/3)*x)/(1+x)^2/(-1+x)^2/x)+1/4*RootOf(_Z^3-4)*l
n((-122113904250156*x^2*(x^4+x^2)^(2/3)-4151872744505304*(x^4+x^2)^(2/3)*x-
122113904250156*(x^4+x^2)^(2/3)+36525817072836*RootOf(RootOf(_Z^3-4)^2+_Z*R
ootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x^2-41772375754200*RootOf(RootOf(_Z^
3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x^2+122536959111840*RootOf(
```

```
_Z^3-4)^4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*(x^4+x^2)^(2/3)
*x^2+2021095211363592*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x^4+1
44074056231742*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x^3-72037028
115871*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x^5+2021095211363592
*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x^2-72037028115871*RootOf(
RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x-2311404791732400*RootOf(_Z^3-4)*
x^4-164768815474900*RootOf(_Z^3-4)*x^3-2311404791732400*RootOf(_Z^3-4)*x^2+
82384407737450*RootOf(_Z^3-4)*x-40584241192040*RootOf(RootOf(_Z^3-4)^2+_Z*R
ootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x+46413750838000*RootOf(RootOf(_Z^3-
4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x+82384407737450*RootOf(_Z^3-
4)*x^5+81168482384080*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*Roo
tOf(_Z^3-4)^2*x^3-92827501676000*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+
_Z^2)*RootOf(_Z^3-4)^3*x^3-40584241192040*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf
(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x^5+46413750838000*RootOf(RootOf(_Z^3-4)^
2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x^5+36525817072836*RootOf(RootOf
(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x^4-41772375754200*Ro
otOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x^4+12253695
9111840*RootOf(_Z^3-4)^4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*
(x^4+x^2)^(2/3)+1207165462000278*RootOf(_Z^3-4)^2*(x^4+x^2)^(1/3)*x^3+18295
9328944392*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2
*(x^4+x^2)^(2/3)+1975361665091364*RootOf(_Z^3-4)^2*(x^4+x^2)^(1/3)*x^2+1207
165462000278*RootOf(_Z^3-4)^2*(x^4+x^2)^(1/3)*x-306342397779600*RootOf(_Z^3
-4)^4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*(x^4+x^2)^(2/3)*x-2
19484629454596*RootOf(_Z^3-4)^4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_
Z^2)*(x^4+x^2)^(1/3)*x^3-112141044506364*RootOf(_Z^3-4)^3*RootOf(RootOf(_Z^
3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*(x^4+x^2)^(1/3)*x^3+548711573636490*RootOf
(_Z^3-4)^4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*(x^4+x^2)^(1/3)*
x^2+280352611265910*RootOf(_Z^3-4)^3*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3
-4)+_Z^2)^2*(x^4+x^2)^(1/3)*x^2-219484629454596*RootOf(_Z^3-4)^4*RootOf(Roo
tOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*(x^4+x^2)^(1/3)*x-112141044506364*Roo
tOf(_Z^3-4)^3*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*(x^4+x^2)^(
1/3)*x+182959328944392*RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_
Z^3-4)+_Z^2)*(x^4+x^2)^(2/3)*x^2-2693697826152060*RootOf(RootOf(_Z^3-4)^2+_
Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2*(x^4+x^2)^(2/3)*x+616775744785002*R
ootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*(x^4+x^2)^(1/
3)*x^3+1009269400557276*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z
^3-4)+_Z^2)*(x^4+x^2)^(1/3)*x^2+616775744785002*RootOf(RootOf(_Z^3-4)^2+_Z*
RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)*(x^4+x^2)^(1/3)*x)/(1+x)^2/(-1+x)^2/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^4+x^2\right)^{\frac{1}{3}}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x^4+x^2)^(1/3),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/((x^4 + x^2)^(1/3)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+1}{\left(x^4+x^2\right)^{1/3}\,\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((x^2 + x^4)^(1/3)*(x^2 - 1)),x)
[Out] int((x^2 + 1)/((x^2 + x^4)^(1/3)*(x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt[3]{x^2\left(x^2+1\right)}\,(x-1)\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-1)/(x**4+x**2)**(1/3),x)
```

```
[Out] Integral((x**2 + 1)/((x**2*(x**2 + 1))**(1/3)*(x - 1)*(x + 1)), x)
```

3.1842 $\int \frac{(-1+x^4)^{2/3}(3+x^4)(-2-x^3+2x^4)}{x^6(-2+3x^3+2x^4)} dx$

Optimal. Leaf size=167

$$-\sqrt[3]{2}\, 3^{2/3} \log\left(\sqrt[3]{2}\, 3^{2/3} \sqrt[3]{x^4-1} + 3x\right)+3\sqrt[3]{2}\,\sqrt[6]{3}\, \tan^{-1}\left(\frac{3^{5/6}x}{\sqrt[3]{3}\,x-2\sqrt[3]{2}\,\sqrt[3]{x^4-1}}\right)+\left(\frac{3}{2}\right)^{2/3} \log\left(-\sqrt[3]{2}\, 3^{2/3} \sqrt[3]{x^4-1}\, x + 2^2\right.$$

Rubi [F] time = 1.49, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^4)^{2/3}(3+x^4)(-2-x^3+2x^4)}{x^6(-2+3x^3+2x^4)} dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^4)^(2/3)*(3 + x^4)*(-2 - x^3 + 2*x^4))/(x^6*(-2 + 3*x^3 + 2*x^
4)),x]
```

```
[Out] (-3*(-1 + x^4)^(2/3))/x^2 + (12*x^2)/(1 + Sqrt[3] + (-1 + x^4)^(1/3)) - (6*
3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + (-1 + x^4)^(1/3))*Sqrt[(1 - (-1 + x^4)^(1/3)
 + (-1 + x^4)^(2/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))^2]*EllipticE[ArcSin[(
1 - Sqrt[3] + (-1 + x^4)^(1/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))], -7 - 4*S
qrt[3]])/(x^2*Sqrt[(1 + (-1 + x^4)^(1/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))^
2]) + (4*Sqrt[2]*3^(3/4)*(1 + (-1 + x^4)^(1/3))*Sqrt[(1 - (-1 + x^4)^(1/3)
+ (-1 + x^4)^(2/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))^2]*EllipticF[ArcSin[(1
 - Sqrt[3] + (-1 + x^4)^(1/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))], -7 - 4*Sq
rt[3]])/(x^2*Sqrt[(1 + (-1 + x^4)^(1/3))/(1 + Sqrt[3] + (-1 + x^4)^(1/3))^2
]) - (3*(-1 + x^4)^(2/3)*Hypergeometric2F1[-5/4, -2/3, -1/4, x^4])/(5*x^5*(
1 - x^4)^(2/3)) - ((-1 + x^4)^(2/3)*Hypergeometric2F1[-2/3, -1/4, 3/4, x^4]
)/(x*(1 - x^4)^(2/3)) - 18*Defer[Int][(-1 + x^4)^(2/3)/(-2 + 3*x^3 + 2*x^4)
, x] - 16*Defer[Int][(x*(-1 + x^4)^(2/3))/(-2 + 3*x^3 + 2*x^4), x]
```

Rubi steps

$$\int \frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)\left(-2-x^3+2x^4\right)}{x^6\left(-2+3x^3+2x^4\right)}\,dx = \int \left(\frac{3\left(-1+x^4\right)^{2/3}}{x^6} + \frac{6\left(-1+x^4\right)^{2/3}}{x^3} + \frac{\left(-1+x^4\right)^{2/3}}{x^2} - \frac{2(9+8x)}{-2+3}\right.$$

$$= -\left(2\int \frac{(9+8x)\left(-1+x^4\right)^{2/3}}{-2+3x^3+2x^4}\,dx\right) + 3\int \frac{\left(-1+x^4\right)^{2/3}}{x^6}\,dx + 6\int$$

$$= -\left(2\int \left(\frac{9\left(-1+x^4\right)^{2/3}}{-2+3x^3+2x^4} + \frac{8x\left(-1+x^4\right)^{2/3}}{-2+3x^3+2x^4}\right)dx\right) + 3\,\text{Subst}\left(\int\right.$$

$$= -\frac{3\left(-1+x^4\right)^{2/3}}{x^2} - \frac{3\left(-1+x^4\right)^{2/3}\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};x^4\right)}{5x^5\left(1-x^4\right)^{2/3}} - \left(-1+\right.$$

$$= -\frac{3\left(-1+x^4\right)^{2/3}}{x^2} - \frac{3\left(-1+x^4\right)^{2/3}\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};x^4\right)}{5x^5\left(1-x^4\right)^{2/3}} - \left(-1+\right.$$

$$= -\frac{3\left(-1+x^4\right)^{2/3}}{x^2} - \frac{3\left(-1+x^4\right)^{2/3}\,{}_2F_1\left(-\frac{5}{4},-\frac{2}{3};-\frac{1}{4};x^4\right)}{5x^5\left(1-x^4\right)^{2/3}} - \left(-1+\right.$$

$$= -\frac{3\left(-1+x^4\right)^{2/3}}{x^2} + \frac{12x^2}{1+\sqrt{3}+\sqrt[3]{-1+x^4}} - \frac{6\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1+\right.}{}$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^4\right)^{2/3}\left(3+x^4\right)\left(-2-x^3+2x^4\right)}{x^6\left(-2+3x^3+2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^4)^(2/3)*(3 + x^4)*(-2 - x^3 + 2*x^4))/(x^6*(-2 + 3*x^3
+ 2*x^4)),x]
```

```
[Out] Integrate[((-1 + x^4)^(2/3)*(3 + x^4)*(-2 - x^3 + 2*x^4))/(x^6*(-2 + 3*x^3
+ 2*x^4)), x]
```

IntegrateAlgebraic [A] time = 3.52, size = 167, normalized size = 1.00

$$-\sqrt[3]{2}\,3^{2/3}\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^4-1}+3x\right)+3\sqrt[3]{2}\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{\sqrt[3]{3}x-2\sqrt[3]{2}\sqrt[3]{x^4-1}}\right)+\left(\frac{3}{2}\right)^{2/3}\log\left(-\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^4-1}\,x+2^{2/3}\sqrt[3]{3}\left(x^4-1\right)^{2/3}+3x^2\right)+\frac{3\left(x^4-1\right)^{2/3}\left(x^4-5x^3-1\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)^(2/3)*(3 + x^4)*(-2 - x^3 + 2*x^4))/(x^6*(-2
 + 3*x^3 + 2*x^4)),x]
```

```
[Out] (3*(-1 + x^4)^(2/3)*(-1 - 5*x^3 + x^4))/(5*x^5) + 3*2^(1/3)*3^(1/6)*ArcTan[
(3^(5/6)*x)/(3^(1/3)*x - 2*2^(1/3)*(-1 + x^4)^(1/3))] - 2^(1/3)*3^(2/3)*Log
[3*x + 2^(1/3)*3^(2/3)*(-1 + x^4)^(1/3)] + (3/2)^(2/3)*Log[3*x^2 - 2^(1/3)*
3^(2/3)*x*(-1 + x^4)^(1/3) + 2^(2/3)*3^(1/3)*(-1 + x^4)^(2/3)]
```

fricas [B] time = 124.26, size = 418, normalized size = 2.50

$$\frac{10\sqrt{3}\,(-18)^{\frac{1}{3}}x^5\arctan\left(\frac{4\sqrt{3}(-18)^{\frac{2}{3}}(2x^9-3x^8-9x^7-4x^5+3x^4+2x)(x^4-1)^{\frac{2}{3}}+6\sqrt{3}(-18)^{\frac{1}{3}}(4x^{10}-42x^9+9x^8-8x^6+42x^5+4x^2)(x^4-1)^{\frac{1}{3}}-\sqrt{3}(8x^{12}-180x^{11}+216x^{10}+27x^9-24x^8+360x^7-216x^6+24x^4-180x^3-8)}{3(8x^{12}+36x^{11}-432x^{10}+27x^9-24x^8-72x^7+432x^6+24x^4+36x^3-8)}\right)+10\,(-18)^{\frac{1}{3}}x^5\log\left(\frac{3(-18)^{\frac{2}{3}}(x^4-1)^{\frac{1}{3}}x^2+18(x^4-1)^{\frac{2}{3}}x-(-18)^{\frac{1}{3}}(2x^4+3x^3-2)}{2x^4+3x^3-2}\right)-5\,(-18)^{\frac{1}{3}}x^5\log\left(\frac{36(-18)^{\frac{1}{3}}(x^5-3x^4-x)(x^4-1)^{\frac{2}{3}}+(-18)^{\frac{2}{3}}(4x^8-42x^7+9x^6-8x^4+42x^3+4)+54(4x^6-3x^5-4x^2)(x^4-1)^{\frac{1}{3}}}{4x^8+12x^7+9x^6-8x^4-12x^3+4}\right)+18(x^4-5x^3-1)(x^4-1)^{\frac{2}{3}}}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)*(2*x^4-x^3-2)/x^6/(2*x^4+3*x^3-2),x, algori
thm="fricas")
```

```
[Out] 1/30*(10*sqrt(3)*(-18)^(1/3)*x^5*arctan(1/3*(4*sqrt(3)*(-18)^(2/3)*(2*x^9 -
 3*x^8 - 9*x^7 - 4*x^5 + 3*x^4 + 2*x)*(x^4 - 1)^(2/3) + 6*sqrt(3)*(-18)^(1/
3)*(4*x^10 - 42*x^9 + 9*x^8 - 8*x^6 + 42*x^5 + 4*x^2)*(x^4 - 1)^(1/3) - sqr
t(3)*(8*x^12 - 180*x^11 + 216*x^10 + 27*x^9 - 24*x^8 + 360*x^7 - 216*x^6 +
24*x^4 - 180*x^3 - 8))/(8*x^12 + 36*x^11 - 432*x^10 + 27*x^9 - 24*x^8 - 72*
x^7 + 432*x^6 + 24*x^4 + 36*x^3 - 8)) + 10*(-18)^(1/3)*x^5*log((3*(-18)^(2/
3)*(x^4 - 1)^(1/3)*x^2 + 18*(x^4 - 1)^(2/3)*x - (-18)^(1/3)*(2*x^4 + 3*x^3
- 2))/(2*x^4 + 3*x^3 - 2)) - 5*(-18)^(1/3)*x^5*log((36*(-18)^(1/3)*(x^5 - 3
*x^4 - x)*(x^4 - 1)^(2/3) + (-18)^(2/3)*(4*x^8 - 42*x^7 + 9*x^6 - 8*x^4 + 4
2*x^3 + 4) + 54*(4*x^6 - 3*x^5 - 4*x^2)*(x^4 - 1)^(1/3))/(4*x^8 + 12*x^7 +
9*x^6 - 8*x^4 - 12*x^3 + 4)) + 18*(x^4 - 5*x^3 - 1)*(x^4 - 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^4-x^3-2)(x^4+3)(x^4-1)^{\frac{2}{3}}}{(2x^4+3x^3-2)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)*(2*x^4-x^3-2)/x^6/(2*x^4+3*x^3-2),x, algori
thm="giac")
```

```
[Out] integrate((2*x^4 - x^3 - 2)*(x^4 + 3)*(x^4 - 1)^(2/3)/((2*x^4 + 3*x^3 - 2)*
x^6), x)
```

maple [C] time = 89.03, size = 808, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(2/3)*(x^4+3)*(2*x^4-x^3-2)/x^6/(2*x^4+3*x^3-2),x)
```

```
[Out] 3/5*(x^8-5*x^7-2*x^4+5*x^3+1)/x^5/(x^4-1)^(1/3)-6*ln(-(-6*RootOf(RootOf(_Z^
3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2)^2*RootOf(_Z^3+18)^2*x^3+2*RootOf(Root
Of(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2)*RootOf(_Z^3+18)^2*(x^4-1)^(2/3)
*x-12*RootOf(_Z^3+18)*(x^4-1)^(1/3)*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z
^3+18)+36*_Z^2)*x^2-RootOf(_Z^3+18)^2*(x^4-1)^(1/3)*x^2-12*RootOf(RootOf(_Z
^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2)*x^4+18*x^3*RootOf(RootOf(_Z^3+18)^2+
6*_Z*RootOf(_Z^3+18)+36*_Z^2)-12*(x^4-1)^(2/3)*x+12*RootOf(RootOf(_Z^3+18)^
2+6*_Z*RootOf(_Z^3+18)+36*_Z^2))/(2*x^4+3*x^3-2))*RootOf(RootOf(_Z^3+18)^2+
6*_Z*RootOf(_Z^3+18)+36*_Z^2)-ln(-(-6*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(
_Z^3+18)+36*_Z^2)^2*RootOf(_Z^3+18)^2*x^3+2*RootOf(RootOf(_Z^3+18)^2+6*_Z*R
ootOf(_Z^3+18)+36*_Z^2)*RootOf(_Z^3+18)^2*(x^4-1)^(2/3)*x-12*RootOf(_Z^3+18
)*(x^4-1)^(1/3)*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2)*x^2-
RootOf(_Z^3+18)^2*(x^4-1)^(1/3)*x^2-12*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf
(_Z^3+18)+36*_Z^2)*x^4+18*x^3*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)
+36*_Z^2)-12*(x^4-1)^(2/3)*x+12*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+1
8)+36*_Z^2))/(2*x^4+3*x^3-2))*RootOf(_Z^3+18)+RootOf(_Z^3+18)*ln((-3*RootOf
(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2)^2*RootOf(_Z^3+18)^2*x^3+Ro
otOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2)*RootOf(_Z^3+18)^2*(x^4
-1)^(2/3)*x+3*RootOf(_Z^3+18)*(x^4-1)^(1/3)*RootOf(RootOf(_Z^3+18)^2+6*_Z*R
```

```
ootOf(_Z^3+18)+36*_Z^2)*x^2+RootOf(_Z^3+18)^2*(x^4-1)^(1/3)*x^2+6*RootOf(Ro
otOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2)*x^4+3*(x^4-1)^(2/3)*x-6*RootO
f(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2))/(2*x^4+3*x^3-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^4 - x^3 - 2\right)\left(x^4 + 3\right)\left(x^4 - 1\right)^{\frac{2}{3}}}{\left(2\,x^4 + 3\,x^3 - 2\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(2/3)*(x^4+3)*(2*x^4-x^3-2)/x^6/(2*x^4+3*x^3-2),x, algori
thm="maxima")
```

```
[Out] integrate((2*x^4 - x^3 - 2)*(x^4 + 3)*(x^4 - 1)^(2/3)/((2*x^4 + 3*x^3 - 2)*
x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^4 - 1\right)^{2/3}\,\left(x^4 + 3\right)\,\left(-2\,x^4 + x^3 + 2\right)}{x^6\,\left(2\,x^4 + 3\,x^3 - 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^4 - 1)^(2/3)*(x^4 + 3)*(x^3 - 2*x^4 + 2))/(x^6*(3*x^3 + 2*x^4 - 2)
),x)
```

```
[Out] int(-((x^4 - 1)^(2/3)*(x^4 + 3)*(x^3 - 2*x^4 + 2))/(x^6*(3*x^3 + 2*x^4 - 2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(2/3)*(x**4+3)*(2*x**4-x**3-2)/x**6/(2*x**4+3*x**3-2),x
)
```

```
[Out] Timed out
```

3.1843 $\int \frac{1}{x^9\left(-b+ax^4\right)^{3/4}}\,dx$

Optimal. Leaf size=167

$$-\frac{21a^2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{ax^4-b}}{\sqrt{ax^4-b}-\sqrt{b}}\right)}{64\sqrt{2}\,b^{11/4}}+\frac{21a^2\tanh^{-1}\left(\frac{\frac{\sqrt{ax^4-b}}{\sqrt{2}\,\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^4-b}}\right)}{64\sqrt{2}\,b^{11/4}}+\frac{\sqrt[4]{ax^4-b}\,\left(7ax^4+4b\right)}{32b^2x^8}$$

Rubi [A] time = 0.25, antiderivative size = 260, normalized size of antiderivative = 1.56, number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 51, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{21a^2\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{ax^4-b}+\sqrt{ax^4-b}+\sqrt{b}\right)}{128\sqrt{2}\,b^{11/4}}+\frac{21a^2\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{ax^4-b}+\sqrt{ax^4-b}+\sqrt{b}\right)}{128\sqrt{2}\,b^{11/4}}-\frac{21a^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{ax^4-b}}{\sqrt[4]{b}}\right)}{64\sqrt{2}\,b^{11/4}}+\frac{21a^2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{ax^4-b}}{\sqrt[4]{b}}+1\right)}{64\sqrt{2}\,b^{11/4}}+\frac{7a\sqrt[4]{ax^4-b}}{32b^2x^4}+\frac{\sqrt[4]{ax^4-b}}{8bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(-b + a*x^4)^(3/4)),x]
```

```
[Out] (-b + a*x^4)^(1/4)/(8*b*x^8) + (7*a*(-b + a*x^4)^(1/4))/(32*b^2*x^4) - (21*
a^2*ArcTan[1 - (Sqrt[2]*(-b + a*x^4)^(1/4))/b^(1/4)])/(64*Sqrt[2]*b^(11/4))
 + (21*a^2*ArcTan[1 + (Sqrt[2]*(-b + a*x^4)^(1/4))/b^(1/4)])/(64*Sqrt[2]*b^
(11/4)) - (21*a^2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^4)^(1/4) + Sqrt[-
b + a*x^4]])/(128*Sqrt[2]*b^(11/4)) + (21*a^2*Log[Sqrt[b] + Sqrt[2]*b^(1/4)
*(-b + a*x^4)^(1/4) + Sqrt[-b + a*x^4]])/(128*Sqrt[2]*b^(11/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^9\left(-b+ax^4\right)^{3/4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^3(-b+ax)^{3/4}}\,dx, x, x^4\right)$$

$$= \frac{\sqrt[4]{-b+ax^4}}{8bx^8} + \frac{(7a)\,\text{Subst}\left(\int \frac{1}{x^2(-b+ax)^{3/4}}\,dx, x, x^4\right)}{32b}$$

$$= \frac{\sqrt[4]{-b+ax^4}}{8bx^8} + \frac{7a\sqrt[4]{-b+ax^4}}{32b^2x^4} + \frac{\left(21a^2\right)\text{Subst}\left(\int \frac{1}{x(-b+ax)^{3/4}}\,dx, x, x^4\right)}{128b^2}$$

$$= \frac{\sqrt[4]{-b+ax^4}}{8bx^8} + \frac{7a\sqrt[4]{-b+ax^4}}{32b^2x^4} + \frac{(21a)\,\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^4}\right)}{32b^2}$$

$$= \frac{\sqrt[4]{-b+ax^4}}{8bx^8} + \frac{7a\sqrt[4]{-b+ax^4}}{32b^2x^4} + \frac{(21a)\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx, x, \sqrt[4]{-b+ax^4}\right)}{64b^{5/2}} + \frac{(21a)\,\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{\sqrt[4]{-b+ax^4}}{8bx^8} + \frac{7a\sqrt[4]{-b+ax^4}}{32b^2x^4} - \frac{\left(21a^2\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}\,x-x^2}\,dx, x, \sqrt[4]{-b+ax^4}\right)}{128\sqrt{2}\,b^{11/4}} - \frac{(21\cdots}{\cdots}$$

$$= \frac{\sqrt[4]{-b+ax^4}}{8bx^8} + \frac{7a\sqrt[4]{-b+ax^4}}{32b^2x^4} - \frac{21a^2\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^4}+\sqrt{-b+ax^4}\right)}{128\sqrt{2}\,b^{11/4}} + \frac{21a\cdots}{\cdots}$$

$$= \frac{\sqrt[4]{-b+ax^4}}{8bx^8} + \frac{7a\sqrt[4]{-b+ax^4}}{32b^2x^4} - \frac{21a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^4}}{\sqrt[4]{b}}\right)}{64\sqrt{2}\,b^{11/4}} + \frac{21a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{-b+a\cdots}}{\sqrt[4]{b}}\right.}{64\sqrt{2}\,b^{11/4}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.23

$$\frac{a^2\sqrt[4]{ax^4-b}\;{}_2F_1\left(\frac{1}{4},3;\frac{5}{4};1-\frac{ax^4}{b}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(-b + a*x^4)^(3/4)),x]
```

```
[Out] (a^2*(-b + a*x^4)^(1/4)*Hypergeometric2F1[1/4, 3, 5/4, 1 - (a*x^4)/b])/b^3
```

IntegrateAlgebraic [A] time = 0.26, size = 166, normalized size = 0.99

$$\frac{21a^2\tan^{-1}\left(\frac{\frac{\sqrt{ax^4-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^4-b}}\right)}{64\sqrt{2}\,b^{11/4}} + \frac{21a^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^4-b}}{\sqrt{ax^4-b}+\sqrt{b}}\right)}{64\sqrt{2}\,b^{11/4}} + \frac{\sqrt[4]{ax^4-b}\left(7ax^4+4b\right)}{32b^2x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^9*(-b + a*x^4)^(3/4)),x]
```

```
[Out] ((-b + a*x^4)^(1/4)*(4*b + 7*a*x^4))/(32*b^2*x^8) + (21*a^2*ArcTan[(-(b^(1/
4)/Sqrt[2]) + Sqrt[-b + a*x^4]/(Sqrt[2]*b^(1/4)))/(-b + a*x^4)^(1/4)])/(64*
Sqrt[2]*b^(11/4)) + (21*a^2*ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^4)^(1/4))/(S
qrt[b] + Sqrt[-b + a*x^4])])/(64*Sqrt[2]*b^(11/4))
```

fricas [A] time = 0.48, size = 234, normalized size = 1.40

$$\frac{84\,b^2x^8\left(-\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(ax^4-b\right)^{\frac{1}{4}}a^2b^8\left(-\frac{a^8}{b^{11}}\right)^{\frac{3}{4}}-\sqrt{b^6\sqrt{-\frac{a^8}{b^{11}}}+\sqrt{ax^4-b}\,a^4}\,b^8\left(-\frac{a^8}{b^{11}}\right)^{\frac{3}{4}}}{a^8}\right)+21\,b^2x^8\left(-\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\log\left(21\,b^3\left(-\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}+21\left(ax^4-b\right)^{\frac{1}{4}}a^2\right)-21\,b^2x^8\left(-\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\log\left(-21\,b^3\left(-\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}+21\left(ax^4-b\right)^{\frac{1}{4}}a^2\right)+4\left(7\,ax^4+4\,b\right)\left(ax^4-b\right)^{\frac{1}{4}}}{128\,b^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(a*x^4-b)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/128*(84*b^2*x^8*(-a^8/b^11)^(1/4)*arctan(-((a*x^4 - b)^(1/4)*a^2*b^8*(-a^
8/b^11)^(3/4) - sqrt(b^6*sqrt(-a^8/b^11) + sqrt(a*x^4 - b)*a^4)*b^8*(-a^8/b
^11)^(3/4))/a^8) + 21*b^2*x^8*(-a^8/b^11)^(1/4)*log(21*b^3*(-a^8/b^11)^(1/4
) + 21*(a*x^4 - b)^(1/4)*a^2) - 21*b^2*x^8*(-a^8/b^11)^(1/4)*log(-21*b^3*(-
a^8/b^11)^(1/4) + 21*(a*x^4 - b)^(1/4)*a^2) + 4*(7*a*x^4 + 4*b)*(a*x^4 - b)
^(1/4))/(b^2*x^8)
```

giac [A] time = 0.27, size = 224, normalized size = 1.34

$$\frac{\frac{42\sqrt{2}a^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{11}{4}}}+\frac{42\sqrt{2}a^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{11}{4}}}+\frac{21\sqrt{2}a^3\log\left(\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{b^{\frac{11}{4}}}-\frac{21\sqrt{2}a^3\log\left(-\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{b^{\frac{11}{4}}}+\frac{8\left(7\left(ax^4-b\right)^{\frac{5}{4}}a^3+11\left(ax^4-b\right)^{\frac{1}{4}}a^3b\right)}{a^2b^2x^8}}{256a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(a*x^4-b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/256*(42*sqrt(2)*a^3*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^4 - b)^(
1/4))/b^(1/4))/b^(11/4) + 42*sqrt(2)*a^3*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/
4) - 2*(a*x^4 - b)^(1/4))/b^(1/4))/b^(11/4) + 21*sqrt(2)*a^3*log(sqrt(2)*(a
*x^4 - b)^(1/4)*b^(1/4) + sqrt(a*x^4 - b) + sqrt(b))/b^(11/4) - 21*sqrt(2)*
a^3*log(-sqrt(2)*(a*x^4 - b)^(1/4)*b^(1/4) + sqrt(a*x^4 - b) + sqrt(b))/b^(
11/4) + 8*(7*(a*x^4 - b)^(5/4)*a^3 + 11*(a*x^4 - b)^(1/4)*a^3*b)/(a^2*b^2*x
^8))/a
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{9}\left(a\,x^{4}-b\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(a*x^4-b)^(3/4),x)
```

```
[Out] int(1/x^9/(a*x^4-b)^(3/4),x)
```

maxima [A] time = 0.42, size = 250, normalized size = 1.50

$$\frac{7\left(ax^4-b\right)^{\frac{5}{4}}a^2+11\left(ax^4-b\right)^{\frac{1}{4}}a^2b}{32\left(\left(ax^4-b\right)^2b^2+2\left(ax^4-b\right)b^3+b^4\right)}+\frac{21\left(\frac{2\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{2\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-2\left(ax^4-b\right)^{\frac{1}{4}}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{\sqrt{2}a^2\log\left(\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}a^2\log\left(-\sqrt{2}\left(ax^4-b\right)^{\frac{1}{4}}b^{\frac{1}{4}}+\sqrt{ax^4-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}\right)}{256b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(a*x^4-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/32*(7*(a*x^4 - b)^(5/4)*a^2 + 11*(a*x^4 - b)^(1/4)*a^2*b)/((a*x^4 - b)^2*
b^2 + 2*(a*x^4 - b)*b^3 + b^4) + 21/256*(2*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(
sqrt(2)*b^(1/4) + 2*(a*x^4 - b)^(1/4))/b^(1/4))/b^(3/4) + 2*sqrt(2)*a^2*arc
tan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^4 - b)^(1/4))/b^(1/4))/b^(3/4) +
 sqrt(2)*a^2*log(sqrt(2)*(a*x^4 - b)^(1/4)*b^(1/4) + sqrt(a*x^4 - b) + sqrt
(b))/b^(3/4) - sqrt(2)*a^2*log(-sqrt(2)*(a*x^4 - b)^(1/4)*b^(1/4) + sqrt(a*
x^4 - b) + sqrt(b))/b^(3/4))/b^2
```

mupad [B] time = 1.39, size = 98, normalized size = 0.59

$$\frac{11\left(a\,x^4-b\right)^{1/4}}{32\,b\,x^8}+\frac{7\left(a\,x^4-b\right)^{5/4}}{32\,b^2\,x^8}-\frac{21\,a^2\,\mathrm{atan}\left(\frac{\left(a\,x^4-b\right)^{1/4}}{(-b)^{1/4}}\right)}{64\,(-b)^{11/4}}+\frac{a^2\,\mathrm{atan}\left(\frac{\left(a\,x^4-b\right)^{1/4}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,21\mathrm{i}}{64\,(-b)^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a*x^4 - b)^(3/4)),x)
```

```
[Out] (11*(a*x^4 - b)^(1/4))/(32*b*x^8) + (7*(a*x^4 - b)^(5/4))/(32*b^2*x^8) - (2
1*a^2*atan((a*x^4 - b)^(1/4)/(-b)^(1/4)))/(64*(-b)^(11/4)) + (a^2*atan(((a*
x^4 - b)^(1/4)*1i)/(-b)^(1/4))*21i)/(64*(-b)^(11/4))
```

sympy [C] time = 1.65, size = 41, normalized size = 0.25

$$-\frac{\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{11}{4}\\\frac{15}{4}\end{matrix}\right|\frac{be^{2i\pi}}{ax^4}\right)}{4a^{\frac{3}{4}}x^{11}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(a*x**4-b)**(3/4),x)
```

```
[Out] -gamma(11/4)*hyper((3/4, 11/4), (15/4,), b*exp_polar(2*I*pi)/(a*x**4))/(4*a
**(3/4)*x**11*gamma(15/4))
```

3.1844 $\int \frac{b^2+cx^2+a^2x^4}{\sqrt{bx+ax^3}\left(-b^2+a^2x^4\right)}\,dx$

Optimal. Leaf size=167

$$-\frac{(2ab+c)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{4\sqrt{2}\,a^{5/4}b^{5/4}}-\frac{(2ab+c)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{4\sqrt{2}\,a^{5/4}b^{5/4}}+\frac{(c-2ab)\sqrt{ax^3+bx}}{2ab\left(ax^2+b\right)}$$

Rubi [C] time = 1.74, antiderivative size = 503, normalized size of antiderivative = 3.01, number of steps used = 18, number of rules used = 12, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2056, 6715, 6725, 220, 1455, 527, 523, 409, 1211, 1699, 206, 203}

$$-\frac{\sqrt{x}(2ab+c)\sqrt{ax^2+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2+b}}\right)}{4\sqrt{2}\,a^{5/4}b^{5/4}\sqrt{ax^3+bx}}-\frac{\sqrt{x}(2ab+c)\sqrt{ax^2+b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{ax^2+b}}\right)}{4\sqrt{2}\,a^{5/4}b^{5/4}\sqrt{ax^3+bx}}-\frac{\sqrt{x}(2ab-c)\left(\sqrt{a}x+\sqrt{b}\right)\sqrt{\frac{ax^2+b}{\left(\sqrt{a}x+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{4a^{5/4}b^{5/4}\sqrt{ax^3+bx}}-\frac{\sqrt{x}(2ab+c)\left(\sqrt{a}x+\sqrt{b}\right)\sqrt{\frac{ax^2+b}{\left(\sqrt{a}x+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{4a^{5/4}b^{5/4}\sqrt{ax^3+bx}}-\frac{x(2ab-c)}{2ab\sqrt{ax^3+bx}}+\frac{\sqrt{x}\left(\sqrt{a}x+\sqrt{b}\right)\sqrt{\frac{ax^2+b}{\left(\sqrt{a}x+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2 + c*x^2 + a^2*x^4)/(Sqrt[b*x + a*x^3]*(-b^2 + a^2*x^4)),x]
```

```
[Out] -1/2*((2*a*b - c)*x)/(a*b*Sqrt[b*x + a*x^3]) - ((2*a*b + c)*Sqrt[x]*Sqrt[b
+ a*x^2]*ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/Sqrt[b + a*x^2]])/(4*Sqrt
[2]*a^(5/4)*b^(5/4)*Sqrt[b*x + a*x^3]) - ((2*a*b + c)*Sqrt[x]*Sqrt[b + a*x^
2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/Sqrt[b + a*x^2]])/(4*Sqrt[2]*a
^(5/4)*b^(5/4)*Sqrt[b*x + a*x^3]) + (Sqrt[x]*(Sqrt[b] + Sqrt[a]*x)*Sqrt[(b
+ a*x^2)/(Sqrt[b] + Sqrt[a]*x)^2]*EllipticF[2*ArcTan[(a^(1/4)*Sqrt[x])/b^(1
/4)], 1/2])/(a^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3]) - ((2*a*b - c)*Sqrt[x]*(Sqr
t[b] + Sqrt[a]*x)*Sqrt[(b + a*x^2)/(Sqrt[b] + Sqrt[a]*x)^2]*EllipticF[2*Arc
Tan[(a^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(4*a^(5/4)*b^(5/4)*Sqrt[b*x + a*x^3])
 - ((2*a*b + c)*Sqrt[x]*(Sqrt[b] + Sqrt[a]*x)*Sqrt[(b + a*x^2)/(Sqrt[b] + S
qrt[a]*x)^2]*EllipticF[2*ArcTan[(a^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(4*a^(5/4
)*b^(5/4)*Sqrt[b*x + a*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1455

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, n, q, r}, x] && EqQ[n2,
 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{b^2+cx^2+a^2x^4}{\sqrt{bx+ax^3}\left(-b^2+a^2x^4\right)}dx = \frac{\left(\sqrt{x}\sqrt{b+ax^2}\right)\int \frac{b^2+cx^2+a^2x^4}{\sqrt{x}\sqrt{b+ax^2}\left(-b^2+a^2x^4\right)}dx}{\sqrt{bx+ax^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b+ax^2}\right)\text{Subst}\left(\int \frac{b^2+cx^4+a^2x^8}{\sqrt{b+ax^4}\left(-b^2+a^2x^8\right)}dx,x,\sqrt{x}\right)}{\sqrt{bx+ax^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b+ax^2}\right)\text{Subst}\left(\int\left(\frac{1}{\sqrt{b+ax^4}}+\frac{2b^2+cx^4}{\sqrt{b+ax^4}\left(-b^2+a^2x^8\right)}\right)dx,x,\sqrt{x}\right)}{\sqrt{bx+ax^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b+ax^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}dx,x,\sqrt{x}\right)}{\sqrt{bx+ax^3}} + \frac{\left(2\sqrt{x}\sqrt{b+ax^2}\right)\text{Subst}\left(\int -\right.}{\sqrt{bx+}}$$

$$= \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} + \frac{\left(2\sqrt{x}\sqrt{b+ax^2}\right)\text{Sul}}{}$$

$$= -\frac{(2ab-c)x}{2ab\sqrt{bx+ax^3}} + \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} + \cdots$$

$$= -\frac{(2ab-c)x}{2ab\sqrt{bx+ax^3}} + \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} - \cdots$$

$$= -\frac{(2ab-c)x}{2ab\sqrt{bx+ax^3}} + \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} - \cdots$$

$$= -\frac{(2ab-c)x}{2ab\sqrt{bx+ax^3}} + \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} - \cdots$$

$$= -\frac{(2ab-c)x}{2ab\sqrt{bx+ax^3}} + \frac{\sqrt{x}\left(\sqrt{b}+\sqrt{a}x\right)\sqrt{\frac{b+ax^2}{\left(\sqrt{b}+\sqrt{a}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx+ax^3}} - \cdots$$

$$= -\frac{(2ab-c)x}{2ab\sqrt{bx+ax^3}} - \frac{(2ab+c)\sqrt{x}\sqrt{b+ax^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{b+ax^2}}\right)}{4\sqrt{2}a^{5/4}b^{5/4}\sqrt{bx+ax^3}} - \frac{(2ab+c)\sqrt{\cdot}}{4}$$

Mathematica [C] time = 0.70, size = 105, normalized size = 0.63

$$-\frac{x\left((2ab+c)\left(ax^2+b\right)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{ax^2}{b},\frac{ax^2}{b}\right)+b(2ab-c)\sqrt{\frac{ax^2}{b}+1}\right)}{2ab^2\sqrt{x\left(ax^2+b\right)}\sqrt{\frac{ax^2}{b}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b^2 + c*x^2 + a^2*x^4)/(Sqrt[b*x + a*x^3]*(-b^2 + a^2*x^4)),x]
```

```
[Out] -1/2*(x*(b*(2*a*b - c)*Sqrt[1 + (a*x^2)/b] + (2*a*b + c)*(b + a*x^2)*Appell
F1[1/4, -1/2, 1, 5/4, -((a*x^2)/b), (a*x^2)/b]))/(a*b^2*Sqrt[x*(b + a*x^2)]
*Sqrt[1 + (a*x^2)/b])
```

IntegrateAlgebraic [A] time = 0.64, size = 167, normalized size = 1.00

$$-\frac{(2ab+c)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{4\sqrt{2}a^{5/4}b^{5/4}}-\frac{(2ab+c)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax^3+bx}}{ax^2+b}\right)}{4\sqrt{2}a^{5/4}b^{5/4}}+\frac{(c-2ab)\sqrt{ax^3+bx}}{2ab\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^2 + c*x^2 + a^2*x^4)/(Sqrt[b*x + a*x^3]*(-b^2 + a^2*x
^4)),x]
```

```
[Out] ((-2*a*b + c)*Sqrt[b*x + a*x^3])/(2*a*b*(b + a*x^2)) - ((2*a*b + c)*ArcTan[
(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3])/(b + a*x^2)])/(4*Sqrt[2]*a^(5/4
)*b^(5/4)) - ((2*a*b + c)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[b*x + a*x^3
])/(b + a*x^2)])/(4*Sqrt[2]*a^(5/4)*b^(5/4))
```

fricas [B] time = 0.92, size = 1675, normalized size = 10.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^4+c*x^2+b^2)/(a*x^3+b*x)^(1/2)/(a^2*x^4-b^2),x, algorithm=
"fricas")
```

```
[Out] -1/16*(4*(1/4)^(1/4)*(a^2*b*x^2 + a*b^2)*((16*a^4*b^4 + 32*a^3*b^3*c + 24*a
^2*b^2*c^2 + 8*a*b*c^3 + c^4)/(a^5*b^5))^(1/4)*arctan(1/2*sqrt(a*x^3 + b*x)
*(4*(1/4)^(3/4)*(8*a^7*b^7 + 12*a^6*b^6*c + 6*a^5*b^5*c^2 + a^4*b^4*c^3)*x*
((16*a^4*b^4 + 32*a^3*b^3*c + 24*a^2*b^2*c^2 + 8*a*b*c^3 + c^4)/(a^5*b^5))^
(3/4) + sqrt(64*a^6*b^6 + 192*a^5*b^5*c + 240*a^4*b^4*c^2 + 160*a^3*b^3*c^3
 + 60*a^2*b^2*c^4 + 12*a*b*c^5 + c^6)*(4*(1/4)^(3/4)*a^4*b^4*x*((16*a^4*b^4
 + 32*a^3*b^3*c + 24*a^2*b^2*c^2 + 8*a*b*c^3 + c^4)/(a^5*b^5))^(3/4) + (1/4
)^(1/4)*(4*a^3*b^4 + 4*a^2*b^3*c + a*b^2*c^2 + (4*a^4*b^3 + 4*a^3*b^2*c + a
^2*b*c^2)*x^2)*((16*a^4*b^4 + 32*a^3*b^3*c + 24*a^2*b^2*c^2 + 8*a*b*c^3 + c
^4)/(a^5*b^5))^(1/4)) - (1/4)^(1/4)*(32*a^6*b^7 + 80*a^5*b^6*c + 80*a^4*b^5
*c^2 + 40*a^3*b^4*c^3 + 10*a^2*b^3*c^4 + a*b^2*c^5 + (32*a^7*b^6 + 80*a^6*b
^5*c + 80*a^5*b^4*c^2 + 40*a^4*b^3*c^3 + 10*a^3*b^2*c^4 + a^2*b*c^5)*x^2)*(
(16*a^4*b^4 + 32*a^3*b^3*c + 24*a^2*b^2*c^2 + 8*a*b*c^3 + c^4)/(a^5*b^5))^(
1/4))/((64*a^7*b^6 + 192*a^6*b^5*c + 240*a^5*b^4*c^2 + 160*a^4*b^3*c^3 + 60
*a^3*b^2*c^4 + 12*a^2*b*c^5 + a*c^6)*x^3 + (64*a^6*b^7 + 192*a^5*b^6*c + 24
0*a^4*b^5*c^2 + 160*a^3*b^4*c^3 + 60*a^2*b^3*c^4 + 12*a*b^2*c^5 + b*c^6)*x)
) + (1/4)^(1/4)*(a^2*b*x^2 + a*b^2)*((16*a^4*b^4 + 32*a^3*b^3*c + 24*a^2*b^
2*c^2 + 8*a*b*c^3 + c^4)/(a^5*b^5))^(1/4)*log((8*a^3*b^5 + 12*a^2*b^4*c + 6
*a*b^3*c^2 + b^2*c^3 + (8*a^5*b^3 + 12*a^4*b^2*c + 6*a^3*b*c^2 + a^2*c^3)*x
^4 + 6*(8*a^4*b^4 + 12*a^3*b^3*c + 6*a^2*b^2*c^2 + a*b*c^3)*x^2 + 8*sqrt(a*
x^3 + b*x)*((1/4)^(1/4)*(4*a^4*b^4 + 4*a^3*b^3*c + a^2*b^2*c^2)*x*((16*a^4*
b^4 + 32*a^3*b^3*c + 24*a^2*b^2*c^2 + 8*a*b*c^3 + c^4)/(a^5*b^5))^(1/4) + (
1/4)^(3/4)*(a^5*b^4*x^2 + a^4*b^5)*((16*a^4*b^4 + 32*a^3*b^3*c + 24*a^2*b^2
*c^2 + 8*a*b*c^3 + c^4)/(a^5*b^5))^(3/4)) + 4*((2*a^5*b^4 + a^4*b^3*c)*x^3
+ (2*a^4*b^5 + a^3*b^4*c)*x)*sqrt((16*a^4*b^4 + 32*a^3*b^3*c + 24*a^2*b^2*c
^2 + 8*a*b*c^3 + c^4)/(a^5*b^5)))/(a^2*x^4 - 2*a*b*x^2 + b^2)) - (1/4)^(1/4
)*(a^2*b*x^2 + a*b^2)*((16*a^4*b^4 + 32*a^3*b^3*c + 24*a^2*b^2*c^2 + 8*a*b*
c^3 + c^4)/(a^5*b^5))^(1/4)*log((8*a^3*b^5 + 12*a^2*b^4*c + 6*a*b^3*c^2 + b
^2*c^3 + (8*a^5*b^3 + 12*a^4*b^2*c + 6*a^3*b*c^2 + a^2*c^3)*x^4 + 6*(8*a^4*
b^4 + 12*a^3*b^3*c + 6*a^2*b^2*c^2 + a*b*c^3)*x^2 - 8*sqrt(a*x^3 + b*x)*((1
/4)^(1/4)*(4*a^4*b^4 + 4*a^3*b^3*c + a^2*b^2*c^2)*x*((16*a^4*b^4 + 32*a^3*b
^3*c + 24*a^2*b^2*c^2 + 8*a*b*c^3 + c^4)/(a^5*b^5))^(1/4) + (1/4)^(3/4)*(a^
5*b^4*x^2 + a^4*b^5)*((16*a^4*b^4 + 32*a^3*b^3*c + 24*a^2*b^2*c^2 + 8*a*b*c
```

```
^3 + c^4)/(a^5*b^5))^(3/4)) + 4*((2*a^5*b^4 + a^4*b^3*c)*x^3 + (2*a^4*b^5 +
 a^3*b^4*c)*x)*sqrt((16*a^4*b^4 + 32*a^3*b^3*c + 24*a^2*b^2*c^2 + 8*a*b*c^3
 + c^4)/(a^5*b^5)))/(a^2*x^4 - 2*a*b*x^2 + b^2)) + 8*sqrt(a*x^3 + b*x)*(2*a
*b - c))/(a^2*b*x^2 + a*b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2x^4+cx^2+b^2}{\left(a^2x^4-b^2\right)\sqrt{ax^3+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^4+c*x^2+b^2)/(a*x^3+b*x)^(1/2)/(a^2*x^4-b^2),x, algorithm=
"giac")
```

```
[Out] integrate((a^2*x^4 + c*x^2 + b^2)/((a^2*x^4 - b^2)*sqrt(a*x^3 + b*x)), x)
```

maple [C] time = 0.02, size = 567, normalized size = 3.40

$$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{ax^3+bx}}+\frac{(2ab+c)\left(\frac{\sqrt{-ab}\,\sqrt{\frac{xa}{\sqrt{-ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{2\sqrt{ab}\,a\sqrt{ax^3+bx}\left(-\frac{\sqrt{-ab}}{a}-\frac{\sqrt{ab}}{a}\right)}-\frac{\sqrt{-ab}\,\sqrt{\frac{xa}{\sqrt{-ab}}+1}\,\sqrt{-\frac{2xa}{\sqrt{-ab}}+2}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},-\frac{\sqrt{-ab}}{a\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)},\frac{\sqrt{2}}{2}\right)}{2\sqrt{ab}\,a\sqrt{ax^3+bx}\left(-\frac{\sqrt{-ab}}{a}+\frac{\sqrt{ab}}{a}\right)}\right)}{2a}+\frac{(-2ab+c)\left(\frac{x}{b\sqrt{\left(x^2+\frac{b}{a}\right)xa}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{xa}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{2ba\sqrt{ax^3+bx}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^4+c*x^2+b^2)/(a*x^3+b*x)^(1/2)/(a^2*x^4-b^2),x)
```

```
[Out] 1/a*(-a*b)^(1/2)*((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x-1/a*(-a
*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2
)*EllipticF(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/2*(2
*a*b+c)/a*(1/2/(a*b)^(1/2)/a*(-a*b)^(1/2)*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*
a/(-a*b)^(1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(
-a*b)^(1/2)-1/a*(a*b)^(1/2))*EllipticPi(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2
))^(1/2),-1/a*(-a*b)^(1/2)/(-1/a*(-a*b)^(1/2)-1/a*(a*b)^(1/2)),1/2*2^(1/2))
-1/2/(a*b)^(1/2)/a*(-a*b)^(1/2)*(x*a/(-a*b)^(1/2)+1)^(1/2)*(-2*x*a/(-a*b)^(
1/2)+2)^(1/2)*(-x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)/(-1/a*(-a*b)^(1/2
)+1/a*(a*b)^(1/2))*EllipticPi(((x+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),-
1/a*(-a*b)^(1/2)/(-1/a*(-a*b)^(1/2)+1/a*(a*b)^(1/2)),1/2*2^(1/2)))+1/2*(-2*
a*b+c)/a*(x/b/((x^2+b/a)*x*a)^(1/2)+1/2/b/a*(-a*b)^(1/2)*((x+1/a*(-a*b)^(1/
2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-
x*a/(-a*b)^(1/2))^(1/2)/(a*x^3+b*x)^(1/2)*EllipticF(((x+1/a*(-a*b)^(1/2))*a
/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2x^4+cx^2+b^2}{\left(a^2x^4-b^2\right)\sqrt{ax^3+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^4+c*x^2+b^2)/(a*x^3+b*x)^(1/2)/(a^2*x^4-b^2),x, algorithm=
"maxima")
```

```
[Out] integrate((a^2*x^4 + c*x^2 + b^2)/((a^2*x^4 - b^2)*sqrt(a*x^3 + b*x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(c*x^2 + b^2 + a^2*x^4)/((b^2 - a^2*x^4)*(b*x + a*x^3)^(1/2)),x)
```

[Out] \text{Hanged}

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2x^4 + b^2 + cx^2}{\sqrt{x\left(ax^2 + b\right)}\left(ax^2 - b\right)\left(ax^2 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**4+c*x**2+b**2)/(a*x**3+b*x)**(1/2)/(a**2*x**4-b**2),x)
```

```
[Out] Integral((a**2*x**4 + b**2 + c*x**2)/(sqrt(x*(a*x**2 + b))*(a*x**2 - b)*(a*
x**2 + b)), x)
```

3.1845 $\int \frac{1}{x^{11}\left(-b+ax^5\right)^{3/4}}\,dx$

Optimal. Leaf size=167

$$-\frac{21a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^5-b}}{\sqrt{ax^5-b}-\sqrt{b}}\right)}{80\sqrt{2}\,b^{11/4}}+\frac{21a^2\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^5-b}}\right)}{80\sqrt{2}\,b^{11/4}}+\frac{\sqrt[4]{ax^5-b}\left(7ax^5+4b\right)}{40b^2x^{10}}$$

Rubi [A] time = 0.25, antiderivative size = 260, normalized size of antiderivative = 1.56, number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 51, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{21a^2\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^5-b}+\sqrt{ax^5-b}+\sqrt{b}\right)}{160\sqrt{2}\,b^{11/4}}+\frac{21a^2\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^5-b}+\sqrt{ax^5-b}+\sqrt{b}\right)}{160\sqrt{2}\,b^{11/4}}-\frac{21a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{ax^5-b}}{\sqrt[4]{b}}\right)}{80\sqrt{2}\,b^{11/4}}+\frac{21a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax^5-b}}{\sqrt[4]{b}}+1\right)}{80\sqrt{2}\,b^{11/4}}+\frac{7a\sqrt[4]{ax^5-b}}{40b^2x^5}+\frac{\sqrt[4]{ax^5-b}}{10bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^11*(-b + a*x^5)^(3/4)),x]
```

```
[Out] (-b + a*x^5)^(1/4)/(10*b*x^10) + (7*a*(-b + a*x^5)^(1/4))/(40*b^2*x^5) - (2
1*a^2*ArcTan[1 - (Sqrt[2]*(-b + a*x^5)^(1/4))/b^(1/4)])/(80*Sqrt[2]*b^(11/4
)) + (21*a^2*ArcTan[1 + (Sqrt[2]*(-b + a*x^5)^(1/4))/b^(1/4)])/(80*Sqrt[2]*
b^(11/4)) - (21*a^2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*(-b + a*x^5)^(1/4) + Sqrt
[-b + a*x^5]])/(160*Sqrt[2]*b^(11/4)) + (21*a^2*Log[Sqrt[b] + Sqrt[2]*b^(1/
4)*(-b + a*x^5)^(1/4) + Sqrt[-b + a*x^5]])/(160*Sqrt[2]*b^(11/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{11}\left(-b+ax^5\right)^{3/4}}\,dx = \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x^3(-b+ax)^{3/4}}\,dx,x,x^5\right)$$

$$= \frac{\sqrt[4]{-b+ax^5}}{10bx^{10}} + \frac{(7a)\,\text{Subst}\left(\int \frac{1}{x^2(-b+ax)^{3/4}}\,dx,x,x^5\right)}{40b}$$

$$= \frac{\sqrt[4]{-b+ax^5}}{10bx^{10}} + \frac{7a\sqrt[4]{-b+ax^5}}{40b^2x^5} + \frac{\left(21a^2\right)\text{Subst}\left(\int \frac{1}{x(-b+ax)^{3/4}}\,dx,x,x^5\right)}{160b^2}$$

$$= \frac{\sqrt[4]{-b+ax^5}}{10bx^{10}} + \frac{7a\sqrt[4]{-b+ax^5}}{40b^2x^5} + \frac{(21a)\,\text{Subst}\left(\int \frac{1}{\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{-b+ax^5}\right)}{40b^2}$$

$$= \frac{\sqrt[4]{-b+ax^5}}{10bx^{10}} + \frac{7a\sqrt[4]{-b+ax^5}}{40b^2x^5} + \frac{(21a)\,\text{Subst}\left(\int \frac{\sqrt{b}-x^2}{\frac{b}{a}+\frac{x^4}{a}}\,dx,x,\sqrt[4]{-b+ax^5}\right)}{80b^{5/2}} + \frac{(21a)\,\text{Sub}}{}$$

$$= \frac{\sqrt[4]{-b+ax^5}}{10bx^{10}} + \frac{7a\sqrt[4]{-b+ax^5}}{40b^2x^5} - \frac{\left(21a^2\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{b}+2x}{-\sqrt{b}-\sqrt{2}\sqrt[4]{b}x-x^2}\,dx,x,\sqrt[4]{-b+ax^5}\right)}{160\sqrt{2}\,b^{11/4}} -$$

$$= \frac{\sqrt[4]{-b+ax^5}}{10bx^{10}} + \frac{7a\sqrt[4]{-b+ax^5}}{40b^2x^5} - \frac{21a^2\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{-b+ax^5}+\sqrt{-b+ax^5}\right)}{160\sqrt{2}\,b^{11/4}} +$$

$$= \frac{\sqrt[4]{-b+ax^5}}{10bx^{10}} + \frac{7a\sqrt[4]{-b+ax^5}}{40b^2x^5} - \frac{21a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{-b+ax^5}}{\sqrt[4]{b}}\right)}{80\sqrt{2}\,b^{11/4}} + \frac{21a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{\ldots}}{\ldots}\right)}{80\sqrt{2}\,b^{11/4}}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.25

$$\frac{4a^2\sqrt[4]{ax^5-b}\;{}_2F_1\left(\frac{1}{4},3;\frac{5}{4};1-\frac{ax^5}{b}\right)}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^11*(-b + a*x^5)^(3/4)),x]
```

```
[Out] (4*a^2*(-b + a*x^5)^(1/4)*Hypergeometric2F1[1/4, 3, 5/4, 1 - (a*x^5)/b])/(5
*b^3)
```

IntegrateAlgebraic [A] time = 0.21, size = 166, normalized size = 0.99

$$\frac{21a^2\tan^{-1}\left(\frac{\frac{\sqrt{ax^5-b}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt[4]{ax^5-b}}\right)}{80\sqrt{2}\,b^{11/4}} + \frac{21a^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{ax^5-b}}{\sqrt{ax^5-b}+\sqrt{b}}\right)}{80\sqrt{2}\,b^{11/4}} + \frac{\sqrt[4]{ax^5-b}\left(7ax^5+4b\right)}{40b^2x^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^11*(-b + a*x^5)^(3/4)),x]
```

```
[Out] ((-b + a*x^5)^(1/4)*(4*b + 7*a*x^5))/(40*b^2*x^10) + (21*a^2*ArcTan[(-(b^(1
/4)/Sqrt[2]) + Sqrt[-b + a*x^5]/(Sqrt[2]*b^(1/4)))/(-b + a*x^5)^(1/4)])/(80
*Sqrt[2]*b^(11/4)) + (21*a^2*ArcTanh[(Sqrt[2]*b^(1/4)*(-b + a*x^5)^(1/4))/(
Sqrt[b] + Sqrt[-b + a*x^5])])/(80*Sqrt[2]*b^(11/4))
```

fricas [A] time = 0.48, size = 234, normalized size = 1.40

$$\frac{84 b^{2} x^{10} \left(-\frac{a^{8}}{b^{11}}\right)^{\frac{1}{4}} \arctan\left(-\frac{\left(a x^{5}-b\right)^{\frac{1}{4}} a^{2} b^{8} \left(-\frac{a^{8}}{b^{11}}\right)^{\frac{3}{4}}-\sqrt{b^{6} \sqrt{-\frac{a^{8}}{b^{11}}}+\sqrt{a x^{5}-b}\, a^{4}}\, b^{8} \left(-\frac{a^{8}}{b^{11}}\right)^{\frac{3}{4}}}{a^{8}}\right)+21 b^{2} x^{10} \left(-\frac{a^{8}}{b^{11}}\right)^{\frac{1}{4}} \log\left(21 b^{3} \left(-\frac{a^{8}}{b^{11}}\right)^{\frac{1}{4}}+21 \left(a x^{5}-b\right)^{\frac{1}{4}} a^{2}\right)-21 b^{2} x^{10} \left(-\frac{a^{8}}{b^{11}}\right)^{\frac{1}{4}} \log\left(-21 b^{3} \left(-\frac{a^{8}}{b^{11}}\right)^{\frac{1}{4}}+21 \left(a x^{5}-b\right)^{\frac{1}{4}} a^{2}\right)+4 \left(7 a x^{5}+4 b\right) \left(a x^{5}-b\right)^{\frac{1}{4}}}{160 b^{2} x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(a*x^5-b)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/160*(84*b^2*x^10*(-a^8/b^11)^(1/4)*arctan(-((a*x^5 - b)^(1/4)*a^2*b^8*(-a
^8/b^11)^(3/4) - sqrt(b^6*sqrt(-a^8/b^11) + sqrt(a*x^5 - b)*a^4)*b^8*(-a^8/
b^11)^(3/4))/a^8) + 21*b^2*x^10*(-a^8/b^11)^(1/4)*log(21*b^3*(-a^8/b^11)^(1
/4) + 21*(a*x^5 - b)^(1/4)*a^2) - 21*b^2*x^10*(-a^8/b^11)^(1/4)*log(-21*b^3
*(-a^8/b^11)^(1/4) + 21*(a*x^5 - b)^(1/4)*a^2) + 4*(7*a*x^5 + 4*b)*(a*x^5 -
 b)^(1/4))/(b^2*x^10)
```

giac [A] time = 0.26, size = 224, normalized size = 1.34

$$\frac{\frac{42 \sqrt{2} a^{3} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} b^{\frac{1}{4}}+2 \left(a x^{5}-b\right)^{\frac{1}{4}}\right)}{2 b^{\frac{1}{4}}}\right)}{b^{\frac{11}{4}}}+\frac{42 \sqrt{2} a^{3} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} b^{\frac{1}{4}}-2 \left(a x^{5}-b\right)^{\frac{1}{4}}\right)}{2 b^{\frac{1}{4}}}\right)}{b^{\frac{11}{4}}}+\frac{21 \sqrt{2} a^{3} \log\left(\sqrt{2} \left(a x^{5}-b\right)^{\frac{1}{4}} b^{\frac{1}{4}}+\sqrt{a x^{5}-b}+\sqrt{b}\right)}{b^{\frac{11}{4}}}-\frac{21 \sqrt{2} a^{3} \log\left(-\sqrt{2} \left(a x^{5}-b\right)^{\frac{1}{4}} b^{\frac{1}{4}}+\sqrt{a x^{5}-b}+\sqrt{b}\right)}{b^{\frac{11}{4}}}+\frac{8 \left(7 \left(a x^{5}-b\right)^{\frac{5}{4}} a^{3}+11 \left(a x^{5}-b\right)^{\frac{1}{4}} a^{3} b\right)}{a^{2} b^{2} x^{10}}}{320 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(a*x^5-b)^(3/4),x, algorithm="giac")
```

```
[Out] 1/320*(42*sqrt(2)*a^3*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(a*x^5 - b)^(
1/4))/b^(1/4))/b^(11/4) + 42*sqrt(2)*a^3*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/
4) - 2*(a*x^5 - b)^(1/4))/b^(1/4))/b^(11/4) + 21*sqrt(2)*a^3*log(sqrt(2)*(a
*x^5 - b)^(1/4)*b^(1/4) + sqrt(a*x^5 - b) + sqrt(b))/b^(11/4) - 21*sqrt(2)*
a^3*log(-sqrt(2)*(a*x^5 - b)^(1/4)*b^(1/4) + sqrt(a*x^5 - b) + sqrt(b))/b^(
11/4) + 8*(7*(a*x^5 - b)^(5/4)*a^3 + 11*(a*x^5 - b)^(1/4)*a^3*b)/(a^2*b^2*x
^10))/a
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{11} \left(a\,x^{5}-b\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^11/(a*x^5-b)^(3/4),x)
```

```
[Out] int(1/x^11/(a*x^5-b)^(3/4),x)
```

maxima [A] time = 0.49, size = 250, normalized size = 1.50

$$\frac{7 \left(a x^{5}-b\right)^{\frac{5}{4}} a^{2}+11 \left(a x^{5}-b\right)^{\frac{1}{4}} a^{2} b}{40 \left(\left(a x^{5}-b\right)^{2} b^{2}+2 \left(a x^{5}-b\right) b^{3}+b^{4}\right)}+\frac{21 \left(\frac{2 \sqrt{2} a^{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} b^{\frac{1}{4}}+2 \left(a x^{5}-b\right)^{\frac{1}{4}}\right)}{2 b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{2 \sqrt{2} a^{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} b^{\frac{1}{4}}-2 \left(a x^{5}-b\right)^{\frac{1}{4}}\right)}{2 b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{\sqrt{2} a^{2} \log\left(\sqrt{2} \left(a x^{5}-b\right)^{\frac{1}{4}} b^{\frac{1}{4}}+\sqrt{a x^{5}-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2} a^{2} \log\left(-\sqrt{2} \left(a x^{5}-b\right)^{\frac{1}{4}} b^{\frac{1}{4}}+\sqrt{a x^{5}-b}+\sqrt{b}\right)}{b^{\frac{3}{4}}}\right)}{320 b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(a*x^5-b)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/40*(7*(a*x^5 - b)^(5/4)*a^2 + 11*(a*x^5 - b)^(1/4)*a^2*b)/((a*x^5 - b)^2*
b^2 + 2*(a*x^5 - b)*b^3 + b^4) + 21/320*(2*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(
sqrt(2)*b^(1/4) + 2*(a*x^5 - b)^(1/4))/b^(1/4))/b^(3/4) + 2*sqrt(2)*a^2*arc
tan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(a*x^5 - b)^(1/4))/b^(1/4))/b^(3/4) +
```

```
sqrt(2)*a^2*log(sqrt(2)*(a*x^5 - b)^(1/4)*b^(1/4) + sqrt(a*x^5 - b) + sqrt
(b))/b^(3/4) - sqrt(2)*a^2*log(-sqrt(2)*(a*x^5 - b)^(1/4)*b^(1/4) + sqrt(a*
x^5 - b) + sqrt(b))/b^(3/4))/b^2
```

mupad [B] time = 1.47, size = 98, normalized size = 0.59

$$\frac{11\left(a\,x^5-b\right)^{1/4}}{40\,b\,x^{10}}+\frac{7\left(a\,x^5-b\right)^{5/4}}{40\,b^2\,x^{10}}-\frac{21\,a^2\,\mathrm{atan}\left(\frac{\left(a\,x^5-b\right)^{1/4}}{(-b)^{1/4}}\right)}{80\,(-b)^{11/4}}+\frac{a^2\,\mathrm{atan}\left(\frac{\left(a\,x^5-b\right)^{1/4}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,21\mathrm{i}}{80\,(-b)^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11*(a*x^5 - b)^(3/4)),x)
```

```
[Out] (11*(a*x^5 - b)^(1/4))/(40*b*x^10) + (7*(a*x^5 - b)^(5/4))/(40*b^2*x^10) -
(21*a^2*atan((a*x^5 - b)^(1/4)/(-b)^(1/4)))/(80*(-b)^(11/4)) + (a^2*atan(((
a*x^5 - b)^(1/4)*1i)/(-b)^(1/4))*21i)/(80*(-b)^(11/4))
```

sympy [C] time = 1.88, size = 42, normalized size = 0.25

$$-\frac{\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\right|\frac{be^{2i\pi}}{ax^5}\right)}{5a^{\frac{3}{4}}x^{\frac{55}{4}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**11/(a*x**5-b)**(3/4),x)
```

```
[Out] -gamma(11/4)*hyper((3/4, 11/4), (15/4,), b*exp_polar(2*I*pi)/(a*x**5))/(5*a
**(3/4)*x**(55/4)*gamma(15/4))
```

3.1846 $\int \frac{1+x^6}{\sqrt[3]{-x^2+x^4}\left(-1+x^6\right)} dx$

Optimal. Leaf size=167

$$-\frac{\left(x^4-x^2\right)^{2/3}}{x\left(x^2-1\right)}-\frac{2\tan^{-1}\left(\frac{\sqrt[6]{3}x}{\sqrt[3]{x^4-x^2}}\right)}{3\sqrt[6]{3}}-\frac{\tan^{-1}\left(\frac{3^{5/6}x\sqrt[3]{x^4-x^2}}{3^{2/3}\left(x^4-x^2\right)^{2/3}-3x^2}\right)}{3\sqrt[6]{3}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[3]{3}}+\frac{\left(x^4-x^2\right)^{2/3}}{3^{2/3}}}{x\sqrt[3]{x^4-x^2}}\right)}{3^{2/3}}$$

Rubi [F] time = 2.64, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x^6}{\sqrt[3]{-x^2+x^4}\left(-1+x^6\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^6)/((-x^2 + x^4)^(1/3)*(-1 + x^6)),x]
```

```
[Out] (-2*x*(1 - x^2)^(1/3)*AppellF1[1/6, 1/3, 1, 7/6, x^2, (-2*x^2)/(1 - I*Sqrt[
3])])/(-x^2 + x^4)^(1/3) - (2*x*(1 - x^2)^(1/3)*AppellF1[1/6, 1/3, 1, 7/6,
x^2, (-2*x^2)/(1 + I*Sqrt[3])])/(-x^2 + x^4)^(1/3) + (3*x*(1 - x^2)^(1/3)*H
ypergeometric2F1[1/6, 1/3, 7/6, x^2])/(-x^2 + x^4)^(1/3) + (x^(2/3)*(-1 + x
^2)^(1/3)*Defer[Subst][Defer[Int][1/((-1 + x)*(-1 + x^6)^(1/3)), x], x, x^(
1/3)])/(3*(-x^2 + x^4)^(1/3)) - (x^(2/3)*(-1 + x^2)^(1/3)*Defer[Subst][Defe
r[Int][1/((1 + x)*(-1 + x^6)^(1/3)), x], x, x^(1/3)])/(3*(-x^2 + x^4)^(1/3)
) + ((1 + I*Sqrt[3])*x^(2/3)*(-1 + x^2)^(1/3)*Defer[Subst][Defer[Int][1/((-
1 - I*Sqrt[3] + 2*x)*(-1 + x^6)^(1/3)), x], x, x^(1/3)])/(3*(-x^2 + x^4)^(1
/3)) - ((1 - I*Sqrt[3])*x^(2/3)*(-1 + x^2)^(1/3)*Defer[Subst][Defer[Int][1/
((1 - I*Sqrt[3] + 2*x)*(-1 + x^6)^(1/3)), x], x, x^(1/3)])/(3*(-x^2 + x^4)^
(1/3)) + ((1 - I*Sqrt[3])*x^(2/3)*(-1 + x^2)^(1/3)*Defer[Subst][Defer[Int][
1/((-1 + I*Sqrt[3] + 2*x)*(-1 + x^6)^(1/3)), x], x, x^(1/3)])/(3*(-x^2 + x^
4)^(1/3)) - ((1 + I*Sqrt[3])*x^(2/3)*(-1 + x^2)^(1/3)*Defer[Subst][Defer[In
t][1/((1 + I*Sqrt[3] + 2*x)*(-1 + x^6)^(1/3)), x], x, x^(1/3)])/(3*(-x^2 +
x^4)^(1/3))
```

Rubi steps

$$\int \frac{1+x^6}{\sqrt[3]{-x^2+x^4}\left(-1+x^6\right)}dx = \frac{\left(x^{2/3}\sqrt[3]{-1+x^2}\right)\int \frac{1+x^6}{x^{2/3}\sqrt[3]{-1+x^2}\left(-1+x^6\right)}dx}{\sqrt[3]{-x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{1+x^{18}}{\sqrt[3]{-1+x^6}\left(-1+x^{18}\right)}dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{-1+x^6}} + \frac{2}{\sqrt[3]{-1+x^6}\left(-1+x^{18}\right)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x^6}}dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-x^2+x^4}} + \frac{\left(6x^{2/3}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt[3]{-}}\right.}{\sqrt[3]{-x^2+x}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{1-x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x^6}}dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-x^2+x^4}} + \frac{\left(6x^{2/3}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \left(\frac{}{9(-1+}\right.\right.}{}$$

$$= \frac{3x\sqrt[3]{1-x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^2\right)}{\sqrt[3]{-x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{-2+x}{\left(1-x+x^2\right)\sqrt[3]{-1+x^6}}dx, x, \sqrt[3]{}\right.}{3\sqrt[3]{-x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1-x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^2\right)}{\sqrt[3]{-x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \left(\frac{1+i\sqrt{3}}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{-1+x^6}} + \right.\right.}{3\sqrt[3]{-x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1-x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^2\right)}{\sqrt[3]{-x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{-1+x^6}}dx, x, \sqrt[3]{x}\right.}{3\sqrt[3]{-x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1-x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^2\right)}{\sqrt[3]{-x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{-1+x^6}}dx, x, \sqrt[3]{x}\right.}{3\sqrt[3]{-x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1-x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^2\right)}{\sqrt[3]{-x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{-1+x^6}}dx, x, \sqrt[3]{x}\right.}{3\sqrt[3]{-x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1-x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^2\right)}{\sqrt[3]{-x^2+x^4}} + \frac{\left(\left(-i-\sqrt{3}\right)\left(1-i\sqrt{3}\right)x^{2/3}\sqrt[3]{1-x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt[3]{1-}}\right.}{2\sqrt[3]{-x^2+x^4}}$$

$$= -\frac{2x\sqrt[3]{1-x^2}\,F_1\left(\frac{1}{6};\frac{1}{3},1;\frac{7}{6};x^2,-\frac{2x^2}{1-i\sqrt{3}}\right)}{\sqrt[3]{-x^2+x^4}} - \frac{2x\sqrt[3]{1-x^2}\,F_1\left(\frac{1}{6};\frac{1}{3},1;\frac{7}{6};x^2,-\frac{2x^2}{1+i\sqrt{3}}\right)}{\sqrt[3]{-x^2+x^4}} +$$

$$= -\frac{2x\sqrt[3]{1-x^2}\,F_1\left(\frac{1}{6};\frac{1}{3},1;\frac{7}{6};x^2,-\frac{2x^2}{1-i\sqrt{3}}\right)}{\sqrt[3]{-x^2+x^4}} - \frac{2x\sqrt[3]{1-x^2}\,F_1\left(\frac{1}{6};\frac{1}{3},1;\frac{7}{6};x^2,-\frac{2x^2}{1+i\sqrt{3}}\right)}{\sqrt[3]{-x^2+x^4}} +$$

Mathematica [F] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{1+x^6}{\sqrt[3]{-x^2+x^4}\left(-1+x^6\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^6)/((-x^2 + x^4)^(1/3)*(-1 + x^6)),x]
```

```
[Out] Integrate[(1 + x^6)/((-x^2 + x^4)^(1/3)*(-1 + x^6)), x]
```

IntegrateAlgebraic [A] time = 0.54, size = 167, normalized size = 1.00

$$-\frac{\left(x^4-x^2\right)^{2/3}}{x\left(x^2-1\right)}-\frac{2\tan^{-1}\left(\frac{\sqrt[6]{3}x}{\sqrt[3]{x^4-x^2}}\right)}{3\sqrt[6]{3}}-\frac{\tan^{-1}\left(\frac{3^{5/6}x\sqrt[3]{x^4-x^2}}{3^{2/3}\left(x^4-x^2\right)^{2/3}-3x^2}\right)}{3\sqrt[6]{3}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[3]{3}}+\frac{\left(x^4-x^2\right)^{2/3}}{3^{2/3}}}{x\sqrt[3]{x^4-x^2}}\right)}{3^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^6)/((-x^2 + x^4)^(1/3)*(-1 + x^6)),x]
```

```
[Out] -((-x^2 + x^4)^(2/3)/(x*(-1 + x^2))) - (2*ArcTan[(3^(1/6)*x)/(-x^2 + x^4)^(
1/3)])/(3*3^(1/6)) - ArcTan[(3^(5/6)*x*(-x^2 + x^4)^(1/3))/(-3*x^2 + 3^(2/3
)*(-x^2 + x^4)^(2/3))]/(3*3^(1/6)) - ArcTanh[(x^2/3^(1/3) + (-x^2 + x^4)^(2
/3)/3^(2/3))/(x*(-x^2 + x^4)^(1/3))]/3^(2/3)
```

fricas [B] time = 6.60, size = 2054, normalized size = 12.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^4-x^2)^(1/3)/(x^6-1),x, algorithm="fricas")
```

```
[Out] 1/72*(16*3^(5/6)*(x^3 - x)*arctan(-1/3*(5200566*3^(5/6)*(x^4 - x^2)^(1/3)*(
960*x^3 + 419*x^2 - 960*x) - 931*3^(5/6)*(2*3^(5/6)*(x^4 - x^2)^(2/3)*(2011
20*x^2 + 1557961*x - 201120) + 2*sqrt(3)*(x^4 - x^2)^(1/3)*(1557961*x^3 - 6
03360*x^2 - 1557961*x) + 175561*3^(1/6)*(x^5 + x^3 + x)) + 5200566*3^(1/6)*
(x^4 - x^2)^(2/3)*(419*x^2 - 2880*x - 419) + 2514*sqrt(3)*(100560*x^5 + 155
7961*x^4 - 502800*x^3 - 1557961*x^2 + 100560*x))/(73560059*x^5 - 9479471040
*x^4 - 367800295*x^3 + 9479471040*x^2 + 73560059*x)) - 8*3^(5/6)*(x^3 - x)*
arctan(1/3*(1862*sqrt(3)*(8*(x^4 - x^2)^(2/3)*(3^(5/6)*(1024558870866960*x^
6 - 2840455578302701*x^5 + 221502398520960*x^4 + 4879497632339105*x^3 - 221
502398520960*x^2 - 2840455578302701*x - 1024558870866960) - 18*3^(5/6)*(305
38688294400*x^6 + 261995401277240*x^5 - 513898276545299*x^4 - 7144764797870
80*x^3 + 513898276545299*x^2 + 261995401277240*x - 30538688294400)) + 2*(x^
4 - x^2)^(1/3)*(sqrt(3)*(9732856787649299*x^7 + 33905739302897760*x^6 - 957
56305292621940*x^5 - 77616433764537120*x^4 + 95756305292621940*x^3 + 339057
39302897760*x^2 - 9732856787649299*x) - 2600283*sqrt(3)*(5229447840*x^7 - 9
925022695*x^6 - 16872865920*x^5 + 22235880413*x^4 + 16872865920*x^3 - 99250
22695*x^2 - 5229447840*x)) + 3^(1/6)*(1509469307073299*x^9 + 65809436013483
840*x^8 - 74031236443268180*x^7 - 233676461263875840*x^6 + 1314583105087300
71*x^5 + 233676461263875840*x^4 - 74031236443268180*x^3 - 65809436013483840
*x^2 + 1509469307073299*x) - 6*3^(1/6)*(864612490925040*x^9 + 4987313340531
01*x^8 + 11166292173984960*x^7 - 27875816538793188*x^6 - 31843321748145360*
x^5 + 27875816538793188*x^4 + 11166292173984960*x^3 - 498731334053101*x^2 +
 864612490925040*x))*sqrt((3^(2/3)*(x^5 + x^3 + x) + 12*3^(2/3)*(x^4 - x^2)
 + 6*(x^4 - x^2)^(2/3)*(3^(1/3)*(x^2 - 1) + 3*3^(1/3)*x) + 18*(x^4 - x^2)^(
1/3)*(x^3 + x^2 - x))/(x^5 + x^3 + x)) + 10401132*(x^4 - x^2)^(2/3)*(3^(1/6
)*(4268995295279*x^6 - 21514334541120*x^5 - 29545286126340*x^4 + 6872675495
9040*x^3 + 29545286126340*x^2 - 21514334541120*x - 4268995295279) - 3*3^(1/
6)*(763467207360*x^6 + 2042237245205*x^5 - 4977855152640*x^4 + 467632693895
3*x^3 + 4977855152640*x^2 + 2042237245205*x - 763467207360)) + 48*sqrt(3)*(
18113631684879580*x^9 + 83939366796831719*x^8 - 489157709353984248O*x^7 -
503636200780990314*x^6 + 1105110840502125612O*x^5 + 503636200780990314*x^4
- 489157709353984248O*x^3 - 83939366796831719*x^2 + 18113631684879588O*x) -
 2600283*sqrt(3)*(3602552045521*x^9 - 9955710305280*x^8 + 17741462866034*x^
7 + 59734261831680*x^6 - 10265061413421*x^5 - 59734261831680*x^4 + 17741462
866034*x^3 + 9955710305280*x^2 + 3602552045521*x) - 10401132*(x^4 - x^2)^(1
```

```
/3)*(3^(5/6)*(2920267143121*x^7 + 2687453530560*x^6 - 2634089693748*x^5 - 1
2246111927360*x^4 + 2634089693748*x^3 + 2687453530560*x^2 - 2920267143121*x
) - 3^(5/6)*(2855342875200*x^7 + 5579433413501*x^6 - 30080363166720*x^5 - 2
3965852712839*x^4 + 30080363166720*x^3 + 5579433413501*x^2 - 2855342875200*
x)))/(6837784281928633319*x^9 - 9417576913539826176O*x^8 - 9681741764124891
7346*x^7 + 565054614812389570560*x^6 + 241499325255998267925*x^5 - 56505461
4812389570560*x^4 - 96817417641248917346*x^3 + 94175769135398261760*x^2 + 6
837784281928633319*x)) - 8*3^(5/6)*(x^3 - x)*arctan(1/3*(1862*sqrt(3)*(8*(x
^4 - x^2)^(2/3)*(3^(5/6)*(1024558870866960*x^6 - 2840455578302701*x^5 + 221
502398520960*x^4 + 4879497632339105*x^3 - 221502398520960*x^2 - 28404555783
02701*x - 1024558870866960) + 18*3^(5/6)*(30538688294400*x^6 + 261995401277
240*x^5 - 513898276545299*x^4 - 714476479787080*x^3 + 513898276545299*x^2 +
 261995401277240*x - 30538688294400)) + 2*(x^4 - x^2)^(1/3)*(sqrt(3)*(97328
56787649299*x^7 + 33905739302897760*x^6 - 95756305292621940*x^5 - 776164337
64537120*x^4 + 95756305292621940*x^3 + 33905739302897760*x^2 - 973285678764
9299*x) + 2600283*sqrt(3)*(5229447840*x^7 - 9925022695*x^6 - 16872865920*x^
5 + 22235880413*x^4 + 16872865920*x^3 - 9925022695*x^2 - 5229447840*x)) + 3
^(1/6)*(1509469307073299*x^9 + 65809436013483840*x^8 - 74031236443268180*x^
7 - 233676461263875840*x^6 + 131458310508730071*x^5 + 233676461263875840*x^
4 - 74031236443268180*x^3 - 65809436013483840*x^2 + 1509469307073299*x) + 6
*3^(1/6)*(864612490925040*x^9 + 498731334053101*x^8 + 11166292173984960*x^7
 - 27875816538793188*x^6 - 31843321748145360*x^5 + 27875816538793188*x^4 +
11166292173984960*x^3 - 498731334053101*x^2 + 864612490925040*x))*sqrt((3^(
2/3)*(x^5 + x^3 + x) - 12*3^(2/3)*(x^4 - x^2) - 6*(x^4 - x^2)^(2/3)*(3^(1/3
)*(x^2 - 1) - 3*3^(1/3)*x) + 18*(x^4 - x^2)^(1/3)*(x^3 - x^2 - x))/(x^5 + x
^3 + x)) + 10401132*(x^4 - x^2)^(2/3)*(3^(1/6)*(4268995295279*x^6 - 2151433
4541120*x^5 - 29545286126340*x^4 + 68726754959040*x^3 + 29545286126340*x^2
- 21514334541120*x - 4268995295279) + 3*3^(1/6)*(763467207360*x^6 + 2042237
245205*x^5 - 4977855152640*x^4 + 4676326938953*x^3 + 4977855152640*x^2 + 20
42237245205*x - 763467207360)) + 48*sqrt(3)*(181136316848795880*x^9 + 83939
366796831719*x^8 - 489157709353984248O*x^7 - 503636200780990314*x^6 + 11051
108405021256120*x^5 + 503636200780990314*x^4 - 4891577093539842480*x^3 - 83
939366796831719*x^2 + 181136316848795880*x) + 2600283*sqrt(3)*(360255204552
1*x^9 - 9955710305280*x^8 + 17741462866034*x^7 + 59734261831680*x^6 - 10265
061413421*x^5 - 59734261831680*x^4 + 17741462866034*x^3 + 9955710305280*x^2
 + 3602552045521*x) + 10401132*(x^4 - x^2)^(1/3)*(3^(5/6)*(2920267143121*x^
7 + 2687453530560*x^6 - 2634089693748*x^5 - 12246111927360*x^4 + 2634089693
748*x^3 + 2687453530560*x^2 - 2920267143121*x) + 3^(5/6)*(2855342875200*x^7
 + 5579433413501*x^6 - 30080363166720*x^5 - 23965852712839*x^4 + 3008036316
6720*x^3 + 5579433413501*x^2 - 2855342875200*x)))/(6837784281928633319*x^9
- 94175769135398261760*x^8 - 96817417641248917346*x^7 + 5650546148123895705
60*x^6 + 241499325255998267925*x^5 - 565054614812389570560*x^4 - 9681741764
1248917346*x^3 + 94175769135398261760*x^2 + 6837784281928633319*x)) - 3*3^(
1/3)*(x^3 - x)*log(10401132*(3^(2/3)*(x^5 + x^3 + x) + 12*3^(2/3)*(x^4 - x^
2) + 6*(x^4 - x^2)^(2/3)*(3^(1/3)*(x^2 - 1) + 3*3^(1/3)*x) + 18*(x^4 - x^2)
^(1/3)*(x^3 + x^2 - x))/(x^5 + x^3 + x)) - 3*3^(1/3)*(x^3 - x)*log(2600283*
(3^(2/3)*(x^5 + x^3 + x) + 12*3^(2/3)*(x^4 - x^2) + 6*(x^4 - x^2)^(2/3)*(3^
(1/3)*(x^2 - 1) + 3*3^(1/3)*x) + 18*(x^4 - x^2)^(1/3)*(x^3 + x^2 - x))/(x^5
 + x^3 + x)) + 3*3^(1/3)*(x^3 - x)*log(10401132*(3^(2/3)*(x^5 + x^3 + x) -
12*3^(2/3)*(x^4 - x^2) - 6*(x^4 - x^2)^(2/3)*(3^(1/3)*(x^2 - 1) - 3*3^(1/3)
*x) + 18*(x^4 - x^2)^(1/3)*(x^3 - x^2 - x))/(x^5 + x^3 + x)) + 3*3^(1/3)*(x
^3 - x)*log(2600283*(3^(2/3)*(x^5 + x^3 + x) - 12*3^(2/3)*(x^4 - x^2) - 6*(
x^4 - x^2)^(2/3)*(3^(1/3)*(x^2 - 1) - 3*3^(1/3)*x) + 18*(x^4 - x^2)^(1/3)*(
x^3 - x^2 - x))/(x^5 + x^3 + x)) - 72*(x^4 - x^2)^(2/3))/(x^3 - x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+1}{\left(x^6-1\right)\left(x^4-x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^4-x^2)^(1/3)/(x^6-1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 1)/((x^6 - 1)*(x^4 - x^2)^(1/3)), x)
```

maple [C] time = 5.16, size = 758, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/(x^4-x^2)^(1/3)/(x^6-1),x)
```

```
[Out] -x/(x^2*(x^2-1))^(1/3)-1/162*ln(-(RootOf(_Z^6+243)^4*x^5+12*RootOf(_Z^6+243
)^4*x^4+RootOf(_Z^6+243)^4*x^3-54*(x^4-x^2)^(1/3)*RootOf(_Z^6+243)^2*x^3-12
*RootOf(_Z^6+243)^4*x^2-54*(x^4-x^2)^(1/3)*RootOf(_Z^6+243)^2*x^2+RootOf(_Z
^6+243)^4*x+162*x^2*(x^4-x^2)^(2/3)+54*RootOf(_Z^6+243)^2*(x^4-x^2)^(1/3)*x
+486*(x^4-x^2)^(2/3)*x-162*(x^4-x^2)^(2/3))/(x^2+x+1)/(x^2-x+1)/x)*RootOf(_
Z^6+243)^4-1/18*ln(-(RootOf(_Z^6+243)^4*x^5+12*RootOf(_Z^6+243)^4*x^4+RootO
f(_Z^6+243)^4*x^3-54*(x^4-x^2)^(1/3)*RootOf(_Z^6+243)^2*x^3-12*RootOf(_Z^6+
243)^4*x^2-54*(x^4-x^2)^(1/3)*RootOf(_Z^6+243)^2*x^2+RootOf(_Z^6+243)^4*x+1
62*x^2*(x^4-x^2)^(2/3)+54*RootOf(_Z^6+243)^2*(x^4-x^2)^(1/3)*x+486*(x^4-x^2
)^(2/3)*x-162*(x^4-x^2)^(2/3))/(x^2+x+1)/(x^2-x+1)/x)*RootOf(_Z^6+243)+1/9*
RootOf(_Z^6+243)*ln((6*(x^4-x^2)^(1/3)*RootOf(_Z^6+243)^5*x^3+RootOf(_Z^6+2
43)^4*x^5-6*(x^4-x^2)^(1/3)*RootOf(_Z^6+243)^5*x^2-12*RootOf(_Z^6+243)^4*x^
4-6*RootOf(_Z^6+243)^5*(x^4-x^2)^(1/3)*x+RootOf(_Z^6+243)^4*x^3-54*(x^4-x^2
)^(1/3)*RootOf(_Z^6+243)^2*x^3+12*RootOf(_Z^6+243)^4*x^2-27*RootOf(_Z^6+243
)*x^5+54*(x^4-x^2)^(1/3)*RootOf(_Z^6+243)^2*x^2+RootOf(_Z^6+243)^4*x+324*Ro
otOf(_Z^6+243)*x^4+324*x^2*(x^4-x^2)^(2/3)+54*RootOf(_Z^6+243)^2*(x^4-x^2)^
(1/3)*x-27*RootOf(_Z^6+243)*x^3-972*(x^4-x^2)^(2/3)*x-324*RootOf(_Z^6+243)*
x^2-324*(x^4-x^2)^(2/3)-27*RootOf(_Z^6+243)*x)/(x^2+x+1)/(x^2-x+1)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+1}{\left(x^6-1\right)\left(x^4-x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^4-x^2)^(1/3)/(x^6-1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + 1)/((x^6 - 1)*(x^4 - x^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6+1}{\left(x^6-1\right)\,{\left(x^4-x^2\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)/((x^6 - 1)*(x^4 - x^2)^(1/3)),x)
```

```
[Out] int((x^6 + 1)/((x^6 - 1)*(x^4 - x^2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}+1\right)\left(x^{4}-x^{2}+1\right)}{\sqrt[3]{x^{2}\left(x-1\right)\left(x+1\right)}\left(x-1\right)\left(x+1\right)\left(x^{2}-x+1\right)\left(x^{2}+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/(x**4-x**2)**(1/3)/(x**6-1),x)
```

```
[Out] Integral((x**2 + 1)*(x**4 - x**2 + 1)/((x**2*(x - 1)*(x + 1))**(1/3)*(x - 1
)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1)), x)
```

3.1847 $\int \frac{-b^6+a^6x^6}{\sqrt{b^2x+a^2x^3}\,(b^6+a^6x^6)}\,dx$

Optimal. Leaf size=167

$$-\frac{2\sqrt{a^2x^3+b^2x}}{3\left(a^2x^2+b^2\right)}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt[4]{3}\sqrt{a}\sqrt{b}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt[4]{3}\sqrt{a}\sqrt{b}}$$

Rubi [C] time = 20.80, antiderivative size = 2697, normalized size of antiderivative = 16.15, number of steps used = 27, number of rules used = 10, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2056, 6715, 6725, 220, 2073, 414, 523, 409, 1217, 1707}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(-b^6 + a^6*x^6)/(Sqrt[b^2*x + a^2*x^3]*(b^6 + a^6*x^6)),x]
```

```
[Out] (-4*a^2*x)/(Sqrt[3]*(Sqrt[3]*a^2 - 3*Sqrt[-a^4])*Sqrt[b^2*x + a^2*x^3]) - (
4*a^2*x)/(Sqrt[3]*(Sqrt[3]*a^2 + 3*Sqrt[-a^4])*Sqrt[b^2*x + a^2*x^3]) - (4*
2^(1/4)*a^6*Sqrt[x]*Sqrt[b^2 + a^2*x^2]*ArcTan[(Sqrt[3*a^2 - Sqrt[3]*Sqrt[-
a^4]]*Sqrt[b]*Sqrt[x])/(2^(1/4)*(a^2 - Sqrt[3]*Sqrt[-a^4])^(1/4)*Sqrt[b^2 +
 a^2*x^2])])/(Sqrt[3]*Sqrt[-a^4]*(a^2 - Sqrt[3]*Sqrt[-a^4])^(3/4)*(3*a^2 -
Sqrt[3]*Sqrt[-a^4])^(3/2)*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) + (4*2^(1/4)*a^6*S
qrt[x]*Sqrt[b^2 + a^2*x^2]*ArcTan[(Sqrt[-3*a^2 + Sqrt[3]*Sqrt[-a^4]]*Sqrt[b
]*Sqrt[x])/(2^(1/4)*(a^2 - Sqrt[3]*Sqrt[-a^4])^(1/4)*Sqrt[b^2 + a^2*x^2])])
/(Sqrt[3]*Sqrt[-a^4]*(a^2 - Sqrt[3]*Sqrt[-a^4])^(3/4)*(-3*a^2 + Sqrt[3]*Sqr
t[-a^4])^(3/2)*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) - (4*2^(1/4)*a^6*Sqrt[x]*Sqrt
[b^2 + a^2*x^2]*ArcTan[(Sqrt[-3*a^2 - Sqrt[3]*Sqrt[-a^4]]*Sqrt[b]*Sqrt[x])/
(2^(1/4)*(a^2 + Sqrt[3]*Sqrt[-a^4])^(1/4)*Sqrt[b^2 + a^2*x^2])])/(Sqrt[3]*S
qrt[-a^4]*(-3*a^2 - Sqrt[3]*Sqrt[-a^4])^(3/2)*(a^2 + Sqrt[3]*Sqrt[-a^4])^(3
/4)*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) + (4*2^(1/4)*a^6*Sqrt[x]*Sqrt[b^2 + a^2*
x^2]*ArcTan[(Sqrt[3*a^2 + Sqrt[3]*Sqrt[-a^4]]*Sqrt[b]*Sqrt[x])/(2^(1/4)*(a^
2 + Sqrt[3]*Sqrt[-a^4])^(1/4)*Sqrt[b^2 + a^2*x^2])])/(Sqrt[3]*Sqrt[-a^4]*(a
^2 + Sqrt[3]*Sqrt[-a^4])^(3/4)*(3*a^2 + Sqrt[3]*Sqrt[-a^4])^(3/2)*Sqrt[b]*S
qrt[b^2*x + a^2*x^3]) + (Sqrt[x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2
]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]], 1/2])/(Sqrt[a]*Sqrt[b]*Sqr
t[b^2*x + a^2*x^3]) - (2*a^(3/2)*Sqrt[x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b
+ a*x)^2]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]], 1/2])/(Sqrt[3]*(Sq
rt[3]*a^2 - 3*Sqrt[-a^4])*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) - (2*a^(3/2)*Sqrt[
x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2]*EllipticF[2*ArcTan[(Sqrt[a]*
Sqrt[x])/Sqrt[b]], 1/2])/(Sqrt[3]*(Sqrt[3]*a^2 + 3*Sqrt[-a^4])*Sqrt[b]*Sqrt
[b^2*x + a^2*x^3]) + (2*a^(7/2)*(1 - (Sqrt[2]*a)/Sqrt[a^2 - Sqrt[3]*Sqrt[-a
^4]])*Sqrt[x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2]*EllipticF[2*ArcTa
n[(Sqrt[a]*Sqrt[x])/Sqrt[b]], 1/2])/(Sqrt[3]*Sqrt[-a^4]*(3*a^2 + Sqrt[3]*Sq
rt[-a^4])*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) + (2*a^(7/2)*(1 + (Sqrt[2]*a)/Sqrt
[a^2 - Sqrt[3]*Sqrt[-a^4]])*Sqrt[x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x
)^2]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]], 1/2])/(Sqrt[3]*Sqrt[-a^
4]*(3*a^2 + Sqrt[3]*Sqrt[-a^4])*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) - (2*a^(7/2)
*(1 - (Sqrt[2]*a)/Sqrt[a^2 + Sqrt[3]*Sqrt[-a^4]])*Sqrt[x]*(b + a*x)*Sqrt[(b
^2 + a^2*x^2)/(b + a*x)^2]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]], 1
/2])/(Sqrt[3]*Sqrt[-a^4]*(3*a^2 - Sqrt[3]*Sqrt[-a^4])*Sqrt[b]*Sqrt[b^2*x +
a^2*x^3]) - (2*a^(7/2)*(1 + (Sqrt[2]*a)/Sqrt[a^2 + Sqrt[3]*Sqrt[-a^4]])*Sqr
t[x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2]*EllipticF[2*ArcTan[(Sqrt[a
]*Sqrt[x])/Sqrt[b]], 1/2])/(Sqrt[3]*Sqrt[-a^4]*(3*a^2 - Sqrt[3]*Sqrt[-a^4])
*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) - ((Sqrt[2]*a + Sqrt[a^2 - Sqrt[3]*Sqrt[-a^
4]])^2*Sqrt[x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2]*EllipticPi[-1/4*
(Sqrt[2]*a - Sqrt[a^2 - Sqrt[3]*Sqrt[-a^4]])^2/(Sqrt[2]*a*Sqrt[a^2 - Sqrt[3
]*Sqrt[-a^4]]), 2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]], 1/2])/(2*Sqrt[3]*Sqrt[
```

```
a]*(Sqrt[3]*a^2 + 3*Sqrt[-a^4])*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) - ((Sqrt[2]*
a - Sqrt[a^2 - Sqrt[3]*Sqrt[-a^4]])^2*Sqrt[x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2
)/(b + a*x)^2]*EllipticPi[(Sqrt[2]*a + Sqrt[a^2 - Sqrt[3]*Sqrt[-a^4]])^2/(4
*Sqrt[2]*a*Sqrt[a^2 - Sqrt[3]*Sqrt[-a^4]]), 2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt
[b]], 1/2])/(2*Sqrt[3]*Sqrt[a]*(Sqrt[3]*a^2 + 3*Sqrt[-a^4])*Sqrt[b]*Sqrt[b^
2*x + a^2*x^3]) - ((Sqrt[2]*a + Sqrt[a^2 + Sqrt[3]*Sqrt[-a^4]])^2*Sqrt[x]*(
b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2]*EllipticPi[-1/4*(Sqrt[2]*a - Sqr
t[a^2 + Sqrt[3]*Sqrt[-a^4]])^2/(Sqrt[2]*a*Sqrt[a^2 + Sqrt[3]*Sqrt[-a^4]]),
2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]], 1/2])/(2*Sqrt[3]*Sqrt[a]*(Sqrt[3]*a^2
- 3*Sqrt[-a^4])*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) - ((Sqrt[2]*a - Sqrt[a^2 + S
qrt[3]*Sqrt[-a^4]])^2*Sqrt[x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2]*E
llipticPi[(Sqrt[2]*a + Sqrt[a^2 + Sqrt[3]*Sqrt[-a^4]])^2/(4*Sqrt[2]*a*Sqrt[
a^2 + Sqrt[3]*Sqrt[-a^4]]), 2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]], 1/2])/(2*S
qrt[3]*Sqrt[a]*(Sqrt[3]*a^2 - 3*Sqrt[-a^4])*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
```

```
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-b^6+a^6x^6}{\sqrt{b^2x+a^2x^3}\left(b^6+a^6x^6\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{b^2+a^2x^2}\right)\int \frac{-b^6+a^6x^6}{\sqrt{x}\sqrt{b^2+a^2x^2}\left(b^6+a^6x^6\right)}\,dx}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{-b^6+a^6x^{12}}{\sqrt{b^2+a^2x^4}\left(b^6+a^6x^{12}\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt{b^2+a^2x^4}} - \frac{2b^6}{\sqrt{b^2+a^2x^4}\left(b^6+a^6x^{12}\right)}\right)dx, x, \sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2+a^2x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}} - \frac{\left(4b^6\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Su}}{\sqrt{}}$$

$$= \frac{\sqrt{x}\,(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\left(4b^6\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\Big(}{}$$

$$= \frac{\sqrt{x}\,(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\left(8\sqrt{-a^4}\,b^4\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Su}}{}$$

$$= \frac{4\sqrt{-a^4}\,x}{\sqrt{3}\left(3a^2-\sqrt{3}\sqrt{-a^4}\right)\sqrt{b^2x+a^2x^3}} - \frac{4\sqrt{-a^4}\,x}{\sqrt{3}\left(3a^2+\sqrt{3}\sqrt{-a^4}\right)\sqrt{b^2x+a^2x^3}} +$$

$$= \frac{4\sqrt{-a^4}\,x}{\sqrt{3}\left(3a^2-\sqrt{3}\sqrt{-a^4}\right)\sqrt{b^2x+a^2x^3}} - \frac{4\sqrt{-a^4}\,x}{\sqrt{3}\left(3a^2+\sqrt{3}\sqrt{-a^4}\right)\sqrt{b^2x+a^2x^3}} +$$

$$= \frac{4\sqrt{-a^4}\,x}{\sqrt{3}\left(3a^2-\sqrt{3}\sqrt{-a^4}\right)\sqrt{b^2x+a^2x^3}} - \frac{4\sqrt{-a^4}\,x}{\sqrt{3}\left(3a^2+\sqrt{3}\sqrt{-a^4}\right)\sqrt{b^2x+a^2x^3}} +$$

$$= \frac{4\sqrt{-a^4}\,x}{\sqrt{3}\left(3a^2-\sqrt{3}\sqrt{-a^4}\right)\sqrt{b^2x+a^2x^3}} - \frac{4\sqrt{-a^4}\,x}{\sqrt{3}\left(3a^2+\sqrt{3}\sqrt{-a^4}\right)\sqrt{b^2x+a^2x^3}} +$$

$$= \frac{4\sqrt{-a^4}\,x}{\sqrt{3}\left(3a^2-\sqrt{3}\sqrt{-a^4}\right)\sqrt{b^2x+a^2x^3}} - \frac{4\sqrt{-a^4}\,x}{\sqrt{3}\left(3a^2+\sqrt{3}\sqrt{-a^4}\right)\sqrt{b^2x+a^2x^3}} -$$

Mathematica [C] time = 2.27, size = 345, normalized size = 2.07

$$\frac{2\left(-x^{3/2}-\frac{ix^2\sqrt{\frac{b^2}{a^2x^2}+1}\left(-\Pi\left(-\frac{i\sqrt{2}a}{\sqrt{\frac{(1-i\sqrt{3})a^2}{b^2}}b};i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(\frac{i\sqrt{2}a}{\sqrt{\frac{(1-i\sqrt{3})a^2}{b^2}}b};i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(-\frac{i\sqrt{2}a}{\sqrt{\frac{(1+i\sqrt{3})a^2}{b^2}}b};i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(\frac{i\sqrt{2}a}{\sqrt{\frac{(1+i\sqrt{3})a^2}{b^2}}b};i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)+2F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{\sqrt{\frac{ib}{a}}}\right)}{3\sqrt{x}\sqrt{x\left(a^2x^2+b^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b^6 + a^6*x^6)/(Sqrt[b^2*x + a^2*x^3]*(b^6 + a^6*x^6)),x]
```

```
[Out] (2*(-x^(3/2) - (I*Sqrt[1 + b^2/(a^2*x^2)]*x^2*(2*EllipticF[I*ArcSinh[Sqrt[(
I*b)/a]/Sqrt[x]], -1] - EllipticPi[((-I)*Sqrt[2]*a)/(Sqrt[((1 - I*Sqrt[3])*
a^2)/b^2]*b), I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] - EllipticPi[(I*Sqrt[2]
*a)/(Sqrt[((1 - I*Sqrt[3])*a^2)/b^2]*b), I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]],
-1] - EllipticPi[((-I)*Sqrt[2]*a)/(Sqrt[((1 + I*Sqrt[3])*a^2)/b^2]*b), I*Ar
cSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] - EllipticPi[(I*Sqrt[2]*a)/(Sqrt[((1 + I*
Sqrt[3])*a^2)/b^2]*b), I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1]))/Sqrt[(I*b)/a
]))/(3*Sqrt[x]*Sqrt[x*(b^2 + a^2*x^2)])
```

IntegrateAlgebraic [A] time = 0.55, size = 167, normalized size = 1.00

$$-\frac{2\sqrt{a^2x^3+b^2x}}{3\left(a^2x^2+b^2\right)}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt[4]{3}\sqrt{a}\sqrt{b}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt[4]{3}\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b^6 + a^6*x^6)/(Sqrt[b^2*x + a^2*x^3]*(b^6 + a^6*x^6))
,x]
```

```
[Out] (-2*Sqrt[b^2*x + a^2*x^3])/(3*(b^2 + a^2*x^2)) - (2*ArcTan[(3^(1/4)*Sqrt[a]
*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])/(b^2 + a^2*x^2)])/(3*3^(1/4)*Sqrt[a]*Sqrt[b
]) - (2*ArcTanh[(3^(1/4)*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])/(b^2 + a^2*
x^2)])/(3*3^(1/4)*Sqrt[a]*Sqrt[b])
```

fricas [B] time = 0.61, size = 456, normalized size = 2.73

$$-\frac{4\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(a^2x^2+b^2\right)\left(\frac{1}{a^2b^2}\right)^{\frac{1}{4}}\arctan\left(\frac{3\left(\frac{1}{3}\right)^{\frac{3}{4}}\sqrt{a^2x^3+b^2x}\,a^2b^2\left(\frac{1}{a^2b^2}\right)^{\frac{3}{4}}}{a^2x^2+b^2}\right)+\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(a^2x^2+b^2\right)\left(\frac{1}{a^2b^2}\right)^{\frac{1}{4}}\log\left(\frac{a^4x^4+5a^2b^2x^2+b^4+6\sqrt{\frac{1}{3}}\left(a^4b^2x^3+a^2b^4x\right)\sqrt{\frac{1}{a^2b^2}}+6\left(\left(\frac{1}{3}\right)^{\frac{1}{4}}a^2b^2x\left(\frac{1}{a^2b^2}\right)^{\frac{1}{4}}+\left(\frac{1}{3}\right)^{\frac{3}{4}}\left(a^4b^2x^2+a^2b^4\right)\left(\frac{1}{a^2b^2}\right)^{\frac{3}{4}}\right)\sqrt{a^2x^3+b^2x}}{a^4x^4-a^2b^2x^2+b^4}\right)-\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(a^2x^2+b^2\right)\left(\frac{1}{a^2b^2}\right)^{\frac{1}{4}}\log\left(\frac{a^4x^4+5a^2b^2x^2+b^4+6\sqrt{\frac{1}{3}}\left(a^4b^2x^3+a^2b^4x\right)\sqrt{\frac{1}{a^2b^2}}-6\left(\left(\frac{1}{3}\right)^{\frac{1}{4}}a^2b^2x\left(\frac{1}{a^2b^2}\right)^{\frac{1}{4}}+\left(\frac{1}{3}\right)^{\frac{3}{4}}\left(a^4b^2x^2+a^2b^4\right)\left(\frac{1}{a^2b^2}\right)^{\frac{3}{4}}\right)\sqrt{a^2x^3+b^2x}}{a^4x^4-a^2b^2x^2+b^4}\right)+4\sqrt{a^2x^3+b^2x}}{6\left(a^2x^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^6*x^6-b^6)/(a^2*x^3+b^2*x)^(1/2)/(a^6*x^6+b^6),x, algorithm="f
ricas")
```

```
[Out] -1/6*(4*(1/3)^(1/4)*(a^2*x^2 + b^2)*(1/(a^2*b^2))^(1/4)*arctan(3*(1/3)^(3/4
)*sqrt(a^2*x^3 + b^2*x)*a^2*b^2*(1/(a^2*b^2))^(3/4)/(a^2*x^2 + b^2)) + (1/3
)^(1/4)*(a^2*x^2 + b^2)*(1/(a^2*b^2))^(1/4)*log((a^4*x^4 + 5*a^2*b^2*x^2 +
b^4 + 6*sqrt(1/3)*(a^4*b^2*x^3 + a^2*b^4*x)*sqrt(1/(a^2*b^2)) + 6*((1/3)^(1
/4)*a^2*b^2*x*(1/(a^2*b^2))^(1/4) + (1/3)^(3/4)*(a^4*b^2*x^2 + a^2*b^4)*(1/
(a^2*b^2))^(3/4))*sqrt(a^2*x^3 + b^2*x))/(a^4*x^4 - a^2*b^2*x^2 + b^4)) - (
1/3)^(1/4)*(a^2*x^2 + b^2)*(1/(a^2*b^2))^(1/4)*log((a^4*x^4 + 5*a^2*b^2*x^2
 + b^4 + 6*sqrt(1/3)*(a^4*b^2*x^3 + a^2*b^4*x)*sqrt(1/(a^2*b^2)) - 6*((1/3)
^(1/4)*a^2*b^2*x*(1/(a^2*b^2))^(1/4) + (1/3)^(3/4)*(a^4*b^2*x^2 + a^2*b^4)*
(1/(a^2*b^2))^(3/4))*sqrt(a^2*x^3 + b^2*x))/(a^4*x^4 - a^2*b^2*x^2 + b^4))
+ 4*sqrt(a^2*x^3 + b^2*x))/(a^2*x^2 + b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a^6x^6-b^6}{\left(a^6x^6+b^6\right)\sqrt{a^2x^3+b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^6*x^6-b^6)/(a^2*x^3+b^2*x)^(1/2)/(a^6*x^6+b^6),x, algorithm="g
iac")
```

```
[Out] integrate((a^6*x^6 - b^6)/((a^6*x^6 + b^6)*sqrt(a^2*x^3 + b^2*x)), x)
```

maple [C] time = 0.13, size = 447, normalized size = 2.68

$$\frac{ib\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{2}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a^2x^3+b^2x}}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(a^4\_Z^4-a^2b^2\_Z^2+b^4\right)}\frac{\left(-\_\alpha^2a^2+2b^2\right)\left(a^3\_\alpha^3-i\_\alpha^2a^2b-2\_\alpha a b^2+2ib^3\right)\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},-\frac{i\_\alpha^3a^3+\_\alpha^2a^2b-2i\_\alpha a b^2-2b^3}{3b^3},\frac{\sqrt{2}}{2}\right)}{\_\alpha\left(2\_\alpha^2a^2-b^2\right)\sqrt{x\left(a^2x^2+b^2\right)}}\right)}{9b\,a^2}-\frac{2b^2\left(\frac{x}{b^2\sqrt{\left(x^2+\frac{b^2}{a^2}\right)x\,a^2}}+\frac{i\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{2}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},\frac{\sqrt{2}}{2}\right)}{2ba\sqrt{a^2x^3+b^2x}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^6*x^6-b^6)/(a^2*x^3+b^2*x)^(1/2)/(a^6*x^6+b^6),x)
```

```
[Out] I*b/a*(-I*(x+I*b/a)/b*a)^(1/2)*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1
/2)/(a^2*x^3+b^2*x)^(1/2)*EllipticF((-I*(x+I*b/a)/b*a)^(1/2),1/2*2^(1/2))-1
/9*I/b/a^2*2^(1/2)*sum((-_alpha^2*a^2+2*b^2)/_alpha/(2*_alpha^2*a^2-b^2)*(-
I*_alpha^2*a^2*b+a^3*_alpha^3+2*I*b^3-2*_alpha*a*b^2)*(-I*(x+I*b/a)/b*a)^(1
/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1/2)/(x*(a^2*x^2+b^2))^(1/2)*Ellipti
cPi((-I*(x+I*b/a)/b*a)^(1/2),-1/3*(I*_alpha^3*a^3+_alpha^2*a^2*b-2*I*_alpha
*a*b^2-2*b^3)/b^3,1/2*2^(1/2)),_alpha=RootOf(_Z^4*a^4-_Z^2*a^2*b^2+b^4))-2/
3*b^2*(x/b^2/((x^2+b^2/a^2)*x*a^2)^(1/2)+1/2*I/b/a*(-I*(x+I*b/a)/b*a)^(1/2)
*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1/2)/(a^2*x^3+b^2*x)^(1/2)*Elli
pticF((-I*(x+I*b/a)/b*a)^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^6x^6-b^6}{\left(a^6x^6+b^6\right)\sqrt{a^2x^3+b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^6*x^6-b^6)/(a^2*x^3+b^2*x)^(1/2)/(a^6*x^6+b^6),x, algorithm="m
axima")
```

```
[Out] integrate((a^6*x^6 - b^6)/((a^6*x^6 + b^6)*sqrt(a^2*x^3 + b^2*x)), x)
```

mupad [B] time = 5.79, size = 201, normalized size = 1.20

$$\frac{3^{3/4}\ln\left(\frac{3^{3/4}b^2-6\sqrt{a}\,\sqrt{b}\,\sqrt{a^2x^3+b^2x}+3^{3/4}a^2x^2+3\,3^{1/4}a\,b\,x}{a^2x^2-\sqrt{3}\,a\,b\,x+b^2}\right)}{9\sqrt{a}\,\sqrt{b}}-\frac{2\sqrt{a^2x^3+b^2x}}{3\left(a^2x^2+b^2\right)}+\frac{3^{3/4}\ln\left(\frac{3^{3/4}b^2+3^{3/4}a^2x^2-3\,3^{1/4}a\,b\,x+\sqrt{a}\,\sqrt{b}\,\sqrt{a^2x^3+b^2x}\,6\mathrm{i}}{a^2x^2+\sqrt{3}\,a\,b\,x+b^2}\right)1\mathrm{i}}{9\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^6 - a^6*x^6)/((b^6 + a^6*x^6)*(b^2*x + a^2*x^3)^(1/2)),x)
```

```
[Out] (3^(3/4)*log((3^(3/4)*b^2 - 6*a^(1/2)*b^(1/2)*(b^2*x + a^2*x^3)^(1/2) + 3^(
3/4)*a^2*x^2 + 3*3^(1/4)*a*b*x)/(b^2 + a^2*x^2 - 3^(1/2)*a*b*x)))/(9*a^(1/2
)*b^(1/2)) - (2*(b^2*x + a^2*x^3)^(1/2))/(3*(b^2 + a^2*x^2)) + (3^(3/4)*log
((3^(3/4)*b^2 + a^(1/2)*b^(1/2)*(b^2*x + a^2*x^3)^(1/2)*6i + 3^(3/4)*a^2*x^
2 - 3*3^(1/4)*a*b*x)/(b^2 + a^2*x^2 + 3^(1/2)*a*b*x))*1i)/(9*a^(1/2)*b^(1/2
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax-b\right)\left(ax+b\right)\left(a^2x^2-abx+b^2\right)\left(a^2x^2+abx+b^2\right)}{\sqrt{x\left(a^2x^2+b^2\right)}\left(a^2x^2+b^2\right)\left(a^4x^4-a^2b^2x^2+b^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**6*x**6-b**6)/(a**2*x**3+b**2*x)**(1/2)/(a**6*x**6+b**6),x)
```

```
[Out] Integral((a*x - b)*(a*x + b)*(a**2*x**2 - a*b*x + b**2)*(a**2*x**2 + a*b*x
+ b**2)/(sqrt(x*(a**2*x**2 + b**2))*(a**2*x**2 + b**2)*(a**4*x**4 - a**2*b*
*2*x**2 + b**4)), x)
```

3.1848 $\int \frac{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{x^2\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$

Optimal. Leaf size=167

$$\frac{2\left(2ax^2-1\right)\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{3bx^2}-\frac{4\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{3x}-\frac{\sqrt{2}\,a\tanh^{-1}\left(\sqrt{2}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+\right.}{b}$$

Rubi [F] time = 1.78, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{x^2\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[-(a/b^2) + (a^2*x^2)/b^2]/(x^2*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a
^2*x^2)/b^2]]),x]
```

```
[Out] Defer[Int][Sqrt[-(a/b^2) + (a^2*x^2)/b^2]/(x^2*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^
2) + (a^2*x^2)/b^2]]), x]
```

Rubi steps

$$\int \frac{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{x^2\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx=\int \frac{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{x^2\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$$

Mathematica [C] time = 5.41, size = 341, normalized size = 2.04

$$\frac{\frac{3\sqrt{\frac{a(ax^2-1)}{b^2}}\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)\left({}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};2x\left(\sqrt{\frac{a(ax^2-1)}{b^2}}b+ax\right)\right)+2bx\sqrt{\frac{a(ax^2-1)}{b^2}}+2ax^2+3\right)}{bx\sqrt{\frac{a(ax^2-1)}{b^2}}+ax^2-1}-\frac{4a\left(3bx\sqrt{\frac{a(ax^2-1)}{b^2}}+3ax^2-1\right)}{bx\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}-\frac{3\sqrt{2}\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(\sqrt{2}\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)^2+a}}{\sqrt{a}}\right)\right)}{b}}{6\sqrt{x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(a/b^2) + (a^2*x^2)/b^2]/(x^2*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2
) + (a^2*x^2)/b^2]]),x]
```

```
[Out] ((-4*a*(-1 + 3*a*x^2 + 3*b*x*Sqrt[(a*(-1 + a*x^2))/b^2]))/(b*x*(a*x + b*Sqr
t[(a*(-1 + a*x^2))/b^2])) - (3*Sqrt[2]*Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^
2))/b^2])]*(Sqrt[2]*Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])] + Sqrt[a
]*ArcTanh[Sqrt[a + (a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])^2]/Sqrt[a]]))/b + (
3*Sqrt[(a*(-1 + a*x^2))/b^2]*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])*(3 + 2*a*
x^2 + 2*b*x*Sqrt[(a*(-1 + a*x^2))/b^2] + Hypergeometric2F1[-1/2, 1, 1/2, 2*
x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])]))/(-1 + a*x^2 + b*x*Sqrt[(a*(-1 + a
*x^2))/b^2]))/(6*Sqrt[x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])])
```

IntegrateAlgebraic [A] time = 3.31, size = 223, normalized size = 1.34

$$\frac{2(2ax^2-1)\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{3bx^2}-\frac{4\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{3x}+\frac{a\log\left(\sqrt{2}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}-1\right)}{\sqrt{2}b}-\frac{a\log\left(\sqrt{2}b\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}+b\right)}{\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-(a/b^2) + (a^2*x^2)/b^2]/(x^2*Sqrt[a*x^2 + b*x*Sqr
t[-(a/b^2) + (a^2*x^2)/b^2]]),x]
```

```
[Out] (2*(-1 + 2*a*x^2)*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]])/(3*b*x^
2) - (4*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^
2*x^2)/b^2]])/(3*x) + (a*Log[-1 + Sqrt[2]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) +
(a^2*x^2)/b^2]]])/(Sqrt[2]*b) - (a*Log[b + Sqrt[2]*b*Sqrt[a*x^2 + b*x*Sqrt[
-(a/b^2) + (a^2*x^2)/b^2]]])/(Sqrt[2]*b)
```

fricas [A] time = 32.33, size = 175, normalized size = 1.05

$$\frac{3\sqrt{2}ax^2\log\left(4ax^2-4bx\sqrt{\frac{a^2x^2-a}{b^2}}+2\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\left(2\sqrt{2}bx\sqrt{\frac{a^2x^2-a}{b^2}}-\sqrt{2}\left(2ax^2-1\right)\right)-1\right)+4\left(2ax^2-2bx\sqrt{\frac{a^2x^2-a}{b^2}}-1\right)\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}}{6bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b^2+a^2*x^2/b^2)^(1/2)/x^2/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1
/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*a*x^2*log(4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b^2) + 2*sqrt(a
*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*(2*sqrt(2)*b*x*sqrt((a^2*x^2 - a)/b^2)
- sqrt(2)*(2*a*x^2 - 1)) - 1) + 4*(2*a*x^2 - 2*b*x*sqrt((a^2*x^2 - a)/b^2)
- 1)*sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2)))/(b*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}}{\sqrt{ax^2+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,bx}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b^2+a^2*x^2/b^2)^(1/2)/x^2/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1
/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*x^2/b^2 - a/b^2)/(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)
*b*x)*x^2), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{x^2\sqrt{a\,x^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a/b^2+a^2*x^2/b^2)^(1/2)/x^2/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(
1/2),x)
```

```
[Out] int((-a/b^2+a^2*x^2/b^2)^(1/2)/x^2/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(
1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}}{\sqrt{ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\,bx\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b^2+a^2*x^2/b^2)^(1/2)/x^2/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1
/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2/b^2 - a/b^2)/(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)
*b*x)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a^2\,x^2}{b^2} - \frac{a}{b^2}}}{x^2\,\sqrt{a\,x^2 + b\,x\,\sqrt{\frac{a^2\,x^2}{b^2} - \frac{a}{b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a^2*x^2)/b^2 - a/b^2)^(1/2)/(x^2*(a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)
^(1/2))^(1/2)),x)
```

```
[Out] int(((a^2*x^2)/b^2 - a/b^2)^(1/2)/(x^2*(a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)
^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a\left(ax^2-1\right)}{b^2}}}{x^2\sqrt{x\left(ax + b\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b**2+a**2*x**2/b**2)**(1/2)/x**2/(a*x**2+b*x*(-a/b**2+a**2*x*
*2/b**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(a*x**2 - 1)/b**2)/(x**2*sqrt(x*(a*x + b*sqrt(a**2*x**2/b**
2 - a/b**2)))), x)
```

3.1849 $\int \frac{(-2q+px^3)\sqrt{q+px^3}}{bx^4+a(q+px^3)^2}\,dx$

Optimal. Leaf size=167

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x\sqrt{px^3+q}}{\sqrt{a}px^3+\sqrt{a}q-\sqrt{b}x^2}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\frac{\sqrt[4]{a}px^3}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{a}q}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}x^2}{\sqrt{2}\sqrt[4]{a}}}{x\sqrt{px^3+q}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

Rubi [A] time = 0.46, antiderivative size = 242, normalized size of antiderivative = 1.45, number of steps used = 10, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {6712, 211, 1165, 628, 1162, 617, 204}

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}\sqrt{px^3+q}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}\sqrt{px^3+q}}+1\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{px^3+q}}+\sqrt{a}+\frac{\sqrt{b}x^2}{px^3+q}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{px^3+q}}+\sqrt{a}+\frac{\sqrt{b}x^2}{px^3+q}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((-2*q + p*x^3)*Sqrt[q + p*x^3])/(b*x^4 + a*(q + p*x^3)^2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a^(1/4)*Sqrt[q + p*x^3])]/(Sqrt[2]*a^(3/4)*
b^(1/4)) - ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a^(1/4)*Sqrt[q + p*x^3])]/(Sqrt[
2]*a^(3/4)*b^(1/4)) + Log[Sqrt[a] + (Sqrt[b]*x^2)/(q + p*x^3) - (Sqrt[2]*a^
(1/4)*b^(1/4)*x)/Sqrt[q + p*x^3]]/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - Log[Sqrt[a]
 + (Sqrt[b]*x^2)/(q + p*x^3) + (Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[q + p*x^3]]
/(2*Sqrt[2]*a^(3/4)*b^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6712

```
Int[(u_)*(v_)^(r_.)*((a_.)*(v_)^(p_.) + (b_.)*(w_)^(q_.))^(m_.), x_Symbol]
:> With[{c = Simplify[u/(p*w*D[v, x] - q*v*D[w, x])]}, -Dist[c*q, Subst[Int
[(a + b*x^q)^m, x], x, v^(m*p + r + 1)*w], x] /; FreeQ[c, x]] /; FreeQ[{a,
b, m, p, q, r}, x] && EqQ[p + q*(m*p + r + 1), 0] && IntegerQ[q] && Integer
Q[m]
```

Rubi steps

$$\int \frac{\left(-2q+px^3\right)\sqrt{q+px^3}}{bx^4+a\left(q+px^3\right)^2}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \frac{x}{\sqrt{q+px^3}}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \frac{x}{\sqrt{q+px^3}}\right)}{\sqrt{a}} - \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \frac{x}{\sqrt{q+px^3}}\right)}{\sqrt{a}}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x}{\sqrt{q+px^3}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x}{\sqrt{q+px^3}}\right.}{2\sqrt{a}\sqrt{b}}$$

$$= \frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\,x^2}{q+px^3}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{q+px^3}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}} - \frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\,x^2}{q+px^3}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{q+px^3}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}} - \text{Subst}\left(\int \frac{}{-1}\right.$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}\sqrt{q+px^3}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}\sqrt{q+px^3}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}} + \frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\,x^2}{q+px^3}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}{\sqrt{q+px^3}}\right.}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}$$

Mathematica [C] time = 6.51, size = 13567, normalized size = 81.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-2*q + p*x^3)*Sqrt[q + p*x^3])/(b*x^4 + a*(q + p*x^3)^2),x]
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 1.19, size = 167, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x\sqrt{px^3+q}}{\sqrt{a}\,px^3+\sqrt{a}\,q-\sqrt{b}\,x^2}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}} - \frac{\tanh^{-1}\left(\frac{\frac{\sqrt[4]{a}\,px^3}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{a}\,q}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}\,x^2}{\sqrt{2}\sqrt[4]{a}}}{x\sqrt{px^3+q}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*q + p*x^3)*Sqrt[q + p*x^3])/(b*x^4 + a*(q + p*x^3)^
2),x]
```

```
[Out] -(ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x*Sqrt[q + p*x^3])/(Sqrt[a]*q - Sqrt[b]*x
^2 + Sqrt[a]*p*x^3)]/(Sqrt[2]*a^(3/4)*b^(1/4))) - ArcTanh[((a^(1/4)*q)/(Sqr
t[2]*b^(1/4)) + (b^(1/4)*x^2)/(Sqrt[2]*a^(1/4)) + (a^(1/4)*p*x^3)/(Sqrt[2]*
b^(1/4)))/(x*Sqrt[q + p*x^3])]/(Sqrt[2]*a^(3/4)*b^(1/4))
```

fricas [B] time = 1.19, size = 354, normalized size = 2.12

$$\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}} \arctan\left(\frac{a^2bx\left(-\frac{1}{a^3b}\right)^{\frac{3}{4}}}{\sqrt{px^3+q}}\right) + \frac{1}{4}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}} \log\left(\frac{ap^2x^6 + 2\,apqx^3 - bx^4 + aq^2 + 2\left(abx^3\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}} + \left(a^3bpx^4 + a^3bqx\right)\left(-\frac{1}{a^3b}\right)^{\frac{3}{4}}\right)\sqrt{px^3+q} - 2\left(a^2bpx^5 + a^2bqx^2\right)\sqrt{-\frac{1}{a^3b}}}{ap^2x^6 + 2\,apqx^3 + bx^4 + aq^2}\right) - \frac{1}{4}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}} \log\left(\frac{ap^2x^6 + 2\,apqx^3 - bx^4 + aq^2 - 2\left(abx^3\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}} + \left(a^3bpx^4 + a^3bqx\right)\left(-\frac{1}{a^3b}\right)^{\frac{3}{4}}\right)\sqrt{px^3+q} - 2\left(a^2bpx^5 + a^2bqx^2\right)\sqrt{-\frac{1}{a^3b}}}{ap^2x^6 + 2\,apqx^3 + bx^4 + aq^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p*x^3+q)^(1/2)/(b*x^4+a*(p*x^3+q)^2),x, algorithm="f
ricas")
```

```
[Out] (-1/(a^3*b))^(1/4)*arctan(a^2*b*x*(-1/(a^3*b))^(3/4)/sqrt(p*x^3 + q)) + 1/4
*(-1/(a^3*b))^(1/4)*log((a*p^2*x^6 + 2*a*p*q*x^3 - b*x^4 + a*q^2 + 2*(a*b*x
^3*(-1/(a^3*b))^(1/4) + (a^3*b*p*x^4 + a^3*b*q*x)*(-1/(a^3*b))^(3/4))*sqrt(
p*x^3 + q) - 2*(a^2*b*p*x^5 + a^2*b*q*x^2)*sqrt(-1/(a^3*b)))/(a*p^2*x^6 + 2
*a*p*q*x^3 + b*x^4 + a*q^2)) - 1/4*(-1/(a^3*b))^(1/4)*log((a*p^2*x^6 + 2*a*
p*q*x^3 - b*x^4 + a*q^2 - 2*(a*b*x^3*(-1/(a^3*b))^(1/4) + (a^3*b*p*x^4 + a^
3*b*q*x)*(-1/(a^3*b))^(3/4))*sqrt(p*x^3 + q) - 2*(a^2*b*p*x^5 + a^2*b*q*x^2
)*sqrt(-1/(a^3*b)))/(a*p^2*x^6 + 2*a*p*q*x^3 + b*x^4 + a*q^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^3+q}\left(px^3-2\,q\right)}{bx^4+\left(px^3+q\right)^2 a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p*x^3+q)^(1/2)/(b*x^4+a*(p*x^3+q)^2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(p*x^3 + q)*(p*x^3 - 2*q)/(b*x^4 + (p*x^3 + q)^2*a), x)
```

maple [C] time = 0.32, size = 1164, normalized size = 6.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^3-2*q)*(p*x^3+q)^(1/2)/(b*x^4+a*(p*x^3+q)^2),x)
```

```
[Out] -2/3*I/a*3^(1/2)/p*(-q*p^2)^(1/3)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I*3^(1/2)/
p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2)*((x-1/p*(-q*p^2)^(1/3))/(
-3/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2)*(-I*(x+1/2/p*(
-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/
2)/(p*x^3+q)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I*3
^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2),(I*3^(1/2)/p*(-q*p
^2)^(1/3)/(-3/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2))-1/
2*I/a/p^2/q^2/b*2^(1/2)*sum((3*_alpha^3*a*p*q+_alpha^4*b+3*a*q^2)/_alpha^2/
(3*_alpha^3*a*p^2+3*a*p*q+2*_alpha*b)*(-q*p^2)^(1/3)*(1/2*I*p*(2*x+1/p*(-I*
3^(1/2)*(-q*p^2)^(1/3)+(-q*p^2)^(1/3)))/(-q*p^2)^(1/3))^(1/2)*(p*(x-1/p*(-q
*p^2)^(1/3))/(-3*(-q*p^2)^(1/3)+I*3^(1/2)*(-q*p^2)^(1/3)))^(1/2)*(-1/2*I*p*
(2*x+1/p*(I*3^(1/2)*(-q*p^2)^(1/3)+(-q*p^2)^(1/3)))/(-q*p^2)^(1/3))^(1/2)/(
p*x^3+q)^(1/2)*(2*q*p^2*(_alpha^4*a*p^2+_alpha*a*p*q+_alpha^2*b)+I*(-q*p^2)
^(2/3)*3^(1/2)*_alpha^3*p*b+(-q*p^2)^(2/3)*_alpha^5*a*p^3+I*(-q*p^2)^(1/3)*
3^(1/2)*a*q^2*p^2+I*(-q*p^2)^(1/3)*3^(1/2)*_alpha*q*p*b+I*(-q*p^2)^(1/3)*p^
3*3^(1/2)*_alpha^3*a*q-(-q*p^2)^(1/3)*_alpha^3*a*p^3*q+(-q*p^2)^(2/3)*_alph
```

```
a^2*a*p^2*q+I*(-q*p^2)^(2/3)*p^3*3^(1/2)*_alpha^5*a+(-q*p^2)^(2/3)*_alpha^3
*b*p+I*(-q*p^2)^(2/3)*3^(1/2)*_alpha^2*a*q*p^2-I*(-q*p^2)^(2/3)*3^(1/2)*q*b
-(-q*p^2)^(1/3)*a*p^2*q^2-(-q*p^2)^(1/3)*_alpha*b*p*q-(-q*p^2)^(2/3)*b*q)*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I*3^(1/2)/p*(-q*p^2)^(
1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2),1/2/p/q*(I*3^(1/2)*_alpha^3*b*p^2-2*I
*(-q*p^2)^(1/3)*3^(1/2)*_alpha^2*b*p-2*I*(-q*p^2)^(1/3)*3^(1/2)*_alpha^4*a*
p^3-3*p^4*_alpha^5*a+I*(-q*p^2)^(2/3)*3^(1/2)*_alpha^3*a*p^2+3*(-q*p^2)^(2/
3)*_alpha^3*a*p^2+I*(-q*p^2)^(2/3)*3^(1/2)*a*p*q+I*3^(1/2)*_alpha^5*a*p^4+I
*3^(1/2)*_alpha^2*a*p^3*q+I*(-q*p^2)^(2/3)*3^(1/2)*_alpha*b-3*p^3*_alpha^2*
a*q-2*I*(-q*p^2)^(1/3)*3^(1/2)*_alpha*a*p^2*q-3*_alpha^3*p^2*b+3*(-q*p^2)^(
2/3)*a*q*p-I*3^(1/2)*b*p*q+3*(-q*p^2)^(2/3)*_alpha*b+3*b*p*q)/b,(I*3^(1/2)/
p*(-q*p^2)^(1/3)/(-3/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1
/2)),_alpha=RootOf(_Z^6*a*p^2+2*_Z^3*a*p*q+_Z^4*b+a*q^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^3+q}\left(px^3-2\,q\right)}{bx^4+\left(px^3+q\right)^2a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p*x^3+q)^(1/2)/(b*x^4+a*(p*x^3+q)^2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(p*x^3 + q)*(p*x^3 - 2*q)/(b*x^4 + (p*x^3 + q)^2*a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{p\,x^3+q}\,\left(2\,q-p\,x^3\right)}{a\,\left(p\,x^3+q\right)^2+b\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q + p*x^3)^(1/2)*(2*q - p*x^3))/(a*(q + p*x^3)^2 + b*x^4),x)
```

```
[Out] int(-((q + p*x^3)^(1/2)*(2*q - p*x^3))/(a*(q + p*x^3)^2 + b*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(px^3-2q\right)\sqrt{px^3+q}}{ap^2x^6+2apqx^3+aq^2+bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**3-2*q)*(p*x**3+q)**(1/2)/(b*x**4+a*(p*x**3+q)**2),x)
```

```
[Out] Integral((p*x**3 - 2*q)*sqrt(p*x**3 + q)/(a*p**2*x**6 + 2*a*p*q*x**3 + a*q*
*2 + b*x**4), x)
```

3.1850 $\int x^4\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=167

$$\frac{x\left(192x^{12}+360x^8+212x^4+39\right)\sqrt{\sqrt{x^4+1}+x^2}+x\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}\left(192x^{10}+264x^6+104x^2\right)}{384\sqrt{x^4+1}\left(4x^4+1\right)+384\left(4x^6+3x^2\right)}-\text{13 tar}$$

Rubi [F] time = 0.35, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^4\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^4*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

```
[Out] Defer[Int][x^4*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]], x]
```

Rubi steps

$$\int x^4\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}\,dx=\int x^4\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}\,dx$$

Mathematica [C] time = 7.29, size = 1550, normalized size = 9.28

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

```
[Out] (5*x^3*(x^2 + Sqrt[1 + x^4])^(7/2)*(1 + x^4 + x^2*Sqrt[1 + x^4])*(-((-1 + 3
*x^4 + 6*x^8 + 6*x^6*Sqrt[1 + x^4])*Hypergeometric2F1[-3/2, -3/2, 3/2, (x^2
 + Sqrt[1 + x^4])^2]) + 6*(1 + 13*x^4 + 28*x^8 + 16*x^12 + 5*x^2*Sqrt[1 + x
^4] + 20*x^6*Sqrt[1 + x^4] + 16*x^10*Sqrt[1 + x^4])*Hypergeometric2F1[-1/2,
 -1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] - 6*(1 + x^4)*(1 + 8*x^4 + 8*x^8 + 4*x
^2*Sqrt[1 + x^4] + 8*x^6*Sqrt[1 + x^4])*HypergeometricPFQ[{-1/2, -1/2, 2},
{1, 5/2}, (x^2 + Sqrt[1 + x^4])^2]))/(6*(-30*x^2*Hypergeometric2F1[-3/2, -3
/2, 3/2, (x^2 + Sqrt[1 + x^4])^2] - 190*x^6*Hypergeometric2F1[-3/2, -3/2, 3
/2, (x^2 + Sqrt[1 + x^4])^2] - 320*x^10*Hypergeometric2F1[-3/2, -3/2, 3/2,
(x^2 + Sqrt[1 + x^4])^2] - 160*x^14*Hypergeometric2F1[-3/2, -3/2, 3/2, (x^2
 + Sqrt[1 + x^4])^2] - 5*Sqrt[1 + x^4]*Hypergeometric2F1[-3/2, -3/2, 3/2, (
x^2 + Sqrt[1 + x^4])^2] - 90*x^4*Sqrt[1 + x^4]*Hypergeometric2F1[-3/2, -3/2
, 3/2, (x^2 + Sqrt[1 + x^4])^2] - 240*x^8*Sqrt[1 + x^4]*Hypergeometric2F1[-
3/2, -3/2, 3/2, (x^2 + Sqrt[1 + x^4])^2] - 160*x^12*Sqrt[1 + x^4]*Hypergeom
etric2F1[-3/2, -3/2, 3/2, (x^2 + Sqrt[1 + x^4])^2] + 220*x^2*Hypergeometric
2F1[-1/2, -1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] + 2840*x^6*Hypergeometric2F1[
-1/2, -1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] + 10000*x^10*Hypergeometric2F1[-1
/2, -1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] + 13120*x^14*Hypergeometric2F1[-1/2
, -1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] + 5760*x^18*Hypergeometric2F1[-1/2, -
1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] + 25*Sqrt[1 + x^4]*Hypergeometric2F1[-1/
2, -1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] + 960*x^4*Sqrt[1 + x^4]*Hypergeometr
ic2F1[-1/2, -1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] + 5600*x^8*Sqrt[1 + x^4]*Hy
pergeometric2F1[-1/2, -1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] + 10240*x^12*Sqrt
```

```
[1 + x^4]*Hypergeometric2F1[-1/2, -1/2, 5/2, (x^2 + Sqrt[1 + x^4])^2] + 576
0*x^16*Sqrt[1 + x^4]*Hypergeometric2F1[-1/2, -1/2, 5/2, (x^2 + Sqrt[1 + x^4
])^2] + 22*x^2*Hypergeometric2F1[1/2, 1/2, 7/2, (x^2 + Sqrt[1 + x^4])^2] +
440*x^6*Hypergeometric2F1[1/2, 1/2, 7/2, (x^2 + Sqrt[1 + x^4])^2] + 2464*x^
10*Hypergeometric2F1[1/2, 1/2, 7/2, (x^2 + Sqrt[1 + x^4])^2] + 5632*x^14*Hy
pergeometric2F1[1/2, 1/2, 7/2, (x^2 + Sqrt[1 + x^4])^2] + 5632*x^18*Hyperge
ometric2F1[1/2, 1/2, 7/2, (x^2 + Sqrt[1 + x^4])^2] + 2048*x^22*Hypergeometr
ic2F1[1/2, 1/2, 7/2, (x^2 + Sqrt[1 + x^4])^2] + 2*Sqrt[1 + x^4]*Hypergeomet
ric2F1[1/2, 1/2, 7/2, (x^2 + Sqrt[1 + x^4])^2] + 120*x^4*Sqrt[1 + x^4]*Hype
rgeometric2F1[1/2, 1/2, 7/2, (x^2 + Sqrt[1 + x^4])^2] + 1120*x^8*Sqrt[1 + x
^4]*Hypergeometric2F1[1/2, 1/2, 7/2, (x^2 + Sqrt[1 + x^4])^2] + 3584*x^12*S
qrt[1 + x^4]*Hypergeometric2F1[1/2, 1/2, 7/2, (x^2 + Sqrt[1 + x^4])^2] + 46
08*x^16*Sqrt[1 + x^4]*Hypergeometric2F1[1/2, 1/2, 7/2, (x^2 + Sqrt[1 + x^4]
)^2] + 2048*x^20*Sqrt[1 + x^4]*Hypergeometric2F1[1/2, 1/2, 7/2, (x^2 + Sqrt
[1 + x^4])^2] - 10*(10*x^2 + 152*x^6 + 592*x^10 + 832*x^14 + 384*x^18 + Sqr
t[1 + x^4] + 48*x^4*Sqrt[1 + x^4] + 320*x^8*Sqrt[1 + x^4] + 640*x^12*Sqrt[1
 + x^4] + 384*x^16*Sqrt[1 + x^4])*HypergeometricPFQ[{-1/2, -1/2, 2}, {1, 5/
2}, (x^2 + Sqrt[1 + x^4])^2] - 4*(10*x^2 + 170*x^6 + 832*x^10 + 1696*x^14 +
 1536*x^18 + 512*x^22 + Sqrt[1 + x^4] + 50*x^4*Sqrt[1 + x^4] + 400*x^8*Sqrt
[1 + x^4] + 1120*x^12*Sqrt[1 + x^4] + 1280*x^16*Sqrt[1 + x^4] + 512*x^20*Sq
rt[1 + x^4])*HypergeometricPFQ[{1/2, 1/2, 3}, {2, 7/2}, (x^2 + Sqrt[1 + x^4
])^2]))
```

IntegrateAlgebraic [A] time = 0.59, size = 167, normalized size = 1.00

$$\frac{x\left(192x^{12}+360x^8+212x^4+39\right)\sqrt{\sqrt{x^4+1}+x^2}+x\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}\left(192x^{10}+264x^6+104x^2\right)}{384\sqrt{x^4+1}\left(4x^4+1\right)+384\left(4x^6+3x^2\right)}-\frac{13\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]],x]
```

```
[Out] (x*Sqrt[1 + x^4]*(104*x^2 + 264*x^6 + 192*x^10)*Sqrt[x^2 + Sqrt[1 + x^4]] +
 x*(39 + 212*x^4 + 360*x^8 + 192*x^12)*Sqrt[x^2 + Sqrt[1 + x^4]])/(384*Sqrt
[1 + x^4]*(1 + 4*x^4) + 384*(3*x^2 + 4*x^6)) - (13*ArcTanh[(Sqrt[2]*x*Sqrt[
x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])])/(64*Sqrt[2])
```

fricas [A] time = 0.88, size = 105, normalized size = 0.63

$$-\frac{1}{384}\left(8x^7+13x^3-\left(56x^5+39x\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}+\frac{13}{512}\sqrt{2}\log\left(4x^4+4\sqrt{x^4+1}x^2-2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/384*(8*x^7 + 13*x^3 - (56*x^5 + 39*x)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4
 + 1)) + 13/512*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 - 2*(sqrt(2)*x^3 +
sqrt(2)*sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}\,x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^4, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int x^4\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int(x^4*(x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}\,x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2),x)
```

```
[Out] int(x^4*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\sqrt{x^2+\sqrt{x^4+1}}\,\sqrt{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**4+1)**(1/2)*(x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(x**2 + sqrt(x**4 + 1))*sqrt(x**4 + 1), x)
```

3.1851 $\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)}{bx^4+a\left(q+px^5\right)^2}\,dx$

Optimal. Leaf size=167

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x\sqrt{px^5+q}}{\sqrt{a}\,px^5+\sqrt{a}\,q-\sqrt{b}\,x^2}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\frac{\sqrt[4]{a}\,px^5}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{a}\,q}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}\,x^2}{\sqrt{2}\sqrt[4]{a}}}{x\sqrt{px^5+q}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}}$$

Rubi [A] time = 0.47, antiderivative size = 242, normalized size of antiderivative = 1.45, number of steps used = 10, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {6712, 211, 1165, 628, 1162, 617, 204}

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}\sqrt{px^5+q}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}\sqrt{px^5+q}}+1\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{px^5+q}}+\sqrt{a}+\frac{\sqrt{b}\,x^2}{px^5+q}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}-\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{px^5+q}}+\sqrt{a}+\frac{\sqrt{b}\,x^2}{px^5+q}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(b*x^4 + a*(q + p*x^5)^2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a^(1/4)*Sqrt[q + p*x^5])]/(Sqrt[2]*a^(3/4)*
b^(1/4)) - ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a^(1/4)*Sqrt[q + p*x^5])]/(Sqrt[
2]*a^(3/4)*b^(1/4)) + Log[Sqrt[a] + (Sqrt[b]*x^2)/(q + p*x^5) - (Sqrt[2]*a^
(1/4)*b^(1/4)*x)/Sqrt[q + p*x^5]]/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - Log[Sqrt[a]
 + (Sqrt[b]*x^2)/(q + p*x^5) + (Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[q + p*x^5]]
/(2*Sqrt[2]*a^(3/4)*b^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6712

```
Int[(u_)*(v_)^(r_.)*((a_.)*(v_)^(p_.) + (b_.)*(w_)^(q_.))^(m_.), x_Symbol]
:> With[{c = Simplify[u/(p*w*D[v, x] - q*v*D[w, x])]}, -Dist[c*q, Subst[Int
[(a + b*x^q)^m, x], x, v^(m*p + r + 1)*w], x] /; FreeQ[c, x]] /; FreeQ[{a,
b, m, p, q, r}, x] && EqQ[p + q*(m*p + r + 1), 0] && IntegerQ[q] && Integer
Q[m]
```

Rubi steps

$$\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)}{bx^4+a\left(q+px^5\right)^2}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \frac{x}{\sqrt{q+px^5}}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \frac{x}{\sqrt{q+px^5}}\right)}{\sqrt{a}} - \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \frac{x}{\sqrt{q+px^5}}\right)}{\sqrt{a}}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x}{\sqrt{q+px^5}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x}{\sqrt{q+px^5}}\right)}{2\sqrt{a}\sqrt{b}}$$

$$= \frac{\log\left(\sqrt{a}+\frac{\sqrt{b}x^2}{q+px^5}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{q+px^5}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{\log\left(\sqrt{a}+\frac{\sqrt{b}x^2}{q+px^5}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{q+px^5}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{\text{Subst}\left(\int \frac{1}{-1-\cdots}\right.}{\cdots}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}\sqrt{q+px^5}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}\sqrt{q+px^5}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{\log\left(\sqrt{a}+\frac{\sqrt{b}x^2}{q+px^5}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{q+px^5}}\right.}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

Mathematica [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)}{bx^4+a\left(q+px^5\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(b*x^4 + a*(q + p*x^5)^2),x]
[Out] Integrate[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(b*x^4 + a*(q + p*x^5)^2), x]
```

IntegrateAlgebraic [A] time = 2.10, size = 167, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x\sqrt{px^5+q}}{\sqrt{a}px^5+\sqrt{a}q-\sqrt{b}x^2}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{\tanh^{-1}\left(\frac{\frac{\sqrt[4]{a}px^5}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{a}q}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}x^2}{\sqrt{2}\sqrt[4]{a}}}{x\sqrt{px^5+q}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(b*x^4 + a*(q + p*x^5
)^2),x]
```

```
[Out] -(ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x*Sqrt[q + p*x^5])/(Sqrt[a]*q - Sqrt[b]*x
^2 + Sqrt[a]*p*x^5)]/(Sqrt[2]*a^(3/4)*b^(1/4))) - ArcTanh[((a^(1/4)*q)/(Sqr
t[2]*b^(1/4)) + (b^(1/4)*x^2)/(Sqrt[2]*a^(1/4)) + (a^(1/4)*p*x^5)/(Sqrt[2]*
b^(1/4)))/(x*Sqrt[q + p*x^5])]/(Sqrt[2]*a^(3/4)*b^(1/4))
```

fricas [B] time = 1.62, size = 354, normalized size = 2.12

$$\left(-\frac{1}{a^{3}b}\right)^{\frac{1}{4}}\arctan\left(\frac{a^{2}bx\left(-\frac{1}{a^{3}b}\right)^{\frac{3}{4}}}{\sqrt{px^{5}+q}}\right)+\frac{1}{4}\left(-\frac{1}{a^{3}b}\right)^{\frac{1}{4}}\log\left(\frac{ap^{2}x^{10}+2apqx^{5}-bx^{4}+aq^{2}+2\sqrt{px^{5}+q}\left(abx^{3}\left(-\frac{1}{a^{3}b}\right)^{\frac{1}{4}}+\left(a^{3}bpx^{6}+a^{3}bqx\right)\left(-\frac{1}{a^{3}b}\right)^{\frac{3}{4}}\right)-2\left(a^{2}bpx^{7}+a^{2}bqx^{2}\right)\sqrt{-\frac{1}{a^{3}b}}}{ap^{2}x^{10}+2apqx^{5}+bx^{4}+aq^{2}}\right)-\frac{1}{4}\left(-\frac{1}{a^{3}b}\right)^{\frac{1}{4}}\log\left(\frac{ap^{2}x^{10}+2apqx^{5}-bx^{4}+aq^{2}-2\sqrt{px^{5}+q}\left(abx^{3}\left(-\frac{1}{a^{3}b}\right)^{\frac{1}{4}}+\left(a^{3}bpx^{6}+a^{3}bqx\right)\left(-\frac{1}{a^{3}b}\right)^{\frac{3}{4}}\right)-2\left(a^{2}bpx^{7}+a^{2}bqx^{2}\right)\sqrt{-\frac{1}{a^{3}b}}}{ap^{2}x^{10}+2apqx^{5}+bx^{4}+aq^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^5+q)^(1/2)*(3*p*x^5-2*q)/(b*x^4+a*(p*x^5+q)^2),x, algorithm=
"fricas")
```

```
[Out] (-1/(a^3*b))^(1/4)*arctan(a^2*b*x*(-1/(a^3*b))^(3/4)/sqrt(p*x^5 + q)) + 1/4
*(-1/(a^3*b))^(1/4)*log((a*p^2*x^10 + 2*a*p*q*x^5 - b*x^4 + a*q^2 + 2*sqrt(
p*x^5 + q)*(a*b*x^3*(-1/(a^3*b))^(1/4) + (a^3*b*p*x^6 + a^3*b*q*x)*(-1/(a^3
*b))^(3/4)) - 2*(a^2*b*p*x^7 + a^2*b*q*x^2)*sqrt(-1/(a^3*b)))/(a*p^2*x^10 +
 2*a*p*q*x^5 + b*x^4 + a*q^2)) - 1/4*(-1/(a^3*b))^(1/4)*log((a*p^2*x^10 + 2
*a*p*q*x^5 - b*x^4 + a*q^2 - 2*sqrt(p*x^5 + q)*(a*b*x^3*(-1/(a^3*b))^(1/4)
+ (a^3*b*p*x^6 + a^3*b*q*x)*(-1/(a^3*b))^(3/4)) - 2*(a^2*b*p*x^7 + a^2*b*q*
x^2)*sqrt(-1/(a^3*b)))/(a*p^2*x^10 + 2*a*p*q*x^5 + b*x^4 + a*q^2))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^5+q)^(1/2)*(3*p*x^5-2*q)/(b*x^4+a*(p*x^5+q)^2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p\,x^{5}+q}\,\left(3p\,x^{5}-2q\right)}{b\,x^{4}+a\left(p\,x^{5}+q\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^5+q)^(1/2)*(3*p*x^5-2*q)/(b*x^4+a*(p*x^5+q)^2),x)
```

```
[Out] int((p*x^5+q)^(1/2)*(3*p*x^5-2*q)/(b*x^4+a*(p*x^5+q)^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,px^{5}-2\,q\right)\sqrt{px^{5}+q}}{bx^{4}+\left(px^{5}+q\right)^{2}a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^5+q)^(1/2)*(3*p*x^5-2*q)/(b*x^4+a*(p*x^5+q)^2),x, algorithm=
"maxima")
```

```
[Out] integrate((3*p*x^5 - 2*q)*sqrt(p*x^5 + q)/(b*x^4 + (p*x^5 + q)^2*a), x)
```

mupad [B] time = 150.61, size = 498, normalized size = 2.98

$$\frac{\ln\left(\frac{\left(2\sqrt{p\,x^5+q}\,{\left(-a^3\,b\right)}^{15/4}-a^{23/2}\,b^{7/2}\,p\,x^4+a^{19/2}\,b^{7/2}\,x\,\sqrt{-a^3\,b}\right)\left(2\,a^{27/2}\,b^{7/2}\,q\,\sqrt{p\,x^5+q}\,{\left(-a^3\,b\right)}^{15/4}-a^{24}\,b^8\,x^3+a^{25}\,b^7\,p\,x^4\,\left(p\,x^5+q\right)+a^{23}\,b^7\,q\,x\,\sqrt{-a^3\,b}+2\,a^{23}\,b^7\,x\,\left(p\,x^5+q\right)\sqrt{-a^3\,b}\right)}{\left(x^2\,\sqrt{-a^3\,b}+a^2\,q+a^2\,p\,x^5\right)\left(4\,q\,{\left(-a^3\,b\right)}^{15/2}+2\,p\,x^5\,{\left(-a^3\,b\right)}^{15/2}+a^{22}\,b^8\,x^2-a^{23}\,b^7\,p^2\,x^8\right)}\right){\left(-a^3\,b\right)}^{1/4}}{2\,a^{3/2}\,\sqrt{b}}+\frac{\ln\left(\frac{\left(-2\sqrt{p\,x^5+q}\,{\left(-a^3\,b\right)}^{15/4}+a^{23/2}\,b^{7/2}\,p\,x^4\,1{}\mathrm{i}+a^{19/2}\,b^{7/2}\,x\,\sqrt{-a^3\,b}\,1{}\mathrm{i}\right)\left(a^{24}\,b^8\,x^3\,1{}\mathrm{i}-2\,a^{27/2}\,b^{7/2}\,q\,\sqrt{p\,x^5+q}\,{\left(-a^3\,b\right)}^{15/4}-a^{25}\,b^7\,p\,x^4\,\left(p\,x^5+q\right)1{}\mathrm{i}+a^{23}\,b^7\,q\,x\,\sqrt{-a^3\,b}\,1{}\mathrm{i}+a^{23}\,b^7\,x\,\left(p\,x^5+q\right)\sqrt{-a^3\,b}\,2{}\mathrm{i}\right)}{\left(a^2\,q-x^2\,\sqrt{-a^3\,b}+a^2\,p\,x^5\right)\left(4\,q\,{\left(-a^3\,b\right)}^{15/2}+2\,p\,x^5\,{\left(-a^3\,b\right)}^{15/2}-a^{22}\,b^8\,x^2+a^{23}\,b^7\,p^2\,x^8\right)}\right){\left(-a^3\,b\right)}^{1/4}\,1{}\mathrm{i}}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q + p*x^5)^(1/2)*(2*q - 3*p*x^5))/(a*(q + p*x^5)^2 + b*x^4),x)
```

```
[Out] (log(((2*(q + p*x^5)^(1/2)*(-a^3*b)^(15/4) - a^(23/2)*b^(7/2)*p*x^4 + a^(19
/2)*b^(7/2)*x*(-a^3*b)^(1/2))*(2*a^(27/2)*b^(7/2)*q*(q + p*x^5)^(1/2)*(-a^3
*b)^(15/4) - a^24*b^8*x^3 + a^25*b^7*p*x^4*(q + p*x^5) + a^23*b^7*q*x*(-a^3
*b)^(1/2) + 2*a^23*b^7*x*(q + p*x^5)*(-a^3*b)^(1/2)))/((x^2*(-a^3*b)^(1/2)
+ a^2*q + a^2*p*x^5)*(4*q*(-a^3*b)^(15/2) + 2*p*x^5*(-a^3*b)^(15/2) + a^22*
b^8*x^2 - a^23*b^7*p^2*x^8)))*(-a^3*b)^(1/4))/(2*a^(3/2)*b^(1/2)) + (log(((
a^(23/2)*b^(7/2)*p*x^4*1i - 2*(q + p*x^5)^(1/2)*(-a^3*b)^(15/4) + a^(19/2)*
b^(7/2)*x*(-a^3*b)^(1/2)*1i)*(a^24*b^8*x^3*1i - 2*a^(27/2)*b^(7/2)*q*(q + p
*x^5)^(1/2)*(-a^3*b)^(15/4) - a^25*b^7*p*x^4*(q + p*x^5)*1i + a^23*b^7*q*x*
(-a^3*b)^(1/2)*1i + a^23*b^7*x*(q + p*x^5)*(-a^3*b)^(1/2)*2i))/((a^2*q - x^
2*(-a^3*b)^(1/2) + a^2*p*x^5)*(4*q*(-a^3*b)^(15/2) + 2*p*x^5*(-a^3*b)^(15/2
) - a^22*b^8*x^2 + a^23*b^7*p^2*x^8)))*(-a^3*b)^(1/4)*1i)/(2*a^(3/2)*b^(1/2
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^5+q}\,\left(3px^5-2q\right)}{ap^2x^{10}+2apqx^5+aq^2+bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**5+q)**(1/2)*(3*p*x**5-2*q)/(b*x**4+a*(p*x**5+q)**2),x)
```

```
[Out] Integral(sqrt(p*x**5 + q)*(3*p*x**5 - 2*q)/(a*p**2*x**10 + 2*a*p*q*x**5 + a
*q**2 + b*x**4), x)
```

3.1852 $\int \frac{-3+x}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$

Optimal. Leaf size=168

$$-\frac{\log\left(2\sqrt[3]{1-x^2}+2^{2/3}x+2^{2/3}\right)}{2^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x^2-2\left(1-x^2\right)^{2/3}+\left(2^{2/3}x+2^{2/3}\right)\sqrt[3]{1-x^2}-2\sqrt[3]{2}x-\sqrt[3]{2}\right)}{2\,2^{2/3}}-\frac{\sqrt{3}\,\text{ta}}{}$$

Rubi [A] time = 0.02, antiderivative size = 95, normalized size of antiderivative = 0.57, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1008}

$$\frac{\log\left(x^2+3\right)}{2\,2^{2/3}}-\frac{3\log\left((x+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{1-x}\right)}{2\,2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(x+1)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 + x)^(2/3))/(Sqrt[3]*(1 - x)^(1/3))
])/2^(2/3) + Log[3 + x^2]/(2*2^(2/3)) - (3*Log[2^(1/3)*(1 - x)^(1/3) + (1 +
 x)^(2/3)])/(2*2^(2/3))
```

Rule 1008

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[(Sqrt[3]*h*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^
(2/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))])/(2^(2/3)*a^(1/3)*f), x] + (-Simp[(
3*h*Log[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2^(5/3)*a^
(1/3)*f), x] + Simp[(h*Log[d + f*x^2])/(2^(5/3)*a^(1/3)*f), x]) /; FreeQ[{a
, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && G
tQ[a, 0]
```

Rubi steps

$$\int \frac{-3+x}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx=\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1+x)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}}+\frac{\log\left(3+x^2\right)}{2\,2^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{1-x}+(1+x)^{2/3}\right)}{2\,2^{2/3}}$$

Mathematica [C] time = 0.22, size = 143, normalized size = 0.85

$$\frac{1}{6}x^2F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)+\frac{27xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3 + x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (x^2*AppellF1[1, 1/3, 1, 2, x^2, -1/3*x^2])/6 + (27*x*AppellF1[1/2, 1/3, 1,
 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)*(-9*AppellF1[1/2, 1/3, 1,
3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2] - Ap
pellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

IntegrateAlgebraic [A] time = 0.24, size = 168, normalized size = 1.00

$$-\frac{\log\left(2\sqrt[3]{1-x^2}+2^{2/3}x+2^{2/3}\right)}{2^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x^2-2\left(1-x^2\right)^{2/3}+\left(2^{2/3}x+2^{2/3}\right)\sqrt[3]{1-x^2}-2\sqrt[3]{2}x-\sqrt[3]{2}\right)}{2\,2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^2}}{\sqrt[3]{1-x^2}-2^{2/3}x-2^{2/3}}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 + x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^2)^(1/3))/(-2^(2/3) - 2^(2/3)*x + (1 - x^
2)^(1/3))])/2^(2/3)) - Log[2^(2/3) + 2^(2/3)*x + 2*(1 - x^2)^(1/3)]/2^(2/3)
 + Log[-2^(1/3) - 2*2^(1/3)*x - 2^(1/3)*x^2 + (2^(2/3) + 2^(2/3)*x)*(1 - x^
2)^(1/3) - 2*(1 - x^2)^(2/3)]/(2*2^(2/3))
```

fricas [B] time = 5.52, size = 315, normalized size = 1.88

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(12\cdot 4^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(x^4+3x^3+3x^2+9x\right)\left(-x^2+1\right)^{\frac{2}{3}}-12(-1)^{\frac{1}{3}}\left(x^5+19x^4+42x^3+6x^2-27x-9\right)\left(-x^2+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\left(x^6-18x^5-117x^4-36x^3+207x^2+54x-27\right)\right)}{6\left(x^6+54x^5+171x^4+108x^3-81x^2-162x-27\right)}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{6\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^2+3x\right)\left(-x^2+1\right)^{\frac{2}{3}}-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^4+18x^3+24x^2-18x-9\right)+6\left(x^3+7x^2+3x-3\right)\left(-x^2+1\right)^{\frac{1}{3}}}{x^4+6x^2+9}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(\frac{6\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}(x+1)-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^2+3\right)+12\left(-x^2+1\right)^{\frac{2}{3}}}{x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^(2/3)*(-1)
^(2/3)*(x^4 + 3*x^3 + 3*x^2 + 9*x)*(-x^2 + 1)^(2/3) - 12*(-1)^(1/3)*(x^5 +
19*x^4 + 42*x^3 + 6*x^2 - 27*x - 9)*(-x^2 + 1)^(1/3) + 4^(1/3)*(x^6 - 18*x^
5 - 117*x^4 - 36*x^3 + 207*x^2 + 54*x - 27))/(x^6 + 54*x^5 + 171*x^4 + 108*
x^3 - 81*x^2 - 162*x - 27)) - 1/24*4^(2/3)*(-1)^(1/3)*log(-(6*4^(2/3)*(-1)^
(1/3)*(x^2 + 3*x)*(-x^2 + 1)^(2/3) - 4^(1/3)*(-1)^(2/3)*(x^4 + 18*x^3 + 24*
x^2 - 18*x - 9) + 6*(x^3 + 7*x^2 + 3*x - 3)*(-x^2 + 1)^(1/3))/(x^4 + 6*x^2
+ 9)) + 1/12*4^(2/3)*(-1)^(1/3)*log((6*4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3)*
(x + 1) - 4^(2/3)*(-1)^(1/3)*(x^2 + 3) + 12*(-x^2 + 1)^(2/3))/(x^2 + 3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-3}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate((x - 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

maple [C] time = 7.91, size = 1553, normalized size = 9.24

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+x)/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] -1/2*ln((2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+
2)^3*x^2-8*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^
3+2)^2*x^2+6*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^
3+2)^3*x-24*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z
^3+2)^2*x-18*(-x^2+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+2*_Z*R
ootOf(_Z^3+2)+4*_Z^2)-6*(-x^2+1)^(1/3)*RootOf(_Z^3+2)^2*x-18*(-x^2+1)^(1/3)
*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x-6*(-x
^2+1)^(1/3)*RootOf(_Z^3+2)^2-18*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z
*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)-RootOf(_Z^3+2)*x^2+4*RootOf(RootOf(_
Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^2+12*(-x^2+1)^(2/3)-3*RootOf(_Z^3+2)
+12*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2))/(2*RootOf(RootOf(_
Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x+3)/(2*RootOf(Root
Of(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x-3))*RootOf(_Z
^3+2)-ln((2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3
+2)^3*x^2-8*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z
^3+2)^2*x^2+6*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z
```

```
^3+2)^3*x-24*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_
Z^3+2)^2*x-18*(-x^2+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+2*_Z*
RootOf(_Z^3+2)+4*_Z^2)-6*(-x^2+1)^(1/3)*RootOf(_Z^3+2)^2*x-18*(-x^2+1)^(1/3
)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x-6*(-
x^2+1)^(1/3)*RootOf(_Z^3+2)^2-18*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_
Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)-RootOf(_Z^3+2)*x^2+4*RootOf(RootOf(
_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^2+12*(-x^2+1)^(2/3)-3*RootOf(_Z^3+2
)+12*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2))/(2*RootOf(RootOf(
_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x+3)/(2*RootOf(Roo
tOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x-3))*RootOf(R
ootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)+RootOf(RootOf(_Z^3+2)^2+2*_Z*Ro
otOf(_Z^3+2)+4*_Z^2)*ln(-(6*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_
Z^2)*RootOf(_Z^3+2)^3*x^2+8*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_
Z^2)^2*RootOf(_Z^3+2)^2*x^2+18*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+
4*_Z^2)*RootOf(_Z^3+2)^3*x+24*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4
*_Z^2)^2*RootOf(_Z^3+2)^2*x-18*(-x^2+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootO
f(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)-3*(-x^2+1)^(1/3)*RootOf(_Z^3+2)^2*x
-18*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*Root
Of(_Z^3+2)*x-3*(-x^2+1)^(1/3)*RootOf(_Z^3+2)^2-18*(-x^2+1)^(1/3)*RootOf(Roo
tOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)-3*RootOf(_Z^3+2)*x
^2-4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^2-18*RootOf(_Z^3
+2)*x-24*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x+6*(-x^2+1)^(
2/3)+9*RootOf(_Z^3+2)+12*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2
))/(2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*
x-x+3)/(2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2
)^2*x-x-3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-3}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
[Out] integrate((x - 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x-3}{\left(1-x^2\right)^{1/3}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3)/((1 - x^2)^(1/3)*(x^2 + 3)),x)
[Out] int((x - 3)/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-3}{\sqrt[3]{-\left(x-1\right)\left(x+1\right)}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(-x**2+1)**(1/3)/(x**2+3),x)
[Out] Integral((x - 3)/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.1853

$$\int \frac{x^3(-3ab+(a+2b)x)}{(-a+x)(-b+x)\sqrt[4]{x(-a+x)(-b+x)^2}\left(-ab^2d+b(2a+b)dx-(a+2b)dx^2+(-1+d)x^3\right)}\,dx$$

Optimal. Leaf size=168

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}}{x}\right)-2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-}}{x}\right.$$

Rubi [F] time = 73.78, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3(-3ab+(a+2b)x)}{(-a+x)(-b+x)\sqrt[4]{x(-a+x)(-b+x)^2}\left(-ab^2d+b(2a+b)dx-(a+2b)dx^2+(-1+d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*(-3*a*b + (a + 2*b)*x))/((-a + x)*(-b + x)*(x*(-a + x)*(-b + x)^2)
^(1/4)*(-(a*b^2*d) + b*(2*a + b)*d*x - (a + 2*b)*d*x^2 + (-1 + d)*x^3)),x]
```

```
[Out] (-4*(a + 2*b)*x^2*(1 - x/a)^(1/4)*Hypergeometric2F1[5/4, 7/4, 11/4, -(((a -
 b)*x)/(a*(b - x)))])/(7*a*b*(1 - d)*(-((a - x)*(b - x)^2*x))^(1/4)*(1 - x/
b)^(5/4)) + (16*(a^2*d + 4*b^2*d + a*b*(3 + d))*x*Gamma[7/4]*(11*b*(7*b - 4
*x)*(a - x)*Hypergeometric2F1[1, 5/4, 11/4, ((a - b)*x)/(b*(a - x))] + 20*(
a - b)*(b - x)*x*Hypergeometric2F1[2, 9/4, 15/4, ((a - b)*x)/(b*(a - x))]))
/(693*b^3*(1 - d)^2*(a - x)^2*(-((a - x)*(b - x)^2*x))^(1/4)*Gamma[3/4]) -
(4*a*b^2*d*(a^2*d + 4*b^2*d + a*b*(3 + d))*x^(1/4)*(-a + x)^(1/4)*Sqrt[-b +
 x]*Defer[Subst][Defer[Int][x^2/((-a + x^4)^(5/4)*(-b + x^4)^(3/2)*(a*b^2*d
 - 2*a*b*(1 + b/(2*a))*d*x^4 + a*(1 + (2*b)/a)*d*x^8 + (1 - d)*x^12)), x],
x, x^(1/4)])/((1 - d)^2*(-((a - x)*(b - x)^2*x))^(1/4)) + (4*b*d*(2*a^3*d +
 4*b^3*d + a^2*b*(7 + 2*d) + a*b^2*(5 + 7*d))*x^(1/4)*(-a + x)^(1/4)*Sqrt[-
b + x]*Defer[Subst][Defer[Int][x^6/((-a + x^4)^(5/4)*(-b + x^4)^(3/2)*(a*b^
2*d - 2*a*b*(1 + b/(2*a))*d*x^4 + a*(1 + (2*b)/a)*d*x^8 + (1 - d)*x^12)), x
], x, x^(1/4)])/((1 - d)^2*(-((a - x)*(b - x)^2*x))^(1/4)) - (4*(a + 2*b)*d
*(a*b*(5 - d) + a^2*d + b^2*(1 + 3*d))*x^(1/4)*(-a + x)^(1/4)*Sqrt[-b + x]*
Defer[Subst][Defer[Int][x^10/((-a + x^4)^(5/4)*(-b + x^4)^(3/2)*(a*b^2*d -
2*a*b*(1 + b/(2*a))*d*x^4 + a*(1 + (2*b)/a)*d*x^8 + (1 - d)*x^12)), x], x,
x^(1/4)])/((1 - d)^2*(-((a - x)*(b - x)^2*x))^(1/4))
```

Rubi steps

$$\int \frac{x^3(-3ab+(a+2b)x)}{(-a+x)(-b+x)\sqrt[4]{x(-a+x)(-b+x)^2}\left(-ab^2d+b(2a+b)dx-(a+2b)dx^2+(-1+d)x^3\right)}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{-a+\right.}{}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-a\right.}{}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-a\right.}{}$$

$$= -\frac{\left(4(a+2b)\right.}{}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-a\right.}{}$$

$$= -\frac{\left(4ab^2d\left(a^2\right.\right.}{}$$

$$= -\frac{4(a+2b)}{7ab(1-d)}$$

Mathematica [F] time = 7.31, size = 0, normalized size = 0.00

$$\int \frac{x^3(-3ab+(a+2b)x)}{(-a+x)(-b+x)\sqrt[4]{x(-a+x)(-b+x)^2}\left(-ab^2d+b(2a+b)dx-(a+2b)dx^2+(-1+d)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(-3*a*b + (a + 2*b)*x))/((-a + x)*(-b + x)*(x*(-a + x)*(-b +
 x)^2)^(1/4)*(-(a*b^2*d) + b*(2*a + b)*d*x - (a + 2*b)*d*x^2 + (-1 + d)*x^3
)),x]
```

```
[Out] Integrate[(x^3*(-3*a*b + (a + 2*b)*x))/((-a + x)*(-b + x)*(x*(-a + x)*(-b +
 x)^2)^(1/4)*(-(a*b^2*d) + b*(2*a + b)*d*x - (a + 2*b)*d*x^2 + (-1 + d)*x^3
)), x]
```

IntegrateAlgebraic [A] time = 1.32, size = 168, normalized size = 1.00

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}}{x}\right)-2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}}{x}\right)+\frac{4\left(-ab^2x+2abx^2-ax^3+b^2x^2-2bx^3+x^4\right)^{3/4}}{(x-a)(x-b)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(-3*a*b + (a + 2*b)*x))/((-a + x)*(-b + x)*(x*(-a +
 x)*(-b + x)^2)^(1/4)*(-(a*b^2*d) + b*(2*a + b)*d*x - (a + 2*b)*d*x^2 + (-1
 + d)*x^3)),x]
```

```
[Out] (4*(-(a*b^2*x) + 2*a*b*x^2 + b^2*x^2 - a*x^3 - 2*b*x^3 + x^4)^(3/4))/((-a +
 x)*(-b + x)^2) + 2*d^(1/4)*ArcTan[(d^(1/4)*(-(a*b^2*x) + (2*a*b + b^2)*x^2
 + (-a - 2*b)*x^3 + x^4)^(1/4))/x] - 2*d^(1/4)*ArcTanh[(d^(1/4)*(-(a*b^2*x)
 + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/4))/x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-3*a*b+(a+2*b)*x)/(-a+x)/(-b+x)/(x*(-a+x)*(-b+x)^2)^(1/4)/(-
a*b^2*d+b*(2*a+b)*d*x-(a+2*b)*d*x^2+(-1+d)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3\,ab - (a+2\,b)x)x^3}{\left(ab^2d - (2\,a+b)bdx + (a+2\,b)dx^2 - (d-1)x^3\right)\left(-(a-x)(b-x)^2x\right)^{\frac{1}{4}}(a-x)(b-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-3*a*b+(a+2*b)*x)/(-a+x)/(-b+x)/(x*(-a+x)*(-b+x)^2)^(1/4)/(-
a*b^2*d+b*(2*a+b)*d*x-(a+2*b)*d*x^2+(-1+d)*x^3),x, algorithm="giac")
```

```
[Out] integrate((3*a*b - (a + 2*b)*x)*x^3/((a*b^2*d - (2*a + b)*b*d*x + (a + 2*b)
*d*x^2 - (d - 1)*x^3)*(-(a - x)*(b - x)^2*x)^(1/4)*(a - x)*(b - x)), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^3\,(-3ab+(a+2b)\,x)}{(-a+x)\,(-b+x)\left(x\,(-a+x)\,(-b+x)^2\right)^{\frac{1}{4}}\left(-a\,b^2d+b\,(2a+b)\,dx-(a+2b)\,d\,x^2+(-1+d)\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-3*a*b+(a+2*b)*x)/(-a+x)/(-b+x)/(x*(-a+x)*(-b+x)^2)^(1/4)/(-a*b^2*
d+b*(2*a+b)*d*x-(a+2*b)*d*x^2+(-1+d)*x^3),x)
```

```
[Out] int(x^3*(-3*a*b+(a+2*b)*x)/(-a+x)/(-b+x)/(x*(-a+x)*(-b+x)^2)^(1/4)/(-a*b^2*
d+b*(2*a+b)*d*x-(a+2*b)*d*x^2+(-1+d)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3\,ab - (a+2\,b)x)x^3}{\left(ab^2d - (2\,a+b)bdx + (a+2\,b)dx^2 - (d-1)x^3\right)\left(-(a-x)(b-x)^2x\right)^{\frac{1}{4}}(a-x)(b-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-3*a*b+(a+2*b)*x)/(-a+x)/(-b+x)/(x*(-a+x)*(-b+x)^2)^(1/4)/(-
a*b^2*d+b*(2*a+b)*d*x-(a+2*b)*d*x^2+(-1+d)*x^3),x, algorithm="maxima")
```

```
[Out] integrate((3*a*b - (a + 2*b)*x)*x^3/((a*b^2*d - (2*a + b)*b*d*x + (a + 2*b)
*d*x^2 - (d - 1)*x^3)*(-(a - x)*(b - x)^2*x)^(1/4)*(a - x)*(b - x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^3\,\left(3\,a\,b-x\,\left(a+2\,b\right)\right)}{\left(a-x\right)\,\left(b-x\right)\,{\left(-x\,\left(a-x\right)\,{\left(b-x\right)}^2\right)}^{1/4}\,\left(x^3\,\left(d-1\right)-d\,x^2\,\left(a+2\,b\right)-a\,b^2\,d+b\,d\,x\,\left(2\,a+b\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(3*a*b - x*(a + 2*b)))/((a - x)*(b - x)*(-x*(a - x)*(b - x)^2)^(1
/4)*(x^3*(d - 1) - d*x^2*(a + 2*b) - a*b^2*d + b*d*x*(2*a + b))),x)
```

```
[Out] -int((x^3*(3*a*b - x*(a + 2*b)))/((a - x)*(b - x)*(-x*(a - x)*(b - x)^2)^(1
/4)*(x^3*(d - 1) - d*x^2*(a + 2*b) - a*b^2*d + b*d*x*(2*a + b))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-3*a*b+(a+2*b)*x)/(-a+x)/(-b+x)/(x*(-a+x)*(-b+x)**2)**(1/4)
/(-a*b**2*d+b*(2*a+b)*d*x-(a+2*b)*d*x**2+(-1+d)*x**3),x)
```

```
[Out] Timed out
```

3.1854 $\int \frac{ax+\sqrt{b^2+a^2x^2}}{b+\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$

Optimal. Leaf size=168

$$\sqrt{a^2x^2+b^2}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{3a}-\frac{b}{2a}\right)+\frac{(ax+3b^2)\sqrt{\sqrt{a^2x^2+b^2}+ax}}{3a}+\frac{b\log\left(\sqrt{a^2x^2+b^2}+ax\right)}{2a}+\frac{(-b^3-b)\,\mathrm{lo}}{}$$

Rubi [F] time = 13.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ax+\sqrt{b^2+a^2x^2}}{b+\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*x + Sqrt[b^2 + a^2*x^2])/(b + Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

```
[Out] x/(4*b) - (b*x)/4 - ((1 + b^2)*x)/(4*b) + Sqrt[b^2 + a^2*x^2]/(4*a*b) + ((1
 - b^2)*Sqrt[b^2 + a^2*x^2])/(4*a*b) - ((1 + b^2)*Sqrt[b^2 + a^2*x^2])/(4*a
*b) - (x*Sqrt[b^2 + a^2*x^2])/(4*b) - ((1 - a*x)*Sqrt[b^2 + a^2*x^2])/(4*a*
b) + 1/(4*a*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]) - b^2/(4*a*Sqrt[a*x + Sqrt[b^2
 + a^2*x^2]]) - (1 + b^2)/(4*a*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]) + ((1 + b^2
)^2*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/(4*a*b^2) - ((1 - b^4)*Sqrt[a*x + Sqrt
[b^2 + a^2*x^2]])/(4*a*b^2) + (a*x + Sqrt[b^2 + a^2*x^2])^(3/2)/(12*a) + ((
1 + b^(-2))*(a*x + Sqrt[b^2 + a^2*x^2])^(3/2))/(12*a) - (a*x + Sqrt[b^2 + a
^2*x^2])^(3/2)/(12*a*b^2) - ((1 + b^2)^2*ArcTan[Sqrt[a*x + Sqrt[b^2 + a^2*x
^2]]])/(4*a*b^2) + ((1 - b^4)*ArcTan[Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]])/(4*a
*b^2) - ArcTanh[(a*x)/Sqrt[b^2 + a^2*x^2]]/(8*a*b) - (b*ArcTanh[(a*x)/Sqrt[
b^2 + a^2*x^2]])/(4*a) - ((1 - b^2)^2*ArcTanh[(a*x)/Sqrt[b^2 + a^2*x^2]])/(
8*a*b) + ((1 + 2*b^2)*ArcTanh[(a*x)/Sqrt[b^2 + a^2*x^2]])/(8*a*b) + ((1 - b
^4)*ArcTanh[(a*x)/Sqrt[b^2 + a^2*x^2]])/(8*a*b) + ((1 + b^2)^2*ArcTanh[(2*b
^2 - a*(1 - b^2)*x)/((1 + b^2)*Sqrt[b^2 + a^2*x^2])])/(8*a*b) - ((1 - b^4)*
ArcTanh[(2*b^2 - a*(1 - b^2)*x)/((1 + b^2)*Sqrt[b^2 + a^2*x^2])])/(8*a*b) -
 ((1 + b^2)^2*ArcTanh[Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]/b])/(4*a*b) + ((1 - b
^4)*ArcTanh[Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]/b])/(4*a*b) - ((1 + b^2)^2*Log[
1 - b^2 + 2*a*x])/(8*a*b) + ((1 - b^4)*Log[1 - b^2 + 2*a*x])/(8*a*b) - Defe
r[Int][(Sqrt[b^2 + a^2*x^2]*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/(-1 + b^2 - 2*
a*x), x]/b^2 + (1 - b^(-2))*Defer[Int][(Sqrt[b^2 + a^2*x^2]*Sqrt[a*x + Sqrt
[b^2 + a^2*x^2]])/(1 - b^2 + 2*a*x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{ax+\sqrt{b^2+a^2x^2}}{b+\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx &= \int\left(\frac{ax}{b+\sqrt{ax+\sqrt{b^2+a^2x^2}}}+\frac{\sqrt{b^2+a^2x^2}}{b+\sqrt{ax+\sqrt{b^2+a^2x^2}}}\right)dx\\
&= a\int\frac{x}{b+\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx+\int\frac{\sqrt{b^2+a^2x^2}}{b+\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx\\
&= a\int\left(\frac{-1-b^2}{4ab}+\frac{x}{2b}+\frac{-1+b^4}{4ab\left(-1+b^2-2ax\right)}-\frac{\sqrt{b^2+a^2x^2}}{2ab}+\frac{\left(1-b^2\right)\sqrt{b^2+a^2x^2}}{2ab\left(1-b^2+2ax\right)}\right.\\
&= \text{rest of steps removed due to Latex formating problem}
\end{aligned}$$

Mathematica [B] time = 3.79, size = 423, normalized size = 2.52

$$\frac{b\left(-2\sqrt{a^2x^2+b^2}+b^2\log\left(b^2\left(\sqrt{a^2x^2+b^2}+ax+2\right)+\sqrt{a^2x^2+b^2}-ax\right)-\left(b^2-1\right)\log\left(\sqrt{a^2x^2+b^2}+ax\right)+\log\left(b^2\left(\sqrt{a^2x^2+b^2}+ax+2\right)+\sqrt{a^2x^2+b^2}-ax\right)-2\left(b^2+1\right)\log\left(2ax-b^2+1\right)-2ax\right)}{4a}+\frac{ab^2x\left(ax\left(6\sqrt{a^2x^2+b^2}+5\right)+3\sqrt{a^2x^2+b^2}+6a^2x^2\right)-3\left(b^2+1\right)b\sqrt{\sqrt{a^2x^2+b^2}+ax}\left(ax\left(\sqrt{a^2x^2+b^2}+ax\right)+b^2\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{b}\right)+b^4\left(3\sqrt{a^2x^2+b^2}+6ax+1\right)+4a^3x^3\left(\sqrt{a^2x^2+b^2}+ax\right)}{3a\sqrt{\sqrt{a^2x^2+b^2}+ax}\left(ax\left(\sqrt{a^2x^2+b^2}+ax\right)+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + Sqrt[b^2 + a^2*x^2])/(b + Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])
,x]
```

```
[Out] (4*a^3*x^3*(a*x + Sqrt[b^2 + a^2*x^2]) + b^4*(1 + 6*a*x + 3*Sqrt[b^2 + a^2*
x^2]) + a*b^2*x*(6*a^2*x^2 + 3*Sqrt[b^2 + a^2*x^2] + a*x*(5 + 6*Sqrt[b^2 +
a^2*x^2])) - 3*b*(1 + b^2)*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]*(b^2 + a*x*(a*x
+ Sqrt[b^2 + a^2*x^2]))*ArcTanh[Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]/b])/(3*a*Sq
rt[a*x + Sqrt[b^2 + a^2*x^2]]*(b^2 + a*x*(a*x + Sqrt[b^2 + a^2*x^2]))) + (b
*(-2*a*x - 2*Sqrt[b^2 + a^2*x^2] - 2*(1 + b^2)*Log[1 - b^2 + 2*a*x] - (-1 +
 b^2)*Log[a*x + Sqrt[b^2 + a^2*x^2]] + Log[-(a*x) + Sqrt[b^2 + a^2*x^2] + b
^2*(2 + a*x + Sqrt[b^2 + a^2*x^2])] + b^2*Log[-(a*x) + Sqrt[b^2 + a^2*x^2]
+ b^2*(2 + a*x + Sqrt[b^2 + a^2*x^2])]))/(4*a)
```

IntegrateAlgebraic [A] time = 0.32, size = 168, normalized size = 1.00

$$\sqrt{a^2x^2+b^2}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{3a}-\frac{b}{2a}\right)+\frac{\left(ax+3b^2\right)\sqrt{\sqrt{a^2x^2+b^2}+ax}}{3a}+\frac{b\log\left(\sqrt{a^2x^2+b^2}+ax\right)}{2a}+\frac{\left(-b^3-b\right)\log\left(\sqrt{\sqrt{a^2x^2+b^2}+ax}+b\right)}{a}-\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x + Sqrt[b^2 + a^2*x^2])/(b + Sqrt[a*x + Sqrt[b^2 + a
^2*x^2]]),x]
```

```
[Out] -1/2*(b*x) + ((3*b^2 + a*x)*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/(3*a) + Sqrt[b
^2 + a^2*x^2]*(-1/2*b/a + Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]/(3*a)) + (b*Log[a
*x + Sqrt[b^2 + a^2*x^2]])/(2*a) + ((-b - b^3)*Log[b + Sqrt[a*x + Sqrt[b^2
+ a^2*x^2]]])/a
```

fricas [A] time = 0.49, size = 122, normalized size = 0.73

$$-\frac{3\,abx+6\left(b^3+b\right)\log\left(b+\sqrt{ax+\sqrt{a^2x^2+b^2}}\right)-6\,b\log\left(\sqrt{ax+\sqrt{a^2x^2+b^2}}\right)-2\left(3\,b^2+ax+\sqrt{a^2x^2+b^2}\right)\sqrt{ax+\sqrt{a^2x^2+b^2}}+3\,\sqrt{a^2x^2+b^2}\,b}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2+b^2)^(1/2))/(b+(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)),x,
algorithm="fricas")
```

```
[Out] -1/6*(3*a*b*x + 6*(b^3 + b)*log(b + sqrt(a*x + sqrt(a^2*x^2 + b^2))) - 6*b*
log(sqrt(a*x + sqrt(a^2*x^2 + b^2))) - 2*(3*b^2 + a*x + sqrt(a^2*x^2 + b^2)
)*sqrt(a*x + sqrt(a^2*x^2 + b^2)) + 3*sqrt(a^2*x^2 + b^2)*b)/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+\sqrt{a^2x^2+b^2}}{b+\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2+b^2)^(1/2))/(b+(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)),x,
algorithm="giac")
```

```
[Out] integrate((a*x + sqrt(a^2*x^2 + b^2))/(b + sqrt(a*x + sqrt(a^2*x^2 + b^2)))
, x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{ax+\sqrt{a^2x^2+b^2}}{b+\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+(a^2*x^2+b^2)^(1/2))/(b+(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)),x)
```

```
[Out] int((a*x+(a^2*x^2+b^2)^(1/2))/(b+(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ax^2}{4\,b}+\frac{\frac{b^2\,\operatorname{arsinh}\left(\frac{ax}{b}\right)}{2\,a}+\frac{1}{2}\,\sqrt{a^2x^2+b^2}\,x}{2\,b}-\int -\frac{ab^2x-2\,a^2x^2-b^2+\sqrt{a^2x^2+b^2}\,\left(b^2-2\,ax\right)}{2\left(b^3+abx+2\,\sqrt{ax+\sqrt{a^2x^2+b^2}}\,b^2+\sqrt{a^2x^2+b^2}\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2+b^2)^(1/2))/(b+(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)),x,
algorithm="maxima")
```

```
[Out] 1/4*a*x^2/b + 1/2*integrate(sqrt(a^2*x^2 + b^2), x)/b - integrate(-1/2*(a*b
^2*x - 2*a^2*x^2 - b^2 + sqrt(a^2*x^2 + b^2)*(b^2 - 2*a*x))/(b^3 + a*b*x +
2*sqrt(a*x + sqrt(a^2*x^2 + b^2))*b^2 + sqrt(a^2*x^2 + b^2)*b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+\sqrt{a^2\,x^2+b^2}}{b+\sqrt{a\,x+\sqrt{a^2\,x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + (b^2 + a^2*x^2)^(1/2))/(b + (a*x + (b^2 + a^2*x^2)^(1/2))^(1/2))
,x)
```

```
[Out] int((a*x + (b^2 + a^2*x^2)^(1/2))/(b + (a*x + (b^2 + a^2*x^2)^(1/2))^(1/2))
, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+\sqrt{a^2x^2+b^2}}{b+\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a**2*x**2+b**2)**(1/2))/(b+(a*x+(a**2*x**2+b**2)**(1/2))**(
1/2)),x)
```

```
[Out] Integral((a*x + sqrt(a**2*x**2 + b**2))/(b + sqrt(a*x + sqrt(a**2*x**2 + b*
*2))), x)
```

3.1855 $\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx$

Optimal. Leaf size=169

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{3x^2+1}+1}\right)}{4\sqrt{3}}-\frac{i\tanh^{-1}\left(\frac{2i\sqrt{3}x-2i\sqrt{3}x\sqrt[3]{3x^2+1}}{3x^2-4(3x^2+1)^{2/3}+2\sqrt[3]{3x^2+1}-1}\right)}{8\sqrt{3}}+\frac{1}{8}\tanh^{-1}\left(\frac{6x\sqrt[3]{3x^2+1}}{3x^2+4\left(3x^2+1\right)^{2/3}-2\sqrt[3]{3x^2+1}+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 0.48, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {394}

$$\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{3x^2+1}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}-\frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{3x^2+1}}{x}\right)+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

```
[Out] ArcTan[x/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 - (1 + 3*x^2)^(1/3))^2/(3*Sqrt[3]
*x)]/(4*Sqrt[3]) - ArcTanh[(1 - (1 + 3*x^2)^(1/3))/x]/4
```

Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(3+x^2\right)\sqrt[3]{1+3x^2}}\,dx=\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}-\frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{1+3x^2}}{x}\right)$$

Mathematica [C] time = 0.10, size = 126, normalized size = 0.75

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-3x^2,-\frac{x^2}{3}\right)}{\left(x^2+3\right)\sqrt[3]{3x^2+1}\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-3x^2,-\frac{x^2}{3}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-3x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-3x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -1/3*x^2])/((3 + x^2)*(1 + 3*x^2)^
(1/3)*(-9*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/
2, 1/3, 2, 5/2, -3*x^2, -1/3*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/2, -3*x^2, -1
/3*x^2])))
```

IntegrateAlgebraic [F] time = 3.98, size = 0, normalized size = 0.00

$$\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((3 + x^2)*(1 + 3*x^2)^(1/3)), x]
```

fricas [B] time = 1.67, size = 345, normalized size = 2.04

$$\frac{1}{36}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\left(3x^4-10x^3-36x^2+18x+9\right)\left(3x^2+1\right)^{\frac{2}{3}}-4\sqrt{3}\left(x^5+15x^4-26x^3-54x^2+9x-9\right)\left(3x^2+1\right)^{\frac{1}{3}}+\sqrt{3}\left(x^6-2x^5-105x^4-28x^3+63x^2+126x+9\right)}{x^6+126x^5-225x^4-828x^3-81x^2-162x+81}\right)-\frac{1}{36}\sqrt{3}\arctan\left(\frac{2\left(2\sqrt{3}\left(23x^3+9x\right)\left(3x^2+1\right)^{\frac{2}{3}}+\sqrt{3}\left(x^5-80x^3-9x\right)\left(3x^2+1\right)^{\frac{1}{3}}+\sqrt{3}\left(11x^5+10x^3-9x\right)\right)}{x^6-657x^4-189x^2-27}\right)+\frac{1}{24}\log\left(\frac{x^6+108x^5+549x^4+360x^3+99x^2+6\left(3x^4+32x^3+42x^2+3\right)\left(3x^2+1\right)^{\frac{2}{3}}+6\left(x^5+27x^4+70x^3+18x^2+9x+3\right)\left(3x^2+1\right)^{\frac{1}{3}}+108x-9}{x^6+9x^4+27x^2+27}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(3)*arctan((4*sqrt(3)*(3*x^4 - 10*x^3 - 36*x^2 + 18*x + 9)*(3*x^2
+ 1)^(2/3) - 4*sqrt(3)*(x^5 + 15*x^4 - 26*x^3 - 54*x^2 + 9*x - 9)*(3*x^2 +
1)^(1/3) + sqrt(3)*(x^6 - 2*x^5 - 105*x^4 - 28*x^3 + 63*x^2 + 126*x + 9))/(
x^6 + 126*x^5 - 225*x^4 - 828*x^3 - 81*x^2 - 162*x + 81)) - 1/36*sqrt(3)*ar
ctan(2*(2*sqrt(3)*(23*x^3 + 9*x)*(3*x^2 + 1)^(2/3) + sqrt(3)*(x^5 - 80*x^3
- 9*x)*(3*x^2 + 1)^(1/3) + sqrt(3)*(11*x^5 + 10*x^3 - 9*x))/(x^6 - 657*x^4
- 189*x^2 - 27)) + 1/24*log((x^6 + 108*x^5 + 549*x^4 + 360*x^3 + 99*x^2 + 6
*(3*x^4 + 32*x^3 + 42*x^2 + 3)*(3*x^2 + 1)^(2/3) + 6*(x^5 + 27*x^4 + 70*x^3
 + 18*x^2 + 9*x + 3)*(3*x^2 + 1)^(1/3) + 108*x - 9)/(x^6 + 9*x^4 + 27*x^2 +
 27))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)
```

maple [C] time = 2.94, size = 446, normalized size = 2.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+3)/(3*x^2+1)^(1/3),x)
```

```
[Out] RootOf(48*_Z^2-12*_Z+1)*ln((24*RootOf(48*_Z^2-12*_Z+1)*(3*x^2+1)^(1/3)*x+12
*RootOf(48*_Z^2-12*_Z+1)*x^2-24*RootOf(48*_Z^2-12*_Z+1)*(3*x^2+1)^(1/3)-48*
RootOf(48*_Z^2-12*_Z+1)*x-2*(3*x^2+1)^(2/3)-4*(3*x^2+1)^(1/3)*x-x^2-12*Root
Of(48*_Z^2-12*_Z+1)+4*(3*x^2+1)^(1/3)+4*x+1)/(x^2+3))+1/4*ln(-(12*RootOf(48
*_Z^2-12*_Z+1)*(3*x^2+1)^(1/3)*x+6*RootOf(48*_Z^2-12*_Z+1)*x^2-12*RootOf(48
*_Z^2-12*_Z+1)*(3*x^2+1)^(1/3)-24*RootOf(48*_Z^2-12*_Z+1)*x+(3*x^2+1)^(2/3)
-(3*x^2+1)^(1/3)*x-x^2-6*RootOf(48*_Z^2-12*_Z+1)+(3*x^2+1)^(1/3)+4*x+1)/(x^
2+3))-ln(-(12*RootOf(48*_Z^2-12*_Z+1)*(3*x^2+1)^(1/3)*x+6*RootOf(48*_Z^2-12
*_Z+1)*x^2-12*RootOf(48*_Z^2-12*_Z+1)*(3*x^2+1)^(1/3)-24*RootOf(48*_Z^2-12*
_Z+1)*x+(3*x^2+1)^(2/3)-(3*x^2+1)^(1/3)*x-x^2-6*RootOf(48*_Z^2-12*_Z+1)+(3*
x^2+1)^(1/3)+4*x+1)/(x^2+3))*RootOf(48*_Z^2-12*_Z+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2+3\right)\,{\left(3\,x^2+1\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 3)*(3*x^2 + 1)^(1/3)),x)
[Out] int(1/((x^2 + 3)*(3*x^2 + 1)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{2}+3\right)\sqrt[3]{3x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+3)/(3*x**2+1)**(1/3),x)
[Out] Integral(1/((x**2 + 3)*(3*x**2 + 1)**(1/3)), x)
```

3.1856 $\int \frac{\left(1+x^2\right)\sqrt[4]{x^3+x^5}}{x^2\left(-1+x^2\right)}\,dx$

Optimal. Leaf size=169

$$\frac{4\sqrt[4]{x^5+x^3}}{x}+\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^5+x^3}}\right)-\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^5+x^3}}\right)-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^5+x^3}}{\sqrt{2}\,x^2-\sqrt{x^5+x^3}}\right)}{\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^5+x^3}}{2^{3/4}}}{x\sqrt[4]{x^5+x^3}}\right)}{\sqrt[4]{2}}$$

Rubi [C] time = 0.18, antiderivative size = 44, normalized size of antiderivative = 0.26, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2056, 466, 510}

$$\frac{4\sqrt[4]{x^5+x^3}\,F_1\left(-\frac{1}{8};1,-\frac{5}{4};\frac{7}{8};x^2,-x^2\right)}{x\sqrt[4]{x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((1 + x^2)*(x^3 + x^5)^(1/4))/(x^2*(-1 + x^2)),x]
```

```
[Out] (4*(x^3 + x^5)^(1/4)*AppellF1[-1/8, 1, -5/4, 7/8, x^2, -x^2])/(x*(1 + x^2)^
(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{(1+x^2)\sqrt[4]{x^3+x^5}}{x^2(-1+x^2)}\,dx = \frac{\sqrt[4]{x^3+x^5}\int \frac{(1+x^2)^{5/4}}{x^{5/4}(-1+x^2)}\,dx}{x^{3/4}\sqrt[4]{1+x^2}}$$
$$= \frac{\left(4\sqrt[4]{x^3+x^5}\right)\text{Subst}\left(\int \frac{(1+x^8)^{5/4}}{x^2(-1+x^8)}\,dx, x, \sqrt[4]{x}\right)}{x^{3/4}\sqrt[4]{1+x^2}}$$
$$= \frac{4\sqrt[4]{x^3+x^5}\,F_1\left(-\frac{1}{8};1,-\frac{5}{4};\frac{7}{8};x^2,-x^2\right)}{x\sqrt[4]{1+x^2}}$$

Mathematica [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(1+x^2)\sqrt[4]{x^3+x^5}}{x^2(-1+x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^2)*(x^3 + x^5)^(1/4))/(x^2*(-1 + x^2)),x]
[Out] Integrate[((1 + x^2)*(x^3 + x^5)^(1/4))/(x^2*(-1 + x^2)), x]
```

IntegrateAlgebraic [A] time = 0.45, size = 169, normalized size = 1.00

$$\frac{4\sqrt[4]{x^5+x^3}}{x} + \sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^5+x^3}}\right) - \sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^5+x^3}}\right) - \frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^5+x^3}}{\sqrt{2}x^2-\sqrt{x^5+x^3}}\right)}{\sqrt[4]{2}} - \frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^5+x^3}}{2^{3/4}}}{x\sqrt[4]{x^5+x^3}}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^2)*(x^3 + x^5)^(1/4))/(x^2*(-1 + x^2)),x]
[Out] (4*(x^3 + x^5)^(1/4))/x + 2^(1/4)*ArcTan[(2^(1/4)*x)/(x^3 + x^5)^(1/4)] - A
rcTan[(2^(3/4)*x*(x^3 + x^5)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^3 + x^5])]/2^(1/4
) - 2^(1/4)*ArcTanh[(2^(1/4)*x)/(x^3 + x^5)^(1/4)] - ArcTanh[(x^2/2^(1/4) +
 Sqrt[x^3 + x^5]/2^(3/4))/(x*(x^3 + x^5)^(1/4))]/2^(1/4)
```

fricas [B] time = 5.67, size = 1053, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^5+x^3)^(1/4)/x^2/(x^2-1),x, algorithm="fricas")
[Out] 1/8*(4*2^(3/4)*x*arctan(1/2*(2*x^6 + 8*x^5 + 12*x^4 + 8*x^3 + 4*2^(3/4)*(x^
5 + x^3)^(3/4)*(x^2 - 6*x + 1) + 8*sqrt(2)*sqrt(x^5 + x^3)*(x^3 + 2*x^2 + x
) + 2*x^2 + sqrt(2)*(32*sqrt(2)*(x^5 + x^3)^(3/4)*x + 2^(3/4)*(x^6 - 16*x^5
 - 2*x^4 - 16*x^3 + x^2) + 4*2^(1/4)*sqrt(x^5 + x^3)*(x^3 - 6*x^2 + x) + 8*
(x^5 + x^3)^(1/4)*(x^4 + 2*x^3 + x^2))*sqrt((4*2^(3/4)*(x^5 + x^3)^(1/4)*x^
2 + sqrt(2)*(x^4 + 2*x^3 + x^2) + 8*sqrt(x^5 + x^3)*x + 4*2^(1/4)*(x^5 + x^
3)^(3/4))/(x^4 + 2*x^3 + x^2)) + 8*2^(1/4)*(x^5 + x^3)^(1/4)*(3*x^4 - 2*x^3
 + 3*x^2))/(x^6 - 28*x^5 + 6*x^4 - 28*x^3 + x^2)) - 4*2^(3/4)*x*arctan(1/2*
(2*x^6 + 8*x^5 + 12*x^4 + 8*x^3 - 4*2^(3/4)*(x^5 + x^3)^(3/4)*(x^2 - 6*x +
1) + 8*sqrt(2)*sqrt(x^5 + x^3)*(x^3 + 2*x^2 + x) + 2*x^2 + sqrt(2)*(32*sqrt
(2)*(x^5 + x^3)^(3/4)*x - 2^(3/4)*(x^6 - 16*x^5 - 2*x^4 - 16*x^3 + x^2) - 4
*2^(1/4)*sqrt(x^5 + x^3)*(x^3 - 6*x^2 + x) + 8*(x^5 + x^3)^(1/4)*(x^4 + 2*x
```

```
^3 + x^2))*sqrt(-(4*2^(3/4)*(x^5 + x^3)^(1/4)*x^2 - sqrt(2)*(x^4 + 2*x^3 +
x^2) - 8*sqrt(x^5 + x^3)*x + 4*2^(1/4)*(x^5 + x^3)^(3/4))/(x^4 + 2*x^3 + x^
2)) - 8*2^(1/4)*(x^5 + x^3)^(1/4)*(3*x^4 - 2*x^3 + 3*x^2))/(x^6 - 28*x^5 +
6*x^4 - 28*x^3 + x^2)) - 2^(3/4)*x*log(2*(4*2^(3/4)*(x^5 + x^3)^(1/4)*x^2 +
 sqrt(2)*(x^4 + 2*x^3 + x^2) + 8*sqrt(x^5 + x^3)*x + 4*2^(1/4)*(x^5 + x^3)^
(3/4))/(x^4 + 2*x^3 + x^2)) + 2^(3/4)*x*log(-2*(4*2^(3/4)*(x^5 + x^3)^(1/4)
*x^2 - sqrt(2)*(x^4 + 2*x^3 + x^2) - 8*sqrt(x^5 + x^3)*x + 4*2^(1/4)*(x^5 +
 x^3)^(3/4))/(x^4 + 2*x^3 + x^2)) - 8*2^(1/4)*x*arctan(-1/2*(4*2^(3/4)*(x^5
 + x^3)^(1/4)*x^2 - 2^(3/4)*(2*2^(3/4)*sqrt(x^5 + x^3)*x + 2^(1/4)*(x^4 + 2
*x^3 + x^2)) + 4*2^(1/4)*(x^5 + x^3)^(3/4))/(x^4 - 2*x^3 + x^2)) - 2*2^(1/4
)*x*log((4*sqrt(2)*(x^5 + x^3)^(1/4)*x^2 + 2^(3/4)*(x^4 + 2*x^3 + x^2) + 4*
2^(1/4)*sqrt(x^5 + x^3)*x + 4*(x^5 + x^3)^(3/4))/(x^4 - 2*x^3 + x^2)) + 2*2
^(1/4)*x*log((4*sqrt(2)*(x^5 + x^3)^(1/4)*x^2 - 2^(3/4)*(x^4 + 2*x^3 + x^2)
 - 4*2^(1/4)*sqrt(x^5 + x^3)*x + 4*(x^5 + x^3)^(3/4))/(x^4 - 2*x^3 + x^2))
+ 32*(x^5 + x^3)^(1/4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5+x^3\right)^{\frac{1}{4}}\left(x^2+1\right)}{\left(x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^5+x^3)^(1/4)/x^2/(x^2-1),x, algorithm="giac")
```

```
[Out] integrate((x^5 + x^3)^(1/4)*(x^2 + 1)/((x^2 - 1)*x^2), x)
```

maple [C] time = 15.26, size = 1756, normalized size = 10.39

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^5+x^3)^(1/4)/x^2/(x^2-1),x)
```

```
[Out] 4*(x^3*(x^2+1))^(1/4)/x+(1/2*RootOf(_Z^4+2)*ln(-(2*(x^7+3*x^5+3*x^3+x)^(1/2
)*RootOf(_Z^4+2)^3*x^2-2*(x^7+3*x^5+3*x^3+x)^(1/4)*RootOf(_Z^4+2)^2*x^4+Roo
tOf(_Z^4+2)*x^6-2*RootOf(_Z^4+2)*x^5+2*RootOf(_Z^4+2)^3*(x^7+3*x^5+3*x^3+x)
^(1/2)-4*(x^7+3*x^5+3*x^3+x)^(1/4)*RootOf(_Z^4+2)^2*x^2+3*RootOf(_Z^4+2)*x^
4-4*RootOf(_Z^4+2)*x^3+4*(x^7+3*x^5+3*x^3+x)^(3/4)-2*RootOf(_Z^4+2)^2*(x^7+
3*x^5+3*x^3+x)^(1/4)+3*RootOf(_Z^4+2)*x^2-2*RootOf(_Z^4+2)*x+RootOf(_Z^4+2)
)/(1+x)^2/(x^2+1)^2)-1/2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*ln(-(2*(x^7+3*x^5+3*
x^3+x)^(1/2)*RootOf(_Z^4+2)^2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^2+2*(x^7+3*x^
5+3*x^3+x)^(1/4)*RootOf(_Z^4+2)^2*x^4-RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^6+2*R
ootOf(_Z^2+RootOf(_Z^4+2)^2)*x^5+2*(x^7+3*x^5+3*x^3+x)^(1/2)*RootOf(_Z^4+2)
^2*RootOf(_Z^2+RootOf(_Z^4+2)^2)+4*(x^7+3*x^5+3*x^3+x)^(1/4)*RootOf(_Z^4+2)
^2*x^2-3*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^4+4*RootOf(_Z^2+RootOf(_Z^4+2)^2)*
x^3+4*(x^7+3*x^5+3*x^3+x)^(3/4)+2*RootOf(_Z^4+2)^2*(x^7+3*x^5+3*x^3+x)^(1/4
)-3*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^2+2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x-Roo
tOf(_Z^2+RootOf(_Z^4+2)^2))/(1+x)^2/(x^2+1)^2)+1/4*ln(-(RootOf(_Z^4+2)^3*x^
6-2*RootOf(_Z^4+2)^3*x^5+4*(x^7+3*x^5+3*x^3+x)^(1/4)*RootOf(_Z^4+2)^2*x^4+3
*RootOf(_Z^4+2)^3*x^4-4*RootOf(_Z^4+2)^3*x^3+8*(x^7+3*x^5+3*x^3+x)^(1/2)*Ro
otOf(_Z^4+2)*x^2+8*(x^7+3*x^5+3*x^3+x)^(1/4)*RootOf(_Z^4+2)^2*x^2+3*RootOf(
_Z^4+2)^3*x^2-2*RootOf(_Z^4+2)^3*x+8*(x^7+3*x^5+3*x^3+x)^(3/4)+8*(x^7+3*x^5
+3*x^3+x)^(1/2)*RootOf(_Z^4+2)+4*RootOf(_Z^4+2)^2*(x^7+3*x^5+3*x^3+x)^(1/4)
+RootOf(_Z^4+2)^3)/(x^2+1)^2/(-1+x)^2)*RootOf(_Z^4+2)^3+1/4*ln(-(RootOf(_Z^
4+2)^3*x^6-2*RootOf(_Z^4+2)^3*x^5+4*(x^7+3*x^5+3*x^3+x)^(1/4)*RootOf(_Z^4+2
)^2*x^4+3*RootOf(_Z^4+2)^3*x^4-4*RootOf(_Z^4+2)^3*x^3+8*(x^7+3*x^5+3*x^3+x)
^(1/2)*RootOf(_Z^4+2)*x^2+8*(x^7+3*x^5+3*x^3+x)^(1/4)*RootOf(_Z^4+2)^2*x^2+
3*RootOf(_Z^4+2)^3*x^2-2*RootOf(_Z^4+2)^3*x+8*(x^7+3*x^5+3*x^3+x)^(3/4)+8*(
```

```
x^7+3*x^5+3*x^3+x)^(1/2)*RootOf(_Z^4+2)+4*RootOf(_Z^4+2)^2*(x^7+3*x^5+3*x^3
+x)^(1/4)+RootOf(_Z^4+2)^3)/(x^2+1)^2/(-1+x)^2)*RootOf(_Z^4+2)^2*RootOf(_Z^
2+RootOf(_Z^4+2)^2)-1/2*RootOf(_Z^4+2)^2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*ln((
-RootOf(_Z^4+2)^3*x^6+RootOf(_Z^4+2)^2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^6-2*
RootOf(_Z^4+2)^3*x^5+2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^4+2)^2*x^5+4
*(x^7+3*x^5+3*x^3+x)^(1/4)*RootOf(_Z^4+2)*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^4
-3*RootOf(_Z^4+2)^3*x^4+3*RootOf(_Z^4+2)^2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^
4-4*RootOf(_Z^4+2)^3*x^3+4*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^4+2)^2*x
^3+4*(x^7+3*x^5+3*x^3+x)^(1/2)*RootOf(_Z^4+2)*x^2+4*(x^7+3*x^5+3*x^3+x)^(1/
2)*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^2+8*(x^7+3*x^5+3*x^3+x)^(1/4)*RootOf(_Z^
4+2)*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^2-3*RootOf(_Z^4+2)^3*x^2+3*RootOf(_Z^4
+2)^2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^2-2*RootOf(_Z^4+2)^3*x+2*RootOf(_Z^2+
RootOf(_Z^4+2)^2)*RootOf(_Z^4+2)^2*x+8*(x^7+3*x^5+3*x^3+x)^(3/4)+4*(x^7+3*x
^5+3*x^3+x)^(1/2)*RootOf(_Z^4+2)+4*(x^7+3*x^5+3*x^3+x)^(1/2)*RootOf(_Z^2+Ro
otOf(_Z^4+2)^2)+4*(x^7+3*x^5+3*x^3+x)^(1/4)*RootOf(_Z^2+RootOf(_Z^4+2)^2)*R
ootOf(_Z^4+2)-RootOf(_Z^4+2)^3+RootOf(_Z^4+2)^2*RootOf(_Z^2+RootOf(_Z^4+2)^
2))/(x^2+1)^2/(-1+x)^2))*(x^3*(x^2+1))^(1/4)/x*(x*(x^2+1)^3)^(1/4)/(x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5+x^3\right)^{\frac{1}{4}}\left(x^2+1\right)}{\left(x^2-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^5+x^3)^(1/4)/x^2/(x^2-1),x, algorithm="maxima")
[Out] integrate((x^5 + x^3)^(1/4)*(x^2 + 1)/((x^2 - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^5+x^3\right)^{1/4}\,\left(x^2+1\right)}{x^2\,\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + x^5)^(1/4)*(x^2 + 1))/(x^2*(x^2 - 1)),x)
[Out] int(((x^3 + x^5)^(1/4)*(x^2 + 1))/(x^2*(x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\left(x^2+1\right)}\,\left(x^2+1\right)}{x^2\,(x-1)\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**5+x**3)**(1/4)/x**2/(x**2-1),x)
[Out] Integral((x**3*(x**2 + 1))**(1/4)*(x**2 + 1)/(x**2*(x - 1)*(x + 1)), x)
```

3.1857 $\int \frac{(4+x^2+x^5)\sqrt[4]{-2-x^2-2x^4+2x^5}}{x^2(-2-x^2+2x^5)}\,dx$

Optimal. Leaf size=170

$$\frac{2\sqrt[4]{2x^5-2x^4-x^2-2}}{x}-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{2x^5-2x^4-x^2-2}}{\sqrt{2}x^2-\sqrt{2x^5-2x^4-x^2-2}}\right)}{\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}x\sqrt[4]{2x^5-2x^4-x^2-2}}{2x^2+\sqrt{2}\sqrt{2x^5-2x^4-x^2-2}}\right)}{\sqrt[4]{2}}$$

Rubi [F] time = 1.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(4+x^2+x^5)\sqrt[4]{-2-x^2-2x^4+2x^5}}{x^2(-2-x^2+2x^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((4 + x^2 + x^5)*(-2 - x^2 - 2*x^4 + 2*x^5)^(1/4))/(x^2*(-2 - x^2 + 2*x
^5)),x]
```

```
[Out] -2*Defer[Int][(-2 - x^2 - 2*x^4 + 2*x^5)^(1/4)/x^2, x] + Defer[Int][(-2 - x
^2 - 2*x^4 + 2*x^5)^(1/4)/(2 + x^2 - 2*x^5), x] + 5*Defer[Int][(x^3*(-2 - x
^2 - 2*x^4 + 2*x^5)^(1/4))/(-2 - x^2 + 2*x^5), x]
```

Rubi steps

$$\begin{aligned}\int \frac{(4+x^2+x^5)\sqrt[4]{-2-x^2-2x^4+2x^5}}{x^2(-2-x^2+2x^5)}\,dx &= \int \left(-\frac{2\sqrt[4]{-2-x^2-2x^4+2x^5}}{x^2}+\frac{(-1+5x^3)\sqrt[4]{-2-x^2-2x^4+2x^5}}{-2-x^2+2x^5}\right)dx\\ &= -\left(2\int \frac{\sqrt[4]{-2-x^2-2x^4+2x^5}}{x^2}\,dx\right)+\int \frac{(-1+5x^3)\sqrt[4]{-2-x^2-2x^4}}{-2-x^2+2x^5}\\ &= -\left(2\int \frac{\sqrt[4]{-2-x^2-2x^4+2x^5}}{x^2}\,dx\right)+\int \left(\frac{\sqrt[4]{-2-x^2-2x^4+2x^5}}{2+x^2-2x^5}+\right.\\ &= -\left(2\int \frac{\sqrt[4]{-2-x^2-2x^4+2x^5}}{x^2}\,dx\right)+5\int \frac{x^3\sqrt[4]{-2-x^2-2x^4+2x^5}}{-2-x^2+2x^5}\end{aligned}$$

Mathematica [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(4+x^2+x^5)\sqrt[4]{-2-x^2-2x^4+2x^5}}{x^2(-2-x^2+2x^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((4 + x^2 + x^5)*(-2 - x^2 - 2*x^4 + 2*x^5)^(1/4))/(x^2*(-2 - x^2
 + 2*x^5)),x]
```

```
[Out] Integrate[((4 + x^2 + x^5)*(-2 - x^2 - 2*x^4 + 2*x^5)^(1/4))/(x^2*(-2 - x^2
 + 2*x^5)), x]
```

IntegrateAlgebraic [A] time = 0.39, size = 170, normalized size = 1.00

$$\frac{2\sqrt[4]{2x^5-2x^4-x^2-2}}{x}-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{2x^5-2x^4-x^2-2}}{\sqrt{2}x^2-\sqrt{2x^5-2x^4-x^2-2}}\right)}{\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}x\sqrt[4]{2x^5-2x^4-x^2-2}}{2x^2+\sqrt{2}\sqrt{2x^5-2x^4-x^2-2}}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((4 + x^2 + x^5)*(-2 - x^2 - 2*x^4 + 2*x^5)^(1/4))/(x^2*
(-2 - x^2 + 2*x^5)),x]
```

```
[Out] (2*(-2 - x^2 - 2*x^4 + 2*x^5)^(1/4))/x - ArcTan[(2^(3/4)*x*(-2 - x^2 - 2*x^
4 + 2*x^5)^(1/4))/(Sqrt[2]*x^2 - Sqrt[-2 - x^2 - 2*x^4 + 2*x^5])]/2^(1/4) -
 ArcTanh[(2*2^(1/4)*x*(-2 - x^2 - 2*x^4 + 2*x^5)^(1/4))/(2*x^2 + Sqrt[2]*Sq
rt[-2 - x^2 - 2*x^4 + 2*x^5])]/2^(1/4)
```

fricas [B] time = 83.42, size = 1274, normalized size = 7.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x^2+4)*(2*x^5-2*x^4-x^2-2)^(1/4)/x^2/(2*x^5-x^2-2),x, algori
thm="fricas")
```

```
[Out] 1/32*(4*8^(3/4)*sqrt(2)*x*arctan(1/8*(32*x^10 - 32*x^7 - 64*x^5 + 8*x^4 + 4
*8^(3/4)*sqrt(2)*(2*x^6 - 8*x^5 - x^3 - 2*x)*(2*x^5 - 2*x^4 - x^2 - 2)^(3/4
) + 16*8^(1/4)*sqrt(2)*(6*x^8 - 8*x^7 - 3*x^5 - 6*x^3)*(2*x^5 - 2*x^4 - x^2
 - 2)^(1/4) + 32*sqrt(2)*(2*x^7 - x^4 - 2*x^2)*sqrt(2*x^5 - 2*x^4 - x^2 - 2
) + 32*x^2 + sqrt(2)*(128*sqrt(2)*(2*x^5 - 2*x^4 - x^2 - 2)^(3/4)*x^5 + 8^(
3/4)*sqrt(2)*(4*x^10 - 40*x^9 + 32*x^8 - 4*x^7 + 20*x^6 - 8*x^5 + 41*x^4 +
4*x^2 + 4) + 8*8^(1/4)*sqrt(2)*(2*x^7 - 8*x^6 - x^4 - 2*x^2)*sqrt(2*x^5 - 2
*x^4 - x^2 - 2) + 32*(2*x^8 - x^5 - 2*x^3)*(2*x^5 - 2*x^4 - x^2 - 2)^(1/4))
*sqrt((8^(3/4)*sqrt(2)*(2*x^5 - 2*x^4 - x^2 - 2)^(1/4)*x^3 + 2*8^(1/4)*sqrt
(2)*(2*x^5 - 2*x^4 - x^2 - 2)^(3/4)*x + 8*sqrt(2*x^5 - 2*x^4 - x^2 - 2)*x^2
 + sqrt(2)*(2*x^5 - x^2 - 2))/(2*x^5 - x^2 - 2)) + 32)/(4*x^10 - 64*x^9 + 6
4*x^8 - 4*x^7 + 32*x^6 - 8*x^5 + 65*x^4 + 4*x^2 + 4)) - 4*8^(3/4)*sqrt(2)*x
*arctan(1/8*(32*x^10 - 32*x^7 - 64*x^5 + 8*x^4 - 4*8^(3/4)*sqrt(2)*(2*x^6 -
 8*x^5 - x^3 - 2*x)*(2*x^5 - 2*x^4 - x^2 - 2)^(3/4) - 16*8^(1/4)*sqrt(2)*(6
*x^8 - 8*x^7 - 3*x^5 - 6*x^3)*(2*x^5 - 2*x^4 - x^2 - 2)^(1/4) + 32*sqrt(2)*
(2*x^7 - x^4 - 2*x^2)*sqrt(2*x^5 - 2*x^4 - x^2 - 2) + 32*x^2 + sqrt(2)*(128
*sqrt(2)*(2*x^5 - 2*x^4 - x^2 - 2)^(3/4)*x^5 - 8^(3/4)*sqrt(2)*(4*x^10 - 40
*x^9 + 32*x^8 - 4*x^7 + 20*x^6 - 8*x^5 + 41*x^4 + 4*x^2 + 4) - 8*8^(1/4)*sq
rt(2)*(2*x^7 - 8*x^6 - x^4 - 2*x^2)*sqrt(2*x^5 - 2*x^4 - x^2 - 2) + 32*(2*x
^8 - x^5 - 2*x^3)*(2*x^5 - 2*x^4 - x^2 - 2)^(1/4))*sqrt(-(8^(3/4)*sqrt(2)*(
2*x^5 - 2*x^4 - x^2 - 2)^(1/4)*x^3 + 2*8^(1/4)*sqrt(2)*(2*x^5 - 2*x^4 - x^2
 - 2)^(3/4)*x - 8*sqrt(2*x^5 - 2*x^4 - x^2 - 2)*x^2 - sqrt(2)*(2*x^5 - x^2
- 2))/(2*x^5 - x^2 - 2)) + 32)/(4*x^10 - 64*x^9 + 64*x^8 - 4*x^7 + 32*x^6 -
 8*x^5 + 65*x^4 + 4*x^2 + 4)) - 8^(3/4)*sqrt(2)*x*log(8*(8^(3/4)*sqrt(2)*(2
*x^5 - 2*x^4 - x^2 - 2)^(1/4)*x^3 + 2*8^(1/4)*sqrt(2)*(2*x^5 - 2*x^4 - x^2
- 2)^(3/4)*x + 8*sqrt(2*x^5 - 2*x^4 - x^2 - 2)*x^2 + sqrt(2)*(2*x^5 - x^2 -
 2))/(2*x^5 - x^2 - 2)) + 8^(3/4)*sqrt(2)*x*log(-8*(8^(3/4)*sqrt(2)*(2*x^5
- 2*x^4 - x^2 - 2)^(1/4)*x^3 + 2*8^(1/4)*sqrt(2)*(2*x^5 - 2*x^4 - x^2 - 2)^
(3/4)*x - 8*sqrt(2*x^5 - 2*x^4 - x^2 - 2)*x^2 - sqrt(2)*(2*x^5 - x^2 - 2))/
(2*x^5 - x^2 - 2)) + 64*(2*x^5 - 2*x^4 - x^2 - 2)^(1/4))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{5}-2\,x^{4}-x^{2}-2\right)^{\frac{1}{4}}\left(x^{5}+x^{2}+4\right)}{\left(2\,x^{5}-x^{2}-2\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x^2+4)*(2*x^5-2*x^4-x^2-2)^(1/4)/x^2/(2*x^5-x^2-2),x, algori
thm="giac")
```

```
[Out] integrate((2*x^5 - 2*x^4 - x^2 - 2)^(1/4)*(x^5 + x^2 + 4)/((2*x^5 - x^2 - 2
)*x^2), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{5}+x^{2}+4\right) \left(2 x^{5}-2 x^{4}-x^{2}-2\right)^{\frac{1}{4}}}{x^{2} \left(2 x^{5}-x^{2}-2\right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+x^2+4)*(2*x^5-2*x^4-x^2-2)^(1/4)/x^2/(2*x^5-x^2-2),x)
```

```
[Out] int((x^5+x^2+4)*(2*x^5-2*x^4-x^2-2)^(1/4)/x^2/(2*x^5-x^2-2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(2 \, x^{5} - 2 \, x^{4} - x^{2} - 2\right)}^{\frac{1}{4}} {\left(x^{5} + x^{2} + 4\right)}}{{\left(2 \, x^{5} - x^{2} - 2\right)} x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x^2+4)*(2*x^5-2*x^4-x^2-2)^(1/4)/x^2/(2*x^5-x^2-2),x, algori
thm="maxima")
```

```
[Out] integrate((2*x^5 - 2*x^4 - x^2 - 2)^(1/4)*(x^5 + x^2 + 4)/((2*x^5 - x^2 - 2
)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(x^5+x^2+4\right)\,{\left(2\,x^5-2\,x^4-x^2-2\right)}^{1/4}}{x^2\,\left(-2\,x^5+x^2+2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^2 + x^5 + 4)*(2*x^5 - 2*x^4 - x^2 - 2)^(1/4))/(x^2*(x^2 - 2*x^5 +
2)),x)
```

```
[Out] int(-((x^2 + x^5 + 4)*(2*x^5 - 2*x^4 - x^2 - 2)^(1/4))/(x^2*(x^2 - 2*x^5 +
2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{5} + x^{2} + 4\right) \sqrt[4]{2 x^{5} - 2 x^{4} - x^{2} - 2}}{x^{2} \left(2 x^{5} - x^{2} - 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+x**2+4)*(2*x**5-2*x**4-x**2-2)**(1/4)/x**2/(2*x**5-x**2-2),
x)
```

```
[Out] Integral((x**5 + x**2 + 4)*(2*x**5 - 2*x**4 - x**2 - 2)**(1/4)/(x**2*(2*x**
5 - x**2 - 2)), x)
```

3.1858 $\int \frac{(-1+x^4)^2\sqrt{x^2+\sqrt{1+x^4}}}{(1+x^4)^2}\,dx$

Optimal. Leaf size=170

$$-\frac{3}{2}\tan^{-1}\left(x\sqrt{\sqrt{x^4+1}+x^2}\right)+\frac{\tan^{-1}\left(\sqrt{2}x\sqrt{\sqrt{x^4+1}+x^2}\right)}{2\sqrt{2}}+\frac{x\left(2x^8+8x^4+4\right)\sqrt{\sqrt{x^4+1}+x^2}+x\sqrt{x^4+1}}{2\left(2x^8+3x^4+1\right)+2\sqrt{x^4+1}\ (}$$

Rubi [F] time = 3.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^4)^2\sqrt{x^2+\sqrt{1+x^4}}}{(1+x^4)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^4)^2*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^4)^2,x]
```

```
[Out] Defer[Int][Sqrt[x^2 + Sqrt[1 + x^4]], x] + (I/4)*Defer[Int][Sqrt[x^2 + Sqrt
[1 + x^4]]/((-1)^(1/4) - x)^2, x] - ((-1)^(1/4)*Defer[Int][Sqrt[x^2 + Sqrt[
1 + x^4]]/((-1)^(1/4) - x), x])/4 - (I/4)*Defer[Int][Sqrt[x^2 + Sqrt[1 + x^
4]]/(-(-1)^(3/4) - x)^2, x] + ((-1)^(3/4)*Defer[Int][Sqrt[x^2 + Sqrt[1 + x^
4]]/(-(-1)^(3/4) - x), x])/4 + (I/4)*Defer[Int][Sqrt[x^2 + Sqrt[1 + x^4]]/(
(-1)^(1/4) + x)^2, x] - ((-1)^(1/4)*Defer[Int][Sqrt[x^2 + Sqrt[1 + x^4]]/((
-1)^(1/4) + x), x])/4 - (I/4)*Defer[Int][Sqrt[x^2 + Sqrt[1 + x^4]]/(-(-1)^(
3/4) + x)^2, x] + ((-1)^(3/4)*Defer[Int][Sqrt[x^2 + Sqrt[1 + x^4]]/(-(-1)^(
3/4) + x), x])/4
```

Rubi steps

$$\int \frac{\left(-1+x^4\right)^2 \sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^4\right)^2} \, dx = \int \left(\sqrt{x^2+\sqrt{1+x^4}}+\frac{4\sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^4\right)^2}-\frac{4\sqrt{x^2+\sqrt{1+x^4}}}{1+x^4}\right) dx$$

$$= 4\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^4\right)^2} \, dx - 4\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+x^4} \, dx + \int \sqrt{x^2+\sqrt{1+x^4}} \, dx$$

$$= -\left(4\int \left(\frac{i\sqrt{x^2+\sqrt{1+x^4}}}{2\left(i-x^2\right)}+\frac{i\sqrt{x^2+\sqrt{1+x^4}}}{2\left(i+x^2\right)}\right) dx\right) + 4\int \left(-\frac{\sqrt{x^2+\sqrt{1+x^4}}}{4\left(i-x^2\right)^2} \cdots\right.$$

$$= -\left(2i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{i-x^2} \, dx\right) - 2i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{i+x^2} \, dx - 2\int \frac{\sqrt{x^2+\sqrt{1+\cdots}}}{-1-x^4} \cdots$$

$$= -\left(2i\int \left(-\frac{(-1)^{3/4}\sqrt{x^2+\sqrt{1+x^4}}}{2\left(\sqrt[4]{-1}-x\right)}-\frac{(-1)^{3/4}\sqrt{x^2+\sqrt{1+x^4}}}{2\left(\sqrt[4]{-1}+x\right)}\right) dx\right) - 2i\int \left(-\cdots\right.$$

$$= \frac{1}{4}i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(\sqrt[4]{-1}-x\right)^2} \, dx - \frac{1}{4}i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(-(-1)^{3/4}-x\right)^2} \, dx + \frac{1}{4}i\int \frac{\sqrt{x^2+\sqrt{1+x\cdots}}}{\left(\sqrt[4]{-1}+x\right)^2} \cdots$$

$$= \frac{1}{4}i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(\sqrt[4]{-1}-x\right)^2} \, dx - \frac{1}{4}i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(-(-1)^{3/4}-x\right)^2} \, dx + \frac{1}{4}i\int \frac{\sqrt{x^2+\sqrt{1+x\cdots}}}{\left(\sqrt[4]{-1}+x\right)^2} \cdots$$

$$= \frac{1}{4}i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(\sqrt[4]{-1}-x\right)^2} \, dx - \frac{1}{4}i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(-(-1)^{3/4}-x\right)^2} \, dx + \frac{1}{4}i\int \frac{\sqrt{x^2+\sqrt{1+x\cdots}}}{\left(\sqrt[4]{-1}+x\right)^2} \cdots$$

Mathematica [F] time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^4\right)^2 \sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^4\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^4)^2*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^4)^2,x]
[Out] Integrate[((-1 + x^4)^2*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^4)^2, x]
```

IntegrateAlgebraic [A] time = 0.45, size = 170, normalized size = 1.00

$$-\frac{3}{2}\tan^{-1}\left(x\sqrt{\sqrt{x^4+1}+x^2}\right)+\frac{\tan^{-1}\left(\sqrt{2}x\sqrt{\sqrt{x^4+1}+x^2}\right)}{2\sqrt{2}}+\frac{x\left(2x^8+8x^4+4\right)\sqrt{\sqrt{x^4+1}+x^2}+x\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}\left(2x^6+7x^2\right)}{2\left(2x^8+3x^4+1\right)+2\sqrt{x^4+1}\left(2x^6+2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)^2*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^4)^2,x]
[Out] (x*Sqrt[1 + x^4]*(7*x^2 + 2*x^6)*Sqrt[x^2 + Sqrt[1 + x^4]] + x*(4 + 8*x^4 +
 2*x^8)*Sqrt[x^2 + Sqrt[1 + x^4]])/(2*Sqrt[1 + x^4]*(2*x^2 + 2*x^6) + 2*(1
+ 3*x^4 + 2*x^8)) - (3*ArcTan[x*Sqrt[x^2 + Sqrt[1 + x^4]]])/2 + ArcTan[Sqrt
[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]]]/(2*Sqrt[2])
```

fricas [A] time = 1.63, size = 164, normalized size = 0.96

$$-\frac{2\sqrt{2}\left(x^4+1\right)\arctan\left(-\frac{\left(\sqrt{2}\,x^2-\sqrt{2}\,\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{2\,x}\right)-3\left(x^4+1\right)\arctan\left(-\frac{4\left(3\,x^9-12\,x^5-\left(3\,x^7-5\,x^3\right)\sqrt{x^4+1}+x\right)\sqrt{x^2+\sqrt{x^4+1}}}{17\,x^8-46\,x^4+1}\right)-4\left(2\,x^5-\sqrt{x^4+1}\,x^3+4\,x\right)\sqrt{x^2+\sqrt{x^4+1}}}{8\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^2,x, algorithm="frica
s")
```

```
[Out] -1/8*(2*sqrt(2)*(x^4 + 1)*arctan(-1/2*(sqrt(2)*x^2 - sqrt(2)*sqrt(x^4 + 1))
*sqrt(x^2 + sqrt(x^4 + 1))/x) - 3*(x^4 + 1)*arctan(-4*(3*x^9 - 12*x^5 - (3*
x^7 - 5*x^3)*sqrt(x^4 + 1) + x)*sqrt(x^2 + sqrt(x^4 + 1))/(17*x^8 - 46*x^4
+ 1)) - 4*(2*x^5 - sqrt(x^4 + 1)*x^3 + 4*x)*sqrt(x^2 + sqrt(x^4 + 1)))/(x^4
 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)^2\sqrt{x^2+\sqrt{x^4+1}}}{\left(x^4+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^2,x, algorithm="giac"
)
```

```
[Out] integrate((x^4 - 1)^2*sqrt(x^2 + sqrt(x^4 + 1))/(x^4 + 1)^2, x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)^2\sqrt{x^2+\sqrt{x^4+1}}}{\left(x^4+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^2,x)
```

```
[Out] int((x^4-1)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)^2\sqrt{x^2+\sqrt{x^4+1}}}{\left(x^4+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^2,x, algorithm="maxim
a")
```

```
[Out] integrate((x^4 - 1)^2*sqrt(x^2 + sqrt(x^4 + 1))/(x^4 + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-1\right)^2\sqrt{\sqrt{x^4+1}+x^2}}{\left(x^4+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 - 1)^2*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 + 1)^2,x)
[Out] int(((x^4 - 1)^2*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)^2 (x+1)^2 \left(x^2+1\right)^2 \sqrt{x^2+\sqrt{x^4+1}}}{\left(x^4+1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**2*(x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**2,x)
[Out] Integral((x - 1)**2*(x + 1)**2*(x**2 + 1)**2*sqrt(x**2 + sqrt(x**4 + 1))/(x
**4 + 1)**2, x)
```

3.1859 $\int \left(\frac{1}{\sqrt{1-\sqrt{x}}}-\sqrt{1-\sqrt{x}-x}\right) dx$

Optimal. Leaf size=171

$$\sqrt{-x-\sqrt{x}+1}\left(\frac{1}{4}\sqrt{\left(2\sqrt{x}+1\right)^2}+\frac{2}{3}\right)-\frac{2}{3}\sqrt{-x-\sqrt{x}+1}\,x-\frac{8\sqrt{1-\sqrt{x}}}{3}-\frac{4}{3}\sqrt{1-\sqrt{x}}\,\sqrt{x}-\frac{2}{3}\sqrt{-x-\sqrt{x}+1}\,\sqrt{x}$$

Rubi [A] time = 0.06, antiderivative size = 102, normalized size of antiderivative = 0.60, number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {190, 43, 1341, 640, 612, 619, 216}

$$\frac{4}{3}\left(1-\sqrt{x}\right)^{3/2}-4\sqrt{1-\sqrt{x}}+\frac{2}{3}\left(-x-\sqrt{x}+1\right)^{3/2}+\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}+\frac{5}{8}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - Sqrt[x]] - Sqrt[1 - Sqrt[x] - x],x]
```

```
[Out] -4*Sqrt[1 - Sqrt[x]] + (4*(1 - Sqrt[x])^(3/2))/3 + ((1 + 2*Sqrt[x])*Sqrt[1
- Sqrt[x] - x])/4 + (2*(1 - Sqrt[x] - x)^(3/2))/3 + (5*ArcSin[(1 + 2*Sqrt[x
])/Sqrt[5]])/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \left(\frac{1}{\sqrt{1-\sqrt{x}}}-\sqrt{1-\sqrt{x}-x}\right) dx = \int \frac{1}{\sqrt{1-\sqrt{x}}}\, dx - \int \sqrt{1-\sqrt{x}-x}\, dx$$
$$= 2\,\text{Subst}\left(\int \frac{x}{\sqrt{1-x}}\, dx, x, \sqrt{x}\right) - 2\,\text{Subst}\left(\int x\sqrt{1-x-x^2}\, dx, x, \sqrt{x}\right)$$
$$= \frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2} + 2\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{1-x}}-\sqrt{1-x}\right) dx, x, \sqrt{x}\right) + \text{Subst}\Big($$
$$= -4\sqrt{1-\sqrt{x}} + \frac{4}{3}\left(1-\sqrt{x}\right)^{3/2} + \frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} + \frac{2}{3}\Big(1-\sqrt{x}-$$
$$= -4\sqrt{1-\sqrt{x}} + \frac{4}{3}\left(1-\sqrt{x}\right)^{3/2} + \frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} + \frac{2}{3}\Big(1-\sqrt{x}-$$
$$= -4\sqrt{1-\sqrt{x}} + \frac{4}{3}\left(1-\sqrt{x}\right)^{3/2} + \frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} + \frac{2}{3}\Big(1-\sqrt{x}-$$

Mathematica [A] time = 0.09, size = 113, normalized size = 0.66

$$2\left(\frac{2}{3}\left(1-\sqrt{x}\right)^{3/2}-2\sqrt{1-\sqrt{x}}\right)-2\left(\frac{1}{2}\left(\frac{1}{4}\sqrt{-x-\sqrt{x}+1}\left(-2\sqrt{x}-1\right)+\frac{5}{8}\sin^{-1}\left(\frac{-2\sqrt{x}-1}{\sqrt{5}}\right)\right)-\frac{1}{3}\left(-x-\sqrt{x}+1\right)^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - Sqrt[x]] - Sqrt[1 - Sqrt[x] - x],x]
```

```
[Out] 2*(-2*Sqrt[1 - Sqrt[x]] + (2*(1 - Sqrt[x])^(3/2))/3) - 2*(-1/3*(1 - Sqrt[x]
 - x)^(3/2) + (((-1 - 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/4 + (5*ArcSin[(-1 -
 2*Sqrt[x])/Sqrt[5]])/8)/2)
```

IntegrateAlgebraic [A] time = 0.62, size = 86, normalized size = 0.50

$$-\frac{4}{3}\sqrt{1-\sqrt{x}}\left(\sqrt{x}+2\right)+\frac{1}{12}\left(-8x-2\sqrt{x}+11\right)\sqrt{-x-\sqrt{x}+1}-\frac{5}{4}\tan^{-1}\left(\frac{\sqrt{-x-\sqrt{x}+1}-1}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[1 - Sqrt[x]] - Sqrt[1 - Sqrt[x] - x],x]
```

```
[Out] (-4*Sqrt[1 - Sqrt[x]]*(2 + Sqrt[x]))/3 + ((11 - 2*Sqrt[x] - 8*x)*Sqrt[1 - S
qrt[x] - x])/12 - (5*ArcTan[(-1 + Sqrt[1 - Sqrt[x] - x])/Sqrt[x]])/4
```

fricas [A] time = 1.34, size = 100, normalized size = 0.58

$$-\frac{1}{12}\left(8x+2\sqrt{x}-11\right)\sqrt{-x-\sqrt{x}+1}-\frac{4}{3}\left(\sqrt{x}+2\right)\sqrt{-\sqrt{x}+1}-\frac{5}{16}\arctan\left(-\frac{\left(8x^2-\left(16x^2-38x+11\right)\sqrt{x}-9x+3\right)\sqrt{-x-\sqrt{x}+1}}{4\left(4x^3-13x^2+7x-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x^(1/2))^(1/2)-(1-x^(1/2)-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(8*x + 2*sqrt(x) - 11)*sqrt(-x - sqrt(x) + 1) - 4/3*(sqrt(x) + 2)*sqr
t(-sqrt(x) + 1) - 5/16*arctan(-1/4*(8*x^2 - (16*x^2 - 38*x + 11)*sqrt(x) -
9*x + 3)*sqrt(-x - sqrt(x) + 1)/(4*x^3 - 13*x^2 + 7*x - 1))
```

giac [A] time = 0.19, size = 66, normalized size = 0.39

$$-\frac{1}{12}\left(2\sqrt{x}\left(4\sqrt{x}+1\right)-11\right)\sqrt{-x-\sqrt{x}+1}+\frac{4}{3}\left(-\sqrt{x}+1\right)^{\frac{3}{2}}-4\sqrt{-\sqrt{x}+1}+\frac{5}{8}\arcsin\left(\frac{1}{5}\sqrt{5}\left(2\sqrt{x}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x^(1/2))^(1/2)-(1-x^(1/2)-x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*(2*sqrt(x)*(4*sqrt(x) + 1) - 11)*sqrt(-x - sqrt(x) + 1) + 4/3*(-sqrt(
x) + 1)^(3/2) - 4*sqrt(-sqrt(x) + 1) + 5/8*arcsin(1/5*sqrt(5)*(2*sqrt(x) +
1))
```

maple [A] time = 0.02, size = 72, normalized size = 0.42

$$\frac{4\left(1-\sqrt{x}\right)^{\frac{3}{2}}}{3}-4\sqrt{1-\sqrt{x}}+\frac{2\left(1-\sqrt{x}-x\right)^{\frac{3}{2}}}{3}-\frac{\left(-2\sqrt{x}-1\right)\sqrt{1-\sqrt{x}-x}}{4}+\frac{5\arcsin\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x^(1/2))^(1/2)-(1-x^(1/2)-x)^(1/2),x)
```

```
[Out] 4/3*(1-x^(1/2))^(3/2)-4*(1-x^(1/2))^(1/2)+2/3*(1-x^(1/2)-x)^(3/2)-1/4*(-2*x
^(1/2)-1)*(1-x^(1/2)-x)^(1/2)+5/8*arcsin(2/5*5^(1/2)*(x^(1/2)+1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4}{3}\left(-\sqrt{x}+1\right)^{\frac{3}{2}}-4\sqrt{-\sqrt{x}+1}-\int\sqrt{-x-\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x^(1/2))^(1/2)-(1-x^(1/2)-x)^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*(-sqrt(x) + 1)^(3/2) - 4*sqrt(-sqrt(x) + 1) - integrate(sqrt(-x - sqrt(
x) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int\sqrt{1-\sqrt{x}-x}-\frac{1}{\sqrt{1-\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x^(1/2))^(1/2) - (1 - x^(1/2) - x)^(1/2),x)
```

```
[Out] -int((1 - x^(1/2) - x)^(1/2) - 1/(1 - x^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{1}{\sqrt{1-\sqrt{x}}}\right)dx-\int\sqrt{-\sqrt{x}-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x**(1/2))**(1/2)-(1-x**(1/2)-x)**(1/2),x)
```

```
[Out] -Integral(-1/sqrt(1 - sqrt(x)), x) - Integral(sqrt(-sqrt(x) - x + 1), x)
```

3.1860 $\int \frac{-5+x}{\sqrt[3]{-2-x+x^2}\left(-3+4x+x^2\right)}\,dx$

Optimal. Leaf size=171

$$\frac{\log\left(2\sqrt[3]{x^2-x-2}+2^{2/3}x+2^{2/3}\right)}{2^{2/3}}-\frac{\log\left(-\sqrt[3]{2}\,x^2-2\left(x^2-x-2\right)^{2/3}+\left(2^{2/3}x+2^{2/3}\right)\sqrt[3]{x^2-x-2}-2\sqrt[3]{2}\,x-\right.}{2\,2^{2/3}}$$

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-5+x}{\sqrt[3]{-2-x+x^2}\left(-3+4x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-5 + x)/((-2 - x + x^2)^(1/3)*(-3 + 4*x + x^2)),x]
```

```
[Out] Defer[Int][(-5 + x)/((-2 - x + x^2)^(1/3)*(-3 + 4*x + x^2)), x]
```

Rubi steps

$$\int \frac{-5+x}{\sqrt[3]{-2-x+x^2}\left(-3+4x+x^2\right)}\,dx=\int \frac{-5+x}{\sqrt[3]{-2-x+x^2}\left(-3+4x+x^2\right)}\,dx$$

Mathematica [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{-5+x}{\sqrt[3]{-2-x+x^2}\left(-3+4x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-5 + x)/((-2 - x + x^2)^(1/3)*(-3 + 4*x + x^2)),x]
```

```
[Out] Integrate[(-5 + x)/((-2 - x + x^2)^(1/3)*(-3 + 4*x + x^2)), x]
```

IntegrateAlgebraic [A] time = 0.24, size = 171, normalized size = 1.00

$$\frac{\log\left(2\sqrt[3]{x^2-x-2}+2^{2/3}x+2^{2/3}\right)}{2^{2/3}}-\frac{\log\left(-\sqrt[3]{2}\,x^2-2\left(x^2-x-2\right)^{2/3}+\left(2^{2/3}x+2^{2/3}\right)\sqrt[3]{x^2-x-2}-2\sqrt[3]{2}\,x-\sqrt[3]{2}\right)}{2\,2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^2-x-2}}{\sqrt[3]{x^2-x-2}-2^{2/3}x-2^{2/3}}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-5 + x)/((-2 - x + x^2)^(1/3)*(-3 + 4*x + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(-2 - x + x^2)^(1/3))/(-2^(2/3) - 2^(2/3)*x + (-2
- x + x^2)^(1/3))])/2^(2/3) + Log[2^(2/3) + 2^(2/3)*x + 2*(-2 - x + x^2)^(1
/3)]/2^(2/3) - Log[-2^(1/3) - 2*2^(1/3)*x - 2^(1/3)*x^2 + (2^(2/3) + 2^(2/3
)*x)*(-2 - x + x^2)^(1/3) - 2*(-2 - x + x^2)^(2/3)]/(2*2^(2/3))
```

fricas [B] time = 7.61, size = 300, normalized size = 1.75

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(12\cdot 4^{\frac{2}{3}}\left(x^4+5x^3+4x^2+9x-9\right)\left(x^2-x-2\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(x^6+30x^5+3x^4+100x^3-45x^2-306x-351\right)+12\left(x^5-9x^4+40x^2+75x+45\right)\left(x^2-x-2\right)^{\frac{1}{3}}\right)}{6\left(x^6-42x^5-69x^4+100x^3+315x^2+486x+81\right)}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(\frac{6\cdot 4^{\frac{2}{3}}\left(x^2+x+3\right)\left(x^2-x-2\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(x^4-10x^3+10x^2+30x+45\right)-6\left(x^3-x^2+7x+9\right)\left(x^2-x-2\right)^{\frac{1}{3}}}{x^4+8x^3+10x^2-24x+9}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}\left(x^2+4x-3\right)+6\cdot 4^{\frac{1}{3}}\left(x^2-x-2\right)^{\frac{1}{3}}(x+1)+12\left(x^2-x-2\right)^{\frac{2}{3}}}{x^2+4x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+x)/(x^2-x-2)^(1/3)/(x^2+4*x-3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^(2/3)*(x^4 + 5*x^3 +
4*x^2 + 9*x - 9)*(x^2 - x - 2)^(2/3) + 4^(1/3)*(x^6 + 30*x^5 + 3*x^4 + 100*
x^3 - 45*x^2 - 306*x - 351) + 12*(x^5 - 9*x^4 + 40*x^2 + 75*x + 45)*(x^2 -
x - 2)^(1/3))/(x^6 - 42*x^5 - 69*x^4 + 100*x^3 + 315*x^2 + 486*x + 81)) - 1
/24*4^(2/3)*log((6*4^(2/3)*(x^2 + x + 3)*(x^2 - x - 2)^(2/3) + 4^(1/3)*(x^4
 - 10*x^3 + 10*x^2 + 30*x + 45) - 6*(x^3 - x^2 + 7*x + 9)*(x^2 - x - 2)^(1/
3))/(x^4 + 8*x^3 + 10*x^2 - 24*x + 9)) + 1/12*4^(2/3)*log((4^(2/3)*(x^2 + 4
*x - 3) + 6*4^(1/3)*(x^2 - x - 2)^(1/3)*(x + 1) + 12*(x^2 - x - 2)^(2/3))/(
x^2 + 4*x - 3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-5}{\left(x^2+4\,x-3\right)\left(x^2-x-2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+x)/(x^2-x-2)^(1/3)/(x^2+4*x-3),x, algorithm="giac")
```

```
[Out] integrate((x - 5)/((x^2 + 4*x - 3)*(x^2 - x - 2)^(1/3)), x)
```

maple [C] time = 9.44, size = 941, normalized size = 5.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+x)/(x^2-x-2)^(1/3)/(x^2+4*x-3),x)
```

```
[Out] RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*ln((48*RootOf(RootOf(_Z
^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^2-356*RootOf(RootOf(
_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^2+120*RootOf(Roo
tOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x-890*RootOf(Roo
tOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x-750*(x^2-x-2
)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^
2-375*(x^2-x-2)^(1/3)*RootOf(_Z^3-2)^2*x+123*(x^2-x-2)^(1/3)*RootOf(RootOf(
_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-375*(x^2-x-2)^(1/3)*
RootOf(_Z^3-2)^2+123*(x^2-x-2)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z
^3-2)+4*_Z^2)*RootOf(_Z^3-2)-12*RootOf(_Z^3-2)*x^2+89*RootOf(RootOf(_Z^3-2)
^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2+96*RootOf(_Z^3-2)*x-712*RootOf(RootOf(_Z
^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x-873*(x^2-x-2)^(2/3)-252*RootOf(_Z^3-2
)+1869*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2))/(x^2+4*x-3))+1/
2*RootOf(_Z^3-2)*ln((356*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2
)*RootOf(_Z^3-2)^3*x^2-192*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z
^2)^2*RootOf(_Z^3-2)^2*x^2+890*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+
4*_Z^2)*RootOf(_Z^3-2)^3*x-480*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+
4*_Z^2)^2*RootOf(_Z^3-2)^2*x+1500*(x^2-x-2)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2
*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2+750*(x^2-x-2)^(1/3)*RootOf(_Z^3
-2)^2*x+1746*(x^2-x-2)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*
_Z^2)*RootOf(_Z^3-2)*x+750*(x^2-x-2)^(1/3)*RootOf(_Z^3-2)^2+1746*(x^2-x-2)^
(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)+44
5*RootOf(_Z^3-2)*x^2-240*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2
)*x^2+178*RootOf(_Z^3-2)*x-96*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4
*_Z^2)*x-246*(x^2-x-2)^(2/3)+1869*RootOf(_Z^3-2)-1008*RootOf(RootOf(_Z^3-2)
^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2))/(x^2+4*x-3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-5}{\left(x^2+4\,x-3\right)\left(x^2-x-2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+x)/(x^2-x-2)^(1/3)/(x^2+4*x-3),x, algorithm="maxima")
[Out] integrate((x - 5)/((x^2 + 4*x - 3)*(x^2 - x - 2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x-5}{\left(x^2-x-2\right)^{1/3}\,\left(x^2+4\,x-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 5)/((x^2 - x - 2)^(1/3)*(4*x + x^2 - 3)),x)
[Out] int((x - 5)/((x^2 - x - 2)^(1/3)*(4*x + x^2 - 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-5}{\sqrt[3]{(x-2)\,(x+1)}\,\left(x^2+4x-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+x)/(x**2-x-2)**(1/3)/(x**2+4*x-3),x)
[Out] Integral((x - 5)/(((x - 2)*(x + 1))**(1/3)*(x**2 + 4*x - 3)), x)
```

3.1861 $\int \frac{(-a+x)(-2a+b+x)}{\left((-a+x)(-b+x)^2\right)^{3/4}\left(a+b^2d-(1+2bd)x+dx^2\right)}\,dx$

Optimal. Leaf size=171

$$-2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{3/4}}{(x-a)(b-x)}\right)+2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2\right.}{(x-a)(b-x)}\right.$$

Rubi [C] time = 2.05, antiderivative size = 325, normalized size of antiderivative = 1.90, number of steps used = 7, number of rules used = 4, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6719, 6728, 137, 136}

$$\frac{4(a-x)^2(b-x)\left(1-\sqrt{-4ad+4bd+1}\right)\sqrt{-\frac{b-x}{a-b}}F_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{a-x}{a-b},-\frac{2d(a-x)}{-2ad+2bd-\sqrt{-4ad+4bd+1}+1}\right)}{5(a-b)\left(-\sqrt{-4ad+4bd+1}-2ad+2bd+1\right)\left(-\left((a-x)(b-x)^2\right)\right)^{3/4}}+\frac{4(a-x)^2(b-x)\left(\sqrt{-4ad+4bd+1}+1\right)\sqrt{-\frac{b-x}{a-b}}F_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{a-x}{a-b},-\frac{2d(a-x)}{-2ad+2bd+\sqrt{-4ad+4bd+1}+1}\right)}{5(a-b)\left(\sqrt{-4ad+4bd+1}-2ad+2bd+1\right)\left(-\left((a-x)(b-x)^2\right)\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-a + x)*(-2*a + b + x))/(((-a + x)*(-b + x)^2)^(3/4)*(a + b^2*d - (1
+ 2*b*d)*x + d*x^2)),x]
```

```
[Out] (4*(1 - Sqrt[1 - 4*a*d + 4*b*d])*(a - x)^2*(b - x)*Sqrt[-((b - x)/(a - b))]
*AppellF1[5/4, 3/2, 1, 9/4, (a - x)/(a - b), (-2*d*(a - x))/(1 - 2*a*d + 2*
b*d - Sqrt[1 - 4*a*d + 4*b*d])])/(5*(a - b)*(1 - 2*a*d + 2*b*d - Sqrt[1 - 4
*a*d + 4*b*d])*(-((a - x)*(b - x)^2))^(3/4)) + (4*(1 + Sqrt[1 - 4*a*d + 4*b
*d])*(a - x)^2*(b - x)*Sqrt[-((b - x)/(a - b))]*AppellF1[5/4, 3/2, 1, 9/4,
(a - x)/(a - b), (-2*d*(a - x))/(1 - 2*a*d + 2*b*d + Sqrt[1 - 4*a*d + 4*b*d
])])/(5*(a - b)*(1 - 2*a*d + 2*b*d + Sqrt[1 - 4*a*d + 4*b*d])*(-((a - x)*(b
 - x)^2))^(3/4))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(-a+x)(-2a+b+x)}{\left((-a+x)(-b+x)^2\right)^{3/4}\left(a+b^2d-(1+2bd)x+dx^2\right)}\,dx = \frac{\left((-a+x)^{3/4}(-b+x)^{3/2}\right)\int \frac{\sqrt[4]{-a+x}\,(-2a+b+x)}{(-b+x)^{3/2}\left(a+b^2d-(1+2bd)x+d\right.}}{\left((-a+x)(-b+x)^2\right)^{3/4}}$$

$$= \frac{\left((-a+x)^{3/4}(-b+x)^{3/2}\right)\int\left(\frac{\left(1+\sqrt{1-4ad+4bd}\right)\sqrt[4]{-}}{(-b+x)^{3/2}\left(-1-2bd-\sqrt{1-4ad}\right.}\right.}{\left((-a+x)(-\right.}$$

$$= \frac{\left(\left(1-\sqrt{1-4ad+4bd}\right)(-a+x)^{3/4}(-b+x)^{3/2}\int\right.}{\left((-a+x)(-b+x)^2\right.}$$

$$= \frac{\left(\left(1-\sqrt{1-4ad+4bd}\right)(-a+x)^{3/4}(-b+x)\sqrt{\frac{-b+x}{a-b}}\right.}{(a-b)\left((-a+x)(-\right.}$$

$$= \frac{4\left(1-\sqrt{1-4ad+4bd}\right)(a-x)^2(b-x)\sqrt{-\frac{b-x}{a-b}}\,F_1\left(\right.}{5(a-b)\left(1-2ad+2bd-\sqrt{1-4ad+4\right.}}$$

Mathematica [C] time = 1.66, size = 492, normalized size = 2.88

$$\frac{2(b-x)\left((x-a)^{3/4}\left(\sqrt[4]{b-a}\sqrt{\frac{x-b}{a-b}}\,\Pi\left(-\frac{\sqrt{2}\sqrt{a-b}\sqrt{d}}{\sqrt{2ad-2bd-\sqrt{-4ad+4bd+1}-1}};\sin^{-1}\left(\frac{\sqrt[4]{x-a}}{\sqrt[4]{b-a}}\right)\middle|-1\right)+\sqrt[4]{b-a}\sqrt{\frac{x-b}{a-b}}\,\Pi\left(\frac{\sqrt{2}\sqrt{a-b}\sqrt{d}}{\sqrt{2ad-2bd-\sqrt{-4ad+4bd+1}-1}};\sin^{-1}\left(\frac{\sqrt[4]{x-a}}{\sqrt[4]{b-a}}\right)\middle|-1\right)+\sqrt[4]{b-a}\sqrt{\frac{x-b}{a-b}}\,\Pi\left(-\frac{\sqrt{2}\sqrt{a-b}\sqrt{d}}{\sqrt{2ad-2bd+\sqrt{-4ad+4bd+1}-1}};\sin^{-1}\left(\frac{\sqrt[4]{x-a}}{\sqrt[4]{b-a}}\right)\middle|-1\right)+\sqrt[4]{b-a}\sqrt{\frac{x-b}{a-b}}\,\Pi\left(\frac{\sqrt{2}\sqrt{a-b}\sqrt{d}}{\sqrt{2ad-2bd+\sqrt{-4ad+4bd+1}-1}};\sin^{-1}\left(\frac{\sqrt[4]{x-a}}{\sqrt[4]{b-a}}\right)\middle|-1\right)+\frac{(b-x)\left(\frac{a-x}{a-b}\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{x-b}{a-b}\right)}{(x-a)^{3/4}}\right)+2(a-x)\right)}{\left((x-a)(b-x)^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-a + x)*(-2*a + b + x))/(((-a + x)*(-b + x)^2)^(3/4)*(a + b^2*d
 - (1 + 2*b*d)*x + d*x^2)),x]
```

```
[Out] (2*(b - x)*(2*(a - x) + (-a + x)^(3/4)*((-a + b)^(1/4)*Sqrt[(-b + x)/(a - b
)]*EllipticPi[-((Sqrt[2]*Sqrt[a - b]*Sqrt[d])/Sqrt[-1 + 2*a*d - 2*b*d - Sqr
t[1 - 4*a*d + 4*b*d]]), ArcSin[(-a + x)^(1/4)/(-a + b)^(1/4)], -1] + (-a +
b)^(1/4)*Sqrt[(-b + x)/(a - b)]*EllipticPi[(Sqrt[2]*Sqrt[a - b]*Sqrt[d])/Sq
rt[-1 + 2*a*d - 2*b*d - Sqrt[1 - 4*a*d + 4*b*d]], ArcSin[(-a + x)^(1/4)/(-a
 + b)^(1/4)], -1] + (-a + b)^(1/4)*Sqrt[(-b + x)/(a - b)]*EllipticPi[-((Sqr
t[2]*Sqrt[a - b]*Sqrt[d])/Sqrt[-1 + 2*a*d - 2*b*d + Sqrt[1 - 4*a*d + 4*b*d]
]), ArcSin[(-a + x)^(1/4)/(-a + b)^(1/4)], -1] + (-a + b)^(1/4)*Sqrt[(-b +
x)/(a - b)]*EllipticPi[(Sqrt[2]*Sqrt[a - b]*Sqrt[d])/Sqrt[-1 + 2*a*d - 2*b*
d + Sqrt[1 - 4*a*d + 4*b*d]], ArcSin[(-a + x)^(1/4)/(-a + b)^(1/4)], -1] +
(((a - x)/(a - b))^(3/4)*(b - x)*Hypergeometric2F1[1/2, 3/4, 3/2, (-b + x)/
(a - b)])/(-a + x)^(3/4))))/((b - x)^2*(-a + x))^(3/4)
```

IntegrateAlgebraic [A] time = 4.01, size = 171, normalized size = 1.00

$$-2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{3/4}}{(x-a)(b-x)}\right)+2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{3/4}}{(x-a)(b-x)}\right)-\frac{4\sqrt[4]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}}{b-x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-a + x)*(-2*a + b + x))/(((-a + x)*(-b + x)^2)^(3/4)*(
a + b^2*d - (1 + 2*b*d)*x + d*x^2)),x]
```

```
[Out] (-4*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/4))/(b - x) - 2*
d^(1/4)*ArcTan[(d^(1/4)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)
^(3/4))/((b - x)*(-a + x))] + 2*d^(1/4)*ArcTanh[(d^(1/4)*(-(a*b^2) + (2*a*b
 + b^2)*x + (-a - 2*b)*x^2 + x^3)^(3/4))/((b - x)*(-a + x))]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-2*a+b+x)/((-a+x)*(-b+x)^2)^(3/4)/(a+b^2*d-(2*b*d+1)*x+d*
x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2a-b-x)(a-x)}{\left(-(a-x)(b-x)^2\right)^{\frac{3}{4}}\left(b^2d+dx^2-(2bd+1)x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-2*a+b+x)/((-a+x)*(-b+x)^2)^(3/4)/(a+b^2*d-(2*b*d+1)*x+d*
x^2),x, algorithm="giac")
```

```
[Out] integrate((2*a - b - x)*(a - x)/((-(a - x)*(b - x)^2)^(3/4)*(b^2*d + d*x^2
- (2*b*d + 1)*x + a)), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{(-a+x)(-2a+b+x)}{\left((-a+x)(-b+x)^2\right)^{\frac{3}{4}}\left(a+b^2d-(2bd+1)x+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x)*(-2*a+b+x)/((-a+x)*(-b+x)^2)^(3/4)/(a+b^2*d-(2*b*d+1)*x+d*x^2),x
)
```

```
[Out] int((-a+x)*(-2*a+b+x)/((-a+x)*(-b+x)^2)^(3/4)/(a+b^2*d-(2*b*d+1)*x+d*x^2),x
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2a-b-x)(a-x)}{\left(-(a-x)(b-x)^2\right)^{\frac{3}{4}}\left(b^2d+dx^2-(2bd+1)x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-2*a+b+x)/((-a+x)*(-b+x)^2)^(3/4)/(a+b^2*d-(2*b*d+1)*x+d*
x^2),x, algorithm="maxima")
```

```
[Out] integrate((2*a - b - x)*(a - x)/((-(a - x)*(b - x)^2)^(3/4)*(b^2*d + d*x^2
- (2*b*d + 1)*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{(a-x)\,(b-2a+x)}{\left(-(a-x)\,(b-x)^2\right)^{3/4}\,\left(a-x\,(2bd+1)+b^2\,d+d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a - x)*(b - 2*a + x))/((-(a - x)*(b - x)^2)^(3/4)*(a - x*(2*b*d + 1)
 + b^2*d + d*x^2)),x)
```

```
[Out] int(-((a - x)*(b - 2*a + x))/((-(a - x)*(b - x)^2)^(3/4)*(a - x*(2*b*d + 1)
 + b^2*d + d*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-2*a+b+x)/((-a+x)*(-b+x)**2)**(3/4)/(a+b**2*d-(2*b*d+1)*x
+d*x**2),x)
```

```
[Out] Timed out
```

3.1862 $\int \frac{1+x}{(1+4x+x^2)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=171

$$\frac{\log\left(2\sqrt[3]{1-x^3}+2^{2/3}x-2^{2/3}\right)}{3\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{1-x^3}-2^{2/3}x+2^{2/3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(-2\left(1-x^3\right)^{2/3}+\left(2^{2/3}x-2^{2/3}\right)\sqrt[3]{1-x^3}-\sqrt[3]{2}x^2+2\right.}{6\,2^{2/3}}$$

Rubi [F] time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x}{(1+4x+x^2)\sqrt[3]{1-x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x)/((1 + 4*x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] ((3 - Sqrt[3])*Defer[Int][1/((4 - 2*Sqrt[3] + 2*x)*(1 - x^3)^(1/3)), x])/3
+ ((3 + Sqrt[3])*Defer[Int][1/((4 + 2*Sqrt[3] + 2*x)*(1 - x^3)^(1/3)), x])/
3
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(1+4x+x^2)\sqrt[3]{1-x^3}}\,dx &= \int \left(\frac{1-\frac{1}{\sqrt{3}}}{\left(4-2\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}+\frac{1+\frac{1}{\sqrt{3}}}{\left(4+2\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\right)dx\\ &= \frac{1}{3}\left(3-\sqrt{3}\right)\int \frac{1}{\left(4-2\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx+\frac{1}{3}\left(3+\sqrt{3}\right)\int \frac{1}{\left(4+2\sqrt{3}+2x\right)\sqrt[3]{1\right.}}\end{aligned}$$

Mathematica [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1+x}{(1+4x+x^2)\sqrt[3]{1-x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x)/((1 + 4*x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] Integrate[(1 + x)/((1 + 4*x + x^2)*(1 - x^3)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 1.14, size = 171, normalized size = 1.00

$$\frac{\log\left(2\sqrt[3]{1-x^3}+2^{2/3}x-2^{2/3}\right)}{3\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{1-x^3}-2^{2/3}x+2^{2/3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(-2\left(1-x^3\right)^{2/3}+\left(2^{2/3}x-2^{2/3}\right)\sqrt[3]{1-x^3}-\sqrt[3]{2}x^2+2\sqrt[3]{2}x-\sqrt[3]{2}\right)}{6\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((1 + 4*x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(2/3) - 2^(2/3)*x + (1 - x^3)^(1/3))]/(
2^(2/3)*Sqrt[3]) + Log[-2^(2/3) + 2^(2/3)*x + 2*(1 - x^3)^(1/3)]/(3*2^(2/3)
) - Log[-2^(1/3) + 2*2^(1/3)*x - 2^(1/3)*x^2 + (-2^(2/3) + 2^(2/3)*x)*(1 -
x^3)^(1/3) - 2*(1 - x^3)^(2/3)]/(6*2^(2/3))
```

fricas [B] time = 10.21, size = 316, normalized size = 1.85

$$\frac{1}{18}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\left(12\cdot 4^{\frac{2}{3}}\sqrt{3}\left(2x^{4}+7x^{3}+7x+2\right)\left(-x^{3}+1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\sqrt{3}\left(91x^{6}-42x^{5}+105x^{4}-92x^{3}+105x^{2}-42x+91\right)-12\sqrt{3}\left(19x^{5}-29x^{4}+28x^{3}-28x^{2}+29x-19\right)\left(-x^{3}+1\right)^{\frac{1}{3}}\right)}{6\left(53x^{6}-174x^{5}+111x^{4}-196x^{3}+111x^{2}-174x+53\right)}\right)-\frac{1}{72}\cdot 4^{\frac{2}{3}}\log\left(\frac{6\cdot 4^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{2}{3}}\left(2x^{2}-x+2\right)+4^{\frac{1}{3}}\left(19x^{4}-10x^{3}+18x^{2}-10x+19\right)-6\left(5x^{3}-3x^{2}+3x-5\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{x^{4}+8x^{3}+18x^{2}+8x+1}\right)+\frac{1}{36}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}\left(x^{2}+4x+1\right)-6\cdot 4^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}(x-1)-12\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}+4x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+4*x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/18*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(12*4^(2/3)*sqrt(3)*(2*x^4 + 7*x^3
+ 7*x + 2)*(-x^3 + 1)^(2/3) + 4^(1/3)*sqrt(3)*(91*x^6 - 42*x^5 + 105*x^4 -
92*x^3 + 105*x^2 - 42*x + 91) - 12*sqrt(3)*(19*x^5 - 29*x^4 + 28*x^3 - 28*x
^2 + 29*x - 19)*(-x^3 + 1)^(1/3))/(53*x^6 - 174*x^5 + 111*x^4 - 196*x^3 + 1
11*x^2 - 174*x + 53)) - 1/72*4^(2/3)*log((6*4^(2/3)*(-x^3 + 1)^(2/3)*(2*x^2
 - x + 2) + 4^(1/3)*(19*x^4 - 10*x^3 + 18*x^2 - 10*x + 19) - 6*(5*x^3 - 3*x
^2 + 3*x - 5)*(-x^3 + 1)^(1/3))/(x^4 + 8*x^3 + 18*x^2 + 8*x + 1)) + 1/36*4^
(2/3)*log((4^(2/3)*(x^2 + 4*x + 1) - 6*4^(1/3)*(-x^3 + 1)^(1/3)*(x - 1) - 1
2*(-x^3 + 1)^(2/3))/(x^2 + 4*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(-x^{3}+1\right)^{\frac{1}{3}}\left(x^{2}+4\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+4*x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/((-x^3 + 1)^(1/3)*(x^2 + 4*x + 1)), x)
```

maple [C] time = 8.92, size = 1158, normalized size = 6.77

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2+4*x+1)/(-x^3+1)^(1/3),x)
```

```
[Out] -1/6*ln((108*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*
_Z^2)*RootOf(_Z^3-2)^2+90*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z
^2)*RootOf(_Z^3-2)^3*x+756*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_
Z^2)^2*RootOf(_Z^3-2)^2*x-33*(-x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x-90*(-x^3+1)^
(1/3)*RootOf(_Z^3-2)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x
+33*(-x^3+1)^(1/3)*RootOf(_Z^3-2)^2+90*(-x^3+1)^(1/3)*RootOf(_Z^3-2)*RootOf
(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-35*RootOf(_Z^3-2)*x^2-294*Ro
otOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^2-30*(-x^3+1)^(2/3)+10
*RootOf(_Z^3-2)*x+84*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x
-35*RootOf(_Z^3-2)-294*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)
)/(x^2+4*x+1))*RootOf(_Z^3-2)-ln((108*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^
2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2+90*RootOf(RootOf(_Z^3-2)^2+
6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x+756*RootOf(RootOf(_Z^3-2)^2
+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x-33*(-x^3+1)^(1/3)*RootOf
(_Z^3-2)^2*x-90*(-x^3+1)^(1/3)*RootOf(_Z^3-2)*RootOf(RootOf(_Z^3-2)^2+6*_Z*
RootOf(_Z^3-2)+36*_Z^2)*x+33*(-x^3+1)^(1/3)*RootOf(_Z^3-2)^2+90*(-x^3+1)^(1
/3)*RootOf(_Z^3-2)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-35*
RootOf(_Z^3-2)*x^2-294*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)
*x^2-30*(-x^3+1)^(2/3)+10*RootOf(_Z^3-2)*x+84*RootOf(RootOf(_Z^3-2)^2+6*_Z*
RootOf(_Z^3-2)+36*_Z^2)*x-35*RootOf(_Z^3-2)-294*RootOf(RootOf(_Z^3-2)^2+6*_
Z*RootOf(_Z^3-2)+36*_Z^2))/(x^2+4*x+1))*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf
(_Z^3-2)+36*_Z^2)+1/6*RootOf(_Z^3-2)*ln(-(108*(-x^3+1)^(2/3)*RootOf(RootOf(
_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2-90*RootOf(RootOf(_Z
^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x+216*RootOf(RootOf(_
```

```
Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x+15*(-x^3+1)^(1/3
)*RootOf(_Z^3-2)^2*x+198*(-x^3+1)^(1/3)*RootOf(_Z^3-2)*RootOf(RootOf(_Z^3-2
)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x-15*(-x^3+1)^(1/3)*RootOf(_Z^3-2)^2-198*(
-x^3+1)^(1/3)*RootOf(_Z^3-2)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36
*_Z^2)-35*RootOf(_Z^3-2)*x^2+84*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)
+36*_Z^2)*x^2+66*(-x^3+1)^(2/3)-20*RootOf(_Z^3-2)*x+48*RootOf(RootOf(_Z^3-2
)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x-35*RootOf(_Z^3-2)+84*RootOf(RootOf(_Z^3-
2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2))/(x^2+4*x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^2+4\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+4*x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
[Out] integrate((x + 1)/((-x^3 + 1)^(1/3)*(x^2 + 4*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+1}{\left(1-x^3\right)^{1/3}\,\left(x^2+4\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((1 - x^3)^(1/3)*(4*x + x^2 + 1)),x)
[Out] int((x + 1)/((1 - x^3)^(1/3)*(4*x + x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt[3]{-\left(x-1\right)\left(x^2+x+1\right)}\,\left(x^2+4x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+4*x+1)/(-x**3+1)**(1/3),x)
[Out] Integral((x + 1)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 + 4*x + 1)), x)
```

3.1863 $\int \frac{x^8}{\sqrt{-1+x^4}\left(-1+x^{16}\right)}\,dx$

Optimal. Leaf size=171

$$-\frac{x}{8\sqrt{x^4-1}}+\frac{1}{32}\tan^{-1}\left(\frac{\frac{x^4}{2}-x^2-\frac{1}{2}}{x\sqrt{x^4-1}}\right)+\frac{\tan^{-1}\left(\frac{-\frac{x^4}{2^{3/4}}+\frac{x^2}{\sqrt[4]{2}}+\frac{1}{2^{3/4}}}{x\sqrt{x^4-1}}\right)}{8\,2^{3/4}}-\frac{1}{32}\tanh^{-1}\left(\frac{\frac{x^4}{2}+x^2-\frac{1}{2}}{x\sqrt{x^4-1}}\right)+\frac{\tanh^{-1}\left(\frac{2^{3/4}x\sqrt{x^4-1}}{x^4+\sqrt{2}x^2-1}\right)}{8\,2^{3/4}}$$

Rubi [C] time = 1.23, antiderivative size = 833, normalized size of antiderivative = 4.87, number of steps used = 57, number of rules used = 19, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.950, Rules used = {6725, 1152, 414, 423, 427, 424, 253, 222, 409, 1211, 1699, 206, 203, 1429, 1215, 1457, 540, 538, 537}

$$-\frac{x\left(1-x^2\right)}{16\sqrt{x^4-1}}+\frac{\sqrt{x^2+1}\,\Pi\left(-\sqrt[4]{-1};\sin^{-1}(x)|-1\right)\sqrt{1-x^2}}{8\sqrt{x^4-1}}+\frac{\sqrt{x^2+1}\,\Pi\left(\sqrt[4]{-1};\sin^{-1}(x)|-1\right)\sqrt{1-x^2}}{8\sqrt{x^4-1}}+\frac{\sqrt{x^2+1}\,\Pi\left(-(-1)^{3/4};\sin^{-1}(x)|-1\right)\sqrt{1-x^2}}{8\sqrt{x^4-1}}+\frac{\sqrt{x^2+1}\,\Pi\left((-1)^{3/4};\sin^{-1}(x)|-1\right)\sqrt{1-x^2}}{8\sqrt{x^4-1}}-\left(\frac{1}{32}-\frac{i}{32}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)-\left(\frac{1}{32}-\frac{i}{32}\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)+\frac{\sqrt{x^2-1}\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{4\left((2-2i)+2\sqrt{2}\right)\sqrt{x^4-1}}-\frac{\sqrt{x^2-1}\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{8\left((1+i)+\sqrt{2}\right)\sqrt{x^4-1}}-\frac{\sqrt{x^2-1}\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{8\left((1-i)+\sqrt{2}\right)\sqrt{x^4-1}}-\frac{\sqrt{x^2-1}\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{8\left((-1+i)+\sqrt{2}\right)\sqrt{x^4-1}}-\frac{\sqrt{x^2-1}\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{8\left((-1-i)+\sqrt{2}\right)\sqrt{x^4-1}}+\frac{\sqrt{x^2-1}\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{8\sqrt{2}\left(1+(-1)^{3/4}\right)\sqrt{x^4-1}}+\frac{\sqrt{x^2-1}\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{8\sqrt{2}\left(1+\sqrt[4]{-1}\right)\sqrt{x^4-1}}+\frac{\sqrt{x^2-1}\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{8\sqrt{2}\left(1-\sqrt[4]{-1}\right)\sqrt{x^4-1}}-\frac{\sqrt{x^2-1}\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{4\sqrt{2}\sqrt{x^4-1}}-\frac{x\left(x^2+1\right)}{16\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(Sqrt[-1 + x^4]*(-1 + x^16)),x]
```

```
[Out] -1/16*(x*(1 - x^2))/Sqrt[-1 + x^4] - (x*(1 + x^2))/(16*Sqrt[-1 + x^4]) - (1
/32 - I/32)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] - (1/32 - I/32)*ArcTanh[((1
+ I)*x)/Sqrt[-1 + x^4]] - (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(S
qrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(4*Sqrt[2]*Sqrt[-1 + x^4]) + (Sqrt[-1 + x^
2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(8*Sqr
t[2]*(1 - (-1)^(1/4))*Sqrt[-1 + x^4]) + (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*Ellip
ticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(8*Sqrt[2]*(1 + (-1)^(1/4))*
Sqrt[-1 + x^4]) + (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x
)/Sqrt[-1 + x^2]], 1/2])/(8*Sqrt[2]*(1 + (-1)^(3/4))*Sqrt[-1 + x^4]) - (Sqr
t[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2
])/(8*((-1 - I) + Sqrt[2])*Sqrt[-1 + x^4]) - (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*
EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(8*((-1 + I) + Sqrt[2])
*Sqrt[-1 + x^4]) - (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*
x)/Sqrt[-1 + x^2]], 1/2])/(8*((1 - I) + Sqrt[2])*Sqrt[-1 + x^4]) - (Sqrt[-1
 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(
8*((1 + I) + Sqrt[2])*Sqrt[-1 + x^4]) + (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*Ellip
ticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(4*((2 - 2*I) + 2*Sqrt[2])*S
qrt[-1 + x^4]) + (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticPi[-(-1)^(1/4), ArcSi
n[x], -1])/(8*Sqrt[-1 + x^4]) + (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticPi[(-1
)^(1/4), ArcSin[x], -1])/(8*Sqrt[-1 + x^4]) + (Sqrt[1 - x^2]*Sqrt[1 + x^2]*
EllipticPi[-(-1)^(3/4), ArcSin[x], -1])/(8*Sqrt[-1 + x^4]) + (Sqrt[1 - x^2]
*Sqrt[1 + x^2]*EllipticPi[(-1)^(3/4), ArcSin[x], -1])/(8*Sqrt[-1 + x^4])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
```

```
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] &&  !GtQ[c, 0]
```

Rule 540

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x
] + Dist[(b*c - a*d)/b, Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1215

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[c/(c*d + e*q), Int[1/Sqrt[a + c*x^4], x], x] + Di
st[e/(c*d + e*q), Int[(q - c*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
 FreeQ[{a, c, d, e}, x] && GtQ[-(a*c), 0] &&  !LtQ[c, 0]
```

Rule 1429

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> W
ith[{r = Rt[-(a*c), 2]}, -Dist[c/(2*r), Int[(d + e*x^n)^q/(r - c*x^n), x],
x] - Dist[c/(2*r), Int[(d + e*x^n)^q/(r + c*x^n), x], x]] /; FreeQ[{a, c, d
, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1457

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + c*x^(2*n))^FracPart[p]/((d +
e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p]), Int[(d + e*x^n)^(p + q)*
(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, n, p,
 q, r}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^8}{\sqrt{-1+x^4}\left(-1+x^{16}\right)}\,dx = \int\left(\frac{1}{8\left(-1+x^2\right)\sqrt{-1+x^4}} - \frac{1}{8\left(1+x^2\right)\sqrt{-1+x^4}} - \frac{1}{4\sqrt{-1+x^4}\left(1+x^4\right)} + \frac{}{2\sqrt{-1+}}\right.$$

$$= \frac{1}{8}\int \frac{1}{\left(-1+x^2\right)\sqrt{-1+x^4}}\,dx - \frac{1}{8}\int \frac{1}{\left(1+x^2\right)\sqrt{-1+x^4}}\,dx - \frac{1}{4}\int \frac{1}{\sqrt{-1+x^4}\left(1+x\right.}$$

$$= \frac{1}{4}i\int \frac{1}{\left(i-x^4\right)\sqrt{-1+x^4}}\,dx + \frac{1}{4}i\int \frac{1}{\sqrt{-1+x^4}\left(i+x^4\right)}\,dx - \frac{1}{8}\int \frac{1}{\left(1-ix^2\right)\sqrt{-1+x}}$$

$$= -\frac{x\left(1-x^2\right)}{16\sqrt{-1+x^4}} - \frac{x\left(1+x^2\right)}{16\sqrt{-1+x^4}} - 2\left(\frac{1}{16}\int \frac{1}{\sqrt{-1+x^4}}\,dx\right) - \frac{1}{16}\int \frac{1-ix^2}{\left(1+ix^2\right)\sqrt{-1+x}}$$

$$= -\frac{x\left(1-x^2\right)}{16\sqrt{-1+x^4}} - \frac{x\left(1+x^2\right)}{16\sqrt{-1+x^4}} - \frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{8\sqrt{2}\,\sqrt{-1+x^4}} - \frac{1}{16}\,\text{Sub}$$

$$= -\frac{x\left(1-x^2\right)}{16\sqrt{-1+x^4}} - \frac{x\left(1+x^2\right)}{16\sqrt{-1+x^4}} - \left(\frac{1}{32}-\frac{i}{32}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right) - \left(\frac{1}{32}-\frac{i}{32}\right)\tanh^{-}$$

$$= -\frac{x\left(1-x^2\right)}{16\sqrt{-1+x^4}} - \frac{x\left(1+x^2\right)}{16\sqrt{-1+x^4}} - \left(\frac{1}{32}-\frac{i}{32}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right) - \left(\frac{1}{32}-\frac{i}{32}\right)\tanh^{-}$$

$$= -\frac{x\left(1-x^2\right)}{16\sqrt{-1+x^4}} - \frac{x\left(1+x^2\right)}{16\sqrt{-1+x^4}} - \left(\frac{1}{32}-\frac{i}{32}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right) - \left(\frac{1}{32}-\frac{i}{32}\right)\tanh^{-}$$

$$= -\frac{x\left(1-x^2\right)}{16\sqrt{-1+x^4}} - \frac{x\left(1+x^2\right)}{16\sqrt{-1+x^4}} - \left(\frac{1}{32}-\frac{i}{32}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right) - \left(\frac{1}{32}-\frac{i}{32}\right)\tanh^{-}$$

Mathematica [C] time = 0.67, size = 161, normalized size = 0.94

$$-\frac{\sqrt{1-x^4}\,F\left(\sin^{-1}(x)|-1\right)+\sqrt{1-x^4}\,\Pi\left(-i;\sin^{-1}(x)|-1\right)+\sqrt{1-x^4}\,\Pi\left(i;\sin^{-1}(x)|-1\right)-\sqrt{1-x^4}\,\Pi\left(-\sqrt[4]{-1};\sin^{-1}(x)|-1\right)-\sqrt{1-x^4}\,\Pi\left(\sqrt[4]{-1};\sin^{-1}(x)|-1\right)-\sqrt{1-x^4}\,\Pi\left(-(-1)^{3/4};\sin^{-1}(x)|-1\right)-\sqrt{1-x^4}\,\Pi\left((-1)^{3/4};\sin^{-1}(x)|-1\right)+x}{8\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(Sqrt[-1 + x^4]*(-1 + x^16)),x]
```

```
[Out] -1/8*(x + Sqrt[1 - x^4]*EllipticF[ArcSin[x], -1] + Sqrt[1 - x^4]*EllipticPi
[-I, ArcSin[x], -1] + Sqrt[1 - x^4]*EllipticPi[I, ArcSin[x], -1] - Sqrt[1 -
 x^4]*EllipticPi[-(-1)^(1/4), ArcSin[x], -1] - Sqrt[1 - x^4]*EllipticPi[(-1
)^(1/4), ArcSin[x], -1] - Sqrt[1 - x^4]*EllipticPi[-(-1)^(3/4), ArcSin[x],
-1] - Sqrt[1 - x^4]*EllipticPi[(-1)^(3/4), ArcSin[x], -1])/Sqrt[-1 + x^4]
```

IntegrateAlgebraic [C] time = 0.81, size = 153, normalized size = 0.89

$$-\frac{x}{8\sqrt{x^4-1}} - \left(\frac{1}{32}-\frac{i}{32}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right) + \left(\frac{1}{32}+\frac{i}{32}\right)\tan^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^4-1}}{x}\right) - \frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt{x^4-1}}{-x^4+\sqrt{2}x^2+1}\right)}{8\,2^{3/4}} + \frac{\tanh^{-1}\left(\frac{2^{3/4}x\sqrt{x^4-1}}{x^4+\sqrt{2}x^2-1}\right)}{8\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/(Sqrt[-1 + x^4]*(-1 + x^16)),x]
```

[Out] -1/8*x/Sqrt[-1 + x^4] - (1/32 - I/32)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] + (1/32 + I/32)*ArcTan[((1/2 + I/2)*Sqrt[-1 + x^4])/x] - ArcTan[(2^(3/4)*x*Sqrt[-1 + x^4])/(1 + Sqrt[2]*x^2 - x^4)]/(8*2^(3/4)) + ArcTanh[(2^(3/4)*x*Sqrt[-1 + x^4])/(-1 + Sqrt[2]*x^2 + x^4)]/(8*2^(3/4))

fricas [B] time = 1.19, size = 740, normalized size = 4.33

$$\frac{4 \cdot 2^{\frac{1}{4}} (x^4-1) \arctan\left(\frac{2 x^{16}+4 x^8+\sqrt{2}\left(2^{\frac{3}{4}}(x^{16}-20 x^{12}+34 x^8-20 x^4+1)+8(x^{11}+x^3+4\sqrt{2}(x^9-x^5))\sqrt{x^4-1}+4 \cdot 2^{\frac{1}{4}}(x^{14}-9 x^{10}+9 x^6-x^2)\right)\sqrt{\frac{8 x^6-8 x^2+\sqrt{2}(x^8+1)+4\sqrt{x^4-1}\left(2^{\frac{3}{4}} x^3+2^{\frac{1}{4}}(x^5-x)\right)}{x^8+1}}+8\sqrt{2}(x^{14}-x^{10}+x^6-x^2)+4\sqrt{x^4-1}\left(2^{\frac{3}{4}}(x^{13}-9 x^9+9 x^5-x)+2 \cdot 2^{\frac{1}{4}}(3 x^{11}-8 x^7+3 x^3)\right)+2}{2(x^{16}-32 x^{12}+66 x^8-32 x^4+1)}\right) - 4 \cdot 2^{\frac{1}{4}} (x^4-1) \arctan\left(\frac{2 x^{16}+4 x^8-\sqrt{2}\left(2^{\frac{3}{4}}(x^{16}-20 x^{12}+34 x^8-20 x^4+1)-8(x^{11}+x^3+4\sqrt{2}(x^9-x^5))\sqrt{x^4-1}+4 \cdot 2^{\frac{1}{4}}(x^{14}-9 x^{10}+9 x^6-x^2)\right)\sqrt{\frac{8 x^6-8 x^2+\sqrt{2}(x^8+1)-4\sqrt{x^4-1}\left(2^{\frac{3}{4}} x^3+2^{\frac{1}{4}}(x^5-x)\right)}{x^8+1}}+8\sqrt{2}(x^{14}-x^{10}+x^6-x^2)-4\sqrt{x^4-1}\left(2^{\frac{3}{4}}(x^{13}-9 x^9+9 x^5-x)+2 \cdot 2^{\frac{1}{4}}(3 x^{11}-8 x^7+3 x^3)\right)+2}{2(x^{16}-32 x^{12}+66 x^8-32 x^4+1)}\right) + 2^{\frac{1}{4}}(x^4-1)\log\left(\frac{8\left(8 x^6-8 x^2+\sqrt{2}(x^8+1)+4\sqrt{x^4-1}\left(2^{\frac{3}{4}} x^3+2^{\frac{1}{4}}(x^5-x)\right)\right)}{x^8+1}\right) - 2^{\frac{1}{4}}(x^4-1)\log\left(\frac{8\left(8 x^6-8 x^2+\sqrt{2}(x^8+1)-4\sqrt{x^4-1}\left(2^{\frac{3}{4}} x^3+2^{\frac{1}{4}}(x^5-x)\right)\right)}{x^8+1}\right) + 4(x^4-1)\arctan\left(\frac{\sqrt{x^4-1}\,x}{x^2+1}\right) + 2(x^4-1)\log\left(\frac{x^4+2 x^2-2\sqrt{x^4-1}\,x-1}{x^4+1}\right) - 8\sqrt{x^4-1}\,x}{64(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^4-1)^(1/2)/(x^16-1),x, algorithm="fricas")

[Out] 1/64*(4*2^(1/4)*(x^4 - 1)*arctan(1/2*(2*x^16 + 4*x^8 + sqrt(2)*(2^(3/4)*(x^16 - 20*x^12 + 34*x^8 - 20*x^4 + 1) + 8*(x^11 + x^3 + 4*sqrt(2)*(x^9 - x^5))*sqrt(x^4 - 1) + 4*2^(1/4)*(x^14 - 9*x^10 + 9*x^6 - x^2))*sqrt((8*x^6 - 8*x^2 + sqrt(2)*(x^8 + 1) + 4*sqrt(x^4 - 1)*(2^(3/4)*x^3 + 2^(1/4)*(x^5 - x)))/(x^8 + 1)) + 8*sqrt(2)*(x^14 - x^10 + x^6 - x^2) + 4*sqrt(x^4 - 1)*(2^(3/4)*(x^13 - 9*x^9 + 9*x^5 - x) + 2*2^(1/4)*(3*x^11 - 8*x^7 + 3*x^3)) + 2)/(x^16 - 32*x^12 + 66*x^8 - 32*x^4 + 1)) - 4*2^(1/4)*(x^4 - 1)*arctan(1/2*(2*x^16 + 4*x^8 - sqrt(2)*(2^(3/4)*(x^16 - 20*x^12 + 34*x^8 - 20*x^4 + 1) - 8*(x^11 + x^3 + 4*sqrt(2)*(x^9 - x^5))*sqrt(x^4 - 1) + 4*2^(1/4)*(x^14 - 9*x^10 + 9*x^6 - x^2))*sqrt((8*x^6 - 8*x^2 + sqrt(2)*(x^8 + 1) - 4*sqrt(x^4 - 1)*(2^(3/4)*x^3 + 2^(1/4)*(x^5 - x)))/(x^8 + 1)) + 8*sqrt(2)*(x^14 - x^10 + x^6 - x^2) - 4*sqrt(x^4 - 1)*(2^(3/4)*(x^13 - 9*x^9 + 9*x^5 - x) + 2*2^(1/4)*(3*x^11 - 8*x^7 + 3*x^3)) + 2)/(x^16 - 32*x^12 + 66*x^8 - 32*x^4 + 1)) + 2^(1/4)*(x^4 - 1)*log(8*(8*x^6 - 8*x^2 + sqrt(2)*(x^8 + 1) + 4*sqrt(x^4 - 1)*(2^(3/4)*x^3 + 2^(1/4)*(x^5 - x)))/(x^8 + 1)) - 2^(1/4)*(x^4 - 1)*log(8*(8*x^6 - 8*x^2 + sqrt(2)*(x^8 + 1) - 4*sqrt(x^4 - 1)*(2^(3/4)*x^3 + 2^(1/4)*(x^5 - x)))/(x^8 + 1)) + 4*(x^4 - 1)*arctan(sqrt(x^4 - 1)*x/(x^2 + 1)) + 2*(x^4 - 1)*log((x^4 + 2*x^2 - 2*sqrt(x^4 - 1)*x - 1)/(x^4 + 1)) - 8*sqrt(x^4 - 1)*x)/(x^4 - 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(x^{16}-1\right)\sqrt{x^4-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^4-1)^(1/2)/(x^16-1),x, algorithm="giac")

[Out] integrate(x^8/((x^16 - 1)*sqrt(x^4 - 1)), x)

maple [C] time = 0.08, size = 281, normalized size = 1.64

$$\frac{i\left(\sum_{\_\alpha=\mathit{RootOf}(\_Z^4+1)}\_\alpha\left(-\sqrt{2}\,\mathrm{arctanh}\left(\frac{\_\alpha^2(\_\alpha^2+x^2)\sqrt{-2}}{2\sqrt{x^4-1}}\right)-\frac{4\_\alpha^3\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\mathrm{EllipticPi}(ix,\_\alpha^2,i)}{\sqrt{x^4-1}}\right)\right)}{64}-\frac{x^3+x^2+x+1}{32\sqrt{(-1+x)(x^3+x^2+x+1)}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\mathrm{EllipticF}(ix,i)}{8\sqrt{x^4-1}}-\frac{x^3-x^2+x-1}{32\sqrt{(1+x)(x^3-x^2+x-1)}}+\frac{(x^2-1)x}{16\sqrt{(x^2-1)(x^2+1)}}-\frac{\sum_{\_\alpha=\mathit{RootOf}(\_Z^8+1)}\_\alpha\left(-\frac{\mathrm{arctanh}\left(\frac{\_\alpha^2(\_\alpha^6+x^2)}{\sqrt{\_\alpha^4-1}\,\sqrt{x^4-1}}\right)}{\sqrt{\_\alpha^4-1}}-\frac{2i\_\alpha^7\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\mathrm{EllipticPi}(ix,\_\alpha^6,i)}{\sqrt{x^4-1}}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(x^4-1)^(1/2)/(x^16-1),x)

[Out] 1/64*I*sum(_alpha*(-2^(1/2)*arctanh(1/2*_alpha^2*(_alpha^2+x^2)*(-2)^(1/2)/(x^4-1)^(1/2))-4*_alpha^3*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*EllipticPi(I*x,_alpha^2,I)),_alpha=RootOf(_Z^4+1))-1/32*(x^3+x^2+x+1)/((-1+x)*(x^3+x^2+x+1))^(1/2)+1/8*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*EllipticF(I*x,I)-1/32*(x^3-x^2+x-1)/((1+x)*(x^3-x^2+x-1))^(1/2)+1/16*(x^2-1)*x/((x^2-1)*(x^2+1))^(1/2)-1/32*sum(_alpha*(-1/(_alpha^4-1)^(1/2)*arctanh(_alpha^2*(_alpha^6+x^2)/(_alpha^4-1)^(1/2)/(x^4-1)^(1/2))-2*I*_alpha^7*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*EllipticPi(I*x,_alpha^6,I)),_alpha=RootOf(_Z^8+1))

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(x^{16}-1)\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^4-1)^(1/2)/(x^16-1),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((x^16 - 1)*sqrt(x^4 - 1)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\sqrt{x^4-1}\,\left(x^{16}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((x^4 - 1)^(1/2)*(x^16 - 1)),x)
```

```
[Out] int(x^8/((x^4 - 1)^(1/2)*(x^16 - 1)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**4-1)**(1/2)/(x**16-1),x)
```

```
[Out] Timed out
```

3.1864 $\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^4}\,dx$

Optimal. Leaf size=171

$$\frac{11x\left(315a^2x^4+798ab^2x^2+611b^4\right)}{7680b^6\left(ax^2+b^2\right)^{5/2}\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{x\left(1155a^2x^4+3102ab^2x^2+2587b^4\right)}{3840b^5\left(ax^2+b^2\right)^3\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{231\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\,\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{512\sqrt{a}\,b^{13/2}}$$

Rubi [F] time = 1.97, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^4,x]
```

```
[Out] (5*Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b - Sqrt[-a]*x), x])/(32*b^7) +
(5*Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b + Sqrt[-a]*x), x])/(32*b^7) +
(a^2*Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(Sqrt[-a]*b - a*x)^4, x])/(16*b
^4) + ((-a)^(3/2)*Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(Sqrt[-a]*b - a*x)
^3, x])/(8*b^5) - (5*a*Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(Sqrt[-a]*b -
 a*x)^2, x])/(32*b^6) + (a^2*Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(Sqrt[-
a]*b + a*x)^4, x])/(16*b^4) + ((-a)^(3/2)*Defer[Int][Sqrt[b + Sqrt[b^2 + a*
x^2]]/(Sqrt[-a]*b + a*x)^3, x])/(8*b^5) - (5*a*Defer[Int][Sqrt[b + Sqrt[b^2
 + a*x^2]]/(Sqrt[-a]*b + a*x)^2, x])/(32*b^6)
```

Rubi steps

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^4}\,dx=\int \left(\frac{a^2\sqrt{b+\sqrt{b^2+ax^2}}}{16b^4\left(\sqrt{-a}\,b-ax\right)^4}+\frac{a^4\sqrt{b+\sqrt{b^2+ax^2}}}{8(-a)^{5/2}b^5\left(\sqrt{-a}\,b-ax\right)^3}-\frac{5a\sqrt{b+\sqrt{b^2+ax^2}}}{32b^6\left(\sqrt{-a}\,b-ax\right)^2}+\frac{a^2\sqrt{b}}{16b^4(}\right.$$

$$=-\frac{(5a)\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(\sqrt{-a}\,b-ax\right)^2}\,dx}{32b^6}-\frac{(5a)\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(\sqrt{-a}\,b+ax\right)^2}\,dx}{32b^6}-\frac{(5a)\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{-ab^2-a^2x^2}\,dx}{16b^6}+\frac{(-a)^{3/2}}{}$$

$$=-\frac{(5a)\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(\sqrt{-a}\,b-ax\right)^2}\,dx}{32b^6}-\frac{(5a)\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(\sqrt{-a}\,b+ax\right)^2}\,dx}{32b^6}-\frac{(5a)\int \left(-\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{2ab\left(b-\sqrt{-a}\,x\right)}-\frac{\sqrt{b+\sqrt{b^2+}}}{2ab\left(b+\sqrt{-}\right.}\right.}{16b^6}$$

$$=\frac{5\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b-\sqrt{-a}\,x}\,dx}{32b^7}+\frac{5\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b+\sqrt{-a}\,x}\,dx}{32b^7}-\frac{(5a)\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(\sqrt{-a}\,b-ax\right)^2}\,dx}{32b^6}-\frac{(5a)\int \frac{\sqrt{b+\sqrt{b^2+}}}{\left(\sqrt{-a}\,b+a\right.}}{32b^6}$$

Mathematica [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\left(b^2+ax^2\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^4,x]
```

```
[Out] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^4, x]
```

IntegrateAlgebraic [A] time = 0.39, size = 171, normalized size = 1.00

$$\frac{11x\left(315a^2x^4+798ab^2x^2+611b^4\right)}{7680b^6\left(ax^2+b^2\right)^{5/2}\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{x\left(1155a^2x^4+3102ab^2x^2+2587b^4\right)}{3840b^5\left(ax^2+b^2\right)^3\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{231\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{512\sqrt{a}\,b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + Sqrt[b^2 + a*x^2]]/(b^2 + a*x^2)^4,x]
```

```
[Out] (11*x*(611*b^4 + 798*a*b^2*x^2 + 315*a^2*x^4))/(7680*b^6*(b^2 + a*x^2)^(5/2
)*Sqrt[b + Sqrt[b^2 + a*x^2]]) + (x*(2587*b^4 + 3102*a*b^2*x^2 + 1155*a^2*x
^4))/(3840*b^5*(b^2 + a*x^2)^3*Sqrt[b + Sqrt[b^2 + a*x^2]]) + (231*ArcTan[(
Sqrt[a]*x)/(Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/(512*Sqrt[a]*b^(13/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\left(ax^2+b^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 + b^2)^4, x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{\left(a\,x^2+b^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^4,x)
```

```
[Out] int((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\left(ax^2+b^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2+b^2)^4,x, algorithm="maxima")
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 + b^2)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b+\sqrt{b^2+a\,x^2}}}{\left(b^2+a\,x^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(a*x^2 + b^2)^4,x)
[Out] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(a*x^2 + b^2)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\left(ax^2+b^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x**2+b**2)**(1/2))**(1/2)/(a*x**2+b**2)**4,x)
[Out] Integral(sqrt(b + sqrt(a*x**2 + b**2))/(a*x**2 + b**2)**4, x)
```

3.1865 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx$

Optimal. Leaf size=171

$$\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)+\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2(\cdots}}{}\right.$$

Rubi [F] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/(1 + x^2),x]
```

```
[Out] (I/2)*Defer[Int][Sqrt[x^2 + Sqrt[1 + x^4]]/(I - x), x] + (I/2)*Defer[Int][S
qrt[x^2 + Sqrt[1 + x^4]]/(I + x), x]
```

Rubi steps

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx=\int \left(\frac{i\sqrt{x^2+\sqrt{1+x^4}}}{2(i-x)}+\frac{i\sqrt{x^2+\sqrt{1+x^4}}}{2(i+x)}\right)dx$$
$$=\frac{1}{2}i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{i-x}\,dx+\frac{1}{2}i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{i+x}\,dx$$

Mathematica [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/(1 + x^2),x]
```

```
[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/(1 + x^2), x]
```

IntegrateAlgebraic [A] time = 0.83, size = 242, normalized size = 1.42

$$\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\,x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)+\sqrt{2}\,\tanh^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\,x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2 + Sqrt[1 + x^4]]/(1 + x^2),x]
```

```
[Out] Sqrt[-1 + Sqrt[2]]*ArcTan[(-Sqrt[1/2 + 1/Sqrt[2]] + Sqrt[1/2 + 1/Sqrt[2]]*x
^2 + Sqrt[1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])] +
Sqrt[2]*ArcTanh[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2])/(x*Sqr
t[x^2 + Sqrt[1 + x^4]])] - Sqrt[1 + Sqrt[2]]*ArcTanh[(-Sqrt[-1/2 + 1/Sqrt[2
```

```
]] + Sqrt[-1/2 + 1/Sqrt[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*
Sqrt[x^2 + Sqrt[1 + x^4]])]
```

fricas [B] time = 1.67, size = 333, normalized size = 1.95

$$\sqrt{\sqrt{2}-1}\arctan\left(\frac{\left(\sqrt{2}x^2+x^2+\sqrt{x^4+1}\left(\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+3}-\sqrt{2}-1\right)-\left(x^2+\sqrt{2}\left(x^2+2\right)+3\right)\sqrt{-2\sqrt{2}+3}+1\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}-1}}{2x}\right)+\frac{1}{4}\sqrt{2}\log\left(4x^4+4\sqrt{x^4+1}x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)-\frac{1}{4}\sqrt{\sqrt{2}+1}\log\left(\frac{\sqrt{2}x^2+2x^2+\left(x^3+\sqrt{2}x-\sqrt{x^4+1}x+x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}+1}+\sqrt{x^4+1}\left(\sqrt{2}+1\right)}{x^2+1}\right)+\frac{1}{4}\sqrt{\sqrt{2}+1}\log\left(\frac{\sqrt{2}x^2+2x^2-\left(x^3+\sqrt{2}x-\sqrt{x^4+1}x+x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}+1}+\sqrt{x^4+1}\left(\sqrt{2}+1\right)}{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] sqrt(sqrt(2) - 1)*arctan(1/2*(sqrt(2)*x^2 + x^2 + sqrt(x^4 + 1)*((sqrt(2) +
 1)*sqrt(-2*sqrt(2) + 3) - sqrt(2) - 1) - (x^2 + sqrt(2)*(x^2 + 2) + 3)*sqr
t(-2*sqrt(2) + 3) + 1)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(sqrt(2) - 1)/x) + 1/4
*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^
4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1) - 1/4*sqrt(sqrt(2) + 1)*log((sqrt(
2)*x^2 + 2*x^2 + (x^3 + sqrt(2)*x - sqrt(x^4 + 1)*x + x)*sqrt(x^2 + sqrt(x^
4 + 1))*sqrt(sqrt(2) + 1) + sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 + 1)) + 1/4*s
qrt(sqrt(2) + 1)*log((sqrt(2)*x^2 + 2*x^2 - (x^3 + sqrt(2)*x - sqrt(x^4 + 1
)*x + x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(sqrt(2) + 1) + sqrt(x^4 + 1)*(sqrt(
2) + 1))/(x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(x^2 + 1), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^2 + 1),x)
```

```
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{x^2 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/(x**2 + 1), x)
```

3.1866 $\int \frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}}}{x} \, dx$

Optimal. Leaf size=172

$$-4\sqrt{1-\sqrt{1-\sqrt{\frac{x^2-1}{x^2}}}}+\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{\sqrt{1-\sqrt{1-\sqrt{\frac{x^2-1}{x^2}}}}}{\sqrt{\sqrt{2}-1}}\right)+2\tanh^{-1}\left(\sqrt{1-\sqrt{1-\sqrt{\frac{x^2-1}{x^2}}}}\right)+\sqrt{1+\sqrt{}}$$

Rubi [A] time = 1.18, antiderivative size = 164, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {6742, 2073, 207, 1166, 203}

$$-4\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}}+\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}}}{\sqrt{\sqrt{2}-1}}\right)+2\tanh^{-1}\left(\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}}\right)+\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}}}{\sqrt{1+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-2)]]]/x,x]
```

```
[Out] -4*Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-2)]]] + Sqrt[-1 + Sqrt[2]]*ArcTan[Sqrt[1
- Sqrt[1 - Sqrt[1 - x^(-2)]]]/Sqrt[-1 + Sqrt[2]]] + 2*ArcTanh[Sqrt[1 - Sqrt
[1 - Sqrt[1 - x^(-2)]]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[1 - Sqrt[1 - Sqrt
[1 - x^(-2)]]]/Sqrt[1 + Sqrt[2]]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}}}{x}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-\sqrt{1-\sqrt{1-x}}}}{x}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \text{Subst}\left(\int \frac{\sqrt{1-\sqrt{1-x}}\,x}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)$$

$$= -\left(2\,\text{Subst}\left(\int \frac{\sqrt{1-x}\,(-1+x^2)}{x(-2+x^2)}\,dx, x, \sqrt{1-\sqrt{1-\frac{1}{x^2}}}\right)\right)$$

$$= -\left(4\,\text{Subst}\left(\int \frac{x^4(-2+x^2)}{1+x^2-3x^4+x^6}\,dx, x, \sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}}\right)\right)$$

$$= -\left(4\,\text{Subst}\left(\int \left(1-\frac{1+x^2-x^4}{1+x^2-3x^4+x^6}\right)dx, x, \sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}}\right)\right)$$

$$= -4\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}} + 4\,\text{Subst}\left(\int \frac{1+x^2-x^4}{1+x^2-3x^4+x^6}\,dx, x, \sqrt{1-\sqrt{1-\sqrt{1-\right.$$

$$= -4\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}} + 4\,\text{Subst}\left(\int \left(-\frac{1}{2(-1+x^2)} + \frac{-1-x^2}{2(-1-2x^2+x^4)}\right)dx, x\right.$$

$$= -4\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}} - 2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}}\right) + 2$$

$$= -4\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}} + 2\tanh^{-1}\left(\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}}\right) + \left(-1-\sqrt{2}\right)\text{Subst}$$

$$= -4\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}} + \sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}}}{\sqrt{-1+\sqrt{2}}}\right) + 2\tanh^{-1}$$

Mathematica [A] time = 0.32, size = 164, normalized size = 0.95

$$-4\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}} + \sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}}}{\sqrt{\sqrt{2}-1}}\right) + 2\tanh^{-1}\left(\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}}\right) + \sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}}}{\sqrt{1+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-2)]]]/x,x]
```

```
[Out] -4*Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-2)]]] + Sqrt[-1 + Sqrt[2]]*ArcTan[Sqrt[1
 - Sqrt[1 - Sqrt[1 - x^(-2)]]]/Sqrt[-1 + Sqrt[2]]] + 2*ArcTanh[Sqrt[1 - Sqrt
[1 - Sqrt[1 - x^(-2)]]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[1 - Sqrt[1 - Sqrt
[1 - x^(-2)]]]/Sqrt[1 + Sqrt[2]]]
```

IntegrateAlgebraic [A] time = 1.24, size = 172, normalized size = 1.00

$$-4\sqrt{1-\sqrt{1-\sqrt{\frac{x^2-1}{x^2}}}}+\sqrt{\sqrt{2}-1}\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{1-\sqrt{1-\sqrt{\frac{x^2-1}{x^2}}}}\right)+2\tanh^{-1}\left(\sqrt{1-\sqrt{1-\sqrt{\frac{x^2-1}{x^2}}}}\right)+\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{1-\sqrt{1-\sqrt{\frac{x^2-1}{x^2}}}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-2)]]]/x,x]
```

```
[Out] -4*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x^2)/x^2]]] + Sqrt[-1 + Sqrt[2]]*ArcTan[Sqr
t[1 + Sqrt[2]]*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x^2)/x^2]]]] + 2*ArcTanh[Sqrt[1
 - Sqrt[1 - Sqrt[(-1 + x^2)/x^2]]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[-1 + S
qrt[2]]*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x^2)/x^2]]]]
```

fricas [B] time = 151.93, size = 1037, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(1-(1-1/x^2)^(1/2))^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(2) - 1)*arctan(-1/45218*(2*sqrt(2)*(21008*x^4 + 608*x^2 + sqrt(2
)*(15192*x^4 - 163*x^2) + (32368*x^4 + 248*x^2 + sqrt(2)*(22856*x^4 + 231*x
^2))*sqrt((x^2 - 1)/x^2))*sqrt(141401*sqrt(2) - 198689)*sqrt(sqrt(2) - 1)*s
qrt(-sqrt((x^2 - 1)/x^2) + 1) + sqrt(2)*(68704*x^4 - 76436*x^2 + sqrt(2)*(4
9408*x^4 - 55516*x^2 - 479) + 4*(9512*x^4 + 17*x^2 + sqrt(2)*(6672*x^4 + 10
7*x^2))*sqrt((x^2 - 1)/x^2) - 710)*sqrt(141401*sqrt(2) - 198689)*sqrt(sqrt(
2) - 1) - 90436*((40*x^4 - 5*x^2 + 4*sqrt(2)*(6*x^4 + x^2) + (40*sqrt(2)*x^
4 + 56*x^4 + x^2)*sqrt((x^2 - 1)/x^2))*sqrt(sqrt(2) - 1)*sqrt(-sqrt((x^2 -
1)/x^2) + 1) + (32*x^4 - 18*x^2 + sqrt(2)*(8*x^4 + 13*x^2) + (32*x^4 + 2*x^
2 + sqrt(2)*(24*x^4 - x^2))*sqrt((x^2 - 1)/x^2))*sqrt(sqrt(2) - 1))*sqrt(-s
qrt(-sqrt((x^2 - 1)/x^2) + 1) + 1))/(64*x^4 - 112*x^2 - 1)) + 1/4*sqrt(sqrt
(2) + 1)*log(4*(479*sqrt(2)*x^2 + 710*x^2 - (479*sqrt(2)*x^2 + 710*x^2)*sqr
t((x^2 - 1)/x^2))*sqrt(sqrt(2) + 1)*sqrt(-sqrt((x^2 - 1)/x^2) + 1) - 2*(191
6*x^2 + sqrt(2)*(1420*x^2 - 231) - 4*(355*sqrt(2)*x^2 + 479*x^2)*sqrt((x^2
- 1)/x^2) - 248)*sqrt(sqrt(2) + 1) - 4*(710*sqrt(2)*x^2 + 958*x^2 - (479*sq
rt(2)*x^2 + 710*x^2 - (479*sqrt(2)*x^2 + 710*x^2)*sqrt((x^2 - 1)/x^2))*sqrt
(-sqrt((x^2 - 1)/x^2) + 1) - 2*(355*sqrt(2)*x^2 + 479*x^2)*sqrt((x^2 - 1)/x
^2))*sqrt(-sqrt(-sqrt((x^2 - 1)/x^2) + 1) + 1)) - 1/4*sqrt(sqrt(2) + 1)*log
(-4*(479*sqrt(2)*x^2 + 710*x^2 - (479*sqrt(2)*x^2 + 710*x^2)*sqrt((x^2 - 1)
/x^2))*sqrt(sqrt(2) + 1)*sqrt(-sqrt((x^2 - 1)/x^2) + 1) + 2*(1916*x^2 + sqr
t(2)*(1420*x^2 - 231) - 4*(355*sqrt(2)*x^2 + 479*x^2)*sqrt((x^2 - 1)/x^2) -
 248)*sqrt(sqrt(2) + 1) - 4*(710*sqrt(2)*x^2 + 958*x^2 - (479*sqrt(2)*x^2 +
 710*x^2 - (479*sqrt(2)*x^2 + 710*x^2)*sqrt((x^2 - 1)/x^2))*sqrt(-sqrt((x^2
 - 1)/x^2) + 1) - 2*(355*sqrt(2)*x^2 + 479*x^2)*sqrt((x^2 - 1)/x^2))*sqrt(-
sqrt(-sqrt((x^2 - 1)/x^2) + 1) + 1)) - 4*sqrt(-sqrt(-sqrt((x^2 - 1)/x^2) +
1) + 1) + log(-2*(x^2*sqrt((x^2 - 1)/x^2) + x^2)*sqrt(-sqrt(-sqrt((x^2 - 1)
/x^2) + 1) + 1)*sqrt(-sqrt((x^2 - 1)/x^2) + 1) - 2*(x^2*sqrt((x^2 - 1)/x^2)
 + x^2)*sqrt(-sqrt((x^2 - 1)/x^2) + 1) + 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(1-(1-1/x^2)^(1/2))^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^8
-4*x^6+4*x^4-4*x^2)]Error index.cc index_gcd Error: Bad Argument ValueError
```

```
index.cc index_gcd Error: Bad Argument ValueError index.cc index_gcd Error
: Bad Argument ValueError index.cc index_gcd Error: Bad Argument ValueDisco
ntinuities at zeroes of x^8-4*x^6+4*x^4-4*x^2 were not checkedEvaluation ti
me: 2.24Done
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(1-(1-1/x^2)^(1/2))^(1/2))^(1/2)/x,x)
[Out] int((1-(1-(1-1/x^2)^(1/2))^(1/2))^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\sqrt{-\sqrt{-\frac{1}{x^2}+1}+1}+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(1-(1-1/x^2)^(1/2))^(1/2))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(-sqrt(-sqrt(-1/x^2 + 1) + 1) + 1)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - (1 - (1 - 1/x^2)^(1/2))^(1/2))^(1/2)/x,x)
[Out] int((1 - (1 - (1 - 1/x^2)^(1/2))^(1/2))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x^2}}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(1-(1-1/x**2)**(1/2))**(1/2))**(1/2)/x,x)
[Out] Integral(sqrt(1 - sqrt(1 - sqrt(1 - 1/x**2)))/x, x)
```

3.1867 $\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=172

$$\frac{\log\left(2\sqrt[3]{1-x^3}+\sqrt[3]{2}x-\sqrt[3]{2}\right)}{2\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{1-x^3}-\sqrt[3]{2}x+\sqrt[3]{2}}\right)}{2\sqrt[3]{2}}-\frac{\log\left(4\left(1-x^3\right)^{2/3}+\left(2\sqrt[3]{2}-2\sqrt[3]{2}x\right)\sqrt[3]{1-x^3}+2\cdots\right.}{4\sqrt[3]{2}}$$

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 0.56, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2148}

$$\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(1
/3) - Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + (3*Log[-1 + x + 2^(2/3)*(1 - x^3
)^(1/3)])/(4*2^(1/3))
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}}+\frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((1 + x)*(1 - x^3)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.99, size = 172, normalized size = 1.00

$$\frac{\log\left(2\sqrt[3]{1-x^3}+\sqrt[3]{2}x-\sqrt[3]{2}\right)}{2\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{1-x^3}-\sqrt[3]{2}x+\sqrt[3]{2}}\right)}{2\sqrt[3]{2}}-\frac{\log\left(4\left(1-x^3\right)^{2/3}+\left(2\sqrt[3]{2}-2\sqrt[3]{2}x\right)\sqrt[3]{1-x^3}+2^{2/3}x^2-2\,2^{2/3}x+2^{2/3}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1 - x^3)^
(1/3))])/(2*2^(1/3)) + Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)]/(2*2^(
1/3)) - Log[2^(2/3) - 2*2^(2/3)*x + 2^(2/3)*x^2 + (2*2^(1/3) - 2*2^(1/3)*x)
*(1 - x^3)^(1/3) + 4*(1 - x^3)^(2/3)]/(4*2^(1/3))
```

fricas [B] time = 3.45, size = 301, normalized size = 1.75

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}\left(13x^{6}+2x^{5}+19x^{4}-4x^{3}+19x^{2}+2x+13\right)-4\sqrt{2}\left(5x^{5}-5x^{4}+6x^{3}-6x^{2}+5x-5\right)\left(-x^{3}+1\right)^{\frac{1}{3}}+16\cdot2^{\frac{1}{6}}\left(x^{4}+2x^{3}+2x^{2}+2x+1\right)\left(-x^{3}+1\right)^{\frac{2}{3}}\right)}{6\left(3x^{6}-18x^{5}-3x^{4}-28x^{3}-3x^{2}-18x+3\right)}\right)-\frac{1}{24}\cdot2^{\frac{2}{3}}\log\left(\frac{4\cdot2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{2}{3}}\left(x^{2}+1\right)+2^{\frac{1}{3}}\left(5x^{4}+6x^{2}+5\right)-2\left(3x^{3}-x^{2}+x-3\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{x^{4}+4x^{3}+6x^{2}+4x+1}\right)+\frac{1}{12}\cdot2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}\left(x^{2}+2x+1\right)-2\cdot2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\left(x-1\right)-4\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}+2x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(13*x^6 + 2*x^5 +
19*x^4 - 4*x^3 + 19*x^2 + 2*x + 13) - 4*sqrt(2)*(5*x^5 - 5*x^4 + 6*x^3 - 6*
x^2 + 5*x - 5)*(-x^3 + 1)^(1/3) + 16*2^(1/6)*(x^4 + 2*x^3 + 2*x^2 + 2*x + 1
)*(-x^3 + 1)^(2/3))/(3*x^6 - 18*x^5 - 3*x^4 - 28*x^3 - 3*x^2 - 18*x + 3)) -
 1/24*2^(2/3)*log((4*2^(2/3)*(-x^3 + 1)^(2/3)*(x^2 + 1) + 2^(1/3)*(5*x^4 +
6*x^2 + 5) - 2*(3*x^3 - x^2 + x - 3)*(-x^3 + 1)^(1/3))/(x^4 + 4*x^3 + 6*x^2
 + 4*x + 1)) + 1/12*2^(2/3)*log((2^(2/3)*(x^2 + 2*x + 1) - 2*2^(1/3)*(-x^3
+ 1)^(1/3)*(x - 1) - 4*(-x^3 + 1)^(2/3))/(x^2 + 2*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^{3}+1\right)^{\frac{1}{3}}\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

maple [C] time = 8.11, size = 1162, normalized size = 6.76

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(-x^3+1)^(1/3),x)
```

```
[Out] 1/2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*ln((4*RootOf(RootOf
(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x-12*RootOf(RootOf(
_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x+8*(-x^3+1)^(2/3)
*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2+9*(-x
^3+1)^(1/3)*RootOf(_Z^3-4)^2*x-8*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_
Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x-9*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2
+8*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootO
f(_Z^3-4)-14*RootOf(_Z^3-4)*x^2+42*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3
-4)+4*_Z^2)*x^2-4*RootOf(_Z^3-4)*x+12*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_
Z^3-4)+4*_Z^2)*x-36*(-x^3+1)^(2/3)-14*RootOf(_Z^3-4)+42*RootOf(RootOf(_Z^3-
4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(1+x)^2)-1/4*ln(-(10*RootOf(RootOf(_Z^3-4
)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x+12*RootOf(RootOf(_Z^3-4)
^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x+8*(-x^3+1)^(2/3)*RootOf
(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2-13*(-x^3+1)^
(1/3)*RootOf(_Z^3-4)^2*x-8*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*Root
Of(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x+13*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2+8*(-
x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^
```

```
3-4)+35*RootOf(_Z^3-4)*x^2+42*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4
*_Z^2)*x^2+30*RootOf(_Z^3-4)*x+36*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-
4)+4*_Z^2)*x+52*(-x^3+1)^(2/3)+35*RootOf(_Z^3-4)+42*RootOf(RootOf(_Z^3-4)^2
+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(1+x)^2)*RootOf(_Z^3-4)-1/2*ln(-(10*RootOf(Ro
otOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x+12*RootOf(Roo
tOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x+8*(-x^3+1)^(
2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2-1
3*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x-8*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^
2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x+13*(-x^3+1)^(1/3)*RootOf(_Z^
3-4)^2+8*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)
*RootOf(_Z^3-4)+35*RootOf(_Z^3-4)*x^2+42*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootO
f(_Z^3-4)+4*_Z^2)*x^2+30*RootOf(_Z^3-4)*x+36*RootOf(RootOf(_Z^3-4)^2+2*_Z*R
ootOf(_Z^3-4)+4*_Z^2)*x+52*(-x^3+1)^(2/3)+35*RootOf(_Z^3-4)+42*RootOf(RootO
f(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(1+x)^2)*RootOf(RootOf(_Z^3-4)^2+2
*_Z*RootOf(_Z^3-4)+4*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(1-x^3\right)^{1/3}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(1/3)*(x + 1)),x)
[Out] int(1/((1 - x^3)^(1/3)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x**3+1)**(1/3),x)
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

3.1868 $\int \frac{(ab-2bx+x^2)(b^2-2bx+x^2)}{(x(-a+x)(-b+x)^3)^{3/4}(bd-(a+d)x+x^2)}\,dx$

Optimal. Leaf size=173

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(ab^3x+x^3(3ab+3b^2)+x^2(-3ab^2-b^3)+x^4(-a-3b)+x^5\right)^{3/4}}{x(x-a)(b-x)^2}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(ab^3x+x^3(3ab+3b^2)+x^2(-3ab^2-b^3)+x^4(-a-3b)+x^5\right)^{3/4}}{x(x-a)(b-x)^2}\right)}{d^{3/4}}$$

Rubi [F] time = 6.83, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ab-2bx+x^2)(b^2-2bx+x^2)}{(x(-a+x)(-b+x)^3)^{3/4}(bd-(a+d)x+x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a*b - 2*b*x + x^2)*(b^2 - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^3)^(3/4
)*(b*d - (a + d)*x + x^2)),x]
```

```
[Out] (4*(b - x)^2*x*(1 - x/a)^(3/4)*(1 - x/b)^(1/4)*AppellF1[1/4, 3/4, 1/4, 5/4,
 x/a, x/b])/((a - x)*(b - x)^3*x)^(3/4) + ((a - 2*b + d + Sqrt[a^2 + 2*a*d
- 4*b*d + d^2])*x^(3/4)*(-a + x)^(3/4)*(-b + x)^(9/4)*Defer[Int][1/(x^(3/4)
*(-a + x)^(3/4)*(-b + x)^(1/4)*(-a - d - Sqrt[a^2 + 2*a*d - 4*b*d + d^2] +
2*x)), x])/((a - x)*(b - x)^3*x)^(3/4) + ((a - 2*b + d - Sqrt[a^2 + 2*a*d -
 4*b*d + d^2])*x^(3/4)*(-a + x)^(3/4)*(-b + x)^(9/4)*Defer[Int][1/(x^(3/4)*
(-a + x)^(3/4)*(-b + x)^(1/4)*(-a - d + Sqrt[a^2 + 2*a*d - 4*b*d + d^2] + 2
*x)), x])/((a - x)*(b - x)^3*x)^(3/4)
```

Rubi steps

$$\int \frac{(ab-2bx+x^2)(b^2-2bx+x^2)}{(x(-a+x)(-b+x)^3)^{3/4}(bd-(a+d)x+x^2)}\,dx = \int \frac{(-b+x)^2(ab-2bx+x^2)}{(x(-a+x)(-b+x)^3)^{3/4}(bd-(a+d)x+x^2)}\,dx$$

$$= \frac{\left(x^{3/4}(-a+x)^{3/4}(-b+x)^{9/4}\right)\int \frac{ab-2bx+x^2}{x^{3/4}(-a+x)^{3/4}\sqrt[4]{-b+x}\,(bd-(a+d)x+}}{(x(-a+x)(-b+x)^3)^{3/4}}$$

$$= \frac{\left(x^{3/4}(-a+x)^{3/4}(-b+x)^{9/4}\right)\int \left(\frac{1}{x^{3/4}(-a+x)^{3/4}\sqrt[4]{-b+x}} + \frac{}{x^{3/4}(-a}\right.}{(x(-a+x)(-b+x)^3)^{3/4}}$$

$$= \frac{\left(x^{3/4}(-a+x)^{3/4}(-b+x)^{9/4}\right)\int \frac{1}{x^{3/4}(-a+x)^{3/4}\sqrt[4]{-b+x}}\,dx}{(x(-a+x)(-b+x)^3)^{3/4}} + \frac{(x^{3/}}{}$$

$$= \frac{\left(x^{3/4}(-a+x)^{3/4}(-b+x)^{9/4}\right)\int \left(\frac{a-2b+d+\sqrt{a^2+2ad-}}{x^{3/4}(-a+x)^{3/4}\sqrt[4]{-b+x}\left(-a-d-\sqrt{a^2}\right.}\right.}{(x(-a+}$$

$$= \frac{\left(\left(a-2b+d-\sqrt{a^2+2ad-4bd+d^2}\right)x^{3/4}(-a+x)^{3/4}(-l\right.}{(x(-a+x)(-}$$

$$= \frac{4(b-x)^2x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1-\frac{x}{b}}\,F_1\left(\frac{1}{4};\frac{3}{4},\frac{1}{4};\frac{5}{4};\frac{x}{a},\frac{x}{b}\right)}{((a-x)(b-x)^3x)^{3/4}} + \frac{((a-}{}$$

Mathematica [F] time = 5.82, size = 0, normalized size = 0.00

$$\int \frac{(ab-2bx+x^2)(b^2-2bx+x^2)}{(x(-a+x)(-b+x)^3)^{3/4}(bd-(a+d)x+x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a*b - 2*b*x + x^2)*(b^2 - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^3
)^(3/4)*(b*d - (a + d)*x + x^2)),x]
```

```
[Out] Integrate[((a*b - 2*b*x + x^2)*(b^2 - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^3
)^(3/4)*(b*d - (a + d)*x + x^2)), x]
```

IntegrateAlgebraic [A] time = 3.85, size = 173, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,(ab^3x+x^3(3ab+3b^2)+x^2(-3ab^2-b^3)+x^4(-a-3b)+x^5)^{3/4}}{x(x-a)(b-x)^2}\right)}{d^{3/4}} - \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,(ab^3x+x^3(3ab+3b^2)+x^2(-3ab^2-b^3)+x^4(-a-3b)+x^5)^{3/4}}{x(x-a)(b-x)^2}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a*b - 2*b*x + x^2)*(b^2 - 2*b*x + x^2))/((x*(-a + x)*(
-b + x)^3)^(3/4)*(b*d - (a + d)*x + x^2)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(a*b^3*x + (-3*a*b^2 - b^3)*x^2 + (3*a*b + 3*b^2)*x^3 +
(-a - 3*b)*x^4 + x^5)^(3/4))/((b - x)^2*x*(-a + x))])/d^(3/4) - (2*ArcTanh[
(d^(1/4)*(a*b^3*x + (-3*a*b^2 - b^3)*x^2 + (3*a*b + 3*b^2)*x^3 + (-a - 3*b)
*x^4 + x^5)^(3/4))/((b - x)^2*x*(-a + x))])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*b*x+x^2)*(b^2-2*b*x+x^2)/(x*(-a+x)*(-b+x)^3)^(3/4)/(b*d-(a
+d)*x+x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ab-2\,bx+x^2)(b^2-2\,bx+x^2)}{\left((a-x)(b-x)^3x\right)^{\frac{3}{4}}\left(bd-(a+d)x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*b*x+x^2)*(b^2-2*b*x+x^2)/(x*(-a+x)*(-b+x)^3)^(3/4)/(b*d-(a
+d)*x+x^2),x, algorithm="giac")
```

```
[Out] integrate((a*b - 2*b*x + x^2)*(b^2 - 2*b*x + x^2)/(((a - x)*(b - x)^3*x)^(3
/4)*(b*d - (a + d)*x + x^2)), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{(ab-2bx+x^2)\,(b^2-2bx+x^2)}{\left(x\,(-a+x)\,(-b+x)^3\right)^{\frac{3}{4}}\left(bd-(a+d)\,x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b-2*b*x+x^2)*(b^2-2*b*x+x^2)/(x*(-a+x)*(-b+x)^3)^(3/4)/(b*d-(a+d)*x+
x^2),x)
```

```
[Out] int((a*b-2*b*x+x^2)*(b^2-2*b*x+x^2)/(x*(-a+x)*(-b+x)^3)^(3/4)/(b*d-(a+d)*x+
x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ab-2\,bx+x^2)(b^2-2\,bx+x^2)}{\left((a-x)(b-x)^3x\right)^{\frac{3}{4}}\left(bd-(a+d)x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*b*x+x^2)*(b^2-2*b*x+x^2)/(x*(-a+x)*(-b+x)^3)^(3/4)/(b*d-(a
+d)*x+x^2),x, algorithm="maxima")
```

```
[Out] integrate((a*b - 2*b*x + x^2)*(b^2 - 2*b*x + x^2)/(((a - x)*(b - x)^3*x)^(3
/4)*(b*d - (a + d)*x + x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b^2-2\,b\,x+x^2\right)\,\left(x^2-2\,b\,x+a\,b\right)}{\left(x^2+(-a-d)\,x+b\,d\right)\,{\left(x\,(a-x)\,{(b-x)}^3\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b^2 - 2*b*x + x^2)*(a*b - 2*b*x + x^2))/((b*d + x^2 - x*(a + d))*(x*(
a - x)*(b - x)^3)^(3/4)),x)
```

```
[Out] int(((b^2 - 2*b*x + x^2)*(a*b - 2*b*x + x^2))/((b*d + x^2 - x*(a + d))*(x*(
a - x)*(b - x)^3)^(3/4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*b*x+x**2)*(b**2-2*b*x+x**2)/(x*(-a+x)*(-b+x)**3)**(3/4)/(b
*d-(a+d)*x+x**2),x)
```

```
[Out] Timed out
```

3.1869 $\int \frac{-2ab+(a+b)x}{\sqrt[3]{x(-a+x)(-b+x)}\left(-ab+(a+b)x+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=173

$$\frac{\log\left(\sqrt[3]{d}\,x\sqrt[3]{x^2(-a-b)+abx+x^3}+\left(x^2(-a-b)+abx+x^3\right)^{2/3}+d^{2/3}x^2\right)}{2\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{x^2(-a-b)+abx+x^3}-\sqrt[3]{d}\,x\right)}{\sqrt[3]{d}}$$

Rubi [F] time = 4.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2ab+(a+b)x}{\sqrt[3]{x(-a+x)(-b+x)}\left(-ab+(a+b)x+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*a*b + (a + b)*x)/((x*(-a + x)*(-b + x))^(1/3)*(-(a*b) + (a + b)*x +
 (-1 + d)*x^2)),x]
```

```
[Out] ((a + b - Sqrt[a^2 - 2*a*b + b^2 + 4*a*b*d])*x^(1/3)*(-a + x)^(1/3)*(-b + x
)^(1/3)*Defer[Int][1/(x^(1/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*(a + b - Sqrt[a
^2 - 2*a*b + b^2 + 4*a*b*d] + 2*(-1 + d)*x)), x])/((a - x)*(b - x)*x)^(1/3)
 + ((a + b + Sqrt[a^2 - 2*a*b + b^2 + 4*a*b*d])*x^(1/3)*(-a + x)^(1/3)*(-b
+ x)^(1/3)*Defer[Int][1/(x^(1/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*(a + b + Sqr
t[a^2 - 2*a*b + b^2 + 4*a*b*d] + 2*(-1 + d)*x)), x])/((a - x)*(b - x)*x)^(1
/3)
```

Rubi steps

$$\int \frac{-2ab+(a+b)x}{\sqrt[3]{x(-a+x)(-b+x)}\left(-ab+(a+b)x+(-1+d)x^2\right)}\,dx=\frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\int \frac{-2ab+(a+b)x}{\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\left(-ab+(a+b)x+\right.}}{\sqrt[3]{x(-a+x)(-b+x)}}$$

$$=\frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\int\left(\frac{a+b-\sqrt{a^2-2ab+}}{\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\left(a+b-\sqrt{a^2-2}\right.}\right.}{}$$

$$=\frac{\left(\left(a+b-\sqrt{a^2-2ab+b^2+4abd}\right)\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b}\right.}{\sqrt[3]{x(-a+}}$$

Mathematica [F] time = 4.93, size = 0, normalized size = 0.00

$$\int \frac{-2ab+(a+b)x}{\sqrt[3]{x(-a+x)(-b+x)}\left(-ab+(a+b)x+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*a*b + (a + b)*x)/((x*(-a + x)*(-b + x))^(1/3)*(-(a*b) + (a +
b)*x + (-1 + d)*x^2)),x]
```

```
[Out] Integrate[(-2*a*b + (a + b)*x)/((x*(-a + x)*(-b + x))^(1/3)*(-(a*b) + (a +
b)*x + (-1 + d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.49, size = 173, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{d}\,x\sqrt[3]{x^2(-a-b)+abx+x^3}+\left(x^2(-a-b)+abx+x^3\right)^{2/3}+d^{2/3}x^2\right)}{2\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{x^2(-a-b)+abx+x^3}-\sqrt[3]{d}\,x\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\,x}{2\sqrt[3]{x^2(-a-b)+abx+x^3}+\sqrt[3]{d}\,x}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*a*b + (a + b)*x)/((x*(-a + x)*(-b + x))^(1/3)*(-(a*b
) + (a + b)*x + (-1 + d)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*x)/(d^(1/3)*x + 2*(a*b*x + (-a - b)*x^2 +
x^3)^(1/3))])/d^(1/3) - Log[-(d^(1/3)*x) + (a*b*x + (-a - b)*x^2 + x^3)^(1/
3)]/d^(1/3) + Log[d^(2/3)*x^2 + d^(1/3)*x*(a*b*x + (-a - b)*x^2 + x^3)^(1/3
) + (a*b*x + (-a - b)*x^2 + x^3)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a*b+(a+b)*x+(-1+d)*x^2
),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,ab-(a+b)x}{\left((a-x)(b-x)x\right)^{\frac{1}{3}}\left((d-1)x^2-ab+(a+b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a*b+(a+b)*x+(-1+d)*x^2
),x, algorithm="giac")
```

```
[Out] integrate(-(2*a*b - (a + b)*x)/(((a - x)*(b - x)*x)^(1/3)*((d - 1)*x^2 - a*
b + (a + b)*x)), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{-2ab+(a+b)\,x}{\left(x\,(-a+x)\,(-b+x)\right)^{\frac{1}{3}}\left(-ab+(a+b)\,x+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a*b+(a+b)*x+(-1+d)*x^2),x)
```

```
[Out] int((-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a*b+(a+b)*x+(-1+d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,ab-(a+b)x}{\left((a-x)(b-x)x\right)^{\frac{1}{3}}\left((d-1)x^2-ab+(a+b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a*b+(a+b)*x+(-1+d)*x^2
),x, algorithm="maxima")
```

```
[Out] -integrate((2*a*b - (a + b)*x)/(((a - x)*(b - x)*x)^(1/3)*((d - 1)*x^2 - a*
b + (a + b)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{2\,a\,b-x\,\left(a+b\right)}{{\left(x\,\left(a-x\right)\,\left(b-x\right)\right)}^{1/3}\,\left(\left(d-1\right)\,x^2+\left(a+b\right)\,x-a\,b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*a*b - x*(a + b))/((x*(a - x)*(b - x))^(1/3)*(x*(a + b) - a*b + x^2*
(d - 1))),x)
```

```
[Out] -int((2*a*b - x*(a + b))/((x*(a - x)*(b - x))^(1/3)*(x*(a + b) - a*b + x^2*
(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))**(1/3)/(-a*b+(a+b)*x+(-1+d)*x*
*2),x)
```

```
[Out] Timed out
```

3.1870 $\int \frac{(-1+x^3)^{2/3}(1+x^3)}{x^6(2+x^3)}\,dx$

Optimal. Leaf size=173

$$-\frac{\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3-1}-3x\right)}{4\,2^{2/3}\sqrt[3]{3}}+\frac{\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}+\sqrt[3]{3}x}\right)}{4\,2^{2/3}}+\frac{(x^3-1)^{2/3}(-x^3-4)}{40x^5}+\frac{\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3-1}\,x+2^{2/}\right.}{8\,2^{2/3}\sqrt[}$$

Rubi [A] time = 0.19, antiderivative size = 168, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {580, 583, 12, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\sqrt[3]{2}-\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3-1}}\right)}{4\,2^{2/3}\sqrt[3]{3}}+\frac{\sqrt[6]{3}\tan^{-1}\left(\frac{2^{2/3}x}{\sqrt[6]{3}\sqrt[3]{x^3-1}}+\frac{1}{\sqrt{3}}\right)}{4\,2^{2/3}}-\frac{(x^3-1)^{2/3}}{10x^5}-\frac{(x^3-1)^{2/3}}{40x^2}+\frac{\log\left(\frac{\sqrt[3]{6}x}{\sqrt[3]{x^3-1}}+\frac{3^{2/3}x^2}{(x^3-1)^{2/3}}+2^{2/3}\right)}{8\,2^{2/3}\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^3)^(2/3)*(1 + x^3))/(x^6*(2 + x^3)),x]
```

```
[Out] -1/10*(-1 + x^3)^(2/3)/x^5 - (-1 + x^3)^(2/3)/(40*x^2) + (3^(1/6)*ArcTan[1/
Sqrt[3] + (2^(2/3)*x)/(3^(1/6)*(-1 + x^3)^(1/3))])/(4*2^(2/3)) - Log[2^(1/3
) - (3^(1/3)*x)/(-1 + x^3)^(1/3)]/(4*2^(2/3)*3^(1/3)) + Log[2^(2/3) + (3^(2
/3)*x^2)/(-1 + x^3)^(2/3) + (6^(1/3)*x)/(-1 + x^3)^(1/3)]/(8*2^(2/3)*3^(1/3
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(-1+x^3)^{2/3}(1+x^3)}{x^6(2+x^3)}\,dx = -\frac{(-1+x^3)^{2/3}}{10x^5} + \frac{1}{10}\int \frac{-1+7x^3}{x^3\sqrt[3]{-1+x^3}\,(2+x^3)}\,dx$$

$$= -\frac{(-1+x^3)^{2/3}}{10x^5} - \frac{(-1+x^3)^{2/3}}{40x^2} + \frac{1}{40}\int \frac{30}{\sqrt[3]{-1+x^3}\,(2+x^3)}\,dx$$

$$= -\frac{(-1+x^3)^{2/3}}{10x^5} - \frac{(-1+x^3)^{2/3}}{40x^2} + \frac{3}{4}\int \frac{1}{\sqrt[3]{-1+x^3}\,(2+x^3)}\,dx$$

$$= -\frac{(-1+x^3)^{2/3}}{10x^5} - \frac{(-1+x^3)^{2/3}}{40x^2} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{2-3x^3}\,dx, x, \frac{x}{\sqrt[3]{-1+x^3}}\right)$$

$$= -\frac{(-1+x^3)^{2/3}}{10x^5} - \frac{(-1+x^3)^{2/3}}{40x^2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{3}x}\,dx, x, \frac{x}{\sqrt[3]{-1+x^3}}\right)}{4\,2^{2/3}} + \frac{\text{Subst}\left(\int \frac{}{2^2}\right.}{}$$

$$= -\frac{(-1+x^3)^{2/3}}{10x^5} - \frac{(-1+x^3)^{2/3}}{40x^2} - \frac{\log\left(\sqrt[3]{2} - \frac{\sqrt[3]{3}x}{\sqrt[3]{-1+x^3}}\right)}{4\,2^{2/3}\sqrt[3]{3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{6}x+3^{2/3}x\right.}{8\sqrt[3]{2}}$$

$$= -\frac{(-1+x^3)^{2/3}}{10x^5} - \frac{(-1+x^3)^{2/3}}{40x^2} - \frac{\log\left(\sqrt[3]{2} - \frac{\sqrt[3]{3}x}{\sqrt[3]{-1+x^3}}\right)}{4\,2^{2/3}\sqrt[3]{3}} + \frac{\log\left(2^{2/3} + \frac{3^{2/3}x^2}{(-1+x^3)^{2/3}} + \right.}{8\,2^{2/3}\sqrt[3]{3}}$$

$$= -\frac{(-1+x^3)^{2/3}}{10x^5} - \frac{(-1+x^3)^{2/3}}{40x^2} + \frac{\sqrt[6]{3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{3}x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{4\,2^{2/3}} - \frac{\log\left(\sqrt[3]{2} - \frac{\sqrt[3]{3}x}{\sqrt[3]{-1+x^3}}\right)}{4\,2^{2/3}\sqrt[3]{3}} +$$

Mathematica [A] time = 0.22, size = 156, normalized size = 0.90

$$\frac{1}{240}\left(5\sqrt[3]{2}\sqrt[6]{3}\left(6\tan^{-1}\left(\frac{2^{2/3}x}{\sqrt[6]{3}\sqrt[3]{1-x^3}}+\frac{1}{\sqrt{3}}\right)+\sqrt{3}\left(\log\left(\frac{2^{2/3}\sqrt[3]{3}x}{\sqrt[3]{1-x^3}}+\frac{\sqrt[3]{2}\,3^{2/3}x^2}{(1-x^3)^{2/3}}+2\right)-2\log\left(2-\frac{2^{2/3}\sqrt[3]{3}x}{\sqrt[3]{1-x^3}}\right)\right)\right)-\frac{6(x^3-1)^{2/3}(x^3+4)}{x^5}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^3)^(2/3)*(1 + x^3))/(x^6*(2 + x^3)),x]
```

```
[Out] ((-6*(-1 + x^3)^(2/3)*(4 + x^3))/x^5 + 5*2^(1/3)*3^(1/6)*(6*ArcTan[1/Sqrt[3
] + (2^(2/3)*x)/(3^(1/6)*(1 - x^3)^(1/3))] + Sqrt[3]*(-2*Log[2 - (2^(2/3)*3
^(1/3)*x)/(1 - x^3)^(1/3)] + Log[2 + (2^(1/3)*3^(2/3)*x^2)/(1 - x^3)^(2/3)
+ (2^(2/3)*3^(1/3)*x)/(1 - x^3)^(1/3)])))/240
```

IntegrateAlgebraic [A] time = 0.40, size = 173, normalized size = 1.00

$$-\frac{\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3-1}-3x\right)}{4\,2^{2/3}\sqrt[3]{3}}+\frac{\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}+\sqrt[3]{3}x}\right)}{4\,2^{2/3}}+\frac{(x^3-1)^{2/3}(-x^3-4)}{40x^5}+\frac{\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3-1}\,x+2^{2/3}\sqrt[3]{3}(x^3-1)^{2/3}+3x^2\right)}{8\,2^{2/3}\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(2/3)*(1 + x^3))/(x^6*(2 + x^3)),x]
```

```
[Out] ((-4 - x^3)*(-1 + x^3)^(2/3))/(40*x^5) + (3^(1/6)*ArcTan[(3^(5/6)*x)/(3^(1/
3)*x + 2*2^(1/3)*(-1 + x^3)^(1/3))])/(4*2^(2/3)) - Log[-3*x + 2^(1/3)*3^(2/
3)*(-1 + x^3)^(1/3)]/(4*2^(2/3)*3^(1/3)) + Log[3*x^2 + 2^(1/3)*3^(2/3)*x*(-
1 + x^3)^(1/3) + 2^(2/3)*3^(1/3)*(-1 + x^3)^(2/3)]/(8*2^(2/3)*3^(1/3))
```

fricas [**B**] time = 5.69, size = 289, normalized size = 1.67

$$\frac{10 \cdot 12^{\frac{2}{3}} (-1)^{\frac{1}{3}} x^5 \log\left(-\frac{18 \cdot 12^{\frac{1}{3}} (-1)^{\frac{2}{3}} (x^3-1)^{\frac{1}{3}} x^2 + 12^{\frac{2}{3}} (-1)^{\frac{1}{3}} (x^3+2) - 36 (x^3-1)^{\frac{2}{3}} x}{x^3+2}\right) - 5 \cdot 12^{\frac{2}{3}} (-1)^{\frac{1}{3}} x^5 \log\left(-\frac{6 \cdot 12^{\frac{2}{3}} (-1)^{\frac{1}{3}} (4x^4-x)(x^3-1)^{\frac{2}{3}} - 12^{\frac{1}{3}} (-1)^{\frac{2}{3}} (55x^6-50x^3+4) - 18 (7x^5-4x^2)(x^3-1)^{\frac{1}{3}}}{x^6+4x^3+4}\right) - 60 \cdot 12^{\frac{1}{6}} (-1)^{\frac{1}{3}} x^5 \arctan\left(\frac{12^{\frac{1}{6}}\left(12 \cdot 12^{\frac{2}{3}} (-1)^{\frac{2}{3}} (4x^7+7x^4-2x)(x^3-1)^{\frac{2}{3}} + 36 (-1)^{\frac{1}{3}} (55x^8-50x^5+4x^2)(x^3-1)^{\frac{1}{3}} - 12^{\frac{1}{3}} (377x^9-600x^6+204x^3-8)\right)}{6(487x^9-480x^6+12x^3+8)}\right) - 36 (x^3+4)(x^3-1)^{\frac{2}{3}}}{1440 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^3+1)/x^6/(x^3+2),x, algorithm="fricas")
```

```
[Out] 1/1440*(10*12^(2/3)*(-1)^(1/3)*x^5*log(-(18*12^(1/3)*(-1)^(2/3)*(x^3 - 1)^(
1/3)*x^2 + 12^(2/3)*(-1)^(1/3)*(x^3 + 2) - 36*(x^3 - 1)^(2/3)*x)/(x^3 + 2))
 - 5*12^(2/3)*(-1)^(1/3)*x^5*log(-(6*12^(2/3)*(-1)^(1/3)*(4*x^4 - x)*(x^3 -
 1)^(2/3) - 12^(1/3)*(-1)^(2/3)*(55*x^6 - 50*x^3 + 4) - 18*(7*x^5 - 4*x^2)*
(x^3 - 1)^(1/3))/(x^6 + 4*x^3 + 4)) - 60*12^(1/6)*(-1)^(1/3)*x^5*arctan(1/6
*12^(1/6)*(12*12^(2/3)*(-1)^(2/3)*(4*x^7 + 7*x^4 - 2*x)*(x^3 - 1)^(2/3) + 3
6*(-1)^(1/3)*(55*x^8 - 50*x^5 + 4*x^2)*(x^3 - 1)^(1/3) - 12^(1/3)*(377*x^9
- 600*x^6 + 204*x^3 - 8))/(487*x^9 - 480*x^6 + 12*x^3 + 8)) - 36*(x^3 + 4)*
(x^3 - 1)^(2/3))/x^5
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^3+1)(x^3-1)^{\frac{2}{3}}}{(x^3+2)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^3+1)/x^6/(x^3+2),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)*(x^3 - 1)^(2/3)/((x^3 + 2)*x^6), x)
```

maple [**C**] time = 2.45, size = 824, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)*(x^3+1)/x^6/(x^3+2),x)
```

```
[Out] -1/40*(x^6+3*x^3-4)/x^5/(x^3-1)^(1/3)+3/4*RootOf(RootOf(_Z^3+18)^2+18*_Z*Ro
otOf(_Z^3+18)+324*_Z^2)*ln((3*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18
)+324*_Z^2)*RootOf(_Z^3+18)^3*x^3-54*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(
_Z^3+18)+324*_Z^2)^2*RootOf(_Z^3+18)^2*x^3+3*RootOf(_Z^3+18)^2*(x^3-1)^(1/3
)*x^2-5*RootOf(_Z^3+18)*x^3+90*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+1
8)+324*_Z^2)*x^3+18*x*(x^3-1)^(2/3)+2*RootOf(_Z^3+18)-36*RootOf(RootOf(_Z^3
+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2))/(x^3+2))-1/24*ln(-(9*RootOf(RootOf(
_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)*RootOf(_Z^3+18)^3*x^3-108*RootOf
(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)^2*RootOf(_Z^3+18)^2*x^3-
42*(x^3-1)^(2/3)*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)*R
ootOf(_Z^3+18)^2*x-RootOf(_Z^3+18)^2*(x^3-1)^(1/3)*x^2+126*(x^3-1)^(1/3)*Ro
otOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)*RootOf(_Z^3+18)*x^2+
6*RootOf(_Z^3+18)*x^3-72*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324
*_Z^2)*x^3-6*x*(x^3-1)^(2/3)-6*RootOf(_Z^3+18)+72*RootOf(RootOf(_Z^3+18)^2+
18*_Z*RootOf(_Z^3+18)+324*_Z^2))/(x^3+2))*RootOf(_Z^3+18)-3/4*ln(-(9*RootOf
(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)*RootOf(_Z^3+18)^3*x^3-10
8*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)^2*RootOf(_Z^3+18
)^2*x^3-42*(x^3-1)^(2/3)*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324
*_Z^2)*RootOf(_Z^3+18)^2*x-RootOf(_Z^3+18)^2*(x^3-1)^(1/3)*x^2+126*(x^3-1)^
(1/3)*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)*RootOf(_Z^3+
18)*x^2+6*RootOf(_Z^3+18)*x^3-72*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3
+18)+324*_Z^2)*x^3-6*x*(x^3-1)^(2/3)-6*RootOf(_Z^3+18)+72*RootOf(RootOf(_Z^
3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2))/(x^3+2))*RootOf(RootOf(_Z^3+18)^2+
18*_Z*RootOf(_Z^3+18)+324*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^3+1)(x^3-1)^{\frac{2}{3}}}{(x^3+2)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^3+1)/x^6/(x^3+2),x, algorithm="maxima")
[Out] integrate((x^3 + 1)*(x^3 - 1)^(2/3)/((x^3 + 2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^3-1)^{2/3}\,(x^3+1)}{x^6\,(x^3+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(x^3 + 1))/(x^6*(x^3 + 2)),x)
[Out] int(((x^3 - 1)^(2/3)*(x^3 + 1))/(x^6*(x^3 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((x-1)\,(x^2+x+1)\right)^{\frac{2}{3}}\,(x+1)\,(x^2-x+1)}{x^6\,(x^3+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)*(x**3+1)/x**6/(x**3+2),x)
[Out] Integral(((x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)/(x**6*(x**3
 + 2)), x)
```

3.1871 $\int \frac{1}{(1+x)\left(-2+3x-2x^2+3x^3-2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{i(x-1)\sqrt{2x^2+x+2}\left(-\frac{i\tanh^{-1}\left(-\frac{\sqrt{2x^2+x+2}}{\sqrt{3}}+\sqrt{\frac{2}{3}}x+\sqrt{\frac{2}{3}}\right)}{12\sqrt{3}}-\frac{91i\tanh^{-1}\left(\frac{\sqrt{2x^2+x+2}}{\sqrt{5}}-\sqrt{\frac{2}{5}}x+\sqrt{\frac{2}{5}}\right)}{200\sqrt{5}}+\frac{i\left(22x^3+73x^2-107x-18\right)}{600(x-1)^2\sqrt{2x^2+x+2}}\right)}{\sqrt{-(x-1)^2\left(2x^2+x+2\right)}}$$

Rubi [F] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(1+x)\left(-2+3x-2x^2+3x^3-2x^4\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 + x)*(-2 + 3*x - 2*x^2 + 3*x^3 - 2*x^4)^(3/2)),x]
[Out] Defer[Int][1/((1 + x)*(-2 + 3*x - 2*x^2 + 3*x^3 - 2*x^4)^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{(1+x)\left(-2+3x-2x^2+3x^3-2x^4\right)^{3/2}}\,dx=\int \frac{1}{(1+x)\left(-2+3x-2x^2+3x^3-2x^4\right)^{3/2}}\,dx$$

Mathematica [A] time = 0.15, size = 146, normalized size = 0.84

$$\frac{-250\sqrt{3}\sqrt{2x^2+x+2}(x-1)^2\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{2\sqrt{2x^2+x+2}}\right)+819\sqrt{5}\sqrt{2x^2+x+2}(x-1)^2\tanh^{-1}\left(\frac{\sqrt{5}(x+1)}{2\sqrt{2x^2+x+2}}\right)-30\left(22x^3+73x^2-107x-18\right)}{18000(x-1)\sqrt{-(x-1)^2\left(2x^2+x+2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(-2 + 3*x - 2*x^2 + 3*x^3 - 2*x^4)^(3/2)),x]
[Out] (-30*(-18 - 107*x + 73*x^2 + 22*x^3) - 250*Sqrt[3]*(-1 + x)^2*Sqrt[2 + x +
2*x^2]*ArcTanh[(Sqrt[3]*(1 - x))/(2*Sqrt[2 + x + 2*x^2])] + 819*Sqrt[5]*(-1
 + x)^2*Sqrt[2 + x + 2*x^2]*ArcTanh[(Sqrt[5]*(1 + x))/(2*Sqrt[2 + x + 2*x^2
])])/(18000*(-1 + x)*Sqrt[-((-1 + x)^2*(2 + x + 2*x^2))])
```

IntegrateAlgebraic [A] time = 10.62, size = 173, normalized size = 1.00

$$\frac{i(x-1)\sqrt{2x^2+x+2}\left(-\frac{i\tanh^{-1}\left(-\frac{\sqrt{2x^2+x+2}}{\sqrt{3}}+\sqrt{\frac{2}{3}}x+\sqrt{\frac{2}{3}}\right)}{12\sqrt{3}}-\frac{91i\tanh^{-1}\left(\frac{\sqrt{2x^2+x+2}}{\sqrt{5}}-\sqrt{\frac{2}{5}}x+\sqrt{\frac{2}{5}}\right)}{200\sqrt{5}}+\frac{i\left(22x^3+73x^2-107x-18\right)}{600(x-1)^2\sqrt{2x^2+x+2}}\right)}{\sqrt{-(x-1)^2\left(2x^2+x+2\right)}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x)*(-2 + 3*x - 2*x^2 + 3*x^3 - 2*x^4)^(3/2)),x]
[Out] (I*(-1 + x)*Sqrt[2 + x + 2*x^2]*(((I/600)*(-18 - 107*x + 73*x^2 + 22*x^3))/
((-1 + x)^2*Sqrt[2 + x + 2*x^2]) - ((I/12)*ArcTanh[Sqrt[2/3] + Sqrt[2/3]*x
```

```
- Sqrt[2 + x + 2*x^2]/Sqrt[3]])/Sqrt[3] - (((91*I)/200)*ArcTanh[Sqrt[2/5] -
 Sqrt[2/5]*x + Sqrt[2 + x + 2*x^2]/Sqrt[5]])/Sqrt[5]))/Sqrt[-((-1 + x)^2*(2
 + x + 2*x^2))]
```

fricas [A] time = 0.79, size = 213, normalized size = 1.23

$$-\frac{819 \sqrt{5} \left(2 x^{5}-5 x^{4}+5 x^{3}-5 x^{2}+5 x-2\right) \arctan\left(\frac{\sqrt{5} \sqrt{-2 x^{4}+3 x^{3}-2 x^{2}+3 x-2} (x+1)}{2 \left(2 x^{3}-x^{2}+x-2\right)}\right)+250 \sqrt{3} \left(2 x^{5}-5 x^{4}+5 x^{3}-5 x^{2}+5 x-2\right) \arctan\left(\frac{\sqrt{3} \sqrt{-2 x^{4}+3 x^{3}-2 x^{2}+3 x-2}}{2 \left(2 x^{2}+x+2\right)}\right)-30 \sqrt{-2 x^{4}+3 x^{3}-2 x^{2}+3 x-2} \left(22 x^{3}+73 x^{2}-107 x-18\right)}{18000 \left(2 x^{5}-5 x^{4}+5 x^{3}-5 x^{2}+5 x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-2*x^4+3*x^3-2*x^2+3*x-2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/18000*(819*sqrt(5)*(2*x^5 - 5*x^4 + 5*x^3 - 5*x^2 + 5*x - 2)*arctan(1/2*
sqrt(5)*sqrt(-2*x^4 + 3*x^3 - 2*x^2 + 3*x - 2)*(x + 1)/(2*x^3 - x^2 + x - 2
)) + 250*sqrt(3)*(2*x^5 - 5*x^4 + 5*x^3 - 5*x^2 + 5*x - 2)*arctan(1/2*sqrt(
3)*sqrt(-2*x^4 + 3*x^3 - 2*x^2 + 3*x - 2)/(2*x^2 + x + 2)) - 30*sqrt(-2*x^4
 + 3*x^3 - 2*x^2 + 3*x - 2)*(22*x^3 + 73*x^2 - 107*x - 18))/(2*x^5 - 5*x^4
+ 5*x^3 - 5*x^2 + 5*x - 2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-2*x^4+3*x^3-2*x^2+3*x-2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.03, size = 287, normalized size = 1.66

$$-\frac{\left(819 \arctan\left(\frac{(1+x) \sqrt{5}}{2 \sqrt{-2x^{2}-x-2}}\right) \sqrt{-2x^{2}-x-2}\, \sqrt{5}\, x^{2}+250 \arctan\left(\frac{(-1+x) \sqrt{3}}{2 \sqrt{-2x^{2}-x-2}}\right) \sqrt{3}\, \sqrt{-2x^{2}-x-2}\, x^{2}-1638 \arctan\left(\frac{(1+x) \sqrt{5}}{2 \sqrt{-2x^{2}-x-2}}\right) \sqrt{-2x^{2}-x-2}\, \sqrt{5}\, x-500 \arctan\left(\frac{(-1+x) \sqrt{3}}{2 \sqrt{-2x^{2}-x-2}}\right) \sqrt{3}\, \sqrt{-2x^{2}-x-2}\, x+819 \sqrt{5}\, \arctan\left(\frac{(1+x) \sqrt{5}}{2 \sqrt{-2x^{2}-x-2}}\right) \sqrt{-2x^{2}-x-2}+250 \sqrt{3}\, \arctan\left(\frac{(-1+x) \sqrt{3}}{2 \sqrt{-2x^{2}-x-2}}\right) \sqrt{-2x^{2}-x-2}-660x^{3}-2190x^{2}+3210x+540\right) (-1+x) \left(2x^{2}+x+2\right)}{18000 \left(-2x^{4}+3x^{3}-2x^{2}+3x-2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(-2*x^4+3*x^3-2*x^2+3*x-2)^(3/2),x)
```

```
[Out] -1/18000*(819*arctan(1/2*(1+x)*5^(1/2)/(-2*x^2-x-2)^(1/2))*(-2*x^2-x-2)^(1/
2)*5^(1/2)*x^2+250*arctan(1/2*(-1+x)*3^(1/2)/(-2*x^2-x-2)^(1/2))*3^(1/2)*(-
2*x^2-x-2)^(1/2)*x^2-1638*arctan(1/2*(1+x)*5^(1/2)/(-2*x^2-x-2)^(1/2))*(-2*
x^2-x-2)^(1/2)*5^(1/2)*x-500*arctan(1/2*(-1+x)*3^(1/2)/(-2*x^2-x-2)^(1/2))*
3^(1/2)*(-2*x^2-x-2)^(1/2)*x+819*5^(1/2)*arctan(1/2*(1+x)*5^(1/2)/(-2*x^2-x
-2)^(1/2))*(-2*x^2-x-2)^(1/2)+250*3^(1/2)*arctan(1/2*(-1+x)*3^(1/2)/(-2*x^2
-x-2)^(1/2))*(-2*x^2-x-2)^(1/2)-660*x^3-2190*x^2+3210*x+540)*(-1+x)*(2*x^2+
x+2)/(-2*x^4+3*x^3-2*x^2+3*x-2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-2 x^{4}+3 x^{3}-2 x^{2}+3 x-2\right)^{\frac{3}{2}} (x+1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-2*x^4+3*x^3-2*x^2+3*x-2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-2*x^4 + 3*x^3 - 2*x^2 + 3*x - 2)^(3/2)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x+1) \left(-2 x^{4}+3 x^{3}-2 x^{2}+3 x-2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)*(3*x - 2*x^2 + 3*x^3 - 2*x^4 - 2)^(3/2)),x)
[Out] int(1/((x + 1)*(3*x - 2*x^2 + 3*x^3 - 2*x^4 - 2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(x-1\right)^{2}\left(2x^{2}+x+2\right)\right)^{\frac{3}{2}}\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-2*x**4+3*x**3-2*x**2+3*x-2)**(3/2),x)
[Out] Integral(1/((-(x - 1)**2*(2*x**2 + x + 2))**(3/2)*(x + 1)), x)
```

3.1872 $\int \frac{\sqrt{-x+x^4}}{-b+ax^6}\,dx$

Optimal. Leaf size=173

$$\frac{\sqrt{\sqrt{b}\left(\sqrt{a}-\sqrt{b}\right)}\tan^{-1}\left(\frac{x\sqrt{x^4-x}\sqrt{\sqrt{a}\sqrt{b}-b}}{\sqrt{b}(x-1)\left(x^2+x+1\right)}\right)}{3\sqrt{a}\,b}-\frac{\sqrt{-\left(\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)\right)}\tan^{-1}\left(\frac{x\sqrt{x^4-x}\sqrt{-\sqrt{a}\sqrt{b}-b}}{\sqrt{b}(x-1)\left(x^2+x+1\right)}\right)}{3\sqrt{a}\,b}$$

Rubi [A] time = 0.28, antiderivative size = 185, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2056, 1493, 1491, 1175, 402, 217, 206, 377, 205, 208}

$$\frac{\sqrt{x^4-x}\sqrt{\sqrt{a}-\sqrt{b}}\,\tan^{-1}\left(\frac{x^{3/2}\sqrt{\sqrt{a}-\sqrt{b}}}{\sqrt[4]{b}\sqrt{x^3-1}}\right)}{3\sqrt{a}\,b^{3/4}\sqrt{x}\sqrt{x^3-1}}+\frac{\sqrt{x^4-x}\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh^{-1}\left(\frac{x^{3/2}\sqrt{\sqrt{a}+\sqrt{b}}}{\sqrt[4]{b}\sqrt{x^3-1}}\right)}{3\sqrt{a}\,b^{3/4}\sqrt{x}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-x + x^4]/(-b + a*x^6),x]
```

```
[Out] (Sqrt[Sqrt[a] - Sqrt[b]]*Sqrt[-x + x^4]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*x^(
3/2))/(b^(1/4)*Sqrt[-1 + x^3])]])/(3*Sqrt[a]*b^(3/4)*Sqrt[x]*Sqrt[-1 + x^3])
 + (Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt[-x + x^4]*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]
*x^(3/2))/(b^(1/4)*Sqrt[-1 + x^3])]])/(3*Sqrt[a]*b^(3/4)*Sqrt[x]*Sqrt[-1 + x
^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
```

```
GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[-(a*c), 2]}, -Dist[c/(2*r), Int[(d + e*x^2)^q/(r - c*x^2), x], x] - Di
st[c/(2*r), Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e, q}
, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1491

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k
 - 1)*(d + e*x^(n/k))^q*(a + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /
; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1493

```
Int[((f_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n
_))^(q_.), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/f, Subst[Int[x^(k
*(m + 1) - 1)*(d + (e*x^(k*n))/f)^q*(a + (c*x^(2*k*n))/f)^p, x], x, (f*x)^(
1/k)], x]] /; FreeQ[{a, c, d, e, f, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
 && FractionQ[m] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\sqrt{-x+x^4}}{-b+ax^6}\,dx = \frac{\sqrt{-x+x^4}\int \frac{\sqrt{x}\sqrt{-1+x^3}}{-b+ax^6}\,dx}{\sqrt{x}\sqrt{-1+x^3}}$$

$$= \frac{\left(2\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{x^2\sqrt{-1+x^6}}{-b+ax^{12}}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{-1+x^3}}$$

$$= \frac{\left(2\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{\sqrt{-1+x^2}}{-b+ax^4}\,dx, x, x^{3/2}\right)}{3\sqrt{x}\sqrt{-1+x^3}}$$

$$= -\frac{\left(\sqrt{a}\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{\sqrt{-1+x^2}}{\sqrt{a}\sqrt{b}-ax^2}\,dx, x, x^{3/2}\right)}{3\sqrt{b}\sqrt{x}\sqrt{-1+x^3}} - \frac{\left(\sqrt{a}\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{\sqrt{-1+x^2}}{\sqrt{a}\sqrt{b}+ax^2}\,dx,\right.}{3\sqrt{b}\sqrt{x}\sqrt{-1+x^3}}$$

$$= -\frac{\left(\sqrt{a}\left(-1+\frac{\sqrt{b}}{\sqrt{a}}\right)\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{a}\sqrt{b}-ax^2\right)}\,dx, x, x^{3/2}\right)}{3\sqrt{b}\sqrt{x}\sqrt{-1+x^3}} + \frac{\left(\sqrt{a}\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\sqrt{-x\right.}{}$$

$$= -\frac{\left(\sqrt{a}\left(-1+\frac{\sqrt{b}}{\sqrt{a}}\right)\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}-\left(-a+\sqrt{a}\sqrt{b}\right)x^2}\,dx, x, \frac{x^{3/2}}{\sqrt{-1+x^3}}\right)}{3\sqrt{b}\sqrt{x}\sqrt{-1+x^3}} + \frac{\left(\sqrt{a}\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\right.}{}$$

$$= \frac{\sqrt{\sqrt{a}-\sqrt{b}}\sqrt{-x+x^4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,x^{3/2}}{\sqrt[4]{b}\sqrt{-1+x^3}}\right)}{3\sqrt{a}\,b^{3/4}\sqrt{x}\sqrt{-1+x^3}} + \frac{\sqrt{\sqrt{a}+\sqrt{b}}\sqrt{-x+x^4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,x^{3/2}}{\sqrt[4]{b}\sqrt{-1+x^3}}\right.}{3\sqrt{a}\,b^{3/4}\sqrt{x}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.26, size = 280, normalized size = 1.62

$$\frac{\sqrt{x}\sqrt{x^3-1}\left(\sqrt{\sqrt{a}-\sqrt{b}}\left(\tan^{-1}\left(\frac{\sqrt[4]{a}-\sqrt[4]{b}x^{3/2}}{\sqrt{x^3-1}\sqrt{\sqrt{a}-\sqrt{b}}}\right)-\tan^{-1}\left(\frac{\sqrt[4]{a}+\sqrt[4]{b}x^{3/2}}{\sqrt{x^3-1}\sqrt{\sqrt{a}-\sqrt{b}}}\right)\right)-\sqrt{\sqrt{a}+\sqrt{b}}\tanh^{-1}\left(\frac{-\sqrt[4]{b}x^{3/2}+i\sqrt[4]{a}}{\sqrt{x^3-1}\sqrt{\sqrt{a}+\sqrt{b}}}\right)+\sqrt{\sqrt{a}+\sqrt{b}}\tanh^{-1}\left(\frac{\sqrt[4]{b}x^{3/2}+i\sqrt[4]{a}}{\sqrt{x^3-1}\sqrt{\sqrt{a}+\sqrt{b}}}\right)\right)}{6\sqrt{a}\,b^{3/4}\sqrt{x\left(x^3-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x + x^4]/(-b + a*x^6),x]
```

```
[Out] (Sqrt[x]*Sqrt[-1 + x^3]*(Sqrt[Sqrt[a] - Sqrt[b]]*(ArcTan[(a^(1/4) - b^(1/4)
*x^(3/2))/(Sqrt[Sqrt[a] - Sqrt[b]]*Sqrt[-1 + x^3])] - ArcTan[(a^(1/4) + b^(
1/4)*x^(3/2))/(Sqrt[Sqrt[a] - Sqrt[b]]*Sqrt[-1 + x^3])]) - Sqrt[Sqrt[a] + S
qrt[b]]*ArcTanh[(I*a^(1/4) - b^(1/4)*x^(3/2))/(Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt
[-1 + x^3])] + Sqrt[Sqrt[a] + Sqrt[b]]*ArcTanh[(I*a^(1/4) + b^(1/4)*x^(3/2)
)/(Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt[-1 + x^3])]))/(6*Sqrt[a]*b^(3/4)*Sqrt[x*(-1
 + x^3)])
```

IntegrateAlgebraic [A] time = 0.80, size = 173, normalized size = 1.00

$$\frac{\sqrt{\sqrt{b}\left(\sqrt{a}-\sqrt{b}\right)}\tan^{-1}\left(\frac{x\sqrt{x^4-x}\sqrt{\sqrt{a}\sqrt{b}-b}}{\sqrt{b}(x-1)\left(x^2+x+1\right)}\right)}{3\sqrt{a}\,b} - \frac{\sqrt{-\left(\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)\right)}\tan^{-1}\left(\frac{x\sqrt{x^4-x}\sqrt{-\sqrt{a}\sqrt{b}-b}}{\sqrt{b}(x-1)\left(x^2+x+1\right)}\right)}{3\sqrt{a}\,b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-x + x^4]/(-b + a*x^6),x]
```

```
[Out] -1/3*(Sqrt[-((Sqrt[a] + Sqrt[b])*Sqrt[b])]*ArcTan[(Sqrt[-(Sqrt[a]*Sqrt[b])
- b]*x*Sqrt[-x + x^4])/(Sqrt[b]*(-1 + x)*(1 + x + x^2))])/(Sqrt[a]*b) + (Sq
```

```
rt[(Sqrt[a] - Sqrt[b])*Sqrt[b]]*ArcTan[(Sqrt[Sqrt[a]*Sqrt[b] - b]*x*Sqrt[-x
 + x^4])/(Sqrt[b]*(-1 + x)*(1 + x + x^2))]])/(3*Sqrt[a]*b)
```

fricas [B] time = 1.18, size = 1083, normalized size = 6.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)/(a*x^6-b),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(-(a*b*sqrt(1/(a*b^3)) - 1)/(a*b))*log((2*((a^2 - 3*a*b + 4*b^2)*
x^4 + 2*(a*b - 2*b^2)*x + (2*(a^2*b^2 - 2*a*b^3)*x^4 + (a^2*b^2 - 3*a*b^3 +
 4*b^4)*x)*sqrt(1/(a*b^3)))*sqrt(x^4 - x) + ((a^2*b - 4*a*b^2)*x^6 + 2*(a^2
*b - 3*a*b^2 + 4*b^3)*x^3 + 3*a*b^2 - 4*b^3 + ((a^3*b^2 - 6*a^2*b^3 + 8*a*b
^4)*x^6 + a^2*b^3 + 4*(a^2*b^3 - 2*a*b^4)*x^3)*sqrt(1/(a*b^3)))*sqrt(-(a*b*
sqrt(1/(a*b^3)) - 1)/(a*b)))/(a*x^6 - b)) + 1/12*sqrt(-(a*b*sqrt(1/(a*b^3))
 - 1)/(a*b))*log((2*((a^2 - 3*a*b + 4*b^2)*x^4 + 2*(a*b - 2*b^2)*x + (2*(a^
2*b^2 - 2*a*b^3)*x^4 + (a^2*b^2 - 3*a*b^3 + 4*b^4)*x)*sqrt(1/(a*b^3)))*sqrt
(x^4 - x) - ((a^2*b - 4*a*b^2)*x^6 + 2*(a^2*b - 3*a*b^2 + 4*b^3)*x^3 + 3*a*
b^2 - 4*b^3 + ((a^3*b^2 - 6*a^2*b^3 + 8*a*b^4)*x^6 + a^2*b^3 + 4*(a^2*b^3 -
 2*a*b^4)*x^3)*sqrt(1/(a*b^3)))*sqrt(-(a*b*sqrt(1/(a*b^3)) - 1)/(a*b)))/(a*
x^6 - b)) - 1/12*sqrt((a*b*sqrt(1/(a*b^3)) + 1)/(a*b))*log((2*((a^2 - 3*a*b
 + 4*b^2)*x^4 + 2*(a*b - 2*b^2)*x - (2*(a^2*b^2 - 2*a*b^3)*x^4 + (a^2*b^2 -
 3*a*b^3 + 4*b^4)*x)*sqrt(1/(a*b^3)))*sqrt(x^4 - x) + ((a^2*b - 4*a*b^2)*x^
6 + 2*(a^2*b - 3*a*b^2 + 4*b^3)*x^3 + 3*a*b^2 - 4*b^3 - ((a^3*b^2 - 6*a^2*b
^3 + 8*a*b^4)*x^6 + a^2*b^3 + 4*(a^2*b^3 - 2*a*b^4)*x^3)*sqrt(1/(a*b^3)))*s
qrt((a*b*sqrt(1/(a*b^3)) + 1)/(a*b)))/(a*x^6 - b)) + 1/12*sqrt((a*b*sqrt(1/
(a*b^3)) + 1)/(a*b))*log((2*((a^2 - 3*a*b + 4*b^2)*x^4 + 2*(a*b - 2*b^2)*x
- (2*(a^2*b^2 - 2*a*b^3)*x^4 + (a^2*b^2 - 3*a*b^3 + 4*b^4)*x)*sqrt(1/(a*b^3
)))*sqrt(x^4 - x) - ((a^2*b - 4*a*b^2)*x^6 + 2*(a^2*b - 3*a*b^2 + 4*b^3)*x^
3 + 3*a*b^2 - 4*b^3 - ((a^3*b^2 - 6*a^2*b^3 + 8*a*b^4)*x^6 + a^2*b^3 + 4*(a
^2*b^3 - 2*a*b^4)*x^3)*sqrt(1/(a*b^3)))*sqrt((a*b*sqrt(1/(a*b^3)) + 1)/(a*b
)))/(a*x^6 - b))
```

giac [A] time = 1.10, size = 207, normalized size = 1.20

$$\frac{\left(4\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,a+5\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,b\right)|b|\arctan\left(\frac{\sqrt{-\frac{1}{x^3}+1}}{\sqrt{-\frac{b+\sqrt{(a-b)b+b^2}}{b}}}\right)}{3\left(4a^2b^3+5ab^4\right)}-\frac{\left(4\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,a+5\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,b\right)|b|\arctan\left(\frac{\sqrt{-\frac{1}{x^3}+1}}{\sqrt{-\frac{b-\sqrt{(a-b)b+b^2}}{b}}}\right)}{3\left(4a^2b^3+5ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)/(a*x^6-b),x, algorithm="giac")
```

```
[Out] 1/3*(4*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a + 5*sqrt(a*b)*sqrt(-b^2 - sqrt(
a*b)*b)*b)*abs(b)*arctan(sqrt(-1/x^3 + 1)/sqrt(-(b + sqrt((a - b)*b + b^2))
/b))/(4*a^2*b^3 + 5*a*b^4) - 1/3*(4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a +
5*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b)*abs(b)*arctan(sqrt(-1/x^3 + 1)/sqrt
(-(b - sqrt((a - b)*b + b^2))/b))/(4*a^2*b^3 + 5*a*b^4)
```

maple [C] time = 0.47, size = 365, normalized size = 2.11

$$-\frac{\sqrt{4}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(a\_Z^6-b\right)}\frac{\left(-\_\alpha^3+1\right)(-1+x)^2\left(\_\alpha^5+\_\alpha^4+\_\alpha^3+\_\alpha^2+\_\alpha+1\right)\left(1-i\sqrt{3}\right)\sqrt{\frac{x\left(-3+i\sqrt{3}\right)}{(-1+x)\left(i\sqrt{3}-1\right)}}\sqrt{\frac{i\sqrt{3}+2x+1}{(-1+x)\left(-1-i\sqrt{3}\right)}}\sqrt{\frac{1+2x-i\sqrt{3}}{(-1+x)\left(i\sqrt{3}-1\right)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)-\frac{\_\alpha^5 a\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)}},\frac{i\_\alpha^5\sqrt{3}\,a-3\_\alpha^5 a-i\sqrt{3}\,b+3b}{6b},\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\right)}{b}\right)}{\_\alpha^4(a-b)\left(-3+i\sqrt{3}\right)\sqrt{x(-1+x)\left(i\sqrt{3}+2x+1\right)\left(1+2x-i\sqrt{3}\right)}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x)^(1/2)/(a*x^6-b),x)
```

```
[Out] -1/3*4^(1/2)*sum((-_alpha^3+1)/_alpha^4*(-1+x)^2*(_alpha^5+_alpha^4+_alpha^
3+_alpha^2+_alpha+1)/(a-b)*(1-I*3^(1/2))*(x/(-1+x)*(-3+I*3^(1/2))/(I*3^(1/2
)-1))^(1/2)*(1/(-1+x)*(I*3^(1/2)+2*x+1)/(-1-I*3^(1/2)))^(1/2)*(1/(-1+x)*(1+
2*x-I*3^(1/2))/(I*3^(1/2)-1))^(1/2)/(-3+I*3^(1/2))/(x*(-1+x)*(I*3^(1/2)+2*x
+1)*(1+2*x-I*3^(1/2)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I
*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-_alpha^5*a/b*EllipticPi(((-3/2+1/2*I*
3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),1/6*(I*_alpha^5*3^(1/2)*a-3*_
alpha^5*a-I*3^(1/2)*b+3*b)/b,((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))),_alpha=RootOf(_Z^6*a-b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4 - x}}{ax^6 - b}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)/(a*x^6-b),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 - x)/(a*x^6 - b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{x^4 - x}}{b - a\,x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - x)^(1/2)/(b - a*x^6),x)
```

```
[Out] -int((x^4 - x)^(1/2)/(b - a*x^6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x (x - 1) \left(x^2 + x + 1\right)}}{ax^6 - b}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x)**(1/2)/(a*x**6-b),x)
```

```
[Out] Integral(sqrt(x*(x - 1)*(x**2 + x + 1))/(a*x**6 - b), x)
```

3.1873 $\int \frac{(1+2x^8)\sqrt[4]{-1-2x^4+2x^8}\,(1-3x^8+4x^{16})}{x^{10}(-1+2x^8)}\,dx$

Optimal. Leaf size=173

$$-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{2x^8-2x^4-1}}{\sqrt{2}x^2-\sqrt{2x^8-2x^4-1}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}x\sqrt[4]{2x^8-2x^4-1}}{2x^2+\sqrt{2}\sqrt{2x^8-2x^4-1}}\right)}{2\sqrt[4]{2}}+\frac{\sqrt[4]{2x^8-2x^4-1}\,(20x^{16}-4x^{12}+9x^8+2x^4+5)}{45x^9}$$

Rubi [F] time = 3.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+2x^8)\sqrt[4]{-1-2x^4+2x^8}\,(1-3x^8+4x^{16})}{x^{10}(-1+2x^8)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + 2*x^8)*(-1 - 2*x^4 + 2*x^8)^(1/4)*(1 - 3*x^8 + 4*x^16))/(x^10*(-1
 + 2*x^8)),x]
```

```
[Out] ((-1 - 2*x^4 + 2*x^8)^(1/4)*AppellF1[-9/4, -1/4, -1/4, -5/4, (2*x^4)/(1 + S
qrt[3]), (2*x^4)/(1 - Sqrt[3])])/(9*x^9*(1 - (2*x^4)/(1 - Sqrt[3]))^(1/4)*(
1 - (2*x^4)/(1 + Sqrt[3]))^(1/4)) + ((-1 - 2*x^4 + 2*x^8)^(1/4)*AppellF1[-1
/4, -1/4, -1/4, 3/4, (2*x^4)/(1 + Sqrt[3]), (2*x^4)/(1 - Sqrt[3])])/(x*(1 -
 (2*x^4)/(1 - Sqrt[3]))^(1/4)*(1 - (2*x^4)/(1 + Sqrt[3]))^(1/4)) + (4*x^7*(
-1 - 2*x^4 + 2*x^8)^(1/4)*AppellF1[7/4, -1/4, -1/4, 11/4, (2*x^4)/(1 + Sqrt
[3]), (2*x^4)/(1 - Sqrt[3])])/(7*(1 - (2*x^4)/(1 - Sqrt[3]))^(1/4)*(1 - (2*
x^4)/(1 + Sqrt[3]))^(1/4)) + ((I/2)*Defer[Int][(-1 - 2*x^4 + 2*x^8)^(1/4)/(
I - 2^(1/8)*x), x])/2^(3/4) - Defer[Int][(-1 - 2*x^4 + 2*x^8)^(1/4)/(1 - 2^
(1/8)*x), x]/(2*2^(3/4)) - ((1/4 - I/4)*Defer[Int][(-1 - 2*x^4 + 2*x^8)^(1/
4)/((-1)^(1/4) - 2^(1/8)*x), x])/2^(1/4) - ((-1)^(1/4)*Defer[Int][(-1 - 2*x
^4 + 2*x^8)^(1/4)/(-(-1)^(3/4) - 2^(1/8)*x), x])/(2*2^(3/4)) + ((I/2)*Defer
[Int][(-1 - 2*x^4 + 2*x^8)^(1/4)/(I + 2^(1/8)*x), x])/2^(3/4) - Defer[Int][
(-1 - 2*x^4 + 2*x^8)^(1/4)/(1 + 2^(1/8)*x), x]/(2*2^(3/4)) - ((1/4 - I/4)*D
efer[Int][(-1 - 2*x^4 + 2*x^8)^(1/4)/((-1)^(1/4) + 2^(1/8)*x), x])/2^(1/4)
- ((-1)^(1/4)*Defer[Int][(-1 - 2*x^4 + 2*x^8)^(1/4)/(-(-1)^(3/4) + 2^(1/8)*
x), x])/(2*2^(3/4))
```

Rubi steps

$$\int \frac{\left(1+2x^8\right)\sqrt[4]{-1-2x^4+2x^8}\left(1-3x^8+4x^{16}\right)}{x^{10}\left(-1+2x^8\right)}dx = \int\left(-\frac{\sqrt[4]{-1-2x^4+2x^8}}{x^{10}} - \frac{\sqrt[4]{-1-2x^4+2x^8}}{x^2} + 4x^6\sqrt[4]{-1-\right.$$

$$= 4\int x^6\sqrt[4]{-1-2x^4+2x^8}\,dx + 4\int \frac{x^6\sqrt[4]{-1-2x^4+2x^8}}{-1+2x^8}\,d$$

$$= 4\int\left(\frac{x^2\sqrt[4]{-1-2x^4+2x^8}}{2\left(-\sqrt{2}+2x^4\right)} + \frac{x^2\sqrt[4]{-1-2x^4+2x^8}}{2\left(\sqrt{2}+2x^4\right)}\right)dx - $$

$$= \frac{\sqrt[4]{-1-2x^4+2x^8}\,F_1\left(-\frac{9}{4};-\frac{1}{4},-\frac{1}{4};-\frac{5}{4};\frac{2x^4}{1+\sqrt{3}},\frac{2x^4}{1-\sqrt{3}}\right)}{9x^9\sqrt[4]{1-\frac{2x^4}{1-\sqrt{3}}}\sqrt[4]{1-\frac{2x^4}{1+\sqrt{3}}}} + $$

$$= \frac{\sqrt[4]{-1-2x^4+2x^8}\,F_1\left(-\frac{9}{4};-\frac{1}{4},-\frac{1}{4};-\frac{5}{4};\frac{2x^4}{1+\sqrt{3}},\frac{2x^4}{1-\sqrt{3}}\right)}{9x^9\sqrt[4]{1-\frac{2x^4}{1-\sqrt{3}}}\sqrt[4]{1-\frac{2x^4}{1+\sqrt{3}}}} + $$

$$= \frac{\sqrt[4]{-1-2x^4+2x^8}\,F_1\left(-\frac{9}{4};-\frac{1}{4},-\frac{1}{4};-\frac{5}{4};\frac{2x^4}{1+\sqrt{3}},\frac{2x^4}{1-\sqrt{3}}\right)}{9x^9\sqrt[4]{1-\frac{2x^4}{1-\sqrt{3}}}\sqrt[4]{1-\frac{2x^4}{1+\sqrt{3}}}} + $$

$$= \frac{\sqrt[4]{-1-2x^4+2x^8}\,F_1\left(-\frac{9}{4};-\frac{1}{4},-\frac{1}{4};-\frac{5}{4};\frac{2x^4}{1+\sqrt{3}},\frac{2x^4}{1-\sqrt{3}}\right)}{9x^9\sqrt[4]{1-\frac{2x^4}{1-\sqrt{3}}}\sqrt[4]{1-\frac{2x^4}{1+\sqrt{3}}}} + $$

$$= \frac{\sqrt[4]{-1-2x^4+2x^8}\,F_1\left(-\frac{9}{4};-\frac{1}{4},-\frac{1}{4};-\frac{5}{4};\frac{2x^4}{1+\sqrt{3}},\frac{2x^4}{1-\sqrt{3}}\right)}{9x^9\sqrt[4]{1-\frac{2x^4}{1-\sqrt{3}}}\sqrt[4]{1-\frac{2x^4}{1+\sqrt{3}}}} + $$

Mathematica [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\left(1+2x^8\right)\sqrt[4]{-1-2x^4+2x^8}\left(1-3x^8+4x^{16}\right)}{x^{10}\left(-1+2x^8\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + 2*x^8)*(-1 - 2*x^4 + 2*x^8)^(1/4)*(1 - 3*x^8 + 4*x^16))/(x^
10*(-1 + 2*x^8)),x]
```

```
[Out] Integrate[((1 + 2*x^8)*(-1 - 2*x^4 + 2*x^8)^(1/4)*(1 - 3*x^8 + 4*x^16))/(x^
10*(-1 + 2*x^8)), x]
```

IntegrateAlgebraic [A] time = 2.87, size = 173, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{2x^8-2x^4-1}}{\sqrt{2}x^2-\sqrt{2x^8-2x^4-1}}\right)}{2\sqrt[4]{2}} - \frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}x\sqrt[4]{2x^8-2x^4-1}}{2x^2+\sqrt{2}\sqrt{2x^8-2x^4-1}}\right)}{2\sqrt[4]{2}} + \frac{\sqrt[4]{2x^8-2x^4-1}\left(20x^{16}-4x^{12}+9x^8+2x^4+5\right)}{45x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + 2*x^8)*(-1 - 2*x^4 + 2*x^8)^(1/4)*(1 - 3*x^8 + 4*x
^16))/(x^10*(-1 + 2*x^8)),x]
```

```
[Out] ((-1 - 2*x^4 + 2*x^8)^(1/4)*(5 + 2*x^4 + 9*x^8 - 4*x^12 + 20*x^16))/(45*x^9
) - ArcTan[(2^(3/4)*x*(-1 - 2*x^4 + 2*x^8)^(1/4))/(Sqrt[2]*x^2 - Sqrt[-1 -
2*x^4 + 2*x^8])]/(2*2^(1/4)) - ArcTanh[(2*2^(1/4)*x*(-1 - 2*x^4 + 2*x^8)^(1
/4))/(2*x^2 + Sqrt[2]*Sqrt[-1 - 2*x^4 + 2*x^8])]/(2*2^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)*(2*x^8-2*x^4-1)^(1/4)*(4*x^16-3*x^8+1)/x^10/(2*x^8-1),x
, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^{16}-3\,x^{8}+1\right)\left(2\,x^{8}-2\,x^{4}-1\right)^{\frac{1}{4}}\left(2\,x^{8}+1\right)}{\left(2\,x^{8}-1\right)x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)*(2*x^8-2*x^4-1)^(1/4)*(4*x^16-3*x^8+1)/x^10/(2*x^8-1),x
, algorithm="giac")
```

```
[Out] integrate((4*x^16 - 3*x^8 + 1)*(2*x^8 - 2*x^4 - 1)^(1/4)*(2*x^8 + 1)/((2*x^
8 - 1)*x^10), x)
```

maple [C] time = 3.25, size = 1128, normalized size = 6.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8+1)*(2*x^8-2*x^4-1)^(1/4)*(4*x^16-3*x^8+1)/x^10/(2*x^8-1),x)
```

```
[Out] 1/45*(40*x^24-48*x^20+6*x^16-10*x^12-3*x^8-12*x^4-5)/x^9/(2*x^8-2*x^4-1)^(3
/4)+(-1/4*RootOf(_Z^4+2)*ln(-(-8*RootOf(_Z^4+2)*x^24+32*RootOf(_Z^4+2)*x^20
-8*(8*x^24-24*x^20+12*x^16+16*x^12-6*x^8-6*x^4-1)^(1/4)*RootOf(_Z^4+2)^2*x^
17-28*RootOf(_Z^4+2)*x^16+16*(8*x^24-24*x^20+12*x^16+16*x^12-6*x^8-6*x^4-1)
^(1/4)*RootOf(_Z^4+2)^2*x^13-4*(8*x^24-24*x^20+12*x^16+16*x^12-6*x^8-6*x^4-
1)^(1/2)*RootOf(_Z^4+2)^3*x^10-16*RootOf(_Z^4+2)*x^12+4*(8*x^24-24*x^20+12*
x^16+16*x^12-6*x^8-6*x^4-1)^(1/2)*RootOf(_Z^4+2)^3*x^6+14*RootOf(_Z^4+2)*x^
8-8*(8*x^24-24*x^20+12*x^16+16*x^12-6*x^8-6*x^4-1)^(1/4)*RootOf(_Z^4+2)^2*x
^5+2*(8*x^24-24*x^20+12*x^16+16*x^12-6*x^8-6*x^4-1)^(1/2)*RootOf(_Z^4+2)^3*
x^2+4*(8*x^24-24*x^20+12*x^16+16*x^12-6*x^8-6*x^4-1)^(3/4)*x^3+8*RootOf(_Z^
4+2)*x^4-2*(8*x^24-24*x^20+12*x^16+16*x^12-6*x^8-6*x^4-1)^(1/4)*RootOf(_Z^4
+2)^2*x+RootOf(_Z^4+2))/(2*x^8-1)/(2*x^8-2*x^4-1)^2)+1/4*RootOf(_Z^2+RootOf
(_Z^4+2)^2)*ln(-(8*x^24*RootOf(_Z^2+RootOf(_Z^4+2)^2)-32*RootOf(_Z^2+RootOf
(_Z^4+2)^2)*x^20+8*(8*x^24-24*x^20+12*x^16+16*x^12-6*x^8-6*x^4-1)^(1/4)*Roo
tOf(_Z^4+2)^2*x^17+28*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^16-16*(8*x^24-24*x^20
+12*x^16+16*x^12-6*x^8-6*x^4-1)^(1/4)*RootOf(_Z^4+2)^2*x^13-4*(8*x^24-24*x^
20+12*x^16+16*x^12-6*x^8-6*x^4-1)^(1/2)*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootO
f(_Z^4+2)^2*x^10+16*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^12+4*(8*x^24-24*x^20+12
*x^16+16*x^12-6*x^8-6*x^4-1)^(1/2)*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^
4+2)^2*x^6-14*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^8+8*(8*x^24-24*x^20+12*x^16+1
6*x^12-6*x^8-6*x^4-1)^(1/4)*RootOf(_Z^4+2)^2*x^5+2*(8*x^24-24*x^20+12*x^16+
16*x^12-6*x^8-6*x^4-1)^(1/2)*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^4+2)^2
*x^2+4*(8*x^24-24*x^20+12*x^16+16*x^12-6*x^8-6*x^4-1)^(3/4)*x^3-8*RootOf(_Z
^2+RootOf(_Z^4+2)^2)*x^4+2*(8*x^24-24*x^20+12*x^16+16*x^12-6*x^8-6*x^4-1)^(
```

```
1/4)*RootOf(_Z^4+2)^2*x-RootOf(_Z^2+RootOf(_Z^4+2)^2))/(2*x^8-1)/(2*x^8-2*x
^4-1)^2))/(2*x^8-2*x^4-1)^(3/4)*((2*x^8-2*x^4-1)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^{16}-3\,x^{8}+1\right)\left(2\,x^{8}-2\,x^{4}-1\right)^{\frac{1}{4}}\left(2\,x^{8}+1\right)}{\left(2\,x^{8}-1\right)x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)*(2*x^8-2*x^4-1)^(1/4)*(4*x^16-3*x^8+1)/x^10/(2*x^8-1),x
, algorithm="maxima")
```

```
[Out] integrate((4*x^16 - 3*x^8 + 1)*(2*x^8 - 2*x^4 - 1)^(1/4)*(2*x^8 + 1)/((2*x^
8 - 1)*x^10), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^8+1\right)\,{\left(2\,x^8-2\,x^4-1\right)}^{1/4}\,\left(4\,x^{16}-3\,x^8+1\right)}{x^{10}\,\left(2\,x^8-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^8 + 1)*(2*x^8 - 2*x^4 - 1)^(1/4)*(4*x^16 - 3*x^8 + 1))/(x^10*(2*x
^8 - 1)),x)
```

```
[Out] int(((2*x^8 + 1)*(2*x^8 - 2*x^4 - 1)^(1/4)*(4*x^16 - 3*x^8 + 1))/(x^10*(2*x
^8 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8+1)*(2*x**8-2*x**4-1)**(1/4)*(4*x**16-3*x**8+1)/x**10/(2*x
**8-1),x)
```

```
[Out] Timed out
```

3.1874 $\int \frac{\left(b^2+ax^2\right)^{3/2}}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$

Optimal. Leaf size=173

$$\frac{2x\sqrt{ax^2+b^2}\,\left(15ax^2+46b^2\right)}{105\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{2\sqrt{2}\,b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\,\sqrt{b}\,\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{\sqrt{2}\,\sqrt{b}}\right)}{\sqrt{a}}-\frac{2x\left(3abx^2+46b^3\right)}{105\sqrt{\sqrt{ax^2+b^2}+b}}$$

Rubi [F] time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(b^2+ax^2\right)^{3/2}}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(b^2 + a*x^2)^(3/2)/Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] Defer[Int][(b^2 + a*x^2)^(3/2)/Sqrt[b + Sqrt[b^2 + a*x^2]], x]
```

Rubi steps

$$\int \frac{\left(b^2+ax^2\right)^{3/2}}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx=\int \frac{\left(b^2+ax^2\right)^{3/2}}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Mathematica [C] time = 0.64, size = 260, normalized size = 1.50

$$\frac{60a^3x^6+232a^2b^2x^4+48a^2bx^4\sqrt{ax^2+b^2}-210ab^4x^2+105\sqrt{2}\,b^{7/2}\sqrt{\sqrt{ax^2+b^2}-b}\,\left(2b\sqrt{ax^2+b^2}+ax^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{ax^2+b^2}-b}}{\sqrt{2}\,\sqrt{b}}\right)-420b^5\sqrt{ax^2+b^2}+210b^4\left(2b\sqrt{ax^2+b^2}+ax^2+2b^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b-\sqrt{b^2+ax^2}}{2b}\right)-420b^6}{210ax\left(\sqrt{ax^2+b^2}+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2 + a*x^2)^(3/2)/Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] (-420*b^6 - 210*a*b^4*x^2 + 232*a^2*b^2*x^4 + 60*a^3*x^6 - 420*b^5*Sqrt[b^2
 + a*x^2] + 48*a^2*b*x^4*Sqrt[b^2 + a*x^2] + 105*Sqrt[2]*b^(7/2)*Sqrt[-b +
Sqrt[b^2 + a*x^2]]*(2*b^2 + a*x^2 + 2*b*Sqrt[b^2 + a*x^2])*ArcTan[Sqrt[-b +
 Sqrt[b^2 + a*x^2]]/(Sqrt[2]*Sqrt[b])] + 210*b^4*(2*b^2 + a*x^2 + 2*b*Sqrt[
b^2 + a*x^2])*Hypergeometric2F1[-1/2, 1, 1/2, (b - Sqrt[b^2 + a*x^2])/(2*b)
])/(210*a*x*(b + Sqrt[b^2 + a*x^2])^(3/2))
```

IntegrateAlgebraic [A] time = 0.27, size = 140, normalized size = 0.81

$$\frac{2x\sqrt{ax^2+b^2}\,\left(15ax^2+46b^2\right)}{105\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{\sqrt{2}\,b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\,\sqrt{b}\,\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}-\frac{2x\left(3abx^2+46b^3\right)}{105\sqrt{\sqrt{ax^2+b^2}+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^2 + a*x^2)^(3/2)/Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] (2*x*Sqrt[b^2 + a*x^2]*(46*b^2 + 15*a*x^2))/(105*Sqrt[b + Sqrt[b^2 + a*x^2]
]) - (2*x*(46*b^3 + 3*a*b*x^2))/(105*Sqrt[b + Sqrt[b^2 + a*x^2]]) + (Sqrt[2
]*b^(7/2)*ArcTan[(Sqrt[a]*x)/(Sqrt[2]*Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])]
)/Sqrt[a]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^(3/2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2 + b^2\right)^{\frac{3}{2}}}{\sqrt{b + \sqrt{ax^2 + b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^(3/2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*x^2 + b^2)^(3/2)/sqrt(b + sqrt(a*x^2 + b^2)), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^2 + b^2\right)^{\frac{3}{2}}}{\sqrt{b + \sqrt{a\,x^2 + b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b^2)^(3/2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x^2+b^2)^(3/2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2 + b^2\right)^{\frac{3}{2}}}{\sqrt{b + \sqrt{ax^2 + b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^(3/2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*x^2 + b^2)^(3/2)/sqrt(b + sqrt(a*x^2 + b^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b^2 + a\,x^2\right)}^{3/2}}{\sqrt{b + \sqrt{b^2 + a\,x^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b^2)^(3/2)/(b + (a*x^2 + b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x^2 + b^2)^(3/2)/(b + (a*x^2 + b^2)^(1/2))^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2 + b^2\right)^{\frac{3}{2}}}{\sqrt{b + \sqrt{ax^2 + b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b**2)**(3/2)/(b+(a*x**2+b**2)**(1/2))**(1/2),x)
```

```
[Out] Integral((a*x**2 + b**2)**(3/2)/sqrt(b + sqrt(a*x**2 + b**2)), x)
```

3.1875 $\int \frac{x(-2q+px^6)\sqrt{q+px^6}}{bx^8+a(q+px^6)^2}\,dx$

Optimal. Leaf size=173

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^2\sqrt{px^6+q}}{\sqrt{a}px^6+\sqrt{a}q-\sqrt{b}x^4}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\frac{\sqrt[4]{a}px^6}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{a}q}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}x^4}{\sqrt{2}\sqrt[4]{a}}}{x^2\sqrt{px^6+q}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

Rubi [A] time = 0.59, antiderivative size = 255, normalized size of antiderivative = 1.47, number of steps used = 10, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {6714, 211, 1165, 628, 1162, 617, 204}

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^2}{\sqrt[4]{a}\sqrt{px^6+q}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x^2}{\sqrt[4]{a}\sqrt{px^6+q}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^2}{\sqrt{px^6+q}}+\sqrt{a}+\frac{\sqrt{b}x^4}{px^6+q}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^2}{\sqrt{px^6+q}}+\sqrt{a}+\frac{\sqrt{b}x^4}{px^6+q}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-2*q + p*x^6)*Sqrt[q + p*x^6])/(b*x^8 + a*(q + p*x^6)^2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*b^(1/4)*x^2)/(a^(1/4)*Sqrt[q + p*x^6])]/(2*Sqrt[2]*a^(3
/4)*b^(1/4)) - ArcTan[1 + (Sqrt[2]*b^(1/4)*x^2)/(a^(1/4)*Sqrt[q + p*x^6])]/
(2*Sqrt[2]*a^(3/4)*b^(1/4)) + Log[Sqrt[a] + (Sqrt[b]*x^4)/(q + p*x^6) - (Sq
rt[2]*a^(1/4)*b^(1/4)*x^2)/Sqrt[q + p*x^6]]/(4*Sqrt[2]*a^(3/4)*b^(1/4)) - L
og[Sqrt[a] + (Sqrt[b]*x^4)/(q + p*x^6) + (Sqrt[2]*a^(1/4)*b^(1/4)*x^2)/Sqrt
[q + p*x^6]]/(4*Sqrt[2]*a^(3/4)*b^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6714

```
Int[(u_)*(v_)^(r_.)*(w_)^(s_.)*((a_.)*(v_)^(p_.) + (b_.)*(w_)^(q_.))^(m_.),
 x_Symbol] :> With[{c = Simplify[u/(p*w*D[v, x] - q*v*D[w, x])]}, -Dist[(c*
q)/(s + 1), Subst[Int[(a + b*x^(q/(s + 1)))^m, x], x, v^(m*p + r + 1)*w^(s
+ 1)], x] /; FreeQ[c, x]] /; FreeQ[{a, b, m, p, q, r, s}, x] && EqQ[p*(s +
1) + q*(m*p + r + 1), 0] && NeQ[s, -1] && IntegerQ[q/(s + 1)] && IntegerQ[m
]
```

Rubi steps

$$\int \frac{x\left(-2q+px^6\right)\sqrt{q+px^6}}{bx^8+a\left(q+px^6\right)^2}\,dx = -\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \frac{x^2}{\sqrt{q+px^6}}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \frac{x^2}{\sqrt{q+px^6}}\right)}{2\sqrt{a}} - \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \frac{x^2}{\sqrt{q+px^6}}\right)}{2\sqrt{a}}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x^2}{\sqrt{q+px^6}}\right)}{4\sqrt{a}\sqrt{b}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x^2}{\sqrt{q+px^6}}\right)}{4\sqrt{a}\sqrt{b}}$$

$$= \frac{\log\left(\sqrt{a}+\frac{\sqrt{b}x^4}{q+px^6}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^2}{\sqrt{q+px^6}}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{\log\left(\sqrt{a}+\frac{\sqrt{b}x^4}{q+px^6}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^2}{\sqrt{q+px^6}}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}} - \text{Subst}\Big(\int$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^2}{\sqrt[4]{a}\sqrt{q+px^6}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x^2}{\sqrt[4]{a}\sqrt{q+px^6}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{\log\left(\sqrt{a}+\frac{\sqrt{b}x^4}{q+px^6}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^2}{\sqrt{q+px^6}}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

Mathematica [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x\left(-2q+px^6\right)\sqrt{q+px^6}}{bx^8+a\left(q+px^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(-2*q + p*x^6)*Sqrt[q + p*x^6])/(b*x^8 + a*(q + p*x^6)^2),x]
[Out] Integrate[(x*(-2*q + p*x^6)*Sqrt[q + p*x^6])/(b*x^8 + a*(q + p*x^6)^2), x]
```

IntegrateAlgebraic [A] time = 27.74, size = 173, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^2\sqrt{px^6+q}}{\sqrt{a}px^6+\sqrt{a}q-\sqrt{b}x^4}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} - \frac{\tanh^{-1}\left(\frac{\frac{\sqrt[4]{a}px^6}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{a}q}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}x^4}{\sqrt{2}\sqrt[4]{a}}}{x^2\sqrt{px^6+q}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-2*q + p*x^6)*Sqrt[q + p*x^6])/(b*x^8 + a*(q + p*x^6
)^2),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x^2*Sqrt[q + p*x^6])/(Sqrt[a]*q - Sqrt
[b]*x^4 + Sqrt[a]*p*x^6)]/(Sqrt[2]*a^(3/4)*b^(1/4)) - ArcTanh[((a^(1/4)*q)/
(Sqrt[2]*b^(1/4)) + (b^(1/4)*x^4)/(Sqrt[2]*a^(1/4)) + (a^(1/4)*p*x^6)/(Sqrt
[2]*b^(1/4)))/(x^2*Sqrt[q + p*x^6])]/(2*Sqrt[2]*a^(3/4)*b^(1/4))
```

fricas [B] time = 1.65, size = 361, normalized size = 2.09

$$\frac{1}{2}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2bx^2\left(-\frac{1}{a^3b}\right)^{\frac{3}{4}}}{\sqrt{px^6+q}}\right)+\frac{1}{8}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(\frac{ap^2x^{12}+2\,apqx^6-bx^8+aq^2+2\left(abx^6\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+\left(a^3bpx^8+a^3bqx^2\right)\left(-\frac{1}{a^3b}\right)^{\frac{3}{4}}\right)\sqrt{px^6+q}-2\left(a^2bpx^{10}+a^2bqx^4\right)\sqrt{-\frac{1}{a^3b}}}{ap^2x^{12}+2\,apqx^6+bx^8+aq^2}\right)-\frac{1}{8}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(\frac{ap^2x^{12}+2\,apqx^6-bx^8+aq^2-2\left(abx^6\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+\left(a^3bpx^8+a^3bqx^2\right)\left(-\frac{1}{a^3b}\right)^{\frac{3}{4}}\right)\sqrt{px^6+q}-2\left(a^2bpx^{10}+a^2bqx^4\right)\sqrt{-\frac{1}{a^3b}}}{ap^2x^{12}+2\,apqx^6+bx^8+aq^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(p*x^6-2*q)*(p*x^6+q)^(1/2)/(b*x^8+a*(p*x^6+q)^2),x, algorithm=
"fricas")
```

```
[Out] 1/2*(-1/(a^3*b))^(1/4)*arctan(a^2*b*x^2*(-1/(a^3*b))^(3/4)/sqrt(p*x^6 + q))
 + 1/8*(-1/(a^3*b))^(1/4)*log((a*p^2*x^12 + 2*a*p*q*x^6 - b*x^8 + a*q^2 + 2
*(a*b*x^6*(-1/(a^3*b))^(1/4) + (a^3*b*p*x^8 + a^3*b*q*x^2)*(-1/(a^3*b))^(3/
4))*sqrt(p*x^6 + q) - 2*(a^2*b*p*x^10 + a^2*b*q*x^4)*sqrt(-1/(a^3*b)))/(a*p
^2*x^12 + 2*a*p*q*x^6 + b*x^8 + a*q^2)) - 1/8*(-1/(a^3*b))^(1/4)*log((a*p^2
*x^12 + 2*a*p*q*x^6 - b*x^8 + a*q^2 - 2*(a*b*x^6*(-1/(a^3*b))^(1/4) + (a^3*
b*p*x^8 + a^3*b*q*x^2)*(-1/(a^3*b))^(3/4))*sqrt(p*x^6 + q) - 2*(a^2*b*p*x^1
0 + a^2*b*q*x^4)*sqrt(-1/(a^3*b)))/(a*p^2*x^12 + 2*a*p*q*x^6 + b*x^8 + a*q^
2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^6+q}\left(px^6-2\,q\right)x}{bx^8+\left(px^6+q\right)^2a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(p*x^6-2*q)*(p*x^6+q)^(1/2)/(b*x^8+a*(p*x^6+q)^2),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(p*x^6 + q)*(p*x^6 - 2*q)*x/(b*x^8 + (p*x^6 + q)^2*a), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x\left(p\,x^6-2q\right)\sqrt{p\,x^6+q}}{b\,x^8+a\left(p\,x^6+q\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(p*x^6-2*q)*(p*x^6+q)^(1/2)/(b*x^8+a*(p*x^6+q)^2),x)
```

```
[Out] int(x*(p*x^6-2*q)*(p*x^6+q)^(1/2)/(b*x^8+a*(p*x^6+q)^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^6+q}\left(px^6-2\,q\right)x}{bx^8+\left(px^6+q\right)^2a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(p*x^6-2*q)*(p*x^6+q)^(1/2)/(b*x^8+a*(p*x^6+q)^2),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(p*x^6 + q)*(p*x^6 - 2*q)*x/(b*x^8 + (p*x^6 + q)^2*a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x\,\sqrt{p\,x^6+q}\,\left(2\,q-p\,x^6\right)}{a\,{\left(p\,x^6+q\right)}^2+b\,x^8} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(q + p*x^6)^(1/2)*(2*q - p*x^6))/(a*(q + p*x^6)^2 + b*x^8),x)
[Out] int(-(x*(q + p*x^6)^(1/2)*(2*q - p*x^6))/(a*(q + p*x^6)^2 + b*x^8), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(px^{6}-2q\right)\sqrt{px^{6}+q}}{ap^{2}x^{12}+2apqx^{6}+aq^{2}+bx^{8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(p*x**6-2*q)*(p*x**6+q)**(1/2)/(b*x**8+a*(p*x**6+q)**2),x)
[Out] Integral(x*(p*x**6 - 2*q)*sqrt(p*x**6 + q)/(a*p**2*x**12 + 2*a*p*q*x**6 + a
*q**2 + b*x**8), x)
```

3.1876 $\int \frac{\sqrt[3]{-1-2x+6x^2}}{-1+6x}\,dx$

Optimal. Leaf size=174

$$\frac{1}{4}\sqrt[3]{6x^2-2x-1}-\frac{1}{12}\sqrt[3]{\frac{7}{6}}\log\left(\sqrt[3]{6}\,7^{2/3}\sqrt[3]{6x^2-2x-1}+7\right)+\frac{1}{24}\sqrt[3]{\frac{7}{6}}\log\left(6^{2/3}\sqrt[3]{7}\left(6x^2-2x-1\right)^{2/3}-\sqrt[3]{6}\,7^{2/3}\sqrt[3]{6}\right.$$

Rubi [A] time = 0.16, antiderivative size = 119, normalized size of antiderivative = 0.68, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {694, 266, 50, 58, 617, 204, 31}

$$\frac{\sqrt[3]{(6x-1)^2-7}}{4\sqrt[3]{6}}+\frac{1}{12}\sqrt[3]{\frac{7}{6}}\log(1-6x)-\frac{1}{8}\sqrt[3]{\frac{7}{6}}\log\left(\sqrt[3]{(6x-1)^2-7}+\sqrt[3]{7}\right)+\frac{\sqrt[3]{\frac{7}{2}}\tan^{-1}\left(\frac{7-2\,7^{2/3}\sqrt[3]{(6x-1)^2-7}}{7\sqrt{3}}\right)}{4\,3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 2*x + 6*x^2)^(1/3)/(-1 + 6*x),x]
```

```
[Out] (-7 + (-1 + 6*x)^2)^(1/3)/(4*6^(1/3)) + ((7/2)^(1/3)*ArcTan[(7 - 2*7^(2/3)*
(-7 + (-1 + 6*x)^2)^(1/3))/(7*Sqrt[3])])/(4*3^(5/6)) + ((7/6)^(1/3)*Log[1 -
 6*x])/12 - ((7/6)^(1/3)*Log[7^(1/3) + (-7 + (-1 + 6*x)^2)^(1/3)])/8
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 694

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Dist[1/e, Subst[Int[x^m*(a - b^2/(4*c) + (c*x^2)/e^2)^p, x], x, d
 + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{-1-2x+6x^2}}{-1+6x}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{\sqrt[3]{-\frac{7}{6}+\frac{x^2}{6}}}{x}\,dx, x, -1+6x\right)$$

$$= \frac{1}{12}\,\text{Subst}\left(\int \frac{\sqrt[3]{-\frac{7}{6}+\frac{x}{6}}}{x}\,dx, x, (-1+6x)^2\right)$$

$$= \frac{\sqrt[3]{-7+(-1+6x)^2}}{4\sqrt[3]{6}} - \frac{7}{72}\,\text{Subst}\left(\int \frac{1}{\left(-\frac{7}{6}+\frac{x}{6}\right)^{2/3}x}\,dx, x, (-1+6x)^2\right)$$

$$= \frac{\sqrt[3]{-7+(-1+6x)^2}}{4\sqrt[3]{6}} + \frac{1}{12}\sqrt[3]{\frac{7}{6}}\log(1-6x) - \frac{1}{8}\sqrt[3]{\frac{7}{6}}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{\frac{7}{6}}+x}\,dx, x, \sqrt[3]{-1-2x+\right.$$

$$= \frac{\sqrt[3]{-7+(-1+6x)^2}}{4\sqrt[3]{6}} + \frac{1}{12}\sqrt[3]{\frac{7}{6}}\log(1-6x) - \frac{1}{8}\sqrt[3]{\frac{7}{6}}\log\left(\sqrt[3]{7}+\sqrt[3]{6}\sqrt[3]{-1-2x+6x^2}\right) - \frac{1}{4}$$

$$= \frac{\sqrt[3]{-7+(-1+6x)^2}}{4\sqrt[3]{6}} + \frac{\sqrt[3]{\frac{7}{2}}\tan^{-1}\left(\frac{7-2\sqrt[3]{6}\,7^{2/3}\sqrt[3]{-1-2x+6x^2}}{7\sqrt{3}}\right)}{4\ 3^{5/6}} + \frac{1}{12}\sqrt[3]{\frac{7}{6}}\log(1-6x) - \frac{1}{8}\sqrt[3]{\frac{7}{6}}\ 1$$

Mathematica [A] time = 0.12, size = 152, normalized size = 0.87

$$\frac{1}{4}\sqrt[3]{6x^2-2x-1} + \frac{\sqrt[3]{\frac{7}{2}}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{\frac{2}{7}}\sqrt[3]{6x^2-2x-1}}{\sqrt[6]{3}}\right)}{4\ 3^{5/6}} - \frac{1}{12}\sqrt[3]{\frac{7}{6}}\log\left(\sqrt[3]{(1-6x)^2-7}+\sqrt[3]{7}\right) + \frac{1}{24}\sqrt[3]{\frac{7}{6}}\log\left(\left((1-6x)^2-7\right)^{2/3} - \sqrt[3]{7}\sqrt[3]{(1-6x)^2-7} + 7^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x + 6*x^2)^(1/3)/(-1 + 6*x),x]
```

```
[Out] (-1 - 2*x + 6*x^2)^(1/3)/4 + ((7/2)^(1/3)*ArcTan[1/Sqrt[3] - (2*(2/7)^(1/3)
*(-1 - 2*x + 6*x^2)^(1/3))/3^(1/6)])/(4*3^(5/6)) - ((7/6)^(1/3)*Log[7^(1/3)
 + (-7 + (1 - 6*x)^2)^(1/3)])/12 + ((7/6)^(1/3)*Log[7^(2/3) - 7^(1/3)*(-7 +
 (1 - 6*x)^2)^(1/3) + (-7 + (1 - 6*x)^2)^(2/3)])/24
```

IntegrateAlgebraic [A] time = 0.44, size = 174, normalized size = 1.00

$$\frac{1}{4}\sqrt[3]{6x^2-2x-1} - \frac{1}{12}\sqrt[3]{\frac{7}{6}}\log\left(\sqrt[3]{6}\,7^{2/3}\sqrt[3]{6x^2-2x-1}+7\right) + \frac{1}{24}\sqrt[3]{\frac{7}{6}}\log\left(6^{2/3}\sqrt[3]{7}\left(6x^2-2x-1\right)^{2/3} - \sqrt[3]{6}\,7^{2/3}\sqrt[3]{6x^2-2x-1}+7\right) + \frac{\sqrt[3]{\frac{7}{2}}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{\frac{2}{7}}\sqrt[3]{6x^2-2x-1}}{\sqrt[6]{3}}\right)}{4\ 3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 2*x + 6*x^2)^(1/3)/(-1 + 6*x),x]
```

```
[Out] (-1 - 2*x + 6*x^2)^(1/3)/4 + ((7/2)^(1/3)*ArcTan[1/Sqrt[3] - (2*(2/7)^(1/3)
*(-1 - 2*x + 6*x^2)^(1/3))/3^(1/6)])/(4*3^(5/6)) - ((7/6)^(1/3)*Log[7 + 6^(
1/3)*7^(2/3)*(-1 - 2*x + 6*x^2)^(1/3)])/12 + ((7/6)^(1/3)*Log[7 - 6^(1/3)*7
^(2/3)*(-1 - 2*x + 6*x^2)^(1/3) + 6^(2/3)*7^(1/3)*(-1 - 2*x + 6*x^2)^(2/3)]
)/24
```

fricas [A] time = 0.84, size = 146, normalized size = 0.84

$$\frac{1}{24} \cdot 6^{\frac{1}{6}} \sqrt{2} (-7)^{\frac{1}{3}} \arctan\left(\frac{1}{42} \cdot 6^{\frac{1}{6}} \left(2 \cdot 6^{\frac{2}{3}} \sqrt{2} (-7)^{\frac{2}{3}} \left(6x^2 - 2x - 1\right)^{\frac{1}{3}} - 7 \cdot 6^{\frac{1}{3}} \sqrt{2}\right)\right) - \frac{1}{144} \cdot 6^{\frac{2}{3}} (-7)^{\frac{1}{3}} \log\left(6^{\frac{2}{3}} (-7)^{\frac{1}{3}} \left(6x^2 - 2x - 1\right)^{\frac{1}{3}} + 6^{\frac{1}{3}} (-7)^{\frac{2}{3}} + 6\left(6x^2 - 2x - 1\right)^{\frac{2}{3}}\right) + \frac{1}{72} \cdot 6^{\frac{2}{3}} (-7)^{\frac{1}{3}} \log\left(-6^{\frac{2}{3}} (-7)^{\frac{1}{3}} + 6\left(6x^2 - 2x - 1\right)^{\frac{1}{3}}\right) + \frac{1}{4} \left(6x^2 - 2x - 1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2-2*x-1)^(1/3)/(-1+6*x),x, algorithm="fricas")
```

```
[Out] 1/24*6^(1/6)*sqrt(2)*(-7)^(1/3)*arctan(1/42*6^(1/6)*(2*6^(2/3)*sqrt(2)*(-7)
^(2/3)*(6*x^2 - 2*x - 1)^(1/3) - 7*6^(1/3)*sqrt(2))) - 1/144*6^(2/3)*(-7)^(
1/3)*log(6^(2/3)*(-7)^(1/3)*(6*x^2 - 2*x - 1)^(1/3) + 6^(1/3)*(-7)^(2/3) +
6*(6*x^2 - 2*x - 1)^(2/3)) + 1/72*6^(2/3)*(-7)^(1/3)*log(-6^(2/3)*(-7)^(1/3
) + 6*(6*x^2 - 2*x - 1)^(1/3)) + 1/4*(6*x^2 - 2*x - 1)^(1/3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(6x^2 - 2x - 1\right)^{\frac{1}{3}}}{6x - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2-2*x-1)^(1/3)/(-1+6*x),x, algorithm="giac")
```

```
[Out] integrate((6*x^2 - 2*x - 1)^(1/3)/(6*x - 1), x)
```

maple [C] time = 8.25, size = 2661, normalized size = 15.29

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x^2-2*x-1)^(1/3)/(-1+6*x),x)
```

```
[Out] 1/4*(6*x^2-2*x-1)^(1/3)+(1/72*RootOf(_Z^3+252)*ln(-(15846*RootOf(_Z^3+252)*
x^2-6194*RootOf(_Z^3+252)*x-2413*RootOf(_Z^3+252)+5334*RootOf(RootOf(_Z^3+2
52)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)-35028*RootOf(RootOf(_Z^3+252)^2+42*
_Z*RootOf(_Z^3+252)+1764*_Z^2)*x^2+13692*RootOf(RootOf(_Z^3+252)^2+42*_Z*Ro
otOf(_Z^3+252)+1764*_Z^2)*x+54432*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z
^3+252)+1764*_Z^2)*x^4-36288*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+25
2)+1764*_Z^2)*x^3-24624*RootOf(_Z^3+252)*x^4+16416*RootOf(_Z^3+252)*x^3-100
548*(36*x^4-24*x^3-8*x^2+4*x+1)^(1/3)*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootO
f(_Z^3+252)+1764*_Z^2)*RootOf(_Z^3+252)*x^2+10584*RootOf(RootOf(_Z^3+252)^2
+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)^2*RootOf(_Z^3+252)^2*x^4-7056*RootOf(Roo
tOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)^2*RootOf(_Z^3+252)^2*x^3-
4788*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*RootOf(_Z^
3+252)^3*x^4+3192*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^
2)*RootOf(_Z^3+252)^3*x^3-588*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+2
52)+1764*_Z^2)^2*RootOf(_Z^3+252)^2*x^2+266*RootOf(RootOf(_Z^3+252)^2+42*_Z
*RootOf(_Z^3+252)+1764*_Z^2)*RootOf(_Z^3+252)^3*x^2+588*RootOf(RootOf(_Z^3+
252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)^2*RootOf(_Z^3+252)^2*x-266*RootOf(
RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*RootOf(_Z^3+252)^3*x+3
9*RootOf(_Z^3+252)^2*(36*x^4-24*x^3-8*x^2+4*x+1)^(1/3)+33516*(36*x^4-24*x^3
-8*x^2+4*x+1)^(1/3)*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_
Z^2)*RootOf(_Z^3+252)*x-16758*(36*x^4-24*x^3-8*x^2+4*x+1)^(2/3)+2520*(36*x^
```

4-24*x^3-8*x^2+4*x+1)^(2/3)*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252
)+1764*_Z^2)*RootOf(_Z^3+252)^2-234*RootOf(_Z^3+252)^2*(36*x^4-24*x^3-8*x^2
+4*x+1)^(1/3)*x^2+78*RootOf(_Z^3+252)^2*(36*x^4-24*x^3-8*x^2+4*x+1)^(1/3)*x
+16758*(36*x^4-24*x^3-8*x^2+4*x+1)^(1/3)*RootOf(RootOf(_Z^3+252)^2+42*_Z*Ro
otOf(_Z^3+252)+1764*_Z^2)*RootOf(_Z^3+252))/(6*x^2-2*x-1)/(-1+6*x)^2)-1/72*
ln((17442*RootOf(_Z^3+252)*x^2-7790*RootOf(_Z^3+252)*x-2413*RootOf(_Z^3+252
)-106680*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)+771120
*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*x^2-344400*Roo
tOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*x-2358720*RootOf(R
ootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*x^4+1572480*RootOf(Root
Of(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*x^3-53352*RootOf(_Z^3+252)
*x^4+35568*RootOf(_Z^3+252)*x^3+9828*(36*x^4-24*x^3-8*x^2+4*x+1)^(1/3)*Root
Of(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*RootOf(_Z^3+252)*x^
2+211680*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)^2*Root
Of(_Z^3+252)^2*x^4-141120*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+
1764*_Z^2)^2*RootOf(_Z^3+252)^2*x^3+4788*RootOf(RootOf(_Z^3+252)^2+42*_Z*Ro
otOf(_Z^3+252)+1764*_Z^2)*RootOf(_Z^3+252)^3*x^4-3192*RootOf(RootOf(_Z^3+25
2)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*RootOf(_Z^3+252)^3*x^3-11760*RootOf(
RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)^2*RootOf(_Z^3+252)^2*x
^2-266*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*RootOf(_
Z^3+252)^3*x^2+11760*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*
_Z^2)^2*RootOf(_Z^3+252)^2*x+266*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^
3+252)+1764*_Z^2)*RootOf(_Z^3+252)^3*x-399*RootOf(_Z^3+252)^2*(36*x^4-24*x^
3-8*x^2+4*x+1)^(1/3)-3276*(36*x^4-24*x^3-8*x^2+4*x+1)^(1/3)*RootOf(RootOf(_
Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*RootOf(_Z^3+252)*x+1638*(36*x^
4-24*x^3-8*x^2+4*x+1)^(2/3)+2520*(36*x^4-24*x^3-8*x^2+4*x+1)^(2/3)*RootOf(R
ootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*RootOf(_Z^3+252)^2+2394
RootOf(_Z^3+252)^2(36*x^4-24*x^3-8*x^2+4*x+1)^(1/3)*x^2-798*RootOf(_Z^3+2
52)^2*(36*x^4-24*x^3-8*x^2+4*x+1)^(1/3)*x-1638*(36*x^4-24*x^3-8*x^2+4*x+1)^
(1/3)*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*RootOf(_Z
^3+252))/(6*x^2-2*x-1)/(-1+6*x)^2)*RootOf(_Z^3+252)-7/12*ln((17442*RootOf(_
Z^3+252)*x^2-7790*RootOf(_Z^3+252)*x-2413*RootOf(_Z^3+252)-106680*RootOf(Ro
otOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)+771120*RootOf(RootOf(_Z^
3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*x^2-344400*RootOf(RootOf(_Z^3+25
2)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*x-2358720*RootOf(RootOf(_Z^3+252)^2+
42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*x^4+1572480*RootOf(RootOf(_Z^3+252)^2+42*
_Z*RootOf(_Z^3+252)+1764*_Z^2)*x^3-53352*RootOf(_Z^3+252)*x^4+35568*RootOf(
_Z^3+252)*x^3+9828*(36*x^4-24*x^3-8*x^2+4*x+1)^(1/3)*RootOf(RootOf(_Z^3+252
)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*RootOf(_Z^3+252)*x^2+211680*RootOf(Ro
otOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)^2*RootOf(_Z^3+252)^2*x^4
-141120*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)^2*RootO
f(_Z^3+252)^2*x^3+4788*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+176
4*_Z^2)*RootOf(_Z^3+252)^3*x^4-3192*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(
_Z^3+252)+1764*_Z^2)*RootOf(_Z^3+252)^3*x^3-11760*RootOf(RootOf(_Z^3+252)^2
+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)^2*RootOf(_Z^3+252)^2*x^2-266*RootOf(Root
Of(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*RootOf(_Z^3+252)^3*x^2+117
60*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)^2*RootOf(_Z^
3+252)^2*x+266*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*
RootOf(_Z^3+252)^3*x-399*RootOf(_Z^3+252)^2*(36*x^4-24*x^3-8*x^2+4*x+1)^(1/
3)-3276*(36*x^4-24*x^3-8*x^2+4*x+1)^(1/3)*RootOf(RootOf(_Z^3+252)^2+42*_Z*R
ootOf(_Z^3+252)+1764*_Z^2)*RootOf(_Z^3+252)*x+1638*(36*x^4-24*x^3-8*x^2+4*x
+1)^(2/3)+2520*(36*x^4-24*x^3-8*x^2+4*x+1)^(2/3)*RootOf(RootOf(_Z^3+252)^2+
42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*RootOf(_Z^3+252)^2+2394*RootOf(_Z^3+252)^
2*(36*x^4-24*x^3-8*x^2+4*x+1)^(1/3)*x^2-798*RootOf(_Z^3+252)^2*(36*x^4-24*x
^3-8*x^2+4*x+1)^(1/3)*x-1638*(36*x^4-24*x^3-8*x^2+4*x+1)^(1/3)*RootOf(RootO
f(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2)*RootOf(_Z^3+252))/(6*x^2-2*
x-1)/(-1+6*x)^2)*RootOf(RootOf(_Z^3+252)^2+42*_Z*RootOf(_Z^3+252)+1764*_Z^2
))/(6*x^2-2*x-1)^(2/3)*((6*x^2-2*x-1)^2)^(1/3)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(6\,x^2-2\,x-1\right)^{\frac{1}{3}}}{6\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2-2*x-1)^(1/3)/(-1+6*x),x, algorithm="maxima")
[Out] integrate((6*x^2 - 2*x - 1)^(1/3)/(6*x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(6\,x^2-2\,x-1\right)^{1/3}}{6\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x^2 - 2*x - 1)^(1/3)/(6*x - 1),x)
[Out] int((6*x^2 - 2*x - 1)^(1/3)/(6*x - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{6x^2-2x-1}}{6x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x**2-2*x-1)**(1/3)/(-1+6*x),x)
[Out] Integral((6*x**2 - 2*x - 1)**(1/3)/(6*x - 1), x)
```

3.1877 $\int \frac{-2abx+(a+b)x^2}{(x(-a+x)(-b+x))^{2/3}\left(-ab+(a+b)x+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=174

$$-\frac{\log\left(\sqrt[3]{x^2(-a-b)+abx+x^3}-\sqrt[3]{d}\,x\right)}{d^{2/3}}+\frac{\log\left(\sqrt[3]{d}\,x\sqrt[3]{x^2(-a-b)+abx+x^3}+\left(x^2(-a-b)+abx+x^3\right)^{2/3}+d^{2/3}x^2\right.}{2d^{2/3}}$$

Rubi [F] time = 4.63, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2abx+(a+b)x^2}{(x(-a+x)(-b+x))^{2/3}\left(-ab+(a+b)x+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*a*b*x + (a + b)*x^2)/((x*(-a + x)*(-b + x))^(2/3)*(-(a*b) + (a + b)
*x + (-1 + d)*x^2)),x]
```

```
[Out] ((a + b - Sqrt[a^2 - 2*a*b + b^2 + 4*a*b*d])*x^(2/3)*(-a + x)^(2/3)*(-b + x
)^(2/3)*Defer[Int][x^(1/3)/((-a + x)^(2/3)*(-b + x)^(2/3)*(a + b - Sqrt[a^2
 - 2*a*b + b^2 + 4*a*b*d] + 2*(-1 + d)*x)), x])/((a - x)*(b - x)*x)^(2/3) +
 ((a + b + Sqrt[a^2 - 2*a*b + b^2 + 4*a*b*d])*x^(2/3)*(-a + x)^(2/3)*(-b +
x)^(2/3)*Defer[Int][x^(1/3)/((-a + x)^(2/3)*(-b + x)^(2/3)*(a + b + Sqrt[a^
2 - 2*a*b + b^2 + 4*a*b*d] + 2*(-1 + d)*x)), x])/((a - x)*(b - x)*x)^(2/3)
```

Rubi steps

$$\int \frac{-2abx+(a+b)x^2}{(x(-a+x)(-b+x))^{2/3}\left(-ab+(a+b)x+(-1+d)x^2\right)}\,dx=\int \frac{x(-2ab+(a+b)x)}{(x(-a+x)(-b+x))^{2/3}\left(-ab+(a+b)x+(-1+d\right.}$$

$$=\frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{\sqrt[3]{x}\,(-2ab+(a+}{(-a+x)^{2/3}(-b+x)^{2/3}\left(-ab+(a\right.}}{(x(-a+x)(-b+x))^{2/3}}$$

$$=\frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int\left(\frac{\left(a+b-\sqrt{a^2-\right.}{(-a+x)^{2/3}(-b+x)^{2/3}\left(a+b-\right.}\right.}{}$$

$$=\frac{\left(\left(a+b-\sqrt{a^2-2ab+b^2+4abd}\right)x^{2/3}(-a+x)^{2/3}(-\right.}{(x(-a}$$

Mathematica [F] time = 10.76, size = 0, normalized size = 0.00

$$\int \frac{-2abx+(a+b)x^2}{(x(-a+x)(-b+x))^{2/3}\left(-ab+(a+b)x+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*a*b*x + (a + b)*x^2)/((x*(-a + x)*(-b + x))^(2/3)*(-(a*b) + (
a + b)*x + (-1 + d)*x^2)),x]
```

```
[Out] Integrate[(-2*a*b*x + (a + b)*x^2)/((x*(-a + x)*(-b + x))^(2/3)*(-(a*b) + (
a + b)*x + (-1 + d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 1.11, size = 174, normalized size = 1.00

$$-\frac{\log\left(\sqrt[3]{x^2(-a-b)+abx+x^3}-\sqrt[3]{d}\,x\right)}{d^{2/3}}+\frac{\log\left(\sqrt[3]{d}\,x\sqrt[3]{x^2(-a-b)+abx+x^3}+\left(x^2(-a-b)+abx+x^3\right)^{2/3}+d^{2/3}x^2\right)}{2d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\,x}{2\sqrt[3]{x^2(-a-b)+abx+x^3}+\sqrt[3]{d}\,x}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*a*b*x + (a + b)*x^2)/((x*(-a + x)*(-b + x))^(2/3)*(-
(a*b) + (a + b)*x + (-1 + d)*x^2)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*x)/(d^(1/3)*x + 2*(a*b*x + (-a - b)*x^2
+ x^3)^(1/3))])/d^(2/3)) - Log[-(d^(1/3)*x) + (a*b*x + (-a - b)*x^2 + x^3)^
(1/3)]/d^(2/3) + Log[d^(2/3)*x^2 + d^(1/3)*x*(a*b*x + (-a - b)*x^2 + x^3)^(
1/3) + (a*b*x + (-a - b)*x^2 + x^3)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b*x+(a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(-a*b+(a+b)*x+(-1+d)
*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,abx-(a+b)x^2}{\left((a-x)(b-x)x\right)^{\frac{2}{3}}\left((d-1)x^2-ab+(a+b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b*x+(a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(-a*b+(a+b)*x+(-1+d)
*x^2),x, algorithm="giac")
```

```
[Out] integrate(-(2*a*b*x - (a + b)*x^2)/(((a - x)*(b - x)*x)^(2/3)*((d - 1)*x^2
- a*b + (a + b)*x)), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{-2abx+(a+b)\,x^2}{\left(x\,(-a+x)\,(-b+x)\right)^{\frac{2}{3}}\left(-ab+(a+b)\,x+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a*b*x+(a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(-a*b+(a+b)*x+(-1+d)*x^2),
x)
```

```
[Out] int((-2*a*b*x+(a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(-a*b+(a+b)*x+(-1+d)*x^2),
x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,abx-(a+b)x^2}{\left((a-x)(b-x)x\right)^{\frac{2}{3}}\left((d-1)x^2-ab+(a+b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b*x+(a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(-a*b+(a+b)*x+(-1+d)
*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((2*a*b*x - (a + b)*x^2)/(((a - x)*(b - x)*x)^(2/3)*((d - 1)*x^2
- a*b + (a + b)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\right)-2\,a\,b\,x}{{\left(x\,\left(a-x\right)\,\left(b-x\right)\right)}^{2/3}\,\left(\left(d-1\right)\,x^2+\left(a+b\right)\,x-a\,b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b) - 2*a*b*x)/((x*(a - x)*(b - x))^(2/3)*(x*(a + b) - a*b + x
^2*(d - 1))),x)
```

```
[Out] int((x^2*(a + b) - 2*a*b*x)/((x*(a - x)*(b - x))^(2/3)*(x*(a + b) - a*b + x
^2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a*b*x+(a+b)*x**2)/(x*(-a+x)*(-b+x))**(2/3)/(-a*b+(a+b)*x+(-1+
d)*x**2),x)
```

```
[Out] Timed out
```

3.1878 $\int \frac{x(-ab+x^2)}{(x^2(-a+x)(-b+x))^{2/3}(abd-(1+ad+bd)x+dx^2)}\,dx$

Optimal. Leaf size=174

$$-\frac{\log\left(d^{2/3}\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+\sqrt[3]{d}\,x\sqrt[3]{x^3(-a-b)+abx^2+x^4}+x^2\right)}{2\sqrt[3]{d}}+\frac{\log\left(x-\sqrt[3]{d}\,\sqrt[3]{x^3(-a-b)+abx\right.}{\sqrt[3]{d}}$$

Rubi [F] time = 9.84, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(-ab+x^2)}{(x^2(-a+x)(-b+x))^{2/3}(abd-(1+ad+bd)x+dx^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-(a*b) + x^2))/((x^2*(-a + x)*(-b + x))^(2/3)*(a*b*d - (1 + a*d + b
*d)*x + d*x^2)),x]
```

```
[Out] (3*x^2*(1 - x/a)^(2/3)*(1 - x/b)^(2/3)*AppellF1[2/3, 2/3, 2/3, 5/3, x/a, x/
b])/(2*d*((a - x)*(b - x)*x^2)^(2/3)) + ((1 + a*d + b*d + Sqrt[a^2*d^2 + 2*
a*d*(1 - b*d) + (1 + b*d)^2])*x^(4/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[I
nt][1/(x^(1/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*(-1 - a*d - b*d - Sqrt[1 + 2*a
*d + 2*b*d + a^2*d^2 - 2*a*b*d^2 + b^2*d^2] + 2*d*x)), x])/(d*((a - x)*(b -
 x)*x^2)^(2/3)) + ((1 + a*d + b*d - Sqrt[a^2*d^2 + 2*a*d*(1 - b*d) + (1 + b
*d)^2])*x^(4/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Int][1/(x^(1/3)*(-a + x
)^(2/3)*(-b + x)^(2/3)*(-1 - a*d - b*d + Sqrt[1 + 2*a*d + 2*b*d + a^2*d^2 -
 2*a*b*d^2 + b^2*d^2] + 2*d*x)), x])/(d*((a - x)*(b - x)*x^2)^(2/3))
```

Rubi steps

$$\int \frac{x(-ab+x^2)}{(x^2(-a+x)(-b+x))^{2/3}(abd-(1+ad+bd)x+dx^2)}\,dx = \frac{\left(x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{-ab}{\sqrt[3]{x}(-a+x)^{2/3}(-b+x)^{2/3}}}{(x^2(-a+x)(-b+x))^{2/3}}$$

$$= \frac{\left(x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int\left(\frac{1}{d\sqrt[3]{x}(-a+x)^{2/3}(-b+x)}\right.}{(x^2(-a+x)}$$

$$= \frac{\left(x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{1}{\sqrt[3]{x}(-a+x)^{2/3}(-b+x)^{2/3}}}{d\,(x^2(-a+x)(-b+x))^{2/3}}$$

$$= -\frac{\left(x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int\left(\frac{-1-}{\sqrt[3]{x}(-a+x)^{2/3}(-b+x}\right.}{}$$

$$= -\frac{\left(\left(-1-ad-bd-\sqrt{a^2d^2+2ad(1-bd)+(1+b\right.\right.}{}$$

$$= \frac{3x^2\left(1-\frac{x}{a}\right)^{2/3}\left(1-\frac{x}{b}\right)^{2/3}F_1\left(\frac{2}{3};\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{x}{a},\frac{x}{b}\right)}{2d\left((a-x)(b-x)x^2\right)^{2/3}} - \frac{((}{}$$

Mathematica [F] time = 7.42, size = 0, normalized size = 0.00

$$\int \frac{x\left(-ab+x^2\right)}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(abd-(1+ad+bd)x+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(-(a*b) + x^2))/((x^2*(-a + x)*(-b + x))^(2/3)*(a*b*d - (1 + a
*d + b*d)*x + d*x^2)),x]
```

```
[Out] Integrate[(x*(-(a*b) + x^2))/((x^2*(-a + x)*(-b + x))^(2/3)*(a*b*d - (1 + a
*d + b*d)*x + d*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.59, size = 174, normalized size = 1.00

$$-\frac{\log\left(d^{2/3}\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+\sqrt[3]{d}\,x\sqrt[3]{x^3(-a-b)+abx^2+x^4}+x^2\right)}{2\sqrt[3]{d}}+\frac{\log\left(x-\sqrt[3]{d}\,\sqrt[3]{x^3(-a-b)+abx^2+x^4}\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{d}\,\sqrt[3]{x^3(-a-b)+abx^2+x^4}+x}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-(a*b) + x^2))/((x^2*(-a + x)*(-b + x))^(2/3)*(a*b*d
 - (1 + a*d + b*d)*x + d*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*d^(1/3)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(
1/3))])/d^(1/3) + Log[x - d^(1/3)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3)]/d^(
1/3) - Log[x^2 + d^(1/3)*x*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3) + d^(2/3)*(
a*b*x^2 + (-a - b)*x^3 + x^4)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(a*b*d-(a*d+b*d+1)*x+d*x^2
),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(ab-x^2\right)x}{\left((a-x)(b-x)x^2\right)^{\frac{2}{3}}\left(abd+dx^2-(ad+bd+1)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(a*b*d-(a*d+b*d+1)*x+d*x^2
),x, algorithm="giac")
```

```
[Out] integrate(-(a*b - x^2)*x/(((a - x)*(b - x)*x^2)^(2/3)*(a*b*d + d*x^2 - (a*d
 + b*d + 1)*x)), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{x\left(-ab+x^2\right)}{\left(x^2(-a+x)(-b+x)\right)^{\frac{2}{3}}\left(abd-(ad+bd+1)x+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(a*b*d-(a*d+b*d+1)*x+d*x^2),x)
```

```
[Out] int(x*(-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(a*b*d-(a*d+b*d+1)*x+d*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ab-x^2)x}{\left((a-x)(b-x)x^2\right)^{\frac{2}{3}}\left(abd+dx^2-(ad+bd+1)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(a*b*d-(a*d+b*d+1)*x+d*x^2
),x, algorithm="maxima")
```

```
[Out] -integrate((a*b - x^2)*x/(((a - x)*(b - x)*x^2)^(2/3)*(a*b*d + d*x^2 - (a*d
 + b*d + 1)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x\,\left(a\,b-x^2\right)}{\left(d\,x^2+\left(-a\,d-b\,d-1\right)\,x+a\,b\,d\right)\,{\left(x^2\,\left(a-x\right)\,\left(b-x\right)\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a*b - x^2))/((d*x^2 - x*(a*d + b*d + 1) + a*b*d)*(x^2*(a - x)*(b -
 x))^(2/3)),x)
```

```
[Out] int(-(x*(a*b - x^2))/((d*x^2 - x*(a*d + b*d + 1) + a*b*d)*(x^2*(a - x)*(b -
 x))^(2/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*b+x**2)/(x**2*(-a+x)*(-b+x))**(2/3)/(a*b*d-(a*d+b*d+1)*x+d*
x**2),x)
```

```
[Out] Timed out
```

3.1879 $\int \frac{(-2+x^3)(-1+x^3)^{2/3}}{x^3(-1+2x^3)}\,dx$

Optimal. Leaf size=174

$$-\frac{1}{6}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{1}{2}\log\left(\sqrt[3]{x^3-1}+x\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-1}-x}\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^3-1}+x}\right)}{2\sqrt{3}}-\frac{\left(x^3-1\right)^{2/3}}{x^2}+\frac{1}{4}\log$$

Rubi [A] time = 0.10, antiderivative size = 143, normalized size of antiderivative = 0.82, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {580, 530, 239, 377, 200, 31, 634, 618, 204, 628}

$$-\frac{1}{2}\log\left(\frac{x}{\sqrt[3]{x^3-1}}+1\right)-\frac{1}{4}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{x^3-1}}}{\sqrt{3}}\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\left(x^3-1\right)^{2/3}}{x^2}+\frac{1}{4}\log\left(-\frac{x}{\sqrt[3]{x^3-1}}+\frac{x^2}{\left(x^3-1\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-2 + x^3)*(-1 + x^3)^(2/3))/(x^3*(-1 + 2*x^3)),x]
```

```
[Out] -((-1 + x^3)^(2/3)/x^2) + (Sqrt[3]*ArcTan[(1 - (2*x)/(-1 + x^3)^(1/3))/Sqrt
[3]])/2 + ArcTan[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) + Log[1
+ x^2/(-1 + x^3)^(2/3) - x/(-1 + x^3)^(1/3)]/4 - Log[1 + x/(-1 + x^3)^(1/3)
]/2 - Log[-x + (-1 + x^3)^(1/3)]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
```

```
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(-2+x^3)(-1+x^3)^{2/3}}{x^3(-1+2x^3)}\,dx = -\frac{(-1+x^3)^{2/3}}{x^2} - \frac{1}{2}\int \frac{-2-2x^3}{\sqrt[3]{-1+x^3}\,(-1+2x^3)}\,dx$$

$$= -\frac{(-1+x^3)^{2/3}}{x^2} + \frac{1}{2}\int \frac{1}{\sqrt[3]{-1+x^3}}\,dx + \frac{3}{2}\int \frac{1}{\sqrt[3]{-1+x^3}\,(-1+2x^3)}\,dx$$

$$= -\frac{(-1+x^3)^{2/3}}{x^2} + \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}\log\left(-x+\sqrt[3]{-1+x^3}\right) + \frac{3}{2}\text{Subst}\left(\int \frac{1}{-1-\dots}\right.$$

$$= -\frac{(-1+x^3)^{2/3}}{x^2} + \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}\log\left(-x+\sqrt[3]{-1+x^3}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-\dots}\right.$$

$$= -\frac{(-1+x^3)^{2/3}}{x^2} + \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\log\left(1+\frac{x}{\sqrt[3]{-1+x^3}}\right) - \frac{1}{4}\log\left(-x+\sqrt[3]{-1+\dots}\right.$$

$$= -\frac{(-1+x^3)^{2/3}}{x^2} + \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{4}\log\left(1+\frac{x^2}{(-1+x^3)^{2/3}}-\frac{x}{\sqrt[3]{-1+x^3}}\right) - \frac{1}{2}\text{lo}\dots$$

$$= -\frac{(-1+x^3)^{2/3}}{x^2} + \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right) + \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{4}\log\left(1+\frac{\dots}{(-1\dots}\right.$$

Mathematica [C] time = 0.24, size = 146, normalized size = 0.84

$$\frac{1}{6}\left(-2\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{1-x^3}}-1}{\sqrt{3}}\right)-\frac{6(x^3-1)^{2/3}}{x^2}+\log\left(-\frac{x}{\sqrt[3]{1-x^3}}+\frac{x^2}{(1-x^3)^{2/3}}+1\right)\right)-\frac{x^4\sqrt[3]{1-x^3}\,F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,2x^3\right)}{4\sqrt[3]{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-2 + x^3)*(-1 + x^3)^(2/3))/(x^3*(-1 + 2*x^3)),x]
```

```
[Out] -1/4*(x^4*(1 - x^3)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, x^3, 2*x^3])/(-1 + x^3
)^(1/3) + ((-6*(-1 + x^3)^(2/3))/x^2 - 2*Sqrt[3]*ArcTan[(-1 + (2*x)/(1 - x^
3)^(1/3))/Sqrt[3]] + Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)] - 2*L
og[1 + x/(1 - x^3)^(1/3)])/6
```

IntegrateAlgebraic [A] time = 0.27, size = 174, normalized size = 1.00

$$-\frac{1}{6}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{1}{2}\log\left(\sqrt[3]{x^3-1}+x\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}-x}\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{2\sqrt{3}}-\frac{(x^3-1)^{2/3}}{x^2}+\frac{1}{4}\log\left(-\sqrt[3]{x^3-1}\,x+(x^3-1)^{2/3}+x^2\right)+\frac{1}{12}\log\left(\sqrt[3]{x^3-1}\,x+(x^3-1)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^3)*(-1 + x^3)^(2/3))/(x^3*(-1 + 2*x^3)),x]
```

```
[Out] -((-1 + x^3)^(2/3)/x^2) - (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(-1 + x^3)^(1
/3))])/2 + ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x^3)^(1/3))]/(2*Sqrt[3]) - Log[-
x + (-1 + x^3)^(1/3)]/6 - Log[x + (-1 + x^3)^(1/3)]/2 + Log[x^2 - x*(-1 + x
^3)^(1/3) + (-1 + x^3)^(2/3)]/4 + Log[x^2 + x*(-1 + x^3)^(1/3) + (-1 + x^3)
^(2/3)]/12
```

fricas [A] time = 6.68, size = 214, normalized size = 1.23

$$\frac{2\sqrt{3}x^2\arctan\left(\frac{383838\sqrt{3}(x^{10}-3x^4-2x)(x^3-1)^{\frac{2}{3}}+13468\sqrt{3}(x^{11}-3x^8+4x^2)(x^3-1)^{\frac{1}{3}}+\sqrt{3}(198653x^{12}+393594x^9+5568x^6-400090x^3-198189)}{3(185185x^{12}+370434x^9-96x^6-370322x^3-185193)}\right)-x^2\log\left(\frac{8x^9-12x^6+6x^3-3(x^{10}-3x^4-2x)(x^3-1)^{\frac{2}{3}}+3(x^{11}-3x^8+4x^2)(x^3-1)^{\frac{1}{3}}-1}{8x^9-12x^6+6x^3-1}\right)-12(x^3-1)^{\frac{2}{3}}}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3-1)^(2/3)/x^3/(2*x^3-1),x, algorithm="fricas")
```

```
[Out] 1/12*(2*sqrt(3)*x^2*arctan(1/3*(383838*sqrt(3)*(x^10 - 3*x^4 - 2*x)*(x^3 -
1)^(2/3) + 13468*sqrt(3)*(x^11 - 3*x^8 + 4*x^2)*(x^3 - 1)^(1/3) + sqrt(3)*(
198653*x^12 + 393594*x^9 + 5568*x^6 - 400090*x^3 - 198189))/(185185*x^12 +
370434*x^9 - 96*x^6 - 370322*x^3 - 185193)) - x^2*log((8*x^9 - 12*x^6 + 6*x
^3 - 3*(x^10 - 3*x^4 - 2*x)*(x^3 - 1)^(2/3) + 3*(x^11 - 3*x^8 + 4*x^2)*(x^3
 - 1)^(1/3) - 1)/(8*x^9 - 12*x^6 + 6*x^3 - 1)) - 12*(x^3 - 1)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^3-1)^{\frac{2}{3}}(x^3-2)}{(2x^3-1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3-1)^(2/3)/x^3/(2*x^3-1),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 1)^(2/3)*(x^3 - 2)/((2*x^3 - 1)*x^3), x)
```

maple [C] time = 8.05, size = 801, normalized size = 4.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2)*(x^3-1)^(2/3)/x^3/(2*x^3-1),x)
```

```
[Out] -(x^3-1)^(2/3)/x^2-1/6*ln((-11+60*x*(x^3-1)^(2/3)+108*x^2*(x^3-1)^(1/3)+22*
x^12-44*x^9+27*x^11*(x^3-1)^(1/3)-81*x^8*(x^3-1)^(1/3)+132*x^6-110*x^3+76*R
ootOf(4*_Z^2-2*_Z+1)^2*x^12-456*RootOf(4*_Z^2-2*_Z+1)^2*x^9+912*RootOf(4*_Z
^2-2*_Z+1)^2*x^6-608*RootOf(4*_Z^2-2*_Z+1)^2*x^3+38*RootOf(4*_Z^2-2*_Z+1)-7
20*RootOf(4*_Z^2-2*_Z+1)*x^6+556*RootOf(4*_Z^2-2*_Z+1)*x^3-98*RootOf(4*_Z^2
-2*_Z+1)*x^12+284*x^9*RootOf(4*_Z^2-2*_Z+1)+54*RootOf(4*_Z^2-2*_Z+1)*(x^3-1
)^(2/3)*x^10+6*RootOf(4*_Z^2-2*_Z+1)*(x^3-1)^(1/3)*x^11-18*RootOf(4*_Z^2-2*
_Z+1)*(x^3-1)^(1/3)*x^8-162*RootOf(4*_Z^2-2*_Z+1)*(x^3-1)^(2/3)*x^4-108*Roo
tOf(4*_Z^2-2*_Z+1)*(x^3-1)^(2/3)*x+24*RootOf(4*_Z^2-2*_Z+1)*(x^3-1)^(1/3)*x
^2-30*(x^3-1)^(2/3)*x^10+90*(x^3-1)^(2/3)*x^4)/(2*x^3-1)^3)+1/3*RootOf(4*_Z
^2-2*_Z+1)*ln(-(19-6*x*(x^3-1)^(2/3)+108*x^2*(x^3-1)^(1/3)-19*x^12-38*x^9+2
7*x^11*(x^3-1)^(1/3)-81*x^8*(x^3-1)^(1/3)+38*x^3+44*RootOf(4*_Z^2-2*_Z+1)^2
*x^12-264*RootOf(4*_Z^2-2*_Z+1)^2*x^9+528*RootOf(4*_Z^2-2*_Z+1)^2*x^6-352*R
ootOf(4*_Z^2-2*_Z+1)^2*x^3-22*RootOf(4*_Z^2-2*_Z+1)-456*RootOf(4*_Z^2-2*_Z+
1)*x^6+260*RootOf(4*_Z^2-2*_Z+1)*x^3-16*RootOf(4*_Z^2-2*_Z+1)*x^12+272*x^9*
RootOf(4*_Z^2-2*_Z+1)+54*RootOf(4*_Z^2-2*_Z+1)*(x^3-1)^(2/3)*x^10-60*RootOf
(4*_Z^2-2*_Z+1)*(x^3-1)^(1/3)*x^11+180*RootOf(4*_Z^2-2*_Z+1)*(x^3-1)^(1/3)*
x^8-162*RootOf(4*_Z^2-2*_Z+1)*(x^3-1)^(2/3)*x^4-108*RootOf(4*_Z^2-2*_Z+1)*(
x^3-1)^(2/3)*x-240*RootOf(4*_Z^2-2*_Z+1)*(x^3-1)^(1/3)*x^2+3*(x^3-1)^(2/3)*
x^10-9*(x^3-1)^(2/3)*x^4)/(2*x^3-1)^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^3-1)^{\frac{2}{3}}(x^3-2)}{(2x^3-1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^3-1)^(2/3)/x^3/(2*x^3-1),x, algorithm="maxima")
[Out] integrate((x^3 - 1)^(2/3)*(x^3 - 2)/((2*x^3 - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3-1\right)^{2/3}\,\left(x^3-2\right)}{x^3\,\left(2\,x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(x^3 - 2))/(x^3*(2*x^3 - 1)),x)
[Out] int(((x^3 - 1)^(2/3)*(x^3 - 2))/(x^3*(2*x^3 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x^3-2\right)}{x^3\left(2x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2)*(x**3-1)**(2/3)/x**3/(2*x**3-1),x)
[Out] Integral(((x - 1)*(x**2 + x + 1))**(2/3)*(x**3 - 2)/(x**3*(2*x**3 - 1)), x)
```

3.1880 $\int \frac{(-b+ax^2)\sqrt[4]{-bx^2+ax^4}}{b+ax^2}\,dx$

Optimal. Leaf size=174

$$\frac{9b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{4a^{3/4}}-\frac{2\sqrt[4]{2}\,b\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{a^{3/4}}-\frac{9b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{4a^{3/4}}+\frac{2\sqrt[4]{2}\,b\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{a^{3/4}}+\frac{1}{2}x\sqrt[4]{ax^4-b}$$

Rubi [C] time = 0.26, antiderivative size = 64, normalized size of antiderivative = 0.37, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2056, 466, 511, 510}

$$-\frac{2x\sqrt[4]{ax^4-bx^2}\,F_1\left(\frac{3}{4};1,-\frac{5}{4};\frac{7}{4};-\frac{ax^2}{b},\frac{ax^2}{b}\right)}{3\sqrt[4]{1-\frac{ax^2}{b}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4))/(b + a*x^2),x]
```

```
[Out] (-2*x*(-(b*x^2) + a*x^4)^(1/4)*AppellF1[3/4, 1, -5/4, 7/4, -((a*x^2)/b), (a
*x^2)/b])/(3*(1 - (a*x^2)/b)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\left(-b+ax^2\right)\sqrt[4]{-bx^2+ax^4}}{b+ax^2}\,dx = \frac{\sqrt[4]{-bx^2+ax^4}\int \frac{\sqrt{x}\left(-b+ax^2\right)^{5/4}}{b+ax^2}dx}{\sqrt{x}\sqrt[4]{-b+ax^2}}$$

$$= \frac{\left(2\sqrt[4]{-bx^2+ax^4}\right)\text{Subst}\left(\int \frac{x^2\left(-b+ax^4\right)^{5/4}}{b+ax^4}dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt[4]{-b+ax^2}}$$

$$= -\frac{\left(2b\sqrt[4]{-bx^2+ax^4}\right)\text{Subst}\left(\int \frac{x^2\left(1-\frac{ax^4}{b}\right)^{5/4}}{b+ax^4}dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt[4]{1-\frac{ax^2}{b}}}$$

$$= -\frac{2x\sqrt[4]{-bx^2+ax^4}\,F_1\left(\frac{3}{4};1,-\frac{5}{4};\frac{7}{4};-\frac{ax^2}{b},\frac{ax^2}{b}\right)}{3\sqrt[4]{1-\frac{ax^2}{b}}}$$

Mathematica [C] time = 0.17, size = 164, normalized size = 0.94

$$\frac{x\sqrt[4]{ax^4-bx^2}\left(27ax^2\left(1-\frac{a^2x^4}{b^2}\right)^{3/4}F_1\left(\frac{7}{4};\frac{3}{4},1;\frac{11}{4};\frac{ax^2}{b},-\frac{ax^2}{b}\right)-49b\left(1-\frac{ax^2}{b}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{2ax^2}{ax^2+b}\right)+21\left(b-ax^2\right)\left(\frac{ax^2}{b}+1\right)^{3/4}\right)}{42\left(b-ax^2\right)\left(\frac{ax^2}{b}+1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4))/(b + a*x^2),x]
```

```
[Out] (x*(-(b*x^2) + a*x^4)^(1/4)*(21*(b - a*x^2)*(1 + (a*x^2)/b)^(3/4) + 27*a*x^
2*(1 - (a^2*x^4)/b^2)^(3/4)*AppellF1[7/4, 3/4, 1, 11/4, (a*x^2)/b, -((a*x^2
)/b)] - 49*b*(1 - (a*x^2)/b)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (2*a*x^
2)/(b + a*x^2)]))/(42*(b - a*x^2)*(1 + (a*x^2)/b)^(3/4))
```

IntegrateAlgebraic [A] time = 0.68, size = 174, normalized size = 1.00

$$\frac{9b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{4a^{3/4}}-\frac{2\sqrt[4]{2}\,b\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{a^{3/4}}-\frac{9b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{4a^{3/4}}+\frac{2\sqrt[4]{2}\,b\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^2}}\right)}{a^{3/4}}+\frac{1}{2}x\sqrt[4]{ax^4-bx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^2)*(-(b*x^2) + a*x^4)^(1/4))/(b + a*x^2),x]
```

```
[Out] (x*(-(b*x^2) + a*x^4)^(1/4))/2 + (9*b*ArcTan[(a^(1/4)*x)/(-(b*x^2) + a*x^4)
^(1/4)])/(4*a^(3/4)) - (2*2^(1/4)*b*ArcTan[(2^(1/4)*a^(1/4)*x)/(-(b*x^2) +
a*x^4)^(1/4)])/a^(3/4) - (9*b*ArcTanh[(a^(1/4)*x)/(-(b*x^2) + a*x^4)^(1/4)]
)/(4*a^(3/4)) + (2*2^(1/4)*b*ArcTanh[(2^(1/4)*a^(1/4)*x)/(-(b*x^2) + a*x^4)
^(1/4)])/a^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(a*x^4-b*x^2)^(1/4)/(a*x^2+b),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.38, size = 403, normalized size = 2.32

$$\frac{1}{2}\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}x^2+\frac{2^{\frac{3}{4}}(-a)^{\frac{1}{4}}b\arctan\left(\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a}+\frac{2^{\frac{3}{4}}(-a)^{\frac{1}{4}}b\arctan\left(-\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a}+\frac{2^{\frac{3}{4}}(-a)^{\frac{1}{4}}b\log\left(2^{\frac{3}{4}}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a-\frac{b}{x^2}}\right)}{2a}-\frac{2^{\frac{3}{4}}(-a)^{\frac{1}{4}}b\log\left(-2^{\frac{3}{4}}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a-\frac{b}{x^2}}\right)}{2a}+\frac{9\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{8(-a)^{\frac{3}{4}}}+\frac{9\sqrt{2}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{8(-a)^{\frac{3}{4}}}+\frac{9\sqrt{2}b\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^2}}\right)}{16(-a)^{\frac{3}{4}}}+\frac{9\sqrt{2}(-a)^{\frac{1}{4}}b\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x^2}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x^2}}\right)}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(a*x^4-b*x^2)^(1/4)/(a*x^2+b),x, algorithm="giac")
```

```
[Out] 1/2*(a - b/x^2)^(1/4)*x^2 + 2^(3/4)*(-a)^(1/4)*b*arctan(1/2*2^(1/4)*(2^(3/4
)*(-a)^(1/4) + 2*(a - b/x^2)^(1/4))/(-a)^(1/4))/a + 2^(3/4)*(-a)^(1/4)*b*ar
ctan(-1/2*2^(1/4)*(2^(3/4)*(-a)^(1/4) - 2*(a - b/x^2)^(1/4))/(-a)^(1/4))/a
+ 1/2*2^(3/4)*(-a)^(1/4)*b*log(2^(3/4)*(-a)^(1/4)*(a - b/x^2)^(1/4) + sqrt(
2)*sqrt(-a) + sqrt(a - b/x^2))/a - 1/2*2^(3/4)*(-a)^(1/4)*b*log(-2^(3/4)*(-
a)^(1/4)*(a - b/x^2)^(1/4) + sqrt(2)*sqrt(-a) + sqrt(a - b/x^2))/a + 9/8*sq
rt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a - b/x^2)^(1/4))/(-a)^
(1/4))/(-a)^(3/4) + 9/8*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) -
 2*(a - b/x^2)^(1/4))/(-a)^(1/4))/(-a)^(3/4) + 9/16*sqrt(2)*b*log(sqrt(2)*(
-a)^(1/4)*(a - b/x^2)^(1/4) + sqrt(-a) + sqrt(a - b/x^2))/(-a)^(3/4) + 9/16
*sqrt(2)*(-a)^(1/4)*b*log(-sqrt(2)*(-a)^(1/4)*(a - b/x^2)^(1/4) + sqrt(-a)
+ sqrt(a - b/x^2))/a
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^2-b\right)\left(a\,x^4-b\,x^2\right)^{\frac{1}{4}}}{a\,x^2+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)*(a*x^4-b*x^2)^(1/4)/(a*x^2+b),x)
```

```
[Out] int((a*x^2-b)*(a*x^4-b*x^2)^(1/4)/(a*x^2+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4-bx^2\right)^{\frac{1}{4}}\left(ax^2-b\right)}{ax^2+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(a*x^4-b*x^2)^(1/4)/(a*x^2+b),x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 - b*x^2)^(1/4)*(a*x^2 - b)/(a*x^2 + b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(b-a\,x^2\right)\left(a\,x^4-b\,x^2\right)^{1/4}}{a\,x^2+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b - a*x^2)*(a*x^4 - b*x^2)^(1/4))/(b + a*x^2),x)
```

```
[Out] int(-((b - a*x^2)*(a*x^4 - b*x^2)^(1/4))/(b + a*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\left(ax^2-b\right)}\left(ax^2-b\right)}{ax^2+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)*(a*x**4-b*x**2)**(1/4)/(a*x**2+b),x)
```

```
[Out] Integral((x**2*(a*x**2 - b))**(1/4)*(a*x**2 - b)/(a*x**2 + b), x)
```

3.1881 $\int \frac{1+x^3}{(-1+x^3)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

Optimal. Leaf size=174

$$\frac{4\tan^{-1}\left(\frac{x\sqrt{a+b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2+\sqrt{a}\,x+\sqrt{a}}\right)}{3\sqrt{a+b-c}}-\frac{2\sqrt{-2a-2b-c}\,\tan^{-1}\left(\frac{x\sqrt{-2a-2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2-2\sqrt{a}\,x+\sqrt{a}}\right)}{3(2a+2b+c)}$$

Rubi [F] time = 1.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x^3}{(-1+x^3)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^3)/((-1 + x^3)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] Defer[Int][1/Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4], x] + (2*Defer[Int][1/((
-1 + x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x])/3 - (2*(1 - I*Sqrt[3])*
Defer[Int][1/((1 - I*Sqrt[3] + 2*x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),
 x])/3 - (2*(1 + I*Sqrt[3])*Defer[Int][1/((1 + I*Sqrt[3] + 2*x)*Sqrt[a + b*
x + c*x^2 + b*x^3 + a*x^4]), x])/3
```

Rubi steps

$$\int \frac{1+x^3}{(-1+x^3)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx=\int\left(\frac{1}{\sqrt{a+bx+cx^2+bx^3+ax^4}}+\frac{2}{(-1+x^3)\sqrt{a+bx+cx^2+bx^3}}\right.$$

$$=2\int\frac{1}{(-1+x^3)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx+\int\frac{1}{\sqrt{a+bx+cx^2}}$$

$$=2\int\left(\frac{1}{3(-1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}+\frac{-2}{3(1+x+x^2)\sqrt{a+}}\right.$$

$$=\frac{2}{3}\int\frac{1}{(-1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx+\frac{2}{3}\int\frac{}{(1+x+x^2)\sqrt{}}$$

$$=\frac{2}{3}\int\frac{1}{(-1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx+\frac{2}{3}\int\left(\frac{}{(1-i\sqrt{3}+2}\right.$$

$$=\frac{2}{3}\int\frac{1}{(-1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx-\frac{1}{3}\left(2\left(1-i\sqrt{3}\right)\right)\int$$

Mathematica [C] time = 4.89, size = 5076, normalized size = 29.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^3)/((-1 + x^3)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 1.31, size = 174, normalized size = 1.00

$$\frac{4\tan^{-1}\left(\frac{x\sqrt{a+b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2+\sqrt{a}\,x+\sqrt{a}}\right)}{3\sqrt{a+b-c}}-\frac{2\sqrt{-2a-2b-c}\,\tan^{-1}\left(\frac{x\sqrt{-2a-2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2-2\sqrt{a}\,x+\sqrt{a}}\right)}{3(2a+2b+c)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)/((-1 + x^3)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x
^4]),x]
```

```
[Out] (-2*Sqrt[-2*a - 2*b - c]*ArcTan[(Sqrt[-2*a - 2*b - c]*x)/(Sqrt[a] - 2*Sqrt[
a]*x + Sqrt[a]*x^2 - Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(3*(2*a + 2*b
 + c)) + (4*ArcTan[(Sqrt[a + b - c]*x)/(Sqrt[a] + Sqrt[a]*x + Sqrt[a]*x^2 -
 Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(3*Sqrt[a + b - c])
```

fricas [A] time = 3.19, size = 1497, normalized size = 8.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [-1/6*(2*(2*a + 2*b + c)*sqrt(-a - b + c)*log(-((8*a*b - b^2 - 4*a*c)*x^4 -
 2*(8*a^2 - 4*a*b - 3*b^2 - 4*(a - b)*c)*x^3 - (24*a^2 + 3*b^2 - 4*(5*a + 2
*b)*c + 8*c^2)*x^2 - 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((2*a - b)*x^2
 + (4*a + b - 2*c)*x + 2*a - b)*sqrt(-a - b + c) + 8*a*b - b^2 - 4*a*c - 2*
(8*a^2 - 4*a*b - 3*b^2 - 4*(a - b)*c)*x)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)) -
 sqrt(2*a + 2*b + c)*(a + b - c)*log(((24*a^2 + 16*a*b + b^2 + 4*a*c)*x^4 -
 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(2*a + b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a -
 2*b)*c + 4*c^2)*x^2 - 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a + b)*x
^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(2*a + 2*b + c) + 24*a^2 + 16*a*b + b
^2 + 4*a*c - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(2*a + b)*c)*x)/(x^4 - 4*x^3 + 6*
x^2 - 4*x + 1)))/(2*a^2 + 4*a*b + 2*b^2 - (a + b)*c - c^2), 1/3*((a + b - c
)*sqrt(-2*a - 2*b - c)*arctan(1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4
*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(-2*a - 2*b - c)/((2*a^2 + 2
*a*b + a*c)*x^4 + (2*a*b + 2*b^2 + b*c)*x^3 + (2*(a + b)*c + c^2)*x^2 + 2*a
^2 + 2*a*b + a*c + (2*a*b + 2*b^2 + b*c)*x)) - (2*a + 2*b + c)*sqrt(-a - b
+ c)*log(-((8*a*b - b^2 - 4*a*c)*x^4 - 2*(8*a^2 - 4*a*b - 3*b^2 - 4*(a - b)
*c)*x^3 - (24*a^2 + 3*b^2 - 4*(5*a + 2*b)*c + 8*c^2)*x^2 - 4*sqrt(a*x^4 + b
*x^3 + c*x^2 + b*x + a)*((2*a - b)*x^2 + (4*a + b - 2*c)*x + 2*a - b)*sqrt(
-a - b + c) + 8*a*b - b^2 - 4*a*c - 2*(8*a^2 - 4*a*b - 3*b^2 - 4*(a - b)*c)
*x)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)))/(2*a^2 + 4*a*b + 2*b^2 - (a + b)*c -
c^2), -1/6*(4*(2*a + 2*b + c)*sqrt(a + b - c)*arctan(-2*sqrt(a*x^4 + b*x^3
+ c*x^2 + b*x + a)*sqrt(a + b - c)/((2*a - b)*x^2 + (4*a + b - 2*c)*x + 2*a
 - b)) - sqrt(2*a + 2*b + c)*(a + b - c)*log(((24*a^2 + 16*a*b + b^2 + 4*a*
c)*x^4 - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(2*a + b)*c)*x^3 + 2*(24*a^2 + 3*b^2
- 4*(a - 2*b)*c + 4*c^2)*x^2 - 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*
a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(2*a + 2*b + c) + 24*a^2 + 16
*a*b + b^2 + 4*a*c - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(2*a + b)*c)*x)/(x^4 - 4*
x^3 + 6*x^2 - 4*x + 1)))/(2*a^2 + 4*a*b + 2*b^2 - (a + b)*c - c^2), 1/3*((a
 + b - c)*sqrt(-2*a - 2*b - c)*arctan(1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x
+ a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(-2*a - 2*b - c)/((2
*a^2 + 2*a*b + a*c)*x^4 + (2*a*b + 2*b^2 + b*c)*x^3 + (2*(a + b)*c + c^2)*x
^2 + 2*a^2 + 2*a*b + a*c + (2*a*b + 2*b^2 + b*c)*x)) - 2*(2*a + 2*b + c)*sq
rt(a + b - c)*arctan(-2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(a + b -
c)/((2*a - b)*x^2 + (4*a + b - 2*c)*x + 2*a - b)))/(2*a^2 + 4*a*b + 2*b^2 -
 (a + b)*c - c^2)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x^{3}+1}{\left(x^{3}-1\right) \sqrt{a \,x^{4}+b \,x^{3}+c \,x^{2}+b x +a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^3-1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((x^3+1)/(x^3-1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} + 1}{\sqrt{a x^{4} + b x^{3} + c x^{2} + b x + a} {\left(x^{3} - 1\right)}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((x^3 + 1)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(x^3 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3+1}{\left(x^3-1\right)\,\sqrt{a\,x^4+b\,x^3+c\,x^2+b\,x+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/((x^3 - 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)),x)
```

```
[Out] int((x^3 + 1)/((x^3 - 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x + 1\right) \left(x^{2} - x + 1\right)}{\left(x - 1\right) \left(x^{2} + x + 1\right) \sqrt{a x^{4} + a + b x^{3} + b x + c x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**3-1)/(a*x**4+b*x**3+c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((x + 1)*(x**2 - x + 1)/((x - 1)*(x**2 + x + 1)*sqrt(a*x**4 + a + b
*x**3 + b*x + c*x**2)), x)
```

3.1882 $\int \frac{1}{\sqrt{d+\sqrt{c+\sqrt{b+ax}}}}\,dx$

Optimal. Leaf size=174

$$\frac{32\left(13cd-12d^3\right)\sqrt{\sqrt{\sqrt{ax+b}+c}+d}}{105a}-\frac{32\left(5c-6d^2\right)\sqrt{\sqrt{ax+b}+c}\sqrt{\sqrt{\sqrt{ax+b}+c}+d}}{105a}+\sqrt{ax+b}\left(\frac{8\sqrt{\sqrt{}}}{}\right.$$

Rubi [A] time = 0.16, antiderivative size = 127, normalized size of antiderivative = 0.73, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {371, 1398, 772}

$$-\frac{8\left(c-3d^2\right)\left(\sqrt{\sqrt{ax+b}+c}+d\right)^{3/2}}{3a}+\frac{8d\left(c-d^2\right)\sqrt{\sqrt{\sqrt{ax+b}+c}+d}}{a}+\frac{8\left(\sqrt{\sqrt{ax+b}+c}+d\right)^{7/2}}{7a}-\frac{24d\left(\sqrt{\sqrt{ax+b}+c}+d\right)^{5/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[d + Sqrt[c + Sqrt[b + a*x]]],x]
```

```
[Out] (8*d*(c - d^2)*Sqrt[d + Sqrt[c + Sqrt[b + a*x]]])/a - (8*(c - 3*d^2)*(d + S
qrt[c + Sqrt[b + a*x]])^(3/2))/(3*a) - (24*d*(d + Sqrt[c + Sqrt[b + a*x]])^
(5/2))/(5*a) + (8*(d + Sqrt[c + Sqrt[b + a*x]])^(7/2))/(7*a)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+\sqrt{c+\sqrt{b+ax}}}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{d+\sqrt{c+x}}}\,dx, x, \sqrt{b+ax}\right)}{a}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{-c+x}{\sqrt{d+\sqrt{x}}}\,dx, x, c+\sqrt{b+ax}\right)}{a}$$

$$= \frac{4\,\text{Subst}\left(\int \frac{x(-c+x^2)}{\sqrt{d+x}}\,dx, x, \sqrt{c+\sqrt{b+ax}}\right)}{a}$$

$$= \frac{4\,\text{Subst}\left(\int \left(-\frac{d(-c+d^2)}{\sqrt{d+x}} + (-c+3d^2)\sqrt{d+x} - 3d(d+x)^{3/2} + (d+x)^{5/2}\right)dx, x, \sqrt{c}\right.}{a}$$

$$= \frac{8d(c-d^2)\sqrt{d+\sqrt{c+\sqrt{b+ax}}}}{a} - \frac{8(c-3d^2)\left(d+\sqrt{c+\sqrt{b+ax}}\right)^{3/2}}{3a} - \frac{24d\left(d\right.}{}$$

Mathematica [A] time = 0.14, size = 114, normalized size = 0.66

$$\frac{8\sqrt{\sqrt{\sqrt{ax+b}+c}+d}\left(24d^2\sqrt{\sqrt{ax+b}+c}-20c\sqrt{\sqrt{ax+b}+c}+15\sqrt{ax+b}\sqrt{\sqrt{ax+b}+c}-18d\sqrt{ax+b}+52cd-48d^3\right)}{105a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[d + Sqrt[c + Sqrt[b + a*x]]],x]
```

```
[Out] (8*Sqrt[d + Sqrt[c + Sqrt[b + a*x]]]*(52*c*d - 48*d^3 - 18*d*Sqrt[b + a*x]
- 20*c*Sqrt[c + Sqrt[b + a*x]] + 24*d^2*Sqrt[c + Sqrt[b + a*x]] + 15*Sqrt[b
 + a*x]*Sqrt[c + Sqrt[b + a*x]]))/(105*a)
```

IntegrateAlgebraic [A] time = 0.14, size = 114, normalized size = 0.66

$$\frac{16\left(-9d\sqrt{ax+b}+26cd-24d^3\right)\sqrt{\sqrt{\sqrt{ax+b}+c}+d}}{105a} - \frac{8\sqrt{\sqrt{ax+b}+c}\left(-15\sqrt{ax+b}+20c-24d^2\right)\sqrt{\sqrt{\sqrt{ax+b}+c}+d}}{105a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[d + Sqrt[c + Sqrt[b + a*x]]],x]
```

```
[Out] (-8*(20*c - 24*d^2 - 15*Sqrt[b + a*x])*Sqrt[c + Sqrt[b + a*x]]*Sqrt[d + Sqr
t[c + Sqrt[b + a*x]]])/(105*a) + (16*(26*c*d - 24*d^3 - 9*d*Sqrt[b + a*x])*
Sqrt[d + Sqrt[c + Sqrt[b + a*x]]])/(105*a)
```

fricas [A] time = 0.61, size = 71, normalized size = 0.41

$$-\frac{8\left(48\,d^3-52\,cd-\left(24\,d^2-20\,c+15\,\sqrt{ax+b}\right)\sqrt{c+\sqrt{ax+b}}+18\,\sqrt{ax+b}\,d\right)\sqrt{d+\sqrt{c+\sqrt{ax+b}}}}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+(c+(a*x+b)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -8/105*(48*d^3 - 52*c*d - (24*d^2 - 20*c + 15*sqrt(a*x + b))*sqrt(c + sqrt(
a*x + b)) + 18*sqrt(a*x + b)*d)*sqrt(d + sqrt(c + sqrt(a*x + b)))/a
```

giac [A] time = 4.50, size = 190, normalized size = 1.09

$$\frac{8\left(15\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{7}{2}}\operatorname{sgn}\left(\sqrt{c+\sqrt{ax+b}}\right)-63\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{5}{2}}d\operatorname{sgn}\left(\sqrt{c+\sqrt{ax+b}}\right)+105\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{3}{2}}d^2\operatorname{sgn}\left(\sqrt{c+\sqrt{ax+b}}\right)-105\sqrt{d+\sqrt{c+\sqrt{ax+b}}}\,d^3\operatorname{sgn}\left(\sqrt{c+\sqrt{ax+b}}\right)-35c\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{3}{2}}\operatorname{sgn}\left(\sqrt{c+\sqrt{ax+b}}\right)+105c\sqrt{d+\sqrt{c+\sqrt{ax+b}}}\,d\operatorname{sgn}\left(\sqrt{c+\sqrt{ax+b}}\right)\right)}{105a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+(c+(a*x+b)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/105*(15*(d + sqrt(c + sqrt(a*x + b)))^(7/2)*sgn(sqrt(c + sqrt(a*x + b)))
- 63*(d + sqrt(c + sqrt(a*x + b)))^(5/2)*d*sgn(sqrt(c + sqrt(a*x + b))) + 1
05*(d + sqrt(c + sqrt(a*x + b)))^(3/2)*d^2*sgn(sqrt(c + sqrt(a*x + b))) - 1
05*sqrt(d + sqrt(c + sqrt(a*x + b)))*d^3*sgn(sqrt(c + sqrt(a*x + b))) - 35*
c*(d + sqrt(c + sqrt(a*x + b)))^(3/2)*sgn(sqrt(c + sqrt(a*x + b))) + 105*c*
sqrt(d + sqrt(c + sqrt(a*x + b)))*d*sgn(sqrt(c + sqrt(a*x + b))))/a
```

maple [A] time = 0.01, size = 93, normalized size = 0.53

$$\frac{\frac{8\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{7}{2}}}{7}-\frac{24d\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{5}{2}}}{5}+\frac{8\left(3d^2-c\right)\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{3}{2}}}{3}-8\left(d^2-c\right)d\sqrt{d+\sqrt{c+\sqrt{ax+b}}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+(c+(a*x+b)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 2/a*(4/7*(d+(c+(a*x+b)^(1/2))^(1/2))^(7/2)-12/5*d*(d+(c+(a*x+b)^(1/2))^(1/2
))^(5/2)+4/3*(3*d^2-c)*(d+(c+(a*x+b)^(1/2))^(1/2))^(3/2)-4*(d^2-c)*d*(d+(c+
(a*x+b)^(1/2))^(1/2))^(1/2))
```

maxima [A] time = 0.31, size = 92, normalized size = 0.53

$$\frac{8\left(15\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{7}{2}}-63\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{5}{2}}d+35\left(3d^2-c\right)\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{3}{2}}-105\left(d^3-cd\right)\sqrt{d+\sqrt{c+\sqrt{ax+b}}}\right)}{105a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+(c+(a*x+b)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/105*(15*(d + sqrt(c + sqrt(a*x + b)))^(7/2) - 63*(d + sqrt(c + sqrt(a*x +
 b)))^(5/2)*d + 35*(3*d^2 - c)*(d + sqrt(c + sqrt(a*x + b)))^(3/2) - 105*(d
^3 - c*d)*sqrt(d + sqrt(c + sqrt(a*x + b))))/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{d+\sqrt{c+\sqrt{b+a\,x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + (c + (b + a*x)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] int(1/(d + (c + (b + a*x)^(1/2))^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+\sqrt{c+\sqrt{ax+b}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+(c+(a*x+b)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(d + sqrt(c + sqrt(a*x + b))), x)
```

3.1883 $\int \frac{1}{x\sqrt[3]{3+3x+x^2}}\,dx$

Optimal. Leaf size=175

$$\frac{\log\left(-3\sqrt[3]{x^2+3x+3}+\sqrt[3]{3}\,x+3\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\log\left(3^{2/3}x^2+9\left(x^2+3x+3\right)^{2/3}+\left(3\sqrt[3]{3}\,x+9\sqrt[3]{3}\right)\sqrt[3]{x^2+3x+3}+6\,3^{2/3}\right.}{6\sqrt[3]{3}}$$

Rubi [A] time = 0.02, antiderivative size = 83, normalized size of antiderivative = 0.47, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {752}

$$\frac{\log\left(3^{2/3}\sqrt[3]{x^2+3x+3}-x-3\right)}{2\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{2(x+3)}{3\sqrt[6]{3}\sqrt[3]{x^2+3x+3}}+\frac{1}{\sqrt{3}}\right)}{3^{5/6}}-\frac{\log(x)}{2\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(3 + 3*x + x^2)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2*(3 + x))/(3*3^(1/6)*(3 + 3*x + x^2)^(1/3))]/3^(5/6)
) - Log[x]/(2*3^(1/3)) + Log[-3 - x + 3^(2/3)*(3 + 3*x + x^2)^(1/3)]/(2*3^(
1/3))
```

Rule 752

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[-3*c*e^2*(2*c*d - b*e), 3]}, -Simp[(Sqrt[3]*c*e*ArcTa
n[1/Sqrt[3] - (2*(c*d - b*e - c*e*x))/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3))])
/q^2, x] + (-Simp[(3*c*e*Log[d + e*x])/(2*q^2), x] + Simp[(3*c*e*Log[c*d -
b*e - c*e*x + q*(a + b*x + c*x^2)^(1/3)])/(2*q^2), x])] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[2*c*d - b*e, 0] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*c
*e^2, 0] && NegQ[c*e^2*(2*c*d - b*e)]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{3+3x+x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2(3+x)}{3\sqrt[6]{3}\sqrt[3]{3+3x+x^2}}\right)}{3^{5/6}}-\frac{\log(x)}{2\sqrt[3]{3}}+\frac{\log\left(-3-x+3^{2/3}\sqrt[3]{3+3x+x^2}\right)}{2\sqrt[3]{3}}$$

Mathematica [C] time = 0.05, size = 114, normalized size = 0.65

$$-\frac{3\sqrt[3]{\frac{2x-i\sqrt{3}+3}{x}}\sqrt[3]{\frac{2x+i\sqrt{3}+3}{x}}\,F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{3+i\sqrt{3}}{2x},\frac{i\left(3i+\sqrt{3}\right)}{2x}\right)}{2\,2^{2/3}\sqrt[3]{x^2+3x+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*(3 + 3*x + x^2)^(1/3)),x]
```

```
[Out] (-3*((3 - I*Sqrt[3] + 2*x)/x)^(1/3)*((3 + I*Sqrt[3] + 2*x)/x)^(1/3)*AppellF
1[2/3, 1/3, 1/3, 5/3, -1/2*(3 + I*Sqrt[3])/x, ((I/2)*(3*I + Sqrt[3]))/x])/(
2*2^(2/3)*(3 + 3*x + x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.24, size = 175, normalized size = 1.00

$$\frac{\log\left(-3\sqrt[3]{x^2+3x+3}+\sqrt[3]{3}x+3\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\log\left(3^{2/3}x^2+9\left(x^2+3x+3\right)^{2/3}+\left(3\sqrt[3]{3}x+9\sqrt[3]{3}\right)\sqrt[3]{x^2+3x+3}+6\,3^{2/3}x+9\,3^{2/3}\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2+3x+3}}{\sqrt{3}}+\frac{2x}{3\sqrt[6]{3}}+\frac{2}{\sqrt[6]{3}}}{\sqrt[3]{x^2+3x+3}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(3 + 3*x + x^2)^(1/3)),x]
```

```
[Out] -(ArcTan[(2/3^(1/6) + (2*x)/(3*3^(1/6)) + (3 + 3*x + x^2)^(1/3)/Sqrt[3])/(3
 + 3*x + x^2)^(1/3)]/3^(5/6)) + Log[3*3^(1/3) + 3^(1/3)*x - 3*(3 + 3*x + x^
2)^(1/3)]/(3*3^(1/3)) - Log[9*3^(2/3) + 6*3^(2/3)*x + 3^(2/3)*x^2 + (9*3^(1
/3) + 3*3^(1/3)*x)*(3 + 3*x + x^2)^(1/3) + 9*(3 + 3*x + x^2)^(2/3)]/(6*3^(1
/3))
```

fricas [A] time = 1.77, size = 156, normalized size = 0.89

$$\frac{1}{9}\cdot 3^{\frac{2}{3}}\log\left(\frac{3^{\frac{1}{3}}(x+3)-3\left(x^2+3x+3\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(\frac{3^{\frac{1}{3}}\left(x^2+6x+9\right)+3\cdot 3^{\frac{2}{3}}\left(x^2+3x+3\right)^{\frac{2}{3}}+3\left(x^2+3x+3\right)^{\frac{1}{3}}(x+3)}{x^2}\right)-\frac{1}{3}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{1}{6}}\left(3^{\frac{1}{3}}x^3+6\cdot 3^{\frac{2}{3}}\left(x^2+3x+3\right)^{\frac{2}{3}}(x+3)-6\left(x^2+6x+9\right)\left(x^2+3x+3\right)^{\frac{1}{3}}\right)}{3\left(x^3+18x^2+54x+54\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3*x+3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/9*3^(2/3)*log((3^(1/3)*(x + 3) - 3*(x^2 + 3*x + 3)^(1/3))/x) - 1/18*3^(2/
3)*log((3^(1/3)*(x^2 + 6*x + 9) + 3*3^(2/3)*(x^2 + 3*x + 3)^(2/3) + 3*(x^2
+ 3*x + 3)^(1/3)*(x + 3))/x^2) - 1/3*3^(1/6)*arctan(1/3*3^(1/6)*(3^(1/3)*x^
3 + 6*3^(2/3)*(x^2 + 3*x + 3)^(2/3)*(x + 3) - 6*(x^2 + 6*x + 9)*(x^2 + 3*x
+ 3)^(1/3))/(x^3 + 18*x^2 + 54*x + 54))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3\,x+3\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3*x+3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3*x + 3)^(1/3)*x), x)
```

maple [C] time = 14.18, size = 2320, normalized size = 13.26

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+3*x+3)^(1/3),x)
```

```
[Out] 1/9*RootOf(_Z^3-9)*ln((18684300290294655*(x^2+3*x+3)^(2/3)+19475041052229*R
ootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^3*x^3-159
4171217561031*RootOf(_Z^3-9)*x^2-4782513652683093*RootOf(_Z^3-9)*x+51850893
6864960*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x^3+13926825976
107285*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x^2+417804779283
21855*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x+417804779283218
55*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)-59352506063936*RootO
f(_Z^3-9)*x^3-4782513652683093*RootOf(_Z^3-9)-170135744908815*RootOf(RootOf
(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+7291531924663
5*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2-83
46446165241*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3
-9)^3+6228100096764885*(x^2+3*x+3)^(2/3)*x-4609623772788309*(x^2+3*x+3)^(1/
```

3)*RootOf(_Z^3-9)^2+24305106415545*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3
-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2*x^2-2782148721747*RootOf(RootOf(_Z^3-9)^2+3*
_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^3*x^2+72915319246635*RootOf(RootOf
(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2*x-8346446165241*R
ootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^3*x+53949
2107992192*(x^2+3*x+3)^(2/3)*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+3*_Z*
RootOf(_Z^3-9)+9*_Z^2)*x-2076033365588295*(x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)*
RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x^2-12456200193529770*(
x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf
(_Z^3-9)*x+1618476323976576*(x^2+3*x+3)^(2/3)*RootOf(_Z^3-9)^2*RootOf(RootO
f(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)-512180419198701*(x^2+3*x+3)^(1/3)*R
ootOf(_Z^3-9)^2*x^2-3073082515192206*(x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^2*x-1
8684300290294655*(x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3
-9)+9*_Z^2)*RootOf(_Z^3-9))/x^3)-1/9*ln(-(-13828871318364927*(x^2+3*x+3)^(2
/3)-19475041052229*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*Root
Of(_Z^3-9)^3*x^3-1585824771395790*RootOf(_Z^3-9)*x^2-4757474314187370*RootO
f(_Z^3-9)*x-1382249059253274*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*
_Z^2)*x^3-18611384971048020*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_
Z^2)*x^2-55834154913144060*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z
^2)*x-55834154913144060*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)
-117777629220623*RootOf(_Z^3-9)*x^3-4757474314187370*RootOf(_Z^3-9)-2285608
68065502*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-
9)^2*x^3+97954657742358*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)
^2*RootOf(_Z^3-9)^2+8346446165241*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-
9)+9*_Z^2)*RootOf(_Z^3-9)^3-4609623772788309*(x^2+3*x+3)^(2/3)*x+6228100096
764885*(x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^2+32651552580786*RootOf(RootOf(_Z^3
-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2*x^2+2782148721747*Root
Of(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^3*x^2+979546
57742358*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-
9)^2*x+8346446165241*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*Ro
otOf(_Z^3-9)^3*x+539492107992192*(x^2+3*x+3)^(2/3)*RootOf(_Z^3-9)^2*RootOf(
RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x+1536541257596103*(x^2+3*x+3)
^(1/3)*RootOf(_Z^3-9)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x
^2+9219247545576618*(x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_
Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)*x+1618476323976576*(x^2+3*x+3)^(2/3)*RootOf(_
Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)+69201112186276
5*(x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^2*x^2+4152066731176590*(x^2+3*x+3)^(1/3)
*RootOf(_Z^3-9)^2*x+13828871318364927*(x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-
9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9))/x^3)*RootOf(_Z^3-9)-1/3*ln
(-(-13828871318364927*(x^2+3*x+3)^(2/3)-19475041052229*RootOf(RootOf(_Z^3-9
)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^3*x^3-1585824771395790*RootO
f(_Z^3-9)*x^2-4757474314187370*RootOf(_Z^3-9)*x-1382249059253274*RootOf(Roo
tOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x^3-18611384971048020*RootOf(Root
Of(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x^2-55834154913144060*RootOf(RootO
f(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x-55834154913144060*RootOf(RootOf(_
Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)-117777629220623*RootOf(_Z^3-9)*x^3-475
7474314187370*RootOf(_Z^3-9)-228560868065502*RootOf(RootOf(_Z^3-9)^2+3*_Z*R
ootOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+97954657742358*RootOf(RootOf(_
Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2+8346446165241*RootO
f(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^3-46096237727
88309*(x^2+3*x+3)^(2/3)*x+6228100096764885*(x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)
^2+32651552580786*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)^2*Roo
tOf(_Z^3-9)^2*x^2+2782148721747*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)
+9*_Z^2)*RootOf(_Z^3-9)^3*x^2+97954657742358*RootOf(RootOf(_Z^3-9)^2+3*_Z*R
ootOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2*x+8346446165241*RootOf(RootOf(_Z^3
-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^3*x+539492107992192*(x^2+3
*x+3)^(2/3)*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*
_Z^2)*x+1536541257596103*(x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)*RootOf(RootOf(_Z^
3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x^2+9219247545576618*(x^2+3*x+3)^(1/3)*R

```
ootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)*x+1618476
323976576*(x^2+3*x+3)^(2/3)*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)^2+3*_Z*R
ootOf(_Z^3-9)+9*_Z^2)+692011121862765*(x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^2*x^
2+4152066731176590*(x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^2*x+13828871318364927*(
x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf
(_Z^3-9))/x^3)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3\,x+3\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3*x+3)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 3*x + 3)^(1/3)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(x^2+3\,x+3\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x + x^2 + 3)^(1/3)),x)
[Out] int(1/(x*(3*x + x^2 + 3)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{x^2+3x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+3*x+3)**(1/3),x)
[Out] Integral(1/(x*(x**2 + 3*x + 3)**(1/3)), x)
```

3.1884 $\int \frac{-ab+x^2}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(abd-(1+ad+bd)x+dx^2\right)}\,dx$

Optimal. Leaf size=175

$$\frac{\log\left(x-\sqrt[3]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}\right)}{d^{2/3}}-\frac{\log\left(d^{2/3}\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+\sqrt[3]{d}\,x\sqrt[3]{x^3(-a-b)+abx^2+x^4}+\right.}{2d^{2/3}}$$

Rubi [F] time = 9.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-ab+x^2}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(abd-(1+ad+bd)x+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a*b) + x^2)/((x^2*(-a + x)*(-b + x))^(1/3)*(a*b*d - (1 + a*d + b*d)*
x + d*x^2)),x]
```

```
[Out] (3*x*(1 - x/a)^(1/3)*(1 - x/b)^(1/3)*AppellF1[1/3, 1/3, 1/3, 4/3, x/a, x/b]
)/(d*((a - x)*(b - x)*x^2)^(1/3)) + ((1 + a*d + b*d + Sqrt[a^2*d^2 + 2*a*d*
(1 - b*d) + (1 + b*d)^2])*x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Int][
1/(x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*(-1 - a*d - b*d - Sqrt[1 + 2*a*d +
 2*b*d + a^2*d^2 - 2*a*b*d^2 + b^2*d^2] + 2*d*x)), x])/(d*((a - x)*(b - x)*
x^2)^(1/3)) + ((1 + a*d + b*d - Sqrt[a^2*d^2 + 2*a*d*(1 - b*d) + (1 + b*d)^
2])*x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Int][1/(x^(2/3)*(-a + x)^(1
/3)*(-b + x)^(1/3)*(-1 - a*d - b*d + Sqrt[1 + 2*a*d + 2*b*d + a^2*d^2 - 2*a
*b*d^2 + b^2*d^2] + 2*d*x)), x])/(d*((a - x)*(b - x)*x^2)^(1/3))
```

Rubi steps

$$\int \frac{-ab+x^2}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(abd-(1+ad+bd)x+dx^2\right)}\,dx = \frac{\left(x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\int \frac{-ab+x^2}{x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\left(abd-(1+ad+b\right.}}{\sqrt[3]{x^2(-a+x)(-b+x)}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\int\left(\frac{1}{dx^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}}-\frac{}{dx^{2/3}\sqrt[3]{}}\right.}{\sqrt[3]{x^2(-a+x)(-b+x}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\int \frac{1}{x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}}\,dx}{d\sqrt[3]{x^2(-a+x)(-b+x)}} - \frac{\left(x^{2/3}\right.}{}$$

$$= -\frac{\left(x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\int\left(\frac{-1-ad-bd-\sqrt{1+2}}{x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\left(-1-ad-bd\right.}\right.}{}$$

$$= -\frac{\left(\left(-1-ad-bd-\sqrt{a^2d^2+2ad(1-bd)+(1+bd)^2}\right)\right.}{}$$

$$= \frac{3x\sqrt[3]{1-\frac{x}{a}}\,\sqrt[3]{1-\frac{x}{b}}\,F_1\left(\frac{1}{3};\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{x}{a},\frac{x}{b}\right)}{d\sqrt[3]{(a-x)(b-x)x^2}} - \frac{\left(\left(-1-ad-\right.\right.}{}$$

Mathematica [F] time = 14.41, size = 0, normalized size = 0.00

$$\int \frac{-ab+x^2}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(abd-(1+ad+bd)x+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-(a*b) + x^2)/((x^2*(-a + x)*(-b + x))^(1/3)*(a*b*d - (1 + a*d +
 b*d)*x + d*x^2)),x]
```

```
[Out] Integrate[(-(a*b) + x^2)/((x^2*(-a + x)*(-b + x))^(1/3)*(a*b*d - (1 + a*d +
 b*d)*x + d*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.44, size = 175, normalized size = 1.00

$$\frac{\log\left(x-\sqrt[3]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}\right)}{d^{2/3}}-\frac{\log\left(d^{2/3}\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+\sqrt[3]{d}\,x\sqrt[3]{x^3(-a-b)+abx^2+x^4}+x^2\right)}{2d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}+x}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*b) + x^2)/((x^2*(-a + x)*(-b + x))^(1/3)*(a*b*d - (
1 + a*d + b*d)*x + d*x^2)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*d^(1/3)*(a*b*x^2 + (-a - b)*x^3 + x^4)
^(1/3))])/d^(2/3)) + Log[x - d^(1/3)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3)]/
d^(2/3) - Log[x^2 + d^(1/3)*x*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3) + d^(2/3
)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(a*b*d-(a*d+b*d+1)*x+d*x^2),
x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ab-x^2}{\left((a-x)(b-x)x^2\right)^{\frac{1}{3}}\left(abd+dx^2-(ad+bd+1)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(a*b*d-(a*d+b*d+1)*x+d*x^2),
x, algorithm="giac")
```

```
[Out] integrate(-(a*b - x^2)/(((a - x)*(b - x)*x^2)^(1/3)*(a*b*d + d*x^2 - (a*d +
 b*d + 1)*x)), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{-ab+x^2}{\left(x^2\left(-a+x\right)\left(-b+x\right)\right)^{\frac{1}{3}}\left(abd-\left(ad+bd+1\right)x+d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(a*b*d-(a*d+b*d+1)*x+d*x^2),x)
```

```
[Out] int((-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(a*b*d-(a*d+b*d+1)*x+d*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ab-x^2}{\left((a-x)(b-x)x^2\right)^{\frac{1}{3}}\left(abd+dx^2-(ad+bd+1)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(a*b*d-(a*d+b*d+1)*x+d*x^2),
x, algorithm="maxima")
```

```
[Out] -integrate((a*b - x^2)/(((a - x)*(b - x)*x^2)^(1/3)*(a*b*d + d*x^2 - (a*d +
 b*d + 1)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a\,b-x^2}{\left(d\,x^2+\left(-a\,d-b\,d-1\right)\,x+a\,b\,d\right)\,{\left(x^2\,\left(a-x\right)\,\left(b-x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*b - x^2)/((d*x^2 - x*(a*d + b*d + 1) + a*b*d)*(x^2*(a - x)*(b - x))
^(1/3)),x)
```

```
[Out] int(-(a*b - x^2)/((d*x^2 - x*(a*d + b*d + 1) + a*b*d)*(x^2*(a - x)*(b - x))
^(1/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+x**2)/(x**2*(-a+x)*(-b+x))**(1/3)/(a*b*d-(a*d+b*d+1)*x+d*x*
*2),x)
```

```
[Out] Timed out
```

3.1885 $\int \frac{(-3+2x)(1-x+x^3)^{2/3}}{x^3(-2+2x+x^3)}\,dx$

Optimal. Leaf size=175

$$-\frac{1}{2}\left(\frac{3}{2}\right)^{2/3}\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3-x+1}-3x\right)+\frac{3\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2}\sqrt[3]{x^3-x+1}+\sqrt[3]{3}x}\right)}{2\,2^{2/3}}-\frac{3\left(x^3-x+1\right)^{2/3}}{4x^2}+\frac{1}{4}\left(\frac{3}{2}\right)^{2/3}\log\left(\sqrt[3]{\right.$$

Rubi [F] time = 2.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-3+2x)\left(1-x+x^3\right)^{2/3}}{x^3\left(-2+2x+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + 2*x)*(1 - x + x^3)^(2/3))/(x^3*(-2 + 2*x + x^3)),x]
```

```
[Out] (27*2^(1/3)*(1 - x + x^3)^(2/3)*Defer[Int][((((18 - 2*Sqrt[69])^(1/3) + (2*
(9 + Sqrt[69]))^(1/3))/6^(2/3) + x)^(2/3)*((-6 + 2^(1/3)*(27 - 3*Sqrt[69])^
(2/3) + 6*3^(1/3)*(2/(9 - Sqrt[69]))^(2/3))/18 - (((9 - Sqrt[69])^(1/3) + (
9 + Sqrt[69])^(1/3))*x)/(2^(1/3)*3^(2/3)) + x^2)^(2/3))/x^3, x])/((6^(1/3)*
((18 - 2*Sqrt[69])^(1/3) + (2*(9 + Sqrt[69]))^(1/3)) + 6*x)^(2/3)*(-6 + 2^(
1/3)*(27 - 3*Sqrt[69])^(2/3) + 6*3^(1/3)*(2/(9 - Sqrt[69]))^(2/3) - 3*2^(2/
3)*3^(1/3)*((9 - Sqrt[69])^(1/3) + (9 + Sqrt[69])^(1/3))*x + 18*x^2)^(2/3))
 + (9*2^(1/3)*(1 - x + x^3)^(2/3)*Defer[Int][((((18 - 2*Sqrt[69])^(1/3) + (
2*(9 + Sqrt[69]))^(1/3))/6^(2/3) + x)^(2/3)*((-6 + 2^(1/3)*(27 - 3*Sqrt[69]
)^(2/3) + 6*3^(1/3)*(2/(9 - Sqrt[69]))^(2/3))/18 - (((9 - Sqrt[69])^(1/3) +
 (9 + Sqrt[69])^(1/3))*x)/(2^(1/3)*3^(2/3)) + x^2)^(2/3))/x^2, x])/((6^(1/3
)*((18 - 2*Sqrt[69])^(1/3) + (2*(9 + Sqrt[69]))^(1/3)) + 6*x)^(2/3)*(-6 + 2
^(1/3)*(27 - 3*Sqrt[69])^(2/3) + 6*3^(1/3)*(2/(9 - Sqrt[69]))^(2/3) - 3*2^(
2/3)*3^(1/3)*((9 - Sqrt[69])^(1/3) + (9 + Sqrt[69])^(1/3))*x + 18*x^2)^(2/3
)) + (9*2^(1/3)*(1 - x + x^3)^(2/3)*Defer[Int][((((18 - 2*Sqrt[69])^(1/3) +
 (2*(9 + Sqrt[69]))^(1/3))/6^(2/3) + x)^(2/3)*((-6 + 2^(1/3)*(27 - 3*Sqrt[6
9])^(2/3) + 6*3^(1/3)*(2/(9 - Sqrt[69]))^(2/3))/18 - (((9 - Sqrt[69])^(1/3)
 + (9 + Sqrt[69])^(1/3))*x)/(2^(1/3)*3^(2/3)) + x^2)^(2/3))/x, x])/((6^(1/3
)*((18 - 2*Sqrt[69])^(1/3) + (2*(9 + Sqrt[69]))^(1/3)) + 6*x)^(2/3)*(-6 + 2
^(1/3)*(27 - 3*Sqrt[69])^(2/3) + 6*3^(1/3)*(2/(9 - Sqrt[69]))^(2/3) - 3*2^(
2/3)*3^(1/3)*((9 - Sqrt[69])^(1/3) + (9 + Sqrt[69])^(1/3))*x + 18*x^2)^(2/3
)) - (5*Defer[Int][(1 - x + x^3)^(2/3)/(-2 + 2*x + x^3), x])/2 - Defer[Int]
[(x*(1 - x + x^3)^(2/3))/(-2 + 2*x + x^3), x]/2 - Defer[Int][(x^2*(1 - x +
x^3)^(2/3))/(-2 + 2*x + x^3), x]/2
```

Rubi steps

$$\int \frac{(-3+2x)\left(1-x+x^3\right)^{2/3}}{x^3\left(-2+2x+x^3\right)}\,dx = \int \left(\frac{3\left(1-x+x^3\right)^{2/3}}{2x^3} + \frac{\left(1-x+x^3\right)^{2/3}}{2x^2} + \frac{\left(1-x+x^3\right)^{2/3}}{2x} + \frac{\left(-5-x-x^2\right)\left(1-x\right.}{2\left(-2+2x+x\right.}\right.$$

$$= \frac{1}{2}\int \frac{\left(1-x+x^3\right)^{2/3}}{x^2}\,dx + \frac{1}{2}\int \frac{\left(1-x+x^3\right)^{2/3}}{x}\,dx + \frac{1}{2}\int \frac{\left(-5-x-x^2\right)\left(1-x+\right.}{-2+2x+x^3}$$

$$= \frac{1}{2}\int \left(-\frac{5\left(1-x+x^3\right)^{2/3}}{-2+2x+x^3} - \frac{x\left(1-x+x^3\right)^{2/3}}{-2+2x+x^3} - \frac{x^2\left(1-x+x^3\right)^{2/3}}{-2+2x+x^3}\right) dx + \frac{(1}{2\left(\sqrt[3]{18-}\right.}$$

$$= -\left(\frac{1}{2}\int \frac{x\left(1-x+x^3\right)^{2/3}}{-2+2x+x^3}\,dx\right) - \frac{1}{2}\int \frac{x^2\left(1-x+x^3\right)^{2/3}}{-2+2x+x^3}\,dx - \frac{5}{2}\int \frac{\left(1-x+x^3\right)^{2/3}}{-2+2x+x^3}$$

Mathematica [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(-3+2x)\left(1-x+x^3\right)^{2/3}}{x^3\left(-2+2x+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + 2*x)*(1 - x + x^3)^(2/3))/(x^3*(-2 + 2*x + x^3)),x]
```

```
[Out] Integrate[((-3 + 2*x)*(1 - x + x^3)^(2/3))/(x^3*(-2 + 2*x + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.60, size = 175, normalized size = 1.00

$$-\frac{1}{2}\left(\frac{3}{2}\right)^{2/3}\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3-x+1}-3x\right)+\frac{3\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2}\sqrt[3]{x^3-x+1}+\sqrt[3]{3}x}\right)}{2\,2^{2/3}}-\frac{3\left(x^3-x+1\right)^{2/3}}{4x^2}+\frac{1}{4}\left(\frac{3}{2}\right)^{2/3}\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3-x+1}\,x+2^{2/3}\sqrt[3]{3}\left(x^3-x+1\right)^{2/3}+3x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + 2*x)*(1 - x + x^3)^(2/3))/(x^3*(-2 + 2*x + x^3)),
x]
```

```
[Out] (-3*(1 - x + x^3)^(2/3))/(4*x^2) + (3*3^(1/6)*ArcTan[(3^(5/6)*x)/(3^(1/3)*x
 + 2*2^(1/3)*(1 - x + x^3)^(1/3))])/(2*2^(2/3)) - ((3/2)^(2/3)*Log[-3*x + 2
^(1/3)*3^(2/3)*(1 - x + x^3)^(1/3)])/2 + ((3/2)^(2/3)*Log[3*x^2 + 2^(1/3)*3
^(2/3)*x*(1 - x + x^3)^(1/3) + 2^(2/3)*3^(1/3)*(1 - x + x^3)^(2/3)])/4
```

fricas [B] time = 15.47, size = 426, normalized size = 2.43

$$-\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-9)^{\frac{1}{3}}x^2\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(4\cdot 4^{\frac{2}{3}}(-9)^{\frac{2}{3}}\left(4x^7+7x^5-7x^4-2x^3+4x^2-2x\right)\left(x^3-x+1\right)^{\frac{2}{3}}+12(-9)^{\frac{1}{3}}\left(55x^8-50x^6+50x^5+4x^4-8x^3+4x^2\right)\left(x^3-x+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\left(377x^9-600x^7+600x^6+204x^5-408x^4+196x^3+24x^2-24x+8\right)\right)}{6\left(487x^9-480x^7+480x^6+12x^5-24x^4+20x^3-24x^2+24x-8\right)}\right)-2\cdot 4^{\frac{2}{3}}(-9)^{\frac{1}{3}}x^2\log\left(-\frac{6\cdot 4^{\frac{1}{3}}(-9)^{\frac{2}{3}}\left(x^3-x+1\right)^{\frac{1}{3}}x^2+4^{\frac{2}{3}}(-9)^{\frac{1}{3}}\left(x^3+2x-2\right)-36\left(x^3-x+1\right)^{\frac{2}{3}}x}{x^3+2x-2}\right)+4^{\frac{2}{3}}(-9)^{\frac{1}{3}}x^2\log\left(-\frac{18\cdot 4^{\frac{2}{3}}(-9)^{\frac{1}{3}}\left(4x^4-x^2+x\right)\left(x^3-x+1\right)^{\frac{2}{3}}-4^{\frac{1}{3}}(-9)^{\frac{2}{3}}\left(55x^6-50x^4+50x^3+4x^2-8x+4\right)-54\left(7x^5-4x^3+4x^2\right)\left(x^3-x+1\right)^{\frac{1}{3}}}{x^6+4x^4-4x^3+4x^2-8x+4}\right)+36\left(x^3-x+1\right)^{\frac{2}{3}}}{48x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^3-x+1)^(2/3)/x^3/(x^3+2*x-2),x, algorithm="fricas")
```

```
[Out] -1/48*(4*4^(1/6)*sqrt(3)*(-9)^(1/3)*x^2*arctan(1/6*4^(1/6)*sqrt(3)*(4*4^(2/
3)*(-9)^(2/3)*(4*x^7 + 7*x^5 - 7*x^4 - 2*x^3 + 4*x^2 - 2*x)*(x^3 - x + 1)^(
```

```
2/3) + 12*(-9)^(1/3)*(55*x^8 - 50*x^6 + 50*x^5 + 4*x^4 - 8*x^3 + 4*x^2)*(x^
3 - x + 1)^(1/3) - 4^(1/3)*(377*x^9 - 600*x^7 + 600*x^6 + 204*x^5 - 408*x^4
 + 196*x^3 + 24*x^2 - 24*x + 8))/(487*x^9 - 480*x^7 + 480*x^6 + 12*x^5 - 24
*x^4 + 20*x^3 - 24*x^2 + 24*x - 8)) - 2*4^(2/3)*(-9)^(1/3)*x^2*log(-(6*4^(1
/3)*(-9)^(2/3)*(x^3 - x + 1)^(1/3)*x^2 + 4^(2/3)*(-9)^(1/3)*(x^3 + 2*x - 2)
 - 36*(x^3 - x + 1)^(2/3)*x)/(x^3 + 2*x - 2)) + 4^(2/3)*(-9)^(1/3)*x^2*log(
-(18*4^(2/3)*(-9)^(1/3)*(4*x^4 - x^2 + x)*(x^3 - x + 1)^(2/3) - 4^(1/3)*(-9
)^(2/3)*(55*x^6 - 50*x^4 + 50*x^3 + 4*x^2 - 8*x + 4) - 54*(7*x^5 - 4*x^3 +
4*x^2)*(x^3 - x + 1)^(1/3))/(x^6 + 4*x^4 - 4*x^3 + 4*x^2 - 8*x + 4)) + 36*(
x^3 - x + 1)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-x+1\right)^{\frac{2}{3}}(2x-3)}{\left(x^3+2x-2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^3-x+1)^(2/3)/x^3/(x^3+2*x-2),x, algorithm="giac")
```

```
[Out] integrate((x^3 - x + 1)^(2/3)*(2*x - 3)/((x^3 + 2*x - 2)*x^3), x)
```

maple [C] time = 4.48, size = 873, normalized size = 4.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*x)*(x^3-x+1)^(2/3)/x^3/(x^3+2*x-2),x)
```

```
[Out] -3/4*(x^3-x+1)^(2/3)/x^2-1/4*ln((2*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^
3+18)+36*_Z^2)*RootOf(_Z^3+18)^3*x^3+6*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf
(_Z^3+18)+36*_Z^2)^2*RootOf(_Z^3+18)^2*x^3-3*RootOf(_Z^3+18)^2*(x^3-x+1)^(1
/3)*x^2+4*RootOf(_Z^3+18)*x^3+12*x^3*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_
Z^3+18)+36*_Z^2)-18*(x^3-x+1)^(2/3)*x-4*RootOf(_Z^3+18)*x-12*RootOf(RootOf(
_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2)*x+4*RootOf(_Z^3+18)+12*RootOf(Root
Of(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2))/(x^3+2*x-2))*RootOf(_Z^3+18)-3
/2*ln((2*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2)*RootOf(_Z^3
+18)^3*x^3+6*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2)^2*RootO
f(_Z^3+18)^2*x^3-3*RootOf(_Z^3+18)^2*(x^3-x+1)^(1/3)*x^2+4*RootOf(_Z^3+18)*
x^3+12*x^3*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2)-18*(x^3-x
+1)^(2/3)*x-4*RootOf(_Z^3+18)*x-12*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^
3+18)+36*_Z^2)*x+4*RootOf(_Z^3+18)+12*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(
_Z^3+18)+36*_Z^2))/(x^3+2*x-2))*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+1
8)+36*_Z^2)+3/2*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2)*ln(-
(5*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2)*RootOf(_Z^3+18)^3
*x^3+12*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2)^2*RootOf(_Z^
3+18)^2*x^3-14*(x^3-x+1)^(2/3)*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18
)+36*_Z^2)*RootOf(_Z^3+18)^2*x+8*RootOf(_Z^3+18)^2*(x^3-x+1)^(1/3)*x^2+42*(
x^3-x+1)^(1/3)*RootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2)*RootO
f(_Z^3+18)*x^2-25*RootOf(_Z^3+18)*x^3-60*x^3*RootOf(RootOf(_Z^3+18)^2+6*_Z*
RootOf(_Z^3+18)+36*_Z^2)+48*(x^3-x+1)^(2/3)*x+10*RootOf(_Z^3+18)*x+24*RootO
f(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2)*x-10*RootOf(_Z^3+18)-24*R
ootOf(RootOf(_Z^3+18)^2+6*_Z*RootOf(_Z^3+18)+36*_Z^2))/(x^3+2*x-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-x+1\right)^{\frac{2}{3}}(2x-3)}{\left(x^3+2x-2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^3-x+1)^(2/3)/x^3/(x^3+2*x-2),x, algorithm="maxima")
[Out] integrate((x^3 - x + 1)^(2/3)*(2*x - 3)/((x^3 + 2*x - 2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x-3)\,\left(x^3-x+1\right)^{2/3}}{x^3\,\left(x^3+2\,x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x - 3)*(x^3 - x + 1)^(2/3))/(x^3*(2*x + x^3 - 2)),x)
[Out] int(((2*x - 3)*(x^3 - x + 1)^(2/3))/(x^3*(2*x + x^3 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x-3)\left(x^3-x+1\right)^{\frac{2}{3}}}{x^3\left(x^3+2x-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x**3-x+1)**(2/3)/x**3/(x**3+2*x-2),x)
[Out] Integral((2*x - 3)*(x**3 - x + 1)**(2/3)/(x**3*(x**3 + 2*x - 2)), x)
```

3.1886 $\int \frac{(-3+4x)\left(-1+2x+x^3\right)^{2/3}}{x^3\left(2-4x+x^3\right)} dx$

Optimal. Leaf size=175

$$\frac{1}{2}\left(\frac{3}{2}\right)^{2/3} \log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3+2x-1}-3x\right)-\frac{3\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2}\sqrt[3]{x^3+2x-1}+\sqrt[3]{3}x}\right)}{2\,2^{2/3}}+\frac{3\left(x^3+2x-1\right)^{2/3}}{4x^2}-\frac{1}{4}\left(\frac{3}{2}\right)^{2/3}\log\left(\right.$$

Rubi [F] time = 2.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-3+4x)\left(-1+2x+x^3\right)^{2/3}}{x^3\left(2-4x+x^3\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + 4*x)*(-1 + 2*x + x^3)^(2/3))/(x^3*(2 - 4*x + x^3)),x]
```

```
[Out] (-27*2^(1/3)*(-1 + 2*x + x^3)^(2/3)*Defer[Int][(((4*(3/(9 + Sqrt[177]))^(1/
3) - (2*(9 + Sqrt[177]))^(1/3))/6^(2/3) + x)^(2/3)*((12 + 24*3^(1/3)*(2/(9
+ Sqrt[177]))^(2/3) + 2^(1/3)*(3*(9 + Sqrt[177]))^(2/3))/18 - ((2*(6/(9 + S
qrt[177]))^(1/3) - ((9 + Sqrt[177])/2)^(1/3))*x)/3^(2/3) + x^2)^(2/3))/x^3,
 x])/((6^(1/3)*(4*(3/(9 + Sqrt[177]))^(1/3) - (2*(9 + Sqrt[177]))^(1/3)) +
6*x)^(2/3)*(12 + 24*3^(1/3)*(2/(9 + Sqrt[177]))^(2/3) + 2^(1/3)*(3*(9 + Sqr
t[177]))^(2/3) - 6*3^(1/3)*(2*(6/(9 + Sqrt[177]))^(1/3) - ((9 + Sqrt[177])/
2)^(1/3))*x + 18*x^2)^(2/3)) - (18*2^(1/3)*(-1 + 2*x + x^3)^(2/3)*Defer[Int
][(((4*(3/(9 + Sqrt[177]))^(1/3) - (2*(9 + Sqrt[177]))^(1/3))/6^(2/3) + x)^
(2/3)*((12 + 24*3^(1/3)*(2/(9 + Sqrt[177]))^(2/3) + 2^(1/3)*(3*(9 + Sqrt[17
7]))^(2/3))/18 - ((2*(6/(9 + Sqrt[177]))^(1/3) - ((9 + Sqrt[177])/2)^(1/3))
*x)/3^(2/3) + x^2)^(2/3))/x^2, x])/((6^(1/3)*(4*(3/(9 + Sqrt[177]))^(1/3) -
 (2*(9 + Sqrt[177]))^(1/3)) + 6*x)^(2/3)*(12 + 24*3^(1/3)*(2/(9 + Sqrt[177]
))^(2/3) + 2^(1/3)*(3*(9 + Sqrt[177]))^(2/3) - 6*3^(1/3)*(2*(6/(9 + Sqrt[17
7]))^(1/3) - ((9 + Sqrt[177])/2)^(1/3))*x + 18*x^2)^(2/3)) - (36*2^(1/3)*(-
1 + 2*x + x^3)^(2/3)*Defer[Int][(((4*(3/(9 + Sqrt[177]))^(1/3) - (2*(9 + Sq
rt[177]))^(1/3))/6^(2/3) + x)^(2/3)*((12 + 24*3^(1/3)*(2/(9 + Sqrt[177]))^(
2/3) + 2^(1/3)*(3*(9 + Sqrt[177]))^(2/3))/18 - ((2*(6/(9 + Sqrt[177]))^(1/3
) - ((9 + Sqrt[177])/2)^(1/3))*x)/3^(2/3) + x^2)^(2/3))/x, x])/((6^(1/3)*(4
*(3/(9 + Sqrt[177]))^(1/3) - (2*(9 + Sqrt[177]))^(1/3)) + 6*x)^(2/3)*(12 +
24*3^(1/3)*(2/(9 + Sqrt[177]))^(2/3) + 2^(1/3)*(3*(9 + Sqrt[177]))^(2/3) -
6*3^(1/3)*(2*(6/(9 + Sqrt[177]))^(1/3) - ((9 + Sqrt[177])/2)^(1/3))*x + 18*
x^2)^(2/3)) - (13*Defer[Int][(-1 + 2*x + x^3)^(2/3)/(2 - 4*x + x^3), x])/2
+ Defer[Int][(x*(-1 + 2*x + x^3)^(2/3))/(2 - 4*x + x^3), x] + 2*Defer[Int][
(x^2*(-1 + 2*x + x^3)^(2/3))/(2 - 4*x + x^3), x]
```

Rubi steps

$$\int \frac{(-3+4x)\left(-1+2x+x^3\right)^{2/3}}{x^3\left(2-4x+x^3\right)}\,dx = \int \left(-\frac{3\left(-1+2x+x^3\right)^{2/3}}{2x^3} - \frac{\left(-1+2x+x^3\right)^{2/3}}{x^2} - \frac{2\left(-1+2x+x^3\right)^{2/3}}{x} + \frac{(-13\right.$$

$$= \frac{1}{2}\int \frac{\left(-13+2x+4x^2\right)\left(-1+2x+x^3\right)^{2/3}}{2-4x+x^3}\,dx - \frac{3}{2}\int \frac{\left(-1+2x+x^3\right)^{2/3}}{x^3}\,dx - 2$$

$$= \frac{1}{2}\int \left(-\frac{13\left(-1+2x+x^3\right)^{2/3}}{2-4x+x^3} + \frac{2x\left(-1+2x+x^3\right)^{2/3}}{2-4x+x^3} + \frac{4x^2\left(-1+2x+x^3\right)^{2/3}}{2-4x+x^3}\right.$$

$$= 2\int \frac{x^2\left(-1+2x+x^3\right)^{2/3}}{2-4x+x^3}\,dx - \frac{13}{2}\int \frac{\left(-1+2x+x^3\right)^{2/3}}{2-4x+x^3}\,dx - \frac{\left(-1+2x+x\right.}{\left(\frac{4\sqrt[3]{\frac{3}{9+\sqrt{177}}}-\sqrt[3]{2(9}}{6^{2/3}}\right.}$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(-3+4x)\left(-1+2x+x^3\right)^{2/3}}{x^3\left(2-4x+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + 4*x)*(-1 + 2*x + x^3)^(2/3))/(x^3*(2 - 4*x + x^3)),x]
```

```
[Out] Integrate[((-3 + 4*x)*(-1 + 2*x + x^3)^(2/3))/(x^3*(2 - 4*x + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.58, size = 175, normalized size = 1.00

$$\frac{1}{2}\left(\frac{3}{2}\right)^{2/3}\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3+2x-1}-3x\right)-\frac{3\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2}\sqrt[3]{x^3+2x-1}+\sqrt[3]{3}x}\right)}{2\,2^{2/3}}+\frac{3\left(x^3+2x-1\right)^{2/3}}{4x^2}-\frac{1}{4}\left(\frac{3}{2}\right)^{2/3}\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3+2x-1}\,x+2^{2/3}\sqrt[3]{3}\left(x^3+2x-1\right)^{2/3}+3x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + 4*x)*(-1 + 2*x + x^3)^(2/3))/(x^3*(2 - 4*x + x^3)
),x]
```

```
[Out] (3*(-1 + 2*x + x^3)^(2/3))/(4*x^2) - (3*3^(1/6)*ArcTan[(3^(5/6)*x)/(3^(1/3)
*x + 2*2^(1/3)*(-1 + 2*x + x^3)^(1/3))])/(2*2^(2/3)) + ((3/2)^(2/3)*Log[-3*
x + 2^(1/3)*3^(2/3)*(-1 + 2*x + x^3)^(1/3)])/2 - ((3/2)^(2/3)*Log[3*x^2 + 2
^(1/3)*3^(2/3)*x*(-1 + 2*x + x^3)^(1/3) + 2^(2/3)*3^(1/3)*(-1 + 2*x + x^3)^
(2/3)])/4
```

fricas [B] time = 17.27, size = 427, normalized size = 2.44

$$-\frac{4\cdot 9^{\frac{1}{3}}4^{\frac{1}{6}}\sqrt{3}x^2\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(4\cdot 9^{\frac{2}{3}}4^{\frac{2}{3}}\left(4x^7-14x^5+7x^4-8x^3+8x^2-2x\right)\left(x^3+2x-1\right)^{\frac{2}{3}}-12\cdot 9^{\frac{1}{3}}\left(55x^8+100x^6-50x^5+16x^4-16x^3+4x^2\right)\left(x^3+2x-1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\left(377x^9+1200x^7-600x^6+816x^5-816x^4+268x^3-96x^2+48x-8\right)\right)}{6\left(487x^9+960x^7-480x^6+48x^5-48x^4-52x^3+96x^2-48x+8\right)}\right)-2\cdot 9^{\frac{1}{3}}4^{\frac{2}{3}}x^2\log\left(-\frac{6\cdot 9^{\frac{2}{3}}4^{\frac{1}{3}}\left(x^3+2x-1\right)^{\frac{1}{3}}x^2-9^{\frac{1}{3}}4^{\frac{2}{3}}\left(x^3-4x+2\right)-36\left(x^3+2x-1\right)^{\frac{2}{3}}x}{x^3-4x+2}\right)+9^{\frac{1}{3}}4^{\frac{2}{3}}x^2\log\left(\frac{18\cdot 9^{\frac{1}{3}}4^{\frac{2}{3}}\left(4x^4+2x^2-x\right)\left(x^3+2x-1\right)^{\frac{2}{3}}+9^{\frac{2}{3}}4^{\frac{1}{3}}\left(55x^6+100x^4-50x^3+16x^2-16x+4\right)+54\left(7x^5+8x^3-4x^2\right)\left(x^3+2x-1\right)^{\frac{1}{3}}}{x^6-8x^4+4x^3+16x^2-16x+4}\right)-36\left(x^3+2x-1\right)^{\frac{2}{3}}}{48x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+4*x)*(x^3+2*x-1)^(2/3)/x^3/(x^3-4*x+2),x, algorithm="fricas")
```

```
[Out] -1/48*(4*9^(1/3)*4^(1/6)*sqrt(3)*x^2*arctan(1/6*4^(1/6)*sqrt(3)*(4*9^(2/3)*
4^(2/3)*(4*x^7 - 14*x^5 + 7*x^4 - 8*x^3 + 8*x^2 - 2*x)*(x^3 + 2*x - 1)^(2/3
) - 12*9^(1/3)*(55*x^8 + 100*x^6 - 50*x^5 + 16*x^4 - 16*x^3 + 4*x^2)*(x^3 +
 2*x - 1)^(1/3) - 4^(1/3)*(377*x^9 + 1200*x^7 - 600*x^6 + 816*x^5 - 816*x^4
 + 268*x^3 - 96*x^2 + 48*x - 8))/(487*x^9 + 960*x^7 - 480*x^6 + 48*x^5 - 48
*x^4 - 52*x^3 + 96*x^2 - 48*x + 8)) - 2*9^(1/3)*4^(2/3)*x^2*log(-(6*9^(2/3)
*4^(1/3)*(x^3 + 2*x - 1)^(1/3)*x^2 - 9^(1/3)*4^(2/3)*(x^3 - 4*x + 2) - 36*(
x^3 + 2*x - 1)^(2/3)*x)/(x^3 - 4*x + 2)) + 9^(1/3)*4^(2/3)*x^2*log((18*9^(1
/3)*4^(2/3)*(4*x^4 + 2*x^2 - x)*(x^3 + 2*x - 1)^(2/3) + 9^(2/3)*4^(1/3)*(55
*x^6 + 100*x^4 - 50*x^3 + 16*x^2 - 16*x + 4) + 54*(7*x^5 + 8*x^3 - 4*x^2)*(
x^3 + 2*x - 1)^(1/3))/(x^6 - 8*x^4 + 4*x^3 + 16*x^2 - 16*x + 4)) - 36*(x^3
+ 2*x - 1)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+2x-1\right)^{\frac{2}{3}}(4x-3)}{\left(x^3-4x+2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+4*x)*(x^3+2*x-1)^(2/3)/x^3/(x^3-4*x+2),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 2*x - 1)^(2/3)*(4*x - 3)/((x^3 - 4*x + 2)*x^3), x)
```

maple [C] time = 4.68, size = 1054, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+4*x)*(x^3+2*x-1)^(2/3)/x^3/(x^3-4*x+2),x)
```

```
[Out] 3/4*(x^3+2*x-1)^(2/3)/x^2+3/2*RootOf(RootOf(_Z^3-18)^2+6*_Z*RootOf(_Z^3-18)
+36*_Z^2)*ln((5*RootOf(RootOf(_Z^3-18)^2+6*_Z*RootOf(_Z^3-18)+36*_Z^2)*Root
Of(_Z^3-18)^3*x^3+12*RootOf(RootOf(_Z^3-18)^2+6*_Z*RootOf(_Z^3-18)+36*_Z^2)
^2*RootOf(_Z^3-18)^2*x^3+14*(x^3+2*x-1)^(2/3)*RootOf(RootOf(_Z^3-18)^2+6*_Z
*RootOf(_Z^3-18)+36*_Z^2)*RootOf(_Z^3-18)^2*x+8*(x^3+2*x-1)^(1/3)*RootOf(_Z
^3-18)^2*x^2+42*(x^3+2*x-1)^(1/3)*RootOf(RootOf(_Z^3-18)^2+6*_Z*RootOf(_Z^3
-18)+36*_Z^2)*RootOf(_Z^3-18)*x^2+25*RootOf(_Z^3-18)*x^3+60*RootOf(RootOf(_
Z^3-18)^2+6*_Z*RootOf(_Z^3-18)+36*_Z^2)*x^3+48*(x^3+2*x-1)^(2/3)*x+20*RootO
f(_Z^3-18)*x+48*RootOf(RootOf(_Z^3-18)^2+6*_Z*RootOf(_Z^3-18)+36*_Z^2)*x-10
*RootOf(_Z^3-18)-24*RootOf(RootOf(_Z^3-18)^2+6*_Z*RootOf(_Z^3-18)+36*_Z^2))
/(x^3-4*x+2))-1/4*ln(-(3*RootOf(RootOf(_Z^3-18)^2+6*_Z*RootOf(_Z^3-18)+36*_
Z^2)*RootOf(_Z^3-18)^3*x^3-12*RootOf(RootOf(_Z^3-18)^2+6*_Z*RootOf(_Z^3-18)
+36*_Z^2)^2*RootOf(_Z^3-18)^2*x^3+14*(x^3+2*x-1)^(2/3)*RootOf(RootOf(_Z^3-1
8)^2+6*_Z*RootOf(_Z^3-18)+36*_Z^2)*RootOf(_Z^3-18)^2*x-(x^3+2*x-1)^(1/3)*Ro
otOf(_Z^3-18)^2*x^2+42*(x^3+2*x-1)^(1/3)*RootOf(RootOf(_Z^3-18)^2+6*_Z*Root
Of(_Z^3-18)+36*_Z^2)*RootOf(_Z^3-18)*x^2-6*RootOf(_Z^3-18)*x^3+24*RootOf(Ro
otOf(_Z^3-18)^2+6*_Z*RootOf(_Z^3-18)+36*_Z^2)*x^3-6*(x^3+2*x-1)^(2/3)*x-12*
RootOf(_Z^3-18)*x+48*RootOf(RootOf(_Z^3-18)^2+6*_Z*RootOf(_Z^3-18)+36*_Z^2)
*x+6*RootOf(_Z^3-18)-24*RootOf(RootOf(_Z^3-18)^2+6*_Z*RootOf(_Z^3-18)+36*_Z
^2))/(x^3-4*x+2))*RootOf(_Z^3-18)-3/2*ln(-(3*RootOf(RootOf(_Z^3-18)^2+6*_Z*
RootOf(_Z^3-18)+36*_Z^2)*RootOf(_Z^3-18)^3*x^3-12*RootOf(RootOf(_Z^3-18)^2+
6*_Z*RootOf(_Z^3-18)+36*_Z^2)^2*RootOf(_Z^3-18)^2*x^3+14*(x^3+2*x-1)^(2/3)*
RootOf(RootOf(_Z^3-18)^2+6*_Z*RootOf(_Z^3-18)+36*_Z^2)*RootOf(_Z^3-18)^2*x-
(x^3+2*x-1)^(1/3)*RootOf(_Z^3-18)^2*x^2+42*(x^3+2*x-1)^(1/3)*RootOf(RootOf(
_Z^3-18)^2+6*_Z*RootOf(_Z^3-18)+36*_Z^2)*RootOf(_Z^3-18)*x^2-6*RootOf(_Z^3-
18)*x^3+24*RootOf(RootOf(_Z^3-18)^2+6*_Z*RootOf(_Z^3-18)+36*_Z^2)*x^3-6*(x^
3+2*x-1)^(2/3)*x-12*RootOf(_Z^3-18)*x+48*RootOf(RootOf(_Z^3-18)^2+6*_Z*Root
Of(_Z^3-18)+36*_Z^2)*x+6*RootOf(_Z^3-18)-24*RootOf(RootOf(_Z^3-18)^2+6*_Z*R
ootOf(_Z^3-18)+36*_Z^2))/(x^3-4*x+2))*RootOf(RootOf(_Z^3-18)^2+6*_Z*RootOf(
_Z^3-18)+36*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+2x-1\right)^{\frac{2}{3}}(4x-3)}{\left(x^3-4x+2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+4*x)*(x^3+2*x-1)^(2/3)/x^3/(x^3-4*x+2),x, algorithm="maxima")
[Out] integrate((x^3 + 2*x - 1)^(2/3)*(4*x - 3)/((x^3 - 4*x + 2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(4x-3)\,\left(x^3+2x-1\right)^{2/3}}{x^3\,\left(x^3-4x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((4*x - 3)*(2*x + x^3 - 1)^(2/3))/(x^3*(x^3 - 4*x + 2)),x)
[Out] int(((4*x - 3)*(2*x + x^3 - 1)^(2/3))/(x^3*(x^3 - 4*x + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+4*x)*(x**3+2*x-1)**(2/3)/x**3/(x**3-4*x+2),x)
[Out] Timed out
```

3.1887 $\int \frac{\sqrt{-81+27x+135x^2-150x^3+65x^4-13x^5+x^6}}{-1+x}\,dx$

Optimal. Leaf size=175

$$\frac{\sqrt{x^6-13x^5+65x^4-150x^3+135x^2+27x-81}\,\left(8x^2-62x+115\right)}{24(x-3)^2}-\frac{77}{16}\log\left(-2x^3+13x^2+2\sqrt{x^6-13x^5+65\right.$$

Rubi [A] time = 0.33, antiderivative size = 208, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {6688, 6719, 1653, 814, 843, 621, 206, 724, 204}

$$\frac{\sqrt{-(3-x)^4\left(-x^2+x+1\right)}(41-18x)}{8(3-x)^2}-\frac{\left(-x^2+x+1\right)\sqrt{-(3-x)^4\left(-x^2+x+1\right)}}{3(3-x)^2}+\frac{4\sqrt{-(3-x)^4\left(-x^2+x+1\right)}\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)}{(3-x)^2\sqrt{x^2-x-1}}-\frac{77\sqrt{-(3-x)^4\left(-x^2+x+1\right)}\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)}{16(3-x)^2\sqrt{x^2-x-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-81 + 27*x + 135*x^2 - 150*x^3 + 65*x^4 - 13*x^5 + x^6]/(-1 + x),x
]
```

```
[Out] ((41 - 18*x)*Sqrt[-((3 - x)^4*(1 + x - x^2))])/(8*(3 - x)^2) - ((1 + x - x^
2)*Sqrt[-((3 - x)^4*(1 + x - x^2))])/(3*(3 - x)^2) + (4*Sqrt[-((3 - x)^4*(1
 + x - x^2))]*ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])])/((3 - x)^2*Sqrt[-1 -
x + x^2]) - (77*Sqrt[-((3 - x)^4*(1 + x - x^2))]*ArcTanh[(1 - 2*x)/(2*Sqrt[
-1 - x + x^2])])/(16*(3 - x)^2*Sqrt[-1 - x + x^2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\sqrt{-81+27x+135x^2-150x^3+65x^4-13x^5+x^6}}{-1+x}\,dx = \int \frac{\sqrt{(-3+x)^4\left(-1-x+x^2\right)}}{-1+x}\,dx$$

$$= \frac{\sqrt{(-3+x)^4\left(-1-x+x^2\right)}\int \frac{(-3+x)^2\sqrt{-1-x+x^2}}{-1+x}\,dx}{(-3+x)^2\sqrt{-1-x+x^2}}$$

$$= -\frac{\left(1+x-x^2\right)\sqrt{-(3-x)^4\left(1+x-x^2\right)}}{3(3-x)^2} + \frac{\sqrt{(-3+}\cdots}{\cdots}$$

$$= \frac{(41-18x)\sqrt{-(3-x)^4\left(1+x-x^2\right)}}{8(3-x)^2} - \frac{\left(1+x-x^2\right)\cdots}{\cdots}$$

$$= \frac{(41-18x)\sqrt{-(3-x)^4\left(1+x-x^2\right)}}{8(3-x)^2} - \frac{\left(1+x-x^2\right)\cdots}{\cdots}$$

$$= \frac{(41-18x)\sqrt{-(3-x)^4\left(1+x-x^2\right)}}{8(3-x)^2} - \frac{\left(1+x-x^2\right)\cdots}{\cdots}$$

$$= \frac{(41-18x)\sqrt{-(3-x)^4\left(1+x-x^2\right)}}{8(3-x)^2} - \frac{\left(1+x-x^2\right)\cdots}{\cdots}$$

Mathematica [A] time = 0.09, size = 161, normalized size = 0.92

$$\frac{(x-3)^2\sqrt{x^2-x-1}\left(2\left(8\sqrt{x^2-x-1}\,x^2-62\sqrt{x^2-x-1}\,x+115\sqrt{x^2-x-1}+96\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)\right)-246\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)-15\tanh^{-1}\left(\frac{2x-1}{2\sqrt{x^2-x-1}}\right)\right)}{48\sqrt{(x-3)^4\left(x^2-x-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-81 + 27*x + 135*x^2 - 150*x^3 + 65*x^4 - 13*x^5 + x^6]/(-1
+ x),x]
```

```
[Out] ((-3 + x)^2*Sqrt[-1 - x + x^2]*(2*(115*Sqrt[-1 - x + x^2] - 62*x*Sqrt[-1 -
x + x^2] + 8*x^2*Sqrt[-1 - x + x^2] + 96*ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^
2])]) - 246*ArcTanh[(1 - 2*x)/(2*Sqrt[-1 - x + x^2])] - 15*ArcTanh[(-1 + 2*
x)/(2*Sqrt[-1 - x + x^2])]))/(48*Sqrt[(-3 + x)^4*(-1 - x + x^2)])
```

IntegrateAlgebraic [A] time = 0.48, size = 175, normalized size = 1.00

$$\frac{\sqrt{x^6-13x^5+65x^4-150x^3+135x^2+27x-81}\left(8x^2-62x+115\right)}{24(x-3)^2}-\frac{77}{16}\log\left(-2x^3+13x^2+2\sqrt{x^6-13x^5+65x^4-150x^3+135x^2+27x-81}-24x+9\right)-8\tan^{-1}\left(\frac{x^2-6x+9}{x^3-7x^2-\sqrt{x^6-13x^5+65x^4-150x^3+135x^2+27x-81}+15x-9}\right)+\frac{77}{8}\log(x-3)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-81 + 27*x + 135*x^2 - 150*x^3 + 65*x^4 - 13*x^5 +
x^6]/(-1 + x),x]
```

```
[Out] ((115 - 62*x + 8*x^2)*Sqrt[-81 + 27*x + 135*x^2 - 150*x^3 + 65*x^4 - 13*x^5
 + x^6])/(24*(-3 + x)^2) - 8*ArcTan[(9 - 6*x + x^2)/(-9 + 15*x - 7*x^2 + x^
3 - Sqrt[-81 + 27*x + 135*x^2 - 150*x^3 + 65*x^4 - 13*x^5 + x^6])] + (77*Lo
g[-3 + x])/8 - (77*Log[9 - 24*x + 13*x^2 - 2*x^3 + 2*Sqrt[-81 + 27*x + 135*
x^2 - 150*x^3 + 65*x^4 - 13*x^5 + x^6]])/16
```

fricas [A] time = 0.43, size = 202, normalized size = 1.15

$$-\frac{205x^2+1536\left(x^2-6x+9\right)\arctan\left(-\frac{x^3-7x^2+15x-\sqrt{x^6-13x^5+65x^4-150x^3+135x^2+27x-81}-9}{x^2-6x+9}\right)+924\left(x^2-6x+9\right)\log\left(-\frac{2x^3-13x^2+24x-2\sqrt{x^6-13x^5+65x^4-150x^3+135x^2+27x-81}-9}{x^2-6x+9}\right)-8\sqrt{x^6-13x^5+65x^4-150x^3+135x^2+27x-81}\left(8x^2-62x+115\right)-1230x+1845}{192\left(x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-13*x^5+65*x^4-150*x^3+135*x^2+27*x-81)^(1/2)/(-1+x),x, algor
ithm="fricas")
```

```
[Out] -1/192*(205*x^2 + 1536*(x^2 - 6*x + 9)*arctan(-(x^3 - 7*x^2 + 15*x - sqrt(x
^6 - 13*x^5 + 65*x^4 - 150*x^3 + 135*x^2 + 27*x - 81) - 9)/(x^2 - 6*x + 9))
 + 924*(x^2 - 6*x + 9)*log(-(2*x^3 - 13*x^2 + 24*x - 2*sqrt(x^6 - 13*x^5 +
65*x^4 - 150*x^3 + 135*x^2 + 27*x - 81) - 9)/(x^2 - 6*x + 9)) - 8*sqrt(x^6
- 13*x^5 + 65*x^4 - 150*x^3 + 135*x^2 + 27*x - 81)*(8*x^2 - 62*x + 115) - 1
230*x + 1845)/(x^2 - 6*x + 9)
```

giac [A] time = 0.38, size = 62, normalized size = 0.35

$$\frac{1}{24}\,(2\,(4\,x-31)x+115)\sqrt{x^2-x-1}-8\,\arctan\left(-x+\sqrt{x^2-x-1}+1\right)-\frac{77}{16}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x-1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-13*x^5+65*x^4-150*x^3+135*x^2+27*x-81)^(1/2)/(-1+x),x, algor
ithm="giac")
```

```
[Out] 1/24*(2*(4*x - 31)*x + 115)*sqrt(x^2 - x - 1) - 8*arctan(-x + sqrt(x^2 - x
- 1) + 1) - 77/16*log(abs(-2*x + 2*sqrt(x^2 - x - 1) + 1))
```

maple [A] time = 0.02, size = 120, normalized size = 0.69

$$\frac{\sqrt{x^6-13x^5+65x^4-150x^3+135x^2+27x-81}\,\left(16\left(x^2-x-1\right)^{\frac{3}{2}}-108x\sqrt{x^2-x-1}+231\ln\left(x-\frac{1}{2}+\sqrt{x^2-x-1}\right)-192\arctan\left(\frac{-3+x}{2\sqrt{x^2-x-1}}\right)+246\sqrt{x^2-x-1}\right)}{48\,(-3+x)^2\,\sqrt{x^2-x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-13*x^5+65*x^4-150*x^3+135*x^2+27*x-81)^(1/2)/(-1+x),x)
```

```
[Out] 1/48*(x^6-13*x^5+65*x^4-150*x^3+135*x^2+27*x-81)^(1/2)*(16*(x^2-x-1)^(3/2)-
108*x*(x^2-x-1)^(1/2)+231*ln(x-1/2+(x^2-x-1)^(1/2))-192*arctan(1/2*(-3+x)/(
x^2-x-1)^(1/2))+246*(x^2-x-1)^(1/2))/(-3+x)^2/(x^2-x-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^6-13\,x^5+65\,x^4-150\,x^3+135\,x^2+27\,x-81}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-13*x^5+65*x^4-150*x^3+135*x^2+27*x-81)^(1/2)/(-1+x),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(x^6 - 13*x^5 + 65*x^4 - 150*x^3 + 135*x^2 + 27*x - 81)/(x -
1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^6-13\,x^5+65\,x^4-150\,x^3+135\,x^2+27\,x-81}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((27*x + 135*x^2 - 150*x^3 + 65*x^4 - 13*x^5 + x^6 - 81)^(1/2)/(x - 1),x
)
```

```
[Out] int((27*x + 135*x^2 - 150*x^3 + 65*x^4 - 13*x^5 + x^6 - 81)^(1/2)/(x - 1),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-3)^4 \left(x^2-x-1\right)}}{x-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-13*x**5+65*x**4-150*x**3+135*x**2+27*x-81)**(1/2)/(-1+x),x)
[Out] Integral(sqrt((x - 3)**4*(x**2 - x - 1))/(x - 1), x)
```

3.1888 $\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^2(aq+bx+apx^3)}\,dx$

Optimal. Leaf size=175

$$\frac{2\sqrt{2a^2pq-b^2}\,\tan^{-1}\left(\frac{x\sqrt{2a^2pq-b^2}}{a\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+apx^3+aq+bx}\right)}{a^2}-\frac{b\log\left(\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+px^3+q\right)}{a^2}+\frac{b\log(x)}{a^2}$$

Rubi [F] time = 2.91, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^2(aq+bx+apx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6])/(x^2*(a*q
+ b*x + a*p*x^3)),x]
```

```
[Out] -(Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x^2, x]/a) + (b*De
fer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x, x])/(a^2*q) - (b^2*
Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/(a*q + b*x + a*p*x^3
), x])/(a^2*q) + 3*p*Defer[Int][(x*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x
^6])/(a*q + b*x + a*p*x^3), x] - (b*p*Defer[Int][(x^2*Sqrt[q^2 - 2*p*q*x^2
+ 2*p*q*x^3 + p^2*x^6])/(a*q + b*x + a*p*x^3), x])/(a*q)
```

Rubi steps

$$\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^2(aq+bx+apx^3)}\,dx = \int \left(-\frac{\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{ax^2}+\frac{b\sqrt{q^2-2pqx^2+2pqx^3+\ldots}}{a^2qx}\right.$$

$$= -\frac{\int \frac{\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^2}\,dx}{a}+\frac{\int \frac{(-b^2+3a^2pqx-abpx^2)\sqrt{q^2-2pqx^2+2pqx\ldots}}{aq+bx+apx^3}}{a^2q}$$

$$= -\frac{\int \frac{\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^2}\,dx}{a}+\frac{\int \left(-\frac{b^2\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{aq+bx+apx^3}+\frac{3a^2\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{\int \frac{\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^2}\,dx}{a}+(3p)\int \frac{x\sqrt{q^2-2pqx^2+2pqx^3+\ldots}}{aq+bx+apx^3}$$

Mathematica [F] time = 1.85, size = 0, normalized size = 0.00

$$\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^2(aq+bx+apx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6])/(x^2
*(a*q + b*x + a*p*x^3)),x]
```

```
[Out] Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6])/(x^2
*(a*q + b*x + a*p*x^3)), x]
```

IntegrateAlgebraic [A] time = 1.00, size = 175, normalized size = 1.00

$$\frac{2\sqrt{2a^2pq-b^2}\tan^{-1}\left(\frac{x\sqrt{2a^2pq-b^2}}{a\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+apx^3+aq+bx}\right)}{a^2}-\frac{b\log\left(\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+px^3+q\right)}{a^2}+\frac{b\log(x)}{a^2}+\frac{\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x
^6])/(x^2*(a*q + b*x + a*p*x^3)),x]
```

```
[Out] Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/(a*x) + (2*Sqrt[-b^2 + 2*a^2*p*
q]*ArcTan[(Sqrt[-b^2 + 2*a^2*p*q]*x)/(a*q + b*x + a*p*x^3 + a*Sqrt[q^2 - 2*
p*q*x^2 + 2*p*q*x^3 + p^2*x^6])])/a^2 + (b*Log[x])/a^2 - (b*Log[q + p*x^3 +
 Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]])/a^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)/x^2/(a*p*x^3+
a*q+b*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6+2\,pqx^3-2\,pqx^2+q^2}\left(2\,px^3-q\right)}{\left(apx^3+aq+bx\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)/x^2/(a*p*x^3+
a*q+b*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)*(2*p*x^3 - q)/((a*p*x
^3 + a*q + b*x)*x^2), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(2p\,x^3-q\right)\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}}{x^2\left(ap\,x^3+aq+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)/x^2/(a*p*x^3+a*q+b*
x),x)
```

```
[Out] int((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)/x^2/(a*p*x^3+a*q+b*
x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6+2\,pqx^3-2\,pqx^2+q^2}\left(2\,px^3-q\right)}{\left(apx^3+aq+bx\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)/x^2/(a*p*x^3+
a*q+b*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)*(2*p*x^3 - q)/((a*p*x
^3 + a*q + b*x)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(q-2\,p\,x^3\right)\,\sqrt{p^2\,x^6+2\,p\,q\,x^3-2\,p\,q\,x^2+q^2}}{x^2\,\left(a\,p\,x^3+b\,x+a\,q\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q - 2*p*x^3)*(p^2*x^6 + q^2 - 2*p*q*x^2 + 2*p*q*x^3)^(1/2))/(x^2*(a*
q + b*x + a*p*x^3)),x)
```

```
[Out] int(-((q - 2*p*x^3)*(p^2*x^6 + q^2 - 2*p*q*x^2 + 2*p*q*x^3)^(1/2))/(x^2*(a*
q + b*x + a*p*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x**3-q)*(p**2*x**6+2*p*q*x**3-2*p*q*x**2+q**2)**(1/2)/x**2/(
a*p*x**3+a*q+b*x),x)
```

```
[Out] Timed out
```

3.1889 $\int \frac{(1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}{(-1+x^2)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=175

$$\sqrt{2\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{2}-1\right)}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\sqrt{2\left(1+\sqrt{2}\right)}\tan$$

Rubi [C] time = 1.08, antiderivative size = 192, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {6725, 2132, 206, 2133, 725}

$$-\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)+\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)+\frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/((-1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] -1/2*(Sqrt[1 - I]*ArcTanh[(1 - I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])]) + (Sqrt
[1 - I]*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/2 + (Sqrt[1 + I]*
ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/2 - (Sqrt[1 + I]*ArcTanh[
(1 + I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/2 + ArcTanh[(Sqrt[2]*x)/Sqrt[x^2
+ Sqrt[1 + x^4]]]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}{\left(-1+x^2\right)\sqrt{1+x^4}}\,dx &= \int \left(\frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}+\frac{2\sqrt{x^2+\sqrt{1+x^4}}}{\left(-1+x^2\right)\sqrt{1+x^4}}\right)dx\\
&= 2\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(-1+x^2\right)\sqrt{1+x^4}}\,dx+\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx\\
&= 2\int \left(-\frac{\sqrt{x^2+\sqrt{1+x^4}}}{2(1-x)\sqrt{1+x^4}}-\frac{\sqrt{x^2+\sqrt{1+x^4}}}{2(1+x)\sqrt{1+x^4}}\right)dx+\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\frac{}{\sqrt{x}}\right.\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}-\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1-x)\sqrt{1+x^4}}\,dx-\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}-\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1-x)\sqrt{1-ix^2}}\,dx-\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{1}}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}-\left(-\frac{1}{2}-\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{(1+i)-x^2}\,dx,x,\frac{-1-ix}{\sqrt{1+ix^2}}\right)-\left(-\right.\\
&= -\frac{1}{2}\sqrt{1-i}\,\tanh^{-1}\left(\frac{1-ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)+\frac{1}{2}\sqrt{1-i}\,\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)+\frac{1}{2}
\end{aligned}
$$

Mathematica [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\left(1+x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}{\left(-1+x^2\right)\sqrt{1+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/((-1 + x^2)*Sqrt[1 + x^4]),
x]
```

```
[Out] Integrate[((1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/((-1 + x^2)*Sqrt[1 + x^4]),
 x]
```

IntegrateAlgebraic [A] time = 0.78, size = 246, normalized size = 1.41

$$\sqrt{2\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-\sqrt{2\left(1+\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/((-1 + x^2)*Sqrt[1
 + x^4]),x]
```

```
[Out] Sqrt[2*(-1 + Sqrt[2])]*ArcTan[(-Sqrt[-1/2 + 1/Sqrt[2]] + Sqrt[-1/2 + 1/Sqrt
[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 + x^4
]])] + Sqrt[2]*ArcTanh[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2])
/(x*Sqrt[x^2 + Sqrt[1 + x^4]])] - Sqrt[2*(1 + Sqrt[2])]*ArcTanh[(-Sqrt[1/2
```

```
+ 1/Sqrt[2]] + Sqrt[1/2 + 1/Sqrt[2]]*x^2 + Sqrt[1/2 + 1/Sqrt[2]]*Sqrt[1 + x
^4])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])]
```

fricas [B] time = 3.95, size = 374, normalized size = 2.14

$$-\sqrt{2\sqrt{2}-2}\arctan\left(-\frac{\left(4x^2+2\sqrt{2}\left(x^2-1\right)-\sqrt{x^4+1}\left(\left(\sqrt{2}+2\right)\sqrt{-8\sqrt{2}+12}+2\sqrt{2}+4\right)+\left(2x^2+\sqrt{2}\left(x^2-3\right)-4\right)\sqrt{-8\sqrt{2}+12}\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{2\sqrt{2}-2}}{8x}\right)+\frac{1}{4}\sqrt{2}\log\left(4x^4+4\sqrt{x^4+1}x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)-\frac{1}{4}\sqrt{2\sqrt{2}+2}\log\left(-\frac{2\sqrt{2}x^2+4x^2+\left(\sqrt{2}\sqrt{x^4+1}x-\sqrt{2}\left(x^3-x\right)+2x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{2\sqrt{2}+2}+2\sqrt{x^4+1}\left(\sqrt{2}+1\right)}{x^2-1}\right)+\frac{1}{4}\sqrt{2\sqrt{2}+2}\log\left(-\frac{2\sqrt{2}x^2+4x^2-\left(\sqrt{2}\sqrt{x^4+1}x-\sqrt{2}\left(x^3-x\right)+2x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{2\sqrt{2}+2}+2\sqrt{x^4+1}\left(\sqrt{2}+1\right)}{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2-1)/(x^4+1)^(1/2),x, algori
thm="fricas")
```

```
[Out] -sqrt(2*sqrt(2) - 2)*arctan(-1/8*(4*x^2 + 2*sqrt(2)*(x^2 - 1) - sqrt(x^4 +
1)*((sqrt(2) + 2)*sqrt(-8*sqrt(2) + 12) + 2*sqrt(2) + 4) + (2*x^2 + sqrt(2)
*(x^2 - 3) - 4)*sqrt(-8*sqrt(2) + 12))*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(2*sqr
t(2) - 2)/x) + 1/4*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3
 + sqrt(2)*sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1) - 1/4*sqrt(2*sqr
t(2) + 2)*log(-(2*sqrt(2)*x^2 + 4*x^2 + (sqrt(2)*sqrt(x^4 + 1)*x - sqrt(2)*
(x^3 - x) + 2*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(2*sqrt(2) + 2) + 2*sqrt(x^4
 + 1)*(sqrt(2) + 1))/(x^2 - 1)) + 1/4*sqrt(2*sqrt(2) + 2)*log(-(2*sqrt(2)*x
^2 + 4*x^2 - (sqrt(2)*sqrt(x^4 + 1)*x - sqrt(2)*(x^3 - x) + 2*x)*sqrt(x^2 +
 sqrt(x^4 + 1))*sqrt(2*sqrt(2) + 2) + 2*sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 -
 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2+1\right)}{\sqrt{x^4+1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2-1)/(x^4+1)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))*(x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)\sqrt{x^2+\sqrt{x^4+1}}}{\left(x^2-1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2-1)/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2-1)/(x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2+1\right)}{\sqrt{x^4+1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2-1)/(x^4+1)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))*(x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+1\right)\sqrt{\sqrt{x^4+1}+x^2}}{\left(x^2-1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*((x^4 + 1)^(1/2) + x^2)^(1/2))/((x^2 - 1)*(x^4 + 1)^(1/2)),x
)
```

```
[Out] int(((x^2 + 1)*((x^4 + 1)^(1/2) + x^2)^(1/2))/((x^2 - 1)*(x^4 + 1)^(1/2)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)\sqrt{x^2+\sqrt{x^4+1}}}{(x-1)\,(x+1)\,\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**2+(x**4+1)**(1/2))**(1/2)/(x**2-1)/(x**4+1)**(1/2),x
)
```

```
[Out] Integral((x**2 + 1)*sqrt(x**2 + sqrt(x**4 + 1))/((x - 1)*(x + 1)*sqrt(x**4
+ 1)), x)
```

3.1890 $\int \frac{2+x}{(-3+x)\sqrt[4]{1-x^2}\,(1+x^2)}\,dx$

Optimal. Leaf size=176

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{1-x^2}-\sqrt[4]{2}x+\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{1-x^2}+\sqrt[4]{2}x-\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\left(2\sqrt[4]{2}x-2\sqrt[4]{2}\right)\sqrt[4]{1-x^2}}{\sqrt{2}x^2+2\sqrt{1-x^2}-2\sqrt{2}x+\sqrt{2}}\right)}{2\sqrt[4]{2}}$$

Rubi [C] time = 0.68, antiderivative size = 378, normalized size of antiderivative = 2.15, number of steps used = 32, number of rules used = 18, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6725, 746, 399, 490, 1213, 537, 444, 63, 297, 1162, 617, 204, 1165, 628, 1010, 298, 203, 206}

$$\frac{\log\left(\sqrt{1-x^2}-2\sqrt[4]{2}\sqrt[4]{1-x^2}+2\sqrt{2}\right)}{8\sqrt[4]{2}}-\frac{\log\left(\sqrt{1-x^2}+2\sqrt[4]{2}\sqrt[4]{1-x^2}+2\sqrt{2}\right)}{8\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}-\frac{\tan^{-1}\left(1-\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{2}}+1\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}-\frac{\sqrt{x^2}\,\Pi\left(-\frac{1}{\sqrt{2}};\sin^{-1}\left(\sqrt[4]{1-x^2}\right)\middle|-1\right)}{2\sqrt{2}x}+\frac{3i\sqrt{x^2}\,\Pi\left(-\frac{i}{2\sqrt{2}};\sin^{-1}\left(\sqrt[4]{1-x^2}\right)\middle|-1\right)}{4\sqrt{2}x}-\frac{3i\sqrt{x^2}\,\Pi\left(\frac{i}{2\sqrt{2}};\sin^{-1}\left(\sqrt[4]{1-x^2}\right)\middle|-1\right)}{4\sqrt{2}x}+\frac{\sqrt{x^2}\,\Pi\left(\frac{1}{\sqrt{2}};\sin^{-1}\left(\sqrt[4]{1-x^2}\right)\middle|-1\right)}{2\sqrt{2}x}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((-3 + x)*(1 - x^2)^(1/4)*(1 + x^2)),x]
```

```
[Out] -1/2*ArcTan[(1 - x^2)^(1/4)/2^(1/4)]/2^(1/4) - ArcTan[1 - (1 - x^2)^(1/4)/2
^(1/4)]/(4*2^(1/4)) + ArcTan[1 + (1 - x^2)^(1/4)/2^(1/4)]/(4*2^(1/4)) + Arc
Tanh[(1 - x^2)^(1/4)/2^(1/4)]/(2*2^(1/4)) - (Sqrt[x^2]*EllipticPi[-(1/Sqrt[
2]), ArcSin[(1 - x^2)^(1/4)], -1])/(2*Sqrt[2]*x) + (((3*I)/4)*Sqrt[x^2]*Ell
ipticPi[(-1/2*I)/Sqrt[2], ArcSin[(1 - x^2)^(1/4)], -1])/(Sqrt[2]*x) - (((3*
I)/4)*Sqrt[x^2]*EllipticPi[(I/2)/Sqrt[2], ArcSin[(1 - x^2)^(1/4)], -1])/(Sq
rt[2]*x) + (Sqrt[x^2]*EllipticPi[1/Sqrt[2], ArcSin[(1 - x^2)^(1/4)], -1])/(
2*Sqrt[2]*x) + Log[2*Sqrt[2] - 2*2^(1/4)*(1 - x^2)^(1/4) + Sqrt[1 - x^2]]/(
8*2^(1/4)) - Log[2*Sqrt[2] + 2*2^(1/4)*(1 - x^2)^(1/4) + Sqrt[1 - x^2]]/(8*
2^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
```

```
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{2+x}{(-3+x)\sqrt[4]{1-x^2}\left(1+x^2\right)}dx = \int \left(\frac{1}{2(-3+x)\sqrt[4]{1-x^2}} + \frac{-1-x}{2\sqrt[4]{1-x^2}\left(1+x^2\right)}\right)dx$$
$$= \frac{1}{2}\int \frac{1}{(-3+x)\sqrt[4]{1-x^2}}dx + \frac{1}{2}\int \frac{-1-x}{\sqrt[4]{1-x^2}\left(1+x^2\right)}dx$$
$$= -\left(\frac{1}{2}\int \frac{x}{\sqrt[4]{1-x^2}\left(9-x^2\right)}dx\right) - \frac{1}{2}\int \frac{1}{\sqrt[4]{1-x^2}\left(1+x^2\right)}dx - \frac{1}{2}\int \frac{x}{\sqrt[4]{1-x^2}\left(1+\right.}$$
$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-x}(9-x)}dx, x, x^2\right)\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-x}(1+x)}dx, x, x\right.$$
$$= \frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{2}-x^2\right)\sqrt{1-x^4}}dx, x, \sqrt[4]{1-x^2}\right)}{2x} - \frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{2}+x^2\right)\sqrt{1-x^4}}dx,\right.}{2x}$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}dx, x, \sqrt[4]{1-x^2}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{2}+x^2}dx, x, \sqrt[4]{1-x^2}\right)$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}} - \frac{\sqrt{x^2}\,\Pi\left(-\frac{1}{\sqrt{2}};\sin^{-1}\left(\sqrt[4]{1-x^2}\right)\middle|-1\right)}{2\sqrt{2}\,x} + $$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}} - \frac{\sqrt{x^2}\,\Pi\left(-\frac{1}{\sqrt{2}};\sin^{-1}\left(\sqrt[4]{1-x^2}\right)\middle|-1\right)}{2\sqrt{2}\,x} + $$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}} - \frac{\tan^{-1}\left(1-\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}} + \frac{\tan^{-1}\left(1+\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{2}}\right)}{2\sqrt[4]{2}}$$

Mathematica [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{2+x}{(-3+x)\sqrt[4]{1-x^2}\left(1+x^2\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2 + x)/((-3 + x)*(1 - x^2)^(1/4)*(1 + x^2)),x]
[Out] Integrate[(2 + x)/((-3 + x)*(1 - x^2)^(1/4)*(1 + x^2)), x]
```

IntegrateAlgebraic [A] time = 0.33, size = 176, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{1-x^2}-\sqrt[4]{2}x+\sqrt[4]{2}}\right)}{2\sqrt[4]{2}} + \frac{\tan^{-1}\left(\frac{\sqrt[4]{1-x^2}}{\sqrt[4]{1-x^2}+\sqrt[4]{2}x-\sqrt[4]{2}}\right)}{2\sqrt[4]{2}} - \frac{\tanh^{-1}\left(\frac{\left(2\sqrt[4]{2}x-2\sqrt[4]{2}\right)\sqrt[4]{1-x^2}}{\sqrt{2}x^2+2\sqrt{1-x^2}-2\sqrt{2}x+\sqrt{2}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + x)/((-3 + x)*(1 - x^2)^(1/4)*(1 + x^2)),x]
[Out] -1/2*ArcTan[(1 - x^2)^(1/4)/(2^(1/4) - 2^(1/4)*x + (1 - x^2)^(1/4))]/2^(1/4
) + ArcTan[(1 - x^2)^(1/4)/(-2^(1/4) + 2^(1/4)*x + (1 - x^2)^(1/4))]/(2*2^(
1/4)) - ArcTanh[((-2*2^(1/4) + 2*2^(1/4)*x)*(1 - x^2)^(1/4))/(Sqrt[2] - 2*S
qrt[2]*x + Sqrt[2]*x^2 + 2*Sqrt[1 - x^2])]/(2*2^(1/4))
```

fricas [B] time = 9.02, size = 986, normalized size = 5.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3+x)/(-x^2+1)^(1/4)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/16*8^(3/4)*sqrt(2)*arctan(-1/2*(2*x^6 - 12*x^5 + 22*x^4 - 24*x^3 + 8^(3/
4)*sqrt(2)*(x^5 - 5*x^4 + 16*x^3 - 16*x^2 - x + 5)*(-x^2 + 1)^(1/4) + 4*8^(
1/4)*sqrt(2)*(3*x^3 - 9*x^2 + 11*x - 1)*(-x^2 + 1)^(3/4) + 8*sqrt(2)*(x^4 -
 4*x^3 + 4*x^2 - 4*x + 3)*sqrt(-x^2 + 1) + 38*x^2 - (8^(3/4)*sqrt(2)*(3*x^4
 - 12*x^3 + 20*x^2 - 12*x + 1)*sqrt(-x^2 + 1) + 32*sqrt(2)*(x^3 - 3*x^2 + 3
*x - 1)*(-x^2 + 1)^(3/4) + 8^(1/4)*sqrt(2)*(x^6 - 6*x^5 - x^4 + 12*x^3 + 11
*x^2 - 46*x + 13) + 8*(x^5 - 5*x^4 + 8*x^3 - 8*x^2 + 7*x - 3)*(-x^2 + 1)^(1
/4))*sqrt(-(2*8^(1/4)*sqrt(2)*(x^2 - 2*x + 1)*(-x^2 + 1)^(1/4) + 8^(3/4)*sq
rt(2)*(-x^2 + 1)^(3/4) - sqrt(2)*(x^3 - 3*x^2 + x - 3) - 8*sqrt(-x^2 + 1)*(
x - 1))/(x^3 - 3*x^2 + x - 3)) - 12*x + 18)/(x^6 - 6*x^5 + 43*x^4 - 76*x^3
+ 19*x^2 + 58*x - 23)) + 1/16*8^(3/4)*sqrt(2)*arctan(-1/2*(2*x^6 - 12*x^5 +
 22*x^4 - 24*x^3 - 8^(3/4)*sqrt(2)*(x^5 - 5*x^4 + 16*x^3 - 16*x^2 - x + 5)*
(-x^2 + 1)^(1/4) - 4*8^(1/4)*sqrt(2)*(3*x^3 - 9*x^2 + 11*x - 1)*(-x^2 + 1)^
(3/4) + 8*sqrt(2)*(x^4 - 4*x^3 + 4*x^2 - 4*x + 3)*sqrt(-x^2 + 1) + 38*x^2 +
 (8^(3/4)*sqrt(2)*(3*x^4 - 12*x^3 + 20*x^2 - 12*x + 1)*sqrt(-x^2 + 1) - 32*
sqrt(2)*(x^3 - 3*x^2 + 3*x - 1)*(-x^2 + 1)^(3/4) + 8^(1/4)*sqrt(2)*(x^6 - 6
*x^5 - x^4 + 12*x^3 + 11*x^2 - 46*x + 13) - 8*(x^5 - 5*x^4 + 8*x^3 - 8*x^2
+ 7*x - 3)*(-x^2 + 1)^(1/4))*sqrt((2*8^(1/4)*sqrt(2)*(x^2 - 2*x + 1)*(-x^2
+ 1)^(1/4) + 8^(3/4)*sqrt(2)*(-x^2 + 1)^(3/4) + sqrt(2)*(x^3 - 3*x^2 + x -
3) + 8*sqrt(-x^2 + 1)*(x - 1))/(x^3 - 3*x^2 + x - 3)) - 12*x + 18)/(x^6 - 6
*x^5 + 43*x^4 - 76*x^3 + 19*x^2 + 58*x - 23)) - 1/64*8^(3/4)*sqrt(2)*log(64
*(2*8^(1/4)*sqrt(2)*(x^2 - 2*x + 1)*(-x^2 + 1)^(1/4) + 8^(3/4)*sqrt(2)*(-x^
2 + 1)^(3/4) + sqrt(2)*(x^3 - 3*x^2 + x - 3) + 8*sqrt(-x^2 + 1)*(x - 1))/(x
^3 - 3*x^2 + x - 3)) + 1/64*8^(3/4)*sqrt(2)*log(-64*(2*8^(1/4)*sqrt(2)*(x^2
 - 2*x + 1)*(-x^2 + 1)^(1/4) + 8^(3/4)*sqrt(2)*(-x^2 + 1)^(3/4) - sqrt(2)*(
x^3 - 3*x^2 + x - 3) - 8*sqrt(-x^2 + 1)*(x - 1))/(x^3 - 3*x^2 + x - 3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\left(x^2+1\right)\left(-x^2+1\right)^{\frac{1}{4}}(x-3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3+x)/(-x^2+1)^(1/4)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate((x + 2)/((x^2 + 1)*(-x^2 + 1)^(1/4)*(x - 3)), x)
```

maple [C] time = 5.28, size = 407, normalized size = 2.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3+x)/(-x^2+1)^(1/4)/(x^2+1),x)
```

```
[Out] -1/4*RootOf(_Z^2+RootOf(_Z^4+2)^2)*ln((2*(-x^2+1)^(1/2)*RootOf(_Z^2+RootOf(
_Z^4+2)^2)*RootOf(_Z^4+2)^2*x-2*(-x^2+1)^(1/2)*RootOf(_Z^2+RootOf(_Z^4+2)^2
)*RootOf(_Z^4+2)^2+2*(-x^2+1)^(1/4)*RootOf(_Z^4+2)^2*x^2-4*(-x^2+1)^(1/4)*R
ootOf(_Z^4+2)^2*x-RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^3+2*(-x^2+1)^(1/4)*RootOf
(_Z^4+2)^2+3*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^2+4*(-x^2+1)^(3/4)-5*RootOf(_Z
^2+RootOf(_Z^4+2)^2)*x-RootOf(_Z^2+RootOf(_Z^4+2)^2))/(-3+x)/(x^2+1))-1/4*R
ootOf(_Z^4+2)*ln(-(2*(-x^2+1)^(1/2)*RootOf(_Z^4+2)^3*x-2*(-x^2+1)^(1/2)*Roo
tOf(_Z^4+2)^3+2*(-x^2+1)^(1/4)*RootOf(_Z^4+2)^2*x^2-4*(-x^2+1)^(1/4)*RootOf
```

```
(_Z^4+2)^2*x+RootOf(_Z^4+2)*x^3+2*(-x^2+1)^(1/4)*RootOf(_Z^4+2)^2-3*RootOf(
_Z^4+2)*x^2-4*(-x^2+1)^(3/4)+5*RootOf(_Z^4+2)*x+RootOf(_Z^4+2))/(-3+x)/(x^2
+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\left(x^2+1\right)\left(-x^2+1\right)^{\frac{1}{4}}(x-3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3+x)/(-x^2+1)^(1/4)/(x^2+1),x, algorithm="maxima")
[Out] integrate((x + 2)/((x^2 + 1)*(-x^2 + 1)^(1/4)*(x - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\left(1-x^2\right)^{1/4}\,\left(x^2+1\right)\,(x-3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((1 - x^2)^(1/4)*(x^2 + 1)*(x - 3)),x)
[Out] int((x + 2)/((1 - x^2)^(1/4)*(x^2 + 1)*(x - 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\sqrt[4]{-(x-1)(x+1)}\,(x-3)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3+x)/(-x**2+1)**(1/4)/(x**2+1),x)
[Out] Integral((x + 2)/((-(x - 1)*(x + 1))**(1/4)*(x - 3)*(x**2 + 1)), x)
```

3.1891 $\int \frac{\sqrt{b^2+a^2x^3}\left(2b^2+cx^3+a^2x^6\right)}{x\left(-b^2+a^2x^6\right)}\,dx$

Optimal. Leaf size=176

$$\frac{\left(c\sqrt{b-a}-3ab\sqrt{b-a}\right)\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{b-a}}\right)}{3a\sqrt{b}}+\frac{\left(-c\sqrt{a+b}-3ab\sqrt{a+b}\right)\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{a+b}}\right)}{3a\sqrt{b}}+\frac{2}{3}\sqrt{a^2x^3+b^2}+\frac{4}{3}b$$

Rubi [A] time = 0.78, antiderivative size = 223, normalized size of antiderivative = 1.27, number of steps used = 14, number of rules used = 7, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {6725, 266, 50, 63, 208, 444, 205}

$$\frac{(3ab-c)\sqrt{a^2x^3+b^2}}{3ab}+\frac{(3ab+c)\sqrt{a^2x^3+b^2}}{3ab}-\frac{\sqrt{a-b}(3ab-c)\tan^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{a-b}}\right)}{3a\sqrt{b}}-\frac{\sqrt{a+b}(3ab+c)\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{a+b}}\right)}{3a\sqrt{b}}-\frac{4}{3}\sqrt{a^2x^3+b^2}+\frac{4}{3}b\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + a^2*x^3]*(2*b^2 + c*x^3 + a^2*x^6))/(x*(-b^2 + a^2*x^6)),x]
```

```
[Out] (-4*Sqrt[b^2 + a^2*x^3])/3 + ((3*a*b - c)*Sqrt[b^2 + a^2*x^3])/(3*a*b) + ((
3*a*b + c)*Sqrt[b^2 + a^2*x^3])/(3*a*b) - (Sqrt[a - b]*(3*a*b - c)*ArcTan[S
qrt[b^2 + a^2*x^3]/(Sqrt[a - b]*Sqrt[b])])/(3*a*Sqrt[b]) + (4*b*ArcTanh[Sqr
t[b^2 + a^2*x^3]/b])/3 - (Sqrt[a + b]*(3*a*b + c)*ArcTanh[Sqrt[b^2 + a^2*x^
3]/(Sqrt[b]*Sqrt[a + b])])/(3*a*Sqrt[b])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{b^2+a^2x^3}\left(2b^2+cx^3+a^2x^6\right)}{x\left(-b^2+a^2x^6\right)}\,dx = \int\left(-\frac{2\sqrt{b^2+a^2x^3}}{x}-\frac{(3ab+c)x^2\sqrt{b^2+a^2x^3}}{2b\left(b-ax^3\right)}+\frac{(3ab-c)x^2\sqrt{b^2+a^2x^3}}{2b\left(b+ax^3\right)}\right.$$

$$=-\left(2\int\frac{\sqrt{b^2+a^2x^3}}{x}\,dx\right)+\frac{(3ab-c)\int\frac{x^2\sqrt{b^2+a^2x^3}}{b+ax^3}\,dx}{2b}-\frac{(3ab+c)\int\frac{x^2\sqrt{b^2+}}{b-ax}}{2b}$$

$$=-\left(\frac{2}{3}\operatorname{Subst}\left(\int\frac{\sqrt{b^2+a^2x}}{x}\,dx,x,x^3\right)\right)+\frac{(3ab-c)\operatorname{Subst}\left(\int\frac{\sqrt{b^2+a^2x}}{b+ax}\,dx,x,\right.}{6b}$$

$$=-\frac{4}{3}\sqrt{b^2+a^2x^3}+\frac{(3ab-c)\sqrt{b^2+a^2x^3}}{3ab}+\frac{(3ab+c)\sqrt{b^2+a^2x^3}}{3ab}-\frac{1}{3}\left(2b^2\right)$$

$$=-\frac{4}{3}\sqrt{b^2+a^2x^3}+\frac{(3ab-c)\sqrt{b^2+a^2x^3}}{3ab}+\frac{(3ab+c)\sqrt{b^2+a^2x^3}}{3ab}-\frac{\left(4b^2\right)S}{}$$

$$=-\frac{4}{3}\sqrt{b^2+a^2x^3}+\frac{(3ab-c)\sqrt{b^2+a^2x^3}}{3ab}+\frac{(3ab+c)\sqrt{b^2+a^2x^3}}{3ab}-\frac{\sqrt{a-b}}{}$$

Mathematica [A] time = 0.23, size = 194, normalized size = 1.10

$$\frac{c\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{a-b}}\right)-\sqrt{a+b}(3ab+c)\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{a+b}}\right)+2a\sqrt{b}\sqrt{a^2x^3+b^2}-3ab\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{a-b}}\right)+4ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{b}\right)}{3a\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + a^2*x^3]*(2*b^2 + c*x^3 + a^2*x^6))/(x*(-b^2 + a^2*x^
6)),x]
```

```
[Out] (2*a*Sqrt[b]*Sqrt[b^2 + a^2*x^3] - 3*a*Sqrt[a - b]*b*ArcTan[Sqrt[b^2 + a^2*
x^3]/(Sqrt[a - b]*Sqrt[b])] + Sqrt[a - b]*c*ArcTan[Sqrt[b^2 + a^2*x^3]/(Sqr
t[a - b]*Sqrt[b])] + 4*a*b^(3/2)*ArcTanh[Sqrt[b^2 + a^2*x^3]/b] - Sqrt[a +
b]*(3*a*b + c)*ArcTanh[Sqrt[b^2 + a^2*x^3]/(Sqrt[b]*Sqrt[a + b])])/(3*a*Sqr
t[b])
```

IntegrateAlgebraic [A] time = 0.26, size = 176, normalized size = 1.00

$$\frac{\left(c\sqrt{a-b}-3ab\sqrt{a-b}\right)\tan^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{a-b}}\right)}{3a\sqrt{b}}+\frac{\left(-c\sqrt{a+b}-3ab\sqrt{a+b}\right)\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{a+b}}\right)}{3a\sqrt{b}}+\frac{2}{3}\sqrt{a^2x^3+b^2}+\frac{4}{3}b\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{b}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[b^2 + a^2*x^3]*(2*b^2 + c*x^3 + a^2*x^6))/(x*(-b^2
 + a^2*x^6)),x]
```

```
[Out] (2*Sqrt[b^2 + a^2*x^3])/3 + ((-3*a*Sqrt[a - b]*b + Sqrt[a - b]*c)*ArcTan[Sq
rt[b^2 + a^2*x^3]/(Sqrt[a - b]*Sqrt[b])])/(3*a*Sqrt[b]) + (4*b*ArcTanh[Sqrt
[b^2 + a^2*x^3]/b])/3 + ((-3*a*b*Sqrt[a + b] - Sqrt[a + b]*c)*ArcTanh[Sqrt[
b^2 + a^2*x^3]/(Sqrt[b]*Sqrt[a + b])])/(3*a*Sqrt[b])
```

fricas [A] time = 0.71, size = 735, normalized size = 4.18

$$\left[\frac{4ab\log\left(b+\sqrt{a^2x^3+b^2}\right)-4ab\log\left(-b+\sqrt{a^2x^3+b^2}\right)-(3ab-c)\sqrt{-\frac{a-b}{b}}\log\left(\frac{a^2x^3-ab+2b^2+2\sqrt{a^2x^3+b^2}\,b\sqrt{-\frac{a-b}{b}}}{ax^3+b}\right)+(3ab+c)\sqrt{\frac{a+b}{b}}\log\left(\frac{a^2x^3+ab+2b^2-2\sqrt{a^2x^3+b^2}\,b\sqrt{\frac{a+b}{b}}}{ax^3-b}\right)+4\sqrt{a^2x^3+b^2}\,a}{6a},\ \frac{4ab\log\left(b+\sqrt{a^2x^3+b^2}\right)-4ab\log\left(-b+\sqrt{a^2x^3+b^2}\right)+2(3ab-c)\sqrt{\frac{a-b}{b}}\arctan\left(\frac{b\sqrt{\frac{a-b}{b}}}{\sqrt{a^2x^3+b^2}}\right)+(3ab+c)\sqrt{\frac{a+b}{b}}\log\left(\frac{a^2x^3+ab+2b^2-2\sqrt{a^2x^3+b^2}\,b\sqrt{\frac{a+b}{b}}}{ax^3-b}\right)+4\sqrt{a^2x^3+b^2}\,a}{6a},\ \frac{4ab\log\left(b+\sqrt{a^2x^3+b^2}\right)-4ab\log\left(-b+\sqrt{a^2x^3+b^2}\right)+2(3ab+c)\sqrt{-\frac{a+b}{b}}\arctan\left(\frac{b\sqrt{-\frac{a+b}{b}}}{\sqrt{a^2x^3+b^2}}\right)-(3ab-c)\sqrt{-\frac{a-b}{b}}\log\left(\frac{a^2x^3-ab+2b^2+2\sqrt{a^2x^3+b^2}\,b\sqrt{-\frac{a-b}{b}}}{ax^3+b}\right)+4\sqrt{a^2x^3+b^2}\,a}{6a},\ \frac{2ab\log\left(b+\sqrt{a^2x^3+b^2}\right)-2ab\log\left(-b+\sqrt{a^2x^3+b^2}\right)+(3ab+c)\sqrt{-\frac{a+b}{b}}\arctan\left(\frac{b\sqrt{-\frac{a+b}{b}}}{\sqrt{a^2x^3+b^2}}\right)+(3ab-c)\sqrt{\frac{a-b}{b}}\arctan\left(\frac{b\sqrt{\frac{a-b}{b}}}{\sqrt{a^2x^3+b^2}}\right)+2\sqrt{a^2x^3+b^2}\,a}{3a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^3+b^2)^(1/2)*(a^2*x^6+c*x^3+2*b^2)/x/(a^2*x^6-b^2),x, algo
rithm="fricas")
```

```
[Out] [1/6*(4*a*b*log(b + sqrt(a^2*x^3 + b^2)) - 4*a*b*log(-b + sqrt(a^2*x^3 + b^
2)) - (3*a*b - c)*sqrt(-(a - b)/b)*log((a^2*x^3 - a*b + 2*b^2 + 2*sqrt(a^2*
x^3 + b^2)*b*sqrt(-(a - b)/b))/(a*x^3 + b)) + (3*a*b + c)*sqrt((a + b)/b)*l
og((a^2*x^3 + a*b + 2*b^2 - 2*sqrt(a^2*x^3 + b^2)*b*sqrt((a + b)/b))/(a*x^3
 - b)) + 4*sqrt(a^2*x^3 + b^2)*a)/a, 1/6*(4*a*b*log(b + sqrt(a^2*x^3 + b^2)
) - 4*a*b*log(-b + sqrt(a^2*x^3 + b^2)) + 2*(3*a*b - c)*sqrt((a - b)/b)*arc
tan(b*sqrt((a - b)/b)/sqrt(a^2*x^3 + b^2)) + (3*a*b + c)*sqrt((a + b)/b)*lo
g((a^2*x^3 + a*b + 2*b^2 - 2*sqrt(a^2*x^3 + b^2)*b*sqrt((a + b)/b))/(a*x^3
- b)) + 4*sqrt(a^2*x^3 + b^2)*a)/a, 1/6*(4*a*b*log(b + sqrt(a^2*x^3 + b^2))
 - 4*a*b*log(-b + sqrt(a^2*x^3 + b^2)) + 2*(3*a*b + c)*sqrt(-(a + b)/b)*arc
tan(b*sqrt(-(a + b)/b)/sqrt(a^2*x^3 + b^2)) - (3*a*b - c)*sqrt(-(a - b)/b)*
log((a^2*x^3 - a*b + 2*b^2 + 2*sqrt(a^2*x^3 + b^2)*b*sqrt(-(a - b)/b))/(a*x
^3 + b)) + 4*sqrt(a^2*x^3 + b^2)*a)/a, 1/3*(2*a*b*log(b + sqrt(a^2*x^3 + b^
2)) - 2*a*b*log(-b + sqrt(a^2*x^3 + b^2)) + (3*a*b + c)*sqrt(-(a + b)/b)*ar
ctan(b*sqrt(-(a + b)/b)/sqrt(a^2*x^3 + b^2)) + (3*a*b - c)*sqrt((a - b)/b)*
arctan(b*sqrt((a - b)/b)/sqrt(a^2*x^3 + b^2)) + 2*sqrt(a^2*x^3 + b^2)*a)/a]
```

giac [A] time = 0.30, size = 183, normalized size = 1.04

$$\frac{2}{3}\,b\log\left(\left|b+\sqrt{a^2x^3+b^2}\right|\right)-\frac{2}{3}\,b\log\left(\left|-b+\sqrt{a^2x^3+b^2}\right|\right)+\frac{2}{3}\,\sqrt{a^2x^3+b^2}-\frac{\left(3\,a^2b-3\,ab^2-ac+bc\right)\arctan\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{ab-b^2}}\right)}{3\,\sqrt{ab-b^2}\,a}+\frac{\left(3\,a^2b+3\,ab^2+ac+bc\right)\arctan\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{-ab-b^2}}\right)}{3\,\sqrt{-ab-b^2}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^3+b^2)^(1/2)*(a^2*x^6+c*x^3+2*b^2)/x/(a^2*x^6-b^2),x, algo
rithm="giac")
```

```
[Out] 2/3*b*log(abs(b + sqrt(a^2*x^3 + b^2))) - 2/3*b*log(abs(-b + sqrt(a^2*x^3 +
 b^2))) + 2/3*sqrt(a^2*x^3 + b^2) - 1/3*(3*a^2*b - 3*a*b^2 - a*c + b*c)*arc
tan(sqrt(a^2*x^3 + b^2)/sqrt(a*b - b^2))/(sqrt(a*b - b^2)*a) + 1/3*(3*a^2*b
 + 3*a*b^2 + a*c + b*c)*arctan(sqrt(a^2*x^3 + b^2)/sqrt(-a*b - b^2))/(sqrt(
-a*b - b^2)*a)
```

maple [C] time = 0.77, size = 943, normalized size = 5.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^3+b^2)^(1/2)*(a^2*x^6+c*x^3+2*b^2)/x/(a^2*x^6-b^2),x)
```

```
[Out] 1/2*(3*a*b-c)/b*(2/3/a*(a^2*x^3+b^2)^(1/2)+1/3*I/a^2*2^(1/2)*sum((-a*b^2)^(
1/3)*(1/2*I*a*(2*x+1/a*(-I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)
^(1/3))^(1/2)*(a*(x-1/a*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^
2)^(1/3)))^(1/2)*(-1/2*I*a*(2*x+1/a*(I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3
```

```
)))/(-a*b^2)^(1/3))^(1/2)/(a^2*x^3+b^2)^(1/2)*(I*(-a*b^2)^(1/3)*_alpha*3^(1
/2)*a-I*3^(1/2)*(-a*b^2)^(2/3)+2*_alpha^2*a^2-(-a*b^2)^(1/3)*_alpha*a-(-a*b
^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/a*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
a*(-a*b^2)^(1/3))*3^(1/2)*a/(-a*b^2)^(1/3))^(1/2),1/2*(2*I*(-a*b^2)^(1/3)*3
^(1/2)*_alpha^2*a-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*b^2-3*(-a*b^2)^
(2/3)*_alpha-3*b^2)/b/(a-b),(I*3^(1/2)/a*(-a*b^2)^(1/3)/(-3/2/a*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/a*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*a+b)))+1/2*(
3*a*b+c)/b*(2/3/a*(a^2*x^3+b^2)^(1/2)+1/3*I/a^2*2^(1/2)*sum((-a*b^2)^(1/3)*
(1/2*I*a*(2*x+1/a*(-I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3
))^(1/2)*(a*(x-1/a*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1
/3)))^(1/2)*(-1/2*I*a*(2*x+1/a*(I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(
-a*b^2)^(1/3))^(1/2)/(a^2*x^3+b^2)^(1/2)*(I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*a
-I*3^(1/2)*(-a*b^2)^(2/3)+2*_alpha^2*a^2-(-a*b^2)^(1/3)*_alpha*a-(-a*b^2)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/a*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/a*(-a
*b^2)^(1/3))*3^(1/2)*a/(-a*b^2)^(1/3))^(1/2),-1/2*(2*I*(-a*b^2)^(1/3)*3^(1/
2)*_alpha^2*a-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*b^2-3*(-a*b^2)^(2/3
)*_alpha-3*b^2)/b/(a+b),(I*3^(1/2)/a*(-a*b^2)^(1/3)/(-3/2/a*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/a*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*a-b)))-4/3*(a^2*
x^3+b^2)^(1/2)+4/3*b^2*arctanh((a^2*x^3+b^2)^(1/2)/(b^2)^(1/2))/(b^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^6 + cx^3 + 2\,b^2\right)\sqrt{a^2x^3 + b^2}}{\left(a^2x^6 - b^2\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^3+b^2)^(1/2)*(a^2*x^6+c*x^3+2*b^2)/x/(a^2*x^6-b^2),x, algo
rithm="maxima")
```

```
[Out] integrate((a^2*x^6 + c*x^3 + 2*b^2)*sqrt(a^2*x^3 + b^2)/((a^2*x^6 - b^2)*x)
, x)
```

mupad [B] time = 9.52, size = 204, normalized size = 1.16

$$\frac{2\,\sqrt{a^2\,x^3+b^2}}{3}+\frac{2\,b\,\ln\left(\frac{{\left(b+\sqrt{a^2\,x^3+b^2}\right)}^3\,\left(b-\sqrt{a^2\,x^3+b^2}\right)}{x^6}\right)}{3}+\frac{\ln\left(\frac{a\,b+2\,b^2+a^2\,x^3-2\,\sqrt{b}\,\sqrt{a^2\,x^3+b^2}\,\sqrt{a+b}}{b-a\,x^3}\right)\,\sqrt{a+b}\,\left(c+3\,a\,b\right)}{6\,a\,\sqrt{b}}+\frac{\ln\left(\frac{2\,b^2-a\,b+a^2\,x^3+\sqrt{b}\,\sqrt{a^2\,x^3+b^2}\,\sqrt{a-b}\,2{}\mathrm{i}}{a\,x^3+b}\right)\,\sqrt{a-b}\,\left(c-3\,a\,b\right)\,1{}\mathrm{i}}{6\,a\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b^2 + a^2*x^3)^(1/2)*(c*x^3 + 2*b^2 + a^2*x^6))/(x*(b^2 - a^2*x^6)),
x)
```

```
[Out] (2*(b^2 + a^2*x^3)^(1/2))/3 + (2*b*log(((b + (b^2 + a^2*x^3)^(1/2))^3*(b -
(b^2 + a^2*x^3)^(1/2)))/x^6))/3 + (log((2*b^2 - a*b + a^2*x^3 + b^(1/2)*(b^
2 + a^2*x^3)^(1/2)*(a - b)^(1/2)*2i)/(b + a*x^3))*(a - b)^(1/2)*(c - 3*a*b)
*1i)/(6*a*b^(1/2)) + (log((a*b + 2*b^2 + a^2*x^3 - 2*b^(1/2)*(b^2 + a^2*x^3
)^(1/2)*(a + b)^(1/2))/(b - a*x^3))*(a + b)^(1/2)*(c + 3*a*b))/(6*a*b^(1/2)
)
```

sympy [A] time = 114.32, size = 162, normalized size = 0.92

$$-\frac{2b\log\left(b-\sqrt{a^2x^3+b^2}\right)}{3}+\frac{2b\log\left(b+\sqrt{a^2x^3+b^2}\right)}{3}+\frac{2\sqrt{a^2x^3+b^2}}{3}-\frac{(a-b)\,(3ab-c)\,\operatorname{atan}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{ab-b^2}}\right)}{3a\sqrt{ab-b^2}}+\frac{(a+b)\,(3ab+c)\,\operatorname{atan}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{-ab-b^2}}\right)}{3a\sqrt{-ab-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**3+b**2)**(1/2)*(a**2*x**6+c*x**3+2*b**2)/x/(a**2*x**6-b*
*2),x)
```

```
[Out] -2*b*log(b - sqrt(a**2*x**3 + b**2))/3 + 2*b*log(b + sqrt(a**2*x**3 + b**2)
)/3 + 2*sqrt(a**2*x**3 + b**2)/3 - (a - b)*(3*a*b - c)*atan(sqrt(a**2*x**3
+ b**2)/sqrt(a*b - b**2))/(3*a*sqrt(a*b - b**2)) + (a + b)*(3*a*b + c)*atan
(sqrt(a**2*x**3 + b**2)/sqrt(-a*b - b**2))/(3*a*sqrt(-a*b - b**2))
```

3.1892 $\int \frac{1-x^4+2x^8}{\sqrt[4]{1+x^4}\left(-1-2x^4+x^8\right)}\,dx$

Optimal. Leaf size=176

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{3\tan^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[8]{2}}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{3\tanh^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[8]{2}}+\frac{3\tan^{-1}\left(\frac{2^{5/8}x\sqrt[4]{x^4+1}}{\sqrt[4]{2}\,x^2-\sqrt{x^4+1}}\right)}{4\,2^{5/8}}-$$

Rubi [A] time = 0.49, antiderivative size = 306, normalized size of antiderivative = 1.74, number of steps used = 31, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6728, 240, 212, 206, 203, 1428, 408, 377, 211, 1165, 628, 1162, 617, 204}

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{3\tan^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[8]{2}}-\frac{3\left(1-\sqrt{2}\right)\tan^{-1}\left(1-\frac{2^{5/8}x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[8]{2}\left(2-\sqrt{2}\right)}+\frac{3\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{2^{5/8}x}{\sqrt[4]{x^4+1}}+1\right)}{4\sqrt[8]{2}\left(2-\sqrt{2}\right)}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{3\tanh^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[8]{2}}-\frac{3\left(1-\sqrt{2}\right)\log\left(-\frac{2x}{\sqrt[4]{x^4+1}}+\frac{2^{5/8}x^2}{\sqrt{x^4+1}}+2^{3/8}\right)}{8\sqrt[8]{2}\left(2-\sqrt{2}\right)}+\frac{3\left(1-\sqrt{2}\right)\log\left(\frac{2^{5/8}x}{\sqrt[4]{x^4+1}}+\frac{\sqrt[4]{2}\,x^2}{\sqrt{x^4+1}}+1\right)}{8\sqrt[8]{2}\left(2-\sqrt{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4 + 2*x^8)/((1 + x^4)^(1/4)*(-1 - 2*x^4 + x^8)),x]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)] - (3*ArcTan[(2^(1/8)*x)/(1 + x^4)^(1/4)])/(4*2^(1
/8)) - (3*(1 - Sqrt[2])*ArcTan[1 - (2^(5/8)*x)/(1 + x^4)^(1/4)])/(4*2^(1/8)
*(2 - Sqrt[2])) + (3*(1 - Sqrt[2])*ArcTan[1 + (2^(5/8)*x)/(1 + x^4)^(1/4)])
/(4*2^(1/8)*(2 - Sqrt[2])) + ArcTanh[x/(1 + x^4)^(1/4)] - (3*ArcTanh[(2^(1/
8)*x)/(1 + x^4)^(1/4)])/(4*2^(1/8)) - (3*(1 - Sqrt[2])*Log[2^(3/8) + (2^(5/
8)*x^2)/Sqrt[1 + x^4] - (2*x)/(1 + x^4)^(1/4)])/(8*2^(1/8)*(2 - Sqrt[2])) +
 (3*(1 - Sqrt[2])*Log[1 + (2^(1/4)*x^2)/Sqrt[1 + x^4] + (2^(5/8)*x)/(1 + x^
4)^(1/4)])/(8*2^(1/8)*(2 - Sqrt[2]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 408

```
Int[((a_) + (b_.)*(x_)^4)^(p_)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d
, Int[(a + b*x^4)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(p
- 1)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[p, 3/4] || EqQ[p, 5/4])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1428

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^
n)^q/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[(d + e*x^n)^q/(b + r + 2
*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1-x^4+2x^8}{\sqrt[4]{1+x^4}\left(-1-2x^4+x^8\right)}\,dx = \int \left(\frac{2}{\sqrt[4]{1+x^4}}+\frac{3\left(1+x^4\right)^{3/4}}{-1-2x^4+x^8}\right)dx$$

$$= 2\int \frac{1}{\sqrt[4]{1+x^4}}\,dx + 3\int \frac{\left(1+x^4\right)^{3/4}}{-1-2x^4+x^8}\,dx$$

$$= 2\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right) + \frac{3\int \frac{\left(1+x^4\right)^{3/4}}{-2-2\sqrt{2}+2x^4}\,dx}{\sqrt{2}} - \frac{3\int \frac{\left(1+x^4\right)^{3/4}}{-2+2\sqrt{2}+2x^4}\,dx}{\sqrt{2}}$$

$$= \left(3\left(1-\sqrt{2}\right)\right)\int \frac{1}{\sqrt[4]{1+x^4}\left(-2+2\sqrt{2}+2x^4\right)}\,dx + \left(3\left(1+\sqrt{2}\right)\right)\int \frac{}{\sqrt[4]{1+x^4}\,(-}$$

$$= \tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) + \tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) + \left(3\left(1-\sqrt{2}\right)\right)\text{Subst}\left(\int \frac{}{-2+2\sqrt{2}-}\right.$$

$$= \tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) + \tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) - \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1-\sqrt[4]{2}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right.$$

$$= \tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) - \frac{3\tan^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{4\sqrt[8]{2}} + \tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) - \frac{3\tanh^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{4\sqrt[8]{2}}$$

$$= \tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) - \frac{3\tan^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{4\sqrt[8]{2}} + \tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) - \frac{3\tanh^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{4\sqrt[8]{2}}$$

$$= \tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right) - \frac{3\tan^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{1+x^4}}\right)}{4\sqrt[8]{2}} - \frac{3\left(1-\sqrt{2}\right)\tan^{-1}\left(1-\frac{2^{5/8}x}{\sqrt[4]{1+x^4}}\right)}{4\sqrt[8]{2}\left(2-\sqrt{2}\right)} + \frac{3\left(1-\right.}{}$$

Mathematica [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1-x^4+2x^8}{\sqrt[4]{1+x^4}\left(-1-2x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^4 + 2*x^8)/((1 + x^4)^(1/4)*(-1 - 2*x^4 + x^8)),x]
[Out] Integrate[(1 - x^4 + 2*x^8)/((1 + x^4)^(1/4)*(-1 - 2*x^4 + x^8)), x]
```

IntegrateAlgebraic [A] time = 0.64, size = 176, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right) - \frac{3\tan^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[8]{2}} + \tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right) - \frac{3\tanh^{-1}\left(\frac{\sqrt[8]{2}\,x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt[8]{2}} + \frac{3\tan^{-1}\left(\frac{2^{5/8}x\sqrt[4]{x^4+1}}{\sqrt[4]{2}\,x^2-\sqrt{x^4+1}}\right)}{4\,2^{5/8}} - \frac{3\tanh^{-1}\left(\frac{2\,2^{3/8}x\sqrt[4]{x^4+1}}{2^{3/4}\sqrt{x^4+1}+2x^2}\right)}{4\,2^{5/8}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^4 + 2*x^8)/((1 + x^4)^(1/4)*(-1 - 2*x^4 + x^8)),x
]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)] - (3*ArcTan[(2^(1/8)*x)/(1 + x^4)^(1/4)])/(4*2^(1
/8)) + (3*ArcTan[(2^(5/8)*x*(1 + x^4)^(1/4))/(2^(1/4)*x^2 - Sqrt[1 + x^4])]
)/(4*2^(5/8)) + ArcTanh[x/(1 + x^4)^(1/4)] - (3*ArcTanh[(2^(1/8)*x)/(1 + x^
4)^(1/4)])/(4*2^(1/8)) - (3*ArcTanh[(2*2^(3/8)*x*(1 + x^4)^(1/4))/(2*x^2 +
2^(3/4)*Sqrt[1 + x^4])])/(4*2^(5/8))
```

fricas [**B**] time = 0.43, size = 353, normalized size = 2.01

$$-\frac{3}{4}\cdot 2^{\frac{7}{8}}\arctan\left(\frac{2^{\frac{7}{8}}x\sqrt{\frac{2^{\frac{1}{4}}x^2+\sqrt{x^4+1}}{x^2}}-2^{\frac{7}{8}}\left(x^4+1\right)^{\frac{1}{4}}}{2x}\right)-\frac{3}{16}\cdot 2^{\frac{7}{8}}\log\left(\frac{2^{\frac{1}{8}}x+\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+\frac{3}{16}\cdot 2^{\frac{7}{8}}\log\left(-\frac{2^{\frac{1}{8}}x-\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-\frac{3}{4}\cdot 2^{\frac{3}{8}}\arctan\left(\frac{2^{\frac{3}{8}}x\sqrt{\frac{2^{\frac{1}{4}}x^2+2^{\frac{5}{8}}\left(x^4+1\right)^{\frac{1}{4}}x+\sqrt{x^4+1}}{x^2}}-x-2^{\frac{3}{8}}\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-\frac{3}{4}\cdot 2^{\frac{3}{8}}\arctan\left(\frac{2^{\frac{3}{8}}x\sqrt{\frac{2^{\frac{1}{4}}x^2-2^{\frac{5}{8}}\left(x^4+1\right)^{\frac{1}{4}}x+\sqrt{x^4+1}}{x^2}}+x-2^{\frac{3}{8}}\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-\frac{3}{16}\cdot 2^{\frac{3}{8}}\log\left(\frac{4\left(2^{\frac{1}{4}}x^2+2^{\frac{5}{8}}\left(x^4+1\right)^{\frac{1}{4}}x+\sqrt{x^4+1}\right)}{x^2}\right)+\frac{3}{16}\cdot 2^{\frac{3}{8}}\log\left(\frac{4\left(2^{\frac{1}{4}}x^2-2^{\frac{5}{8}}\left(x^4+1\right)^{\frac{1}{4}}x+\sqrt{x^4+1}\right)}{x^2}\right)-\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\log\left(\frac{x+\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{2}\log\left(-\frac{x-\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-x^4+1)/(x^4+1)^(1/4)/(x^8-2*x^4-1),x, algorithm="fricas")
```

```
[Out] -3/4*2^(7/8)*arctan(1/2*(2^(7/8)*x*sqrt((2^(1/4)*x^2 + sqrt(x^4 + 1))/x^2)
- 2^(7/8)*(x^4 + 1)^(1/4))/x) - 3/16*2^(7/8)*log((2^(1/8)*x + (x^4 + 1)^(1/
4))/x) + 3/16*2^(7/8)*log(-(2^(1/8)*x - (x^4 + 1)^(1/4))/x) - 3/4*2^(3/8)*a
rctan((2^(3/8)*x*sqrt((2^(1/4)*x^2 + 2^(5/8)*(x^4 + 1)^(1/4)*x + sqrt(x^4 +
 1))/x^2) - x - 2^(3/8)*(x^4 + 1)^(1/4))/x) - 3/4*2^(3/8)*arctan((2^(3/8)*x
*sqrt((2^(1/4)*x^2 - 2^(5/8)*(x^4 + 1)^(1/4)*x + sqrt(x^4 + 1))/x^2) + x -
2^(3/8)*(x^4 + 1)^(1/4))/x) - 3/16*2^(3/8)*log(4*(2^(1/4)*x^2 + 2^(5/8)*(x^
4 + 1)^(1/4)*x + sqrt(x^4 + 1))/x^2) + 3/16*2^(3/8)*log(4*(2^(1/4)*x^2 - 2^
(5/8)*(x^4 + 1)^(1/4)*x + sqrt(x^4 + 1))/x^2) - arctan((x^4 + 1)^(1/4)/x) +
 1/2*log((x + (x^4 + 1)^(1/4))/x) - 1/2*log(-(x - (x^4 + 1)^(1/4))/x)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-x^4+1)/(x^4+1)^(1/4)/(x^8-2*x^4-1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [**F**] time = 4.20, size = 0, normalized size = 0.00

$$\int \frac{2x^8-x^4+1}{\left(x^4+1\right)^{\frac{1}{4}}\left(x^8-2x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8-x^4+1)/(x^4+1)^(1/4)/(x^8-2*x^4-1),x)
```

```
[Out] int((2*x^8-x^4+1)/(x^4+1)^(1/4)/(x^8-2*x^4-1),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^8-x^4+1}{\left(x^8-2\,x^4-1\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-x^4+1)/(x^4+1)^(1/4)/(x^8-2*x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^8 - x^4 + 1)/((x^8 - 2*x^4 - 1)*(x^4 + 1)^(1/4)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{2\,x^8-x^4+1}{\left(x^4+1\right)^{1/4}\,\left(-x^8+2\,x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^8 - x^4 + 1)/((x^4 + 1)^(1/4)*(2*x^4 - x^8 + 1)),x)
```

```
[Out] int(-(2*x^8 - x^4 + 1)/((x^4 + 1)^(1/4)*(2*x^4 - x^8 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8-x**4+1)/(x**4+1)**(1/4)/(x**8-2*x**4-1),x)
```

```
[Out] Timed out
```

3.1893 $\int \frac{(-1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}{(1+x^2)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=176

$$-\sqrt{2\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\sqrt{2\left(1+\sqrt{2}\right)}\,\text{ta}$$

Rubi [C] time = 1.01, antiderivative size = 180, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {6725, 2132, 206, 2133, 725}

$$\frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-x}{\sqrt{1+i}\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{x+1}{\sqrt{1+i}\sqrt{1-ix^2}}\right)+\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1-x}{\sqrt{1-i}\sqrt{1+ix^2}}\right)-\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{x+1}{\sqrt{1-i}\sqrt{1+ix^2}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] (Sqrt[1 + I]*ArcTanh[(1 - x)/(Sqrt[1 + I]*Sqrt[1 - I*x^2])])/2 - (Sqrt[1 +
I]*ArcTanh[(1 + x)/(Sqrt[1 + I]*Sqrt[1 - I*x^2])])/2 + (Sqrt[1 - I]*ArcTanh
[(1 - x)/(Sqrt[1 - I]*Sqrt[1 + I*x^2])])/2 - (Sqrt[1 - I]*ArcTanh[(1 + x)/(
Sqrt[1 - I]*Sqrt[1 + I*x^2])])/2 + ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 +
x^4]]]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}{(1+x^2)\sqrt{1+x^4}}\,dx &= \int \left(\frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}} - \frac{2\sqrt{x^2+\sqrt{1+x^4}}}{(1+x^2)\sqrt{1+x^4}}\right)dx \\
&= -\left(2\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x^2)\sqrt{1+x^4}}\,dx\right) + \int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx \\
&= -\left(2\int \left(\frac{i\sqrt{x^2+\sqrt{1+x^4}}}{2(i-x)\sqrt{1+x^4}} + \frac{i\sqrt{x^2+\sqrt{1+x^4}}}{2(i+x)\sqrt{1+x^4}}\right)dx\right) + \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x\right. \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}} - i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(i-x)\sqrt{1+x^4}}\,dx - i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(i+x)\sqrt{1+x^4}}\,dx \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}} - \left(-\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(i-x)\sqrt{1+ix^2}}\,dx - \left(-\frac{1}{2}+\frac{i}{2}\right)\int \frac{}{(i+x} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}} - \left(-\frac{1}{2}-\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{(1+i)-x^2}\,dx, x, \frac{-1-x}{\sqrt{1-ix^2}}\right) - \Big(\\
&= \frac{1}{2}\sqrt{1+i}\,\tanh^{-1}\left(\frac{1-x}{\sqrt{1+i}\sqrt{1-ix^2}}\right) - \frac{1}{2}\sqrt{1+i}\,\tanh^{-1}\left(\frac{1+x}{\sqrt{1+i}\sqrt{1-ix^2}}\right) + \frac{1}{2}
\end{aligned}
$$

Mathematica [F] time = 0.23, size = 0, normalized size = 0.00

$$
\int \frac{(-1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}{(1+x^2)\sqrt{1+x^4}}\,dx
$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/((1 + x^2)*Sqrt[1 + x^4]),
x]
```

```
[Out] Integrate[((-1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/((1 + x^2)*Sqrt[1 + x^4]),
 x]
```

IntegrateAlgebraic [A] time = 0.93, size = 247, normalized size = 1.40

$$
-\sqrt{2\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-\sqrt{2\left(1+\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/((1 + x^2)*Sqrt[1
 + x^4]),x]
```

```
[Out] -(Sqrt[2*(-1 + Sqrt[2])]*ArcTan[(-Sqrt[1/2 + 1/Sqrt[2]] + Sqrt[1/2 + 1/Sqrt
[2]]*x^2 + Sqrt[1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 + x^4]
])]) + Sqrt[2]*ArcTanh[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2])
```

```
/(x*Sqrt[x^2 + Sqrt[1 + x^4]])] - Sqrt[2*(1 + Sqrt[2])]*ArcTanh[(-Sqrt[-1/2
 + 1/Sqrt[2]] + Sqrt[-1/2 + 1/Sqrt[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*Sqrt[1
+ x^4])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])]
```

fricas [B] time = 3.95, size = 368, normalized size = 2.09

$$-\sqrt{2\sqrt{2}-2}\arctan\left(\frac{\left(4x^2+2\sqrt{2}\left(x^2+1\right)+\sqrt{x^4+1}\left(\left(\sqrt{2}+2\right)\sqrt{-8\sqrt{2}+12}-2\sqrt{2}-4\right)-\left(2x^2+\sqrt{2}\left(x^2+3\right)+4\right)\sqrt{-8\sqrt{2}+12}\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{2\sqrt{2}-2}}{8x}\right)+\frac{1}{4}\sqrt{2}\log\left(4x^4+4\sqrt{x^4+1}x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)+\frac{1}{4}\sqrt{2\sqrt{2}+2}\log\left(\frac{2\sqrt{2}x^2+4x^2+\left(\sqrt{2}\sqrt{x^4+1}x-\sqrt{2}\left(x^3+x\right)-2x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{2\sqrt{2}+2}+2\sqrt{x^4+1}\left(\sqrt{2}+1\right)}{x^2+1}\right)-\frac{1}{4}\sqrt{2\sqrt{2}+2}\log\left(\frac{2\sqrt{2}x^2+4x^2-\left(\sqrt{2}\sqrt{x^4+1}x-\sqrt{2}\left(x^3+x\right)-2x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{2\sqrt{2}+2}+2\sqrt{x^4+1}\left(\sqrt{2}+1\right)}{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)/(x^4+1)^(1/2),x, algori
thm="fricas")
```

```
[Out] -sqrt(2*sqrt(2) - 2)*arctan(1/8*(4*x^2 + 2*sqrt(2)*(x^2 + 1) + sqrt(x^4 + 1
)*((sqrt(2) + 2)*sqrt(-8*sqrt(2) + 12) - 2*sqrt(2) - 4) - (2*x^2 + sqrt(2)*
(x^2 + 3) + 4)*sqrt(-8*sqrt(2) + 12))*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(2*sqrt
(2) - 2)/x) + 1/4*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3
+ sqrt(2)*sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1) + 1/4*sqrt(2*sqrt
(2) + 2)*log((2*sqrt(2)*x^2 + 4*x^2 + (sqrt(2)*sqrt(x^4 + 1)*x - sqrt(2)*(x
^3 + x) - 2*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(2*sqrt(2) + 2) + 2*sqrt(x^4 +
 1)*(sqrt(2) + 1))/(x^2 + 1)) - 1/4*sqrt(2*sqrt(2) + 2)*log((2*sqrt(2)*x^2
+ 4*x^2 - (sqrt(2)*sqrt(x^4 + 1)*x - sqrt(2)*(x^3 + x) - 2*x)*sqrt(x^2 + sq
rt(x^4 + 1))*sqrt(2*sqrt(2) + 2) + 2*sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 + 1)
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-1\right)}{\sqrt{x^4+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)/(x^4+1)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-1\right)\sqrt{x^2+\sqrt{x^4+1}}}{\left(x^2+1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)/(x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-1\right)}{\sqrt{x^4+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)/(x^4+1)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2-1\right)\sqrt{\sqrt{x^4+1}+x^2}}{\left(x^2+1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*((x^4 + 1)^(1/2) + x^2)^(1/2))/((x^2 + 1)*(x^4 + 1)^(1/2)),x
)
```

```
[Out] int(((x^2 - 1)*((x^4 + 1)^(1/2) + x^2)^(1/2))/((x^2 + 1)*(x^4 + 1)^(1/2)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\sqrt{x^2+\sqrt{x^4+1}}}{\left(x^2+1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**2+(x**4+1)**(1/2))**(1/2)/(x**2+1)/(x**4+1)**(1/2),x
)
```

```
[Out] Integral((x - 1)*(x + 1)*sqrt(x**2 + sqrt(x**4 + 1))/((x**2 + 1)*sqrt(x**4
+ 1)), x)
```

3.1894 $\int \frac{1}{(1+x)\sqrt[3]{1-x+x^2}}\,dx$

Optimal. Leaf size=177

$$\frac{\log\left(3\sqrt[3]{x^2-x+1}+\sqrt[3]{3}x-2\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\log\left(3^{2/3}x^2+9\left(x^2-x+1\right)^{2/3}+\left(6\sqrt[3]{3}-3\sqrt[3]{3}x\right)\sqrt[3]{x^2-x+1}-4\,3^{2/3}x+\right.}{6\sqrt[3]{3}}$$

Rubi [A] time = 0.02, antiderivative size = 88, normalized size of antiderivative = 0.50, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {750}

$$\frac{\log\left(-3^{2/3}\sqrt[3]{x^2-x+1}-x+2\right)}{2\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{2(2-x)}{3\sqrt[6]{3}\sqrt[3]{x^2-x+1}}+\frac{1}{\sqrt{3}}\right)}{3^{5/6}}-\frac{\log(x+1)}{2\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(1 - x + x^2)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2*(2 - x))/(3*3^(1/6)*(1 - x + x^2)^(1/3))]/3^(5/6))
- Log[1 + x]/(2*3^(1/3)) + Log[2 - x - 3^(2/3)*(1 - x + x^2)^(1/3)]/(2*3^(1
/3))
```

Rule 750

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[3*c*e^2*(2*c*d - b*e), 3]}, -Simp[(Sqrt[3]*c*e*ArcTan
[1/Sqrt[3] + (2*(c*d - b*e - c*e*x))/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3))])/
q^2, x] + (-Simp[(3*c*e*Log[d + e*x])/(2*q^2), x] + Simp[(3*c*e*Log[c*d - b
*e - c*e*x - q*(a + b*x + c*x^2)^(1/3)])/(2*q^2), x])] /; FreeQ[{a, b, c, d
, e}, x] && NeQ[2*c*d - b*e, 0] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*c*
e^2, 0] && PosQ[c*e^2*(2*c*d - b*e)]
```

Rubi steps

$$\int \frac{1}{(1+x)\sqrt[3]{1-x+x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2(2-x)}{3\sqrt[6]{3}\sqrt[3]{1-x+x^2}}\right)}{3^{5/6}}-\frac{\log(1+x)}{2\sqrt[3]{3}}+\frac{\log\left(2-x-3^{2/3}\sqrt[3]{1-x+x^2}\right)}{2\sqrt[3]{3}}$$

Mathematica [C] time = 0.06, size = 120, normalized size = 0.68

$$-\frac{3\sqrt[3]{\frac{2x-i\sqrt{3}-1}{x+1}}\sqrt[3]{\frac{2x+i\sqrt{3}-1}{x+1}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{3-i\sqrt{3}}{2x+2},\frac{3+i\sqrt{3}}{2x+2}\right)}{2\,2^{2/3}\sqrt[3]{x^2-x+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + x)*(1 - x + x^2)^(1/3)),x]
```

```
[Out] (-3*((-1 - I*Sqrt[3] + 2*x)/(1 + x))^(1/3)*((-1 + I*Sqrt[3] + 2*x)/(1 + x))
^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, (3 - I*Sqrt[3])/(2 + 2*x), (3 + I*Sqrt[
3])/(2 + 2*x)])/(2*2^(2/3)*(1 - x + x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.20, size = 177, normalized size = 1.00

$$\frac{\log\left(3\sqrt[3]{x^2-x+1}+\sqrt[3]{3}x-2\sqrt[3]{3}\right)}{3\sqrt[3]{3}}-\frac{\log\left(3^{2/3}x^2+9\left(x^2-x+1\right)^{2/3}+\left(6\sqrt[3]{3}-3\sqrt[3]{3}x\right)\sqrt[3]{x^2-x+1}-4\,3^{2/3}x+4\,3^{2/3}\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2-x+1}}{\sqrt{3}}-\frac{2x}{3\sqrt[6]{3}}+\frac{4}{3\sqrt[6]{3}}}{\sqrt[3]{x^2-x+1}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x)*(1 - x + x^2)^(1/3)),x]
```

```
[Out] -(ArcTan[(4/(3*3^(1/6)) - (2*x)/(3*3^(1/6)) + (1 - x + x^2)^(1/3)/Sqrt[3]])/
(1 - x + x^2)^(1/3)]/3^(5/6)) + Log[-2*3^(1/3) + 3^(1/3)*x + 3*(1 - x + x^2
)^(1/3)]/(3*3^(1/3)) - Log[4*3^(2/3) - 4*3^(2/3)*x + 3^(2/3)*x^2 + (6*3^(1/
3) - 3*3^(1/3)*x)*(1 - x + x^2)^(1/3) + 9*(1 - x + x^2)^(2/3)]/(6*3^(1/3))
```

fricas [A] time = 2.25, size = 175, normalized size = 0.99

$$-\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(\frac{3\cdot 3^{\frac{2}{3}}\left(x^2-x+1\right)^{\frac{2}{3}}+3^{\frac{1}{3}}\left(x^2-4x+4\right)-3\left(x^2-x+1\right)^{\frac{1}{3}}(x-2)}{x^2+2x+1}\right)+\frac{1}{9}\cdot 3^{\frac{2}{3}}\log\left(\frac{3^{\frac{1}{3}}(x-2)+3\left(x^2-x+1\right)^{\frac{1}{3}}}{x+1}\right)-\frac{1}{3}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{1}{6}}\left(6\cdot 3^{\frac{2}{3}}\left(x^2-x+1\right)^{\frac{2}{3}}(x-2)+3^{\frac{1}{3}}\left(x^3+3x^2+3x+1\right)+6\left(x^2-x+1\right)^{\frac{1}{3}}\left(x^2-4x+4\right)\right)}{3\left(x^3-15x^2+21x-17\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2-x+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/18*3^(2/3)*log((3*3^(2/3)*(x^2 - x + 1)^(2/3) + 3^(1/3)*(x^2 - 4*x + 4)
- 3*(x^2 - x + 1)^(1/3)*(x - 2))/(x^2 + 2*x + 1)) + 1/9*3^(2/3)*log((3^(1/3
)*(x - 2) + 3*(x^2 - x + 1)^(1/3))/(x + 1)) - 1/3*3^(1/6)*arctan(1/3*3^(1/6
)*(6*3^(2/3)*(x^2 - x + 1)^(2/3)*(x - 2) + 3^(1/3)*(x^3 + 3*x^2 + 3*x + 1)
+ 6*(x^2 - x + 1)^(1/3)*(x^2 - 4*x + 4))/(x^3 - 15*x^2 + 21*x - 17))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2-x+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(1/3)*(x + 1)), x)
```

maple [C] time = 14.20, size = 1417, normalized size = 8.01

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^2-x+1)^(1/3),x)
```

```
[Out] 1/9*RootOf(_Z^3-9)*ln((61590*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*
_Z^2)*RootOf(_Z^3-9)^3*x^3+65997*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9
)+9*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+123180*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf
(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^3*x^2+131994*RootOf(RootOf(_Z^3-9)^2+3*_Z*R
ootOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2*x^2+967776*(x^2-x+1)^(2/3)*RootOf(
RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^2*x+246360*Root
Of(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^3*x+263988*R
ootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2*x-193
5552*(x^2-x+1)^(2/3)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*Ro
otOf(_Z^3-9)^2+322592*(x^2-x+1)^(1/3)*RootOf(_Z^3-9)^2*x^2+1579341*(x^2-x+1
)^(1/3)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)*
x^2-1290368*(x^2-x+1)^(1/3)*RootOf(_Z^3-9)^2*x-6317364*(x^2-x+1)^(1/3)*Root
Of(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)*x+349010*Roo
```

```
tOf(_Z^3-9)*x^3+373983*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*
x^3+1290368*(x^2-x+1)^(1/3)*RootOf(_Z^3-9)^2+6317364*(x^2-x+1)^(1/3)*RootOf
(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)-3756990*RootOf
(_Z^3-9)*x^2-4025817*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x^
2-1834695*(x^2-x+1)^(2/3)*x+5851050*RootOf(_Z^3-9)*x+6269715*RootOf(RootOf(
_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x+3669390*(x^2-x+1)^(2/3)-4455010*Roo
tOf(_Z^3-9)-4773783*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2))/(1
+x)^3)+1/3*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*ln(-(21999*R
ootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^3*x^3+184
770*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2*
x^3+43998*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9
)^3*x^2+369540*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)^2*RootOf
(_Z^3-9)^2*x^2+967776*(x^2-x+1)^(2/3)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_
Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^2*x+87996*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf
(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^3*x+739080*RootOf(RootOf(_Z^3-9)^2+3*_Z*Roo
tOf(_Z^3-9)+9*_Z^2)^2*RootOf(_Z^3-9)^2*x-1935552*(x^2-x+1)^(2/3)*RootOf(Roo
tOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)^2+322592*(x^2-x+1)
^(1/3)*RootOf(_Z^3-9)^2*x^2-611565*(x^2-x+1)^(1/3)*RootOf(RootOf(_Z^3-9)^2+
3*_Z*RootOf(_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)*x^2-1290368*(x^2-x+1)^(1/3)*Root
Of(_Z^3-9)^2*x+2446260*(x^2-x+1)^(1/3)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(
_Z^3-9)+9*_Z^2)*RootOf(_Z^3-9)*x-58664*RootOf(_Z^3-9)*x^3-492720*RootOf(Roo
tOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x^3+1290368*(x^2-x+1)^(1/3)*RootO
f(_Z^3-9)^2-2446260*(x^2-x+1)^(1/3)*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^
3-9)+9*_Z^2)*RootOf(_Z^3-9)+1473933*RootOf(_Z^3-9)*x^2+12379590*RootOf(Root
Of(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2)*x^2+4738023*(x^2-x+1)^(2/3)*x-1825
917*RootOf(_Z^3-9)*x-15335910*RootOf(RootOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9
*_Z^2)*x-9476046*(x^2-x+1)^(2/3)+1591261*RootOf(_Z^3-9)+13365030*RootOf(Roo
tOf(_Z^3-9)^2+3*_Z*RootOf(_Z^3-9)+9*_Z^2))/(1+x)^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-x+1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2-x+1)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^2 - x + 1)^(1/3)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x+1)\,\left(x^2-x+1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)*(x^2 - x + 1)^(1/3)),x)
[Out] int(1/((x + 1)*(x^2 - x + 1)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)\,\sqrt[3]{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2-x+1)**(1/3),x)
[Out] Integral(1/((x + 1)*(x**2 - x + 1)**(1/3)), x)
```

3.1895 $\int \frac{\sqrt[3]{bx+ax^3}\left(b+ax^4\right)}{x^4}\,dx$

Optimal. Leaf size=177

$$\frac{1}{12}\sqrt[3]{a}\,b\log\left(a^{2/3}x^2+\sqrt[3]{a}\,x\sqrt[3]{ax^3+bx}+\left(ax^3+bx\right)^{2/3}\right)-\frac{1}{6}\sqrt[3]{a}\,b\log\left(\sqrt[3]{ax^3+bx}-\sqrt[3]{a}\,x\right)-\frac{\sqrt[3]{a}\,b\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{a}\,x}{2\sqrt[3]{ax^3+bx}+\sqrt[3]{\ldots}}\right.}{2\sqrt{3}}$$

Rubi [A] time = 0.41, antiderivative size = 267, normalized size of antiderivative = 1.51, number of steps used = 14, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {2052, 2004, 2032, 329, 275, 331, 292, 31, 634, 617, 204, 628, 2014}

$$\frac{\sqrt[3]{a}\,bx^{2/3}\left(ax^2+b\right)^{2/3}\log\left(\frac{a^{2/3}x^{4/3}}{\left(ax^2+b\right)^{2/3}}+\frac{\sqrt[3]{a}\,x^{2/3}}{\sqrt[3]{ax^2+b}}+1\right)}{12\left(ax^3+bx\right)^{2/3}}+\frac{1}{2}ax\sqrt[3]{ax^3+bx}-\frac{3\left(ax^3+bx\right)^{4/3}}{8x^4}-\frac{\sqrt[3]{a}\,bx^{2/3}\left(ax^2+b\right)^{2/3}\log\left(1-\frac{\sqrt[3]{a}\,x^{2/3}}{\sqrt[3]{ax^2+b}}\right)}{6\left(ax^3+bx\right)^{2/3}}-\frac{\sqrt[3]{a}\,bx^{2/3}\left(ax^2+b\right)^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a}\,x^{2/3}}{\sqrt[3]{ax^2+b}}+1}{\sqrt{3}}\right)}{2\sqrt{3}\left(ax^3+bx\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*x + a*x^3)^(1/3)*(b + a*x^4))/x^4,x]
```

```
[Out] (a*x*(b*x + a*x^3)^(1/3))/2 - (3*(b*x + a*x^3)^(4/3))/(8*x^4) - (a^(1/3)*b*
x^(2/3)*(b + a*x^2)^(2/3)*ArcTan[(1 + (2*a^(1/3)*x^(2/3))/(b + a*x^2)^(1/3)
)/Sqrt[3]])/(2*Sqrt[3]*(b*x + a*x^3)^(2/3)) - (a^(1/3)*b*x^(2/3)*(b + a*x^2
)^(2/3)*Log[1 - (a^(1/3)*x^(2/3))/(b + a*x^2)^(1/3)])/(6*(b*x + a*x^3)^(2/3
)) + (a^(1/3)*b*x^(2/3)*(b + a*x^2)^(2/3)*Log[1 + (a^(2/3)*x^(4/3))/(b + a*
x^2)^(2/3) + (a^(1/3)*x^(2/3))/(b + a*x^2)^(1/3)])/(12*(b*x + a*x^3)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{bx+ax^3}\left(b+ax^4\right)}{x^4}\,dx &= \int\left(a\sqrt[3]{bx+ax^3}+\frac{b\sqrt[3]{bx+ax^3}}{x^4}\right)dx\\
&= a\int \sqrt[3]{bx+ax^3}\,dx + b\int \frac{\sqrt[3]{bx+ax^3}}{x^4}\,dx\\
&= \frac{1}{2}ax\sqrt[3]{bx+ax^3} - \frac{3\left(bx+ax^3\right)^{4/3}}{8x^4} + \frac{1}{3}(ab)\int \frac{x}{\left(bx+ax^3\right)^{2/3}}\,dx\\
&= \frac{1}{2}ax\sqrt[3]{bx+ax^3} - \frac{3\left(bx+ax^3\right)^{4/3}}{8x^4} + \frac{\left(abx^{2/3}\left(b+ax^2\right)^{2/3}\right)\int \frac{\sqrt[3]{x}}{\left(b+ax^2\right)^{2/3}}\,dx}{3\left(bx+ax^3\right)^{2/3}}\\
&= \frac{1}{2}ax\sqrt[3]{bx+ax^3} - \frac{3\left(bx+ax^3\right)^{4/3}}{8x^4} + \frac{\left(abx^{2/3}\left(b+ax^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x^3}{\left(b+ax^6\right)^{2/3}}\,dx, x, \sqrt[3]{\cdots}\right.}{\left(bx+ax^3\right)^{2/3}}\\
&= \frac{1}{2}ax\sqrt[3]{bx+ax^3} - \frac{3\left(bx+ax^3\right)^{4/3}}{8x^4} + \frac{\left(abx^{2/3}\left(b+ax^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{\left(b+ax^3\right)^{2/3}}\,dx, x, x\cdots\right.}{2\left(bx+ax^3\right)^{2/3}}\\
&= \frac{1}{2}ax\sqrt[3]{bx+ax^3} - \frac{3\left(bx+ax^3\right)^{4/3}}{8x^4} + \frac{\left(abx^{2/3}\left(b+ax^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{1-ax^3}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{b+a\cdots}}\right.}{2\left(bx+ax^3\right)^{2/3}}\\
&= \frac{1}{2}ax\sqrt[3]{bx+ax^3} - \frac{3\left(bx+ax^3\right)^{4/3}}{8x^4} + \frac{\left(a^{2/3}bx^{2/3}\left(b+ax^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{a}\,x}\,dx, x, \cdots\right.}{6\left(bx+ax^3\right)^{2/3}}\\
&= \frac{1}{2}ax\sqrt[3]{bx+ax^3} - \frac{3\left(bx+ax^3\right)^{4/3}}{8x^4} - \frac{\sqrt[3]{a}\,bx^{2/3}\left(b+ax^2\right)^{2/3}\log\left(1-\frac{\sqrt[3]{a}\,x^{2/3}}{\sqrt[3]{b+ax^2}}\right)}{6\left(bx+ax^3\right)^{2/3}} + \left(\sqrt[3]{a}\,b\cdots\right.\\
&= \frac{1}{2}ax\sqrt[3]{bx+ax^3} - \frac{3\left(bx+ax^3\right)^{4/3}}{8x^4} - \frac{\sqrt[3]{a}\,bx^{2/3}\left(b+ax^2\right)^{2/3}\log\left(1-\frac{\sqrt[3]{a}\,x^{2/3}}{\sqrt[3]{b+ax^2}}\right)}{6\left(bx+ax^3\right)^{2/3}} + \frac{\sqrt[3]{a}\,bx\cdots}{\cdots}\\
&= \frac{1}{2}ax\sqrt[3]{bx+ax^3} - \frac{3\left(bx+ax^3\right)^{4/3}}{8x^4} - \frac{\sqrt[3]{a}\,bx^{2/3}\left(b+ax^2\right)^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a}\,x^{2/3}}{\sqrt[3]{b+ax^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\left(bx+ax^3\right)^{2/3}} - \frac{\sqrt[3]{a}\,bx\cdots}{\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.14, size = 69, normalized size = 0.39

$$\frac{3\sqrt[3]{x\left(ax^2+b\right)}\left(\frac{2ax^4\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{ax^2}{b}\right)}{\sqrt[3]{\frac{ax^2}{b}+1}}-ax^2-b\right)}{8x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*x + a*x^3)^(1/3)*(b + a*x^4))/x^4,x]
```

```
[Out] (3*(x*(b + a*x^2))^(1/3)*(-b - a*x^2 + (2*a*x^4*Hypergeometric2F1[-1/3, 2/3
, 5/3, -((a*x^2)/b)])/(1 + (a*x^2)/b)^(1/3)))/(8*x^3)
```

IntegrateAlgebraic [A] time = 0.56, size = 177, normalized size = 1.00

$$\frac{1}{12}\sqrt[3]{a}\,b\log\left(a^{2/3}x^2+\sqrt[3]{a}\,x\sqrt[3]{ax^3+bx}+\left(ax^3+bx\right)^{2/3}\right)-\frac{1}{6}\sqrt[3]{a}\,b\log\left(\sqrt[3]{ax^3+bx}-\sqrt[3]{a}\,x\right)-\frac{\sqrt[3]{a}\,b\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a}\,x}{2\sqrt[3]{ax^3+bx}+\sqrt[3]{a}\,x}\right)}{2\sqrt{3}}+\frac{\sqrt[3]{ax^3+bx}\left(4ax^4-3ax^2-3b\right)}{8x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b*x + a*x^3)^(1/3)*(b + a*x^4))/x^4,x]
```

```
[Out] ((b*x + a*x^3)^(1/3)*(-3*b - 3*a*x^2 + 4*a*x^4))/(8*x^3) - (a^(1/3)*b*ArcTa
n[(Sqrt[3]*a^(1/3)*x)/(a^(1/3)*x + 2*(b*x + a*x^3)^(1/3))])/(2*Sqrt[3]) - (
a^(1/3)*b*Log[-(a^(1/3)*x) + (b*x + a*x^3)^(1/3)])/6 + (a^(1/3)*b*Log[a^(2/
3)*x^2 + a^(1/3)*x*(b*x + a*x^3)^(1/3) + (b*x + a*x^3)^(2/3)])/12
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x)^(1/3)*(a*x^4+b)/x^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x)^(1/3)*(a*x^4+b)/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^3+bx\right)^{\frac{1}{3}}\left(a\,x^4+b\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b*x)^(1/3)*(a*x^4+b)/x^4,x)
```

```
[Out] int((a*x^3+b*x)^(1/3)*(a*x^4+b)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4+b\right)\left(ax^3+bx\right)^{\frac{1}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x)^(1/3)*(a*x^4+b)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + b)*(a*x^3 + b*x)^(1/3)/x^4, x)
```

mupad [B] time = 1.85, size = 65, normalized size = 0.37

$$\frac{3\,a\,x\left(a\,x^3+b\,x\right)^{1/3}{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{2}{3};\ \frac{5}{3};\ -\frac{a\,x^2}{b}\right)}{4\left(\frac{a\,x^2}{b}+1\right)^{1/3}}-\frac{3\left(a\,x^3+b\,x\right)^{1/3}\left(a\,x^2+b\right)}{8\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x + a*x^3)^(1/3)*(b + a*x^4))/x^4,x)
```

```
[Out] (3*a*x*(b*x + a*x^3)^(1/3)*hypergeom([-1/3, 2/3], 5/3, -(a*x^2)/b))/(4*((a*
x^2)/b + 1)^(1/3)) - (3*(b*x + a*x^3)^(1/3)*(b + a*x^2))/(8*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x\left(ax^2+b\right)}\left(ax^4+b\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b*x)**(1/3)*(a*x**4+b)/x**4,x)
```

```
[Out] Integral((x*(a*x**2 + b))**(1/3)*(a*x**4 + b)/x**4, x)
```

3.1896 $\int \frac{b^4+a^4x^4}{\sqrt{-b^2x+a^2x^3}\left(-b^4+a^4x^4\right)}\,dx$

Optimal. Leaf size=177

$$\frac{\sqrt{a^2x^3-b^2x}}{b^2-a^2x^2}-\frac{\tan^{-1}\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{a^2x^3-b^2x}}{a^2x^2-2abx-b^2}\right)}{4\sqrt{a}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\frac{a^{3/2}x^2}{2\sqrt{b}}-\frac{b^{3/2}}{2\sqrt{a}}+\sqrt{a}\sqrt{b}\,x}{\sqrt{a^2x^3-b^2x}}\right)}{4\sqrt{a}\sqrt{b}}$$

Rubi [A] time = 1.63, antiderivative size = 234, normalized size of antiderivative = 1.32, number of steps used = 22, number of rules used = 13, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {2056, 6715, 6725, 224, 221, 1404, 414, 523, 409, 1211, 1699, 203, 206}

$$-\frac{x}{\sqrt{a^2x^3-b^2x}}-\frac{\sqrt{x}\sqrt{a^2x^2-b^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^2}\sqrt{b}\sqrt{x}}{\sqrt{a^2x^2-b^2}}\right)}{2\sqrt{2}\sqrt[4]{-a^2}\sqrt{b}\sqrt{a^2x^3-b^2x}}-\frac{\sqrt{x}\sqrt{a^2x^2-b^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^2}\sqrt{b}\sqrt{x}}{\sqrt{a^2x^2-b^2}}\right)}{2\sqrt{2}\sqrt[4]{-a^2}\sqrt{b}\sqrt{a^2x^3-b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b^4 + a^4*x^4)/(Sqrt[-(b^2*x) + a^2*x^3]*(-b^4 + a^4*x^4)),x]
```

```
[Out] -(x/Sqrt[-(b^2*x) + a^2*x^3]) - (Sqrt[x]*Sqrt[-b^2 + a^2*x^2]*ArcTan[(Sqrt[
2]*(-a^2)^(1/4)*Sqrt[b]*Sqrt[x])/Sqrt[-b^2 + a^2*x^2]])/(2*Sqrt[2]*(-a^2)^(
1/4)*Sqrt[b]*Sqrt[-(b^2*x) + a^2*x^3]) - (Sqrt[x]*Sqrt[-b^2 + a^2*x^2]*ArcT
anh[(Sqrt[2]*(-a^2)^(1/4)*Sqrt[b]*Sqrt[x])/Sqrt[-b^2 + a^2*x^2]])/(2*Sqrt[2
]*(-a^2)^(1/4)*Sqrt[b]*Sqrt[-(b^2*x) + a^2*x^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1404

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{b^4+a^4x^4}{\sqrt{-b^2x+a^2x^3}\left(-b^4+a^4x^4\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\int \frac{b^4+a^4x^4}{\sqrt{x}\sqrt{-b^2+a^2x^2}\left(-b^4+a^4x^4\right)}\,dx}{\sqrt{-b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{b^4+a^4x^8}{\sqrt{-b^2+a^2x^4}\left(-b^4+a^4x^8\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{-b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt{-b^2+a^2x^4}} + \frac{2b^4}{\sqrt{-b^2+a^2x^4}\left(-b^4+a^4x^8\right)}\right)dx, x, \sqrt{\right.}{\sqrt{-b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b^2+a^2x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{-b^2x+a^2x^3}} + \frac{\left(4b^4\sqrt{x}\sqrt{-b^2+a^2\right.}{}$$

$$= \frac{\left(4b^4\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{1}{\left(-b^2+a^2x^4\right)^{3/2}\left(b^2+a^2x^4\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{-b^2x+a^2x^3}} + \frac{\left(2\sqrt{x}\right.}{}$$

$$= -\frac{x}{\sqrt{-b^2x+a^2x^3}} + \frac{2\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{-b^2x+a^2x^3}} + \frac{\left(\sqrt{x}\sqrt{-b\right.}{}$$

$$= -\frac{x}{\sqrt{-b^2x+a^2x^3}} + \frac{2\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{-b^2x+a^2x^3}} - \frac{\left(\sqrt{x}\sqrt{-b\right.}{}$$

$$= -\frac{x}{\sqrt{-b^2x+a^2x^3}} + \frac{2\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{-b^2x+a^2x^3}} - \frac{\left(\sqrt{x}\sqrt{-b\right.}{}$$

$$= -\frac{x}{\sqrt{-b^2x+a^2x^3}} + \frac{\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{-b^2x+a^2x^3}} - 2\frac{\left(\sqrt{x}\sqrt{-b\right.}{}$$

$$= -\frac{x}{\sqrt{-b^2x+a^2x^3}} + \frac{\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{-b^2x+a^2x^3}} - \frac{\left(\sqrt{x}\sqrt{-b^2\right.}{}$$

$$= -\frac{x}{\sqrt{-b^2x+a^2x^3}} - \frac{\sqrt{x}\sqrt{-b^2+a^2x^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^2}\sqrt{b}\sqrt{x}}{\sqrt{-b^2+a^2x^2}}\right)}{2\sqrt{2}\sqrt[4]{-a^2}\sqrt{b}\sqrt{-b^2x+a^2x^3}} - \frac{\sqrt{x}\sqrt{-b^2+}{2\sqrt{2}\sqrt[4]{}}$$

Mathematica [C] time = 0.56, size = 73, normalized size = 0.41

$$\frac{x\left(-\sqrt{1-\frac{a^2x^2}{b^2}}\,F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{a^2x^2}{b^2},-\frac{a^2x^2}{b^2}\right)-1\right)}{\sqrt{a^2x^3-b^2x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b^4 + a^4*x^4)/(Sqrt[-(b^2*x) + a^2*x^3]*(-b^4 + a^4*x^4)),x]
```

```
[Out] (x*(-1 - Sqrt[1 - (a^2*x^2)/b^2]*AppellF1[1/4, -1/2, 1, 5/4, (a^2*x^2)/b^2,
 -((a^2*x^2)/b^2)]))/Sqrt[-(b^2*x) + a^2*x^3]
```

IntegrateAlgebraic [A] time = 0.49, size = 177, normalized size = 1.00

$$\frac{\sqrt{a^2x^3-b^2x}}{b^2-a^2x^2}-\frac{\tan^{-1}\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{a^2x^3-b^2x}}{a^2x^2-2abx-b^2}\right)}{4\sqrt{a}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\frac{a^{3/2}x^2}{2\sqrt{b}}-\frac{b^{3/2}}{2\sqrt{a}}+\sqrt{a}\sqrt{b}x}{\sqrt{a^2x^3-b^2x}}\right)}{4\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^4 + a^4*x^4)/(Sqrt[-(b^2*x) + a^2*x^3]*(-b^4 + a^4*x^
4)),x]
```

```
[Out] Sqrt[-(b^2*x) + a^2*x^3]/(b^2 - a^2*x^2) - ArcTan[(2*Sqrt[a]*Sqrt[b]*Sqrt[-
(b^2*x) + a^2*x^3])/(-b^2 - 2*a*b*x + a^2*x^2)]/(4*Sqrt[a]*Sqrt[b]) - ArcTa
nh[(-1/2*b^(3/2)/Sqrt[a] + Sqrt[a]*Sqrt[b]*x + (a^(3/2)*x^2)/(2*Sqrt[b]))/S
qrt[-(b^2*x) + a^2*x^3]]/(4*Sqrt[a]*Sqrt[b])
```

fricas [B] time = 0.64, size = 1141, normalized size = 6.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4+b^4)/(a^2*x^3-b^2*x)^(1/2)/(a^4*x^4-b^4),x, algorithm="f
ricas")
```

```
[Out] -1/16*(4*sqrt(2)*(1/4)^(1/4)*(a^2*x^2 - b^2)*(1/(a^2*b^2))^(1/4)*arctan(1/2
*((4*sqrt(2)*(1/4)^(3/4)*a^2*b^2*x*(1/(a^2*b^2))^(3/4) - sqrt(2)*(1/4)^(1/4
)*(a^2*x^2 - b^2)*(1/(a^2*b^2))^(1/4))*sqrt(a^2*x^3 - b^2*x) - (2*a^2*x^3 -
 2*b^2*x - (4*sqrt(2)*(1/4)^(3/4)*a^2*b^2*x*(1/(a^2*b^2))^(3/4) + sqrt(2)*(
1/4)^(1/4)*(a^2*x^2 - b^2)*(1/(a^2*b^2))^(1/4))*sqrt(a^2*x^3 - b^2*x))*sqrt
((a^4*x^4 + 2*a^2*b^2*x^2 + b^4 + 8*(sqrt(2)*(1/4)^(1/4)*a^2*b^2*x*(1/(a^2*
b^2))^(1/4) + sqrt(2)*(1/4)^(3/4)*(a^4*b^2*x^2 - a^2*b^4)*(1/(a^2*b^2))^(3/
4))*sqrt(a^2*x^3 - b^2*x) + 8*(a^4*b^2*x^3 - a^2*b^4*x)*sqrt(1/(a^2*b^2)))/
(a^4*x^4 + 2*a^2*b^2*x^2 + b^4)))/(a^2*x^3 - b^2*x)) + 4*sqrt(2)*(1/4)^(1/4
)*(a^2*x^2 - b^2)*(1/(a^2*b^2))^(1/4)*arctan(1/2*((4*sqrt(2)*(1/4)^(3/4)*a^
2*b^2*x*(1/(a^2*b^2))^(3/4) - sqrt(2)*(1/4)^(1/4)*(a^2*x^2 - b^2)*(1/(a^2*b
^2))^(1/4))*sqrt(a^2*x^3 - b^2*x) + (2*a^2*x^3 - 2*b^2*x + (4*sqrt(2)*(1/4)
^(3/4)*a^2*b^2*x*(1/(a^2*b^2))^(3/4) + sqrt(2)*(1/4)^(1/4)*(a^2*x^2 - b^2)*
(1/(a^2*b^2))^(1/4))*sqrt(a^2*x^3 - b^2*x))*sqrt((a^4*x^4 + 2*a^2*b^2*x^2 +
 b^4 - 8*(sqrt(2)*(1/4)^(1/4)*a^2*b^2*x*(1/(a^2*b^2))^(1/4) + sqrt(2)*(1/4)
^(3/4)*(a^4*b^2*x^2 - a^2*b^4)*(1/(a^2*b^2))^(3/4))*sqrt(a^2*x^3 - b^2*x) +
 8*(a^4*b^2*x^3 - a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 + 2*a^2*b^2*x^2 +
b^4)))/(a^2*x^3 - b^2*x)) + sqrt(2)*(1/4)^(1/4)*(a^2*x^2 - b^2)*(1/(a^2*b^2
))^(1/4)*log((a^4*x^4 + 2*a^2*b^2*x^2 + b^4 + 8*(sqrt(2)*(1/4)^(1/4)*a^2*b^
2*x*(1/(a^2*b^2))^(1/4) + sqrt(2)*(1/4)^(3/4)*(a^4*b^2*x^2 - a^2*b^4)*(1/(a
^2*b^2))^(3/4))*sqrt(a^2*x^3 - b^2*x) + 8*(a^4*b^2*x^3 - a^2*b^4*x)*sqrt(1/
(a^2*b^2)))/(a^4*x^4 + 2*a^2*b^2*x^2 + b^4)) - sqrt(2)*(1/4)^(1/4)*(a^2*x^2
 - b^2)*(1/(a^2*b^2))^(1/4)*log((a^4*x^4 + 2*a^2*b^2*x^2 + b^4 - 8*(sqrt(2)
*(1/4)^(1/4)*a^2*b^2*x*(1/(a^2*b^2))^(1/4) + sqrt(2)*(1/4)^(3/4)*(a^4*b^2*x
^2 - a^2*b^4)*(1/(a^2*b^2))^(3/4))*sqrt(a^2*x^3 - b^2*x) + 8*(a^4*b^2*x^3 -
 a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 + 2*a^2*b^2*x^2 + b^4)) + 16*sqrt(a
^2*x^3 - b^2*x))/(a^2*x^2 - b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^4x^4+b^4}{\left(a^4x^4-b^4\right)\sqrt{a^2x^3-b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4+b^4)/(a^2*x^3-b^2*x)^(1/2)/(a^4*x^4-b^4),x, algorithm="g
iac")
```

```
[Out] integrate((a^4*x^4 + b^4)/((a^4*x^4 - b^4)*sqrt(a^2*x^3 - b^2*x)), x)
```

maple [C] time = 0.10, size = 787, normalized size = 4.45

$$\frac{b\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{2\left(x-\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{ax}{b}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a^2x^3-b^2x}}+\frac{b\left(-\frac{a^2x^2+abx}{b^2a\sqrt{\left(x-\frac{b}{a}\right)\left(a^2x^2+abx\right)}}-\frac{\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{2\left(x-\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{ax}{b}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}},\frac{\sqrt{2}}{2}\right)}{2a\sqrt{a^2x^3-b^2x}}+\frac{\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{2\left(x-\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{ax}{b}}\left(-\frac{2b\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}},\frac{\sqrt{2}}{2}\right)}{a}+\frac{b\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}},\frac{\sqrt{2}}{2}\right)}{a}\right)}{2b\sqrt{a^2x^3-b^2x}}\right)}{2}-b^2\left(-\frac{i\sqrt{1+\frac{ax}{b}}\,\sqrt{-\frac{2ax}{b}+2}\,\sqrt{-\frac{ax}{b}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}},-\frac{b}{a\left(-\frac{ib}{a}-\frac{b}{a}\right)},\frac{\sqrt{2}}{2}\right)}{2a^2\sqrt{a^2x^3-b^2x}\left(-\frac{ib}{a}-\frac{b}{a}\right)}+\frac{i\sqrt{1+\frac{ax}{b}}\,\sqrt{-\frac{2ax}{b}+2}\,\sqrt{-\frac{ax}{b}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}},-\frac{b}{a\left(-\frac{b}{a}+\frac{ib}{a}\right)},\frac{\sqrt{2}}{2}\right)}{2a^2\sqrt{a^2x^3-b^2x}\left(-\frac{b}{a}+\frac{ib}{a}\right)}\right)-\frac{b\left(-\frac{a^2x^2-abx}{b^2a\sqrt{\left(x+\frac{b}{a}\right)\left(a^2x^2-abx\right)}}+\frac{\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{2\left(x-\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{ax}{b}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}},\frac{\sqrt{2}}{2}\right)}{2a\sqrt{a^2x^3-b^2x}}+\frac{\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{2\left(x-\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{ax}{b}}\left(-\frac{2b\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}},\frac{\sqrt{2}}{2}\right)}{a}+\frac{b\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}},\frac{\sqrt{2}}{2}\right)}{a}\right)}{2b\sqrt{a^2x^3-b^2x}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^4*x^4+b^4)/(a^2*x^3-b^2*x)^(1/2)/(a^4*x^4-b^4),x)
```

```
[Out] b/a*((x+b/a)/b*a)^(1/2)*(-2*(x-b/a)/b*a)^(1/2)*(-a*x/b)^(1/2)/(a^2*x^3-b^2*
x)^(1/2)*EllipticF(((x+b/a)/b*a)^(1/2),1/2*2^(1/2))+1/2*b*(-(a^2*x^2+a*b*x)
/b^2/a/((x-b/a)*(a^2*x^2+a*b*x))^(1/2)-1/2/a*((x+b/a)/b*a)^(1/2)*(-2*(x-b/a
)/b*a)^(1/2)*(-a*x/b)^(1/2)/(a^2*x^3-b^2*x)^(1/2)*EllipticF(((x+b/a)/b*a)^(
1/2),1/2*2^(1/2))+1/2/b*((x+b/a)/b*a)^(1/2)*(-2*(x-b/a)/b*a)^(1/2)*(-a*x/b)
^(1/2)/(a^2*x^3-b^2*x)^(1/2)*(-2*b/a*EllipticE(((x+b/a)/b*a)^(1/2),1/2*2^(1
/2))+b/a*EllipticF(((x+b/a)/b*a)^(1/2),1/2*2^(1/2))))-b^2*(-1/2*I/a^2*(1+a*
x/b)^(1/2)*(-2*a*x/b+2)^(1/2)*(-a*x/b)^(1/2)/(a^2*x^3-b^2*x)^(1/2)/(-I*b/a-
b/a)*EllipticPi(((x+b/a)/b*a)^(1/2),-b/a/(-I*b/a-b/a),1/2*2^(1/2))+1/2*I/a^
2*(1+a*x/b)^(1/2)*(-2*a*x/b+2)^(1/2)*(-a*x/b)^(1/2)/(a^2*x^3-b^2*x)^(1/2)/(
-b/a+I*b/a)*EllipticPi(((x+b/a)/b*a)^(1/2),-b/a/(-b/a+I*b/a),1/2*2^(1/2)))-
1/2*b*(-(a^2*x^2-a*b*x)/b^2/a/((x+b/a)*(a^2*x^2-a*b*x))^(1/2)+1/2/a*((x+b/a
)/b*a)^(1/2)*(-2*(x-b/a)/b*a)^(1/2)*(-a*x/b)^(1/2)/(a^2*x^3-b^2*x)^(1/2)*El
lipticF(((x+b/a)/b*a)^(1/2),1/2*2^(1/2))+1/2/b*((x+b/a)/b*a)^(1/2)*(-2*(x-b
/a)/b*a)^(1/2)*(-a*x/b)^(1/2)/(a^2*x^3-b^2*x)^(1/2)*(-2*b/a*EllipticE(((x+b
/a)/b*a)^(1/2),1/2*2^(1/2))+b/a*EllipticF(((x+b/a)/b*a)^(1/2),1/2*2^(1/2)))
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^4x^4 + b^4}{\left(a^4x^4 - b^4\right)\sqrt{a^2x^3 - b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4+b^4)/(a^2*x^3-b^2*x)^(1/2)/(a^4*x^4-b^4),x, algorithm="m
axima")
```

```
[Out] integrate((a^4*x^4 + b^4)/((a^4*x^4 - b^4)*sqrt(a^2*x^3 - b^2*x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^4 + a^4*x^4)/((b^4 - a^4*x^4)*(a^2*x^3 - b^2*x)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^4x^4 + b^4}{\sqrt{x\left(ax - b\right)\left(ax + b\right)}\,\left(ax - b\right)\left(ax + b\right)\left(a^2x^2 + b^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**4*x**4+b**4)/(a**2*x**3-b**2*x)**(1/2)/(a**4*x**4-b**4),x)
```

```
[Out] Integral((a**4*x**4 + b**4)/(sqrt(x*(a*x - b)*(a*x + b))*(a*x - b)*(a*x + b
)*(a**2*x**2 + b**2)), x)
```

3.1897 $\int \frac{cx^6(-4b+ax^5)}{(b+ax^5)^{3/4}(b^2+2abx^5-c^2x^8+a^2x^{10})} dx$

Optimal. Leaf size=177

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^5+b}}\right)}{c^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^5+b}}\right)}{c^{3/4}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{ax^5+b}}{\sqrt{2}\sqrt[4]{c}}-\frac{\sqrt[4]{c}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5+b}}\right)}{\sqrt{2}\,c^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x\sqrt[4]{ax^5+b}}{\sqrt{ax^5+b}+\sqrt{c}\,x^2}\right)}{\sqrt{2}\,c^{3/4}}$$

Rubi [F] time = 4.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{cx^6\left(-4b+ax^5\right)}{\left(b+ax^5\right)^{3/4}\left(b^2+2abx^5-c^2x^8+a^2x^{10}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c*x^6*(-4*b + a*x^5))/((b + a*x^5)^(3/4)*(b^2 + 2*a*b*x^5 - c^2*x^8 +
a^2*x^10)),x]
```

```
[Out] (c*x^2*(1 + (a*x^5)/b)^(3/4)*Hypergeometric2F1[2/5, 3/4, 7/5, -((a*x^5)/b)]
)/(2*a*(b + a*x^5)^(3/4)) - (c^3*Defer[Int][x^4/((-b + c*x^4 - a*x^5)*(b +
a*x^5)^(3/4)), x])/(2*a^2) - (b*c^2*Defer[Int][1/((b + a*x^5)^(3/4)*(b - c*
x^4 + a*x^5)), x])/(2*a^2) - (b*c*Defer[Int][x/((b + a*x^5)^(3/4)*(b - c*x^
4 + a*x^5)), x])/(2*a) - (5*b*Defer[Int][x^2/((b + a*x^5)^(3/4)*(b - c*x^4
+ a*x^5)), x])/2 + (b*c^2*Defer[Int][1/((b + a*x^5)^(3/4)*(b + c*x^4 + a*x^
5)), x])/(2*a^2) - (b*c*Defer[Int][x/((b + a*x^5)^(3/4)*(b + c*x^4 + a*x^5)
), x])/(2*a) + (5*b*Defer[Int][x^2/((b + a*x^5)^(3/4)*(b + c*x^4 + a*x^5)),
 x])/2 + (c^3*Defer[Int][x^4/((b + a*x^5)^(3/4)*(b + c*x^4 + a*x^5)), x])/(
2*a^2)
```

Rubi steps

$$\int \frac{cx^6\left(-4b+ax^5\right)}{\left(b+ax^5\right)^{3/4}\left(b^2+2abx^5-c^2x^8+a^2x^{10}\right)}dx = c\int \frac{x^6\left(-4b+ax^5\right)}{\left(b+ax^5\right)^{3/4}\left(b^2+2abx^5-c^2x^8+a^2x^{10}\right)}dx$$

$$= c\int \left(\frac{x}{a\left(b+ax^5\right)^{3/4}} + \frac{x\left(-b^2-6abx^5+c^2x^8\right)}{a\left(b+ax^5\right)^{3/4}\left(b^2+2abx^5-c^2x^8+a^2x^{10}\right)}\right.$$

$$= \frac{c\int \frac{x}{\left(b+ax^5\right)^{3/4}}dx}{a} + \frac{c\int \frac{x\left(-b^2-6abx^5+c^2x^8\right)}{\left(b+ax^5\right)^{3/4}\left(b^2+2abx^5-c^2x^8+a^2x^{10}\right)}dx}{a}$$

$$= \frac{c\int \left(\frac{-bc^2-abcx-5a^2bx^2+c^3x^4}{2ac\left(b+ax^5\right)^{3/4}\left(b-cx^4+ax^5\right)} + \frac{bc^2-abcx+5a^2bx^2+c^3x^4}{2ac\left(b+ax^5\right)^{3/4}\left(b+cx^4+ax^5\right)}\right)dx}{a} + \frac{\left(c\left(1+\right.\right.}{}$$

$$= \frac{cx^2\left(1+\frac{ax^5}{b}\right)^{3/4} {}_2F_1\left(\frac{2}{5},\frac{3}{4};\frac{7}{5};-\frac{ax^5}{b}\right)}{2a\left(b+ax^5\right)^{3/4}} + \frac{\int \frac{-bc^2-abcx-5a^2bx^2+c^3x^4}{\left(b+ax^5\right)^{3/4}\left(b-cx^4+ax^5\right)}dx}{2a^2} -$$

$$= \frac{cx^2\left(1+\frac{ax^5}{b}\right)^{3/4} {}_2F_1\left(\frac{2}{5},\frac{3}{4};\frac{7}{5};-\frac{ax^5}{b}\right)}{2a\left(b+ax^5\right)^{3/4}} + \frac{\int\left(-\frac{c^3x^4}{\left(-b+cx^4-ax^5\right)\left(b+ax^5\right)^{3/4}} -\right.}{}$$

$$= \frac{cx^2\left(1+\frac{ax^5}{b}\right)^{3/4} {}_2F_1\left(\frac{2}{5},\frac{3}{4};\frac{7}{5};-\frac{ax^5}{b}\right)}{2a\left(b+ax^5\right)^{3/4}} - \frac{1}{2}(5b)\int \frac{x^2}{\left(b+ax^5\right)^{3/4}\left(b-\right.}$$

Mathematica [F] time = 1.21, size = 56, normalized size = 0.32

$$c\int \frac{x^6\left(ax^5-4b\right)}{\left(ax^5+b\right)^{3/4}\left(a^2x^{10}+2abx^5+b^2-c^2x^8\right)}dx$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^6*(-4*b + a*x^5))/((b + a*x^5)^(3/4)*(b^2 + 2*a*b*x^5 - c^2*
x^8 + a^2*x^10)),x]
```

```
[Out] c*Integrate[(x^6*(-4*b + a*x^5))/((b + a*x^5)^(3/4)*(b^2 + 2*a*b*x^5 - c^2*
x^8 + a^2*x^10)), x]
```

IntegrateAlgebraic [A] time = 14.27, size = 176, normalized size = 0.99

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^5+b}}\right)}{c^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{ax^5+b}}\right)}{c^{3/4}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x\sqrt[4]{ax^5+b}}{\sqrt{ax^5+b}-\sqrt{c}\,x^2}\right)}{\sqrt{2}\,c^{3/4}} - \frac{\tanh^{-1}\left(\frac{\frac{\sqrt{ax^5+b}}{\sqrt{2}\sqrt[4]{c}}+\frac{\sqrt[4]{c}\,x^2}{\sqrt{2}}}{x\sqrt[4]{ax^5+b}}\right)}{\sqrt{2}\,c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^6*(-4*b + a*x^5))/((b + a*x^5)^(3/4)*(b^2 + 2*a*b*x
^5 - c^2*x^8 + a^2*x^10)),x]
```

```
[Out] ArcTan[(c^(1/4)*x)/(b + a*x^5)^(1/4)]/c^(3/4) + ArcTan[(Sqrt[2]*c^(1/4)*x*(
b + a*x^5)^(1/4))/(-(Sqrt[c]*x^2) + Sqrt[b + a*x^5])]/(Sqrt[2]*c^(3/4)) - A
rcTanh[(c^(1/4)*x)/(b + a*x^5)^(1/4)]/c^(3/4) - ArcTanh[((c^(1/4)*x^2)/Sqrt
```

```
[2] + Sqrt[b + a*x^5]/(Sqrt[2]*c^(1/4)))/(x*(b + a*x^5)^(1/4))]/(Sqrt[2]*c^
(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^6*(a*x^5-4*b)/(a*x^5+b)^(3/4)/(a^2*x^10-c^2*x^8+2*a*b*x^5+b^2
),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^5 - 4\,b)cx^6}{(a^2x^{10} - c^2x^8 + 2\,abx^5 + b^2)(ax^5 + b)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^6*(a*x^5-4*b)/(a*x^5+b)^(3/4)/(a^2*x^10-c^2*x^8+2*a*b*x^5+b^2
),x, algorithm="giac")
```

```
[Out] integrate((a*x^5 - 4*b)*c*x^6/((a^2*x^10 - c^2*x^8 + 2*a*b*x^5 + b^2)*(a*x^
5 + b)^(3/4)), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{c\,x^6\,(a\,x^5 - 4b)}{(a\,x^5 + b)^{\frac{3}{4}}\,(a^2x^{10} - c^2x^8 + 2ab\,x^5 + b^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^6*(a*x^5-4*b)/(a*x^5+b)^(3/4)/(a^2*x^10-c^2*x^8+2*a*b*x^5+b^2),x)
```

```
[Out] int(c*x^6*(a*x^5-4*b)/(a*x^5+b)^(3/4)/(a^2*x^10-c^2*x^8+2*a*b*x^5+b^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c\int \frac{(ax^5 - 4\,b)x^6}{(a^2x^{10} - c^2x^8 + 2\,abx^5 + b^2)(ax^5 + b)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^6*(a*x^5-4*b)/(a*x^5+b)^(3/4)/(a^2*x^10-c^2*x^8+2*a*b*x^5+b^2
),x, algorithm="maxima")
```

```
[Out] c*integrate((a*x^5 - 4*b)*x^6/((a^2*x^10 - c^2*x^8 + 2*a*b*x^5 + b^2)*(a*x^
5 + b)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{c\,x^6\,(4\,b - a\,x^5)}{(a\,x^5 + b)^{3/4}\,(a^2\,x^{10} + 2\,a\,b\,x^5 + b^2 - c^2\,x^8)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(c*x^6*(4*b - a*x^5))/((b + a*x^5)^(3/4)*(b^2 + a^2*x^10 - c^2*x^8 + 2
*a*b*x^5)),x)
```

```
[Out] -int((c*x^6*(4*b - a*x^5))/((b + a*x^5)^(3/4)*(b^2 + a^2*x^10 - c^2*x^8 + 2
*a*b*x^5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c\left(\int \frac{ax^{11}}{a^2x^{10}\left(ax^5+b\right)^{\frac{3}{4}}+2abx^5\left(ax^5+b\right)^{\frac{3}{4}}+b^2\left(ax^5+b\right)^{\frac{3}{4}}-c^2x^8\left(ax^5+b\right)^{\frac{3}{4}}}\,dx+\int \left(-\frac{4bx^6}{a^2x^{10}\left(ax^5+b\right)^{\frac{3}{4}}+2abx^5\left(ax^5+b\right)^{\frac{3}{4}}+b^2\left(ax^5+b\right)^{\frac{3}{4}}-c^2x^8\left(ax^5+b\right)^{\frac{3}{4}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**6*(a*x**5-4*b)/(a*x**5+b)**(3/4)/(a**2*x**10-c**2*x**8+2*a*b
*x**5+b**2),x)
```

```
[Out] c*(Integral(a*x**11/(a**2*x**10*(a*x**5 + b)**(3/4) + 2*a*b*x**5*(a*x**5 +
b)**(3/4) + b**2*(a*x**5 + b)**(3/4) - c**2*x**8*(a*x**5 + b)**(3/4)), x) +
 Integral(-4*b*x**6/(a**2*x**10*(a*x**5 + b)**(3/4) + 2*a*b*x**5*(a*x**5 +
b)**(3/4) + b**2*(a*x**5 + b)**(3/4) - c**2*x**8*(a*x**5 + b)**(3/4)), x))
```

3.1898 $\int \frac{x^2\sqrt{ax+\sqrt{-b+ax}}}{\sqrt{-b+ax}}\,dx$

Optimal. Leaf size=177

$$\frac{\left(-320b^3-16b^2-60b+21\right)\log\left(2\sqrt{ax-b}-2\sqrt{\sqrt{ax-b}+ax}+1\right)}{512a^3}+\frac{\sqrt{\sqrt{ax-b}+ax}\left(128a^2x^2-160abx+1\right.}{3840a^3}$$

Rubi [A] time = 0.90, antiderivative size = 255, normalized size of antiderivative = 1.44, number of steps used = 8, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1661, 640, 612, 621, 206}

$$\frac{\left(80b^2+24b+21\right)\left(2\sqrt{ax-b}+1\right)\sqrt{\sqrt{ax-b}+ax}}{256a^3}-\frac{(1-4b)\left(80b^2+24b+21\right)\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{2\sqrt{\sqrt{ax-b}+ax}}\right)}{512a^3}+\frac{(ax-b)^{3/2}\left(\sqrt{ax-b}+ax\right)^{3/2}}{3a^3}-\frac{(68b+35)\left(\sqrt{ax-b}+ax\right)^{3/2}}{160a^3}+\frac{3(b-ax)\left(\sqrt{ax-b}+ax\right)^{3/2}}{10a^3}+\frac{3(20b+7)\sqrt{ax-b}\left(\sqrt{ax-b}+ax\right)^{3/2}}{80a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a*x + Sqrt[-b + a*x]])/Sqrt[-b + a*x],x]
```

```
[Out] -1/160*((35 + 68*b)*(a*x + Sqrt[-b + a*x])^(3/2))/a^3 + (3*(b - a*x)*(a*x +
 Sqrt[-b + a*x])^(3/2))/(10*a^3) + (3*(7 + 20*b)*Sqrt[-b + a*x]*(a*x + Sqrt
[-b + a*x])^(3/2))/(80*a^3) + ((-b + a*x)^(3/2)*(a*x + Sqrt[-b + a*x])^(3/2
))/(3*a^3) + ((21 + 24*b + 80*b^2)*Sqrt[a*x + Sqrt[-b + a*x]]*(1 + 2*Sqrt[-
b + a*x]))/(256*a^3) - ((1 - 4*b)*(21 + 24*b + 80*b^2)*ArcTanh[(1 + 2*Sqrt[
-b + a*x])/(2*Sqrt[a*x + Sqrt[-b + a*x]])])/(512*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
```

```
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2\sqrt{ax+\sqrt{-b+ax}}}{\sqrt{-b+ax}}\,dx = \frac{2\,\text{Subst}\left(\int \left(b+x^2\right)^2\sqrt{b+x+x^2}\,dx, x, \sqrt{-b+ax}\right)}{a^3}$$

$$= \frac{(-b+ax)^{3/2}\left(ax+\sqrt{-b+ax}\right)^{3/2}}{3a^3} + \frac{\text{Subst}\left(\int \sqrt{b+x+x^2}\left(6b^2+9bx^2-\frac{9x^3}{2}\right)dx, x, \right.}{3a^3}$$

$$= \frac{3(b-ax)\left(ax+\sqrt{-b+ax}\right)^{3/2}}{10a^3} + \frac{(-b+ax)^{3/2}\left(ax+\sqrt{-b+ax}\right)^{3/2}}{3a^3} + \frac{\text{Subst}\left(\int \sqrt{b+\right.}{}$$

$$= \frac{3(b-ax)\left(ax+\sqrt{-b+ax}\right)^{3/2}}{10a^3} + \frac{3(7+20b)\sqrt{-b+ax}\left(ax+\sqrt{-b+ax}\right)^{3/2}}{80a^3} + \frac{(-b+}{}$$

$$= -\frac{(35+68b)\left(ax+\sqrt{-b+ax}\right)^{3/2}}{160a^3} + \frac{3(b-ax)\left(ax+\sqrt{-b+ax}\right)^{3/2}}{10a^3} + \frac{3(7+20b)\sqrt{-b}}{}$$

$$= -\frac{(35+68b)\left(ax+\sqrt{-b+ax}\right)^{3/2}}{160a^3} + \frac{3(b-ax)\left(ax+\sqrt{-b+ax}\right)^{3/2}}{10a^3} + \frac{3(7+20b)\sqrt{-b}}{}$$

$$= -\frac{(35+68b)\left(ax+\sqrt{-b+ax}\right)^{3/2}}{160a^3} + \frac{3(b-ax)\left(ax+\sqrt{-b+ax}\right)^{3/2}}{10a^3} + \frac{3(7+20b)\sqrt{-b}}{}$$

$$= -\frac{(35+68b)\left(ax+\sqrt{-b+ax}\right)^{3/2}}{160a^3} + \frac{3(b-ax)\left(ax+\sqrt{-b+ax}\right)^{3/2}}{10a^3} + \frac{3(7+20b)\sqrt{-b}}{}$$

Mathematica [A] time = 0.37, size = 199, normalized size = 1.12

$$\frac{2\sqrt{\sqrt{ax-b}+ax}\left(128a^2x^2\left(10\sqrt{ax-b}+1\right)+400b^2\left(6\sqrt{ax-b}-1\right)+8b\left(20ax\left(10\sqrt{ax-b}-1\right)+30\sqrt{ax-b}-81\right)-24ax\left(6\sqrt{ax-b}-7\right)-210\sqrt{ax-b}+315\right)+15\left(320b^3+16b^2+60b-21\right)\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{2\sqrt{\sqrt{ax-b}+ax}}\right)}{7680a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a*x + Sqrt[-b + a*x]])/Sqrt[-b + a*x],x]
```

```
[Out] (2*Sqrt[a*x + Sqrt[-b + a*x]]*(315 - 210*Sqrt[-b + a*x] - 24*a*x*(-7 + 6*Sq
rt[-b + a*x]) + 400*b^2*(-1 + 6*Sqrt[-b + a*x]) + 128*a^2*x^2*(1 + 10*Sqrt[
-b + a*x]) + 8*b*(-81 + 30*Sqrt[-b + a*x] + 20*a*x*(-1 + 10*Sqrt[-b + a*x])
)) + 15*(-21 + 60*b + 16*b^2 + 320*b^3)*ArcTanh[(1 + 2*Sqrt[-b + a*x])/(2*S
qrt[a*x + Sqrt[-b + a*x]])]))/(7680*a^3)
```

IntegrateAlgebraic [A] time = 0.53, size = 209, normalized size = 1.18

$$\frac{\sqrt{\sqrt{ax-b}+ax}\left(5280b^2\sqrt{ax-b}+1280(ax-b)^{5/2}+128(ax-b)^2+4160b(ax-b)^{3/2}-144(ax-b)^{3/2}+96b(ax-b)+168(ax-b)+96b\sqrt{ax-b}-210\sqrt{ax-b}-432b^2-480b+315\right)}{3840a^3}+\frac{\left(-320b^3-16b^2-60b+21\right)\log\left(-2\sqrt{ax-b}+2\sqrt{\sqrt{ax-b}+ax}-1\right)}{512a^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[a*x + Sqrt[-b + a*x]])/Sqrt[-b + a*x],x]
```

```
[Out] (Sqrt[a*x + Sqrt[-b + a*x]]*(315 - 480*b - 432*b^2 - 210*Sqrt[-b + a*x] + 9
6*b*Sqrt[-b + a*x] + 5280*b^2*Sqrt[-b + a*x] + 168*(-b + a*x) + 96*b*(-b +
a*x) - 144*(-b + a*x)^(3/2) + 4160*b*(-b + a*x)^(3/2) + 128*(-b + a*x)^2 +
1280*(-b + a*x)^(5/2)))/(3840*a^3) + ((21 - 60*b - 16*b^2 - 320*b^3)*Log[-1
 - 2*Sqrt[-b + a*x] + 2*Sqrt[a*x + Sqrt[-b + a*x]]])/(512*a^3)
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x+(a*x-b)^(1/2))^(1/2)/(a*x-b)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [**A**] time = 2.09, size = 224, normalized size = 1.27

$$\frac{2\sqrt{ax+\sqrt{ax-b}}\left(2\sqrt{ax-b}\left(4\sqrt{ax-b}\left(2\sqrt{ax-b}\left(8\sqrt{ax-b}\left(\frac{10\sqrt{ax-b}}{a^2}+\frac{1}{a^2}\right)+\frac{260a^{12}b-9a^{12}}{a^{14}}\right)+\frac{3\left(4a^{12}b+7a^{12}\right)}{a^{14}}\right)+\frac{3\left(880a^{12}b^2+16a^{12}b-35a^{12}\right)}{a^{14}}\right)-\frac{3\left(144a^{12}b^2+160a^{12}b-105a^{12}\right)}{a^{14}}\right)-\frac{15\left(320b^3+16b^2+60b-21\right)\log\left(\left|-2\sqrt{ax-b}+2\sqrt{ax+\sqrt{ax-b}}-1\right|\right)}{a^2}}{7680a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x+(a*x-b)^(1/2))^(1/2)/(a*x-b)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7680*(2*sqrt(a*x + sqrt(a*x - b))*(2*sqrt(a*x - b)*(4*sqrt(a*x - b)*(2*sq
rt(a*x - b)*(8*sqrt(a*x - b)*(10*sqrt(a*x - b)/a^2 + 1/a^2) + (260*a^12*b -
 9*a^12)/a^14) + 3*(4*a^12*b + 7*a^12)/a^14) + 3*(880*a^12*b^2 + 16*a^12*b
- 35*a^12)/a^14) - 3*(144*a^12*b^2 + 160*a^12*b - 105*a^12)/a^14) - 15*(320
*b^3 + 16*b^2 + 60*b - 21)*log(abs(-2*sqrt(a*x - b) + 2*sqrt(a*x + sqrt(a*x
 - b)) - 1))/a^2)/a
```

maple [**B**] time = 0.01, size = 442, normalized size = 2.50

$$\frac{5b^2\sqrt{ax+\sqrt{ax-b}}}{16a^3}+\frac{5\ln\left(\frac{1}{2}+\sqrt{ax-b}+\sqrt{ax+\sqrt{ax-b}}\right)b^3}{8a^3}-\frac{b\left(ax+\sqrt{ax-b}\right)^{\frac{3}{2}}}{8a^3}+\frac{3b\sqrt{ax+\sqrt{ax-b}}}{32a^3}+\frac{\ln\left(\frac{1}{2}+\sqrt{ax-b}+\sqrt{ax+\sqrt{ax-b}}\right)b^2}{32a^3}+\frac{15\ln\left(\frac{1}{2}+\sqrt{ax-b}+\sqrt{ax+\sqrt{ax-b}}\right)b}{128a^3}+\frac{21\sqrt{ax+\sqrt{ax-b}}\,\sqrt{ax-b}}{128a^3}+\frac{21\sqrt{ax+\sqrt{ax-b}}}{256a^3}+\frac{(ax-b)^{\frac{3}{2}}\left(ax+\sqrt{ax-b}\right)^{\frac{3}{2}}}{3a^3}-\frac{3\left(ax+\sqrt{ax-b}\right)^{\frac{3}{2}}x}{10a^2}+\frac{21\sqrt{ax-b}\,\left(ax+\sqrt{ax-b}\right)^{\frac{3}{2}}}{80a^3}+\frac{3b\sqrt{ax+\sqrt{ax-b}}\,\sqrt{ax-b}}{16a^3}+\frac{5b^2\sqrt{ax+\sqrt{ax-b}}\,\sqrt{ax-b}}{8a^3}+\frac{3b\sqrt{ax-b}\,\left(ax+\sqrt{ax-b}\right)^{\frac{3}{2}}}{4a^3}-\frac{7\left(ax+\sqrt{ax-b}\right)^{\frac{3}{2}}}{32a^3}-\frac{21\ln\left(\frac{1}{2}+\sqrt{ax-b}+\sqrt{ax+\sqrt{ax-b}}\right)}{512a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x+(a*x-b)^(1/2))^(1/2)/(a*x-b)^(1/2),x)
```

```
[Out] 5/16/a^3*b^2*(a*x+(a*x-b)^(1/2))^(1/2)+5/8/a^3*ln(1/2+(a*x-b)^(1/2)+(a*x+(a
*x-b)^(1/2))^(1/2))*b^3-1/8/a^3*b*(a*x+(a*x-b)^(1/2))^(3/2)+3/32/a^3*b*(a*x
+(a*x-b)^(1/2))^(1/2)+1/32/a^3*ln(1/2+(a*x-b)^(1/2)+(a*x+(a*x-b)^(1/2))^(1/
2))*b^2+15/128/a^3*ln(1/2+(a*x-b)^(1/2)+(a*x+(a*x-b)^(1/2))^(1/2))*b+21/128
/a^3*(a*x+(a*x-b)^(1/2))^(1/2)*(a*x-b)^(1/2)+21/256/a^3*(a*x+(a*x-b)^(1/2))
^(1/2)+1/3/a^3*(a*x-b)^(3/2)*(a*x+(a*x-b)^(1/2))^(3/2)-3/10/a^2*(a*x+(a*x-b
)^(1/2))^(3/2)*x+21/80/a^3*(a*x-b)^(1/2)*(a*x+(a*x-b)^(1/2))^(3/2)+3/16/a^3
*b*(a*x+(a*x-b)^(1/2))^(1/2)*(a*x-b)^(1/2)+5/8/a^3*b^2*(a*x+(a*x-b)^(1/2))^
(1/2)*(a*x-b)^(1/2)+3/4/a^3*b*(a*x-b)^(1/2)*(a*x+(a*x-b)^(1/2))^(3/2)-7/32/
a^3*(a*x+(a*x-b)^(1/2))^(3/2)-21/512/a^3*ln(1/2+(a*x-b)^(1/2)+(a*x+(a*x-b)^
(1/2))^(1/2))
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+\sqrt{ax-b}}\,x^2}{\sqrt{ax-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x+(a*x-b)^(1/2))^(1/2)/(a*x-b)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*x + sqrt(a*x - b))*x^2/sqrt(a*x - b), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\sqrt{a\,x+\sqrt{a\,x-b}}}{\sqrt{a\,x-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + (a*x - b)^(1/2))^(1/2))/(a*x - b)^(1/2),x)
[Out] int((x^2*(a*x + (a*x - b)^(1/2))^(1/2))/(a*x - b)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{ax + \sqrt{ax - b}}}{\sqrt{ax - b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x+(a*x-b)**(1/2))**(1/2)/(a*x-b)**(1/2),x)
[Out] Integral(x**2*sqrt(a*x + sqrt(a*x - b))/sqrt(a*x - b), x)
```

3.1899 $\int \frac{a+bx^2+ak^4x^4}{\sqrt{(1-x)x(1-k^2x)}\,(-1+k^4x^4)}\,dx$

Optimal. Leaf size=178

$$\frac{(-2ak^2-b)\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^3+(-k^2-1)x^2+x}}\right)}{4(k-1)k^2}+\frac{(-2ak^2-b)\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^3+(-k^2-1)x^2+x}}\right)}{4k^2(k+1)}+\frac{(b-2ak^2)\tan^{-1}\left(\frac{\sqrt{k^2+1}\sqrt{k^2x^3+\ldots}}{(x-1)(k\ldots}\right)}{2k^2\sqrt{k^2+1}}$$

Rubi [C] time = 7.96, antiderivative size = 495, normalized size of antiderivative = 2.78, number of steps used = 28, number of rules used = 8, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6718, 6725, 115, 6688, 934, 12, 168, 537}

$$-\frac{(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\,(2ak^2+b)\,\Pi\left(-\frac{1}{k};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{2(-k^2)^{3/2}\sqrt{x-x^2}\sqrt{(1-x)x(1-k^2x)}}-\frac{(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\,(2ak^2+b)\,\Pi\left(\frac{1}{k};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{2(-k^2)^{3/2}\sqrt{x-x^2}\sqrt{(1-x)x(1-k^2x)}}+\frac{(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\,(b-2ak^2)\,\Pi\left(-\frac{1}{\sqrt{-k^2}};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{2(-k^2)^{3/2}\sqrt{x-x^2}\sqrt{(1-x)x(1-k^2x)}}+\frac{(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\,(b-2ak^2)\,\Pi\left(\frac{1}{\sqrt{-k^2}};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{2(-k^2)^{3/2}\sqrt{x-x^2}\sqrt{(1-x)x(1-k^2x)}}+\frac{2a\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x(1-k^2x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^2 + a*k^4*x^4)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^4*x^4)),x]
```

```
[Out] (2*a*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticF[ArcSin[Sqrt[x]], k^2])/S
qrt[(1 - x)*x*(1 - k^2*x)] - ((b + 2*a*k^2)*(1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1
 - k^2*x]*EllipticPi[-k^(-1), ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/(2*(-k^
2)^(3/2)*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2]) - ((b + 2*a*k^2)*(1 - x
)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticPi[k^(-1), ArcSin[Sqrt[-k^2]*Sqr
t[-x]], k^(-2)])/(2*(-k^2)^(3/2)*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2])
 + ((b - 2*a*k^2)*(1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticPi[-(1/S
qrt[-k^2]), ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/(2*(-k^2)^(3/2)*Sqrt[(1 -
 x)*x*(1 - k^2*x)]*Sqrt[x - x^2]) + ((b - 2*a*k^2)*(1 - x)*Sqrt[-x]*Sqrt[x]
*Sqrt[1 - k^2*x]*EllipticPi[1/Sqrt[-k^2], ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-
2)])/(2*(-k^2)^(3/2)*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{a+bx^2+ak^4x^4}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(-1+k^4x^4\right)}\,dx = \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{a+bx^2+ak^4x^4}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\left(-1+k^4x^4\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \left(\frac{a}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}} + \frac{2a+bx^2}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\left(-1+k^4x^4\right)}\right) \cdots}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{2a+bx^2}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\left(-1+k^4x^4\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(a\sqrt{1-x}\,\sqrt{x} \cdots\right.}{\sqrt{\cdots}}$$

$$= \frac{2a\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right) \cdots}{\cdots}$$

$$= \frac{2a\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(\left(b-2ak^2\right)\sqrt{1-x}\,\sqrt{x} \cdots\right.}{2k^2\sqrt{\cdots}}$$

$$= \frac{2a\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(\left(b-2ak^2\right)\sqrt{1-x}\,\sqrt{x} \cdots\right.}{2k^2\sqrt{(\cdots}}$$

$$= \frac{2a\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(\left(b-2ak^2\right)\sqrt{1-x}\,\sqrt{x} \cdots\right.}{\cdots}$$

$$= \frac{2a\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(\left(b-2ak^2\right)\sqrt{1-x}\,\sqrt{x} \cdots\right.}{4k^2\sqrt{\cdots}}$$

$$= \frac{2a\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(\left(b-2ak^2\right)\sqrt{2-2x}\,\sqrt{1 \cdots}\right.}{2\cdots}$$

$$= \frac{2a\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(\left(b-2ak^2\right)\sqrt{2-2x}\,\sqrt{1 \cdots}\right.}{4\cdots}$$

$$= \frac{2a\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\left(b-2ak^2\right)\sqrt{2-2x}\,\sqrt{1 \cdots}\right.}{\cdots}$$

$$= \frac{2a\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(b+2ak^2\right)(1-x)\sqrt{-x} \cdots}{2\left(-k^2\right)^{3/2} \cdots}$$

Mathematica [C] time = 5.00, size = 282, normalized size = 1.58

$$\frac{i\sqrt{\frac{1}{x-1}+1}\,(x-1)^{3/2}\sqrt{\frac{1-\frac{1}{k^2}}{x-1}+1}\left(-(k+1)\left(\left(k^2+1\right)\left(2ak^2+b\right)\Pi\left(\frac{k-1}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+(k-1)\left(b-2ak^2\right)\left((1-ik)\Pi\left(\frac{k-i}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+(1+ik)\Pi\left(\frac{k+i}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)\right)+4k^2\left(ak^4+a+b\right)F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+\left(k^3-k^2+k-1\right)\left(2ak^2+b\right)\Pi\left(1+\frac{1}{k};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)}{2k^2\left(k^4-1\right)\sqrt{(x-1)x\left(k^2x-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + a*k^4*x^4)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1 + k^4*x^
4)),x]
```

```
[Out] ((I/2)*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (1 - k^(-2))/(-1 + x)]*(-1 + x)^(3/
2)*(4*k^2*(a + b + a*k^4)*EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)]
+ (b + 2*a*k^2)*(-1 + k - k^2 + k^3)*EllipticPi[1 + k^(-1), I*ArcSinh[1/Sqr
t[-1 + x]], 1 - k^(-2)] - (1 + k)*((1 + k^2)*(b + 2*a*k^2)*EllipticPi[(-1 +
 k)/k, I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] + (-1 + k)*(b - 2*a*k^2)*((1
- I*k)*EllipticPi[(-I + k)/k, I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] + (1 +
 I*k)*EllipticPi[(I + k)/k, I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)]))))/(k^2
*(-1 + k^4)*Sqrt[(-1 + x)*x*(-1 + k^2*x)])
```

IntegrateAlgebraic [A] time = 0.40, size = 178, normalized size = 1.00

$$\frac{\left(-2ak^2-b\right)\tan^{-1}\left(\frac{(k-1)x}{\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}\right)}{4(k-1)k^2}+\frac{\left(-2ak^2-b\right)\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}\right)}{4k^2(k+1)}+\frac{\left(b-2ak^2\right)\tan^{-1}\left(\frac{\sqrt{k^2+1}\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{(x-1)\left(k^2x-1\right)}\right)}{2k^2\sqrt{k^2+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2 + a*k^4*x^4)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(-1
 + k^4*x^4)),x]
```

```
[Out] ((-b - 2*a*k^2)*ArcTan[((-1 + k)*x)/Sqrt[x + (-1 - k^2)*x^2 + k^2*x^3]])/(4
*(-1 + k)*k^2) + ((-b - 2*a*k^2)*ArcTan[((1 + k)*x)/Sqrt[x + (-1 - k^2)*x^2
 + k^2*x^3]])/(4*k^2*(1 + k)) + ((b - 2*a*k^2)*ArcTan[(Sqrt[1 + k^2]*Sqrt[x
 + (-1 - k^2)*x^2 + k^2*x^3])/((-1 + x)*(-1 + k^2*x))])/(2*k^2*Sqrt[1 + k^2
])
```

fricas [B] time = 1.59, size = 356, normalized size = 2.00

$$\frac{2\left(2\,ak^4-(2\,a+b)k^2+b\right)\sqrt{k^2+1}\,\arctan\left(\frac{\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\left(k^2x^2-2\left(k^2+1\right)x+1\right)\sqrt{k^2+1}}{2\left(\left(k^4+k^2\right)x^3-\left(k^4+2\,k^2+1\right)x^2+\left(k^2+1\right)x\right)}\right)+\left(2\,ak^5-2\,ak^4+(2\,a+b)k^3-(2\,a+b)k^2+bk-b\right)\arctan\left(\frac{\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\left(k^2x^2-2\left(k^2+k+1\right)x+1\right)}{2\left(\left(k^3+k^2\right)x^3-\left(k^3+k^2+k+1\right)x^2+(k+1)x\right)}\right)+\left(2\,ak^5+2\,ak^4+(2\,a+b)k^3+(2\,a+b)k^2+bk+b\right)\arctan\left(\frac{\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\left(k^2x^2-2\left(k^2-k+1\right)x+1\right)}{2\left(\left(k^3-k^2\right)x^3-\left(k^3-k^2+k-1\right)x^2+(k-1)x\right)}\right)}{8\left(k^6-k^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*k^4*x^4+b*x^2+a)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^4*x^4-1),x, alg
orithm="fricas")
```

```
[Out] 1/8*(2*(2*a*k^4 - (2*a + b)*k^2 + b)*sqrt(k^2 + 1)*arctan(1/2*sqrt(k^2*x^3
- (k^2 + 1)*x^2 + x)*(k^2*x^2 - 2*(k^2 + 1)*x + 1)*sqrt(k^2 + 1)/((k^4 + k^
2)*x^3 - (k^4 + 2*k^2 + 1)*x^2 + (k^2 + 1)*x)) + (2*a*k^5 - 2*a*k^4 + (2*a
+ b)*k^3 - (2*a + b)*k^2 + b*k - b)*arctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2
 + x)*(k^2*x^2 - 2*(k^2 + k + 1)*x + 1)/((k^3 + k^2)*x^3 - (k^3 + k^2 + k +
 1)*x^2 + (k + 1)*x)) + (2*a*k^5 + 2*a*k^4 + (2*a + b)*k^3 + (2*a + b)*k^2
+ b*k + b)*arctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2 - 2*(k^2 -
 k + 1)*x + 1)/((k^3 - k^2)*x^3 - (k^3 - k^2 + k - 1)*x^2 + (k - 1)*x)))/(k
^6 - k^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ak^4x^4+bx^2+a}{\left(k^4x^4-1\right)\sqrt{\left(k^2x-1\right)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*k^4*x^4+b*x^2+a)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^4*x^4-1),x, alg
orithm="giac")
```

```
[Out] integrate((a*k^4*x^4 + b*x^2 + a)/((k^4*x^4 - 1)*sqrt((k^2*x - 1)*(x - 1)*x
)), x)
```

maple [C] time = 0.04, size = 604, normalized size = 3.39

$$-\frac{2a\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\operatorname{EllipticF}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^2\sqrt{k^2x^3-k^2x^2-x^2+x}}-\frac{\left(2a\,k^2+b\right)\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\operatorname{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}-\frac{1}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{2k^5\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}-\frac{1}{k}\right)}-\frac{\left(-2a\,k^2-b\right)\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\operatorname{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}+\frac{1}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{2k^5\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}+\frac{1}{k}\right)}+\frac{\left(-2a\,k^2+b\right)\left(\frac{i\sqrt{-k^2x+1}\,\sqrt{\frac{x}{\frac{1}{k^2}-1}-\frac{1}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\operatorname{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}-\frac{i}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^3\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}-\frac{i}{k}\right)}-\frac{i\sqrt{-k^2x+1}\,\sqrt{\frac{x}{\frac{1}{k^2}-1}-\frac{1}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\operatorname{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{1}{k^2\left(\frac{1}{k^2}+\frac{i}{k}\right)},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{k^3\sqrt{k^2x^3-k^2x^2-x^2+x}\,\left(\frac{1}{k^2}+\frac{i}{k}\right)}\right)}{2k^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*k^4*x^4+b*x^2+a)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^4*x^4-1),x)
```

```
[Out] -2*a/k^2*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2
*x^3-k^2*x^2-x^2+x)^(1/2)*EllipticF((-(x-1/k^2)*k^2)^(1/2),(1/k^2/(1/k^2-1)
)^(1/2))-1/2*(2*a*k^2+b)/k^5*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2
)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2-1/k)*EllipticPi((-(x-1
/k^2)*k^2)^(1/2),1/k^2/(1/k^2-1/k),(1/k^2/(1/k^2-1))^(1/2))-1/2*(-2*a*k^2-b
)/k^5*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^
3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+1/k)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/
(1/k^2+1/k),(1/k^2/(1/k^2-1))^(1/2))+1/2/k^2*(-2*a*k^2+b)*(I/k^3*(-k^2*x+1)
^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2
+x)^(1/2)/(1/k^2-I/k)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-I/k),(
1/k^2/(1/k^2-1))^(1/2))-I/k^3*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^
(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)/(1/k^2+I/k)*EllipticPi((-
(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+I/k),(1/k^2/(1/k^2-1))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ak^4x^4 + bx^2 + a}{\left(k^4x^4 - 1\right)\sqrt{\left(k^2x - 1\right)(x - 1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*k^4*x^4+b*x^2+a)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^4*x^4-1),x, alg
orithm="maxima")
```

```
[Out] integrate((a*k^4*x^4 + b*x^2 + a)/((k^4*x^4 - 1)*sqrt((k^2*x - 1)*(x - 1)*x
)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + a*k^4*x^4)/((k^4*x^4 - 1)*(x*(k^2*x - 1)*(x - 1))^(1/2)),x
)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ak^4x^4 + a + bx^2}{\sqrt{x\,(x-1)\left(k^2x-1\right)}\,(kx-1)\,(kx+1)\left(k^2x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*k**4*x**4+b*x**2+a)/((1-x)*x*(-k**2*x+1))**(1/2)/(k**4*x**4-1)
,x)
```

```
[Out] Integral((a*k**4*x**4 + a + b*x**2)/(sqrt(x*(x - 1)*(k**2*x - 1))*(k**x - 1)
*(k*x + 1)*(k**2*x**2 + 1)), x)
```

3.1900 $\int \frac{(-2q+px^3)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\,(aq^2+2apqx^3+bx^4+ap^2x^6)}{x^9}\,dx$

Optimal. Leaf size=178

$$\frac{1}{2}\left(-ap^2q^2-2bpq\right)\log\left(\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}+px^3+q\right)+\log(x)\left(ap^2q^2+2bpq\right)+\frac{\sqrt{p^2x^6-2pqx^4+2pqx}}{}$$

Rubi [F] time = 1.61, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2q+px^3)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\,(aq^2+2apqx^3+bx^4+ap^2x^6)}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(a*q^2 + 2*
a*p*q*x^3 + b*x^4 + a*p^2*x^6))/x^9,x]
```

```
[Out] a*p^3*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6], x] - 2*a*q^3*
Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^9, x] - 3*a*p*q^2*
Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^6, x] - 2*b*q*Defe
r[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^5, x] + b*p*Defer[Int]
[Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^2, x]
```

Rubi steps

$$\int \frac{(-2q+px^3)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\,(aq^2+2apqx^3+bx^4+ap^2x^6)}{x^9}\,dx = \int \left(ap^3\sqrt{q^2+2pqx^3-2pqx^4+}\right.$$

$$= (bp)\int \frac{\sqrt{q^2+2pqx^3-2pqx^4+}}{x^2}$$

Mathematica [F] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{(-2q+px^3)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\,(aq^2+2apqx^3+bx^4+ap^2x^6)}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(a*q^
2 + 2*a*p*q*x^3 + b*x^4 + a*p^2*x^6))/x^9,x]
```

```
[Out] Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(a*q^
2 + 2*a*p*q*x^3 + b*x^4 + a*p^2*x^6))/x^9, x]
```

IntegrateAlgebraic [A] time = 0.41, size = 178, normalized size = 1.00

$$\frac{1}{2}\left(-ap^2q^2-2bpq\right)\log\left(\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}+px^3+q\right)+\log(x)\left(ap^2q^2+2bpq\right)+\frac{\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}\left(ap^3x^9-ap^2qx^7+3ap^2qx^6-apq^2x^4+3apq^2x^3+aq^3+2bpx^7+2bqx^4\right)}{4x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x
^6]*(a*q^2 + 2*a*p*q*x^3 + b*x^4 + a*p^2*x^6))/x^9,x]
```

```
[Out] (Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(a*q^3 + 3*a*p*q^2*x^3 + 2*b*q
*x^4 - a*p*q^2*x^4 + 3*a*p^2*q*x^6 + 2*b*p*x^7 - a*p^2*q*x^7 + a*p^3*x^9))/
```

```
(4*x^8) + (2*b*p*q + a*p^2*q^2)*Log[x] + ((-2*b*p*q - a*p^2*q^2)*Log[q + p*
x^3 + Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]])/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(a*p^2*x^6+2*
a*p*q*x^3+b*x^4+a*q^2)/x^9,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ap^2x^6 + 2\,apqx^3 + bx^4 + aq^2\right)\sqrt{p^2x^6 - 2\,pqx^4 + 2\,pqx^3 + q^2}\,\left(px^3 - 2\,q\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(a*p^2*x^6+2*
a*p*q*x^3+b*x^4+a*q^2)/x^9,x, algorithm="giac")
```

```
[Out] integrate((a*p^2*x^6 + 2*a*p*q*x^3 + b*x^4 + a*q^2)*sqrt(p^2*x^6 - 2*p*q*x^
4 + 2*p*q*x^3 + q^2)*(p*x^3 - 2*q)/x^9, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\left(p\,x^3 - 2q\right)\sqrt{p^2x^6 - 2pq\,x^4 + 2pq\,x^3 + q^2}\;\left(a\,p^2x^6 + 2apq\,x^3 + b\,x^4 + a\,q^2\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(a*p^2*x^6+2*a*p*q*
x^3+b*x^4+a*q^2)/x^9,x)
```

```
[Out] int((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(a*p^2*x^6+2*a*p*q*
x^3+b*x^4+a*q^2)/x^9,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ap^2x^6 + 2\,apqx^3 + bx^4 + aq^2\right)\sqrt{p^2x^6 - 2\,pqx^4 + 2\,pqx^3 + q^2}\,\left(px^3 - 2\,q\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(a*p^2*x^6+2*
a*p*q*x^3+b*x^4+a*q^2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate((a*p^2*x^6 + 2*a*p*q*x^3 + b*x^4 + a*q^2)*sqrt(p^2*x^6 - 2*p*q*x^
4 + 2*p*q*x^3 + q^2)*(p*x^3 - 2*q)/x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(2\,q - p\,x^3\right)\,\sqrt{p^2\,x^6 - 2\,p\,q\,x^4 + 2\,p\,q\,x^3 + q^2}\;\left(a\,p^2\,x^6 + 2\,a\,p\,q\,x^3 + a\,q^2 + b\,x^4\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*q - p*x^3)*(p^2*x^6 + q^2 + 2*p*q*x^3 - 2*p*q*x^4)^(1/2)*(a*q^2 +
b*x^4 + a*p^2*x^6 + 2*a*p*q*x^3))/x^9,x)
```

```
[Out] int(-((2*q - p*x^3)*(p^2*x^6 + q^2 + 2*p*q*x^3 - 2*p*q*x^4)^(1/2)*(a*q^2 +
b*x^4 + a*p^2*x^6 + 2*a*p*q*x^3))/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(px^3-2q\right)\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}\,\left(ap^2x^6+2apqx^3+aq^2+bx^4\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**3-2*q)*(p**2*x**6-2*p*q*x**4+2*p*q*x**3+q**2)**(1/2)*(a*p**
2*x**6+2*a*p*q*x**3+b*x**4+a*q**2)/x**9,x)
```

```
[Out] Integral((p*x**3 - 2*q)*sqrt(p**2*x**6 - 2*p*q*x**4 + 2*p*q*x**3 + q**2)*(a
*p**2*x**6 + 2*a*p*q*x**3 + a*q**2 + b*x**4)/x**9, x)
```

3.1901 $\int \frac{-1+x^3}{(1+x^3)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

Optimal. Leaf size=179

$$\frac{4\tan^{-1}\left(\frac{x\sqrt{a-b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2-\sqrt{a}\,x+\sqrt{a}}\right)}{3\sqrt{a-b-c}}-\frac{2\sqrt{-2a+2b-c}\,\tan^{-1}\left(\frac{x\sqrt{-2a+2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2+2\sqrt{a}\,x+\sqrt{a}}\right)}{3(2a-2b+c)}$$

Rubi [F] time = 1.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+x^3}{(1+x^3)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x^3)/((1 + x^3)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] Defer[Int][1/Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4], x] - (2*Defer[Int][1/((
1 + x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x])/3 + (2*(1 + I*Sqrt[3])*D
efer[Int][1/((-1 - I*Sqrt[3] + 2*x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),
 x])/3 + (2*(1 - I*Sqrt[3])*Defer[Int][1/((-1 + I*Sqrt[3] + 2*x)*Sqrt[a + b
*x + c*x^2 + b*x^3 + a*x^4]), x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^3}{(1+x^3)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx &= \int \left(\frac{1}{\sqrt{a+bx+cx^2+bx^3+ax^4}}-\frac{2}{(1+x^3)\sqrt{a+bx+cx^2+bx^3}\right.\\
&= -\left(2\int \frac{1}{(1+x^3)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx\right)+\int \frac{}{\sqrt{a+bx+}}\\
&= -\left(2\int \left(\frac{1}{3(1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}+\frac{}{3(1-x+x^2)\sqrt{a}}\right.\right.\\
&= -\left(\frac{2}{3}\int \frac{1}{(1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx\right)-\frac{2}{3}\int \frac{}{(1-x+x^2}\\
&= -\left(\frac{2}{3}\int \frac{1}{(1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx\right)-\frac{2}{3}\int \left(\frac{}{(-1-i\sqrt{3}}\right.\\
&= -\left(\frac{2}{3}\int \frac{1}{(1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx\right)+\frac{1}{3}\left(2\left(1-i\sqrt{3}\right)\right)
\end{aligned}$$

Mathematica [C] time = 3.44, size = 4912, normalized size = 27.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + x^3)/((1 + x^3)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] (2*(x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])^2*Sqrt[((Root[a +
b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 +
a*#1^4 & , 2])*(x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3]))/((x -
```

```
Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 +
 b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3]))
]*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2
 + b*#1^3 + a*#1^4 & , 4])*Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#
1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#
1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(x - Root[a + b*#1 + c*#1^2 + b*#1^3 +
 a*#1^4 & , 4])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a +
 b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*
#1^3 + a*#1^4 & , 2])^2*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] -
Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])^2)]*((2*(-1)^(2/3)*(Ellipt
icF[ArcSin[Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Roo
t[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#
1^3 + a*#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2]
)*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2
 + b*#1^3 + a*#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4
 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 +
 c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4
 & , 4]))/((-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#
1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#
1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*((-1)^(1/3
) - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1]) + EllipticPi[(((-1)^(1
/3) - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#
1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & ,
 4]))/(((-1)^(1/3) - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root
[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1
^3 + a*#1^4 & , 4])), ArcSin[Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a
*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b
*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1
^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root
[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2
 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3]
)*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2
 + b*#1^3 + a*#1^4 & , 4]))/((-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & ,
 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#
1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & ,
 4])))]*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 +
c*#1^2 + b*#1^3 + a*#1^4 & , 2])))/((1 + (-1)^(1/3))^2*((-1)^(1/3) - Root[a
 + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*((-1)^(1/3) - Root[a + b*#1 + c*
#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
, 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])) - (2*(EllipticF[Ar
cSin[Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a +
b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 +
a*#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Roo
t[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#
1^3 + a*#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2
] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^
2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4
]))/((-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*
#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
, 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*(1 + Root[a + b*
#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1]) + EllipticPi[((1 + Root[a + b*#1 + c*
#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
, 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((1 + Root[a + b*#
1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#
1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])), ArcSin[Sqrt
[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*
#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
, 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#
1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#
```

```
1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[
a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^
3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((-Ro
ot[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*
#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Ro
ot[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*(-Root[a + b*#1 + c*#1^2
+ b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])
))/((-1 + (-1)^(1/3))*(1 + (-1)^(1/3))^2*(1 + Root[a + b*#1 + c*#1^2 + b*#1
^3 + a*#1^4 & , 1])*(1 + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(
Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 +
b*#1^3 + a*#1^4 & , 4])) - (2*(-1)^(2/3)*(EllipticF[ArcSin[Sqrt[((x - Root[
a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^
3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((x -
 Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 +
 b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))
]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c
*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
 , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((-Root[a + b*#1
+ c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^
4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1
+ c*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*((-1)^(2/3) + Root[a + b*#1 + c*#1^2
+ b*#1^3 + a*#1^4 & , 1]) + EllipticPi[(((-1)^(2/3) + Root[a + b*#1 + c*#1^
2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1
] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/(((-1)^(2/3) + Root[a
 + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3
 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])), ArcSi
n[Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#
1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#
1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a
 + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3
 + a*#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] -
 Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 +
 b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))
/((-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^
2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2
] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*(-Root[a + b*#1 + c
*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
 , 2])))/((-1 + (-1)^(1/3))*(1 + (-1)^(1/3))^2*((-1)^(2/3) + Root[a + b*#1
+ c*#1^2 + b*#1^3 + a*#1^4 & , 1])*((-1)^(2/3) + Root[a + b*#1 + c*#1^2 + b
*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - R
oot[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])) + EllipticF[ArcSin[Sqrt[((
x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^
2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4
]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 +
 c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4
 & , 4]))]], ((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b
*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a
*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((Root[a +
 b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 +
 a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a +
 b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))]/(-Root[a + b*#1 + c*#1^2 + b*#1^
3 + a*#1^4 & , 2] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])))/(Sqr
t[x*(b + c*x + b*x^2) + a*(1 + x^4)]*(-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*
#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2]))
```

IntegrateAlgebraic [A] time = 1.29, size = 179, normalized size = 1.00

$$\frac{4\tan^{-1}\left(\frac{x\sqrt{a-b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2-\sqrt{a}\,x+\sqrt{a}}\right)}{3\sqrt{a-b-c}}-\frac{2\sqrt{-2a+2b-c}\,\tan^{-1}\left(\frac{x\sqrt{-2a+2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2+2\sqrt{a}\,x+\sqrt{a}}\right)}{3(2a-2b+c)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)/((1 + x^3)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x
^4]),x]
```

```
[Out] (4*ArcTan[(Sqrt[a - b - c]*x)/(Sqrt[a] - Sqrt[a]*x + Sqrt[a]*x^2 - Sqrt[a +
 b*x + c*x^2 + b*x^3 + a*x^4])])/(3*Sqrt[a - b - c]) - (2*Sqrt[-2*a + 2*b -
 c]*ArcTan[(Sqrt[-2*a + 2*b - c]*x)/(Sqrt[a] + 2*Sqrt[a]*x + Sqrt[a]*x^2 -
Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(3*(2*a - 2*b + c))
```

fricas [A] time = 3.22, size = 1508, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^3+1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/6*(sqrt(2*a - 2*b + c)*(a - b - c)*log(((24*a^2 - 16*a*b + b^2 + 4*a*c)*
x^4 + 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4
*(a + 2*b)*c + 4*c^2)*x^2 + 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a -
 b)*x^2 + 2*(2*a + b - c)*x + 4*a - b)*sqrt(2*a - 2*b + c) + 24*a^2 - 16*a*
b + b^2 + 4*a*c + 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x)/(x^4 + 4*x^3
 + 6*x^2 + 4*x + 1)) - 2*(2*a - 2*b + c)*sqrt(-a + b + c)*log(((8*a*b + b^2
 + 4*a*c)*x^4 - 2*(8*a^2 + 4*a*b - 3*b^2 - 4*(a + b)*c)*x^3 + (24*a^2 + 3*b
^2 - 4*(5*a - 2*b)*c + 8*c^2)*x^2 - 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)
*((2*a + b)*x^2 - (4*a - b - 2*c)*x + 2*a + b)*sqrt(-a + b + c) + 8*a*b + b
^2 + 4*a*c - 2*(8*a^2 + 4*a*b - 3*b^2 - 4*(a + b)*c)*x)/(x^4 - 2*x^3 + 3*x^
2 - 2*x + 1)))/(2*a^2 - 4*a*b + 2*b^2 - (a - b)*c - c^2), -1/6*(4*(2*a - 2*
b + c)*sqrt(a - b - c)*arctan(2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(
a - b - c)/((2*a + b)*x^2 - (4*a - b - 2*c)*x + 2*a + b)) - sqrt(2*a - 2*b
+ c)*(a - b - c)*log(((24*a^2 - 16*a*b + b^2 + 4*a*c)*x^4 + 4*(8*a^2 + 4*a*
b - 3*b^2 - 2*(2*a - b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a + 2*b)*c + 4*c^2)
*x^2 + 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x^2 + 2*(2*a + b
- c)*x + 4*a - b)*sqrt(2*a - 2*b + c) + 24*a^2 - 16*a*b + b^2 + 4*a*c + 4*(
8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1)))
/(2*a^2 - 4*a*b + 2*b^2 - (a - b)*c - c^2), 1/3*((a - b - c)*sqrt(-2*a + 2*
b - c)*arctan(-1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x^2 + 2
*(2*a + b - c)*x + 4*a - b)*sqrt(-2*a + 2*b - c)/((2*a^2 - 2*a*b + a*c)*x^4
 + (2*a*b - 2*b^2 + b*c)*x^3 + (2*(a - b)*c + c^2)*x^2 + 2*a^2 - 2*a*b + a*
c + (2*a*b - 2*b^2 + b*c)*x)) - (2*a - 2*b + c)*sqrt(-a + b + c)*log(((8*a*
b + b^2 + 4*a*c)*x^4 - 2*(8*a^2 + 4*a*b - 3*b^2 - 4*(a + b)*c)*x^3 + (24*a^
2 + 3*b^2 - 4*(5*a - 2*b)*c + 8*c^2)*x^2 - 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b
*x + a)*((2*a + b)*x^2 - (4*a - b - 2*c)*x + 2*a + b)*sqrt(-a + b + c) + 8*
a*b + b^2 + 4*a*c - 2*(8*a^2 + 4*a*b - 3*b^2 - 4*(a + b)*c)*x)/(x^4 - 2*x^3
 + 3*x^2 - 2*x + 1)))/(2*a^2 - 4*a*b + 2*b^2 - (a - b)*c - c^2), 1/3*((a -
b - c)*sqrt(-2*a + 2*b - c)*arctan(-1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x +
a)*((4*a - b)*x^2 + 2*(2*a + b - c)*x + 4*a - b)*sqrt(-2*a + 2*b - c)/((2*a
^2 - 2*a*b + a*c)*x^4 + (2*a*b - 2*b^2 + b*c)*x^3 + (2*(a - b)*c + c^2)*x^2
 + 2*a^2 - 2*a*b + a*c + (2*a*b - 2*b^2 + b*c)*x)) - 2*(2*a - 2*b + c)*sqrt
(a - b - c)*arctan(2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(a - b - c)/
((2*a + b)*x^2 - (4*a - b - 2*c)*x + 2*a + b)))/(2*a^2 - 4*a*b + 2*b^2 - (a
 - b)*c - c^2)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^3+1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{x^3 - 1}{\left(x^3 + 1\right) \sqrt{a\,x^4 + b\,x^3 + c\,x^2 + bx + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x^3+1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
[Out] int((x^3-1)/(x^3+1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 - 1}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a}\left(x^3 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^3+1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
[Out] integrate((x^3 - 1)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(x^3 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 - 1}{\left(x^3 + 1\right) \sqrt{a\,x^4 + b\,x^3 + c\,x^2 + b\,x + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/((x^3 + 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)),x)
[Out] int((x^3 - 1)/((x^3 + 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\left(x^2 + x + 1\right)}{(x+1)\left(x^2 - x + 1\right)\sqrt{ax^4 + a + bx^3 + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(x**3+1)/(a*x**4+b*x**3+c*x**2+b*x+a)**(1/2),x)
[Out] Integral((x - 1)*(x**2 + x + 1)/((x + 1)*(x**2 - x + 1)*sqrt(a*x**4 + a + b
*x**3 + b*x + c*x**2)), x)
```

3.1902 $\int \frac{5x-4(1+k)x^2+3kx^3}{\sqrt[3]{(1-x)x(1-kx)}\left(-1+(1+k)x-kx^2+bx^5\right)}\,dx$

Optimal. Leaf size=179

$$-\frac{\log\left(b^{2/3}x^4+\sqrt[3]{b}\,x^2\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[3]{b}\,x^2\right)}{\sqrt[3]{b}}$$

Rubi [F] time = 13.58, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{5x-4(1+k)x^2+3kx^3}{\sqrt[3]{(1-x)x(1-kx)}\left(-1+(1+k)x-kx^2+bx^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(5*x - 4*(1 + k)*x^2 + 3*k*x^3)/(((1 - x)*x*(1 - k*x))^(1/3)*(-1 + (1 +
 k)*x - k*x^2 + b*x^5)),x]
```

```
[Out] (12*(1 + k)*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][x
^7/((1 - x^3)^(1/3)*(1 - k*x^3)^(1/3)*(1 - (1 + k)*x^3 + k*x^6 - b*x^15)),
x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3) + (15*(1 - x)^(1/3)*x^(1/3)*(1
 - k*x)^(1/3)*Defer[Subst][Defer[Int][x^4/((1 - x^3)^(1/3)*(1 - k*x^3)^(1/3
)*(-1 + (1 + k)*x^3 - k*x^6 + b*x^15)), x], x, x^(1/3)])/((1 - x)*x*(1 - k*
x))^(1/3) + (9*k*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[I
nt][x^10/((1 - x^3)^(1/3)*(1 - k*x^3)^(1/3)*(-1 + (1 + k)*x^3 - k*x^6 + b*x
^15)), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\int \frac{5x-4(1+k)x^2+3kx^3}{\sqrt[3]{(1-x)x(1-kx)}\left(-1+(1+k)x-kx^2+bx^5\right)}\,dx=\int \frac{x\left(5-4(1+k)x+3kx^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\left(-1+(1+k)x-kx^2+bx^5\right)}\,dx$$

$$=\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{x^{2/3}\left(5-4(1+k)x+3kx^2\right)}{\sqrt[3]{1-x}\sqrt[3]{1-kx}\left(-1+(1+k)x-kx^2+bx^5\right)}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$=\frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}\left(\int \frac{x^4\left(5-4(1+k)x^3+3kx^6\right)}{\sqrt[3]{1-x^3}\sqrt[3]{1-kx^3}\left(-1+(1+k)x^3-k\right.}\right.}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$=\frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}\left(\int\left(\frac{4(1+k)x^7}{\sqrt[3]{1-x^3}\sqrt[3]{1-kx^3}\left(1-(1+k)x^3+k\right.}\right.\right.}{}$$

$$=\frac{\left(15\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt[3]{1-x^3}\sqrt[3]{1-kx^3}\left(-1+(1+k)x^3-\right.}\right.}{\sqrt[3]{(1-x)x(1-kx)}}$$

Mathematica [F] time = 3.13, size = 0, normalized size = 0.00

$$\int \frac{5x-4(1+k)x^2+3kx^3}{\sqrt[3]{(1-x)x(1-kx)}\left(-1+(1+k)x-kx^2+bx^5\right)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(5*x - 4*(1 + k)*x^2 + 3*k*x^3)/(((1 - x)*x*(1 - k*x))^(1/3)*(-1 + (1 + k)*x - k*x^2 + b*x^5)),x]

[Out] Integrate[(5*x - 4*(1 + k)*x^2 + 3*k*x^3)/(((1 - x)*x*(1 - k*x))^(1/3)*(-1 + (1 + k)*x - k*x^2 + b*x^5)), x]

IntegrateAlgebraic [A] time = 1.11, size = 179, normalized size = 1.00

$$-\frac{\log \left(b^{2/3}x^4+\sqrt[3]{b}\,x^2\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2\sqrt[3]{b}}+\frac{\log \left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[3]{b}\,x^2\right)}{\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{kx^3+(-k-1)x^2+x}}{2\sqrt[3]{b}\,x^2+\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(5*x - 4*(1 + k)*x^2 + 3*k*x^3)/(((1 - x)*x*(1 - k*x))^(1/3)*(-1 + (1 + k)*x - k*x^2 + b*x^5)),x]

[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(2*b^(1/3)*x^2 + (x + (-1 - k)*x^2 + k*x^3)^(1/3))])/b^(1/3) + Log[-(b^(1/3)*x^2) + (x + (-1 - k)*x^2 + k*x^3)^(1/3)]/b^(1/3) - Log[b^(2/3)*x^4 + b^(1/3)*x^2*(x + (-1 - k)*x^2 + k*x^3)^(1/3) + (x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x-4*(1+k)*x^2+3*k*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(-1+(1+k)*x-k*x^2+b*x^5),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,kx^3-4\,(k+1)x^2+5\,x}{\left(bx^5-kx^2+(k+1)x-1\right)\left((kx-1)(x-1)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x-4*(1+k)*x^2+3*k*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(-1+(1+k)*x-k*x^2+b*x^5),x, algorithm="giac")

[Out] integrate((3*k*x^3 - 4*(k + 1)*x^2 + 5*x)/((b*x^5 - k*x^2 + (k + 1)*x - 1)*((k*x - 1)*(x - 1)*x)^(1/3)), x)

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{5x-4\,(1+k)\,x^2+3k\,x^3}{\left((1-x)\,x\,(-kx+1)\right)^{\frac{1}{3}}\left(-1+(1+k)\,x-k\,x^2+b\,x^5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x-4*(1+k)*x^2+3*k*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(-1+(1+k)*x-k*x^2+b*x^5),x)

[Out] int((5*x-4*(1+k)*x^2+3*k*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(-1+(1+k)*x-k*x^2+b*x^5),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,kx^3-4\,(k+1)x^2+5\,x}{\left(bx^5-kx^2+(k+1)x-1\right)\left((kx-1)(x-1)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x-4*(1+k)*x^2+3*k*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(-1+(1+k)*x-k*
x^2+b*x^5),x, algorithm="maxima")
```

```
[Out] integrate((3*k*x^3 - 4*(k + 1)*x^2 + 5*x)/((b*x^5 - k*x^2 + (k + 1)*x - 1)*
((k*x - 1)*(x - 1)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x-4\,x^2\,\left(k+1\right)+3\,k\,x^3}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/3}\,\left(b\,x^5-k\,x^2+\left(k+1\right)\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x - 4*x^2*(k + 1) + 3*k*x^3)/((x*(k*x - 1)*(x - 1))^(1/3)*(b*x^5 + x
*(k + 1) - k*x^2 - 1)),x)
```

```
[Out] int((5*x - 4*x^2*(k + 1) + 3*k*x^3)/((x*(k*x - 1)*(x - 1))^(1/3)*(b*x^5 + x
*(k + 1) - k*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(3 k x^{2} - 4 k x - 4 x + 5\right)}{\sqrt[3]{x \left(x - 1\right) \left(k x - 1\right)} \left(b x^{5} - k x^{2} + k x + x - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x-4*(1+k)*x**2+3*k*x**3)/((1-x)*x*(-k*x+1))**(1/3)/(-1+(1+k)*x
-k*x**2+b*x**5),x)
```

```
[Out] Integral(x*(3*k*x**2 - 4*k*x - 4*x + 5)/((x*(x - 1)*(k*x - 1))**(1/3)*(b*x*
*5 - k*x**2 + k*x + x - 1)), x)
```

3.1903 $\int \frac{x^2(8-7(1+k)x+6kx^2)}{\sqrt[3]{(1-x)x(1-kx)}\left(-1+(1+k)x-kx^2+bx^8\right)}dx$

Optimal. Leaf size=179

$$-\frac{\log\left(b^{2/3}x^6+\sqrt[3]{b}x^3\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[3]{b}\,x\right.}{\sqrt[3]{b}}$$

Rubi [F] time = 14.97, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(8-7(1+k)x+6kx^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\left(-1+(1+k)x-kx^2+bx^8\right)}dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(8 - 7*(1 + k)*x + 6*k*x^2))/(((1 - x)*x*(1 - k*x))^(1/3)*(-1 + (1
 + k)*x - k*x^2 + b*x^8)),x]
```

```
[Out] (21*(1 + k)*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][x
^10/((1 - x^3)^(1/3)*(1 - k*x^3)^(1/3)*(1 - (1 + k)*x^3 + k*x^6 - b*x^24)),
 x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3) + (24*(1 - x)^(1/3)*x^(1/3)*(
1 - k*x)^(1/3)*Defer[Subst][Defer[Int][x^7/((1 - x^3)^(1/3)*(1 - k*x^3)^(1/
3)*(-1 + (1 + k)*x^3 - k*x^6 + b*x^24)), x], x, x^(1/3)])/((1 - x)*x*(1 - k
*x))^(1/3) + (18*k*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer
[Int][x^13/((1 - x^3)^(1/3)*(1 - k*x^3)^(1/3)*(-1 + (1 + k)*x^3 - k*x^6 + b
*x^24)), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\int \frac{x^2\left(8-7(1+k)x+6kx^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\left(-1+(1+k)x-kx^2+bx^8\right)}dx = \frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{x^{5/3}\left(8-7(1+k)x+6kx^2\right)}{\sqrt[3]{1-x}\sqrt[3]{1-kx}\left(-1+(1+k)x-kx^2+bx^8\right)}d}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$= \frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}\left(\int \frac{x^7\left(8-7(1+k)x^3+6k\right.}{\sqrt[3]{1-x^3}\sqrt[3]{1-kx^3}\left(-1+(1+k)x\right.}\right.}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$= \frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}\left(\int\left(\frac{7(1+k)x^{10}}{\sqrt[3]{1-x^3}\sqrt[3]{1-kx^3}\left(1-(1+k)x\right.}\right.\right.}{}$$

$$= \frac{\left(24\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}\left(\int \frac{x^7}{\sqrt[3]{1-x^3}\sqrt[3]{1-kx^3}\left(-1+(1+k\right.}\right.}{\sqrt[3]{(1-x)x(1-kx)}}$$

Mathematica [F] time = 5.93, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(8-7(1+k)x+6kx^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\left(-1+(1+k)x-kx^2+bx^8\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(8 - 7*(1 + k)*x + 6*k*x^2))/(((1 - x)*x*(1 - k*x))^(1/3)*(-
1 + (1 + k)*x - k*x^2 + b*x^8)),x]
```

[Out] Integrate[(x^2*(8 - 7*(1 + k)*x + 6*k*x^2))/(((1 - x)*x*(1 - k*x))^(1/3)*(-1 + (1 + k)*x - k*x^2 + b*x^8)), x]

IntegrateAlgebraic [A] time = 2.97, size = 179, normalized size = 1.00

$$-\frac{\log \left(b^{2/3}x^6+\sqrt[3]{b}\,x^3\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2\sqrt[3]{b}}+\frac{\log \left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[3]{b}\,x^3\right)}{\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{kx^3+(-k-1)x^2+x}}{2\sqrt[3]{b}\,x^3+\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2*(8 - 7*(1 + k)*x + 6*k*x^2))/(((1 - x)*x*(1 - k*x))^(1/3)*(-1 + (1 + k)*x - k*x^2 + b*x^8)),x]

[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(2*b^(1/3)*x^3 + (x + (-1 - k)*x^2 + k*x^3)^(1/3))])/b^(1/3) + Log[-(b^(1/3)*x^3) + (x + (-1 - k)*x^2 + k*x^3)^(1/3)]/b^(1/3) - Log[b^(2/3)*x^6 + b^(1/3)*x^3*(x + (-1 - k)*x^2 + k*x^3)^(1/3) + (x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(8-7*(1+k)*x+6*k*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(-1+(1+k)*x-k*x^2+b*x^8),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(6\,kx^2-7\,(k+1)x+8\right)x^2}{\left(bx^8-kx^2+(k+1)x-1\right)\left((kx-1)(x-1)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(8-7*(1+k)*x+6*k*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(-1+(1+k)*x-k*x^2+b*x^8),x, algorithm="giac")

[Out] integrate((6*k*x^2 - 7*(k + 1)*x + 8)*x^2/((b*x^8 - k*x^2 + (k + 1)*x - 1)*((k*x - 1)*(x - 1)*x)^(1/3)), x)

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(8-7\,(1+k)\,x+6k\,x^2\right)}{\left((1-x)\,x\,(-kx+1)\right)^{\frac{1}{3}}\left(-1+(1+k)\,x-k\,x^2+b\,x^8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(8-7*(1+k)*x+6*k*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(-1+(1+k)*x-k*x^2+b*x^8),x)

[Out] int(x^2*(8-7*(1+k)*x+6*k*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(-1+(1+k)*x-k*x^2+b*x^8),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(6\,kx^2-7\,(k+1)x+8\right)x^2}{\left(bx^8-kx^2+(k+1)x-1\right)\left((kx-1)(x-1)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(8-7*(1+k)*x+6*k*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(-1+(1+k)*x-k*
x^2+b*x^8),x, algorithm="maxima")
```

```
[Out] integrate((6*k*x^2 - 7*(k + 1)*x + 8)*x^2/((b*x^8 - k*x^2 + (k + 1)*x - 1)*
((k*x - 1)*(x - 1)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(6\,k\,x^2-7\,x\,\left(k+1\right)+8\right)}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/3}\,\left(b\,x^8-k\,x^2+\left(k+1\right)\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(6*k*x^2 - 7*x*(k + 1) + 8))/((x*(k*x - 1)*(x - 1))^(1/3)*(b*x^8 +
 x*(k + 1) - k*x^2 - 1)),x)
```

```
[Out] int((x^2*(6*k*x^2 - 7*x*(k + 1) + 8))/((x*(k*x - 1)*(x - 1))^(1/3)*(b*x^8 +
 x*(k + 1) - k*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(6 k x^{2} - 7 k x - 7 x + 8\right)}{\sqrt[3]{x \left(x - 1\right) \left(k x - 1\right)} \left(b x^{8} - k x^{2} + k x + x - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(8-7*(1+k)*x+6*k*x**2)/((1-x)*x*(-k*x+1))**(1/3)/(-1+(1+k)*x
-k*x**2+b*x**8),x)
```

```
[Out] Integral(x**2*(6*k*x**2 - 7*k*x - 7*x + 8)/((x*(x - 1)*(k*x - 1))**(1/3)*(b
*x**8 - k*x**2 + k*x + x - 1)), x)
```

3.1904 $\int \frac{x(-ab+x^2)}{(x^2(-a+x)(-b+x))^{2/3}(ab-(a+b+d)x+x^2)}\,dx$

Optimal. Leaf size=180

$$\frac{\log\left(\sqrt[3]{x^3(-a-b)+abx^2+x^4}-\sqrt[3]{d}\,x\right)}{d^{2/3}}-\frac{\log\left(\sqrt[3]{d}\,x\sqrt[3]{x^3(-a-b)+abx^2+x^4}+\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+d^{2/3}\right.}{2d^{2/3}}$$

Rubi [F] time = 6.86, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\left(-ab+x^2\right)}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(ab-(a+b+d)x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-(a*b) + x^2))/((x^2*(-a + x)*(-b + x))^(2/3)*(a*b - (a + b + d)*x
+ x^2)),x]
```

```
[Out] (3*x^2*(1 - x/a)^(2/3)*(1 - x/b)^(2/3)*AppellF1[2/3, 2/3, 2/3, 5/3, x/a, x/
b])/(2*((a - x)*(b - x)*x^2)^(2/3)) + ((a + b + d + Sqrt[a^2 - 2*a*(b - d)
+ (b + d)^2])*x^(4/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Int][1/(x^(1/3)*(
-a + x)^(2/3)*(-b + x)^(2/3)*(-a - b - d - Sqrt[a^2 - 2*a*b + b^2 + 2*a*d +
 2*b*d + d^2] + 2*x)), x])/((a - x)*(b - x)*x^2)^(2/3) + ((a + b + d - Sqrt
[a^2 - 2*a*(b - d) + (b + d)^2])*x^(4/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defe
r[Int][1/(x^(1/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*(-a - b - d + Sqrt[a^2 - 2*
a*b + b^2 + 2*a*d + 2*b*d + d^2] + 2*x)), x])/((a - x)*(b - x)*x^2)^(2/3)
```

Rubi steps

$$\int \frac{x\left(-ab+x^2\right)}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(ab-(a+b+d)x+x^2\right)}\,dx = \frac{\left(x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{-ab+x^2}{\sqrt[3]{x}\,(-a+x)^{2/3}(-b+x)^{2/3}\left(ab-(a+b\right.}}{\left(x^2(-a+x)(-b+x)\right)^{2/3}}$$

$$= \frac{\left(x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int\left(\frac{1}{\sqrt[3]{x}\,(-a+x)^{2/3}(-b+x)^{2/3}}-\frac{}{\sqrt[3]{x}\,(}\right.}{\left(x^2(-a+x)(-b+x)\right)^{2/}}$$

$$= \frac{\left(x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{1}{\sqrt[3]{x}\,(-a+x)^{2/3}(-b+x)^{2/3}}\,dx}{\left(x^2(-a+x)(-b+x)\right)^{2/3}}-\frac{(}{}$$

$$= -\frac{\left(x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int\left(\frac{-a-b-d-\sqrt{a^2-}}{\sqrt[3]{x}\,(-a+x)^{2/3}(-b+x)^{2/3}\left(-a-b-\right.}\right.}{}$$

$$= -\frac{\left(\left(-a-b-d-\sqrt{a^2-2a(b-d)+(b+d)^2}\right)x^{4/3}(-a+\right.}{(x^2}$$

$$= \frac{3x^2\left(1-\frac{x}{a}\right)^{2/3}\left(1-\frac{x}{b}\right)^{2/3}F_1\left(\frac{2}{3};\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{x}{a},\frac{x}{b}\right)}{2\left((a-x)(b-x)x^2\right)^{2/3}}-\frac{\left(\left(-a-b\right.\right.}{}$$

Mathematica [F] time = 5.48, size = 0, normalized size = 0.00

$$\int \frac{x\left(-ab+x^2\right)}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(ab-(a+b+d)x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(-(a*b) + x^2))/((x^2*(-a + x)*(-b + x))^(2/3)*(a*b - (a + b +
 d)*x + x^2)),x]
```

```
[Out] Integrate[(x*(-(a*b) + x^2))/((x^2*(-a + x)*(-b + x))^(2/3)*(a*b - (a + b +
 d)*x + x^2)), x]
```

IntegrateAlgebraic [A] time = 0.54, size = 180, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{x^3(-a-b)+abx^2+x^4}-\sqrt[3]{d}\,x\right)}{d^{2/3}}-\frac{\log\left(\sqrt[3]{d}\,x\sqrt[3]{x^3(-a-b)+abx^2+x^4}+\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+d^{2/3}x^2\right)}{2d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\,x}{2\sqrt[3]{x^3(-a-b)+abx^2+x^4}+\sqrt[3]{d}\,x}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-(a*b) + x^2))/((x^2*(-a + x)*(-b + x))^(2/3)*(a*b -
 (a + b + d)*x + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*x)/(d^(1/3)*x + 2*(a*b*x^2 + (-a - b)*x^3
+ x^4)^(1/3))])/d^(2/3) + Log[-(d^(1/3)*x) + (a*b*x^2 + (-a - b)*x^3 + x^4)
^(1/3)]/d^(2/3) - Log[d^(2/3)*x^2 + d^(1/3)*x*(a*b*x^2 + (-a - b)*x^3 + x^4
)^(1/3) + (a*b*x^2 + (-a - b)*x^3 + x^4)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(a*b-(a+b+d)*x+x^2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(ab-x^2\right)x}{\left((a-x)(b-x)x^2\right)^{\frac{2}{3}}\left(ab-(a+b+d)x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(a*b-(a+b+d)*x+x^2),x, alg
orithm="giac")
```

```
[Out] integrate(-(a*b - x^2)*x/(((a - x)*(b - x)*x^2)^(2/3)*(a*b - (a + b + d)*x
+ x^2)), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x\left(-ab+x^2\right)}{\left(x^2\left(-a+x\right)\left(-b+x\right)\right)^{\frac{2}{3}}\left(ab-\left(a+b+d\right)x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(a*b-(a+b+d)*x+x^2),x)
```

```
[Out] int(x*(-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(a*b-(a+b+d)*x+x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ab - x^2)x}{((a-x)(b-x)x^2)^{\frac{2}{3}}(ab - (a+b+d)x + x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(a*b-(a+b+d)*x+x^2),x, alg
orithm="maxima")
```

```
[Out] -integrate((a*b - x^2)*x/(((a - x)*(b - x)*x^2)^(2/3)*(a*b - (a + b + d)*x
+ x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x\,(a\,b - x^2)}{(x^2\,(a-x)\,(b-x))^{2/3}\,(x^2 + (-a-b-d)\,x + a\,b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a*b - x^2))/((x^2*(a - x)*(b - x))^(2/3)*(a*b - x*(a + b + d) + x^
2)),x)
```

```
[Out] int(-(x*(a*b - x^2))/((x^2*(a - x)*(b - x))^(2/3)*(a*b - x*(a + b + d) + x^
2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*b+x**2)/(x**2*(-a+x)*(-b+x))**(2/3)/(a*b-(a+b+d)*x+x**2),x)
```

```
[Out] Timed out
```

3.1905 $\int \frac{(b+x^3)(c+x^3)}{\sqrt[3]{a+x^3}}\,dx$

Optimal. Leaf size=180

$$\frac{1}{27}\left(-2a^2+3ab+3ac-9bc\right)\log\left(\sqrt[3]{a+x^3}-x\right)+\frac{1}{27}\left(2\sqrt{3}\,a^2-3\sqrt{3}\,ab-3\sqrt{3}\,ac+9\sqrt{3}\,bc\right)\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{a+x^3}}\right.$$

Rubi [A] time = 0.09, antiderivative size = 124, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {528, 388, 239}

$$-\frac{1}{18}\left(2a^2-3a(b+c)+9bc\right)\log\left(\sqrt[3]{a+x^3}-x\right)+\frac{\left(2a^2-3a(b+c)+9bc\right)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{a+x^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}}-\frac{1}{18}x\left(a+x^3\right)^{2/3}(4a-3b-6c)+\frac{1}{6}x\left(a+x^3\right)^{2/3}\left(b+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[((b + x^3)*(c + x^3))/(a + x^3)^(1/3),x]
```

```
[Out] -1/18*((4*a - 3*b - 6*c)*x*(a + x^3)^(2/3)) + (x*(a + x^3)^(2/3)*(b + x^3))
/6 + ((2*a^2 + 9*b*c - 3*a*(b + c))*ArcTan[(1 + (2*x)/(a + x^3)^(1/3))/Sqrt
[3]])/(9*Sqrt[3]) - ((2*a^2 + 9*b*c - 3*a*(b + c))*Log[-x + (a + x^3)^(1/3)
])/18
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(b+x^3\right)\left(c+x^3\right)}{\sqrt[3]{a+x^3}}\,dx=\frac{1}{6}x\left(a+x^3\right)^{2/3}\left(b+x^3\right)+\frac{1}{6}\int \frac{-b(a-6c)-(4a-3b-6c)x^3}{\sqrt[3]{a+x^3}}\,dx$$

$$=-\frac{1}{18}(4a-3b-6c)x\left(a+x^3\right)^{2/3}+\frac{1}{6}x\left(a+x^3\right)^{2/3}\left(b+x^3\right)+\frac{1}{9}\left(2a^2+9bc-3a(b+c)\right)$$

$$=-\frac{1}{18}(4a-3b-6c)x\left(a+x^3\right)^{2/3}+\frac{1}{6}x\left(a+x^3\right)^{2/3}\left(b+x^3\right)+\frac{\left(2a^2+9bc-3a(b+c)\right)\text{t}}{9\sqrt{3}}$$

Mathematica [A] time = 0.15, size = 259, normalized size = 1.44

$$\frac{1}{54}\left(\left(-4a^2+6a(b+c)-18bc\right)\log\left(1-\frac{x}{\sqrt[3]{a+x^3}}\right)+2\sqrt{3}\left(2a^2-3a(b+c)+9bc\right)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{a+x^3}}+1}{\sqrt{3}}\right)+2a^2\log\left(\frac{x}{\sqrt[3]{a+x^3}}+\frac{x^2}{\left(a+x^3\right)^{2/3}}+1\right)+9bc\log\left(\frac{x}{\sqrt[3]{a+x^3}}+\frac{x^2}{\left(a+x^3\right)^{2/3}}+1\right)+18bx\left(a+x^3\right)^{2/3}-3ab\log\left(\frac{x}{\sqrt[3]{a+x^3}}+\frac{x^2}{\left(a+x^3\right)^{2/3}}+1\right)+18cx\left(a+x^3\right)^{2/3}-3ac\log\left(\frac{x}{\sqrt[3]{a+x^3}}+\frac{x^2}{\left(a+x^3\right)^{2/3}}+1\right)-12ax\left(a+x^3\right)^{2/3}+9x^4\left(a+x^3\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((b + x^3)*(c + x^3))/(a + x^3)^(1/3),x]
```

```
[Out] (-12*a*x*(a + x^3)^(2/3) + 18*b*x*(a + x^3)^(2/3) + 18*c*x*(a + x^3)^(2/3)
+ 9*x^4*(a + x^3)^(2/3) + 2*Sqrt[3]*(2*a^2 + 9*b*c - 3*a*(b + c))*ArcTan[(1
 + (2*x)/(a + x^3)^(1/3))/Sqrt[3]] + (-4*a^2 - 18*b*c + 6*a*(b + c))*Log[1
- x/(a + x^3)^(1/3)] + 2*a^2*Log[1 + x^2/(a + x^3)^(2/3) + x/(a + x^3)^(1/3
)] - 3*a*b*Log[1 + x^2/(a + x^3)^(2/3) + x/(a + x^3)^(1/3)] - 3*a*c*Log[1 +
 x^2/(a + x^3)^(2/3) + x/(a + x^3)^(1/3)] + 9*b*c*Log[1 + x^2/(a + x^3)^(2/
3) + x/(a + x^3)^(1/3)])/54
```

IntegrateAlgebraic [A] time = 0.73, size = 180, normalized size = 1.00

$$\frac{1}{27}\left(-2a^2+3ab+3ac-9bc\right)\log\left(\sqrt[3]{a+x^3}-x\right)+\frac{1}{27}\left(2\sqrt{3}a^2-3\sqrt{3}ab-3\sqrt{3}ac+9\sqrt{3}bc\right)\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{a+x^3}+x}\right)+\frac{1}{54}\left(2a^2-3ab-3ac+9bc\right)\log\left(x\sqrt[3]{a+x^3}+\left(a+x^3\right)^{2/3}+x^2\right)+\frac{1}{18}\left(a+x^3\right)^{2/3}\left(-4ax+6bx+6cx+3x^4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b + x^3)*(c + x^3))/(a + x^3)^(1/3),x]
```

```
[Out] ((a + x^3)^(2/3)*(-4*a*x + 6*b*x + 6*c*x + 3*x^4))/18 + ((2*Sqrt[3]*a^2 - 3
*Sqrt[3]*a*b - 3*Sqrt[3]*a*c + 9*Sqrt[3]*b*c)*ArcTan[(Sqrt[3]*x)/(x + 2*(a
+ x^3)^(1/3))])/27 + ((-2*a^2 + 3*a*b + 3*a*c - 9*b*c)*Log[-x + (a + x^3)^(
1/3)])/27 + ((2*a^2 - 3*a*b - 3*a*c + 9*b*c)*Log[x^2 + x*(a + x^3)^(1/3) +
(a + x^3)^(2/3)])/54
```

fricas [A] time = 0.79, size = 158, normalized size = 0.88

$$-\frac{1}{27}\sqrt{3}\left(2a^2-3ab-3(a-3b)c\right)\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+a\right)^{\frac{1}{3}}}{3x}\right)-\frac{1}{27}\left(2a^2-3ab-3(a-3b)c\right)\log\left(-\frac{x-\left(x^3+a\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{54}\left(2a^2-3ab-3(a-3b)c\right)\log\left(\frac{x^2+\left(x^3+a\right)^{\frac{1}{3}}x+\left(x^3+a\right)^{\frac{2}{3}}}{x^2}\right)+\frac{1}{18}\left(3x^4-2(2a-3b-3c)x\right)\left(x^3+a\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+b)*(x^3+c)/(x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/27*sqrt(3)*(2*a^2 - 3*a*b - 3*(a - 3*b)*c)*arctan(1/3*(sqrt(3)*x + 2*sqr
t(3)*(x^3 + a)^(1/3))/x) - 1/27*(2*a^2 - 3*a*b - 3*(a - 3*b)*c)*log(-(x - (
x^3 + a)^(1/3))/x) + 1/54*(2*a^2 - 3*a*b - 3*(a - 3*b)*c)*log((x^2 + (x^3 +
 a)^(1/3)*x + (x^3 + a)^(2/3))/x^2) + 1/18*(3*x^4 - 2*(2*a - 3*b - 3*c)*x)*
(x^3 + a)^(2/3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+b\right)\left(x^3+c\right)}{\left(x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+b)*(x^3+c)/(x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 + b)*(x^3 + c)/(x^3 + a)^(1/3), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+b\right)\left(x^3+c\right)}{\left(x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+b)*(x^3+c)/(x^3+a)^(1/3),x)
```

```
[Out] int((x^3+b)*(x^3+c)/(x^3+a)^(1/3),x)
```

maxima [B] time = 0.43, size = 412, normalized size = 2.29

$$-\frac{2}{27}\sqrt{3}a^2\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+a\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{1}{6}\left(2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+a\right)^{\frac{1}{3}}}{x}+1\right)\right)-\log\left(\frac{\left(x^3+a\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+a\right)^{\frac{2}{3}}}{x^2}+1\right)+2\log\left(\frac{\left(x^3+a\right)^{\frac{1}{3}}}{x}-1\right)\right)bc+\frac{1}{27}a^2\log\left(\frac{\left(x^3+a\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+a\right)^{\frac{2}{3}}}{x^2}+1\right)-\frac{2}{27}a^2\log\left(\frac{\left(x^3+a\right)^{\frac{1}{3}}}{x}-1\right)+\frac{1}{18}\left(2\sqrt{3}a\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+a\right)^{\frac{1}{3}}}{x}+1\right)\right)-a\log\left(\frac{\left(x^3+a\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+a\right)^{\frac{2}{3}}}{x^2}+1\right)+2a\log\left(\frac{\left(x^3+a\right)^{\frac{1}{3}}}{x}-1\right)+\frac{6\left(x^3+a\right)^{\frac{2}{3}}a}{x^2\left(\frac{x^3+a}{x^3}-1\right)}\right)b+\frac{1}{18}\left(2\sqrt{3}a\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+a\right)^{\frac{1}{3}}}{x}+1\right)\right)-a\log\left(\frac{\left(x^3+a\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+a\right)^{\frac{2}{3}}}{x^2}+1\right)+2a\log\left(\frac{\left(x^3+a\right)^{\frac{1}{3}}}{x}-1\right)+\frac{6\left(x^3+a\right)^{\frac{2}{3}}a}{x^2\left(\frac{x^3+a}{x^3}-1\right)}\right)c-\frac{\frac{7\left(x^3+a\right)^{\frac{2}{3}}a^2}{x^2}-\frac{4\left(x^3+a\right)^{\frac{5}{3}}a^2}{x^5}}{18\left(\frac{2\left(x^3+a\right)}{x^3}-\frac{\left(x^3+a\right)^2}{x^6}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+b)*(x^3+c)/(x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -2/27*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*(x^3 + a)^(1/3)/x + 1)) - 1/6*(2*sq
rt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + a)^(1/3)/x + 1)) - log((x^3 + a)^(1/3)/x
 + (x^3 + a)^(2/3)/x^2 + 1) + 2*log((x^3 + a)^(1/3)/x - 1))*b*c + 1/27*a^2*
log((x^3 + a)^(1/3)/x + (x^3 + a)^(2/3)/x^2 + 1) - 2/27*a^2*log((x^3 + a)^(
1/3)/x - 1) + 1/18*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*(x^3 + a)^(1/3)/x + 1
)) - a*log((x^3 + a)^(1/3)/x + (x^3 + a)^(2/3)/x^2 + 1) + 2*a*log((x^3 + a)
^(1/3)/x - 1) + 6*(x^3 + a)^(2/3)*a/(x^2*((x^3 + a)/x^3 - 1)))*b + 1/18*(2*
sqrt(3)*a*arctan(1/3*sqrt(3)*(2*(x^3 + a)^(1/3)/x + 1)) - a*log((x^3 + a)^(
1/3)/x + (x^3 + a)^(2/3)/x^2 + 1) + 2*a*log((x^3 + a)^(1/3)/x - 1) + 6*(x^3
 + a)^(2/3)*a/(x^2*((x^3 + a)/x^3 - 1)))*c - 1/18*(7*(x^3 + a)^(2/3)*a^2/x^
2 - 4*(x^3 + a)^(5/3)*a^2/x^5)/(2*(x^3 + a)/x^3 - (x^3 + a)^2/x^6 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^3+b\right)\,\left(x^3+c\right)}{{\left(x^3+a\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + x^3)*(c + x^3))/(a + x^3)^(1/3),x)
```

```
[Out] int(((b + x^3)*(c + x^3))/(a + x^3)^(1/3), x)
```

sympy [C] time = 3.68, size = 153, normalized size = 0.85

$$\frac{bcx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{x^{3}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{bx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{x^{3}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{7}{3}\right)}+\frac{cx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{x^{3}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{7}{3}\right)}+\frac{x^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{x^{3}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+b)*(x**3+c)/(x**3+a)**(1/3),x)
```

```
[Out] b*c*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_polar(I*pi)/a)/(3*a**(1
/3)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), x**3*exp_pola
r(I*pi)/a)/(3*a**(1/3)*gamma(7/3)) + c*x**4*gamma(4/3)*hyper((1/3, 4/3), (7
/3,), x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3)) + x**7*gamma(7/3)*hyp
er((1/3, 7/3), (10/3,), x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(10/3))
```

3.1906 $$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^3\left(aq+bx^2+apx^3\right)}\,dx$$

Optimal. Leaf size=180

$$\frac{2\sqrt{2a^2pq-b^2}\tan^{-1}\left(\frac{x^2\sqrt{2a^2pq-b^2}}{a\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}+apx^3+aq+bx^2}\right)}{a^2}-\frac{b\log\left(\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}+px^3+q\right)}{a^2}+\frac{2b\log(}{a^2}$$

Rubi [F] time = 2.92, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^3\left(aq+bx^2+apx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6])/(x^3*(a*q
+ b*x^2 + a*p*x^3)),x]
```

```
[Out] (-2*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^3, x])/a + (2*
b*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x, x])/(a^2*q) + 3
*p*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/(a*q + b*x^2 + a*
p*x^3), x] - (2*b^2*Defer[Int][(x*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^
6])/(a*q + b*x^2 + a*p*x^3), x])/(a^2*q) - (2*b*p*Defer[Int][(x^2*Sqrt[q^2
+ 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6])/(a*q + b*x^2 + a*p*x^3), x])/(a*q)
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^3\left(aq+bx^2+apx^3\right)}\,dx &= \int \left(-\frac{2\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{ax^3}+\frac{2b\sqrt{q^2+2pqx^3-2pqx}}{a^2qx}\right.\\
&= -\frac{2\int \frac{\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^3}\,dx}{a}+\frac{\int \frac{\left(3a^2pq-2b^2x-2abpx^2\right)\sqrt{q^2+2pqx^3-2}}{aq+bx^2+apx^3}}{a^2q}\\
&= -\frac{2\int \frac{\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^3}\,dx}{a}+\frac{\int \left(\frac{3a^2pq\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{aq+bx^2+apx^3}-\right.}{}\\
&= -\frac{2\int \frac{\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^3}\,dx}{a}+(3p)\int \frac{\sqrt{q^2+2pqx^3-2pqx^4+}}{aq+bx^2+apx^3}
\end{aligned}$$

Mathematica [F] time = 3.27, size = 0, normalized size = 0.00

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^3\left(aq+bx^2+apx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6])/(x^3
*(a*q + b*x^2 + a*p*x^3)),x]
```

```
[Out] Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6])/(x^3
*(a*q + b*x^2 + a*p*x^3)), x]
```

IntegrateAlgebraic [A] time = 1.01, size = 180, normalized size = 1.00

$$\frac{2\sqrt{2a^2pq-b^2}\tan^{-1}\left(\frac{x^2\sqrt{2a^2pq-b^2}}{a\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}+apx^3+aq+bx^2}\right)}{a^2}-\frac{b\log\left(\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}+px^3+q\right)}{a^2}+\frac{2b\log(x)}{a^2}+\frac{\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x
^6])/(x^3*(a*q + b*x^2 + a*p*x^3)),x]
```

```
[Out] Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/(a*x^2) + (2*Sqrt[-b^2 + 2*a^2*
p*q]*ArcTan[(Sqrt[-b^2 + 2*a^2*p*q]*x^2)/(a*q + b*x^2 + a*p*x^3 + a*Sqrt[q^
2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6])])/a^2 + (2*b*Log[x])/a^2 - (b*Log[q +
 p*x^3 + Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]])/a^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)/x^3/(a*p*x^3+
b*x^2+a*q),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6-2\,pqx^4+2\,pqx^3+q^2}\left(px^3-2\,q\right)}{\left(apx^3+bx^2+aq\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)/x^3/(a*p*x^3+
b*x^2+a*q),x, algorithm="giac")
```

```
[Out] integrate(sqrt(p^2*x^6 - 2*p*q*x^4 + 2*p*q*x^3 + q^2)*(p*x^3 - 2*q)/((a*p*x
^3 + b*x^2 + a*q)*x^3), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(p\,x^3-2q\right)\sqrt{p^2x^6-2pq\,x^4+2pq\,x^3+q^2}}{x^3\left(ap\,x^3+b\,x^2+aq\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)/x^3/(a*p*x^3+b*x^2+
a*q),x)
```

```
[Out] int((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)/x^3/(a*p*x^3+b*x^2+
a*q),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6-2\,pqx^4+2\,pqx^3+q^2}\left(px^3-2\,q\right)}{\left(apx^3+bx^2+aq\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)/x^3/(a*p*x^3+
b*x^2+a*q),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(p^2*x^6 - 2*p*q*x^4 + 2*p*q*x^3 + q^2)*(p*x^3 - 2*q)/((a*p*x
^3 + b*x^2 + a*q)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(2\,q-p\,x^3\right)\,\sqrt{p^2\,x^6-2\,p\,q\,x^4+2\,p\,q\,x^3+q^2}}{x^3\,\left(a\,p\,x^3+b\,x^2+a\,q\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*q - p*x^3)*(p^2*x^6 + q^2 + 2*p*q*x^3 - 2*p*q*x^4)^(1/2))/(x^3*(a*
q + b*x^2 + a*p*x^3)),x)
```

```
[Out] int(-((2*q - p*x^3)*(p^2*x^6 + q^2 + 2*p*q*x^3 - 2*p*q*x^4)^(1/2))/(x^3*(a*
q + b*x^2 + a*p*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**3-2*q)*(p**2*x**6-2*p*q*x**4+2*p*q*x**3+q**2)**(1/2)/x**3/(
a*p*x**3+b*x**2+a*q),x)
```

```
[Out] Timed out
```

3.1907 $\int \frac{\sqrt[4]{x^2+x^6}\left(1+x^8\right)}{x^4\left(-1+x^4\right)}\,dx$

Optimal. Leaf size=180

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^6+x^2}}\right)}{2^{3/4}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}\,x^2-\sqrt{x^6+x^2}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^6+x^2}}\right)}{2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[4]{2}}+\frac{2\sqrt[4]{x^6+x^2}\left(x^4+1\right)}{5x^3}$$

Rubi [C] time = 0.52, antiderivative size = 121, normalized size of antiderivative = 0.67, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2056, 6725, 277, 329, 364, 279, 466, 510}

$$\frac{4\sqrt[4]{x^6+x^2}\,F_1\left(-\frac{5}{8};1,-\frac{1}{4};\frac{3}{8};x^4,-x^4\right)}{5\sqrt[4]{x^4+1}\,x^3}+\frac{8\sqrt[4]{x^6+x^2}\,x\,{}_2F_1\left(\frac{3}{8},\frac{3}{4};\frac{11}{8};-x^4\right)}{15\sqrt[4]{x^4+1}}+\frac{2}{5}\sqrt[4]{x^6+x^2}\,x-\frac{2\sqrt[4]{x^6+x^2}}{5x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((x^2 + x^6)^(1/4)*(1 + x^8))/(x^4*(-1 + x^4)),x]
```

```
[Out] (-2*(x^2 + x^6)^(1/4))/(5*x^3) + (2*x*(x^2 + x^6)^(1/4))/5 + (4*(x^2 + x^6)
^(1/4)*AppellF1[-5/8, 1, -1/4, 3/8, x^4, -x^4])/(5*x^3*(1 + x^4)^(1/4)) + (
8*x*(x^2 + x^6)^(1/4)*Hypergeometric2F1[3/8, 3/4, 11/8, -x^4])/(15*(1 + x^4
)^(1/4))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{x^2+x^6}\left(1+x^8\right)}{x^4\left(-1+x^4\right)}\,dx = \frac{\sqrt[4]{x^2+x^6}\int \frac{\sqrt[4]{1+x^4}\left(1+x^8\right)}{x^{7/2}\left(-1+x^4\right)}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\sqrt[4]{x^2+x^6}\int \left(\frac{\sqrt[4]{1+x^4}}{x^{7/2}}+\sqrt{x}\sqrt[4]{1+x^4}+\frac{2\sqrt[4]{1+x^4}}{x^{7/2}\left(-1+x^4\right)}\right)dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\sqrt[4]{x^2+x^6}\int \frac{\sqrt[4]{1+x^4}}{x^{7/2}}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\sqrt[4]{x^2+x^6}\int \sqrt{x}\sqrt[4]{1+x^4}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{\sqrt[4]{1+x^4}}{x^{7/2}\left(-1+x^4\right)}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= -\frac{2\sqrt[4]{x^2+x^6}}{5x^3} + \frac{2}{5}x\sqrt[4]{x^2+x^6} + 2\frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{\sqrt{x}}{\left(1+x^4\right)^{3/4}}\,dx}{5\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(4\sqrt[4]{x^2+x^6}\right)\text{Subst}\Big(\int$$

$$= -\frac{2\sqrt[4]{x^2+x^6}}{5x^3} + \frac{2}{5}x\sqrt[4]{x^2+x^6} + \frac{4\sqrt[4]{x^2+x^6}\,F_1\left(-\frac{5}{8};1,-\frac{1}{4};\frac{3}{8};x^4,-x^4\right)}{5x^3\sqrt[4]{1+x^4}} + 2\frac{\left(4\sqrt[4]{x^2+x^6}\right)}{}$$

$$= -\frac{2\sqrt[4]{x^2+x^6}}{5x^3} + \frac{2}{5}x\sqrt[4]{x^2+x^6} + \frac{4\sqrt[4]{x^2+x^6}\,F_1\left(-\frac{5}{8};1,-\frac{1}{4};\frac{3}{8};x^4,-x^4\right)}{5x^3\sqrt[4]{1+x^4}} + \frac{8x\sqrt[4]{x^2+x^6}\;{}_2F}{15\sqrt[4]{1}}$$

Mathematica [C] time = 0.08, size = 63, normalized size = 0.35

$$\frac{2\sqrt[4]{x^6+x^2}\left(3\left(x^4+1\right)^{5/4}-10x^4F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};-x^4,x^4\right)\right)}{15x^3\sqrt[4]{x^4+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((x^2 + x^6)^(1/4)*(1 + x^8))/(x^4*(-1 + x^4)),x]
```

```
[Out] (2*(x^2 + x^6)^(1/4)*(3*(1 + x^4)^(5/4) - 10*x^4*AppellF1[3/8, -1/4, 1, 11/
8, -x^4, x^4]))/(15*x^3*(1 + x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.75, size = 180, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2^{3/4}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[4]{2}}+\frac{2\sqrt[4]{x^6+x^2}\left(x^4+1\right)}{5x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((x^2 + x^6)^(1/4)*(1 + x^8))/(x^4*(-1 + x^4)),x]
```

```
[Out] (2*(1 + x^4)*(x^2 + x^6)^(1/4))/(5*x^3) + ArcTan[(2^(1/4)*x)/(x^2 + x^6)^(1
/4)]/2^(3/4) + ArcTan[(2^(3/4)*x*(x^2 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^2
 + x^6])]/(2*2^(1/4)) - ArcTanh[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/2^(3/4) + Ar
cTanh[(x^2/2^(1/4) + Sqrt[x^2 + x^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))]/(2*2^(
1/4))
```

fricas [B] time = 12.68, size = 1099, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^2)^(1/4)*(x^8+1)/x^4/(x^4-1),x, algorithm="fricas")
```

```
[Out] 1/320*(20*8^(3/4)*sqrt(2)*x^3*arctan(-1/8*(8*x^9 + 32*x^7 + 48*x^5 + 4*8^(3
/4)*sqrt(2)*(x^6 + x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 32*x^3 + 16*8^(1/4)*sqrt(
2)*(3*x^6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 32*sqrt(2)*sqrt(x^6 + x^2)*(
x^5 + 2*x^3 + x) - sqrt(2)*(128*sqrt(2)*(x^6 + x^2)^(3/4)*x^2 + 8^(3/4)*sqr
t(2)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3 + x) + 8*8^(1/4)*sqrt(2)*sqrt(x^6 + x^2
)*(x^5 - 6*x^3 + x) + 32*(x^6 + 2*x^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt((8^(3/
4)*sqrt(2)*(x^6 + x^2)^(1/4)*x^2 + 2*8^(1/4)*sqrt(2)*(x^6 + x^2)^(3/4) + sq
rt(2)*(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x)/(x^5 + 2*x^3 + x)) + 8*x)/(x
^9 - 28*x^7 + 6*x^5 - 28*x^3 + x)) - 20*8^(3/4)*sqrt(2)*x^3*arctan(-1/8*(8*
x^9 + 32*x^7 + 48*x^5 - 4*8^(3/4)*sqrt(2)*(x^6 + x^2)^(3/4)*(x^4 - 6*x^2 +
1) + 32*x^3 - 16*8^(1/4)*sqrt(2)*(3*x^6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4)
+ 32*sqrt(2)*sqrt(x^6 + x^2)*(x^5 + 2*x^3 + x) - sqrt(2)*(128*sqrt(2)*(x^6
+ x^2)^(3/4)*x^2 - 8^(3/4)*sqrt(2)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3 + x) - 8*
8^(1/4)*sqrt(2)*sqrt(x^6 + x^2)*(x^5 - 6*x^3 + x) + 32*(x^6 + 2*x^4 + x^2)*
(x^6 + x^2)^(1/4))*sqrt(-(8^(3/4)*sqrt(2)*(x^6 + x^2)^(1/4)*x^2 + 2*8^(1/4)
*sqrt(2)*(x^6 + x^2)^(3/4) - sqrt(2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*
x)/(x^5 + 2*x^3 + x)) + 8*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3 + x)) + 5*8^(3/
4)*sqrt(2)*x^3*log(8*(8^(3/4)*sqrt(2)*(x^6 + x^2)^(1/4)*x^2 + 2*8^(1/4)*sqr
t(2)*(x^6 + x^2)^(3/4) + sqrt(2)*(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x)/(
x^5 + 2*x^3 + x)) - 5*8^(3/4)*sqrt(2)*x^3*log(-8*(8^(3/4)*sqrt(2)*(x^6 + x^
2)^(1/4)*x^2 + 2*8^(1/4)*sqrt(2)*(x^6 + x^2)^(3/4) - sqrt(2)*(x^5 + 2*x^3 +
 x) - 8*sqrt(x^6 + x^2)*x)/(x^5 + 2*x^3 + x)) + 40*8^(3/4)*x^3*arctan(1/8*(
16*8^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 2^(3/4)*(8^(3/4)*(x^5 + 2*x^3 + x) + 8*8
^(1/4)*sqrt(x^6 + x^2)*x) + 4*8^(3/4)*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x))
 - 10*8^(3/4)*x^3*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4)*x^2 + 8^(3/4)*sqrt(x^6
+ x^2)*x + 8^(1/4)*(x^5 + 2*x^3 + x) + 4*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 +
x)) + 10*8^(3/4)*x^3*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4)*x^2 - 8^(3/4)*sqrt(x
^6 + x^2)*x - 8^(1/4)*(x^5 + 2*x^3 + x) + 4*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3
 + x)) + 128*(x^6 + x^2)^(1/4)*(x^4 + 1))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+1\right)\left(x^6+x^2\right)^{\frac{1}{4}}}{\left(x^4-1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^2)^(1/4)*(x^8+1)/x^4/(x^4-1),x, algorithm="giac")
```

```
[Out] integrate((x^8 + 1)*(x^6 + x^2)^(1/4)/((x^4 - 1)*x^4), x)
```

maple [C] time = 30.05, size = 1588, normalized size = 8.82

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+x^2)^(1/4)*(x^8+1)/x^4/(x^4-1),x)
```

```
[Out] 2/5*(x^8+2*x^4+1)/x^3*(x^2*(x^4+1))^(1/4)/(x^4+1)+(-1/4*RootOf(_Z^4+2)*ln(-
(RootOf(_Z^4+2)*x^12-2*(x^14+3*x^10+3*x^6+x^2)^(1/4)*RootOf(_Z^4+2)^2*x^8-2
*RootOf(_Z^4+2)*x^10+2*(x^14+3*x^10+3*x^6+x^2)^(1/2)*RootOf(_Z^4+2)^3*x^4+3
*RootOf(_Z^4+2)*x^8-4*(x^14+3*x^10+3*x^6+x^2)^(1/4)*RootOf(_Z^4+2)^2*x^4-4*
RootOf(_Z^4+2)*x^6+2*RootOf(_Z^4+2)^3*(x^14+3*x^10+3*x^6+x^2)^(1/2)+3*RootO
f(_Z^4+2)*x^4+4*(x^14+3*x^10+3*x^6+x^2)^(3/4)-2*(x^14+3*x^10+3*x^6+x^2)^(1/
4)*RootOf(_Z^4+2)^2-2*RootOf(_Z^4+2)*x^2+RootOf(_Z^4+2))/(x^2+1)^2/(x^4+1)^
2)-1/4*RootOf(_Z^2+RootOf(_Z^4+2)^2)*ln(-(RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^1
2+2*(x^14+3*x^10+3*x^6+x^2)^(1/4)*RootOf(_Z^4+2)^2*x^8-2*RootOf(_Z^2+RootOf
(_Z^4+2)^2)*x^10-2*(x^14+3*x^10+3*x^6+x^2)^(1/2)*RootOf(_Z^4+2)^2*RootOf(_Z
^2+RootOf(_Z^4+2)^2)*x^4+3*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^8+4*(x^14+3*x^10
+3*x^6+x^2)^(1/4)*RootOf(_Z^4+2)^2*x^4-4*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^6-
2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^4+2)^2*(x^14+3*x^10+3*x^6+x^2)^(1
/2)+3*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^4+4*(x^14+3*x^10+3*x^6+x^2)^(3/4)+2*(
x^14+3*x^10+3*x^6+x^2)^(1/4)*RootOf(_Z^4+2)^2-2*RootOf(_Z^2+RootOf(_Z^4+2)^
2)*x^2+RootOf(_Z^2+RootOf(_Z^4+2)^2))/(x^2+1)^2/(x^4+1)^2)-1/4*ln((RootOf(_
Z^4+2)^3*x^10+2*RootOf(_Z^4+2)^3*x^6-(x^14+3*x^10+3*x^6+x^2)^(1/2)*RootOf(_
Z^4+2)*x^4+RootOf(_Z^4+2)^3*x^2+2*(x^14+3*x^10+3*x^6+x^2)^(3/4)-(x^14+3*x^1
0+3*x^6+x^2)^(1/2)*RootOf(_Z^4+2))/(x^4+1)^2/(1+x)/x/(-1+x))*RootOf(_Z^4+2)
^3-1/4*ln((RootOf(_Z^4+2)^3*x^10+2*RootOf(_Z^4+2)^3*x^6-(x^14+3*x^10+3*x^6+
x^2)^(1/2)*RootOf(_Z^4+2)*x^4+RootOf(_Z^4+2)^3*x^2+2*(x^14+3*x^10+3*x^6+x^2
)^(3/4)-(x^14+3*x^10+3*x^6+x^2)^(1/2)*RootOf(_Z^4+2))/(x^4+1)^2/(1+x)/x/(-1
+x))*RootOf(_Z^4+2)^2*RootOf(_Z^2+RootOf(_Z^4+2)^2)+1/4*RootOf(_Z^4+2)^2*Ro
otOf(_Z^2+RootOf(_Z^4+2)^2)*ln((RootOf(_Z^4+2)^3*x^12-x^12*RootOf(_Z^2+Root
Of(_Z^4+2)^2)*RootOf(_Z^4+2)^2+2*RootOf(_Z^4+2)^3*x^10-2*RootOf(_Z^4+2)^2*R
ootOf(_Z^2+RootOf(_Z^4+2)^2)*x^10+4*(x^14+3*x^10+3*x^6+x^2)^(1/4)*RootOf(_Z
^4+2)*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^8+3*RootOf(_Z^4+2)^3*x^8-3*RootOf(_Z^
4+2)^2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^8+4*RootOf(_Z^4+2)^3*x^6-4*RootOf(_Z
^4+2)^2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^6-4*(x^14+3*x^10+3*x^6+x^2)^(1/2)*R
ootOf(_Z^4+2)*x^4-4*(x^14+3*x^10+3*x^6+x^2)^(1/2)*RootOf(_Z^2+RootOf(_Z^4+2
)^2)*x^4+8*(x^14+3*x^10+3*x^6+x^2)^(1/4)*RootOf(_Z^4+2)*RootOf(_Z^2+RootOf(
_Z^4+2)^2)*x^4+3*RootOf(_Z^4+2)^3*x^4-3*RootOf(_Z^4+2)^2*RootOf(_Z^2+RootOf
(_Z^4+2)^2)*x^4+2*RootOf(_Z^4+2)^3*x^2-2*RootOf(_Z^4+2)^2*RootOf(_Z^2+RootO
f(_Z^4+2)^2)*x^2+8*(x^14+3*x^10+3*x^6+x^2)^(3/4)-4*(x^14+3*x^10+3*x^6+x^2)^
(1/2)*RootOf(_Z^4+2)-4*(x^14+3*x^10+3*x^6+x^2)^(1/2)*RootOf(_Z^2+RootOf(_Z^
4+2)^2)+4*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^4+2)*(x^14+3*x^10+3*x^6+x
^2)^(1/4)+RootOf(_Z^4+2)^3-RootOf(_Z^4+2)^2*RootOf(_Z^2+RootOf(_Z^4+2)^2))/
(1+x)^2/(-1+x)^2/(x^4+1)^2))*(x^2*(x^4+1))^(1/4)/x*(x^2*(x^4+1)^3)^(1/4)/(x
^4+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+1\right)\left(x^6+x^2\right)^{\frac{1}{4}}}{\left(x^4-1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^2)^(1/4)*(x^8+1)/x^4/(x^4-1),x, algorithm="maxima")
[Out] integrate((x^8 + 1)*(x^6 + x^2)^(1/4)/((x^4 - 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6+x^2\right)^{1/4}\,\left(x^8+1\right)}{x^4\,\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + x^6)^(1/4)*(x^8 + 1))/(x^4*(x^4 - 1)),x)
[Out] int(((x^2 + x^6)^(1/4)*(x^8 + 1))/(x^4*(x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\left(x^4+1\right)}\,\left(x^8+1\right)}{x^4\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+x**2)**(1/4)*(x**8+1)/x**4/(x**4-1),x)
[Out] Integral((x**2*(x**4 + 1))**(1/4)*(x**8 + 1)/(x**4*(x - 1)*(x + 1)*(x**2 +
1)), x)
```

3.1908 $\int \frac{\sqrt[4]{x^2+x^6}\left(1+x^8\right)}{x^4\left(-1+x^4\right)}\,dx$

Optimal. Leaf size=180

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2^{3/4}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[4]{2}}+\frac{2\sqrt[4]{x^6+x^2}\left(x^4+1\right)}{5x^3}$$

Rubi [C] time = 0.39, antiderivative size = 121, normalized size of antiderivative = 0.67, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2056, 6725, 277, 329, 364, 279, 466, 510}

$$\frac{4\sqrt[4]{x^6+x^2}F_1\left(-\frac{5}{8};1,-\frac{1}{4};\frac{3}{8};x^4,-x^4\right)}{5\sqrt[4]{x^4+1}x^3}+\frac{8\sqrt[4]{x^6+x^2}x\,{}_2F_1\left(\frac{3}{8},\frac{3}{4};\frac{11}{8};-x^4\right)}{15\sqrt[4]{x^4+1}}+\frac{2}{5}\sqrt[4]{x^6+x^2}x-\frac{2\sqrt[4]{x^6+x^2}}{5x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((x^2 + x^6)^(1/4)*(1 + x^8))/(x^4*(-1 + x^4)),x]
```

```
[Out] (-2*(x^2 + x^6)^(1/4))/(5*x^3) + (2*x*(x^2 + x^6)^(1/4))/5 + (4*(x^2 + x^6)
^(1/4)*AppellF1[-5/8, 1, -1/4, 3/8, x^4, -x^4])/(5*x^3*(1 + x^4)^(1/4)) + (
8*x*(x^2 + x^6)^(1/4)*Hypergeometric2F1[3/8, 3/4, 11/8, -x^4])/(15*(1 + x^4
)^(1/4))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{x^2+x^6}\left(1+x^8\right)}{x^4\left(-1+x^4\right)}\,dx = \frac{\sqrt[4]{x^2+x^6}\int \frac{\sqrt[4]{1+x^4}\left(1+x^8\right)}{x^{7/2}\left(-1+x^4\right)}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\sqrt[4]{x^2+x^6}\int \left(\frac{\sqrt[4]{1+x^4}}{x^{7/2}}+\sqrt{x}\sqrt[4]{1+x^4}+\frac{2\sqrt[4]{1+x^4}}{x^{7/2}\left(-1+x^4\right)}\right)dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\sqrt[4]{x^2+x^6}\int \frac{\sqrt[4]{1+x^4}}{x^{7/2}}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\sqrt[4]{x^2+x^6}\int \sqrt{x}\sqrt[4]{1+x^4}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{\sqrt[4]{1+x^4}}{x^{7/2}\left(-1+x^4\right)}\,d}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= -\frac{2\sqrt[4]{x^2+x^6}}{5x^3} + \frac{2}{5}x\sqrt[4]{x^2+x^6} + 2\frac{\left(2\sqrt[4]{x^2+x^6}\right)\int \frac{\sqrt{x}}{\left(1+x^4\right)^{3/4}}\,dx}{5\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(4\sqrt[4]{x^2+x^6}\right)\text{Subst}}{\sqrt{x}\sqrt[4]{}}$$

$$= -\frac{2\sqrt[4]{x^2+x^6}}{5x^3} + \frac{2}{5}x\sqrt[4]{x^2+x^6} + \frac{4\sqrt[4]{x^2+x^6}\,F_1\left(-\frac{5}{8};1,-\frac{1}{4};\frac{3}{8};x^4,-x^4\right)}{5x^3\sqrt[4]{1+x^4}} + 2\frac{\left(4\sqrt[4]{x^2+x^6}\right.}{}$$

$$= -\frac{2\sqrt[4]{x^2+x^6}}{5x^3} + \frac{2}{5}x\sqrt[4]{x^2+x^6} + \frac{4\sqrt[4]{x^2+x^6}\,F_1\left(-\frac{5}{8};1,-\frac{1}{4};\frac{3}{8};x^4,-x^4\right)}{5x^3\sqrt[4]{1+x^4}} + \frac{8x\sqrt[4]{x^2+x^6}}{15}$$

Mathematica [C] time = 0.04, size = 63, normalized size = 0.35

$$\frac{2\sqrt[4]{x^6+x^2}\left(3\left(x^4+1\right)^{5/4}-10x^4F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};-x^4,x^4\right)\right)}{15x^3\sqrt[4]{x^4+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((x^2 + x^6)^(1/4)*(1 + x^8))/(x^4*(-1 + x^4)),x]
```

```
[Out] (2*(x^2 + x^6)^(1/4)*(3*(1 + x^4)^(5/4) - 10*x^4*AppellF1[3/8, -1/4, 1, 11/
8, -x^4, x^4]))/(15*x^3*(1 + x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.00, size = 180, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2^{3/4}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{2\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\sqrt[4]{2}}+\frac{2\sqrt[4]{x^6+x^2}\left(x^4+1\right)}{5x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((x^2 + x^6)^(1/4)*(1 + x^8))/(x^4*(-1 + x^4)),x]
```

```
[Out] (2*(1 + x^4)*(x^2 + x^6)^(1/4))/(5*x^3) + ArcTan[(2^(1/4)*x)/(x^2 + x^6)^(1
/4)]/2^(3/4) + ArcTan[(2^(3/4)*x*(x^2 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^2
 + x^6])]/(2*2^(1/4)) - ArcTanh[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/2^(3/4) + Ar
cTanh[(x^2/2^(1/4) + Sqrt[x^2 + x^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))]/(2*2^(
1/4))
```

fricas [B] time = 13.08, size = 1099, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^2)^(1/4)*(x^8+1)/x^4/(x^4-1),x, algorithm="fricas")
```

```
[Out] 1/320*(20*8^(3/4)*sqrt(2)*x^3*arctan(-1/8*(8*x^9 + 32*x^7 + 48*x^5 + 4*8^(3
/4)*sqrt(2)*(x^6 + x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 32*x^3 + 16*8^(1/4)*sqrt(
2)*(3*x^6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 32*sqrt(2)*sqrt(x^6 + x^2)*(
x^5 + 2*x^3 + x) - sqrt(2)*(128*sqrt(2)*(x^6 + x^2)^(3/4)*x^2 + 8^(3/4)*sqr
t(2)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3 + x) + 8*8^(1/4)*sqrt(2)*sqrt(x^6 + x^2
)*(x^5 - 6*x^3 + x) + 32*(x^6 + 2*x^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt((8^(3/
4)*sqrt(2)*(x^6 + x^2)^(1/4)*x^2 + 2*8^(1/4)*sqrt(2)*(x^6 + x^2)^(3/4) + sq
rt(2)*(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x)/(x^5 + 2*x^3 + x)) + 8*x)/(x
^9 - 28*x^7 + 6*x^5 - 28*x^3 + x)) - 20*8^(3/4)*sqrt(2)*x^3*arctan(-1/8*(8*
x^9 + 32*x^7 + 48*x^5 - 4*8^(3/4)*sqrt(2)*(x^6 + x^2)^(3/4)*(x^4 - 6*x^2 +
1) + 32*x^3 - 16*8^(1/4)*sqrt(2)*(3*x^6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4)
+ 32*sqrt(2)*sqrt(x^6 + x^2)*(x^5 + 2*x^3 + x) - sqrt(2)*(128*sqrt(2)*(x^6
+ x^2)^(3/4)*x^2 - 8^(3/4)*sqrt(2)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3 + x) - 8*
8^(1/4)*sqrt(2)*sqrt(x^6 + x^2)*(x^5 - 6*x^3 + x) + 32*(x^6 + 2*x^4 + x^2)*
(x^6 + x^2)^(1/4))*sqrt(-(8^(3/4)*sqrt(2)*(x^6 + x^2)^(1/4)*x^2 + 2*8^(1/4)
*sqrt(2)*(x^6 + x^2)^(3/4) - sqrt(2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*
x)/(x^5 + 2*x^3 + x)) + 8*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3 + x)) + 5*8^(3/
4)*sqrt(2)*x^3*log(8*(8^(3/4)*sqrt(2)*(x^6 + x^2)^(1/4)*x^2 + 2*8^(1/4)*sqr
t(2)*(x^6 + x^2)^(3/4) + sqrt(2)*(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x)/(
x^5 + 2*x^3 + x)) - 5*8^(3/4)*sqrt(2)*x^3*log(-8*(8^(3/4)*sqrt(2)*(x^6 + x^
2)^(1/4)*x^2 + 2*8^(1/4)*sqrt(2)*(x^6 + x^2)^(3/4) - sqrt(2)*(x^5 + 2*x^3 +
 x) - 8*sqrt(x^6 + x^2)*x)/(x^5 + 2*x^3 + x)) + 40*8^(3/4)*x^3*arctan(1/8*(
16*8^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 2^(3/4)*(8^(3/4)*(x^5 + 2*x^3 + x) + 8*8
^(1/4)*sqrt(x^6 + x^2)*x) + 4*8^(3/4)*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x))
 - 10*8^(3/4)*x^3*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4)*x^2 + 8^(3/4)*sqrt(x^6
+ x^2)*x + 8^(1/4)*(x^5 + 2*x^3 + x) + 4*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 +
x)) + 10*8^(3/4)*x^3*log(-(4*sqrt(2)*(x^6 + x^2)^(1/4)*x^2 - 8^(3/4)*sqrt(x
^6 + x^2)*x - 8^(1/4)*(x^5 + 2*x^3 + x) + 4*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3
 + x)) + 128*(x^6 + x^2)^(1/4)*(x^4 + 1))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+1\right)\left(x^6+x^2\right)^{\frac{1}{4}}}{\left(x^4-1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^2)^(1/4)*(x^8+1)/x^4/(x^4-1),x, algorithm="giac")
```

```
[Out] integrate((x^8 + 1)*(x^6 + x^2)^(1/4)/((x^4 - 1)*x^4), x)
```

maple [C] time = 28.21, size = 1591, normalized size = 8.84

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+x^2)^(1/4)*(x^8+1)/x^4/(x^4-1),x)
```

```
[Out] 2/5*(x^8+2*x^4+1)/x^3*(x^2*(x^4+1))^(1/4)/(x^4+1)+(1/4*RootOf(_Z^2+RootOf(_
Z^4+2)^2)*ln(-(-RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^12+2*(x^14+3*x^10+3*x^6+x^2
)^(1/4)*RootOf(_Z^4+2)^2*x^8+2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^10+2*(x^14+3
*x^10+3*x^6+x^2)^(1/2)*RootOf(_Z^4+2)^2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^4-3
*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^8+4*(x^14+3*x^10+3*x^6+x^2)^(1/4)*RootOf(_
Z^4+2)^2*x^4+4*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^6+2*RootOf(_Z^2+RootOf(_Z^4+
2)^2)*RootOf(_Z^4+2)^2*(x^14+3*x^10+3*x^6+x^2)^(1/2)-3*RootOf(_Z^2+RootOf(_
Z^4+2)^2)*x^4+2*(x^14+3*x^10+3*x^6+x^2)^(1/4)*RootOf(_Z^4+2)^2+4*(x^14+3*x^
10+3*x^6+x^2)^(3/4)+2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^2-RootOf(_Z^2+RootOf(
_Z^4+2)^2))/(x^2+1)^2/(x^4+1)^2)-1/4*RootOf(_Z^4+2)*ln(-(RootOf(_Z^4+2)*x^1
2-2*(x^14+3*x^10+3*x^6+x^2)^(1/4)*RootOf(_Z^4+2)^2*x^8-2*RootOf(_Z^4+2)*x^1
0+2*(x^14+3*x^10+3*x^6+x^2)^(1/2)*RootOf(_Z^4+2)^3*x^4+3*RootOf(_Z^4+2)*x^8
-4*(x^14+3*x^10+3*x^6+x^2)^(1/4)*RootOf(_Z^4+2)^2*x^4-4*RootOf(_Z^4+2)*x^6+
2*RootOf(_Z^4+2)^3*(x^14+3*x^10+3*x^6+x^2)^(1/2)+3*RootOf(_Z^4+2)*x^4+4*(x^
14+3*x^10+3*x^6+x^2)^(3/4)-2*(x^14+3*x^10+3*x^6+x^2)^(1/4)*RootOf(_Z^4+2)^2
-2*RootOf(_Z^4+2)*x^2+RootOf(_Z^4+2))/(x^2+1)^2/(x^4+1)^2)-1/4*ln((RootOf(_
Z^4+2)^3*x^10+2*RootOf(_Z^4+2)^3*x^6-(x^14+3*x^10+3*x^6+x^2)^(1/2)*RootOf(_
Z^4+2)*x^4+RootOf(_Z^4+2)^3*x^2+2*(x^14+3*x^10+3*x^6+x^2)^(3/4)-(x^14+3*x^1
0+3*x^6+x^2)^(1/2)*RootOf(_Z^4+2))/(x^4+1)^2/(1+x)/x/(-1+x))*RootOf(_Z^4+2)
^3-1/4*ln((RootOf(_Z^4+2)^3*x^10+2*RootOf(_Z^4+2)^3*x^6-(x^14+3*x^10+3*x^6+
x^2)^(1/2)*RootOf(_Z^4+2)*x^4+RootOf(_Z^4+2)^3*x^2+2*(x^14+3*x^10+3*x^6+x^2
)^(3/4)-(x^14+3*x^10+3*x^6+x^2)^(1/2)*RootOf(_Z^4+2))/(x^4+1)^2/(1+x)/x/(-1
+x))*RootOf(_Z^4+2)^2*RootOf(_Z^2+RootOf(_Z^4+2)^2)+1/4*RootOf(_Z^4+2)^2*Ro
otOf(_Z^2+RootOf(_Z^4+2)^2)*ln((RootOf(_Z^4+2)^3*x^12-x^12*RootOf(_Z^2+Root
Of(_Z^4+2)^2)*RootOf(_Z^4+2)^2+2*RootOf(_Z^4+2)^3*x^10-2*RootOf(_Z^4+2)^2*R
ootOf(_Z^2+RootOf(_Z^4+2)^2)*x^10+4*(x^14+3*x^10+3*x^6+x^2)^(1/4)*RootOf(_Z
^4+2)*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^8+3*RootOf(_Z^4+2)^3*x^8-3*RootOf(_Z^
4+2)^2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^8+4*RootOf(_Z^4+2)^3*x^6-4*RootOf(_Z
^4+2)^2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*x^6-4*(x^14+3*x^10+3*x^6+x^2)^(1/2)*R
ootOf(_Z^4+2)*x^4-4*(x^14+3*x^10+3*x^6+x^2)^(1/2)*RootOf(_Z^2+RootOf(_Z^4+2
)^2)*x^4+8*(x^14+3*x^10+3*x^6+x^2)^(1/4)*RootOf(_Z^4+2)*RootOf(_Z^2+RootOf(
_Z^4+2)^2)*x^4+3*RootOf(_Z^4+2)^3*x^4-3*RootOf(_Z^4+2)^2*RootOf(_Z^2+RootOf
(_Z^4+2)^2)*x^4+2*RootOf(_Z^4+2)^3*x^2-2*RootOf(_Z^4+2)^2*RootOf(_Z^2+RootO
f(_Z^4+2)^2)*x^2+8*(x^14+3*x^10+3*x^6+x^2)^(3/4)-4*(x^14+3*x^10+3*x^6+x^2)^
(1/2)*RootOf(_Z^4+2)-4*(x^14+3*x^10+3*x^6+x^2)^(1/2)*RootOf(_Z^2+RootOf(_Z^
4+2)^2)+4*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^4+2)*(x^14+3*x^10+3*x^6+x
^2)^(1/4)+RootOf(_Z^4+2)^3-RootOf(_Z^4+2)^2*RootOf(_Z^2+RootOf(_Z^4+2)^2))/
(1+x)^2/(-1+x)^2/(x^4+1)^2))*(x^2*(x^4+1))^(1/4)/x*(x^2*(x^4+1)^3)^(1/4)/(x
^4+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^8+1)(x^6+x^2)^{\frac{1}{4}}}{(x^4-1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^2)^(1/4)*(x^8+1)/x^4/(x^4-1),x, algorithm="maxima")
[Out] integrate((x^8 + 1)*(x^6 + x^2)^(1/4)/((x^4 - 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^6+x^2)^{1/4}\,(x^8+1)}{x^4\,(x^4-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + x^6)^(1/4)*(x^8 + 1))/(x^4*(x^4 - 1)),x)
[Out] int(((x^2 + x^6)^(1/4)*(x^8 + 1))/(x^4*(x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\,(x^4+1)}\,(x^8+1)}{x^4\,(x-1)\,(x+1)\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+x**2)**(1/4)*(x**8+1)/x**4/(x**4-1),x)
[Out] Integral((x**2*(x**4 + 1))**(1/4)*(x**8 + 1)/(x**4*(x - 1)*(x + 1)*(x**2 +
1)), x)
```

3.1909 $\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx$

Optimal. Leaf size=181

$$\frac{\log\left(2\sqrt[3]{x^2-3x+2}+\sqrt[3]{2}x-2\sqrt[3]{2}\right)}{2\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+4\left(x^2-3x+2\right)^{2/3}+\left(4\sqrt[3]{2}-2\sqrt[3]{2}x\right)\sqrt[3]{x^2-3x+2}-4\,2^{2/3}\right.}{4\sqrt[3]{2}}$$

Rubi [A] time = 0.03, antiderivative size = 176, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {755, 123}

$$\frac{3\sqrt[3]{x-2}\sqrt[3]{x-1}\log\left(-\frac{(x-2)^{2/3}}{\sqrt[3]{2}}-\sqrt[3]{2}\sqrt[3]{x-1}\right)}{4\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}-\frac{\sqrt[3]{x-2}\sqrt[3]{x-1}\log(x)}{2\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}-\frac{\sqrt{3}\sqrt[3]{x-2}\sqrt[3]{x-1}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2}(x-2)^{2/3}}{\sqrt{3}\sqrt[3]{x-1}}\right)}{2\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*(-2 + x)^(1/3)*(-1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2^(1/3)*(-2
 + x)^(2/3))/(Sqrt[3]*(-1 + x)^(1/3))])/(2^(1/3)*(2 - 3*x + x^2)^(1/3)) + (
3*(-2 + x)^(1/3)*(-1 + x)^(1/3)*Log[-((-2 + x)^(2/3)/2^(1/3)) - 2^(1/3)*(-1
 + x)^(1/3)])/(4*2^(1/3)*(2 - 3*x + x^2)^(1/3)) - ((-2 + x)^(1/3)*(-1 + x)^
(1/3)*Log[x])/(2*2^(1/3)*(2 - 3*x + x^2)^(1/3))
```

Rule 123

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)*((e_.) + (f_.)*(x_))
^(1/3)), x_Symbol] :> With[{q = Rt[(b*(b*e - a*f))/(b*c - a*d)^2, 3]}, -Sim
p[Log[a + b*x]/(2*q*(b*c - a*d)), x] + (-Simp[(Sqrt[3]*ArcTan[1/Sqrt[3] + (
2*q*(c + d*x)^(2/3))/(Sqrt[3]*(e + f*x)^(1/3))])/(2*q*(b*c - a*d)), x] + Si
mp[(3*Log[q*(c + d*x)^(2/3) - (e + f*x)^(1/3)])/(4*q*(b*c - a*d)), x])] /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - b*c*f - a*d*f, 0]
```

Rule 755

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[((b + q + 2*c*x)^(1/3)*(b - q
+ 2*c*x)^(1/3))/(a + b*x + c*x^2)^(1/3), Int[1/((d + e*x)*(b + q + 2*c*x)^(
1/3)*(b - q + 2*c*x)^(1/3)), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^
2 - 4*a*c, 0] && EqQ[c^2*d^2 - b*c*d*e - 2*b^2*e^2 + 9*a*c*e^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx &= \frac{\left(\sqrt[3]{-4+2x}\sqrt[3]{-2+2x}\right)\int \frac{1}{x\sqrt[3]{-4+2x}\sqrt[3]{-2+2x}}\,dx}{\sqrt[3]{2-3x+x^2}}\\ &= -\frac{\sqrt{3}\sqrt[3]{-2+x}\sqrt[3]{-1+x}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2}(-2+x)^{2/3}}{\sqrt{3}\sqrt[3]{-1+x}}\right)}{2\sqrt[3]{2}\sqrt[3]{2-3x+x^2}}+\frac{3\sqrt[3]{-2+x}\sqrt[3]{-1+x}\log\left(-\frac{(-2+x)^{2/3}}{\sqrt[3]{2}}\right.}{4\sqrt[3]{2}\sqrt[3]{2-3x+x^2}}\end{aligned}$$

Mathematica [C] time = 0.03, size = 59, normalized size = 0.33

$$-\frac{3\sqrt[3]{1-\frac{2}{x}}\sqrt[3]{1-\frac{1}{x}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{1}{x},\frac{2}{x}\right)}{2\sqrt[3]{x^2-3x+2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

```
[Out] (-3*(1 - 2/x)^(1/3)*(1 - x^(-1))^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, x^(-1),
 2/x])/(2*(2 - 3*x + x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.23, size = 181, normalized size = 1.00

$$\frac{\log\left(2\sqrt[3]{x^2-3x+2}+\sqrt[3]{2}x-2\sqrt[3]{2}\right)}{2\sqrt[3]{2}}-\frac{\log\left(2^{2/3}x^2+4\left(x^2-3x+2\right)^{2/3}+\left(4\sqrt[3]{2}-2\sqrt[3]{2}x\right)\sqrt[3]{x^2-3x+2}-4\,2^{2/3}x+4\,2^{2/3}\right)}{4\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^2-3x+2}}{\sqrt[3]{x^2-3x+2}-\sqrt[3]{2}x+2\sqrt[3]{2}}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(2 - 3*x + x^2)^(1/3))/(2*2^(1/3) - 2^(1/3)*x + (2
 - 3*x + x^2)^(1/3))]])/(2*2^(1/3)) + Log[-2*2^(1/3) + 2^(1/3)*x + 2*(2 - 3*
x + x^2)^(1/3)]/(2*2^(1/3)) - Log[4*2^(2/3) - 4*2^(2/3)*x + 2^(2/3)*x^2 + (
4*2^(1/3) - 2*2^(1/3)*x)*(2 - 3*x + x^2)^(1/3) + 4*(2 - 3*x + x^2)^(2/3)]/(
4*2^(1/3))
```

fricas [A] time = 1.81, size = 277, normalized size = 1.53

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}\left(x^6+36x^5-612x^4+2880x^3-5760x^2+5184x-1728\right)+12\sqrt{2}\left(x^5-38x^4+252x^3-648x^2+720x-288\right)\left(x^2-3x+2\right)^{\frac{1}{3}}+48\cdot 2^{\frac{1}{6}}\left(x^4-6x^3+6x^2\right)\left(x^2-3x+2\right)^{\frac{2}{3}}\right)}{6\left(x^6-108x^5+972x^4-3456x^3+6048x^2-5184x+1728\right)}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}x^2+6\cdot 2^{\frac{1}{3}}\left(x^2-3x+2\right)^{\frac{1}{3}}(x-2)+12\left(x^2-3x+2\right)^{\frac{2}{3}}}{x^2}\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(\frac{12\cdot 2^{\frac{2}{3}}\left(x^2-3x+2\right)^{\frac{2}{3}}\left(x^2-6x+6\right)+2^{\frac{1}{3}}\left(x^4-36x^3+180x^2-288x+144\right)-6\left(x^3-14x^2+36x-24\right)\left(x^2-3x+2\right)^{\frac{1}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^6 + 36*x^5 - 6
12*x^4 + 2880*x^3 - 5760*x^2 + 5184*x - 1728) + 12*sqrt(2)*(x^5 - 38*x^4 +
252*x^3 - 648*x^2 + 720*x - 288)*(x^2 - 3*x + 2)^(1/3) + 48*2^(1/6)*(x^4 -
6*x^3 + 6*x^2)*(x^2 - 3*x + 2)^(2/3))/(x^6 - 108*x^5 + 972*x^4 - 3456*x^3 +
 6048*x^2 - 5184*x + 1728)) + 1/12*2^(2/3)*log((2^(2/3)*x^2 + 6*2^(1/3)*(x^
2 - 3*x + 2)^(1/3)*(x - 2) + 12*(x^2 - 3*x + 2)^(2/3))/x^2) - 1/24*2^(2/3)*
log((12*2^(2/3)*(x^2 - 3*x + 2)^(2/3)*(x^2 - 6*x + 6) + 2^(1/3)*(x^4 - 36*x
^3 + 180*x^2 - 288*x + 144) - 6*(x^3 - 14*x^2 + 36*x - 24)*(x^2 - 3*x + 2)^
(1/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-3x+2\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)
```

maple [C] time = 8.62, size = 1612, normalized size = 8.91

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2-3*x+2)^(1/3),x)
```

```
[Out] -1/4*ln(-(68*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^
3-4)^3*x^2+24*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(
_Z^3-4)^2*x^2+216*(x^2-3*x+2)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^
```

```
3-4)+4*_Z^2)*RootOf(_Z^3-4)^2-306*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-
4)+4*_Z^2)*RootOf(_Z^3-4)^3*x-108*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-
4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x+129*(x^2-3*x+2)^(1/3)*RootOf(_Z^3-4)^2*x-21
6*(x^2-3*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*Roo
tOf(_Z^3-4)*x+306*RootOf(_Z^3-4)^3*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3
-4)+4*_Z^2)+108*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootO
f(_Z^3-4)^2-258*(x^2-3*x+2)^(1/3)*RootOf(_Z^3-4)^2+432*(x^2-3*x+2)^(1/3)*Ro
otOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)+17*RootOf(
_Z^3-4)*x^2+6*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-516*(
x^2-3*x+2)^(2/3)+408*RootOf(_Z^3-4)*x+144*RootOf(RootOf(_Z^3-4)^2+2*_Z*Root
Of(_Z^3-4)+4*_Z^2)*x-408*RootOf(_Z^3-4)-144*RootOf(RootOf(_Z^3-4)^2+2*_Z*Ro
otOf(_Z^3-4)+4*_Z^2))/x^2)*RootOf(_Z^3-4)-1/2*ln(-(68*RootOf(RootOf(_Z^3-4)
^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x^2+24*RootOf(RootOf(_Z^3-4
)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+216*(x^2-3*x+2)^(2/3
)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2-306*
RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x-108*
RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x+12
9*(x^2-3*x+2)^(1/3)*RootOf(_Z^3-4)^2*x-216*(x^2-3*x+2)^(1/3)*RootOf(RootOf(
_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x+306*RootOf(_Z^3-4)^3
*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+108*RootOf(RootOf(_Z^3
-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2-258*(x^2-3*x+2)^(1/3)*
RootOf(_Z^3-4)^2+432*(x^2-3*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(
_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)+17*RootOf(_Z^3-4)*x^2+6*RootOf(RootOf(_Z^3-4
)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-516*(x^2-3*x+2)^(2/3)+408*RootOf(_Z^3-4
)*x+144*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-408*RootOf(_Z
^3-4)-144*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/x^2)*RootOf(
RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+1/2*RootOf(RootOf(_Z^3-4)^2+2*
_Z*RootOf(_Z^3-4)+4*_Z^2)*ln((56*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4
)+4*_Z^2)*RootOf(_Z^3-4)^3*x^2-24*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-
4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+216*(x^2-3*x+2)^(2/3)*RootOf(RootOf(_Z^3-
4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2-252*RootOf(RootOf(_Z^3-4)
^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x+108*RootOf(RootOf(_Z^3-4)
^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-237*(x^2-3*x+2)^(1/3)*R
ootOf(_Z^3-4)^2*x-216*(x^2-3*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf
(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x+252*RootOf(_Z^3-4)^3*RootOf(RootOf(_Z^3-4
)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)-108*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^
3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2+474*(x^2-3*x+2)^(1/3)*RootOf(_Z^3-4)^2+432*
(x^2-3*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootO
f(_Z^3-4)+98*RootOf(_Z^3-4)*x^2-42*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3
-4)+4*_Z^2)*x^2+948*(x^2-3*x+2)^(2/3)-840*RootOf(_Z^3-4)*x+360*RootOf(RootO
f(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+840*RootOf(_Z^3-4)-360*RootOf(Roo
tOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-3\,x+2\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(x^2-3\,x+2\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 - 3*x + 2)^(1/3)),x)
[Out] int(1/(x*(x^2 - 3*x + 2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{(x-2)(x-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2-3*x+2)**(1/3),x)
[Out] Integral(1/(x*((x - 2)*(x - 1))**(1/3)), x)
```

3.1910 $\int \frac{ab-x^2}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(ab-(a+b+d)x+x^2\right)}\,dx$

Optimal. Leaf size=181

$$\frac{\log\left(\sqrt[3]{d}\,x\sqrt[3]{x^3(-a-b)+abx^2+x^4}+\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+d^{2/3}x^2\right)}{2\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{x^3(-a-b)+abx^2+x^4}\right.-}{\sqrt[3]{d}}$$

Rubi [F] time = 6.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ab-x^2}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(ab-(a+b+d)x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*b - x^2)/((x^2*(-a + x)*(-b + x))^(1/3)*(a*b - (a + b + d)*x + x^2))
,x]
```

```
[Out] (-3*x*(1 - x/a)^(1/3)*(1 - x/b)^(1/3)*AppellF1[1/3, 1/3, 1/3, 4/3, x/a, x/b
])/((a - x)*(b - x)*x^2)^(1/3) - ((a + b + d + Sqrt[a^2 - 2*a*(b - d) + (b
+ d)^2])*x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Int][1/(x^(2/3)*(-a +
x)^(1/3)*(-b + x)^(1/3)*(-a - b - d - Sqrt[a^2 - 2*a*b + b^2 + 2*a*d + 2*b*
d + d^2] + 2*x)), x])/((a - x)*(b - x)*x^2)^(1/3) - ((a + b + d - Sqrt[a^2
- 2*a*(b - d) + (b + d)^2])*x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Int
][1/(x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*(-a - b - d + Sqrt[a^2 - 2*a*b +
 b^2 + 2*a*d + 2*b*d + d^2] + 2*x)), x])/((a - x)*(b - x)*x^2)^(1/3)
```

Rubi steps

$$\int \frac{ab-x^2}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(ab-(a+b+d)x+x^2\right)}\,dx=\frac{\left(x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\int \frac{ab-x^2}{x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\left(ab-(a+b+d)x+x\right.}}{\sqrt[3]{x^2(-a+x)(-b+x)}}$$

$$=\frac{\left(x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\int\left(-\frac{1}{x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}}+\frac{}{x^{2/3}\sqrt[3]{-a}}\right.}{\sqrt[3]{x^2(-a+x)(-b+x)}}$$

$$=-\frac{\left(x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\int\frac{1}{x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}}\,dx}{\sqrt[3]{x^2(-a+x)(-b+x)}}+\frac{\left(x^{2/3}\right.}{}$$

$$=\frac{\left(x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\int\left(\frac{-a-b-d-\sqrt{a^2-2ab+b^2}}{x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\left(-a-b-d-\sqrt{a^2-}\right.}\right.}{}$$

$$=\frac{\left(\left(-a-b-d-\sqrt{a^2-2a(b-d)+(b+d)^2}\right)x^{2/3}\sqrt[3]{-a+}\right.}{\sqrt[3]{x^2(}}$$

$$=-\frac{3x\sqrt[3]{1-\frac{x}{a}}\sqrt[3]{1-\frac{x}{b}}\,F_1\left(\frac{1}{3};\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{x}{a},\frac{x}{b}\right)}{\sqrt[3]{(a-x)(b-x)x^2}}+\frac{\left(\left(-a-b-d\right.\right.}{}$$

Mathematica [F] time = 13.53, size = 0, normalized size = 0.00

$$\int \frac{ab - x^2}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(ab - (a+b+d)x + x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*b - x^2)/((x^2*(-a + x)*(-b + x))^(1/3)*(a*b - (a + b + d)*x +
 x^2)),x]
```

```
[Out] Integrate[(a*b - x^2)/((x^2*(-a + x)*(-b + x))^(1/3)*(a*b - (a + b + d)*x +
 x^2)), x]
```

IntegrateAlgebraic [A] time = 0.40, size = 181, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{d}\,x\sqrt[3]{x^3(-a-b)+abx^2+x^4}+\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+d^{2/3}x^2\right)}{2\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{x^3(-a-b)+abx^2+x^4}-\sqrt[3]{d}\,x\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\,x}{2\sqrt[3]{x^3(-a-b)+abx^2+x^4}+\sqrt[3]{d}\,x}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b - x^2)/((x^2*(-a + x)*(-b + x))^(1/3)*(a*b - (a + b
 + d)*x + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*x)/(d^(1/3)*x + 2*(a*b*x^2 + (-a - b)*x^3
+ x^4)^(1/3))])/d^(1/3) - Log[-(d^(1/3)*x) + (a*b*x^2 + (-a - b)*x^3 + x^4)
^(1/3)]/d^(1/3) + Log[d^(2/3)*x^2 + d^(1/3)*x*(a*b*x^2 + (-a - b)*x^3 + x^4
)^(1/3) + (a*b*x^2 + (-a - b)*x^3 + x^4)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(a*b-(a+b+d)*x+x^2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab - x^2}{\left((a-x)(b-x)x^2\right)^{\frac{1}{3}}\left(ab - (a+b+d)x + x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(a*b-(a+b+d)*x+x^2),x, algori
thm="giac")
```

```
[Out] integrate((a*b - x^2)/(((a - x)*(b - x)*x^2)^(1/3)*(a*b - (a + b + d)*x + x
^2)), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{ab - x^2}{\left(x^2\,(-a+x)\,(-b+x)\right)^{\frac{1}{3}}\left(ab - (a+b+d)\,x + x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b-x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(a*b-(a+b+d)*x+x^2),x)
```

```
[Out] int((a*b-x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(a*b-(a+b+d)*x+x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab-x^2}{\left((a-x)(b-x)x^2\right)^{\frac{1}{3}}\left(ab-(a+b+d)x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(a*b-(a+b+d)*x+x^2),x, algori
thm="maxima")
```

```
[Out] integrate((a*b - x^2)/(((a - x)*(b - x)*x^2)^(1/3)*(a*b - (a + b + d)*x + x
^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,b-x^2}{{\left(x^2\,\left(a-x\right)\,\left(b-x\right)\right)}^{1/3}\,\left(x^2+\left(-a-b-d\right)\,x+a\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b - x^2)/((x^2*(a - x)*(b - x))^(1/3)*(a*b - x*(a + b + d) + x^2)),x
)
```

```
[Out] int((a*b - x^2)/((x^2*(a - x)*(b - x))^(1/3)*(a*b - x*(a + b + d) + x^2)),
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-x**2)/(x**2*(-a+x)*(-b+x))**(1/3)/(a*b-(a+b+d)*x+x**2),x)
```

```
[Out] Timed out
```

3.1911 $\int \frac{\left(-q+2px^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^2+a\left(q+px^3\right)^2\right)}{x^5}\,dx$

Optimal. Leaf size=181

$$\frac{1}{2}\left(-ap^2q^2-2bpq\right)\log\left(\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+px^3+q\right)+\frac{1}{2}\log(x)\left(ap^2q^2+2bpq\right)+\frac{\sqrt{p^2x^6+2pqx^3-2p\cdots}}{}$$

Rubi [F] time = 1.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-q+2px^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^2+a\left(q+px^3\right)^2\right)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*x^2 + a*
(q + p*x^3)^2))/x^5,x]
```

```
[Out] 2*b*p*Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6], x] - a*q^3*De
fer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x^5, x] - b*q*Defer[In
t][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x^3, x] + 3*a*p^2*q*Defer[In
t][x*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6], x] + 2*a*p^3*Defer[Int][x
^4*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6], x]
```

Rubi steps

$$\int \frac{\left(-q+2px^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^2+a\left(q+px^3\right)^2\right)}{x^5}\,dx=\int \left(2bp\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}-\frac{a\cdots}{\cdots}\right.$$

$$=(2bp)\int \sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\,dx\cdots$$

Mathematica [F] time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{\left(-q+2px^3\right)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^2+a\left(q+px^3\right)^2\right)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*x^
2 + a*(q + p*x^3)^2))/x^5,x]
```

```
[Out] Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*x^
2 + a*(q + p*x^3)^2))/x^5, x]
```

IntegrateAlgebraic [A] time = 0.37, size = 181, normalized size = 1.00

$$\frac{1}{2}\left(-ap^2q^2-2bpq\right)\log\left(\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+px^3+q\right)+\frac{1}{2}\log(x)\left(ap^2q^2+2bpq\right)+\frac{\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}\left(ap^3x^9+3ap^2qx^6-ap^2qx^5+3apq^2x^3-apq^2x^2+aq^3+2bpx^5+2bqx^2\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x
^6]*(b*x^2 + a*(q + p*x^3)^2))/x^5,x]
```

```
[Out] (Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(a*q^3 + 2*b*q*x^2 - a*p*q^2*x
^2 + 3*a*p*q^2*x^3 + 2*b*p*x^5 - a*p^2*q*x^5 + 3*a*p^2*q*x^6 + a*p^3*x^9))/
(4*x^4) + ((2*b*p*q + a*p^2*q^2)*Log[x])/2 + ((-2*b*p*q - a*p^2*q^2)*Log[q
+ p*x^3 + Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]])/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^2+a*(p*x
^3+q)^2)/x^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6 + 2\,pqx^3 - 2\,pqx^2 + q^2}\,\left(2\,px^3 - q\right)\left(\left(px^3 + q\right)^2 a + bx^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^2+a*(p*x
^3+q)^2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)*(2*p*x^3 - q)*((p*x^3
 + q)^2*a + b*x^2)/x^5, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\left(2p\,x^3 - q\right)\sqrt{p^2x^6 + 2pq\,x^3 - 2pq\,x^2 + q^2}\,\left(b\,x^2 + a\left(p\,x^3 + q\right)^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^2+a*(p*x^3+q)^
2)/x^5,x)
```

```
[Out] int((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^2+a*(p*x^3+q)^
2)/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6 + 2\,pqx^3 - 2\,pqx^2 + q^2}\,\left(2\,px^3 - q\right)\left(\left(px^3 + q\right)^2 a + bx^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^2+a*(p*x
^3+q)^2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)*(2*p*x^3 - q)*((p*x^3
 + q)^2*a + b*x^2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(q - 2\,p\,x^3\right)\,\left(a\,\left(p\,x^3 + q\right)^2 + b\,x^2\right)\,\sqrt{p^2\,x^6 + 2\,p\,q\,x^3 - 2\,p\,q\,x^2 + q^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q - 2*p*x^3)*(a*(q + p*x^3)^2 + b*x^2)*(p^2*x^6 + q^2 - 2*p*q*x^2 +
2*p*q*x^3)^(1/2))/x^5,x)
```

```
[Out] -int(((q - 2*p*x^3)*(a*(q + p*x^3)^2 + b*x^2)*(p^2*x^6 + q^2 - 2*p*q*x^2 +
2*p*q*x^3)^(1/2))/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2px^3-q\right)\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}\,\left(ap^2x^6+2apqx^3+aq^2+bx^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x**3-q)*(p**2*x**6+2*p*q*x**3-2*p*q*x**2+q**2)**(1/2)*(b*x**
2+a*(p*x**3+q)**2)/x**5,x)
```

```
[Out] Integral((2*p*x**3 - q)*sqrt(p**2*x**6 + 2*p*q*x**3 - 2*p*q*x**2 + q**2)*(a
*p**2*x**6 + 2*a*p*q*x**3 + a*q**2 + b*x**2)/x**5, x)
```

3.1912 $\int \frac{-d+cx}{x^7 \sqrt[3]{-b+ax^3}}\,dx$

Optimal. Leaf size=184

$$\frac{2a^2 d\log\left(\sqrt[3]{ax^3-b}+\sqrt[3]{b}\right)}{27b^{7/3}}-\frac{a^2 d\log\left(-\sqrt[3]{b}\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}+b^{2/3}\right)}{27b^{7/3}}+\frac{2a^2 d\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{b}}\right)}{9\sqrt{3}\,b^{7/3}}+\frac{(ax^3}{}$$

Rubi [A] time = 0.25, antiderivative size = 198, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1844, 266, 51, 56, 617, 204, 31, 271, 264}

$$\frac{a^2 d\log\left(\sqrt[3]{ax^3-b}+\sqrt[3]{b}\right)}{9b^{7/3}}+\frac{2a^2 d\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{b}}\right)}{9\sqrt{3}\,b^{7/3}}-\frac{a^2 d\log(x)}{9b^{7/3}}+\frac{3ac\left(ax^3-b\right)^{2/3}}{10b^2x^2}-\frac{2ad\left(ax^3-b\right)^{2/3}}{9b^2x^3}+\frac{c\left(ax^3-b\right)^{2/3}}{5bx^5}-\frac{d\left(ax^3-b\right)^{2/3}}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(-d + c*x)/(x^7*(-b + a*x^3)^(1/3)),x]
```

```
[Out] -1/6*(d*(-b + a*x^3)^(2/3))/(b*x^6) + (c*(-b + a*x^3)^(2/3))/(5*b*x^5) - (2
*a*d*(-b + a*x^3)^(2/3))/(9*b^2*x^3) + (3*a*c*(-b + a*x^3)^(2/3))/(10*b^2*x
^2) + (2*a^2*d*ArcTan[(b^(1/3) - 2*(-b + a*x^3)^(1/3))/(Sqrt[3]*b^(1/3))])/
(9*Sqrt[3]*b^(7/3)) - (a^2*d*Log[x])/(9*b^(7/3)) + (a^2*d*Log[b^(1/3) + (-b
 + a*x^3)^(1/3)])/(9*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{-d+cx}{x^7\sqrt[3]{-b+ax^3}}\,dx = \int \left(-\frac{d}{x^7\sqrt[3]{-b+ax^3}}+\frac{c}{x^6\sqrt[3]{-b+ax^3}}\right)dx$$

$$= c\int \frac{1}{x^6\sqrt[3]{-b+ax^3}}\,dx - d\int \frac{1}{x^7\sqrt[3]{-b+ax^3}}\,dx$$

$$= \frac{c\left(-b+ax^3\right)^{2/3}}{5bx^5}+\frac{(3ac)\int \frac{1}{x^3\sqrt[3]{-b+ax^3}}\,dx}{5b}-\frac{1}{3}d\,\text{Subst}\left(\int \frac{1}{x^3\sqrt[3]{-b+ax}}\,dx,x,x^3\right)$$

$$= -\frac{d\left(-b+ax^3\right)^{2/3}}{6bx^6}+\frac{c\left(-b+ax^3\right)^{2/3}}{5bx^5}+\frac{3ac\left(-b+ax^3\right)^{2/3}}{10b^2x^2}-\frac{(2ad)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{-b+ax}}\,dx,x,x\right.}{9b}$$

$$= -\frac{d\left(-b+ax^3\right)^{2/3}}{6bx^6}+\frac{c\left(-b+ax^3\right)^{2/3}}{5bx^5}-\frac{2ad\left(-b+ax^3\right)^{2/3}}{9b^2x^3}+\frac{3ac\left(-b+ax^3\right)^{2/3}}{10b^2x^2}-\frac{\left(2a^2d\right)\text{Sub}}{}$$

$$= -\frac{d\left(-b+ax^3\right)^{2/3}}{6bx^6}+\frac{c\left(-b+ax^3\right)^{2/3}}{5bx^5}-\frac{2ad\left(-b+ax^3\right)^{2/3}}{9b^2x^3}+\frac{3ac\left(-b+ax^3\right)^{2/3}}{10b^2x^2}-\frac{a^2d\log(x)}{9b^{7/3}}$$

$$= -\frac{d\left(-b+ax^3\right)^{2/3}}{6bx^6}+\frac{c\left(-b+ax^3\right)^{2/3}}{5bx^5}-\frac{2ad\left(-b+ax^3\right)^{2/3}}{9b^2x^3}+\frac{3ac\left(-b+ax^3\right)^{2/3}}{10b^2x^2}-\frac{a^2d\log(x)}{9b^{7/3}}$$

$$= -\frac{d\left(-b+ax^3\right)^{2/3}}{6bx^6}+\frac{c\left(-b+ax^3\right)^{2/3}}{5bx^5}-\frac{2ad\left(-b+ax^3\right)^{2/3}}{9b^2x^3}+\frac{3ac\left(-b+ax^3\right)^{2/3}}{10b^2x^2}+\frac{2a^2d\tan^{-1}}{9\sqrt{}}$$

Mathematica [C] time = 0.07, size = 65, normalized size = 0.35

$$\frac{\left(ax^3-b\right)^{2/3}\left(bc\left(3ax^3+2b\right)-5a^2dx^5\,{}_2F_1\left(\frac{2}{3},3;\frac{5}{3};1-\frac{ax^3}{b}\right)\right)}{10b^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-d + c*x)/(x^7*(-b + a*x^3)^(1/3)),x]
```

```
[Out] ((-b + a*x^3)^(2/3)*(b*c*(2*b + 3*a*x^3) - 5*a^2*d*x^5*Hypergeometric2F1[2/
3, 3, 5/3, 1 - (a*x^3)/b]))/(10*b^3*x^5)
```

IntegrateAlgebraic [A] time = 18.22, size = 184, normalized size = 1.00

$$\frac{2a^2d\log\left(\sqrt[3]{ax^3-b}+\sqrt[3]{b}\right)}{27b^{7/3}}-\frac{a^2d\log\left(-\sqrt[3]{b}\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}+b^{2/3}\right)}{27b^{7/3}}+\frac{2a^2d\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{b}}\right)}{9\sqrt{3}\,b^{7/3}}+\frac{\left(ax^3-b\right)^{2/3}\left(27acx^4-20adx^3+18bcx-15bd\right)}{90b^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-d + c*x)/(x^7*(-b + a*x^3)^(1/3)),x]
```

```
[Out] ((-b + a*x^3)^(2/3)*(-15*b*d + 18*b*c*x - 20*a*d*x^3 + 27*a*c*x^4))/(90*b^2
*x^6) + (2*a^2*d*ArcTan[1/Sqrt[3] - (2*(-b + a*x^3)^(1/3))/(Sqrt[3]*b^(1/3)
)])/(9*Sqrt[3]*b^(7/3)) + (2*a^2*d*Log[b^(1/3) + (-b + a*x^3)^(1/3)])/(27*b
^(7/3)) - (a^2*d*Log[b^(2/3) - b^(1/3)*(-b + a*x^3)^(1/3) + (-b + a*x^3)^(2
/3)])/(27*b^(7/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x-d)/x^7/(a*x^3-b)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx-d}{\left(ax^3-b\right)^{\frac{1}{3}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x-d)/x^7/(a*x^3-b)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*x - d)/((a*x^3 - b)^(1/3)*x^7), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{cx-d}{x^7\left(a\,x^3-b\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x-d)/x^7/(a*x^3-b)^(1/3),x)
```

```
[Out] int((c*x-d)/x^7/(a*x^3-b)^(1/3),x)
```

maxima [A] time = 0.67, size = 219, normalized size = 1.19

$$-\frac{1}{54}\left(\frac{4\sqrt{3}a^{2}\arctan\left(\frac{\sqrt{3}\left(2\left(ax^{3}-b\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}+\frac{2a^{2}\log\left(\left(ax^{3}-b\right)^{\frac{2}{3}}-\left(ax^{3}-b\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{7}{3}}}-\frac{4a^{2}\log\left(\left(ax^{3}-b\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{b^{\frac{7}{3}}}+\frac{3\left(4\left(ax^{3}-b\right)^{\frac{5}{3}}a^{2}+7\left(ax^{3}-b\right)^{\frac{2}{3}}a^{2}b\right)}{\left(ax^{3}-b\right)^{2}b^{2}+2\left(ax^{3}-b\right)b^{3}+b^{4}}\right)d+\frac{c\left(\frac{5\left(ax^{3}-b\right)^{\frac{2}{3}}a}{x^{2}}-\frac{2\left(ax^{3}-b\right)^{\frac{5}{3}}}{x^{5}}\right)}{10b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x-d)/x^7/(a*x^3-b)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/54*(4*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*(a*x^3 - b)^(1/3) - b^(1/3))/b^(
1/3))/b^(7/3) + 2*a^2*log((a*x^3 - b)^(2/3) - (a*x^3 - b)^(1/3)*b^(1/3) + b
^(2/3))/b^(7/3) - 4*a^2*log((a*x^3 - b)^(1/3) + b^(1/3))/b^(7/3) + 3*(4*(a*
x^3 - b)^(5/3)*a^2 + 7*(a*x^3 - b)^(2/3)*a^2*b)/((a*x^3 - b)^2*b^2 + 2*(a*x
^3 - b)*b^3 + b^4))*d + 1/10*c*(5*(a*x^3 - b)^(2/3)*a/x^2 - 2*(a*x^3 - b)^(
5/3)/x^5)/b^2
```

mupad [B] time = 2.28, size = 250, normalized size = 1.36

$$\frac{2a^{2}d\ln\left(\left(ax^{3}-b\right)^{1/3}+b^{1/3}\right)}{27b^{7/3}}-\frac{\frac{7a^{2}d\left(ax^{3}-b\right)^{2/3}}{18b}+\frac{2a^{2}d\left(ax^{3}-b\right)^{5/3}}{9b^{2}}}{\left(b-ax^{3}\right)^{2}-2b\left(b-ax^{3}\right)+b^{2}}+\frac{2a^{2}d\ln\left(\frac{4a^{4}d^{2}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)^{2}}{81b^{11/3}}+\frac{4a^{4}d^{2}\left(ax^{3}-b\right)^{1/3}}{81b^{4}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{27b^{7/3}}-\frac{2a^{2}d\ln\left(\frac{4a^{4}d^{2}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)^{2}}{81b^{11/3}}+\frac{4a^{4}d^{2}\left(ax^{3}-b\right)^{1/3}}{81b^{4}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{27b^{7/3}}+\frac{c\left(ax^{3}-b\right)^{2/3}\left(3ax^{3}+2b\right)}{10b^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(d - c*x)/(x^7*(a*x^3 - b)^(1/3)),x)
```

```
[Out] (2*a^2*d*log((a*x^3 - b)^(1/3) + b^(1/3)))/(27*b^(7/3)) - ((7*a^2*d*(a*x^3
- b)^(2/3))/(18*b) + (2*a^2*d*(a*x^3 - b)^(5/3))/(9*b^2))/((b - a*x^3)^2 -
2*b*(b - a*x^3) + b^2) + (2*a^2*d*log((4*a^4*d^2*((3^(1/2)*1i)/2 - 1/2)^2)/
(81*b^(11/3)) + (4*a^4*d^2*(a*x^3 - b)^(1/3))/(81*b^4))*((3^(1/2)*1i)/2 - 1
/2))/(27*b^(7/3)) - (2*a^2*d*log((4*a^4*d^2*((3^(1/2)*1i)/2 + 1/2)^2)/(81*b
^(11/3)) + (4*a^4*d^2*(a*x^3 - b)^(1/3))/(81*b^4))*((3^(1/2)*1i)/2 + 1/2))/
(27*b^(7/3)) + (c*(a*x^3 - b)^(2/3)*(2*b + 3*a*x^3))/(10*b^2*x^5)
```

sympy [C] time = 2.96, size = 320, normalized size = 1.74

$$c\left(\begin{cases}-\frac{a^{\frac{5}{3}}\left(-1+\frac{b}{ax^{3}}\right)^{\frac{2}{3}}e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{5}{3}\right)}{3b^{2}\Gamma\left(\frac{1}{3}\right)}-\frac{2a^{\frac{2}{3}}\left(-1+\frac{b}{ax^{3}}\right)^{\frac{2}{3}}e^{-\frac{i\pi}{3}}\Gamma\left(-\frac{5}{3}\right)}{9bx^{3}\Gamma\left(\frac{1}{3}\right)} & \text{for}\ \left|\frac{b}{ax^{3}}\right|>1\\ \frac{3a^{\frac{11}{3}}x^{6}\left(1-\frac{b}{ax^{3}}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{9a^{2}b^{2}x^{6}\Gamma\left(\frac{1}{3}\right)-9ab^{3}x^{3}\Gamma\left(\frac{1}{3}\right)}-\frac{a^{\frac{8}{3}}bx^{3}\left(1-\frac{b}{ax^{3}}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{9a^{2}b^{2}x^{6}\Gamma\left(\frac{1}{3}\right)-9ab^{3}x^{3}\Gamma\left(\frac{1}{3}\right)}-\frac{2a^{\frac{5}{3}}b^{2}\left(1-\frac{b}{ax^{3}}\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{9a^{2}b^{2}x^{6}\Gamma\left(\frac{1}{3}\right)-9ab^{3}x^{3}\Gamma\left(\frac{1}{3}\right)} & \text{otherwise}\end{cases}\right)+\frac{d\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^{3}}\right)}{3\sqrt[3]{a}x^{7}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x-d)/x**7/(a*x**3-b)**(1/3),x)
```

```
[Out] c*Piecewise((-a**(5/3)*(-1 + b/(a*x**3))**(2/3)*exp(-I*pi/3)*gamma(-5/3)/(3
*b**2*gamma(1/3)) - 2*a**(2/3)*(-1 + b/(a*x**3))**(2/3)*exp(-I*pi/3)*gamma(
-5/3)/(9*b*x**3*gamma(1/3)), Abs(b/(a*x**3)) > 1), (3*a**(11/3)*x**6*(1 - b
/(a*x**3))**(2/3)*gamma(-5/3)/(9*a**2*b**2*x**6*gamma(1/3) - 9*a*b**3*x**3*
gamma(1/3)) - a**(8/3)*b*x**3*(1 - b/(a*x**3))**(2/3)*gamma(-5/3)/(9*a**2*b
**2*x**6*gamma(1/3) - 9*a*b**3*x**3*gamma(1/3)) - 2*a**(5/3)*b**2*(1 - b/(a
*x**3))**(2/3)*gamma(-5/3)/(9*a**2*b**2*x**6*gamma(1/3) - 9*a*b**3*x**3*gam
ma(1/3)), True)) + d*gamma(7/3)*hyper((1/3, 7/3), (10/3,), b*exp_polar(2*I*
pi)/(a*x**3))/(3*a**(1/3)*x**7*gamma(10/3))
```

3.1913 $\int \frac{1+x^2+x^4}{(1-x^4)\sqrt[4]{x^3+x^5}}\,dx$

Optimal. Leaf size=184

$$\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{4\sqrt[4]{2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{4\sqrt[4]{2}}+\frac{(x^5+x^3)^{3/4}}{x^2(x^2+1)}-\frac{3\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^5+x^3}}{\sqrt{2}x^2-\sqrt{x^5+x^3}}\right)}{4\,2^{3/4}}+\frac{3\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^5+x^3}}{2^{3/4}}}{x\sqrt[4]{x^5+x^3}}\right)}{4\,2^{3/4}}$$

Rubi [C] time = 0.65, antiderivative size = 94, normalized size of antiderivative = 0.51, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2056, 6715, 6725, 245, 1455, 527, 530, 429}

$$\frac{6\sqrt[4]{x^2+1}\,xF_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^2,-x^2\right)}{\sqrt[4]{x^5+x^3}}-\frac{3\sqrt[4]{x^2+1}\,x\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^2\right)}{\sqrt[4]{x^5+x^3}}+\frac{x}{\sqrt[4]{x^5+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^2 + x^4)/((1 - x^4)*(x^3 + x^5)^(1/4)),x]
```

```
[Out] x/(x^3 + x^5)^(1/4) + (6*x*(1 + x^2)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8, x^2,
-x^2])/(x^3 + x^5)^(1/4) - (3*x*(1 + x^2)^(1/4)*Hypergeometric2F1[1/8, 1/4,
 9/8, -x^2])/(x^3 + x^5)^(1/4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1455

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*
```

```
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, n, q, r}, x] && EqQ[n2,
 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^2+x^4}{\left(1-x^4\right)\sqrt[4]{x^3+x^5}}\,dx = \frac{\left(x^{3/4}\sqrt[4]{1+x^2}\right)\int \frac{1+x^2+x^4}{x^{3/4}\sqrt[4]{1+x^2}\left(1-x^4\right)}\,dx}{\sqrt[4]{x^3+x^5}}$$

$$= \frac{\left(4x^{3/4}\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{1+x^8+x^{16}}{\sqrt[4]{1+x^8}\left(1-x^{16}\right)}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{x^3+x^5}}$$

$$= \frac{\left(4x^{3/4}\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \left(-\frac{1}{\sqrt[4]{1+x^8}}+\frac{2+x^8}{\sqrt[4]{1+x^8}\left(1-x^{16}\right)}\right)dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{x^3+x^5}}$$

$$= -\frac{\left(4x^{3/4}\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{x^3+x^5}} + \frac{\left(4x^{3/4}\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{2+x^8}{\sqrt[4]{1+x^8}\left(1-x^{16}\right)}\right.}{\sqrt[4]{x^3+x^5}}$$

$$= -\frac{4x\sqrt[4]{1+x^2}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^2\right)}{\sqrt[4]{x^3+x^5}} + \frac{\left(4x^{3/4}\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{2+x^8}{\left(1-x^8\right)\left(1+x^8\right)^{5/4}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{x^3+x^5}}$$

$$= \frac{x}{\sqrt[4]{x^3+x^5}} - \frac{4x\sqrt[4]{1+x^2}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^2\right)}{\sqrt[4]{x^3+x^5}} - \frac{\left(x^{3/4}\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{-7+x^8}{\left(1-x^8\right)\sqrt[4]{1+x^8}}\,dx, x\right.}{\sqrt[4]{x^3+x^5}}$$

$$= \frac{x}{\sqrt[4]{x^3+x^5}} - \frac{4x\sqrt[4]{1+x^2}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^2\right)}{\sqrt[4]{x^3+x^5}} + \frac{\left(x^{3/4}\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}}\,dx, x, \sqrt[4]{x}\right.}{\sqrt[4]{x^3+x^5}}$$

$$= \frac{x}{\sqrt[4]{x^3+x^5}} + \frac{6x\sqrt[4]{1+x^2}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^2,-x^2\right)}{\sqrt[4]{x^3+x^5}} - \frac{3x\sqrt[4]{1+x^2}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^2\right)}{\sqrt[4]{x^3+x^5}}$$

Mathematica [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1+x^2+x^4}{\left(1-x^4\right)\sqrt[4]{x^3+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^2 + x^4)/((1 - x^4)*(x^3 + x^5)^(1/4)),x]
```

```
[Out] Integrate[(1 + x^2 + x^4)/((1 - x^4)*(x^3 + x^5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.66, size = 184, normalized size = 1.00

$$\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{4\sqrt[4]{2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{4\sqrt[4]{2}}+\frac{\left(x^5+x^3\right)^{3/4}}{x^2\left(x^2+1\right)}-\frac{3\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^5+x^3}}{\sqrt{2}x^2-\sqrt{x^5+x^3}}\right)}{4\,2^{3/4}}+\frac{3\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^5+x^3}}{2^{3/4}}}{x\sqrt[4]{x^5+x^3}}\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2 + x^4)/((1 - x^4)*(x^3 + x^5)^(1/4)),x]
```

```
[Out] (x^3 + x^5)^(3/4)/(x^2*(1 + x^2)) + (3*ArcTan[(2^(1/4)*x)/(x^3 + x^5)^(1/4)
])/(4*2^(1/4)) - (3*ArcTan[(2^(3/4)*x*(x^3 + x^5)^(1/4))/(Sqrt[2]*x^2 - Sqr
t[x^3 + x^5])])/(4*2^(3/4)) + (3*ArcTanh[(2^(1/4)*x)/(x^3 + x^5)^(1/4)])/(4
*2^(1/4)) + (3*ArcTanh[(x^2/2^(1/4) + Sqrt[x^3 + x^5]/2^(3/4))/(x*(x^3 + x^
5)^(1/4))])/(4*2^(3/4))
```

fricas [B] time = 15.41, size = 1102, normalized size = 5.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(-x^4+1)/(x^5+x^3)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/32*(12*2^(3/4)*(x^4 + x^2)*arctan(-1/2*(4*2^(3/4)*(x^5 + x^3)^(1/4)*x^2
- 2^(3/4)*(2*2^(3/4)*sqrt(x^5 + x^3)*x + 2^(1/4)*(x^4 + 2*x^3 + x^2)) + 4*2
^(1/4)*(x^5 + x^3)^(3/4))/(x^4 - 2*x^3 + x^2)) - 3*2^(3/4)*(x^4 + x^2)*log(
(4*sqrt(2)*(x^5 + x^3)^(1/4)*x^2 + 2^(3/4)*(x^4 + 2*x^3 + x^2) + 4*2^(1/4)*
sqrt(x^5 + x^3)*x + 4*(x^5 + x^3)^(3/4))/(x^4 - 2*x^3 + x^2)) + 3*2^(3/4)*(
x^4 + x^2)*log((4*sqrt(2)*(x^5 + x^3)^(1/4)*x^2 - 2^(3/4)*(x^4 + 2*x^3 + x^
2) - 4*2^(1/4)*sqrt(x^5 + x^3)*x + 4*(x^5 + x^3)^(3/4))/(x^4 - 2*x^3 + x^2)
) - 12*2^(1/4)*(x^4 + x^2)*arctan(1/2*(2*x^6 + 8*x^5 + 12*x^4 + 8*x^3 + 4*2
^(3/4)*(x^5 + x^3)^(3/4)*(x^2 - 6*x + 1) + 8*sqrt(2)*sqrt(x^5 + x^3)*(x^3 +
 2*x^2 + x) + 2*x^2 + sqrt(2)*(32*sqrt(2)*(x^5 + x^3)^(3/4)*x + 2^(3/4)*(x^
6 - 16*x^5 - 2*x^4 - 16*x^3 + x^2) + 4*2^(1/4)*sqrt(x^5 + x^3)*(x^3 - 6*x^2
 + x) + 8*(x^5 + x^3)^(1/4)*(x^4 + 2*x^3 + x^2))*sqrt((4*2^(3/4)*(x^5 + x^3
)^(1/4)*x^2 + sqrt(2)*(x^4 + 2*x^3 + x^2) + 8*sqrt(x^5 + x^3)*x + 4*2^(1/4)
*(x^5 + x^3)^(3/4))/(x^4 + 2*x^3 + x^2)) + 8*2^(1/4)*(x^5 + x^3)^(1/4)*(3*x
^4 - 2*x^3 + 3*x^2))/(x^6 - 28*x^5 + 6*x^4 - 28*x^3 + x^2)) + 12*2^(1/4)*(x
^4 + x^2)*arctan(1/2*(2*x^6 + 8*x^5 + 12*x^4 + 8*x^3 - 4*2^(3/4)*(x^5 + x^3
)^(3/4)*(x^2 - 6*x + 1) + 8*sqrt(2)*sqrt(x^5 + x^3)*(x^3 + 2*x^2 + x) + 2*x
^2 + sqrt(2)*(32*sqrt(2)*(x^5 + x^3)^(3/4)*x - 2^(3/4)*(x^6 - 16*x^5 - 2*x^
4 - 16*x^3 + x^2) - 4*2^(1/4)*sqrt(x^5 + x^3)*(x^3 - 6*x^2 + x) + 8*(x^5 +
x^3)^(1/4)*(x^4 + 2*x^3 + x^2))*sqrt(-(4*2^(3/4)*(x^5 + x^3)^(1/4)*x^2 - sq
rt(2)*(x^4 + 2*x^3 + x^2) - 8*sqrt(x^5 + x^3)*x + 4*2^(1/4)*(x^5 + x^3)^(3/
4))/(x^4 + 2*x^3 + x^2)) - 8*2^(1/4)*(x^5 + x^3)^(1/4)*(3*x^4 - 2*x^3 + 3*x
^2))/(x^6 - 28*x^5 + 6*x^4 - 28*x^3 + x^2)) - 3*2^(1/4)*(x^4 + x^2)*log(8*(
4*2^(3/4)*(x^5 + x^3)^(1/4)*x^2 + sqrt(2)*(x^4 + 2*x^3 + x^2) + 8*sqrt(x^5
+ x^3)*x + 4*2^(1/4)*(x^5 + x^3)^(3/4))/(x^4 + 2*x^3 + x^2)) + 3*2^(1/4)*(x
^4 + x^2)*log(-8*(4*2^(3/4)*(x^5 + x^3)^(1/4)*x^2 - sqrt(2)*(x^4 + 2*x^3 +
```

```
x^2) - 8*sqrt(x^5 + x^3)*x + 4*2^(1/4)*(x^5 + x^3)^(3/4))/(x^4 + 2*x^3 + x^
2)) - 32*(x^5 + x^3)^(3/4))/(x^4 + x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^4 + x^2 + 1}{\left(x^5 + x^3\right)^{\frac{1}{4}}\left(x^4 - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(-x^4+1)/(x^5+x^3)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-(x^4 + x^2 + 1)/((x^5 + x^3)^(1/4)*(x^4 - 1)), x)
```

maple [C] time = 13.33, size = 733, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)/(-x^4+1)/(x^5+x^3)^(1/4),x)
```

```
[Out] x/(x^3*(x^2+1))^(1/4)-3/16*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln(((x^5+x^3)^(1/2
)*RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*x+RootOf(_Z^2+RootOf(_Z^4+
8)^2)*x^4-2*(x^5+x^3)^(1/4)*RootOf(_Z^4+8)^2*x^2-2*RootOf(_Z^2+RootOf(_Z^4+
8)^2)*x^3+RootOf(_Z^2+RootOf(_Z^4+8)^2)*x^2-4*(x^5+x^3)^(3/4))/(1+x)^2/x^2)
-3/16*RootOf(_Z^4+8)*ln(-(RootOf(_Z^4+8)^3*(x^5+x^3)^(1/2)*x-2*(x^5+x^3)^(1
/4)*RootOf(_Z^4+8)^2*x^2-RootOf(_Z^4+8)*x^4+2*RootOf(_Z^4+8)*x^3+4*(x^5+x^3
)^(3/4)-RootOf(_Z^4+8)*x^2)/(1+x)^2/x^2)+3/64*ln(-(RootOf(_Z^4+8)^3*x^4-2*R
ootOf(_Z^4+8)^3*x^3+8*(x^5+x^3)^(1/4)*RootOf(_Z^4+8)^2*x^2+RootOf(_Z^4+8)^3
*x^2-16*(x^5+x^3)^(1/2)*RootOf(_Z^4+8)*x+16*(x^5+x^3)^(3/4))/(-1+x)^2/x^2)*
RootOf(_Z^4+8)^3+3/64*ln(-(RootOf(_Z^4+8)^3*x^4-2*RootOf(_Z^4+8)^3*x^3+8*(x
^5+x^3)^(1/4)*RootOf(_Z^4+8)^2*x^2+RootOf(_Z^4+8)^3*x^2-16*(x^5+x^3)^(1/2)*
RootOf(_Z^4+8)*x+16*(x^5+x^3)^(3/4))/(-1+x)^2/x^2)*RootOf(_Z^4+8)^2*RootOf(
_Z^2+RootOf(_Z^4+8)^2)-3/32*RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*
ln(-(RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^4-RootOf(_Z^4+8)^3*x^
4+2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^3-2*RootOf(_Z^4+8)^3*x
^3-8*RootOf(_Z^2+RootOf(_Z^4+8)^2)*(x^5+x^3)^(1/4)*RootOf(_Z^4+8)*x^2+RootO
f(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^2-RootOf(_Z^4+8)^3*x^2+8*RootOf
(_Z^2+RootOf(_Z^4+8)^2)*(x^5+x^3)^(1/2)*x+8*(x^5+x^3)^(1/2)*RootOf(_Z^4+8)*
x-16*(x^5+x^3)^(3/4))/(-1+x)^2/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4 + x^2 + 1}{\left(x^5 + x^3\right)^{\frac{1}{4}}\left(x^4 - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(-x^4+1)/(x^5+x^3)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 + x^2 + 1)/((x^5 + x^3)^(1/4)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^4 + x^2 + 1}{{\left(x^5 + x^3\right)}^{1/4}\,\left(x^4 - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 + x^4 + 1)/((x^3 + x^5)^(1/4)*(x^4 - 1)),x)
```

```
[Out] int(-(x^2 + x^4 + 1)/((x^3 + x^5)^(1/4)*(x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^4\sqrt[4]{x^5+x^3}-\sqrt[4]{x^5+x^3}}\,dx-\int \frac{x^4}{x^4\sqrt[4]{x^5+x^3}-\sqrt[4]{x^5+x^3}}\,dx-\int \frac{1}{x^4\sqrt[4]{x^5+x^3}-\sqrt[4]{x^5+x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)/(-x**4+1)/(x**5+x**3)**(1/4),x)
```

```
[Out] -Integral(x**2/(x**4*(x**5 + x**3)**(1/4) - (x**5 + x**3)**(1/4)), x) - Int
egral(x**4/(x**4*(x**5 + x**3)**(1/4) - (x**5 + x**3)**(1/4)), x) - Integra
l(1/(x**4*(x**5 + x**3)**(1/4) - (x**5 + x**3)**(1/4)), x)
```

3.1914 $\int \frac{\sqrt{1+x}\,\sqrt{1+\sqrt{1+x}}}{x\sqrt{1+\sqrt{1+\sqrt{1+x}}}}\,dx$

Optimal. Leaf size=184

$$\frac{8}{5}\sqrt{x+1}\sqrt{\sqrt{\sqrt{x+1}+1}+1}-\frac{32}{15}\sqrt{\sqrt{x+1}+1}\sqrt{\sqrt{\sqrt{x+1}+1}+1}+\frac{88}{15}\sqrt{\sqrt{\sqrt{x+1}+1}+1}-2\sqrt{2\left(1+\sqrt{2}\right)}\ \text{ta}$$

Rubi [A] time = 1.75, antiderivative size = 162, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {1586, 898, 1287, 1093, 207, 203}

$$\frac{8}{5}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{5/2}-\frac{16}{3}\left(\sqrt{\sqrt{x+1}+1}+1\right)^{3/2}+8\sqrt{\sqrt{\sqrt{x+1}+1}+1}-2\sqrt{2\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{\sqrt{\sqrt{2}-1}}\right)-2\sqrt{2\left(\sqrt{2}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{\sqrt{1+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x]*Sqrt[1 + Sqrt[1 + x]])/(x*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]])
,x]
```

```
[Out] 8*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]] - (16*(1 + Sqrt[1 + Sqrt[1 + x]])^(3/2))/
3 + (8*(1 + Sqrt[1 + Sqrt[1 + x]])^(5/2))/5 - 2*Sqrt[2*(1 + Sqrt[2])]*ArcTa
n[Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]/Sqrt[-1 + Sqrt[2]]] - 2*Sqrt[2*(-1 + Sqrt
[2])]*ArcTanh[Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]/Sqrt[1 + Sqrt[2]]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
```

```
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+x}\sqrt{1+\sqrt{1+x}}}{x\sqrt{1+\sqrt{1+\sqrt{1+x}}}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^2\sqrt{1+x}}{\left(-1+x^2\right)\sqrt{1+\sqrt{1+x}}}\,dx, x, \sqrt{1+x}\right)\\
&= 2\,\text{Subst}\left(\int \frac{x^2}{(-1+x)\sqrt{1+x}\sqrt{1+\sqrt{1+x}}}\,dx, x, \sqrt{1+x}\right)\\
&= 4\,\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{\sqrt{1+x}\left(-2+x^2\right)}\,dx, x, \sqrt{1+\sqrt{1+x}}\right)\\
&= 4\,\text{Subst}\left(\int \frac{(-1+x)^2(1+x)^{3/2}}{-2+x^2}\,dx, x, \sqrt{1+\sqrt{1+x}}\right)\\
&= 8\,\text{Subst}\left(\int \frac{x^4\left(-2+x^2\right)^2}{-1-2x^2+x^4}\,dx, x, \sqrt{1+\sqrt{1+\sqrt{1+x}}}\right)\\
&= 8\,\text{Subst}\left(\int \left(1-2x^2+x^4+\frac{1}{-1-2x^2+x^4}\right)dx, x, \sqrt{1+\sqrt{1+\sqrt{1+x}}}\right)\\
&= 8\sqrt{1+\sqrt{1+\sqrt{1+x}}} - \frac{16}{3}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{3/2} + \frac{8}{5}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{5/2}\\
&= 8\sqrt{1+\sqrt{1+\sqrt{1+x}}} - \frac{16}{3}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{3/2} + \frac{8}{5}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{5/2}\\
&= 8\sqrt{1+\sqrt{1+\sqrt{1+x}}} - \frac{16}{3}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{3/2} + \frac{8}{5}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{5/2}
\end{aligned}
$$

Mathematica [A] time = 3.53, size = 146, normalized size = 0.79

$$2\left(\frac{4}{15}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\left(3\sqrt{x+1}-4\sqrt{\sqrt{x+1}+1}+11\right)-\sqrt{2\left(1+\sqrt{2}\right)}\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)-\sqrt{2\left(\sqrt{2}-1\right)}\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x]*Sqrt[1 + Sqrt[1 + x]])/(x*Sqrt[1 + Sqrt[1 + Sqrt[1 +
 x]]]),x]
```

```
[Out] 2*((4*(11 + 3*Sqrt[1 + x] - 4*Sqrt[1 + Sqrt[1 + x]])*Sqrt[1 + Sqrt[1 + Sqrt
[1 + x]]])/15 - Sqrt[2*(1 + Sqrt[2])]*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[1 + Sqr
t[1 + Sqrt[1 + x]]]] - Sqrt[2*(-1 + Sqrt[2])]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sq
rt[1 + Sqrt[1 + Sqrt[1 + x]]]])
```

IntegrateAlgebraic [A] time = 0.31, size = 165, normalized size = 0.90

$$\frac{8}{15}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\left(3\sqrt{x+1}+11\right)-\frac{32}{15}\sqrt{\sqrt{x+1}+1}\sqrt{\sqrt{\sqrt{x+1}+1}+1}-2\sqrt{2\left(1+\sqrt{2}\right)}\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)-2\sqrt{2\left(\sqrt{2}-1\right)}\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x]*Sqrt[1 + Sqrt[1 + x]])/(x*Sqrt[1 + Sqrt[1 +
 Sqrt[1 + x]]]),x]
```

```
[Out] (-32*Sqrt[1 + Sqrt[1 + x]]*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]])/15 + (8*(11 + 3
*Sqrt[1 + x])*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]])/15 - 2*Sqrt[2*(1 + Sqrt[2])]
*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]] - 2*Sqrt[2*(-1 +
 Sqrt[2])]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]]
```

fricas [A] time = 0.44, size = 186, normalized size = 1.01

$$4\sqrt{2}\sqrt{\sqrt{2}+1}\arctan\left(\sqrt{\sqrt{2}+\sqrt{\sqrt{x+1}+1}}\sqrt{\sqrt{2}+1}-\sqrt{\sqrt{2}+1}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)-\sqrt{2}\sqrt{\sqrt{2}-1}\log\left(2\sqrt{2}\left(\sqrt{2}+2\right)\sqrt{\sqrt{2}-1}+4\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)+\sqrt{2}\sqrt{\sqrt{2}-1}\log\left(-2\sqrt{2}\left(\sqrt{2}+2\right)\sqrt{\sqrt{2}-1}+4\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)+\frac{8}{15}\left(3\sqrt{x+1}-4\sqrt{\sqrt{x+1}+1}+11\right)\sqrt{\sqrt{\sqrt{x+1}+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(1+(1+x)^(1/2))^(1/2)/x/(1+(1+(1+x)^(1/2))^(1/2))^(1/
2),x, algorithm="fricas")
```

```
[Out] 4*sqrt(2)*sqrt(sqrt(2) + 1)*arctan(sqrt(sqrt(2) + sqrt(sqrt(x + 1) + 1))*sq
rt(sqrt(2) + 1) - sqrt(sqrt(2) + 1)*sqrt(sqrt(sqrt(x + 1) + 1) + 1)) - sqrt
(2)*sqrt(sqrt(2) - 1)*log(2*sqrt(2)*(sqrt(2) + 2)*sqrt(sqrt(2) - 1) + 4*sqr
t(sqrt(sqrt(x + 1) + 1) + 1)) + sqrt(2)*sqrt(sqrt(2) - 1)*log(-2*sqrt(2)*(s
qrt(2) + 2)*sqrt(sqrt(2) - 1) + 4*sqrt(sqrt(sqrt(x + 1) + 1) + 1)) + 8/15*(
3*sqrt(x + 1) - 4*sqrt(sqrt(x + 1) + 1) + 11)*sqrt(sqrt(sqrt(x + 1) + 1) +
1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(1+(1+x)^(1/2))^(1/2)/x/(1+(1+(1+x)^(1/2))^(1/2))^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-2,[1]%%%}+%%%{-4,[0]%%%},%%%{-4,[1]%%%}+%%%{
-4,[0]%%%},%%%{1,[2]%%%}+%%%{-1,[1]%%%}+%%%{-1,[0]%%%}] at parameters value
s [-72]Warning, choosing root of [1,0,%%%{-2,[1]%%%}+%%%{-4,[0]%%%},%%%{-4,
[1]%%%}+%%%{-4,[0]%%%},%%%{1,[2]%%%}+%%%{-1,[1]%%%}+%%%{-1,[0]%%%}] at para
meters values [11]Warning, need to choose a branch for the root of a polyno
mial with parameters. This might be wrong.The choice was done assuming [x]=
[91]4*(2/5*sqrt(sqrt(sqrt(x+1)+1)+1)*(sqrt(sqrt(x+1)+1)+1)^2-4/3*sqrt(sqrt(
sqrt(x+1)+1)+1)*(sqrt(sqrt(x+1)+1)+1)+2*sqrt(sqrt(sqrt(x+1)+1)+1)+sqrt(2*(s
qrt(2)-1))/4*ln(abs(sqrt(sqrt(sqrt(x+1)+1)+1)-sqrt((2+2*sqrt(2))/2)))-sqrt(
2*(sqrt(2)-1))/4*ln(sqrt(sqrt(sqrt(x+1)+1)+1)+sqrt((2+2*sqrt(2))/2))-sqrt(2
*(sqrt(2)+1))/2*atan(sqrt(sqrt(sqrt(x+1)+1)+1)/sqrt(-(2-2*sqrt(2))/2)))
```

maple [A] time = 0.05, size = 115, normalized size = 0.62

$$\frac{8\left(1+\sqrt{1+\sqrt{1+x}}\right)^{\frac{5}{2}}}{5}-\frac{16\left(1+\sqrt{1+\sqrt{1+x}}\right)^{\frac{3}{2}}}{3}+8\sqrt{1+\sqrt{1+\sqrt{1+x}}}-\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{1+\sqrt{1+\sqrt{1+x}}}}{\sqrt{\sqrt{2}-1}}\right)}{\sqrt{\sqrt{2}-1}}-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\sqrt{1+\sqrt{1+x}}}}{\sqrt{1+\sqrt{2}}}\right)}{\sqrt{1+\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)*(1+(1+x)^(1/2))^(1/2)/x/(1+(1+(1+x)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 8/5*(1+(1+(1+x)^(1/2))^(1/2))^(5/2)-16/3*(1+(1+(1+x)^(1/2))^(1/2))^(3/2)+8*
(1+(1+(1+x)^(1/2))^(1/2))^(1/2)-2*2^(1/2)/(2^(1/2)-1)^(1/2)*arctan((1+(1+(1
+x)^(1/2))^(1/2))^(1/2)/(2^(1/2)-1)^(1/2))-2*2^(1/2)/(1+2^(1/2))^(1/2)*arct
anh((1+(1+(1+x)^(1/2))^(1/2))^(1/2)/(1+2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}\sqrt{\sqrt{x+1}+1}}{x\sqrt{\sqrt{\sqrt{x+1}+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(1+(1+x)^(1/2))^(1/2)/x/(1+(1+(1+x)^(1/2))^(1/2))^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + 1)*sqrt(sqrt(x + 1) + 1)/(x*sqrt(sqrt(sqrt(x + 1) + 1) +
 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{x+1}+1}\,\sqrt{x+1}}{x\sqrt{\sqrt{\sqrt{x+1}+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((x + 1)^(1/2) + 1)^(1/2)*(x + 1)^(1/2))/(x*(((x + 1)^(1/2) + 1)^(1/2)
 + 1)^(1/2)),x)
```

```
[Out] int((((x + 1)^(1/2) + 1)^(1/2)*(x + 1)^(1/2))/(x*(((x + 1)^(1/2) + 1)^(1/2)
 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}\sqrt{\sqrt{x+1}+1}}{x\sqrt{\sqrt{\sqrt{x+1}+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(1+(1+x)**(1/2))**(1/2)/x/(1+(1+(1+x)**(1/2))**(1/2)
)**(1/2),x)
```

```
[Out] Integral(sqrt(x + 1)*sqrt(sqrt(x + 1) + 1)/(x*sqrt(sqrt(sqrt(x + 1) + 1) +
1)), x)
```

3.1915 $\int \frac{1+x}{(3+x)(1+2x)\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=185

$$\frac{\log\left(5\sqrt[3]{x^2+1}+\sqrt[3]{10}\,x-2\sqrt[3]{10}\right)}{5\sqrt[3]{10}}-\frac{\log\left(10^{2/3}x^2+25\left(x^2+1\right)^{2/3}+\left(10\sqrt[3]{10}-5\sqrt[3]{10}\,x\right)\sqrt[3]{x^2+1}-4\,10^{2/3}x+4\,10^{2/}\right.}{10\sqrt[3]{10}}$$

Rubi [C] time = 0.57, antiderivative size = 238, normalized size of antiderivative = 1.29, number of steps used = 18, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {6742, 757, 429, 444, 55, 617, 204, 31}

$$\frac{2}{15}xF_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};\frac{x^2}{9},-x^2\right)+\frac{1}{5}xF_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};4x^2,-x^2\right)-\frac{\log\left(1-4x^2\right)}{20\sqrt[3]{10}}-\frac{\log\left(9-x^2\right)}{10\sqrt[3]{10}}+\frac{3\log\left(\sqrt[3]{10}-2\sqrt[3]{x^2+1}\right)}{20\sqrt[3]{10}}+\frac{3\log\left(\sqrt[3]{10}-\sqrt[3]{x^2+1}\right)}{10\sqrt[3]{10}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{x^2+1}+\sqrt[3]{5}}{\sqrt{3}\sqrt[3]{5}}\right)}{5\sqrt[3]{10}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\,2^{2/3}\sqrt[3]{x^2+1}+\sqrt[3]{5}}{\sqrt{3}\sqrt[3]{5}}\right)}{10\sqrt[3]{10}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x)/((3 + x)*(1 + 2*x)*(1 + x^2)^(1/3)),x]
```

```
[Out] (2*x*AppellF1[1/2, 1, 1/3, 3/2, x^2/9, -x^2])/15 + (x*AppellF1[1/2, 1, 1/3,
 3/2, 4*x^2, -x^2])/5 + (Sqrt[3]*ArcTan[(5^(1/3) + 2^(2/3)*(1 + x^2)^(1/3))
/(Sqrt[3]*5^(1/3))])/(5*10^(1/3)) + (Sqrt[3]*ArcTan[(5^(1/3) + 2*2^(2/3)*(1
 + x^2)^(1/3))/(Sqrt[3]*5^(1/3))])/(10*10^(1/3)) - Log[1 - 4*x^2]/(20*10^(1
/3)) - Log[9 - x^2]/(10*10^(1/3)) + (3*Log[10^(1/3) - 2*(1 + x^2)^(1/3)])/(
20*10^(1/3)) + (3*Log[10^(1/3) - (1 + x^2)^(1/3)])/(10*10^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1+x}{(3+x)(1+2x)\sqrt[3]{1+x^2}}\,dx = \int \left(\frac{2}{5(3+x)\sqrt[3]{1+x^2}}+\frac{1}{5(1+2x)\sqrt[3]{1+x^2}}\right)dx$$

$$= \frac{1}{5}\int \frac{1}{(1+2x)\sqrt[3]{1+x^2}}\,dx+\frac{2}{5}\int \frac{1}{(3+x)\sqrt[3]{1+x^2}}\,dx$$

$$= \frac{1}{5}\int \left(\frac{1}{\left(1-4x^2\right)\sqrt[3]{1+x^2}}+\frac{2x}{\sqrt[3]{1+x^2}\left(-1+4x^2\right)}\right)dx+\frac{2}{5}\int \left(-\frac{3}{\left(-9+x^2\right)\sqrt[3]{1}\right.$$

$$= \frac{1}{5}\int \frac{1}{\left(1-4x^2\right)\sqrt[3]{1+x^2}}\,dx+\frac{2}{5}\int \frac{x}{\left(-9+x^2\right)\sqrt[3]{1+x^2}}\,dx+\frac{2}{5}\int \frac{x}{\sqrt[3]{1+x^2}\left(-1\right.}$$

$$= \frac{2}{15}xF_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};\frac{x^2}{9},-x^2\right)+\frac{1}{5}xF_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};4x^2,-x^2\right)+\frac{1}{5}\,\mathrm{Subst}\left(\int \frac{}{(-9}\right.$$

$$= \frac{2}{15}xF_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};\frac{x^2}{9},-x^2\right)+\frac{1}{5}xF_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};4x^2,-x^2\right)-\frac{\log\left(1-4x^2\right)}{20\sqrt[3]{10}}-\frac{\mathrm{l}}{}$$

$$= \frac{2}{15}xF_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};\frac{x^2}{9},-x^2\right)+\frac{1}{5}xF_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};4x^2,-x^2\right)-\frac{\log\left(1-4x^2\right)}{20\sqrt[3]{10}}-\frac{\mathrm{l}}{}$$

$$= \frac{2}{15}xF_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};\frac{x^2}{9},-x^2\right)+\frac{1}{5}xF_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};4x^2,-x^2\right)+\frac{\sqrt{3}\tan^{-1}\left(\frac{5+10^{2/3}}{5}\right.}{5\sqrt[3]{10}}$$

Mathematica [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1+x}{(3+x)(1+2x)\sqrt[3]{1+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x)/((3 + x)*(1 + 2*x)*(1 + x^2)^(1/3)),x]

[Out] Integrate[(1 + x)/((3 + x)*(1 + 2*x)*(1 + x^2)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.27, size = 185, normalized size = 1.00

$$\frac{\log\left(5\sqrt[3]{x^2+1}+\sqrt[3]{10}\,x-2\sqrt[3]{10}\right)}{5\sqrt[3]{10}}-\frac{\log\left(10^{2/3}x^2+25\left(x^2+1\right)^{2/3}+\left(10\sqrt[3]{10}-5\sqrt[3]{10}\,x\right)\sqrt[3]{x^2+1}-4\,10^{2/3}x+4\,10^{2/3}\right)}{10\sqrt[3]{10}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2+1}}{\sqrt{3}}-\frac{2\sqrt[3]{2}\,x}{\sqrt{3}\,5^{2/3}}+\frac{4\sqrt[3]{2}}{\sqrt{3}\,5^{2/3}}}{\sqrt[3]{x^2+1}}\right)}{5\sqrt[3]{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((3 + x)*(1 + 2*x)*(1 + x^2)^(1/3)),x]
```

```
[Out] -1/5*(Sqrt[3]*ArcTan[((4*2^(1/3))/(Sqrt[3]*5^(2/3)) - (2*2^(1/3)*x)/(Sqrt[3
]*5^(2/3)) + (1 + x^2)^(1/3)/Sqrt[3])/(1 + x^2)^(1/3)])/10^(1/3) + Log[-2*1
0^(1/3) + 10^(1/3)*x + 5*(1 + x^2)^(1/3)]/(5*10^(1/3)) - Log[4*10^(2/3) - 4
*10^(2/3)*x + 10^(2/3)*x^2 + (10*10^(1/3) - 5*10^(1/3)*x)*(1 + x^2)^(1/3) +
 25*(1 + x^2)^(2/3)]/(10*10^(1/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(3+x)/(1+2*x)/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(x^2+1\right)^{\frac{1}{3}}(2\,x+1)(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(3+x)/(1+2*x)/(x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/((x^2 + 1)^(1/3)*(2*x + 1)*(x + 3)), x)
```

maple [C] time = 16.31, size = 1371, normalized size = 7.41

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(3+x)/(1+2*x)/(x^2+1)^(1/3),x)
```

```
[Out] RootOf(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2)*ln(-(2389977875
*RootOf(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-10
0)^3*x^3-34881392500*RootOf(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*
_Z^2)^2*RootOf(_Z^3-100)^2*x^3+4779955750*RootOf(RootOf(_Z^3-100)^2+50*_Z*R
ootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^3*x^2-69762785000*RootOf(RootOf
(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2)^2*RootOf(_Z^3-100)^2*x^2-197
503327200*(x^2+1)^(2/3)*RootOf(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+25
00*_Z^2)*RootOf(_Z^3-100)^2*x+28679734500*RootOf(RootOf(_Z^3-100)^2+50*_Z*R
ootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^3*x-418576710000*RootOf(RootOf(
_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2)^2*RootOf(_Z^3-100)^2*x+395006
654400*(x^2+1)^(2/3)*RootOf(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*
_Z^2)*RootOf(_Z^3-100)^2-7900133088*(x^2+1)^(1/3)*RootOf(_Z^3-100)^2*x^2-25
9900176450*(x^2+1)^(1/3)*RootOf(_Z^3-100)*RootOf(RootOf(_Z^3-100)^2+50*_Z*R
ootOf(_Z^3-100)+2500*_Z^2)*x^2+31600532352*(x^2+1)^(1/3)*RootOf(_Z^3-100)^2
*x+1039600705800*(x^2+1)^(1/3)*RootOf(_Z^3-100)*RootOf(RootOf(_Z^3-100)^2+5
```

```
0*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x-3059171680*RootOf(_Z^3-100)*x^3+44648182
400*RootOf(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x^3-3160053
2352*(x^2+1)^(1/3)*RootOf(_Z^3-100)^2-1039600705800*(x^2+1)^(1/3)*RootOf(_Z
^3-100)*RootOf(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2)+1163441
22955*RootOf(_Z^3-100)*x^2-1698026186900*RootOf(RootOf(_Z^3-100)^2+50*_Z*Ro
otOf(_Z^3-100)+2500*_Z^2)*x^2-135106477950*(x^2+1)^(2/3)*x-36710060160*Root
Of(_Z^3-100)*x+535778188800*RootOf(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100
)+2500*_Z^2)*x+270212955900*(x^2+1)^(2/3)+122462466315*RootOf(_Z^3-100)-178
7322551700*RootOf(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2))/(1+
2*x)/(3+x)^2)+1/50*RootOf(_Z^3-100)*ln(-(348813925*RootOf(RootOf(_Z^3-100)^
2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^3*x^3-59749446875*Root
Of(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2)^2*RootOf(_Z^3-100)^
2*x^3+697627850*RootOf(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2)
*RootOf(_Z^3-100)^3*x^2-119498893750*RootOf(RootOf(_Z^3-100)^2+50*_Z*RootOf
(_Z^3-100)+2500*_Z^2)^2*RootOf(_Z^3-100)^2*x^2+98751663600*(x^2+1)^(2/3)*Ro
otOf(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^
2*x+4185767100*RootOf(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2)*
RootOf(_Z^3-100)^3*x-716993362500*RootOf(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z
^3-100)+2500*_Z^2)^2*RootOf(_Z^3-100)^2*x-197503327200*(x^2+1)^(2/3)*RootOf
(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^2+39
50066544*(x^2+1)^(1/3)*RootOf(_Z^3-100)^2*x^2+67553238975*(x^2+1)^(1/3)*Roo
tOf(_Z^3-100)*RootOf(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x
^2-15800266176*(x^2+1)^(1/3)*RootOf(_Z^3-100)^2*x-270212955900*(x^2+1)^(1/3
)*RootOf(_Z^3-100)*RootOf(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z
^2)*x+1144109674*RootOf(_Z^3-100)*x^3-195978185750*RootOf(RootOf(_Z^3-100)^
2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x^3+15800266176*(x^2+1)^(1/3)*RootOf(_Z
^3-100)^2+270212955900*(x^2+1)^(1/3)*RootOf(_Z^3-100)*RootOf(RootOf(_Z^3-10
0)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2)-15585006169*RootOf(_Z^3-100)*x^2+266
9605286375*RootOf(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x^2+
129950088225*(x^2+1)^(2/3)*x+13729316088*RootOf(_Z^3-100)*x-2351738229000*R
ootOf(RootOf(_Z^3-100)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x-259900176450*(
x^2+1)^(2/3)-17873225517*RootOf(_Z^3-100)+3061561657875*RootOf(RootOf(_Z^3-
100)^2+50*_Z*RootOf(_Z^3-100)+2500*_Z^2))/(1+2*x)/(3+x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(x^2+1\right)^{\frac{1}{3}}(2\,x+1)(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(3+x)/(1+2*x)/(x^2+1)^(1/3),x, algorithm="maxima")
[Out] integrate((x + 1)/((x^2 + 1)^(1/3)*(2*x + 1)*(x + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+1}{(2\,x+1)\,\left(x^2+1\right)^{1/3}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((2*x + 1)*(x^2 + 1)^(1/3)*(x + 3)),x)
[Out] int((x + 1)/((2*x + 1)*(x^2 + 1)^(1/3)*(x + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{(x+3)\,(2x+1)\,\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(3+x)/(1+2*x)/(x**2+1)**(1/3),x)
[Out] Integral((x + 1)/((x + 3)*(2*x + 1)*(x**2 + 1)**(1/3)), x)
```

3.1916 $\int \frac{x(5-4(1+k)x+3kx^2)}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+(b+bk)x-bkx^2+x^5\right)}\,dx$

Optimal. Leaf size=185

$$\frac{\log\left(x^2-\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{b^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+2x^2}\right)}{b^{2/3}}-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right.}{2}$$

Rubi [F] time = 15.98, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(5-4(1+k)x+3kx^2)}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+(b+bk)x-bkx^2+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(5 - 4*(1 + k)*x + 3*k*x^2))/(((1 - x)*x*(1 - k*x))^(1/3)*(-b + (b +
 b*k)*x - b*k*x^2 + x^5)),x]
```

```
[Out] (12*(1 + k)*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][x
^7/((1 - x^3)^(1/3)*(1 - k*x^3)^(1/3)*(b - b*(1 + k)*x^3 + b*k*x^6 - x^15))
, x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3) + (15*(1 - x)^(1/3)*x^(1/3)*
(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][x^4/((1 - x^3)^(1/3)*(1 - k*x^3)^(1
/3)*(-b + b*(1 + k)*x^3 - b*k*x^6 + x^15)), x], x, x^(1/3)])/((1 - x)*x*(1
- k*x))^(1/3) + (9*k*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Def
er[Int][x^10/((1 - x^3)^(1/3)*(1 - k*x^3)^(1/3)*(-b + b*(1 + k)*x^3 - b*k*x
^6 + x^15)), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\int \frac{x(5-4(1+k)x+3kx^2)}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+(b+bk)x-bkx^2+x^5\right)}\,dx = \frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{x^{2/3}(5-4(1+k)x+3kx^2)}{\sqrt[3]{1-x}\sqrt[3]{1-kx}\left(-b+(b+bk)x-bkx^2+x^5\right)}}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$= \frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{x^{2/3}(5-4(1+k)x+3kx^2)}{\sqrt[3]{1-x}\sqrt[3]{1-kx}\left(x^5-b(-1+x)(-1+kx)\right)}\,d}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$= \frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}\left(\int \frac{x^4(5-4(1+k)x^3+}{\sqrt[3]{1-x^3}\sqrt[3]{1-kx^3}\left(x^{15}-b(-1\right.}\right.}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$= \frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}\left(\int\left(\frac{4(1+k)x^7}{\sqrt[3]{1-x^3}\sqrt[3]{1-kx^3}\left(b-b(1+\right.}\right.\right.}{}$$

$$= \frac{\left(15\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt[3]{1-x^3}\sqrt[3]{1-kx^3}\left(-b+b(1\right.}\right.}{\sqrt[3]{(1-x)x(1-kx)}}$$

Mathematica [F] time = 2.07, size = 0, normalized size = 0.00

$$\int \frac{x(5-4(1+k)x+3kx^2)}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+(b+bk)x-bkx^2+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(5 - 4*(1 + k)*x + 3*k*x^2))/(((1 - x)*x*(1 - k*x))^(1/3)*(-b
+ (b + b*k)*x - b*k*x^2 + x^5)),x]
```

```
[Out] Integrate[(x*(5 - 4*(1 + k)*x + 3*k*x^2))/(((1 - x)*x*(1 - k*x))^(1/3)*(-b
+ (b + b*k)*x - b*k*x^2 + x^5)), x]
```

IntegrateAlgebraic [A] time = 2.90, size = 185, normalized size = 1.00

$$\frac{\log\left(x^2-\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{b^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+2x^2}\right)}{b^{2/3}}-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{b}x^2\sqrt[3]{kx^3+(-k-1)x^2+x}+x^4\right)}{2b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(5 - 4*(1 + k)*x + 3*k*x^2))/(((1 - x)*x*(1 - k*x))^(
1/3)*(-b + (b + b*k)*x - b*k*x^2 + x^5)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(2*x^2 +
 b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))])/b^(2/3) + Log[x^2 - b^(1/3)*(x
 + (-1 - k)*x^2 + k*x^3)^(1/3)]/b^(2/3) - Log[x^4 + b^(1/3)*x^2*(x + (-1 -
k)*x^2 + k*x^3)^(1/3) + b^(2/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(2/3
))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5-4*(1+k)*x+3*k*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(-b+(b*k+b)*x-b*
k*x^2+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,kx^2-4\,(k+1)x+5\right)x}{\left(x^5-bkx^2+(bk+b)x-b\right)\left((kx-1)(x-1)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5-4*(1+k)*x+3*k*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(-b+(b*k+b)*x-b*
k*x^2+x^5),x, algorithm="giac")
```

```
[Out] integrate((3*k*x^2 - 4*(k + 1)*x + 5)*x/((x^5 - b*k*x^2 + (b*k + b)*x - b)*
((k*x - 1)*(x - 1)*x)^(1/3)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x\left(5-4\,(1+k)\,x+3k\,x^2\right)}{\left((1-x)\,x\,(-kx+1)\right)^{\frac{1}{3}}\left(-b+(bk+b)\,x-bk\,x^2+x^5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(5-4*(1+k)*x+3*k*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(-b+(b*k+b)*x-b*k*x^2+
x^5),x)
```

```
[Out] int(x*(5-4*(1+k)*x+3*k*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(-b+(b*k+b)*x-b*k*x^2+
x^5),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,kx^2-4\,(k+1)x+5\right)x}{\left(x^5-bkx^2+(bk+b)x-b\right)\left((kx-1)(x-1)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5-4*(1+k)*x+3*k*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(-b+(b*k+b)*x-b*
k*x^2+x^5),x, algorithm="maxima")
```

```
[Out] integrate((3*k*x^2 - 4*(k + 1)*x + 5)*x/((x^5 - b*k*x^2 + (b*k + b)*x - b)*
((k*x - 1)*(x - 1)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x\,\left(3\,k\,x^2-4\,x\,\left(k+1\right)+5\right)}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/3}\,\left(-x^5+b\,k\,x^2+\left(-b-b\,k\right)\,x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(3*k*x^2 - 4*x*(k + 1) + 5))/((x*(k*x - 1)*(x - 1))^(1/3)*(b - x*(b
 + b*k) - x^5 + b*k*x^2)),x)
```

```
[Out] int(-(x*(3*k*x^2 - 4*x*(k + 1) + 5))/((x*(k*x - 1)*(x - 1))^(1/3)*(b - x*(b
 + b*k) - x^5 + b*k*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(3kx^2-4kx-4x+5\right)}{\sqrt[3]{x\,(x-1)\,(kx-1)}\,\left(-bkx^2+bkx+bx-b+x^5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5-4*(1+k)*x+3*k*x**2)/((1-x)*x*(-k*x+1))**(1/3)/(-b+(b*k+b)*x-
b*k*x**2+x**5),x)
```

```
[Out] Integral(x*(3*k*x**2 - 4*k*x - 4*x + 5)/((x*(x - 1)*(k*x - 1))**(1/3)*(-b*k
*x**2 + b*k*x + b*x - b + x**5)), x)
```

3.1917 $\int \frac{1+x^4}{\left(1-x^4\right)\sqrt[4]{x^3+x^5}}\,dx$

Optimal. Leaf size=185

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{2\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{2\sqrt[4]{2}}+\frac{2\left(x^5+x^3\right)^{3/4}}{x^2\left(x^2+1\right)}-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^5+x^3}}{\sqrt{2}x^2-\sqrt{x^5+x^3}}\right)}{2\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^5+x^3}}{2^{3/4}}}{x\sqrt[4]{x^5+x^3}}\right)}{2\,2^{3/4}}$$

Rubi [C] time = 0.58, antiderivative size = 81, normalized size of antiderivative = 0.44, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2056, 6715, 6725, 245, 1404, 429}

$$\frac{8x\sqrt[4]{x^2+1}F_1\left(\frac{1}{8};1,\frac{5}{4};\frac{9}{8};x^2,-x^2\right)}{\sqrt[4]{x^5+x^3}}-\frac{4x\sqrt[4]{x^2+1}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^2\right)}{\sqrt[4]{x^5+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^4)/((1 - x^4)*(x^3 + x^5)^(1/4)),x]
```

```
[Out] (8*x*(1 + x^2)^(1/4)*AppellF1[1/8, 1, 5/4, 9/8, x^2, -x^2])/(x^3 + x^5)^(1/
4) - (4*x*(1 + x^2)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -x^2])/(x^3 + x^
5)^(1/4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1404

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^4}{\left(1-x^4\right)\sqrt[4]{x^3+x^5}}\,dx = \frac{\left(x^{3/4}\sqrt[4]{1+x^2}\right)\int \frac{1+x^4}{x^{3/4}\sqrt[4]{1+x^2}\left(1-x^4\right)}\,dx}{\sqrt[4]{x^3+x^5}}$$

$$= \frac{\left(4x^{3/4}\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{1+x^{16}}{\sqrt[4]{1+x^8}\left(1-x^{16}\right)}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{x^3+x^5}}$$

$$= \frac{\left(4x^{3/4}\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \left(-\frac{1}{\sqrt[4]{1+x^8}}+\frac{2}{\sqrt[4]{1+x^8}\left(1-x^{16}\right)}\right)dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{x^3+x^5}}$$

$$= -\frac{\left(4x^{3/4}\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{x^3+x^5}} + \frac{\left(8x^{3/4}\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}\left(1-\right.}\right.}{\sqrt[4]{x^3+x^5}}$$

$$= -\frac{4x\sqrt[4]{1+x^2}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^2\right)}{\sqrt[4]{x^3+x^5}} + \frac{\left(8x^{3/4}\sqrt[4]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\left(1-x^8\right)\left(1+x^8\right)^{5/4}}\,dx, x, \sqrt[4]{x}\right.}{\sqrt[4]{x^3+x^5}}$$

$$= \frac{8x\sqrt[4]{1+x^2}\,F_1\left(\frac{1}{8};1,\frac{5}{4};\frac{9}{8};x^2,-x^2\right)}{\sqrt[4]{x^3+x^5}} - \frac{4x\sqrt[4]{1+x^2}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^2\right)}{\sqrt[4]{x^3+x^5}}$$

Mathematica [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{\left(1-x^4\right)\sqrt[4]{x^3+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^4)/((1 - x^4)*(x^3 + x^5)^(1/4)),x]
[Out] Integrate[(1 + x^4)/((1 - x^4)*(x^3 + x^5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.63, size = 185, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{2\sqrt[4]{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{2\sqrt[4]{2}} + \frac{2\left(x^5+x^3\right)^{3/4}}{x^2\left(x^2+1\right)} - \frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^5+x^3}}{\sqrt{2}x^2-\sqrt{x^5+x^3}}\right)}{2\,2^{3/4}} + \frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^5+x^3}}{2^{3/4}}}{x\sqrt[4]{x^5+x^3}}\right)}{2\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)/((1 - x^4)*(x^3 + x^5)^(1/4)),x]
[Out] (2*(x^3 + x^5)^(3/4))/(x^2*(1 + x^2)) + ArcTan[(2^(1/4)*x)/(x^3 + x^5)^(1/4
)]/(2*2^(1/4)) - ArcTan[(2^(3/4)*x*(x^3 + x^5)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x
^3 + x^5])]/(2*2^(3/4)) + ArcTanh[(2^(1/4)*x)/(x^3 + x^5)^(1/4)]/(2*2^(1/4)
) + ArcTanh[(x^2/2^(1/4) + Sqrt[x^3 + x^5]/2^(3/4))/(x*(x^3 + x^5)^(1/4))]/
(2*2^(3/4))
```

fricas [B] time = 13.50, size = 1100, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(-x^4+1)/(x^5+x^3)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/16*(4*2^(3/4)*(x^4 + x^2)*arctan(-1/2*(4*2^(3/4)*(x^5 + x^3)^(1/4)*x^2 -
 2^(3/4)*(2*2^(3/4)*sqrt(x^5 + x^3)*x + 2^(1/4)*(x^4 + 2*x^3 + x^2)) + 4*2^
(1/4)*(x^5 + x^3)^(3/4))/(x^4 - 2*x^3 + x^2)) - 2^(3/4)*(x^4 + x^2)*log((4*
sqrt(2)*(x^5 + x^3)^(1/4)*x^2 + 2^(3/4)*(x^4 + 2*x^3 + x^2) + 4*2^(1/4)*sqr
t(x^5 + x^3)*x + 4*(x^5 + x^3)^(3/4))/(x^4 - 2*x^3 + x^2)) + 2^(3/4)*(x^4 +
 x^2)*log((4*sqrt(2)*(x^5 + x^3)^(1/4)*x^2 - 2^(3/4)*(x^4 + 2*x^3 + x^2) -
4*2^(1/4)*sqrt(x^5 + x^3)*x + 4*(x^5 + x^3)^(3/4))/(x^4 - 2*x^3 + x^2)) - 4
*2^(1/4)*(x^4 + x^2)*arctan(1/2*(2*x^6 + 8*x^5 + 12*x^4 + 8*x^3 + 4*2^(3/4)
*(x^5 + x^3)^(3/4)*(x^2 - 6*x + 1) + 8*sqrt(2)*sqrt(x^5 + x^3)*(x^3 + 2*x^2
 + x) + 2*x^2 + sqrt(2)*(32*sqrt(2)*(x^5 + x^3)^(3/4)*x + 2^(3/4)*(x^6 - 16
*x^5 - 2*x^4 - 16*x^3 + x^2) + 4*2^(1/4)*sqrt(x^5 + x^3)*(x^3 - 6*x^2 + x)
+ 8*(x^5 + x^3)^(1/4)*(x^4 + 2*x^3 + x^2))*sqrt((4*2^(3/4)*(x^5 + x^3)^(1/4
)*x^2 + sqrt(2)*(x^4 + 2*x^3 + x^2) + 8*sqrt(x^5 + x^3)*x + 4*2^(1/4)*(x^5
+ x^3)^(3/4))/(x^4 + 2*x^3 + x^2)) + 8*2^(1/4)*(x^5 + x^3)^(1/4)*(3*x^4 - 2
*x^3 + 3*x^2))/(x^6 - 28*x^5 + 6*x^4 - 28*x^3 + x^2)) + 4*2^(1/4)*(x^4 + x^
2)*arctan(1/2*(2*x^6 + 8*x^5 + 12*x^4 + 8*x^3 - 4*2^(3/4)*(x^5 + x^3)^(3/4)
*(x^2 - 6*x + 1) + 8*sqrt(2)*sqrt(x^5 + x^3)*(x^3 + 2*x^2 + x) + 2*x^2 + sq
rt(2)*(32*sqrt(2)*(x^5 + x^3)^(3/4)*x - 2^(3/4)*(x^6 - 16*x^5 - 2*x^4 - 16*
x^3 + x^2) - 4*2^(1/4)*sqrt(x^5 + x^3)*(x^3 - 6*x^2 + x) + 8*(x^5 + x^3)^(1
/4)*(x^4 + 2*x^3 + x^2))*sqrt(-(4*2^(3/4)*(x^5 + x^3)^(1/4)*x^2 - sqrt(2)*(
x^4 + 2*x^3 + x^2) - 8*sqrt(x^5 + x^3)*x + 4*2^(1/4)*(x^5 + x^3)^(3/4))/(x^
4 + 2*x^3 + x^2)) - 8*2^(1/4)*(x^5 + x^3)^(1/4)*(3*x^4 - 2*x^3 + 3*x^2))/(x
^6 - 28*x^5 + 6*x^4 - 28*x^3 + x^2)) - 2^(1/4)*(x^4 + x^2)*log(8*(4*2^(3/4)
*(x^5 + x^3)^(1/4)*x^2 + sqrt(2)*(x^4 + 2*x^3 + x^2) + 8*sqrt(x^5 + x^3)*x
+ 4*2^(1/4)*(x^5 + x^3)^(3/4))/(x^4 + 2*x^3 + x^2)) + 2^(1/4)*(x^4 + x^2)*l
og(-8*(4*2^(3/4)*(x^5 + x^3)^(1/4)*x^2 - sqrt(2)*(x^4 + 2*x^3 + x^2) - 8*sq
rt(x^5 + x^3)*x + 4*2^(1/4)*(x^5 + x^3)^(3/4))/(x^4 + 2*x^3 + x^2)) - 32*(x
^5 + x^3)^(3/4))/(x^4 + x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^4+1}{\left(x^5+x^3\right)^{\frac{1}{4}}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(-x^4+1)/(x^5+x^3)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-(x^4 + 1)/((x^5 + x^3)^(1/4)*(x^4 - 1)), x)
```

maple [C] time = 13.52, size = 737, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(-x^4+1)/(x^5+x^3)^(1/4),x)
```

```
[Out] 2*x/(x^3*(x^2+1))^(1/4)+1/8*RootOf(_Z^4+8)*ln(-(-RootOf(_Z^4+8)^3*(x^5+x^3)
^(1/2)*x-2*(x^5+x^3)^(1/4)*RootOf(_Z^4+8)^2*x^2+RootOf(_Z^4+8)*x^4-2*RootOf
(_Z^4+8)*x^3+4*(x^5+x^3)^(3/4)+RootOf(_Z^4+8)*x^2)/(1+x)^2/x^2)+1/8*RootOf(
_Z^2+RootOf(_Z^4+8)^2)*ln(-((x^5+x^3)^(1/2)*RootOf(_Z^4+8)^2*RootOf(_Z^2+Ro
otOf(_Z^4+8)^2)*x+2*(x^5+x^3)^(1/4)*RootOf(_Z^4+8)^2*x^2+RootOf(_Z^2+RootOf
(_Z^4+8)^2)*x^4-2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*x^3+4*(x^5+x^3)^(3/4)+RootO
f(_Z^2+RootOf(_Z^4+8)^2)*x^2)/(1+x)^2/x^2)-1/32*ln(-(-RootOf(_Z^4+8)^3*x^4+
```

```
2*RootOf(_Z^4+8)^3*x^3+8*(x^5+x^3)^(1/4)*RootOf(_Z^4+8)^2*x^2-RootOf(_Z^4+8
)^3*x^2+16*(x^5+x^3)^(1/2)*RootOf(_Z^4+8)*x+16*(x^5+x^3)^(3/4))/(-1+x)^2/x^
2)*RootOf(_Z^4+8)^3-1/32*ln(-(-RootOf(_Z^4+8)^3*x^4+2*RootOf(_Z^4+8)^3*x^3+
8*(x^5+x^3)^(1/4)*RootOf(_Z^4+8)^2*x^2-RootOf(_Z^4+8)^3*x^2+16*(x^5+x^3)^(1
/2)*RootOf(_Z^4+8)*x+16*(x^5+x^3)^(3/4))/(-1+x)^2/x^2)*RootOf(_Z^4+8)^2*Roo
tOf(_Z^2+RootOf(_Z^4+8)^2)+1/16*RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z^4+8)
^2)*ln((RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^4-RootOf(_Z^4+8)^3
*x^4+2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^3-2*RootOf(_Z^4+8)^
3*x^3+8*RootOf(_Z^2+RootOf(_Z^4+8)^2)*(x^5+x^3)^(1/4)*RootOf(_Z^4+8)*x^2+Ro
otOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^2-RootOf(_Z^4+8)^3*x^2+8*Roo
tOf(_Z^2+RootOf(_Z^4+8)^2)*(x^5+x^3)^(1/2)*x+8*(x^5+x^3)^(1/2)*RootOf(_Z^4+
8)*x+16*(x^5+x^3)^(3/4))/(-1+x)^2/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4+1}{\left(x^5+x^3\right)^{\frac{1}{4}}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(-x^4+1)/(x^5+x^3)^(1/4),x, algorithm="maxima")
[Out] -integrate((x^4 + 1)/((x^5 + x^3)^(1/4)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^4+1}{\left(x^5+x^3\right)^{1/4}\,\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 + 1)/((x^3 + x^5)^(1/4)*(x^4 - 1)),x)
[Out] int(-(x^4 + 1)/((x^3 + x^5)^(1/4)*(x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{x^4\sqrt[4]{x^5+x^3}-\sqrt[4]{x^5+x^3}}\,dx-\int \frac{1}{x^4\sqrt[4]{x^5+x^3}-\sqrt[4]{x^5+x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(-x**4+1)/(x**5+x**3)**(1/4),x)
[Out] -Integral(x**4/(x**4*(x**5 + x**3)**(1/4) - (x**5 + x**3)**(1/4)), x) - Int
egral(1/(x**4*(x**5 + x**3)**(1/4) - (x**5 + x**3)**(1/4)), x)
```

3.1918 $\int \frac{(1+x^3)^{2/3}(2+x^3+x^6)}{x^6(-2+x^3)^2}\,dx$

Optimal. Leaf size=185

$$-\frac{35\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3+1}-3x\right)}{36\ 2^{2/3}\sqrt[3]{3}}+\frac{35\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2}\sqrt[3]{x^3+1}+\sqrt[3]{3}x}\right)}{12\ 2^{2/3}3^{5/6}}+\frac{35\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3+1}\,x+2^{2/3}\sqrt[3]{3}\left(x^3+1\right)^{2/3}+3x\right.}{72\ 2^{2/3}\sqrt[3]{3}}$$

Rubi [C] time = 0.50, antiderivative size = 264, normalized size of antiderivative = 1.43, number of steps used = 14, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {6742, 264, 277, 239, 378, 377, 200, 31, 634, 617, 204, 628, 429}

$$\frac{3}{8}xF_1\left(\frac{1}{3};1,-\frac{2}{3};\frac{4}{3};\frac{x^3}{2},-x^3\right)+\frac{x\left(x^3+1\right)^{2/3}}{3\left(2-x^3\right)}-\frac{1}{9}\sqrt[3]{\frac{2}{3}}\log\left(\sqrt[3]{2}-\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+1}}\right)-\frac{3}{8}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{2^{2/3}x}{\sqrt[6]{3}\sqrt[3]{x^3+1}}+\frac{1}{\sqrt{3}}\right)}{3\ 3^{5/6}}-\frac{\left(x^3+1\right)^{5/3}}{10x^5}-\frac{3\left(x^3+1\right)^{2/3}}{8x^2}+\frac{\log\left(\frac{\sqrt[3]{6}x}{\sqrt[3]{x^3+1}}+\frac{3^{2/3}x^2}{\left(x^3+1\right)^{2/3}}+2^{2/3}\right)}{9\ 2^{2/3}\sqrt[3]{3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((1 + x^3)^(2/3)*(2 + x^3 + x^6))/(x^6*(-2 + x^3)^2),x]
```

```
[Out] (-3*(1 + x^3)^(2/3))/(8*x^2) + (x*(1 + x^3)^(2/3))/(3*(2 - x^3)) - (1 + x^3
)^(5/3)/(10*x^5) + (3*x*AppellF1[1/3, 1, -2/3, 4/3, x^3/2, -x^3])/8 + (Sqrt
[3]*ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]])/4 + (2^(1/3)*ArcTan[1/Sqrt
[3] + (2^(2/3)*x)/(3^(1/6)*(1 + x^3)^(1/3))])/(3*3^(5/6)) - ((2/3)^(1/3)*Lo
g[2^(1/3) - (3^(1/3)*x)/(1 + x^3)^(1/3)])/9 + Log[2^(2/3) + (3^(2/3)*x^2)/(
1 + x^3)^(2/3) + (6^(1/3)*x)/(1 + x^3)^(1/3)]/(9*2^(2/3)*3^(1/3)) - (3*Log[
-x + (1 + x^3)^(1/3)])/8
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(1+x^3\right)^{2/3}\left(2+x^3+x^6\right)}{x^6\left(-2+x^3\right)^2}dx = \int \left(\frac{\left(1+x^3\right)^{2/3}}{2x^6}+\frac{3\left(1+x^3\right)^{2/3}}{4x^3}+\frac{2\left(1+x^3\right)^{2/3}}{\left(-2+x^3\right)^2}-\frac{3\left(1+x^3\right)^{2/3}}{4\left(-2+x^3\right)}\right)dx$$

$$= \frac{1}{2}\int \frac{\left(1+x^3\right)^{2/3}}{x^6}dx+\frac{3}{4}\int \frac{\left(1+x^3\right)^{2/3}}{x^3}dx-\frac{3}{4}\int \frac{\left(1+x^3\right)^{2/3}}{-2+x^3}dx+2\int \frac{\left(1+x^3\right)^{2}}{\left(-2+x^3\right.}$$

$$= -\frac{3\left(1+x^3\right)^{2/3}}{8x^2}+\frac{x\left(1+x^3\right)^{2/3}}{3\left(2-x^3\right)}-\frac{\left(1+x^3\right)^{5/3}}{10x^5}+\frac{3}{8}xF_1\left(\frac{1}{3};1,-\frac{2}{3};\frac{4}{3};\frac{x^3}{2},-x^3\right)-\frac{2}{3}$$

$$= -\frac{3\left(1+x^3\right)^{2/3}}{8x^2}+\frac{x\left(1+x^3\right)^{2/3}}{3\left(2-x^3\right)}-\frac{\left(1+x^3\right)^{5/3}}{10x^5}+\frac{3}{8}xF_1\left(\frac{1}{3};1,-\frac{2}{3};\frac{4}{3};\frac{x^3}{2},-x^3\right)+\frac{1}{4}$$

$$= -\frac{3\left(1+x^3\right)^{2/3}}{8x^2}+\frac{x\left(1+x^3\right)^{2/3}}{3\left(2-x^3\right)}-\frac{\left(1+x^3\right)^{5/3}}{10x^5}+\frac{3}{8}xF_1\left(\frac{1}{3};1,-\frac{2}{3};\frac{4}{3};\frac{x^3}{2},-x^3\right)+\frac{1}{4}$$

$$= -\frac{3\left(1+x^3\right)^{2/3}}{8x^2}+\frac{x\left(1+x^3\right)^{2/3}}{3\left(2-x^3\right)}-\frac{\left(1+x^3\right)^{5/3}}{10x^5}+\frac{3}{8}xF_1\left(\frac{1}{3};1,-\frac{2}{3};\frac{4}{3};\frac{x^3}{2},-x^3\right)+\frac{1}{4}$$

$$= -\frac{3\left(1+x^3\right)^{2/3}}{8x^2}+\frac{x\left(1+x^3\right)^{2/3}}{3\left(2-x^3\right)}-\frac{\left(1+x^3\right)^{5/3}}{10x^5}+\frac{3}{8}xF_1\left(\frac{1}{3};1,-\frac{2}{3};\frac{4}{3};\frac{x^3}{2},-x^3\right)+\frac{1}{4}$$

$$= -\frac{3\left(1+x^3\right)^{2/3}}{8x^2}+\frac{x\left(1+x^3\right)^{2/3}}{3\left(2-x^3\right)}-\frac{\left(1+x^3\right)^{5/3}}{10x^5}+\frac{3}{8}xF_1\left(\frac{1}{3};1,-\frac{2}{3};\frac{4}{3};\frac{x^3}{2},-x^3\right)+\frac{1}{4}$$

<u>**Mathematica**</u> [A] time = 0.30, size = 162, normalized size = 0.88

$$\frac{35\left(6\tan^{-1}\left(\frac{2^{2/3}x}{\sqrt[6]{3}\sqrt[3]{x^3+1}}+\frac{1}{\sqrt{3}}\right)+\sqrt{3}\left(\log\left(\frac{2^{2/3}\sqrt[3]{3}x}{\sqrt[3]{x^3+1}}+\frac{\sqrt[3]{2}3^{2/3}x^2}{\left(x^3+1\right)^{2/3}}+2\right)-2\log\left(2-\frac{2^{2/3}\sqrt[3]{3}x}{\sqrt[3]{x^3+1}}\right)\right)\right)}{72\,2^{2/3}3^{5/6}}+\frac{\left(x^3+1\right)^{2/3}\left(-97x^6+102x^3+24\right)}{120x^5\left(x^3-2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)^(2/3)*(2 + x^3 + x^6))/(x^6*(-2 + x^3)^2),x]
```

```
[Out] ((1 + x^3)^(2/3)*(24 + 102*x^3 - 97*x^6))/(120*x^5*(-2 + x^3)) + (35*(6*Arc
Tan[1/Sqrt[3] + (2^(2/3)*x)/(3^(1/6)*(1 + x^3)^(1/3))] + Sqrt[3]*(-2*Log[2
- (2^(2/3)*3^(1/3)*x)/(1 + x^3)^(1/3)] + Log[2 + (2^(1/3)*3^(2/3)*x^2)/(1 +
 x^3)^(2/3) + (2^(2/3)*3^(1/3)*x)/(1 + x^3)^(1/3)])))/(72*2^(2/3)*3^(5/6))
```

<u>**IntegrateAlgebraic**</u> [A] time = 0.60, size = 185, normalized size = 1.00

$$-\frac{35\log\left(\sqrt[3]{2}3^{2/3}\sqrt[3]{x^3+1}-3x\right)}{36\,2^{2/3}\sqrt[3]{3}}+\frac{35\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2}\sqrt[3]{x^3+1}+\sqrt[3]{3}x}\right)}{12\,2^{2/3}3^{5/6}}+\frac{35\log\left(\sqrt[3]{2}3^{2/3}\sqrt[3]{x^3+1}x+2^{2/3}\sqrt[3]{3}\left(x^3+1\right)^{2/3}+3x^2\right)}{72\,2^{2/3}\sqrt[3]{3}}+\frac{\left(x^3+1\right)^{2/3}\left(-97x^6+102x^3+24\right)}{120x^5\left(x^3-2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)^(2/3)*(2 + x^3 + x^6))/(x^6*(-2 + x^3)^2),x]
```

```
[Out] ((1 + x^3)^(2/3)*(24 + 102*x^3 - 97*x^6))/(120*x^5*(-2 + x^3)) + (35*ArcTan
[(3^(5/6)*x)/(3^(1/3)*x + 2*2^(1/3)*(1 + x^3)^(1/3))])/(12*2^(2/3)*3^(5/6))
```

```
 - (35*Log[-3*x + 2^(1/3)*3^(2/3)*(1 + x^3)^(1/3)])/(36*2^(2/3)*3^(1/3)) +
(35*Log[3*x^2 + 2^(1/3)*3^(2/3)*x*(1 + x^3)^(1/3) + 2^(2/3)*3^(1/3)*(1 + x^
3)^(2/3)])/(72*2^(2/3)*3^(1/3))
```

fricas [B] time = 3.57, size = 319, normalized size = 1.72

$$\frac{350 \cdot 12^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} \left(x^{8} - 2 x^{5}\right) \log\left(\frac{18 \cdot 12^{\frac{1}{3}} \left(-1\right)^{\frac{2}{3}} \left(x^{3}+1\right)^{\frac{1}{3}} x^{2} + 12^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} \left(x^{3}-2\right) - 36 \left(x^{3}+1\right)^{\frac{2}{3}} x}{x^{3}-2}\right) - 175 \cdot 12^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} \left(x^{8} - 2 x^{5}\right) \log\left(-\frac{6 \cdot 12^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} \left(4 x^{4}+x\right) \left(x^{3}+1\right)^{\frac{2}{3}} - 12^{\frac{1}{3}} \left(-1\right)^{\frac{2}{3}} \left(55 x^{6}+50 x^{3}+4\right) - 18 \left(7 x^{5}+4 x^{2}\right) \left(x^{3}+1\right)^{\frac{1}{3}}}{x^{6}-4 x^{3}+4}\right) - 2100 \cdot 12^{\frac{1}{6}} \left(-1\right)^{\frac{1}{3}} \left(x^{8} - 2 x^{5}\right) \arctan\left(\frac{12^{\frac{1}{6}} \left(12 \cdot 12^{\frac{2}{3}} \left(-1\right)^{\frac{2}{3}} \left(4 x^{7}-7 x^{4}-2 x\right) \left(x^{3}+1\right)^{\frac{2}{3}} + 36 \left(-1\right)^{\frac{1}{3}} \left(55 x^{8}+50 x^{5}+4 x^{2}\right) \left(x^{3}+1\right)^{\frac{1}{3}} - 12^{\frac{1}{3}} \left(377 x^{9}+600 x^{6}+204 x^{3}+8\right)\right)}{6 \left(487 x^{9}+480 x^{6}+12 x^{3}-8\right)}\right) - 108 \left(97 x^{6} - 102 x^{3} - 24\right) \left(x^{3}+1\right)^{\frac{2}{3}}}{12960 \left(x^{8} - 2 x^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^6+x^3+2)/x^6/(x^3-2)^2,x, algorithm="fricas")
```

```
[Out] 1/12960*(350*12^(2/3)*(-1)^(1/3)*(x^8 - 2*x^5)*log((18*12^(1/3)*(-1)^(2/3)*
(x^3 + 1)^(1/3)*x^2 + 12^(2/3)*(-1)^(1/3)*(x^3 - 2) - 36*(x^3 + 1)^(2/3)*x)
/(x^3 - 2)) - 175*12^(2/3)*(-1)^(1/3)*(x^8 - 2*x^5)*log(-(6*12^(2/3)*(-1)^(
1/3)*(4*x^4 + x)*(x^3 + 1)^(2/3) - 12^(1/3)*(-1)^(2/3)*(55*x^6 + 50*x^3 + 4
) - 18*(7*x^5 + 4*x^2)*(x^3 + 1)^(1/3))/(x^6 - 4*x^3 + 4)) - 2100*12^(1/6)*
(-1)^(1/3)*(x^8 - 2*x^5)*arctan(1/6*12^(1/6)*(12*12^(2/3)*(-1)^(2/3)*(4*x^7
 - 7*x^4 - 2*x)*(x^3 + 1)^(2/3) + 36*(-1)^(1/3)*(55*x^8 + 50*x^5 + 4*x^2)*(
x^3 + 1)^(1/3) - 12^(1/3)*(377*x^9 + 600*x^6 + 204*x^3 + 8))/(487*x^9 + 480
*x^6 + 12*x^3 - 8)) - 108*(97*x^6 - 102*x^3 - 24)*(x^3 + 1)^(2/3))/(x^8 - 2
*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6} + x^{3} + 2\right) \left(x^{3} + 1\right)^{\frac{2}{3}}}{\left(x^{3} - 2\right)^{2} x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^6+x^3+2)/x^6/(x^3-2)^2,x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^3 + 2)*(x^3 + 1)^(2/3)/((x^3 - 2)^2*x^6), x)
```

maple [C] time = 2.98, size = 900, normalized size = 4.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)*(x^6+x^3+2)/x^6/(x^3-2)^2,x)
```

```
[Out] -1/120*(97*x^9-5*x^6-126*x^3-24)/(x^3-2)/(x^3+1)^(1/3)/x^5+35/216*RootOf(_Z
^3+18)*ln((3*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)*RootO
f(_Z^3+18)^3*x^3+135*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^
2)^2*RootOf(_Z^3+18)^2*x^3-21*(x^3+1)^(2/3)*RootOf(RootOf(_Z^3+18)^2+18*_Z*
RootOf(_Z^3+18)+324*_Z^2)*RootOf(_Z^3+18)^2*x+4*RootOf(_Z^3+18)^2*(x^3+1)^(
1/3)*x^2+9*(x^3+1)^(1/3)*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324
*_Z^2)*RootOf(_Z^3+18)*x^2+2*RootOf(_Z^3+18)*x^3+90*RootOf(RootOf(_Z^3+18)^
2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)*x^3-3*x*(x^3+1)^(2/3)+2*RootOf(_Z^3+18)+9
0*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2))/(x^3-2))-35/216
*ln((6*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)*RootOf(_Z^3
+18)^3*x^3-162*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)^2*R
ootOf(_Z^3+18)^2*x^3+42*(x^3+1)^(2/3)*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf
(_Z^3+18)+324*_Z^2)*RootOf(_Z^3+18)^2*x+RootOf(_Z^3+18)^2*(x^3+1)^(1/3)*x^2
+144*(x^3+1)^(1/3)*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)
*RootOf(_Z^3+18)*x^2-10*RootOf(_Z^3+18)*x^3+270*RootOf(RootOf(_Z^3+18)^2+18
*_Z*RootOf(_Z^3+18)+324*_Z^2)*x^3-48*x*(x^3+1)^(2/3)-4*RootOf(_Z^3+18)+108*
RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2))/(x^3-2))*RootOf(_
Z^3+18)-35/12*ln((6*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2
)*RootOf(_Z^3+18)^3*x^3-162*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+
```

```
324*_Z^2)^2*RootOf(_Z^3+18)^2*x^3+42*(x^3+1)^(2/3)*RootOf(RootOf(_Z^3+18)^2
+18*_Z*RootOf(_Z^3+18)+324*_Z^2)*RootOf(_Z^3+18)^2*x+RootOf(_Z^3+18)^2*(x^3
+1)^(1/3)*x^2+144*(x^3+1)^(1/3)*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+
18)+324*_Z^2)*RootOf(_Z^3+18)*x^2-10*RootOf(_Z^3+18)*x^3+270*RootOf(RootOf(
_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)*x^3-48*x*(x^3+1)^(2/3)-4*RootOf(
_Z^3+18)+108*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2))/(x^3
-2))*RootOf(RootOf(_Z^3+18)^2+18*_Z*RootOf(_Z^3+18)+324*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^6 + x^3 + 2)(x^3 + 1)^{\frac{2}{3}}}{(x^3 - 2)^2 x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^6+x^3+2)/x^6/(x^3-2)^2,x, algorithm="maxima")
```

```
[Out] integrate((x^6 + x^3 + 2)*(x^3 + 1)^(2/3)/((x^3 - 2)^2*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^3 + 1)^{2/3}\,(x^6 + x^3 + 2)}{x^6\,(x^3 - 2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)^(2/3)*(x^3 + x^6 + 2))/(x^6*(x^3 - 2)^2),x)
```

```
[Out] int(((x^3 + 1)^(2/3)*(x^3 + x^6 + 2))/(x^6*(x^3 - 2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)*(x**6+x**3+2)/x**6/(x**3-2)**2,x)
```

```
[Out] Timed out
```

3.1919 $\int \frac{1+x^8}{\sqrt[4]{x^2+x^6}\left(-1+x^8\right)} dx$

Optimal. Leaf size=185

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{4\,2^{3/4}}-\frac{\left(x^6+x^2\right)^{3/4}}{x\left(x^4+1\right)}$$

Rubi [C] time = 0.54, antiderivative size = 81, normalized size of antiderivative = 0.44, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2056, 6715, 6725, 245, 1404, 429}

$$\frac{2x\sqrt[4]{x^4+1}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^6+x^2}}-\frac{4x\sqrt[4]{x^4+1}\,F_1\left(\frac{1}{8};1,\frac{5}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^8)/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] (-4*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, 5/4, 9/8, x^4, -x^4])/(x^2 + x^6)^(1
/4) + (2*x*(1 + x^4)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -x^4])/(x^2 + x
^6)^(1/4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1404

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^8}{\sqrt[4]{x^2+x^6}\left(-1+x^8\right)}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{1+x^8}{\sqrt{x}\sqrt[4]{1+x^4}\left(-1+x^8\right)}\,dx}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1+x^{16}}{\sqrt[4]{1+x^8}\left(-1+x^{16}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[4]{1+x^8}}+\frac{2}{\sqrt[4]{1+x^8}\left(-1+x^{16}\right)}\right)dx,x,\sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left(4\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}\left(-1+x^{16}\right)}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left(4\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(-1+x^8\right)\left(1+x^8\right)^{5/4}}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= -\frac{4x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,\frac{5}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^2+x^6}} + \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}}$$

Mathematica [C] time = 0.37, size = 45, normalized size = 0.24

$$-\frac{x\left(\sqrt[4]{x^4+1}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};-x^4,x^4\right)+1\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^8)/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -((x*(1 + (1 + x^4)^(1/4)*AppellF1[1/8, -3/4, 1, 9/8, -x^4, x^4]))/(x^2 + x
^6)^(1/4))
```

IntegrateAlgebraic [A] time = 0.78, size = 185, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}} + \frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{4\,2^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}} - \frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{4\,2^{3/4}} - \frac{\left(x^6+x^2\right)^{3/4}}{x\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^8)/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -((x^2 + x^6)^(3/4)/(x*(1 + x^4))) - ArcTan[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/
(4*2^(1/4)) + ArcTan[(2^(3/4)*x*(x^2 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^2
+ x^6])]/(4*2^(3/4)) - ArcTanh[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/(4*2^(1/4)) -
```

```
ArcTanh[(x^2/2^(1/4) + Sqrt[x^2 + x^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))]/(4*
2^(3/4))
```

fricas [B] time = 51.81, size = 1055, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="fricas")
```

```
[Out] -1/32*(4*2^(3/4)*(x^5 + x)*arctan(1/2*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + 2^
(3/4)*(2*2^(3/4)*sqrt(x^6 + x^2)*x + 2^(1/4)*(x^5 + 2*x^3 + x)) + 4*2^(1/4)
*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 2^(3/4)*(x^5 + x)*log(-(4*sqrt(2)*
(x^6 + x^2)^(1/4)*x^2 + 2^(3/4)*(x^5 + 2*x^3 + x) + 4*2^(1/4)*sqrt(x^6 + x^
2)*x + 4*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) - 2^(3/4)*(x^5 + x)*log(-(4*
sqrt(2)*(x^6 + x^2)^(1/4)*x^2 - 2^(3/4)*(x^5 + 2*x^3 + x) - 4*2^(1/4)*sqrt(
x^6 + x^2)*x + 4*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) - 4*2^(1/4)*(x^5 + x
)*arctan(-1/2*(2*x^9 + 8*x^7 + 12*x^5 + 8*x^3 + 4*2^(3/4)*(x^6 + x^2)^(3/4)
*(x^4 - 6*x^2 + 1) + 8*sqrt(2)*sqrt(x^6 + x^2)*(x^5 + 2*x^3 + x) - sqrt(2)*
(32*sqrt(2)*(x^6 + x^2)^(3/4)*x^2 + 2^(3/4)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3
+ x) + 4*2^(1/4)*sqrt(x^6 + x^2)*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x^4 + x^2)*
(x^6 + x^2)^(1/4))*sqrt((4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2
*x^3 + x) + 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3
 + x)) + 8*2^(1/4)*(3*x^6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2*x)/(x^9 -
28*x^7 + 6*x^5 - 28*x^3 + x)) + 4*2^(1/4)*(x^5 + x)*arctan(-1/2*(2*x^9 + 8*
x^7 + 12*x^5 + 8*x^3 - 4*2^(3/4)*(x^6 + x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 8*sq
rt(2)*sqrt(x^6 + x^2)*(x^5 + 2*x^3 + x) - sqrt(2)*(32*sqrt(2)*(x^6 + x^2)^(
3/4)*x^2 - 2^(3/4)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3 + x) - 4*2^(1/4)*sqrt(x^6
 + x^2)*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt(-
(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 +
 x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x)) - 8*2^(1/4)*(3*x^
6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3
+ x)) + 2^(1/4)*(x^5 + x)*log(8*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*
(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5
 + 2*x^3 + x)) - 2^(1/4)*(x^5 + x)*log(-8*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2
- sqrt(2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(
3/4))/(x^5 + 2*x^3 + x)) + 32*(x^6 + x^2)^(3/4))/(x^5 + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 + 1}{\left(x^8 - 1\right)\left(x^6 + x^2\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="giac")
```

```
[Out] integrate((x^8 + 1)/((x^8 - 1)*(x^6 + x^2)^(1/4)), x)
```

maple [C] time = 19.42, size = 651, normalized size = 3.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8+1)/(x^6+x^2)^(1/4)/(x^8-1),x)
```

```
[Out] -x/(x^2*(x^4+1))^(1/4)-1/16*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln(-(RootOf(_Z^2+
RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*(x^6+x^2)^(1/2)*x+RootOf(_Z^2+RootOf(_Z^
4+8)^2)*x^5+2*RootOf(_Z^4+8)^2*(x^6+x^2)^(1/4)*x^2-2*RootOf(_Z^2+RootOf(_Z^
4+8)^2)*x^3+4*(x^6+x^2)^(3/4)+RootOf(_Z^2+RootOf(_Z^4+8)^2)*x)/x/(x^2+1)^2)
```

```
+1/16*RootOf(_Z^4+8)*ln(-(RootOf(_Z^4+8)^3*(x^6+x^2)^(1/2)*x-RootOf(_Z^4+8)
*x^5-2*RootOf(_Z^4+8)^2*(x^6+x^2)^(1/4)*x^2+2*RootOf(_Z^4+8)*x^3+4*(x^6+x^2
)^(3/4)-RootOf(_Z^4+8)*x)/x/(x^2+1)^2)-1/32*ln(-(RootOf(_Z^4+8)^2*x^2-2*Roo
tOf(_Z^4+8)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))/(1+x)/x/(-1+x))*RootOf(_Z^
4+8)^3-1/32*ln(-(RootOf(_Z^4+8)^2*x^2-2*RootOf(_Z^4+8)*(x^6+x^2)^(1/4)*x+2*
(x^6+x^2)^(1/2))/(1+x)/x/(-1+x))*RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z^4+8
)^2)+1/32*RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln((RootOf(_Z^4+8)
^3*x^5-RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^5+2*RootOf(_Z^4+8)^
3*x^3-2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^3+8*RootOf(_Z^4+8)
*(x^6+x^2)^(1/4)*RootOf(_Z^2+RootOf(_Z^4+8)^2)*x^2+RootOf(_Z^4+8)^3*x-RootO
f(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x-8*(x^6+x^2)^(1/2)*RootOf(_Z^4+8
)*x-8*RootOf(_Z^2+RootOf(_Z^4+8)^2)*(x^6+x^2)^(1/2)*x+16*(x^6+x^2)^(3/4))/(
1+x)^2/(-1+x)^2/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8+1}{\left(x^8-1\right)\left(x^6+x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="maxima")
```

```
[Out] integrate((x^8 + 1)/((x^8 - 1)*(x^6 + x^2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8+1}{\left(x^6+x^2\right)^{1/4}\,\left(x^8-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 + 1)/((x^2 + x^6)^(1/4)*(x^8 - 1)),x)
```

```
[Out] int((x^8 + 1)/((x^2 + x^6)^(1/4)*(x^8 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8+1}{\sqrt[4]{x^2\left(x^4+1\right)}\,\left(x-1\right)\left(x+1\right)\left(x^2+1\right)\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8+1)/(x**6+x**2)**(1/4)/(x**8-1),x)
```

```
[Out] Integral((x**8 + 1)/((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)*(x
**4 + 1)), x)
```

3.1920 $\int \frac{1+x^8}{\sqrt[4]{x^2+x^6}\left(-1+x^8\right)}\,dx$

Optimal. Leaf size=185

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{4\,2^{3/4}}-\frac{\left(x^6+x^2\right)^{3/4}}{x\left(x^4+1\right)}$$

Rubi [C] time = 0.41, antiderivative size = 81, normalized size of antiderivative = 0.44, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2056, 6715, 6725, 245, 1404, 429}

$$\frac{2x\sqrt[4]{x^4+1}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^6+x^2}}-\frac{4x\sqrt[4]{x^4+1}\,F_1\left(\frac{1}{8};1,\frac{5}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^8)/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] (-4*x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, 5/4, 9/8, x^4, -x^4])/(x^2 + x^6)^(1
/4) + (2*x*(1 + x^4)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -x^4])/(x^2 + x
^6)^(1/4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1404

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^8}{\sqrt[4]{x^2+x^6}\left(-1+x^8\right)}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{1+x^8}{\sqrt{x}\sqrt[4]{1+x^4}\left(-1+x^8\right)}\,dx}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1+x^{16}}{\sqrt[4]{1+x^8}\left(-1+x^{16}\right)}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[4]{1+x^8}}+\frac{2}{\sqrt[4]{1+x^8}\left(-1+x^{16}\right)}\right)dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left(4\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}\left(-1+x^{16}\right)}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left(4\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(-1+x^8\right)\left(1+x^8\right)^{5/4}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= -\frac{4x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,\frac{5}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^2+x^6}} + \frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}}$$

Mathematica [C] time = 0.29, size = 45, normalized size = 0.24

$$-\frac{x\left(\sqrt[4]{x^4+1}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};-x^4,x^4\right)+1\right)}{\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^8)/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -((x*(1 + (1 + x^4)^(1/4)*AppellF1[1/8, -3/4, 1, 9/8, -x^4, x^4]))/(x^2 + x
^6)^(1/4))
```

IntegrateAlgebraic [A] time = 0.00, size = 185, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}\,x^2-\sqrt{x^6+x^2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^6+x^2}}\right)}{4\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{4\,2^{3/4}}-\frac{\left(x^6+x^2\right)^{3/4}}{x\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^8)/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -((x^2 + x^6)^(3/4)/(x*(1 + x^4))) - ArcTan[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/
(4*2^(1/4)) + ArcTan[(2^(3/4)*x*(x^2 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^2
+ x^6])]/(4*2^(3/4)) - ArcTanh[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/(4*2^(1/4)) -
```

```
ArcTanh[(x^2/2^(1/4) + Sqrt[x^2 + x^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))]/(4*
2^(3/4))
```

fricas [B] time = 52.56, size = 1055, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="fricas")
```

```
[Out] -1/32*(4*2^(3/4)*(x^5 + x)*arctan(1/2*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + 2^
(3/4)*(2*2^(3/4)*sqrt(x^6 + x^2)*x + 2^(1/4)*(x^5 + 2*x^3 + x)) + 4*2^(1/4)
*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 2^(3/4)*(x^5 + x)*log(-(4*sqrt(2)*
(x^6 + x^2)^(1/4)*x^2 + 2^(3/4)*(x^5 + 2*x^3 + x) + 4*2^(1/4)*sqrt(x^6 + x^
2)*x + 4*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) - 2^(3/4)*(x^5 + x)*log(-(4*
sqrt(2)*(x^6 + x^2)^(1/4)*x^2 - 2^(3/4)*(x^5 + 2*x^3 + x) - 4*2^(1/4)*sqrt(
x^6 + x^2)*x + 4*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) - 4*2^(1/4)*(x^5 + x
)*arctan(-1/2*(2*x^9 + 8*x^7 + 12*x^5 + 8*x^3 + 4*2^(3/4)*(x^6 + x^2)^(3/4)
*(x^4 - 6*x^2 + 1) + 8*sqrt(2)*sqrt(x^6 + x^2)*(x^5 + 2*x^3 + x) - sqrt(2)*
(32*sqrt(2)*(x^6 + x^2)^(3/4)*x^2 + 2^(3/4)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3
+ x) + 4*2^(1/4)*sqrt(x^6 + x^2)*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x^4 + x^2)*
(x^6 + x^2)^(1/4))*sqrt((4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2
*x^3 + x) + 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3
 + x)) + 8*2^(1/4)*(3*x^6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2*x)/(x^9 -
28*x^7 + 6*x^5 - 28*x^3 + x)) + 4*2^(1/4)*(x^5 + x)*arctan(-1/2*(2*x^9 + 8*
x^7 + 12*x^5 + 8*x^3 - 4*2^(3/4)*(x^6 + x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 8*sq
rt(2)*sqrt(x^6 + x^2)*(x^5 + 2*x^3 + x) - sqrt(2)*(32*sqrt(2)*(x^6 + x^2)^(
3/4)*x^2 - 2^(3/4)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3 + x) - 4*2^(1/4)*sqrt(x^6
 + x^2)*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt(-
(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 +
 x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x)) - 8*2^(1/4)*(3*x^
6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3
+ x)) + 2^(1/4)*(x^5 + x)*log(8*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*
(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5
 + 2*x^3 + x)) - 2^(1/4)*(x^5 + x)*log(-8*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2
- sqrt(2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(
3/4))/(x^5 + 2*x^3 + x)) + 32*(x^6 + x^2)^(3/4))/(x^5 + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8+1}{\left(x^8-1\right)\left(x^6+x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="giac")
```

```
[Out] integrate((x^8 + 1)/((x^8 - 1)*(x^6 + x^2)^(1/4)), x)
```

maple [C] time = 18.62, size = 646, normalized size = 3.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8+1)/(x^6+x^2)^(1/4)/(x^8-1),x)
```

```
[Out] -x/(x^2*(x^4+1))^(1/4)-1/16*RootOf(_Z^4-8)*ln((RootOf(_Z^4-8)^3*(x^6+x^2)^(
1/2)*x+RootOf(_Z^4-8)*x^5+2*(x^6+x^2)^(1/4)*RootOf(_Z^4-8)^2*x^2+2*RootOf(_
Z^4-8)*x^3+4*(x^6+x^2)^(3/4)+RootOf(_Z^4-8)*x)/(1+x)^2/(-1+x)^2/x)+1/16*Roo
tOf(_Z^2+RootOf(_Z^4-8)^2)*ln((RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)
```

```
^2*(x^6+x^2)^(1/2)*x-RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^5-2*(x^6+x^2)^(1/4)*Ro
otOf(_Z^4-8)^2*x^2-2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^3+4*(x^6+x^2)^(3/4)-Ro
otOf(_Z^2+RootOf(_Z^4-8)^2)*x)/(1+x)^2/(-1+x)^2/x)+1/32*ln(-(RootOf(_Z^4-8)
^2*x^2-2*RootOf(_Z^4-8)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))/x/(x^2+1))*Roo
tOf(_Z^4-8)^3+1/32*ln(-(RootOf(_Z^4-8)^2*x^2-2*RootOf(_Z^4-8)*(x^6+x^2)^(1/
4)*x+2*(x^6+x^2)^(1/2))/x/(x^2+1))*RootOf(_Z^4-8)^2*RootOf(_Z^2+RootOf(_Z^4
-8)^2)-1/32*RootOf(_Z^4-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln((-RootOf(_Z^2
+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x^5+RootOf(_Z^4-8)^3*x^5+2*RootOf(_Z^2+
RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x^3-2*RootOf(_Z^4-8)^3*x^3-8*RootOf(_Z^2
+RootOf(_Z^4-8)^2)*(x^6+x^2)^(1/4)*RootOf(_Z^4-8)*x^2+8*RootOf(_Z^2+RootOf(
_Z^4-8)^2)*(x^6+x^2)^(1/2)*x-RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2
*x+8*(x^6+x^2)^(1/2)*RootOf(_Z^4-8)*x+RootOf(_Z^4-8)^3*x-16*(x^6+x^2)^(3/4)
)/x/(x^2+1)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8+1}{(x^8-1)(x^6+x^2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="maxima")
```

```
[Out] integrate((x^8 + 1)/((x^8 - 1)*(x^6 + x^2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8+1}{(x^6+x^2)^{1/4}\,(x^8-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 + 1)/((x^2 + x^6)^(1/4)*(x^8 - 1)),x)
```

```
[Out] int((x^8 + 1)/((x^2 + x^6)^(1/4)*(x^8 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8+1}{\sqrt[4]{x^2\left(x^4+1\right)}\,(x-1)\,(x+1)\left(x^2+1\right)\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8+1)/(x**6+x**2)**(1/4)/(x**8-1),x)
```

```
[Out] Integral((x**8 + 1)/((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)*(x
**4 + 1)), x)
```

3.1921 $\int \frac{x^2(8-7(1+k)x+6kx^2)}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+b(1+k)x-bkx^2+x^8\right)}\,dx$

Optimal. Leaf size=185

$$\frac{\log\left(x^3-\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{b^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+2x^3}\right)}{b^{2/3}}-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right.}{2}$$

Rubi [F] time = 20.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(8-7(1+k)x+6kx^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+b(1+k)x-bkx^2+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(8 - 7*(1 + k)*x + 6*k*x^2))/(((1 - x)*x*(1 - k*x))^(1/3)*(-b + b*
(1 + k)*x - b*k*x^2 + x^8)),x]
```

```
[Out] (21*(1 + k)*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][x
^10/((1 - x^3)^(1/3)*(1 - k*x^3)^(1/3)*(b - b*(1 + k)*x^3 + b*k*x^6 - x^24)
), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3) + (24*(1 - x)^(1/3)*x^(1/3)
*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][x^7/((1 - x^3)^(1/3)*(1 - k*x^3)^(
1/3)*(-b + b*(1 + k)*x^3 - b*k*x^6 + x^24)), x], x, x^(1/3)])/((1 - x)*x*(1
 - k*x))^(1/3) + (18*k*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][D
efer[Int][x^13/((1 - x^3)^(1/3)*(1 - k*x^3)^(1/3)*(-b + b*(1 + k)*x^3 - b*k
*x^6 + x^24)), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\int \frac{x^2\left(8-7(1+k)x+6kx^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+b(1+k)x-bkx^2+x^8\right)}\,dx=\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int\frac{x^{5/3}\left(8-7(1+k)x+6kx^2\right)}{\sqrt[3]{1-x}\sqrt[3]{1-kx}\left(-b+b(1+k)x-bkx^2+x^8\right)}}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$=\frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}\left(\int\frac{x^7\left(8-7(1+k)x^3+\right.}{\sqrt[3]{1-x^3}\sqrt[3]{1-kx^3}\left(-b+b(1+\right.}\right.}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$=\frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}\left(\int\left(\frac{7(1+k)x^{10}}{\sqrt[3]{1-x^3}\sqrt[3]{1-kx^3}\left(b-b(1+\right.}\right.\right.}{}$$

$$=\frac{\left(24\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}\left(\int\frac{x^7}{\sqrt[3]{1-x^3}\sqrt[3]{1-kx^3}\left(-b+b(1\right.}\right.}{\sqrt[3]{(1-x)x(1-kx)}}$$

Mathematica [F] time = 4.39, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(8-7(1+k)x+6kx^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+b(1+k)x-bkx^2+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(8 - 7*(1 + k)*x + 6*k*x^2))/(((1 - x)*x*(1 - k*x))^(1/3)*(-
b + b*(1 + k)*x - b*k*x^2 + x^8)),x]
```

```
[Out] Integrate[(x^2*(8 - 7*(1 + k)*x + 6*k*x^2))/(((1 - x)*x*(1 - k*x))^(1/3)*(-
b + b*(1 + k)*x - b*k*x^2 + x^8)), x]
```

IntegrateAlgebraic [A] time = 3.00, size = 185, normalized size = 1.00

$$\frac{\log\left(x^3-\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{b^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+2x^3}\right)}{b^{2/3}}-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{b}x^3\sqrt[3]{kx^3+(-k-1)x^2+x}+x^6\right)}{2b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(8 - 7*(1 + k)*x + 6*k*x^2))/(((1 - x)*x*(1 - k*x))
^(1/3)*(-b + b*(1 + k)*x - b*k*x^2 + x^8)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(2*x^3 +
 b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))])/b^(2/3) + Log[x^3 - b^(1/3)*(x
 + (-1 - k)*x^2 + k*x^3)^(1/3)]/b^(2/3) - Log[x^6 + b^(1/3)*x^3*(x + (-1 -
k)*x^2 + k*x^3)^(1/3) + b^(2/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(2/3
))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(8-7*(1+k)*x+6*k*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(-b+b*(1+k)*x-
b*k*x^2+x^8),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(6\,kx^2-7\,(k+1)x+8\right)x^2}{\left(x^8-bkx^2+b(k+1)x-b\right)\left((kx-1)(x-1)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(8-7*(1+k)*x+6*k*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(-b+b*(1+k)*x-
b*k*x^2+x^8),x, algorithm="giac")
```

```
[Out] integrate((6*k*x^2 - 7*(k + 1)*x + 8)*x^2/((x^8 - b*k*x^2 + b*(k + 1)*x - b
)*((k*x - 1)*(x - 1)*x)^(1/3)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(8-7\,(1+k)\,x+6k\,x^2\right)}{\left((1-x)\,x\,(-kx+1)\right)^{\frac{1}{3}}\left(-b+b\,(1+k)\,x-bk\,x^2+x^8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(8-7*(1+k)*x+6*k*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(-b+b*(1+k)*x-b*k*x^
2+x^8),x)
```

```
[Out] int(x^2*(8-7*(1+k)*x+6*k*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(-b+b*(1+k)*x-b*k*x^
2+x^8),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(6\,kx^2-7\,(k+1)x+8\right)x^2}{\left(x^8-bkx^2+b(k+1)x-b\right)\left((kx-1)(x-1)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(8-7*(1+k)*x+6*k*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(-b+b*(1+k)*x-
b*k*x^2+x^8),x, algorithm="maxima")
```

```
[Out] integrate((6*k*x^2 - 7*(k + 1)*x + 8)*x^2/((x^8 - b*k*x^2 + b*(k + 1)*x - b
)*((k*x - 1)*(x - 1)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2\,\left(6\,k\,x^2-7\,x\,\left(k+1\right)+8\right)}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/3}\,\left(-x^8+b\,k\,x^2-b\,\left(k+1\right)\,x+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(6*k*x^2 - 7*x*(k + 1) + 8))/((x*(k*x - 1)*(x - 1))^(1/3)*(b - x^
8 - b*x*(k + 1) + b*k*x^2)),x)
```

```
[Out] -int((x^2*(6*k*x^2 - 7*x*(k + 1) + 8))/((x*(k*x - 1)*(x - 1))^(1/3)*(b - x^
8 - b*x*(k + 1) + b*k*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(6 k x^{2} - 7 k x - 7 x + 8\right)}{\sqrt[3]{x \left(x - 1\right) \left(k x - 1\right)} \left(- b k x^{2} + b k x + b x - b + x^{8}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(8-7*(1+k)*x+6*k*x**2)/((1-x)*x*(-k*x+1))**(1/3)/(-b+b*(1+k)
*x-b*k*x**2+x**8),x)
```

```
[Out] Integral(x**2*(6*k*x**2 - 7*k*x - 7*x + 8)/((x*(x - 1)*(k*x - 1))**(1/3)*(-
b*k*x**2 + b*k*x + b*x - b + x**8)), x)
```

3.1922 $\int \frac{1}{\left(1+x^2\right)^2 \sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=185

$$\frac{\left(\sqrt{x^2+1}+x\right)^{5/2}}{8\left(x^2+\sqrt{x^2+1}\,x+1\right)^2}-\frac{3\sqrt{\sqrt{x^2+1}+x}}{8\left(x^2+\sqrt{x^2+1}\,x+1\right)^2}+\frac{3\tan^{-1}\left(\frac{\frac{\sqrt{x^2+1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2+1}+x}}\right)}{4\sqrt{2}}+\frac{3\tanh^{-1}\left(\frac{\frac{\sqrt{x^2+1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2+1}+x}}\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.16, antiderivative size = 225, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2122, 288, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt{\sqrt{x^2+1}+x}}{2\left(\left(\sqrt{x^2+1}+x\right)^2+1\right)}-\frac{2\sqrt{\sqrt{x^2+1}+x}}{\left(\left(\sqrt{x^2+1}+x\right)^2+1\right)^2}-\frac{3\log\left(\sqrt{x^2+1}-\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right)}{8\sqrt{2}}+\frac{3\log\left(\sqrt{x^2+1}+\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right)}{8\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{\sqrt{x^2+1}+x}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^2*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] (-2*Sqrt[x + Sqrt[1 + x^2]])/(1 + (x + Sqrt[1 + x^2])^2)^2 + Sqrt[x + Sqrt[
1 + x^2]]/(2*(1 + (x + Sqrt[1 + x^2])^2)) - (3*ArcTan[1 - Sqrt[2]*Sqrt[x +
Sqrt[1 + x^2]]])/(4*Sqrt[2]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[x + Sqrt[1 + x^2]
]])/(4*Sqrt[2]) - (3*Log[1 + x + Sqrt[1 + x^2] - Sqrt[2]*Sqrt[x + Sqrt[1 +
x^2]]])/(8*Sqrt[2]) + (3*Log[1 + x + Sqrt[1 + x^2] + Sqrt[2]*Sqrt[x + Sqrt[
1 + x^2]]])/(8*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{1}{\left(1+x^2\right)^2 \sqrt{x+\sqrt{1+x^2}}}\,dx = 8\,\text{Subst}\left(\int \frac{x^{3/2}}{\left(1+x^2\right)^3}\,dx, x, x+\sqrt{1+x^2}\right)$$

$$= -\frac{2\sqrt{x+\sqrt{1+x^2}}}{\left(1+\left(x+\sqrt{1+x^2}\right)^2\right)^2} + \text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1+x^2\right)^2}\,dx, x, x+\sqrt{1+x^2}\right)$$

$$= -\frac{2\sqrt{x+\sqrt{1+x^2}}}{\left(1+\left(x+\sqrt{1+x^2}\right)^2\right)^2} + \frac{\sqrt{x+\sqrt{1+x^2}}}{2\left(1+\left(x+\sqrt{1+x^2}\right)^2\right)} + \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\right.$$

$$= -\frac{2\sqrt{x+\sqrt{1+x^2}}}{\left(1+\left(x+\sqrt{1+x^2}\right)^2\right)^2} + \frac{\sqrt{x+\sqrt{1+x^2}}}{2\left(1+\left(x+\sqrt{1+x^2}\right)^2\right)} + \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x\right.$$

$$= -\frac{2\sqrt{x+\sqrt{1+x^2}}}{\left(1+\left(x+\sqrt{1+x^2}\right)^2\right)^2} + \frac{\sqrt{x+\sqrt{1+x^2}}}{2\left(1+\left(x+\sqrt{1+x^2}\right)^2\right)} + \frac{3}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x\right.$$

$$= -\frac{2\sqrt{x+\sqrt{1+x^2}}}{\left(1+\left(x+\sqrt{1+x^2}\right)^2\right)^2} + \frac{\sqrt{x+\sqrt{1+x^2}}}{2\left(1+\left(x+\sqrt{1+x^2}\right)^2\right)} + \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+}\right.$$

$$= -\frac{2\sqrt{x+\sqrt{1+x^2}}}{\left(1+\left(x+\sqrt{1+x^2}\right)^2\right)^2} + \frac{\sqrt{x+\sqrt{1+x^2}}}{2\left(1+\left(x+\sqrt{1+x^2}\right)^2\right)} - \frac{3\log\left(1+x+\sqrt{1+x^2}-\right.}{8\sqrt{2}}$$

$$= -\frac{2\sqrt{x+\sqrt{1+x^2}}}{\left(1+\left(x+\sqrt{1+x^2}\right)^2\right)^2} + \frac{\sqrt{x+\sqrt{1+x^2}}}{2\left(1+\left(x+\sqrt{1+x^2}\right)^2\right)} - \frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{x+\sqrt{}}\right.}{4\sqrt{2}}$$

Mathematica [A] time = 0.23, size = 220, normalized size = 1.19

$$\frac{1}{16}\left(\frac{8\sqrt{\sqrt{x^2+1}+x}}{\left(\sqrt{x^2+1}+x\right)^2+1}-\frac{8\sqrt{\sqrt{x^2+1}+x}}{\left(x^2+\sqrt{x^2+1}\,x+1\right)^2}-3\sqrt{2}\log\left(\sqrt{x^2+1}-\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right)+3\sqrt{2}\log\left(\sqrt{x^2+1}+\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right)-6\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\sqrt{x^2+1}+x}\right)+6\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)^2*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] ((-8*Sqrt[x + Sqrt[1 + x^2]])/(1 + x^2 + x*Sqrt[1 + x^2])^2 + (8*Sqrt[x + S
qrt[1 + x^2]])/(1 + (x + Sqrt[1 + x^2])^2) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*S
qrt[x + Sqrt[1 + x^2]]] + 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[x + Sqrt[1 + x^
2]]] - 3*Sqrt[2]*Log[1 + x + Sqrt[1 + x^2] - Sqrt[2]*Sqrt[x + Sqrt[1 + x^2]
]] + 3*Sqrt[2]*Log[1 + x + Sqrt[1 + x^2] + Sqrt[2]*Sqrt[x + Sqrt[1 + x^2]]]
)/16
```

IntegrateAlgebraic [A] time = 0.33, size = 185, normalized size = 1.00

$$\frac{\left(\sqrt{x^2+1}+x\right)^{5/2}}{8\left(x^2+\sqrt{x^2+1}\,x+1\right)^2}-\frac{3\sqrt{\sqrt{x^2+1}+x}}{8\left(x^2+\sqrt{x^2+1}\,x+1\right)^2}+\frac{3\tan^{-1}\left(\frac{\frac{\sqrt{x^2+1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2+1}+x}}\right)}{4\sqrt{2}}+\frac{3\tanh^{-1}\left(\frac{\frac{\sqrt{x^2+1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2+1}+x}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x^2)^2*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] (-3*Sqrt[x + Sqrt[1 + x^2]])/(8*(1 + x^2 + x*Sqrt[1 + x^2])^2) + (x + Sqrt[
1 + x^2])^(5/2)/(8*(1 + x^2 + x*Sqrt[1 + x^2])^2) + (3*ArcTan[(-(1/Sqrt[2])
 + x/Sqrt[2] + Sqrt[1 + x^2]/Sqrt[2])/Sqrt[x + Sqrt[1 + x^2]]])/(4*Sqrt[2])
 + (3*ArcTanh[(1/Sqrt[2] + x/Sqrt[2] + Sqrt[1 + x^2]/Sqrt[2])/Sqrt[x + Sqrt
[1 + x^2]]])/(4*Sqrt[2])
```

fricas [A] time = 0.45, size = 248, normalized size = 1.34

$$-\frac{12\sqrt{2}\left(x^2+1\right)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+x+\sqrt{x^2+1}+1}-\sqrt{2}\sqrt{x+\sqrt{x^2+1}}-1\right)+12\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+4x+4\sqrt{x^2+1}+4}-\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+1\right)-3\sqrt{2}\left(x^2+1\right)\log\left(4\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+4x+4\sqrt{x^2+1}+4\right)+3\sqrt{2}\left(x^2+1\right)\log\left(-4\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+4x+4\sqrt{x^2+1}+4\right)+4\left(3x^2-3\sqrt{x^2+1}\,x+1\right)\sqrt{x+\sqrt{x^2+1}}}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(12*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x + sqrt(x^2 +
 1)) + x + sqrt(x^2 + 1) + 1) - sqrt(2)*sqrt(x + sqrt(x^2 + 1)) - 1) + 12*s
qrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*sqrt(x + sqrt(x^2 + 1))
 + 4*x + 4*sqrt(x^2 + 1) + 4) - sqrt(2)*sqrt(x + sqrt(x^2 + 1)) + 1) - 3*sq
rt(2)*(x^2 + 1)*log(4*sqrt(2)*sqrt(x + sqrt(x^2 + 1)) + 4*x + 4*sqrt(x^2 +
1) + 4) + 3*sqrt(2)*(x^2 + 1)*log(-4*sqrt(2)*sqrt(x + sqrt(x^2 + 1)) + 4*x
+ 4*sqrt(x^2 + 1) + 4) + 4*(3*x^2 - 3*sqrt(x^2 + 1)*x + 1)*sqrt(x + sqrt(x^
2 + 1)))/(x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+1\right)^2\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 1)^2*sqrt(x + sqrt(x^2 + 1))), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+1\right)^2\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2/(x+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int(1/(x^2+1)^2/(x+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+1\right)^2\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 1)^2*sqrt(x + sqrt(x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2+1\right)^2 \sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^2*(x + (x^2 + 1)^(1/2))^(1/2)),x)
```

```
[Out] int(1/((x^2 + 1)^2*(x + (x^2 + 1)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+\sqrt{x^2+1}}\,\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/(x+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x + sqrt(x**2 + 1))*(x**2 + 1)**2), x)
```

3.1923 $\int \frac{x(-a+x)(ab+(a-2b)x)}{(x(-a+x)(-b+x)^2)^{3/4}(b^2d+(a-2bd)x+(-1+d)x^2)}\,dx$

Optimal. Leaf size=186

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{3/4}}{x(x-a)(b-x)}\right)-2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(\cdots}{x(x-a)(b-x)}\right.$$

Rubi [F] time = 4.78, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(-a+x)(ab+(a-2b)x)}{\left(x(-a+x)(-b+x)^2\right)^{3/4}\left(b^2d+(a-2bd)x+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-a + x)*(a*b + (a - 2*b)*x))/((x*(-a + x)*(-b + x)^2)^(3/4)*(b^2*d
+ (a - 2*b*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] ((a - 2*b - Sqrt[a^2 - 4*a*b*d + 4*b^2*d])*x^(3/4)*(-a + x)^(3/4)*(-b + x)^
(3/2)*Defer[Int][(x^(1/4)*(-a + x)^(1/4))/((-b + x)^(3/2)*(a - 2*b*d - Sqrt
[a^2 - 4*a*b*d + 4*b^2*d] + 2*(-1 + d)*x)), x])/(-((a - x)*(b - x)^2*x))^(3
/4) + ((a - 2*b + Sqrt[a^2 - 4*a*b*d + 4*b^2*d])*x^(3/4)*(-a + x)^(3/4)*(-b
 + x)^(3/2)*Defer[Int][(x^(1/4)*(-a + x)^(1/4))/((-b + x)^(3/2)*(a - 2*b*d
+ Sqrt[a^2 - 4*a*b*d + 4*b^2*d] + 2*(-1 + d)*x)), x])/(-((a - x)*(b - x)^2*
x))^(3/4)
```

Rubi steps

$$\begin{aligned}\int \frac{x(-a+x)(ab+(a-2b)x)}{\left(x(-a+x)(-b+x)^2\right)^{3/4}\left(b^2d+(a-2bd)x+(-1+d)x^2\right)}\,dx &= \frac{\left(x^{3/4}(-a+x)^{3/4}(-b+x)^{3/2}\right)\int \frac{\sqrt[4]{x}\sqrt[4]{-a+x}(\cdots}{(-b+x)^{3/2}\left(b^2d+(a-\cdots\right.}}{\left(x(-a+x)(-b+x)^2\right)^{3/4}}\\ &= \frac{\left(x^{3/4}(-a+x)^{3/4}(-b+x)^{3/2}\right)\int\left(\frac{\left(a-2b-\sqrt{a^2-\cdots}\right.}{(-b+x)^{3/2}\left(a-2bd-\cdots\right.}\right.}{(\cdots}\\ &= \frac{\left(\left(a-2b-\sqrt{a^2-4abd+4b^2d}\right)x^{3/4}(-a+x)^{\cdots}\right.}{(x(-a\cdots}\end{aligned}$$

Mathematica [F] time = 15.46, size = 0, normalized size = 0.00

$$\int \frac{x(-a+x)(ab+(a-2b)x)}{\left(x(-a+x)(-b+x)^2\right)^{3/4}\left(b^2d+(a-2bd)x+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(-a + x)*(a*b + (a - 2*b)*x))/((x*(-a + x)*(-b + x)^2)^(3/4)*(
b^2*d + (a - 2*b*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] Integrate[(x*(-a + x)*(a*b + (a - 2*b)*x))/((x*(-a + x)*(-b + x)^2)^(3/4)*(
b^2*d + (a - 2*b*d)*x + (-1 + d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 4.22, size = 186, normalized size = 1.00

$$2\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt[4]{d}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{3/4}}{x(x-a)(b-x)}\right)-2\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{3/4}}{x(x-a)(b-x)}\right)+\frac{4\sqrt[4]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}}{b-x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-a + x)*(a*b + (a - 2*b)*x))/((x*(-a + x)*(-b + x)^2
)^(3/4)*(b^2*d + (a - 2*b*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] (4*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/4))/(b - x) +
 2*d^(1/4)*ArcTan[(d^(1/4)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3
 + x^4)^(3/4))/((b - x)*x*(-a + x))] - 2*d^(1/4)*ArcTanh[(d^(1/4)*(-(a*b^2*
x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(3/4))/((b - x)*x*(-a + x))]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+x)*(a*b+(a-2*b)*x)/(x*(-a+x)*(-b+x)^2)^(3/4)/(b^2*d+(-2*b*d
+a)*x+(-1+d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ab+(a-2\,b)x)(a-x)x}{\left(-(a-x)(b-x)^2x\right)^{\frac{3}{4}}\left(b^2d+(d-1)x^2-(2\,bd-a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+x)*(a*b+(a-2*b)*x)/(x*(-a+x)*(-b+x)^2)^(3/4)/(b^2*d+(-2*b*d
+a)*x+(-1+d)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-(a*b + (a - 2*b)*x)*(a - x)*x/((-(a - x)*(b - x)^2*x)^(3/4)*(b^2
*d + (d - 1)*x^2 - (2*b*d - a)*x)), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x\left(-a+x\right)\left(ab+\left(a-2b\right)x\right)}{\left(x\left(-a+x\right)\left(-b+x\right)^{2}\right)^{\frac{3}{4}}\left(b^{2}d+\left(-2bd+a\right)x+\left(-1+d\right)x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a+x)*(a*b+(a-2*b)*x)/(x*(-a+x)*(-b+x)^2)^(3/4)/(b^2*d+(-2*b*d+a)*x+
(-1+d)*x^2),x)
```

```
[Out] int(x*(-a+x)*(a*b+(a-2*b)*x)/(x*(-a+x)*(-b+x)^2)^(3/4)/(b^2*d+(-2*b*d+a)*x+
(-1+d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ab+(a-2\,b)x)(a-x)x}{\left(-(a-x)(b-x)^2x\right)^{\frac{3}{4}}\left(b^2d+(d-1)x^2-(2\,bd-a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+x)*(a*b+(a-2*b)*x)/(x*(-a+x)*(-b+x)^2)^(3/4)/(b^2*d+(-2*b*d
+a)*x+(-1+d)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((a*b + (a - 2*b)*x)*(a - x)*x/((-(a - x)*(b - x)^2*x)^(3/4)*(b^2
*d + (d - 1)*x^2 - (2*b*d - a)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x\,\left(a-x\right)\,\left(a\,b+x\,\left(a-2\,b\right)\right)}{{\left(-x\,\left(a-x\right)\,{\left(b-x\right)}^2\right)}^{3/4}\,\left(b^2\,d+x\,\left(a-2\,b\,d\right)+x^2\,\left(d-1\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a - x)*(a*b + x*(a - 2*b)))/((-x*(a - x)*(b - x)^2)^(3/4)*(b^2*d +
 x*(a - 2*b*d) + x^2*(d - 1))),x)
```

```
[Out] int(-(x*(a - x)*(a*b + x*(a - 2*b)))/((-x*(a - x)*(b - x)^2)^(3/4)*(b^2*d +
 x*(a - 2*b*d) + x^2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+x)*(a*b+(a-2*b)*x)/(x*(-a+x)*(-b+x)**2)**(3/4)/(b**2*d+(-2*
b*d+a)*x+(-1+d)*x**2),x)
```

```
[Out] Timed out
```

3.1924 $\int \frac{(1+x^2)(1-3x^2+x^4)}{x^2\sqrt{\frac{-2+x+2x^2}{-1+x+x^2}}(1-x-3x^2+x^3+x^4)}\,dx$

Optimal. Leaf size=186

$$\frac{\sqrt{\frac{2x^2+x-2}{x^2+x-1}}\,(x^2+x-1)}{2x}-\frac{3\tanh^{-1}\left(\frac{\sqrt{\frac{2x^2+x-2}{x^2+x-1}}}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{1}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}+\frac{1}{2\sqrt{5}}}\sqrt{\frac{2x^2+x-2}{x^2+x-1}}\right)+\frac{1}{5}\sqrt{2\left(5+\right.}$$

Rubi [F] time = 6.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+x^2)(1-3x^2+x^4)}{x^2\sqrt{\frac{-2+x+2x^2}{-1+x+x^2}}\,(1-x-3x^2+x^3+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^2)*(1 - 3*x^2 + x^4))/(x^2*Sqrt[(-2 + x + 2*x^2)/(-1 + x + x^2)
]*(1 - x - 3*x^2 + x^3 + x^4)),x]
```

```
[Out] (Sqrt[-2 + x + 2*x^2]*Defer[Int][Sqrt[-1 + x + x^2]/Sqrt[-2 + x + 2*x^2], x
])/(Sqrt[(2 - x - 2*x^2)/(1 - x - x^2)]*Sqrt[-1 + x + x^2]) + (Sqrt[-2 + x
+ 2*x^2]*Defer[Int][Sqrt[-1 + x + x^2]/(x^2*Sqrt[-2 + x + 2*x^2]), x])/(Sqr
t[(2 - x - 2*x^2)/(1 - x - x^2)]*Sqrt[-1 + x + x^2]) + (Sqrt[-2 + x + 2*x^2
]*Defer[Int][Sqrt[-1 + x + x^2]/(x*Sqrt[-2 + x + 2*x^2]), x])/(Sqrt[(2 - x
- 2*x^2)/(1 - x - x^2)]*Sqrt[-1 + x + x^2]) + (Sqrt[-2 + x + 2*x^2]*Defer[I
nt][Sqrt[-1 + x + x^2]/(Sqrt[-2 + x + 2*x^2]*(1 - x - 3*x^2 + x^3 + x^4)),
x])/(Sqrt[(2 - x - 2*x^2)/(1 - x - x^2)]*Sqrt[-1 + x + x^2]) + (3*Sqrt[-2 +
 x + 2*x^2]*Defer[Int][(x*Sqrt[-1 + x + x^2])/(Sqrt[-2 + x + 2*x^2]*(1 - x
- 3*x^2 + x^3 + x^4)), x])/(Sqrt[(2 - x - 2*x^2)/(1 - x - x^2)]*Sqrt[-1 + x
 + x^2]) - (Sqrt[-2 + x + 2*x^2]*Defer[Int][(x^2*Sqrt[-1 + x + x^2])/(Sqrt[
-2 + x + 2*x^2]*(1 - x - 3*x^2 + x^3 + x^4)), x])/(Sqrt[(2 - x - 2*x^2)/(1
- x - x^2)]*Sqrt[-1 + x + x^2]) - (2*Sqrt[-2 + x + 2*x^2]*Defer[Int][(x^3*S
qrt[-1 + x + x^2])/(Sqrt[-2 + x + 2*x^2]*(1 - x - 3*x^2 + x^3 + x^4)), x])/
(Sqrt[(2 - x - 2*x^2)/(1 - x - x^2)]*Sqrt[-1 + x + x^2])
```

Rubi steps

$$\int \frac{(1+x^2)(1-3x^2+x^4)}{x^2\sqrt{\frac{-2+x+2x^2}{-1+x+x^2}}\,(1-x-3x^2+x^3+x^4)}\,dx = \frac{\sqrt{-2+x+2x^2}\int \frac{(1+x^2)\sqrt{-1+x+x^2}\,(1-3x^2+x^4)}{x^2\sqrt{-2+x+2x^2}\,(1-x-3x^2+x^3+x^4)}\,dx}{\sqrt{-1+x+x^2}\sqrt{\frac{-2+x+2x^2}{-1+x+x^2}}}$$

$$= \frac{\sqrt{-2+x+2x^2}\int\left(\frac{\sqrt{-1+x+x^2}}{\sqrt{-2+x+2x^2}}+\frac{\sqrt{-1+x+x^2}}{x^2\sqrt{-2+x+2x^2}}+\frac{\sqrt{-1+x+x^2}}{x\sqrt{-2+x+2x^2}}+\cdots\right.}{\sqrt{-1+x+x^2}\sqrt{\frac{-2+x+2x^2}{-1+x+x^2}}}$$

$$= \frac{\sqrt{-2+x+2x^2}\int\frac{\sqrt{-1+x+x^2}}{\sqrt{-2+x+2x^2}}\,dx}{\sqrt{-1+x+x^2}\sqrt{\frac{-2+x+2x^2}{-1+x+x^2}}}+\frac{\sqrt{-2+x+2x^2}\int\frac{\sqrt{-1+x+x^2}}{x^2\sqrt{-2+x+2x^2}}\cdots}{\sqrt{-1+x+x^2}\sqrt{\frac{-2+x+2x^2}{-1+x+x^2}}}$$

$$= \frac{\sqrt{-2+x+2x^2}\int\frac{\sqrt{-1+x+x^2}}{\sqrt{-2+x+2x^2}}\,dx}{\sqrt{-1+x+x^2}\sqrt{\frac{-2+x+2x^2}{-1+x+x^2}}}+\frac{\sqrt{-2+x+2x^2}\int\frac{\sqrt{-1+x+x^2}}{x^2\sqrt{-2+x+2x^2}}\cdots}{\sqrt{-1+x+x^2}\sqrt{\frac{-2+x+2x^2}{-1+x+x^2}}}$$

$$= \frac{\sqrt{-2+x+2x^2}\int\frac{\sqrt{-1+x+x^2}}{\sqrt{-2+x+2x^2}}\,dx}{\sqrt{-1+x+x^2}\sqrt{\frac{-2+x+2x^2}{-1+x+x^2}}}+\frac{\sqrt{-2+x+2x^2}\int\frac{\sqrt{-1+x+x^2}}{x^2\sqrt{-2+x+2x^2}}\cdots}{\sqrt{-1+x+x^2}\sqrt{\frac{-2+x+2x^2}{-1+x+x^2}}}$$

Mathematica [C] time = 6.66, size = 26530, normalized size = 142.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + x^2)*(1 - 3*x^2 + x^4))/(x^2*Sqrt[(-2 + x + 2*x^2)/(-1 + x
+ x^2)]*(1 - x - 3*x^2 + x^3 + x^4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.74, size = 188, normalized size = 1.01

$$\frac{\sqrt{\frac{2x^2+x-2}{x^2+x-1}}\,(x^2+x-1)}{2x}-\frac{3\tanh^{-1}\left(\frac{\sqrt{\frac{2x^2+x-2}{x^2+x-1}}}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{1}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}-\frac{1}{2\sqrt{5}}}\sqrt{\frac{2x^2+x-2}{x^2+x-1}}\right)+\frac{1}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}+\frac{1}{2\sqrt{5}}}\sqrt{\frac{2x^2+x-2}{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^2)*(1 - 3*x^2 + x^4))/(x^2*Sqrt[(-2 + x + 2*x^2)
/(-1 + x + x^2)]*(1 - x - 3*x^2 + x^3 + x^4)),x]
```

```
[Out] ((-1 + x + x^2)*Sqrt[(-2 + x + 2*x^2)/(-1 + x + x^2)])/(2*x) - (3*ArcTanh[S
qrt[(-2 + x + 2*x^2)/(-1 + x + x^2)]/Sqrt[2]])/(2*Sqrt[2]) + (Sqrt[2*(5 + S
qrt[5])]*ArcTanh[Sqrt[1/2 - 1/(2*Sqrt[5])]*Sqrt[(-2 + x + 2*x^2)/(-1 + x +
x^2)]])/5 + (Sqrt[2*(5 - Sqrt[5])]*ArcTanh[Sqrt[1/2 + 1/(2*Sqrt[5])]*Sqrt[(
-2 + x + 2*x^2)/(-1 + x + x^2)]])/5
```

fricas [B] time = 0.77, size = 723, normalized size = 3.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4-3*x^2+1)/x^2/((2*x^2+x-2)/(x^2+x-1))^(1/2)/(x^4+x^3-
3*x^2-x+1),x, algorithm="fricas")
```

```
[Out] 1/80*(4*x*sqrt(2*sqrt(5) + 10)*log(((20575*x^4 + 50235*x^3 - 15795*x^2 - sq
rt(5)*(10237*x^4 + 22677*x^3 - 9661*x^2 - 22677*x + 10237) - 50235*x + 2057
5)*sqrt(2*sqrt(5) + 10) + 20*(1627*x^4 + 4593*x^3 - 288*x^2 - sqrt(5)*(861*
x^4 + 2105*x^3 - 478*x^2 - 2105*x + 861) - 4593*x + 1627)*sqrt((2*x^2 + x -
 2)/(x^2 + x - 1)))/(x^4 + x^3 - 3*x^2 - x + 1)) - 4*x*sqrt(2*sqrt(5) + 10)
*log(-((20575*x^4 + 50235*x^3 - 15795*x^2 - sqrt(5)*(10237*x^4 + 22677*x^3
- 9661*x^2 - 22677*x + 10237) - 50235*x + 20575)*sqrt(2*sqrt(5) + 10) - 20*
(1627*x^4 + 4593*x^3 - 288*x^2 - sqrt(5)*(861*x^4 + 2105*x^3 - 478*x^2 - 21
05*x + 861) - 4593*x + 1627)*sqrt((2*x^2 + x - 2)/(x^2 + x - 1)))/(x^4 + x^
3 - 3*x^2 - x + 1)) + 4*x*sqrt(-2*sqrt(5) + 10)*log(((20575*x^4 + 50235*x^3
 - 15795*x^2 + sqrt(5)*(10237*x^4 + 22677*x^3 - 9661*x^2 - 22677*x + 10237)
 - 50235*x + 20575)*sqrt(-2*sqrt(5) + 10) + 20*(1627*x^4 + 4593*x^3 - 288*x
^2 + sqrt(5)*(861*x^4 + 2105*x^3 - 478*x^2 - 2105*x + 861) - 4593*x + 1627)
*sqrt((2*x^2 + x - 2)/(x^2 + x - 1)))/(x^4 + x^3 - 3*x^2 - x + 1)) - 4*x*sq
rt(-2*sqrt(5) + 10)*log(-((20575*x^4 + 50235*x^3 - 15795*x^2 + sqrt(5)*(102
37*x^4 + 22677*x^3 - 9661*x^2 - 22677*x + 10237) - 50235*x + 20575)*sqrt(-2
*sqrt(5) + 10) - 20*(1627*x^4 + 4593*x^3 - 288*x^2 + sqrt(5)*(861*x^4 + 210
5*x^3 - 478*x^2 - 2105*x + 861) - 4593*x + 1627)*sqrt((2*x^2 + x - 2)/(x^2
+ x - 1)))/(x^4 + x^3 - 3*x^2 - x + 1)) + 15*sqrt(2)*x*log(-(32*x^4 + 48*x^
3 - 47*x^2 - 4*sqrt(2)*(4*x^4 + 7*x^3 - 5*x^2 - 7*x + 4)*sqrt((2*x^2 + x -
2)/(x^2 + x - 1)) - 48*x + 32)/x^2) + 40*(x^2 + x - 1)*sqrt((2*x^2 + x - 2)
/(x^2 + x - 1)))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4 - 3x^2 + 1)(x^2 + 1)}{(x^4 + x^3 - 3x^2 - x + 1)x^2\sqrt{\frac{2x^2+x-2}{x^2+x-1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4-3*x^2+1)/x^2/((2*x^2+x-2)/(x^2+x-1))^(1/2)/(x^4+x^3-
3*x^2-x+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 3*x^2 + 1)*(x^2 + 1)/((x^4 + x^3 - 3*x^2 - x + 1)*x^2*sqrt
((2*x^2 + x - 2)/(x^2 + x - 1))), x)
```

maple [C] time = 0.57, size = 14911, normalized size = 80.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4-3*x^2+1)/x^2/((2*x^2+x-2)/(x^2+x-1))^(1/2)/(x^4+x^3-3*x^2-
x+1),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^4 - 3x^2 + 1)(x^2 + 1)}{(x^4 + x^3 - 3x^2 - x + 1)x^2\sqrt{\frac{2x^2+x-2}{x^2+x-1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4-3*x^2+1)/x^2/((2*x^2+x-2)/(x^2+x-1))^(1/2)/(x^4+x^3-
3*x^2-x+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 3*x^2 + 1)*(x^2 + 1)/((x^4 + x^3 - 3*x^2 - x + 1)*x^2*sqrt
((2*x^2 + x - 2)/(x^2 + x - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+1\right)\,\left(x^4-3\,x^2+1\right)}{x^2\,\sqrt{\frac{2\,x^2+x-2}{x^2+x-1}}\,\left(x^4+x^3-3\,x^2-x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*(x^4 - 3*x^2 + 1))/(x^2*((x + 2*x^2 - 2)/(x + x^2 - 1))^(1/2
)*(x^3 - 3*x^2 - x + x^4 + 1)),x)
```

```
[Out] int(((x^2 + 1)*(x^4 - 3*x^2 + 1))/(x^2*((x + 2*x^2 - 2)/(x + x^2 - 1))^(1/2
)*(x^3 - 3*x^2 - x + x^4 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4-3*x**2+1)/x**2/((2*x**2+x-2)/(x**2+x-1))**(1/2)/(x
**4+x**3-3*x**2-x+1),x)
```

```
[Out] Timed out
```

3.1925 $\int \frac{x^3(5-4(1+k)x+3kx^2)}{((1-x)x(1-kx))^{2/3}(-1+(1+k)x-kx^2+bx^5)}\,dx$

Optimal. Leaf size=186

$$\frac{\log\left(bx^2-b^{2/3}\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{b^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{kx^3+(-k-1)x^2+x}}{2\sqrt[3]{b}\,x^2+\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{b^{2/3}}-\frac{\log\left(b^{5/3}x^2\sqrt[3]{kx^3+(-k-1)x^2+x}+\right.}{2\ldots}$$

Rubi [F] time = 18.27, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3\left(5-4(1+k)x+3kx^2\right)}{((1-x)x(1-kx))^{2/3}\left(-1+(1+k)x-kx^2+bx^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*(5 - 4*(1 + k)*x + 3*k*x^2))/(((1 - x)*x*(1 - k*x))^(2/3)*(-1 + (1
 + k)*x - k*x^2 + b*x^5)),x]

[Out] (9*k*x*((1 - x)/(1 - k*x))^(2/3)*(1 - k*x)*Hypergeometric2F1[1/3, 2/3, 4/3,
 ((1 - k)*x)/(1 - k*x)])/(b*((1 - x)*x*(1 - k*x))^(2/3)) + (9*k*(1 + k)*(1
- x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst][Defer[Int][x^3/((1 - x^3)^(
2/3)*(1 - k*x^3)^(2/3)*(1 - (1 + k)*x^3 + k*x^6 - b*x^15)), x], x, x^(1/3)]
)/(b*((1 - x)*x*(1 - k*x))^(2/3)) + (12*(1 + k)*(1 - x)^(2/3)*x^(2/3)*(1 -
k*x)^(2/3)*Defer[Subst][Defer[Int][x^12/((1 - x^3)^(2/3)*(1 - k*x^3)^(2/3)*
(1 - (1 + k)*x^3 + k*x^6 - b*x^15)), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))
^(2/3) + (9*k*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst][Defer[Int]
[1/((1 - x^3)^(2/3)*(1 - k*x^3)^(2/3)*(-1 + (1 + k)*x^3 - k*x^6 + b*x^15)),
 x], x, x^(1/3)])/(b*((1 - x)*x*(1 - k*x))^(2/3)) + (9*k^2*(1 - x)^(2/3)*x^
(2/3)*(1 - k*x)^(2/3)*Defer[Subst][Defer[Int][x^6/((1 - x^3)^(2/3)*(1 - k*x
^3)^(2/3)*(-1 + (1 + k)*x^3 - k*x^6 + b*x^15)), x], x, x^(1/3)])/(b*((1 - x
)*x*(1 - k*x))^(2/3)) + (15*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Sub
st][Defer[Int][x^9/((1 - x^3)^(2/3)*(1 - k*x^3)^(2/3)*(-1 + (1 + k)*x^3 - k
*x^6 + b*x^15)), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(2/3)
```

Rubi steps

$$\int \frac{x^3\left(5-4(1+k)x+3kx^2\right)}{((1-x)x(1-kx))^{2/3}\left(-1+(1+k)x-kx^2+bx^5\right)}\,dx = \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{x^{7/3}\left(5-4(1+k)x+3kx^2\right)}{(1-x)^{2/3}(1-kx)^{2/3}\left(-1+(1+k)x-kx\right.}}{((1-x)x(1-kx))^{2/3}}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\text{Subst}\left(\int \frac{x^9\left(5-4(1\right.}{\left(1-x^3\right)^{2/3}\left(1-kx^3\right)^{2/3}\left(\right.}\right.}{((1-x)x(1-kx))^{2/3}}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\text{Subst}\left(\int\left(\frac{3k}{b\left(1-x^3\right)^{2/3}\left(1-kx^3\right)^{2}}\right.\right.}{((1-x)x(1}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\text{Subst}\left(\int \frac{3k-3k(1+k)x^3+3k^2}{\left(1-x^3\right)^{2/3}\left(1-kx^3\right)^{2/3}\left(\right.}\right.}{b((1-x)x(1-kx))^{2/3}}$$

$$= \frac{9kx\left(\frac{1-x}{1-kx}\right)^{2/3}(1-kx)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{(1-k)x}{1-kx}\right)}{b((1-x)x(1-kx))^{2/3}} + \frac{\left(3(1-x)\right.}{}$$

$$= \frac{9kx\left(\frac{1-x}{1-kx}\right)^{2/3}(1-kx)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{(1-k)x}{1-kx}\right)}{b((1-x)x(1-kx))^{2/3}} + \frac{\left(15(1-x\right.}{}$$

Mathematica [F] time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(5-4(1+k)x+3kx^2\right)}{((1-x)x(1-kx))^{2/3}\left(-1+(1+k)x-kx^2+bx^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(5 - 4*(1 + k)*x + 3*k*x^2))/(((1 - x)*x*(1 - k*x))^(2/3)*(-
1 + (1 + k)*x - k*x^2 + b*x^5)),x]
```

```
[Out] Integrate[(x^3*(5 - 4*(1 + k)*x + 3*k*x^2))/(((1 - x)*x*(1 - k*x))^(2/3)*(-
1 + (1 + k)*x - k*x^2 + b*x^5)), x]
```

IntegrateAlgebraic [A] time = 8.95, size = 186, normalized size = 1.00

$$\frac{\log\left(bx^2-b^{2/3}\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{b^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{kx^3+(-k-1)x^2+x}}{2\sqrt[3]{b}x^2+\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{b^{2/3}}-\frac{\log\left(b^{5/3}x^2\sqrt[3]{kx^3+(-k-1)x^2+x}+b^{4/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+b^2x^4\right)}{2b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(5 - 4*(1 + k)*x + 3*k*x^2))/(((1 - x)*x*(1 - k*x))
^(2/3)*(-1 + (1 + k)*x - k*x^2 + b*x^5)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(2*b^(1/3)*x^2
 + (x + (-1 - k)*x^2 + k*x^3)^(1/3))])/b^(2/3)) + Log[b*x^2 - b^(2/3)*(x +
(-1 - k)*x^2 + k*x^3)^(1/3)]/b^(2/3) - Log[b^2*x^4 + b^(5/3)*x^2*(x + (-1 -
 k)*x^2 + k*x^3)^(1/3) + b^(4/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(2/
3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5-4*(1+k)*x+3*k*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-1+(1+k)*x-k*
x^2+b*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,kx^2 - 4\,(k+1)x + 5\right)x^3}{\left(bx^5 - kx^2 + (k+1)x - 1\right)\left((kx-1)(x-1)x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5-4*(1+k)*x+3*k*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-1+(1+k)*x-k*
x^2+b*x^5),x, algorithm="giac")
```

```
[Out] integrate((3*k*x^2 - 4*(k + 1)*x + 5)*x^3/((b*x^5 - k*x^2 + (k + 1)*x - 1)*
((k*x - 1)*(x - 1)*x)^(2/3)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(5-4\left(1+k\right)x+3k\,x^{2}\right)}{\left(\left(1-x\right)x\left(-kx+1\right)\right)^{\frac{2}{3}}\left(-1+\left(1+k\right)x-k\,x^{2}+b\,x^{5}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(5-4*(1+k)*x+3*k*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-1+(1+k)*x-k*x^2+b*
x^5),x)
```

```
[Out] int(x^3*(5-4*(1+k)*x+3*k*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-1+(1+k)*x-k*x^2+b*
x^5),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,kx^2 - 4\,(k+1)x + 5\right)x^3}{\left(bx^5 - kx^2 + (k+1)x - 1\right)\left((kx-1)(x-1)x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5-4*(1+k)*x+3*k*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-1+(1+k)*x-k*
x^2+b*x^5),x, algorithm="maxima")
```

```
[Out] integrate((3*k*x^2 - 4*(k + 1)*x + 5)*x^3/((b*x^5 - k*x^2 + (k + 1)*x - 1)*
((k*x - 1)*(x - 1)*x)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(3\,k\,x^2-4\,x\,\left(k+1\right)+5\right)}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{2/3}\,\left(b\,x^5-k\,x^2+\left(k+1\right)\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(3*k*x^2 - 4*x*(k + 1) + 5))/((x*(k*x - 1)*(x - 1))^(2/3)*(b*x^5 +
 x*(k + 1) - k*x^2 - 1)),x)
```

```
[Out] int((x^3*(3*k*x^2 - 4*x*(k + 1) + 5))/((x*(k*x - 1)*(x - 1))^(2/3)*(b*x^5 +
 x*(k + 1) - k*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(3kx^{2} - 4kx - 4x + 5\right)}{\left(x\left(x - 1\right)\left(kx - 1\right)\right)^{\frac{2}{3}}\left(bx^{5} - kx^{2} + kx + x - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(5-4*(1+k)*x+3*k*x**2)/((1-x)*x*(-k*x+1))**(2/3)/(-1+(1+k)*x
-k*x**2+b*x**5),x)
```

```
[Out] Integral(x**3*(3*k*x**2 - 4*k*x - 4*x + 5)/((x*(x - 1)*(k*x - 1))**(2/3)*(b
*x**5 - k*x**2 + k*x + x - 1)), x)
```

3.1926 $\int \frac{\sqrt[3]{-1-x+5x^2+2x^3-10x^4+2x^5+7x^6-5x^7+x^8}}{x^2}\,dx$

Optimal. Leaf size=186

$$-\frac{\left(\sqrt[3]{x-1}+1\right)\left((x-1)^{2/3}-\sqrt[3]{x-1}+1\right)^2(x-1)^{2/3}\sqrt[3]{(x-1)^2\left(x^2-x-1\right)^3}\left(\frac{(x-1)^{2/3}\left(6x^2-21x+10\right)}{10x}-\frac{1}{3}\log\left(\sqrt[3]{x-1}+\right.\right.}{\left(-x+(x-1)^{2/3}-\sqrt[3]{x-1}+1\right)x\left(x^3-2x^2+1\right)}$$

Rubi [A] time = 0.36, antiderivative size = 299, normalized size of antiderivative = 1.61, number of steps used = 12, number of rules used = 11, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.239, Rules used = {6688, 6719, 897, 1482, 1489, 292, 31, 634, 618, 204, 628}

$$\frac{3\sqrt[3]{-(1-x)^2\left(-x^2+x+1\right)^3}(1-x)}{5\left(-x^2+x+1\right)}-\frac{\sqrt[3]{-(1-x)^2\left(-x^2+x+1\right)^3}}{x\left(-x^2+x+1\right)}+\frac{3\sqrt[3]{-(1-x)^2\left(-x^2+x+1\right)^3}}{2\left(-x^2+x+1\right)}+\frac{\sqrt[3]{-(1-x)^2\left(-x^2+x+1\right)^3}\log\left(\sqrt[3]{x-1}+1\right)}{3(x-1)^{2/3}\left(-x^2+x+1\right)}-\frac{\sqrt[3]{-(1-x)^2\left(-x^2+x+1\right)^3}\log\left((x-1)^{2/3}-\sqrt[3]{x-1}+1\right)}{6(x-1)^{2/3}\left(-x^2+x+1\right)}+\frac{\sqrt[3]{-(1-x)^2\left(-x^2+x+1\right)^3}\tan^{-1}\left(\frac{1-2\sqrt[3]{x-1}}{\sqrt{3}}\right)}{\sqrt{3}(x-1)^{2/3}\left(-x^2+x+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - x + 5*x^2 + 2*x^3 - 10*x^4 + 2*x^5 + 7*x^6 - 5*x^7 + x^8)^(1/3)/x
^2,x]
```

```
[Out] (3*(-((1 - x)^2*(1 + x - x^2)^3))^(1/3))/(2*(1 + x - x^2)) + (3*(1 - x)*(-(
(1 - x)^2*(1 + x - x^2)^3))^(1/3))/(5*(1 + x - x^2)) - (-((1 - x)^2*(1 + x
- x^2)^3))^(1/3)/(x*(1 + x - x^2)) + ((-((1 - x)^2*(1 + x - x^2)^3))^(1/3)*
ArcTan[(1 - 2*(-1 + x)^(1/3))/Sqrt[3]])/(Sqrt[3]*(-1 + x)^(2/3)*(1 + x - x^
2)) + ((-((1 - x)^2*(1 + x - x^2)^3))^(1/3)*Log[1 + (-1 + x)^(1/3)])/(3*(-1
 + x)^(2/3)*(1 + x - x^2)) - ((-((1 - x)^2*(1 + x - x^2)^3))^(1/3)*Log[1 -
(-1 + x)^(1/3) + (-1 + x)^(2/3)])/(6*(-1 + x)^(2/3)*(1 + x - x^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((-d)^((m - Mod[m, n])/n - 1)*(c*d^
2 - b*d*e + a*e^2)^p*x^(Mod[m, n] + 1)*(d + e*x^n)^(q + 1))/(n*e^(2*p + (m
- Mod[m, n])/n)*(q + 1)), x] + Dist[1/(n*e^(2*p + (m - Mod[m, n])/n)*(q + 1
)), Int[x^Mod[m, n]*(d + e*x^n)^(q + 1)*ExpandToSum[Together[(1*(n*e^(2*p +
 (m - Mod[m, n])/n)*(q + 1)*x^(m - Mod[m, n])*(a + b*x^n + c*x^(2*n))^p - (
-d)^((m - Mod[m, n])/n - 1)*(c*d^2 - b*d*e + a*e^2)^p*(d*(Mod[m, n] + 1) +
e*(Mod[m, n] + n*(q + 1) + 1)*x^n)))/(d + e*x^n)], x], x], x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IGt
Q[p, 0] && ILtQ[q, -1] && IGtQ[m, 0]
```

Rule 1489

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && EqQ[n2, 2*n] && IG
tQ[n, 0] && IGtQ[p, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{-1-x+5x^2+2x^3-10x^4+2x^5+7x^6-5x^7+x^8}}{x^2}\,dx = \int \frac{\sqrt[3]{(-1+x)^2\left(-1-x+x^2\right)^3}}{x^2}\,dx$$

$$= \frac{\sqrt[3]{(-1+x)^2\left(-1-x+x^2\right)^3}\int \frac{(-1+x)^{2/3}\left(-1-x+x^2\right)}{x^2}\,dx}{(-1+x)^{2/3}\left(-1-x+x^2\right)}$$

$$= \frac{\left(3\sqrt[3]{(-1+x)^2\left(-1-x+x^2\right)^3}\right)\text{Subst}\left(\int \frac{x^4\left(-1+x^3+x^6\right.}{\left(1+x^3\right)^2}\right.}{(-1+x)^{2/3}\left(-1-x+x^2\right)}$$

$$= -\frac{\sqrt[3]{-(1-x)^2\left(1+x-x^2\right)^3}}{x\left(1+x-x^2\right)} - \frac{\sqrt[3]{(-1+x)^2\left(-1-x+\right.}}{(-1}$$

$$= -\frac{\sqrt[3]{-(1-x)^2\left(1+x-x^2\right)^3}}{x\left(1+x-x^2\right)} - \frac{\sqrt[3]{(-1+x)^2\left(-1-x+\right.}}{}$$

$$= \frac{3\sqrt[3]{-(1-x)^2\left(1+x-x^2\right)^3}}{2\left(1+x-x^2\right)} + \frac{3(1-x)\sqrt[3]{-(1-x)^2\left(\right.}}{5\left(1+x-x\right.}$$

$$= \frac{3\sqrt[3]{-(1-x)^2\left(1+x-x^2\right)^3}}{2\left(1+x-x^2\right)} + \frac{3(1-x)\sqrt[3]{-(1-x)^2\left(\right.}}{5\left(1+x-x\right.}$$

$$= \frac{3\sqrt[3]{-(1-x)^2\left(1+x-x^2\right)^3}}{2\left(1+x-x^2\right)} + \frac{3(1-x)\sqrt[3]{-(1-x)^2\left(\right.}}{5\left(1+x-x\right.}$$

$$= \frac{3\sqrt[3]{-(1-x)^2\left(1+x-x^2\right)^3}}{2\left(1+x-x^2\right)} + \frac{3(1-x)\sqrt[3]{-(1-x)^2\left(\right.}}{5\left(1+x-x\right.}$$

$$= \frac{3\sqrt[3]{-(1-x)^2\left(1+x-x^2\right)^3}}{2\left(1+x-x^2\right)} + \frac{3(1-x)\sqrt[3]{-(1-x)^2\left(\right.}}{5\left(1+x-x\right.}$$

Mathematica [C] time = 0.03, size = 63, normalized size = 0.34

$$\frac{\sqrt[3]{(x-1)^2\left(x^2-x-1\right)^3}\left(5x\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-x\right)+6x^2-21x+10\right)}{10x\left(x^2-x-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - x + 5*x^2 + 2*x^3 - 10*x^4 + 2*x^5 + 7*x^6 - 5*x^7 + x^8)^(
1/3)/x^2,x]
```

```
[Out] (((-1 + x)^2*(-1 - x + x^2)^3)^(1/3)*(10 - 21*x + 6*x^2 + 5*x*Hypergeometri
c2F1[2/3, 1, 5/3, 1 - x]))/(10*x*(-1 - x + x^2))
```

IntegrateAlgebraic [A] time = 23.38, size = 190, normalized size = 1.02

$$-\frac{\left(\sqrt[3]{x-1}+1\right)\left((x-1)^{2/3}-\sqrt[3]{x-1}+1\right)^2(x-1)^{2/3}\sqrt[3]{(x-1)^2\left(x^2-x-1\right)^3}\left(\frac{(x-1)^{2/3}\left(6(x-1)^2-9(x-1)-5\right)}{10x}-\frac{1}{3}\log\left(\sqrt[3]{x-1}+1\right)+\frac{1}{6}\log\left((x-1)^{2/3}-\sqrt[3]{x-1}+1\right)-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x-1}}{\sqrt{3}}\right)}{\sqrt{3}}\right)}{\left(-x+(x-1)^{2/3}-\sqrt[3]{x-1}+1\right)x\left(x^3-2x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - x + 5*x^2 + 2*x^3 - 10*x^4 + 2*x^5 + 7*x^6 - 5*x^7
 + x^8)^(1/3)/x^2,x]
```

```
[Out] -(((1 + (-1 + x)^(1/3))*(1 - (-1 + x)^(1/3) + (-1 + x)^(2/3))^2*(-1 + x)^(2
/3)*((-1 + x)^2*(-1 - x + x^2)^3)^(1/3)*(((-5 - 9*(-1 + x) + 6*(-1 + x)^2)*
(-1 + x)^(2/3))/(10*x) - ArcTan[1/Sqrt[3] - (2*(-1 + x)^(1/3))/Sqrt[3]]/Sqr
t[3] - Log[1 + (-1 + x)^(1/3)]/3 + Log[1 - (-1 + x)^(1/3) + (-1 + x)^(2/3)]
/6))/((1 - (-1 + x)^(1/3) + (-1 + x)^(2/3) - x)*x*(1 - 2*x^2 + x^3)))
```

fricas [B] time = 0.48, size = 399, normalized size = 2.15

$$-\frac{10\,\sqrt{3}\left(x^{3}-x^{2}-x\right)\arctan\left(-\frac{\sqrt{3}\left(x^{3}-2\,x^{2}+1\right)-2\,\sqrt{3}\left(x^{8}-5\,x^{7}+7\,x^{6}+2\,x^{5}-10\,x^{4}+2\,x^{3}+5\,x^{2}-x-1\right)^{\frac{1}{3}}}{3\left(x^{3}-2\,x^{2}+1\right)}\right)-5\left(x^{3}-x^{2}-x\right)\log\left(\frac{x^{6}-4\,x^{5}+4\,x^{4}+2\,x^{3}-4\,x^{2}-\left(x^{8}-5\,x^{7}+7\,x^{6}+2\,x^{5}-10\,x^{4}+2\,x^{3}+5\,x^{2}-x-1\right)^{\frac{1}{3}}\left(x^{3}-2\,x^{2}+1\right)+\left(x^{8}-5\,x^{7}+7\,x^{6}+2\,x^{5}-10\,x^{4}+2\,x^{3}+5\,x^{2}-x-1\right)^{\frac{2}{3}}+1}{x^{6}-4\,x^{5}+4\,x^{4}+2\,x^{3}-4\,x^{2}+1}\right)+10\left(x^{3}-x^{2}-x\right)\log\left(\frac{x^{3}-2\,x^{2}+\left(x^{8}-5\,x^{7}+7\,x^{6}+2\,x^{5}-10\,x^{4}+2\,x^{3}+5\,x^{2}-x-1\right)^{\frac{1}{3}}+1}{x^{3}-2\,x^{2}+1}\right)-3\left(x^{8}-5\,x^{7}+7\,x^{6}+2\,x^{5}-10\,x^{4}+2\,x^{3}+5\,x^{2}-x-1\right)^{\frac{1}{3}}\left(6\,x^{2}-21\,x+10\right)}{30\left(x^{3}-x^{2}-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-5*x^7+7*x^6+2*x^5-10*x^4+2*x^3+5*x^2-x-1)^(1/3)/x^2,x, algor
ithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*(x^3 - x^2 - x)*arctan(-1/3*(sqrt(3)*(x^3 - 2*x^2 + 1) -
2*sqrt(3)*(x^8 - 5*x^7 + 7*x^6 + 2*x^5 - 10*x^4 + 2*x^3 + 5*x^2 - x - 1)^(1
/3))/(x^3 - 2*x^2 + 1)) - 5*(x^3 - x^2 - x)*log((x^6 - 4*x^5 + 4*x^4 + 2*x^
3 - 4*x^2 - (x^8 - 5*x^7 + 7*x^6 + 2*x^5 - 10*x^4 + 2*x^3 + 5*x^2 - x - 1)^
(1/3)*(x^3 - 2*x^2 + 1) + (x^8 - 5*x^7 + 7*x^6 + 2*x^5 - 10*x^4 + 2*x^3 + 5
*x^2 - x - 1)^(2/3) + 1)/(x^6 - 4*x^5 + 4*x^4 + 2*x^3 - 4*x^2 + 1)) + 10*(x
^3 - x^2 - x)*log((x^3 - 2*x^2 + (x^8 - 5*x^7 + 7*x^6 + 2*x^5 - 10*x^4 + 2*
x^3 + 5*x^2 - x - 1)^(1/3) + 1)/(x^3 - 2*x^2 + 1)) - 3*(x^8 - 5*x^7 + 7*x^6
 + 2*x^5 - 10*x^4 + 2*x^3 + 5*x^2 - x - 1)^(1/3)*(6*x^2 - 21*x + 10))/(x^3
- x^2 - x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{8}-5\,x^{7}+7\,x^{6}+2\,x^{5}-10\,x^{4}+2\,x^{3}+5\,x^{2}-x-1\right)^{\frac{1}{3}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-5*x^7+7*x^6+2*x^5-10*x^4+2*x^3+5*x^2-x-1)^(1/3)/x^2,x, algor
ithm="giac")
```

```
[Out] integrate((x^8 - 5*x^7 + 7*x^6 + 2*x^5 - 10*x^4 + 2*x^3 + 5*x^2 - x - 1)^(1
/3)/x^2, x)
```

maple [A] time = 0.04, size = 137, normalized size = 0.74

$$\frac{\left(6x^{3}-27x^{2}+31x-10\right)\left(\left(-1+x\right)^{2}\left(x^{2}-x-1\right)^{3}\right)^{\frac{1}{3}}}{10x\left(-1+x\right)\left(x^{2}-x-1\right)}+\frac{\left(-\frac{\ln\left(1+\left(-1+x\right)^{\frac{1}{3}}\right)}{3}+\frac{\ln\left(1-\left(-1+x\right)^{\frac{1}{3}}+\left(-1+x\right)^{\frac{2}{3}}\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(-1+2\left(-1+x\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)}{3}\right)\left(\left(-1+x\right)^{2}\left(x^{2}-x-1\right)^{3}\right)^{\frac{1}{3}}}{\left(-1+x\right)^{\frac{2}{3}}\left(x^{2}-x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-5*x^7+7*x^6+2*x^5-10*x^4+2*x^3+5*x^2-x-1)^(1/3)/x^2,x)
```

```
[Out] 1/10*(6*x^3-27*x^2+31*x-10)/x/(-1+x)*((-1+x)^2*(x^2-x-1)^3)^(1/3)/(x^2-x-1)
+(-1/3*ln(1+(-1+x)^(1/3))+1/6*ln(1-(-1+x)^(1/3)+(-1+x)^(2/3))+1/3*3^(1/2)*a
rctan(1/3*(-1+2*(-1+x)^(1/3))*3^(1/2)))*((-1+x)^2*(x^2-x-1)^3)^(1/3)/(-1+x)
^(2/3)/(x^2-x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{8}-5\,x^{7}+7\,x^{6}+2\,x^{5}-10\,x^{4}+2\,x^{3}+5\,x^{2}-x-1\right)^{\frac{1}{3}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-5*x^7+7*x^6+2*x^5-10*x^4+2*x^3+5*x^2-x-1)^(1/3)/x^2,x, algor
ithm="maxima")
```

```
[Out] integrate((x^8 - 5*x^7 + 7*x^6 + 2*x^5 - 10*x^4 + 2*x^3 + 5*x^2 - x - 1)^(1
/3)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^8-5\,x^7+7\,x^6+2\,x^5-10\,x^4+2\,x^3+5\,x^2-x-1\right)^{1/3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 - x + 2*x^3 - 10*x^4 + 2*x^5 + 7*x^6 - 5*x^7 + x^8 - 1)^(1/3)/x^
2,x)
```

```
[Out] int((5*x^2 - x + 2*x^3 - 10*x^4 + 2*x^5 + 7*x^6 - 5*x^7 + x^8 - 1)^(1/3)/x^
2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\left(x-1\right)^{2}\left(x^{2}-x-1\right)^{3}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-5*x**7+7*x**6+2*x**5-10*x**4+2*x**3+5*x**2-x-1)**(1/3)/x**2
,x)
```

```
[Out] Integral(((x - 1)**2*(x**2 - x - 1)**3)**(1/3)/x**2, x)
```

3.1927 $\int \frac{(-b+a^2x^2)\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx$

Optimal. Leaf size=186

$$\frac{1}{2}ax\sqrt{\sqrt{a^2x^4+b}+ax^2}-\frac{b\log\left(\sqrt{a^2x^4+b}+\sqrt{2}\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}+ax^2\right)}{\sqrt{2}\sqrt{a}}-\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a^2x^4+b}}{\sqrt{b}}+\ldots\right.}{\sqrt{2}\ldots}$$

Rubi [F] time = 1.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-b+a^2x^2)\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-b + a^2*x^2)*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]])/Sqrt[b + a^2*x^4],x]
```

```
[Out] -((b*ArcTanh[(Sqrt[2]*Sqrt[a]*x)/Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]])/(Sqrt[2]
*Sqrt[a])) + a^2*Defer[Int][(x^2*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]])/Sqrt[b +
a^2*x^4], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-b+a^2x^2)\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx &= \int \left(-\frac{b\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}+\frac{a^2x^2\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\right)dx\\
&= a^2\int \frac{x^2\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx-b\int \frac{\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx\\
&= a^2\int \frac{x^2\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx-b\,\text{Subst}\left(\int \frac{1}{1-2ax^2}\,dx,x,\frac{\ldots}{\sqrt{ax^2+\ldots}}\right.\\
&= -\frac{b\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x}{\sqrt{ax^2+\sqrt{b+a^2x^4}}}\right)}{\sqrt{2}\sqrt{a}}+a^2\int \frac{x^2\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx
\end{aligned}$$

Mathematica [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(-b+a^2x^2)\sqrt{ax^2+\sqrt{b+a^2x^4}}}{\sqrt{b+a^2x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-b + a^2*x^2)*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]])/Sqrt[b + a^2*x^4
],x]
```

```
[Out] Integrate[((-b + a^2*x^2)*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]])/Sqrt[b + a^2*x^4
], x]
```

IntegrateAlgebraic [A] time = 0.56, size = 186, normalized size = 1.00

$$\frac{1}{2}ax\sqrt{\sqrt{a^2x^4+b}+ax^2}+\frac{b\log\left(\sqrt{a^2x^4+b}-\sqrt{2}\,\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}+ax^2\right)}{\sqrt{2}\,\sqrt{a}}+\frac{\sqrt{a}\,\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a^2x^4+b}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}}{\sqrt{b}}+\frac{ax^2}{\sqrt{b}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a^2*x^2)*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]])/Sqrt[b
+ a^2*x^4],x]
```

```
[Out] (a*x*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]])/2 + (Sqrt[a]*Sqrt[b]*ArcTan[(a*x^2)/S
qrt[b] + Sqrt[b + a^2*x^4]/Sqrt[b] - (Sqrt[2]*Sqrt[a]*x*Sqrt[a*x^2 + Sqrt[b
 + a^2*x^4]])/Sqrt[b]])/Sqrt[2] + (b*Log[a*x^2 + Sqrt[b + a^2*x^4] - Sqrt[2
]*Sqrt[a]*x*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]])/(Sqrt[2]*Sqrt[a])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/(a^2*x^4+b)^(1/2),x,
algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-b\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{\sqrt{a^2x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/(a^2*x^4+b)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - b)*sqrt(a*x^2 + sqrt(a^2*x^4 + b))/sqrt(a^2*x^4 + b),
x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-b\right)\sqrt{a\,x^2+\sqrt{a^2x^4+b}}}{\sqrt{a^2x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/(a^2*x^4+b)^(1/2),x)
```

```
[Out] int((a^2*x^2-b)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/(a^2*x^4+b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-b\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{\sqrt{a^2x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2)/(a^2*x^4+b)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 - b)*sqrt(a*x^2 + sqrt(a^2*x^4 + b))/sqrt(a^2*x^4 + b),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{\sqrt{a^2\,x^4+b}+a\,x^2}\,\left(b-a^2\,x^2\right)}{\sqrt{a^2\,x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(((b + a^2*x^4)^(1/2) + a*x^2)^(1/2)*(b - a^2*x^2))/(b + a^2*x^4)^(1/2
),x)
```

```
[Out] int(-(((b + a^2*x^4)^(1/2) + a*x^2)^(1/2)*(b - a^2*x^2))/(b + a^2*x^4)^(1/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2+\sqrt{a^2x^4+b}}\,\left(a^2x^2-b\right)}{\sqrt{a^2x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b)*(a*x**2+(a**2*x**4+b)**(1/2))**(1/2)/(a**2*x**4+b)*
*(1/2),x)
```

```
[Out] Integral(sqrt(a*x**2 + sqrt(a**2*x**4 + b))*(a**2*x**2 - b)/sqrt(a**2*x**4
+ b), x)
```

3.1928 $\int \frac{-i+\sqrt{k}x}{(i+\sqrt{k}x)\sqrt{(1-x^2)(1-k^2x^2)}}\,dx$

Optimal. Leaf size=187

$$\frac{\tan^{-1}\left(\frac{(-k-2\sqrt{k}-1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}+kx^2-1}\right)}{k+1}+\frac{\tan^{-1}\left(\frac{(-k+2\sqrt{k}-1)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}+kx^2-1}\right)}{k+1}+\frac{i\tanh^{-1}\left(\frac{(2k^{3/2}+2\sqrt{k})x^2}{k^2x^4+(kx^2-1)\sqrt{k^2x^4+(-k^2-1)x^2+1}+2kx^2+1}\right)}{k+1}$$

Rubi [C] time = 1.45, antiderivative size = 201, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.170$, Rules used = {6719, 6742, 419, 2113, 537, 571, 93, 208}

$$\frac{2i\sqrt{1-x^2}\sqrt{1-k^2x^2}\tanh^{-1}\left(\frac{\sqrt{k}\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{(k+1)\sqrt{(1-x^2)(1-k^2x^2)}}+\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}-\frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\Pi\left(-k;\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-I + Sqrt[k]*x)/((I + Sqrt[k]*x)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),x]
```

```
[Out] ((2*I)*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*ArcTanh[(Sqrt[k]*Sqrt[1 - x^2])/Sqrt
[1 - k^2*x^2]])/((1 + k)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) + (Sqrt[1 - x^2]*Sq
rt[1 - k^2*x^2]*EllipticF[ArcSin[x], k^2])/Sqrt[(1 - x^2)*(1 - k^2*x^2)] -
(2*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[-k, ArcSin[x], k^2])/Sqrt[(1
- x^2)*(1 - k^2*x^2)]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
```

```
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-i+\sqrt{k}x}{\left(i+\sqrt{k}x\right)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}\,dx = \frac{\left(\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \frac{-i+\sqrt{k}x}{\left(i+\sqrt{k}x\right)\sqrt{1-x^2}\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \left(\frac{1}{\sqrt{1-x^2}\sqrt{1-k^2x^2}}-\frac{2i}{\left(i+\sqrt{k}x\right)\sqrt{1-x^2}\sqrt{1-k^2x^2}}\right)dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= -\frac{\left(2i\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \frac{1}{\left(i+\sqrt{k}x\right)\sqrt{1-x^2}\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(\sqrt{1-x^2}\sqrt{1-\right.}{\sqrt{(1-}}$$

$$= \frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(2\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \frac{}{\sqrt{1-x^2}(}}{\sqrt{\left(1-x^2\right)\left(1-k^2x\right.}}$$

$$= \frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi\left(-k;\sin^{-}\right.}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi\left(-k;\sin^{-}\right.}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{2i\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{k}\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{(1+k)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-}\right.}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right.}}$$

Mathematica [C] time = 0.36, size = 208, normalized size = 1.11

$$\frac{-2i\sqrt{k}\sqrt{x^2-1}\sqrt{k^2x^2-1}\tanh^{-1}\left(\frac{\sqrt{k(k+1)}\sqrt{x^2-1}}{\sqrt{k+1}\sqrt{k^2x^2-1}}\right)+\sqrt{k+1}\sqrt{k(k+1)}\sqrt{1-x^2}\sqrt{1-k^2x^2}F\left(\sin^{-1}(x)|k^2\right)-2\sqrt{k+1}\sqrt{k(k+1)}\sqrt{1-x^2}\sqrt{1-k^2x^2}\Pi\left(-k;\sin^{-1}(x)|k^2\right)}{\sqrt{k+1}\sqrt{k(k+1)}\sqrt{(x^2-1)(k^2x^2-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-I + Sqrt[k]*x)/((I + Sqrt[k]*x)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),
x]
```

```
[Out] ((-2*I)*Sqrt[k]*Sqrt[-1 + x^2]*Sqrt[-1 + k^2*x^2]*ArcTanh[(Sqrt[k*(1 + k)]*
Sqrt[-1 + x^2])/(Sqrt[1 + k]*Sqrt[-1 + k^2*x^2])] + Sqrt[1 + k]*Sqrt[k*(1 +
 k)]*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticF[ArcSin[x], k^2] - 2*Sqrt[1 +
 k]*Sqrt[k*(1 + k)]*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[-k, ArcSin[x
], k^2])/(Sqrt[1 + k]*Sqrt[k*(1 + k)]*Sqrt[(-1 + x^2)*(-1 + k^2*x^2)])
```

IntegrateAlgebraic [A] time = 3.66, size = 56, normalized size = 0.30

$$-\frac{2\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2+2i\sqrt{k}\,x-1}\right)}{k+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-I + Sqrt[k]*x)/((I + Sqrt[k]*x)*Sqrt[(1 - x^2)*(1 - k^
2*x^2)]),x]
```

```
[Out] (-2*ArcTan[((1 + k)*x)/(-1 + (2*I)*Sqrt[k]*x + k*x^2 + Sqrt[1 + (-1 - k^2)*
x^2 + k^2*x^4])])/(1 + k)
```

fricas [A] time = 1.04, size = 249, normalized size = 1.33

$$\frac{i\log\left(\frac{\left(-ik^6-5ik^5-10ik^4-10ik^3-5ik^2-ik\right)x^3+\left(ik^5+5ik^4+10ik^3+10ik^2+5ik+i\right)x+\sqrt{k^2x^4-\left(k^2+1\right)x^2+1}\left(k^4+4k^3-\left(k^5+4k^4+6k^3+4k^2+k\right)x^2+\left(2ik^4+8ik^3+12ik^2+8ik+2i\right)\sqrt{k}\,x+6k^2+4k+1\right)+2\left(\left(k^5+3k^4+3k^3+k^2\right)x^4+k^3-\left(k^5+3k^4+4k^3+4k^2+3k+1\right)x^2+3k^2+3k+1\right)\sqrt{k}}{4\left(k^5x^4+2k^4x^2+k^3\right)}\right)}{k+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-I+k^(1/2)*x)/(I+k^(1/2)*x)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, alg
orithm="fricas")
```

```
[Out] I*log(1/4*((-I*k^6 - 5*I*k^5 - 10*I*k^4 - 10*I*k^3 - 5*I*k^2 - I*k)*x^3 + (
I*k^5 + 5*I*k^4 + 10*I*k^3 + 10*I*k^2 + 5*I*k + I)*x + sqrt(k^2*x^4 - (k^2
+ 1)*x^2 + 1)*(k^4 + 4*k^3 - (k^5 + 4*k^4 + 6*k^3 + 4*k^2 + k)*x^2 + (2*I*k
^4 + 8*I*k^3 + 12*I*k^2 + 8*I*k + 2*I)*sqrt(k)*x + 6*k^2 + 4*k + 1) + 2*((k
^5 + 3*k^4 + 3*k^3 + k^2)*x^4 + k^3 - (k^5 + 3*k^4 + 4*k^3 + 4*k^2 + 3*k +
1)*x^2 + 3*k^2 + 3*k + 1)*sqrt(k))/(k^5*x^4 + 2*k^4*x^2 + k^3))/(k + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{k}\,x-i}{\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}\left(\sqrt{k}\,x+i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-I+k^(1/2)*x)/(I+k^(1/2)*x)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((sqrt(k)*x - I)/(sqrt((k^2*x^2 - 1)*(x^2 - 1))*(sqrt(k)*x + I)),
x)
```

maple [C] time = 0.17, size = 211, normalized size = 1.13

$$\frac{\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\operatorname{EllipticF}(x,k)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}-\frac{2\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\operatorname{EllipticPi}(x,-k,k)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}+\frac{i\ln\left(\frac{2k^2+4k+2+\left(-k^3-2k^2-k\right)\left(x^2+\frac{1}{k}\right)+2\sqrt{(1+k)^2}\,\sqrt{k^3\left(x^2+\frac{1}{k}\right)^2+\left(-k^3-2k^2-k\right)\left(x^2+\frac{1}{k}\right)+k^2+2k+1}}{x^2+\frac{1}{k}}\right)}{\sqrt{(1+k)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I+k^(1/2)*x)/(I+k^(1/2)*x)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x)
```

```
[Out] (-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*EllipticF(x
,k)-2*(-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*Ellip
ticPi(x,-k,k)+I/((1+k)^2)^(1/2)*ln((2*k^2+4*k+2+(-k^3-2*k^2-k)*(x^2+1/k)+2*
((1+k)^2)^(1/2)*(k^3*(x^2+1/k)^2+(-k^3-2*k^2-k)*(x^2+1/k)+k^2+2*k+1)^(1/2))
/(x^2+1/k))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{k}\,x-i}{\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}\left(\sqrt{k}\,x+i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-I+k^(1/2)*x)/(I+k^(1/2)*x)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((sqrt(k)*x - I)/(sqrt((k^2*x^2 - 1)*(x^2 - 1))*(sqrt(k)*x + I)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{k}\;x-\mathrm{i}}{\left(\sqrt{k}\;x+1\mathrm{i}\right)\sqrt{\left(x^2-1\right)\left(k^2\,x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^(1/2)*x - 1i)/((k^(1/2)*x + 1i)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)),x)
```

```
[Out] int((k^(1/2)*x - 1i)/((k^(1/2)*x + 1i)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{k}\,x-i}{\sqrt{(x-1)(x+1)(kx-1)(kx+1)}\left(\sqrt{k}\,x+i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-I+k**(1/2)*x)/(I+k**(1/2)*x)/((-x**2+1)*(-k**2*x**2+1))**(1/2),
x)
```

```
[Out] Integral((sqrt(k)*x - I)/(sqrt((x - 1)*(x + 1)*(k*x - 1)*(k*x + 1))*(sqrt(k
)*x + I)), x)
```

3.1929 $\int \frac{i+\sqrt{k}x}{\left(-i+\sqrt{k}x\right)\sqrt{(1-x^2)(1-k^2x^2)}}\,dx$

Optimal. Leaf size=187

$$\frac{\tan^{-1}\left(\frac{\left(-k-2\sqrt{k}-1\right)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}+kx^2-1}\right)}{k+1}+\frac{\tan^{-1}\left(\frac{\left(-k+2\sqrt{k}-1\right)x}{\sqrt{k^2x^4+(-k^2-1)x^2+1}+kx^2-1}\right)}{k+1}-\frac{i\tanh^{-1}\left(\frac{\left(2k^{3/2}+2\sqrt{k}\right)x^2}{k^2x^4+(kx^2-1)\sqrt{k^2x^4+(-k^2-1)x^2+1}+2kx^2+1}\right)}{k+1}$$

Rubi [C] time = 1.22, antiderivative size = 201, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.170, Rules used = {6719, 6742, 419, 2113, 537, 571, 93, 208}

$$-\frac{2i\sqrt{1-x^2}\sqrt{1-k^2x^2}\tanh^{-1}\left(\frac{\sqrt{k}\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{(k+1)\sqrt{(1-x^2)(1-k^2x^2)}}+\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}-\frac{2\sqrt{1-x^2}\sqrt{1-k^2x^2}\Pi\left(-k;\sin^{-1}(x)|k^2\right)}{\sqrt{(1-x^2)(1-k^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Int[(I + Sqrt[k]*x)/((-I + Sqrt[k]*x)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),x]
```

```
[Out] ((-2*I)*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*ArcTanh[(Sqrt[k]*Sqrt[1 - x^2])/Sqr
t[1 - k^2*x^2]])/((1 + k)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) + (Sqrt[1 - x^2]*S
qrt[1 - k^2*x^2]*EllipticF[ArcSin[x], k^2])/Sqrt[(1 - x^2)*(1 - k^2*x^2)] -
 (2*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[-k, ArcSin[x], k^2])/Sqrt[(1
 - x^2)*(1 - k^2*x^2)]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
```

```
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{i+\sqrt{k}\,x}{\left(-i+\sqrt{k}\,x\right)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}\,dx = \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{i+\sqrt{k}\,x}{\left(-i+\sqrt{k}\,x\right)\sqrt{1-x^2}\,\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \left(\frac{1}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}}+\frac{2i}{\left(-i+\sqrt{k}\,x\right)\sqrt{1-x^2}\,\sqrt{1-k^2x^2}}\right)dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(2i\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1}{\left(-i+\sqrt{k}\,x\right)\sqrt{1-x^2}\,\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(\sqrt{1-x^2}\,\sqrt{1-\right.}{\sqrt{(1}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{}{\sqrt{1-x^2}}}{\sqrt{\left(1-x^2\right)\left(1-k\right.}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(-k;\sin\right.}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{2\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(-k;\sin\right.}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= -\frac{2i\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{k}\,\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{(1+k)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\mathrm{s}\right.}{\sqrt{\left(1-x^2\right)\left(1-k\right.}}$$

Mathematica [C] time = 0.26, size = 208, normalized size = 1.11

$$\frac{2i\sqrt{k}\sqrt{x^2-1}\sqrt{k^2x^2-1}\tanh^{-1}\left(\frac{\sqrt{k(k+1)}\sqrt{x^2-1}}{\sqrt{k+1}\sqrt{k^2x^2-1}}\right)+\sqrt{k+1}\sqrt{k(k+1)}\sqrt{1-x^2}\sqrt{1-k^2x^2}F\left(\sin^{-1}(x)|k^2\right)-2\sqrt{k+1}\sqrt{k(k+1)}\sqrt{1-x^2}\sqrt{1-k^2x^2}\Pi\left(-k;\sin^{-1}(x)|k^2\right)}{\sqrt{k+1}\sqrt{k(k+1)}\sqrt{\left(x^2-1\right)\left(k^2x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I + Sqrt[k]*x)/((-I + Sqrt[k]*x)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),
x]
```

```
[Out] ((2*I)*Sqrt[k]*Sqrt[-1 + x^2]*Sqrt[-1 + k^2*x^2]*ArcTanh[(Sqrt[k*(1 + k)]*S
qrt[-1 + x^2])/(Sqrt[1 + k]*Sqrt[-1 + k^2*x^2])] + Sqrt[1 + k]*Sqrt[k*(1 +
k)]*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticF[ArcSin[x], k^2] - 2*Sqrt[1 +
k]*Sqrt[k*(1 + k)]*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[-k, ArcSin[x]
, k^2])/(Sqrt[1 + k]*Sqrt[k*(1 + k)]*Sqrt[(-1 + x^2)*(-1 + k^2*x^2)])
```

IntegrateAlgebraic [A] time = 3.70, size = 56, normalized size = 0.30

$$-\frac{2\tan^{-1}\left(\frac{(k+1)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2-2i\sqrt{k}\,x-1}\right)}{k+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(I + Sqrt[k]*x)/((-I + Sqrt[k]*x)*Sqrt[(1 - x^2)*(1 - k^
2*x^2)]),x]
```

```
[Out] (-2*ArcTan[((1 + k)*x)/(-1 - (2*I)*Sqrt[k]*x + k*x^2 + Sqrt[1 + (-1 - k^2)*
x^2 + k^2*x^4])])/(1 + k)
```

fricas [A] time = 1.07, size = 249, normalized size = 1.33

$$\frac{i\log\left(\frac{\left(-ik^6-5ik^5-10ik^4-10ik^3-5ik^2-ik\right)x^3+\left(ik^5+5ik^4+10ik^3+10ik^2+5ik+i\right)x+\sqrt{k^2x^4-\left(k^2+1\right)x^2+1}\left(k^4+4k^3-\left(k^5+4k^4+6k^3+4k^2+k\right)x^2+\left(-2ik^4-8ik^3-12ik^2-8ik-2i\right)\sqrt{k}\,x+6k^2+4k+1\right)-2\left(\left(k^5+3k^4+3k^3+k^2\right)x^4+k^3-\left(k^5+3k^4+4k^3+4k^2+3k+1\right)x^2+3k^2+3k+1\right)\sqrt{k}}{4\left(k^5x^4+2k^4x^2+k^3\right)}\right)}{k+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I+k^(1/2)*x)/(-I+k^(1/2)*x)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, alg
orithm="fricas")
```

```
[Out] I*log(1/4*((-I*k^6 - 5*I*k^5 - 10*I*k^4 - 10*I*k^3 - 5*I*k^2 - I*k)*x^3 + (
I*k^5 + 5*I*k^4 + 10*I*k^3 + 10*I*k^2 + 5*I*k + I)*x + sqrt(k^2*x^4 - (k^2
+ 1)*x^2 + 1)*(k^4 + 4*k^3 - (k^5 + 4*k^4 + 6*k^3 + 4*k^2 + k)*x^2 + (-2*I*
k^4 - 8*I*k^3 - 12*I*k^2 - 8*I*k - 2*I)*sqrt(k)*x + 6*k^2 + 4*k + 1) - 2*((
k^5 + 3*k^4 + 3*k^3 + k^2)*x^4 + k^3 - (k^5 + 3*k^4 + 4*k^3 + 4*k^2 + 3*k +
 1)*x^2 + 3*k^2 + 3*k + 1)*sqrt(k))/(k^5*x^4 + 2*k^4*x^2 + k^3))/(k + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{k}\,x+i}{\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}\left(\sqrt{k}\,x-i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I+k^(1/2)*x)/(-I+k^(1/2)*x)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((sqrt(k)*x + I)/(sqrt((k^2*x^2 - 1)*(x^2 - 1))*(sqrt(k)*x - I)),
x)
```

maple [C] time = 0.16, size = 211, normalized size = 1.13

$$\frac{\sqrt{-x^{2}+1}\,\sqrt{-k^{2}x^{2}+1}\,\operatorname{EllipticF}(x,k)}{\sqrt{k^{2}x^{4}-k^{2}x^{2}-x^{2}+1}}-\frac{2\sqrt{-x^{2}+1}\,\sqrt{-k^{2}x^{2}+1}\,\operatorname{EllipticPi}(x,-k,k)}{\sqrt{k^{2}x^{4}-k^{2}x^{2}-x^{2}+1}}-\frac{i\ln\left(\frac{2k^{2}+4k+2+(-k^{3}-2k^{2}-k)\left(x^{2}+\frac{1}{k}\right)+2\sqrt{(1+k)^{2}}\,\sqrt{k^{3}\left(x^{2}+\frac{1}{k}\right)^{2}+(-k^{3}-2k^{2}-k)\left(x^{2}+\frac{1}{k}\right)+k^{2}+2k+1}}{x^{2}+\frac{1}{k}}\right)}{\sqrt{(1+k)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((I+k^(1/2)*x)/(-I+k^(1/2)*x)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x)
```

```
[Out] (-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*EllipticF(x
,k)-2*(-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*Ellip
ticPi(x,-k,k)-I/((1+k)^2)^(1/2)*ln((2*k^2+4*k+2+(-k^3-2*k^2-k)*(x^2+1/k)+2*
((1+k)^2)^(1/2)*(k^3*(x^2+1/k)^2+(-k^3-2*k^2-k)*(x^2+1/k)+k^2+2*k+1)^(1/2))
/(x^2+1/k))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{k}\,x+i}{\sqrt{(k^{2}x^{2}-1)(x^{2}-1)}\,(\sqrt{k}\,x-i)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I+k^(1/2)*x)/(-I+k^(1/2)*x)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((sqrt(k)*x + I)/(sqrt((k^2*x^2 - 1)*(x^2 - 1))*(sqrt(k)*x - I)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{k}\,x+1\mathrm{i}}{(\sqrt{k}\,x-\mathrm{i})\,\sqrt{(x^{2}-1)\,(k^{2}\,x^{2}-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^(1/2)*x + 1i)/((k^(1/2)*x - 1i)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)),x)
```

```
[Out] int((k^(1/2)*x + 1i)/((k^(1/2)*x - 1i)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{k}\,x+i}{\sqrt{(x-1)(x+1)(kx-1)(kx+1)}\,(\sqrt{k}\,x-i)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I+k**(1/2)*x)/(-I+k**(1/2)*x)/((-x**2+1)*(-k**2*x**2+1))**(1/2),
x)
```

```
[Out] Integral((sqrt(k)*x + I)/(sqrt((x - 1)*(x + 1)*(k*x - 1)*(k*x + 1))*(sqrt(k
)*x - I)), x)
```

3.1930 $\int \frac{x^4}{\sqrt[4]{x^2+x^6}\left(-1+x^8\right)}\,dx$

Optimal. Leaf size=187

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{8\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{8\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{8\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{8\,2^{3/4}}+\frac{\left(x^6+x^2\right)^{3/4}}{2x\left(x^4+1\right)}$$

Rubi [C] time = 0.15, antiderivative size = 46, normalized size of antiderivative = 0.25, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2056, 1479, 466, 510}

$$-\frac{2x^5\sqrt[4]{x^4+1}F_1\left(\frac{9}{8};1,\frac{5}{4};\frac{17}{8};x^4,-x^4\right)}{9\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^4/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] (-2*x^5*(1 + x^4)^(1/4)*AppellF1[9/8, 1, 5/4, 17/8, x^4, -x^4])/(9*(x^2 + x
^6)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1479

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n
2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*(a/d + (c*x^n)/e)^
p, x] /; FreeQ[{a, c, d, e, f, q, m, n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2
 + a*e^2, 0] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{x^2+x^6}\left(-1+x^8\right)}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{x^{7/2}}{\sqrt[4]{1+x^4}\left(-1+x^8\right)}\,dx}{\sqrt[4]{x^2+x^6}}$$
$$= \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{x^{7/2}}{\left(-1+x^4\right)\left(1+x^4\right)^{5/4}}\,dx}{\sqrt[4]{x^2+x^6}}$$
$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{x^8}{\left(-1+x^8\right)\left(1+x^8\right)^{5/4}}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$
$$= -\frac{2x^5\sqrt[4]{1+x^4}\,F_1\left(\frac{9}{8};1,\frac{5}{4};\frac{17}{8};x^4,-x^4\right)}{9\sqrt[4]{x^2+x^6}}$$

Mathematica [C] time = 1.13, size = 48, normalized size = 0.26

$$\frac{x-x\sqrt[4]{x^4+1}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};-x^4,x^4\right)}{2\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] (x - x*(1 + x^4)^(1/4)*AppellF1[1/8, -3/4, 1, 9/8, -x^4, x^4])/(2*(x^2 + x^
6)^(1/4))
```

IntegrateAlgebraic [A] time = 0.69, size = 187, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{8\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{8\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{8\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{8\,2^{3/4}}+\frac{\left(x^6+x^2\right)^{3/4}}{2x\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] (x^2 + x^6)^(3/4)/(2*x*(1 + x^4)) - ArcTan[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/(
8*2^(1/4)) + ArcTan[(2^(3/4)*x*(x^2 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^2 +
 x^6])]/(8*2^(3/4)) - ArcTanh[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/(8*2^(1/4)) -
ArcTanh[(x^2/2^(1/4) + Sqrt[x^2 + x^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))]/(8*2
^(3/4))
```

fricas [B] time = 6.42, size = 765, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="fricas")
```

```
[Out] 1/64*(4*2^(3/4)*(x^5 + x)*arctan(1/2*2^(3/4)*(x^6 + x^2)^(1/4)*(x^4 + 1)/(x
^5 + x)) - 2^(3/4)*(x^5 + x)*log((4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 2*2^(3/
4)*(x^6 + x^2)^(3/4) + sqrt(2)*(x^5 + 2*x^3 + x) + 4*sqrt(x^6 + x^2)*x)/(x^
5 - 2*x^3 + x)) + 2^(3/4)*(x^5 + x)*log(-(4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 +
 2*2^(3/4)*(x^6 + x^2)^(3/4) - sqrt(2)*(x^5 + 2*x^3 + x) - 4*sqrt(x^6 + x^2
)*x)/(x^5 - 2*x^3 + x)) - 4*2^(1/4)*(x^5 + x)*arctan(1/2*(4*2^(1/4)*(x^6 +
```

```
x^2)^(1/4)*x^2 + sqrt(2)*(2*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 +
2*x^3 + x) - 4*sqrt(x^6 + x^2)*x + 2*2^(1/4)*(x^6 + x^2)^(3/4))*sqrt((x^5 +
 2*x^3 + 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x + 2*
2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x)) + 2*2^(3/4)*(x^6 + x^2)^(
3/4))/(x^5 - 2*x^3 + x)) - 4*2^(1/4)*(x^5 + x)*arctan(1/2*(4*2^(1/4)*(x^6 +
 x^2)^(1/4)*x^2 + sqrt(2)*(2*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 +
 2*x^3 + x) + 4*sqrt(x^6 + x^2)*x + 2*2^(1/4)*(x^6 + x^2)^(3/4))*sqrt((x^5
+ 2*x^3 - 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x - 2
*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x)) + 2*2^(3/4)*(x^6 + x^2)^
(3/4))/(x^5 - 2*x^3 + x)) - 2^(1/4)*(x^5 + x)*log(2*(x^5 + 2*x^3 + 4*2^(1/4
)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x + 2*2^(3/4)*(x^6 + x^
2)^(3/4) + x)/(x^5 + 2*x^3 + x)) + 2^(1/4)*(x^5 + x)*log(2*(x^5 + 2*x^3 - 4
*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x - 2*2^(3/4)*(x
^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x)) + 32*(x^6 + x^2)^(3/4))/(x^5 + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(x^8-1)(x^6+x^2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="giac")
```

```
[Out] integrate(x^4/((x^8 - 1)*(x^6 + x^2)^(1/4)), x)
```

maple [C] time = 18.70, size = 651, normalized size = 3.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6+x^2)^(1/4)/(x^8-1),x)
```

```
[Out] 1/2*x/(x^2*(x^4+1))^(1/4)-1/32*RootOf(_Z^4+8)*ln(-(-RootOf(_Z^4+8)^3*(x^6+x
^2)^(1/2)*x+RootOf(_Z^4+8)*x^5-2*RootOf(_Z^4+8)^2*(x^6+x^2)^(1/4)*x^2-2*Roo
tOf(_Z^4+8)*x^3+4*(x^6+x^2)^(3/4)+RootOf(_Z^4+8)*x)/x/(x^2+1)^2)+1/32*RootO
f(_Z^2+RootOf(_Z^4+8)^2)*ln(-(-RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)
^2*(x^6+x^2)^(1/2)*x-RootOf(_Z^2+RootOf(_Z^4+8)^2)*x^5+2*RootOf(_Z^4+8)^2*(
x^6+x^2)^(1/4)*x^2+2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*x^3+4*(x^6+x^2)^(3/4)-Ro
otOf(_Z^2+RootOf(_Z^4+8)^2)*x)/x/(x^2+1)^2)+1/64*ln((RootOf(_Z^4+8)^2*x^2+2
*RootOf(_Z^4+8)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))/(1+x)/x/(-1+x))*RootOf
(_Z^4+8)^3+1/64*ln((RootOf(_Z^4+8)^2*x^2+2*RootOf(_Z^4+8)*(x^6+x^2)^(1/4)*x
+2*(x^6+x^2)^(1/2))/(1+x)/x/(-1+x))*RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z^
4+8)^2)-1/64*RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln((-RootOf(_Z^
4+8)^3*x^5+RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^5-2*RootOf(_Z^4
+8)^3*x^3+2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^3+8*RootOf(_Z^
4+8)*(x^6+x^2)^(1/4)*RootOf(_Z^2+RootOf(_Z^4+8)^2)*x^2+8*(x^6+x^2)^(1/2)*Ro
otOf(_Z^4+8)*x+8*RootOf(_Z^2+RootOf(_Z^4+8)^2)*(x^6+x^2)^(1/2)*x-RootOf(_Z^
4+8)^3*x+RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x+16*(x^6+x^2)^(3/4
))/(1+x)^2/(-1+x)^2/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(x^8-1)(x^6+x^2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((x^8 - 1)*(x^6 + x^2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(x^6+x^2\right)^{1/4}\,\left(x^8-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((x^2 + x^6)^(1/4)*(x^8 - 1)),x)
```

```
[Out] int(x^4/((x^2 + x^6)^(1/4)*(x^8 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt[4]{x^2\left(x^4+1\right)}\,(x-1)\,(x+1)\left(x^2+1\right)\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6+x**2)**(1/4)/(x**8-1),x)
```

```
[Out] Integral(x**4/((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)*(x**4 +
1)), x)
```

3.1931 $\int \frac{x^4}{\sqrt[4]{x^2+x^6}\left(-1+x^8\right)}\,dx$

Optimal. Leaf size=187

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^6+x^2}}\right)}{8\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}\,x^2-\sqrt{x^6+x^2}}\right)}{8\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^6+x^2}}\right)}{8\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{8\,2^{3/4}}+\frac{\left(x^6+x^2\right)^{3/4}}{2x\left(x^4+1\right)}$$

Rubi [C] time = 0.15, antiderivative size = 46, normalized size of antiderivative = 0.25, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2056, 1479, 466, 510}

$$-\frac{2x^5\sqrt[4]{x^4+1}\,F_1\left(\frac{9}{8};1,\frac{5}{4};\frac{17}{8};x^4,-x^4\right)}{9\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^4/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] (-2*x^5*(1 + x^4)^(1/4)*AppellF1[9/8, 1, 5/4, 17/8, x^4, -x^4])/(9*(x^2 + x
^6)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1479

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n
2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*(a/d + (c*x^n)/e)^
p, x] /; FreeQ[{a, c, d, e, f, q, m, n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2
 + a*e^2, 0] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{x^2+x^6}\left(-1+x^8\right)}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{x^{7/2}}{\sqrt[4]{1+x^4}\left(-1+x^8\right)}\,dx}{\sqrt[4]{x^2+x^6}}$$
$$= \frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{x^{7/2}}{\left(-1+x^4\right)\left(1+x^4\right)^{5/4}}\,dx}{\sqrt[4]{x^2+x^6}}$$
$$= \frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{x^8}{\left(-1+x^8\right)\left(1+x^8\right)^{5/4}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$
$$= -\frac{2x^5\sqrt[4]{1+x^4}\,F_1\left(\frac{9}{8};1,\frac{5}{4};\frac{17}{8};x^4,-x^4\right)}{9\sqrt[4]{x^2+x^6}}$$

Mathematica [C] time = 0.89, size = 48, normalized size = 0.26

$$\frac{x - x\sqrt[4]{x^4+1}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};-x^4,x^4\right)}{2\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] (x - x*(1 + x^4)^(1/4)*AppellF1[1/8, -3/4, 1, 9/8, -x^4, x^4])/(2*(x^2 + x^
6)^(1/4))
```

IntegrateAlgebraic [A] time = 0.00, size = 187, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{8\sqrt[4]{2}} + \frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{8\,2^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{8\sqrt[4]{2}} - \frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{8\,2^{3/4}} + \frac{\left(x^6+x^2\right)^{3/4}}{2x\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] (x^2 + x^6)^(3/4)/(2*x*(1 + x^4)) - ArcTan[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/(
8*2^(1/4)) + ArcTan[(2^(3/4)*x*(x^2 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^2 +
 x^6])]/(8*2^(3/4)) - ArcTanh[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/(8*2^(1/4)) -
ArcTanh[(x^2/2^(1/4) + Sqrt[x^2 + x^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))]/(8*2
^(3/4))
```

fricas [B] time = 6.42, size = 765, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="fricas")
```

```
[Out] 1/64*(4*2^(3/4)*(x^5 + x)*arctan(1/2*2^(3/4)*(x^6 + x^2)^(1/4)*(x^4 + 1)/(x
^5 + x)) - 2^(3/4)*(x^5 + x)*log((4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 2*2^(3/
4)*(x^6 + x^2)^(3/4) + sqrt(2)*(x^5 + 2*x^3 + x) + 4*sqrt(x^6 + x^2)*x)/(x^
5 - 2*x^3 + x)) + 2^(3/4)*(x^5 + x)*log(-(4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 +
 2*2^(3/4)*(x^6 + x^2)^(3/4) - sqrt(2)*(x^5 + 2*x^3 + x) - 4*sqrt(x^6 + x^2
)*x)/(x^5 - 2*x^3 + x)) - 4*2^(1/4)*(x^5 + x)*arctan(1/2*(4*2^(1/4)*(x^6 +
```

```
x^2)^(1/4)*x^2 + sqrt(2)*(2*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 +
2*x^3 + x) - 4*sqrt(x^6 + x^2)*x + 2*2^(1/4)*(x^6 + x^2)^(3/4))*sqrt((x^5 +
 2*x^3 + 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x + 2*
2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x)) + 2*2^(3/4)*(x^6 + x^2)^(
3/4))/(x^5 - 2*x^3 + x)) - 4*2^(1/4)*(x^5 + x)*arctan(1/2*(4*2^(1/4)*(x^6 +
 x^2)^(1/4)*x^2 + sqrt(2)*(2*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 +
 2*x^3 + x) + 4*sqrt(x^6 + x^2)*x + 2*2^(1/4)*(x^6 + x^2)^(3/4))*sqrt((x^5
+ 2*x^3 - 4*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x - 2
*2^(3/4)*(x^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x)) + 2*2^(3/4)*(x^6 + x^2)^
(3/4))/(x^5 - 2*x^3 + x)) - 2^(1/4)*(x^5 + x)*log(2*(x^5 + 2*x^3 + 4*2^(1/4
)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x + 2*2^(3/4)*(x^6 + x^
2)^(3/4) + x)/(x^5 + 2*x^3 + x)) + 2^(1/4)*(x^5 + x)*log(2*(x^5 + 2*x^3 - 4
*2^(1/4)*(x^6 + x^2)^(1/4)*x^2 + 4*sqrt(2)*sqrt(x^6 + x^2)*x - 2*2^(3/4)*(x
^6 + x^2)^(3/4) + x)/(x^5 + 2*x^3 + x)) + 32*(x^6 + x^2)^(3/4))/(x^5 + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^8-1\right)\left(x^6+x^2\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="giac")
```

```
[Out] integrate(x^4/((x^8 - 1)*(x^6 + x^2)^(1/4)), x)
```

maple [C] time = 19.63, size = 648, normalized size = 3.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6+x^2)^(1/4)/(x^8-1),x)
```

```
[Out] 1/2*x/(x^2*(x^4+1))^(1/4)+1/32*RootOf(_Z^4-8)*ln(-(-RootOf(_Z^4-8)^3*(x^6+x
^2)^(1/2)*x-RootOf(_Z^4-8)*x^5+2*(x^6+x^2)^(1/4)*RootOf(_Z^4-8)^2*x^2-2*Roo
tOf(_Z^4-8)*x^3+4*(x^6+x^2)^(3/4)-RootOf(_Z^4-8)*x)/(1+x)^2/(-1+x)^2/x)-1/3
2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln((-RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_
Z^4-8)^2*(x^6+x^2)^(1/2)*x+RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^5-2*(x^6+x^2)^(1
/4)*RootOf(_Z^4-8)^2*x^2+2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^3+4*(x^6+x^2)^(3
/4)+RootOf(_Z^2+RootOf(_Z^4-8)^2)*x)/(1+x)^2/(-1+x)^2/x)-1/64*ln((RootOf(_Z
^4-8)^2*x^2+2*RootOf(_Z^4-8)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))/x/(x^2+1)
)*RootOf(_Z^4-8)^3-1/64*ln((RootOf(_Z^4-8)^2*x^2+2*RootOf(_Z^4-8)*(x^6+x^2)
^(1/4)*x+2*(x^6+x^2)^(1/2))/x/(x^2+1))*RootOf(_Z^4-8)^2*RootOf(_Z^2+RootOf(
_Z^4-8)^2)+1/64*RootOf(_Z^4-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln(-(-RootOf
(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x^5+RootOf(_Z^4-8)^3*x^5+2*RootOf(
_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x^3-2*RootOf(_Z^4-8)^3*x^3+8*RootOf
(_Z^2+RootOf(_Z^4-8)^2)*(x^6+x^2)^(1/4)*RootOf(_Z^4-8)*x^2+8*RootOf(_Z^2+Ro
otOf(_Z^4-8)^2)*(x^6+x^2)^(1/2)*x+8*(x^6+x^2)^(1/2)*RootOf(_Z^4-8)*x-RootOf
(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x+RootOf(_Z^4-8)^3*x+16*(x^6+x^2)^
(3/4))/x/(x^2+1)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^8-1\right)\left(x^6+x^2\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((x^8 - 1)*(x^6 + x^2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(x^6+x^2\right)^{1/4}\,\left(x^8-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((x^2 + x^6)^(1/4)*(x^8 - 1)),x)
```

```
[Out] int(x^4/((x^2 + x^6)^(1/4)*(x^8 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt[4]{x^2\left(x^4+1\right)}\,(x-1)\,(x+1)\left(x^2+1\right)\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6+x**2)**(1/4)/(x**8-1),x)
```

```
[Out] Integral(x**4/((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)*(x**4 +
1)), x)
```

3.1932 $\int \frac{(-1+x^4)\sqrt[4]{x^2+x^6}}{1-x^4+x^8}\,dx$

Optimal. Leaf size=187

$$\frac{\tan^{-1}\left(\frac{\sqrt[8]{3}x}{\sqrt[4]{x^6+x^2}}\right)}{2\,3^{3/8}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,3^{7/8}x\sqrt[4]{x^6+x^2}}{3^{3/4}\sqrt{x^6+x^2}-3x^2}\right)}{2\sqrt{2}\,3^{3/8}}-\frac{\tanh^{-1}\left(\frac{\sqrt[8]{3}x}{\sqrt[4]{x^6+x^2}}\right)}{2\,3^{3/8}}+\frac{\tanh^{-1}\left(\frac{\frac{\sqrt[4]{3}x^2}{\sqrt{2}}+\frac{\sqrt{x^6+x^2}}{\sqrt{2}\sqrt[8]{3}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\sqrt{2}\,3^{3/8}}$$

Rubi [C] time = 0.51, antiderivative size = 163, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2056, 6728, 466, 510}

$$-\frac{2\left(\sqrt{3}+3i\right)x\sqrt[4]{x^6+x^2}\,F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};-x^4,\frac{2x^4}{1-i\sqrt{3}}\right)}{9\left(\sqrt{3}+i\right)\sqrt[4]{x^4+1}}-\frac{2\left(-\sqrt{3}+3i\right)x\sqrt[4]{x^6+x^2}\,F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};-x^4,\frac{2x^4}{1+i\sqrt{3}}\right)}{9\left(-\sqrt{3}+i\right)\sqrt[4]{x^4+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 - x^4 + x^8),x]
```

```
[Out] (-2*(3*I + Sqrt[3])*x*(x^2 + x^6)^(1/4)*AppellF1[3/8, -1/4, 1, 11/8, -x^4,
(2*x^4)/(1 - I*Sqrt[3])])/(9*(I + Sqrt[3])*(1 + x^4)^(1/4)) - (2*(3*I - Sqr
t[3])*x*(x^2 + x^6)^(1/4)*AppellF1[3/8, -1/4, 1, 11/8, -x^4, (2*x^4)/(1 + I
*Sqrt[3])])/(9*(I - Sqrt[3])*(1 + x^4)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(-1+x^4)\sqrt[4]{x^2+x^6}}{1-x^4+x^8}\,dx = \frac{\sqrt[4]{x^2+x^6}\int \frac{\sqrt{x}(-1+x^4)\sqrt[4]{1+x^4}}{1-x^4+x^8}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\sqrt[4]{x^2+x^6}\int \left(\frac{\left(1+\frac{i}{\sqrt{3}}\right)\sqrt{x}\sqrt[4]{1+x^4}}{-1-i\sqrt{3}+2x^4} + \frac{\left(1-\frac{i}{\sqrt{3}}\right)\sqrt{x}\sqrt[4]{1+x^4}}{-1+i\sqrt{3}+2x^4}\right)dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\left(\left(3-i\sqrt{3}\right)\sqrt[4]{x^2+x^6}\right)\int \frac{\sqrt{x}\sqrt[4]{1+x^4}}{-1+i\sqrt{3}+2x^4}\,dx}{3\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(\left(3+i\sqrt{3}\right)\sqrt[4]{x^2+x^6}\right)\int \frac{\sqrt{x}\sqrt[4]{1+x^4}}{-1-i\sqrt{3}+2x^4}\,dx}{3\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\left(2\left(3-i\sqrt{3}\right)\sqrt[4]{x^2+x^6}\right)\text{Subst}\left(\int \frac{x^2\sqrt[4]{1+x^8}}{-1+i\sqrt{3}+2x^8}\,dx, x, \sqrt{x}\right)}{3\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(2\left(3+i\sqrt{3}\right)\sqrt[4]{x^2+x\right.}{}$$

$$= -\frac{2\left(3i+\sqrt{3}\right)x\sqrt[4]{x^2+x^6}\,F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};-x^4,\frac{2x^4}{1-i\sqrt{3}}\right)}{9\left(i+\sqrt{3}\right)\sqrt[4]{1+x^4}} - \frac{2\left(3i-\sqrt{3}\right)x\sqrt[4]{x^2+x^6}\,F_1}{9\left(i-\sqrt{3}\right.}$$

Mathematica [F] time = 3.86, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^4)\sqrt[4]{x^2+x^6}}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 - x^4 + x^8),x]
[Out] Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 - x^4 + x^8), x]
```

IntegrateAlgebraic [A] time = 0.97, size = 187, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[8]{3}x}{\sqrt[4]{x^6+x^2}}\right)}{2\,3^{3/8}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\,3^{7/8}x\sqrt[4]{x^6+x^2}}{3^{3/4}\sqrt{x^6+x^2}-3x^2}\right)}{2\sqrt{2}\,3^{3/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{3}x}{\sqrt[4]{x^6+x^2}}\right)}{2\,3^{3/8}} + \frac{\tanh^{-1}\left(\frac{\frac{\sqrt[8]{3}x^2}{\sqrt{2}}+\frac{\sqrt{x^6+x^2}}{\sqrt{2}\sqrt[8]{3}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\sqrt{2}\,3^{3/8}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 - x^4 + x^8),x]
[Out] ArcTan[(3^(1/8)*x)/(x^2 + x^6)^(1/4)]/(2*3^(3/8)) - ArcTan[(Sqrt[2]*3^(7/8)
*x*(x^2 + x^6)^(1/4))/(-3*x^2 + 3^(3/4)*Sqrt[x^2 + x^6])]/(2*Sqrt[2]*3^(3/8
)) - ArcTanh[(3^(1/8)*x)/(x^2 + x^6)^(1/4)]/(2*3^(3/8)) + ArcTanh[((3^(1/8)
*x^2)/Sqrt[2] + Sqrt[x^2 + x^6]/(Sqrt[2]*3^(1/8)))/(x*(x^2 + x^6)^(1/4))]/(
2*Sqrt[2]*3^(3/8))
```

fricas [B] time = 18.30, size = 1348, normalized size = 7.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8-x^4+1),x, algorithm="fricas")
[Out] -1/108*27^(7/8)*sqrt(2)*arctan(1/81*(sqrt(3)*(27^(3/4)*(x^9 + 5*x^5 + x) +
3*(x^6 + x^2)^(3/4)*(27^(5/8)*sqrt(2)*x^2 + 3*27^(1/8)*sqrt(2)*(x^4 + 1)) +
 18*sqrt(x^6 + x^2)*(3*x^3 + sqrt(3)*(x^5 + x)) + 18*27^(1/4)*(x^7 + x^3) +
```

```
(x^6 + x^2)^(1/4)*(9*27^(3/8)*sqrt(2)*x^4 + 27^(7/8)*sqrt(2)*(x^6 + x^2)))
*sqrt((2*(x^6 + x^2)^(3/4)*(9*27^(3/8)*sqrt(2)*x^2 - 27^(7/8)*sqrt(2)*(x^4
+ 1)) + 9*sqrt(3)*(x^9 - x^5 + x) - 12*sqrt(x^6 + x^2)*(27^(3/4)*x^3 - 3*27
^(1/4)*(x^5 + x)) + 6*(x^6 + x^2)^(1/4)*(9*27^(1/8)*sqrt(2)*x^4 - 27^(5/8)*
sqrt(2)*(x^6 + x^2)))/(x^9 - x^5 + x)) + 9*(x^6 + x^2)^(3/4)*(27^(7/8)*sqrt
(2)*x^2 + 3*27^(3/8)*sqrt(2)*(x^4 + 1)) + 27*(x^6 + x^2)^(1/4)*(27^(5/8)*sq
rt(2)*x^4 + 3*27^(1/8)*sqrt(2)*(x^6 + x^2)))/(x^9 - x^5 + x)) - 1/108*27^(7
/8)*sqrt(2)*arctan(-1/81*(sqrt(3)*(27^(3/4)*(x^9 + 5*x^5 + x) - 3*(x^6 + x^
2)^(3/4)*(27^(5/8)*sqrt(2)*x^2 + 3*27^(1/8)*sqrt(2)*(x^4 + 1)) + 18*sqrt(x^
6 + x^2)*(3*x^3 + sqrt(3)*(x^5 + x)) + 18*27^(1/4)*(x^7 + x^3) - (x^6 + x^2
)^(1/4)*(9*27^(3/8)*sqrt(2)*x^4 + 27^(7/8)*sqrt(2)*(x^6 + x^2)))*sqrt(-(2*(
x^6 + x^2)^(3/4)*(9*27^(3/8)*sqrt(2)*x^2 - 27^(7/8)*sqrt(2)*(x^4 + 1)) - 9*
sqrt(3)*(x^9 - x^5 + x) + 12*sqrt(x^6 + x^2)*(27^(3/4)*x^3 - 3*27^(1/4)*(x^
5 + x)) + 6*(x^6 + x^2)^(1/4)*(9*27^(1/8)*sqrt(2)*x^4 - 27^(5/8)*sqrt(2)*(x
^6 + x^2)))/(x^9 - x^5 + x)) - 9*(x^6 + x^2)^(3/4)*(27^(7/8)*sqrt(2)*x^2 +
3*27^(3/8)*sqrt(2)*(x^4 + 1)) - 27*(x^6 + x^2)^(1/4)*(27^(5/8)*sqrt(2)*x^4
+ 3*27^(1/8)*sqrt(2)*(x^6 + x^2)))/(x^9 - x^5 + x)) - 1/432*27^(7/8)*sqrt(2
)*log(3*(2*(x^6 + x^2)^(3/4)*(9*27^(3/8)*sqrt(2)*x^2 - 27^(7/8)*sqrt(2)*(x^
4 + 1)) + 9*sqrt(3)*(x^9 - x^5 + x) - 12*sqrt(x^6 + x^2)*(27^(3/4)*x^3 - 3*
27^(1/4)*(x^5 + x)) + 6*(x^6 + x^2)^(1/4)*(9*27^(1/8)*sqrt(2)*x^4 - 27^(5/8
)*sqrt(2)*(x^6 + x^2)))/(x^9 - x^5 + x)) + 1/432*27^(7/8)*sqrt(2)*log(-3*(2
*(x^6 + x^2)^(3/4)*(9*27^(3/8)*sqrt(2)*x^2 - 27^(7/8)*sqrt(2)*(x^4 + 1)) -
9*sqrt(3)*(x^9 - x^5 + x) + 12*sqrt(x^6 + x^2)*(27^(3/4)*x^3 - 3*27^(1/4)*(
x^5 + x)) + 6*(x^6 + x^2)^(1/4)*(9*27^(1/8)*sqrt(2)*x^4 - 27^(5/8)*sqrt(2)*
(x^6 + x^2)))/(x^9 - x^5 + x)) - 1/54*27^(7/8)*arctan(1/18*(27^(5/8)*(x^6 +
 x^2)^(1/4)*(x^4 + 1) + 3^(3/4)*(27^(3/8)*(x^6 + x^2)^(1/4)*(x^4 + 1) + 3*2
7^(1/8)*(x^6 + x^2)^(3/4)) - 3*27^(3/8)*(x^6 + x^2)^(3/4))/(x^5 + x)) - 1/2
16*27^(7/8)*log((2*27^(3/4)*(x^7 + x^3) + 2*(x^6 + x^2)^(3/4)*(9*27^(1/8)*x
^2 + 27^(5/8)*(x^4 + 1)) + 18*sqrt(x^6 + x^2)*(x^5 + sqrt(3)*x^3 + x) + 3*2
7^(1/4)*(x^9 + 5*x^5 + x) + 2*(x^6 + x^2)^(1/4)*(27^(7/8)*x^4 + 3*27^(3/8)*
(x^6 + x^2)))/(x^9 - x^5 + x)) + 1/216*27^(7/8)*log((2*27^(3/4)*(x^7 + x^3)
 - 2*(x^6 + x^2)^(3/4)*(9*27^(1/8)*x^2 + 27^(5/8)*(x^4 + 1)) + 18*sqrt(x^6
+ x^2)*(x^5 + sqrt(3)*x^3 + x) + 3*27^(1/4)*(x^9 + 5*x^5 + x) - 2*(x^6 + x^
2)^(1/4)*(27^(7/8)*x^4 + 3*27^(3/8)*(x^6 + x^2)))/(x^9 - x^5 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^2)^(1/4)*(x^4 - 1)/(x^8 - x^4 + 1), x)
```

maple [C] time = 74.33, size = 1509, normalized size = 8.07

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^6+x^2)^(1/4)/(x^8-x^4+1),x)
```

```
[Out] 1/12*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*ln(-(Roo
tOf(_Z^2+RootOf(_Z^8-243)^2)*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootO
f(_Z^8-243)^2))*RootOf(_Z^8-243)^9*x^5-2*RootOf(_Z^2+RootOf(_Z^8-243)^2)*Ro
otOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-243
)^9*x^3+RootOf(_Z^2+RootOf(_Z^8-243)^2)*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf
(_Z^2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-243)^9*x+9*RootOf(_Z^2+RootOf(_Z^8-2
43)^2)*RootOf(_Z^8-243)^5*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_
```

```
Z^8-243)^2))*x^3+54*(x^6+x^2)^(1/2)*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^
2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-243)^4*x-324*RootOf(_Z^2+RootOf(_Z^8-243
)^2)*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*RootOf(_
Z^8-243)*x^5+486*RootOf(_Z^2+RootOf(_Z^8-243)^2)*RootOf(_Z^2-RootOf(_Z^8-24
3)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-243)*x^3+486*RootOf(_Z^8-24
3)*(x^6+x^2)^(1/4)*RootOf(_Z^2+RootOf(_Z^8-243)^2)*x^2-324*RootOf(_Z^2+Root
Of(_Z^8-243)^2)*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2
))*RootOf(_Z^8-243)*x+1458*(x^6+x^2)^(3/4))/(RootOf(_Z^8-243)^4*x^4-2*RootO
f(_Z^8-243)^4*x^2+RootOf(_Z^8-243)^4-18*x^4+27*x^2-18)/x)+1/12*RootOf(_Z^2+
RootOf(_Z^8-243)^2)*ln((RootOf(_Z^8-243)^10*RootOf(_Z^2+RootOf(_Z^8-243)^2)
*x^5-2*RootOf(_Z^8-243)^10*RootOf(_Z^2+RootOf(_Z^8-243)^2)*x^3+RootOf(_Z^8-
243)^10*RootOf(_Z^2+RootOf(_Z^8-243)^2)*x-9*RootOf(_Z^8-243)^6*RootOf(_Z^2+
RootOf(_Z^8-243)^2)*x^3+54*RootOf(_Z^2+RootOf(_Z^8-243)^2)*(x^6+x^2)^(1/2)*
RootOf(_Z^8-243)^4*x-324*RootOf(_Z^2+RootOf(_Z^8-243)^2)*RootOf(_Z^8-243)^2
*x^5+486*RootOf(_Z^2+RootOf(_Z^8-243)^2)*RootOf(_Z^8-243)^2*x^3+486*RootOf(
_Z^8-243)^2*(x^6+x^2)^(1/4)*x^2-324*RootOf(_Z^2+RootOf(_Z^8-243)^2)*RootOf(
_Z^8-243)^2*x-1458*(x^6+x^2)^(3/4))/(RootOf(_Z^8-243)^4*x^4-2*RootOf(_Z^8-2
43)^4*x^2+RootOf(_Z^8-243)^4+18*x^4-27*x^2+18)/x)+1/2916*RootOf(_Z^8-243)^7
*RootOf(_Z^2+RootOf(_Z^8-243)^2)*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+R
ootOf(_Z^8-243)^2))*ln(-(RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z
^8-243)^2))*RootOf(_Z^8-243)^10*x^5-2*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_
Z^2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-243)^10*x^3+RootOf(_Z^2-RootOf(_Z^8-24
3)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-243)^10*x+9*RootOf(_Z^2-Roo
tOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-243)^6*x^3+54*(x
^6+x^2)^(1/2)*RootOf(_Z^2+RootOf(_Z^8-243)^2)*RootOf(_Z^2-RootOf(_Z^8-243)*
RootOf(_Z^2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-243)^3*x-324*RootOf(_Z^2-RootO
f(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-243)^2*x^5+486*Roo
tOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-243)
^2*x^3-486*RootOf(_Z^8-243)*(x^6+x^2)^(1/4)*RootOf(_Z^2+RootOf(_Z^8-243)^2)
*x^2-324*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*Root
Of(_Z^8-243)^2*x+1458*(x^6+x^2)^(3/4))/(RootOf(_Z^8-243)^4*x^4-2*RootOf(_Z^
8-243)^4*x^2+RootOf(_Z^8-243)^4-18*x^4+27*x^2-18)/x)+1/12*RootOf(_Z^8-243)*
ln(-(RootOf(_Z^8-243)^11*x^5-2*RootOf(_Z^8-243)^11*x^3+RootOf(_Z^8-243)^11*
x-9*RootOf(_Z^8-243)^7*x^3-54*RootOf(_Z^8-243)^5*(x^6+x^2)^(1/2)*x-324*Root
Of(_Z^8-243)^3*x^5+486*RootOf(_Z^8-243)^3*x^3+486*RootOf(_Z^8-243)^2*(x^6+x
^2)^(1/4)*x^2-324*RootOf(_Z^8-243)^3*x+1458*(x^6+x^2)^(3/4))/(RootOf(_Z^8-2
43)^4*x^4-2*RootOf(_Z^8-243)^4*x^2+RootOf(_Z^8-243)^4+18*x^4-27*x^2+18)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8-x^4+1),x, algorithm="maxima")
[Out] integrate((x^6 + x^2)^(1/4)*(x^4 - 1)/(x^8 - x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6+x^2\right)^{1/4}\,\left(x^4-1\right)}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + x^6)^(1/4)*(x^4 - 1))/(x^8 - x^4 + 1),x)
[Out] int(((x^2 + x^6)^(1/4)*(x^4 - 1))/(x^8 - x^4 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2 \left(x^4+1\right)}\, (x-1)\, (x+1) \left(x^2+1\right)}{x^8-x^4+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**6+x**2)**(1/4)/(x**8-x**4+1),x)
```

```
[Out] Integral((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)/(x**8 - x**4 +
 1), x)
```

3.1933 $\int \frac{\left(-1+x^4\right)\sqrt[4]{x^2+x^6}}{1-x^4+x^8}\,dx$

Optimal. Leaf size=187

$$\frac{\tan^{-1}\left(\frac{\sqrt[8]{3}\,x}{\sqrt[4]{x^6+x^2}}\right)}{2\,3^{3/8}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,3^{7/8}x\sqrt[4]{x^6+x^2}}{3^{3/4}\sqrt{x^6+x^2}-3x^2}\right)}{2\sqrt{2}\,3^{3/8}}-\frac{\tanh^{-1}\left(\frac{\sqrt[8]{3}\,x}{\sqrt[4]{x^6+x^2}}\right)}{2\,3^{3/8}}+\frac{\tanh^{-1}\left(\frac{\frac{\sqrt[8]{3}\,x^2}{\sqrt{2}}+\frac{\sqrt{x^6+x^2}}{\sqrt{2}\sqrt[8]{3}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\sqrt{2}\,3^{3/8}}$$

Rubi [C] time = 0.44, antiderivative size = 163, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2056, 6728, 466, 510}

$$-\frac{2\left(\sqrt{3}+3i\right)x\sqrt[4]{x^6+x^2}\,F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};-x^4,\frac{2x^4}{1-i\sqrt{3}}\right)}{9\left(\sqrt{3}+i\right)\sqrt[4]{x^4+1}}-\frac{2\left(-\sqrt{3}+3i\right)x\sqrt[4]{x^6+x^2}\,F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};-x^4,\frac{2x^4}{1+i\sqrt{3}}\right)}{9\left(-\sqrt{3}+i\right)\sqrt[4]{x^4+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 - x^4 + x^8),x]
```

```
[Out] (-2*(3*I + Sqrt[3])*x*(x^2 + x^6)^(1/4)*AppellF1[3/8, -1/4, 1, 11/8, -x^4,
(2*x^4)/(1 - I*Sqrt[3])])/(9*(I + Sqrt[3])*(1 + x^4)^(1/4)) - (2*(3*I - Sqr
t[3])*x*(x^2 + x^6)^(1/4)*AppellF1[3/8, -1/4, 1, 11/8, -x^4, (2*x^4)/(1 + I
*Sqrt[3])])/(9*(I - Sqrt[3])*(1 + x^4)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(-1+x^4)\sqrt[4]{x^2+x^6}}{1-x^4+x^8}\,dx = \frac{\sqrt[4]{x^2+x^6}\int \frac{\sqrt{x}(-1+x^4)\sqrt[4]{1+x^4}}{1-x^4+x^8}\,dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\sqrt[4]{x^2+x^6}\int \left(\frac{\left(1+\frac{i}{\sqrt{3}}\right)\sqrt{x}\sqrt[4]{1+x^4}}{-1-i\sqrt{3}+2x^4} + \frac{\left(1-\frac{i}{\sqrt{3}}\right)\sqrt{x}\sqrt[4]{1+x^4}}{-1+i\sqrt{3}+2x^4}\right)dx}{\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\left((3-i\sqrt{3})\sqrt[4]{x^2+x^6}\right)\int \frac{\sqrt{x}\sqrt[4]{1+x^4}}{-1+i\sqrt{3}+2x^4}\,dx}{3\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left((3+i\sqrt{3})\sqrt[4]{x^2+x^6}\right)\int \frac{\sqrt{x}\sqrt[4]{1+x^4}}{-1-i\sqrt{3}+2x^4}\,dx}{3\sqrt{x}\sqrt[4]{1+x^4}}$$

$$= \frac{\left(2(3-i\sqrt{3})\sqrt[4]{x^2+x^6}\right)\text{Subst}\left(\int \frac{x^2\sqrt[4]{1+x^8}}{-1+i\sqrt{3}+2x^8}\,dx, x, \sqrt{x}\right)}{3\sqrt{x}\sqrt[4]{1+x^4}} + \frac{\left(2(3+i\sqrt{3})\sqrt[4]{x^2+x^6}\right)}{3\sqrt{}}$$

$$= -\frac{2(3i+\sqrt{3})x\sqrt[4]{x^2+x^6}F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};-x^4,\frac{2x^4}{1-i\sqrt{3}}\right)}{9(i+\sqrt{3})\sqrt[4]{1+x^4}} - \frac{2(3i-\sqrt{3})x\sqrt[4]{x^2+x^6}F_1\left(\frac{3}{8}\right.}{9(i-\sqrt{3})\sqrt[4]{}}$$

Mathematica [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^4)\sqrt[4]{x^2+x^6}}{1-x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 - x^4 + x^8),x]
```

```
[Out] Integrate[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 - x^4 + x^8), x]
```

IntegrateAlgebraic [A] time = 0.00, size = 187, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[8]{3}x}{\sqrt[4]{x^6+x^2}}\right)}{2\,3^{3/8}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\,3^{7/8}x\sqrt[4]{x^6+x^2}}{3^{3/4}\sqrt{x^6+x^2}-3x^2}\right)}{2\sqrt{2}\,3^{3/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{3}x}{\sqrt[4]{x^6+x^2}}\right)}{2\,3^{3/8}} + \frac{\tanh^{-1}\left(\frac{\frac{\sqrt[8]{3}x^2}{\sqrt{2}}+\frac{\sqrt{x^6+x^2}}{\sqrt{2}\sqrt[8]{3}}}{x\sqrt[4]{x^6+x^2}}\right)}{2\sqrt{2}\,3^{3/8}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(x^2 + x^6)^(1/4))/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(3^(1/8)*x)/(x^2 + x^6)^(1/4)]/(2*3^(3/8)) - ArcTan[(Sqrt[2]*3^(7/8)
*x*(x^2 + x^6)^(1/4))/(-3*x^2 + 3^(3/4)*Sqrt[x^2 + x^6])]/(2*Sqrt[2]*3^(3/8
)) - ArcTanh[(3^(1/8)*x)/(x^2 + x^6)^(1/4)]/(2*3^(3/8)) + ArcTanh[((3^(1/8)
*x^2)/Sqrt[2] + Sqrt[x^2 + x^6]/(Sqrt[2]*3^(1/8)))/(x*(x^2 + x^6)^(1/4))]/(
2*Sqrt[2]*3^(3/8))
```

fricas [B] time = 17.94, size = 1348, normalized size = 7.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/108*27^(7/8)*sqrt(2)*arctan(1/81*(sqrt(3)*(27^(3/4)*(x^9 + 5*x^5 + x) +
3*(x^6 + x^2)^(3/4)*(27^(5/8)*sqrt(2)*x^2 + 3*27^(1/8)*sqrt(2)*(x^4 + 1)) +
 18*sqrt(x^6 + x^2)*(3*x^3 + sqrt(3)*(x^5 + x)) + 18*27^(1/4)*(x^7 + x^3) +
```

```
(x^6 + x^2)^(1/4)*(9*27^(3/8)*sqrt(2)*x^4 + 27^(7/8)*sqrt(2)*(x^6 + x^2)))
*sqrt((2*(x^6 + x^2)^(3/4)*(9*27^(3/8)*sqrt(2)*x^2 - 27^(7/8)*sqrt(2)*(x^4
+ 1)) + 9*sqrt(3)*(x^9 - x^5 + x) - 12*sqrt(x^6 + x^2)*(27^(3/4)*x^3 - 3*27
^(1/4)*(x^5 + x)) + 6*(x^6 + x^2)^(1/4)*(9*27^(1/8)*sqrt(2)*x^4 - 27^(5/8)*
sqrt(2)*(x^6 + x^2)))/(x^9 - x^5 + x)) + 9*(x^6 + x^2)^(3/4)*(27^(7/8)*sqrt
(2)*x^2 + 3*27^(3/8)*sqrt(2)*(x^4 + 1)) + 27*(x^6 + x^2)^(1/4)*(27^(5/8)*sq
rt(2)*x^4 + 3*27^(1/8)*sqrt(2)*(x^6 + x^2)))/(x^9 - x^5 + x)) - 1/108*27^(7
/8)*sqrt(2)*arctan(-1/81*(sqrt(3)*(27^(3/4)*(x^9 + 5*x^5 + x) - 3*(x^6 + x^
2)^(3/4)*(27^(5/8)*sqrt(2)*x^2 + 3*27^(1/8)*sqrt(2)*(x^4 + 1)) + 18*sqrt(x^
6 + x^2)*(3*x^3 + sqrt(3)*(x^5 + x)) + 18*27^(1/4)*(x^7 + x^3) - (x^6 + x^2
)^(1/4)*(9*27^(3/8)*sqrt(2)*x^4 + 27^(7/8)*sqrt(2)*(x^6 + x^2)))*sqrt(-(2*(
x^6 + x^2)^(3/4)*(9*27^(3/8)*sqrt(2)*x^2 - 27^(7/8)*sqrt(2)*(x^4 + 1)) - 9*
sqrt(3)*(x^9 - x^5 + x) + 12*sqrt(x^6 + x^2)*(27^(3/4)*x^3 - 3*27^(1/4)*(x^
5 + x)) + 6*(x^6 + x^2)^(1/4)*(9*27^(1/8)*sqrt(2)*x^4 - 27^(5/8)*sqrt(2)*(x
^6 + x^2)))/(x^9 - x^5 + x)) - 9*(x^6 + x^2)^(3/4)*(27^(7/8)*sqrt(2)*x^2 +
3*27^(3/8)*sqrt(2)*(x^4 + 1)) - 27*(x^6 + x^2)^(1/4)*(27^(5/8)*sqrt(2)*x^4
+ 3*27^(1/8)*sqrt(2)*(x^6 + x^2)))/(x^9 - x^5 + x)) - 1/432*27^(7/8)*sqrt(2
)*log(3*(2*(x^6 + x^2)^(3/4)*(9*27^(3/8)*sqrt(2)*x^2 - 27^(7/8)*sqrt(2)*(x^
4 + 1)) + 9*sqrt(3)*(x^9 - x^5 + x) - 12*sqrt(x^6 + x^2)*(27^(3/4)*x^3 - 3*
27^(1/4)*(x^5 + x)) + 6*(x^6 + x^2)^(1/4)*(9*27^(1/8)*sqrt(2)*x^4 - 27^(5/8
)*sqrt(2)*(x^6 + x^2)))/(x^9 - x^5 + x)) + 1/432*27^(7/8)*sqrt(2)*log(-3*(2
*(x^6 + x^2)^(3/4)*(9*27^(3/8)*sqrt(2)*x^2 - 27^(7/8)*sqrt(2)*(x^4 + 1)) -
9*sqrt(3)*(x^9 - x^5 + x) + 12*sqrt(x^6 + x^2)*(27^(3/4)*x^3 - 3*27^(1/4)*(
x^5 + x)) + 6*(x^6 + x^2)^(1/4)*(9*27^(1/8)*sqrt(2)*x^4 - 27^(5/8)*sqrt(2)*
(x^6 + x^2)))/(x^9 - x^5 + x)) - 1/54*27^(7/8)*arctan(1/18*(27^(5/8)*(x^6 +
 x^2)^(1/4)*(x^4 + 1) + 3^(3/4)*(27^(3/8)*(x^6 + x^2)^(1/4)*(x^4 + 1) + 3*2
7^(1/8)*(x^6 + x^2)^(3/4)) - 3*27^(3/8)*(x^6 + x^2)^(3/4))/(x^5 + x)) - 1/2
16*27^(7/8)*log((2*27^(3/4)*(x^7 + x^3) + 2*(x^6 + x^2)^(3/4)*(9*27^(1/8)*x
^2 + 27^(5/8)*(x^4 + 1)) + 18*sqrt(x^6 + x^2)*(x^5 + sqrt(3)*x^3 + x) + 3*2
7^(1/4)*(x^9 + 5*x^5 + x) + 2*(x^6 + x^2)^(1/4)*(27^(7/8)*x^4 + 3*27^(3/8)*
(x^6 + x^2)))/(x^9 - x^5 + x)) + 1/216*27^(7/8)*log((2*27^(3/4)*(x^7 + x^3)
 - 2*(x^6 + x^2)^(3/4)*(9*27^(1/8)*x^2 + 27^(5/8)*(x^4 + 1)) + 18*sqrt(x^6
+ x^2)*(x^5 + sqrt(3)*x^3 + x) + 3*27^(1/4)*(x^9 + 5*x^5 + x) - 2*(x^6 + x^
2)^(1/4)*(27^(7/8)*x^4 + 3*27^(3/8)*(x^6 + x^2)))/(x^9 - x^5 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^2)^(1/4)*(x^4 - 1)/(x^8 - x^4 + 1), x)
```

maple [C] time = 71.04, size = 1508, normalized size = 8.06

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^6+x^2)^(1/4)/(x^8-x^4+1),x)
```

```
[Out] 1/12*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*ln(-(Roo
tOf(_Z^2+RootOf(_Z^8-243)^2)*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootO
f(_Z^8-243)^2))*RootOf(_Z^8-243)^9*x^5-2*RootOf(_Z^2+RootOf(_Z^8-243)^2)*Ro
otOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-243
)^9*x^3+RootOf(_Z^2+RootOf(_Z^8-243)^2)*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf
(_Z^2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-243)^9*x+9*RootOf(_Z^2+RootOf(_Z^8-2
43)^2)*RootOf(_Z^8-243)^5*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_
```

```
Z^8-243)^2))*x^3+54*(x^6+x^2)^(1/2)*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^
2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-243)^4*x-324*RootOf(_Z^2+RootOf(_Z^8-243
)^2)*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*RootOf(_
Z^8-243)*x^5+486*RootOf(_Z^2+RootOf(_Z^8-243)^2)*RootOf(_Z^2-RootOf(_Z^8-24
3)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-243)*x^3+486*RootOf(_Z^8-24
3)*(x^6+x^2)^(1/4)*RootOf(_Z^2+RootOf(_Z^8-243)^2)*x^2-324*RootOf(_Z^2+Root
Of(_Z^8-243)^2)*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2
))*RootOf(_Z^8-243)*x+1458*(x^6+x^2)^(3/4))/(RootOf(_Z^8-243)^4*x^4-2*RootO
f(_Z^8-243)^4*x^2+RootOf(_Z^8-243)^4-18*x^4+27*x^2-18)/x)-1/12*RootOf(_Z^8-
243)*ln((RootOf(_Z^8-243)^11*x^5-2*RootOf(_Z^8-243)^11*x^3+RootOf(_Z^8-243)
^11*x-9*RootOf(_Z^8-243)^7*x^3-54*RootOf(_Z^8-243)^5*(x^6+x^2)^(1/2)*x-324*
RootOf(_Z^8-243)^3*x^5+486*RootOf(_Z^8-243)^3*x^3-486*RootOf(_Z^8-243)^2*(x
^6+x^2)^(1/4)*x^2-324*RootOf(_Z^8-243)^3*x-1458*(x^6+x^2)^(3/4))/(RootOf(_Z
^8-243)^4*x^4-2*RootOf(_Z^8-243)^4*x^2+RootOf(_Z^8-243)^4+18*x^4-27*x^2+18)
/x)-1/2916*RootOf(_Z^8-243)^7*RootOf(_Z^2+RootOf(_Z^8-243)^2)*RootOf(_Z^2-R
ootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*ln((RootOf(_Z^2-RootOf(_Z^
8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-243)^10*x^5-2*RootOf(_Z
^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-243)^10*x^
3+RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*RootOf(_Z^8
-243)^10*x+9*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*
RootOf(_Z^8-243)^6*x^3+54*(x^6+x^2)^(1/2)*RootOf(_Z^2+RootOf(_Z^8-243)^2)*R
ootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-24
3)^3*x-324*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z^8-243)^2))*Ro
otOf(_Z^8-243)^2*x^5+486*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z^2+RootOf(_Z
^8-243)^2))*RootOf(_Z^8-243)^2*x^3+486*RootOf(_Z^8-243)*(x^6+x^2)^(1/4)*Roo
tOf(_Z^2+RootOf(_Z^8-243)^2)*x^2-324*RootOf(_Z^2-RootOf(_Z^8-243)*RootOf(_Z
^2+RootOf(_Z^8-243)^2))*RootOf(_Z^8-243)^2*x-1458*(x^6+x^2)^(3/4))/(RootOf(
_Z^8-243)^4*x^4-2*RootOf(_Z^8-243)^4*x^2+RootOf(_Z^8-243)^4-18*x^4+27*x^2-1
8)/x)-1/12*RootOf(_Z^2+RootOf(_Z^8-243)^2)*ln(-(RootOf(_Z^8-243)^10*RootOf(
_Z^2+RootOf(_Z^8-243)^2)*x^5-2*RootOf(_Z^8-243)^10*RootOf(_Z^2+RootOf(_Z^8-
243)^2)*x^3+RootOf(_Z^8-243)^10*RootOf(_Z^2+RootOf(_Z^8-243)^2)*x-9*RootOf(
_Z^8-243)^6*RootOf(_Z^2+RootOf(_Z^8-243)^2)*x^3+54*RootOf(_Z^2+RootOf(_Z^8-
243)^2)*(x^6+x^2)^(1/2)*RootOf(_Z^8-243)^4*x-324*RootOf(_Z^2+RootOf(_Z^8-24
3)^2)*RootOf(_Z^8-243)^2*x^5+486*RootOf(_Z^2+RootOf(_Z^8-243)^2)*RootOf(_Z^
8-243)^2*x^3-486*RootOf(_Z^8-243)^2*(x^6+x^2)^(1/4)*x^2-324*RootOf(_Z^2+Roo
tOf(_Z^8-243)^2)*RootOf(_Z^8-243)^2*x+1458*(x^6+x^2)^(3/4))/(RootOf(_Z^8-24
3)^4*x^4-2*RootOf(_Z^8-243)^4*x^2+RootOf(_Z^8-243)^4+18*x^4-27*x^2+18)/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^2\right)^{\frac{1}{4}}\left(x^4-1\right)}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^6+x^2)^(1/4)/(x^8-x^4+1),x, algorithm="maxima")
[Out] integrate((x^6 + x^2)^(1/4)*(x^4 - 1)/(x^8 - x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^6+x^2\right)^{1/4}\,\left(x^4-1\right)}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + x^6)^(1/4)*(x^4 - 1))/(x^8 - x^4 + 1),x)
[Out] int(((x^2 + x^6)^(1/4)*(x^4 - 1))/(x^8 - x^4 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2 \left(x^4+1\right)}\,(x-1)\,(x+1)\left(x^2+1\right)}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**6+x**2)**(1/4)/(x**8-x**4+1),x)
```

```
[Out] Integral((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)/(x**8 - x**4 +
 1), x)
```

3.1934 $\int \frac{1-x^4+x^8}{\sqrt[4]{x^2+x^6}\left(-1+x^8\right)} dx$

Optimal. Leaf size=187

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{8\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{8\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{8\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{8\,2^{3/4}}-\frac{3\left(x^6+x^2\right)^{3/4}}{2x\left(x^4+1\right)}$$

Rubi [C] time = 0.56, antiderivative size = 99, normalized size of antiderivative = 0.53, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2056, 6715, 6725, 245, 1455, 527, 530, 429}

$$-\frac{\sqrt[4]{x^4+1}\,xF_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^6+x^2}}+\frac{\sqrt[4]{x^4+1}\,x\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{2\sqrt[4]{x^6+x^2}}-\frac{3x}{2\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x^4 + x^8)/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] (-3*x)/(2*(x^2 + x^6)^(1/4)) - (x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8
, x^4, -x^4])/(x^2 + x^6)^(1/4) + (x*(1 + x^4)^(1/4)*Hypergeometric2F1[1/8,
 1/4, 9/8, -x^4])/(2*(x^2 + x^6)^(1/4))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1455

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*
```

```
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, n, q, r}, x] && EqQ[n2,
 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1-x^4+x^8}{\sqrt[4]{x^2+x^6}\left(-1+x^8\right)}\,dx = \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^4}\right)\int \frac{1-x^4+x^8}{\sqrt{x}\,\sqrt[4]{1+x^4}\left(-1+x^8\right)}\,dx}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1-x^8+x^{16}}{\sqrt[4]{1+x^8}\left(-1+x^{16}\right)}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[4]{1+x^8}}+\frac{2-x^8}{\sqrt[4]{1+x^8}\left(-1+x^{16}\right)}\right)dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{2-x^8}{\sqrt[4]{1+x^8}\left(-1\right.}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= \frac{2x\sqrt[4]{1+x^4}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left(2\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{2-x^8}{\left(-1+x^8\right)\left(1+x^8\right)^{5/4}}\,dx, x, \sqrt{x}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= -\frac{3x}{2\sqrt[4]{x^2+x^6}} + \frac{2x\sqrt[4]{1+x^4}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{5-3x^8}{\left(-1+x^8\right)\sqrt[4]{}}\right.}{2\sqrt[4]{x^2+x^6}}$$

$$= -\frac{3x}{2\sqrt[4]{x^2+x^6}} + \frac{2x\sqrt[4]{1+x^4}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}} + \frac{\left(\sqrt{x}\,\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(-1+x^8\right)\sqrt[4]{}}\right.}{\sqrt[4]{x^2+x^6}}$$

$$= -\frac{3x}{2\sqrt[4]{x^2+x^6}} - \frac{x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^2+x^6}} + \frac{x\sqrt[4]{1+x^4}\;{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{2\sqrt[4]{x^2+x^6}}$$

Mathematica [C] time = 0.43, size = 47, normalized size = 0.25

$$-\frac{x\left(\sqrt[4]{x^4+1}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};-x^4,x^4\right)+3\right)}{2\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^4 + x^8)/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -1/2*(x*(3 + (1 + x^4)^(1/4)*AppellF1[1/8, -3/4, 1, 9/8, -x^4, x^4]))/(x^2
+ x^6)^(1/4)
```

IntegrateAlgebraic [A] time = 0.95, size = 187, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{8\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{8\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{8\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{8\,2^{3/4}}-\frac{3\left(x^6+x^2\right)^{3/4}}{2x\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^4 + x^8)/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] (-3*(x^2 + x^6)^(3/4))/(2*x*(1 + x^4)) - ArcTan[(2^(1/4)*x)/(x^2 + x^6)^(1/
4)]/(8*2^(1/4)) + ArcTan[(2^(3/4)*x*(x^2 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[
x^2 + x^6])]/(8*2^(3/4)) - ArcTanh[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/(8*2^(1/4
)) - ArcTanh[(x^2/2^(1/4) + Sqrt[x^2 + x^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))]
/(8*2^(3/4))
```

fricas [B] time = 53.13, size = 1055, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-x^4+1)/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="fricas")
```

```
[Out] -1/64*(4*2^(3/4)*(x^5 + x)*arctan(1/2*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + 2^
(3/4)*(2*2^(3/4)*sqrt(x^6 + x^2)*x + 2^(1/4)*(x^5 + 2*x^3 + x)) + 4*2^(1/4)
*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 2^(3/4)*(x^5 + x)*log(-(4*sqrt(2)*
(x^6 + x^2)^(1/4)*x^2 + 2^(3/4)*(x^5 + 2*x^3 + x) + 4*2^(1/4)*sqrt(x^6 + x^
2)*x + 4*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) - 2^(3/4)*(x^5 + x)*log(-(4*
sqrt(2)*(x^6 + x^2)^(1/4)*x^2 - 2^(3/4)*(x^5 + 2*x^3 + x) - 4*2^(1/4)*sqrt(
x^6 + x^2)*x + 4*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) - 4*2^(1/4)*(x^5 + x
)*arctan(-1/2*(2*x^9 + 8*x^7 + 12*x^5 + 8*x^3 + 4*2^(3/4)*(x^6 + x^2)^(3/4)
*(x^4 - 6*x^2 + 1) + 8*sqrt(2)*sqrt(x^6 + x^2)*(x^5 + 2*x^3 + x) - sqrt(2)*
(32*sqrt(2)*(x^6 + x^2)^(3/4)*x^2 + 2^(3/4)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3
+ x) + 4*2^(1/4)*sqrt(x^6 + x^2)*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x^4 + x^2)*
(x^6 + x^2)^(1/4))*sqrt((4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2
*x^3 + x) + 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3
 + x)) + 8*2^(1/4)*(3*x^6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2*x)/(x^9 -
28*x^7 + 6*x^5 - 28*x^3 + x)) + 4*2^(1/4)*(x^5 + x)*arctan(-1/2*(2*x^9 + 8*
x^7 + 12*x^5 + 8*x^3 - 4*2^(3/4)*(x^6 + x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 8*sq
rt(2)*sqrt(x^6 + x^2)*(x^5 + 2*x^3 + x) - sqrt(2)*(32*sqrt(2)*(x^6 + x^2)^(
3/4)*x^2 - 2^(3/4)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3 + x) - 4*2^(1/4)*sqrt(x^6
 + x^2)*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt(-
(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 +
 x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x)) - 8*2^(1/4)*(3*x^
6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3
+ x)) + 2^(1/4)*(x^5 + x)*log(8*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*
(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5
 + 2*x^3 + x)) - 2^(1/4)*(x^5 + x)*log(-8*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2
```

```
- sqrt(2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(
3/4))/(x^5 + 2*x^3 + x)) + 96*(x^6 + x^2)^(3/4))/(x^5 + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 - x^4 + 1}{\left(x^8 - 1\right)\left(x^6 + x^2\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-x^4+1)/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="giac")
```

```
[Out] integrate((x^8 - x^4 + 1)/((x^8 - 1)*(x^6 + x^2)^(1/4)), x)
```

maple [C] time = 18.79, size = 649, normalized size = 3.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-x^4+1)/(x^6+x^2)^(1/4)/(x^8-1),x)
```

```
[Out] -3/2*x/(x^2*(x^4+1))^(1/4)-1/32*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln(-(RootOf(_
Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*(x^6+x^2)^(1/2)*x+RootOf(_Z^2+RootOf
(_Z^4+8)^2)*x^5+2*RootOf(_Z^4+8)^2*(x^6+x^2)^(1/4)*x^2-2*RootOf(_Z^2+RootOf
(_Z^4+8)^2)*x^3+4*(x^6+x^2)^(3/4)+RootOf(_Z^2+RootOf(_Z^4+8)^2)*x)/x/(x^2+1
)^2)+1/32*RootOf(_Z^4+8)*ln(-(RootOf(_Z^4+8)^3*(x^6+x^2)^(1/2)*x-RootOf(_Z^
4+8)*x^5-2*RootOf(_Z^4+8)^2*(x^6+x^2)^(1/4)*x^2+2*RootOf(_Z^4+8)*x^3+4*(x^6
+x^2)^(3/4)-RootOf(_Z^4+8)*x)/x/(x^2+1)^2)+1/64*ln((RootOf(_Z^4+8)^2*x^2+2*
RootOf(_Z^4+8)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))/(1+x)/x/(-1+x))*RootOf(
_Z^4+8)^3+1/64*ln((RootOf(_Z^4+8)^2*x^2+2*RootOf(_Z^4+8)*(x^6+x^2)^(1/4)*x+
2*(x^6+x^2)^(1/2))/(1+x)/x/(-1+x))*RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z^4
+8)^2)-1/64*RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln((RootOf(_Z^2+
RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^5-RootOf(_Z^4+8)^3*x^5+2*RootOf(_Z^2+R
ootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^3-2*RootOf(_Z^4+8)^3*x^3+8*RootOf(_Z^4+
8)*(x^6+x^2)^(1/4)*RootOf(_Z^2+RootOf(_Z^4+8)^2)*x^2+RootOf(_Z^2+RootOf(_Z^
4+8)^2)*RootOf(_Z^4+8)^2*x+8*RootOf(_Z^2+RootOf(_Z^4+8)^2)*(x^6+x^2)^(1/2)*
x-RootOf(_Z^4+8)^3*x+8*(x^6+x^2)^(1/2)*RootOf(_Z^4+8)*x+16*(x^6+x^2)^(3/4))
/(1+x)^2/(-1+x)^2/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 - x^4 + 1}{\left(x^8 - 1\right)\left(x^6 + x^2\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-x^4+1)/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="maxima")
```

```
[Out] integrate((x^8 - x^4 + 1)/((x^8 - 1)*(x^6 + x^2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8 - x^4 + 1}{{\left(x^6 + x^2\right)}^{1/4}\,\left(x^8 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 - x^4 + 1)/((x^2 + x^6)^(1/4)*(x^8 - 1)),x)
```

```
[Out] int((x^8 - x^4 + 1)/((x^2 + x^6)^(1/4)*(x^8 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 - x^4 + 1}{\sqrt[4]{x^2 \left(x^4 + 1\right)} \left(x - 1\right) \left(x + 1\right) \left(x^2 + 1\right) \left(x^4 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-x**4+1)/(x**6+x**2)**(1/4)/(x**8-1),x)
```

```
[Out] Integral((x**8 - x**4 + 1)/((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2
+ 1)*(x**4 + 1)), x)
```

3.1935 $\int \frac{1-x^4+x^8}{\sqrt[4]{x^2+x^6}\left(-1+x^8\right)} dx$

Optimal. Leaf size=187

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{8\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{8\ 2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{8\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{8\ 2^{3/4}}-\frac{3\left(x^6+x^2\right)^{3/4}}{2x\left(x^4+1\right)}$$

Rubi [C] time = 0.45, antiderivative size = 99, normalized size of antiderivative = 0.53, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2056, 6715, 6725, 245, 1455, 527, 530, 429}

$$-\frac{\sqrt[4]{x^4+1}\,xF_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^6+x^2}}+\frac{\sqrt[4]{x^4+1}\,x\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{2\sqrt[4]{x^6+x^2}}-\frac{3x}{2\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x^4 + x^8)/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] (-3*x)/(2*(x^2 + x^6)^(1/4)) - (x*(1 + x^4)^(1/4)*AppellF1[1/8, 1, 1/4, 9/8
, x^4, -x^4])/(x^2 + x^6)^(1/4) + (x*(1 + x^4)^(1/4)*Hypergeometric2F1[1/8,
 1/4, 9/8, -x^4])/(2*(x^2 + x^6)^(1/4))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1455

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*
```

```
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, n, q, r}, x] && EqQ[n2,
 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1-x^4+x^8}{\sqrt[4]{x^2+x^6}\left(-1+x^8\right)}\,dx=\frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\int \frac{1-x^4+x^8}{\sqrt{x}\sqrt[4]{1+x^4}\left(-1+x^8\right)}\,dx}{\sqrt[4]{x^2+x^6}}$$

$$=\frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1-x^8+x^{16}}{\sqrt[4]{1+x^8}\left(-1+x^{16}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$=\frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[4]{1+x^8}}+\frac{2-x^8}{\sqrt[4]{1+x^8}\left(-1+x^{16}\right)}\right)dx,x,\sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$=\frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^8}}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}+\frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{2-x^8}{\sqrt[4]{1+x^8}\left(-1+x^{16}\right.}\right.}{\sqrt[4]{x^2+x^6}}$$

$$=\frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}}+\frac{\left(2\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{2-x^8}{\left(-1+x^8\right)\left(1+x^8\right)^{5/4}}\,dx,x,\sqrt{x}\right)}{\sqrt[4]{x^2+x^6}}$$

$$=-\frac{3x}{2\sqrt[4]{x^2+x^6}}+\frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}}+\frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{5-3x^8}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\right.}{2\sqrt[4]{x^2+x^6}}$$

$$=-\frac{3x}{2\sqrt[4]{x^2+x^6}}+\frac{2x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{\sqrt[4]{x^2+x^6}}+\frac{\left(\sqrt{x}\sqrt[4]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\left(-1+x^8\right)\sqrt[4]{1+x^8}}\right.}{\sqrt[4]{x^2+x^6}}$$

$$=-\frac{3x}{2\sqrt[4]{x^2+x^6}}-\frac{x\sqrt[4]{1+x^4}\,F_1\left(\frac{1}{8};1,\frac{1}{4};\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^2+x^6}}+\frac{x\sqrt[4]{1+x^4}\,{}_2F_1\left(\frac{1}{8},\frac{1}{4};\frac{9}{8};-x^4\right)}{2\sqrt[4]{x^2+x^6}}$$

Mathematica [C] time = 0.28, size = 47, normalized size = 0.25

$$-\frac{x\left(\sqrt[4]{x^4+1}F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};-x^4,x^4\right)+3\right)}{2\sqrt[4]{x^6+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^4 + x^8)/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] -1/2*(x*(3 + (1 + x^4)^(1/4)*AppellF1[1/8, -3/4, 1, 9/8, -x^4, x^4]))/(x^2
+ x^6)^(1/4)
```

IntegrateAlgebraic [A] time = 0.00, size = 187, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{8\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^6+x^2}}{\sqrt{2}x^2-\sqrt{x^6+x^2}}\right)}{8\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^6+x^2}}\right)}{8\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^6+x^2}}{2^{3/4}}}{x\sqrt[4]{x^6+x^2}}\right)}{8\,2^{3/4}}-\frac{3\left(x^6+x^2\right)^{3/4}}{2x\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^4 + x^8)/((x^2 + x^6)^(1/4)*(-1 + x^8)),x]
```

```
[Out] (-3*(x^2 + x^6)^(3/4))/(2*x*(1 + x^4)) - ArcTan[(2^(1/4)*x)/(x^2 + x^6)^(1/
4)]/(8*2^(1/4)) + ArcTan[(2^(3/4)*x*(x^2 + x^6)^(1/4))/(Sqrt[2]*x^2 - Sqrt[
x^2 + x^6])]/(8*2^(3/4)) - ArcTanh[(2^(1/4)*x)/(x^2 + x^6)^(1/4)]/(8*2^(1/4
)) - ArcTanh[(x^2/2^(1/4) + Sqrt[x^2 + x^6]/2^(3/4))/(x*(x^2 + x^6)^(1/4))]
/(8*2^(3/4))
```

fricas [B] time = 53.59, size = 1055, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-x^4+1)/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="fricas")
```

```
[Out] -1/64*(4*2^(3/4)*(x^5 + x)*arctan(1/2*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + 2^
(3/4)*(2*2^(3/4)*sqrt(x^6 + x^2)*x + 2^(1/4)*(x^5 + 2*x^3 + x)) + 4*2^(1/4)
*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) + 2^(3/4)*(x^5 + x)*log(-(4*sqrt(2)*
(x^6 + x^2)^(1/4)*x^2 + 2^(3/4)*(x^5 + 2*x^3 + x) + 4*2^(1/4)*sqrt(x^6 + x^
2)*x + 4*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) - 2^(3/4)*(x^5 + x)*log(-(4*
sqrt(2)*(x^6 + x^2)^(1/4)*x^2 - 2^(3/4)*(x^5 + 2*x^3 + x) - 4*2^(1/4)*sqrt(
x^6 + x^2)*x + 4*(x^6 + x^2)^(3/4))/(x^5 - 2*x^3 + x)) - 4*2^(1/4)*(x^5 + x
)*arctan(-1/2*(2*x^9 + 8*x^7 + 12*x^5 + 8*x^3 + 4*2^(3/4)*(x^6 + x^2)^(3/4)
*(x^4 - 6*x^2 + 1) + 8*sqrt(2)*sqrt(x^6 + x^2)*(x^5 + 2*x^3 + x) - sqrt(2)*
(32*sqrt(2)*(x^6 + x^2)^(3/4)*x^2 + 2^(3/4)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3
+ x) + 4*2^(1/4)*sqrt(x^6 + x^2)*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x^4 + x^2)*
(x^6 + x^2)^(1/4))*sqrt((4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*(x^5 + 2
*x^3 + x) + 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3
 + x)) + 8*2^(1/4)*(3*x^6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2*x)/(x^9 -
28*x^7 + 6*x^5 - 28*x^3 + x)) + 4*2^(1/4)*(x^5 + x)*arctan(-1/2*(2*x^9 + 8*
x^7 + 12*x^5 + 8*x^3 - 4*2^(3/4)*(x^6 + x^2)^(3/4)*(x^4 - 6*x^2 + 1) + 8*sq
rt(2)*sqrt(x^6 + x^2)*(x^5 + 2*x^3 + x) - sqrt(2)*(32*sqrt(2)*(x^6 + x^2)^(
3/4)*x^2 - 2^(3/4)*(x^9 - 16*x^7 - 2*x^5 - 16*x^3 + x) - 4*2^(1/4)*sqrt(x^6
 + x^2)*(x^5 - 6*x^3 + x) + 8*(x^6 + 2*x^4 + x^2)*(x^6 + x^2)^(1/4))*sqrt(-
(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 - sqrt(2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 +
 x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5 + 2*x^3 + x)) - 8*2^(1/4)*(3*x^
6 - 2*x^4 + 3*x^2)*(x^6 + x^2)^(1/4) + 2*x)/(x^9 - 28*x^7 + 6*x^5 - 28*x^3
+ x)) + 2^(1/4)*(x^5 + x)*log(8*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2 + sqrt(2)*
(x^5 + 2*x^3 + x) + 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(3/4))/(x^5
 + 2*x^3 + x)) - 2^(1/4)*(x^5 + x)*log(-8*(4*2^(3/4)*(x^6 + x^2)^(1/4)*x^2
```

```
- sqrt(2)*(x^5 + 2*x^3 + x) - 8*sqrt(x^6 + x^2)*x + 4*2^(1/4)*(x^6 + x^2)^(
3/4))/(x^5 + 2*x^3 + x)) + 96*(x^6 + x^2)^(3/4))/(x^5 + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 - x^4 + 1}{(x^8 - 1)(x^6 + x^2)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-x^4+1)/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="giac")
```

```
[Out] integrate((x^8 - x^4 + 1)/((x^8 - 1)*(x^6 + x^2)^(1/4)), x)
```

maple [C] time = 18.84, size = 649, normalized size = 3.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-x^4+1)/(x^6+x^2)^(1/4)/(x^8-1),x)
```

```
[Out] -3/2*x/(x^2*(x^4+1))^(1/4)+1/32*RootOf(_Z^4+8)*ln(-(RootOf(_Z^4+8)^3*(x^6+x
^2)^(1/2)*x-RootOf(_Z^4+8)*x^5-2*RootOf(_Z^4+8)^2*(x^6+x^2)^(1/4)*x^2+2*Roo
tOf(_Z^4+8)*x^3+4*(x^6+x^2)^(3/4)-RootOf(_Z^4+8)*x)/x/(x^2+1)^2)-1/32*RootO
f(_Z^2+RootOf(_Z^4+8)^2)*ln(-(RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^
2*(x^6+x^2)^(1/2)*x+RootOf(_Z^2+RootOf(_Z^4+8)^2)*x^5+2*RootOf(_Z^4+8)^2*(x
^6+x^2)^(1/4)*x^2-2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*x^3+4*(x^6+x^2)^(3/4)+Roo
tOf(_Z^2+RootOf(_Z^4+8)^2)*x)/x/(x^2+1)^2)-1/64*ln((RootOf(_Z^4+8)^2*x^2-2*
RootOf(_Z^4+8)*(x^6+x^2)^(1/4)*x+2*(x^6+x^2)^(1/2))/(1+x)/x/(-1+x))*RootOf(
_Z^4+8)^3-1/64*ln((RootOf(_Z^4+8)^2*x^2-2*RootOf(_Z^4+8)*(x^6+x^2)^(1/4)*x+
2*(x^6+x^2)^(1/2))/(1+x)/x/(-1+x))*RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z^4
+8)^2)+1/64*RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln((RootOf(_Z^4+
8)^3*x^5-RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^5+2*RootOf(_Z^4+8
)^3*x^3-2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^3+8*RootOf(_Z^4+
8)*(x^6+x^2)^(1/4)*RootOf(_Z^2+RootOf(_Z^4+8)^2)*x^2+RootOf(_Z^4+8)^3*x-Roo
tOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x-8*(x^6+x^2)^(1/2)*RootOf(_Z^4
+8)*x-8*RootOf(_Z^2+RootOf(_Z^4+8)^2)*(x^6+x^2)^(1/2)*x+16*(x^6+x^2)^(3/4))
/(1+x)^2/(-1+x)^2/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 - x^4 + 1}{(x^8 - 1)(x^6 + x^2)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-x^4+1)/(x^6+x^2)^(1/4)/(x^8-1),x, algorithm="maxima")
```

```
[Out] integrate((x^8 - x^4 + 1)/((x^8 - 1)*(x^6 + x^2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8 - x^4 + 1}{(x^6 + x^2)^{1/4}\,(x^8 - 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 - x^4 + 1)/((x^2 + x^6)^(1/4)*(x^8 - 1)),x)
```

```
[Out] int((x^8 - x^4 + 1)/((x^2 + x^6)^(1/4)*(x^8 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 - x^4 + 1}{\sqrt[4]{x^2 \left(x^4 + 1\right)} \left(x - 1\right) \left(x + 1\right) \left(x^2 + 1\right) \left(x^4 + 1\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-x**4+1)/(x**6+x**2)**(1/4)/(x**8-1),x)
```

```
[Out] Integral((x**8 - x**4 + 1)/((x**2*(x**4 + 1))**(1/4)*(x - 1)*(x + 1)*(x**2
+ 1)*(x**4 + 1)), x)
```

3.1936 $\int \frac{1}{(-b+ax)\sqrt[4]{-x^3+x^4}}\,dx$

Optimal. Leaf size=188

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x\sqrt[4]{x^4-x^3}\sqrt[4]{a-b}}{x^2\sqrt{a-b}-\sqrt{b}\sqrt{x^4-x^3}}\right)}{b^{3/4}\sqrt[4]{a-b}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\frac{x^2\sqrt[4]{a-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}\sqrt{x^4-x^3}}{\sqrt{2}\sqrt[4]{a-b}}}{x\sqrt[4]{x^4-x^3}}\right)}{b^{3/4}\sqrt[4]{a-b}}$$

Rubi [A] time = 0.11, antiderivative size = 139, normalized size of antiderivative = 0.74, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2056, 93, 212, 208, 205}

$$-\frac{2\sqrt[4]{x-1}\,x^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{x}\sqrt[4]{b-a}}{\sqrt[4]{b}\sqrt[4]{x-1}}\right)}{b^{3/4}\sqrt[4]{x^4-x^3}\sqrt[4]{b-a}}-\frac{2\sqrt[4]{x-1}\,x^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{x}\sqrt[4]{b-a}}{\sqrt[4]{b}\sqrt[4]{x-1}}\right)}{b^{3/4}\sqrt[4]{x^4-x^3}\sqrt[4]{b-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-b + a*x)*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (-2*(-1 + x)^(1/4)*x^(3/4)*ArcTan[((-a + b)^(1/4)*x^(1/4))/(b^(1/4)*(-1 + x
)^(1/4))])/(b^(3/4)*(-a + b)^(1/4)*(-x^3 + x^4)^(1/4)) - (2*(-1 + x)^(1/4)*
x^(3/4)*ArcTanh[((-a + b)^(1/4)*x^(1/4))/(b^(1/4)*(-1 + x)^(1/4))])/(b^(3/4
)*(-a + b)^(1/4)*(-x^3 + x^4)^(1/4))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{(-b+ax)\sqrt[4]{-x^3+x^4}}\,dx = \frac{\left(\sqrt[4]{-1+x}\,x^{3/4}\right)\int \frac{1}{\sqrt[4]{-1+x}\,x^{3/4}(-b+ax)}\,dx}{\sqrt[4]{-x^3+x^4}}$$
$$= \frac{\left(4\sqrt[4]{-1+x}\,x^{3/4}\right)\text{Subst}\left(\int \frac{1}{-b-(a-b)x^4}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt[4]{-1+x}}\right)}{\sqrt[4]{-x^3+x^4}}$$
$$= -\frac{\left(2\sqrt[4]{-1+x}\,x^{3/4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{-a+b}\,x^2}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt[4]{-1+x}}\right)}{\sqrt{b}\,\sqrt[4]{-x^3+x^4}} - \frac{\left(2\sqrt[4]{-1+x}\,x^{3/4}\right)\text{Subst}(\ldots}{\sqrt{b}\,\sqrt[4]{\ldots}}$$
$$= -\frac{2\sqrt[4]{-1+x}\,x^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{-a+b}\,\sqrt[4]{x}}{\sqrt[4]{b}\,\sqrt[4]{-1+x}}\right)}{b^{3/4}\sqrt[4]{-a+b}\,\sqrt[4]{-x^3+x^4}} - \frac{2\sqrt[4]{-1+x}\,x^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{-a+b}\,\sqrt[4]{x}}{\sqrt[4]{b}\,\sqrt[4]{-1+x}}\right)}{b^{3/4}\sqrt[4]{-a+b}\,\sqrt[4]{-x^3+x^4}}$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.26

$$\frac{4(x-1)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{b(x-1)}{(a-b)x}\right)}{3\sqrt[4]{(x-1)x^3}\,(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-b + a*x)*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (4*(-1 + x)*Hypergeometric2F1[3/4, 1, 7/4, -((b*(-1 + x))/((a - b)*x))])/(3
*(a - b)*((-1 + x)*x^3)^(1/4))
```

IntegrateAlgebraic [A] time = 1.37, size = 188, normalized size = 1.00

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x\sqrt[4]{x^4-x^3}\sqrt[4]{a-b}}{x^2\sqrt{a-b}-\sqrt{b}\sqrt{x^4-x^3}}\right)}{b^{3/4}\sqrt[4]{a-b}} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\frac{x^2\sqrt[4]{a-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}\sqrt{x^4-x^3}}{\sqrt{2}\sqrt[4]{a-b}}}{x\sqrt[4]{x^4-x^3}}\right)}{b^{3/4}\sqrt[4]{a-b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-b + a*x)*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[2]*(a - b)^(1/4)*b^(1/4)*x*(-x^3 + x^4)^(1/4))/(Sqrt[
a - b]*x^2 - Sqrt[b]*Sqrt[-x^3 + x^4]])])/((a - b)^(1/4)*b^(3/4)) - (Sqrt[2]
*ArcTanh[(((a - b)^(1/4)*x^2)/(Sqrt[2]*b^(1/4)) + (b^(1/4)*Sqrt[-x^3 + x^4]
)/(Sqrt[2]*(a - b)^(1/4)))/(x*(-x^3 + x^4)^(1/4))])/((a - b)^(1/4)*b^(3/4))
```

fricas [A] time = 0.44, size = 262, normalized size = 1.39

$$-4\left(-\frac{1}{ab^3-b^4}\right)^{\frac{1}{4}}\arctan\left(\frac{bx\left(-\frac{1}{ab^3-b^4}\right)^{\frac{1}{4}}\sqrt{-\frac{(ab-b^2)x^2\sqrt{-\frac{1}{ab^3-b^4}}-\sqrt{x^4-x^3}}{x^2}}-\left(x^4-x^3\right)^{\frac{1}{4}}b\left(-\frac{1}{ab^3-b^4}\right)^{\frac{1}{4}}}{x}\right)+\left(-\frac{1}{ab^3-b^4}\right)^{\frac{1}{4}}\log\left(\frac{\left(ab^2-b^3\right)x\left(-\frac{1}{ab^3-b^4}\right)^{\frac{3}{4}}+\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)-\left(-\frac{1}{ab^3-b^4}\right)^{\frac{1}{4}}\log\left(-\frac{\left(ab^2-b^3\right)x\left(-\frac{1}{ab^3-b^4}\right)^{\frac{3}{4}}-\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-b)/(x^4-x^3)^(1/4),x, algorithm="fricas")
```

```
[Out] -4*(-1/(a*b^3 - b^4))^(1/4)*arctan((b*x*(-1/(a*b^3 - b^4))^(1/4)*sqrt(-((a*
b - b^2)*x^2*sqrt(-1/(a*b^3 - b^4)) - sqrt(x^4 - x^3))/x^2) - (x^4 - x^3)^(
1/4)*b*(-1/(a*b^3 - b^4))^(1/4))/x) + (-1/(a*b^3 - b^4))^(1/4)*log(((a*b^2
- b^3)*x*(-1/(a*b^3 - b^4))^(3/4) + (x^4 - x^3)^(1/4))/x) - (-1/(a*b^3 - b^
```

```
4))^(1/4)*log(-((a*b^2 - b^3)*x*(-1/(a*b^3 - b^4))^(3/4) - (x^4 - x^3)^(1/4
))/x)
```

giac [B] time = 0.22, size = 317, normalized size = 1.69

$$-\frac{2\left(ab^3-b^4\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a-b}{b}\right)^{\frac{1}{4}}+2\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)}{2\left(\frac{a-b}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,ab^3-\sqrt{2}\,b^4}-\frac{2\left(ab^3-b^4\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a-b}{b}\right)^{\frac{1}{4}}-2\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)}{2\left(\frac{a-b}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,ab^3-\sqrt{2}\,b^4}+\frac{\left(ab^3-b^4\right)^{\frac{3}{4}}\log\left(\sqrt{2}\left(\frac{a-b}{b}\right)^{\frac{1}{4}}\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+\sqrt{\frac{a-b}{b}}+\sqrt{-\frac{1}{x}+1}\right)}{\sqrt{2}\,ab^3-\sqrt{2}\,b^4}-\frac{\left(ab^3-b^4\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\left(\frac{a-b}{b}\right)^{\frac{1}{4}}\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+\sqrt{\frac{a-b}{b}}+\sqrt{-\frac{1}{x}+1}\right)}{\sqrt{2}\,ab^3-\sqrt{2}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-b)/(x^4-x^3)^(1/4),x, algorithm="giac")
```

```
[Out] -2*(a*b^3 - b^4)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*((a - b)/b)^(1/4) + 2*(-
1/x + 1)^(1/4))/((a - b)/b)^(1/4))/(sqrt(2)*a*b^3 - sqrt(2)*b^4) - 2*(a*b^3
 - b^4)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*((a - b)/b)^(1/4) - 2*(-1/x + 1)
^(1/4))/((a - b)/b)^(1/4))/(sqrt(2)*a*b^3 - sqrt(2)*b^4) + (a*b^3 - b^4)^(3
/4)*log(sqrt(2)*((a - b)/b)^(1/4)*(-1/x + 1)^(1/4) + sqrt((a - b)/b) + sqrt
(-1/x + 1))/(sqrt(2)*a*b^3 - sqrt(2)*b^4) - (a*b^3 - b^4)^(3/4)*log(-sqrt(2
)*((a - b)/b)^(1/4)*(-1/x + 1)^(1/4) + sqrt((a - b)/b) + sqrt(-1/x + 1))/(s
qrt(2)*a*b^3 - sqrt(2)*b^4)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax-b)\left(x^4-x^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-b)/(x^4-x^3)^(1/4),x)
```

```
[Out] int(1/(a*x-b)/(x^4-x^3)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4-x^3\right)^{\frac{1}{4}}(ax-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-b)/(x^4-x^3)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 - x^3)^(1/4)*(a*x - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(x^4-x^3\right)^{1/4}\,(b-a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^4 - x^3)^(1/4)*(b - a*x)),x)
```

```
[Out] -int(1/((x^4 - x^3)^(1/4)*(b - a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^3\,(x-1)}\,(ax-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-b)/(x**4-x**3)**(1/4),x)
```

```
[Out] Integral(1/((x**3*(x - 1))**(1/4)*(a*x - b)), x)
```

3.1937 $\int \frac{-x+x^2}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(1-2x+k^2x^2\right)}\,dx$

Optimal. Leaf size=189

$$\frac{i\tan^{-1}\left(\frac{\sqrt{k^2+2i\sqrt{k^2-1}-2}\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{k^2x-1}\right)}{2\sqrt{k^2-1}\sqrt{k^2+2i\sqrt{k^2-1}-2}}-\frac{i\tan^{-1}\left(\frac{\sqrt{k^2-2i\sqrt{k^2-1}-2}\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{k^2x-1}\right)}{2\sqrt{k^2-1}\sqrt{k^2-2i\sqrt{k^2-1}-2}}$$

Rubi [C] time = 2.39, antiderivative size = 455, normalized size of antiderivative = 2.41, number of steps used = 26, number of rules used = 14, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.342, Rules used = {1593, 6718, 21, 6688, 6728, 922, 934, 12, 168, 537, 843, 714, 110, 115}

$$\frac{\left(-k^2-\sqrt{1-k^2}+1\right)(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\,\Pi\left(\frac{1}{1-\sqrt{1-k^2}};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{\left(-k^2\right)^{3/2}\sqrt{1-k^2}\sqrt{x-x^2}\sqrt{(1-x)x\left(1-k^2x\right)}}-\frac{\left(-k^2+\sqrt{1-k^2}+1\right)(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\,\Pi\left(\frac{1}{\sqrt{1-k^2}+1};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{\left(-k^2\right)^{3/2}\sqrt{1-k^2}\sqrt{x-x^2}\sqrt{(1-x)x\left(1-k^2x\right)}}-\frac{\left(-k^2-\sqrt{1-k^2}+2\right)\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{k^2\sqrt{1-k^2}\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{\left(-k^2+\sqrt{1-k^2}+2\right)\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{k^2\sqrt{1-k^2}\sqrt{(1-x)x\left(1-k^2x\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-x + x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(1 - 2*x + k^2*x^2)),x]
```

```
[Out] -(((2 - k^2 - Sqrt[1 - k^2])*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticF[
ArcSin[Sqrt[x]], k^2])/(k^2*Sqrt[1 - k^2]*Sqrt[(1 - x)*x*(1 - k^2*x)])) + (
(2 - k^2 + Sqrt[1 - k^2])*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticF[Arc
Sin[Sqrt[x]], k^2])/(k^2*Sqrt[1 - k^2]*Sqrt[(1 - x)*x*(1 - k^2*x)]) + ((1 -
 k^2 - Sqrt[1 - k^2])*(1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticPi[(
1 - Sqrt[1 - k^2])^(-1), ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/((-k^2)^(3/2
)*Sqrt[1 - k^2]*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2]) - ((1 - k^2 + Sq
rt[1 - k^2])*(1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticPi[(1 + Sqrt[
1 - k^2])^(-1), ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/((-k^2)^(3/2)*Sqrt[1
- k^2]*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 110

```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_S
ymbol] :> Simp[(2*Sqrt[e]*Rt[-(b/d), 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]
*Rt[-(b/d), 2])], (c*f)/(d*e)])/b, x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[d
*e - c*f, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !LtQ[-(b/d), 0]
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 714

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c*x)/b]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0]
 && RationalQ[b]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 922

```
Int[Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]/(((d_.) + (e_.)*(x_))*Sqrt[(f_.
) + (g_.)*(x_)]), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[1/((d
+ e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] - Dist[1/e^2, Int[(c*d
- b*e - c*e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d
^2 - b*d*e + a*e^2, 0]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-x+x^2}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(1-2x+k^2x^2\right)}\,dx = \int \frac{(-1+x)x}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(1-2x+k^2x^2\right)}\,dx$$

$$= \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{(-1+x)\sqrt{x}}{\sqrt{1-x}\sqrt{1-k^2x}\left(1-2x+k^2x^2\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= -\frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{\sqrt{1-x}\sqrt{x}}{\sqrt{1-k^2x}\left(1-2x+k^2x^2\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= -\frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{\sqrt{x-x^2}}{\sqrt{1-k^2x}\left(1-2x+k^2x^2\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= -\frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \left(-\frac{k^2\sqrt{x-x^2}}{\sqrt{1-k^2}\left(2+2\sqrt{1-k^2}-2k^2x\right)\sqrt{1-k^2x}} - \frac{}{\sqrt{1-k^2}\sqrt{1-\cdots}}\right.}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(k^2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{\sqrt{x-x^2}}{\left(2+2\sqrt{1-k^2}-2k^2x\right)\sqrt{1-k^2x}}\,dx}{\sqrt{1-k^2}\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(k^2\sqrt{1-x}\cdots\right.}{}$$

$$= -\frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{-2+2k^2-2\sqrt{1-k^2}-2k^2x}{\sqrt{1-k^2x}\sqrt{x-x^2}}\,dx}{4k^2\sqrt{1-k^2}\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{1-x}\sqrt{x}\sqrt{1-\cdots}\right.}{4k^2\sqrt{1-\cdots}}$$

$$= -\frac{\left(\left(2-k^2-\sqrt{1-k^2}\right)\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-k^2x}\sqrt{x-x^2}}\,dx}{2k^2\sqrt{1-k^2}\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(\left(\cdots\right.\right.}{}$$

$$= -\frac{\left(\left(2-k^2-\sqrt{1-k^2}\right)\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}}\,dx}{2k^2\sqrt{1-k^2}\sqrt{(1-x)x\left(1-k^2x\right)}} + \cdots$$

$$= -\frac{\left(2-k^2-\sqrt{1-k^2}\right)\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{k^2\sqrt{1-k^2}\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2-\cdots\right.}{}$$

$$= -\frac{\left(2-k^2-\sqrt{1-k^2}\right)\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{k^2\sqrt{1-k^2}\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2-\cdots\right.}{}$$

Mathematica [C] time = 2.22, size = 227, normalized size = 1.20

$$-\frac{i\sqrt{x-1}\sqrt{x}\sqrt{\frac{k^2x-1}{k^2-1}}\left(2\sqrt{1-k^2}\,F\left(i\sinh^{-1}\left(\sqrt{x-1}\right)|\frac{k^2}{k^2-1}\right)-\left(\sqrt{1-k^2}+1\right)\Pi\left(\frac{k^2}{k^2-\sqrt{1-k^2}-1};i\sinh^{-1}\left(\sqrt{x-1}\right)|\frac{k^2}{k^2-1}\right)-\left(\sqrt{1-k^2}-1\right)\Pi\left(\frac{k^2}{k^2+\sqrt{1-k^2}-1};i\sinh^{-1}\left(\sqrt{x-1}\right)|\frac{k^2}{k^2-1}\right)\right)}{k^2\sqrt{1-k^2}\sqrt{(x-1)x\left(k^2x-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(1 - 2*x + k^2*x^2)),x]
```

```
[Out] ((-I)*Sqrt[-1 + x]*Sqrt[x]*Sqrt[(-1 + k^2*x)/(-1 + k^2)]*(2*Sqrt[1 - k^2]*E
llipticF[I*ArcSinh[Sqrt[-1 + x]], k^2/(-1 + k^2)] - (1 + Sqrt[1 - k^2])*Ell
```

```
ipticPi[k^2/(-1 + k^2 - Sqrt[1 - k^2]), I*ArcSinh[Sqrt[-1 + x]], k^2/(-1 +
k^2)] - (-1 + Sqrt[1 - k^2])*EllipticPi[k^2/(-1 + k^2 + Sqrt[1 - k^2]), I*A
rcSinh[Sqrt[-1 + x]], k^2/(-1 + k^2)]))/(k^2*Sqrt[1 - k^2]*Sqrt[(-1 + x)*x*
(-1 + k^2*x)])
```

IntegrateAlgebraic [A] time = 0.55, size = 189, normalized size = 1.00

$$\frac{i\tan^{-1}\left(\frac{\sqrt{k^2+2i\sqrt{k^2-1}-2}\sqrt{k^2x^3+(-k^2-1)x^2+x}}{k^2x-1}\right)}{2\sqrt{k^2-1}\sqrt{k^2+2i\sqrt{k^2-1}-2}}-\frac{i\tan^{-1}\left(\frac{\sqrt{k^2-2i\sqrt{k^2-1}-2}\sqrt{k^2x^3+(-k^2-1)x^2+x}}{k^2x-1}\right)}{2\sqrt{k^2-1}\sqrt{k^2-2i\sqrt{k^2-1}-2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x + x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(1 - 2*x + k^2*x
^2)),x]
```

```
[Out] ((-1/2*I)*ArcTan[(Sqrt[-2 + k^2 - (2*I)*Sqrt[-1 + k^2]]*Sqrt[x + (-1 - k^2)
*x^2 + k^2*x^3])/(-1 + k^2*x)])/(Sqrt[-1 + k^2]*Sqrt[-2 + k^2 - (2*I)*Sqrt[
-1 + k^2]]) + ((I/2)*ArcTan[(Sqrt[-2 + k^2 + (2*I)*Sqrt[-1 + k^2]]*Sqrt[x +
 (-1 - k^2)*x^2 + k^2*x^3])/(-1 + k^2*x)])/(Sqrt[-1 + k^2]*Sqrt[-2 + k^2 +
(2*I)*Sqrt[-1 + k^2]])
```

fricas [A] time = 0.67, size = 467, normalized size = 2.47

$$\left[\frac{(k^2-1)\log\left(\frac{k^4x^4+4k^2x^3-2(3k^2+2)x^2-4\sqrt{k^2x^3-(k^2+1)x^2+x}(k^2x^2-1)+4x+1}{k^4x^4-4k^2x^3+2(k^2+2)x^2-4x+1}\right)-\sqrt{-k^2+1}\log\left(\frac{k^4x^4-4(2k^4-k^2)x^3+2(4k^4+k^2-2)x^2-4\sqrt{k^2x^3-(k^2+1)x^2+x}(k^2x^2-2k^2x+1)\sqrt{-k^2+1}-4(2k^2-1)x+1}{k^4x^4-4k^2x^3+2(k^2+2)x^2-4x+1}\right)}{4(k^4-k^2)}, \frac{(k^2-1)\log\left(\frac{k^4x^4+4k^2x^3-2(3k^2+2)x^2-4\sqrt{k^2x^3-(k^2+1)x^2+x}(k^2x^2-1)+4x+1}{k^4x^4-4k^2x^3+2(k^2+2)x^2-4x+1}\right)+2\sqrt{k^2-1}\arctan\left(\frac{\sqrt{k^2x^3-(k^2+1)x^2+x}(k^2x^2-2k^2x+1)\sqrt{k^2-1}}{2((k^4-k^2)x^3-(k^4-1)x^2+(k^2-1)x)}\right)}{4(k^4-k^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-2*x+1),x, algorithm="
fricas")
```

```
[Out] [1/4*((k^2 - 1)*log((k^4*x^4 + 4*k^2*x^3 - 2*(3*k^2 + 2)*x^2 - 4*sqrt(k^2*x
^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2 - 1) + 4*x + 1)/(k^4*x^4 - 4*k^2*x^3 + 2*(
k^2 + 2)*x^2 - 4*x + 1)) - sqrt(-k^2 + 1)*log((k^4*x^4 - 4*(2*k^4 - k^2)*x^
3 + 2*(4*k^4 + k^2 - 2)*x^2 - 4*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2
- 2*k^2*x + 1)*sqrt(-k^2 + 1) - 4*(2*k^2 - 1)*x + 1)/(k^4*x^4 - 4*k^2*x^3 +
 2*(k^2 + 2)*x^2 - 4*x + 1)))/(k^4 - k^2), 1/4*((k^2 - 1)*log((k^4*x^4 + 4*
k^2*x^3 - 2*(3*k^2 + 2)*x^2 - 4*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2
- 1) + 4*x + 1)/(k^4*x^4 - 4*k^2*x^3 + 2*(k^2 + 2)*x^2 - 4*x + 1)) + 2*sqrt
(k^2 - 1)*arctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2 - 2*k^2*x +
 1)*sqrt(k^2 - 1)/((k^4 - k^2)*x^3 - (k^4 - 1)*x^2 + (k^2 - 1)*x)))/(k^4 -
k^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-x}{(k^2x^2-2x+1)\sqrt{(k^2x-1)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-2*x+1),x, algorithm="
giac")
```

```
[Out] integrate((x^2 - x)/((k^2*x^2 - 2*x + 1)*sqrt((k^2*x - 1)*(x - 1)*x)), x)
```

maple [C] time = 0.06, size = 1121, normalized size = 5.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-2*x+1),x)
```

```
[Out] -2/k^4*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x
^3-k^2*x^2-x^2+x)^(1/2)*EllipticF((-(x-1/k^2)*k^2)^(1/2),(1/k^2/(1/k^2-1))^
(1/2))+1/k^2*(-2/(-k^2+1)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2
)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)*EllipticPi((-(x-1/k^2)*k^2)^(
1/2),1/k^2/(1/k^2-(1+(-k^2+1)^(1/2))/k^2),(1/k^2/(1/k^2-1))^(1/2))+2/(-k^2+
1)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^
3-k^2*x^2-x^2+x)^(1/2)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-(1+(-
k^2+1)^(1/2))/k^2),(1/k^2/(1/k^2-1))^(1/2))/k^2-1/(-k^2+1)^(1/2)*(-k^2*x+1)
^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2
+x)^(1/2)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-(1+(-k^2+1)^(1/2))
/k^2),(1/k^2/(1/k^2-1))^(1/2))+2/(-k^2+1)^(1/2)*(-k^2*x+1)^(1/2)*(1/(1/k^2-
1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)*Ellipti
cPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-(1+(-k^2+1)^(1/2))/k^2),(1/k^2/(1/k
^2-1))^(1/2))/k^2-2/(-k^2+1)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(
1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)*EllipticPi((-(x-1/k^2)*k^2
)^(1/2),1/k^2/(1/k^2+(-1+(-k^2+1)^(1/2))/k^2),(1/k^2/(1/k^2-1))^(1/2))+2/(-
k^2+1)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^
2*x^3-k^2*x^2-x^2+x)^(1/2)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+(
-1+(-k^2+1)^(1/2))/k^2),(1/k^2/(1/k^2-1))^(1/2))/k^2+1/(-k^2+1)^(1/2)*(-k^2
*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^
2-x^2+x)^(1/2)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+(-1+(-k^2+1)^
(1/2))/k^2),(1/k^2/(1/k^2-1))^(1/2))-2/(-k^2+1)^(1/2)*(-k^2*x+1)^(1/2)*(1/(
1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)*E
llipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+(-1+(-k^2+1)^(1/2))/k^2),(1/k
^2/(1/k^2-1))^(1/2))/k^2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-2*x+1),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(k-1>0)', see `assume?` for more det
ails)Is k-1 positive, negative or zero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x-x^2}{\left(k^2\,x^2-2\,x+1\right)\,\sqrt{x\,\left(k^2\,x-1\right)\,\left(x-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - x^2)/((k^2*x^2 - 2*x + 1)*(x*(k^2*x - 1)*(x - 1))^(1/2)),x)
```

```
[Out] int(-(x - x^2)/((k^2*x^2 - 2*x + 1)*(x*(k^2*x - 1)*(x - 1))^(1/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x)/((1-x)*x*(-k**2*x+1))**(1/2)/(k**2*x**2-2*x+1),x)
```

```
[Out] Timed out
```

3.1938 $\int \frac{1+x^2}{(-1+x^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

Optimal. Leaf size=189

$$-\frac{\sqrt{-2a-2b-c}\tan^{-1}\left(\frac{x\sqrt{-2a-2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}x^2-2\sqrt{a}x+\sqrt{a}}\right)}{2a+2b+c}-\frac{\sqrt{-2a+2b-c}\tan^{-1}\left(\frac{x\sqrt{-2a+2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}x^2+2}\right.}{2a-2b+c}$$

Rubi [F] time = 0.77, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x^2}{(-1+x^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^2)/((-1 + x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] Defer[Int][1/Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4], x] + Defer[Int][1/((-1
+ x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x] - Defer[Int][1/((1 + x)*Sqr
t[a + b*x + c*x^2 + b*x^3 + a*x^4]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{(-1+x^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx &= \int \left(\frac{1}{\sqrt{a+bx+cx^2+bx^3+ax^4}}+\frac{2}{(-1+x^2)\sqrt{a+bx+cx^2+b}}\right.\\
&= 2\int \frac{1}{(-1+x^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx+\int \frac{}{\sqrt{a+bx+c}}\\
&= 2\int \left(\frac{1}{2(-1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}-\frac{1}{2(1+x)\sqrt{a+bx}}\right.\\
&= \int \frac{1}{\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx+\int \frac{1}{(-1+x)\sqrt{a+bx+cx^2}}
\end{aligned}$$

Mathematica [C] time = 2.12, size = 3575, normalized size = 18.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2)/((-1 + x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] (2*(x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])^2*Sqrt[((Root[a +
b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 +
a*#1^4 & , 2])*(x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3]))/((x -
 Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 +
 b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3]))
]*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2
 + b*#1^3 + a*#1^4 & , 4])*Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#
1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#
1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(x - Root[a + b*#1 + c*#1^2 + b*#1^3 +
 a*#1^4 & , 4])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a +
 b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*
#1^3 + a*#1^4 & , 2])^2*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] -
```

```
Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])^2)]*(-((EllipticF[ArcSin[S
qrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 +
 c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4
 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a +
b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 +
a*#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Ro
ot[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*
#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((
-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 +
 b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] -
 Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*(-1 + Root[a + b*#1 +
c*#1^2 + b*#1^3 + a*#1^4 & , 1]) + EllipticPi[((-1 + Root[a + b*#1 + c*#1^2
 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1]
 - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((-1 + Root[a + b*#1 +
 c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4
 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])), ArcSin[Sqrt[((
x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^
2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4
]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 +
 c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4
 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a +
 b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 +
 a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((-Root[
a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^
3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[
a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*(-Root[a + b*#1 + c*#1^2 + b
*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2]))/(
(-1 + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(-1 + Root[a + b*#1
+ c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^
4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))) + (EllipticF[
ArcSin[Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a
+ b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3
+ a*#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(R
oot[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b
*#1^3 + a*#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & ,
 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#
1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & ,
 4]))/((-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 +
c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4
& , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*(1 + Root[a +
b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1]) + EllipticPi[((1 + Root[a + b*#1 +
c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4
& , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((1 + Root[a + b
*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a
*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])), ArcSin[Sq
rt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 +
c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4
& , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b
*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a
*#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Roo
t[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#
1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((-
Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 +
b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] -
Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*(-Root[a + b*#1 + c*#1^
2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2
]))/((1 + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(1 + Root[a + b*
#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*
#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])) + EllipticF
```

```
[ArcSin[Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a
 + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3
 + a*#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(
Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 +
b*#1^3 + a*#1^4 & , 4]))]], ((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & ,
2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1
^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & ,
4]))/((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*
#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
, 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))]/(-Root[a + b*#1 +
 c*#1^2 + b*#1^3 + a*#1^4 & , 2] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4
 & , 4])))/(Sqrt[x*(b + c*x + b*x^2) + a*(1 + x^4)]*(-Root[a + b*#1 + c*#1^
2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2
]))
```

IntegrateAlgebraic [A] time = 1.32, size = 189, normalized size = 1.00

$$-\frac{\sqrt{-2a-2b-c}\tan^{-1}\left(\frac{x\sqrt{-2a-2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}x^2-2\sqrt{a}x+\sqrt{a}}\right)}{2a+2b+c}-\frac{\sqrt{-2a+2b-c}\tan^{-1}\left(\frac{x\sqrt{-2a+2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}x^2+2\sqrt{a}x+\sqrt{a}}\right)}{2a-2b+c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 + x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x
^4]),x]
```

```
[Out] -((Sqrt[-2*a - 2*b - c]*ArcTan[(Sqrt[-2*a - 2*b - c]*x)/(Sqrt[a] - 2*Sqrt[a
]*x + Sqrt[a]*x^2 - Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(2*a + 2*b + c
)) - (Sqrt[-2*a + 2*b - c]*ArcTan[(Sqrt[-2*a + 2*b - c]*x)/(Sqrt[a] + 2*Sqr
t[a]*x + Sqrt[a]*x^2 - Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(2*a - 2*b
+ c)
```

fricas [B] time = 0.98, size = 1659, normalized size = 8.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/4*((2*a + 2*b + c)*sqrt(2*a - 2*b + c)*log(((24*a^2 - 16*a*b + b^2 + 4*a
*c)*x^4 + 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x^3 + 2*(24*a^2 + 3*b^2
 - 4*(a + 2*b)*c + 4*c^2)*x^2 + 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4
*a - b)*x^2 + 2*(2*a + b - c)*x + 4*a - b)*sqrt(2*a - 2*b + c) + 24*a^2 - 1
6*a*b + b^2 + 4*a*c + 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x)/(x^4 + 4
*x^3 + 6*x^2 + 4*x + 1)) + sqrt(2*a + 2*b + c)*(2*a - 2*b + c)*log(((24*a^2
 + 16*a*b + b^2 + 4*a*c)*x^4 - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(2*a + b)*c)*x^
3 + 2*(24*a^2 + 3*b^2 - 4*(a - 2*b)*c + 4*c^2)*x^2 - 4*sqrt(a*x^4 + b*x^3 +
 c*x^2 + b*x + a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(2*a +
2*b + c) + 24*a^2 + 16*a*b + b^2 + 4*a*c - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(2*
a + b)*c)*x)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)))/(4*a^2 - 4*b^2 + 4*a*c + c^2
), 1/4*(2*(2*a - 2*b + c)*sqrt(-2*a - 2*b - c)*arctan(1/2*sqrt(a*x^4 + b*x^
3 + c*x^2 + b*x + a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(-2*
a - 2*b - c)/((2*a^2 + 2*a*b + a*c)*x^4 + (2*a*b + 2*b^2 + b*c)*x^3 + (2*(a
 + b)*c + c^2)*x^2 + 2*a^2 + 2*a*b + a*c + (2*a*b + 2*b^2 + b*c)*x)) + (2*a
 + 2*b + c)*sqrt(2*a - 2*b + c)*log(((24*a^2 - 16*a*b + b^2 + 4*a*c)*x^4 +
4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a +
2*b)*c + 4*c^2)*x^2 + 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x^
2 + 2*(2*a + b - c)*x + 4*a - b)*sqrt(2*a - 2*b + c) + 24*a^2 - 16*a*b + b^
2 + 4*a*c + 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x)/(x^4 + 4*x^3 + 6*x
^2 + 4*x + 1)))/(4*a^2 - 4*b^2 + 4*a*c + c^2), 1/4*(2*(2*a + 2*b + c)*sqrt(
-2*a + 2*b - c)*arctan(-1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b
```

```
)*x^2 + 2*(2*a + b - c)*x + 4*a - b)*sqrt(-2*a + 2*b - c)/((2*a^2 - 2*a*b +
 a*c)*x^4 + (2*a*b - 2*b^2 + b*c)*x^3 + (2*(a - b)*c + c^2)*x^2 + 2*a^2 - 2
*a*b + a*c + (2*a*b - 2*b^2 + b*c)*x)) + sqrt(2*a + 2*b + c)*(2*a - 2*b + c
)*log(((24*a^2 + 16*a*b + b^2 + 4*a*c)*x^4 - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(
2*a + b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a - 2*b)*c + 4*c^2)*x^2 - 4*sqrt(a
*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a +
b)*sqrt(2*a + 2*b + c) + 24*a^2 + 16*a*b + b^2 + 4*a*c - 4*(8*a^2 - 4*a*b -
 3*b^2 - 2*(2*a + b)*c)*x)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)))/(4*a^2 - 4*b^2
 + 4*a*c + c^2), 1/2*((2*a + 2*b + c)*sqrt(-2*a + 2*b - c)*arctan(-1/2*sqrt
(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x^2 + 2*(2*a + b - c)*x + 4*a
- b)*sqrt(-2*a + 2*b - c)/((2*a^2 - 2*a*b + a*c)*x^4 + (2*a*b - 2*b^2 + b*c
)*x^3 + (2*(a - b)*c + c^2)*x^2 + 2*a^2 - 2*a*b + a*c + (2*a*b - 2*b^2 + b*
c)*x)) + (2*a - 2*b + c)*sqrt(-2*a - 2*b - c)*arctan(1/2*sqrt(a*x^4 + b*x^3
 + c*x^2 + b*x + a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(-2*a
 - 2*b - c)/((2*a^2 + 2*a*b + a*c)*x^4 + (2*a*b + 2*b^2 + b*c)*x^3 + (2*(a
+ b)*c + c^2)*x^2 + 2*a^2 + 2*a*b + a*c + (2*a*b + 2*b^2 + b*c)*x)))/(4*a^2
 - 4*b^2 + 4*a*c + c^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{ax^4+bx^3+cx^2+bx+a}\,(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((x^2 + 1)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(x^2 - 1)), x)
```

maple [C] time = 0.03, size = 4541, normalized size = 24.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 2*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c
+_Z*b+a,index=1))*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4
*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,ind
ex=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z
^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2
)*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))^2*((RootOf(_Z^4*a+_Z^3*b+
_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-Root
Of(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+
a,index=3)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z
^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,in
dex=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-RootOf(_Z^4*a+_Z^3*b
+_Z^2*c+_Z*b+a,index=4))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootO
f(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b
+a,index=2)))^(1/2)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^
4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,inde
x=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(a*x^4+b*x^3+c*x^2+b*x+a)
^(1/2)*EllipticF(((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*
a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,inde
x=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^
2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)
,((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+
_Z*b+a,index=3))*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*
a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index
```

=3)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^
2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2))+2*
(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_
Z*b+a,index=1))*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a
+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index
=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2
*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)*
(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))^2*((RootOf(_Z^4*a+_Z^3*b+_Z
^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-RootOf
(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,
index=3)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3
*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,inde
x=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-RootOf(_Z^4*a+_Z^3*b+_
Z^2*c+_Z*b+a,index=4))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(
_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a
,index=2)))^(1/2)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*
a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=
2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(a*x^4+b*x^3+c*x^2+b*x+a)^(
1/2)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-1)*(EllipticF(((RootOf(_Z
^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index
=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+
_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-Root
Of(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2),((RootOf(_Z^4*a+_Z^3*b+_Z^2
*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))*(-RootOf(_Z
^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index
=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3)+RootOf(_Z^4*a+_Z^3*b+_Z^
2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_
Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2))+(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_
Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+
_Z^3*b+_Z^2*c+_Z*b+a,index=1)-1)*EllipticPi(((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_
Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*
a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=
4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^
2*c+_Z*b+a,index=2)))^(1/2),(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-1)
*(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_
Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1)-1)/(RootOf(_Z^
4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=
2)),((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2
*c+_Z*b+a,index=3))*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z
^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,in
dex=3)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+
_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2))
)-2*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2
*c+_Z*b+a,index=1))*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z
^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,i
ndex=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+
_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1
/2)*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))^2*((RootOf(_Z^4*a+_Z^3*
b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-Ro
otOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*
b+a,index=3)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+
_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,
index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-RootOf(_Z^4*a+_Z^3
*b+_Z^2*c+_Z*b+a,index=4))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-Roo
tOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z
*b+a,index=2)))^(1/2)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_
Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,in
dex=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(a*x^4+b*x^3+c*x^2+b*x+
a)^(1/2)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)+1)*(EllipticF(((RootO

```
f(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,i
ndex=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^
3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-
RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2),((RootOf(_Z^4*a+_Z^3*b+
_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))*(-RootO
f(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,i
ndex=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3)+RootOf(_Z^4*a+_Z^3*b
+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+Root
Of(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2))+(RootOf(_Z^4*a+_Z^3*b+_Z^2
*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^
4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1)+1)*EllipticPi(((RootOf(_Z^4*a+_Z^3*b+_Z^2
*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_
Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,in
dex=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b
+_Z^2*c+_Z*b+a,index=2)))^(1/2),(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2
)+1)*(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2
*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1)+1)/(RootOf
(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,in
dex=2)),((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+
_Z^2*c+_Z*b+a,index=3))*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootO
f(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+
a,index=3)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^
3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1
/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{ax^4+bx^3+cx^2+bx+a}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((x^2 + 1)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+1}{\left(x^2-1\right)\,\sqrt{a\,x^4+b\,x^3+c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((x^2 - 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)),x)
```

```
[Out] int((x^2 + 1)/((x^2 - 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{(x-1)\,(x+1)\,\sqrt{ax^4+a+bx^3+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-1)/(a*x**4+b*x**3+c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)/((x - 1)*(x + 1)*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**
2)), x)
```

3.1939 $\int \frac{(-2q+px^3)\sqrt{q+px^3}}{cx^4+bx^2(q+px^3)+a(q+px^3)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{\sqrt{2}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{x\sqrt{b-\sqrt{b^2-4ac}}}{\sqrt{2}\sqrt{a}\sqrt{px^3+q}}\right)}{\sqrt{a}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b}}{\sqrt{2}\sqrt{a}\sqrt{px^3+q}}\right)}{\sqrt{a}\sqrt{b^2-4ac}}$$

Rubi [F] time = 1.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2q+px^3)\sqrt{q+px^3}}{cx^4+bx^2(q+px^3)+a(q+px^3)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*q + p*x^3)*Sqrt[q + p*x^3])/(c*x^4 + b*x^2*(q + p*x^3) + a*(q + p*
x^3)^2),x]
```

```
[Out] -2*q*Defer[Int][Sqrt[q + p*x^3]/(a*q^2 + b*q*x^2 + 2*a*p*q*x^3 + c*x^4 + b*
p*x^5 + a*p^2*x^6), x] + p*Defer[Int][(x^3*Sqrt[q + p*x^3])/(a*q^2 + b*q*x^
2 + 2*a*p*q*x^3 + c*x^4 + b*p*x^5 + a*p^2*x^6), x]
```

Rubi steps

$$\int \frac{(-2q+px^3)\sqrt{q+px^3}}{cx^4+bx^2(q+px^3)+a(q+px^3)^2}\,dx=\int\left(-\frac{2q\sqrt{q+px^3}}{aq^2+bqx^2+2apqx^3+cx^4+bpx^5+ap^2x^6}+\frac{}{aq^2+bqx^2+2}\right.$$

$$=p\int\frac{x^3\sqrt{q+px^3}}{aq^2+bqx^2+2apqx^3+cx^4+bpx^5+ap^2x^6}\,dx-(2q)\int\frac{}{aq^2+}$$

Mathematica [C] time = 6.68, size = 16759, normalized size = 88.67

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-2*q + p*x^3)*Sqrt[q + p*x^3])/(c*x^4 + b*x^2*(q + p*x^3) + a*(
q + p*x^3)^2),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 1.65, size = 189, normalized size = 1.00

$$-\frac{\sqrt{2}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{x\sqrt{b-\sqrt{b^2-4ac}}}{\sqrt{2}\sqrt{a}\sqrt{px^3+q}}\right)}{\sqrt{a}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b}}{\sqrt{2}\sqrt{a}\sqrt{px^3+q}}\right)}{\sqrt{a}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*q + p*x^3)*Sqrt[q + p*x^3])/(c*x^4 + b*x^2*(q + p*x
^3) + a*(q + p*x^3)^2),x]
```

```
[Out] -((Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b - Sqrt[b^2 - 4*a*c]]*x)/
(Sqrt[2]*Sqrt[a]*Sqrt[q + p*x^3])])/(Sqrt[a]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqr
t[b^2 - 4*a*c]])) - (Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b + S
qrt[b^2 - 4*a*c]]*x)/(Sqrt[2]*Sqrt[a]*Sqrt[q + p*x^3])])/(Sqrt[a]*Sqrt[b^2
- 4*a*c])
```

fricas [B] time = 4.53, size = 1321, normalized size = 6.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p*x^3+q)^(1/2)/(c*x^4+b*x^2*(p*x^3+q)+a*(p*x^3+q)^2)
,x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 -
 4*a^2*c))*log((2*a*p^2*x^6 + 4*a*p*q*x^3 - 2*c*x^4 + 2*a*q^2 + sqrt(2)*((b
^2 - 4*a*c)*x^3 - (2*(a^2*b^2 - 4*a^3*c)*p*x^4 + (a*b^3 - 4*a^2*b*c)*x^3 +
2*(a^2*b^2 - 4*a^3*c)*q*x)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(p*x^3 + q)*sqrt(-(
b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c)) - 2*((a*b
^2 - 4*a^2*c)*p*x^5 + (a*b^2 - 4*a^2*c)*q*x^2)/sqrt(a^2*b^2 - 4*a^3*c))/(a*
p^2*x^6 + b*p*x^5 + 2*a*p*q*x^3 + c*x^4 + b*q*x^2 + a*q^2)) + 1/4*sqrt(2)*s
qrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log
((2*a*p^2*x^6 + 4*a*p*q*x^3 - 2*c*x^4 + 2*a*q^2 - sqrt(2)*((b^2 - 4*a*c)*x^
3 - (2*(a^2*b^2 - 4*a^3*c)*p*x^4 + (a*b^3 - 4*a^2*b*c)*x^3 + 2*(a^2*b^2 - 4
*a^3*c)*q*x)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(p*x^3 + q)*sqrt(-(b + (a*b^2 - 4
*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c)) - 2*((a*b^2 - 4*a^2*c)*
p*x^5 + (a*b^2 - 4*a^2*c)*q*x^2)/sqrt(a^2*b^2 - 4*a^3*c))/(a*p^2*x^6 + b*p*
x^5 + 2*a*p*q*x^3 + c*x^4 + b*q*x^2 + a*q^2)) - 1/4*sqrt(2)*sqrt(-(b - (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log((2*a*p^2*x^6
+ 4*a*p*q*x^3 - 2*c*x^4 + 2*a*q^2 + sqrt(2)*((b^2 - 4*a*c)*x^3 + (2*(a^2*b^
2 - 4*a^3*c)*p*x^4 + (a*b^3 - 4*a^2*b*c)*x^3 + 2*(a^2*b^2 - 4*a^3*c)*q*x)/s
qrt(a^2*b^2 - 4*a^3*c))*sqrt(p*x^3 + q)*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a
^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c)) + 2*((a*b^2 - 4*a^2*c)*p*x^5 + (a*b^2
 - 4*a^2*c)*q*x^2)/sqrt(a^2*b^2 - 4*a^3*c))/(a*p^2*x^6 + b*p*x^5 + 2*a*p*q*
x^3 + c*x^4 + b*q*x^2 + a*q^2)) + 1/4*sqrt(2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/
sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log((2*a*p^2*x^6 + 4*a*p*q*x^3
- 2*c*x^4 + 2*a*q^2 - sqrt(2)*((b^2 - 4*a*c)*x^3 + (2*(a^2*b^2 - 4*a^3*c)*p
*x^4 + (a*b^3 - 4*a^2*b*c)*x^3 + 2*(a^2*b^2 - 4*a^3*c)*q*x)/sqrt(a^2*b^2 -
4*a^3*c))*sqrt(p*x^3 + q)*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3
*c))/(a*b^2 - 4*a^2*c)) + 2*((a*b^2 - 4*a^2*c)*p*x^5 + (a*b^2 - 4*a^2*c)*q*
x^2)/sqrt(a^2*b^2 - 4*a^3*c))/(a*p^2*x^6 + b*p*x^5 + 2*a*p*q*x^3 + c*x^4 +
b*q*x^2 + a*q^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p*x^3+q)^(1/2)/(c*x^4+b*x^2*(p*x^3+q)+a*(p*x^3+q)^2)
,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 0.45, size = 1362, normalized size = 7.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^3-2*q)*(p*x^3+q)^(1/2)/(c*x^4+b*x^2*(p*x^3+q)+a*(p*x^3+q)^2),x)
```

```
[Out] -2/3*I/a*3^(1/2)/p*(-q*p^2)^(1/3)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I*3^(1/2)/
p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2)*((x-1/p*(-q*p^2)^(1/3))/(
```

```
-3/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2)*(-I*(x+1/2/p*(
-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/
2)/(p*x^3+q)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I*3
^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-q*p^2)^(1/3))^(1/2),(I*3^(1/2)/p*(-q*p
^2)^(1/3)/(-3/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2))-I/
a/p^2/q^2/c*2^(1/2)*sum((_alpha^5*b*p+3*_alpha^3*a*p*q+_alpha^4*c+_alpha^2*
b*q+3*a*q^2)/_alpha/(6*_alpha^4*a*p^2+5*_alpha^3*b*p+6*_alpha*a*p*q+4*_alph
a^2*c+2*b*q)*(-q*p^2)^(1/3)*(1/2*I*p*(2*x+1/p*(-I*3^(1/2)*(-q*p^2)^(1/3)+(-
q*p^2)^(1/3)))/(-q*p^2)^(1/3))^(1/2)*(p*(x-1/p*(-q*p^2)^(1/3))/(-3*(-q*p^2)
^(1/3)+I*3^(1/2)*(-q*p^2)^(1/3)))^(1/2)*(-1/2*I*p*(2*x+1/p*(I*3^(1/2)*(-q*p
^2)^(1/3)+(-q*p^2)^(1/3)))/(-q*p^2)^(1/3))^(1/2)/(p*x^3+q)^(1/2)*(2*q*p^2*(
_alpha^4*a*p^2+_alpha^3*b*p+_alpha*a*p*q+_alpha^2*c)+I*(-q*p^2)^(1/3)*3^(1/
2)*a*q^2*p^2+I*(-q*p^2)^(2/3)*3^(1/2)*_alpha^2*a*q*p^2+(-q*p^2)^(2/3)*_alph
a^5*a*p^3+I*(-q*p^2)^(1/3)*p^3*3^(1/2)*_alpha^3*a*q+I*(-q*p^2)^(2/3)*3^(1/2
)*_alpha^4*b*p^2+I*(-q*p^2)^(2/3)*3^(1/2)*_alpha^3*p*c+(-q*p^2)^(2/3)*_alph
a^4*b*p^2-I*(-q*p^2)^(2/3)*3^(1/2)*q*c-(-q*p^2)^(1/3)*_alpha^3*a*p^3*q+(-q*
p^2)^(2/3)*_alpha^2*a*p^2*q+I*(-q*p^2)^(2/3)*p^3*3^(1/2)*_alpha^5*a+(-q*p^2
)^(2/3)*_alpha^3*c*p+I*(-q*p^2)^(1/3)*3^(1/2)*_alpha*q*p*c-(-q*p^2)^(1/3)*_
alpha^2*b*p^2*q+I*(-q*p^2)^(1/3)*3^(1/2)*_alpha^2*b*q*p^2-(-q*p^2)^(1/3)*a*
p^2*q^2-(-q*p^2)^(1/3)*_alpha*c*p*q-(-q*p^2)^(2/3)*c*q)*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I*3^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-
q*p^2)^(1/3))^(1/2),1/2/p/q*(I*3^(1/2)*_alpha^3*c*p^2+I*(-q*p^2)^(2/3)*3^(1
/2)*_alpha*c-2*I*(-q*p^2)^(1/3)*3^(1/2)*_alpha^3*b*p^2+I*(-q*p^2)^(2/3)*3^(
1/2)*a*p*q-3*p^4*_alpha^5*a-I*3^(1/2)*c*p*q+I*3^(1/2)*_alpha^5*a*p^4+I*(-q*
p^2)^(2/3)*3^(1/2)*_alpha^2*b*p+3*(-q*p^2)^(2/3)*_alpha^3*a*p^2-2*I*(-q*p^2
)^(1/3)*3^(1/2)*_alpha^4*a*p^3+I*(-q*p^2)^(2/3)*3^(1/2)*_alpha^3*a*p^2-3*b*
p^3*_alpha^4+I*3^(1/2)*_alpha^4*b*p^3-2*I*(-q*p^2)^(1/3)*3^(1/2)*_alpha^2*c
*p-3*p^3*_alpha^2*a*q-2*I*(-q*p^2)^(1/3)*3^(1/2)*_alpha*a*p^2*q+3*(-q*p^2)^
(2/3)*_alpha^2*b*p-3*_alpha^3*p^2*c+3*(-q*p^2)^(2/3)*a*q*p+I*3^(1/2)*_alpha
^2*a*p^3*q+3*(-q*p^2)^(2/3)*_alpha*c+3*q*c*p)/c,(I*3^(1/2)/p*(-q*p^2)^(1/3)
/(-3/2/p*(-q*p^2)^(1/3)+1/2*I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2)),_alpha=Root
Of(_Z^6*a*p^2+_Z^5*b*p+2*_Z^3*a*p*q+_Z^4*c+_Z^2*b*q+a*q^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^3+q}\left(px^3-2\,q\right)}{cx^4+\left(px^3+q\right)bx^2+\left(px^3+q\right)^2a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p*x^3+q)^(1/2)/(c*x^4+b*x^2*(p*x^3+q)+a*(p*x^3+q)^2)
,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(p*x^3 + q)*(p*x^3 - 2*q)/(c*x^4 + (p*x^3 + q)*b*x^2 + (p*x^3
 + q)^2*a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{p\,x^3+q}\,\left(2\,q-p\,x^3\right)}{a\,{\left(p\,x^3+q\right)}^2+c\,x^4+b\,x^2\,\left(p\,x^3+q\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q + p*x^3)^(1/2)*(2*q - p*x^3))/(a*(q + p*x^3)^2 + c*x^4 + b*x^2*(q
+ p*x^3)),x)
```

```
[Out] int(-((q + p*x^3)^(1/2)*(2*q - p*x^3))/(a*(q + p*x^3)^2 + c*x^4 + b*x^2*(q
+ p*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(px^3 - 2q\right)\sqrt{px^3 + q}}{ap^2x^6 + 2apqx^3 + aq^2 + bpx^5 + bqx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**3-2*q)*(p*x**3+q)**(1/2)/(c*x**4+b*x**2*(p*x**3+q)+a*(p*x**
3+q)**2),x)
```

```
[Out] Integral((p*x**3 - 2*q)*sqrt(p*x**3 + q)/(a*p**2*x**6 + 2*a*p*q*x**3 + a*q*
*2 + b*p*x**5 + b*q*x**2 + c*x**4), x)
```

3.1940 $\int \frac{(-1+x^4)^2}{(1+x^4)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal. Leaf size=189

$$-\frac{3}{2}\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)}{\sqrt{2}}+\frac{(6x^8+12x^4+4)x^2+\sqrt{x^4+1}\,(6x^6}{2x(2x^8+3x^4+1)\sqrt{\sqrt{x^4+1}+x^2}+2x\sqrt{x^4+1}\sqrt{}}$$

Rubi [F] time = 2.75, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^4)^2}{(1+x^4)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x^4)^2/((1 + x^4)^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] -4*ArcTanh[x/Sqrt[x^2 + Sqrt[1 + x^4]]] + Defer[Int][1/Sqrt[x^2 + Sqrt[1 +
x^4]], x] + (I/4)*Defer[Int][1/(((-1)^(1/4) - x)^2*Sqrt[x^2 + Sqrt[1 + x^4]
]), x] + (3*(-1)^(1/4)*Defer[Int][1/(((-1)^(1/4) - x)*Sqrt[x^2 + Sqrt[1 + x
^4]]), x])/4 - (I/4)*Defer[Int][1/((-(-1)^(3/4) - x)^2*Sqrt[x^2 + Sqrt[1 +
x^4]]), x] - (3*(-1)^(3/4)*Defer[Int][1/((-(-1)^(3/4) - x)*Sqrt[x^2 + Sqrt[
1 + x^4]]), x])/4 + (I/4)*Defer[Int][1/(((-1)^(1/4) + x)^2*Sqrt[x^2 + Sqrt[
1 + x^4]]), x] + (3*(-1)^(1/4)*Defer[Int][1/(((-1)^(1/4) + x)*Sqrt[x^2 + Sq
rt[1 + x^4]]), x])/4 - (I/4)*Defer[Int][1/((-(-1)^(3/4) + x)^2*Sqrt[x^2 + S
qrt[1 + x^4]]), x] - (3*(-1)^(3/4)*Defer[Int][1/((-(-1)^(3/4) + x)*Sqrt[x^2
 + Sqrt[1 + x^4]]), x])/4
```

Rubi steps

$$\int \frac{\left(-1+x^4\right)^2}{\left(1+x^4\right)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx = \int \left(\frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}+\frac{4}{\left(1+x^4\right)^2\sqrt{x^2+\sqrt{1+x^4}}}-\frac{4}{\left(1+x^4\right)\sqrt{x^2+\sqrt{1+x^4}}}\right)$$

$$= 4\int \frac{1}{\left(1+x^4\right)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx - 4\int \frac{1}{\left(1+x^4\right)\sqrt{x^2+\sqrt{1+x^4}}}\,dx + \int \frac{}{\sqrt{x}}$$

$$= 4\int \left(-\frac{1}{4\left(i-x^2\right)^2\sqrt{x^2+\sqrt{1+x^4}}}-\frac{1}{4\left(i+x^2\right)^2\sqrt{x^2+\sqrt{1+x^4}}}-\frac{}{2\left(-1-x^4\right)}\right.$$

$$= -4\tanh^{-1}\left(\frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\right) - 2\int \frac{1}{\left(-1-x^4\right)\sqrt{x^2+\sqrt{1+x^4}}}\,dx + \int \frac{}{\sqrt{x^2+}}$$

$$= -4\tanh^{-1}\left(\frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\right) - 2\int \left(-\frac{i}{2\left(i-x^2\right)\sqrt{x^2+\sqrt{1+x^4}}}-\frac{}{2\left(i+x^2\right)}\right.$$

$$= -4\tanh^{-1}\left(\frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\right) + \frac{1}{4}i\int \frac{1}{\left(\sqrt[4]{-1}-x\right)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx - \frac{1}{4}i\int$$

$$= -4\tanh^{-1}\left(\frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\right) + \frac{1}{4}i\int \frac{1}{\left(\sqrt[4]{-1}-x\right)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx - \frac{1}{4}i\int$$

$$= -4\tanh^{-1}\left(\frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\right) + \frac{1}{4}i\int \frac{1}{\left(\sqrt[4]{-1}-x\right)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx - \frac{1}{4}i\int$$

Mathematica [F] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^4\right)^2}{\left(1+x^4\right)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^4)^2/((1 + x^4)^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
[Out] Integrate[(-1 + x^4)^2/((1 + x^4)^2*Sqrt[x^2 + Sqrt[1 + x^4]]), x]
```

IntegrateAlgebraic [A] time = 0.77, size = 202, normalized size = 1.07

$$-\frac{3}{2}\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)+\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}+\frac{\left(6x^8+12x^4+4\right)x^2+\sqrt{x^4+1}\left(6x^6+9x^2\right)x^2}{2x\left(2x^8+3x^4+1\right)\sqrt{\sqrt{x^4+1}+x^2}+2x\sqrt{x^4+1}\sqrt{\sqrt{x^4+1}+x^2}\left(2x^6+2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)^2/((1 + x^4)^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
[Out] (x^2*Sqrt[1 + x^4]*(9*x^2 + 6*x^6) + x^2*(4 + 12*x^4 + 6*x^8))/(2*x*Sqrt[1
+ x^4]*(2*x^2 + 2*x^6)*Sqrt[x^2 + Sqrt[1 + x^4]] + 2*x*(1 + 3*x^4 + 2*x^8)*
Sqrt[x^2 + Sqrt[1 + x^4]]) - (3*ArcTanh[x/Sqrt[x^2 + Sqrt[1 + x^4]]])/2 + A
rcTanh[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2])/(x*Sqrt[x^2 + S
qrt[1 + x^4]])]/Sqrt[2]
```

fricas [A] time = 1.03, size = 180, normalized size = 0.95

$$\frac{\sqrt{2}\left(x^4+1\right)\log\left(4x^4+4\sqrt{x^4+1}\,x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)+3\left(x^4+1\right)\log\left(-\frac{9x^4+8\sqrt{x^4+1}\,x^2-4\left(2x^3+\sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}}+1}{x^4+1}\right)-4\left(x^7+3x^3-\left(x^5+4x\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{8\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^2/(x^4+1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/8*(sqrt(2)*(x^4 + 1)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + s
qrt(2)*sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1) + 3*(x^4 + 1)*log(-(
9*x^4 + 8*sqrt(x^4 + 1)*x^2 - 4*(2*x^3 + sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x
^4 + 1)) + 1)/(x^4 + 1)) - 4*(x^7 + 3*x^3 - (x^5 + 4*x)*sqrt(x^4 + 1))*sqrt
(x^2 + sqrt(x^4 + 1)))/(x^4 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)^2}{\left(x^4+1\right)^2\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^2/(x^4+1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((x^4 - 1)^2/((x^4 + 1)^2*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)^2}{\left(x^4+1\right)^2\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^2/(x^4+1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int((x^4-1)^2/(x^4+1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)^2}{\left(x^4+1\right)^2\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^2/(x^4+1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((x^4 - 1)^2/((x^4 + 1)^2*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^4-1\right)^2}{\left(x^4+1\right)^2\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)^2/((x^4 + 1)^2*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

```
[Out] int((x^4 - 1)^2/((x^4 + 1)^2*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)^2 (x+1)^2 \left(x^2+1\right)^2}{\sqrt{x^2+\sqrt{x^4+1}} \left(x^4+1\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**2/(x**4+1)**2/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] Integral((x - 1)**2*(x + 1)**2*(x**2 + 1)**2/(sqrt(x**2 + sqrt(x**4 + 1))*(
x**4 + 1)**2), x)
```

3.1941 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^2\right)^2 \sqrt{1+x^4}}\,dx$

Optimal. Leaf size=189

$$\frac{-\left(\left(x^2-1\right)x^2\right)-\sqrt{x^4+1}\,x^2}{4x\left(x^2+1\right)\sqrt{\sqrt{x^4+1}+x^2}}+\frac{1}{4}\sqrt{5\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)+\frac{1}{4}\sqrt{1+5\sqrt{2}}\,\tanh^{-1}\left(\sqrt{2}\right.$$

Rubi [C] time = 1.46, antiderivative size = 405, normalized size of antiderivative = 2.14, number of steps used = 28, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6742, 2133, 731, 725, 206, 6725}

$$\frac{i\sqrt{1-ix^2}}{8(-x+i)}-\frac{i\sqrt{1-ix^2}}{8(x+i)}-\frac{i\sqrt{1+ix^2}}{8(-x+i)}+\frac{i\sqrt{1+ix^2}}{8(x+i)}-\frac{1}{8}\sqrt{1+i}\,\tanh^{-1}\left(\frac{1-x}{\sqrt{1+i}\sqrt{1-ix^2}}\right)+\frac{\tanh^{-1}\left(\frac{1-x}{\sqrt{1+i}\sqrt{1-ix^2}}\right)}{4(1+i)^{5/2}}+\frac{1}{8}\sqrt{1+i}\,\tanh^{-1}\left(\frac{x+1}{\sqrt{1+i}\sqrt{1-ix^2}}\right)-\frac{\tanh^{-1}\left(\frac{x+1}{\sqrt{1+i}\sqrt{1-ix^2}}\right)}{4(1+i)^{5/2}}-\frac{1}{8}\sqrt{1-i}\,\tanh^{-1}\left(\frac{1-x}{\sqrt{1-i}\sqrt{1+ix^2}}\right)+\frac{\tanh^{-1}\left(\frac{1-x}{\sqrt{1-i}\sqrt{1+ix^2}}\right)}{4(1-i)^{5/2}}+\frac{1}{8}\sqrt{1-i}\,\tanh^{-1}\left(\frac{x+1}{\sqrt{1-i}\sqrt{1+ix^2}}\right)-\frac{\tanh^{-1}\left(\frac{x+1}{\sqrt{1-i}\sqrt{1+ix^2}}\right)}{4(1-i)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x^2)^2*Sqrt[1 + x^4]),x]
```

```
[Out] ((I/8)*Sqrt[1 - I*x^2])/(I - x) - ((I/8)*Sqrt[1 - I*x^2])/(I + x) - ((I/8)*
Sqrt[1 + I*x^2])/(I - x) + ((I/8)*Sqrt[1 + I*x^2])/(I + x) + ArcTanh[(1 - x
)/(Sqrt[1 + I]*Sqrt[1 - I*x^2])]/(4*(1 + I)^(5/2)) - (Sqrt[1 + I]*ArcTanh[(
1 - x)/(Sqrt[1 + I]*Sqrt[1 - I*x^2])])/8 - ArcTanh[(1 + x)/(Sqrt[1 + I]*Sqr
t[1 - I*x^2])]/(4*(1 + I)^(5/2)) + (Sqrt[1 + I]*ArcTanh[(1 + x)/(Sqrt[1 + I
]*Sqrt[1 - I*x^2])])/8 + ArcTanh[(1 - x)/(Sqrt[1 - I]*Sqrt[1 + I*x^2])]/(4*
(1 - I)^(5/2)) - (Sqrt[1 - I]*ArcTanh[(1 - x)/(Sqrt[1 - I]*Sqrt[1 + I*x^2])
])/8 - ArcTanh[(1 + x)/(Sqrt[1 - I]*Sqrt[1 + I*x^2])]/(4*(1 - I)^(5/2)) + (
Sqrt[1 - I]*ArcTanh[(1 + x)/(Sqrt[1 - I]*Sqrt[1 + I*x^2])])/8
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^2\right)^2\sqrt{1+x^4}}\,dx = \int \left(-\frac{\sqrt{x^2+\sqrt{1+x^4}}}{4(i-x)^2\sqrt{1+x^4}} - \frac{\sqrt{x^2+\sqrt{1+x^4}}}{4(i+x)^2\sqrt{1+x^4}} - \frac{\sqrt{x^2+\sqrt{1+x^4}}}{2\left(-1-x^2\right)\sqrt{1+x^4}}\right) dx$$

$$= -\left(\frac{1}{4}\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(i-x)^2\sqrt{1+x^4}}\,dx\right) - \frac{1}{4}\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(i+x)^2\sqrt{1+x^4}}\,dx - \frac{1}{2}\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(-1-x^2\right)\sqrt{1+x^4}}\,dx$$

$$= -\left(\left(\frac{1}{8}-\frac{i}{8}\right)\int \frac{1}{(i-x)^2\sqrt{1-ix^2}}\,dx\right) - \left(\frac{1}{8}-\frac{i}{8}\right)\int \frac{1}{(i+x)^2\sqrt{1-ix^2}}\,dx - \left(\frac{1}{8}+\frac{i}{8}\right)\int \frac{}{(i}$$

$$= \frac{i\sqrt{1-ix^2}}{8(i-x)} - \frac{i\sqrt{1-ix^2}}{8(i+x)} - \frac{i\sqrt{1+ix^2}}{8(i-x)} + \frac{i\sqrt{1+ix^2}}{8(i+x)} + \frac{1}{8}i\int \frac{1}{(i-x)\sqrt{1-ix^2}}\,dx + \frac{1}{8}i\int$$

$$= \frac{i\sqrt{1-ix^2}}{8(i-x)} - \frac{i\sqrt{1-ix^2}}{8(i+x)} - \frac{i\sqrt{1+ix^2}}{8(i-x)} + \frac{i\sqrt{1+ix^2}}{8(i+x)} + \left(-\frac{1}{8}+\frac{i}{8}\right)\int \frac{1}{(i-x)\sqrt{1+ix^2}}\,dx +$$

$$= \frac{i\sqrt{1-ix^2}}{8(i-x)} - \frac{i\sqrt{1-ix^2}}{8(i+x)} - \frac{i\sqrt{1+ix^2}}{8(i-x)} + \frac{i\sqrt{1+ix^2}}{8(i+x)} - \frac{1}{16}(1+i)^{3/2}\tanh^{-1}\left(\frac{1-x}{\sqrt{1+i}\sqrt{1-i}}\right.$$

$$= \frac{i\sqrt{1-ix^2}}{8(i-x)} - \frac{i\sqrt{1-ix^2}}{8(i+x)} - \frac{i\sqrt{1+ix^2}}{8(i-x)} + \frac{i\sqrt{1+ix^2}}{8(i+x)} - \frac{1}{8}\sqrt{1+i}\tanh^{-1}\left(\frac{1-x}{\sqrt{1+i}\sqrt{1-ix^2}}\right.$$

Mathematica [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^2\right)^2\sqrt{1+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x^2)^2*Sqrt[1 + x^4]),x]
```

```
[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x^2)^2*Sqrt[1 + x^4]), x]
```

IntegrateAlgebraic [A] time = 1.46, size = 247, normalized size = 1.31

$$\frac{-\left(\left(x^2-1\right)x^2\right)-\sqrt{x^4+1}\,x^2}{4x\left(x^2+1\right)\sqrt{\sqrt{x^4+1}+x^2}}+\frac{1}{4}\sqrt{5\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\,x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)+\frac{1}{4}\sqrt{1+5\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\,x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x^2)^2*Sqrt[1 + x^4]),x]
```

```
[Out] (-(x^2*(-1 + x^2)) - x^2*Sqrt[1 + x^4])/(4*x*(1 + x^2)*Sqrt[x^2 + Sqrt[1 +
x^4]]) + (Sqrt[-1 + 5*Sqrt[2]]*ArcTan[(-Sqrt[1/2 + 1/Sqrt[2]] + Sqrt[1/2 +
```

```
1/Sqrt[2]]*x^2 + Sqrt[1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4]])/(x*Sqrt[x^2 + Sqrt[1
+ x^4]])])/4 + (Sqrt[1 + 5*Sqrt[2]]*ArcTanh[(-Sqrt[-1/2 + 1/Sqrt[2]] + Sqrt
[-1/2 + 1/Sqrt[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2
+ Sqrt[1 + x^4]])])/4
```

fricas [B] time = 4.28, size = 384, normalized size = 2.03

$$\frac{4\left(x^2+1\right)\sqrt{5\sqrt{2}-1}\arctan\left(\frac{\left(21x^2+7\sqrt{2}\left(x^2+2\right)+\sqrt{x^4+1}\left(\left(\sqrt{2}+3\right)\sqrt{-98\sqrt{2}+147}-7\sqrt{2}-21\right)-\left(3x^2+\sqrt{2}\left(x^2+4\right)+5\right)\sqrt{-98\sqrt{2}+147}-7\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{5\sqrt{2}-1}}{98x}\right)+\left(x^2+1\right)\sqrt{5\sqrt{2}+1}\log\left(\frac{7\sqrt{2}x^2+14x^2+\left(x^3+\sqrt{2}\left(2x^3+3x\right)-\sqrt{x^4+1}\left(2\sqrt{2}x+x\right)+5x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{5\sqrt{2}+1}+7\sqrt{x^4+1}\left(\sqrt{2}+1\right)}{x^2+1}\right)-\left(x^2+1\right)\sqrt{5\sqrt{2}+1}\log\left(\frac{7\sqrt{2}x^2+14x^2-\left(x^3+\sqrt{2}\left(2x^3+3x\right)-\sqrt{x^4+1}\left(2\sqrt{2}x+x\right)+5x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{5\sqrt{2}+1}+7\sqrt{x^4+1}\left(\sqrt{2}+1\right)}{x^2+1}\right)-4\left(x^3-\sqrt{x^4+1}\,x+x\right)\sqrt{x^2+\sqrt{x^4+1}}}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)^2/(x^4+1)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/16*(4*(x^2 + 1)*sqrt(5*sqrt(2) - 1)*arctan(1/98*(21*x^2 + 7*sqrt(2)*(x^2
+ 2) + sqrt(x^4 + 1)*((sqrt(2) + 3)*sqrt(-98*sqrt(2) + 147) - 7*sqrt(2) - 2
1) - (3*x^2 + sqrt(2)*(x^2 + 4) + 5)*sqrt(-98*sqrt(2) + 147) - 7)*sqrt(x^2
+ sqrt(x^4 + 1))*sqrt(5*sqrt(2) - 1)/x) + (x^2 + 1)*sqrt(5*sqrt(2) + 1)*log
((7*sqrt(2)*x^2 + 14*x^2 + (x^3 + sqrt(2)*(2*x^3 + 3*x) - sqrt(x^4 + 1)*(2*
sqrt(2)*x + x) + 5*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(5*sqrt(2) + 1) + 7*sqr
t(x^4 + 1)*(sqrt(2) + 1))/(x^2 + 1)) - (x^2 + 1)*sqrt(5*sqrt(2) + 1)*log((7
*sqrt(2)*x^2 + 14*x^2 - (x^3 + sqrt(2)*(2*x^3 + 3*x) - sqrt(x^4 + 1)*(2*sqr
t(2)*x + x) + 5*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(5*sqrt(2) + 1) + 7*sqrt(x
^4 + 1)*(sqrt(2) + 1))/(x^2 + 1)) - 4*(x^3 - sqrt(x^4 + 1)*x + x)*sqrt(x^2
+ sqrt(x^4 + 1)))/(x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)^2/(x^4+1)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x^2 + 1)^2), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\left(x^2+1\right)^2\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)^2/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)^2/(x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)^2/(x^4+1)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x^2 + 1)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\left(x^2+1\right)^2 \sqrt{x^4+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^2 + 1)^2*(x^4 + 1)^(1/2)),x)
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^2 + 1)^2*(x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\left(x^2+1\right)^2 \sqrt{x^4+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**2+1)**2/(x**4+1)**(1/2),x)
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x**2 + 1)**2*sqrt(x**4 + 1)), x)
```

3.1942 $\int \frac{\sqrt{q+px^5}(-2q+3px^5)}{cx^4+bx^2(q+px^5)+a(q+px^5)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{\sqrt{2}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{x\sqrt{b-\sqrt{b^2-4ac}}}{\sqrt{2}\sqrt{a}\sqrt{px^5+q}}\right)}{\sqrt{a}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b}}{\sqrt{2}\sqrt{a}\sqrt{px^5+q}}\right)}{\sqrt{a}\sqrt{b^2-4ac}}$$

Rubi [F] time = 1.37, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)}{cx^4+bx^2\left(q+px^5\right)+a\left(q+px^5\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(c*x^4 + b*x^2*(q + p*x^5) + a*(q +
p*x^5)^2),x]
```

```
[Out] -2*q*Defer[Int][Sqrt[q + p*x^5]/(a*q^2 + b*q*x^2 + c*x^4 + 2*a*p*q*x^5 + b*
p*x^7 + a*p^2*x^10), x] + 3*p*Defer[Int][(x^5*Sqrt[q + p*x^5])/(a*q^2 + b*q
*x^2 + c*x^4 + 2*a*p*q*x^5 + b*p*x^7 + a*p^2*x^10), x]
```

Rubi steps

$$\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)}{cx^4+bx^2\left(q+px^5\right)+a\left(q+px^5\right)^2}\,dx=\int\left(-\frac{2q\sqrt{q+px^5}}{aq^2+bqx^2+cx^4+2apqx^5+bpx^7+ap^2x^{10}}+\frac{}{aq^2+bqx^2+c}\right.$$

$$=(3p)\int\frac{x^5\sqrt{q+px^5}}{aq^2+bqx^2+cx^4+2apqx^5+bpx^7+ap^2x^{10}}\,dx-(2q)\int\frac{}{ac}$$

Mathematica [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{q+px^5}\left(-2q+3px^5\right)}{cx^4+bx^2\left(q+px^5\right)+a\left(q+px^5\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(c*x^4 + b*x^2*(q + p*x^5) + a
*(q + p*x^5)^2),x]
```

```
[Out] Integrate[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(c*x^4 + b*x^2*(q + p*x^5) + a
*(q + p*x^5)^2), x]
```

IntegrateAlgebraic [A] time = 13.42, size = 189, normalized size = 1.00

$$-\frac{\sqrt{2}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{x\sqrt{b-\sqrt{b^2-4ac}}}{\sqrt{2}\sqrt{a}\sqrt{px^5+q}}\right)}{\sqrt{a}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b}}{\sqrt{2}\sqrt{a}\sqrt{px^5+q}}\right)}{\sqrt{a}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[q + p*x^5]*(-2*q + 3*p*x^5))/(c*x^4 + b*x^2*(q + p
*x^5) + a*(q + p*x^5)^2),x]
```

```
[Out] -((Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b - Sqrt[b^2 - 4*a*c]]*x)/
(Sqrt[2]*Sqrt[a]*Sqrt[q + p*x^5])])/(Sqrt[a]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqr
t[b^2 - 4*a*c]])) - (Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b + S
qrt[b^2 - 4*a*c]]*x)/(Sqrt[2]*Sqrt[a]*Sqrt[q + p*x^5])])/(Sqrt[a]*Sqrt[b^2
- 4*a*c])
```

fricas [B] time = 86.73, size = 1321, normalized size = 6.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^5+q)^(1/2)*(3*p*x^5-2*q)/(c*x^4+b*x^2*(p*x^5+q)+a*(p*x^5+q)^
2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 -
 4*a^2*c))*log((2*a*p^2*x^10 + 4*a*p*q*x^5 - 2*c*x^4 + 2*a*q^2 + sqrt(2)*sq
rt(p*x^5 + q)*((b^2 - 4*a*c)*x^3 - (2*(a^2*b^2 - 4*a^3*c)*p*x^6 + (a*b^3 -
4*a^2*b*c)*x^3 + 2*(a^2*b^2 - 4*a^3*c)*q*x)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-
(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c)) - 2*((a*
b^2 - 4*a^2*c)*p*x^7 + (a*b^2 - 4*a^2*c)*q*x^2)/sqrt(a^2*b^2 - 4*a^3*c))/(a
*p^2*x^10 + b*p*x^7 + 2*a*p*q*x^5 + c*x^4 + b*q*x^2 + a*q^2)) + 1/4*sqrt(2)
*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*l
og((2*a*p^2*x^10 + 4*a*p*q*x^5 - 2*c*x^4 + 2*a*q^2 - sqrt(2)*sqrt(p*x^5 + q
)*((b^2 - 4*a*c)*x^3 - (2*(a^2*b^2 - 4*a^3*c)*p*x^6 + (a*b^3 - 4*a^2*b*c)*x
^3 + 2*(a^2*b^2 - 4*a^3*c)*q*x)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2
- 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c)) - 2*((a*b^2 - 4*a^2*
c)*p*x^7 + (a*b^2 - 4*a^2*c)*q*x^2)/sqrt(a^2*b^2 - 4*a^3*c))/(a*p^2*x^10 +
b*p*x^7 + 2*a*p*q*x^5 + c*x^4 + b*q*x^2 + a*q^2)) - 1/4*sqrt(2)*sqrt(-(b -
(a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log((2*a*p^2*
x^10 + 4*a*p*q*x^5 - 2*c*x^4 + 2*a*q^2 + sqrt(2)*sqrt(p*x^5 + q)*((b^2 - 4*
a*c)*x^3 + (2*(a^2*b^2 - 4*a^3*c)*p*x^6 + (a*b^3 - 4*a^2*b*c)*x^3 + 2*(a^2*
b^2 - 4*a^3*c)*q*x)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/s
qrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c)) + 2*((a*b^2 - 4*a^2*c)*p*x^7 + (
a*b^2 - 4*a^2*c)*q*x^2)/sqrt(a^2*b^2 - 4*a^3*c))/(a*p^2*x^10 + b*p*x^7 + 2*
a*p*q*x^5 + c*x^4 + b*q*x^2 + a*q^2)) + 1/4*sqrt(2)*sqrt(-(b - (a*b^2 - 4*a
^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log((2*a*p^2*x^10 + 4*a*p
*q*x^5 - 2*c*x^4 + 2*a*q^2 - sqrt(2)*sqrt(p*x^5 + q)*((b^2 - 4*a*c)*x^3 + (
2*(a^2*b^2 - 4*a^3*c)*p*x^6 + (a*b^3 - 4*a^2*b*c)*x^3 + 2*(a^2*b^2 - 4*a^3*
c)*q*x)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2
- 4*a^3*c))/(a*b^2 - 4*a^2*c)) + 2*((a*b^2 - 4*a^2*c)*p*x^7 + (a*b^2 - 4*a^
2*c)*q*x^2)/sqrt(a^2*b^2 - 4*a^3*c))/(a*p^2*x^10 + b*p*x^7 + 2*a*p*q*x^5 +
c*x^4 + b*q*x^2 + a*q^2))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^5+q)^(1/2)*(3*p*x^5-2*q)/(c*x^4+b*x^2*(p*x^5+q)+a*(p*x^5+q)^
2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 8.88, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p\,x^{5}+q}\,\left(3 p\,x^{5}-2 q\right)}{c\,x^{4}+b\,x^{2} \left(p\,x^{5}+q\right)+a \left(p\,x^{5}+q\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^5+q)^(1/2)*(3*p*x^5-2*q)/(c*x^4+b*x^2*(p*x^5+q)+a*(p*x^5+q)^2),x)
```

```
[Out] int((p*x^5+q)^(1/2)*(3*p*x^5-2*q)/(c*x^4+b*x^2*(p*x^5+q)+a*(p*x^5+q)^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,px^5 - 2\,q\right)\sqrt{px^5 + q}}{cx^4 + \left(px^5 + q\right)bx^2 + \left(px^5 + q\right)^2 a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^5+q)^(1/2)*(3*p*x^5-2*q)/(c*x^4+b*x^2*(p*x^5+q)+a*(p*x^5+q)^
2),x, algorithm="maxima")
```

```
[Out] integrate((3*p*x^5 - 2*q)*sqrt(p*x^5 + q)/(c*x^4 + (p*x^5 + q)*b*x^2 + (p*x
^5 + q)^2*a), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q + p*x^5)^(1/2)*(2*q - 3*p*x^5))/(a*(q + p*x^5)^2 + c*x^4 + b*x^2*(
q + p*x^5)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^5 + q}\,\left(3px^5 - 2q\right)}{ap^2x^{10} + 2apqx^5 + aq^2 + bpx^7 + bqx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**5+q)**(1/2)*(3*p*x**5-2*q)/(c*x**4+b*x**2*(p*x**5+q)+a*(p*x
**5+q)**2),x)
```

```
[Out] Integral(sqrt(p*x**5 + q)*(3*p*x**5 - 2*q)/(a*p**2*x**10 + 2*a*p*q*x**5 + a
*q**2 + b*p*x**7 + b*q*x**2 + c*x**4), x)
```

3.1943 $\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx$

Optimal. Leaf size=190

$$\frac{1}{12}\tan^{-1}\left(\frac{x}{2\sqrt[3]{x^2+1}+1}\right)+\frac{i\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2+1}}{\sqrt{3}}-\frac{ix}{\sqrt{3}}-\frac{1}{\sqrt{3}}}{\sqrt[3]{x^2+1}}\right)}{8\sqrt{3}}-\frac{i\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2+1}}{\sqrt{3}}+\frac{ix}{\sqrt{3}}-\frac{1}{\sqrt{3}}}{\sqrt[3]{x^2+1}}\right)}{8\sqrt{3}}-\frac{1}{24}i\tanh^{-1}\left(\frac{2ix-2ix\sqrt[3]{x^2}\cdots}{x^2-4\left(x^2+1\right)^{2/3}+2\cdots}\right)$$

Rubi [A] time = 0.01, antiderivative size = 70, normalized size of antiderivative = 0.37, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {394}

$$\frac{1}{12}\tan^{-1}\left(\frac{\left(1-\sqrt[3]{x^2+1}\right)^2}{3x}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{x^2+1}\right)}{x}\right)}{4\sqrt{3}}+\frac{1}{12}\tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^(1/3)*(9 + x^2)),x]
```

```
[Out] ArcTan[x/3]/12 + ArcTan[(1 - (1 + x^2)^(1/3))^2/(3*x)]/12 - ArcTanh[(Sqrt[3
]*(1 - (1 + x^2)^(1/3)))/x]/(4*Sqrt[3])
```

Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx=\frac{1}{12}\tan^{-1}\left(\frac{x}{3}\right)+\frac{1}{12}\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+x^2}\right)^2}{3x}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1+x^2}\right)}{x}\right)}{4\sqrt{3}}$$

Mathematica [C] time = 0.09, size = 124, normalized size = 0.65

$$-\frac{27xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,-\frac{x^2}{9}\right)}{\sqrt[3]{x^2+1}\left(x^2+9\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,-\frac{x^2}{9}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,-\frac{x^2}{9}\right)\right)-27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,-\frac{x^2}{9}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + x^2)^(1/3)*(9 + x^2)),x]
```

```
[Out] (-27*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, -1/9*x^2])/((1 + x^2)^(1/3)*(9 + x^
2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -x^2, -1/9*x^2] + 2*x^2*(AppellF1[3/2, 1
/3, 2, 5/2, -x^2, -1/9*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/2, -x^2, -1/9*x^2])
))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((1 + x^2)^(1/3)*(9 + x^2)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((1 + x^2)^(1/3)*(9 + x^2)), x]
```

fricas [B] time = 1.75, size = 1395, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="fricas")
```

```
[Out] 1/144*sqrt(3)*log(4*(x^6 + 1647*x^4 + 891*x^2 + 18*(3*x^4 + 32*sqrt(3)*x^3
+ 126*x^2 + 27)*(x^2 + 1)^(2/3) + 108*sqrt(3)*(x^5 + 10*x^3 + 9*x) + 6*(81*
x^4 + 162*x^2 + sqrt(3)*(x^5 + 210*x^3 + 81*x) + 81)*(x^2 + 1)^(1/3) - 243)
/(x^6 + 27*x^4 + 243*x^2 + 729)) - 1/144*sqrt(3)*log(4*(x^6 + 1647*x^4 + 89
1*x^2 + 18*(3*x^4 - 32*sqrt(3)*x^3 + 126*x^2 + 27)*(x^2 + 1)^(2/3) - 108*sq
rt(3)*(x^5 + 10*x^3 + 9*x) + 6*(81*x^4 + 162*x^2 - sqrt(3)*(x^5 + 210*x^3 +
 81*x) + 81)*(x^2 + 1)^(1/3) - 243)/(x^6 + 27*x^4 + 243*x^2 + 729)) - 1/36*
arctan((384*x^11 - 130320*x^9 + 2379456*x^7 - 629856*x^5 - 1259712*x^3 + 36
*(388*x^9 - 27864*x^7 + 303264*x^5 + 17496*x^3 + sqrt(3)*(x^10 + 549*x^8 -
8046*x^6 + 129762*x^4 - 19683*x^2 + 59049) - 236196*x)*(x^2 + 1)^(2/3) + sq
rt(3)*(x^12 - 234*x^10 + 229311*x^8 - 1214028*x^6 + 6816879*x^4 + 6022998*x
^2 + 531441) + 2*(x^12 + 50616*x^10 - 1869399*x^8 - 3773304*x^6 - 6908733*x
^4 + 72*(x^10 + 1620*x^8 - 63666*x^6 - 43740*x^4 + 59049*x^2 + 12*sqrt(3)*(
11*x^9 - 261*x^7 - 6075*x^5 - 2187*x^3))*(x^2 + 1)^(2/3) + 6*sqrt(3)*(43*x^
11 + 14055*x^9 - 563922*x^7 - 1307826*x^5 - 898857*x^3 + 177147*x) + 6*(453
*x^10 + 21141*x^8 - 1483758*x^6 - 1404054*x^4 - 885735*x^2 + sqrt(3)*(x^11
+ 8985*x^9 - 349110*x^7 + 118098*x^5 + 32805*x^3 - 177147*x) + 531441)*(x^2
 + 1)^(1/3) + 1594323)*sqrt((x^6 + 1647*x^4 + 891*x^2 + 18*(3*x^4 - 32*sqrt
(3)*x^3 + 126*x^2 + 27)*(x^2 + 1)^(2/3) - 108*sqrt(3)*(x^5 + 10*x^3 + 9*x)
+ 6*(81*x^4 + 162*x^2 - sqrt(3)*(x^5 + 210*x^3 + 81*x) + 81)*(x^2 + 1)^(1/3
) - 243)/(x^6 + 27*x^4 + 243*x^2 + 729)) + 12*(x^11 - 6423*x^9 + 225018*x^7
 - 1106622*x^5 - 1541835*x^3 + 3*sqrt(3)*(37*x^10 - 675*x^8 + 34722*x^6 - 9
7686*x^4 + 59049*x^2 + 59049) - 177147*x)*(x^2 + 1)^(1/3) - 8503056*x)/(x^1
2 - 48978*x^10 + 2332071*x^8 - 16419996*x^6 - 24151041*x^4 - 9565938*x^2 +
4782969)) + 1/36*arctan(-(384*x^11 - 130320*x^9 + 2379456*x^7 - 629856*x^5
- 1259712*x^3 + 36*(388*x^9 - 27864*x^7 + 303264*x^5 + 17496*x^3 - sqrt(3)*
(x^10 + 549*x^8 - 8046*x^6 + 129762*x^4 - 19683*x^2 + 59049) - 236196*x)*(x
^2 + 1)^(2/3) - sqrt(3)*(x^12 - 234*x^10 + 229311*x^8 - 1214028*x^6 + 68168
79*x^4 + 6022998*x^2 + 531441) + 2*(x^12 + 50616*x^10 - 1869399*x^8 - 37733
04*x^6 - 6908733*x^4 + 72*(x^10 + 1620*x^8 - 63666*x^6 - 43740*x^4 + 59049*
x^2 - 12*sqrt(3)*(11*x^9 - 261*x^7 - 6075*x^5 - 2187*x^3))*(x^2 + 1)^(2/3)
- 6*sqrt(3)*(43*x^11 + 14055*x^9 - 563922*x^7 - 1307826*x^5 - 898857*x^3 +
177147*x) + 6*(453*x^10 + 21141*x^8 - 1483758*x^6 - 1404054*x^4 - 885735*x^
2 - sqrt(3)*(x^11 + 8985*x^9 - 349110*x^7 + 118098*x^5 + 32805*x^3 - 177147
*x) + 531441)*(x^2 + 1)^(1/3) + 1594323)*sqrt((x^6 + 1647*x^4 + 891*x^2 + 1
8*(3*x^4 + 32*sqrt(3)*x^3 + 126*x^2 + 27)*(x^2 + 1)^(2/3) + 108*sqrt(3)*(x^
5 + 10*x^3 + 9*x) + 6*(81*x^4 + 162*x^2 + sqrt(3)*(x^5 + 210*x^3 + 81*x) +
81)*(x^2 + 1)^(1/3) - 243)/(x^6 + 27*x^4 + 243*x^2 + 729)) + 12*(x^11 - 642
3*x^9 + 225018*x^7 - 1106622*x^5 - 1541835*x^3 - 3*sqrt(3)*(37*x^10 - 675*x
^8 + 34722*x^6 - 97686*x^4 + 59049*x^2 + 59049) - 177147*x)*(x^2 + 1)^(1/3)
 - 8503056*x)/(x^12 - 48978*x^10 + 2332071*x^8 - 16419996*x^6 - 24151041*x^
4 - 9565938*x^2 + 4782969)) - 1/36*arctan(6*(11*x^5 + 30*x^3 + 6*(23*x^3 +
```

```
27*x)*(x^2 + 1)^(2/3) + (x^5 - 240*x^3 - 81*x)*(x^2 + 1)^(1/3) - 81*x)/(x^6
 - 1971*x^4 - 1701*x^2 - 729))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2+9)(x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 9)*(x^2 + 1)^(1/3)), x)
```

maple [C] time = 10.57, size = 512, normalized size = 2.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/3)/(x^2+9),x)
```

```
[Out] 1/12*RootOf(_Z^2+1)*ln((24*RootOf(_Z^2+1)^2*RootOf(12*_Z*RootOf(_Z^2+1)+144
*_Z^2-1)*(x^2+1)^(1/3)*x+576*RootOf(_Z^2+1)*RootOf(12*_Z*RootOf(_Z^2+1)+144
*_Z^2-1)^2*(x^2+1)^(1/3)*x-48*RootOf(_Z^2+1)^2*RootOf(12*_Z*RootOf(_Z^2+1)+
144*_Z^2-1)*x-1152*RootOf(_Z^2+1)*RootOf(12*_Z*RootOf(_Z^2+1)+144*_Z^2-1)^2
*x-12*RootOf(12*_Z*RootOf(_Z^2+1)+144*_Z^2-1)*RootOf(_Z^2+1)*x^2+72*(x^2+1)
^(1/3)*RootOf(12*_Z*RootOf(_Z^2+1)+144*_Z^2-1)*RootOf(_Z^2+1)+36*RootOf(12*
_Z*RootOf(_Z^2+1)+144*_Z^2-1)*RootOf(_Z^2+1)+4*RootOf(_Z^2+1)*x-6*(x^2+1)^(
2/3)+96*RootOf(12*_Z*RootOf(_Z^2+1)+144*_Z^2-1)*x+x^2-3)/(x^2+9))-1/12*ln((
2*RootOf(_Z^2+1)*(x^2+1)^(1/3)*x+48*RootOf(12*_Z*RootOf(_Z^2+1)+144*_Z^2-1)
*(x^2+1)^(1/3)*x+4*RootOf(_Z^2+1)*x-6*(x^2+1)^(2/3)+96*RootOf(12*_Z*RootOf(
_Z^2+1)+144*_Z^2-1)*x-x^2-6*(x^2+1)^(1/3)+3)/(x^2+9))*RootOf(_Z^2+1)-ln((2*
RootOf(_Z^2+1)*(x^2+1)^(1/3)*x+48*RootOf(12*_Z*RootOf(_Z^2+1)+144*_Z^2-1)*(
x^2+1)^(1/3)*x+4*RootOf(_Z^2+1)*x-6*(x^2+1)^(2/3)+96*RootOf(12*_Z*RootOf(_Z
^2+1)+144*_Z^2-1)*x-x^2-6*(x^2+1)^(1/3)+3)/(x^2+9))*RootOf(12*_Z*RootOf(_Z^
2+1)+144*_Z^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2+9)(x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 9)*(x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x^2+1)^{1/3}\,(x^2+9)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^(1/3)*(x^2 + 9)),x)
```

```
[Out] int(1/((x^2 + 1)^(1/3)*(x^2 + 9)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x^2+1}\,(x^2+9)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/3)/(x**2+9),x)
```

```
[Out] Integral(1/((x**2 + 1)**(1/3)*(x**2 + 9)), x)
```

3.1944 $\int \frac{-3-2(1+k^2)x+(1+k^2)x^2+4k^2x^3+k^2x^4}{\left((1-x^2)(1-k^2x^2)\right)^{2/3}\left(-1+d-(2+d)x-(1+dk^2)x^2+dk^2x^3\right)}\,dx$

Optimal. Leaf size=190

$$-\frac{\log\left(d^{2/3}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+\left(\sqrt[3]{d}\,x+\sqrt[3]{d}\right)\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+x^2+2x+1\right)}{2\sqrt[3]{d}}+\frac{\log\left(-\sqrt[3]{d}\,\sqrt[3]{k^2\ldots\right.}{}$$

Rubi [F] time = 8.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3-2\left(1+k^2\right)x+\left(1+k^2\right)x^2+4k^2x^3+k^2x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(-1+d-(2+d)x-\left(1+dk^2\right)x^2+dk^2x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3 - 2*(1 + k^2)*x + (1 + k^2)*x^2 + 4*k^2*x^3 + k^2*x^4)/(((1 - x^2)*
(1 - k^2*x^2))^(2/3)*(-1 + d - (2 + d)*x - (1 + d*k^2)*x^2 + d*k^2*x^3)),x]
```

```
[Out] ((5*d + k^(-2))*x*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*AppellF1[1/2, 2/3, 2/
3, 3/2, x^2, k^2*x^2])/(d^2*((1 - x^2)*(1 - k^2*x^2))^(2/3)) - (3^(3/4)*Sqr
t[2 + Sqrt[3]]*Sqrt[(-1 - k^2 + 2*k^2*x^2)^2]*((-1 + k^2)^(2/3) + 2^(2/3)*k
^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))*Sqrt[((-1 + k^2)^(4/3) - 2^(2/3)*k^
(2/3)*(-1 + k^2)^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3) + 2*2^(1/3)*k^(4/3)*
((1 - x^2)*(1 - k^2*x^2))^(2/3))/((1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2/3)*
k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))/((1 +
Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3)
)], -7 - 4*Sqrt[3]])/(2^(2/3)*d*k^(2/3)*(1 + k^2 - 2*k^2*x^2)*Sqrt[(-1 - k^
2*(1 - 2*x^2))^2]*Sqrt[((-1 + k^2)^(2/3)*((-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3
)*((1 - x^2)*(1 - k^2*x^2))^(1/3)))/((1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2/
3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))^2]) - ((1 - d + 5*d*k^2 - 8*d^2
*k^2)*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][1/((1 - x^2)^(2/3)*(1
- k^2*x^2)^(2/3)*(1 - d + (2 + d)*x + (1 + d*k^2)*x^2 - d*k^2*x^3)), x])/(d
^2*k^2*((1 - x^2)*(1 - k^2*x^2))^(2/3)) - ((2 + d + 11*d*k^2 + 2*d^2*k^2*(1
 - k^2))*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][x/((1 - x^2)^(2/3)*
(1 - k^2*x^2)^(2/3)*(1 - d + (2 + d)*x + (1 + d*k^2)*x^2 - d*k^2*x^3)), x])
/(d^2*k^2*((1 - x^2)*(1 - k^2*x^2))^(2/3)) - ((1 + 8*d*k^2 + 2*d^2*(k^2 + 3
*k^4))*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][x^2/((1 - x^2)^(2/3)*
(1 - k^2*x^2)^(2/3)*(1 - d + (2 + d)*x + (1 + d*k^2)*x^2 - d*k^2*x^3)), x])
/(d^2*k^2*((1 - x^2)*(1 - k^2*x^2))^(2/3))
```

Rubi steps

$$\int \frac{-3-2\left(1+k^2\right)x+\left(1+k^2\right)x^2+4k^2x^3+k^2x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(-1+d-(2+d)x-\left(1+dk^2\right)x^2+dk^2x^3\right)}\,dx = \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int \frac{\ }{\left(1-x^2\right)^2 \cdots}}{\left(\left(1-x^2\right) \cdots\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int\left(\frac{\ }{d^2\left(1-\cdots\right.}\right.}{\cdots}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int \frac{\ }{\left(1-x^2\right)^2 \cdots}}{d\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^2 \cdots}$$

$$= \frac{\left(5d+\frac{1}{k^2}\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3} \cdots}{d^2\left(\left(1-x^2\right)\left(1-k \cdots\right.\right.}$$

$$= \frac{\left(5d+\frac{1}{k^2}\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3} \cdots}{d^2\left(\left(1-x^2\right)\left(1-k \cdots\right.\right.}$$

$$= \frac{\left(5d+\frac{1}{k^2}\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3} \cdots}{d^2\left(\left(1-x^2\right)\left(1-k \cdots\right.\right.}$$

$$= \frac{\left(5d+\frac{1}{k^2}\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3} \cdots}{d^2\left(\left(1-x^2\right)\left(1-k \cdots\right.\right.}$$

Mathematica [F] time = 3.59, size = 0, normalized size = 0.00

$$\int \frac{-3-2\left(1+k^2\right)x+\left(1+k^2\right)x^2+4k^2x^3+k^2x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(-1+d-(2+d)x-\left(1+dk^2\right)x^2+dk^2x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3 - 2*(1 + k^2)*x + (1 + k^2)*x^2 + 4*k^2*x^3 + k^2*x^4)/(((1 -
 x^2)*(1 - k^2*x^2))^(2/3)*(-1 + d - (2 + d)*x - (1 + d*k^2)*x^2 + d*k^2*x^
3)),x]
```

```
[Out] Integrate[(-3 - 2*(1 + k^2)*x + (1 + k^2)*x^2 + 4*k^2*x^3 + k^2*x^4)/(((1 -
 x^2)*(1 - k^2*x^2))^(2/3)*(-1 + d - (2 + d)*x - (1 + d*k^2)*x^2 + d*k^2*x^
3)), x]
```

IntegrateAlgebraic [A] time = 8.12, size = 190, normalized size = 1.00

$$-\frac{\log\left(d^{2/3}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+\left(\sqrt[3]{d}\,x+\sqrt[3]{d}\right)\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+x^2+2x+1\right)}{2\sqrt[3]{d}}+\frac{\log\left(-\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+x+1\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x+\sqrt{3}}{2\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+x+1}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 - 2*(1 + k^2)*x + (1 + k^2)*x^2 + 4*k^2*x^3 + k^2*x^
4)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)*(-1 + d - (2 + d)*x - (1 + d*k^2)*x^2 +
 d*k^2*x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3] + Sqrt[3]*x)/(1 + x + 2*d^(1/3)*(1 + (-1 - k^2)*x^
2 + k^2*x^4)^(1/3))])/d^(1/3) + Log[1 + x - d^(1/3)*(1 + (-1 - k^2)*x^2 + k
^2*x^4)^(1/3)]/d^(1/3) - Log[1 + 2*x + x^2 + (d^(1/3) + d^(1/3)*x)*(1 + (-1
 - k^2)*x^2 + k^2*x^4)^(1/3) + d^(2/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)
]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-2*(k^2+1)*x+(k^2+1)*x^2+4*k^2*x^3+k^2*x^4)/((-x^2+1)*(-k^2*x^
2+1))^(2/3)/(-1+d-(2+d)*x-(d*k^2+1)*x^2+d*k^2*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^4 + 4\,k^2x^3 + \left(k^2+1\right)x^2 - 2\left(k^2+1\right)x - 3}{\left(dk^2x^3 - \left(dk^2+1\right)x^2 - (d+2)x + d - 1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-2*(k^2+1)*x+(k^2+1)*x^2+4*k^2*x^3+k^2*x^4)/((-x^2+1)*(-k^2*x^
2+1))^(2/3)/(-1+d-(2+d)*x-(d*k^2+1)*x^2+d*k^2*x^3),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^4 + 4*k^2*x^3 + (k^2 + 1)*x^2 - 2*(k^2 + 1)*x - 3)/((d*k^2
*x^3 - (d*k^2 + 1)*x^2 - (d + 2)*x + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(2/3)
), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{-3-2\left(k^2+1\right)x+\left(k^2+1\right)x^2+4k^2x^3+k^2x^4}{\left(\left(-x^2+1\right)\left(-k^2x^2+1\right)\right)^{\frac{2}{3}}\left(-1+d-(2+d)\,x-\left(d\,k^2+1\right)x^2+d\,k^2x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-2*(k^2+1)*x+(k^2+1)*x^2+4*k^2*x^3+k^2*x^4)/((-x^2+1)*(-k^2*x^2+1))^
(2/3)/(-1+d-(2+d)*x-(d*k^2+1)*x^2+d*k^2*x^3),x)
```

```
[Out] int((-3-2*(k^2+1)*x+(k^2+1)*x^2+4*k^2*x^3+k^2*x^4)/((-x^2+1)*(-k^2*x^2+1))^
(2/3)/(-1+d-(2+d)*x-(d*k^2+1)*x^2+d*k^2*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^4 + 4\,k^2x^3 + \left(k^2+1\right)x^2 - 2\left(k^2+1\right)x - 3}{\left(dk^2x^3 - \left(dk^2+1\right)x^2 - (d+2)x + d - 1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-2*(k^2+1)*x+(k^2+1)*x^2+4*k^2*x^3+k^2*x^4)/((-x^2+1)*(-k^2*x^
2+1))^(2/3)/(-1+d-(2+d)*x-(d*k^2+1)*x^2+d*k^2*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^4 + 4*k^2*x^3 + (k^2 + 1)*x^2 - 2*(k^2 + 1)*x - 3)/((d*k^2
*x^3 - (d*k^2 + 1)*x^2 - (d + 2)*x + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(2/3)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{4\,k^2\,x^3-2\,x\,\left(k^2+1\right)+k^2\,x^4+x^2\,\left(k^2+1\right)-3}{{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)}^{2/3}\,\left(x^2\,\left(d\,k^2+1\right)-d+x\,\left(d+2\right)-d\,k^2\,x^3+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*k^2*x^3 - 2*x*(k^2 + 1) + k^2*x^4 + x^2*(k^2 + 1) - 3)/(((x^2 - 1)*
(k^2*x^2 - 1))^(2/3)*(x^2*(d*k^2 + 1) - d + x*(d + 2) - d*k^2*x^3 + 1)),x)
```

```
[Out] int(-(4*k^2*x^3 - 2*x*(k^2 + 1) + k^2*x^4 + x^2*(k^2 + 1) - 3)/(((x^2 - 1)*
(k^2*x^2 - 1))^(2/3)*(x^2*(d*k^2 + 1) - d + x*(d + 2) - d*k^2*x^3 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-2*(k**2+1)*x+(k**2+1)*x**2+4*k**2*x**3+k**2*x**4)/((-x**2+1)*
(-k**2*x**2+1))**(2/3)/(-1+d-(2+d)*x-(d*k**2+1)*x**2+d*k**2*x**3),x)
```

```
[Out] Timed out
```

3.1945 $\int \frac{\sqrt[4]{x^3+x^5}\left(1+x^4+x^8\right)}{x^4\left(-1+x^4\right)}\,dx$

Optimal. Leaf size=190

$$\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^5+x^3}}\right)}{2\,2^{3/4}}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^5+x^3}}\right)}{2\,2^{3/4}}-\frac{3\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^5+x^3}}{\sqrt{2}\,x^2-\sqrt{x^5+x^3}}\right)}{4\sqrt[4]{2}}-\frac{3\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^5+x^3}}{2^{3/4}}}{x\sqrt[4]{x^5+x^3}}\right)}{4\sqrt[4]{2}}+\frac{4\sqrt[4]{x^5+x^3}\,\left(x^4+2x^2+1\right.}{9x^3}$$

Rubi [C] time = 0.61, antiderivative size = 211, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2056, 1586, 6725, 364, 466, 510}

$$\frac{4\sqrt[4]{x^5+x^3}\,F_1\left(-\frac{9}{8};1,\frac{3}{4};-\frac{1}{8};x^2,-x^2\right)}{3\sqrt[4]{x^2+1}\,x^3}+\frac{4\sqrt[4]{x^5+x^3}\,x^3\,{}_2F_1\left(\frac{3}{4},\frac{15}{8};\frac{23}{8};-x^2\right)}{15\sqrt[4]{x^2+1}}+\frac{4\sqrt[4]{x^5+x^3}\,x\,{}_2F_1\left(\frac{3}{4},\frac{7}{8};\frac{15}{8};-x^2\right)}{7\sqrt[4]{x^2+1}}-\frac{8\sqrt[4]{x^5+x^3}\,{}_2F_1\left(-\frac{1}{8},\frac{3}{4};\frac{7}{8};-x^2\right)}{\sqrt[4]{x^2+1}\,x}-\frac{8\sqrt[4]{x^5+x^3}\,{}_2F_1\left(-\frac{9}{8},\frac{3}{4};-\frac{1}{8};-x^2\right)}{9\sqrt[4]{x^2+1}\,x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((x^3 + x^5)^(1/4)*(1 + x^4 + x^8))/(x^4*(-1 + x^4)),x]
```

```
[Out] (4*(x^3 + x^5)^(1/4)*AppellF1[-9/8, 1, 3/4, -1/8, x^2, -x^2])/(3*x^3*(1 + x
^2)^(1/4)) - (8*(x^3 + x^5)^(1/4)*Hypergeometric2F1[-9/8, 3/4, -1/8, -x^2])
/(9*x^3*(1 + x^2)^(1/4)) - (8*(x^3 + x^5)^(1/4)*Hypergeometric2F1[-1/8, 3/4
, 7/8, -x^2])/(x*(1 + x^2)^(1/4)) + (4*x*(x^3 + x^5)^(1/4)*Hypergeometric2F
1[3/4, 7/8, 15/8, -x^2])/(7*(1 + x^2)^(1/4)) + (4*x^3*(x^3 + x^5)^(1/4)*Hyp
ergeometric2F1[3/4, 15/8, 23/8, -x^2])/(15*(1 + x^2)^(1/4))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
```

```
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{x^3+x^5}\left(1+x^4+x^8\right)}{x^4\left(-1+x^4\right)}\,dx = \frac{\sqrt[4]{x^3+x^5}\int \frac{\sqrt[4]{1+x^2}\left(1+x^4+x^8\right)}{x^{13/4}\left(-1+x^4\right)}\,dx}{x^{3/4}\sqrt[4]{1+x^2}}$$

$$= \frac{\sqrt[4]{x^3+x^5}\int \frac{1+x^4+x^8}{x^{13/4}\left(-1+x^2\right)\left(1+x^2\right)^{3/4}}\,dx}{x^{3/4}\sqrt[4]{1+x^2}}$$

$$= \frac{\sqrt[4]{x^3+x^5}\int \left(\frac{2}{x^{13/4}\left(1+x^2\right)^{3/4}}+\frac{2}{x^{5/4}\left(1+x^2\right)^{3/4}}+\frac{x^{3/4}}{\left(1+x^2\right)^{3/4}}+\frac{x^{11/4}}{\left(1+x^2\right)^{3/4}}+\frac{3}{x^{13/4}\left(-1+x^2\right)\left(1+x^2\right.}\right.}{x^{3/4}\sqrt[4]{1+x^2}}$$

$$= \frac{\sqrt[4]{x^3+x^5}\int \frac{x^{3/4}}{\left(1+x^2\right)^{3/4}}\,dx}{x^{3/4}\sqrt[4]{1+x^2}}+\frac{\sqrt[4]{x^3+x^5}\int \frac{x^{11/4}}{\left(1+x^2\right)^{3/4}}\,dx}{x^{3/4}\sqrt[4]{1+x^2}}+\frac{\left(2\sqrt[4]{x^3+x^5}\right)\int \frac{1}{x^{13/4}\left(1+x^2\right.}}{x^{3/4}\sqrt[4]{1+x^2}}$$

$$= -\frac{8\sqrt[4]{x^3+x^5}\,{}_2F_1\left(-\frac{9}{8},\frac{3}{4};-\frac{1}{8};-x^2\right)}{9x^3\sqrt[4]{1+x^2}}-\frac{8\sqrt[4]{x^3+x^5}\,{}_2F_1\left(-\frac{1}{8},\frac{3}{4};\frac{7}{8};-x^2\right)}{x\sqrt[4]{1+x^2}}+\frac{4x\sqrt[4]{x^3}}{}$$

$$= \frac{4\sqrt[4]{x^3+x^5}\,F_1\left(-\frac{9}{8};1,\frac{3}{4};-\frac{1}{8};x^2,-x^2\right)}{3x^3\sqrt[4]{1+x^2}}-\frac{8\sqrt[4]{x^3+x^5}\,{}_2F_1\left(-\frac{9}{8},\frac{3}{4};-\frac{1}{8};-x^2\right)}{9x^3\sqrt[4]{1+x^2}}-\frac{8\sqrt[4]{x}}{}$$

Mathematica [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3+x^5}\left(1+x^4+x^8\right)}{x^4\left(-1+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((x^3 + x^5)^(1/4)*(1 + x^4 + x^8))/(x^4*(-1 + x^4)),x]
[Out] Integrate[((x^3 + x^5)^(1/4)*(1 + x^4 + x^8))/(x^4*(-1 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.69, size = 190, normalized size = 1.00

$$\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^5+x^3}}\right)}{2\,2^{3/4}}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^5+x^3}}\right)}{2\,2^{3/4}}-\frac{3\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^5+x^3}}{\sqrt{2}\,x^2-\sqrt{x^5+x^3}}\right)}{4\sqrt[4]{2}}-\frac{3\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^5+x^3}}{2^{3/4}}}{x\sqrt[4]{x^5+x^3}}\right)}{4\sqrt[4]{2}}+\frac{4\sqrt[4]{x^5+x^3}\left(x^4+2x^2+1\right)}{9x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((x^3 + x^5)^(1/4)*(1 + x^4 + x^8))/(x^4*(-1 + x^4)),x]
[Out] (4*(1 + 2*x^2 + x^4)*(x^3 + x^5)^(1/4))/(9*x^3) + (3*ArcTan[(2^(1/4)*x)/(x^
3 + x^5)^(1/4)])/(2*2^(3/4)) - (3*ArcTan[(2^(3/4)*x*(x^3 + x^5)^(1/4))/(Sqr
```

```
t[2]*x^2 - Sqrt[x^3 + x^5]])]/(4*2^(1/4)) - (3*ArcTanh[(2^(1/4)*x)/(x^3 + x
^5)^(1/4)])/(2*2^(3/4)) - (3*ArcTanh[(x^2/2^(1/4) + Sqrt[x^3 + x^5]/2^(3/4)
)/(x*(x^3 + x^5)^(1/4))])/(4*2^(1/4))
```

fricas [B] time = 6.08, size = 1133, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x^3)^(1/4)*(x^8+x^4+1)/x^4/(x^4-1),x, algorithm="fricas")
```

```
[Out] 1/1152*(108*8^(3/4)*sqrt(2)*x^3*arctan(1/8*(8*x^6 + 32*x^5 + 48*x^4 + 4*8^(
3/4)*sqrt(2)*(x^5 + x^3)^(3/4)*(x^2 - 6*x + 1) + 32*x^3 + 16*8^(1/4)*sqrt(2
)*(x^5 + x^3)^(1/4)*(3*x^4 - 2*x^3 + 3*x^2) + 32*sqrt(2)*sqrt(x^5 + x^3)*(x
^3 + 2*x^2 + x) + 8*x^2 + sqrt(2)*(8^(3/4)*sqrt(2)*(x^6 - 16*x^5 - 2*x^4 -
16*x^3 + x^2) + 8*8^(1/4)*sqrt(2)*sqrt(x^5 + x^3)*(x^3 - 6*x^2 + x) + 128*s
qrt(2)*(x^5 + x^3)^(3/4)*x + 32*(x^5 + x^3)^(1/4)*(x^4 + 2*x^3 + x^2))*sqrt
((8^(3/4)*sqrt(2)*(x^5 + x^3)^(1/4)*x^2 + 2*8^(1/4)*sqrt(2)*(x^5 + x^3)^(3/
4) + sqrt(2)*(x^4 + 2*x^3 + x^2) + 8*sqrt(x^5 + x^3)*x)/(x^4 + 2*x^3 + x^2)
))/(x^6 - 28*x^5 + 6*x^4 - 28*x^3 + x^2)) - 108*8^(3/4)*sqrt(2)*x^3*arctan(
1/8*(8*x^6 + 32*x^5 + 48*x^4 - 4*8^(3/4)*sqrt(2)*(x^5 + x^3)^(3/4)*(x^2 - 6
*x + 1) + 32*x^3 - 16*8^(1/4)*sqrt(2)*(x^5 + x^3)^(1/4)*(3*x^4 - 2*x^3 + 3*
x^2) + 32*sqrt(2)*sqrt(x^5 + x^3)*(x^3 + 2*x^2 + x) + 8*x^2 - sqrt(2)*(8^(3
/4)*sqrt(2)*(x^6 - 16*x^5 - 2*x^4 - 16*x^3 + x^2) + 8*8^(1/4)*sqrt(2)*sqrt(
x^5 + x^3)*(x^3 - 6*x^2 + x) - 128*sqrt(2)*(x^5 + x^3)^(3/4)*x - 32*(x^5 +
x^3)^(1/4)*(x^4 + 2*x^3 + x^2))*sqrt(-(8^(3/4)*sqrt(2)*(x^5 + x^3)^(1/4)*x^
2 + 2*8^(1/4)*sqrt(2)*(x^5 + x^3)^(3/4) - sqrt(2)*(x^4 + 2*x^3 + x^2) - 8*s
qrt(x^5 + x^3)*x)/(x^4 + 2*x^3 + x^2)))/(x^6 - 28*x^5 + 6*x^4 - 28*x^3 + x^
2)) - 27*8^(3/4)*sqrt(2)*x^3*log(8*(8^(3/4)*sqrt(2)*(x^5 + x^3)^(1/4)*x^2 +
 2*8^(1/4)*sqrt(2)*(x^5 + x^3)^(3/4) + sqrt(2)*(x^4 + 2*x^3 + x^2) + 8*sqrt
(x^5 + x^3)*x)/(x^4 + 2*x^3 + x^2)) + 27*8^(3/4)*sqrt(2)*x^3*log(-8*(8^(3/4
)*sqrt(2)*(x^5 + x^3)^(1/4)*x^2 + 2*8^(1/4)*sqrt(2)*(x^5 + x^3)^(3/4) - sqr
t(2)*(x^4 + 2*x^3 + x^2) - 8*sqrt(x^5 + x^3)*x)/(x^4 + 2*x^3 + x^2)) - 216*
8^(3/4)*x^3*arctan(-1/8*(16*8^(1/4)*(x^5 + x^3)^(1/4)*x^2 - 2^(3/4)*(8^(3/4
)*(x^4 + 2*x^3 + x^2) + 8*8^(1/4)*sqrt(x^5 + x^3)*x) + 4*8^(3/4)*(x^5 + x^3
)^(3/4))/(x^4 - 2*x^3 + x^2)) - 54*8^(3/4)*x^3*log((4*sqrt(2)*(x^5 + x^3)^(
1/4)*x^2 + 8^(3/4)*sqrt(x^5 + x^3)*x + 8^(1/4)*(x^4 + 2*x^3 + x^2) + 4*(x^5
 + x^3)^(3/4))/(x^4 - 2*x^3 + x^2)) + 54*8^(3/4)*x^3*log((4*sqrt(2)*(x^5 +
x^3)^(1/4)*x^2 - 8^(3/4)*sqrt(x^5 + x^3)*x - 8^(1/4)*(x^4 + 2*x^3 + x^2) +
4*(x^5 + x^3)^(3/4))/(x^4 - 2*x^3 + x^2)) + 512*(x^5 + x^3)^(1/4)*(x^4 + 2*
x^2 + 1))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8 + x^4 + 1\right)\left(x^5 + x^3\right)^{\frac{1}{4}}}{\left(x^4 - 1\right)x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x^3)^(1/4)*(x^8+x^4+1)/x^4/(x^4-1),x, algorithm="giac")
```

```
[Out] integrate((x^8 + x^4 + 1)*(x^5 + x^3)^(1/4)/((x^4 - 1)*x^4), x)
```

maple [C] time = 16.10, size = 1780, normalized size = 9.37

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+x^3)^(1/4)*(x^8+x^4+1)/x^4/(x^4-1),x)
```

```
[Out] 4/9*(x^6+3*x^4+3*x^2+1)/x^3*(x^3*(x^2+1))^(1/4)/(x^2+1)+(-3/8*RootOf(_Z^2+R
ootOf(_Z^4-2)^2)*ln((-2*(x^7+3*x^5+3*x^3+x)^(1/2)*RootOf(_Z^4-2)^2*RootOf(_
Z^2+RootOf(_Z^4-2)^2)*x^2-2*(x^7+3*x^5+3*x^3+x)^(1/4)*RootOf(_Z^4-2)^2*x^4+
RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^6+2*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^5-2*(x^
7+3*x^5+3*x^3+x)^(1/2)*RootOf(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)-4*(x^
7+3*x^5+3*x^3+x)^(1/4)*RootOf(_Z^4-2)^2*x^2+3*RootOf(_Z^2+RootOf(_Z^4-2)^2)
*x^4+4*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^3+4*(x^7+3*x^5+3*x^3+x)^(3/4)-2*Root
Of(_Z^4-2)^2*(x^7+3*x^5+3*x^3+x)^(1/4)+3*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^2+
2*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x+RootOf(_Z^2+RootOf(_Z^4-2)^2))/(x^2+1)^2/
(-1+x)^2)+3/8*RootOf(_Z^4-2)*ln((-2*(x^7+3*x^5+3*x^3+x)^(1/2)*RootOf(_Z^4-2
)^3*x^2+2*(x^7+3*x^5+3*x^3+x)^(1/4)*RootOf(_Z^4-2)^2*x^4-RootOf(_Z^4-2)*x^6
-2*RootOf(_Z^4-2)*x^5-2*RootOf(_Z^4-2)^3*(x^7+3*x^5+3*x^3+x)^(1/2)+4*(x^7+3
*x^5+3*x^3+x)^(1/4)*RootOf(_Z^4-2)^2*x^2-3*RootOf(_Z^4-2)*x^4-4*RootOf(_Z^4
-2)*x^3+4*(x^7+3*x^5+3*x^3+x)^(3/4)+2*RootOf(_Z^4-2)^2*(x^7+3*x^5+3*x^3+x)^
(1/4)-3*RootOf(_Z^4-2)*x^2-2*RootOf(_Z^4-2)*x-RootOf(_Z^4-2))/(x^2+1)^2/(-1
+x)^2)+3/16*ln((-RootOf(_Z^4-2)^3*x^6-2*RootOf(_Z^4-2)^3*x^5+4*(x^7+3*x^5+3
*x^3+x)^(1/4)*RootOf(_Z^4-2)^2*x^4-3*RootOf(_Z^4-2)^3*x^4-4*RootOf(_Z^4-2)^
3*x^3-8*(x^7+3*x^5+3*x^3+x)^(1/2)*RootOf(_Z^4-2)*x^2+8*(x^7+3*x^5+3*x^3+x)^
(1/4)*RootOf(_Z^4-2)^2*x^2-3*RootOf(_Z^4-2)^3*x^2-2*RootOf(_Z^4-2)^3*x+8*(x
^7+3*x^5+3*x^3+x)^(3/4)-8*(x^7+3*x^5+3*x^3+x)^(1/2)*RootOf(_Z^4-2)+4*RootOf
(_Z^4-2)^2*(x^7+3*x^5+3*x^3+x)^(1/4)-RootOf(_Z^4-2)^3)/(1+x)^2/(x^2+1)^2)*R
ootOf(_Z^4-2)^3+3/16*ln((-RootOf(_Z^4-2)^3*x^6-2*RootOf(_Z^4-2)^3*x^5+4*(x^
7+3*x^5+3*x^3+x)^(1/4)*RootOf(_Z^4-2)^2*x^4-3*RootOf(_Z^4-2)^3*x^4-4*RootOf
(_Z^4-2)^3*x^3-8*(x^7+3*x^5+3*x^3+x)^(1/2)*RootOf(_Z^4-2)*x^2+8*(x^7+3*x^5+
3*x^3+x)^(1/4)*RootOf(_Z^4-2)^2*x^2-3*RootOf(_Z^4-2)^3*x^2-2*RootOf(_Z^4-2)
^3*x+8*(x^7+3*x^5+3*x^3+x)^(3/4)-8*(x^7+3*x^5+3*x^3+x)^(1/2)*RootOf(_Z^4-2)
+4*RootOf(_Z^4-2)^2*(x^7+3*x^5+3*x^3+x)^(1/4)-RootOf(_Z^4-2)^3)/(1+x)^2/(x^
2+1)^2)*RootOf(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)-3/8*RootOf(_Z^4-2)^2
*RootOf(_Z^2+RootOf(_Z^4-2)^2)*ln(-(RootOf(_Z^4-2)^3*x^6-RootOf(_Z^4-2)^2*R
ootOf(_Z^2+RootOf(_Z^4-2)^2)*x^6-2*RootOf(_Z^4-2)^3*x^5+2*RootOf(_Z^4-2)^2*
RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^5+4*(x^7+3*x^5+3*x^3+x)^(1/4)*RootOf(_Z^4-2
)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^4+3*RootOf(_Z^4-2)^3*x^4-3*RootOf(_Z^2+Ro
otOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2*x^4-4*RootOf(_Z^4-2)^3*x^3+4*RootOf(_Z^4-2
)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^3-4*(x^7+3*x^5+3*x^3+x)^(1/2)*RootOf(_Z
^4-2)*x^2-4*(x^7+3*x^5+3*x^3+x)^(1/2)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^2+8*(
x^7+3*x^5+3*x^3+x)^(1/4)*RootOf(_Z^4-2)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^2+3
*RootOf(_Z^4-2)^3*x^2-3*RootOf(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^2-
2*RootOf(_Z^4-2)^3*x+2*RootOf(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x+8*(
x^7+3*x^5+3*x^3+x)^(3/4)-4*(x^7+3*x^5+3*x^3+x)^(1/2)*RootOf(_Z^4-2)-4*RootO
f(_Z^2+RootOf(_Z^4-2)^2)*(x^7+3*x^5+3*x^3+x)^(1/2)+4*RootOf(_Z^2+RootOf(_Z^
4-2)^2)*RootOf(_Z^4-2)*(x^7+3*x^5+3*x^3+x)^(1/4)+RootOf(_Z^4-2)^3-RootOf(_Z
^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2))/(1+x)^2/(x^2+1)^2))*(x^3*(x^2+1))^(1
/4)/x*(x*(x^2+1)^3)^(1/4)/(x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+x^4+1\right)\left(x^5+x^3\right)^{\frac{1}{4}}}{\left(x^4-1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x^3)^(1/4)*(x^8+x^4+1)/x^4/(x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((x^8 + x^4 + 1)*(x^5 + x^3)^(1/4)/((x^4 - 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(x^5+x^3\right)}^{1/4}\,\left(x^8+x^4+1\right)}{x^4\,\left(x^4-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + x^5)^(1/4)*(x^4 + x^8 + 1))/(x^4*(x^4 - 1)),x)
```

```
[Out] int(((x^3 + x^5)^(1/4)*(x^4 + x^8 + 1))/(x^4*(x^4 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\left(x^2+1\right)}\left(x^2-x+1\right)\left(x^2+x+1\right)\left(x^4-x^2+1\right)}{x^4\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+x**3)**(1/4)*(x**8+x**4+1)/x**4/(x**4-1),x)
```

```
[Out] Integral((x**3*(x**2 + 1))**(1/4)*(x**2 - x + 1)*(x**2 + x + 1)*(x**4 - x**
2 + 1)/(x**4*(x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.1946 $\int \frac{1}{(b+ax^2)\sqrt[3]{x+x^3}}\,dx$

Optimal. Leaf size=191

$$\frac{\log\left(x\sqrt[3]{a-b}+\sqrt[3]{b}\sqrt[3]{x^3+x}\right)}{2b^{2/3}\sqrt[3]{a-b}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x\sqrt[3]{a-b}}{x\sqrt[3]{a-b}-2\sqrt[3]{b}\sqrt[3]{x^3+x}}\right)}{2b^{2/3}\sqrt[3]{a-b}}-\frac{\log\left(-\sqrt[3]{b}\sqrt[3]{x^3+x}\,x\sqrt[3]{a-b}+x^2(a-b)^{2/3}+b^{2/3}\right.}{4b^{2/3}\sqrt[3]{a-b}}$$

Rubi [A] time = 0.33, antiderivative size = 264, normalized size of antiderivative = 1.38, number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {2056, 466, 465, 377, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{x}\sqrt[3]{x^2+1}\log\left(\frac{x^{2/3}\sqrt[3]{a-b}}{\sqrt[3]{x^2+1}}+\sqrt[3]{b}\right)}{2b^{2/3}\sqrt[3]{x^3+x}\sqrt[3]{a-b}}-\frac{\sqrt[3]{x}\sqrt[3]{x^2+1}\log\left(\frac{x^{4/3}(a-b)^{2/3}}{(x^2+1)^{2/3}}-\frac{\sqrt[3]{b}x^{2/3}\sqrt[3]{a-b}}{\sqrt[3]{x^2+1}}+b^{2/3}\right)}{4b^{2/3}\sqrt[3]{x^3+x}\sqrt[3]{a-b}}-\frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{x^2+1}\tan^{-1}\left(\frac{\sqrt[3]{b}-\frac{2x^{2/3}\sqrt[3]{a-b}}{\sqrt[3]{x^2+1}}}{\sqrt{3}\sqrt[3]{b}}\right)}{2b^{2/3}\sqrt[3]{x^3+x}\sqrt[3]{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b + a*x^2)*(x + x^3)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*x^(1/3)*(1 + x^2)^(1/3)*ArcTan[(b^(1/3) - (2*(a - b)^(1/3)*x^
(2/3))/(1 + x^2)^(1/3))/(Sqrt[3]*b^(1/3))])/((a - b)^(1/3)*b^(2/3)*(x + x^3
)^(1/3)) + (x^(1/3)*(1 + x^2)^(1/3)*Log[b^(1/3) + ((a - b)^(1/3)*x^(2/3))/(
1 + x^2)^(1/3)])/(2*(a - b)^(1/3)*b^(2/3)*(x + x^3)^(1/3)) - (x^(1/3)*(1 +
x^2)^(1/3)*Log[b^(2/3) + ((a - b)^(2/3)*x^(4/3))/(1 + x^2)^(2/3) - ((a - b)
^(1/3)*b^(1/3)*x^(2/3))/(1 + x^2)^(1/3)])/(4*(a - b)^(1/3)*b^(2/3)*(x + x^3
)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{\left(b+ax^2\right)\sqrt[3]{x+x^3}}\,dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\int \frac{1}{\sqrt[3]{x}\sqrt[3]{1+x^2}\left(b+ax^2\right)}\,dx}{\sqrt[3]{x+x^3}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{1+x^6}\left(b+ax^6\right)}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x+x^3}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^3}\left(b+ax^3\right)}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{x+x^3}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{b-(-a+b)x^3}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2\sqrt[3]{x+x^3}}$$

$$= \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a-b}\,x}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2b^{2/3}\sqrt[3]{x+x^3}} + \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{2}{b^{2/3}-\sqrt[3]{\ldots}}\right.}{2b^{2/3}\sqrt[3]{\ldots}}$$

$$= \frac{\sqrt[3]{x}\sqrt[3]{1+x^2}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{a-b}\,x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2\sqrt[3]{a-b}\,b^{2/3}\sqrt[3]{x+x^3}} - \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{-\sqrt[3]{a-b}\sqrt[3]{b}+2(a-b)^{2/3}x}{b^{2/3}-\sqrt[3]{a-b}\sqrt[3]{b}\,x+(a-b)^{2/3}x^2}\right.}{4\sqrt[3]{a-b}\,b^{2/3}\sqrt[3]{x+x^3}}$$

$$= \frac{\sqrt[3]{x}\sqrt[3]{1+x^2}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{a-b}\,x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2\sqrt[3]{a-b}\,b^{2/3}\sqrt[3]{x+x^3}} - \frac{\sqrt[3]{x}\sqrt[3]{1+x^2}\log\left(b^{2/3}+\frac{(a-b)^{2/3}x^{4/3}}{\left(1+x^2\right)^{2/3}}-\frac{\sqrt[3]{a-b}\sqrt[3]{b}\,x^{2/3}}{\sqrt[3]{1+x^2}}\right.}{4\sqrt[3]{a-b}\,b^{2/3}\sqrt[3]{x+x^3}}$$

$$= -\frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{1+x^2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a-b}\,x^{2/3}}{\sqrt[3]{b}\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{a-b}\,b^{2/3}\sqrt[3]{x+x^3}} + \frac{\sqrt[3]{x}\sqrt[3]{1+x^2}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{a-b}\,x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2\sqrt[3]{a-b}\,b^{2/3}\sqrt[3]{x+x^3}} - \frac{\sqrt[3]{x}\sqrt[3]{\ldots}}{\ldots}$$

Mathematica [C] time = 0.03, size = 68, normalized size = 0.36

$$\frac{3x\sqrt[3]{x^2+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{(a-b)x^2}{ax^2+b}\right)}{2b\sqrt[3]{x^3+x}\sqrt[3]{\frac{ax^2}{b}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((b + a*x^2)*(x + x^3)^(1/3)),x]
```

```
[Out] (3*x*(1 + x^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, ((a - b)*x^2)/(b + a*
x^2)])/(2*b*(1 + (a*x^2)/b)^(1/3)*(x + x^3)^(1/3))
```

IntegrateAlgebraic [A] time = 0.48, size = 191, normalized size = 1.00

$$\frac{\log\left(x\sqrt[3]{a-b}+\sqrt[3]{b}\sqrt[3]{x^3+x}\right)}{2b^{2/3}\sqrt[3]{a-b}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x\sqrt[3]{a-b}}{x\sqrt[3]{a-b}-2\sqrt[3]{b}\sqrt[3]{x^3+x}}\right)}{2b^{2/3}\sqrt[3]{a-b}} - \frac{\log\left(-\sqrt[3]{b}\sqrt[3]{x^3+x}\,x\sqrt[3]{a-b}+x^2(a-b)^{2/3}+b^{2/3}\left(x^3+x\right)^{2/3}\right)}{4b^{2/3}\sqrt[3]{a-b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((b + a*x^2)*(x + x^3)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*(a - b)^(1/3)*x)/((a - b)^(1/3)*x - 2*b^(1/3)
*(x + x^3)^(1/3))])/((a - b)^(1/3)*b^(2/3)) + Log[(a - b)^(1/3)*x + b^(1/3)
*(x + x^3)^(1/3)]/(2*(a - b)^(1/3)*b^(2/3)) - Log[(a - b)^(2/3)*x^2 - (a -
```

```
b)^(1/3)*b^(1/3)*x*(x + x^3)^(1/3) + b^(2/3)*(x + x^3)^(2/3)]/(4*(a - b)^(1
/3)*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b)/(x^3+x)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.38, size = 187, normalized size = 0.98

$$\frac{3\left(-ab^{2}+b^{3}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a-b}{b}\right)^{\frac{1}{3}}+2\left(\frac{1}{x^{2}}+1\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a-b}{b}\right)^{\frac{1}{3}}}\right)}{2\left(\sqrt{3}\,ab^{2}-\sqrt{3}\,b^{3}\right)}-\frac{\left(-ab^{2}+b^{3}\right)^{\frac{2}{3}}\log\left(\left(-\frac{a-b}{b}\right)^{\frac{2}{3}}+\left(-\frac{a-b}{b}\right)^{\frac{1}{3}}\left(\frac{1}{x^{2}}+1\right)^{\frac{1}{3}}+\left(\frac{1}{x^{2}}+1\right)^{\frac{2}{3}}\right)}{4\left(ab^{2}-b^{3}\right)}+\frac{\left(-\frac{a-b}{b}\right)^{\frac{2}{3}}\log\left(\left|-\left(-\frac{a-b}{b}\right)^{\frac{1}{3}}+\left(\frac{1}{x^{2}}+1\right)^{\frac{1}{3}}\right|\right)}{2\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b)/(x^3+x)^(1/3),x, algorithm="giac")
[Out] 3/2*(-a*b^2 + b^3)^(2/3)*arctan(1/3*sqrt(3)*((-(a - b)/b)^(1/3) + 2*(1/x^2
+ 1)^(1/3))/(-(a - b)/b)^(1/3))/(sqrt(3)*a*b^2 - sqrt(3)*b^3) - 1/4*(-a*b^2
 + b^3)^(2/3)*log((-(a - b)/b)^(2/3) + (-(a - b)/b)^(1/3)*(1/x^2 + 1)^(1/3)
 + (1/x^2 + 1)^(2/3))/(a*b^2 - b^3) + 1/2*(-(a - b)/b)^(2/3)*log(abs(-(-(a
- b)/b)^(1/3) + (1/x^2 + 1)^(1/3)))/(a - b)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,x^{2}+b\right)\left(x^{3}+x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+b)/(x^3+x)^(1/3),x)
[Out] int(1/(a*x^2+b)/(x^3+x)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3\left(x^{3}+x\right)}{4\left(ax^{\frac{7}{3}}+bx^{\frac{1}{3}}\right)\left(x^{2}+1\right)^{\frac{1}{3}}}+\int \frac{3\left(bx^{2}+b\right)}{2\left(a^{2}x^{\frac{13}{3}}+2\,abx^{\frac{7}{3}}+b^{2}x^{\frac{1}{3}}\right)\left(x^{2}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b)/(x^3+x)^(1/3),x, algorithm="maxima")
[Out] -3/4*(x^3 + x)/((a*x^(7/3) + b*x^(1/3))*(x^2 + 1)^(1/3)) + integrate(3/2*(b
*x^2 + b)/((a^2*x^(13/3) + 2*a*b*x^(7/3) + b^2*x^(1/3))*(x^2 + 1)^(1/3)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\,x^{2}+b\right)\,{\left(x^{3}+x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b + a*x^2)*(x + x^3)^(1/3)),x)
```

```
[Out] int(1/((b + a*x^2)*(x + x^3)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x\left(x^{2}+1\right)}\left(ax^{2}+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+b)/(x**3+x)**(1/3),x)
```

```
[Out] Integral(1/((x*(x**2 + 1))**(1/3)*(a*x**2 + b)), x)
```

3.1947 $\int \frac{\left(-1+2x^6\right)\sqrt[3]{x+x^7}}{\left(1-2x^2+x^6\right)\left(1-x^2+x^6\right)}\,dx$

Optimal. Leaf size=191

$$-\frac{1}{2}\log\left(\sqrt[3]{x^7+x}-x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^7+x}-2x\right)}{2^{2/3}}-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^7+x}+x}\right)+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^7+x}+x}\right)}{2^{2/3}}+\frac{1}{4}\log\left(\right.$$

Rubi [F] time = 6.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1+2x^6\right)\sqrt[3]{x+x^7}}{\left(1-2x^2+x^6\right)\left(1-x^2+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + 2*x^6)*(x + x^7)^(1/3))/((1 - 2*x^2 + x^6)*(1 - x^2 + x^6)),x]

[Out] ((x + x^7)^(1/3)*Defer[Subst][Defer[Int][(1 + x^9)^(1/3)/(-1 + x), x], x, x
^(2/3)])/(2*x^(1/3)*(1 + x^6)^(1/3)) - ((1 + I*Sqrt[3])*(x + x^7)^(1/3)*Def
er[Subst][Defer[Int][(1 + x^9)^(1/3)/(1 - I*Sqrt[3] + 2*x), x], x, x^(2/3)]
)/(2*x^(1/3)*(1 + x^6)^(1/3)) - ((1 - I*Sqrt[3])*(x + x^7)^(1/3)*Defer[Subs
t][Defer[Int][(1 + x^9)^(1/3)/(1 + I*Sqrt[3] + 2*x), x], x, x^(2/3)])/(2*x^
(1/3)*(1 + x^6)^(1/3)) - ((-1 - Sqrt[5])^(2/3)*(x + x^7)^(1/3)*Defer[Subst]
[Defer[Int][(1 + x^9)^(1/3)/((1 + Sqrt[5])^(1/3) - (-2)^(1/3)*x), x], x, x^
(2/3)])/(2^(1/3)*Sqrt[5]*x^(1/3)*(1 + x^6)^(1/3)) + ((-1)^(2/3)*(x + x^7)^(
1/3)*Defer[Subst][Defer[Int][(1 + x^9)^(1/3)/((1 + Sqrt[5])^(1/3) - (-2)^(1
/3)*x), x], x, x^(2/3)])/(Sqrt[5]*(2*(1 + Sqrt[5]))^(1/3)*x^(1/3)*(1 + x^6)
^(1/3)) - ((1 - Sqrt[5])^(2/3)*(x + x^7)^(1/3)*Defer[Subst][Defer[Int][(1 +
 x^9)^(1/3)/((-1 + Sqrt[5])^(1/3) + (-2)^(1/3)*x), x], x, x^(2/3)])/(2^(1/3
)*Sqrt[5]*x^(1/3)*(1 + x^6)^(1/3)) - ((-1)^(2/3)*(x + x^7)^(1/3)*Defer[Subs
t][Defer[Int][(1 + x^9)^(1/3)/((-1 + Sqrt[5])^(1/3) + (-2)^(1/3)*x), x], x,
 x^(2/3)])/(Sqrt[5]*(2*(-1 + Sqrt[5]))^(1/3)*x^(1/3)*(1 + x^6)^(1/3)) - ((-
1 + Sqrt[5])^(2/3)*(x + x^7)^(1/3)*Defer[Subst][Defer[Int][(1 + x^9)^(1/3)/
((-1 + Sqrt[5])^(1/3) - 2^(1/3)*x), x], x, x^(2/3)])/(2^(1/3)*Sqrt[5]*x^(1/
3)*(1 + x^6)^(1/3)) - ((x + x^7)^(1/3)*Defer[Subst][Defer[Int][(1 + x^9)^(1
/3)/((-1 + Sqrt[5])^(1/3) - 2^(1/3)*x), x], x, x^(2/3)])/(Sqrt[5]*(2*(-1 +
Sqrt[5]))^(1/3)*x^(1/3)*(1 + x^6)^(1/3)) - ((1 + Sqrt[5])^(2/3)*(x + x^7)^(
1/3)*Defer[Subst][Defer[Int][(1 + x^9)^(1/3)/((1 + Sqrt[5])^(1/3) + 2^(1/3)
*x), x], x, x^(2/3)])/(2^(1/3)*Sqrt[5]*x^(1/3)*(1 + x^6)^(1/3)) + ((x + x^7
)^(1/3)*Defer[Subst][Defer[Int][(1 + x^9)^(1/3)/((1 + Sqrt[5])^(1/3) + 2^(1
/3)*x), x], x, x^(2/3)])/(Sqrt[5]*(2*(1 + Sqrt[5]))^(1/3)*x^(1/3)*(1 + x^6)
^(1/3)) + ((-1 - Sqrt[5])^(1/3)*(x + x^7)^(1/3)*Defer[Subst][Defer[Int][(1
+ x^9)^(1/3)/((-1 + Sqrt[5])^(1/3) - (-1)^(2/3)*2^(1/3)*x), x], x, x^(2/3)]
)/(2*Sqrt[5]*x^(1/3)*(1 + x^6)^(1/3)) + ((-1 - Sqrt[5])^(1/3)*(5 - Sqrt[5])
*(x + x^7)^(1/3)*Defer[Subst][Defer[Int][(1 + x^9)^(1/3)/((-1 + Sqrt[5])^(1
/3) - (-1)^(2/3)*2^(1/3)*x), x], x, x^(2/3)])/(10*x^(1/3)*(1 + x^6)^(1/3))
- ((1 - Sqrt[5])^(1/3)*(x + x^7)^(1/3)*Defer[Subst][Defer[Int][(1 + x^9)^(1
/3)/((1 + Sqrt[5])^(1/3) + (-1)^(2/3)*2^(1/3)*x), x], x, x^(2/3)])/(2*Sqrt[
5]*x^(1/3)*(1 + x^6)^(1/3)) + ((1 - Sqrt[5])^(1/3)*(5 + Sqrt[5])*(x + x^7)^
(1/3)*Defer[Subst][Defer[Int][(1 + x^9)^(1/3)/((1 + Sqrt[5])^(1/3) + (-1)^(
2/3)*2^(1/3)*x), x], x, x^(2/3)])/(10*x^(1/3)*(1 + x^6)^(1/3)) + (3*(x + x^
7)^(1/3)*Defer[Subst][Defer[Int][(x*(1 + x^9)^(1/3))/(1 - x^3 + x^9), x], x
, x^(2/3)])/(2*x^(1/3)*(1 + x^6)^(1/3)) - (9*(x + x^7)^(1/3)*Defer[Subst][D
efer[Int][(x^7*(1 + x^9)^(1/3))/(1 - x^3 + x^9), x], x, x^(2/3)])/(2*x^(1/3
)*(1 + x^6)^(1/3))
```

Rubi steps

$$\int \frac{\left(-1+2x^6\right)\sqrt[3]{x+x^7}}{\left(1-2x^2+x^6\right)\left(1-x^2+x^6\right)}\,dx = \frac{\sqrt[3]{x+x^7}\int \frac{\sqrt[3]{x}\sqrt[3]{1+x^6}\left(-1+2x^6\right)}{\left(1-2x^2+x^6\right)\left(1-x^2+x^6\right)}\,dx}{\sqrt[3]{x}\sqrt[3]{1+x^6}}$$

$$= \frac{\left(3\sqrt[3]{x+x^7}\right)\text{Subst}\left(\int \frac{x^3\sqrt[3]{1+x^{18}}\left(-1+2x^{18}\right)}{\left(1-2x^6+x^{18}\right)\left(1-x^6+x^{18}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1+x^6}}$$

$$= \frac{\left(3\sqrt[3]{x+x^7}\right)\text{Subst}\left(\int \frac{x\sqrt[3]{1+x^9}\left(-1+2x^9\right)}{\left(1-2x^3+x^9\right)\left(1-x^3+x^9\right)}\,dx,x,x^{2/3}\right)}{2\sqrt[3]{x}\sqrt[3]{1+x^6}}$$

$$= \frac{\left(3\sqrt[3]{x+x^7}\right)\text{Subst}\left(\int\left(\frac{\sqrt[3]{1+x^9}}{3(-1+x)}+\frac{(1-x)\sqrt[3]{1+x^9}}{3\left(1+x+x^2\right)}+\frac{x\left(-1-2x^3\right)\sqrt[3]{1+x^9}}{1-x^3-x^6}+\frac{x\left(1-3x^6\right)\sqrt[3]{1}}{1-x^3+x}\right.\right.}{2\sqrt[3]{x}\sqrt[3]{1+x^6}}$$

$$= \frac{\sqrt[3]{x+x^7}\,\text{Subst}\left(\int \frac{\sqrt[3]{1+x^9}}{-1+x}\,dx,x,x^{2/3}\right)}{2\sqrt[3]{x}\sqrt[3]{1+x^6}} + \frac{\sqrt[3]{x+x^7}\,\text{Subst}\left(\int \frac{(1-x)\sqrt[3]{1+x^9}}{1+x+x^2}\,dx,\right.}{2\sqrt[3]{x}\sqrt[3]{1+x^6}}$$

$$= \frac{\sqrt[3]{x+x^7}\,\text{Subst}\left(\int \frac{\sqrt[3]{1+x^9}}{-1+x}\,dx,x,x^{2/3}\right)}{2\sqrt[3]{x}\sqrt[3]{1+x^6}} + \frac{\sqrt[3]{x+x^7}\,\text{Subst}\left(\int\left(\frac{\left(-1-i\sqrt{3}\right)\sqrt[3]{1+x^9}}{1-i\sqrt{3}+2x}\right.\right.}{2\sqrt[3]{x}\sqrt[3]{1}}$$

$$= \frac{\sqrt[3]{x+x^7}\,\text{Subst}\left(\int \frac{\sqrt[3]{1+x^9}}{-1+x}\,dx,x,x^{2/3}\right)}{2\sqrt[3]{x}\sqrt[3]{1+x^6}} + \frac{\left(3\sqrt[3]{x+x^7}\right)\text{Subst}\left(\int \frac{x\sqrt[3]{1+x^9}}{-1+x^3+x^6}\,dx\right.}{2\sqrt[3]{x}\sqrt[3]{1+x^6}}$$

$$= \frac{\sqrt[3]{x+x^7}\,\text{Subst}\left(\int \frac{\sqrt[3]{1+x^9}}{-1+x}\,dx,x,x^{2/3}\right)}{2\sqrt[3]{x}\sqrt[3]{1+x^6}} + \frac{\left(3\sqrt[3]{x+x^7}\right)\text{Subst}\left(\int \frac{x\sqrt[3]{1+x^9}}{1-x^3+x^9}\,dx,\right.}{2\sqrt[3]{x}\sqrt[3]{1+x^6}}$$

$$= \frac{\sqrt[3]{x+x^7}\,\text{Subst}\left(\int \frac{\sqrt[3]{1+x^9}}{-1+x}\,dx,x,x^{2/3}\right)}{2\sqrt[3]{x}\sqrt[3]{1+x^6}} + \frac{\left(3\sqrt[3]{x+x^7}\right)\text{Subst}\left(\int \frac{x\sqrt[3]{1+x^9}}{1-x^3+x^9}\,dx,\right.}{2\sqrt[3]{x}\sqrt[3]{1+x^6}}$$

$$= \frac{\sqrt[3]{x+x^7}\,\text{Subst}\left(\int \frac{\sqrt[3]{1+x^9}}{-1+x}\,dx,x,x^{2/3}\right)}{2\sqrt[3]{x}\sqrt[3]{1+x^6}} + \frac{\left(3\sqrt[3]{x+x^7}\right)\text{Subst}\left(\int \frac{x\sqrt[3]{1+x^9}}{1-x^3+x^9}\,dx,\right.}{2\sqrt[3]{x}\sqrt[3]{1+x^6}}$$

$$= \frac{\sqrt[3]{x+x^7}\,\text{Subst}\left(\int \frac{\sqrt[3]{1+x^9}}{-1+x}\,dx,x,x^{2/3}\right)}{2\sqrt[3]{x}\sqrt[3]{1+x^6}} + \frac{\left(3\sqrt[3]{x+x^7}\right)\text{Subst}\left(\int \frac{x\sqrt[3]{1+x^9}}{1-x^3+x^9}\,dx,\right.}{2\sqrt[3]{x}\sqrt[3]{1+x^6}}$$

Mathematica [F] time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+2x^6\right)\sqrt[3]{x+x^7}}{\left(1-2x^2+x^6\right)\left(1-x^2+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + 2*x^6)*(x + x^7)^(1/3))/((1 - 2*x^2 + x^6)*(1 - x^2 + x^6)
),x]
```

```
[Out] Integrate[((-1 + 2*x^6)*(x + x^7)^(1/3))/((1 - 2*x^2 + x^6)*(1 - x^2 + x^6)
), x]
```

IntegrateAlgebraic [A] time = 3.03, size = 191, normalized size = 1.00

$$-\frac{1}{2}\log\left(\sqrt[3]{x^7+x}-x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^7+x}-2x\right)}{2^{2/3}}-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^7+x}+x}\right)+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^7+x}+x}\right)}{2^{2/3}}+\frac{1}{4}\log\left(\sqrt[3]{x^7+x}\,x+\left(x^7+x\right)^{2/3}+x^2\right)-\frac{\log\left(2^{2/3}\sqrt[3]{x^7+x}\,x+\sqrt[3]{2}\left(x^7+x\right)^{2/3}+2x^2\right)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + 2*x^6)*(x + x^7)^(1/3))/((1 - 2*x^2 + x^6)*(1 - x
^2 + x^6)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(x + x^7)^(1/3))]) + (Sqrt[3]*ArcTa
n[(Sqrt[3]*x)/(x + 2^(2/3)*(x + x^7)^(1/3))])/2^(2/3) - Log[-x + (x + x^7)^
(1/3)]/2 + Log[-2*x + 2^(2/3)*(x + x^7)^(1/3)]/2^(2/3) + Log[x^2 + x*(x + x
^7)^(1/3) + (x + x^7)^(2/3)]/4 - Log[2*x^2 + 2^(2/3)*x*(x + x^7)^(1/3) + 2^
(1/3)*(x + x^7)^(2/3)]/(2*2^(2/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6-1)*(x^7+x)^(1/3)/(x^6-2*x^2+1)/(x^6-x^2+1),x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^7+x\right)^{\frac{1}{3}}\left(2\,x^6-1\right)}{\left(x^6-x^2+1\right)\left(x^6-2\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6-1)*(x^7+x)^(1/3)/(x^6-2*x^2+1)/(x^6-x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate((x^7 + x)^(1/3)*(2*x^6 - 1)/((x^6 - x^2 + 1)*(x^6 - 2*x^2 + 1)),
x)
```

maple [F] time = 2.45, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^6-1\right)\left(x^7+x\right)^{\frac{1}{3}}}{\left(x^6-2x^2+1\right)\left(x^6-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6-1)*(x^7+x)^(1/3)/(x^6-2*x^2+1)/(x^6-x^2+1),x)
```

```
[Out] int((2*x^6-1)*(x^7+x)^(1/3)/(x^6-2*x^2+1)/(x^6-x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^7+x\right)^{\frac{1}{3}}\left(2\,x^6-1\right)}{\left(x^6-x^2+1\right)\left(x^6-2\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6-1)*(x^7+x)^(1/3)/(x^6-2*x^2+1)/(x^6-x^2+1),x, algorithm="m
axima")
```

```
[Out] integrate((x^7 + x)^(1/3)*(2*x^6 - 1)/((x^6 - x^2 + 1)*(x^6 - 2*x^2 + 1)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^6-1\right)\,{\left(x^7+x\right)}^{1/3}}{\left(x^6-x^2+1\right)\,\left(x^6-2\,x^2+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^6 - 1)*(x + x^7)^(1/3))/((x^6 - x^2 + 1)*(x^6 - 2*x^2 + 1)),x)
```

```
[Out] int(((2*x^6 - 1)*(x + x^7)^(1/3))/((x^6 - x^2 + 1)*(x^6 - 2*x^2 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**6-1)*(x**7+x)**(1/3)/(x**6-2*x**2+1)/(x**6-x**2+1),x)
```

```
[Out] Timed out
```

3.1948 $\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^3+a(q+px^3)^3\right)}{x^6}\,dx$

Optimal. Leaf size=191

$$\frac{\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}\left(6ap^4x^{12}+24ap^3qx^9-4ap^3qx^8+36ap^2q^2x^6-8ap^2q^2x^5-16ap^2q^2x^4+24apq^3x^3-\right.}{30x^5}$$

Rubi [F] time = 1.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^3+a(q+px^3)^3\right)}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*x^3 + a*
(q + p*x^3)^3))/x^6,x]
```

```
[Out] p*(2*b + 3*a*p*q^2)*Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6],
 x] - a*q^4*Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x^6, x]
- q*(b + a*p*q^2)*Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x^
3, x] + 5*a*p^3*q*Defer[Int][x^3*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6
], x] + 2*a*p^4*Defer[Int][x^6*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6],
 x]
```

Rubi steps

$$\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^3+a(q+px^3)^3\right)}{x^6}\,dx = \int \left(p\left(2b+3apq^2\right)\sqrt{q^2-2pqx^2+2pqx^3}\right.$$

$$= \left(2ap^4\right)\int x^6\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}$$

Mathematica [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^3+a(q+px^3)^3\right)}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*x^
3 + a*(q + p*x^3)^3))/x^6,x]
```

```
[Out] Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*x^
3 + a*(q + p*x^3)^3))/x^6, x]
```

IntegrateAlgebraic [A] time = 0.49, size = 191, normalized size = 1.00

$$\frac{\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}\left(6ap^4x^{12}+24ap^3qx^9-4ap^3qx^8+36ap^2q^2x^6-8ap^2q^2x^5-16ap^2q^2x^4+24apq^3x^3-4apq^3x^2+6aq^4+15bpx^6+15bqx^3\right)}{30x^5}-bpq\log\left(\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+px^3+q\right)+bpq\log(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x
^6]*(b*x^3 + a*(q + p*x^3)^3))/x^6,x]
```

```
[Out] (Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(6*a*q^4 - 4*a*p*q^3*x^2 + 15*
b*q*x^3 + 24*a*p*q^3*x^3 - 16*a*p^2*q^2*x^4 - 8*a*p^2*q^2*x^5 + 15*b*p*x^6
+ 36*a*p^2*q^2*x^6 - 4*a*p^3*q*x^8 + 24*a*p^3*q*x^9 + 6*a*p^4*x^12))/(30*x^
5) + b*p*q*Log[x] - b*p*q*Log[q + p*x^3 + Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3
+ p^2*x^6]]
```

fricas [A] time = 55.10, size = 188, normalized size = 0.98

$$-\frac{30\,bpqx^5\log\left(\frac{px^3+q+\sqrt{p^2x^6+2\,pqx^3-2\,pqx^2+q^2}}{x}\right)-\left(6\,ap^4x^{12}+24\,ap^3qx^9-4\,ap^3qx^8-8\,ap^2q^2x^5-16\,ap^2q^2x^4-4\,apq^3x^2+3\left(12\,ap^2q^2+5\,bp\right)x^6+6\,aq^4+3\left(8\,apq^3+5\,bq\right)x^3\right)\sqrt{p^2x^6+2\,pqx^3-2\,pqx^2+q^2}}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^3+a*(p*x
^3+q)^3)/x^6,x, algorithm="fricas")
```

```
[Out] -1/30*(30*b*p*q*x^5*log((p*x^3 + q + sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 +
 q^2))/x) - (6*a*p^4*x^12 + 24*a*p^3*q*x^9 - 4*a*p^3*q*x^8 - 8*a*p^2*q^2*x^
5 - 16*a*p^2*q^2*x^4 - 4*a*p*q^3*x^2 + 3*(12*a*p^2*q^2 + 5*b*p)*x^6 + 6*a*q
^4 + 3*(8*a*p*q^3 + 5*b*q)*x^3)*sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)
)/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6+2\,pqx^3-2\,pqx^2+q^2}\left(\left(px^3+q\right)^3a+bx^3\right)\left(2\,px^3-q\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^3+a*(p*x
^3+q)^3)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)*((p*x^3 + q)^3*a + b*
x^3)*(2*p*x^3 - q)/x^6, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(2p\,x^3-q\right)\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}\,\left(b\,x^3+a\left(p\,x^3+q\right)^3\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^3+a*(p*x^3+q)^
3)/x^6,x)
```

```
[Out] int((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^3+a*(p*x^3+q)^
3)/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6+2\,pqx^3-2\,pqx^2+q^2}\left(\left(px^3+q\right)^3a+bx^3\right)\left(2\,px^3-q\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^3+a*(p*x
^3+q)^3)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)*((p*x^3 + q)^3*a + b*
x^3)*(2*p*x^3 - q)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(q-2\,p\,x^3\right)\,\left(a\,{\left(p\,x^3+q\right)}^3+b\,x^3\right)\,\sqrt{p^2\,x^6+2\,p\,q\,x^3-2\,p\,q\,x^2+q^2}}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q - 2*p*x^3)*(a*(q + p*x^3)^3 + b*x^3)*(p^2*x^6 + q^2 - 2*p*q*x^2 +
2*p*q*x^3)^(1/2))/x^6,x)
```

```
[Out] -int(((q - 2*p*x^3)*(a*(q + p*x^3)^3 + b*x^3)*(p^2*x^6 + q^2 - 2*p*q*x^2 +
2*p*q*x^3)^(1/2))/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 p x^{3} - q\right) \sqrt{p^{2} x^{6} + 2 p q x^{3} - 2 p q x^{2} + q^{2}} \left(a p^{3} x^{9} + 3 a p^{2} q x^{6} + 3 a p q^{2} x^{3} + a q^{3} + b x^{3}\right)}{x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x**3-q)*(p**2*x**6+2*p*q*x**3-2*p*q*x**2+q**2)**(1/2)*(b*x**
3+a*(p*x**3+q)**3)/x**6,x)
```

```
[Out] Integral((2*p*x**3 - q)*sqrt(p**2*x**6 + 2*p*q*x**3 - 2*p*q*x**2 + q**2)*(a
*p**3*x**9 + 3*a*p**2*q*x**6 + 3*a*p*q**2*x**3 + a*q**3 + b*x**3)/x**6, x)
```

3.1949 $\int \frac{-1+(-1+2k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(2+b)x+(1+bk)x^2)}\,dx$

Optimal. Leaf size=192

$$\frac{\log\left(\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+x-1\right)}{b^{2/3}}-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\left(\sqrt[3]{b}-\sqrt[3]{b}\,x\right)\sqrt[3]{kx^3+(-k-1)x\right.}}{2b^{2/3}}$$

Rubi [F] time = 2.70, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+(-1+2k)x}{\sqrt[3]{(1-x)x(1-kx)}\,\left(1-(2+b)x+(1+bk)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + (-1 + 2*k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (2 + b)*x + (1 +
b*k)*x^2)),x]
```

```
[Out] -(((1 + Sqrt[4 + b - 4*k]/Sqrt[b] - 2*k)*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1
/3)*Defer[Int][1/((1 - k*x)^(1/3)*(-2 - b - Sqrt[b]*Sqrt[4 + b - 4*k] + 2*(
1 + b*k)*x)*(x - x^2)^(1/3)), x])/((1 - x)*x*(1 - k*x))^(1/3)) - ((1 - Sqrt
[4 + b - 4*k]/Sqrt[b] - 2*k)*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[In
t][1/((1 - k*x)^(1/3)*(-2 - b + Sqrt[b]*Sqrt[4 + b - 4*k] + 2*(1 + b*k)*x)*
(x - x^2)^(1/3)), x])/((1 - x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\begin{aligned}\int \frac{-1+(-1+2k)x}{\sqrt[3]{(1-x)x(1-kx)}\,\left(1-(2+b)x+(1+bk)x^2\right)}\,dx &= \frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{-1+(-1+2k)x}{\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\left(1-(2+b)x+(1+bk)x^2\right)}}{\sqrt[3]{(1-x)x(1-kx)}}\\ &= \frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{-1+(-1+2k)x}{\sqrt[3]{1-kx}\sqrt[3]{x-x^2}\left(1-(2+b)x+(1+bk)x^2\right)}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}\\ &= \frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int\left(\frac{-1-\frac{\sqrt{4+b-4k}}{\sqrt{b}}+2k}{\sqrt[3]{1-kx}\left(-2-b-\sqrt{b}\sqrt{4+b-4k}+2(1+bk)x\right.}\right.}{\sqrt[3]{(1-x)x(1\right.}}\\ &= \frac{\left(\left(-1-\frac{\sqrt{4+b-4k}}{\sqrt{b}}+2k\right)\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{}{\sqrt[3]{1-kx}\left(-2-\right.}}{\sqrt[3]{(1-x)x(1-kx)}}\end{aligned}$$

Mathematica [F] time = 7.52, size = 0, normalized size = 0.00

$$\int \frac{-1+(-1+2k)x}{\sqrt[3]{(1-x)x(1-kx)}\,\left(1-(2+b)x+(1+bk)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + (-1 + 2*k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (2 + b)*x +
 (1 + b*k)*x^2)),x]
```

```
[Out] Integrate[(-1 + (-1 + 2*k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (2 + b)*x +
 (1 + b*k)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.71, size = 192, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{b}\,\sqrt[3]{kx^3+(-k-1)x^2+x}+x-1\right)}{b^{2/3}}-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\left(\sqrt[3]{b}-\sqrt[3]{b}\,x\right)\sqrt[3]{kx^3+(-k-1)x^2+x}+x^2-2x+1\right)}{2b^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}-2x+2}\right)}{b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + (-1 + 2*k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (2
 + b)*x + (1 + b*k)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(2 - 2*x
 + b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))]])/b^(2/3) + Log[-1 + x + b^(1/
3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3)]/b^(2/3) - Log[1 - 2*x + x^2 + (b^(1/3)
 - b^(1/3)*x)*(x + (-1 - k)*x^2 + k*x^3)^(1/3) + b^(2/3)*(x + (-1 - k)*x^2
+ k*x^3)^(2/3)]/(2*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(2+b)*x+(b*k+1)*x^2),
x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2\,k-1)x-1}{((kx-1)(x-1)x)^{\frac{1}{3}}\left((bk+1)x^2-(b+2)x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(2+b)*x+(b*k+1)*x^2),
x, algorithm="giac")
```

```
[Out] integrate(((2*k - 1)*x - 1)/(((k*x - 1)*(x - 1)*x)^(1/3)*((b*k + 1)*x^2 - (
b + 2)*x + 1)), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{-1+(-1+2k)\,x}{((1-x)\,x\,(-kx+1))^{\frac{1}{3}}\left(1-(2+b)\,x+(bk+1)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(2+b)*x+(b*k+1)*x^2),x)
```

```
[Out] int((-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(2+b)*x+(b*k+1)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2\,k-1)x-1}{((kx-1)(x-1)x)^{\frac{1}{3}}\left((bk+1)x^2-(b+2)x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(2+b)*x+(b*k+1)*x^2),
x, algorithm="maxima")
```

[Out] integrate(((2*k - 1)*x - 1)/(((k*x - 1)*(x - 1)*x)^(1/3)*((b*k + 1)*x^2 - (
b + 2)*x + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(2\,k-1\right)-1}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/3}\,\left(\left(b\,k+1\right)\,x^2+\left(-b-2\right)\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(2*k - 1) - 1)/((x*(k*x - 1)*(x - 1))^(1/3)*(x^2*(b*k + 1) - x*(b +
2) + 1)),x)

[Out] int((x*(2*k - 1) - 1)/((x*(k*x - 1)*(x - 1))^(1/3)*(x^2*(b*k + 1) - x*(b +
2) + 1)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(1-(2+b)*x+(b*k+1)*x**2
),x)

[Out] Timed out

3.1950 $\int \frac{-3+2(1+k^2)x+(1+k^2)x^2-4k^2x^3+k^2x^4}{((1-x^2)(1-k^2x^2))^{2/3}(1-d-(2+d)x+(1+dk^2)x^2+dk^2x^3)}\,dx$

Optimal. Leaf size=192

$$-\frac{\log\left(d^{2/3}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+\left(\sqrt[3]{d}-\sqrt[3]{d}\,x\right)\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+x^2-2x+1\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{d}\,\sqrt[3]{k^2x^4\right.}}{}$$

Rubi [F] time = 7.83, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3+2\left(1+k^2\right)x+\left(1+k^2\right)x^2-4k^2x^3+k^2x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d-(2+d)x+\left(1+dk^2\right)x^2+dk^2x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3 + 2*(1 + k^2)*x + (1 + k^2)*x^2 - 4*k^2*x^3 + k^2*x^4)/(((1 - x^2)*
(1 - k^2*x^2))^(2/3)*(1 - d - (2 + d)*x + (1 + d*k^2)*x^2 + d*k^2*x^3)),x]
```

```
[Out] -(((1 + 5*d*k^2)*x*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*AppellF1[1/2, 2/3, 2
/3, 3/2, x^2, k^2*x^2])/(d^2*k^2*((1 - x^2)*(1 - k^2*x^2))^(2/3))) - (3^(3/
4)*Sqrt[2 + Sqrt[3]]*Sqrt[(-1 - k^2 + 2*k^2*x^2)^2]*((-1 + k^2)^(2/3) + 2^(
2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))*Sqrt[((-1 + k^2)^(4/3) - 2^(2
/3)*k^(2/3)*(-1 + k^2)^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3) + 2*2^(1/3)*k^
(4/3)*((1 - x^2)*(1 - k^2*x^2))^(2/3))/((1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^
(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))/
((1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))
^(1/3))], -7 - 4*Sqrt[3]])/(2^(2/3)*d*k^(2/3)*(1 + k^2 - 2*k^2*x^2)*Sqrt[(-
1 - k^2*(1 - 2*x^2))^2]*Sqrt[((-1 + k^2)^(2/3)*((-1 + k^2)^(2/3) + 2^(2/3)*
k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3)))/((1 + Sqrt[3])*(-1 + k^2)^(2/3) +
 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))^2]) - ((1 - d + 5*d*k^2 -
 8*d^2*k^2)*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][1/((1 - x^2)^(2/
3)*(1 - k^2*x^2)^(2/3)*(-1 + d + (2 + d)*x - (1 + d*k^2)*x^2 - d*k^2*x^3)),
 x])/(d^2*k^2*((1 - x^2)*(1 - k^2*x^2))^(2/3)) - ((2 + d + 11*d*k^2 + 2*d^2
*k^2*(1 - k^2))*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][x/((1 - x^2)
^(2/3)*(1 - k^2*x^2)^(2/3)*(1 - d - (2 + d)*x + (1 + d*k^2)*x^2 + d*k^2*x^3
)), x])/(d^2*k^2*((1 - x^2)*(1 - k^2*x^2))^(2/3)) + ((1 + 8*d*k^2 + 2*d^2*(
k^2 + 3*k^4))*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][x^2/((1 - x^2)
^(2/3)*(1 - k^2*x^2)^(2/3)*(1 - d - (2 + d)*x + (1 + d*k^2)*x^2 + d*k^2*x^3
)), x])/(d^2*k^2*((1 - x^2)*(1 - k^2*x^2))^(2/3))
```

Rubi steps

$$\int \frac{-3+2\left(1+k^2\right)x+\left(1+k^2\right)x^2-4k^2x^3+k^2x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d-(2+d)x+\left(1+dk^2\right)x^2+dk^2x^3\right)}\,dx = \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int \frac{-}{\left(1-x^2\right)^{2/3}}}{\left(\left(1-x^2\right)\left(1\right.\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int\left(-\frac{}{d^2k^2(1}\right.}{}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int \frac{}{\left(1-x^2\right)^{2/3}}}{d\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}}$$

$$= -\frac{\left(1+5dk^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2}}{d^2k^2\left(\left(1-x^2\right)\left(1-\right.\right.}$$

$$= -\frac{\left(1+5dk^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2}}{d^2k^2\left(\left(1-x^2\right)\left(1-\right.\right.}$$

$$= -\frac{\left(1+5dk^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2}}{d^2k^2\left(\left(1-x^2\right)\left(1-\right.\right.}$$

$$= -\frac{\left(1+5dk^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2}}{d^2k^2\left(\left(1-x^2\right)\left(1-\right.\right.}$$

Mathematica [F] time = 3.53, size = 0, normalized size = 0.00

$$\int \frac{-3+2\left(1+k^2\right)x+\left(1+k^2\right)x^2-4k^2x^3+k^2x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d-(2+d)x+\left(1+dk^2\right)x^2+dk^2x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3 + 2*(1 + k^2)*x + (1 + k^2)*x^2 - 4*k^2*x^3 + k^2*x^4)/(((1 -
 x^2)*(1 - k^2*x^2))^(2/3)*(1 - d - (2 + d)*x + (1 + d*k^2)*x^2 + d*k^2*x^3
)),x]
```

```
[Out] Integrate[(-3 + 2*(1 + k^2)*x + (1 + k^2)*x^2 - 4*k^2*x^3 + k^2*x^4)/(((1 -
 x^2)*(1 - k^2*x^2))^(2/3)*(1 - d - (2 + d)*x + (1 + d*k^2)*x^2 + d*k^2*x^3
)), x]
```

IntegrateAlgebraic [A] time = 8.09, size = 192, normalized size = 1.00

$$-\frac{\log\left(d^{2/3}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+\left(\sqrt[3]{d}-\sqrt[3]{d}\,x\right)\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+x^2-2x+1\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+x-1\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x-\sqrt{3}}{-2\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+x-1}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 + 2*(1 + k^2)*x + (1 + k^2)*x^2 - 4*k^2*x^3 + k^2*x^
4)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)*(1 - d - (2 + d)*x + (1 + d*k^2)*x^2 +
d*k^2*x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(-Sqrt[3] + Sqrt[3]*x)/(-1 + x - 2*d^(1/3)*(1 + (-1 - k^2)*
x^2 + k^2*x^4)^(1/3))]])/d^(1/3) + Log[-1 + x + d^(1/3)*(1 + (-1 - k^2)*x^2
+ k^2*x^4)^(1/3)]/d^(1/3) - Log[1 - 2*x + x^2 + (d^(1/3) - d^(1/3)*x)*(1 +
(-1 - k^2)*x^2 + k^2*x^4)^(1/3) + d^(2/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(2
/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*(k^2+1)*x+(k^2+1)*x^2-4*k^2*x^3+k^2*x^4)/((-x^2+1)*(-k^2*x^
2+1))^(2/3)/(1-d-(2+d)*x+(d*k^2+1)*x^2+d*k^2*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^4 - 4k^2x^3 + (k^2+1)x^2 + 2(k^2+1)x - 3}{\left(dk^2x^3 + (dk^2+1)x^2 - (d+2)x - d + 1\right)\left((k^2x^2-1)(x^2-1)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*(k^2+1)*x+(k^2+1)*x^2-4*k^2*x^3+k^2*x^4)/((-x^2+1)*(-k^2*x^
2+1))^(2/3)/(1-d-(2+d)*x+(d*k^2+1)*x^2+d*k^2*x^3),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^4 - 4*k^2*x^3 + (k^2 + 1)*x^2 + 2*(k^2 + 1)*x - 3)/((d*k^2
*x^3 + (d*k^2 + 1)*x^2 - (d + 2)*x - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(2/3)
), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{-3 + 2(k^2+1)x + (k^2+1)x^2 - 4k^2x^3 + k^2x^4}{\left((-x^2+1)(-k^2x^2+1)\right)^{\frac{2}{3}}\left(1 - d - (2+d)x + (dk^2+1)x^2 + dk^2x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*(k^2+1)*x+(k^2+1)*x^2-4*k^2*x^3+k^2*x^4)/((-x^2+1)*(-k^2*x^2+1))^
(2/3)/(1-d-(2+d)*x+(d*k^2+1)*x^2+d*k^2*x^3),x)
```

```
[Out] int((-3+2*(k^2+1)*x+(k^2+1)*x^2-4*k^2*x^3+k^2*x^4)/((-x^2+1)*(-k^2*x^2+1))^
(2/3)/(1-d-(2+d)*x+(d*k^2+1)*x^2+d*k^2*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^4 - 4k^2x^3 + (k^2+1)x^2 + 2(k^2+1)x - 3}{\left(dk^2x^3 + (dk^2+1)x^2 - (d+2)x - d + 1\right)\left((k^2x^2-1)(x^2-1)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*(k^2+1)*x+(k^2+1)*x^2-4*k^2*x^3+k^2*x^4)/((-x^2+1)*(-k^2*x^
2+1))^(2/3)/(1-d-(2+d)*x+(d*k^2+1)*x^2+d*k^2*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^4 - 4*k^2*x^3 + (k^2 + 1)*x^2 + 2*(k^2 + 1)*x - 3)/((d*k^2
*x^3 + (d*k^2 + 1)*x^2 - (d + 2)*x - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(2/3)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x\,\left(k^2+1\right)-4\,k^2\,x^3+k^2\,x^4+x^2\,\left(k^2+1\right)-3}{{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)}^{2/3}\,\left(x^2\,\left(d\,k^2+1\right)-d-x\,\left(d+2\right)+d\,k^2\,x^3+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x*(k^2 + 1) - 4*k^2*x^3 + k^2*x^4 + x^2*(k^2 + 1) - 3)/(((x^2 - 1)*(
k^2*x^2 - 1))^(2/3)*(x^2*(d*k^2 + 1) - d - x*(d + 2) + d*k^2*x^3 + 1)),x)
```

```
[Out] int((2*x*(k^2 + 1) - 4*k^2*x^3 + k^2*x^4 + x^2*(k^2 + 1) - 3)/(((x^2 - 1)*(
k^2*x^2 - 1))^(2/3)*(x^2*(d*k^2 + 1) - d - x*(d + 2) + d*k^2*x^3 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*(k**2+1)*x+(k**2+1)*x**2-4*k**2*x**3+k**2*x**4)/((-x**2+1)*
(-k**2*x**2+1))**(2/3)/(1-d-(2+d)*x+(d*k**2+1)*x**2+d*k**2*x**3),x)
```

```
[Out] Timed out
```

3.1951 $\int \frac{b-3ax^3+3x^6}{x^6(-b+2ax^3)\sqrt[4]{-bx+ax^4}}\,dx$

Optimal. Leaf size=192

$$\frac{\sqrt{2}\left(2a^2-3b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x\sqrt[4]{ax^4-bx}}{\sqrt{ax^4-bx}-\sqrt{a}\,x^2}\right)}{3\sqrt[4]{a}\,b^2}+\frac{\sqrt{2}\left(2a^2-3b\right)\tanh^{-1}\left(\frac{\sqrt{ax^4-bx}+\sqrt{a}\,x^2}{\sqrt{2}\sqrt[4]{a}\,x\sqrt[4]{ax^4-bx}}\right)}{3\sqrt[4]{a}\,b^2}-\frac{4\left(ax^4-bx\right)^{3/4}\left(b-ax^3\right)}{21b^2x^6}$$

Rubi [B] time = 1.64, antiderivative size = 617, normalized size of antiderivative = 3.21, number of steps used = 20, number of rules used = 14, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.326, Rules used = {2056, 6725, 271, 264, 466, 465, 494, 461, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{x}\left(2a^2-3b\right)\sqrt[4]{ax^3-b}\log\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3-b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3-b}}+1\right)}{3\sqrt{2}\sqrt[4]{a}\,b^2\sqrt[4]{ax^4-bx}}+\frac{\sqrt[4]{x}\left(2a^2-3b\right)\sqrt[4]{ax^3-b}\log\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3-b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3-b}}+1\right)}{3\sqrt{2}\sqrt[4]{a}\,b^2\sqrt[4]{ax^4-bx}}-\frac{\sqrt{2}\sqrt[4]{x}\left(2a^2-3b\right)\sqrt[4]{ax^3-b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3-b}}\right)}{3\sqrt[4]{a}\,b^2\sqrt[4]{ax^4-bx}}+\frac{\sqrt{2}\sqrt[4]{x}\left(2a^2-3b\right)\sqrt[4]{ax^3-b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{ax^3-b}}+1\right)}{3\sqrt[4]{a}\,b^2\sqrt[4]{ax^4-bx}}-\frac{\left(2a^2-3b\right)\left(b-ax^3\right)^2}{21a^2b^2x^5\sqrt[4]{ax^4-bx}}+\frac{4a\left(2-\frac{b}{a^2}\right)\left(b-ax^3\right)}{21b^2x^2\sqrt[4]{ax^4-bx}}-\frac{\left(2a^2-3b\right)\left(b-ax^3\right)}{3ab^2x^2\sqrt[4]{ax^4-bx}}+\frac{\left(2-\frac{b}{a^2}\right)\left(b-ax^3\right)}{7bx^5\sqrt[4]{ax^4-bx}}-\frac{2\left(b-ax^3\right)}{3abx^2\sqrt[4]{ax^4-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(b - 3*a*x^3 + 3*x^6)/(x^6*(-b + 2*a*x^3)*(-(b*x) + a*x^4)^(1/4)),x]
```

```
[Out] ((2 - b/a^2)*(b - a*x^3))/(7*b*x^5*(-(b*x) + a*x^4)^(1/4)) - ((2*a^2 - 3*b)
*(b - a*x^3))/(3*a*b^2*x^2*(-(b*x) + a*x^4)^(1/4)) - (2*(b - a*x^3))/(3*a*b
*x^2*(-(b*x) + a*x^4)^(1/4)) + (4*a*(2 - b/a^2)*(b - a*x^3))/(21*b^2*x^2*(-
(b*x) + a*x^4)^(1/4)) - ((2*a^2 - 3*b)*(b - a*x^3)^2)/(21*a^2*b^2*x^5*(-(b*
x) + a*x^4)^(1/4)) - (Sqrt[2]*(2*a^2 - 3*b)*x^(1/4)*(-b + a*x^3)^(1/4)*ArcT
an[1 - (Sqrt[2]*a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)])/(3*a^(1/4)*b^2*(-(b*x
) + a*x^4)^(1/4)) + (Sqrt[2]*(2*a^2 - 3*b)*x^(1/4)*(-b + a*x^3)^(1/4)*ArcTa
n[1 + (Sqrt[2]*a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)])/(3*a^(1/4)*b^2*(-(b*x)
 + a*x^4)^(1/4)) - ((2*a^2 - 3*b)*x^(1/4)*(-b + a*x^3)^(1/4)*Log[1 + (Sqrt[
a]*x^(3/2))/Sqrt[-b + a*x^3] - (Sqrt[2]*a^(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)
])/(3*Sqrt[2]*a^(1/4)*b^2*(-(b*x) + a*x^4)^(1/4)) + ((2*a^2 - 3*b)*x^(1/4)*
(-b + a*x^3)^(1/4)*Log[1 + (Sqrt[a]*x^(3/2))/Sqrt[-b + a*x^3] + (Sqrt[2]*a^
(1/4)*x^(3/4))/(-b + a*x^3)^(1/4)])/(3*Sqrt[2]*a^(1/4)*b^2*(-(b*x) + a*x^4)
^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{b-3ax^3+3x^6}{x^6\left(-b+2ax^3\right)\sqrt[4]{-bx+ax^4}}\,dx = \frac{\left(\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\int \frac{b-3ax^3+3x^6}{x^{25/4}\sqrt[4]{-b+ax^3}\left(-b+2ax^3\right)}\,dx}{\sqrt[4]{-bx+ax^4}}$$

$$= \frac{\left(\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\int\left(-\frac{3\left(2-\frac{b}{a^2}\right)}{4x^{25/4}\sqrt[4]{-b+ax^3}}+\frac{3}{2ax^{13/4}\sqrt[4]{-b+ax^3}}+\frac{-2a^2b+3b^2}{4a^2x^{25/4}\sqrt[4]{-b+ax^3}\left(-b+\cdots\right.}\cdots\right.}{\sqrt[4]{-bx+ax^4}}$$

$$= \frac{\left(3\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\int\frac{1}{x^{13/4}\sqrt[4]{-b+ax^3}}\,dx}{2a\sqrt[4]{-bx+ax^4}} - \frac{\left(3\left(2-\frac{b}{a^2}\right)\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\int\frac{\cdots}{x^{25/4}\cdots}}{4\sqrt[4]{-bx+ax^4}}\cdots$$

$$= \frac{\left(2-\frac{b}{a^2}\right)\left(b-ax^3\right)}{7bx^5\sqrt[4]{-bx+ax^4}} - \frac{2\left(b-ax^3\right)}{3abx^2\sqrt[4]{-bx+ax^4}} - \frac{\left(3a\left(2-\frac{b}{a^2}\right)\sqrt[4]{x}\,\sqrt[4]{-b+ax^3}\right)\int\cdots}{7b\sqrt[4]{-bx+ax^4}}\cdots$$

$$= \frac{\left(2-\frac{b}{a^2}\right)\left(b-ax^3\right)}{7bx^5\sqrt[4]{-bx+ax^4}} - \frac{2\left(b-ax^3\right)}{3abx^2\sqrt[4]{-bx+ax^4}} + \frac{4a\left(2-\frac{b}{a^2}\right)\left(b-ax^3\right)}{21b^2x^2\sqrt[4]{-bx+ax^4}} + \frac{\left(\left(-2a\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(2-\frac{b}{a^2}\right)\left(b-ax^3\right)}{7bx^5\sqrt[4]{-bx+ax^4}} - \frac{2\left(b-ax^3\right)}{3abx^2\sqrt[4]{-bx+ax^4}} + \frac{4a\left(2-\frac{b}{a^2}\right)\left(b-ax^3\right)}{21b^2x^2\sqrt[4]{-bx+ax^4}} + \frac{\left(\left(-2a\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(2-\frac{b}{a^2}\right)\left(b-ax^3\right)}{7bx^5\sqrt[4]{-bx+ax^4}} - \frac{2\left(b-ax^3\right)}{3abx^2\sqrt[4]{-bx+ax^4}} + \frac{4a\left(2-\frac{b}{a^2}\right)\left(b-ax^3\right)}{21b^2x^2\sqrt[4]{-bx+ax^4}} + \frac{\left(\left(-2a\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(2-\frac{b}{a^2}\right)\left(b-ax^3\right)}{7bx^5\sqrt[4]{-bx+ax^4}} - \frac{\left(2a^2-3b\right)\left(b-ax^3\right)}{3ab^2x^2\sqrt[4]{-bx+ax^4}} - \frac{2\left(b-ax^3\right)}{3abx^2\sqrt[4]{-bx+ax^4}} + \frac{4a\left(2-\cdots\right.}{21b^2x^{\cdots}}$$

$$= \frac{\left(2-\frac{b}{a^2}\right)\left(b-ax^3\right)}{7bx^5\sqrt[4]{-bx+ax^4}} - \frac{\left(2a^2-3b\right)\left(b-ax^3\right)}{3ab^2x^2\sqrt[4]{-bx+ax^4}} - \frac{2\left(b-ax^3\right)}{3abx^2\sqrt[4]{-bx+ax^4}} + \frac{4a\left(2-\cdots\right.}{21b^2x^{\cdots}}$$

$$= \frac{\left(2-\frac{b}{a^2}\right)\left(b-ax^3\right)}{7bx^5\sqrt[4]{-bx+ax^4}} - \frac{\left(2a^2-3b\right)\left(b-ax^3\right)}{3ab^2x^2\sqrt[4]{-bx+ax^4}} - \frac{2\left(b-ax^3\right)}{3abx^2\sqrt[4]{-bx+ax^4}} + \frac{4a\left(2-\cdots\right.}{21b^2x^{\cdots}}$$

$$= \frac{\left(2-\frac{b}{a^2}\right)\left(b-ax^3\right)}{7bx^5\sqrt[4]{-bx+ax^4}} - \frac{\left(2a^2-3b\right)\left(b-ax^3\right)}{3ab^2x^2\sqrt[4]{-bx+ax^4}} - \frac{2\left(b-ax^3\right)}{3abx^2\sqrt[4]{-bx+ax^4}} + \frac{4a\left(2-\cdots\right.}{21b^2x^{\cdots}}$$

$$= \frac{\left(2-\frac{b}{a^2}\right)\left(b-ax^3\right)}{7bx^5\sqrt[4]{-bx+ax^4}} - \frac{\left(2a^2-3b\right)\left(b-ax^3\right)}{3ab^2x^2\sqrt[4]{-bx+ax^4}} - \frac{2\left(b-ax^3\right)}{3abx^2\sqrt[4]{-bx+ax^4}} + \frac{4a\left(2-\cdots\right.}{21b^2x^{\cdots}}$$

Mathematica [C] time = 0.92, size = 244, normalized size = 1.27

$$\frac{\frac{\left(2a^2-3b\right)\left(-16ax^3\left(b-2ax^3\right)^2{}_3F_2\left(\frac{5}{4},2,2;1,\frac{9}{4};\frac{ax^3}{b-ax^3}\right)+8ax^3\left(-24a^2x^6+10abx^3+b^2\right){}_2F_1\left(\frac{5}{4},2;\frac{9}{4};\frac{ax^3}{b-ax^3}\right)+5\left(128a^3x^9-144a^2bx^6+13ab^2x^3+3b^3\right){}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{ax^3}{b-ax^3}\right)\right)}{a^2\left(ax^3-b\right)}+15\left(2-\frac{b}{a^2}\right)\left(b-ax^3\right)\left(4ax^3+3b\right)+\frac{210bx^3\left(ax^3-b\right)}{a}}{315b^2x^5\sqrt[4]{ax^4-bx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b - 3*a*x^3 + 3*x^6)/(x^6*(-b + 2*a*x^3)*(-(b*x) + a*x^4)^(1/4))
,x]
```

```
[Out] ((210*b*x^3*(-b + a*x^3))/a + 15*(2 - b/a^2)*(b - a*x^3)*(3*b + 4*a*x^3) +
((2*a^2 - 3*b)*(5*(3*b^3 + 13*a*b^2*x^3 - 144*a^2*b*x^6 + 128*a^3*x^9)*Hype
```

```
rgeometric2F1[1/4, 1, 5/4, (a*x^3)/(b - a*x^3)] + 8*a*x^3*(b^2 + 10*a*b*x^3
 - 24*a^2*x^6)*Hypergeometric2F1[5/4, 2, 9/4, (a*x^3)/(b - a*x^3)] - 16*a*x
^3*(b - 2*a*x^3)^2*HypergeometricPFQ[{5/4, 2, 2}, {1, 9/4}, (a*x^3)/(b - a*
x^3)]))/(a^2*(-b + a*x^3)))/(315*b^2*x^5*(-(b*x) + a*x^4)^(1/4))
```

IntegrateAlgebraic [A] time = 1.09, size = 192, normalized size = 1.00

$$\frac{\sqrt{2}\left(2a^2-3b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x\sqrt[4]{ax^4-bx}}{\sqrt{ax^4-bx}-\sqrt{a}\,x^2}\right)}{3\sqrt[4]{a}\,b^2}+\frac{\sqrt{2}\left(2a^2-3b\right)\tanh^{-1}\left(\frac{\sqrt{ax^4-bx}+\sqrt{a}\,x^2}{\sqrt{2}\sqrt[4]{a}\,x\sqrt[4]{ax^4-bx}}\right)}{3\sqrt[4]{a}\,b^2}-\frac{4\left(ax^4-bx\right)^{3/4}\left(b-ax^3\right)}{21b^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b - 3*a*x^3 + 3*x^6)/(x^6*(-b + 2*a*x^3)*(-(b*x) + a*x^
4)^(1/4)),x]
```

```
[Out] (-4*(b - a*x^3)*(-(b*x) + a*x^4)^(3/4))/(21*b^2*x^6) + (Sqrt[2]*(2*a^2 - 3*
b)*ArcTan[(Sqrt[2]*a^(1/4)*x*(-(b*x) + a*x^4)^(1/4))/(-(Sqrt[a]*x^2) + Sqrt
[-(b*x) + a*x^4])])/(3*a^(1/4)*b^2) + (Sqrt[2]*(2*a^2 - 3*b)*ArcTanh[(Sqrt[
a]*x^2 + Sqrt[-(b*x) + a*x^4])/(Sqrt[2]*a^(1/4)*x*(-(b*x) + a*x^4)^(1/4))])
/(3*a^(1/4)*b^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-3*a*x^3+b)/x^6/(2*a*x^3-b)/(a*x^4-b*x)^(1/4),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.51, size = 237, normalized size = 1.23

$$\frac{4\left(a-\frac{b}{x^3}\right)^{\frac{7}{4}}}{21\,b^2}+\frac{\sqrt{2}\left(2\,a^{\frac{11}{4}}-3\,a^{\frac{3}{4}}b\right)\log\left(\sqrt{2}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}a^{\frac{1}{4}}+\sqrt{a-\frac{b}{x^3}}+\sqrt{a}\right)}{6\,ab^2}-\frac{\sqrt{2}\left(2\,a^{\frac{11}{4}}-3\,a^{\frac{3}{4}}b\right)\log\left(-\sqrt{2}\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}a^{\frac{1}{4}}+\sqrt{a-\frac{b}{x^3}}+\sqrt{a}\right)}{6\,ab^2}-\frac{\left(2\,\sqrt{2}\,a^{\frac{11}{4}}-3\,\sqrt{2}\,a^{\frac{3}{4}}b\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}+2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2\,a^{\frac{1}{4}}}\right)}{3\,ab^2}-\frac{\left(2\,\sqrt{2}\,a^{\frac{11}{4}}-3\,\sqrt{2}\,a^{\frac{3}{4}}b\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}-2\left(a-\frac{b}{x^3}\right)^{\frac{1}{4}}\right)}{2\,a^{\frac{1}{4}}}\right)}{3\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-3*a*x^3+b)/x^6/(2*a*x^3-b)/(a*x^4-b*x)^(1/4),x, algorithm=
"giac")
```

```
[Out] 4/21*(a - b/x^3)^(7/4)/b^2 + 1/6*sqrt(2)*(2*a^(11/4) - 3*a^(3/4)*b)*log(sqr
t(2)*(a - b/x^3)^(1/4)*a^(1/4) + sqrt(a - b/x^3) + sqrt(a))/(a*b^2) - 1/6*s
qrt(2)*(2*a^(11/4) - 3*a^(3/4)*b)*log(-sqrt(2)*(a - b/x^3)^(1/4)*a^(1/4) +
sqrt(a - b/x^3) + sqrt(a))/(a*b^2) - 1/3*(2*sqrt(2)*a^(11/4) - 3*sqrt(2)*a^
(3/4)*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4) + 2*(a - b/x^3)^(1/4))/a^(1/4)
)/(a*b^2) - 1/3*(2*sqrt(2)*a^(11/4) - 3*sqrt(2)*a^(3/4)*b)*arctan(-1/2*sqrt
(2)*(sqrt(2)*a^(1/4) - 2*(a - b/x^3)^(1/4))/a^(1/4))/(a*b^2)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{3x^6-3a\,x^3+b}{x^6\left(2a\,x^3-b\right)\left(a\,x^4-bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^6-3*a*x^3+b)/x^6/(2*a*x^3-b)/(a*x^4-b*x)^(1/4),x)
```

```
[Out] int((3*x^6-3*a*x^3+b)/x^6/(2*a*x^3-b)/(a*x^4-b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^6 - 3\,ax^3 + b}{\left(ax^4 - bx\right)^{\frac{1}{4}}\left(2\,ax^3 - b\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-3*a*x^3+b)/x^6/(2*a*x^3-b)/(a*x^4-b*x)^(1/4),x, algorithm=
"maxima")
```

```
[Out] integrate((3*x^6 - 3*a*x^3 + b)/((a*x^4 - b*x)^(1/4)*(2*a*x^3 - b)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{3\,x^6 - 3\,a\,x^3 + b}{x^6\,\left(a\,x^4 - b\,x\right)^{1/4}\,\left(b - 2\,a\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - 3*a*x^3 + 3*x^6)/(x^6*(a*x^4 - b*x)^(1/4)*(b - 2*a*x^3)),x)
```

```
[Out] -int((b - 3*a*x^3 + 3*x^6)/(x^6*(a*x^4 - b*x)^(1/4)*(b - 2*a*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3ax^3 + b + 3x^6}{x^6\sqrt[4]{x\left(ax^3 - b\right)}\,\left(2ax^3 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**6-3*a*x**3+b)/x**6/(2*a*x**3-b)/(a*x**4-b*x)**(1/4),x)
```

```
[Out] Integral((-3*a*x**3 + b + 3*x**6)/(x**6*(x*(a*x**3 - b))**(1/4)*(2*a*x**3 -
 b)), x)
```

3.1952 $\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^6+a\left(q+px^3\right)^3\right)}{x^{11}}\,dx$

Optimal. Leaf size=192

$$\frac{\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}\left(6ap^4x^{12}-4ap^3qx^{10}+24ap^3qx^9-16ap^2q^2x^8-8ap^2q^2x^7+36ap^2q^2x^6-4apq^3x^4+\right.}{30x^{10}}$$

Rubi [F] time = 1.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^6+a\left(q+px^3\right)^3\right)}{x^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(b*x^6 + a*
(q + p*x^3)^3))/x^11,x]
```

```
[Out] -2*a*q^4*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^11, x] -
5*a*p*q^3*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^8, x] -
q*(2*b + 3*a*p^2*q)*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/
x^5, x] + p*(b + a*p^2*q)*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2
*x^6]/x^2, x] + a*p^4*Defer[Int][x*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x
^6], x]
```

Rubi steps

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^6+a\left(q+px^3\right)^3\right)}{x^{11}}\,dx = \int \left(-\frac{2aq^4\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^{11}}-\right.$$

$$= \left(ap^4\right)\int x\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\,d$$

Mathematica [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^6+a\left(q+px^3\right)^3\right)}{x^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(b*x^
6 + a*(q + p*x^3)^3))/x^11,x]
```

```
[Out] Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(b*x^
6 + a*(q + p*x^3)^3))/x^11, x]
```

IntegrateAlgebraic [A] time = 0.58, size = 192, normalized size = 1.00

$$\frac{\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}\left(6ap^4x^{12}-4ap^3qx^{10}+24ap^3qx^9-16ap^2q^2x^8-8ap^2q^2x^7+36ap^2q^2x^6-4apq^3x^4+24apq^3x^3+6aq^4+15bpx^9+15bqx^6\right)}{30x^{10}}-bpq\log\left(\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}+px^3+q\right)+2bpq\log(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x
^6]*(b*x^6 + a*(q + p*x^3)^3))/x^11,x]
```

```
[Out] (Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(6*a*q^4 + 24*a*p*q^3*x^3 - 4*
a*p*q^3*x^4 + 15*b*q*x^6 + 36*a*p^2*q^2*x^6 - 8*a*p^2*q^2*x^7 - 16*a*p^2*q^
2*x^8 + 15*b*p*x^9 + 24*a*p^3*q*x^9 - 4*a*p^3*q*x^10 + 6*a*p^4*x^12))/(30*x
^10) + 2*b*p*q*Log[x] - b*p*q*Log[q + p*x^3 + Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*
x^4 + p^2*x^6]]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(b*x^6+a*(p*x
^3+q)^3)/x^11,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6 - 2\,pqx^4 + 2\,pqx^3 + q^2}\left(bx^6 + \left(px^3 + q\right)^3 a\right)\left(px^3 - 2\,q\right)}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(b*x^6+a*(p*x
^3+q)^3)/x^11,x, algorithm="giac")
```

```
[Out] integrate(sqrt(p^2*x^6 - 2*p*q*x^4 + 2*p*q*x^3 + q^2)*(b*x^6 + (p*x^3 + q)^
3*a)*(p*x^3 - 2*q)/x^11, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(p\,x^3 - 2q\right)\sqrt{p^2x^6 - 2pq\,x^4 + 2pq\,x^3 + q^2}\,\left(b\,x^6 + a\left(p\,x^3 + q\right)^3\right)}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(b*x^6+a*(p*x^3+q)^
3)/x^11,x)
```

```
[Out] int((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(b*x^6+a*(p*x^3+q)^
3)/x^11,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6 - 2\,pqx^4 + 2\,pqx^3 + q^2}\left(bx^6 + \left(px^3 + q\right)^3 a\right)\left(px^3 - 2\,q\right)}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(b*x^6+a*(p*x
^3+q)^3)/x^11,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(p^2*x^6 - 2*p*q*x^4 + 2*p*q*x^3 + q^2)*(b*x^6 + (p*x^3 + q)^
3*a)*(p*x^3 - 2*q)/x^11, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(a\left(p\,x^3 + q\right)^3 + b\,x^6\right)\,\left(2\,q - p\,x^3\right)\,\sqrt{p^2\,x^6 - 2\,p\,q\,x^4 + 2\,p\,q\,x^3 + q^2}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*(q + p*x^3)^3 + b*x^6)*(2*q - p*x^3)*(p^2*x^6 + q^2 + 2*p*q*x^3 -
2*p*q*x^4)^(1/2))/x^11,x)
```

```
[Out] int(-((a*(q + p*x^3)^3 + b*x^6)*(2*q - p*x^3)*(p^2*x^6 + q^2 + 2*p*q*x^3 -
2*p*q*x^4)^(1/2))/x^11, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(px^3-2q\right)\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}\left(ap^3x^9+3ap^2qx^6+3apq^2x^3+aq^3+bx^6\right)}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**3-2*q)*(p**2*x**6-2*p*q*x**4+2*p*q*x**3+q**2)**(1/2)*(b*x**
6+a*(p*x**3+q)**3)/x**11,x)
```

```
[Out] Integral((p*x**3 - 2*q)*sqrt(p**2*x**6 - 2*p*q*x**4 + 2*p*q*x**3 + q**2)*(a
*p**3*x**9 + 3*a*p**2*q*x**6 + 3*a*p*q**2*x**3 + a*q**3 + b*x**6)/x**11, x)
```

3.1953 $\int \frac{x^2}{\sqrt{\frac{b+ax}{d+cx}}} \, dx$

Optimal. Leaf size=193

$$\frac{\sqrt{\frac{ax+b}{cx+d}}\left(8a^2c^3x^3+10a^2c^2dx^2-a^2cd^2x-3a^2d^3-10abc^3x^2-14abc^2dx-4abcd^2+15b^2c^3x+15b^2c^2d\right)}{24a^3c^2}+\frac{\left(a^3d^3\right.}{}$$

Rubi [A] time = 0.28, antiderivative size = 266, normalized size of antiderivative = 1.38, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1960, 413, 385, 199, 208}

$$-\frac{(3ad+5bc)(bc-ad)^2\sqrt{\frac{ax+b}{cx+d}}}{12a^2c^2\left(a-\frac{c(ax+b)}{cx+d}\right)^2}-\frac{\left(a^2d^2+2abcd+5b^2c^2\right)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ax+b}{cx+d}}}{\sqrt{a}}\right)}{8a^{7/2}c^{5/2}}+\frac{(cx+d)\left(a^2d^2+2abcd+5b^2c^2\right)\sqrt{\frac{ax+b}{cx+d}}}{8a^3c^2}-\frac{(bc-ad)^2\sqrt{\frac{ax+b}{cx+d}}\left(b-\frac{d(ax+b)}{cx+d}\right)}{3ac\left(a-\frac{c(ax+b)}{cx+d}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[(b + a*x)/(d + c*x)],x]
```

```
[Out] ((5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b + a*x)/(d + c*x)]*(d + c*x))/(8*
a^3*c^2) - ((b*c - a*d)^2*(5*b*c + 3*a*d)*Sqrt[(b + a*x)/(d + c*x)])/(12*a^
2*c^2*(a - (c*(b + a*x))/(d + c*x))^2) - ((b*c - a*d)^2*Sqrt[(b + a*x)/(d +
 c*x)]*(b - (d*(b + a*x))/(d + c*x)))/(3*a*c*(a - (c*(b + a*x))/(d + c*x))^
3) - ((b*c - a*d)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[(
b + a*x)/(d + c*x)])/Sqrt[a]])/(8*a^(7/2)*c^(5/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{\frac{b+ax}{d+cx}}}\,dx = -\left((2(bc-ad))\,\text{Subst}\left(\int \frac{\left(-b+dx^2\right)^2}{\left(a-cx^2\right)^4}\,dx, x, \sqrt{\frac{b+ax}{d+cx}}\right)\right)$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{b+ax}{d+cx}}\left(b-\frac{d(b+ax)}{d+cx}\right)}{3ac\left(a-\frac{c(b+ax)}{d+cx}\right)^3} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-b(5bc+ad)+3d(bc+ad)x^2}{\left(a-cx^2\right)^3}\,dx, x, \sqrt{\frac{b+ax}{d+cx}}\right)}{3ac}$$

$$= -\frac{(bc-ad)^2(5bc+3ad)\sqrt{\frac{b+ax}{d+cx}}}{12a^2c^2\left(a-\frac{c(b+ax)}{d+cx}\right)^2} - \frac{(bc-ad)^2\sqrt{\frac{b+ax}{d+cx}}\left(b-\frac{d(b+ax)}{d+cx}\right)}{3ac\left(a-\frac{c(b+ax)}{d+cx}\right)^3} - \frac{\left((bc-ad)\left(5b^2c^2+2abcd+a^2\right.\right.}{}$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{\frac{b+ax}{d+cx}}\,(d+cx)}{8a^3c^2} - \frac{(bc-ad)^2(5bc+3ad)\sqrt{\frac{b+ax}{d+cx}}}{12a^2c^2\left(a-\frac{c(b+ax)}{d+cx}\right)^2} - \frac{(bc-ad)^2\sqrt{\frac{b+ax}{d+cx}}\left(b-\right.}{3ac\left(a-\frac{c(b+ax)}{d+cx}\right)^3}$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{\frac{b+ax}{d+cx}}\,(d+cx)}{8a^3c^2} - \frac{(bc-ad)^2(5bc+3ad)\sqrt{\frac{b+ax}{d+cx}}}{12a^2c^2\left(a-\frac{c(b+ax)}{d+cx}\right)^2} - \frac{(bc-ad)^2\sqrt{\frac{b+ax}{d+cx}}\left(b-\right.}{3ac\left(a-\frac{c(b+ax)}{d+cx}\right)^3}$$

Mathematica [A] time = 0.44, size = 201, normalized size = 1.04

$$\frac{\sqrt{c}\,(ax+b)\sqrt{\frac{a(cx+d)}{ad-bc}}\left(a^2\left(8c^2x^2+2cdx-3d^2\right)-2abc(5cx+2d)+15b^2c^2\right)+3\sqrt{ax+b}\,\sqrt{ad-bc}\left(a^2d^2+2abcd+5b^2c^2\right)\sinh^{-1}\left(\frac{\sqrt{c}\,\sqrt{ax+b}}{\sqrt{ad-bc}}\right)}{24a^3c^{5/2}\sqrt{\frac{ax+b}{cx+d}}\,\sqrt{\frac{a(cx+d)}{ad-bc}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[(b + a*x)/(d + c*x)],x]
```

```
[Out] (Sqrt[c]*(b + a*x)*Sqrt[(a*(d + c*x))/(-(b*c) + a*d)]*(15*b^2*c^2 - 2*a*b*c
*(2*d + 5*c*x) + a^2*(-3*d^2 + 2*c*d*x + 8*c^2*x^2)) + 3*Sqrt[-(b*c) + a*d]
*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[b + a*x]*ArcSinh[(Sqrt[c]*Sqrt[b +
a*x])/Sqrt[-(b*c) + a*d]])/(24*a^3*c^(5/2)*Sqrt[(b + a*x)/(d + c*x)]*Sqrt[(
a*(d + c*x))/(-(b*c) + a*d)])
```

IntegrateAlgebraic [A] time = 0.31, size = 193, normalized size = 1.00

$$\frac{\sqrt{\frac{ax+b}{cx+d}}\left(8a^2c^3x^3+10a^2c^2dx^2-a^2cd^2x-3a^2d^3-10abc^3x^2-14abc^2dx-4abcd^2+15b^2c^3x+15b^2c^2d\right)}{24a^3c^2}+\frac{\left(a^3d^3+a^2bcd^2+3ab^2c^2d-5b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ax+b}{cx+d}}}{\sqrt{a}}\right)}{8a^{7/2}c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[(b + a*x)/(d + c*x)],x]
```

```
[Out] (Sqrt[(b + a*x)/(d + c*x)]*(15*b^2*c^2*d - 4*a*b*c*d^2 - 3*a^2*d^3 + 15*b^2
*c^3*x - 14*a*b*c^2*d*x - a^2*c*d^2*x - 10*a*b*c^3*x^2 + 10*a^2*c^2*d*x^2 +
```

8*a^2*c^3*x^3))/(24*a^3*c^2) + ((-5*b^3*c^3 + 3*a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*ArcTanh[(Sqrt[c]*Sqrt[(b + a*x)/(d + c*x)])/Sqrt[a]])/(8*a^(7/2)*c^(5/2))

fricas [A] time = 0.63, size = 419, normalized size = 2.17

$$\left[-\frac{3\left(5b^3c^3-3ab^2c^2d-a^2bcd^2-a^3d^3\right)\sqrt{ac}\log\left(-2acx-bc-ad-2\sqrt{ac}(cx+d)\sqrt{\frac{ax+b}{cx+d}}\right)-2\left(8a^3c^4x^3+15ab^2c^3d-4a^2bc^2d^2-3a^3cd^3-10\left(a^2bc^4-a^3c^3d\right)x^2+\left(15ab^2c^4-14a^2bc^3d-a^3c^2d^2\right)x\right)\sqrt{\frac{ax+b}{cx+d}}}{48a^4c^3},\ \frac{3\left(5b^3c^3-3ab^2c^2d-a^2bcd^2-a^3d^3\right)\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}(cx+d)\sqrt{\frac{ax+b}{cx+d}}}{acx+bc}\right)+\left(8a^3c^4x^3+15ab^2c^3d-4a^2bc^2d^2-3a^3cd^3-10\left(a^2bc^4-a^3c^3d\right)x^2+\left(15ab^2c^4-14a^2bc^3d-a^3c^2d^2\right)x\right)\sqrt{\frac{ax+b}{cx+d}}}{24a^4c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((a*x+b)/(c*x+d))^(1/2),x, algorithm="fricas")

[Out] [-1/48*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(a*c)*log(-2*a*c*x - b*c - a*d - 2*sqrt(a*c)*(c*x + d)*sqrt((a*x + b)/(c*x + d))) - 2*(8*a^3*c^4*x^3 + 15*a*b^2*c^3*d - 4*a^2*b*c^2*d^2 - 3*a^3*c*d^3 - 10*(a^2*b*c^4 - a^3*c^3*d)*x^2 + (15*a*b^2*c^4 - 14*a^2*b*c^3*d - a^3*c^2*d^2)*x)*sqrt((a*x + b)/(c*x + d)))/(a^4*c^3), 1/24*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(-a*c)*arctan(sqrt(-a*c)*(c*x + d)*sqrt((a*x + b)/(c*x + d))/(a*c*x + b*c)) + (8*a^3*c^4*x^3 + 15*a*b^2*c^3*d - 4*a^2*b*c^2*d^2 - 3*a^3*c*d^3 - 10*(a^2*b*c^4 - a^3*c^3*d)*x^2 + (15*a*b^2*c^4 - 14*a^2*b*c^3*d - a^3*c^2*d^2)*x)*sqrt((a*x + b)/(c*x + d)))/(a^4*c^3)]

giac [B] time = 0.36, size = 551, normalized size = 2.85

$$\frac{1}{24}\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)\left(\frac{3\left(5b^4c^4-8ab^3c^3d+2a^2b^2c^2d^2+a^4d^4\right)\arctan\left(\frac{c\sqrt{\frac{ax+b}{cx+d}}}{\sqrt{-ac}}\right)}{\sqrt{-ac}\,a^3c^2}-\frac{33a^2b^4c^4\sqrt{\frac{ax+b}{cx+d}}-\frac{40(ax+b)ab^4c^5\sqrt{\frac{ax+b}{cx+d}}}{cx+d}+\frac{15(ax+b)^2b^4c^6\sqrt{\frac{ax+b}{cx+d}}}{(cx+d)^2}-72a^3b^3c^3d\sqrt{\frac{ax+b}{cx+d}}+\frac{64(ax+b)a^2b^3c^4d\sqrt{\frac{ax+b}{cx+d}}}{cx+d}-\frac{24(ax+b)^2ab^3c^5d\sqrt{\frac{ax+b}{cx+d}}}{(cx+d)^2}+42a^4b^2c^2d^2\sqrt{\frac{ax+b}{cx+d}}+\frac{6(ax+b)^2a^2b^2c^4d^2\sqrt{\frac{ax+b}{cx+d}}}{(cx+d)^2}-\frac{32(ax+b)a^4bc^2d^3\sqrt{\frac{ax+b}{cx+d}}}{cx+d}-3a^6d^4\sqrt{\frac{ax+b}{cx+d}}+\frac{8(ax+b)a^5cd^4\sqrt{\frac{ax+b}{cx+d}}}{cx+d}+\frac{3(ax+b)^2a^4c^2d^4\sqrt{\frac{ax+b}{cx+d}}}{(cx+d)^2}}{\left(a-\frac{(ax+b)c}{cx+d}\right)^3a^3c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((a*x+b)/(c*x+d))^(1/2),x, algorithm="giac")

[Out] 1/24*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)*(3*(5*b^4*c^4 - 8*a*b^3*c^3*d + 2*a^2*b^2*c^2*d^2 + a^4*d^4)*arctan(c*sqrt((a*x + b)/(c*x + d))/sqrt(-a*c))/(sqrt(-a*c)*a^3*c^2) - (33*a^2*b^4*c^4*sqrt((a*x + b)/(c*x + d)) - 40*(a*x + b)*a*b^4*c^5*sqrt((a*x + b)/(c*x + d))/(c*x + d) + 15*(a*x + b)^2*b^4*c^6*sqrt((a*x + b)/(c*x + d))/(c*x + d)^2 - 72*a^3*b^3*c^3*d*sqrt((a*x + b)/(c*x + d)) + 64*(a*x + b)*a^2*b^3*c^4*d*sqrt((a*x + b)/(c*x + d))/(c*x + d) - 24*(a*x + b)^2*a*b^3*c^5*d*sqrt((a*x + b)/(c*x + d))/(c*x + d)^2 + 42*a^4*b^2*c^2*d^2*sqrt((a*x + b)/(c*x + d)) + 6*(a*x + b)^2*a^2*b^2*c^4*d^2*sqrt((a*x + b)/(c*x + d))/(c*x + d)^2 - 32*(a*x + b)*a^4*b*c^2*d^3*sqrt((a*x + b)/(c*x + d))/(c*x + d) - 3*a^6*d^4*sqrt((a*x + b)/(c*x + d)) + 8*(a*x + b)*a^5*c*d^4*sqrt((a*x + b)/(c*x + d))/(c*x + d) + 3*(a*x + b)^2*a^4*c^2*d^4*sqrt((a*x + b)/(c*x + d))/(c*x + d)^2)/((a - (a*x + b)*c/(c*x + d))^3*a^3*c^2))

maple [B] time = 0.05, size = 588, normalized size = 3.05

$$\frac{(ax+b)\left(3\ln\left(\frac{2acx+2\sqrt{acx^2+adx+bcx+bd}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)a^3d^3+3\ln\left(\frac{2acx+2\sqrt{acx^2+adx+bcx+bd}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)a^2bcd^2-15\ln\left(\frac{2acx+2\sqrt{acx^2+adx+bcx+bd}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)ab^2c^2d+9\ln\left(\frac{2acx+2\sqrt{acx^2+adx+bcx+bd}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)b^3c^3+24\ln\left(\frac{2acx+2\sqrt{(cx+d)(ax+b)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)ab^2c^2d-24\ln\left(\frac{2acx+2\sqrt{(cx+d)(ax+b)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)b^3c^3-12\sqrt{ac}\,\sqrt{acx^2+adx+bcx+bd}\,xa^2cd-36\sqrt{ac}\,\sqrt{acx^2+adx+bcx+bd}\,xabc^2+48\sqrt{(cx+d)(ax+b)}\,\sqrt{ac}\,b^2c^2+16\left(acx^2+adx+bcx+bd\right)^{\frac{3}{2}}ac\sqrt{ac}-6\sqrt{ac}\,\sqrt{acx^2+adx+bcx+bd}\,a^2d^2-24\sqrt{ac}\,\sqrt{acx^2+adx+bcx+bd}\,abcd-18\sqrt{ac}\,\sqrt{acx^2+adx+bcx+bd}\,b^2c^2\right)}{48a^3\sqrt{\frac{ax+b}{cx+d}}\,\sqrt{(cx+d)(ax+b)}\,c^2\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a*x+b)/(c*x+d))^(1/2),x)

[Out] 1/48*(a*x+b)/a^3*(3*ln(1/2*(2*a*c*x+2*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a^3*d^3+3*ln(1/2*(2*a*c*x+2*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a^2*b*c*d^2-15*ln(1/2*(2*a*c*x+2*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*b^2*c^2*d+9*ln(1/2*(2*a*c*x+2*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b^3*c^3+24*ln(1/2*(2*a*c*x+2*((c*x+d)*(a*x+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*b^2*c^2*d-24*ln(1/2*(2*a*c*x+2*((c*x+d)*(a*x+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b^3*c^3-12*(a*c)^(1/2)*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*x*a^2*c*d-36*(a*c)^(1/2)*(a*c*x^2+a*d*x+b*c*x+b

```
*d)^(1/2)*x*a*b*c^2+48*((c*x+d)*(a*x+b))^(1/2)*(a*c)^(1/2)*b^2*c^2+16*(a*c*
x^2+a*d*x+b*c*x+b*d)^(3/2)*a*c*(a*c)^(1/2)-6*(a*c)^(1/2)*(a*c*x^2+a*d*x+b*c
*x+b*d)^(1/2)*a^2*d^2-24*(a*c)^(1/2)*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*a*b*c*
d-18*(a*c)^(1/2)*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*b^2*c^2)/((a*x+b)/(c*x+d))
^(1/2)/((c*x+d)*(a*x+b))^(1/2)/c^2/(a*c)^(1/2)
```

maxima [A] time = 0.45, size = 355, normalized size = 1.84

$$-\frac{3\left(5\,b^{3}c^{5}-3\,ab^{2}c^{4}d-a^{2}bc^{3}d^{2}-a^{3}c^{2}d^{3}\right)\left(\frac{ax+b}{cx+d}\right)^{\frac{5}{2}}-8\left(5\,ab^{3}c^{4}-3\,a^{2}b^{2}c^{3}d-3\,a^{3}bc^{2}d^{2}+a^{4}cd^{3}\right)\left(\frac{ax+b}{cx+d}\right)^{\frac{3}{2}}+3\left(11\,a^{2}b^{3}c^{3}-13\,a^{3}b^{2}c^{2}d+a^{4}bcd^{2}+a^{5}d^{3}\right)\sqrt{\frac{ax+b}{cx+d}}}{24\left(a^{6}c^{2}-\frac{3\,(ax+b)a^{5}c^{3}}{cx+d}+\frac{3\,(ax+b)^{2}a^{4}c^{4}}{(cx+d)^{2}}-\frac{(ax+b)^{3}a^{3}c^{5}}{(cx+d)^{3}}\right)}+\frac{\left(5\,b^{3}c^{3}-3\,ab^{2}c^{2}d-a^{2}bcd^{2}-a^{3}d^{3}\right)\log\left(\frac{c\sqrt{\frac{ax+b}{cx+d}}-\sqrt{ac}}{c\sqrt{\frac{ax+b}{cx+d}}+\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^{3}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x+b)/(c*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*(5*b^3*c^5 - 3*a*b^2*c^4*d - a^2*b*c^3*d^2 - a^3*c^2*d^3)*((a*x +
b)/(c*x + d))^(5/2) - 8*(5*a*b^3*c^4 - 3*a^2*b^2*c^3*d - 3*a^3*b*c^2*d^2 +
a^4*c*d^3)*((a*x + b)/(c*x + d))^(3/2) + 3*(11*a^2*b^3*c^3 - 13*a^3*b^2*c^2
*d + a^4*b*c*d^2 + a^5*d^3)*sqrt((a*x + b)/(c*x + d)))/(a^6*c^2 - 3*(a*x +
b)*a^5*c^3/(c*x + d) + 3*(a*x + b)^2*a^4*c^4/(c*x + d)^2 - (a*x + b)^3*a^3*
c^5/(c*x + d)^3) + 1/16*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)
*log((c*sqrt((a*x + b)/(c*x + d)) - sqrt(a*c))/(c*sqrt((a*x + b)/(c*x + d))
 + sqrt(a*c)))/(sqrt(a*c)*a^3*c^2)
```

mupad [B] time = 0.86, size = 305, normalized size = 1.58

$$\frac{\frac{\left(\frac{b+a\,x}{d+c\,x}\right)^{5/2}\left(\frac{a^{3}\,d^{3}}{8}+\frac{a^{2}\,b\,c\,d^{2}}{8}+\frac{3\,a\,b^{2}\,c^{2}\,d}{8}-\frac{5\,b^{3}\,c^{3}}{8}\right)}{a^{6}}+\frac{\left(\frac{b+a\,x}{d+c\,x}\right)^{3/2}\left(\frac{a^{3}\,d^{3}}{3}-a^{2}\,b\,c\,d^{2}-a\,b^{2}\,c^{2}\,d+\frac{5\,b^{3}\,c^{3}}{3}\right)}{a^{5}\,c}-\frac{\sqrt{\frac{b+a\,x}{d+c\,x}}\left(\frac{a^{3}\,d^{3}}{8}+\frac{a^{2}\,b\,c\,d^{2}}{8}-\frac{13\,a\,b^{2}\,c^{2}\,d}{8}+\frac{11\,b^{3}\,c^{3}}{8}\right)}{a^{4}\,c^{2}}}{\frac{3\,c^{2}\,(b+a\,x)^{2}}{a^{2}\,(d+c\,x)^{2}}-\frac{c^{3}\,(b+a\,x)^{3}}{a^{3}\,(d+c\,x)^{3}}-\frac{3\,c\,(b+a\,x)}{a\,(d+c\,x)}+1}+\frac{\mathrm{atanh}\left(\frac{\sqrt{c}\,\sqrt{\frac{b+a\,x}{d+c\,x}}}{\sqrt{a}}\right)\,(a\,d-b\,c)\,\left(a^{2}\,d^{2}+2\,a\,b\,c\,d+5\,b^{2}\,c^{2}\right)}{8\,a^{7/2}\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b + a*x)/(d + c*x))^(1/2),x)
```

```
[Out] ((((b + a*x)/(d + c*x))^(5/2)*((a^3*d^3)/8 - (5*b^3*c^3)/8 + (3*a*b^2*c^2*d
)/8 + (a^2*b*c*d^2)/8))/a^6 + (((b + a*x)/(d + c*x))^(3/2)*((a^3*d^3)/3 + (
5*b^3*c^3)/3 - a*b^2*c^2*d - a^2*b*c*d^2))/(a^5*c) - (((b + a*x)/(d + c*x))
^(1/2)*((a^3*d^3)/8 + (11*b^3*c^3)/8 - (13*a*b^2*c^2*d)/8 + (a^2*b*c*d^2)/8
))/(a^4*c^2))/((3*c^2*(b + a*x)^2)/(a^2*(d + c*x)^2) - (c^3*(b + a*x)^3)/(a
^3*(d + c*x)^3) - (3*c*(b + a*x))/(a*(d + c*x)) + 1) + (atanh((c^(1/2)*((b
+ a*x)/(d + c*x))^(1/2))/a^(1/2))*(a*d - b*c)*(a^2*d^2 + 5*b^2*c^2 + 2*a*b*
c*d))/(8*a^(7/2)*c^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\sqrt{\frac{ax+b}{cx+d}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((a*x+b)/(c*x+d))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt((a*x + b)/(c*x + d)), x)
```

3.1954 $\int \frac{b+2ax}{(-b+ax)(2b+ax)\sqrt[4]{-1+bx+ax^2}}\,dx$

Optimal. Leaf size=193

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{a-2b^2}\sqrt[4]{ax^2+bx-1}}{\sqrt{a-2b^2}-\sqrt{a}\sqrt{ax^2+bx-1}}\right)}{a^{3/4}\sqrt[4]{a-2b^2}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{ax^2+bx-1}}{\sqrt{2}\sqrt[4]{a-2b^2}}+\frac{\sqrt[4]{a-2b^2}}{\sqrt{2}\sqrt[4]{a}}}{\sqrt[4]{ax^2+bx-1}}\right)}{a^{3/4}\sqrt[4]{a-2b^2}}$$

Rubi [A] time = 2.28, antiderivative size = 245, normalized size of antiderivative = 1.27, number of steps used = 30, number of rules used = 13, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 749, 748, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$\frac{2\sqrt[4]{4a+b^2}\sqrt[4]{\frac{a\left(-ax^2-bx+1\right)}{4a+b^2}}\tan^{-1}\left(\frac{\sqrt[4]{4a+b^2}\sqrt[4]{1-\frac{(2ax+b)^2}{4a+b^2}}}{\sqrt{2}\sqrt[4]{a-2b^2}}\right)}{a\sqrt[4]{a-2b^2}\sqrt[4]{ax^2+bx-1}}-\frac{2\sqrt[4]{4a+b^2}\sqrt[4]{\frac{a\left(-ax^2-bx+1\right)}{4a+b^2}}\tanh^{-1}\left(\frac{\sqrt[4]{4a+b^2}\sqrt[4]{1-\frac{(2ax+b)^2}{4a+b^2}}}{\sqrt{2}\sqrt[4]{a-2b^2}}\right)}{a\sqrt[4]{a-2b^2}\sqrt[4]{ax^2+bx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*a*x)/((-b + a*x)*(2*b + a*x)*(-1 + b*x + a*x^2)^(1/4)),x]
```

```
[Out] (2*(4*a + b^2)^(1/4)*((a*(1 - b*x - a*x^2))/(4*a + b^2))^(1/4)*ArcTan[((4*a
 + b^2)^(1/4)*(1 - (b + 2*a*x)^2/(4*a + b^2))^(1/4))/(Sqrt[2]*(a - 2*b^2)^(
1/4))])/(a*(a - 2*b^2)^(1/4)*(-1 + b*x + a*x^2)^(1/4)) - (2*(4*a + b^2)^(1/
4)*((a*(1 - b*x - a*x^2))/(4*a + b^2))^(1/4)*ArcTanh[((4*a + b^2)^(1/4)*(1
- (b + 2*a*x)^2/(4*a + b^2))^(1/4))/(Sqrt[2]*(a - 2*b^2)^(1/4))])/(a*(a - 2
*b^2)^(1/4)*(-1 + b*x + a*x^2)^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
```

```
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 748

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[1/((-4*c)/(b^2 - 4*a*c))^p, Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c),
 x]^p/Simp[2*c*d - b*e + e*x, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && GtQ[4*a - b^2/c, 0] && IntegerQ[4*p]
```

Rule 749

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Dist[(a + b*x + c*x^2)^p/(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^p, I
nt[(-((a*c)/(b^2 - 4*a*c)) - (b*c*x)/(b^2 - 4*a*c) - (c^2*x^2)/(b^2 - 4*a*c
))^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&  !GtQ[4*a - b^2/c
, 0] && IntegerQ[4*p]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{b+2ax}{(-b+ax)(2b+ax)\sqrt[4]{-1+bx+ax^2}}\,dx = \int \left(\frac{1}{(-b+ax)\sqrt[4]{-1+bx+ax^2}}+\frac{1}{(2b+ax)\sqrt[4]{-1+bx+ax^2}}\right)dx$$

$$= \int \frac{1}{(-b+ax)\sqrt[4]{-1+bx+ax^2}}\,dx + \int \frac{1}{(2b+ax)\sqrt[4]{-1+bx+ax^2}}\,d$$

$$= \frac{\sqrt[4]{-\frac{a\left(-1+bx+ax^2\right)}{4a+b^2}}\int \frac{1}{(-b+ax)\sqrt[4]{\frac{a}{4a+b^2}-\frac{abx}{4a+b^2}-\frac{a^2x^2}{4a+b^2}}}\,dx}{\sqrt[4]{-1+bx+ax^2}} + \frac{\sqrt[4]{-\frac{a\left(-1+bx+ax^2\right)}{4a+b^2}}}{}$$

$$= \frac{\left(\sqrt{2}\sqrt[4]{-\frac{a\left(-1+bx+ax^2\right)}{4a+b^2}}\right)\text{Subst}\left(\int \frac{1}{\left(-\frac{3a^2b}{4a+b^2}+ax\right)\sqrt[4]{1-\frac{\left(4a+b^2\right)x^2}{a^2}}}\,dx, x, -\frac{ab}{4a+}\right.}{\sqrt[4]{-1+bx+ax^2}}$$

$$= -2\frac{\left(\sqrt{2}\,a\sqrt[4]{-\frac{a\left(-1+bx+ax^2\right)}{4a+b^2}}\right)\text{Subst}\left(\int \frac{x}{\left(\frac{9a^4b^2}{\left(4a+b^2\right)^2}-a^2x^2\right)\sqrt[4]{1-\frac{\left(4a+b^2\right)x^2}{a^2}}}\,dx, x\right.}{\sqrt[4]{-1+bx+ax^2}}$$

$$= -2\frac{\left(a\sqrt[4]{-\frac{a\left(-1+bx+ax^2\right)}{4a+b^2}}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{9a^4b^2}{\left(4a+b^2\right)^2}-a^2x\right)\sqrt[4]{1-\frac{\left(4a+b^2\right)x}{a^2}}}\,dx, x, \left(-\right.\right.}{\sqrt{2}\sqrt[4]{-1+bx+ax^2}}$$

$$= 2\frac{\left(2\sqrt{2}\,a^3\sqrt[4]{-\frac{a\left(-1+bx+ax^2\right)}{4a+b^2}}\right)\text{Subst}\left(\int \frac{x^2}{\frac{9a^4b^2}{\left(4a+b^2\right)^2}-\frac{a^4}{4a+b^2}+\frac{a^4x^4}{4a+b^2}}\,dx, x, \sqrt[4]{1}\right.}{\left(4a+b^2\right)\sqrt[4]{-1+bx+ax^2}}$$

$$= 2\left(-\frac{\left(\sqrt{2}\sqrt{4a+b^2}\sqrt[4]{-\frac{a\left(-1+bx+ax^2\right)}{4a+b^2}}\right)\text{Subst}\left(\int \frac{1}{2\sqrt{a-2b^2}-\sqrt{4a+b^2}\,x^2}\,dx\right.}{a\sqrt[4]{-1+bx+ax^2}}\right.$$

$$= 2\left(\frac{\sqrt[4]{4a+b^2}\sqrt[4]{\frac{a\left(1-bx-ax^2\right)}{4a+b^2}}\tan^{-1}\left(\frac{\sqrt[4]{4a+b^2}\sqrt[4]{1-\frac{(b+2ax)^2}{4a+b^2}}}{\sqrt{2}\sqrt[4]{a-2b^2}}\right)}{a\sqrt[4]{a-2b^2}\sqrt[4]{-1+bx+ax^2}}-\frac{\sqrt[4]{4a+b^2}}{}\right.$$

Mathematica [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{b+2ax}{(-b+ax)(2b+ax)\sqrt[4]{-1+bx+ax^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(b + 2*a*x)/((-b + a*x)*(2*b + a*x)*(-1 + b*x + a*x^2)^(1/4)),x]
[Out] Integrate[(b + 2*a*x)/((-b + a*x)*(2*b + a*x)*(-1 + b*x + a*x^2)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 0.60, size = 193, normalized size = 1.00

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{a-2b^2}\sqrt[4]{ax^2+bx-1}}{\sqrt{a-2b^2}-\sqrt{a}\sqrt{ax^2+bx-1}}\right)}{a^{3/4}\sqrt[4]{a-2b^2}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{ax^2+bx-1}}{\sqrt{2}\sqrt[4]{a-2b^2}}+\frac{\sqrt[4]{a-2b^2}}{\sqrt{2}\sqrt[4]{a}}}{\sqrt[4]{ax^2+bx-1}}\right)}{a^{3/4}\sqrt[4]{a-2b^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + 2*a*x)/((-b + a*x)*(2*b + a*x)*(-1 + b*x + a*x^2)^(
1/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[2]*a^(1/4)*(a - 2*b^2)^(1/4)*(-1 + b*x + a*x^2)^(1/4)
)/(Sqrt[a - 2*b^2] - Sqrt[a]*Sqrt[-1 + b*x + a*x^2])]])/(a^(3/4)*(a - 2*b^2)
^(1/4)) - (Sqrt[2]*ArcTanh[((a - 2*b^2)^(1/4)/(Sqrt[2]*a^(1/4)) + (a^(1/4)*
Sqrt[-1 + b*x + a*x^2])/(Sqrt[2]*(a - 2*b^2)^(1/4)))/(-1 + b*x + a*x^2)^(1/
4)]])/(a^(3/4)*(a - 2*b^2)^(1/4))
```

fricas [A] time = 1.24, size = 240, normalized size = 1.24

$$-\frac{4\arctan\left(\frac{a\sqrt{\frac{2ab^2-a^2}{\sqrt{2a^3b^2-a^4}}+\sqrt{ax^2+bx-1}}}{\left(2a^3b^2-a^4\right)^{\frac{1}{4}}}-\frac{\left(ax^2+bx-1\right)^{\frac{1}{4}}a}{\left(2a^3b^2-a^4\right)^{\frac{1}{4}}}\right)}{\left(2a^3b^2-a^4\right)^{\frac{1}{4}}}-\frac{\log\left(\frac{2a^2b^2-a^3}{\left(2a^3b^2-a^4\right)^{\frac{3}{4}}}+\left(ax^2+bx-1\right)^{\frac{1}{4}}\right)}{\left(2a^3b^2-a^4\right)^{\frac{1}{4}}}+\frac{\log\left(-\frac{2a^2b^2-a^3}{\left(2a^3b^2-a^4\right)^{\frac{3}{4}}}+\left(ax^2+bx-1\right)^{\frac{1}{4}}\right)}{\left(2a^3b^2-a^4\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+b)/(a*x-b)/(a*x+2*b)/(a*x^2+b*x-1)^(1/4),x, algorithm="fri
cas")
```

```
[Out] -4*arctan(a*sqrt((2*a*b^2 - a^2)/sqrt(2*a^3*b^2 - a^4) + sqrt(a*x^2 + b*x -
 1))/(2*a^3*b^2 - a^4)^(1/4) - (a*x^2 + b*x - 1)^(1/4)*a/(2*a^3*b^2 - a^4)^
(1/4))/(2*a^3*b^2 - a^4)^(1/4) - log((2*a^2*b^2 - a^3)/(2*a^3*b^2 - a^4)^(3
/4) + (a*x^2 + b*x - 1)^(1/4))/(2*a^3*b^2 - a^4)^(1/4) + log(-(2*a^2*b^2 -
a^3)/(2*a^3*b^2 - a^4)^(3/4) + (a*x^2 + b*x - 1)^(1/4))/(2*a^3*b^2 - a^4)^(
1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2\,ax+b}{\left(ax^2+bx-1\right)^{\frac{1}{4}}(ax+2\,b)(ax-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+b)/(a*x-b)/(a*x+2*b)/(a*x^2+b*x-1)^(1/4),x, algorithm="gia
c")
```

```
[Out] integrate((2*a*x + b)/((a*x^2 + b*x - 1)^(1/4)*(a*x + 2*b)*(a*x - b)), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int\frac{2ax+b}{(ax-b)\,(ax+2b)\left(a\,x^2+bx-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x+b)/(a*x-b)/(a*x+2*b)/(a*x^2+b*x-1)^(1/4),x)
```

```
[Out] int((2*a*x+b)/(a*x-b)/(a*x+2*b)/(a*x^2+b*x-1)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,ax+b}{\left(ax^2+bx-1\right)^{\frac{1}{4}}(ax+2\,b)(ax-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+b)/(a*x-b)/(a*x+2*b)/(a*x^2+b*x-1)^(1/4),x, algorithm="max
ima")
```

```
[Out] integrate((2*a*x + b)/((a*x^2 + b*x - 1)^(1/4)*(a*x + 2*b)*(a*x - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{b+2\,a\,x}{(2\,b+a\,x)\,\left(b-a\,x\right)\,{\left(a\,x^2+b\,x-1\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + 2*a*x)/((2*b + a*x)*(b - a*x)*(b*x + a*x^2 - 1)^(1/4)),x)
```

```
[Out] -int((b + 2*a*x)/((2*b + a*x)*(b - a*x)*(b*x + a*x^2 - 1)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2ax+b}{(ax-b)\,(ax+2b)\,\sqrt[4]{ax^2+bx-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x+b)/(a*x-b)/(a*x+2*b)/(a*x**2+b*x-1)**(1/4),x)
```

```
[Out] Integral((2*a*x + b)/((a*x - b)*(a*x + 2*b)*(a*x**2 + b*x - 1)**(1/4)), x)
```

3.1955 $\int \frac{-1+akx+kx^2}{(1+kx^2)\sqrt{(1-x^2)(1-k^2x^2)}}\,dx$

Optimal. Leaf size=193

$$-\frac{a\sqrt{k}\tanh^{-1}\left(\frac{\left(2k^{3/2}+2\sqrt{k}\right)x^2}{k^2x^4+\left(kx^2-1\right)\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+2kx^2+1}\right)}{2(k+1)}+\frac{\tan^{-1}\left(\frac{\left(-k-2\sqrt{k}-1\right)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2-1}\right)}{k+1}+\frac{\tan^{-1}\left(\frac{\left(-k+2\sqrt{k}-1\right)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+k}\right)}{k+1}$$

Rubi [C] time = 2.80, antiderivative size = 411, normalized size of antiderivative = 2.13, number of steps used = 16, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {6719, 6725, 419, 2113, 537, 571, 93, 208}

$$\frac{\sqrt{1-x^2}\left(2\sqrt{-k}-ak\right)\sqrt{1-k^2x^2}\tanh^{-1}\left(\frac{\sqrt{k}\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{2\sqrt{k}(k+1)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}-\frac{\sqrt{1-x^2}\left(ak+2\sqrt{-k}\right)\sqrt{1-k^2x^2}\tanh^{-1}\left(\frac{\sqrt{k}\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{2\sqrt{k}(k+1)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}-\frac{\sqrt{1-x^2}\left(2-a\sqrt{-k}\right)\sqrt{1-k^2x^2}\,\Pi\left(-k;\sin^{-1}(x)|k^2\right)}{2\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}-\frac{\sqrt{1-x^2}\left(a\sqrt{-k}+2\right)\sqrt{1-k^2x^2}\,\Pi\left(-k;\sin^{-1}(x)|k^2\right)}{2\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}+\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + a*k*x + k*x^2)/((1 + k*x^2)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),x]
```

```
[Out] ((2*Sqrt[-k] - a*k)*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*ArcTanh[(Sqrt[k]*Sqrt[1
 - x^2])/Sqrt[1 - k^2*x^2]])/(2*Sqrt[k]*(1 + k)*Sqrt[(1 - x^2)*(1 - k^2*x^2
)]) - ((2*Sqrt[-k] + a*k)*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*ArcTanh[(Sqrt[k]*
Sqrt[1 - x^2])/Sqrt[1 - k^2*x^2]])/(2*Sqrt[k]*(1 + k)*Sqrt[(1 - x^2)*(1 - k
^2*x^2)]) + (Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticF[ArcSin[x], k^2])/Sqr
t[(1 - x^2)*(1 - k^2*x^2)] - ((2 - a*Sqrt[-k])*Sqrt[1 - x^2]*Sqrt[1 - k^2*x
^2]*EllipticPi[-k, ArcSin[x], k^2])/(2*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) - ((2
 + a*Sqrt[-k])*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[-k, ArcSin[x], k^
2])/(2*Sqrt[(1 - x^2)*(1 - k^2*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+akx+kx^2}{\left(1+kx^2\right)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}\,dx = \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{-1+akx+kx^2}{\sqrt{1-x^2}\left(1+kx^2\right)\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \left(\frac{1}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}} - \frac{2-akx}{\sqrt{1-x^2}\left(1+kx^2\right)\sqrt{1-k^2x^2}}\right)dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{1}{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \frac{\cdots}{\sqrt{1-x^2}\,(\cdots}}{\sqrt{\left(1-x^2\right)\left(1-k^2\cdots\right.}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \left(\frac{2-\cdots}{2\left(1-\sqrt{-k}\,x\right)\sqrt{\cdots}}\right.}{\sqrt{\left(1-x\cdots\right.}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(\left(2-a\sqrt{-k}\right)\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \cdots}{2\sqrt{\left(1-x^2\right)\left(1-\cdots\right.}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(\left(2-a\sqrt{-k}\right)\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\right)\int \cdots}{2\sqrt{\left(1-x^2\right)\left(1-k\cdots\right.}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(2-a\sqrt{-k}\right)\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(-\cdots\right.}{2\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(2-a\sqrt{-k}\right)\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(-\cdots\right.}{2\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(2\sqrt{-k}-ak\right)\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{k}\,\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{2\sqrt{k}\,(1+k)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(2\sqrt{-k}+ak\right)\sqrt{1-\cdots}}{2\sqrt{k}\,(1+\cdots}$$

Mathematica [C] time = 0.50, size = 202, normalized size = 1.05

$$\frac{ak\sqrt{x^2-1}\,\sqrt{k^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{k(k+1)}\,\sqrt{x^2-1}}{\sqrt{k+1}\,\sqrt{k^2x^2-1}}\right)+\sqrt{k+1}\,\sqrt{k(k+1)}\,\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)-2\sqrt{k+1}\,\sqrt{k(k+1)}\,\sqrt{1-x^2}\,\sqrt{1-k^2x^2}\,\Pi\left(-k;\sin^{-1}(x)|k^2\right)}{\sqrt{k+1}\,\sqrt{k(k+1)}\,\sqrt{\left(x^2-1\right)\left(k^2x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + a*k*x + k*x^2)/((1 + k*x^2)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]), 
x]
```

```
[Out] (a*k*Sqrt[-1 + x^2]*Sqrt[-1 + k^2*x^2]*ArcTanh[(Sqrt[k*(1 + k)]*Sqrt[-1 + x
^2])/(Sqrt[1 + k]*Sqrt[-1 + k^2*x^2])] + Sqrt[1 + k]*Sqrt[k*(1 + k)]*Sqrt[1
 - x^2]*Sqrt[1 - k^2*x^2]*EllipticF[ArcSin[x], k^2] - 2*Sqrt[1 + k]*Sqrt[k*
(1 + k)]*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[-k, ArcSin[x], k^2])/(S
qrt[1 + k]*Sqrt[k*(1 + k)]*Sqrt[(-1 + x^2)*(-1 + k^2*x^2)])
```

IntegrateAlgebraic [A] time = 3.79, size = 193, normalized size = 1.00

$$-\frac{a\sqrt{k}\,\tanh^{-1}\left(\frac{\left(2k^{3/2}+2\sqrt{k}\right)x^2}{k^2x^4+\left(kx^2-1\right)\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+2kx^2+1}\right)}{2(k+1)}+\frac{\tan^{-1}\left(\frac{\left(-k-2\sqrt{k}-1\right)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2-1}\right)}{k+1}+\frac{\tan^{-1}\left(\frac{\left(-k+2\sqrt{k}-1\right)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2-1}\right)}{k+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + a*k*x + k*x^2)/((1 + k*x^2)*Sqrt[(1 - x^2)*(1 - k^
2*x^2)]),x]
```

```
[Out] ArcTan[((-1 - 2*Sqrt[k] - k)*x)/(-1 + k*x^2 + Sqrt[1 + (-1 - k^2)*x^2 + k^2
*x^4])]/(1 + k) + ArcTan[((-1 + 2*Sqrt[k] - k)*x)/(-1 + k*x^2 + Sqrt[1 + (-
1 - k^2)*x^2 + k^2*x^4])]/(1 + k) - (a*Sqrt[k]*ArcTanh[((2*Sqrt[k] + 2*k^(3
/2))*x^2)/(1 + 2*k*x^2 + k^2*x^4 + (-1 + k*x^2)*Sqrt[1 + (-1 - k^2)*x^2 + k
^2*x^4])])/(2*(1 + k))
```

fricas [B] time = 4.11, size = 1809, normalized size = 9.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*k*x+k*x^2-1)/(k*x^2+1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algori
thm="fricas")
```

```
[Out] -1/8*sqrt((a^2*k - 4*sqrt(-a^2*k/(k^4 + 4*k^3 + 6*k^2 + 4*k + 1))*(k^2 + 2*
k + 1) - 4)/(k^2 + 2*k + 1))*log(2*(sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)*(a^3*
k - (a^3*k^2 + 4*a*k)*x^2 + 2*(a^2*k^3 + 2*(a^2 + 2)*k^2 + (a^2 + 8)*k + 4)
*sqrt(-a^2*k/(k^4 + 4*k^3 + 6*k^2 + 4*k + 1))*x + 4*a) + (2*a^2*k^3*x^4 + 2
*(a*k^3 + 2*a*k^2 + a*k)*x^3 + 2*a^2*k - 2*(a^2*k^3 + a^2*k)*x^2 - 2*(a*k^2
 + 2*a*k + a)*x + (4*(k^4 + 2*k^3 + k^2)*x^4 - (a*k^5 + 4*a*k^4 + 6*a*k^3 +
 4*a*k^2 + a*k)*x^3 - 4*(k^4 + 2*k^3 + 2*k^2 + 2*k + 1)*x^2 + 4*k^2 + (a*k^
4 + 4*a*k^3 + 6*a*k^2 + 4*a*k + a)*x + 8*k + 4)*sqrt(-a^2*k/(k^4 + 4*k^3 +
6*k^2 + 4*k + 1)))*sqrt((a^2*k - 4*sqrt(-a^2*k/(k^4 + 4*k^3 + 6*k^2 + 4*k +
 1))*(k^2 + 2*k + 1) - 4)/(k^2 + 2*k + 1)))/(k^2*x^4 + 2*k*x^2 + 1)) + 1/8*
sqrt((a^2*k - 4*sqrt(-a^2*k/(k^4 + 4*k^3 + 6*k^2 + 4*k + 1))*(k^2 + 2*k + 1
) - 4)/(k^2 + 2*k + 1))*log(2*(sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)*(a^3*k - (
a^3*k^2 + 4*a*k)*x^2 + 2*(a^2*k^3 + 2*(a^2 + 2)*k^2 + (a^2 + 8)*k + 4)*sqrt
(-a^2*k/(k^4 + 4*k^3 + 6*k^2 + 4*k + 1))*x + 4*a) - (2*a^2*k^3*x^4 + 2*(a*k
^3 + 2*a*k^2 + a*k)*x^3 + 2*a^2*k - 2*(a^2*k^3 + a^2*k)*x^2 - 2*(a*k^2 + 2*
a*k + a)*x + (4*(k^4 + 2*k^3 + k^2)*x^4 - (a*k^5 + 4*a*k^4 + 6*a*k^3 + 4*a*
k^2 + a*k)*x^3 - 4*(k^4 + 2*k^3 + 2*k^2 + 2*k + 1)*x^2 + 4*k^2 + (a*k^4 + 4
*a*k^3 + 6*a*k^2 + 4*a*k + a)*x + 8*k + 4)*sqrt(-a^2*k/(k^4 + 4*k^3 + 6*k^2
 + 4*k + 1)))*sqrt((a^2*k - 4*sqrt(-a^2*k/(k^4 + 4*k^3 + 6*k^2 + 4*k + 1))*
(k^2 + 2*k + 1) - 4)/(k^2 + 2*k + 1)))/(k^2*x^4 + 2*k*x^2 + 1)) - 1/8*sqrt(
(a^2*k + 4*sqrt(-a^2*k/(k^4 + 4*k^3 + 6*k^2 + 4*k + 1))*(k^2 + 2*k + 1) - 4
)/(k^2 + 2*k + 1))*log(2*(sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)*(a^3*k - (a^3*k
^2 + 4*a*k)*x^2 - 2*(a^2*k^3 + 2*(a^2 + 2)*k^2 + (a^2 + 8)*k + 4)*sqrt(-a^2
*k/(k^4 + 4*k^3 + 6*k^2 + 4*k + 1))*x + 4*a) + (2*a^2*k^3*x^4 + 2*(a*k^3 +
2*a*k^2 + a*k)*x^3 + 2*a^2*k - 2*(a^2*k^3 + a^2*k)*x^2 - 2*(a*k^2 + 2*a*k +
 a)*x - (4*(k^4 + 2*k^3 + k^2)*x^4 - (a*k^5 + 4*a*k^4 + 6*a*k^3 + 4*a*k^2 +
 a*k)*x^3 - 4*(k^4 + 2*k^3 + 2*k^2 + 2*k + 1)*x^2 + 4*k^2 + (a*k^4 + 4*a*k^
3 + 6*a*k^2 + 4*a*k + a)*x + 8*k + 4)*sqrt(-a^2*k/(k^4 + 4*k^3 + 6*k^2 + 4*
k + 1)))*sqrt((a^2*k + 4*sqrt(-a^2*k/(k^4 + 4*k^3 + 6*k^2 + 4*k + 1))*(k^2
+ 2*k + 1) - 4)/(k^2 + 2*k + 1)))/(k^2*x^4 + 2*k*x^2 + 1)) + 1/8*sqrt((a^2*
k + 4*sqrt(-a^2*k/(k^4 + 4*k^3 + 6*k^2 + 4*k + 1))*(k^2 + 2*k + 1) - 4)/(k^
2 + 2*k + 1))*log(2*(sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)*(a^3*k - (a^3*k^2 +
4*a*k)*x^2 - 2*(a^2*k^3 + 2*(a^2 + 2)*k^2 + (a^2 + 8)*k + 4)*sqrt(-a^2*k/(k
^4 + 4*k^3 + 6*k^2 + 4*k + 1))*x + 4*a) - (2*a^2*k^3*x^4 + 2*(a*k^3 + 2*a*k
^2 + a*k)*x^3 + 2*a^2*k - 2*(a^2*k^3 + a^2*k)*x^2 - 2*(a*k^2 + 2*a*k + a)*x
 - (4*(k^4 + 2*k^3 + k^2)*x^4 - (a*k^5 + 4*a*k^4 + 6*a*k^3 + 4*a*k^2 + a*k)
```

```
*x^3 - 4*(k^4 + 2*k^3 + 2*k^2 + 2*k + 1)*x^2 + 4*k^2 + (a*k^4 + 4*a*k^3 + 6
*a*k^2 + 4*a*k + a)*x + 8*k + 4)*sqrt(-a^2*k/(k^4 + 4*k^3 + 6*k^2 + 4*k + 1
)))*sqrt((a^2*k + 4*sqrt(-a^2*k/(k^4 + 4*k^3 + 6*k^2 + 4*k + 1))*(k^2 + 2*k
 + 1) - 4)/(k^2 + 2*k + 1)))/(k^2*x^4 + 2*k*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{akx + kx^2 - 1}{(kx^2 + 1)\sqrt{(k^2x^2 - 1)(x^2 - 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*k*x+k*x^2-1)/(k*x^2+1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((a*k*x + k*x^2 - 1)/((k*x^2 + 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))),
x)
```

maple [C] time = 0.05, size = 337, normalized size = 1.75

$$\frac{\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\operatorname{EllipticF}(x,k)}{\sqrt{k^2x^4-k^2x^2-x^2+1}} + \frac{a\,\operatorname{arctanh}\left(\frac{x^2k}{\sqrt{\frac{1}{k}+2+k}\,\sqrt{k^2x^4-k^2x^2-x^2+1}} + \frac{x^2k^2}{2\sqrt{\frac{1}{k}+2+k}\,\sqrt{k^2x^4-k^2x^2-x^2+1}} - \frac{k}{2\sqrt{\frac{1}{k}+2+k}\,\sqrt{k^2x^4-k^2x^2-x^2+1}} + \frac{x^2}{2\sqrt{\frac{1}{k}+2+k}\,\sqrt{k^2x^4-k^2x^2-x^2+1}} - \frac{1}{2\sqrt{\frac{1}{k}+2+k}\,\sqrt{k^2x^4-k^2x^2-x^2+1}\,k} - \frac{1}{\sqrt{\frac{1}{k}+2+k}\,\sqrt{k^2x^4-k^2x^2-x^2+1}}\right)}{2\sqrt{\frac{1}{k}+2+k}} - \frac{2\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\operatorname{EllipticPi}(x,-k,k)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*k*x+k*x^2-1)/(k*x^2+1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x)
```

```
[Out] (-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*EllipticF(x
,k)+1/2*a/(1/k+2+k)^(1/2)*arctanh(1/(1/k+2+k)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)
^(1/2)*x^2*k+1/2/(1/k+2+k)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*x^2*k^2-1/2/
(1/k+2+k)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*k+1/2/(1/k+2+k)^(1/2)/(k^2*x^
4-k^2*x^2-x^2+1)^(1/2)*x^2-1/2/(1/k+2+k)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2
)/k-1/(1/k+2+k)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2))-2*(-x^2+1)^(1/2)*(-k^2
*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*EllipticPi(x,-k,k)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{akx + kx^2 - 1}{(kx^2 + 1)\sqrt{(k^2x^2 - 1)(x^2 - 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*k*x+k*x^2-1)/(k*x^2+1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((a*k*x + k*x^2 - 1)/((k*x^2 + 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{k\,x^2 + a\,k\,x - 1}{(k\,x^2 + 1)\,\sqrt{(x^2 - 1)\,(k^2\,x^2 - 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^2 + a*k*x - 1)/((k*x^2 + 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)),x)
```

```
[Out] int((k*x^2 + a*k*x - 1)/((k*x^2 + 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{akx + kx^2 - 1}{\sqrt{(x - 1)(x + 1)(kx - 1)(kx + 1)}\,(kx^2 + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*k*x+k*x**2-1)/(k*x**2+1)/((-x**2+1)*(-k**2*x**2+1))**(1/2),x)
```

```
[Out] Integral((a*k*x + k*x**2 - 1)/(sqrt((x - 1)*(x + 1)*(k*x - 1)*(k*x + 1))*(k
*x**2 + 1)), x)
```

3.1956 $\int \frac{(-b+x^3)(b+x^3)}{\sqrt[3]{ax^2+x^3}}\,dx$

Optimal. Leaf size=193

$$\frac{(6561b^2-728a^6)\log\left(\sqrt[3]{ax^2+x^3}-x\right)}{6561}+\frac{(728a^6-6561b^2)\log\left(x\sqrt[3]{ax^2+x^3}+\left(ax^2+x^3\right)^{2/3}+x^2\right)}{13122}+\frac{(728\sqrt{3}\,a^6-$$

Rubi [B] time = 0.42, antiderivative size = 416, normalized size of antiderivative = 2.16, number of steps used = 12, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2053, 2011, 59, 2024}

$$-\frac{364a^6x^{2/3}\sqrt[3]{a+x}\log(x)}{6561\sqrt[3]{ax^2+x^3}}-\frac{364a^6x^{2/3}\sqrt[3]{a+x}\log\left(\frac{\sqrt[3]{a+x}}{\sqrt[3]{x}}-1\right)}{2187\sqrt[3]{ax^2+x^3}}-\frac{728a^6x^{2/3}\sqrt[3]{a+x}\tan^{-1}\left(\frac{2\sqrt[3]{a+x}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{2187\sqrt{3}\sqrt[3]{ax^2+x^3}}-\frac{728a^5\left(ax^2+x^3\right)^{2/3}}{2187x}+\frac{182}{729}a^4\left(ax^2+x^3\right)^{2/3}-\frac{52}{243}a^3x\left(ax^2+x^3\right)^{2/3}+\frac{26}{135}a^2x^2\left(ax^2+x^3\right)^{2/3}+\frac{b^2x^{2/3}\sqrt[3]{a+x}\log(x)}{2\sqrt[3]{ax^2+x^3}}+\frac{3b^2x^{2/3}\sqrt[3]{a+x}\log\left(\frac{\sqrt[3]{a+x}}{\sqrt[3]{x}}-1\right)}{2\sqrt[3]{ax^2+x^3}}+\frac{\sqrt{3}b^2x^{2/3}\sqrt[3]{a+x}\tan^{-1}\left(\frac{2\sqrt[3]{a+x}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{ax^2+x^3}}-\frac{8}{45}ax^3\left(ax^2+x^3\right)^{2/3}+\frac{1}{6}x^4\left(ax^2+x^3\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + x^3)*(b + x^3))/(a*x^2 + x^3)^(1/3),x]
```

```
[Out] (182*a^4*(a*x^2 + x^3)^(2/3))/729 - (728*a^5*(a*x^2 + x^3)^(2/3))/(2187*x)
- (52*a^3*x*(a*x^2 + x^3)^(2/3))/243 + (26*a^2*x^2*(a*x^2 + x^3)^(2/3))/135
 - (8*a*x^3*(a*x^2 + x^3)^(2/3))/45 + (x^4*(a*x^2 + x^3)^(2/3))/6 - (728*a^
6*x^(2/3)*(a + x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(a + x)^(1/3))/(Sqrt[3]*x^(1/
3))])/(2187*Sqrt[3]*(a*x^2 + x^3)^(1/3)) + (Sqrt[3]*b^2*x^(2/3)*(a + x)^(1/
3)*ArcTan[1/Sqrt[3] + (2*(a + x)^(1/3))/(Sqrt[3]*x^(1/3))])/(a*x^2 + x^3)^(
1/3) - (364*a^6*x^(2/3)*(a + x)^(1/3)*Log[x])/(6561*(a*x^2 + x^3)^(1/3)) +
(b^2*x^(2/3)*(a + x)^(1/3)*Log[x])/(2*(a*x^2 + x^3)^(1/3)) - (364*a^6*x^(2/
3)*(a + x)^(1/3)*Log[-1 + (a + x)^(1/3)/x^(1/3)])/(2187*(a*x^2 + x^3)^(1/3)
) + (3*b^2*x^(2/3)*(a + x)^(1/3)*Log[-1 + (a + x)^(1/3)/x^(1/3)])/(2*(a*x^2
 + x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2053

```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Expan
dIntegrand[Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && (Po
lyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IntegerQ[p] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-b+x^3\right)\left(b+x^3\right)}{\sqrt[3]{ax^2+x^3}}\,dx &= \int \left(-\frac{b^2}{\sqrt[3]{ax^2+x^3}}+\frac{x^6}{\sqrt[3]{ax^2+x^3}}\right) dx \\
&= -\left(b^2 \int \frac{1}{\sqrt[3]{ax^2+x^3}}\,dx\right)+\int \frac{x^6}{\sqrt[3]{ax^2+x^3}}\,dx \\
&= \frac{1}{6}x^4\left(ax^2+x^3\right)^{2/3}-\frac{1}{9}(8a)\int \frac{x^5}{\sqrt[3]{ax^2+x^3}}\,dx-\frac{\left(b^2x^{2/3}\sqrt[3]{a+x}\right)\int \frac{1}{x^{2/3}\sqrt[3]{a+x}}\,dx}{\sqrt[3]{ax^2+x^3}} \\
&= -\frac{8}{45}ax^3\left(ax^2+x^3\right)^{2/3}+\frac{1}{6}x^4\left(ax^2+x^3\right)^{2/3}+\frac{\sqrt{3}\,b^2x^{2/3}\sqrt[3]{a+x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+x}}{\sqrt{3}\sqrt[3]{x}}\right)}{\sqrt[3]{ax^2+x^3}} \\
&= \frac{26}{135}a^2x^2\left(ax^2+x^3\right)^{2/3}-\frac{8}{45}ax^3\left(ax^2+x^3\right)^{2/3}+\frac{1}{6}x^4\left(ax^2+x^3\right)^{2/3}+\frac{\sqrt{3}\,b^2x^{2/3}\sqrt[3]{a+}}{} \\
&= -\frac{52}{243}a^3x\left(ax^2+x^3\right)^{2/3}+\frac{26}{135}a^2x^2\left(ax^2+x^3\right)^{2/3}-\frac{8}{45}ax^3\left(ax^2+x^3\right)^{2/3}+\frac{1}{6}x^4\left(ax^2\right. \\
&= \frac{182}{729}a^4\left(ax^2+x^3\right)^{2/3}-\frac{52}{243}a^3x\left(ax^2+x^3\right)^{2/3}+\frac{26}{135}a^2x^2\left(ax^2+x^3\right)^{2/3}-\frac{8}{45}ax^3\left(ax^2\right. \\
&= \frac{182}{729}a^4\left(ax^2+x^3\right)^{2/3}-\frac{728a^5\left(ax^2+x^3\right)^{2/3}}{2187x}-\frac{52}{243}a^3x\left(ax^2+x^3\right)^{2/3}+\frac{26}{135}a^2x^2\left(ax^2\right. \\
&= \frac{182}{729}a^4\left(ax^2+x^3\right)^{2/3}-\frac{728a^5\left(ax^2+x^3\right)^{2/3}}{2187x}-\frac{52}{243}a^3x\left(ax^2+x^3\right)^{2/3}+\frac{26}{135}a^2x^2\left(ax^2\right. \\
&= \frac{182}{729}a^4\left(ax^2+x^3\right)^{2/3}-\frac{728a^5\left(ax^2+x^3\right)^{2/3}}{2187x}-\frac{52}{243}a^3x\left(ax^2+x^3\right)^{2/3}+\frac{26}{135}a^2x^2\left(ax^2\right.
\end{aligned}
$$

Mathematica [C] time = 0.10, size = 192, normalized size = 0.99

$$
\frac{3x\sqrt[3]{\frac{a+x}{a}}\left(a^6\,{}_2F_1\left(-\frac{17}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right)-6a^6\,{}_2F_1\left(-\frac{14}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right)+15a^6\,{}_2F_1\left(-\frac{11}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right)-20a^6\,{}_2F_1\left(-\frac{8}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right)+15a^6\,{}_2F_1\left(-\frac{5}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right)-6a^6\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right)+a^6\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right)-b^2\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right)\right)}{\sqrt[3]{x^2(a+x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + x^3)*(b + x^3))/(a*x^2 + x^3)^(1/3),x]
```

```
[Out] (3*x*((a + x)/a)^(1/3)*(a^6*Hypergeometric2F1[-17/3, 1/3, 4/3, -(x/a)] - 6*
a^6*Hypergeometric2F1[-14/3, 1/3, 4/3, -(x/a)] + 15*a^6*Hypergeometric2F1[-
11/3, 1/3, 4/3, -(x/a)] - 20*a^6*Hypergeometric2F1[-8/3, 1/3, 4/3, -(x/a)]
+ 15*a^6*Hypergeometric2F1[-5/3, 1/3, 4/3, -(x/a)] - 6*a^6*Hypergeometric2F
1[-2/3, 1/3, 4/3, -(x/a)] + a^6*Hypergeometric2F1[1/3, 1/3, 4/3, -(x/a)] -
b^2*Hypergeometric2F1[1/3, 1/3, 4/3, -(x/a)]))/(x^2*(a + x))^(1/3)
```

IntegrateAlgebraic [A] time = 0.77, size = 193, normalized size = 1.00

$$
\frac{\left(6561b^2-728a^6\right)\log\left(\sqrt[3]{ax^2+x^3}-x\right)}{6561}+\frac{\left(728a^6-6561b^2\right)\log\left(x\sqrt[3]{ax^2+x^3}+\left(ax^2+x^3\right)^{2/3}+x^2\right)}{13122}+\frac{\left(728\sqrt{3}\,a^6-6561\sqrt{3}\,b^2\right)\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{ax^2+x^3}+x}\right)}{6561}+\frac{\left(ax^2+x^3\right)^{2/3}\left(-7280a^5+5460a^4x-4680a^3x^2+4212a^2x^3-3888ax^4+3645x^5\right)}{21870x}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + x^3)*(b + x^3))/(a*x^2 + x^3)^(1/3),x]
```

```
[Out] ((a*x^2 + x^3)^(2/3)*(-7280*a^5 + 5460*a^4*x - 4680*a^3*x^2 + 4212*a^2*x^3
- 3888*a*x^4 + 3645*x^5))/(21870*x) + ((728*Sqrt[3]*a^6 - 6561*Sqrt[3]*b^2)
*ArcTan[(Sqrt[3]*x)/(x + 2*(a*x^2 + x^3)^(1/3))])/6561 + ((-728*a^6 + 6561*
b^2)*Log[-x + (a*x^2 + x^3)^(1/3)])/6561 + ((728*a^6 - 6561*b^2)*Log[x^2 +
x*(a*x^2 + x^3)^(1/3) + (a*x^2 + x^3)^(2/3)])/13122
```

fricas [A] time = 1.14, size = 185, normalized size = 0.96

$$-\frac{10\,\sqrt{3}\left(728\,a^{6}-6561\,b^{2}\right)x\arctan\left(\frac{\sqrt{3}x+2\,\sqrt{3}\left(ax^{2}+x^{3}\right)^{\frac{1}{3}}}{3\,x}\right)+10\left(728\,a^{6}-6561\,b^{2}\right)x\log\left(-\frac{x-\left(ax^{2}+x^{3}\right)^{\frac{1}{3}}}{x}\right)-5\left(728\,a^{6}-6561\,b^{2}\right)x\log\left(\frac{x^{2}+\left(ax^{2}+x^{3}\right)^{\frac{1}{3}}x+\left(ax^{2}+x^{3}\right)^{\frac{2}{3}}}{x^{2}}\right)+3\left(7280\,a^{5}-5460\,a^{4}x+4680\,a^{3}x^{2}-4212\,a^{2}x^{3}+3888\,ax^{4}-3645\,x^{5}\right)\left(ax^{2}+x^{3}\right)^{\frac{2}{3}}}{65610\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-b)*(x^3+b)/(a*x^2+x^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/65610*(10*sqrt(3)*(728*a^6 - 6561*b^2)*x*arctan(1/3*(sqrt(3)*x + 2*sqrt(
3)*(a*x^2 + x^3)^(1/3))/x) + 10*(728*a^6 - 6561*b^2)*x*log(-(x - (a*x^2 + x
^3)^(1/3))/x) - 5*(728*a^6 - 6561*b^2)*x*log((x^2 + (a*x^2 + x^3)^(1/3)*x +
 (a*x^2 + x^3)^(2/3))/x^2) + 3*(7280*a^5 - 5460*a^4*x + 4680*a^3*x^2 - 4212
*a^2*x^3 + 3888*a*x^4 - 3645*x^5)*(a*x^2 + x^3)^(2/3))/x
```

giac [A] time = 0.61, size = 197, normalized size = 1.02

$$-\frac{10\,\sqrt{3}\left(728\,a^{7}-6561\,ab^{2}\right)\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{a}{x}+1\right)^{\frac{1}{3}}+1\right)\right)-5\left(728\,a^{7}-6561\,ab^{2}\right)\log\left(\left(\frac{a}{x}+1\right)^{\frac{2}{3}}+\left(\frac{a}{x}+1\right)^{\frac{1}{3}}+1\right)+10\left(728\,a^{7}-6561\,ab^{2}\right)\log\left(\left|\left(\frac{a}{x}+1\right)^{\frac{1}{3}}-1\right|\right)+\frac{3\left(7280\,a^{7}\left(\frac{a}{x}+1\right)^{\frac{17}{3}}-41860\,a^{7}\left(\frac{a}{x}+1\right)^{\frac{14}{3}}+99320\,a^{7}\left(\frac{a}{x}+1\right)^{\frac{11}{3}}-123812\,a^{7}\left(\frac{a}{x}+1\right)^{\frac{8}{3}}+84592\,a^{7}\left(\frac{a}{x}+1\right)^{\frac{5}{3}}-29165\,a^{7}\left(\frac{a}{x}+1\right)^{\frac{2}{3}}\right)x^{6}}{a^{6}}}{65610\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-b)*(x^3+b)/(a*x^2+x^3)^(1/3),x, algorithm="giac")
```

```
[Out] -1/65610*(10*sqrt(3)*(728*a^7 - 6561*a*b^2)*arctan(1/3*sqrt(3)*(2*(a/x + 1)
^(1/3) + 1)) - 5*(728*a^7 - 6561*a*b^2)*log((a/x + 1)^(2/3) + (a/x + 1)^(1/
3) + 1) + 10*(728*a^7 - 6561*a*b^2)*log(abs((a/x + 1)^(1/3) - 1)) + 3*(7280
*a^7*(a/x + 1)^(17/3) - 41860*a^7*(a/x + 1)^(14/3) + 99320*a^7*(a/x + 1)^(1
1/3) - 123812*a^7*(a/x + 1)^(8/3) + 84592*a^7*(a/x + 1)^(5/3) - 29165*a^7*(
a/x + 1)^(2/3))*x^6/a^6)/a
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}-b\right)\left(x^{3}+b\right)}{\left(a\,x^{2}+x^{3}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-b)*(x^3+b)/(a*x^2+x^3)^(1/3),x)
```

```
[Out] int((x^3-b)*(x^3+b)/(a*x^2+x^3)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(x^{3}+b\right)}{\left(x^{3}-b\right)}}{{\left(ax^{2}+x^{3}\right)}^{\frac{1}{3}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-b)*(x^3+b)/(a*x^2+x^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + b)*(x^3 - b)/(a*x^2 + x^3)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(x^3+b\right)\,\left(b-x^3\right)}{{\left(x^3+a\,x^2\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b + x^3)*(b - x^3))/(a*x^2 + x^3)^(1/3),x)
[Out] -int(((b + x^3)*(b - x^3))/(a*x^2 + x^3)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-b+x^{3}\right)\left(b+x^{3}\right)}{\sqrt[3]{x^{2}\left(a+x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-b)*(x**3+b)/(a*x**2+x**3)**(1/3),x)
[Out] Integral((-b + x**3)*(b + x**3)/(x**2*(a + x))**(1/3), x)
```

3.1957 $\int \frac{x}{(1-x^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

Optimal. Leaf size=193

$$\frac{\sqrt{-2a-2b-c}\,\tan^{-1}\left(\frac{x\sqrt{-2a-2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2-2\sqrt{a}\,x+\sqrt{a}}\right)}{2(2a+2b+c)}-\frac{\sqrt{-2a+2b-c}\,\tan^{-1}\left(\frac{x\sqrt{-2a+2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2+2\sqrt{a}\,x+\right.}{2(2a-2b+c)}$$

Rubi [F] time = 0.75, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{(1-x^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/((1 - x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] -1/2*Defer[Int][1/((-1 + x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x] - De
fer[Int][1/((1 + x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x]/2
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1-x^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx &= \int \left(-\frac{1}{2(-1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}-\frac{1}{2(1+x)\sqrt{a+bx+cx^2}}\right.\\ &= -\left(\frac{1}{2}\int \frac{1}{(-1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx\right)-\frac{1}{2}\int \frac{}{(1+x)\sqrt{a+}}\end{aligned}$$

Mathematica [C] time = 1.49, size = 3036, normalized size = 15.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 - x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] ((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])^2*((-(EllipticF[ArcS
in[Sqrt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*
#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*
#1^4 & , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[
a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^
3 + a*#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2]
- Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2
+ b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])
)/((-Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1
^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & ,
2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])))]*(-1 + Root[a + b*#
1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])) + EllipticPi[((-1 + Root[a + b*#1 + c
*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &
 , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((-1 + Root[a + b
*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a
*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])), ArcSin[Sq
rt[((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 +
c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4
& , 4]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b
```

```
*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a
*#1^4 & , 4]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Roo
t[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#
1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((-
Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 +
b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] -
Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))))]*(Root[a + b*#1 + c*#1^2
 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2]
))/((-1 + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(-1 + Root[a + b
*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])) + (-(EllipticF[ArcSin[Sqrt[((x - Ro
ot[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*
#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((
x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^
2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4
]))]], -(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1
+ c*#1^2 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^
4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((-Root[a + b*
#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*
#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*
#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))))]*(1 + Root[a + b*#1 + c*#1^2 + b*#1
^3 + a*#1^4 & , 1])) + EllipticPi[((1 + Root[a + b*#1 + c*#1^2 + b*#1^3 + a
*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b
*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((1 + Root[a + b*#1 + c*#1^2 + b*#1
^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root
[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])), ArcSin[Sqrt[((x - Root[a + b
*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a
*#1^4 & , 2] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((x - Root
[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1
^3 + a*#1^4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))]], -
(((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#1^2
 + b*#1^3 + a*#1^4 & , 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1]
 - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((-Root[a + b*#1 + c*#
1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & ,
 3])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 + c*#
1^2 + b*#1^3 + a*#1^4 & , 4]))))]*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4
& , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2]))/((1 + Root[a + b
*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1])*(1 + Root[a + b*#1 + c*#1^2 + b*#1^3
 + a*#1^4 & , 2]))))*Sqrt[((Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1]
- Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(x - Root[a + b*#1 + c*#
1^2 + b*#1^3 + a*#1^4 & , 3]))/((x - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1
^4 & , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - Root[a + b*#1
 + c*#1^2 + b*#1^3 + a*#1^4 & , 3]))]*Sqrt[((x - Root[a + b*#1 + c*#1^2 + b
*#1^3 + a*#1^4 & , 1])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] - R
oot[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2])*(x - Root[a + b*#1 + c*#1^2
 + b*#1^3 + a*#1^4 & , 4])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2]
 - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4]))/((x - Root[a + b*#1 +
c*#1^2 + b*#1^3 + a*#1^4 & , 2])^2*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^
4 & , 1] - Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 4])^2)]*(-Root[a +
b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 +
a*#1^4 & , 4]))/(Sqrt[x*(b + c*x + b*x^2) + a*(1 + x^4)]*(-Root[a + b*#1 +
c*#1^2 + b*#1^3 + a*#1^4 & , 1] + Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4
& , 2])*(Root[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , 2] - Root[a + b*#1 +
c*#1^2 + b*#1^3 + a*#1^4 & , 4]))
```

IntegrateAlgebraic [A] time = 1.29, size = 193, normalized size = 1.00

$$\frac{\sqrt{-2a-2b-c}\,\tan^{-1}\left(\frac{x\sqrt{-2a-2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2-2\sqrt{a}\,x+\sqrt{a}}\right)}{2(2a+2b+c)}-\frac{\sqrt{-2a+2b-c}\,\tan^{-1}\left(\frac{x\sqrt{-2a+2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2+2\sqrt{a}\,x+\sqrt{a}}\right)}{2(2a-2b+c)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((1 - x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] (Sqrt[-2*a - 2*b - c]*ArcTan[(Sqrt[-2*a - 2*b - c]*x)/(Sqrt[a] - 2*Sqrt[a]*
x + Sqrt[a]*x^2 - Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(2*(2*a + 2*b +
c)) - (Sqrt[-2*a + 2*b - c]*ArcTan[(Sqrt[-2*a + 2*b - c]*x)/(Sqrt[a] + 2*Sq
rt[a]*x + Sqrt[a]*x^2 - Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(2*(2*a -
2*b + c))
```

fricas [B] time = 1.35, size = 1661, normalized size = 8.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((2*a + 2*b + c)*sqrt(2*a - 2*b + c)*log(((24*a^2 - 16*a*b + b^2 + 4*a
*c)*x^4 + 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x^3 + 2*(24*a^2 + 3*b^2
 - 4*(a + 2*b)*c + 4*c^2)*x^2 + 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4
*a - b)*x^2 + 2*(2*a + b - c)*x + 4*a - b)*sqrt(2*a - 2*b + c) + 24*a^2 - 1
6*a*b + b^2 + 4*a*c + 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x)/(x^4 + 4
*x^3 + 6*x^2 + 4*x + 1)) + sqrt(2*a + 2*b + c)*(2*a - 2*b + c)*log(((24*a^2
 + 16*a*b + b^2 + 4*a*c)*x^4 - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(2*a + b)*c)*x^
3 + 2*(24*a^2 + 3*b^2 - 4*(a - 2*b)*c + 4*c^2)*x^2 + 4*sqrt(a*x^4 + b*x^3 +
 c*x^2 + b*x + a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(2*a +
2*b + c) + 24*a^2 + 16*a*b + b^2 + 4*a*c - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(2*
a + b)*c)*x)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)))/(4*a^2 - 4*b^2 + 4*a*c + c^2
), -1/8*(2*(2*a - 2*b + c)*sqrt(-2*a - 2*b - c)*arctan(1/2*sqrt(a*x^4 + b*x
^3 + c*x^2 + b*x + a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(-2
*a - 2*b - c)/((2*a^2 + 2*a*b + a*c)*x^4 + (2*a*b + 2*b^2 + b*c)*x^3 + (2*(
a + b)*c + c^2)*x^2 + 2*a^2 + 2*a*b + a*c + (2*a*b + 2*b^2 + b*c)*x)) - (2*
a + 2*b + c)*sqrt(2*a - 2*b + c)*log(((24*a^2 - 16*a*b + b^2 + 4*a*c)*x^4 +
 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a +
 2*b)*c + 4*c^2)*x^2 + 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x
^2 + 2*(2*a + b - c)*x + 4*a - b)*sqrt(2*a - 2*b + c) + 24*a^2 - 16*a*b + b
^2 + 4*a*c + 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x)/(x^4 + 4*x^3 + 6*
x^2 + 4*x + 1)))/(4*a^2 - 4*b^2 + 4*a*c + c^2), 1/8*(2*(2*a + 2*b + c)*sqrt
(-2*a + 2*b - c)*arctan(-1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a -
b)*x^2 + 2*(2*a + b - c)*x + 4*a - b)*sqrt(-2*a + 2*b - c)/((2*a^2 - 2*a*b
+ a*c)*x^4 + (2*a*b - 2*b^2 + b*c)*x^3 + (2*(a - b)*c + c^2)*x^2 + 2*a^2 -
2*a*b + a*c + (2*a*b - 2*b^2 + b*c)*x)) + sqrt(2*a + 2*b + c)*(2*a - 2*b +
c)*log(((24*a^2 + 16*a*b + b^2 + 4*a*c)*x^4 - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*
(2*a + b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a - 2*b)*c + 4*c^2)*x^2 + 4*sqrt(
a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a +
 b)*sqrt(2*a + 2*b + c) + 24*a^2 + 16*a*b + b^2 + 4*a*c - 4*(8*a^2 - 4*a*b
- 3*b^2 - 2*(2*a + b)*c)*x)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)))/(4*a^2 - 4*b^
2 + 4*a*c + c^2), 1/4*((2*a + 2*b + c)*sqrt(-2*a + 2*b - c)*arctan(-1/2*sqr
t(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x^2 + 2*(2*a + b - c)*x + 4*a
 - b)*sqrt(-2*a + 2*b - c)/((2*a^2 - 2*a*b + a*c)*x^4 + (2*a*b - 2*b^2 + b*
c)*x^3 + (2*(a - b)*c + c^2)*x^2 + 2*a^2 - 2*a*b + a*c + (2*a*b - 2*b^2 + b
*c)*x)) - (2*a - 2*b + c)*sqrt(-2*a - 2*b - c)*arctan(1/2*sqrt(a*x^4 + b*x^
3 + c*x^2 + b*x + a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(-2*
a - 2*b - c)/((2*a^2 + 2*a*b + a*c)*x^4 + (2*a*b + 2*b^2 + b*c)*x^3 + (2*(a
 + b)*c + c^2)*x^2 + 2*a^2 + 2*a*b + a*c + (2*a*b + 2*b^2 + b*c)*x)))/(4*a^
2 - 4*b^2 + 4*a*c + c^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a}\left(x^2 - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(-x/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(x^2 - 1)), x)
```

maple [C] time = 0.02, size = 3458, normalized size = 17.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
[Out] -(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+
_Z*b+a,index=1))*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*
a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,inde
x=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^
2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)
*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))^2*((RootOf(_Z^4*a+_Z^3*b+_
Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-RootO
f(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a
,index=3)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^
3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,ind
ex=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-RootOf(_Z^4*a+_Z^3*b+
_Z^2*c+_Z*b+a,index=4))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf
(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+
a,index=2)))^(1/2)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4
*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index
=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(a*x^4+b*x^3+c*x^2+b*x+a)^
(1/2)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-1)*(EllipticF(((RootOf(_
Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,inde
x=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b
+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-Roo
tOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2),((RootOf(_Z^4*a+_Z^3*b+_Z^
2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))*(-RootOf(_
Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,inde
x=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3)+RootOf(_Z^4*a+_Z^3*b+_Z
^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(
_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2))+(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+
_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a
+_Z^3*b+_Z^2*c+_Z*b+a,index=1)-1)*EllipticPi(((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+
_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4
*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index
=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z
^2*c+_Z*b+a,index=2)))^(1/2),(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-1
)*(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+
_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1)-1)/(RootOf(_Z
^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index
=2)),((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^
2*c+_Z*b+a,index=3))*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_
Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,i
ndex=3)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b
+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)
))-(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*
c+_Z*b+a,index=1))*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^
4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,in
dex=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_
Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/
2)*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))^2*((RootOf(_Z^4*a+_Z^3*b
+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-Roo
tOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b
+a,index=3)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_
```

```
Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2)*((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,i
ndex=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))*(x-RootOf(_Z^4*a+_Z^3*
b+_Z^2*c+_Z*b+a,index=4))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-Root
Of(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*
b+a,index=2)))^(1/2)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z
^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,ind
ex=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(a*x^4+b*x^3+c*x^2+b*x+a
)^(1/2)/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)+1)*(EllipticF(((RootOf
(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,in
dex=2))*(x-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3
*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-R
ootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2),((RootOf(_Z^4*a+_Z^3*b+_
Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3))*(-RootOf
(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,in
dex=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=3)+RootOf(_Z^4*a+_Z^3*b+
_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootO
f(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/2))+(RootOf(_Z^4*a+_Z^3*b+_Z^2*
c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4
*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1)+1)*EllipticPi(((RootOf(_Z^4*a+_Z^3*b+_Z^2*
c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2))*(x-RootOf(_Z
^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,ind
ex=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(x-RootOf(_Z^4*a+_Z^3*b+
_Z^2*c+_Z*b+a,index=2)))^(1/2),(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)
+1)*(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*
c+_Z*b+a,index=1))/(RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1)+1)/(RootOf(
_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,ind
ex=2)),((RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)-RootOf(_Z^4*a+_Z^3*b+_
Z^2*c+_Z*b+a,index=3))*(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=4)+RootOf
(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a
,index=3)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=1))/(-RootOf(_Z^4*a+_Z^3
*b+_Z^2*c+_Z*b+a,index=4)+RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a,index=2)))^(1/
2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{ax^4+bx^3+cx^2+bx+a}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x}{\left(x^2-1\right)\,\sqrt{a\,x^4+b\,x^3+c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((x^2 - 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)),x)
```

```
[Out] -int(x/((x^2 - 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^2\sqrt{ax^4+a+bx^3+bx+cx^2}-\sqrt{ax^4+a+bx^3+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)/(a*x**4+b*x**3+c*x**2+b*x+a)**(1/2),x)
```

```
[Out] -Integral(x/(x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) - sqrt(a*x**4 +
a + b*x**3 + b*x + c*x**2)), x)
```

3.1958 $\int \frac{1}{\sqrt[4]{-1+x^4}\left(-1-x^4+x^8\right)}\,dx$

Optimal. Leaf size=193

$$-\frac{1}{2}\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}}}{\sqrt[4]{x^4}}\right.$$

Rubi [A] time = 0.20, antiderivative size = 209, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1428, 377, 212, 206, 203}

$$-\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{5}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((-1 + x^4)^(1/4)*(-1 - x^4 + x^8)),x]
```

```
[Out] -1/2*(((3 - Sqrt[5])/2)^(1/4)*ArcTan[((2/(3 + Sqrt[5]))^(1/4)*x)/(-1 + x^4)
^(1/4)])/Sqrt[5] - (((3 + Sqrt[5])/2)^(1/4)*ArcTan[(((3 + Sqrt[5])/2)^(1/4)
*x)/(-1 + x^4)^(1/4)])/(2*Sqrt[5]) - (((3 - Sqrt[5])/2)^(1/4)*ArcTanh[((2/(
3 + Sqrt[5]))^(1/4)*x)/(-1 + x^4)^(1/4)])/(2*Sqrt[5]) - (((3 + Sqrt[5])/2)^
(1/4)*ArcTanh[(((3 + Sqrt[5])/2)^(1/4)*x)/(-1 + x^4)^(1/4)])/(2*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1428

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^
n)^q/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[(d + e*x^n)^q/(b + r + 2
*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{-1+x^4}\left(-1-x^4+x^8\right)}\,dx = \frac{2\int \frac{1}{\sqrt[4]{-1+x^4}\left(-1-\sqrt{5}+2x^4\right)}\,dx}{\sqrt{5}} - \frac{2\int \frac{1}{\sqrt[4]{-1+x^4}\left(-1+\sqrt{5}+2x^4\right)}\,dx}{\sqrt{5}}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{-1-\sqrt{5}-\left(1-\sqrt{5}\right)x^4}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{5}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{-1+\sqrt{5}-\left(1+\sqrt{5}\right)x^4}\,dx,\right.}{\sqrt{5}}$$

$$= -\frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{10}} - \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-1+}}\right.}{\sqrt{10}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{-1+x^4}}\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}} - \frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{-1+x^4}}\right)}{2\sqrt{5}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{-1+x^4}}\right.}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}}$$

Mathematica [A] time = 0.40, size = 214, normalized size = 1.11

$$\frac{-\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4-1}}\right)}{1+\sqrt{5}}+\frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4-1}}\right)}{1-\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4-1}}\right)}{1+\sqrt{5}}+\frac{\sqrt[4]{3-\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4-1}}\right)}{1-\sqrt{5}}}{\sqrt[4]{2}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x^4)^(1/4)*(-1 - x^4 + x^8)),x]
```

```
[Out] (-(((3 + Sqrt[5])^(1/4)*ArcTan[((2/(3 + Sqrt[5]))^(1/4)*x)/(-1 + x^4)^(1/4)
])/(1 + Sqrt[5])) + ((3 - Sqrt[5])^(1/4)*ArcTan[(((3 + Sqrt[5])/2)^(1/4)*x)
/(-1 + x^4)^(1/4)])/(1 - Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*ArcTanh[((2/(3 + S
qrt[5]))^(1/4)*x)/(-1 + x^4)^(1/4)])/(1 + Sqrt[5]) + ((3 - Sqrt[5])^(1/4)*A
rcTanh[(((3 + Sqrt[5])/2)^(1/4)*x)/(-1 + x^4)^(1/4)])/(1 - Sqrt[5]))/(2^(1/
4)*Sqrt[5])
```

IntegrateAlgebraic [A] time = 0.65, size = 193, normalized size = 1.00

$$-\frac{1}{2}\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x^4)^(1/4)*(-1 - x^4 + x^8)),x]
```

```
[Out] -1/2*(Sqrt[(-1 + Sqrt[5])/10]*ArcTan[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(-1 + x^4)^
(1/4)]) - (Sqrt[(1 + Sqrt[5])/10]*ArcTan[(Sqrt[1/2 + Sqrt[5]/2]*x)/(-1 + x^
4)^(1/4)])/2 - (Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[(Sqrt[-1/2 + Sqrt[5]/2]*x)/
(-1 + x^4)^(1/4)])/2 - (Sqrt[(1 + Sqrt[5])/10]*ArcTanh[(Sqrt[1/2 + Sqrt[5]/
2]*x)/(-1 + x^4)^(1/4)])/2
```

fricas [B] time = 28.67, size = 953, normalized size = 4.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)^(1/4)/(x^8-x^4-1),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(10)*sqrt(sqrt(5) + 1)*arctan(-1/40*(sqrt(2)*(2*sqrt(10)*(5*x^6 -
 sqrt(5)*(x^6 + 2*x^2))*sqrt(x^4 - 1) - sqrt(10)*(5*x^8 + 5*x^4 - sqrt(5)*(
5*x^8 - 3*x^4 - 1) - 5))*(sqrt(5) + 1) + 4*(sqrt(10)*(5*x^5 - sqrt(5)*(x^5
+ 2*x))*(x^4 - 1)^(3/4) - sqrt(10)*(x^4 - 1)^(1/4)*(5*x^3 - sqrt(5)*(2*x^7
- x^3)))*sqrt(sqrt(5) + 1))/(x^8 - x^4 - 1)) + 1/20*sqrt(10)*sqrt(sqrt(5) -
 1)*arctan(1/40*(sqrt(2)*(2*sqrt(10)*(5*x^6 + sqrt(5)*(x^6 + 2*x^2))*sqrt(x
^4 - 1) + sqrt(10)*(5*x^8 + 5*x^4 + sqrt(5)*(5*x^8 - 3*x^4 - 1) - 5))*(sqrt
(5) - 1) - 4*(sqrt(10)*(5*x^5 + sqrt(5)*(x^5 + 2*x))*(x^4 - 1)^(3/4) + sqrt
(10)*(x^4 - 1)^(1/4)*(5*x^3 + sqrt(5)*(2*x^7 - x^3)))*sqrt(sqrt(5) - 1))/(x
^8 - x^4 - 1)) - 1/80*sqrt(10)*sqrt(sqrt(5) - 1)*log((10*(2*x^5 + sqrt(5)*x
 - x)*(x^4 - 1)^(3/4) + (sqrt(10)*sqrt(x^4 - 1)*(5*x^2 + sqrt(5)*(2*x^6 - x
^2)) + sqrt(10)*(5*x^8 - 5*x^4 + sqrt(5)*(2*x^4 - 1)))*sqrt(sqrt(5) - 1) -
10*(x^7 - 3*x^3 - sqrt(5)*(x^7 - x^3))*(x^4 - 1)^(1/4))/(x^8 - x^4 - 1)) +
1/80*sqrt(10)*sqrt(sqrt(5) - 1)*log((10*(2*x^5 + sqrt(5)*x - x)*(x^4 - 1)^(
3/4) - (sqrt(10)*sqrt(x^4 - 1)*(5*x^2 + sqrt(5)*(2*x^6 - x^2)) + sqrt(10)*(
5*x^8 - 5*x^4 + sqrt(5)*(2*x^4 - 1)))*sqrt(sqrt(5) - 1) - 10*(x^7 - 3*x^3 -
 sqrt(5)*(x^7 - x^3))*(x^4 - 1)^(1/4))/(x^8 - x^4 - 1)) + 1/80*sqrt(10)*sqr
t(sqrt(5) + 1)*log((10*(2*x^5 - sqrt(5)*x - x)*(x^4 - 1)^(3/4) + (sqrt(10)*
sqrt(x^4 - 1)*(5*x^2 - sqrt(5)*(2*x^6 - x^2)) - sqrt(10)*(5*x^8 - 5*x^4 - s
qrt(5)*(2*x^4 - 1)))*sqrt(sqrt(5) + 1) + 10*(x^7 - 3*x^3 + sqrt(5)*(x^7 - x
^3))*(x^4 - 1)^(1/4))/(x^8 - x^4 - 1)) - 1/80*sqrt(10)*sqrt(sqrt(5) + 1)*lo
g((10*(2*x^5 - sqrt(5)*x - x)*(x^4 - 1)^(3/4) - (sqrt(10)*sqrt(x^4 - 1)*(5*
x^2 - sqrt(5)*(2*x^6 - x^2)) - sqrt(10)*(5*x^8 - 5*x^4 - sqrt(5)*(2*x^4 - 1
)))*sqrt(sqrt(5) + 1) + 10*(x^7 - 3*x^3 + sqrt(5)*(x^7 - x^3))*(x^4 - 1)^(1
/4))/(x^8 - x^4 - 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)^(1/4)/(x^8-x^4-1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [C] time = 11.05, size = 1722, normalized size = 8.92

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-1)^(1/4)/(x^8-x^4-1),x)
```

```
[Out] 1/20*RootOf(_Z^2+400*RootOf(6400*_Z^4-80*_Z^2-1)^2-5)*ln(-(6400*RootOf(_Z^2
+400*RootOf(6400*_Z^4-80*_Z^2-1)^2-5)*RootOf(6400*_Z^4-80*_Z^2-1)^4*x^4-160
*(x^4-1)^(1/2)*RootOf(_Z^2+400*RootOf(6400*_Z^4-80*_Z^2-1)^2-5)*RootOf(6400
*_Z^4-80*_Z^2-1)^2*x^2-320*RootOf(_Z^2+400*RootOf(6400*_Z^4-80*_Z^2-1)^2-5)
*RootOf(6400*_Z^4-80*_Z^2-1)^2*x^4+480*RootOf(6400*_Z^4-80*_Z^2-1)^2*(x^4-1
)^(3/4)*x+640*(x^4-1)^(1/4)*RootOf(6400*_Z^4-80*_Z^2-1)^2*x^3+4*(x^4-1)^(1/
2)*RootOf(_Z^2+400*RootOf(6400*_Z^4-80*_Z^2-1)^2-5)*x^2+3*RootOf(_Z^2+400*R
ootOf(6400*_Z^4-80*_Z^2-1)^2-5)*x^4-8*(x^4-1)^(3/4)*x-14*x^3*(x^4-1)^(1/4)+
80*RootOf(6400*_Z^4-80*_Z^2-1)^2*RootOf(_Z^2+400*RootOf(6400*_Z^4-80*_Z^2-1
)^2-5)-RootOf(_Z^2+400*RootOf(6400*_Z^4-80*_Z^2-1)^2-5))/(320*RootOf(6400*_
Z^4-80*_Z^2-1)^3*x^2-12*RootOf(6400*_Z^4-80*_Z^2-1)*x^2-1)/(320*RootOf(6400
*_Z^4-80*_Z^2-1)^3*x^2-12*RootOf(6400*_Z^4-80*_Z^2-1)*x^2+1))+RootOf(6400*_
Z^4-80*_Z^2-1)*ln(-(64000*RootOf(6400*_Z^4-80*_Z^2-1)^5*x^4+1600*RootOf(640
0*_Z^4-80*_Z^2-1)^3*(x^4-1)^(1/2)*x^2+1600*x^4*RootOf(6400*_Z^4-80*_Z^2-1)^
3-240*RootOf(6400*_Z^4-80*_Z^2-1)^2*(x^4-1)^(3/4)*x-320*(x^4-1)^(1/4)*RootO
```

```
f(6400*_Z^4-80*_Z^2-1)^2*x^3+20*(x^4-1)^(1/2)*RootOf(6400*_Z^4-80*_Z^2-1)*x
^2-(x^4-1)^(3/4)*x-3*x^3*(x^4-1)^(1/4)-800*RootOf(6400*_Z^4-80*_Z^2-1)^3)/(
80*RootOf(6400*_Z^4-80*_Z^2-1)^2*x^4+1))+4*RootOf(6400*_Z^4-80*_Z^2-1)^2*Ro
otOf(_Z^2+400*RootOf(6400*_Z^4-80*_Z^2-1)^2-5)*ln(-(12800*RootOf(_Z^2+400*R
ootOf(6400*_Z^4-80*_Z^2-1)^2-5)*RootOf(6400*_Z^4-80*_Z^2-1)^4*x^4-480*(x^4-
1)^(1/2)*RootOf(_Z^2+400*RootOf(6400*_Z^4-80*_Z^2-1)^2-5)*RootOf(6400*_Z^4-
80*_Z^2-1)^2*x^2+400*RootOf(_Z^2+400*RootOf(6400*_Z^4-80*_Z^2-1)^2-5)*RootO
f(6400*_Z^4-80*_Z^2-1)^2*x^4-640*RootOf(6400*_Z^4-80*_Z^2-1)^2*(x^4-1)^(3/4
)*x+1120*(x^4-1)^(1/4)*RootOf(6400*_Z^4-80*_Z^2-1)^2*x^3-4*(x^4-1)^(1/2)*Ro
otOf(_Z^2+400*RootOf(6400*_Z^4-80*_Z^2-1)^2-5)*x^2+2*RootOf(_Z^2+400*RootOf
(6400*_Z^4-80*_Z^2-1)^2-5)*x^4-6*(x^4-1)^(3/4)*x+8*x^3*(x^4-1)^(1/4)-160*Ro
otOf(6400*_Z^4-80*_Z^2-1)^2*RootOf(_Z^2+400*RootOf(6400*_Z^4-80*_Z^2-1)^2-5
)-RootOf(_Z^2+400*RootOf(6400*_Z^4-80*_Z^2-1)^2-5))/(80*RootOf(6400*_Z^4-80
*_Z^2-1)^2*x^4+1))+80*RootOf(6400*_Z^4-80*_Z^2-1)^3*ln((64000*RootOf(6400*_
Z^4-80*_Z^2-1)^5*x^4+3200*RootOf(6400*_Z^4-80*_Z^2-1)^3*(x^4-1)^(1/2)*x^2-4
000*x^4*RootOf(6400*_Z^4-80*_Z^2-1)^3-240*RootOf(6400*_Z^4-80*_Z^2-1)^2*(x^
4-1)^(3/4)*x+320*(x^4-1)^(1/4)*RootOf(6400*_Z^4-80*_Z^2-1)^2*x^3-60*(x^4-1)
^(1/2)*RootOf(6400*_Z^4-80*_Z^2-1)*x^2+60*RootOf(6400*_Z^4-80*_Z^2-1)*x^4+4
*(x^4-1)^(3/4)*x-7*x^3*(x^4-1)^(1/4)+800*RootOf(6400*_Z^4-80*_Z^2-1)^3-20*R
ootOf(6400*_Z^4-80*_Z^2-1))/(320*RootOf(6400*_Z^4-80*_Z^2-1)^3*x^2-12*RootO
f(6400*_Z^4-80*_Z^2-1)*x^2-1)/(320*RootOf(6400*_Z^4-80*_Z^2-1)^3*x^2-12*Roo
tOf(6400*_Z^4-80*_Z^2-1)*x^2+1))-RootOf(6400*_Z^4-80*_Z^2-1)*ln((64000*Root
Of(6400*_Z^4-80*_Z^2-1)^5*x^4+3200*RootOf(6400*_Z^4-80*_Z^2-1)^3*(x^4-1)^(1
/2)*x^2-4000*x^4*RootOf(6400*_Z^4-80*_Z^2-1)^3-240*RootOf(6400*_Z^4-80*_Z^2
-1)^2*(x^4-1)^(3/4)*x+320*(x^4-1)^(1/4)*RootOf(6400*_Z^4-80*_Z^2-1)^2*x^3-6
0*(x^4-1)^(1/2)*RootOf(6400*_Z^4-80*_Z^2-1)*x^2+60*RootOf(6400*_Z^4-80*_Z^2
-1)*x^4+4*(x^4-1)^(3/4)*x-7*x^3*(x^4-1)^(1/4)+800*RootOf(6400*_Z^4-80*_Z^2-
1)^3-20*RootOf(6400*_Z^4-80*_Z^2-1))/(320*RootOf(6400*_Z^4-80*_Z^2-1)^3*x^2
-12*RootOf(6400*_Z^4-80*_Z^2-1)*x^2-1)/(320*RootOf(6400*_Z^4-80*_Z^2-1)^3*x
^2-12*RootOf(6400*_Z^4-80*_Z^2-1)*x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^8 - x^4 - 1\right)\left(x^4 - 1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)^(1/4)/(x^8-x^4-1),x, algorithm="maxima")
[Out] integrate(1/((x^8 - x^4 - 1)*(x^4 - 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(x^4-1\right)^{1/4}\,\left(-x^8+x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^4 - 1)^(1/4)*(x^4 - x^8 + 1)),x)
[Out] -int(1/((x^4 - 1)^(1/4)*(x^4 - x^8 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{\left(x - 1\right)\left(x + 1\right)\left(x^{2} + 1\right)}\,\left(x^{8} - x^{4} - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-1)**(1/4)/(x**8-x**4-1),x)
```

```
[Out] Integral(1/(((x - 1)*(x + 1)*(x**2 + 1))**(1/4)*(x**8 - x**4 - 1)), x)
```

3.1959 $\int \frac{-1+2x^4}{\sqrt[4]{-1+x^4}\left(-1-x^4+x^8\right)}\,dx$

Optimal. Leaf size=193

$$-\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}\cdots}}{\sqrt[4]{x^4\cdots}}\right.$$

Rubi [A] time = 0.29, antiderivative size = 189, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {6728, 377, 212, 206, 203}

$$-\frac{1}{2}\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{2}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{2}\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{2}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4-1}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + 2*x^4)/((-1 + x^4)^(1/4)*(-1 - x^4 + x^8)),x]
```

```
[Out] -1/2*(((3 - Sqrt[5])/2)^(1/4)*ArcTan[((2/(3 + Sqrt[5]))^(1/4)*x)/(-1 + x^4)
^(1/4)]) + (((3 + Sqrt[5])/2)^(1/4)*ArcTan[(((3 + Sqrt[5])/2)^(1/4)*x)/(-1
+ x^4)^(1/4)])/2 - (((3 - Sqrt[5])/2)^(1/4)*ArcTanh[((2/(3 + Sqrt[5]))^(1/4
)*x)/(-1 + x^4)^(1/4)])/2 + (((3 + Sqrt[5])/2)^(1/4)*ArcTanh[(((3 + Sqrt[5]
)/2)^(1/4)*x)/(-1 + x^4)^(1/4)])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-1+2x^4}{\sqrt[4]{-1+x^4}\,(-1-x^4+x^8)}\,dx = \int \left(\frac{2}{\sqrt[4]{-1+x^4}\,(-1-\sqrt{5}+2x^4)} + \frac{2}{\sqrt[4]{-1+x^4}\,(-1+\sqrt{5}+2x^4)}\right)dx$$

$$= 2\int \frac{1}{\sqrt[4]{-1+x^4}\,(-1-\sqrt{5}+2x^4)}\,dx + 2\int \frac{1}{\sqrt[4]{-1+x^4}\,(-1+\sqrt{5}+2x^4)}\,dx$$

$$= 2\,\mathrm{Subst}\left(\int \frac{1}{-1-\sqrt{5}-(1-\sqrt{5})x^4}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right) + 2\,\mathrm{Subst}\left(\int \frac{}{-1+\sqrt{5}} \right.$$

$$= \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{2}} - \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{2}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{-1+x^4}}\right)}{2^{3/4}\sqrt[4]{3+\sqrt{5}}} + \frac{1}{2}\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}\,x}{\sqrt[4]{-1+x^4}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+}}}{\sqrt[4]{-1+}}\right)}{2^{3/4}\sqrt[4]{3+}}$$

Mathematica [A] time = 0.16, size = 172, normalized size = 0.89

$$\frac{-\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}x}{\sqrt[4]{x^4-1}}\right)+\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}x}{\sqrt[4]{x^4-1}}\right)-\sqrt[4]{3-\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}x}{\sqrt[4]{x^4-1}}\right)+\sqrt[4]{3+\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt[4]{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 2*x^4)/((-1 + x^4)^(1/4)*(-1 - x^4 + x^8)),x]
```

```
[Out] (-((3 - Sqrt[5])^(1/4)*ArcTan[((2/(3 + Sqrt[5]))^(1/4)*x)/(-1 + x^4)^(1/4)]
) + (3 + Sqrt[5])^(1/4)*ArcTan[(((3 + Sqrt[5])/2)^(1/4)*x)/(-1 + x^4)^(1/4)
] - (3 - Sqrt[5])^(1/4)*ArcTanh[((2/(3 + Sqrt[5]))^(1/4)*x)/(-1 + x^4)^(1/4
)] + (3 + Sqrt[5])^(1/4)*ArcTanh[(((3 + Sqrt[5])/2)^(1/4)*x)/(-1 + x^4)^(1/
4)])/(2*2^(1/4))
```

IntegrateAlgebraic [A] time = 0.74, size = 193, normalized size = 1.00

$$-\frac{1}{2}\sqrt{\frac{1}{2}(\sqrt{5}-1)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}(1+\sqrt{5})}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}(\sqrt{5}-1)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}(1+\sqrt{5})}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}x}{\sqrt[4]{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x^4)/((-1 + x^4)^(1/4)*(-1 - x^4 + x^8)),x]
```

```
[Out] -1/2*(Sqrt[(-1 + Sqrt[5])/2]*ArcTan[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(-1 + x^4)^(
1/4)]) + (Sqrt[(1 + Sqrt[5])/2]*ArcTan[(Sqrt[1/2 + Sqrt[5]/2]*x)/(-1 + x^4)
^(1/4)])/2 - (Sqrt[(-1 + Sqrt[5])/2]*ArcTanh[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(-1
 + x^4)^(1/4)])/2 + (Sqrt[(1 + Sqrt[5])/2]*ArcTanh[(Sqrt[1/2 + Sqrt[5]/2]*x
)/(-1 + x^4)^(1/4)])/2
```

fricas [B] time = 31.72, size = 990, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)/(x^4-1)^(1/4)/(x^8-x^4-1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt(sqrt(5) + 1)*arctan(-1/8*((sqrt(5)*sqrt(2)*(x^8 + x^4 - 1
) - sqrt(2)*(5*x^8 - 3*x^4 - 1) - 2*(sqrt(5)*sqrt(2)*x^6 - sqrt(2)*(x^6 + 2
```

```
*x^2))*sqrt(x^4 - 1))*sqrt(2*sqrt(5) + 2)*sqrt(sqrt(5) + 1) - 4*((sqrt(5)*s
qrt(2)*x^5 - sqrt(2)*(x^5 + 2*x))*(x^4 - 1)^(3/4) - (sqrt(5)*sqrt(2)*x^3 -
sqrt(2)*(2*x^7 - x^3))*(x^4 - 1)^(1/4))*sqrt(sqrt(5) + 1))/(x^8 - x^4 - 1))
 + 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*arctan(1/8*((sqrt(5)*sqrt(2)*(x^8 + x^4 -
1) + sqrt(2)*(5*x^8 - 3*x^4 - 1) + 2*(sqrt(5)*sqrt(2)*x^6 + sqrt(2)*(x^6 +
2*x^2))*sqrt(x^4 - 1))*sqrt(2*sqrt(5) - 2)*sqrt(sqrt(5) - 1) - 4*((sqrt(5)*
sqrt(2)*x^5 + sqrt(2)*(x^5 + 2*x))*(x^4 - 1)^(3/4) + (sqrt(5)*sqrt(2)*x^3 +
 sqrt(2)*(2*x^7 - x^3))*(x^4 - 1)^(1/4))*sqrt(sqrt(5) - 1))/(x^8 - x^4 - 1)
) - 1/16*sqrt(2)*sqrt(sqrt(5) - 1)*log((2*(2*x^5 + sqrt(5)*x - x)*(x^4 - 1)
^(3/4) + (sqrt(5)*sqrt(2)*(x^8 - x^4) + sqrt(2)*(2*x^4 - 1) + sqrt(x^4 - 1)
*(sqrt(5)*sqrt(2)*x^2 + sqrt(2)*(2*x^6 - x^2)))*sqrt(sqrt(5) - 1) - 2*(x^7
- 3*x^3 - sqrt(5)*(x^7 - x^3))*(x^4 - 1)^(1/4))/(x^8 - x^4 - 1)) + 1/16*sqr
t(2)*sqrt(sqrt(5) - 1)*log((2*(2*x^5 + sqrt(5)*x - x)*(x^4 - 1)^(3/4) - (sq
rt(5)*sqrt(2)*(x^8 - x^4) + sqrt(2)*(2*x^4 - 1) + sqrt(x^4 - 1)*(sqrt(5)*sq
rt(2)*x^2 + sqrt(2)*(2*x^6 - x^2)))*sqrt(sqrt(5) - 1) - 2*(x^7 - 3*x^3 - sq
rt(5)*(x^7 - x^3))*(x^4 - 1)^(1/4))/(x^8 - x^4 - 1)) + 1/16*sqrt(2)*sqrt(sq
rt(5) + 1)*log((2*(2*x^5 - sqrt(5)*x - x)*(x^4 - 1)^(3/4) + (sqrt(5)*sqrt(2
)*(x^8 - x^4) - sqrt(2)*(2*x^4 - 1) - sqrt(x^4 - 1)*(sqrt(5)*sqrt(2)*x^2 -
sqrt(2)*(2*x^6 - x^2)))*sqrt(sqrt(5) + 1) + 2*(x^7 - 3*x^3 + sqrt(5)*(x^7 -
 x^3))*(x^4 - 1)^(1/4))/(x^8 - x^4 - 1)) - 1/16*sqrt(2)*sqrt(sqrt(5) + 1)*l
og((2*(2*x^5 - sqrt(5)*x - x)*(x^4 - 1)^(3/4) - (sqrt(5)*sqrt(2)*(x^8 - x^4
) - sqrt(2)*(2*x^4 - 1) - sqrt(x^4 - 1)*(sqrt(5)*sqrt(2)*x^2 - sqrt(2)*(2*x
^6 - x^2)))*sqrt(sqrt(5) + 1) + 2*(x^7 - 3*x^3 + sqrt(5)*(x^7 - x^3))*(x^4
- 1)^(1/4))/(x^8 - x^4 - 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)/(x^4-1)^(1/4)/(x^8-x^4-1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [C] time = 10.62, size = 1753, normalized size = 9.08

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-1)/(x^4-1)^(1/4)/(x^8-x^4-1),x)
```

```
[Out] RootOf(256*_Z^4-16*_Z^2-1)*ln((512*RootOf(256*_Z^4-16*_Z^2-1)^5*x^4+192*Roo
tOf(256*_Z^4-16*_Z^2-1)^3*(x^4-1)^(1/2)*x^2+128*x^4*RootOf(256*_Z^4-16*_Z^2
-1)^3+48*RootOf(256*_Z^4-16*_Z^2-1)^2*(x^4-1)^(3/4)*x+64*(x^4-1)^(1/4)*Root
Of(256*_Z^4-16*_Z^2-1)^2*x^3+4*RootOf(256*_Z^4-16*_Z^2-1)*(x^4-1)^(1/2)*x^2
+8*x^4*RootOf(256*_Z^4-16*_Z^2-1)+(x^4-1)^(3/4)*x+3*x^3*(x^4-1)^(1/4)-32*Ro
otOf(256*_Z^4-16*_Z^2-1)^3-4*RootOf(256*_Z^4-16*_Z^2-1))/(16*RootOf(256*_Z^
4-16*_Z^2-1)^2*x^4+1))+1/4*RootOf(_Z^2+16*RootOf(256*_Z^4-16*_Z^2-1)^2-1)*l
n((256*RootOf(_Z^2+16*RootOf(256*_Z^4-16*_Z^2-1)^2-1)*RootOf(256*_Z^4-16*_Z
^2-1)^4*x^4-96*(x^4-1)^(1/2)*RootOf(_Z^2+16*RootOf(256*_Z^4-16*_Z^2-1)^2-1)
*RootOf(256*_Z^4-16*_Z^2-1)^2*x^2-96*RootOf(_Z^2+16*RootOf(256*_Z^4-16*_Z^2
-1)^2-1)*RootOf(256*_Z^4-16*_Z^2-1)^2*x^4-96*RootOf(256*_Z^4-16*_Z^2-1)^2*(
x^4-1)^(3/4)*x-128*(x^4-1)^(1/4)*RootOf(256*_Z^4-16*_Z^2-1)^2*x^3+8*(x^4-1)
^(1/2)*RootOf(_Z^2+16*RootOf(256*_Z^4-16*_Z^2-1)^2-1)*x^2+9*RootOf(_Z^2+16*
RootOf(256*_Z^4-16*_Z^2-1)^2-1)*x^4+8*(x^4-1)^(3/4)*x+14*x^3*(x^4-1)^(1/4)+
16*RootOf(256*_Z^4-16*_Z^2-1)^2*RootOf(_Z^2+16*RootOf(256*_Z^4-16*_Z^2-1)^2
-1)-3*RootOf(_Z^2+16*RootOf(256*_Z^4-16*_Z^2-1)^2-1))/(64*RootOf(256*_Z^4-1
```

```
6*_Z^2-1)^3*x^2-4*RootOf(256*_Z^4-16*_Z^2-1)*x^2-1)/(64*RootOf(256*_Z^4-16*
_Z^2-1)^3*x^2-4*RootOf(256*_Z^4-16*_Z^2-1)*x^2+1))+4*RootOf(256*_Z^4-16*_Z^
2-1)^2*RootOf(_Z^2+16*RootOf(256*_Z^4-16*_Z^2-1)^2-1)*ln((768*RootOf(_Z^2+1
6*RootOf(256*_Z^4-16*_Z^2-1)^2-1)*RootOf(256*_Z^4-16*_Z^2-1)^4*x^4-128*(x^4
-1)^(1/2)*RootOf(_Z^2+16*RootOf(256*_Z^4-16*_Z^2-1)^2-1)*RootOf(256*_Z^4-16
*_Z^2-1)^2*x^2+112*RootOf(_Z^2+16*RootOf(256*_Z^4-16*_Z^2-1)^2-1)*RootOf(25
6*_Z^4-16*_Z^2-1)^2*x^4+96*RootOf(256*_Z^4-16*_Z^2-1)^2*(x^4-1)^(3/4)*x-128
*(x^4-1)^(1/4)*RootOf(256*_Z^4-16*_Z^2-1)^2*x^3-6*(x^4-1)^(1/2)*RootOf(_Z^2
+16*RootOf(256*_Z^4-16*_Z^2-1)^2-1)*x^2+2*RootOf(_Z^2+16*RootOf(256*_Z^4-16
*_Z^2-1)^2-1)*x^4+2*(x^4-1)^(3/4)*x-6*x^3*(x^4-1)^(1/4)-48*RootOf(256*_Z^4-
16*_Z^2-1)^2*RootOf(_Z^2+16*RootOf(256*_Z^4-16*_Z^2-1)^2-1)-RootOf(_Z^2+16*
RootOf(256*_Z^4-16*_Z^2-1)^2-1))/(16*RootOf(256*_Z^4-16*_Z^2-1)^2*x^4+1))+1
6*RootOf(256*_Z^4-16*_Z^2-1)^3*ln(-(1536*RootOf(256*_Z^4-16*_Z^2-1)^5*x^4+2
56*RootOf(256*_Z^4-16*_Z^2-1)^3*(x^4-1)^(1/2)*x^2-416*x^4*RootOf(256*_Z^4-1
6*_Z^2-1)^3+48*RootOf(256*_Z^4-16*_Z^2-1)^2*(x^4-1)^(3/4)*x-64*(x^4-1)^(1/4
)*RootOf(256*_Z^4-16*_Z^2-1)^2*x^3-28*RootOf(256*_Z^4-16*_Z^2-1)*(x^4-1)^(1
/2)*x^2+24*x^4*RootOf(256*_Z^4-16*_Z^2-1)-4*(x^4-1)^(3/4)*x+7*x^3*(x^4-1)^(
1/4)+96*RootOf(256*_Z^4-16*_Z^2-1)^3-8*RootOf(256*_Z^4-16*_Z^2-1))/(64*Root
Of(256*_Z^4-16*_Z^2-1)^3*x^2-4*RootOf(256*_Z^4-16*_Z^2-1)*x^2-1)/(64*RootOf
(256*_Z^4-16*_Z^2-1)^3*x^2-4*RootOf(256*_Z^4-16*_Z^2-1)*x^2+1))-RootOf(256*
_Z^4-16*_Z^2-1)*ln(-(1536*RootOf(256*_Z^4-16*_Z^2-1)^5*x^4+256*RootOf(256*_
Z^4-16*_Z^2-1)^3*(x^4-1)^(1/2)*x^2-416*x^4*RootOf(256*_Z^4-16*_Z^2-1)^3+48*
RootOf(256*_Z^4-16*_Z^2-1)^2*(x^4-1)^(3/4)*x-64*(x^4-1)^(1/4)*RootOf(256*_Z
^4-16*_Z^2-1)^2*x^3-28*RootOf(256*_Z^4-16*_Z^2-1)*(x^4-1)^(1/2)*x^2+24*x^4*
RootOf(256*_Z^4-16*_Z^2-1)-4*(x^4-1)^(3/4)*x+7*x^3*(x^4-1)^(1/4)+96*RootOf(
256*_Z^4-16*_Z^2-1)^3-8*RootOf(256*_Z^4-16*_Z^2-1))/(64*RootOf(256*_Z^4-16*
_Z^2-1)^3*x^2-4*RootOf(256*_Z^4-16*_Z^2-1)*x^2-1)/(64*RootOf(256*_Z^4-16*_Z
^2-1)^3*x^2-4*RootOf(256*_Z^4-16*_Z^2-1)*x^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^4-1}{\left(x^8-x^4-1\right)\left(x^4-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)/(x^4-1)^(1/4)/(x^8-x^4-1),x, algorithm="maxima")
[Out] integrate((2*x^4 - 1)/((x^8 - x^4 - 1)*(x^4 - 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{2\,x^4-1}{{\left(x^4-1\right)}^{1/4}\,\left(-x^8+x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^4 - 1)/((x^4 - 1)^(1/4)*(x^4 - x^8 + 1)),x)
[Out] -int((2*x^4 - 1)/((x^4 - 1)^(1/4)*(x^4 - x^8 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-1)/(x**4-1)**(1/4)/(x**8-x**4-1),x)
[Out] Timed out
```

3.1960 $\int \frac{1}{\sqrt[4]{1+x^4}\left(-1+x^4+x^8\right)} \, dx$

Optimal. Leaf size=193

$$-\frac{1}{2}\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\cdots\right.$$

Rubi [A] time = 0.12, antiderivative size = 209, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1428, 377, 212, 206, 203}

$$-\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt{5}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((1 + x^4)^(1/4)*(-1 + x^4 + x^8)),x]
```

```
[Out] -1/2*(((3 - Sqrt[5])/2)^(1/4)*ArcTan[((2/(3 + Sqrt[5]))^(1/4)*x)/(1 + x^4)^
(1/4)])/Sqrt[5] - (((3 + Sqrt[5])/2)^(1/4)*ArcTan[(((3 + Sqrt[5])/2)^(1/4)*
x)/(1 + x^4)^(1/4)])/(2*Sqrt[5]) - (((3 - Sqrt[5])/2)^(1/4)*ArcTanh[((2/(3
+ Sqrt[5]))^(1/4)*x)/(1 + x^4)^(1/4)])/(2*Sqrt[5]) - (((3 + Sqrt[5])/2)^(1/
4)*ArcTanh[(((3 + Sqrt[5])/2)^(1/4)*x)/(1 + x^4)^(1/4)])/(2*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1428

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^
n)^q/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[(d + e*x^n)^q/(b + r + 2
*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{1+x^4}\left(-1+x^4+x^8\right)}dx = \frac{2\int \frac{1}{\sqrt[4]{1+x^4}\left(1-\sqrt{5}+2x^4\right)}dx}{\sqrt{5}} - \frac{2\int \frac{1}{\sqrt[4]{1+x^4}\left(1+\sqrt{5}+2x^4\right)}dx}{\sqrt{5}}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{5}-\left(-1-\sqrt{5}\right)x^4}dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{5}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{5}-\left(-1+\sqrt{5}\right)x^4}dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{5}}$$

$$= -\frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{10}} - \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{10}} - $$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}x}{\sqrt[4]{1+x^4}}\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}} - \frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt{5}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}x}{\sqrt[4]{1+x^4}}\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}} - $$

Mathematica [A] time = 0.30, size = 214, normalized size = 1.11

$$\frac{-\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}x}{\sqrt[4]{x^4+1}}\right)}{1+\sqrt{5}} + \frac{\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x}{\sqrt[4]{x^4+1}}\right)}{1-\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}x}{\sqrt[4]{x^4+1}}\right)}{1+\sqrt{5}} + \frac{\sqrt[4]{3-\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x}{\sqrt[4]{x^4+1}}\right)}{1-\sqrt{5}}}{\sqrt[4]{2}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^4)^(1/4)*(-1 + x^4 + x^8)),x]
```

```
[Out] (-(((3 + Sqrt[5])^(1/4)*ArcTan[((2/(3 + Sqrt[5]))^(1/4)*x)/(1 + x^4)^(1/4)]
)/(1 + Sqrt[5])) + ((3 - Sqrt[5])^(1/4)*ArcTan[(((3 + Sqrt[5])/2)^(1/4)*x)/
(1 + x^4)^(1/4)])/(1 - Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*ArcTanh[((2/(3 + Sqr
t[5]))^(1/4)*x)/(1 + x^4)^(1/4)])/(1 + Sqrt[5]) + ((3 - Sqrt[5])^(1/4)*ArcT
anh[(((3 + Sqrt[5])/2)^(1/4)*x)/(1 + x^4)^(1/4)])/(1 - Sqrt[5]))/(2^(1/4)*S
qrt[5])
```

IntegrateAlgebraic [A] time = 0.61, size = 193, normalized size = 1.00

$$-\frac{1}{2}\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}x}{\sqrt[4]{x^4+1}}\right) - \frac{1}{2}\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}x}{\sqrt[4]{x^4+1}}\right) - \frac{1}{2}\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}x}{\sqrt[4]{x^4+1}}\right) - \frac{1}{2}\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}x}{\sqrt[4]{x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x^4)^(1/4)*(-1 + x^4 + x^8)),x]
```

```
[Out] -1/2*(Sqrt[(-1 + Sqrt[5])/10]*ArcTan[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(1 + x^4)^(
1/4)]) - (Sqrt[(1 + Sqrt[5])/10]*ArcTan[(Sqrt[1/2 + Sqrt[5]/2]*x)/(1 + x^4)
^(1/4)])/2 - (Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(1
 + x^4)^(1/4)])/2 - (Sqrt[(1 + Sqrt[5])/10]*ArcTanh[(Sqrt[1/2 + Sqrt[5]/2]*
x)/(1 + x^4)^(1/4)])/2
```

fricas [B] time = 32.04, size = 913, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4)/(x^8+x^4-1),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(10)*sqrt(sqrt(5) + 1)*arctan(-1/40*(sqrt(2)*(2*sqrt(10)*(5*x^6 -
 sqrt(5)*(x^6 - 2*x^2))*sqrt(x^4 + 1) - sqrt(10)*(5*x^8 - 5*x^4 - sqrt(5)*(
5*x^8 + 3*x^4 - 1) - 5))*(sqrt(5) + 1) + 4*(sqrt(10)*(5*x^5 - sqrt(5)*(x^5
- 2*x))*(x^4 + 1)^(3/4) + sqrt(10)*(x^4 + 1)^(1/4)*(5*x^3 + sqrt(5)*(2*x^7
+ x^3)))*sqrt(sqrt(5) + 1))/(x^8 + x^4 - 1)) + 1/20*sqrt(10)*sqrt(sqrt(5) -
 1)*arctan(1/40*(sqrt(2)*(2*sqrt(10)*(5*x^6 + sqrt(5)*(x^6 - 2*x^2))*sqrt(x
^4 + 1) + sqrt(10)*(5*x^8 - 5*x^4 + sqrt(5)*(5*x^8 + 3*x^4 - 1) - 5))*(sqrt
(5) - 1) - 4*(sqrt(10)*(5*x^5 + sqrt(5)*(x^5 - 2*x))*(x^4 + 1)^(3/4) - sqrt
(10)*(x^4 + 1)^(1/4)*(5*x^3 - sqrt(5)*(2*x^7 + x^3)))*sqrt(sqrt(5) - 1))/(x
^8 + x^4 - 1)) - 1/80*sqrt(10)*sqrt(sqrt(5) + 1)*log((10*(2*x^5 + sqrt(5)*x
 + x)*(x^4 + 1)^(3/4) + (sqrt(10)*sqrt(x^4 + 1)*(5*x^2 + sqrt(5)*(2*x^6 + x
^2)) + sqrt(10)*(5*x^8 + 5*x^4 + sqrt(5)*(2*x^4 + 1)))*sqrt(sqrt(5) + 1) +
10*(x^7 + 3*x^3 + sqrt(5)*(x^7 + x^3))*(x^4 + 1)^(1/4))/(x^8 + x^4 - 1)) +
1/80*sqrt(10)*sqrt(sqrt(5) + 1)*log((10*(2*x^5 + sqrt(5)*x + x)*(x^4 + 1)^(
3/4) - (sqrt(10)*sqrt(x^4 + 1)*(5*x^2 + sqrt(5)*(2*x^6 + x^2)) + sqrt(10)*(
5*x^8 + 5*x^4 + sqrt(5)*(2*x^4 + 1)))*sqrt(sqrt(5) + 1) + 10*(x^7 + 3*x^3 +
 sqrt(5)*(x^7 + x^3))*(x^4 + 1)^(1/4))/(x^8 + x^4 - 1)) + 1/80*sqrt(10)*sqr
t(sqrt(5) - 1)*log((10*(2*x^5 - sqrt(5)*x + x)*(x^4 + 1)^(3/4) + (sqrt(10)*
sqrt(x^4 + 1)*(5*x^2 - sqrt(5)*(2*x^6 + x^2)) - sqrt(10)*(5*x^8 + 5*x^4 - s
qrt(5)*(2*x^4 + 1)))*sqrt(sqrt(5) - 1) - 10*(x^7 + 3*x^3 - sqrt(5)*(x^7 + x
^3))*(x^4 + 1)^(1/4))/(x^8 + x^4 - 1)) - 1/80*sqrt(10)*sqrt(sqrt(5) - 1)*lo
g((10*(2*x^5 - sqrt(5)*x + x)*(x^4 + 1)^(3/4) - (sqrt(10)*sqrt(x^4 + 1)*(5*
x^2 - sqrt(5)*(2*x^6 + x^2)) - sqrt(10)*(5*x^8 + 5*x^4 - sqrt(5)*(2*x^4 + 1
)))*sqrt(sqrt(5) - 1) - 10*(x^7 + 3*x^3 - sqrt(5)*(x^7 + x^3))*(x^4 + 1)^(1
/4))/(x^8 + x^4 - 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4)/(x^8+x^4-1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [C] time = 11.27, size = 1722, normalized size = 8.92

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1)^(1/4)/(x^8+x^4-1),x)
```

```
[Out] -1/20*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*ln(-(-3200*RootOf(_Z
^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*RootOf(6400*_Z^4+80*_Z^2-1)^4*x^4+1
60*(x^4+1)^(1/2)*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*RootOf(64
00*_Z^4+80*_Z^2-1)^2*x^2-200*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+
5)*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^4-320*(x^4+1)^(3/4)*RootOf(6400*_Z^4+80*
_Z^2-1)^2*x+560*(x^4+1)^(1/4)*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^3+3*(x^4+1)^(
1/2)*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*x^2-3*RootOf(_Z^2+400
*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*x^4-7*(x^4+1)^(3/4)*x+11*x^3*(x^4+1)^(1/4
)-40*RootOf(6400*_Z^4+80*_Z^2-1)^2*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2
-1)^2+5)-RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5))/(320*RootOf(6400
*_Z^4+80*_Z^2-1)^3*x^2+12*RootOf(6400*_Z^4+80*_Z^2-1)*x^2+1)/(320*RootOf(64
00*_Z^4+80*_Z^2-1)^3*x^2+12*RootOf(6400*_Z^4+80*_Z^2-1)*x^2-1))-RootOf(6400
*_Z^4+80*_Z^2-1)*ln(-(-64000*RootOf(6400*_Z^4+80*_Z^2-1)^5*x^4-1600*RootOf(
6400*_Z^4+80*_Z^2-1)^3*(x^4+1)^(1/2)*x^2+1600*x^4*RootOf(6400*_Z^4+80*_Z^2-
1)^3+240*(x^4+1)^(3/4)*RootOf(6400*_Z^4+80*_Z^2-1)^2*x-320*(x^4+1)^(1/4)*Ro
```

```
otOf(6400*_Z^4+80*_Z^2-1)^2*x^3+20*(x^4+1)^(1/2)*RootOf(6400*_Z^4+80*_Z^2-1
)*x^2-(x^4+1)^(3/4)*x+3*x^3*(x^4+1)^(1/4)+800*RootOf(6400*_Z^4+80*_Z^2-1)^3
)/(80*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^4+1))-4*RootOf(6400*_Z^4+80*_Z^2-1)^2
*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*ln((-6400*RootOf(_Z^2+400
*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*RootOf(6400*_Z^4+80*_Z^2-1)^4*x^4+320*(x^
4+1)^(1/2)*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*RootOf(6400*_Z^
4+80*_Z^2-1)^2*x^2+240*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*Roo
tOf(6400*_Z^4+80*_Z^2-1)^2*x^4-480*(x^4+1)^(3/4)*RootOf(6400*_Z^4+80*_Z^2-1
)^2*x-640*(x^4+1)^(1/4)*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^3-2*(x^4+1)^(1/2)*R
ootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*x^2-2*RootOf(_Z^2+400*RootO
f(6400*_Z^4+80*_Z^2-1)^2+5)*x^4+2*(x^4+1)^(3/4)*x+6*x^3*(x^4+1)^(1/4)+80*Ro
otOf(6400*_Z^4+80*_Z^2-1)^2*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5
)-RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5))/(80*RootOf(6400*_Z^4+80
*_Z^2-1)^2*x^4+1))-80*RootOf(6400*_Z^4+80*_Z^2-1)^3*ln((128000*RootOf(6400*
_Z^4+80*_Z^2-1)^5*x^4+4800*RootOf(6400*_Z^4+80*_Z^2-1)^3*(x^4+1)^(1/2)*x^2+
7200*x^4*RootOf(6400*_Z^4+80*_Z^2-1)^3+320*(x^4+1)^(3/4)*RootOf(6400*_Z^4+8
0*_Z^2-1)^2*x+560*(x^4+1)^(1/4)*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^3+100*(x^4+
1)^(1/2)*RootOf(6400*_Z^4+80*_Z^2-1)*x^2+90*RootOf(6400*_Z^4+80*_Z^2-1)*x^4
+7*(x^4+1)^(3/4)*x+11*x^3*(x^4+1)^(1/4)+1600*RootOf(6400*_Z^4+80*_Z^2-1)^3+
30*RootOf(6400*_Z^4+80*_Z^2-1))/(320*RootOf(6400*_Z^4+80*_Z^2-1)^3*x^2+12*R
ootOf(6400*_Z^4+80*_Z^2-1)*x^2+1)/(320*RootOf(6400*_Z^4+80*_Z^2-1)^3*x^2+12
*RootOf(6400*_Z^4+80*_Z^2-1)*x^2-1))-RootOf(6400*_Z^4+80*_Z^2-1)*ln((128000
*RootOf(6400*_Z^4+80*_Z^2-1)^5*x^4+4800*RootOf(6400*_Z^4+80*_Z^2-1)^3*(x^4+
1)^(1/2)*x^2+7200*x^4*RootOf(6400*_Z^4+80*_Z^2-1)^3+320*(x^4+1)^(3/4)*RootO
f(6400*_Z^4+80*_Z^2-1)^2*x+560*(x^4+1)^(1/4)*RootOf(6400*_Z^4+80*_Z^2-1)^2*
x^3+100*(x^4+1)^(1/2)*RootOf(6400*_Z^4+80*_Z^2-1)*x^2+90*RootOf(6400*_Z^4+8
0*_Z^2-1)*x^4+7*(x^4+1)^(3/4)*x+11*x^3*(x^4+1)^(1/4)+1600*RootOf(6400*_Z^4+
80*_Z^2-1)^3+30*RootOf(6400*_Z^4+80*_Z^2-1))/(320*RootOf(6400*_Z^4+80*_Z^2-
1)^3*x^2+12*RootOf(6400*_Z^4+80*_Z^2-1)*x^2+1)/(320*RootOf(6400*_Z^4+80*_Z^
2-1)^3*x^2+12*RootOf(6400*_Z^4+80*_Z^2-1)*x^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^8 + x^4 - 1\right)\left(x^4 + 1\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4)/(x^8+x^4-1),x, algorithm="maxima")
[Out] integrate(1/((x^8 + x^4 - 1)*(x^4 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^4 + 1\right)^{1/4}\left(x^8 + x^4 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^4 + 1)^(1/4)*(x^4 + x^8 - 1)),x)
[Out] int(1/((x^4 + 1)^(1/4)*(x^4 + x^8 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^4 + 1}\left(x^8 + x^4 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)**(1/4)/(x**8+x**4-1),x)
[Out] Integral(1/((x**4 + 1)**(1/4)*(x**8 + x**4 - 1)), x)
```

3.1961 $\int \frac{-2+x^4}{\sqrt[4]{1+x^4}\left(-1+x^4+x^8\right)}\,dx$

Optimal. Leaf size=193

$$\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-}}{\sqrt[4]{x^4+}}\right.$$

Rubi [A] time = 0.31, antiderivative size = 189, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6728, 377, 212, 206, 203}

$$\frac{1}{2}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^4)/((1 + x^4)^(1/4)*(-1 + x^4 + x^8)),x]
```

```
[Out] (((3 + Sqrt[5])/2)^(1/4)*ArcTan[((2/(3 + Sqrt[5]))^(1/4)*x)/(1 + x^4)^(1/4)
])/2 + (((3 - Sqrt[5])/2)^(1/4)*ArcTan[(((3 + Sqrt[5])/2)^(1/4)*x)/(1 + x^4
)^(1/4)])/2 + (((3 + Sqrt[5])/2)^(1/4)*ArcTanh[((2/(3 + Sqrt[5]))^(1/4)*x)/
(1 + x^4)^(1/4)])/2 + (((3 - Sqrt[5])/2)^(1/4)*ArcTanh[(((3 + Sqrt[5])/2)^(
1/4)*x)/(1 + x^4)^(1/4)])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-2+x^4}{\sqrt[4]{1+x^4}\left(-1+x^4+x^8\right)}dx = \int \left(\frac{1-\sqrt{5}}{\sqrt[4]{1+x^4}\left(1-\sqrt{5}+2x^4\right)}+\frac{1+\sqrt{5}}{\sqrt[4]{1+x^4}\left(1+\sqrt{5}+2x^4\right)}\right)dx$$
$$= \left(1-\sqrt{5}\right)\int \frac{1}{\sqrt[4]{1+x^4}\left(1-\sqrt{5}+2x^4\right)}dx+\left(1+\sqrt{5}\right)\int \frac{1}{\sqrt[4]{1+x^4}\left(1+\sqrt{5}+2x^4\right.}$$
$$= \left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{5}-\left(-1-\sqrt{5}\right)x^4}dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)+\left(1+\sqrt{5}\right)\text{Subst}\Big($$
$$= -\frac{\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt{2}}-\frac{\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{\cdot}}\right.}{2\sqrt{2}}$$
$$= \frac{1}{2}\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{2}\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x}{\sqrt[4]{1+x^4}}\right)+$$

Mathematica [A] time = 0.12, size = 170, normalized size = 0.88

$$\frac{\sqrt[4]{3+\sqrt{5}}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}x}{\sqrt[4]{x^4+1}}\right)+\sqrt[4]{3-\sqrt{5}}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x}{\sqrt[4]{x^4+1}}\right)+\sqrt[4]{3+\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}x}{\sqrt[4]{x^4+1}}\right)+\sqrt[4]{3-\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^4)/((1 + x^4)^(1/4)*(-1 + x^4 + x^8)),x]
```

```
[Out] ((3 + Sqrt[5])^(1/4)*ArcTan[((2/(3 + Sqrt[5]))^(1/4)*x)/(1 + x^4)^(1/4)] +
(3 - Sqrt[5])^(1/4)*ArcTan[(((3 + Sqrt[5])/2)^(1/4)*x)/(1 + x^4)^(1/4)] + (
3 + Sqrt[5])^(1/4)*ArcTanh[((2/(3 + Sqrt[5]))^(1/4)*x)/(1 + x^4)^(1/4)] + (
3 - Sqrt[5])^(1/4)*ArcTanh[(((3 + Sqrt[5])/2)^(1/4)*x)/(1 + x^4)^(1/4)])/(2
*2^(1/4))
```

IntegrateAlgebraic [A] time = 0.72, size = 193, normalized size = 1.00

$$\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}x}{\sqrt[4]{x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2 + x^4)/((1 + x^4)^(1/4)*(-1 + x^4 + x^8)),x]
```

```
[Out] (Sqrt[(1 + Sqrt[5])/2]*ArcTan[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(1 + x^4)^(1/4)])/
2 + (Sqrt[(-1 + Sqrt[5])/2]*ArcTan[(Sqrt[1/2 + Sqrt[5]/2]*x)/(1 + x^4)^(1/4
)])/2 + (Sqrt[(1 + Sqrt[5])/2]*ArcTanh[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(1 + x^4)
^(1/4)])/2 + (Sqrt[(-1 + Sqrt[5])/2]*ArcTanh[(Sqrt[1/2 + Sqrt[5]/2]*x)/(1 +
 x^4)^(1/4)])/2
```

fricas [B] time = 31.33, size = 985, normalized size = 5.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2)/(x^4+1)^(1/4)/(x^8+x^4-1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt(sqrt(5) - 1)*arctan(1/4*(sqrt(2)*(sqrt(5)*sqrt(2)*(x^8 +
x^4) + sqrt(2)*(2*x^4 + 1) + sqrt(x^4 + 1)*(sqrt(5)*sqrt(2)*x^2 + sqrt(2)*(
```

```
2*x^6 + x^2)))*sqrt(sqrt(5) + 1)*sqrt(sqrt(5) - 1) + 2*((x^4 + 1)^(3/4)*(sq
rt(5)*sqrt(2)*x + sqrt(2)*(2*x^5 + x)) + (x^4 + 1)^(1/4)*(sqrt(5)*sqrt(2)*(
x^7 + x^3) + sqrt(2)*(x^7 + 3*x^3)))*sqrt(sqrt(5) - 1))/(x^8 + x^4 - 1)) +
1/4*sqrt(2)*sqrt(sqrt(5) + 1)*arctan(-1/4*(sqrt(2)*(sqrt(5)*sqrt(2)*(x^8 +
x^4) - sqrt(2)*(2*x^4 + 1) - sqrt(x^4 + 1)*(sqrt(5)*sqrt(2)*x^2 - sqrt(2)*(
2*x^6 + x^2)))*sqrt(sqrt(5) + 1)*sqrt(sqrt(5) - 1) + 2*((x^4 + 1)^(3/4)*(sq
rt(5)*sqrt(2)*x - sqrt(2)*(2*x^5 + x)) - (x^4 + 1)^(1/4)*(sqrt(5)*sqrt(2)*(
x^7 + x^3) - sqrt(2)*(x^7 + 3*x^3)))*sqrt(sqrt(5) + 1))/(x^8 + x^4 - 1)) +
1/16*sqrt(2)*sqrt(sqrt(5) - 1)*log((4*(2*x^5 + sqrt(5)*x + x)*(x^4 + 1)^(3/
4) + (sqrt(5)*sqrt(2)*(x^8 + 3*x^4 + 1) + sqrt(2)*(5*x^8 + 7*x^4 + 1) + 2*s
qrt(x^4 + 1)*(sqrt(5)*sqrt(2)*(x^6 + x^2) + sqrt(2)*(x^6 + 3*x^2)))*sqrt(sq
rt(5) - 1) + 4*(x^7 + 3*x^3 + sqrt(5)*(x^7 + x^3))*(x^4 + 1)^(1/4))/(x^8 +
x^4 - 1)) - 1/16*sqrt(2)*sqrt(sqrt(5) - 1)*log((4*(2*x^5 + sqrt(5)*x + x)*(
x^4 + 1)^(3/4) - (sqrt(5)*sqrt(2)*(x^8 + 3*x^4 + 1) + sqrt(2)*(5*x^8 + 7*x^
4 + 1) + 2*sqrt(x^4 + 1)*(sqrt(5)*sqrt(2)*(x^6 + x^2) + sqrt(2)*(x^6 + 3*x^
2)))*sqrt(sqrt(5) - 1) + 4*(x^7 + 3*x^3 + sqrt(5)*(x^7 + x^3))*(x^4 + 1)^(1
/4))/(x^8 + x^4 - 1)) - 1/16*sqrt(2)*sqrt(sqrt(5) + 1)*log((4*(2*x^5 - sqrt
(5)*x + x)*(x^4 + 1)^(3/4) + (sqrt(5)*sqrt(2)*(x^8 + 3*x^4 + 1) - sqrt(2)*(
5*x^8 + 7*x^4 + 1) - 2*sqrt(x^4 + 1)*(sqrt(5)*sqrt(2)*(x^6 + x^2) - sqrt(2)
*(x^6 + 3*x^2)))*sqrt(sqrt(5) + 1) - 4*(x^7 + 3*x^3 - sqrt(5)*(x^7 + x^3))*
(x^4 + 1)^(1/4))/(x^8 + x^4 - 1)) + 1/16*sqrt(2)*sqrt(sqrt(5) + 1)*log((4*(
2*x^5 - sqrt(5)*x + x)*(x^4 + 1)^(3/4) - (sqrt(5)*sqrt(2)*(x^8 + 3*x^4 + 1)
 - sqrt(2)*(5*x^8 + 7*x^4 + 1) - 2*sqrt(x^4 + 1)*(sqrt(5)*sqrt(2)*(x^6 + x^
2) - sqrt(2)*(x^6 + 3*x^2)))*sqrt(sqrt(5) + 1) - 4*(x^7 + 3*x^3 - sqrt(5)*(
x^7 + x^3))*(x^4 + 1)^(1/4))/(x^8 + x^4 - 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2)/(x^4+1)^(1/4)/(x^8+x^4-1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [C] time = 10.34, size = 1692, normalized size = 8.77

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2)/(x^4+1)^(1/4)/(x^8+x^4-1),x)
```

```
[Out] -1/4*RootOf(_Z^2+16*RootOf(256*_Z^4+16*_Z^2-1)^2+1)*ln(-(768*RootOf(_Z^2+16
*RootOf(256*_Z^4+16*_Z^2-1)^2+1)*RootOf(256*_Z^4+16*_Z^2-1)^4*x^4-128*(x^4+
1)^(1/2)*RootOf(256*_Z^4+16*_Z^2-1)^2*RootOf(_Z^2+16*RootOf(256*_Z^4+16*_Z^
2-1)^2+1)*x^2-112*RootOf(_Z^2+16*RootOf(256*_Z^4+16*_Z^2-1)^2+1)*RootOf(256
*_Z^4+16*_Z^2-1)^2*x^4+96*(x^4+1)^(3/4)*RootOf(256*_Z^4+16*_Z^2-1)^2*x+128*
(x^4+1)^(1/4)*RootOf(256*_Z^4+16*_Z^2-1)^2*x^3+6*(x^4+1)^(1/2)*RootOf(_Z^2+
16*RootOf(256*_Z^4+16*_Z^2-1)^2+1)*x^2+2*RootOf(_Z^2+16*RootOf(256*_Z^4+16*
_Z^2-1)^2+1)*x^4-2*(x^4+1)^(3/4)*x-6*x^3*(x^4+1)^(1/4)-48*RootOf(256*_Z^4+1
6*_Z^2-1)^2*RootOf(_Z^2+16*RootOf(256*_Z^4+16*_Z^2-1)^2+1)+RootOf(_Z^2+16*R
ootOf(256*_Z^4+16*_Z^2-1)^2+1))/(16*RootOf(256*_Z^4+16*_Z^2-1)^2*x^4+1))+Ro
otOf(256*_Z^4+16*_Z^2-1)*ln((2048*RootOf(256*_Z^4+16*_Z^2-1)^5*x^4+448*Root
Of(256*_Z^4+16*_Z^2-1)^3*(x^4+1)^(1/2)*x^2+608*x^4*RootOf(256*_Z^4+16*_Z^2-
1)^3+64*(x^4+1)^(3/4)*RootOf(256*_Z^4+16*_Z^2-1)^2*x+112*(x^4+1)^(1/4)*Root
Of(256*_Z^4+16*_Z^2-1)^2*x^3+44*(x^4+1)^(1/2)*RootOf(256*_Z^4+16*_Z^2-1)*x^
2+42*RootOf(256*_Z^4+16*_Z^2-1)*x^4+7*(x^4+1)^(3/4)*x+11*x^3*(x^4+1)^(1/4)+
```

```
128*RootOf(256*_Z^4+16*_Z^2-1)^3+14*RootOf(256*_Z^4+16*_Z^2-1))/(64*RootOf(
256*_Z^4+16*_Z^2-1)^3*x^2+4*RootOf(256*_Z^4+16*_Z^2-1)*x^2+1)/(64*RootOf(25
6*_Z^4+16*_Z^2-1)^3*x^2+4*RootOf(256*_Z^4+16*_Z^2-1)*x^2-1))+4*RootOf(256*_
Z^4+16*_Z^2-1)^2*RootOf(_Z^2+16*RootOf(256*_Z^4+16*_Z^2-1)^2+1)*ln((256*Roo
tOf(_Z^2+16*RootOf(256*_Z^4+16*_Z^2-1)^2+1)*RootOf(256*_Z^4+16*_Z^2-1)^4*x^
4-96*(x^4+1)^(1/2)*RootOf(256*_Z^4+16*_Z^2-1)^2*RootOf(_Z^2+16*RootOf(256*_
Z^4+16*_Z^2-1)^2+1)*x^2+96*RootOf(_Z^2+16*RootOf(256*_Z^4+16*_Z^2-1)^2+1)*R
ootOf(256*_Z^4+16*_Z^2-1)^2*x^4+96*(x^4+1)^(3/4)*RootOf(256*_Z^4+16*_Z^2-1)
^2*x-128*(x^4+1)^(1/4)*RootOf(256*_Z^4+16*_Z^2-1)^2*x^3-8*(x^4+1)^(1/2)*Roo
tOf(_Z^2+16*RootOf(256*_Z^4+16*_Z^2-1)^2+1)*x^2+9*RootOf(_Z^2+16*RootOf(256
*_Z^4+16*_Z^2-1)^2+1)*x^4+8*(x^4+1)^(3/4)*x-14*x^3*(x^4+1)^(1/4)+16*RootOf(
256*_Z^4+16*_Z^2-1)^2*RootOf(_Z^2+16*RootOf(256*_Z^4+16*_Z^2-1)^2+1)+3*Root
Of(_Z^2+16*RootOf(256*_Z^4+16*_Z^2-1)^2+1))/(64*RootOf(256*_Z^4+16*_Z^2-1)^
3*x^2+4*RootOf(256*_Z^4+16*_Z^2-1)*x^2+1)/(64*RootOf(256*_Z^4+16*_Z^2-1)^3*
x^2+4*RootOf(256*_Z^4+16*_Z^2-1)*x^2-1))+16*RootOf(256*_Z^4+16*_Z^2-1)^3*ln
(-(1024*RootOf(256*_Z^4+16*_Z^2-1)^5*x^4+192*RootOf(256*_Z^4+16*_Z^2-1)^3*(
x^4+1)^(1/2)*x^2-160*x^4*RootOf(256*_Z^4+16*_Z^2-1)^3+48*(x^4+1)^(3/4)*Root
Of(256*_Z^4+16*_Z^2-1)^2*x-80*(x^4+1)^(1/4)*RootOf(256*_Z^4+16*_Z^2-1)^2*x^
3-8*(x^4+1)^(1/2)*RootOf(256*_Z^4+16*_Z^2-1)*x^2+4*RootOf(256*_Z^4+16*_Z^2-
1)*x^4-2*(x^4+1)^(3/4)*x+3*x^3*(x^4+1)^(1/4)-64*RootOf(256*_Z^4+16*_Z^2-1)^
3+2*RootOf(256*_Z^4+16*_Z^2-1))/(16*RootOf(256*_Z^4+16*_Z^2-1)^2*x^4+1))+Ro
otOf(256*_Z^4+16*_Z^2-1)*ln(-(1024*RootOf(256*_Z^4+16*_Z^2-1)^5*x^4+192*Roo
tOf(256*_Z^4+16*_Z^2-1)^3*(x^4+1)^(1/2)*x^2-160*x^4*RootOf(256*_Z^4+16*_Z^2
-1)^3+48*(x^4+1)^(3/4)*RootOf(256*_Z^4+16*_Z^2-1)^2*x-80*(x^4+1)^(1/4)*Root
Of(256*_Z^4+16*_Z^2-1)^2*x^3-8*(x^4+1)^(1/2)*RootOf(256*_Z^4+16*_Z^2-1)*x^2
+4*RootOf(256*_Z^4+16*_Z^2-1)*x^4-2*(x^4+1)^(3/4)*x+3*x^3*(x^4+1)^(1/4)-64*
RootOf(256*_Z^4+16*_Z^2-1)^3+2*RootOf(256*_Z^4+16*_Z^2-1))/(16*RootOf(256*_
Z^4+16*_Z^2-1)^2*x^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-2}{\left(x^8+x^4-1\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2)/(x^4+1)^(1/4)/(x^8+x^4-1),x, algorithm="maxima")
[Out] integrate((x^4 - 2)/((x^8 + x^4 - 1)*(x^4 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4-2}{\left(x^4+1\right)^{1/4}\,\left(x^8+x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 2)/((x^4 + 1)^(1/4)*(x^4 + x^8 - 1)),x)
[Out] int((x^4 - 2)/((x^4 + 1)^(1/4)*(x^4 + x^8 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2)/(x**4+1)**(1/4)/(x**8+x**4-1),x)
[Out] Timed out
```

3.1962 $\int \frac{\sqrt[4]{bx^3+ax^4}\left(-d+cx^8\right)}{x^8}\,dx$

Optimal. Leaf size=193

$$\frac{3b^2c\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{16a^{7/4}}-\frac{3b^2c\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{16a^{7/4}}+\sqrt[4]{ax^4+bx^3}\left(-262144a^7dx^6+65536a^6bdx^5-40960a^5b^2dx^4\right.$$

Rubi [A] time = 0.61, antiderivative size = 341, normalized size of antiderivative = 1.77, number of steps used = 16, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {2052, 2004, 2024, 2032, 63, 331, 298, 203, 206, 2016, 2014}

$$\frac{3b^2cx^{9/4}(ax+b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{16a^{7/4}\left(ax^4+bx^3\right)^{3/4}}-\frac{3b^2cx^{9/4}(ax+b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{16a^{7/4}\left(ax^4+bx^3\right)^{3/4}}-\frac{32768a^5d\left(ax^4+bx^3\right)^{5/4}}{348075b^6x^5}+\frac{8192a^4d\left(ax^4+bx^3\right)^{5/4}}{69615b^5x^6}-\frac{1024a^3d\left(ax^4+bx^3\right)^{5/4}}{7735b^4x^7}+\frac{256a^2d\left(ax^4+bx^3\right)^{5/4}}{1785b^3x^8}-\frac{16ad\left(ax^4+bx^3\right)^{5/4}}{105b^2x^9}+\frac{1}{2}cx\sqrt[4]{ax^4+bx^3}+\frac{bc\sqrt[4]{ax^4+bx^3}}{8a}+\frac{4d\left(ax^4+bx^3\right)^{5/4}}{25bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*x^3 + a*x^4)^(1/4)*(-d + c*x^8))/x^8,x]
```

```
[Out] (b*c*(b*x^3 + a*x^4)^(1/4))/(8*a) + (c*x*(b*x^3 + a*x^4)^(1/4))/2 + (4*d*(b
*x^3 + a*x^4)^(5/4))/(25*b*x^10) - (16*a*d*(b*x^3 + a*x^4)^(5/4))/(105*b^2*
x^9) + (256*a^2*d*(b*x^3 + a*x^4)^(5/4))/(1785*b^3*x^8) - (1024*a^3*d*(b*x^
3 + a*x^4)^(5/4))/(7735*b^4*x^7) + (8192*a^4*d*(b*x^3 + a*x^4)^(5/4))/(6961
5*b^5*x^6) - (32768*a^5*d*(b*x^3 + a*x^4)^(5/4))/(348075*b^6*x^5) + (3*b^2*
c*x^(9/4)*(b + a*x)^(3/4)*ArcTan[(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(16*a^
(7/4)*(b*x^3 + a*x^4)^(3/4)) - (3*b^2*c*x^(9/4)*(b + a*x)^(3/4)*ArcTanh[(a^
(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(16*a^(7/4)*(b*x^3 + a*x^4)^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{bx^3+ax^4}\,\left(-d+cx^8\right)}{x^8}\,dx &= \int \left(c\sqrt[4]{bx^3+ax^4}-\frac{d\sqrt[4]{bx^3+ax^4}}{x^8}\right)dx \\
&= c\int \sqrt[4]{bx^3+ax^4}\,dx-d\int \frac{\sqrt[4]{bx^3+ax^4}}{x^8}\,dx \\
&= \frac{1}{2}cx\sqrt[4]{bx^3+ax^4}+\frac{4d\left(bx^3+ax^4\right)^{5/4}}{25bx^{10}}+\frac{1}{8}(bc)\int \frac{x^3}{\left(bx^3+ax^4\right)^{3/4}}\,dx+\frac{(4ad)\int
\\
&= \frac{bc\sqrt[4]{bx^3+ax^4}}{8a}+\frac{1}{2}cx\sqrt[4]{bx^3+ax^4}+\frac{4d\left(bx^3+ax^4\right)^{5/4}}{25bx^{10}}-\frac{16ad\left(bx^3+ax^4\right)^{5/4}}{105b^2x^9}-
\\
&= \frac{bc\sqrt[4]{bx^3+ax^4}}{8a}+\frac{1}{2}cx\sqrt[4]{bx^3+ax^4}+\frac{4d\left(bx^3+ax^4\right)^{5/4}}{25bx^{10}}-\frac{16ad\left(bx^3+ax^4\right)^{5/4}}{105b^2x^9}-
\\
&= \frac{bc\sqrt[4]{bx^3+ax^4}}{8a}+\frac{1}{2}cx\sqrt[4]{bx^3+ax^4}+\frac{4d\left(bx^3+ax^4\right)^{5/4}}{25bx^{10}}-\frac{16ad\left(bx^3+ax^4\right)^{5/4}}{105b^2x^9}-
\\
&= \frac{bc\sqrt[4]{bx^3+ax^4}}{8a}+\frac{1}{2}cx\sqrt[4]{bx^3+ax^4}+\frac{4d\left(bx^3+ax^4\right)^{5/4}}{25bx^{10}}-\frac{16ad\left(bx^3+ax^4\right)^{5/4}}{105b^2x^9}-
\\
&= \frac{bc\sqrt[4]{bx^3+ax^4}}{8a}+\frac{1}{2}cx\sqrt[4]{bx^3+ax^4}+\frac{4d\left(bx^3+ax^4\right)^{5/4}}{25bx^{10}}-\frac{16ad\left(bx^3+ax^4\right)^{5/4}}{105b^2x^9}-
\\
&= \frac{bc\sqrt[4]{bx^3+ax^4}}{8a}+\frac{1}{2}cx\sqrt[4]{bx^3+ax^4}+\frac{4d\left(bx^3+ax^4\right)^{5/4}}{25bx^{10}}-\frac{16ad\left(bx^3+ax^4\right)^{5/4}}{105b^2x^9}-
\end{aligned}
$$

Mathematica [C] time = 0.44, size = 424, normalized size = 2.20

$$
\frac{4\sqrt[4]{x^3(ax+b)}\left(-8192a^{14}dx^6\sqrt[4]{\frac{ax}{b}+1}+2048a^{13}bdx^5\sqrt[4]{\frac{ax}{b}+1}-1280a^{12}b^2dx^4\sqrt[4]{\frac{ax}{b}+1}+960a^{11}b^3dx^3\sqrt[4]{\frac{ax}{b}+1}-780a^{10}b^4dx^2\sqrt[4]{\frac{ax}{b}+1}+663a^9b^5dx\sqrt[4]{\frac{ax}{b}+1}+13923a^8b^6d\sqrt[4]{\frac{ax}{b}+1}+1092048a^6b^8cx^6\sqrt[4]{\frac{ax}{b}+1}+4600038a^5b^9cx^5\sqrt[4]{\frac{ax}{b}+1}+8698470a^4b^{10}cx^4\sqrt[4]{\frac{ax}{b}+1}+9313560a^3b^{11}cx^3\sqrt[4]{\frac{ax}{b}+1}+5833620a^2b^{12}cx^2\sqrt[4]{\frac{ax}{b}+1}-13923b^{14}c\,{}_2F_1\left(-\frac{33}{4},-\frac{25}{4};-\frac{21}{4};-\frac{ax}{b}\right)+111384b^{14}c\,{}_2F_1\left(-\frac{29}{4},-\frac{25}{4};-\frac{21}{4};-\frac{ax}{b}\right)-389844b^{14}c\,{}_2F_1\left(-\frac{25}{4},-\frac{25}{4};-\frac{21}{4};-\frac{ax}{b}\right)+292383b^{14}c\sqrt[4]{\frac{ax}{b}+1}+2002923ab^{13}cx\sqrt[4]{\frac{ax}{b}+1}\right)}{348075a^8b^6x^7\sqrt[4]{\frac{ax}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*x^3 + a*x^4)^(1/4)*(-d + c*x^8))/x^8,x]
```

```
[Out] (4*(x^3*(b + a*x))^(1/4)*(292383*b^14*c*(1 + (a*x)/b)^(1/4) + 13923*a^8*b^6
*d*(1 + (a*x)/b)^(1/4) + 2002923*a*b^13*c*x*(1 + (a*x)/b)^(1/4) + 663*a^9*b
^5*d*x*(1 + (a*x)/b)^(1/4) + 5833620*a^2*b^12*c*x^2*(1 + (a*x)/b)^(1/4) - 7
80*a^10*b^4*d*x^2*(1 + (a*x)/b)^(1/4) + 9313560*a^3*b^11*c*x^3*(1 + (a*x)/b
)^(1/4) + 960*a^11*b^3*d*x^3*(1 + (a*x)/b)^(1/4) + 8698470*a^4*b^10*c*x^4*(
1 + (a*x)/b)^(1/4) - 1280*a^12*b^2*d*x^4*(1 + (a*x)/b)^(1/4) + 4600038*a^5*
b^9*c*x^5*(1 + (a*x)/b)^(1/4) + 2048*a^13*b*d*x^5*(1 + (a*x)/b)^(1/4) + 109
2048*a^6*b^8*c*x^6*(1 + (a*x)/b)^(1/4) - 8192*a^14*d*x^6*(1 + (a*x)/b)^(1/4
) - 13923*b^14*c*Hypergeometric2F1[-33/4, -25/4, -21/4, -((a*x)/b)] + 11138
4*b^14*c*Hypergeometric2F1[-29/4, -25/4, -21/4, -((a*x)/b)] - 389844*b^14*c
*Hypergeometric2F1[-25/4, -25/4, -21/4, -((a*x)/b)]))/(348075*a^8*b^6*x^7*(
1 + (a*x)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 2.76, size = 193, normalized size = 1.00

$$
\frac{3b^2c\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{16a^{7/4}}-\frac{3b^2c\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{16a^{7/4}}+\frac{\sqrt[4]{ax^4+bx^3}\left(-262144a^7dx^6+65536a^6bdx^5-40960a^5b^2dx^4+30720a^4b^3dx^3-24960a^3b^4dx^2+21216a^2b^5dx+1392300ab^6cx^8+445536ab^6d+348075b^7cx^7\right)}{2784600ab^6x^7}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b*x^3 + a*x^4)^(1/4)*(-d + c*x^8))/x^8,x]
```

```
[Out] ((b*x^3 + a*x^4)^(1/4)*(445536*a*b^6*d + 21216*a^2*b^5*d*x - 24960*a^3*b^4*
d*x^2 + 30720*a^4*b^3*d*x^3 - 40960*a^5*b^2*d*x^4 + 65536*a^6*b*d*x^5 - 262
144*a^7*d*x^6 + 348075*b^7*c*x^7 + 1392300*a*b^6*c*x^8))/(2784600*a*b^6*x^7
) + (3*b^2*c*ArcTan[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(16*a^(7/4)) - (3*b
^2*c*ArcTanh[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(16*a^(7/4))
```

fricas [B] time = 0.61, size = 377, normalized size = 1.95

$$\frac{4176900 \left(\frac{b^8c^4}{a^7}\right)^{\frac{1}{4}} ab^6x^7 \arctan\left(-\frac{\left(\frac{b^8c^4}{a^7}\right)^{\frac{3}{4}}\left(ax^4+bx^3\right)^{\frac{1}{4}}a^5b^2c-\left(\frac{b^8c^4}{a^7}\right)^{\frac{3}{4}}a^5x\sqrt{\frac{\sqrt{ax^4+bx^3}\,b^4c^2+\sqrt{\frac{b^8c^4}{a^7}}\,a^4x^2}{x^2}}}{b^8c^4x}\right) - 1044225 \left(\frac{b^8c^4}{a^7}\right)^{\frac{1}{4}} ab^6x^7 \log\left(\frac{3\left(\left(ax^4+bx^3\right)^{\frac{1}{4}}b^2c+\left(\frac{b^8c^4}{a^7}\right)^{\frac{1}{4}}a^2x\right)}{x}\right) + 1044225 \left(\frac{b^8c^4}{a^7}\right)^{\frac{1}{4}} ab^6x^7 \log\left(\frac{3\left(\left(ax^4+bx^3\right)^{\frac{1}{4}}b^2c-\left(\frac{b^8c^4}{a^7}\right)^{\frac{1}{4}}a^2x\right)}{x}\right) + 4\left(1392300\,ab^6cx^8 + 348075\,b^7cx^7 - 262144\,a^7dx^6 + 65536\,a^6bdx^5 - 40960\,a^5b^2dx^4 + 30720\,a^4b^3dx^3 - 24960\,a^3b^4dx^2 + 21216\,a^2b^5dx + 445536\,ab^6d\right)\left(ax^4+bx^3\right)^{\frac{1}{4}}}{11138400\,ab^6x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^3)^(1/4)*(c*x^8-d)/x^8,x, algorithm="fricas")
```

```
[Out] 1/11138400*(4176900*(b^8*c^4/a^7)^(1/4)*a*b^6*x^7*arctan(-((b^8*c^4/a^7)^(3
/4)*(a*x^4 + b*x^3)^(1/4)*a^5*b^2*c - (b^8*c^4/a^7)^(3/4)*a^5*x*sqrt((sqrt(
a*x^4 + b*x^3)*b^4*c^2 + sqrt(b^8*c^4/a^7)*a^4*x^2)/x^2))/(b^8*c^4*x)) - 10
44225*(b^8*c^4/a^7)^(1/4)*a*b^6*x^7*log(3*((a*x^4 + b*x^3)^(1/4)*b^2*c + (b
^8*c^4/a^7)^(1/4)*a^2*x)/x) + 1044225*(b^8*c^4/a^7)^(1/4)*a*b^6*x^7*log(3*(
(a*x^4 + b*x^3)^(1/4)*b^2*c - (b^8*c^4/a^7)^(1/4)*a^2*x)/x) + 4*(1392300*a*
b^6*c*x^8 + 348075*b^7*c*x^7 - 262144*a^7*d*x^6 + 65536*a^6*b*d*x^5 - 40960
*a^5*b^2*d*x^4 + 30720*a^4*b^3*d*x^3 - 24960*a^3*b^4*d*x^2 + 21216*a^2*b^5*
d*x + 445536*a*b^6*d)*(a*x^4 + b*x^3)^(1/4))/(a*b^6*x^7)
```

giac [B] time = 0.67, size = 358, normalized size = 1.85

$$\frac{\frac{2088450\,\sqrt{2}\,b^3c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a+\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}a} + \frac{2088450\,\sqrt{2}\,b^3c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a+\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}a} + \frac{1044225\,\sqrt{2}\,b^3c\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{(-a)^{\frac{3}{4}}a} + \frac{1044225\,\sqrt{2}(-a)^{\frac{1}{4}}b^3c\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a+\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{a^2} + \frac{2784600\left(\left(a+\frac{b}{x}\right)^{\frac{5}{4}}b^3c+3\left(a+\frac{b}{x}\right)^{\frac{1}{4}}ab^3c\right)x^2}{ab^2} + \frac{256\left(13923\left(a+\frac{b}{x}\right)^{\frac{25}{4}}b^{120}d-82875\left(a+\frac{b}{x}\right)^{\frac{21}{4}}ab^{120}d+204750\left(a+\frac{b}{x}\right)^{\frac{17}{4}}a^2b^{120}d-267750\left(a+\frac{b}{x}\right)^{\frac{13}{4}}a^3b^{120}d+193375\left(a+\frac{b}{x}\right)^{\frac{9}{4}}a^4b^{120}d-69615\left(a+\frac{b}{x}\right)^{\frac{5}{4}}a^5b^{120}d\right)}{b^{125}}}{22276800\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^3)^(1/4)*(c*x^8-d)/x^8,x, algorithm="giac")
```

```
[Out] 1/22276800*(2088450*sqrt(2)*b^3*c*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) +
2*(a + b/x)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a) + 2088450*sqrt(2)*b^3*c*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(a + b/x)^(1/4))/(-a)^(1/4))/((-a)^(
3/4)*a) + 1044225*sqrt(2)*b^3*c*log(sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sq
rt(-a) + sqrt(a + b/x))/((-a)^(3/4)*a) + 1044225*sqrt(2)*(-a)^(1/4)*b^3*c*l
og(-sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a + b/x))/a^2 + 27
84600*((a + b/x)^(5/4)*b^3*c + 3*(a + b/x)^(1/4)*a*b^3*c)*x^2/(a*b^2) + 256
*(13923*(a + b/x)^(25/4)*b^120*d - 82875*(a + b/x)^(21/4)*a*b^120*d + 20475
0*(a + b/x)^(17/4)*a^2*b^120*d - 267750*(a + b/x)^(13/4)*a^3*b^120*d + 1933
75*(a + b/x)^(9/4)*a^4*b^120*d - 69615*(a + b/x)^(5/4)*a^5*b^120*d)/b^125)/
b
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{4}+b\,x^{3}\right)^{\frac{1}{4}} \left(c\,x^{8}-d\right)}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b*x^3)^(1/4)*(c*x^8-d)/x^8,x)
```

```
[Out] int((a*x^4+b*x^3)^(1/4)*(c*x^8-d)/x^8,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^8 - d)(ax^4 + bx^3)^{\frac{1}{4}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^3)^(1/4)*(c*x^8-d)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((c*x^8 - d)*(a*x^4 + b*x^3)^(1/4)/x^8, x)
```

mupad [B] time = 4.81, size = 207, normalized size = 1.07

$$\frac{4\,d\,(a\,x^4+b\,x^3)^{1/4}}{25\,x^7}-\frac{16\,a^2\,d\,(a\,x^4+b\,x^3)^{1/4}}{1785\,b^2\,x^5}+\frac{256\,a^3\,d\,(a\,x^4+b\,x^3)^{1/4}}{23205\,b^3\,x^4}-\frac{1024\,a^4\,d\,(a\,x^4+b\,x^3)^{1/4}}{69615\,b^4\,x^3}+\frac{8192\,a^5\,d\,(a\,x^4+b\,x^3)^{1/4}}{348075\,b^5\,x^2}-\frac{32768\,a^6\,d\,(a\,x^4+b\,x^3)^{1/4}}{348075\,b^6\,x}+\frac{4\,c\,x\,(a\,x^4+b\,x^3)^{1/4}\,{}_2F_1\left(-\frac{1}{4},\frac{7}{4};\frac{11}{4};-\frac{a\,x}{b}\right)}{7\left(\frac{a\,x}{b}+1\right)^{1/4}}+\frac{4\,a\,d\,(a\,x^4+b\,x^3)^{1/4}}{525\,b\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((d - c*x^8)*(a*x^4 + b*x^3)^(1/4))/x^8,x)
```

```
[Out] (4*d*(a*x^4 + b*x^3)^(1/4))/(25*x^7) - (16*a^2*d*(a*x^4 + b*x^3)^(1/4))/(17
85*b^2*x^5) + (256*a^3*d*(a*x^4 + b*x^3)^(1/4))/(23205*b^3*x^4) - (1024*a^4
*d*(a*x^4 + b*x^3)^(1/4))/(69615*b^4*x^3) + (8192*a^5*d*(a*x^4 + b*x^3)^(1/
4))/(348075*b^5*x^2) - (32768*a^6*d*(a*x^4 + b*x^3)^(1/4))/(348075*b^6*x) +
 (4*c*x*(a*x^4 + b*x^3)^(1/4)*hypergeom([-1/4, 7/4], 11/4, -(a*x)/b))/(7*((
a*x)/b + 1)^(1/4)) + (4*a*d*(a*x^4 + b*x^3)^(1/4))/(525*b*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3(ax+b)}\,(cx^8 - d)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b*x**3)**(1/4)*(c*x**8-d)/x**8,x)
```

```
[Out] Integral((x**3*(a*x + b))**(1/4)*(c*x**8 - d)/x**8, x)
```

3.1963 $\int \frac{x^4(2+x^5)}{\sqrt{1+x^5}\,(-1-x^5+ax^{10})}\,dx$

Optimal. Leaf size=193

$$\frac{\sqrt{2}\left(4a+\sqrt{4a+1}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{-2a-\sqrt{4a+1}-1}}\right)}{5\sqrt{a}\sqrt{4a+1}\sqrt{-2a-\sqrt{4a+1}-1}}+\frac{\sqrt{2}\left(-4a+\sqrt{4a+1}-1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{-2a+\sqrt{4a+1}-1}}\right)}{5\sqrt{a}\sqrt{4a+1}\sqrt{-2a+\sqrt{4a+1}-1}}$$

Rubi [A] time = 0.40, antiderivative size = 73, normalized size of antiderivative = 0.38, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {6715, 826, 1161, 618, 206}

$$\frac{2\tanh^{-1}\left(\sqrt{4a+1}-2\sqrt{a}\sqrt{x^5+1}\right)}{5\sqrt{a}}-\frac{2\tanh^{-1}\left(2\sqrt{a}\sqrt{x^5+1}+\sqrt{4a+1}\right)}{5\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(2 + x^5))/(Sqrt[1 + x^5]*(-1 - x^5 + a*x^10)),x]
```

```
[Out] (2*ArcTanh[Sqrt[1 + 4*a] - 2*Sqrt[a]*Sqrt[1 + x^5]])/(5*Sqrt[a]) - (2*ArcTa
nh[Sqrt[1 + 4*a] + 2*Sqrt[a]*Sqrt[1 + x^5]])/(5*Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{x^4\left(2+x^5\right)}{\sqrt{1+x^5}\left(-1-x^5+ax^{10}\right)}\,dx = \frac{1}{5}\text{Subst}\left(\int \frac{2+x}{\sqrt{1+x}\left(-1-x+ax^2\right)}\,dx,x,x^5\right)$$
$$= \frac{2}{5}\text{Subst}\left(\int \frac{1+x^2}{a+(-1-2a)x^2+ax^4}\,dx,x,\sqrt{1+x^5}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\frac{\sqrt{1+4a}\,x}{\sqrt{a}}+x^2}\,dx,x,\sqrt{1+x^5}\right)}{5a} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{\sqrt{1+4a}\,x}{\sqrt{a}}+x^2}\,dx,x,\sqrt{1+x}\right.}{5a}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-x^2}\,dx,x,-\frac{\sqrt{1+4a}}{\sqrt{a}}+2\sqrt{1+x^5}\right)}{5a} - \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{1}{a}-x^2}\,dx,x,\frac{\sqrt{1+4}}{\sqrt{a}}\right.}{5a}$$
$$= \frac{2\tanh^{-1}\left(\sqrt{a}\left(\frac{\sqrt{1+4a}}{\sqrt{a}}-2\sqrt{1+x^5}\right)\right)}{5\sqrt{a}} - \frac{2\tanh^{-1}\left(\sqrt{a}\left(\frac{\sqrt{1+4a}}{\sqrt{a}}+2\sqrt{1+x^5}\right)\right)}{5\sqrt{a}}$$

Mathematica [A] time = 0.15, size = 158, normalized size = 0.82

$$\frac{\sqrt{2a-\sqrt{4a+1}+1}\left(\sqrt{4a+1}+1\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{2a-\sqrt{4a+1}+1}}\right)-\left(\sqrt{4a+1}-1\right)\sqrt{2a+\sqrt{4a+1}+1}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{2a+\sqrt{4a+1}+1}}\right)}{5\sqrt{2}\,a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(2 + x^5))/(Sqrt[1 + x^5]*(-1 - x^5 + a*x^10)),x]
```

```
[Out] (Sqrt[1 + 2*a - Sqrt[1 + 4*a]]*(1 + Sqrt[1 + 4*a])*ArcTanh[(Sqrt[2]*Sqrt[a]
*Sqrt[1 + x^5])/Sqrt[1 + 2*a - Sqrt[1 + 4*a]]] - (-1 + Sqrt[1 + 4*a])*Sqrt[
1 + 2*a + Sqrt[1 + 4*a]]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[1 + x^5])/Sqrt[1 + 2
*a + Sqrt[1 + 4*a]]])/(5*Sqrt[2]*a^(3/2))
```

IntegrateAlgebraic [A] time = 0.33, size = 193, normalized size = 1.00

$$\frac{\sqrt{2}\left(4a+\sqrt{4a+1}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{-2a-\sqrt{4a+1}-1}}\right)}{5\sqrt{a}\sqrt{4a+1}\sqrt{-2a-\sqrt{4a+1}-1}}+\frac{\sqrt{2}\left(-4a+\sqrt{4a+1}-1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{-2a+\sqrt{4a+1}-1}}\right)}{5\sqrt{a}\sqrt{4a+1}\sqrt{-2a+\sqrt{4a+1}-1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(2 + x^5))/(Sqrt[1 + x^5]*(-1 - x^5 + a*x^10)),x]
```

```
[Out] (Sqrt[2]*(1 + 4*a + Sqrt[1 + 4*a])*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[1 + x^5])/S
qrt[-1 - 2*a - Sqrt[1 + 4*a]]])/(5*Sqrt[a]*Sqrt[1 + 4*a]*Sqrt[-1 - 2*a - Sq
rt[1 + 4*a]]) + (Sqrt[2]*(-1 - 4*a + Sqrt[1 + 4*a])*ArcTan[(Sqrt[2]*Sqrt[a]
*Sqrt[1 + x^5])/Sqrt[-1 - 2*a + Sqrt[1 + 4*a]]])/(5*Sqrt[a]*Sqrt[1 + 4*a]*S
qrt[-1 - 2*a + Sqrt[1 + 4*a]])
```

fricas [A] time = 0.51, size = 74, normalized size = 0.38

$$\left[\frac{\log\left(\frac{ax^{10}-2\sqrt{x^5+1}\sqrt{a}\,x^5+x^5+1}{ax^{10}-x^5-1}\right)}{5\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\,x^5}{\sqrt{x^5+1}}\right)}{5a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+2)/(x^5+1)^(1/2)/(a*x^10-x^5-1),x, algorithm="fricas")
```

```
[Out] [1/5*log((a*x^10 - 2*sqrt(x^5 + 1)*sqrt(a)*x^5 + x^5 + 1)/(a*x^10 - x^5 - 1
))/sqrt(a), 2/5*sqrt(-a)*arctan(sqrt(-a)*x^5/sqrt(x^5 + 1))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+2)/(x^5+1)^(1/2)/(a*x^10-x^5-1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:a: recursive definition (in sto)  Error: Bad
 Argument ValueWarning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a]=[6]
a: recursive definition (in sto)  Error: Bad Argument ValueWarning, need to
 choose a branch for the root of a polynomial with parameters. This might b
e wrong.The choice was done assuming [a]=[-31]Done
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(x^{5}+2\right)}{\sqrt{x^{5}+1}\, \left(a\,x^{10}-x^{5}-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^5+2)/(x^5+1)^(1/2)/(a*x^10-x^5-1),x)
```

```
[Out] int(x^4*(x^5+2)/(x^5+1)^(1/2)/(a*x^10-x^5-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{5}+2\right)x^{4}}{\left(ax^{10}-x^{5}-1\right)\sqrt{x^{5}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+2)/(x^5+1)^(1/2)/(a*x^10-x^5-1),x, algorithm="maxima")
```

```
[Out] integrate((x^5 + 2)*x^4/((a*x^10 - x^5 - 1)*sqrt(x^5 + 1)), x)
```

mupad [B] time = 2.29, size = 47, normalized size = 0.24

$$\frac{\ln\left(\frac{a\,x^{10}+x^5-2\,\sqrt{a}\,x^5\,\sqrt{x^5+1}+1}{-4\,a\,x^{10}+4\,x^5+4}\right)}{5\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(x^5 + 2))/((x^5 + 1)^(1/2)*(x^5 - a*x^10 + 1)),x)
```

```
[Out] log((a*x^10 + x^5 - 2*a^(1/2)*x^5*(x^5 + 1)^(1/2) + 1)/(4*x^5 - 4*a*x^10 +
4))/(5*a^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**5+2)/(x**5+1)**(1/2)/(a*x**10-x**5-1),x)
```

```
[Out] Timed out
```

3.1964 $\int \frac{x^4(-2+x^5)}{\sqrt{-1+x^5}\,(1-x^5+ax^{10})}\,dx$

Optimal. Leaf size=193

$$\frac{\sqrt{2}\left(-4a+\sqrt{1-4a}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5-1}}{\sqrt{2a-\sqrt{1-4a}-1}}\right)}{5\sqrt{1-4a}\sqrt{a}\sqrt{2a-\sqrt{1-4a}-1}}+\frac{\sqrt{2}\left(4a+\sqrt{1-4a}-1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5-1}}{\sqrt{2a+\sqrt{1-4a}-1}}\right)}{5\sqrt{1-4a}\sqrt{a}\sqrt{2a+\sqrt{1-4a}-1}}$$

Rubi [A] time = 0.38, antiderivative size = 81, normalized size of antiderivative = 0.42, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6715, 826, 1164, 628}

$$\frac{\log\left(-\sqrt{a}\left(1-x^5\right)+\sqrt{a}-\sqrt{x^5-1}\right)}{5\sqrt{a}}-\frac{\log\left(-\sqrt{a}\left(1-x^5\right)+\sqrt{a}+\sqrt{x^5-1}\right)}{5\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(-2 + x^5))/(Sqrt[-1 + x^5]*(1 - x^5 + a*x^10)),x]
```

```
[Out] Log[Sqrt[a] - Sqrt[a]*(1 - x^5) - Sqrt[-1 + x^5]]/(5*Sqrt[a]) - Log[Sqrt[a]
 - Sqrt[a]*(1 - x^5) + Sqrt[-1 + x^5]]/(5*Sqrt[a])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{x^4\left(-2+x^5\right)}{\sqrt{-1+x^5}\left(1-x^5+ax^{10}\right)}\,dx = \frac{1}{5}\operatorname{Subst}\left(\int \frac{-2+x}{\sqrt{-1+x}\left(1-x+ax^2\right)}\,dx, x, x^5\right)$$
$$= \frac{2}{5}\operatorname{Subst}\left(\int \frac{-1+x^2}{a+(-1+2a)x^2+ax^4}\,dx, x, \sqrt{-1+x^5}\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{\sqrt{a}}+2x}{-1-\frac{x}{\sqrt{a}}-x^2}\,dx, x, \sqrt{-1+x^5}\right)}{5\sqrt{a}} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{\sqrt{a}}-2x}{-1+\frac{x}{\sqrt{a}}-x^2}\,dx, x, \sqrt{-1+x^5}\right)}{5\sqrt{a}}$$
$$= \frac{\log\left(\sqrt{a}-\sqrt{a}\left(1-x^5\right)-\sqrt{-1+x^5}\right)}{5\sqrt{a}} - \frac{\log\left(\sqrt{a}-\sqrt{a}\left(1-x^5\right)+\sqrt{-1+x^5}\right)}{5\sqrt{a}}$$

Mathematica [A] time = 0.19, size = 158, normalized size = 0.82

$$\frac{\left(\sqrt{1-4a}-1\right)\sqrt{-2a+\sqrt{1-4a}+1}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5-1}}{\sqrt{-2a+\sqrt{1-4a}+1}}\right)-\left(\sqrt{1-4a}+1\right)\sqrt{-2a-\sqrt{1-4a}+1}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5-1}}{\sqrt{-2a-\sqrt{1-4a}+1}}\right)}{5\sqrt{2}a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(-2 + x^5))/(Sqrt[-1 + x^5]*(1 - x^5 + a*x^10)),x]
```

```
[Out] (-((1 + Sqrt[1 - 4*a])*Sqrt[1 - Sqrt[1 - 4*a] - 2*a]*ArcTanh[(Sqrt[2]*Sqrt[
a]*Sqrt[-1 + x^5])/Sqrt[1 - Sqrt[1 - 4*a] - 2*a]]) + (-1 + Sqrt[1 - 4*a])*S
qrt[1 + Sqrt[1 - 4*a] - 2*a]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[-1 + x^5])/Sqrt[
1 + Sqrt[1 - 4*a] - 2*a]])/(5*Sqrt[2]*a^(3/2))
```

IntegrateAlgebraic [A] time = 0.32, size = 193, normalized size = 1.00

$$\frac{\sqrt{2}\left(-4a+\sqrt{1-4a}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5-1}}{\sqrt{2a-\sqrt{1-4a}-1}}\right)}{5\sqrt{1-4a}\sqrt{a}\sqrt{2a-\sqrt{1-4a}-1}} + \frac{\sqrt{2}\left(4a+\sqrt{1-4a}-1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5-1}}{\sqrt{2a+\sqrt{1-4a}-1}}\right)}{5\sqrt{1-4a}\sqrt{a}\sqrt{2a+\sqrt{1-4a}-1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(-2 + x^5))/(Sqrt[-1 + x^5]*(1 - x^5 + a*x^10)),x]
```

```
[Out] (Sqrt[2]*(1 + Sqrt[1 - 4*a] - 4*a)*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[-1 + x^5])/
Sqrt[-1 - Sqrt[1 - 4*a] + 2*a]])/(5*Sqrt[1 - 4*a]*Sqrt[a]*Sqrt[-1 - Sqrt[1
- 4*a] + 2*a]) + (Sqrt[2]*(-1 + Sqrt[1 - 4*a] + 4*a)*ArcTan[(Sqrt[2]*Sqrt[a
]*Sqrt[-1 + x^5])/Sqrt[-1 + Sqrt[1 - 4*a] + 2*a]])/(5*Sqrt[1 - 4*a]*Sqrt[a]
*Sqrt[-1 + Sqrt[1 - 4*a] + 2*a])
```

fricas [A] time = 0.54, size = 74, normalized size = 0.38

$$\left[\frac{\log\left(\frac{ax^{10}-2\sqrt{x^5-1}\sqrt{a}x^5+x^5-1}{ax^{10}-x^5+1}\right)}{5\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x^5}{\sqrt{x^5-1}}\right)}{5a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5-2)/(x^5-1)^(1/2)/(a*x^10-x^5+1),x, algorithm="fricas")
```

```
[Out] [1/5*log((a*x^10 - 2*sqrt(x^5 - 1)*sqrt(a)*x^5 + x^5 - 1)/(a*x^10 - x^5 + 1
))/sqrt(a), 2/5*sqrt(-a)*arctan(sqrt(-a)*x^5/sqrt(x^5 - 1))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5-2)/(x^5-1)^(1/2)/(a*x^10-x^5+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:a: recursive definition (in sto)  Error: Bad
 Argument ValueWarning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a]=[6]
a: recursive definition (in sto)  Error: Bad Argument ValueWarning, need to
 choose a branch for the root of a polynomial with parameters. This might b
e wrong.The choice was done assuming [a]=[-31]Done
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(x^{5}-2\right)}{\sqrt{x^{5}-1}\, \left(a\,x^{10}-x^{5}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^5-2)/(x^5-1)^(1/2)/(a*x^10-x^5+1),x)
```

```
[Out] int(x^4*(x^5-2)/(x^5-1)^(1/2)/(a*x^10-x^5+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{5}-2\right)x^{4}}{\left(ax^{10}-x^{5}+1\right)\sqrt{x^{5}-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5-2)/(x^5-1)^(1/2)/(a*x^10-x^5+1),x, algorithm="maxima")
```

```
[Out] integrate((x^5 - 2)*x^4/((a*x^10 - x^5 + 1)*sqrt(x^5 - 1)), x)
```

mupad [B] time = 2.24, size = 47, normalized size = 0.24

$$\frac{\ln\left(\frac{a\,x^{10}+x^5-2\,\sqrt{a}\,x^5\,\sqrt{x^5-1}-1}{4\,a\,x^{10}-4\,x^5+4}\right)}{5\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x^5 - 2))/((x^5 - 1)^(1/2)*(a*x^10 - x^5 + 1)),x)
```

```
[Out] log((a*x^10 + x^5 - 2*a^(1/2)*x^5*(x^5 - 1)^(1/2) - 1)/(4*a*x^10 - 4*x^5 +
4))/(5*a^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**5-2)/(x**5-1)**(1/2)/(a*x**10-x**5+1),x)
```

```
[Out] Timed out
```

3.1965 $\int \frac{x^4(3+x^5)}{\sqrt{1+x^5}\left(-1+a-(1+2a)x^5+ax^{10}\right)}\,dx$

Optimal. Leaf size=193

$$\frac{\sqrt{2}\left(8a+\sqrt{8a+1}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{-4a-\sqrt{8a+1}-1}}\right)}{5\sqrt{a}\sqrt{8a+1}\sqrt{-4a-\sqrt{8a+1}-1}}+\frac{\sqrt{2}\left(-8a+\sqrt{8a+1}-1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{-4a+\sqrt{8a+1}-1}}\right)}{5\sqrt{a}\sqrt{8a+1}\sqrt{-4a+\sqrt{8a+1}-1}}$$

Rubi [A] time = 0.47, antiderivative size = 73, normalized size of antiderivative = 0.38, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {6715, 826, 1161, 618, 206}

$$\frac{2\tanh^{-1}\left(\sqrt{8a+1}-2\sqrt{a}\sqrt{x^5+1}\right)}{5\sqrt{a}}-\frac{2\tanh^{-1}\left(2\sqrt{a}\sqrt{x^5+1}+\sqrt{8a+1}\right)}{5\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(3 + x^5))/(Sqrt[1 + x^5]*(-1 + a - (1 + 2*a)*x^5 + a*x^10)),x]
```

```
[Out] (2*ArcTanh[Sqrt[1 + 8*a] - 2*Sqrt[a]*Sqrt[1 + x^5]])/(5*Sqrt[a]) - (2*ArcTa
nh[Sqrt[1 + 8*a] + 2*Sqrt[a]*Sqrt[1 + x^5]])/(5*Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{x^4\left(3+x^5\right)}{\sqrt{1+x^5}\left(-1+a-(1+2a)x^5+ax^{10}\right)}\,dx=\frac{1}{5}\operatorname{Subst}\left(\int\frac{3+x}{\sqrt{1+x}\left(-1+a+(-1-2a)x+ax^2\right)}\,dx,x,x^5\right)$$
$$=\frac{2}{5}\operatorname{Subst}\left(\int\frac{2+x^2}{4a+(-1-4a)x^2+ax^4}\,dx,x,\sqrt{1+x^5}\right)$$
$$=\frac{\operatorname{Subst}\left(\int\frac{1}{2-\frac{\sqrt{1+8a}\,x}{\sqrt{a}}+x^2}\,dx,x,\sqrt{1+x^5}\right)}{5a}+\frac{\operatorname{Subst}\left(\int\frac{1}{2+\frac{\sqrt{1+8a}\,x}{\sqrt{a}}+x^2}\,d\right.}{5a}$$
$$=-\frac{2\operatorname{Subst}\left(\int\frac{1}{\frac{1}{a}-x^2}\,dx,x,-\frac{\sqrt{1+8a}}{\sqrt{a}}+2\sqrt{1+x^5}\right)}{5a}-\frac{2\operatorname{Subst}\left(\int\frac{1}{\frac{1}{a}-x}\right.}{5a}$$
$$=\frac{2\tanh^{-1}\left(\sqrt{a}\left(\frac{\sqrt{1+8a}}{\sqrt{a}}-2\sqrt{1+x^5}\right)\right)}{5\sqrt{a}}-\frac{2\tanh^{-1}\left(\sqrt{a}\left(\frac{\sqrt{1+8a}}{\sqrt{a}}+\right.\right.}{5\sqrt{a}}$$

Mathematica [A] time = 0.21, size = 158, normalized size = 0.82

$$\frac{\sqrt{4a-\sqrt{8a+1}+1}\left(\sqrt{8a+1}+1\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{4a-\sqrt{8a+1}+1}}\right)-\left(\sqrt{8a+1}-1\right)\sqrt{4a+\sqrt{8a+1}+1}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{4a+\sqrt{8a+1}+1}}\right)}{10\sqrt{2}\,a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(3 + x^5))/(Sqrt[1 + x^5]*(-1 + a - (1 + 2*a)*x^5 + a*x^10))
,x]
```

```
[Out] (Sqrt[1 + 4*a - Sqrt[1 + 8*a]]*(1 + Sqrt[1 + 8*a])*ArcTanh[(Sqrt[2]*Sqrt[a]
*Sqrt[1 + x^5])/Sqrt[1 + 4*a - Sqrt[1 + 8*a]]] - (-1 + Sqrt[1 + 8*a])*Sqrt[
1 + 4*a + Sqrt[1 + 8*a]]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[1 + x^5])/Sqrt[1 + 4
*a + Sqrt[1 + 8*a]]])/(10*Sqrt[2]*a^(3/2))
```

IntegrateAlgebraic [A] time = 0.33, size = 193, normalized size = 1.00

$$\frac{\sqrt{2}\left(8a+\sqrt{8a+1}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{-4a-\sqrt{8a+1}-1}}\right)}{5\sqrt{a}\sqrt{8a+1}\sqrt{-4a-\sqrt{8a+1}-1}}+\frac{\sqrt{2}\left(-8a+\sqrt{8a+1}-1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{-4a+\sqrt{8a+1}-1}}\right)}{5\sqrt{a}\sqrt{8a+1}\sqrt{-4a+\sqrt{8a+1}-1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(3 + x^5))/(Sqrt[1 + x^5]*(-1 + a - (1 + 2*a)*x^5 +
 a*x^10)),x]
```

```
[Out] (Sqrt[2]*(1 + 8*a + Sqrt[1 + 8*a])*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[1 + x^5])/S
qrt[-1 - 4*a - Sqrt[1 + 8*a]]])/(5*Sqrt[a]*Sqrt[1 + 8*a]*Sqrt[-1 - 4*a - Sq
rt[1 + 8*a]]) + (Sqrt[2]*(-1 - 8*a + Sqrt[1 + 8*a])*ArcTan[(Sqrt[2]*Sqrt[a]
*Sqrt[1 + x^5])/Sqrt[-1 - 4*a + Sqrt[1 + 8*a]]])/(5*Sqrt[a]*Sqrt[1 + 8*a]*S
qrt[-1 - 4*a + Sqrt[1 + 8*a]])
```

fricas [A] time = 0.63, size = 92, normalized size = 0.48

$$\left[\frac{\log\left(\frac{ax^{10}-(2a-1)x^5-2\sqrt{x^5+1}\left(x^5-1\right)\sqrt{a}+a+1}{ax^{10}-(2a+1)x^5+a-1}\right)}{5\sqrt{a}},\ \frac{2\sqrt{-a}\arctan\left(\frac{\left(x^5-1\right)\sqrt{-a}}{\sqrt{x^5+1}}\right)}{5a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+3)/(x^5+1)^(1/2)/(-1+a-(1+2*a)*x^5+a*x^10),x, algorithm=
"fricas")
```

```
[Out] [1/5*log((a*x^10 - (2*a - 1)*x^5 - 2*sqrt(x^5 + 1)*(x^5 - 1)*sqrt(a) + a +
1)/(a*x^10 - (2*a + 1)*x^5 + a - 1))/sqrt(a), 2/5*sqrt(-a)*arctan((x^5 - 1)
*sqrt(-a)/sqrt(x^5 + 1))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+3)/(x^5+1)^(1/2)/(-1+a-(1+2*a)*x^5+a*x^10),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:a: recursive definition (in sto)  Error: Bad
 Argument ValueWarning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a]=[6]
a: recursive definition (in sto)  Error: Bad Argument ValueWarning, need to
 choose a branch for the root of a polynomial with parameters. This might b
e wrong.The choice was done assuming [a]=[-31]a: recursive definition (in s
to)  Error: Bad Argument ValueWarning, need to choose a branch for the root
 of a polynomial with parameters. This might be wrong.The choice was done a
ssuming [a]=[-5]Warning, choosing root of [1,0,%%%{16,[2]%%%}+%%%{-12,[1]%%
%}+%%%{-2,[0]%%%},%%%{64,[2]%%%}+%%%{8,[1]%%%},%%%{64,[4]%%%}+%%%{128,[3]%%
%}+%%%{112,[2]%%%}+%%%{20,[1]%%%}+%%%{1,[0]%%%}] at parameters values [-86]
a: recursive definition (in sto)  Error: Bad Argument ValueWarning, need to
 choose a branch for the root of a polynomial with parameters. This might b
e wrong.The choice was done assuming [a]=[-96]Done
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(x^{5}+3\right)}{\sqrt{x^{5}+1}\, \left(-1+a-\left(1+2 a \right) x^{5}+a \,x^{10}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^5+3)/(x^5+1)^(1/2)/(-1+a-(1+2*a)*x^5+a*x^10),x)
```

```
[Out] int(x^4*(x^5+3)/(x^5+1)^(1/2)/(-1+a-(1+2*a)*x^5+a*x^10),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(x^{5} + 3\right)} x^{4}}{{\left(a x^{10} - {\left(2 \, a + 1\right)} x^{5} + a - 1\right)} \sqrt{x^{5} + 1}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+3)/(x^5+1)^(1/2)/(-1+a-(1+2*a)*x^5+a*x^10),x, algorithm=
"maxima")
```

```
[Out] integrate((x^5 + 3)*x^4/((a*x^10 - (2*a + 1)*x^5 + a - 1)*sqrt(x^5 + 1)), x
)
```

mupad [B] time = 2.67, size = 73, normalized size = 0.38

$$\frac{\ln\left(\frac{a-2\,a\,x^5+a\,x^{10}+2\,\sqrt{a}\,\sqrt{x^5+1}+x^5-2\,\sqrt{a}\,x^5\,\sqrt{x^5+1}+1}{2\,a\,x^5-a-a\,x^{10}+x^5+1}\right)}{5\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x^5 + 3))/((x^5 + 1)^(1/2)*(a - x^5*(2*a + 1) + a*x^10 - 1)),x)
```

```
[Out] log((a - 2*a*x^5 + a*x^10 + 2*a^(1/2)*(x^5 + 1)^(1/2) + x^5 - 2*a^(1/2)*x^5
*(x^5 + 1)^(1/2) + 1)/(2*a*x^5 - a - a*x^10 + x^5 + 1))/(5*a^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**5+3)/(x**5+1)**(1/2)/(-1+a-(1+2*a)*x**5+a*x**10),x)
```

```
[Out] Timed out
```

3.1966 $\int \frac{1}{\sqrt{-b+a^2x^2}\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx$

Optimal. Leaf size=193

$$\frac{4\log\left(\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}-\sqrt[3]{c}\right)}{ac^{2/3}}-\frac{2\log\left(\sqrt[3]{c}\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}+c^{2/3}\right)}{ac^{2/3}}$$

Rubi [F] time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{-b+a^2x^2}\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[-b + a^2*x^2]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3)),x]
```

```
[Out] Defer[Int][1/(Sqrt[-b + a^2*x^2]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/
3)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{-b+a^2x^2}\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx=\int \frac{1}{\sqrt{-b+a^2x^2}\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx$$

Mathematica [A] time = 0.25, size = 174, normalized size = 0.90

$$-\frac{2\left(\log\left(\sqrt[3]{c}\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[3]{c}}+1}{\sqrt{3}}\right)\right)}{ac^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-b + a^2*x^2]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3
)),x]
```

```
[Out] (-2*(2*Sqrt[3]*ArcTan[(1 + (2*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3))
/c^(1/3))/Sqrt[3]] - 2*Log[c^(1/3) - (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))
^(1/3)] + Log[c^(2/3) + c^(1/3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3
) + (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3)]))/(a*c^(2/3))
```

IntegrateAlgebraic [A] time = 0.72, size = 193, normalized size = 1.00

$$\frac{4\log\left(\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}-\sqrt[3]{c}\right)}{ac^{2/3}}-\frac{2\log\left(\sqrt[3]{c}\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}+c^{2/3}\right)}{ac^{2/3}}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{3}\sqrt[3]{c}}+\frac{1}{\sqrt{3}}\right)}{ac^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-b + a^2*x^2]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1
/4))^(2/3)),x]
```

[Out] (-4*Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1
/3))/(Sqrt[3]*c^(1/3))])/(a*c^(2/3)) + (4*Log[-c^(1/3) + (c + (a*x + Sqrt[-
b + a^2*x^2])^(1/4))^(1/3)])/(a*c^(2/3)) - (2*Log[c^(2/3) + c^(1/3)*(c + (a
*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + (c + (a*x + Sqrt[-b + a^2*x^2])^(1/
4))^(2/3)])/(a*c^(2/3))

fricas [A] time = 1.18, size = 180, normalized size = 0.93

$$-\frac{2\left(2\sqrt{3}\left(c^{2}\right)^{\frac{1}{6}}c\arctan\left(\frac{\sqrt{3}\sqrt{c^{2}}c+2\sqrt{3}\left(c^{2}\right)^{\frac{5}{6}}\left(c+\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}{3c^{2}}\right)+\left(c^{2}\right)^{\frac{2}{3}}\log\left(\left(c+\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{2}{3}}c+\left(c^{2}\right)^{\frac{1}{3}}c+\left(c^{2}\right)^{\frac{2}{3}}\left(c+\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}\right)-2\left(c^{2}\right)^{\frac{2}{3}}\log\left(\left(c+\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}c-\left(c^{2}\right)^{\frac{2}{3}}\right)\right)}{ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(2/3),x, al
gorithm="fricas")

[Out] -2*(2*sqrt(3)*(c^2)^(1/6)*c*arctan(1/3*(sqrt(3)*sqrt(c^2)*c + 2*sqrt(3)*(c^
2)^(5/6)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3))/c^2) + (c^2)^(2/3)*lo
g((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3)*c + (c^2)^(1/3)*c + (c^2)^(2/
3)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)) - 2*(c^2)^(2/3)*log((c + (a
*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)*c - (c^2)^(2/3)))/(a*c^2)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(2/3),x, al
gorithm="giac")

[Out] Timed out

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^{2}x^{2}-b}\,\left(c+\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(2/3),x)

[Out] int(1/(a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(2/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^{2}x^{2}-b}\left(c+\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(2/3),x, al
gorithm="maxima")

[Out] integrate(1/(sqrt(a^2*x^2 - b)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3))
, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(c+\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/4}\right)^{2/3}\,\sqrt{a^2\,x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(2/3)*(a^2*x^2 - b)^(1/2)),x
)
```

```
[Out] int(1/((c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(2/3)*(a^2*x^2 - b)^(1/2)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}\right)^{\frac{2}{3}}\sqrt{a^2x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*x**2-b)**(1/2)/(c+(a*x+(a**2*x**2-b)**(1/2))**(1/4))**(2/
3),x)
```

```
[Out] Integral(1/((c + (a*x + sqrt(a**2*x**2 - b))**(1/4))**(2/3)*sqrt(a**2*x**2
- b)), x)
```

3.1967 $\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=193

$$-\frac{2\log\left(\sqrt[3]{c}\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}+c^{2/3}\right)}{a\sqrt[3]{c}}+\frac{4\log\left(\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}-\right.}{a\sqrt[3]{c}}$$

Rubi [F] time = 0.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(Sqrt[-b + a^2*x^2]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)),x]

[Out] Defer[Int][1/(Sqrt[-b + a^2*x^2]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)), x]

Rubi steps

$$\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Mathematica [A] time = 0.11, size = 127, normalized size = 0.66

$$\frac{12\log\left(\sqrt[3]{c}-\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}\right)+8\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[3]{c}}+1}{\sqrt{3}}\right)-\log\left(\sqrt{a^2x^2-b}+ax\right)}{2a\sqrt[3]{c}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[-b + a^2*x^2]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)),x]

[Out] (8*Sqrt[3]*ArcTan[(1 + (2*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3))/c^(1/3))/Sqrt[3]] - Log[a*x + Sqrt[-b + a^2*x^2]] + 12*Log[c^(1/3) - (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)])/(2*a*c^(1/3))

IntegrateAlgebraic [A] time = 0.65, size = 193, normalized size = 1.00

$$-\frac{2\log\left(\sqrt[3]{c}\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}+c^{2/3}\right)}{a\sqrt[3]{c}}+\frac{4\log\left(\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}-\sqrt[3]{c}\right)}{a\sqrt[3]{c}}+\frac{4\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{3}\sqrt[3]{c}}+\frac{1}{\sqrt{3}}\right)}{a\sqrt[3]{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-b + a^2*x^2]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1
/4))^(1/3)),x]
```

```
[Out] (4*Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/
3))/(Sqrt[3]*c^(1/3))])/(a*c^(1/3)) + (4*Log[-c^(1/3) + (c + (a*x + Sqrt[-b
 + a^2*x^2])^(1/4))^(1/3)])/(a*c^(1/3)) - (2*Log[c^(2/3) + c^(1/3)*(c + (a*
x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4
))^(2/3)])/(a*c^(1/3))
```

fricas [A] time = 1.06, size = 532, normalized size = 2.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3),x, al
gorithm="fricas")
```

```
[Out] [2*(sqrt(3)*c*sqrt(-1/c^(2/3))*log(2*sqrt(3)*(a*c^(2/3)*x - sqrt(a^2*x^2 -
b)*c^(2/3))*(a*x + sqrt(a^2*x^2 - b))^(3/4)*(c + (a*x + sqrt(a^2*x^2 - b))^
(1/4))^(2/3)*sqrt(-1/c^(2/3)) - (3*a*c^(2/3)*x + sqrt(3)*(a*c*x - sqrt(a^2*
x^2 - b)*c)*sqrt(-1/c^(2/3)) - 3*sqrt(a^2*x^2 - b)*c^(2/3))*(a*x + sqrt(a^2
*x^2 - b))^(3/4)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3) + (3*a*c*x - s
qrt(3)*(a*c^(4/3)*x - sqrt(a^2*x^2 - b)*c^(4/3))*sqrt(-1/c^(2/3)) - 3*sqrt(
a^2*x^2 - b)*c)*(a*x + sqrt(a^2*x^2 - b))^(3/4) + 2*b) - c^(2/3)*log((c + (
a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3) + (c + (a*x + sqrt(a^2*x^2 - b))^(1/4
))^(1/3)*c^(1/3) + c^(2/3)) + 2*c^(2/3)*log((c + (a*x + sqrt(a^2*x^2 - b))^
(1/4))^(1/3) - c^(1/3)))/(a*c), 2*(2*sqrt(3)*c^(2/3)*arctan(1/3*sqrt(3) + 2
/3*sqrt(3)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)/c^(1/3)) - c^(2/3)*l
og((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3) + (c + (a*x + sqrt(a^2*x^2 -
 b))^(1/4))^(1/3)*c^(1/3) + c^(2/3)) + 2*c^(2/3)*log((c + (a*x + sqrt(a^2*x
^2 - b))^(1/4))^(1/3) - c^(1/3)))/(a*c)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3),x, al
gorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^{2} x^{2}-b}\,\left(c+\left(a x+\sqrt{a^{2} x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3),x)
```

```
[Out] int(1/(a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^{2} x^{2}-b}\left(c+\left(a x+\sqrt{a^{2} x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3),x, al
gorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 - b)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(c+\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/4}\right)^{1/3}\,\sqrt{a^2\,x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(1/3)*(a^2*x^2 - b)^(1/2)),x
)
```

```
[Out] int(1/((c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(1/3)*(a^2*x^2 - b)^(1/2)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}\,\sqrt{a^2x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*x**2-b)**(1/2)/(c+(a*x+(a**2*x**2-b)**(1/2))**(1/4))**(1/
3),x)
```

```
[Out] Integral(1/((c + (a*x + sqrt(a**2*x**2 - b))**(1/4))**(1/3)*sqrt(a**2*x**2
- b)), x)
```

3.1968 $\int \frac{1}{\sqrt[3]{1-3x^2}\,(-3+x^2)}\,dx$

Optimal. Leaf size=194

$$-\frac{1}{8}\tan^{-1}\left(\frac{-\frac{\sqrt[3]{1-3x^2}}{\sqrt{3}}+x+\frac{1}{\sqrt{3}}}{\sqrt[3]{1-3x^2}}\right)-\frac{1}{8}\tan^{-1}\left(\frac{\frac{\sqrt[3]{1-3x^2}}{\sqrt{3}}+x-\frac{1}{\sqrt{3}}}{\sqrt[3]{1-3x^2}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{1-3x^2}+1}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2\sqrt{3}x\sqrt[3]{1-3x^2}-2\sqrt{3}x}{3x^2+4(1-3x^2)^{2/3}-2\sqrt[3]{1-3x}}\right.}{8\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 0.42, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {395}

$$-\frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)+\frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - 3*x^2)^(1/3)*(-3 + x^2)),x]
```

```
[Out] -1/4*ArcTan[(1 - (1 - 3*x^2)^(1/3))/x] - ArcTanh[x/Sqrt[3]]/(4*Sqrt[3]) + A
rcTanh[(1 - (1 - 3*x^2)^(1/3))^2/(3*Sqrt[3]*x)]/(4*Sqrt[3])
```

Rule 395

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, -Simp[(q*ArcTanh[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Si
mp[(q*ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[
a, 3]*d), x] - Simp[(q*ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(-3+x^2)}\,dx=-\frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}$$

Mathematica [C] time = 0.10, size = 126, normalized size = 0.65

$$\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,\frac{x^2}{3}\right)}{\sqrt[3]{1-3x^2}\,(x^2-3)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};3x^2,\frac{x^2}{3}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};3x^2,\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - 3*x^2)^(1/3)*(-3 + x^2)),x]
```

```
[Out] (9*x*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3])/((1 - 3*x^2)^(1/3)*(-3 + x^2
)*(9*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2
, 5/2, 3*x^2, x^2/3] + 3*AppellF1[3/2, 4/3, 1, 5/2, 3*x^2, x^2/3])))
```

IntegrateAlgebraic [F] time = 4.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{1-3x^2}\left(-3+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((1 - 3*x^2)^(1/3)*(-3 + x^2)),x]
[Out] Defer[IntegrateAlgebraic][1/((1 - 3*x^2)^(1/3)*(-3 + x^2)), x]
```

fricas [B] time = 3.29, size = 1790, normalized size = 9.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(x^2-3),x, algorithm="fricas")
[Out] -1/72*sqrt(6)*sqrt(3)*sqrt(2)*arctan(1/9*(36*sqrt(6)*sqrt(3)*sqrt(2)*(3*x^1
1 - 1117*x^9 + 3918*x^7 - 1866*x^5 + 255*x^3 - 9*x) + sqrt(3)*(sqrt(6)*sqrt
(3)*sqrt(2)*(x^12 + 2184*x^10 - 211215*x^8 + 94152*x^6 - 13581*x^4 + 432*x^
2 + 27) + 12*(sqrt(6)*sqrt(3)*sqrt(2)*(x^10 - 107*x^8 - 7262*x^6 + 2322*x^4
 - 243*x^2 + 9) - 48*sqrt(3)*(5*x^9 - 245*x^7 + 183*x^5 - 15*x^3))*(-3*x^2
+ 1)^(2/3) - 12*sqrt(3)*(29*x^11 + 293*x^9 - 2670*x^7 + 4986*x^5 - 1215*x^3
 + 81*x) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(49*x^10 - 5043*x^8 + 3658*x^6 + 378*
x^4 - 171*x^2 + 9) - 2*sqrt(3)*(x^11 + 917*x^9 - 40566*x^7 + 15786*x^5 - 20
43*x^3 + 81*x))*(-3*x^2 + 1)^(1/3))*sqrt((x^6 - 93*x^4 + 4*sqrt(6)*sqrt(2)*
(x^5 + 13*x^3) - 117*x^2 - 2*(4*sqrt(6)*sqrt(2)*x^3 - 3*x^4 - 18*x^2 + 9)*(
-3*x^2 + 1)^(2/3) + (6*x^4 - sqrt(6)*sqrt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^
2 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 12*(2*sqrt(6
)*sqrt(3)*sqrt(2)*(35*x^9 - 4860*x^7 + 2106*x^5 - 396*x^3 + 27*x) - 3*sqrt(
3)*(x^10 + 589*x^8 + 3946*x^6 - 774*x^4 - 27*x^2 + 9))*(-3*x^2 + 1)^(2/3) -
 3*sqrt(3)*(x^12 + 3150*x^10 + 77991*x^8 + 4260*x^6 - 14337*x^4 + 2862*x^2
- 135) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(x^11 - 1591*x^9 + 42426*x^7 - 15102*x^
5 + 1269*x^3 - 27*x) - 6*sqrt(3)*(27*x^10 + 2307*x^8 + 4574*x^6 - 2538*x^4
+ 279*x^2 - 9))*(-3*x^2 + 1)^(1/3))/(x^12 - 4986*x^10 + 327519*x^8 - 159660
*x^6 + 25839*x^4 - 2106*x^2 + 81)) - 1/72*sqrt(6)*sqrt(3)*sqrt(2)*arctan(1/
9*(36*sqrt(6)*sqrt(3)*sqrt(2)*(3*x^11 - 1117*x^9 + 3918*x^7 - 1866*x^5 + 25
5*x^3 - 9*x) + sqrt(3)*(sqrt(6)*sqrt(3)*sqrt(2)*(x^12 + 2184*x^10 - 211215*
x^8 + 94152*x^6 - 13581*x^4 + 432*x^2 + 27) + 12*(sqrt(6)*sqrt(3)*sqrt(2)*(
x^10 - 107*x^8 - 7262*x^6 + 2322*x^4 - 243*x^2 + 9) + 48*sqrt(3)*(5*x^9 - 2
45*x^7 + 183*x^5 - 15*x^3))*(-3*x^2 + 1)^(2/3) + 12*sqrt(3)*(29*x^11 + 293*
x^9 - 2670*x^7 + 4986*x^5 - 1215*x^3 + 81*x) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(
49*x^10 - 5043*x^8 + 3658*x^6 + 378*x^4 - 171*x^2 + 9) + 2*sqrt(3)*(x^11 +
917*x^9 - 40566*x^7 + 15786*x^5 - 2043*x^3 + 81*x))*(-3*x^2 + 1)^(1/3))*sqr
t((x^6 - 93*x^4 - 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3) - 117*x^2 + 2*(4*sqrt(6)
*sqrt(2)*x^3 + 3*x^4 + 18*x^2 - 9)*(-3*x^2 + 1)^(2/3) + (6*x^4 + sqrt(6)*sq
rt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 -
9*x^4 + 27*x^2 - 27)) + 12*(2*sqrt(6)*sqrt(3)*sqrt(2)*(35*x^9 - 4860*x^7 +
2106*x^5 - 396*x^3 + 27*x) + 3*sqrt(3)*(x^10 + 589*x^8 + 3946*x^6 - 774*x^4
 - 27*x^2 + 9))*(-3*x^2 + 1)^(2/3) + 3*sqrt(3)*(x^12 + 3150*x^10 + 77991*x^
8 + 4260*x^6 - 14337*x^4 + 2862*x^2 - 135) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(x^
11 - 1591*x^9 + 42426*x^7 - 15102*x^5 + 1269*x^3 - 27*x) + 6*sqrt(3)*(27*x^
10 + 2307*x^8 + 4574*x^6 - 2538*x^4 + 279*x^2 - 9))*(-3*x^2 + 1)^(1/3))/(x^
12 - 4986*x^10 + 327519*x^8 - 159660*x^6 + 25839*x^4 - 2106*x^2 + 81)) + 1/
288*sqrt(6)*sqrt(2)*log(12*(x^6 - 93*x^4 + 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3)
 - 117*x^2 - 2*(4*sqrt(6)*sqrt(2)*x^3 - 3*x^4 - 18*x^2 + 9)*(-3*x^2 + 1)^(2
/3) + (6*x^4 - sqrt(6)*sqrt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^
2 + 1)^(1/3) + 9)/(x^6 - 9*x^4 + 27*x^2 - 27)) - 1/288*sqrt(6)*sqrt(2)*log(
12*(x^6 - 93*x^4 - 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3) - 117*x^2 + 2*(4*sqrt(6
)*sqrt(2)*x^3 + 3*x^4 + 18*x^2 - 9)*(-3*x^2 + 1)^(2/3) + (6*x^4 + sqrt(6)*s
```

```
qrt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 -
 9*x^4 + 27*x^2 - 27)) + 1/72*sqrt(3)*log(-(x^12 + 2598*x^10 + 55143*x^8 +
114228*x^6 - 22113*x^4 - 7290*x^2 + 8*(3*x^10 + 576*x^8 + 5598*x^6 + 5832*x
^4 - 729*x^2 + sqrt(3)*(41*x^9 + 1368*x^7 + 4482*x^5 + 864*x^3 - 243*x))*(-
3*x^2 + 1)^(2/3) + 4*sqrt(3)*(25*x^11 + 2359*x^9 + 15426*x^7 + 6966*x^5 - 4
347*x^3 + 243*x) - 4*(84*x^10 + 4536*x^8 + 20880*x^6 + 5832*x^4 - 2916*x^2
+ sqrt(3)*(x^11 + 521*x^9 + 7362*x^7 + 10746*x^5 - 1971*x^3 - 243*x))*(-3*x
^2 + 1)^(1/3) + 729)/(x^12 - 18*x^10 + 135*x^8 - 540*x^6 + 1215*x^4 - 1458*
x^2 + 729))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-3\right)\left(-3\,x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(x^2-3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```

maple [C] time = 6.24, size = 643, normalized size = 3.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+1)^(1/3)/(x^2-3),x)
```

```
[Out] 48*RootOf(2304*_Z^4+48*_Z^2+1)^3*ln((-18432*(-3*x^2+1)^(1/3)*RootOf(2304*_Z
^4+48*_Z^2+1)^5*x+36864*RootOf(2304*_Z^4+48*_Z^2+1)^5*x-768*(-3*x^2+1)^(1/3
)*RootOf(2304*_Z^4+48*_Z^2+1)^3*x+2304*RootOf(2304*_Z^4+48*_Z^2+1)^3*x+48*R
ootOf(2304*_Z^4+48*_Z^2+1)^2*x^2+96*RootOf(2304*_Z^4+48*_Z^2+1)^2*(-3*x^2+1
)^(1/3)+2*(-3*x^2+1)^(2/3)+48*RootOf(2304*_Z^4+48*_Z^2+1)^2+32*RootOf(2304*
_Z^4+48*_Z^2+1)*x+x^2+1)/(x^2-3))+RootOf(2304*_Z^4+48*_Z^2+1)*ln((-18432*(-
3*x^2+1)^(1/3)*RootOf(2304*_Z^4+48*_Z^2+1)^5*x+36864*RootOf(2304*_Z^4+48*_Z
^2+1)^5*x-768*(-3*x^2+1)^(1/3)*RootOf(2304*_Z^4+48*_Z^2+1)^3*x+2304*RootOf(
2304*_Z^4+48*_Z^2+1)^3*x+48*RootOf(2304*_Z^4+48*_Z^2+1)^2*x^2+96*RootOf(230
4*_Z^4+48*_Z^2+1)^2*(-3*x^2+1)^(1/3)+2*(-3*x^2+1)^(2/3)+48*RootOf(2304*_Z^4
+48*_Z^2+1)^2+32*RootOf(2304*_Z^4+48*_Z^2+1)*x+x^2+1)/(x^2-3))+RootOf(2304*
_Z^4+48*_Z^2+1)*ln((9216*(-3*x^2+1)^(1/3)*RootOf(2304*_Z^4+48*_Z^2+1)^5*x-1
8432*RootOf(2304*_Z^4+48*_Z^2+1)^5*x+576*(-3*x^2+1)^(1/3)*RootOf(2304*_Z^4+
48*_Z^2+1)^3*x-768*RootOf(2304*_Z^4+48*_Z^2+1)^3*x-24*RootOf(2304*_Z^4+48*_
Z^2+1)^2*x^2-48*RootOf(2304*_Z^4+48*_Z^2+1)^2*(-3*x^2+1)^(1/3)+8*(-3*x^2+1)
^(1/3)*RootOf(2304*_Z^4+48*_Z^2+1)*x+(-3*x^2+1)^(2/3)-24*RootOf(2304*_Z^4+4
8*_Z^2+1)^2-(-3*x^2+1)^(1/3))/(x^2-3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-3\right)\left(-3\,x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(x^2-3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2-3\right)\,{\left(1-3\,x^2\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 3)*(1 - 3*x^2)^(1/3)),x)
```

```
[Out] int(1/((x^2 - 3)*(1 - 3*x^2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(x^2-3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+1)**(1/3)/(x**2-3),x)
```

```
[Out] Integral(1/((1 - 3*x**2)**(1/3)*(x**2 - 3)), x)
```

3.1969 $\int \frac{-a+x}{\sqrt[3]{x^2(-a+x)}\left(a^2d-2adx+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=194

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{x-2\sqrt[6]{d}\sqrt[3]{x^3-ax^2}}\right)}{2ad^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[6]{d}\sqrt[3]{x^3-ax^2}+x}\right)}{2ad^{5/6}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{d}\sqrt[3]{x^3-ax^2}}\right)}{ad^{5/6}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{d}\left(x^3-ax^2\right)^{2/3}+\frac{x^2}{\sqrt[6]{d}}}{x\sqrt[3]{x^3-ax^2}}\right)}{2ad^{5/6}}$$

Rubi [B] time = 0.65, antiderivative size = 418, normalized size of antiderivative = 2.15, number of steps used = 9, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {6719, 911, 105, 59, 91}

$$-\frac{x^{2/3}\sqrt[3]{x-a}\log\left(-2a\sqrt{d}\left(\sqrt{d}+1\right)-2(1-d)x\right)}{4ad^{5/6}\sqrt[3]{-\left(x^2(a-x)\right)}}+\frac{x^{2/3}\sqrt[3]{x-a}\log\left(2(1-d)x-2a\left(1-\sqrt{d}\right)\sqrt{d}\right)}{4ad^{5/6}\sqrt[3]{-\left(x^2(a-x)\right)}}-\frac{3x^{2/3}\sqrt[3]{x-a}\log\left(-\sqrt[6]{d}\sqrt[3]{x-a}-\sqrt[3]{x}\right)}{4ad^{5/6}\sqrt[3]{-\left(x^2(a-x)\right)}}+\frac{3x^{2/3}\sqrt[3]{x-a}\log\left(\sqrt[6]{d}\sqrt[3]{x-a}-\sqrt[3]{x}\right)}{4ad^{5/6}\sqrt[3]{-\left(x^2(a-x)\right)}}-\frac{\sqrt{3}x^{2/3}\sqrt[3]{x-a}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{x}}\right)}{2ad^{5/6}\sqrt[3]{-\left(x^2(a-x)\right)}}+\frac{\sqrt{3}x^{2/3}\sqrt[3]{x-a}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{2ad^{5/6}\sqrt[3]{-\left(x^2(a-x)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + x)/((x^2*(-a + x))^(1/3)*(a^2*d - 2*a*d*x + (-1 + d)*x^2)),x]
```

```
[Out] -1/2*(Sqrt[3]*x^(2/3)*(-a + x)^(1/3)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(-a + x)
^(1/3))/(Sqrt[3]*x^(1/3))])/(a*d^(5/6)*(-((a - x)*x^2))^(1/3)) + (Sqrt[3]*x
^(2/3)*(-a + x)^(1/3)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(-a + x)^(1/3))/(Sqrt[3
]*x^(1/3))])/(2*a*d^(5/6)*(-((a - x)*x^2))^(1/3)) - (x^(2/3)*(-a + x)^(1/3)
*Log[-2*a*(1 + Sqrt[d])*Sqrt[d] - 2*(1 - d)*x])/(4*a*d^(5/6)*(-((a - x)*x^2
))^(1/3)) + (x^(2/3)*(-a + x)^(1/3)*Log[-2*a*(1 - Sqrt[d])*Sqrt[d] + 2*(1 -
 d)*x])/(4*a*d^(5/6)*(-((a - x)*x^2))^(1/3)) - (3*x^(2/3)*(-a + x)^(1/3)*Lo
g[-x^(1/3) - d^(1/6)*(-a + x)^(1/3)])/(4*a*d^(5/6)*(-((a - x)*x^2))^(1/3))
+ (3*x^(2/3)*(-a + x)^(1/3)*Log[-x^(1/3) + d^(1/6)*(-a + x)^(1/3)])/(4*a*d^
(5/6)*(-((a - x)*x^2))^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
```

```
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[m] && !
IntegerQ[n]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{-a+x}{\sqrt[3]{x^2(-a+x)}\left(a^2d-2adx+(-1+d)x^2\right)}dx = \frac{\left(x^{2/3}\sqrt[3]{-a+x}\right)\int \frac{(-a+x)^{2/3}}{x^{2/3}\left(a^2d-2adx+(-1+d)x^2\right)}dx}{\sqrt[3]{x^2(-a+x)}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{-a+x}\right)\int \left(\frac{(-1+d)(-a+x)^{2/3}}{a\sqrt{d}\,x^{2/3}\left(-2a\sqrt{d}-2ad-2(1-d)x\right)} + \frac{(-1+d)(-a}{a\sqrt{d}\,x^{2/3}\left(-2a\sqrt{d}+\right.}\right.}{\sqrt[3]{x^2(-a+x)}}$$

$$= -\frac{\left((1-d)x^{2/3}\sqrt[3]{-a+x}\right)\int \frac{(-a+x)^{2/3}}{x^{2/3}\left(-2a\sqrt{d}-2ad-2(1-d)x\right)}dx}{a\sqrt{d}\sqrt[3]{x^2(-a+x)}} - \frac{\left((1-d)\right.}{}$$

$$= \frac{\left((1-d)x^{2/3}\sqrt[3]{-a+x}\right)\int \frac{1}{x^{2/3}\sqrt[3]{-a+x}\left(-2a\sqrt{d}-2ad-2(1-d)x\right)}dx}{\left(1-\sqrt{d}\right)\sqrt{d}\sqrt[3]{x^2(-a+x)}} + \frac{\left((1\right.}{}$$

$$= -\frac{\sqrt{3}x^{2/3}\sqrt[3]{-a+x}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[3]{-a+x}}{\sqrt{3}\sqrt[3]{x}}\right)}{2ad^{5/6}\sqrt[3]{-\left((a-x)x^2\right)}} + \frac{\sqrt{3}x^{2/3}\sqrt[3]{-a+}}{2ad^{5}}$$

Mathematica [C] time = 0.15, size = 69, normalized size = 0.36

$$-\frac{3x\left({}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{x}{\sqrt{d}(a-x)}\right)+{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{x}{\sqrt{d}(x-a)}\right)\right)}{2ad\sqrt[3]{x^2(x-a)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + x)/((x^2*(-a + x))^(1/3)*(a^2*d - 2*a*d*x + (-1 + d)*x^2)),
x]
```

```
[Out] (-3*x*(Hypergeometric2F1[1/3, 1, 4/3, x/(Sqrt[d]*(a - x))] + Hypergeometric
2F1[1/3, 1, 4/3, x/(Sqrt[d]*(-a + x))]))/(2*a*d*(x^2*(-a + x))^(1/3))
```

IntegrateAlgebraic [A] time = 0.59, size = 194, normalized size = 1.00

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{x-2\sqrt[6]{d}\sqrt[3]{x^3-ax^2}}\right)}{2ad^{5/6}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[6]{d}\sqrt[3]{x^3-ax^2}+x}\right)}{2ad^{5/6}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{d}\sqrt[3]{x^3-ax^2}}\right)}{ad^{5/6}} - \frac{\tanh^{-1}\left(\frac{\sqrt[6]{d}\left(x^3-ax^2\right)^{2/3}+\frac{x^2}{\sqrt[6]{d}}}{x\sqrt[3]{x^3-ax^2}}\right)}{2ad^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + x)/((x^2*(-a + x))^(1/3)*(a^2*d - 2*a*d*x + (-1 +
d)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*d^(1/6)*(-(a*x^2) + x^3)^(1/3))])/(2*a*d
^(5/6)) - (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*d^(1/6)*(-(a*x^2) + x^3)^(1/3)
)])/(2*a*d^(5/6)) - ArcTanh[x/(d^(1/6)*(-(a*x^2) + x^3)^(1/3))]/(a*d^(5/6))
 - ArcTanh[(x^2/d^(1/6) + d^(1/6)*(-(a*x^2) + x^3)^(2/3))/(x*(-(a*x^2) + x^
3)^(1/3))]/(2*a*d^(5/6))
```

fricas [B] time = 0.61, size = 535, normalized size = 2.76

$$-\sqrt{3}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{3}\,adx\sqrt{\frac{(-ax^2+x^3)^{\frac{1}{3}}a^5d^4x\left(\frac{1}{a^6d^5}\right)^{\frac{5}{6}}+a^4d^3x^2\left(\frac{1}{a^6d^5}\right)^{\frac{2}{3}}+(-ax^2+x^3)^{\frac{2}{3}}}{x^2}}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}-2\sqrt{3}\left(-ax^2+x^3\right)^{\frac{1}{3}}ad\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}-\sqrt{3}\,x}{3\,x}\right)-\sqrt{3}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{3}\,adx\sqrt{-\frac{(-ax^2+x^3)^{\frac{1}{3}}a^5d^4x\left(\frac{1}{a^6d^5}\right)^{\frac{5}{6}}-a^4d^3x^2\left(\frac{1}{a^6d^5}\right)^{\frac{2}{3}}-(-ax^2+x^3)^{\frac{2}{3}}}{x^2}}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}-2\sqrt{3}\left(-ax^2+x^3\right)^{\frac{1}{3}}ad\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}+\sqrt{3}\,x}{3\,x}\right)-\frac{1}{2}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\log\left(\frac{a^5d^4x\left(\frac{1}{a^6d^5}\right)^{\frac{5}{6}}+\left(-ax^2+x^3\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{2}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\log\left(-\frac{a^5d^4x\left(\frac{1}{a^6d^5}\right)^{\frac{5}{6}}-\left(-ax^2+x^3\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{4}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\log\left(\frac{(-ax^2+x^3)^{\frac{1}{3}}a^5d^4x\left(\frac{1}{a^6d^5}\right)^{\frac{5}{6}}+a^4d^3x^2\left(\frac{1}{a^6d^5}\right)^{\frac{2}{3}}+(-ax^2+x^3)^{\frac{2}{3}}}{x^2}\right)+\frac{1}{4}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\log\left(-\frac{(-ax^2+x^3)^{\frac{1}{3}}a^5d^4x\left(\frac{1}{a^6d^5}\right)^{\frac{5}{6}}-a^4d^3x^2\left(\frac{1}{a^6d^5}\right)^{\frac{2}{3}}-(-ax^2+x^3)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)/(x^2*(-a+x))^(1/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x, algorithm
="fricas")
```

```
[Out] -sqrt(3)*(1/(a^6*d^5))^(1/6)*arctan(1/3*(2*sqrt(3)*a*d*x*sqrt(((-a*x^2 + x^
3)^(1/3)*a^5*d^4*x*(1/(a^6*d^5))^(5/6) + a^4*d^3*x^2*(1/(a^6*d^5))^(2/3) +
(-a*x^2 + x^3)^(2/3))/x^2)*(1/(a^6*d^5))^(1/6) - 2*sqrt(3)*(-a*x^2 + x^3)^(
1/3)*a*d*(1/(a^6*d^5))^(1/6) - sqrt(3)*x)/x) - sqrt(3)*(1/(a^6*d^5))^(1/6)*
arctan(1/3*(2*sqrt(3)*a*d*x*sqrt(-((-a*x^2 + x^3)^(1/3)*a^5*d^4*x*(1/(a^6*d
^5))^(5/6) - a^4*d^3*x^2*(1/(a^6*d^5))^(2/3) - (-a*x^2 + x^3)^(2/3))/x^2)*(
1/(a^6*d^5))^(1/6) - 2*sqrt(3)*(-a*x^2 + x^3)^(1/3)*a*d*(1/(a^6*d^5))^(1/6)
 + sqrt(3)*x)/x) - 1/2*(1/(a^6*d^5))^(1/6)*log((a^5*d^4*x*(1/(a^6*d^5))^(5/
6) + (-a*x^2 + x^3)^(1/3))/x) + 1/2*(1/(a^6*d^5))^(1/6)*log(-(a^5*d^4*x*(1/
(a^6*d^5))^(5/6) - (-a*x^2 + x^3)^(1/3))/x) - 1/4*(1/(a^6*d^5))^(1/6)*log((
(-a*x^2 + x^3)^(1/3)*a^5*d^4*x*(1/(a^6*d^5))^(5/6) + a^4*d^3*x^2*(1/(a^6*d^
5))^(2/3) + (-a*x^2 + x^3)^(2/3))/x^2) + 1/4*(1/(a^6*d^5))^(1/6)*log(-((-a*
x^2 + x^3)^(1/3)*a^5*d^4*x*(1/(a^6*d^5))^(5/6) - a^4*d^3*x^2*(1/(a^6*d^5))^
(2/3) - (-a*x^2 + x^3)^(2/3))/x^2)
```

giac [A] time = 5.28, size = 234, normalized size = 1.21

$$-\frac{\left(-\frac{1}{d}\right)^{\frac{5}{6}}\arctan\left(\frac{\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}}{\left(-\frac{1}{d}\right)^{\frac{1}{6}}}\right)}{a}-\frac{\sqrt{3}\log\left(\sqrt{3}\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}\left(-\frac{1}{d}\right)^{\frac{1}{6}}+\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{d}\right)^{\frac{1}{3}}\right)}{4\left(-d^5\right)^{\frac{1}{6}}a}+\frac{\sqrt{3}\log\left(-\sqrt{3}\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}\left(-\frac{1}{d}\right)^{\frac{1}{6}}+\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{d}\right)^{\frac{1}{3}}\right)}{4\left(-d^5\right)^{\frac{1}{6}}a}+\frac{\arctan\left(\frac{\sqrt{3}\left(-\frac{1}{d}\right)^{\frac{1}{6}}+2\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}}{\left(-\frac{1}{d}\right)^{\frac{1}{6}}}\right)}{2\left(-d^5\right)^{\frac{1}{6}}a}+\frac{\arctan\left(-\frac{\sqrt{3}\left(-\frac{1}{d}\right)^{\frac{1}{6}}-2\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}}{\left(-\frac{1}{d}\right)^{\frac{1}{6}}}\right)}{2\left(-d^5\right)^{\frac{1}{6}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)/(x^2*(-a+x))^(1/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x, algorithm
="giac")
```

```
[Out] -(-1/d)^(5/6)*arctan((-a/x + 1)^(1/3)/(-1/d)^(1/6))/a - 1/4*sqrt(3)*log(sqr
t(3)*(-a/x + 1)^(1/3)*(-1/d)^(1/6) + (-a/x + 1)^(2/3) + (-1/d)^(1/3))/((-d^
5)^(1/6)*a) + 1/4*sqrt(3)*log(-sqrt(3)*(-a/x + 1)^(1/3)*(-1/d)^(1/6) + (-a/
x + 1)^(2/3) + (-1/d)^(1/3))/((-d^5)^(1/6)*a) + 1/2*arctan((sqrt(3)*(-1/d)^
(1/6) + 2*(-a/x + 1)^(1/3))/(-1/d)^(1/6))/((-d^5)^(1/6)*a) + 1/2*arctan(-(s
qrt(3)*(-1/d)^(1/6) - 2*(-a/x + 1)^(1/3))/(-1/d)^(1/6))/((-d^5)^(1/6)*a)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{-a+x}{\left(x^{2}\left(-a+x\right)\right)^{\frac{1}{3}}\left(a^{2}d-2adx+\left(-1+d\right)x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x)/(x^2*(-a+x))^(1/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x)
```

```
[Out] int((-a+x)/(x^2*(-a+x))^(1/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a-x}{\left(a^{2}d-2\,adx+(d-1)x^{2}\right)\left(-(a-x)x^{2}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)/(x^2*(-a+x))^(1/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x, algorithm
="maxima")
```

```
[Out] -integrate((a - x)/((a^2*d - 2*a*d*x + (d - 1)*x^2)*(-(a - x)*x^2)^(1/3)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a-x}{\left(-x^2\,\left(a-x\right)\right)^{1/3}\,\left(d\,a^2-2\,d\,a\,x+\left(d-1\right)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a - x)/((-x^2*(a - x))^(1/3)*(a^2*d + x^2*(d - 1) - 2*a*d*x)),x)
```

```
[Out] int(-(a - x)/((-x^2*(a - x))^(1/3)*(a^2*d + x^2*(d - 1) - 2*a*d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)/(x**2*(-a+x))**(1/3)/(a**2*d-2*a*d*x+(-1+d)*x**2),x)
```

```
[Out] Timed out
```

3.1970 $\int \frac{b-ax^4+2x^8}{\sqrt[4]{-b+ax^4}\left(b+3ax^4\right)}\,dx$

Optimal. Leaf size=194

$$\frac{x\left(ax^4-b\right)^{3/4}}{6a^2}+\frac{\left(-6a^2-b\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{36a^{9/4}}+\frac{\left(6a^2+b\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{9\sqrt{2}\,a^{9/4}}+\frac{\left(-6a^2-b\right)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{36a^{9/4}}+\frac{\left(6a^2+b\right)\text{ta}}{9\sqrt{}}$$

Rubi [A] time = 0.43, antiderivative size = 256, normalized size of antiderivative = 1.32, number of steps used = 15, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {6725, 240, 212, 206, 203, 321, 377}

$$\frac{b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{12a^{9/4}}+\frac{b\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{12a^{9/4}}+\frac{x\left(ax^4-b\right)^{3/4}}{6a^2}-\frac{\left(3a^2+2b\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{18a^{9/4}}+\frac{\left(6a^2+b\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{9\sqrt{2}\,a^{9/4}}-\frac{\left(3a^2+2b\right)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{18a^{9/4}}+\frac{\left(6a^2+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{9\sqrt{2}\,a^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b - a*x^4 + 2*x^8)/((-b + a*x^4)^(1/4)*(b + 3*a*x^4)),x]
```

```
[Out] (x*(-b + a*x^4)^(3/4))/(6*a^2) + (b*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])
/(12*a^(9/4)) - ((3*a^2 + 2*b)*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(18*
a^(9/4)) + ((6*a^2 + b)*ArcTan[(Sqrt[2]*a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(9*
Sqrt[2]*a^(9/4)) + (b*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(12*a^(9/4))
 - ((3*a^2 + 2*b)*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(18*a^(9/4)) + (
(6*a^2 + b)*ArcTanh[(Sqrt[2]*a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(9*Sqrt[2]*a^(
9/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{b-ax^4+2x^8}{\sqrt[4]{-b+ax^4}\left(b+3ax^4\right)}\,dx = \int \left(\frac{-3-\frac{2b}{a^2}}{9\sqrt[4]{-b+ax^4}} + \frac{2x^4}{3a\sqrt[4]{-b+ax^4}} + \frac{2\left(6a^2b+b^2\right)}{9a^2\sqrt[4]{-b+ax^4}\left(b+3ax^4\right)}\right) dx$$

$$= \frac{2\int \frac{x^4}{\sqrt[4]{-b+ax^4}}\,dx}{3a} + \frac{\left(2b\left(6a^2+b\right)\right)\int \frac{1}{\sqrt[4]{-b+ax^4}\left(b+3ax^4\right)}\,dx}{9a^2} + \frac{1}{9}\left(-3-\frac{2b}{a^2}\right)\int \frac{1}{\sqrt[4]{-b}\ldots}$$

$$= \frac{x\left(-b+ax^4\right)^{3/4}}{6a^2} + \frac{b\int \frac{1}{\sqrt[4]{-b+ax^4}}\,dx}{6a^2} + \frac{\left(2b\left(6a^2+b\right)\right)\text{Subst}\left(\int \frac{1}{b-4abx^4}\,dx, x, \frac{x}{\sqrt[4]{-b\ldots}}\ldots\right)}{9a^2}\ldots$$

$$= \frac{x\left(-b+ax^4\right)^{3/4}}{6a^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{6a^2} + \frac{\left(6a^2+b\right)\text{Subst}\left(\int \frac{1}{1-2\sqrt{a}\ldots}\ldots\right)}{9a^2}\ldots$$

$$= \frac{x\left(-b+ax^4\right)^{3/4}}{6a^2} - \frac{\left(3a^2+2b\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{18a^{9/4}} + \frac{\left(6a^2+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{9\sqrt{2}\,a^{9/4}} - \ldots$$

$$= \frac{x\left(-b+ax^4\right)^{3/4}}{6a^2} + \frac{b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{12a^{9/4}} - \frac{\left(3a^2+2b\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{18a^{9/4}} + \frac{\left(6a^2+b\ldots\right)}{\ldots}$$

Mathematica [A] time = 0.17, size = 221, normalized size = 1.14

$$\frac{-6a^2\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+12\sqrt{2}\,a^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)-\left(6a^2+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+2\sqrt{2}\left(6a^2+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+6\sqrt[4]{a}\,x\left(ax^4-b\right)^{3/4}-b\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)+2\sqrt{2}\,b\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{36a^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b - a*x^4 + 2*x^8)/((-b + a*x^4)^(1/4)*(b + 3*a*x^4)),x]
```

```
[Out] (6*a^(1/4)*x*(-b + a*x^4)^(3/4) - 6*a^2*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/
4)] - b*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)] + 12*Sqrt[2]*a^2*ArcTan[(Sqr
t[2]*a^(1/4)*x)/(-b + a*x^4)^(1/4)] + 2*Sqrt[2]*b*ArcTan[(Sqrt[2]*a^(1/4)*x
)/(-b + a*x^4)^(1/4)] - (6*a^2 + b)*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)]
 + 2*Sqrt[2]*(6*a^2 + b)*ArcTanh[(Sqrt[2]*a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(
36*a^(9/4))
```

IntegrateAlgebraic [A] time = 1.17, size = 194, normalized size = 1.00

$$\frac{x\left(ax^4-b\right)^{3/4}}{6a^2}+\frac{\left(-6a^2-b\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{36a^{9/4}}+\frac{\left(6a^2+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{9\sqrt{2}\,a^{9/4}}+\frac{\left(-6a^2-b\right)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{36a^{9/4}}+\frac{\left(6a^2+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{9\sqrt{2}\,a^{9/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b - a*x^4 + 2*x^8)/((-b + a*x^4)^(1/4)*(b + 3*a*x^4)),x
]
```

```
[Out] (x*(-b + a*x^4)^(3/4))/(6*a^2) + ((-6*a^2 - b)*ArcTan[(a^(1/4)*x)/(-b + a*x
^4)^(1/4)])/(36*a^(9/4)) + ((6*a^2 + b)*ArcTan[(Sqrt[2]*a^(1/4)*x)/(-b + a*
x^4)^(1/4)])/(9*Sqrt[2]*a^(9/4)) + ((-6*a^2 - b)*ArcTanh[(a^(1/4)*x)/(-b +
a*x^4)^(1/4)])/(36*a^(9/4)) + ((6*a^2 + b)*ArcTanh[(Sqrt[2]*a^(1/4)*x)/(-b
+ a*x^4)^(1/4)])/(9*Sqrt[2]*a^(9/4))
```

fricas [B] time = 0.66, size = 1231, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-a*x^4+b)/(a*x^4-b)^(1/4)/(3*a*x^4+b),x, algorithm="fricas"
)
```

```
[Out] 1/72*(16*(1/4)^(1/4)*a^2*((1296*a^8 + 864*a^6*b + 216*a^4*b^2 + 24*a^2*b^3
+ b^4)/a^9)^(1/4)*arctan(((1/4)^(1/4)*a^2*x*sqrt((2*(1296*a^13 + 864*a^11*b
 + 216*a^9*b^2 + 24*a^7*b^3 + a^5*b^4)*x^2*sqrt((1296*a^8 + 864*a^6*b + 216
*a^4*b^2 + 24*a^2*b^3 + b^4)/a^9) + (46656*a^12 + 46656*a^10*b + 19440*a^8*
b^2 + 4320*a^6*b^3 + 540*a^4*b^4 + 36*a^2*b^5 + b^6)*sqrt(a*x^4 - b))/x^2)*
((1296*a^8 + 864*a^6*b + 216*a^4*b^2 + 24*a^2*b^3 + b^4)/a^9)^(1/4) - (1/4)
^(1/4)*(216*a^8 + 108*a^6*b + 18*a^4*b^2 + a^2*b^3)*(a*x^4 - b)^(1/4)*((129
6*a^8 + 864*a^6*b + 216*a^4*b^2 + 24*a^2*b^3 + b^4)/a^9)^(1/4))/((1296*a^8
+ 864*a^6*b + 216*a^4*b^2 + 24*a^2*b^3 + b^4)*x)) + 4*(1/4)^(1/4)*a^2*((129
6*a^8 + 864*a^6*b + 216*a^4*b^2 + 24*a^2*b^3 + b^4)/a^9)^(1/4)*log((4*(1/4)
^(3/4)*a^7*x*((1296*a^8 + 864*a^6*b + 216*a^4*b^2 + 24*a^2*b^3 + b^4)/a^9)^
(3/4) + (216*a^6 + 108*a^4*b + 18*a^2*b^2 + b^3)*(a*x^4 - b)^(1/4))/x) - 4*
(1/4)^(1/4)*a^2*((1296*a^8 + 864*a^6*b + 216*a^4*b^2 + 24*a^2*b^3 + b^4)/a^
9)^(1/4)*log(-(4*(1/4)^(3/4)*a^7*x*((1296*a^8 + 864*a^6*b + 216*a^4*b^2 + 2
4*a^2*b^3 + b^4)/a^9)^(3/4) - (216*a^6 + 108*a^4*b + 18*a^2*b^2 + b^3)*(a*x
^4 - b)^(1/4))/x) - 4*a^2*((1296*a^8 + 864*a^6*b + 216*a^4*b^2 + 24*a^2*b^3
 + b^4)/a^9)^(1/4)*arctan((a^2*x*sqrt(((1296*a^13 + 864*a^11*b + 216*a^9*b^
2 + 24*a^7*b^3 + a^5*b^4)*x^2*sqrt((1296*a^8 + 864*a^6*b + 216*a^4*b^2 + 24
*a^2*b^3 + b^4)/a^9) + (46656*a^12 + 46656*a^10*b + 19440*a^8*b^2 + 4320*a^
6*b^3 + 540*a^4*b^4 + 36*a^2*b^5 + b^6)*sqrt(a*x^4 - b))/x^2)*((1296*a^8 +
864*a^6*b + 216*a^4*b^2 + 24*a^2*b^3 + b^4)/a^9)^(1/4) - (216*a^8 + 108*a^6
*b + 18*a^4*b^2 + a^2*b^3)*(a*x^4 - b)^(1/4)*((1296*a^8 + 864*a^6*b + 216*a
^4*b^2 + 24*a^2*b^3 + b^4)/a^9)^(1/4))/((1296*a^8 + 864*a^6*b + 216*a^4*b^2
 + 24*a^2*b^3 + b^4)*x)) - a^2*((1296*a^8 + 864*a^6*b + 216*a^4*b^2 + 24*a^
2*b^3 + b^4)/a^9)^(1/4)*log((a^7*x*((1296*a^8 + 864*a^6*b + 216*a^4*b^2 + 2
4*a^2*b^3 + b^4)/a^9)^(3/4) + (216*a^6 + 108*a^4*b + 18*a^2*b^2 + b^3)*(a*x
^4 - b)^(1/4))/x) + a^2*((1296*a^8 + 864*a^6*b + 216*a^4*b^2 + 24*a^2*b^3 +
 b^4)/a^9)^(1/4)*log(-(a^7*x*((1296*a^8 + 864*a^6*b + 216*a^4*b^2 + 24*a^2*
b^3 + b^4)/a^9)^(3/4) - (216*a^6 + 108*a^4*b + 18*a^2*b^2 + b^3)*(a*x^4 - b
)^(1/4))/x) + 12*(a*x^4 - b)^(3/4)*x)/a^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^{8} - ax^{4} + b}{\left(3\,ax^{4} + b\right)\left(ax^{4} - b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-a*x^4+b)/(a*x^4-b)^(1/4)/(3*a*x^4+b),x, algorithm="giac")
```

```
[Out] integrate((2*x^8 - a*x^4 + b)/((3*a*x^4 + b)*(a*x^4 - b)^(1/4)), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{2x^8 - a\,x^4 + b}{\left(a\,x^4 - b\right)^{\frac{1}{4}} \left(3a\,x^4 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8-a*x^4+b)/(a*x^4-b)^(1/4)/(3*a*x^4+b),x)
```

```
[Out] int((2*x^8-a*x^4+b)/(a*x^4-b)^(1/4)/(3*a*x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^8 - ax^4 + b}{\left(3\,ax^4 + b\right)\left(ax^4 - b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-a*x^4+b)/(a*x^4-b)^(1/4)/(3*a*x^4+b),x, algorithm="maxima"
)
```

```
[Out] integrate((2*x^8 - a*x^4 + b)/((3*a*x^4 + b)*(a*x^4 - b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x^8 - a\,x^4 + b}{\left(a\,x^4 - b\right)^{1/4}\,\left(3\,a\,x^4 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a*x^4 + 2*x^8)/((a*x^4 - b)^(1/4)*(b + 3*a*x^4)),x)
```

```
[Out] int((b - a*x^4 + 2*x^8)/((a*x^4 - b)^(1/4)*(b + 3*a*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-ax^4 + b + 2x^8}{\sqrt[4]{ax^4 - b}\,\left(3ax^4 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8-a*x**4+b)/(a*x**4-b)**(1/4)/(3*a*x**4+b),x)
```

```
[Out] Integral((-a*x**4 + b + 2*x**8)/((a*x**4 - b)**(1/4)*(3*a*x**4 + b)), x)
```

3.1971 $\int \frac{d+cx^2}{\sqrt{ax+\sqrt{b^2+a^2x^2}}} \, dx$

Optimal. Leaf size=194

$$\frac{2\sqrt{a^2x^2+b^2}\left(12a^4cx^5+60a^4dx^3-3a^2b^2cx^3+25a^2b^2dx-4b^4cx\right)}{15a^2\left(\sqrt{a^2x^2+b^2}+ax\right)^{7/2}}+\frac{2\left(84a^6cx^6+420a^6dx^4+21a^4b^2cx^4+385a^4\right.}{105a^3\left(\sqrt{a^2x^2+b\right.}$$

Rubi [A] time = 0.32, antiderivative size = 196, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {6742, 2117, 14, 2119, 448}

$$-\frac{b^2d}{3a\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}+\frac{d\sqrt{\sqrt{a^2x^2+b^2}+ax}}{a}-\frac{b^2c\sqrt{\sqrt{a^2x^2+b^2}+ax}}{4a^3}+\frac{c\left(\sqrt{a^2x^2+b^2}+ax\right)^{5/2}}{20a^3}-\frac{b^6c}{28a^3\left(\sqrt{a^2x^2+b^2}+ax\right)^{7/2}}+\frac{b^4c}{12a^3\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*x^2)/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] -1/28*(b^6*c)/(a^3*(a*x + Sqrt[b^2 + a^2*x^2])^(7/2)) + (b^4*c)/(12*a^3*(a*
x + Sqrt[b^2 + a^2*x^2])^(3/2)) - (b^2*d)/(3*a*(a*x + Sqrt[b^2 + a^2*x^2])^
(3/2)) - (b^2*c*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/(4*a^3) + (d*Sqrt[a*x + Sq
rt[b^2 + a^2*x^2]])/a + (c*(a*x + Sqrt[b^2 + a^2*x^2])^(5/2))/(20*a^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{d+cx^2}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx = \int \left(\frac{d}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}+\frac{cx^2}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}\right)dx$$

$$= c\int \frac{x^2}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx + d\int \frac{1}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$$

$$= \frac{c\,\text{Subst}\left(\int \frac{(-b^2+x^2)^2(b^2+x^2)}{x^{9/2}}\,dx, x, ax+\sqrt{b^2+a^2x^2}\right)}{8a^3} + \frac{d\,\text{Subst}\left(\int \frac{b^2+x^2}{x^{5/2}}\,dx, x, ax+ \right.}{2a}$$

$$= \frac{c\,\text{Subst}\left(\int \left(\frac{b^6}{x^{9/2}}-\frac{b^4}{x^{5/2}}-\frac{b^2}{\sqrt{x}}+x^{3/2}\right)dx, x, ax+\sqrt{b^2+a^2x^2}\right)}{8a^3} + \frac{d\,\text{Subst}\left(\int \left(\frac{b^2}{x^{5/2}}+\right.\right.}{}$$

$$= -\frac{b^6c}{28a^3\left(ax+\sqrt{b^2+a^2x^2}\right)^{7/2}} + \frac{b^4c}{12a^3\left(ax+\sqrt{b^2+a^2x^2}\right)^{3/2}} - \frac{b^2d}{3a\left(ax+\sqrt{b^2+a^2x^2}\right.}$$

Mathematica [B] time = 7.39, size = 952, normalized size = 4.91

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*x^2)/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] (2*Sqrt[b^2 + a^2*x^2]*((-21*a^2*d*(a*x + Sqrt[b^2 + a^2*x^2])^2*(2*b^4 + 6
*a^3*x^3*(a*x + Sqrt[b^2 + a^2*x^2]) + 3*a*b^2*x*(2*a*x + Sqrt[b^2 + a^2*x^
2])))/(b^2 + a*x*(a*x + Sqrt[b^2 + a^2*x^2])) + (c*(16*b^8 - 280*a^7*x^7*(a
*x + Sqrt[b^2 + a^2*x^2]) + 14*a*b^6*x*(7*a*x + 4*Sqrt[b^2 + a^2*x^2]) - 28
*a^5*b^2*x^5*(11*a*x + 6*Sqrt[b^2 + a^2*x^2]) + 7*a^3*b^4*x^3*(4*a*x + 11*S
qrt[b^2 + a^2*x^2])))/(b^2 + a*x*(a*x + Sqrt[b^2 + a^2*x^2])) + (5*c*(a*x +
 Sqrt[b^2 + a^2*x^2])^2*(b^2 + 2*a*x*(a*x + Sqrt[b^2 + a^2*x^2]))^4*(-8*b^8
 + 7*a^3*b^4*x^3*(a*x - 4*Sqrt[b^2 + a^2*x^2]) + 56*a^7*x^7*(a*x + Sqrt[b^2
 + a^2*x^2]) + 28*a^5*b^2*x^5*(4*a*x + 3*Sqrt[b^2 + a^2*x^2]) - 7*a*b^6*x*(
7*a*x + 4*Sqrt[b^2 + a^2*x^2])))/(b^12 + 1024*a^11*x^11*(a*x + Sqrt[b^2 + a
^2*x^2]) + 256*a^9*b^2*x^9*(13*a*x + 11*Sqrt[b^2 + a^2*x^2]) + 256*a^7*b^4*
x^7*(16*a*x + 11*Sqrt[b^2 + a^2*x^2]) + 112*a^5*b^6*x^5*(21*a*x + 11*Sqrt[b
^2 + a^2*x^2]) + 20*a^3*b^8*x^3*(31*a*x + 11*Sqrt[b^2 + a^2*x^2]) + a*b^10*
x*(61*a*x + 11*Sqrt[b^2 + a^2*x^2])) + (21*a^2*d*(b^2 + 2*a*x*(a*x + Sqrt[b
^2 + a^2*x^2]))^5*(7*b^4 + 6*a^3*x^3*(a*x + Sqrt[b^2 + a^2*x^2]) + 3*a*b^2*
x*(7*a*x + 6*Sqrt[b^2 + a^2*x^2])))/(b^10 + 256*a^9*x^9*(a*x + Sqrt[b^2 + a
^2*x^2]) + 40*a^3*b^6*x^3*(7*a*x + 3*Sqrt[b^2 + a^2*x^2]) + 64*a^7*b^2*x^7*
(11*a*x + 9*Sqrt[b^2 + a^2*x^2]) + a*b^8*x*(41*a*x + 9*Sqrt[b^2 + a^2*x^2])
 + 16*a^5*b^4*x^5*(43*a*x + 27*Sqrt[b^2 + a^2*x^2]))))/(315*a^3*b^2*(a*x +
Sqrt[b^2 + a^2*x^2])^(5/2))
```

IntegrateAlgebraic [A] time = 0.30, size = 194, normalized size = 1.00

$$\frac{2\sqrt{a^2x^2+b^2}\left(12a^4cx^5+60a^4dx^3-3a^2b^2cx^3+25a^2b^2dx-4b^4cx\right)}{15a^2\left(\sqrt{a^2x^2+b^2}+ax\right)^{7/2}}+\frac{2\left(84a^6cx^6+420a^6dx^4+21a^4b^2cx^4+385a^4b^2dx^2-49a^2b^4cx^2+35a^2b^4d-8b^6c\right)}{105a^3\left(\sqrt{a^2x^2+b^2}+ax\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + c*x^2)/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] (2*Sqrt[b^2 + a^2*x^2]*(-4*b^4*c*x + 25*a^2*b^2*d*x - 3*a^2*b^2*c*x^3 + 60*
a^4*d*x^3 + 12*a^4*c*x^5))/(15*a^2*(a*x + Sqrt[b^2 + a^2*x^2])^(7/2)) + (2*
```

```
(-8*b^6*c + 35*a^2*b^4*d - 49*a^2*b^4*c*x^2 + 385*a^4*b^2*d*x^2 + 21*a^4*b^
2*c*x^4 + 420*a^6*d*x^4 + 84*a^6*c*x^6))/(105*a^3*(a*x + Sqrt[b^2 + a^2*x^2
])^(7/2))
```

fricas [A] time = 0.47, size = 112, normalized size = 0.58

$$-\frac{2\left(15\,a^{4}cx^{4}+8\,b^{4}c-35\,a^{2}b^{2}d+\left(a^{2}b^{2}c+35\,a^{4}d\right)x^{2}-\left(15\,a^{3}cx^{3}+\left(4\,ab^{2}c+35\,a^{3}d\right)x\right)\sqrt{a^{2}x^{2}+b^{2}}\right)\sqrt{ax+\sqrt{a^{2}x^{2}+b^{2}}}}{105\,a^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/105*(15*a^4*c*x^4 + 8*b^4*c - 35*a^2*b^2*d + (a^2*b^2*c + 35*a^4*d)*x^2
- (15*a^3*c*x^3 + (4*a*b^2*c + 35*a^3*d)*x)*sqrt(a^2*x^2 + b^2))*sqrt(a*x +
 sqrt(a^2*x^2 + b^2))/(a^3*b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^{2}+d}{\sqrt{ax+\sqrt{a^{2}x^{2}+b^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + d)/sqrt(a*x + sqrt(a^2*x^2 + b^2)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{c\,x^{2}+d}{\sqrt{ax+\sqrt{a^{2}x^{2}+b^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] int((c*x^2+d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^{2}+d}{\sqrt{ax+\sqrt{a^{2}x^{2}+b^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + d)/sqrt(a*x + sqrt(a^2*x^2 + b^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,x^{2}+d}{\sqrt{a\,x+\sqrt{a^{2}\,x^{2}+b^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + c*x^2)/(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int((d + c*x^2)/(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^2 + d}{\sqrt{ax + \sqrt{a^2x^2 + b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+d)/(a*x+(a**2*x**2+b**2)**(1/2))**(1/2),x)
[Out] Integral((c*x**2 + d)/sqrt(a*x + sqrt(a**2*x**2 + b**2)), x)
```

3.1972 $\int \frac{\sqrt[4]{bx^3+ax^4}\left(-c+dx^8\right)}{x^4}\,dx$

Optimal. Leaf size=195

$$\frac{1463b^6d\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{32768a^{23/4}}-\frac{1463b^6d\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{32768a^{23/4}}+\frac{\sqrt[4]{ax^4+bx^3}\left(-262144a^7cx^2+65536a^6bcx+327680a^5b\right.}{}$$

Rubi [A] time = 0.75, antiderivative size = 341, normalized size of antiderivative = 1.75, number of steps used = 16, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {2052, 2016, 2014, 2021, 2024, 2032, 63, 331, 298, 203, 206}

$$\frac{1463b^6dx^{9/4}(ax+b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{32768a^{23/4}\left(ax^4+bx^3\right)^{3/4}}-\frac{1463b^6dx^{9/4}(ax+b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{32768a^{23/4}\left(ax^4+bx^3\right)^{3/4}}+\frac{1463b^5d\sqrt[4]{ax^4+bx^3}}{49152a^5}-\frac{209b^4dx\sqrt[4]{ax^4+bx^3}}{12288a^4}+\frac{19b^3dx^2\sqrt[4]{ax^4+bx^3}}{1536a^3}-\frac{19b^2dx^3\sqrt[4]{ax^4+bx^3}}{1920a^2}-\frac{16ac\left(ax^4+bx^3\right)^{5/4}}{45b^2x^5}+\frac{4c\left(ax^4+bx^3\right)^{5/4}}{9bx^6}+\frac{bdx^4\sqrt[4]{ax^4+bx^3}}{120a}+\frac{1}{6}dx^5\sqrt[4]{ax^4+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[((b*x^3 + a*x^4)^(1/4)*(-c + d*x^8))/x^4,x]
```

```
[Out] (1463*b^5*d*(b*x^3 + a*x^4)^(1/4))/(49152*a^5) - (209*b^4*d*x*(b*x^3 + a*x^
4)^(1/4))/(12288*a^4) + (19*b^3*d*x^2*(b*x^3 + a*x^4)^(1/4))/(1536*a^3) - (
19*b^2*d*x^3*(b*x^3 + a*x^4)^(1/4))/(1920*a^2) + (b*d*x^4*(b*x^3 + a*x^4)^(
1/4))/(120*a) + (d*x^5*(b*x^3 + a*x^4)^(1/4))/6 + (4*c*(b*x^3 + a*x^4)^(5/4
))/(9*b*x^6) - (16*a*c*(b*x^3 + a*x^4)^(5/4))/(45*b^2*x^5) + (1463*b^6*d*x^
(9/4)*(b + a*x)^(3/4)*ArcTan[(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(32768*a^(
23/4)*(b*x^3 + a*x^4)^(3/4)) - (1463*b^6*d*x^(9/4)*(b + a*x)^(3/4)*ArcTanh[
(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(32768*a^(23/4)*(b*x^3 + a*x^4)^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\sqrt[4]{bx^3+ax^4}\left(-c+dx^8\right)}{x^4}\,dx = \int \left(-\frac{c\sqrt[4]{bx^3+ax^4}}{x^4}+dx^4\sqrt[4]{bx^3+ax^4}\right)dx$$

$$= -\left(c\int \frac{\sqrt[4]{bx^3+ax^4}}{x^4}\,dx\right)+d\int x^4\sqrt[4]{bx^3+ax^4}\,dx$$

$$= \frac{1}{6}dx^5\sqrt[4]{bx^3+ax^4}+\frac{4c\left(bx^3+ax^4\right)^{5/4}}{9bx^6}+\frac{(4ac)\int \frac{\sqrt[4]{bx^3+ax^4}}{x^3}\,dx}{9b}+\frac{1}{24}(bd)\int \frac{x}{\left(bx^3+\right.}$$

$$= \frac{bdx^4\sqrt[4]{bx^3+ax^4}}{120a}+\frac{1}{6}dx^5\sqrt[4]{bx^3+ax^4}+\frac{4c\left(bx^3+ax^4\right)^{5/4}}{9bx^6}-\frac{16ac\left(bx^3+ax^4\right)^{5/4}}{45b^2x^5}$$

$$= -\frac{19b^2dx^3\sqrt[4]{bx^3+ax^4}}{1920a^2}+\frac{bdx^4\sqrt[4]{bx^3+ax^4}}{120a}+\frac{1}{6}dx^5\sqrt[4]{bx^3+ax^4}+\frac{4c\left(bx^3+ax^4\right)^{5}}{9bx^6}$$

$$= \frac{19b^3dx^2\sqrt[4]{bx^3+ax^4}}{1536a^3}-\frac{19b^2dx^3\sqrt[4]{bx^3+ax^4}}{1920a^2}+\frac{bdx^4\sqrt[4]{bx^3+ax^4}}{120a}+\frac{1}{6}dx^5\sqrt[4]{bx^3+}$$

$$= -\frac{209b^4dx\sqrt[4]{bx^3+ax^4}}{12288a^4}+\frac{19b^3dx^2\sqrt[4]{bx^3+ax^4}}{1536a^3}-\frac{19b^2dx^3\sqrt[4]{bx^3+ax^4}}{1920a^2}+\frac{bdx^4\sqrt[4]{b}}{12}$$

$$= \frac{1463b^5d\sqrt[4]{bx^3+ax^4}}{49152a^5}-\frac{209b^4dx\sqrt[4]{bx^3+ax^4}}{12288a^4}+\frac{19b^3dx^2\sqrt[4]{bx^3+ax^4}}{1536a^3}-\frac{19b^2dx^3\sqrt[4]{}}{192}$$

$$= \frac{1463b^5d\sqrt[4]{bx^3+ax^4}}{49152a^5}-\frac{209b^4dx\sqrt[4]{bx^3+ax^4}}{12288a^4}+\frac{19b^3dx^2\sqrt[4]{bx^3+ax^4}}{1536a^3}-\frac{19b^2dx^3\sqrt[4]{}}{192}$$

$$= \frac{1463b^5d\sqrt[4]{bx^3+ax^4}}{49152a^5}-\frac{209b^4dx\sqrt[4]{bx^3+ax^4}}{12288a^4}+\frac{19b^3dx^2\sqrt[4]{bx^3+ax^4}}{1536a^3}-\frac{19b^2dx^3\sqrt[4]{}}{192}$$

$$= \frac{1463b^5d\sqrt[4]{bx^3+ax^4}}{49152a^5}-\frac{209b^4dx\sqrt[4]{bx^3+ax^4}}{12288a^4}+\frac{19b^3dx^2\sqrt[4]{bx^3+ax^4}}{1536a^3}-\frac{19b^2dx^3\sqrt[4]{}}{192}$$

$$= \frac{1463b^5d\sqrt[4]{bx^3+ax^4}}{49152a^5}-\frac{209b^4dx\sqrt[4]{bx^3+ax^4}}{12288a^4}+\frac{19b^3dx^2\sqrt[4]{bx^3+ax^4}}{1536a^3}-\frac{19b^2dx^3\sqrt[4]{}}{192}$$

$$= \frac{1463b^5d\sqrt[4]{bx^3+ax^4}}{49152a^5}-\frac{209b^4dx\sqrt[4]{bx^3+ax^4}}{12288a^4}+\frac{19b^3dx^2\sqrt[4]{bx^3+ax^4}}{1536a^3}-\frac{19b^2dx^3\sqrt[4]{}}{192}$$

Mathematica [C] time = 0.36, size = 323, normalized size = 1.66

$$\frac{4\sqrt[4]{x^3(ax+b)}\left(-4a^{10}cx^2\sqrt[4]{\frac{ax}{b}+1}+a^9bcx\sqrt[4]{\frac{ax}{b}+1}+5a^8b^2c\sqrt[4]{\frac{ax}{b}+1}+44a^2b^8dx^2\sqrt[4]{\frac{ax}{b}+1}-5b^{10}d\,{}_2F_1\left(-\frac{33}{4},-\frac{9}{4};-\frac{5}{4};-\frac{ax}{b}\right)+40b^{10}d\,{}_2F_1\left(-\frac{29}{4},-\frac{9}{4};-\frac{5}{4};-\frac{ax}{b}\right)-140b^{10}d\,{}_2F_1\left(-\frac{25}{4},-\frac{9}{4};-\frac{5}{4};-\frac{ax}{b}\right)+280b^{10}d\,{}_2F_1\left(-\frac{21}{4},-\frac{9}{4};-\frac{5}{4};-\frac{ax}{b}\right)-350b^{10}d\,{}_2F_1\left(-\frac{17}{4},-\frac{9}{4};-\frac{5}{4};-\frac{ax}{b}\right)+280b^{10}d\,{}_2F_1\left(-\frac{13}{4},-\frac{9}{4};-\frac{5}{4};-\frac{ax}{b}\right)-140b^{10}d\,{}_2F_1\left(-\frac{9}{4},-\frac{9}{4};-\frac{5}{4};-\frac{ax}{b}\right)+35b^{10}d\sqrt[4]{\frac{ax}{b}+1}+79ab^9dx\sqrt[4]{\frac{ax}{b}+1}\right)}{45a^8b^2x^3\sqrt[4]{\frac{ax}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*x^3 + a*x^4)^(1/4)*(-c + d*x^8))/x^4,x]
```

```
[Out] (4*(x^3*(b + a*x))^(1/4)*(5*a^8*b^2*c*(1 + (a*x)/b)^(1/4) + 35*b^10*d*(1 +
(a*x)/b)^(1/4) + a^9*b*c*x*(1 + (a*x)/b)^(1/4) + 79*a*b^9*d*x*(1 + (a*x)/b)
^(1/4) - 4*a^10*c*x^2*(1 + (a*x)/b)^(1/4) + 44*a^2*b^8*d*x^2*(1 + (a*x)/b)^
```

```
(1/4) - 5*b^10*d*Hypergeometric2F1[-33/4, -9/4, -5/4, -((a*x)/b)] + 40*b^10
*d*Hypergeometric2F1[-29/4, -9/4, -5/4, -((a*x)/b)] - 140*b^10*d*Hypergeome
tric2F1[-25/4, -9/4, -5/4, -((a*x)/b)] + 280*b^10*d*Hypergeometric2F1[-21/4
, -9/4, -5/4, -((a*x)/b)] - 350*b^10*d*Hypergeometric2F1[-17/4, -9/4, -5/4,
 -((a*x)/b)] + 280*b^10*d*Hypergeometric2F1[-13/4, -9/4, -5/4, -((a*x)/b)]
- 140*b^10*d*Hypergeometric2F1[-9/4, -9/4, -5/4, -((a*x)/b)]))/(45*a^8*b^2*
x^3*(1 + (a*x)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 2.12, size = 195, normalized size = 1.00

$$\frac{1463b^6d\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+bx^3}}\right)}{32768a^{23/4}}-\frac{1463b^6d\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+bx^3}}\right)}{32768a^{23/4}}+\frac{\sqrt[4]{ax^4+bx^3}\left(-262144a^7cx^2+65536a^6bcx+327680a^5b^2c+122880a^5b^2dx^8+6144a^4b^3dx^7-7296a^3b^4dx^6+9120a^2b^5dx^5-12540ab^6dx^4+21945b^7dx^3\right)}{737280a^5b^2x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b*x^3 + a*x^4)^(1/4)*(-c + d*x^8))/x^4,x]
```

```
[Out] ((b*x^3 + a*x^4)^(1/4)*(327680*a^5*b^2*c + 65536*a^6*b*c*x - 262144*a^7*c*x
^2 + 21945*b^7*d*x^3 - 12540*a*b^6*d*x^4 + 9120*a^2*b^5*d*x^5 - 7296*a^3*b^
4*d*x^6 + 6144*a^4*b^3*d*x^7 + 122880*a^5*b^2*d*x^8))/(737280*a^5*b^2*x^3)
+ (1463*b^6*d*ArcTan[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(32768*a^(23/4)) -
 (1463*b^6*d*ArcTanh[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(32768*a^(23/4))
```

fricas [B] time = 0.66, size = 385, normalized size = 1.97

$$\frac{263340\left(\frac{b^{24}d^4}{a^{23}}\right)^{\frac14}a^5b^2x^3\arctan\left(-\frac{\left(\frac{b^{24}d^4}{a^{23}}\right)^{\frac34}(ax^4+bx^3)^{\frac14}a^{17}b^6d-\left(\frac{b^{24}d^4}{a^{23}}\right)^{\frac34}a^{17}x\sqrt{\frac{\sqrt{ax^4+bx^3}\,b^{12}d^2+\sqrt{\frac{b^{24}d^4}{a^{23}}}\,a^{12}x^2}{x^2}}}{b^{24}d^4x}\right)-65835\left(\frac{b^{24}d^4}{a^{23}}\right)^{\frac14}a^5b^2x^3\log\left(\frac{1463\left((ax^4+bx^3)^{\frac14}b^6d+\left(\frac{b^{24}d^4}{a^{23}}\right)^{\frac14}a^6x\right)}{x}\right)+65835\left(\frac{b^{24}d^4}{a^{23}}\right)^{\frac14}a^5b^2x^3\log\left(\frac{1463\left((ax^4+bx^3)^{\frac14}b^6d-\left(\frac{b^{24}d^4}{a^{23}}\right)^{\frac14}a^6x\right)}{x}\right)+4\left(122880a^5b^2dx^8+6144a^4b^3dx^7-7296a^3b^4dx^6+9120a^2b^5dx^5-12540ab^6dx^4+21945b^7dx^3-262144a^7cx^2+65536a^6bcx+327680a^5b^2c\right)(ax^4+bx^3)^{\frac14}}{2949120a^5b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^3)^(1/4)*(d*x^8-c)/x^4,x, algorithm="fricas")
```

```
[Out] 1/2949120*(263340*(b^24*d^4/a^23)^(1/4)*a^5*b^2*x^3*arctan(-((b^24*d^4/a^23
)^(3/4)*(a*x^4 + b*x^3)^(1/4)*a^17*b^6*d - (b^24*d^4/a^23)^(3/4)*a^17*x*sqr
t((sqrt(a*x^4 + b*x^3)*b^12*d^2 + sqrt(b^24*d^4/a^23)*a^12*x^2)/x^2))/(b^24
*d^4*x)) - 65835*(b^24*d^4/a^23)^(1/4)*a^5*b^2*x^3*log(1463*((a*x^4 + b*x^3
)^(1/4)*b^6*d + (b^24*d^4/a^23)^(1/4)*a^6*x)/x) + 65835*(b^24*d^4/a^23)^(1/
4)*a^5*b^2*x^3*log(1463*((a*x^4 + b*x^3)^(1/4)*b^6*d - (b^24*d^4/a^23)^(1/4
)*a^6*x)/x) + 4*(122880*a^5*b^2*d*x^8 + 6144*a^4*b^3*d*x^7 - 7296*a^3*b^4*d
*x^6 + 9120*a^2*b^5*d*x^5 - 12540*a*b^6*d*x^4 + 21945*b^7*d*x^3 - 262144*a^
7*c*x^2 + 65536*a^6*b*c*x + 327680*a^5*b^2*c)*(a*x^4 + b*x^3)^(1/4))/(a^5*b
^2*x^3)
```

giac [B] time = 0.86, size = 359, normalized size = 1.84

$$\frac{\frac{131670\sqrt{2}b^7d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac14}+2\left(a+\frac{b}{x}\right)^{\frac14}\right)}{2(-a)^{\frac14}}\right)}{(-a)^{\frac34}a^5}+\frac{131670\sqrt{2}b^7d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac14}-2\left(a+\frac{b}{x}\right)^{\frac14}\right)}{2(-a)^{\frac14}}\right)}{(-a)^{\frac34}a^5}+\frac{65835\sqrt{2}b^7d\log\left(\sqrt{2}(-a)^{\frac14}\left(a+\frac{b}{x}\right)^{\frac14}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{(-a)^{\frac34}a^5}+\frac{65835\sqrt{2}(-a)^{\frac14}b^7d\log\left(-\sqrt{2}(-a)^{\frac14}\left(a+\frac{b}{x}\right)^{\frac14}+\sqrt{-a}+\sqrt{a+\frac{b}{x}}\right)}{a^6}+\frac{24\left(7315\left(a+\frac{b}{x}\right)^{\frac{21}{4}}b^7d-40755\left(a+\frac{b}{x}\right)^{\frac{17}{4}}ab^7d+92910\left(a+\frac{b}{x}\right)^{\frac{13}{4}}a^2b^7d-109782\left(a+\frac{b}{x}\right)^{\frac94}a^3b^7d+69327\left(a+\frac{b}{x}\right)^{\frac54}a^4b^7d+21945\left(a+\frac{b}{x}\right)^{\frac14}a^5b^7d\right)x^6}{a^5b^6}+\frac{524288\left(5\left(a+\frac{b}{x}\right)^{\frac94}b^8c-9\left(a+\frac{b}{x}\right)^{\frac54}ab^8c\right)}{b^9}}{5898240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^3)^(1/4)*(d*x^8-c)/x^4,x, algorithm="giac")
```

```
[Out] 1/5898240*(131670*sqrt(2)*b^7*d*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*
(a + b/x)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a^5) + 131670*sqrt(2)*b^7*d*arctan
(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(a + b/x)^(1/4))/(-a)^(1/4))/((-a)^(3
/4)*a^5) + 65835*sqrt(2)*b^7*d*log(sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqr
t(-a) + sqrt(a + b/x))/((-a)^(3/4)*a^5) + 65835*sqrt(2)*(-a)^(1/4)*b^7*d*lo
g(-sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a + b/x))/a^6 + 24*
(7315*(a + b/x)^(21/4)*b^7*d - 40755*(a + b/x)^(17/4)*a*b^7*d + 92910*(a +
b/x)^(13/4)*a^2*b^7*d - 109782*(a + b/x)^(9/4)*a^3*b^7*d + 69327*(a + b/x)^
(5/4)*a^4*b^7*d + 21945*(a + b/x)^(1/4)*a^5*b^7*d)*x^6/(a^5*b^6) + 524288*(
5*(a + b/x)^(9/4)*b^8*c - 9*(a + b/x)^(5/4)*a*b^8*c)/b^9)/b
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{4}+b\,x^{3}\right)^{\frac{1}{4}} \left(d\,x^{8}-c\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b*x^3)^(1/4)*(d*x^8-c)/x^4,x)
[Out] int((a*x^4+b*x^3)^(1/4)*(d*x^8-c)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^{8}-c\right)\left(ax^{4}+bx^{3}\right)^{\frac{1}{4}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b*x^3)^(1/4)*(d*x^8-c)/x^4,x, algorithm="maxima")
[Out] integrate((d*x^8 - c)*(a*x^4 + b*x^3)^(1/4)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(c-d\,x^{8}\right)\,\left(a\,x^{4}+b\,x^{3}\right)^{1/4}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((c - d*x^8)*(a*x^4 + b*x^3)^(1/4))/x^4,x)
[Out] int(-((c - d*x^8)*(a*x^4 + b*x^3)^(1/4))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^{3}\left(ax+b\right)}\,\left(-c+dx^{8}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b*x**3)**(1/4)*(d*x**8-c)/x**4,x)
[Out] Integral((x**3*(a*x + b))**(1/4)*(-c + d*x**8)/x**4, x)
```

3.1973 $\int \frac{\left(b^2+ax^2\right)^{5/2}}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$

Optimal. Leaf size=195

$$\frac{2x\sqrt{ax^2+b^2}\left(45a^2x^4+155ab^2x^2+247b^4\right)}{495\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{2x\left(5a^2bx^4+31ab^3x^2+247b^5\right)}{495\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{2\sqrt{2}\,b^{11/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\,\sqrt{b}\,\sqrt{\sqrt{ax^2+b^2}}}\right.}{\sqrt{a}}$$

Rubi [F] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(b^2+ax^2\right)^{5/2}}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(b^2 + a*x^2)^(5/2)/Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] Defer[Int][(b^2 + a*x^2)^(5/2)/Sqrt[b + Sqrt[b^2 + a*x^2]], x]
```

Rubi steps

$$\int \frac{\left(b^2+ax^2\right)^{5/2}}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx=\int \frac{\left(b^2+ax^2\right)^{5/2}}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Mathematica [C] time = 0.69, size = 388, normalized size = 1.99

$$\frac{2\left(90a^4x^8\sqrt{ax^2+b^2}+170a^4bx^8+718a^3b^3x^6+470a^3b^2x^6\sqrt{ax^2+b^2}+990a^2b^5x^4+990a^2b^4x^4\sqrt{ax^2+b^2}-1485ab^7x^2-1980b^8\sqrt{ax^2+b^2}-495ab^6x^2\sqrt{ax^2+b^2}-1980b^9\right)+495\sqrt{2}\,b^{11/2}\sqrt{\sqrt{ax^2+b^2}-b}\left(4b^2\sqrt{ax^2+b^2}+ax^2\sqrt{ax^2+b^2}+3abx^2+4b^3\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{ax^2+b^2}-b}}{\sqrt{2}\sqrt{b}}\right)+990b^6\left(4b^2\sqrt{ax^2+b^2}+ax^2\sqrt{ax^2+b^2}+3abx^2+4b^3\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b-\sqrt{b^2+ax^2}}{2b}\right)}{990ax\left(\sqrt{ax^2+b^2}+b\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2 + a*x^2)^(5/2)/Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] (2*(-1980*b^9 - 1485*a*b^7*x^2 + 990*a^2*b^5*x^4 + 718*a^3*b^3*x^6 + 170*a^
4*b*x^8 - 1980*b^8*Sqrt[b^2 + a*x^2] - 495*a*b^6*x^2*Sqrt[b^2 + a*x^2] + 99
0*a^2*b^4*x^4*Sqrt[b^2 + a*x^2] + 470*a^3*b^2*x^6*Sqrt[b^2 + a*x^2] + 90*a^
4*x^8*Sqrt[b^2 + a*x^2]) + 495*Sqrt[2]*b^(11/2)*Sqrt[-b + Sqrt[b^2 + a*x^2]
]*(4*b^3 + 3*a*b*x^2 + 4*b^2*Sqrt[b^2 + a*x^2] + a*x^2*Sqrt[b^2 + a*x^2])*A
rcTan[Sqrt[-b + Sqrt[b^2 + a*x^2]]/(Sqrt[2]*Sqrt[b])] + 990*b^6*(4*b^3 + 3*
a*b*x^2 + 4*b^2*Sqrt[b^2 + a*x^2] + a*x^2*Sqrt[b^2 + a*x^2])*Hypergeometric
2F1[-1/2, 1, 1/2, (b - Sqrt[b^2 + a*x^2])/(2*b)])/(990*a*x*(b + Sqrt[b^2 +
a*x^2])^(5/2))
```

IntegrateAlgebraic [A] time = 0.30, size = 162, normalized size = 0.83

$$\frac{2x\sqrt{ax^2+b^2}\left(45a^2x^4+155ab^2x^2+247b^4\right)}{495\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{2x\left(5a^2bx^4+31ab^3x^2+247b^5\right)}{495\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{\sqrt{2}\,b^{11/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\,\sqrt{b}\,\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^2 + a*x^2)^(5/2)/Sqrt[b + Sqrt[b^2 + a*x^2]],x]
```

```
[Out] (2*x*Sqrt[b^2 + a*x^2]*(247*b^4 + 155*a*b^2*x^2 + 45*a^2*x^4))/(495*Sqrt[b
+ Sqrt[b^2 + a*x^2]]) - (2*x*(247*b^5 + 31*a*b^3*x^2 + 5*a^2*b*x^4))/(495*S
qrt[b + Sqrt[b^2 + a*x^2]]) + (Sqrt[2]*b^(11/2)*ArcTan[(Sqrt[a]*x)/(Sqrt[2]
*Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/Sqrt[a]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^(5/2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2+b^2\right)^{\frac{5}{2}}}{\sqrt{b+\sqrt{ax^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^(5/2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((a*x^2 + b^2)^(5/2)/sqrt(b + sqrt(a*x^2 + b^2)), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^2+b^2\right)^{\frac{5}{2}}}{\sqrt{b+\sqrt{a\,x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b^2)^(5/2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x^2+b^2)^(5/2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2+b^2\right)^{\frac{5}{2}}}{\sqrt{b+\sqrt{ax^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^(5/2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((a*x^2 + b^2)^(5/2)/sqrt(b + sqrt(a*x^2 + b^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b^2+a\,x^2\right)^{5/2}}{\sqrt{b+\sqrt{b^2+a\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b^2)^(5/2)/(b + (a*x^2 + b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x^2 + b^2)^(5/2)/(b + (a*x^2 + b^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2+b^2\right)^{\frac{5}{2}}}{\sqrt{b+\sqrt{ax^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b**2)**(5/2)/(b+(a*x**2+b**2)**(1/2))**(1/2),x)
```

```
[Out] Integral((a*x**2 + b**2)**(5/2)/sqrt(b + sqrt(a*x**2 + b**2)), x)
```

3.1974 $\int \sqrt{x+\sqrt{1+x^2}} \sqrt{1+\sqrt{x+\sqrt{1+x^2}}} \, dx$

Optimal. Leaf size=195

$$\frac{\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\,(6x+16)+\sqrt{x^2+1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\,(60x-8)+6\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{}\right.}{105\sqrt{\sqrt{x^2+1}+x}}$$

Rubi [F] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{x+\sqrt{1+x^2}} \sqrt{1+\sqrt{x+\sqrt{1+x^2}}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]],x]
[Out] Defer[Int][Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]], x]
```

Rubi steps

$$\int \sqrt{x+\sqrt{1+x^2}} \sqrt{1+\sqrt{x+\sqrt{1+x^2}}} \, dx = \int \sqrt{x+\sqrt{1+x^2}} \sqrt{1+\sqrt{x+\sqrt{1+x^2}}} \, dx$$

Mathematica [F] time = 1.90, size = 0, normalized size = 0.00

$$\int \sqrt{x+\sqrt{1+x^2}} \sqrt{1+\sqrt{x+\sqrt{1+x^2}}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]],x]
[Out] Integrate[Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]], x]
```

IntegrateAlgebraic [A] time = 0.25, size = 195, normalized size = 1.00

$$\frac{\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\,(6x+16)+\sqrt{x^2+1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\,(60x-8)+6\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)+\left(60x^2-8x-75\right)\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}}{105\sqrt{\sqrt{x^2+1}+x}}-\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]
],x]
[Out] ((-75 - 8*x + 60*x^2)*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] + (16 + 6*x)*Sqrt[x
 + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] + Sqrt[1 + x^2]*((-8 +
60*x)*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] + 6*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1
+ Sqrt[x + Sqrt[1 + x^2]]]))/(105*Sqrt[x + Sqrt[1 + x^2]]) - ArcTanh[Sqrt[1
 + Sqrt[x + Sqrt[1 + x^2]]]]
```

fricas [A] time = 0.81, size = 98, normalized size = 0.50

$$\frac{1}{105}\left(\left(135x-75\sqrt{x^2+1}-8\right)\sqrt{x+\sqrt{x^2+1}}+6x+6\sqrt{x^2+1}+16\right)\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}-\frac{1}{2}\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}+1\right)+\frac{1}{2}\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+1)^(1/2))^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2),x, algo
rithm="fricas")
```

```
[Out] 1/105*((135*x - 75*sqrt(x^2 + 1) - 8)*sqrt(x + sqrt(x^2 + 1)) + 6*x + 6*sqr
t(x^2 + 1) + 16)*sqrt(sqrt(x + sqrt(x^2 + 1)) + 1) - 1/2*log(sqrt(sqrt(x +
sqrt(x^2 + 1)) + 1) + 1) + 1/2*log(sqrt(sqrt(x + sqrt(x^2 + 1)) + 1) - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+1)^(1/2))^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \sqrt{x+\sqrt{x^2+1}}\,\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+1)^(1/2))^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] int((x+(x^2+1)^(1/2))^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x+\sqrt{x^2+1}}\,\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+1)^(1/2))^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x^2 + 1))*sqrt(sqrt(x + sqrt(x^2 + 1)) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sqrt{x+\sqrt{x^2+1}}+1}\,\sqrt{x+\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2)*(x + (x^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int(((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2)*(x + (x^2 + 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x+\sqrt{x^2+1}}\,\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+1)**(1/2))**(1/2)*(1+(x+(x**2+1)**(1/2))**(1/2))**(1/2),
x)
```

```
[Out] Integral(sqrt(x + sqrt(x**2 + 1))*sqrt(sqrt(x + sqrt(x**2 + 1)) + 1), x)
```

3.1975 $\int \frac{1+x^6}{\sqrt[3]{x+x^5}\left(-1+x^6\right)}\,dx$

Optimal. Leaf size=196

$$-\frac{1}{3}\log\left(\sqrt[3]{x^5+x}+x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^5+x}-2x\right)}{6\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^5+x}-x}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^5+x}+x}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{1}{6}\log\left(-\sqrt[3]{x^5+x}x+\left(\right.\right.$$

Rubi [F] time = 1.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x^6}{\sqrt[3]{x+x^5}\left(-1+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^6)/((x + x^5)^(1/3)*(-1 + x^6)),x]
```

```
[Out] -((x*(1 + x^4)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6, (-2*x^4)/(1 - I*Sqrt[3]), -
x^4])/(x + x^5)^(1/3)) - (x*(1 + x^4)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6, (-2*
x^4)/(1 + I*Sqrt[3]), -x^4])/(x + x^5)^(1/3) - ((I - Sqrt[3])*x^3*(1 + x^4)
^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -x^4, (-2*x^4)/(1 - I*Sqrt[3])])/(4*(I +
Sqrt[3])*(x + x^5)^(1/3)) - ((I + Sqrt[3])*x^3*(1 + x^4)^(1/3)*AppellF1[2/3
, 1/3, 1, 5/3, -x^4, (-2*x^4)/(1 + I*Sqrt[3])])/(4*(I - Sqrt[3])*(x + x^5)^
(1/3)) + (3*x*(1 + x^4)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -x^4])/(2*(x
 + x^5)^(1/3)) + (x^(1/3)*(1 + x^4)^(1/3)*Defer[Subst][Defer[Int][1/((-1 +
x)*(1 + x^6)^(1/3)), x], x, x^(2/3)])/(3*(x + x^5)^(1/3)) - ((1 - I*Sqrt[3]
)*x^(1/3)*(1 + x^4)^(1/3)*Defer[Subst][Defer[Int][1/((1 - I*Sqrt[3] + 2*x)*
(1 + x^6)^(1/3)), x], x, x^(2/3)])/(3*(x + x^5)^(1/3)) - ((1 + I*Sqrt[3])*x
^(1/3)*(1 + x^4)^(1/3)*Defer[Subst][Defer[Int][1/((1 + I*Sqrt[3] + 2*x)*(1
+ x^6)^(1/3)), x], x, x^(2/3)])/(3*(x + x^5)^(1/3))
```

Rubi steps

$$\int \frac{1+x^6}{\sqrt[3]{x+x^5}\left(-1+x^6\right)}\,dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\int \frac{1+x^6}{\sqrt[3]{x}\sqrt[3]{1+x^4}\left(-1+x^6\right)}\,dx}{\sqrt[3]{x+x^5}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1+x^9}{\sqrt[3]{1+x^6}\left(-1+x^9\right)}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{x+x^5}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1+x^6}}+\frac{2}{\sqrt[3]{1+x^6}\left(-1+x^9\right)}\right)dx, x, x^{2/3}\right)}{2\sqrt[3]{x+x^5}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{x+x^5}} + \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^6}\left(-1+\right.}\right.}{\sqrt[3]{x+x^5}}$$

$$= \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} + \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{9(-1+x)\sqrt[3]{1+x^6}}+\frac{-2}{9\left(1+x+x^2\right.}\right.\right.}{\sqrt[3]{x+x^5}}$$

$$= \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} + \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{3\sqrt[3]{x+x^5}} +$$

$$= \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} + \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{3\sqrt[3]{x+x^5}} +$$

$$= \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} + \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{3\sqrt[3]{x+x^5}} +$$

$$= \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} + \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{3\sqrt[3]{x+x^5}} +$$

$$= \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} + \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{3\sqrt[3]{x+x^5}} +$$

$$= -\frac{x\sqrt[3]{1+x^4}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^4}{1-i\sqrt{3}},-x^4\right)}{\sqrt[3]{x+x^5}} - \frac{x\sqrt[3]{1+x^4}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^4}{1+i\sqrt{3}},-x^4\right)}{\sqrt[3]{x+x^5}} + 3$$

$$= -\frac{x\sqrt[3]{1+x^4}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^4}{1-i\sqrt{3}},-x^4\right)}{\sqrt[3]{x+x^5}} - \frac{x\sqrt[3]{1+x^4}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^4}{1+i\sqrt{3}},-x^4\right)}{\sqrt[3]{x+x^5}} - ($$

Mathematica [F] time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{1+x^6}{\sqrt[3]{x+x^5}\left(-1+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^6)/((x + x^5)^(1/3)*(-1 + x^6)),x]
[Out] Integrate[(1 + x^6)/((x + x^5)^(1/3)*(-1 + x^6)), x]
```

IntegrateAlgebraic [A] time = 1.21, size = 196, normalized size = 1.00

$$-\frac{1}{3}\log\left(\sqrt[3]{x^5+x}+x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^5+x}-2x\right)}{6\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^5+x}-x}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^5+x}+x}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{1}{6}\log\left(-\sqrt[3]{x^5+x}x+\left(x^5+x\right)^{2/3}+x^2\right)-\frac{\log\left(2^{2/3}\sqrt[3]{x^5+x}x+\sqrt[3]{2}\left(x^5+x\right)^{2/3}+2x^2\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^6)/((x + x^5)^(1/3)*(-1 + x^6)),x]
```

```
[Out] -(ArcTan[(Sqrt[3]*x)/(-x + 2*(x + x^5)^(1/3))]/Sqrt[3]) - ArcTan[(Sqrt[3]*x
)/(x + 2^(2/3)*(x + x^5)^(1/3))]/(2*2^(1/3)*Sqrt[3]) - Log[x + (x + x^5)^(1
/3)]/3 + Log[-2*x + 2^(2/3)*(x + x^5)^(1/3)]/(6*2^(1/3)) + Log[x^2 - x*(x +
 x^5)^(1/3) + (x + x^5)^(2/3)]/6 - Log[2*x^2 + 2^(2/3)*x*(x + x^5)^(1/3) +
2^(1/3)*(x + x^5)^(2/3)]/(12*2^(1/3))
```

fricas [B] time = 6.21, size = 368, normalized size = 1.88

$$-\frac{1}{36}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}\left(x^{8}+14x^{6}+6x^{4}+14x^{2}+1\right)\left(x^{5}+x\right)^{\frac{2}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(x^{12}-24x^{10}-57x^{8}-56x^{6}-57x^{4}-24x^{2}+1\right)-24\sqrt{6}\left(x^{9}-x^{7}-x^{3}+x\right)\left(x^{5}+x\right)^{\frac{1}{3}}\right)}{6\left(x^{12}+48x^{10}+15x^{8}+88x^{6}+15x^{4}+48x^{2}+1\right)}\right)-\frac{1}{72}\,2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}\left(x^{8}+14x^{6}+6x^{4}+14x^{2}+1\right)+12\cdot 2^{\frac{1}{3}}\left(x^{5}+x^{3}+x\right)\left(x^{5}+x\right)^{\frac{1}{3}}+6\left(x^{5}+x\right)^{\frac{2}{3}}\left(x^{4}+4x^{2}+1\right)}{x^{8}-4x^{6}+6x^{4}-4x^{2}+1}\right)+\frac{1}{36}\,2^{\frac{2}{3}}\log\left(\frac{3\cdot 2^{\frac{2}{3}}\left(x^{5}+x\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(x^{4}-2x^{2}+1\right)-6\left(x^{5}+x\right)^{\frac{1}{3}}x}{x^{4}-2x^{2}+1}\right)+\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(x^{5}+x\right)^{\frac{1}{3}}}{3x}\right)-\frac{1}{6}\log\left(\frac{x^{4}+x^{2}+3\left(x^{5}+x\right)^{\frac{1}{3}}x+3\left(x^{5}+x\right)^{\frac{2}{3}}+1}{x^{4}+x^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^5+x)^(1/3)/(x^6-1),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(x^8 + 14*x^6 +
 6*x^4 + 14*x^2 + 1)*(x^5 + x)^(2/3) - sqrt(6)*2^(1/3)*(x^12 - 24*x^10 - 57
*x^8 - 56*x^6 - 57*x^4 - 24*x^2 + 1) - 24*sqrt(6)*(x^9 - x^7 - x^3 + x)*(x^
5 + x)^(1/3))/(x^12 + 48*x^10 + 15*x^8 + 88*x^6 + 15*x^4 + 48*x^2 + 1)) - 1
/72*2^(2/3)*log((2^(2/3)*(x^8 + 14*x^6 + 6*x^4 + 14*x^2 + 1) + 12*2^(1/3)*(
x^5 + x^3 + x)*(x^5 + x)^(1/3) + 6*(x^5 + x)^(2/3)*(x^4 + 4*x^2 + 1))/(x^8
- 4*x^6 + 6*x^4 - 4*x^2 + 1)) + 1/36*2^(2/3)*log((3*2^(2/3)*(x^5 + x)^(2/3)
 - 2^(1/3)*(x^4 - 2*x^2 + 1) - 6*(x^5 + x)^(1/3)*x)/(x^4 - 2*x^2 + 1)) + 1/
3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(x^5 + x)^(1/3))/x) - 1/6*log(
(x^4 + x^2 + 3*(x^5 + x)^(1/3)*x + 3*(x^5 + x)^(2/3) + 1)/(x^4 + x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+1}{\left(x^6-1\right)\left(x^5+x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^5+x)^(1/3)/(x^6-1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 1)/((x^6 - 1)*(x^5 + x)^(1/3)), x)
```

maple [F] time = 33.10, size = 0, normalized size = 0.00

$$\int \frac{x^6+1}{\left(x^5+x\right)^{\frac{1}{3}}\left(x^6-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/(x^5+x)^(1/3)/(x^6-1),x)
```

```
[Out] int((x^6+1)/(x^5+x)^(1/3)/(x^6-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+1}{\left(x^6-1\right)\left(x^5+x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^5+x)^(1/3)/(x^6-1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + 1)/((x^6 - 1)*(x^5 + x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6+1}{\left(x^6-1\right)\left(x^5+x\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)/((x^6 - 1)*(x + x^5)^(1/3)),x)
```

```
[Out] int((x^6 + 1)/((x^6 - 1)*(x + x^5)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)\left(x^4-x^2+1\right)}{\sqrt[3]{x\left(x^4+1\right)}\,(x-1)\,(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/(x**5+x)**(1/3)/(x**6-1),x)
```

```
[Out] Integral((x**2 + 1)*(x**4 - x**2 + 1)/((x*(x**4 + 1))**(1/3)*(x - 1)*(x + 1
)*(x**2 - x + 1)*(x**2 + x + 1)), x)
```

3.1976 $\int \frac{b^8+a^8x^8}{\sqrt{b^4+a^4x^4}\left(-b^8+a^8x^8\right)}\,dx$

Optimal. Leaf size=196

$$-\frac{x}{2\sqrt{a^4x^4+b^4}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{a^4x^4+b^4}+a^2x^2+b^2}\right)}{2\sqrt{2}\,ab}+\frac{\tanh^{-1}\left(\frac{\sqrt{6-4\sqrt{2}}\,abx}{\sqrt{a^4x^4+b^4}+a^2x^2+b^2}\right)}{4\sqrt{2}\,ab}-\frac{\tanh^{-1}\left(\frac{\sqrt{6+4\sqrt{2}}\,abx}{\sqrt{a^4x^4+b^4}+a^2x^2+b^2}\right)}{4\sqrt{2}\,ab}$$

Rubi [A] time = 0.64, antiderivative size = 101, normalized size of antiderivative = 0.52, number of steps used = 16, number of rules used = 10, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6725, 220, 1404, 414, 523, 409, 1211, 1699, 206, 203}

$$-\frac{x}{2\sqrt{a^4x^4+b^4}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{a^4x^4+b^4}}\right)}{4\sqrt{2}\,ab}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{a^4x^4+b^4}}\right)}{4\sqrt{2}\,ab}$$

Antiderivative was successfully verified.

```
[In] Int[(b^8 + a^8*x^8)/(Sqrt[b^4 + a^4*x^4]*(-b^8 + a^8*x^8)),x]
```

```
[Out] -1/2*x/Sqrt[b^4 + a^4*x^4] - ArcTan[(Sqrt[2]*a*b*x)/Sqrt[b^4 + a^4*x^4]]/(4
*Sqrt[2]*a*b) - ArcTanh[(Sqrt[2]*a*b*x)/Sqrt[b^4 + a^4*x^4]]/(4*Sqrt[2]*a*b
)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1404

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{b^8+a^8x^8}{\sqrt{b^4+a^4x^4}\left(-b^8+a^8x^8\right)}dx = \int \left(\frac{1}{\sqrt{b^4+a^4x^4}}+\frac{2b^8}{\sqrt{b^4+a^4x^4}\left(-b^8+a^8x^8\right)}\right)dx$$

$$= \left(2b^8\right)\int \frac{1}{\sqrt{b^4+a^4x^4}\left(-b^8+a^8x^8\right)}dx+\int \frac{1}{\sqrt{b^4+a^4x^4}}dx$$

$$= \frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{2ab\sqrt{b^4+a^4x^4}}+\left(2b^8\right)\int \frac{1}{\left(-b^4+a^4x^4\right)\left(b^4+a^4x^4\right)}$$

$$= -\frac{x}{2\sqrt{b^4+a^4x^4}}+\frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{2ab\sqrt{b^4+a^4x^4}}+\frac{\int \frac{3a^4b^4-a^8x^4}{\left(-b^4+a^4x^4\right)\sqrt{b^4+}}}{2a^4}$$

$$= -\frac{x}{2\sqrt{b^4+a^4x^4}}+\frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{2ab\sqrt{b^4+a^4x^4}}-\frac{1}{2}\int \frac{1}{\sqrt{b^4+a^4x}}$$

$$= -\frac{x}{2\sqrt{b^4+a^4x^4}}+\frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{4ab\sqrt{b^4+a^4x^4}}-\frac{1}{2}\int \frac{}{\left(1-\frac{a^2x^2}{b^2}\right)}$$

$$= -\frac{x}{2\sqrt{b^4+a^4x^4}}+\frac{\left(b^2+a^2x^2\right)\sqrt{\frac{b^4+a^4x^4}{\left(b^2+a^2x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{ax}{b}\right)|\frac{1}{2}\right)}{4ab\sqrt{b^4+a^4x^4}}-2\left(\frac{1}{4}\int \frac{1}{\sqrt{b^4+a}}\right.$$

$$= -\frac{x}{2\sqrt{b^4+a^4x^4}}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{1-2a^2b^2x^2}dx,x,\frac{x}{\sqrt{b^4+a^4x^4}}\right)-\frac{1}{4}\text{Subst}\left(\int \frac{}{1}\right.$$

$$= -\frac{x}{2\sqrt{b^4+a^4x^4}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{b^4+a^4x^4}}\right)}{4\sqrt{2}\,ab}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{b^4+a^4x^4}}\right)}{4\sqrt{2}\,ab}$$

Mathematica [C] time = 0.58, size = 199, normalized size = 1.02

$$\frac{x\left(-\frac{5\left(a^4b^4x^4+b^8\right)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{a^4x^4}{b^4},\frac{a^4x^4}{b^4}\right)}{\left(b^4-a^4x^4\right)\left(5b^4F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{a^4x^4}{b^4},\frac{a^4x^4}{b^4}\right)+2a^4x^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-\frac{a^4x^4}{b^4},\frac{a^4x^4}{b^4}\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{a^4x^4}{b^4},\frac{a^4x^4}{b^4}\right)\right)\right)}-1\right)}{2\sqrt{a^4x^4+b^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b^8 + a^8*x^8)/(Sqrt[b^4 + a^4*x^4]*(-b^8 + a^8*x^8)),x]
```

```
[Out] (x*(-1 - (5*(b^8 + a^4*b^4*x^4)*AppellF1[1/4, -1/2, 1, 5/4, -((a^4*x^4)/b^4
), (a^4*x^4)/b^4])/((b^4 - a^4*x^4)*(5*b^4*AppellF1[1/4, -1/2, 1, 5/4, -((a
^4*x^4)/b^4), (a^4*x^4)/b^4] + 2*a^4*x^4*(2*AppellF1[5/4, -1/2, 2, 9/4, -((
a^4*x^4)/b^4), (a^4*x^4)/b^4] + AppellF1[5/4, 1/2, 1, 9/4, -((a^4*x^4)/b^4)
, (a^4*x^4)/b^4])))))/(2*Sqrt[b^4 + a^4*x^4])
```

IntegrateAlgebraic [A] time = 0.89, size = 114, normalized size = 0.58

$$-\frac{x}{2\sqrt{a^4x^4+b^4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{a^4x^4+b^4}}\right)}{4\sqrt{2}\,ab}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,abx}{\sqrt{a^4x^4+b^4}+a^2x^2+b^2}\right)}{2\sqrt{2}\,ab}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^8 + a^8*x^8)/(Sqrt[b^4 + a^4*x^4]*(-b^8 + a^8*x^8)),x
]
```

```
[Out] -1/2*x/Sqrt[b^4 + a^4*x^4] - ArcTan[(Sqrt[2]*a*b*x)/(b^2 + a^2*x^2 + Sqrt[b
^4 + a^4*x^4])]/(2*Sqrt[2]*a*b) - ArcTanh[(Sqrt[2]*a*b*x)/Sqrt[b^4 + a^4*x^
4]]/(4*Sqrt[2]*a*b)
```

fricas [A] time = 0.71, size = 159, normalized size = 0.81

$$-\frac{8\,\sqrt{a^4x^4+b^4}\,abx+2\,\sqrt{2}\left(a^4x^4+b^4\right)\arctan\left(\frac{\sqrt{2}\,abx}{\sqrt{a^4x^4+b^4}}\right)-\sqrt{2}\left(a^4x^4+b^4\right)\log\left(\frac{a^4x^4+2\,a^2b^2x^2+b^4-2\,\sqrt{2}\,\sqrt{a^4x^4+b^4}\,abx}{a^4x^4-2\,a^2b^2x^2+b^4}\right)}{16\left(a^5bx^4+ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^8*x^8+b^8)/(a^4*x^4+b^4)^(1/2)/(a^8*x^8-b^8),x, algorithm="fri
cas")
```

```
[Out] -1/16*(8*sqrt(a^4*x^4 + b^4)*a*b*x + 2*sqrt(2)*(a^4*x^4 + b^4)*arctan(sqrt(
2)*a*b*x/sqrt(a^4*x^4 + b^4)) - sqrt(2)*(a^4*x^4 + b^4)*log((a^4*x^4 + 2*a^
2*b^2*x^2 + b^4 - 2*sqrt(2)*sqrt(a^4*x^4 + b^4)*a*b*x)/(a^4*x^4 - 2*a^2*b^2
*x^2 + b^4)))/(a^5*b*x^4 + a*b^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^8x^8+b^8}{\left(a^8x^8-b^8\right)\sqrt{a^4x^4+b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^8*x^8+b^8)/(a^4*x^4+b^4)^(1/2)/(a^8*x^8-b^8),x, algorithm="gia
c")
```

```
[Out] integrate((a^8*x^8 + b^8)/((a^8*x^8 - b^8)*sqrt(a^4*x^4 + b^4)), x)
```

maple [C] time = 0.06, size = 595, normalized size = 3.04

$$\frac{\sqrt{1-\frac{ia^2x^2}{b^2}}\,\sqrt{1+\frac{ia^2x^2}{b^2}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{ia^2}{b^2}},i\right)}{\sqrt{\frac{ia^2}{b^2}}\,\sqrt{a^4x^4+b^4}}+\frac{b\left(-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\left(2a^2b^2x^2+2b^4\right)\sqrt{2}}{4\sqrt{b^4}\,\sqrt{a^4x^4+b^4}}\right)}{4\sqrt{b^4}}-\frac{a\sqrt{1-\frac{ia^2x^2}{b^2}}\,\sqrt{1+\frac{ia^2x^2}{b^2}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{ia^2}{b^2}},-i,\frac{\sqrt{-\frac{ia^2}{b^2}}}{\sqrt{\frac{ia^2}{b^2}}}\right)}{\sqrt{\frac{ia^2}{b^2}}\,b\sqrt{a^4x^4+b^4}}\right)}{4a}-\frac{\sqrt{1-\frac{ia^2x^2}{b^2}}\,\sqrt{1+\frac{ia^2x^2}{b^2}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{ia^2}{b^2}},i,\frac{\sqrt{-\frac{ia^2}{b^2}}}{\sqrt{\frac{ia^2}{b^2}}}\right)}{2\sqrt{\frac{ia^2}{b^2}}\,\sqrt{a^4x^4+b^4}}-b^4\left(\frac{x}{2b^4\sqrt{\left(x^4+\frac{b^4}{a^4}\right)a^4}}+\frac{\sqrt{1-\frac{ia^2x^2}{b^2}}\,\sqrt{1+\frac{ia^2x^2}{b^2}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{ia^2}{b^2}},i\right)}{2b^4\sqrt{\frac{ia^2}{b^2}}\,\sqrt{a^4x^4+b^4}}\right)-\frac{b\left(-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\left(2a^2b^2x^2+2b^4\right)\sqrt{2}}{4\sqrt{b^4}\,\sqrt{a^4x^4+b^4}}\right)}{4\sqrt{b^4}}+\frac{a\sqrt{1-\frac{ia^2x^2}{b^2}}\,\sqrt{1+\frac{ia^2x^2}{b^2}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{ia^2}{b^2}},-i,\frac{\sqrt{-\frac{ia^2}{b^2}}}{\sqrt{\frac{ia^2}{b^2}}}\right)}{\sqrt{\frac{ia^2}{b^2}}\,b\sqrt{a^4x^4+b^4}}\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^8*x^8+b^8)/(a^4*x^4+b^4)^(1/2)/(a^8*x^8-b^8),x)
```

```
[Out] 1/(I*a^2/b^2)^(1/2)*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/b^2*x^2)^(1/2)/(a^4*x^
4+b^4)^(1/2)*EllipticF(x*(I*a^2/b^2)^(1/2),I)+1/4*b/a*(-1/4*2^(1/2)/(b^4)^(
1/2)*arctanh(1/4*(2*a^2*b^2*x^2+2*b^4)*2^(1/2)/(b^4)^(1/2)/(a^4*x^4+b^4)^(1
/2))-1/(I*a^2/b^2)^(1/2)/b*a*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/b^2*x^2)^(1/2
)/(a^4*x^4+b^4)^(1/2)*EllipticPi(x*(I*a^2/b^2)^(1/2),-I,(-I*a^2/b^2)^(1/2)/
(I*a^2/b^2)^(1/2)))-1/2/(I*a^2/b^2)^(1/2)*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/
b^2*x^2)^(1/2)/(a^4*x^4+b^4)^(1/2)*EllipticPi(x*(I*a^2/b^2)^(1/2),I,(-I*a^2
/b^2)^(1/2)/(I*a^2/b^2)^(1/2))-b^4*(1/2/b^4*x/((x^4+b^4/a^4)*a^4)^(1/2)+1/2
/b^4/(I*a^2/b^2)^(1/2)*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/b^2*x^2)^(1/2)/(a^4
*x^4+b^4)^(1/2)*EllipticF(x*(I*a^2/b^2)^(1/2),I))-1/4*b/a*(-1/4*2^(1/2)/(b^
4)^(1/2)*arctanh(1/4*(2*a^2*b^2*x^2+2*b^4)*2^(1/2)/(b^4)^(1/2)/(a^4*x^4+b^4
)^(1/2))+1/(I*a^2/b^2)^(1/2)/b*a*(1-I*a^2/b^2*x^2)^(1/2)*(1+I*a^2/b^2*x^2)^
(1/2)/(a^4*x^4+b^4)^(1/2)*EllipticPi(x*(I*a^2/b^2)^(1/2),-I,(-I*a^2/b^2)^(1
/2)/(I*a^2/b^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^8x^8 + b^8}{\left(a^8x^8 - b^8\right)\sqrt{a^4x^4 + b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^8*x^8+b^8)/(a^4*x^4+b^4)^(1/2)/(a^8*x^8-b^8),x, algorithm="max
ima")
```

```
[Out] integrate((a^8*x^8 + b^8)/((a^8*x^8 - b^8)*sqrt(a^4*x^4 + b^4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a^8\,x^8 + b^8}{\sqrt{a^4\,x^4 + b^4}\,\left(b^8 - a^8\,x^8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^8 + a^8*x^8)/((b^4 + a^4*x^4)^(1/2)*(b^8 - a^8*x^8)),x)
```

```
[Out] int(-(b^8 + a^8*x^8)/((b^4 + a^4*x^4)^(1/2)*(b^8 - a^8*x^8)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^8x^8 + b^8}{(ax - b)\,(ax + b)\left(a^2x^2 + b^2\right)\left(a^4x^4 + b^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**8*x**8+b**8)/(a**4*x**4+b**4)**(1/2)/(a**8*x**8-b**8),x)
```

```
[Out] Integral((a**8*x**8 + b**8)/((a*x - b)*(a*x + b)*(a**2*x**2 + b**2)*(a**4*x
**4 + b**4)**(3/2)), x)
```

3.1977 $$\int \frac{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{x\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$$

Optimal. Leaf size=196

$$\frac{\left(-ax^2-2\right)\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{bx}+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}-\frac{\sqrt{a}\log\left(b\left(-\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)+\sqrt{2}\sqrt{a}\ldots\right.}{\sqrt{2}\,b\ldots}$$

Rubi [F] time = 1.71, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{x\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[-(a/b^2) + (a^2*x^2)/b^2]/(x*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2
*x^2)/b^2]]),x]
```

```
[Out] Defer[Int][Sqrt[-(a/b^2) + (a^2*x^2)/b^2]/(x*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2)
 + (a^2*x^2)/b^2]]), x]
```

Rubi steps

$$\int \frac{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{x\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx=\int \frac{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{x\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$$

Mathematica [A] time = 11.66, size = 384, normalized size = 1.96

$$\frac{\sqrt{x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(-8a\left(10a^2x^4+2ax^2\left(5bx\sqrt{\frac{a(ax^2-1)}{b^2}}-6\right)-7bx\sqrt{\frac{a(ax^2-1)}{b^2}}+2\right)-5\sqrt{2}\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(2a^2x^3+2ax\left(bx\sqrt{\frac{a(ax^2-1)}{b^2}}-1\right)-b\sqrt{\frac{a(ax^2-1)}{b^2}}\right)\tanh^{-1}\left(\frac{\sqrt{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}}{\sqrt{2}\,ax}\right)\right)+9\sqrt{2}\,a^{3/2}x\left(4a^2x^4+ax^2\left(4bx\sqrt{\frac{a(ax^2-1)}{b^2}}-5\right)-3bx\sqrt{\frac{a(ax^2-1)}{b^2}}+1\right)\sinh^{-1}\left(\frac{b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax}{\sqrt{a}}\right)}{8bx\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)^2\left(bx\sqrt{\frac{a(ax^2-1)}{b^2}}+ax^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(a/b^2) + (a^2*x^2)/b^2]/(x*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2)
+ (a^2*x^2)/b^2]]),x]
```

```
[Out] (9*Sqrt[2]*a^(3/2)*x*(1 + 4*a^2*x^4 - 3*b*x*Sqrt[(a*(-1 + a*x^2))/b^2] + a*
x^2*(-5 + 4*b*x*Sqrt[(a*(-1 + a*x^2))/b^2]))*ArcSinh[(a*x + b*Sqrt[(a*(-1 +
 a*x^2))/b^2])/Sqrt[a]] + Sqrt[x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*(-8*
a*(2 + 10*a^2*x^4 - 7*b*x*Sqrt[(a*(-1 + a*x^2))/b^2] + 2*a*x^2*(-6 + 5*b*x*
Sqrt[(a*(-1 + a*x^2))/b^2])) - 5*Sqrt[2]*Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*
x^2))/b^2])]*(2*a^2*x^3 - b*Sqrt[(a*(-1 + a*x^2))/b^2] + 2*a*x*(-1 + b*x*Sq
rt[(a*(-1 + a*x^2))/b^2]))*ArcTanh[Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/
b^2])]/(Sqrt[2]*a*x)]))/(8*b*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])^2*(-1 +
 a*x^2 + b*x*Sqrt[(a*(-1 + a*x^2))/b^2]))
```

IntegrateAlgebraic [A] time = 3.26, size = 196, normalized size = 1.00

$$\frac{(-ax^2-2)\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{bx}+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}-\frac{\sqrt{a}\log\left(b\left(-\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)+\sqrt{2}\sqrt{a}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}-ax\right)}{\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-(a/b^2) + (a^2*x^2)/b^2]/(x*Sqrt[a*x^2 + b*x*Sqrt[
-(a/b^2) + (a^2*x^2)/b^2]]),x]
```

```
[Out] ((-2 - a*x^2)*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]])/(b*x) + Sqr
t[-(a/b^2) + (a^2*x^2)/b^2]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]
] - (Sqrt[a]*Log[-(a*x) - b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2] + Sqrt[2]*Sqrt[a
]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]])/(Sqrt[2]*b)
```

fricas [A] time = 35.11, size = 295, normalized size = 1.51

$$\left[\frac{\sqrt{2}\sqrt{a}x\log\left(-4a^2x^2-4abx\sqrt{\frac{a^2x^2-a}{b^2}}-2\left(\sqrt{2}a^{\frac{3}{2}}x+\sqrt{2}\sqrt{a}b\sqrt{\frac{a^2x^2-a}{b^2}}\right)\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}+a\right)-4\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\left(ax^2-bx\sqrt{\frac{a^2x^2-a}{b^2}}+2\right)}{4bx},-\frac{\sqrt{2}\sqrt{-a}x\arctan\left(\frac{\sqrt{2}\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\sqrt{-a}}{2ax}\right)+2\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\left(ax^2-bx\sqrt{\frac{a^2x^2-a}{b^2}}+2\right)}{2bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b^2+a^2*x^2/b^2)^(1/2)/x/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2
))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(a)*x*log(-4*a^2*x^2 - 4*a*b*x*sqrt((a^2*x^2 - a)/b^2) -
2*(sqrt(2)*a^(3/2)*x + sqrt(2)*sqrt(a)*b*sqrt((a^2*x^2 - a)/b^2))*sqrt(a*x^
2 + b*x*sqrt((a^2*x^2 - a)/b^2)) + a) - 4*sqrt(a*x^2 + b*x*sqrt((a^2*x^2 -
a)/b^2))*(a*x^2 - b*x*sqrt((a^2*x^2 - a)/b^2) + 2))/(b*x), -1/2*(sqrt(2)*sq
rt(-a)*x*arctan(1/2*sqrt(2)*sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*sqrt(
-a)/(a*x)) + 2*sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*(a*x^2 - b*x*sqrt(
(a^2*x^2 - a)/b^2) + 2))/(b*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}}{\sqrt{ax^2+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,bx}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b^2+a^2*x^2/b^2)^(1/2)/x/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2
))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2*x^2/b^2 - a/b^2)/(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)
*b*x)*x), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{x\sqrt{a\,x^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a/b^2+a^2*x^2/b^2)^(1/2)/x/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/
2),x)
```

```
[Out] int((-a/b^2+a^2*x^2/b^2)^(1/2)/x/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/
2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}}{\sqrt{ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\,bx}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b^2+a^2*x^2/b^2)^(1/2)/x/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2
))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2/b^2 - a/b^2)/(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)
*b*x)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a^2\,x^2}{b^2} - \frac{a}{b^2}}}{x\,\sqrt{a\,x^2 + b\,x\,\sqrt{\frac{a^2\,x^2}{b^2} - \frac{a}{b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a^2*x^2)/b^2 - a/b^2)^(1/2)/(x*(a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(
1/2))^(1/2)),x)
```

```
[Out] int(((a^2*x^2)/b^2 - a/b^2)^(1/2)/(x*(a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(
1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a(ax^2-1)}{b^2}}}{x\sqrt{x\left(ax + b\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b**2+a**2*x**2/b**2)**(1/2)/x/(a*x**2+b*x*(-a/b**2+a**2*x**2/
b**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(a*x**2 - 1)/b**2)/(x*sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 -
 a/b**2)))), x)
```

3.1978 $\int \sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=196

$$\frac{\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\,(8x-15)+\sqrt{x^2+1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\,(48x-16)+8\sqrt{\sqrt{x^2+1}+x}\,\sqrt{\right.}{60\sqrt{x^2+1}+60x}$$

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]],x]
```

```
[Out] Defer[Int][Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]], x]
```

Rubi steps

$$\int \sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx = \int \sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Mathematica [A] time = 0.27, size = 179, normalized size = 0.91

$$\frac{1}{120}\left(48\left(\sqrt{\sqrt{x^2+1}+x}+1\right)^{5/2}-80\left(\sqrt{\sqrt{x^2+1}+x}+1\right)^{3/2}-\frac{30\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}}{\sqrt{\sqrt{x^2+1}+x}}-\frac{60\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}}{\sqrt{x^2+1}+x}-15\log\left(1-\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)+15\log\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]],x]
```

```
[Out] ((-60*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/(x + Sqrt[1 + x^2]) - (30*Sqrt[1 +
 Sqrt[x + Sqrt[1 + x^2]]])/Sqrt[x + Sqrt[1 + x^2]] - 80*(1 + Sqrt[x + Sqrt[
1 + x^2]])^(3/2) + 48*(1 + Sqrt[x + Sqrt[1 + x^2]])^(5/2) - 15*Log[1 - Sqrt
[1 + Sqrt[x + Sqrt[1 + x^2]]]] + 15*Log[1 + Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]
]]])/120
```

IntegrateAlgebraic [A] time = 0.20, size = 196, normalized size = 1.00

$$\frac{\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\,(8x-15)+\sqrt{x^2+1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\,(48x-16)+8\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)+\left(48x^2-16x-6\right)\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}}{60\sqrt{x^2+1}+60x}+\frac{1}{4}\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]],x]
```

```
[Out] ((-6 - 16*x + 48*x^2)*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] + (-15 + 8*x)*Sqrt[
x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] + Sqrt[1 + x^2]*((-16
+ 48*x)*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] + 8*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[
1 + Sqrt[x + Sqrt[1 + x^2]]]))/(60*x + 60*Sqrt[1 + x^2]) + ArcTanh[Sqrt[1 +
 Sqrt[x + Sqrt[1 + x^2]]]]/4
```

fricas [A] time = 0.55, size = 98, normalized size = 0.50

$$\frac{1}{60}\left(\left(15\,x-15\,\sqrt{x^2+1}+8\right)\sqrt{x+\sqrt{x^2+1}}+54\,x-6\,\sqrt{x^2+1}-16\right)\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}+\frac{1}{8}\,\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}+1\right)-\frac{1}{8}\,\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
[Out] 1/60*((15*x - 15*sqrt(x^2 + 1) + 8)*sqrt(x + sqrt(x^2 + 1)) + 54*x - 6*sqrt
(x^2 + 1) - 16)*sqrt(sqrt(x + sqrt(x^2 + 1)) + 1) + 1/8*log(sqrt(sqrt(x + s
qrt(x^2 + 1)) + 1) + 1) - 1/8*log(sqrt(sqrt(x + sqrt(x^2 + 1)) + 1) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sqrt(x + sqrt(x^2 + 1)) + 1), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{1+\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2),x)
[Out] int((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(x + sqrt(x^2 + 1)) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2),x)
[Out] int(((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x+(x**2+1)**(1/2))**(1/2))**(1/2),x)
[Out] Integral(sqrt(sqrt(x + sqrt(x**2 + 1)) + 1), x)
```

3.1979 $\int \frac{2+3x}{\sqrt[3]{4+3x^2}\left(-12+52x+9x^2\right)}\,dx$

Optimal. Leaf size=197

$$\frac{\log\left(14\sqrt[3]{3x^2+4}+3\sqrt[3]{14}\,x-10\sqrt[3]{14}\right)}{14\sqrt[3]{14}}-\frac{\log\left(9\,14^{2/3}x^2+196\left(3x^2+4\right)^{2/3}+\left(140\sqrt[3]{14}-42\sqrt[3]{14}\,x\right)\sqrt[3]{3x^2+4}-60\right.}{28\sqrt[3]{14}}$$

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2+3x}{\sqrt[3]{4+3x^2}\left(-12+52x+9x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2 + 3*x)/((4 + 3*x^2)^(1/3)*(-12 + 52*x + 9*x^2)),x]
```

```
[Out] Defer[Int][(2 + 3*x)/((4 + 3*x^2)^(1/3)*(-12 + 52*x + 9*x^2)), x]
```

Rubi steps

$$\int \frac{2+3x}{\sqrt[3]{4+3x^2}\left(-12+52x+9x^2\right)}\,dx=\int \frac{2+3x}{\sqrt[3]{4+3x^2}\left(-12+52x+9x^2\right)}\,dx$$

Mathematica [F] time = 1.22, size = 0, normalized size = 0.00

$$\int \frac{2+3x}{\sqrt[3]{4+3x^2}\left(-12+52x+9x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2 + 3*x)/((4 + 3*x^2)^(1/3)*(-12 + 52*x + 9*x^2)),x]
```

```
[Out] Integrate[(2 + 3*x)/((4 + 3*x^2)^(1/3)*(-12 + 52*x + 9*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.30, size = 197, normalized size = 1.00

$$\frac{\log\left(14\sqrt[3]{3x^2+4}+3\sqrt[3]{14}\,x-10\sqrt[3]{14}\right)}{14\sqrt[3]{14}}-\frac{\log\left(9\,14^{2/3}x^2+196\left(3x^2+4\right)^{2/3}+\left(140\sqrt[3]{14}-42\sqrt[3]{14}\,x\right)\sqrt[3]{3x^2+4}-60\,14^{2/3}x+100\,14^{2/3}\right)}{28\sqrt[3]{14}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{3x^2+4}}{\sqrt{3}}-\frac{\sqrt[3]{2}\sqrt{3}x}{7^{2/3}}+\frac{10\sqrt[3]{2}}{\sqrt{3}\,7^{2/3}}}{\sqrt[3]{3x^2+4}}\right)}{14\sqrt[3]{14}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x)/((4 + 3*x^2)^(1/3)*(-12 + 52*x + 9*x^2)),x]
```

```
[Out] -1/14*(Sqrt[3]*ArcTan[((10*2^(1/3))/(Sqrt[3]*7^(2/3)) - (2^(1/3)*Sqrt[3]*x)
/7^(2/3) + (4 + 3*x^2)^(1/3)/Sqrt[3])/(4 + 3*x^2)^(1/3)])/14^(1/3) + Log[-1
0*14^(1/3) + 3*14^(1/3)*x + 14*(4 + 3*x^2)^(1/3)]/(14*14^(1/3)) - Log[100*1
4^(2/3) - 60*14^(2/3)*x + 9*14^(2/3)*x^2 + (140*14^(1/3) - 42*14^(1/3)*x)*(
4 + 3*x^2)^(1/3) + 196*(4 + 3*x^2)^(2/3)]/(28*14^(1/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(3*x^2+4)^(1/3)/(9*x^2+52*x-12),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x+2}{\left(9x^2+52x-12\right)\left(3x^2+4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(3*x^2+4)^(1/3)/(9*x^2+52*x-12),x, algorithm="giac")
```

```
[Out] integrate((3*x + 2)/((9*x^2 + 52*x - 12)*(3*x^2 + 4)^(1/3)), x)
```

maple [C] time = 16.64, size = 1411, normalized size = 7.16

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)/(3*x^2+4)^(1/3)/(9*x^2+52*x-12),x)
```

```
[Out] 1/2*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2)*ln(-(-68411
000939778*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2)*RootO
f(_Z^3-196)^3*x^3-4149815744330613*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(_
Z^3-196)+9604*_Z^2)^2*RootOf(_Z^3-196)^2*x^3+1605300200763000*(3*x^2+4)^(2/
3)*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2)*RootOf(_Z^3-
196)^2*x-1216195572262720*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+
9604*_Z^2)*RootOf(_Z^3-196)^3*x^2-73774502121433120*RootOf(RootOf(_Z^3-196)
^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2)^2*RootOf(_Z^3-196)^2*x^2-5351000669210
000*(3*x^2+4)^(2/3)*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+9604*_
Z^2)*RootOf(_Z^3-196)^2+49141842880500*(3*x^2+4)^(1/3)*RootOf(_Z^3-196)^2*x
^2+15109141162947363*(3*x^2+4)^(1/3)*RootOf(_Z^3-196)*RootOf(RootOf(_Z^3-19
6)^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2)*x^2-2280366697992600*RootOf(RootOf(_
Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2)*RootOf(_Z^3-196)^3*x-138327191
477687100*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2)^2*Roo
tOf(_Z^3-196)^2*x-327612285870000*(3*x^2+4)^(1/3)*RootOf(_Z^3-196)^2*x-1007
27607752982420*(3*x^2+4)^(1/3)*RootOf(_Z^3-196)*RootOf(RootOf(_Z^3-196)^2+9
8*_Z*RootOf(_Z^3-196)+9604*_Z^2)*x+174517859540250*RootOf(_Z^3-196)*x^3+105
86264653904625*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2)*
x^3-6862160373772242*(3*x^2+4)^(2/3)*x+546020476450000*(3*x^2+4)^(1/3)*Root
Of(_Z^3-196)^2+167879346254970700*(3*x^2+4)^(1/3)*RootOf(_Z^3-196)*RootOf(R
ootOf(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2)-8525468910767164*RootOf
(_Z^3-196)*x^2-517155495866035894*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(_Z
^3-196)+9604*_Z^2)*x^2+22873867912574140*(3*x^2+4)^(2/3)+5817261984675000*R
ootOf(_Z^3-196)*x+352875488463487500*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf
(_Z^3-196)+9604*_Z^2)*x-15504011514569552*RootOf(_Z^3-196)-9404743418398611
92*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2))/(6+x)^2/(9*
x-2))+1/196*RootOf(_Z^3-196)*ln((-84690117231237*RootOf(RootOf(_Z^3-196)^2+
98*_Z*RootOf(_Z^3-196)+9604*_Z^2)*RootOf(_Z^3-196)^3*x^3-13408556184196488*
RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2)^2*RootOf(_Z^3-1
96)^2*x^3+3210600401526000*(3*x^2+4)^(2/3)*RootOf(RootOf(_Z^3-196)^2+98*_Z*
RootOf(_Z^3-196)+9604*_Z^2)*RootOf(_Z^3-196)^2*x-1505602084110880*RootOf(Ro
otOf(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2)*RootOf(_Z^3-196)^3*x^2-2
38374332163493120*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^
2)^2*RootOf(_Z^3-196)^2*x^2-10702001338420000*(3*x^2+4)^(2/3)*RootOf(RootOf
```

```
(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2)*RootOf(_Z^3-196)^2+982836857
61000*(3*x^2+4)^(1/3)*RootOf(_Z^3-196)^2*x^2-20586481121316726*(3*x^2+4)^(1
/3)*RootOf(_Z^3-196)*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+9604*
_Z^2)*x^2-2823003907707900*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)
+9604*_Z^2)*RootOf(_Z^3-196)^3*x-446951872806549600*RootOf(RootOf(_Z^3-196)
^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2)^2*RootOf(_Z^3-196)^2*x-655224571740000
*(3*x^2+4)^(1/3)*RootOf(_Z^3-196)^2*x+137243207475444840*(3*x^2+4)^(1/3)*Ro
otOf(_Z^3-196)*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2)*
x-385426451889099*RootOf(_Z^3-196)*x^3-61022612838281976*RootOf(RootOf(_Z^3
-196)^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2)*x^3+20145521550596484*(3*x^2+4)^(
2/3)*x+1092040952900000*(3*x^2+4)^(1/3)*RootOf(_Z^3-196)^2-2287386791257414
00*(3*x^2+4)^(1/3)*RootOf(_Z^3-196)*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(
_Z^3-196)+9604*_Z^2)+7542989624962646*RootOf(_Z^3-196)*x^2+1194243242183377
904*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2)*x^2-6715173
8501988280*(3*x^2+4)^(2/3)-12847548396303300*RootOf(_Z^3-196)*x-20340870946
09399200*RootOf(RootOf(_Z^3-196)^2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2)*x+1919
3353915099208*RootOf(_Z^3-196)+3038786256855632192*RootOf(RootOf(_Z^3-196)^
2+98*_Z*RootOf(_Z^3-196)+9604*_Z^2))/(6+x)^2/(9*x-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x+2}{\left(9\,x^2+52\,x-12\right)\left(3\,x^2+4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(3*x^2+4)^(1/3)/(9*x^2+52*x-12),x, algorithm="maxima")
[Out] integrate((3*x + 2)/((9*x^2 + 52*x - 12)*(3*x^2 + 4)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x+2}{{\left(3\,x^2+4\right)}^{1/3}\,\left(9\,x^2+52\,x-12\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)/((3*x^2 + 4)^(1/3)*(52*x + 9*x^2 - 12)),x)
[Out] int((3*x + 2)/((3*x^2 + 4)^(1/3)*(52*x + 9*x^2 - 12)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x+2}{(x+6)\,(9x-2)\,\sqrt[3]{3x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(3*x**2+4)**(1/3)/(9*x**2+52*x-12),x)
[Out] Integral((3*x + 2)/((x + 6)*(9*x - 2)*(3*x**2 + 4)**(1/3)), x)
```

3.1980 $\int \left(c+bx+ax^2\right)^{5/2}\,dx$

Optimal. Leaf size=197

$$\frac{\sqrt{ax^2+bx+c}\,\left(256a^5x^5+640a^4bx^4+832a^4cx^3+432a^3b^2x^3+1248a^3bcx^2+1056a^3c^2x+8a^2b^3x^2+96a^2b^2\right.}{1536a^3}$$

Rubi [A] time = 0.07, antiderivative size = 149, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 621, 206}

$$-\frac{5\left(b^2-4ac\right)^3\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{ax^2+bx+c}}\right)}{1024a^{7/2}}+\frac{5\left(b^2-4ac\right)^2(2ax+b)\sqrt{ax^2+bx+c}}{512a^3}-\frac{5\left(b^2-4ac\right)(2ax+b)\left(ax^2+bx+c\right)^{3/2}}{192a^2}+\frac{(2ax+b)\left(ax^2+bx+c\right)^{5/2}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + b*x + a*x^2)^(5/2),x]
```

```
[Out] (5*(b^2 - 4*a*c)^2*(b + 2*a*x)*Sqrt[c + b*x + a*x^2])/(512*a^3) - (5*(b^2 -
 4*a*c)*(b + 2*a*x)*(c + b*x + a*x^2)^(3/2))/(192*a^2) + ((b + 2*a*x)*(c +
b*x + a*x^2)^(5/2))/(12*a) - (5*(b^2 - 4*a*c)^3*ArcTanh[(b + 2*a*x)/(2*Sqrt
[a]*Sqrt[c + b*x + a*x^2])])/(1024*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(c+bx+ax^2\right)^{5/2}dx=\frac{(b+2ax)\left(c+bx+ax^2\right)^{5/2}}{12a}-\frac{\left(5\left(b^2-4ac\right)\right)\int\left(c+bx+ax^2\right)^{3/2}dx}{24a}$$

$$=-\frac{5\left(b^2-4ac\right)(b+2ax)\left(c+bx+ax^2\right)^{3/2}}{192a^2}+\frac{(b+2ax)\left(c+bx+ax^2\right)^{5/2}}{12a}+\frac{\left(5\left(b^2-4\right.\right.}{}$$

$$=\frac{5\left(b^2-4ac\right)^2(b+2ax)\sqrt{c+bx+ax^2}}{512a^3}-\frac{5\left(b^2-4ac\right)(b+2ax)\left(c+bx+ax^2\right)^{3/2}}{192a^2}+$$

$$=\frac{5\left(b^2-4ac\right)^2(b+2ax)\sqrt{c+bx+ax^2}}{512a^3}-\frac{5\left(b^2-4ac\right)(b+2ax)\left(c+bx+ax^2\right)^{3/2}}{192a^2}+$$

$$=\frac{5\left(b^2-4ac\right)^2(b+2ax)\sqrt{c+bx+ax^2}}{512a^3}-\frac{5\left(b^2-4ac\right)(b+2ax)\left(c+bx+ax^2\right)^{3/2}}{192a^2}+$$

Mathematica [A] time = 0.64, size = 162, normalized size = 0.82

$$\frac{\sqrt{x(ax+b)+c}\left(2(2ax+b)\left(32a^2bx\left(8ax^2+13c\right)+16a^2\left(8a^2x^4+26acx^2+33c^2\right)-40ab^3x+8ab^2\left(11ax^2-20c\right)+15b^4\right)+\frac{15\left(b^2-4ac\right)^{5/2}\sin^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{a(x(ax+b)+c)}{4ac-b^2}}}\right)}{3072a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + b*x + a*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + x*(b + a*x)]*(2*(b + 2*a*x)*(15*b^4 - 40*a*b^3*x + 32*a^2*b*x*(13
*c + 8*a*x^2) + 8*a*b^2*(-20*c + 11*a*x^2) + 16*a^2*(33*c^2 + 26*a*c*x^2 +
8*a^2*x^4)) + (15*(b^2 - 4*a*c)^(5/2)*ArcSin[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]]
)/Sqrt[(a*(c + x*(b + a*x)))/(-b^2 + 4*a*c)]))/(3072*a^3)
```

IntegrateAlgebraic [A] time = 0.69, size = 197, normalized size = 1.00

$$\frac{\sqrt{ax^2+bx+c}\left(256a^5x^5+640a^4bx^4+832a^4cx^3+432a^3b^2x^3+1248a^3bcx^2+1056a^3c^2x+8a^2b^3x^2+96a^2b^2cx+528a^2bc^2-10ab^4x-160ab^3c+15b^5\right)}{1536a^3}-\frac{5\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)\log\left(-2\sqrt{a}\sqrt{ax^2+bx+c}+2ax+b\right)}{1024a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + b*x + a*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + b*x + a*x^2]*(15*b^5 - 160*a*b^3*c + 528*a^2*b*c^2 - 10*a*b^4*x +
 96*a^2*b^2*c*x + 1056*a^3*c^2*x + 8*a^2*b^3*x^2 + 1248*a^3*b*c*x^2 + 432*a
^3*b^2*x^3 + 832*a^4*c*x^3 + 640*a^4*b*x^4 + 256*a^5*x^5))/(1536*a^3) - (5*
(-b^6 + 12*a*b^4*c - 48*a^2*b^2*c^2 + 64*a^3*c^3)*Log[b + 2*a*x - 2*Sqrt[a]
*Sqrt[c + b*x + a*x^2]])/(1024*a^(7/2))
```

fricas [A] time = 0.83, size = 425, normalized size = 2.16

$$\left[-\frac{15\left(b^6-12ab^4c+48a^2b^2c^2-64a^3c^3\right)\sqrt{a}\log\left(-8a^2x^2-8abx-4\sqrt{ax^2+bx+c}(2ax+b)\sqrt{a}-b^2-4ac\right)-4\left(256a^6x^5+640a^5bx^4+15ab^5-160a^2b^3c+528a^3bc^2+16\left(27a^4b^2+52a^5c\right)x^3+8\left(a^3b^3+156a^4bc\right)x^2-2\left(5a^2b^4-48a^3b^2c-528a^4c^2\right)x\right)\sqrt{ax^2+bx+c}}{6144a^4}, \frac{15\left(b^6-12ab^4c+48a^2b^2c^2-64a^3c^3\right)\sqrt{-a}\arctan\left(\frac{\sqrt{ax^2+bx+c}(2ax+b)\sqrt{-a}}{2\left(a^2x^2+abx+ac\right)}\right)+2\left(256a^6x^5+640a^5bx^4+15ab^5-160a^2b^3c+528a^3bc^2+16\left(27a^4b^2+52a^5c\right)x^3+8\left(a^3b^3+156a^4bc\right)x^2-2\left(5a^2b^4-48a^3b^2c-528a^4c^2\right)x\right)\sqrt{ax^2+bx+c}}{3072a^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6144*(15*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(a)*log(-
8*a^2*x^2 - 8*a*b*x - 4*sqrt(a*x^2 + b*x + c)*(2*a*x + b)*sqrt(a) - b^2 - 4
*a*c) - 4*(256*a^6*x^5 + 640*a^5*b*x^4 + 15*a*b^5 - 160*a^2*b^3*c + 528*a^3
*b*c^2 + 16*(27*a^4*b^2 + 52*a^5*c)*x^3 + 8*(a^3*b^3 + 156*a^4*b*c)*x^2 - 2
*(5*a^2*b^4 - 48*a^3*b^2*c - 528*a^4*c^2)*x)*sqrt(a*x^2 + b*x + c))/a^4, 1/
3072*(15*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-a)*arctan(1
/2*sqrt(a*x^2 + b*x + c)*(2*a*x + b)*sqrt(-a)/(a^2*x^2 + a*b*x + a*c)) + 2*
(256*a^6*x^5 + 640*a^5*b*x^4 + 15*a*b^5 - 160*a^2*b^3*c + 528*a^3*b*c^2 + 1
6*(27*a^4*b^2 + 52*a^5*c)*x^3 + 8*(a^3*b^3 + 156*a^4*b*c)*x^2 - 2*(5*a^2*b^
4 - 48*a^3*b^2*c - 528*a^4*c^2)*x)*sqrt(a*x^2 + b*x + c))/a^4]
```

giac [A] time = 0.49, size = 208, normalized size = 1.06

$$\frac{1}{1536}\sqrt{ax^2+bx+c}\left(2\left(4\left(2\left(8\left(2a^2x+5ab\right)x+\frac{27a^5b^2+52a^6c}{a^5}\right)x+\frac{a^4b^3+156a^5bc}{a^5}\right)x-\frac{5a^3b^4-48a^4b^2c-528a^5c^2}{a^5}\right)x+\frac{15a^2b^5-160a^3b^3c+528a^4bc^2}{a^5}\right)+\frac{5\left(b^6-12ab^4c+48a^2b^2c^2-64a^3c^3\right)\log\left(\left|-2\left(\sqrt{a}x-\sqrt{ax^2+bx+c}\right)\sqrt{a}-b\right|\right)}{1024a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/1536*sqrt(a*x^2 + b*x + c)*(2*(4*(2*(8*(2*a^2*x + 5*a*b)*x + (27*a^5*b^2
+ 52*a^6*c)/a^5)*x + (a^4*b^3 + 156*a^5*b*c)/a^5)*x - (5*a^3*b^4 - 48*a^4*b
^2*c - 528*a^5*c^2)/a^5)*x + (15*a^2*b^5 - 160*a^3*b^3*c + 528*a^4*b*c^2)/a
^5) + 5/1024*(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*log(abs(-2*(s
qrt(a)*x - sqrt(a*x^2 + b*x + c))*sqrt(a) - b))/a^(7/2)
```

maple [A] time = 0.00, size = 360, normalized size = 1.83

$$\frac{(2ax+b)\left(ax^2+bx+c\right)^{\frac{5}{2}}}{12a}+\frac{5\left(ax^2+bx+c\right)^{\frac{3}{2}}xc}{24}-\frac{5\left(ax^2+bx+c\right)^{\frac{3}{2}}x b^2}{96a}+\frac{5\left(ax^2+bx+c\right)^{\frac{3}{2}}bc}{48a}-\frac{5\left(ax^2+bx+c\right)^{\frac{3}{2}}b^3}{192a^2}+\frac{5\sqrt{ax^2+bx+c}\,x c^2}{16}-\frac{5\sqrt{ax^2+bx+c}\,x c b^2}{32a}+\frac{5\sqrt{ax^2+bx+c}\,x b^4}{256a^2}+\frac{5\sqrt{ax^2+bx+c}\,b c^2}{32a}-\frac{5\sqrt{ax^2+bx+c}\,b^3 c}{64a^2}+\frac{5\sqrt{ax^2+bx+c}\,b^5}{512a^3}+\frac{5\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{ax^2+bx+c}\right)c^3}{16\sqrt{a}}-\frac{15\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{ax^2+bx+c}\right)c^2 b^2}{64a^{\frac{3}{2}}}+\frac{15\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{ax^2+bx+c}\right)b^4 c}{256a^{\frac{5}{2}}}-\frac{5\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{ax^2+bx+c}\right)b^6}{1024a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x+c)^(5/2),x)
```

```
[Out] 1/12*(2*a*x+b)/a*(a*x^2+b*x+c)^(5/2)+5/24*(a*x^2+b*x+c)^(3/2)*x*c-5/96/a*(a
*x^2+b*x+c)^(3/2)*x*b^2+5/48/a*(a*x^2+b*x+c)^(3/2)*b*c-5/192/a^2*(a*x^2+b*x
+c)^(3/2)*b^3+5/16*(a*x^2+b*x+c)^(1/2)*x*c^2-5/32/a*(a*x^2+b*x+c)^(1/2)*x*c
*b^2+5/256/a^2*(a*x^2+b*x+c)^(1/2)*x*b^4+5/32/a*(a*x^2+b*x+c)^(1/2)*b*c^2-5
/64/a^2*(a*x^2+b*x+c)^(1/2)*b^3*c+5/512/a^3*(a*x^2+b*x+c)^(1/2)*b^5+5/16/a^
(1/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x+c)^(1/2))*c^3-15/64/a^(3/2)*ln((1/2
*b+a*x)/a^(1/2)+(a*x^2+b*x+c)^(1/2))*c^2*b^2+15/256/a^(5/2)*ln((1/2*b+a*x)/
a^(1/2)+(a*x^2+b*x+c)^(1/2))*b^4*c-5/1024/a^(7/2)*ln((1/2*b+a*x)/a^(1/2)+(a
*x^2+b*x+c)^(1/2))*b^6
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 1.76, size = 143, normalized size = 0.73

$$\frac{\left(\frac{b}{2}+a\,x\right)\,{\left(a\,x^2+b\,x+c\right)}^{5/2}}{6\,a}+\frac{\left(5\,a\,c-\frac{5\,b^2}{4}\right)\,\left(\frac{\left(\left(\frac{x}{2}+\frac{b}{4\,a}\right)\,\sqrt{a\,x^2+b\,x+c}+\frac{\ln\left(\frac{\frac{b}{2}+a\,x}{\sqrt{a}}+\sqrt{a\,x^2+b\,x+c}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,a^{3/2}}\right)\,\left(3\,a\,c-\frac{3\,b^2}{4}\right)}{4\,a}+\frac{\left(\frac{b}{2}+a\,x\right)\,{\left(a\,x^2+b\,x+c\right)}^{3/2}}{4\,a}\right)}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + b*x + a*x^2)^(5/2),x)
```

```
[Out] ((b/2 + a*x)*(c + b*x + a*x^2)^(5/2))/(6*a) + ((5*a*c - (5*b^2)/4)*((((x/2
+ b/(4*a))*(c + b*x + a*x^2)^(1/2) + (log((b/2 + a*x)/a^(1/2) + (c + b*x +
a*x^2)^(1/2))*(a*c - b^2/4))/(2*a^(3/2)))*(3*a*c - (3*b^2)/4))/(4*a) + ((b/
2 + a*x)*(c + b*x + a*x^2)^(3/2))/(4*a)))/(6*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^{2} + bx + c\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x+c)**(5/2),x)
```

```
[Out] Integral((a*x**2 + b*x + c)**(5/2), x)
```

3.1981 $\int \frac{1-x^4+x^8}{x^2\left(-1+x^4\right)^{3/4}\left(-1-x^4+x^8\right)}\,dx$

Optimal. Leaf size=197

$$-\frac{\sqrt[4]{x^4-1}}{x}+\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-1}}\right)-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4-1}}\right)-\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\right.$$

Rubi [C] time = 0.55, antiderivative size = 157, normalized size of antiderivative = 0.80, number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6728, 264, 1528, 511, 510}

$$\frac{4\sqrt[4]{x^4-1}\,x^3F_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};x^4,\frac{2x^4}{1-\sqrt{5}}\right)}{3\sqrt{5}\left(1-\sqrt{5}\right)\sqrt[4]{1-x^4}}-\frac{4\sqrt[4]{x^4-1}\,x^3F_1\left(\frac{3}{4};1,-\frac{1}{4};\frac{7}{4};\frac{2x^4}{1+\sqrt{5}},x^4\right)}{3\sqrt{5}\left(1+\sqrt{5}\right)\sqrt[4]{1-x^4}}-\frac{\sqrt[4]{x^4-1}}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x^4 + x^8)/(x^2*(-1 + x^4)^(3/4)*(-1 - x^4 + x^8)),x]
```

```
[Out] -((-1 + x^4)^(1/4)/x) + (4*x^3*(-1 + x^4)^(1/4)*AppellF1[3/4, -1/4, 1, 7/4,
 x^4, (2*x^4)/(1 - Sqrt[5])])/(3*Sqrt[5]*(1 - Sqrt[5])*(1 - x^4)^(1/4)) - (
4*x^3*(-1 + x^4)^(1/4)*AppellF1[3/4, 1, -1/4, 7/4, (2*x^4)/(1 + Sqrt[5]), x
^4])/(3*Sqrt[5]*(1 + Sqrt[5])*(1 - x^4)^(1/4))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1528

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q, (
f*x)^m/(a + b*x^n + c*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, e, f, q, n}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&  !IntegerQ[q] &&
IntegerQ[m]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1-x^4+x^8}{x^2\left(-1+x^4\right)^{3/4}\left(-1-x^4+x^8\right)}\,dx = \int \left(-\frac{1}{x^2\left(-1+x^4\right)^{3/4}}+\frac{2x^2\sqrt[4]{-1+x^4}}{-1-x^4+x^8}\right)dx$$

$$= 2\int \frac{x^2\sqrt[4]{-1+x^4}}{-1-x^4+x^8}\,dx-\int \frac{1}{x^2\left(-1+x^4\right)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{-1+x^4}}{x}+2\int \left(-\frac{2x^2\sqrt[4]{-1+x^4}}{\sqrt{5}\left(1+\sqrt{5}-2x^4\right)}-\frac{2x^2\sqrt[4]{-1+x^4}}{\sqrt{5}\left(-1+\sqrt{5}+2x^4\right)}\right)dx$$

$$= -\frac{\sqrt[4]{-1+x^4}}{x}-\frac{4\int \frac{x^2\sqrt[4]{-1+x^4}}{1+\sqrt{5}-2x^4}\,dx}{\sqrt{5}}-\frac{4\int \frac{x^2\sqrt[4]{-1+x^4}}{-1+\sqrt{5}+2x^4}\,dx}{\sqrt{5}}$$

$$= -\frac{\sqrt[4]{-1+x^4}}{x}-\frac{\left(4\sqrt[4]{-1+x^4}\right)\int \frac{x^2\sqrt[4]{1-x^4}}{1+\sqrt{5}-2x^4}\,dx}{\sqrt{5}\sqrt[4]{1-x^4}}-\frac{\left(4\sqrt[4]{-1+x^4}\right)\int \frac{x^2\sqrt[4]{1-x^4}}{-1+\sqrt{5}+2x}}{\sqrt{5}\sqrt[4]{1-x^4}}$$

$$= -\frac{\sqrt[4]{-1+x^4}}{x}+\frac{4x^3\sqrt[4]{-1+x^4}\,F_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};x^4,\frac{2x^4}{1-\sqrt{5}}\right)}{3\sqrt{5}\left(1-\sqrt{5}\right)\sqrt[4]{1-x^4}}-\frac{4x^3\sqrt[4]{-1+x^4}\,F}{3\sqrt{5}\,(1}$$

Mathematica [F] time = 7.22, size = 0, normalized size = 0.00

$$\int \frac{1-x^4+x^8}{x^2\left(-1+x^4\right)^{3/4}\left(-1-x^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^4 + x^8)/(x^2*(-1 + x^4)^(3/4)*(-1 - x^4 + x^8)),x]
```

```
[Out] Integrate[(1 - x^4 + x^8)/(x^2*(-1 + x^4)^(3/4)*(-1 - x^4 + x^8)), x]
```

IntegrateAlgebraic [A] time = 0.97, size = 197, normalized size = 1.00

$$-\frac{\sqrt[4]{x^4-1}}{x}+\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-1}}\right)-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4-1}}\right)-\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-1}}\right)+\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^4 + x^8)/(x^2*(-1 + x^4)^(3/4)*(-1 - x^4 + x^8)),
x]
```

```
[Out] -((-1 + x^4)^(1/4)/x) + Sqrt[(-1 + Sqrt[5])/10]*ArcTan[(Sqrt[-1/2 + Sqrt[5]
/2]*x)/(-1 + x^4)^(1/4)] - Sqrt[(1 + Sqrt[5])/10]*ArcTan[(Sqrt[1/2 + Sqrt[5
]/2]*x)/(-1 + x^4)^(1/4)] - Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[(Sqrt[-1/2 + Sq
rt[5]/2]*x)/(-1 + x^4)^(1/4)] + Sqrt[(1 + Sqrt[5])/10]*ArcTanh[(Sqrt[1/2 +
Sqrt[5]/2]*x)/(-1 + x^4)^(1/4)]
```

fricas [B] time = 18.20, size = 971, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-x^4+1)/x^2/(x^4-1)^(3/4)/(x^8-x^4-1),x, algorithm="fricas")
```

```
[Out] -1/40*(4*sqrt(10)*x*sqrt(sqrt(5) + 1)*arctan(-1/40*(sqrt(2)*(2*sqrt(10)*(5*
x^6 - sqrt(5)*(x^6 + 2*x^2))*sqrt(x^4 - 1) - sqrt(10)*(5*x^8 + 5*x^4 - sqrt
(5)*(5*x^8 - 3*x^4 - 1) - 5))*(sqrt(5) + 1) + 4*(sqrt(10)*(5*x^5 - sqrt(5)*
(x^5 + 2*x))*(x^4 - 1)^(3/4) - sqrt(10)*(x^4 - 1)^(1/4)*(5*x^3 - sqrt(5)*(2
*x^7 - x^3)))*sqrt(sqrt(5) + 1))/(x^8 - x^4 - 1)) + 4*sqrt(10)*x*sqrt(sqrt(
5) - 1)*arctan(1/40*(sqrt(2)*(2*sqrt(10)*(5*x^6 + sqrt(5)*(x^6 + 2*x^2))*sq
rt(x^4 - 1) + sqrt(10)*(5*x^8 + 5*x^4 + sqrt(5)*(5*x^8 - 3*x^4 - 1) - 5))*(
sqrt(5) - 1) - 4*(sqrt(10)*(5*x^5 + sqrt(5)*(x^5 + 2*x))*(x^4 - 1)^(3/4) +
sqrt(10)*(x^4 - 1)^(1/4)*(5*x^3 + sqrt(5)*(2*x^7 - x^3)))*sqrt(sqrt(5) - 1)
)/(x^8 - x^4 - 1)) + sqrt(10)*x*sqrt(sqrt(5) - 1)*log((10*(2*x^5 + sqrt(5)*
x - x)*(x^4 - 1)^(3/4) + (sqrt(10)*sqrt(x^4 - 1)*(5*x^2 + sqrt(5)*(2*x^6 -
x^2)) + sqrt(10)*(5*x^8 - 5*x^4 + sqrt(5)*(2*x^4 - 1)))*sqrt(sqrt(5) - 1) -
 10*(x^7 - 3*x^3 - sqrt(5)*(x^7 - x^3))*(x^4 - 1)^(1/4))/(x^8 - x^4 - 1)) -
 sqrt(10)*x*sqrt(sqrt(5) - 1)*log((10*(2*x^5 + sqrt(5)*x - x)*(x^4 - 1)^(3/
4) - (sqrt(10)*sqrt(x^4 - 1)*(5*x^2 + sqrt(5)*(2*x^6 - x^2)) + sqrt(10)*(5*
x^8 - 5*x^4 + sqrt(5)*(2*x^4 - 1)))*sqrt(sqrt(5) - 1) - 10*(x^7 - 3*x^3 - s
qrt(5)*(x^7 - x^3))*(x^4 - 1)^(1/4))/(x^8 - x^4 - 1)) + sqrt(10)*x*sqrt(sqr
t(5) + 1)*log((10*(2*x^5 - sqrt(5)*x - x)*(x^4 - 1)^(3/4) + (sqrt(10)*sqrt(
x^4 - 1)*(5*x^2 - sqrt(5)*(2*x^6 - x^2)) - sqrt(10)*(5*x^8 - 5*x^4 - sqrt(5
)*(2*x^4 - 1)))*sqrt(sqrt(5) + 1) + 10*(x^7 - 3*x^3 + sqrt(5)*(x^7 - x^3))*
(x^4 - 1)^(1/4))/(x^8 - x^4 - 1)) - sqrt(10)*x*sqrt(sqrt(5) + 1)*log((10*(2
*x^5 - sqrt(5)*x - x)*(x^4 - 1)^(3/4) - (sqrt(10)*sqrt(x^4 - 1)*(5*x^2 - sq
rt(5)*(2*x^6 - x^2)) - sqrt(10)*(5*x^8 - 5*x^4 - sqrt(5)*(2*x^4 - 1)))*sqrt
(sqrt(5) + 1) + 10*(x^7 - 3*x^3 + sqrt(5)*(x^7 - x^3))*(x^4 - 1)^(1/4))/(x^
8 - x^4 - 1)) + 40*(x^4 - 1)^(1/4))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-x^4+1)/x^2/(x^4-1)^(3/4)/(x^8-x^4-1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [C] time = 11.69, size = 3500, normalized size = 17.77

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-x^4+1)/x^2/(x^4-1)^(3/4)/(x^8-x^4-1),x)
```

```
[Out] -(x^4-1)^(1/4)/x+(1/10*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*ln(
-(-32000*RootOf(6400*_Z^4+80*_Z^2-1)^4*x^12-1600*RootOf(6400*_Z^4+80*_Z^2-1
)^2*x^12-160*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*RootOf(6400*_
Z^4+80*_Z^2-1)^2*(x^12-3*x^8+3*x^4-1)^(1/4)*x^9+64000*RootOf(6400*_Z^4+80*_
Z^2-1)^4*x^8-15*x^12-6*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*(x^
12-3*x^8+3*x^4-1)^(1/4)*x^9+800*(x^12-3*x^8+3*x^4-1)^(1/2)*RootOf(6400*_Z^4
+80*_Z^2-1)^2*x^6+3600*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^8+480*RootOf(_Z^2+40
0*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*RootOf(6400*_Z^4+80*_Z^2-1)^2*(x^12-3*x^
8+3*x^4-1)^(3/4)*x^3+320*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*R
ootOf(6400*_Z^4+80*_Z^2-1)^2*(x^12-3*x^8+3*x^4-1)^(1/4)*x^5+20*(x^12-3*x^8+
3*x^4-1)^(1/2)*x^6-32000*RootOf(6400*_Z^4+80*_Z^2-1)^4*x^4+35*x^8+8*RootOf(
_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*(x^12-3*x^8+3*x^4-1)^(3/4)*x^3+12
*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*(x^12-3*x^8+3*x^4-1)^(1/4
)*x^5-800*(x^12-3*x^8+3*x^4-1)^(1/2)*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^2-2400
```

*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^4-160*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*RootOf(6400*_Z^4+80*_Z^2-1)^2*(x^12-3*x^8+3*x^4-1)^(1/4)*x-20*(x^12-3*x^8+3*x^4-1)^(1/2)*x^2-25*x^4-6*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*(x^12-3*x^8+3*x^4-1)^(1/4)*x+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)/(80*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^4+x^4+1)/(1+x)^2/(x^2+1)^2/(-1+x)^2)+2*RootOf(6400*_Z^4+80*_Z^2-1)*ln((3200*RootOf(6400*_Z^4+80*_Z^2-1)^4*x^12-80*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^12-320*RootOf(6400*_Z^4+80*_Z^2-1)^3*(x^12-3*x^8+3*x^4-1)^(1/4)*x^9-6400*RootOf(6400*_Z^4+80*_Z^2-1)^4*x^8+8*RootOf(6400*_Z^4+80*_Z^2-1)*(x^12-3*x^8+3*x^4-1)^(1/4)*x^9+80*(x^12-3*x^8+3*x^4-1)^(1/2)*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^6+200*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^8+960*RootOf(6400*_Z^4+80*_Z^2-1)^3*(x^12-3*x^8+3*x^4-1)^(3/4)*x^3+640*RootOf(6400*_Z^4+80*_Z^2-1)^3*(x^12-3*x^8+3*x^4-1)^(1/4)*x^5-(x^12-3*x^8+3*x^4-1)^(1/2)*x^6+3200*RootOf(6400*_Z^4+80*_Z^2-1)^4*x^4-4*RootOf(6400*_Z^4+80*_Z^2-1)*(x^12-3*x^8+3*x^4-1)^(3/4)*x^3-16*RootOf(6400*_Z^4+80*_Z^2-1)*(x^12-3*x^8+3*x^4-1)^(1/4)*x^5-80*(x^12-3*x^8+3*x^4-1)^(1/2)*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^2-160*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^4-320*RootOf(6400*_Z^4+80*_Z^2-1)^3*(x^12-3*x^8+3*x^4-1)^(1/4)*x+(x^12-3*x^8+3*x^4-1)^(1/2)*x^2+8*RootOf(6400*_Z^4+80*_Z^2-1)*(x^12-3*x^8+3*x^4-1)^(1/4)*x+40*RootOf(6400*_Z^4+80*_Z^2-1)^2)/(640*RootOf(6400*_Z^4+80*_Z^2-1)^3*x^2+4*RootOf(6400*_Z^4+80*_Z^2-1)*x^2-1)/(640*RootOf(6400*_Z^4+80*_Z^2-1)^3*x^2+4*RootOf(6400*_Z^4+80*_Z^2-1)*x^2+1)/(1+x)^2/(x^2+1)^2/(-1+x)^2)-8*RootOf(6400*_Z^4+80*_Z^2-1)^2*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*ln((-16000*RootOf(6400*_Z^4+80*_Z^2-1)^4*x^12+400*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^12+240*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*RootOf(6400*_Z^4+80*_Z^2-1)^2*(x^12-3*x^8+3*x^4-1)^(1/4)*x^9+32000*RootOf(6400*_Z^4+80*_Z^2-1)^4*x^8-RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*(x^12-3*x^8+3*x^4-1)^(1/4)*x^9+400*(x^12-3*x^8+3*x^4-1)^(1/2)*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^6-1000*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^8+320*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*RootOf(6400*_Z^4+80*_Z^2-1)^2*(x^12-3*x^8+3*x^4-1)^(3/4)*x^3-480*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*RootOf(6400*_Z^4+80*_Z^2-1)^2*(x^12-3*x^8+3*x^4-1)^(1/4)*x^5-5*(x^12-3*x^8+3*x^4-1)^(1/2)*x^6-16000*RootOf(6400*_Z^4+80*_Z^2-1)^4*x^4-3*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*(x^12-3*x^8+3*x^4-1)^(3/4)*x^3+2*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*(x^12-3*x^8+3*x^4-1)^(1/4)*x^5-400*(x^12-3*x^8+3*x^4-1)^(1/2)*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^2+800*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^4+240*RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*RootOf(6400*_Z^4+80*_Z^2-1)^2*(x^12-3*x^8+3*x^4-1)^(1/4)*x+5*(x^12-3*x^8+3*x^4-1)^(1/2)*x^2-RootOf(_Z^2+400*RootOf(6400*_Z^4+80*_Z^2-1)^2+5)*(x^12-3*x^8+3*x^4-1)^(1/4)*x-200*RootOf(6400*_Z^4+80*_Z^2-1)^2)/(640*RootOf(6400*_Z^4+80*_Z^2-1)^3*x^2+4*RootOf(6400*_Z^4+80*_Z^2-1)*x^2-1)/(640*RootOf(6400*_Z^4+80*_Z^2-1)^3*x^2+4*RootOf(6400*_Z^4+80*_Z^2-1)*x^2+1)/(1+x)^2/(x^2+1)^2/(-1+x)^2)-160*ln((-6400*RootOf(6400*_Z^4+80*_Z^2-1)^4*x^12-320*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^12+1920*RootOf(6400*_Z^4+80*_Z^2-1)^3*(x^12-3*x^8+3*x^4-1)^(1/4)*x^9+12800*RootOf(6400*_Z^4+80*_Z^2-1)^4*x^8-3*x^12+32*RootOf(6400*_Z^4+80*_Z^2-1)*(x^12-3*x^8+3*x^4-1)^(1/4)*x^9-160*(x^12-3*x^8+3*x^4-1)^(1/2)*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^6+720*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^8+2560*RootOf(6400*_Z^4+80*_Z^2-1)^3*(x^12-3*x^8+3*x^4-1)^(3/4)*x^3-3840*RootOf(6400*_Z^4+80*_Z^2-1)^3*(x^12-3*x^8+3*x^4-1)^(1/4)*x^5-4*(x^12-3*x^8+3*x^4-1)^(1/2)*x^6-6400*RootOf(6400*_Z^4+80*_Z^2-1)^4*x^4+7*x^8+56*RootOf(6400*_Z^4+80*_Z^2-1)*(x^12-3*x^8+3*x^4-1)^(3/4)*x^3-64*RootOf(6400*_Z^4+80*_Z^2-1)*(x^12-3*x^8+3*x^4-1)^(1/4)*x^5+160*(x^12-3*x^8+3*x^4-1)^(1/2)*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^2-480*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^4+1920*RootOf(6400*_Z^4+80*_Z^2-1)^3*(x^12-3*x^8+3*x^4-1)^(1/4)*x+4*(x^12-3*x^8+3*x^4-1)^(1/2)*x^2-5*x^4+32*RootOf(6400*_Z^4+80*_Z^2-1)*(x^12-3*x^8+3*x^4-1)^(1/4)*x+80*RootOf(6400*_Z^4+80*_Z^2-1)^2+1)/(80*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^4+x^4+1)/(1+x)^2/(x^2+1)^2/(-1+x)^2)*RootOf(6400*_Z^4+80*_Z^2-1)^3-2*ln((-6400*RootOf(6400*_Z^4+80*_Z^2-1)^4*x^12-320*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^12+1920*RootOf(6400*_Z^4+80*_Z^2-1)^3*(x^12-3*x^8+3*x^4-1)^(1/4)*x^9+12800*RootOf(6400*_Z^4+80*_Z^2-1)^4*x^8-3*x^12+32*RootOf(6400*_Z^4+80*_Z^2-1)*(x^12-3*x^8+3*x^4-1)^(1/4)*x^9-

```
160*(x^12-3*x^8+3*x^4-1)^(1/2)*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^6+720*RootOf
(6400*_Z^4+80*_Z^2-1)^2*x^8+2560*RootOf(6400*_Z^4+80*_Z^2-1)^3*(x^12-3*x^8+
3*x^4-1)^(3/4)*x^3-3840*RootOf(6400*_Z^4+80*_Z^2-1)^3*(x^12-3*x^8+3*x^4-1)^
(1/4)*x^5-4*(x^12-3*x^8+3*x^4-1)^(1/2)*x^6-6400*RootOf(6400*_Z^4+80*_Z^2-1)
^4*x^4+7*x^8+56*RootOf(6400*_Z^4+80*_Z^2-1)*(x^12-3*x^8+3*x^4-1)^(3/4)*x^3-
64*RootOf(6400*_Z^4+80*_Z^2-1)*(x^12-3*x^8+3*x^4-1)^(1/4)*x^5+160*(x^12-3*x
^8+3*x^4-1)^(1/2)*RootOf(6400*_Z^4+80*_Z^2-1)^2*x^2-480*RootOf(6400*_Z^4+80
*_Z^2-1)^2*x^4+1920*RootOf(6400*_Z^4+80*_Z^2-1)^3*(x^12-3*x^8+3*x^4-1)^(1/4
)*x+4*(x^12-3*x^8+3*x^4-1)^(1/2)*x^2-5*x^4+32*RootOf(6400*_Z^4+80*_Z^2-1)*(
x^12-3*x^8+3*x^4-1)^(1/4)*x+80*RootOf(6400*_Z^4+80*_Z^2-1)^2+1)/(80*RootOf(
6400*_Z^4+80*_Z^2-1)^2*x^4+x^4+1)/(1+x)^2/(x^2+1)^2/(-1+x)^2)*RootOf(6400*_
Z^4+80*_Z^2-1))/(x^4-1)^(3/4)*((x^4-1)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 - x^4 + 1}{\left(x^8 - x^4 - 1\right)\left(x^4 - 1\right)^{\frac{3}{4}} x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-x^4+1)/x^2/(x^4-1)^(3/4)/(x^8-x^4-1),x, algorithm="maxima")
[Out] integrate((x^8 - x^4 + 1)/((x^8 - x^4 - 1)*(x^4 - 1)^(3/4)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^8 - x^4 + 1}{x^2 \left(x^4 - 1\right)^{3/4} \left(-x^8 + x^4 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^8 - x^4 + 1)/(x^2*(x^4 - 1)^(3/4)*(x^4 - x^8 + 1)),x)
[Out] int(-(x^8 - x^4 + 1)/(x^2*(x^4 - 1)^(3/4)*(x^4 - x^8 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-x**4+1)/x**2/(x**4-1)**(3/4)/(x**8-x**4-1),x)
[Out] Timed out
```

3.1982 $\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{x+\sqrt{1+x^2}}} \, dx$

Optimal. Leaf size=197

$$\frac{\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}(16x-2)+\sqrt{x^2+1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}(32x+3)+16\sqrt{\sqrt{x^2+1}+x}\right.}{24\left(\sqrt{x^2+1}+x\right)^{3/2}}$$

Rubi [F] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{x+\sqrt{1+x^2}}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[x + Sqrt[1 + x^2]],x]
[Out] Defer[Int][Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[x + Sqrt[1 + x^2]], x]
```

Rubi steps

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{x+\sqrt{1+x^2}}} \, dx = \int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{x+\sqrt{1+x^2}}} \, dx$$

Mathematica [F] time = 2.97, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{x+\sqrt{1+x^2}}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[x + Sqrt[1 + x^2]],x]
[Out] Integrate[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[x + Sqrt[1 + x^2]], x]
```

IntegrateAlgebraic [A] time = 0.25, size = 197, normalized size = 1.00

$$\frac{\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}(16x-2)+\sqrt{x^2+1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}(32x+3)+16\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)+\left(32x^2+3x+8\right)\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}}{24\left(\sqrt{x^2+1}+x\right)^{3/2}}-\frac{1}{8}\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[x + Sqrt[1 + x^2]
],x]
```

```
[Out] ((8 + 3*x + 32*x^2)*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] + (-2 + 16*x)*Sqrt[x
+ Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] + Sqrt[1 + x^2]*((3 + 32
*x)*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] + 16*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 +
 Sqrt[x + Sqrt[1 + x^2]]]))/(24*(x + Sqrt[1 + x^2])^(3/2)) - ArcTanh[Sqrt[1
 + Sqrt[x + Sqrt[1 + x^2]]]]/8
```

fricas [A] time = 0.64, size = 108, normalized size = 0.55

$$-\frac{1}{24}\left(\left(16x^2-\sqrt{x^2+1}\,(16x+3)+3x-8\right)\sqrt{x+\sqrt{x^2+1}}-2x+2\sqrt{x^2+1}-16\right)\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}-\frac{1}{16}\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}+1\right)+\frac{1}{16}\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x+(x^2+1)^(1/2))^(1/2),x, algo
rithm="fricas")
```

```
[Out] -1/24*((16*x^2 - sqrt(x^2 + 1)*(16*x + 3) + 3*x - 8)*sqrt(x + sqrt(x^2 + 1)
) - 2*x + 2*sqrt(x^2 + 1) - 16)*sqrt(sqrt(x + sqrt(x^2 + 1)) + 1) - 1/16*lo
g(sqrt(sqrt(x + sqrt(x^2 + 1)) + 1) + 1) + 1/16*log(sqrt(sqrt(x + sqrt(x^2
+ 1)) + 1) - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x+(x^2+1)^(1/2))^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}}{\sqrt{x+\sqrt{x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}}{\sqrt{x+\sqrt{x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x+(x^2+1)^(1/2))^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x + sqrt(x^2 + 1)) + 1)/sqrt(x + sqrt(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}}{\sqrt{x+\sqrt{x^2+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2)/(x + (x^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int(((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2)/(x + (x^2 + 1)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x+(x**2+1)**(1/2))**(1/2))**(1/2)/(x+(x**2+1)**(1/2))**(1/2),
x)
```

```
[Out] Integral(sqrt(sqrt(x + sqrt(x**2 + 1)) + 1)/sqrt(x + sqrt(x**2 + 1)), x)
```

3.1983 $\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx$

Optimal. Leaf size=198

$$\frac{\log\left(2\sqrt[3]{3x^2-6x+4}+2^{2/3}x-2\,2^{2/3}\right)}{3\,2^{2/3}}-\frac{\log\left(-\sqrt[3]{2}\,x^2-2\left(3x^2-6x+4\right)^{2/3}+\left(2^{2/3}x-2\,2^{2/3}\right)\sqrt[3]{3x^2-6x+4}+4\sqrt[3]{\cdots}\right)}{6\,2^{2/3}}$$

Rubi [A] time = 0.01, antiderivative size = 97, normalized size of antiderivative = 0.49, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {750}

$$\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{3x^2-6x+4}-3x+6\right)}{2\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{3x^2-6x+4}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2^(2/3)*(2 - x))/(Sqrt[3]*(4 - 6*x + 3*x^2)^(1/3))]/(
2^(2/3)*Sqrt[3])) - Log[x]/(2*2^(2/3)) + Log[6 - 3*x - 3*2^(1/3)*(4 - 6*x +
 3*x^2)^(1/3)]/(2*2^(2/3))
```

Rule 750

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[3*c*e^2*(2*c*d - b*e), 3]}, -Simp[(Sqrt[3]*c*e*ArcTan
[1/Sqrt[3] + (2*(c*d - b*e - c*e*x))/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3))])/
q^2, x] + (-Simp[(3*c*e*Log[d + e*x])/(2*q^2), x] + Simp[(3*c*e*Log[c*d - b
*e - c*e*x - q*(a + b*x + c*x^2)^(1/3)])/(2*q^2), x])] /; FreeQ[{a, b, c, d
, e}, x] && NeQ[2*c*d - b*e, 0] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*c*
e^2, 0] && PosQ[c*e^2*(2*c*d - b*e)]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{4-6x+3x^2}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\,2^{2/3}}+\frac{\log\left(6-3x-3\sqrt[3]{2}\sqrt[3]{4-6x+3x^2}\right)}{2\,2^{2/3}}$$

Mathematica [C] time = 0.06, size = 111, normalized size = 0.56

$$-\frac{\sqrt[3]{\frac{3x+i\sqrt{3}-3}{x}}\sqrt[3]{\frac{9x-3i\sqrt{3}-9}{x}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{3-i\sqrt{3}}{3x},\frac{3+i\sqrt{3}}{3x}\right)}{2\sqrt[3]{3x^2-6x+4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

```
[Out] -1/2*(((-3 + I*Sqrt[3] + 3*x)/x)^(1/3)*((-9 - (3*I)*Sqrt[3] + 9*x)/x)^(1/3)
*AppellF1[2/3, 1/3, 1/3, 5/3, (3 - I*Sqrt[3])/(3*x), (3 + I*Sqrt[3])/(3*x)]
)/(4 - 6*x + 3*x^2)^(1/3)
```

IntegrateAlgebraic [A] time = 0.23, size = 198, normalized size = 1.00

$$\frac{\log\left(2\sqrt[3]{3x^2-6x+4}+2^{2/3}x-2\,2^{2/3}\right)}{3\,2^{2/3}}-\frac{\log\left(-\sqrt[3]{2}x^2-2\left(3x^2-6x+4\right)^{2/3}+\left(2^{2/3}x-2\,2^{2/3}\right)\sqrt[3]{3x^2-6x+4}+4\sqrt[3]{2}x-4\sqrt[3]{2}\right)}{6\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{3x^2-6x+4}}{\sqrt{3}}-\frac{2^{2/3}x}{\sqrt{3}}+\frac{2\,2^{2/3}}{\sqrt{3}}}{\sqrt[3]{3x^2-6x+4}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

```
[Out] -(ArcTan[((2*2^(2/3))/Sqrt[3] - (2^(2/3)*x)/Sqrt[3] + (4 - 6*x + 3*x^2)^(1/
3)/Sqrt[3])/(4 - 6*x + 3*x^2)^(1/3)]/(2^(2/3)*Sqrt[3])) + Log[-2*2^(2/3) +
2^(2/3)*x + 2*(4 - 6*x + 3*x^2)^(1/3)]/(3*2^(2/3)) - Log[-4*2^(1/3) + 4*2^(
1/3)*x - 2^(1/3)*x^2 + (-2*2^(2/3) + 2^(2/3)*x)*(4 - 6*x + 3*x^2)^(1/3) - 2
*(4 - 6*x + 3*x^2)^(2/3)]/(6*2^(2/3))
```

fricas [A] time = 2.48, size = 171, normalized size = 0.86

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}x^3+2\cdot 4^{\frac{2}{3}}\left(3x^2-6x+4\right)^{\frac{2}{3}}(x-2)+4\left(3x^2-6x+4\right)^{\frac{1}{3}}\left(x^2-4x+4\right)\right)}{6\left(x^3-12x^2+24x-16\right)}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{1}{3}}(x-2)+2\left(3x^2-6x+4\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}\left(3x^2-6x+4\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(x^2-4x+4\right)-2\left(3x^2-6x+4\right)^{\frac{1}{3}}(x-2)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3)*x^3 + 2*4^(2/3)*(3
*x^2 - 6*x + 4)^(2/3)*(x - 2) + 4*(3*x^2 - 6*x + 4)^(1/3)*(x^2 - 4*x + 4))/
(x^3 - 12*x^2 + 24*x - 16)) + 1/12*4^(2/3)*log((4^(1/3)*(x - 2) + 2*(3*x^2
- 6*x + 4)^(1/3))/x) - 1/24*4^(2/3)*log((4^(2/3)*(3*x^2 - 6*x + 4)^(2/3) +
4^(1/3)*(x^2 - 4*x + 4) - 2*(3*x^2 - 6*x + 4)^(1/3)*(x - 2))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-6\,x+4\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)
```

maple [C] time = 13.52, size = 2395, normalized size = 12.10

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^2-6*x+4)^(1/3),x)
```

```
[Out] -1/6*ln((-120*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(
_Z^3-2)^2*x-320*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-36*(3*x
^2-6*x+4)^(2/3)+RootOf(_Z^3-2)*x^3-24*RootOf(_Z^3-2)*x^2+48*RootOf(_Z^3-2)*
x+10*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^3+480*RootOf(Roo
tOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x-32*RootOf(_Z^3-2)-240*RootOf(Ro
otOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2+8*RootOf(_Z^3-2)^3*RootOf(Ro
otOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+80*RootOf(RootOf(_Z^3-2)^2+2*_Z*
RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2+60*RootOf(RootOf(_Z^3-2)^2+2*_Z*R
ootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^2+6*RootOf(RootOf(_Z^3-2)^2+2*_Z
*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^2-20*RootOf(RootOf(_Z^3-2)^2+2*_
Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-2*RootOf(RootOf(_Z^3-2)^2+2
*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3+48*(3*x^2-6*x+4)^(2/3)*Root
```

```
Of(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x-48*(3*x^
2-6*x+4)^(1/3)*RootOf(_Z^3-2)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4
*_Z^2)*x^2+192*(3*x^2-6*x+4)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3
-2)+4*_Z^2)*RootOf(_Z^3-2)*x-12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)
+4*_Z^2)*RootOf(_Z^3-2)^3*x+18*(3*x^2-6*x+4)^(2/3)*x-36*RootOf(_Z^3-2)^2*(3
*x^2-6*x+4)^(1/3)-96*(3*x^2-6*x+4)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^
3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-9*(3*x^2-6*x+4)^(1/3)*RootOf(_Z^3-2)^2*x
^2+36*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)*x-192*(3*x^2-6*x+4)^(1/3)*RootOf
(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2))/x^3)*RootOf(_
Z^3-2)-1/3*ln((-120*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*R
ootOf(_Z^3-2)^2*x-320*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-3
6*(3*x^2-6*x+4)^(2/3)+RootOf(_Z^3-2)*x^3-24*RootOf(_Z^3-2)*x^2+48*RootOf(_Z
^3-2)*x+10*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^3+480*Root
Of(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x-32*RootOf(_Z^3-2)-240*Roo
tOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2+8*RootOf(_Z^3-2)^3*Roo
tOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+80*RootOf(RootOf(_Z^3-2)^2
+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2+60*RootOf(RootOf(_Z^3-2)^2+
2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^2+6*RootOf(RootOf(_Z^3-2)^
2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^2-20*RootOf(RootOf(_Z^3-2)
^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-2*RootOf(RootOf(_Z^3-
2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3+48*(3*x^2-6*x+4)^(2/3
)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x-48
*(3*x^2-6*x+4)^(1/3)*RootOf(_Z^3-2)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^
3-2)+4*_Z^2)*x^2+192*(3*x^2-6*x+4)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootO
f(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_
Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x+18*(3*x^2-6*x+4)^(2/3)*x-36*RootOf(_Z^3-2
)^2*(3*x^2-6*x+4)^(1/3)-96*(3*x^2-6*x+4)^(2/3)*RootOf(_Z^3-2)^2*RootOf(Root
Of(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-9*(3*x^2-6*x+4)^(1/3)*RootOf(_Z^3-
2)^2*x^2+36*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)*x-192*(3*x^2-6*x+4)^(1/3)*
RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2))/x^3)*Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+1/3*RootOf(RootOf(_Z^3-2)
^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*ln(-(120*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf
(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x-400*RootOf(RootOf(_Z^3-2)^2+2*_Z*Root
Of(_Z^3-2)+4*_Z^2)-60*(3*x^2-6*x+4)^(2/3)+12*RootOf(_Z^3-2)*x^3-120*RootOf(
_Z^3-2)*x^2+240*RootOf(_Z^3-2)*x+30*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^
3-2)+4*_Z^2)*x^3+600*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x-
160*RootOf(_Z^3-2)-300*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*
x^2-32*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)
-80*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2-
60*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x
^2-24*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*
x^2+20*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)
^2*x^3+8*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)
^3*x^3+48*(3*x^2-6*x+4)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+2*_Z
*RootOf(_Z^3-2)+4*_Z^2)*x-48*(3*x^2-6*x+4)^(1/3)*RootOf(_Z^3-2)*RootOf(Root
Of(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2+192*(3*x^2-6*x+4)^(1/3)*RootOf
(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x+48*RootOf(Ro
otOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x+30*(3*x^2-6*x
+4)^(2/3)*x-60*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)-96*(3*x^2-6*x+4)^(2/3)*
RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-15*(3*
x^2-6*x+4)^(1/3)*RootOf(_Z^3-2)^2*x^2+60*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/
3)*x-192*(3*x^2-6*x+4)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*
_Z^2)*RootOf(_Z^3-2))/x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^{2}-6\,x+4\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(3x^2-6x+4\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x^2 - 6*x + 4)^(1/3)),x)
```

```
[Out] int(1/(x*(3*x^2 - 6*x + 4)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{3x^2-6x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**2-6*x+4)**(1/3),x)
```

```
[Out] Integral(1/(x*(3*x**2 - 6*x + 4)**(1/3)), x)
```

3.1984 $\int \frac{x^2 \sqrt[4]{bx^3+ax^4}}{-b+ax}\,dx$

Optimal. Leaf size=198

$$-\frac{155b^3\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{64a^{15/4}}+\frac{2\sqrt[4]{2}\,b^3\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{a^{15/4}}+\frac{155b^3\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{64a^{15/4}}-\frac{2\sqrt[4]{2}\,b^3\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{a^{15/4}}+(32a$$

Rubi [A] time = 0.46, antiderivative size = 339, normalized size of antiderivative = 1.71, number of steps used = 27, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {2042, 101, 157, 50, 63, 331, 298, 203, 206, 105, 93}

$$-\frac{155b^3\sqrt[4]{ax^4+bx^3}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{64a^{15/4}x^{3/4}\sqrt[4]{ax+b}}+\frac{2\sqrt[4]{2}\,b^3\sqrt[4]{ax^4+bx^3}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{a^{15/4}x^{3/4}\sqrt[4]{ax+b}}+\frac{155b^3\sqrt[4]{ax^4+bx^3}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{64a^{15/4}x^{3/4}\sqrt[4]{ax+b}}-\frac{2\sqrt[4]{2}\,b^3\sqrt[4]{ax^4+bx^3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{a^{15/4}x^{3/4}\sqrt[4]{ax+b}}+\frac{101b^2\sqrt[4]{ax^4+bx^3}}{96a^3}+\frac{13bx\sqrt[4]{ax^4+bx^3}}{24a^2}+\frac{x^2\sqrt[4]{ax^4+bx^3}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b*x^3 + a*x^4)^(1/4))/(-b + a*x),x]
```

```
[Out] (101*b^2*(b*x^3 + a*x^4)^(1/4))/(96*a^3) + (13*b*x*(b*x^3 + a*x^4)^(1/4))/(
24*a^2) + (x^2*(b*x^3 + a*x^4)^(1/4))/(3*a) - (155*b^3*(b*x^3 + a*x^4)^(1/4
)*ArcTan[(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(64*a^(15/4)*x^(3/4)*(b + a*x)
^(1/4)) + (2*2^(1/4)*b^3*(b*x^3 + a*x^4)^(1/4)*ArcTan[(2^(1/4)*a^(1/4)*x^(1
/4))/(b + a*x)^(1/4)])/(a^(15/4)*x^(3/4)*(b + a*x)^(1/4)) + (155*b^3*(b*x^3
 + a*x^4)^(1/4)*ArcTanh[(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(64*a^(15/4)*x^
(3/4)*(b + a*x)^(1/4)) - (2*2^(1/4)*b^3*(b*x^3 + a*x^4)^(1/4)*ArcTanh[(2^(1
/4)*a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(a^(15/4)*x^(3/4)*(b + a*x)^(1/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
```

```
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\int \frac{x^2 \sqrt[4]{bx^3+ax^4}}{-b+ax}\,dx = \frac{\sqrt[4]{bx^3+ax^4}\int \frac{x^{11/4}\sqrt[4]{b+ax}}{-b+ax}\,dx}{x^{3/4}\sqrt[4]{b+ax}}$$

$$= \frac{x^2\sqrt[4]{bx^3+ax^4}}{3a} - \frac{\sqrt[4]{bx^3+ax^4}\int \frac{x^{7/4}\left(-\frac{11b^2}{4}-\frac{13abx}{4}\right)}{(-b+ax)(b+ax)^{3/4}}\,dx}{3ax^{3/4}\sqrt[4]{b+ax}}$$

$$= \frac{x^2\sqrt[4]{bx^3+ax^4}}{3a} + \frac{\left(13b\sqrt[4]{bx^3+ax^4}\right)\int \frac{x^{7/4}}{(b+ax)^{3/4}}\,dx}{12ax^{3/4}\sqrt[4]{b+ax}} + \frac{\left(2b^2\sqrt[4]{bx^3+ax^4}\right)\int \frac{x^{7/4}}{(-b+ax)(b+ax)^{3/4}}\,dx}{ax^{3/4}\sqrt[4]{b+ax}}$$

$$= \frac{13bx\sqrt[4]{bx^3+ax^4}}{24a^2} + \frac{x^2\sqrt[4]{bx^3+ax^4}}{3a} - \frac{\left(91b^2\sqrt[4]{bx^3+ax^4}\right)\int \frac{x^{3/4}}{(b+ax)^{3/4}}\,dx}{96a^2x^{3/4}\sqrt[4]{b+ax}} + \frac{\left(2b^2\sqrt[4]{bx^3+ax^4}\right.}{a^2x^{3/4}\sqrt[4]{b}}$$

$$= \frac{101b^2\sqrt[4]{bx^3+ax^4}}{96a^3} + \frac{13bx\sqrt[4]{bx^3+ax^4}}{24a^2} + \frac{x^2\sqrt[4]{bx^3+ax^4}}{3a} + \frac{\left(91b^3\sqrt[4]{bx^3+ax^4}\right)\int \frac{1}{\sqrt[4]{x}\,(b+ax)^{3/}}}{128a^3x^{3/4}\sqrt[4]{b+ax}}$$

$$= \frac{101b^2\sqrt[4]{bx^3+ax^4}}{96a^3} + \frac{13bx\sqrt[4]{bx^3+ax^4}}{24a^2} + \frac{x^2\sqrt[4]{bx^3+ax^4}}{3a} + \frac{\left(91b^3\sqrt[4]{bx^3+ax^4}\right)\text{Subst}\left(\int \frac{}{(b}\right.}{32a^3x^{3/4}\sqrt[4]{b+a}}$$

$$= \frac{101b^2\sqrt[4]{bx^3+ax^4}}{96a^3} + \frac{13bx\sqrt[4]{bx^3+ax^4}}{24a^2} + \frac{x^2\sqrt[4]{bx^3+ax^4}}{3a} - \frac{\left(2\sqrt{2}\,b^3\sqrt[4]{bx^3+ax^4}\right)\text{Subst}\left(\int\right.}{a^{7/2}x^{3/4}\sqrt[4]{b}}$$

$$= \frac{101b^2\sqrt[4]{bx^3+ax^4}}{96a^3} + \frac{13bx\sqrt[4]{bx^3+ax^4}}{24a^2} + \frac{x^2\sqrt[4]{bx^3+ax^4}}{3a} + \frac{2\sqrt[4]{2}\,b^3\sqrt[4]{bx^3+ax^4}\tan^{-1}\left(\frac{\sqrt[4]{2}}{\sqrt[4]{b}}\right.}{a^{15/4}x^{3/4}\sqrt[4]{b+ax}}$$

$$= \frac{101b^2\sqrt[4]{bx^3+ax^4}}{96a^3} + \frac{13bx\sqrt[4]{bx^3+ax^4}}{24a^2} + \frac{x^2\sqrt[4]{bx^3+ax^4}}{3a} - \frac{155b^3\sqrt[4]{bx^3+ax^4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{b+ax}}\right.}{64a^{15/4}x^{3/4}\sqrt[4]{b+ax}}$$

Mathematica [C] time = 0.17, size = 183, normalized size = 0.92

$$\frac{4x^3\left(21a^2x^2(ax+b)\,{}_2F_1\left(-\frac{1}{4},\frac{11}{4};\frac{15}{4};-\frac{ax}{b}\right)+77b^2(ax+b)\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{ax}{b}\right)+77b^2\left((ax+b)\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{ax}{b}\right)-2b\sqrt[4]{\frac{ax}{b}+1}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{2ax}{b+ax}\right)\right)+33abx(ax+b)\,{}_2F_1\left(-\frac{1}{4},\frac{7}{4};\frac{11}{4};-\frac{ax}{b}\right)\right)}{231a^3\left(x^3(ax+b)\right)^{3/4}\sqrt[4]{\frac{ax}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b*x^3 + a*x^4)^(1/4))/(-b + a*x),x]
```

```
[Out] (4*x^3*(77*b^2*(b + a*x)*Hypergeometric2F1[-1/4, 3/4, 7/4, -((a*x)/b)] + 33
*a*b*x*(b + a*x)*Hypergeometric2F1[-1/4, 7/4, 11/4, -((a*x)/b)] + 21*a^2*x^
2*(b + a*x)*Hypergeometric2F1[-1/4, 11/4, 15/4, -((a*x)/b)] + 77*b^2*((b +
a*x)*Hypergeometric2F1[3/4, 3/4, 7/4, -((a*x)/b)] - 2*b*(1 + (a*x)/b)^(1/4)
*Hypergeometric2F1[3/4, 1, 7/4, (2*a*x)/(b + a*x)])))/(231*a^3*(x^3*(b + a*
x))^(3/4)*(1 + (a*x)/b)^(1/4))
```

IntegrateAlgebraic [A] time = 0.86, size = 198, normalized size = 1.00

$$-\frac{155b^3\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{64a^{15/4}}+\frac{2\sqrt[4]{2}\,b^3\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{a^{15/4}}+\frac{155b^3\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{64a^{15/4}}-\frac{2\sqrt[4]{2}\,b^3\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{a^{15/4}}+\frac{\left(32a^2x^2+52abx+101b^2\right)\sqrt[4]{ax^4+bx^3}}{96a^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(b*x^3 + a*x^4)^(1/4))/(-b + a*x),x]
```

```
[Out] ((101*b^2 + 52*a*b*x + 32*a^2*x^2)*(b*x^3 + a*x^4)^(1/4))/(96*a^3) - (155*b
^3*ArcTan[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(64*a^(15/4)) + (2*2^(1/4)*b^
```

```
3*ArcTan[(2^(1/4)*a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/a^(15/4) + (155*b^3*Ar
cTanh[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(64*a^(15/4)) - (2*2^(1/4)*b^3*Ar
cTanh[(2^(1/4)*a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/a^(15/4)
```

fricas [B] time = 0.66, size = 489, normalized size = 2.47

$$\frac{1536\cdot 2^{\frac{1}{4}} a^{3} \left(\frac{b^{12}}{a^{15}}\right)^{\frac{1}{4}} \arctan\left(-\frac{2^{\frac{3}{4}} \left(ax^{4}+bx^{3}\right)^{\frac{1}{4}} a^{11} b^{3} \left(\frac{b^{12}}{a^{15}}\right)^{\frac{3}{4}} - 2^{\frac{3}{4}} a^{11} x \sqrt{\frac{\sqrt{2} a^{8} x^{2} \sqrt{\frac{b^{12}}{a^{15}}} + \sqrt{ax^{4}+bx^{3}}\, b^{6}}{x^{2}}} \left(\frac{b^{12}}{a^{15}}\right)^{\frac{3}{4}}}{2 b^{12} x}\right) - 384\cdot 2^{\frac{1}{4}} a^{3} \left(\frac{b^{12}}{a^{15}}\right)^{\frac{1}{4}} \log\left(\frac{2^{\frac{1}{4}} a^{4} x \left(\frac{b^{12}}{a^{15}}\right)^{\frac{1}{4}} + \left(ax^{4}+bx^{3}\right)^{\frac{1}{4}} b^{3}}{x}\right) + 384\cdot 2^{\frac{1}{4}} a^{3} \left(\frac{b^{12}}{a^{15}}\right)^{\frac{1}{4}} \log\left(-\frac{2^{\frac{1}{4}} a^{4} x \left(\frac{b^{12}}{a^{15}}\right)^{\frac{1}{4}} - \left(ax^{4}+bx^{3}\right)^{\frac{1}{4}} b^{3}}{x}\right) - 1860\, a^{3} \left(\frac{b^{12}}{a^{15}}\right)^{\frac{1}{4}} \arctan\left(-\frac{\left(ax^{4}+bx^{3}\right)^{\frac{1}{4}} a^{11} b^{3} \left(\frac{b^{12}}{a^{15}}\right)^{\frac{3}{4}} - a^{11} x \sqrt{\frac{a^{8} x^{2} \sqrt{\frac{b^{12}}{a^{15}}} + \sqrt{ax^{4}+bx^{3}}\, b^{6}}{x^{2}}} \left(\frac{b^{12}}{a^{15}}\right)^{\frac{3}{4}}}{b^{12} x}\right) + 465\, a^{3} \left(\frac{b^{12}}{a^{15}}\right)^{\frac{1}{4}} \log\left(\frac{155\left(a^{4} x \left(\frac{b^{12}}{a^{15}}\right)^{\frac{1}{4}} + \left(ax^{4}+bx^{3}\right)^{\frac{1}{4}} b^{3}\right)}{x}\right) - 465\, a^{3} \left(\frac{b^{12}}{a^{15}}\right)^{\frac{1}{4}} \log\left(-\frac{155\left(a^{4} x \left(\frac{b^{12}}{a^{15}}\right)^{\frac{1}{4}} - \left(ax^{4}+bx^{3}\right)^{\frac{1}{4}} b^{3}\right)}{x}\right) + 4\left(ax^{4}+bx^{3}\right)^{\frac{1}{4}} \left(32\, a^{2} x^{2} + 52\, abx + 101\, b^{2}\right)}{384\, a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^4+b*x^3)^(1/4)/(a*x-b),x, algorithm="fricas")
```

```
[Out] 1/384*(1536*2^(1/4)*a^3*(b^12/a^15)^(1/4)*arctan(-1/2*(2^(3/4)*(a*x^4 + b*x
^3)^(1/4)*a^11*b^3*(b^12/a^15)^(3/4) - 2^(3/4)*a^11*x*sqrt((sqrt(2)*a^8*x^2
*sqrt(b^12/a^15) + sqrt(a*x^4 + b*x^3)*b^6)/x^2)*(b^12/a^15)^(3/4))/(b^12*x
)) - 384*2^(1/4)*a^3*(b^12/a^15)^(1/4)*log((2^(1/4)*a^4*x*(b^12/a^15)^(1/4)
 + (a*x^4 + b*x^3)^(1/4)*b^3)/x) + 384*2^(1/4)*a^3*(b^12/a^15)^(1/4)*log(-(
2^(1/4)*a^4*x*(b^12/a^15)^(1/4) - (a*x^4 + b*x^3)^(1/4)*b^3)/x) - 1860*a^3*
(b^12/a^15)^(1/4)*arctan(-((a*x^4 + b*x^3)^(1/4)*a^11*b^3*(b^12/a^15)^(3/4)
 - a^11*x*sqrt((a^8*x^2*sqrt(b^12/a^15) + sqrt(a*x^4 + b*x^3)*b^6)/x^2)*(b^
12/a^15)^(3/4))/(b^12*x)) + 465*a^3*(b^12/a^15)^(1/4)*log(155*(a^4*x*(b^12/
a^15)^(1/4) + (a*x^4 + b*x^3)^(1/4)*b^3)/x) - 465*a^3*(b^12/a^15)^(1/4)*log
(-155*(a^4*x*(b^12/a^15)^(1/4) - (a*x^4 + b*x^3)^(1/4)*b^3)/x) + 4*(a*x^4 +
 b*x^3)^(1/4)*(32*a^2*x^2 + 52*a*b*x + 101*b^2))/a^3
```

giac [B] time = 2.86, size = 461, normalized size = 2.33

$$-\frac{2^{\frac{3}{4}} (-a)^{\frac{1}{4}} b^{3} \arctan\left(\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}} (-a)^{\frac{1}{4}} + 2\left(a+\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^{4}} - \frac{2^{\frac{3}{4}} (-a)^{\frac{1}{4}} b^{3} \arctan\left(-\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}} (-a)^{\frac{1}{4}} - 2\left(a+\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^{4}} - \frac{2^{\frac{3}{4}} (-a)^{\frac{1}{4}} b^{3} \log\left(2^{\frac{3}{4}} (-a)^{\frac{1}{4}} \left(a+\frac{b}{x}\right)^{\frac{1}{4}} + \sqrt{2}\sqrt{-a} + \sqrt{a+\frac{b}{x}}\right)}{2a^{4}} + \frac{2^{\frac{3}{4}} (-a)^{\frac{1}{4}} b^{3} \log\left(-2^{\frac{3}{4}} (-a)^{\frac{1}{4}} \left(a+\frac{b}{x}\right)^{\frac{1}{4}} + \sqrt{2}\sqrt{-a} + \sqrt{a+\frac{b}{x}}\right)}{2a^{4}} + \frac{155\sqrt{2} (-a)^{\frac{1}{4}} b^{3} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2} (-a)^{\frac{1}{4}} + 2\left(a+\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{128a^{4}} + \frac{155\sqrt{2} (-a)^{\frac{1}{4}} b^{3} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2} (-a)^{\frac{1}{4}} - 2\left(a+\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{128a^{4}} + \frac{155\sqrt{2} (-a)^{\frac{1}{4}} b^{3} \log\left(\sqrt{2} (-a)^{\frac{1}{4}} \left(a+\frac{b}{x}\right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a+\frac{b}{x}}\right)}{256a^{4}} + \frac{155\sqrt{2} b^{3} \log\left(-\sqrt{2} (-a)^{\frac{1}{4}} \left(a+\frac{b}{x}\right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a+\frac{b}{x}}\right)}{256(-a)^{\frac{3}{4}} a^{3}} + \frac{\left(101\left(a+\frac{b}{x}\right)^{\frac{9}{4}} b^{3} - 150\left(a+\frac{b}{x}\right)^{\frac{5}{4}} ab^{3} + 81\left(a+\frac{b}{x}\right)^{\frac{1}{4}} a^{2} b^{3}\right) x^{3}}{96a^{3}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^4+b*x^3)^(1/4)/(a*x-b),x, algorithm="giac")
```

```
[Out] -2^(3/4)*(-a)^(1/4)*b^3*arctan(1/2*2^(1/4)*(2^(3/4)*(-a)^(1/4) + 2*(a + b/x
)^(1/4))/(-a)^(1/4))/a^4 - 2^(3/4)*(-a)^(1/4)*b^3*arctan(-1/2*2^(1/4)*(2^(3
/4)*(-a)^(1/4) - 2*(a + b/x)^(1/4))/(-a)^(1/4))/a^4 - 1/2*2^(3/4)*(-a)^(1/4
)*b^3*log(2^(3/4)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(2)*sqrt(-a) + sqrt(a +
b/x))/a^4 + 1/2*2^(3/4)*(-a)^(1/4)*b^3*log(-2^(3/4)*(-a)^(1/4)*(a + b/x)^(1
/4) + sqrt(2)*sqrt(-a) + sqrt(a + b/x))/a^4 + 155/128*sqrt(2)*(-a)^(1/4)*b^
3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x)^(1/4))/(-a)^(1/4))/a
^4 + 155/128*sqrt(2)*(-a)^(1/4)*b^3*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4)
 - 2*(a + b/x)^(1/4))/(-a)^(1/4))/a^4 + 155/256*sqrt(2)*(-a)^(1/4)*b^3*log(
sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a + b/x))/a^4 + 155/25
6*sqrt(2)*b^3*log(-sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a +
 b/x))/((-a)^(3/4)*a^3) + 1/96*(101*(a + b/x)^(9/4)*b^3 - 150*(a + b/x)^(5/
4)*a*b^3 + 81*(a + b/x)^(1/4)*a^2*b^3)*x^3/(a^3*b^3)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a\,x^{4}+b\,x^{3}\right)^{\frac{1}{4}}}{ax-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^4+b*x^3)^(1/4)/(a*x-b),x)
```

```
[Out] int(x^2*(a*x^4+b*x^3)^(1/4)/(a*x-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{4}+bx^{3}\right)^{\frac{1}{4}} x^{2}}{ax-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x^4+b*x^3)^(1/4)/(a*x-b),x, algorithm="maxima")
[Out] integrate((a*x^4 + b*x^3)^(1/4)*x^2/(a*x - b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2\,{\left(a\,x^4+b\,x^3\right)}^{1/4}}{b-a\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a*x^4 + b*x^3)^(1/4))/(b - a*x),x)
[Out] -int((x^2*(a*x^4 + b*x^3)^(1/4))/(b - a*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\sqrt[4]{x^{3}\left(ax+b\right)}}{ax-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x**4+b*x**3)**(1/4)/(a*x-b),x)
[Out] Integral(x**2*(x**3*(a*x + b))**(1/4)/(a*x - b), x)
```

3.1985 $\int \frac{-1+x^6}{\sqrt[3]{x+x^5}\,(1+x^6)}\,dx$

Optimal. Leaf size=198

$$\frac{1}{3}\log\left(\sqrt[3]{x^5+x}-x\right)-\frac{\log\left(2^{2/3}\sqrt[3]{x^5+x}+2x\right)}{6\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^5+x}+x}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^5+x}-x}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{1}{6}\log\left(\sqrt[3]{x^5+x}x+(\right.$$

Rubi [F] time = 1.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+x^6}{\sqrt[3]{x+x^5}\,(1+x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x^6)/((x + x^5)^(1/3)*(1 + x^6)),x]
```

```
[Out] -((x*(1 + x^4)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6, (-2*x^4)/(1 - I*Sqrt[3]), -
x^4])/(x + x^5)^(1/3)) - (x*(1 + x^4)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6, (-2*
x^4)/(1 + I*Sqrt[3]), -x^4])/(x + x^5)^(1/3) + ((I - Sqrt[3])*x^3*(1 + x^4)
^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -x^4, (-2*x^4)/(1 - I*Sqrt[3])])/(4*(I +
Sqrt[3])*(x + x^5)^(1/3)) + ((I + Sqrt[3])*x^3*(1 + x^4)^(1/3)*AppellF1[2/3
, 1/3, 1, 5/3, -x^4, (-2*x^4)/(1 + I*Sqrt[3])])/(4*(I - Sqrt[3])*(x + x^5)^
(1/3)) + (3*x*(1 + x^4)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -x^4])/(2*(x
 + x^5)^(1/3)) - (x^(1/3)*(1 + x^4)^(1/3)*Defer[Subst][Defer[Int][1/((1 + x
)*(1 + x^6)^(1/3)), x], x, x^(2/3)])/(3*(x + x^5)^(1/3)) + ((1 + I*Sqrt[3])
*x^(1/3)*(1 + x^4)^(1/3)*Defer[Subst][Defer[Int][1/((-1 - I*Sqrt[3] + 2*x)*
(1 + x^6)^(1/3)), x], x, x^(2/3)])/(3*(x + x^5)^(1/3)) + ((1 - I*Sqrt[3])*x
^(1/3)*(1 + x^4)^(1/3)*Defer[Subst][Defer[Int][1/((-1 + I*Sqrt[3] + 2*x)*(1
 + x^6)^(1/3)), x], x, x^(2/3)])/(3*(x + x^5)^(1/3))
```

Rubi steps

$$\int \frac{-1+x^6}{\sqrt[3]{x+x^5}\left(1+x^6\right)}\,dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\int \frac{-1+x^6}{\sqrt[3]{x}\sqrt[3]{1+x^4}\left(1+x^6\right)}\,dx}{\sqrt[3]{x+x^5}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{-1+x^9}{\sqrt[3]{1+x^6}\left(1+x^9\right)}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{x+x^5}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1+x^6}} - \frac{2}{\sqrt[3]{1+x^6}\left(1+x^9\right)}\right)dx, x, x^{2/3}\right)}{2\sqrt[3]{x+x^5}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{x+x^5}} - \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^6}\left(1+x^9\right)}\,dx\right.}{\sqrt[3]{x+x^5}}$$

$$= \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} - \frac{\left(3\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \left(\frac{1}{9(1+x)\sqrt[3]{1+x^6}} + \frac{2-x}{9\left(1-x+x^2\right)\sqrt[3]{1+x^6}}\right.\right.}{\sqrt[3]{x+x^5}}$$

$$= \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} - \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{3\sqrt[3]{x+x^5}} - \frac{\left(\sqrt[3]{x}\right.}{}$$

$$= \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} - \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{3\sqrt[3]{x+x^5}} - \frac{\left(\sqrt[3]{x}\right.}{}$$

$$= \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} - \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{3\sqrt[3]{x+x^5}} - \frac{\left(\left(-1\right.\right.}{}$$

$$= \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} - \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{3\sqrt[3]{x+x^5}} - \frac{\left(\left(-1\right.\right.}{}$$

$$= \frac{3x\sqrt[3]{1+x^4}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^4\right)}{2\sqrt[3]{x+x^5}} - \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^4}\right)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt[3]{1+x^6}}\,dx, x, x^{2/3}\right)}{3\sqrt[3]{x+x^5}} - \frac{\left(\left(-1\right.\right.}{}$$

$$= -\frac{x\sqrt[3]{1+x^4}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^4}{1-i\sqrt{3}},-x^4\right)}{\sqrt[3]{x+x^5}} - \frac{x\sqrt[3]{1+x^4}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^4}{1+i\sqrt{3}},-x^4\right)}{\sqrt[3]{x+x^5}} + \frac{3x\sqrt[3]{1}}{}$$

$$= -\frac{x\sqrt[3]{1+x^4}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^4}{1-i\sqrt{3}},-x^4\right)}{\sqrt[3]{x+x^5}} - \frac{x\sqrt[3]{1+x^4}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^4}{1+i\sqrt{3}},-x^4\right)}{\sqrt[3]{x+x^5}} + \frac{\left(i-\right.}{}$$

<u>**Mathematica**</u> **[F]** time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{-1+x^6}{\sqrt[3]{x+x^5}\left(1+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^6)/((x + x^5)^(1/3)*(1 + x^6)),x]
[Out] Integrate[(-1 + x^6)/((x + x^5)^(1/3)*(1 + x^6)), x]
```

IntegrateAlgebraic [A] time = 1.16, size = 198, normalized size = 1.00

$$\frac{1}{3}\log\left(\sqrt[3]{x^5+x}-x\right)-\frac{\log\left(2^{2/3}\sqrt[3]{x^5+x}+2x\right)}{6\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^5+x}+x}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^5+x}-x}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{1}{6}\log\left(\sqrt[3]{x^5+x}\,x+\left(x^5+x\right)^{2/3}+x^2\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^5+x}\,x-\sqrt[3]{2}\left(x^5+x\right)^{2/3}-2x^2\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^6)/((x + x^5)^(1/3)*(1 + x^6)),x]
```

```
[Out] -(ArcTan[(Sqrt[3]*x)/(x + 2*(x + x^5)^(1/3))]/Sqrt[3]) - ArcTan[(Sqrt[3]*x)
/(-x + 2^(2/3)*(x + x^5)^(1/3))]/(2*2^(1/3)*Sqrt[3]) + Log[-x + (x + x^5)^(
1/3)]/3 - Log[2*x + 2^(2/3)*(x + x^5)^(1/3)]/(6*2^(1/3)) - Log[x^2 + x*(x +
 x^5)^(1/3) + (x + x^5)^(2/3)]/6 + Log[-2*x^2 + 2^(2/3)*x*(x + x^5)^(1/3) -
 2^(1/3)*(x + x^5)^(2/3)]/(12*2^(1/3))
```

fricas [B] time = 5.08, size = 397, normalized size = 2.01

$$\frac{1}{36}\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(x^8-14x^6+6x^4-14x^2+1\right)\left(x^5+x\right)^{\frac{2}{3}}-24\sqrt{6}(-1)^{\frac{1}{3}}\left(x^9+x^7+x^3+x\right)\left(x^5+x\right)^{\frac{1}{3}}+\sqrt{6}\,2^{\frac{1}{3}}\left(x^{12}+24x^{10}-57x^8+56x^6-57x^4+24x^2+1\right)\right)}{6\left(x^{12}-48x^{10}+15x^8-88x^6+15x^4-48x^2+1\right)}\right)-\frac{1}{72}2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(\frac{12\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^5-x^3+x\right)\left(x^5+x\right)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^8-14x^6+6x^4-14x^2+1\right)-6\left(x^5+x\right)^{\frac{2}{3}}\left(x^4-4x^2+1\right)}{x^8+4x^6+6x^4+4x^2+1}\right)+\frac{1}{36}2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^4+2x^2+1\right)-3\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^5+x\right)^{\frac{2}{3}}+6\left(x^5+x\right)^{\frac{1}{3}}x}{x^4+2x^2+1}\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^5+x\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{6}\log\left(\frac{x^4-x^2+3\left(x^5+x\right)^{\frac{1}{3}}x-3\left(x^5+x\right)^{\frac{2}{3}}+1}{x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^5+x)^(1/3)/(x^6+1),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(-1)^
(2/3)*(x^8 - 14*x^6 + 6*x^4 - 14*x^2 + 1)*(x^5 + x)^(2/3) - 24*sqrt(6)*(-1)
^(1/3)*(x^9 + x^7 + x^3 + x)*(x^5 + x)^(1/3) + sqrt(6)*2^(1/3)*(x^12 + 24*x
^10 - 57*x^8 + 56*x^6 - 57*x^4 + 24*x^2 + 1))/(x^12 - 48*x^10 + 15*x^8 - 88
*x^6 + 15*x^4 - 48*x^2 + 1)) - 1/72*2^(2/3)*(-1)^(1/3)*log((12*2^(1/3)*(-1)
^(2/3)*(x^5 - x^3 + x)*(x^5 + x)^(1/3) - 2^(2/3)*(-1)^(1/3)*(x^8 - 14*x^6 +
 6*x^4 - 14*x^2 + 1) - 6*(x^5 + x)^(2/3)*(x^4 - 4*x^2 + 1))/(x^8 + 4*x^6 +
6*x^4 + 4*x^2 + 1)) + 1/36*2^(2/3)*(-1)^(1/3)*log(-(2^(1/3)*(-1)^(2/3)*(x^4
 + 2*x^2 + 1) - 3*2^(2/3)*(-1)^(1/3)*(x^5 + x)^(2/3) + 6*(x^5 + x)^(1/3)*x)
/(x^4 + 2*x^2 + 1)) + 1/3*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^5 +
x)^(1/3))/x) + 1/6*log((x^4 - x^2 + 3*(x^5 + x)^(1/3)*x - 3*(x^5 + x)^(2/3)
 + 1)/(x^4 - x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6-1}{\left(x^6+1\right)\left(x^5+x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^5+x)^(1/3)/(x^6+1),x, algorithm="giac")
```

```
[Out] integrate((x^6 - 1)/((x^6 + 1)*(x^5 + x)^(1/3)), x)
```

maple [F] time = 22.59, size = 0, normalized size = 0.00

$$\int \frac{x^6-1}{\left(x^5+x\right)^{\frac{1}{3}}\left(x^6+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)/(x^5+x)^(1/3)/(x^6+1),x)
```

```
[Out] int((x^6-1)/(x^5+x)^(1/3)/(x^6+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6-1}{\left(x^6+1\right)\left(x^5+x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^5+x)^(1/3)/(x^6+1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 - 1)/((x^6 + 1)*(x^5 + x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6-1}{\left(x^6+1\right)\left(x^5+x\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)/((x^6 + 1)*(x + x^5)^(1/3)),x)
```

```
[Out] int((x^6 - 1)/((x^6 + 1)*(x + x^5)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}{\sqrt[3]{x\left(x^4+1\right)}\left(x^2+1\right)\left(x^4-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)/(x**5+x)**(1/3)/(x**6+1),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1)/((x*(x**4 + 1))**(1/
3)*(x**2 + 1)*(x**4 - x**2 + 1)), x)
```

3.1986 $\int \frac{\sqrt[4]{-bx^3+ax^4}\left(-d+cx^8\right)}{x^4}\,dx$

Optimal. Leaf size=198

$$\frac{1463b^6c\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^3}}\right)}{32768a^{23/4}}-\frac{1463b^6c\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-bx^3}}\right)}{32768a^{23/4}}+\frac{\sqrt[4]{ax^4-bx^3}\left(-262144a^7dx^2-65536a^6bdx+122880a\right.}{}$$

Rubi [A] time = 0.80, antiderivative size = 359, normalized size of antiderivative = 1.81, number of steps used = 16, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2052, 2016, 2014, 2021, 2024, 2032, 63, 331, 298, 203, 206}

$$\frac{1463b^6cx^{9/4}(ax-b)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax-b}}\right)}{32768a^{23/4}\left(ax^4-bx^3\right)^{3/4}}-\frac{1463b^6cx^{9/4}(ax-b)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax-b}}\right)}{32768a^{23/4}\left(ax^4-bx^3\right)^{3/4}}-\frac{1463b^5c\sqrt[4]{ax^4-bx^3}}{49152a^5}-\frac{209b^4cx\sqrt[4]{ax^4-bx^3}}{12288a^4}-\frac{19b^3cx^2\sqrt[4]{ax^4-bx^3}}{1536a^3}-\frac{19b^2cx^3\sqrt[4]{ax^4-bx^3}}{1920a^2}-\frac{16ad\left(ax^4-bx^3\right)^{5/4}}{45b^2x^5}-\frac{bcx^4\sqrt[4]{ax^4-bx^3}}{120a}+\frac{1}{6}cx^5\sqrt[4]{ax^4-bx^3}-\frac{4d\left(ax^4-bx^3\right)^{5/4}}{9bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-(b*x^3) + a*x^4)^(1/4)*(-d + c*x^8))/x^4,x]
```

```
[Out] (-1463*b^5*c*(-(b*x^3) + a*x^4)^(1/4))/(49152*a^5) - (209*b^4*c*x*(-(b*x^3)
 + a*x^4)^(1/4))/(12288*a^4) - (19*b^3*c*x^2*(-(b*x^3) + a*x^4)^(1/4))/(153
6*a^3) - (19*b^2*c*x^3*(-(b*x^3) + a*x^4)^(1/4))/(1920*a^2) - (b*c*x^4*(-(b
*x^3) + a*x^4)^(1/4))/(120*a) + (c*x^5*(-(b*x^3) + a*x^4)^(1/4))/6 - (4*d*(
-(b*x^3) + a*x^4)^(5/4))/(9*b*x^6) - (16*a*d*(-(b*x^3) + a*x^4)^(5/4))/(45*
b^2*x^5) + (1463*b^6*c*x^(9/4)*(-b + a*x)^(3/4)*ArcTan[(a^(1/4)*x^(1/4))/(-
b + a*x)^(1/4)])/(32768*a^(23/4)*(-(b*x^3) + a*x^4)^(3/4)) - (1463*b^6*c*x^
(9/4)*(-b + a*x)^(3/4)*ArcTanh[(a^(1/4)*x^(1/4))/(-b + a*x)^(1/4)])/(32768*
a^(23/4)*(-(b*x^3) + a*x^4)^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2052

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_S
ymbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ
[{a, b, c, j, m, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !Integer
Q[p] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{-bx^3+ax^4}\left(-d+cx^8\right)}{x^4}\,dx &= \int \left(-\frac{d\sqrt[4]{-bx^3+ax^4}}{x^4}+cx^4\sqrt[4]{-bx^3+ax^4}\right)dx \\
&= c\int x^4\sqrt[4]{-bx^3+ax^4}\,dx-d\int \frac{\sqrt[4]{-bx^3+ax^4}}{x^4}\,dx \\
&= \frac{1}{6}cx^5\sqrt[4]{-bx^3+ax^4}-\frac{4d\left(-bx^3+ax^4\right)^{5/4}}{9bx^6}-\frac{1}{24}(bc)\int \frac{x^7}{\left(-bx^3+ax^4\right)^{3/4}}\,dx-\cdots \\
&= -\frac{bcx^4\sqrt[4]{-bx^3+ax^4}}{120a}+\frac{1}{6}cx^5\sqrt[4]{-bx^3+ax^4}-\frac{4d\left(-bx^3+ax^4\right)^{5/4}}{9bx^6}-\cdots \\
&= -\frac{19b^2cx^3\sqrt[4]{-bx^3+ax^4}}{1920a^2}-\frac{bcx^4\sqrt[4]{-bx^3+ax^4}}{120a}+\frac{1}{6}cx^5\sqrt[4]{-bx^3+ax^4}-\cdots \\
&= -\frac{19b^3cx^2\sqrt[4]{-bx^3+ax^4}}{1536a^3}-\frac{19b^2cx^3\sqrt[4]{-bx^3+ax^4}}{1920a^2}-\frac{bcx^4\sqrt[4]{-bx^3+ax^4}}{120a}+\cdots \\
&= -\frac{209b^4cx\sqrt[4]{-bx^3+ax^4}}{12288a^4}-\frac{19b^3cx^2\sqrt[4]{-bx^3+ax^4}}{1536a^3}-\frac{19b^2cx^3\sqrt[4]{-bx^3+ax^4}}{1920a^2}-\cdots \\
&= -\frac{1463b^5c\sqrt[4]{-bx^3+ax^4}}{49152a^5}-\frac{209b^4cx\sqrt[4]{-bx^3+ax^4}}{12288a^4}-\frac{19b^3cx^2\sqrt[4]{-bx^3+ax^4}}{1536a^3}-\cdots \\
&= -\frac{1463b^5c\sqrt[4]{-bx^3+ax^4}}{49152a^5}-\frac{209b^4cx\sqrt[4]{-bx^3+ax^4}}{12288a^4}-\frac{19b^3cx^2\sqrt[4]{-bx^3+ax^4}}{1536a^3}-\cdots \\
&= -\frac{1463b^5c\sqrt[4]{-bx^3+ax^4}}{49152a^5}-\frac{209b^4cx\sqrt[4]{-bx^3+ax^4}}{12288a^4}-\frac{19b^3cx^2\sqrt[4]{-bx^3+ax^4}}{1536a^3}-\cdots \\
&= -\frac{1463b^5c\sqrt[4]{-bx^3+ax^4}}{49152a^5}-\frac{209b^4cx\sqrt[4]{-bx^3+ax^4}}{12288a^4}-\frac{19b^3cx^2\sqrt[4]{-bx^3+ax^4}}{1536a^3}-\cdots \\
&= -\frac{1463b^5c\sqrt[4]{-bx^3+ax^4}}{49152a^5}-\frac{209b^4cx\sqrt[4]{-bx^3+ax^4}}{12288a^4}-\frac{19b^3cx^2\sqrt[4]{-bx^3+ax^4}}{1536a^3}-\cdots \\
&= -\frac{1463b^5c\sqrt[4]{-bx^3+ax^4}}{49152a^5}-\frac{209b^4cx\sqrt[4]{-bx^3+ax^4}}{12288a^4}-\frac{19b^3cx^2\sqrt[4]{-bx^3+ax^4}}{1536a^3}-\cdots
\end{aligned}
$$

Mathematica [C] time = 0.39, size = 325, normalized size = 1.64

$$
-\frac{4\sqrt[4]{x^3(ax-b)}\left(4a^{10}dx^2\sqrt[4]{1-\frac{ax}{b}}+a^9bdx\sqrt[4]{1-\frac{ax}{b}}-5a^8b^2d\sqrt[4]{1-\frac{ax}{b}}-44a^2b^8cx^2\sqrt[4]{1-\frac{ax}{b}}+5b^{10}c\,{}_2F_1\left(-\frac{33}{4},-\frac{9}{4};-\frac{5}{4};\frac{ax}{b}\right)-40b^{10}c\,{}_2F_1\left(-\frac{29}{4},-\frac{9}{4};-\frac{5}{4};\frac{ax}{b}\right)+140b^{10}c\,{}_2F_1\left(-\frac{25}{4},-\frac{9}{4};-\frac{5}{4};\frac{ax}{b}\right)-280b^{10}c\,{}_2F_1\left(-\frac{21}{4},-\frac{9}{4};-\frac{5}{4};\frac{ax}{b}\right)+350b^{10}c\,{}_2F_1\left(-\frac{17}{4},-\frac{9}{4};-\frac{5}{4};\frac{ax}{b}\right)-280b^{10}c\,{}_2F_1\left(-\frac{13}{4},-\frac{9}{4};-\frac{5}{4};\frac{ax}{b}\right)+140b^{10}c\,{}_2F_1\left(-\frac{9}{4},-\frac{9}{4};-\frac{5}{4};\frac{ax}{b}\right)-35b^{10}c\sqrt[4]{1-\frac{ax}{b}}+79ab^9cx\sqrt[4]{1-\frac{ax}{b}}\right)}{45a^8b^2x^3\sqrt[4]{1-\frac{ax}{b}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((-(b*x^3) + a*x^4)^(1/4)*(-d + c*x^8))/x^4,x]
```

```
[Out] (-4*(x^3*(-b + a*x))^(1/4)*(-35*b^10*c*(1 - (a*x)/b)^(1/4) - 5*a^8*b^2*d*(1
 - (a*x)/b)^(1/4) + 79*a*b^9*c*x*(1 - (a*x)/b)^(1/4) + a^9*b*d*x*(1 - (a*x)
/b)^(1/4) - 44*a^2*b^8*c*x^2*(1 - (a*x)/b)^(1/4) + 4*a^10*d*x^2*(1 - (a*x)/
b)^(1/4) + 5*b^10*c*Hypergeometric2F1[-33/4, -9/4, -5/4, (a*x)/b] - 40*b^10
```

```
*c*Hypergeometric2F1[-29/4, -9/4, -5/4, (a*x)/b] + 140*b^10*c*Hypergeometri
c2F1[-25/4, -9/4, -5/4, (a*x)/b] - 280*b^10*c*Hypergeometric2F1[-21/4, -9/4
, -5/4, (a*x)/b] + 350*b^10*c*Hypergeometric2F1[-17/4, -9/4, -5/4, (a*x)/b]
 - 280*b^10*c*Hypergeometric2F1[-13/4, -9/4, -5/4, (a*x)/b] + 140*b^10*c*Hy
pergeometric2F1[-9/4, -9/4, -5/4, (a*x)/b]))/(45*a^8*b^2*x^3*(1 - (a*x)/b)^
(1/4))
```

IntegrateAlgebraic [A] time = 1.48, size = 198, normalized size = 1.00

$$\frac{1463b^6c\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-bx^3}}\right)}{32768a^{23/4}}-\frac{1463b^6c\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-bx^3}}\right)}{32768a^{23/4}}+\frac{\sqrt[4]{ax^4-bx^3}\left(-262144a^7dx^2-65536a^6bdx+122880a^5b^2cx^8+327680a^5b^2d-6144a^4b^3cx^7-7296a^3b^4cx^6-9120a^2b^5cx^5-12540ab^6cx^4-21945b^7cx^3\right)}{737280a^5b^2x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-(b*x^3) + a*x^4)^(1/4)*(-d + c*x^8))/x^4,x]
```

```
[Out] ((-(b*x^3) + a*x^4)^(1/4)*(327680*a^5*b^2*d - 65536*a^6*b*d*x - 262144*a^7*
d*x^2 - 21945*b^7*c*x^3 - 12540*a*b^6*c*x^4 - 9120*a^2*b^5*c*x^5 - 7296*a^3
*b^4*c*x^6 - 6144*a^4*b^3*c*x^7 + 122880*a^5*b^2*c*x^8))/(737280*a^5*b^2*x^
3) + (1463*b^6*c*ArcTan[(a^(1/4)*x)/(-(b*x^3) + a*x^4)^(1/4)])/(32768*a^(23
/4)) - (1463*b^6*c*ArcTanh[(a^(1/4)*x)/(-(b*x^3) + a*x^4)^(1/4)])/(32768*a^
(23/4))
```

fricas [B] time = 0.62, size = 390, normalized size = 1.97

$$\frac{263340\left(\frac{b^{24}c^4}{a^{23}}\right)^{\frac{1}{4}}a^5b^2x^3\arctan\left(-\frac{\left(\frac{b^{24}c^4}{a^{23}}\right)^{\frac{3}{4}}\left(ax^4-bx^3\right)^{\frac{1}{4}}a^{17}b^6c-\left(\frac{b^{24}c^4}{a^{23}}\right)^{\frac{3}{4}}a^{17}x\sqrt{\frac{\sqrt{ax^4-bx^3}b^{12}c^2+\sqrt{\frac{b^{24}c^4}{a^{23}}}a^{12}x^2}{x^2}}}{b^{24}c^4x}\right)-65835\left(\frac{b^{24}c^4}{a^{23}}\right)^{\frac{1}{4}}a^5b^2x^3\log\left(\frac{1463\left(\left(ax^4-bx^3\right)^{\frac{1}{4}}b^6c+\left(\frac{b^{24}c^4}{a^{23}}\right)^{\frac{1}{4}}a^6x\right)}{x}\right)+65835\left(\frac{b^{24}c^4}{a^{23}}\right)^{\frac{1}{4}}a^5b^2x^3\log\left(\frac{1463\left(\left(ax^4-bx^3\right)^{\frac{1}{4}}b^6c-\left(\frac{b^{24}c^4}{a^{23}}\right)^{\frac{1}{4}}a^6x\right)}{x}\right)+4\left(122880a^5b^2cx^8-6144a^4b^3cx^7-7296a^3b^4cx^6-9120a^2b^5cx^5-12540ab^6cx^4-21945b^7cx^3-262144a^7dx^2-65536a^6bdx+327680a^5b^2d\right)\left(ax^4-bx^3\right)^{\frac{1}{4}}}{2949120a^5b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x^3)^(1/4)*(c*x^8-d)/x^4,x, algorithm="fricas")
```

```
[Out] 1/2949120*(263340*(b^24*c^4/a^23)^(1/4)*a^5*b^2*x^3*arctan(-((b^24*c^4/a^23
)^(3/4)*(a*x^4 - b*x^3)^(1/4)*a^17*b^6*c - (b^24*c^4/a^23)^(3/4)*a^17*x*sqr
t((sqrt(a*x^4 - b*x^3)*b^12*c^2 + sqrt(b^24*c^4/a^23)*a^12*x^2)/x^2))/(b^24
*c^4*x)) - 65835*(b^24*c^4/a^23)^(1/4)*a^5*b^2*x^3*log(1463*((a*x^4 - b*x^3
)^(1/4)*b^6*c + (b^24*c^4/a^23)^(1/4)*a^6*x)/x) + 65835*(b^24*c^4/a^23)^(1/
4)*a^5*b^2*x^3*log(1463*((a*x^4 - b*x^3)^(1/4)*b^6*c - (b^24*c^4/a^23)^(1/4
)*a^6*x)/x) + 4*(122880*a^5*b^2*c*x^8 - 6144*a^4*b^3*c*x^7 - 7296*a^3*b^4*c
*x^6 - 9120*a^2*b^5*c*x^5 - 12540*a*b^6*c*x^4 - 21945*b^7*c*x^3 - 262144*a^
7*d*x^2 - 65536*a^6*b*d*x + 327680*a^5*b^2*d)*(a*x^4 - b*x^3)^(1/4))/(a^5*b
^2*x^3)
```

giac [B] time = 0.67, size = 373, normalized size = 1.88

$$\frac{\frac{131670\sqrt{2}b^7c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}a^5}+\frac{131670\sqrt{2}b^7c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}a^5}+\frac{65835\sqrt{2}b^7c\log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x}}\right)}{(-a)^{\frac{3}{4}}a^5}+\frac{65835\sqrt{2}(-a)^{\frac{1}{4}}b^7c\log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x}}\right)}{a^6}+\frac{24\left(7315\left(a-\frac{b}{x}\right)^{\frac{21}{4}}b^7c-40755\left(a-\frac{b}{x}\right)^{\frac{17}{4}}ab^7c+92910\left(a-\frac{b}{x}\right)^{\frac{13}{4}}a^2b^7c-109782\left(a-\frac{b}{x}\right)^{\frac{9}{4}}a^3b^7c+69327\left(a-\frac{b}{x}\right)^{\frac{5}{4}}a^4b^7c+21945\left(a-\frac{b}{x}\right)^{\frac{1}{4}}a^5b^7c\right)x^6}{a^5b^6}+\frac{524288\left(5\left(a-\frac{b}{x}\right)^{\frac{9}{4}}b^8d-9\left(a-\frac{b}{x}\right)^{\frac{5}{4}}ab^8d\right)}{b^9}}{5898240b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x^3)^(1/4)*(c*x^8-d)/x^4,x, algorithm="giac")
```

```
[Out] 1/5898240*(131670*sqrt(2)*b^7*c*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*
(a - b/x)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a^5) + 131670*sqrt(2)*b^7*c*arctan
(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(a - b/x)^(1/4))/(-a)^(1/4))/((-a)^(3
/4)*a^5) + 65835*sqrt(2)*b^7*c*log(sqrt(2)*(-a)^(1/4)*(a - b/x)^(1/4) + sqr
t(-a) + sqrt(a - b/x))/((-a)^(3/4)*a^5) + 65835*sqrt(2)*(-a)^(1/4)*b^7*c*lo
g(-sqrt(2)*(-a)^(1/4)*(a - b/x)^(1/4) + sqrt(-a) + sqrt(a - b/x))/a^6 + 24*
(7315*(a - b/x)^(21/4)*b^7*c - 40755*(a - b/x)^(17/4)*a*b^7*c + 92910*(a -
b/x)^(13/4)*a^2*b^7*c - 109782*(a - b/x)^(9/4)*a^3*b^7*c + 69327*(a - b/x)^
(5/4)*a^4*b^7*c + 21945*(a - b/x)^(1/4)*a^5*b^7*c)*x^6/(a^5*b^6) + 524288*(
5*(a - b/x)^(9/4)*b^8*d - 9*(a - b/x)^(5/4)*a*b^8*d)/b^9)/b
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{4}-b\,x^{3}\right)^{\frac{1}{4}} \left(c\,x^{8}-d\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b*x^3)^(1/4)*(c*x^8-d)/x^4,x)
[Out] int((a*x^4-b*x^3)^(1/4)*(c*x^8-d)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{8}-d\right)\left(ax^{4}-bx^{3}\right)^{\frac{1}{4}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x^3)^(1/4)*(c*x^8-d)/x^4,x, algorithm="maxima")
[Out] integrate((c*x^8 - d)*(a*x^4 - b*x^3)^(1/4)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(d-c\,x^{8}\right)\,\left(a\,x^{4}-b\,x^{3}\right)^{1/4}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((d - c*x^8)*(a*x^4 - b*x^3)^(1/4))/x^4,x)
[Out] -int(((d - c*x^8)*(a*x^4 - b*x^3)^(1/4))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^{3}\,(ax-b)}\,\left(cx^{8}-d\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b*x**3)**(1/4)*(c*x**8-d)/x**4,x)
[Out] Integral((x**3*(a*x - b))**(1/4)*(c*x**8 - d)/x**4, x)
```

3.1987 $\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^6} dx$

Optimal. Leaf size=198

$$\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{\sqrt{2}\sqrt{b}}\right)}{64\sqrt{2}\,b^{9/2}}+\frac{\sqrt{\sqrt{ax^2+b^2}+b}\,\left(a^{3/2}\left(14b^2x^2-70bx^2\sqrt{ax^2+b^2}\right)+105a^{5/2}x^4+\right.}{1920\sqrt{a}\,b^4x^5}$$

Rubi [F] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^6} dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b + Sqrt[b^2 + a*x^2]]/x^6,x]
```

```
[Out] Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/x^6, x]
```

Rubi steps

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^6} dx = \int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^6} dx$$

Mathematica [C] time = 0.25, size = 95, normalized size = 0.48

$$\frac{\sqrt{\sqrt{ax^2+b^2}+b}\left(\left(2b\sqrt{ax^2+b^2}+ax^2+2b^2\right){}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};\frac{b-\sqrt{b^2+ax^2}}{2b}\right)-20b^2\right)}{80b^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/x^6,x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 + a*x^2]]*(-20*b^2 + (2*b^2 + a*x^2 + 2*b*Sqrt[b^2 + a*x
^2])*Hypergeometric2F1[-5/2, 2, -3/2, (b - Sqrt[b^2 + a*x^2])/(2*b)]))/(80*
b^2*x^5)
```

IntegrateAlgebraic [A] time = 0.43, size = 166, normalized size = 0.84

$$\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{128\sqrt{2}\,b^{9/2}}+\frac{\sqrt{\sqrt{ax^2+b^2}+b}\,\left(a^{3/2}\left(14b^2x^2-70bx^2\sqrt{ax^2+b^2}\right)+105a^{5/2}x^4+\sqrt{a}\left(48b^3\sqrt{ax^2+b^2}-432b^4\right)\right)}{1920\sqrt{a}\,b^4x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + Sqrt[b^2 + a*x^2]]/x^6,x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 + a*x^2]]*(105*a^(5/2)*x^4 + Sqrt[a]*(-432*b^4 + 48*b^3*
Sqrt[b^2 + a*x^2]) + a^(3/2)*(14*b^2*x^2 - 70*b*x^2*Sqrt[b^2 + a*x^2])))/(1
920*Sqrt[a]*b^4*x^5) + (7*a^(5/2)*ArcTan[(Sqrt[a]*x)/(Sqrt[2]*Sqrt[b]*Sqrt[
b + Sqrt[b^2 + a*x^2]])])/(128*Sqrt[2]*b^(9/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^6,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^6,x, algorithm="giac")
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/x^6, x)
```

maple [C] time = 0.04, size = 31, normalized size = 0.16

$$-\frac{\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\ \mathrm{hypergeom}\left(\left[-\frac{5}{2},-\frac{1}{4},\frac{1}{4}\right],\left[-\frac{3}{2},\frac{1}{2}\right],-\frac{x^2a}{b^2}\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+(a*x^2+b^2)^(1/2))^(1/2)/x^6,x)
[Out] -1/5*(b^2)^(1/4)*2^(1/2)/x^5*hypergeom([-5/2,-1/4,1/4],[-3/2,1/2],-x^2*a/b^
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^6,x, algorithm="maxima")
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b+\sqrt{b^2+a\,x^2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/x^6,x)
[Out] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/x^6, x)
```

sympy [C] time = 1.58, size = 51, normalized size = 0.26

$$\frac{\sqrt{b}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_3F_2\left(\begin{matrix}-\frac{5}{2},-\frac{1}{4},\frac{1}{4}\\-\frac{3}{2},\frac{1}{2}\end{matrix}\middle|\frac{ax^2e^{i\pi}}{b^2}\right)}{20\pi x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x**2+b**2)**(1/2))**(1/2)/x**6,x)
```

```
[Out] sqrt(b)*gamma(-1/4)*gamma(1/4)*hyper((-5/2, -1/4, 1/4), (-3/2, 1/2), a*x**2
*exp_polar(I*pi)/b**2)/(20*pi*x**5)
```

3.1988 $\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^6\sqrt{b^2+ax^2}}\,dx$

Optimal. Leaf size=198

$$\frac{\sqrt{\sqrt{ax^2+b^2}+b}\left(a^{3/2}\left(210bx^2\sqrt{ax^2+b^2}-42b^2x^2\right)-315a^{5/2}x^4+\sqrt{a}\left(16b^4-144b^3\sqrt{ax^2+b^2}\right)\right)}{640\sqrt{a}\,b^5x^5}-\frac{63a^{5/2}\tan}{}$$

Rubi [F] time = 0.68, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^6\sqrt{b^2+ax^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b + Sqrt[b^2 + a*x^2]]/(x^6*Sqrt[b^2 + a*x^2]),x]
[Out] Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(x^6*Sqrt[b^2 + a*x^2]), x]
```

Rubi steps

$$\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^6\sqrt{b^2+ax^2}}\,dx=\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{x^6\sqrt{b^2+ax^2}}\,dx$$

Mathematica [C] time = 0.25, size = 81, normalized size = 0.41

$$-\frac{a^3x\,{}_2F_1\left(-\frac{5}{2},3;-\frac{3}{2};\frac{b-\sqrt{b^2+ax^2}}{2b}\right)}{20b^3\left(\sqrt{ax^2+b^2}-b\right)^3\sqrt{\sqrt{ax^2+b^2}+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b + Sqrt[b^2 + a*x^2]]/(x^6*Sqrt[b^2 + a*x^2]),x]
[Out] -1/20*(a^3*x*Hypergeometric2F1[-5/2, 3, -3/2, (b - Sqrt[b^2 + a*x^2])/(2*b)
])/(b^3*(-b + Sqrt[b^2 + a*x^2])^3*Sqrt[b + Sqrt[b^2 + a*x^2]])
```

IntegrateAlgebraic [A] time = 0.40, size = 166, normalized size = 0.84

$$\frac{\sqrt{\sqrt{ax^2+b^2}+b}\left(a^{3/2}\left(210bx^2\sqrt{ax^2+b^2}-42b^2x^2\right)-315a^{5/2}x^4+\sqrt{a}\left(16b^4-144b^3\sqrt{ax^2+b^2}\right)\right)}{640\sqrt{a}\,b^5x^5}-\frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{128\sqrt{2}\,b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + Sqrt[b^2 + a*x^2]]/(x^6*Sqrt[b^2 + a*x^2]),x]
[Out] (Sqrt[b + Sqrt[b^2 + a*x^2]]*(-315*a^(5/2)*x^4 + Sqrt[a]*(16*b^4 - 144*b^3*
Sqrt[b^2 + a*x^2]) + a^(3/2)*(-42*b^2*x^2 + 210*b*x^2*Sqrt[b^2 + a*x^2])))/
(640*Sqrt[a]*b^5*x^5) - (63*a^(5/2)*ArcTan[(Sqrt[a]*x)/(Sqrt[2]*Sqrt[b]*Sqr
t[b + Sqrt[b^2 + a*x^2]])])/(128*Sqrt[2]*b^(11/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^6/(a*x^2+b^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\sqrt{ax^2+b^2}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^6/(a*x^2+b^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(sqrt(a*x^2 + b^2)*x^6), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{a\,x^2+b^2}}}{x^6\sqrt{a\,x^2+b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+(a*x^2+b^2)^(1/2))^(1/2)/x^6/(a*x^2+b^2)^(1/2),x)
```

```
[Out] int((b+(a*x^2+b^2)^(1/2))^(1/2)/x^6/(a*x^2+b^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}}{\sqrt{ax^2+b^2}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x^2+b^2)^(1/2))^(1/2)/x^6/(a*x^2+b^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(b + sqrt(a*x^2 + b^2))/(sqrt(a*x^2 + b^2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b+\sqrt{b^2+a\,x^2}}}{x^6\,\sqrt{b^2+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(x^6*(a*x^2 + b^2)^(1/2)),x)
```

```
[Out] int((b + (a*x^2 + b^2)^(1/2))^(1/2)/(x^6*(a*x^2 + b^2)^(1/2)), x)
```

sympy [C] time = 1.99, size = 49, normalized size = 0.25

$$-\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_{3}F_{2}\left(\begin{matrix}-\frac{5}{2},\frac{1}{4},\frac{3}{4}\\-\frac{3}{2},\frac{1}{2}\end{matrix}\middle|\frac{ax^2e^{i\pi}}{b^2}\right)}{5\pi\sqrt{b}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+(a*x**2+b**2)**(1/2))**(1/2)/x**6/(a*x**2+b**2)**(1/2),x)
```

```
[Out] -gamma(1/4)*gamma(3/4)*hyper((-5/2, 1/4, 3/4), (-3/2, 1/2), a*x**2*exp_pola
r(I*pi)/b**2)/(5*pi*sqrt(b)*x**5)
```

3.1989 $\int \sqrt{d+\sqrt{c+\sqrt{b+ax}}}\,dx$

Optimal. Leaf size=198

$$\frac{8\left(35ax+35b-28c^2+36cd^2-16d^4\right)\sqrt{\sqrt{\sqrt{ax+b}+c}+d}}{315a}-\frac{64\left(2cd-d^3\right)\sqrt{\sqrt{ax+b}+c}\sqrt{\sqrt{\sqrt{ax+b}+c}+d}}{315a}$$

Rubi [A] time = 0.15, antiderivative size = 129, normalized size of antiderivative = 0.65, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {371, 1398, 772}

$$-\frac{8\left(c-3d^2\right)\left(\sqrt{\sqrt{ax+b}+c}+d\right)^{5/2}}{5a}+\frac{8d\left(c-d^2\right)\left(\sqrt{\sqrt{ax+b}+c}+d\right)^{3/2}}{3a}+\frac{8\left(\sqrt{\sqrt{ax+b}+c}+d\right)^{9/2}}{9a}-\frac{24d\left(\sqrt{\sqrt{ax+b}+c}+d\right)^{7/2}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + Sqrt[c + Sqrt[b + a*x]]],x]
```

```
[Out] (8*d*(c - d^2)*(d + Sqrt[c + Sqrt[b + a*x]])^(3/2))/(3*a) - (8*(c - 3*d^2)*
(d + Sqrt[c + Sqrt[b + a*x]])^(5/2))/(5*a) - (24*d*(d + Sqrt[c + Sqrt[b + a
*x]])^(7/2))/(7*a) + (8*(d + Sqrt[c + Sqrt[b + a*x]])^(9/2))/(9*a)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \sqrt{d+\sqrt{c+\sqrt{b+ax}}}\,dx = \frac{2\,\text{Subst}\left(\int x\sqrt{d+\sqrt{c+x}}\,dx, x, \sqrt{b+ax}\right)}{a}$$
$$= \frac{2\,\text{Subst}\left(\int \sqrt{d+\sqrt{x}}\,(-c+x)\,dx, x, c+\sqrt{b+ax}\right)}{a}$$
$$= \frac{4\,\text{Subst}\left(\int x\sqrt{d+x}\,\left(-c+x^2\right)\,dx, x, \sqrt{c+\sqrt{b+ax}}\right)}{a}$$
$$= \frac{4\,\text{Subst}\left(\int \left(-d\left(-c+d^2\right)\sqrt{d+x}+\left(-c+3d^2\right)(d+x)^{3/2}-3d(d+x)^{5/2}+(d+\right.\right.}{a}$$
$$= \frac{8d\left(c-d^2\right)\left(d+\sqrt{c+\sqrt{b+ax}}\right)^{3/2}}{3a} - \frac{8\left(c-3d^2\right)\left(d+\sqrt{c+\sqrt{b+ax}}\right)^{5/2}}{5a} - \frac{24}{}$$

Mathematica [A] time = 0.13, size = 114, normalized size = 0.58

$$\frac{8\left(\sqrt{\sqrt{ax+b}+c}+d\right)^{3/2}\left(24d^2\sqrt{\sqrt{ax+b}+c}-28c\sqrt{\sqrt{ax+b}+c}+35\sqrt{ax+b}\sqrt{\sqrt{ax+b}+c}-30d\sqrt{ax+b}+12cd-16d^3\right)}{315a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + Sqrt[c + Sqrt[b + a*x]]],x]
```

```
[Out] (8*(d + Sqrt[c + Sqrt[b + a*x]])^(3/2)*(12*c*d - 16*d^3 - 30*d*Sqrt[b + a*x
] - 28*c*Sqrt[c + Sqrt[b + a*x]] + 24*d^2*Sqrt[c + Sqrt[b + a*x]] + 35*Sqrt
[b + a*x]*Sqrt[c + Sqrt[b + a*x]]))/(315*a)
```

IntegrateAlgebraic [A] time = 0.13, size = 144, normalized size = 0.73

$$-\frac{8\left(-7c\sqrt{ax+b}+6d^2\sqrt{ax+b}-35(ax+b)+28c^2-36cd^2+16d^4\right)\sqrt{\sqrt{\sqrt{ax+b}+c}+d}}{315a}-\frac{8\sqrt{\sqrt{ax+b}+c}\sqrt{\sqrt{\sqrt{ax+b}+c}+d}\left(-5d\sqrt{ax+b}+16cd-8d^3\right)}{315a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[d + Sqrt[c + Sqrt[b + a*x]]],x]
```

```
[Out] (-8*Sqrt[c + Sqrt[b + a*x]]*(16*c*d - 8*d^3 - 5*d*Sqrt[b + a*x])*Sqrt[d + S
qrt[c + Sqrt[b + a*x]]])/(315*a) - (8*(28*c^2 - 36*c*d^2 + 16*d^4 - 7*c*Sqr
t[b + a*x] + 6*d^2*Sqrt[b + a*x] - 35*(b + a*x))*Sqrt[d + Sqrt[c + Sqrt[b +
 a*x]]])/(315*a)
```

fricas [A] time = 0.79, size = 94, normalized size = 0.47

$$-\frac{8\left(16\,d^4-36\,cd^2+28\,c^2-35\,ax+\left(6\,d^2-7\,c\right)\sqrt{ax+b}-\left(8\,d^3-16\,cd+5\,\sqrt{ax+b}\,d\right)\sqrt{c+\sqrt{ax+b}}-35\,b\right)\sqrt{d+\sqrt{c+\sqrt{ax+b}}}}{315\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+(c+(a*x+b)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -8/315*(16*d^4 - 36*c*d^2 + 28*c^2 - 35*a*x + (6*d^2 - 7*c)*sqrt(a*x + b) -
 (8*d^3 - 16*c*d + 5*sqrt(a*x + b)*d)*sqrt(c + sqrt(a*x + b)) - 35*b)*sqrt(
d + sqrt(c + sqrt(a*x + b)))/a
```

giac [B] time = 2.66, size = 444, normalized size = 2.24

$$\frac{8\left(35\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{9}{2}}-180\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{7}{2}}d+378\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{5}{2}}d^{2}-420\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{3}{2}}d^{3}+315\sqrt{d+\sqrt{c+\sqrt{ax+b}}}\,d^{4}-126c\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{5}{2}}+420c\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{3}{2}}d-630c\sqrt{d+\sqrt{c+\sqrt{ax+b}}}\,d^{2}+315c^{2}\sqrt{d+\sqrt{c+\sqrt{ax+b}}}+21\left(3\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{5}{2}}-10\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{3}{2}}d+15\sqrt{d+\sqrt{c+\sqrt{ax+b}}}\,d^{2}-15c\sqrt{d+\sqrt{c+\sqrt{ax+b}}}\right)c+3\left(15\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{7}{2}}\mathrm{sgn}\left(\sqrt{c+\sqrt{ax+b}}\right)-63\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{5}{2}}d\,\mathrm{sgn}\left(\sqrt{c+\sqrt{ax+b}}\right)+105\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{3}{2}}d^{2}\mathrm{sgn}\left(\sqrt{c+\sqrt{ax+b}}\right)-105\sqrt{d+\sqrt{c+\sqrt{ax+b}}}\,d^{3}\mathrm{sgn}\left(\sqrt{c+\sqrt{ax+b}}\right)-35c\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{3}{2}}\mathrm{sgn}\left(\sqrt{c+\sqrt{ax+b}}\right)+105c\sqrt{d+\sqrt{c+\sqrt{ax+b}}}\,d\,\mathrm{sgn}\left(\sqrt{c+\sqrt{ax+b}}\right)\right)d\right)}{315a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+(c+(a*x+b)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/315*(35*(d + sqrt(c + sqrt(a*x + b)))^(9/2) - 180*(d + sqrt(c + sqrt(a*x
+ b)))^(7/2)*d + 378*(d + sqrt(c + sqrt(a*x + b)))^(5/2)*d^2 - 420*(d + sqr
t(c + sqrt(a*x + b)))^(3/2)*d^3 + 315*sqrt(d + sqrt(c + sqrt(a*x + b)))*d^4
 - 126*c*(d + sqrt(c + sqrt(a*x + b)))^(5/2) + 420*c*(d + sqrt(c + sqrt(a*x
 + b)))^(3/2)*d - 630*c*sqrt(d + sqrt(c + sqrt(a*x + b)))*d^2 + 315*c^2*sqr
t(d + sqrt(c + sqrt(a*x + b))) + 21*(3*(d + sqrt(c + sqrt(a*x + b)))^(5/2)
- 10*(d + sqrt(c + sqrt(a*x + b)))^(3/2)*d + 15*sqrt(d + sqrt(c + sqrt(a*x
+ b)))*d^2 - 15*c*sqrt(d + sqrt(c + sqrt(a*x + b))))*c + 3*(15*(d + sqrt(c
+ sqrt(a*x + b)))^(7/2)*sgn(sqrt(c + sqrt(a*x + b))) - 63*(d + sqrt(c + sqr
t(a*x + b)))^(5/2)*d*sgn(sqrt(c + sqrt(a*x + b))) + 105*(d + sqrt(c + sqrt(
a*x + b)))^(3/2)*d^2*sgn(sqrt(c + sqrt(a*x + b))) - 105*sqrt(d + sqrt(c + s
qrt(a*x + b)))*d^3*sgn(sqrt(c + sqrt(a*x + b))) - 35*c*(d + sqrt(c + sqrt(a
*x + b)))^(3/2)*sgn(sqrt(c + sqrt(a*x + b))) + 105*c*sqrt(d + sqrt(c + sqrt
(a*x + b)))*d*sgn(sqrt(c + sqrt(a*x + b))))*d)/a
```

maple [A] time = 0.01, size = 93, normalized size = 0.47

$$\frac{\frac{8\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{9}{2}}}{9}-\frac{24d\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{7}{2}}}{7}+\frac{8\left(3d^{2}-c\right)\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{5}{2}}}{5}-\frac{8\left(d^{2}-c\right)d\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{3}{2}}}{3}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+(c+(a*x+b)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] 2/a*(4/9*(d+(c+(a*x+b)^(1/2))^(1/2))^(9/2)-12/7*d*(d+(c+(a*x+b)^(1/2))^(1/2
))^(7/2)+4/5*(3*d^2-c)*(d+(c+(a*x+b)^(1/2))^(1/2))^(5/2)-4/3*(d^2-c)*d*(d+(
c+(a*x+b)^(1/2))^(1/2))^(3/2))
```

maxima [A] time = 0.34, size = 92, normalized size = 0.46

$$\frac{8\left(35\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{9}{2}}-135\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{7}{2}}d+63\left(3d^{2}-c\right)\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{5}{2}}-105\left(d^{3}-cd\right)\left(d+\sqrt{c+\sqrt{ax+b}}\right)^{\frac{3}{2}}\right)}{315a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+(c+(a*x+b)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/315*(35*(d + sqrt(c + sqrt(a*x + b)))^(9/2) - 135*(d + sqrt(c + sqrt(a*x
+ b)))^(7/2)*d + 63*(3*d^2 - c)*(d + sqrt(c + sqrt(a*x + b)))^(5/2) - 105*(
d^3 - c*d)*(d + sqrt(c + sqrt(a*x + b)))^(3/2))/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d+\sqrt{c+\sqrt{b+a\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + (c + (b + a*x)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] int((d + (c + (b + a*x)^(1/2))^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d+\sqrt{c+\sqrt{ax+b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+(c+(a*x+b)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(d + sqrt(c + sqrt(a*x + b))), x)
```

3.1990 $\int \frac{1}{(-b+ax^2)\sqrt[3]{-x+x^3}}\,dx$

Optimal. Leaf size=199

$$-\frac{\log\left(x\sqrt[3]{a-b}+\sqrt[3]{b}\sqrt[3]{x^3-x}\right)}{2b^{2/3}\sqrt[3]{a-b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x\sqrt[3]{a-b}}{x\sqrt[3]{a-b}-2\sqrt[3]{b}\sqrt[3]{x^3-x}}\right)}{2b^{2/3}\sqrt[3]{a-b}}+\frac{\log\left(-\sqrt[3]{b}\sqrt[3]{x^3-x}\,x\sqrt[3]{a-b}+x^2(a-b)^{2/3}+b^{2/3}\right.}{4b^{2/3}\sqrt[3]{a-b}}$$

Rubi [A] time = 0.34, antiderivative size = 270, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2056, 466, 465, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{x}\sqrt[3]{x^2-1}\log\left(\frac{x^{2/3}\sqrt[3]{a-b}}{\sqrt[3]{x^2-1}}+\sqrt[3]{b}\right)}{2b^{2/3}\sqrt[3]{x^3-x}\sqrt[3]{a-b}}+\frac{\sqrt[3]{x}\sqrt[3]{x^2-1}\log\left(\frac{x^{4/3}(a-b)^{2/3}}{(x^2-1)^{2/3}}-\frac{\sqrt[3]{b}x^{2/3}\sqrt[3]{a-b}}{\sqrt[3]{x^2-1}}+b^{2/3}\right)}{4b^{2/3}\sqrt[3]{x^3-x}\sqrt[3]{a-b}}+\frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{x^2-1}\tan^{-1}\left(\frac{\sqrt[3]{b}-\frac{2x^{2/3}\sqrt[3]{a-b}}{\sqrt[3]{x^2-1}}}{\sqrt{3}\sqrt[3]{b}}\right)}{2b^{2/3}\sqrt[3]{x^3-x}\sqrt[3]{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-b + a*x^2)*(-x + x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*x^(1/3)*(-1 + x^2)^(1/3)*ArcTan[(b^(1/3) - (2*(a - b)^(1/3)*x^(2/3
))/(-1 + x^2)^(1/3))/(Sqrt[3]*b^(1/3))])/(2*(a - b)^(1/3)*b^(2/3)*(-x + x^3
)^(1/3)) - (x^(1/3)*(-1 + x^2)^(1/3)*Log[b^(1/3) + ((a - b)^(1/3)*x^(2/3))/
(-1 + x^2)^(1/3)])/(2*(a - b)^(1/3)*b^(2/3)*(-x + x^3)^(1/3)) + (x^(1/3)*(-
1 + x^2)^(1/3)*Log[b^(2/3) + ((a - b)^(2/3)*x^(4/3))/(-1 + x^2)^(2/3) - ((a
 - b)^(1/3)*b^(1/3)*x^(2/3))/(-1 + x^2)^(1/3)]])/(4*(a - b)^(1/3)*b^(2/3)*(-
x + x^3)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{\left(-b+ax^2\right)\sqrt[3]{-x+x^3}}\,dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\int \frac{1}{\sqrt[3]{x}\sqrt[3]{-1+x^2}\left(-b+ax^2\right)}\,dx}{\sqrt[3]{-x+x^3}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{-1+x^6}\left(-b+ax^6\right)}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-x+x^3}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x^3}\left(-b+ax^3\right)}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{-x+x^3}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{-b-(a-b)x^3}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2\sqrt[3]{-x+x^3}}$$

$$= \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{-\sqrt[3]{b}-\sqrt[3]{a-b}\,x}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2b^{2/3}\sqrt[3]{-x+x^3}} + \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{\cdots}{b\cdots}\right.}{2b^{2\cdots}}$$

$$= -\frac{\sqrt[3]{x}\sqrt[3]{-1+x^2}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{a-b}\,x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2\sqrt[3]{a-b}\,b^{2/3}\sqrt[3]{-x+x^3}} + \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{-\sqrt[3]{a-b}\sqrt[3]{b}+2(a-b\cdots}{b^{2/3}-\sqrt[3]{a-b}\sqrt[3]{b}\,x+(a-\cdots}\right.}{4\sqrt[3]{a-b}\,b^{2/3}\sqrt[3]{-x+x^3}}$$

$$= -\frac{\sqrt[3]{x}\sqrt[3]{-1+x^2}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{a-b}\,x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2\sqrt[3]{a-b}\,b^{2/3}\sqrt[3]{-x+x^3}} + \frac{\sqrt[3]{x}\sqrt[3]{-1+x^2}\log\left(b^{2/3}+\frac{(a-b)^{2/3}x^{4/3}}{\left(-1+x^2\right)^{2/3}}-\frac{\sqrt[3]{a-b}\cdots}{\sqrt[3]{-\cdots}}\right.}{4\sqrt[3]{a-b}\,b^{2/3}\sqrt[3]{-x+x^3}}$$

$$= \frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{-1+x^2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a-b}\,x^{2/3}}{\sqrt[3]{b}\sqrt[3]{-1+x^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{a-b}\,b^{2/3}\sqrt[3]{-x+x^3}} - \frac{\sqrt[3]{x}\sqrt[3]{-1+x^2}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{a-b}\,x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2\sqrt[3]{a-b}\,b^{2/3}\sqrt[3]{-x+x^3}} + \frac{\sqrt[3]{x}\cdots}{\cdots}$$

Mathematica [C] time = 0.03, size = 75, normalized size = 0.38

$$-\frac{3x\sqrt[3]{1-x^2}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{(a-b)x^2}{ax^2-b}\right)}{2b\sqrt[3]{x\left(x^2-1\right)}\sqrt[3]{1-\frac{ax^2}{b}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-b + a*x^2)*(-x + x^3)^(1/3)),x]
```

```
[Out] (-3*x*(1 - x^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, ((a - b)*x^2)/(-b +
a*x^2)])/(2*b*(x*(-1 + x^2))^(1/3)*(1 - (a*x^2)/b)^(1/3))
```

IntegrateAlgebraic [A] time = 0.51, size = 199, normalized size = 1.00

$$-\frac{\log\left(x\sqrt[3]{a-b}+\sqrt[3]{b}\sqrt[3]{x^3-x}\right)}{2b^{2/3}\sqrt[3]{a-b}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x\sqrt[3]{a-b}}{x\sqrt[3]{a-b}-2\sqrt[3]{b}\sqrt[3]{x^3-x}}\right)}{2b^{2/3}\sqrt[3]{a-b}} + \frac{\log\left(-\sqrt[3]{b}\sqrt[3]{x^3-x}\,x\sqrt[3]{a-b}+x^2(a-b)^{2/3}+b^{2/3}\left(x^3-x\right)^{2/3}\right)}{4b^{2/3}\sqrt[3]{a-b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-b + a*x^2)*(-x + x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(a - b)^(1/3)*x)/((a - b)^(1/3)*x - 2*b^(1/3)*(-x
+ x^3)^(1/3))])/(2*(a - b)^(1/3)*b^(2/3)) - Log[(a - b)^(1/3)*x + b^(1/3)*(
-x + x^3)^(1/3)]/(2*(a - b)^(1/3)*b^(2/3)) + Log[(a - b)^(2/3)*x^2 - (a - b
```

```
)^(1/3)*b^(1/3)*x*(-x + x^3)^(1/3) + b^(2/3)*(-x + x^3)^(2/3)]/(4*(a - b)^(
1/3)*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2-b)/(x^3-x)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.52, size = 195, normalized size = 0.98

$$-\frac{3\left(-ab^2+b^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a-b}{b}\right)^{\frac{1}{3}}+2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a-b}{b}\right)^{\frac{1}{3}}}\right)}{2\left(\sqrt{3}\,ab^2-\sqrt{3}\,b^3\right)}+\frac{\left(-ab^2+b^3\right)^{\frac{2}{3}}\log\left(\left(-\frac{a-b}{b}\right)^{\frac{2}{3}}+\left(-\frac{a-b}{b}\right)^{\frac{1}{3}}\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}\right)}{4\left(ab^2-b^3\right)}-\frac{\left(-\frac{a-b}{b}\right)^{\frac{2}{3}}\log\left(\left|-\left(-\frac{a-b}{b}\right)^{\frac{1}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right|\right)}{2\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2-b)/(x^3-x)^(1/3),x, algorithm="giac")
```

```
[Out] -3/2*(-a*b^2 + b^3)^(2/3)*arctan(1/3*sqrt(3)*((-(a - b)/b)^(1/3) + 2*(-1/x^
2 + 1)^(1/3))/(-(a - b)/b)^(1/3))/(sqrt(3)*a*b^2 - sqrt(3)*b^3) + 1/4*(-a*b
^2 + b^3)^(2/3)*log((-(a - b)/b)^(2/3) + (-(a - b)/b)^(1/3)*(-1/x^2 + 1)^(1
/3) + (-1/x^2 + 1)^(2/3))/(a*b^2 - b^3) - 1/2*(-(a - b)/b)^(2/3)*log(abs(-(
-(a - b)/b)^(1/3) + (-1/x^2 + 1)^(1/3)))/(a - b)
```

maple [F] time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,x^2-b\right)\left(x^3-x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2-b)/(x^3-x)^(1/3),x)
```

```
[Out] int(1/(a*x^2-b)/(x^3-x)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^2-b\right)\left(x^3-x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2-b)/(x^3-x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x^2 - b)*(x^3 - x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(x^3-x\right)^{1/3}\,\left(b-a\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^3 - x)^(1/3)*(b - a*x^2)),x)
```

```
[Out] -int(1/((x^3 - x)^(1/3)*(b - a*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x\left(x-1\right)\left(x+1\right)}\,\left(ax^{2}-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2-b)/(x**3-x)**(1/3),x)
[Out] Integral(1/((x*(x - 1)*(x + 1))**(1/3)*(a*x**2 - b)), x)
```

3.1991 $\int \frac{b+ax^4}{\left(-b+ax^4\right)\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx$

Optimal. Leaf size=199

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tan^{-1}\left(\frac{(1+i)x\sqrt[4]{2ab+c}\,\sqrt[4]{a^2x^8+b^2+cx^4}}{x^2\sqrt{2ab+c}-i\sqrt{a^2x^8+b^2+cx^4}}\right)}{\sqrt[4]{2ab+c}}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a^2x^8+b^2+cx^4}}{\sqrt[4]{2ab+c}}+\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{2ab+c}}{x\sqrt[4]{a^2x^8+b^2+cx^4}}\right)}{\sqrt[4]{2ab+c}}$$

Rubi [F] time = 0.62, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{b+ax^4}{\left(-b+ax^4\right)\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(b + a*x^4)/((-b + a*x^4)*(b^2 + c*x^4 + a^2*x^8)^(1/4)),x]
```

```
[Out] (x*(1 + (2*a^2*x^4)/(c - Sqrt[-4*a^2*b^2 + c^2]))^(1/4)*(1 + (2*a^2*x^4)/(c
 + Sqrt[-4*a^2*b^2 + c^2]))^(1/4)*AppellF1[1/4, 1/4, 1/4, 5/4, (-2*a^2*x^4)
/(c - Sqrt[-4*a^2*b^2 + c^2]), (-2*a^2*x^4)/(c + Sqrt[-4*a^2*b^2 + c^2])])/
(b^2 + c*x^4 + a^2*x^8)^(1/4) + 2*b*Defer[Int][1/((-b + a*x^4)*(b^2 + c*x^4
 + a^2*x^8)^(1/4)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{b+ax^4}{\left(-b+ax^4\right)\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx &= \int \left(\frac{1}{\sqrt[4]{b^2+cx^4+a^2x^8}}+\frac{2b}{\left(-b+ax^4\right)\sqrt[4]{b^2+cx^4+a^2x^8}}\right)dx\\
&= (2b)\int \frac{1}{\left(-b+ax^4\right)\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx+\int \frac{1}{\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx\\
&= (2b)\int \frac{1}{\left(-b+ax^4\right)\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx+\frac{\left(\sqrt[4]{1+\frac{2a^2x^4}{c-\sqrt{-4a^2b^2+c^2}}}\,\sqrt[4]{1+\frac{\cdots}{c+\cdots}}\cdots\right.}{\cdots}\\
&= \frac{x\sqrt[4]{1+\frac{2a^2x^4}{c-\sqrt{-4a^2b^2+c^2}}}\,\sqrt[4]{1+\frac{2a^2x^4}{c+\sqrt{-4a^2b^2+c^2}}}\,F_1\left(\frac{1}{4};\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{2a^2x^4}{c-\sqrt{-4a^2b^2+c^2}},-\cdots\right)}{\sqrt[4]{b^2+cx^4+a^2x^8}}
\end{aligned}$$

Mathematica [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{b+ax^4}{\left(-b+ax^4\right)\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(b + a*x^4)/((-b + a*x^4)*(b^2 + c*x^4 + a^2*x^8)^(1/4)),x]
```

```
[Out] Integrate[(b + a*x^4)/((-b + a*x^4)*(b^2 + c*x^4 + a^2*x^8)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 3.22, size = 207, normalized size = 1.04

$$-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tan^{-1}\left(\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{2ab+c}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a^2x^8+b^2+cx^4}}{\sqrt[4]{2ab+c}}}{x\sqrt[4]{a^2x^8+b^2+cx^4}}\right)}{\sqrt[4]{2ab+c}}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a^2x^8+b^2+cx^4}}{\sqrt[4]{2ab+c}}+\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{2ab+c}}{x\sqrt[4]{a^2x^8+b^2+cx^4}}\right)}{\sqrt[4]{2ab+c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^4)/((-b + a*x^4)*(b^2 + c*x^4 + a^2*x^8)^(1/4))
,x]
```

```
[Out] ((-1/4 - I/4)*ArcTan[((1/2 - I/2)*(2*a*b + c)^(1/4)*x^2 - ((1/2 + I/2)*Sqrt
[b^2 + c*x^4 + a^2*x^8])/(2*a*b + c)^(1/4))/(x*(b^2 + c*x^4 + a^2*x^8)^(1/4
))])/(2*a*b + c)^(1/4) - ((1/4 + I/4)*ArcTanh[((1/2 - I/2)*(2*a*b + c)^(1/4
)*x^2 + ((1/2 + I/2)*Sqrt[b^2 + c*x^4 + a^2*x^8])/(2*a*b + c)^(1/4))/(x*(b^
2 + c*x^4 + a^2*x^8)^(1/4))])/(2*a*b + c)^(1/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)/(a*x^4-b)/(a^2*x^8+c*x^4+b^2)^(1/4),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4+b}{\left(a^2x^8+cx^4+b^2\right)^{\frac{1}{4}}\left(ax^4-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)/(a*x^4-b)/(a^2*x^8+c*x^4+b^2)^(1/4),x, algorithm="giac"
)
```

```
[Out] integrate((a*x^4 + b)/((a^2*x^8 + c*x^4 + b^2)^(1/4)*(a*x^4 - b)), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{a\,x^4+b}{\left(a\,x^4-b\right)\left(a^2x^8+c\,x^4+b^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b)/(a*x^4-b)/(a^2*x^8+c*x^4+b^2)^(1/4),x)
```

```
[Out] int((a*x^4+b)/(a*x^4-b)/(a^2*x^8+c*x^4+b^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4+b}{\left(a^2x^8+cx^4+b^2\right)^{\frac{1}{4}}\left(ax^4-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)/(a*x^4-b)/(a^2*x^8+c*x^4+b^2)^(1/4),x, algorithm="maxim
a")
```

```
[Out] integrate((a*x^4 + b)/((a^2*x^8 + c*x^4 + b^2)^(1/4)*(a*x^4 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a\,x^4+b}{\left(b-a\,x^4\right)\,{\left(a^2\,x^8+b^2+c\,x^4\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x^4)/((b - a*x^4)*(c*x^4 + b^2 + a^2*x^8)^(1/4)),x)
```

```
[Out] int(-(b + a*x^4)/((b - a*x^4)*(c*x^4 + b^2 + a^2*x^8)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4+b}{\left(ax^4-b\right)\sqrt[4]{a^2x^8+b^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b)/(a*x**4-b)/(a**2*x**8+c*x**4+b**2)**(1/4),x)
```

```
[Out] Integral((a*x**4 + b)/((a*x**4 - b)*(a**2*x**8 + b**2 + c*x**4)**(1/4)), x)
```

3.1992 $\int \frac{2x^4-x^9}{\sqrt{-1+x^5}\,(a-ax^5+x^{10})}\,dx$

Optimal. Leaf size=199

$$-\frac{\sqrt{2}\left(a+\sqrt{a-4}\sqrt{a}-4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x^5-1}}{\sqrt{-a-\sqrt{a-4}\sqrt{a}+2}}\right)}{5\sqrt{-a-\sqrt{a-4}\sqrt{a}+2}\sqrt{a-4}\sqrt{a}}-\frac{\sqrt{2}\left(-a+\sqrt{a-4}\sqrt{a}+4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x^5-1}}{\sqrt{-a+\sqrt{a-4}\sqrt{a}+2}}\right)}{5\sqrt{-a+\sqrt{a-4}\sqrt{a}+2}\sqrt{a-4}\sqrt{a}}$$

Rubi [A] time = 0.45, antiderivative size = 60, normalized size of antiderivative = 0.30, number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1593, 6715, 826, 1164, 628}

$$\frac{\log\left(\sqrt{a}\sqrt{x^5-1}+x^5\right)}{5\sqrt{a}}-\frac{\log\left(x^5-\sqrt{a}\sqrt{x^5-1}\right)}{5\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x^4 - x^9)/(Sqrt[-1 + x^5]*(a - a*x^5 + x^10)),x]
```

```
[Out] -1/5*Log[x^5 - Sqrt[a]*Sqrt[-1 + x^5]]/Sqrt[a] + Log[x^5 + Sqrt[a]*Sqrt[-1
+ x^5]]/(5*Sqrt[a])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2x^4-x^9}{\sqrt{-1+x^5}\left(a-ax^5+x^{10}\right)}\,dx &= \int \frac{x^4\left(2-x^5\right)}{\sqrt{-1+x^5}\left(a-ax^5+x^{10}\right)}\,dx \\
&= \frac{1}{5}\operatorname{Subst}\left(\int \frac{2-x}{\sqrt{-1+x}\left(a-ax+x^2\right)}\,dx, x, x^5\right) \\
&= \frac{2}{5}\operatorname{Subst}\left(\int \frac{1-x^2}{1+(2-a)x^2+x^4}\,dx, x, \sqrt{-1+x^5}\right) \\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{a}+2x}{-1-\sqrt{a}\,x-x^2}\,dx, x, \sqrt{-1+x^5}\right)}{5\sqrt{a}} - \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a}-2x}{-1+\sqrt{a}\,x-x^2}\,dx, x, \sqrt{-1+\right.}{5\sqrt{a}} \\
&= -\frac{\log\left(x^5-\sqrt{a}\sqrt{-1+x^5}\right)}{5\sqrt{a}} + \frac{\log\left(x^5+\sqrt{a}\sqrt{-1+x^5}\right)}{5\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.27, size = 161, normalized size = 0.81

$$
-\frac{1}{5}\sqrt{2}\left(\frac{\left(\sqrt{\frac{a-4}{a}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x^5-1}}{\sqrt{-a-\sqrt{a-4}\sqrt{a}+2}}\right)}{\sqrt{-a-\sqrt{a-4}\sqrt{a}+2}} - \frac{\left(\sqrt{\frac{a-4}{a}}-1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x^5-1}}{\sqrt{-a+\sqrt{a-4}\sqrt{a}+2}}\right)}{\sqrt{-a+\sqrt{a-4}\sqrt{a}+2}}\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*x^4 - x^9)/(Sqrt[-1 + x^5]*(a - a*x^5 + x^10)),x]
```

```
[Out] -1/5*(Sqrt[2]*(((1 + Sqrt[(-4 + a)/a])*ArcTan[(Sqrt[2]*Sqrt[-1 + x^5])/Sqrt
[2 - Sqrt[-4 + a]*Sqrt[a] - a]])/Sqrt[2 - Sqrt[-4 + a]*Sqrt[a] - a] - ((-1
+ Sqrt[(-4 + a)/a])*ArcTan[(Sqrt[2]*Sqrt[-1 + x^5])/Sqrt[2 + Sqrt[-4 + a]*S
qrt[a] - a]])/Sqrt[2 + Sqrt[-4 + a]*Sqrt[a] - a]))
```

IntegrateAlgebraic [A] time = 0.40, size = 199, normalized size = 1.00

$$
-\frac{\sqrt{2}\left(a+\sqrt{a-4}\sqrt{a}-4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x^5-1}}{\sqrt{-a-\sqrt{a-4}\sqrt{a}+2}}\right)}{5\sqrt{-a-\sqrt{a-4}\sqrt{a}+2}\sqrt{a-4}\sqrt{a}} - \frac{\sqrt{2}\left(-a+\sqrt{a-4}\sqrt{a}+4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x^5-1}}{\sqrt{-a+\sqrt{a-4}\sqrt{a}+2}}\right)}{5\sqrt{-a+\sqrt{a-4}\sqrt{a}+2}\sqrt{a-4}\sqrt{a}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*x^4 - x^9)/(Sqrt[-1 + x^5]*(a - a*x^5 + x^10)),x]
```

```
[Out] -1/5*(Sqrt[2]*(-4 + Sqrt[-4 + a]*Sqrt[a] + a)*ArcTan[(Sqrt[2]*Sqrt[-1 + x^5
])/Sqrt[2 - Sqrt[-4 + a]*Sqrt[a] - a]])/(Sqrt[2 - Sqrt[-4 + a]*Sqrt[a] - a]
*Sqrt[-4 + a]*Sqrt[a]) - (Sqrt[2]*(4 + Sqrt[-4 + a]*Sqrt[a] - a)*ArcTan[(Sq
rt[2]*Sqrt[-1 + x^5])/Sqrt[2 + Sqrt[-4 + a]*Sqrt[a] - a]])/(5*Sqrt[2 + Sqrt
[-4 + a]*Sqrt[a] - a]*Sqrt[-4 + a]*Sqrt[a])
```

fricas [A] time = 0.90, size = 86, normalized size = 0.43

$$
\left[\frac{\log\left(\frac{x^{10}+2\sqrt{x^5-1}\sqrt{a}\,x^5+ax^5-a}{x^{10}-ax^5+a}\right)}{5\sqrt{a}}, -\frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{x^5-1}\sqrt{-a}\,x^5}{ax^5-a}\right)}{5\,a}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^9+2*x^4)/(x^5-1)^(1/2)/(x^10-a*x^5+a),x, algorithm="fricas")
```

```
[Out] [1/5*log((x^10 + 2*sqrt(x^5 - 1)*sqrt(a)*x^5 + a*x^5 - a)/(x^10 - a*x^5 + a
))/sqrt(a), -2/5*sqrt(-a)*arctan(sqrt(x^5 - 1)*sqrt(-a)*x^5/(a*x^5 - a))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^9+2*x^4)/(x^5-1)^(1/2)/(x^10-a*x^5+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a]=[88]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [a]=[
-46]Undef/Unsigned Inf encountered in limitLimit: Max order reached or unab
le to make series expansion Error: Bad Argument Value
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{-x^{9}+2x^{4}}{\sqrt{x^{5}-1}\,\left(x^{10}-a\,x^{5}+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^9+2*x^4)/(x^5-1)^(1/2)/(x^10-a*x^5+a),x)
```

```
[Out] int((-x^9+2*x^4)/(x^5-1)^(1/2)/(x^10-a*x^5+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{9}-2\,x^{4}}{\left(x^{10}-ax^{5}+a\right)\sqrt{x^{5}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^9+2*x^4)/(x^5-1)^(1/2)/(x^10-a*x^5+a),x, algorithm="maxima")
```

```
[Out] -integrate((x^9 - 2*x^4)/((x^10 - a*x^5 + a)*sqrt(x^5 - 1)), x)
```

mupad [B] time = 2.24, size = 47, normalized size = 0.24

$$\frac{\ln\left(\frac{a\,x^5-a+x^{10}+2\,\sqrt{a}\,x^5\,\sqrt{x^5-1}}{x^{10}-a\,x^5+a}\right)}{5\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4 - x^9)/((x^5 - 1)^(1/2)*(a - a*x^5 + x^10)),x)
```

```
[Out] log((a*x^5 - a + x^10 + 2*a^(1/2)*x^5*(x^5 - 1)^(1/2))/(a - a*x^5 + x^10))/
(5*a^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**9+2*x**4)/(x**5-1)**(1/2)/(x**10-a*x**5+a),x)
```

```
[Out] Timed out
```

3.1993 $\int \frac{1+x^2}{(-1+x^2)\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=199

$$\sqrt{\sqrt{x^2+1}+x}-\frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}}+2\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{\sqrt{2}-1}}\right)-2\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{1+\sqrt{2}}}\right)$$

Rubi [A] time = 0.56, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6725, 2117, 14, 2119, 1628, 828, 826, 1166, 204, 206, 207, 203}

$$\sqrt{\sqrt{x^2+1}+x}-\frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}}-\frac{2\tan^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{1+\sqrt{2}}}+\frac{2\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{\sqrt{2}-1}}-\frac{2\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{1+\sqrt{2}}}+\frac{2\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{\sqrt{2}-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/((-1 + x^2)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/3*1/(x + Sqrt[1 + x^2])^(3/2) + Sqrt[x + Sqrt[1 + x^2]] - (2*ArcTan[Sqrt
[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]])/Sqrt[1 + Sqrt[2]] + (2*ArcTan[Sqrt
[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]])/Sqrt[-1 + Sqrt[2]] - (2*ArcTanh[Sqr
t[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]])/Sqrt[1 + Sqrt[2]] + (2*ArcTanh[Sq
rt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]])/Sqrt[-1 + Sqrt[2]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^2}{\left(-1+x^2\right)\sqrt{x+\sqrt{1+x^2}}}\,dx = \int \left(\frac{1}{\sqrt{x+\sqrt{1+x^2}}} + \frac{2}{\left(-1+x^2\right)\sqrt{x+\sqrt{1+x^2}}}\right) dx$$

$$= 2\int \frac{1}{\left(-1+x^2\right)\sqrt{x+\sqrt{1+x^2}}}\,dx + \int \frac{1}{\sqrt{x+\sqrt{1+x^2}}}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1+x^2}{x^{5/2}}\,dx, x, x+\sqrt{1+x^2}\right) + 2\int \left(-\frac{1}{2(1-x)\sqrt{x+\sqrt{1+x^2}}} - \right.$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{x^{5/2}} + \frac{1}{\sqrt{x}}\right) dx, x, x+\sqrt{1+x^2}\right) - \int \frac{1}{(1-x)\sqrt{x+\sqrt{1+x^2}}}\,d$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} - \text{Subst}\left(\int \frac{1+x^2}{x^{3/2}\left(1+2x-x^2\right)}\,dx, x,\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} - \text{Subst}\left(\int \left(-\frac{1}{x^{3/2}} + \frac{2(1+x)}{x^{3/2}\left(1+2x-\right.}\right.\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} - 2\,\text{Subst}\left(\int \frac{1+x}{x^{3/2}\left(1+2x-x^2\right)}\,dx,\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} - 2\,\text{Subst}\left(\int \frac{-1+x}{\sqrt{x}\left(1+2x-x^2\right)}\,dx,\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} - 4\,\text{Subst}\left(\int \frac{-1+x^2}{1+2x^2-x^4}\,dx, x, \sqrt{x}\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} - 2\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}-x^2}\,dx, x, \sqrt{x}\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} + \frac{2\tan^{-1}\left(\frac{\sqrt{x+\sqrt{1+x^2}}}{\sqrt{-1+\sqrt{2}}}\right)}{\sqrt{-1+\sqrt{2}}} - \frac{2\tan^{-1}\left(\sqrt{\right.}}{\sqrt{1+}}$$

Mathematica [A] time = 0.46, size = 233, normalized size = 1.17

$$\sqrt{\sqrt{x^2+1}+x} - \frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}} - \left(\left(\sqrt{2}-2\right)\sqrt{2\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{1}{\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}}\right)\right) - \sqrt{2\left(\sqrt{2}-1\right)}\left(2+\sqrt{2}\right)\tan^{-1}\left(\frac{1}{\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}}\right) + \left(\sqrt{2}-2\right)\sqrt{2\left(1+\sqrt{2}\right)}\tanh^{-1}\left(\frac{1}{\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}}\right) + \sqrt{2\left(\sqrt{2}-1\right)}\left(2+\sqrt{2}\right)\tanh^{-1}\left(\frac{1}{\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2)/((-1 + x^2)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/3*1/(x + Sqrt[1 + x^2])^(3/2) + Sqrt[x + Sqrt[1 + x^2]] - (-2 + Sqrt[2])
*Sqrt[2*(1 + Sqrt[2])]*ArcTan[1/(Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]
)] - Sqrt[2*(-1 + Sqrt[2])]*(2 + Sqrt[2])*ArcTan[1/(Sqrt[1 + Sqrt[2]]*Sqrt[
x + Sqrt[1 + x^2]])] + (-2 + Sqrt[2])*Sqrt[2*(1 + Sqrt[2])]*ArcTanh[1/(Sqrt
[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]])] + Sqrt[2*(-1 + Sqrt[2])]*(2 + Sqrt
[2])*ArcTanh[1/(Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]])]
```

IntegrateAlgebraic [A] time = 0.32, size = 199, normalized size = 1.00

$$\sqrt{\sqrt{x^2+1}+x} - \frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}} - 2\sqrt{\sqrt{2}-1}\tan^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right) + 2\sqrt{1+\sqrt{2}}\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right) - 2\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right) + 2\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 + x^2)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/3*1/(x + Sqrt[1 + x^2])^(3/2) + Sqrt[x + Sqrt[1 + x^2]] - 2*Sqrt[-1 + Sq
rt[2]]*ArcTan[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]] + 2*Sqrt[1 + Sqrt
[2]]*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]] - 2*Sqrt[-1 + Sqrt[2
]]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]] + 2*Sqrt[1 + Sqrt[2]
]*ArcTanh[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]]
```

fricas [B] time = 1.19, size = 287, normalized size = 1.44

$$-\frac{2}{3}\left(x^2-\sqrt{x^2+1}\,x-1\right)\sqrt{x+\sqrt{x^2+1}}-4\sqrt{\sqrt{2}+1}\arctan\left(\sqrt{x+\sqrt{2}+\sqrt{x^2+1}-1}\sqrt{\sqrt{2}+1}-\sqrt{x+\sqrt{x^2+1}}\sqrt{\sqrt{2}+1}\right)+4\sqrt{\sqrt{2}-1}\arctan\left(\sqrt{x+\sqrt{2}+\sqrt{x^2+1}+1}\sqrt{\sqrt{2}-1}-\sqrt{x+\sqrt{x^2+1}}\sqrt{\sqrt{2}-1}\right)-\sqrt{\sqrt{2}-1}\log\left(2\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+2\sqrt{x+\sqrt{x^2+1}}\right)+\sqrt{\sqrt{2}-1}\log\left(-2\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+2\sqrt{x+\sqrt{x^2+1}}\right)+\sqrt{\sqrt{2}+1}\log\left(2\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+2\sqrt{x+\sqrt{x^2+1}}\right)-\sqrt{\sqrt{2}+1}\log\left(-2\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)+2\sqrt{x+\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(x^2 - sqrt(x^2 + 1)*x - 1)*sqrt(x + sqrt(x^2 + 1)) - 4*sqrt(sqrt(2) +
 1)*arctan(sqrt(x + sqrt(2) + sqrt(x^2 + 1) - 1)*sqrt(sqrt(2) + 1) - sqrt(x
 + sqrt(x^2 + 1))*sqrt(sqrt(2) + 1)) + 4*sqrt(sqrt(2) - 1)*arctan(sqrt(x +
sqrt(2) + sqrt(x^2 + 1) + 1)*sqrt(sqrt(2) - 1) - sqrt(x + sqrt(x^2 + 1))*sq
rt(sqrt(2) - 1)) - sqrt(sqrt(2) - 1)*log(2*(sqrt(2) + 1)*sqrt(sqrt(2) - 1)
+ 2*sqrt(x + sqrt(x^2 + 1))) + sqrt(sqrt(2) - 1)*log(-2*(sqrt(2) + 1)*sqrt(
sqrt(2) - 1) + 2*sqrt(x + sqrt(x^2 + 1))) + sqrt(sqrt(2) + 1)*log(2*sqrt(sq
rt(2) + 1)*(sqrt(2) - 1) + 2*sqrt(x + sqrt(x^2 + 1))) - sqrt(sqrt(2) + 1)*l
og(-2*sqrt(sqrt(2) + 1)*(sqrt(2) - 1) + 2*sqrt(x + sqrt(x^2 + 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^2-1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/((x^2 - 1)*sqrt(x + sqrt(x^2 + 1))), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^2-1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-1)/(x+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int((x^2+1)/(x^2-1)/(x+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^2-1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/((x^2 - 1)*sqrt(x + sqrt(x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+1}{\left(x^2-1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((x^2 - 1)*(x + (x^2 + 1)^(1/2))^(1/2)),x)
[Out] int((x^2 + 1)/((x^2 - 1)*(x + (x^2 + 1)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{(x-1)\,(x+1)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-1)/(x+(x**2+1)**(1/2))**(1/2),x)
[Out] Integral((x**2 + 1)/((x - 1)*(x + 1)*sqrt(x + sqrt(x**2 + 1))), x)
```

3.1994 $\int \frac{(-q+px^2)(aq+bx+apx^2)\sqrt{q^2+p^2x^4}}{x^3(cq+dx+cpx^2)}\,dx$

Optimal. Leaf size=200

$$\frac{\sqrt{p^2x^4+q^2}\,\left(acpx^2+acq-2adx+2bcx\right)}{2c^2x^2}+\frac{\log\left(\sqrt{p^2x^4+q^2}+px^2+q\right)\left(-ac^2pq+ad^2-bcd\right)}{c^3}-\frac{2(ad-bc)\sqrt{2c^2pq}}{}$$

Rubi [C] time = 9.28, antiderivative size = 1405, normalized size of antiderivative = 7.02, number of steps used = 46, number of rules used = 21, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6728, 275, 277, 217, 206, 305, 220, 1196, 266, 50, 63, 208, 1729, 1209, 1198, 1217, 1707, 1248, 735, 844, 725}

Antiderivative was successfully verified.

```
[In] Int[((-q + p*x^2)*(a*q + b*x + a*p*x^2)*Sqrt[q^2 + p^2*x^4])/(x^3*(c*q + d*
x + c*p*x^2)),x]
```

```
[Out] ((b*c*d - a*d^2 + a*c^2*p*q)*Sqrt[q^2 + p^2*x^4])/(2*c^3*q) - ((b*c - a*d)*
(d - Sqrt[d^2 - 4*c^2*p*q])*Sqrt[q^2 + p^2*x^4])/(4*c^3*q) - ((b*c - a*d)*(
d + Sqrt[d^2 - 4*c^2*p*q])*Sqrt[q^2 + p^2*x^4])/(4*c^3*q) + (a*q*Sqrt[q^2 +
 p^2*x^4])/(2*c*x^2) + ((b*c - a*d)*Sqrt[q^2 + p^2*x^4])/(c^2*x) + ((b*c -
a*d)*Sqrt[-d^2 + 2*c^2*p*q]*ArcTan[(Sqrt[-d^2 + 2*c^2*p*q]*x)/(c*Sqrt[q^2 +
 p^2*x^4])])/c^3 - (a*p*q*ArcTanh[(p*x^2)/Sqrt[q^2 + p^2*x^4]])/(2*c) - ((b
*c - a*d)*(d - Sqrt[d^2 - 4*c^2*p*q])*ArcTanh[(p*x^2)/Sqrt[q^2 + p^2*x^4]])
/(4*c^3) - ((b*c - a*d)*(d + Sqrt[d^2 - 4*c^2*p*q])*ArcTanh[(p*x^2)/Sqrt[q^
2 + p^2*x^4]])/(4*c^3) + ((b*c - a*d)*Sqrt[d^2 - 2*c^2*p*q]*(d + Sqrt[d^2 -
 4*c^2*p*q])*Sqrt[d^2 - 2*c^2*p*q - d*Sqrt[d^2 - 4*c^2*p*q]]*ArcTanh[(p*(4*
c^2*q^2 + (d - Sqrt[d^2 - 4*c^2*p*q])^2*x^2))/(2*Sqrt[2]*Sqrt[d^2 - 2*c^2*p
*q]*Sqrt[d^2 - 2*c^2*p*q - d*Sqrt[d^2 - 4*c^2*p*q]]*Sqrt[q^2 + p^2*x^4])])/
(4*Sqrt[2]*c^5*p*q) + ((b*c - a*d)*Sqrt[d^2 - 2*c^2*p*q]*(d - Sqrt[d^2 - 4*
c^2*p*q])*Sqrt[d^2 - 2*c^2*p*q + d*Sqrt[d^2 - 4*c^2*p*q]]*ArcTanh[(p*(4*c^2
*q^2 + (d + Sqrt[d^2 - 4*c^2*p*q])^2*x^2))/(2*Sqrt[2]*Sqrt[d^2 - 2*c^2*p*q]
*Sqrt[d^2 - 2*c^2*p*q + d*Sqrt[d^2 - 4*c^2*p*q]]*Sqrt[q^2 + p^2*x^4])])/(4*
Sqrt[2]*c^5*p*q) - ((b*c*d - a*d^2 + a*c^2*p*q)*ArcTanh[Sqrt[q^2 + p^2*x^4]
/q])/(2*c^3) - ((b*c - a*d)*Sqrt[p]*Sqrt[q]*(q + p*x^2)*Sqrt[(q^2 + p^2*x^4
)/(q + p*x^2)^2]*EllipticF[2*ArcTan[(Sqrt[p]*x)/Sqrt[q]], 1/2])/(c^2*Sqrt[q
^2 + p^2*x^4]) + (d*(b*c - a*d)*(d - Sqrt[d^2 - 4*c^2*p*q])*(q + p*x^2)*Sqr
t[(q^2 + p^2*x^4)/(q + p*x^2)^2]*EllipticF[2*ArcTan[(Sqrt[p]*x)/Sqrt[q]], 1
/2])/(4*c^4*Sqrt[p]*Sqrt[q]*Sqrt[q^2 + p^2*x^4]) - ((b*c - a*d)*(d^2 - 2*c^
2*p*q)*(d - Sqrt[d^2 - 4*c^2*p*q])*(q + p*x^2)*Sqrt[(q^2 + p^2*x^4)/(q + p*
x^2)^2]*EllipticF[2*ArcTan[(Sqrt[p]*x)/Sqrt[q]], 1/2])/(4*c^4*d*Sqrt[p]*Sqr
t[q]*Sqrt[q^2 + p^2*x^4]) + (d*(b*c - a*d)*(d + Sqrt[d^2 - 4*c^2*p*q])*(q +
 p*x^2)*Sqrt[(q^2 + p^2*x^4)/(q + p*x^2)^2]*EllipticF[2*ArcTan[(Sqrt[p]*x)/
Sqrt[q]], 1/2])/(4*c^4*Sqrt[p]*Sqrt[q]*Sqrt[q^2 + p^2*x^4]) - ((b*c - a*d)*
(d^2 - 2*c^2*p*q)*(d + Sqrt[d^2 - 4*c^2*p*q])*(q + p*x^2)*Sqrt[(q^2 + p^2*x
^4)/(q + p*x^2)^2]*EllipticF[2*ArcTan[(Sqrt[p]*x)/Sqrt[q]], 1/2])/(4*c^4*d*
Sqrt[p]*Sqrt[q]*Sqrt[q^2 + p^2*x^4])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
```

```
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Dist[d, I
nt[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, Int[(x*(a + c*x^4)^p)/(d
^2 - e^2*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(-q+px^2)(aq+bx+apx^2)\sqrt{q^2+p^2x^4}}{x^3(cq+dx+cpx^2)}dx = \int \left(-\frac{aq\sqrt{q^2+p^2x^4}}{cx^3} + \frac{(-bc+ad)\sqrt{q^2+p^2x^4}}{c^2x^2} + \frac{(bcd-ad^2+a\ldots}{\ldots}\right.$$

$$= -\frac{(bc-ad)\int \frac{\sqrt{q^2+p^2x^4}}{x^2}dx}{c^2} + \frac{(bc-ad)\int \frac{(-d^2+2c^2pq-cdpx)\sqrt{q^2+p^2x^4}}{cq+dx+cpx^2}}{c^3q}$$

$$= \frac{(bc-ad)\sqrt{q^2+p^2x^4}}{c^2x} - \frac{(2(bc-ad)p^2)\int \frac{x^2}{\sqrt{q^2+p^2x^4}}dx}{c^2} + \frac{(bc-a\ldots}{\ldots}$$

$$= \frac{(bcd-ad^2+ac^2pq)\sqrt{q^2+p^2x^4}}{2c^3q} + \frac{aq\sqrt{q^2+p^2x^4}}{2cx^2} + \frac{(bc-ad)\ldots}{c\ldots}$$

$$= \frac{(bcd-ad^2+ac^2pq)\sqrt{q^2+p^2x^4}}{2c^3q} + \frac{aq\sqrt{q^2+p^2x^4}}{2cx^2} + \frac{(bc-ad)\ldots}{c\ldots}$$

$$= \frac{(bcd-ad^2+ac^2pq)\sqrt{q^2+p^2x^4}}{2c^3q} + \frac{aq\sqrt{q^2+p^2x^4}}{2cx^2} + \frac{(bc-ad)\ldots}{c\ldots}$$

$$= \frac{(bcd-ad^2+ac^2pq)\sqrt{q^2+p^2x^4}}{2c^3q} - \frac{(bc-ad)\left(d-\sqrt{d^2-4c^2pq}\ldots\right)}{4c^3q}$$

$$= \frac{(bcd-ad^2+ac^2pq)\sqrt{q^2+p^2x^4}}{2c^3q} - \frac{(bc-ad)\left(d-\sqrt{d^2-4c^2pq}\ldots\right)}{4c^3q}$$

$$= \frac{(bcd-ad^2+ac^2pq)\sqrt{q^2+p^2x^4}}{2c^3q} - \frac{(bc-ad)\left(d-\sqrt{d^2-4c^2pq}\ldots\right)}{4c^3q}$$

$$= \frac{(bcd-ad^2+ac^2pq)\sqrt{q^2+p^2x^4}}{2c^3q} - \frac{(bc-ad)\left(d-\sqrt{d^2-4c^2pq}\ldots\right)}{4c^3q}$$

Mathematica [C] time = 9.15, size = 7717, normalized size = 38.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-q + p*x^2)*(a*q + b*x + a*p*x^2)*Sqrt[q^2 + p^2*x^4])/(x^3*(c*
q + d*x + c*p*x^2)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 4.28, size = 200, normalized size = 1.00

$$\frac{\sqrt{p^2x^4+q^2}\left(acpx^2+acq-2adx+2bcx\right)}{2c^2x^2}+\frac{\log\left(\sqrt{p^2x^4+q^2}+px^2+q\right)\left(-ac^2pq+ad^2-bcd\right)}{c^3}-\frac{2(ad-bc)\sqrt{2c^2pq-d^2}\tan^{-1}\left(\frac{x\sqrt{2c^2pq-d^2}}{c\sqrt{p^2x^4+q^2}+cpx^2+cq+dx}\right)}{c^3}+\frac{\log(x)\left(ac^2pq-ad^2+bcd\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-q + p*x^2)*(a*q + b*x + a*p*x^2)*Sqrt[q^2 + p^2*x^4])
/(x^3*(c*q + d*x + c*p*x^2)),x]
```

```
[Out] ((a*c*q + 2*b*c*x - 2*a*d*x + a*c*p*x^2)*Sqrt[q^2 + p^2*x^4])/(2*c^2*x^2) -
 (2*(-(b*c) + a*d)*Sqrt[-d^2 + 2*c^2*p*q]*ArcTan[(Sqrt[-d^2 + 2*c^2*p*q]*x)
/(c*q + d*x + c*p*x^2 + c*Sqrt[q^2 + p^2*x^4])]])/c^3 + ((b*c*d - a*d^2 + a*
c^2*p*q)*Log[x])/c^3 + ((-(b*c*d) + a*d^2 - a*c^2*p*q)*Log[q + p*x^2 + Sqrt
[q^2 + p^2*x^4]])/c^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^2-q)*(a*p*x^2+a*q+b*x)*(p^2*x^4+q^2)^(1/2)/x^3/(c*p*x^2+c*q+
d*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^4+q^2}\left(apx^2+aq+bx\right)\left(px^2-q\right)}{\left(cpx^2+cq+dx\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^2-q)*(a*p*x^2+a*q+b*x)*(p^2*x^4+q^2)^(1/2)/x^3/(c*p*x^2+c*q+
d*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(p^2*x^4 + q^2)*(a*p*x^2 + a*q + b*x)*(p*x^2 - q)/((c*p*x^2 +
 c*q + d*x)*x^3), x)
```

maple [C] time = 0.11, size = 13623, normalized size = 68.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^2-q)*(a*p*x^2+a*q+b*x)*(p^2*x^4+q^2)^(1/2)/x^3/(c*p*x^2+c*q+d*x),x
)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^4+q^2}\left(apx^2+aq+bx\right)\left(px^2-q\right)}{\left(cpx^2+cq+dx\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^2-q)*(a*p*x^2+a*q+b*x)*(p^2*x^4+q^2)^(1/2)/x^3/(c*p*x^2+c*q+
d*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(p^2*x^4 + q^2)*(a*p*x^2 + a*q + b*x)*(p*x^2 - q)/((c*p*x^2 +
 c*q + d*x)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\sqrt{p^2\,x^4+q^2}\,\left(q-p\,x^2\right)\,\left(a\,p\,x^2+b\,x+a\,q\right)}{x^3\,\left(c\,p\,x^2+d\,x+c\,q\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((p^2*x^4 + q^2)^(1/2)*(q - p*x^2)*(a*q + b*x + a*p*x^2))/(x^3*(c*q +
d*x + c*p*x^2)),x)
```

```
[Out] int(-((p^2*x^4 + q^2)^(1/2)*(q - p*x^2)*(a*q + b*x + a*p*x^2))/(x^3*(c*q +
d*x + c*p*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(px^2-q\right)\sqrt{p^2x^4+q^2}\,\left(apx^2+aq+bx\right)}{x^3\left(cpx^2+cq+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**2-q)*(a*p*x**2+a*q+b*x)*(p**2*x**4+q**2)**(1/2)/x**3/(c*p*x
**2+c*q+d*x),x)
```

```
[Out] Integral((p*x**2 - q)*sqrt(p**2*x**4 + q**2)*(a*p*x**2 + a*q + b*x)/(x**3*(
c*p*x**2 + c*q + d*x)), x)
```

3.1995 $\int \frac{x(3+7x^4)}{\sqrt[3]{1+x^4}(-4+x^3+x^7)}\,dx$

Optimal. Leaf size=200

$$\sqrt[3]{2}\tanh^{-1}\left(1-\sqrt[3]{2}x\sqrt[3]{x^4+1}\right)-\frac{\sqrt{3}\tan^{-1}\left(\frac{3\sqrt{3}x\sqrt[3]{x^4+1}-6x^2\sqrt[3]{x^4+1}}{-3\sqrt[3]{x^4+1}x+2\sqrt{3}\sqrt[3]{x^4+1}x^2+4\,2^{2/3}\sqrt{3}x-6\,2^{2/3}}\right)}{2^{2/3}}-\frac{\tanh^{-1}\left(\frac{2^{2/3}\sqrt[3]{x^4+1}x+2}{2^{2/3}\sqrt[3]{x^4+1}x+2(x^4+1)^{2}}\right)}{2^{2/3}}$$

Rubi [F] time = 0.49, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(3+7x^4)}{\sqrt[3]{1+x^4}(-4+x^3+x^7)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(3 + 7*x^4))/((1 + x^4)^(1/3)*(-4 + x^3 + x^7)),x]
```

```
[Out] 3*Defer[Int][x/((1 + x^4)^(1/3)*(-4 + x^3 + x^7)), x] + 7*Defer[Int][x^5/((
1 + x^4)^(1/3)*(-4 + x^3 + x^7)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{x(3+7x^4)}{\sqrt[3]{1+x^4}(-4+x^3+x^7)}\,dx &= \int \left(\frac{3x}{\sqrt[3]{1+x^4}(-4+x^3+x^7)}+\frac{7x^5}{\sqrt[3]{1+x^4}(-4+x^3+x^7)}\right)dx\\ &= 3\int \frac{x}{\sqrt[3]{1+x^4}(-4+x^3+x^7)}\,dx+7\int \frac{x^5}{\sqrt[3]{1+x^4}(-4+x^3+x^7)}\,dx\end{aligned}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x(3+7x^4)}{\sqrt[3]{1+x^4}(-4+x^3+x^7)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(3 + 7*x^4))/((1 + x^4)^(1/3)*(-4 + x^3 + x^7)),x]
```

```
[Out] Integrate[(x*(3 + 7*x^4))/((1 + x^4)^(1/3)*(-4 + x^3 + x^7)), x]
```

IntegrateAlgebraic [A] time = 19.99, size = 200, normalized size = 1.00

$$\sqrt[3]{2}\tanh^{-1}\left(1-\sqrt[3]{2}x\sqrt[3]{x^4+1}\right)-\frac{\sqrt{3}\tan^{-1}\left(\frac{3\sqrt{3}x\sqrt[3]{x^4+1}-6x^2\sqrt[3]{x^4+1}}{-3\sqrt[3]{x^4+1}x+2\sqrt{3}\sqrt[3]{x^4+1}x^2+4\,2^{2/3}\sqrt{3}x-6\,2^{2/3}}\right)}{2^{2/3}}-\frac{\tanh^{-1}\left(\frac{2^{2/3}\sqrt[3]{x^4+1}x+2\sqrt[3]{2}}{2^{2/3}\sqrt[3]{x^4+1}x+2(x^4+1)^{2/3}x^2+2\sqrt[3]{2}}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(3 + 7*x^4))/((1 + x^4)^(1/3)*(-4 + x^3 + x^7)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(3*Sqrt[3]*x*(1 + x^4)^(1/3) - 6*x^2*(1 + x^4)^(1/3))/(-6
*2^(2/3) + 4*2^(2/3)*Sqrt[3]*x - 3*x*(1 + x^4)^(1/3) + 2*Sqrt[3]*x^2*(1 + x
^4)^(1/3))])/2^(2/3)) + 2^(1/3)*ArcTanh[1 - 2^(1/3)*x*(1 + x^4)^(1/3)] - Ar
cTanh[(2*2^(1/3) + 2^(2/3)*x*(1 + x^4)^(1/3))/(2*2^(1/3) + 2^(2/3)*x*(1 + x
^4)^(1/3) + 2*x^2*(1 + x^4)^(2/3))]/2^(2/3)
```

fricas [B] time = 82.37, size = 344, normalized size = 1.72

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(6\cdot 4^{\frac{2}{3}}\left(x^{16}+2x^{12}+4x^{9}+x^{8}+4x^{5}-32x^{2}\right)\left(x^{4}+1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(x^{21}+3x^{17}+60x^{14}+3x^{13}+120x^{10}+x^{9}+192x^{7}+60x^{6}+192x^{3}-64\right)+24\left(x^{15}+2x^{11}+28x^{8}+x^{7}+28x^{4}+16x\right)\left(x^{4}+1\right)^{\frac{1}{3}}\right)}{6\left(x^{21}+3x^{17}-12x^{14}+3x^{13}-24x^{10}+x^{9}-384x^{7}-12x^{6}-384x^{3}-64\right)}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(-\frac{12\left(x^{4}+1\right)^{\frac{2}{3}}x^{2}-4^{\frac{2}{3}}\left(x^{7}+x^{3}-4\right)-12\cdot 4^{\frac{1}{3}}\left(x^{4}+1\right)^{\frac{1}{3}}x}{x^{7}+x^{3}-4}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(\frac{3\cdot 4^{\frac{2}{3}}\left(x^{9}+x^{5}+8x^{2}\right)\left(x^{4}+1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(x^{14}+2x^{10}+28x^{7}+x^{6}+28x^{3}+16\right)+24\left(x^{8}+x^{4}+2x\right)\left(x^{4}+1\right)^{\frac{1}{3}}}{x^{14}+2x^{10}-8x^{7}+x^{6}-8x^{3}+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(7*x^4+3)/(x^4+1)^(1/3)/(x^7+x^3-4),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(6*4^(2/3)*(x^16 + 2*x^12 +
 4*x^9 + x^8 + 4*x^5 - 32*x^2)*(x^4 + 1)^(2/3) + 4^(1/3)*(x^21 + 3*x^17 + 6
0*x^14 + 3*x^13 + 120*x^10 + x^9 + 192*x^7 + 60*x^6 + 192*x^3 - 64) + 24*(x
^15 + 2*x^11 + 28*x^8 + x^7 + 28*x^4 + 16*x)*(x^4 + 1)^(1/3))/(x^21 + 3*x^1
7 - 12*x^14 + 3*x^13 - 24*x^10 + x^9 - 384*x^7 - 12*x^6 - 384*x^3 - 64)) +
1/12*4^(2/3)*log(-(12*(x^4 + 1)^(2/3)*x^2 - 4^(2/3)*(x^7 + x^3 - 4) - 12*4^
(1/3)*(x^4 + 1)^(1/3)*x)/(x^7 + x^3 - 4)) - 1/24*4^(2/3)*log((3*4^(2/3)*(x^
9 + x^5 + 8*x^2)*(x^4 + 1)^(2/3) + 4^(1/3)*(x^14 + 2*x^10 + 28*x^7 + x^6 +
28*x^3 + 16) + 24*(x^8 + x^4 + 2*x)*(x^4 + 1)^(1/3))/(x^14 + 2*x^10 - 8*x^7
 + x^6 - 8*x^3 + 16))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(7\,x^{4}+3\right)x}{\left(x^{7}+x^{3}-4\right)\left(x^{4}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(7*x^4+3)/(x^4+1)^(1/3)/(x^7+x^3-4),x, algorithm="giac")
```

```
[Out] integrate((7*x^4 + 3)*x/((x^7 + x^3 - 4)*(x^4 + 1)^(1/3)), x)
```

maple [C] time = 50.93, size = 933, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(7*x^4+3)/(x^4+1)^(1/3)/(x^7+x^3-4),x)
```

```
[Out] -1/2*ln((RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)
^3*x^7+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-
2)^2*x^7+RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)
^3*x^3+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-
2)^2*x^3+6*(x^4+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+2*_Z*Root
Of(_Z^3-2)+4*_Z^2)*x^2+6*(x^4+1)^(1/3)*RootOf(_Z^3-2)^2*x-4*RootOf(_Z^3-2)-
16*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2))/(x^7+x^3-4))*RootOf
(_Z^3-2)-ln((RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^
3-2)^3*x^7+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_
Z^3-2)^2*x^7+RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^
3-2)^3*x^3+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_
Z^3-2)^2*x^3+6*(x^4+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+2*_Z*
RootOf(_Z^3-2)+4*_Z^2)*x^2+6*(x^4+1)^(1/3)*RootOf(_Z^3-2)^2*x-4*RootOf(_Z^3
-2)-16*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2))/(x^7+x^3-4))*Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+1/2*RootOf(_Z^3-2)*ln((Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^7-2*Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^7+x^
7*RootOf(_Z^3-2)-2*x^7*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+
RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3-2*
RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-
6*(x^4+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2
)+4*_Z^2)*x^2+12*(x^4+1)^(1/3)*RootOf(_Z^3-2)*RootOf(RootOf(_Z^3-2)^2+2*_Z*
RootOf(_Z^3-2)+4*_Z^2)*x+RootOf(_Z^3-2)*x^3-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*
```

```
RootOf(_Z^3-2)+4*_Z^2)*x^3-6*x^2*(x^4+1)^(2/3)+4*RootOf(_Z^3-2)-8*RootOf(Ro
otOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2))/(x^7+x^3-4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(7x^4+3)x}{(x^7+x^3-4)(x^4+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(7*x^4+3)/(x^4+1)^(1/3)/(x^7+x^3-4),x, algorithm="maxima")
[Out] integrate((7*x^4 + 3)*x/((x^7 + x^3 - 4)*(x^4 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(7x^4+3)}{(x^4+1)^{1/3}\,(x^7+x^3-4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(7*x^4 + 3))/((x^4 + 1)^(1/3)*(x^3 + x^7 - 4)),x)
[Out] int((x*(7*x^4 + 3))/((x^4 + 1)^(1/3)*(x^3 + x^7 - 4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(7x^4+3)}{\sqrt[3]{x^4+1}\,(x^7+x^3-4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(7*x**4+3)/(x**4+1)**(1/3)/(x**7+x**3-4),x)
[Out] Integral(x*(7*x**4 + 3)/((x**4 + 1)**(1/3)*(x**7 + x**3 - 4)), x)
```

3.1996 $\int \frac{\sqrt{-bx+a^2x^2}}{\sqrt{ax^2+x\sqrt{-bx+a^2x^2}}}\,dx$

Optimal. Leaf size=200

$$\frac{\sqrt{a^2x^2-bx}\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(8a^2x-9b\right)}{12abx}+\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(\frac{19b-8a^2x}{12b}+\frac{3\sqrt{b}\sqrt{\sqrt{a^2x^2-bx}-ax}}{4\sqrt{\ldots}}\right.$$

Rubi [F] time = 3.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-bx+a^2x^2}}{\sqrt{ax^2+x\sqrt{-bx+a^2x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[-(b*x) + a^2*x^2]/Sqrt[a*x^2 + x*Sqrt[-(b*x) + a^2*x^2]],x]
```

```
[Out] (2*Sqrt[-(b*x) + a^2*x^2]*Defer[Subst][Defer[Int][(x^2*Sqrt[-b + a^2*x^2])/
Sqrt[a*x^4 + x^2*Sqrt[-(b*x^2) + a^2*x^4]], x], x, Sqrt[x]])/(Sqrt[x]*Sqrt[
-b + a^2*x])
```

Rubi steps

$$\int \frac{\sqrt{-bx+a^2x^2}}{\sqrt{ax^2+x\sqrt{-bx+a^2x^2}}}\,dx=\frac{\sqrt{-bx+a^2x^2}\int \frac{\sqrt{x}\sqrt{-b+a^2x}}{\sqrt{ax^2+x\sqrt{-bx+a^2x^2}}}\,dx}{\sqrt{x}\sqrt{-b+a^2x}}$$

$$=\frac{\left(2\sqrt{-bx+a^2x^2}\right)\text{Subst}\left(\int \frac{x^2\sqrt{-b+a^2x^2}}{\sqrt{ax^4+x^2\sqrt{-bx^2+a^2x^4}}}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{-b+a^2x}}$$

Mathematica [C] time = 1.81, size = 205, normalized size = 1.02

$$\frac{b\sqrt{x\left(a^2x-b\right)}\left(\sqrt{x\left(a^2x-b\right)}+ax\right)^2\left(4\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{b}{2a\left(ax+\sqrt{x\left(a^2x-b\right)}\right)}\right)-{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};1-\frac{b}{2a\left(ax+\sqrt{x\left(a^2x-b\right)}\right)}\right)-4\right)}{3\sqrt{x\left(\sqrt{x\left(a^2x-b\right)}+ax\right)}\left(a\left(\sqrt{x\left(a^2x-b\right)}+ax\right)-b\right)\left(2a\left(\sqrt{x\left(a^2x-b\right)}+ax\right)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(b*x) + a^2*x^2]/Sqrt[a*x^2 + x*Sqrt[-(b*x) + a^2*x^2]],x]
```

```
[Out] (b*Sqrt[x*(-b + a^2*x)]*(a*x + Sqrt[x*(-b + a^2*x)])^2*(-4 + 4*Hypergeometr
ic2F1[-3/2, 1, -1/2, 1 - b/(2*a*(a*x + Sqrt[x*(-b + a^2*x)]))] - Hypergeome
tric2F1[-3/2, 2, -1/2, 1 - b/(2*a*(a*x + Sqrt[x*(-b + a^2*x)]))]))/(3*Sqrt[
x*(a*x + Sqrt[x*(-b + a^2*x)])]*(-b + a*(a*x + Sqrt[x*(-b + a^2*x)]))*(-b +
 2*a*(a*x + Sqrt[x*(-b + a^2*x)])))
```

IntegrateAlgebraic [A] time = 5.69, size = 200, normalized size = 1.00

$$\frac{\sqrt{a^2x^2-bx}\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(8a^2x-9b\right)}{12abx}+\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(\frac{19b-8a^2x}{12b}+\frac{3\sqrt{b}\sqrt{\sqrt{a^2x^2-bx}-ax}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{\sqrt{a^2x^2-bx}-ax}}{\sqrt{b}}\right)}{4\sqrt{2}a^{3/2}x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-(b*x) + a^2*x^2]/Sqrt[a*x^2 + x*Sqrt[-(b*x) + a^2*
x^2]],x]
```

```
[Out] ((-9*b + 8*a^2*x)*Sqrt[-(b*x) + a^2*x^2]*Sqrt[x*(a*x + Sqrt[-(b*x) + a^2*x^
2])])/(12*a*b*x) + Sqrt[x*(a*x + Sqrt[-(b*x) + a^2*x^2])]*((19*b - 8*a^2*x)
/(12*b) + (3*Sqrt[b]*Sqrt[-(a*x) + Sqrt[-(b*x) + a^2*x^2]]*ArcTan[(Sqrt[2]*
Sqrt[a]*Sqrt[-(a*x) + Sqrt[-(b*x) + a^2*x^2]])/Sqrt[b]])/(4*Sqrt[2]*a^(3/2)
*x))
```

fricas [A] time = 0.46, size = 323, normalized size = 1.62

$$\left[\frac{9\sqrt{2}\sqrt{a}b^2x\log\left(-\frac{4a^2x^2+4\sqrt{a^2x^2-bx}\,ax-bx-2\left(\sqrt{2}a^{\frac{3}{2}}x+\sqrt{2}\sqrt{a^2x^2-bx}\sqrt{a}\right)\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}}{x}\right)-4\left(8a^4x^2-19a^2bx-\left(8a^3x-9ab\right)\sqrt{a^2x^2-bx}\right)\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}}{48a^2bx},\ \frac{9\sqrt{2}\sqrt{-a}b^2x\arctan\left(\frac{\sqrt{2}\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}\sqrt{-a}}{2ax}\right)-2\left(8a^4x^2-19a^2bx-\left(8a^3x-9ab\right)\sqrt{a^2x^2-bx}\right)\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}}{24a^2bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(1/2),x, algori
thm="fricas")
```

```
[Out] [1/48*(9*sqrt(2)*sqrt(a)*b^2*x*log(-(4*a^2*x^2 + 4*sqrt(a^2*x^2 - b*x)*a*x
- b*x - 2*(sqrt(2)*a^(3/2)*x + sqrt(2)*sqrt(a^2*x^2 - b*x)*sqrt(a))*sqrt(a*
x^2 + sqrt(a^2*x^2 - b*x)*x))/x) - 4*(8*a^4*x^2 - 19*a^2*b*x - (8*a^3*x - 9
*a*b)*sqrt(a^2*x^2 - b*x))*sqrt(a*x^2 + sqrt(a^2*x^2 - b*x)*x))/(a^2*b*x),
1/24*(9*sqrt(2)*sqrt(-a)*b^2*x*arctan(1/2*sqrt(2)*sqrt(a*x^2 + sqrt(a^2*x^2
 - b*x)*x)*sqrt(-a)/(a*x)) - 2*(8*a^4*x^2 - 19*a^2*b*x - (8*a^3*x - 9*a*b)*
sqrt(a^2*x^2 - b*x))*sqrt(a*x^2 + sqrt(a^2*x^2 - b*x)*x))/(a^2*b*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2-bx}}{\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate(sqrt(a^2*x^2 - b*x)/sqrt(a*x^2 + sqrt(a^2*x^2 - b*x)*x), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2-bx}}{\sqrt{a\,x^2+x\sqrt{a^2x^2-bx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(1/2),x)
```

```
[Out] int((a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2 - bx}}{\sqrt{ax^2 + \sqrt{a^2x^2 - bx}\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 - b*x)/sqrt(a*x^2 + sqrt(a^2*x^2 - b*x)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a^2\,x^2 - b\,x}}{\sqrt{a\,x^2 + x\,\sqrt{a^2\,x^2 - b\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 - b*x)^(1/2)/(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(1/2),x)
```

```
[Out] int((a^2*x^2 - b*x)^(1/2)/(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(a^2x - b\right)}}{\sqrt{x\left(ax + \sqrt{a^2x^2 - bx}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b*x)**(1/2)/(a*x**2+x*(a**2*x**2-b*x)**(1/2))**(1/2),x
)
```

```
[Out] Integral(sqrt(x*(a**2*x - b))/sqrt(x*(a*x + sqrt(a**2*x**2 - b*x))), x)
```

3.1997 $\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\left(a^2+bd-(2a+d)x+x^2\right)}\,dx$

Optimal. Leaf size=201

$$-\frac{\log\left(a^2+d^{2/3}\left(x(-a-b)+ab+x^2\right)^{2/3}+\sqrt[3]{x(-a-b)+ab+x^2}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)-2ax+x^2\right)}{2d^{2/3}}+\frac{\log\left(\sqrt[3]{d}\,\sqrt[3]{x(-a\ldots\right.}}{}$$

Rubi [F] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\left(a^2+bd-(2a+d)x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a - 2*b + x)/(((-a + x)*(-b + x))^(1/3)*(a^2 + b*d - (2*a + d)*x + x^2
)),x]
```

```
[Out] Defer[Int][(a - 2*b + x)/((a*b + (-a - b)*x + x^2)^(1/3)*(a^2 + b*d + (-2*a
 - d)*x + x^2)), x]
```

Rubi steps

$$\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\left(a^2+bd-(2a+d)x+x^2\right)}\,dx=\int \frac{a-2b+x}{\sqrt[3]{ab+(-a-b)x+x^2}\left(a^2+bd+(-2a-d)x+x^2\right)}\,d\ldots$$

Mathematica [F] time = 11.11, size = 0, normalized size = 0.00

$$\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\left(a^2+bd-(2a+d)x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a - 2*b + x)/(((-a + x)*(-b + x))^(1/3)*(a^2 + b*d - (2*a + d)*x
 + x^2)),x]
```

```
[Out] Integrate[(a - 2*b + x)/(((-a + x)*(-b + x))^(1/3)*(a^2 + b*d - (2*a + d)*x
 + x^2)), x]
```

IntegrateAlgebraic [A] time = 0.48, size = 201, normalized size = 1.00

$$-\frac{\log\left(a^2+d^{2/3}\left(x(-a-b)+ab+x^2\right)^{2/3}+\sqrt[3]{x(-a-b)+ab+x^2}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)-2ax+x^2\right)}{2d^{2/3}}+\frac{\log\left(\sqrt[3]{d}\,\sqrt[3]{x(-a-b)+ab+x^2}+a-x\right)}{d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\sqrt[3]{x(-a-b)+ab+x^2}}{\sqrt[3]{d}\sqrt[3]{x(-a-b)+ab+x^2}-2a+2x}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - 2*b + x)/(((-a + x)*(-b + x))^(1/3)*(a^2 + b*d - (2
*a + d)*x + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(a*b + (-a - b)*x + x^2)^(1/3))/(-2*a + 2*
x + d^(1/3)*(a*b + (-a - b)*x + x^2)^(1/3))])/d^(2/3) + Log[a - x + d^(1/3)
*(a*b + (-a - b)*x + x^2)^(1/3)]/d^(2/3) - Log[a^2 - 2*a*x + x^2 + (-(a*d^(
1/3)) + d^(1/3)*x)*(a*b + (-a - b)*x + x^2)^(1/3) + d^(2/3)*(a*b + (-a - b)
*x + x^2)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)/((-a+x)*(-b+x))^(1/3)/(a^2+b*d-(2*a+d)*x+x^2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a-2\,b+x}{\left(a^2+bd-(2\,a+d)x+x^2\right)((a-x)(b-x))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)/((-a+x)*(-b+x))^(1/3)/(a^2+b*d-(2*a+d)*x+x^2),x, algori
thm="giac")
```

```
[Out] integrate((a - 2*b + x)/((a^2 + b*d - (2*a + d)*x + x^2)*((a - x)*(b - x))^
(1/3)), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{a-2b+x}{\left((-a+x)\,(-b+x)\right)^{\frac{1}{3}}\left(a^2+bd-(2a+d)\,x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-2*b+x)/((-a+x)*(-b+x))^(1/3)/(a^2+b*d-(2*a+d)*x+x^2),x)
```

```
[Out] int((a-2*b+x)/((-a+x)*(-b+x))^(1/3)/(a^2+b*d-(2*a+d)*x+x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a-2\,b+x}{\left(a^2+bd-(2\,a+d)x+x^2\right)((a-x)(b-x))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)/((-a+x)*(-b+x))^(1/3)/(a^2+b*d-(2*a+d)*x+x^2),x, algori
thm="maxima")
```

```
[Out] integrate((a - 2*b + x)/((a^2 + b*d - (2*a + d)*x + x^2)*((a - x)*(b - x))^
(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a-2\,b+x}{{\left(\left(a-x\right)\,\left(b-x\right)\right)}^{1/3}\,\left(b\,d-x\,\left(2\,a+d\right)+a^2+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - 2*b + x)/(((a - x)*(b - x))^(1/3)*(b*d - x*(2*a + d) + a^2 + x^2))
,x)
```

```
[Out] int((a - 2*b + x)/(((a - x)*(b - x))^(1/3)*(b*d - x*(2*a + d) + a^2 + x^2))
, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)/((-a+x)*(-b+x))**(1/3)/(a**2+b*d-(2*a+d)*x+x**2),x)
```

```
[Out] Timed out
```

3.1998 $\int \frac{-a(a-2b)-2bx+x^2}{((-a+x)(-b+x))^{2/3}\left(b+a^2d-(1+2ad)x+dx^2\right)}\,dx$

Optimal. Leaf size=201

$$-\frac{\log\left(d^{2/3}\left(x(-a-b)+ab+x^2\right)^{4/3}+\left(x(-a-b)+ab+x^2\right)^{2/3}\left(\sqrt[3]{d}\,x-b\sqrt[3]{d}\right)+b^2-2bx+x^2\right)}{2d^{2/3}}+\frac{\log\left(\sqrt[3]{d}\left(x(-a-\right.\right.}{}$$

Rubi [C] time = 1.56, antiderivative size = 191, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 5, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6719, 1586, 6728, 137, 136}

$$-\frac{3(a-x)^2\left(-\frac{b-x}{a-b}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};\frac{a-x}{a-b},-\frac{2d(a-x)}{1-\sqrt{4ad-4bd+1}}\right)}{4((a-x)(b-x))^{2/3}}-\frac{3(a-x)^2\left(-\frac{b-x}{a-b}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};\frac{a-x}{a-b},-\frac{2d(a-x)}{\sqrt{4ad-4bd+1}+1}\right)}{4((a-x)(b-x))^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-(a*(a - 2*b)) - 2*b*x + x^2)/(((-a + x)*(-b + x))^(2/3)*(b + a^2*d -
(1 + 2*a*d)*x + d*x^2)),x]
```

```
[Out] (-3*(a - x)^2*(-((b - x)/(a - b)))^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, (a - x)
/(a - b), (-2*d*(a - x))/(1 - Sqrt[1 + 4*a*d - 4*b*d])])/(4*((a - x)*(b - x
))^(2/3)) - (3*(a - x)^2*(-((b - x)/(a - b)))^(2/3)*AppellF1[4/3, 2/3, 1, 7
/3, (a - x)/(a - b), (-2*d*(a - x))/(1 + Sqrt[1 + 4*a*d - 4*b*d])])/(4*((a
- x)*(b - x))^(2/3))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-a(a-2b)-2bx+x^2}{((-a+x)(-b+x))^{2/3}\left(b+a^2d-(1+2ad)x+dx^2\right)}\,dx = \frac{\left((-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{-a(a-2b)-2bx+x^2}{(-a+x)^{2/3}(-b+x)^{2/3}\left(b+a^2d-(1+\right.}}{((-a+x)(-b+x))^{2/3}}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{\sqrt[3]{-a+x}\,(a-2b+x)}{(-b+x)^{2/3}\left(b+a^2d-(1+2ad)x+dx\right.}}{((-a+x)(-b+x))^{2/3}}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{2/3}\right)\int\left(\frac{\left(1+\sqrt{1+4ad-4bd}\right)\sqrt[3]{-a}}{(-b+x)^{2/3}\left(-1-2ad-\sqrt{1+4ad-\right.}}\right.}{((-a+x)(-b}$$

$$= \frac{\left(\left(1-\sqrt{1+4ad-4bd}\right)(-a+x)^{2/3}(-b+x)^{2/3}\right)\int -}{((-a+x)(-b+x))^{2/}}$$

$$= \frac{\left(\left(1-\sqrt{1+4ad-4bd}\right)(-a+x)^{2/3}\left(\frac{-b+x}{a-b}\right)^{2/3}\right)\int -}{((-a+x)(-b+x)}$$

$$= -\frac{3(a-x)^2\left(-\frac{b-x}{a-b}\right)^{2/3} F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};\frac{a-x}{a-b},-\frac{2d(a-x)}{1-\sqrt{1+4ad-}}\right.}{4((a-x)(b-x))^{2/3}}$$

Mathematica [F] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{-a(a-2b)-2bx+x^2}{((-a+x)(-b+x))^{2/3}\left(b+a^2d-(1+2ad)x+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-(a*(a - 2*b)) - 2*b*x + x^2)/(((-a + x)*(-b + x))^(2/3)*(b + a^
2*d - (1 + 2*a*d)*x + d*x^2)),x]
```

```
[Out] Integrate[(-(a*(a - 2*b)) - 2*b*x + x^2)/(((-a + x)*(-b + x))^(2/3)*(b + a^
2*d - (1 + 2*a*d)*x + d*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.77, size = 201, normalized size = 1.00

$$-\frac{\log\left(d^{2/3}\left(x(-a-b)+ab+x^2\right)^{4/3}+\left(x(-a-b)+ab+x^2\right)^{2/3}\left(\sqrt[3]{d}x-b\sqrt[3]{d}\right)+b^2-2bx+x^2\right)}{2d^{2/3}}+\frac{\log\left(\sqrt[3]{d}\left(x(-a-b)+ab+x^2\right)^{2/3}+b-x\right)}{d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\left(x(-a-b)+ab+x^2\right)^{2/3}}{\sqrt[3]{d}\left(x(-a-b)+ab+x^2\right)^{2/3}-2b+2x}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*(a - 2*b)) - 2*b*x + x^2)/(((-a + x)*(-b + x))^(2/3
)*(b + a^2*d - (1 + 2*a*d)*x + d*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(a*b + (-a - b)*x + x^2)^(2/3))/(-2*b + 2*
x + d^(1/3)*(a*b + (-a - b)*x + x^2)^(2/3))])/d^(2/3) + Log[b - x + d^(1/3)
*(a*b + (-a - b)*x + x^2)^(2/3)]/d^(2/3) - Log[b^2 - 2*b*x + x^2 + (-(b*d^(
1/3)) + d^(1/3)*x)*(a*b + (-a - b)*x + x^2)^(2/3) + d^(2/3)*(a*b + (-a - b)
*x + x^2)^(4/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a-2*b)-2*b*x+x^2)/((-a+x)*(-b+x))^(2/3)/(b+a^2*d-(2*a*d+1)*x
+d*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(a-2\,b)a+2\,bx-x^2}{\left(a^2d+dx^2-(2\,ad+1)x+b\right)((a-x)(b-x))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a-2*b)-2*b*x+x^2)/((-a+x)*(-b+x))^(2/3)/(b+a^2*d-(2*a*d+1)*x
+d*x^2),x, algorithm="giac")
```

```
[Out] integrate(-((a - 2*b)*a + 2*b*x - x^2)/((a^2*d + d*x^2 - (2*a*d + 1)*x + b)
*((a - x)*(b - x))^(2/3)), x)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{-a\,(a-2b)-2bx+x^2}{((-a+x)\,(-b+x))^{\frac{2}{3}}\left(b+a^2d-(2ad+1)\,x+d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*(a-2*b)-2*b*x+x^2)/((-a+x)*(-b+x))^(2/3)/(b+a^2*d-(2*a*d+1)*x+d*x^2
),x)
```

```
[Out] int((-a*(a-2*b)-2*b*x+x^2)/((-a+x)*(-b+x))^(2/3)/(b+a^2*d-(2*a*d+1)*x+d*x^2
),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(a-2\,b)a+2\,bx-x^2}{\left(a^2d+dx^2-(2\,ad+1)x+b\right)((a-x)(b-x))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a-2*b)-2*b*x+x^2)/((-a+x)*(-b+x))^(2/3)/(b+a^2*d-(2*a*d+1)*x
+d*x^2),x, algorithm="maxima")
```

```
[Out] -integrate(((a - 2*b)*a + 2*b*x - x^2)/((a^2*d + d*x^2 - (2*a*d + 1)*x + b)
*((a - x)*(b - x))^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{-x^2+2\,b\,x+a\,\left(a-2\,b\right)}{{\left(\left(a-x\right)\,\left(b-x\right)\right)}^{2/3}\,\left(b-x\,\left(2\,a\,d+1\right)+a^2\,d+d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*b*x + a*(a - 2*b) - x^2)/(((a - x)*(b - x))^(2/3)*(b - x*(2*a*d + 1
) + a^2*d + d*x^2)),x)
```

```
[Out] -int((2*b*x + a*(a - 2*b) - x^2)/(((a - x)*(b - x))^(2/3)*(b - x*(2*a*d + 1
) + a^2*d + d*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a-2*b)-2*b*x+x**2)/((-a+x)*(-b+x))**(2/3)/(b+a**2*d-(2*a*d+1
)*x+d*x**2),x)
```

```
[Out] Timed out
```

3.1999 $\int \frac{-1+(2-k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(b+2k)x+\left(b+k^2\right)x^2\right)}\,dx$

Optimal. Leaf size=201

$$-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{kx^3+(-k-1)x^2+x}\left(\sqrt[3]{b}-\sqrt[3]{b}\,kx\right)+k^2x^2-2kx+1\right)}{2b^{2/3}}+\frac{\log\left(\sqrt[3]{b}\sqrt[3]{kx^3+\cdots\right.}$$

Rubi [F] time = 3.67, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+(2-k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(b+2k)x+\left(b+k^2\right)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + (2 - k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (b + 2*k)*x + (b + k
^2)*x^2)),x]
```

```
[Out] ((2 - k*(1 + Sqrt[-4 + b + 4*k]/Sqrt[b]))*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(
1/3)*Defer[Int][1/((1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*(-b - 2*k - Sqrt[b
]*Sqrt[-4 + b + 4*k] + 2*(b + k^2)*x)), x])/((1 - x)*x*(1 - k*x))^(1/3) + (
(2 - k*(1 - Sqrt[-4 + b + 4*k]/Sqrt[b]))*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1
/3)*Defer[Int][1/((1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*(-b - 2*k + Sqrt[b]
*Sqrt[-4 + b + 4*k] + 2*(b + k^2)*x)), x])/((1 - x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\int \frac{-1+(2-k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(b+2k)x+\left(b+k^2\right)x^2\right)}\,dx = \frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{-1+(2-k)x}{\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\left(1-(b+2k)x+\left(b+k^2\right)x^2\right)}\cdots}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$= \frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int\left(\frac{2-k-\frac{k\sqrt{-4+b+4k}}{\sqrt{b}}}{\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\left(-b-2k-\sqrt{b}\sqrt{-4+b+4k}\cdots\right.}\cdots\right.}{\sqrt[3]{(1-x}\cdots}$$

$$= \frac{\left(\left(2-k\left(1-\frac{\sqrt{-4+b+4k}}{\sqrt{b}}\right)\right)\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{\cdots}{\sqrt[3]{1-x}\sqrt[3]{x}\cdots}}{\sqrt[3]{(1-x)x(1-kx)}}$$

Mathematica [F] time = 5.20, size = 0, normalized size = 0.00

$$\int \frac{-1+(2-k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(b+2k)x+\left(b+k^2\right)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + (2 - k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (b + 2*k)*x +
(b + k^2)*x^2)),x]
```

```
[Out] Integrate[(-1 + (2 - k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (b + 2*k)*x +
(b + k^2)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.67, size = 201, normalized size = 1.00

$$-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{kx^3+(-k-1)x^2+x}\left(\sqrt[3]{b}-\sqrt[3]{b}\,kx\right)+k^2x^2-2kx+1\right)}{2b^{2/3}}+\frac{\log\left(\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+kx-1\right)}{b^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}-2kx+2}\right)}{b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + (2 - k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (b +
2*k)*x + (b + k^2)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(2 - 2*k
*x + b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))])/b^(2/3) + Log[-1 + k*x + b
^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3)]/b^(2/3) - Log[1 - 2*k*x + k^2*x^2
+ (b^(1/3) - b^(1/3)*k*x)*(x + (-1 - k)*x^2 + k*x^3)^(1/3) + b^(2/3)*(x + (
-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(2-k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(b+2*k)*x+(k^2+b)*x^2),x
, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(k-2)x+1}{((kx-1)(x-1)x)^{\frac{1}{3}}\left(\left(k^2+b\right)x^2-(b+2k)x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(2-k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(b+2*k)*x+(k^2+b)*x^2),x
, algorithm="giac")
```

```
[Out] integrate(-((k - 2)*x + 1)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k^2 + b)*x^2 - (b
 + 2*k)*x + 1)), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{-1+(2-k)\,x}{((1-x)\,x\,(-kx+1))^{\frac{1}{3}}\left(1-(b+2k)\,x+\left(k^2+b\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+(2-k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(b+2*k)*x+(k^2+b)*x^2),x)
```

```
[Out] int((-1+(2-k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(b+2*k)*x+(k^2+b)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(k-2)x+1}{((kx-1)(x-1)x)^{\frac{1}{3}}\left(\left(k^2+b\right)x^2-(b+2k)x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(2-k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(b+2*k)*x+(k^2+b)*x^2),x
, algorithm="maxima")
```

```
[Out] -integrate(((k - 2)*x + 1)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k^2 + b)*x^2 - (b
 + 2*k)*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x\,\left(k-2\right)+1}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/3}\,\left(\left(k^2+b\right)\,x^2+\left(-b-2\,k\right)\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(k - 2) + 1)/((x*(k*x - 1)*(x - 1))^(1/3)*(x^2*(b + k^2) - x*(b + 2
*k) + 1)),x)
```

```
[Out] int(-(x*(k - 2) + 1)/((x*(k*x - 1)*(x - 1))^(1/3)*(x^2*(b + k^2) - x*(b + 2
*k) + 1)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(2-k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(1-(b+2*k)*x+(k**2+b)*x**2
),x)
```

```
[Out] Timed out
```

3.2000 $$\int \frac{-3k-2(1+k^2)x+k(1+k^2)x^2+4k^2x^3+k^3x^4}{\left((1-x^2)(1-k^2x^2)\right)^{2/3}\left(-1+d-(2+d)kx-(d+k^2)x^2+dkx^3\right)}\,dx$$

Optimal. Leaf size=201

$$-\frac{\log\left(d^{2/3}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\left(\sqrt[3]{d}\,kx+\sqrt[3]{d}\right)+k^2x^2+2kx+1\right)}{2\sqrt[3]{d}}+\frac{\log\left(-\right.}{}$$

Rubi [F] time = 8.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3k-2\left(1+k^2\right)x+k\left(1+k^2\right)x^2+4k^2x^3+k^3x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(-1+d-(2+d)kx-\left(d+k^2\right)x^2+dkx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3*k - 2*(1 + k^2)*x + k*(1 + k^2)*x^2 + 4*k^2*x^3 + k^3*x^4)/(((1 - x
^2)*(1 - k^2*x^2))^(2/3)*(-1 + d - (2 + d)*k*x - (d + k^2)*x^2 + d*k*x^3)),
x]
```

```
[Out] (k*(5*d + k^2)*x*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*AppellF1[1/2, 2/3, 2/3
, 3/2, x^2, k^2*x^2])/(d^2*((1 - x^2)*(1 - k^2*x^2))^(2/3)) - (3^(3/4)*Sqrt
[2 + Sqrt[3]]*k^(4/3)*Sqrt[(-1 - k^2 + 2*k^2*x^2)^2]*((-1 + k^2)^(2/3) + 2^
(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))*Sqrt[((-1 + k^2)^(4/3) - 2^(
2/3)*k^(2/3)*(-1 + k^2)^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3) + 2*2^(1/3)*k
^(4/3)*((1 - x^2)*(1 - k^2*x^2))^(2/3))/((1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2
^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))^2]*EllipticF[ArcSin[((1 - S
qrt[3])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))
/((1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2)
)^(1/3))], -7 - 4*Sqrt[3]])/(2^(2/3)*d*(1 + k^2 - 2*k^2*x^2)*Sqrt[(-1 - k^2
*(1 - 2*x^2))^2]*Sqrt[((-1 + k^2)^(2/3)*((-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)
*((1 - x^2)*(1 - k^2*x^2))^(1/3)))/((1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2/3
)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))^2]) + (k*(8*d^2 - k^2 - d*(5 - k
^2))*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][1/((1 - x^2)^(2/3)*(1 -
 k^2*x^2)^(2/3)*(1 - d + (2 + d)*k*x + (d + k^2)*x^2 - d*k*x^3)), x])/(d^2*
((1 - x^2)*(1 - k^2*x^2))^(2/3)) - ((2*k^4 - 2*d^2*(1 - k^2) + d*k^2*(11 +
k^2))*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][x/((1 - x^2)^(2/3)*(1
- k^2*x^2)^(2/3)*(1 - d + (2 + d)*k*x + (d + k^2)*x^2 - d*k*x^3)), x])/(d^2
*((1 - x^2)*(1 - k^2*x^2))^(2/3)) - (k*(8*d*k^2 + k^4 + 2*d^2*(3 + k^2))*(1
 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][x^2/((1 - x^2)^(2/3)*(1 - k^2*
x^2)^(2/3)*(1 - d + (2 + d)*k*x + (d + k^2)*x^2 - d*k*x^3)), x])/(d^2*((1 -
 x^2)*(1 - k^2*x^2))^(2/3))
```

Rubi steps

$$\int \frac{-3k-2\left(1+k^2\right)x+k\left(1+k^2\right)x^2+4k^2x^3+k^3x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(-1+d-(2+d)kx-\left(d+k^2\right)x^2+dkx^3\right)}\,dx = \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int \frac{-3k-}{\left(1-x^2\right)^{2/3}\left(1-\right.}}{\left(\left(1-x^2\right)\left(1-\right.\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int\left(\frac{k\left(5d\right.}{d^2\left(1-x^2\right)^{2/3}}\right.}{}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int \frac{-k\left(8d^2-k^2-\right.}{}}{d^2\left(\left(\right.\right.}$$

$$= \frac{k\left(5d+k^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}F}{d^2\left(\left(1-x^2\right)\left(1-k^2x^2\right.\right.}$$

$$= \frac{k\left(5d+k^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}F}{d^2\left(\left(1-x^2\right)\left(1-k^2x^2\right.\right.}$$

$$= \frac{k\left(5d+k^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}F}{d^2\left(\left(1-x^2\right)\left(1-k^2x^2\right.\right.}$$

$$= \frac{k\left(5d+k^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}F}{d^2\left(\left(1-x^2\right)\left(1-k^2x^2\right.\right.}$$

Mathematica [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{-3k-2\left(1+k^2\right)x+k\left(1+k^2\right)x^2+4k^2x^3+k^3x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(-1+d-(2+d)kx-\left(d+k^2\right)x^2+dkx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3*k - 2*(1 + k^2)*x + k*(1 + k^2)*x^2 + 4*k^2*x^3 + k^3*x^4)/((
(1 - x^2)*(1 - k^2*x^2))^(2/3)*(-1 + d - (2 + d)*k*x - (d + k^2)*x^2 + d*k*
x^3)),x]
```

```
[Out] Integrate[(-3*k - 2*(1 + k^2)*x + k*(1 + k^2)*x^2 + 4*k^2*x^3 + k^3*x^4)/((
(1 - x^2)*(1 - k^2*x^2))^(2/3)*(-1 + d - (2 + d)*k*x - (d + k^2)*x^2 + d*k*
x^3)), x]
```

IntegrateAlgebraic [A] time = 8.11, size = 201, normalized size = 1.00

$$-\frac{\log\left(d^{2/3}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\left(\sqrt[3]{d}\,kx+\sqrt[3]{d}\right)+k^2x^2+2kx+1\right)}{2\sqrt[3]{d}}+\frac{\log\left(-\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+kx+1\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}kx+\sqrt{3}}{2\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+kx+1}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*k - 2*(1 + k^2)*x + k*(1 + k^2)*x^2 + 4*k^2*x^3 + k^
3*x^4)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)*(-1 + d - (2 + d)*k*x - (d + k^2)*x
^2 + d*k*x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3] + Sqrt[3]*k*x)/(1 + k*x + 2*d^(1/3)*(1 + (-1 - k^2
)*x^2 + k^2*x^4)^(1/3))])/d^(1/3) + Log[1 + k*x - d^(1/3)*(1 + (-1 - k^2)*x
^2 + k^2*x^4)^(1/3)]/d^(1/3) - Log[1 + 2*k*x + k^2*x^2 + (d^(1/3) + d^(1/3)
*k*x)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3) + d^(2/3)*(1 + (-1 - k^2)*x^2 +
k^2*x^4)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*k-2*(k^2+1)*x+k*(k^2+1)*x^2+4*k^2*x^3+k^3*x^4)/((-x^2+1)*(-k^
2*x^2+1))^(2/3)/(-1+d-(2+d)*k*x-(k^2+d)*x^2+d*k*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^3x^4 + 4\,k^2x^3 + \left(k^2+1\right)kx^2 - 2\left(k^2+1\right)x - 3\,k}{\left(dkx^3 - (d+2)kx - \left(k^2+d\right)x^2 + d - 1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*k-2*(k^2+1)*x+k*(k^2+1)*x^2+4*k^2*x^3+k^3*x^4)/((-x^2+1)*(-k^
2*x^2+1))^(2/3)/(-1+d-(2+d)*k*x-(k^2+d)*x^2+d*k*x^3),x, algorithm="giac")
```

```
[Out] integrate((k^3*x^4 + 4*k^2*x^3 + (k^2 + 1)*k*x^2 - 2*(k^2 + 1)*x - 3*k)/((d
*k*x^3 - (d + 2)*k*x - (k^2 + d)*x^2 + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(2/
3)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{-3k - 2\left(k^2+1\right)x + k\left(k^2+1\right)x^2 + 4k^2x^3 + k^3x^4}{\left(\left(-x^2+1\right)\left(-k^2x^2+1\right)\right)^{\frac{2}{3}}\left(-1 + d - (2+d)\,kx - \left(k^2+d\right)x^2 + dk\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*k-2*(k^2+1)*x+k*(k^2+1)*x^2+4*k^2*x^3+k^3*x^4)/((-x^2+1)*(-k^2*x^2+
1))^(2/3)/(-1+d-(2+d)*k*x-(k^2+d)*x^2+d*k*x^3),x)
```

```
[Out] int((-3*k-2*(k^2+1)*x+k*(k^2+1)*x^2+4*k^2*x^3+k^3*x^4)/((-x^2+1)*(-k^2*x^2+
1))^(2/3)/(-1+d-(2+d)*k*x-(k^2+d)*x^2+d*k*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^3x^4 + 4\,k^2x^3 + \left(k^2+1\right)kx^2 - 2\left(k^2+1\right)x - 3\,k}{\left(dkx^3 - (d+2)kx - \left(k^2+d\right)x^2 + d - 1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*k-2*(k^2+1)*x+k*(k^2+1)*x^2+4*k^2*x^3+k^3*x^4)/((-x^2+1)*(-k^
2*x^2+1))^(2/3)/(-1+d-(2+d)*k*x-(k^2+d)*x^2+d*k*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k^3*x^4 + 4*k^2*x^3 + (k^2 + 1)*k*x^2 - 2*(k^2 + 1)*x - 3*k)/((d
*k*x^3 - (d + 2)*k*x - (k^2 + d)*x^2 + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(2/
3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{4\,k^2\,x^3-2\,x\,\left(k^2+1\right)-3\,k+k^3\,x^4+k\,x^2\,\left(k^2+1\right)}{{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)}^{2/3}\,\left(-d\,k\,x^3+\left(k^2+d\right)\,x^2+k\,\left(d+2\right)\,x-d+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*k^2*x^3 - 2*x*(k^2 + 1) - 3*k + k^3*x^4 + k*x^2*(k^2 + 1))/(((x^2 -
 1)*(k^2*x^2 - 1))^(2/3)*(x^2*(d + k^2) - d + k*x*(d + 2) - d*k*x^3 + 1)),x
)
```

```
[Out] int(-(4*k^2*x^3 - 2*x*(k^2 + 1) - 3*k + k^3*x^4 + k*x^2*(k^2 + 1))/(((x^2 -
 1)*(k^2*x^2 - 1))^(2/3)*(x^2*(d + k^2) - d + k*x*(d + 2) - d*k*x^3 + 1)),
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*k-2*(k**2+1)*x+k*(k**2+1)*x**2+4*k**2*x**3+k**3*x**4)/((-x**2
+1)*(-k**2*x**2+1))**(2/3)/(-1+d-(2+d)*k*x-(k**2+d)*x**2+d*k*x**3),x)
```

```
[Out] Timed out
```

3.2001 $\int \frac{-1+2x^4}{\sqrt[4]{1+x^4}\left(-1+x^4+x^8\right)}\,dx$

Optimal. Leaf size=201

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(11+5\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(5\sqrt{5}-11\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(11+5\sqrt{5}\right)}\tanh^{-}$$

Rubi [A] time = 0.40, antiderivative size = 213, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6728, 377, 212, 206, 203}

$$\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt{5}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + 2*x^4)/((1 + x^4)^(1/4)*(-1 + x^4 + x^8)),x]
```

```
[Out] (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTan[((2/(3 + Sqrt[5]))^(1/4)*x)/(1 + x^4)^
(1/4)])/(2*Sqrt[5]) - (((123 - 55*Sqrt[5])/2)^(1/4)*ArcTan[(((3 + Sqrt[5])/
2)^(1/4)*x)/(1 + x^4)^(1/4)])/(2*Sqrt[5]) + (((123 + 55*Sqrt[5])/2)^(1/4)*A
rcTanh[((2/(3 + Sqrt[5]))^(1/4)*x)/(1 + x^4)^(1/4)])/(2*Sqrt[5]) - (((123 -
 55*Sqrt[5])/2)^(1/4)*ArcTanh[(((3 + Sqrt[5])/2)^(1/4)*x)/(1 + x^4)^(1/4)])
/(2*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+2x^4}{\sqrt[4]{1+x^4}\left(-1+x^4+x^8\right)}\,dx &= \int \left(\frac{2-\frac{4}{\sqrt{5}}}{\sqrt[4]{1+x^4}\left(1-\sqrt{5}+2x^4\right)}+\frac{2+\frac{4}{\sqrt{5}}}{\sqrt[4]{1+x^4}\left(1+\sqrt{5}+2x^4\right)}\right)dx\\
&= \frac{1}{5}\left(2\left(5-2\sqrt{5}\right)\right)\int \frac{1}{\sqrt[4]{1+x^4}\left(1-\sqrt{5}+2x^4\right)}\,dx+\frac{1}{5}\left(2\left(5+2\sqrt{5}\right)\right)\int \frac{}{\sqrt[4]{1+x^4}}\\
&= \frac{1}{5}\left(2\left(5-2\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{5}-\left(-1-\sqrt{5}\right)x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{1}{5}\left(2\left(5+\right.\right.\\
&= \frac{\left(2-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{10}}+\frac{\left(2-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}}\right.}{\sqrt{10}}\\
&= \frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt{5}}+
\end{aligned}
$$

Mathematica [A] time = 0.28, size = 181, normalized size = 0.90

$$
\frac{\sqrt[4]{123+55\sqrt{5}}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}x}{\sqrt[4]{x^4+1}}\right)-\sqrt[4]{123-55\sqrt{5}}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x}{\sqrt[4]{x^4+1}}\right)+\sqrt[4]{123+55\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}x}{\sqrt[4]{x^4+1}}\right)-\sqrt[4]{123-55\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt[4]{2}\sqrt{5}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + 2*x^4)/((1 + x^4)^(1/4)*(-1 + x^4 + x^8)),x]
```

```
[Out] ((123 + 55*Sqrt[5])^(1/4)*ArcTan[((2/(3 + Sqrt[5]))^(1/4)*x)/(1 + x^4)^(1/4
)] - (123 - 55*Sqrt[5])^(1/4)*ArcTan[(((3 + Sqrt[5])/2)^(1/4)*x)/(1 + x^4)^
(1/4)] + (123 + 55*Sqrt[5])^(1/4)*ArcTanh[((2/(3 + Sqrt[5]))^(1/4)*x)/(1 +
x^4)^(1/4)] - (123 - 55*Sqrt[5])^(1/4)*ArcTanh[(((3 + Sqrt[5])/2)^(1/4)*x)/
(1 + x^4)^(1/4)])/(2*2^(1/4)*Sqrt[5])
```

IntegrateAlgebraic [A] time = 1.13, size = 201, normalized size = 1.00

$$
\frac{1}{2}\sqrt{\frac{1}{10}\left(11+5\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(5\sqrt{5}-11\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}x}{\sqrt[4]{x^4+1}}\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(11+5\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(5\sqrt{5}-11\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}x}{\sqrt[4]{x^4+1}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x^4)/((1 + x^4)^(1/4)*(-1 + x^4 + x^8)),x]
```

```
[Out] (Sqrt[(11 + 5*Sqrt[5])/10]*ArcTan[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(1 + x^4)^(1/4
)])/2 - (Sqrt[(-11 + 5*Sqrt[5])/10]*ArcTan[(Sqrt[1/2 + Sqrt[5]/2]*x)/(1 + x
^4)^(1/4)])/2 + (Sqrt[(11 + 5*Sqrt[5])/10]*ArcTanh[(Sqrt[-1/2 + Sqrt[5]/2]*
x)/(1 + x^4)^(1/4)])/2 - (Sqrt[(-11 + 5*Sqrt[5])/10]*ArcTanh[(Sqrt[1/2 + Sq
rt[5]/2]*x)/(1 + x^4)^(1/4)])/2
```

fricas [B] time = 43.78, size = 1049, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)/(x^4+1)^(1/4)/(x^8+x^4-1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(10)*sqrt(5*sqrt(5) - 11)*arctan(1/40*(sqrt(2)*(2*sqrt(10)*(5*x^6
+ 10*x^2 + sqrt(5)*(3*x^6 + 4*x^2))*sqrt(x^4 + 1) + sqrt(10)*(15*x^8 + 25*x
^4 + sqrt(5)*(5*x^8 + 11*x^4 + 3) + 5))*sqrt(5*sqrt(5) - 11)*sqrt(sqrt(5) +
 1) + 4*(sqrt(10)*(5*x^5 + sqrt(5)*(3*x^5 + 4*x) + 10*x)*(x^4 + 1)^(3/4) +
sqrt(10)*(10*x^7 + 15*x^3 + sqrt(5)*(4*x^7 + 7*x^3))*(x^4 + 1)^(1/4))*sqrt(
5*sqrt(5) - 11))/(x^8 + x^4 - 1)) - 1/20*sqrt(10)*sqrt(5*sqrt(5) + 11)*arct
an(-1/40*(sqrt(2)*(2*sqrt(10)*(5*x^6 + 10*x^2 - sqrt(5)*(3*x^6 + 4*x^2))*sq
rt(x^4 + 1) - sqrt(10)*(15*x^8 + 25*x^4 - sqrt(5)*(5*x^8 + 11*x^4 + 3) + 5)
)*sqrt(5*sqrt(5) + 11)*sqrt(sqrt(5) - 1) - 4*(sqrt(10)*(5*x^5 - sqrt(5)*(3*
x^5 + 4*x) + 10*x)*(x^4 + 1)^(3/4) - sqrt(10)*(10*x^7 + 15*x^3 - sqrt(5)*(4
*x^7 + 7*x^3))*(x^4 + 1)^(1/4))*sqrt(5*sqrt(5) + 11))/(x^8 + x^4 - 1)) - 1/
80*sqrt(10)*sqrt(5*sqrt(5) - 11)*log((10*(2*x^5 + sqrt(5)*x + x)*(x^4 + 1)^
(3/4) + (sqrt(10)*(10*x^6 + 15*x^2 + sqrt(5)*(4*x^6 + 7*x^2))*sqrt(x^4 + 1)
 + sqrt(10)*(10*x^8 + 20*x^4 + sqrt(5)*(5*x^8 + 9*x^4 + 2) + 5))*sqrt(5*sqr
t(5) - 11) + 10*(x^7 + 3*x^3 + sqrt(5)*(x^7 + x^3))*(x^4 + 1)^(1/4))/(x^8 +
 x^4 - 1)) + 1/80*sqrt(10)*sqrt(5*sqrt(5) - 11)*log((10*(2*x^5 + sqrt(5)*x
+ x)*(x^4 + 1)^(3/4) - (sqrt(10)*(10*x^6 + 15*x^2 + sqrt(5)*(4*x^6 + 7*x^2)
)*sqrt(x^4 + 1) + sqrt(10)*(10*x^8 + 20*x^4 + sqrt(5)*(5*x^8 + 9*x^4 + 2) +
 5))*sqrt(5*sqrt(5) - 11) + 10*(x^7 + 3*x^3 + sqrt(5)*(x^7 + x^3))*(x^4 + 1
)^(1/4))/(x^8 + x^4 - 1)) + 1/80*sqrt(10)*sqrt(5*sqrt(5) + 11)*log((10*(2*x
^5 - sqrt(5)*x + x)*(x^4 + 1)^(3/4) + (sqrt(10)*(10*x^6 + 15*x^2 - sqrt(5)*
(4*x^6 + 7*x^2))*sqrt(x^4 + 1) - sqrt(10)*(10*x^8 + 20*x^4 - sqrt(5)*(5*x^8
 + 9*x^4 + 2) + 5))*sqrt(5*sqrt(5) + 11) - 10*(x^7 + 3*x^3 - sqrt(5)*(x^7 +
 x^3))*(x^4 + 1)^(1/4))/(x^8 + x^4 - 1)) - 1/80*sqrt(10)*sqrt(5*sqrt(5) + 1
1)*log((10*(2*x^5 - sqrt(5)*x + x)*(x^4 + 1)^(3/4) - (sqrt(10)*(10*x^6 + 15
*x^2 - sqrt(5)*(4*x^6 + 7*x^2))*sqrt(x^4 + 1) - sqrt(10)*(10*x^8 + 20*x^4 -
 sqrt(5)*(5*x^8 + 9*x^4 + 2) + 5))*sqrt(5*sqrt(5) + 11) - 10*(x^7 + 3*x^3 -
 sqrt(5)*(x^7 + x^3))*(x^4 + 1)^(1/4))/(x^8 + x^4 - 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)/(x^4+1)^(1/4)/(x^8+x^4-1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [C] time = 11.30, size = 1608, normalized size = 8.00

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-1)/(x^4+1)^(1/4)/(x^8+x^4-1),x)
```

```
[Out] -1/20*RootOf(_Z^2+400*RootOf(6400*_Z^4-880*_Z^2-1)^2-55)*ln(5*(19200*RootOf
(_Z^2+400*RootOf(6400*_Z^4-880*_Z^2-1)^2-55)*RootOf(6400*_Z^4-880*_Z^2-1)^4
*x^4-4000*(x^4+1)^(1/2)*RootOf(6400*_Z^4-880*_Z^2-1)^2*RootOf(_Z^2+400*Root
Of(6400*_Z^4-880*_Z^2-1)^2-55)*x^2+1760*RootOf(_Z^2+400*RootOf(6400*_Z^4-88
0*_Z^2-1)^2-55)*RootOf(6400*_Z^4-880*_Z^2-1)^2*x^4+2400*(x^4+1)^(3/4)*RootO
f(6400*_Z^4-880*_Z^2-1)^2*x-3200*(x^4+1)^(1/4)*RootOf(6400*_Z^4-880*_Z^2-1)
^2*x^3+7*RootOf(_Z^2+400*RootOf(6400*_Z^4-880*_Z^2-1)^2-55)*x^4-40*(x^4+1)^
(3/4)*x-30*x^3*(x^4+1)^(1/4)+1200*RootOf(6400*_Z^4-880*_Z^2-1)^2*RootOf(_Z^
2+400*RootOf(6400*_Z^4-880*_Z^2-1)^2-55)+5*RootOf(_Z^2+400*RootOf(6400*_Z^4
-880*_Z^2-1)^2-55))/(960*RootOf(6400*_Z^4-880*_Z^2-1)^3*x^2-116*RootOf(6400
*_Z^4-880*_Z^2-1)*x^2+5)/(960*RootOf(6400*_Z^4-880*_Z^2-1)^3*x^2-116*RootOf
(6400*_Z^4-880*_Z^2-1)*x^2-5))+RootOf(6400*_Z^4-880*_Z^2-1)*ln(-(38400*x^4*
```

```
RootOf(6400*_Z^4-880*_Z^2-1)^5+8000*RootOf(6400*_Z^4-880*_Z^2-1)^3*(x^4+1)^
(1/2)*x^2-14080*x^4*RootOf(6400*_Z^4-880*_Z^2-1)^3+240*(x^4+1)^(3/4)*RootOf
(6400*_Z^4-880*_Z^2-1)^2*x-320*(x^4+1)^(1/4)*RootOf(6400*_Z^4-880*_Z^2-1)^2
*x^3-1100*(x^4+1)^(1/2)*RootOf(6400*_Z^4-880*_Z^2-1)*x^2+1224*x^4*RootOf(64
00*_Z^4-880*_Z^2-1)-29*(x^4+1)^(3/4)*x+47*x^3*(x^4+1)^(1/4)-2400*RootOf(640
0*_Z^4-880*_Z^2-1)^3+340*RootOf(6400*_Z^4-880*_Z^2-1))/(80*RootOf(6400*_Z^4
-880*_Z^2-1)^2*x^4-8*x^4+5))-4*RootOf(6400*_Z^4-880*_Z^2-1)^2*RootOf(_Z^2+4
00*RootOf(6400*_Z^4-880*_Z^2-1)^2-55)*ln(-(6400*RootOf(_Z^2+400*RootOf(6400
*_Z^4-880*_Z^2-1)^2-55)*RootOf(6400*_Z^4-880*_Z^2-1)^4*x^4-1680*RootOf(_Z^2
+400*RootOf(6400*_Z^4-880*_Z^2-1)^2-55)*RootOf(6400*_Z^4-880*_Z^2-1)^2*x^4+
2400*(x^4+1)^(3/4)*RootOf(6400*_Z^4-880*_Z^2-1)^2*x+3200*(x^4+1)^(1/4)*Root
Of(6400*_Z^4-880*_Z^2-1)^2*x^3+50*(x^4+1)^(1/2)*RootOf(_Z^2+400*RootOf(6400
*_Z^4-880*_Z^2-1)^2-55)*x^2+54*RootOf(_Z^2+400*RootOf(6400*_Z^4-880*_Z^2-1)
^2-55)*x^4-290*(x^4+1)^(3/4)*x-470*x^3*(x^4+1)^(1/4)-400*RootOf(6400*_Z^4-8
80*_Z^2-1)^2*RootOf(_Z^2+400*RootOf(6400*_Z^4-880*_Z^2-1)^2-55)+15*RootOf(_
Z^2+400*RootOf(6400*_Z^4-880*_Z^2-1)^2-55))/(80*RootOf(6400*_Z^4-880*_Z^2-1
)^2*x^4-8*x^4+5))+80*RootOf(6400*_Z^4-880*_Z^2-1)^3*ln(5*(-1600*RootOf(6400
*_Z^4-880*_Z^2-1)^3*(x^4+1)^(1/2)*x^2-800*x^4*RootOf(6400*_Z^4-880*_Z^2-1)^
3+320*(x^4+1)^(3/4)*RootOf(6400*_Z^4-880*_Z^2-1)^2*x+560*(x^4+1)^(1/4)*Root
Of(6400*_Z^4-880*_Z^2-1)^2*x^3+60*(x^4+1)^(1/2)*RootOf(6400*_Z^4-880*_Z^2-1
)*x^2-70*x^4*RootOf(6400*_Z^4-880*_Z^2-1)+3*(x^4+1)^(3/4)*x-x^3*(x^4+1)^(1/
4)-50*RootOf(6400*_Z^4-880*_Z^2-1))/(960*RootOf(6400*_Z^4-880*_Z^2-1)^3*x^2
-116*RootOf(6400*_Z^4-880*_Z^2-1)*x^2+5)/(960*RootOf(6400*_Z^4-880*_Z^2-1)^
3*x^2-116*RootOf(6400*_Z^4-880*_Z^2-1)*x^2-5))-11*RootOf(6400*_Z^4-880*_Z^2
-1)*ln(5*(-1600*RootOf(6400*_Z^4-880*_Z^2-1)^3*(x^4+1)^(1/2)*x^2-800*x^4*Ro
otOf(6400*_Z^4-880*_Z^2-1)^3+320*(x^4+1)^(3/4)*RootOf(6400*_Z^4-880*_Z^2-1)
^2*x+560*(x^4+1)^(1/4)*RootOf(6400*_Z^4-880*_Z^2-1)^2*x^3+60*(x^4+1)^(1/2)*
RootOf(6400*_Z^4-880*_Z^2-1)*x^2-70*x^4*RootOf(6400*_Z^4-880*_Z^2-1)+3*(x^4
+1)^(3/4)*x-x^3*(x^4+1)^(1/4)-50*RootOf(6400*_Z^4-880*_Z^2-1))/(960*RootOf(
6400*_Z^4-880*_Z^2-1)^3*x^2-116*RootOf(6400*_Z^4-880*_Z^2-1)*x^2+5)/(960*Ro
otOf(6400*_Z^4-880*_Z^2-1)^3*x^2-116*RootOf(6400*_Z^4-880*_Z^2-1)*x^2-5))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^4-1}{\left(x^8+x^4-1\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-1)/(x^4+1)^(1/4)/(x^8+x^4-1),x, algorithm="maxima")
[Out] integrate((2*x^4 - 1)/((x^8 + x^4 - 1)*(x^4 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{2\,x^4-1}{{\left(x^4+1\right)}^{1/4}\,\left(x^8+x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4 - 1)/((x^4 + 1)^(1/4)*(x^4 + x^8 - 1)),x)
[Out] int((2*x^4 - 1)/((x^4 + 1)^(1/4)*(x^4 + x^8 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-1)/(x**4+1)**(1/4)/(x**8+x**4-1),x)
[Out] Timed out
```

3.2002 $$\int \frac{(-2x+(1+k)x^2)(a-a(1+k)x+(1+ak)x^2)}{(-1+x)((1-x)x(1-kx))^{2/3}(-1+kx)(b-b(1+k)x+(-1+bk)x^2)}\,dx$$

Optimal. Leaf size=203

$$\frac{(-a-b)\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+x^2\right)}{2\sqrt[3]{b}}+\frac{(a+b)\log\left(x-\sqrt[3]{b}\,\sqrt[3]{kx^3+(\right.}{\sqrt[3]{b}}$$

Rubi [F] time = 54.85, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2x+(1+k)x^2)(a-a(1+k)x+(1+ak)x^2)}{(-1+x)((1-x)x(1-kx))^{2/3}(-1+kx)(b-b(1+k)x+(-1+bk)x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*x + (1 + k)*x^2)*(a - a*(1 + k)*x + (1 + a*k)*x^2))/((-1 + x)*((1
- x)*x*(1 - k*x))^(2/3)*(-1 + k*x)*(b - b*(1 + k)*x + (-1 + b*k)*x^2)),x]
```

```
[Out] (-3*(1 + k)*(1 + a*k)*(1 - x)*x)/((1 - k)^2*(1 - b*k)*((1 - x)*x*(1 - k*x))
^(2/3)) + (3*(1 + k)*(2 + a + b + 4*a*k + a*(1 - 2*b)*k^2 + b*k^2)*(1 - x)*
x)/(2*(1 - k)^2*(1 - b*k)^2*((1 - x)*x*(1 - k*x))^(2/3)) - (3*(1 + k)*(1 +
a*k)*x^2)/(2*(1 - k)*(1 - b*k)*((1 - x)*x*(1 - k*x))^(2/3)) + (3*(2 + a + b
 + 4*a*k + a*(1 - 2*b)*k^2 + b*k^2)*x^2)/(2*(1 - k)*(1 - b*k)^2*((1 - x)*x*
(1 - k*x))^(2/3)) - (3*(1 + k)*(1 + a*k)*(1 - x)*(((1 - k)*x)/(1 - k*x))^(2
/3)*(1 - k*x)*Hypergeometric2F1[1/3, 2/3, 4/3, (1 - x)/(1 - k*x)])/((1 - k)
^3*(1 - b*k)*((1 - x)*x*(1 - k*x))^(2/3)) + (3*(1 + k)*(2 + a + b + 4*a*k +
 a*(1 - 2*b)*k^2 + b*k^2)*(1 - x)*(((1 - k)*x)/(1 - k*x))^(2/3)*(1 - k*x)*H
ypergeometric2F1[1/3, 2/3, 4/3, (1 - x)/(1 - k*x)])/(2*(1 - k)^3*(1 - b*k)^
2*((1 - x)*x*(1 - k*x))^(2/3)) + ((a + b)*(3 + b + 3*k + b*k^3 + (4 + b^2*(
1 - k)^2*(1 + k + k^2) + b*(5 + 2*k + 5*k^2))/(Sqrt[b]*Sqrt[4 + b*(1 - k)^2
]))*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Int][x^(1/3)/((1 - x)^(5/3)
*(1 - k*x)^(5/3)*(-(b*(1 + k)) - Sqrt[b]*Sqrt[4 + b - 2*b*k + b*k^2] + 2*(-
1 + b*k)*x)), x])/((1 - b*k)^2*((1 - x)*x*(1 - k*x))^(2/3)) + ((a + b)*(3*(
1 + k) + b*(1 + k^3) - (4 + b*(5 + 2*k + 5*k^2) + b^2*(1 - k - k^3 + k^4))/
(Sqrt[b]*Sqrt[4 + b*(1 - k)^2]))*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defe
r[Int][x^(1/3)/((1 - x)^(5/3)*(1 - k*x)^(5/3)*(-(b*(1 + k)) + Sqrt[b]*Sqrt[
4 + b - 2*b*k + b*k^2] + 2*(-1 + b*k)*x)), x])/((1 - b*k)^2*((1 - x)*x*(1 -
 k*x))^(2/3))
```

Rubi steps

$$\int \frac{\left(-2x+(1+k)x^2\right)\left(a-a(1+k)x+(1+ak)x^2\right)}{(-1+x)((1-x)x(1-kx))^{2/3}(-1+kx)\left(b-b(1+k)x+(-1+bk)x^2\right)}\,dx = \int \frac{x(-2+(1+k)x)\left(a-\right.}{(-1+x)((1-x)x(1-kx))^{2/3}(-1+}$$

$$= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{\sqrt[3]{}}{(1-x)^{2/3}(-}}{((1-x)x}$$

$$= -\frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{\sqrt[3]{x}}{(1-x)^{5/3}(}}{((1-x)x(}$$

$$= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{\sqrt[3]{x}(-2+(}{(1-x)^{5/3}(1-}}{((1-x)x(1-k}$$

$$= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \left(\frac{(2+a+b+}{(1-b}\right.}{}$$

$$= -\frac{\left((1+k)(1+ak)(1-x)^{2/3}x^{2/3}(1-k\right.}{(1-bk)((1-x)x(1}$$

$$= -\frac{3(1+k)(1+ak)x^2}{2(1-k)(1-bk)((1-x)x(1-kx))^{2/}}$$

$$= -\frac{3(1+k)(1+ak)(1-x)x}{(1-k)^2(1-bk)((1-x)x(1-kx))^{2/3}}$$

$$= -\frac{3(1+k)(1+ak)(1-x)x}{(1-k)^2(1-bk)((1-x)x(1-kx))^{2/3}}$$

Mathematica [F] time = 14.56, size = 0, normalized size = 0.00

$$\int \frac{\left(-2x+(1+k)x^2\right)\left(a-a(1+k)x+(1+ak)x^2\right)}{(-1+x)((1-x)x(1-kx))^{2/3}(-1+kx)\left(b-b(1+k)x+(-1+bk)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2*x + (1 + k)*x^2)*(a - a*(1 + k)*x + (1 + a*k)*x^2))/((-1 + x
)*((1 - x)*x*(1 - k*x))^(2/3)*(-1 + k*x)*(b - b*(1 + k)*x + (-1 + b*k)*x^2)
),x]
```

```
[Out] Integrate[((-2*x + (1 + k)*x^2)*(a - a*(1 + k)*x + (1 + a*k)*x^2))/((-1 + x
)*((1 - x)*x*(1 - k*x))^(2/3)*(-1 + k*x)*(b - b*(1 + k)*x + (-1 + b*k)*x^2)
), x]
```

IntegrateAlgebraic [A] time = 0.81, size = 203, normalized size = 1.00

$$\frac{(-a-b)\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+x^2\right)}{2\sqrt[3]{b}}+\frac{(a+b)\log\left(x-\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{\sqrt[3]{b}}+\frac{\left(\sqrt{3}\,a+\sqrt{3}\,b\right)\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+x}\right)}{\sqrt[3]{b}}+\frac{3x^2}{2\left(kx^3-kx^2-x^2+x\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*x + (1 + k)*x^2)*(a - a*(1 + k)*x + (1 + a*k)*x^2))
/((-1 + x)*((1 - x)*x*(1 - k*x))^(2/3)*(-1 + k*x)*(b - b*(1 + k)*x + (-1 +
b*k)*x^2)),x]
```

```
[Out] (3*x^2)/(2*(x - x^2 - k*x^2 + k*x^3)^(2/3)) + ((Sqrt[3]*a + Sqrt[3]*b)*ArcT
an[(Sqrt[3]*x)/(x + 2*b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))])/b^(1/3) +
 ((a + b)*Log[x - b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3)])/b^(1/3) + ((-a
 - b)*Log[x^2 + b^(1/3)*x*(x + (-1 - k)*x^2 + k*x^3)^(1/3) + b^(2/3)*(x + (
-1 - k)*x^2 + k*x^3)^(2/3)])/(2*b^(1/3))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x+(1+k)*x^2)*(a-a*(1+k)*x+(a*k+1)*x^2)/(-1+x)/((1-x)*x*(-k*x+
1))^(2/3)/(k*x-1)/(b-b*(1+k)*x+(b*k-1)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a(k+1)x - (ak+1)x^2 - a\right)\left((k+1)x^2 - 2\,x\right)}{\left(b(k+1)x - (bk-1)x^2 - b\right)\left((kx-1)(x-1)x\right)^{\frac{2}{3}}(kx-1)(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x+(1+k)*x^2)*(a-a*(1+k)*x+(a*k+1)*x^2)/(-1+x)/((1-x)*x*(-k*x+
1))^(2/3)/(k*x-1)/(b-b*(1+k)*x+(b*k-1)*x^2),x, algorithm="giac")
```

```
[Out] integrate((a*(k + 1)*x - (a*k + 1)*x^2 - a)*((k + 1)*x^2 - 2*x)/((b*(k + 1)
*x - (b*k - 1)*x^2 - b)*((k*x - 1)*(x - 1)*x)^(2/3)*(k*x - 1)*(x - 1)), x)
```

maple [**F**] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{\left(-2x + (1+k)\,x^2\right)\left(a - a\,(1+k)\,x + (ak+1)\,x^2\right)}{(-1+x)\left((1-x)\,x\,(-kx+1)\right)^{\frac{2}{3}}(kx-1)\left(b - b\,(1+k)\,x + (bk-1)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x+(1+k)*x^2)*(a-a*(1+k)*x+(a*k+1)*x^2)/(-1+x)/((1-x)*x*(-k*x+1))^(2
/3)/(k*x-1)/(b-b*(1+k)*x+(b*k-1)*x^2),x)
```

```
[Out] int((-2*x+(1+k)*x^2)*(a-a*(1+k)*x+(a*k+1)*x^2)/(-1+x)/((1-x)*x*(-k*x+1))^(2
/3)/(k*x-1)/(b-b*(1+k)*x+(b*k-1)*x^2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a(k+1)x - (ak+1)x^2 - a\right)\left((k+1)x^2 - 2\,x\right)}{\left(b(k+1)x - (bk-1)x^2 - b\right)\left((kx-1)(x-1)x\right)^{\frac{2}{3}}(kx-1)(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x+(1+k)*x^2)*(a-a*(1+k)*x+(a*k+1)*x^2)/(-1+x)/((1-x)*x*(-k*x+
1))^(2/3)/(k*x-1)/(b-b*(1+k)*x+(b*k-1)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((a*(k + 1)*x - (a*k + 1)*x^2 - a)*((k + 1)*x^2 - 2*x)/((b*(k + 1)
*x - (b*k - 1)*x^2 - b)*((k*x - 1)*(x - 1)*x)^(2/3)*(k*x - 1)*(x - 1)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(2\,x - x^2\,(k+1)\right)\left((a\,k+1)\,x^2 - a\,(k+1)\,x + a\right)}{(k\,x-1)\,(x-1)\,\left(x\,(k\,x-1)\,(x-1)\right)^{2/3}\left((b\,k-1)\,x^2 - b\,(k+1)\,x + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*x - x^2*(k + 1))*(a + x^2*(a*k + 1) - a*x*(k + 1)))/((k*x - 1)*(x
- 1)*(x*(k*x - 1)*(x - 1))^(2/3)*(b + x^2*(b*k - 1) - b*x*(k + 1))),x)
```

```
[Out] -int(((2*x - x^2*(k + 1))*(a + x^2*(a*k + 1) - a*x*(k + 1)))/((k*x - 1)*(x
- 1)*(x*(k*x - 1)*(x - 1))^(2/3)*(b + x^2*(b*k - 1) - b*x*(k + 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x+(1+k)*x**2)*(a-a*(1+k)*x+(a*k+1)*x**2)/(-1+x)/((1-x)*x*(-k*
x+1))**(2/3)/(k*x-1)/(b-b*(1+k)*x+(b*k-1)*x**2),x)
```

```
[Out] Timed out
```

3.2003 $\int \frac{-3k+2(1+k^2)x+k(1+k^2)x^2-4k^2x^3+k^3x^4}{((1-x^2)(1-k^2x^2))^{2/3}(1-d-(2+d)kx+(d+k^2)x^2+dkx^3)}\,dx$

Optimal. Leaf size=203

$$-\frac{\log\left(d^{2/3}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\left(\sqrt[3]{d}-\sqrt[3]{d}\,kx\right)+k^2x^2-2kx+1\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{d}\right.}{}$$

Rubi [F] time = 8.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3k+2\left(1+k^2\right)x+k\left(1+k^2\right)x^2-4k^2x^3+k^3x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d-(2+d)kx+\left(d+k^2\right)x^2+dkx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3*k + 2*(1 + k^2)*x + k*(1 + k^2)*x^2 - 4*k^2*x^3 + k^3*x^4)/(((1 - x
^2)*(1 - k^2*x^2))^(2/3)*(1 - d - (2 + d)*k*x + (d + k^2)*x^2 + d*k*x^3)),x
]
```

```
[Out] -((k*(5*d + k^2)*x*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*AppellF1[1/2, 2/3, 2
/3, 3/2, x^2, k^2*x^2])/(d^2*((1 - x^2)*(1 - k^2*x^2))^(2/3))) - (3^(3/4)*S
qrt[2 + Sqrt[3]]*k^(4/3)*Sqrt[(-1 - k^2 + 2*k^2*x^2)^2]*((-1 + k^2)^(2/3) +
 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))*Sqrt[((-1 + k^2)^(4/3) -
2^(2/3)*k^(2/3)*(-1 + k^2)^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3) + 2*2^(1/3
)*k^(4/3)*((1 - x^2)*(1 - k^2*x^2))^(2/3))/((1 + Sqrt[3])*(-1 + k^2)^(2/3)
+ 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))^2]*EllipticF[ArcSin[((1
- Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/
3))/((1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x
^2))^(1/3))], -7 - 4*Sqrt[3]])/(2^(2/3)*d*(1 + k^2 - 2*k^2*x^2)*Sqrt[(-1 -
k^2*(1 - 2*x^2))^2]*Sqrt[((-1 + k^2)^(2/3)*((-1 + k^2)^(2/3) + 2^(2/3)*k^(2
/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3)))/((1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^(
2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))^2]) - (k*(8*d^2 - k^2 - d*(5
- k^2))*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][1/((1 - x^2)^(2/3)*(
1 - k^2*x^2)^(2/3)*(1 - d - (2 + d)*k*x + (d + k^2)*x^2 + d*k*x^3)), x])/(d
^2*((1 - x^2)*(1 - k^2*x^2))^(2/3)) - ((2*k^4 - 2*d^2*(1 - k^2) + d*k^2*(11
 + k^2))*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][x/((1 - x^2)^(2/3)*
(1 - k^2*x^2)^(2/3)*(1 - d - (2 + d)*k*x + (d + k^2)*x^2 + d*k*x^3)), x])/(
d^2*((1 - x^2)*(1 - k^2*x^2))^(2/3)) + (k*(8*d*k^2 + k^4 + 2*d^2*(3 + k^2))
*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][x^2/((1 - x^2)^(2/3)*(1 - k
^2*x^2)^(2/3)*(1 - d - (2 + d)*k*x + (d + k^2)*x^2 + d*k*x^3)), x])/(d^2*((
1 - x^2)*(1 - k^2*x^2))^(2/3))
```

Rubi steps

$$\int \frac{-3k+2\left(1+k^2\right)x+k\left(1+k^2\right)x^2-4k^2x^3+k^3x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d-(2+d)kx+\left(d+k^2\right)x^2+dkx^3\right)}\,dx = \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int \frac{-3k+2(1}{\left(1-x^2\right)^{2/3}\left(1-k^2\right.}}{\left(\left(1-x^2\right)\left(1-k\right.\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int\left(-\frac{k(5d}{d^2\left(1-x^2\right)^{2/3}}\right.}{}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int \frac{-k\left(8d^2-k^2-d(5\right.}{(}}{d^2\left(\left(1\right.\right.}$$

$$= -\frac{k\left(5d+k^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}F}{d^2\left(\left(1-x^2\right)\left(1-k^2x^2\right.\right.}$$

$$= -\frac{k\left(5d+k^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}F}{d^2\left(\left(1-x^2\right)\left(1-k^2x^2\right.\right.}$$

$$= -\frac{k\left(5d+k^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}F}{d^2\left(\left(1-x^2\right)\left(1-k^2x^2\right.\right.}$$

$$= -\frac{k\left(5d+k^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}F}{d^2\left(\left(1-x^2\right)\left(1-k^2x^2\right.\right.}$$

Mathematica [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{-3k+2\left(1+k^2\right)x+k\left(1+k^2\right)x^2-4k^2x^3+k^3x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d-(2+d)kx+\left(d+k^2\right)x^2+dkx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3*k + 2*(1 + k^2)*x + k*(1 + k^2)*x^2 - 4*k^2*x^3 + k^3*x^4)/((
(1 - x^2)*(1 - k^2*x^2))^(2/3)*(1 - d - (2 + d)*k*x + (d + k^2)*x^2 + d*k*x
^3)),x]
```

```
[Out] Integrate[(-3*k + 2*(1 + k^2)*x + k*(1 + k^2)*x^2 - 4*k^2*x^3 + k^3*x^4)/((
(1 - x^2)*(1 - k^2*x^2))^(2/3)*(1 - d - (2 + d)*k*x + (d + k^2)*x^2 + d*k*x
^3)), x]
```

IntegrateAlgebraic [A] time = 8.12, size = 203, normalized size = 1.00

$$-\frac{\log\left(d^{2/3}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\left(\sqrt[3]{d}-\sqrt[3]{d}kx\right)+k^2x^2-2kx+1\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+kx-1\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}kx-\sqrt{3}}{-2\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+kx-1}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*k + 2*(1 + k^2)*x + k*(1 + k^2)*x^2 - 4*k^2*x^3 + k^
3*x^4)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)*(1 - d - (2 + d)*k*x + (d + k^2)*x^
2 + d*k*x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(-Sqrt[3] + Sqrt[3]*k*x)/(-1 + k*x - 2*d^(1/3)*(1 + (-1 - k
^2)*x^2 + k^2*x^4)^(1/3))]])/d^(1/3) + Log[-1 + k*x + d^(1/3)*(1 + (-1 - k^2
)*x^2 + k^2*x^4)^(1/3)]/d^(1/3) - Log[1 - 2*k*x + k^2*x^2 + (d^(1/3) - d^(1
/3)*k*x)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3) + d^(2/3)*(1 + (-1 - k^2)*x^2
 + k^2*x^4)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*k+2*(k^2+1)*x+k*(k^2+1)*x^2-4*k^2*x^3+k^3*x^4)/((-x^2+1)*(-k^
2*x^2+1))^(2/3)/(1-d-(2+d)*k*x+(k^2+d)*x^2+d*k*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^3x^4 - 4\,k^2x^3 + \left(k^2+1\right)kx^2 + 2\left(k^2+1\right)x - 3\,k}{\left(dkx^3 - (d+2)kx + \left(k^2+d\right)x^2 - d + 1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*k+2*(k^2+1)*x+k*(k^2+1)*x^2-4*k^2*x^3+k^3*x^4)/((-x^2+1)*(-k^
2*x^2+1))^(2/3)/(1-d-(2+d)*k*x+(k^2+d)*x^2+d*k*x^3),x, algorithm="giac")
```

```
[Out] integrate((k^3*x^4 - 4*k^2*x^3 + (k^2 + 1)*k*x^2 + 2*(k^2 + 1)*x - 3*k)/((d
*k*x^3 - (d + 2)*k*x + (k^2 + d)*x^2 - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(2/
3)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{-3k + 2\left(k^2+1\right)x + k\left(k^2+1\right)x^2 - 4k^2x^3 + k^3x^4}{\left(\left(-x^2+1\right)\left(-k^2x^2+1\right)\right)^{\frac{2}{3}}\left(1 - d - (2+d)\,kx + \left(k^2+d\right)x^2 + dk\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*k+2*(k^2+1)*x+k*(k^2+1)*x^2-4*k^2*x^3+k^3*x^4)/((-x^2+1)*(-k^2*x^2+
1))^(2/3)/(1-d-(2+d)*k*x+(k^2+d)*x^2+d*k*x^3),x)
```

```
[Out] int((-3*k+2*(k^2+1)*x+k*(k^2+1)*x^2-4*k^2*x^3+k^3*x^4)/((-x^2+1)*(-k^2*x^2+
1))^(2/3)/(1-d-(2+d)*k*x+(k^2+d)*x^2+d*k*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^3x^4 - 4\,k^2x^3 + \left(k^2+1\right)kx^2 + 2\left(k^2+1\right)x - 3\,k}{\left(dkx^3 - (d+2)kx + \left(k^2+d\right)x^2 - d + 1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*k+2*(k^2+1)*x+k*(k^2+1)*x^2-4*k^2*x^3+k^3*x^4)/((-x^2+1)*(-k^
2*x^2+1))^(2/3)/(1-d-(2+d)*k*x+(k^2+d)*x^2+d*k*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k^3*x^4 - 4*k^2*x^3 + (k^2 + 1)*k*x^2 + 2*(k^2 + 1)*x - 3*k)/((d
*k*x^3 - (d + 2)*k*x + (k^2 + d)*x^2 - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(2/
3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{2\,x\,\left(k^2+1\right)-3\,k-4\,k^2\,x^3+k^3\,x^4+k\,x^2\,\left(k^2+1\right)}{{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)}^{2/3}\,\left(d\,k\,x^3+\left(k^2+d\right)\,x^2-k\,\left(d+2\right)\,x-d+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x*(k^2 + 1) - 3*k - 4*k^2*x^3 + k^3*x^4 + k*x^2*(k^2 + 1))/(((x^2 -
1)*(k^2*x^2 - 1))^(2/3)*(x^2*(d + k^2) - d - k*x*(d + 2) + d*k*x^3 + 1)),x)
```

```
[Out] int((2*x*(k^2 + 1) - 3*k - 4*k^2*x^3 + k^3*x^4 + k*x^2*(k^2 + 1))/(((x^2 -
1)*(k^2*x^2 - 1))^(2/3)*(x^2*(d + k^2) - d - k*x*(d + 2) + d*k*x^3 + 1)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*k+2*(k**2+1)*x+k*(k**2+1)*x**2-4*k**2*x**3+k**3*x**4)/((-x**2
+1)*(-k**2*x**2+1))**(2/3)/(1-d-(2+d)*k*x+(k**2+d)*x**2+d*k*x**3),x)
```

```
[Out] Timed out
```

3.2004 $\int \frac{x(4ab-3(a+b)x+2x^2)}{\sqrt[3]{x^2(-a+x)(-b+x)}\,(-ab+(a+b)x-x^2+dx^4)}\,dx$

Optimal. Leaf size=204

$$-\frac{\log\left(\sqrt[3]{d}\,x^2\sqrt[3]{x^3(-a-b)+abx^2+x^4}+\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+d^{2/3}x^4\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{x^3(-a-b)+abx^2+x^4}\right.}{\sqrt[3]{d}}$$

Rubi [F] time = 12.29, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\left(4ab-3(a+b)x+2x^2\right)}{\sqrt[3]{x^2(-a+x)(-b+x)}\,\left(-ab+(a+b)x-x^2+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(4*a*b - 3*(a + b)*x + 2*x^2))/((x^2*(-a + x)*(-b + x))^(1/3)*(-(a*b
) + (a + b)*x - x^2 + d*x^4)),x]
```

```
[Out] (9*(a + b)*x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][Defer[Int][x^
6/((-a + x^3)^(1/3)*(-b + x^3)^(1/3)*(a*b - a*(1 + b/a)*x^3 + x^6 - d*x^12)
), x], x, x^(1/3)])/((a - x)*(b - x)*x^2)^(1/3) + (12*a*b*x^(2/3)*(-a + x)^
(1/3)*(-b + x)^(1/3)*Defer[Subst][Defer[Int][x^3/((-a + x^3)^(1/3)*(-b + x^
3)^(1/3)*(-(a*b) + a*(1 + b/a)*x^3 - x^6 + d*x^12)), x], x, x^(1/3)])/((a -
 x)*(b - x)*x^2)^(1/3) + (6*x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Sub
st][Defer[Int][x^9/((-a + x^3)^(1/3)*(-b + x^3)^(1/3)*(-(a*b) + a*(1 + b/a)
*x^3 - x^6 + d*x^12)), x], x, x^(1/3)])/((a - x)*(b - x)*x^2)^(1/3)
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(4ab-3(a+b)x+2x^2\right)}{\sqrt[3]{x^2(-a+x)(-b+x)}\,\left(-ab+(a+b)x-x^2+dx^4\right)}\,dx &= \frac{\left(x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\int \frac{\sqrt[3]{x}\left(4ab-3(a+b)x+2x^2\right)}{\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\left(-ab+(a+b)x-x\right.}}{\sqrt[3]{x^2(-a+x)(-b+x)}}\\ &= \frac{\left(3x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\text{Subst}\left(\int \frac{x^3\left(4ab-3\right.}{\sqrt[3]{-a+x^3}\,\sqrt[3]{-b+x^3}\left(\right.}\right.}{\sqrt[3]{x^2(-a+x)(-b+x)}}\\ &= \frac{\left(3x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\text{Subst}\left(\int\left(\frac{3}{\sqrt[3]{-a+x^3}\,\sqrt[3]{-b+x^3}}\right.\right.}{}\\ &= \frac{\left(6x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt[3]{-a+x^3}\,\sqrt[3]{-b+x^3}\left(\right.}\right.}{\sqrt[3]{x^2(-a+x)(-b+x)}}\end{aligned}$$

Mathematica [F] time = 3.09, size = 0, normalized size = 0.00

$$\int \frac{x\left(4ab-3(a+b)x+2x^2\right)}{\sqrt[3]{x^2(-a+x)(-b+x)}\,\left(-ab+(a+b)x-x^2+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(4*a*b - 3*(a + b)*x + 2*x^2))/((x^2*(-a + x)*(-b + x))^(1/3)*
(-(a*b) + (a + b)*x - x^2 + d*x^4)),x]
```

```
[Out] Integrate[(x*(4*a*b - 3*(a + b)*x + 2*x^2))/((x^2*(-a + x)*(-b + x))^(1/3)*
(-(a*b) + (a + b)*x - x^2 + d*x^4)), x]
```

IntegrateAlgebraic [A] time = 0.93, size = 204, normalized size = 1.00

$$-\frac{\log\left(\sqrt[3]{d}\,x^2\sqrt[3]{x^3(-a-b)+abx^2+x^4}+\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+d^{2/3}x^4\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{x^3(-a-b)+abx^2+x^4}-\sqrt[3]{d}\,x^2\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^3(-a-b)+abx^2+x^4}}{\sqrt[3]{x^3(-a-b)+abx^2+x^4}+2\sqrt[3]{d}\,x^2}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(4*a*b - 3*(a + b)*x + 2*x^2))/((x^2*(-a + x)*(-b + x
))^(1/3)*(-(a*b) + (a + b)*x - x^2 + d*x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3))/(2*d^(1/3)*x
^2 + (a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3))])/d^(1/3) + Log[-(d^(1/3)*x^2) +
 (a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3)]/d^(1/3) - Log[d^(2/3)*x^4 + d^(1/3)*
x^2*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3) + (a*b*x^2 + (-a - b)*x^3 + x^4)^(
2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a*b+(a+b)*x
-x^2+d*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,ab-3\,(a+b)x+2\,x^2\right)x}{\left(dx^4-ab+(a+b)x-x^2\right)\left((a-x)(b-x)x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a*b+(a+b)*x
-x^2+d*x^4),x, algorithm="giac")
```

```
[Out] integrate((4*a*b - 3*(a + b)*x + 2*x^2)*x/((d*x^4 - a*b + (a + b)*x - x^2)*
((a - x)*(b - x)*x^2)^(1/3)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x\left(4ab-3\,(a+b)\,x+2x^2\right)}{\left(x^2\,(-a+x)\,(-b+x)\right)^{\frac{1}{3}}\left(-ab+(a+b)\,x-x^2+d\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a*b+(a+b)*x-x^2+d
*x^4),x)
```

```
[Out] int(x*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a*b+(a+b)*x-x^2+d
*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,ab-3\,(a+b)x+2\,x^2\right)x}{\left(dx^4-ab+(a+b)x-x^2\right)\left((a-x)(b-x)x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a*b+(a+b)*x
-x^2+d*x^4),x, algorithm="maxima")
```

```
[Out] integrate((4*a*b - 3*(a + b)*x + 2*x^2)*x/((d*x^4 - a*b + (a + b)*x - x^2)*
((a - x)*(b - x)*x^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x\left(4ab+2x^2-3x(a+b)\right)}{\left(x^2(a-x)(b-x)\right)^{1/3}\left(-dx^4+x^2+(-a-b)x+ab\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(4*a*b + 2*x^2 - 3*x*(a + b)))/((x^2*(a - x)*(b - x))^(1/3)*(a*b -
d*x^4 + x^2 - x*(a + b))),x)
```

```
[Out] -int((x*(4*a*b + 2*x^2 - 3*x*(a + b)))/((x^2*(a - x)*(b - x))^(1/3)*(a*b -
d*x^4 + x^2 - x*(a + b))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*a*b-3*(a+b)*x+2*x**2)/(x**2*(-a+x)*(-b+x))**(1/3)/(-a*b+(a+b
)*x-x**2+d*x**4),x)
```

```
[Out] Timed out
```

3.2005 $\int \frac{x^2}{\left(x^2(-a+x)\right)^{2/3}\left(-a^2+2ax+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=205

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\,x}{\sqrt[6]{d}\,x-2\sqrt[3]{x^3-ax^2}}\right)}{2ad^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\,x}{2\sqrt[3]{x^3-ax^2}+\sqrt[6]{d}\,x}\right)}{2ad^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{d}\,x}{\sqrt[3]{x^3-ax^2}}\right)}{ad^{5/6}}+\frac{\tanh^{-1}\left(\frac{\frac{\left(x^3-ax^2\right)^{2/3}}{\sqrt[6]{d}}+\sqrt[6]{d}\,x^2}{x\sqrt[3]{x^3-ax^2}}\right)}{2ad^{5/6}}$$

Rubi [A] time = 0.83, antiderivative size = 408, normalized size of antiderivative = 1.99, number of steps used = 9, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {6719, 911, 105, 59, 91}

$$-\frac{x^{4/3}(x-a)^{2/3}\log\left(2a\left(1-\sqrt{d}\right)-2(1-d)x\right)}{4ad^{5/6}\left(-\left(x^2(a-x)\right)\right)^{2/3}}+\frac{x^{4/3}(x-a)^{2/3}\log\left(2(1-d)x-2a\left(\sqrt{d}+1\right)\right)}{4ad^{5/6}\left(-\left(x^2(a-x)\right)\right)^{2/3}}+\frac{3x^{4/3}(x-a)^{2/3}\log\left(-\sqrt[3]{x-a}-\sqrt[6]{d}\sqrt[3]{x}\right)}{4ad^{5/6}\left(-\left(x^2(a-x)\right)\right)^{2/3}}-\frac{3x^{4/3}(x-a)^{2/3}\log\left(\sqrt[6]{d}\sqrt[3]{x}-\sqrt[3]{x-a}\right)}{4ad^{5/6}\left(-\left(x^2(a-x)\right)\right)^{2/3}}+\frac{\sqrt{3}x^{4/3}(x-a)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{x-a}}\right)}{2ad^{5/6}\left(-\left(x^2(a-x)\right)\right)^{2/3}}-\frac{\sqrt{3}x^{4/3}(x-a)^{2/3}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{x-a}}+\frac{1}{\sqrt{3}}\right)}{2ad^{5/6}\left(-\left(x^2(a-x)\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((x^2*(-a + x))^(2/3)*(-a^2 + 2*a*x + (-1 + d)*x^2)),x]
```

```
[Out] (Sqrt[3]*x^(4/3)*(-a + x)^(2/3)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*x^(1/3))/(Sqr
t[3]*(-a + x)^(1/3))])/(2*a*d^(5/6)*(-((a - x)*x^2))^(2/3)) - (Sqrt[3]*x^(4
/3)*(-a + x)^(2/3)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*x^(1/3))/(Sqrt[3]*(-a + x)
^(1/3))])/(2*a*d^(5/6)*(-((a - x)*x^2))^(2/3)) - (x^(4/3)*(-a + x)^(2/3)*Lo
g[2*a*(1 - Sqrt[d]) - 2*(1 - d)*x])/(4*a*d^(5/6)*(-((a - x)*x^2))^(2/3)) +
(x^(4/3)*(-a + x)^(2/3)*Log[-2*a*(1 + Sqrt[d]) + 2*(1 - d)*x])/(4*a*d^(5/6)
*(-((a - x)*x^2))^(2/3)) + (3*x^(4/3)*(-a + x)^(2/3)*Log[-(d^(1/6)*x^(1/3))
 - (-a + x)^(1/3)])/(4*a*d^(5/6)*(-((a - x)*x^2))^(2/3)) - (3*x^(4/3)*(-a +
 x)^(2/3)*Log[d^(1/6)*x^(1/3) - (-a + x)^(1/3)])/(4*a*d^(5/6)*(-((a - x)*x^
2))^(2/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
```

```
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[m] && !
IntegerQ[n]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{x^2}{\left(x^2(-a+x)\right)^{2/3}\left(-a^2+2ax+(-1+d)x^2\right)}\,dx = \frac{\left(x^{4/3}(-a+x)^{2/3}\right)\int \frac{x^{2/3}}{(-a+x)^{2/3}\left(-a^2+2ax+(-1+d)x^2\right)}\,dx}{\left(x^2(-a+x)\right)^{2/3}}$$

$$= \frac{\left(x^{4/3}(-a+x)^{2/3}\right)\int \left(\frac{(-1+d)x^{2/3}}{a\sqrt{d}\,(-a+x)^{2/3}\left(2a-2a\sqrt{d}-2(1-d)x\right)}+\frac{}{a\sqrt{d}\,(-a+x)}\right.}{\left(x^2(-a+x)\right)^{2/3}}$$

$$= -\frac{\left((1-d)x^{4/3}(-a+x)^{2/3}\right)\int \frac{x^{2/3}}{(-a+x)^{2/3}\left(2a-2a\sqrt{d}-2(1-d)x\right)}\,dx}{a\sqrt{d}\,\left(x^2(-a+x)\right)^{2/3}} - \frac{((}{}$$

$$= -\frac{\left((1-d)x^{4/3}(-a+x)^{2/3}\right)\int \frac{1}{\sqrt[3]{x}\,(-a+x)^{2/3}\left(-2a-2a\sqrt{d}+2(1-d)x\right)}\,dx}{\left(1-\sqrt{d}\right)\sqrt{d}\,\left(x^2(-a+x)\right)^{2/3}} -$$

$$= \frac{\sqrt{3}\,x^{4/3}(-a+x)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{-a+x}}\right)}{2ad^{5/6}\left(-\left((a-x)x^2\right)\right)^{2/3}} - \frac{\sqrt{3}\,x^{4/3}(-a+}{2ad^{5/}}$$

Mathematica [C] time = 0.15, size = 75, normalized size = 0.37

$$\frac{3x^2\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{\sqrt{d}\,x}{x-a}\right)-{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{\sqrt{d}\,x}{a-x}\right)\right)}{4a\sqrt{d}\,\left(x^2(x-a)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((x^2*(-a + x))^(2/3)*(-a^2 + 2*a*x + (-1 + d)*x^2)),x]
```

```
[Out] (3*x^2*(-Hypergeometric2F1[2/3, 1, 5/3, (Sqrt[d]*x)/(a - x)] + Hypergeometr
ic2F1[2/3, 1, 5/3, (Sqrt[d]*x)/(-a + x)]))/(4*a*Sqrt[d]*(x^2*(-a + x))^(2/3
))
```

IntegrateAlgebraic [A] time = 0.69, size = 205, normalized size = 1.00

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\,x}{\sqrt[6]{d}\,x-2\sqrt[3]{x^3-ax^2}}\right)}{2ad^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\,x}{2\sqrt[3]{x^3-ax^2}+\sqrt[6]{d}\,x}\right)}{2ad^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{d}\,x}{\sqrt[3]{x^3-ax^2}}\right)}{ad^{5/6}}+\frac{\tanh^{-1}\left(\frac{\frac{\left(x^3-ax^2\right)^{2/3}}{\sqrt[6]{d}}+\sqrt[6]{d}\,x^2}{x\sqrt[3]{x^3-ax^2}}\right)}{2ad^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((x^2*(-a + x))^(2/3)*(-a^2 + 2*a*x + (-1 + d)*x^2))
,x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/6)*x)/(d^(1/6)*x - 2*(-(a*x^2) + x^3)^(1/3))]
)/(2*a*d^(5/6)) - (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/6)*x)/(d^(1/6)*x + 2*(-(a*x
^2) + x^3)^(1/3))])/(2*a*d^(5/6)) + ArcTanh[(d^(1/6)*x)/(-(a*x^2) + x^3)^(1
/3)]/(a*d^(5/6)) + ArcTanh[(d^(1/6)*x^2 + (-(a*x^2) + x^3)^(2/3)/d^(1/6))/(
x*(-(a*x^2) + x^3)^(1/3))]/(2*a*d^(5/6))
```

fricas [B] time = 0.46, size = 521, normalized size = 2.54

$$-\sqrt{3}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{3}a^5d^4x\sqrt{\frac{a^2d^2x^2\left(\frac{1}{a^6d^5}\right)^{\frac{1}{3}}+\left(-ax^2+x^3\right)^{\frac{1}{3}}adx\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}+\left(-ax^2+x^3\right)^{\frac{2}{3}}}{x^2}}\left(\frac{1}{a^6d^5}\right)^{\frac{5}{6}}-2\sqrt{3}\left(-ax^2+x^3\right)^{\frac{1}{3}}a^5d^4\left(\frac{1}{a^6d^5}\right)^{\frac{5}{6}}-\sqrt{3}x}{3x}\right)-\sqrt{3}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{3}a^5d^4x\sqrt{\frac{a^2d^2x^2\left(\frac{1}{a^6d^5}\right)^{\frac{1}{3}}-\left(-ax^2+x^3\right)^{\frac{1}{3}}adx\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}+\left(-ax^2+x^3\right)^{\frac{2}{3}}}{x^2}}\left(\frac{1}{a^6d^5}\right)^{\frac{5}{6}}-2\sqrt{3}\left(-ax^2+x^3\right)^{\frac{1}{3}}a^5d^4\left(\frac{1}{a^6d^5}\right)^{\frac{5}{6}}+\sqrt{3}x}{3x}\right)+\frac{1}{2}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\log\left(\frac{adx\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}+\left(-ax^2+x^3\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{2}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\log\left(-\frac{adx\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}-\left(-ax^2+x^3\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{4}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\log\left(\frac{a^2d^2x^2\left(\frac{1}{a^6d^5}\right)^{\frac{1}{3}}+\left(-ax^2+x^3\right)^{\frac{1}{3}}adx\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}+\left(-ax^2+x^3\right)^{\frac{2}{3}}}{x^2}\right)-\frac{1}{4}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\log\left(\frac{a^2d^2x^2\left(\frac{1}{a^6d^5}\right)^{\frac{1}{3}}-\left(-ax^2+x^3\right)^{\frac{1}{3}}adx\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}+\left(-ax^2+x^3\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2*(-a+x))^(2/3)/(-a^2+2*a*x+(-1+d)*x^2),x, algorithm="fric
as")
```

```
[Out] -sqrt(3)*(1/(a^6*d^5))^(1/6)*arctan(1/3*(2*sqrt(3)*a^5*d^4*x*sqrt((a^2*d^2*
x^2*(1/(a^6*d^5))^(1/3) + (-a*x^2 + x^3)^(1/3)*a*d*x*(1/(a^6*d^5))^(1/6) +
(-a*x^2 + x^3)^(2/3))/x^2)*(1/(a^6*d^5))^(5/6) - 2*sqrt(3)*(-a*x^2 + x^3)^(
1/3)*a^5*d^4*(1/(a^6*d^5))^(5/6) - sqrt(3)*x)/x) - sqrt(3)*(1/(a^6*d^5))^(1
/6)*arctan(1/3*(2*sqrt(3)*a^5*d^4*x*sqrt((a^2*d^2*x^2*(1/(a^6*d^5))^(1/3) -
 (-a*x^2 + x^3)^(1/3)*a*d*x*(1/(a^6*d^5))^(1/6) + (-a*x^2 + x^3)^(2/3))/x^2
)*(1/(a^6*d^5))^(5/6) - 2*sqrt(3)*(-a*x^2 + x^3)^(1/3)*a^5*d^4*(1/(a^6*d^5)
)^(5/6) + sqrt(3)*x)/x) + 1/2*(1/(a^6*d^5))^(1/6)*log((a*d*x*(1/(a^6*d^5))^
(1/6) + (-a*x^2 + x^3)^(1/3))/x) - 1/2*(1/(a^6*d^5))^(1/6)*log(-(a*d*x*(1/(
a^6*d^5))^(1/6) - (-a*x^2 + x^3)^(1/3))/x) + 1/4*(1/(a^6*d^5))^(1/6)*log((a
^2*d^2*x^2*(1/(a^6*d^5))^(1/3) + (-a*x^2 + x^3)^(1/3)*a*d*x*(1/(a^6*d^5))^(
1/6) + (-a*x^2 + x^3)^(2/3))/x^2) - 1/4*(1/(a^6*d^5))^(1/6)*log((a^2*d^2*x^
2*(1/(a^6*d^5))^(1/3) - (-a*x^2 + x^3)^(1/3)*a*d*x*(1/(a^6*d^5))^(1/6) + (-
a*x^2 + x^3)^(2/3))/x^2)
```

giac [A] time = 0.22, size = 209, normalized size = 1.02

$$-\frac{\sqrt{3}\log\left(\sqrt{3}\,(-d)^{\frac{1}{6}}\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}+\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}+(-d)^{\frac{1}{3}}\right)}{4a(-d)^{\frac{5}{6}}}-\frac{\sqrt{3}\,(-d)^{\frac{1}{6}}\log\left(-\sqrt{3}\,(-d)^{\frac{1}{6}}\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}+\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}+(-d)^{\frac{1}{3}}\right)}{4ad}-\frac{\arctan\left(\frac{\sqrt{3}(-d)^{\frac{1}{6}}+2\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}}{(-d)^{\frac{1}{6}}}\right)}{2a(-d)^{\frac{5}{6}}}-\frac{\arctan\left(-\frac{\sqrt{3}(-d)^{\frac{1}{6}}-2\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}}{(-d)^{\frac{1}{6}}}\right)}{2a(-d)^{\frac{5}{6}}}-\frac{\arctan\left(\frac{\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}}{(-d)^{\frac{1}{6}}}\right)}{a(-d)^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2*(-a+x))^(2/3)/(-a^2+2*a*x+(-1+d)*x^2),x, algorithm="giac
")
```

```
[Out] -1/4*sqrt(3)*log(sqrt(3)*(-d)^(1/6)*(-a/x + 1)^(1/3) + (-a/x + 1)^(2/3) + (
-d)^(1/3))/(a*(-d)^(5/6)) - 1/4*sqrt(3)*(-d)^(1/6)*log(-sqrt(3)*(-d)^(1/6)*
(-a/x + 1)^(1/3) + (-a/x + 1)^(2/3) + (-d)^(1/3))/(a*d) - 1/2*arctan((sqrt(
3)*(-d)^(1/6) + 2*(-a/x + 1)^(1/3))/(-d)^(1/6))/(a*(-d)^(5/6)) - 1/2*arctan
(-(sqrt(3)*(-d)^(1/6) - 2*(-a/x + 1)^(1/3))/(-d)^(1/6))/(a*(-d)^(5/6)) - ar
ctan((-a/x + 1)^(1/3)/(-d)^(1/6))/(a*(-d)^(5/6))
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2(-a+x)\right)^{\frac{2}{3}}\left(-a^2+2ax+(-1+d)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2*(-a+x))^(2/3)/(-a^2+2*a*x+(-1+d)*x^2),x)
```

```
[Out] int(x^2/(x^2*(-a+x))^(2/3)/(-a^2+2*a*x+(-1+d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-(a-x)x^2\right)^{\frac{2}{3}}\left((d-1)x^2-a^2+2\,ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2*(-a+x))^(2/3)/(-a^2+2*a*x+(-1+d)*x^2),x, algorithm="maxi
ma")
```

```
[Out] integrate(x^2/((-(a - x)*x^2)^(2/3)*((d - 1)*x^2 - a^2 + 2*a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(-x^2\,\left(a-x\right)\right)^{2/3}\,\left(-a^2+2\,a\,x+\left(d-1\right)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((-x^2*(a - x))^(2/3)*(2*a*x - a^2 + x^2*(d - 1))),x)
```

```
[Out] int(x^2/((-x^2*(a - x))^(2/3)*(2*a*x - a^2 + x^2*(d - 1))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2\left(-a+x\right)\right)^{\frac{2}{3}}\left(-a^2+2ax+dx^2-x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2*(-a+x))**(2/3)/(-a**2+2*a*x+(-1+d)*x**2),x)
```

```
[Out] Integral(x**2/((x**2*(-a + x))**(2/3)*(-a**2 + 2*a*x + d*x**2 - x**2)), x)
```

3.2006 $\int \frac{(-2+(1+k)x)\left(1-(1+k)x+(a+k)x^2\right)}{x((1-x)x(1-kx))^{2/3}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$

Optimal. Leaf size=205

$$\frac{(a+b)\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{(-a-b)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1\right.\right.}{2b^{2/3}}$$

Rubi [F] time = 25.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+(1+k)x)\left(1-(1+k)x+(a+k)x^2\right)}{x((1-x)x(1-kx))^{2/3}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + (1 + k)*x)*(1 - (1 + k)*x + (a + k)*x^2))/(x*((1 - x)*x*(1 - k*x
))^(2/3)*(1 - (1 + k)*x + (-b + k)*x^2)),x]
```

```
[Out] (3*(2*k^2 - a*(1 + 2*b + k^2) - b*(1 + 4*k + k^2))*(1 - x)*(1 - k*x))/(2*(b
 - k)^2*((1 - x)*x*(1 - k*x))^(2/3)) + (3*(1 + k)*(a + k)*(1 - x)*(((1 - k)
*x)/(1 - k*x))^(2/3)*(1 - k*x)*Hypergeometric2F1[1/3, 2/3, 4/3, (1 - x)/(1
- k*x)])/((1 - k)*(b - k)*((1 - x)*x*(1 - k*x))^(2/3)) + (3*(1 + k)*(2*k^2
- a*(1 + 2*b + k^2) - b*(1 + 4*k + k^2))*(1 - x)*(((1 - k)*x)/(1 - k*x))^(2
/3)*(1 - k*x)*Hypergeometric2F1[1/3, 2/3, 4/3, (1 - x)/(1 - k*x)])/(2*(1 -
k)*(b - k)^2*((1 - x)*x*(1 - k*x))^(2/3)) + ((a + b)*(1 + k^3 + 3*b*(1 + k)
 + (4*b^2 + (1 - k)^2*(1 + k + k^2) + b*(5 + 2*k + 5*k^2))/Sqrt[4*b + (-1 +
 k)^2])*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Int][1/((1 - x)^(2/3)*x
^(5/3)*(1 - k*x)^(2/3)*(-1 - k - Sqrt[1 + 4*b - 2*k + k^2] + 2*(-b + k)*x))
, x])/((b - k)^2*((1 - x)*x*(1 - k*x))^(2/3)) + ((a + b)*(1 + k^3 + 3*b*(1
+ k) - (1 + 4*b^2 - k - k^3 + k^4 + b*(5 + 2*k + 5*k^2))/Sqrt[4*b + (-1 + k
)^2])*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Int][1/((1 - x)^(2/3)*x^(
5/3)*(1 - k*x)^(2/3)*(-1 - k + Sqrt[1 + 4*b - 2*k + k^2] + 2*(-b + k)*x)),
x])/((b - k)^2*((1 - x)*x*(1 - k*x))^(2/3))
```

Rubi steps

$$\int \frac{(-2+(1+k)x)\left(1-(1+k)x+(a+k)x^2\right)}{x((1-x)x(1-kx))^{2/3}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx=\frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{(-2+(1+k)x)\left(1-(1+k)x+(a+k)x^2\right)}{(1-x)^{2/3}x^{5/3}(1-kx)^{2/3}\left(1-(1+k)x+(-\right.}}{((1-x)x(1-kx))^{2/3}}$$

$$=\frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int\left(-\frac{2k^2-a\left(1+2b+k^2\right)-b\left(1+4k+k^2\right)}{(b-k)^2(1-x)^{2/3}x^{5/3}(1-kx)^{2/3}}\right.}{((1}$$

$$=-\frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int\frac{(a+b)\left(1+2b+k^2\right)-(a+b)(1+k)\left(1+\right.}{(1-x)^{2/3}x^{5/3}(1-kx)^{2/3}\left(1+(-1-k)x\right.}}{(b-k)^2((1-x)x(1-kx))^{2/3}}$$

$$=\frac{3\left(2k^2-a\left(1+2b+k^2\right)-b\left(1+4k+k^2\right)\right)(1-x)(1-k}{2(b-k)^2((1-x)x(1-kx))^{2/3}}$$

$$=\frac{3\left(2k^2-a\left(1+2b+k^2\right)-b\left(1+4k+k^2\right)\right)(1-x)(1-k}{2(b-k)^2((1-x)x(1-kx))^{2/3}}$$

Mathematica [F] time = 14.01, size = 0, normalized size = 0.00

$$\int \frac{(-2+(1+k)x)\left(1-(1+k)x+(a+k)x^2\right)}{x((1-x)x(1-kx))^{2/3}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + (1 + k)*x)*(1 - (1 + k)*x + (a + k)*x^2))/(x*((1 - x)*x*(1
 - k*x))^(2/3)*(1 - (1 + k)*x + (-b + k)*x^2)),x]
```

```
[Out] Integrate[((-2 + (1 + k)*x)*(1 - (1 + k)*x + (a + k)*x^2))/(x*((1 - x)*x*(1
 - k*x))^(2/3)*(1 - (1 + k)*x + (-b + k)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.47, size = 205, normalized size = 1.00

$$\frac{(a+b)\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{(-a-b)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2b^{2/3}}+\frac{\left(\sqrt{3}\,a+\sqrt{3}\,b\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{b^{2/3}}+\frac{3\sqrt[3]{kx^3+(-k-1)x^2+x}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + (1 + k)*x)*(1 - (1 + k)*x + (a + k)*x^2))/(x*((1
- x)*x*(1 - k*x))^(2/3)*(1 - (1 + k)*x + (-b + k)*x^2)),x]
```

```
[Out] (3*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/x + ((Sqrt[3]*a + Sqrt[3]*b)*ArcTan[(S
qrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(x + (-1 - k)*x^2 + k*x^3)^(1/3))])/b^(2/3
) + ((a + b)*Log[-(b^(1/3)*x) + (x + (-1 - k)*x^2 + k*x^3)^(1/3)])/b^(2/3)
+ ((-a - b)*Log[b^(2/3)*x^2 + b^(1/3)*x*(x + (-1 - k)*x^2 + k*x^3)^(1/3) +
(x + (-1 - k)*x^2 + k*x^3)^(2/3)])/(2*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x^2)/x/((1-x)*x*(-k*x+1))^(2/3)/(1-
(1+k)*x+(-b+k)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left((a+k)x^2-(k+1)x+1\right)((k+1)x-2)}{((kx-1)(x-1)x)^{\frac{2}{3}}\left((b-k)x^2+(k+1)x-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x^2)/x/((1-x)*x*(-k*x+1))^(2/3)/(1-
(1+k)*x+(-b+k)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-((a + k)*x^2 - (k + 1)*x + 1)*((k + 1)*x - 2)/(((k*x - 1)*(x - 1
)*x)^(2/3)*((b - k)*x^2 + (k + 1)*x - 1)*x), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(-2+(1+k)\,x)\left(1-(1+k)\,x+(a+k)\,x^2\right)}{x\left((1-x)\,x\,(-kx+1)\right)^{\frac{2}{3}}\left(1-(1+k)\,x+(-b+k)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x^2)/x/((1-x)*x*(-k*x+1))^(2/3)/(1-(1+k)*
x+(-b+k)*x^2),x)
```

```
[Out] int((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x^2)/x/((1-x)*x*(-k*x+1))^(2/3)/(1-(1+k)*
x+(-b+k)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left((a+k)x^2-(k+1)x+1\right)((k+1)x-2)}{((kx-1)(x-1)x)^{\frac{2}{3}}\left((b-k)x^2+(k+1)x-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x^2)/x/((1-x)*x*(-k*x+1))^(2/3)/(1-
(1+k)*x+(-b+k)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate(((a + k)*x^2 - (k + 1)*x + 1)*((k + 1)*x - 2)/(((k*x - 1)*(x - 1
)*x)^(2/3)*((b - k)*x^2 + (k + 1)*x - 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(x\,(k+1)-2)\,\left((a+k)\,x^2+(-k-1)\,x+1\right)}{x\,(x\,(k\,x-1)\,(x-1))^{2/3}\,\left((b-k)\,x^2+(k+1)\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x*(k + 1) - 2)*(x^2*(a + k) - x*(k + 1) + 1))/(x*(x*(k*x - 1)*(x - 1
))^(2/3)*(x*(k + 1) + x^2*(b - k) - 1)),x)
```

```
[Out] int(-((x*(k + 1) - 2)*(x^2*(a + k) - x*(k + 1) + 1))/(x*(x*(k*x - 1)*(x - 1
))^(2/3)*(x*(k + 1) + x^2*(b - k) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x**2)/x/((1-x)*x*(-k*x+1))**(2/3)/(
1-(1+k)*x+(-b+k)*x**2),x)
```

```
[Out] Timed out
```

3.2007 $\int \frac{-2a^2bx+a(3a+2b)x^2-4ax^3+x^4}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(-a^2+2ax-(1+bd)x^2+dx^3\right)}\,dx$

Optimal. Leaf size=206

$$-\frac{\log\left(a^2+d^{2/3}\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+\sqrt[3]{x^3(-a-b)+abx^2+x^4}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)-2ax+x^2\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{d}\ \sqrt[3]{\ldots}\right.}{}$$

Rubi [F] time = 11.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int \frac{-2a^2bx+a(3a+2b)x^2-4ax^3+x^4}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(-a^2+2ax-(1+bd)x^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*a^2*b*x + a*(3*a + 2*b)*x^2 - 4*a*x^3 + x^4)/((x^2*(-a + x)*(-b + x
))^(2/3)*(-a^2 + 2*a*x - (1 + b*d)*x^2 + d*x^3)),x]
```

```
[Out] (9*a*x^(4/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x^4*(-a
 + x^3)^(1/3))/((-b + x^3)^(2/3)*(a^2 - 2*a*x^3 + (1 + b*d)*x^6 - d*x^9)),
x], x, x^(1/3)])/((a - x)*(b - x)*x^2)^(2/3) + (6*a*b*x^(4/3)*(-a + x)^(2/3
)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x*(-a + x^3)^(1/3))/((-b + x^3)^(
2/3)*(-a^2 + 2*a*x^3 - (1 + b*d)*x^6 + d*x^9)), x], x, x^(1/3)])/((a - x)*(
b - x)*x^2)^(2/3) + (3*x^(4/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Subst][D
efer[Int][(x^7*(-a + x^3)^(1/3))/((-b + x^3)^(2/3)*(-a^2 + 2*a*x^3 - (1 + b
*d)*x^6 + d*x^9)), x], x, x^(1/3)])/((a - x)*(b - x)*x^2)^(2/3)
```

Rubi steps

$$\int \frac{-2a^2bx+a(3a+2b)x^2-4ax^3+x^4}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(-a^2+2ax-(1+bd)x^2+dx^3\right)}\,dx = \frac{\left(x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{-2a^2bx+a\ldots}{x^{4/3}(-a+x)^{2/3}(-b+x\ldots}}{\left(x^2(-a+x)(-b+x)\right)^{\ldots}}$$

$$= \frac{\left(x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{-2a^2b+(3\ldots}{\sqrt[3]{x}\,(-a+x)^{2/3}(-b+x\ldots}}{\left(x^2(-a+x)(-b+x)\right)^{\ldots}}$$

$$= \frac{\left(x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{\sqrt[3]{-a+x}\,(2a\ldots}{\sqrt[3]{x}\,(-b+x)^{2/3}\left(-a^2+\ldots\right.}}{\left(x^2(-a+x)(-b+x)\right)^{2/3}}$$

$$= \frac{\left(3x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\text{Subst}\left(\int \frac{x\ldots}{(-b+x^3)\ldots}\right.}{\left(x^2(-a+x)(-b+\ldots\right.}$$

$$= \frac{\left(3x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\text{Subst}\left(\int\left(\frac{\ldots}{(-b+x^3\ldots}\right.\right.}{\ldots}$$

$$= \frac{\left(3x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\text{Subst}\left(\int \frac{\ldots}{(-b+x^3)\ldots}\right.}{\left(x^2(-a+x)(-b+\ldots\right.}$$

Mathematica [F] time = 4.49, size = 0, normalized size = 0.00

$$\int \frac{-2a^2bx + a(3a + 2b)x^2 - 4ax^3 + x^4}{\left(x^2(-a + x)(-b + x)\right)^{2/3} \left(-a^2 + 2ax - (1 + bd)x^2 + dx^3\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*a^2*b*x + a*(3*a + 2*b)*x^2 - 4*a*x^3 + x^4)/((x^2*(-a + x)*(
-b + x))^(2/3)*(-a^2 + 2*a*x - (1 + b*d)*x^2 + d*x^3)),x]
```

```
[Out] Integrate[(-2*a^2*b*x + a*(3*a + 2*b)*x^2 - 4*a*x^3 + x^4)/((x^2*(-a + x)*(
-b + x))^(2/3)*(-a^2 + 2*a*x - (1 + b*d)*x^2 + d*x^3)), x]
```

IntegrateAlgebraic [A] time = 4.20, size = 206, normalized size = 1.00

$$-\frac{\log \left(a^2 + d^{2/3} \left(x^3(-a - b) + abx^2 + x^4\right)^{2/3} + \sqrt[3]{x^3(-a - b) + abx^2 + x^4} \left(\sqrt[3]{d}\, x - a\sqrt[3]{d}\right) - 2ax + x^2\right)}{2\sqrt[3]{d}} + \frac{\log \left(\sqrt[3]{d}\, \sqrt[3]{x^3(-a - b) + abx^2 + x^4} + a - x\right)}{\sqrt[3]{d}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt{3}\, a - \sqrt{3}\, x}{-2\sqrt[3]{d}\, \sqrt[3]{x^3(-a-b)+abx^2+x^4} + a - x}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*a^2*b*x + a*(3*a + 2*b)*x^2 - 4*a*x^3 + x^4)/((x^2*(
-a + x)*(-b + x))^(2/3)*(-a^2 + 2*a*x - (1 + b*d)*x^2 + d*x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*a - Sqrt[3]*x)/(a - x - 2*d^(1/3)*(a*b*x^2 + (-a -
 b)*x^3 + x^4)^(1/3))]])/d^(1/3) + Log[a - x + d^(1/3)*(a*b*x^2 + (-a - b)*x
^3 + x^4)^(1/3)]/d^(1/3) - Log[a^2 - 2*a*x + x^2 + (-(a*d^(1/3)) + d^(1/3)*
x)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3) + d^(2/3)*(a*b*x^2 + (-a - b)*x^3 +
 x^4)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a^2*b*x+a*(3*a+2*b)*x^2-4*a*x^3+x^4)/(x^2*(-a+x)*(-b+x))^(2/3
)/(-a^2+2*a*x-(b*d+1)*x^2+d*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,a^2bx - (3\,a + 2\,b)ax^2 + 4\,ax^3 - x^4}{\left((a - x)(b - x)x^2\right)^{\frac{2}{3}} \left(dx^3 - (bd + 1)x^2 - a^2 + 2\,ax\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a^2*b*x+a*(3*a+2*b)*x^2-4*a*x^3+x^4)/(x^2*(-a+x)*(-b+x))^(2/3
)/(-a^2+2*a*x-(b*d+1)*x^2+d*x^3),x, algorithm="giac")
```

```
[Out] integrate(-(2*a^2*b*x - (3*a + 2*b)*a*x^2 + 4*a*x^3 - x^4)/(((a - x)*(b - x
)*x^2)^(2/3)*(d*x^3 - (b*d + 1)*x^2 - a^2 + 2*a*x)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{-2a^2bx + a\,(3a + 2b)\,x^2 - 4a\,x^3 + x^4}{\left(x^2\,(-a + x)\,(-b + x)\right)^{\frac{2}{3}} \left(-a^2 + 2ax - (bd + 1)\,x^2 + d\,x^3\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a^2*b*x+a*(3*a+2*b)*x^2-4*a*x^3+x^4)/(x^2*(-a+x)*(-b+x))^(2/3)/(-a^
2+2*a*x-(b*d+1)*x^2+d*x^3),x)
```

```
[Out] int((-2*a^2*b*x+a*(3*a+2*b)*x^2-4*a*x^3+x^4)/(x^2*(-a+x)*(-b+x))^(2/3)/(-a^
2+2*a*x-(b*d+1)*x^2+d*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,a^2bx - (3\,a + 2\,b)ax^2 + 4\,ax^3 - x^4}{\left((a-x)(b-x)x^2\right)^{\frac{2}{3}}\left(dx^3 - (bd+1)x^2 - a^2 + 2\,ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a^2*b*x+a*(3*a+2*b)*x^2-4*a*x^3+x^4)/(x^2*(-a+x)*(-b+x))^(2/3
)/(-a^2+2*a*x-(b*d+1)*x^2+d*x^3),x, algorithm="maxima")
```

```
[Out] -integrate((2*a^2*b*x - (3*a + 2*b)*a*x^2 + 4*a*x^3 - x^4)/(((a - x)*(b - x
)*x^2)^(2/3)*(d*x^3 - (b*d + 1)*x^2 - a^2 + 2*a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{4\,a\,x^3 - x^4 - a\,x^2\,\left(3\,a + 2\,b\right) + 2\,a^2\,b\,x}{{\left(x^2\,\left(a-x\right)\,\left(b-x\right)\right)}^{2/3}\,\left(-a^2 + 2\,a\,x + d\,x^3 + \left(-b\,d - 1\right)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*a*x^3 - x^4 - a*x^2*(3*a + 2*b) + 2*a^2*b*x)/((x^2*(a - x)*(b - x))
^(2/3)*(2*a*x + d*x^3 - x^2*(b*d + 1) - a^2)),x)
```

```
[Out] int(-(4*a*x^3 - x^4 - a*x^2*(3*a + 2*b) + 2*a^2*b*x)/((x^2*(a - x)*(b - x))
^(2/3)*(2*a*x + d*x^3 - x^2*(b*d + 1) - a^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a**2*b*x+a*(3*a+2*b)*x**2-4*a*x**3+x**4)/(x**2*(-a+x)*(-b+x))
**(2/3)/(-a**2+2*a*x-(b*d+1)*x**2+d*x**3),x)
```

```
[Out] Timed out
```

3.2008 $\int \frac{1}{\sqrt[3]{-8+12x+54x^2-135x^3+81x^4}}\,dx$

Optimal. Leaf size=206

$$\frac{\log\left(3^{2/3}\sqrt[3]{81x^4-135x^3+54x^2+12x-8}-9x+6\right)}{3\sqrt[3]{3}}-\frac{\log\left(27x^2+\sqrt[3]{3}\left(81x^4-135x^3+54x^2+12x-8\right)^{2/3}+(3\ 3\right.}{6\sqrt[3]{}}$$

Rubi [A] time = 0.07, antiderivative size = 170, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6688, 6719, 55, 617, 204, 31}

$$\frac{(2-3x)\sqrt[3]{3x+1}\log(2-3x)}{6\sqrt[3]{3}\sqrt[3]{-(2-3x)^3(3x+1)}}-\frac{(2-3x)\sqrt[3]{3x+1}\log\left(\sqrt[3]{3}-\sqrt[3]{3x+1}\right)}{2\sqrt[3]{3}\sqrt[3]{-(2-3x)^3(3x+1)}}-\frac{(2-3x)\sqrt[3]{3x+1}\tan^{-1}\left(\frac{2\sqrt[3]{3x+1}+\sqrt[3]{3}}{3^{5/6}}\right)}{3^{5/6}\sqrt[3]{-(2-3x)^3(3x+1)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-8 + 12*x + 54*x^2 - 135*x^3 + 81*x^4)^(-1/3),x]
```

```
[Out] -(((2 - 3*x)*(1 + 3*x)^(1/3)*ArcTan[(3^(1/3) + 2*(1 + 3*x)^(1/3))/3^(5/6)])
/(3^(5/6)*(-((2 - 3*x)^3*(1 + 3*x)))^(1/3))) + ((2 - 3*x)*(1 + 3*x)^(1/3)*L
og[2 - 3*x])/(6*3^(1/3)*(-((2 - 3*x)^3*(1 + 3*x)))^(1/3)) - ((2 - 3*x)*(1 +
 3*x)^(1/3)*Log[3^(1/3) - (1 + 3*x)^(1/3)])/(2*3^(1/3)*(-((2 - 3*x)^3*(1 +
3*x)))^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-8+12x+54x^2-135x^3+81x^4}}\,dx = \int \frac{1}{\sqrt[3]{(-2+3x)^3(1+3x)}}\,dx$$

$$= \frac{\left((-2+3x)\sqrt[3]{1+3x}\right)\int \frac{1}{(-2+3x)\sqrt[3]{1+3x}}\,dx}{\sqrt[3]{(-2+3x)^3(1+3x)}}$$

$$= \frac{(2-3x)\sqrt[3]{1+3x}\,\log(2-3x)}{6\sqrt[3]{3}\,\sqrt[3]{-(2-3x)^3(1+3x)}} + \frac{\left((-2+3x)\sqrt[3]{1+3x}\right)\operatorname{Subst}\left(\int \frac{\cdots}{3^{2/3}\cdots}\cdots\right)}{2\sqrt[3]{(-2+3x)^3(1\cdots}}$$

$$= \frac{(2-3x)\sqrt[3]{1+3x}\,\log(2-3x)}{6\sqrt[3]{3}\,\sqrt[3]{-(2-3x)^3(1+3x)}} - \frac{(2-3x)\sqrt[3]{1+3x}\,\log\left(\sqrt[3]{3}-\sqrt[3]{1+\cdots}\right.}{2\sqrt[3]{3}\,\sqrt[3]{-(2-3x)^3(1+3x)}}\cdots$$

$$= -\frac{(2-3x)\sqrt[3]{1+3x}\,\tan^{-1}\left(\frac{1}{3}\left(\sqrt{3}+2\sqrt[6]{3}\,\sqrt[3]{1+3x}\right)\right)}{3^{5/6}\sqrt[3]{-(2-3x)^3(1+3x)}} + \frac{(2-3x)\sqrt[3]{1\cdots}}{6\sqrt[3]{3}\,\sqrt[3]{-(2\cdots}}\cdots$$

Mathematica [A] time = 0.09, size = 100, normalized size = 0.49

$$-\frac{(3x-2)\sqrt[3]{3x+1}\left(\sqrt{3}\left(\log(2-3x)-3\log\left(\sqrt[3]{3}-\sqrt[3]{3x+1}\right)\right)-6\tan^{-1}\left(\frac{2\sqrt[3]{3x+1}+\sqrt[3]{3}}{3^{5/6}}\right)\right)}{6\,3^{5/6}\sqrt[3]{(3x-2)^3(3x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-8 + 12*x + 54*x^2 - 135*x^3 + 81*x^4)^(-1/3),x]
```

```
[Out] -1/6*((-2 + 3*x)*(1 + 3*x)^(1/3)*(-6*ArcTan[(3^(1/3) + 2*(1 + 3*x)^(1/3))/3
^(5/6)] + Sqrt[3]*(Log[2 - 3*x] - 3*Log[3^(1/3) - (1 + 3*x)^(1/3)])))/(3^(5
/6)*((-2 + 3*x)^3*(1 + 3*x))^(1/3))
```

IntegrateAlgebraic [A] time = 0.32, size = 206, normalized size = 1.00

$$\frac{\log\left(3^{2/3}\sqrt[3]{81x^4-135x^3+54x^2+12x-8}-9x+6\right)}{3\sqrt[3]{3}} - \frac{\log\left(27x^2+\sqrt[3]{3}\left(81x^4-135x^3+54x^2+12x-8\right)^{2/3}+\left(3\,3^{2/3}x-2\,3^{2/3}\right)\sqrt[3]{81x^4-135x^3+54x^2+12x-8}-36x+12\right)}{6\sqrt[3]{3}} - \frac{\tan^{-1}\left(\frac{3\,3^{5/6}x-2\,3^{5/6}}{2\sqrt[3]{81x^4-135x^3+54x^2+12x-8}+3\sqrt[3]{3}x-2\sqrt[3]{3}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-8 + 12*x + 54*x^2 - 135*x^3 + 81*x^4)^(-1/3),x]
```

```
[Out] -(ArcTan[(-2*3^(5/6) + 3*3^(5/6)*x)/(-2*3^(1/3) + 3*3^(1/3)*x + 2*(-8 + 12*
x + 54*x^2 - 135*x^3 + 81*x^4)^(1/3))]/3^(5/6)) + Log[6 - 9*x + 3^(2/3)*(-8
 + 12*x + 54*x^2 - 135*x^3 + 81*x^4)^(1/3)]/(3*3^(1/3)) - Log[12 - 36*x + 2
7*x^2 + (-2*3^(2/3) + 3*3^(2/3)*x)*(-8 + 12*x + 54*x^2 - 135*x^3 + 81*x^4)^
(1/3) + 3^(1/3)*(-8 + 12*x + 54*x^2 - 135*x^3 + 81*x^4)^(2/3)]/(6*3^(1/3))
```

fricas [A] time = 0.83, size = 189, normalized size = 0.92

$$-\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(\frac{3^{\frac{2}{3}}\left(9x^2-12x+4\right)+3^{\frac{1}{3}}\left(81x^4-135x^3+54x^2+12x-8\right)^{\frac{1}{3}}(3x-2)+\left(81x^4-135x^3+54x^2+12x-8\right)^{\frac{2}{3}}}{9x^2-12x+4}\right)+\frac{1}{9}\cdot 3^{\frac{2}{3}}\log\left(-\frac{3^{\frac{1}{3}}(3x-2)-\left(81x^4-135x^3+54x^2+12x-8\right)^{\frac{1}{3}}}{3x-2}\right)+\frac{1}{3}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{3^{\frac{1}{6}}\left(3^{\frac{1}{3}}(3x-2)+2\left(81x^4-135x^3+54x^2+12x-8\right)^{\frac{1}{3}}\right)}{3(3x-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(81*x^4-135*x^3+54*x^2+12*x-8)^(1/3),x, algorithm="fricas")

[Out] -1/18*3^(2/3)*log((3^(2/3)*(9*x^2 - 12*x + 4) + 3^(1/3)*(81*x^4 - 135*x^3 +
 54*x^2 + 12*x - 8)^(1/3)*(3*x - 2) + (81*x^4 - 135*x^3 + 54*x^2 + 12*x - 8
)^(2/3))/(9*x^2 - 12*x + 4)) + 1/9*3^(2/3)*log(-(3^(1/3)*(3*x - 2) - (81*x^
4 - 135*x^3 + 54*x^2 + 12*x - 8)^(1/3))/(3*x - 2)) + 1/3*3^(1/6)*arctan(1/3
*3^(1/6)*(3^(1/3)*(3*x - 2) + 2*(81*x^4 - 135*x^3 + 54*x^2 + 12*x - 8)^(1/3
))/(3*x - 2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(81\,x^4-135\,x^3+54\,x^2+12\,x-8\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(81*x^4-135*x^3+54*x^2+12*x-8)^(1/3),x, algorithm="giac")

[Out] integrate((81*x^4 - 135*x^3 + 54*x^2 + 12*x - 8)^(-1/3), x)
```

maple [C] time = 2.58, size = 2039, normalized size = 9.90

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(81*x^4-135*x^3+54*x^2+12*x-8)^(1/3),x)

[Out] 1/9*RootOf(_Z^3-9)*ln(-(-729*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9
)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3-1134*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*Roo
tOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3+972*RootOf(16*RootOf(_Z^3-9)^2+36
*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2+1512*RootOf(16*RootOf(_Z
^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^2+270*(81*x^4-135*
x^3+54*x^2+12*x-8)^(2/3)*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81
*_Z^2)*RootOf(_Z^3-9)^2-324*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)
+81*_Z^2)^2*RootOf(_Z^3-9)^2*x-504*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(
_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x-1134*(81*x^4-135*x^3+54*x^2+12*x-8)^(1/
3)*RootOf(_Z^3-9)*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)*
x+576*(81*x^4-135*x^3+54*x^2+12*x-8)^(1/3)*RootOf(_Z^3-9)^2*x-3888*RootOf(1
6*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3-6048*RootOf(_Z^3-9)*x^
3+756*(81*x^4-135*x^3+54*x^2+12*x-8)^(1/3)*RootOf(_Z^3-9)*RootOf(16*RootOf(
_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)-384*(81*x^4-135*x^3+54*x^2+12*x-8)^
(1/3)*RootOf(_Z^3-9)^2+972*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+
81*_Z^2)*x^2+1512*RootOf(_Z^3-9)*x^2+504*(81*x^4-135*x^3+54*x^2+12*x-8)^(2/
3)+3888*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)*x+6048*Roo
tOf(_Z^3-9)*x-1872*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)
-2912*RootOf(_Z^3-9))/(-2+3*x)^3)-1/9*ln((3645*RootOf(16*RootOf(_Z^3-9)^2+3
6*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+2268*RootOf(16*RootOf(_
Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3-4860*RootOf(16*
RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2-3024*
RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x
^2+540*(81*x^4-135*x^3+54*x^2+12*x-8)^(2/3)*RootOf(16*RootOf(_Z^3-9)^2+36*_
Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2+1620*RootOf(16*RootOf(_Z^3-9)^2+
36*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x+1008*RootOf(16*RootOf(_Z
^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x-2592*(81*x^4-135*x
^3+54*x^2+12*x-8)^(1/3)*RootOf(_Z^3-9)*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*Roo
tOf(_Z^3-9)+81*_Z^2)*x+1008*(81*x^4-135*x^3+54*x^2+12*x-8)^(1/3)*RootOf(_Z^
3-9)^2*x-4860*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3-
3024*RootOf(_Z^3-9)*x^3+1728*(81*x^4-135*x^3+54*x^2+12*x-8)^(1/3)*RootOf(_Z
^3-9)*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)-672*(81*x^4-
135*x^3+54*x^2+12*x-8)^(1/3)*RootOf(_Z^3-9)^2-14580*RootOf(16*RootOf(_Z^3-9
```

```
)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2-9072*RootOf(_Z^3-9)*x^2+1152*(81*x^4-
135*x^3+54*x^2+12*x-8)^(2/3)+25920*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(
_Z^3-9)+81*_Z^2)*x+16128*RootOf(_Z^3-9)*x-9360*RootOf(16*RootOf(_Z^3-9)^2+3
6*_Z*RootOf(_Z^3-9)+81*_Z^2)-5824*RootOf(_Z^3-9))/(-2+3*x)^3)*RootOf(_Z^3-9
)-1/4*ln((3645*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*R
ootOf(_Z^3-9)^2*x^3+2268*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81
*_Z^2)*RootOf(_Z^3-9)^3*x^3-4860*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z
^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2-3024*RootOf(16*RootOf(_Z^3-9)^2+36*_Z
*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^2+540*(81*x^4-135*x^3+54*x^2+12
*x-8)^(2/3)*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf
(_Z^3-9)^2+1620*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*
RootOf(_Z^3-9)^2*x+1008*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*
_Z^2)*RootOf(_Z^3-9)^3*x-2592*(81*x^4-135*x^3+54*x^2+12*x-8)^(1/3)*RootOf(_
Z^3-9)*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)*x+1008*(81*
x^4-135*x^3+54*x^2+12*x-8)^(1/3)*RootOf(_Z^3-9)^2*x-4860*RootOf(16*RootOf(_
Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3-3024*RootOf(_Z^3-9)*x^3+1728*(81
*x^4-135*x^3+54*x^2+12*x-8)^(1/3)*RootOf(_Z^3-9)*RootOf(16*RootOf(_Z^3-9)^2
+36*_Z*RootOf(_Z^3-9)+81*_Z^2)-672*(81*x^4-135*x^3+54*x^2+12*x-8)^(1/3)*Roo
tOf(_Z^3-9)^2-14580*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2
)*x^2-9072*RootOf(_Z^3-9)*x^2+1152*(81*x^4-135*x^3+54*x^2+12*x-8)^(2/3)+259
20*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)*x+16128*RootOf(
_Z^3-9)*x-9360*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-9)+81*_Z^2)-582
4*RootOf(_Z^3-9))/(-2+3*x)^3)*RootOf(16*RootOf(_Z^3-9)^2+36*_Z*RootOf(_Z^3-
9)+81*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(81\,x^4-135\,x^3+54\,x^2+12\,x-8\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(81*x^4-135*x^3+54*x^2+12*x-8)^(1/3),x, algorithm="maxima")
[Out] integrate((81*x^4 - 135*x^3 + 54*x^2 + 12*x - 8)^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(81\,x^4-135\,x^3+54\,x^2+12\,x-8\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(12*x + 54*x^2 - 135*x^3 + 81*x^4 - 8)^(1/3),x)
[Out] int(1/(12*x + 54*x^2 - 135*x^3 + 81*x^4 - 8)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{81x^4-135x^3+54x^2+12x-8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(81*x**4-135*x**3+54*x**2+12*x-8)**(1/3),x)
[Out] Integral((81*x**4 - 135*x**3 + 54*x**2 + 12*x - 8)**(-1/3), x)
```

3.2009 $\int \frac{1+x^2}{\left(-1-x+x^2\right)\sqrt[3]{-1+x^6}}\,dx$

Optimal. Leaf size=206

$$\frac{\log\left(-2\sqrt[3]{x^6-1}+2^{2/3}x^2+2^{2/3}x-2^{2/3}\right)}{3\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^6-1}}{\sqrt[3]{x^6-1}+2^{2/3}x^2+2^{2/3}x-2^{2/3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(2\left(x^6-1\right)^{2/3}+\sqrt[3]{2}x^4+2\sqrt[3]{2}x^3-\ldots\right.}{}$$

Rubi [F] time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x^2}{\left(-1-x+x^2\right)\sqrt[3]{-1+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^2)/((-1 - x + x^2)*(-1 + x^6)^(1/3)),x]
```

```
[Out] (x*(1 - x^6)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, x^6])/(-1 + x^6)^(1/3)
+ (1 + Sqrt[5])*Defer[Int][1/((-1 - Sqrt[5] + 2*x)*(-1 + x^6)^(1/3)), x] +
(1 - Sqrt[5])*Defer[Int][1/((-1 + Sqrt[5] + 2*x)*(-1 + x^6)^(1/3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{\left(-1-x+x^2\right)\sqrt[3]{-1+x^6}}\,dx &= \int \left(\frac{1}{\sqrt[3]{-1+x^6}}+\frac{2+x}{\left(-1-x+x^2\right)\sqrt[3]{-1+x^6}}\right)dx\\
&= \int \frac{1}{\sqrt[3]{-1+x^6}}\,dx+\int \frac{2+x}{\left(-1-x+x^2\right)\sqrt[3]{-1+x^6}}\,dx\\
&= \frac{\sqrt[3]{1-x^6}\int \frac{1}{\sqrt[3]{1-x^6}}\,dx}{\sqrt[3]{-1+x^6}}+\int \left(\frac{1+\sqrt{5}}{\left(-1-\sqrt{5}+2x\right)\sqrt[3]{-1+x^6}}+\frac{1-\sqrt{5}}{\left(-1+\sqrt{5}+2x\right)\sqrt[3]{-1\ldots}}\right.\\
&= \frac{x\sqrt[3]{1-x^6}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^6\right)}{\sqrt[3]{-1+x^6}}+\left(1-\sqrt{5}\right)\int \frac{1}{\left(-1+\sqrt{5}+2x\right)\sqrt[3]{-1+x^6}}\,dx+\left(1+\ldots\right.
\end{aligned}$$

Mathematica [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{\left(-1-x+x^2\right)\sqrt[3]{-1+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^2)/((-1 - x + x^2)*(-1 + x^6)^(1/3)),x]
```

```
[Out] Integrate[(1 + x^2)/((-1 - x + x^2)*(-1 + x^6)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 6.83, size = 206, normalized size = 1.00

$$\frac{\log\left(-2\sqrt[3]{x^6-1}+2^{2/3}x^2+2^{2/3}x-2^{2/3}\right)}{3\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^6-1}}{\sqrt[3]{x^6-1}+2^{2/3}x^2+2^{2/3}x-2^{2/3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(2\left(x^6-1\right)^{2/3}+\sqrt[3]{2}x^4+2\sqrt[3]{2}x^3-\sqrt[3]{2}x^2+\left(2^{2/3}x^2+2^{2/3}x-2^{2/3}\right)\sqrt[3]{x^6-1}-2\sqrt[3]{2}x+\sqrt[3]{2}\right)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 - x + x^2)*(-1 + x^6)^(1/3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*(-1 + x^6)^(1/3))/(-2^(2/3) + 2^(2/3)*x + 2^(2/3)*x^2 + (-1
 + x^6)^(1/3))]/(2^(2/3)*Sqrt[3]) + Log[-2^(2/3) + 2^(2/3)*x + 2^(2/3)*x^2
- 2*(-1 + x^6)^(1/3)]/(3*2^(2/3)) - Log[2^(1/3) - 2*2^(1/3)*x - 2^(1/3)*x^2
 + 2*2^(1/3)*x^3 + 2^(1/3)*x^4 + (-2^(2/3) + 2^(2/3)*x + 2^(2/3)*x^2)*(-1 +
 x^6)^(1/3) + 2*(-1 + x^6)^(2/3)]/(6*2^(2/3))
```

fricas [A] time = 33.83, size = 223, normalized size = 1.08

$$-\frac{1}{6} \cdot 4^{\frac{1}{6}} \sqrt{3} \arctan\left(\frac{4^{\frac{1}{6}} \sqrt{3} \left(2 \cdot 4^{\frac{2}{3}} \left(x^{6}-1\right)^{\frac{2}{3}} \left(x^{2}+x-1\right)-4^{\frac{1}{3}} \left(x^{6}-3 x^{5}+5 x^{3}-3 x-1\right)-4 \left(x^{6}-1\right)^{\frac{1}{3}} \left(x^{4}+2 x^{3}-x^{2}-2 x+1\right)\right)}{6 \left(3 x^{6}+3 x^{5}-5 x^{3}+3 x-3\right)}\right)-\frac{1}{24} \cdot 4^{\frac{2}{3}} \log\left(\frac{4^{\frac{2}{3}} \left(x^{6}-1\right)^{\frac{2}{3}}+4^{\frac{1}{3}} \left(x^{4}+2 x^{3}-x^{2}-2 x+1\right)+2 \left(x^{6}-1\right)^{\frac{1}{3}} \left(x^{2}+x-1\right)}{x^{4}-2 x^{3}-x^{2}+2 x+1}\right)+\frac{1}{12} \cdot 4^{\frac{2}{3}} \log\left(-\frac{4^{\frac{1}{3}} \left(x^{2}+x-1\right)-2 \left(x^{6}-1\right)^{\frac{1}{3}}}{x^{2}-x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x-1)/(x^6-1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(2*4^(2/3)*(x^6 - 1)^(2/3)*
(x^2 + x - 1) - 4^(1/3)*(x^6 - 3*x^5 + 5*x^3 - 3*x - 1) - 4*(x^6 - 1)^(1/3)
*(x^4 + 2*x^3 - x^2 - 2*x + 1))/(3*x^6 + 3*x^5 - 5*x^3 + 3*x - 3)) - 1/24*4
^(2/3)*log((4^(2/3)*(x^6 - 1)^(2/3) + 4^(1/3)*(x^4 + 2*x^3 - x^2 - 2*x + 1)
 + 2*(x^6 - 1)^(1/3)*(x^2 + x - 1))/(x^4 - 2*x^3 - x^2 + 2*x + 1)) + 1/12*4
^(2/3)*log(-(4^(1/3)*(x^2 + x - 1) - 2*(x^6 - 1)^(1/3))/(x^2 - x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}+1}{\left(x^{6}-1\right)^{\frac{1}{3}} \left(x^{2}-x-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x-1)/(x^6-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/((x^6 - 1)^(1/3)*(x^2 - x - 1)), x)
```

maple [C] time = 30.58, size = 1807, normalized size = 8.77

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-x-1)/(x^6-1)^(1/3),x)
```

```
[Out] 1/3*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*ln(-(6*RootOf(RootO
f(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x+9*x^2*(x^6-1)^
(2/3)+7*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+9*x*(x^6-1)^(2/
3)-9*(x^6-1)^(2/3)-10*RootOf(_Z^3-2)*x^3+6*RootOf(_Z^3-2)*x+5*RootOf(RootOf
(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^3-7*x^6*RootOf(RootOf(_Z^3-2)^2+2*
_Z*RootOf(_Z^3-2)+4*_Z^2)-3*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_
Z^2)*x^5-3*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x-14*RootOf(
_Z^3-2)+14*RootOf(_Z^3-2)*x^6+6*RootOf(_Z^3-2)*x^5+24*(x^6-1)^(2/3)*RootOf(
RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x^2+6*RootOf(
RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^5-12*Root
Of(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^5-10*Roo
tOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^3+20*
RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3-12
*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x-15*
(x^6-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z
^3-2)*x^4+24*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z
^2)*RootOf(_Z^3-2)^2*x-30*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf
(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^3+15*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3-2)^
2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2+30*(x^6-1)^(1/3)*RootOf(Ro
```

```
otOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-12*RootOf(_Z^3-
2)^2*(x^6-1)^(1/3)-12*RootOf(_Z^3-2)^2*(x^6-1)^(1/3)*x^4-24*RootOf(_Z^3-2)^
2*(x^6-1)^(1/3)*x^3-24*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z
^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2+12*RootOf(_Z^3-2)^2*(x^6-1)^(1/3)*x^2+24*Roo
tOf(_Z^3-2)^2*(x^6-1)^(1/3)*x-15*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z
*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2))/(x^2-x-1)^3)+1/6*RootOf(_Z^3-2)*ln(
(-48*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2
*x+30*x^2*(x^6-1)^(2/3)+56*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z
^2)+30*x*(x^6-1)^(2/3)-30*(x^6-1)^(2/3)-15*RootOf(_Z^3-2)*x^3+9*RootOf(_Z^3
-2)*x+120*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^3-56*x^6*Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-72*RootOf(RootOf(_Z^3-2)^
2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^5-72*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z
^3-2)+4*_Z^2)*x-7*RootOf(_Z^3-2)+7*RootOf(_Z^3-2)*x^6+9*RootOf(_Z^3-2)*x^5+
48*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf
(_Z^3-2)^2*x^2-48*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*Roo
tOf(_Z^3-2)^2*x^5+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*Roo
tOf(_Z^3-2)^3*x^5+80*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*
RootOf(_Z^3-2)^2*x^3-10*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)
*RootOf(_Z^3-2)^3*x^3+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)
*RootOf(_Z^3-2)^3*x-18*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z
^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^4+48*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2
*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x-36*(x^6-1)^(1/3)*RootOf(RootO
f(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^3+18*(x^6-1)^(1/3)
*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2+36*
(x^6-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z
^3-2)*x-24*RootOf(_Z^3-2)^2*(x^6-1)^(1/3)-24*RootOf(_Z^3-2)^2*(x^6-1)^(1/3)
*x^4-48*RootOf(_Z^3-2)^2*(x^6-1)^(1/3)*x^3-48*(x^6-1)^(2/3)*RootOf(RootOf(_
Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2+24*RootOf(_Z^3-2)^2*(
x^6-1)^(1/3)*x^2+48*RootOf(_Z^3-2)^2*(x^6-1)^(1/3)*x-18*(x^6-1)^(1/3)*RootO
f(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2))/(x^2-x-1)^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^6-1\right)^{\frac{1}{3}}\left(x^2-x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x-1)/(x^6-1)^(1/3),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/((x^6 - 1)^(1/3)*(x^2 - x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x^2+1}{\left(x^6-1\right)^{1/3}\left(-x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 + 1)/((x^6 - 1)^(1/3)*(x - x^2 + 1)),x)
[Out] int(-(x^2 + 1)/((x^6 - 1)^(1/3)*(x - x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt[3]{(x-1)(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\left(x^2-x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-x-1)/(x**6-1)**(1/3),x)
```

```
[Out] Integral((x**2 + 1)/(((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1))**(1/3)
*(x**2 - x - 1)), x)
```

3.2010 $\int \frac{1+x^2}{(-1-x+x^2)\sqrt[3]{-1+x^6}}\,dx$

Optimal. Leaf size=206

$$\frac{\log\left(-2\sqrt[3]{x^6-1}+2^{2/3}x^2+2^{2/3}x-2^{2/3}\right)}{3\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^6-1}}{\sqrt[3]{x^6-1}+2^{2/3}x^2+2^{2/3}x-2^{2/3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(2\left(x^6-1\right)^{2/3}+\sqrt[3]{2}x^4+2\sqrt[3]{2}x^3-\sqrt[3]{}\right.}{}$$

Rubi [F] time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x^2}{(-1-x+x^2)\sqrt[3]{-1+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^2)/((-1 - x + x^2)*(-1 + x^6)^(1/3)),x]
```

```
[Out] (x*(1 - x^6)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, x^6])/(-1 + x^6)^(1/3)
+ (1 + Sqrt[5])*Defer[Int][1/((-1 - Sqrt[5] + 2*x)*(-1 + x^6)^(1/3)), x] +
(1 - Sqrt[5])*Defer[Int][1/((-1 + Sqrt[5] + 2*x)*(-1 + x^6)^(1/3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{(-1-x+x^2)\sqrt[3]{-1+x^6}}\,dx &= \int \left(\frac{1}{\sqrt[3]{-1+x^6}}+\frac{2+x}{(-1-x+x^2)\sqrt[3]{-1+x^6}}\right)dx\\
&= \int \frac{1}{\sqrt[3]{-1+x^6}}\,dx+\int \frac{2+x}{(-1-x+x^2)\sqrt[3]{-1+x^6}}\,dx\\
&= \frac{\sqrt[3]{1-x^6}\int \frac{1}{\sqrt[3]{1-x^6}}\,dx}{\sqrt[3]{-1+x^6}}+\int \left(\frac{1+\sqrt{5}}{(-1-\sqrt{5}+2x)\sqrt[3]{-1+x^6}}+\frac{1-\sqrt{5}}{(-1+\sqrt{5}+2x)\sqrt[3]{-1}}\right.\\
&= \frac{x\sqrt[3]{1-x^6}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^6\right)}{\sqrt[3]{-1+x^6}}+\left(1-\sqrt{5}\right)\int \frac{1}{(-1+\sqrt{5}+2x)\sqrt[3]{-1+x^6}}\,dx+(1+
\end{aligned}$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{(-1-x+x^2)\sqrt[3]{-1+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^2)/((-1 - x + x^2)*(-1 + x^6)^(1/3)),x]
```

```
[Out] Integrate[(1 + x^2)/((-1 - x + x^2)*(-1 + x^6)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.00, size = 206, normalized size = 1.00

$$\frac{\log\left(-2\sqrt[3]{x^6-1}+2^{2/3}x^2+2^{2/3}x-2^{2/3}\right)}{3\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^6-1}}{\sqrt[3]{x^6-1}+2^{2/3}x^2+2^{2/3}x-2^{2/3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(2\left(x^6-1\right)^{2/3}+\sqrt[3]{2}x^4+2\sqrt[3]{2}x^3-\sqrt[3]{2}x^2+\left(2^{2/3}x^2+2^{2/3}x-2^{2/3}\right)\sqrt[3]{x^6-1}-2\sqrt[3]{2}x+\sqrt[3]{2}\right)}{6\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 - x + x^2)*(-1 + x^6)^(1/3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*(-1 + x^6)^(1/3))/(-2^(2/3) + 2^(2/3)*x + 2^(2/3)*x^2 + (-1
 + x^6)^(1/3))]/(2^(2/3)*Sqrt[3]) + Log[-2^(2/3) + 2^(2/3)*x + 2^(2/3)*x^2
- 2*(-1 + x^6)^(1/3)]/(3*2^(2/3)) - Log[2^(1/3) - 2*2^(1/3)*x - 2^(1/3)*x^2
 + 2*2^(1/3)*x^3 + 2^(1/3)*x^4 + (-2^(2/3) + 2^(2/3)*x + 2^(2/3)*x^2)*(-1 +
 x^6)^(1/3) + 2*(-1 + x^6)^(2/3)]/(6*2^(2/3))
```

fricas [A] time = 37.05, size = 223, normalized size = 1.08

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(2\cdot 4^{\frac{2}{3}}\left(x^6-1\right)^{\frac{2}{3}}\left(x^2+x-1\right)-4^{\frac{1}{3}}\left(x^6-3x^5+5x^3-3x-1\right)-4\left(x^6-1\right)^{\frac{1}{3}}\left(x^4+2x^3-x^2-2x+1\right)\right)}{6\left(3x^6+3x^5-5x^3+3x-3\right)}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}\left(x^6-1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(x^4+2x^3-x^2-2x+1\right)+2\left(x^6-1\right)^{\frac{1}{3}}\left(x^2+x-1\right)}{x^4-2x^3-x^2+2x+1}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(-\frac{4^{\frac{1}{3}}\left(x^2+x-1\right)-2\left(x^6-1\right)^{\frac{1}{3}}}{x^2-x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x-1)/(x^6-1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(2*4^(2/3)*(x^6 - 1)^(2/3)*
(x^2 + x - 1) - 4^(1/3)*(x^6 - 3*x^5 + 5*x^3 - 3*x - 1) - 4*(x^6 - 1)^(1/3)
*(x^4 + 2*x^3 - x^2 - 2*x + 1))/(3*x^6 + 3*x^5 - 5*x^3 + 3*x - 3)) - 1/24*4
^(2/3)*log((4^(2/3)*(x^6 - 1)^(2/3) + 4^(1/3)*(x^4 + 2*x^3 - x^2 - 2*x + 1)
 + 2*(x^6 - 1)^(1/3)*(x^2 + x - 1))/(x^4 - 2*x^3 - x^2 + 2*x + 1)) + 1/12*4
^(2/3)*log(-(4^(1/3)*(x^2 + x - 1) - 2*(x^6 - 1)^(1/3))/(x^2 - x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^6-1\right)^{\frac{1}{3}}\left(x^2-x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x-1)/(x^6-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/((x^6 - 1)^(1/3)*(x^2 - x - 1)), x)
```

maple [C] time = 30.29, size = 2702, normalized size = 13.12

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-x-1)/(x^6-1)^(1/3),x)
```

```
[Out] 1/6*RootOf(_Z^3-2)*ln(-(48*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z
^2)^2*RootOf(_Z^3-2)^2*x+18*x^2*(x^6-1)^(2/3)+56*RootOf(RootOf(_Z^3-2)^2+2*
_Z*RootOf(_Z^3-2)+4*_Z^2)+18*x*(x^6-1)^(2/3)-18*(x^6-1)^(2/3)+25*RootOf(_Z^
3-2)*x^3-15*RootOf(_Z^3-2)*x+40*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)
+4*_Z^2)*x^3-56*x^6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-24*
RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^5-24*RootOf(RootOf(_Z
^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+35*RootOf(_Z^3-2)-35*RootOf(_Z^3-2)*x
^6-15*RootOf(_Z^3-2)*x^5+48*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*Root
Of(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x^2+48*RootOf(RootOf(_Z^3-2)^2+2*_Z*Roo
tOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^5+30*RootOf(RootOf(_Z^3-2)^2+2*_Z*
RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^5-80*RootOf(RootOf(_Z^3-2)^2+2*_Z
*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-50*RootOf(RootOf(_Z^3-2)^2+2
*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3+30*RootOf(RootOf(_Z^3-2)^2+
2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x-18*(x^6-1)^(1/3)*RootOf(Root
Of(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^4+48*(x^6-1)^(2/3
)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x-36
*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_
Z^3-2)*x^3+18*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_
Z^2)*RootOf(_Z^3-2)*x^2+36*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootO
```

```
f(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x+15*RootOf(_Z^3-2)^2*(x^6-1)^(1/3)+15*Roo
tOf(_Z^3-2)^2*(x^6-1)^(1/3)*x^4+30*RootOf(_Z^3-2)^2*(x^6-1)^(1/3)*x^3-48*(x
^6-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3
-2)^2-15*RootOf(_Z^3-2)^2*(x^6-1)^(1/3)*x^2-30*RootOf(_Z^3-2)^2*(x^6-1)^(1/
3)*x-18*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*R
ootOf(_Z^3-2))/(x^2-x-1)^3)-1/6*ln(-(12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf
(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x-30*x^2*(x^6-1)^(2/3)-14*RootOf(RootOf
(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-30*x*(x^6-1)^(2/3)+30*(x^6-1)^(2/3)-
75*RootOf(_Z^3-2)*x^3+45*RootOf(_Z^3-2)*x-30*RootOf(RootOf(_Z^3-2)^2+2*_Z*R
ootOf(_Z^3-2)+4*_Z^2)*x^3+14*x^6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2
)+4*_Z^2)+18*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^5+18*Roo
tOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x-35*RootOf(_Z^3-2)+35*Roo
tOf(_Z^3-2)*x^6+45*RootOf(_Z^3-2)*x^5-48*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3-2
)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x^2+12*RootOf(RootOf(_Z^3-
2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^5+30*RootOf(RootOf(_Z
^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^5-20*RootOf(RootOf(_
Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-50*RootOf(RootO
f(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3+30*RootOf(Root
Of(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x+30*(x^6-1)^(1/3
)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^4-48
*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_
Z^3-2)^2*x+60*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_
Z^2)*RootOf(_Z^3-2)*x^3-30*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootO
f(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2-60*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3-2)
^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-9*RootOf(_Z^3-2)^2*(x^6-1)^
(1/3)-9*RootOf(_Z^3-2)^2*(x^6-1)^(1/3)*x^4-18*RootOf(_Z^3-2)^2*(x^6-1)^(1/3
)*x^3+48*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*
RootOf(_Z^3-2)^2+9*RootOf(_Z^3-2)^2*(x^6-1)^(1/3)*x^2+18*RootOf(_Z^3-2)^2*(
x^6-1)^(1/3)*x+30*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)
+4*_Z^2)*RootOf(_Z^3-2))/(x^2-x-1)^3)*RootOf(_Z^3-2)-1/3*ln(-(12*RootOf(Roo
tOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x-30*x^2*(x^6-
1)^(2/3)-14*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-30*x*(x^6-1
)^(2/3)+30*(x^6-1)^(2/3)-75*RootOf(_Z^3-2)*x^3+45*RootOf(_Z^3-2)*x-30*RootO
f(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^3+14*x^6*RootOf(RootOf(_Z^
3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+18*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_
Z^3-2)+4*_Z^2)*x^5+18*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x
-35*RootOf(_Z^3-2)+35*RootOf(_Z^3-2)*x^6+45*RootOf(_Z^3-2)*x^5-48*(x^6-1)^(
2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x
^2+12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^
2*x^5+30*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)
^3*x^5-20*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3
-2)^2*x^3-50*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^
3-2)^3*x^3+30*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z
^3-2)^3*x+30*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z
^2)*RootOf(_Z^3-2)*x^4-48*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf
(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x+60*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3-2)^
2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^3-30*(x^6-1)^(1/3)*RootOf(Ro
otOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2-60*(x^6-1)^(1
/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-9*
RootOf(_Z^3-2)^2*(x^6-1)^(1/3)-9*RootOf(_Z^3-2)^2*(x^6-1)^(1/3)*x^4-18*Root
Of(_Z^3-2)^2*(x^6-1)^(1/3)*x^3+48*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_
Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2+9*RootOf(_Z^3-2)^2*(x^6-1)^(1/3)*
x^2+18*RootOf(_Z^3-2)^2*(x^6-1)^(1/3)*x+30*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3
-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2))/(x^2-x-1)^3)*RootOf(RootO
f(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^6-1\right)^{\frac{1}{3}}\left(x^2-x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x-1)/(x^6-1)^(1/3),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/((x^6 - 1)^(1/3)*(x^2 - x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x^2+1}{\left(x^6-1\right)^{1/3}\,\left(-x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 + 1)/((x^6 - 1)^(1/3)*(x - x^2 + 1)),x)
[Out] int(-(x^2 + 1)/((x^6 - 1)^(1/3)*(x - x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt[3]{(x-1)\,(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,\left(x^2-x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-x-1)/(x**6-1)**(1/3),x)
[Out] Integral((x**2 + 1)/(((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1))**(1/3)
*(x**2 - x - 1)), x)
```

3.2011 $\int \frac{(2-2x^4+3x^5+4x^6)\sqrt[3]{-x+2x^3-x^5+x^6+x^7}}{(-1+x^2-x^4+x^5+x^6)^2}\,dx$

Optimal. Leaf size=206

$$\frac{1}{3}\log\left(\sqrt[3]{x^7+x^6-x^5+2x^3-x}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^7+x^6-x^5+2x^3-x}}{\sqrt[3]{x^7+x^6-x^5+2x^3-x}+2x}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^2+\sqrt[3]{x^7+x^6-x^5+2x^3-x}\,x+\left(x^7-\right.\right.$$

Rubi [F] time = 4.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(2-2x^4+3x^5+4x^6)\sqrt[3]{-x+2x^3-x^5+x^6+x^7}}{(-1+x^2-x^4+x^5+x^6)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((2 - 2*x^4 + 3*x^5 + 4*x^6)*(-x + 2*x^3 - x^5 + x^6 + x^7)^(1/3))/(-1
+ x^2 - x^4 + x^5 + x^6)^2,x]
```

```
[Out] (3*(-x + 2*x^3 - x^5 + x^6 + x^7)^(1/3)*Defer[Subst][Defer[Int][(-1 + 2*x^6
 - x^12 + x^15 + x^18)^(1/3)/(1 - x^6 + x^12 - x^15 - x^18), x], x, x^(1/3)
])/(x^(1/3)*(-1 + 2*x^2 - x^4 + x^5 + x^6)^(1/3)) - (3*(-x + 2*x^3 - x^5 +
x^6 + x^7)^(1/3)*Defer[Subst][Defer[Int][(-1 + 2*x^6 - x^12 + x^15 + x^18)^
(1/3)/(-1 + x^6 - x^12 + x^15 + x^18)^2, x], x, x^(1/3)])/(x^(1/3)*(-1 + 2*
x^2 - x^4 + x^5 + x^6)^(1/3)) + (18*(-x + 2*x^3 - x^5 + x^6 + x^7)^(1/3)*De
fer[Subst][Defer[Int][(x^3*(-1 + 2*x^6 - x^12 + x^15 + x^18)^(1/3))/(-1 + x
^6 - x^12 + x^15 + x^18)^2, x], x, x^(1/3)])/(x^(1/3)*(-1 + 2*x^2 - x^4 + x
^5 + x^6)^(1/3)) + (3*(-x + 2*x^3 - x^5 + x^6 + x^7)^(1/3)*Defer[Subst][Def
er[Int][(x^6*(-1 + 2*x^6 - x^12 + x^15 + x^18)^(1/3))/(-1 + x^6 - x^12 + x^
15 + x^18)^2, x], x, x^(1/3)])/(x^(1/3)*(-1 + 2*x^2 - x^4 + x^5 + x^6)^(1/3
)) - (12*(-x + 2*x^3 - x^5 + x^6 + x^7)^(1/3)*Defer[Subst][Defer[Int][(x^9*
(-1 + 2*x^6 - x^12 + x^15 + x^18)^(1/3))/(-1 + x^6 - x^12 + x^15 + x^18)^2,
 x], x, x^(1/3)])/(x^(1/3)*(-1 + 2*x^2 - x^4 + x^5 + x^6)^(1/3)) - (3*(-x +
 2*x^3 - x^5 + x^6 + x^7)^(1/3)*Defer[Subst][Defer[Int][(x^12*(-1 + 2*x^6 -
 x^12 + x^15 + x^18)^(1/3))/(-1 + x^6 - x^12 + x^15 + x^18)^2, x], x, x^(1/
3)])/(x^(1/3)*(-1 + 2*x^2 - x^4 + x^5 + x^6)^(1/3)) + (9*(-x + 2*x^3 - x^5
+ x^6 + x^7)^(1/3)*Defer[Subst][Defer[Int][(x^15*(-1 + 2*x^6 - x^12 + x^15
+ x^18)^(1/3))/(-1 + x^6 - x^12 + x^15 + x^18)^2, x], x, x^(1/3)])/(x^(1/3)
*(-1 + 2*x^2 - x^4 + x^5 + x^6)^(1/3)) + (12*(-x + 2*x^3 - x^5 + x^6 + x^7)
^(1/3)*Defer[Subst][Defer[Int][(x^3*(-1 + 2*x^6 - x^12 + x^15 + x^18)^(1/3)
)/(-1 + x^6 - x^12 + x^15 + x^18), x], x, x^(1/3)])/(x^(1/3)*(-1 + 2*x^2 -
x^4 + x^5 + x^6)^(1/3))
```

Rubi steps

$$\int \frac{\left(2-2x^4+3x^5+4x^6\right)\sqrt[3]{-x+2x^3-x^5+x^6+x^7}}{\left(-1+x^2-x^4+x^5+x^6\right)^2}\,dx = \frac{\sqrt[3]{-x+2x^3-x^5+x^6+x^7}\int \frac{\sqrt[3]{x}\sqrt[3]{-1+2x^2-x^4+x^5+x^6}\left(2-\cdots\right.}{\left(-1+x^2-x^4+x^5+x\cdots\right.}}{\sqrt[3]{x}\sqrt[3]{-1+2x^2-x^4+x^5+x^6}}$$

$$= \frac{\left(3\sqrt[3]{-x+2x^3-x^5+x^6+x^7}\right)\text{Subst}\left(\int \frac{x^3\sqrt[3]{-1+2x^6-x\cdots}}{\left(-\cdots\right.}\right.}{\sqrt[3]{x}\sqrt[3]{-1+2x^2-x^4+\cdots}}$$

$$= \frac{\left(3\sqrt[3]{-x+2x^3-x^5+x^6+x^7}\right)\text{Subst}\left(\int\left(\frac{\left(-1+6x^3+x^6-\cdots\right.}{\cdots}\right.\right.}{\sqrt[3]{x}\cdots}$$

$$= \frac{\left(3\sqrt[3]{-x+2x^3-x^5+x^6+x^7}\right)\text{Subst}\left(\int \frac{\left(-1+6x^3+x^6-4\cdots\right.}{\cdots}\right.}{\sqrt[3]{x}\sqrt[3]{-1+2x^2-x^4\cdots}}$$

$$= \frac{\left(3\sqrt[3]{-x+2x^3-x^5+x^6+x^7}\right)\text{Subst}\left(\int\left(-\frac{\sqrt[3]{-1+2x^6-\cdots}}{\left(-1+x^6-x^{1\cdots}\right.}\right.\right.}{\cdots}$$

$$= \frac{\left(3\sqrt[3]{-x+2x^3-x^5+x^6+x^7}\right)\text{Subst}\left(\int \frac{\sqrt[3]{-1+2x^6-x^{12}\cdots}}{1-x^6+x^{12}-x\cdots}\right.}{\sqrt[3]{x}\sqrt[3]{-1+2x^2-x^4+x^5+x^6}}$$

Mathematica [F] time = 2.16, size = 0, normalized size = 0.00

$$\int \frac{\left(2-2x^4+3x^5+4x^6\right)\sqrt[3]{-x+2x^3-x^5+x^6+x^7}}{\left(-1+x^2-x^4+x^5+x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((2 - 2*x^4 + 3*x^5 + 4*x^6)*(-x + 2*x^3 - x^5 + x^6 + x^7)^(1/3)
)/(-1 + x^2 - x^4 + x^5 + x^6)^2,x]
```

```
[Out] Integrate[((2 - 2*x^4 + 3*x^5 + 4*x^6)*(-x + 2*x^3 - x^5 + x^6 + x^7)^(1/3)
)/(-1 + x^2 - x^4 + x^5 + x^6)^2, x]
```

IntegrateAlgebraic [A] time = 5.56, size = 206, normalized size = 1.00

$$\frac{1}{3}\log\left(\sqrt[3]{x^7+x^6-x^5+2x^3-x}-x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^7+x^6-x^5+2x^3-x}}{\sqrt[3]{x^7+x^6-x^5+2x^3-x}+2x}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^2+\sqrt[3]{x^7+x^6-x^5+2x^3-x}\,x+\left(x^7+x^6-x^5+2x^3-x\right)^{2/3}\right)-\frac{\sqrt[3]{x^7+x^6-x^5+2x^3-x}\,x}{x^6+x^5-x^4+x^2-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 - 2*x^4 + 3*x^5 + 4*x^6)*(-x + 2*x^3 - x^5 + x^6 + x
^7)^(1/3))/(-1 + x^2 - x^4 + x^5 + x^6)^2,x]
```

```
[Out] -((x*(-x + 2*x^3 - x^5 + x^6 + x^7)^(1/3))/(-1 + x^2 - x^4 + x^5 + x^6)) -
ArcTan[(Sqrt[3]*(-x + 2*x^3 - x^5 + x^6 + x^7)^(1/3))/(2*x + (-x + 2*x^3 -
x^5 + x^6 + x^7)^(1/3))]/Sqrt[3] + Log[-x + (-x + 2*x^3 - x^5 + x^6 + x^7)^
(1/3)]/3 - Log[x^2 + x*(-x + 2*x^3 - x^5 + x^6 + x^7)^(1/3) + (-x + 2*x^3 -
 x^5 + x^6 + x^7)^(2/3)]/6
```

fricas [A] time = 11.41, size = 268, normalized size = 1.30

$$\frac{2\sqrt{3}\left(x^6+x^5-x^4+x^2-1\right)\arctan\left(-\frac{2\sqrt{3}\left(x^7+x^6-x^5+2x^3-x\right)^{\frac{1}{3}}x+\sqrt{3}\left(x^6+x^5-x^4+x^2-1\right)-2\sqrt{3}\left(x^7+x^6-x^5+2x^3-x\right)^{\frac{2}{3}}}{3\left(x^6+x^5-x^4+3x^2-1\right)}\right)+\left(x^6+x^5-x^4+x^2-1\right)\log\left(\frac{x^6+x^5-x^4+x^2+3\left(x^7+x^6-x^5+2x^3-x\right)^{\frac{1}{3}}x-3\left(x^7+x^6-x^5+2x^3-x\right)^{\frac{2}{3}}-1}{x^6+x^5-x^4+x^2-1}\right)-6\left(x^7+x^6-x^5+2x^3-x\right)^{\frac{1}{3}}x}{6\left(x^6+x^5-x^4+x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6+3*x^5-2*x^4+2)*(x^7+x^6-x^5+2*x^3-x)^(1/3)/(x^6+x^5-x^4+x^
2-1)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(x^6 + x^5 - x^4 + x^2 - 1)*arctan(-1/3*(2*sqrt(3)*(x^7 + x^
6 - x^5 + 2*x^3 - x)^(1/3)*x + sqrt(3)*(x^6 + x^5 - x^4 + x^2 - 1) - 2*sqrt
(3)*(x^7 + x^6 - x^5 + 2*x^3 - x)^(2/3))/(x^6 + x^5 - x^4 + 3*x^2 - 1)) + (
x^6 + x^5 - x^4 + x^2 - 1)*log((x^6 + x^5 - x^4 + x^2 + 3*(x^7 + x^6 - x^5
+ 2*x^3 - x)^(1/3)*x - 3*(x^7 + x^6 - x^5 + 2*x^3 - x)^(2/3) - 1)/(x^6 + x^
5 - x^4 + x^2 - 1)) - 6*(x^7 + x^6 - x^5 + 2*x^3 - x)^(1/3)*x)/(x^6 + x^5 -
 x^4 + x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^7+x^6-x^5+2x^3-x\right)^{\frac{1}{3}}\left(4x^6+3x^5-2x^4+2\right)}{\left(x^6+x^5-x^4+x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6+3*x^5-2*x^4+2)*(x^7+x^6-x^5+2*x^3-x)^(1/3)/(x^6+x^5-x^4+x^
2-1)^2,x, algorithm="giac")
```

```
[Out] integrate((x^7 + x^6 - x^5 + 2*x^3 - x)^(1/3)*(4*x^6 + 3*x^5 - 2*x^4 + 2)/(
x^6 + x^5 - x^4 + x^2 - 1)^2, x)
```

maple [C] time = 12.62, size = 3644, normalized size = 17.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^6+3*x^5-2*x^4+2)*(x^7+x^6-x^5+2*x^3-x)^(1/3)/(x^6+x^5-x^4+x^2-1)^2
,x)
```

```
[Out] -x/(x^6+x^5-x^4+x^2-1)*(x*(x^6+x^5-x^4+2*x^2-1))^(1/3)+(1/6*RootOf(_Z^2+2*_
Z+4)*ln(-(-852+372*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-
4*x^4+x^2)^(1/3)*RootOf(_Z^2+2*_Z+4)*x^5-372*(x^14+2*x^13-x^12-2*x^11+5*x^1
0+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)*RootOf(_Z^2+2*_Z+4)*x^4+744*(x^1
4+2*x^13-x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)*RootOf
(_Z^2+2*_Z+4)*x^2+372*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x
^6-4*x^4+x^2)^(1/3)*RootOf(_Z^2+2*_Z+4)*x^6-1704*x^11+46*RootOf(_Z^2+2*_Z+4
)^2*x^4-46*RootOf(_Z^2+2*_Z+4)^2*x^2-314*RootOf(_Z^2+2*_Z+4)*x^2-852*x^12-3
692*x^7+852*x^10+1704*x^9+1704*x^5+5396*x^6+3692*x^2-4544*x^8-5680*x^4-1044
*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)*x
^6-1044*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^
(1/3)*x^5+1044*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^
4+x^2)^(1/3)*x^4+372*RootOf(_Z^2+2*_Z+4)*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*
x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(2/3)-2088*(x^14+2*x^13-x^12-2*x^11+5*x^10
+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)*x^2-372*RootOf(_Z^2+2*_Z+4)*(x^14
+2*x^13-x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)+23*Root
Of(_Z^2+2*_Z+4)^2*x^12+46*RootOf(_Z^2+2*_Z+4)^2*x^11-23*RootOf(_Z^2+2*_Z+4)
^2*x^10-46*RootOf(_Z^2+2*_Z+4)^2*x^9+69*RootOf(_Z^2+2*_Z+4)^2*x^8+46*RootOf
(_Z^2+2*_Z+4)^2*x^7-92*RootOf(_Z^2+2*_Z+4)^2*x^6-46*RootOf(_Z^2+2*_Z+4)^2*x
^5-4*RootOf(_Z^2+2*_Z+4)-4*RootOf(_Z^2+2*_Z+4)*x^12-8*RootOf(_Z^2+2*_Z+4)*x
^11+4*RootOf(_Z^2+2*_Z+4)*x^10+8*RootOf(_Z^2+2*_Z+4)*x^9+314*RootOf(_Z^2+2*
_Z+4)*x^7-306*RootOf(_Z^2+2*_Z+4)*x^6+8*RootOf(_Z^2+2*_Z+4)*x^5+23*RootOf(_
Z^2+2*_Z+4)^2+310*RootOf(_Z^2+2*_Z+4)*x^8+636*RootOf(_Z^2+2*_Z+4)*x^4+1044*
(x^14+2*x^13-x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)-10
44*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(2/3)
```

```
)/(1+x)/(x^5-x^3+x^2+x-1)/(x^6+x^5-x^4+x^2-1))-1/6*ln((752+372*(x^14+2*x^13
-x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)*RootOf(_Z^2+2*
_Z+4)*x^5-372*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4
+x^2)^(1/3)*RootOf(_Z^2+2*_Z+4)*x^4+744*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*x
^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)*RootOf(_Z^2+2*_Z+4)*x^2+372*(x^14+2*x
^13-x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)*RootOf(_Z^2
+2*_Z+4)*x^6+1504*x^11-46*RootOf(_Z^2+2*_Z+4)^2*x^4+46*RootOf(_Z^2+2*_Z+4)^
2*x^2-130*RootOf(_Z^2+2*_Z+4)*x^2+752*x^12+4136*x^7-752*x^10-1504*x^9-1504*
x^5-5640*x^6-4136*x^2+4888*x^8+6768*x^4+1788*(x^14+2*x^13-x^12-2*x^11+5*x^1
0+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)*x^6+1788*(x^14+2*x^13-x^12-2*x^1
1+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)*x^5-1788*(x^14+2*x^13-x^1
2-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)*x^4+372*RootOf(_Z^
2+2*_Z+4)*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2
)^(2/3)+3576*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+
x^2)^(1/3)*x^2-372*RootOf(_Z^2+2*_Z+4)*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*x^
9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)-23*RootOf(_Z^2+2*_Z+4)^2*x^12-46*RootO
f(_Z^2+2*_Z+4)^2*x^11+23*RootOf(_Z^2+2*_Z+4)^2*x^10+46*RootOf(_Z^2+2*_Z+4)^
2*x^9-69*RootOf(_Z^2+2*_Z+4)^2*x^8-46*RootOf(_Z^2+2*_Z+4)^2*x^7+92*RootOf(_
Z^2+2*_Z+4)^2*x^6+46*RootOf(_Z^2+2*_Z+4)^2*x^5-96*RootOf(_Z^2+2*_Z+4)-96*Ro
otOf(_Z^2+2*_Z+4)*x^12-192*RootOf(_Z^2+2*_Z+4)*x^11+96*RootOf(_Z^2+2*_Z+4)*
x^10+192*RootOf(_Z^2+2*_Z+4)*x^9+130*RootOf(_Z^2+2*_Z+4)*x^7+62*RootOf(_Z^2
+2*_Z+4)*x^6+192*RootOf(_Z^2+2*_Z+4)*x^5-23*RootOf(_Z^2+2*_Z+4)^2+34*RootOf
(_Z^2+2*_Z+4)*x^8+452*RootOf(_Z^2+2*_Z+4)*x^4-1788*(x^14+2*x^13-x^12-2*x^11
+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)+1788*(x^14+2*x^13-x^12-2*x
^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(2/3))/(1+x)/(x^5-x^3+x^2+x-1
)/(x^6+x^5-x^4+x^2-1))*RootOf(_Z^2+2*_Z+4)-1/3*ln((752+372*(x^14+2*x^13-x^1
2-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)*RootOf(_Z^2+2*_Z+4
)*x^5-372*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2
)^(1/3)*RootOf(_Z^2+2*_Z+4)*x^4+744*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*x^9-6
*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)*RootOf(_Z^2+2*_Z+4)*x^2+372*(x^14+2*x^13-
x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)*RootOf(_Z^2+2*_
Z+4)*x^6+1504*x^11-46*RootOf(_Z^2+2*_Z+4)^2*x^4+46*RootOf(_Z^2+2*_Z+4)^2*x^
2-130*RootOf(_Z^2+2*_Z+4)*x^2+752*x^12+4136*x^7-752*x^10-1504*x^9-1504*x^5-
5640*x^6-4136*x^2+4888*x^8+6768*x^4+1788*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*
x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)*x^6+1788*(x^14+2*x^13-x^12-2*x^11+5*
x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)*x^5-1788*(x^14+2*x^13-x^12-2*
x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)*x^4+372*RootOf(_Z^2+2*
_Z+4)*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(2
/3)+3576*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)
^(1/3)*x^2-372*RootOf(_Z^2+2*_Z+4)*(x^14+2*x^13-x^12-2*x^11+5*x^10+4*x^9-6*
x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)-23*RootOf(_Z^2+2*_Z+4)^2*x^12-46*RootOf(_Z
^2+2*_Z+4)^2*x^11+23*RootOf(_Z^2+2*_Z+4)^2*x^10+46*RootOf(_Z^2+2*_Z+4)^2*x^
9-69*RootOf(_Z^2+2*_Z+4)^2*x^8-46*RootOf(_Z^2+2*_Z+4)^2*x^7+92*RootOf(_Z^2+
2*_Z+4)^2*x^6+46*RootOf(_Z^2+2*_Z+4)^2*x^5-96*RootOf(_Z^2+2*_Z+4)-96*RootOf
(_Z^2+2*_Z+4)*x^12-192*RootOf(_Z^2+2*_Z+4)*x^11+96*RootOf(_Z^2+2*_Z+4)*x^10
+192*RootOf(_Z^2+2*_Z+4)*x^9+130*RootOf(_Z^2+2*_Z+4)*x^7+62*RootOf(_Z^2+2*_
Z+4)*x^6+192*RootOf(_Z^2+2*_Z+4)*x^5-23*RootOf(_Z^2+2*_Z+4)^2+34*RootOf(_Z^
2+2*_Z+4)*x^8+452*RootOf(_Z^2+2*_Z+4)*x^4-1788*(x^14+2*x^13-x^12-2*x^11+5*x
^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(1/3)+1788*(x^14+2*x^13-x^12-2*x^11+
5*x^10+4*x^9-6*x^8-2*x^7+6*x^6-4*x^4+x^2)^(2/3))/(1+x)/(x^5-x^3+x^2+x-1)/(x
^6+x^5-x^4+x^2-1)))*(x*(x^6+x^5-x^4+2*x^2-1))^(1/3)*(x^2*(x^6+x^5-x^4+2*x^2
-1)^2)^(1/3)/x/(x^6+x^5-x^4+2*x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{7}+x^{6}-x^{5}+2\,x^{3}-x\right)^{\frac{1}{3}}\left(4\,x^{6}+3\,x^{5}-2\,x^{4}+2\right)}{\left(x^{6}+x^{5}-x^{4}+x^{2}-1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^6+3*x^5-2*x^4+2)*(x^7+x^6-x^5+2*x^3-x)^(1/3)/(x^6+x^5-x^4+x^
2-1)^2,x, algorithm="maxima")
```

```
[Out] integrate((x^7 + x^6 - x^5 + 2*x^3 - x)^(1/3)*(4*x^6 + 3*x^5 - 2*x^4 + 2)/(
x^6 + x^5 - x^4 + x^2 - 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(4\,x^6+3\,x^5-2\,x^4+2\right)\,{\left(x^7+x^6-x^5+2\,x^3-x\right)}^{1/3}}{{\left(x^6+x^5-x^4+x^2-1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^5 - 2*x^4 + 4*x^6 + 2)*(2*x^3 - x - x^5 + x^6 + x^7)^(1/3))/(x^2
- x^4 + x^5 + x^6 - 1)^2,x)
```

```
[Out] int(((3*x^5 - 2*x^4 + 4*x^6 + 2)*(2*x^3 - x - x^5 + x^6 + x^7)^(1/3))/(x^2
- x^4 + x^5 + x^6 - 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x \left(x + 1\right) \left(x^{5} - x^{3} + x^{2} + x - 1\right)} \left(4 x^{6} + 3 x^{5} - 2 x^{4} + 2\right)}{\left(x^{6} + x^{5} - x^{4} + x^{2} - 1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**6+3*x**5-2*x**4+2)*(x**7+x**6-x**5+2*x**3-x)**(1/3)/(x**6+x
**5-x**4+x**2-1)**2,x)
```

```
[Out] Integral((x*(x + 1)*(x**5 - x**3 + x**2 + x - 1))**(1/3)*(4*x**6 + 3*x**5 -
 2*x**4 + 2)/(x**6 + x**5 - x**4 + x**2 - 1)**2, x)
```

3.2012 $\int \frac{-1+(-1+2k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(b-(1+2b)x+(b+k)x^2\right)}\,dx$

Optimal. Leaf size=207

$$-\frac{\log\left(b^{2/3}x^2-2b^{2/3}x+b^{2/3}+\left(\sqrt[3]{b}-\sqrt[3]{b}\,x\right)\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{b}\,x-\right.}{}$$

Rubi [F] time = 2.65, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+(-1+2k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(b-(1+2b)x+(b+k)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + (-1 + 2*k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(b - (1 + 2*b)*x + (b
+ k)*x^2)),x]
```

```
[Out] -(((1 - 2*k + Sqrt[1 + 4*b - 4*b*k])*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*
Defer[Int][1/((1 - k*x)^(1/3)*(-1 - 2*b - Sqrt[1 + 4*b - 4*b*k] + 2*(b + k)
*x)*(x - x^2)^(1/3)), x])/((1 - x)*x*(1 - k*x))^(1/3)) - ((1 - 2*k - Sqrt[1
 + 4*b - 4*b*k])*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Int][1/((1 - k
*x)^(1/3)*(-1 - 2*b + Sqrt[1 + 4*b - 4*b*k] + 2*(b + k)*x)*(x - x^2)^(1/3))
, x])/((1 - x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+(-1+2k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(b-(1+2b)x+(b+k)x^2\right)}\,dx &= \frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{-1+(-1+2k)x}{\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\left(b-(1+2b)x+(b+k)x^2\right)}}{\sqrt[3]{(1-x)x(1-kx)}}\\
&= \frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{-1+(-1+2k)x}{\sqrt[3]{1-kx}\sqrt[3]{x-x^2}\left(b-(1+2b)x+(b+k)x^2\right)}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}\\
&= \frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int\left(\frac{-1+2k-\sqrt{1+4b-4bk}}{\sqrt[3]{1-kx}\left(-1-2b-\sqrt{1+4b-4bk}+2(b+k)x\right)}\right.}{\sqrt[3]{(1-x)x(1}}\\
&= \frac{\left(\left(-1+2k-\sqrt{1+4b-4bk}\right)\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{}{\sqrt[3]{1-}}}{\sqrt[3]{(1-x)x(1-kx)}}
\end{aligned}$$

Mathematica [F] time = 5.66, size = 0, normalized size = 0.00

$$\int \frac{-1+(-1+2k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(b-(1+2b)x+(b+k)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + (-1 + 2*k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(b - (1 + 2*b)*x
 + (b + k)*x^2)),x]
```

```
[Out] Integrate[(-1 + (-1 + 2*k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(b - (1 + 2*b)*x
 + (b + k)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.73, size = 207, normalized size = 1.00

$$-\frac{\log\left(b^{2/3}x^2-2b^{2/3}x+b^{2/3}+\left(\sqrt[3]{b}-\sqrt[3]{b}\,x\right)\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{b}\,x-\sqrt[3]{b}+\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{kx^3+(-k-1)x^2+x}}{-2\sqrt[3]{b}\,x+2\sqrt[3]{b}+\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + (-1 + 2*k)*x)/((((1 - x)*x*(1 - k*x))^(1/3)*(b - (1
 + 2*b)*x + (b + k)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(2*b^(1/3) - 2*b
^(1/3)*x + (x + (-1 - k)*x^2 + k*x^3)^(1/3))])/b^(1/3) + Log[-b^(1/3) + b^(
1/3)*x + (x + (-1 - k)*x^2 + k*x^3)^(1/3)]/b^(1/3) - Log[b^(2/3) - 2*b^(2/3
)*x + b^(2/3)*x^2 + (b^(1/3) - b^(1/3)*x)*(x + (-1 - k)*x^2 + k*x^3)^(1/3)
+ (x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-(1+2*b)*x+(b+k)*x^2),
x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2k-1)x-1}{((kx-1)(x-1)x)^{\frac{1}{3}}\left((b+k)x^2-(2b+1)x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-(1+2*b)*x+(b+k)*x^2),
x, algorithm="giac")
```

```
[Out] integrate(((2*k - 1)*x - 1)/((((k*x - 1)*(x - 1)*x)^(1/3)*((b + k)*x^2 - (2*
b + 1)*x + b)), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{-1+(-1+2k)\,x}{((1-x)\,x\,(-kx+1))^{\frac{1}{3}}\left(b-(1+2b)\,x+(b+k)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-(1+2*b)*x+(b+k)*x^2),x)
```

```
[Out] int((-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-(1+2*b)*x+(b+k)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2k-1)x-1}{((kx-1)(x-1)x)^{\frac{1}{3}}\left((b+k)x^2-(2b+1)x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-(1+2*b)*x+(b+k)*x^2),
x, algorithm="maxima")
```

```
[Out] integrate(((2*k - 1)*x - 1)/(((k*x - 1)*(x - 1)*x)^(1/3)*((b + k)*x^2 - (2*
b + 1)*x + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(2\,k-1\right)-1}{\left(\left(b+k\right)\,x^2+\left(-2\,b-1\right)\,x+b\right)\,{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*k - 1) - 1)/((b + x^2*(b + k) - x*(2*b + 1))*(x*(k*x - 1)*(x - 1)
)^(1/3)),x)
```

```
[Out] int((x*(2*k - 1) - 1)/((b + x^2*(b + k) - x*(2*b + 1))*(x*(k*x - 1)*(x - 1)
)^(1/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(b-(1+2*b)*x+(b+k)*x**2
),x)
```

```
[Out] Timed out
```

3.2013 $\int \frac{\sqrt{b^2+a^2x^3}\left(2b^2+cx^3+a^2x^6\right)}{x^7\left(-b^2+a^2x^6\right)}\,dx$

Optimal. Leaf size=207

$$\frac{\sqrt{a^2x^3+b^2}\,\left(a^2x^3+2b^2+2cx^3\right)}{6b^2x^6}-\frac{\sqrt{b-a}\,\left(3a^2b-ac\right)\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\,\sqrt{b-a}}\right)}{3b^{5/2}}-\frac{\sqrt{a+b}\,\left(3a^2b+ac\right)\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\,\sqrt{a+b}}\right)}{3b^{5/2}}$$

Rubi [A] time = 0.91, antiderivative size = 359, normalized size of antiderivative = 1.73, number of steps used = 23, number of rules used = 9, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.173, Rules used = {6725, 266, 47, 51, 63, 208, 50, 444, 205}

$$\frac{c\sqrt{a^2x^3+b^2}}{3b^2x^3}-\frac{2a^2\sqrt{a^2x^3+b^2}}{b^2}+\frac{a^2\sqrt{a^2x^3+b^2}}{6b^2x^3}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{b}\right)}{b}+\frac{\sqrt{a^2x^3+b^2}}{3x^6}-\frac{a\sqrt{a-b}\,(3ab-c)\tan^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\,\sqrt{a-b}}\right)}{3b^{5/2}}-\frac{a\sqrt{a+b}\,(3ab+c)\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\,\sqrt{a+b}}\right)}{3b^{5/2}}+\frac{a(3ab-c)\sqrt{a^2x^3+b^2}}{3b^3}+\frac{a(3ab+c)\sqrt{a^2x^3+b^2}}{3b^3}+\frac{a^2c\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{b}\right)}{3b^3}-\frac{a^4\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{b}\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + a^2*x^3]*(2*b^2 + c*x^3 + a^2*x^6))/(x^7*(-b^2 + a^2*x^6)),
x]
```

```
[Out] (-2*a^2*Sqrt[b^2 + a^2*x^3])/b^2 + (a*(3*a*b - c)*Sqrt[b^2 + a^2*x^3])/(3*b
^3) + (a*(3*a*b + c)*Sqrt[b^2 + a^2*x^3])/(3*b^3) + Sqrt[b^2 + a^2*x^3]/(3*
x^6) + (a^2*Sqrt[b^2 + a^2*x^3])/(6*b^2*x^3) + (c*Sqrt[b^2 + a^2*x^3])/(3*b
^2*x^3) - (a*Sqrt[a - b]*(3*a*b - c)*ArcTan[Sqrt[b^2 + a^2*x^3]/(Sqrt[a - b
]*Sqrt[b])])/(3*b^(5/2)) - (a^4*ArcTanh[Sqrt[b^2 + a^2*x^3]/b])/(6*b^3) + (
2*a^2*ArcTanh[Sqrt[b^2 + a^2*x^3]/b])/b + (a^2*c*ArcTanh[Sqrt[b^2 + a^2*x^3
]/b])/(3*b^3) - (a*Sqrt[a + b]*(3*a*b + c)*ArcTanh[Sqrt[b^2 + a^2*x^3]/(Sqr
t[b]*Sqrt[a + b])])/(3*b^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{b^2+a^2x^3}\left(2b^2+cx^3+a^2x^6\right)}{x^7\left(-b^2+a^2x^6\right)}\,dx = \int\left(-\frac{2\sqrt{b^2+a^2x^3}}{x^7}-\frac{c\sqrt{b^2+a^2x^3}}{b^2x^4}-\frac{3a^2\sqrt{b^2+a^2x^3}}{b^2x}-\frac{a^2(3ab+c)x^2\sqrt{b}}{2b^3\left(b-ax\right.}\right.$$

$$= -\left(2\int\frac{\sqrt{b^2+a^2x^3}}{x^7}\,dx\right)-\frac{\left(3a^2\right)\int\frac{\sqrt{b^2+a^2x^3}}{x}\,dx}{b^2}+\frac{\left(a^2(3ab-c)\right)\int\frac{x^2\sqrt{b^2+a^2}}{b+ax^3}}{2b^3}$$

$$= -\left(\frac{2}{3}\operatorname{Subst}\left(\int\frac{\sqrt{b^2+a^2x}}{x^3}\,dx,x,x^3\right)\right)-\frac{a^2\operatorname{Subst}\left(\int\frac{\sqrt{b^2+a^2x}}{x}\,dx,x,x^3\right)}{b^2}+$$

$$= -\frac{2a^2\sqrt{b^2+a^2x^3}}{b^2}+\frac{a(3ab-c)\sqrt{b^2+a^2x^3}}{3b^3}+\frac{a(3ab+c)\sqrt{b^2+a^2x^3}}{3b^3}+$$

$$= -\frac{2a^2\sqrt{b^2+a^2x^3}}{b^2}+\frac{a(3ab-c)\sqrt{b^2+a^2x^3}}{3b^3}+\frac{a(3ab+c)\sqrt{b^2+a^2x^3}}{3b^3}+$$

$$= -\frac{2a^2\sqrt{b^2+a^2x^3}}{b^2}+\frac{a(3ab-c)\sqrt{b^2+a^2x^3}}{3b^3}+\frac{a(3ab+c)\sqrt{b^2+a^2x^3}}{3b^3}+$$

$$= -\frac{2a^2\sqrt{b^2+a^2x^3}}{b^2}+\frac{a(3ab-c)\sqrt{b^2+a^2x^3}}{3b^3}+\frac{a(3ab+c)\sqrt{b^2+a^2x^3}}{3b^3}+$$

Mathematica [C] time = 1.09, size = 321, normalized size = 1.55

$$\frac{\frac{3b^4c\left(a^2x^3\sqrt{\frac{a^2x^3}{b^2}+1}\tanh^{-1}\left(\sqrt{\frac{a^2x^3}{b^2}+1}\right)+a^2x^3+b^2\right)}{x^3\sqrt{a^2x^3+b^2}}+18a^2b^4\left(b\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{b}\right)-\sqrt{a^2x^3+b^2}\right)-3ab^3(3ab-c)\left(\sqrt{b}\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{a-b}}\right)-\sqrt{a^2x^3+b^2}\right)-3ab^3(3ab+c)\left(\sqrt{b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{a+b}}\right)-\sqrt{a^2x^3+b^2}\right)+4a^4\left(a^2x^3+b^2\right)^{3/2}{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{a^2x^3}{b^2}+1\right)}{9b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + a^2*x^3]*(2*b^2 + c*x^3 + a^2*x^6))/(x^7*(-b^2 + a^2*
x^6)),x]
```

```
[Out] (-3*a*b^3*(3*a*b - c)*(-Sqrt[b^2 + a^2*x^3] + Sqrt[a - b]*Sqrt[b]*ArcTan[Sq
rt[b^2 + a^2*x^3]/(Sqrt[a - b]*Sqrt[b])]) + 18*a^2*b^4*(-Sqrt[b^2 + a^2*x^3
] + b*ArcTanh[Sqrt[b^2 + a^2*x^3]/b]) - 3*a*b^3*(3*a*b + c)*(-Sqrt[b^2 + a^
2*x^3] + Sqrt[b]*Sqrt[a + b]*ArcTanh[Sqrt[b^2 + a^2*x^3]/(Sqrt[b]*Sqrt[a +
b])]) + (3*b^4*c*(b^2 + a^2*x^3 + a^2*x^3*Sqrt[1 + (a^2*x^3)/b^2]*ArcTanh[S
qrt[1 + (a^2*x^3)/b^2]]))/(x^3*Sqrt[b^2 + a^2*x^3]) + 4*a^4*(b^2 + a^2*x^3)
^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (a^2*x^3)/b^2])/(9*b^6)
```

IntegrateAlgebraic [A] time = 0.70, size = 207, normalized size = 1.00

$$\frac{\sqrt{a^2x^3+b^2}\left(a^2x^3+2b^2+2cx^3\right)}{6b^2x^6}-\frac{\sqrt{a-b}\left(3a^2b-ac\right)\tan^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{a-b}}\right)}{3b^{5/2}}-\frac{\sqrt{a+b}\left(3a^2b+ac\right)\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{a+b}}\right)}{3b^{5/2}}+\frac{\left(-a^4+12a^2b^2+2a^2c\right)\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{b}\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[b^2 + a^2*x^3]*(2*b^2 + c*x^3 + a^2*x^6))/(x^7*(-b
^2 + a^2*x^6)),x]
```

```
[Out] (Sqrt[b^2 + a^2*x^3]*(2*b^2 + a^2*x^3 + 2*c*x^3))/(6*b^2*x^6) - (Sqrt[a - b
]*(3*a^2*b - a*c)*ArcTan[Sqrt[b^2 + a^2*x^3]/(Sqrt[a - b]*Sqrt[b])])/(3*b^(
5/2)) + ((-a^4 + 12*a^2*b^2 + 2*a^2*c)*ArcTanh[Sqrt[b^2 + a^2*x^3]/b])/(6*b
^3) - (Sqrt[a + b]*(3*a^2*b + a*c)*ArcTanh[Sqrt[b^2 + a^2*x^3]/(Sqrt[b]*Sqr
t[a + b])])/(3*b^(5/2))
```

fricas [A] time = 1.28, size = 1051, normalized size = 5.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^3+b^2)^(1/2)*(a^2*x^6+c*x^3+2*b^2)/x^7/(a^2*x^6-b^2),x, al
gorithm="fricas")
```

```
[Out] [-1/12*(2*(3*a^2*b^2 - a*b*c)*x^6*sqrt(-(a - b)/b)*log((a^2*x^3 - a*b + 2*b
^2 + 2*sqrt(a^2*x^3 + b^2)*b*sqrt(-(a - b)/b))/(a*x^3 + b)) - 2*(3*a^2*b^2
+ a*b*c)*x^6*sqrt((a + b)/b)*log((a^2*x^3 + a*b + 2*b^2 - 2*sqrt(a^2*x^3 +
b^2)*b*sqrt((a + b)/b))/(a*x^3 - b)) + (a^4 - 12*a^2*b^2 - 2*a^2*c)*x^6*log
(b + sqrt(a^2*x^3 + b^2)) - (a^4 - 12*a^2*b^2 - 2*a^2*c)*x^6*log(-b + sqrt(
a^2*x^3 + b^2)) - 2*sqrt(a^2*x^3 + b^2)*((a^2*b + 2*b*c)*x^3 + 2*b^3))/(b^3
*x^6), 1/12*(4*(3*a^2*b^2 - a*b*c)*x^6*sqrt((a - b)/b)*arctan(b*sqrt((a - b
)/b)/sqrt(a^2*x^3 + b^2)) + 2*(3*a^2*b^2 + a*b*c)*x^6*sqrt((a + b)/b)*log((
a^2*x^3 + a*b + 2*b^2 - 2*sqrt(a^2*x^3 + b^2)*b*sqrt((a + b)/b))/(a*x^3 - b
)) - (a^4 - 12*a^2*b^2 - 2*a^2*c)*x^6*log(b + sqrt(a^2*x^3 + b^2)) + (a^4 -
 12*a^2*b^2 - 2*a^2*c)*x^6*log(-b + sqrt(a^2*x^3 + b^2)) + 2*sqrt(a^2*x^3 +
 b^2)*((a^2*b + 2*b*c)*x^3 + 2*b^3))/(b^3*x^6), 1/12*(4*(3*a^2*b^2 + a*b*c)
*x^6*sqrt(-(a + b)/b)*arctan(b*sqrt(-(a + b)/b)/sqrt(a^2*x^3 + b^2)) - 2*(3
*a^2*b^2 - a*b*c)*x^6*sqrt(-(a - b)/b)*log((a^2*x^3 - a*b + 2*b^2 + 2*sqrt(
a^2*x^3 + b^2)*b*sqrt(-(a - b)/b))/(a*x^3 + b)) - (a^4 - 12*a^2*b^2 - 2*a^2
*c)*x^6*log(b + sqrt(a^2*x^3 + b^2)) + (a^4 - 12*a^2*b^2 - 2*a^2*c)*x^6*log
(-b + sqrt(a^2*x^3 + b^2)) + 2*sqrt(a^2*x^3 + b^2)*((a^2*b + 2*b*c)*x^3 + 2
*b^3))/(b^3*x^6), 1/12*(4*(3*a^2*b^2 + a*b*c)*x^6*sqrt(-(a + b)/b)*arctan(b
*sqrt(-(a + b)/b)/sqrt(a^2*x^3 + b^2)) + 4*(3*a^2*b^2 - a*b*c)*x^6*sqrt((a
- b)/b)*arctan(b*sqrt((a - b)/b)/sqrt(a^2*x^3 + b^2)) - (a^4 - 12*a^2*b^2 -
 2*a^2*c)*x^6*log(b + sqrt(a^2*x^3 + b^2)) + (a^4 - 12*a^2*b^2 - 2*a^2*c)*x
^6*log(-b + sqrt(a^2*x^3 + b^2)) + 2*sqrt(a^2*x^3 + b^2)*((a^2*b + 2*b*c)*x
^3 + 2*b^3))/(b^3*x^6)]
```

giac [A] time = 0.19, size = 308, normalized size = 1.49

$$-\frac{\left(3\,a^{3}b-3\,a^{2}b^{2}-a^{2}c+abc\right)\arctan\left(\frac{\sqrt{a^{2}x^{3}+b^{2}}}{\sqrt{ab-b^{2}}}\right)}{3\,\sqrt{ab-b^{2}}\,b^{2}}+\frac{\left(3\,a^{3}b+3\,a^{2}b^{2}+a^{2}c+abc\right)\arctan\left(\frac{\sqrt{a^{2}x^{3}+b^{2}}}{\sqrt{-ab-b^{2}}}\right)}{3\,\sqrt{-ab-b^{2}}\,b^{2}}-\frac{\left(a^{4}-12\,a^{2}b^{2}-2\,a^{2}c\right)\log\left(\left|b+\sqrt{a^{2}x^{3}+b^{2}}\right|\right)}{12\,b^{3}}+\frac{\left(a^{4}-12\,a^{2}b^{2}-2\,a^{2}c\right)\log\left(\left|-b+\sqrt{a^{2}x^{3}+b^{2}}\right|\right)}{12\,b^{3}}+\frac{\sqrt{a^{2}x^{3}+b^{2}}\,a^{4}b^{2}+\left(a^{2}x^{3}+b^{2}\right)^{\frac{3}{2}}a^{4}-2\,\sqrt{a^{2}x^{3}+b^{2}}\,a^{2}b^{2}c+2\left(a^{2}x^{3}+b^{2}\right)^{\frac{3}{2}}a^{2}c}{6\,a^{4}b^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^3+b^2)^(1/2)*(a^2*x^6+c*x^3+2*b^2)/x^7/(a^2*x^6-b^2),x, al
gorithm="giac")
```

```
[Out] -1/3*(3*a^3*b - 3*a^2*b^2 - a^2*c + a*b*c)*arctan(sqrt(a^2*x^3 + b^2)/sqrt(
a*b - b^2))/(sqrt(a*b - b^2)*b^2) + 1/3*(3*a^3*b + 3*a^2*b^2 + a^2*c + a*b*
c)*arctan(sqrt(a^2*x^3 + b^2)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*b^2) - 1/
12*(a^4 - 12*a^2*b^2 - 2*a^2*c)*log(abs(b + sqrt(a^2*x^3 + b^2)))/b^3 + 1/1
2*(a^4 - 12*a^2*b^2 - 2*a^2*c)*log(abs(-b + sqrt(a^2*x^3 + b^2)))/b^3 + 1/6
*(sqrt(a^2*x^3 + b^2)*a^4*b^2 + (a^2*x^3 + b^2)^(3/2)*a^4 - 2*sqrt(a^2*x^3
+ b^2)*a^2*b^2*c + 2*(a^2*x^3 + b^2)^(3/2)*a^2*c)/(a^4*b^2*x^6)
```

maple [C] time = 0.09, size = 1088, normalized size = 5.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^3+b^2)^(1/2)*(a^2*x^6+c*x^3+2*b^2)/x^7/(a^2*x^6-b^2),x)
```

```
[Out] -c/b^2*(-1/3*(a^2*x^3+b^2)^(1/2)/x^3-1/3*a^2*arctanh((a^2*x^3+b^2)^(1/2)/(b
^2)^(1/2))/(b^2)^(1/2))+1/2*a^2*(3*a*b-c)/b^3*(2/3/a*(a^2*x^3+b^2)^(1/2)+1/
3*I/a^2*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*a*(2*x+1/a*(-I*3^(1/2)*(-a*b^2)^(
1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(a*(x-1/a*(-a*b^2)^(1/3))/(-3*(
-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*a*(2*x+1/a*(I*3^(1/2
```

```
)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(a^2*x^3+b^2)^(1/2)
*(I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*a-I*3^(1/2)*(-a*b^2)^(2/3)+2*_alpha^2*a^2
-(-a*b^2)^(1/3)*_alpha*a-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/a
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/a*(-a*b^2)^(1/3))*3^(1/2)*a/(-a*b^2)^(1/3))^(
1/2),1/2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*a-I*(-a*b^2)^(2/3)*3^(1/2)*_a
lpha+I*3^(1/2)*b^2-3*(-a*b^2)^(2/3)*_alpha-3*b^2)/b/(a-b),(I*3^(1/2)/a*(-a*
b^2)^(1/3)/(-3/2/a*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/a*(-a*b^2)^(1/3)))^(1/2)),_
alpha=RootOf(_Z^3*a+b)))+1/3*(a^2*x^3+b^2)^(1/2)/x^6+1/6*a^2/b^2*(a^2*x^3+b
^2)^(1/2)/x^3-1/6*a^4/b^2*arctanh((a^2*x^3+b^2)^(1/2)/(b^2)^(1/2))/(b^2)^(1
/2)+1/2*a^2*(3*a*b+c)/b^3*(2/3/a*(a^2*x^3+b^2)^(1/2)+1/3*I/a^2*2^(1/2)*sum(
(-a*b^2)^(1/3)*(1/2*I*a*(2*x+1/a*(-I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))
)/(-a*b^2)^(1/3))^(1/2)*(a*(x-1/a*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1
/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*a*(2*x+1/a*(I*3^(1/2)*(-a*b^2)^(1/3)+(-a
*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(a^2*x^3+b^2)^(1/2)*(I*(-a*b^2)^(1/3)*_
alpha*3^(1/2)*a-I*3^(1/2)*(-a*b^2)^(2/3)+2*_alpha^2*a^2-(-a*b^2)^(1/3)*_alp
ha*a-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/a*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/a*(-a*b^2)^(1/3))*3^(1/2)*a/(-a*b^2)^(1/3))^(1/2),-1/2*(2*I*(-a*b
^2)^(1/3)*3^(1/2)*_alpha^2*a-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*b^2-
3*(-a*b^2)^(2/3)*_alpha-3*b^2)/b/(a+b),(I*3^(1/2)/a*(-a*b^2)^(1/3)/(-3/2/a*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/a*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*a
-b)))-3*a^2/b^2*(2/3*(a^2*x^3+b^2)^(1/2)-2/3*b^2*arctanh((a^2*x^3+b^2)^(1/2
)/(b^2)^(1/2))/(b^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^6 + cx^3 + 2\,b^2\right)\sqrt{a^2x^3 + b^2}}{\left(a^2x^6 - b^2\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^3+b^2)^(1/2)*(a^2*x^6+c*x^3+2*b^2)/x^7/(a^2*x^6-b^2),x, al
gorithm="maxima")
```

```
[Out] integrate((a^2*x^6 + c*x^3 + 2*b^2)*sqrt(a^2*x^3 + b^2)/((a^2*x^6 - b^2)*x^
7), x)
```

mupad [B] time = 9.50, size = 248, normalized size = 1.20

$$\frac{\sqrt{a^2\,x^3+b^2}}{3\,x^6}+\frac{a^2\,\ln\left(\frac{{\left(b+\sqrt{a^2\,x^3+b^2}\right)}^3\,\left(b-\sqrt{a^2\,x^3+b^2}\right)}{x^6}\right)\,\left(-a^2+12\,b^2+2\,c\right)}{12\,b^3}+\frac{\sqrt{a^2\,x^3+b^2}\,\left(a^2+2\,c\right)}{6\,b^2\,x^3}+\frac{a\,\ln\left(\frac{a\,b+2\,b^2+a^2\,x^3-2\,\sqrt{b}\,\sqrt{a^2\,x^3+b^2}\,\sqrt{a+b}}{b-a\,x^3}\right)\,\sqrt{a+b}\,\left(c+3\,a\,b\right)}{6\,b^{5/2}}+\frac{a\,\ln\left(\frac{2\,b^2-a\,b+a^2\,x^3+2\,\sqrt{b}\,\sqrt{a^2\,x^3+b^2}\,\sqrt{b-a}}{a\,x^3+b}\right)\,\sqrt{b-a}\,\left(c-3\,a\,b\right)}{6\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b^2 + a^2*x^3)^(1/2)*(c*x^3 + 2*b^2 + a^2*x^6))/(x^7*(b^2 - a^2*x^6)
),x)
```

```
[Out] (b^2 + a^2*x^3)^(1/2)/(3*x^6) + (a^2*log(((b + (b^2 + a^2*x^3)^(1/2))^3*(b
- (b^2 + a^2*x^3)^(1/2)))/x^6)*(2*c - a^2 + 12*b^2))/(12*b^3) + ((b^2 + a^2
*x^3)^(1/2)*(2*c + a^2))/(6*b^2*x^3) + (a*log((a*b + 2*b^2 + a^2*x^3 - 2*b^
(1/2)*(b^2 + a^2*x^3)^(1/2)*(a + b)^(1/2))/(b - a*x^3))*(a + b)^(1/2)*(c +
3*a*b))/(6*b^(5/2)) + (a*log((2*b^2 - a*b + a^2*x^3 + 2*b^(1/2)*(b^2 + a^2*
x^3)^(1/2)*(b - a)^(1/2))/(b + a*x^3))*(b - a)^(1/2)*(c - 3*a*b))/(6*b^(5/2
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**3+b**2)**(1/2)*(a**2*x**6+c*x**3+2*b**2)/x**7/(a**2*x**6
-b**2),x)
```

```
[Out] Timed out
```

3.2014 $\int \frac{x^7(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}\left(-a^2+2ax-x^2+dx^8\right)}\,dx$

Optimal. Leaf size=207

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}x^2}{\sqrt[6]{d}x^2-2\sqrt[3]{x^3-ax^2}}\right)}{2d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}x^2}{2\sqrt[3]{x^3-ax^2}+\sqrt[6]{d}x^2}\right)}{2d^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{d}\left(x^3-ax^2\right)^{2/3}}{a-x}\right)}{d^{5/6}}-\frac{\tanh^{-1}\left(\frac{\frac{\left(x^3-ax^2\right)^{2/3}}{\sqrt[6]{d}}+\sqrt[6]{d}x^4}{x^2\sqrt[3]{x^3-ax^2}}\right)}{2d^{5/6}}$$

Rubi [F] time = 3.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^7(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}\left(-a^2+2ax-x^2+dx^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^7*(-4*a + 3*x))/((x^2*(-a + x))^(2/3)*(-a^2 + 2*a*x - x^2 + d*x^8)),
x]
```

```
[Out] (12*a*x^(4/3)*(-a + x)^(2/3)*Defer[Subst][Defer[Int][x^19/((-a + x^3)^(2/3)
*(a^2 - 2*a*x^3 + x^6 - d*x^24)), x], x, x^(1/3)])/(-((a - x)*x^2))^(2/3) +
 (9*x^(4/3)*(-a + x)^(2/3)*Defer[Subst][Defer[Int][x^22/((-a + x^3)^(2/3)*(
-a^2 + 2*a*x^3 - x^6 + d*x^24)), x], x, x^(1/3)])/(-((a - x)*x^2))^(2/3)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}\left(-a^2+2ax-x^2+dx^8\right)}\,dx &= \frac{\left(x^{4/3}(-a+x)^{2/3}\right)\int \frac{x^{17/3}(-4a+3x)}{(-a+x)^{2/3}\left(-a^2+2ax-x^2+dx^8\right)}\,dx}{\left(x^2(-a+x)\right)^{2/3}}\\
&= \frac{\left(3x^{4/3}(-a+x)^{2/3}\right)\text{Subst}\left(\int \frac{x^{19}\left(-4a+3x^3\right)}{\left(-a+x^3\right)^{2/3}\left(-a^2+2ax^3-x^6+dx^{24}\right)}\,dx, x, \sqrt[3]{\ }\right.}{\left(x^2(-a+x)\right)^{2/3}}\\
&= \frac{\left(3x^{4/3}(-a+x)^{2/3}\right)\text{Subst}\left(\int \left(\frac{4ax^{19}}{\left(-a+x^3\right)^{2/3}\left(a^2-2ax^3+x^6-dx^{24}\right)}+\frac{}{\left(-a+x^3\right.}\right.\right.}{\left(x^2(-a+x)\right)^{2/3}}\\
&= \frac{\left(9x^{4/3}(-a+x)^{2/3}\right)\text{Subst}\left(\int \frac{x^{22}}{\left(-a+x^3\right)^{2/3}\left(-a^2+2ax^3-x^6+dx^{24}\right)}\,dx, x, \sqrt[3]{\ }\right.}{\left(x^2(-a+x)\right)^{2/3}}
\end{aligned}$$

Mathematica [F] time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{x^7(-4a+3x)}{\left(x^2(-a+x)\right)^{2/3}\left(-a^2+2ax-x^2+dx^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^7*(-4*a + 3*x))/((x^2*(-a + x))^(2/3)*(-a^2 + 2*a*x - x^2 + d*
x^8)),x]
```

```
[Out] Integrate[(x^7*(-4*a + 3*x))/((x^2*(-a + x))^(2/3)*(-a^2 + 2*a*x - x^2 + d*
x^8)), x]
```

IntegrateAlgebraic [A] time = 0.89, size = 207, normalized size = 1.00

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}x^2}{\sqrt[6]{d}x^2-2\sqrt[3]{x^3-ax^2}}\right)}{2d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}x^2}{2\sqrt[3]{x^3-ax^2}+\sqrt[6]{d}x^2}\right)}{2d^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{d}\left(x^3-ax^2\right)^{2/3}}{a-x}\right)}{d^{5/6}}-\frac{\tanh^{-1}\left(\frac{\frac{\left(x^3-ax^2\right)^{2/3}}{\sqrt[6]{d}}+\sqrt[6]{d}x^4}{x^2\sqrt[3]{x^3-ax^2}}\right)}{2d^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^7*(-4*a + 3*x))/((x^2*(-a + x))^(2/3)*(-a^2 + 2*a*x -
 x^2 + d*x^8)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/6)*x^2)/(d^(1/6)*x^2 - 2*(-(a*x^2) + x^3
)^(1/3))])/d^(5/6) + (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/6)*x^2)/(d^(1/6)*x^2 + 2
*(-(a*x^2) + x^3)^(1/3))])/(2*d^(5/6)) + ArcTanh[(d^(1/6)*(-(a*x^2) + x^3)^
(2/3))/(a - x)]/d^(5/6) - ArcTanh[(d^(1/6)*x^4 + (-(a*x^2) + x^3)^(2/3)/d^(
1/6))/(x^2*(-(a*x^2) + x^3)^(1/3))]/(2*d^(5/6))
```

fricas [B] time = 0.74, size = 431, normalized size = 2.08

$$-\sqrt{3}\left(\frac{1}{d^5}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{3}d^4\left(\frac{1}{d^5}\right)^{\frac{5}{6}}x^2\sqrt{\frac{d^2\left(\frac{1}{d^5}\right)^{\frac{1}{3}}x^4+\left(-ax^2+x^3\right)^{\frac{1}{3}}d\left(\frac{1}{d^5}\right)^{\frac{1}{6}}x^2+\left(-ax^2+x^3\right)^{\frac{2}{3}}}{x^4}}+2\sqrt{3}\left(-ax^2+x^3\right)^{\frac{1}{3}}d^4\left(\frac{1}{d^5}\right)^{\frac{5}{6}}+\sqrt{3}x^2}{3x^2}\right)-\sqrt{3}\left(\frac{1}{d^5}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{3}d^4\left(\frac{1}{d^5}\right)^{\frac{5}{6}}x^2\sqrt{\frac{d^2\left(\frac{1}{d^5}\right)^{\frac{1}{3}}x^4-\left(-ax^2+x^3\right)^{\frac{1}{3}}d\left(\frac{1}{d^5}\right)^{\frac{1}{6}}x^2+\left(-ax^2+x^3\right)^{\frac{2}{3}}}{x^4}}+2\sqrt{3}\left(-ax^2+x^3\right)^{\frac{1}{3}}d^4\left(\frac{1}{d^5}\right)^{\frac{5}{6}}-\sqrt{3}x^2}{3x^2}\right)-\frac{1}{2}\left(\frac{1}{d^5}\right)^{\frac{1}{6}}\log\left(-\frac{d\left(\frac{1}{d^5}\right)^{\frac{1}{6}}x^2+\left(-ax^2+x^3\right)^{\frac{1}{3}}}{x^2}\right)+\frac{1}{2}\left(\frac{1}{d^5}\right)^{\frac{1}{6}}\log\left(\frac{d\left(\frac{1}{d^5}\right)^{\frac{1}{6}}x^2-\left(-ax^2+x^3\right)^{\frac{1}{3}}}{x^2}\right)-\frac{1}{4}\left(\frac{1}{d^5}\right)^{\frac{1}{6}}\log\left(\frac{d^2\left(\frac{1}{d^5}\right)^{\frac{1}{3}}x^4+\left(-ax^2+x^3\right)^{\frac{1}{3}}d\left(\frac{1}{d^5}\right)^{\frac{1}{6}}x^2+\left(-ax^2+x^3\right)^{\frac{2}{3}}}{x^4}\right)+\frac{1}{4}\left(\frac{1}{d^5}\right)^{\frac{1}{6}}\log\left(\frac{d^2\left(\frac{1}{d^5}\right)^{\frac{1}{3}}x^4-\left(-ax^2+x^3\right)^{\frac{1}{3}}d\left(\frac{1}{d^5}\right)^{\frac{1}{6}}x^2+\left(-ax^2+x^3\right)^{\frac{2}{3}}}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-4*a+3*x)/(x^2*(-a+x))^(2/3)/(d*x^8-a^2+2*a*x-x^2),x, algori
thm="fricas")
```

```
[Out] -sqrt(3)*(d^(-5))^(1/6)*arctan(1/3*(2*sqrt(3)*d^4*(d^(-5))^(5/6)*x^2*sqrt((
d^2*(d^(-5))^(1/3)*x^4 + (-a*x^2 + x^3)^(1/3)*d*(d^(-5))^(1/6)*x^2 + (-a*x^
2 + x^3)^(2/3))/x^4) + 2*sqrt(3)*(-a*x^2 + x^3)^(1/3)*d^4*(d^(-5))^(5/6) +
sqrt(3)*x^2)/x^2) - sqrt(3)*(d^(-5))^(1/6)*arctan(1/3*(2*sqrt(3)*d^4*(d^(-5
))^(5/6)*x^2*sqrt((d^2*(d^(-5))^(1/3)*x^4 - (-a*x^2 + x^3)^(1/3)*d*(d^(-5))
^(1/6)*x^2 + (-a*x^2 + x^3)^(2/3))/x^4) + 2*sqrt(3)*(-a*x^2 + x^3)^(1/3)*d^
4*(d^(-5))^(5/6) - sqrt(3)*x^2)/x^2) - 1/2*(d^(-5))^(1/6)*log(-(d*(d^(-5))^
(1/6)*x^2 + (-a*x^2 + x^3)^(1/3))/x^2) + 1/2*(d^(-5))^(1/6)*log((d*(d^(-5))
^(1/6)*x^2 - (-a*x^2 + x^3)^(1/3))/x^2) - 1/4*(d^(-5))^(1/6)*log((d^2*(d^(-
5))^(1/3)*x^4 + (-a*x^2 + x^3)^(1/3)*d*(d^(-5))^(1/6)*x^2 + (-a*x^2 + x^3)^
(2/3))/x^4) + 1/4*(d^(-5))^(1/6)*log((d^2*(d^(-5))^(1/3)*x^4 - (-a*x^2 + x^
3)^(1/3)*d*(d^(-5))^(1/6)*x^2 + (-a*x^2 + x^3)^(2/3))/x^4)
```

giac [B] time = 0.96, size = 399, normalized size = 1.93

$$\frac{1}{2}\sqrt{3}\left(\frac{1}{d^5}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}-\sqrt{3}ad^{\frac{1}{6}}-\sqrt{3}\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}}{\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}+ad^{\frac{1}{6}}-\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}}\right)+\frac{1}{2}\sqrt{3}\left(\frac{1}{d^5}\right)^{\frac{1}{6}}\arctan\left(-\frac{\sqrt{3}\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}+\sqrt{3}ad^{\frac{1}{6}}-\sqrt{3}\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}}{\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}-ad^{\frac{1}{6}}-\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}}\right)+\frac{1}{4}\left(\frac{1}{d^5}\right)^{\frac{1}{6}}\log\left(\left(\sqrt{3}\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}+\sqrt{3}ad^{\frac{1}{6}}-\sqrt{3}\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}\right)^2+\left(\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}-ad^{\frac{1}{6}}-\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}\right)^2\right)-\frac{1}{4}\left(\frac{1}{d^5}\right)^{\frac{1}{6}}\log\left(\left(\sqrt{3}\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}-\sqrt{3}ad^{\frac{1}{6}}-\sqrt{3}\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}\right)^2+\left(\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}+ad^{\frac{1}{6}}-\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}\right)^2\right)+\frac{1}{2}\left(\frac{1}{d^5}\right)^{\frac{1}{6}}\log\left(\left|\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}+ad^{\frac{1}{6}}-\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}\right|\right)-\frac{1}{2}\left(\frac{1}{d^5}\right)^{\frac{1}{6}}\log\left(\left|\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}-ad^{\frac{1}{6}}-\left(-\frac{a}{x}+1\right)^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-4*a+3*x)/(x^2*(-a+x))^(2/3)/(d*x^8-a^2+2*a*x-x^2),x, algori
thm="giac")
```

```
[Out] 1/2*sqrt(3)*(d^(-5))^(1/6)*arctan((sqrt(3)*(-a/x + 1)^(4/3) - sqrt(3)*a*d^(
1/6) - sqrt(3)*(-a/x + 1)^(1/3))/((-a/x + 1)^(4/3) + a*d^(1/6) - (-a/x + 1)
^(1/3))) + 1/2*sqrt(3)*(d^(-5))^(1/6)*arctan(-(sqrt(3)*(-a/x + 1)^(4/3) + s
qrt(3)*a*d^(1/6) - sqrt(3)*(-a/x + 1)^(1/3))/((-a/x + 1)^(4/3) - a*d^(1/6)
- (-a/x + 1)^(1/3))) + 1/4*(d^(-5))^(1/6)*log((sqrt(3)*(-a/x + 1)^(4/3) + s
qrt(3)*a*d^(1/6) - sqrt(3)*(-a/x + 1)^(1/3))^2 + ((-a/x + 1)^(4/3) - a*d^(1
/6) - (-a/x + 1)^(1/3))^2) - 1/4*(d^(-5))^(1/6)*log((sqrt(3)*(-a/x + 1)^(4/
3) - sqrt(3)*a*d^(1/6) - sqrt(3)*(-a/x + 1)^(1/3))^2 + ((-a/x + 1)^(4/3) +
a*d^(1/6) - (-a/x + 1)^(1/3))^2) + 1/2*(d^(-5))^(1/6)*log(abs((-a/x + 1)^(4
```

```
/3) + a*d^(1/6) - (-a/x + 1)^(1/3))) - 1/2*(d^(-5))^(1/6)*log(abs((-a/x + 1
)^(4/3) - a*d^(1/6) - (-a/x + 1)^(1/3)))
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{x^7(-4a+3x)}{(x^2(-a+x))^{\frac{2}{3}}(d\,x^8-a^2+2ax-x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(-4*a+3*x)/(x^2*(-a+x))^(2/3)/(d*x^8-a^2+2*a*x-x^2),x)
```

```
[Out] int(x^7*(-4*a+3*x)/(x^2*(-a+x))^(2/3)/(d*x^8-a^2+2*a*x-x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(4\,a-3\,x)x^7}{(dx^8-a^2+2\,ax-x^2)(-(a-x)x^2)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-4*a+3*x)/(x^2*(-a+x))^(2/3)/(d*x^8-a^2+2*a*x-x^2),x, algori
thm="maxima")
```

```
[Out] -integrate((4*a - 3*x)*x^7/((d*x^8 - a^2 + 2*a*x - x^2)*(-(a - x)*x^2)^(2/3
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x^7\,(4\,a-3\,x)}{(-x^2\,(a-x))^{2/3}\,(-a^2+2\,a\,x+d\,x^8-x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^7*(4*a - 3*x))/((-x^2*(a - x))^(2/3)*(2*a*x + d*x^8 - a^2 - x^2)),x
)
```

```
[Out] int(-(x^7*(4*a - 3*x))/((-x^2*(a - x))^(2/3)*(2*a*x + d*x^8 - a^2 - x^2)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7(-4a+3x)}{(x^2(-a+x))^{\frac{2}{3}}(-a^2+2ax+dx^8-x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(-4*a+3*x)/(x**2*(-a+x))**(2/3)/(d*x**8-a**2+2*a*x-x**2),x)
```

```
[Out] Integral(x**7*(-4*a + 3*x)/((x**2*(-a + x))**(2/3)*(-a**2 + 2*a*x + d*x**8
- x**2)), x)
```

3.2015 $\int \frac{-1+2x+(-2k+k^2)x^2}{((1-x)x(1-kx))^{2/3}(b-(1+2bk)x+(1+bk^2)x^2)}\,dx$

Optimal. Leaf size=208

$$\frac{\log\left(\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+x^2-x\right)}{b^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}}{\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}-2x^2+2x}\right)}{b^{2/3}}-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2\right.\right.}{}$$

Rubi [F] time = 3.51, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+2x+(-2k+k^2)x^2}{((1-x)x(1-kx))^{2/3}\left(b-(1+2bk)x+(1+bk^2)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 2*x + (-2*k + k^2)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(b - (1 + 2*
b*k)*x + (1 + b*k^2)*x^2)),x]
```

```
[Out] ((2 - k - Sqrt[1 - 4*b*(1 - k)]*k)*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*De
fer[Int][(1 - k*x)^(1/3)/((1 - x)^(2/3)*x^(2/3)*(-1 - 2*b*k - Sqrt[1 - 4*b
+ 4*b*k] + 2*(1 + b*k^2)*x)), x])/((1 - x)*x*(1 - k*x))^(2/3) + ((2 - (1 -
Sqrt[1 - 4*b*(1 - k)])*k)*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Int][
(1 - k*x)^(1/3)/((1 - x)^(2/3)*x^(2/3)*(-1 - 2*b*k + Sqrt[1 - 4*b + 4*b*k]
+ 2*(1 + b*k^2)*x)), x])/((1 - x)*x*(1 - k*x))^(2/3)
```

Rubi steps

$$\begin{aligned}\int \frac{-1+2x+(-2k+k^2)x^2}{((1-x)x(1-kx))^{2/3}\left(b-(1+2bk)x+(1+bk^2)x^2\right)}\,dx &= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{-1+2x+(-2k+k^2)x}{(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}(b-(1+2bk)}}{((1-x)x(1-kx))^{2/3}}\\ &= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{(-1+(2-k)x)\sqrt[3]{1-kx}}{(1-x)^{2/3}x^{2/3}(b-(1+2bk)x+(1+bk}}{((1-x)x(1-kx))^{2/3}}\\ &= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \left(\frac{\left(2-k-k\sqrt{1-4b+4bk}\right)}{(1-x)^{2/3}x^{2/3}\left(-1-2bk-\sqrt{1-4b+}\right.}\right.}{((1-x)}\\ &= \frac{\left(\left(2-\left(1-\sqrt{1-4b(1-k)}\right)k\right)(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}}{((1-x)x(1-k}\end{aligned}$$

Mathematica [F] time = 5.50, size = 0, normalized size = 0.00

$$\int \frac{-1+2x+(-2k+k^2)x^2}{((1-x)x(1-kx))^{2/3}\left(b-(1+2bk)x+(1+bk^2)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + 2*x + (-2*k + k^2)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(b - (
1 + 2*b*k)*x + (1 + b*k^2)*x^2)),x]
```

```
[Out] Integrate[(-1 + 2*x + (-2*k + k^2)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(b - (
1 + 2*b*k)*x + (1 + b*k^2)*x^2)), x]
```

IntegrateAlgebraic [A] time = 1.02, size = 208, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+x^2-x\right)}{b^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}}{\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}-2x^2+2x}\right)}{b^{2/3}}-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{4/3}+\left(\sqrt[3]{b}\,x-\sqrt[3]{b}\,x^2\right)\left(kx^3+(-k-1)x^2+x\right)^{2/3}+x^4-2x^3+x^2\right)}{2b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x + (-2*k + k^2)*x^2)/(((1 - x)*x*(1 - k*x))^(2/
3)*(b - (1 + 2*b*k)*x + (1 + b*k^2)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3))/(2*x - 2
*x^2 + b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3))])/b^(2/3) + Log[-x + x^2 +
 b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3)]/b^(2/3) - Log[x^2 - 2*x^3 + x^4
+ (b^(1/3)*x - b^(1/3)*x^2)*(x + (-1 - k)*x^2 + k*x^3)^(2/3) + b^(2/3)*(x +
 (-1 - k)*x^2 + k*x^3)^(4/3)]/(2*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x+(k^2-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-(2*b*k+1)*x+(b
*k^2+1)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(k^2-2\,k\right)x^2+2\,x-1}{\left((kx-1)(x-1)x\right)^{\frac{2}{3}}\left(\left(bk^2+1\right)x^2-(2\,bk+1)x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x+(k^2-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-(2*b*k+1)*x+(b
*k^2+1)*x^2),x, algorithm="giac")
```

```
[Out] integrate(((k^2 - 2*k)*x^2 + 2*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((b*k^2
+ 1)*x^2 - (2*b*k + 1)*x + b)), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{-1+2x+\left(k^2-2k\right)x^2}{\left((1-x)\,x\,(-kx+1)\right)^{\frac{2}{3}}\left(b-(2bk+1)\,x+\left(b\,k^2+1\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x+(k^2-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-(2*b*k+1)*x+(b*k^2+1
)*x^2),x)
```

```
[Out] int((-1+2*x+(k^2-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-(2*b*k+1)*x+(b*k^2+1
)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(k^2-2\,k\right)x^2+2\,x-1}{\left((kx-1)(x-1)x\right)^{\frac{2}{3}}\left(\left(bk^2+1\right)x^2-(2\,bk+1)x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x+(k^2-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-(2*b*k+1)*x+(b
*k^2+1)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(((k^2 - 2*k)*x^2 + 2*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((b*k^2
+ 1)*x^2 - (2*b*k + 1)*x + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(2\,k-k^2\right)\,x^2-2\,x+1}{\left(\left(b\,k^2+1\right)\,x^2+\left(-2\,b\,k-1\right)\,x+b\right)\,{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(2*k - k^2) - 2*x + 1)/((b + x^2*(b*k^2 + 1) - x*(2*b*k + 1))*(x*
(k*x - 1)*(x - 1))^(2/3)),x)
```

```
[Out] int(-(x^2*(2*k - k^2) - 2*x + 1)/((b + x^2*(b*k^2 + 1) - x*(2*b*k + 1))*(x*
(k*x - 1)*(x - 1))^(2/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x+(k**2-2*k)*x**2)/((1-x)*x*(-k*x+1))**(2/3)/(b-(2*b*k+1)*x
+(b*k**2+1)*x**2),x)
```

```
[Out] Timed out
```

3.2016 $\int \frac{\sqrt[3]{-x^2+x^4}}{x(1+x^2)}\,dx$

Optimal. Leaf size=208

$$\frac{\log\left(-2x^2+2^{2/3}\sqrt{3}\sqrt[3]{x^4-x^2}\,x-\sqrt[3]{2}\left(x^4-x^2\right)^{2/3}\right)}{4\,2^{2/3}}-\frac{\log\left(2x^2+\sqrt[3]{2}\left(x^4-x^2\right)^{2/3}\right)}{2\,2^{2/3}}+\frac{\log\left(2x^2+2^{2/3}\sqrt{3}\sqrt[3]{x^4-x^2}\right.}{4\,2^{2/3}}$$

Rubi [C] time = 0.60, antiderivative size = 370, normalized size of antiderivative = 1.78, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1311, 2013, 622, 619, 236, 219, 2034, 758, 133}

$$\frac{3\left(\frac{x^2}{x^2+1}\right)^{2/3}\left(-\frac{1-x^2}{x^2+1}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},\frac{2}{3};\frac{7}{3};\frac{2}{x^2+1},\frac{1}{x^2+1}\right)}{4\left(x^4-x^2\right)^{2/3}}-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(x^2-x^4\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{x^2-x^4}\right)\sqrt{\frac{2\sqrt[3]{2}\left(x^2-x^4\right)^{2/3}+2^{2/3}\sqrt[3]{x^2-x^4}+1}{\left(-2^{2/3}\sqrt[3]{x^2-x^4}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{x^2-x^4}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{x^2-x^4}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2^{2/3}\left(1-2x^2\right)\left(x^4-x^2\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{x^2-x^4}}{\left(-2^{2/3}\sqrt[3]{x^2-x^4}-\sqrt{3}+1\right)^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-x^2 + x^4)^(1/3)/(x*(1 + x^2)),x]
```

```
[Out] (3*(x^2/(1 + x^2))^(2/3)*(-((1 - x^2)/(1 + x^2)))^(2/3)*AppellF1[4/3, 2/3,
2/3, 7/3, 2/(1 + x^2), (1 + x^2)^(-1)])/(4*(-x^2 + x^4)^(2/3)) - (3^(3/4)*S
qrt[2 - Sqrt[3]]*(x^2 - x^4)^(2/3)*(1 - 2^(2/3)*(x^2 - x^4)^(1/3))*Sqrt[(1
+ 2^(2/3)*(x^2 - x^4)^(1/3) + 2*2^(1/3)*(x^2 - x^4)^(2/3))/(1 - Sqrt[3] - 2
^(2/3)*(x^2 - x^4)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(x^2 -
 x^4)^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(x^2 - x^4)^(1/3))], -7 + 4*Sqrt[3]])/(
2^(2/3)*(1 - 2*x^2)*(-x^2 + x^4)^(2/3)*Sqrt[-((1 - 2^(2/3)*(x^2 - x^4)^(1/3
))/(1 - Sqrt[3] - 2^(2/3)*(x^2 - x^4)^(1/3))^2)])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 1311

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] - Dist[(c*d^2 - b*d*e + a*e^2)/(d*e*f^2),
Int[((f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1))/(d + e*x^2), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m, 0]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{\sqrt[3]{-x^2+x^4}}{x\left(1+x^2\right)}\,dx = -\left(2\int \frac{x}{\left(1+x^2\right)\left(-x^2+x^4\right)^{2/3}}\,dx\right)+\int \frac{x}{\left(-x^2+x^4\right)^{2/3}}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(-x+x^2\right)^{2/3}}\,dx,x,x^2\right)-\text{Subst}\left(\int \frac{1}{(1+x)\left(-x+x^2\right)^{2/3}}\,dx,x,x^2\right)$$

$$= \frac{\left(\left(\frac{x^2}{1+x^2}\right)^{2/3}\left(\frac{-1+x^2}{1+x^2}\right)^{2/3}\right)\text{Subst}\left(\int \frac{\sqrt[3]{x}}{(1-2x)^{2/3}(1-x)^{2/3}}\,dx,x,\frac{1}{1+x^2}\right)}{\left(\frac{1}{1+x^2}\right)^{4/3}\left(-x^2+x^4\right)^{2/3}}+\frac{\left(x^2-x^4\right)^{2/3}\text{Subst}\left(\int \frac{1}{\left(x-x^2\right)^{2/3}} \cdots\right.}{2\left(-x^2+x^4\right)^{2/3}}$$

$$= \frac{3\left(\frac{x^2}{1+x^2}\right)^{2/3}\left(-\frac{1-x^2}{1+x^2}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},\frac{2}{3};\frac{7}{3};\frac{2}{1+x^2},\frac{1}{1+x^2}\right)}{4\left(-x^2+x^4\right)^{2/3}}-\frac{\left(x^2-x^4\right)^{2/3}\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{2/3}}\,dx,x,1-\cdots\right.}{2^{2/3}\left(-x^2+x^4\right)^{2/3}}$$

$$= \frac{3\left(\frac{x^2}{1+x^2}\right)^{2/3}\left(-\frac{1-x^2}{1+x^2}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},\frac{2}{3};\frac{7}{3};\frac{2}{1+x^2},\frac{1}{1+x^2}\right)}{4\left(-x^2+x^4\right)^{2/3}}+\frac{\left(3\sqrt{-\left(1-2x^2\right)^2}\left(x^2-x^4\right)^{2/3}\right)\text{Subst}\left(\int \cdots\right.}{2\,2^{2/3}\left(1-2x^2\right)\left(\cdots\right.}$$

$$= \frac{3\left(\frac{x^2}{1+x^2}\right)^{2/3}\left(-\frac{1-x^2}{1+x^2}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},\frac{2}{3};\frac{7}{3};\frac{2}{1+x^2},\frac{1}{1+x^2}\right)}{4\left(-x^2+x^4\right)^{2/3}}-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\sqrt{-\left(1-2x^2\right)^2}\left(x^2-x^4\right)^{2/\cdots}}{2^{2/3}\left(1\cdots\right.}$$

Mathematica [C] time = 0.03, size = 47, normalized size = 0.23

$$\frac{3\sqrt[3]{x^2\left(x^2-1\right)}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^2,-x^2\right)}{2\sqrt[3]{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-x^2 + x^4)^(1/3)/(x*(1 + x^2)),x]
```

```
[Out] (3*(x^2*(-1 + x^2))^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, x^2, -x^2])/(2*(1 - x
^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.43, size = 208, normalized size = 1.00

$$\frac{\log\left(-2x^2+2^{2/3}\sqrt{3}\sqrt[3]{x^4-x^2}\,x-\sqrt[3]{2}\left(x^4-x^2\right)^{2/3}\right)}{4\,2^{2/3}}-\frac{\log\left(2x^2+\sqrt[3]{2}\left(x^4-x^2\right)^{2/3}\right)}{2\,2^{2/3}}+\frac{\log\left(2x^2+2^{2/3}\sqrt{3}\sqrt[3]{x^4-x^2}\,x+\sqrt[3]{2}\left(x^4-x^2\right)^{2/3}\right)}{4\,2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x^2}{\sqrt[3]{2}\left(x^4-x^2\right)^{2/3}-x^2}\right)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^2 + x^4)^(1/3)/(x*(1 + x^2)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*x^2)/(-x^2 + 2^(1/3)*(-x^2 + x^4)^(2/3))])/2^
(2/3) + Log[-2*x^2 + 2^(2/3)*Sqrt[3]*x*(-x^2 + x^4)^(1/3) - 2^(1/3)*(-x^2 +
 x^4)^(2/3)]/(4*2^(2/3)) - Log[2*x^2 + 2^(1/3)*(-x^2 + x^4)^(2/3)]/(2*2^(2/
3)) + Log[2*x^2 + 2^(2/3)*Sqrt[3]*x*(-x^2 + x^4)^(1/3) + 2^(1/3)*(-x^2 + x^
4)^(2/3)]/(4*2^(2/3))
```

fricas [B] time = 1.93, size = 365, normalized size = 1.75

$$-\frac{1}{12}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(-\frac{4^{\frac{1}{6}}\sqrt{3}\left(6\cdot 4^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(x^{10}-33x^8+110x^6-110x^4+33x^2-1\right)\left(x^4-x^2\right)^{\frac{1}{3}}-48\,(-1)^{\frac{1}{3}}\left(x^8-2x^6-6x^4-2x^2+1\right)\left(x^4-x^2\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(x^{12}+42x^{10}-417x^8+812x^6-417x^4+42x^2+1\right)\right)}{6\left(x^{12}-102x^{10}+447x^8-628x^6+447x^4-102x^2+1\right)}\right)-\frac{1}{48}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(\frac{24\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^4-x^2\right)^{\frac{2}{3}}\left(x^4-4x^2+1\right)-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^8-32x^6+78x^4-32x^2+1\right)-12\left(x^6-11x^4+11x^2-1\right)\left(x^4-x^2\right)^{\frac{1}{3}}}{x^8+4x^6+6x^4+4x^2+1}\right)+\frac{1}{24}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{3\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^4-x^2\right)^{\frac{1}{3}}\left(x^2-1\right)-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^4+2x^2+1\right)-12\left(x^4-x^2\right)^{\frac{2}{3}}}{x^4+2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2)^(1/3)/x/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/12*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(-1/6*4^(1/6)*sqrt(3)*(6*4^(2/3)*(-1
)^(2/3)*(x^10 - 33*x^8 + 110*x^6 - 110*x^4 + 33*x^2 - 1)*(x^4 - x^2)^(1/3)
- 48*(-1)^(1/3)*(x^8 - 2*x^6 - 6*x^4 - 2*x^2 + 1)*(x^4 - x^2)^(2/3) + 4^(1/
3)*(x^12 + 42*x^10 - 417*x^8 + 812*x^6 - 417*x^4 + 42*x^2 + 1))/(x^12 - 102
*x^10 + 447*x^8 - 628*x^6 + 447*x^4 - 102*x^2 + 1)) - 1/48*4^(2/3)*(-1)^(1/
3)*log((24*4^(1/3)*(-1)^(2/3)*(x^4 - x^2)^(2/3)*(x^4 - 4*x^2 + 1) - 4^(2/3)
*(-1)^(1/3)*(x^8 - 32*x^6 + 78*x^4 - 32*x^2 + 1) - 12*(x^6 - 11*x^4 + 11*x^
2 - 1)*(x^4 - x^2)^(1/3))/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1)) + 1/24*4^(2/3)
*(-1)^(1/3)*log(-(3*4^(2/3)*(-1)^(1/3)*(x^4 - x^2)^(1/3)*(x^2 - 1) - 4^(1/3
)*(-1)^(2/3)*(x^4 + 2*x^2 + 1) - 12*(x^4 - x^2)^(2/3))/(x^4 + 2*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^2\right)^{\frac{1}{3}}}{\left(x^2+1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2)^(1/3)/x/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 - x^2)^(1/3)/((x^2 + 1)*x), x)
```

maple [C] time = 19.81, size = 1684, normalized size = 8.10

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^2)^(1/3)/x/(x^2+1),x)
```

```
[Out] -1/4*ln(-(5842000*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootO
f(_Z^3+2)^4*x^4+77152*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2
*RootOf(_Z^3+2)^3*x^4-24828500*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+
4*_Z^2)*RootOf(_Z^3+2)^4*x^2-327896*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^
3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^3*x^2-3286125*x^4*RootOf(_Z^3+2)^2-43398*x^4*
RootOf(_Z^3+2)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)+5842000*
RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^4+77152*
RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^3-3361
0788*(x^4-x^2)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*Ro
otOf(_Z^3+2)^2-3651250*x^2*RootOf(_Z^3+2)^2-48220*x^2*RootOf(RootOf(_Z^3+2)
^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)-16805394*RootOf(_Z^3+2)*(x^4-
x^2)^(1/3)*x^2+2679900*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*
(x^4-x^2)^(1/3)*x^2-3286125*RootOf(_Z^3+2)^2-43398*RootOf(RootOf(_Z^3+2)^2+
2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)+16805394*RootOf(_Z^3+2)*(x^4-x^2
)^(1/3)-2679900*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*(x^4-x^
2)^(1/3)+36290688*(x^4-x^2)^(2/3))/(x^2+1)^2)*RootOf(_Z^3+2)-1/2*ln(-(58420
00*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^4*x^4
+77152*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)
^3*x^4-24828500*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(
_Z^3+2)^4*x^2-327896*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*
RootOf(_Z^3+2)^3*x^2-3286125*x^4*RootOf(_Z^3+2)^2-43398*x^4*RootOf(_Z^3+2)*
RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)+5842000*RootOf(RootOf(_
Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^4+77152*RootOf(RootOf(_
Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^3-33610788*(x^4-x^2)^
(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2-
3651250*x^2*RootOf(_Z^3+2)^2-48220*x^2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(
_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)-16805394*RootOf(_Z^3+2)*(x^4-x^2)^(1/3)*x^2+
```

```
2679900*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*(x^4-x^2)^(1/3)
*x^2-3286125*RootOf(_Z^3+2)^2-43398*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^
3+2)+4*_Z^2)*RootOf(_Z^3+2)+16805394*RootOf(_Z^3+2)*(x^4-x^2)^(1/3)-2679900
*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*(x^4-x^2)^(1/3)+362906
88*(x^4-x^2)^(2/3))/(x^2+1)^2)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+
4*_Z^2)+1/4*RootOf(_Z^3+2)*ln((38576*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z
^3+2)+4*_Z^2)*RootOf(_Z^3+2)^4*x^4+11684000*RootOf(RootOf(_Z^3+2)^2+2*_Z*Ro
otOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^3*x^4-163948*RootOf(RootOf(_Z^3+2)^2+
2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^4*x^2-49657000*RootOf(RootOf(_Z^
3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^3*x^2-16877*x^4*RootOf(
_Z^3+2)^2-5111750*x^4*RootOf(_Z^3+2)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z
^3+2)+4*_Z^2)+38576*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*Roo
tOf(_Z^3+2)^4+11684000*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^
2*RootOf(_Z^3+2)^3-33610788*(x^4-x^2)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*Ro
otOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2+188058*x^2*RootOf(_Z^3+2)^2+56959500*
x^2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)-1680
5394*RootOf(_Z^3+2)*(x^4-x^2)^(1/3)*x^2-36290688*RootOf(RootOf(_Z^3+2)^2+2*
_Z*RootOf(_Z^3+2)+4*_Z^2)*(x^4-x^2)^(1/3)*x^2-16877*RootOf(_Z^3+2)^2-511175
0*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)+168053
94*RootOf(_Z^3+2)*(x^4-x^2)^(1/3)+36290688*RootOf(RootOf(_Z^3+2)^2+2*_Z*Roo
tOf(_Z^3+2)+4*_Z^2)*(x^4-x^2)^(1/3)-2679900*(x^4-x^2)^(2/3))/(x^2+1)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4 - x^2\right)^{\frac{1}{3}}}{\left(x^2 + 1\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^2)^(1/3)/x/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^2)^(1/3)/((x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^4 - x^2\right)^{1/3}}{x\,\left(x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - x^2)^(1/3)/(x*(x^2 + 1)),x)
```

```
[Out] int((x^4 - x^2)^(1/3)/(x*(x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x^2 (x-1)(x+1)}}{x\left(x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**2)**(1/3)/x/(x**2+1),x)
```

```
[Out] Integral((x**2*(x - 1)*(x + 1))**(1/3)/(x*(x**2 + 1)), x)
```

3.2017 $\int \frac{b+ax^6}{x^3\left(-b+ax^3\right)\sqrt[4]{bx+ax^4}}\,dx$

Optimal. Leaf size=208

$$\frac{4\left(ax^4+bx\right)^{3/4}}{9bx^3}+\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}-\frac{2^{3/4}(a+b)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}\,b}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}-\frac{2^{3/4}(a+b}{}$$

Rubi [A] time = 1.15, antiderivative size = 343, normalized size of antiderivative = 1.65, number of steps used = 17, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {2056, 6725, 264, 329, 275, 240, 212, 206, 203, 466, 465, 494, 453}

$$\frac{2\sqrt[4]{x}\sqrt[4]{ax^3+b}\tan^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{a}\sqrt[4]{ax^4+bx}}-\frac{2^{3/4}\sqrt[4]{x}(a+b)\sqrt[4]{ax^3+b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{a}\,b\sqrt[4]{ax^4+bx}}+\frac{2\sqrt[4]{x}\sqrt[4]{ax^3+b}\tanh^{-1}\left(\frac{\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{a}\sqrt[4]{ax^4+bx}}-\frac{2^{3/4}\sqrt[4]{x}(a+b)\sqrt[4]{ax^3+b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}x^{3/4}}{\sqrt[4]{ax^3+b}}\right)}{3\sqrt[4]{a}\,b\sqrt[4]{ax^4+bx}}+\frac{4(a+b)\left(ax^3+b\right)}{9abx^2\sqrt[4]{ax^4+bx}}-\frac{4\left(ax^3+b\right)}{9ax^2\sqrt[4]{ax^4+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^6)/(x^3*(-b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (-4*(b + a*x^3))/(9*a*x^2*(b*x + a*x^4)^(1/4)) + (4*(a + b)*(b + a*x^3))/(9
*a*b*x^2*(b*x + a*x^4)^(1/4)) + (2*x^(1/4)*(b + a*x^3)^(1/4)*ArcTan[(a^(1/4
)*x^(3/4))/(b + a*x^3)^(1/4)])/(3*a^(1/4)*(b*x + a*x^4)^(1/4)) - (2^(3/4)*(
a + b)*x^(1/4)*(b + a*x^3)^(1/4)*ArcTan[(2^(1/4)*a^(1/4)*x^(3/4))/(b + a*x^
3)^(1/4)])/(3*a^(1/4)*b*(b*x + a*x^4)^(1/4)) + (2*x^(1/4)*(b + a*x^3)^(1/4)
*ArcTanh[(a^(1/4)*x^(3/4))/(b + a*x^3)^(1/4)])/(3*a^(1/4)*(b*x + a*x^4)^(1/
4)) - (2^(3/4)*(a + b)*x^(1/4)*(b + a*x^3)^(1/4)*ArcTanh[(2^(1/4)*a^(1/4)*x
^(3/4))/(b + a*x^3)^(1/4)])/(3*a^(1/4)*b*(b*x + a*x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

[n, 0]

Rubi steps

$$\int \frac{b+ax^6}{x^3\left(-b+ax^3\right)\sqrt[4]{bx+ax^4}}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\int \frac{b+ax^6}{x^{13/4}\left(-b+ax^3\right)\sqrt[4]{b+ax^3}}\,dx}{\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\int \left(\frac{b}{ax^{13/4}\sqrt[4]{b+ax^3}}+\frac{1}{\sqrt[4]{x}\sqrt[4]{b+ax^3}}+\frac{ab+b^2}{ax^{13/4}\left(-b+ax^3\right)\sqrt[4]{b+ax^3}}\right)dx}{\sqrt[4]{bx+ax^4}}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\int \frac{1}{\sqrt[4]{x}\sqrt[4]{b+ax^3}}\,dx}{\sqrt[4]{bx+ax^4}} + \frac{\left(b\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\int \frac{1}{x^{13/4}\sqrt[4]{b+ax^3}}\,dx}{a\sqrt[4]{bx+ax^4}} + \frac{\left(b(a+b\right.}{}$$

$$= -\frac{4\left(b+ax^3\right)}{9ax^2\sqrt[4]{bx+ax^4}} + \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{b+ax^{12}}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^4}} + \frac{\left(4b(a+b\right.}{}$$

$$= -\frac{4\left(b+ax^3\right)}{9ax^2\sqrt[4]{bx+ax^4}} + \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx, x, x^{3/4}\right)}{3\sqrt[4]{bx+ax^4}} + \frac{\left(4b(a+b\right.}{}$$

$$= -\frac{4\left(b+ax^3\right)}{9ax^2\sqrt[4]{bx+ax^4}} + \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx, x, \frac{x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{bx+ax^4}} + \frac{\left(4(a+b\right.}{}$$

$$= -\frac{4\left(b+ax^3\right)}{9ax^2\sqrt[4]{bx+ax^4}} + \frac{4(a+b)\left(b+ax^3\right)}{9abx^2\sqrt[4]{bx+ax^4}} + \frac{\left(2\sqrt[4]{x}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx\right.}{3\sqrt[4]{bx+ax^4}}$$

$$= -\frac{4\left(b+ax^3\right)}{9ax^2\sqrt[4]{bx+ax^4}} + \frac{4(a+b)\left(b+ax^3\right)}{9abx^2\sqrt[4]{bx+ax^4}} + \frac{2\sqrt[4]{x}\sqrt[4]{b+ax^3}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{a}\sqrt[4]{bx+ax^4}} + \frac{2\sqrt[4]{}}{}$$

$$= -\frac{4\left(b+ax^3\right)}{9ax^2\sqrt[4]{bx+ax^4}} + \frac{4(a+b)\left(b+ax^3\right)}{9abx^2\sqrt[4]{bx+ax^4}} + \frac{2\sqrt[4]{x}\sqrt[4]{b+ax^3}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,x^{3/4}}{\sqrt[4]{b+ax^3}}\right)}{3\sqrt[4]{a}\sqrt[4]{bx+ax^4}} - \frac{2^{3/4}}{}$$

Mathematica [C] time = 3.24, size = 188, normalized size = 0.90

$$\frac{4\left(\frac{4\Gamma\left(\frac{5}{4}\right)(a+b)\left(ax^3+b\right)\left(5\left(-4a^2x^6+3abx^3+b^2\right){}_2F_1\left(1,1;\frac{5}{4};-\frac{2ax^3}{b-ax^3}\right)-32ax^3\left(ax^3+b\right){}_2F_1\left(2,2;\frac{9}{4};-\frac{2ax^3}{b-ax^3}\right)\right)}{ab\Gamma\left(\frac{1}{4}\right)\left(b-ax^3\right)^2}+15x^3\sqrt[4]{\frac{ax^3}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{ax^3}{b}\right)-\frac{5\left(ax^3+b\right)}{a}\right)}{45x^2\sqrt[4]{x\left(ax^3+b\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b + a*x^6)/(x^3*(-b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*((-5*(b + a*x^3))/a + 15*x^3*(1 + (a*x^3)/b)^(1/4)*Hypergeometric2F1[1/4
, 1/4, 5/4, -((a*x^3)/b)] + (4*(a + b)*(b + a*x^3)*Gamma[5/4]*(5*(b^2 + 3*a
*b*x^3 - 4*a^2*x^6)*Hypergeometric2F1[1, 1, 5/4, (-2*a*x^3)/(b - a*x^3)] -
32*a*x^3*(b + a*x^3)*Hypergeometric2F1[2, 2, 9/4, (-2*a*x^3)/(b - a*x^3)]))
/(a*b*(b - a*x^3)^2*Gamma[1/4])))/(45*x^2*(x*(b + a*x^3))^(1/4))
```

IntegrateAlgebraic [A] time = 1.02, size = 208, normalized size = 1.00

$$\frac{4\left(ax^4+bx\right)^{3/4}}{9bx^3}+\frac{2\tan^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}-\frac{2^{3/4}(a+b)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}\,b}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}}-\frac{2^{3/4}(a+b)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\left(ax^4+bx\right)^{3/4}}{ax^3+b}\right)}{3\sqrt[4]{a}\,b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^6)/(x^3*(-b + a*x^3)*(b*x + a*x^4)^(1/4)),x]
```

```
[Out] (4*(b*x + a*x^4)^(3/4))/(9*b*x^3) + (2*ArcTan[(a^(1/4)*(b*x + a*x^4)^(3/4))
/(b + a*x^3)])/(3*a^(1/4)) - (2^(3/4)*(a + b)*ArcTan[(2^(1/4)*a^(1/4)*(b*x
+ a*x^4)^(3/4))/(b + a*x^3)])/(3*a^(1/4)*b) + (2*ArcTanh[(a^(1/4)*(b*x + a*
x^4)^(3/4))/(b + a*x^3)])/(3*a^(1/4)) - (2^(3/4)*(a + b)*ArcTanh[(2^(1/4)*a
^(1/4)*(b*x + a*x^4)^(3/4))/(b + a*x^3)])/(3*a^(1/4)*b)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^3/(a*x^3-b)/(a*x^4+b*x)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.21, size = 437, normalized size = 2.10

$$\frac{\sqrt{2}\left(-a\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}+2\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)}{3a}+\frac{\sqrt{2}\left(-a\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}-2\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)}{3a}-\frac{\sqrt{2}\left(-a\right)^{\frac{3}{4}}\log\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^{3}}}\right)}{6a}+\frac{\sqrt{2}\left(-a\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\left(-a\right)^{\frac{1}{4}}\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a+\frac{b}{x^{3}}}\right)}{6a}-\frac{2^{\frac{1}{4}}(a+b)\log\left(2^{\frac{3}{4}}\left(-a\right)^{\frac{1}{4}}\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a+\frac{b}{x^{3}}}\right)}{6\left(-a\right)^{\frac{1}{4}}b}+\frac{2^{\frac{1}{4}}(a+b)\log\left(-2^{\frac{3}{4}}\left(-a\right)^{\frac{1}{4}}\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{-a}+\sqrt{a+\frac{b}{x^{3}}}\right)}{6\left(-a\right)^{\frac{1}{4}}b}-\frac{\sqrt{2}\left(2^{\frac{3}{4}}\left(-a\right)^{\frac{3}{4}}a+2^{\frac{3}{4}}\left(-a\right)^{\frac{3}{4}}b\right)\arctan\left(\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}\left(-a\right)^{\frac{1}{4}}+2\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)}{6ab}-\frac{\sqrt{2}\left(2^{\frac{3}{4}}\left(-a\right)^{\frac{3}{4}}a+2^{\frac{3}{4}}\left(-a\right)^{\frac{3}{4}}b\right)\arctan\left(-\frac{2^{\frac{1}{4}}\left(2^{\frac{3}{4}}\left(-a\right)^{\frac{1}{4}}-2\left(a+\frac{b}{x^{3}}\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)}{6ab}+\frac{4\left(a+\frac{b}{x^{3}}\right)^{\frac{3}{4}}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^3/(a*x^3-b)/(a*x^4+b*x)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a + b/x^
3)^(1/4))/(-a)^(1/4))/a + 1/3*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(
2)*(-a)^(1/4) - 2*(a + b/x^3)^(1/4))/(-a)^(1/4))/a - 1/6*sqrt(2)*(-a)^(3/4)
*log(sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a) + sqrt(a + b/x^3))/a +
 1/6*sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(-a
) + sqrt(a + b/x^3))/a - 1/6*2^(1/4)*(a + b)*log(2^(3/4)*(-a)^(1/4)*(a + b/
x^3)^(1/4) + sqrt(2)*sqrt(-a) + sqrt(a + b/x^3))/((-a)^(1/4)*b) + 1/6*2^(1/
4)*(a + b)*log(-2^(3/4)*(-a)^(1/4)*(a + b/x^3)^(1/4) + sqrt(2)*sqrt(-a) + s
qrt(a + b/x^3))/((-a)^(1/4)*b) - 1/6*sqrt(2)*(2^(3/4)*(-a)^(3/4)*a + 2^(3/4
)*(-a)^(3/4)*b)*arctan(1/2*2^(1/4)*(2^(3/4)*(-a)^(1/4) + 2*(a + b/x^3)^(1/4
))/(-a)^(1/4))/(a*b) - 1/6*sqrt(2)*(2^(3/4)*(-a)^(3/4)*a + 2^(3/4)*(-a)^(3/
4)*b)*arctan(-1/2*2^(1/4)*(2^(3/4)*(-a)^(1/4) - 2*(a + b/x^3)^(1/4))/(-a)^(
1/4))/(a*b) + 4/9*(a + b/x^3)^(3/4)/b
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{a\,x^{6}+b}{x^{3}\left(a\,x^{3}-b\right)\left(a\,x^{4}+bx\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6+b)/x^3/(a*x^3-b)/(a*x^4+b*x)^(1/4),x)
```

```
[Out] int((a*x^6+b)/x^3/(a*x^3-b)/(a*x^4+b*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{6}+b}{\left(ax^{4}+bx\right)^{\frac{1}{4}}\left(ax^{3}-b\right)x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6+b)/x^3/(a*x^3-b)/(a*x^4+b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x^6 + b)/((a*x^4 + b*x)^(1/4)*(a*x^3 - b)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{a\,x^6+b}{x^3\,{\left(a\,x^4+b\,x\right)}^{1/4}\,\left(b-a\,x^3\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x^6)/(x^3*(b*x + a*x^4)^(1/4)*(b - a*x^3)),x)
[Out] -int((b + a*x^6)/(x^3*(b*x + a*x^4)^(1/4)*(b - a*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^{6} + b}{x^{3} \sqrt[4]{x \left(ax^{3} + b\right)} \left(ax^{3} - b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6+b)/x**3/(a*x**3-b)/(a*x**4+b*x)**(1/4),x)
[Out] Integral((a*x**6 + b)/(x**3*(x*(a*x**3 + b))**(1/4)*(a*x**3 - b)), x)
```

3.2018 $\int \frac{3+x}{\sqrt[3]{-1+x^2}\left(5-x+2x^2\right)}\,dx$

Optimal. Leaf size=209

$$\frac{\log \left(3 \sqrt[3]{x^2-1}+\sqrt[3]{2}\, 3^{2/3} x-\sqrt[3]{2}\, 3^{2/3}\right)}{\sqrt[3]{2}\, 3^{2/3}}-\frac{\log \left(2^{2/3} \sqrt[3]{3}\, x^2+3 \left(x^2-1\right)^{2/3}+\left(\sqrt[3]{2}\, 3^{2/3}-\sqrt[3]{2}\, 3^{2/3} x\right) \sqrt[3]{x^2-1}-2\, 2^{2/3} \sqrt[3]{\cdots}\right.}{2 \sqrt[3]{2}\, 3^{2/3}}$$

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3+x}{\sqrt[3]{-1+x^2}\left(5-x+2x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3 + x)/((-1 + x^2)^(1/3)*(5 - x + 2*x^2)),x]
```

```
[Out] Defer[Int][(3 + x)/((-1 + x^2)^(1/3)*(5 - x + 2*x^2)), x]
```

Rubi steps

$$\int \frac{3+x}{\sqrt[3]{-1+x^2}\left(5-x+2x^2\right)}\,dx=\int \frac{3+x}{\sqrt[3]{-1+x^2}\left(5-x+2x^2\right)}\,dx$$

Mathematica [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{3+x}{\sqrt[3]{-1+x^2}\left(5-x+2x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3 + x)/((-1 + x^2)^(1/3)*(5 - x + 2*x^2)),x]
```

```
[Out] Integrate[(3 + x)/((-1 + x^2)^(1/3)*(5 - x + 2*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.30, size = 209, normalized size = 1.00

$$\frac{\log \left(3 \sqrt[3]{x^2-1}+\sqrt[3]{2}\, 3^{2/3} x-\sqrt[3]{2}\, 3^{2/3}\right)}{\sqrt[3]{2}\, 3^{2/3}}-\frac{\log \left(2^{2/3} \sqrt[3]{3}\, x^2+3 \left(x^2-1\right)^{2/3}+\left(\sqrt[3]{2}\, 3^{2/3}-\sqrt[3]{2}\, 3^{2/3} x\right) \sqrt[3]{x^2-1}-2\, 2^{2/3} \sqrt[3]{3}\, x+2^{2/3} \sqrt[3]{3}\right)}{2 \sqrt[3]{2}\, 3^{2/3}}+\frac{\tan ^{-1}\left(\frac{3^{5/6} \sqrt[3]{x^2-1}}{\sqrt[3]{3} \sqrt[3]{x^2-1}-2 \sqrt[3]{2}\, x+2 \sqrt[3]{2}}\right)}{\sqrt[3]{2} \sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + x)/((-1 + x^2)^(1/3)*(5 - x + 2*x^2)),x]
```

```
[Out] ArcTan[(3^(5/6)*(-1 + x^2)^(1/3))/(2*2^(1/3) - 2*2^(1/3)*x + 3^(1/3)*(-1 + 
x^2)^(1/3))]/(2^(1/3)*3^(1/6)) + Log[-(2^(1/3)*3^(2/3)) + 2^(1/3)*3^(2/3)*x
 + 3*(-1 + x^2)^(1/3)]/(2^(1/3)*3^(2/3)) - Log[2^(2/3)*3^(1/3) - 2*2^(2/3)*
3^(1/3)*x + 2^(2/3)*3^(1/3)*x^2 + (2^(1/3)*3^(2/3) - 2^(1/3)*3^(2/3)*x)*(-1
 + x^2)^(1/3) + 3*(-1 + x^2)^(2/3)]/(2*2^(1/3)*3^(2/3))
```

fricas [B] time = 9.62, size = 313, normalized size = 1.50

$$-\frac{1}{18} \cdot 18^{\frac{1}{6}} \sqrt{6} \arctan\left(\frac{18^{\frac{1}{6}} \left(6 \cdot 18^{\frac{2}{3}} \sqrt{6} \left(8 x^4-26 x^3+33 x^2-56 x+5\right) \left(x^2-1\right)^{\frac{2}{3}}+18^{\frac{1}{3}} \sqrt{6} \left(8 x^6+96 x^5-582 x^4+155 x^3+1029 x^2-399 x-91\right)+36 \sqrt{6} \left(4 x^5-62 x^4+133 x^3-31 x^2-73 x+29\right) \left(x^2-1\right)^{\frac{1}{3}}\right)}{18 \left(8 x^6-336 x^5+1038 x^4-709 x^3-483 x^2+897 x-199\right)}\right)-\frac{1}{108} \cdot 18^{\frac{2}{3}} \log\left(\frac{3 \cdot 18^{\frac{2}{3}} \left(4 x^2-11 x+1\right) \left(x^2-1\right)^{\frac{2}{3}}+18^{\frac{1}{3}} \left(4 x^4-58 x^3+75 x^2+44 x-29\right)-36 \left(x^3-6 x^2+3 x+2\right) \left(x^2-1\right)^{\frac{1}{3}}}{4 x^4-4 x^3+21 x^2-10 x+25}\right)+\frac{1}{54} \cdot 18^{\frac{2}{3}} \log\left(\frac{18^{\frac{2}{3}} \left(2 x^2-x+5\right)+18 \cdot 18^{\frac{1}{3}} \left(x^2-1\right)^{\frac{1}{3}} (x-1)+54 \left(x^2-1\right)^{\frac{2}{3}}}{2 x^2-x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2-1)^(1/3)/(2*x^2-x+5),x, algorithm="fricas")
```

```
[Out] -1/18*18^(1/6)*sqrt(6)*arctan(1/18*18^(1/6)*(6*18^(2/3)*sqrt(6)*(8*x^4 - 26
*x^3 + 33*x^2 - 56*x + 5)*(x^2 - 1)^(2/3) + 18^(1/3)*sqrt(6)*(8*x^6 + 96*x^
5 - 582*x^4 + 155*x^3 + 1029*x^2 - 399*x - 91) + 36*sqrt(6)*(4*x^5 - 62*x^4
 + 133*x^3 - 31*x^2 - 73*x + 29)*(x^2 - 1)^(1/3))/(8*x^6 - 336*x^5 + 1038*x
^4 - 709*x^3 - 483*x^2 + 897*x - 199)) - 1/108*18^(2/3)*log((3*18^(2/3)*(4*
x^2 - 11*x + 1)*(x^2 - 1)^(2/3) + 18^(1/3)*(4*x^4 - 58*x^3 + 75*x^2 + 44*x
- 29) - 36*(x^3 - 6*x^2 + 3*x + 2)*(x^2 - 1)^(1/3))/(4*x^4 - 4*x^3 + 21*x^2
 - 10*x + 25)) + 1/54*18^(2/3)*log((18^(2/3)*(2*x^2 - x + 5) + 18*18^(1/3)*
(x^2 - 1)^(1/3)*(x - 1) + 54*(x^2 - 1)^(2/3))/(2*x^2 - x + 5))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+3}{\left(2x^2-x+5\right)\left(x^2-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2-1)^(1/3)/(2*x^2-x+5),x, algorithm="giac")
```

```
[Out] integrate((x + 3)/((2*x^2 - x + 5)*(x^2 - 1)^(1/3)), x)
```

maple [C] time = 9.64, size = 1374, normalized size = 6.57

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+x)/(x^2-1)^(1/3)/(2*x^2-x+5),x)
```

```
[Out] RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*ln((303*RootOf(RootO
f(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)^3*x^2+1800*RootO
f(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)^2*RootOf(_Z^3-12)^2*x^2+1
215*(x^2-1)^(2/3)*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*Ro
otOf(_Z^3-12)^2-909*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*
RootOf(_Z^3-12)^3*x-5400*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_
Z^2)^2*RootOf(_Z^3-12)^2*x+96*(x^2-1)^(1/3)*RootOf(_Z^3-12)^2*x-2430*(x^2-1
)^(1/3)*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-
12)*x-96*(x^2-1)^(1/3)*RootOf(_Z^3-12)^2+2430*(x^2-1)^(1/3)*RootOf(RootOf(_
Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)+202*RootOf(_Z^3-12)
*x^2+1200*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x^2-576*(x
^2-1)^(2/3)-1313*RootOf(_Z^3-12)*x-7800*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootO
f(_Z^3-12)+36*_Z^2)*x-707*RootOf(_Z^3-12)-4200*RootOf(RootOf(_Z^3-12)^2+6*_
Z*RootOf(_Z^3-12)+36*_Z^2))/(2*x^2-x+5))-1/6*ln(-(3*RootOf(RootOf(_Z^3-12)^
2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)^3*x^2-1800*RootOf(RootOf(_Z
^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)^2*RootOf(_Z^3-12)^2*x^2+1215*(x^2-1)
^(2/3)*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-1
2)^2-9*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-1
2)^3*x+5400*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)^2*RootOf
(_Z^3-12)^2*x-501*(x^2-1)^(1/3)*RootOf(_Z^3-12)^2*x-2430*(x^2-1)^(1/3)*Root
Of(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)*x+501*(x
^2-1)^(1/3)*RootOf(_Z^3-12)^2+2430*(x^2-1)^(1/3)*RootOf(RootOf(_Z^3-12)^2+6
*_Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)+4*RootOf(_Z^3-12)*x^2-2400*Roo
tOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x^2+3006*(x^2-1)^(2/3)-
5*RootOf(_Z^3-12)*x+3000*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_
Z^2)*x+7*RootOf(_Z^3-12)-4200*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)
+36*_Z^2))/(2*x^2-x+5))*RootOf(_Z^3-12)-ln(-(3*RootOf(RootOf(_Z^3-12)^2+6*_
Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)^3*x^2-1800*RootOf(RootOf(_Z^3-12
)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)^2*RootOf(_Z^3-12)^2*x^2+1215*(x^2-1)^(2/3
)*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)^2-
```

```
9*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)^3*
x+5400*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)^2*RootOf(_Z^3
-12)^2*x-501*(x^2-1)^(1/3)*RootOf(_Z^3-12)^2*x-2430*(x^2-1)^(1/3)*RootOf(Ro
otOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)*x+501*(x^2-1)
^(1/3)*RootOf(_Z^3-12)^2+2430*(x^2-1)^(1/3)*RootOf(RootOf(_Z^3-12)^2+6*_Z*R
ootOf(_Z^3-12)+36*_Z^2)*RootOf(_Z^3-12)+4*RootOf(_Z^3-12)*x^2-2400*RootOf(R
ootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*x^2+3006*(x^2-1)^(2/3)-5*Roo
tOf(_Z^3-12)*x+3000*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)*
x+7*RootOf(_Z^3-12)-4200*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_
Z^2))/(2*x^2-x+5))*RootOf(RootOf(_Z^3-12)^2+6*_Z*RootOf(_Z^3-12)+36*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+3}{\left(2\,x^2-x+5\right)\left(x^2-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2-1)^(1/3)/(2*x^2-x+5),x, algorithm="maxima")
[Out] integrate((x + 3)/((2*x^2 - x + 5)*(x^2 - 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x+3}{\left(x^2-1\right)^{1/3}\,\left(2\,x^2-x+5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3)/((x^2 - 1)^(1/3)*(2*x^2 - x + 5)),x)
[Out] int((x + 3)/((x^2 - 1)^(1/3)*(2*x^2 - x + 5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+3}{\sqrt[3]{(x-1)\,(x+1)}\,\left(2x^2-x+5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x**2-1)**(1/3)/(2*x**2-x+5),x)
[Out] Integral((x + 3)/(((x - 1)*(x + 1))**(1/3)*(2*x**2 - x + 5)), x)
```

3.2019 $\int \frac{(-2+(1+k)x)\left(1-(1+k)x+(a+k)x^2\right)}{x^2\sqrt[3]{(1-x)x(1-kx)}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$

Optimal. Leaf size=209

$$\frac{(-a-b)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2\sqrt[3]{b}}+\frac{(a+b)\log\left(\sqrt[3]{kx^3+(-k-1)x^2\right.}}{\sqrt[3]{b}}$$

Rubi [F] time = 23.85, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+(1+k)x)\left(1-(1+k)x+(a+k)x^2\right)}{x^2\sqrt[3]{(1-x)x(1-kx)}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + (1 + k)*x)*(1 - (1 + k)*x + (a + k)*x^2))/(x^2*((1 - x)*x*(1 - k
*x))^(1/3)*(1 - (1 + k)*x + (-b + k)*x^2)),x]
```

```
[Out] (3*(2*k^2 - a*(1 + 2*b + k^2) - b*(1 + 4*k + k^2))*(1 - x)*(1 - k*x))/(4*(b
 - k)^2*x*((1 - x)*x*(1 - k*x))^(1/3)) + (3*(1 + k)*(a + k)*(1 - x)*(((1 -
k)*x)/(1 - k*x))^(4/3)*(1 - k*x)*Hypergeometric2F1[2/3, 4/3, 5/3, (1 - x)/(
1 - k*x)])/(2*(1 - k)*(b - k)*x*((1 - x)*x*(1 - k*x))^(1/3)) + (3*(1 + k)*(
2*k^2 - a*(1 + 2*b + k^2) - b*(1 + 4*k + k^2))*(1 - x)*(((1 - k)*x)/(1 - k*
x))^(4/3)*(1 - k*x)*Hypergeometric2F1[2/3, 4/3, 5/3, (1 - x)/(1 - k*x)])/(4
*(1 - k)*(b - k)^2*x*((1 - x)*x*(1 - k*x))^(1/3)) + ((a + b)*(1 + k^3 + 3*b
*(1 + k) + (4*b^2 + (1 - k)^2*(1 + k + k^2) + b*(5 + 2*k + 5*k^2))/Sqrt[4*b
 + (-1 + k)^2])*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Int][1/((1 - x)
^(1/3)*x^(7/3)*(1 - k*x)^(1/3)*(-1 - k - Sqrt[1 + 4*b - 2*k + k^2] + 2*(-b
+ k)*x)), x])/((b - k)^2*((1 - x)*x*(1 - k*x))^(1/3)) + ((a + b)*(1 + k^3 +
 3*b*(1 + k) - (1 + 4*b^2 - k - k^3 + k^4 + b*(5 + 2*k + 5*k^2))/Sqrt[4*b +
 (-1 + k)^2])*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Int][1/((1 - x)^(
1/3)*x^(7/3)*(1 - k*x)^(1/3)*(-1 - k + Sqrt[1 + 4*b - 2*k + k^2] + 2*(-b +
k)*x)), x])/((b - k)^2*((1 - x)*x*(1 - k*x))^(1/3))
```

Rubi steps

$$\int \frac{(-2+(1+k)x)\left(1-(1+k)x+(a+k)x^2\right)}{x^2\sqrt[3]{(1-x)x(1-kx)}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx=\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int\frac{(-2+(1+k)x)\left(1-(1+k)x+(a+k)x^2\right)}{\sqrt[3]{1-x}\,x^{7/3}\sqrt[3]{1-kx}\left(1-(1+k)x+(-b+k)x^2\right)}}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$=\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int\left(-\frac{2k^2-a\left(1+2b+k^2\right)-b\left(1+4k+k^2\right)}{(b-k)^2\sqrt[3]{1-x}\,x^{7/3}\sqrt[3]{1-kx}}-\frac{}{(b-}\right.}{\sqrt[3]{(1-x}}$$

$$=-\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int\frac{(a+b)\left(1+2b+k^2\right)-(a+b)(1+k)\left(1+3b-k+\right.}{\sqrt[3]{1-x}\,x^{7/3}\sqrt[3]{1-kx}\left(1+(-1-k)x+(-b+k)\right.}}{(b-k)^2\sqrt[3]{(1-x)x(1-kx)}}$$

$$=\frac{3\left(2k^2-a\left(1+2b+k^2\right)-b\left(1+4k+k^2\right)\right)(1-x)(1-kx}{4(b-k)^2x\sqrt[3]{(1-x)x(1-kx)}}$$

$$=\frac{3\left(2k^2-a\left(1+2b+k^2\right)-b\left(1+4k+k^2\right)\right)(1-x)(1-kx}{4(b-k)^2x\sqrt[3]{(1-x)x(1-kx)}}$$

Mathematica [F] time = 12.81, size = 0, normalized size = 0.00

$$\int \frac{(-2+(1+k)x)\left(1-(1+k)x+(a+k)x^2\right)}{x^2\sqrt[3]{(1-x)x(1-kx)}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + (1 + k)*x)*(1 - (1 + k)*x + (a + k)*x^2))/(x^2*((1 - x)*x*
(1 - k*x))^(1/3)*(1 - (1 + k)*x + (-b + k)*x^2)),x]
```

```
[Out] Integrate[((-2 + (1 + k)*x)*(1 - (1 + k)*x + (a + k)*x^2))/(x^2*((1 - x)*x*
(1 - k*x))^(1/3)*(1 - (1 + k)*x + (-b + k)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.47, size = 209, normalized size = 1.00

$$\frac{(-a-b)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2\sqrt[3]{b}}+\frac{(a+b)\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}+\frac{\left(-\sqrt{3}a-\sqrt{3}b\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{\sqrt[3]{b}}+\frac{3\left(kx^3+(-k-1)x^2+x\right)^{2/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + (1 + k)*x)*(1 - (1 + k)*x + (a + k)*x^2))/(x^2*((
1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x + (-b + k)*x^2)),x]
```

```
[Out] (3*(x + (-1 - k)*x^2 + k*x^3)^(2/3))/(2*x^2) + ((-(Sqrt[3]*a) - Sqrt[3]*b)*
ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(x + (-1 - k)*x^2 + k*x^3)^(1/3))
])/b^(1/3) + ((a + b)*Log[-(b^(1/3)*x) + (x + (-1 - k)*x^2 + k*x^3)^(1/3)])
/b^(1/3) + ((-a - b)*Log[b^(2/3)*x^2 + b^(1/3)*x*(x + (-1 - k)*x^2 + k*x^3)
^(1/3) + (x + (-1 - k)*x^2 + k*x^3)^(2/3)])/(2*b^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x^2)/x^2/((1-x)*x*(-k*x+1))^(1/3)/(
1-(1+k)*x+(-b+k)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left((a+k)x^2-(k+1)x+1\right)((k+1)x-2)}{((kx-1)(x-1)x)^{\frac{1}{3}}\left((b-k)x^2+(k+1)x-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x^2)/x^2/((1-x)*x*(-k*x+1))^(1/3)/(
1-(1+k)*x+(-b+k)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-((a + k)*x^2 - (k + 1)*x + 1)*((k + 1)*x - 2)/(((k*x - 1)*(x - 1
)*x)^(1/3)*((b - k)*x^2 + (k + 1)*x - 1)*x^2), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(-2+(1+k)\,x)\left(1-(1+k)\,x+(a+k)\,x^2\right)}{x^2\left((1-x)\,x\,(-kx+1)\right)^{\frac{1}{3}}\left(1-(1+k)\,x+(-b+k)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x^2)/x^2/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k
)*x+(-b+k)*x^2),x)
```

```
[Out] int((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x^2)/x^2/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k
)*x+(-b+k)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left((a+k)x^2-(k+1)x+1\right)((k+1)x-2)}{((kx-1)(x-1)x)^{\frac{1}{3}}\left((b-k)x^2+(k+1)x-1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x^2)/x^2/((1-x)*x*(-k*x+1))^(1/3)/(
1-(1+k)*x+(-b+k)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate(((a + k)*x^2 - (k + 1)*x + 1)*((k + 1)*x - 2)/(((k*x - 1)*(x - 1
)*x)^(1/3)*((b - k)*x^2 + (k + 1)*x - 1)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(x\,(k+1)-2)\,\left((a+k)\,x^2+(-k-1)\,x+1\right)}{x^2\,(x\,(k\,x-1)\,(x-1))^{1/3}\,\left((b-k)\,x^2+(k+1)\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x*(k + 1) - 2)*(x^2*(a + k) - x*(k + 1) + 1))/(x^2*(x*(k*x - 1)*(x -
 1))^(1/3)*(x*(k + 1) + x^2*(b - k) - 1)),x)
```

```
[Out] int(-((x*(k + 1) - 2)*(x^2*(a + k) - x*(k + 1) + 1))/(x^2*(x*(k*x - 1)*(x -
 1))^(1/3)*(x*(k + 1) + x^2*(b - k) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x**2)/x**2/((1-x)*x*(-k*x+1))**(1/3
)/(1-(1+k)*x+(-b+k)*x**2),x)
```

```
[Out] Timed out
```

3.2020 $\int \frac{(-2+x^3)\sqrt[3]{x+x^3+x^4}}{(1+x^3)(1-x^2+x^3)}\,dx$

Optimal. Leaf size=209

$$-\log\left(\sqrt[3]{x^4+x^3+x}-x\right)+\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^4+x^3+x}-2x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+x^3+x}+x}\right)+\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\right.$$

Rubi [F] time = 6.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+x^3)\sqrt[3]{x+x^3+x^4}}{(1+x^3)(1-x^2+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + x^3)*(x + x^3 + x^4)^(1/3))/((1 + x^3)*(1 - x^2 + x^3)),x]
```

```
[Out] (3*(x + x^3 + x^4)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6 + x^9)^(1/3), x],
 x, x^(1/3)])/(x^(1/3)*(1 + x^2 + x^3)^(1/3)) - (3*(1 - I*Sqrt[3])*(x + x^3
 + x^4)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6 + x^9)^(1/3), x], x, x^(1/3)
])/(2*x^(1/3)*(1 + x^2 + x^3)^(1/3)) - (3*(1 + I*Sqrt[3])*(x + x^3 + x^4)^(
1/3)*Defer[Subst][Defer[Int][(1 + x^6 + x^9)^(1/3), x], x, x^(1/3)])/(2*x^(
1/3)*(1 + x^2 + x^3)^(1/3)) - ((x + x^3 + x^4)^(1/3)*Defer[Subst][Defer[Int
][(1 + x^6 + x^9)^(1/3)/(1 + x), x], x, x^(1/3)])/(x^(1/3)*(1 + x^2 + x^3)^
(1/3)) + ((1 + I*Sqrt[3])*(x + x^3 + x^4)^(1/3)*Defer[Subst][Defer[Int][(1
+ x^6 + x^9)^(1/3)/(-1 - I*Sqrt[3] + 2*x), x], x, x^(1/3)])/(x^(1/3)*(1 + x
^2 + x^3)^(1/3)) + ((1 - I*Sqrt[3])*(x + x^3 + x^4)^(1/3)*Defer[Subst][Defe
r[Int][(1 + x^6 + x^9)^(1/3)/(-1 + I*Sqrt[3] + 2*x), x], x, x^(1/3)])/(x^(1
/3)*(1 + x^2 + x^3)^(1/3)) + ((1 - I*Sqrt[3])^(1/3)*(1 + I*Sqrt[3])*(x + x^
3 + x^4)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6 + x^9)^(1/3)/((1 - I*Sqrt[3
])^(1/3) + (-2)^(1/3)*x), x], x, x^(1/3)])/(2*x^(1/3)*(1 + x^2 + x^3)^(1/3)
) + ((1 - I*Sqrt[3])*(1 + I*Sqrt[3])^(1/3)*(x + x^3 + x^4)^(1/3)*Defer[Subs
t][Defer[Int][(1 + x^6 + x^9)^(1/3)/((1 + I*Sqrt[3])^(1/3) + (-2)^(1/3)*x),
 x], x, x^(1/3)])/(2*x^(1/3)*(1 + x^2 + x^3)^(1/3)) + ((1 - I*Sqrt[3])^(1/3
)*(1 + I*Sqrt[3])*(x + x^3 + x^4)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6 +
x^9)^(1/3)/((1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x), x], x, x^(1/3)])/(2*x^(1/3)
*(1 + x^2 + x^3)^(1/3)) + ((1 - I*Sqrt[3])*(1 + I*Sqrt[3])^(1/3)*(x + x^3 +
 x^4)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6 + x^9)^(1/3)/((1 + I*Sqrt[3])^
(1/3) - 2^(1/3)*x), x], x, x^(1/3)])/(2*x^(1/3)*(1 + x^2 + x^3)^(1/3)) + ((
1 - I*Sqrt[3])^(1/3)*(1 + I*Sqrt[3])*(x + x^3 + x^4)^(1/3)*Defer[Subst][Def
er[Int][(1 + x^6 + x^9)^(1/3)/((1 - I*Sqrt[3])^(1/3) - (-1)^(2/3)*2^(1/3)*x
), x], x, x^(1/3)])/(2*x^(1/3)*(1 + x^2 + x^3)^(1/3)) + ((1 - I*Sqrt[3])*(1
 + I*Sqrt[3])^(1/3)*(x + x^3 + x^4)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6
+ x^9)^(1/3)/((1 + I*Sqrt[3])^(1/3) - (-1)^(2/3)*2^(1/3)*x), x], x, x^(1/3)
])/(2*x^(1/3)*(1 + x^2 + x^3)^(1/3)) - (6*(x + x^3 + x^4)^(1/3)*Defer[Subst
][Defer[Int][(x^3*(1 + x^6 + x^9)^(1/3))/(1 - x^6 + x^9), x], x, x^(1/3)])/
(x^(1/3)*(1 + x^2 + x^3)^(1/3)) + (9*(x + x^3 + x^4)^(1/3)*Defer[Subst][Def
er[Int][(x^6*(1 + x^6 + x^9)^(1/3))/(1 - x^6 + x^9), x], x, x^(1/3)])/(x^(1
/3)*(1 + x^2 + x^3)^(1/3))
```

Rubi steps

$$\int \frac{\left(-2+x^3\right)\sqrt[3]{x+x^3+x^4}}{\left(1+x^3\right)\left(1-x^2+x^3\right)}\,dx = \frac{\sqrt[3]{x+x^3+x^4}\int \frac{\sqrt[3]{x}\left(-2+x^3\right)\sqrt[3]{1+x^2+x^3}}{\left(1+x^3\right)\left(1-x^2+x^3\right)}\,dx}{\sqrt[3]{x}\sqrt[3]{1+x^2+x^3}}$$

$$= \frac{\sqrt[3]{x+x^3+x^4}\int \left(\frac{\sqrt[3]{x}\sqrt[3]{1+x^2+x^3}}{1+x}+\frac{(-1-x)\sqrt[3]{x}\sqrt[3]{1+x^2+x^3}}{1-x+x^2}+\frac{\sqrt[3]{x}(-2+3x)\sqrt[3]{1+x^2+x^3}}{1-x^2+x^3}\right)dx}{\sqrt[3]{x}\sqrt[3]{1+x^2+x^3}}$$

$$= \frac{\sqrt[3]{x+x^3+x^4}\int \frac{\sqrt[3]{x}\sqrt[3]{1+x^2+x^3}}{1+x}\,dx}{\sqrt[3]{x}\sqrt[3]{1+x^2+x^3}} + \frac{\sqrt[3]{x+x^3+x^4}\int \frac{(-1-x)\sqrt[3]{x}\sqrt[3]{1+x^2+x^3}}{1-x+x^2}\,dx}{\sqrt[3]{x}\sqrt[3]{1+x^2+x^3}} + \frac{\sqrt[3]{x+}}{}$$

$$= \frac{\sqrt[3]{x+x^3+x^4}\int \left(\frac{\left(-1+i\sqrt{3}\right)\sqrt[3]{x}\sqrt[3]{1+x^2+x^3}}{-1-i\sqrt{3}+2x}+\frac{\left(-1-i\sqrt{3}\right)\sqrt[3]{x}\sqrt[3]{1+x^2+x^3}}{-1+i\sqrt{3}+2x}\right)dx}{\sqrt[3]{x}\sqrt[3]{1+x^2+x^3}} + \frac{\left(3\sqrt[3]{x+x^3}\right.}{}$$

$$= \frac{\left(3\sqrt[3]{x+x^3+x^4}\right)\text{Subst}\left(\int \left(\sqrt[3]{1+x^6+x^9}-\frac{\sqrt[3]{1+x^6+x^9}}{1+x^3}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1+x^2+x^3}} + \frac{\left(3\sqrt[3]{x+x^3}\right.}{}$$

$$= \frac{\left(3\sqrt[3]{x+x^3+x^4}\right)\text{Subst}\left(\int \sqrt[3]{1+x^6+x^9}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1+x^2+x^3}} - \frac{\left(3\sqrt[3]{x+x^3+x^4}\right)\text{Subst}\left(\right.}{\sqrt[3]{x}\sqrt[3]{1+}}$$

$$= \frac{\left(3\sqrt[3]{x+x^3+x^4}\right)\text{Subst}\left(\int \sqrt[3]{1+x^6+x^9}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1+x^2+x^3}} - \frac{\left(3\sqrt[3]{x+x^3+x^4}\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{\sqrt[3]{x+x^3+x^4}\,\text{Subst}\left(\int \frac{\sqrt[3]{1+x^6+x^9}}{1+x}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1+x^2+x^3}} - \frac{\sqrt[3]{x+x^3+x^4}\,\text{Subst}\left(\int \frac{(2-x)\sqrt[3]{1}}{1-x}\right.}{\sqrt[3]{x}\sqrt[3]{1+x^2+x}}$$

$$= -\frac{\sqrt[3]{x+x^3+x^4}\,\text{Subst}\left(\int \frac{\sqrt[3]{1+x^6+x^9}}{1+x}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1+x^2+x^3}} - \frac{\sqrt[3]{x+x^3+x^4}\,\text{Subst}\left(\int \left(\frac{(-1-i}{-1}\right.\right.}{}$$

$$= -\frac{\sqrt[3]{x+x^3+x^4}\,\text{Subst}\left(\int \frac{\sqrt[3]{1+x^6+x^9}}{1+x}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1+x^2+x^3}} + \frac{\left(3\sqrt[3]{x+x^3+x^4}\right)\text{Subst}\left(\int \sqrt[3]{1}\right.}{\sqrt[3]{x}\sqrt[3]{1+x^2+}}$$

Mathematica [F] time = 2.60, size = 0, normalized size = 0.00

$$\int \frac{\left(-2+x^3\right)\sqrt[3]{x+x^3+x^4}}{\left(1+x^3\right)\left(1-x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + x^3)*(x + x^3 + x^4)^(1/3))/((1 + x^3)*(1 - x^2 + x^3)),x]
```

```
[Out] Integrate[((-2 + x^3)*(x + x^3 + x^4)^(1/3))/((1 + x^3)*(1 - x^2 + x^3)), x
]
```

IntegrateAlgebraic [A] time = 0.51, size = 209, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^4+x^3+x}-x\right)+\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^4+x^3+x}-2x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+x^3+x}+x}\right)+\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^3+x}+x}\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^4+x^3+x}\,x+\left(x^4+x^3+x\right)^{2/3}\right)-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^4+x^3+x}\,x+\sqrt[3]{2}\left(x^4+x^3+x\right)^{2/3}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^3)*(x + x^3 + x^4)^(1/3))/((1 + x^3)*(1 - x^2 +
 x^3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(x + x^3 + x^4)^(1/3))]) + 2^(1/3)*Sqrt
[3]*ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(x + x^3 + x^4)^(1/3))] - Log[-x + (x +
 x^3 + x^4)^(1/3)] + 2^(1/3)*Log[-2*x + 2^(2/3)*(x + x^3 + x^4)^(1/3)] + Lo
g[x^2 + x*(x + x^3 + x^4)^(1/3) + (x + x^3 + x^4)^(2/3)]/2 - Log[2*x^2 + 2^
(2/3)*x*(x + x^3 + x^4)^(1/3) + 2^(1/3)*(x + x^3 + x^4)^(2/3)]/2^(2/3)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^4+x^3+x)^(1/3)/(x^3+1)/(x^3-x^2+1),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3+x\right)^{\frac{1}{3}}\left(x^3-2\right)}{\left(x^3-x^2+1\right)\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^4+x^3+x)^(1/3)/(x^3+1)/(x^3-x^2+1),x, algorithm="giac"
)
```

```
[Out] integrate((x^4 + x^3 + x)^(1/3)*(x^3 - 2)/((x^3 - x^2 + 1)*(x^3 + 1)), x)
```

maple [F] time = 1.44, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-2\right)\left(x^4+x^3+x\right)^{\frac{1}{3}}}{\left(x^3+1\right)\left(x^3-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2)*(x^4+x^3+x)^(1/3)/(x^3+1)/(x^3-x^2+1),x)
```

```
[Out] int((x^3-2)*(x^4+x^3+x)^(1/3)/(x^3+1)/(x^3-x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3+x\right)^{\frac{1}{3}}\left(x^3-2\right)}{\left(x^3-x^2+1\right)\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^4+x^3+x)^(1/3)/(x^3+1)/(x^3-x^2+1),x, algorithm="maxim
a")
```

```
[Out] integrate((x^4 + x^3 + x)^(1/3)*(x^3 - 2)/((x^3 - x^2 + 1)*(x^3 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^3-2\right)\left(x^4+x^3+x\right)^{1/3}}{\left(x^3+1\right)\left(x^3-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 2)*(x + x^3 + x^4)^(1/3))/((x^3 + 1)*(x^3 - x^2 + 1)),x)
```

```
[Out] int(((x^3 - 2)*(x + x^3 + x^4)^(1/3))/((x^3 + 1)*(x^3 - x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x\left(x^3+x^2+1\right)}\left(x^3-2\right)}{(x+1)\left(x^2-x+1\right)\left(x^3-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2)*(x**4+x**3+x)**(1/3)/(x**3+1)/(x**3-x**2+1),x)
```

```
[Out] Integral((x*(x**3 + x**2 + 1))**(1/3)*(x**3 - 2)/((x + 1)*(x**2 - x + 1)*(x
**3 - x**2 + 1)), x)
```

3.2021 $\int \frac{(1+x^3)^{2/3}(-1+2x^6)}{x^6(-1+2x^3)}\,dx$

Optimal. Leaf size=209

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-x\right)-\frac{\log\left(3^{2/3}\sqrt[3]{x^3+1}-3x\right)}{\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}+\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{x^3+1}+\sqrt[3]{3}x}\right)+\frac{(x^3+1)^2}{}$$

Rubi [C] time = 0.39, antiderivative size = 99, normalized size of antiderivative = 0.47, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {6725, 264, 277, 239, 429}

$$2xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-x^3,2x^3\right)-\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{2\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{(x^3+1)^{5/3}}{5x^5}-\frac{(x^3+1)^{2/3}}{x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((1 + x^3)^(2/3)*(-1 + 2*x^6))/(x^6*(-1 + 2*x^3)),x]
```

```
[Out] -((1 + x^3)^(2/3)/x^2) - (1 + x^3)^(5/3)/(5*x^5) + 2*x*AppellF1[1/3, -2/3,
1, 4/3, -x^3, 2*x^3] + (2*ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]])/Sqrt
[3] - Log[-x + (1 + x^3)^(1/3)]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(1+x^3\right)^{2/3}\left(-1+2x^6\right)}{x^6\left(-1+2x^3\right)}\,dx = \int \left(\frac{\left(1+x^3\right)^{2/3}}{x^6}+\frac{2\left(1+x^3\right)^{2/3}}{x^3}-\frac{2\left(1+x^3\right)^{2/3}}{-1+2x^3}\right)dx$$
$$= 2\int \frac{\left(1+x^3\right)^{2/3}}{x^3}\,dx - 2\int \frac{\left(1+x^3\right)^{2/3}}{-1+2x^3}\,dx + \int \frac{\left(1+x^3\right)^{2/3}}{x^6}\,dx$$
$$= -\frac{\left(1+x^3\right)^{2/3}}{x^2} - \frac{\left(1+x^3\right)^{5/3}}{5x^5} + 2xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-x^3,2x^3\right) + 2\int \frac{1}{\sqrt[3]{1+x^3}}\,dx$$
$$= -\frac{\left(1+x^3\right)^{2/3}}{x^2} - \frac{\left(1+x^3\right)^{5/3}}{5x^5} + 2xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-x^3,2x^3\right) + \frac{2\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - 1$$

Mathematica [C] time = 0.41, size = 164, normalized size = 0.78

$$\frac{1}{45}\left(-20\,3^{2/3}\log\left(1-\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+1}}\right)+60\sqrt[6]{3}\tan^{-1}\left(\frac{2x}{\sqrt[6]{3}\sqrt[3]{x^3+1}}+\frac{1}{\sqrt{3}}\right)-\frac{9\left(x^3+1\right)^{2/3}}{x^5}-\frac{54\left(x^3+1\right)^{2/3}}{x^2}+10\,3^{2/3}\log\left(\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+1}}+\frac{3^{2/3}x^2}{\left(x^3+1\right)^{2/3}}+1\right)\right)-\frac{1}{2}x^4F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-x^3,2x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + x^3)^(2/3)*(-1 + 2*x^6))/(x^6*(-1 + 2*x^3)),x]
```

```
[Out] -1/2*(x^4*AppellF1[4/3, 1/3, 1, 7/3, -x^3, 2*x^3]) + ((-9*(1 + x^3)^(2/3))/
x^5 - (54*(1 + x^3)^(2/3))/x^2 + 60*3^(1/6)*ArcTan[1/Sqrt[3] + (2*x)/(3^(1/
6)*(1 + x^3)^(1/3))] - 20*3^(2/3)*Log[1 - (3^(1/3)*x)/(1 + x^3)^(1/3)] + 10
*3^(2/3)*Log[1 + (3^(2/3)*x^2)/(1 + x^3)^(2/3) + (3^(1/3)*x)/(1 + x^3)^(1/3
)])/45
```

IntegrateAlgebraic [A] time = 0.45, size = 209, normalized size = 1.00

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-x\right)-\frac{\log\left(3^{2/3}\sqrt[3]{x^3+1}-3x\right)}{\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}+\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{x^3+1}+\sqrt[3]{3}x}\right)+\frac{\left(x^3+1\right)^{2/3}\left(-6x^3-1\right)}{5x^5}+\frac{1}{6}\log\left(\sqrt[3]{x^3+1}x+\left(x^3+1\right)^{2/3}+x^2\right)+\frac{\log\left(3^{2/3}\sqrt[3]{x^3+1}x+\sqrt[3]{3}\left(x^3+1\right)^{2/3}+3x^2\right)}{2\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)^(2/3)*(-1 + 2*x^6))/(x^6*(-1 + 2*x^3)),x]
```

```
[Out] ((-1 - 6*x^3)*(1 + x^3)^(2/3))/(5*x^5) + ArcTan[(Sqrt[3]*x)/(x + 2*(1 + x^3
)^(1/3))]/Sqrt[3] + 3^(1/6)*ArcTan[(3^(5/6)*x)/(3^(1/3)*x + 2*(1 + x^3)^(1/
3))] - Log[-x + (1 + x^3)^(1/3)]/3 - Log[-3*x + 3^(2/3)*(1 + x^3)^(1/3)]/3^
(1/3) + Log[x^2 + x*(1 + x^3)^(1/3) + (1 + x^3)^(2/3)]/6 + Log[3*x^2 + 3^(2
/3)*x*(1 + x^3)^(1/3) + 3^(1/3)*(1 + x^3)^(2/3)]/(2*3^(1/3))
```

fricas [B] time = 29.76, size = 382, normalized size = 1.83

$$\frac{10\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^5\log\left(\frac{9\cdot 3^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^3+1\right)^{\frac{1}{3}}x^2+3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(2x^3-1\right)-9\left(x^3+1\right)^{\frac{2}{3}}x}{2x^3-1}\right)-5\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^5\log\left(-\frac{3\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(7x^4+x\right)\left(x^3+1\right)^{\frac{2}{3}}-3^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(31x^6+23x^3+1\right)-9\left(5x^5+2x^2\right)\left(x^3+1\right)^{\frac{1}{3}}}{4x^6-4x^3+1}\right)-30\cdot 3^{\frac{1}{6}}(-1)^{\frac{1}{3}}x^5\arctan\left(\frac{3^{\frac{1}{6}}\left(6\cdot 3^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(14x^7-5x^4-x\right)\left(x^3+1\right)^{\frac{2}{3}}+18(-1)^{\frac{1}{3}}\left(31x^8+23x^5+x^2\right)\left(x^3+1\right)^{\frac{1}{3}}-3^{\frac{1}{3}}\left(127x^9+201x^6+48x^3+1\right)\right)}{3\left(251x^9+231x^6+6x^3-1\right)}\right)+30\sqrt{3}x^5\arctan\left(-\frac{25382\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}x^2-13720\sqrt{3}\left(x^3+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(5831x^3+7200\right)}{58653x^3+8000}\right)-15x^5\log\left(3\left(x^3+1\right)^{\frac{1}{3}}x^2-3\left(x^3+1\right)^{\frac{2}{3}}x+1\right)-18\left(6x^3+1\right)\left(x^3+1\right)^{\frac{2}{3}}}{90x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6-1)/x^6/(2*x^3-1),x, algorithm="fricas")
```

```
[Out] 1/90*(10*3^(2/3)*(-1)^(1/3)*x^5*log((9*3^(1/3)*(-1)^(2/3)*(x^3 + 1)^(1/3)*x
^2 + 3^(2/3)*(-1)^(1/3)*(2*x^3 - 1) - 9*(x^3 + 1)^(2/3)*x)/(2*x^3 - 1)) - 5
*3^(2/3)*(-1)^(1/3)*x^5*log(-(3*3^(2/3)*(-1)^(1/3)*(7*x^4 + x)*(x^3 + 1)^(2
/3) - 3^(1/3)*(-1)^(2/3)*(31*x^6 + 23*x^3 + 1) - 9*(5*x^5 + 2*x^2)*(x^3 + 1
)^(1/3))/(4*x^6 - 4*x^3 + 1)) - 30*3^(1/6)*(-1)^(1/3)*x^5*arctan(1/3*3^(1/6
)*(6*3^(2/3)*(-1)^(2/3)*(14*x^7 - 5*x^4 - x)*(x^3 + 1)^(2/3) + 18*(-1)^(1/3
)*(31*x^8 + 23*x^5 + x^2)*(x^3 + 1)^(1/3) - 3^(1/3)*(127*x^9 + 201*x^6 + 48
```

```
*x^3 + 1))/(251*x^9 + 231*x^6 + 6*x^3 - 1)) + 30*sqrt(3)*x^5*arctan(-(25382
*sqrt(3)*(x^3 + 1)^(1/3)*x^2 - 13720*sqrt(3)*(x^3 + 1)^(2/3)*x + sqrt(3)*(5
831*x^3 + 7200))/(58653*x^3 + 8000)) - 15*x^5*log(3*(x^3 + 1)^(1/3)*x^2 - 3
*(x^3 + 1)^(2/3)*x + 1) - 18*(6*x^3 + 1)*(x^3 + 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{6}-1\right)\left(x^{3}+1\right)^{\frac{2}{3}}}{\left(2\,x^{3}-1\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6-1)/x^6/(2*x^3-1),x, algorithm="giac")
[Out] integrate((2*x^6 - 1)*(x^3 + 1)^(2/3)/((2*x^3 - 1)*x^6), x)
```

maple [F] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}+1\right)^{\frac{2}{3}}\left(2x^{6}-1\right)}{x^{6}\left(2x^{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)*(2*x^6-1)/x^6/(2*x^3-1),x)
[Out] int((x^3+1)^(2/3)*(2*x^6-1)/x^6/(2*x^3-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{6}-1\right)\left(x^{3}+1\right)^{\frac{2}{3}}}{\left(2\,x^{3}-1\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6-1)/x^6/(2*x^3-1),x, algorithm="maxima")
[Out] integrate((2*x^6 - 1)*(x^3 + 1)^(2/3)/((2*x^3 - 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(x^3+1\right)}^{2/3}\,\left(2\,x^6-1\right)}{x^6\,\left(2\,x^3-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)^(2/3)*(2*x^6 - 1))/(x^6*(2*x^3 - 1)),x)
[Out] int(((x^3 + 1)^(2/3)*(2*x^6 - 1))/(x^6*(2*x^3 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x + 1\right) \left(x^{2} - x + 1\right)\right)^{\frac{2}{3}} \left(2 x^{6} - 1\right)}{x^{6} \left(2 x^{3} - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)*(2*x**6-1)/x**6/(2*x**3-1),x)
[Out] Integral(((x + 1)*(x**2 - x + 1))**(2/3)*(2*x**6 - 1)/(x**6*(2*x**3 - 1)),
x)
```

3.2022 $\int \frac{-b+ax^4}{\left(b+ax^4\right)\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx$

Optimal. Leaf size=209

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{2ab-c}\,\sqrt[4]{a^2x^8+b^2+cx^4}}{x^2\sqrt{2ab-c}-\sqrt{a^2x^8+b^2+cx^4}}\right)}{2\sqrt{2}\sqrt[4]{2ab-c}}-\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{a^2x^8+b^2+cx^4}}{\sqrt{2}\sqrt[4]{2ab-c}}+\frac{x^2\sqrt[4]{2ab-c}}{\sqrt{2}}}{x\sqrt[4]{a^2x^8+b^2+cx^4}}\right)}{2\sqrt{2}\sqrt[4]{2ab-c}}$$

Rubi [F] time = 0.63, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-b+ax^4}{\left(b+ax^4\right)\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-b + a*x^4)/((b + a*x^4)*(b^2 + c*x^4 + a^2*x^8)^(1/4)),x]
```

```
[Out] (x*(1 + (2*a^2*x^4)/(c - Sqrt[-4*a^2*b^2 + c^2]))^(1/4)*(1 + (2*a^2*x^4)/(c
 + Sqrt[-4*a^2*b^2 + c^2]))^(1/4)*AppellF1[1/4, 1/4, 1/4, 5/4, (-2*a^2*x^4)
/(c - Sqrt[-4*a^2*b^2 + c^2]), (-2*a^2*x^4)/(c + Sqrt[-4*a^2*b^2 + c^2])])/
(b^2 + c*x^4 + a^2*x^8)^(1/4) - 2*b*Defer[Int][1/((b + a*x^4)*(b^2 + c*x^4
+ a^2*x^8)^(1/4)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{-b+ax^4}{\left(b+ax^4\right)\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx &= \int \left(\frac{1}{\sqrt[4]{b^2+cx^4+a^2x^8}}-\frac{2b}{\left(b+ax^4\right)\sqrt[4]{b^2+cx^4+a^2x^8}}\right)dx\\ &= -\left((2b)\int \frac{1}{\left(b+ax^4\right)\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx\right)+\int \frac{1}{\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx\\ &= -\left((2b)\int \frac{1}{\left(b+ax^4\right)\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx\right)+\frac{\left(\sqrt[4]{1+\frac{2a^2x^4}{c-\sqrt{-4a^2b^2+c^2}}}\,\sqrt[4]{1+\frac{\ldots}{c+\sqrt{\ldots}}}\ldots\right)}{\ldots}\\ &= \frac{x\sqrt[4]{1+\frac{2a^2x^4}{c-\sqrt{-4a^2b^2+c^2}}}\,\sqrt[4]{1+\frac{2a^2x^4}{c+\sqrt{-4a^2b^2+c^2}}}\,F_1\left(\frac{1}{4};\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{2a^2x^4}{c-\sqrt{-4a^2b^2+c^2}},-\frac{2a\ldots}{c+\sqrt{-4a\ldots}}\ldots\right)}{\sqrt[4]{b^2+cx^4+a^2x^8}}\end{aligned}$$

Mathematica [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{-b+ax^4}{\left(b+ax^4\right)\sqrt[4]{b^2+cx^4+a^2x^8}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-b + a*x^4)/((b + a*x^4)*(b^2 + c*x^4 + a^2*x^8)^(1/4)),x]
```

```
[Out] Integrate[(-b + a*x^4)/((b + a*x^4)*(b^2 + c*x^4 + a^2*x^8)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 3.62, size = 209, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\,\sqrt[4]{2ab-c}\,\sqrt[4]{a^2x^8+b^2+cx^4}}{x^2\sqrt{2ab-c}-\sqrt{a^2x^8+b^2+cx^4}}\right)}{2\sqrt{2}\,\sqrt[4]{2ab-c}}-\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{a^2x^8+b^2+cx^4}}{\sqrt{2}\,\sqrt[4]{2ab-c}}+\frac{x^2\sqrt[4]{2ab-c}}{\sqrt{2}}}{x\sqrt[4]{a^2x^8+b^2+cx^4}}\right)}{2\sqrt{2}\,\sqrt[4]{2ab-c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^4)/((b + a*x^4)*(b^2 + c*x^4 + a^2*x^8)^(1/4))
,x]
```

```
[Out] ArcTan[(Sqrt[2]*(2*a*b - c)^(1/4)*x*(b^2 + c*x^4 + a^2*x^8)^(1/4))/(Sqrt[2*
a*b - c]*x^2 - Sqrt[b^2 + c*x^4 + a^2*x^8])]/(2*Sqrt[2]*(2*a*b - c)^(1/4))
- ArcTanh[(((2*a*b - c)^(1/4)*x^2)/Sqrt[2] + Sqrt[b^2 + c*x^4 + a^2*x^8]/(S
qrt[2]*(2*a*b - c)^(1/4)))/(x*(b^2 + c*x^4 + a^2*x^8)^(1/4))]/(2*Sqrt[2]*(2
*a*b - c)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)/(a*x^4+b)/(a^2*x^8+c*x^4+b^2)^(1/4),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4-b}{\left(a^2x^8+cx^4+b^2\right)^{\frac{1}{4}}\left(ax^4+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)/(a*x^4+b)/(a^2*x^8+c*x^4+b^2)^(1/4),x, algorithm="giac"
)
```

```
[Out] integrate((a*x^4 - b)/((a^2*x^8 + c*x^4 + b^2)^(1/4)*(a*x^4 + b)), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{a\,x^4-b}{\left(a\,x^4+b\right)\left(a^2x^8+c\,x^4+b^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b)/(a*x^4+b)/(a^2*x^8+c*x^4+b^2)^(1/4),x)
```

```
[Out] int((a*x^4-b)/(a*x^4+b)/(a^2*x^8+c*x^4+b^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4-b}{\left(a^2x^8+cx^4+b^2\right)^{\frac{1}{4}}\left(ax^4+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)/(a*x^4+b)/(a^2*x^8+c*x^4+b^2)^(1/4),x, algorithm="maxim
a")
```

```
[Out] integrate((a*x^4 - b)/((a^2*x^8 + c*x^4 + b^2)^(1/4)*(a*x^4 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{b-a\,x^4}{\left(a\,x^4+b\right)\,{\left(a^2\,x^8+b^2+c\,x^4\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^4)/((b + a*x^4)*(c*x^4 + b^2 + a^2*x^8)^(1/4)),x)
```

```
[Out] int(-(b - a*x^4)/((b + a*x^4)*(c*x^4 + b^2 + a^2*x^8)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4-b}{\left(ax^4+b\right)\sqrt[4]{a^2x^8+b^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b)/(a*x**4+b)/(a**2*x**8+c*x**4+b**2)**(1/4),x)
```

```
[Out] Integral((a*x**4 - b)/((a*x**4 + b)*(a**2*x**8 + b**2 + c*x**4)**(1/4)), x)
```

3.2023 $\int \frac{(2+x)^2 \sqrt[3]{-19+66x-30x^2+9x^3}}{(-3+2x)^2(-5+6x-6x^2+x^3)}\,dx$

Optimal. Leaf size=210

$$\frac{\sqrt[3]{9x^3-30x^2+66x-19}}{2x-3}+\frac{1}{3}\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{9x^3-30x^2+66x-19}-4x+6\right)-\frac{\log\left(8x^2+\sqrt[3]{2}\left(9x^3-30x^2+66\right.\right.}{}$$

Rubi [F] time = 1.51, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(2+x)^2\sqrt[3]{-19+66x-30x^2+9x^3}}{(-3+2x)^2\left(-5+6x-6x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((2 + x)^2*(-19 + 66*x - 30*x^2 + 9*x^3)^(1/3))/((-3 + 2*x)^2*(-5 + 6*x
 - 6*x^2 + x^3)),x]
```

```
[Out] ((-19 + 66*x - 30*x^2 + 9*x^3)^(1/3)*Defer[Subst][Defer[Int][((7 + 9*x)^(1/
3)*(343 - 63*x + 81*x^2)^(1/3))/(-35/9 + x), x], x, -10/9 + x])/(63*3^(1/3)
*(-1 + 3*x)^(1/3)*(19 - 9*x + 3*x^2)^(1/3)) - (2*(-19 + 66*x - 30*x^2 + 9*x
^3)^(1/3)*Defer[Subst][Defer[Int][((7 + 9*x)^(1/3)*(343 - 63*x + 81*x^2)^(1
/3))/(-7/9 + 2*x)^2, x], x, -10/9 + x])/(3*3^(1/3)*(-1 + 3*x)^(1/3)*(19 - 9
*x + 3*x^2)^(1/3)) + (2*(-19 + 66*x - 30*x^2 + 9*x^3)^(1/3)*Defer[Subst][De
fer[Int][((7 + 9*x)^(1/3)*(343 - 63*x + 81*x^2)^(1/3))/(-7/9 + 2*x), x], x,
 -10/9 + x])/(21*3^(1/3)*(-1 + 3*x)^(1/3)*(19 - 9*x + 3*x^2)^(1/3)) - (2*(2
 + I*Sqrt[3])*(-19 + 66*x - 30*x^2 + 9*x^3)^(1/3)*Defer[Subst][Defer[Int][(
(7 + 9*x)^(1/3)*(343 - 63*x + 81*x^2)^(1/3))/((60 + 27*(-1 - I*Sqrt[3]))/27
 + 2*x), x], x, -10/9 + x])/(63*3^(1/3)*(-1 + 3*x)^(1/3)*(19 - 9*x + 3*x^2)
^(1/3)) - (2*(2 - I*Sqrt[3])*(-19 + 66*x - 30*x^2 + 9*x^3)^(1/3)*Defer[Subs
t][Defer[Int][((7 + 9*x)^(1/3)*(343 - 63*x + 81*x^2)^(1/3))/((60 + 27*(-1 +
 I*Sqrt[3]))/27 + 2*x), x], x, -10/9 + x])/(63*3^(1/3)*(-1 + 3*x)^(1/3)*(19
 - 9*x + 3*x^2)^(1/3))
```

Rubi steps

$$\int \frac{(2+x)^2\sqrt[3]{-19+66x-30x^2+9x^3}}{(-3+2x)^2\left(-5+6x-6x^2+x^3\right)}\,dx = \int \left(\frac{\sqrt[3]{-19+66x-30x^2+9x^3}}{21(-5+x)} - \frac{2\sqrt[3]{-19+66x-30x^2+9x^3}}{(-3+2x)^2} + \frac{2\sqrt[3]{-}\cdots}{\cdots}\right.$$

$$= \frac{1}{21}\int \frac{\sqrt[3]{-19+66x-30x^2+9x^3}}{-5+x}\,dx + \frac{1}{21}\int \frac{(5-4x)\sqrt[3]{-19+66x-3\cdots}}{1-x+x^2}$$

$$= \frac{1}{21}\int \left(\frac{\left(-4-2i\sqrt{3}\right)\sqrt[3]{-19+66x-30x^2+9x^3}}{-1-i\sqrt{3}+2x} + \frac{\left(-4+2i\sqrt{3}\right)\sqrt[3]{-19\cdots}}{-1+i\cdots}\right.$$

$$= -\left(\frac{1}{21}\left(2\left(2-i\sqrt{3}\right)\right)\int \frac{\sqrt[3]{-19+66x-30x^2+9x^3}}{-1+i\sqrt{3}+2x}\,dx\right) - \frac{1}{21}\left(2\left(2+i\cdots\right.\right.$$

$$= -\left(\frac{1}{21}\left(2\left(2-i\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{\sqrt[3]{\frac{2401}{81}+\frac{98x}{3}+9x^3}}{\frac{1}{27}\left(60+27\left(-1+i\sqrt{3}\right)\right)+2x}\,dx, x, -\cdots\right.\right.$$

$$= \frac{\sqrt[3]{-19+66x-30x^2+9x^3}\ \text{Subst}\left(\int \frac{\sqrt[3]{7+9x}\ \sqrt[3]{343-63x+81x^2}}{-\frac{35}{9}+x}\,dx, x, -\frac{10}{9}+\cdots\right.}{63\sqrt[3]{-3+9x}\ \sqrt[3]{19-9x+3x^2}}$$

Mathematica [F] time = 1.90, size = 0, normalized size = 0.00

$$\int \frac{(2+x)^2\sqrt[3]{-19+66x-30x^2+9x^3}}{(-3+2x)^2\left(-5+6x-6x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((2 + x)^2*(-19 + 66*x - 30*x^2 + 9*x^3)^(1/3))/((-3 + 2*x)^2*(-5
 + 6*x - 6*x^2 + x^3)),x]
```

```
[Out] Integrate[((2 + x)^2*(-19 + 66*x - 30*x^2 + 9*x^3)^(1/3))/((-3 + 2*x)^2*(-5
 + 6*x - 6*x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 1.14, size = 210, normalized size = 1.00

$$\frac{\sqrt[3]{9x^3-30x^2+66x-19}}{2x-3}+\frac{1}{3}\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{9x^3-30x^2+66x-19}-4x+6\right)-\frac{\log\left(8x^2+\sqrt[3]{2}\left(9x^3-30x^2+66x-19\right)^{2/3}+\left(2\ 2^{2/3}x-3\ 2^{2/3}\right)\sqrt[3]{9x^3-30x^2+66x-19}-24x+18\right)}{3\ 2^{2/3}}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{2\sqrt{3}x-3\sqrt{3}}{2^{2/3}\sqrt[3]{9x^3-30x^2+66x-19}+2x-3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + x)^2*(-19 + 66*x - 30*x^2 + 9*x^3)^(1/3))/((-3 + 2
*x)^2*(-5 + 6*x - 6*x^2 + x^3)),x]
```

```
[Out] (-19 + 66*x - 30*x^2 + 9*x^3)^(1/3)/(-3 + 2*x) + (2^(1/3)*ArcTan[(-3*Sqrt[3
] + 2*Sqrt[3]*x)/(-3 + 2*x + 2^(2/3)*(-19 + 66*x - 30*x^2 + 9*x^3)^(1/3))])
/Sqrt[3] + (2^(1/3)*Log[6 - 4*x + 2^(2/3)*(-19 + 66*x - 30*x^2 + 9*x^3)^(1/
3)])/3 - Log[18 - 24*x + 8*x^2 + (-3*2^(2/3) + 2*2^(2/3)*x)*(-19 + 66*x - 3
0*x^2 + 9*x^3)^(1/3) + 2^(1/3)*(-19 + 66*x - 30*x^2 + 9*x^3)^(2/3)]/(3*2^(2
/3))
```

fricas [B] time = 25.53, size = 532, normalized size = 2.53

$$\frac{2\sqrt{3}\,2^{\frac{1}{3}}(2x-3)\arctan\left(-\frac{6\sqrt{3}\,2^{\frac{2}{3}}\left(5380x^8-59100x^7+301161x^6-909412x^5+1740060x^4-2110416x^3+1545376x^2-606864x+94131\right)\left(9x^3-30x^2+66x-19\right)^{\frac{1}{3}}-42\sqrt{3}\,2^{\frac{1}{3}}\left(82x^7-963x^6+4404x^5-10852x^4+15852x^3-14316x^2+7786x-1905\right)\left(9x^3-30x^2+66x-19\right)^{\frac{2}{3}}+\sqrt{3}\left(43721x^9-510066x^8+2889414x^7-10065027x^6+23187528x^5-35703864x^4+35637567x^3-21385926x^2+6711858x-806653\right)}{3\left(62551x^9-773406x^8+4465170x^7-15587817x^6+35620200x^5-54275256x^4+54133401x^3-33459498x^2+11334294x-1538783\right)}\right)-2^{\frac{1}{3}}(2x-3)\log\left(\frac{3\cdot 2^{\frac{2}{3}}\left(82x^4-471x^3+1086x^2-1100x+381\right)\left(9x^3-30x^2+66x-19\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(1345x^6-10740x^5+40044x^4-83056x^3+95748x^2-53484x+10459\right)+12\left(68x^5-468x^4+1425x^3-2218x^2+1632x-414\right)\left(9x^3-30x^2+66x-19\right)^{\frac{1}{3}}}{x^6-12x^5+48x^4-82x^3+96x^2-60x+25}\right)+2\cdot 2^{\frac{1}{3}}(2x-3)\log\left(\frac{7\cdot 2^{\frac{2}{3}}\left(x^3-6x^2+6x-5\right)-6\cdot 2^{\frac{1}{3}}\left(9x^3-30x^2+66x-19\right)^{\frac{1}{3}}\left(4x^2-12x+9\right)+6\left(9x^3-30x^2+66x-19\right)^{\frac{2}{3}}(2x-3)}{x^3-6x^2+6x-5}\right)+18\left(9x^3-30x^2+66x-19\right)^{\frac{1}{3}}}{18\,(2x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)^2*(9*x^3-30*x^2+66*x-19)^(1/3)/(-3+2*x)^2/(x^3-6*x^2+6*x-5)
,x, algorithm="fricas")
```

```
[Out] 1/18*(2*sqrt(3)*2^(1/3)*(2*x - 3)*arctan(-1/3*(6*sqrt(3)*2^(2/3)*(5380*x^8
- 59100*x^7 + 301161*x^6 - 909412*x^5 + 1740060*x^4 - 2110416*x^3 + 1545376
*x^2 - 606864*x + 94131)*(9*x^3 - 30*x^2 + 66*x - 19)^(1/3) - 42*sqrt(3)*2^
(1/3)*(82*x^7 - 963*x^6 + 4404*x^5 - 10852*x^4 + 15852*x^3 - 14316*x^2 + 77
86*x - 1905)*(9*x^3 - 30*x^2 + 66*x - 19)^(2/3) + sqrt(3)*(43721*x^9 - 5100
66*x^8 + 2889414*x^7 - 10065027*x^6 + 23187528*x^5 - 35703864*x^4 + 3563756
7*x^3 - 21385926*x^2 + 6711858*x - 806653))/(62551*x^9 - 773406*x^8 + 44651
70*x^7 - 15587817*x^6 + 35620200*x^5 - 54275256*x^4 + 54133401*x^3 - 334594
98*x^2 + 11334294*x - 1538783)) - 2^(1/3)*(2*x - 3)*log((3*2^(2/3)*(82*x^4
- 471*x^3 + 1086*x^2 - 1100*x + 381)*(9*x^3 - 30*x^2 + 66*x - 19)^(2/3) + 2
^(1/3)*(1345*x^6 - 10740*x^5 + 40044*x^4 - 83056*x^3 + 95748*x^2 - 53484*x
+ 10459) + 12*(68*x^5 - 468*x^4 + 1425*x^3 - 2218*x^2 + 1632*x - 414)*(9*x^
3 - 30*x^2 + 66*x - 19)^(1/3))/(x^6 - 12*x^5 + 48*x^4 - 82*x^3 + 96*x^2 - 6
0*x + 25)) + 2*2^(1/3)*(2*x - 3)*log((7*2^(2/3)*(x^3 - 6*x^2 + 6*x - 5) - 6
*2^(1/3)*(9*x^3 - 30*x^2 + 66*x - 19)^(1/3)*(4*x^2 - 12*x + 9) + 6*(9*x^3 -
 30*x^2 + 66*x - 19)^(2/3)*(2*x - 3))/(x^3 - 6*x^2 + 6*x - 5)) + 18*(9*x^3
- 30*x^2 + 66*x - 19)^(1/3))/(2*x - 3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(9 x^3 - 30 x^2 + 66 x - 19\right)^{\frac{1}{3}} (x+2)^2}{\left(x^3 - 6 x^2 + 6 x - 5\right)(2 x - 3)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)^2*(9*x^3-30*x^2+66*x-19)^(1/3)/(-3+2*x)^2/(x^3-6*x^2+6*x-5)
,x, algorithm="giac")
```

```
[Out] integrate((9*x^3 - 30*x^2 + 66*x - 19)^(1/3)*(x + 2)^2/((x^3 - 6*x^2 + 6*x
- 5)*(2*x - 3)^2), x)
```

maple [C] time = 15.18, size = 5143, normalized size = 24.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)^2*(9*x^3-30*x^2+66*x-19)^(1/3)/(-3+2*x)^2/(x^3-6*x^2+6*x-5),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(9 x^3 - 30 x^2 + 66 x - 19\right)^{\frac{1}{3}} (x+2)^2}{\left(x^3 - 6 x^2 + 6 x - 5\right)(2 x - 3)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)^2*(9*x^3-30*x^2+66*x-19)^(1/3)/(-3+2*x)^2/(x^3-6*x^2+6*x-5)
,x, algorithm="maxima")
```

```
[Out] integrate((9*x^3 - 30*x^2 + 66*x - 19)^(1/3)*(x + 2)^2/((x^3 - 6*x^2 + 6*x
- 5)*(2*x - 3)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x+2)^2 \left(9\,x^3-30\,x^2+66\,x-19\right)^{1/3}}{(2\,x-3)^2 \left(x^3-6\,x^2+6\,x-5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 2)^2*(66*x - 30*x^2 + 9*x^3 - 19)^(1/3))/((2*x - 3)^2*(6*x - 6*x^
2 + x^3 - 5)),x)
```

```
[Out] int(((x + 2)^2*(66*x - 30*x^2 + 9*x^3 - 19)^(1/3))/((2*x - 3)^2*(6*x - 6*x^
2 + x^3 - 5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{(3x-1)\left(3x^2-9x+19\right)}\,(x+2)^2}{(x-5)\,(2x-3)^2\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)**2*(9*x**3-30*x**2+66*x-19)**(1/3)/(-3+2*x)**2/(x**3-6*x**2
+6*x-5),x)
```

```
[Out] Integral(((3*x - 1)*(3*x**2 - 9*x + 19))**(1/3)*(x + 2)**2/((x - 5)*(2*x -
3)**2*(x**2 - x + 1)), x)
```

3.2024 $\int \frac{1+x^3}{\left(-1+x^3\right)\sqrt[3]{x^2+x^4}}\,dx$

Optimal. Leaf size=210

$$-\frac{2}{3}\log\left(\sqrt[3]{x^4+x^2}+x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}-2x\right)}{3\sqrt[3]{2}}+\frac{1}{3}\log\left(x^2-\sqrt[3]{x^4+x^2}\,x+\left(x^4+x^2\right)^{2/3}\right)-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x\right.}$$

Rubi [F] time = 1.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x^3}{\left(-1+x^3\right)\sqrt[3]{x^2+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^3)/((-1 + x^3)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] (-2*x*(1 + x^2)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6, (-2*x^2)/(1 - I*Sqrt[3]),
-x^2])/(x^2 + x^4)^(1/3) - (2*x*(1 + x^2)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6,
(-2*x^2)/(1 + I*Sqrt[3]), -x^2])/(x^2 + x^4)^(1/3) - ((I - Sqrt[3])*x^2*(1
+ x^2)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -x^2, (-2*x^2)/(1 - I*Sqrt[3])])/(2
*(I + Sqrt[3])*(x^2 + x^4)^(1/3)) - ((I + Sqrt[3])*x^2*(1 + x^2)^(1/3)*Appe
llF1[2/3, 1/3, 1, 5/3, -x^2, (-2*x^2)/(1 + I*Sqrt[3])])/(2*(I - Sqrt[3])*(x
^2 + x^4)^(1/3)) + (3*x*(1 + x^2)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -x
^2])/(x^2 + x^4)^(1/3) + (2*x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int]
[1/((-1 + x)*(1 + x^6)^(1/3)), x], x, x^(1/3)])/(3*(x^2 + x^4)^(1/3)) - (2*
(1 - I*Sqrt[3])*x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][1/((1 - I*S
qrt[3] + 2*x)*(1 + x^6)^(1/3)), x], x, x^(1/3)])/(3*(x^2 + x^4)^(1/3)) - (2
*(1 + I*Sqrt[3])*x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][1/((1 + I*
Sqrt[3] + 2*x)*(1 + x^6)^(1/3)), x], x, x^(1/3)])/(3*(x^2 + x^4)^(1/3))
```

Rubi steps

$$\int \frac{1+x^3}{\left(-1+x^3\right)\sqrt[3]{x^2+x^4}}\,dx = \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{1+x^3}{x^{2/3}\sqrt[3]{1+x^2}\left(-1+x^3\right)}\,dx}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1+x^9}{\sqrt[3]{1+x^6}\left(-1+x^9\right)}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1+x^6}} + \frac{2}{\sqrt[3]{1+x^6}\left(-1+x^9\right)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(6x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^6}\left(-1+x^9\right.}\right.}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(6x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int\left(\frac{1}{9(-1+x)\sqrt[3]{1+x^6}} + \frac{-2-x}{9\left(1+x+x^2\right)\sqrt[3]{}}\right.\right.}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(2x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{3\sqrt[3]{x^2+x^4}} + \cdots$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(2x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{3\sqrt[3]{x^2+x^4}} + \cdots$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(2x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{3\sqrt[3]{x^2+x^4}} + \cdots$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(2x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{3\sqrt[3]{x^2+x^4}} + \cdots$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(2x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{3\sqrt[3]{x^2+x^4}} + \cdots$$

$$= -\frac{2x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^2}{1-i\sqrt{3}},-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{2x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^2}{1+i\sqrt{3}},-x^2\right)}{\sqrt[3]{x^2+x^4}} + \cdots$$

$$= -\frac{2x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^2}{1-i\sqrt{3}},-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{2x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^2}{1+i\sqrt{3}},-x^2\right)}{\sqrt[3]{x^2+x^4}} - \cdots$$

Mathematica [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{1+x^3}{\left(-1+x^3\right)\sqrt[3]{x^2+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^3)/((-1 + x^3)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] Integrate[(1 + x^3)/((-1 + x^3)*(x^2 + x^4)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.78, size = 210, normalized size = 1.00

$$-\frac{2}{3}\log\left(\sqrt[3]{x^4+x^2}+x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}-2x\right)}{3\sqrt[3]{2}}+\frac{1}{3}\log\left(x^2-\sqrt[3]{x^4+x^2}\,x+\left(x^4+x^2\right)^{2/3}\right)-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^4+x^2}\,x+\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{6\sqrt[3]{2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+x^2}-x}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}+x}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3)/((-1 + x^3)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3]*x)/(-x + 2*(x^2 + x^4)^(1/3))])/Sqrt[3] - ArcTan[(Sqrt[
3]*x)/(x + 2^(2/3)*(x^2 + x^4)^(1/3))]/(2^(1/3)*Sqrt[3]) - (2*Log[x + (x^2
+ x^4)^(1/3)])/3 + Log[-2*x + 2^(2/3)*(x^2 + x^4)^(1/3)]/(3*2^(1/3)) + Log[
x^2 - x*(x^2 + x^4)^(1/3) + (x^2 + x^4)^(2/3)]/3 - Log[2*x^2 + 2^(2/3)*x*(x
^2 + x^4)^(1/3) + 2^(1/3)*(x^2 + x^4)^(2/3)]/(6*2^(1/3))
```

fricas [B] time = 5.95, size = 424, normalized size = 2.02

$$-\frac{1}{18}\cdot 4^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{3\cdot 4^{\frac{2}{3}}\sqrt{3}\left(x^4+2x^3-6x^2+2x+1\right)\left(x^4+x^2\right)^{\frac{2}{3}}+6\cdot 4^{\frac{1}{3}}\sqrt{3}\left(x^5+14x^4+6x^3+14x^2+x\right)\left(x^4+x^2\right)^{\frac{1}{3}}+\sqrt{3}\left(x^7+30x^6+51x^5+52x^4+51x^3+30x^2+x\right)}{3\left(x^7-6x^6-93x^5-20x^4-93x^3-6x^2+x\right)}\right)-\frac{2}{3}\sqrt{3}\arctan\left(-\frac{1078\sqrt{3}\left(x^4+x^2\right)^{\frac{1}{3}}x+\sqrt{3}\left(32x^3-605x^2+32x\right)-196\sqrt{3}\left(x^4+x^2\right)^{\frac{2}{3}}}{8x^3+1331x^2+8x}\right)-\frac{1}{36}\cdot 4^{\frac{1}{3}}\log\left(\frac{6\cdot 4^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{2}{3}}\left(x^2+4x+1\right)+4^{\frac{2}{3}}\left(x^5+14x^4+6x^3+14x^2+x\right)+24\left(x^4+x^2\right)^{\frac{1}{3}}\left(x^3+x^2+x\right)}{x^5-4x^4+6x^3-4x^2+x}\right)+\frac{1}{18}\cdot 4^{\frac{1}{3}}\log\left(-\frac{3\cdot 4^{\frac{2}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}x+4^{\frac{1}{3}}\left(x^3-2x^2+x\right)-6\left(x^4+x^2\right)^{\frac{2}{3}}}{x^3-2x^2+x}\right)-\frac{1}{3}\log\left(\frac{x^3+x^2+3\left(x^4+x^2\right)^{\frac{1}{3}}x+x+3\left(x^4+x^2\right)^{\frac{2}{3}}}{x^3+x^2+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-1)/(x^4+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/18*4^(1/3)*sqrt(3)*arctan(1/3*(3*4^(2/3)*sqrt(3)*(x^4 + 2*x^3 - 6*x^2 +
2*x + 1)*(x^4 + x^2)^(2/3) + 6*4^(1/3)*sqrt(3)*(x^5 + 14*x^4 + 6*x^3 + 14*x
^2 + x)*(x^4 + x^2)^(1/3) + sqrt(3)*(x^7 + 30*x^6 + 51*x^5 + 52*x^4 + 51*x^
3 + 30*x^2 + x))/(x^7 - 6*x^6 - 93*x^5 - 20*x^4 - 93*x^3 - 6*x^2 + x)) - 2/
3*sqrt(3)*arctan(-(1078*sqrt(3)*(x^4 + x^2)^(1/3)*x + sqrt(3)*(32*x^3 - 605
*x^2 + 32*x) - 196*sqrt(3)*(x^4 + x^2)^(2/3))/(8*x^3 + 1331*x^2 + 8*x)) - 1
/36*4^(1/3)*log((6*4^(1/3)*(x^4 + x^2)^(2/3)*(x^2 + 4*x + 1) + 4^(2/3)*(x^5
 + 14*x^4 + 6*x^3 + 14*x^2 + x) + 24*(x^4 + x^2)^(1/3)*(x^3 + x^2 + x))/(x^
5 - 4*x^4 + 6*x^3 - 4*x^2 + x)) + 1/18*4^(1/3)*log(-(3*4^(2/3)*(x^4 + x^2)^
(1/3)*x + 4^(1/3)*(x^3 - 2*x^2 + x) - 6*(x^4 + x^2)^(2/3))/(x^3 - 2*x^2 + x
)) - 1/3*log((x^3 + x^2 + 3*(x^4 + x^2)^(1/3)*x + x + 3*(x^4 + x^2)^(2/3))/
(x^3 + x^2 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3+1}{\left(x^4+x^2\right)^{\frac{1}{3}}\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-1)/(x^4+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)/((x^4 + x^2)^(1/3)*(x^3 - 1)), x)
```

maple [C] time = 21.35, size = 2327, normalized size = 11.08

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^3-1)/(x^4+x^2)^(1/3),x)
```

```
[Out] -1/6*ln((33000*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*RootOf
(_Z^3-4)^3*x^3+37494*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)^
2*RootOf(_Z^3-4)^2*x^3-82500*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+
9*_Z^2)*RootOf(_Z^3-4)^3*x^2-93735*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z
^3-4)+9*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+33000*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*R
ootOf(_Z^3-4)+9*_Z^2)*RootOf(_Z^3-4)^3*x+158472*(x^4+x^2)^(2/3)*RootOf(_Z^3
-4)^2*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)+37494*RootOf(4*
```

```
RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)^2*RootOf(_Z^3-4)^2*x+302664*Ro
otOf(_Z^3-4)^2*(x^4+x^2)^(1/3)*x+316944*(x^4+x^2)^(1/3)*RootOf(_Z^3-4)*Root
Of(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*x+198000*RootOf(_Z^3-4)*x
^3+224964*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*x^3+176000*
RootOf(_Z^3-4)*x^2+199968*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_
Z^2)*x^2+198000*RootOf(_Z^3-4)*x+605328*(x^4+x^2)^(2/3)+224964*RootOf(4*Roo
tOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*x)/(-1+x)^2/x)*RootOf(_Z^3-4)-1/4
*ln((33000*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*RootOf(_Z^
3-4)^3*x^3+37494*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)^2*Ro
otOf(_Z^3-4)^2*x^3-82500*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z
^2)*RootOf(_Z^3-4)^3*x^2-93735*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4
)+9*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+33000*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootO
f(_Z^3-4)+9*_Z^2)*RootOf(_Z^3-4)^3*x+158472*(x^4+x^2)^(2/3)*RootOf(_Z^3-4)^
2*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)+37494*RootOf(4*Root
Of(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)^2*RootOf(_Z^3-4)^2*x+302664*RootOf
(_Z^3-4)^2*(x^4+x^2)^(1/3)*x+316944*(x^4+x^2)^(1/3)*RootOf(_Z^3-4)*RootOf(4
*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*x+198000*RootOf(_Z^3-4)*x^3+2
24964*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*x^3+176000*Root
Of(_Z^3-4)*x^2+199968*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)
*x^2+198000*RootOf(_Z^3-4)*x+605328*(x^4+x^2)^(2/3)+224964*RootOf(4*RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*x)/(-1+x)^2/x)*RootOf(4*RootOf(_Z^3-4
)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)+1/4*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_
Z^3-4)+9*_Z^2)*ln(-(8004*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z
^2)*RootOf(_Z^3-4)^3*x^3-37494*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4
)+9*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-20010*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootO
f(_Z^3-4)+9*_Z^2)*RootOf(_Z^3-4)^3*x^2+93735*RootOf(4*RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+9*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+8004*RootOf(4*RootOf(_Z^3-4)
^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*RootOf(_Z^3-4)^3*x+158472*(x^4+x^2)^(2/3)*Ro
otOf(_Z^3-4)^2*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)-37494*
RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)^2*RootOf(_Z^3-4)^2*x-
91368*RootOf(_Z^3-4)^2*(x^4+x^2)^(1/3)*x+316944*(x^4+x^2)^(1/3)*RootOf(_Z^3
-4)*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*x-26680*RootOf(_Z
^3-4)*x^3+124980*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*x^3-
96048*RootOf(_Z^3-4)*x^2+449928*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-
4)+9*_Z^2)*x^2-26680*RootOf(_Z^3-4)*x-182736*(x^4+x^2)^(2/3)+124980*RootOf(
4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*x)/(-1+x)^2/x)-2/3*ln((3582*
RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)^2*RootOf(_Z^3-4)^4*x^
3-8955*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)^2*RootOf(_Z^3-
4)^4*x^2+3582*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)^2*RootO
f(_Z^3-4)^4*x+41832*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*R
ootOf(_Z^3-4)^2*x^3+29232*(x^4+x^2)^(2/3)*RootOf(_Z^3-4)^2*RootOf(4*RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)-38376*(x^4+x^2)^(1/3)*RootOf(4*RootOf
(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*RootOf(_Z^3-4)^2*x-59208*RootOf(4*Ro
otOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*RootOf(_Z^3-4)^2*x^2+41832*RootO
f(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*RootOf(_Z^3-4)^2*x+119680*
x^3+180288*(x^4+x^2)^(2/3)+77952*x*(x^4+x^2)^(1/3)-71808*x^2+119680*x)/x/(x
^2+x+1))-1/4*RootOf(_Z^3-4)^2*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+9*_Z^2)*ln(-(3366*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)^2*
RootOf(_Z^3-4)^4*x^3-8415*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_
Z^2)^2*RootOf(_Z^3-4)^4*x^2+3366*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3
-4)+9*_Z^2)^2*RootOf(_Z^3-4)^4*x-8688*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf
(_Z^3-4)+9*_Z^2)*RootOf(_Z^3-4)^2*x^3+14616*(x^4+x^2)^(2/3)*RootOf(_Z^3-4)^
2*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)+33804*(x^4+x^2)^(1/
3)*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*RootOf(_Z^3-4)^2*x
-20916*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*RootOf(_Z^3-4)
^2*x^2-8688*RootOf(4*RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+9*_Z^2)*RootOf(_Z
^3-4)^2*x-19104*x^3-51168*(x^4+x^2)^(2/3)+38976*x*(x^4+x^2)^(1/3)-12736*x^2
-19104*x)/x/(x^2+x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3+1}{\left(x^4+x^2\right)^{\frac{1}{3}}\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-1)/(x^4+x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 1)/((x^4 + x^2)^(1/3)*(x^3 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3+1}{\left(x^4+x^2\right)^{1/3}\,\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/((x^2 + x^4)^(1/3)*(x^3 - 1)),x)
```

```
[Out] int((x^3 + 1)/((x^2 + x^4)^(1/3)*(x^3 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)\left(x^2-x+1\right)}{\sqrt[3]{x^2\left(x^2+1\right)}\,(x-1)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**3-1)/(x**4+x**2)**(1/3),x)
```

```
[Out] Integral((x + 1)*(x**2 - x + 1)/((x**2*(x**2 + 1))**(1/3)*(x - 1)*(x**2 + x
 + 1)), x)
```

3.2025 $\int \frac{x(4ab-3(a+b)x+2x^2)}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-abd+(a+b)dx-dx^2+x^4\right)}\,dx$

Optimal. Leaf size=210

$$\frac{\log\left(x^2-\sqrt[3]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}\right)}{d^{2/3}}-\frac{\log\left(d^{2/3}\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+\sqrt[3]{d}\,x^2\sqrt[3]{x^3(-a-b)+abx^2+x^4}\right.}{2d^{2/3}}$$

Rubi [F] time = 16.81, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\left(4ab-3(a+b)x+2x^2\right)}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-abd+(a+b)dx-dx^2+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(4*a*b - 3*(a + b)*x + 2*x^2))/((x^2*(-a + x)*(-b + x))^(1/3)*(-(a*b
*d) + (a + b)*d*x - d*x^2 + x^4)),x]
```

```
[Out] (9*(a + b)*x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][Defer[Int][x^
6/((-a + x^3)^(1/3)*(-b + x^3)^(1/3)*(a*b*d - a*(1 + b/a)*d*x^3 + d*x^6 - x
^12)), x], x, x^(1/3)])/((a - x)*(b - x)*x^2)^(1/3) + (12*a*b*x^(2/3)*(-a +
 x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][Defer[Int][x^3/((-a + x^3)^(1/3)*(-b
+ x^3)^(1/3)*(-(a*b*d) + a*(1 + b/a)*d*x^3 - d*x^6 + x^12)), x], x, x^(1/3)
])/((a - x)*(b - x)*x^2)^(1/3) + (6*x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*D
efer[Subst][Defer[Int][x^9/((-a + x^3)^(1/3)*(-b + x^3)^(1/3)*(-(a*b*d) + a
*(1 + b/a)*d*x^3 - d*x^6 + x^12)), x], x, x^(1/3)])/((a - x)*(b - x)*x^2)^(
1/3)
```

Rubi steps

$$\int \frac{x\left(4ab-3(a+b)x+2x^2\right)}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-abd+(a+b)dx-dx^2+x^4\right)}\,dx = \frac{\left(x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\int \frac{\sqrt[3]{x}\left(4ab-3(a+b)x+2x^2\right)}{\sqrt[3]{-a+x}\sqrt[3]{-b+x}\left(-abd+(a+b)dx-\right.}}{\sqrt[3]{x^2(-a+x)(-b+x)}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\text{Subst}\left(\int \frac{x^3\left(4ab-3\right.}{\sqrt[3]{-a+x^3}\sqrt[3]{-b+x^3}\left(-\right.}\right.}{\sqrt[3]{x^2(-a+x)(-b+x)}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\text{Subst}\left(\int\left(\frac{3}{\sqrt[3]{-a+x^3}\sqrt[3]{-b+x^3}\left(\right.}\right.\right.}{}$$

$$= \frac{\left(6x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt[3]{-a+x^3}\sqrt[3]{-b+x^3}\left(-\right.}\right.}{\sqrt[3]{x^2(-a+x)(-b+x)}}$$

Mathematica [F] time = 3.61, size = 0, normalized size = 0.00

$$\int \frac{x\left(4ab-3(a+b)x+2x^2\right)}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-abd+(a+b)dx-dx^2+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(4*a*b - 3*(a + b)*x + 2*x^2))/((x^2*(-a + x)*(-b + x))^(1/3)*
(-(a*b*d) + (a + b)*d*x - d*x^2 + x^4)),x]
```

```
[Out] Integrate[(x*(4*a*b - 3*(a + b)*x + 2*x^2))/((x^2*(-a + x)*(-b + x))^(1/3)*
(-(a*b*d) + (a + b)*d*x - d*x^2 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.94, size = 210, normalized size = 1.00

$$\frac{\log \left(x^2-\sqrt[3]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}\right)}{d^{2/3}}-\frac{\log \left(d^{2/3}\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+\sqrt[3]{d}\,x^2\sqrt[3]{x^3(-a-b)+abx^2+x^4}+x^4\right)}{2d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}}{\sqrt[3]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}+2x^2}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(4*a*b - 3*(a + b)*x + 2*x^2))/((x^2*(-a + x)*(-b + x
))^(1/3)*(-(a*b*d) + (a + b)*d*x - d*x^2 + x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3))/(2*x
^2 + d^(1/3)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3))]])/d^(2/3) + Log[x^2 - d^
(1/3)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3)]/d^(2/3) - Log[x^4 + d^(1/3)*x^2
*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3) + d^(2/3)*(a*b*x^2 + (-a - b)*x^3 + x
^4)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a*b*d+(a+b)
*d*x-d*x^2+x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,ab-3\,(a+b)x+2\,x^2\right)x}{\left((a-x)(b-x)x^2\right)^{\frac{1}{3}}\left(x^4-abd+(a+b)dx-dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a*b*d+(a+b)
*d*x-d*x^2+x^4),x, algorithm="giac")
```

```
[Out] integrate((4*a*b - 3*(a + b)*x + 2*x^2)*x/(((a - x)*(b - x)*x^2)^(1/3)*(x^4
 - a*b*d + (a + b)*d*x - d*x^2)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x\left(4ab-3\,(a+b)\,x+2x^2\right)}{\left(x^2\,(-a+x)\,(-b+x)\right)^{\frac{1}{3}}\left(-abd+(a+b)\,dx-d\,x^2+x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a*b*d+(a+b)*d*x-d
*x^2+x^4),x)
```

```
[Out] int(x*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a*b*d+(a+b)*d*x-d
*x^2+x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,ab-3\,(a+b)x+2\,x^2\right)x}{\left((a-x)(b-x)x^2\right)^{\frac{1}{3}}\left(x^4-abd+(a+b)dx-dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a*b*d+(a+b)
*d*x-d*x^2+x^4),x, algorithm="maxima")
```

```
[Out] integrate((4*a*b - 3*(a + b)*x + 2*x^2)*x/(((a - x)*(b - x)*x^2)^(1/3)*(x^4
 - a*b*d + (a + b)*d*x - d*x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x\,\left(4\,a\,b+2\,x^2-3\,x\,\left(a+b\right)\right)}{{\left(x^2\,\left(a-x\right)\,\left(b-x\right)\right)}^{1/3}\,\left(-x^4+d\,x^2-d\,\left(a+b\right)\,x+a\,b\,d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(4*a*b + 2*x^2 - 3*x*(a + b)))/((x^2*(a - x)*(b - x))^(1/3)*(d*x^2
- x^4 - d*x*(a + b) + a*b*d)),x)
```

```
[Out] -int((x*(4*a*b + 2*x^2 - 3*x*(a + b)))/((x^2*(a - x)*(b - x))^(1/3)*(d*x^2
- x^4 - d*x*(a + b) + a*b*d)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*a*b-3*(a+b)*x+2*x**2)/(x**2*(-a+x)*(-b+x))**(1/3)/(-a*b*d+(a
+b)*d*x-d*x**2+x**4),x)
```

```
[Out] Timed out
```

3.2026 $\int \frac{\sqrt{b+ax^4}}{-b+ax^4}\,dx$

Optimal. Leaf size=210

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{ax^4+b}+\sqrt{a}x^2+\sqrt{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{6-4\sqrt{2}}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{ax^4+b}+\sqrt{a}x^2+\sqrt{b}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{6+4\sqrt{2}}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{ax^4+b}+\sqrt{a}x^2+\sqrt{b}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}$$

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {404, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{ax^4+b}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{ax^4+b}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b + a*x^4]/(-b + a*x^4),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[b + a*x^4]]/(Sqrt[2]*a^(1/4)*b
^(1/4)) - ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[b + a*x^4]]/(2*Sqrt[2]*a
^(1/4)*b^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 404

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[a/c,
 Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{b+ax^4}}{-b+ax^4}\,dx &= -\text{Subst}\left(\int \frac{1}{1-4abx^4}\,dx, x, \frac{x}{\sqrt{b+ax^4}}\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2\sqrt{a}\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt{b+ax^4}}\right)\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2\sqrt{a}\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt{b+ax^4}}\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{b+ax^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{b+ax^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}
\end{aligned}
$$

Mathematica [C] time = 0.21, size = 152, normalized size = 0.72

$$
-\frac{5bx\sqrt{ax^4+b}\,F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{ax^4}{b},\frac{ax^4}{b}\right)}{\left(b-ax^4\right)\left(2ax^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-\frac{ax^4}{b},\frac{ax^4}{b}\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{ax^4}{b},\frac{ax^4}{b}\right)\right)+5bF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{ax^4}{b},\frac{ax^4}{b}\right)\right)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[b + a*x^4]/(-b + a*x^4),x]
```

```
[Out] (-5*b*x*Sqrt[b + a*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((a*x^4)/b), (a*x^4)/b
])/((b - a*x^4)*(5*b*AppellF1[1/4, -1/2, 1, 5/4, -((a*x^4)/b), (a*x^4)/b] +
 2*a*x^4*(2*AppellF1[5/4, -1/2, 2, 9/4, -((a*x^4)/b), (a*x^4)/b] + AppellF1
[5/4, 1/2, 1, 9/4, -((a*x^4)/b), (a*x^4)/b])))
```

IntegrateAlgebraic [A] time = 0.37, size = 97, normalized size = 0.46

$$
-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{ax^4+b}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{ax^4+b}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + a*x^4]/(-b + a*x^4),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[b + a*x^4]]/(Sqrt[2]*a^(1/4)*b
^(1/4)) - ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[b + a*x^4]]/(2*Sqrt[2]*a
^(1/4)*b^(1/4))
```

fricas [A] time = 0.77, size = 259, normalized size = 1.23

$$
\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{ax^4+b}\left(\frac{1}{ab}\right)^{\frac{1}{4}}-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}ax^2\left(\frac{1}{ab}\right)^{\frac{1}{4}}+2\left(\frac{1}{4}\right)^{\frac{3}{4}}ab\left(\frac{1}{ab}\right)^{\frac{3}{4}}}{\sqrt{a}}}{x}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abx^3\left(\frac{1}{ab}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}bx\left(\frac{1}{ab}\right)^{\frac{1}{4}}+\sqrt{ax^4+b}\left(x^2+b\sqrt{\frac{1}{ab}}\right)}{ax^4-b}\right)+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abx^3\left(\frac{1}{ab}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}bx\left(\frac{1}{ab}\right)^{\frac{1}{4}}-\sqrt{ax^4+b}\left(x^2+b\sqrt{\frac{1}{ab}}\right)}{ax^4-b}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(1/2)/(a*x^4-b),x, algorithm="fricas")
```

```
[Out] (1/4)^(1/4)*(1/(a*b))^(1/4)*arctan(((1/4)^(1/4)*sqrt(a*x^4 + b)*(1/(a*b))^(
1/4) - ((1/4)^(1/4)*a*x^2*(1/(a*b))^(1/4) + 2*(1/4)^(3/4)*a*b*(1/(a*b))^(3/
4))/sqrt(a))/x) - 1/4*(1/4)^(1/4)*(1/(a*b))^(1/4)*log((4*(1/4)^(3/4)*a*b*x^
3*(1/(a*b))^(3/4) + 2*(1/4)^(1/4)*b*x*(1/(a*b))^(1/4) + sqrt(a*x^4 + b)*(x^
2 + b*sqrt(1/(a*b))))/(a*x^4 - b)) + 1/4*(1/4)^(1/4)*(1/(a*b))^(1/4)*log(-(
4*(1/4)^(3/4)*a*b*x^3*(1/(a*b))^(3/4) + 2*(1/4)^(1/4)*b*x*(1/(a*b))^(1/4) -
 sqrt(a*x^4 + b)*(x^2 + b*sqrt(1/(a*b))))/(a*x^4 - b))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\sqrt{ax^4+b}}{ax^4-b}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(1/2)/(a*x^4-b),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^4 + b)/(a*x^4 - b), x)
```

maple [A] time = 0.04, size = 97, normalized size = 0.46

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{a\,x^{4}+b}\,\sqrt{2}}{2x(ab)^{\frac{1}{4}}}\right)}{4\,(ab)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{a\,x^{4}+b}\,\sqrt{2}}{2x}+(ab)^{\frac{1}{4}}}{\frac{\sqrt{a\,x^{4}+b}\,\sqrt{2}}{2x}-(ab)^{\frac{1}{4}}}\right)}{8\,(ab)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b)^(1/2)/(a*x^4-b),x)
```

```
[Out] 1/4*2^(1/2)/(a*b)^(1/4)*arctan(1/2*(a*x^4+b)^(1/2)*2^(1/2)/x/(a*b)^(1/4))-1
/8*2^(1/2)/(a*b)^(1/4)*ln((1/2*(a*x^4+b)^(1/2)*2^(1/2)/x+(a*b)^(1/4))/(1/2*
(a*x^4+b)^(1/2)*2^(1/2)/x-(a*b)^(1/4)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{4}+b}}{ax^{4}-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(1/2)/(a*x^4-b),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^4 + b)/(a*x^4 - b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\sqrt{a\,x^{4}+b}}{b-a\,x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x^4)^(1/2)/(b - a*x^4),x)
```

```
[Out] -int((b + a*x^4)^(1/2)/(b - a*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{4}+b}}{ax^{4}-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b)**(1/2)/(a*x**4-b),x)
```

```
[Out] Integral(sqrt(a*x**4 + b)/(a*x**4 - b), x)
```

3.2027 $\int \frac{1+x^2}{\left(-1+x+x^2\right)\sqrt[3]{-1+x^6}}\,dx$

Optimal. Leaf size=210

$$-\frac{\log\left(-2\sqrt[3]{x^6-1}+2^{2/3}x^2-2^{2/3}x-2^{2/3}\right)}{3\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^6-1}}{\sqrt[3]{x^6-1}+2^{2/3}x^2-2^{2/3}x-2^{2/3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(2\left(x^6-1\right)^{2/3}+\sqrt[3]{2}x^4-2\sqrt[3]{2}x^3-\right.}{}$$

Rubi [F] time = 0.41, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x^2}{\left(-1+x+x^2\right)\sqrt[3]{-1+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^2)/((-1 + x + x^2)*(-1 + x^6)^(1/3)),x]
```

```
[Out] (x*(1 - x^6)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, x^6])/(-1 + x^6)^(1/3)
- (1 - Sqrt[5])*Defer[Int][1/((1 - Sqrt[5] + 2*x)*(-1 + x^6)^(1/3)), x] - (
1 + Sqrt[5])*Defer[Int][1/((1 + Sqrt[5] + 2*x)*(-1 + x^6)^(1/3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{\left(-1+x+x^2\right)\sqrt[3]{-1+x^6}}\,dx &= \int \left(\frac{1}{\sqrt[3]{-1+x^6}}+\frac{2-x}{\left(-1+x+x^2\right)\sqrt[3]{-1+x^6}}\right)dx\\
&= \int \frac{1}{\sqrt[3]{-1+x^6}}\,dx+\int \frac{2-x}{\left(-1+x+x^2\right)\sqrt[3]{-1+x^6}}\,dx\\
&= \frac{\sqrt[3]{1-x^6}\int \frac{1}{\sqrt[3]{1-x^6}}\,dx}{\sqrt[3]{-1+x^6}}+\int \left(\frac{-1+\sqrt{5}}{\left(1-\sqrt{5}+2x\right)\sqrt[3]{-1+x^6}}+\frac{-1-\sqrt{5}}{\left(1+\sqrt{5}+2x\right)\sqrt[3]{-1+x}\right.}\\
&= \frac{x\sqrt[3]{1-x^6}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^6\right)}{\sqrt[3]{-1+x^6}}+\left(-1-\sqrt{5}\right)\int \frac{1}{\left(1+\sqrt{5}+2x\right)\sqrt[3]{-1+x^6}}\,dx+\left(-1+\right.
\end{aligned}$$

Mathematica [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{\left(-1+x+x^2\right)\sqrt[3]{-1+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^2)/((-1 + x + x^2)*(-1 + x^6)^(1/3)),x]
```

```
[Out] Integrate[(1 + x^2)/((-1 + x + x^2)*(-1 + x^6)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 6.95, size = 210, normalized size = 1.00

$$-\frac{\log\left(-2\sqrt[3]{x^6-1}+2^{2/3}x^2-2^{2/3}x-2^{2/3}\right)}{3\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^6-1}}{\sqrt[3]{x^6-1}+2^{2/3}x^2-2^{2/3}x-2^{2/3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(2\left(x^6-1\right)^{2/3}+\sqrt[3]{2}x^4-2\sqrt[3]{2}x^3-\sqrt[3]{2}x^2+\left(2^{2/3}x^2-2^{2/3}x-2^{2/3}\right)\sqrt[3]{x^6-1}+2\sqrt[3]{2}x+\sqrt[3]{2}\right)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 + x + x^2)*(-1 + x^6)^(1/3)),x]
```

```
[Out] -(ArcTan[(Sqrt[3]*(-1 + x^6)^(1/3))/(-2^(2/3) - 2^(2/3)*x + 2^(2/3)*x^2 + (
-1 + x^6)^(1/3))]/(2^(2/3)*Sqrt[3])) - Log[-2^(2/3) - 2^(2/3)*x + 2^(2/3)*x
^2 - 2*(-1 + x^6)^(1/3)]/(3*2^(2/3)) + Log[2^(1/3) + 2*2^(1/3)*x - 2^(1/3)*
x^2 - 2*2^(1/3)*x^3 + 2^(1/3)*x^4 + (-2^(2/3) - 2^(2/3)*x + 2^(2/3)*x^2)*(-
1 + x^6)^(1/3) + 2*(-1 + x^6)^(2/3)]/(6*2^(2/3))
```

fricas [A] time = 24.13, size = 251, normalized size = 1.20

$$-\frac{1}{6} \cdot 4^{\frac{1}{6}} \sqrt{3} \left(-1\right)^{\frac{1}{3}} \arctan\left(\frac{4^{\frac{1}{6}} \sqrt{3} \left(2 \cdot 4^{\frac{2}{3}} \left(-1\right)^{\frac{2}{3}} \left(x^{6}-1\right)^{\frac{2}{3}} \left(x^{2}-x-1\right)+4 \left(-1\right)^{\frac{1}{3}} \left(x^{6}-1\right)^{\frac{1}{3}} \left(x^{4}-2 x^{3}-x^{2}+2 x+1\right)-4^{\frac{1}{3}} \left(x^{6}+3 x^{5}-5 x^{3}+3 x-1\right)\right)}{6 \left(3 x^{6}-3 x^{5}+5 x^{3}-3 x-3\right)}\right)-\frac{1}{24} \cdot 4^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} \log\left(\frac{4^{\frac{1}{3}} \left(-1\right)^{\frac{2}{3}} \left(x^{4}-2 x^{3}-x^{2}+2 x+1\right)-4^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} \left(x^{6}-1\right)^{\frac{2}{3}}+2 \left(x^{6}-1\right)^{\frac{1}{3}} \left(x^{2}-x-1\right)}{x^{4}+2 x^{3}-x^{2}-2 x+1}\right)+\frac{1}{12} \cdot 4^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} \log\left(\frac{4^{\frac{1}{3}} \left(x^{2}-x-1\right)+2 \left(-1\right)^{\frac{1}{3}} \left(x^{6}-1\right)^{\frac{1}{3}}}{x^{2}+x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x-1)/(x^6-1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*sqrt(3)*(2*4^(2/3)*(-1)^
(2/3)*(x^6 - 1)^(2/3)*(x^2 - x - 1) + 4*(-1)^(1/3)*(x^6 - 1)^(1/3)*(x^4 - 2
*x^3 - x^2 + 2*x + 1) - 4^(1/3)*(x^6 + 3*x^5 - 5*x^3 + 3*x - 1))/(3*x^6 - 3
*x^5 + 5*x^3 - 3*x - 3)) - 1/24*4^(2/3)*(-1)^(1/3)*log((4^(1/3)*(-1)^(2/3)*
(x^4 - 2*x^3 - x^2 + 2*x + 1) - 4^(2/3)*(-1)^(1/3)*(x^6 - 1)^(2/3) + 2*(x^6
 - 1)^(1/3)*(x^2 - x - 1))/(x^4 + 2*x^3 - x^2 - 2*x + 1)) + 1/12*4^(2/3)*(-
1)^(1/3)*log((4^(1/3)*(x^2 - x - 1) + 2*(-1)^(1/3)*(x^6 - 1)^(1/3))/(x^2 +
x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}+1}{\left(x^{6}-1\right)^{\frac{1}{3}} \left(x^{2}+x-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x-1)/(x^6-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/((x^6 - 1)^(1/3)*(x^2 + x - 1)), x)
```

maple [C] time = 31.39, size = 2712, normalized size = 12.91

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2+x-1)/(x^6-1)^(1/3),x)
```

```
[Out] -1/6*ln((-18*x^2*(x^6-1)^(2/3)+7*RootOf(_Z^3+2)*x^6-3*RootOf(_Z^3+2)*x-3*Ro
otOf(_Z^3+2)*x^5+18*x*(x^6-1)^(2/3)+18*(x^6-1)^(2/3)+5*RootOf(_Z^3+2)*x^3-7
0*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)-30*RootOf(RootOf(_Z^3
+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x-60*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(
_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x-7*RootOf(_Z^3+2)+70*RootOf(RootOf(_Z^3
+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^6+50*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootO
f(_Z^3+2)+4*_Z^2)*x^3-60*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2
)^2*RootOf(_Z^3+2)^2*x^5-6*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z
^2)*RootOf(_Z^3+2)^3*x^5+48*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*Root
Of(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x^2-30*x^5*RootOf(RootOf(_Z^3+2)^2+2*_Z
*RootOf(_Z^3+2)+4*_Z^2)+10*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z
^2)*RootOf(_Z^3+2)^3*x^3+100*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*
_Z^2)^2*RootOf(_Z^3+2)^2*x^3-6*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+
4*_Z^2)*RootOf(_Z^3+2)^3*x-48*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*Ro
otOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x^4-48*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3
+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x+96*(x^6-1)^(1/3)*RootO
f(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x^3+48*(x^6-1
```

)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*
x^2-96*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*Ro
otOf(_Z^3+2)*x-9*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)*x^4-9*RootOf(_Z^3+2)^2*(x^6
-1)^(1/3)+18*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)*x^3-48*(x^6-1)^(2/3)*RootOf(Roo
tOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2+9*RootOf(_Z^3+2)
^2*(x^6-1)^(1/3)*x^2-18*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)*x-48*(x^6-1)^(1/3)*R
ootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2))/(x^2+x-1
)^3)*RootOf(_Z^3+2)-1/3*ln((-18*x^2*(x^6-1)^(2/3)+7*RootOf(_Z^3+2)*x^6-3*Ro
otOf(_Z^3+2)*x-3*RootOf(_Z^3+2)*x^5+18*x*(x^6-1)^(2/3)+18*(x^6-1)^(2/3)+5*R
ootOf(_Z^3+2)*x^3-70*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)-30
*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x-60*RootOf(RootOf(_Z^
3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x-7*RootOf(_Z^3+2)+70
*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^6+50*RootOf(RootOf(_
Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^3-60*RootOf(RootOf(_Z^3+2)^2+2*_Z*Ro
otOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^5-6*RootOf(RootOf(_Z^3+2)^2+2*_Z*
RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^5+48*(x^6-1)^(2/3)*RootOf(RootOf(
_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x^2-30*x^5*RootOf(Ro
otOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)+10*RootOf(RootOf(_Z^3+2)^2+2*_Z*
RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^3+100*RootOf(RootOf(_Z^3+2)^2+2*_
Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^3-6*RootOf(RootOf(_Z^3+2)^2+2
*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x-48*(x^6-1)^(1/3)*RootOf(RootO
f(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x^4-48*(x^6-1)^(2/3)
*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x+96*
(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z
^3+2)*x^3+48*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z
^2)*RootOf(_Z^3+2)*x^2-96*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf
(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x-9*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)*x^4-9*Ro
otOf(_Z^3+2)^2*(x^6-1)^(1/3)+18*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)*x^3-48*(x^6-
1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)
^2+9*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)*x^2-18*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)*x
-48*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootO
f(_Z^3+2))/(x^2+x-1)^3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)
+1/3*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*ln(-(-30*x^2*(x^6-
1)^(2/3)+28*RootOf(_Z^3+2)*x^6-36*RootOf(_Z^3+2)*x-36*RootOf(_Z^3+2)*x^5+30
x(x^6-1)^(2/3)+30*(x^6-1)^(2/3)+60*RootOf(_Z^3+2)*x^3-70*RootOf(RootOf(_Z
^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)-90*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(
_Z^3+2)+4*_Z^2)*x+60*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*
RootOf(_Z^3+2)^2*x-28*RootOf(_Z^3+2)+70*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf
(_Z^3+2)+4*_Z^2)*x^6+150*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2
)*x^3+60*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+
2)^2*x^5+24*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3
+2)^3*x^5+48*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z
^2)*RootOf(_Z^3+2)^2*x^2-90*x^5*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)
+4*_Z^2)-40*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3
+2)^3*x^3-100*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(
_Z^3+2)^2*x^3+24*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf
(_Z^3+2)^3*x-48*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4
*_Z^2)*RootOf(_Z^3+2)*x^4-48*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*Roo
tOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x+96*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+
2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x^3+48*(x^6-1)^(1/3)*RootOf
(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x^2-96*(x^6-1)
^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x
-15*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)*x^4-15*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)+30
RootOf(_Z^3+2)^2(x^6-1)^(1/3)*x^3-48*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3+2)^
2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2+15*RootOf(_Z^3+2)^2*(x^6-1)^
(1/3)*x^2-30*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)*x-48*(x^6-1)^(1/3)*RootOf(RootO
f(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2))/(x^2+x-1)^3)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^6-1\right)^{\frac{1}{3}}\left(x^2+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x-1)/(x^6-1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/((x^6 - 1)^(1/3)*(x^2 + x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2+1}{\left(x^6-1\right)^{1/3}\,\left(x^2+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((x^6 - 1)^(1/3)*(x + x^2 - 1)),x)
```

```
[Out] int((x^2 + 1)/((x^6 - 1)^(1/3)*(x + x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt[3]{(x-1)\,(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,\left(x^2+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2+x-1)/(x**6-1)**(1/3),x)
```

```
[Out] Integral((x**2 + 1)/(((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1))**(1/3)
*(x**2 + x - 1)), x)
```

3.2028 $\int \frac{1+x^2}{\left(-1+x+x^2\right)\sqrt[3]{-1+x^6}}\,dx$

Optimal. Leaf size=210

$$-\frac{\log\left(-2\sqrt[3]{x^6-1}+2^{2/3}x^2-2^{2/3}x-2^{2/3}\right)}{3\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^6-1}}{\sqrt[3]{x^6-1}+2^{2/3}x^2-2^{2/3}x-2^{2/3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(2\left(x^6-1\right)^{2/3}+\sqrt[3]{2}x^4-2\sqrt[3]{2}x^3-\right.}{}$$

Rubi [F] time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x^2}{\left(-1+x+x^2\right)\sqrt[3]{-1+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^2)/((-1 + x + x^2)*(-1 + x^6)^(1/3)),x]
```

```
[Out] (x*(1 - x^6)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, x^6])/(-1 + x^6)^(1/3)
- (1 - Sqrt[5])*Defer[Int][1/((1 - Sqrt[5] + 2*x)*(-1 + x^6)^(1/3)), x] - (
1 + Sqrt[5])*Defer[Int][1/((1 + Sqrt[5] + 2*x)*(-1 + x^6)^(1/3)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{\left(-1+x+x^2\right)\sqrt[3]{-1+x^6}}\,dx &= \int\left(\frac{1}{\sqrt[3]{-1+x^6}}+\frac{2-x}{\left(-1+x+x^2\right)\sqrt[3]{-1+x^6}}\right)dx\\
&= \int \frac{1}{\sqrt[3]{-1+x^6}}\,dx+\int \frac{2-x}{\left(-1+x+x^2\right)\sqrt[3]{-1+x^6}}\,dx\\
&= \frac{\sqrt[3]{1-x^6}\int \frac{1}{\sqrt[3]{1-x^6}}\,dx}{\sqrt[3]{-1+x^6}}+\int\left(\frac{-1+\sqrt{5}}{\left(1-\sqrt{5}+2x\right)\sqrt[3]{-1+x^6}}+\frac{-1-\sqrt{5}}{\left(1+\sqrt{5}+2x\right)\sqrt[3]{-1+x\right.}}\right.\\
&= \frac{x\sqrt[3]{1-x^6}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};x^6\right)}{\sqrt[3]{-1+x^6}}+\left(-1-\sqrt{5}\right)\int \frac{1}{\left(1+\sqrt{5}+2x\right)\sqrt[3]{-1+x^6}}\,dx+\left(-1+\right.
\end{aligned}$$

Mathematica [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{\left(-1+x+x^2\right)\sqrt[3]{-1+x^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^2)/((-1 + x + x^2)*(-1 + x^6)^(1/3)),x]
```

```
[Out] Integrate[(1 + x^2)/((-1 + x + x^2)*(-1 + x^6)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.00, size = 210, normalized size = 1.00

$$-\frac{\log\left(-2\sqrt[3]{x^6-1}+2^{2/3}x^2-2^{2/3}x-2^{2/3}\right)}{3\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^6-1}}{\sqrt[3]{x^6-1}+2^{2/3}x^2-2^{2/3}x-2^{2/3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(2\left(x^6-1\right)^{2/3}+\sqrt[3]{2}x^4-2\sqrt[3]{2}x^3-\sqrt[3]{2}x^2+\left(2^{2/3}x^2-2^{2/3}x-2^{2/3}\right)\sqrt[3]{x^6-1}+2\sqrt[3]{2}x+\sqrt[3]{2}\right)}{6\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)/((-1 + x + x^2)*(-1 + x^6)^(1/3)),x]
```

```
[Out] -(ArcTan[(Sqrt[3]*(-1 + x^6)^(1/3))/(-2^(2/3) - 2^(2/3)*x + 2^(2/3)*x^2 + (
-1 + x^6)^(1/3))]/(2^(2/3)*Sqrt[3])) - Log[-2^(2/3) - 2^(2/3)*x + 2^(2/3)*x
^2 - 2*(-1 + x^6)^(1/3)]/(3*2^(2/3)) + Log[2^(1/3) + 2*2^(1/3)*x - 2^(1/3)*
x^2 - 2*2^(1/3)*x^3 + 2^(1/3)*x^4 + (-2^(2/3) - 2^(2/3)*x + 2^(2/3)*x^2)*(-
1 + x^6)^(1/3) + 2*(-1 + x^6)^(2/3)]/(6*2^(2/3))
```

fricas [A] time = 20.64, size = 251, normalized size = 1.20

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(2\cdot 4^{\frac{2}{3}}\,(-1)^{\frac{2}{3}}\left(x^{6}-1\right)^{\frac{2}{3}}\left(x^{2}-x-1\right)+4\,(-1)^{\frac{1}{3}}\left(x^{6}-1\right)^{\frac{1}{3}}\left(x^{4}-2x^{3}-x^{2}+2x+1\right)-4^{\frac{1}{3}}\left(x^{6}+3x^{5}-5x^{3}+3x-1\right)\right)}{6\left(3x^{6}-3x^{5}+5x^{3}-3x-3\right)}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\log\left(\frac{4^{\frac{1}{3}}\,(-1)^{\frac{2}{3}}\left(x^{4}-2x^{3}-x^{2}+2x+1\right)-4^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\left(x^{6}-1\right)^{\frac{2}{3}}+2\left(x^{6}-1\right)^{\frac{1}{3}}\left(x^{2}-x-1\right)}{x^{4}+2x^{3}-x^{2}-2x+1}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\log\left(\frac{4^{\frac{1}{3}}\left(x^{2}-x-1\right)+2\,(-1)^{\frac{1}{3}}\left(x^{6}-1\right)^{\frac{1}{3}}}{x^{2}+x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x-1)/(x^6-1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*sqrt(3)*(2*4^(2/3)*(-1)^
(2/3)*(x^6 - 1)^(2/3)*(x^2 - x - 1) + 4*(-1)^(1/3)*(x^6 - 1)^(1/3)*(x^4 - 2
*x^3 - x^2 + 2*x + 1) - 4^(1/3)*(x^6 + 3*x^5 - 5*x^3 + 3*x - 1))/(3*x^6 - 3
*x^5 + 5*x^3 - 3*x - 3)) - 1/24*4^(2/3)*(-1)^(1/3)*log((4^(1/3)*(-1)^(2/3)*
(x^4 - 2*x^3 - x^2 + 2*x + 1) - 4^(2/3)*(-1)^(1/3)*(x^6 - 1)^(2/3) + 2*(x^6
 - 1)^(1/3)*(x^2 - x - 1))/(x^4 + 2*x^3 - x^2 - 2*x + 1)) + 1/12*4^(2/3)*(-
1)^(1/3)*log((4^(1/3)*(x^2 - x - 1) + 2*(-1)^(1/3)*(x^6 - 1)^(1/3))/(x^2 +
x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}+1}{\left(x^{6}-1\right)^{\frac{1}{3}}\left(x^{2}+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x-1)/(x^6-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/((x^6 - 1)^(1/3)*(x^2 + x - 1)), x)
```

maple [C] time = 30.97, size = 2695, normalized size = 12.83

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2+x-1)/(x^6-1)^(1/3),x)
```

```
[Out] 1/6*RootOf(_Z^3+2)*ln((-18*x^2*(x^6-1)^(2/3)-35*RootOf(_Z^3+2)*x^6+15*RootO
f(_Z^3+2)*x+15*RootOf(_Z^3+2)*x^5+18*x*(x^6-1)^(2/3)+18*(x^6-1)^(2/3)-25*Ro
otOf(_Z^3+2)*x^3+56*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)+24*
RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x+48*RootOf(RootOf(_Z^3
+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x+35*RootOf(_Z^3+2)-56
*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^6-40*RootOf(RootOf(_
Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^3+48*RootOf(RootOf(_Z^3+2)^2+2*_Z*Ro
otOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^5+30*RootOf(RootOf(_Z^3+2)^2+2*_Z
*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^5+48*(x^6-1)^(2/3)*RootOf(RootOf
(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x^2+24*x^5*RootOf(R
ootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)-50*RootOf(RootOf(_Z^3+2)^2+2*_Z
*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^3-80*RootOf(RootOf(_Z^3+2)^2+2*_
Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^3+30*RootOf(RootOf(_Z^3+2)^2+
2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x+18*(x^6-1)^(1/3)*RootOf(Root
Of(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x^4-48*(x^6-1)^(2/3
)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-36
*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_
```

```
Z^3+2)*x^3-18*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_
Z^2)*RootOf(_Z^3+2)*x^2+36*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootO
f(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x-15*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)*x^4-15
*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)+30*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)*x^3-48*(x
^6-1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3
+2)^2+15*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)*x^2-30*RootOf(_Z^3+2)^2*(x^6-1)^(1/
3)*x+18*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*R
ootOf(_Z^3+2))/(x^2+x-1)^3)-1/6*ln(-(-30*x^2*(x^6-1)^(2/3)-35*RootOf(_Z^3+2
)*x^6+45*RootOf(_Z^3+2)*x+45*RootOf(_Z^3+2)*x^5+30*x*(x^6-1)^(2/3)+30*(x^6-
1)^(2/3)-75*RootOf(_Z^3+2)*x^3+14*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+
2)+4*_Z^2)+18*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x-12*Root
Of(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x+35*Roo
tOf(_Z^3+2)-14*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^6-30*R
ootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^3-12*RootOf(RootOf(_Z^
3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^5-30*RootOf(RootOf(
_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^5+48*(x^6-1)^(2/3)
*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x^2+1
8*x^5*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)+50*RootOf(RootOf(
_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^3+20*RootOf(RootOf
(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^3-30*RootOf(Roo
tOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x+30*(x^6-1)^(1/
3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x^4-4
8*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(
_Z^3+2)^2*x-60*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*
_Z^2)*RootOf(_Z^3+2)*x^3-30*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*Root
Of(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x^2+60*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+2
)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x-9*RootOf(_Z^3+2)^2*(x^6-1)
^(1/3)*x^4-9*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)+18*RootOf(_Z^3+2)^2*(x^6-1)^(1/
3)*x^3-48*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)
*RootOf(_Z^3+2)^2+9*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)*x^2-18*RootOf(_Z^3+2)^2*
(x^6-1)^(1/3)*x+30*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2
)+4*_Z^2)*RootOf(_Z^3+2))/(x^2+x-1)^3)*RootOf(_Z^3+2)-1/3*ln(-(-30*x^2*(x^6
-1)^(2/3)-35*RootOf(_Z^3+2)*x^6+45*RootOf(_Z^3+2)*x+45*RootOf(_Z^3+2)*x^5+3
0*x*(x^6-1)^(2/3)+30*(x^6-1)^(2/3)-75*RootOf(_Z^3+2)*x^3+14*RootOf(RootOf(_
Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)+18*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf
(_Z^3+2)+4*_Z^2)*x-12*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2
*RootOf(_Z^3+2)^2*x+35*RootOf(_Z^3+2)-14*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootO
f(_Z^3+2)+4*_Z^2)*x^6-30*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2
)*x^3-12*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+
2)^2*x^5-30*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3
+2)^3*x^5+48*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z
^2)*RootOf(_Z^3+2)^2*x^2+18*x^5*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)
+4*_Z^2)+50*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3
+2)^3*x^3+20*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_
Z^3+2)^2*x^3-30*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(
_Z^3+2)^3*x+30*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*
_Z^2)*RootOf(_Z^3+2)*x^4-48*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*Root
Of(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-60*(x^6-1)^(1/3)*RootOf(RootOf(_Z^3+2
)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x^3-30*(x^6-1)^(1/3)*RootOf(
RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x^2+60*(x^6-1)^
(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x-
9*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)*x^4-9*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)+18*Ro
otOf(_Z^3+2)^2*(x^6-1)^(1/3)*x^3-48*(x^6-1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2
*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2+9*RootOf(_Z^3+2)^2*(x^6-1)^(1/3
)*x^2-18*RootOf(_Z^3+2)^2*(x^6-1)^(1/3)*x+30*(x^6-1)^(1/3)*RootOf(RootOf(_Z
^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2))/(x^2+x-1)^3)*RootOf(Roo
tOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\left(x^6-1\right)^{\frac{1}{3}}\left(x^2+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x-1)/(x^6-1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/((x^6 - 1)^(1/3)*(x^2 + x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2+1}{\left(x^6-1\right)^{1/3}\,\left(x^2+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/((x^6 - 1)^(1/3)*(x + x^2 - 1)),x)
```

```
[Out] int((x^2 + 1)/((x^6 - 1)^(1/3)*(x + x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt[3]{(x-1)(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,\left(x^2+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2+x-1)/(x**6-1)**(1/3),x)
```

```
[Out] Integral((x**2 + 1)/(((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1))**(1/3)
*(x**2 + x - 1)), x)
```

3.2029 $\int \frac{(-b^2+ax^2)^2}{(b^2+ax^2)^2\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$

Optimal. Leaf size=211

$$\frac{2x\left(ax^2+2b^2\right)}{\left(ax^2+b^2\right)\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{bx}{\sqrt{ax^2+b^2}\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}-\frac{2\sqrt{2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{2}\sqrt{b}\sqrt{}}\right.}{\sqrt{}}$$

Rubi [F] time = 2.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=$ 0.000, Rules used = {}

$$\int \frac{(-b^2+ax^2)^2}{(b^2+ax^2)^2\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-b^2 + a*x^2)^2/((b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

```
[Out] Defer[Int][1/Sqrt[b + Sqrt[b^2 + a*x^2]], x] - b*Defer[Int][1/((b - Sqrt[-a
]*x)*Sqrt[b + Sqrt[b^2 + a*x^2]]), x] - b*Defer[Int][1/((b + Sqrt[-a]*x)*Sq
rt[b + Sqrt[b^2 + a*x^2]]), x] - a*b^2*Defer[Int][1/((Sqrt[-a]*b - a*x)^2*S
qrt[b + Sqrt[b^2 + a*x^2]]), x] - a*b^2*Defer[Int][1/((Sqrt[-a]*b + a*x)^2*
Sqrt[b + Sqrt[b^2 + a*x^2]]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-b^2+ax^2)^2}{(b^2+ax^2)^2\sqrt{b+\sqrt{b^2+ax^2}}}\,dx &= \int \left(\frac{1}{\sqrt{b+\sqrt{b^2+ax^2}}}+\frac{4b^4}{(b^2+ax^2)^2\sqrt{b+\sqrt{b^2+ax^2}}}-\frac{4b^2}{(b^2+ax^2)\sqrt{b+}}\right.\\
&= -\left((4b^2)\int \frac{1}{(b^2+ax^2)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx\right)+(4b^4)\int \frac{1}{(b^2+ax^2)^2\sqrt{b+}}\\
&= -\left((4b^2)\int \left(\frac{1}{2b\left(b-\sqrt{-a}\,x\right)\sqrt{b+\sqrt{b^2+ax^2}}}+\frac{1}{2b\left(b+\sqrt{-a}\,x\right)\sqrt{b+\sqrt{b^2}}}\right.\right.\\
&= -\left((2b)\int \frac{1}{\left(b-\sqrt{-a}\,x\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx\right)-(2b)\int \frac{1}{\left(b+\sqrt{-a}\,x\right)\sqrt{b+}}\\
&= -\left((2b)\int \frac{1}{\left(b-\sqrt{-a}\,x\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx\right)-(2b)\int \frac{1}{\left(b+\sqrt{-a}\,x\right)\sqrt{b+}}\\
&= b\int \frac{1}{\left(b-\sqrt{-a}\,x\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx+b\int \frac{1}{\left(b+\sqrt{-a}\,x\right)\sqrt{b+\sqrt{b^2+ax^2}}}
\end{aligned}$$

Mathematica [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{(-b^2+ax^2)^2}{(b^2+ax^2)^2\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-b^2 + a*x^2)^2/((b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

```
[Out] Integrate[(-b^2 + a*x^2)^2/((b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]]), x
]
```

IntegrateAlgebraic [A] time = 0.48, size = 179, normalized size = 0.85

$$\frac{2x\left(ax^2+2b^2\right)}{\left(ax^2+b^2\right)\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{bx}{\sqrt{ax^2+b^2}\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b^2 + a*x^2)^2/((b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*
x^2]]),x]
```

```
[Out] -((b*x)/(Sqrt[b^2 + a*x^2]*Sqrt[b + Sqrt[b^2 + a*x^2]])) + (2*x*(2*b^2 + a*
x^2))/((b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]]) - (Sqrt[b]*ArcTan[(Sqrt[a
]*x)/(Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/Sqrt[a] - (Sqrt[2]*Sqrt[b]*Arc
Tan[(Sqrt[a]*x)/(Sqrt[2]*Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/Sqrt[a]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b^2)^2/(a*x^2+b^2)^2/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2-b^2\right)^2}{\left(ax^2+b^2\right)^2\sqrt{b+\sqrt{ax^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b^2)^2/(a*x^2+b^2)^2/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((a*x^2 - b^2)^2/((a*x^2 + b^2)^2*sqrt(b + sqrt(a*x^2 + b^2))), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^2-b^2\right)^2}{\left(a\,x^2+b^2\right)^2\sqrt{b+\sqrt{a\,x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b^2)^2/(a*x^2+b^2)^2/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x^2-b^2)^2/(a*x^2+b^2)^2/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2-b^2\right)^2}{\left(ax^2+b^2\right)^2\sqrt{b+\sqrt{ax^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b^2)^2/(a*x^2+b^2)^2/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((a*x^2 - b^2)^2/((a*x^2 + b^2)^2*sqrt(b + sqrt(a*x^2 + b^2))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x^2-b^2\right)^2}{\left(b^2+a\,x^2\right)^2\,\sqrt{b+\sqrt{b^2+a\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 - b^2)^2/((a*x^2 + b^2)^2*(b + (a*x^2 + b^2)^(1/2))^(1/2)),x)
```

```
[Out] int((a*x^2 - b^2)^2/((a*x^2 + b^2)^2*(b + (a*x^2 + b^2)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2-b^2\right)^2}{\sqrt{b+\sqrt{ax^2+b^2}}\,\left(ax^2+b^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b**2)**2/(a*x**2+b**2)**2/(b+(a*x**2+b**2)**(1/2))**(1/2)
,x)
```

```
[Out] Integral((a*x**2 - b**2)**2/(sqrt(b + sqrt(a*x**2 + b**2))*(a*x**2 + b**2)*
*2), x)
```

3.2030 $\int \frac{(-2+(1+k)x)\left(a-a(1+k)x+(1+ak)x^2\right)}{(-1+x)\sqrt[3]{(1-x)x(1-kx)}\,(-1+kx)\left(b-b(1+k)x+(-1+bk)x^2\right)}\,dx$

Optimal. Leaf size=212

$$\frac{(a+b)\log\left(x-\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{b^{2/3}}+\frac{(-a-b)\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-\right.}{2b^{2/3}}$$

Rubi [F] time = 55.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+(1+k)x)\left(a-a(1+k)x+(1+ak)x^2\right)}{(-1+x)\sqrt[3]{(1-x)x(1-kx)}\,(-1+kx)\left(b-b(1+k)x+(-1+bk)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + (1 + k)*x)*(a - a*(1 + k)*x + (1 + a*k)*x^2))/((-1 + x)*((1 - x)
*x*(1 - k*x))^(1/3)*(-1 + k*x)*(b - b*(1 + k)*x + (-1 + b*k)*x^2)),x]
```

```
[Out] (-3*(1 + k)*(1 + a*k)*x)/((1 - k)*(1 - b*k)*((1 - x)*x*(1 - k*x))^(1/3)) +
(3*(2 + a + b + 4*a*k + a*(1 - 2*b)*k^2 + b*k^2)*x)/((1 - k)*(1 - b*k)^2*((
1 - x)*x*(1 - k*x))^(1/3)) - (3*(1 + k)*(1 + a*k)*(1 - x)*(((1 - k)*x)/(1 -
 k*x))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, (1 - x)/(1 - k*x)])/((1 - k)^
2*(1 - b*k)*((1 - x)*x*(1 - k*x))^(1/3)) + (3*(1 + k)*(2 + a + b + 4*a*k +
a*(1 - 2*b)*k^2 + b*k^2)*(1 - x)*(((1 - k)*x)/(1 - k*x))^(1/3)*Hypergeometr
ic2F1[1/3, 2/3, 5/3, (1 - x)/(1 - k*x)])/(2*(1 - k)^2*(1 - b*k)^2*((1 - x)*
x*(1 - k*x))^(1/3)) + ((a + b)*(3 + b + 3*k + b*k^3 + (4 + b^2*(1 - k)^2*(1
 + k + k^2) + b*(5 + 2*k + 5*k^2))/(Sqrt[b]*Sqrt[4 + b*(1 - k)^2]))*(1 - x)
^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Int][1/((1 - x)^(4/3)*x^(1/3)*(1 - k*x
)^(4/3)*(-(b*(1 + k)) - Sqrt[b]*Sqrt[4 + b - 2*b*k + b*k^2] + 2*(-1 + b*k)*
x)), x])/((1 - b*k)^2*((1 - x)*x*(1 - k*x))^(1/3)) + ((a + b)*(3*(1 + k) +
b*(1 + k^3) - (4 + b*(5 + 2*k + 5*k^2) + b^2*(1 - k - k^3 + k^4))/(Sqrt[b]*
Sqrt[4 + b*(1 - k)^2]))*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Int][1/
((1 - x)^(4/3)*x^(1/3)*(1 - k*x)^(4/3)*(-(b*(1 + k)) + Sqrt[b]*Sqrt[4 + b -
 2*b*k + b*k^2] + 2*(-1 + b*k)*x)), x])/((1 - b*k)^2*((1 - x)*x*(1 - k*x))^
(1/3))
```

Rubi steps

$$\int \frac{(-2+(1+k)x)\left(a-a(1+k)x+(1+ak)x^2\right)}{(-1+x)\sqrt[3]{(1-x)x(1-kx)}\,(-1+kx)\left(b-b(1+k)x+(-1+bk)x^2\right)}\,dx = \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{(-2+(1}{\sqrt[3]{1-x}\,(-1+x)\,\sqrt[3]{x}}}{\sqrt[3]{(1-x)x(1}}$$

$$= -\frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{(-2+(1}{(1-x)^{4/3}\,\sqrt[3]{x}\,\sqrt[3]{1}}}{\sqrt[3]{(1-x)x(1}}$$

$$= \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{(-2+(1+k)x)}{(1-x)^{4/3}\,\sqrt[3]{x}\,(1-kx}}{\sqrt[3]{(1-x)x(1-k}}$$

$$= \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \left(\frac{2+a+b+4ak+a(1}{(1-bk)^2(1-x)^{4/3}}\right.}{}$$

$$= -\frac{\left((1+k)(1+ak)\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)}{(1-bk)\sqrt[3]{(1-x)x(1-}}$$

$$= -\frac{3(1+k)(1+ak)x}{(1-k)(1-bk)\sqrt[3]{(1-x)x(1-kx)}} + \cdots$$

$$= -\frac{3(1+k)(1+ak)x}{(1-k)(1-bk)\sqrt[3]{(1-x)x(1-kx)}} + \cdots$$

Mathematica [F] time = 9.44, size = 0, normalized size = 0.00

$$\int \frac{(-2+(1+k)x)\left(a-a(1+k)x+(1+ak)x^2\right)}{(-1+x)\sqrt[3]{(1-x)x(1-kx)}\,(-1+kx)\left(b-b(1+k)x+(-1+bk)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + (1 + k)*x)*(a - a*(1 + k)*x + (1 + a*k)*x^2))/((-1 + x)*((
1 - x)*x*(1 - k*x))^(1/3)*(-1 + k*x)*(b - b*(1 + k)*x + (-1 + b*k)*x^2)),x]
```

```
[Out] Integrate[((-2 + (1 + k)*x)*(a - a*(1 + k)*x + (1 + a*k)*x^2))/((-1 + x)*((
1 - x)*x*(1 - k*x))^(1/3)*(-1 + k*x)*(b - b*(1 + k)*x + (-1 + b*k)*x^2)), x
]
```

IntegrateAlgebraic [A] time = 0.50, size = 212, normalized size = 1.00

$$\frac{(a+b)\log\left(x-\sqrt[3]{b}\,\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{b^{2/3}} + \frac{(-a-b)\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+x^2\right)}{2b^{2/3}} + \frac{\left(-\sqrt{3}a-\sqrt{3}b\right)\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+x}\right)}{b^{2/3}} + \frac{3\left(kx^3-kx^2-x^2+x\right)^{2/3}}{(x-1)(kx-1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + (1 + k)*x)*(a - a*(1 + k)*x + (1 + a*k)*x^2))/((-
1 + x)*((1 - x)*x*(1 - k*x))^(1/3)*(-1 + k*x)*(b - b*(1 + k)*x + (-1 + b*k)
*x^2)),x]
```

```
[Out] (3*(x - x^2 - k*x^2 + k*x^3)^(2/3))/((-1 + x)*(-1 + k*x)) + ((-(Sqrt[3]*a)
- Sqrt[3]*b)*ArcTan[(Sqrt[3]*x)/(x + 2*b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(
1/3))])/b^(2/3) + ((a + b)*Log[x - b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3)
])/b^(2/3) + ((-a - b)*Log[x^2 + b^(1/3)*x*(x + (-1 - k)*x^2 + k*x^3)^(1/3)
 + b^(2/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3)])/(2*b^(2/3))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(a-a*(1+k)*x+(a*k+1)*x^2)/(-1+x)/((1-x)*x*(-k*x+1))^
(1/3)/(k*x-1)/(b-b*(1+k)*x+(b*k-1)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a(k+1)x-(ak+1)x^2-a\right)((k+1)x-2)}{\left(b(k+1)x-(bk-1)x^2-b\right)((kx-1)(x-1)x)^{\frac{1}{3}}(kx-1)(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(a-a*(1+k)*x+(a*k+1)*x^2)/(-1+x)/((1-x)*x*(-k*x+1))^
(1/3)/(k*x-1)/(b-b*(1+k)*x+(b*k-1)*x^2),x, algorithm="giac")
```

```
[Out] integrate((a*(k + 1)*x - (a*k + 1)*x^2 - a)*((k + 1)*x - 2)/((b*(k + 1)*x -
 (b*k - 1)*x^2 - b)*((k*x - 1)*(x - 1)*x)^(1/3)*(k*x - 1)*(x - 1)), x)
```

maple [**F**] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{(-2+(1+k)x)\left(a-a(1+k)x+(ka+1)x^2\right)}{(-1+x)((1-x)x(-kx+1))^{\frac{1}{3}}(kx-1)\left(b-b(1+k)x+(bk-1)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+(1+k)*x)*(a-a*(1+k)*x+(a*k+1)*x^2)/(-1+x)/((1-x)*x*(-k*x+1))^(1/3)/
(k*x-1)/(b-b*(1+k)*x+(b*k-1)*x^2),x)
```

```
[Out] int((-2+(1+k)*x)*(a-a*(1+k)*x+(a*k+1)*x^2)/(-1+x)/((1-x)*x*(-k*x+1))^(1/3)/
(k*x-1)/(b-b*(1+k)*x+(b*k-1)*x^2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a(k+1)x-(ak+1)x^2-a\right)((k+1)x-2)}{\left(b(k+1)x-(bk-1)x^2-b\right)((kx-1)(x-1)x)^{\frac{1}{3}}(kx-1)(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(a-a*(1+k)*x+(a*k+1)*x^2)/(-1+x)/((1-x)*x*(-k*x+1))^
(1/3)/(k*x-1)/(b-b*(1+k)*x+(b*k-1)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((a*(k + 1)*x - (a*k + 1)*x^2 - a)*((k + 1)*x - 2)/((b*(k + 1)*x -
 (b*k - 1)*x^2 - b)*((k*x - 1)*(x - 1)*x)^(1/3)*(k*x - 1)*(x - 1)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x\,(k+1)-2)\,\left((a\,k+1)\,x^2-a\,(k+1)\,x+a\right)}{(k\,x-1)\,(x-1)\,(x\,(k\,x-1)\,(x-1))^{1/3}\,\left((b\,k-1)\,x^2-b\,(k+1)\,x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x*(k + 1) - 2)*(a + x^2*(a*k + 1) - a*x*(k + 1)))/((k*x - 1)*(x - 1)*
(x*(k*x - 1)*(x - 1))^(1/3)*(b + x^2*(b*k - 1) - b*x*(k + 1))),x)
```

```
[Out] int(((x*(k + 1) - 2)*(a + x^2*(a*k + 1) - a*x*(k + 1)))/((k*x - 1)*(x - 1)*
(x*(k*x - 1)*(x - 1))^(1/3)*(b + x^2*(b*k - 1) - b*x*(k + 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(a-a*(1+k)*x+(a*k+1)*x**2)/(-1+x)/((1-x)*x*(-k*x+1))
**(1/3)/(k*x-1)/(b-b*(1+k)*x+(b*k-1)*x**2),x)
```

```
[Out] Timed out
```

3.2031 $\int \frac{(b+x^3)^3}{\sqrt[3]{a+x^3}}\, dx$

Optimal. Leaf size=212

$$\frac{1}{162}\left(a+x^3\right)^{2/3}\left(28a^2x-108abx-21ax^4+162b^2x+81bx^4+18x^7\right)+\frac{1}{243}\left(14a^3-54a^2b+81ab^2-81b^3\right)\log\Big($$

Rubi [A] time = 0.12, antiderivative size = 171, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {416, 528, 388, 239}

$$\frac{1}{162}x\left(28a^2-87ab+99b^2\right)\left(a+x^3\right)^{2/3}+\frac{1}{162}\left(14a^3-54a^2b+81ab^2-81b^3\right)\log\left(\sqrt[3]{a+x^3}-x\right)-\frac{\left(14a^3-54a^2b+81ab^2-81b^3\right)\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{a+x^3}}+1}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{1}{9}x\left(a+x^3\right)^{2/3}\left(b+x^3\right)^2-\frac{1}{54}x(7a-15b)\left(a+x^3\right)^{2/3}\left(b+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + x^3)^3/(a + x^3)^(1/3),x]
```

```
[Out] ((28*a^2 - 87*a*b + 99*b^2)*x*(a + x^3)^(2/3))/162 - ((7*a - 15*b)*x*(a + x
^3)^(2/3)*(b + x^3))/54 + (x*(a + x^3)^(2/3)*(b + x^3)^2)/9 - ((14*a^3 - 54
*a^2*b + 81*a*b^2 - 81*b^3)*ArcTan[(1 + (2*x)/(a + x^3)^(1/3))/Sqrt[3]])/(8
1*Sqrt[3]) + ((14*a^3 - 54*a^2*b + 81*a*b^2 - 81*b^3)*Log[-x + (a + x^3)^(1
/3)])/162
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(b+x^3)^3}{\sqrt[3]{a+x^3}}\,dx = \frac{1}{9}x\left(a+x^3\right)^{2/3}\left(b+x^3\right)^2+\frac{1}{9}\int \frac{\left(b+x^3\right)\left(-((a-9b)b)+(-7a+15b)x^3\right)}{\sqrt[3]{a+x^3}}\,dx$$

$$= -\frac{1}{54}(7a-15b)x\left(a+x^3\right)^{2/3}\left(b+x^3\right)+\frac{1}{9}x\left(a+x^3\right)^{2/3}\left(b+x^3\right)^2+\frac{1}{54}\int \frac{b\left(7a^2-21ab+54b^2\right)}{\sqrt[3]{a}}$$

$$= \frac{1}{162}\left(28a^2-87ab+99b^2\right)x\left(a+x^3\right)^{2/3}-\frac{1}{54}(7a-15b)x\left(a+x^3\right)^{2/3}\left(b+x^3\right)+\frac{1}{9}x\left(a+x^3\right)^{2/3}($$

$$= \frac{1}{162}\left(28a^2-87ab+99b^2\right)x\left(a+x^3\right)^{2/3}-\frac{1}{54}(7a-15b)x\left(a+x^3\right)^{2/3}\left(b+x^3\right)+\frac{1}{9}x\left(a+x^3\right)^{2/3}($$

Mathematica [A] time = 5.15, size = 151, normalized size = 0.71

$$\frac{1}{486}\left(3x\left(a+x^3\right)^{2/3}\left(28a^2-3a\left(36b+7x^3\right)+9\left(18b^2+9bx^3+2x^6\right)\right)+\left(-14a^3+54a^2b-81ab^2+81b^3\right)\left(-2\log\left(1-\frac{x}{\sqrt[3]{a+x^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{a+x^3}}+1}{\sqrt{3}}\right)+\log\left(\frac{x}{\sqrt[3]{a+x^3}}+\frac{x^2}{\left(a+x^3\right)^{2/3}}+1\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + x^3)^3/(a + x^3)^(1/3),x]
```

```
[Out] (3*x*(a + x^3)^(2/3)*(28*a^2 - 3*a*(36*b + 7*x^3) + 9*(18*b^2 + 9*b*x^3 + 2
*x^6)) + (-14*a^3 + 54*a^2*b - 81*a*b^2 + 81*b^3)*(2*Sqrt[3]*ArcTan[(1 + (2
*x)/(a + x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - x/(a + x^3)^(1/3)] + Log[1 + x^2/
(a + x^3)^(2/3) + x/(a + x^3)^(1/3)]))/486
```

IntegrateAlgebraic [A] time = 1.34, size = 212, normalized size = 1.00

$$\frac{1}{162}\left(a+x^3\right)^{2/3}\left(28a^2x-108abx-21ax^4+162b^2x+81bx^4+18x^7\right)+\frac{1}{243}\left(14a^3-54a^2b+81ab^2-81b^3\right)\log\left(\sqrt[3]{a+x^3}-x\right)+\frac{1}{243}\left(-14\sqrt{3}a^3+54\sqrt{3}a^2b-81\sqrt{3}ab^2+81\sqrt{3}b^3\right)\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{a+x^3}+x}\right)+\frac{1}{486}\left(-14a^3+54a^2b-81ab^2+81b^3\right)\log\left(x\sqrt[3]{a+x^3}+\left(a+x^3\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + x^3)^3/(a + x^3)^(1/3),x]
```

```
[Out] ((a + x^3)^(2/3)*(28*a^2*x - 108*a*b*x + 162*b^2*x - 21*a*x^4 + 81*b*x^4 +
18*x^7))/162 + ((-14*Sqrt[3]*a^3 + 54*Sqrt[3]*a^2*b - 81*Sqrt[3]*a*b^2 + 81
*Sqrt[3]*b^3)*ArcTan[(Sqrt[3]*x)/(x + 2*(a + x^3)^(1/3))])/243 + ((14*a^3 -
 54*a^2*b + 81*a*b^2 - 81*b^3)*Log[-x + (a + x^3)^(1/3)])/243 + ((-14*a^3 +
 54*a^2*b - 81*a*b^2 + 81*b^3)*Log[x^2 + x*(a + x^3)^(1/3) + (a + x^3)^(2/3
)])/486
```

fricas [A] time = 0.77, size = 190, normalized size = 0.90

$$\frac{1}{243}\sqrt{3}\left(14a^3-54a^2b+81ab^2-81b^3\right)\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+a\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{243}\left(14a^3-54a^2b+81ab^2-81b^3\right)\log\left(-\frac{x-\left(x^3+a\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{486}\left(14a^3-54a^2b+81ab^2-81b^3\right)\log\left(\frac{x^2+\left(x^3+a\right)^{\frac{1}{3}}x+\left(x^3+a\right)^{\frac{2}{3}}}{x^2}\right)+\frac{1}{162}\left(18x^7-3(7a-27b)x^4+2\left(14a^2-54ab+81b^2\right)x\right)\left(x^3+a\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+b)^3/(x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/243*sqrt(3)*(14*a^3 - 54*a^2*b + 81*a*b^2 - 81*b^3)*arctan(1/3*(sqrt(3)*x
 + 2*sqrt(3)*(x^3 + a)^(1/3))/x) + 1/243*(14*a^3 - 54*a^2*b + 81*a*b^2 - 81
*b^3)*log(-(x - (x^3 + a)^(1/3))/x) - 1/486*(14*a^3 - 54*a^2*b + 81*a*b^2 -
 81*b^3)*log((x^2 + (x^3 + a)^(1/3)*x + (x^3 + a)^(2/3))/x^2) + 1/162*(18*x
^7 - 3*(7*a - 27*b)*x^4 + 2*(14*a^2 - 54*a*b + 81*b^2)*x)*(x^3 + a)^(2/3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+b\right)^3}{\left(x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+b)^3/(x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 + b)^3/(x^3 + a)^(1/3), x)
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}+b\right)^{3}}{\left(x^{3}+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+b)^3/(x^3+a)^(1/3),x)
```

```
[Out] int((x^3+b)^3/(x^3+a)^(1/3),x)
```

maxima [B] time = 0.60, size = 480, normalized size = 2.26

$$\frac{14}{243}\sqrt{3}a^{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^{3}+a\right)^{\frac{1}{3}}}{x}+1\right)\right)-\frac{1}{6}\left(2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^{3}+a\right)^{\frac{1}{3}}}{x}+1\right)\right)-\log\left(\frac{\left(x^{3}+a\right)^{\frac{1}{3}}}{x}+\frac{\left(x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}+1\right)+2\log\left(\frac{\left(x^{3}+a\right)^{\frac{1}{3}}}{x}-1\right)\right)b^{3}-\frac{7}{243}a^{3}\log\left(\frac{\left(x^{3}+a\right)^{\frac{1}{3}}}{x}+\frac{\left(x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}+1\right)+\frac{14}{243}a^{3}\log\left(\frac{\left(x^{3}+a\right)^{\frac{1}{3}}}{x}-1\right)+\frac{1}{6}\left(2\sqrt{3}a\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^{3}+a\right)^{\frac{1}{3}}}{x}+1\right)\right)-a\log\left(\frac{\left(x^{3}+a\right)^{\frac{1}{3}}}{x}+\frac{\left(x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}+1\right)+2a\log\left(\frac{\left(x^{3}+a\right)^{\frac{1}{3}}}{x}-1\right)+\frac{6\left(x^{3}+a\right)^{\frac{2}{3}}a}{x^{2}\left(\frac{x^{3}+a}{x^{3}}-1\right)}\right)b^{2}-\frac{1}{18}\left(4\sqrt{3}a^{2}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^{3}+a\right)^{\frac{1}{3}}}{x}+1\right)\right)-2a^{2}\log\left(\frac{\left(x^{3}+a\right)^{\frac{1}{3}}}{x}+\frac{\left(x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}+1\right)+4a^{2}\log\left(\frac{\left(x^{3}+a\right)^{\frac{1}{3}}}{x}-1\right)+\frac{3\left(\frac{7\left(x^{3}+a\right)^{\frac{2}{3}}a^{2}}{x^{2}}-\frac{4\left(x^{3}+a\right)^{\frac{5}{3}}a^{2}}{x^{5}}\right)}{\frac{2\left(x^{3}+a\right)}{x^{3}}-\frac{\left(x^{3}+a\right)^{2}}{x^{6}}-1}\right)b+\frac{\frac{67\left(x^{3}+a\right)^{\frac{2}{3}}a^{3}}{x^{2}}-\frac{77\left(x^{3}+a\right)^{\frac{5}{3}}a^{3}}{x^{5}}+\frac{28\left(x^{3}+a\right)^{\frac{8}{3}}a^{3}}{x^{8}}}{162\left(\frac{3\left(x^{3}+a\right)}{x^{3}}-\frac{3\left(x^{3}+a\right)^{2}}{x^{6}}+\frac{\left(x^{3}+a\right)^{3}}{x^{9}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+b)^3/(x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 14/243*sqrt(3)*a^3*arctan(1/3*sqrt(3)*(2*(x^3 + a)^(1/3)/x + 1)) - 1/6*(2*s
qrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + a)^(1/3)/x + 1)) - log((x^3 + a)^(1/3)/
x + (x^3 + a)^(2/3)/x^2 + 1) + 2*log((x^3 + a)^(1/3)/x - 1))*b^3 - 7/243*a^
3*log((x^3 + a)^(1/3)/x + (x^3 + a)^(2/3)/x^2 + 1) + 14/243*a^3*log((x^3 +
a)^(1/3)/x - 1) + 1/6*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*(x^3 + a)^(1/3)/x
+ 1)) - a*log((x^3 + a)^(1/3)/x + (x^3 + a)^(2/3)/x^2 + 1) + 2*a*log((x^3 +
 a)^(1/3)/x - 1) + 6*(x^3 + a)^(2/3)*a/(x^2*((x^3 + a)/x^3 - 1)))*b^2 - 1/1
8*(4*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*(x^3 + a)^(1/3)/x + 1)) - 2*a^2*log(
(x^3 + a)^(1/3)/x + (x^3 + a)^(2/3)/x^2 + 1) + 4*a^2*log((x^3 + a)^(1/3)/x
- 1) + 3*(7*(x^3 + a)^(2/3)*a^2/x^2 - 4*(x^3 + a)^(5/3)*a^2/x^5)/(2*(x^3 +
a)/x^3 - (x^3 + a)^2/x^6 - 1))*b + 1/162*(67*(x^3 + a)^(2/3)*a^3/x^2 - 77*(
x^3 + a)^(5/3)*a^3/x^5 + 28*(x^3 + a)^(8/3)*a^3/x^8)/(3*(x^3 + a)/x^3 - 3*(
x^3 + a)^2/x^6 + (x^3 + a)^3/x^9 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^{3}+b\right)^{3}}{\left(x^{3}+a\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + x^3)^3/(a + x^3)^(1/3),x)
```

```
[Out] int((b + x^3)^3/(a + x^3)^(1/3), x)
```

sympy [C] time = 3.99, size = 151, normalized size = 0.71

$$\frac{b^{3}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{x^{3}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{b^{2}x^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{x^{3}e^{i\pi}}{a}\right)}{\sqrt[3]{a}\,\Gamma\left(\frac{7}{3}\right)}+\frac{bx^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{x^{3}e^{i\pi}}{a}\right)}{\sqrt[3]{a}\,\Gamma\left(\frac{10}{3}\right)}+\frac{x^{10}\Gamma\left(\frac{10}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{10}{3}\\\frac{13}{3}\end{matrix}\middle|\frac{x^{3}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+b)**3/(x**3+a)**(1/3),x)
```

```
[Out] b**3*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_polar(I*pi)/a)/(3*a**(
1/3)*gamma(4/3)) + b**2*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), x**3*exp_
polar(I*pi)/a)/(a**(1/3)*gamma(7/3)) + b*x**7*gamma(7/3)*hyper((1/3, 7/3),
(10/3,), x**3*exp_polar(I*pi)/a)/(a**(1/3)*gamma(10/3)) + x**10*gamma(10/3)
*hyper((1/3, 10/3), (13/3,), x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(13/3
))
```

3.2032 $\int \frac{-3+(1-2k^2)x+3k^2x^2+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\left(-1+d-(2+d)x-(1+dk^2)x^2+dk^2x^3\right)}\,dx$

Optimal. Leaf size=212

$$\frac{\log\left(-\sqrt[3]{d}\,\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}+x+1\right)}{d^{2/3}}-\frac{\log\left(d^{2/3}\left(k^2x^4+(-k^2-1)x^2+1\right)^{2/3}+\left(\sqrt[3]{d}\,x+\sqrt[3]{d}\right)\sqrt[3]{k^2x^4+\cdots}\right.}{2d^{2/3}}$$

Rubi [F] time = 6.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3+(1-2k^2)x+3k^2x^2+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\left(-1+d-(2+d)x-(1+dk^2)x^2+dk^2x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3 + (1 - 2*k^2)*x + 3*k^2*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(
1/3)*(-1 + d - (2 + d)*x - (1 + d*k^2)*x^2 + d*k^2*x^3)),x]
```

```
[Out] (x*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*AppellF1[1/2, 1/3, 1/3, 3/2, x^2, k^
2*x^2])/(d*((1 - x^2)*(1 - k^2*x^2))^(1/3)) - ((1 - 4*d)*(1 - x^2)^(1/3)*(1
 - k^2*x^2)^(1/3)*Defer[Int][1/((1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*(1 - d
+ (2 + d)*x + (1 + d*k^2)*x^2 - d*k^2*x^3)), x])/(d*((1 - x^2)*(1 - k^2*x^2
))^(1/3)) - (2*(1 + d - d*k^2)*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*Defer[In
t][x/((1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*(1 - d + (2 + d)*x + (1 + d*k^2)*
x^2 - d*k^2*x^3)), x])/(d*((1 - x^2)*(1 - k^2*x^2))^(1/3)) - ((1 + 4*d*k^2)
*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*Defer[Int][x^2/((1 - x^2)^(1/3)*(1 - k
^2*x^2)^(1/3)*(1 - d + (2 + d)*x + (1 + d*k^2)*x^2 - d*k^2*x^3)), x])/(d*((
1 - x^2)*(1 - k^2*x^2))^(1/3))
```

Rubi steps

$$\int \frac{-3+\left(1-2k^2\right)x+3k^2x^2+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d-(2+d)x-\left(1+dk^2\right)x^2+dk^2x^3\right)}\,dx = \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{-3+(1-\cdots}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,(-\cdots}}{\sqrt[3]{\left(1-x^2\right)\left(1-k\cdots\right.}}$$

$$= \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int\left(\frac{1}{d\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}}\cdots\right.}{\sqrt[3]{(1-\cdots}}$$

$$= \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{1}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}}\,d\cdots}{d\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{x\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,k^2\cdots\right.}{d\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{x\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,k^2\cdots\right.}{d\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{x\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,k^2\cdots\right.}{d\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

Mathematica [F] time = 4.04, size = 0, normalized size = 0.00

$$\int \frac{-3+\left(1-2k^2\right)x+3k^2x^2+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d-(2+d)x-\left(1+dk^2\right)x^2+dk^2x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3 + (1 - 2*k^2)*x + 3*k^2*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x
^2))^(1/3)*(-1 + d - (2 + d)*x - (1 + d*k^2)*x^2 + d*k^2*x^3)),x]
```

```
[Out] Integrate[(-3 + (1 - 2*k^2)*x + 3*k^2*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x
^2))^(1/3)*(-1 + d - (2 + d)*x - (1 + d*k^2)*x^2 + d*k^2*x^3)), x]
```

IntegrateAlgebraic [A] time = 5.64, size = 212, normalized size = 1.00

$$\frac{\log\left(-\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+x+1\right)}{d^{2/3}}-\frac{\log\left(d^{2/3}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+\left(\sqrt[3]{d}x+\sqrt[3]{d}\right)\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+x^2+2x+1\right)}{2d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}{\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+2x+2}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 + (1 - 2*k^2)*x + 3*k^2*x^2 + k^2*x^3)/(((1 - x^2)*(
1 - k^2*x^2))^(1/3)*(-1 + d - (2 + d)*x - (1 + d*k^2)*x^2 + d*k^2*x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))/(2 +
 2*x + d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))])/d^(2/3) + Log[1 + x
- d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3)]/d^(2/3) - Log[1 + 2*x + x^2
 + (d^(1/3) + d^(1/3)*x)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3) + d^(2/3)*(1
+ (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+(-2*k^2+1)*x+3*k^2*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)
/(-1+d-(2+d)*x-(d*k^2+1)*x^2+d*k^2*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3 + 3\,k^2x^2 - \left(2\,k^2 - 1\right)x - 3}{\left(dk^2x^3 - \left(dk^2 + 1\right)x^2 - (d+2)x + d - 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+(-2*k^2+1)*x+3*k^2*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)
/(-1+d-(2+d)*x-(d*k^2+1)*x^2+d*k^2*x^3),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^3 + 3*k^2*x^2 - (2*k^2 - 1)*x - 3)/((d*k^2*x^3 - (d*k^2 +
1)*x^2 - (d + 2)*x + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{-3 + \left(-2k^2 + 1\right)x + 3k^2x^2 + k^2x^3}{\left(\left(-x^2 + 1\right)\left(-k^2x^2 + 1\right)\right)^{\frac{1}{3}}\left(-1 + d - (2+d)\,x - \left(d\,k^2 + 1\right)x^2 + d\,k^2x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+(-2*k^2+1)*x+3*k^2*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d
-(2+d)*x-(d*k^2+1)*x^2+d*k^2*x^3),x)
```

```
[Out] int((-3+(-2*k^2+1)*x+3*k^2*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d
-(2+d)*x-(d*k^2+1)*x^2+d*k^2*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3 + 3\,k^2x^2 - \left(2\,k^2 - 1\right)x - 3}{\left(dk^2x^3 - \left(dk^2 + 1\right)x^2 - (d+2)x + d - 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+(-2*k^2+1)*x+3*k^2*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)
/(-1+d-(2+d)*x-(d*k^2+1)*x^2+d*k^2*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^3 + 3*k^2*x^2 - (2*k^2 - 1)*x - 3)/((d*k^2*x^3 - (d*k^2 +
1)*x^2 - (d + 2)*x + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{3\,k^2\,x^2 + k^2\,x^3 - x\,\left(2\,k^2 - 1\right) - 3}{\left(\left(x^2 - 1\right)\,\left(k^2\,x^2 - 1\right)\right)^{1/3}\,\left(x^2\,\left(d\,k^2 + 1\right) - d + x\,\left(d+2\right) - d\,k^2\,x^3 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*k^2*x^2 + k^2*x^3 - x*(2*k^2 - 1) - 3)/(((x^2 - 1)*(k^2*x^2 - 1))^(
1/3)*(x^2*(d*k^2 + 1) - d + x*(d + 2) - d*k^2*x^3 + 1)),x)
```

```
[Out] int(-(3*k^2*x^2 + k^2*x^3 - x*(2*k^2 - 1) - 3)/(((x^2 - 1)*(k^2*x^2 - 1))^(
1/3)*(x^2*(d*k^2 + 1) - d + x*(d + 2) - d*k^2*x^3 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+(-2*k**2+1)*x+3*k**2*x**2+k**2*x**3)/((-x**2+1)*(-k**2*x**2+1
))**(1/3)/(-1+d-(2+d)*x-(d*k**2+1)*x**2+d*k**2*x**3),x)
```

```
[Out] Timed out
```

3.2033 $\int \frac{x^2}{(-b+ax^4)\sqrt{b+ax^4}}\,dx$

Optimal. Leaf size=212

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{ax^4+b}+\sqrt{a}x^2+\sqrt{b}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{6-4\sqrt{2}}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{ax^4+b}+\sqrt{a}x^2+\sqrt{b}}\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{6+4\sqrt{2}}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{ax^4+b}+\sqrt{a}x^2+\sqrt{b}}\right)}{4\sqrt{2}a^{3/4}b^{3/4}}$$

Rubi [A] time = 0.30, antiderivative size = 97, normalized size of antiderivative = 0.46, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {490, 1211, 220, 1699, 205, 208}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{ax^4+b}}\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{ax^4+b}}\right)}{4\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-b + a*x^4)*Sqrt[b + a*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[b + a*x^4]]/(4*Sqrt[2]*a^(3/4)*b^(3
/4)) - ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[b + a*x^4]]/(4*Sqrt[2]*a^(3
/4)*b^(3/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
```

```
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(-b+ax^4)\sqrt{b+ax^4}}\,dx &= -\frac{\int \frac{1}{(\sqrt{b}-\sqrt{a}x^2)\sqrt{b+ax^4}}\,dx}{2\sqrt{a}}+\frac{\int \frac{1}{(\sqrt{b}+\sqrt{a}x^2)\sqrt{b+ax^4}}\,dx}{2\sqrt{a}}\\
&= \frac{\int \frac{\sqrt{b}-\sqrt{a}x^2}{(\sqrt{b}+\sqrt{a}x^2)\sqrt{b+ax^4}}\,dx}{4\sqrt{a}\sqrt{b}}-\frac{\int \frac{\sqrt{b}+\sqrt{a}x^2}{(\sqrt{b}-\sqrt{a}x^2)\sqrt{b+ax^4}}\,dx}{4\sqrt{a}\sqrt{b}}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{b}-2\sqrt{a}bx^2}\,dx,x,\frac{x}{\sqrt{b+ax^4}}\right)}{4\sqrt{a}}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{b}+2\sqrt{a}bx^2}\,dx,x,\frac{x}{\sqrt{b+ax^4}}\right)}{4\sqrt{a}}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{b+ax^4}}\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{b+ax^4}}\right)}{4\sqrt{2}a^{3/4}b^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 64, normalized size = 0.30

$$-\frac{x^3\sqrt{\frac{ax^4+b}{b}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{ax^4}{b},\frac{ax^4}{b}\right)}{3b\sqrt{ax^4+b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-b + a*x^4)*Sqrt[b + a*x^4]),x]
```

```
[Out] -1/3*(x^3*Sqrt[(b + a*x^4)/b]*AppellF1[3/4, 1/2, 1, 7/4, -((a*x^4)/b), (a*x
^4)/b])/(b*Sqrt[b + a*x^4])
```

IntegrateAlgebraic [A] time = 0.51, size = 97, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{ax^4+b}}\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{ax^4+b}}\right)}{4\sqrt{2}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-b + a*x^4)*Sqrt[b + a*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[b + a*x^4]]/(4*Sqrt[2]*a^(3/4)*b^(3
/4)) - ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[b + a*x^4]]/(4*Sqrt[2]*a^(3
/4)*b^(3/4))
```

fricas [B] time = 1.16, size = 288, normalized size = 1.36

$$-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{ax^4+b}\,a^2b^2\left(\frac{1}{a^3b^3}\right)^{\frac{3}{4}}-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}a^3b^2x^2\left(\frac{1}{a^3b^3}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}ab\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}}{\sqrt{a}}}{x}\right)-\frac{1}{8}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}a^2b^3x\left(\frac{1}{a^3b^3}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}abx^3\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}+\sqrt{ax^4+b}\left(ab^2\sqrt{\frac{1}{a^3b^3}}+x^2\right)}{ax^4-b}\right)+\frac{1}{8}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}a^2b^3x\left(\frac{1}{a^3b^3}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}abx^3\left(\frac{1}{a^3b^3}\right)^{\frac{1}{4}}-\sqrt{ax^4+b}\left(ab^2\sqrt{\frac{1}{a^3b^3}}+x^2\right)}{ax^4-b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4-b)/(a*x^4+b)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(1/4)^(1/4)*(1/(a^3*b^3))^(1/4)*arctan((2*(1/4)^(3/4)*sqrt(a*x^4 + b)*
a^2*b^2*(1/(a^3*b^3))^(3/4) - (2*(1/4)^(3/4)*a^3*b^2*x^2*(1/(a^3*b^3))^(3/4
) + (1/4)^(1/4)*a*b*(1/(a^3*b^3))^(1/4))/sqrt(a))/x) - 1/8*(1/4)^(1/4)*(1/(
```

```
a^3*b^3))^(1/4)*log((4*(1/4)^(3/4)*a^2*b^3*x*(1/(a^3*b^3))^(3/4) + 2*(1/4)^
(1/4)*a*b*x^3*(1/(a^3*b^3))^(1/4) + sqrt(a*x^4 + b)*(a*b^2*sqrt(1/(a^3*b^3)
) + x^2))/(a*x^4 - b)) + 1/8*(1/4)^(1/4)*(1/(a^3*b^3))^(1/4)*log(-(4*(1/4)^
(3/4)*a^2*b^3*x*(1/(a^3*b^3))^(3/4) + 2*(1/4)^(1/4)*a*b*x^3*(1/(a^3*b^3))^(
1/4) - sqrt(a*x^4 + b)*(a*b^2*sqrt(1/(a^3*b^3)) + x^2))/(a*x^4 - b))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{ax^4+b}\left(ax^4-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4-b)/(a*x^4+b)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(a*x^4 + b)*(a*x^4 - b)), x)
```

maple [A] time = 0.03, size = 109, normalized size = 0.51

$$-\frac{\sqrt{2}\,(ab)^{\frac{1}{4}}\ln\left(\frac{\frac{\sqrt{a\,x^4+b}\,\sqrt{2}}{2x}+(ab)^{\frac{1}{4}}}{\frac{\sqrt{a\,x^4+b}\,\sqrt{2}}{2x}-(ab)^{\frac{1}{4}}}\right)}{16ab}-\frac{\sqrt{2}\,(ab)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a\,x^4+b}\,\sqrt{2}}{2x(ab)^{\frac{1}{4}}}\right)}{8ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^4-b)/(a*x^4+b)^(1/2),x)
```

```
[Out] -1/16*2^(1/2)*(a*b)^(1/4)/a/b*ln((1/2*(a*x^4+b)^(1/2)*2^(1/2)/x+(a*b)^(1/4)
)/(1/2*(a*x^4+b)^(1/2)*2^(1/2)/x-(a*b)^(1/4)))-1/8*2^(1/2)*(a*b)^(1/4)/a/b*
arctan(1/2*(a*x^4+b)^(1/2)*2^(1/2)/x/(a*b)^(1/4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{ax^4+b}\left(ax^4-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4-b)/(a*x^4+b)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(a*x^4 + b)*(a*x^4 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^2}{\sqrt{a\,x^4+b}\,\left(b-a\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((b + a*x^4)^(1/2)*(b - a*x^4)),x)
```

```
[Out] -int(x^2/((b + a*x^4)^(1/2)*(b - a*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax^4-b\right)\sqrt{ax^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**4-b)/(a*x**4+b)**(1/2),x)
```

```
[Out] Integral(x**2/((a*x**4 - b)*sqrt(a*x**4 + b)), x)
```

3.2034

$$\int \frac{(-1+x)(-1+kx)(-2+(1+k)x)}{\sqrt[3]{(1-x)x(1-kx)}\left(b-2(b+bk)x+\left(b+4bk+bk^2\right)x^2-2bk(1+k)x^3+\left(-1+bk^2\right)x^4\right)}\,dx$$

Optimal. Leaf size=212

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{x-2\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{2b^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+x}\right)}{2b^{5/6}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{b^{5/6}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{b}(}{}\right.}{}$$

Rubi [F] time = 18.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x)(-1+kx)(-2+(1+k)x)}{\sqrt[3]{(1-x)x(1-kx)}\left(b-2(b+bk)x+\left(b+4bk+bk^2\right)x^2-2bk(1+k)x^3+\left(-1+bk^2\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x)*(-1 + k*x)*(-2 + (1 + k)*x))/(((1 - x)*x*(1 - k*x))^(1/3)*(b
- 2*(b + b*k)*x + (b + 4*b*k + b*k^2)*x^2 - 2*b*k*(1 + k)*x^3 + (-1 + b*k^2
)*x^4)),x]
```

```
[Out] (6*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][(x*(1 - x^
3)^(2/3)*(1 - k*x^3)^(2/3))/(x^12 - b*(-1 + x^3)^2*(-1 + k*x^3)^2), x], x,
x^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3) + (3*(1 + k)*(1 - x)^(1/3)*x^(1/3)*(1
 - k*x)^(1/3)*Defer[Subst][Defer[Int][(x^4*(1 - x^3)^(2/3)*(1 - k*x^3)^(2/3
))/(-x^12 + b*(-1 + x^3)^2*(-1 + k*x^3)^2), x], x, x^(1/3)])/((1 - x)*x*(1
- k*x))^(1/3)
```

Rubi steps

$$\int \frac{(-1+x)(-1+kx)(-2+(1+k)x)}{\sqrt[3]{(1-x)x(1-kx)}\left(b-2(b+bk)x+\left(b+4bk+bk^2\right)x^2-2bk(1+k)x^3+\left(-1+bk^2\right)x^4\right)}\,dx = \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{\cdots}\right.}{\cdots}$$

$$= -\frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\cdots\right.}{\cdots}$$

$$= \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{\cdots}\right.}{\cdots}$$

$$= \frac{\left(3\sqrt[3]{1-x}\,\sqrt[3]{x}\cdots\right.}{\cdots}$$

$$= \frac{\left(3\sqrt[3]{1-x}\,\sqrt[3]{x}\cdots\right.}{\cdots}$$

$$= \frac{\left(3\sqrt[3]{1-x}\,\sqrt[3]{x}\cdots\right.}{\cdots}$$

$$= \frac{\left(6\sqrt[3]{1-x}\,\sqrt[3]{x}\cdots\right.}{\cdots}$$

$$= \frac{\left(6\sqrt[3]{1-x}\,\sqrt[3]{x}\cdots\right.}{\cdots}$$

Mathematica [F] time = 4.55, size = 0, normalized size = 0.00

$$\int \frac{(-1+x)(-1+kx)(-2+(1+k)x)}{\sqrt[3]{(1-x)x(1-kx)}\left(b-2(b+bk)x+\left(b+4bk+bk^2\right)x^2-2bk(1+k)x^3+\left(-1+bk^2\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x)*(-1 + k*x)*(-2 + (1 + k)*x))/(((1 - x)*x*(1 - k*x))^(1/
3)*(b - 2*(b + b*k)*x + (b + 4*b*k + b*k^2)*x^2 - 2*b*k*(1 + k)*x^3 + (-1 +
 b*k^2)*x^4)),x]
```

```
[Out] Integrate[((-1 + x)*(-1 + k*x)*(-2 + (1 + k)*x))/(((1 - x)*x*(1 - k*x))^(1/
3)*(b - 2*(b + b*k)*x + (b + 4*b*k + b*k^2)*x^2 - 2*b*k*(1 + k)*x^3 + (-1 +
 b*k^2)*x^4)), x]
```

IntegrateAlgebraic [A] time = 1.04, size = 212, normalized size = 1.00

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{x-2\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{2b^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+x}\right)}{2b^{5/6}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{b^{5/6}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\frac{x^2}{\sqrt[6]{b}}}{x\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{2b^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)*(-1 + k*x)*(-2 + (1 + k)*x))/(((1 - x)*x*(1 -
k*x))^(1/3)*(b - 2*(b + b*k)*x + (b + 4*b*k + b*k^2)*x^2 - 2*b*k*(1 + k)*x^
3 + (-1 + b*k^2)*x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*b^(1/6)*(x + (-1 - k)*x^2 + k*x^3)^(1/3)
)])/(2*b^(5/6)) - (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*b^(1/6)*(x + (-1 - k)*
x^2 + k*x^3)^(1/3))])/(2*b^(5/6)) - ArcTanh[x/(b^(1/6)*(x + (-1 - k)*x^2 +
k*x^3)^(1/3))]/b^(5/6) - ArcTanh[(x^2/b^(1/6) + b^(1/6)*(x + (-1 - k)*x^2 +
 k*x^3)^(2/3))/(x*(x + (-1 - k)*x^2 + k*x^3)^(1/3))]/(2*b^(5/6))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(k*x-1)*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-2*(b*k+b)
*x+(b*k^2+4*b*k+b)*x^2-2*b*k*(1+k)*x^3+(b*k^2-1)*x^4),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [A] time = 0.48, size = 290, normalized size = 1.37

$$\frac{\sqrt{3}\left(-b^{5}\right)^{\frac{5}{6}}\log\left(\sqrt{3}\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^{2}}\right)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{6}}+\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^{2}}\right)^{\frac{2}{3}}+\left(-\frac{1}{b}\right)^{\frac{1}{3}}\right)}{4\,b^{5}}-\frac{\sqrt{3}\left(-b^{5}\right)^{\frac{5}{6}}\log\left(-\sqrt{3}\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^{2}}\right)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{6}}+\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^{2}}\right)^{\frac{2}{3}}+\left(-\frac{1}{b}\right)^{\frac{1}{3}}\right)}{4\,b^{5}}-\frac{\left(-b^{5}\right)^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(-\frac{1}{b}\right)^{\frac{1}{6}}+2\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^{2}}\right)^{\frac{1}{3}}}{\left(-\frac{1}{b}\right)^{\frac{1}{6}}}\right)}{2\,b^{5}}-\frac{\left(-b^{5}\right)^{\frac{5}{6}}\arctan\left(-\frac{\sqrt{3}\left(-\frac{1}{b}\right)^{\frac{1}{6}}-2\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^{2}}\right)^{\frac{1}{3}}}{\left(-\frac{1}{b}\right)^{\frac{1}{6}}}\right)}{2\,b^{5}}-\frac{\left(-b^{5}\right)^{\frac{5}{6}}\arctan\left(\frac{\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^{2}}\right)^{\frac{1}{3}}}{\left(-\frac{1}{b}\right)^{\frac{1}{6}}}\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(k*x-1)*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-2*(b*k+b)
*x+(b*k^2+4*b*k+b)*x^2-2*b*k*(1+k)*x^3+(b*k^2-1)*x^4),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(3)*(-b^5)^(5/6)*log(sqrt(3)*(k - k/x - 1/x + 1/x^2)^(1/3)*(-1/b)^(
1/6) + (k - k/x - 1/x + 1/x^2)^(2/3) + (-1/b)^(1/3))/b^5 - 1/4*sqrt(3)*(-b^
5)^(5/6)*log(-sqrt(3)*(k - k/x - 1/x + 1/x^2)^(1/3)*(-1/b)^(1/6) + (k - k/x
 - 1/x + 1/x^2)^(2/3) + (-1/b)^(1/3))/b^5 - 1/2*(-b^5)^(5/6)*arctan((sqrt(3
)*(-1/b)^(1/6) + 2*(k - k/x - 1/x + 1/x^2)^(1/3))/(-1/b)^(1/6))/b^5 - 1/2*(
-b^5)^(5/6)*arctan(-(sqrt(3)*(-1/b)^(1/6) - 2*(k - k/x - 1/x + 1/x^2)^(1/3)
)/(-1/b)^(1/6))/b^5 - (-b^5)^(5/6)*arctan((k - k/x - 1/x + 1/x^2)^(1/3)/(-1
/b)^(1/6))/b^5
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x\right)\left(kx-1\right)\left(-2+\left(1+k\right)x\right)}{\left(\left(1-x\right)x\left(-kx+1\right)\right)^{\frac{1}{3}}\left(b-2\left(bk+b\right)x+\left(b\,k^{2}+4bk+b\right)x^{2}-2bk\left(1+k\right)x^{3}+\left(b\,k^{2}-1\right)x^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)*(k*x-1)*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-2*(b*k+b)*x+(b*
k^2+4*b*k+b)*x^2-2*b*k*(1+k)*x^3+(b*k^2-1)*x^4),x)
```

```
[Out] int((-1+x)*(k*x-1)*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-2*(b*k+b)*x+(b*
k^2+4*b*k+b)*x^2-2*b*k*(1+k)*x^3+(b*k^2-1)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{((k+1)x-2)(kx-1)(x-1)}{\left(2\,b(k+1)kx^{3}-\left(bk^{2}-1\right)x^{4}-\left(bk^{2}+4\,bk+b\right)x^{2}+2\,(bk+b)x-b\right)((kx-1)(x-1)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(k*x-1)*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-2*(b*k+b)
*x+(b*k^2+4*b*k+b)*x^2-2*b*k*(1+k)*x^3+(b*k^2-1)*x^4),x, algorithm="maxima"
)
```

```
[Out] -integrate(((k + 1)*x - 2)*(k*x - 1)*(x - 1)/((2*b*(k + 1)*k*x^3 - (b*k^2 -
 1)*x^4 - (b*k^2 + 4*b*k + b)*x^2 + 2*(b*k + b)*x - b)*((k*x - 1)*(x - 1)*x
)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x\ (k+1)-2)\ (k\,x-1)\ (x-1)}{(x\ (k\,x-1)\ (x-1))^{1/3}\ \left(b+x^4\ \left(b\,k^2-1\right)+x^2\ \left(b\,k^2+4\,b\,k+b\right)-2\,x\ (b+b\,k)-2\,b\,k\,x^3\ (k+1)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x*(k + 1) - 2)*(k*x - 1)*(x - 1))/((x*(k*x - 1)*(x - 1))^(1/3)*(b + x
^4*(b*k^2 - 1) + x^2*(b + 4*b*k + b*k^2) - 2*x*(b + b*k) - 2*b*k*x^3*(k + 1
))),x)
```

```
[Out] int(((x*(k + 1) - 2)*(k*x - 1)*(x - 1))/((x*(k*x - 1)*(x - 1))^(1/3)*(b + x
^4*(b*k^2 - 1) + x^2*(b + 4*b*k + b*k^2) - 2*x*(b + b*k) - 2*b*k*x^3*(k + 1
))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(k*x-1)*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(b-2*(b*k+b
)*x+(b*k**2+4*b*k+b)*x**2-2*b*k*(1+k)*x**3+(b*k**2-1)*x**4),x)
```

```
[Out] Timed out
```

3.2035 $\int \frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}{x^2}\,dx$

Optimal. Leaf size=213

$$\sqrt{\frac{x-1}{x}}\left(\frac{8}{63}\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\sqrt{1-\sqrt{\frac{x-1}{x}}}-\frac{8}{315}\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\right)-\frac{64}{315}\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}+\frac{64}{315}\sqrt{}$$

Rubi [A] time = 0.35, antiderivative size = 94, normalized size of antiderivative = 0.44, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {6715, 371, 1398, 772}

$$\frac{8}{9}\left(1-\sqrt{1-\sqrt{\frac{x-1}{x}}}\right)^{9/2}-\frac{24}{7}\left(1-\sqrt{1-\sqrt{\frac{x-1}{x}}}\right)^{7/2}+\frac{16}{5}\left(1-\sqrt{1-\sqrt{\frac{x-1}{x}}}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]]/x^2,x]
```

```
[Out] (16*(1 - Sqrt[1 - Sqrt[(-1 + x)/x]])^(5/2))/5 - (24*(1 - Sqrt[1 - Sqrt[(-1
+ x)/x]])^(7/2))/7 + (8*(1 - Sqrt[1 - Sqrt[(-1 + x)/x]])^(9/2))/9
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}{x^2}\,dx = -\operatorname{Subst}\left(\int \sqrt{1-\sqrt{1-\sqrt{1-x}}}\,dx, x, \frac{1}{x}\right)$$
$$= 2\operatorname{Subst}\left(\int \sqrt{1-\sqrt{1-x}}\,x\,dx, x, \sqrt{\frac{-1+x}{x}}\right)$$
$$= 2\operatorname{Subst}\left(\int \sqrt{1-\sqrt{x}}\,(-1+x)\,dx, x, 1-\sqrt{\frac{-1+x}{x}}\right)$$
$$= 4\operatorname{Subst}\left(\int \sqrt{1-x}\,x\left(-1+x^2\right)\,dx, x, \sqrt{1-\sqrt{\frac{-1+x}{x}}}\right)$$
$$= 4\operatorname{Subst}\left(\int \left(-2(1-x)^{3/2}+3(1-x)^{5/2}-(1-x)^{7/2}\right)dx, x, \sqrt{1-\sqrt{\frac{-1+x}{x}}}\right)$$
$$= \frac{16}{5}\left(1-\sqrt{1-\sqrt{-\frac{1-x}{x}}}\right)^{5/2} - \frac{24}{7}\left(1-\sqrt{1-\sqrt{-\frac{1-x}{x}}}\right)^{7/2} + \frac{8}{9}\left(1-\sqrt{1-\sqrt{-\frac{1}{}}}\right.$$

Mathematica [A] time = 0.14, size = 67, normalized size = 0.31

$$\frac{8}{315}\left(1-\sqrt{1-\sqrt{\frac{x-1}{x}}}\right)^{5/2}\left(65\sqrt{1-\sqrt{\frac{x-1}{x}}}-35\sqrt{\frac{x-1}{x}}+61\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]]/x^2,x]
```

```
[Out] (8*(1 - Sqrt[1 - Sqrt[(-1 + x)/x]])^(5/2)*(61 + 65*Sqrt[1 - Sqrt[(-1 + x)/x
]] - 35*Sqrt[(-1 + x)/x]))/315
```

IntegrateAlgebraic [A] time = 2.22, size = 184, normalized size = 0.86

$$\sqrt{\frac{x-1}{x}}\left(\frac{8}{63}\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\sqrt{1-\sqrt{\frac{x-1}{x}}}-\frac{8}{315}\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\right)+\frac{8\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\,(27x-35)}{315x}+\frac{64}{315}\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\sqrt{1-\sqrt{\frac{x-1}{x}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]]/x^2,x]
```

```
[Out] (64*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]]*Sqrt[1 - Sqrt[(-1 + x)/x]])/315 +
((-8*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]])/315 + (8*Sqrt[1 - Sqrt[1 - Sqrt[
(-1 + x)/x]]]*Sqrt[1 - Sqrt[(-1 + x)/x]])/63)*Sqrt[(-1 + x)/x] + (8*Sqrt[1
- Sqrt[1 - Sqrt[(-1 + x)/x]]]*(-35 + 27*x))/(315*x)
```

fricas [A] time = 0.83, size = 75, normalized size = 0.35

$$\frac{8\left(\left(5\,x\sqrt{\frac{x-1}{x}}+8\,x\right)\sqrt{-\sqrt{\frac{x-1}{x}}+1}-x\sqrt{\frac{x-1}{x}}+27\,x-35\right)\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}}{315\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(1-(1-1/x)^(1/2))^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 8/315*((5*x*sqrt((x - 1)/x) + 8*x)*sqrt(-sqrt((x - 1)/x) + 1) - x*sqrt((x -
 1)/x) + 27*x - 35)*sqrt(-sqrt(-sqrt((x - 1)/x) + 1) + 1)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\sqrt{-\sqrt{-\frac{1}{x}+1}+1}+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(1-(1-1/x)^(1/2))^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-sqrt(-sqrt(-1/x + 1) + 1) + 1)/x^2, x)
```

maple [A] time = 0.06, size = 71, normalized size = 0.33

$$\frac{8\left(1-\sqrt{1-\sqrt{1-\frac{1}{x}}}\right)^{\frac{9}{2}}}{9}-\frac{24\left(1-\sqrt{1-\sqrt{1-\frac{1}{x}}}\right)^{\frac{7}{2}}}{7}+\frac{16\left(1-\sqrt{1-\sqrt{1-\frac{1}{x}}}\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(1-(1-1/x)^(1/2))^(1/2))^(1/2)/x^2,x)
```

```
[Out] 8/9*(1-(1-(1-1/x)^(1/2))^(1/2))^(9/2)-24/7*(1-(1-(1-1/x)^(1/2))^(1/2))^(7/2
)+16/5*(1-(1-(1-1/x)^(1/2))^(1/2))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\sqrt{-\sqrt{-\frac{1}{x}+1}+1}+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(1-(1-1/x)^(1/2))^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-sqrt(-sqrt(-1/x + 1) + 1) + 1)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - (1 - (1 - 1/x)^(1/2))^(1/2))^(1/2)/x^2,x)
```

```
[Out] int((1 - (1 - (1 - 1/x)^(1/2))^(1/2))^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(1-(1-1/x)**(1/2))**(1/2))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(1 - sqrt(1 - sqrt(1 - 1/x)))/x**2, x)
```

3.2036 $\int \frac{\sqrt[3]{x^2+x^4}}{x\left(-1+x^2\right)}\,dx$

Optimal. Leaf size=213

$$\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}-2x\right)}{2\,2^{2/3}}+\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}+2x\right)}{2\,2^{2/3}}-\frac{\log\left(-2x^2+2^{2/3}\sqrt[3]{x^4+x^2}\,x-\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{4\,2^{2/3}}-\frac{\log\left(2x\right.}{}$$

Rubi [C] time = 0.68, antiderivative size = 382, normalized size of antiderivative = 1.79, number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1311, 2013, 622, 619, 236, 219, 2034, 758, 133}

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(-x^4-x^2\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{-x^2\left(x^2+1\right)}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-x^2\left(x^2+1\right)\right)^{2/3}+2^{2/3}\sqrt[3]{-x^2\left(x^2+1\right)}+1}{\left(-2^{2/3}\sqrt[3]{-x^2\left(x^2+1\right)}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-x^2\left(x^2+1\right)}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-x^2\left(x^2+1\right)}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2^{2/3}\left(2x^2+1\right)\left(x^4+x^2\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-x^2\left(x^2+1\right)}}{\left(-2^{2/3}\sqrt[3]{-x^2\left(x^2+1\right)}-\sqrt{3}+1\right)^2}}}-\frac{3\left(-\frac{x^2}{1-x^2}\right)^{2/3}\left(-\frac{x^2+1}{1-x^2}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},\frac{2}{3};\frac{7}{3};\frac{1}{1-x^2},\frac{2}{1-x^2}\right)}{4\left(x^4+x^2\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^2 + x^4)^(1/3)/(x*(-1 + x^2)),x]
```

```
[Out] (-3*(-(x^2/(1 - x^2)))^(2/3)*(-((1 + x^2)/(1 - x^2)))^(2/3)*AppellF1[4/3, 2
/3, 2/3, 7/3, (1 - x^2)^(-1), 2/(1 - x^2)])/(4*(x^2 + x^4)^(2/3)) + (3^(3/4
)*Sqrt[2 - Sqrt[3]]*(-x^2 - x^4)^(2/3)*(1 - 2^(2/3)*(-(x^2*(1 + x^2)))^(1/3
))*Sqrt[(1 + 2^(2/3)*(-(x^2*(1 + x^2)))^(1/3) + 2*2^(1/3)*(-(x^2*(1 + x^2))
)^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-(x^2*(1 + x^2)))^(1/3))^2]*EllipticF[ArcS
in[(1 + Sqrt[3] - 2^(2/3)*(-(x^2*(1 + x^2)))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*
(-(x^2*(1 + x^2)))^(1/3))], -7 + 4*Sqrt[3]])/(2^(2/3)*(1 + 2*x^2)*(x^2 + x^
4)^(2/3)*Sqrt[-((1 - 2^(2/3)*(-(x^2*(1 + x^2)))^(1/3))/(1 - Sqrt[3] - 2^(2/
3)*(-(x^2*(1 + x^2)))^(1/3))^2)])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Dist[((1/(d + e*x))^(2*p)*(a + b*
x + c*x^2)^p)/(e*((e*(b - q + 2*c*x))/(2*c*(d + e*x)))^p*((e*(b + q + 2*c*x
))/(2*c*(d + e*x)))^p), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - (e*(b -
q))/(2*c))*x, x]^p*Simp[1 - (d - (e*(b + q))/(2*c))*x, x]^p, x], x, 1/(d +
e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m,
0]
```

Rule 1311

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] - Dist[(c*d^2 - b*d*e + a*e^2)/(d*e*f^2),
Int[((f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1))/(d + e*x^2), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m, 0]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{\sqrt[3]{x^2+x^4}}{x\left(-1+x^2\right)}\,dx = 2\int \frac{x}{\left(-1+x^2\right)\left(x^2+x^4\right)^{2/3}}\,dx + \int \frac{x}{\left(x^2+x^4\right)^{2/3}}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(x+x^2\right)^{2/3}}\,dx, x, x^2\right) + \text{Subst}\left(\int \frac{1}{(-1+x)\left(x+x^2\right)^{2/3}}\,dx, x, x^2\right)$$

$$= -\frac{\left(\left(\frac{x^2}{-1+x^2}\right)^{2/3}\left(\frac{1+x^2}{-1+x^2}\right)^{2/3}\right)\text{Subst}\left(\int \frac{\sqrt[3]{x}}{(1+x)^{2/3}(1+2x)^{2/3}}\,dx, x, \frac{1}{-1+x^2}\right)}{\left(\frac{1}{-1+x^2}\right)^{4/3}\left(x^2+x^4\right)^{2/3}} + \frac{\left(-x^2-x^4\right)^{2/3}\text{Subst}\left(\int \cdots\right.}{2\left(x^2+x^4\right)\cdots}$$

$$= -\frac{3\left(-\frac{x^2}{1-x^2}\right)^{2/3}\left(-\frac{1+x^2}{1-x^2}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},\frac{2}{3};\frac{7}{3};\frac{1}{1-x^2},\frac{2}{1-x^2}\right)}{4\left(x^2+x^4\right)^{2/3}} - \frac{\left(-x^2-x^4\right)^{2/3}\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{2/3}}\,dx, x, \cdots\right.}{2^{2/3}\left(x^2+x^4\right)^{2/3}}$$

$$= -\frac{3\left(-\frac{x^2}{1-x^2}\right)^{2/3}\left(-\frac{1+x^2}{1-x^2}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},\frac{2}{3};\frac{7}{3};\frac{1}{1-x^2},\frac{2}{1-x^2}\right)}{4\left(x^2+x^4\right)^{2/3}} + \frac{\left(3\sqrt{-\left(-1-2x^2\right)^2}\left(-x^2-x^4\right)^{2/3}\right)\text{Su}\cdots}{2\,2^{2/3}\left(-1-\cdots\right.}$$

$$= -\frac{3\left(-\frac{x^2}{1-x^2}\right)^{2/3}\left(-\frac{1+x^2}{1-x^2}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},\frac{2}{3};\frac{7}{3};\frac{1}{1-x^2},\frac{2}{1-x^2}\right)}{4\left(x^2+x^4\right)^{2/3}} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}\sqrt{-\left(1+2x^2\right)^2}\left(-x^2-\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.06, size = 46, normalized size = 0.22

$$-\frac{3\sqrt[3]{x^4+x^2}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{1}{x^2},\frac{1}{x^2}\right)}{2\sqrt[3]{\frac{1}{x^2}+1}\,x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2 + x^4)^(1/3)/(x*(-1 + x^2)),x]
```

```
[Out] (-3*(x^2 + x^4)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -x^(-2), x^(-2)])/(2*(1 +
 x^(-2))^(1/3)*x^2)
```

IntegrateAlgebraic [A] time = 0.49, size = 213, normalized size = 1.00

$$\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}-2x\right)}{2\,2^{2/3}} + \frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}+2x\right)}{2\,2^{2/3}} - \frac{\log\left(-2x^2+2^{2/3}\sqrt[3]{x^4+x^2}\,x-\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{4\,2^{2/3}} - \frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^4+x^2}\,x+\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{4\,2^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x^2}{x^2+\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}}\right)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2 + x^4)^(1/3)/(x*(-1 + x^2)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*x^2)/(x^2 + 2^(1/3)*(x^2 + x^4)^(2/3))])/2^(2
/3) + Log[-2*x + 2^(2/3)*(x^2 + x^4)^(1/3)]/(2*2^(2/3)) + Log[2*x + 2^(2/3)
*(x^2 + x^4)^(1/3)]/(2*2^(2/3)) - Log[-2*x^2 + 2^(2/3)*x*(x^2 + x^4)^(1/3)
- 2^(1/3)*(x^2 + x^4)^(2/3)]/(4*2^(2/3)) - Log[2*x^2 + 2^(2/3)*x*(x^2 + x^4
)^(1/3) + 2^(1/3)*(x^2 + x^4)^(2/3)]/(4*2^(2/3))
```

fricas [B] time = 3.45, size = 326, normalized size = 1.53

$$\frac{1}{12}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(-\frac{4^{\frac{1}{6}}\sqrt{3}\left(6\cdot 4^{\frac{2}{3}}\left(x^{10}+33x^{8}+110x^{6}+110x^{4}+33x^{2}+1\right)\left(x^{4}+x^{2}\right)^{\frac{1}{3}}-48\left(x^{8}+2x^{6}-6x^{4}+2x^{2}+1\right)\left(x^{4}+x^{2}\right)^{\frac{2}{3}}-4^{\frac{1}{3}}\left(x^{12}-42x^{10}-417x^{8}-812x^{6}-417x^{4}-42x^{2}+1\right)\right)}{6\left(x^{12}+102x^{10}+447x^{8}+628x^{6}+447x^{4}+102x^{2}+1\right)}\right) - \frac{1}{48}\cdot 4^{\frac{2}{3}}\log\left(\frac{24\cdot 4^{\frac{1}{3}}\left(x^{4}+4x^{2}+1\right)\left(x^{4}+x^{2}\right)^{\frac{2}{3}}+4^{\frac{2}{3}}\left(x^{8}+32x^{6}+78x^{4}+32x^{2}+1\right)+12\left(x^{6}+11x^{4}+11x^{2}+1\right)\left(x^{4}+x^{2}\right)^{\frac{1}{3}}}{x^{8}-4x^{6}+6x^{4}-4x^{2}+1}\right) + \frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(-\frac{3\cdot 4^{\frac{2}{3}}\left(x^{4}+x^{2}\right)^{\frac{1}{3}}\left(x^{2}+1\right)-4^{\frac{1}{3}}\left(x^{4}-2x^{2}+1\right)-12\left(x^{4}+x^{2}\right)^{\frac{2}{3}}}{x^{4}-2x^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2)^(1/3)/x/(x^2-1),x, algorithm="fricas")
```

```
[Out] 1/12*4^(1/6)*sqrt(3)*arctan(-1/6*4^(1/6)*sqrt(3)*(6*4^(2/3)*(x^10 + 33*x^8
+ 110*x^6 + 110*x^4 + 33*x^2 + 1)*(x^4 + x^2)^(1/3) - 48*(x^8 + 2*x^6 - 6*x
^4 + 2*x^2 + 1)*(x^4 + x^2)^(2/3) - 4^(1/3)*(x^12 - 42*x^10 - 417*x^8 - 812
*x^6 - 417*x^4 - 42*x^2 + 1))/(x^12 + 102*x^10 + 447*x^8 + 628*x^6 + 447*x^
4 + 102*x^2 + 1)) - 1/48*4^(2/3)*log((24*4^(1/3)*(x^4 + 4*x^2 + 1)*(x^4 + x
^2)^(2/3) + 4^(2/3)*(x^8 + 32*x^6 + 78*x^4 + 32*x^2 + 1) + 12*(x^6 + 11*x^4
 + 11*x^2 + 1)*(x^4 + x^2)^(1/3))/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)) + 1/24
*4^(2/3)*log(-(3*4^(2/3)*(x^4 + x^2)^(1/3)*(x^2 + 1) - 4^(1/3)*(x^4 - 2*x^2
 + 1) - 12*(x^4 + x^2)^(2/3))/(x^4 - 2*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4 + x^2\right)^{\frac{1}{3}}}{\left(x^2 - 1\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2)^(1/3)/x/(x^2-1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + x^2)^(1/3)/((x^2 - 1)*x), x)
```

maple [C] time = 20.08, size = 1110, normalized size = 5.21

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2)^(1/3)/x/(x^2-1),x)
```

```
[Out] 1/2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*ln((2247248*RootOf(
RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^4*x^4+2390992*R
ootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^3*x^4-9
550804*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^4
*x^2-10161716*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(
_Z^3-2)^3*x^2+5758573*RootOf(_Z^3-2)^2*x^4+6126917*RootOf(RootOf(_Z^3-2)^2+
2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^4+2247248*RootOf(RootOf(_Z^3-2
)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^4+2390992*RootOf(RootOf(_Z^3
-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^3-34713150*(x^4+x^2)^(2/
3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2+205
06138*RootOf(_Z^3-2)^2*x^2+21817802*RootOf(_Z^3-2)*RootOf(RootOf(_Z^3-2)^2+
2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2-7334688*RootOf(_Z^3-2)*(x^4+x^2)^(1/3)*x^2+
20043774*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*(x^4+x^2)^(1/3
)*x^2+5758573*RootOf(_Z^3-2)^2+6126917*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(
_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)-7334688*(x^4+x^2)^(1/3)*RootOf(_Z^3-2)+20043
774*(x^4+x^2)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-200
43774*(x^4+x^2)^(2/3))/(1+x)^2/(-1+x)^2)+1/4*RootOf(_Z^3-2)*ln((2247248*Roo
tOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^4*x^4+21035
04*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^3*x
^4-9550804*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-
2)^4*x^2-8939892*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*Root
Of(_Z^3-2)^3*x^2-3511325*RootOf(_Z^3-2)^2*x^4-3286725*RootOf(RootOf(_Z^3-2)
^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^4+2247248*RootOf(RootOf(_Z^
3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^4+2103504*RootOf(RootOf(_
Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^3+34713150*(x^4+x^2)^
(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2-
30056942*RootOf(_Z^3-2)^2*x^2-28134366*RootOf(_Z^3-2)*RootOf(RootOf(_Z^3-2)
^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2+10021887*RootOf(_Z^3-2)*(x^4+x^2)^(1/3)*
```

```
x^2-14669376*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*(x^4+x^2)^
(1/3)*x^2-3511325*RootOf(_Z^3-2)^2-3286725*RootOf(RootOf(_Z^3-2)^2+2*_Z*Roo
tOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)+10021887*(x^4+x^2)^(1/3)*RootOf(_Z^3-2)-
14669376*(x^4+x^2)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2
)+14669376*(x^4+x^2)^(2/3))/(1+x)^2/(-1+x)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^2\right)^{\frac{1}{3}}}{\left(x^2-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2)^(1/3)/x/(x^2-1),x, algorithm="maxima")
[Out] integrate((x^4 + x^2)^(1/3)/((x^2 - 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(x^4+x^2\right)^{1/3}}{x-x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4)^(1/3)/(x*(x^2 - 1)),x)
[Out] -int((x^2 + x^4)^(1/3)/(x - x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x^2\left(x^2+1\right)}}{x\left(x-1\right)\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2)**(1/3)/x/(x**2-1),x)
[Out] Integral((x**2*(x**2 + 1))**(1/3)/(x*(x - 1)*(x + 1)), x)
```

3.2037 $$\int \frac{1}{(c+dx)\sqrt{b+a^2x^2}\sqrt{ax-\sqrt{b+a^2x^2}}}\,dx$$

Optimal. Leaf size=213

$$\frac{2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{ax-\sqrt{a^2x^2+b}}}{\sqrt{\sqrt{a^2c^2+bd^2}+ac}}\right)}{\sqrt{a^2c^2+bd^2}\sqrt{\sqrt{a^2c^2+bd^2}+ac}}-\frac{2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{ax-\sqrt{a^2x^2+b}}}{\sqrt{ac-\sqrt{a^2c^2+bd^2}}}\right)}{\sqrt{a^2c^2+bd^2}\sqrt{ac-\sqrt{a^2c^2+bd^2}}}$$

Rubi [F] time = 0.88, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+dx)\sqrt{b+a^2x^2}\sqrt{ax-\sqrt{b+a^2x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*Sqrt[b + a^2*x^2]*Sqrt[a*x - Sqrt[b + a^2*x^2]]),x]
```

```
[Out] Defer[Int][1/((c + d*x)*Sqrt[b + a^2*x^2]*Sqrt[a*x - Sqrt[b + a^2*x^2]]), x
]
```

Rubi steps

$$\int \frac{1}{(c+dx)\sqrt{b+a^2x^2}\sqrt{ax-\sqrt{b+a^2x^2}}}\,dx=\int \frac{1}{(c+dx)\sqrt{b+a^2x^2}\sqrt{ax-\sqrt{b+a^2x^2}}}\,dx$$

Mathematica [A] time = 0.74, size = 317, normalized size = 1.49

$$\frac{2\left(ax\left(ax-\sqrt{a^2x^2+b}\right)+b\right)\left(\left(\sqrt{a^2c^2+bd^2}+ac\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{d}}{\sqrt{ax-\sqrt{a^2x^2+b}}\sqrt{-\sqrt{a^2c^2+bd^2}-ac}}\right)+\sqrt{-\sqrt{a^2c^2+bd^2}-ac}\sqrt{\sqrt{a^2c^2+bd^2}-ac}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{d}}{\sqrt{ax-\sqrt{a^2x^2+b}}\sqrt{\sqrt{a^2c^2+bd^2}-ac}}\right)\right)}{\sqrt{b}\sqrt{d}\sqrt{a^2x^2+b}\left(ax-\sqrt{a^2x^2+b}\right)\sqrt{a^2c^2+bd^2}\sqrt{-\sqrt{a^2c^2+bd^2}-ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*Sqrt[b + a^2*x^2]*Sqrt[a*x - Sqrt[b + a^2*x^2]]),x]
```

```
[Out] (2*(b + a*x*(a*x - Sqrt[b + a^2*x^2]))*((a*c + Sqrt[a^2*c^2 + b*d^2])*ArcTa
n[(Sqrt[b]*Sqrt[d])/(Sqrt[-(a*c) - Sqrt[a^2*c^2 + b*d^2]]*Sqrt[a*x - Sqrt[b
 + a^2*x^2]])] + Sqrt[-(a*c) - Sqrt[a^2*c^2 + b*d^2]]*Sqrt[-(a*c) + Sqrt[a^
2*c^2 + b*d^2]]*ArcTan[(Sqrt[b]*Sqrt[d])/(Sqrt[-(a*c) + Sqrt[a^2*c^2 + b*d^
2]]*Sqrt[a*x - Sqrt[b + a^2*x^2]])]))/(Sqrt[b]*Sqrt[d]*Sqrt[a^2*c^2 + b*d^2
]*Sqrt[-(a*c) - Sqrt[a^2*c^2 + b*d^2]]*Sqrt[b + a^2*x^2]*(a*x - Sqrt[b + a^
2*x^2]))
```

IntegrateAlgebraic [A] time = 0.60, size = 213, normalized size = 1.00

$$\frac{2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{ax-\sqrt{a^2x^2+b}}}{\sqrt{\sqrt{a^2c^2+bd^2}+ac}}\right)}{\sqrt{a^2c^2+bd^2}\sqrt{\sqrt{a^2c^2+bd^2}+ac}}-\frac{2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{ax-\sqrt{a^2x^2+b}}}{\sqrt{ac-\sqrt{a^2c^2+bd^2}}}\right)}{\sqrt{a^2c^2+bd^2}\sqrt{ac-\sqrt{a^2c^2+bd^2}}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((c + d*x)*Sqrt[b + a^2*x^2]*Sqrt[a*x - Sqrt[b + a^2*x
^2]]),x]

[Out] (-2*Sqrt[d]*ArcTan[(Sqrt[d]*Sqrt[a*x - Sqrt[b + a^2*x^2]])/Sqrt[a*c - Sqrt[
a^2*c^2 + b*d^2]]])/(Sqrt[a^2*c^2 + b*d^2]*Sqrt[a*c - Sqrt[a^2*c^2 + b*d^2]
]) + (2*Sqrt[d]*ArcTan[(Sqrt[d]*Sqrt[a*x - Sqrt[b + a^2*x^2]])/Sqrt[a*c + S
qrt[a^2*c^2 + b*d^2]]])/(Sqrt[a^2*c^2 + b*d^2]*Sqrt[a*c + Sqrt[a^2*c^2 + b*
d^2]])

fricas [B] time = 0.60, size = 877, normalized size = 4.12

$$\sqrt{\frac{ac + \frac{a^2bc^2d+b^2d^3}{\sqrt{a^2b^2c^2d^2+b^3d^4}}}{a^2bc^2d + b^2d^3}} \log\left(2\sqrt{ax - \sqrt{a^2x^2+b}}\, d + 2\left(a^2c^2 + bd^2 - \frac{a^3bc^3d + ab^2cd^3}{\sqrt{a^2b^2c^2d^2+b^3d^4}}\right)\sqrt{\frac{ac + \frac{a^2bc^2d+b^2d^3}{\sqrt{a^2b^2c^2d^2+b^3d^4}}}{a^2bc^2d + b^2d^3}}\right) - \sqrt{\frac{ac + \frac{a^2bc^2d+b^2d^3}{\sqrt{a^2b^2c^2d^2+b^3d^4}}}{a^2bc^2d + b^2d^3}} \log\left(2\sqrt{ax - \sqrt{a^2x^2+b}}\, d - 2\left(a^2c^2 + bd^2 - \frac{a^3bc^3d + ab^2cd^3}{\sqrt{a^2b^2c^2d^2+b^3d^4}}\right)\sqrt{\frac{ac + \frac{a^2bc^2d+b^2d^3}{\sqrt{a^2b^2c^2d^2+b^3d^4}}}{a^2bc^2d + b^2d^3}}\right) + \sqrt{\frac{ac - \frac{a^2bc^2d+b^2d^3}{\sqrt{a^2b^2c^2d^2+b^3d^4}}}{a^2bc^2d + b^2d^3}} \log\left(2\sqrt{ax - \sqrt{a^2x^2+b}}\, d + 2\left(a^2c^2 + bd^2 + \frac{a^3bc^3d + ab^2cd^3}{\sqrt{a^2b^2c^2d^2+b^3d^4}}\right)\sqrt{\frac{ac - \frac{a^2bc^2d+b^2d^3}{\sqrt{a^2b^2c^2d^2+b^3d^4}}}{a^2bc^2d + b^2d^3}}\right) - \sqrt{\frac{ac - \frac{a^2bc^2d+b^2d^3}{\sqrt{a^2b^2c^2d^2+b^3d^4}}}{a^2bc^2d + b^2d^3}} \log\left(2\sqrt{ax - \sqrt{a^2x^2+b}}\, d - 2\left(a^2c^2 + bd^2 + \frac{a^3bc^3d + ab^2cd^3}{\sqrt{a^2b^2c^2d^2+b^3d^4}}\right)\sqrt{\frac{ac - \frac{a^2bc^2d+b^2d^3}{\sqrt{a^2b^2c^2d^2+b^3d^4}}}{a^2bc^2d + b^2d^3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a^2*x^2+b)^(1/2)/(a*x-(a^2*x^2+b)^(1/2))^(1/2),x, algo
rithm="fricas")

[Out] sqrt((a*c + (a^2*b*c^2*d + b^2*d^3)/sqrt(a^2*b^2*c^2*d^2 + b^3*d^4))/(a^2*b
*c^2*d + b^2*d^3))*log(2*sqrt(a*x - sqrt(a^2*x^2 + b))*d + 2*(a^2*c^2 + b*d
^2 - (a^3*b*c^3*d + a*b^2*c*d^3)/sqrt(a^2*b^2*c^2*d^2 + b^3*d^4))*sqrt((a*c
 + (a^2*b*c^2*d + b^2*d^3)/sqrt(a^2*b^2*c^2*d^2 + b^3*d^4))/(a^2*b*c^2*d +
b^2*d^3))) - sqrt((a*c + (a^2*b*c^2*d + b^2*d^3)/sqrt(a^2*b^2*c^2*d^2 + b^3
*d^4))/(a^2*b*c^2*d + b^2*d^3))*log(2*sqrt(a*x - sqrt(a^2*x^2 + b))*d - 2*(
a^2*c^2 + b*d^2 - (a^3*b*c^3*d + a*b^2*c*d^3)/sqrt(a^2*b^2*c^2*d^2 + b^3*d^
4))*sqrt((a*c + (a^2*b*c^2*d + b^2*d^3)/sqrt(a^2*b^2*c^2*d^2 + b^3*d^4))/(a
^2*b*c^2*d + b^2*d^3))) + sqrt((a*c - (a^2*b*c^2*d + b^2*d^3)/sqrt(a^2*b^2*
c^2*d^2 + b^3*d^4))/(a^2*b*c^2*d + b^2*d^3))*log(2*sqrt(a*x - sqrt(a^2*x^2
+ b))*d + 2*(a^2*c^2 + b*d^2 + (a^3*b*c^3*d + a*b^2*c*d^3)/sqrt(a^2*b^2*c^2
*d^2 + b^3*d^4))*sqrt((a*c - (a^2*b*c^2*d + b^2*d^3)/sqrt(a^2*b^2*c^2*d^2 +
 b^3*d^4))/(a^2*b*c^2*d + b^2*d^3))) - sqrt((a*c - (a^2*b*c^2*d + b^2*d^3)/
sqrt(a^2*b^2*c^2*d^2 + b^3*d^4))/(a^2*b*c^2*d + b^2*d^3))*log(2*sqrt(a*x -
sqrt(a^2*x^2 + b))*d - 2*(a^2*c^2 + b*d^2 + (a^3*b*c^3*d + a*b^2*c*d^3)/sqr
t(a^2*b^2*c^2*d^2 + b^3*d^4))*sqrt((a*c - (a^2*b*c^2*d + b^2*d^3)/sqrt(a^2*
b^2*c^2*d^2 + b^3*d^4))/(a^2*b*c^2*d + b^2*d^3)))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2 + b}\,\sqrt{ax - \sqrt{a^2x^2 + b}}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a^2*x^2+b)^(1/2)/(a*x-(a^2*x^2+b)^(1/2))^(1/2),x, algo
rithm="giac")

[Out] integrate(1/(sqrt(a^2*x^2 + b)*sqrt(a*x - sqrt(a^2*x^2 + b))*(d*x + c)), x)

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)\sqrt{a^2x^2 + b}\,\sqrt{ax - \sqrt{a^2x^2 + b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x+c)/(a^2*x^2+b)^(1/2)/(a*x-(a^2*x^2+b)^(1/2))^(1/2),x)

[Out] int(1/(d*x+c)/(a^2*x^2+b)^(1/2)/(a*x-(a^2*x^2+b)^(1/2))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2 + b}\,\sqrt{ax - \sqrt{a^2x^2 + b}}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a^2*x^2+b)^(1/2)/(a*x-(a^2*x^2+b)^(1/2))^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 + b)*sqrt(a*x - sqrt(a^2*x^2 + b))*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a\,x-\sqrt{a^2\,x^2+b}}\,\sqrt{a^2\,x^2+b}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x - (b + a^2*x^2)^(1/2))^(1/2)*(b + a^2*x^2)^(1/2)*(c + d*x)),x)
```

```
[Out] int(1/((a*x - (b + a^2*x^2)^(1/2))^(1/2)*(b + a^2*x^2)^(1/2)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\sqrt{ax-\sqrt{a^2x^2+b}}\,\sqrt{a^2x^2+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a**2*x**2+b)**(1/2)/(a*x-(a**2*x**2+b)**(1/2))**(1/2),
x)
```

```
[Out] Integral(1/((c + d*x)*sqrt(a*x - sqrt(a**2*x**2 + b))*sqrt(a**2*x**2 + b)),
 x)
```

3.2038 $\int \frac{\sqrt{1+x}\sqrt{1+\sqrt{1+x}}}{x^2\sqrt{1+\sqrt{1+\sqrt{1+x}}}}\,dx$

Optimal. Leaf size=213

$$-\frac{\left(\sqrt{\sqrt{x+1}+1}+1\right)^{5/2}}{2\left(\sqrt{x+1}-1\right)\sqrt{\sqrt{x+1}+1}}+\frac{5\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{2\left(\sqrt{x+1}-1\right)\sqrt{\sqrt{x+1}+1}}-\frac{1}{4}\sqrt{17+25\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{\sqrt{\sqrt{2}-1}}\right)$$

Rubi [B] time = 3.40, antiderivative size = 432, normalized size of antiderivative = 2.03, number of steps used = 19, number of rules used = 9, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {1586, 2102, 1594, 28, 2073, 207, 1178, 1166, 203}

$$\frac{8\left(\sqrt{\sqrt{x+1}+1}+1\right)^{3/2}}{3\left(1-\sqrt{x+1}\right)\sqrt{\sqrt{x+1}+1}}-\frac{\left(50-17\sqrt{\sqrt{x+1}+1}\right)\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{30\left(1-\sqrt{x+1}\right)}-\frac{24\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{5\left(1-\sqrt{x+1}\right)\sqrt{\sqrt{x+1}+1}}-\frac{1}{30\left(1-\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)}+\frac{1}{30\left(\sqrt{\sqrt{\sqrt{x+1}+1}+1}+1\right)}-\frac{1}{60}\sqrt{10961+8989\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{\sqrt{\sqrt{2}-1}}\right)+\frac{1}{15}\sqrt{\frac{1}{2}\left(97+113\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{\sqrt{\sqrt{2}-1}}\right)+\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)-\frac{1}{60}\sqrt{8989\sqrt{2}-10961}\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{\sqrt{1+\sqrt{2}}}\right)-\frac{1}{15}\sqrt{\frac{1}{2}\left(113\sqrt{2}-97\right)}\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{\sqrt{1+\sqrt{2}}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[1 + x]*Sqrt[1 + Sqrt[1 + x]])/(x^2*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]
]),x]
```

```
[Out] (-24*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]])/(5*(1 - Sqrt[1 + x])*Sqrt[1 + Sqrt[1
+ x]]) - ((50 - 17*Sqrt[1 + Sqrt[1 + x]])*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]])/
(30*(1 - Sqrt[1 + x])) + (8*(1 + Sqrt[1 + Sqrt[1 + x]])^(3/2))/(3*(1 - Sqrt
[1 + x])*Sqrt[1 + Sqrt[1 + x]]) - 1/(30*(1 - Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]
])) + 1/(30*(1 + Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]])) + (Sqrt[(97 + 113*Sqrt[2
])/2]*ArcTan[Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]/Sqrt[-1 + Sqrt[2]]])/15 - (Sqr
t[10961 + 8989*Sqrt[2]]*ArcTan[Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]/Sqrt[-1 + Sq
rt[2]]])/60 + ArcTanh[Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]] - (Sqrt[(-97 + 113*S
qrt[2])/2]*ArcTanh[Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]/Sqrt[1 + Sqrt[2]]])/15 -
 (Sqrt[-10961 + 8989*Sqrt[2]]*ArcTanh[Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]/Sqrt[
1 + Sqrt[2]]])/60
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 2102

```
Int[(Pm_)*(Qn_)^(p_.), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x
]}, Simp[(Coeff[Pm, x, m]*x^(m - n + 1)*Qn^(p + 1))/((m + n*p + 1)*Coeff[Qn
, x, n]), x] + Dist[1/((m + n*p + 1)*Coeff[Qn, x, n]), Int[ExpandToSum[(m +
 n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn +
(p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 < 0
] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{1+x}\sqrt{1+\sqrt{1+x}}}{x^2\sqrt{1+\sqrt{1+\sqrt{1+x}}}}\,dx = 2\,\text{Subst}\left(\int \frac{x^2\sqrt{1+x}}{\left(-1+x^2\right)^2\sqrt{1+\sqrt{1+x}}}\,dx, x, \sqrt{1+x}\right)$$

$$= 2\,\text{Subst}\left(\int \frac{x^2}{(-1+x)^2(1+x)^{3/2}\sqrt{1+\sqrt{1+x}}}\,dx, x, \sqrt{1+x}\right)$$

$$= 4\,\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^2\sqrt{1+x}\left(-2+x^2\right)^2}\,dx, x, \sqrt{1+\sqrt{1+x}}\right)$$

$$= 4\,\text{Subst}\left(\int \frac{(-1+x)^2(1+x)^{3/2}}{x^2\left(-2+x^2\right)^2}\,dx, x, \sqrt{1+\sqrt{1+x}}\right)$$

$$= 8\,\text{Subst}\left(\int \frac{x^4\left(-2+x^2\right)^2}{\left(1+x^2-3x^4+x^6\right)^2}\,dx, x, \sqrt{1+\sqrt{1+\sqrt{1+x}}}\right)$$

$$= -\frac{8\left(1+\sqrt{1+\sqrt{1+x}}\right)^{3/2}}{3\left(2+\sqrt{1+\sqrt{1+x}}-3\left(1+\sqrt{1+\sqrt{1+x}}\right)^2+\left(1+\sqrt{1+\sqrt{1+x}}\right)^3\right)} - \frac{8}{3}\cdots$$

$$= -\frac{8\left(1+\sqrt{1+\sqrt{1+x}}\right)^{3/2}}{3\left(2+\sqrt{1+\sqrt{1+x}}-3\left(1+\sqrt{1+\sqrt{1+x}}\right)^2+\left(1+\sqrt{1+\sqrt{1+x}}\right)^3\right)} - \frac{8}{3}\cdots$$

$$= \frac{24\sqrt{1+\sqrt{1+\sqrt{1+x}}}}{5\left(2+\sqrt{1+\sqrt{1+x}}-3\left(1+\sqrt{1+\sqrt{1+x}}\right)^2+\left(1+\sqrt{1+\sqrt{1+x}}\right)^3\right)} - \frac{\cdots}{3\left(2\cdots\right.}$$

$$= \frac{24\sqrt{1+\sqrt{1+\sqrt{1+x}}}}{5\left(2+\sqrt{1+\sqrt{1+x}}-3\left(1+\sqrt{1+\sqrt{1+x}}\right)^2+\left(1+\sqrt{1+\sqrt{1+x}}\right)^3\right)} - \frac{\cdots}{3\left(2\cdots\right.}$$

$$= \frac{24\sqrt{1+\sqrt{1+\sqrt{1+x}}}}{5\left(2+\sqrt{1+\sqrt{1+x}}-3\left(1+\sqrt{1+\sqrt{1+x}}\right)^2+\left(1+\sqrt{1+\sqrt{1+x}}\right)^3\right)} - \frac{\cdots}{3\left(2\cdots\right.}$$

$$= -\frac{1}{30\left(1-\sqrt{1+\sqrt{1+\sqrt{1+x}}}\right)} + \frac{1}{30\left(1+\sqrt{1+\sqrt{1+\sqrt{1+x}}}\right)} + \frac{\cdots}{5\left(2+\sqrt{1\cdots}\right.}$$

$$= -\frac{1}{30\left(1-\sqrt{1+\sqrt{1+\sqrt{1+x}}}\right)} + \frac{1}{30\left(1+\sqrt{1+\sqrt{1+\sqrt{1+x}}}\right)} - \frac{\left(50-\cdots\right.}{30\left(1+2\left(\cdots\right.\right.}$$

$$= -\frac{1}{30\left(1-\sqrt{1+\sqrt{1+\sqrt{1+x}}}\right)} + \frac{1}{30\left(1+\sqrt{1+\sqrt{1+\sqrt{1+x}}}\right)} - \frac{\left(50-\cdots\right.}{30\left(1+2\left(\cdots\right.\right.}$$

$$\left(50-\cdots\right.$$

Mathematica [A] time = 1.04, size = 357, normalized size = 1.68

$$\frac{1}{4}\left(\frac{\left(\sqrt{2}-1\right)\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{\sqrt{2}-\sqrt{\sqrt{x+1}+1}}+\frac{\left(1+\sqrt{2}\right)\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{\sqrt{\sqrt{x+1}+1}+\sqrt{2}}-\frac{2}{\sqrt{\sqrt{\sqrt{x+1}+1}+1}-1}-\frac{2}{\sqrt{\sqrt{\sqrt{x+1}+1}+1}+1}-5\sqrt{2\left(1+\sqrt{2}\right)}\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)+\frac{\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)}{\left(\sqrt{2}-1\right)^{3/2}}+4\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)+\frac{\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)}{\left(1+\sqrt{2}\right)^{3/2}}-5\sqrt{2\left(\sqrt{2}-1\right)}\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x]*Sqrt[1 + Sqrt[1 + x]])/(x^2*Sqrt[1 + Sqrt[1 + Sqrt[1
 + x]]]),x]
```

```
[Out] (((-1 + Sqrt[2])*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]])/(Sqrt[2] - Sqrt[1 + Sqrt[
1 + x]]) + ((1 + Sqrt[2])*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]])/(Sqrt[2] + Sqrt[
1 + Sqrt[1 + x]]) - 2/(-1 + Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]) - 2/(1 + Sqrt[
1 + Sqrt[1 + Sqrt[1 + x]]]) + ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[1 + Sqrt[1 + Sq
rt[1 + x]]]]/(-1 + Sqrt[2])^(3/2) - 5*Sqrt[2*(1 + Sqrt[2])]*ArcTan[Sqrt[1 +
 Sqrt[2]]*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]] + 4*ArcTanh[Sqrt[1 + Sqrt[1 + Sq
rt[1 + x]]]] - 5*Sqrt[2*(-1 + Sqrt[2])]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[1 +
 Sqrt[1 + Sqrt[1 + x]]]] + ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[1 + Sqrt[1 + Sqr
t[1 + x]]]]/(1 + Sqrt[2])^(3/2))/4
```

IntegrateAlgebraic [A] time = 1.05, size = 210, normalized size = 0.99

$$\frac{\sqrt{\sqrt{\sqrt{x+1}+1}+1}\left(3-\sqrt{x+1}\right)}{2\left(\sqrt{x+1}-1\right)\sqrt{\sqrt{x+1}+1}}+\frac{\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{1-\sqrt{x+1}}-\frac{1}{4}\sqrt{17+25\sqrt{2}}\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)+\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)-\frac{1}{4}\sqrt{25\sqrt{2}-17}\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x]*Sqrt[1 + Sqrt[1 + x]])/(x^2*Sqrt[1 + Sqrt[1
 + Sqrt[1 + x]]]),x]
```

```
[Out] Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]/(1 - Sqrt[1 + x]) + ((3 - Sqrt[1 + x])*Sqrt
[1 + Sqrt[1 + Sqrt[1 + x]]])/(2*(-1 + Sqrt[1 + x])*Sqrt[1 + Sqrt[1 + x]]) -
 (Sqrt[17 + 25*Sqrt[2]]*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[1 + Sqrt[1 + Sqrt[1 +
 x]]]])/4 + ArcTanh[Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]] - (Sqrt[-17 + 25*Sqrt[
2]]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[1 + Sqrt[1 + Sqrt[1 + x]]]])/4
```

fricas [A] time = 0.48, size = 255, normalized size = 1.20

$$\frac{4x\sqrt{25\sqrt{2}+17}\arctan\left(\frac{1}{31}\sqrt{25\sqrt{2}+17}\left(4\sqrt{2}+1\right)\sqrt{\sqrt{2}+\sqrt{\sqrt{x+1}+1}}-\frac{1}{31}\sqrt{25\sqrt{2}+17}\left(4\sqrt{2}+1\right)\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)-x\sqrt{25\sqrt{2}-17}\log\left(\sqrt{25\sqrt{2}-17}\left(3\sqrt{2}+7\right)+31\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)+x\sqrt{25\sqrt{2}-17}\log\left(-\sqrt{25\sqrt{2}-17}\left(3\sqrt{2}+7\right)+31\sqrt{\sqrt{\sqrt{x+1}+1}+1}\right)+4x\log\left(\sqrt{\sqrt{\sqrt{x+1}+1}+1}+1\right)-4x\log\left(\sqrt{\sqrt{\sqrt{x+1}+1}+1}-1\right)-4\left(\sqrt{\sqrt{x+1}+1}\left(\sqrt{x+1}-3\right)+2\sqrt{x+1}+2\right)\sqrt{\sqrt{\sqrt{x+1}+1}+1}}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(1+(1+x)^(1/2))^(1/2)/x^2/(1+(1+(1+x)^(1/2))^(1/2))^(
1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*x*sqrt(25*sqrt(2) + 17)*arctan(1/31*sqrt(25*sqrt(2) + 17)*(4*sqrt(2)
 + 1)*sqrt(sqrt(2) + sqrt(sqrt(x + 1) + 1)) - 1/31*sqrt(25*sqrt(2) + 17)*(4
*sqrt(2) + 1)*sqrt(sqrt(sqrt(x + 1) + 1) + 1)) - x*sqrt(25*sqrt(2) - 17)*lo
g(sqrt(25*sqrt(2) - 17)*(3*sqrt(2) + 7) + 31*sqrt(sqrt(sqrt(x + 1) + 1) + 1
)) + x*sqrt(25*sqrt(2) - 17)*log(-sqrt(25*sqrt(2) - 17)*(3*sqrt(2) + 7) + 3
1*sqrt(sqrt(sqrt(x + 1) + 1) + 1)) + 4*x*log(sqrt(sqrt(sqrt(x + 1) + 1) + 1
) + 1) - 4*x*log(sqrt(sqrt(sqrt(x + 1) + 1) + 1) - 1) - 4*(sqrt(sqrt(x + 1)
 + 1)*(sqrt(x + 1) - 3) + 2*sqrt(x + 1) + 2)*sqrt(sqrt(sqrt(x + 1) + 1) + 1
))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(1+(1+x)^(1/2))^(1/2)/x^2/(1+(1+(1+x)^(1/2))^(1/2))^(
1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-2,[1]%%%}+%%%{-4,[0]%%%},%%%{-4,[1]%%%}+%%%{
-4,[0]%%%},%%%{1,[2]%%%}+%%%{-1,[1]%%%}+%%%{-1,[0]%%%}] at parameters value
s [60]Warning, choosing root of [1,0,%%%{-2,[1]%%%}+%%%{-4,[0]%%%},%%%{-4,[
1]%%%}+%%%{-4,[0]%%%},%%%{1,[2]%%%}+%%%{-1,[1]%%%}+%%%{-1,[0]%%%}] at param
eters values [-9]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [x]=[
44]4*(-1/8*ln(sqrt(sqrt(sqrt(x+1)+1)+1)-1)+1/8*ln(sqrt(sqrt(sqrt(x+1)+1)+1)
+1)+sqrt(25*sqrt(2)-17)/32*ln(abs(sqrt(sqrt(sqrt(x+1)+1)+1)-sqrt((16+16*sqr
t(2))/2/8)))-sqrt(25*sqrt(2)-17)/32*ln(sqrt(sqrt(sqrt(x+1)+1)+1)+sqrt((16+1
6*sqrt(2))/2/8))-sqrt(25*sqrt(2)+17)/16*atan(sqrt(sqrt(sqrt(x+1)+1)+1)/sqrt
(-(16-16*sqrt(2))/2/8))+(-sqrt(sqrt(sqrt(x+1)+1)+1)*(sqrt(sqrt(x+1)+1)+1)^2
+5*sqrt(sqrt(sqrt(x+1)+1)+1))/8/((sqrt(sqrt(x+1)+1)+1)^3-3*(sqrt(sqrt(x+1)+
1)+1)^2+sqrt(sqrt(x+1)+1)+2))
```

maple [A] time = 0.06, size = 267, normalized size = 1.25

$$-\frac{1}{2\left(\sqrt{1+\sqrt{1+\sqrt{1+x}}}-1\right)}-\frac{\ln\left(\sqrt{1+\sqrt{1+\sqrt{1+x}}}-1\right)}{2}-\frac{1}{2\left(1+\sqrt{1+\sqrt{1+\sqrt{1+x}}}\right)}+\frac{\ln\left(1+\sqrt{1+\sqrt{1+\sqrt{1+x}}}\right)}{2}+\frac{\frac{\left(1+\sqrt{1+\sqrt{1+x}}\right)^{\frac{3}{2}}}{2}-\frac{3\sqrt{1+\sqrt{1+\sqrt{1+x}}}}{2}}{\left(1+\sqrt{1+\sqrt{1+x}}\right)^{2}-2\sqrt{1+\sqrt{1+x}}-3}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1+\sqrt{1+\sqrt{1+x}}}}{\sqrt{1+\sqrt{2}}}\right)}{\sqrt{1+\sqrt{2}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\sqrt{1+\sqrt{1+x}}}}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{1+\sqrt{1+\sqrt{1+x}}}}{\sqrt{\sqrt{2}-1}}\right)}{\sqrt{\sqrt{2}-1}}+\frac{\arctan\left(\frac{\sqrt{1+\sqrt{1+\sqrt{1+x}}}}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)*(1+(1+x)^(1/2))^(1/2)/x^2/(1+(1+(1+x)^(1/2))^(1/2))^(1/2),x
)
```

```
[Out] -1/2/((1+(1+(1+x)^(1/2))^(1/2))^(1/2)-1)-1/2*ln((1+(1+(1+x)^(1/2))^(1/2))^(
1/2)-1)-1/2/(1+(1+(1+(1+x)^(1/2))^(1/2))^(1/2))+1/2*ln(1+(1+(1+(1+x)^(1/2))
^(1/2))^(1/2))+2*(1/4*(1+(1+(1+x)^(1/2))^(1/2))^(3/2)-3/4*(1+(1+(1+x)^(1/2)
)^(1/2))^(1/2))/((1+(1+(1+x)^(1/2))^(1/2))^2-2*(1+(1+x)^(1/2))^(1/2)-3)-2^(
1/2)/(1+2^(1/2))^(1/2)*arctanh((1+(1+(1+x)^(1/2))^(1/2))^(1/2)/(1+2^(1/2))^
(1/2))-1/4/(1+2^(1/2))^(1/2)*arctanh((1+(1+(1+x)^(1/2))^(1/2))^(1/2)/(1+2^(
1/2))^(1/2))-2^(1/2)/(2^(1/2)-1)^(1/2)*arctan((1+(1+(1+x)^(1/2))^(1/2))^(1/
2)/(2^(1/2)-1)^(1/2))+1/4/(2^(1/2)-1)^(1/2)*arctan((1+(1+(1+x)^(1/2))^(1/2)
)^(1/2)/(2^(1/2)-1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}\,\sqrt{\sqrt{x+1}+1}}{x^{2}\sqrt{\sqrt{\sqrt{x+1}+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(1+(1+x)^(1/2))^(1/2)/x^2/(1+(1+(1+x)^(1/2))^(1/2))^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + 1)*sqrt(sqrt(x + 1) + 1)/(x^2*sqrt(sqrt(sqrt(x + 1) + 1)
 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\sqrt{x+1}+1}\,\sqrt{x+1}}{x^{2}\,\sqrt{\sqrt{\sqrt{x+1}+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((x + 1)^(1/2) + 1)^(1/2)*(x + 1)^(1/2))/(x^2*(((x + 1)^(1/2) + 1)^(1/
2) + 1)^(1/2)),x)
```

```
[Out] int((((x + 1)^(1/2) + 1)^(1/2)*(x + 1)^(1/2))/(x^2*(((x + 1)^(1/2) + 1)^(1/
2) + 1)^(1/2)), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(1+(1+x)**(1/2))**(1/2)/x**2/(1+(1+(1+x)**(1/2))**(1
/2))**(1/2),x)
```

```
[Out] Timed out
```

3.2039 $\int \frac{-1+(2-k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(b-(1+2bk)x+\left(1+bk^2\right)x^2\right)}\,dx$

Optimal. Leaf size=214

$$-\frac{\log\left(b^{2/3}k^2x^2-2b^{2/3}kx+b^{2/3}+\sqrt[3]{kx^3+(-k-1)x^2+x}\left(\sqrt[3]{b}-\sqrt[3]{b}\,kx\right)+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{b}\right.}{}$$

Rubi [F] time = 4.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+(2-k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(b-(1+2bk)x+\left(1+bk^2\right)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + (2 - k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(b - (1 + 2*b*k)*x + (1 +
 b*k^2)*x^2)),x]
```

```
[Out] ((2 - k - Sqrt[1 - 4*b*(1 - k)]*k)*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*De
fer[Int][1/((1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*(-1 - 2*b*k - Sqrt[1 - 4*
b + 4*b*k] + 2*(1 + b*k^2)*x)), x])/((1 - x)*x*(1 - k*x))^(1/3) + ((2 - (1
- Sqrt[1 - 4*b*(1 - k)])*k)*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Int
][1/((1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*(-1 - 2*b*k + Sqrt[1 - 4*b + 4*b
*k] + 2*(1 + b*k^2)*x)), x])/((1 - x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\begin{aligned}\int \frac{-1+(2-k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(b-(1+2bk)x+\left(1+bk^2\right)x^2\right)}\,dx &= \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{-1+(2-k)x}{\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\left(b-(1+2bk)x+(1+\right.}}{\sqrt[3]{(1-x)x(1-kx)}}\\ &= \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int\left(\frac{2-k-k\sqrt{1-4b+4b}}{\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\left(-1-2bk-\sqrt{1-4}\right.}\right.}{\sqrt[3]{(}}\\ &= \frac{\left(\left(2-\left(1-\sqrt{1-4b(1-k)}\right)k\right)\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)}{\sqrt[3]{(1-x)x(1}}\end{aligned}$$

Mathematica [F] time = 5.75, size = 0, normalized size = 0.00

$$\int \frac{-1+(2-k)x}{\sqrt[3]{(1-x)x(1-kx)}\left(b-(1+2bk)x+\left(1+bk^2\right)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + (2 - k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(b - (1 + 2*b*k)*x
+ (1 + b*k^2)*x^2)),x]
```

```
[Out] Integrate[(-1 + (2 - k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(b - (1 + 2*b*k)*x
+ (1 + b*k^2)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.65, size = 214, normalized size = 1.00

$$-\frac{\log\left(b^{2/3}k^2x^2-2b^{2/3}kx+b^{2/3}+\sqrt[3]{kx^3+(-k-1)x^2+x}\left(\sqrt[3]{b}-\sqrt[3]{b}\,kx\right)+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{b}\,kx-\sqrt[3]{b}+\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{kx^3+(-k-1)x^2+x}}{-2\sqrt[3]{b}\,kx+2\sqrt[3]{b}+\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + (2 - k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(b - (1 +
2*b*k)*x + (1 + b*k^2)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(2*b^(1/3) - 2*b
^(1/3)*k*x + (x + (-1 - k)*x^2 + k*x^3)^(1/3))])/b^(1/3) + Log[-b^(1/3) + b
^(1/3)*k*x + (x + (-1 - k)*x^2 + k*x^3)^(1/3)]/b^(1/3) - Log[b^(2/3) - 2*b^
(2/3)*k*x + b^(2/3)*k^2*x^2 + (b^(1/3) - b^(1/3)*k*x)*(x + (-1 - k)*x^2 + k
*x^3)^(1/3) + (x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(2-k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-(2*b*k+1)*x+(b*k^2+1)*x^
2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(k-2)x+1}{((kx-1)(x-1)x)^{\frac{1}{3}}\left(\left(bk^2+1\right)x^2-(2\,bk+1)x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(2-k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-(2*b*k+1)*x+(b*k^2+1)*x^
2),x, algorithm="giac")
```

```
[Out] integrate(-((k - 2)*x + 1)/(((k*x - 1)*(x - 1)*x)^(1/3)*((b*k^2 + 1)*x^2 -
(2*b*k + 1)*x + b)), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{-1+(2-k)\,x}{((1-x)\,x\,(-kx+1))^{\frac{1}{3}}\left(b-(2bk+1)\,x+\left(b\,k^2+1\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+(2-k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-(2*b*k+1)*x+(b*k^2+1)*x^2),x)
```

```
[Out] int((-1+(2-k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-(2*b*k+1)*x+(b*k^2+1)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(k-2)x+1}{((kx-1)(x-1)x)^{\frac{1}{3}}\left(\left(bk^2+1\right)x^2-(2\,bk+1)x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(2-k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(b-(2*b*k+1)*x+(b*k^2+1)*x^
2),x, algorithm="maxima")
```

```
[Out] -integrate(((k - 2)*x + 1)/(((k*x - 1)*(x - 1)*x)^(1/3)*((b*k^2 + 1)*x^2 -
(2*b*k + 1)*x + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x\,\left(k-2\right)+1}{\left(\left(b\,k^2+1\right)\,x^2+\left(-2\,b\,k-1\right)\,x+b\right)\,{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(k - 2) + 1)/((b + x^2*(b*k^2 + 1) - x*(2*b*k + 1))*(x*(k*x - 1)*(x
 - 1))^(1/3)),x)
```

```
[Out] int(-(x*(k - 2) + 1)/((b + x^2*(b*k^2 + 1) - x*(2*b*k + 1))*(x*(k*x - 1)*(x
 - 1))^(1/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(2-k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(b-(2*b*k+1)*x+(b*k**2+1)*
x**2),x)
```

```
[Out] Timed out
```

3.2040 $\int \frac{3+(1-2k^2)x-3k^2x^2+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(1-d-(2+d)x+(1+dk^2)x^2+dk^2x^3)}\,dx$

Optimal. Leaf size=214

$$-\frac{\log\left(\sqrt[3]{d}\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}+x-1\right)}{d^{2/3}}+\frac{\log\left(d^{2/3}\left(k^2x^4+(-k^2-1)x^2+1\right)^{2/3}+\left(\sqrt[3]{d}-\sqrt[3]{d}x\right)\sqrt[3]{k^2x^4+(-k}\right.}{2d^{2/3}}$$

Rubi [F] time = 6.41, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3+(1-2k^2)x-3k^2x^2+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(1-d-(2+d)x+(1+dk^2)x^2+dk^2x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3 + (1 - 2*k^2)*x - 3*k^2*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1
/3)*(1 - d - (2 + d)*x + (1 + d*k^2)*x^2 + d*k^2*x^3)),x]
```

```
[Out] (x*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*AppellF1[1/2, 1/3, 1/3, 3/2, x^2, k^
2*x^2])/(d*((1 - x^2)*(1 - k^2*x^2))^(1/3)) + ((1 - 4*d)*(1 - x^2)^(1/3)*(1
 - k^2*x^2)^(1/3)*Defer[Int][1/((1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*(-1 + d
 + (2 + d)*x - (1 + d*k^2)*x^2 - d*k^2*x^3)), x])/(d*((1 - x^2)*(1 - k^2*x^
2))^(1/3)) + (2*(1 + d - d*k^2)*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*Defer[I
nt][x/((1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*(1 - d - (2 + d)*x + (1 + d*k^2)
*x^2 + d*k^2*x^3)), x])/(d*((1 - x^2)*(1 - k^2*x^2))^(1/3)) - ((1 + 4*d*k^2
)*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*Defer[Int][x^2/((1 - x^2)^(1/3)*(1 -
k^2*x^2)^(1/3)*(1 - d - (2 + d)*x + (1 + d*k^2)*x^2 + d*k^2*x^3)), x])/(d*(
(1 - x^2)*(1 - k^2*x^2))^(1/3))
```

Rubi steps

$$\int \frac{3+\left(1-2k^2\right)x-3k^2x^2+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(1-d-(2+d)x+\left(1+dk^2\right)x^2+dk^2x^3\right)}\,dx = \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{3+(1-}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,(}}{\sqrt[3]{\left(1-x^2\right)\left(1-\right.}}$$

$$= \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int\left(\frac{1}{d\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x}}\right.}{\sqrt[3]{(1-}}$$

$$= \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{1}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}}}{d\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{x\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,k\right.}{d\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{x\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,k\right.}{d\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{x\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,k\right.}{d\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

Mathematica [F] time = 3.87, size = 0, normalized size = 0.00

$$\int \frac{3+\left(1-2k^2\right)x-3k^2x^2+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(1-d-(2+d)x+\left(1+dk^2\right)x^2+dk^2x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3 + (1 - 2*k^2)*x - 3*k^2*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^
2))^(1/3)*(1 - d - (2 + d)*x + (1 + d*k^2)*x^2 + d*k^2*x^3)),x]
```

```
[Out] Integrate[(3 + (1 - 2*k^2)*x - 3*k^2*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^
2))^(1/3)*(1 - d - (2 + d)*x + (1 + d*k^2)*x^2 + d*k^2*x^3)), x]
```

IntegrateAlgebraic [A] time = 5.64, size = 214, normalized size = 1.00

$$-\frac{\log\left(\sqrt[3]{d}\,\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+x-1\right)}{d^{2/3}}+\frac{\log\left(d^{2/3}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+\left(\sqrt[3]{d}-\sqrt[3]{d}\,x\right)\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+x^2-2x+1\right)}{2d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}{\sqrt[3]{d}\,\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}-2x+2}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + (1 - 2*k^2)*x - 3*k^2*x^2 + k^2*x^3)/(((1 - x^2)*(1
 - k^2*x^2))^(1/3)*(1 - d - (2 + d)*x + (1 + d*k^2)*x^2 + d*k^2*x^3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))/(2
 - 2*x + d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))])/d^(2/3)) - Log[-1
+ x + d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3)]/d^(2/3) + Log[1 - 2*x +
 x^2 + (d^(1/3) - d^(1/3)*x)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3) + d^(2/3)
*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+(-2*k^2+1)*x-3*k^2*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/
(1-d-(2+d)*x+(d*k^2+1)*x^2+d*k^2*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3 - 3\,k^2x^2 - \left(2\,k^2 - 1\right)x + 3}{\left(dk^2x^3 + \left(dk^2 + 1\right)x^2 - (d+2)x - d + 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+(-2*k^2+1)*x-3*k^2*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/
(1-d-(2+d)*x+(d*k^2+1)*x^2+d*k^2*x^3),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^3 - 3*k^2*x^2 - (2*k^2 - 1)*x + 3)/((d*k^2*x^3 + (d*k^2 +
1)*x^2 - (d + 2)*x - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{3 + \left(-2k^2 + 1\right)x - 3k^2x^2 + k^2x^3}{\left(\left(-x^2 + 1\right)\left(-k^2x^2 + 1\right)\right)^{\frac{1}{3}}\left(1 - d - (2+d)\,x + \left(d\,k^2 + 1\right)x^2 + d\,k^2x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+(-2*k^2+1)*x-3*k^2*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d-(
2+d)*x+(d*k^2+1)*x^2+d*k^2*x^3),x)
```

```
[Out] int((3+(-2*k^2+1)*x-3*k^2*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d-(
2+d)*x+(d*k^2+1)*x^2+d*k^2*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3 - 3\,k^2x^2 - \left(2\,k^2 - 1\right)x + 3}{\left(dk^2x^3 + \left(dk^2 + 1\right)x^2 - (d+2)x - d + 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+(-2*k^2+1)*x-3*k^2*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/
(1-d-(2+d)*x+(d*k^2+1)*x^2+d*k^2*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^3 - 3*k^2*x^2 - (2*k^2 - 1)*x + 3)/((d*k^2*x^3 + (d*k^2 +
1)*x^2 - (d + 2)*x - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{3\,k^2\,x^2 - k^2\,x^3 + x\,\left(2\,k^2 - 1\right) - 3}{\left(\left(x^2 - 1\right)\,\left(k^2\,x^2 - 1\right)\right)^{1/3}\,\left(x^2\,\left(d\,k^2 + 1\right) - d - x\,\left(d + 2\right) + d\,k^2\,x^3 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*k^2*x^2 - k^2*x^3 + x*(2*k^2 - 1) - 3)/(((x^2 - 1)*(k^2*x^2 - 1))^(
1/3)*(x^2*(d*k^2 + 1) - d - x*(d + 2) + d*k^2*x^3 + 1)),x)
```

```
[Out] -int((3*k^2*x^2 - k^2*x^3 + x*(2*k^2 - 1) - 3)/(((x^2 - 1)*(k^2*x^2 - 1))^(
1/3)*(x^2*(d*k^2 + 1) - d - x*(d + 2) + d*k^2*x^3 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+(-2*k**2+1)*x-3*k**2*x**2+k**2*x**3)/((-x**2+1)*(-k**2*x**2+1)
)**(1/3)/(1-d-(2+d)*x+(d*k**2+1)*x**2+d*k**2*x**3),x)
```

```
[Out] Timed out
```

3.2041 $\int \frac{(1+x^3)^{2/3}(8-4x^3+x^6)}{x^6(2+x^3)}\,dx$

Optimal. Leaf size=214

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{5\log\left(\sqrt[3]{2}\sqrt[3]{x^3+1}-x\right)}{3\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}-\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3+1}+x}\right)}{2^{2/3}\sqrt{3}}+\frac{2\left(x^3+1\right)^{2/3}\left(3x^3-\right.}{5x^5}$$

Rubi [C] time = 0.39, antiderivative size = 103, normalized size of antiderivative = 0.48, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6725, 264, 277, 239, 429}

$$\frac{5}{2}xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-x^3,-\frac{x^3}{2}\right)+2\log\left(\sqrt[3]{x^3+1}-x\right)-\frac{4\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{4\left(x^3+1\right)^{5/3}}{5x^5}+\frac{2\left(x^3+1\right)^{2/3}}{x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((1 + x^3)^(2/3)*(8 - 4*x^3 + x^6))/(x^6*(2 + x^3)),x]
```

```
[Out] (2*(1 + x^3)^(2/3))/x^2 - (4*(1 + x^3)^(5/3))/(5*x^5) + (5*x*AppellF1[1/3,
-2/3, 1, 4/3, -x^3, -1/2*x^3])/2 - (4*ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sq
rt[3]])/Sqrt[3] + 2*Log[-x + (1 + x^3)^(1/3)]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(1+x^3\right)^{2/3}\left(8-4x^3+x^6\right)}{x^6\left(2+x^3\right)}\,dx = \int \left(\frac{4\left(1+x^3\right)^{2/3}}{x^6}-\frac{4\left(1+x^3\right)^{2/3}}{x^3}+\frac{5\left(1+x^3\right)^{2/3}}{2+x^3}\right)dx$$

$$= 4\int \frac{\left(1+x^3\right)^{2/3}}{x^6}\,dx-4\int \frac{\left(1+x^3\right)^{2/3}}{x^3}\,dx+5\int \frac{\left(1+x^3\right)^{2/3}}{2+x^3}\,dx$$

$$= \frac{2\left(1+x^3\right)^{2/3}}{x^2}-\frac{4\left(1+x^3\right)^{5/3}}{5x^5}+\frac{5}{2}xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-x^3,-\frac{x^3}{2}\right)-4\int \frac{1}{\sqrt[3]{1+x^3}}$$

$$= \frac{2\left(1+x^3\right)^{2/3}}{x^2}-\frac{4\left(1+x^3\right)^{5/3}}{5x^5}+\frac{5}{2}xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-x^3,-\frac{x^3}{2}\right)-\frac{4\tan^{-1}\left(\frac{1+\cdots}{\cdots}\right)}{\sqrt{3}}$$

Mathematica [C] time = 0.23, size = 154, normalized size = 0.72

$$\frac{1}{8}\left(x^4F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-x^3,-\frac{x^3}{2}\right)+\frac{16\left(x^3+1\right)^{2/3}\left(3x^3-2\right)}{5x^5}-2\sqrt[3]{2}\left(-2\log\left(2-\frac{2^{2/3}x}{\sqrt[3]{x^3+1}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2^{2/3}x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)+\log\left(\frac{2^{2/3}x}{\sqrt[3]{x^3+1}}+\frac{\sqrt[3]{2}x^2}{\left(x^3+1\right)^{2/3}}+2\right)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + x^3)^(2/3)*(8 - 4*x^3 + x^6))/(x^6*(2 + x^3)),x]
```

```
[Out] ((16*(1 + x^3)^(2/3)*(-2 + 3*x^3))/(5*x^5) + x^4*AppellF1[4/3, 1/3, 1, 7/3,
 -x^3, -1/2*x^3] - 2*2^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2^(2/3)*x)/(1 + x^3)^(
1/3))/Sqrt[3]] - 2*Log[2 - (2^(2/3)*x)/(1 + x^3)^(1/3)] + Log[2 + (2^(1/3)*
x^2)/(1 + x^3)^(2/3) + (2^(2/3)*x)/(1 + x^3)^(1/3)]))/8
```

IntegrateAlgebraic [A] time = 0.48, size = 214, normalized size = 1.00

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3+1}-x\right)+\frac{5\log\left(\sqrt[3]{2}\sqrt[3]{x^3+1}-x\right)}{3\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}+x}\right)}{\sqrt{3}}-\frac{5\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3+1}+x}\right)}{2^{2/3}\sqrt{3}}+\frac{2\left(x^3+1\right)^{2/3}\left(3x^3-2\right)}{5x^5}+\frac{1}{6}\log\left(\sqrt[3]{x^3+1}x+\left(x^3+1\right)^{2/3}+x^2\right)-\frac{5\log\left(\sqrt[3]{2}\sqrt[3]{x^3+1}x+2^{2/3}\left(x^3+1\right)^{2/3}+x^2\right)}{6\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)^(2/3)*(8 - 4*x^3 + x^6))/(x^6*(2 + x^3)),x]
```

```
[Out] (2*(1 + x^3)^(2/3)*(-2 + 3*x^3))/(5*x^5) + ArcTan[(Sqrt[3]*x)/(x + 2*(1 + x
^3)^(1/3))]/Sqrt[3] - (5*ArcTan[(Sqrt[3]*x)/(x + 2*2^(1/3)*(1 + x^3)^(1/3))
])/(2^(2/3)*Sqrt[3]) - Log[-x + (1 + x^3)^(1/3)]/3 + (5*Log[-x + 2^(1/3)*(1
 + x^3)^(1/3)])/(3*2^(2/3)) + Log[x^2 + x*(1 + x^3)^(1/3) + (1 + x^3)^(2/3)
]/6 - (5*Log[x^2 + 2^(1/3)*x*(1 + x^3)^(1/3) + 2^(2/3)*(1 + x^3)^(2/3)])/(6
*2^(2/3))
```

fricas [B] time = 27.89, size = 361, normalized size = 1.69

$$\frac{100\cdot 4^{\frac{1}{6}}\sqrt{3}x^5\arctan\left(\frac{4^{\frac{1}{6}}\left(12\cdot 4^{\frac{2}{3}}\sqrt{3}\left(2x^7+5x^4+2x\right)\left(x^3+1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\sqrt{3}\left(91x^9+168x^6+84x^3+8\right)+12\sqrt{3}\left(19x^8+22x^5+4x^2\right)\left(x^3+1\right)^{\frac{1}{3}}\right)}{6\left(53x^9+48x^6-12x^3-8\right)}\right)+50\cdot 4^{\frac{2}{3}}x^5\log\left(-\frac{6\cdot 4^{\frac{1}{3}}\left(x^3+1\right)^{\frac{1}{3}}x^2+4^{\frac{2}{3}}\left(x^3+2\right)-12\left(x^3+1\right)^{\frac{2}{3}}x}{x^3+2}\right)-25\cdot 4^{\frac{2}{3}}x^5\log\left(\frac{6\cdot 4^{\frac{2}{3}}\left(2x^4+x\right)\left(x^3+1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(19x^6+22x^3+4\right)+6\left(5x^5+4x^2\right)\left(x^3+1\right)^{\frac{1}{3}}}{x^6+4x^3+4}\right)+120\sqrt{3}x^5\arctan\left(-\frac{25382\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}x^2-13720\sqrt{3}\left(x^3+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(5831x^3+7200\right)}{58653x^3+8000}\right)-60x^5\log\left(3\left(x^3+1\right)^{\frac{1}{3}}x^2-3\left(x^3+1\right)^{\frac{2}{3}}x+1\right)+144\left(3x^3-2\right)\left(x^3+1\right)^{\frac{2}{3}}}{360x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^6-4*x^3+8)/x^6/(x^3+2),x, algorithm="fricas")
```

```
[Out] 1/360*(100*4^(1/6)*sqrt(3)*x^5*arctan(1/6*4^(1/6)*(12*4^(2/3)*sqrt(3)*(2*x^
7 + 5*x^4 + 2*x)*(x^3 + 1)^(2/3) + 4^(1/3)*sqrt(3)*(91*x^9 + 168*x^6 + 84*x
^3 + 8) + 12*sqrt(3)*(19*x^8 + 22*x^5 + 4*x^2)*(x^3 + 1)^(1/3))/(53*x^9 + 4
8*x^6 - 12*x^3 - 8)) + 50*4^(2/3)*x^5*log(-(6*4^(1/3)*(x^3 + 1)^(1/3)*x^2 +
 4^(2/3)*(x^3 + 2) - 12*(x^3 + 1)^(2/3)*x)/(x^3 + 2)) - 25*4^(2/3)*x^5*log(
(6*4^(2/3)*(2*x^4 + x)*(x^3 + 1)^(2/3) + 4^(1/3)*(19*x^6 + 22*x^3 + 4) + 6*
```

```
(5*x^5 + 4*x^2)*(x^3 + 1)^(1/3))/(x^6 + 4*x^3 + 4)) + 120*sqrt(3)*x^5*arcta
n(-(25382*sqrt(3)*(x^3 + 1)^(1/3)*x^2 - 13720*sqrt(3)*(x^3 + 1)^(2/3)*x + s
qrt(3)*(5831*x^3 + 7200))/(58653*x^3 + 8000)) - 60*x^5*log(3*(x^3 + 1)^(1/3
)*x^2 - 3*(x^3 + 1)^(2/3)*x + 1) + 144*(3*x^3 - 2)*(x^3 + 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6-4\,x^3+8\right)\left(x^3+1\right)^{\frac{2}{3}}}{\left(x^3+2\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^6-4*x^3+8)/x^6/(x^3+2),x, algorithm="giac")
[Out] integrate((x^6 - 4*x^3 + 8)*(x^3 + 1)^(2/3)/((x^3 + 2)*x^6), x)
```

maple [F] time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}+1\right)^{\frac{2}{3}} \left(x^{6}-4 x^{3}+8\right)}{x^{6} \left(x^{3}+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)*(x^6-4*x^3+8)/x^6/(x^3+2),x)
[Out] int((x^3+1)^(2/3)*(x^6-4*x^3+8)/x^6/(x^3+2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6-4\,x^3+8\right)\left(x^3+1\right)^{\frac{2}{3}}}{\left(x^3+2\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^6-4*x^3+8)/x^6/(x^3+2),x, algorithm="maxima")
[Out] integrate((x^6 - 4*x^3 + 8)*(x^3 + 1)^(2/3)/((x^3 + 2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(x^3+1\right)}^{2/3}\,\left(x^6-4\,x^3+8\right)}{x^6\,\left(x^3+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)^(2/3)*(x^6 - 4*x^3 + 8))/(x^6*(x^3 + 2)),x)
[Out] int(((x^3 + 1)^(2/3)*(x^6 - 4*x^3 + 8))/(x^6*(x^3 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x + 1\right) \left(x^{2} - x + 1\right)\right)^{\frac{2}{3}} \left(x^{2} + 2 x + 2\right) \left(x^{4} - 2 x^{3} + 2 x^{2} - 4 x + 4\right)}{x^{6} \left(x^{3} + 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)*(x**6-4*x**3+8)/x**6/(x**3+2),x)
[Out] Integral(((x + 1)*(x**2 - x + 1))**(2/3)*(x**2 + 2*x + 2)*(x**4 - 2*x**3 +
2*x**2 - 4*x + 4)/(x**6*(x**3 + 2)), x)
```

3.2042 $\int \frac{(-1+x^3)^{2/3}(8+2x^3+x^6)}{x^6(-2+x^3)}\,dx$

Optimal. Leaf size=214

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{2}{3}\sqrt[3]{2}\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}-\frac{2\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}+\frac{(x^3-1)^{2/}}{10}$$

Rubi [C] time = 0.39, antiderivative size = 123, normalized size of antiderivative = 0.57, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6725, 264, 277, 239, 430, 429}

$$-\frac{2x(x^3-1)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,\frac{x^3}{2}\right)}{(1-x^3)^{2/3}}+\frac{3}{2}\log\left(\sqrt[3]{x^3-1}-x\right)-\sqrt{3}\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)-\frac{4(x^3-1)^{5/3}}{5x^5}+\frac{3(x^3-1)^{2/3}}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^3)^(2/3)*(8 + 2*x^3 + x^6))/(x^6*(-2 + x^3)),x]
```

```
[Out] (3*(-1 + x^3)^(2/3))/(2*x^2) - (4*(-1 + x^3)^(5/3))/(5*x^5) - (2*x*(-1 + x^
3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, x^3, x^3/2])/(1 - x^3)^(2/3) - Sqrt[3]
*ArcTan[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]] + (3*Log[-x + (-1 + x^3)^(1/3
)])/2
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^3\right)^{2/3}\left(8+2x^3+x^6\right)}{x^6\left(-2+x^3\right)}\,dx &= \int\left(-\frac{4\left(-1+x^3\right)^{2/3}}{x^6}-\frac{3\left(-1+x^3\right)^{2/3}}{x^3}+\frac{4\left(-1+x^3\right)^{2/3}}{-2+x^3}\right)dx\\
&= -\left(3\int\frac{\left(-1+x^3\right)^{2/3}}{x^3}\,dx\right)-4\int\frac{\left(-1+x^3\right)^{2/3}}{x^6}\,dx+4\int\frac{\left(-1+x^3\right)^{2/3}}{-2+x^3}\,dx\\
&= \frac{3\left(-1+x^3\right)^{2/3}}{2x^2}-\frac{4\left(-1+x^3\right)^{5/3}}{5x^5}-3\int\frac{1}{\sqrt[3]{-1+x^3}}\,dx+\frac{\left(4\left(-1+x^3\right)^{2/3}\right)\int\frac{(1-\ldots}{-2\ldots}}{\left(1-x^3\right)^{2/3}}\\
&= \frac{3\left(-1+x^3\right)^{2/3}}{2x^2}-\frac{4\left(-1+x^3\right)^{5/3}}{5x^5}-\frac{2x\left(-1+x^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,\frac{x^3}{2}\right)}{\left(1-x^3\right)^{2/3}}-\sqrt{3}\ldots
\end{aligned}$$

Mathematica [C] time = 0.27, size = 181, normalized size = 0.85

$$-\frac{\sqrt[3]{1-x^3}\,x^4F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,\frac{x^3}{2}\right)}{8\sqrt[3]{x^3-1}}+\frac{\left(x^3-1\right)^{2/3}\left(7x^3+8\right)}{10x^5}-\frac{-2\log\left(2-\frac{2^{2/3}x}{\sqrt[3]{1-x^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2^{2/3}x}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)+\log\left(\frac{2^{2/3}x}{\sqrt[3]{1-x^3}}+\frac{\sqrt[3]{2}x^2}{\left(1-x^3\right)^{2/3}}+2\right)}{3\,2^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^3)^(2/3)*(8 + 2*x^3 + x^6))/(x^6*(-2 + x^3)),x]
```

```
[Out] ((-1 + x^3)^(2/3)*(8 + 7*x^3))/(10*x^5) - (x^4*(1 - x^3)^(1/3)*AppellF1[4/3
, 1/3, 1, 7/3, x^3, x^3/2])/(8*(-1 + x^3)^(1/3)) - (2*Sqrt[3]*ArcTan[(1 + (
2^(2/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - 2*Log[2 - (2^(2/3)*x)/(1 - x^3)^(1/3
)] + Log[2 + (2^(1/3)*x^2)/(1 - x^3)^(2/3) + (2^(2/3)*x)/(1 - x^3)^(1/3)])/
(3*2^(2/3))
```

IntegrateAlgebraic [A] time = 0.50, size = 214, normalized size = 1.00

$$-\frac{1}{3}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{2}{3}\sqrt[3]{2}\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}-\frac{2\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}+\frac{\left(x^3-1\right)^{2/3}\left(7x^3+8\right)}{10x^5}+\frac{1}{6}\log\left(\sqrt[3]{x^3-1}\,x+\left(x^3-1\right)^{2/3}+x^2\right)-\frac{1}{3}\sqrt[3]{2}\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}\,x+2^{2/3}\left(x^3-1\right)^{2/3}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(2/3)*(8 + 2*x^3 + x^6))/(x^6*(-2 + x^3)),x]
```

```
[Out] ((-1 + x^3)^(2/3)*(8 + 7*x^3))/(10*x^5) + ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x
^3)^(1/3))]/Sqrt[3] - (2*2^(1/3)*ArcTan[(Sqrt[3]*x)/(x + 2*2^(1/3)*(-1 + x^
3)^(1/3))])/Sqrt[3] - Log[-x + (-1 + x^3)^(1/3)]/3 + (2*2^(1/3)*Log[-x + 2^
(1/3)*(-1 + x^3)^(1/3)])/3 + Log[x^2 + x*(-1 + x^3)^(1/3) + (-1 + x^3)^(2/3
)]/6 - (2^(1/3)*Log[x^2 + 2^(1/3)*x*(-1 + x^3)^(1/3) + 2^(2/3)*(-1 + x^3)^(
2/3)])/3
```

fricas [B] time = 26.72, size = 360, normalized size = 1.68

$$\frac{20\sqrt{3}\,2^{\frac{1}{3}}x^5\arctan\left(\frac{12\sqrt{3}\,2^{\frac{2}{3}}\left(2x^7-5x^4+2x\right)\left(x^3-1\right)^{\frac{2}{3}}+6\sqrt{3}\,2^{\frac{1}{3}}\left(19x^8-22x^5+4x^2\right)\left(x^3-1\right)^{\frac{1}{3}}+\sqrt{3}\left(91x^9-168x^6+84x^3-8\right)}{3\left(53x^9-48x^6-12x^3+8\right)}\right)+30\sqrt{3}x^5\arctan\left(-\frac{25382\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}x^2-13720\sqrt{3}\left(x^3-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(5831x^3-7200\right)}{58653x^3-8000}\right)+20\cdot2^{\frac{1}{3}}x^5\log\left(-\frac{3\cdot2^{\frac{2}{3}}\left(x^3-1\right)^{\frac{1}{3}}x^2-6\left(x^3-1\right)^{\frac{2}{3}}x+2^{\frac{1}{3}}\left(x^3-2\right)}{x^3-2}\right)-10\cdot2^{\frac{1}{3}}x^5\log\left(\frac{12\cdot2^{\frac{1}{3}}\left(2x^4-x\right)\left(x^3-1\right)^{\frac{2}{3}}+2^{\frac{2}{3}}\left(19x^6-22x^3+4\right)+6\left(5x^5-4x^2\right)\left(x^3-1\right)^{\frac{1}{3}}}{x^6-4x^3+4}\right)-15x^5\log\left(-3\left(x^3-1\right)^{\frac{1}{3}}x^2+3\left(x^3-1\right)^{\frac{2}{3}}x+1\right)+9\left(7x^3+8\right)\left(x^3-1\right)^{\frac{2}{3}}}{90x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+2*x^3+8)/x^6/(x^3-2),x, algorithm="fricas")
```

```
[Out] 1/90*(20*sqrt(3)*2^(1/3)*x^5*arctan(1/3*(12*sqrt(3)*2^(2/3)*(2*x^7 - 5*x^4
+ 2*x)*(x^3 - 1)^(2/3) + 6*sqrt(3)*2^(1/3)*(19*x^8 - 22*x^5 + 4*x^2)*(x^3 -
 1)^(1/3) + sqrt(3)*(91*x^9 - 168*x^6 + 84*x^3 - 8))/(53*x^9 - 48*x^6 - 12*
x^3 + 8)) + 30*sqrt(3)*x^5*arctan(-(25382*sqrt(3)*(x^3 - 1)^(1/3)*x^2 - 137
20*sqrt(3)*(x^3 - 1)^(2/3)*x + sqrt(3)*(5831*x^3 - 7200))/(58653*x^3 - 8000
)) + 20*2^(1/3)*x^5*log(-(3*2^(2/3)*(x^3 - 1)^(1/3)*x^2 - 6*(x^3 - 1)^(2/3)
*x + 2^(1/3)*(x^3 - 2))/(x^3 - 2)) - 10*2^(1/3)*x^5*log((12*2^(1/3)*(2*x^4
- x)*(x^3 - 1)^(2/3) + 2^(2/3)*(19*x^6 - 22*x^3 + 4) + 6*(5*x^5 - 4*x^2)*(x
^3 - 1)^(1/3))/(x^6 - 4*x^3 + 4)) - 15*x^5*log(-3*(x^3 - 1)^(1/3)*x^2 + 3*(
x^3 - 1)^(2/3)*x + 1) + 9*(7*x^3 + 8)*(x^3 - 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+2\,x^3+8\right)\left(x^3-1\right)^{\frac{2}{3}}}{\left(x^3-2\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+2*x^3+8)/x^6/(x^3-2),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 2*x^3 + 8)*(x^3 - 1)^(2/3)/((x^3 - 2)*x^6), x)
```

maple [F] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}-1\right)^{\frac{2}{3}} \left(x^{6}+2 x^{3}+8\right)}{x^{6} \left(x^{3}-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)*(x^6+2*x^3+8)/x^6/(x^3-2),x)
```

```
[Out] int((x^3-1)^(2/3)*(x^6+2*x^3+8)/x^6/(x^3-2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+2\,x^3+8\right)\left(x^3-1\right)^{\frac{2}{3}}}{\left(x^3-2\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+2*x^3+8)/x^6/(x^3-2),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + 2*x^3 + 8)*(x^3 - 1)^(2/3)/((x^3 - 2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(x^3-1\right)}^{2/3}\,\left(x^6+2\,x^3+8\right)}{x^6\,\left(x^3-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(2*x^3 + x^6 + 8))/(x^6*(x^3 - 2)),x)
```

```
[Out] int(((x^3 - 1)^(2/3)*(2*x^3 + x^6 + 8))/(x^6*(x^3 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((x-1)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}\left(x^{6}+2x^{3}+8\right)}{x^{6}\left(x^{3}-2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)*(x**6+2*x**3+8)/x**6/(x**3-2),x)
```

```
[Out] Integral(((x - 1)*(x**2 + x + 1))**(2/3)*(x**6 + 2*x**3 + 8)/(x**6*(x**3 -
2)), x)
```

3.2043 $$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(cx^4+bx^2\left(q+px^3\right)+a\left(q+px^3\right)^2\right)}{x^9}\,dx$$

Optimal. Leaf size=214

$$\frac{\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}\,\left(3ap^3x^9-3ap^2qx^7+9ap^2qx^6-3apq^2x^4+9apq^2x^3+3aq^3+4bp^2x^8-8bpqx^6+\right.}{12x^8}$$

Rubi [F] time = 1.91, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(cx^4+bx^2\left(q+px^3\right)+a\left(q+px^3\right)^2\right)}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(c*x^4 + b*
x^2*(q + p*x^3) + a*(q + p*x^3)^2))/x^9,x]
```

```
[Out] a*p^3*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6], x] - 2*a*q^3*
Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^9, x] - 2*b*q^2*De
fer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^7, x] - 3*a*p*q^2*De
fer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^6, x] - 2*c*q*Defer[
Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^5, x] - b*p*q*Defer[Int]
[Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^4, x] + c*p*Defer[Int][Sqrt[
q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^2, x] + b*p^2*Defer[Int][Sqrt[q^2
+ 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x, x]
```

Rubi steps

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(cx^4+bx^2\left(q+px^3\right)+a\left(q+px^3\right)^2\right)}{x^9}\,dx = \int \left(ap^3\sqrt{q^2+2pqx^3-\right.$$

$$= (cp)\int \frac{\sqrt{q^2+2pqx^3-}}{x}$$

Mathematica [F] time = 1.27, size = 0, normalized size = 0.00

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(cx^4+bx^2\left(q+px^3\right)+a\left(q+px^3\right)^2\right)}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(c*x^
4 + b*x^2*(q + p*x^3) + a*(q + p*x^3)^2))/x^9,x]
```

```
[Out] Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(c*x^
4 + b*x^2*(q + p*x^3) + a*(q + p*x^3)^2))/x^9, x]
```

IntegrateAlgebraic [A] time = 0.74, size = 214, normalized size = 1.00

$$\frac{\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}\left(3ap^3x^9-3ap^2qx^7+9ap^2qx^6-3apq^2x^4+9apq^2x^3+3aq^3+4bp^2x^8-8bpqx^6+8bpqx^5+4bq^2x^2+6cpx^7+6cqx^4\right)}{12x^8}+\frac{1}{2}\left(-ap^2q^2-2cpq\right)\log\left(\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}+px^3+q\right)+\log(x)\left(ap^2q^2+2cpq\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x
^6]*(c*x^4 + b*x^2*(q + p*x^3) + a*(q + p*x^3)^2))/x^9,x]
```

```
[Out] (Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(3*a*q^3 + 4*b*q^2*x^2 + 9*a*p
*q^2*x^3 + 6*c*q*x^4 - 3*a*p*q^2*x^4 + 8*b*p*q*x^5 - 8*b*p*q*x^6 + 9*a*p^2*
q*x^6 + 6*c*p*x^7 - 3*a*p^2*q*x^7 + 4*b*p^2*x^8 + 3*a*p^3*x^9))/(12*x^8) +
(2*c*p*q + a*p^2*q^2)*Log[x] + ((-2*c*p*q - a*p^2*q^2)*Log[q + p*x^3 + Sqrt
[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]])/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(c*x^4+b*x^2*
(p*x^3+q)+a*(p*x^3+q)^2)/x^9,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6 - 2\,pqx^4 + 2\,pqx^3 + q^2}\left(cx^4 + \left(px^3 + q\right)bx^2 + \left(px^3 + q\right)^2 a\right)\left(px^3 - 2\,q\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(c*x^4+b*x^2*
(p*x^3+q)+a*(p*x^3+q)^2)/x^9,x, algorithm="giac")
```

```
[Out] integrate(sqrt(p^2*x^6 - 2*p*q*x^4 + 2*p*q*x^3 + q^2)*(c*x^4 + (p*x^3 + q)*
b*x^2 + (p*x^3 + q)^2*a)*(p*x^3 - 2*q)/x^9, x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(p\,x^3 - 2q\right)\sqrt{p^2x^6 - 2pq\,x^4 + 2pq\,x^3 + q^2}\,\left(c\,x^4 + b\,x^2\left(p\,x^3 + q\right) + a\left(p\,x^3 + q\right)^2\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(c*x^4+b*x^2*(p*x^3
+q)+a*(p*x^3+q)^2)/x^9,x)
```

```
[Out] int((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(c*x^4+b*x^2*(p*x^3
+q)+a*(p*x^3+q)^2)/x^9,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6 - 2\,pqx^4 + 2\,pqx^3 + q^2}\left(cx^4 + \left(px^3 + q\right)bx^2 + \left(px^3 + q\right)^2 a\right)\left(px^3 - 2\,q\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(c*x^4+b*x^2*
(p*x^3+q)+a*(p*x^3+q)^2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(p^2*x^6 - 2*p*q*x^4 + 2*p*q*x^3 + q^2)*(c*x^4 + (p*x^3 + q)*
b*x^2 + (p*x^3 + q)^2*a)*(p*x^3 - 2*q)/x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(2\,q - p\,x^3\right)\,\left(a\,\left(p\,x^3 + q\right)^2 + c\,x^4 + b\,x^2\,\left(p\,x^3 + q\right)\right)\,\sqrt{p^2\,x^6 - 2\,p\,q\,x^4 + 2\,p\,q\,x^3 + q^2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*q - p*x^3)*(a*(q + p*x^3)^2 + c*x^4 + b*x^2*(q + p*x^3))*(p^2*x^6
+ q^2 + 2*p*q*x^3 - 2*p*q*x^4)^(1/2))/x^9,x)
```

```
[Out] int(-((2*q - p*x^3)*(a*(q + p*x^3)^2 + c*x^4 + b*x^2*(q + p*x^3))*(p^2*x^6
+ q^2 + 2*p*q*x^3 - 2*p*q*x^4)^(1/2))/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(px^3 - 2q)\sqrt{p^2x^6 - 2pqx^4 + 2pqx^3 + q^2}\,(ap^2x^6 + 2apqx^3 + aq^2 + bpx^5 + bqx^2 + cx^4)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**3-2*q)*(p**2*x**6-2*p*q*x**4+2*p*q*x**3+q**2)**(1/2)*(c*x**
4+b*x**2*(p*x**3+q)+a*(p*x**3+q)**2)/x**9,x)
```

```
[Out] Integral((p*x**3 - 2*q)*sqrt(p**2*x**6 - 2*p*q*x**4 + 2*p*q*x**3 + q**2)*(a
*p**2*x**6 + 2*a*p*q*x**3 + a*q**2 + b*p*x**5 + b*q*x**2 + c*x**4)/x**9, x)
```

3.2044 $\int \frac{-b+ax^2}{\sqrt[3]{b^2x^2+a^3x^3}}\,dx$

Optimal. Leaf size=215

$$\frac{\left(a^3x^3+b^2x^2\right)^{2/3}\left(3a^3x-4b^2\right)}{6a^5x}+\frac{\left(9a^5b-2b^4\right)\log\left(\sqrt[3]{a^3x^3+b^2x^2}-ax\right)}{9a^6}-\frac{\left(9a^5b-2b^4\right)\tan^{-1}\left(\frac{\sqrt{3}\,ax}{2\sqrt[3]{a^3x^3+b^2x^2}+ax}\right)}{3\sqrt{3}\,a^6}+\frac{(2b$$

Rubi [B] time = 0.34, antiderivative size = 466, normalized size of antiderivative = 2.17, number of steps used = 8, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2053, 2011, 59, 2024}

$$\frac{bx^{2/3}\log(x)\sqrt[3]{a^3x+b^2}}{2a\sqrt[3]{a^3x^3+b^2x^2}}+\frac{3bx^{2/3}\sqrt[3]{a^3x+b^2}\log\left(\frac{\sqrt[3]{a^3x+b^2}}{a\sqrt[3]{x}}-1\right)}{2a\sqrt[3]{a^3x^3+b^2x^2}}+\frac{\sqrt{3}\,bx^{2/3}\sqrt[3]{a^3x+b^2}\tan^{-1}\left(\frac{2\sqrt[3]{a^3x+b^2}}{\sqrt{3}\,a\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{a\sqrt[3]{a^3x^3+b^2x^2}}-\frac{b^4x^{2/3}\log(x)\sqrt[3]{a^3x+b^2}}{9a^6\sqrt[3]{a^3x^3+b^2x^2}}-\frac{b^4x^{2/3}\sqrt[3]{a^3x+b^2}\log\left(\frac{\sqrt[3]{a^3x+b^2}}{a\sqrt[3]{x}}-1\right)}{3a^6\sqrt[3]{a^3x^3+b^2x^2}}-\frac{2b^4x^{2/3}\sqrt[3]{a^3x+b^2}\tan^{-1}\left(\frac{2\sqrt[3]{a^3x+b^2}}{\sqrt{3}\,a\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,a^6\sqrt[3]{a^3x^3+b^2x^2}}-\frac{2b^2\left(a^3x^3+b^2x^2\right)^{2/3}}{3a^5x}+\frac{\left(a^3x^3+b^2x^2\right)^{2/3}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^2)/(b^2*x^2 + a^3*x^3)^(1/3),x]
```

```
[Out] (b^2*x^2 + a^3*x^3)^(2/3)/(2*a^2) - (2*b^2*(b^2*x^2 + a^3*x^3)^(2/3))/(3*a^
5*x) + (Sqrt[3]*b*x^(2/3)*(b^2 + a^3*x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(b^2 +
a^3*x)^(1/3))/(Sqrt[3]*a*x^(1/3))])/(a*(b^2*x^2 + a^3*x^3)^(1/3)) - (2*b^4*
x^(2/3)*(b^2 + a^3*x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(b^2 + a^3*x)^(1/3))/(Sqr
t[3]*a*x^(1/3))])/(3*Sqrt[3]*a^6*(b^2*x^2 + a^3*x^3)^(1/3)) + (b*x^(2/3)*(b
^2 + a^3*x)^(1/3)*Log[x])/(2*a*(b^2*x^2 + a^3*x^3)^(1/3)) - (b^4*x^(2/3)*(b
^2 + a^3*x)^(1/3)*Log[x])/(9*a^6*(b^2*x^2 + a^3*x^3)^(1/3)) + (3*b*x^(2/3)*
(b^2 + a^3*x)^(1/3)*Log[-1 + (b^2 + a^3*x)^(1/3)/(a*x^(1/3))])/(2*a*(b^2*x^
2 + a^3*x^3)^(1/3)) - (b^4*x^(2/3)*(b^2 + a^3*x)^(1/3)*Log[-1 + (b^2 + a^3*
x)^(1/3)/(a*x^(1/3))])/(3*a^6*(b^2*x^2 + a^3*x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2053

```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Expan
dIntegrand[Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && (Po
lyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IntegerQ[p] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{-b+ax^2}{\sqrt[3]{b^2x^2+a^3x^3}}\,dx &= \int \left(-\frac{b}{\sqrt[3]{b^2x^2+a^3x^3}}+\frac{ax^2}{\sqrt[3]{b^2x^2+a^3x^3}}\right)dx\\
&= a\int \frac{x^2}{\sqrt[3]{b^2x^2+a^3x^3}}\,dx - b\int \frac{1}{\sqrt[3]{b^2x^2+a^3x^3}}\,dx\\
&= \frac{\left(b^2x^2+a^3x^3\right)^{2/3}}{2a^2} - \frac{\left(2b^2\right)\int \frac{x}{\sqrt[3]{b^2x^2+a^3x^3}}\,dx}{3a^2} - \frac{\left(bx^{2/3}\sqrt[3]{b^2+a^3x}\right)\int \frac{1}{x^{2/3}\sqrt[3]{b^2+a^3x}}\,dx}{\sqrt[3]{b^2x^2+a^3x^3}}\\
&= \frac{\left(b^2x^2+a^3x^3\right)^{2/3}}{2a^2} - \frac{2b^2\left(b^2x^2+a^3x^3\right)^{2/3}}{3a^5x} + \frac{\sqrt{3}\,bx^{2/3}\sqrt[3]{b^2+a^3x}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b^2+a^3x}}{\sqrt{3}\,a\sqrt[3]{x}}\right)}{a\sqrt[3]{b^2x^2+a^3x^3}}\\
&= \frac{\left(b^2x^2+a^3x^3\right)^{2/3}}{2a^2} - \frac{2b^2\left(b^2x^2+a^3x^3\right)^{2/3}}{3a^5x} + \frac{\sqrt{3}\,bx^{2/3}\sqrt[3]{b^2+a^3x}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b^2+a^3x}}{\sqrt{3}\,a\sqrt[3]{x}}\right)}{a\sqrt[3]{b^2x^2+a^3x^3}}\\
&= \frac{\left(b^2x^2+a^3x^3\right)^{2/3}}{2a^2} - \frac{2b^2\left(b^2x^2+a^3x^3\right)^{2/3}}{3a^5x} + \frac{\sqrt{3}\,bx^{2/3}\sqrt[3]{b^2+a^3x}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b^2+a^3x}}{\sqrt{3}\,a\sqrt[3]{x}}\right)}{a\sqrt[3]{b^2x^2+a^3x^3}}
\end{aligned}$$

Mathematica [C] time = 0.12, size = 119, normalized size = 0.55

$$\frac{3\left(x^2\left(a^3x+b^2\right)\right)^{2/3}\left(b^3\,{}_2F_1\left(-\frac{5}{3},\frac{1}{3};\frac{4}{3};-\frac{a^3x}{b^2}\right)-2b^3\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};-\frac{a^3x}{b^2}\right)+\left(b^3-a^5\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{a^3x}{b^2}\right)\right)}{a^5bx\left(\frac{a^3x}{b^2}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^2)/(b^2*x^2 + a^3*x^3)^(1/3),x]
```

```
[Out] (3*(x^2*(b^2 + a^3*x))^(2/3)*(b^3*Hypergeometric2F1[-5/3, 1/3, 4/3, -((a^3*
x)/b^2)] - 2*b^3*Hypergeometric2F1[-2/3, 1/3, 4/3, -((a^3*x)/b^2)] + (-a^5
+ b^3)*Hypergeometric2F1[1/3, 1/3, 4/3, -((a^3*x)/b^2)]))/(a^5*b*x*(1 + (a^
3*x)/b^2)^(2/3))
```

IntegrateAlgebraic [A] time = 0.62, size = 215, normalized size = 1.00

$$\frac{\left(a^3x^3+b^2x^2\right)^{2/3}\left(3a^3x-4b^2\right)}{6a^5x}+\frac{\left(9a^5b-2b^4\right)\log\left(\sqrt[3]{a^3x^3+b^2x^2}-ax\right)}{9a^6}-\frac{\left(9a^5b-2b^4\right)\tan^{-1}\left(\frac{\sqrt{3}\,ax}{2\sqrt[3]{a^3x^3+b^2x^2}+ax}\right)}{3\sqrt{3}\,a^6}+\frac{\left(2b^4-9a^5b\right)\log\left(ax\sqrt[3]{a^3x^3+b^2x^2}+\left(a^3x^3+b^2x^2\right)^{2/3}+a^2x^2\right)}{18a^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^2)/(b^2*x^2 + a^3*x^3)^(1/3),x]
```

```
[Out] ((-4*b^2 + 3*a^3*x)*(b^2*x^2 + a^3*x^3)^(2/3))/(6*a^5*x) - ((9*a^5*b - 2*b^
4)*ArcTan[(Sqrt[3]*a*x)/(a*x + 2*(b^2*x^2 + a^3*x^3)^(1/3))])/(3*Sqrt[3]*a^
6) + ((9*a^5*b - 2*b^4)*Log[-(a*x) + (b^2*x^2 + a^3*x^3)^(1/3)])/(9*a^6) +
((-9*a^5*b + 2*b^4)*Log[a^2*x^2 + a*x*(b^2*x^2 + a^3*x^3)^(1/3) + (b^2*x^2
+ a^3*x^3)^(2/3)])/(18*a^6)
```

fricas [A] time = 0.49, size = 206, normalized size = 0.96

$$\frac{2\sqrt{3}\left(9a^5b-2b^4\right)x\arctan\left(\frac{\sqrt{3}\,ax+2\sqrt{3}\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}}{3ax}\right)+2\left(9a^5b-2b^4\right)x\log\left(-\frac{ax-\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}}{x}\right)-\left(9a^5b-2b^4\right)x\log\left(\frac{a^2x^2+\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}ax+\left(a^3x^3+b^2x^2\right)^{\frac{2}{3}}}{x^2}\right)+3\left(a^3x^3+b^2x^2\right)^{\frac{2}{3}}\left(3a^4x-4ab^2\right)}{18a^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/18*(2*sqrt(3)*(9*a^5*b - 2*b^4)*x*arctan(1/3*(sqrt(3)*a*x + 2*sqrt(3)*(a^
3*x^3 + b^2*x^2)^(1/3))/(a*x)) + 2*(9*a^5*b - 2*b^4)*x*log(-(a*x - (a^3*x^3
 + b^2*x^2)^(1/3))/x) - (9*a^5*b - 2*b^4)*x*log((a^2*x^2 + (a^3*x^3 + b^2*x
^2)^(1/3)*a*x + (a^3*x^3 + b^2*x^2)^(2/3))/x^2) + 3*(a^3*x^3 + b^2*x^2)^(2/
3)*(3*a^4*x - 4*a*b^2))/(a^6*x)
```

giac [A] time = 0.31, size = 195, normalized size = 0.91

$$\frac{\frac{2\sqrt{3}\left(9a^5b^3-2b^6\right)\arctan\left(\frac{\sqrt{3}\left(a+2\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^6}-\frac{\left(9a^5b^3-2b^6\right)\log\left(a^2+\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}a+\left(a^3+\frac{b^2}{x}\right)^{\frac{2}{3}}\right)}{a^6}+\frac{2\left(9a^5b^3-2b^6\right)\log\left(\left|-a+\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}\right|\right)}{a^6}+\frac{3\left(7\left(a^3+\frac{b^2}{x}\right)^{\frac{2}{3}}a^3b^6-4\left(a^3+\frac{b^2}{x}\right)^{\frac{5}{3}}b^6\right)x^2}{a^5b^4}}{18b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="giac")
```

```
[Out] 1/18*(2*sqrt(3)*(9*a^5*b^3 - 2*b^6)*arctan(1/3*sqrt(3)*(a + 2*(a^3 + b^2/x)
^(1/3))/a)/a^6 - (9*a^5*b^3 - 2*b^6)*log(a^2 + (a^3 + b^2/x)^(1/3)*a + (a^3
 + b^2/x)^(2/3))/a^6 + 2*(9*a^5*b^3 - 2*b^6)*log(abs(-a + (a^3 + b^2/x)^(1/
3)))/a^6 + 3*(7*(a^3 + b^2/x)^(2/3)*a^3*b^6 - 4*(a^3 + b^2/x)^(5/3)*b^6)*x^
2/(a^5*b^4))/b^2
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2-b}{\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)/(a^3*x^3+b^2*x^2)^(1/3),x)
```

```
[Out] int((a*x^2-b)/(a^3*x^3+b^2*x^2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2-b}{\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 - b)/(a^3*x^3 + b^2*x^2)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{b-a\,x^2}{{\left(a^3\,x^3+b^2\,x^2\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^2)/(a^3*x^3 + b^2*x^2)^(1/3),x)
```

```
[Out] int(-(b - a*x^2)/(a^3*x^3 + b^2*x^2)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2-b}{\sqrt[3]{x^2\left(a^3x+b^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)/(a**3*x**3+b**2*x**2)**(1/3),x)
```

```
[Out] Integral((a*x**2 - b)/(x**2*(a**3*x + b**2))**(1/3), x)
```

3.2045 $\int \frac{b+ax^2}{\sqrt[3]{b^2x^2+a^3x^3}} \, dx$

Optimal. Leaf size=215

$$\frac{\left(a^3x^3+b^2x^2\right)^{2/3}\left(3a^3x-4b^2\right)}{6a^5x}+\frac{\left(-9a^5b-2b^4\right)\log\left(\sqrt[3]{a^3x^3+b^2x^2}-ax\right)}{9a^6}+\frac{\left(9a^5b+2b^4\right)\tan^{-1}\left(\frac{\sqrt{3}\,ax}{2\sqrt[3]{a^3x^3+b^2x^2}+ax}\right)}{3\sqrt{3}\,a^6}+$$

Rubi [B] time = 0.36, antiderivative size = 467, normalized size of antiderivative = 2.17, number of steps used = 8, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2053, 2011, 59, 2024}

$$-\frac{bx^{2/3}\log(x)\sqrt[3]{a^3x+b^2}}{2a\sqrt[3]{a^3x^3+b^2x^2}}-\frac{3bx^{2/3}\sqrt[3]{a^3x+b^2}\log\left(\frac{\sqrt[3]{a^3x+b^2}}{a\sqrt[3]{x}}-1\right)}{2a\sqrt[3]{a^3x^3+b^2x^2}}-\frac{\sqrt{3}\,bx^{2/3}\sqrt[3]{a^3x+b^2}\tan^{-1}\left(\frac{2\sqrt[3]{a^3x+b^2}}{\sqrt{3}\,a\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{a\sqrt[3]{a^3x^3+b^2x^2}}-\frac{b^4x^{2/3}\log(x)\sqrt[3]{a^3x+b^2}}{9a^6\sqrt[3]{a^3x^3+b^2x^2}}-\frac{b^4x^{2/3}\sqrt[3]{a^3x+b^2}\log\left(\frac{\sqrt[3]{a^3x+b^2}}{a\sqrt[3]{x}}-1\right)}{3a^6\sqrt[3]{a^3x^3+b^2x^2}}-\frac{2b^4x^{2/3}\sqrt[3]{a^3x+b^2}\tan^{-1}\left(\frac{2\sqrt[3]{a^3x+b^2}}{\sqrt{3}\,a\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,a^6\sqrt[3]{a^3x^3+b^2x^2}}-\frac{2b^2\left(a^3x^3+b^2x^2\right)^{2/3}}{3a^5x}+\frac{\left(a^3x^3+b^2x^2\right)^{2/3}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^2)/(b^2*x^2 + a^3*x^3)^(1/3),x]
```

```
[Out] (b^2*x^2 + a^3*x^3)^(2/3)/(2*a^2) - (2*b^2*(b^2*x^2 + a^3*x^3)^(2/3))/(3*a^
5*x) - (Sqrt[3]*b*x^(2/3)*(b^2 + a^3*x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(b^2 +
a^3*x)^(1/3))/(Sqrt[3]*a*x^(1/3))])/(a*(b^2*x^2 + a^3*x^3)^(1/3)) - (2*b^4*
x^(2/3)*(b^2 + a^3*x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(b^2 + a^3*x)^(1/3))/(Sqr
t[3]*a*x^(1/3))])/(3*Sqrt[3]*a^6*(b^2*x^2 + a^3*x^3)^(1/3)) - (b*x^(2/3)*(b
^2 + a^3*x)^(1/3)*Log[x])/(2*a*(b^2*x^2 + a^3*x^3)^(1/3)) - (b^4*x^(2/3)*(b
^2 + a^3*x)^(1/3)*Log[x])/(9*a^6*(b^2*x^2 + a^3*x^3)^(1/3)) - (3*b*x^(2/3)*
(b^2 + a^3*x)^(1/3)*Log[-1 + (b^2 + a^3*x)^(1/3)/(a*x^(1/3))])/(2*a*(b^2*x^
2 + a^3*x^3)^(1/3)) - (b^4*x^(2/3)*(b^2 + a^3*x)^(1/3)*Log[-1 + (b^2 + a^3*
x)^(1/3)/(a*x^(1/3))])/(3*a^6*(b^2*x^2 + a^3*x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2053

```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Expan
dIntegrand[Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && (Po
lyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IntegerQ[p] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{b+ax^2}{\sqrt[3]{b^2x^2+a^3x^3}}\,dx &= \int \left(\frac{b}{\sqrt[3]{b^2x^2+a^3x^3}}+\frac{ax^2}{\sqrt[3]{b^2x^2+a^3x^3}}\right)dx \\
&= a\int \frac{x^2}{\sqrt[3]{b^2x^2+a^3x^3}}\,dx + b\int \frac{1}{\sqrt[3]{b^2x^2+a^3x^3}}\,dx \\
&= \frac{\left(b^2x^2+a^3x^3\right)^{2/3}}{2a^2} - \frac{\left(2b^2\right)\int \frac{x}{\sqrt[3]{b^2x^2+a^3x^3}}\,dx}{3a^2} + \frac{\left(bx^{2/3}\sqrt[3]{b^2+a^3x}\right)\int \frac{1}{x^{2/3}\sqrt[3]{b^2+a^3x}}\,dx}{\sqrt[3]{b^2x^2+a^3x^3}} \\
&= \frac{\left(b^2x^2+a^3x^3\right)^{2/3}}{2a^2} - \frac{2b^2\left(b^2x^2+a^3x^3\right)^{2/3}}{3a^5x} - \frac{\sqrt{3}\,bx^{2/3}\sqrt[3]{b^2+a^3x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b^2+a^3x}}{\sqrt{3}\,a\sqrt[3]{x}}\right)}{a\sqrt[3]{b^2x^2+a^3x^3}} \\
&= \frac{\left(b^2x^2+a^3x^3\right)^{2/3}}{2a^2} - \frac{2b^2\left(b^2x^2+a^3x^3\right)^{2/3}}{3a^5x} - \frac{\sqrt{3}\,bx^{2/3}\sqrt[3]{b^2+a^3x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b^2+a^3x}}{\sqrt{3}\,a\sqrt[3]{x}}\right)}{a\sqrt[3]{b^2x^2+a^3x^3}} \\
&= \frac{\left(b^2x^2+a^3x^3\right)^{2/3}}{2a^2} - \frac{2b^2\left(b^2x^2+a^3x^3\right)^{2/3}}{3a^5x} - \frac{\sqrt{3}\,bx^{2/3}\sqrt[3]{b^2+a^3x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b^2+a^3x}}{\sqrt{3}\,a\sqrt[3]{x}}\right)}{a\sqrt[3]{b^2x^2+a^3x^3}}
\end{aligned}
$$

Mathematica [C] time = 0.08, size = 117, normalized size = 0.54

$$
\frac{3\left(x^2\left(a^3x+b^2\right)\right)^{2/3}\left(b^3\,{}_2F_1\left(-\frac{5}{3},\frac{1}{3};\frac{4}{3};-\frac{a^3x}{b^2}\right)-2b^3\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};-\frac{a^3x}{b^2}\right)+\left(a^5+b^3\right)\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{a^3x}{b^2}\right)\right)}{a^5bx\left(\frac{a^3x}{b^2}+1\right)^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^2)/(b^2*x^2 + a^3*x^3)^(1/3),x]
```

```
[Out] (3*(x^2*(b^2 + a^3*x))^(2/3)*(b^3*Hypergeometric2F1[-5/3, 1/3, 4/3, -((a^3*
x)/b^2)] - 2*b^3*Hypergeometric2F1[-2/3, 1/3, 4/3, -((a^3*x)/b^2)] + (a^5 +
 b^3)*Hypergeometric2F1[1/3, 1/3, 4/3, -((a^3*x)/b^2)]))/(a^5*b*x*(1 + (a^3
*x)/b^2)^(2/3))
```

IntegrateAlgebraic [A] time = 0.63, size = 221, normalized size = 1.03

$$
\frac{\left(a^3x^3+b^2x^2\right)^{2/3}\left(3a^3x-4b^2\right)}{6a^5x}+\frac{\left(9a^5b+2b^4\right)\tan^{-1}\left(\frac{\sqrt{3}\,ax}{2\sqrt[3]{a^3x^3+b^2x^2}+ax}\right)}{3\sqrt{3}\,a^6}+\frac{\left(-9a^5b-2b^4\right)\log\left(a^7x-a^6\sqrt[3]{a^3x^3+b^2x^2}\right)}{9a^6}+\frac{\left(9a^5b+2b^4\right)\log\left(ax\sqrt[3]{a^3x^3+b^2x^2}+\left(a^3x^3+b^2x^2\right)^{2/3}+a^2x^2\right)}{18a^6}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^2)/(b^2*x^2 + a^3*x^3)^(1/3),x]
```

```
[Out] ((-4*b^2 + 3*a^3*x)*(b^2*x^2 + a^3*x^3)^(2/3))/(6*a^5*x) + ((9*a^5*b + 2*b^
4)*ArcTan[(Sqrt[3]*a*x)/(a*x + 2*(b^2*x^2 + a^3*x^3)^(1/3))])/(3*Sqrt[3]*a^
6) + ((-9*a^5*b - 2*b^4)*Log[a^7*x - a^6*(b^2*x^2 + a^3*x^3)^(1/3)])/(9*a^6
) + ((9*a^5*b + 2*b^4)*Log[a^2*x^2 + a*x*(b^2*x^2 + a^3*x^3)^(1/3) + (b^2*x
^2 + a^3*x^3)^(2/3)])/(18*a^6)
```

fricas [A] time = 0.46, size = 206, normalized size = 0.96

$$
-\frac{2\sqrt{3}\left(9a^5b+2b^4\right)x\arctan\left(\frac{\sqrt{3}\,ax+2\sqrt{3}\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}}{3ax}\right)+2\left(9a^5b+2b^4\right)x\log\left(-\frac{ax-\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}}{x}\right)-\left(9a^5b+2b^4\right)x\log\left(\frac{a^2x^2+\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}ax+\left(a^3x^3+b^2x^2\right)^{\frac{2}{3}}}{x^2}\right)-3\left(a^3x^3+b^2x^2\right)^{\frac{2}{3}}\left(3a^4x-4ab^2\right)}{18a^6x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*(9*a^5*b + 2*b^4)*x*arctan(1/3*(sqrt(3)*a*x + 2*sqrt(3)*(a
^3*x^3 + b^2*x^2)^(1/3))/(a*x)) + 2*(9*a^5*b + 2*b^4)*x*log(-(a*x - (a^3*x^
3 + b^2*x^2)^(1/3))/x) - (9*a^5*b + 2*b^4)*x*log((a^2*x^2 + (a^3*x^3 + b^2*
x^2)^(1/3)*a*x + (a^3*x^3 + b^2*x^2)^(2/3))/x^2) - 3*(a^3*x^3 + b^2*x^2)^(2
/3)*(3*a^4*x - 4*a*b^2))/(a^6*x)
```

giac [A] time = 0.29, size = 195, normalized size = 0.91

$$-\frac{\frac{2\sqrt{3}\left(9a^5b^3+2b^6\right)\arctan\left(\frac{\sqrt{3}\left(a+2\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^6}-\frac{\left(9a^5b^3+2b^6\right)\log\left(a^2+\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}a+\left(a^3+\frac{b^2}{x}\right)^{\frac{2}{3}}\right)}{a^6}+\frac{2\left(9a^5b^3+2b^6\right)\log\left(\left|-a+\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}\right|\right)}{a^6}-\frac{3\left(7\left(a^3+\frac{b^2}{x}\right)^{\frac{2}{3}}a^3b^6-4\left(a^3+\frac{b^2}{x}\right)^{\frac{5}{3}}b^6\right)x^2}{a^5b^4}}{18b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="giac")
```

```
[Out] -1/18*(2*sqrt(3)*(9*a^5*b^3 + 2*b^6)*arctan(1/3*sqrt(3)*(a + 2*(a^3 + b^2/x
)^(1/3))/a)/a^6 - (9*a^5*b^3 + 2*b^6)*log(a^2 + (a^3 + b^2/x)^(1/3)*a + (a^
3 + b^2/x)^(2/3))/a^6 + 2*(9*a^5*b^3 + 2*b^6)*log(abs(-a + (a^3 + b^2/x)^(1
/3)))/a^6 - 3*(7*(a^3 + b^2/x)^(2/3)*a^3*b^6 - 4*(a^3 + b^2/x)^(5/3)*b^6)*x
^2/(a^5*b^4))/b^2
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2+b}{\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)/(a^3*x^3+b^2*x^2)^(1/3),x)
```

```
[Out] int((a*x^2+b)/(a^3*x^3+b^2*x^2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2+b}{\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 + b)/(a^3*x^3 + b^2*x^2)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x^2+b}{{\left(a^3\,x^3+b^2\,x^2\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^2)/(a^3*x^3 + b^2*x^2)^(1/3),x)
```

```
[Out] int((b + a*x^2)/(a^3*x^3 + b^2*x^2)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2+b}{\sqrt[3]{x^2\left(a^3x+b^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b)/(a**3*x**3+b**2*x**2)**(1/3),x)
```

```
[Out] Integral((a*x**2 + b)/(x**2*(a**3*x + b**2))**(1/3), x)
```

3.2046 $$\int \frac{-a^2b+4abx-(2a+3b)x^2+2x^3}{\sqrt[4]{x(-a+x)^2(-b+x)^3}\left(b+\left(-1+a^2d\right)x-2adx^2+dx^3\right)}\,dx$$

Optimal. Leaf size=215

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{-a^2b^3x+x^4\left(a^2+6ab+3b^2\right)+x^3\left(-3a^2b-6ab^2-b^3\right)+x^2\left(3a^2b^2+2ab^3\right)+x^5(-2a-3b)+x^6}}{b-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{-a^2b^3x+x^4\left(a^2+6ab+3b^2\right)\cdots}}{\cdots}\right)}{\cdots}$$

Rubi [F] time = 14.67, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-a^2b+4abx-(2a+3b)x^2+2x^3}{\sqrt[4]{x(-a+x)^2(-b+x)^3}\left(b+\left(-1+a^2d\right)x-2adx^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a^2*b) + 4*a*b*x - (2*a + 3*b)*x^2 + 2*x^3)/((x*(-a + x)^2*(-b + x)^
3)^(1/4)*(b + (-1 + a^2*d)*x - 2*a*d*x^2 + d*x^3)),x]
```

```
[Out] (12*b*x^(1/4)*Sqrt[-a + x]*(-b + x)^(3/4)*Defer[Subst][Defer[Int][(x^6*Sqrt
[-a + x^4])/((-b + x^4)^(3/4)*(-b + (1 - a^2*d)*x^4 + 2*a*d*x^8 - d*x^12)),
 x], x, x^(1/4)])/(-((a - x)^2*(b - x)^3*x))^(1/4) + (4*a*b*x^(1/4)*Sqrt[-a
 + x]*(-b + x)^(3/4)*Defer[Subst][Defer[Int][(x^2*Sqrt[-a + x^4])/((-b + x^
4)^(3/4)*(b - (1 - a^2*d)*x^4 - 2*a*d*x^8 + d*x^12)), x], x, x^(1/4)])/(-((
a - x)^2*(b - x)^3*x))^(1/4) + (8*x^(1/4)*Sqrt[-a + x]*(-b + x)^(3/4)*Defer
[Subst][Defer[Int][(x^10*Sqrt[-a + x^4])/((-b + x^4)^(3/4)*(b - (1 - a^2*d)
*x^4 - 2*a*d*x^8 + d*x^12)), x], x, x^(1/4)])/(-((a - x)^2*(b - x)^3*x))^(1
/4)
```

Rubi steps

$$\int \frac{-a^2b+4abx-(2a+3b)x^2+2x^3}{\sqrt[4]{x(-a+x)^2(-b+x)^3}\left(b+\left(-1+a^2d\right)x-2adx^2+dx^3\right)}\,dx = \frac{\left(\sqrt[4]{x}\sqrt{-a+x}(-b+x)^{3/4}\right)\int \frac{-a^2b+4abx-(2\cdots}{\sqrt[4]{x}\sqrt{-a+x}(-b+x)^{3/4}\left(b+\cdots\right.}}{\sqrt[4]{x(-a+x)^2(-b+x)^3}}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt{-a+x}(-b+x)^{3/4}\right)\int \frac{\sqrt{-a+x}\left(ab-3b\cdots\right.}{\sqrt[4]{x}(-b+x)^{3/4}\left(b+\left(-1+a^2\cdots\right.\right.}}{\sqrt[4]{x(-a+x)^2(-b+x)^3}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt{-a+x}(-b+x)^{3/4}\right)\text{Subst}\left(\int \frac{x^2\sqrt{-\cdots}}{\left(-b+x^4\right)^{3/4}\left(b\cdots\right.}\right.}{\sqrt[4]{x(-a+x)^2(-b+\cdots}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt{-a+x}(-b+x)^{3/4}\right)\text{Subst}\left(\int\left(\frac{\cdots}{\left(-b+x^4\right)^{3/4}\left(\cdots\right.}\right.\right.}{\cdots}$$

$$= \frac{\left(8\sqrt[4]{x}\sqrt{-a+x}(-b+x)^{3/4}\right)\text{Subst}\left(\int \frac{\cdots}{\left(-b+x^4\right)^{3/4}\left(b\cdots\right.}\right.}{\sqrt[4]{x(-a+x)^2(-b+\cdots}}$$

Mathematica [F] time = 4.03, size = 0, normalized size = 0.00

$$\int \frac{-a^2b+4abx-(2a+3b)x^2+2x^3}{\sqrt[4]{x(-a+x)^2(-b+x)^3}\left(b+\left(-1+a^2d\right)x-2adx^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-(a^2*b) + 4*a*b*x - (2*a + 3*b)*x^2 + 2*x^3)/((x*(-a + x)^2*(-b
 + x)^3)^(1/4)*(b + (-1 + a^2*d)*x - 2*a*d*x^2 + d*x^3)),x]
```

```
[Out] Integrate[(-(a^2*b) + 4*a*b*x - (2*a + 3*b)*x^2 + 2*x^3)/((x*(-a + x)^2*(-b
 + x)^3)^(1/4)*(b + (-1 + a^2*d)*x - 2*a*d*x^2 + d*x^3)), x]
```

IntegrateAlgebraic [A] time = 0.56, size = 215, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{-a^2b^3x+x^4\left(a^2+6ab+3b^2\right)+x^3\left(-3a^2b-6ab^2-b^3\right)+x^2\left(3a^2b^2+2ab^3\right)+x^5(-2a-3b)+x^6}}{b-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{-a^2b^3x+x^4\left(a^2+6ab+3b^2\right)+x^3\left(-3a^2b-6ab^2-b^3\right)+x^2\left(3a^2b^2+2ab^3\right)+x^5(-2a-3b)+x^6}}{b-x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a^2*b) + 4*a*b*x - (2*a + 3*b)*x^2 + 2*x^3)/((x*(-a +
 x)^2*(-b + x)^3)^(1/4)*(b + (-1 + a^2*d)*x - 2*a*d*x^2 + d*x^3)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(-(a^2*b^3*x) + (3*a^2*b^2 + 2*a*b^3)*x^2 + (-3*a^2*b -
 6*a*b^2 - b^3)*x^3 + (a^2 + 6*a*b + 3*b^2)*x^4 + (-2*a - 3*b)*x^5 + x^6)^(
1/4))/(b - x)])/d^(3/4) + (2*ArcTanh[(d^(1/4)*(-(a^2*b^3*x) + (3*a^2*b^2 +
2*a*b^3)*x^2 + (-3*a^2*b - 6*a*b^2 - b^3)*x^3 + (a^2 + 6*a*b + 3*b^2)*x^4 +
 (-2*a - 3*b)*x^5 + x^6)^(1/4))/(b - x)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*b+4*a*b*x-(2*a+3*b)*x^2+2*x^3)/(x*(-a+x)^2*(-b+x)^3)^(1/4)/
(b+(a^2*d-1)*x-2*a*d*x^2+d*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2b - 4\,abx + (2\,a + 3\,b)x^2 - 2\,x^3}{\left(-(a - x)^2(b - x)^3x\right)^{\frac{1}{4}}\left(2\,adx^2 - dx^3 - \left(a^2d - 1\right)x - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*b+4*a*b*x-(2*a+3*b)*x^2+2*x^3)/(x*(-a+x)^2*(-b+x)^3)^(1/4)/
(b+(a^2*d-1)*x-2*a*d*x^2+d*x^3),x, algorithm="giac")
```

```
[Out] integrate((a^2*b - 4*a*b*x + (2*a + 3*b)*x^2 - 2*x^3)/((-(a - x)^2*(b - x)^
3*x)^(1/4)*(2*a*d*x^2 - d*x^3 - (a^2*d - 1)*x - b)), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{-a^2b + 4abx - (2a + 3b)\,x^2 + 2x^3}{\left(x\,(-a + x)^2\,(-b + x)^3\right)^{\frac{1}{4}}\left(b + \left(a^2d - 1\right)x - 2ad\,x^2 + d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*b+4*a*b*x-(2*a+3*b)*x^2+2*x^3)/(x*(-a+x)^2*(-b+x)^3)^(1/4)/(b+(a^
2*d-1)*x-2*a*d*x^2+d*x^3),x)
```

```
[Out] int((-a^2*b+4*a*b*x-(2*a+3*b)*x^2+2*x^3)/(x*(-a+x)^2*(-b+x)^3)^(1/4)/(b+(a^
2*d-1)*x-2*a*d*x^2+d*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2b - 4\,abx + (2\,a + 3\,b)x^2 - 2\,x^3}{\left(-(a-x)^2(b-x)^3x\right)^{\frac{1}{4}}\left(2\,adx^2 - dx^3 - \left(a^2d - 1\right)x - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*b+4*a*b*x-(2*a+3*b)*x^2+2*x^3)/(x*(-a+x)^2*(-b+x)^3)^(1/4)/
(b+(a^2*d-1)*x-2*a*d*x^2+d*x^3),x, algorithm="maxima")
```

```
[Out] integrate((a^2*b - 4*a*b*x + (2*a + 3*b)*x^2 - 2*x^3)/((-(a - x)^2*(b - x)^
3*x)^(1/4)*(2*a*d*x^2 - d*x^3 - (a^2*d - 1)*x - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x^2\,\left(2\,a+3\,b\right)+a^2\,b-2\,x^3-4\,a\,b\,x}{{\left(-x\,{\left(a-x\right)}^2\,{\left(b-x\right)}^3\right)}^{1/4}\,\left(d\,x^3-2\,a\,d\,x^2+\left(a^2\,d-1\right)\,x+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(2*a + 3*b) + a^2*b - 2*x^3 - 4*a*b*x)/((-x*(a - x)^2*(b - x)^3)^
(1/4)*(b + d*x^3 + x*(a^2*d - 1) - 2*a*d*x^2)),x)
```

```
[Out] int(-(x^2*(2*a + 3*b) + a^2*b - 2*x^3 - 4*a*b*x)/((-x*(a - x)^2*(b - x)^3)^
(1/4)*(b + d*x^3 + x*(a^2*d - 1) - 2*a*d*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*b+4*a*b*x-(2*a+3*b)*x**2+2*x**3)/(x*(-a+x)**2*(-b+x)**3)**
(1/4)/(b+(a**2*d-1)*x-2*a*d*x**2+d*x**3),x)
```

```
[Out] Timed out
```

3.2047 $$\int \frac{ab^3-(6a-b)b^2x+9abx^2-(4a+3b)x^3+2x^4}{\sqrt[4]{x(-a+x)^2(-b+x)^3}\left(-a^2+\left(2a-b^3d\right)x+\left(-1+3b^2d\right)x^2-3bdx^3+dx^4\right)}\,dx$$

Optimal. Leaf size=215

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{-a^2b^3x+x^4\left(a^2+6ab+3b^2\right)+x^3\left(-3a^2b-6ab^2-b^3\right)+x^2\left(3a^2b^2+2ab^3\right)+x^5(-2a-3b)+x^6}}{a-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{-a^2b^3x+x^4\left(a^2+6ab+\right.}}{}\right.}{}$$

Rubi [F] time = 27.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ab^3-(6a-b)b^2x+9abx^2-(4a+3b)x^3+2x^4}{\sqrt[4]{x(-a+x)^2(-b+x)^3}\left(-a^2+\left(2a-b^3d\right)x+\left(-1+3b^2d\right)x^2-3bdx^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*b^3 - (6*a - b)*b^2*x + 9*a*b*x^2 - (4*a + 3*b)*x^3 + 2*x^4)/((x*(-a
 + x)^2*(-b + x)^3)^(1/4)*(-a^2 + (2*a - b^3*d)*x + (-1 + 3*b^2*d)*x^2 - 3*
b*d*x^3 + d*x^4)),x]
```

```
[Out] (-4*a*b*x^(1/4)*Sqrt[-a + x]*(-b + x)^(3/4)*Defer[Subst][Defer[Int][(x^2*(-
b + x^4)^(5/4))/(Sqrt[-a + x^4]*(a^2 - 2*a*(1 - (b^3*d)/(2*a))*x^4 + (1 - 3
*b^2*d)*x^8 + 3*b*d*x^12 - d*x^16)), x], x, x^(1/4)]])/(-((a - x)^2*(b - x)^
3*x))^(1/4) + (4*(4*a - b)*x^(1/4)*Sqrt[-a + x]*(-b + x)^(3/4)*Defer[Subst]
[Defer[Int][(x^6*(-b + x^4)^(5/4))/(Sqrt[-a + x^4]*(a^2 - 2*a*(1 - (b^3*d)/
(2*a))*x^4 + (1 - 3*b^2*d)*x^8 + 3*b*d*x^12 - d*x^16)), x], x, x^(1/4)]])/(-
((a - x)^2*(b - x)^3*x))^(1/4) - (8*x^(1/4)*Sqrt[-a + x]*(-b + x)^(3/4)*Def
er[Subst][Defer[Int][(x^10*(-b + x^4)^(5/4))/(Sqrt[-a + x^4]*(a^2 - 2*a*(1
- (b^3*d)/(2*a))*x^4 + (1 - 3*b^2*d)*x^8 + 3*b*d*x^12 - d*x^16)), x], x, x^
(1/4)]])/(-((a - x)^2*(b - x)^3*x))^(1/4)
```

Rubi steps

$$\int \frac{ab^3-(6a-b)b^2x+9abx^2-(4a+3b)x^3+2x^4}{\sqrt[4]{x(-a+x)^2(-b+x)^3}\left(-a^2+\left(2a-b^3d\right)x+\left(-1+3b^2d\right)x^2-3bdx^3+dx^4\right)}\,dx = \frac{\left(\sqrt[4]{x}\sqrt{-a+x}(-b+x)^{3/4}\right)\int \cdots}{\cdots}$$

$$= \frac{\left(\sqrt[4]{x}\sqrt{-a+x}(-b+x)^{3/4}\right)\int \cdots}{\sqrt[4]{\cdots}}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt{-a+x}(-b+x)^{3/4}\right)\cdots}{\cdots}$$

$$= \frac{\left(4\sqrt[4]{x}\sqrt{-a+x}(-b+x)^{3/4}\right)\cdots}{\cdots}$$

$$= -\frac{\left(4\sqrt[4]{x}\sqrt{-a+x}(-b+x)^{3/4}\cdots\right.}{\cdots}$$

$$= -\frac{\left(4\sqrt[4]{x}\sqrt{-a+x}(-b+x)^{3/4}\cdots\right.}{\cdots}$$

$$= -\frac{\left(8\sqrt[4]{x}\sqrt{-a+x}(-b+x)^{3/4}\cdots\right.}{\cdots}$$

Mathematica [F] time = 4.74, size = 0, normalized size = 0.00

$$\int \frac{ab^3-(6a-b)b^2x+9abx^2-(4a+3b)x^3+2x^4}{\sqrt[4]{x(-a+x)^2(-b+x)^3}\left(-a^2+\left(2a-b^3d\right)x+\left(-1+3b^2d\right)x^2-3bdx^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*b^3 - (6*a - b)*b^2*x + 9*a*b*x^2 - (4*a + 3*b)*x^3 + 2*x^4)/(
(x*(-a + x)^2*(-b + x)^3)^(1/4)*(-a^2 + (2*a - b^3*d)*x + (-1 + 3*b^2*d)*x^
2 - 3*b*d*x^3 + d*x^4)),x]
```

```
[Out] Integrate[(a*b^3 - (6*a - b)*b^2*x + 9*a*b*x^2 - (4*a + 3*b)*x^3 + 2*x^4)/(
(x*(-a + x)^2*(-b + x)^3)^(1/4)*(-a^2 + (2*a - b^3*d)*x + (-1 + 3*b^2*d)*x^
2 - 3*b*d*x^3 + d*x^4)), x]
```

IntegrateAlgebraic [A] time = 0.62, size = 215, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{-a^2b^3x+x^4\left(a^2+6ab+3b^2\right)+x^3\left(-3a^2b-6ab^2-b^3\right)+x^2\left(3a^2b^2+2ab^3\right)+x^5(-2a-3b)+x^6}}{a-x}\right)}{d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{-a^2b^3x+x^4\left(a^2+6ab+3b^2\right)+x^3\left(-3a^2b-6ab^2-b^3\right)+x^2\left(3a^2b^2+2ab^3\right)+x^5(-2a-3b)+x^6}}{a-x}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b^3 - (6*a - b)*b^2*x + 9*a*b*x^2 - (4*a + 3*b)*x^3 +
 2*x^4)/((x*(-a + x)^2*(-b + x)^3)^(1/4)*(-a^2 + (2*a - b^3*d)*x + (-1 + 3*
b^2*d)*x^2 - 3*b*d*x^3 + d*x^4)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(-(a^2*b^3*x) + (3*a^2*b^2 + 2*a*b^3)*x^2 + (-3*a^2*b -
 6*a*b^2 - b^3)*x^3 + (a^2 + 6*a*b + 3*b^2)*x^4 + (-2*a - 3*b)*x^5 + x^6)^(
```

```
1/4))/(a - x)])/d^(3/4) + (2*ArcTanh[(d^(1/4)*(-(a^2*b^3*x) + (3*a^2*b^2 +
2*a*b^3)*x^2 + (-3*a^2*b - 6*a*b^2 - b^3)*x^3 + (a^2 + 6*a*b + 3*b^2)*x^4 +
 (-2*a - 3*b)*x^5 + x^6)^(1/4))/(a - x)])/d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b^3-(6*a-b)*b^2*x+9*a*b*x^2-(4*a+3*b)*x^3+2*x^4)/(x*(-a+x)^2*(
-b+x)^3)^(1/4)/(-a^2+(-b^3*d+2*a)*x+(3*b^2*d-1)*x^2-3*b*d*x^3+d*x^4),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ab^3 - (6a-b)b^2x + 9abx^2 - (4a+3b)x^3 + 2x^4}{\left(-(a-x)^2(b-x)^3x\right)^{\frac{1}{4}}\left(3bdx^3 - dx^4 - \left(3b^2d-1\right)x^2 + a^2 + \left(b^3d-2a\right)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b^3-(6*a-b)*b^2*x+9*a*b*x^2-(4*a+3*b)*x^3+2*x^4)/(x*(-a+x)^2*(
-b+x)^3)^(1/4)/(-a^2+(-b^3*d+2*a)*x+(3*b^2*d-1)*x^2-3*b*d*x^3+d*x^4),x, alg
orithm="giac")
```

```
[Out] integrate(-(a*b^3 - (6*a - b)*b^2*x + 9*a*b*x^2 - (4*a + 3*b)*x^3 + 2*x^4)/
((-(a - x)^2*(b - x)^3*x)^(1/4)*(3*b*d*x^3 - d*x^4 - (3*b^2*d - 1)*x^2 + a^
2 + (b^3*d - 2*a)*x)), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{a\,b^3 - (6a-b)\,b^2x + 9ab\,x^2 - (4a+3b)\,x^3 + 2x^4}{\left(x\,(-a+x)^2\,(-b+x)^3\right)^{\frac{1}{4}}\left(-a^2 + \left(-b^3d+2a\right)x + \left(3b^2d-1\right)x^2 - 3bd\,x^3 + d\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b^3-(6*a-b)*b^2*x+9*a*b*x^2-(4*a+3*b)*x^3+2*x^4)/(x*(-a+x)^2*(-b+x)^
3)^(1/4)/(-a^2+(-b^3*d+2*a)*x+(3*b^2*d-1)*x^2-3*b*d*x^3+d*x^4),x)
```

```
[Out] int((a*b^3-(6*a-b)*b^2*x+9*a*b*x^2-(4*a+3*b)*x^3+2*x^4)/(x*(-a+x)^2*(-b+x)^
3)^(1/4)/(-a^2+(-b^3*d+2*a)*x+(3*b^2*d-1)*x^2-3*b*d*x^3+d*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ab^3 - (6a-b)b^2x + 9abx^2 - (4a+3b)x^3 + 2x^4}{\left(-(a-x)^2(b-x)^3x\right)^{\frac{1}{4}}\left(3bdx^3 - dx^4 - \left(3b^2d-1\right)x^2 + a^2 + \left(b^3d-2a\right)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b^3-(6*a-b)*b^2*x+9*a*b*x^2-(4*a+3*b)*x^3+2*x^4)/(x*(-a+x)^2*(
-b+x)^3)^(1/4)/(-a^2+(-b^3*d+2*a)*x+(3*b^2*d-1)*x^2-3*b*d*x^3+d*x^4),x, alg
orithm="maxima")
```

```
[Out] -integrate((a*b^3 - (6*a - b)*b^2*x + 9*a*b*x^2 - (4*a + 3*b)*x^3 + 2*x^4)/
((-(a - x)^2*(b - x)^3*x)^(1/4)*(3*b*d*x^3 - d*x^4 - (3*b^2*d - 1)*x^2 + a^
2 + (b^3*d - 2*a)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,b^3-x^3\,\left(4\,a+3\,b\right)+2\,x^4-b^2\,x\,\left(6\,a-b\right)+9\,a\,b\,x^2}{{\left(-x\,{\left(a-x\right)}^2\,{\left(b-x\right)}^3\right)}^{1/4}\,\left(x^2\,\left(3\,b^2\,d-1\right)+x\,\left(2\,a-b^3\,d\right)+d\,x^4-a^2-3\,b\,d\,x^3\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b^3 - x^3*(4*a + 3*b) + 2*x^4 - b^2*x*(6*a - b) + 9*a*b*x^2)/((-x*(a
 - x)^2*(b - x)^3)^(1/4)*(x^2*(3*b^2*d - 1) + x*(2*a - b^3*d) + d*x^4 - a^2
 - 3*b*d*x^3)),x)
```

```
[Out] int((a*b^3 - x^3*(4*a + 3*b) + 2*x^4 - b^2*x*(6*a - b) + 9*a*b*x^2)/((-x*(a
 - x)^2*(b - x)^3)^(1/4)*(x^2*(3*b^2*d - 1) + x*(2*a - b^3*d) + d*x^4 - a^2
 - 3*b*d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b**3-(6*a-b)*b**2*x+9*a*b*x**2-(4*a+3*b)*x**3+2*x**4)/(x*(-a+x
)**2*(-b+x)**3)**(1/4)/(-a**2+(-b**3*d+2*a)*x+(3*b**2*d-1)*x**2-3*b*d*x**3+
d*x**4),x)
```

```
[Out] Timed out
```

3.2048 $\int \frac{1}{(d+cx)\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$

Optimal. Leaf size=215

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}\right)}{\sqrt{c}\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}\right)}{\sqrt{c}\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}+\frac{2}{c\sqrt{\sqrt{a^2x^2+b^2}+ax}}$$

Rubi [A] time = 0.50, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2119, 1628, 828, 826, 1166, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}\right)}{\sqrt{c}\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}\right)}{\sqrt{c}\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}+\frac{2}{c\sqrt{\sqrt{a^2x^2+b^2}+ax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + c*x)*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

```
[Out] 2/(c*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]) + (2*ArcTan[(Sqrt[c]*Sqrt[a*x + Sqrt[
b^2 + a^2*x^2]])/Sqrt[a*d - Sqrt[b^2*c^2 + a^2*d^2]]])/(Sqrt[c]*Sqrt[a*d -
Sqrt[b^2*c^2 + a^2*d^2]]) + (2*ArcTan[(Sqrt[c]*Sqrt[a*x + Sqrt[b^2 + a^2*x^
2]])/Sqrt[a*d + Sqrt[b^2*c^2 + a^2*d^2]]])/(Sqrt[c]*Sqrt[a*d + Sqrt[b^2*c^2
 + a^2*d^2]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$\int \frac{1}{(d+cx)\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx = \text{Subst}\left(\int \frac{b^2+x^2}{x^{3/2}\left(-b^2c+2adx+cx^2\right)}\,dx, x, ax+\sqrt{b^2+a^2x^2}\right)$$

$$= \text{Subst}\left(\int \left(\frac{1}{cx^{3/2}}+\frac{2\left(b^2c-adx\right)}{cx^{3/2}\left(-b^2c+2adx+cx^2\right)}\right)dx, x, ax+\sqrt{b^2+a^2x^2}\right)$$

$$= -\frac{2}{c\sqrt{ax+\sqrt{b^2+a^2x^2}}}+\frac{2\,\text{Subst}\left(\int \frac{b^2c-adx}{x^{3/2}\left(-b^2c+2adx+cx^2\right)}\,dx, x, ax+\sqrt{b^2+a^2x^2}\right)}{c}$$

$$= \frac{2}{c\sqrt{ax+\sqrt{b^2+a^2x^2}}}-\frac{2\,\text{Subst}\left(\int \frac{-ab^2cd-b^2c^2x}{\sqrt{x}\left(-b^2c+2adx+cx^2\right)}\,dx, x, ax+\sqrt{b^2+a^2x^2}\right)}{b^2c^2}$$

$$= \frac{2}{c\sqrt{ax+\sqrt{b^2+a^2x^2}}}-\frac{4\,\text{Subst}\left(\int \frac{-ab^2cd-b^2c^2x^2}{-b^2c+2adx^2+cx^4}\,dx, x, \sqrt{ax+\sqrt{b^2+a^2x^2}}\right)}{b^2c^2}$$

$$= \frac{2}{c\sqrt{ax+\sqrt{b^2+a^2x^2}}}+2\,\text{Subst}\left(\int \frac{1}{ad-\sqrt{b^2c^2+a^2d^2}+cx^2}\,dx, x, \sqrt{ax+\sqrt{}}\right.$$

$$= \frac{2}{c\sqrt{ax+\sqrt{b^2+a^2x^2}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+\sqrt{b^2+a^2x^2}}}{\sqrt{ad-\sqrt{b^2c^2+a^2d^2}}}\right)}{\sqrt{c}\sqrt{ad-\sqrt{b^2c^2+a^2d^2}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+\sqrt{b^2+a^2}}}{\sqrt{ad+\sqrt{b^2c^2+a^2d}}}\right.}{\sqrt{c}\sqrt{ad+\sqrt{b^2c^2+a^2}}}$$

Mathematica [A] time = 0.64, size = 333, normalized size = 1.55

$$-\frac{2\left(-\frac{\left(ad\left(\sqrt{a^2d^2+b^2c^2}+ad\right)+b^2c^2\right)\tan^{-1}\left(\frac{b\sqrt{c}}{\sqrt{\sqrt{a^2x^2+b^2}+ax}\sqrt{-\sqrt{a^2d^2+b^2c^2}-ad}}\right)}{b\sqrt{a^2d^2+b^2c^2}\sqrt{-\sqrt{a^2d^2+b^2c^2}-ad}}+\frac{\left(ad\left(ad-\sqrt{a^2d^2+b^2c^2}\right)+b^2c^2\right)\tan^{-1}\left(\frac{b\sqrt{c}}{\sqrt{\sqrt{a^2x^2+b^2}+ax}\sqrt{\sqrt{a^2d^2+b^2c^2}-ad}}\right)}{b\sqrt{a^2d^2+b^2c^2}\sqrt{\sqrt{a^2d^2+b^2c^2}-ad}}-\frac{\sqrt{c}}{\sqrt{\sqrt{a^2x^2+b^2}+ax}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + c*x)*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

```
[Out] (-2*(-(Sqrt[c]/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]) - ((b^2*c^2 + a*d*(a*d + Sq
rt[b^2*c^2 + a^2*d^2]))*ArcTan[(b*Sqrt[c])/(Sqrt[-(a*d) - Sqrt[b^2*c^2 + a^
2*d^2]]*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])])/(b*Sqrt[b^2*c^2 + a^2*d^2]*Sqrt[
```

```
-(a*d) - Sqrt[b^2*c^2 + a^2*d^2]]) + ((b^2*c^2 + a*d*(a*d - Sqrt[b^2*c^2 +
a^2*d^2]))*ArcTan[(b*Sqrt[c])/(Sqrt[-(a*d) + Sqrt[b^2*c^2 + a^2*d^2]]*Sqrt[
a*x + Sqrt[b^2 + a^2*x^2]])])/(b*Sqrt[b^2*c^2 + a^2*d^2]*Sqrt[-(a*d) + Sqrt
[b^2*c^2 + a^2*d^2]])))/c^(3/2)
```

IntegrateAlgebraic [A] time = 0.70, size = 215, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}\right)}{\sqrt{c}\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}\right)}{\sqrt{c}\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}+\frac{2}{c\sqrt{\sqrt{a^2x^2+b^2}+ax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + c*x)*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

```
[Out] 2/(c*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]) + (2*ArcTan[(Sqrt[c]*Sqrt[a*x + Sqrt[
b^2 + a^2*x^2]])/Sqrt[a*d - Sqrt[b^2*c^2 + a^2*d^2]]])/(Sqrt[c]*Sqrt[a*d -
Sqrt[b^2*c^2 + a^2*d^2]]) + (2*ArcTan[(Sqrt[c]*Sqrt[a*x + Sqrt[b^2 + a^2*x^
2]])/Sqrt[a*d + Sqrt[b^2*c^2 + a^2*d^2]]])/(Sqrt[c]*Sqrt[a*d + Sqrt[b^2*c^2
 + a^2*d^2]])
```

fricas [B] time = 0.55, size = 673, normalized size = 3.13

$$-\frac{b^2c\sqrt{\frac{b^2c^3\sqrt{\frac{b^2c^2+a^2d^2}{b^4c^6}}+ad}{b^2c^3}}\log\left(4\left(b^2c^3\sqrt{\frac{b^2c^2+a^2d^2}{b^4c^6}}-ad\right)\sqrt{\frac{b^2c^3\sqrt{\frac{b^2c^2+a^2d^2}{b^4c^6}}+ad}{b^2c^3}}+4\sqrt{ax+\sqrt{a^2x^2+b^2}}\right)-b^2c\sqrt{\frac{b^2c^3\sqrt{\frac{b^2c^2+a^2d^2}{b^4c^6}}+ad}{b^2c^3}}\log\left(-4\left(b^2c^3\sqrt{\frac{b^2c^2+a^2d^2}{b^4c^6}}-ad\right)\sqrt{\frac{b^2c^3\sqrt{\frac{b^2c^2+a^2d^2}{b^4c^6}}+ad}{b^2c^3}}+4\sqrt{ax+\sqrt{a^2x^2+b^2}}\right)-b^2c\sqrt{-\frac{b^2c^3\sqrt{\frac{b^2c^2+a^2d^2}{b^4c^6}}-ad}{b^2c^3}}\log\left(4\left(b^2c^3\sqrt{\frac{b^2c^2+a^2d^2}{b^4c^6}}+ad\right)\sqrt{-\frac{b^2c^3\sqrt{\frac{b^2c^2+a^2d^2}{b^4c^6}}-ad}{b^2c^3}}+4\sqrt{ax+\sqrt{a^2x^2+b^2}}\right)+b^2c\sqrt{-\frac{b^2c^3\sqrt{\frac{b^2c^2+a^2d^2}{b^4c^6}}-ad}{b^2c^3}}\log\left(-4\left(b^2c^3\sqrt{\frac{b^2c^2+a^2d^2}{b^4c^6}}+ad\right)\sqrt{-\frac{b^2c^3\sqrt{\frac{b^2c^2+a^2d^2}{b^4c^6}}-ad}{b^2c^3}}+4\sqrt{ax+\sqrt{a^2x^2+b^2}}\right)+2\sqrt{ax+\sqrt{a^2x^2+b^2}}\left(ax-\sqrt{a^2x^2+b^2}\right)}{b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x+d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -(b^2*c*sqrt((b^2*c^3*sqrt((b^2*c^2 + a^2*d^2)/(b^4*c^6)) + a*d)/(b^2*c^3))
*log(4*(b^2*c^3*sqrt((b^2*c^2 + a^2*d^2)/(b^4*c^6)) - a*d)*sqrt((b^2*c^3*sq
rt((b^2*c^2 + a^2*d^2)/(b^4*c^6)) + a*d)/(b^2*c^3)) + 4*sqrt(a*x + sqrt(a^2
*x^2 + b^2))) - b^2*c*sqrt((b^2*c^3*sqrt((b^2*c^2 + a^2*d^2)/(b^4*c^6)) + a
*d)/(b^2*c^3))*log(-4*(b^2*c^3*sqrt((b^2*c^2 + a^2*d^2)/(b^4*c^6)) - a*d)*s
qrt((b^2*c^3*sqrt((b^2*c^2 + a^2*d^2)/(b^4*c^6)) + a*d)/(b^2*c^3)) + 4*sqrt
(a*x + sqrt(a^2*x^2 + b^2))) - b^2*c*sqrt(-(b^2*c^3*sqrt((b^2*c^2 + a^2*d^2
)/(b^4*c^6)) - a*d)/(b^2*c^3))*log(4*(b^2*c^3*sqrt((b^2*c^2 + a^2*d^2)/(b^4
*c^6)) + a*d)*sqrt(-(b^2*c^3*sqrt((b^2*c^2 + a^2*d^2)/(b^4*c^6)) - a*d)/(b^
2*c^3)) + 4*sqrt(a*x + sqrt(a^2*x^2 + b^2))) + b^2*c*sqrt(-(b^2*c^3*sqrt((b
^2*c^2 + a^2*d^2)/(b^4*c^6)) - a*d)/(b^2*c^3))*log(-4*(b^2*c^3*sqrt((b^2*c^
2 + a^2*d^2)/(b^4*c^6)) + a*d)*sqrt(-(b^2*c^3*sqrt((b^2*c^2 + a^2*d^2)/(b^4
*c^6)) - a*d)/(b^2*c^3)) + 4*sqrt(a*x + sqrt(a^2*x^2 + b^2))) + 2*sqrt(a*x
+ sqrt(a^2*x^2 + b^2))*(a*x - sqrt(a^2*x^2 + b^2)))/(b^2*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+\sqrt{a^2x^2+b^2}}\,(cx+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x+d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + sqrt(a^2*x^2 + b^2))*(c*x + d)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx+d)\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x+d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
[Out] int(1/(c*x+d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + \sqrt{a^2x^2 + b^2}}\,(cx + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x+d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*x + sqrt(a^2*x^2 + b^2))*(c*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a\,x + \sqrt{a^2\,x^2 + b^2}}\,\left(d + c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x + (b^2 + a^2*x^2)^(1/2))^(1/2)*(d + c*x)),x)
[Out] int(1/((a*x + (b^2 + a^2*x^2)^(1/2))^(1/2)*(d + c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + \sqrt{a^2x^2 + b^2}}\,(cx + d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x+d)/(a*x+(a**2*x**2+b**2)**(1/2))**(1/2),x)
[Out] Integral(1/(sqrt(a*x + sqrt(a**2*x**2 + b**2))*(c*x + d)), x)
```

3.2049 $\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bqx+cx^2+bpx^4+a(q+px^3)^2\right)}{x^5}\,dx$

Optimal. Leaf size=215

$$\frac{\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}\,\left(3ap^3x^9+9ap^2qx^6-3ap^2qx^5+9apq^2x^3-3apq^2x^2+3aq^3+4bp^2x^7+8bpqx^4-\right.}{12x^4}$$

Rubi [F] time = 1.88, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bqx+cx^2+bpx^4+a(q+px^3)^2\right)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*q*x + c*
x^2 + b*p*x^4 + a*(q + p*x^3)^2))/x^5,x]
```

```
[Out] 2*c*p*Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6], x] - a*q^3*De
fer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x^5, x] - b*q^2*Defer[
Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x^4, x] - c*q*Defer[Int][S
qrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x^3, x] + b*p*q*Defer[Int][Sqrt[
q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x, x] + 3*a*p^2*q*Defer[Int][x*Sqrt[
q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6], x] + 2*b*p^2*Defer[Int][x^2*Sqrt[q^
2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6], x] + 2*a*p^3*Defer[Int][x^4*Sqrt[q^2
- 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6], x]
```

Rubi steps

$$\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bqx+cx^2+bpx^4+a(q+px^3)^2\right)}{x^5}\,dx = \int \left(2cp\sqrt{q^2-2pqx^2+2\right.$$

$$= (2cp)\int \sqrt{q^2-2pqx^2+}$$

Mathematica [F] time = 1.15, size = 0, normalized size = 0.00

$$\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bqx+cx^2+bpx^4+a(q+px^3)^2\right)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*q*
x + c*x^2 + b*p*x^4 + a*(q + p*x^3)^2))/x^5,x]
```

```
[Out] Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*q*
x + c*x^2 + b*p*x^4 + a*(q + p*x^3)^2))/x^5, x]
```

IntegrateAlgebraic [A] time = 0.65, size = 215, normalized size = 1.00

$$\frac{\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}\,\left(3ap^3x^9+9ap^2qx^6-3ap^2qx^5+9apq^2x^3-3apq^2x^2+3aq^3+4bp^2x^7+8bpqx^4-8bpqx^3+4bq^2x+6cpx^5+6cqx^2\right)}{12x^4}+\frac{1}{2}\left(-ap^2q^2-2cpq\right)\log\left(\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+px^3+q\right)+\frac{1}{2}\log(x)\left(ap^2q^2+2cpq\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x
^6]*(b*q*x + c*x^2 + b*p*x^4 + a*(q + p*x^3)^2))/x^5,x]
```

```
[Out] (Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(3*a*q^3 + 4*b*q^2*x + 6*c*q*x
^2 - 3*a*p*q^2*x^2 - 8*b*p*q*x^3 + 9*a*p*q^2*x^3 + 8*b*p*q*x^4 + 6*c*p*x^5
- 3*a*p^2*q*x^5 + 9*a*p^2*q*x^6 + 4*b*p^2*x^7 + 3*a*p^3*x^9))/(12*x^4) + ((
2*c*p*q + a*p^2*q^2)*Log[x])/2 + ((-2*c*p*q - a*p^2*q^2)*Log[q + p*x^3 + Sq
rt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]])/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*q*x+c*x^2+
b*p*x^4+a*(p*x^3+q)^2)/x^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6 + 2\,pqx^3 - 2\,pqx^2 + q^2}\left(bpx^4 + \left(px^3 + q\right)^2 a + bqx + cx^2\right)\left(2\,px^3 - q\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*q*x+c*x^2+
b*p*x^4+a*(p*x^3+q)^2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)*(b*p*x^4 + (p*x^3 + q
)^2*a + b*q*x + c*x^2)*(2*p*x^3 - q)/x^5, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\left(2p\,x^3 - q\right)\sqrt{p^2x^6 + 2pq\,x^3 - 2pq\,x^2 + q^2}\,\left(bqx + c\,x^2 + bp\,x^4 + a\left(p\,x^3 + q\right)^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*q*x+c*x^2+b*p*x^
4+a*(p*x^3+q)^2)/x^5,x)
```

```
[Out] int((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*q*x+c*x^2+b*p*x^
4+a*(p*x^3+q)^2)/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6 + 2\,pqx^3 - 2\,pqx^2 + q^2}\left(bpx^4 + \left(px^3 + q\right)^2 a + bqx + cx^2\right)\left(2\,px^3 - q\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*q*x+c*x^2+
b*p*x^4+a*(p*x^3+q)^2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)*(b*p*x^4 + (p*x^3 + q
)^2*a + b*q*x + c*x^2)*(2*p*x^3 - q)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(q - 2\,p\,x^3\right)\,\left(a\,\left(p\,x^3 + q\right)^2 + c\,x^2 + b\,p\,x^4 + b\,q\,x\right)\,\sqrt{p^2\,x^6 + 2\,p\,q\,x^3 - 2\,p\,q\,x^2 + q^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q - 2*p*x^3)*(a*(q + p*x^3)^2 + c*x^2 + b*p*x^4 + b*q*x)*(p^2*x^6 +
q^2 - 2*p*q*x^2 + 2*p*q*x^3)^(1/2))/x^5,x)
```

```
[Out] -int(((q - 2*p*x^3)*(a*(q + p*x^3)^2 + c*x^2 + b*p*x^4 + b*q*x)*(p^2*x^6 +
q^2 - 2*p*q*x^2 + 2*p*q*x^3)^(1/2))/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2px^3 - q\right)\sqrt{p^2x^6 + 2pqx^3 - 2pqx^2 + q^2}\,\left(ap^2x^6 + 2apqx^3 + aq^2 + bpx^4 + bqx + cx^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x**3-q)*(p**2*x**6+2*p*q*x**3-2*p*q*x**2+q**2)**(1/2)*(b*q*x
+c*x**2+b*p*x**4+a*(p*x**3+q)**2)/x**5,x)
```

```
[Out] Integral((2*p*x**3 - q)*sqrt(p**2*x**6 + 2*p*q*x**3 - 2*p*q*x**2 + q**2)*(a
*p**2*x**6 + 2*a*p*q*x**3 + a*q**2 + b*p*x**4 + b*q*x + c*x**2)/x**5, x)
```

3.2050 $\int \frac{-a(a-2b)-2bx+x^2}{((-a+x)(-b+x))^{2/3}\left(a^2+bd-(2a+d)x+x^2\right)}\,dx$

Optimal. Leaf size=216

$$-\frac{\log\left(\left(x(-a-b)+ab+x^2\right)^{2/3}\left(d^{2/3}x-bd^{2/3}\right)+\sqrt[3]{d}\left(x(-a-b)+ab+x^2\right)^{4/3}+b^2d-2bdx+dx^2\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[6]{d}\left(x(-\right.\right.}{}$$

Rubi [C] time = 1.66, antiderivative size = 209, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 5, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.096, Rules used = {6719, 1586, 6728, 137, 136}

$$-\frac{3(a-x)^2\left(-\frac{b-x}{a-b}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};\frac{a-x}{a-b},-\frac{2(a-x)}{\sqrt{d}\left(\sqrt{d}-\sqrt{4a-4b+d}\right)}\right)}{4d((a-x)(b-x))^{2/3}}-\frac{3(a-x)^2\left(-\frac{b-x}{a-b}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};\frac{a-x}{a-b},-\frac{2(a-x)}{\sqrt{d}\left(\sqrt{d}+\sqrt{4a-4b+d}\right)}\right)}{4d((a-x)(b-x))^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-(a*(a - 2*b)) - 2*b*x + x^2)/(((-a + x)*(-b + x))^(2/3)*(a^2 + b*d -
(2*a + d)*x + x^2)),x]
```

```
[Out] (-3*(a - x)^2*(-((b - x)/(a - b)))^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, (a - x)
/(a - b), (-2*(a - x))/(Sqrt[d]*(Sqrt[d] - Sqrt[4*a - 4*b + d]))])/(4*d*((a
 - x)*(b - x))^(2/3)) - (3*(a - x)^2*(-((b - x)/(a - b)))^(2/3)*AppellF1[4/
3, 2/3, 1, 7/3, (a - x)/(a - b), (-2*(a - x))/(Sqrt[d]*(Sqrt[d] + Sqrt[4*a
- 4*b + d]))])/(4*d*((a - x)*(b - x))^(2/3))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-a(a-2b)-2bx+x^2}{((-a+x)(-b+x))^{2/3}\left(a^2+bd-(2a+d)x+x^2\right)}dx = \frac{\left((-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{-a(a-2b)-2bx+x^2}{(-a+x)^{2/3}(-b+x)^{2/3}\left(a^2+bd-(2a+d)\right.}}{((-a+x)(-b+x))^{2/3}}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{\sqrt[3]{-a+x}\,(a-2b+x)}{(-b+x)^{2/3}\left(a^2+bd-(2a+d)x+x^2\right)}dx}{((-a+x)(-b+x))^{2/3}}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{2/3}\right)\int \left(\frac{\left(1+\frac{\sqrt{4a-4b+d}}{\sqrt{d}}\right)\sqrt[3]{-a+x}}{(-b+x)^{2/3}\left(-2a-d-\sqrt{d}\,\sqrt{4a-4b+d}\right.}\right.}{((-a+x)(-b+x}$$

$$= \frac{\left(\left(1-\frac{\sqrt{4a-4b+d}}{\sqrt{d}}\right)(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{}{(-b+x)^{2/3}\left(-2a\right.}}{((-a+x)(-b+x))^{2/3}}$$

$$= \frac{\left(\left(1-\frac{\sqrt{4a-4b+d}}{\sqrt{d}}\right)(-a+x)^{2/3}\left(\frac{-b+x}{a-b}\right)^{2/3}\right)\int \frac{}{\left(-2a-d+\sqrt{d}\,\sqrt{4\right.}}}{((-a+x)(-b+x))^{2/3}}$$

$$= -\frac{3(a-x)^2\left(-\frac{b-x}{a-b}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};\frac{a-x}{a-b},-\frac{2(a-x)}{\sqrt{d}\left(\sqrt{d}-\sqrt{4a-4\right.}}\right.}{4d((a-x)(b-x))^{2/3}}$$

Mathematica [F] time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{-a(a-2b)-2bx+x^2}{((-a+x)(-b+x))^{2/3}\left(a^2+bd-(2a+d)x+x^2\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-(a*(a - 2*b)) - 2*b*x + x^2)/(((-a + x)*(-b + x))^(2/3)*(a^2 +
b*d - (2*a + d)*x + x^2)),x]
```

```
[Out] Integrate[(-(a*(a - 2*b)) - 2*b*x + x^2)/(((-a + x)*(-b + x))^(2/3)*(a^2 +
b*d - (2*a + d)*x + x^2)), x]
```

IntegrateAlgebraic [A] time = 0.65, size = 216, normalized size = 1.00

$$-\frac{\log\left(\left(x(-a-b)+ab+x^2\right)^{2/3}\left(d^{2/3}x-bd^{2/3}\right)+\sqrt[3]{d}\left(x(-a-b)+ab+x^2\right)^{4/3}+b^2d-2bdx+dx^2\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[6]{d}\left(x(-a-b)+ab+x^2\right)^{2/3}+b\sqrt{d}-\sqrt{d}\,x\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(x(-a-b)+ab+x^2\right)^{2/3}}{\left(x(-a-b)+ab+x^2\right)^{2/3}-2b\sqrt[3]{d}+2\sqrt[3]{d}\,x}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*(a - 2*b)) - 2*b*x + x^2)/(((-a + x)*(-b + x))^(2/3
)*(a^2 + b*d - (2*a + d)*x + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(a*b + (-a - b)*x + x^2)^(2/3))/(-2*b*d^(1/3) + 2*
d^(1/3)*x + (a*b + (-a - b)*x + x^2)^(2/3))])/d^(1/3) + Log[b*Sqrt[d] - Sqr
t[d]*x + d^(1/6)*(a*b + (-a - b)*x + x^2)^(2/3)]/d^(1/3) - Log[b^2*d - 2*b*
d*x + d*x^2 + (-(b*d^(2/3)) + d^(2/3)*x)*(a*b + (-a - b)*x + x^2)^(2/3) + d
^(1/3)*(a*b + (-a - b)*x + x^2)^(4/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a-2*b)-2*b*x+x^2)/((-a+x)*(-b+x))^(2/3)/(a^2+b*d-(2*a+d)*x+x
^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(a-2\,b)a+2\,bx-x^2}{\left(a^2+bd-(2\,a+d)x+x^2\right)((a-x)(b-x))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a-2*b)-2*b*x+x^2)/((-a+x)*(-b+x))^(2/3)/(a^2+b*d-(2*a+d)*x+x
^2),x, algorithm="giac")
```

```
[Out] integrate(-((a - 2*b)*a + 2*b*x - x^2)/((a^2 + b*d - (2*a + d)*x + x^2)*((a
 - x)*(b - x))^(2/3)), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{-a\,(a-2b)-2bx+x^2}{((-a+x)\,(-b+x))^{\frac{2}{3}}\left(a^2+bd-(2a+d)\,x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*(a-2*b)-2*b*x+x^2)/((-a+x)*(-b+x))^(2/3)/(a^2+b*d-(2*a+d)*x+x^2),x)
```

```
[Out] int((-a*(a-2*b)-2*b*x+x^2)/((-a+x)*(-b+x))^(2/3)/(a^2+b*d-(2*a+d)*x+x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(a-2\,b)a+2\,bx-x^2}{\left(a^2+bd-(2\,a+d)x+x^2\right)((a-x)(b-x))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a-2*b)-2*b*x+x^2)/((-a+x)*(-b+x))^(2/3)/(a^2+b*d-(2*a+d)*x+x
^2),x, algorithm="maxima")
```

```
[Out] -integrate(((a - 2*b)*a + 2*b*x - x^2)/((a^2 + b*d - (2*a + d)*x + x^2)*((a
 - x)*(b - x))^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{-x^2+2\,b\,x+a\,(a-2\,b)}{((a-x)\,(b-x))^{2/3}\,\left(b\,d-x\,(2\,a+d)+a^2+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*b*x + a*(a - 2*b) - x^2)/(((a - x)*(b - x))^(2/3)*(b*d - x*(2*a + d
) + a^2 + x^2)),x)
```

```
[Out] -int((2*b*x + a*(a - 2*b) - x^2)/(((a - x)*(b - x))^(2/3)*(b*d - x*(2*a + d
) + a^2 + x^2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a-2*b)-2*b*x+x**2)/((-a+x)*(-b+x))**(2/3)/(a**2+b*d-(2*a+d)*
x+x**2),x)
```

```
[Out] Timed out
```

3.2051 $\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\left(b+a^2d+(-1-2ad)x+dx^2\right)}\,dx$

Optimal. Leaf size=216

$$-\frac{\log\left(a^2d^{2/3}+\sqrt[3]{x(-a-b)+ab+x^2}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)+\left(x(-a-b)+ab+x^2\right)^{2/3}-2ad^{2/3}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{x(-a\right.}}{}$$

Rubi [F] time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\left(b+a^2d+(-1-2ad)x+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a - 2*b + x)/(((-a + x)*(-b + x))^(1/3)*(b + a^2*d + (-1 - 2*a*d)*x +
d*x^2)),x]
```

```
[Out] Defer[Int][(a - 2*b + x)/((a*b + (-a - b)*x + x^2)^(1/3)*(b + a^2*d + (-1 -
 2*a*d)*x + d*x^2)), x]
```

Rubi steps

$$\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\left(b+a^2d+(-1-2ad)x+dx^2\right)}\,dx=\int \frac{a-2b+x}{\sqrt[3]{ab+(-a-b)x+x^2}\left(b+a^2d+(-1-2ad)x+dx\right.}$$

Mathematica [F] time = 11.46, size = 0, normalized size = 0.00

$$\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\left(b+a^2d+(-1-2ad)x+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a - 2*b + x)/(((-a + x)*(-b + x))^(1/3)*(b + a^2*d + (-1 - 2*a*d
)*x + d*x^2)),x]
```

```
[Out] Integrate[(a - 2*b + x)/(((-a + x)*(-b + x))^(1/3)*(b + a^2*d + (-1 - 2*a*d
)*x + d*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.57, size = 216, normalized size = 1.00

$$-\frac{\log\left(a^2d^{2/3}+\sqrt[3]{x(-a-b)+ab+x^2}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)+\left(x(-a-b)+ab+x^2\right)^{2/3}-2ad^{2/3}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{x(-a-b)+ab+x^2}+a\sqrt[3]{d}-\sqrt[3]{d}\,x\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x(-a-b)+ab+x^2}}{\sqrt[3]{x(-a-b)+ab+x^2}-2a\sqrt[3]{d}+2\sqrt[3]{d}\,x}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - 2*b + x)/(((-a + x)*(-b + x))^(1/3)*(b + a^2*d + (-
1 - 2*a*d)*x + d*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(a*b + (-a - b)*x + x^2)^(1/3))/(-2*a*d^(1/3) + 2*
d^(1/3)*x + (a*b + (-a - b)*x + x^2)^(1/3))])/d^(1/3) + Log[a*d^(1/3) - d^(
1/3)*x + (a*b + (-a - b)*x + x^2)^(1/3)]/d^(1/3) - Log[a^2*d^(2/3) - 2*a*d^
(2/3)*x + d^(2/3)*x^2 + (-(a*d^(1/3)) + d^(1/3)*x)*(a*b + (-a - b)*x + x^2)
^(1/3) + (a*b + (-a - b)*x + x^2)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)/((-a+x)*(-b+x))^(1/3)/(b+a^2*d+(-2*a*d-1)*x+d*x^2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a-2\,b+x}{\left(a^2d+dx^2-(2\,ad+1)x+b\right)((a-x)(b-x))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)/((-a+x)*(-b+x))^(1/3)/(b+a^2*d+(-2*a*d-1)*x+d*x^2),x, a
lgorithm="giac")
```

```
[Out] integrate((a - 2*b + x)/((a^2*d + d*x^2 - (2*a*d + 1)*x + b)*((a - x)*(b -
x))^(1/3)), x)
```

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{a-2b+x}{\left(\left(-a+x\right)\left(-b+x\right)\right)^{\frac{1}{3}}\left(b+a^{2}d+\left(-2ad-1\right)x+d\,x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-2*b+x)/((-a+x)*(-b+x))^(1/3)/(b+a^2*d+(-2*a*d-1)*x+d*x^2),x)
```

```
[Out] int((a-2*b+x)/((-a+x)*(-b+x))^(1/3)/(b+a^2*d+(-2*a*d-1)*x+d*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a-2\,b+x}{\left(a^2d+dx^2-(2\,ad+1)x+b\right)((a-x)(b-x))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)/((-a+x)*(-b+x))^(1/3)/(b+a^2*d+(-2*a*d-1)*x+d*x^2),x, a
lgorithm="maxima")
```

```
[Out] integrate((a - 2*b + x)/((a^2*d + d*x^2 - (2*a*d + 1)*x + b)*((a - x)*(b -
x))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a-2\,b+x}{{\left(\left(a-x\right)\,\left(b-x\right)\right)}^{1/3}\,\left(b-x\,\left(2\,a\,d+1\right)+a^2\,d+d\,x^2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - 2*b + x)/(((a - x)*(b - x))^(1/3)*(b - x*(2*a*d + 1) + a^2*d + d*x
^2)),x)
```

```
[Out] int((a - 2*b + x)/(((a - x)*(b - x))^(1/3)*(b - x*(2*a*d + 1) + a^2*d + d*x
^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)/((-a+x)*(-b+x))**(1/3)/(b+a**2*d+(-2*a*d-1)*x+d*x**2),x
)
```

```
[Out] Timed out
```

3.2052 $\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\left(b+a^2d-(1+2ad)x+dx^2\right)}\,dx$

Optimal. Leaf size=216

$$-\frac{\log\left(a^2d^{2/3}+\sqrt[3]{x(-a-b)+ab+x^2}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)+\left(x(-a-b)+ab+x^2\right)^{2/3}-2ad^{2/3}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{x(\right.}$$

Rubi [F] time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\left(b+a^2d-(1+2ad)x+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a - 2*b + x)/(((-a + x)*(-b + x))^(1/3)*(b + a^2*d - (1 + 2*a*d)*x + d
*x^2)),x]
```

```
[Out] Defer[Int][(a - 2*b + x)/((a*b + (-a - b)*x + x^2)^(1/3)*(b + a^2*d + (-1 -
 2*a*d)*x + d*x^2)), x]
```

Rubi steps

$$\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\left(b+a^2d-(1+2ad)x+dx^2\right)}\,dx=\int \frac{a-2b+x}{\sqrt[3]{ab+(-a-b)x+x^2}\left(b+a^2d+(-1-2ad)x+d\right.}$$

Mathematica [F] time = 11.06, size = 0, normalized size = 0.00

$$\int \frac{a-2b+x}{\sqrt[3]{(-a+x)(-b+x)}\left(b+a^2d-(1+2ad)x+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a - 2*b + x)/(((-a + x)*(-b + x))^(1/3)*(b + a^2*d - (1 + 2*a*d)
*x + d*x^2)),x]
```

```
[Out] Integrate[(a - 2*b + x)/(((-a + x)*(-b + x))^(1/3)*(b + a^2*d - (1 + 2*a*d)
*x + d*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.54, size = 216, normalized size = 1.00

$$-\frac{\log\left(a^2d^{2/3}+\sqrt[3]{x(-a-b)+ab+x^2}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)+\left(x(-a-b)+ab+x^2\right)^{2/3}-2ad^{2/3}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{x(-a-b)+ab+x^2}+a\sqrt[3]{d}-\sqrt[3]{d}\,x\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x(-a-b)+ab+x^2}}{\sqrt[3]{x(-a-b)+ab+x^2}-2a\sqrt[3]{d}+2\sqrt[3]{d}\,x}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - 2*b + x)/(((-a + x)*(-b + x))^(1/3)*(b + a^2*d - (1
 + 2*a*d)*x + d*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(a*b + (-a - b)*x + x^2)^(1/3))/(-2*a*d^(1/3) + 2*
d^(1/3)*x + (a*b + (-a - b)*x + x^2)^(1/3))])/d^(1/3) + Log[a*d^(1/3) - d^(
1/3)*x + (a*b + (-a - b)*x + x^2)^(1/3)]/d^(1/3) - Log[a^2*d^(2/3) - 2*a*d^
(2/3)*x + d^(2/3)*x^2 + (-(a*d^(1/3)) + d^(1/3)*x)*(a*b + (-a - b)*x + x^2)
^(1/3) + (a*b + (-a - b)*x + x^2)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)/((-a+x)*(-b+x))^(1/3)/(b+a^2*d-(2*a*d+1)*x+d*x^2),x, al
gorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a-2\,b+x}{\left(a^2d+dx^2-(2\,ad+1)x+b\right)((a-x)(b-x))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)/((-a+x)*(-b+x))^(1/3)/(b+a^2*d-(2*a*d+1)*x+d*x^2),x, al
gorithm="giac")
```

```
[Out] integrate((a - 2*b + x)/((a^2*d + d*x^2 - (2*a*d + 1)*x + b)*((a - x)*(b -
x))^(1/3)), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{a-2b+x}{\left((-a+x)\,(-b+x)\right)^{\frac{1}{3}}\left(b+a^{2}d-(2ad+1)\,x+d\,x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-2*b+x)/((-a+x)*(-b+x))^(1/3)/(b+a^2*d-(2*a*d+1)*x+d*x^2),x)
```

```
[Out] int((a-2*b+x)/((-a+x)*(-b+x))^(1/3)/(b+a^2*d-(2*a*d+1)*x+d*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a-2\,b+x}{\left(a^2d+dx^2-(2\,ad+1)x+b\right)((a-x)(b-x))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)/((-a+x)*(-b+x))^(1/3)/(b+a^2*d-(2*a*d+1)*x+d*x^2),x, al
gorithm="maxima")
```

```
[Out] integrate((a - 2*b + x)/((a^2*d + d*x^2 - (2*a*d + 1)*x + b)*((a - x)*(b -
x))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a-2\,b+x}{{\left(\left(a-x\right)\,\left(b-x\right)\right)}^{1/3}\,\left(b-x\,\left(2\,a\,d+1\right)+a^2\,d+d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - 2*b + x)/(((a - x)*(b - x))^(1/3)*(b - x*(2*a*d + 1) + a^2*d + d*x
^2)),x)
```

```
[Out] int((a - 2*b + x)/(((a - x)*(b - x))^(1/3)*(b - x*(2*a*d + 1) + a^2*d + d*x
^2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)/((-a+x)*(-b+x))**(1/3)/(b+a**2*d-(2*a*d+1)*x+d*x**2),x)
```

```
[Out] Timed out
```

3.2053 $\int \frac{-1+2kx+(1-2k)x^2}{((1-x)x(1-kx))^{2/3}\left(b-(1+2b)x+(b+k)x^2\right)}\,dx$

Optimal. Leaf size=217

$$-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{4/3}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\left(\sqrt[3]{b}\,x-\sqrt[3]{b}\,kx^2\right)+k^2x^4-2kx^3+x^2\right)}{2b^{2/3}}+\frac{\log\left(\sqrt[3]{b}\right.}{}$$

Rubi [F] time = 3.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+2kx+(1-2k)x^2}{((1-x)x(1-kx))^{2/3}\left(b-(1+2b)x+(b+k)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 2*k*x + (1 - 2*k)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(b - (1 + 2*b
)*x + (b + k)*x^2)),x]
```

```
[Out] -(((1 - 2*k + Sqrt[1 + 4*b - 4*b*k])*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*
Defer[Int][(1 - x)^(1/3)/(x^(2/3)*(1 - k*x)^(2/3)*(-1 - 2*b - Sqrt[1 + 4*b
- 4*b*k] + 2*(b + k)*x)), x])/((1 - x)*x*(1 - k*x))^(2/3)) - ((1 - 2*k - Sq
rt[1 + 4*b - 4*b*k])*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Int][(1 -
x)^(1/3)/(x^(2/3)*(1 - k*x)^(2/3)*(-1 - 2*b + Sqrt[1 + 4*b - 4*b*k] + 2*(b
+ k)*x)), x])/((1 - x)*x*(1 - k*x))^(2/3)
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+2kx+(1-2k)x^2}{((1-x)x(1-kx))^{2/3}\left(b-(1+2b)x+(b+k)x^2\right)}\,dx &= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{-1+2kx+(1-2k)x^2}{(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\left(b-(1+2b)x+(b+\right.}}{((1-x)x(1-kx))^{2/3}}\\
&= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{\sqrt[3]{1-x}\,(-1+(-1+2k)x)}{x^{2/3}(1-kx)^{2/3}\left(b-(1+2b)x+(b+k)x^2\right)}\,d}{((1-x)x(1-kx))^{2/3}}\\
&= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \left(\frac{\left(-1+2k-\sqrt{1+4b-4bk}\right)\sqrt[3]{1-x}}{x^{2/3}(1-kx)^{2/3}\left(-1-2b-\sqrt{1+4b-4bk}+\right.}\right.}{((1-x)x(1-}\\
&= \frac{\left(\left(-1+2k-\sqrt{1+4b-4bk}\right)(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int -}{((1-x)x(1-kx))^{2/3}}
\end{aligned}$$

Mathematica [F] time = 8.40, size = 0, normalized size = 0.00

$$\int \frac{-1+2kx+(1-2k)x^2}{((1-x)x(1-kx))^{2/3}\left(b-(1+2b)x+(b+k)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + 2*k*x + (1 - 2*k)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(b - (1
 + 2*b)*x + (b + k)*x^2)),x]
```

```
[Out] Integrate[(-1 + 2*k*x + (1 - 2*k)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(b - (1
 + 2*b)*x + (b + k)*x^2)), x]
```

IntegrateAlgebraic [A] time = 1.78, size = 217, normalized size = 1.00

$$-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{4/3}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\left(\sqrt[3]{b}\,x-\sqrt[3]{b}\,kx^2\right)+k^2x^4-2kx^3+x^2\right)}{2b^{2/3}}+\frac{\log\left(\sqrt[3]{b}\,\left(kx^3+(-k-1)x^2+x\right)^{2/3}+kx^2-x\right)}{b^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}}{\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}-2kx^2+2x}\right)}{b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*k*x + (1 - 2*k)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3
)*(b - (1 + 2*b)*x + (b + k)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3))/(2*x - 2
*k*x^2 + b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3))])/b^(2/3) + Log[-x + k*x
^2 + b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3)]/b^(2/3) - Log[x^2 - 2*k*x^3
+ k^2*x^4 + (b^(1/3)*x - b^(1/3)*k*x^2)*(x + (-1 - k)*x^2 + k*x^3)^(2/3) +
b^(2/3)*(x + (-1 - k)*x^2 + k*x^3)^(4/3)]/(2*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*k*x+(1-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-(1+2*b)*x+(b+k
)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(2\,k-1)x^2-2\,kx+1}{((kx-1)(x-1)x)^{\frac{2}{3}}\left((b+k)x^2-(2\,b+1)x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*k*x+(1-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-(1+2*b)*x+(b+k
)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-((2*k - 1)*x^2 - 2*k*x + 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((b + k
)*x^2 - (2*b + 1)*x + b)), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{-1+2kx+(1-2k)\,x^2}{((1-x)\,x\,(-kx+1))^{\frac{2}{3}}\left(b-(1+2b)\,x+(b+k)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*k*x+(1-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-(1+2*b)*x+(b+k)*x^2)
,x)
```

```
[Out] int((-1+2*k*x+(1-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-(1+2*b)*x+(b+k)*x^2)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(2\,k-1)x^2-2\,kx+1}{((kx-1)(x-1)x)^{\frac{2}{3}}\left((b+k)x^2-(2\,b+1)x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*k*x+(1-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-(1+2*b)*x+(b+k
)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate(((2*k - 1)*x^2 - 2*k*x + 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((b + k
)*x^2 - (2*b + 1)*x + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(2\,k-1)\,x^2-2\,k\,x+1}{\left((b+k)\,x^2+(-2\,b-1)\,x+b\right)\,\left(x\,(k\,x-1)\,(x-1)\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(2*k - 1) - 2*k*x + 1)/((b + x^2*(b + k) - x*(2*b + 1))*(x*(k*x -
 1)*(x - 1))^(2/3)),x)
```

```
[Out] int(-(x^2*(2*k - 1) - 2*k*x + 1)/((b + x^2*(b + k) - x*(2*b + 1))*(x*(k*x -
 1)*(x - 1))^(2/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*k*x+(1-2*k)*x**2)/((1-x)*x*(-k*x+1))**(2/3)/(b-(1+2*b)*x+(b
+k)*x**2),x)
```

```
[Out] Timed out
```

3.2054 $\int \frac{-1+x}{(1+x)\sqrt[3]{-x^2+x^3}}\,dx$

Optimal. Leaf size=217

$$-\log\left(\sqrt[3]{x^3-x^2}-x\right)+2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^3-x^2}-2x\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right)-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{\right.}}{}$$

Rubi [A] time = 0.10, antiderivative size = 291, normalized size of antiderivative = 1.34, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2056, 105, 59, 91}

$$-\frac{3\sqrt[3]{x-1}\,x^{2/3}\log\left(\frac{\sqrt[3]{x-1}}{\sqrt[3]{x}}-1\right)}{2\sqrt[3]{x^3-x^2}}+\frac{3\sqrt[3]{x-1}\,x^{2/3}\log\left(\frac{\sqrt[3]{x-1}}{\sqrt[3]{2}}-\sqrt[3]{x}\right)}{\sqrt[3]{2}\sqrt[3]{x^3-x^2}}-\frac{\sqrt[3]{x-1}\,x^{2/3}\log(x)}{2\sqrt[3]{x^3-x^2}}-\frac{\sqrt[3]{x-1}\,x^{2/3}\log(x+1)}{\sqrt[3]{2}\sqrt[3]{x^3-x^2}}-\frac{\sqrt{3}\sqrt[3]{x-1}\,x^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{x-1}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{x^3-x^2}}+\frac{2^{2/3}\sqrt{3}\sqrt[3]{x-1}\,x^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{x-1}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{x^3-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/((1 + x)*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*(-1 + x)^(1/3)*x^(2/3)*ArcTan[1/Sqrt[3] + (2*(-1 + x)^(1/3))/(Sq
rt[3]*x^(1/3))])/(-x^2 + x^3)^(1/3)) + (2^(2/3)*Sqrt[3]*(-1 + x)^(1/3)*x^(2
/3)*ArcTan[1/Sqrt[3] + (2^(2/3)*(-1 + x)^(1/3))/(Sqrt[3]*x^(1/3))])/(-x^2 +
 x^3)^(1/3) - (3*(-1 + x)^(1/3)*x^(2/3)*Log[-1 + (-1 + x)^(1/3)/x^(1/3)])/(
2*(-x^2 + x^3)^(1/3)) + (3*(-1 + x)^(1/3)*x^(2/3)*Log[(-1 + x)^(1/3)/2^(1/3
) - x^(1/3)])/(2^(1/3)*(-x^2 + x^3)^(1/3)) - ((-1 + x)^(1/3)*x^(2/3)*Log[x]
)/(2*(-x^2 + x^3)^(1/3)) - ((-1 + x)^(1/3)*x^(2/3)*Log[1 + x])/(2^(1/3)*(-x
^2 + x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x}{(1+x)\sqrt[3]{-x^2+x^3}}\,dx &= \frac{\left(\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{(-1+x)^{2/3}}{x^{2/3}(1+x)}\,dx}{\sqrt[3]{-x^2+x^3}} \\
&= \frac{\left(\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{1}{\sqrt[3]{-1+x}\,x^{2/3}}\,dx}{\sqrt[3]{-x^2+x^3}} - \frac{\left(2\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{1}{\sqrt[3]{-1+x}\,x^{2/3}(1+x)}\,dx}{\sqrt[3]{-x^2+x^3}} \\
&= -\frac{\sqrt{3}\,\sqrt[3]{-1+x}\,x^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-1+x}}{\sqrt{3}\,\sqrt[3]{x}}\right)}{\sqrt[3]{-x^2+x^3}} + \frac{2^{2/3}\sqrt{3}\,\sqrt[3]{-1+x}\,x^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}\sqrt[3]{-1+x}}{\sqrt{3}\,\sqrt[3]{x}}\right)}{\sqrt[3]{-x^2+x^3}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.28

$$
\frac{3\left((x-1)x^2\right)^{2/3}\left(x^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};1-x\right)-{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x-1}{2x}\right)\right)}{2x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/((1 + x)*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (3*((-1 + x)*x^2)^(2/3)*(x^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, 1 - x] -
Hypergeometric2F1[2/3, 1, 5/3, (-1 + x)/(2*x)]))/(2*x^2)
```

IntegrateAlgebraic [A] time = 0.55, size = 217, normalized size = 1.00

$$
-\log\left(\sqrt[3]{x^3-x^2}-x\right)+2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^3-x^2}-2x\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right)-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^3-x^2}\,x+\sqrt[3]{2}\left(x^3-x^2\right)^{2/3}\right)}{\sqrt[3]{2}}+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x^2}+x}\right)-2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3-x^2}+x}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/((1 + x)*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-x^2 + x^3)^(1/3))] - 2^(2/3)*Sqrt[3]*Ar
cTan[(Sqrt[3]*x)/(x + 2^(2/3)*(-x^2 + x^3)^(1/3))] - Log[-x + (-x^2 + x^3)^
(1/3)] + 2^(2/3)*Log[-2*x + 2^(2/3)*(-x^2 + x^3)^(1/3)] + Log[x^2 + x*(-x^2
 + x^3)^(1/3) + (-x^2 + x^3)^(2/3)]/2 - Log[2*x^2 + 2^(2/3)*x*(-x^2 + x^3)^
(1/3) + 2^(1/3)*(-x^2 + x^3)^(2/3)]/2^(1/3)
```

fricas [A] time = 0.80, size = 206, normalized size = 0.95

$$
4^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+4^{\frac{1}{3}}\sqrt{3}\left(x^3-x^2\right)^{\frac{1}{3}}}{3x}\right)-\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-x^2\right)^{\frac{1}{3}}}{3x}\right)+4^{\frac{1}{3}}\log\left(-\frac{4^{\frac{2}{3}}x-2\left(x^3-x^2\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{2}\cdot 4^{\frac{1}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}x^2+4^{\frac{2}{3}}\left(x^3-x^2\right)^{\frac{1}{3}}x+2\left(x^3-x^2\right)^{\frac{2}{3}}}{x^2}\right)-\log\left(-\frac{x-\left(x^3-x^2\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{2}\log\left(\frac{x^2+\left(x^3-x^2\right)^{\frac{1}{3}}x+\left(x^3-x^2\right)^{\frac{2}{3}}}{x^2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3-x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] 4^(1/3)*sqrt(3)*arctan(1/3*(sqrt(3)*x + 4^(1/3)*sqrt(3)*(x^3 - x^2)^(1/3))/
x) - sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 - x^2)^(1/3))/x) + 4^(1
/3)*log(-(4^(2/3)*x - 2*(x^3 - x^2)^(1/3))/x) - 1/2*4^(1/3)*log((2*4^(1/3)*
x^2 + 4^(2/3)*(x^3 - x^2)^(1/3)*x + 2*(x^3 - x^2)^(2/3))/x^2) - log(-(x - (
x^3 - x^2)^(1/3))/x) + 1/2*log((x^2 + (x^3 - x^2)^(1/3)*x + (x^3 - x^2)^(2/
3))/x^2)
```

giac [A] time = 0.25, size = 147, normalized size = 0.68

$$
\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{2}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}\right)+2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}\right|\right)-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)-\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3-x^2)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-1/x + 1)^(1/3)))
- 1/2*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-1/x + 1)^(1/3) + (-1/x + 1)^(2/3)) +
2^(2/3)*log(abs(-2^(1/3) + (-1/x + 1)^(1/3))) - sqrt(3)*arctan(1/3*sqrt(3)*
(2*(-1/x + 1)^(1/3) + 1)) + 1/2*log((-1/x + 1)^(2/3) + (-1/x + 1)^(1/3) + 1
) - log(abs((-1/x + 1)^(1/3) - 1))
```

maple [C] time = 5.41, size = 2015, normalized size = 9.29

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(1+x)/(x^3-x^2)^(1/3),x)
```

```
[Out] 1/2*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*ln((2*RootOf(4*Root
Of(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x^2+3*RootOf(4*Root
Of(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x^2-4*RootOf(4*Ro
otOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x-6*RootOf(4*Root
Of(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x-48*RootOf(4*Roo
tOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2*(x^3-x^2)^(2/3)-12
0*RootOf(_Z^3-4)^2*(x^3-x^2)^(1/3)*x-96*(x^3-x^2)^(1/3)*RootOf(4*RootOf(_Z^
3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)*x-88*RootOf(_Z^3-4)*x^2-132
*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*x^2+8*RootOf(_Z^3-4)*x
+12*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*x-240*(x^3-x^2)^(2/
3))/x/(1+x))-ln((4*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*Root
Of(_Z^3-4)^3*x^2+3*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)^2*Ro
otOf(_Z^3-4)^2*x^2-8*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*Ro
otOf(_Z^3-4)^3*x-6*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)^2*Ro
otOf(_Z^3-4)^2*x+48*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*Roo
tOf(_Z^3-4)^2*(x^3-x^2)^(2/3)+72*RootOf(_Z^3-4)^2*(x^3-x^2)^(1/3)*x+96*(x^3
-x^2)^(1/3)*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3
-4)*x+208*RootOf(_Z^3-4)*x^2+156*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3
-4)+_Z^2)*x^2-80*RootOf(_Z^3-4)*x-60*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(
_Z^3-4)+_Z^2)*x+144*(x^3-x^2)^(2/3))/x/(1+x))*RootOf(_Z^3-4)-1/2*ln((4*Root
Of(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x^2+3*Root
Of(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x^2-8*Ro
otOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x-6*Root
Of(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x+48*Roo
tOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2*(x^3-x^2)
^(2/3)+72*RootOf(_Z^3-4)^2*(x^3-x^2)^(1/3)*x+96*(x^3-x^2)^(1/3)*RootOf(4*Ro
otOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)*x+208*RootOf(_Z^3-4
)*x^2+156*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*x^2-80*RootOf
(_Z^3-4)*x-60*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*x+144*(x^
3-x^2)^(2/3))/x/(1+x))*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)-
ln(-(RootOf(_Z^3-4)^4*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)^2
*x^2-2*RootOf(_Z^3-4)^4*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)
^2*x-192*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)
^2*(x^3-x^2)^(2/3)+120*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*
RootOf(_Z^3-4)^2*(x^3-x^2)^(1/3)*x+80*RootOf(_Z^3-4)^2*RootOf(4*RootOf(_Z^3
-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*x^2-104*RootOf(_Z^3-4)^2*RootOf(4*RootOf(_Z
^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*x-960*(x^3-x^2)^(2/3)-576*x*(x^3-x^2)^(1/
3)+1600*x^2-960*x)/x)+ln(-(RootOf(_Z^3-4)^4*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*
RootOf(_Z^3-4)+_Z^2)^2*x^2-2*RootOf(_Z^3-4)^4*RootOf(4*RootOf(_Z^3-4)^2+2*_
Z*RootOf(_Z^3-4)+_Z^2)^2*x+48*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)
+_Z^2)*RootOf(_Z^3-4)^2*(x^3-x^2)^(2/3)-18*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*R
ootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2*(x^3-x^2)^(1/3)*x-22*RootOf(_Z^3-4)^2*
RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*x^2-12*RootOf(_Z^3-4)^2
```

```
*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*x+144*(x^3-x^2)^(2/3)+
240*x*(x^3-x^2)^(1/3)-320*x^2+80*x)/x)+1/8*ln(-(RootOf(_Z^3-4)^4*RootOf(4*R
ootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)^2*x^2-2*RootOf(_Z^3-4)^4*RootOf(4
*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)^2*x+48*RootOf(4*RootOf(_Z^3-4)^
2+2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2*(x^3-x^2)^(2/3)-18*RootOf(4*Ro
otOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2*(x^3-x^2)^(1/3)*x
-22*RootOf(_Z^3-4)^2*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*x^
2-12*RootOf(_Z^3-4)^2*RootOf(4*RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*x
+144*(x^3-x^2)^(2/3)+240*x*(x^3-x^2)^(1/3)-320*x^2+80*x)/x)*RootOf(4*RootOf
(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^3-x^2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3-x^2)^(1/3),x, algorithm="maxima")
[Out] integrate((x - 1)/((x^3 - x^2)^(1/3)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x-1}{\left(x^3-x^2\right)^{1/3}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x^3 - x^2)^(1/3)*(x + 1)),x)
[Out] int((x - 1)/((x^3 - x^2)^(1/3)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt[3]{x^2(x-1)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x**3-x**2)**(1/3),x)
[Out] Integral((x - 1)/((x**2*(x - 1))**(1/3)*(x + 1)), x)
```

3.2055 $\int \frac{\left(-1+x+x^3+x^6\right)^{2/3}\left(3-2x+3x^6\right)}{\left(-1+x+x^6\right)\left(-1+x-x^3+x^6\right)}\,dx$

Optimal. Leaf size=217

$$-\log\left(\sqrt[3]{x^6+x^3+x-1}-x\right)+2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^6+x^3+x-1}-2x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^6+x^3+x-1}+x}\right)-2^{2/3}\cdots$$

Rubi [F] time = 2.29, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1+x+x^3+x^6\right)^{2/3}\left(3-2x+3x^6\right)}{\left(-1+x+x^6\right)\left(-1+x-x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x + x^3 + x^6)^(2/3)*(3 - 2*x + 3*x^6))/((-1 + x + x^6)*(-1 + x
- x^3 + x^6)),x]
```

```
[Out] Defer[Int][(-1 + x + x^3 + x^6)^(2/3)/(-1 + x), x] + Defer[Int][(-1 + x + x
^3 + x^6)^(2/3)/(1 + x), x] + 2*Defer[Int][(-1 + x + x^3 + x^6)^(2/3)/(1 -
x + x^2 + x^4), x] + 3*Defer[Int][(x*(-1 + x + x^3 + x^6)^(2/3))/(1 - x + x
^2 + x^4), x] + Defer[Int][(x^2*(-1 + x + x^3 + x^6)^(2/3))/(1 - x + x^2 +
x^4), x] - Defer[Int][(x^3*(-1 + x + x^3 + x^6)^(2/3))/(1 - x + x^2 + x^4),
 x] + Defer[Int][(-1 + x + x^3 + x^6)^(2/3)/(1 - x - x^6), x] - 6*Defer[Int
][(x^3*(-1 + x + x^3 + x^6)^(2/3))/(-1 + x + x^6), x] - Defer[Int][(x^4*(-1
 + x + x^3 + x^6)^(2/3))/(-1 + x + x^6), x] - Defer[Int][(x^5*(-1 + x + x^3
 + x^6)^(2/3))/(-1 + x + x^6), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x+x^3+x^6\right)^{2/3}\left(3-2x+3x^6\right)}{\left(-1+x+x^6\right)\left(-1+x-x^3+x^6\right)}\,dx &= \int \left(\frac{\left(-1+x+x^3+x^6\right)^{2/3}}{-1+x}+\frac{\left(-1+x+x^3+x^6\right)^{2/3}}{1+x}+\frac{(2+3x+\cdots}{\cdots}\right. \\
&= \int \frac{\left(-1+x+x^3+x^6\right)^{2/3}}{-1+x}\,dx+\int \frac{\left(-1+x+x^3+x^6\right)^{2/3}}{1+x}\,dx+\int \cdots \\
&= \int \frac{\left(-1+x+x^3+x^6\right)^{2/3}}{-1+x}\,dx+\int \frac{\left(-1+x+x^3+x^6\right)^{2/3}}{1+x}\,dx+\int \cdots \\
&= 2\int \frac{\left(-1+x+x^3+x^6\right)^{2/3}}{1-x+x^2+x^4}\,dx+3\int \frac{x\left(-1+x+x^3+x^6\right)^{2/3}}{1-x+x^2+x^4}\,dx-\cdots
\end{aligned}$$

Mathematica [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x+x^3+x^6\right)^{2/3}\left(3-2x+3x^6\right)}{\left(-1+x+x^6\right)\left(-1+x-x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x + x^3 + x^6)^(2/3)*(3 - 2*x + 3*x^6))/((-1 + x + x^6)*(-
1 + x - x^3 + x^6)),x]
```

```
[Out] Integrate[((-1 + x + x^3 + x^6)^(2/3)*(3 - 2*x + 3*x^6))/((-1 + x + x^6)*(-
1 + x - x^3 + x^6)), x]
```

IntegrateAlgebraic [A] time = 0.80, size = 217, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^6+x^3+x-1}-x\right)+2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^6+x^3+x-1}-2x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^6+x^3+x-1}+x}\right)-2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^6+x^3+x-1}+x}\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^6+x^3+x-1}x+\left(x^6+x^3+x-1\right)^{2/3}\right)-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^6+x^3+x-1}x+\sqrt[3]{2}\left(x^6+x^3+x-1\right)^{2/3}\right)}{\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x + x^3 + x^6)^(2/3)*(3 - 2*x + 3*x^6))/((-1 + x
+ x^6)*(-1 + x - x^3 + x^6)),x]
```

```
[Out] Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x + x^3 + x^6)^(1/3))] - 2^(2/3)*Sq
rt[3]*ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(-1 + x + x^3 + x^6)^(1/3))] - Log[-x
 + (-1 + x + x^3 + x^6)^(1/3)] + 2^(2/3)*Log[-2*x + 2^(2/3)*(-1 + x + x^3 +
 x^6)^(1/3)] + Log[x^2 + x*(-1 + x + x^3 + x^6)^(1/3) + (-1 + x + x^3 + x^6
)^(2/3)]/2 - Log[2*x^2 + 2^(2/3)*x*(-1 + x + x^3 + x^6)^(1/3) + 2^(1/3)*(-1
 + x + x^3 + x^6)^(2/3)]/2^(1/3)
```

fricas [B] time = 52.34, size = 615, normalized size = 2.83

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+x-1)^(2/3)*(3*x^6-2*x+3)/(x^6+x-1)/(x^6-x^3+x-1),x, algo
rithm="fricas")
```

```
[Out] -1/3*4^(1/3)*sqrt(3)*arctan(1/3*(3*4^(2/3)*sqrt(3)*(x^13 + 4*x^10 + 2*x^8 -
 7*x^7 + 4*x^5 - 4*x^4 + x^3 - 2*x^2 + x)*(x^6 + x^3 + x - 1)^(2/3) + 6*4^(
1/3)*sqrt(3)*(x^14 + 16*x^11 + 2*x^9 + 17*x^8 + 16*x^6 - 16*x^5 + x^4 - 2*x
^3 + x^2)*(x^6 + x^3 + x - 1)^(1/3) + sqrt(3)*(x^18 + 33*x^15 + 3*x^13 + 10
8*x^12 + 66*x^10 + 5*x^9 + 3*x^8 + 105*x^7 - 108*x^6 + 33*x^5 - 66*x^4 + 34
*x^3 - 3*x^2 + 3*x - 1))/(x^18 - 3*x^15 + 3*x^13 - 108*x^12 - 6*x^10 - 103*
x^9 + 3*x^8 - 111*x^7 + 108*x^6 - 3*x^5 + 6*x^4 - 2*x^3 - 3*x^2 + 3*x - 1))
 + sqrt(3)*arctan(-(682*sqrt(3)*(x^6 + x^3 + x - 1)^(1/3)*x^2 - 248*sqrt(3)
*(x^6 + x^3 + x - 1)^(2/3)*x + sqrt(3)*(96*x^6 + 217*x^3 + 96*x - 96))/(64*
x^6 + 1395*x^3 + 64*x - 64)) + 1/3*4^(1/3)*log((3*4^(2/3)*(x^6 + x^3 + x -
1)^(1/3)*x^2 - 6*(x^6 + x^3 + x - 1)^(2/3)*x + 4^(1/3)*(x^6 - x^3 + x - 1))
/(x^6 - x^3 + x - 1)) - 1/6*4^(1/3)*log((6*4^(1/3)*(x^7 + 5*x^4 + x^2 - x)*
(x^6 + x^3 + x - 1)^(2/3) + 4^(2/3)*(x^12 + 16*x^9 + 2*x^7 + 17*x^6 + 16*x^
4 - 16*x^3 + x^2 - 2*x + 1) + 24*(x^8 + 2*x^5 + x^3 - x^2)*(x^6 + x^3 + x -
 1)^(1/3))/(x^12 - 2*x^9 + 2*x^7 - x^6 - 2*x^4 + 2*x^3 + x^2 - 2*x + 1)) -
1/2*log((x^6 + 3*(x^6 + x^3 + x - 1)^(1/3)*x^2 - 3*(x^6 + x^3 + x - 1)^(2/3
)*x + x - 1)/(x^6 + x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^6-2\,x+3\right)\left(x^6+x^3+x-1\right)^{\frac{2}{3}}}{\left(x^6-x^3+x-1\right)\left(x^6+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+x-1)^(2/3)*(3*x^6-2*x+3)/(x^6+x-1)/(x^6-x^3+x-1),x, algo
rithm="giac")
```

```
[Out] integrate((3*x^6 - 2*x + 3)*(x^6 + x^3 + x - 1)^(2/3)/((x^6 - x^3 + x - 1)*
(x^6 + x - 1)), x)
```

maple [C] time = 54.07, size = 2035, normalized size = 9.38

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+x^3+x-1)^(2/3)*(3*x^6-2*x+3)/(x^6+x-1)/(x^6-x^3+x-1),x)
```

```
[Out] -ln((4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x^3
+5*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x^3-8
*RootOf(_Z^3-4)*x^6-10*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x^6-
6*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2*(x^6+x^3
+x-1)^(2/3)*x-18*RootOf(_Z^3-4)^2*(x^6+x^3+x-1)^(1/3)*x^2-12*(x^6+x^3+x-1)^
(1/3)*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)*x^2-8*
RootOf(_Z^3-4)*x^3-10*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x^3-3
6*(x^6+x^3+x-1)^(2/3)*x-8*RootOf(_Z^3-4)*x-10*RootOf(RootOf(_Z^3-4)^2+_Z*Ro
otOf(_Z^3-4)+_Z^2)*x+8*RootOf(_Z^3-4)+10*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(
_Z^3-4)+_Z^2))/(-1+x)/(1+x)/(x^4+x^2-x+1))*RootOf(_Z^3-4)-ln((4*RootOf(Root
Of(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x^3+5*RootOf(RootOf(_
Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x^3-8*RootOf(_Z^3-4)*x^
6-10*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x^6-6*RootOf(RootOf(_Z
^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2*(x^6+x^3+x-1)^(2/3)*x-18*R
ootOf(_Z^3-4)^2*(x^6+x^3+x-1)^(1/3)*x^2-12*(x^6+x^3+x-1)^(1/3)*RootOf(RootO
f(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)*x^2-8*RootOf(_Z^3-4)*x^3
-10*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x^3-36*(x^6+x^3+x-1)^(2
/3)*x-8*RootOf(_Z^3-4)*x-10*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)
*x+8*RootOf(_Z^3-4)+10*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2))/(-1
+x)/(1+x)/(x^4+x^2-x+1))*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)+Ro
otOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*ln((RootOf(RootOf(_Z^3-4)^2+_
Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x^3+5*RootOf(RootOf(_Z^3-4)^2+_Z*Ro
otOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x^3+2*RootOf(_Z^3-4)*x^6+10*RootOf(Ro
otOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x^6+6*RootOf(RootOf(_Z^3-4)^2+_Z*Roo
tOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2*(x^6+x^3+x-1)^(2/3)*x-6*RootOf(_Z^3-4)^2
*(x^6+x^3+x-1)^(1/3)*x^2+12*(x^6+x^3+x-1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+_Z*
RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)*x^2+6*RootOf(_Z^3-4)*x^3+30*RootOf(Root
Of(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x^3-12*(x^6+x^3+x-1)^(2/3)*x+2*RootOf(
_Z^3-4)*x+10*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x-2*RootOf(_Z^
3-4)-10*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2))/(-1+x)/(1+x)/(x^4+
x^2-x+1))-1/4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4
)^2*ln((RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^4*
x^3+4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2*x^6+
4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2*(x^6+x^3
+x-1)^(2/3)*x+4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3
-4)^2*(x^6+x^3+x-1)^(1/3)*x^2+8*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_
Z^2)*RootOf(_Z^3-4)^2*x^3+4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)
*RootOf(_Z^3-4)^2*x-4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootO
f(_Z^3-4)^2-16*(x^6+x^3+x-1)^(2/3)*x-16*(x^6+x^3+x-1)^(1/3)*x^2)/(x^6+x-1))
+1/4*ln((RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^4
*x^3-4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2*x^6
-4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2*(x^6+x^
3+x-1)^(2/3)*x-4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^
3-4)^2*(x^6+x^3+x-1)^(1/3)*x^2-16*x^6-4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_
Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2*x+4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)
+_Z^2)*RootOf(_Z^3-4)^2-32*(x^6+x^3+x-1)^(2/3)*x-32*(x^6+x^3+x-1)^(1/3)*x^2
-16*x^3-16*x+16)/(x^6+x-1))*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)
*RootOf(_Z^3-4)^2+ln((RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*Roo
tOf(_Z^3-4)^4*x^3-4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(
_Z^3-4)^2*x^6-4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3
-4)^2*(x^6+x^3+x-1)^(2/3)*x-4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^
2)*RootOf(_Z^3-4)^2*(x^6+x^3+x-1)^(1/3)*x^2-16*x^6-4*RootOf(RootOf(_Z^3-4)^
2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2*x+4*RootOf(RootOf(_Z^3-4)^2+_Z*R
ootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2-32*(x^6+x^3+x-1)^(2/3)*x-32*(x^6+x^3+x
-1)^(1/3)*x^2-16*x^3-16*x+16)/(x^6+x-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^{6}-2\,x+3\right)\left(x^{6}+x^{3}+x-1\right)^{\frac{2}{3}}}{\left(x^{6}-x^{3}+x-1\right)\left(x^{6}+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+x-1)^(2/3)*(3*x^6-2*x+3)/(x^6+x-1)/(x^6-x^3+x-1),x, algo
rithm="maxima")
```

```
[Out] integrate((3*x^6 - 2*x + 3)*(x^6 + x^3 + x - 1)^(2/3)/((x^6 - x^3 + x - 1)*
(x^6 + x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(3\,x^6-2\,x+3\right)\,{\left(x^6+x^3+x-1\right)}^{2/3}}{\left(x^6+x-1\right)\,\left(x^6-x^3+x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^6 - 2*x + 3)*(x + x^3 + x^6 - 1)^(2/3))/((x + x^6 - 1)*(x - x^3 +
 x^6 - 1)),x)
```

```
[Out] int(((3*x^6 - 2*x + 3)*(x + x^3 + x^6 - 1)^(2/3))/((x + x^6 - 1)*(x - x^3 +
 x^6 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+x**3+x-1)**(2/3)*(3*x**6-2*x+3)/(x**6+x-1)/(x**6-x**3+x-1),
x)
```

```
[Out] Timed out
```

3.2056 $\int \frac{-1-2x^4+2x^8}{\sqrt[4]{-1+x^4}\,(-1-x^4+x^8)}\,dx$

Optimal. Leaf size=217

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4-1}}\right)+\tanh^{-1}\left(\frac{}{\sqrt[4]{x^4}}\right.$$

Rubi [A] time = 0.38, antiderivative size = 233, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {6728, 240, 212, 206, 203, 1428, 377}

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{5}}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4-1}}\right)}{2\sqrt{5}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 - 2*x^4 + 2*x^8)/((-1 + x^4)^(1/4)*(-1 - x^4 + x^8)),x]
```

```
[Out] ArcTan[x/(-1 + x^4)^(1/4)] - (((3 - Sqrt[5])/2)^(1/4)*ArcTan[((2/(3 + Sqrt[
5]))^(1/4)*x)/(-1 + x^4)^(1/4)])/(2*Sqrt[5]) - (((3 + Sqrt[5])/2)^(1/4)*Arc
Tan[(((3 + Sqrt[5])/2)^(1/4)*x)/(-1 + x^4)^(1/4)])/(2*Sqrt[5]) + ArcTanh[x/
(-1 + x^4)^(1/4)] - (((3 - Sqrt[5])/2)^(1/4)*ArcTanh[((2/(3 + Sqrt[5]))^(1/
4)*x)/(-1 + x^4)^(1/4)])/(2*Sqrt[5]) - (((3 + Sqrt[5])/2)^(1/4)*ArcTanh[(((
3 + Sqrt[5])/2)^(1/4)*x)/(-1 + x^4)^(1/4)])/(2*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1428

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^
n)^q/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[(d + e*x^n)^q/(b + r + 2
*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-1-2x^4+2x^8}{\sqrt[4]{-1+x^4}\left(-1-x^4+x^8\right)}\,dx = \int \left(\frac{2}{\sqrt[4]{-1+x^4}}+\frac{1}{\sqrt[4]{-1+x^4}\left(-1-x^4+x^8\right)}\right)dx$$

$$= 2\int \frac{1}{\sqrt[4]{-1+x^4}}\,dx+\int \frac{1}{\sqrt[4]{-1+x^4}\left(-1-x^4+x^8\right)}\,dx$$

$$= 2\,\mathrm{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)+\frac{2\int \frac{1}{\sqrt[4]{-1+x^4}\left(-1-\sqrt{5}+2x^4\right)}\,dx}{\sqrt{5}}-\frac{2\int \frac{1}{\sqrt[4]{-1+x^4}}\cdots}{}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{-1-\sqrt{5}-\left(1-\sqrt{5}\right)x^4}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{5}}-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{-1+\sqrt{5}-\left(1+\sqrt{5}\right)x^4}\,dx, x, \cdots\right.}{\sqrt{5}}$$

$$= \tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)-\frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx, x, \frac{x}{\sqrt[4]{-1+\cdots}}\right.}{\sqrt{10}}$$

$$= \tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{-1+x^4}}\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{-1+x^4}}\right)}{2\sqrt{5}}+\cdots$$

Mathematica [A] time = 0.55, size = 239, normalized size = 1.10

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4-1}}\right)}{5+\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4-1}}\right)}{\sqrt{5}-5}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4-1}}\right)}{5+\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4-1}}\right)}{\sqrt{5}-5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x^4 + 2*x^8)/((-1 + x^4)^(1/4)*(-1 - x^4 + x^8)),x]
```

```
[Out] ArcTan[x/(-1 + x^4)^(1/4)] - (((3 + Sqrt[5])/2)^(1/4)*ArcTan[((2/(3 + Sqrt[
5]))^(1/4)*x)/(-1 + x^4)^(1/4)])/(5 + Sqrt[5]) + (((3 - Sqrt[5])/2)^(1/4)*A
rcTan[(((3 + Sqrt[5])/2)^(1/4)*x)/(-1 + x^4)^(1/4)])/(-5 + Sqrt[5]) + ArcTa
nh[x/(-1 + x^4)^(1/4)] - (((3 + Sqrt[5])/2)^(1/4)*ArcTanh[((2/(3 + Sqrt[5])
)^(1/4)*x)/(-1 + x^4)^(1/4)])/(5 + Sqrt[5]) + (((3 - Sqrt[5])/2)^(1/4)*ArcT
anh[(((3 + Sqrt[5])/2)^(1/4)*x)/(-1 + x^4)^(1/4)])/(-5 + Sqrt[5])
```

IntegrateAlgebraic [A] time = 0.73, size = 217, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4-1}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 - 2*x^4 + 2*x^8)/((-1 + x^4)^(1/4)*(-1 - x^4 + x^8))
,x]
```

```
[Out] ArcTan[x/(-1 + x^4)^(1/4)] - (Sqrt[(-1 + Sqrt[5])/10]*ArcTan[(Sqrt[-1/2 + S
qrt[5]/2]*x)/(-1 + x^4)^(1/4)])/2 - (Sqrt[(1 + Sqrt[5])/10]*ArcTan[(Sqrt[1/
2 + Sqrt[5]/2]*x)/(-1 + x^4)^(1/4)])/2 + ArcTanh[x/(-1 + x^4)^(1/4)] - (Sqr
t[(-1 + Sqrt[5])/10]*ArcTanh[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(-1 + x^4)^(1/4)])/
2 - (Sqrt[(1 + Sqrt[5])/10]*ArcTanh[(Sqrt[1/2 + Sqrt[5]/2]*x)/(-1 + x^4)^(1
/4)])/2
```

fricas [B] time = 0.46, size = 406, normalized size = 1.87

$$\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+1}\arctan\left(\frac{\sqrt{10}\sqrt{2}\left(\sqrt{5}x-5x\right)\sqrt{\sqrt{5}+1}\sqrt{\frac{\sqrt{5}x^2+x^2+2\sqrt{x^4-1}}{x^2}}-2\sqrt{10}\left(x^4-1\right)^{\frac{1}{4}}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)}{40x}\right)-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}-1}\arctan\left(\frac{\sqrt{10}\sqrt{2}\left(\sqrt{5}x+5x\right)\sqrt{\sqrt{5}-1}\sqrt{\frac{\sqrt{5}x^2-x^2+2\sqrt{x^4-1}}{x^2}}-2\sqrt{10}\left(x^4-1\right)^{\frac{1}{4}}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}}{40x}\right)-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(\frac{\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}+1}+10\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(-\frac{\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}+1}-10\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(\frac{\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}-1}+10\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(-\frac{\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}-1}-10\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)-\arctan\left(\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\log\left(\frac{x+\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{2}\log\left(-\frac{x-\left(x^4-1\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-2*x^4-1)/(x^4-1)^(1/4)/(x^8-x^4-1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(10)*sqrt(sqrt(5) + 1)*arctan(1/40*(sqrt(10)*sqrt(2)*(sqrt(5)*x -
5*x)*sqrt(sqrt(5) + 1)*sqrt((sqrt(5)*x^2 + x^2 + 2*sqrt(x^4 - 1))/x^2) - 2*
sqrt(10)*(x^4 - 1)^(1/4)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5))/x) - 1/10*sqrt(10
)*sqrt(sqrt(5) - 1)*arctan(1/40*(sqrt(10)*sqrt(2)*(sqrt(5)*x + 5*x)*sqrt(sq
rt(5) - 1)*sqrt((sqrt(5)*x^2 - x^2 + 2*sqrt(x^4 - 1))/x^2) - 2*sqrt(10)*(x^
4 - 1)^(1/4)*(sqrt(5) + 5)*sqrt(sqrt(5) - 1))/x) - 1/40*sqrt(10)*sqrt(sqrt(
5) + 1)*log((sqrt(10)*sqrt(5)*x*sqrt(sqrt(5) + 1) + 10*(x^4 - 1)^(1/4))/x)
+ 1/40*sqrt(10)*sqrt(sqrt(5) + 1)*log(-(sqrt(10)*sqrt(5)*x*sqrt(sqrt(5) + 1
) - 10*(x^4 - 1)^(1/4))/x) - 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log((sqrt(10)*
sqrt(5)*x*sqrt(sqrt(5) - 1) + 10*(x^4 - 1)^(1/4))/x) + 1/40*sqrt(10)*sqrt(s
qrt(5) - 1)*log(-(sqrt(10)*sqrt(5)*x*sqrt(sqrt(5) - 1) - 10*(x^4 - 1)^(1/4)
)/x) - arctan((x^4 - 1)^(1/4)/x) + 1/2*log((x + (x^4 - 1)^(1/4))/x) - 1/2*l
og(-(x - (x^4 - 1)^(1/4))/x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-2*x^4-1)/(x^4-1)^(1/4)/(x^8-x^4-1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [F] time = 2.88, size = 0, normalized size = 0.00

$$\int \frac{2x^{8}-2x^{4}-1}{\left(x^{4}-1\right)^{\frac{1}{4}}\left(x^{8}-x^{4}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8-2*x^4-1)/(x^4-1)^(1/4)/(x^8-x^4-1),x)
```

```
[Out] int((2*x^8-2*x^4-1)/(x^4-1)^(1/4)/(x^8-x^4-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^{8}-2\,x^{4}-1}{\left(x^{8}-x^{4}-1\right)\left(x^{4}-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-2*x^4-1)/(x^4-1)^(1/4)/(x^8-x^4-1),x, algorithm="maxima")
[Out] integrate((2*x^8 - 2*x^4 - 1)/((x^8 - x^4 - 1)*(x^4 - 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{-2x^8 + 2x^4 + 1}{\left(x^4 - 1\right)^{1/4}\left(-x^8 + x^4 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4 - 2*x^8 + 1)/((x^4 - 1)^(1/4)*(x^4 - x^8 + 1)),x)
[Out] int((2*x^4 - 2*x^8 + 1)/((x^4 - 1)^(1/4)*(x^4 - x^8 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8-2*x**4-1)/(x**4-1)**(1/4)/(x**8-x**4-1),x)
[Out] Timed out
```

3.2057 $\int \frac{-1+2x^4+2x^8}{\sqrt[4]{1+x^4}\left(-1+x^4+x^8\right)}\,dx$

Optimal. Leaf size=217

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4+1}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right.$$

Rubi [A] time = 0.30, antiderivative size = 233, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {6728, 240, 212, 206, 203, 1428, 377}

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt{5}}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4+1}}\right)}{2\sqrt{5}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + 2*x^4 + 2*x^8)/((1 + x^4)^(1/4)*(-1 + x^4 + x^8)),x]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)] - (((3 - Sqrt[5])/2)^(1/4)*ArcTan[((2/(3 + Sqrt[5
]))^(1/4)*x)/(1 + x^4)^(1/4)])/(2*Sqrt[5]) - (((3 + Sqrt[5])/2)^(1/4)*ArcTa
n[(((3 + Sqrt[5])/2)^(1/4)*x)/(1 + x^4)^(1/4)])/(2*Sqrt[5]) + ArcTanh[x/(1
+ x^4)^(1/4)] - (((3 - Sqrt[5])/2)^(1/4)*ArcTanh[((2/(3 + Sqrt[5]))^(1/4)*x
)/(1 + x^4)^(1/4)])/(2*Sqrt[5]) - (((3 + Sqrt[5])/2)^(1/4)*ArcTanh[(((3 + S
qrt[5])/2)^(1/4)*x)/(1 + x^4)^(1/4)])/(2*Sqrt[5])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1428

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^
n)^q/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[(d + e*x^n)^q/(b + r + 2
*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+2x^4+2x^8}{\sqrt[4]{1+x^4}\left(-1+x^4+x^8\right)}\,dx &= \int \left(\frac{2}{\sqrt[4]{1+x^4}}+\frac{1}{\sqrt[4]{1+x^4}\left(-1+x^4+x^8\right)}\right)dx\\
&= 2\int \frac{1}{\sqrt[4]{1+x^4}}\,dx+\int \frac{1}{\sqrt[4]{1+x^4}\left(-1+x^4+x^8\right)}\,dx\\
&= 2\,\mathrm{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)+\frac{2\int \frac{1}{\sqrt[4]{1+x^4}\left(1-\sqrt{5}+2x^4\right)}\,dx}{\sqrt{5}}-\frac{2\int \frac{1}{\sqrt[4]{1+x^4}\left(1+\sqrt{5}\right.}}{\sqrt{5}}\\
&= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{5}-\left(-1-\sqrt{5}\right)x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{5}}-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{5}-\left(-1+\sqrt{5}\right)x^4}\,dx,x,\right.}{\sqrt{5}}\\
&= \tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{10}}-\\
&= \tan^{-1}\left(\frac{x}{\sqrt[4]{1+x^4}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{1+x^4}}\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt{5}}+\tan
\end{aligned}$$

Mathematica [A] time = 0.49, size = 239, normalized size = 1.10

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4+1}}\right)}{5+\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4+1}}\right)}{\sqrt{5}-5}+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x}{\sqrt[4]{x^4+1}}\right)}{5+\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x}{\sqrt[4]{x^4+1}}\right)}{\sqrt{5}-5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x^4 + 2*x^8)/((1 + x^4)^(1/4)*(-1 + x^4 + x^8)),x]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)] - (((3 + Sqrt[5])/2)^(1/4)*ArcTan[((2/(3 + Sqrt[5
]))^(1/4)*x)/(1 + x^4)^(1/4)])/(5 + Sqrt[5]) + (((3 - Sqrt[5])/2)^(1/4)*Arc
Tan[(((3 + Sqrt[5])/2)^(1/4)*x)/(1 + x^4)^(1/4)])/(-5 + Sqrt[5]) + ArcTanh[
x/(1 + x^4)^(1/4)] - (((3 + Sqrt[5])/2)^(1/4)*ArcTanh[((2/(3 + Sqrt[5]))^(1
/4)*x)/(1 + x^4)^(1/4)])/(5 + Sqrt[5]) + (((3 - Sqrt[5])/2)^(1/4)*ArcTanh[(
((3 + Sqrt[5])/2)^(1/4)*x)/(1 + x^4)^(1/4)])/(-5 + Sqrt[5])
```

IntegrateAlgebraic [A] time = 0.71, size = 217, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4+1}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4+1}}\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x^4 + 2*x^8)/((1 + x^4)^(1/4)*(-1 + x^4 + x^8)),
x]
```

```
[Out] ArcTan[x/(1 + x^4)^(1/4)] - (Sqrt[(-1 + Sqrt[5])/10]*ArcTan[(Sqrt[-1/2 + Sq
rt[5]/2]*x)/(1 + x^4)^(1/4)])/2 - (Sqrt[(1 + Sqrt[5])/10]*ArcTan[(Sqrt[1/2
+ Sqrt[5]/2]*x)/(1 + x^4)^(1/4)])/2 + ArcTanh[x/(1 + x^4)^(1/4)] - (Sqrt[(-
1 + Sqrt[5])/10]*ArcTanh[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(1 + x^4)^(1/4)])/2 - (
Sqrt[(1 + Sqrt[5])/10]*ArcTanh[(Sqrt[1/2 + Sqrt[5]/2]*x)/(1 + x^4)^(1/4)])/
2
```

fricas [B] time = 0.44, size = 406, normalized size = 1.87

$$\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+1}\arctan\left(\frac{\sqrt{10}\sqrt{2}\left(\sqrt{5}x-5x\right)\sqrt{\sqrt{5}+1}\sqrt{\frac{\sqrt{5}x^2+x^2+2\sqrt{x^4+1}}{x^2}}-2\sqrt{10}\left(x^4+1\right)^{\frac{1}{4}}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)}{40x}\right)-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}-1}\arctan\left(\frac{\sqrt{10}\sqrt{2}\left(\sqrt{5}x+5x\right)\sqrt{\sqrt{5}-1}\sqrt{\frac{\sqrt{5}x^2-x^2+2\sqrt{x^4+1}}{x^2}}-2\sqrt{10}\left(x^4+1\right)^{\frac{1}{4}}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}}{40x}\right)-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(\frac{\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}+1}+10\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(-\frac{\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}+1}-10\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(\frac{\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}-1}+10\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(-\frac{\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}-1}-10\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-\arctan\left(\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\log\left(\frac{x+\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{2}\log\left(-\frac{x-\left(x^4+1\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+2*x^4-1)/(x^4+1)^(1/4)/(x^8+x^4-1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(10)*sqrt(sqrt(5) + 1)*arctan(1/40*(sqrt(10)*sqrt(2)*(sqrt(5)*x -
5*x)*sqrt(sqrt(5) + 1)*sqrt((sqrt(5)*x^2 + x^2 + 2*sqrt(x^4 + 1))/x^2) - 2*
sqrt(10)*(x^4 + 1)^(1/4)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5))/x) - 1/10*sqrt(10
)*sqrt(sqrt(5) - 1)*arctan(1/40*(sqrt(10)*sqrt(2)*(sqrt(5)*x + 5*x)*sqrt(sq
rt(5) - 1)*sqrt((sqrt(5)*x^2 - x^2 + 2*sqrt(x^4 + 1))/x^2) - 2*sqrt(10)*(x^
4 + 1)^(1/4)*(sqrt(5) + 5)*sqrt(sqrt(5) - 1))/x) - 1/40*sqrt(10)*sqrt(sqrt(
5) + 1)*log((sqrt(10)*sqrt(5)*x*sqrt(sqrt(5) + 1) + 10*(x^4 + 1)^(1/4))/x)
+ 1/40*sqrt(10)*sqrt(sqrt(5) + 1)*log(-(sqrt(10)*sqrt(5)*x*sqrt(sqrt(5) + 1
) - 10*(x^4 + 1)^(1/4))/x) - 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log((sqrt(10)*
sqrt(5)*x*sqrt(sqrt(5) - 1) + 10*(x^4 + 1)^(1/4))/x) + 1/40*sqrt(10)*sqrt(s
qrt(5) - 1)*log(-(sqrt(10)*sqrt(5)*x*sqrt(sqrt(5) - 1) - 10*(x^4 + 1)^(1/4)
)/x) - arctan((x^4 + 1)^(1/4)/x) + 1/2*log((x + (x^4 + 1)^(1/4))/x) - 1/2*l
og(-(x - (x^4 + 1)^(1/4))/x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+2*x^4-1)/(x^4+1)^(1/4)/(x^8+x^4-1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to convert to real 1/4 Error: Bad Arg
ument ValueUnable to convert to real 1/4 Error: Bad Argument Value
```

maple [F] time = 2.81, size = 0, normalized size = 0.00

$$\int \frac{2x^8+2x^4-1}{\left(x^4+1\right)^{\frac{1}{4}}\left(x^8+x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8+2*x^4-1)/(x^4+1)^(1/4)/(x^8+x^4-1),x)
```

```
[Out] int((2*x^8+2*x^4-1)/(x^4+1)^(1/4)/(x^8+x^4-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^8+2\,x^4-1}{\left(x^8+x^4-1\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+2*x^4-1)/(x^4+1)^(1/4)/(x^8+x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^8 + 2*x^4 - 1)/((x^8 + x^4 - 1)*(x^4 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{2\,x^8+2\,x^4-1}{{\left(x^4+1\right)}^{1/4}\,\left(x^8+x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4 + 2*x^8 - 1)/((x^4 + 1)^(1/4)*(x^4 + x^8 - 1)),x)
```

```
[Out] int((2*x^4 + 2*x^8 - 1)/((x^4 + 1)^(1/4)*(x^4 + x^8 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8+2*x**4-1)/(x**4+1)**(1/4)/(x**8+x**4-1),x)
```

```
[Out] Timed out
```

3.2058 $\int \frac{-ab+(2a-b)x}{\sqrt[3]{x(-a+x)(-b+x)}\left(-a^2+(2a-bd)x+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=218

$$\frac{\log\left(a^2+d^{2/3}\left(x^2(-a-b)+abx+x^3\right)^{2/3}+\sqrt[3]{x^2(-a-b)+abx+x^3}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)-2ax+x^2\right)}{2d^{2/3}}-\frac{\log\left(\sqrt[3]{d}\,\sqrt[3]{x^2(\right.}{}$$

Rubi [F] time = 3.70, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-ab+(2a-b)x}{\sqrt[3]{x(-a+x)(-b+x)}\left(-a^2+(2a-bd)x+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a*b) + (2*a - b)*x)/((x*(-a + x)*(-b + x))^(1/3)*(-a^2 + (2*a - b*d)
*x + (-1 + d)*x^2)),x]
```

```
[Out] ((2*a - b - Sqrt[4*a^2 - 4*a*b + b^2*d]/Sqrt[d])*x^(1/3)*(-a + x)^(1/3)*(-b
 + x)^(1/3)*Defer[Int][1/(x^(1/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*(2*a - b*d
- Sqrt[d]*Sqrt[4*a^2 - 4*a*b + b^2*d] + 2*(-1 + d)*x)), x])/((a - x)*(b - x
)*x)^(1/3) + ((2*a - b + Sqrt[4*a^2 - 4*a*b + b^2*d]/Sqrt[d])*x^(1/3)*(-a +
 x)^(1/3)*(-b + x)^(1/3)*Defer[Int][1/(x^(1/3)*(-a + x)^(1/3)*(-b + x)^(1/3
)*(2*a - b*d + Sqrt[d]*Sqrt[4*a^2 - 4*a*b + b^2*d] + 2*(-1 + d)*x)), x])/((
a - x)*(b - x)*x)^(1/3)
```

Rubi steps

$$\int \frac{-ab+(2a-b)x}{\sqrt[3]{x(-a+x)(-b+x)}\left(-a^2+(2a-bd)x+(-1+d)x^2\right)}\,dx=\frac{\left(\sqrt[3]{x}\,\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\int\frac{-ab+(2a-b}{\sqrt[3]{x}\,\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\left(-a^2+(2\right.}}{\sqrt[3]{x(-a+x)(-b+x)}}$$

$$=\frac{\left(\sqrt[3]{x}\,\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\int\left(\frac{2a-b-}{\sqrt[3]{x}\,\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\left(2a-b\right.}\right.}{}$$

$$=\frac{\left(\left(2a-b-\frac{\sqrt{4a^2-4ab+b^2d}}{\sqrt{d}}\right)\sqrt[3]{x}\,\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right.}{\sqrt[3]{x(-a}}$$

Mathematica [F] time = 6.19, size = 0, normalized size = 0.00

$$\int \frac{-ab+(2a-b)x}{\sqrt[3]{x(-a+x)(-b+x)}\left(-a^2+(2a-bd)x+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-(a*b) + (2*a - b)*x)/((x*(-a + x)*(-b + x))^(1/3)*(-a^2 + (2*a
- b*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] Integrate[(-(a*b) + (2*a - b)*x)/((x*(-a + x)*(-b + x))^(1/3)*(-a^2 + (2*a
- b*d)*x + (-1 + d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 3.02, size = 218, normalized size = 1.00

$$\frac{\log\left(a^2+d^{2/3}\left(x^2(-a-b)+abx+x^3\right)^{2/3}+\sqrt[3]{x^2(-a-b)+abx+x^3}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)-2ax+x^2\right)}{2d^{2/3}}-\frac{\log\left(\sqrt[3]{d}\,\sqrt[3]{x^2(-a-b)+abx+x^3}+a-x\right)}{d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{x^2(-a-b)+abx+x^3}}{\sqrt[3]{d}\,\sqrt[3]{x^2(-a-b)+abx+x^3}-2a+2x}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*b) + (2*a - b)*x)/((x*(-a + x)*(-b + x))^(1/3)*(-a^
2 + (2*a - b*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(a*b*x + (-a - b)*x^2 + x^3)^(1/3))/(-2*
a + 2*x + d^(1/3)*(a*b*x + (-a - b)*x^2 + x^3)^(1/3))])/d^(2/3)) - Log[a -
x + d^(1/3)*(a*b*x + (-a - b)*x^2 + x^3)^(1/3)]/d^(2/3) + Log[a^2 - 2*a*x +
 x^2 + (-(a*d^(1/3)) + d^(1/3)*x)*(a*b*x + (-a - b)*x^2 + x^3)^(1/3) + d^(2
/3)*(a*b*x + (-a - b)*x^2 + x^3)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+(2*a-b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a^2+(-b*d+2*a)*x+(-1+d
)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ab-(2\,a-b)x}{((a-x)(b-x)x)^{\frac{1}{3}}\left((d-1)x^2-a^2-(bd-2\,a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+(2*a-b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a^2+(-b*d+2*a)*x+(-1+d
)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-(a*b - (2*a - b)*x)/(((a - x)*(b - x)*x)^(1/3)*((d - 1)*x^2 - a^
2 - (b*d - 2*a)*x)), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{-ab+(2a-b)\,x}{\left(x\,(-a+x)\,(-b+x)\right)^{\frac{1}{3}}\left(-a^2+(-bd+2a)\,x+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*b+(2*a-b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a^2+(-b*d+2*a)*x+(-1+d)*x^2)
,x)
```

```
[Out] int((-a*b+(2*a-b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a^2+(-b*d+2*a)*x+(-1+d)*x^2)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ab-(2\,a-b)x}{((a-x)(b-x)x)^{\frac{1}{3}}\left((d-1)x^2-a^2-(bd-2\,a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+(2*a-b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a^2+(-b*d+2*a)*x+(-1+d
)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((a*b - (2*a - b)*x)/(((a - x)*(b - x)*x)^(1/3)*((d - 1)*x^2 - a^
2 - (b*d - 2*a)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{a\,b-x\,\left(2\,a-b\right)}{{\left(x\,\left(a-x\right)\,\left(b-x\right)\right)}^{1/3}\,\left(x\,\left(2\,a-b\,d\right)-a^2+x^2\,\left(d-1\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*b - x*(2*a - b))/((x*(a - x)*(b - x))^(1/3)*(x*(2*a - b*d) - a^2 +
x^2*(d - 1))),x)
```

```
[Out] -int((a*b - x*(2*a - b))/((x*(a - x)*(b - x))^(1/3)*(x*(2*a - b*d) - a^2 +
x^2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+(2*a-b)*x)/(x*(-a+x)*(-b+x))**(1/3)/(-a**2+(-b*d+2*a)*x+(-1
+d)*x**2),x)
```

```
[Out] Timed out
```

3.2059 $\int \frac{x}{(1+x)\sqrt[3]{-x^2+x^3}}\,dx$

Optimal. Leaf size=219

$$-\log\left(\sqrt[3]{x^3-x^2}-x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^3-x^2}-2x\right)}{\sqrt[3]{2}}+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right)-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^3-x^2}\right.}{2\sqrt[3]{}}$$

Rubi [A] time = 0.10, antiderivative size = 295, normalized size of antiderivative = 1.35, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2042, 105, 59, 91}

$$-\frac{3\sqrt[3]{x-1}\,x^{2/3}\log\left(\frac{\sqrt[3]{x-1}}{\sqrt[3]{x}}-1\right)}{2\sqrt[3]{x^3-x^2}}+\frac{3\sqrt[3]{x-1}\,x^{2/3}\log\left(\frac{\sqrt[3]{x-1}}{\sqrt[3]{2}}-\sqrt[3]{x}\right)}{2\sqrt[3]{2}\sqrt[3]{x^3-x^2}}-\frac{\sqrt[3]{x-1}\,x^{2/3}\log(x)}{2\sqrt[3]{x^3-x^2}}-\frac{\sqrt[3]{x-1}\,x^{2/3}\log(x+1)}{2\sqrt[3]{2}\sqrt[3]{x^3-x^2}}-\frac{\sqrt{3}\sqrt[3]{x-1}\,x^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{x-1}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{x^3-x^2}}+\frac{\sqrt{3}\sqrt[3]{x-1}\,x^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{x-1}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{x^3-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*(-1 + x)^(1/3)*x^(2/3)*ArcTan[1/Sqrt[3] + (2*(-1 + x)^(1/3))/(Sq
rt[3]*x^(1/3))])/(-x^2 + x^3)^(1/3)) + (Sqrt[3]*(-1 + x)^(1/3)*x^(2/3)*ArcT
an[1/Sqrt[3] + (2^(2/3)*(-1 + x)^(1/3))/(Sqrt[3]*x^(1/3))])/(2^(1/3)*(-x^2
+ x^3)^(1/3)) - (3*(-1 + x)^(1/3)*x^(2/3)*Log[-1 + (-1 + x)^(1/3)/x^(1/3)])
/(2*(-x^2 + x^3)^(1/3)) + (3*(-1 + x)^(1/3)*x^(2/3)*Log[(-1 + x)^(1/3)/2^(1
/3) - x^(1/3)])/(2*2^(1/3)*(-x^2 + x^3)^(1/3)) - ((-1 + x)^(1/3)*x^(2/3)*Lo
g[x])/(2*(-x^2 + x^3)^(1/3)) - ((-1 + x)^(1/3)*x^(2/3)*Log[1 + x])/(2*2^(1/
3)*(-x^2 + x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
```

```
&& NeQ[b*c - a*d, 0] && !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+x)\sqrt[3]{-x^2+x^3}}\,dx &= \frac{\left(\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{\sqrt[3]{x}}{\sqrt[3]{-1+x}\,(1+x)}\,dx}{\sqrt[3]{-x^2+x^3}}\\ &= \frac{\left(\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{1}{\sqrt[3]{-1+x}\,x^{2/3}}\,dx}{\sqrt[3]{-x^2+x^3}} - \frac{\left(\sqrt[3]{-1+x}\,x^{2/3}\right)\int \frac{1}{\sqrt[3]{-1+x}\,x^{2/3}(1+x)}\,dx}{\sqrt[3]{-x^2+x^3}}\\ &= -\frac{\sqrt{3}\,\sqrt[3]{-1+x}\,x^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-1+x}}{\sqrt{3}\sqrt[3]{x}}\right)}{\sqrt[3]{-x^2+x^3}} + \frac{\sqrt{3}\,\sqrt[3]{-1+x}\,x^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}\sqrt[3]{-1+x}}{\sqrt{3}\sqrt[3]{x}}\right.}{\sqrt[3]{2}\,\sqrt[3]{-x^2+x^3}}\end{aligned}$$

Mathematica [C] time = 0.04, size = 61, normalized size = 0.28

$$\frac{3\left((x-1)x^2\right)^{2/3}\left(2x^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};1-x\right)-{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x-1}{2x}\right)\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] (3*((-1 + x)*x^2)^(2/3)*(2*x^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, 1 - x]
 - Hypergeometric2F1[2/3, 1, 5/3, (-1 + x)/(2*x)]))/(4*x^2)
```

IntegrateAlgebraic [A] time = 0.49, size = 219, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^3-x^2}-x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^3-x^2}-2x\right)}{\sqrt[3]{2}}+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right)-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^3-x^2}\,x+\sqrt[3]{2}\left(x^3-x^2\right)^{2/3}\right)}{2\sqrt[3]{2}}+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x^2}+x}\right)-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3-x^2}+x}\right)}{\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((1 + x)*(-x^2 + x^3)^(1/3)),x]
```

```
[Out] Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-x^2 + x^3)^(1/3))] - (Sqrt[3]*ArcTan[(S
qrt[3]*x)/(x + 2^(2/3)*(-x^2 + x^3)^(1/3))])/2^(1/3) - Log[-x + (-x^2 + x^3
)^(1/3)] + Log[-2*x + 2^(2/3)*(-x^2 + x^3)^(1/3)]/2^(1/3) + Log[x^2 + x*(-x
^2 + x^3)^(1/3) + (-x^2 + x^3)^(2/3)]/2 - Log[2*x^2 + 2^(2/3)*x*(-x^2 + x^3
)^(1/3) + 2^(1/3)*(-x^2 + x^3)^(2/3)]/(2*2^(1/3))
```

fricas [A] time = 2.72, size = 241, normalized size = 1.10

$$\frac{1}{4}\left(2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}}-2^{\frac{2}{3}}\right)\log\left(\frac{3\left(2^{\frac{2}{3}}\sqrt{\frac{3}{2}}x\sqrt{-2^{\frac{1}{3}}}+2^{\frac{1}{3}}x+2\left(x^3-x^2\right)^{\frac{1}{3}}\right)}{2x}\right)-\frac{1}{4}\left(2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}}+2^{\frac{2}{3}}\right)\log\left(-\frac{3\left(2^{\frac{2}{3}}\sqrt{\frac{3}{2}}x\sqrt{-2^{\frac{1}{3}}}-2^{\frac{1}{3}}x-2\left(x^3-x^2\right)^{\frac{1}{3}}\right)}{2x}\right)+\frac{1}{2}\cdot 2^{\frac{2}{3}}\log\left(-\frac{3\left(2^{\frac{1}{3}}x-\left(x^3-x^2\right)^{\frac{1}{3}}\right)}{x}\right)-\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-x^2\right)^{\frac{1}{3}}}{3x}\right)-\log\left(-\frac{3\left(x-\left(x^3-x^2\right)^{\frac{1}{3}}\right)}{x}\right)+\frac{1}{2}\log\left(\frac{x^2+\left(x^3-x^2\right)^{\frac{1}{3}}x+\left(x^3-x^2\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x^3-x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3/2)*sqrt(-2^(1/3)) - 2^(2/3))*log(3/2*(2^(2/3)*sqrt(3/2)*x*sqr
t(-2^(1/3)) + 2^(1/3)*x + 2*(x^3 - x^2)^(1/3))/x) - 1/4*(2*sqrt(3/2)*sqrt(-
2^(1/3)) + 2^(2/3))*log(-3/2*(2^(2/3)*sqrt(3/2)*x*sqrt(-2^(1/3)) - 2^(1/3)*
x - 2*(x^3 - x^2)^(1/3))/x) + 1/2*2^(2/3)*log(-3*(2^(1/3)*x - (x^3 - x^2)^(
1/3))/x) - sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 - x^2)^(1/3))/x)
- log(-3*(x - (x^3 - x^2)^(1/3))/x) + 1/2*log((x^2 + (x^3 - x^2)^(1/3)*x +
(x^3 - x^2)^(2/3))/x^2)
```

giac [A] time = 0.32, size = 149, normalized size = 0.68

$$\frac{1}{2}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{4}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}\right)+\frac{1}{2}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}\right|\right)-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)-\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x^3-x^2)^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-1/x + 1)^(1/3
))) - 1/4*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-1/x + 1)^(1/3) + (-1/x + 1)^(2/3)
) + 1/2*2^(2/3)*log(abs(-2^(1/3) + (-1/x + 1)^(1/3))) - sqrt(3)*arctan(1/3*
sqrt(3)*(2*(-1/x + 1)^(1/3) + 1)) + 1/2*log((-1/x + 1)^(2/3) + (-1/x + 1)^(
1/3) + 1) - log(abs((-1/x + 1)^(1/3) - 1))
```

maple [C] time = 2.86, size = 1508, normalized size = 6.89

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/(x^3-x^2)^(1/3),x)
```

```
[Out] 1/2*RootOf(_Z^2-2*_Z+4)*ln((5*RootOf(_Z^2-2*_Z+4)^2*x^2-10*RootOf(_Z^2-2*_Z
+4)^2*x+48*RootOf(_Z^2-2*_Z+4)*(x^3-x^2)^(2/3)+48*RootOf(_Z^2-2*_Z+4)*(x^3-
x^2)^(1/3)*x+38*RootOf(_Z^2-2*_Z+4)*x^2-6*RootOf(_Z^2-2*_Z+4)*x-36*(x^3-x^2
)^(2/3)-36*x*(x^3-x^2)^(1/3)-16*x^2+4*x)/x)-1/2*ln((5*RootOf(_Z^2-2*_Z+4)^2
*x^2-10*RootOf(_Z^2-2*_Z+4)^2*x-48*RootOf(_Z^2-2*_Z+4)*(x^3-x^2)^(2/3)-48*R
ootOf(_Z^2-2*_Z+4)*(x^3-x^2)^(1/3)*x-58*RootOf(_Z^2-2*_Z+4)*x^2+46*RootOf(_
Z^2-2*_Z+4)*x+60*(x^3-x^2)^(2/3)+60*x*(x^3-x^2)^(1/3)+80*x^2-48*x)/x)*RootO
f(_Z^2-2*_Z+4)+ln((5*RootOf(_Z^2-2*_Z+4)^2*x^2-10*RootOf(_Z^2-2*_Z+4)^2*x-4
8*RootOf(_Z^2-2*_Z+4)*(x^3-x^2)^(2/3)-48*RootOf(_Z^2-2*_Z+4)*(x^3-x^2)^(1/3
)*x-58*RootOf(_Z^2-2*_Z+4)*x^2+46*RootOf(_Z^2-2*_Z+4)*x+60*(x^3-x^2)^(2/3)+
60*x*(x^3-x^2)^(1/3)+80*x^2-48*x)/x)-1/2*ln((3*RootOf(RootOf(_Z^3-4)^2+_Z*R
ootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x^2+2*RootOf(RootOf(_Z^3-4)^2+_Z*Roo
tOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x^2-6*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(
_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x-4*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3
-4)+_Z^2)*RootOf(_Z^3-4)^3*x+24*(x^3-x^2)^(2/3)*RootOf(RootOf(_Z^3-4)^2+_Z*
RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2+48*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(
_Z^3-4)+_Z^2)*RootOf(_Z^3-4)*(x^3-x^2)^(1/3)*x+18*RootOf(_Z^3-4)^2*(x^3-x^2
)^(1/3)*x+78*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x^2+52*RootOf(
_Z^3-4)*x^2-30*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*x-20*RootOf(
_Z^3-4)*x+36*(x^3-x^2)^(2/3))/x/(1+x))*RootOf(_Z^3-4)-1/2*ln((3*RootOf(Root
Of(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x^2+2*RootOf(RootOf
(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x^2-6*RootOf(RootOf(_Z^
3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x-4*RootOf(RootOf(_Z^3-4)
^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x+24*(x^3-x^2)^(2/3)*RootOf(Roo
tOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2+48*RootOf(RootOf(_Z^
3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)*(x^3-x^2)^(1/3)*x+18*RootOf(_
Z^3-4)^2*(x^3-x^2)^(1/3)*x+78*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^
2)*x^2+52*RootOf(_Z^3-4)*x^2-30*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_
Z^2)*x-20*RootOf(_Z^3-4)*x+36*(x^3-x^2)^(2/3))/x/(1+x))*RootOf(RootOf(_Z^3-
4)^2+_Z*RootOf(_Z^3-4)+_Z^2)+1/2*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+
_Z^2)*ln((3*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4
)^2*x^2+RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x^
2-6*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)^2*RootOf(_Z^3-4)^2*x-2*
RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^3*x-24*(x^3-
x^2)^(2/3)*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)^2
-48*RootOf(RootOf(_Z^3-4)^2+_Z*RootOf(_Z^3-4)+_Z^2)*RootOf(_Z^3-4)*(x^3-x^2
)^(1/3)*x-30*RootOf(_Z^3-4)^2*(x^3-x^2)^(1/3)*x-66*RootOf(RootOf(_Z^3-4)^2+
_Z*RootOf(_Z^3-4)+_Z^2)*x^2-22*RootOf(_Z^3-4)*x^2+6*RootOf(RootOf(_Z^3-4)^2
+_Z*RootOf(_Z^3-4)+_Z^2)*x+2*RootOf(_Z^3-4)*x-60*(x^3-x^2)^(2/3))/x/(1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3-x^2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x^3-x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 - x^2)^(1/3)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(x^3-x^2\right)^{1/3}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 - x^2)^(1/3)*(x + 1)),x)
```

```
[Out] int(x/((x^3 - x^2)^(1/3)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{x^2\,(x-1)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x**3-x**2)**(1/3),x)
```

```
[Out] Integral(x/((x**2*(x - 1))**(1/3)*(x + 1)), x)
```

3.2060 $\int \frac{\sqrt[3]{-x^2+x^3}}{-1+x^2}\,dx$

Optimal. Leaf size=219

$$-\log\left(\sqrt[3]{x^3-x^2}-x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^3-x^2}-2x\right)}{2^{2/3}}+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right)-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^3-x^2}\ldots\right.}{2\,2\ldots}$$

Rubi [A] time = 0.08, antiderivative size = 298, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2056, 848, 105, 59, 91}

$$\frac{3\sqrt[3]{x^3-x^2}\log\left(\sqrt[3]{2}\sqrt[3]{x}-\sqrt[3]{x-1}\right)}{2\,2^{2/3}\sqrt[3]{x-1}\,x^{2/3}}-\frac{3\sqrt[3]{x^3-x^2}\log\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x-1}}-1\right)}{2\sqrt[3]{x-1}\,x^{2/3}}-\frac{\sqrt[3]{x^3-x^2}\log(x-1)}{2\sqrt[3]{x-1}\,x^{2/3}}-\frac{\sqrt[3]{x^3-x^2}\log(x+1)}{2\,2^{2/3}\sqrt[3]{x-1}\,x^{2/3}}-\frac{\sqrt{3}\sqrt[3]{x^3-x^2}\tan^{-1}\left(\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{x-1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{x-1}\,x^{2/3}}+\frac{\sqrt{3}\sqrt[3]{x^3-x^2}\tan^{-1}\left(\frac{2\sqrt[3]{2}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{x-1}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt[3]{x-1}\,x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + x^3)^(1/3)/(-1 + x^2),x]
```

```
[Out] -((Sqrt[3]*(-x^2 + x^3)^(1/3)*ArcTan[1/Sqrt[3] + (2*x^(1/3))/(Sqrt[3]*(-1 +
 x)^(1/3))])/((-1 + x)^(1/3)*x^(2/3))) + (Sqrt[3]*(-x^2 + x^3)^(1/3)*ArcTan
[1/Sqrt[3] + (2*2^(1/3)*x^(1/3))/(Sqrt[3]*(-1 + x)^(1/3))])/(2^(2/3)*(-1 +
x)^(1/3)*x^(2/3)) + (3*(-x^2 + x^3)^(1/3)*Log[-(-1 + x)^(1/3) + 2^(1/3)*x^(
1/3)])/(2*2^(2/3)*(-1 + x)^(1/3)*x^(2/3)) - (3*(-x^2 + x^3)^(1/3)*Log[-1 +
x^(1/3)/(-1 + x)^(1/3)])/(2*(-1 + x)^(1/3)*x^(2/3)) - ((-x^2 + x^3)^(1/3)*L
og[-1 + x])/(2*(-1 + x)^(1/3)*x^(2/3)) - ((-x^2 + x^3)^(1/3)*Log[1 + x])/(2
*2^(2/3)*(-1 + x)^(1/3)*x^(2/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{-x^2+x^3}}{-1+x^2}\,dx &= \frac{\sqrt[3]{-x^2+x^3}\int \frac{\sqrt[3]{-1+x}\,x^{2/3}}{-1+x^2}\,dx}{\sqrt[3]{-1+x}\,x^{2/3}}\\
&= \frac{\sqrt[3]{-x^2+x^3}\int \frac{x^{2/3}}{(-1+x)^{2/3}(1+x)}\,dx}{\sqrt[3]{-1+x}\,x^{2/3}}\\
&= \frac{\sqrt[3]{-x^2+x^3}\int \frac{1}{(-1+x)^{2/3}\sqrt[3]{x}}\,dx}{\sqrt[3]{-1+x}\,x^{2/3}} - \frac{\sqrt[3]{-x^2+x^3}\int \frac{1}{(-1+x)^{2/3}\sqrt[3]{x}\,(1+x)}\,dx}{\sqrt[3]{-1+x}\,x^{2/3}}\\
&= -\frac{\sqrt{3}\sqrt[3]{-x^2+x^3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{-1+x}}\right)}{\sqrt[3]{-1+x}\,x^{2/3}} + \frac{\sqrt{3}\sqrt[3]{-x^2+x^3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{2}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{-1+x}}\right)}{2^{2/3}\sqrt[3]{-1+x}\,x^{2/3}} + 3\sqrt[3]{\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.28

$$
\frac{3\sqrt[3]{(x-1)x^2}\left(2\sqrt[3]{x}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};1-x\right)-{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{x-1}{2x}\right)\right)}{2x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + x^3)^(1/3)/(-1 + x^2),x]
```

```
[Out] (3*((-1 + x)*x^2)^(1/3)*(2*x^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, 1 - x]
- Hypergeometric2F1[1/3, 1, 4/3, (-1 + x)/(2*x)]))/(2*x)
```

IntegrateAlgebraic [A] time = 0.47, size = 219, normalized size = 1.00

$$
-\log\left(\sqrt[3]{x^3-x^2}-x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^3-x^2}-2x\right)}{2^{2/3}}+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right)-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^3-x^2}\,x+\sqrt[3]{2}\left(x^3-x^2\right)^{2/3}\right)}{2\,2^{2/3}}-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x^2}+x}\right)+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3-x^2}+x}\right)}{2^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^2 + x^3)^(1/3)/(-1 + x^2),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-x^2 + x^3)^(1/3))]) + (Sqrt[3]*ArcTan
[(Sqrt[3]*x)/(x + 2^(2/3)*(-x^2 + x^3)^(1/3))])/2^(2/3) - Log[-x + (-x^2 +
x^3)^(1/3)] + Log[-2*x + 2^(2/3)*(-x^2 + x^3)^(1/3)]/2^(2/3) + Log[x^2 + x*
(-x^2 + x^3)^(1/3) + (-x^2 + x^3)^(2/3)]/2 - Log[2*x^2 + 2^(2/3)*x*(-x^2 +
x^3)^(1/3) + 2^(1/3)*(-x^2 + x^3)^(2/3)]/(2*2^(2/3))
```

fricas [A] time = 0.72, size = 210, normalized size = 0.96

$$
-\frac{1}{2}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}x+4^{\frac{2}{3}}\left(x^3-x^2\right)^{\frac{1}{3}}\right)}{6x}\right)+\frac{1}{4}\cdot 4^{\frac{2}{3}}\log\left(-\frac{4^{\frac{2}{3}}x-2\left(x^3-x^2\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{8}\cdot 4^{\frac{2}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}x^2+4^{\frac{2}{3}}\left(x^3-x^2\right)^{\frac{1}{3}}x+2\left(x^3-x^2\right)^{\frac{2}{3}}}{x^2}\right)+\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-x^2\right)^{\frac{1}{3}}}{3x}\right)-\log\left(-\frac{x-\left(x^3-x^2\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{2}\log\left(\frac{x^2+\left(x^3-x^2\right)^{\frac{1}{3}}x+\left(x^3-x^2\right)^{\frac{2}{3}}}{x^2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)^(1/3)/(x^2-1),x, algorithm="fricas")
```

```
[Out] -1/2*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3)*x + 4^(2/3)*(x^3 -
 x^2)^(1/3))/x) + 1/4*4^(2/3)*log(-(4^(2/3)*x - 2*(x^3 - x^2)^(1/3))/x) - 1
/8*4^(2/3)*log((2*4^(1/3)*x^2 + 4^(2/3)*(x^3 - x^2)^(1/3)*x + 2*(x^3 - x^2)
^(2/3))/x^2) + sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 - x^2)^(1/3))
/x) - log(-(x - (x^3 - x^2)^(1/3))/x) + 1/2*log((x^2 + (x^3 - x^2)^(1/3)*x
+ (x^3 - x^2)^(2/3))/x^2)
```

giac [A] time = 0.21, size = 148, normalized size = 0.68

$$-\frac{1}{2}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}\right)\right)+\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}\right)+\frac{1}{2}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}\right|\right)+\frac{1}{2}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)-\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)^(1/3)/(x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-1/x + 1)^(1/
3))) + sqrt(3)*arctan(1/3*sqrt(3)*(2*(-1/x + 1)^(1/3) + 1)) - 1/4*2^(1/3)*l
og(2^(2/3) + 2^(1/3)*(-1/x + 1)^(1/3) + (-1/x + 1)^(2/3)) + 1/2*2^(1/3)*log
(abs(-2^(1/3) + (-1/x + 1)^(1/3))) + 1/2*log((-1/x + 1)^(2/3) + (-1/x + 1)^
(1/3) + 1) - log(abs((-1/x + 1)^(1/3) - 1))
```

maple [C] time = 3.06, size = 1215, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x^2)^(1/3)/(x^2-1),x)
```

```
[Out] RootOf(_Z^2-_Z+1)*ln(-(-5*RootOf(_Z^2-_Z+1)^2*x^2+24*RootOf(_Z^2-_Z+1)*(x^3
-x^2)^(2/3)+24*(x^3-x^2)^(1/3)*RootOf(_Z^2-_Z+1)*x+10*RootOf(_Z^2-_Z+1)^2*x
+29*RootOf(_Z^2-_Z+1)*x^2-15*(x^3-x^2)^(2/3)-15*x*(x^3-x^2)^(1/3)-23*RootOf
(_Z^2-_Z+1)*x-20*x^2+12*x)/x)-ln((5*RootOf(_Z^2-_Z+1)^2*x^2+24*RootOf(_Z^2-
_Z+1)*(x^3-x^2)^(2/3)+24*(x^3-x^2)^(1/3)*RootOf(_Z^2-_Z+1)*x-10*RootOf(_Z^2
-_Z+1)^2*x+19*RootOf(_Z^2-_Z+1)*x^2-9*(x^3-x^2)^(2/3)-9*x*(x^3-x^2)^(1/3)-3
*RootOf(_Z^2-_Z+1)*x-4*x^2+x)/x)*RootOf(_Z^2-_Z+1)+ln((5*RootOf(_Z^2-_Z+1)^
2*x^2+24*RootOf(_Z^2-_Z+1)*(x^3-x^2)^(2/3)+24*(x^3-x^2)^(1/3)*RootOf(_Z^2-_
Z+1)*x-10*RootOf(_Z^2-_Z+1)^2*x+19*RootOf(_Z^2-_Z+1)*x^2-9*(x^3-x^2)^(2/3)-
9*x*(x^3-x^2)^(1/3)-3*RootOf(_Z^2-_Z+1)*x-4*x^2+x)/x)+1/2*RootOf(_Z^3-2)*ln
(-(6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^4*x
^2+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^3
*x^2-12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^
4*x-8*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^
3*x-48*(x^3-x^2)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf
(_Z^3-2)+4*_Z^2)+39*RootOf(_Z^3-2)^2*x^2+26*RootOf(_Z^3-2)*RootOf(RootOf(_Z
^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2-15*RootOf(_Z^3-2)^2*x-10*RootOf(Roo
tOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-30*(x^3-x^2)^(1/
3)*RootOf(_Z^3-2)*x+36*(x^3-x^2)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(
_Z^3-2)+4*_Z^2)*x-18*(x^3-x^2)^(2/3))/x/(1+x))+RootOf(RootOf(_Z^3-2)^2+2*_Z
*RootOf(_Z^3-2)+4*_Z^2)*ln(-(6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+
4*_Z^2)*RootOf(_Z^3-2)^4*x^2+8*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+
4*_Z^2)^2*RootOf(_Z^3-2)^3*x^2-12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-
2)+4*_Z^2)*RootOf(_Z^3-2)^4*x-16*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2
)+4*_Z^2)^2*RootOf(_Z^3-2)^3*x+48*(x^3-x^2)^(2/3)*RootOf(_Z^3-2)^2*RootOf(R
ootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-33*RootOf(_Z^3-2)^2*x^2-44*Root
Of(_Z^3-2)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2+3*RootOf
(_Z^3-2)^2*x+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_
Z^3-2)*x+18*(x^3-x^2)^(1/3)*RootOf(_Z^3-2)*x-60*(x^3-x^2)^(1/3)*RootOf(Root
Of(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+30*(x^3-x^2)^(2/3))/x/(1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^3 - x^2)^{\frac{1}{3}}}{x^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)^(1/3)/(x^2-1),x, algorithm="maxima")
[Out] integrate((x^3 - x^2)^(1/3)/(x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x^3 - x^2)^{1/3}}{x^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - x^2)^(1/3)/(x^2 - 1),x)
[Out] int((x^3 - x^2)^(1/3)/(x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x^2 (x-1)}}{(x-1)(x+1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x**2)**(1/3)/(x**2-1),x)
[Out] Integral((x**2*(x - 1))**(1/3)/((x - 1)*(x + 1)), x)
```

3.2061 $\int x^2\sqrt{b+a^2x^4}\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$

Optimal. Leaf size=219

$$\frac{\sqrt{a}\,x\left(16a^4x^8+36a^2bx^4+9b^2\right)\sqrt{\sqrt{a^2x^4+b}+ax^2}+\sqrt{a}\,x\sqrt{a^2x^4+b}\sqrt{\sqrt{a^2x^4+b}+ax^2}\left(16a^3x^6+28abx^2\right)}{48a^{3/2}b+96a^{7/2}x^4+96a^{5/2}x^2\sqrt{a^2x^4+b}}-\frac{3b^{\cdot}}{}$$

Rubi [F] time = 0.67, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\sqrt{b+a^2x^4}\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
```

```
[Out] Defer[Int][x^2*Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]], x]
```

Rubi steps

$$\int x^2\sqrt{b+a^2x^4}\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx=\int x^2\sqrt{b+a^2x^4}\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Mathematica [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x^2\sqrt{b+a^2x^4}\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
```

```
[Out] Integrate[x^2*Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]], x]
```

IntegrateAlgebraic [A] time = 0.40, size = 219, normalized size = 1.00

$$\frac{\sqrt{a}\,x\left(16a^4x^8+36a^2bx^4+9b^2\right)\sqrt{\sqrt{a^2x^4+b}+ax^2}+\sqrt{a}\,x\sqrt{a^2x^4+b}\sqrt{\sqrt{a^2x^4+b}+ax^2}\left(16a^3x^6+28abx^2\right)}{48a^{3/2}b+96a^{7/2}x^4+96a^{5/2}x^2\sqrt{a^2x^4+b}}-\frac{3b^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}}{\sqrt{b}}\right)}{16\sqrt{2}\,a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
```

```
[Out] (Sqrt[a]*x*Sqrt[b + a^2*x^4]*(28*a*b*x^2 + 16*a^3*x^6)*Sqrt[a*x^2 + Sqrt[b
+ a^2*x^4]] + Sqrt[a]*x*(9*b^2 + 36*a^2*b*x^4 + 16*a^4*x^8)*Sqrt[a*x^2 + Sq
rt[b + a^2*x^4]])/(48*a^(3/2)*b + 96*a^(7/2)*x^4 + 96*a^(5/2)*x^2*Sqrt[b +
a^2*x^4]) - (3*b^(3/2)*ArcTan[(Sqrt[2]*Sqrt[a]*x*Sqrt[a*x^2 + Sqrt[b + a^2*
x^4]])/Sqrt[b]])/(16*Sqrt[2]*a^(3/2))
```

fricas [A] time = 5.58, size = 319, normalized size = 1.46

$$\left[\frac{9\sqrt{\frac{1}{2}}\,b\sqrt{-\frac{b}{a}}\log\left(4a^2bx^4-4\sqrt{a^2x^4+b}\,abx^2+b^2-4\left(2\sqrt{\frac{1}{2}}\sqrt{a^2x^4+b}\,a^2x^3\sqrt{-\frac{b}{a}}-\sqrt{\frac{1}{2}}\left(2a^3x^5+abx\right)\sqrt{-\frac{b}{a}}\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}\right)-2\left(2a^2x^5-10\sqrt{a^2x^4+b}\,ax^3-9bx\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{96a},\ \frac{9\sqrt{\frac{1}{2}}\,b\sqrt{\frac{b}{a}}\arctan\left(-\frac{\left(\sqrt{\frac{1}{2}}\,ax^2\sqrt{\frac{b}{a}}-\sqrt{\frac{1}{2}}\sqrt{a^2x^4+b}\sqrt{\frac{b}{a}}\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{bx}\right)-\left(2a^2x^5-10\sqrt{a^2x^4+b}\,ax^3-9bx\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{48a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*x^4+b)^(1/2)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorith
m="fricas")
```

```
[Out] [1/96*(9*sqrt(1/2)*b*sqrt(-b/a)*log(4*a^2*b*x^4 - 4*sqrt(a^2*x^4 + b)*a*b*x
^2 + b^2 - 4*(2*sqrt(1/2)*sqrt(a^2*x^4 + b)*a^2*x^3*sqrt(-b/a) - sqrt(1/2)*
(2*a^3*x^5 + a*b*x)*sqrt(-b/a))*sqrt(a*x^2 + sqrt(a^2*x^4 + b))) - 2*(2*a^2
*x^5 - 10*sqrt(a^2*x^4 + b)*a*x^3 - 9*b*x)*sqrt(a*x^2 + sqrt(a^2*x^4 + b)))
/a, 1/48*(9*sqrt(1/2)*b*sqrt(b/a)*arctan(-(sqrt(1/2)*a*x^2*sqrt(b/a) - sqrt
(1/2)*sqrt(a^2*x^4 + b)*sqrt(b/a))*sqrt(a*x^2 + sqrt(a^2*x^4 + b))/(b*x)) -
 (2*a^2*x^5 - 10*sqrt(a^2*x^4 + b)*a*x^3 - 9*b*x)*sqrt(a*x^2 + sqrt(a^2*x^4
 + b)))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*x^4+b)^(1/2)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorith
m="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, need to choose a branch for the root of a polyno
mial with parameters. This might be wrong.The choice was done assuming [a,b
,x]=[0,-68,-97]Precision problem choosing root in common_EXT, current preci
sion 14Warning, need to choose a branch for the root of a polynomial with p
arameters. This might be wrong.The choice was done assuming [a,b,x]=[-67,8,
-61]schur row 3 -1.00112e-08Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b,t_nostep]=[24,49,-35]Warning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming [a,b,t_nostep]=[20,8,5]Warning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming [a,b,t_nostep]=[3,-23,44]Warning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming [a,b,t_nostep]=[-92,-93,-41]schur row 3 -1.33253e-09Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.The choice was done assuming [a,b,t_nostep]=[93,-2,-73]Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b,t_nostep]=[40,96,-96]W
arning, need to choose a branch for the root of a polynomial with parameter
s. This might be wrong.The choice was done assuming [a,b,t_nostep]=[-24,-84
,-66]schur row 3 -9.75014e-10Warning, need to choose a branch for the root
of a polynomial with parameters. This might be wrong.The choice was done as
suming [a,b,t_nostep]=[66,44,-64]schur row 3 3.13659e-08Warning, need to ch
oose a branch for the root of a polynomial with parameters. This might be w
rong.The choice was done assuming [a,b,t_nostep]=[-95,-41,-48]Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [a,b,t_nostep]=[1,69,4]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [a,b,t_nostep]=[99,-63,-95]Warning
, need to choose a branch for the root of a polynomial with parameters. Thi
s might be wrong.The choice was done assuming [a,b,t_nostep]=[81,83,-9]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming [a,b,t_nostep]=[-89,27,4]W
arning, need to choose a branch for the root of a polynomial with parameter
s. This might be wrong.The choice was done assuming [a,b,t_nostep]=[56,75,-
77]Warning, need to choose a branch for the root of a polynomial with param
eters. This might be wrong.The choice was done assuming [a,b,t_nostep]=[37,
```

```
-64,-25]Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.The choice was done assuming [a,b,t_nostep]
=[-98,-33,59]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b,t_no
step]=[92,64,-69]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [a,b,
t_nostep]=[21,-17,-9]Warning, need to choose a branch for the root of a pol
ynomial with parameters. This might be wrong.The choice was done assuming [
a,b,t_nostep]=[-4,96,-65]Warning, need to choose a branch for the root of a
 polynomial with parameters. This might be wrong.The choice was done assumi
ng [a,b,t_nostep]=[-85,-1,-30]schur row 3 -5.59763e-10Warning, need to choo
se a branch for the root of a polynomial with parameters. This might be wro
ng.The choice was done assuming [a,b,t_nostep]=[-75,7,-7]Warning, need to c
hoose a branch for the root of a polynomial with parameters. This might be
wrong.The choice was done assuming [a,b,t_nostep]=[-4,-6,-56]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b,t_nostep]=[-5,-69,82]schur row
3 1.56409e-09Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b,t_no
step]=[-71,32,48]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [a,b,
t_nostep]=[73,53,-20]Warning, need to choose a branch for the root of a pol
ynomial with parameters. This might be wrong.The choice was done assuming [
a,b,t_nostep]=[16,51,70]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.The choice was done assumin
g [a,b,t_nostep]=[-53,-39,82]schur row 3 -6.64448e-11Warning, need to choos
e a branch for the root of a polynomial with parameters. This might be wron
g.The choice was done assuming [a,b,t_nostep]=[-15,91,-72]Warning, need to
choose a branch for the root of a polynomial with parameters. This might be
 wrong.The choice was done assuming [a,b,t_nostep]=[74,86,-86]schur row 1 6
.32223e-07Francis algorithm not precise enough for[1.0,0.0,-599084431612,-1
.19816667401e+12,8.97248832883e+22]Warning, need to choose a branch for the
 root of a polynomial with parameters. This might be wrong.The choice was d
one assuming [a,b,t_nostep]=[-28,72,-21]Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
was done assuming [a,b,t_nostep]=[64,-73,13]Warning, need to choose a branc
h for the root of a polynomial with parameters. This might be wrong.The cho
ice was done assuming [a,b,t_nostep]=[-74,31,29]Warning, need to choose a b
ranch for the root of a polynomial with parameters. This might be wrong.The
 choice was done assuming [a,b,t_nostep]=[95,-80,-59]schur row 3 7.64423e-0
9Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [a,b,t_nostep]=[-12,5
3,99]Warning, need to choose a branch for the root of a polynomial with par
ameters. This might be wrong.The choice was done assuming [a,b,t_nostep]=[6
9,-96,-89]Warning, need to choose a branch for the root of a polynomial wit
h parameters. This might be wrong.The choice was done assuming [a,b,t_noste
p]=[26,-78,-6]Unable to divide, perhaps due to rounding error%%%{%%%{%%{[-2
,%%%{4,[1]%%%}]:[1,0,%%%{1,[1]%%%}]%%},[0]%%%}/%%%{%%{[2,1]:[1,0,%%%{1,[1]%
%%}]%%},[0]%%%},[1]%%%}+%%%{%%%{1,[1]%%%},[0]%%%} / %%%{%%%{1/%%%{%%{[2,1]:
[1,0,%%%{1,[1]%%%}]%%},[0]%%%},[0]%%%},[0]%%%} Error: Bad Argument ValueUna
ble to divide, perhaps due to rounding error%%%{%%%{%%{[2,%%%{4,[1]%%%}]:[1
,0,%%%{1,[1]%%%}]%%},[0]%%%}/%%%{%%{[-2,1]:[1,0,%%%{1,[1]%%%}]%%},[0]%%%},[
1]%%%}+%%%{%%%{1,[1]%%%},[0]%%%} / %%%{%%%{1/%%%{%%{[-2,1]:[1,0,%%%{1,[1]%%
%}]%%},[0]%%%},[0]%%%},[0]%%%} Error: Bad Argument ValueEvaluation time: 10
.71integrate((4*a^2*x^6*sqrt(sqrt(a^2*x^4+b)+a*x^2)*sqrt(a^2*x^4+b)-2*a^2*x
^6-4*a*x^4*sqrt(sqrt(a^2*x^4+b)+a*x^2)*sqrt(a^2*x^4+b)+4*b*x^2*sqrt(sqrt(a^
2*x^4+b)+a*x^2)*sqrt(a^2*x^4+b)-2*b*x^2+x^2*sqrt(sqrt(a^2*x^4+b)+a*x^2)*sqr
t(a^2*x^4+b)+2*x^2*(a^2*x^4+b))/(4*a^2*x^4-4*a*x^2+4*b+1),x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a^2x^4+b}\,\sqrt{a\,x^2+\sqrt{a^2x^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*x^4+b)^(1/2)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x)
```

```
[Out] int(x^2*(a^2*x^4+b)^(1/2)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2x^4+b}\,\sqrt{ax^2+\sqrt{a^2x^4+b}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*x^4+b)^(1/2)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(a^2*x^4 + b)*sqrt(a*x^2 + sqrt(a^2*x^4 + b))*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{\sqrt{a^2\,x^4+b}+a\,x^2}\,\sqrt{a^2\,x^4+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b + a^2*x^4)^(1/2) + a*x^2)^(1/2)*(b + a^2*x^4)^(1/2),x)
```

```
[Out] int(x^2*((b + a^2*x^4)^(1/2) + a*x^2)^(1/2)*(b + a^2*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{ax^2+\sqrt{a^2x^4+b}}\,\sqrt{a^2x^4+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*x**4+b)**(1/2)*(a*x**2+(a**2*x**4+b)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*x**2 + sqrt(a**2*x**4 + b))*sqrt(a**2*x**4 + b), x)
```

3.2062 $\int \frac{(-bx+a^2x^2)^{3/2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=220

$$\frac{\sqrt{a^2x^2-bx}\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(32a^4x^2-88a^2bx+115b^2\right)}{40b^2x}+\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(\frac{9\sqrt{b}\sqrt{\sqrt{a^2x^2-bx}-a}}{8}\right.$$

Rubi [F] time = 4.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-bx+a^2x^2)^{3/2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(b*x) + a^2*x^2)^(3/2)/(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2),x]
```

```
[Out] (2*Sqrt[-(b*x) + a^2*x^2]*Defer[Subst][Defer[Int][(x^4*(-b + a^2*x^2)^(3/2)
)/(a*x^4 + x^2*Sqrt[-(b*x^2) + a^2*x^4])^(3/2), x], x, Sqrt[x]])/(Sqrt[x]*S
qrt[-b + a^2*x])
```

Rubi steps

$$\begin{aligned}
\int \frac{(-bx+a^2x^2)^{3/2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx &= \frac{\sqrt{-bx+a^2x^2}\int \frac{x^{3/2}(-b+a^2x)^{3/2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx}{\sqrt{x}\sqrt{-b+a^2x}}\\
&= \frac{\left(2\sqrt{-bx+a^2x^2}\right)\text{Subst}\left(\int \frac{x^4(-b+a^2x^2)^{3/2}}{\left(ax^4+x^2\sqrt{-bx^2+a^2x^4}\right)^{3/2}}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{-b+a^2x}}
\end{aligned}$$

Mathematica [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{(-bx+a^2x^2)^{3/2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-(b*x) + a^2*x^2)^(3/2)/(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2)
,x]
```

```
[Out] Integrate[(-(b*x) + a^2*x^2)^(3/2)/(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2)
, x]
```

IntegrateAlgebraic [A] time = 7.44, size = 220, normalized size = 1.00

$$\frac{\sqrt{a^2x^2-bx}\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(32a^4x^2-88a^2bx+115b^2\right)}{40b^2x}+\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(\frac{9\sqrt{b}\sqrt{\sqrt{a^2x^2-bx}-ax}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{\sqrt{a^2x^2-bx}-ax}}{\sqrt{b}}\right)}{8\sqrt{2}\sqrt{a}\,x}+\frac{-32a^5x^2+104a^3bx-145ab^2}{40b^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(b*x) + a^2*x^2)^(3/2)/(a*x^2 + x*Sqrt[-(b*x) + a^2*x^
2])^(3/2),x]
```

```
[Out] (Sqrt[-(b*x) + a^2*x^2]*(115*b^2 - 88*a^2*b*x + 32*a^4*x^2)*Sqrt[x*(a*x + S
qrt[-(b*x) + a^2*x^2])])/(40*b^2*x) + Sqrt[x*(a*x + Sqrt[-(b*x) + a^2*x^2])
]*((-145*a*b^2 + 104*a^3*b*x - 32*a^5*x^2)/(40*b^2) + (9*Sqrt[b]*Sqrt[-(a*x
) + Sqrt[-(b*x) + a^2*x^2]]*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[-(a*x) + Sqrt[-(b*
x) + a^2*x^2]])/Sqrt[b]])/(8*Sqrt[2]*Sqrt[a]*x))
```

fricas [A] time = 0.49, size = 367, normalized size = 1.67

$$\left[\frac{45\sqrt{2}\sqrt{a}\,b^3x\log\left(-\frac{4a^2x^2+4\sqrt{a^2x^2-bx}\,ax-bx-2\left(\sqrt{2}a^{\frac{3}{2}}x+\sqrt{2}\sqrt{a^2x^2-bx}\sqrt{a}\right)\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}}{x}\right)-4\left(32a^6x^3-104a^4bx^2+145a^2b^2x-\left(32a^5x^2-88a^3bx+115ab^2\right)\sqrt{a^2x^2-bx}\right)\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}}{160ab^2x},\ \frac{45\sqrt{2}\sqrt{-a}\,b^3x\arctan\left(\frac{\sqrt{2}\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}\sqrt{-a}}{2ax}\right)-2\left(32a^6x^3-104a^4bx^2+145a^2b^2x-\left(32a^5x^2-88a^3bx+115ab^2\right)\sqrt{a^2x^2-bx}\right)\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}}{80ab^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b*x)^(3/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algori
thm="fricas")
```

```
[Out] [1/160*(45*sqrt(2)*sqrt(a)*b^3*x*log(-(4*a^2*x^2 + 4*sqrt(a^2*x^2 - b*x)*a*
x - b*x - 2*(sqrt(2)*a^(3/2)*x + sqrt(2)*sqrt(a^2*x^2 - b*x)*sqrt(a))*sqrt(
a*x^2 + sqrt(a^2*x^2 - b*x)*x))/x) - 4*(32*a^6*x^3 - 104*a^4*b*x^2 + 145*a^
2*b^2*x - (32*a^5*x^2 - 88*a^3*b*x + 115*a*b^2)*sqrt(a^2*x^2 - b*x))*sqrt(a
*x^2 + sqrt(a^2*x^2 - b*x)*x))/(a*b^2*x), 1/80*(45*sqrt(2)*sqrt(-a)*b^3*x*a
rctan(1/2*sqrt(2)*sqrt(a*x^2 + sqrt(a^2*x^2 - b*x)*x)*sqrt(-a)/(a*x)) - 2*(
32*a^6*x^3 - 104*a^4*b*x^2 + 145*a^2*b^2*x - (32*a^5*x^2 - 88*a^3*b*x + 115
*a*b^2)*sqrt(a^2*x^2 - b*x))*sqrt(a*x^2 + sqrt(a^2*x^2 - b*x)*x))/(a*b^2*x)
]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-bx\right)^{\frac{3}{2}}}{\left(ax^2+\sqrt{a^2x^2-bx}\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b*x)^(3/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algori
thm="giac")
```

```
[Out] integrate((a^2*x^2 - b*x)^(3/2)/(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-bx\right)^{\frac{3}{2}}}{\left(a\,x^2+x\sqrt{a^2x^2-bx}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b*x)^(3/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

```
[Out] int((a^2*x^2-b*x)^(3/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2 - bx\right)^{\frac{3}{2}}}{\left(ax^2 + \sqrt{a^2x^2 - bx}\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b*x)^(3/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algori
thm="maxima")
```

```
[Out] integrate((a^2*x^2 - b*x)^(3/2)/(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2\,x^2-b\,x\right)^{3/2}}{\left(a\,x^2+x\,\sqrt{a^2\,x^2-b\,x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 - b*x)^(3/2)/(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2),x)
```

```
[Out] int((a^2*x^2 - b*x)^(3/2)/(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x\left(a^2x - b\right)\right)^{\frac{3}{2}}}{\left(x\left(ax + \sqrt{a^2x^2 - bx}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b*x)**(3/2)/(a*x**2+x*(a**2*x**2-b*x)**(1/2))**(3/2),x
)
```

```
[Out] Integral((x*(a**2*x - b))**(3/2)/(x*(a*x + sqrt(a**2*x**2 - b*x)))**(3/2),
x)
```

3.2063 $\int \frac{-3k+\left(-2+k^2\right)x+3kx^2+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d-(2+d)kx-\left(d+k^2\right)x^2+dkx^3\right)}\,dx$

Optimal. Leaf size=221

$$\frac{\log\left(-\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+kx+1\right)}{d^{2/3}}-\frac{\log\left(d^{2/3}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2}\right.}{2d^{2/3}}$$

Rubi [F] time = 7.55, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3k+\left(-2+k^2\right)x+3kx^2+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d-(2+d)kx-\left(d+k^2\right)x^2+dkx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3*k + (-2 + k^2)*x + 3*k*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1
/3)*(-1 + d - (2 + d)*k*x - (d + k^2)*x^2 + d*k*x^3)),x]
```

```
[Out] (k*x*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*AppellF1[1/2, 1/3, 1/3, 3/2, x^2,
k^2*x^2])/(d*((1 - x^2)*(1 - k^2*x^2))^(1/3)) - ((1 - 4*d)*k*(1 - x^2)^(1/3
)*(1 - k^2*x^2)^(1/3)*Defer[Int][1/((1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*(1
- d + (2 + d)*k*x + (d + k^2)*x^2 - d*k*x^3)), x])/(d*((1 - x^2)*(1 - k^2*x
^2))^(1/3)) - (2*(k^2 - d*(1 - k^2))*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*De
fer[Int][x/((1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*(1 - d + (2 + d)*k*x + (d +
 k^2)*x^2 - d*k*x^3)), x])/(d*((1 - x^2)*(1 - k^2*x^2))^(1/3)) - (k*(4*d +
k^2)*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*Defer[Int][x^2/((1 - x^2)^(1/3)*(1
 - k^2*x^2)^(1/3)*(1 - d + (2 + d)*k*x + (d + k^2)*x^2 - d*k*x^3)), x])/(d*
((1 - x^2)*(1 - k^2*x^2))^(1/3))
```

Rubi steps

$$\int \frac{-3k+\left(-2+k^2\right)x+3kx^2+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d-(2+d)kx-\left(d+k^2\right)x^2+dkx^3\right)}\,dx = \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{-3k+(-2\cdots}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,(-1\cdots}}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2\cdots\right.}}$$

$$= \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{3k+(2-k\cdots}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,(1-\cdots}}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x\cdots\right.}}$$

$$= \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int\left(\frac{k}{d\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}}\cdots\right.}{\sqrt[3]{(1-x\cdots}}$$

$$= -\frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{k-4dk+2(k^2-\cdots}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,(1\cdots}}{d\sqrt[3]{\left(1-x^2\right)\left(1-k\cdots\right.}}$$

$$= \frac{kx\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,k^2\cdots\right.}{d\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{kx\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,k^2\cdots\right.}{d\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{kx\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,k^2\cdots\right.}{d\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

Mathematica [F] time = 1.82, size = 0, normalized size = 0.00

$$\int \frac{-3k+\left(-2+k^2\right)x+3kx^2+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d-(2+d)kx-\left(d+k^2\right)x^2+dkx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3*k + (-2 + k^2)*x + 3*k*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^
2))^(1/3)*(-1 + d - (2 + d)*k*x - (d + k^2)*x^2 + d*k*x^3)),x]
```

```
[Out] Integrate[(-3*k + (-2 + k^2)*x + 3*k*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^
2))^(1/3)*(-1 + d - (2 + d)*k*x - (d + k^2)*x^2 + d*k*x^3)), x]
```

IntegrateAlgebraic [A] time = 5.67, size = 221, normalized size = 1.00

$$\frac{\log\left(-\sqrt[3]{d}\,\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+kx+1\right)}{d^{2/3}}-\frac{\log\left(d^{2/3}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\left(\sqrt[3]{d}\,kx+\sqrt[3]{d}\right)+k^2x^2+2kx+1\right)}{2d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}{\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+2kx+2}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*k + (-2 + k^2)*x + 3*k*x^2 + k^2*x^3)/(((1 - x^2)*(1
 - k^2*x^2))^(1/3)*(-1 + d - (2 + d)*k*x - (d + k^2)*x^2 + d*k*x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))/(2 +
 2*k*x + d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))])/d^(2/3) + Log[1 +
k*x - d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3)]/d^(2/3) - Log[1 + 2*k*x
```

```
+ k^2*x^2 + (d^(1/3) + d^(1/3)*k*x)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3) +
d^(2/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*k+(k^2-2)*x+3*k*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-
1+d-(2+d)*k*x-(k^2+d)*x^2+d*k*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3 + 3\,kx^2 + \left(k^2 - 2\right)x - 3\,k}{\left(dkx^3 - (d+2)kx - \left(k^2 + d\right)x^2 + d - 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*k+(k^2-2)*x+3*k*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-
1+d-(2+d)*k*x-(k^2+d)*x^2+d*k*x^3),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^3 + 3*k*x^2 + (k^2 - 2)*x - 3*k)/((d*k*x^3 - (d + 2)*k*x -
(k^2 + d)*x^2 + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{-3k + \left(k^2 - 2\right)x + 3k\,x^2 + k^2x^3}{\left(\left(-x^2 + 1\right)\left(-k^2x^2 + 1\right)\right)^{\frac{1}{3}}\left(-1 + d - (2+d)\,kx - \left(k^2 + d\right)x^2 + dk\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*k+(k^2-2)*x+3*k*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d-(2
+d)*k*x-(k^2+d)*x^2+d*k*x^3),x)
```

```
[Out] int((-3*k+(k^2-2)*x+3*k*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d-(2
+d)*k*x-(k^2+d)*x^2+d*k*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3 + 3\,kx^2 + \left(k^2 - 2\right)x - 3\,k}{\left(dkx^3 - (d+2)kx - \left(k^2 + d\right)x^2 + d - 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*k+(k^2-2)*x+3*k*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-
1+d-(2+d)*k*x-(k^2+d)*x^2+d*k*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^3 + 3*k*x^2 + (k^2 - 2)*x - 3*k)/((d*k*x^3 - (d + 2)*k*x -
(k^2 + d)*x^2 + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x\,\left(k^2 - 2\right) - 3\,k + k^2\,x^3 + 3\,k\,x^2}{{\left(\left(x^2 - 1\right)\,\left(k^2\,x^2 - 1\right)\right)}^{1/3}\,\left(-d\,k\,x^3 + \left(k^2 + d\right)\,x^2 + k\,\left(d + 2\right)\,x - d + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(k^2 - 2) - 3*k + k^2*x^3 + 3*k*x^2)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/
3)*(x^2*(d + k^2) - d + k*x*(d + 2) - d*k*x^3 + 1)),x)
```

```
[Out] int(-(x*(k^2 - 2) - 3*k + k^2*x^3 + 3*k*x^2)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/
3)*(x^2*(d + k^2) - d + k*x*(d + 2) - d*k*x^3 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*k+(k**2-2)*x+3*k*x**2+k**2*x**3)/((-x**2+1)*(-k**2*x**2+1))**
(1/3)/(-1+d-(2+d)*k*x-(k**2+d)*x**2+d*k*x**3),x)
```

```
[Out] Timed out
```

3.2064 $\int \frac{a+bx}{x(-d+cx)\sqrt[4]{-x^3+x^4}}\,dx$

Optimal. Leaf size=221

$$\frac{\sqrt{2}\,(ac+bd)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,x\,\sqrt[4]{x^4-x^3}\,\sqrt[4]{c-d}}{x^2\sqrt{c-d}-\sqrt{d}\,\sqrt{x^4-x^3}}\right)}{d^{7/4}\sqrt[4]{c-d}}-\frac{\sqrt{2}\,(ac+bd)\tanh^{-1}\left(\frac{x^2\sqrt{c-d}+\sqrt{d}\,\sqrt{x^4-x^3}}{\sqrt{2}\,\sqrt[4]{d}\,x\,\sqrt[4]{x^4-x^3}\,\sqrt[4]{c-d}}\right)}{d^{7/4}\sqrt[4]{c-d}}-\frac{4a\left(x^4-x^3\right)^{3/4}}{3dx^3}$$

Rubi [A] time = 0.34, antiderivative size = 179, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2056, 155, 12, 93, 212, 208, 205}

$$-\frac{2\sqrt[4]{x-1}\,x^{3/4}(ac+bd)\tan^{-1}\left(\frac{\sqrt[4]{x}\,\sqrt[4]{d-c}}{\sqrt[4]{d}\,\sqrt[4]{x-1}}\right)}{d^{7/4}\sqrt[4]{x^4-x^3}\,\sqrt[4]{d-c}}-\frac{2\sqrt[4]{x-1}\,x^{3/4}(ac+bd)\tanh^{-1}\left(\frac{\sqrt[4]{x}\,\sqrt[4]{d-c}}{\sqrt[4]{d}\,\sqrt[4]{x-1}}\right)}{d^{7/4}\sqrt[4]{x^4-x^3}\,\sqrt[4]{d-c}}+\frac{4a(1-x)}{3d\sqrt[4]{x^4-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x*(-d + c*x)*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (4*a*(1 - x))/(3*d*(-x^3 + x^4)^(1/4)) - (2*(a*c + b*d)*(-1 + x)^(1/4)*x^(3
/4)*ArcTan[((-c + d)^(1/4)*x^(1/4))/(d^(1/4)*(-1 + x)^(1/4))])/(d^(7/4)*(-c
 + d)^(1/4)*(-x^3 + x^4)^(1/4)) - (2*(a*c + b*d)*(-1 + x)^(1/4)*x^(3/4)*Arc
Tanh[((-c + d)^(1/4)*x^(1/4))/(d^(1/4)*(-1 + x)^(1/4))])/(d^(7/4)*(-c + d)^
(1/4)*(-x^3 + x^4)^(1/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{a+bx}{x(-d+cx)\sqrt[4]{-x^3+x^4}}\,dx = \frac{\left(\sqrt[4]{-1+x}\,x^{3/4}\right)\int \frac{a+bx}{\sqrt[4]{-1+x}\,x^{7/4}(-d+cx)}\,dx}{\sqrt[4]{-x^3+x^4}}$$

$$= \frac{4a(1-x)}{3d\sqrt[4]{-x^3+x^4}} - \frac{\left(4\sqrt[4]{-1+x}\,x^{3/4}\right)\int -\frac{3(ac+bd)}{4\sqrt[4]{-1+x}\,x^{3/4}(-d+cx)}\,dx}{3d\sqrt[4]{-x^3+x^4}}$$

$$= \frac{4a(1-x)}{3d\sqrt[4]{-x^3+x^4}} + \frac{\left((ac+bd)\sqrt[4]{-1+x}\,x^{3/4}\right)\int \frac{1}{\sqrt[4]{-1+x}\,x^{3/4}(-d+cx)}\,dx}{d\sqrt[4]{-x^3+x^4}}$$

$$= \frac{4a(1-x)}{3d\sqrt[4]{-x^3+x^4}} + \frac{\left(4(ac+bd)\sqrt[4]{-1+x}\,x^{3/4}\right)\text{Subst}\left(\int \frac{1}{-d-(c-d)x^4}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt[4]{-1+x}}\right)}{d\sqrt[4]{-x^3+x^4}}$$

$$= \frac{4a(1-x)}{3d\sqrt[4]{-x^3+x^4}} - \frac{\left(2(ac+bd)\sqrt[4]{-1+x}\,x^{3/4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d}-\sqrt{-c+d}\,x^2}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt[4]{-1+x}}\right)}{d^{3/2}\sqrt[4]{-x^3+x^4}} - \cdots$$

$$= \frac{4a(1-x)}{3d\sqrt[4]{-x^3+x^4}} - \frac{2(ac+bd)\sqrt[4]{-1+x}\,x^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{-c+d}\,\sqrt[4]{x}}{\sqrt[4]{d}\,\sqrt[4]{-1+x}}\right)}{d^{7/4}\sqrt[4]{-c+d}\,\sqrt[4]{-x^3+x^4}} - \frac{2(ac+bd)\sqrt[4]{-1+x}\,x \cdots}{d^{7/4}\sqrt[4]{-c+\cdots}}$$

Mathematica [C] time = 0.05, size = 69, normalized size = 0.31

$$-\frac{4\left((x-1)x^3\right)^{3/4}\left((ac+bd)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{d-dx}{cx-dx}\right)+a(d-c)\right)}{3dx^3(d-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x*(-d + c*x)*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (-4*((-1 + x)*x^3)^(3/4)*(a*(-c + d) + (a*c + b*d)*Hypergeometric2F1[3/4, 1
, 7/4, (d - d*x)/(c*x - d*x)]))/(3*d*(-c + d)*x^3)
```

IntegrateAlgebraic [A] time = 3.14, size = 221, normalized size = 1.00

$$\frac{\sqrt{2}\,(ac+bd)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,x\,\sqrt[4]{x^4-x^3}\,\sqrt[4]{c-d}}{x^2\sqrt{c-d}-\sqrt{d}\,\sqrt{x^4-x^3}}\right)}{d^{7/4}\sqrt[4]{c-d}} - \frac{\sqrt{2}\,(ac+bd)\tanh^{-1}\left(\frac{x^2\sqrt{c-d}+\sqrt{d}\,\sqrt{x^4-x^3}}{\sqrt{2}\,\sqrt[4]{d}\,x\,\sqrt[4]{x^4-x^3}\,\sqrt[4]{c-d}}\right)}{d^{7/4}\sqrt[4]{c-d}} - \frac{4a\left(x^4-x^3\right)^{3/4}}{3dx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x*(-d + c*x)*(-x^3 + x^4)^(1/4)),x]
```

```
[Out] (-4*a*(-x^3 + x^4)^(3/4))/(3*d*x^3) + (Sqrt[2]*(a*c + b*d)*ArcTan[(Sqrt[2]*
(c - d)^(1/4)*d^(1/4)*x*(-x^3 + x^4)^(1/4))/(Sqrt[c - d]*x^2 - Sqrt[d]*Sqrt
[-x^3 + x^4])])/((c - d)^(1/4)*d^(7/4)) - (Sqrt[2]*(a*c + b*d)*ArcTanh[(Sqr
t[c - d]*x^2 + Sqrt[d]*Sqrt[-x^3 + x^4])/(Sqrt[2]*(c - d)^(1/4)*d^(1/4)*x*(
-x^3 + x^4)^(1/4))])/((c - d)^(1/4)*d^(7/4))
```

fricas [B] time = 1.54, size = 1021, normalized size = 4.62

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x-d)/(x^4-x^3)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/3*(12*d*x^3*(-(a^4*c^4 + 4*a^3*b*c^3*d + 6*a^2*b^2*c^2*d^2 + 4*a*b^3*c*d
^3 + b^4*d^4)/(c*d^7 - d^8))^(1/4)*arctan((d^2*x*(-(a^4*c^4 + 4*a^3*b*c^3*d
 + 6*a^2*b^2*c^2*d^2 + 4*a*b^3*c*d^3 + b^4*d^4)/(c*d^7 - d^8))^(1/4)*sqrt(-
((a^4*c^5*d^3 - b^4*d^8 - (a^4 - 4*a^3*b)*c^4*d^4 - 2*(2*a^3*b - 3*a^2*b^2)
*c^3*d^5 - 2*(3*a^2*b^2 - 2*a*b^3)*c^2*d^6 - (4*a*b^3 - b^4)*c*d^7)*x^2*sqr
t(-(a^4*c^4 + 4*a^3*b*c^3*d + 6*a^2*b^2*c^2*d^2 + 4*a*b^3*c*d^3 + b^4*d^4)/
(c*d^7 - d^8)) - (a^6*c^6 + 6*a^5*b*c^5*d + 15*a^4*b^2*c^4*d^2 + 20*a^3*b^3
*c^3*d^3 + 15*a^2*b^4*c^2*d^4 + 6*a*b^5*c*d^5 + b^6*d^6)*sqrt(x^4 - x^3))/x
^2) - (a^3*c^3*d^2 + 3*a^2*b*c^2*d^3 + 3*a*b^2*c*d^4 + b^3*d^5)*(x^4 - x^3)
^(1/4)*(-(a^4*c^4 + 4*a^3*b*c^3*d + 6*a^2*b^2*c^2*d^2 + 4*a*b^3*c*d^3 + b^4
*d^4)/(c*d^7 - d^8))^(1/4))/((a^4*c^4 + 4*a^3*b*c^3*d + 6*a^2*b^2*c^2*d^2 +
 4*a*b^3*c*d^3 + b^4*d^4)*x)) - 3*d*x^3*(-(a^4*c^4 + 4*a^3*b*c^3*d + 6*a^2*
b^2*c^2*d^2 + 4*a*b^3*c*d^3 + b^4*d^4)/(c*d^7 - d^8))^(1/4)*log(((c*d^5 - d
^6)*x*(-(a^4*c^4 + 4*a^3*b*c^3*d + 6*a^2*b^2*c^2*d^2 + 4*a*b^3*c*d^3 + b^4*
d^4)/(c*d^7 - d^8))^(3/4) + (a^3*c^3 + 3*a^2*b*c^2*d + 3*a*b^2*c*d^2 + b^3*
d^3)*(x^4 - x^3)^(1/4))/x) + 3*d*x^3*(-(a^4*c^4 + 4*a^3*b*c^3*d + 6*a^2*b^2
*c^2*d^2 + 4*a*b^3*c*d^3 + b^4*d^4)/(c*d^7 - d^8))^(1/4)*log(-((c*d^5 - d^6
)*x*(-(a^4*c^4 + 4*a^3*b*c^3*d + 6*a^2*b^2*c^2*d^2 + 4*a*b^3*c*d^3 + b^4*d^
4)/(c*d^7 - d^8))^(3/4) - (a^3*c^3 + 3*a^2*b*c^2*d + 3*a*b^2*c*d^2 + b^3*d^
3)*(x^4 - x^3)^(1/4))/x) + 4*(x^4 - x^3)^(3/4)*a)/(d*x^3)
```

giac [B] time = 0.21, size = 386, normalized size = 1.75

$$-\frac{\sqrt{2}\,(ac+bd)\log\left(-\sqrt{2}\left(\frac{c-d}{d}\right)^{\frac{1}{4}}\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+\sqrt{\frac{c-d}{d}}+\sqrt{-\frac{1}{x}+1}\right)}{2\left(cd^3-d^4\right)^{\frac{1}{4}}d}-\frac{\left(\sqrt{2}\left(cd^3-d^4\right)^{\frac{3}{4}}ac+\sqrt{2}\left(cd^3-d^4\right)^{\frac{3}{4}}bd\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c-d}{d}\right)^{\frac{1}{4}}+2\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)}{2\left(\frac{c-d}{d}\right)^{\frac{1}{4}}}\right)}{cd^4-d^5}-\frac{\left(\sqrt{2}\left(cd^3-d^4\right)^{\frac{3}{4}}ac+\sqrt{2}\left(cd^3-d^4\right)^{\frac{3}{4}}bd\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c-d}{d}\right)^{\frac{1}{4}}-2\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)}{2\left(\frac{c-d}{d}\right)^{\frac{1}{4}}}\right)}{cd^4-d^5}+\frac{\left(\left(cd^3-d^4\right)^{\frac{3}{4}}ac+\left(cd^3-d^4\right)^{\frac{3}{4}}bd\right)\log\left(\sqrt{2}\left(\frac{c-d}{d}\right)^{\frac{1}{4}}\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+\sqrt{\frac{c-d}{d}}+\sqrt{-\frac{1}{x}+1}\right)}{\sqrt{2}\,cd^4-\sqrt{2}\,d^5}+\frac{4a\left(-\frac{1}{x}+1\right)^{\frac{3}{4}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x-d)/(x^4-x^3)^(1/4),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(a*c + b*d)*log(-sqrt(2)*((c - d)/d)^(1/4)*(-1/x + 1)^(1/4) +
sqrt((c - d)/d) + sqrt(-1/x + 1))/((c*d^3 - d^4)^(1/4)*d) - (sqrt(2)*(c*d^3
 - d^4)^(3/4)*a*c + sqrt(2)*(c*d^3 - d^4)^(3/4)*b*d)*arctan(1/2*sqrt(2)*(sq
rt(2)*((c - d)/d)^(1/4) + 2*(-1/x + 1)^(1/4))/((c - d)/d)^(1/4))/(c*d^4 - d
^5) - (sqrt(2)*(c*d^3 - d^4)^(3/4)*a*c + sqrt(2)*(c*d^3 - d^4)^(3/4)*b*d)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*((c - d)/d)^(1/4) - 2*(-1/x + 1)^(1/4))/((c - d
)/d)^(1/4))/(c*d^4 - d^5) + ((c*d^3 - d^4)^(3/4)*a*c + (c*d^3 - d^4)^(3/4)*
b*d)*log(sqrt(2)*((c - d)/d)^(1/4)*(-1/x + 1)^(1/4) + sqrt((c - d)/d) + sqr
t(-1/x + 1))/(sqrt(2)*c*d^4 - sqrt(2)*d^5) + 4/3*a*(-1/x + 1)^(3/4)/d
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{x\,(cx-d)\left(x^4-x^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x/(c*x-d)/(x^4-x^3)^(1/4),x)
```

```
[Out] int((b*x+a)/x/(c*x-d)/(x^4-x^3)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(x^4-x^3\right)^{\frac{1}{4}}(cx-d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x-d)/(x^4-x^3)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)/((x^4 - x^3)^(1/4)*(c*x - d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{a+b\,x}{x\,\left(x^4-x^3\right)^{1/4}\,\left(d-c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x)/(x*(x^4 - x^3)^(1/4)*(d - c*x)),x)
```

```
[Out] int(-(a + b*x)/(x*(x^4 - x^3)^(1/4)*(d - c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{x\sqrt[4]{x^3\,(x-1)}\,(cx-d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x-d)/(x**4-x**3)**(1/4),x)
```

```
[Out] Integral((a + b*x)/(x*(x**3*(x - 1))**(1/4)*(c*x - d)), x)
```

3.2065 $\int \frac{(-4+3x)\sqrt[4]{\frac{-a+ax+bx^4}{-c+cx+dx^4}}}{(-1+x)x}\,dx$

Optimal. Leaf size=221

$$-\frac{2\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt[4]{\frac{ax-a+bx^4}{cx-c+dx^4}}}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}+\frac{2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{\frac{ax-a+bx^4}{cx-c+dx^4}}}{\sqrt[4]{b}}\right)}{\sqrt[4]{d}}-\frac{2\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt[4]{\frac{ax-a+bx^4}{cx-c+dx^4}}}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}+\frac{2\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{\frac{ax-a+}{cx-c+}}}{\sqrt[4]{b}}\right)}{\sqrt[4]{d}}$$

Rubi [F] time = 3.72, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-4+3x)\sqrt[4]{\frac{-a+ax+bx^4}{-c+cx+dx^4}}}{(-1+x)x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-4 + 3*x)*((-a + a*x + b*x^4)/(-c + c*x + d*x^4))^(1/4))/((-1 + x)*x)
,x]
```

```
[Out] (((a - a*x - b*x^4)/(c - c*x - d*x^4))^(1/4)*(-c + c*x + d*x^4)^(1/4)*Defer
[Int][(-a + a*x + b*x^4)^(1/4)/((1 - x)*(-c + c*x + d*x^4)^(1/4)), x])/(-a
+ a*x + b*x^4)^(1/4) + (4*((a - a*x - b*x^4)/(c - c*x - d*x^4))^(1/4)*(-c +
 c*x + d*x^4)^(1/4)*Defer[Int][(-a + a*x + b*x^4)^(1/4)/(x*(-c + c*x + d*x^
4)^(1/4)), x])/(-a + a*x + b*x^4)^(1/4)
```

Rubi steps

$$\begin{aligned}
\int \frac{(-4+3x)\sqrt[4]{\frac{-a+ax+bx^4}{-c+cx+dx^4}}}{(-1+x)x}\,dx &= \frac{\left(\sqrt[4]{\frac{-a+ax+bx^4}{-c+cx+dx^4}}\sqrt[4]{-c+cx+dx^4}\right)\int \frac{(-4+3x)\sqrt[4]{-a+ax+bx^4}}{(-1+x)x\sqrt[4]{-c+cx+dx^4}}\,dx}{\sqrt[4]{-a+ax+bx^4}}\\
&= \frac{\left(\sqrt[4]{\frac{-a+ax+bx^4}{-c+cx+dx^4}}\sqrt[4]{-c+cx+dx^4}\right)\int \left(\frac{\sqrt[4]{-a+ax+bx^4}}{(1-x)\sqrt[4]{-c+cx+dx^4}}+\frac{4\sqrt[4]{-a+ax+bx^4}}{x\sqrt[4]{-c+cx+dx^4}}\right)dx}{\sqrt[4]{-a+ax+bx^4}}\\
&= \frac{\left(\sqrt[4]{\frac{-a+ax+bx^4}{-c+cx+dx^4}}\sqrt[4]{-c+cx+dx^4}\right)\int \frac{\sqrt[4]{-a+ax+bx^4}}{(1-x)\sqrt[4]{-c+cx+dx^4}}\,dx}{\sqrt[4]{-a+ax+bx^4}}+\frac{\left(4\sqrt[4]{\frac{-a+ax+bx^4}{-c+cx+dx^4}}\sqrt[4]{-c+cx}\right.}{\sqrt[4]{-a+a}}
\end{aligned}$$

Mathematica [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(-4+3x)\sqrt[4]{\frac{-a+ax+bx^4}{-c+cx+dx^4}}}{(-1+x)x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-4 + 3*x)*((-a + a*x + b*x^4)/(-c + c*x + d*x^4))^(1/4))/((-1 +
 x)*x),x]
```

```
[Out] Integrate[((-4 + 3*x)*((-a + a*x + b*x^4)/(-c + c*x + d*x^4))^(1/4))/((-1 +
 x)*x), x]
```

IntegrateAlgebraic [A] time = 1.09, size = 221, normalized size = 1.00

$$-\frac{2\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt[4]{\frac{ax-a+bx^4}{cx-c+dx^4}}}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}+\frac{2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{\frac{ax-a+bx^4}{cx-c+dx^4}}}{\sqrt[4]{b}}\right)}{\sqrt[4]{d}}-\frac{2\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt[4]{\frac{ax-a+bx^4}{cx-c+dx^4}}}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}+\frac{2\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{\frac{ax-a+bx^4}{cx-c+dx^4}}}{\sqrt[4]{b}}\right)}{\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 + 3*x)*((-a + a*x + b*x^4)/(-c + c*x + d*x^4))^(1/4
))/((-1 + x)*x),x]
```

```
[Out] (-2*a^(1/4)*ArcTan[(c^(1/4)*((-a + a*x + b*x^4)/(-c + c*x + d*x^4))^(1/4))/
a^(1/4)])/c^(1/4) + (2*b^(1/4)*ArcTan[(d^(1/4)*((-a + a*x + b*x^4)/(-c + c*
x + d*x^4))^(1/4))/b^(1/4)])/d^(1/4) - (2*a^(1/4)*ArcTanh[(c^(1/4)*((-a + a
*x + b*x^4)/(-c + c*x + d*x^4))^(1/4))/a^(1/4)])/c^(1/4) + (2*b^(1/4)*ArcTa
nh[(d^(1/4)*((-a + a*x + b*x^4)/(-c + c*x + d*x^4))^(1/4))/b^(1/4)])/d^(1/4
)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)*((b*x^4+a*x-a)/(d*x^4+c*x-c))^(1/4)/(-1+x)/x,x, algorith
m="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x-4)\left(\frac{bx^4+ax-a}{dx^4+cx-c}\right)^{\frac{1}{4}}}{(x-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)*((b*x^4+a*x-a)/(d*x^4+c*x-c))^(1/4)/(-1+x)/x,x, algorith
m="giac")
```

```
[Out] integrate((3*x - 4)*((b*x^4 + a*x - a)/(d*x^4 + c*x - c))^(1/4)/((x - 1)*x)
, x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{(-4+3x)\left(\frac{b\,x^4+ax-a}{d\,x^4+cx-c}\right)^{\frac{1}{4}}}{(-1+x)\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4+3*x)*((b*x^4+a*x-a)/(d*x^4+c*x-c))^(1/4)/(-1+x)/x,x)
```

```
[Out] int((-4+3*x)*((b*x^4+a*x-a)/(d*x^4+c*x-c))^(1/4)/(-1+x)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x-4)\left(\frac{bx^4+ax-a}{dx^4+cx-c}\right)^{\frac{1}{4}}}{(x-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)*((b*x^4+a*x-a)/(d*x^4+c*x-c))^(1/4)/(-1+x)/x,x, algorith
m="maxima")
```

```
[Out] integrate((3*x - 4)*((b*x^4 + a*x - a)/(d*x^4 + c*x - c))^(1/4)/((x - 1)*x)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(3x-4)\left(\frac{b x^4+a x-a}{d x^4+c x-c}\right)^{1/4}}{x (x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x - 4)*((a*x - a + b*x^4)/(c*x - c + d*x^4))^(1/4))/(x*(x - 1)),x)
```

```
[Out] int(((3*x - 4)*((a*x - a + b*x^4)/(c*x - c + d*x^4))^(1/4))/(x*(x - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)*((b*x**4+a*x-a)/(d*x**4+c*x-c))**(1/4)/(-1+x)/x,x)
```

```
[Out] Timed out
```

3.2066 $\int \frac{(1+x^3)^{2/3}(1-2x^3+2x^6)}{x^6(-1-x^3+2x^6)}\,dx$

Optimal. Leaf size=221

$$-\frac{10}{9}\log\left(\sqrt[3]{x^3+1}+x\right)+\frac{1}{9}2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^3+1}-2x\right)-\frac{10\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}-x}\right)}{3\sqrt{3}}-\frac{2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3+1}+x}\right)}{3\sqrt{3}}+\frac{(x^3+1)^{2}}{1}$$

Rubi [F] time = 0.78, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+x^3)^{2/3}(1-2x^3+2x^6)}{x^6(-1-x^3+2x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + x^3)^(2/3)*(1 - 2*x^3 + 2*x^6))/(x^6*(-1 - x^3 + 2*x^6)),x]
```

```
[Out] (-3*(1 + x^3)^(2/3))/(2*x^2) + (1 + x^3)^(5/3)/(5*x^5) - (20*x*AppellF1[1/3
, -2/3, 1, 4/3, -x^3, -2*x^3])/3 + Sqrt[3]*ArcTan[(1 + (2*x)/(1 + x^3)^(1/3
))/Sqrt[3]] - (3*Log[-x + (1 + x^3)^(1/3)])/2 + Defer[Int][(1 + x^3)^(2/3)/
(-1 + x), x]/9 - ((1 - I*Sqrt[3])*Defer[Int][(1 + x^3)^(2/3)/(1 - I*Sqrt[3]
 + 2*x), x])/9 - ((1 + I*Sqrt[3])*Defer[Int][(1 + x^3)^(2/3)/(1 + I*Sqrt[3]
 + 2*x), x])/9
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^3)^{2/3}(1-2x^3+2x^6)}{x^6(-1-x^3+2x^6)}\,dx &= \int \left(\frac{(1+x^3)^{2/3}}{9(-1+x)}-\frac{(1+x^3)^{2/3}}{x^6}+\frac{3(1+x^3)^{2/3}}{x^3}+\frac{(-2-x)(1+x^3)^{2/3}}{9(1+x+x^2)}-\frac{20(1+}{3(1+}\right.\\
&= \frac{1}{9}\int \frac{(1+x^3)^{2/3}}{-1+x}\,dx+\frac{1}{9}\int \frac{(-2-x)(1+x^3)^{2/3}}{1+x+x^2}\,dx+3\int \frac{(1+x^3)^{2/3}}{x^3}\,dx-\frac{20}{3}\\
&= -\frac{3(1+x^3)^{2/3}}{2x^2}+\frac{(1+x^3)^{5/3}}{5x^5}-\frac{20}{3}xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-x^3,-2x^3\right)+\frac{1}{9}\int \frac{(1+x^3}{-1+}\\
&= -\frac{3(1+x^3)^{2/3}}{2x^2}+\frac{(1+x^3)^{5/3}}{5x^5}-\frac{20}{3}xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-x^3,-2x^3\right)+\sqrt{3}\tan^{-1}\left(\frac{1}{-}\right.
\end{aligned}$$

Mathematica [A] time = 0.52, size = 214, normalized size = 0.97

$$\frac{1}{18}\left(-20\log\left(\frac{x}{\sqrt[3]{x^3+1}}+1\right)+2\,2^{2/3}\log\left(1-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3+1}}\right)+20\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{x^3+1}}}{\sqrt{3}}\right)-2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)+10\log\left(-\frac{x}{\sqrt[3]{x^3+1}}+\frac{x^2}{(x^3+1)^{2/3}}+1\right)-2^{2/3}\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3+1}}+\frac{2^{2/3}x^2}{(x^3+1)^{2/3}}+1\right)\right)+(x^3+1)^{2/3}\left(\frac{1}{5x^5}-\frac{13}{10x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)^(2/3)*(1 - 2*x^3 + 2*x^6))/(x^6*(-1 - x^3 + 2*x^6)),x]
```

```
[Out] (1/(5*x^5) - 13/(10*x^2))*(1 + x^3)^(2/3) + (20*Sqrt[3]*ArcTan[(1 - (2*x)/(
1 + x^3)^(1/3))/Sqrt[3]] - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + (2*2^(1/3)*x)/(1 +
 x^3)^(1/3))/Sqrt[3]] + 10*Log[1 + x^2/(1 + x^3)^(2/3) - x/(1 + x^3)^(1/3)]
 - 20*Log[1 + x/(1 + x^3)^(1/3)] + 2*2^(2/3)*Log[1 - (2^(1/3)*x)/(1 + x^3)^
(1/3)] - 2^(2/3)*Log[1 + (2^(2/3)*x^2)/(1 + x^3)^(2/3) + (2^(1/3)*x)/(1 + x
^3)^(1/3)])/18
```

IntegrateAlgebraic [A] time = 0.54, size = 221, normalized size = 1.00

$$-\frac{10}{9}\log\left(\sqrt[3]{x^3+1}+x\right)+\frac{1}{9}2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^3+1}-2x\right)-\frac{10\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+1}-x}\right)}{3\sqrt{3}}-\frac{2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3+1}+x}\right)}{3\sqrt{3}}+\frac{\left(x^3+1\right)^{2/3}\left(2-13x^3\right)}{10x^5}+\frac{5}{9}\log\left(-\sqrt[3]{x^3+1}\,x+\left(x^3+1\right)^{2/3}+x^2\right)-\frac{\log\left(2^{2/3}\sqrt[3]{x^3+1}\,x+\sqrt[3]{2}\left(x^3+1\right)^{2/3}+2x^2\right)}{9\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)^(2/3)*(1 - 2*x^3 + 2*x^6))/(x^6*(-1 - x^3 + 2
*x^6)),x]
```

```
[Out] ((2 - 13*x^3)*(1 + x^3)^(2/3))/(10*x^5) - (10*ArcTan[(Sqrt[3]*x)/(-x + 2*(1
 + x^3)^(1/3))])/(3*Sqrt[3]) - (2^(2/3)*ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(1
+ x^3)^(1/3))])/(3*Sqrt[3]) - (10*Log[x + (1 + x^3)^(1/3)])/9 + (2^(2/3)*Lo
g[-2*x + 2^(2/3)*(1 + x^3)^(1/3)])/9 + (5*Log[x^2 - x*(1 + x^3)^(1/3) + (1
+ x^3)^(2/3)])/9 - Log[2*x^2 + 2^(2/3)*x*(1 + x^3)^(1/3) + 2^(1/3)*(1 + x^3
)^(2/3)]/(9*2^(1/3))
```

fricas [B] time = 4.98, size = 367, normalized size = 1.66

$$-\frac{10\cdot 4^{\frac{1}{3}}\sqrt{3}x^5\arctan\left(\frac{3\cdot 4^{\frac{2}{3}}\sqrt{3}\left(5x^7-4x^4-x\right)\left(x^3+1\right)^{\frac{2}{3}}-6\cdot 4^{\frac{1}{3}}\sqrt{3}\left(19x^8+16x^5+x^2\right)\left(x^3+1\right)^{\frac{1}{3}}-\sqrt{3}\left(71x^9+111x^6+33x^3+1\right)}{3\left(109x^9+105x^6+3x^3-1\right)}\right)-300\sqrt{3}x^5\arctan\left(\frac{4\sqrt{3}\left(x^3+1\right)^{\frac{1}{3}}x^2+2\sqrt{3}\left(x^3+1\right)^{\frac{2}{3}}x+\sqrt{3}\left(x^3+1\right)}{7x^3-1}\right)-10\cdot 4^{\frac{1}{3}}x^5\log\left(\frac{3\cdot 4^{\frac{2}{3}}\left(x^3+1\right)^{\frac{1}{3}}x^2-6\left(x^3+1\right)^{\frac{2}{3}}x-4^{\frac{1}{3}}\left(x^3-1\right)}{x^3-1}\right)+5\cdot 4^{\frac{1}{3}}x^5\log\left(\frac{6\cdot 4^{\frac{1}{3}}\left(5x^4+x\right)\left(x^3+1\right)^{\frac{2}{3}}+4^{\frac{2}{3}}\left(19x^6+16x^3+1\right)+24\left(2x^5+x^2\right)\left(x^3+1\right)^{\frac{1}{3}}}{x^6-2x^3+1}\right)+150x^5\log\left(\frac{2x^3+3\left(x^3+1\right)^{\frac{1}{3}}x^2+3\left(x^3+1\right)^{\frac{2}{3}}x+1}{2x^3+1}\right)+27\left(13x^3-2\right)\left(x^3+1\right)^{\frac{2}{3}}}{270x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6-2*x^3+1)/x^6/(2*x^6-x^3-1),x, algorithm="fri
cas")
```

```
[Out] -1/270*(10*4^(1/3)*sqrt(3)*x^5*arctan(1/3*(3*4^(2/3)*sqrt(3)*(5*x^7 - 4*x^4
 - x)*(x^3 + 1)^(2/3) - 6*4^(1/3)*sqrt(3)*(19*x^8 + 16*x^5 + x^2)*(x^3 + 1)
^(1/3) - sqrt(3)*(71*x^9 + 111*x^6 + 33*x^3 + 1))/(109*x^9 + 105*x^6 + 3*x^
3 - 1)) - 300*sqrt(3)*x^5*arctan((4*sqrt(3)*(x^3 + 1)^(1/3)*x^2 + 2*sqrt(3)
*(x^3 + 1)^(2/3)*x + sqrt(3)*(x^3 + 1))/(7*x^3 - 1)) - 10*4^(1/3)*x^5*log((
3*4^(2/3)*(x^3 + 1)^(1/3)*x^2 - 6*(x^3 + 1)^(2/3)*x - 4^(1/3)*(x^3 - 1))/(x
^3 - 1)) + 5*4^(1/3)*x^5*log((6*4^(1/3)*(5*x^4 + x)*(x^3 + 1)^(2/3) + 4^(2/
3)*(19*x^6 + 16*x^3 + 1) + 24*(2*x^5 + x^2)*(x^3 + 1)^(1/3))/(x^6 - 2*x^3 +
 1)) + 150*x^5*log((2*x^3 + 3*(x^3 + 1)^(1/3)*x^2 + 3*(x^3 + 1)^(2/3)*x + 1
)/(2*x^3 + 1)) + 27*(13*x^3 - 2)*(x^3 + 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^6-2x^3+1\right)\left(x^3+1\right)^{\frac{2}{3}}}{\left(2x^6-x^3-1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6-2*x^3+1)/x^6/(2*x^6-x^3-1),x, algorithm="gia
c")
```

```
[Out] integrate((2*x^6 - 2*x^3 + 1)*(x^3 + 1)^(2/3)/((2*x^6 - x^3 - 1)*x^6), x)
```

maple [C] time = 5.62, size = 1246, normalized size = 5.64

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)*(2*x^6-2*x^3+1)/x^6/(2*x^6-x^3-1),x)
```

```
[Out] -1/10*(13*x^6+11*x^3-2)/x^5/(x^3+1)^(1/3)-1/9*ln(-(324*RootOf(RootOf(_Z^3-4
)^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-9*RootOf(RootOf(_
Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)*RootOf(_Z^3-4)^3*x^3+36*(x^3+1)^(2/
3)*RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)*
x-90*(x^3+1)^(1/3)*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3
```

```
-4)+324*_Z^2)*x^2-RootOf(_Z^3-4)^2*(x^3+1)^(1/3)*x^2+108*RootOf(RootOf(_Z^3
-4)^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)*x^3-3*RootOf(_Z^3-4)*x^3+10*x*(x^3+1)^
(2/3)+36*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)-RootOf(_Z^3
-4))/(-1+x)/(x^2+x+1))*RootOf(_Z^3-4)-2*ln(-(324*RootOf(RootOf(_Z^3-4)^2+18
*_Z*RootOf(_Z^3-4)+324*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-9*RootOf(RootOf(_Z^3-4)
^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)*RootOf(_Z^3-4)^3*x^3+36*(x^3+1)^(2/3)*Roo
tOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)*x-90*(
x^3+1)^(1/3)*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+32
4*_Z^2)*x^2-RootOf(_Z^3-4)^2*(x^3+1)^(1/3)*x^2+108*RootOf(RootOf(_Z^3-4)^2+
18*_Z*RootOf(_Z^3-4)+324*_Z^2)*x^3-3*RootOf(_Z^3-4)*x^3+10*x*(x^3+1)^(2/3)+
36*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)-RootOf(_Z^3-4))/(
-1+x)/(x^2+x+1))*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)+1/9
*RootOf(_Z^3-4)*ln(-(-162*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+324*
_Z^2)^2*RootOf(_Z^3-4)^2*x^3+9*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)
+324*_Z^2)*RootOf(_Z^3-4)^3*x^3-54*(x^3+1)^(1/3)*RootOf(_Z^3-4)*RootOf(Root
Of(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)*x^2-3*RootOf(_Z^3-4)^2*(x^3+1)^
(1/3)*x^2+18*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)*x^3-Roo
tOf(_Z^3-4)*x^3+6*x*(x^3+1)^(2/3)+18*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_
Z^3-4)+324*_Z^2)-RootOf(_Z^3-4))/(-1+x)/(x^2+x+1))-10/9*ln(-(9*(x^3+1)^(2/3
)*RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)*x
+18*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)*RootOf(_Z^3-4)^2
*(x^3+1)^(1/3)*x^2+9*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)
*RootOf(_Z^3-4)^2*x^3-2*x*(x^3+1)^(2/3)+2*x^2*(x^3+1)^(1/3)-2)/(2*x^3+1))-5
*RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)*ln
((81*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)^2*RootOf(_Z^3-4
)^4*x^3-27*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)*RootOf(_Z
^3-4)^2*(x^3+1)^(1/3)*x^2+27*RootOf(RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+3
24*_Z^2)*RootOf(_Z^3-4)^2*x^3+6*x*(x^3+1)^(2/3)+18*RootOf(_Z^3-4)^2*RootOf(
RootOf(_Z^3-4)^2+18*_Z*RootOf(_Z^3-4)+324*_Z^2)+2*x^3+2)/(2*x^3+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{6}-2\,x^{3}+1\right)\left(x^{3}+1\right)^{\frac{2}{3}}}{\left(2\,x^{6}-x^{3}-1\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(2*x^6-2*x^3+1)/x^6/(2*x^6-x^3-1),x, algorithm="max
ima")
```

```
[Out] integrate((2*x^6 - 2*x^3 + 1)*(x^3 + 1)^(2/3)/((2*x^6 - x^3 - 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(x^{3}+1\right)^{2/3}\,\left(2\,x^{6}-2\,x^{3}+1\right)}{x^{6}\,\left(-2\,x^{6}+x^{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3 + 1)^(2/3)*(2*x^6 - 2*x^3 + 1))/(x^6*(x^3 - 2*x^6 + 1)),x)
```

```
[Out] int(-((x^3 + 1)^(2/3)*(2*x^6 - 2*x^3 + 1))/(x^6*(x^3 - 2*x^6 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)*(2*x**6-2*x**3+1)/x**6/(2*x**6-x**3-1),x)
```

```
[Out] Timed out
```

3.2067 $\int \frac{2}{(3+x)\left(2-8x+8x^2\right)^{2/3}}\,dx$

Optimal. Leaf size=222

$$-\frac{3\sqrt[3]{2}\,\sqrt[3]{4x^2-4x+1}}{7(2x-1)}+\frac{1}{7}\sqrt[3]{\frac{2}{7}}\,\log\left(\left(4x^2-4x+1\right)^{2/3}+2\sqrt[3]{7}\,x-\sqrt[3]{7}\right)-\frac{\log\left(\left(4x^2-4x+1\right)^{2/3}+7^{2/3}\sqrt[3]{4x^2-4x}\right.}{7\,2^{2/3}\sqrt[3]{7}}$$

Rubi [A] time = 0.11, antiderivative size = 196, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 646, 51, 56, 617, 204, 31}

$$\frac{3\sqrt[3]{2}(1-2x)}{7\left(4x^2-4x+1\right)^{2/3}}-\frac{(2x-1)^{4/3}\log(x+3)}{7\,2^{2/3}\sqrt[3]{7}\left(4x^2-4x+1\right)^{2/3}}+\frac{3(2x-1)^{4/3}\log\left(\sqrt[3]{8x-4}+2^{2/3}\sqrt[3]{7}\right)}{7\,2^{2/3}\sqrt[3]{7}\left(4x^2-4x+1\right)^{2/3}}+\frac{\sqrt[3]{\frac{2}{7}}\sqrt{3}(2x-1)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{2x-1}}{\sqrt{3}\sqrt[3]{7}}\right)}{7\left(4x^2-4x+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[2/((3 + x)*(2 - 8*x + 8*x^2)^(2/3)),x]
```

```
[Out] (3*2^(1/3)*(1 - 2*x))/(7*(1 - 4*x + 4*x^2)^(2/3)) + ((2/7)^(1/3)*Sqrt[3]*(-
1 + 2*x)^(4/3)*ArcTan[1/Sqrt[3] - (2*(-1 + 2*x)^(1/3))/(Sqrt[3]*7^(1/3))])/
(7*(1 - 4*x + 4*x^2)^(2/3)) - ((-1 + 2*x)^(4/3)*Log[3 + x])/(7*2^(2/3)*7^(1
/3)*(1 - 4*x + 4*x^2)^(2/3)) + (3*(-1 + 2*x)^(4/3)*Log[2^(2/3)*7^(1/3) + (-
4 + 8*x)^(1/3)])/(7*2^(2/3)*7^(1/3)*(1 - 4*x + 4*x^2)^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2}{(3+x)\left(2-8x+8x^2\right)^{2/3}}\,dx &= 2\int \frac{1}{(3+x)\left(2-8x+8x^2\right)^{2/3}}\,dx\\
&= \frac{\left(2(-4+8x)^{4/3}\right)\int \frac{1}{(3+x)(-4+8x)^{4/3}}\,dx}{\left(2-8x+8x^2\right)^{2/3}}\\
&= \frac{3\sqrt[3]{2}\,(1-2x)}{7\left(1-4x+4x^2\right)^{2/3}} - \frac{(-4+8x)^{4/3}\int \frac{1}{(3+x)\sqrt[3]{-4+8x}}\,dx}{14\left(2-8x+8x^2\right)^{2/3}}\\
&= \frac{3\sqrt[3]{2}\,(1-2x)}{7\left(1-4x+4x^2\right)^{2/3}} - \frac{(-1+2x)^{4/3}\log(3+x)}{7\,2^{2/3}\sqrt[3]{7}\left(1-4x+4x^2\right)^{2/3}} - \frac{\left(3(-4+8x)^{4/3}\right)\operatorname{Subst}\left(\int \frac{}{2\cdots}\right.}{28\left(2-\cdots\right.}\\
&= \frac{3\sqrt[3]{2}\,(1-2x)}{7\left(1-4x+4x^2\right)^{2/3}} - \frac{(-1+2x)^{4/3}\log(3+x)}{7\,2^{2/3}\sqrt[3]{7}\left(1-4x+4x^2\right)^{2/3}} + \frac{3(-1+2x)^{4/3}\log\left(2^{2/3}\sqrt[3]{7}+\cdots\right.}{7\,2^{2/3}\sqrt[3]{7}\left(1-4x+4\cdots\right.}\\
&= \frac{3\sqrt[3]{2}\,(1-2x)}{7\left(1-4x+4x^2\right)^{2/3}} + \frac{\sqrt[3]{\frac{2}{7}}\sqrt{3}\,(-1+2x)^{4/3}\tan^{-1}\left(\frac{7-2\,7^{2/3}\sqrt[3]{-1+2x}}{7\sqrt{3}}\right)}{7\left(1-4x+4x^2\right)^{2/3}} - \frac{(-1+2x)\cdots}{7\,2^{2/3}\sqrt[3]{7}\left(1\cdots\right.}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 40, normalized size = 0.18

$$
-\frac{6(2x-1)\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{7}(1-2x)\right)}{7\left(8x^2-8x+2\right)^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[2/((3 + x)*(2 - 8*x + 8*x^2)^(2/3)),x]
```

```
[Out] (-6*(-1 + 2*x)*Hypergeometric2F1[-1/3, 1, 2/3, (1 - 2*x)/7])/(7*(2 - 8*x +
8*x^2)^(2/3))
```

IntegrateAlgebraic [A] time = 0.66, size = 222, normalized size = 1.00

$$
-\frac{3\sqrt[3]{2}\sqrt[3]{4x^2-4x+1}}{7(2x-1)} + \frac{1}{7}\sqrt[3]{\frac{2}{7}}\log\left(\left(4x^2-4x+1\right)^{2/3}+2\sqrt[3]{7}x-\sqrt[3]{7}\right) - \frac{\log\left(\left(4x^2-4x+1\right)^{2/3}+7^{2/3}\sqrt[3]{4x^2-4x+1}-2\sqrt[3]{7}x+\sqrt[3]{7}\right)}{7\,2^{2/3}\sqrt[3]{7}} + \frac{1}{7}\sqrt[3]{\frac{2}{7}}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(4x^2-4x+1\right)^{2/3}}{\left(4x^2-4x+1\right)^{2/3}-4\sqrt[3]{7}x+2\sqrt[3]{7}}\right) - \frac{2}{21}\sqrt[3]{\frac{2}{7}}\log(2x-1)
$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[2/((3 + x)*(2 - 8*x + 8*x^2)^(2/3)),x]

[Out] (-3*2^(1/3)*(1 - 4*x + 4*x^2)^(1/3))/(7*(-1 + 2*x)) + ((2/7)^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - 4*x + 4*x^2)^(2/3))/(2*7^(1/3) - 4*7^(1/3)*x + (1 - 4*x + 4*x^2)^(2/3))]])/7 - (2*(2/7)^(1/3)*Log[-1 + 2*x])/21 + ((2/7)^(1/3)*Log[-7^(1/3) + 2*7^(1/3)*x + (1 - 4*x + 4*x^2)^(2/3)])/7 - Log[7^(1/3) - 2*7^(1/3)*x + 7^(2/3)*(1 - 4*x + 4*x^2)^(1/3) + (1 - 4*x + 4*x^2)^(2/3)]/(7*2^(2/3)*7^(1/3))

fricas [A] time = 0.54, size = 220, normalized size = 0.99

$$\frac{2\cdot 7^{\frac{2}{3}}\sqrt{3}\,2^{\frac{1}{3}}(2x-1)\arctan\left(-\frac{7^{\frac{1}{6}}\sqrt{3}\left(7^{\frac{5}{6}}(2x-1)-7\cdot 7^{\frac{1}{6}}2^{\frac{2}{3}}\left(8x^2-8x+2\right)^{\frac{1}{3}}\right)}{21(2x-1)}\right)-7^{\frac{2}{3}}2^{\frac{1}{3}}(2x-1)\log\left(-\frac{7^{\frac{2}{3}}2^{\frac{1}{3}}\left(8x^2-8x+2\right)^{\frac{1}{3}}(2x-1)-7^{\frac{1}{3}}2^{\frac{2}{3}}\left(4x^2-4x+1\right)-7\left(8x^2-8x+2\right)^{\frac{2}{3}}}{4x^2-4x+1}\right)+2\cdot 7^{\frac{2}{3}}2^{\frac{1}{3}}(2x-1)\log\left(\frac{7^{\frac{2}{3}}2^{\frac{1}{3}}(2x-1)+7\left(8x^2-8x+2\right)^{\frac{1}{3}}}{2x-1}\right)-42\left(8x^2-8x+2\right)^{\frac{1}{3}}}{98(2x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(3+x)/(8*x^2-8*x+2)^(2/3),x, algorithm="fricas")

[Out] 1/98*(2*7^(2/3)*sqrt(3)*2^(1/3)*(2*x - 1)*arctan(-1/21*7^(1/6)*sqrt(3)*(7^(5/6)*(2*x - 1) - 7*7^(1/6)*2^(2/3)*(8*x^2 - 8*x + 2)^(1/3))/(2*x - 1)) - 7^(2/3)*2^(1/3)*(2*x - 1)*log(-(7^(2/3)*2^(1/3)*(8*x^2 - 8*x + 2)^(1/3)*(2*x - 1) - 7^(1/3)*2^(2/3)*(4*x^2 - 4*x + 1) - 7*(8*x^2 - 8*x + 2)^(2/3))/(4*x^2 - 4*x + 1)) + 2*7^(2/3)*2^(1/3)*(2*x - 1)*log((7^(2/3)*2^(1/3)*(2*x - 1) + 7*(8*x^2 - 8*x + 2)^(1/3))/(2*x - 1)) - 42*(8*x^2 - 8*x + 2)^(1/3))/(2*x - 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2}{\left(8x^2-8x+2\right)^{\frac{2}{3}}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(3+x)/(8*x^2-8*x+2)^(2/3),x, algorithm="giac")

[Out] integrate(2/((8*x^2 - 8*x + 2)^(2/3)*(x + 3)), x)

maple [A] time = 0.04, size = 116, normalized size = 0.52

$$-\frac{3(-1+2x)2^{\frac{1}{3}}}{7\left((-1+2x)^2\right)^{\frac{2}{3}}}+\frac{\left(\frac{7^{\frac{2}{3}}\ln\left((-1+2x)^{\frac{1}{3}}+7^{\frac{1}{3}}\right)}{49}-\frac{7^{\frac{2}{3}}\ln\left((-1+2x)^{\frac{2}{3}}-7^{\frac{1}{3}}(-1+2x)^{\frac{1}{3}}+7^{\frac{2}{3}}\right)}{98}-\frac{\sqrt{3}\,7^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,7^{\frac{2}{3}}(-1+2x)^{\frac{1}{3}}}{7}-1\right)}{3}\right)}{49}\right)2^{\frac{1}{3}}(-1+2x)^{\frac{4}{3}}}{\left((-1+2x)^2\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2/(3+x)/(8*x^2-8*x+2)^(2/3),x)

[Out] -3/7*(-1+2*x)*2^(1/3)/((-1+2*x)^2)^(2/3)+(1/49*7^(2/3)*ln((-1+2*x)^(1/3)+7^(1/3))-1/98*7^(2/3)*ln((-1+2*x)^(2/3)-7^(1/3)*(-1+2*x)^(1/3)+7^(2/3))-1/49*3^(1/2)*7^(2/3)*arctan(1/3*3^(1/2)*(2/7*7^(2/3)*(-1+2*x)^(1/3)-1)))*2^(1/3)/((-1+2*x)^2)^(2/3)*(-1+2*x)^(4/3)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\int \frac{1}{\left(8x^2-8x+2\right)^{\frac{2}{3}}(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3+x)/(8*x^2-8*x+2)^(2/3),x, algorithm="maxima")
```

```
[Out] 2*integrate(1/((8*x^2 - 8*x + 2)^(2/3)*(x + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{2}{(x+3)\left(8x^2-8x+2\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/((x + 3)*(8*x^2 - 8*x + 2)^(2/3)),x)
```

```
[Out] int(2/((x + 3)*(8*x^2 - 8*x + 2)^(2/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\sqrt[3]{2}\int \frac{1}{x\left(4x^2-4x+1\right)^{\frac{2}{3}}+3\left(4x^2-4x+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3+x)/(8*x**2-8*x+2)**(2/3),x)
```

```
[Out] 2**(1/3)*Integral(1/(x*(4*x**2 - 4*x + 1)**(2/3) + 3*(4*x**2 - 4*x + 1)**(2
/3)), x)
```

3.2068 $\int \frac{1-(-3+2k)x-(4+k)x^2+3kx^3}{\sqrt[3]{(1-x)x(1-kx)}\left(-1+(5+b)x-(10+bk)x^2+10x^3-5x^4+x^5\right)}dx$

Optimal. Leaf size=222

$$\frac{\log\left(-\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+x^2-2x+1\right)}{b^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+2x^2-4x+2}\right)}{b^{2/3}}-\frac{\log\left(b^{2/3}\left(kx^3+(-\right.\right.}{}$$

Rubi [F] time = 13.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1-(-3+2k)x-(4+k)x^2+3kx^3}{\sqrt[3]{(1-x)x(1-kx)}\left(-1+(5+b)x-(10+bk)x^2+10x^3-5x^4+x^5\right)}dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - (-3 + 2*k)*x - (4 + k)*x^2 + 3*k*x^3)/(((1 - x)*x*(1 - k*x))^(1/3)
*(-1 + (5 + b)*x - (10 + b*k)*x^2 + 10*x^3 - 5*x^4 + x^5)),x]
```

```
[Out] (-6*(2 - k)*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][(
x^4*(1 - x^3)^(2/3))/((1 - k*x^3)^(1/3)*(1 - 5*(1 + b/5)*x^3 + 10*(1 + (b*k
)/10)*x^6 - 10*x^9 + 5*x^12 - x^15)), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x)
)^(1/3) + (9*k*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int
][(x^7*(1 - x^3)^(2/3))/((1 - k*x^3)^(1/3)*(1 - 5*(1 + b/5)*x^3 + 10*(1 + (
b*k)/10)*x^6 - 10*x^9 + 5*x^12 - x^15)), x], x, x^(1/3)])/((1 - x)*x*(1 - k
*x))^(1/3) + (3*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[In
t][(x*(1 - x^3)^(2/3))/((1 - k*x^3)^(1/3)*(-1 + 5*(1 + b/5)*x^3 - 10*(1 + (
b*k)/10)*x^6 + 10*x^9 - 5*x^12 + x^15)), x], x, x^(1/3)])/((1 - x)*x*(1 - k
*x))^(1/3)
```

Rubi steps

$$\int \frac{1-(-3+2k)x-(4+k)x^2+3kx^3}{\sqrt[3]{(1-x)x(1-kx)}\left(-1+(5+b)x-(10+bk)x^2+10x^3-5x^4+x^5\right)}dx = \frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{}{\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{}}}{\sqrt[3]{(1-x}}$$

$$= \frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{}{\sqrt[3]{x}\sqrt[3]{1-kx}(}}{\sqrt[3]{(1-x)x}}$$

$$= \frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}\left(\int\left(\right.\right.}{}$$

$$= \frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}\left(\int\right.}{}$$

Mathematica [F] time = 4.36, size = 0, normalized size = 0.00

$$\int \frac{1-(-3+2k)x-(4+k)x^2+3kx^3}{\sqrt[3]{(1-x)x(1-kx)}\left(-1+(5+b)x-(10+bk)x^2+10x^3-5x^4+x^5\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - (-3 + 2*k)*x - (4 + k)*x^2 + 3*k*x^3)/(((1 - x)*x*(1 - k*x))
^(1/3)*(-1 + (5 + b)*x - (10 + b*k)*x^2 + 10*x^3 - 5*x^4 + x^5)),x]
```

```
[Out] Integrate[(1 - (-3 + 2*k)*x - (4 + k)*x^2 + 3*k*x^3)/(((1 - x)*x*(1 - k*x))
^(1/3)*(-1 + (5 + b)*x - (10 + b*k)*x^2 + 10*x^3 - 5*x^4 + x^5)), x]
```

IntegrateAlgebraic [A] time = 3.02, size = 222, normalized size = 1.00

$$\frac{\log\left(-\sqrt[3]{b}\,\sqrt[3]{kx^3+(-k-1)x^2+x}+x^2-2x+1\right)}{b^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{\sqrt[3]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+2x^2-4x+2}\right)}{b^{2/3}}-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\left(\sqrt[3]{b}x^2-2\sqrt[3]{b}x+\sqrt[3]{b}\right)\sqrt[3]{kx^3+(-k-1)x^2+x}+x^4-4x^3+6x^2-4x+1\right)}{2b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - (-3 + 2*k)*x - (4 + k)*x^2 + 3*k*x^3)/(((1 - x)*x*(
1 - k*x))^(1/3)*(-1 + (5 + b)*x - (10 + b*k)*x^2 + 10*x^3 - 5*x^4 + x^5)),x
]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(2 - 4*x
 + 2*x^2 + b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))])/b^(2/3) + Log[1 - 2*
x + x^2 - b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3)]/b^(2/3) - Log[1 - 4*x +
 6*x^2 - 4*x^3 + x^4 + (b^(1/3) - 2*b^(1/3)*x + b^(1/3)*x^2)*(x + (-1 - k)*
x^2 + k*x^3)^(1/3) + b^(2/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-3+2*k)*x-(4+k)*x^2+3*k*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(-1+(5+
b)*x-(b*k+10)*x^2+10*x^3-5*x^4+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,kx^3-(k+4)x^2-(2\,k-3)x+1}{\left(x^5-5\,x^4-(bk+10)x^2+10\,x^3+(b+5)x-1\right)\left((kx-1)(x-1)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-3+2*k)*x-(4+k)*x^2+3*k*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(-1+(5+
b)*x-(b*k+10)*x^2+10*x^3-5*x^4+x^5),x, algorithm="giac")
```

```
[Out] integrate((3*k*x^3 - (k + 4)*x^2 - (2*k - 3)*x + 1)/((x^5 - 5*x^4 - (b*k +
10)*x^2 + 10*x^3 + (b + 5)*x - 1)*((k*x - 1)*(x - 1)*x)^(1/3)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1-(-3+2k)\,x-(4+k)\,x^2+3k\,x^3}{\left((1-x)\,x\,(-kx+1)\right)^{\frac{1}{3}}\left(-1+(5+b)\,x-(bk+10)\,x^2+10x^3-5x^4+x^5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(-3+2*k)*x-(4+k)*x^2+3*k*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(-1+(5+b)*x-(
b*k+10)*x^2+10*x^3-5*x^4+x^5),x)
```

```
[Out] int((1-(-3+2*k)*x-(4+k)*x^2+3*k*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(-1+(5+b)*x-(
b*k+10)*x^2+10*x^3-5*x^4+x^5),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,kx^3 - (k+4)x^2 - (2\,k-3)x + 1}{\left(x^5 - 5\,x^4 - (bk+10)x^2 + 10\,x^3 + (b+5)x - 1\right)((kx-1)(x-1)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-3+2*k)*x-(4+k)*x^2+3*k*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(-1+(5+
b)*x-(b*k+10)*x^2+10*x^3-5*x^4+x^5),x, algorithm="maxima")
```

```
[Out] integrate((3*k*x^3 - (k + 4)*x^2 - (2*k - 3)*x + 1)/((x^5 - 5*x^4 - (b*k +
10)*x^2 + 10*x^3 + (b + 5)*x - 1)*((k*x - 1)*(x - 1)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{-3\,k\,x^3 + (k+4)\,x^2 + (2\,k-3)\,x - 1}{(x\,(k\,x-1)\,(x-1))^{1/3}\,\left(x^5 - 5\,x^4 + 10\,x^3 + (-b\,k-10)\,x^2 + (b+5)\,x - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(2*k - 3) + x^2*(k + 4) - 3*k*x^3 - 1)/((x*(k*x - 1)*(x - 1))^(1/3)
*(x*(b + 5) - x^2*(b*k + 10) + 10*x^3 - 5*x^4 + x^5 - 1)),x)
```

```
[Out] int(-(x*(2*k - 3) + x^2*(k + 4) - 3*k*x^3 - 1)/((x*(k*x - 1)*(x - 1))^(1/3)
*(x*(b + 5) - x^2*(b*k + 10) + 10*x^3 - 5*x^4 + x^5 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\left(3kx^2 + 2kx - 4x - 1\right)}{\sqrt[3]{x\,(x-1)\,(kx-1)}\,\left(-bkx^2 + bx + x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-3+2*k)*x-(4+k)*x**2+3*k*x**3)/((1-x)*x*(-k*x+1))**(1/3)/(-1+
(5+b)*x-(b*k+10)*x**2+10*x**3-5*x**4+x**5),x)
```

```
[Out] Integral((x - 1)*(3*k*x**2 + 2*k*x - 4*x - 1)/((x*(x - 1)*(k*x - 1))**(1/3)
*(-b*k*x**2 + b*x + x**5 - 5*x**4 + 10*x**3 - 10*x**2 + 5*x - 1)), x)
```

3.2069 $\int \frac{1}{x\sqrt[3]{(1+x)(q+2qx+x^2)}}\,dx$

Optimal. Leaf size=223

$$-\frac{\log\left(q^{2/3}x^2+2q^{2/3}x+q^{2/3}+\left((2q+1)x^2+3qx+q+x^3\right)^{2/3}+\left(\sqrt[3]{q}\,x+\sqrt[3]{q}\right)\sqrt[3]{(2q+1)x^2+3qx+q+x^3}\right)}{4\sqrt[3]{q}}+\frac{\log(}{}$$

Rubi [F] time = 22.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt[3]{(1+x)\left(q+2qx+x^2\right)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*((1 + x)*(q + 2*q*x + x^2))^(1/3)),x]
```

```
[Out] ((1 + 2*q + (1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 - 3*Sqrt[3]*Sqrt[-
((-1 + q)^3*q)])^(2/3))/(1 + 6*q - 15*q^2 + 8*q^3 - 3*Sqrt[3]*Sqrt[-((-1 +
q)^3*q)])^(1/3) + 3*x)^(1/3)*(-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(1
 + 6*q - 15*q^2 + 8*q^3 - 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q - 1
5*q^2 + 8*q^3 - 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) - (3*(1 - 5*q + 4*q^2 +
(1 + 6*q - 15*q^2 + 8*q^3 - 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*((1 + 2*q)/
3 + x))/(1 + 6*q - 15*q^2 + 8*q^3 - 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3) + 9*
((1 + 2*q)/3 + x)^2)^(1/3)*Defer[Subst][Defer[Int][1/(((-1 - 2*q)/3 + x)*((
1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 - 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^
(2/3))/(3*(1 + 6*q - 15*q^2 + 8*q^3 - 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3)) +
 x)^(1/3)*((-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(1 + 6*q - 15*q^2 +
8*q^3 - 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q - 15*q^2 + 8*q^3 - 3*
Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))/9 - ((1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2
 + 8*q^3 - 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*x)/(3*(1 + 6*q - 15*q^2 + 8*
q^3 - 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3)) + x^2)^(1/3)), x], x, (1 + 2*q)/3
 + x])/(3*(q + 3*q*x + (1 + 2*q)*x^2 + x^3)^(1/3))
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{(1+x)\left(q+2qx+x^2\right)}}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{3}(-1-2q)+x\right)\sqrt[3]{\frac{2}{27}(1-q)^2(1+8q)-\frac{1}{3}(1-4q)(1-q)x+x^3}}\,dx, x\right.$$

$$=\frac{\left(\sqrt[3]{1+2q+\frac{1-5q+4q^2+\left(1+6q-15q^2+8q^3-3\sqrt{3}\sqrt{-(-1+q)^3q}\right)^{2/3}}{\sqrt[3]{1+6q-15q^2+8q^3-3\sqrt{3}\sqrt{-(-1+q)^3q}}}+3x}\sqrt[3]{-1+5q-4q^2+}\right.}{}$$

Mathematica [C] time = 0.23, size = 55, normalized size = 0.25

$$-\frac{3\left((x+1)\left(2qx+q+x^2\right)\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x^2+2qx+q}{q(x+1)^2}\right)}{4q(x+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*((1 + x)*(q + 2*q*x + x^2))^(1/3)),x]
```

```
[Out] (-3*((1 + x)*(q + 2*q*x + x^2))^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (q + 2
*q*x + x^2)/(q*(1 + x)^2)])/(4*q*(1 + x)^2)
```

IntegrateAlgebraic [A] time = 0.53, size = 223, normalized size = 1.00

$$-\frac{\log\left(q^{2/3}x^2+2q^{2/3}x+q^{2/3}+\left((2q+1)x^2+3qx+q+x^3\right)^{2/3}+\left(\sqrt[3]{q}\,x+\sqrt[3]{q}\right)\sqrt[3]{(2q+1)x^2+3qx+q+x^3}\right)}{4\sqrt[3]{q}}+\frac{\log\left(\sqrt[3]{(2q+1)x^2+3qx+q+x^3}-\sqrt[3]{q}\,x-\sqrt[3]{q}\right)}{2\sqrt[3]{q}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{(2q+1)x^2+3qx+q+x^3}}{\sqrt[3]{(2q+1)x^2+3qx+q+x^3}+2\sqrt[3]{q}\,x+2\sqrt[3]{q}}\right)}{2\sqrt[3]{q}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*((1 + x)*(q + 2*q*x + x^2))^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(q + 3*q*x + (1 + 2*q)*x^2 + x^3)^(1/3))/(2*q^(1/3
) + 2*q^(1/3)*x + (q + 3*q*x + (1 + 2*q)*x^2 + x^3)^(1/3))])/(2*q^(1/3)) +
Log[-q^(1/3) - q^(1/3)*x + (q + 3*q*x + (1 + 2*q)*x^2 + x^3)^(1/3)]/(2*q^(1
/3)) - Log[q^(2/3) + 2*q^(2/3)*x + q^(2/3)*x^2 + (q^(1/3) + q^(1/3)*x)*(q +
 3*q*x + (1 + 2*q)*x^2 + x^3)^(1/3) + (q + 3*q*x + (1 + 2*q)*x^2 + x^3)^(2/
3)]/(4*q^(1/3))
```

fricas [B] time = 46.43, size = 1383, normalized size = 6.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((1+x)*(2*q*x+x^2+q))^(1/3),x, algorithm="fricas")
```

```
[Out] [1/12*(sqrt(3)*q*sqrt(-1/q^(2/3))*log(-((q^3 - 30*q^2 - 51*q - 1)*x^6 - 54*
(q^3 + 6*q^2 + 2*q)*x^5 - 27*(17*q^3 + 26*q^2 + 2*q)*x^4 - 486*q^3*x - 540*
(2*q^3 + q^2)*x^3 - 81*q^3 - 135*(8*q^3 + q^2)*x^2 - 9*((2*q^2 - q - 1)*x^4
 + 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*((2*q + 1)*x^2 + x^3 + 3*q*x + q)^(2/
3)*q^(1/3) + 9*((q^2 + 7*q + 1)*x^5 + (19*q^2 + 25*q + 1)*x^4 + 9*(7*q^2 +
3*q)*x^3 + 45*q^2*x + 9*(9*q^2 + q)*x^2 + 9*q^2)*((2*q + 1)*x^2 + x^3 + 3*q
*x + q)^(1/3)*q^(2/3) - sqrt(3)*(3*((4*q^2 + 13*q + 1)*x^4 + 6*(7*q^2 + 5*q
)*x^3 + 72*q^2*x + 3*(31*q^2 + 5*q)*x^2 + 18*q^2)*((2*q + 1)*x^2 + x^3 + 3*
q*x + q)^(2/3)*q^(2/3) + 3*((q^3 - 5*q^2 - 5*q)*x^5 - 5*(q^3 + 7*q^2 + q)*x
^4 - 45*q^3*x - 45*(q^3 + q^2)*x^3 - 9*q^3 - 15*(5*q^3 + q^2)*x^2)*((2*q +
1)*x^2 + x^3 + 3*q*x + q)^(1/3) - ((q^3 + 24*q^2 + 3*q - 1)*x^6 + 54*(q^3 +
 2*q^2)*x^5 + 81*(3*q^3 + 2*q^2)*x^4 + 162*q^3*x + 108*(4*q^3 + q^2)*x^3 +
27*q^3 + 27*(14*q^3 + q^2)*x^2)*q^(1/3))*sqrt(-1/q^(2/3)))/x^6) + 2*q^(2/3)
*log(((q - 1)*q^(2/3)*x^2 - 3*((2*q + 1)*x^2 + x^3 + 3*q*x + q)^(1/3)*(q*x
+ q)*q^(1/3) + 3*((2*q + 1)*x^2 + x^3 + 3*q*x + q)^(2/3)*q)/x^2) - q^(2/3)*
log((3*((2*q + 1)*x^2 + x^3 + 3*q*x + q)^(2/3)*((2*q + 1)*x^2 + 6*q*x + 3*q
)*q^(2/3) + 3*((q^2 + 2*q)*x^3 + 9*q^2*x + (7*q^2 + 2*q)*x^2 + 3*q^2)*((2*q
 + 1)*x^2 + x^3 + 3*q*x + q)^(1/3) + ((q^2 + 7*q + 1)*x^4 + 18*(q^2 + q)*x^
3 + 36*q^2*x + 9*(5*q^2 + q)*x^2 + 9*q^2)*q^(1/3))/x^4))/q, -1/12*(2*sqrt(3
)*q^(2/3)*arctan(1/3*sqrt(3)*(6*((2*q^2 - q - 1)*x^4 + 6*(q^2 - q)*x^3 + 3*
(q^2 - q)*x^2)*((2*q + 1)*x^2 + x^3 + 3*q*x + q)^(2/3)*q^(2/3) - 6*((q^3 +
7*q^2 + q)*x^5 + (19*q^3 + 25*q^2 + q)*x^4 + 45*q^3*x + 9*(7*q^3 + 3*q^2)*x
^3 + 9*q^3 + 9*(9*q^3 + q^2)*x^2)*((2*q + 1)*x^2 + x^3 + 3*q*x + q)^(1/3) +
 ((q^3 - 12*q^2 - 15*q - 1)*x^6 - 18*(q^3 + 6*q^2 + 2*q)*x^5 - 9*(17*q^3 +
26*q^2 + 2*q)*x^4 - 162*q^3*x - 180*(2*q^3 + q^2)*x^3 - 27*q^3 - 45*(8*q^3
+ q^2)*x^2)*q^(1/3))/(((q^3 + 24*q^2 + 3*q - 1)*x^6 + 54*(q^3 + 2*q^2)*x^5
+ 81*(3*q^3 + 2*q^2)*x^4 + 162*q^3*x + 108*(4*q^3 + q^2)*x^3 + 27*q^3 + 27*
(14*q^3 + q^2)*x^2)*q^(1/3))) - 2*q^(2/3)*log(((q - 1)*q^(2/3)*x^2 - 3*((2*
q + 1)*x^2 + x^3 + 3*q*x + q)^(1/3)*(q*x + q)*q^(1/3) + 3*((2*q + 1)*x^2 +
x^3 + 3*q*x + q)^(2/3)*q)/x^2) + q^(2/3)*log((3*((2*q + 1)*x^2 + x^3 + 3*q*
x + q)^(2/3)*((2*q + 1)*x^2 + 6*q*x + 3*q)*q^(2/3) + 3*((q^2 + 2*q)*x^3 + 9
*q^2*x + (7*q^2 + 2*q)*x^2 + 3*q^2)*((2*q + 1)*x^2 + x^3 + 3*q*x + q)^(1/3)
```

```
+ ((q^2 + 7*q + 1)*x^4 + 18*(q^2 + q)*x^3 + 36*q^2*x + 9*(5*q^2 + q)*x^2 +
9*q^2)*q^(1/3))/x^4))/q]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(2\,qx + x^2 + q\right)(x+1)\right)^{\frac{1}{3}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((1+x)*(2*q*x+x^2+q))^(1/3),x, algorithm="giac")
[Out] integrate(1/(((2*q*x + x^2 + q)*(x + 1))^(1/3)*x), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left((1+x)\left(2qx + x^2 + q\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((1+x)*(2*q*x+x^2+q))^(1/3),x)
[Out] int(1/x/((1+x)*(2*q*x+x^2+q))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(2\,qx + x^2 + q\right)(x+1)\right)^{\frac{1}{3}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((1+x)*(2*q*x+x^2+q))^(1/3),x, algorithm="maxima")
[Out] integrate(1/(((2*q*x + x^2 + q)*(x + 1))^(1/3)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left((x+1)\,\left(x^2 + 2\,q\,x + q\right)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((x + 1)*(q + 2*q*x + x^2))^(1/3)),x)
[Out] int(1/(x*((x + 1)*(q + 2*q*x + x^2))^(1/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((1+x)*(2*q*x+x**2+q))**(1/3),x)
[Out] Timed out
```

3.2070 $\int \frac{-1+2x+\left(-2k+k^2\right)x^2}{((1-x)x(1-kx))^{2/3}\left(1-(b+2k)x+\left(b+k^2\right)x^2\right)}\,dx$

Optimal. Leaf size=223

$$-\frac{\log\left(\left(b^{2/3}x-b^{2/3}x^2\right)\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{4/3}+bx^4-2bx^3+bx^2\right)}{2\sqrt[3]{b}}+\frac{\log\left(\sqrt[6]{b}\right.}{}$$

Rubi [F] time = 3.60, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+2x+\left(-2k+k^2\right)x^2}{((1-x)x(1-kx))^{2/3}\left(1-(b+2k)x+\left(b+k^2\right)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 2*x + (-2*k + k^2)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(1 - (b + 2*
k)*x + (b + k^2)*x^2)),x]
```

```
[Out] ((2 - k*(1 + Sqrt[-4 + b + 4*k]/Sqrt[b]))*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(
2/3)*Defer[Int][(1 - k*x)^(1/3)/((1 - x)^(2/3)*x^(2/3)*(-b - 2*k - Sqrt[b]*
Sqrt[-4 + b + 4*k] + 2*(b + k^2)*x)), x])/((1 - x)*x*(1 - k*x))^(2/3) + ((2
 - k*(1 - Sqrt[-4 + b + 4*k]/Sqrt[b]))*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3
)*Defer[Int][(1 - k*x)^(1/3)/((1 - x)^(2/3)*x^(2/3)*(-b - 2*k + Sqrt[b]*Sqr
t[-4 + b + 4*k] + 2*(b + k^2)*x)), x])/((1 - x)*x*(1 - k*x))^(2/3)
```

Rubi steps

$$\int \frac{-1+2x+\left(-2k+k^2\right)x^2}{((1-x)x(1-kx))^{2/3}\left(1-(b+2k)x+\left(b+k^2\right)x^2\right)}\,dx = \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{-1+2x+\left(-2k+k^2\right)x}{(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\left(1-(b+2k)\right.}}{((1-x)x(1-kx))^{2/3}}$$

$$= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{(-1+(2-k)x)\sqrt[3]{1-kx}}{(1-x)^{2/3}x^{2/3}\left(1-(b+2k)x+\left(b+k^2\right.\right.}}{((1-x)x(1-kx))^{2/3}}$$

$$= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int\left(\frac{\left(2-k-\frac{k\sqrt{-4+b+4k}}{\sqrt{b}}\right)}{(1-x)^{2/3}x^{2/3}\left(-b-2k-\sqrt{b}\sqrt{-4+}\right.}\right.}{((1-x}$$

$$= \frac{\left(\left(2-k\left(1-\frac{\sqrt{-4+b+4k}}{\sqrt{b}}\right)\right)(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int}{((1-x)x(1-kx)}$$

Mathematica [F] time = 4.67, size = 0, normalized size = 0.00

$$\int \frac{-1+2x+\left(-2k+k^2\right)x^2}{((1-x)x(1-kx))^{2/3}\left(1-(b+2k)x+\left(b+k^2\right)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + 2*x + (-2*k + k^2)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(1 - (
b + 2*k)*x + (b + k^2)*x^2)),x]
```

```
[Out] Integrate[(-1 + 2*x + (-2*k + k^2)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(1 - (
b + 2*k)*x + (b + k^2)*x^2)), x]
```

IntegrateAlgebraic [A] time = 1.00, size = 223, normalized size = 1.00

$$-\frac{\log\left(\left(b^{2/3}x-b^{2/3}x^2\right)\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{4/3}+bx^4-2bx^3+bx^2\right)}{2\sqrt[3]{b}}+\frac{\log\left(\sqrt[6]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\sqrt{b}x^2-\sqrt{b}x\right)}{\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}}{-2\sqrt[3]{b}x^2+2\sqrt[3]{b}x+\left(kx^3+(-k-1)x^2+x\right)^{2/3}}\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*x + (-2*k + k^2)*x^2)/(((1 - x)*x*(1 - k*x))^(2/
3)*(1 - (b + 2*k)*x + (b + k^2)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(x + (-1 - k)*x^2 + k*x^3)^(2/3))/(2*b^(1/3)*x - 2
*b^(1/3)*x^2 + (x + (-1 - k)*x^2 + k*x^3)^(2/3))])/b^(1/3) + Log[-(Sqrt[b]*
x) + Sqrt[b]*x^2 + b^(1/6)*(x + (-1 - k)*x^2 + k*x^3)^(2/3)]/b^(1/3) - Log[
b*x^2 - 2*b*x^3 + b*x^4 + (b^(2/3)*x - b^(2/3)*x^2)*(x + (-1 - k)*x^2 + k*x
^3)^(2/3) + b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(4/3)]/(2*b^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x+(k^2-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-(b+2*k)*x+(k^2
+b)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(k^2-2\,k\right)x^2+2\,x-1}{\left((kx-1)(x-1)x\right)^{\frac{2}{3}}\left(\left(k^2+b\right)x^2-(b+2\,k)x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x+(k^2-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-(b+2*k)*x+(k^2
+b)*x^2),x, algorithm="giac")
```

```
[Out] integrate(((k^2 - 2*k)*x^2 + 2*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k^2 +
b)*x^2 - (b + 2*k)*x + 1)), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{-1+2x+\left(k^2-2k\right)x^2}{\left((1-x)\,x\,(-kx+1)\right)^{\frac{2}{3}}\left(1-(b+2k)\,x+\left(k^2+b\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x+(k^2-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-(b+2*k)*x+(k^2+b)*x^
2),x)
```

```
[Out] int((-1+2*x+(k^2-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-(b+2*k)*x+(k^2+b)*x^
2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(k^2-2\,k\right)x^2+2\,x-1}{\left((kx-1)(x-1)x\right)^{\frac{2}{3}}\left(\left(k^2+b\right)x^2-(b+2\,k)x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x+(k^2-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-(b+2*k)*x+(k^2
+b)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(((k^2 - 2*k)*x^2 + 2*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k^2 +
b)*x^2 - (b + 2*k)*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(2\,k-k^2\right)\,x^2-2\,x+1}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{2/3}\,\left(\left(k^2+b\right)\,x^2+\left(-b-2\,k\right)\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(2*k - k^2) - 2*x + 1)/((x*(k*x - 1)*(x - 1))^(2/3)*(x^2*(b + k^2
) - x*(b + 2*k) + 1)),x)
```

```
[Out] int(-(x^2*(2*k - k^2) - 2*x + 1)/((x*(k*x - 1)*(x - 1))^(2/3)*(x^2*(b + k^2
) - x*(b + 2*k) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x+(k**2-2*k)*x**2)/((1-x)*x*(-k*x+1))**(2/3)/(1-(b+2*k)*x+(
k**2+b)*x**2),x)
```

```
[Out] Timed out
```

3.2071 $\int \frac{3k+\left(-2+k^2\right)x-3kx^2+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(1-d-(2+d)kx+\left(d+k^2\right)x^2+dkx^3\right)}\,dx$

Optimal. Leaf size=223

$$-\frac{\log\left(\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+kx-1\right)}{d^{2/3}}+\frac{\log\left(d^{2/3}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\right.}{2d^{2/3}}$$

Rubi [F] time = 7.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3k+\left(-2+k^2\right)x-3kx^2+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(1-d-(2+d)kx+\left(d+k^2\right)x^2+dkx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3*k + (-2 + k^2)*x - 3*k*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/
3)*(1 - d - (2 + d)*k*x + (d + k^2)*x^2 + d*k*x^3)),x]
```

```
[Out] (k*x*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*AppellF1[1/2, 1/3, 1/3, 3/2, x^2,
k^2*x^2])/(d*((1 - x^2)*(1 - k^2*x^2))^(1/3)) - ((1 - 4*d)*k*(1 - x^2)^(1/3
)*(1 - k^2*x^2)^(1/3)*Defer[Int][1/((1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*(1
- d - (2 + d)*k*x + (d + k^2)*x^2 + d*k*x^3)), x])/(d*((1 - x^2)*(1 - k^2*x
^2))^(1/3)) + (2*(k^2 - d*(1 - k^2))*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*De
fer[Int][x/((1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*(1 - d - (2 + d)*k*x + (d +
 k^2)*x^2 + d*k*x^3)), x])/(d*((1 - x^2)*(1 - k^2*x^2))^(1/3)) - (k*(4*d +
k^2)*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*Defer[Int][x^2/((1 - x^2)^(1/3)*(1
 - k^2*x^2)^(1/3)*(1 - d - (2 + d)*k*x + (d + k^2)*x^2 + d*k*x^3)), x])/(d*
((1 - x^2)*(1 - k^2*x^2))^(1/3))
```

Rubi steps

$$\int \frac{3k+(-2+k^2)x-3kx^2+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(1-d-(2+d)kx+(d+k^2)x^2+dkx^3)}\,dx = \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{3k+(-2}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,(1}}{\sqrt[3]{(1-x^2)(1-k}}$$

$$= \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{3k-(2}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,(1}}{\sqrt[3]{(1-x^2)(1-k}}$$

$$= \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \left(\frac{k}{d\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}}\right.}{\sqrt[3]{(1-}}$$

$$= -\frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{k-4dk-2(k}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}}}{d\sqrt[3]{(1-x^2)(1-}}$$

$$= \frac{kx\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,\right.}{d\sqrt[3]{(1-x^2)(1-k^2x^2)}}$$

$$= \frac{kx\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,\right.}{d\sqrt[3]{(1-x^2)(1-k^2x^2)}}$$

$$= \frac{kx\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,\right.}{d\sqrt[3]{(1-x^2)(1-k^2x^2)}}$$

Mathematica [F] time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{3k+(-2+k^2)x-3kx^2+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(1-d-(2+d)kx+(d+k^2)x^2+dkx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3*k + (-2 + k^2)*x - 3*k*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2
))^(1/3)*(1 - d - (2 + d)*k*x + (d + k^2)*x^2 + d*k*x^3)),x]
```

```
[Out] Integrate[(3*k + (-2 + k^2)*x - 3*k*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2
))^(1/3)*(1 - d - (2 + d)*k*x + (d + k^2)*x^2 + d*k*x^3)), x]
```

IntegrateAlgebraic [A] time = 5.67, size = 223, normalized size = 1.00

$$-\frac{\log\left(\sqrt[3]{d}\,\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}+kx-1\right)}{d^{2/3}}+\frac{\log\left(d^{2/3}\left(k^2x^4+(-k^2-1)x^2+1\right)^{2/3}+\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}\left(\sqrt[3]{d}-\sqrt[3]{d}\,kx\right)+k^2x^2-2kx+1\right)}{2d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}}{\sqrt[3]{d}\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}-2kx+2}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*k + (-2 + k^2)*x - 3*k*x^2 + k^2*x^3)/(((1 - x^2)*(1
- k^2*x^2))^(1/3)*(1 - d - (2 + d)*k*x + (d + k^2)*x^2 + d*k*x^3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))/(2
 - 2*k*x + d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))])/d^(2/3)) - Log[-
1 + k*x + d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3)]/d^(2/3) + Log[1 - 2
```

```
*k*x + k^2*x^2 + (d^(1/3) - d^(1/3)*k*x)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/
3) + d^(2/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*k+(k^2-2)*x-3*k*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-
d-(2+d)*k*x+(k^2+d)*x^2+d*k*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3 - 3\,kx^2 + \left(k^2 - 2\right)x + 3\,k}{\left(dkx^3 - (d+2)kx + \left(k^2 + d\right)x^2 - d + 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*k+(k^2-2)*x-3*k*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-
d-(2+d)*k*x+(k^2+d)*x^2+d*k*x^3),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^3 - 3*k*x^2 + (k^2 - 2)*x + 3*k)/((d*k*x^3 - (d + 2)*k*x +
 (k^2 + d)*x^2 - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{3k + \left(k^2 - 2\right)x - 3k\,x^2 + k^2x^3}{\left(\left(-x^2 + 1\right)\left(-k^2x^2 + 1\right)\right)^{\frac{1}{3}}\left(1 - d - (2+d)\,kx + \left(k^2 + d\right)x^2 + dk\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*k+(k^2-2)*x-3*k*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d-(2+d
)*k*x+(k^2+d)*x^2+d*k*x^3),x)
```

```
[Out] int((3*k+(k^2-2)*x-3*k*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d-(2+d
)*k*x+(k^2+d)*x^2+d*k*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3 - 3\,kx^2 + \left(k^2 - 2\right)x + 3\,k}{\left(dkx^3 - (d+2)kx + \left(k^2 + d\right)x^2 - d + 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*k+(k^2-2)*x-3*k*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-
d-(2+d)*k*x+(k^2+d)*x^2+d*k*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^3 - 3*k*x^2 + (k^2 - 2)*x + 3*k)/((d*k*x^3 - (d + 2)*k*x +
 (k^2 + d)*x^2 - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{3\,k + x\,\left(k^2 - 2\right) + k^2\,x^3 - 3\,k\,x^2}{\left(\left(x^2 - 1\right)\,\left(k^2\,x^2 - 1\right)\right)^{1/3}\,\left(d\,k\,x^3 + \left(k^2 + d\right)\,x^2 - k\,\left(d + 2\right)\,x - d + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*k + x*(k^2 - 2) + k^2*x^3 - 3*k*x^2)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/3
)*(x^2*(d + k^2) - d - k*x*(d + 2) + d*k*x^3 + 1)),x)
```

```
[Out] int((3*k + x*(k^2 - 2) + k^2*x^3 - 3*k*x^2)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/3
)*(x^2*(d + k^2) - d - k*x*(d + 2) + d*k*x^3 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*k+(k**2-2)*x-3*k*x**2+k**2*x**3)/((-x**2+1)*(-k**2*x**2+1))**(
1/3)/(1-d-(2+d)*k*x+(k**2+d)*x**2+d*k*x**3),x)
```

```
[Out] Timed out
```

3.2072 $\int \frac{3+2(1+k^2)x-(1+k^2)x^2-4k^2x^3-k^2x^4}{\left((1-x^2)(1-k^2x^2)\right)^{2/3}\left(1-d-(1+2d)x-(d+k^2)x^2+k^2x^3\right)}\,dx$

Optimal. Leaf size=223

$$-\frac{\log\left(-\sqrt[3]{d}\,x-\sqrt[3]{d}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\right)}{d^{2/3}}+\frac{\log\left(d^{2/3}x^2+2d^{2/3}x+d^{2/3}+\left(\sqrt[3]{d}\,x+\sqrt[3]{d}\right)\sqrt[3]{k^2x^4+\left(-k^2-1\right)}\right.}{2d^{2/3}}$$

Rubi [F] time = 7.94, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3+2\left(1+k^2\right)x-\left(1+k^2\right)x^2-4k^2x^3-k^2x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d-(1+2d)x-\left(d+k^2\right)x^2+k^2x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3 + 2*(1 + k^2)*x - (1 + k^2)*x^2 - 4*k^2*x^3 - k^2*x^4)/(((1 - x^2)*(
1 - k^2*x^2))^(2/3)*(1 - d - (1 + 2*d)*x - (d + k^2)*x^2 + k^2*x^3)),x]
```

```
[Out] -(((5 + d/k^2)*x*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*AppellF1[1/2, 2/3, 2/3
, 3/2, x^2, k^2*x^2])/((1 - x^2)*(1 - k^2*x^2))^(2/3)) + (3^(3/4)*Sqrt[2 +
Sqrt[3]]*Sqrt[(-1 - k^2 + 2*k^2*x^2)^2]*((-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)
*((1 - x^2)*(1 - k^2*x^2))^(1/3))*Sqrt[((-1 + k^2)^(4/3) - 2^(2/3)*k^(2/3)*
(-1 + k^2)^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3) + 2*2^(1/3)*k^(4/3)*((1 -
x^2)*(1 - k^2*x^2))^(2/3))/((1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3
)*((1 - x^2)*(1 - k^2*x^2))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(-1 +
 k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))/((1 + Sqrt[3
])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))], -7
 - 4*Sqrt[3]])/(2^(2/3)*k^(2/3)*(1 + k^2 - 2*k^2*x^2)*Sqrt[(-1 - k^2*(1 - 2
*x^2))^2]*Sqrt[((-1 + k^2)^(2/3)*((-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 -
x^2)*(1 - k^2*x^2))^(1/3)))/((1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/
3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))^2]) + ((d^2 - 8*k^2 - d*(1 - 5*k^2))*(1
 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][1/((1 - x^2)^(2/3)*(1 - k^2*x^
2)^(2/3)*(-1 + d + (1 + 2*d)*x + (d + k^2)*x^2 - k^2*x^3)), x])/(k^2*((1 -
x^2)*(1 - k^2*x^2))^(2/3)) - ((d + 2*d^2 + 2*k^2 + 11*d*k^2 - 2*k^4)*(1 - x
^2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][x/((1 - x^2)^(2/3)*(1 - k^2*x^2)^(
2/3)*(1 - d - (1 + 2*d)*x - (d + k^2)*x^2 + k^2*x^3)), x])/(k^2*((1 - x^2)*
(1 - k^2*x^2))^(2/3)) - ((d^2 + 2*k^2 + 8*d*k^2 + 6*k^4)*(1 - x^2)^(2/3)*(1
 - k^2*x^2)^(2/3)*Defer[Int][x^2/((1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*(1 -
d - (1 + 2*d)*x - (d + k^2)*x^2 + k^2*x^3)), x])/(k^2*((1 - x^2)*(1 - k^2*x
^2))^(2/3))
```

Rubi steps

$$\int \frac{3+2\left(1+k^2\right)x-\left(1+k^2\right)x^2-4k^2x^3-k^2x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d-(1+2d)x-\left(d+k^2\right)x^2+k^2x^3\right)}dx = \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int \frac{3+}{\left(1-x^2\right)^{2/3}(1}}{\left(\left(1-x^2\right)(1-\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int\left(\frac{-5}{\left(1-x^2\right)^{2/3}(}\right.}{}$$

$$= -\frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int \frac{}{\left(1-x^2\right)^{2/3}}}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}}$$

$$= -\frac{\left(5+\frac{d}{k^2}\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}F}{\left(\left(1-x^2\right)\left(1-k^2x^2\right.\right.}$$

$$= -\frac{\left(5+\frac{d}{k^2}\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}F}{\left(\left(1-x^2\right)\left(1-k^2x^2\right.\right.}$$

$$= -\frac{\left(5+\frac{d}{k^2}\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}F}{\left(\left(1-x^2\right)\left(1-k^2x^2\right.\right.}$$

$$= -\frac{\left(5+\frac{d}{k^2}\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}F}{\left(\left(1-x^2\right)\left(1-k^2x^2\right.\right.}$$

Mathematica [F] time = 2.56, size = 0, normalized size = 0.00

$$\int \frac{3+2\left(1+k^2\right)x-\left(1+k^2\right)x^2-4k^2x^3-k^2x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d-(1+2d)x-\left(d+k^2\right)x^2+k^2x^3\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3 + 2*(1 + k^2)*x - (1 + k^2)*x^2 - 4*k^2*x^3 - k^2*x^4)/(((1 -
x^2)*(1 - k^2*x^2))^(2/3)*(1 - d - (1 + 2*d)*x - (d + k^2)*x^2 + k^2*x^3)),
x]
```

```
[Out] Integrate[(3 + 2*(1 + k^2)*x - (1 + k^2)*x^2 - 4*k^2*x^3 - k^2*x^4)/(((1 -
x^2)*(1 - k^2*x^2))^(2/3)*(1 - d - (1 + 2*d)*x - (d + k^2)*x^2 + k^2*x^3)),
 x]
```

IntegrateAlgebraic [A] time = 8.17, size = 223, normalized size = 1.00

$$-\frac{\log\left(-\sqrt[3]{d}\,x-\sqrt[3]{d}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\right)}{d^{2/3}}+\frac{\log\left(d^{2/3}x^2+2d^{2/3}x+d^{2/3}+\left(\sqrt[3]{d}\,x+\sqrt[3]{d}\right)\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}\right)}{2d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\,x+\sqrt{3}\sqrt[3]{d}}{\sqrt[3]{d}\,x+\sqrt[3]{d}+2\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + 2*(1 + k^2)*x - (1 + k^2)*x^2 - 4*k^2*x^3 - k^2*x^4
)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)*(1 - d - (1 + 2*d)*x - (d + k^2)*x^2 + k
^2*x^3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3) + Sqrt[3]*d^(1/3)*x)/(d^(1/3) + d^(1/3)*
x + 2*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))])/d^(2/3)) - Log[-d^(1/3) - d^(
1/3)*x + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3)]/d^(2/3) + Log[d^(2/3) + 2*d^
(2/3)*x + d^(2/3)*x^2 + (d^(1/3) + d^(1/3)*x)*(1 + (-1 - k^2)*x^2 + k^2*x^4
)^(1/3) + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*(k^2+1)*x-(k^2+1)*x^2-4*k^2*x^3-k^2*x^4)/((-x^2+1)*(-k^2*x^2
+1))^(2/3)/(1-d-(1+2*d)*x-(k^2+d)*x^2+k^2*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{k^2x^4 + 4\,k^2x^3 + \left(k^2+1\right)x^2 - 2\left(k^2+1\right)x - 3}{\left(k^2x^3 - \left(k^2+d\right)x^2 - (2\,d+1)x - d + 1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*(k^2+1)*x-(k^2+1)*x^2-4*k^2*x^3-k^2*x^4)/((-x^2+1)*(-k^2*x^2
+1))^(2/3)/(1-d-(1+2*d)*x-(k^2+d)*x^2+k^2*x^3),x, algorithm="giac")
```

```
[Out] integrate(-(k^2*x^4 + 4*k^2*x^3 + (k^2 + 1)*x^2 - 2*(k^2 + 1)*x - 3)/((k^2*
x^3 - (k^2 + d)*x^2 - (2*d + 1)*x - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(2/3))
, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{3+2\left(k^2+1\right)x-\left(k^2+1\right)x^2-4k^2x^3-k^2x^4}{\left(\left(-x^2+1\right)\left(-k^2x^2+1\right)\right)^{\frac{2}{3}}\left(1-d-(1+2d)\,x-\left(k^2+d\right)x^2+k^2x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*(k^2+1)*x-(k^2+1)*x^2-4*k^2*x^3-k^2*x^4)/((-x^2+1)*(-k^2*x^2+1))^(
2/3)/(1-d-(1+2*d)*x-(k^2+d)*x^2+k^2*x^3),x)
```

```
[Out] int((3+2*(k^2+1)*x-(k^2+1)*x^2-4*k^2*x^3-k^2*x^4)/((-x^2+1)*(-k^2*x^2+1))^(
2/3)/(1-d-(1+2*d)*x-(k^2+d)*x^2+k^2*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{k^2x^4 + 4\,k^2x^3 + \left(k^2+1\right)x^2 - 2\left(k^2+1\right)x - 3}{\left(k^2x^3 - \left(k^2+d\right)x^2 - (2\,d+1)x - d + 1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*(k^2+1)*x-(k^2+1)*x^2-4*k^2*x^3-k^2*x^4)/((-x^2+1)*(-k^2*x^2
+1))^(2/3)/(1-d-(1+2*d)*x-(k^2+d)*x^2+k^2*x^3),x, algorithm="maxima")
```

```
[Out] -integrate((k^2*x^4 + 4*k^2*x^3 + (k^2 + 1)*x^2 - 2*(k^2 + 1)*x - 3)/((k^2*
x^3 - (k^2 + d)*x^2 - (2*d + 1)*x - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(2/3))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{4\,k^2\,x^3-2\,x\,\left(k^2+1\right)+k^2\,x^4+x^2\,\left(k^2+1\right)-3}{{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)}^{2/3}\,\left(d-k^2\,x^3+x^2\,\left(k^2+d\right)+x\,\left(2\,d+1\right)-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*k^2*x^3 - 2*x*(k^2 + 1) + k^2*x^4 + x^2*(k^2 + 1) - 3)/(((x^2 - 1)*(
k^2*x^2 - 1))^(2/3)*(d - k^2*x^3 + x^2*(d + k^2) + x*(2*d + 1) - 1)),x)
```

```
[Out] int((4*k^2*x^3 - 2*x*(k^2 + 1) + k^2*x^4 + x^2*(k^2 + 1) - 3)/(((x^2 - 1)*(
k^2*x^2 - 1))^(2/3)*(d - k^2*x^3 + x^2*(d + k^2) + x*(2*d + 1) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*(k**2+1)*x-(k**2+1)*x**2-4*k**2*x**3-k**2*x**4)/((-x**2+1)*(
-k**2*x**2+1))**(2/3)/(1-d-(1+2*d)*x-(k**2+d)*x**2+k**2*x**3),x)
```

```
[Out] Timed out
```

3.2073 $\int \frac{\sqrt[4]{\frac{-1+x}{1+2x}}-3\left(\frac{-1+x}{1+2x}\right)^{3/4}}{(-1+x)(1+x)^2(-1+2x)}\,dx$

Optimal. Leaf size=223

$$\frac{\left(\frac{x-1}{2x+1}\right)^{3/4}(-2x-1)}{2(x+1)}+\frac{\sqrt[4]{\frac{x-1}{2x+1}}\,(2x+1)}{6(x+1)}-\frac{1}{72}\sqrt{24420+55819\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt[4]{\frac{x-1}{2x+1}}}{\sqrt[4]{2}}\right)+\frac{2}{9}\tan^{-1}\left(\frac{2\sqrt[4]{\frac{x-1}{2x+1}}}{2\sqrt{\frac{x-1}{2x+1}}-1}\right)+\frac{1}{72}\sqrt{558$$

Rubi [A] time = 1.33, antiderivative size = 373, normalized size of antiderivative = 1.67, number of steps used = 18, number of rules used = 9, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6725, 634, 617, 204, 628, 1179, 1167, 207, 203}

$$\frac{\sqrt[4]{-\frac{1-x}{2x+1}}\,(2x+1)\left(1-3\sqrt{-\frac{1-x}{2x+1}}\right)}{6(x+1)}-\frac{5}{9}\log\left(2\sqrt{-\frac{1-x}{2x+1}}-2\sqrt[4]{-\frac{1-x}{2x+1}}+1\right)+\frac{5}{9}\log\left(2\sqrt{-\frac{1-x}{2x+1}}+2\sqrt[4]{-\frac{1-x}{2x+1}}+1\right)-\frac{8}{9}\sqrt[4]{2}\left(1+3\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt[4]{-\frac{1-x}{2x+1}}}{\sqrt[4]{2}}\right)+\frac{3\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt[4]{-\frac{1-x}{2x+1}}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}+\frac{2}{9}\tan^{-1}\left(1-2\sqrt[4]{-\frac{1-x}{2x+1}}\right)-\frac{2}{9}\tan^{-1}\left(2\sqrt[4]{-\frac{1-x}{2x+1}}+1\right)+\frac{3\left(1-\sqrt{2}\right)\tanh^{-1}\left(\frac{\sqrt[4]{-\frac{1-x}{2x+1}}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}-\frac{8}{9}\sqrt[4]{2}\left(1-3\sqrt{2}\right)\tanh^{-1}\left(\frac{\sqrt[4]{-\frac{1-x}{2x+1}}}{\sqrt[4]{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(((-1 + x)/(1 + 2*x))^(1/4) - 3*((-1 + x)/(1 + 2*x))^(3/4))/((-1 + x)*(
1 + x)^2*(-1 + 2*x)),x]
```

```
[Out] ((-((1 - x)/(1 + 2*x)))^(1/4)*(1 + 2*x)*(1 - 3*Sqrt[-((1 - x)/(1 + 2*x))]))
/(6*(1 + x)) + (3*(1 + Sqrt[2])*ArcTan[(-((1 - x)/(1 + 2*x)))^(1/4)/2^(1/4)
])/(4*2^(3/4)) - (8*2^(1/4)*(1 + 3*Sqrt[2])*ArcTan[(-((1 - x)/(1 + 2*x)))^(
1/4)/2^(1/4)])/9 + (2*ArcTan[1 - 2*(-((1 - x)/(1 + 2*x)))^(1/4)])/9 - (2*Ar
cTan[1 + 2*(-((1 - x)/(1 + 2*x)))^(1/4)])/9 - (8*2^(1/4)*(1 - 3*Sqrt[2])*Ar
cTanh[(-((1 - x)/(1 + 2*x)))^(1/4)/2^(1/4)])/9 + (3*(1 - Sqrt[2])*ArcTanh[(
-((1 - x)/(1 + 2*x)))^(1/4)/2^(1/4)])/(4*2^(3/4)) - (5*Log[1 - 2*(-((1 - x)
/(1 + 2*x)))^(1/4) + 2*Sqrt[-((1 - x)/(1 + 2*x))]])/9 + (5*Log[1 + 2*(-((1
- x)/(1 + 2*x)))^(1/4) + 2*Sqrt[-((1 - x)/(1 + 2*x))]])/9
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{\frac{-1+x}{1+2x}}-3\left(\frac{-1+x}{1+2x}\right)^{3/4}}{(-1+x)(1+x)^2(-1+2x)}\,dx = -\left(4\,\text{Subst}\left(\int \frac{\left(-1+3x^2\right)\left(1-2x^4\right)^2}{\left(-2+x^4\right)^2\left(1+4x^4\right)}\,dx, x, \sqrt[4]{\frac{-1+x}{1+2x}}\right)\right)$$

$$= -\left(4\,\text{Subst}\left(\int\left(\frac{-2+5x}{9\left(1-2x+2x^2\right)}+\frac{-2-5x}{9\left(1+2x+2x^2\right)}+\frac{-1+3x^2}{\left(-2+x^4\right)^2}+\frac{8\left(-1\right.}{9\left(-2\right.}\right.\right.\right.$$

$$= -\left(\frac{4}{9}\,\text{Subst}\left(\int \frac{-2+5x}{1-2x+2x^2}\,dx, x, \sqrt[4]{\frac{-1+x}{1+2x}}\right)\right)-\frac{4}{9}\,\text{Subst}\left(\int \frac{-2-5x}{1+2x+2x^2}\right.$$

$$= \frac{\sqrt[4]{-\frac{1-x}{1+2x}}\,(1+2x)\left(1-3\sqrt{-\frac{1-x}{1+2x}}\right)}{6(1+x)}-\frac{2}{9}\,\text{Subst}\left(\int \frac{1}{1-2x+2x^2}\,dx, x, \sqrt[4]{\frac{-1+}{1+2}}\right.$$

$$= \frac{\sqrt[4]{-\frac{1-x}{1+2x}}\,(1+2x)\left(1-3\sqrt{-\frac{1-x}{1+2x}}\right)}{6(1+x)}-\frac{8}{9}\sqrt[4]{2}\left(1+3\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt[4]{-\frac{1-x}{1+2x}}}{\sqrt[4]{2}}\right)-\frac{8}{9}$$

$$= \frac{\sqrt[4]{-\frac{1-x}{1+2x}}\,(1+2x)\left(1-3\sqrt{-\frac{1-x}{1+2x}}\right)}{6(1+x)}+\frac{3\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt[4]{-\frac{1-x}{1+2x}}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}-\frac{8}{9}\sqrt[4]{2}\,\left(1\right.$$

Mathematica [C] time = 0.60, size = 398, normalized size = 1.78

$$\frac{1}{18}\left(-18\left(\frac{x-1}{2x+1}\right)^{3/4}{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};\frac{x-1}{4x+2}\right)+\frac{3\sqrt[4]{\frac{x-1}{2x+1}}(2x+1)}{x+1}+8\sqrt[4]{2}\left(1-3\sqrt{2}\right)\log\left(\sqrt[4]{2}-\sqrt[4]{\frac{x-1}{2x+1}}\right)-8i\sqrt[4]{2}\left(1+3\sqrt{2}\right)\log\left(\sqrt[4]{2}-i\sqrt[4]{\frac{x-1}{2x+1}}\right)+8i\sqrt[4]{2}\left(1+3\sqrt{2}\right)\log\left(\sqrt[4]{2}+i\sqrt[4]{\frac{x-1}{2x+1}}\right)+8\sqrt[4]{2}\left(3\sqrt{2}-1\right)\log\left(\sqrt[4]{\frac{x-1}{2x+1}}+\sqrt[4]{2}\right)-10\log\left(2\sqrt{\frac{x-1}{2x+1}}-2\sqrt[4]{\frac{x-1}{2x+1}}+1\right)+10\log\left(2\sqrt{\frac{x-1}{2x+1}}+2\sqrt[4]{\frac{x-1}{2x+1}}+1\right)+4\tan^{-1}\left(1-2\sqrt[4]{\frac{x-1}{2x+1}}\right)-4\tan^{-1}\left(2\sqrt[4]{\frac{x-1}{2x+1}}+1\right)+\frac{27\left(\tan^{-1}\left(\sqrt[4]{\frac{x-1}{4x+2}}\right)+\tanh^{-1}\left(\sqrt[4]{\frac{x-1}{4x+2}}\right)\right)}{2\,2^{3/4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(((-1 + x)/(1 + 2*x))^(1/4) - 3*((-1 + x)/(1 + 2*x))^(3/4))/((-1
+ x)*(1 + x)^2*(-1 + 2*x)),x]
```

```
[Out] ((3*((-1 + x)/(1 + 2*x))^(1/4)*(1 + 2*x))/(1 + x) + 4*ArcTan[1 - 2*((-1 + x
)/(1 + 2*x))^(1/4)] - 4*ArcTan[1 + 2*((-1 + x)/(1 + 2*x))^(1/4)] + (27*(Arc
Tan[((-1 + x)/(2 + 4*x))^(1/4)] + ArcTanh[((-1 + x)/(2 + 4*x))^(1/4)]))/(2*
2^(3/4)) - 18*((-1 + x)/(1 + 2*x))^(3/4)*Hypergeometric2F1[3/4, 2, 7/4, (-1
 + x)/(2 + 4*x)] + 8*2^(1/4)*(1 - 3*Sqrt[2])*Log[2^(1/4) - ((-1 + x)/(1 + 2
*x))^(1/4)] - (8*I)*2^(1/4)*(1 + 3*Sqrt[2])*Log[2^(1/4) - I*((-1 + x)/(1 +
2*x))^(1/4)] + (8*I)*2^(1/4)*(1 + 3*Sqrt[2])*Log[2^(1/4) + I*((-1 + x)/(1 +
 2*x))^(1/4)] + 8*2^(1/4)*(-1 + 3*Sqrt[2])*Log[2^(1/4) + ((-1 + x)/(1 + 2*x
))^(1/4)] - 10*Log[1 - 2*((-1 + x)/(1 + 2*x))^(1/4) + 2*Sqrt[(-1 + x)/(1 +
2*x)]] + 10*Log[1 + 2*((-1 + x)/(1 + 2*x))^(1/4) + 2*Sqrt[(-1 + x)/(1 + 2*x
)]])/18
```

IntegrateAlgebraic [A] time = 0.64, size = 216, normalized size = 0.97

$$\frac{3\left(\frac{x-1}{2x+1}\right)^{3/4}-\sqrt[4]{\frac{x-1}{2x+1}}}{2\left(\frac{x-1}{2x+1}-2\right)}-\frac{1}{72}\sqrt{24420+55819\sqrt{2}}\tan^{-1}\left(\frac{\sqrt[4]{\frac{x-1}{2x+1}}}{\sqrt[4]{2}}\right)-\frac{2}{9}\tan^{-1}\left(\frac{\sqrt{\frac{x-1}{2x+1}}-\frac{1}{2}}{\sqrt[4]{\frac{x-1}{2x+1}}}\right)+\frac{1}{72}\sqrt{55819\sqrt{2}-24420}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{x-1}{2x+1}}}{\sqrt[4]{2}}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt[4]{\frac{x-1}{2x+1}}}{2\sqrt{\frac{x-1}{2x+1}}+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(((-1 + x)/(1 + 2*x))^(1/4) - 3*((-1 + x)/(1 + 2*x))^(3/
4))/((-1 + x)*(1 + x)^2*(-1 + 2*x)),x]
```

```
[Out] (-((-1 + x)/(1 + 2*x))^(1/4) + 3*((-1 + x)/(1 + 2*x))^(3/4))/(2*(-2 + (-1 +
 x)/(1 + 2*x))) - (Sqrt[24420 + 55819*Sqrt[2]]*ArcTan[((-1 + x)/(1 + 2*x))^
(1/4)/2^(1/4)])/72 - (2*ArcTan[(-1/2 + Sqrt[(-1 + x)/(1 + 2*x)])/((-1 + x)/
(1 + 2*x))^(1/4)])/9 + (Sqrt[-24420 + 55819*Sqrt[2]]*ArcTanh[((-1 + x)/(1 +
 2*x))^(1/4)/2^(1/4)])/72 + (10*ArcTanh[(2*((-1 + x)/(1 + 2*x))^(1/4))/(1 +
 2*Sqrt[(-1 + x)/(1 + 2*x)]])])/9
```

fricas [A] time = 1.08, size = 350, normalized size = 1.57

$$-\frac{4(x+1)\sqrt{55819\sqrt{2}+24420}\arctan\left(\frac{1}{106162}\sqrt{55819\sqrt{2}+24420}\left(37\sqrt{2}-330\right)\sqrt{\sqrt{2}+\sqrt{\frac{x-1}{2x+1}}}-\frac{1}{106162}\sqrt{55819\sqrt{2}+24420}\left(37\sqrt{2}-330\right)\left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}\right)-(x+1)\sqrt{55819\sqrt{2}-24420}\log\left(\sqrt{55819\sqrt{2}-24420}\left(165\sqrt{2}+37\right)+53081\left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}\right)+(x+1)\sqrt{55819\sqrt{2}-24420}\log\left(-\sqrt{55819\sqrt{2}-24420}\left(165\sqrt{2}+37\right)+53081\left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}\right)+32(x+1)\arctan\left(2\left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}+1\right)+32(x+1)\arctan\left(2\left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}-1\right)-80(x+1)\log\left(2\sqrt{\frac{x-1}{2x+1}}+2\left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}+1\right)+80(x+1)\log\left(2\sqrt{\frac{x-1}{2x+1}}-2\left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}+1\right)+72(2x+1)\left(\frac{x-1}{2x+1}\right)^{\frac{3}{4}}-24(2x+1)\left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}}{144(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((-1+x)/(1+2*x))^(1/4)-3*((-1+x)/(1+2*x))^(3/4))/(-1+x)/(1+x)^2/
(-1+2*x),x, algorithm="fricas")
```

```
[Out] -1/144*(4*(x + 1)*sqrt(55819*sqrt(2) + 24420)*arctan(1/106162*sqrt(55819*sq
rt(2) + 24420)*(37*sqrt(2) - 330)*sqrt(sqrt(2) + sqrt((x - 1)/(2*x + 1))) -
 1/106162*sqrt(55819*sqrt(2) + 24420)*(37*sqrt(2) - 330)*((x - 1)/(2*x + 1)
)^(1/4)) - (x + 1)*sqrt(55819*sqrt(2) - 24420)*log(sqrt(55819*sqrt(2) - 244
20)*(165*sqrt(2) + 37) + 53081*((x - 1)/(2*x + 1))^(1/4)) + (x + 1)*sqrt(55
819*sqrt(2) - 24420)*log(-sqrt(55819*sqrt(2) - 24420)*(165*sqrt(2) + 37) +
53081*((x - 1)/(2*x + 1))^(1/4)) + 32*(x + 1)*arctan(2*((x - 1)/(2*x + 1))^
(1/4) + 1) + 32*(x + 1)*arctan(2*((x - 1)/(2*x + 1))^(1/4) - 1) - 80*(x + 1
)*log(2*sqrt((x - 1)/(2*x + 1)) + 2*((x - 1)/(2*x + 1))^(1/4) + 1) + 80*(x
+ 1)*log(2*sqrt((x - 1)/(2*x + 1)) - 2*((x - 1)/(2*x + 1))^(1/4) + 1) + 72*
(2*x + 1)*((x - 1)/(2*x + 1))^(3/4) - 24*(2*x + 1)*((x - 1)/(2*x + 1))^(1/4
))/(x + 1)
```

giac [A] time = 0.65, size = 250, normalized size = 1.12

$$-\frac{1}{72}\sqrt{55819\sqrt{2}+24420}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}\right)+\frac{1}{144}\sqrt{55819\sqrt{2}-24420}\log\left(2^{\frac{1}{4}}+\left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}\right)-\frac{1}{144}\sqrt{55819\sqrt{2}-24420}\log\left(\left|-2^{\frac{1}{4}}+\left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}\right|\right)+\frac{3\left(\frac{x-1}{2x+1}\right)^{\frac{3}{4}}-\left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}}{2\left(\frac{x-1}{2x+1}-2\right)}-\frac{2}{9}\arctan\left(2\left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}+1\right)-\frac{2}{9}\arctan\left(2\left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}-1\right)+\frac{5}{9}\log\left(2\sqrt{\frac{x-1}{2x+1}}+2\left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}+1\right)-\frac{5}{9}\log\left(2\sqrt{\frac{x-1}{2x+1}}-2\left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((-1+x)/(1+2*x))^(1/4)-3*((-1+x)/(1+2*x))^(3/4))/(-1+x)/(1+x)^2/
(-1+2*x),x, algorithm="giac")
```

```
[Out] -1/72*sqrt(55819*sqrt(2) + 24420)*arctan(1/2*2^(3/4)*((x - 1)/(2*x + 1))^(1
/4)) + 1/144*sqrt(55819*sqrt(2) - 24420)*log(2^(1/4) + ((x - 1)/(2*x + 1))^
(1/4)) - 1/144*sqrt(55819*sqrt(2) - 24420)*log(abs(-2^(1/4) + ((x - 1)/(2*x
 + 1))^(1/4))) + 1/2*(3*((x - 1)/(2*x + 1))^(3/4) - ((x - 1)/(2*x + 1))^(1/
4))/((x - 1)/(2*x + 1) - 2) - 2/9*arctan(2*((x - 1)/(2*x + 1))^(1/4) + 1) -
 2/9*arctan(2*((x - 1)/(2*x + 1))^(1/4) - 1) + 5/9*log(2*sqrt((x - 1)/(2*x
+ 1)) + 2*((x - 1)/(2*x + 1))^(1/4) + 1) - 5/9*log(2*sqrt((x - 1)/(2*x + 1)
) - 2*((x - 1)/(2*x + 1))^(1/4) + 1)
```

maple [C] time = 5.28, size = 1759, normalized size = 7.89

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((-1+x)/(1+2*x))^(1/4)-3*((-1+x)/(1+2*x))^(3/4))/(-1+x)/(1+x)^2/(-1+2*
x),x)
```

```
[Out] 1/6*(1+2*x)/(1+x)*(-(1-x)/(1+2*x))^(1/4)-1/2*(1+2*x)/(1+x)*(-(1-x)/(1+2*x))
^(3/4)-10/9*ln((8*(-(1-x)/(1+2*x))^(3/4)*x+24*RootOf(18*_Z^2+30*_Z+13)*(-(1
-x)/(1+2*x))^(1/2)*x+4*(-(1-x)/(1+2*x))^(3/4)+12*RootOf(18*_Z^2+30*_Z+13)*(
-(1-x)/(1+2*x))^(1/2)+16*(-(1-x)/(1+2*x))^(1/2)*x-24*RootOf(18*_Z^2+30*_Z+1
3)*(-(1-x)/(1+2*x))^(1/4)*x+8*(-(1-x)/(1+2*x))^(1/2)-12*RootOf(18*_Z^2+30*_
Z+13)*(-(1-x)/(1+2*x))^(1/4)-20*(-(1-x)/(1+2*x))^(1/4)*x-6*RootOf(18*_Z^2+3
0*_Z+13)*x-10*(-(1-x)/(1+2*x))^(1/4)+15*RootOf(18*_Z^2+30*_Z+13)-6*x+15)/(-
1+2*x))-2/3*ln((8*(-(1-x)/(1+2*x))^(3/4)*x+24*RootOf(18*_Z^2+30*_Z+13)*(-(1
-x)/(1+2*x))^(1/2)*x+4*(-(1-x)/(1+2*x))^(3/4)+12*RootOf(18*_Z^2+30*_Z+13)*(
-(1-x)/(1+2*x))^(1/2)+16*(-(1-x)/(1+2*x))^(1/2)*x-24*RootOf(18*_Z^2+30*_Z+1
3)*(-(1-x)/(1+2*x))^(1/4)*x+8*(-(1-x)/(1+2*x))^(1/2)-12*RootOf(18*_Z^2+30*_
Z+13)*(-(1-x)/(1+2*x))^(1/4)-20*(-(1-x)/(1+2*x))^(1/4)*x-6*RootOf(18*_Z^2+3
0*_Z+13)*x-10*(-(1-x)/(1+2*x))^(1/4)+15*RootOf(18*_Z^2+30*_Z+13)-6*x+15)/(-
1+2*x))*RootOf(18*_Z^2+30*_Z+13)+2/3*RootOf(18*_Z^2+30*_Z+13)*ln((8*(-(1-x)
/(1+2*x))^(3/4)*x-24*RootOf(18*_Z^2+30*_Z+13)*(-(1-x)/(1+2*x))^(1/2)*x+4*(-
(1-x)/(1+2*x))^(3/4)-12*RootOf(18*_Z^2+30*_Z+13)*(-(1-x)/(1+2*x))^(1/2)-24*
(-(1-x)/(1+2*x))^(1/2)*x+24*RootOf(18*_Z^2+30*_Z+13)*(-(1-x)/(1+2*x))^(1/4)
*x-12*(-(1-x)/(1+2*x))^(1/2)+12*RootOf(18*_Z^2+30*_Z+13)*(-(1-x)/(1+2*x))^(
1/4)+20*(-(1-x)/(1+2*x))^(1/4)*x+6*RootOf(18*_Z^2+30*_Z+13)*x+10*(-(1-x)/(1
+2*x))^(1/4)-15*RootOf(18*_Z^2+30*_Z+13)+4*x-10)/(-1+2*x))+2/3*RootOf(42467
328*_Z^4+225054720*_Z^2-2817592561)*ln((291962880*RootOf(42467328*_Z^4+2250
54720*_Z^2-2817592561)^3*(-(1-x)/(1+2*x))^(1/2)*x+145981440*RootOf(42467328
*_Z^4+225054720*_Z^2-2817592561)^3*(-(1-x)/(1+2*x))^(1/2)+40919040*RootOf(4
2467328*_Z^4+225054720*_Z^2-2817592561)^3*x+978388992*(-(1-x)/(1+2*x))^(1/4
)*RootOf(42467328*_Z^4+225054720*_Z^2-2817592561)^2*x+1170163776*RootOf(424
67328*_Z^4+225054720*_Z^2-2817592561)*(-(1-x)/(1+2*x))^(1/2)*x+5925856678*(
-(1-x)/(1+2*x))^(3/4)*x+8183808*RootOf(42467328*_Z^4+225054720*_Z^2-2817592
561)^3+489194496*(-(1-x)/(1+2*x))^(1/4)*RootOf(42467328*_Z^4+225054720*_Z^2
-2817592561)^2+585081888*RootOf(42467328*_Z^4+225054720*_Z^2-2817592561)*(-
(1-x)/(1+2*x))^(1/2)+2962928339*(-(1-x)/(1+2*x))^(3/4)+2318857200*RootOf(42
467328*_Z^4+225054720*_Z^2-2817592561)*x+2592476040*(-(1-x)/(1+2*x))^(1/4)*
x+463771440*RootOf(42467328*_Z^4+225054720*_Z^2-2817592561)+1296238020*(-(1
-x)/(1+2*x))^(1/4))/(1+x))+1/72*RootOf(_Z^2+2304*RootOf(42467328*_Z^4+22505
4720*_Z^2-2817592561)^2+12210)*ln(-(6082560*RootOf(_Z^2+2304*RootOf(4246732
8*_Z^4+225054720*_Z^2-2817592561)^2+12210)*(-(1-x)/(1+2*x))^(1/2)*RootOf(42
467328*_Z^4+225054720*_Z^2-2817592561)^2*x+3041280*RootOf(_Z^2+2304*RootOf(
42467328*_Z^4+225054720*_Z^2-2817592561)^2+12210)*(-(1-x)/(1+2*x))^(1/2)*Ro
otOf(42467328*_Z^4+225054720*_Z^2-2817592561)^2+978388992*(-(1-x)/(1+2*x))^
```

```
(1/4)*RootOf(42467328*_Z^4+225054720*_Z^2-2817592561)^2*x+852480*RootOf(424
67328*_Z^4+225054720*_Z^2-2817592561)^2*RootOf(_Z^2+2304*RootOf(42467328*_Z
^4+225054720*_Z^2-2817592561)^2+12210)*x-5925856678*(-(1-x)/(1+2*x))^(3/4)*
x+7855988*RootOf(_Z^2+2304*RootOf(42467328*_Z^4+225054720*_Z^2-2817592561)^
2+12210)*(-(1-x)/(1+2*x))^(1/2)*x+489194496*(-(1-x)/(1+2*x))^(1/4)*RootOf(4
2467328*_Z^4+225054720*_Z^2-2817592561)^2+170496*RootOf(42467328*_Z^4+22505
4720*_Z^2-2817592561)^2*RootOf(_Z^2+2304*RootOf(42467328*_Z^4+225054720*_Z^
2-2817592561)^2+12210)-2962928339*(-(1-x)/(1+2*x))^(3/4)+3927994*RootOf(_Z^
2+2304*RootOf(42467328*_Z^4+225054720*_Z^2-2817592561)^2+12210)*(-(1-x)/(1+
2*x))^(1/2)+2592476040*(-(1-x)/(1+2*x))^(1/4)*x-43791825*RootOf(_Z^2+2304*R
ootOf(42467328*_Z^4+225054720*_Z^2-2817592561)^2+12210)*x+1296238020*(-(1-x
)/(1+2*x))^(1/4)-8758365*RootOf(_Z^2+2304*RootOf(42467328*_Z^4+225054720*_Z
^2-2817592561)^2+12210))/(1+x))
```

maxima [A] time = 0.42, size = 243, normalized size = 1.09

$$-\frac{1}{72} \cdot 2^{\frac{1}{4}} \left(165 \sqrt{2} + 37\right) \arctan\left(\frac{1}{2} \cdot 2^{\frac{3}{4}} \left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}\right) - \frac{1}{144} \cdot 2^{\frac{1}{4}} \left(165 \sqrt{2} - 37\right) \log\left(-\frac{2^{\frac{1}{4}} - \left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}}{2^{\frac{1}{4}} + \left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}}\right) + \frac{3 \left(\frac{x-1}{2x+1}\right)^{\frac{3}{4}} - \left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}}}{2 \left(\frac{x-1}{2x+1} - 2\right)} - \frac{2}{9} \arctan\left(2 \left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}} + 1\right) - \frac{2}{9} \arctan\left(2 \left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}} - 1\right) + \frac{5}{9} \log\left(2 \sqrt{\frac{x-1}{2x+1}} + 2 \left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}} + 1\right) - \frac{5}{9} \log\left(2 \sqrt{\frac{x-1}{2x+1}} - 2 \left(\frac{x-1}{2x+1}\right)^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((-1+x)/(1+2*x))^(1/4)-3*((-1+x)/(1+2*x))^(3/4))/(-1+x)/(1+x)^2/
(-1+2*x),x, algorithm="maxima")
```

```
[Out] -1/72*2^(1/4)*(165*sqrt(2) + 37)*arctan(1/2*2^(3/4)*((x - 1)/(2*x + 1))^(1/
4)) - 1/144*2^(1/4)*(165*sqrt(2) - 37)*log(-(2^(1/4) - ((x - 1)/(2*x + 1))^
(1/4))/(2^(1/4) + ((x - 1)/(2*x + 1))^(1/4))) + 1/2*(3*((x - 1)/(2*x + 1))^
(3/4) - ((x - 1)/(2*x + 1))^(1/4))/((x - 1)/(2*x + 1) - 2) - 2/9*arctan(2*(
(x - 1)/(2*x + 1))^(1/4) + 1) - 2/9*arctan(2*((x - 1)/(2*x + 1))^(1/4) - 1)
 + 5/9*log(2*sqrt((x - 1)/(2*x + 1)) + 2*((x - 1)/(2*x + 1))^(1/4) + 1) - 5
/9*log(2*sqrt((x - 1)/(2*x + 1)) - 2*((x - 1)/(2*x + 1))^(1/4) + 1)
```

mupad [B] time = 2.27, size = 1310, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((x - 1)/(2*x + 1))^(1/4) - 3*((x - 1)/(2*x + 1))^(3/4))/((2*x - 1)*(x
 - 1)*(x + 1)^2),x)
```

```
[Out] (8/27 + 10i/81)^(1/2)*atan((8/27 + 10i/81)^(1/2)*((x - 1)/(2*x + 1))^(1/4)*
(18/13 - 27i/13))*2i - (8/27 - 10i/81)^(1/2)*atan((8/27 - 10i/81)^(1/2)*((x
 - 1)/(2*x + 1))^(1/4)*(18/13 + 27i/13))*2i - (((x - 1)/(2*x + 1))^(1/4) -
3*((x - 1)/(2*x + 1))^(3/4))/((2*x - 2)/(2*x + 1) - 4) - (2^(3/4)*atan(((2^
(3/4)*((13937028229*2^(3/4)*((x - 1)/(2*x + 1))^(1/4))/4 - (2^(3/4)*(37*2^(
1/2) - 330)*((25646402817*2^(3/4))/4 + (2^(3/4)*(37*2^(1/2) - 330)*(4159421
28*2^(3/4)*((x - 1)/(2*x + 1))^(1/4) - (2^(3/4)*(37*2^(1/2) - 330)*((700891
947*2^(3/4))/8 + (2^(3/4)*(37*2^(1/2) - 330)*(3601989*2^(1/2)*(37*2^(1/2) -
 330) - 634967019*2^(3/4)*((x - 1)/(2*x + 1))^(1/4)))/288))/288))/288))/288
)*(37*2^(1/2) - 330)*1i)/288 + (2^(3/4)*((13937028229*2^(3/4)*((x - 1)/(2*x
 + 1))^(1/4))/4 + (2^(3/4)*(37*2^(1/2) - 330)*((25646402817*2^(3/4))/4 - (2
^(3/4)*(37*2^(1/2) - 330)*(415942128*2^(3/4)*((x - 1)/(2*x + 1))^(1/4) + (2
^(3/4)*(37*2^(1/2) - 330)*((700891947*2^(3/4))/8 + (2^(3/4)*(37*2^(1/2) - 3
30)*(3601989*2^(1/2)*(37*2^(1/2) - 330) + 634967019*2^(3/4)*((x - 1)/(2*x +
 1))^(1/4)))/288))/288))/288))/288)*(37*2^(1/2) - 330)*1i)/288)/((118806425
01*2^(3/4))/2 + (2^(3/4)*((13937028229*2^(3/4)*((x - 1)/(2*x + 1))^(1/4))/4
 - (2^(3/4)*(37*2^(1/2) - 330)*((25646402817*2^(3/4))/4 + (2^(3/4)*(37*2^(1
/2) - 330)*(415942128*2^(3/4)*((x - 1)/(2*x + 1))^(1/4) - (2^(3/4)*(37*2^(1
/2) - 330)*((700891947*2^(3/4))/8 + (2^(3/4)*(37*2^(1/2) - 330)*(3601989*2^
(1/2)*(37*2^(1/2) - 330) - 634967019*2^(3/4)*((x - 1)/(2*x + 1))^(1/4)))/28
8))/288))/288))/288)*(37*2^(1/2) - 330))/288 - (2^(3/4)*((13937028229*2^(3/
4)*((x - 1)/(2*x + 1))^(1/4))/4 + (2^(3/4)*(37*2^(1/2) - 330)*((25646402817
```

```
*2^(3/4))/4 - (2^(3/4)*(37*2^(1/2) - 330)*(415942128*2^(3/4)*((x - 1)/(2*x
+ 1))^(1/4) + (2^(3/4)*(37*2^(1/2) - 330)*((700891947*2^(3/4))/8 + (2^(3/4)
*(37*2^(1/2) - 330)*(3601989*2^(1/2)*(37*2^(1/2) - 330) + 634967019*2^(3/4)
*((x - 1)/(2*x + 1))^(1/4)))/288))/288))/288))/288)*(37*2^(1/2) - 330))/288
))*(37*2^(1/2) - 330)*1i)/144 - (2^(3/4)*atan(((2^(3/4)*((13937028229*2^(3/
4)*((x - 1)/(2*x + 1))^(1/4))/4 - (2^(3/4)*(37*2^(1/2) + 330)*((25646402817
*2^(3/4))/4 + (2^(3/4)*(37*2^(1/2) + 330)*(415942128*2^(3/4)*((x - 1)/(2*x
+ 1))^(1/4) - (2^(3/4)*(37*2^(1/2) + 330)*((700891947*2^(3/4))/8 + (2^(3/4)
*(37*2^(1/2) + 330)*(2^(1/2)*(37*2^(1/2) + 330)*3601989i - 634967019*2^(3/4
)*((x - 1)/(2*x + 1))^(1/4))*1i)/288)*1i)/288)*1i)/288)*1i)/288)*(37*2^(1/2
) + 330))/288 + (2^(3/4)*((13937028229*2^(3/4)*((x - 1)/(2*x + 1))^(1/4))/4
 + (2^(3/4)*(37*2^(1/2) + 330)*((25646402817*2^(3/4))/4 - (2^(3/4)*(37*2^(1
/2) + 330)*(415942128*2^(3/4)*((x - 1)/(2*x + 1))^(1/4) + (2^(3/4)*(37*2^(1
/2) + 330)*((700891947*2^(3/4))/8 + (2^(3/4)*(37*2^(1/2) + 330)*(2^(1/2)*(3
7*2^(1/2) + 330)*3601989i + 634967019*2^(3/4)*((x - 1)/(2*x + 1))^(1/4))*1i
)/288)*1i)/288)*1i)/288)*1i)/288)*(37*2^(1/2) + 330))/288)/((11880642501*2^
(3/4))/2 + (2^(3/4)*((13937028229*2^(3/4)*((x - 1)/(2*x + 1))^(1/4))/4 - (2
^(3/4)*(37*2^(1/2) + 330)*((25646402817*2^(3/4))/4 + (2^(3/4)*(37*2^(1/2) +
 330)*(415942128*2^(3/4)*((x - 1)/(2*x + 1))^(1/4) - (2^(3/4)*(37*2^(1/2) +
 330)*((700891947*2^(3/4))/8 + (2^(3/4)*(37*2^(1/2) + 330)*(2^(1/2)*(37*2^(
1/2) + 330)*3601989i - 634967019*2^(3/4)*((x - 1)/(2*x + 1))^(1/4))*1i)/288
)*1i)/288)*1i)/288)*1i)/288)*(37*2^(1/2) + 330)*1i)/288 - (2^(3/4)*((139370
28229*2^(3/4)*((x - 1)/(2*x + 1))^(1/4))/4 + (2^(3/4)*(37*2^(1/2) + 330)*((
25646402817*2^(3/4))/4 - (2^(3/4)*(37*2^(1/2) + 330)*(415942128*2^(3/4)*((x
 - 1)/(2*x + 1))^(1/4) + (2^(3/4)*(37*2^(1/2) + 330)*((700891947*2^(3/4))/8
 + (2^(3/4)*(37*2^(1/2) + 330)*(2^(1/2)*(37*2^(1/2) + 330)*3601989i + 63496
7019*2^(3/4)*((x - 1)/(2*x + 1))^(1/4))*1i)/288)*1i)/288)*1i)/288)*1i)/288)
*(37*2^(1/2) + 330)*1i)/288))*(37*2^(1/2) + 330))/144
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((-1+x)/(1+2*x))**(1/4)-3*((-1+x)/(1+2*x))**(3/4))/(-1+x)/(1+x)*
*2/(-1+2*x),x)
```

```
[Out] Timed out
```

3.2074 $\int \frac{(-1+x^2)^2}{(1+x^2)^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal. Leaf size=223

$$\frac{(x^2+1)x^2+\sqrt{x^4+1}\,x^2}{x(x^2+1)\sqrt{\sqrt{x^4+1}+x^2}}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2(1+\sqrt{2})}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right.$$

Rubi [F] time = 3.36, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^2)^2}{(1+x^2)^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x^2)^2/((1 + x^2)^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] Defer[Int][1/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]), x] - Defer[Int][1/(
(I - x)^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]), x] - I*Defer[Int][1/((I
 - x)*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]), x] - Defer[Int][1/((I + x)^
2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]), x] - I*Defer[Int][1/((I + x)*Sq
rt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^2)^2}{(1+x^2)^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx &= \int \left(\frac{1}{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}+\frac{4}{(1+x^2)^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}-\right.\\
&= 4\int \frac{1}{(1+x^2)^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx-4\int \frac{1}{(1+x^2)\sqrt{1+x^4}\sqrt{x}}\\
&= -\left(4\int \left(\frac{i}{2(i-x)\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}+\frac{i}{2(i+x)\sqrt{1+x^4}\sqrt{x^2+\sqrt{}}}\right.\right.\\
&= -\left(2i\int \frac{1}{(i-x)\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx\right)-2i\int \frac{1}{(i+x)\sqrt{1+x^4}\sqrt{x}}\\
&= -\left(2i\int \frac{1}{(i-x)\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx\right)-2i\int \frac{1}{(i+x)\sqrt{1+x^4}\sqrt{x}}\\
&= i\int \frac{1}{(i-x)\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx+i\int \frac{1}{(i+x)\sqrt{1+x^4}\sqrt{x^2+\sqrt{}}}
\end{aligned}$$

Mathematica [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^2)^2}{(1+x^2)^2\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^2)^2/((1 + x^2)^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]
),x]
```

```
[Out] Integrate[(-1 + x^2)^2/((1 + x^2)^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]
), x]
```

IntegrateAlgebraic [A] time = 1.09, size = 294, normalized size = 1.32

$$\frac{(x^2+1)x^2+\sqrt{x^4+1}\,x^2}{x(x^2+1)\sqrt{\sqrt{x^4+1}+x^2}}+\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)^2/((1 + x^2)^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[
1 + x^4]]),x]
```

```
[Out] (x^2*(1 + x^2) + x^2*Sqrt[1 + x^4])/(x*(1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])
 + Sqrt[2]*ArcTan[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2])/(x*S
qrt[x^2 + Sqrt[1 + x^4]])] - Sqrt[1 + Sqrt[2]]*ArcTan[(-Sqrt[1/2 + 1/Sqrt[2
]] + Sqrt[1/2 + 1/Sqrt[2]]*x^2 + Sqrt[1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sq
rt[x^2 + Sqrt[1 + x^4]])] - Sqrt[-1 + Sqrt[2]]*ArcTanh[(-Sqrt[-1/2 + 1/Sqrt
[2]] + Sqrt[-1/2 + 1/Sqrt[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(
x*Sqrt[x^2 + Sqrt[1 + x^4]])]
```

fricas [B] time = 5.29, size = 385, normalized size = 1.73

$$-\frac{4(x^2+1)\sqrt{\sqrt{2}+1}\arctan\left(\frac{\left(x^2-(x^2+\sqrt{2}+1)\sqrt{-2\sqrt{2}+3}+\sqrt{x^4+1}\left(\sqrt{-2\sqrt{2}+3}-1\right)+\sqrt{2}-1\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}+1}}{2x}\right)+2\sqrt{2}(x^2+1)\arctan\left(-\frac{\left(\sqrt{2}x^2-\sqrt{2}\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{2x}\right)+(x^2+1)\sqrt{\sqrt{2}-1}\log\left(\frac{\sqrt{2}x^2+2x^2+\left(x^3+\sqrt{2}(x^3+2x)-\sqrt{x^4+1}(\sqrt{2}x+x)+3x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}-1}+\sqrt{x^4+1}(\sqrt{2}+1)}{x^2+1}\right)-(x^2+1)\sqrt{\sqrt{2}-1}\log\left(\frac{\sqrt{2}x^2+2x^2-\left(x^3+\sqrt{2}(x^3+2x)-\sqrt{x^4+1}(\sqrt{2}x+x)+3x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}-1}+\sqrt{x^4+1}(\sqrt{2}+1)}{x^2+1}\right)+4\left(x^3-\sqrt{x^4+1}\,x-x\right)\sqrt{x^2+\sqrt{x^4+1}}}{4(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2/(x^2+1)^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, al
gorithm="fricas")
```

```
[Out] -1/4*(4*(x^2 + 1)*sqrt(sqrt(2) + 1)*arctan(1/2*(x^2 - (x^2 + sqrt(2) + 1)*s
qrt(-2*sqrt(2) + 3) + sqrt(x^4 + 1)*(sqrt(-2*sqrt(2) + 3) - 1) + sqrt(2) -
1)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(sqrt(2) + 1)/x) + 2*sqrt(2)*(x^2 + 1)*arc
tan(-1/2*(sqrt(2)*x^2 - sqrt(2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))/x)
 + (x^2 + 1)*sqrt(sqrt(2) - 1)*log((sqrt(2)*x^2 + 2*x^2 + (x^3 + sqrt(2)*(x
^3 + 2*x) - sqrt(x^4 + 1)*(sqrt(2)*x + x) + 3*x)*sqrt(x^2 + sqrt(x^4 + 1))*
sqrt(sqrt(2) - 1) + sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 + 1)) - (x^2 + 1)*sqr
t(sqrt(2) - 1)*log((sqrt(2)*x^2 + 2*x^2 - (x^3 + sqrt(2)*(x^3 + 2*x) - sqrt
(x^4 + 1)*(sqrt(2)*x + x) + 3*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(sqrt(2) - 1
) + sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 + 1)) + 4*(x^3 - sqrt(x^4 + 1)*x - x)
*sqrt(x^2 + sqrt(x^4 + 1)))/(x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2-1\right)^2}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2/(x^2+1)^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, al
gorithm="giac")
```

```
[Out] integrate((x^2 - 1)^2/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*(x^2 + 1)^2)
, x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-1\right)^2}{\left(x^2+1\right)^2 \sqrt{x^4+1}\, \sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^2/(x^2+1)^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int((x^2-1)^2/(x^2+1)^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-1\right)^2}{\sqrt{x^4+1}\, \sqrt{x^2+\sqrt{x^4+1}}\, \left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2/(x^2+1)^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, al
gorithm="maxima")
```

```
[Out] integrate((x^2 - 1)^2/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*(x^2 + 1)^2)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^2-1\right)^2}{\left(x^2+1\right)^2 \sqrt{x^4+1}\, \sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^2/((x^2 + 1)^2*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2))
,x)
```

```
[Out] int((x^2 - 1)^2/((x^2 + 1)^2*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2))
, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-1\right)^2 \left(x+1\right)^2}{\left(x^2+1\right)^2 \sqrt{x^2+\sqrt{x^4+1}}\, \sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**2/(x**2+1)**2/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(
1/2),x)
```

```
[Out] Integral((x - 1)**2*(x + 1)**2/((x**2 + 1)**2*sqrt(x**2 + sqrt(x**4 + 1))*s
qrt(x**4 + 1)), x)
```

3.2075 $$\int \frac{-3+2(1+k^2)x+(1+k^2)x^2-4k^2x^3+k^2x^4}{((1-x^2)(1-k^2x^2))^{2/3}(-1+d+(-1-2d)x+(d+k^2)x^2+k^2x^3)}\,dx$$

Optimal. Leaf size=224

$$\frac{\log\left(\sqrt[3]{d}\,x-\sqrt[3]{d}+\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}\right)}{d^{2/3}}-\frac{\log\left(d^{2/3}x^2-2d^{2/3}x+d^{2/3}+\left(\sqrt[3]{d}-\sqrt[3]{d}\,x\right)\sqrt[3]{k^2x^4+(-k^2-1)\right.}}{2d^{2/3}}$$

Rubi [F] time = 8.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3+2(1+k^2)x+(1+k^2)x^2-4k^2x^3+k^2x^4}{((1-x^2)(1-k^2x^2))^{2/3}(-1+d+(-1-2d)x+(d+k^2)x^2+k^2x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3 + 2*(1 + k^2)*x + (1 + k^2)*x^2 - 4*k^2*x^3 + k^2*x^4)/(((1 - x^2)*
(1 - k^2*x^2))^(2/3)*(-1 + d + (-1 - 2*d)*x + (d + k^2)*x^2 + k^2*x^3)),x]
```

```
[Out] -(((5 + d/k^2)*x*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*AppellF1[1/2, 2/3, 2/3
, 3/2, x^2, k^2*x^2])/((1 - x^2)*(1 - k^2*x^2))^(2/3)) - (3^(3/4)*Sqrt[2 +
Sqrt[3]]*Sqrt[(-1 - k^2 + 2*k^2*x^2)^2]*((-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)
*((1 - x^2)*(1 - k^2*x^2))^(1/3))*Sqrt[((-1 + k^2)^(4/3) - 2^(2/3)*k^(2/3)*
(-1 + k^2)^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3) + 2*2^(1/3)*k^(4/3)*((1 -
x^2)*(1 - k^2*x^2))^(2/3))/((1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3
)*((1 - x^2)*(1 - k^2*x^2))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(-1 +
 k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))/((1 + Sqrt[3
])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))], -7
 - 4*Sqrt[3]])/(2^(2/3)*k^(2/3)*(1 + k^2 - 2*k^2*x^2)*Sqrt[(-1 - k^2*(1 - 2
*x^2))^2]*Sqrt[((-1 + k^2)^(2/3)*((-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 -
x^2)*(1 - k^2*x^2))^(1/3)))/((1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/
3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))^2]) - ((d^2 - 8*k^2 - d*(1 - 5*k^2))*(1
 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][1/((1 - x^2)^(2/3)*(1 - k^2*x^
2)^(2/3)*(1 - d + (1 + 2*d)*x - (d + k^2)*x^2 - k^2*x^3)), x])/(k^2*((1 - x
^2)*(1 - k^2*x^2))^(2/3)) + ((d + 2*d^2 + 2*k^2 + 11*d*k^2 - 2*k^4)*(1 - x^
2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][x/((1 - x^2)^(2/3)*(1 - k^2*x^2)^(2
/3)*(1 - d + (1 + 2*d)*x - (d + k^2)*x^2 - k^2*x^3)), x])/(k^2*((1 - x^2)*(
1 - k^2*x^2))^(2/3)) - ((d^2 + 2*k^2 + 8*d*k^2 + 6*k^4)*(1 - x^2)^(2/3)*(1
- k^2*x^2)^(2/3)*Defer[Int][x^2/((1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*(1 - d
 + (1 + 2*d)*x - (d + k^2)*x^2 - k^2*x^3)), x])/(k^2*((1 - x^2)*(1 - k^2*x^
2))^(2/3))
```

Rubi steps

$$\int \frac{-3+2\left(1+k^2\right)x+\left(1+k^2\right)x^2-4k^2x^3+k^2x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(-1+d+(-1-2d)x+\left(d+k^2\right)x^2+k^2x^3\right)}\,dx = \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int \frac{-3}{\left(1-x^2\right)^{2/3}\left(1\right.}}{\left(\left(1-x^2\right)\left(1\right.\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int \frac{3-}{\left(1-x^2\right)^{2/3}\left(1\right.}}{\left(\left(1-x^2\right)\left(1-\right.\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int\left(\frac{-5}{\left(1-x^2\right)^{2/3}}\right.}{}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int \frac{x}{\left(1-x^2\right)^{2/3}\left(1\right.}}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}}$$

$$= -\frac{\left(5+\frac{d}{k^2}\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}F}{\left(\left(1-x^2\right)\left(1-k^2x^2\right.\right.}$$

$$= -\frac{\left(5+\frac{d}{k^2}\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}F}{\left(\left(1-x^2\right)\left(1-k^2x^2\right.\right.}$$

$$= -\frac{\left(5+\frac{d}{k^2}\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}F}{\left(\left(1-x^2\right)\left(1-k^2x^2\right.\right.}$$

$$= -\frac{\left(5+\frac{d}{k^2}\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}F}{\left(\left(1-x^2\right)\left(1-k^2x^2\right.\right.}$$

Mathematica [F] time = 3.61, size = 0, normalized size = 0.00

$$\int \frac{-3+2\left(1+k^2\right)x+\left(1+k^2\right)x^2-4k^2x^3+k^2x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(-1+d+(-1-2d)x+\left(d+k^2\right)x^2+k^2x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3 + 2*(1 + k^2)*x + (1 + k^2)*x^2 - 4*k^2*x^3 + k^2*x^4)/(((1 -
 x^2)*(1 - k^2*x^2))^(2/3)*(-1 + d + (-1 - 2*d)*x + (d + k^2)*x^2 + k^2*x^3
)),x]
```

```
[Out] Integrate[(-3 + 2*(1 + k^2)*x + (1 + k^2)*x^2 - 4*k^2*x^3 + k^2*x^4)/(((1 -
 x^2)*(1 - k^2*x^2))^(2/3)*(-1 + d + (-1 - 2*d)*x + (d + k^2)*x^2 + k^2*x^3
)), x]
```

IntegrateAlgebraic [A] time = 8.34, size = 224, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{d}\,x-\sqrt[3]{d}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\right)}{d^{2/3}}-\frac{\log\left(d^{2/3}x^2-2d^{2/3}x+d^{2/3}+\left(\sqrt[3]{d}-\sqrt[3]{d}\,x\right)\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}\right)}{2d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\,x-\sqrt{3}\sqrt[3]{d}}{\sqrt[3]{d}\,x-\sqrt[3]{d}-2\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3 + 2*(1 + k^2)*x + (1 + k^2)*x^2 - 4*k^2*x^3 + k^2*x^
4)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)*(-1 + d + (-1 - 2*d)*x + (d + k^2)*x^2
+ k^2*x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(-(Sqrt[3]*d^(1/3)) + Sqrt[3]*d^(1/3)*x)/(-d^(1/3) + d^(1/3
)*x - 2*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))])/d^(2/3) + Log[-d^(1/3) + d^
(1/3)*x + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3)]/d^(2/3) - Log[d^(2/3) - 2*d
^(2/3)*x + d^(2/3)*x^2 + (d^(1/3) - d^(1/3)*x)*(1 + (-1 - k^2)*x^2 + k^2*x^
4)^(1/3) + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*(k^2+1)*x+(k^2+1)*x^2-4*k^2*x^3+k^2*x^4)/((-x^2+1)*(-k^2*x^
2+1))^(2/3)/(-1+d+(-1-2*d)*x+(k^2+d)*x^2+k^2*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^4 - 4\,k^2x^3 + \left(k^2+1\right)x^2 + 2\left(k^2+1\right)x - 3}{\left(k^2x^3 + \left(k^2+d\right)x^2 - (2\,d+1)x + d - 1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*(k^2+1)*x+(k^2+1)*x^2-4*k^2*x^3+k^2*x^4)/((-x^2+1)*(-k^2*x^
2+1))^(2/3)/(-1+d+(-1-2*d)*x+(k^2+d)*x^2+k^2*x^3),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^4 - 4*k^2*x^3 + (k^2 + 1)*x^2 + 2*(k^2 + 1)*x - 3)/((k^2*x
^3 + (k^2 + d)*x^2 - (2*d + 1)*x + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(2/3)),
 x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{-3+2\left(k^2+1\right)x+\left(k^2+1\right)x^2-4k^2x^3+k^2x^4}{\left(\left(-x^2+1\right)\left(-k^2x^2+1\right)\right)^{\frac{2}{3}}\left(-1+d+(-1-2d)\,x+\left(k^2+d\right)x^2+k^2x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*(k^2+1)*x+(k^2+1)*x^2-4*k^2*x^3+k^2*x^4)/((-x^2+1)*(-k^2*x^2+1))^
(2/3)/(-1+d+(-1-2*d)*x+(k^2+d)*x^2+k^2*x^3),x)
```

```
[Out] int((-3+2*(k^2+1)*x+(k^2+1)*x^2-4*k^2*x^3+k^2*x^4)/((-x^2+1)*(-k^2*x^2+1))^
(2/3)/(-1+d+(-1-2*d)*x+(k^2+d)*x^2+k^2*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^4 - 4\,k^2x^3 + \left(k^2+1\right)x^2 + 2\left(k^2+1\right)x - 3}{\left(k^2x^3 + \left(k^2+d\right)x^2 - (2\,d+1)x + d - 1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*(k^2+1)*x+(k^2+1)*x^2-4*k^2*x^3+k^2*x^4)/((-x^2+1)*(-k^2*x^
2+1))^(2/3)/(-1+d+(-1-2*d)*x+(k^2+d)*x^2+k^2*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^4 - 4*k^2*x^3 + (k^2 + 1)*x^2 + 2*(k^2 + 1)*x - 3)/((k^2*x
^3 + (k^2 + d)*x^2 - (2*d + 1)*x + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(2/3)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{2\,x\,\left(k^2+1\right)-4\,k^2\,x^3+k^2\,x^4+x^2\,\left(k^2+1\right)-3}{{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)}^{2/3}\,\left(d+k^2\,x^3+x^2\,\left(k^2+d\right)-x\,\left(2\,d+1\right)-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x*(k^2 + 1) - 4*k^2*x^3 + k^2*x^4 + x^2*(k^2 + 1) - 3)/(((x^2 - 1)*(
k^2*x^2 - 1))^(2/3)*(d + k^2*x^3 + x^2*(d + k^2) - x*(2*d + 1) - 1)),x)
```

```
[Out] int((2*x*(k^2 + 1) - 4*k^2*x^3 + k^2*x^4 + x^2*(k^2 + 1) - 3)/(((x^2 - 1)*(
k^2*x^2 - 1))^(2/3)*(d + k^2*x^3 + x^2*(d + k^2) - x*(2*d + 1) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*(k**2+1)*x+(k**2+1)*x**2-4*k**2*x**3+k**2*x**4)/((-x**2+1)*
(-k**2*x**2+1))**(2/3)/(-1+d+(-1-2*d)*x+(k**2+d)*x**2+k**2*x**3),x)
```

```
[Out] Timed out
```

3.2076

$$\int \frac{x^3(-2+(1+k)x)}{((1-x)x(1-kx))^{2/3}\left(1-(2+2k)x+\left(1+4k+k^2\right)x^2-\left(2k+2k^2\right)x^3+\left(-b+k^2\right)x^4\right)}\,dx$$

Optimal. Leaf size=224

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}x}{\sqrt[6]{b}x-2\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{2b^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}x}{\sqrt[6]{b}x+2\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{2b^{5/6}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{b}x}{\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{b^{5/6}}-\frac{\tanh^{-1}\left(\ldots\right)}{\ldots}$$

Rubi [F] time = 20.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3(-2+(1+k)x)}{((1-x)x(1-kx))^{2/3}\left(1-(2+2k)x+\left(1+4k+k^2\right)x^2-\left(2k+2k^2\right)x^3+\left(-b+k^2\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*(-2 + (1 + k)*x))/(((1 - x)*x*(1 - k*x))^(2/3)*(1 - (2 + 2*k)*x +
(1 + 4*k + k^2)*x^2 - (2*k + 2*k^2)*x^3 + (-b + k^2)*x^4)),x]
```

```
[Out] (-3*(1 + k)*x*((1 - x)/(1 - k*x))^(2/3)*(1 - k*x)*Hypergeometric2F1[1/3, 2/
3, 4/3, ((1 - k)*x)/(1 - k*x)])/((b - k^2)*((1 - x)*x*(1 - k*x))^(2/3)) + (
3*(1 + k)*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst][Defer[Int][1/(
(1 - x^3)^(2/3)*(1 - k*x^3)^(2/3)*(1 - 2*(1 + k)*x^3 + (1 + k*(4 + k))*x^6
- 2*k*(1 + k)*x^9 - b*(1 - k^2/b)*x^12)), x], x, x^(1/3)])/((b - k^2)*((1 -
 x)*x*(1 - k*x))^(2/3)) + (3*(1 + k)*(1 + 4*k + k^2)*(1 - x)^(2/3)*x^(2/3)*
(1 - k*x)^(2/3)*Defer[Subst][Defer[Int][x^6/((1 - x^3)^(2/3)*(1 - k*x^3)^(2
/3)*(1 - 2*(1 + k)*x^3 + (1 + k*(4 + k))*x^6 - 2*k*(1 + k)*x^9 - b*(1 - k^2
/b)*x^12)), x], x, x^(1/3)])/((b - k^2)*((1 - x)*x*(1 - k*x))^(2/3)) - (6*(
b + k + k^2 + k^3)*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst][Defer
[Int][x^9/((1 - x^3)^(2/3)*(1 - k*x^3)^(2/3)*(1 - 2*(1 + k)*x^3 + (1 + k*(4
 + k))*x^6 - 2*k*(1 + k)*x^9 - b*(1 - k^2/b)*x^12)), x], x, x^(1/3)])/((b -
 k^2)*((1 - x)*x*(1 - k*x))^(2/3)) + (6*(1 + k)^2*(1 - x)^(2/3)*x^(2/3)*(1
- k*x)^(2/3)*Defer[Subst][Defer[Int][x^3/((1 - x^3)^(2/3)*(1 - k*x^3)^(2/3)
*(-1 + 2*(1 + k)*x^3 - (1 + k*(4 + k))*x^6 + 2*k*(1 + k)*x^9 + b*(1 - k^2/b
)*x^12)), x], x, x^(1/3)])/((b - k^2)*((1 - x)*x*(1 - k*x))^(2/3))
```

Rubi steps

$$\int \frac{x^3(-2+(1+k)x)}{((1-x)x(1-kx))^{2/3}\left(1-(2+2k)x+\left(1+4k+k^2\right)x^2-\left(2k+2k^2\right)x^3+\left(-b+k^2\right)x^4\right)}\,dx = \frac{\left((1-x)^{2/3}x^{2/3}(1-\right.}{}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}(1-\right.}{}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}(1-\right.}{}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}(1-\right.}{}$$

$$= -\frac{3(1+k)x\left(\frac{1-x}{1-kx}\right)^{2}}{\left(b-k^2\right)}$$

$$= -\frac{3(1+k)x\left(\frac{1-x}{1-kx}\right)^{2}}{\left(b-k^2\right)}$$

Mathematica [F] time = 1.96, size = 0, normalized size = 0.00

$$\int \frac{x^3(-2+(1+k)x)}{((1-x)x(1-kx))^{2/3}\left(1-(2+2k)x+\left(1+4k+k^2\right)x^2-\left(2k+2k^2\right)x^3+\left(-b+k^2\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(-2 + (1 + k)*x))/(((1 - x)*x*(1 - k*x))^(2/3)*(1 - (2 + 2*k
)*x + (1 + 4*k + k^2)*x^2 - (2*k + 2*k^2)*x^3 + (-b + k^2)*x^4)),x]
```

```
[Out] Integrate[(x^3*(-2 + (1 + k)*x))/(((1 - x)*x*(1 - k*x))^(2/3)*(1 - (2 + 2*k
)*x + (1 + 4*k + k^2)*x^2 - (2*k + 2*k^2)*x^3 + (-b + k^2)*x^4)), x]
```

IntegrateAlgebraic [A] time = 1.53, size = 224, normalized size = 1.00

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}x}{\sqrt[6]{b}x-2\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{2b^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}x}{\sqrt[6]{b}x+2\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{2b^{5/6}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{b}x}{\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{b^{5/6}}-\frac{\tanh^{-1}\left(\frac{\frac{\left(kx^3+(-k-1)x^2+x\right)^{2/3}}{\sqrt[6]{b}}+\sqrt[6]{b}x^2}{x\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{2b^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(-2 + (1 + k)*x))/(((1 - x)*x*(1 - k*x))^(2/3)*(1 -
 (2 + 2*k)*x + (1 + 4*k + k^2)*x^2 - (2*k + 2*k^2)*x^3 + (-b + k^2)*x^4)),x
]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/6)*x)/(b^(1/6)*x - 2*(x + (-1 - k)*x^2 +
 k*x^3)^(1/3))])/b^(5/6) + (Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/6)*x)/(b^(1/6)*x +
 2*(x + (-1 - k)*x^2 + k*x^3)^(1/3))])/(2*b^(5/6)) - ArcTanh[(b^(1/6)*x)/(x
 + (-1 - k)*x^2 + k*x^3)^(1/3)]/b^(5/6) - ArcTanh[(b^(1/6)*x^2 + (x + (-1 -
 k)*x^2 + k*x^3)^(2/3)/b^(1/6))/(x*(x + (-1 - k)*x^2 + k*x^3)^(1/3))]/(2*b^
(5/6))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(2/3)/(1-(2+2*k)*x+(k^2+4*k+1
)*x^2-(2*k^2+2*k)*x^3+(k^2-b)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.76, size = 262, normalized size = 1.17

$$-\frac{\sqrt{3}\left(-b\right)^{\frac{1}{6}}\log\left(\sqrt{3}\left(-b\right)^{\frac{1}{6}}\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^{2}}\right)^{\frac{1}{3}}+\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^{2}}\right)^{\frac{2}{3}}+\left(-b\right)^{\frac{1}{3}}\right)}{4\,b}+\frac{\sqrt{3}\left(-b\right)^{\frac{1}{6}}\log\left(-\sqrt{3}\left(-b\right)^{\frac{1}{6}}\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^{2}}\right)^{\frac{1}{3}}+\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^{2}}\right)^{\frac{2}{3}}+\left(-b\right)^{\frac{1}{3}}\right)}{4\,b}-\frac{\left(-b\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(-b\right)^{\frac{1}{6}}+2\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^{2}}\right)^{\frac{1}{3}}}{\left(-b\right)^{\frac{1}{6}}}\right)}{2\,b}-\frac{\left(-b\right)^{\frac{1}{6}}\arctan\left(-\frac{\sqrt{3}\left(-b\right)^{\frac{1}{6}}-2\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^{2}}\right)^{\frac{1}{3}}}{\left(-b\right)^{\frac{1}{6}}}\right)}{2\,b}-\frac{\left(-b\right)^{\frac{1}{6}}\arctan\left(\frac{\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^{2}}\right)^{\frac{1}{3}}}{\left(-b\right)^{\frac{1}{6}}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(2/3)/(1-(2+2*k)*x+(k^2+4*k+1
)*x^2-(2*k^2+2*k)*x^3+(k^2-b)*x^4),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(3)*(-b)^(1/6)*log(sqrt(3)*(-b)^(1/6)*(k - k/x - 1/x + 1/x^2)^(1/3
) + (k - k/x - 1/x + 1/x^2)^(2/3) + (-b)^(1/3))/b + 1/4*sqrt(3)*(-b)^(1/6)*
log(-sqrt(3)*(-b)^(1/6)*(k - k/x - 1/x + 1/x^2)^(1/3) + (k - k/x - 1/x + 1/
x^2)^(2/3) + (-b)^(1/3))/b - 1/2*(-b)^(1/6)*arctan((sqrt(3)*(-b)^(1/6) + 2*
(k - k/x - 1/x + 1/x^2)^(1/3))/(-b)^(1/6))/b - 1/2*(-b)^(1/6)*arctan(-(sqrt
(3)*(-b)^(1/6) - 2*(k - k/x - 1/x + 1/x^2)^(1/3))/(-b)^(1/6))/b - (-b)^(1/6
)*arctan((k - k/x - 1/x + 1/x^2)^(1/3)/(-b)^(1/6))/b
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(-2+\left(1+k\right)x\right)}{\left(\left(1-x\right)x\left(-kx+1\right)\right)^{\frac{2}{3}}\left(1-\left(2+2k\right)x+\left(k^{2}+4k+1\right)x^{2}-\left(2k^{2}+2k\right)x^{3}+\left(k^{2}-b\right)x^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(2/3)/(1-(2+2*k)*x+(k^2+4*k+1)*x^2-
(2*k^2+2*k)*x^3+(k^2-b)*x^4),x)
```

```
[Out] int(x^3*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(2/3)/(1-(2+2*k)*x+(k^2+4*k+1)*x^2-
(2*k^2+2*k)*x^3+(k^2-b)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left({\left(k+1\right)}x-2\right)}x^{3}}{{\left({\left(k^{2}-b\right)}x^{4}-2\,{\left(k^{2}+k\right)}x^{3}+{\left(k^{2}+4\,k+1\right)}x^{2}-2\,{\left(k+1\right)}x+1\right)}\left({\left(kx-1\right)}{\left(x-1\right)}x\right)^{\frac{2}{3}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(2/3)/(1-(2+2*k)*x+(k^2+4*k+1
)*x^2-(2*k^2+2*k)*x^3+(k^2-b)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(((k + 1)*x - 2)*x^3/(((k^2 - b)*x^4 - 2*(k^2 + k)*x^3 + (k^2 + 4*
k + 1)*x^2 - 2*(k + 1)*x + 1)*((k*x - 1)*(x - 1)*x)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x^3\,\left(x\,\left(k+1\right)-2\right)}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{2/3}\,\left(\left(b-k^2\right)\,x^4+\left(2\,k^2+2\,k\right)\,x^3+\left(-k^2-4\,k-1\right)\,x^2+\left(2\,k+2\right)\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(x*(k + 1) - 2))/((x*(k*x - 1)*(x - 1))^(2/3)*(x*(2*k + 2) + x^4*
(b - k^2) - x^2*(4*k + k^2 + 1) + x^3*(2*k + 2*k^2) - 1)),x)
```

```
[Out] int(-(x^3*(x*(k + 1) - 2))/((x*(k*x - 1)*(x - 1))^(2/3)*(x*(2*k + 2) + x^4*
(b - k^2) - x^2*(4*k + k^2 + 1) + x^3*(2*k + 2*k^2) - 1)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))**(2/3)/(1-(2+2*k)*x+(k**2+4*
k+1)*x**2-(2*k**2+2*k)*x**3+(k**2-b)*x**4),x)
```

```
[Out] Timed out
```

3.2077 $\int \frac{\left(1+x^2\right)^2}{\left(-1+x^2\right)^2 \sqrt{1+x^4} \sqrt{x^2+\sqrt{1+x^4}}} \, dx$

Optimal. Leaf size=224

$$\frac{-\left(\left(x^2-1\right)x^2\right)-\sqrt{x^4+1}\,x^2}{x\left(x^2-1\right)\sqrt{\sqrt{x^4+1}+x^2}}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{2}-1\right)}\,x\sqrt{\sqrt{x^4+1}+}{\sqrt{x^4+1}+x^2+1}\right.$$

Rubi [F] time = 1.89, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+x^2\right)^2}{\left(-1+x^2\right)^2 \sqrt{1+x^4} \sqrt{x^2+\sqrt{1+x^4}}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^2)^2/((-1 + x^2)^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] Defer[Int][1/(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]), x] + Defer[Int][1/(
(-1 - x)*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]), x] + Defer[Int][1/((-1 +
 x)^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]), x] + Defer[Int][1/((-1 + x)
*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]), x] + Defer[Int][1/((1 + x)^2*Sqr
t[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]), x]
```

Rubi steps

$$\begin{aligned} \int \frac{\left(1+x^2\right)^2}{\left(-1+x^2\right)^2 \sqrt{1+x^4} \sqrt{x^2+\sqrt{1+x^4}}} \, dx &= \int \left(\frac{1}{\sqrt{1+x^4} \sqrt{x^2+\sqrt{1+x^4}}}+\frac{1}{(-1-x)\sqrt{1+x^4} \sqrt{x^2+\sqrt{1+x^4}}}\right. \\ &= \int \frac{1}{\sqrt{1+x^4} \sqrt{x^2+\sqrt{1+x^4}}} \, dx+\int \frac{1}{(-1-x)\sqrt{1+x^4} \sqrt{x^2+\sqrt{1-}}} \end{aligned}$$

Mathematica [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\left(1+x^2\right)^2}{\left(-1+x^2\right)^2 \sqrt{1+x^4} \sqrt{x^2+\sqrt{1+x^4}}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^2)^2/((-1 + x^2)^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]
),x]
```

```
[Out] Integrate[(1 + x^2)^2/((-1 + x^2)^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]]
), x]
```

IntegrateAlgebraic [A] time = 1.31, size = 295, normalized size = 1.32

$$\frac{-\left(\left(x^2-1\right)x^2\right)-\sqrt{x^4+1}\,x^2}{x\left(x^2-1\right)\sqrt{\sqrt{x^4+1}+x^2}}+\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)+\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)^2/((-1 + x^2)^2*Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[
1 + x^4]]),x]
```

```
[Out] (-(x^2*(-1 + x^2)) - x^2*Sqrt[1 + x^4])/(x*(-1 + x^2)*Sqrt[x^2 + Sqrt[1 + x
^4]]) + Sqrt[2]*ArcTan[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2])
/(x*Sqrt[x^2 + Sqrt[1 + x^4]])] - Sqrt[1 + Sqrt[2]]*ArcTan[(-Sqrt[-1/2 + 1/
Sqrt[2]] + Sqrt[-1/2 + 1/Sqrt[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4
])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])] + Sqrt[-1 + Sqrt[2]]*ArcTanh[(-Sqrt[1/2 +
 1/Sqrt[2]] + Sqrt[1/2 + 1/Sqrt[2]]*x^2 + Sqrt[1/2 + 1/Sqrt[2]]*Sqrt[1 + x^
4])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])]
```

fricas [B] time = 5.01, size = 389, normalized size = 1.74

$$\frac{4\left(x^2-1\right)\sqrt{\sqrt{2}+1}\arctan\left(-\frac{\left(x^2+\left(x^2-\sqrt{2}-1\right)\sqrt{-2\sqrt{2}+3}-\sqrt{x^4+1}\left(\sqrt{-2\sqrt{2}+3}+1\right)-\sqrt{2}+1\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}+1}}{2x}\right)-2\sqrt{2}\left(x^2-1\right)\arctan\left(-\frac{\left(\sqrt{2}x^2-\sqrt{2}\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{2x}\right)-\left(x^2-1\right)\sqrt{\sqrt{2}-1}\log\left(-\frac{\sqrt{2}x^2+2x^2+\left(x^3+\sqrt{2}\left(x^3-2x\right)-\sqrt{x^4+1}\left(\sqrt{2}x+x\right)-3x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}-1}+\sqrt{x^4+1}\left(\sqrt{2}+1\right)}{x^2-1}\right)+\left(x^2-1\right)\sqrt{\sqrt{2}-1}\log\left(-\frac{\sqrt{2}x^2+2x^2-\left(x^3+\sqrt{2}\left(x^3-2x\right)-\sqrt{x^4+1}\left(\sqrt{2}x+x\right)-3x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}-1}+\sqrt{x^4+1}\left(\sqrt{2}+1\right)}{x^2-1}\right)-4\left(x^3-\sqrt{x^4+1}x+x\right)\sqrt{x^2+\sqrt{x^4+1}}}{4\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^2-1)^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, al
gorithm="fricas")
```

```
[Out] 1/4*(4*(x^2 - 1)*sqrt(sqrt(2) + 1)*arctan(-1/2*(x^2 + (x^2 - sqrt(2) - 1)*s
qrt(-2*sqrt(2) + 3) - sqrt(x^4 + 1)*(sqrt(-2*sqrt(2) + 3) + 1) - sqrt(2) +
1)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(sqrt(2) + 1)/x) - 2*sqrt(2)*(x^2 - 1)*arc
tan(-1/2*(sqrt(2)*x^2 - sqrt(2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))/x)
 - (x^2 - 1)*sqrt(sqrt(2) - 1)*log(-(sqrt(2)*x^2 + 2*x^2 + (x^3 + sqrt(2)*(
x^3 - 2*x) - sqrt(x^4 + 1)*(sqrt(2)*x + x) - 3*x)*sqrt(x^2 + sqrt(x^4 + 1))
*sqrt(sqrt(2) - 1) + sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 - 1)) + (x^2 - 1)*sq
rt(sqrt(2) - 1)*log(-(sqrt(2)*x^2 + 2*x^2 - (x^3 + sqrt(2)*(x^3 - 2*x) - sq
rt(x^4 + 1)*(sqrt(2)*x + x) - 3*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(sqrt(2) -
 1) + sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 - 1)) - 4*(x^3 - sqrt(x^4 + 1)*x +
x)*sqrt(x^2 + sqrt(x^4 + 1)))/(x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^2-1)^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, al
gorithm="giac")
```

```
[Out] integrate((x^2 + 1)^2/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - 1)^2)
, x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2}{\left(x^2-1\right)^2\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2/(x^2-1)^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int((x^2+1)^2/(x^2-1)^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2}{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^2-1)^2/(x^4+1)^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2),x, al
gorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^2/(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - 1)^2)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^2+1\right)^2}{\left(x^2-1\right)^2\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^2/((x^2 - 1)^2*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2))
,x)
```

```
[Out] int((x^2 + 1)^2/((x^2 - 1)^2*(x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2))
, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2}{(x-1)^2\,(x+1)^2\sqrt{x^2+\sqrt{x^4+1}}\,\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2/(x**2-1)**2/(x**4+1)**(1/2)/(x**2+(x**4+1)**(1/2))**(
1/2),x)
```

```
[Out] Integral((x**2 + 1)**2/((x - 1)**2*(x + 1)**2*sqrt(x**2 + sqrt(x**4 + 1))*s
qrt(x**4 + 1)), x)
```

3.2078 $\int \frac{(-1+x^2)^2\sqrt{x^2+\sqrt{1+x^4}}}{(1+x^2)^2\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=224

$$\frac{-\left(\left(x^2-1\right)x^2\right)-\sqrt{x^4+1}\,x^2}{x\left(x^2+1\right)\sqrt{\sqrt{x^4+1}+x^2}}+\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+\ldots}}{\sqrt{x^4+1}+x^2+\ldots}\right.$$

Rubi [C] time = 2.61, antiderivative size = 426, normalized size of antiderivative = 1.90, number of steps used = 44, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {6742, 2132, 206, 2133, 731, 725, 6725}

$$\frac{i\sqrt{1-ix^2}}{2(-x+i)}-\frac{i\sqrt{1-ix^2}}{2(x+i)}-\frac{i\sqrt{1+ix^2}}{2(-x+i)}+\frac{i\sqrt{1+ix^2}}{2(x+i)}+\frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-x}{\sqrt{1+i}\sqrt{1-ix^2}}\right)+\frac{\tanh^{-1}\left(\frac{1-x}{\sqrt{1+i}\sqrt{1-ix^2}}\right)}{(1+i)^{5/2}}-\frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{x+1}{\sqrt{1+i}\sqrt{1-ix^2}}\right)-\frac{\tanh^{-1}\left(\frac{x+1}{\sqrt{1+i}\sqrt{1-ix^2}}\right)}{(1+i)^{5/2}}+\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1-x}{\sqrt{1-i}\sqrt{1+ix^2}}\right)+\frac{\tanh^{-1}\left(\frac{1-x}{\sqrt{1-i}\sqrt{1+ix^2}}\right)}{(1-i)^{5/2}}-\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{x+1}{\sqrt{1-i}\sqrt{1+ix^2}}\right)-\frac{\tanh^{-1}\left(\frac{x+1}{\sqrt{1-i}\sqrt{1+ix^2}}\right)}{(1-i)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^2)^2*Sqrt[x^2 + Sqrt[1 + x^4]])/((1 + x^2)^2*Sqrt[1 + x^4]),x]
```

```
[Out] ((I/2)*Sqrt[1 - I*x^2])/(I - x) - ((I/2)*Sqrt[1 - I*x^2])/(I + x) - ((I/2)*
Sqrt[1 + I*x^2])/(I - x) + ((I/2)*Sqrt[1 + I*x^2])/(I + x) + ArcTanh[(1 - x
)/(Sqrt[1 + I]*Sqrt[1 - I*x^2])]/(1 + I)^(5/2) + (Sqrt[1 + I]*ArcTanh[(1 -
x)/(Sqrt[1 + I]*Sqrt[1 - I*x^2])])/2 - ArcTanh[(1 + x)/(Sqrt[1 + I]*Sqrt[1
- I*x^2])]/(1 + I)^(5/2) - (Sqrt[1 + I]*ArcTanh[(1 + x)/(Sqrt[1 + I]*Sqrt[1
 - I*x^2])])/2 + ArcTanh[(1 - x)/(Sqrt[1 - I]*Sqrt[1 + I*x^2])]/(1 - I)^(5/
2) + (Sqrt[1 - I]*ArcTanh[(1 - x)/(Sqrt[1 - I]*Sqrt[1 + I*x^2])])/2 - ArcTa
nh[(1 + x)/(Sqrt[1 - I]*Sqrt[1 + I*x^2])]/(1 - I)^(5/2) - (Sqrt[1 - I]*ArcT
anh[(1 + x)/(Sqrt[1 - I]*Sqrt[1 + I*x^2])])/2 + ArcTanh[(Sqrt[2]*x)/Sqrt[x^
2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(-1+x^2\right)^2 \sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^2\right)^2 \sqrt{1+x^4}}\,dx = \int \left(\frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}} + \frac{4\sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^2\right)^2 \sqrt{1+x^4}} - \frac{4\sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^2\right) \sqrt{1+x^4}}\right) dx$$

$$= 4\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^2\right)^2 \sqrt{1+x^4}}\,dx - 4\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^2\right) \sqrt{1+x^4}}\,dx + \int \frac{\sqrt{x^2+\sqrt{1+x}}}{\sqrt{1+x^4}}$$

$$= -\left(4\int \left(\frac{i\sqrt{x^2+\sqrt{1+x^4}}}{2(i-x)\sqrt{1+x^4}} + \frac{i\sqrt{x^2+\sqrt{1+x^4}}}{2(i+x)\sqrt{1+x^4}}\right) dx\right) + 4\int \left(-\frac{\sqrt{x^2+\sqrt{1}}}{4(i-x)^2\sqrt{1}}\right.$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}} - 2i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(i-x)\sqrt{1+x^4}}\,dx - 2i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(i+x)\sqrt{1+x^4}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}} - (-1+i)\int \frac{1}{(i-x)\sqrt{1+ix^2}}\,dx - (-1+i)\int \frac{}{(i+x}$$

$$= \frac{i\sqrt{1-ix^2}}{2(i-x)} - \frac{i\sqrt{1-ix^2}}{2(i+x)} - \frac{i\sqrt{1+ix^2}}{2(i-x)} + \frac{i\sqrt{1+ix^2}}{2(i+x)} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

$$= \frac{i\sqrt{1-ix^2}}{2(i-x)} - \frac{i\sqrt{1-ix^2}}{2(i+x)} - \frac{i\sqrt{1+ix^2}}{2(i-x)} + \frac{i\sqrt{1+ix^2}}{2(i+x)} + \sqrt{1+i}\tanh^{-1}\left(\frac{}{\sqrt{1+}}\right.$$

$$= \frac{i\sqrt{1-ix^2}}{2(i-x)} - \frac{i\sqrt{1-ix^2}}{2(i+x)} - \frac{i\sqrt{1+ix^2}}{2(i-x)} + \frac{i\sqrt{1+ix^2}}{2(i+x)} + \sqrt{1+i}\tanh^{-1}\left(\frac{}{\sqrt{1+}}\right.$$

$$= \frac{i\sqrt{1-ix^2}}{2(i-x)} - \frac{i\sqrt{1-ix^2}}{2(i+x)} - \frac{i\sqrt{1+ix^2}}{2(i-x)} + \frac{i\sqrt{1+ix^2}}{2(i+x)} + \frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{}{\sqrt{1}}\right.$$

Mathematica [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^2\right)^2 \sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^2\right)^2 \sqrt{1+x^4}}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^2)^2*Sqrt[x^2 + Sqrt[1 + x^4]])/((1 + x^2)^2*Sqrt[1 + x^
4]),x]
```

```
[Out] Integrate[((-1 + x^2)^2*Sqrt[x^2 + Sqrt[1 + x^4]])/((1 + x^2)^2*Sqrt[1 + x^
4]), x]
```

IntegrateAlgebraic [A] time = 1.27, size = 295, normalized size = 1.32

$$\frac{-\left(\left(x^2-1\right)x^2\right)-\sqrt{x^4+1}\,x^2}{x\left(x^2+1\right)\sqrt{\sqrt{x^4+1}+x^2}}+\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\,x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\,x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)^2*Sqrt[x^2 + Sqrt[1 + x^4]])/((1 + x^2)^2*Sq
rt[1 + x^4]),x]
```

```
[Out] (-(x^2*(-1 + x^2)) - x^2*Sqrt[1 + x^4])/(x*(1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^
4]]) + Sqrt[-1 + Sqrt[2]]*ArcTan[(-Sqrt[1/2 + 1/Sqrt[2]] + Sqrt[1/2 + 1/Sqr
t[2]]*x^2 + Sqrt[1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 + x^4
]])] + Sqrt[2]*ArcTanh[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2])
/(x*Sqrt[x^2 + Sqrt[1 + x^4]])] - Sqrt[1 + Sqrt[2]]*ArcTanh[(-Sqrt[-1/2 + 1
/Sqrt[2]] + Sqrt[-1/2 + 1/Sqrt[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*Sqrt[1 + x^
4])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])]
```

fricas [B] time = 4.04, size = 391, normalized size = 1.75

$$\frac{4\left(x^2+1\right)\sqrt{\sqrt{2}-1}\arctan\left(\frac{\left(\sqrt{2}x^2+x^2+\sqrt{x^4+1}\left(\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+3}-\sqrt{2}-1\right)-\left(x^2+\sqrt{2}\left(x^2+2\right)+3\right)\sqrt{-2\sqrt{2}+3}+1\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}-1}}{2x}\right)+\sqrt{2}\left(x^2+1\right)\log\left(4x^4+4\sqrt{x^4+1}\,x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)-\left(x^2+1\right)\sqrt{\sqrt{2}+1}\log\left(\frac{\sqrt{2}x^2+2x^2+\left(x^3+\sqrt{2}x-\sqrt{x^4+1}\,x+x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}+1}+\sqrt{x^4+1}\left(\sqrt{2}+1\right)}{x^2+1}\right)+\left(x^2+1\right)\sqrt{\sqrt{2}+1}\log\left(\frac{\sqrt{2}x^2+2x^2-\left(x^3+\sqrt{2}x-\sqrt{x^4+1}\,x+x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}+1}+\sqrt{x^4+1}\left(\sqrt{2}+1\right)}{x^2+1}\right)-4\left(x^3-\sqrt{x^4+1}\,x+x\right)\sqrt{x^2+\sqrt{x^4+1}}}{4\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)^2/(x^4+1)^(1/2),x, al
gorithm="fricas")
```

```
[Out] 1/4*(4*(x^2 + 1)*sqrt(sqrt(2) - 1)*arctan(1/2*(sqrt(2)*x^2 + x^2 + sqrt(x^4
 + 1)*((sqrt(2) + 1)*sqrt(-2*sqrt(2) + 3) - sqrt(2) - 1) - (x^2 + sqrt(2)*(
x^2 + 2) + 3)*sqrt(-2*sqrt(2) + 3) + 1)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(sqrt
(2) - 1)/x) + sqrt(2)*(x^2 + 1)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2
)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1) - (x^2 + 1)
*sqrt(sqrt(2) + 1)*log((sqrt(2)*x^2 + 2*x^2 + (x^3 + sqrt(2)*x - sqrt(x^4 +
 1)*x + x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(sqrt(2) + 1) + sqrt(x^4 + 1)*(sqr
t(2) + 1))/(x^2 + 1)) + (x^2 + 1)*sqrt(sqrt(2) + 1)*log((sqrt(2)*x^2 + 2*x^
2 - (x^3 + sqrt(2)*x - sqrt(x^4 + 1)*x + x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(
sqrt(2) + 1) + sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 + 1)) - 4*(x^3 - sqrt(x^4
+ 1)*x + x)*sqrt(x^2 + sqrt(x^4 + 1)))/(x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-1\right)^2}{\sqrt{x^4+1}\left(x^2+1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)^2/(x^4+1)^(1/2),x, al
gorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - 1)^2/(sqrt(x^4 + 1)*(x^2 + 1)^2)
, x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}-1\right)^{2} \sqrt{x^{2}+\sqrt{x^{4}+1}}}{\left(x^{2}+1\right)^{2} \sqrt{x^{4}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)^2/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2-1)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)^2/(x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{2}+\sqrt{x^{4}+1}}\left(x^{2}-1\right)^{2}}{\sqrt{x^{4}+1}\left(x^{2}+1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1)^2/(x^4+1)^(1/2),x, al
gorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - 1)^2/(sqrt(x^4 + 1)*(x^2 + 1)^2)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^{2}-1\right)^{2} \sqrt{\sqrt{x^{4}+1}+x^{2}}}{\left(x^{2}+1\right)^{2} \sqrt{x^{4}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)^2*((x^4 + 1)^(1/2) + x^2)^(1/2))/((x^2 + 1)^2*(x^4 + 1)^(1/2
)),x)
```

```
[Out] int(((x^2 - 1)^2*((x^4 + 1)^(1/2) + x^2)^(1/2))/((x^2 + 1)^2*(x^4 + 1)^(1/2
)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-1\right)^{2} \left(x+1\right)^{2} \sqrt{x^{2}+\sqrt{x^{4}+1}}}{\left(x^{2}+1\right)^{2} \sqrt{x^{4}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**2*(x**2+(x**4+1)**(1/2))**(1/2)/(x**2+1)**2/(x**4+1)**(
1/2),x)
```

```
[Out] Integral((x - 1)**2*(x + 1)**2*sqrt(x**2 + sqrt(x**4 + 1))/((x**2 + 1)**2*s
qrt(x**4 + 1)), x)
```

3.2079 $\int \frac{(-2+x)\sqrt[3]{x-x^2+x^3}}{(-1+x)(-1+x+x^2)}\,dx$

Optimal. Leaf size=225

$$-\log\left(\sqrt[3]{x^3-x^2+x}-x\right)+\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^3-x^2+x}-2x\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2+x}\,x+\left(x^3-x^2+x\right)^{2/3}\right)-\frac{\log}{}$$

Rubi [F] time = 1.74, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+x)\sqrt[3]{x-x^2+x^3}}{(-1+x)\left(-1+x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + x)*(x - x^2 + x^3)^(1/3))/((-1 + x)*(-1 + x + x^2)),x]

[Out] -(((x - x^2 + x^3)^(1/3)*Defer[Subst][Defer[Int][(1 - x^3 + x^6)^(1/3)/(-1
+ x), x], x, x^(1/3)])/(x^(1/3)*(1 - x + x^2)^(1/3))) + ((1 - I*Sqrt[3])*(x
 - x^2 + x^3)^(1/3)*Defer[Subst][Defer[Int][(1 - x^3 + x^6)^(1/3)/(1 - I*Sq
rt[3] + 2*x), x], x, x^(1/3)])/(x^(1/3)*(1 - x + x^2)^(1/3)) + ((1 + I*Sqrt
[3])*(x - x^2 + x^3)^(1/3)*Defer[Subst][Defer[Int][(1 - x^3 + x^6)^(1/3)/(1
 + I*Sqrt[3] + 2*x), x], x, x^(1/3)])/(x^(1/3)*(1 - x + x^2)^(1/3)) + (6*(x
 - x^2 + x^3)^(1/3)*Defer[Subst][Defer[Int][(1 - x^3 + x^6)^(1/3)/(1 - Sqrt
[5] + 2*x^3), x], x, x^(1/3)])/(x^(1/3)*(1 - x + x^2)^(1/3)) + (6*(x - x^2
+ x^3)^(1/3)*Defer[Subst][Defer[Int][(1 - x^3 + x^6)^(1/3)/(1 + Sqrt[5] + 2
*x^3), x], x, x^(1/3)])/(x^(1/3)*(1 - x + x^2)^(1/3))
```

Rubi steps

$$\begin{aligned}
\int \frac{(-2+x)\sqrt[3]{x-x^2+x^3}}{(-1+x)\left(-1+x+x^2\right)}\,dx &= \frac{\sqrt[3]{x-x^2+x^3}\int \frac{(-2+x)\sqrt[3]{x}\sqrt[3]{1-x+x^2}}{(-1+x)\left(-1+x+x^2\right)}\,dx}{\sqrt[3]{x}\sqrt[3]{1-x+x^2}}\\
&= \frac{\sqrt[3]{x-x^2+x^3}\int \frac{(-2+x)\sqrt[3]{x}\sqrt[3]{1-x+x^2}}{1-2x+x^3}\,dx}{\sqrt[3]{x}\sqrt[3]{1-x+x^2}}\\
&= \frac{\left(3\sqrt[3]{x-x^2+x^3}\right)\text{Subst}\left(\int \frac{x^3\left(-2+x^3\right)\sqrt[3]{1-x^3+x^6}}{1-2x^3+x^9}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1-x+x^2}}\\
&= \frac{\left(3\sqrt[3]{x-x^2+x^3}\right)\text{Subst}\left(\int \left(-\frac{\sqrt[3]{1-x^3+x^6}}{3(-1+x)}+\frac{(2+x)\sqrt[3]{1-x^3+x^6}}{3\left(1+x+x^2\right)}+\frac{\left(1+2x^3\right)\sqrt[3]{1-x^3+x^6}}{-1+x^3+x^6}\right)dx,x,\right.}{\sqrt[3]{x}\sqrt[3]{1-x+x^2}}\\
&= -\frac{\sqrt[3]{x-x^2+x^3}\,\text{Subst}\left(\int \frac{\sqrt[3]{1-x^3+x^6}}{-1+x}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1-x+x^2}}+\frac{\sqrt[3]{x-x^2+x^3}\,\text{Subst}\left(\int \frac{(2+x)\sqrt[3]{1-x}}{1+x+x}\right.}{\sqrt[3]{x}\sqrt[3]{1-x+x^2}}\\
&= -\frac{\sqrt[3]{x-x^2+x^3}\,\text{Subst}\left(\int \frac{\sqrt[3]{1-x^3+x^6}}{-1+x}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1-x+x^2}}+\frac{\sqrt[3]{x-x^2+x^3}\,\text{Subst}\left(\int\left(\frac{\left(1-i\sqrt{3}\right)}{1-i\sqrt{}}\right.\right.}{\sqrt[3]{x}}\\
&= -\frac{\sqrt[3]{x-x^2+x^3}\,\text{Subst}\left(\int \frac{\sqrt[3]{1-x^3+x^6}}{-1+x}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1-x+x^2}}+\frac{\left(6\sqrt[3]{x-x^2+x^3}\right)\text{Subst}\left(\int \frac{\sqrt[3]{1-x^3}}{1-\sqrt{5}}\right.}{\sqrt[3]{x}\sqrt[3]{1-x+x^2}}
\end{aligned}$$

Mathematica [F] time = 1.63, size = 0, normalized size = 0.00

$$\int \frac{(-2+x)\sqrt[3]{x-x^2+x^3}}{(-1+x)\left(-1+x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + x)*(x - x^2 + x^3)^(1/3))/((-1 + x)*(-1 + x + x^2)),x]
```

```
[Out] Integrate[((-2 + x)*(x - x^2 + x^3)^(1/3))/((-1 + x)*(-1 + x + x^2)), x]
```

IntegrateAlgebraic [A] time = 0.59, size = 225, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^3-x^2+x}-x\right)+\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^3-x^2+x}-2x\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^3-x^2+x}\,x+\left(x^3-x^2+x\right)^{2/3}\right)-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^3-x^2+x}\,x+\sqrt[3]{2}\left(x^3-x^2+x\right)^{2/3}\right)}{2^{2/3}}-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x^2+x}+x}\right)+\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3-x^2+x}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x)*(x - x^2 + x^3)^(1/3))/((-1 + x)*(-1 + x + x^2
)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(x - x^2 + x^3)^(1/3))]) + 2^(1/3)*Sqrt
[3]*ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(x - x^2 + x^3)^(1/3))] - Log[-x + (x -
 x^2 + x^3)^(1/3)] + 2^(1/3)*Log[-2*x + 2^(2/3)*(x - x^2 + x^3)^(1/3)] + Lo
g[x^2 + x*(x - x^2 + x^3)^(1/3) + (x - x^2 + x^3)^(2/3)]/2 - Log[2*x^2 + 2^
(2/3)*x*(x - x^2 + x^3)^(1/3) + 2^(1/3)*(x - x^2 + x^3)^(2/3)]/2^(2/3)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)*(x^3-x^2+x)^(1/3)/(-1+x)/(x^2+x-1),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-x^2+x\right)^{\frac{1}{3}}(x-2)}{\left(x^2+x-1\right)(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)*(x^3-x^2+x)^(1/3)/(-1+x)/(x^2+x-1),x, algorithm="giac")
```

```
[Out] integrate((x^3 - x^2 + x)^(1/3)*(x - 2)/((x^2 + x - 1)*(x - 1)), x)
```

maple [C] time = 10.95, size = 1645, normalized size = 7.31

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+x)*(x^3-x^2+x)^(1/3)/(-1+x)/(x^2+x-1),x)
```

```
[Out] RootOf(_Z^3-2)*ln((19*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*Roo
tOf(_Z^3-2)^3*x^2+5*RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3
-2)+_Z^2)*x^2-76*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_
Z^3-2)^3*x-20*RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z
```

```
^2)*x+76*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^3
+20*RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)+513*Ro
otOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x^2+135*RootOf
(_Z^3-2)^2*x^2-582*(x^3-x^2+x)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3
-2)+_Z^2)*x-582*(x^3-x^2+x)^(1/3)*RootOf(_Z^3-2)*x-209*RootOf(RootOf(_Z^3-2
)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x-55*RootOf(_Z^3-2)^2*x+582*(x^3
-x^2+x)^(2/3)+209*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z
^3-2)+55*RootOf(_Z^3-2)^2)/(x^2+x-1))+RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^
3-2)+_Z^2)*ln((-14*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf
(_Z^3-2)^3*x^2+5*RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)
+_Z^2)*x^2+56*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3
-2)^3*x-20*RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)
*x-56*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^3+20
*RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)+350*RootO
f(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x^2-125*RootOf(_Z
^3-2)^2*x^2-582*(x^3-x^2+x)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)
+_Z^2)*x-582*(x^3-x^2+x)^(1/3)*RootOf(_Z^3-2)*x-42*RootOf(RootOf(_Z^3-2)^2+
_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x+15*RootOf(_Z^3-2)^2*x+582*(x^3-x^2
+x)^(2/3)+42*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)
-15*RootOf(_Z^3-2)^2)/(x^2+x-1))+ln((-RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^
3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^2+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-
2)+_Z^2)^2*RootOf(_Z^3-2)^4*x-4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_
Z^2)^2*RootOf(_Z^3-2)^4-2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*R
ootOf(_Z^3-2)^2*x^2+82*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*Root
Of(_Z^3-2)^2*x-82*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z
^3-2)^2+444*(x^3-x^2+x)^(2/3)+444*x*(x^3-x^2+x)^(1/3)+440*x^2-132*x+132)/(-
1+x))+1/2*ln((-RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^
3-2)^4*x^2+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-
2)^4*x-4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4
-2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^2+82*
RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x-82*RootO
f(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2+444*(x^3-x^2+x)
^(2/3)+444*x*(x^3-x^2+x)^(1/3)+440*x^2-132*x+132)/(-1+x))*RootOf(RootOf(_Z^
3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2-1/2*RootOf(RootOf(_Z^3-2)^2
+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*ln((-RootOf(RootOf(_Z^3-2)^2+_Z*R
ootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^2+4*RootOf(RootOf(_Z^3-2)^2+_Z*Roo
tOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x-4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(
_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4-2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2
)+_Z^2)*RootOf(_Z^3-2)^2*x^2-66*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_
Z^2)*RootOf(_Z^3-2)^2*x+66*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*
RootOf(_Z^3-2)^2+444*(x^3-x^2+x)^(2/3)+444*x*(x^3-x^2+x)^(1/3)+440*x^2-280*
x+280)/(-1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3 - x^2 + x\right)^{\frac{1}{3}}(x-2)}{\left(x^2 + x - 1\right)(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)*(x^3-x^2+x)^(1/3)/(-1+x)/(x^2+x-1),x, algorithm="maxima")
[Out] integrate((x^3 - x^2 + x)^(1/3)*(x - 2)/((x^2 + x - 1)*(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x-2)\,\left(x^3 - x^2 + x\right)^{1/3}}{(x-1)\,\left(x^2 + x - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 2)*(x - x^2 + x^3)^(1/3))/((x - 1)*(x + x^2 - 1)),x)
[Out] int(((x - 2)*(x - x^2 + x^3)^(1/3))/((x - 1)*(x + x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x\left(x^2-x+1\right)}\,(x-2)}{(x-1)\left(x^2+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)*(x**3-x**2+x)**(1/3)/(-1+x)/(x**2+x-1),x)
[Out] Integral((x*(x**2 - x + 1))**(1/3)*(x - 2)/((x - 1)*(x**2 + x - 1)), x)
```

3.2080 $\int \frac{(1+x^2)\sqrt[3]{x^2+x^3}}{-1+x^2}\,dx$

Optimal. Leaf size=225

$$\frac{1}{6}\sqrt[3]{x^3+x^2}\,(3x+1)-\frac{17}{9}\log\left(\sqrt[3]{x^3+x^2}-x\right)+\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^3+x^2}-2x\right)+\frac{17}{18}\log\left(x^2+\sqrt[3]{x^3+x^2}\,x+\left(x^3+x^2\right)^{2/3}\right.$$

Rubi [A] time = 1.01, antiderivative size = 369, normalized size of antiderivative = 1.64, number of steps used = 36, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {2056, 1586, 6733, 6725, 331, 292, 31, 634, 618, 204, 628, 321, 494, 617}

$$\frac{1}{2}\sqrt[3]{x^3+x^2}\,x+\frac{1}{6}\sqrt[3]{x^3+x^2}-\frac{17\sqrt[3]{x^3+x^2}\log\left(1-\frac{\sqrt[3]{x}}{\sqrt[3]{x+1}}\right)}{9\sqrt[3]{x+1}\,x^{2/3}}+\frac{17\sqrt[3]{x^3+x^2}\log\left(\frac{x^{2/3}}{(x+1)^{2/3}}+\frac{\sqrt[3]{x}}{\sqrt[3]{x+1}}+1\right)}{18\sqrt[3]{x+1}\,x^{2/3}}+\frac{\sqrt[3]{2}\sqrt[3]{x^3+x^2}\log\left(1-\frac{\sqrt[3]{2}\sqrt[3]{x}}{\sqrt[3]{x+1}}\right)}{\sqrt[3]{x+1}\,x^{2/3}}-\frac{\sqrt[3]{x^3+x^2}\log\left(\frac{2^{2/3}x^{2/3}}{(x+1)^{2/3}}+\frac{\sqrt[3]{2}\sqrt[3]{x}}{\sqrt[3]{x+1}}+1\right)}{2^{2/3}\sqrt[3]{x+1}\,x^{2/3}}-\frac{17\sqrt[3]{x^3+x^2}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{x}}{\sqrt[3]{x+1}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{x+1}\,x^{2/3}}+\frac{\sqrt[3]{2}\sqrt{3}\sqrt[3]{x^3+x^2}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{x}}{\sqrt[3]{x+1}}+1}{\sqrt{3}}\right)}{\sqrt[3]{x+1}\,x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(x^2 + x^3)^(1/3))/(-1 + x^2),x]
```

```
[Out] (x^2 + x^3)^(1/3)/6 + (x*(x^2 + x^3)^(1/3))/2 - (17*(x^2 + x^3)^(1/3)*ArcTa
n[(1 + (2*x^(1/3))/(1 + x)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*x^(2/3)*(1 + x)^(1/3
)) + (2^(1/3)*Sqrt[3]*(x^2 + x^3)^(1/3)*ArcTan[(1 + (2*2^(1/3)*x^(1/3))/(1
+ x)^(1/3))/Sqrt[3]])/(x^(2/3)*(1 + x)^(1/3)) - (17*(x^2 + x^3)^(1/3)*Log[1
 - x^(1/3)/(1 + x)^(1/3)])/(9*x^(2/3)*(1 + x)^(1/3)) + (17*(x^2 + x^3)^(1/3
)*Log[1 + x^(2/3)/(1 + x)^(2/3) + x^(1/3)/(1 + x)^(1/3)])/(18*x^(2/3)*(1 +
x)^(1/3)) + (2^(1/3)*(x^2 + x^3)^(1/3)*Log[1 - (2^(1/3)*x^(1/3))/(1 + x)^(1
/3)])/(x^(2/3)*(1 + x)^(1/3)) - ((x^2 + x^3)^(1/3)*Log[1 + (2^(2/3)*x^(2/3)
)/(1 + x)^(2/3) + (2^(1/3)*x^(1/3))/(1 + x)^(1/3)])/(2^(2/3)*x^(2/3)*(1 + x
)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{\left(1+x^2\right)\sqrt[3]{x^2+x^3}}{-1+x^2}\,dx = \frac{\sqrt[3]{x^2+x^3}\int \frac{x^{2/3}\sqrt[3]{1+x}\left(1+x^2\right)}{-1+x^2}\,dx}{x^{2/3}\sqrt[3]{1+x}}$$

$$= \frac{\sqrt[3]{x^2+x^3}\int \frac{x^{2/3}\left(1+x^2\right)}{(-1+x)(1+x)^{2/3}}\,dx}{x^{2/3}\sqrt[3]{1+x}}$$

$$= \frac{\left(3\sqrt[3]{x^2+x^3}\right)\text{Subst}\left(\int \frac{x^4\left(1+x^6\right)}{\left(-1+x^3\right)\left(1+x^3\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{x^{2/3}\sqrt[3]{1+x}}$$

$$= \frac{\left(3\sqrt[3]{x^2+x^3}\right)\text{Subst}\left(\int \left(\frac{2x}{\left(1+x^3\right)^{2/3}}+\frac{x^4}{\left(1+x^3\right)^{2/3}}+\frac{x^7}{\left(1+x^3\right)^{2/3}}+\frac{2x}{\left(-1+x^3\right)\left(1+x^3\right)^{2/3}}\right)dx, x, \sqrt[3]{x}\right)}{x^{2/3}\sqrt[3]{1+x}}$$

$$= \frac{\left(3\sqrt[3]{x^2+x^3}\right)\text{Subst}\left(\int \frac{x^4}{\left(1+x^3\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{x^{2/3}\sqrt[3]{1+x}} + \frac{\left(3\sqrt[3]{x^2+x^3}\right)\text{Subst}\left(\int \frac{x^7}{\left(1+x^3\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right.}{x^{2/3}\sqrt[3]{1+x}}$$

$$= \sqrt[3]{x^2+x^3} + \frac{1}{2}x\sqrt[3]{x^2+x^3} - \frac{\left(2\sqrt[3]{x^2+x^3}\right)\text{Subst}\left(\int \frac{x}{\left(1+x^3\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{x^{2/3}\sqrt[3]{1+x}} - \frac{\left(5\sqrt[3]{x^2+x^3}\right.}{}$$

$$= \frac{1}{6}\sqrt[3]{x^2+x^3} + \frac{1}{2}x\sqrt[3]{x^2+x^3} + \frac{\left(5\sqrt[3]{x^2+x^3}\right)\text{Subst}\left(\int \frac{x}{\left(1+x^3\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{3x^{2/3}\sqrt[3]{1+x}} + \frac{\left(2\sqrt[3]{x^2+x\right.}{}$$

$$= \frac{1}{6}\sqrt[3]{x^2+x^3} + \frac{1}{2}x\sqrt[3]{x^2+x^3} - \frac{2\sqrt[3]{x^2+x^3}\log\left(1-\frac{\sqrt[3]{x}}{\sqrt[3]{1+x}}\right)}{x^{2/3}\sqrt[3]{1+x}} + \frac{\sqrt[3]{2}\sqrt[3]{x^2+x^3}\log\left(1-\frac{\sqrt[3]{2}}{\sqrt[3]{1}}\right.}{x^{2/3}\sqrt[3]{1+x}}$$

$$= \frac{1}{6}\sqrt[3]{x^2+x^3} + \frac{1}{2}x\sqrt[3]{x^2+x^3} - \frac{4\sqrt[3]{x^2+x^3}\log\left(1-\frac{\sqrt[3]{x}}{\sqrt[3]{1+x}}\right)}{3x^{2/3}\sqrt[3]{1+x}} + \frac{\sqrt[3]{x^2+x^3}\log\left(1+\frac{x^{2/3}}{(1+x)^{2/3}}\right.}{x^{2/3}\sqrt[3]{1+x}}$$

$$= \frac{1}{6}\sqrt[3]{x^2+x^3} + \frac{1}{2}x\sqrt[3]{x^2+x^3} - \frac{2\sqrt{3}\sqrt[3]{x^2+x^3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{x}}{\sqrt[3]{1+x}}}{\sqrt{3}}\right)}{x^{2/3}\sqrt[3]{1+x}} + \frac{\sqrt[3]{2}\sqrt{3}\sqrt[3]{x^2+x^3}\tan^{-}}{x^{2/3}\sqrt[3]{1+x}}$$

$$= \frac{1}{6}\sqrt[3]{x^2+x^3} + \frac{1}{2}x\sqrt[3]{x^2+x^3} + \frac{2\sqrt[3]{x^2+x^3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{x}}{\sqrt[3]{1+x}}}{\sqrt{3}}\right)}{\sqrt{3}x^{2/3}\sqrt[3]{1+x}} - \frac{2\sqrt{3}\sqrt[3]{x^2+x^3}\tan^{-1}\left(1+\right.}{x^{2/3}\sqrt[3]{1+x}}$$

$$= \frac{1}{6}\sqrt[3]{x^2+x^3} + \frac{1}{2}x\sqrt[3]{x^2+x^3} + \frac{\sqrt[3]{x^2+x^3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{x}}{\sqrt[3]{1+x}}}{\sqrt{3}}\right)}{3\sqrt{3}x^{2/3}\sqrt[3]{1+x}} - \frac{2\sqrt{3}\sqrt[3]{x^2+x^3}\tan^{-1}\left(1+\right.}{x^{2/3}\sqrt[3]{1+x}}$$

Mathematica [C] time = 0.08, size = 84, normalized size = 0.37

$$\frac{3\sqrt[3]{x^2(x+1)}\left(x(x+1)\,{}_2F_1\left(-\frac{1}{3},\frac{5}{3};\frac{8}{3};-x\right)+5(x+1)\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-x\right)-5\sqrt[3]{x+1}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2x}{x+1}\right)\right)}{5(x+1)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(x^2 + x^3)^(1/3))/(-1 + x^2),x]
```

```
[Out] (3*(x^2*(1 + x))^(1/3)*(x*(1 + x)*Hypergeometric2F1[-1/3, 5/3, 8/3, -x] + 5
*(1 + x)*Hypergeometric2F1[2/3, 2/3, 5/3, -x] - 5*(1 + x)^(1/3)*Hypergeomet
ric2F1[2/3, 1, 5/3, (2*x)/(1 + x)]))/(5*(1 + x)^(4/3))
```

IntegrateAlgebraic [A] time = 0.75, size = 225, normalized size = 1.00

$$\frac{1}{6}\sqrt[3]{x^3+x^2}\,(3x+1)-\frac{17}{9}\log\left(\sqrt[3]{x^3+x^2}-x\right)+\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^3+x^2}-2x\right)+\frac{17}{18}\log\left(x^2+\sqrt[3]{x^3+x^2}\,x+\left(x^3+x^2\right)^{2/3}\right)-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^3+x^2}\,x+\sqrt[3]{2}\left(x^3+x^2\right)^{2/3}\right)}{2^{2/3}}-\frac{17\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x^2}+x}\right)}{3\sqrt{3}}+\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3+x^2}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^2)*(x^2 + x^3)^(1/3))/(-1 + x^2),x]
```

```
[Out] ((1 + 3*x)*(x^2 + x^3)^(1/3))/6 - (17*ArcTan[(Sqrt[3]*x)/(x + 2*(x^2 + x^3)
^(1/3))])/(3*Sqrt[3]) + 2^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(x^
2 + x^3)^(1/3))] - (17*Log[-x + (x^2 + x^3)^(1/3)])/9 + 2^(1/3)*Log[-2*x +
2^(2/3)*(x^2 + x^3)^(1/3)] + (17*Log[x^2 + x*(x^2 + x^3)^(1/3) + (x^2 + x^3
)^(2/3)])/18 - Log[2*x^2 + 2^(2/3)*x*(x^2 + x^3)^(1/3) + 2^(1/3)*(x^2 + x^3
)^(2/3)]/2^(2/3)
```

fricas [A] time = 0.45, size = 204, normalized size = 0.91

$$-\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(x^3+x^2\right)^{\frac{1}{3}}+\sqrt{3}x}{3x}\right)+\frac{17}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+x^2\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{6}\left(x^3+x^2\right)^{\frac{1}{3}}(3x+1)+2^{\frac{1}{3}}\log\left(-\frac{2^{\frac{1}{3}}x-\left(x^3+x^2\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{2}\cdot 2^{\frac{1}{3}}\log\left(\frac{2^{\frac{2}{3}}x^2+2^{\frac{1}{3}}\left(x^3+x^2\right)^{\frac{1}{3}}x+\left(x^3+x^2\right)^{\frac{2}{3}}}{x^2}\right)-\frac{17}{9}\log\left(-\frac{x-\left(x^3+x^2\right)^{\frac{1}{3}}}{x}\right)+\frac{17}{18}\log\left(\frac{x^2+\left(x^3+x^2\right)^{\frac{1}{3}}x+\left(x^3+x^2\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^3+x^2)^(1/3)/(x^2-1),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*2^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(x^3 + x^2)^(1/3) + sqrt(3)*x)
/x) + 17/9*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 + x^2)^(1/3))/x)
+ 1/6*(x^3 + x^2)^(1/3)*(3*x + 1) + 2^(1/3)*log(-(2^(1/3)*x - (x^3 + x^2)^(
1/3))/x) - 1/2*2^(1/3)*log((2^(2/3)*x^2 + 2^(1/3)*(x^3 + x^2)^(1/3)*x + (x^
3 + x^2)^(2/3))/x^2) - 17/9*log(-(x - (x^3 + x^2)^(1/3))/x) + 17/18*log((x^
2 + (x^3 + x^2)^(1/3)*x + (x^3 + x^2)^(2/3))/x^2)
```

giac [A] time = 0.32, size = 154, normalized size = 0.68

$$\frac{1}{6}\left(\left(\frac{1}{x}+1\right)^{\frac{4}{3}}+2\left(\frac{1}{x}+1\right)^{\frac{1}{3}}\right)x^2-\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(\frac{1}{x}+1\right)^{\frac{1}{3}}\right)\right)+\frac{17}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{2}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+\left(\frac{1}{x}+1\right)^{\frac{2}{3}}\right)+2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(\frac{1}{x}+1\right)^{\frac{1}{3}}\right|\right)+\frac{17}{18}\log\left(\left(\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)-\frac{17}{9}\log\left(\left|\left(\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^3+x^2)^(1/3)/(x^2-1),x, algorithm="giac")
```

```
[Out] 1/6*((1/x + 1)^(4/3) + 2*(1/x + 1)^(1/3))*x^2 - sqrt(3)*2^(1/3)*arctan(1/6*
sqrt(3)*2^(2/3)*(2^(1/3) + 2*(1/x + 1)^(1/3))) + 17/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*(1/x + 1)^(1/3) + 1)) - 1/2*2^(1/3)*log(2^(2/3) + 2^(1/3)*(1/x + 1
)^(1/3) + (1/x + 1)^(2/3)) + 2^(1/3)*log(abs(-2^(1/3) + (1/x + 1)^(1/3))) +
 17/18*log((1/x + 1)^(2/3) + (1/x + 1)^(1/3) + 1) - 17/9*log(abs((1/x + 1)^
(1/3) - 1))
```

maple [C] time = 3.37, size = 2348, normalized size = 10.44

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^3+x^2)^(1/3)/(x^2-1),x)
```

```
[Out] 1/6*(1+3*x)*(x^2*(1+x))^(1/3)+(1/27*RootOf(_Z^2-51*_Z+2601)*ln((8*RootOf(_Z
^2-51*_Z+2601)^2*x^2-8*RootOf(_Z^2-51*_Z+2601)^2*x+3213*RootOf(_Z^2-51*_Z+2
601)*(x^3+2*x^2+x)^(2/3)+3213*RootOf(_Z^2-51*_Z+2601)*(x^3+2*x^2+x)^(1/3)*x
+2805*RootOf(_Z^2-51*_Z+2601)*x^2-16*RootOf(_Z^2-51*_Z+2601)^2+3213*RootOf(
_Z^2-51*_Z+2601)*(x^3+2*x^2+x)^(1/3)+4947*RootOf(_Z^2-51*_Z+2601)*x-39015*(
x^3+2*x^2+x)^(2/3)-39015*(x^3+2*x^2+x)^(1/3)*x-18207*x^2+2142*RootOf(_Z^2-5
1*_Z+2601)-39015*(x^3+2*x^2+x)^(1/3)-31212*x-13005)/(1+x))-1/27*ln((8*RootO
f(_Z^2-51*_Z+2601)^2*x^2-8*RootOf(_Z^2-51*_Z+2601)^2*x-3213*RootOf(_Z^2-51*
_Z+2601)*(x^3+2*x^2+x)^(2/3)-3213*RootOf(_Z^2-51*_Z+2601)*(x^3+2*x^2+x)^(1/
3)*x-3621*RootOf(_Z^2-51*_Z+2601)*x^2-16*RootOf(_Z^2-51*_Z+2601)^2-3213*Roo
tOf(_Z^2-51*_Z+2601)*(x^3+2*x^2+x)^(1/3)-4131*RootOf(_Z^2-51*_Z+2601)*x+124
848*(x^3+2*x^2+x)^(2/3)+124848*(x^3+2*x^2+x)^(1/3)*x+145656*x^2-510*RootOf(
_Z^2-51*_Z+2601)+124848*(x^3+2*x^2+x)^(1/3)+200277*x+54621)/(1+x))*RootOf(_
Z^2-51*_Z+2601)+17/9*ln((8*RootOf(_Z^2-51*_Z+2601)^2*x^2-8*RootOf(_Z^2-51*_
Z+2601)^2*x-3213*RootOf(_Z^2-51*_Z+2601)*(x^3+2*x^2+x)^(2/3)-3213*RootOf(_Z
^2-51*_Z+2601)*(x^3+2*x^2+x)^(1/3)*x-3621*RootOf(_Z^2-51*_Z+2601)*x^2-16*Ro
otOf(_Z^2-51*_Z+2601)^2-3213*RootOf(_Z^2-51*_Z+2601)*(x^3+2*x^2+x)^(1/3)-41
31*RootOf(_Z^2-51*_Z+2601)*x+124848*(x^3+2*x^2+x)^(2/3)+124848*(x^3+2*x^2+x
)^(1/3)*x+145656*x^2-510*RootOf(_Z^2-51*_Z+2601)+124848*(x^3+2*x^2+x)^(1/3)
+200277*x+54621)/(1+x))+RootOf(_Z^3-2)*ln(-(270*RootOf(_Z^3-2)^3*RootOf(729
*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)*x^2-RootOf(_Z^3-2)^2*RootOf(72
9*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)^2*x^2-270*RootOf(729*RootOf(_
Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x+RootOf(729*RootOf(_Z
^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x-2673*(x^3+2*x^2+x)^
(2/3)*RootOf(_Z^3-2)^2*RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^
2)-540*RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2
)^3+2*RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-
2)^2+15309*(x^3+2*x^2+x)^(1/3)*RootOf(_Z^3-2)^2*x+3240*(x^3+2*x^2+x)^(1/3)*
RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x+153
09*(x^3+2*x^2+x)^(1/3)*RootOf(_Z^3-2)^2+3240*(x^3+2*x^2+x)^(1/3)*RootOf(729
*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)+123930*RootOf(_
Z^3-2)*x^2-459*RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)*x^2+1
45800*RootOf(_Z^3-2)*x-540*RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)
+_Z^2)*x-174960*(x^3+2*x^2+x)^(2/3)+21870*RootOf(_Z^3-2)-81*RootOf(729*Root
Of(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2))/(-1+x)/(1+x))-ln(-(270*RootOf(_Z^3
-2)^3*RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)*x^2+11*RootOf(
_Z^3-2)^2*RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)^2*x^2-270*
RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x-1
1*RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2
*x+2673*(x^3+2*x^2+x)^(2/3)*RootOf(_Z^3-2)^2*RootOf(729*RootOf(_Z^3-2)^2+27
*_Z*RootOf(_Z^3-2)+_Z^2)-540*RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-
2)+_Z^2)*RootOf(_Z^3-2)^3-22*RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-
2)+_Z^2)^2*RootOf(_Z^3-2)^2+87480*(x^3+2*x^2+x)^(1/3)*RootOf(_Z^3-2)^2*x+56
7*(x^3+2*x^2+x)^(1/3)*RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2
)*RootOf(_Z^3-2)*x+87480*(x^3+2*x^2+x)^(1/3)*RootOf(_Z^3-2)^2+567*(x^3+2*x^
2+x)^(1/3)*RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z
^3-2)-109350*RootOf(_Z^3-2)*x^2-4455*RootOf(729*RootOf(_Z^3-2)^2+27*_Z*Root
Of(_Z^3-2)+_Z^2)*x^2-160380*RootOf(_Z^3-2)*x-6534*RootOf(729*RootOf(_Z^3-2)
^2+27*_Z*RootOf(_Z^3-2)+_Z^2)*x-30618*(x^3+2*x^2+x)^(2/3)-51030*RootOf(_Z^3
-2)-2079*RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2))/(-1+x)/(1+
x))*RootOf(_Z^3-2)-1/27*ln(-(270*RootOf(_Z^3-2)^3*RootOf(729*RootOf(_Z^3-2)
^2+27*_Z*RootOf(_Z^3-2)+_Z^2)*x^2+11*RootOf(_Z^3-2)^2*RootOf(729*RootOf(_Z^
3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)^2*x^2-270*RootOf(729*RootOf(_Z^3-2)^2+27*
_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x-11*RootOf(729*RootOf(_Z^3-2)^2+2
7*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x+2673*(x^3+2*x^2+x)^(2/3)*Roo
tOf(_Z^3-2)^2*RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)-540*Ro
```

```
otOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3-22*Ro
otOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2+874
80*(x^3+2*x^2+x)^(1/3)*RootOf(_Z^3-2)^2*x+567*(x^3+2*x^2+x)^(1/3)*RootOf(72
9*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x+87480*(x^3+2
*x^2+x)^(1/3)*RootOf(_Z^3-2)^2+567*(x^3+2*x^2+x)^(1/3)*RootOf(729*RootOf(_Z
^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)-109350*RootOf(_Z^3-2)*x^2
-4455*RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)*x^2-160380*Roo
tOf(_Z^3-2)*x-6534*RootOf(729*RootOf(_Z^3-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2)*x
-30618*(x^3+2*x^2+x)^(2/3)-51030*RootOf(_Z^3-2)-2079*RootOf(729*RootOf(_Z^3
-2)^2+27*_Z*RootOf(_Z^3-2)+_Z^2))/(-1+x)/(1+x))*RootOf(729*RootOf(_Z^3-2)^2
+27*_Z*RootOf(_Z^3-2)+_Z^2))*(x^2*(1+x))^(1/3)*(x*(1+x)^2)^(1/3)/x/(1+x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+x^2\right)^{\frac{1}{3}}\left(x^2+1\right)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^3+x^2)^(1/3)/(x^2-1),x, algorithm="maxima")
[Out] integrate((x^3 + x^2)^(1/3)*(x^2 + 1)/(x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^3+x^2\right)^{1/3}\,\left(x^2+1\right)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + x^3)^(1/3)*(x^2 + 1))/(x^2 - 1),x)
[Out] int(((x^2 + x^3)^(1/3)*(x^2 + 1))/(x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x^2\,(x+1)}\,\left(x^2+1\right)}{(x-1)\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**3+x**2)**(1/3)/(x**2-1),x)
[Out] Integral((x**2*(x + 1))**(1/3)*(x**2 + 1)/((x - 1)*(x + 1)), x)
```

3.2081 $\int \frac{-b+ax}{x\sqrt[3]{-b^3+a^3x^3}}\,dx$

Optimal. Leaf size=225

$$-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a^3x^3-b^3}}{\sqrt{3}a}+\frac{x}{\sqrt{3}}}{x}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a^3x^3-b^3}}{\sqrt{3}b}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}\left(\frac{-ax-b}{2\sqrt[3]{a^3x^3-b^3}-ax+b}\right)-\frac{1}{6}\log\left(-b\sqrt[3]{a^3x^3-b^3}+\left(a\right.\right.$$

Rubi [A] time = 0.14, antiderivative size = 131, normalized size of antiderivative = 0.58, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1844, 239, 266, 56, 617, 204, 31}

$$\frac{1}{2}\log\left(\sqrt[3]{a^3x^3-b^3}+b\right)-\frac{1}{2}\log\left(\sqrt[3]{a^3x^3-b^3}-ax\right)+\frac{\tan^{-1}\left(\frac{\frac{2ax}{\sqrt[3]{a^3x^3-b^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{b-2\sqrt[3]{a^3x^3-b^3}}{\sqrt{3}b}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x)/(x*(-b^3 + a^3*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*a*x)/(-b^3 + a^3*x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + ArcTan[(b -
2*(-b^3 + a^3*x^3)^(1/3))/(Sqrt[3]*b)]/Sqrt[3] - Log[x]/2 + Log[b + (-b^3 +
 a^3*x^3)^(1/3)]/2 - Log[-(a*x) + (-b^3 + a^3*x^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-b+ax}{x\sqrt[3]{-b^3+a^3x^3}}\,dx &= \int \left(\frac{a}{\sqrt[3]{-b^3+a^3x^3}} - \frac{b}{x\sqrt[3]{-b^3+a^3x^3}}\right)dx \\
&= a\int \frac{1}{\sqrt[3]{-b^3+a^3x^3}}\,dx - b\int \frac{1}{x\sqrt[3]{-b^3+a^3x^3}}\,dx \\
&= \frac{\tan^{-1}\left(\frac{1+\frac{2ax}{\sqrt[3]{-b^3+a^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{2}\log\left(-ax+\sqrt[3]{-b^3+a^3x^3}\right) - \frac{1}{3}b\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt[3]{-b^3+a^3x}}\,dx, x\right. \\
&= \frac{\tan^{-1}\left(\frac{1+\frac{2ax}{\sqrt[3]{-b^3+a^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} - \frac{1}{2}\log\left(-ax+\sqrt[3]{-b^3+a^3x^3}\right) + \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{b+x}\,dx, x\right. \\
&= \frac{\tan^{-1}\left(\frac{1+\frac{2ax}{\sqrt[3]{-b^3+a^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(b+\sqrt[3]{-b^3+a^3x^3}\right) - \frac{1}{2}\log\left(-ax+\sqrt[3]{-b^3+a^3x\right. \\
&= \frac{\tan^{-1}\left(\frac{1+\frac{2ax}{\sqrt[3]{-b^3+a^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-b^3+a^3x^3}}{b}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(b+\sqrt[3]{-b^3+a^3x^3}\right) - \frac{1}{2}
\end{aligned}$$

Mathematica [C] time = 0.18, size = 158, normalized size = 0.70

$$\frac{1}{6}\left(-2\log\left(1-\frac{ax}{\sqrt[3]{a^3x^3-b^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2ax}{\sqrt[3]{a^3x^3-b^3}}+1}{\sqrt{3}}\right)-\frac{3\left(a^3x^3-b^3\right)^{2/3}{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-\frac{a^3x^3}{b^3}\right)}{b^2}+\log\left(\frac{ax}{\sqrt[3]{a^3x^3-b^3}}+\frac{a^2x^2}{\left(a^3x^3-b^3\right)^{2/3}}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x)/(x*(-b^3 + a^3*x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*a*x)/(-b^3 + a^3*x^3)^(1/3))/Sqrt[3]] - (3*(-b^3
+ a^3*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, 1 - (a^3*x^3)/b^3])/b^2 - 2
*Log[1 - (a*x)/(-b^3 + a^3*x^3)^(1/3)] + Log[1 + (a^2*x^2)/(-b^3 + a^3*x^3)
^(2/3) + (a*x)/(-b^3 + a^3*x^3)^(1/3)])/6
```

IntegrateAlgebraic [A] time = 5.11, size = 225, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a^3x^3-b^3}}{\sqrt{3}a}+\frac{x}{\sqrt{3}}}{x}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{a^3x^3-b^3}}{\sqrt{3}b}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}\left(\frac{-ax-b}{2\sqrt[3]{a^3x^3-b^3}-ax+b}\right)-\frac{1}{6}\log\left(-b\sqrt[3]{a^3x^3-b^3}+\left(a^3x^3-b^3\right)^{2/3}+b^2\right)+\frac{1}{6}\log\left(ax\sqrt[3]{a^3x^3-b^3}+\left(a^3x^3-b^3\right)^{2/3}+a^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x)/(x*(-b^3 + a^3*x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[(x/Sqrt[3] + (2*(-b^3 + a^3*x^3)^(1/3))/(Sqrt[3]*a))/x]/Sqrt[3]) +
 ArcTan[1/Sqrt[3] - (2*(-b^3 + a^3*x^3)^(1/3))/(Sqrt[3]*b)]/Sqrt[3] - (2*Ar
cTanh[(-b - a*x)/(b - a*x + 2*(-b^3 + a^3*x^3)^(1/3))])/3 - Log[b^2 - b*(-b
^3 + a^3*x^3)^(1/3) + (-b^3 + a^3*x^3)^(2/3)]/6 + Log[a^2*x^2 + a*x*(-b^3 +
 a^3*x^3)^(1/3) + (-b^3 + a^3*x^3)^(2/3)]/6
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/x/(a^3*x^3-b^3)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-b}{\left(a^3x^3-b^3\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/x/(a^3*x^3-b^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*x - b)/((a^3*x^3 - b^3)^(1/3)*x), x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{ax-b}{x\left(a^3x^3-b^3\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-b)/x/(a^3*x^3-b^3)^(1/3),x)
```

```
[Out] int((a*x-b)/x/(a^3*x^3-b^3)^(1/3),x)
```

maxima [A] time = 0.42, size = 227, normalized size = 1.01

$$-\frac{1}{6}\,a\left(\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(a+\frac{2\left(a^3x^3-b^3\right)^{\frac{1}{3}}}{x}\right)}{3\,a}\right)}{a}-\frac{\log\left(a^2+\frac{\left(a^3x^3-b^3\right)^{\frac{1}{3}}a}{x}+\frac{\left(a^3x^3-b^3\right)^{\frac{2}{3}}}{x^2}\right)}{a}+\frac{2\,\log\left(-a+\frac{\left(a^3x^3-b^3\right)^{\frac{1}{3}}}{x}\right)}{a}\right)-\frac{1}{6}\,b\left(\frac{2\,\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\left(b-2\left(a^3x^3-b^3\right)^{\frac{1}{3}}\right)}{3\,b}\right)}{b}+\frac{\log\left(b^2-\left(a^3x^3-b^3\right)^{\frac{1}{3}}b+\left(a^3x^3-b^3\right)^{\frac{2}{3}}\right)}{b}-\frac{2\,\log\left(b+\left(a^3x^3-b^3\right)^{\frac{1}{3}}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/x/(a^3*x^3-b^3)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(a^3*x^3 - b^3)^(1/3)/x)/a)/a -
 log(a^2 + (a^3*x^3 - b^3)^(1/3)*a/x + (a^3*x^3 - b^3)^(2/3)/x^2)/a + 2*log
(-a + (a^3*x^3 - b^3)^(1/3)/x)/a) - 1/6*b*(2*sqrt(3)*arctan(-1/3*sqrt(3)*(b
 - 2*(a^3*x^3 - b^3)^(1/3))/b)/b + log(b^2 - (a^3*x^3 - b^3)^(1/3)*b + (a^3
*x^3 - b^3)^(2/3))/b - 2*log(b + (a^3*x^3 - b^3)^(1/3))/b)
```

mupad [B] time = 2.02, size = 166, normalized size = 0.74

$$\frac{\ln\left(b^3+b^2\left(a^3\,x^3-b^3\right)^{1/3}\right)}{3}+\ln\left(9\,b^3\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)^2+b^2\left(a^3\,x^3-b^3\right)^{1/3}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)-\ln\left(9\,b^3\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)^2+b^2\left(a^3\,x^3-b^3\right)^{1/3}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)+\frac{a\,x\left(1-\frac{a^3\,x^3}{b^3}\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ \frac{a^3\,x^3}{b^3}\right)}{\left(a^3\,x^3-b^3\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x)/(x*(a^3*x^3 - b^3)^(1/3)),x)
```

```
[Out] log(b^3 + b^2*(a^3*x^3 - b^3)^(1/3))/3 + log(9*b^3*((3^(1/2)*1i)/6 - 1/6)^2
 + b^2*(a^3*x^3 - b^3)^(1/3))*((3^(1/2)*1i)/6 - 1/6) - log(9*b^3*((3^(1/2)*
1i)/6 + 1/6)^2 + b^2*(a^3*x^3 - b^3)^(1/3))*((3^(1/2)*1i)/6 + 1/6) + (a*x*(
1 - (a^3*x^3)/b^3)^(1/3)*hypergeom([1/3, 1/3], 4/3, (a^3*x^3)/b^3))/(a^3*x^
3 - b^3)^(1/3)
```

sympy [C] time = 4.55, size = 80, normalized size = 0.36

$$\frac{axe^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{a^3x^3}{b^3}\right)}{3b\Gamma\left(\frac{4}{3}\right)}+\frac{b\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{b^3e^{2i\pi}}{a^3x^3}\right)}{3ax\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/x/(a**3*x**3-b**3)**(1/3),x)
```

```
[Out] a*x*exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), a**3*x**3/b**3)/(3*b*
gamma(4/3)) + b*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b**3*exp_polar(2*I*pi)
/(a**3*x**3))/(3*a*x*gamma(4/3))
```

3.2082 $\int \frac{(-2+x^3)\sqrt[3]{x+2x^3+x^4}}{(1+x^3)(1+x^2+x^3)}\,dx$

Optimal. Leaf size=225

$$-\log\left(\sqrt[3]{x^4+2x^3+x}-x\right)+\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^4+2x^3+x}-2x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^4+2x^3+x}+x}\right)+\sqrt[3]{2}\sqrt{3}\tan^{-1}\Bigg($$

Rubi [F] time = 6.27, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+x^3)\sqrt[3]{x+2x^3+x^4}}{(1+x^3)(1+x^2+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + x^3)*(x + 2*x^3 + x^4)^(1/3))/((1 + x^3)*(1 + x^2 + x^3)),x]
```

```
[Out] (-3*(x + 2*x^3 + x^4)^(1/3)*Defer[Subst][Defer[Int][(1 + 2*x^6 + x^9)^(1/3)
, x], x, x^(1/3)])/(x^(1/3)*(1 + 2*x^2 + x^3)^(1/3)) + (3*(1 - I*Sqrt[3])*(
x + 2*x^3 + x^4)^(1/3)*Defer[Subst][Defer[Int][(1 + 2*x^6 + x^9)^(1/3), x],
 x, x^(1/3)])/(2*x^(1/3)*(1 + 2*x^2 + x^3)^(1/3)) + (3*(1 + I*Sqrt[3])*(x +
 2*x^3 + x^4)^(1/3)*Defer[Subst][Defer[Int][(1 + 2*x^6 + x^9)^(1/3), x], x,
 x^(1/3)])/(2*x^(1/3)*(1 + 2*x^2 + x^3)^(1/3)) + ((x + 2*x^3 + x^4)^(1/3)*D
efer[Subst][Defer[Int][(1 + 2*x^6 + x^9)^(1/3)/(1 + x), x], x, x^(1/3)])/(x
^(1/3)*(1 + 2*x^2 + x^3)^(1/3)) - ((1 + I*Sqrt[3])*(x + 2*x^3 + x^4)^(1/3)*
Defer[Subst][Defer[Int][(1 + 2*x^6 + x^9)^(1/3)/(-1 - I*Sqrt[3] + 2*x), x],
 x, x^(1/3)])/(x^(1/3)*(1 + 2*x^2 + x^3)^(1/3)) - ((1 - I*Sqrt[3])*(x + 2*x
^3 + x^4)^(1/3)*Defer[Subst][Defer[Int][(1 + 2*x^6 + x^9)^(1/3)/(-1 + I*Sqr
t[3] + 2*x), x], x, x^(1/3)])/(x^(1/3)*(1 + 2*x^2 + x^3)^(1/3)) - ((1 - I*S
qrt[3])^(1/3)*(1 + I*Sqrt[3])*(x + 2*x^3 + x^4)^(1/3)*Defer[Subst][Defer[In
t][(1 + 2*x^6 + x^9)^(1/3)/((1 - I*Sqrt[3])^(1/3) + (-2)^(1/3)*x), x], x, x
^(1/3)])/(2*x^(1/3)*(1 + 2*x^2 + x^3)^(1/3)) - ((1 - I*Sqrt[3])*(1 + I*Sqrt
[3])^(1/3)*(x + 2*x^3 + x^4)^(1/3)*Defer[Subst][Defer[Int][(1 + 2*x^6 + x^9
)^(1/3)/((1 + I*Sqrt[3])^(1/3) + (-2)^(1/3)*x), x], x, x^(1/3)])/(2*x^(1/3)
*(1 + 2*x^2 + x^3)^(1/3)) - ((1 - I*Sqrt[3])^(1/3)*(1 + I*Sqrt[3])*(x + 2*x
^3 + x^4)^(1/3)*Defer[Subst][Defer[Int][(1 + 2*x^6 + x^9)^(1/3)/((1 - I*Sqr
t[3])^(1/3) - 2^(1/3)*x), x], x, x^(1/3)])/(2*x^(1/3)*(1 + 2*x^2 + x^3)^(1/
3)) - ((1 - I*Sqrt[3])*(1 + I*Sqrt[3])^(1/3)*(x + 2*x^3 + x^4)^(1/3)*Defer[
Subst][Defer[Int][(1 + 2*x^6 + x^9)^(1/3)/((1 + I*Sqrt[3])^(1/3) - 2^(1/3)*
x), x], x, x^(1/3)])/(2*x^(1/3)*(1 + 2*x^2 + x^3)^(1/3)) - ((1 - I*Sqrt[3])
^(1/3)*(1 + I*Sqrt[3])*(x + 2*x^3 + x^4)^(1/3)*Defer[Subst][Defer[Int][(1 +
 2*x^6 + x^9)^(1/3)/((1 - I*Sqrt[3])^(1/3) - (-1)^(2/3)*2^(1/3)*x), x], x,
x^(1/3)])/(2*x^(1/3)*(1 + 2*x^2 + x^3)^(1/3)) - ((1 - I*Sqrt[3])*(1 + I*Sqr
t[3])^(1/3)*(x + 2*x^3 + x^4)^(1/3)*Defer[Subst][Defer[Int][(1 + 2*x^6 + x^
9)^(1/3)/((1 + I*Sqrt[3])^(1/3) - (-1)^(2/3)*2^(1/3)*x), x], x, x^(1/3)])/(
2*x^(1/3)*(1 + 2*x^2 + x^3)^(1/3)) - (6*(x + 2*x^3 + x^4)^(1/3)*Defer[Subst
][Defer[Int][(x^3*(1 + 2*x^6 + x^9)^(1/3))/(1 + x^6 + x^9), x], x, x^(1/3)]
)/(x^(1/3)*(1 + 2*x^2 + x^3)^(1/3)) - (9*(x + 2*x^3 + x^4)^(1/3)*Defer[Subs
t][Defer[Int][(x^6*(1 + 2*x^6 + x^9)^(1/3))/(1 + x^6 + x^9), x], x, x^(1/3)
])/(x^(1/3)*(1 + 2*x^2 + x^3)^(1/3))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-2+x^3\right)\sqrt[3]{x+2x^3+x^4}}{\left(1+x^3\right)\left(1+x^2+x^3\right)}\,dx &= \frac{\sqrt[3]{x+2x^3+x^4}\int \frac{\sqrt[3]{x}\left(-2+x^3\right)\sqrt[3]{1+2x^2+x^3}}{\left(1+x^3\right)\left(1+x^2+x^3\right)}\,dx}{\sqrt[3]{x}\sqrt[3]{1+2x^2+x^3}}\\
&= \frac{\sqrt[3]{x+2x^3+x^4}\int \left(\frac{\sqrt[3]{x}\sqrt[3]{1+2x^2+x^3}}{-1-x}+\frac{\sqrt[3]{x}(1+x)\sqrt[3]{1+2x^2+x^3}}{1-x+x^2}+\frac{(-2-3x)\sqrt[3]{x}\sqrt[3]{1+2x^2+x^3}}{1+x^2+x^3}\right)}{\sqrt[3]{x}\sqrt[3]{1+2x^2+x^3}}\\
&= \frac{\sqrt[3]{x+2x^3+x^4}\int \frac{\sqrt[3]{x}\sqrt[3]{1+2x^2+x^3}}{-1-x}\,dx}{\sqrt[3]{x}\sqrt[3]{1+2x^2+x^3}}+\frac{\sqrt[3]{x+2x^3+x^4}\int \frac{\sqrt[3]{x}(1+x)\sqrt[3]{1+2x^2+x^3}}{1-x+x^2}\,dx}{\sqrt[3]{x}\sqrt[3]{1+2x^2+x^3}}\\
&= \frac{\sqrt[3]{x+2x^3+x^4}\int \left(\frac{\left(1-i\sqrt{3}\right)\sqrt[3]{x}\sqrt[3]{1+2x^2+x^3}}{-1-i\sqrt{3}+2x}+\frac{\left(1+i\sqrt{3}\right)\sqrt[3]{x}\sqrt[3]{1+2x^2+x^3}}{-1+i\sqrt{3}+2x}\right)dx}{\sqrt[3]{x}\sqrt[3]{1+2x^2+x^3}}+\frac{\left(3\sqrt[3]{x\ldots\right.}{}\\
&= \frac{\left(3\sqrt[3]{x+2x^3+x^4}\right)\text{Subst}\left(\int \left(-\sqrt[3]{1+2x^6+x^9}-\frac{\sqrt[3]{1+2x^6+x^9}}{-1-x^3}\right)dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1+2x^2+x^3}}+\ldots\\
&= -\frac{\left(3\sqrt[3]{x+2x^3+x^4}\right)\text{Subst}\left(\int \sqrt[3]{1+2x^6+x^9}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1+2x^2+x^3}}-\frac{\left(3\sqrt[3]{x+2x^3+x^4}\ldots\right.}{}\\
&= -\frac{\left(3\sqrt[3]{x+2x^3+x^4}\right)\text{Subst}\left(\int \sqrt[3]{1+2x^6+x^9}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1+2x^2+x^3}}-\frac{\left(3\sqrt[3]{x+2x^3+x^4}\ldots\right.}{}\\
&= \frac{\sqrt[3]{x+2x^3+x^4}\,\text{Subst}\left(\int \frac{\sqrt[3]{1+2x^6+x^9}}{1+x}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1+2x^2+x^3}}-\frac{\sqrt[3]{x+2x^3+x^4}\,\text{Subst}\left(\int \ldots\right.}{\sqrt[3]{x}\sqrt[3]{1+\ldots}}\\
&= \frac{\sqrt[3]{x+2x^3+x^4}\,\text{Subst}\left(\int \frac{\sqrt[3]{1+2x^6+x^9}}{1+x}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1+2x^2+x^3}}-\frac{\sqrt[3]{x+2x^3+x^4}\,\text{Subst}\left(\int \left(\ldots\right.\right.}{}\\
&= \frac{\sqrt[3]{x+2x^3+x^4}\,\text{Subst}\left(\int \frac{\sqrt[3]{1+2x^6+x^9}}{1+x}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{1+2x^2+x^3}}-\frac{\left(3\sqrt[3]{x+2x^3+x^4}\right)\text{Subst}\left(\ldots\right.}{\sqrt[3]{x}\sqrt[3]{1-\ldots}}
\end{aligned}
$$

Mathematica [F] time = 2.52, size = 0, normalized size = 0.00

$$\int \frac{\left(-2+x^3\right)\sqrt[3]{x+2x^3+x^4}}{\left(1+x^3\right)\left(1+x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + x^3)*(x + 2*x^3 + x^4)^(1/3))/((1 + x^3)*(1 + x^2 + x^3)),
x]
```

```
[Out] Integrate[((-2 + x^3)*(x + 2*x^3 + x^4)^(1/3))/((1 + x^3)*(1 + x^2 + x^3)),
 x]
```

IntegrateAlgebraic [A] time = 0.61, size = 225, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^4+2x^3+x}-x\right)+\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^4+2x^3+x}-2x\right)-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+2x^3+x}+x}\right)+\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+2x^3+x}+x}\right)+\frac{1}{2}\log\left(x^2+\sqrt[3]{x^4+2x^3+x}\,x+\left(x^4+2x^3+x\right)^{2/3}\right)-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^4+2x^3+x}\,x+\sqrt[3]{2}\left(x^4+2x^3+x\right)^{2/3}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^3)*(x + 2*x^3 + x^4)^(1/3))/((1 + x^3)*(1 + x^2
 + x^3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(x + 2*x^3 + x^4)^(1/3))]) + 2^(1/3)*Sq
rt[3]*ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(x + 2*x^3 + x^4)^(1/3))] - Log[-x +
(x + 2*x^3 + x^4)^(1/3)] + 2^(1/3)*Log[-2*x + 2^(2/3)*(x + 2*x^3 + x^4)^(1/
3)] + Log[x^2 + x*(x + 2*x^3 + x^4)^(1/3) + (x + 2*x^3 + x^4)^(2/3)]/2 - Lo
g[2*x^2 + 2^(2/3)*x*(x + 2*x^3 + x^4)^(1/3) + 2^(1/3)*(x + 2*x^3 + x^4)^(2/
3)]/2^(2/3)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^4+2*x^3+x)^(1/3)/(x^3+1)/(x^3+x^2+1),x, algorithm="fri
cas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+2\,x^3+x\right)^{\frac{1}{3}}\left(x^3-2\right)}{\left(x^3+x^2+1\right)\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^4+2*x^3+x)^(1/3)/(x^3+1)/(x^3+x^2+1),x, algorithm="gia
c")
```

```
[Out] integrate((x^4 + 2*x^3 + x)^(1/3)*(x^3 - 2)/((x^3 + x^2 + 1)*(x^3 + 1)), x)
```

maple [C] time = 31.24, size = 2504, normalized size = 11.13

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2)*(x^4+2*x^3+x)^(1/3)/(x^3+1)/(x^3+x^2+1),x)
```

```
[Out] -ln((20*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*
x^3-40*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x
^2+20*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4-47
*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^3-318*(
x^4+2*x^3+x)^(2/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_
Z^3-2)^2+897*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)
^2*(x^4+2*x^3+x)^(1/3)*x-746*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2
)*RootOf(_Z^3-2)^2*x^2-47*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*R
ootOf(_Z^3-2)^2-904*x^3+1158*(x^4+2*x^3+x)^(2/3)+636*x*(x^4+2*x^3+x)^(1/3)-
2938*x^2-904)/(x^3+x^2+1))+RootOf(_Z^3-2)*ln((19681*RootOf(RootOf(_Z^3-2)^2
+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^3*x^3+104831*RootOf(_Z^3-2)^4*Roo
tOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^3-39362*RootOf(RootOf(_Z^3-2
)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^3*x^2-209662*RootOf(_Z^3-2)^4*
RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2+19681*RootOf(RootOf(_Z^
3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^3-641838*(x^4+2*x^3+x)^(2/3
)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2+104831*R
```

ootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)-78724*RootO
f(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x^3-419324*RootOf
(_Z^3-2)^2*x^3-393620*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootO
f(_Z^3-2)*x^2-2096620*RootOf(_Z^3-2)^2*x^2+2017896*RootOf(RootOf(_Z^3-2)^2+
_Z*RootOf(_Z^3-2)+_Z^2)*(x^4+2*x^3+x)^(1/3)*x+1283676*RootOf(_Z^3-2)*(x^4+2
*x^3+x)^(1/3)*x-78724*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootO
f(_Z^3-2)+734220*(x^4+2*x^3+x)^(2/3)-419324*RootOf(_Z^3-2)^2)/(1+x)/(x^2-x+
1))-ln(-(104831*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z
^3-2)^3*x^3+19681*RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2
)+_Z^2)*x^3-209662*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf
(_Z^3-2)^3*x^2-39362*RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^
3-2)+_Z^2)*x^2+104831*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*Roo
tOf(_Z^3-2)^3-641838*(x^4+2*x^3+x)^(2/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(
_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2+19681*RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)
^2+_Z*RootOf(_Z^3-2)+_Z^2)+628986*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)
+_Z^2)*RootOf(_Z^3-2)*x^3+118086*RootOf(_Z^3-2)^2*x^3+1677296*RootOf(RootOf
(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x^2+314896*RootOf(_Z^3-2)
^2*x^2-734220*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*(x^4+2*x^3+x)
^(1/3)*x+1283676*RootOf(_Z^3-2)*(x^4+2*x^3+x)^(1/3)*x+628986*RootOf(RootOf(
_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)-2017896*(x^4+2*x^3+x)^(2/3
)+118086*RootOf(_Z^3-2)^2)/(1+x)/(x^2-x+1))*RootOf(RootOf(_Z^3-2)^2+_Z*Root
Of(_Z^3-2)+_Z^2)-ln(-(104831*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2
)^2*RootOf(_Z^3-2)^3*x^3+19681*RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+_Z*
RootOf(_Z^3-2)+_Z^2)*x^3-209662*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_
Z^2)^2*RootOf(_Z^3-2)^3*x^2-39362*RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+
_Z*RootOf(_Z^3-2)+_Z^2)*x^2+104831*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2
)+_Z^2)^2*RootOf(_Z^3-2)^3-641838*(x^4+2*x^3+x)^(2/3)*RootOf(RootOf(_Z^3-2)
^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2+19681*RootOf(_Z^3-2)^4*RootOf(R
ootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)+628986*RootOf(RootOf(_Z^3-2)^2+_Z*R
ootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x^3+118086*RootOf(_Z^3-2)^2*x^3+1677296*
RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x^2+314896*R
ootOf(_Z^3-2)^2*x^2-734220*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*
(x^4+2*x^3+x)^(1/3)*x+1283676*RootOf(_Z^3-2)*(x^4+2*x^3+x)^(1/3)*x+628986*R
ootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)-2017896*(x^4+
2*x^3+x)^(2/3)+118086*RootOf(_Z^3-2)^2)/(1+x)/(x^2-x+1))*RootOf(_Z^3-2)+ln(
-(11647*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*
x^3-23294*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^
4*x^2+11647*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2
)^4+124118*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2
x^3-70932(x^4+2*x^3+x)^(2/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z
^2)*RootOf(_Z^3-2)^2-115770*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)
RootOf(_Z^3-2)^2(x^4+2*x^3+x)^(1/3)*x+240938*RootOf(RootOf(_Z^3-2)^2+_Z*R
ootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^2+124118*RootOf(RootOf(_Z^3-2)^2+_Z*
RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2+76480*x^3-373404*(x^4+2*x^3+x)^(2/3)+
141864*x*(x^4+2*x^3+x)^(1/3)+168256*x^2+76480)/(x^3+x^2+1))+1/2*ln(-(11647*
RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^3-2329
4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^2+11
647*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4+1241
18*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^3-709
32*(x^4+2*x^3+x)^(2/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*Root
Of(_Z^3-2)^2-115770*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(
_Z^3-2)^2*(x^4+2*x^3+x)^(1/3)*x+240938*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z
^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^2+124118*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_
Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2+76480*x^3-373404*(x^4+2*x^3+x)^(2/3)+141864*x
*(x^4+2*x^3+x)^(1/3)+168256*x^2+76480)/(x^3+x^2+1))*RootOf(RootOf(_Z^3-2)^2
+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+2\,x^3+x\right)^{\frac{1}{3}}\left(x^3-2\right)}{\left(x^3+x^2+1\right)\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2)*(x^4+2*x^3+x)^(1/3)/(x^3+1)/(x^3+x^2+1),x, algorithm="max
ima")
```

```
[Out] integrate((x^4 + 2*x^3 + x)^(1/3)*(x^3 - 2)/((x^3 + x^2 + 1)*(x^3 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^3-2\right)\,\left(x^4+2\,x^3+x\right)^{1/3}}{\left(x^3+1\right)\,\left(x^3+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 2)*(x + 2*x^3 + x^4)^(1/3))/((x^3 + 1)*(x^2 + x^3 + 1)),x)
```

```
[Out] int(((x^3 - 2)*(x + 2*x^3 + x^4)^(1/3))/((x^3 + 1)*(x^2 + x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x\left(x^3+2x^2+1\right)}\,\left(x^3-2\right)}{(x+1)\left(x^2-x+1\right)\left(x^3+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2)*(x**4+2*x**3+x)**(1/3)/(x**3+1)/(x**3+x**2+1),x)
```

```
[Out] Integral((x*(x**3 + 2*x**2 + 1))**(1/3)*(x**3 - 2)/((x + 1)*(x**2 - x + 1)*
(x**3 + x**2 + 1)), x)
```

3.2083 $\int \frac{x^4}{\sqrt[4]{-1+x^4}\left(1-2x^4+2x^8\right)}\,dx$

Optimal. Leaf size=225

$$-\frac{1}{8}\sqrt{2+\sqrt{2}}\ \tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\ x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}\ -x^2}\right)+\frac{1}{8}\sqrt{2-\sqrt{2}}\ \tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\ x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}\ -x^2}\right)-\frac{1}{8}\sqrt{2+\sqrt{2}}\ \tanh^{-1}\left(\sqrt{2}\right.$$

Rubi [A] time = 0.20, antiderivative size = 242, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {1528, 377, 211, 1165, 628, 1162, 617, 204, 212, 206, 203}

$$\frac{1}{4}(-1)^{5/8}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{x^4-1}}\right)+\frac{(-1)^{5/8}\tan^{-1}\left(1-\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt{2}}-\frac{(-1)^{5/8}\tan^{-1}\left(\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4-1}}+1\right)}{4\sqrt{2}}+\frac{1}{4}(-1)^{5/8}\tanh^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{x^4-1}}\right)+\frac{(-1)^{5/8}\log\left(\frac{\sqrt[8]{-1}\sqrt{2}x}{\sqrt[4]{x^4-1}}+\frac{x^2}{\sqrt{x^4-1}}+\sqrt[4]{-1}\right)}{8\sqrt{2}}-\frac{(-1)^{5/8}\log\left(\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4-1}}-\frac{(-1)^{3/4}x^2}{\sqrt{x^4-1}}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((-1 + x^4)^(1/4)*(1 - 2*x^4 + 2*x^8)),x]
```

```
[Out] ((-1)^(5/8)*ArcTan[((-1)^(7/8)*x)/(-1 + x^4)^(1/4)])/4 + ((-1)^(5/8)*ArcTan
[1 - ((-1)^(7/8)*Sqrt[2]*x)/(-1 + x^4)^(1/4)])/(4*Sqrt[2]) - ((-1)^(5/8)*Ar
cTan[1 + ((-1)^(7/8)*Sqrt[2]*x)/(-1 + x^4)^(1/4)])/(4*Sqrt[2]) + ((-1)^(5/8
)*ArcTanh[((-1)^(7/8)*x)/(-1 + x^4)^(1/4)])/4 + ((-1)^(5/8)*Log[(-1)^(1/4)
+ x^2/Sqrt[-1 + x^4] + ((-1)^(1/8)*Sqrt[2]*x)/(-1 + x^4)^(1/4)])/(8*Sqrt[2]
) - ((-1)^(5/8)*Log[1 - ((-1)^(3/4)*x^2)/Sqrt[-1 + x^4] + ((-1)^(7/8)*Sqrt[
2]*x)/(-1 + x^4)^(1/4)])/(8*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1528

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q, (
f*x)^m/(a + b*x^n + c*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, e, f, q, n}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&  !IntegerQ[q] &&
IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\sqrt[4]{-1+x^4}\left(1-2x^4+2x^8\right)}\,dx &= \int \left(\frac{1-i}{\sqrt[4]{-1+x^4}\left((-2-2i)+4x^4\right)}+\frac{1+i}{\sqrt[4]{-1+x^4}\left((-2+2i)+4x^4\right)}\right)dx \\
&= (1-i)\int \frac{1}{\sqrt[4]{-1+x^4}\left((-2-2i)+4x^4\right)}\,dx+(1+i)\int \frac{1}{\sqrt[4]{-1+x^4}\left((-2+2i)\right.} \\
&= (1-i)\,\text{Subst}\left(\int \frac{1}{(-2-2i)-(2-2i)x^4}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)+(1+i)\,\text{Subst}\Big(\\
&= -\frac{i\,\text{Subst}\left(\int \frac{\sqrt[4]{-1}-x^2}{(-2-2i)-(2-2i)x^4}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{2}}-\frac{i\,\text{Subst}\left(\int \frac{\sqrt[4]{-1}+x^2}{(-2-2i)-(2-2i)x^4}\,dx,x,\right.}{\sqrt{2}} \\
&= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)(-1)^{7/8}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{2}}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(-1)^{7/8}\tanh^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{2}}-\left(\left(\frac{1}{16}\right.\right. \\
&= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)(-1)^{7/8}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{2}}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(-1)^{7/8}\tanh^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{2}}+\left(\frac{1}{16}\right. \\
&= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)(-1)^{7/8}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{2}}+\left(\frac{1}{8}-\frac{i}{8}\right)(-1)^{7/8}\tan^{-1}\left(1-\frac{(-1)^{7/8}\sqrt{2}\,x}{\sqrt[4]{-1+x^4}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.18, size = 201, normalized size = 0.89

$$\left(\frac{1}{16}-\frac{i}{16}\right)(-1)^{5/8}\left((2+2i)\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{x^4-1}}\right)+(2+2i)\tanh^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{x^4-1}}\right)+\sqrt[4]{-1}\left(2\tan^{-1}\left(1-\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4-1}}\right)-2\tan^{-1}\left(\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4-1}}+1\right)-\log\left(-\frac{\sqrt[8]{-1}\sqrt{2}x}{\sqrt[4]{x^4-1}}+\frac{x^2}{\sqrt{x^4-1}}+\sqrt[4]{-1}\right)+\log\left(\frac{\sqrt[8]{-1}\sqrt{2}x}{\sqrt[4]{x^4-1}}+\frac{x^2}{\sqrt{x^4-1}}+\sqrt[4]{-1}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((-1 + x^4)^(1/4)*(1 - 2*x^4 + 2*x^8)),x]
```

```
[Out] (1/16 - I/16)*(-1)^(5/8)*((2 + 2*I)*ArcTan[((-1)^(7/8)*x)/(-1 + x^4)^(1/4)]
 + (2 + 2*I)*ArcTanh[((-1)^(7/8)*x)/(-1 + x^4)^(1/4)] + (-1)^(1/4)*(2*ArcTa
n[1 - ((-1)^(7/8)*Sqrt[2]*x)/(-1 + x^4)^(1/4)] - 2*ArcTan[1 + ((-1)^(7/8)*S
qrt[2]*x)/(-1 + x^4)^(1/4)] - Log[(-1)^(1/4) + x^2/Sqrt[-1 + x^4] - ((-1)^(
1/8)*Sqrt[2]*x)/(-1 + x^4)^(1/4)] + Log[(-1)^(1/4) + x^2/Sqrt[-1 + x^4] + (
(-1)^(1/8)*Sqrt[2]*x)/(-1 + x^4)^(1/4)]))
```

IntegrateAlgebraic [A] time = 0.90, size = 225, normalized size = 1.00

$$-\frac{1}{8}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}-x^2}\right)+\frac{1}{8}\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}-x^2}\right)-\frac{1}{8}\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+x^2}\right)+\frac{1}{8}\sqrt{2-\sqrt{2}}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((-1 + x^4)^(1/4)*(1 - 2*x^4 + 2*x^8)),x]
```

```
[Out] -1/8*(Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]]*x*(-1 + x^4)^(1/4))/(-x^2
 + Sqrt[-1 + x^4])]) + (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]]*x*(-1 +
 x^4)^(1/4))/(-x^2 + Sqrt[-1 + x^4])])/8 - (Sqrt[2 + Sqrt[2]]*ArcTanh[(Sqrt
[2 - Sqrt[2]]*x*(-1 + x^4)^(1/4))/(x^2 + Sqrt[-1 + x^4])])/8 + (Sqrt[2 - Sq
rt[2]]*ArcTanh[(Sqrt[2 + Sqrt[2]]*x*(-1 + x^4)^(1/4))/(x^2 + Sqrt[-1 + x^4]
)])/8
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1)^(1/4)/(2*x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(2\,x^8-2\,x^4+1\right)\left(x^4-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1)^(1/4)/(2*x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x^4/((2*x^8 - 2*x^4 + 1)*(x^4 - 1)^(1/4)), x)
```

maple [C] time = 5.60, size = 544, normalized size = 2.42

$$-\frac{\mathrm{RootOf}\left(\_Z^8+1\right)^7\ln\left(\frac{\mathrm{RootOf}(\_Z^8+1)^7x^4-2\sqrt{x^4-1}\,\mathrm{RootOf}(\_Z^8+1)^5x^2+\mathrm{RootOf}(\_Z^8+1)^3x^4+2(x^4-1)^{\frac14}\mathrm{RootOf}(\_Z^8+1)^2x^3-2(x^4-1)^{\frac34}x-\mathrm{RootOf}(\_Z^8+1)^3}{\mathrm{RootOf}(\_Z^8+1)^4x^4-x^4+1}\right)}{8}+\frac{\mathrm{RootOf}\left(\_Z^8+1\right)^5\ln\left(-\frac{2\sqrt{x^4-1}\,\mathrm{RootOf}(\_Z^8+1)^7x^2-2(x^4-1)^{\frac14}\mathrm{RootOf}(\_Z^8+1)^6x^3+\mathrm{RootOf}(\_Z^8+1)^5x^4-\mathrm{RootOf}(\_Z^8+1)x^4+2(x^4-1)^{\frac34}x+\mathrm{RootOf}(\_Z^8+1)}{\mathrm{RootOf}(\_Z^8+1)^4x^4+x^4-1}\right)}{8}-\frac{\mathrm{RootOf}\left(\_Z^8+1\right)\ln\left(-\frac{\mathrm{RootOf}(\_Z^8+1)^9x^4+2(x^4-1)^{\frac14}\mathrm{RootOf}(\_Z^8+1)^6x^3-\mathrm{RootOf}(\_Z^8+1)^5x^4+2\sqrt{x^4-1}\,\mathrm{RootOf}(\_Z^8+1)^3x^2+\mathrm{RootOf}(\_Z^8+1)^5+2(x^4-1)^{\frac34}x}{\mathrm{RootOf}(\_Z^8+1)^4x^4+x^4-1}\right)}{8}-\frac{\mathrm{RootOf}\left(\_Z^8+1\right)^3\ln\left(-\frac{\mathrm{RootOf}(\_Z^8+1)^{11}x^4+2\mathrm{RootOf}(\_Z^8+1)^7x^4-2(x^4-1)^{\frac14}\mathrm{RootOf}(\_Z^8+1)^6x^3+2\sqrt{x^4-1}\,\mathrm{RootOf}(\_Z^8+1)^5x^2-2(x^4-1)^{\frac34}\mathrm{RootOf}(\_Z^8+1)^4x-\mathrm{RootOf}(\_Z^8+1)^7+\mathrm{RootOf}(\_Z^8+1)^3x^4+2(x^4-1)^{\frac14}\mathrm{RootOf}(\_Z^8+1)^2x^3-2\sqrt{x^4-1}\,\mathrm{RootOf}(\_Z^8+1)x^2+2(x^4-1)^{\frac34}x-\mathrm{RootOf}(\_Z^8+1)^3}{\mathrm{RootOf}(\_Z^8+1)^4x^4-x^4+1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4-1)^(1/4)/(2*x^8-2*x^4+1),x)
```

```
[Out] -1/8*RootOf(_Z^8+1)^7*ln((RootOf(_Z^8+1)^7*x^4-2*(x^4-1)^(1/2)*RootOf(_Z^8+
1)^5*x^2+RootOf(_Z^8+1)^3*x^4+2*(x^4-1)^(1/4)*RootOf(_Z^8+1)^2*x^3-2*(x^4-1
)^(3/4)*x-RootOf(_Z^8+1)^3)/(RootOf(_Z^8+1)^4*x^4-x^4+1))+1/8*RootOf(_Z^8+1
)^5*ln(-(2*(x^4-1)^(1/2)*RootOf(_Z^8+1)^7*x^2-2*(x^4-1)^(1/4)*RootOf(_Z^8+1
)^6*x^3+RootOf(_Z^8+1)^5*x^4-RootOf(_Z^8+1)*x^4+2*(x^4-1)^(3/4)*x+RootOf(_Z
^8+1))/(RootOf(_Z^8+1)^4*x^4+x^4-1))-1/8*RootOf(_Z^8+1)*ln(-(RootOf(_Z^8+1)
^9*x^4+2*(x^4-1)^(1/4)*RootOf(_Z^8+1)^6*x^3-RootOf(_Z^8+1)^5*x^4+2*(x^4-1)^
(1/2)*RootOf(_Z^8+1)^3*x^2+RootOf(_Z^8+1)^5+2*(x^4-1)^(3/4)*x)/(RootOf(_Z^8
+1)^4*x^4+x^4-1))-1/8*RootOf(_Z^8+1)^3*ln(-(RootOf(_Z^8+1)^11*x^4+2*RootOf(
_Z^8+1)^7*x^4-2*(x^4-1)^(1/4)*RootOf(_Z^8+1)^6*x^3+2*(x^4-1)^(1/2)*RootOf(_
Z^8+1)^5*x^2-2*(x^4-1)^(3/4)*RootOf(_Z^8+1)^4*x-RootOf(_Z^8+1)^7+RootOf(_Z^
8+1)^3*x^4+2*(x^4-1)^(1/4)*RootOf(_Z^8+1)^2*x^3-2*(x^4-1)^(1/2)*RootOf(_Z^8
+1)*x^2+2*(x^4-1)^(3/4)*x-RootOf(_Z^8+1)^3)/(RootOf(_Z^8+1)^4*x^4-x^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(2\,x^8-2\,x^4+1\right)\left(x^4-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1)^(1/4)/(2*x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((2*x^8 - 2*x^4 + 1)*(x^4 - 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(x^4-1\right)}^{1/4}\,\left(2\,x^8-2\,x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((x^4 - 1)^(1/4)*(2*x^8 - 2*x^4 + 1)),x)
```

```
[Out] int(x^4/((x^4 - 1)^(1/4)*(2*x^8 - 2*x^4 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4-1)**(1/4)/(2*x**8-2*x**4+1),x)
```

```
[Out] Timed out
```

3.2084 $\int \frac{(-1+x^4)^{3/4}}{1-2x^4+2x^8}\,dx$

Optimal. Leaf size=225

$$-\frac{1}{8}\sqrt{2-\sqrt{2}}\ \tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\ x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}\ -x^2}\right)-\frac{1}{8}\sqrt{2+\sqrt{2}}\ \tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\ x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}\ -x^2}\right)-\frac{1}{8}\sqrt{2-\sqrt{2}}\ \tanh^{-1}\left(\frac{\sqrt{2-}}{\sqrt{x}}\right.$$

Rubi [A] time = 0.19, antiderivative size = 242, normalized size of antiderivative = 1.08, number of steps used = 25, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {1428, 408, 240, 212, 206, 203, 377, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{4}\sqrt[8]{-1}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{x^4-1}}\right)-\frac{\sqrt[8]{-1}\tan^{-1}\left(1-\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4-1}}\right)}{4\sqrt{2}}+\frac{\sqrt[8]{-1}\tan^{-1}\left(\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4-1}}+1\right)}{4\sqrt{2}}+\frac{1}{4}\sqrt[8]{-1}\tanh^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{x^4-1}}\right)-\frac{\sqrt[8]{-1}\log\left(\frac{\sqrt[8]{-1}\sqrt{2}x}{\sqrt[4]{x^4-1}}+\frac{x^2}{\sqrt{x^4-1}}+\sqrt[4]{-1}\right)}{8\sqrt{2}}+\frac{\sqrt[8]{-1}\log\left(\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4-1}}-\frac{(-1)^{3/4}x^2}{\sqrt{x^4-1}}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)^(3/4)/(1 - 2*x^4 + 2*x^8),x]
```

```
[Out] ((-1)^(1/8)*ArcTan[((-1)^(7/8)*x)/(-1 + x^4)^(1/4)])/4 - ((-1)^(1/8)*ArcTan
[1 - ((-1)^(7/8)*Sqrt[2]*x)/(-1 + x^4)^(1/4)])/(4*Sqrt[2]) + ((-1)^(1/8)*Ar
cTan[1 + ((-1)^(7/8)*Sqrt[2]*x)/(-1 + x^4)^(1/4)])/(4*Sqrt[2]) + ((-1)^(1/8
)*ArcTanh[((-1)^(7/8)*x)/(-1 + x^4)^(1/4)])/4 - ((-1)^(1/8)*Log[(-1)^(1/4)
+ x^2/Sqrt[-1 + x^4] + ((-1)^(1/8)*Sqrt[2]*x)/(-1 + x^4)^(1/4)])/(8*Sqrt[2]
) + ((-1)^(1/8)*Log[1 - ((-1)^(3/4)*x^2)/Sqrt[-1 + x^4] + ((-1)^(7/8)*Sqrt[
2]*x)/(-1 + x^4)^(1/4)])/(8*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 408

```
Int[((a_) + (b_.)*(x_)^4)^(p_)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d
, Int[(a + b*x^4)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(p
- 1)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[p, 3/4] || EqQ[p, 5/4])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1428

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^
n)^q/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[(d + e*x^n)^q/(b + r + 2
*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{(-1+x^4)^{3/4}}{1-2x^4+2x^8}\,dx = -\left(2i\int \frac{(-1+x^4)^{3/4}}{(-2-2i)+4x^4}\,dx\right) + 2i\int \frac{(-1+x^4)^{3/4}}{(-2+2i)+4x^4}\,dx$$

$$= -\left((-1-i)\int \frac{1}{\sqrt[4]{-1+x^4}\left((-2-2i)+4x^4\right)}\,dx\right) + (1-i)\int \frac{1}{\sqrt[4]{-1+x^4}\left((-2+2i)+4x^4\right)}\,dx$$

$$= -\left((-1-i)\,\mathrm{Subst}\left(\int \frac{1}{(-2-2i)-(2-2i)x^4}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)\right) + (1-i)\,\mathrm{Subst}\left(\int \frac{}{(-2+2}\right.$$

$$= -\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{-1}-x^2}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{2}} + -\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{-1}+x^2}\,dx, x, \frac{x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{2}} + \cdots$$

$$= -\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-1)^{7/8}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{2}} - \frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-1)^{7/8}\tanh^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{2}} + \left(\left(-\frac{1}{16}-\frac{i}{16}\right)(-1)^{7}\right.$$

$$= -\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-1)^{7/8}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{2}} - \frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-1)^{7/8}\tanh^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{2}} + \left(\frac{1}{16}+\frac{i}{16}\right)(-1)^{7/8}\,1$$

$$= -\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-1)^{7/8}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{-1+x^4}}\right)}{\sqrt{2}} + \left(\frac{1}{8}+\frac{i}{8}\right)(-1)^{7/8}\tan^{-1}\left(1-\frac{(-1)^{7/8}\sqrt{2}\,x}{\sqrt[4]{-1+x^4}}\right) - \left(\frac{1}{8}+\frac{i}{8}\right)(-1$$

Mathematica [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^4)^{3/4}}{1-2x^4+2x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^4)^(3/4)/(1 - 2*x^4 + 2*x^8),x]
```

```
[Out] Integrate[(-1 + x^4)^(3/4)/(1 - 2*x^4 + 2*x^8), x]
```

IntegrateAlgebraic [A] time = 0.80, size = 225, normalized size = 1.00

$$-\frac{1}{8}\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}-x^2}\right)-\frac{1}{8}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}-x^2}\right)-\frac{1}{8}\sqrt{2-\sqrt{2}}\tanh^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+x^2}\right)-\frac{1}{8}\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)^(3/4)/(1 - 2*x^4 + 2*x^8),x]
```

```
[Out] -1/8*(Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]]*x*(-1 + x^4)^(1/4))/(-x^2
 + Sqrt[-1 + x^4])]) - (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]]*x*(-1 +
 x^4)^(1/4))/(-x^2 + Sqrt[-1 + x^4])])/8 - (Sqrt[2 - Sqrt[2]]*ArcTanh[(Sqrt
[2 - Sqrt[2]]*x*(-1 + x^4)^(1/4))/(x^2 + Sqrt[-1 + x^4])])/8 - (Sqrt[2 + Sq
rt[2]]*ArcTanh[(Sqrt[2 + Sqrt[2]]*x*(-1 + x^4)^(1/4))/(x^2 + Sqrt[-1 + x^4]
)])/8
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(3/4)/(2*x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)^{\frac{3}{4}}}{2\,x^8-2\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(3/4)/(2*x^8-2*x^4+1),x, algorithm="giac")
[Out] integrate((x^4 - 1)^(3/4)/(2*x^8 - 2*x^4 + 1), x)
```

maple [C] time = 4.98, size = 463, normalized size = 2.06

$$-\frac{\mathrm{RootOf}\left(\_Z^8+1\right)^3 \ln\left(-\frac{\mathrm{RootOf}(\_Z^8+1)^7 x^4-2\sqrt{x^4-1}\,\mathrm{RootOf}(\_Z^8+1)^5 x^2+\mathrm{RootOf}(\_Z^8+1)^3 x^4-2(x^4-1)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^8+1)^2 x^3+2(x^4-1)^{\frac{3}{4}}x-\mathrm{RootOf}(\_Z^8+1)^3}{\mathrm{RootOf}(\_Z^8+1)^4 x^4-x^4+1}\right)}{8}-\frac{\mathrm{RootOf}\left(\_Z^8+1\right) \ln\left(\frac{2\sqrt{x^4-1}\,\mathrm{RootOf}(\_Z^8+1)^7 x^2+2(x^4-1)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^8+1)^6 x^3+\mathrm{RootOf}(\_Z^8+1)^5 x^4-\mathrm{RootOf}(\_Z^8+1) x^4-2(x^4-1)^{\frac{3}{4}}x+\mathrm{RootOf}(\_Z^8+1)}{\mathrm{RootOf}(\_Z^8+1)^4 x^4+x^4-1}\right)}{8}-\frac{\mathrm{RootOf}\left(\_Z^8+1\right)^7 \ln\left(-\frac{\mathrm{RootOf}(\_Z^8+1)^{11} x^4+\mathrm{RootOf}(\_Z^8+1)^7 x^4-\mathrm{RootOf}(\_Z^8+1)^7+2(x^4-1)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^8+1)^2 x^3-2\sqrt{x^4-1}\,\mathrm{RootOf}(\_Z^8+1) x^2+2(x^4-1)^{\frac{3}{4}}x}{\mathrm{RootOf}(\_Z^8+1)^4 x^4-x^4+1}\right)}{8}+\frac{\mathrm{RootOf}\left(\_Z^8+1\right)^5 \ln\left(-\frac{\mathrm{RootOf}(\_Z^8+1)^9 x^4+2(x^4-1)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^8+1)^6 x^3-\mathrm{RootOf}(\_Z^8+1)^5 x^4+2\sqrt{x^4-1}\,\mathrm{RootOf}(\_Z^8+1)^3 x^2+\mathrm{RootOf}(\_Z^8+1)^5+2(x^4-1)^{\frac{3}{4}}x}{\mathrm{RootOf}(\_Z^8+1)^4 x^4+x^4-1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)^(3/4)/(2*x^8-2*x^4+1),x)
[Out] -1/8*RootOf(_Z^8+1)^3*ln(-(RootOf(_Z^8+1)^7*x^4-2*(x^4-1)^(1/2)*RootOf(_Z^8
+1)^5*x^2+RootOf(_Z^8+1)^3*x^4-2*(x^4-1)^(1/4)*RootOf(_Z^8+1)^2*x^3+2*(x^4-
1)^(3/4)*x-RootOf(_Z^8+1)^3)/(RootOf(_Z^8+1)^4*x^4-x^4+1))-1/8*RootOf(_Z^8+
1)*ln((2*(x^4-1)^(1/2)*RootOf(_Z^8+1)^7*x^2+2*(x^4-1)^(1/4)*RootOf(_Z^8+1)^
6*x^3+RootOf(_Z^8+1)^5*x^4-RootOf(_Z^8+1)*x^4-2*(x^4-1)^(3/4)*x+RootOf(_Z^8
+1))/(RootOf(_Z^8+1)^4*x^4+x^4-1))-1/8*RootOf(_Z^8+1)^7*ln(-(RootOf(_Z^8+1)
^11*x^4+RootOf(_Z^8+1)^7*x^4-RootOf(_Z^8+1)^7+2*(x^4-1)^(1/4)*RootOf(_Z^8+1
)^2*x^3-2*(x^4-1)^(1/2)*RootOf(_Z^8+1)*x^2+2*(x^4-1)^(3/4)*x)/(RootOf(_Z^8+
1)^4*x^4-x^4+1))+1/8*RootOf(_Z^8+1)^5*ln(-(RootOf(_Z^8+1)^9*x^4+2*(x^4-1)^(
1/4)*RootOf(_Z^8+1)^6*x^3-RootOf(_Z^8+1)^5*x^4+2*(x^4-1)^(1/2)*RootOf(_Z^8+
1)^3*x^2+RootOf(_Z^8+1)^5+2*(x^4-1)^(3/4)*x)/(RootOf(_Z^8+1)^4*x^4+x^4-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-1\right)^{\frac{3}{4}}}{2\,x^8-2\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)^(3/4)/(2*x^8-2*x^4+1),x, algorithm="maxima")
[Out] integrate((x^4 - 1)^(3/4)/(2*x^8 - 2*x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(x^4-1\right)}^{3/4}}{2\,x^8-2\,x^4+1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)^(3/4)/(2*x^8 - 2*x^4 + 1),x)
[Out] int((x^4 - 1)^(3/4)/(2*x^8 - 2*x^4 + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)**(3/4)/(2*x**8-2*x**4+1),x)
[Out] Timed out
```

3.2085 $\int \frac{x^4}{\sqrt[4]{1+x^4}\left(1+2x^4+2x^8\right)}\,dx$

Optimal. Leaf size=225

$$\frac{1}{8}\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}-x^2}\right)-\frac{1}{8}\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}-x^2}\right)+\frac{1}{8}\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2-\sqrt{}}}{\sqrt{x^4}}\right.$$

Rubi [A] time = 0.18, antiderivative size = 242, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {1528, 377, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$-\frac{1}{4}(-1)^{5/8}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{x^4+1}}\right)-\frac{(-1)^{5/8}\tan^{-1}\left(1-\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt{2}}+\frac{(-1)^{5/8}\tan^{-1}\left(\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4+1}}+1\right)}{4\sqrt{2}}-\frac{1}{4}(-1)^{5/8}\tanh^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{x^4+1}}\right)-\frac{(-1)^{5/8}\log\left(\frac{\sqrt[8]{-1}\sqrt{2}x}{\sqrt[4]{x^4+1}}+\frac{x^2}{\sqrt{x^4+1}}+\sqrt[4]{-1}\right)}{8\sqrt{2}}+\frac{(-1)^{5/8}\log\left(\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4+1}}-\frac{(-1)^{3/4}x^2}{\sqrt{x^4+1}}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 + x^4)^(1/4)*(1 + 2*x^4 + 2*x^8)),x]
```

```
[Out] -1/4*((-1)^(5/8)*ArcTan[((-1)^(7/8)*x)/(1 + x^4)^(1/4)]) - ((-1)^(5/8)*ArcT
an[1 - ((-1)^(7/8)*Sqrt[2]*x)/(1 + x^4)^(1/4)])/(4*Sqrt[2]) + ((-1)^(5/8)*A
rcTan[1 + ((-1)^(7/8)*Sqrt[2]*x)/(1 + x^4)^(1/4)])/(4*Sqrt[2]) - ((-1)^(5/8
)*ArcTanh[((-1)^(7/8)*x)/(1 + x^4)^(1/4)])/4 - ((-1)^(5/8)*Log[(-1)^(1/4) +
 x^2/Sqrt[1 + x^4] + ((-1)^(1/8)*Sqrt[2]*x)/(1 + x^4)^(1/4)])/(8*Sqrt[2]) +
 ((-1)^(5/8)*Log[1 - ((-1)^(3/4)*x^2)/Sqrt[1 + x^4] + ((-1)^(7/8)*Sqrt[2]*x
)/(1 + x^4)^(1/4)])/(8*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1528

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q, (
f*x)^m/(a + b*x^n + c*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, e, f, q, n}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&  !IntegerQ[q] &&
IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{1+x^4}\left(1+2x^4+2x^8\right)}\,dx = \int \left(\frac{1+i}{\sqrt[4]{1+x^4}\left((2-2i)+4x^4\right)}+\frac{1-i}{\sqrt[4]{1+x^4}\left((2+2i)+4x^4\right)}\right)dx$$
$$= (1-i)\int \frac{1}{\sqrt[4]{1+x^4}\left((2+2i)+4x^4\right)}\,dx+(1+i)\int \frac{1}{\sqrt[4]{1+x^4}\left((2-2i)+4x^4\right)}\,dx$$
$$= (1-i)\,\text{Subst}\left(\int \frac{1}{(2+2i)+(2-2i)x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)+(1+i)\,\text{Subst}\left(\int \frac{1}{(2-\ldots}\right.$$
$$= -\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-1}-x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{2}}+-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-1}+x^2}\,dx, x, \ldots\right.}{\sqrt{2}}$$
$$= -\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(-1)^{7/8}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{2}}-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(-1)^{7/8}\tanh^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{2}}-\left(\left(-\frac{1}{16}+\ldots\right.\right.$$
$$= -\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(-1)^{7/8}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{2}}-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(-1)^{7/8}\tanh^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{2}}-\left(\frac{1}{16}-\frac{i}{16}\ldots\right.$$
$$= -\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(-1)^{7/8}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{2}}-\left(\frac{1}{8}-\frac{i}{8}\right)(-1)^{7/8}\tan^{-1}\left(1-\frac{(-1)^{7/8}\sqrt{2}\,x}{\sqrt[4]{1+x^4}}\right)+\left(\ldots\right.$$

Mathematica [C] time = 0.17, size = 201, normalized size = 0.89

$$\left(-\frac{1}{16}+\frac{i}{16}\right)(-1)^{5/8}\left((2+2i)\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{x^4+1}}\right)+(2+2i)\tanh^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{x^4+1}}\right)+\sqrt[4]{-1}\left(2\tan^{-1}\left(1-\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4+1}}\right)-2\tan^{-1}\left(\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4+1}}+1\right)-\log\left(-\frac{\sqrt[8]{-1}\sqrt{2}x}{\sqrt[4]{x^4+1}}+\frac{x^2}{\sqrt{x^4+1}}+\sqrt[4]{-1}\right)+\log\left(\frac{\sqrt[8]{-1}\sqrt{2}x}{\sqrt[4]{x^4+1}}+\frac{x^2}{\sqrt{x^4+1}}+\sqrt[4]{-1}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((1 + x^4)^(1/4)*(1 + 2*x^4 + 2*x^8)),x]
```

```
[Out] (-1/16 + I/16)*(-1)^(5/8)*((2 + 2*I)*ArcTan[((-1)^(7/8)*x)/(1 + x^4)^(1/4)]
 + (2 + 2*I)*ArcTanh[((-1)^(7/8)*x)/(1 + x^4)^(1/4)] + (-1)^(1/4)*(2*ArcTan
[1 - ((-1)^(7/8)*Sqrt[2]*x)/(1 + x^4)^(1/4)] - 2*ArcTan[1 + ((-1)^(7/8)*Sqr
t[2]*x)/(1 + x^4)^(1/4)] - Log[(-1)^(1/4) + x^2/Sqrt[1 + x^4] - ((-1)^(1/8)
*Sqrt[2]*x)/(1 + x^4)^(1/4)] + Log[(-1)^(1/4) + x^2/Sqrt[1 + x^4] + ((-1)^(
1/8)*Sqrt[2]*x)/(1 + x^4)^(1/4)]))
```

IntegrateAlgebraic [A] time = 0.83, size = 225, normalized size = 1.00

$$\frac{1}{8}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}-x^2}\right)-\frac{1}{8}\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}-x^2}\right)+\frac{1}{8}\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}+x^2}\right)-\frac{1}{8}\sqrt{2-\sqrt{2}}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((1 + x^4)^(1/4)*(1 + 2*x^4 + 2*x^8)),x]
```

```
[Out] (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]]*x*(1 + x^4)^(1/4))/(-x^2 + Sqr
t[1 + x^4])])/8 - (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]]*x*(1 + x^4)^
(1/4))/(-x^2 + Sqrt[1 + x^4])])/8 + (Sqrt[2 + Sqrt[2]]*ArcTanh[(Sqrt[2 - Sq
rt[2]]*x*(1 + x^4)^(1/4))/(x^2 + Sqrt[1 + x^4])])/8 - (Sqrt[2 - Sqrt[2]]*Ar
cTanh[(Sqrt[2 + Sqrt[2]]*x*(1 + x^4)^(1/4))/(x^2 + Sqrt[1 + x^4])])/8
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+1)^(1/4)/(2*x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(2\,x^8+2\,x^4+1\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+1)^(1/4)/(2*x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] integrate(x^4/((2*x^8 + 2*x^4 + 1)*(x^4 + 1)^(1/4)), x)
```

maple [C] time = 5.48, size = 529, normalized size = 2.35

$$-\frac{\mathrm{RootOf}\left(\_Z^8+1\right)^5 \ln\left(-\frac{\mathrm{RootOf}(\_Z^8+1)^9 x^4+2\sqrt{x^4+1}\,\mathrm{RootOf}(\_Z^8+1)^7 x^2-2\left(x^4+1\right)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^8+1)^6 x^3+2\left(x^4+1\right)^{\frac{3}{4}}\mathrm{RootOf}(\_Z^8+1)^4 x-2\sqrt{x^4+1}\,\mathrm{RootOf}(\_Z^8+1)^3 x^2+2\left(x^4+1\right)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^8+1)^2 x^3-\mathrm{RootOf}(\_Z^8+1)^5-\mathrm{RootOf}(\_Z^8+1) x^4+2\left(x^4+1\right)^{\frac{3}{4}} x-\mathrm{RootOf}(\_Z^8+1)}{\mathrm{RootOf}(\_Z^8+1)^4 x^4+x^4+1}\right)}{8}-\frac{\mathrm{RootOf}\left(\_Z^8+1\right)^3 \ln\left(\frac{\mathrm{RootOf}(\_Z^8+1)^{11} x^4+\mathrm{RootOf}(\_Z^8+1)^7 x^4+\mathrm{RootOf}(\_Z^8+1)^7-2\left(x^4+1\right)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^8+1)^2 x^3-2\sqrt{x^4+1}\,\mathrm{RootOf}(\_Z^8+1) x^2-2\left(x^4+1\right)^{\frac{3}{4}} x}{\mathrm{RootOf}(\_Z^8+1)^4 x^4-x^4-1}\right)}{8}-\frac{\mathrm{RootOf}\left(\_Z^8+1\right) \ln\left(\frac{\mathrm{RootOf}(\_Z^8+1)^9 x^4-2\left(x^4+1\right)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^8+1)^6 x^3-\mathrm{RootOf}(\_Z^8+1)^5 x^4+2\sqrt{x^4+1}\,\mathrm{RootOf}(\_Z^8+1)^3 x^2-\mathrm{RootOf}(\_Z^8+1)^5-2\left(x^4+1\right)^{\frac{3}{4}} x}{\mathrm{RootOf}(\_Z^8+1)^4 x^4+x^4+1}\right)}{8}+\frac{\mathrm{RootOf}\left(\_Z^8+1\right)^7 \ln\left(\frac{\mathrm{RootOf}(\_Z^8+1)^7 x^4-2\sqrt{x^4+1}\,\mathrm{RootOf}(\_Z^8+1)^5 x^2+\mathrm{RootOf}(\_Z^8+1)^3 x^4+2\left(x^4+1\right)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^8+1)^2 x^3-2\left(x^4+1\right)^{\frac{3}{4}} x+\mathrm{RootOf}(\_Z^8+1)^3}{\mathrm{RootOf}(\_Z^8+1)^4 x^4-x^4-1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4+1)^(1/4)/(2*x^8+2*x^4+1),x)
```

```
[Out] -1/8*RootOf(_Z^8+1)^5*ln(-(RootOf(_Z^8+1)^9*x^4+2*(x^4+1)^(1/2)*RootOf(_Z^8
+1)^7*x^2-2*(x^4+1)^(1/4)*RootOf(_Z^8+1)^6*x^3+2*(x^4+1)^(3/4)*RootOf(_Z^8+
1)^4*x-2*(x^4+1)^(1/2)*RootOf(_Z^8+1)^3*x^2+2*(x^4+1)^(1/4)*RootOf(_Z^8+1)^
2*x^3-RootOf(_Z^8+1)^5-RootOf(_Z^8+1)*x^4+2*(x^4+1)^(3/4)*x-RootOf(_Z^8+1))
/(RootOf(_Z^8+1)^4*x^4+x^4+1))-1/8*RootOf(_Z^8+1)^3*ln((RootOf(_Z^8+1)^11*x
^4+RootOf(_Z^8+1)^7*x^4+RootOf(_Z^8+1)^7-2*(x^4+1)^(1/4)*RootOf(_Z^8+1)^2*x
^3-2*(x^4+1)^(1/2)*RootOf(_Z^8+1)*x^2-2*(x^4+1)^(3/4)*x)/(RootOf(_Z^8+1)^4*
x^4-x^4-1))-1/8*RootOf(_Z^8+1)*ln((RootOf(_Z^8+1)^9*x^4-2*(x^4+1)^(1/4)*Roo
tOf(_Z^8+1)^6*x^3-RootOf(_Z^8+1)^5*x^4+2*(x^4+1)^(1/2)*RootOf(_Z^8+1)^3*x^2
-RootOf(_Z^8+1)^5-2*(x^4+1)^(3/4)*x)/(RootOf(_Z^8+1)^4*x^4+x^4+1))+1/8*Root
Of(_Z^8+1)^7*ln((RootOf(_Z^8+1)^7*x^4-2*(x^4+1)^(1/2)*RootOf(_Z^8+1)^5*x^2+
RootOf(_Z^8+1)^3*x^4+2*(x^4+1)^(1/4)*RootOf(_Z^8+1)^2*x^3-2*(x^4+1)^(3/4)*x
+RootOf(_Z^8+1)^3)/(RootOf(_Z^8+1)^4*x^4-x^4-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(2\,x^8+2\,x^4+1\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+1)^(1/4)/(2*x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((2*x^8 + 2*x^4 + 1)*(x^4 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(x^4+1\right)}^{1/4}\,\left(2\,x^8+2\,x^4+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((x^4 + 1)^(1/4)*(2*x^4 + 2*x^8 + 1)),x)
```

```
[Out] int(x^4/((x^4 + 1)^(1/4)*(2*x^4 + 2*x^8 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4+1)**(1/4)/(2*x**8+2*x**4+1),x)
```

```
[Out] Timed out
```

3.2086 $\int \frac{(1+x^4)^{3/4}}{1+2x^4+2x^8}\,dx$

Optimal. Leaf size=225

$$\frac{1}{8}\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}-x^2}\right)+\frac{1}{8}\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}-x^2}\right)+\frac{1}{8}\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2}\ldots}{\ldots}\right.$$

Rubi [A] time = 0.19, antiderivative size = 242, normalized size of antiderivative = 1.08, number of steps used = 25, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {1428, 408, 240, 212, 206, 203, 377, 211, 1165, 628, 1162, 617, 204}

$$-\frac{1}{4}\sqrt[8]{-1}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{x^4+1}}\right)+\frac{\sqrt[8]{-1}\tan^{-1}\left(1-\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4+1}}\right)}{4\sqrt{2}}-\frac{\sqrt[8]{-1}\tan^{-1}\left(\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4+1}}+1\right)}{4\sqrt{2}}-\frac{1}{4}\sqrt[8]{-1}\tanh^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{x^4+1}}\right)+\frac{\sqrt[8]{-1}\log\left(\frac{\sqrt[8]{-1}\sqrt{2}x}{\sqrt[4]{x^4+1}}+\frac{x^2}{\sqrt{x^4+1}}+\sqrt[4]{-1}\right)}{8\sqrt{2}}-\frac{\sqrt[8]{-1}\log\left(\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4+1}}-\frac{(-1)^{3/4}x^2}{\sqrt{x^4+1}}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)^(3/4)/(1 + 2*x^4 + 2*x^8),x]
```

```
[Out] -1/4*((-1)^(1/8)*ArcTan[((-1)^(7/8)*x)/(1 + x^4)^(1/4)]) + ((-1)^(1/8)*ArcT
an[1 - ((-1)^(7/8)*Sqrt[2]*x)/(1 + x^4)^(1/4)])/(4*Sqrt[2]) - ((-1)^(1/8)*A
rcTan[1 + ((-1)^(7/8)*Sqrt[2]*x)/(1 + x^4)^(1/4)])/(4*Sqrt[2]) - ((-1)^(1/8
)*ArcTanh[((-1)^(7/8)*x)/(1 + x^4)^(1/4)])/4 + ((-1)^(1/8)*Log[(-1)^(1/4) +
 x^2/Sqrt[1 + x^4] + ((-1)^(1/8)*Sqrt[2]*x)/(1 + x^4)^(1/4)])/(8*Sqrt[2]) -
 ((-1)^(1/8)*Log[1 - ((-1)^(3/4)*x^2)/Sqrt[1 + x^4] + ((-1)^(7/8)*Sqrt[2]*x
)/(1 + x^4)^(1/4)])/(8*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 408

```
Int[((a_) + (b_.)*(x_)^4)^(p_)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d
, Int[(a + b*x^4)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(p
- 1)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[p, 3/4] || EqQ[p, 5/4])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1428

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^
n)^q/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[(d + e*x^n)^q/(b + r + 2
*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{\left(1+x^4\right)^{3/4}}{1+2x^4+2x^8}\,dx = -\left(2i\int \frac{\left(1+x^4\right)^{3/4}}{(2-2i)+4x^4}\,dx\right) + 2i\int \frac{\left(1+x^4\right)^{3/4}}{(2+2i)+4x^4}\,dx$$

$$= -\left((-1+i)\int \frac{1}{\sqrt[4]{1+x^4}\,\left((2-2i)+4x^4\right)}\,dx\right) + (1+i)\int \frac{1}{\sqrt[4]{1+x^4}\,\left((2+2i)+4x^4\right)}\,dx$$

$$= -\left((-1+i)\,\text{Subst}\left(\int \frac{1}{(2-2i)+(2+2i)x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)\right) + (1+i)\,\text{Subst}\left(\int \frac{}{(2+2i}\right.$$

$$= -\left(-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-1}-x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{2}}\right) - -\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-1}+x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right.}{\sqrt{2}}$$

$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-1)^{7/8}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{2}} + \frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-1)^{7/8}\tanh^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{2}} + \left(\left(\frac{1}{16}+\frac{i}{16}\right)(-1)^{7/8}\right.$$

$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-1)^{7/8}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{2}} + \frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-1)^{7/8}\tanh^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{2}} - \left(\frac{1}{16}+\frac{i}{16}\right)(-1)^{7/8}$$

$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-1)^{7/8}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{1+x^4}}\right)}{\sqrt{2}} - \left(\frac{1}{8}+\frac{i}{8}\right)(-1)^{7/8}\tan^{-1}\left(1-\frac{(-1)^{7/8}\sqrt{2}\,x}{\sqrt[4]{1+x^4}}\right) + \left(\frac{1}{8}+\frac{i}{8}\right)(-$$

Mathematica [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(1+x^4\right)^{3/4}}{1+2x^4+2x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^4)^(3/4)/(1 + 2*x^4 + 2*x^8),x]
[Out] Integrate[(1 + x^4)^(3/4)/(1 + 2*x^4 + 2*x^8), x]
```

IntegrateAlgebraic [A] time = 0.79, size = 225, normalized size = 1.00

$$\frac{1}{8}\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}-x^2}\right)+\frac{1}{8}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}-x^2}\right)+\frac{1}{8}\sqrt{2-\sqrt{2}}\tanh^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}+x^2}\right)+\frac{1}{8}\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^4+1}}{\sqrt{x^4+1}+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)^(3/4)/(1 + 2*x^4 + 2*x^8),x]
[Out] (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]]*x*(1 + x^4)^(1/4))/(-x^2 + Sqr
t[1 + x^4])])/8 + (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]]*x*(1 + x^4)^
(1/4))/(-x^2 + Sqrt[1 + x^4])])/8 + (Sqrt[2 - Sqrt[2]]*ArcTanh[(Sqrt[2 - Sq
rt[2]]*x*(1 + x^4)^(1/4))/(x^2 + Sqrt[1 + x^4])])/8 + (Sqrt[2 + Sqrt[2]]*Ar
cTanh[(Sqrt[2 + Sqrt[2]]*x*(1 + x^4)^(1/4))/(x^2 + Sqrt[1 + x^4])])/8
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(3/4)/(2*x^8+2*x^4+1),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)^{\frac{3}{4}}}{2\,x^8+2\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(3/4)/(2*x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)^(3/4)/(2*x^8 + 2*x^4 + 1), x)
```

maple [C] time = 5.30, size = 463, normalized size = 2.06

$$\frac{\operatorname{RootOf}\left(\_Z^8+1\right)\ln\left(\frac{2\sqrt{x^4+1}\,\operatorname{RootOf}\left(\_Z^8+1\right)^7 x^2+2\left(x^4+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^8+1\right)^6 x^3+\operatorname{RootOf}\left(\_Z^8+1\right)^5 x^4-\operatorname{RootOf}\left(\_Z^8+1\right)x^4-2\left(x^4+1\right)^{\frac{3}{4}}x-\operatorname{RootOf}\left(\_Z^8+1\right)}{\operatorname{RootOf}\left(\_Z^8+1\right)^4 x^4+x^4+1}\right)}{8}+\frac{\operatorname{RootOf}\left(\_Z^8+1\right)^3\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^8+1\right)^7 x^4-2\sqrt{x^4+1}\,\operatorname{RootOf}\left(\_Z^8+1\right)^5 x^2+\operatorname{RootOf}\left(\_Z^8+1\right)^3 x^4-2\left(x^4+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^8+1\right)^2 x^3+2\left(x^4+1\right)^{\frac{3}{4}}x+\operatorname{RootOf}\left(\_Z^8+1\right)^3}{\operatorname{RootOf}\left(\_Z^8+1\right)^4 x^4-x^4-1}\right)}{8}+\frac{\operatorname{RootOf}\left(\_Z^8+1\right)^7\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^8+1\right)^{11}x^4+\operatorname{RootOf}\left(\_Z^8+1\right)^7 x^4+\operatorname{RootOf}\left(\_Z^8+1\right)^7+2\left(x^4+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^8+1\right)^2 x^3-2\sqrt{x^4+1}\,\operatorname{RootOf}\left(\_Z^8+1\right)x^2+2\left(x^4+1\right)^{\frac{3}{4}}x}{\operatorname{RootOf}\left(\_Z^8+1\right)^4 x^4-x^4-1}\right)}{8}-\frac{\operatorname{RootOf}\left(\_Z^8+1\right)^5\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^8+1\right)^9 x^4+2\left(x^4+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^8+1\right)^6 x^3-\operatorname{RootOf}\left(\_Z^8+1\right)^5 x^4+2\sqrt{x^4+1}\,\operatorname{RootOf}\left(\_Z^8+1\right)^3 x^2-\operatorname{RootOf}\left(\_Z^8+1\right)^5+2\left(x^4+1\right)^{\frac{3}{4}}x}{\operatorname{RootOf}\left(\_Z^8+1\right)^4 x^4+x^4+1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(3/4)/(2*x^8+2*x^4+1),x)
```

```
[Out] 1/8*RootOf(_Z^8+1)*ln((2*(x^4+1)^(1/2)*RootOf(_Z^8+1)^7*x^2+2*(x^4+1)^(1/4)
*RootOf(_Z^8+1)^6*x^3+RootOf(_Z^8+1)^5*x^4-RootOf(_Z^8+1)*x^4-2*(x^4+1)^(3/
4)*x-RootOf(_Z^8+1))/(RootOf(_Z^8+1)^4*x^4+x^4+1))+1/8*RootOf(_Z^8+1)^3*ln(
-(RootOf(_Z^8+1)^7*x^4-2*(x^4+1)^(1/2)*RootOf(_Z^8+1)^5*x^2+RootOf(_Z^8+1)^
3*x^4-2*(x^4+1)^(1/4)*RootOf(_Z^8+1)^2*x^3+2*(x^4+1)^(3/4)*x+RootOf(_Z^8+1)
^3)/(RootOf(_Z^8+1)^4*x^4-x^4-1))+1/8*RootOf(_Z^8+1)^7*ln(-(RootOf(_Z^8+1)^
11*x^4+RootOf(_Z^8+1)^7*x^4+RootOf(_Z^8+1)^7+2*(x^4+1)^(1/4)*RootOf(_Z^8+1)
^2*x^3-2*(x^4+1)^(1/2)*RootOf(_Z^8+1)*x^2+2*(x^4+1)^(3/4)*x)/(RootOf(_Z^8+1
)^4*x^4-x^4-1))-1/8*RootOf(_Z^8+1)^5*ln(-(RootOf(_Z^8+1)^9*x^4+2*(x^4+1)^(1
/4)*RootOf(_Z^8+1)^6*x^3-RootOf(_Z^8+1)^5*x^4+2*(x^4+1)^(1/2)*RootOf(_Z^8+1
)^3*x^2-RootOf(_Z^8+1)^5+2*(x^4+1)^(3/4)*x)/(RootOf(_Z^8+1)^4*x^4+x^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)^{\frac{3}{4}}}{2\,x^8+2\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(3/4)/(2*x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)^(3/4)/(2*x^8 + 2*x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^4+1\right)^{3/4}}{2\,x^8+2\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)^(3/4)/(2*x^4 + 2*x^8 + 1),x)
```

```
[Out] int((x^4 + 1)^(3/4)/(2*x^4 + 2*x^8 + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(3/4)/(2*x**8+2*x**4+1),x)
```

```
[Out] Timed out
```

3.2087 $\int \frac{x^2\sqrt{-bx+a^2x^2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=225

$$\frac{\sqrt{a^2x^2-bx}\,\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(96a^4x^2-104a^2bx-15b^2\right)}{120a^2b^2x}+\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(\frac{\sqrt{b}\sqrt{\sqrt{a^2x^2-bx}-\cdots}}{\cdots}\right.$$

Rubi [F] time = 4.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\sqrt{-bx+a^2x^2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*Sqrt[-(b*x) + a^2*x^2])/(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2),x
]
```

```
[Out] (2*Sqrt[-(b*x) + a^2*x^2]*Defer[Subst][Defer[Int][(x^6*Sqrt[-b + a^2*x^2])/
(a*x^4 + x^2*Sqrt[-(b*x^2) + a^2*x^4])^(3/2), x], x, Sqrt[x]])/(Sqrt[x]*Sqr
t[-b + a^2*x])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{-bx+a^2x^2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx &= \frac{\sqrt{-bx+a^2x^2}\int \frac{x^{5/2}\sqrt{-b+a^2x}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx}{\sqrt{x}\sqrt{-b+a^2x}}\\ &= \frac{\left(2\sqrt{-bx+a^2x^2}\right)\text{Subst}\left(\int \frac{x^6\sqrt{-b+a^2x^2}}{\left(ax^4+x^2\sqrt{-bx^2+a^2x^4}\right)^{3/2}}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{-b+a^2x}}\end{aligned}$$

Mathematica [F] time = 1.48, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-bx+a^2x^2}}{\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*Sqrt[-(b*x) + a^2*x^2])/(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(
3/2),x]
```

```
[Out] Integrate[(x^2*Sqrt[-(b*x) + a^2*x^2])/(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(
3/2), x]
```

IntegrateAlgebraic [A] time = 7.04, size = 225, normalized size = 1.00

$$\frac{\sqrt{a^2x^2-bx}\,\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(96a^4x^2-104a^2bx-15b^2\right)}{120a^2b^2x}+\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(\frac{\sqrt{b}\sqrt{\sqrt{a^2x^2-bx}-ax}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{\sqrt{a^2x^2-bx}-ax}}{\sqrt{b}}\right)}{8\sqrt{2}\,a^{5/2}x}+\frac{-96a^4x^2+152a^2bx+5b^2}{120ab^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[-(b*x) + a^2*x^2])/(a*x^2 + x*Sqrt[-(b*x) + a^
2*x^2])^(3/2),x]
```

```
[Out] (Sqrt[-(b*x) + a^2*x^2]*(-15*b^2 - 104*a^2*b*x + 96*a^4*x^2)*Sqrt[x*(a*x +
Sqrt[-(b*x) + a^2*x^2])])/(120*a^2*b^2*x) + Sqrt[x*(a*x + Sqrt[-(b*x) + a^2
*x^2])]*((5*b^2 + 152*a^2*b*x - 96*a^4*x^2)/(120*a*b^2) + (Sqrt[b]*Sqrt[-(a
*x) + Sqrt[-(b*x) + a^2*x^2]]*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[-(a*x) + Sqrt[-(
b*x) + a^2*x^2]])/Sqrt[b]])/(8*Sqrt[2]*a^(5/2)*x))
```

fricas [A] time = 0.87, size = 367, normalized size = 1.63

$$\left[\frac{15\sqrt{2}\sqrt{a}b^3x\log\left(-\frac{4a^2x^2+4\sqrt{a^2x^2-bx}\,ax-bx-2\left(\sqrt{2}a^{\frac{3}{2}}x+\sqrt{2}\sqrt{a^2x^2-bx}\sqrt{a}\right)\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}}{x}\right)-4\left(96a^6x^3-152a^4bx^2-5a^2b^2x-\left(96a^5x^2-104a^3bx-15ab^2\right)\sqrt{a^2x^2-bx}\right)\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}}{480a^3b^2x}, \frac{15\sqrt{2}\sqrt{-a}b^3x\arctan\left(\frac{\sqrt{2}\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}\sqrt{-a}}{2ax}\right)-2\left(96a^6x^3-152a^4bx^2-5a^2b^2x-\left(96a^5x^2-104a^3bx-15ab^2\right)\sqrt{a^2x^2-bx}\right)\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}}{240a^3b^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, al
gorithm="fricas")
```

```
[Out] [1/480*(15*sqrt(2)*sqrt(a)*b^3*x*log(-(4*a^2*x^2 + 4*sqrt(a^2*x^2 - b*x)*a*
x - b*x - 2*(sqrt(2)*a^(3/2)*x + sqrt(2)*sqrt(a^2*x^2 - b*x)*sqrt(a))*sqrt(
a*x^2 + sqrt(a^2*x^2 - b*x)*x))/x) - 4*(96*a^6*x^3 - 152*a^4*b*x^2 - 5*a^2*
b^2*x - (96*a^5*x^2 - 104*a^3*b*x - 15*a*b^2)*sqrt(a^2*x^2 - b*x))*sqrt(a*x
^2 + sqrt(a^2*x^2 - b*x)*x))/(a^3*b^2*x), 1/240*(15*sqrt(2)*sqrt(-a)*b^3*x*
arctan(1/2*sqrt(2)*sqrt(a*x^2 + sqrt(a^2*x^2 - b*x)*x)*sqrt(-a)/(a*x)) - 2*
(96*a^6*x^3 - 152*a^4*b*x^2 - 5*a^2*b^2*x - (96*a^5*x^2 - 104*a^3*b*x - 15*
a*b^2)*sqrt(a^2*x^2 - b*x))*sqrt(a*x^2 + sqrt(a^2*x^2 - b*x)*x))/(a^3*b^2*x
)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2 - bx}\,x^2}{\left(ax^2 + \sqrt{a^2x^2 - bx}\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, al
gorithm="giac")
```

```
[Out] integrate(sqrt(a^2*x^2 - b*x)*x^2/(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{a^2x^2 - bx}}{\left(a\,x^2 + x\sqrt{a^2x^2 - bx}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

```
[Out] int(x^2*(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2 - bx}\,x^2}{\left(ax^2 + \sqrt{a^2x^2 - bx}\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2*x^2-b*x)^(1/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, al
gorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 - b*x)*x^2/(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{a^2\,x^2-b\,x}}{{\left(a\,x^2+x\,\sqrt{a^2\,x^2-b\,x}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a^2*x^2 - b*x)^(1/2))/(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2),x)
```

```
[Out] int((x^2*(a^2*x^2 - b*x)^(1/2))/(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \sqrt{x \left(a^{2} x - b\right)}}{\left(x \left(a x + \sqrt{a^{2} x^{2} - b x}\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2*x**2-b*x)**(1/2)/(a*x**2+x*(a**2*x**2-b*x)**(1/2))**(3
/2),x)
```

```
[Out] Integral(x**2*sqrt(x*(a**2*x - b))/(x*(a*x + sqrt(a**2*x**2 - b*x)))**(3/2)
, x)
```

3.2088 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=225

$$\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{\sqrt{2}-1}}\right)-\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{2}-1\right)}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\ \mathrm{t}$$

Rubi [C] time = 0.18, antiderivative size = 81, normalized size of antiderivative = 0.36, number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2133, 725, 206}

$$-\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] -1/2*(Sqrt[1 - I]*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])]) - (Sqrt
[1 + I]*ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx &= \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{1-ix^2}}\,dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{1+ix^2}}\,dx\\
&= \left(-\frac{1}{2}-\frac{i}{2}\right)\mathrm{Subst}\left(\int \frac{1}{(1+i)-x^2}\,dx,x,\frac{1-ix}{\sqrt{1+ix^2}}\right)+\left(-\frac{1}{2}+\frac{i}{2}\right)\mathrm{Subst}\left(\int \frac{1}{(1-i)-x^2}\,dx\right.\\
&= -\frac{1}{2}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)
\end{aligned}$$

Mathematica [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]), x]
```

IntegrateAlgebraic [A] time = 1.62, size = 225, normalized size = 1.00

$$\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^4+1}+x^2}\right)-\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{2\sqrt{2}-2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^4+1}+x^2}\right)+\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{2+2\sqrt{2}}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] Sqrt[(-1 + Sqrt[2])/2]*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]]
- Sqrt[(-1 + Sqrt[2])/2]*ArcTan[(Sqrt[-2 + 2*Sqrt[2]]*x*Sqrt[x^2 + Sqrt[1 +
 x^4]])/(1 + x^2 + Sqrt[1 + x^4])] - Sqrt[(1 + Sqrt[2])/2]*ArcTanh[Sqrt[-1
+ Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]] + Sqrt[(1 + Sqrt[2])/2]*ArcTanh[(Sqrt
[2 + 2*Sqrt[2]]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])]
```

fricas [B] time = 5.51, size = 369, normalized size = 1.64

$$\frac{1}{2}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{\left(2x^2-\sqrt{2}\left(x^3-x^2+x+1\right)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2\right)-2x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{2\sqrt{2}-2}+\left(x^2+\sqrt{2}\sqrt{x^4+1}+1\right)\sqrt{2\sqrt{2}+2}\sqrt{2\sqrt{2}-2}}{2\left(x^2-2x+1\right)}\right)-\frac{1}{8}\sqrt{2\sqrt{2}+2}\log\left(-\frac{\left(2x^3-\sqrt{2}\left(x^3-x^2-x-1\right)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2x\right)-2\right)\sqrt{x^2+\sqrt{x^4+1}}+\left(x^2-\sqrt{2}\left(x^2+1\right)+\sqrt{x^4+1}\left(\sqrt{2}-2\right)+1\right)\sqrt{2\sqrt{2}+2}}{x^2+2x+1}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+2}\log\left(-\frac{\left(2x^3-\sqrt{2}\left(x^3-x^2-x-1\right)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2x\right)-2\right)\sqrt{x^2+\sqrt{x^4+1}}-\left(x^2-\sqrt{2}\left(x^2+1\right)+\sqrt{x^4+1}\left(\sqrt{2}-2\right)+1\right)\sqrt{2\sqrt{2}+2}}{x^2+2x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*sqrt(2*sqrt(2) - 2)*arctan(1/2*((2*x^2 - sqrt(2)*(x^3 - x^2 + x + 1) +
sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2) - 2*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(2
*sqrt(2) - 2) + (x^2 + sqrt(2)*sqrt(x^4 + 1) + 1)*sqrt(2*sqrt(2) + 2)*sqrt(
2*sqrt(2) - 2))/(x^2 - 2*x + 1)) - 1/8*sqrt(2*sqrt(2) + 2)*log(-((2*x^3 - s
qrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2*x) - 2)*sqr
t(x^2 + sqrt(x^4 + 1)) + (x^2 - sqrt(2)*(x^2 + 1) + sqrt(x^4 + 1)*(sqrt(2)
- 2) + 1)*sqrt(2*sqrt(2) + 2))/(x^2 + 2*x + 1)) + 1/8*sqrt(2*sqrt(2) + 2)*l
og(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1)
- 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - (x^2 - sqrt(2)*(x^2 + 1) + sqrt(x^4
 + 1)*(sqrt(2) - 2) + 1)*sqrt(2*sqrt(2) + 2))/(x^2 + 2*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(1+x)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}\,\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)),x)
```

```
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)/(x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)*sqrt(x**4 + 1)), x)
```

3.2089 $\int \frac{2abx-3ax^2+x^3}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-a^2+2ax-(1+bd)x^2+dx^3\right)}\,dx$

Optimal. Leaf size=226

$$-\frac{\log\left(a^2+d^{2/3}\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+\sqrt[3]{x^3(-a-b)+abx^2+x^4}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)-2ax+x^2\right)}{2d^{2/3}}+\frac{\log\left(\sqrt[3]{d}\ \sqrt[3]{\right.}}{}$$

Rubi [F] time = 13.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2abx-3ax^2+x^3}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-a^2+2ax-(1+bd)x^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2*a*b*x - 3*a*x^2 + x^3)/((x^2*(-a + x)*(-b + x))^(1/3)*(-a^2 + 2*a*x
- (1 + b*d)*x^2 + d*x^3)),x]
```

```
[Out] (3*x*(1 - x/a)^(1/3)*(1 - x/b)^(1/3)*AppellF1[1/3, 1/3, 1/3, 4/3, x/a, x/b]
)/(d*((a - x)*(b - x)*x^2)^(1/3)) + (6*a*(1 - b*d)*x^(2/3)*(-a + x)^(1/3)*(
-b + x)^(1/3)*Defer[Subst][Defer[Int][x^3/((-a + x^3)^(1/3)*(-b + x^3)^(1/3
)*(a^2 - 2*a*x^3 + (1 + b*d)*x^6 - d*x^9)), x], x, x^(1/3)])/(d*((a - x)*(b
 - x)*x^2)^(1/3)) - (3*(1 - 3*a*d + b*d)*x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1
/3)*Defer[Subst][Defer[Int][x^6/((-a + x^3)^(1/3)*(-b + x^3)^(1/3)*(a^2 - 2
*a*x^3 + (1 + b*d)*x^6 - d*x^9)), x], x, x^(1/3)])/(d*((a - x)*(b - x)*x^2)
^(1/3)) + (3*a^2*x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][Defer[I
nt][1/((-a + x^3)^(1/3)*(-b + x^3)^(1/3)*(-a^2 + 2*a*x^3 - (1 + b*d)*x^6 +
d*x^9)), x], x, x^(1/3)])/(d*((a - x)*(b - x)*x^2)^(1/3))
```

Rubi steps

$$\int \frac{2abx-3ax^2+x^3}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-a^2+2ax-(1+bd)x^2+dx^3\right)}dx = \int \frac{x\left(2ab-3ax+x^2\right)}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-a^2+2ax-(1+bd)x^2+\right.}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\int \frac{\sqrt[3]{x}\left(2ab-3ax+x^2\right)}{\sqrt[3]{-a+x}\sqrt[3]{-b+x}\left(-a^2+2ax-(1+\right.}}{\sqrt[3]{x^2(-a+x)(-b+x)}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\text{Subst}\left(\int \frac{x^3(2a}{\sqrt[3]{-a+x^3}\sqrt[3]{-b+x^3}(}\right.}{\sqrt[3]{x^2(-a+x)(-b+x}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\text{Subst}\left(\int\left(\frac{1}{d\sqrt[3]{-a+x^3}\sqrt[3]{-b+x}}\right.\right.}{\sqrt[3]{x^2(-a+}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-a+x^3}\sqrt[3]{-b+x^3}}\right.}{d\sqrt[3]{x^2(-a+x)(-b+x)}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\text{Subst}\left(\int\left(\frac{2a}{\sqrt[3]{-a+x^3}\sqrt[3]{-b+x^3}}\right.\right.}{}$$

$$= \frac{\left(3a^2x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\text{Subst}\left(\int \frac{}{\sqrt[3]{-a+x^3}\sqrt[3]{-b+x}}\right.}{d\sqrt[3]{x^2(-a+x)(-b+}}$$

$$= \frac{3x\sqrt[3]{1-\frac{x}{a}}\sqrt[3]{1-\frac{x}{b}}F_1\left(\frac{1}{3};\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{x}{a},\frac{x}{b}\right)}{d\sqrt[3]{(a-x)(b-x)x^2}} + \frac{\left(3a^2x^{2/3}\sqrt[3]{}\right.}{}$$

Mathematica [F] time = 4.24, size = 0, normalized size = 0.00

$$\int \frac{2abx-3ax^2+x^3}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-a^2+2ax-(1+bd)x^2+dx^3\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2*a*b*x - 3*a*x^2 + x^3)/((x^2*(-a + x)*(-b + x))^(1/3)*(-a^2 +
2*a*x - (1 + b*d)*x^2 + d*x^3)),x]
```

```
[Out] Integrate[(2*a*b*x - 3*a*x^2 + x^3)/((x^2*(-a + x)*(-b + x))^(1/3)*(-a^2 +
2*a*x - (1 + b*d)*x^2 + d*x^3)), x]
```

IntegrateAlgebraic [A] time = 1.51, size = 226, normalized size = 1.00

$$-\frac{\log\left(a^2+d^{2/3}\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+\sqrt[3]{x^3(-a-b)+abx^2+x^4}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)-2ax+x^2\right)}{2d^{2/3}}+\frac{\log\left(\sqrt[3]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}+a-x\right)}{d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}}{\sqrt[3]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}-2a+2x}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*a*b*x - 3*a*x^2 + x^3)/((x^2*(-a + x)*(-b + x))^(1/3)
*(-a^2 + 2*a*x - (1 + b*d)*x^2 + d*x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3))/(-2*
a + 2*x + d^(1/3)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3))])/d^(2/3) + Log[a -
```

```
x + d^(1/3)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3)]/d^(2/3) - Log[a^2 - 2*a*
x + x^2 + (-(a*d^(1/3)) + d^(1/3)*x)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3) +
 d^(2/3)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b*x-3*a*x^2+x^3)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a^2+2*a*x-(b*d+
1)*x^2+d*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,abx - 3\,ax^2 + x^3}{\left((a-x)(b-x)x^2\right)^{\frac{1}{3}}\left(dx^3 - (bd+1)x^2 - a^2 + 2\,ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b*x-3*a*x^2+x^3)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a^2+2*a*x-(b*d+
1)*x^2+d*x^3),x, algorithm="giac")
```

```
[Out] integrate((2*a*b*x - 3*a*x^2 + x^3)/(((a - x)*(b - x)*x^2)^(1/3)*(d*x^3 - (
b*d + 1)*x^2 - a^2 + 2*a*x)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{2abx - 3a\,x^2 + x^3}{\left(x^2\left(-a+x\right)\left(-b+x\right)\right)^{\frac{1}{3}}\left(-a^2 + 2ax - \left(bd+1\right)x^2 + d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*b*x-3*a*x^2+x^3)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a^2+2*a*x-(b*d+1)*x^2
+d*x^3),x)
```

```
[Out] int((2*a*b*x-3*a*x^2+x^3)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a^2+2*a*x-(b*d+1)*x^2
+d*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,abx - 3\,ax^2 + x^3}{\left((a-x)(b-x)x^2\right)^{\frac{1}{3}}\left(dx^3 - (bd+1)x^2 - a^2 + 2\,ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b*x-3*a*x^2+x^3)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a^2+2*a*x-(b*d+
1)*x^2+d*x^3),x, algorithm="maxima")
```

```
[Out] integrate((2*a*b*x - 3*a*x^2 + x^3)/(((a - x)*(b - x)*x^2)^(1/3)*(d*x^3 - (
b*d + 1)*x^2 - a^2 + 2*a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 - 3\,a\,x^2 + 2\,a\,b\,x}{{\left(x^2\,\left(a-x\right)\,\left(b-x\right)\right)}^{1/3}\,\left(-a^2 + 2\,a\,x + d\,x^3 + \left(-b\,d-1\right)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 - 3*a*x^2 + 2*a*b*x)/((x^2*(a - x)*(b - x))^(1/3)*(2*a*x + d*x^3 - x^2*(b*d + 1) - a^2)),x)

[Out] int((x^3 - 3*a*x^2 + 2*a*b*x)/((x^2*(a - x)*(b - x))^(1/3)*(2*a*x + d*x^3 - x^2*(b*d + 1) - a^2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a*b*x-3*a*x**2+x**3)/(x**2*(-a+x)*(-b+x))**(1/3)/(-a**2+2*a*x-(b*d+1)*x**2+d*x**3),x)

[Out] Timed out

3.2090 $\int \frac{(-2+k^2)x+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(1-d+(-2+dk^2)x^2+x^4)}\,dx$

Optimal. Leaf size=226

$$\frac{\log\left(\sqrt[3]{d}\,\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}+x^2-1\right)}{2d^{2/3}}-\frac{\log\left(d^{2/3}\left(k^2x^4+(-k^2-1)x^2+1\right)^{2/3}+\left(\sqrt[3]{d}-\sqrt[3]{d}\,x^2\right)\sqrt[3]{k^2x^4+\cdots}\right.}{4d^{2/3}}$$

Rubi [F] time = 0.80, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+k^2)x+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(1-d+(-2+dk^2)x^2+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3)*(1 - d + (-2
+ d*k^2)*x^2 + x^4)),x]
```

```
[Out] Defer[Subst][Defer[Int][(-2 + k^2 + k^2*x)/((1 - d + (-2 + d*k^2)*x + x^2)*
(1 + (-1 - k^2)*x + k^2*x^2)^(1/3)), x], x, x^2]/2
```

Rubi steps

$$\begin{aligned}\int \frac{(-2+k^2)x+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(1-d+(-2+dk^2)x^2+x^4)}\,dx &= \int \frac{x(-2+k^2+k^2x^2)}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(1-d+(-2+dk^2)x^2+\cdots}\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{-2+k^2+k^2x}{\sqrt[3]{(1-x)(1-k^2x)}\,(1-d+(-2+dk^2\cdots}\right.\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{-2+k^2+k^2x}{(1-d+(-2+dk^2)x+x^2)\sqrt[3]{1+(-1\cdots}}\right.\end{aligned}$$

Mathematica [F] time = 12.42, size = 0, normalized size = 0.00

$$\int \frac{(-2+k^2)x+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(1-d+(-2+dk^2)x^2+x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3)*(1 - d
+ (-2 + d*k^2)*x^2 + x^4)),x]
```

```
[Out] Integrate[((-2 + k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3)*(1 - d
+ (-2 + d*k^2)*x^2 + x^4)), x]
```

IntegrateAlgebraic [A] time = 3.99, size = 226, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{d}\,\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}+x^2-1\right)}{2d^{2/3}}-\frac{\log\left(d^{2/3}\left(k^2x^4+(-k^2-1)x^2+1\right)^{2/3}+\left(\sqrt[3]{d}-\sqrt[3]{d}\,x^2\right)\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}+x^4-2x^2+1\right)}{4d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}}{\sqrt[3]{d}\,\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}-2x^2+2}\right)}{2d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3
)*(1 - d + (-2 + d*k^2)*x^2 + x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))/(2 -
 2*x^2 + d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))])/(2*d^(2/3)) + Log[
-1 + x^2 + d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3)]/(2*d^(2/3)) - Log[
1 - 2*x^2 + x^4 + (d^(1/3) - d^(1/3)*x^2)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1
/3) + d^(2/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(4*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((k^2-2)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d+(d*k^2-2)*
x^2+x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3 + \left(k^2 - 2\right)x}{\left(x^4 + \left(dk^2 - 2\right)x^2 - d + 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((k^2-2)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d+(d*k^2-2)*
x^2+x^4),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^3 + (k^2 - 2)*x)/((x^4 + (d*k^2 - 2)*x^2 - d + 1)*((k^2*x^
2 - 1)*(x^2 - 1))^(1/3)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(k^2 - 2\right) x + k^2x^3}{\left(\left(-x^2 + 1\right)\left(-k^2x^2 + 1\right)\right)^{\frac{1}{3}}\left(1 - d + \left(d\,k^2 - 2\right)x^2 + x^4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((k^2-2)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d+(d*k^2-2)*x^2+x^
4),x)
```

```
[Out] int(((k^2-2)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d+(d*k^2-2)*x^2+x^
4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3 + \left(k^2 - 2\right)x}{\left(x^4 + \left(dk^2 - 2\right)x^2 - d + 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((k^2-2)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d+(d*k^2-2)*
x^2+x^4),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^3 + (k^2 - 2)*x)/((x^4 + (d*k^2 - 2)*x^2 - d + 1)*((k^2*x^
2 - 1)*(x^2 - 1))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(k^2-2\right)+k^2 x^3}{\left(\left(x^2-1\right)\left(k^2 x^2-1\right)\right)^{1/3}\left(x^4+\left(d k^2-2\right) x^2-d+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(k^2 - 2) + k^2*x^3)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/3)*(x^2*(d*k^2 -
2) - d + x^4 + 1)),x)
```

```
[Out] int((x*(k^2 - 2) + k^2*x^3)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/3)*(x^2*(d*k^2 -
2) - d + x^4 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((k**2-2)*x+k**2*x**3)/((-x**2+1)*(-k**2*x**2+1))**(1/3)/(1-d+(d*
k**2-2)*x**2+x**4),x)
```

```
[Out] Timed out
```

3.2091 $\int \frac{(1+2x)\sqrt[4]{x^3+x^4}}{-1+x+x^2}\,dx$

Optimal. Leaf size=226

$$2\sqrt[4]{x^4+x^3}+\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)-\sqrt{2\left(1+\sqrt{5}\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4+x^3}}\right)+\sqrt{2\left(\sqrt{5}-1\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4+x^3}}\right)-\tanh^{-1}\Bigg($$

Rubi [B] time = 0.74, antiderivative size = 496, normalized size of antiderivative = 2.19, number of steps used = 25, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2056, 6728, 101, 157, 63, 331, 298, 203, 206, 93}

$$2\sqrt[4]{x^4+x^3}+\frac{\left(1+2\sqrt{5}\right)\sqrt[4]{x^4+x^3}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{2x^{3/4}\sqrt[4]{x+1}}+\frac{\left(1-2\sqrt{5}\right)\sqrt[4]{x^4+x^3}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{2x^{3/4}\sqrt[4]{x+1}}-\frac{2^{3/4}\sqrt[4]{3+\sqrt{5}}\sqrt[4]{x^4+x^3}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{x^{3/4}\sqrt[4]{x+1}}+\frac{2^{3/4}\sqrt[4]{3-\sqrt{5}}\sqrt[4]{x^4+x^3}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{x^{3/4}\sqrt[4]{x+1}}-\frac{\left(1+2\sqrt{5}\right)\sqrt[4]{x^4+x^3}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{2x^{3/4}\sqrt[4]{x+1}}-\frac{\left(1-2\sqrt{5}\right)\sqrt[4]{x^4+x^3}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{2x^{3/4}\sqrt[4]{x+1}}+\frac{2^{3/4}\sqrt[4]{3+\sqrt{5}}\sqrt[4]{x^4+x^3}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{x^{3/4}\sqrt[4]{x+1}}-\frac{2^{3/4}\sqrt[4]{3-\sqrt{5}}\sqrt[4]{x^4+x^3}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{x^{3/4}\sqrt[4]{x+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((1 + 2*x)*(x^3 + x^4)^(1/4))/(-1 + x + x^2),x]
```

```
[Out] 2*(x^3 + x^4)^(1/4) + ((1 - 2*Sqrt[5])*(x^3 + x^4)^(1/4)*ArcTan[x^(1/4)/(1
+ x)^(1/4)])/(2*x^(3/4)*(1 + x)^(1/4)) + ((1 + 2*Sqrt[5])*(x^3 + x^4)^(1/4)
*ArcTan[x^(1/4)/(1 + x)^(1/4)])/(2*x^(3/4)*(1 + x)^(1/4)) - (2^(3/4)*(3 + S
qrt[5])^(1/4)*(x^3 + x^4)^(1/4)*ArcTan[((2/(3 + Sqrt[5]))^(1/4)*x^(1/4))/(1
 + x)^(1/4)])/(x^(3/4)*(1 + x)^(1/4)) + (2^(3/4)*(3 - Sqrt[5])^(1/4)*(x^3 +
 x^4)^(1/4)*ArcTan[(((3 + Sqrt[5])/2)^(1/4)*x^(1/4))/(1 + x)^(1/4)])/(x^(3/
4)*(1 + x)^(1/4)) - ((1 - 2*Sqrt[5])*(x^3 + x^4)^(1/4)*ArcTanh[x^(1/4)/(1 +
 x)^(1/4)])/(2*x^(3/4)*(1 + x)^(1/4)) - ((1 + 2*Sqrt[5])*(x^3 + x^4)^(1/4)*
ArcTanh[x^(1/4)/(1 + x)^(1/4)])/(2*x^(3/4)*(1 + x)^(1/4)) + (2^(3/4)*(3 + S
qrt[5])^(1/4)*(x^3 + x^4)^(1/4)*ArcTanh[((2/(3 + Sqrt[5]))^(1/4)*x^(1/4))/(
1 + x)^(1/4)])/(x^(3/4)*(1 + x)^(1/4)) - (2^(3/4)*(3 - Sqrt[5])^(1/4)*(x^3
+ x^4)^(1/4)*ArcTanh[(((3 + Sqrt[5])/2)^(1/4)*x^(1/4))/(1 + x)^(1/4)])/(x^(
3/4)*(1 + x)^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+2x)\sqrt[4]{x^3+x^4}}{-1+x+x^2}\,dx &= \frac{\sqrt[4]{x^3+x^4}\int \frac{x^{3/4}\sqrt[4]{1+x}\,(1+2x)}{-1+x+x^2}\,dx}{x^{3/4}\sqrt[4]{1+x}} \\
&= \frac{\sqrt[4]{x^3+x^4}\int \left(\frac{2x^{3/4}\sqrt[4]{1+x}}{1-\sqrt{5}+2x}+\frac{2x^{3/4}\sqrt[4]{1+x}}{1+\sqrt{5}+2x}\right)dx}{x^{3/4}\sqrt[4]{1+x}} \\
&= \frac{\left(2\sqrt[4]{x^3+x^4}\right)\int \frac{x^{3/4}\sqrt[4]{1+x}}{1-\sqrt{5}+2x}\,dx}{x^{3/4}\sqrt[4]{1+x}} + \frac{\left(2\sqrt[4]{x^3+x^4}\right)\int \frac{x^{3/4}\sqrt[4]{1+x}}{1+\sqrt{5}+2x}\,dx}{x^{3/4}\sqrt[4]{1+x}} \\
&= 2\sqrt[4]{x^3+x^4} - \frac{\sqrt[4]{x^3+x^4}\int \frac{\frac{3}{4}\left(1-\sqrt{5}\right)+\frac{1}{2}\left(1-2\sqrt{5}\right)x}{\sqrt[4]{x}\,(1+x)^{3/4}\left(1-\sqrt{5}+2x\right)}\,dx}{x^{3/4}\sqrt[4]{1+x}} - \frac{\sqrt[4]{x^3+x^4}\int \frac{\frac{3}{4}\left(1+\sqrt{5}\right)+\frac{1}{2}\left(1+2\sqrt{5}\right)x}{\sqrt[4]{x}\,(1+x)^{3/4}\left(1+\sqrt{5}+2x\right)}\,dx}{x^{3/4}\sqrt[4]{1+x}} \\
&= 2\sqrt[4]{x^3+x^4} + \frac{\left(2\sqrt[4]{x^3+x^4}\right)\int \frac{1}{\sqrt[4]{x}\,(1+x)^{3/4}\left(1-\sqrt{5}+2x\right)}\,dx}{x^{3/4}\sqrt[4]{1+x}} + \frac{\left(2\sqrt[4]{x^3+x^4}\right)\int \frac{1}{\sqrt[4]{x}\,(1+x)^{3/4}\left(1+\sqrt{5}+\cdots\right.}}{x^{3/4}\sqrt[4]{1+x}} \\
&= 2\sqrt[4]{x^3+x^4} + \frac{\left(8\sqrt[4]{x^3+x^4}\right)\text{Subst}\left(\int \frac{x^2}{1-\sqrt{5}-\left(-1-\sqrt{5}\right)x^4}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}} + \frac{\left(8\sqrt[4]{x^3+x^4}\right)\text{Sub}\cdots}{\cdots} \\
&= 2\sqrt[4]{x^3+x^4} - \frac{\left(\left(1-2\sqrt{5}\right)\sqrt[4]{x^3+x^4}\right)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}} - \frac{\left(4\sqrt{2}\sqrt[4]{x^3+x^4}\right)\text{Su}\cdots}{\left(1-\cdots\right.} \\
&= 2\sqrt[4]{x^3+x^4} - \frac{2^{3/4}\sqrt[4]{3+\sqrt{5}}\sqrt[4]{x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}} + \frac{2^{3/4}\sqrt[4]{3-\sqrt{5}}\sqrt[4]{x^3+x^4}\tan\cdots}{x^{3/4}\sqrt[4]{1+\cdots}} \\
&= 2\sqrt[4]{x^3+x^4} + \frac{\left(1-2\sqrt{5}\right)\sqrt[4]{x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{2x^{3/4}\sqrt[4]{1+x}} + \frac{\left(1+2\sqrt{5}\right)\sqrt[4]{x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\cdots\right.}{2x^{3/4}\sqrt[4]{1+x}}
\end{aligned}
$$

Mathematica [C] time = 0.25, size = 147, normalized size = 0.65

$$
\frac{2\sqrt[4]{x^3(x+1)}\left(4(x+1)\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-x\right)-2(x+1)\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-x\right)+\sqrt[4]{x+1}\left(\left(\sqrt{5}-1\right){}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{\left(-1+\sqrt{5}\right)x}{\left(1+\sqrt{5}\right)(x+1)}\right)-\left(1+\sqrt{5}\right){}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{\left(1+\sqrt{5}\right)x}{\left(-1+\sqrt{5}\right)(x+1)}\right)\right)\right)}{3(x+1)^{5/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(x^3 + x^4)^(1/4))/(-1 + x + x^2),x]
```

```
[Out] (2*(x^3*(1 + x))^(1/4)*(4*(1 + x)*Hypergeometric2F1[-1/4, 3/4, 7/4, -x] - 2
*(1 + x)*Hypergeometric2F1[3/4, 3/4, 7/4, -x] + (1 + x)^(1/4)*((-1 + Sqrt[5
])*Hypergeometric2F1[3/4, 1, 7/4, ((-1 + Sqrt[5])*x)/((1 + Sqrt[5])*(1 + x)
)] - (1 + Sqrt[5])*Hypergeometric2F1[3/4, 1, 7/4, ((1 + Sqrt[5])*x)/((-1 +
Sqrt[5])*(1 + x))])))/(3*(1 + x)^(5/4))
```

IntegrateAlgebraic [A] time = 1.10, size = 226, normalized size = 1.00

$$
2\sqrt[4]{x^4+x^3}+\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)-\sqrt{2\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4+x^3}}\right)+\sqrt{2\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4+x^3}}\right)-\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)+\sqrt{2\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4+x^3}}\right)-\sqrt{2\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4+x^3}}\right)
$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 2*x)*(x^3 + x^4)^(1/4))/(-1 + x + x^2),x]

[Out] 2*(x^3 + x^4)^(1/4) + ArcTan[x/(x^3 + x^4)^(1/4)] - Sqrt[2*(1 + Sqrt[5])]*ArcTan[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(x^3 + x^4)^(1/4)] + Sqrt[2*(-1 + Sqrt[5])]*ArcTan[(Sqrt[1/2 + Sqrt[5]/2]*x)/(x^3 + x^4)^(1/4)] - ArcTanh[x/(x^3 + x^4)^(1/4)] + Sqrt[2*(1 + Sqrt[5])]*ArcTanh[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(x^3 + x^4)^(1/4)] - Sqrt[2*(-1 + Sqrt[5])]*ArcTanh[(Sqrt[1/2 + Sqrt[5]/2]*x)/(x^3 + x^4)^(1/4)]

fricas [B] time = 0.58, size = 415, normalized size = 1.84

$$2\sqrt{2\sqrt{5}-2}\arctan\left(\frac{\sqrt{2}x\sqrt{2\sqrt{5}-2}\sqrt{\frac{\sqrt{5}x^2+x^2+2\sqrt{x^4+x^3}}{x^2}}-2\left(x^4+x^3\right)^{\frac{1}{4}}\sqrt{2\sqrt{5}-2}}{4x}\right)-2\sqrt{2\sqrt{5}+2}\arctan\left(\frac{\sqrt{2}x\sqrt{2\sqrt{5}+2}\sqrt{\frac{\sqrt{5}x^2-x^2+2\sqrt{x^4+x^3}}{x^2}}-2\left(x^4+x^3\right)^{\frac{1}{4}}\sqrt{2\sqrt{5}+2}}{4x}\right)+\frac{1}{2}\sqrt{2\sqrt{5}+2}\log\left(\frac{\left(\sqrt{5}x-x\right)\sqrt{2\sqrt{5}+2}+4\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{2}\sqrt{2\sqrt{5}+2}\log\left(-\frac{\left(\sqrt{5}x-x\right)\sqrt{2\sqrt{5}+2}-4\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{2}\sqrt{2\sqrt{5}-2}\log\left(\frac{\left(\sqrt{5}x+x\right)\sqrt{2\sqrt{5}-2}+4\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\sqrt{2\sqrt{5}-2}\log\left(-\frac{\left(\sqrt{5}x+x\right)\sqrt{2\sqrt{5}-2}-4\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)+2\left(x^4+x^3\right)^{\frac{1}{4}}-\arctan\left(\frac{\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{2}\log\left(\frac{x+\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\log\left(-\frac{x-\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)*(x^4+x^3)^(1/4)/(x^2+x-1),x, algorithm="fricas")

[Out] 2*sqrt(2*sqrt(5) - 2)*arctan(1/4*(sqrt(2)*x*sqrt(2*sqrt(5) - 2)*sqrt((sqrt(5)*x^2 + x^2 + 2*sqrt(x^4 + x^3))/x^2) - 2*(x^4 + x^3)^(1/4)*sqrt(2*sqrt(5) - 2))/x) - 2*sqrt(2*sqrt(5) + 2)*arctan(1/4*(sqrt(2)*x*sqrt(2*sqrt(5) + 2)*sqrt((sqrt(5)*x^2 - x^2 + 2*sqrt(x^4 + x^3))/x^2) - 2*(x^4 + x^3)^(1/4)*sqrt(2*sqrt(5) + 2))/x) + 1/2*sqrt(2*sqrt(5) + 2)*log(((sqrt(5)*x - x)*sqrt(2*sqrt(5) + 2) + 4*(x^4 + x^3)^(1/4))/x) - 1/2*sqrt(2*sqrt(5) + 2)*log(-((sqrt(5)*x - x)*sqrt(2*sqrt(5) + 2) - 4*(x^4 + x^3)^(1/4))/x) - 1/2*sqrt(2*sqrt(5) - 2)*log(((sqrt(5)*x + x)*sqrt(2*sqrt(5) - 2) + 4*(x^4 + x^3)^(1/4))/x) + 1/2*sqrt(2*sqrt(5) - 2)*log(-((sqrt(5)*x + x)*sqrt(2*sqrt(5) - 2) - 4*(x^4 + x^3)^(1/4))/x) + 2*(x^4 + x^3)^(1/4) - arctan((x^4 + x^3)^(1/4)/x) - 1/2*log((x + (x^4 + x^3)^(1/4))/x) + 1/2*log(-(x - (x^4 + x^3)^(1/4))/x)

giac [A] time = 0.40, size = 225, normalized size = 1.00

$$-\sqrt{2\sqrt{5}-2}\arctan\left(\frac{\left(\frac{1}{x}+1\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\sqrt{2\sqrt{5}+2}\arctan\left(\frac{\left(\frac{1}{x}+1\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{2}\sqrt{2\sqrt{5}-2}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}+\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right)+\frac{1}{2}\sqrt{2\sqrt{5}+2}\log\left(\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}+\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right)+\frac{1}{2}\sqrt{2\sqrt{5}-2}\log\left(\left|-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}+\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right|\right)-\frac{1}{2}\sqrt{2\sqrt{5}+2}\log\left(\left|-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}+\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right|\right)+2x\left(\frac{1}{x}+1\right)^{\frac{1}{4}}-\arctan\left(\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right)-\frac{1}{2}\log\left(\left(\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{2}\log\left(\left|\left(\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)*(x^4+x^3)^(1/4)/(x^2+x-1),x, algorithm="giac")

[Out] -sqrt(2*sqrt(5) - 2)*arctan((1/x + 1)^(1/4)/sqrt(1/2*sqrt(5) + 1/2)) + sqrt(2*sqrt(5) + 2)*arctan((1/x + 1)^(1/4)/sqrt(1/2*sqrt(5) - 1/2)) - 1/2*sqrt(2*sqrt(5) - 2)*log(sqrt(1/2*sqrt(5) + 1/2) + (1/x + 1)^(1/4)) + 1/2*sqrt(2*sqrt(5) + 2)*log(sqrt(1/2*sqrt(5) - 1/2) + (1/x + 1)^(1/4)) + 1/2*sqrt(2*sqrt(5) - 2)*log(abs(-sqrt(1/2*sqrt(5) + 1/2) + (1/x + 1)^(1/4))) - 1/2*sqrt(2*sqrt(5) + 2)*log(abs(-sqrt(1/2*sqrt(5) - 1/2) + (1/x + 1)^(1/4))) + 2*x*(1/x + 1)^(1/4) - arctan((1/x + 1)^(1/4)) - 1/2*log((1/x + 1)^(1/4) + 1) + 1/2*log(abs((1/x + 1)^(1/4) - 1))

maple [C] time = 10.83, size = 3822, normalized size = 16.91

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2*x)*(x^4+x^3)^(1/4)/(x^2+x-1),x)

[Out] 2*(x^3*(1+x))^(1/4)+(1/2*RootOf(_Z^2+1)*ln((2*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^2+1)*x-2*RootOf(_Z^2+1)*x^3+2*(x^4+3*x^3+3*x^2+x)^(3/4)+2*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^2+1)-2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2-5*RootOf(_Z^2+1)*x^2-4*(x^4+3*x^3+3*x^2+x)^(1/4)*x-4*RootOf(_Z^2+1)*x-2*(x^4+3*x^3+3*x^2+x)^(1/4)-RootOf(_Z^2+1))/(1+x)^2)+1/2*ln((2*(x^4+3*x^3+3*x^2+x)^(3/4)-2*(x^4+3*x^3+3*x^2+x)^(1/2)*x+2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2-2*x^3-2*(x^4+3*x^3+3*x^2+x)^(1/2)+4*(x^4+3*x^3+3*x^2+x)^(1/4)*x-5*x^2+2*(x^4+3*x^3+3*x^2+x)^(1/4)-4*x-1)/(1+x)^2)+1/8*RootOf(_Z^4+64*_Z^2-4096)*ln(-(3*x^3*RootOf(_Z^4

+64*_Z^2-4096)^5+3*x^2*RootOf(_Z^4+64*_Z^2-4096)^5-3*x*RootOf(_Z^4+64*_Z^2-
4096)^5+1440*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^4+64*_Z^2-4096)^3*x+1840*R
ootOf(_Z^4+64*_Z^2-4096)^3*x^3-3*RootOf(_Z^4+64*_Z^2-4096)^5+1440*(x^4+3*x^
3+3*x^2+x)^(1/2)*RootOf(_Z^4+64*_Z^2-4096)^3+3952*RootOf(_Z^4+64*_Z^2-4096)
^3*x^2-11520*(x^4+3*x^3+3*x^2+x)^(3/4)*RootOf(_Z^4+64*_Z^2-4096)^2-7424*Roo
tOf(_Z^4+64*_Z^2-4096)^2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2+2384*RootOf(_Z^4+64*
_Z^2-4096)^3*x-14848*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(_Z^4+64*_Z^2-4096)^2*
x+151552*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^4+64*_Z^2-4096)*x+155648*RootO
f(_Z^4+64*_Z^2-4096)*x^3+272*RootOf(_Z^4+64*_Z^2-4096)^3-7424*(x^4+3*x^3+3*
x^2+x)^(1/4)*RootOf(_Z^4+64*_Z^2-4096)^2+151552*(x^4+3*x^3+3*x^2+x)^(1/2)*R
ootOf(_Z^4+64*_Z^2-4096)+369664*RootOf(_Z^4+64*_Z^2-4096)*x^2-1212416*(x^4+
3*x^3+3*x^2+x)^(3/4)-737280*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2+272384*RootOf(_Z^
4+64*_Z^2-4096)*x-1474560*(x^4+3*x^3+3*x^2+x)^(1/4)*x+58368*RootOf(_Z^4+64*
_Z^2-4096)-737280*(x^4+3*x^3+3*x^2+x)^(1/4))/(x*RootOf(_Z^4+64*_Z^2-4096)^2
-RootOf(_Z^4+64*_Z^2-4096)^2+128*x-64)/(1+x)^2)-1/8*RootOf(RootOf(_Z^4+64*_
Z^2-4096)^2+_Z^2+64)*ln((3*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*Root
Of(_Z^4+64*_Z^2-4096)^4*x^3+3*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*R
ootOf(_Z^4+64*_Z^2-4096)^4*x^2-3*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64
)*RootOf(_Z^4+64*_Z^2-4096)^4*x-1440*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^4+
64*_Z^2-4096)^2*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*x-1456*RootOf(R
ootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*RootOf(_Z^4+64*_Z^2-4096)^2*x^3-3*RootO
f(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*RootOf(_Z^4+64*_Z^2-4096)^4-1440*(x^
4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^4+64*_Z^2-4096)^2*RootOf(RootOf(_Z^4+64*_Z
^2-4096)^2+_Z^2+64)-3568*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*RootOf
(_Z^4+64*_Z^2-4096)^2*x^2-11520*(x^4+3*x^3+3*x^2+x)^(3/4)*RootOf(_Z^4+64*_Z
^2-4096)^2-7424*RootOf(_Z^4+64*_Z^2-4096)^2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2-2
768*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*RootOf(_Z^4+64*_Z^2-4096)^2
x+59392(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+
64)*x+50176*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*x^3-14848*(x^4+3*x^
3+3*x^2+x)^(1/4)*RootOf(_Z^4+64*_Z^2-4096)^2*x-656*RootOf(_Z^4+64*_Z^2-4096
)^2*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)+59392*(x^4+3*x^3+3*x^2+x)^(
1/2)*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)+129024*RootOf(RootOf(_Z^4+
64*_Z^2-4096)^2+_Z^2+64)*x^2-7424*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(_Z^4+64*
_Z^2-4096)^2+475136*(x^4+3*x^3+3*x^2+x)^(3/4)+262144*(x^4+3*x^3+3*x^2+x)^(1
/4)*x^2+107520*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*x+524288*(x^4+3*
x^3+3*x^2+x)^(1/4)*x+28672*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)+2621
44*(x^4+3*x^3+3*x^2+x)^(1/4))/(x*RootOf(_Z^4+64*_Z^2-4096)^2-RootOf(_Z^4+64
*_Z^2-4096)^2-64*x)/(1+x)^2)-1/512*ln(-(7*x^3*RootOf(_Z^4+64*_Z^2-4096)^5+7
*x^2*RootOf(_Z^4+64*_Z^2-4096)^5-7*x*RootOf(_Z^4+64*_Z^2-4096)^5+3712*(x^4+
3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^4+64*_Z^2-4096)^3*x-3456*RootOf(_Z^4+64*_Z^2
-4096)^3*x^3-7*RootOf(_Z^4+64*_Z^2-4096)^5+3712*(x^4+3*x^3+3*x^2+x)^(1/2)*R
ootOf(_Z^4+64*_Z^2-4096)^3-8384*RootOf(_Z^4+64*_Z^2-4096)^3*x^2+46080*(x^4+
3*x^3+3*x^2+x)^(3/4)*RootOf(_Z^4+64*_Z^2-4096)^2-29696*RootOf(_Z^4+64*_Z^2-
4096)^2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2-6400*RootOf(_Z^4+64*_Z^2-4096)^3*x-59
392*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(_Z^4+64*_Z^2-4096)^2*x-131072*(x^4+3*x
^3+3*x^2+x)^(1/2)*RootOf(_Z^4+64*_Z^2-4096)*x+143360*RootOf(_Z^4+64*_Z^2-40
96)*x^3-1472*RootOf(_Z^4+64*_Z^2-4096)^3-29696*(x^4+3*x^3+3*x^2+x)^(1/4)*Ro
otOf(_Z^4+64*_Z^2-4096)^2-131072*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^4+64*_
Z^2-4096)+368640*RootOf(_Z^4+64*_Z^2-4096)*x^2-1900544*(x^4+3*x^3+3*x^2+x)^
(3/4)+1048576*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2+307200*RootOf(_Z^4+64*_Z^2-4096
)*x+2097152*(x^4+3*x^3+3*x^2+x)^(1/4)*x+81920*RootOf(_Z^4+64*_Z^2-4096)+104
8576*(x^4+3*x^3+3*x^2+x)^(1/4))/(x*RootOf(_Z^4+64*_Z^2-4096)^2-RootOf(_Z^4+
64*_Z^2-4096)^2-64*x)/(1+x)^2)*RootOf(_Z^4+64*_Z^2-4096)^3-1/8*ln(-(7*x^3*R
ootOf(_Z^4+64*_Z^2-4096)^5+7*x^2*RootOf(_Z^4+64*_Z^2-4096)^5-7*x*RootOf(_Z^
4+64*_Z^2-4096)^5+3712*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^4+64*_Z^2-4096)^
3*x-3456*RootOf(_Z^4+64*_Z^2-4096)^3*x^3-7*RootOf(_Z^4+64*_Z^2-4096)^5+3712
*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^4+64*_Z^2-4096)^3-8384*RootOf(_Z^4+64*
_Z^2-4096)^3*x^2+46080*(x^4+3*x^3+3*x^2+x)^(3/4)*RootOf(_Z^4+64*_Z^2-4096)^
2-29696*RootOf(_Z^4+64*_Z^2-4096)^2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2-6400*Root

```
Of(_Z^4+64*_Z^2-4096)^3*x-59392*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(_Z^4+64*_Z
^2-4096)^2*x-131072*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^4+64*_Z^2-4096)*x+1
43360*RootOf(_Z^4+64*_Z^2-4096)*x^3-1472*RootOf(_Z^4+64*_Z^2-4096)^3-29696*
(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(_Z^4+64*_Z^2-4096)^2-131072*(x^4+3*x^3+3*x
^2+x)^(1/2)*RootOf(_Z^4+64*_Z^2-4096)+368640*RootOf(_Z^4+64*_Z^2-4096)*x^2-
1900544*(x^4+3*x^3+3*x^2+x)^(3/4)+1048576*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2+307
200*RootOf(_Z^4+64*_Z^2-4096)*x+2097152*(x^4+3*x^3+3*x^2+x)^(1/4)*x+81920*R
ootOf(_Z^4+64*_Z^2-4096)+1048576*(x^4+3*x^3+3*x^2+x)^(1/4))/(x*RootOf(_Z^4+
64*_Z^2-4096)^2-RootOf(_Z^4+64*_Z^2-4096)^2-64*x)/(1+x)^2)*RootOf(_Z^4+64*_
Z^2-4096)-1/512*RootOf(_Z^4+64*_Z^2-4096)^2*RootOf(RootOf(_Z^4+64*_Z^2-4096
)^2+_Z^2+64)*ln((7*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*RootOf(_Z^4+
64*_Z^2-4096)^4*x^3+7*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*RootOf(_Z
^4+64*_Z^2-4096)^4*x^2-7*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*RootOf
(_Z^4+64*_Z^2-4096)^4*x-3712*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^4+64*_Z^2-
4096)^2*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*x+4352*RootOf(RootOf(_Z
^4+64*_Z^2-4096)^2+_Z^2+64)*RootOf(_Z^4+64*_Z^2-4096)^2*x^3-7*RootOf(RootOf
(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*RootOf(_Z^4+64*_Z^2-4096)^4-3712*(x^4+3*x^3+
3*x^2+x)^(1/2)*RootOf(_Z^4+64*_Z^2-4096)^2*RootOf(RootOf(_Z^4+64*_Z^2-4096)
^2+_Z^2+64)+9280*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*RootOf(_Z^4+64
*_Z^2-4096)^2*x^2+46080*(x^4+3*x^3+3*x^2+x)^(3/4)*RootOf(_Z^4+64*_Z^2-4096)
^2-29696*RootOf(_Z^4+64*_Z^2-4096)^2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2+5504*Roo
tOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*RootOf(_Z^4+64*_Z^2-4096)^2*x-3686
40*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*x+
393216*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*x^3-59392*(x^4+3*x^3+3*x
^2+x)^(1/4)*RootOf(_Z^4+64*_Z^2-4096)^2*x+576*RootOf(_Z^4+64*_Z^2-4096)^2*R
ootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)-368640*(x^4+3*x^3+3*x^2+x)^(1/2)
*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)+933888*RootOf(RootOf(_Z^4+64*_
Z^2-4096)^2+_Z^2+64)*x^2-29696*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(_Z^4+64*_Z^
2-4096)^2+4849664*(x^4+3*x^3+3*x^2+x)^(3/4)-2949120*(x^4+3*x^3+3*x^2+x)^(1/
4)*x^2+688128*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)*x-5898240*(x^4+3*
x^3+3*x^2+x)^(1/4)*x+147456*RootOf(RootOf(_Z^4+64*_Z^2-4096)^2+_Z^2+64)-294
9120*(x^4+3*x^3+3*x^2+x)^(1/4))/(x*RootOf(_Z^4+64*_Z^2-4096)^2-RootOf(_Z^4+
64*_Z^2-4096)^2+128*x-64)/(1+x)^2))*(x^3*(1+x))^(1/4)*(x*(1+x)^3)^(1/4)/x/(
1+x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3\right)^{\frac{1}{4}}(2x+1)}{x^2+x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^4+x^3)^(1/4)/(x^2+x-1),x, algorithm="maxima")
[Out] integrate((x^4 + x^3)^(1/4)*(2*x + 1)/(x^2 + x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^4+x^3\right)^{1/4}\,(2x+1)}{x^2+x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + x^4)^(1/4)*(2*x + 1))/(x + x^2 - 1),x)
[Out] int(((x^3 + x^4)^(1/4)*(2*x + 1))/(x + x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\,(x+1)}\,(2x+1)}{x^2+x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x**4+x**3)**(1/4)/(x**2+x-1),x)
```

```
[Out] Integral((x**3*(x + 1))**(1/4)*(2*x + 1)/(x**2 + x - 1), x)
```

3.2092 $\int \frac{(-1+2k^2)x-2k^4x^3+k^4x^5}{((1-x^2)(1-k^2x^2))^{2/3}(-1+d+(1-2dk^2)x^2+dk^4x^4)}\,dx$

Optimal. Leaf size=226

$$-\frac{\log\left(\sqrt[3]{d}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+x^2-1\right)}{2d^{2/3}}+\frac{\log\left(d^{2/3}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{4/3}+\left(\sqrt[3]{d}-\sqrt[3]{d}\,x^2\right)\left(k^2x^4\right.\right.}{4d^{2/3}}$$

Rubi [C] time = 2.91, antiderivative size = 423, normalized size of antiderivative = 1.87, number of steps used = 10, number of rules used = 7, integrand size = 73, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.096, Rules used = {1594, 6715, 6719, 1586, 6728, 137, 136}

$$\frac{3k^2(1-x^2)\left(1-\sqrt{1-4dk^2(1-k^2)}\right)(1-k^2x^2)F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{k^2(1-x^2)}{1-k^2},\frac{2dk^4(1-x^2)}{-2d(1-k^2)k^2-\sqrt{1-4dk^2(1-k^2)}+1}\right)}{2\left(-2d(1-k^2)k^2-\sqrt{1-4dk^2(1-k^2)}+1\right)\sqrt[3]{\frac{1-k^2x^2}{1-k^2}}\left((1-x^2)(1-k^2x^2)\right)^{2/3}}+\frac{3k^2(1-x^2)\left(\sqrt{1-4dk^2(1-k^2)}+1\right)(1-k^2x^2)F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{k^2(1-x^2)}{1-k^2},\frac{2dk^4(1-x^2)}{-2d(1-k^2)k^2+\sqrt{1-4dk^2(1-k^2)}+1}\right)}{2\left(-2d(1-k^2)k^2+\sqrt{1-4dk^2(1-k^2)}+1\right)\sqrt[3]{\frac{1-k^2x^2}{1-k^2}}\left((1-x^2)(1-k^2x^2)\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + 2*k^2)*x - 2*k^4*x^3 + k^4*x^5)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)
*(-1 + d + (1 - 2*d*k^2)*x^2 + d*k^4*x^4)),x]
```

```
[Out] (3*k^2*(1 - Sqrt[1 - 4*d*k^2*(1 - k^2)])*(1 - x^2)*(1 - k^2*x^2)*AppellF1[1
/3, -1/3, 1, 4/3, -((k^2*(1 - x^2))/(1 - k^2)), (2*d*k^4*(1 - x^2))/(1 - 2*
d*k^2*(1 - k^2) - Sqrt[1 - 4*d*k^2*(1 - k^2)])])/(2*(1 - 2*d*k^2*(1 - k^2)
- Sqrt[1 - 4*d*k^2*(1 - k^2)])*((1 - k^2*x^2)/(1 - k^2))^(1/3)*((1 - x^2)*(
1 - k^2*x^2))^(2/3)) + (3*k^2*(1 + Sqrt[1 - 4*d*k^2*(1 - k^2)])*(1 - x^2)*(
1 - k^2*x^2)*AppellF1[1/3, -1/3, 1, 4/3, -((k^2*(1 - x^2))/(1 - k^2)), (2*d
*k^4*(1 - x^2))/(1 - 2*d*k^2*(1 - k^2) + Sqrt[1 - 4*d*k^2*(1 - k^2)])])/(2*
(1 - 2*d*k^2*(1 - k^2) + Sqrt[1 - 4*d*k^2*(1 - k^2)])*((1 - k^2*x^2)/(1 - k
^2))^(1/3)*((1 - x^2)*(1 - k^2*x^2))^(2/3))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
```

```
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(-1+2k^2\right)x-2k^4x^3+k^4x^5}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(-1+d+\left(1-2dk^2\right)x^2+dk^4x^4\right)}\,dx = \int \frac{x\left(-1+2k^2-2k^4x^2+k^4x^4\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(-1+d+\left(1-2dk^2\right)\right.}$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{-1+2k^2-2k^4x+}{\left(\left(1-x\right)\left(1-k^2x\right)\right)^{2/3}\left(-1+d+\left(1\right.\right.}\right.$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{}{(1-x)^{2/3}\left(1-k^2\right.}\right.}{2\left(\left(1-x^2\right)\left(1-k^2x\right.\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{\sqrt[3]{1-k^2x}}{(1-x)^{2/3}\left(-1+d\right.}\right.}{2\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2}}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\text{Subst}\left(\int\left(\frac{\left(-k^2+k^2\right.}{(1-x)^{2/3}\left(1-2\right.}\right.\right.}{2}$$

$$= -\frac{\left(k^2\left(1-\sqrt{1-4dk^2\left(1-k^2\right)}\right)\left(1-x^2\right)^{2/3}\left(1-\right.\right.}{2\left(\left(1\right.\right.}$$

$$= -\frac{\left(k^2\left(1-\sqrt{1-4dk^2\left(1-k^2\right)}\right)\left(1-x^2\right)^{2/3}\left(1-\right.\right.}{2\left(\left(1-x^2\right)\right.}$$

$$= \frac{3k^2\left(1-\sqrt{1-4dk^2\left(1-k^2\right)}\right)\left(1-x^2\right)\left(1-k^2x^2\right.}{2\left(1-2dk^2\left(1-k^2\right)-\sqrt{1-4dk^2\left(1\right.}\right.}$$

Mathematica [F] time = 1.83, size = 0, normalized size = 0.00

$$\int \frac{(-1+2k^2)\,x-2k^4x^3+k^4x^5}{\left((1-x^2)\,(1-k^2x^2)\right)^{2/3}\left(-1+d+(1-2dk^2)\,x^2+dk^4x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + 2*k^2)*x - 2*k^4*x^3 + k^4*x^5)/(((1 - x^2)*(1 - k^2*x^2))
^(2/3)*(-1 + d + (1 - 2*d*k^2)*x^2 + d*k^4*x^4)),x]
```

```
[Out] Integrate[((-1 + 2*k^2)*x - 2*k^4*x^3 + k^4*x^5)/(((1 - x^2)*(1 - k^2*x^2))
^(2/3)*(-1 + d + (1 - 2*d*k^2)*x^2 + d*k^4*x^4)), x]
```

IntegrateAlgebraic [A] time = 5.63, size = 226, normalized size = 1.00

$$-\frac{\log\left(\sqrt[3]{d}\,\left(k^2x^4+(-k^2-1)\,x^2+1\right)^{2/3}+x^2-1\right)}{2d^{2/3}}+\frac{\log\left(d^{2/3}\,\left(k^2x^4+(-k^2-1)\,x^2+1\right)^{4/3}+\left(\sqrt[3]{d}-\sqrt[3]{d}\,x^2\right)\left(k^2x^4+(-k^2-1)\,x^2+1\right)^{2/3}+x^4-2x^2+1\right)}{4d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{d}\,\left(k^2x^4+(-k^2-1)x^2+1\right)^{2/3}}{\sqrt[3]{d}\,\left(k^2x^4+(-k^2-1)x^2+1\right)^{2/3}-2x^2+2}\right)}{2d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + 2*k^2)*x - 2*k^4*x^3 + k^4*x^5)/(((1 - x^2)*(1 -
k^2*x^2))^(2/3)*(-1 + d + (1 - 2*d*k^2)*x^2 + d*k^4*x^4)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3))
/(2 - 2*x^2 + d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3))])/d^(2/3) - Log
[-1 + x^2 + d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(2*d^(2/3)) + Log
[1 - 2*x^2 + x^4 + (d^(1/3) - d^(1/3)*x^2)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(
2/3) + d^(2/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(4/3)]/(4*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2*k^2-1)*x-2*k^4*x^3+k^4*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(-1
+d+(-2*d*k^2+1)*x^2+d*k^4*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^4x^5-2\,k^4x^3+(2\,k^2-1)x}{\left(dk^4x^4-(2\,dk^2-1)x^2+d-1\right)\left((k^2x^2-1)(x^2-1)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2*k^2-1)*x-2*k^4*x^3+k^4*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(-1
+d+(-2*d*k^2+1)*x^2+d*k^4*x^4),x, algorithm="giac")
```

```
[Out] integrate((k^4*x^5 - 2*k^4*x^3 + (2*k^2 - 1)*x)/((d*k^4*x^4 - (2*d*k^2 - 1)
*x^2 + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(2/3)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(2k^2-1)\,x-2k^4x^3+k^4x^5}{\left((-x^2+1)\,(-k^2x^2+1)\right)^{\frac{2}{3}}\left(-1+d+(-2d\,k^2+1)\,x^2+d\,k^4x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((2*k^2-1)*x-2*k^4*x^3+k^4*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(-1+d+(-2
*d*k^2+1)*x^2+d*k^4*x^4),x)
```

```
[Out] int((((2*k^2-1)*x-2*k^4*x^3+k^4*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(-1+d+(-2
*d*k^2+1)*x^2+d*k^4*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^4x^5 - 2\,k^4x^3 + \left(2\,k^2 - 1\right)x}{\left(dk^4x^4 - \left(2\,dk^2 - 1\right)x^2 + d - 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((2*k^2-1)*x-2*k^4*x^3+k^4*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(-1
+d+(-2*d*k^2+1)*x^2+d*k^4*x^4),x, algorithm="maxima")
```

```
[Out] integrate((k^4*x^5 - 2*k^4*x^3 + (2*k^2 - 1)*x)/((d*k^4*x^4 - (2*d*k^2 - 1)
*x^2 + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{k^4\,x^5 - 2\,k^4\,x^3 + x\,\left(2\,k^2 - 1\right)}{\left(\left(x^2 - 1\right)\,\left(k^2\,x^2 - 1\right)\right)^{2/3}\,\left(d - x^2\,\left(2\,d\,k^2 - 1\right) + d\,k^4\,x^4 - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^4*x^5 - 2*k^4*x^3 + x*(2*k^2 - 1))/(((x^2 - 1)*(k^2*x^2 - 1))^(2/3)*
(d - x^2*(2*d*k^2 - 1) + d*k^4*x^4 - 1)),x)
```

```
[Out] int((k^4*x^5 - 2*k^4*x^3 + x*(2*k^2 - 1))/(((x^2 - 1)*(k^2*x^2 - 1))^(2/3)*
(d - x^2*(2*d*k^2 - 1) + d*k^4*x^4 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((2*k**2-1)*x-2*k**4*x**3+k**4*x**5)/((-x**2+1)*(-k**2*x**2+1))**
(2/3)/(-1+d+(-2*d*k**2+1)*x**2+d*k**4*x**4),x)
```

```
[Out] Timed out
```

3.2093

$$\int \frac{(-a+x)(-b+x)(-2ab+(a+b)x)}{\sqrt[3]{x(-a+x)(-b+x)}\left(a^2b^2d-2ab(a+b)dx+\left(a^2+4ab+b^2\right)dx^2-2(a+b)dx^3+(-1+d)x^4\right)}\,dx$$

Optimal. Leaf size=227

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{x-2\sqrt[6]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}}\right)}{2d^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[6]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}+x}\right)}{2d^{5/6}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}}\right)}{d^{5/6}}-\frac{\tanh^{-1}}{}$$

Rubi [F] time = 41.62, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-a+x)(-b+x)(-2ab+(a+b)x)}{\sqrt[3]{x(-a+x)(-b+x)}\left(a^2b^2d-2ab(a+b)dx+\left(a^2+4ab+b^2\right)dx^2-2(a+b)dx^3+(-1+d)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-a + x)*(-b + x)*(-2*a*b + (a + b)*x))/((x*(-a + x)*(-b + x))^(1/3)*(
a^2*b^2*d - 2*a*b*(a + b)*d*x + (a^2 + 4*a*b + b^2)*d*x^2 - 2*(a + b)*d*x^3
 + (-1 + d)*x^4)),x]
```

```
[Out] (3*(a + b)*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][Defer[Int][(x
^4*(-a + x^3)^(2/3)*(-b + x^3)^(2/3))/(a^2*b^2*d - 2*a^2*b*(1 + b/a)*d*x^3
+ a^2*(1 + (b*(4*a + b))/a^2)*d*x^6 - 2*a*(1 + b/a)*d*x^9 - (1 - d)*x^12),
x], x, x^(1/3)])/((a - x)*(b - x)*x)^(1/3) + (6*a*b*x^(1/3)*(-a + x)^(1/3)*
(-b + x)^(1/3)*Defer[Subst][Defer[Int][(x*(-a + x^3)^(2/3)*(-b + x^3)^(2/3)
)/(-(a^2*b^2*d) + 2*a^2*b*(1 + b/a)*d*x^3 - a^2*(1 + (b*(4*a + b))/a^2)*d*x
^6 + 2*a*(1 + b/a)*d*x^9 + (1 - d)*x^12), x], x, x^(1/3)])/((a - x)*(b - x)
*x)^(1/3)
```

Rubi steps

$$\int \frac{(-a+x)(-b+x)(-2ab+(a+b)x)}{\sqrt[3]{x(-a+x)(-b+x)}\left(a^2b^2d-2ab(a+b)dx+\left(a^2+4ab+b^2\right)dx^2-2(a+b)dx^3+(-1+d)x^4\right)}\,dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{}\right.}{}$$

$$= \frac{\left(3\sqrt[3]{x}\right.}{}$$

$$= \frac{\left(3\sqrt[3]{x}\right.}{}$$

$$= \frac{\left(6ab\sqrt[3]{}\right.}{}$$

Mathematica [F] time = 3.71, size = 0, normalized size = 0.00

$$\int \frac{(-a+x)(-b+x)(-2ab+(a+b)x)}{\sqrt[3]{x(-a+x)(-b+x)}\left(a^2b^2d-2ab(a+b)dx+\left(a^2+4ab+b^2\right)dx^2-2(a+b)dx^3+(-1+d)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-a + x)*(-b + x)*(-2*a*b + (a + b)*x))/((x*(-a + x)*(-b + x))^(
1/3)*(a^2*b^2*d - 2*a*b*(a + b)*d*x + (a^2 + 4*a*b + b^2)*d*x^2 - 2*(a + b)
*d*x^3 + (-1 + d)*x^4)),x]
```

```
[Out] Integrate[((-a + x)*(-b + x)*(-2*a*b + (a + b)*x))/((x*(-a + x)*(-b + x))^(
1/3)*(a^2*b^2*d - 2*a*b*(a + b)*d*x + (a^2 + 4*a*b + b^2)*d*x^2 - 2*(a + b)
*d*x^3 + (-1 + d)*x^4)), x]
```

IntegrateAlgebraic [A] time = 1.32, size = 227, normalized size = 1.00

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{x-2\sqrt[6]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}}\right)}{2d^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[6]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}+x}\right)}{2d^{5/6}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}}\right)}{d^{5/6}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{d}\left(x^2(-a-b)+abx+x^3\right)^{2/3}+\frac{x^2}{\sqrt[6]{d}}}{x\sqrt[3]{x^2(-a-b)+abx+x^3}}\right)}{2d^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-a + x)*(-b + x)*(-2*a*b + (a + b)*x))/((x*(-a + x)*(-
b + x))^(1/3)*(a^2*b^2*d - 2*a*b*(a + b)*d*x + (a^2 + 4*a*b + b^2)*d*x^2 -
2*(a + b)*d*x^3 + (-1 + d)*x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*d^(1/6)*(a*b*x + (-a - b)*x^2 + x^3)^(1/
3))])/(2*d^(5/6)) - (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*d^(1/6)*(a*b*x + (-a
 - b)*x^2 + x^3)^(1/3))])/(2*d^(5/6)) - ArcTanh[x/(d^(1/6)*(a*b*x + (-a - b
)*x^2 + x^3)^(1/3))]/d^(5/6) - ArcTanh[(x^2/d^(1/6) + d^(1/6)*(a*b*x + (-a
- b)*x^2 + x^3)^(2/3))/(x*(a*b*x + (-a - b)*x^2 + x^3)^(1/3))]/(2*d^(5/6))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(a^2*b^2*d
-2*a*b*(a+b)*d*x+(a^2+4*a*b+b^2)*d*x^2-2*(a+b)*d*x^3+(-1+d)*x^4),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.59, size = 318, normalized size = 1.40

$$\frac{\sqrt{3}\left(-d^5\right)^{\frac{5}{6}}\log\left(\sqrt{3}\left(\frac{ab}{x^2}-\frac{a}{x}-\frac{b}{x}+1\right)^{\frac{1}{3}}\left(-\frac{1}{d}\right)^{\frac{1}{6}}+\left(\frac{ab}{x^2}-\frac{a}{x}-\frac{b}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{d}\right)^{\frac{1}{3}}\right)}{4d^5}-\frac{\sqrt{3}\left(-d^5\right)^{\frac{5}{6}}\log\left(-\sqrt{3}\left(\frac{ab}{x^2}-\frac{a}{x}-\frac{b}{x}+1\right)^{\frac{1}{3}}\left(-\frac{1}{d}\right)^{\frac{1}{6}}+\left(\frac{ab}{x^2}-\frac{a}{x}-\frac{b}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{d}\right)^{\frac{1}{3}}\right)}{4d^5}-\frac{\left(-d^5\right)^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(-\frac{1}{d}\right)^{\frac{1}{6}}+2\left(\frac{ab}{x^2}-\frac{a}{x}-\frac{b}{x}+1\right)^{\frac{1}{3}}}{\left(-\frac{1}{d}\right)^{\frac{1}{6}}}\right)}{2d^5}-\frac{\left(-d^5\right)^{\frac{5}{6}}\arctan\left(-\frac{\sqrt{3}\left(-\frac{1}{d}\right)^{\frac{1}{6}}-2\left(\frac{ab}{x^2}-\frac{a}{x}-\frac{b}{x}+1\right)^{\frac{1}{3}}}{\left(-\frac{1}{d}\right)^{\frac{1}{6}}}\right)}{2d^5}-\frac{\left(-d^5\right)^{\frac{5}{6}}\arctan\left(\frac{\left(\frac{ab}{x^2}-\frac{a}{x}-\frac{b}{x}+1\right)^{\frac{1}{3}}}{\left(-\frac{1}{d}\right)^{\frac{1}{6}}}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(a^2*b^2*d
-2*a*b*(a+b)*d*x+(a^2+4*a*b+b^2)*d*x^2-2*(a+b)*d*x^3+(-1+d)*x^4),x, algorit
hm="giac")
```

```
[Out] 1/4*sqrt(3)*(-d^5)^(5/6)*log(sqrt(3)*(a*b/x^2 - a/x - b/x + 1)^(1/3)*(-1/d)
^(1/6) + (a*b/x^2 - a/x - b/x + 1)^(2/3) + (-1/d)^(1/3))/d^5 - 1/4*sqrt(3)*
(-d^5)^(5/6)*log(-sqrt(3)*(a*b/x^2 - a/x - b/x + 1)^(1/3)*(-1/d)^(1/6) + (a
*b/x^2 - a/x - b/x + 1)^(2/3) + (-1/d)^(1/3))/d^5 - 1/2*(-d^5)^(5/6)*arctan
((sqrt(3)*(-1/d)^(1/6) + 2*(a*b/x^2 - a/x - b/x + 1)^(1/3))/(-1/d)^(1/6))/d
^5 - 1/2*(-d^5)^(5/6)*arctan(-(sqrt(3)*(-1/d)^(1/6) - 2*(a*b/x^2 - a/x - b/
x + 1)^(1/3))/(-1/d)^(1/6))/d^5 - (-d^5)^(5/6)*arctan((a*b/x^2 - a/x - b/x
+ 1)^(1/3)/(-1/d)^(1/6))/d^5
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(-a+x)(-b+x)(-2ab+(a+b)x)}{(x(-a+x)(-b+x))^{\frac{1}{3}}\left(a^2b^2d-2ab(a+b)dx+\left(a^2+4ab+b^2\right)dx^2-2(a+b)dx^3+(-1+d)x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x)*(-b+x)*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(a^2*b^2*d-2*a*b
*(a+b)*d*x+(a^2+4*a*b+b^2)*d*x^2-2*(a+b)*d*x^3+(-1+d)*x^4),x)
```

```
[Out] int((-a+x)*(-b+x)*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(a^2*b^2*d-2*a*b
*(a+b)*d*x+(a^2+4*a*b+b^2)*d*x^2-2*(a+b)*d*x^3+(-1+d)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(2\,ab - (a+b)x)(a-x)(b-x)}{\left(a^2b^2d - 2\,(a+b)abdx - 2\,(a+b)dx^3 + (d-1)x^4 + \left(a^2 + 4\,ab + b^2\right)dx^2\right)((a-x)(b-x)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(a^2*b^2*d
-2*a*b*(a+b)*d*x+(a^2+4*a*b+b^2)*d*x^2-2*(a+b)*d*x^3+(-1+d)*x^4),x, algorit
hm="maxima")
```

```
[Out] -integrate((2*a*b - (a + b)*x)*(a - x)*(b - x)/((a^2*b^2*d - 2*(a + b)*a*b*
d*x - 2*(a + b)*d*x^3 + (d - 1)*x^4 + (a^2 + 4*a*b + b^2)*d*x^2)*((a - x)*(
b - x)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{(2\,a\,b - x\,(a+b))\,(a-x)\,(b-x)}{(x\,(a-x)\,(b-x))^{1/3}\,\left(x^4\,(d-1) + a^2\,b^2\,d + d\,x^2\,\left(a^2 + 4\,a\,b + b^2\right) - 2\,d\,x^3\,(a+b) - 2\,a\,b\,d\,x\,(a+b)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*a*b - x*(a + b))*(a - x)*(b - x))/((x*(a - x)*(b - x))^(1/3)*(x^4*
(d - 1) + a^2*b^2*d + d*x^2*(4*a*b + a^2 + b^2) - 2*d*x^3*(a + b) - 2*a*b*d
*x*(a + b))),x)
```

```
[Out] -int(((2*a*b - x*(a + b))*(a - x)*(b - x))/((x*(a - x)*(b - x))^(1/3)*(x^4*
(d - 1) + a^2*b^2*d + d*x^2*(4*a*b + a^2 + b^2) - 2*d*x^3*(a + b) - 2*a*b*d
*x*(a + b))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))**(1/3)/(a**2*b**
2*d-2*a*b*(a+b)*d*x+(a**2+4*a*b+b**2)*d*x**2-2*(a+b)*d*x**3+(-1+d)*x**4),x)
```

```
[Out] Timed out
```

3.2094 $\int \frac{\sqrt{b^2+a^2x^3}\left(2b^2+cx^3+a^2x^6\right)}{x\left(b^2+a^2x^6\right)}\,dx$

Optimal. Leaf size=227

$$\frac{\left(\sqrt[4]{-1}\,c\sqrt{a-ib}-(-1)^{3/4}ab\sqrt{a-ib}\right)\tan^{-1}\left(\frac{\sqrt[4]{-1}\,\sqrt{a^2x^3+b^2}}{\sqrt{b}\,\sqrt{a-ib}}\right)}{3a\sqrt{b}}+\frac{\left(-\sqrt[4]{-1}\,c\sqrt{a+ib}-(-1)^{3/4}ab\sqrt{a+ib}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt{a^2}}{\sqrt{2}\,\sqrt{b}\,\sqrt{}}\right.}{3a\sqrt{b}}$$

Rubi [B] time = 3.16, antiderivative size = 609, normalized size of antiderivative = 2.68, number of steps used = 18, number of rules used = 13, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.260, Rules used = {6725, 266, 50, 63, 208, 6715, 825, 827, 1169, 634, 618, 206, 628}

$$\frac{2}{3}\sqrt{a^2x^3+b^2}-\frac{4}{3}b\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{b}\right)-\frac{\left(\sqrt{a^2+b^2}\left(b^2-c\right)+a^2b+b^3\right)\log\left(-\sqrt{2}\sqrt{b}\sqrt{\sqrt{a^2+b^2}+b}\sqrt{a^2x^3+b^2}+b\left(\sqrt{a^2+b^2}+b\right)+a^2x^3\right)}{6\sqrt{2}\sqrt{b}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+b}}+\frac{\left(\sqrt{a^2+b^2}\left(b^2-c\right)+a^2b+b^3\right)\log\left(\sqrt{2}\sqrt{b}\sqrt{\sqrt{a^2+b^2}+b}\sqrt{a^2x^3+b^2}+b\left(\sqrt{a^2+b^2}+b\right)+a^2x^3\right)}{6\sqrt{2}\sqrt{b}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+b}}+\frac{\left(-\sqrt{a^2+b^2}\left(b^2-c\right)+a^2b+b^3\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\sqrt{a^2+b^2}+b}-\sqrt{2}\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{b-\sqrt{a^2+b^2}}}\right)}{3\sqrt{2}\sqrt{b}\sqrt{a^2+b^2}\sqrt{b-\sqrt{a^2+b^2}}}-\frac{\left(-\sqrt{a^2+b^2}\left(b^2-c\right)+a^2b+b^3\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a^2x^3+b^2}+\sqrt{b}\sqrt{\sqrt{a^2+b^2}+b}}{\sqrt{b}\sqrt{b-\sqrt{a^2+b^2}}}\right)}{3\sqrt{2}\sqrt{b}\sqrt{a^2+b^2}\sqrt{b-\sqrt{a^2+b^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + a^2*x^3]*(2*b^2 + c*x^3 + a^2*x^6))/(x*(b^2 + a^2*x^6)),x]
```

```
[Out] (2*Sqrt[b^2 + a^2*x^3])/3 - (4*b*ArcTanh[Sqrt[b^2 + a^2*x^3]/b])/3 + ((a^2*
b + b^3 - Sqrt[a^2 + b^2]*(b^2 - c))*ArcTanh[(Sqrt[b]*Sqrt[b + Sqrt[a^2 + b
^2]] - Sqrt[2]*Sqrt[b^2 + a^2*x^3])/(Sqrt[b]*Sqrt[b - Sqrt[a^2 + b^2]])])/(
3*Sqrt[2]*Sqrt[b]*Sqrt[a^2 + b^2]*Sqrt[b - Sqrt[a^2 + b^2]]) - ((a^2*b + b^
3 - Sqrt[a^2 + b^2]*(b^2 - c))*ArcTanh[(Sqrt[b]*Sqrt[b + Sqrt[a^2 + b^2]] +
 Sqrt[2]*Sqrt[b^2 + a^2*x^3])/(Sqrt[b]*Sqrt[b - Sqrt[a^2 + b^2]])])/(3*Sqrt
[2]*Sqrt[b]*Sqrt[a^2 + b^2]*Sqrt[b - Sqrt[a^2 + b^2]]) - ((a^2*b + b^3 + Sq
rt[a^2 + b^2]*(b^2 - c))*Log[b*(b + Sqrt[a^2 + b^2]) + a^2*x^3 - Sqrt[2]*Sq
rt[b]*Sqrt[b + Sqrt[a^2 + b^2]]*Sqrt[b^2 + a^2*x^3]])/(6*Sqrt[2]*Sqrt[b]*Sq
rt[a^2 + b^2]*Sqrt[b + Sqrt[a^2 + b^2]]) + ((a^2*b + b^3 + Sqrt[a^2 + b^2]*
(b^2 - c))*Log[b*(b + Sqrt[a^2 + b^2]) + a^2*x^3 + Sqrt[2]*Sqrt[b]*Sqrt[b +
 Sqrt[a^2 + b^2]]*Sqrt[b^2 + a^2*x^3]])/(6*Sqrt[2]*Sqrt[b]*Sqrt[a^2 + b^2]*
Sqrt[b + Sqrt[a^2 + b^2]])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{b^2+a^2x^3}\left(2b^2+cx^3+a^2x^6\right)}{x\left(b^2+a^2x^6\right)}\,dx = \int \left(\frac{2\sqrt{b^2+a^2x^3}}{x} - \frac{x^2\sqrt{b^2+a^2x^3}\left(-c+a^2x^3\right)}{b^2+a^2x^6}\right)dx$$

$$= 2\int \frac{\sqrt{b^2+a^2x^3}}{x}\,dx - \int \frac{x^2\sqrt{b^2+a^2x^3}\left(-c+a^2x^3\right)}{b^2+a^2x^6}\,dx$$

$$= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{b^2+a^2x}\left(-c+a^2x\right)}{b^2+a^2x^2}\,dx, x, x^3\right)\right) + \frac{2}{3}\operatorname{Subst}\left(\int \frac{\sqrt{b^2+a^2 \cdots}}{x} \cdots\right.$$

$$= \frac{2}{3}\sqrt{b^2+a^2x^3} - \frac{\operatorname{Subst}\left(\int \frac{-a^2b^2\left(a^2+c\right)+a^4\left(b^2-c\right)x}{\sqrt{b^2+a^2x}\left(b^2+a^2x^2\right)}\,dx, x, x^3\right)}{3a^2} + \frac{1}{3}\left(2b^2\right)\operatorname{Subst}\left(\cdots\right.$$

$$= \frac{2}{3}\sqrt{b^2+a^2x^3} - \frac{2\operatorname{Subst}\left(\int \frac{-a^4b^2\left(b^2-c\right)-a^4b^2\left(a^2+c\right)+a^4\left(b^2-c\right)x^2}{a^4b^2+a^2b^4-2a^2b^2x^2+a^2x^4}\,dx, x, \sqrt{b^2+a^2 \cdots}\right.}{3a^2}$$

$$= \frac{2}{3}\sqrt{b^2+a^2x^3} - \frac{4}{3}b\tanh^{-1}\left(\frac{\sqrt{b^2+a^2x^3}}{b}\right) - \frac{\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}\sqrt{b+\sqrt{a^2+b^2}}\left(-a \cdots\right.}{\cdots}\right.}{\cdots}$$

$$= \frac{2}{3}\sqrt{b^2+a^2x^3} - \frac{4}{3}b\tanh^{-1}\left(\frac{\sqrt{b^2+a^2x^3}}{b}\right) + \frac{\left(a^2b+b^3-\sqrt{a^2+b^2}\left(b^2-c \cdots\right.\right.}{\cdots}$$

$$= \frac{2}{3}\sqrt{b^2+a^2x^3} - \frac{4}{3}b\tanh^{-1}\left(\frac{\sqrt{b^2+a^2x^3}}{b}\right) - \frac{\left(a^2b+b^3+\sqrt{a^2+b^2}\left(b^2-c \cdots\right.\right.}{\cdots}$$

$$= \frac{2}{3}\sqrt{b^2+a^2x^3} - \frac{4}{3}b\tanh^{-1}\left(\frac{\sqrt{b^2+a^2x^3}}{b}\right) + \frac{\left(a^2b+b^3-\sqrt{a^2+b^2}\left(b^2-c \cdots\right.\right.}{3\sqrt{2}\sqrt{b}\sqrt{a^2 \cdots}}$$

Mathematica [A] time = 0.44, size = 267, normalized size = 1.18

$$\frac{-\sqrt{-a^2}\,c\sqrt{\sqrt{-a^2}-b}\,\tan^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{\sqrt{-a^2}-b}}\right)+\sqrt{\sqrt{-a^2}+b}\left(a^2b+\sqrt{-a^2}\,c\right)\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{\sqrt{-a^2}+b}}\right)+2a^2\sqrt{b}\sqrt{a^2x^3+b^2}+a^2b\sqrt{\sqrt{-a^2}-b}\,\tan^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{\sqrt{-a^2}-b}}\right)-4a^2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{b}\right)}{3a^2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + a^2*x^3]*(2*b^2 + c*x^3 + a^2*x^6))/(x*(b^2 + a^2*x^6
)),x]
```

```
[Out] (2*a^2*Sqrt[b]*Sqrt[b^2 + a^2*x^3] + a^2*Sqrt[Sqrt[-a^2] - b]*b*ArcTan[Sqrt
[b^2 + a^2*x^3]/(Sqrt[Sqrt[-a^2] - b]*Sqrt[b])] - Sqrt[-a^2]*Sqrt[Sqrt[-a^2
] - b]*c*ArcTan[Sqrt[b^2 + a^2*x^3]/(Sqrt[Sqrt[-a^2] - b]*Sqrt[b])] - 4*a^2
*b^(3/2)*ArcTanh[Sqrt[b^2 + a^2*x^3]/b] + Sqrt[Sqrt[-a^2] + b]*(a^2*b + Sqr
```

```
t[-a^2]*c)*ArcTanh[Sqrt[b^2 + a^2*x^3]/(Sqrt[b]*Sqrt[Sqrt[-a^2] + b])])/(3*
a^2*Sqrt[b])
```

IntegrateAlgebraic [A] time = 0.51, size = 223, normalized size = 0.98

$$\frac{\left(\sqrt[4]{-1}\,c\sqrt{a-ib}-(-1)^{3/4}ab\sqrt{a-ib}\right)\tan^{-1}\left(\frac{\sqrt[4]{-1}\,\sqrt{a^2x^3+b^2}}{\sqrt{b}\,\sqrt{a-ib}}\right)}{3a\sqrt{b}}+\frac{\left((-1)^{3/4}c\sqrt{a+ib}-\sqrt[4]{-1}\,ab\sqrt{a+ib}\right)\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a^2x^3+b^2}}{\sqrt{b}\,\sqrt{a+ib}}\right)}{3a\sqrt{b}}+\frac{2}{3}\sqrt{a^2x^3+b^2}-\frac{4}{3}b\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{b}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[b^2 + a^2*x^3]*(2*b^2 + c*x^3 + a^2*x^6))/(x*(b^2
+ a^2*x^6)),x]
```

```
[Out] (2*Sqrt[b^2 + a^2*x^3])/3 + ((-((-1)^(3/4)*a*Sqrt[a - I*b]*b) + (-1)^(1/4)*
Sqrt[a - I*b]*c)*ArcTan[((-1)^(1/4)*Sqrt[b^2 + a^2*x^3])/(Sqrt[a - I*b]*Sqr
t[b])])/(3*a*Sqrt[b]) + ((-((-1)^(1/4)*a*Sqrt[a + I*b]*b) + (-1)^(3/4)*Sqrt
[a + I*b]*c)*ArcTan[((-1)^(3/4)*Sqrt[b^2 + a^2*x^3])/(Sqrt[a + I*b]*Sqrt[b]
)])/(3*a*Sqrt[b]) - (4*b*ArcTanh[Sqrt[b^2 + a^2*x^3]/b])/3
```

fricas [B] time = 20.95, size = 9711, normalized size = 42.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^3+b^2)^(1/2)*(a^2*x^6+c*x^3+2*b^2)/x/(a^2*x^6+b^2),x, algo
rithm="fricas")
```

```
[Out] 1/12*(4*sqrt(2)*a^4*b^2*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*
b^2 + a^2*b^4)*c^2 - (a^4*b^4 - 2*a^4*b^2*c - a^2*b^2*c^2)*sqrt((a^6*b^4 +
a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2)))/(a^6*b^4
 + 4*a^4*b^4*c - 4*a^2*b^2*c^3 + a^2*c^4 - 2*(a^4*b^2 - 2*a^2*b^4)*c^2))*sq
rt((a^4*b^4 + 4*a^2*b^4*c - 4*b^2*c^3 + c^4 - 2*(a^2*b^2 - 2*b^4)*c^2)/(a^2
*b^2))*((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(
a^4*b^2))^(3/4)*arctan((sqrt(2)*sqrt(a^2*x^3 + b^2)*((a^10*b^8 + 2*a^8*b^8*
c + 2*a^6*b^6*c^3 - a^6*b^4*c^4)*sqrt((a^4*b^4 + 4*a^2*b^4*c - 4*b^2*c^3 +
c^4 - 2*(a^2*b^2 - 2*b^4)*c^2)/(a^2*b^2))*sqrt((a^6*b^4 + a^4*b^6 + (a^2 +
b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2)) + (a^10*b^10 - a^8*b^8*c^2
 - 5*a^6*b^6*c^4 - 3*a^4*b^4*c^6 + a^4*b^2*c^7 + (a^6*b^4 + 2*a^4*b^6)*c^5
- (a^8*b^6 - 4*a^6*b^8)*c^3 - (a^10*b^8 - 2*a^8*b^10)*c)*sqrt((a^4*b^4 + 4*
a^2*b^4*c - 4*b^2*c^3 + c^4 - 2*(a^2*b^2 - 2*b^4)*c^2)/(a^2*b^2)))*sqrt((a^
6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2 - (a^4*b^4 -
2*a^4*b^2*c - a^2*b^2*c^2)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a
^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2)))/(a^6*b^4 + 4*a^4*b^4*c - 4*a^2*b^2*c^3 +
 a^2*c^4 - 2*(a^4*b^2 - 2*a^2*b^4)*c^2))*((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*
c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2))^(3/4) + sqrt(2)*(a^6*b^4*sqrt((
a^4*b^4 + 4*a^2*b^4*c - 4*b^2*c^3 + c^4 - 2*(a^2*b^2 - 2*b^4)*c^2)/(a^2*b^2
))*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(
a^4*b^2)) + (a^6*b^6 - a^6*b^4*c + a^4*b^4*c^2 - a^4*b^2*c^3)*sqrt((a^4*b^4
 + 4*a^2*b^4*c - 4*b^2*c^3 + c^4 - 2*(a^2*b^2 - 2*b^4)*c^2)/(a^2*b^2)))*sqr
t((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2 - (a^4*b
^4 - 2*a^4*b^2*c - a^2*b^2*c^2)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 +
 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2)))/(a^6*b^4 + 4*a^4*b^4*c - 4*a^2*b^2*
c^3 + a^2*c^4 - 2*(a^4*b^2 - 2*a^2*b^4)*c^2))*sqrt((a^10*b^10 + a^8*b^12 +
(a^2*b^2 + b^4)*c^8 - 4*(a^2*b^4 + b^6)*c^7 + 4*(a^2*b^6 + b^8)*c^6 - 4*(a^
4*b^6 + a^2*b^8)*c^5 - 2*(a^6*b^6 - 3*a^4*b^8 - 4*a^2*b^10)*c^4 + 4*(a^6*b^
8 + a^4*b^10)*c^3 + (a^12*b^8 + a^10*b^10 + (a^4 + a^2*b^2)*c^8 - 4*(a^4*b^
2 + a^2*b^4)*c^7 + 4*(a^4*b^4 + a^2*b^6)*c^6 - 4*(a^6*b^4 + a^4*b^6)*c^5 -
2*(a^8*b^4 - 3*a^6*b^6 - 4*a^4*b^8)*c^4 + 4*(a^8*b^6 + a^6*b^8)*c^3 + 4*(a^
8*b^8 + a^6*b^10)*c^2 + 4*(a^10*b^8 + a^8*b^10)*c)*x^3 + 4*(a^6*b^10 + a^4*
b^12)*c^2 + sqrt(2)*(a^10*b^8 + a^8*b^10 + (a^4*b^2 + a^2*b^4)*c^6 - 4*(a^4
*b^4 + a^2*b^6)*c^5 - (a^6*b^4 - 3*a^4*b^6 - 4*a^2*b^8)*c^4 - (a^8*b^6 - 3*
```

a^6*b^8 - 4*a^4*b^10)*c^2 + 4*(a^8*b^8 + a^6*b^10)*c + (a^8*b^8 + 5*a^4*b^4
*c^4 - a^4*b^2*c^5 + 2*(a^6*b^4 - 4*a^4*b^6)*c^3 - 2*(3*a^6*b^6 - 2*a^4*b^8
)*c^2 - (a^8*b^6 - 4*a^6*b^8)*c)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4
+ 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2)))*sqrt(a^2*x^3 + b^2)*sqrt((a^6*b^4
+ a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2 - (a^4*b^4 - 2*a^4*
b^2*c - a^2*b^2*c^2)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2
 + a^2*b^4)*c^2)/(a^4*b^2)))/(a^6*b^4 + 4*a^4*b^4*c - 4*a^2*b^2*c^3 + a^2*c
^4 - 2*(a^4*b^2 - 2*a^2*b^4)*c^2))*((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 +
2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2))^(1/4) + 4*(a^8*b^10 + a^6*b^12)*c + (
a^10*b^8 + a^8*b^10 + (a^4*b^2 + a^2*b^4)*c^6 - 4*(a^4*b^4 + a^2*b^6)*c^5 -
 (a^6*b^4 - 3*a^4*b^6 - 4*a^2*b^8)*c^4 - (a^8*b^6 - 3*a^6*b^8 - 4*a^4*b^10)
c^2 + 4(a^8*b^8 + a^6*b^10)*c)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4
+ 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2)))/(a^2 + b^2))*((a^6*b^4 + a^4*b^6 +
 (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2))^(3/4) + (a^12*b^10
 + a^10*b^12 - (a^4*b^2 + a^2*b^4)*c^8 + 2*(a^4*b^4 + a^2*b^6)*c^7 - 2*(a^6
*b^4 + a^4*b^6)*c^6 + 6*(a^6*b^6 + a^4*b^8)*c^5 + 6*(a^8*b^8 + a^6*b^10)*c^
3 + 2*(a^10*b^8 + a^8*b^10)*c^2 + 2*(a^10*b^10 + a^8*b^12)*c)*sqrt((a^4*b^4
 + 4*a^2*b^4*c - 4*b^2*c^3 + c^4 - 2*(a^2*b^2 - 2*b^4)*c^2)/(a^2*b^2))*sqrt
((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2
)) + (a^12*b^12 + a^10*b^14 - (a^2*b^2 + b^4)*c^10 + 2*(a^2*b^4 + b^6)*c^9
- 3*(a^4*b^4 + a^2*b^6)*c^8 + 8*(a^4*b^6 + a^2*b^8)*c^7 - 2*(a^6*b^6 + a^4*
b^8)*c^6 + 12*(a^6*b^8 + a^4*b^10)*c^5 + 2*(a^8*b^8 + a^6*b^10)*c^4 + 8*(a^
8*b^10 + a^6*b^12)*c^3 + 3*(a^10*b^10 + a^8*b^12)*c^2 + 2*(a^10*b^12 + a^8*
b^14)*c)*sqrt((a^4*b^4 + 4*a^2*b^4*c - 4*b^2*c^3 + c^4 - 2*(a^2*b^2 - 2*b^4
)*c^2)/(a^2*b^2)))/(a^14*b^12 + a^12*b^14 + (a^2 + b^2)*c^12 - 4*(a^2*b^2 +
 b^4)*c^11 + 2*(a^4*b^2 + 3*a^2*b^4 + 2*b^6)*c^10 - 12*(a^4*b^4 + a^2*b^6)*
c^9 - (a^6*b^4 - 15*a^4*b^6 - 16*a^2*b^8)*c^8 - 8*(a^6*b^6 + a^4*b^8)*c^7 -
 4*(a^8*b^6 - 5*a^6*b^8 - 6*a^4*b^10)*c^6 + 8*(a^8*b^8 + a^6*b^10)*c^5 - (a
^10*b^8 - 15*a^8*b^10 - 16*a^6*b^12)*c^4 + 12*(a^10*b^10 + a^8*b^12)*c^3 +
2*(a^12*b^10 + 3*a^10*b^12 + 2*a^8*b^14)*c^2 + 4*(a^12*b^12 + a^10*b^14)*c)
) + 4*sqrt(2)*a^4*b^2*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^
2 + a^2*b^4)*c^2 - (a^4*b^4 - 2*a^4*b^2*c - a^2*b^2*c^2)*sqrt((a^6*b^4 + a^
4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2)))/(a^6*b^4 +
 4*a^4*b^4*c - 4*a^2*b^2*c^3 + a^2*c^4 - 2*(a^4*b^2 - 2*a^2*b^4)*c^2))*sqrt
((a^4*b^4 + 4*a^2*b^4*c - 4*b^2*c^3 + c^4 - 2*(a^2*b^2 - 2*b^4)*c^2)/(a^2*b
^2))*((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^
4*b^2))^(3/4)*arctan((sqrt(2)*sqrt(a^2*x^3 + b^2)*((a^10*b^8 + 2*a^8*b^8*c
+ 2*a^6*b^6*c^3 - a^6*b^4*c^4)*sqrt((a^4*b^4 + 4*a^2*b^4*c - 4*b^2*c^3 + c^
4 - 2*(a^2*b^2 - 2*b^4)*c^2)/(a^2*b^2))*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^
2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2)) + (a^10*b^10 - a^8*b^8*c^2 -
 5*a^6*b^6*c^4 - 3*a^4*b^4*c^6 + a^4*b^2*c^7 + (a^6*b^4 + 2*a^4*b^6)*c^5 -
(a^8*b^6 - 4*a^6*b^8)*c^3 - (a^10*b^8 - 2*a^8*b^10)*c)*sqrt((a^4*b^4 + 4*a^
2*b^4*c - 4*b^2*c^3 + c^4 - 2*(a^2*b^2 - 2*b^4)*c^2)/(a^2*b^2)))*sqrt((a^6*
b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2 - (a^4*b^4 - 2*
a^4*b^2*c - a^2*b^2*c^2)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4
*b^2 + a^2*b^4)*c^2)/(a^4*b^2)))/(a^6*b^4 + 4*a^4*b^4*c - 4*a^2*b^2*c^3 + a
^2*c^4 - 2*(a^4*b^2 - 2*a^2*b^4)*c^2))*((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^
4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2))^(3/4) + sqrt(2)*(a^6*b^4*sqrt((a^
4*b^4 + 4*a^2*b^4*c - 4*b^2*c^3 + c^4 - 2*(a^2*b^2 - 2*b^4)*c^2)/(a^2*b^2))
*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^
4*b^2)) + (a^6*b^6 - a^6*b^4*c + a^4*b^4*c^2 - a^4*b^2*c^3)*sqrt((a^4*b^4 +
 4*a^2*b^4*c - 4*b^2*c^3 + c^4 - 2*(a^2*b^2 - 2*b^4)*c^2)/(a^2*b^2)))*sqrt(
(a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2 - (a^4*b^4
 - 2*a^4*b^2*c - a^2*b^2*c^2)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2
*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2)))/(a^6*b^4 + 4*a^4*b^4*c - 4*a^2*b^2*c^
3 + a^2*c^4 - 2*(a^4*b^2 - 2*a^2*b^4)*c^2))*sqrt((a^10*b^10 + a^8*b^12 + (a
^2*b^2 + b^4)*c^8 - 4*(a^2*b^4 + b^6)*c^7 + 4*(a^2*b^6 + b^8)*c^6 - 4*(a^4*
b^6 + a^2*b^8)*c^5 - 2*(a^6*b^6 - 3*a^4*b^8 - 4*a^2*b^10)*c^4 + 4*(a^6*b^8
+ a^4*b^10)*c^3 + (a^12*b^8 + a^10*b^10 + (a^4 + a^2*b^2)*c^8 - 4*(a^4*b^2

+ a^2*b^4)*c^7 + 4*(a^4*b^4 + a^2*b^6)*c^6 - 4*(a^6*b^4 + a^4*b^6)*c^5 - 2*
(a^8*b^4 - 3*a^6*b^6 - 4*a^4*b^8)*c^4 + 4*(a^8*b^6 + a^6*b^8)*c^3 + 4*(a^8*
b^8 + a^6*b^10)*c^2 + 4*(a^10*b^8 + a^8*b^10)*c)*x^3 + 4*(a^6*b^10 + a^4*b^
12)*c^2 - sqrt(2)*(a^10*b^8 + a^8*b^10 + (a^4*b^2 + a^2*b^4)*c^6 - 4*(a^4*b
^4 + a^2*b^6)*c^5 - (a^6*b^4 - 3*a^4*b^6 - 4*a^2*b^8)*c^4 - (a^8*b^6 - 3*a^
6*b^8 - 4*a^4*b^10)*c^2 + 4*(a^8*b^8 + a^6*b^10)*c + (a^8*b^8 + 5*a^4*b^4*c
^4 - a^4*b^2*c^5 + 2*(a^6*b^4 - 4*a^4*b^6)*c^3 - 2*(3*a^6*b^6 - 2*a^4*b^8)*
c^2 - (a^8*b^6 - 4*a^6*b^8)*c)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 +
2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2)))*sqrt(a^2*x^3 + b^2)*sqrt((a^6*b^4 +
a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2 - (a^4*b^4 - 2*a^4*b^
2*c - a^2*b^2*c^2)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 +
a^2*b^4)*c^2)/(a^4*b^2)))/(a^6*b^4 + 4*a^4*b^4*c - 4*a^2*b^2*c^3 + a^2*c^4
- 2*(a^4*b^2 - 2*a^2*b^4)*c^2))*((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*
(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2))^(1/4) + 4*(a^8*b^10 + a^6*b^12)*c + (a^
10*b^8 + a^8*b^10 + (a^4*b^2 + a^2*b^4)*c^6 - 4*(a^4*b^4 + a^2*b^6)*c^5 - (
a^6*b^4 - 3*a^4*b^6 - 4*a^2*b^8)*c^4 - (a^8*b^6 - 3*a^6*b^8 - 4*a^4*b^10)*c
^2 + 4*(a^8*b^8 + a^6*b^10)*c)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 +
2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2)))/(a^2 + b^2))*((a^6*b^4 + a^4*b^6 + (
a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2))^(3/4) - (a^12*b^10 +
a^10*b^12 - (a^4*b^2 + a^2*b^4)*c^8 + 2*(a^4*b^4 + a^2*b^6)*c^7 - 2*(a^6*b
^4 + a^4*b^6)*c^6 + 6*(a^6*b^6 + a^4*b^8)*c^5 + 6*(a^8*b^8 + a^6*b^10)*c^3
+ 2*(a^10*b^8 + a^8*b^10)*c^2 + 2*(a^10*b^10 + a^8*b^12)*c)*sqrt((a^4*b^4 +
4*a^2*b^4*c - 4*b^2*c^3 + c^4 - 2*(a^2*b^2 - 2*b^4)*c^2)/(a^2*b^2))*sqrt((
a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2))
- (a^12*b^12 + a^10*b^14 - (a^2*b^2 + b^4)*c^10 + 2*(a^2*b^4 + b^6)*c^9 -
3*(a^4*b^4 + a^2*b^6)*c^8 + 8*(a^4*b^6 + a^2*b^8)*c^7 - 2*(a^6*b^6 + a^4*b^
8)*c^6 + 12*(a^6*b^8 + a^4*b^10)*c^5 + 2*(a^8*b^8 + a^6*b^10)*c^4 + 8*(a^8*
b^10 + a^6*b^12)*c^3 + 3*(a^10*b^10 + a^8*b^12)*c^2 + 2*(a^10*b^12 + a^8*b^
14)*c)*sqrt((a^4*b^4 + 4*a^2*b^4*c - 4*b^2*c^3 + c^4 - 2*(a^2*b^2 - 2*b^4)*
c^2)/(a^2*b^2)))/(a^14*b^12 + a^12*b^14 + (a^2 + b^2)*c^12 - 4*(a^2*b^2 + b
^4)*c^11 + 2*(a^4*b^2 + 3*a^2*b^4 + 2*b^6)*c^10 - 12*(a^4*b^4 + a^2*b^6)*c^
9 - (a^6*b^4 - 15*a^4*b^6 - 16*a^2*b^8)*c^8 - 8*(a^6*b^6 + a^4*b^8)*c^7 - 4
*(a^8*b^6 - 5*a^6*b^8 - 6*a^4*b^10)*c^6 + 8*(a^8*b^8 + a^6*b^10)*c^5 - (a^1
0*b^8 - 15*a^8*b^10 - 16*a^6*b^12)*c^4 + 12*(a^10*b^10 + a^8*b^12)*c^3 + 2*
(a^12*b^10 + 3*a^10*b^12 + 2*a^8*b^14)*c^2 + 4*(a^12*b^12 + a^10*b^14)*c))
+ sqrt(2)*(a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2
+ (a^4*b^4 - 2*a^4*b^2*c - a^2*b^2*c^2)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^
2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2)))*sqrt((a^6*b^4 + a^4*b^6 + (
a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2 - (a^4*b^4 - 2*a^4*b^2*c - a^2*b
^2*c^2)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c
^2)/(a^4*b^2)))/(a^6*b^4 + 4*a^4*b^4*c - 4*a^2*b^2*c^3 + a^2*c^4 - 2*(a^4*b
^2 - 2*a^2*b^4)*c^2))*((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 +
a^2*b^4)*c^2)/(a^4*b^2))^(1/4)*log((a^10*b^10 + a^8*b^12 + (a^2*b^2 + b^4)*
c^8 - 4*(a^2*b^4 + b^6)*c^7 + 4*(a^2*b^6 + b^8)*c^6 - 4*(a^4*b^6 + a^2*b^8)
c^5 - 2(a^6*b^6 - 3*a^4*b^8 - 4*a^2*b^10)*c^4 + 4*(a^6*b^8 + a^4*b^10)*c^
3 + (a^12*b^8 + a^10*b^10 + (a^4 + a^2*b^2)*c^8 - 4*(a^4*b^2 + a^2*b^4)*c^7
+ 4*(a^4*b^4 + a^2*b^6)*c^6 - 4*(a^6*b^4 + a^4*b^6)*c^5 - 2*(a^8*b^4 - 3*a
^6*b^6 - 4*a^4*b^8)*c^4 + 4*(a^8*b^6 + a^6*b^8)*c^3 + 4*(a^8*b^8 + a^6*b^10
)*c^2 + 4*(a^10*b^8 + a^8*b^10)*c)*x^3 + 4*(a^6*b^10 + a^4*b^12)*c^2 + sqrt
(2)*(a^10*b^8 + a^8*b^10 + (a^4*b^2 + a^2*b^4)*c^6 - 4*(a^4*b^4 + a^2*b^6)*
c^5 - (a^6*b^4 - 3*a^4*b^6 - 4*a^2*b^8)*c^4 - (a^8*b^6 - 3*a^6*b^8 - 4*a^4*
b^10)*c^2 + 4*(a^8*b^8 + a^6*b^10)*c + (a^8*b^8 + 5*a^4*b^4*c^4 - a^4*b^2*c
^5 + 2*(a^6*b^4 - 4*a^4*b^6)*c^3 - 2*(3*a^6*b^6 - 2*a^4*b^8)*c^2 - (a^8*b^6
- 4*a^6*b^8)*c)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a
^2*b^4)*c^2)/(a^4*b^2)))*sqrt(a^2*x^3 + b^2)*sqrt((a^6*b^4 + a^4*b^6 + (a^2
+ b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2 - (a^4*b^4 - 2*a^4*b^2*c - a^2*b^2*
c^2)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)
/(a^4*b^2)))/(a^6*b^4 + 4*a^4*b^4*c - 4*a^2*b^2*c^3 + a^2*c^4 - 2*(a^4*b^2
- 2*a^2*b^4)*c^2))*((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2

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*b^4)*c^2)/(a^4*b^2))^(1/4) + 4*(a^8*b^10 + a^6*b^12)*c + (a^10*b^8 + a^8*b
^10 + (a^4*b^2 + a^2*b^4)*c^6 - 4*(a^4*b^4 + a^2*b^6)*c^5 - (a^6*b^4 - 3*a^
4*b^6 - 4*a^2*b^8)*c^4 - (a^8*b^6 - 3*a^6*b^8 - 4*a^4*b^10)*c^2 + 4*(a^8*b^
8 + a^6*b^10)*c)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a
^2*b^4)*c^2)/(a^4*b^2)))/(a^2 + b^2)) - sqrt(2)*(a^6*b^4 + a^4*b^6 + (a^2 +
 b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2 + (a^4*b^4 - 2*a^4*b^2*c - a^2*b^2*c^
2)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(
a^4*b^2)))*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4
)*c^2 - (a^4*b^4 - 2*a^4*b^2*c - a^2*b^2*c^2)*sqrt((a^6*b^4 + a^4*b^6 + (a^
2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2)))/(a^6*b^4 + 4*a^4*b^4*
c - 4*a^2*b^2*c^3 + a^2*c^4 - 2*(a^4*b^2 - 2*a^2*b^4)*c^2))*((a^6*b^4 + a^4
*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2))^(1/4)*log((a
^10*b^10 + a^8*b^12 + (a^2*b^2 + b^4)*c^8 - 4*(a^2*b^4 + b^6)*c^7 + 4*(a^2*
b^6 + b^8)*c^6 - 4*(a^4*b^6 + a^2*b^8)*c^5 - 2*(a^6*b^6 - 3*a^4*b^8 - 4*a^2
*b^10)*c^4 + 4*(a^6*b^8 + a^4*b^10)*c^3 + (a^12*b^8 + a^10*b^10 + (a^4 + a^
2*b^2)*c^8 - 4*(a^4*b^2 + a^2*b^4)*c^7 + 4*(a^4*b^4 + a^2*b^6)*c^6 - 4*(a^6
*b^4 + a^4*b^6)*c^5 - 2*(a^8*b^4 - 3*a^6*b^6 - 4*a^4*b^8)*c^4 + 4*(a^8*b^6
+ a^6*b^8)*c^3 + 4*(a^8*b^8 + a^6*b^10)*c^2 + 4*(a^10*b^8 + a^8*b^10)*c)*x^
3 + 4*(a^6*b^10 + a^4*b^12)*c^2 - sqrt(2)*(a^10*b^8 + a^8*b^10 + (a^4*b^2 +
 a^2*b^4)*c^6 - 4*(a^4*b^4 + a^2*b^6)*c^5 - (a^6*b^4 - 3*a^4*b^6 - 4*a^2*b^
8)*c^4 - (a^8*b^6 - 3*a^6*b^8 - 4*a^4*b^10)*c^2 + 4*(a^8*b^8 + a^6*b^10)*c
+ (a^8*b^8 + 5*a^4*b^4*c^4 - a^4*b^2*c^5 + 2*(a^6*b^4 - 4*a^4*b^6)*c^3 - 2*
(3*a^6*b^6 - 2*a^4*b^8)*c^2 - (a^8*b^6 - 4*a^6*b^8)*c)*sqrt((a^6*b^4 + a^4*
b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2)))*sqrt(a^2*x^3
 + b^2)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c
^2 - (a^4*b^4 - 2*a^4*b^2*c - a^2*b^2*c^2)*sqrt((a^6*b^4 + a^4*b^6 + (a^2 +
 b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2)))/(a^6*b^4 + 4*a^4*b^4*c -
 4*a^2*b^2*c^3 + a^2*c^4 - 2*(a^4*b^2 - 2*a^2*b^4)*c^2))*((a^6*b^4 + a^4*b^
6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2))^(1/4) + 4*(a^8*
b^10 + a^6*b^12)*c + (a^10*b^8 + a^8*b^10 + (a^4*b^2 + a^2*b^4)*c^6 - 4*(a^
4*b^4 + a^2*b^6)*c^5 - (a^6*b^4 - 3*a^4*b^6 - 4*a^2*b^8)*c^4 - (a^8*b^6 - 3
*a^6*b^8 - 4*a^4*b^10)*c^2 + 4*(a^8*b^8 + a^6*b^10)*c)*sqrt((a^6*b^4 + a^4*
b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)/(a^4*b^2)))/(a^2 + b^2))
 - 8*(a^6*b^5 + a^4*b^7 + (a^2*b + b^3)*c^4 + 2*(a^4*b^3 + a^2*b^5)*c^2)*lo
g(b + sqrt(a^2*x^3 + b^2)) + 8*(a^6*b^5 + a^4*b^7 + (a^2*b + b^3)*c^4 + 2*(
a^4*b^3 + a^2*b^5)*c^2)*log(-b + sqrt(a^2*x^3 + b^2)) + 8*(a^6*b^4 + a^4*b^
6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)*sqrt(a^2*x^3 + b^2))/(a^6*
b^4 + a^4*b^6 + (a^2 + b^2)*c^4 + 2*(a^4*b^2 + a^2*b^4)*c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^3+b^2)^(1/2)*(a^2*x^6+c*x^3+2*b^2)/x/(a^2*x^6+b^2),x, algo
rithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueWarning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [abs(b)]=[-25,89]sym2poly/r2sym(co
nst gen & e,const index_m & i,const vecteur & l) Error: Bad Argument ValueW
arning, need to choose a branch for the root of a polynomial with parameter
s. This might be wrong.The choice was done assuming [abs(b)]=[11,12]sym2pol
y/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argum
ent ValueWarning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [abs(b)]=[-33
,-91]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Erro
```

```
r: Bad Argument ValueWarning, need to choose a branch for the root of a pol
ynomial with parameters. This might be wrong.The choice was done assuming [
abs(b)]=[-45,-7]sym2poly/r2sym(const gen & e,const index_m & i,const vecteu
r & l) Error: Bad Argument ValueWarning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming [abs(b)]=[-14,70]sym2poly/r2sym(const gen & e,const index_m & i,c
onst vecteur & l) Error: Bad Argument ValueWarning, need to choose a branch
 for the root of a polynomial with parameters. This might be wrong.The choi
ce was done assuming [abs(b)]=[84,29]sym2poly/r2sym(const gen & e,const ind
ex_m & i,const vecteur & l) Error: Bad Argument ValueWarning, need to choos
e a branch for the root of a polynomial with parameters. This might be wron
g.The choice was done assuming [abs(b)]=[-27,-21]sym2poly/r2sym(const gen &
 e,const index_m & i,const vecteur & l) Error: Bad Argument ValueWarning, n
eed to choose a branch for the root of a polynomial with parameters. This m
ight be wrong.The choice was done assuming [abs(b)]=[-43,4]sym2poly/r2sym(c
onst gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [abs(b)]=[-60,90]sym2p
oly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Arg
ument ValueWarning, need to choose a branch for the root of a polynomial wi
th parameters. This might be wrong.The choice was done assuming [abs(b)]=[-
10,65]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Err
or: Bad Argument ValueWarning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[abs(b)]=[-37,53]sym2poly/r2sym(const gen & e,const index_m & i,const vecte
ur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e,const index_m
 & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen &
 e,const index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r
2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument
 Valuesym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Err
or: Bad Argument ValueEvaluation time: 6.42Done
```

maple [C] time = 0.16, size = 703, normalized size = 3.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^3+b^2)^(1/2)*(a^2*x^6+c*x^3+2*b^2)/x/(a^2*x^6+b^2),x)
```

```
[Out] 2/3*(a^2*x^3+b^2)^(1/2)-4/3*b^2*arctanh((a^2*x^3+b^2)^(1/2)/(b^2)^(1/2))/(b
^2)^(1/2)-1/6*I/a^3/b^2*2^(1/2)*sum((-_alpha^3*a^2*b^2+_alpha^3*a^2*c+a^2*b
^2+b^2*c)/_alpha^3/(a^2+b^2)*(-a*b^2)^(1/3)*(1/2*I*a*(2*x+1/a*(-I*3^(1/2)*(
-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(a*(x-1/a*(-a*b^2)^(1/
3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*a*(2*x+1/a*
(I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(a^2*x^3+b
^2)^(1/2)*(2*a^2*(_alpha^5*a^2-_alpha^2*b^2)+I*a^3*(-a*b^2)^(1/3)*_alpha^4*
3^(1/2)-I*a^2*_alpha^3*3^(1/2)*(-a*b^2)^(2/3)-a^3*(-a*b^2)^(1/3)*_alpha^4-a
^2*_alpha^3*(-a*b^2)^(2/3)-I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b^2*a+I*(-a*b^2)
^(2/3)*3^(1/2)*b^2+(-a*b^2)^(1/3)*_alpha*b^2*a+(-a*b^2)^(2/3)*b^2)*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2/a*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/a*(-a*b^2)^(1/3))*3
^(1/2)*a/(-a*b^2)^(1/3))^(1/2),1/2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^5*a^3
-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha^4*a^2+I*3^(1/2)*_alpha^3*a^2*b^2-3*(-a*b^2
)^(2/3)*_alpha^4*a^2-2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*a*b^2+I*(-a*b^2)^(
2/3)*3^(1/2)*_alpha*b^2-3*_alpha^3*a^2*b^2-I*3^(1/2)*b^4+3*(-a*b^2)^(2/3)*_
alpha*b^2+3*b^4)/b^2/(a^2+b^2),(I*3^(1/2)/a*(-a*b^2)^(1/3)/(-3/2/a*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/a*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^6*a^2+b^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{2} x^{6} + c x^{3} + 2 \, b^{2}\right) \sqrt{a^{2} x^{3} + b^{2}}}{\left(a^{2} x^{6} + b^{2}\right) x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^3+b^2)^(1/2)*(a^2*x^6+c*x^3+2*b^2)/x/(a^2*x^6+b^2),x, algo
rithm="maxima")
```

```
[Out] integrate((a^2*x^6 + c*x^3 + 2*b^2)*sqrt(a^2*x^3 + b^2)/((a^2*x^6 + b^2)*x)
, x)
```

mupad [B] time = 13.68, size = 246, normalized size = 1.08

$$\frac{2\,\sqrt{a^2\,x^3+b^2}}{3}+\frac{2\,b\,\ln\left(\frac{\left(b+\sqrt{a^2\,x^3+b^2}\right)\,{\left(b-\sqrt{a^2\,x^3+b^2}\right)}^3}{x^6}\right)}{3}+\frac{\sqrt{\frac{1}{36}\mathrm{i}}\,\ln\left(\frac{2\,{\left(-1\right)}^{1/4}\,b^2+{\left(-1\right)}^{1/4}\,a^2\,x^3-2\,\sqrt{b}\,\sqrt{a^2\,x^3+b^2}\,\sqrt{a+b\,1\mathrm{i}}-{\left(-1\right)}^{3/4}\,a\,b}{a\,x^3+b\,1\mathrm{i}}\right)\,\sqrt{a+b\,1\mathrm{i}}\,\left(c+a\,b\,1\mathrm{i}\right)}{a\,\sqrt{b}}+\frac{\sqrt{\frac{1}{36}\mathrm{i}}\,\ln\left(\frac{2\,{\left(-1\right)}^{1/4}\,b^2+{\left(-1\right)}^{1/4}\,a^2\,x^3+{\left(-1\right)}^{3/4}\,a\,b+2\,\sqrt{b}\,\sqrt{a^2\,x^3+b^2}\,\sqrt{-a+b\,1\mathrm{i}}}{-a\,x^3+b\,1\mathrm{i}}\right)\,\sqrt{-a+b\,1\mathrm{i}}\,\left(c-a\,b\,1\mathrm{i}\right)}{a\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b^2 + a^2*x^3)^(1/2)*(c*x^3 + 2*b^2 + a^2*x^6))/(x*(b^2 + a^2*x^6)),x
)
```

```
[Out] (2*(b^2 + a^2*x^3)^(1/2))/3 + (2*b*log(((b + (b^2 + a^2*x^3)^(1/2))*(b - (b
^2 + a^2*x^3)^(1/2))^3)/x^6))/3 + ((1i/36)^(1/2)*log((2*(-1)^(1/4)*b^2 + (-
1)^(1/4)*a^2*x^3 - 2*b^(1/2)*(b^2 + a^2*x^3)^(1/2)*(a + b*1i)^(1/2) - (-1)^
(3/4)*a*b)/(b*1i + a*x^3))*(a + b*1i)^(1/2)*(c + a*b*1i))/(a*b^(1/2)) + ((1
i/36)^(1/2)*log((2*(-1)^(1/4)*b^2 + (-1)^(1/4)*a^2*x^3 + (-1)^(3/4)*a*b + 2
*b^(1/2)*(b^2 + a^2*x^3)^(1/2)*(b*1i - a)^(1/2))/(b*1i - a*x^3))*(b*1i - a)
^(1/2)*(c - a*b*1i))/(a*b^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**3+b**2)**(1/2)*(a**2*x**6+c*x**3+2*b**2)/x/(a**2*x**6+b*
*2),x)
```

```
[Out] Timed out
```

3.2095 $\int \frac{\sqrt{x+x^4}\left(b+ax^6\right)}{-d+cx^6} dx$

Optimal. Leaf size=227

$$\frac{\sqrt{-\left(\sqrt{d}\left(\sqrt{c}+\sqrt{d}\right)\right)}\,(ad+bc)\tan^{-1}\left(\frac{x\sqrt{x^4+x}\sqrt{-\sqrt{c}\sqrt{d}-d}}{\sqrt{d}\,(x+1)\left(x^2-x+1\right)}\right)}{3c^{3/2}d}-\frac{\sqrt{\sqrt{d}\left(\sqrt{c}-\sqrt{d}\right)}\,(ad+bc)\tan^{-1}\left(\frac{x\sqrt{x^4+x}\sqrt{\sqrt{c}\sqrt{d}-d}}{\sqrt{d}\,(x+1)\left(x^2-x+1\right)}\right)}{3c^{3/2}d}$$

Rubi [A] time = 0.74, antiderivative size = 250, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2056, 6715, 1693, 195, 215, 1175, 402, 377, 208, 205}

$$-\frac{\sqrt{x^4+x}\sqrt{\sqrt{c}-\sqrt{d}}\,(ad+bc)\tan^{-1}\left(\frac{x^{3/2}\sqrt{\sqrt{c}-\sqrt{d}}}{\sqrt[4]{d}\sqrt{x^3+1}}\right)}{3c^{3/2}d^{3/4}\sqrt{x^3+1}\sqrt{x}}-\frac{\sqrt{x^4+x}\sqrt{\sqrt{c}+\sqrt{d}}\,(ad+bc)\tanh^{-1}\left(\frac{x^{3/2}\sqrt{\sqrt{c}+\sqrt{d}}}{\sqrt[4]{d}\sqrt{x^3+1}}\right)}{3c^{3/2}d^{3/4}\sqrt{x^3+1}\sqrt{x}}+\frac{a\sqrt{x^4+x}\,x}{3c}+\frac{a\sqrt{x^4+x}\sinh^{-1}\left(x^{3/2}\right)}{3c\sqrt{x^3+1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x + x^4]*(b + a*x^6))/(-d + c*x^6),x]
```

```
[Out] (a*x*Sqrt[x + x^4])/(3*c) + (a*Sqrt[x + x^4]*ArcSinh[x^(3/2)])/(3*c*Sqrt[x]
*Sqrt[1 + x^3]) - (Sqrt[Sqrt[c] - Sqrt[d]]*(b*c + a*d)*Sqrt[x + x^4]*ArcTan
[(Sqrt[Sqrt[c] - Sqrt[d]]*x^(3/2))/(d^(1/4)*Sqrt[1 + x^3])])/(3*c^(3/2)*d^(
3/4)*Sqrt[x]*Sqrt[1 + x^3]) - (Sqrt[Sqrt[c] + Sqrt[d]]*(b*c + a*d)*Sqrt[x +
 x^4]*ArcTanh[(Sqrt[Sqrt[c] + Sqrt[d]]*x^(3/2))/(d^(1/4)*Sqrt[1 + x^3])])/(
3*c^(3/2)*d^(3/4)*Sqrt[x]*Sqrt[1 + x^3])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
```

```
- 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[-(a*c), 2]}, -Dist[c/(2*r), Int[(d + e*x^2)^q/(r - c*x^2), x], x] - Di
st[c/(2*r), Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e, q}
, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1693

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\sqrt{x+x^4}\left(b+ax^6\right)}{-d+cx^6}\,dx = \frac{\sqrt{x+x^4}\int \frac{\sqrt{x}\sqrt{1+x^3}\left(b+ax^6\right)}{-d+cx^6}dx}{\sqrt{x}\sqrt{1+x^3}}$$

$$= \frac{\left(2\sqrt{x+x^4}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}\left(b+ax^4\right)}{-d+cx^4}dx, x, x^{3/2}\right)}{3\sqrt{x}\sqrt{1+x^3}}$$

$$= \frac{\left(2\sqrt{x+x^4}\right)\operatorname{Subst}\left(\int \left(\frac{a\sqrt{1+x^2}}{c}+\frac{(bc+ad)\sqrt{1+x^2}}{c\left(-d+cx^4\right)}\right)dx, x, x^{3/2}\right)}{3\sqrt{x}\sqrt{1+x^3}}$$

$$= \frac{\left(2a\sqrt{x+x^4}\right)\operatorname{Subst}\left(\int \sqrt{1+x^2}\,dx, x, x^{3/2}\right)}{3c\sqrt{x}\sqrt{1+x^3}} + \frac{\left(2(bc+ad)\sqrt{x+x^4}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{-d+cx}\right.}{3c\sqrt{x}\sqrt{1+x^3}}$$

$$= \frac{ax\sqrt{x+x^4}}{3c} + \frac{\left(a\sqrt{x+x^4}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}dx, x, x^{3/2}\right)}{3c\sqrt{x}\sqrt{1+x^3}} - \frac{\left((bc+ad)\sqrt{x+x^4}\right)\operatorname{Subs}}{3\sqrt{c}\sqrt{d}\sqrt{}}$$

$$= \frac{ax\sqrt{x+x^4}}{3c} + \frac{a\sqrt{x+x^4}\sinh^{-1}\left(x^{3/2}\right)}{3c\sqrt{x}\sqrt{1+x^3}} + \frac{\left(\left(-1+\frac{\sqrt{d}}{\sqrt{c}}\right)(bc+ad)\sqrt{x+x^4}\right)\operatorname{Subst}\left(\int\right.}{3\sqrt{c}\sqrt{d}\sqrt{x}\sqrt{1+}}$$

$$= \frac{ax\sqrt{x+x^4}}{3c} + \frac{a\sqrt{x+x^4}\sinh^{-1}\left(x^{3/2}\right)}{3c\sqrt{x}\sqrt{1+x^3}} + \frac{\left(\left(-1+\frac{\sqrt{d}}{\sqrt{c}}\right)(bc+ad)\sqrt{x+x^4}\right)\operatorname{Subst}\left(\int\right.}{3\sqrt{c}\sqrt{d}\sqrt{x}\sqrt{1}}$$

$$= \frac{ax\sqrt{x+x^4}}{3c} + \frac{a\sqrt{x+x^4}\sinh^{-1}\left(x^{3/2}\right)}{3c\sqrt{x}\sqrt{1+x^3}} - \frac{\sqrt{\sqrt{c}-\sqrt{d}}\,(bc+ad)\sqrt{x+x^4}\tan^{-1}\left(\frac{\sqrt{\sqrt{c}}}{\sqrt[4]{d}}\right.}{3c^{3/2}d^{3/4}\sqrt{x}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.79, size = 441, normalized size = 1.94

$$\frac{\sqrt{x^4+x}\left(-\left(x^3+1\right)(ad+bc)\left(\sqrt[4]{d}\sinh^{-1}\left(x^{3/2}\right)+\sqrt{\sqrt{c}-\sqrt{d}}\tan^{-1}\left(\frac{-\sqrt[4]{d}x^{3/2}+i\sqrt[4]{c}}{\sqrt{x^3+1}\sqrt{\sqrt{c}-\sqrt{d}}}\right)\right)-\left(x^3+1\right)(ad+bc)\left(\sqrt[4]{d}\sinh^{-1}\left(x^{3/2}\right)-\sqrt{\sqrt{c}-\sqrt{d}}\tan^{-1}\left(\frac{\sqrt[4]{d}x^{3/2}+i\sqrt[4]{c}}{\sqrt{x^3+1}\sqrt{\sqrt{c}-\sqrt{d}}}\right)\right)+\left(x^3+1\right)(ad+bc)\left(\sqrt{\sqrt{c}+\sqrt{d}}\tanh^{-1}\left(\frac{\sqrt[4]{c}-\sqrt[4]{d}x^{3/2}}{\sqrt{x^3+1}\sqrt{\sqrt{c}+\sqrt{d}}}\right)+\sqrt[4]{d}\sinh^{-1}\left(x^{3/2}\right)\right)+\left(x^3+1\right)(ad+bc)\left(\sqrt[4]{d}\sinh^{-1}\left(x^{3/2}\right)-\sqrt{\sqrt{c}+\sqrt{d}}\tanh^{-1}\left(\frac{\sqrt[4]{c}+\sqrt[4]{d}x^{3/2}}{\sqrt{x^3+1}\sqrt{\sqrt{c}+\sqrt{d}}}\right)\right)+2a\sqrt{c}d^{3/4}\left(x^{3/2}\left(x^3+1\right)^{3/2}+\left(x^3+1\right)\sinh^{-1}\left(x^{3/2}\right)\right)\right)}{6c^{3/2}d^{3/4}\sqrt{x}\left(x^3+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x + x^4]*(b + a*x^6))/(-d + c*x^6),x]
```

```
[Out] (Sqrt[x + x^4]*(2*a*Sqrt[c]*d^(3/4)*(x^(3/2)*(1 + x^3)^(3/2) + (1 + x^3)*Ar
cSinh[x^(3/2)]) - (b*c + a*d)*(1 + x^3)*(d^(1/4)*ArcSinh[x^(3/2)] + Sqrt[Sq
rt[c] - Sqrt[d]]*ArcTan[(I*c^(1/4) - d^(1/4)*x^(3/2))/(Sqrt[Sqrt[c] - Sqrt[
d]]*Sqrt[1 + x^3])]) - (b*c + a*d)*(1 + x^3)*(d^(1/4)*ArcSinh[x^(3/2)] - Sq
rt[Sqrt[c] - Sqrt[d]]*ArcTan[(I*c^(1/4) + d^(1/4)*x^(3/2))/(Sqrt[Sqrt[c] -
Sqrt[d]]*Sqrt[1 + x^3])]) + (b*c + a*d)*(1 + x^3)*(d^(1/4)*ArcSinh[x^(3/2)]
 + Sqrt[Sqrt[c] + Sqrt[d]]*ArcTanh[(c^(1/4) - d^(1/4)*x^(3/2))/(Sqrt[Sqrt[c
] + Sqrt[d]]*Sqrt[1 + x^3])]) + (b*c + a*d)*(1 + x^3)*(d^(1/4)*ArcSinh[x^(3
/2)] - Sqrt[Sqrt[c] + Sqrt[d]]*ArcTanh[(c^(1/4) + d^(1/4)*x^(3/2))/(Sqrt[Sq
rt[c] + Sqrt[d]]*Sqrt[1 + x^3])])))/(6*c^(3/2)*d^(3/4)*Sqrt[x]*(1 + x^3)^(3
/2))
```

IntegrateAlgebraic [A] time = 5.87, size = 227, normalized size = 1.00

$$\frac{\sqrt{-\left(\sqrt{d}\left(\sqrt{c}+\sqrt{d}\right)\right)}(ad+bc)\tan^{-1}\left(\frac{x\sqrt{x^4+x}\sqrt{-\sqrt{c}\sqrt{d}-d}}{\sqrt{d}(x+1)\left(x^2-x+1\right)}\right)}{3c^{3/2}d} - \frac{\sqrt{\sqrt{d}\left(\sqrt{c}-\sqrt{d}\right)}(ad+bc)\tan^{-1}\left(\frac{x\sqrt{x^4+x}\sqrt{\sqrt{c}\sqrt{d}-d}}{\sqrt{d}(x+1)\left(x^2-x+1\right)}\right)}{3c^{3/2}d} + \frac{a\sqrt{x^4+x}\,x}{3c} + \frac{a\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)}{3c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x + x^4]*(b + a*x^6))/(-d + c*x^6),x]
```

```
[Out] (a*x*Sqrt[x + x^4])/(3*c) + (Sqrt[-((Sqrt[c] + Sqrt[d])*Sqrt[d])]*(b*c + a*
d)*ArcTan[(Sqrt[-(Sqrt[c]*Sqrt[d]) - d]*x*Sqrt[x + x^4])/(Sqrt[d]*(1 + x)*(
1 - x + x^2))]])/(3*c^(3/2)*d) - (Sqrt[(Sqrt[c] - Sqrt[d])*Sqrt[d]]*(b*c + a
*d)*ArcTan[(Sqrt[Sqrt[c]*Sqrt[d] - d]*x*Sqrt[x + x^4])/(Sqrt[d]*(1 + x)*(1
- x + x^2))]])/(3*c^(3/2)*d) + (a*ArcTanh[x^2/Sqrt[x + x^4]])/(3*c)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/2)*(a*x^6+b)/(c*x^6-d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/2)*(a*x^6+b)/(c*x^6-d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [c,d]=[19,-71]Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [c,d]=[-57,-3]Warning, need to choo
se a branch for the root of a polynomial with parameters. This might be wro
ng.The choice was done assuming [c,d]=[-66,-2]Warning, need to choose a bra
nch for the root of a polynomial with parameters. This might be wrong.The c
hoice was done assuming [c,d]=[22,71]Warning, need to choose a branch for t
he root of a polynomial with parameters. This might be wrong.The choice was
 done assuming [c,d]=[-36,13]-a/6/c*ln(abs(sqrt((1/x)^3+1)-1))+a/6/c*ln(sqr
t((1/x)^3+1)+1)+((2*c*d^3-4*c*d*sqrt(-d^2-d*sqrt(c*d))*sqrt(c*d)-5*d^2*sqrt
(-d^2-d*sqrt(c*d))*sqrt(c*d)-2*d^2*c*d)*a*c^2*abs(d)+(-2*c^2*d^4+4*c^2*d^2*
sqrt(-d^2-d*sqrt(c*d))*sqrt(c*d)+5*c*d^3*sqrt(-d^2-d*sqrt(c*d))*sqrt(c*d)+2
*c*d^3*c*d)*a*abs(d)+(2*c^2*d^2-4*c^2*sqrt(-d^2-d*sqrt(c*d))*sqrt(c*d)-5*c*
d*sqrt(-d^2-d*sqrt(c*d))*sqrt(c*d)-2*c*d*c*d)*b*c^2*abs(d)+(-2*c^3*d^3+4*c^
3*d*sqrt(-d^2-d*sqrt(c*d))*sqrt(c*d)+5*c^2*d^2*sqrt(-d^2-d*sqrt(c*d))*sqrt(
c*d)+2*c^2*d^2*c*d)*b*abs(d))/(12*c^4*d^3+3*c^3*d^4-15*c^2*d^5)/abs(c)*atan
(sqrt((1/x)^3+1)/sqrt(-(6*c*d+sqrt(6*c*d*6*c*d-12*c*d*(-3*c^2+3*c*d)))/2/3/
c/d))-((2*c*d^3-4*c*d*sqrt(-d^2+d*sqrt(c*d))*sqrt(c*d)-5*d^2*sqrt(-d^2+d*sq
rt(c*d))*sqrt(c*d)-2*d^2*c*d)*a*c^2*abs(d)+(-2*c^2*d^4+4*c^2*d^2*sqrt(-d^2+
d*sqrt(c*d))*sqrt(c*d)+5*c*d^3*sqrt(-d^2+d*sqrt(c*d))*sqrt(c*d)+2*c*d^3*c*d
)*a*abs(d)+(2*c^2*d^2-4*c^2*sqrt(-d^2+d*sqrt(c*d))*sqrt(c*d)-5*c*d*sqrt(-d^
2+d*sqrt(c*d))*sqrt(c*d)-2*c*d*c*d)*b*c^2*abs(d)+(-2*c^3*d^3+4*c^3*d*sqrt(-
d^2+d*sqrt(c*d))*sqrt(c*d)+5*c^2*d^2*sqrt(-d^2+d*sqrt(c*d))*sqrt(c*d)+2*c^2
*d^2*c*d)*b*abs(d))/(12*c^4*d^3+3*c^3*d^4-15*c^2*d^5)/abs(c)*atan(sqrt((1/x
)^3+1)/sqrt(-(6*c*d-sqrt(6*c*d*6*c*d-12*c*d*(-3*c^2+3*c*d)))/2/3/c/d))+8*a*
c*1/24/c^2*x*sqrt(x^4+x)
```

maple [C] time = 0.57, size = 686, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x)^(1/2)*(a*x^6+b)/(c*x^6-d),x)
```

```
[Out] a/c*(1/3*x*(x^4+x)^(1/2)-(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1
/2*I*3^(1/2))/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/
2))/(1+x))^(1/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(
3/2+1/2*I*3^(1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1
/2)*(-EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-
3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2))+EllipticPi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))
^(1/2),(1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-
1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))+1/3*(a*d
+b*c)/c*4^(1/2)*sum((-_alpha^3-1)/_alpha^4*(1+x)^2*(_alpha^5-_alpha^4+_alph
a^3-_alpha^2+_alpha-1)/(c-d)*(-1-I*3^(1/2))*(x/(1+x)*(I*3^(1/2)+3)/(1+I*3^(
1/2)))^(1/2)*(-1/(1+x)*(-1+2*x+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)*(-1/(1+x)*(-
1+2*x-I*3^(1/2))/(1+I*3^(1/2)))^(1/2)/(I*3^(1/2)+3)/(x*(1+x)*(-1+2*x+I*3^(1
/2))*(-1+2*x-I*3^(1/2)))^(1/2)*(EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I
*3^(1/2))/(1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2
*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+_alpha^5*c/d*EllipticPi(((3/2+1/2*
I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),1/6*(I*_alpha^5*3^(1/2)*c+3*_
alpha^5*c+I*3^(1/2)*d+3*d)/d,((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))),_alpha=RootOf(_Z^6*c-d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^6+b\right)\sqrt{x^4+x}}{cx^6-d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x)^(1/2)*(a*x^6+b)/(c*x^6-d),x, algorithm="maxima")
[Out] integrate((a*x^6 + b)*sqrt(x^4 + x)/(c*x^6 - d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a\,x^6+b\right)\,\sqrt{x^4+x}}{d-c\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b + a*x^6)*(x + x^4)^(1/2))/(d - c*x^6),x)
[Out] int(-((b + a*x^6)*(x + x^4)^(1/2))/(d - c*x^6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(x+1\right)\left(x^2-x+1\right)}\,\left(ax^6+b\right)}{cx^6-d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x)**(1/2)*(a*x**6+b)/(c*x**6-d),x)
[Out] Integral(sqrt(x*(x + 1)*(x**2 - x + 1))*(a*x**6 + b)/(c*x**6 - d), x)
```

3.2096 $\int \frac{x(-1+kx)\left(1-2kx+(-1+2k)x^2\right)}{((1-x)x(1-kx))^{2/3}\left(-1+4x+(-6+b)x^2+(4-2bk)x^3+\left(-1+bk^2\right)x^4\right)}\,dx$

Optimal. Leaf size=228

$$-\frac{\log\left(-\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+x^2-2x+1\right)}{2b^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}}{\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+2x^2-4x+2}\right)}{2b^{2/3}}+\frac{\log\left(b^{2/3}\left(kx^3+\right.\right.}{}$$

Rubi [F] time = 10.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(-1+kx)\left(1-2kx+(-1+2k)x^2\right)}{((1-x)x(1-kx))^{2/3}\left(-1+4x+(-6+b)x^2+(4-2bk)x^3+\left(-1+bk^2\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-1 + k*x)*(1 - 2*k*x + (-1 + 2*k)*x^2))/(((1 - x)*x*(1 - k*x))^(2/3
)*(-1 + 4*x + (-6 + b)*x^2 + (4 - 2*b*k)*x^3 + (-1 + b*k^2)*x^4)),x]
```

```
[Out] (-3*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst][Defer[Int][(x^3*(1 -
 x^3)^(1/3)*(1 - k*x^3)^(1/3))/(-1 + 4*x^3 - 6*(1 - b/6)*x^6 + 4*(1 - (b*k)
/2)*x^9 - (1 - b*k^2)*x^12), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(2/3) +
 (3*(1 - 2*k)*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst][Defer[Int]
[(x^6*(1 - x^3)^(1/3)*(1 - k*x^3)^(1/3))/(1 - 4*x^3 + 6*(1 - b/6)*x^6 - 4*(
1 - (b*k)/2)*x^9 + (1 - b*k^2)*x^12), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x)
)^(2/3)
```

Rubi steps

$$\int \frac{x(-1+kx)\left(1-2kx+(-1+2k)x^2\right)}{((1-x)x(1-kx))^{2/3}\left(-1+4x+(-6+b)x^2+(4-2bk)x^3+\left(-1+bk^2\right)x^4\right)}\,dx=\frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{}{(1}}{(}$$

$$=-\frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int}{((1}$$

$$=-\frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int}{((1-x}$$

$$=-\frac{\left(3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\text{S}}{}$$

$$=-\frac{\left(3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\text{S}}{}$$

$$=-\frac{\left(3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\text{S}}{}$$

Mathematica [F] time = 3.19, size = 0, normalized size = 0.00

$$\int \frac{x(-1+kx)\left(1-2kx+(-1+2k)x^2\right)}{((1-x)x(1-kx))^{2/3}\left(-1+4x+(-6+b)x^2+(4-2bk)x^3+\left(-1+bk^2\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(-1 + k*x)*(1 - 2*k*x + (-1 + 2*k)*x^2))/(((1 - x)*x*(1 - k*x)
)^(2/3)*(-1 + 4*x + (-6 + b)*x^2 + (4 - 2*b*k)*x^3 + (-1 + b*k^2)*x^4)),x]
```

```
[Out] Integrate[(x*(-1 + k*x)*(1 - 2*k*x + (-1 + 2*k)*x^2))/(((1 - x)*x*(1 - k*x)
)^(2/3)*(-1 + 4*x + (-6 + b)*x^2 + (4 - 2*b*k)*x^3 + (-1 + b*k^2)*x^4)), x]
```

IntegrateAlgebraic [A] time = 3.16, size = 228, normalized size = 1.00

$$-\frac{\log \left(-\sqrt[3]{b} \left(kx^3+(-k-1)x^2+x\right)^{2/3}+x^2-2x+1\right)}{2b^{2/3}}-\frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt{3} \sqrt[3]{b} \left(kx^3+(-k-1)x^2+x\right)^{2/3}}{\sqrt[3]{b} \left(kx^3+(-k-1)x^2+x\right)^{2/3}+2x^2-4x+2}\right)}{2b^{2/3}}+\frac{\log \left(b^{2/3} \left(kx^3+(-k-1)x^2+x\right)^{4/3}+\left(\sqrt[3]{b}\, x^2-2\sqrt[3]{b}\, x+\sqrt[3]{b}\right) \left(kx^3+(-k-1)x^2+x\right)^{2/3}+x^4-4x^3+6x^2-4x+1\right)}{4b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-1 + k*x)*(1 - 2*k*x + (-1 + 2*k)*x^2))/(((1 - x)*x*
(1 - k*x))^(2/3)*(-1 + 4*x + (-6 + b)*x^2 + (4 - 2*b*k)*x^3 + (-1 + b*k^2)*
x^4)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3))/(2
- 4*x + 2*x^2 + b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3))])/b^(2/3) - Log[1
 - 2*x + x^2 - b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(2/3)) + Log[
1 - 4*x + 6*x^2 - 4*x^3 + x^4 + (b^(1/3) - 2*b^(1/3)*x + b^(1/3)*x^2)*(x +
(-1 - k)*x^2 + k*x^3)^(2/3) + b^(2/3)*(x + (-1 - k)*x^2 + k*x^3)^(4/3)]/(4*
b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(k*x-1)*(1-2*k*x+(-1+2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-1+4*x
+(-6+b)*x^2+(-2*b*k+4)*x^3+(b*k^2-1)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((2\,k-1)x^2-2\,kx+1\right)(kx-1)x}{\left(\left(bk^2-1\right)x^4-2\,(bk-2)x^3+(b-6)x^2+4\,x-1\right)\left((kx-1)(x-1)x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(k*x-1)*(1-2*k*x+(-1+2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-1+4*x
+(-6+b)*x^2+(-2*b*k+4)*x^3+(b*k^2-1)*x^4),x, algorithm="giac")
```

```
[Out] integrate(((2*k - 1)*x^2 - 2*k*x + 1)*(k*x - 1)*x/(((b*k^2 - 1)*x^4 - 2*(b*
k - 2)*x^3 + (b - 6)*x^2 + 4*x - 1)*((k*x - 1)*(x - 1)*x)^(2/3)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x\,(kx-1)\left(1-2kx+(-1+2k)\,x^2\right)}{\left((1-x)\,x\,(-kx+1)\right)^{\frac{2}{3}}\left(-1+4x+(-6+b)\,x^2+(-2bk+4)\,x^3+\left(b\,k^2-1\right)x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(k*x-1)*(1-2*k*x+(-1+2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-1+4*x+(-6+b
)*x^2+(-2*b*k+4)*x^3+(b*k^2-1)*x^4),x)
```

```
[Out] int(x*(k*x-1)*(1-2*k*x+(-1+2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-1+4*x+(-6+b
)*x^2+(-2*b*k+4)*x^3+(b*k^2-1)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((2\,k-1)x^2-2\,kx+1\right)(kx-1)x}{\left((bk^2-1)x^4-2\,(bk-2)x^3+(b-6)x^2+4\,x-1\right)((kx-1)(x-1)x)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(k*x-1)*(1-2*k*x+(-1+2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(-1+4*x
+(-6+b)*x^2+(-2*b*k+4)*x^3+(b*k^2-1)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(((2*k - 1)*x^2 - 2*k*x + 1)*(k*x - 1)*x/(((b*k^2 - 1)*x^4 - 2*(b*
k - 2)*x^3 + (b - 6)*x^2 + 4*x - 1)*((k*x - 1)*(x - 1)*x)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(k\,x-1)\,\left((2\,k-1)\,x^2-2\,k\,x+1\right)}{(x\,(k\,x-1)\,(x-1))^{2/3}\,\left((b\,k^2-1)\,x^4+(4-2\,b\,k)\,x^3+(b-6)\,x^2+4\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(k*x - 1)*(x^2*(2*k - 1) - 2*k*x + 1))/((x*(k*x - 1)*(x - 1))^(2/3)*
(4*x + x^4*(b*k^2 - 1) - x^3*(2*b*k - 4) + x^2*(b - 6) - 1)),x)
```

```
[Out] int((x*(k*x - 1)*(x^2*(2*k - 1) - 2*k*x + 1))/((x*(k*x - 1)*(x - 1))^(2/3)*
(4*x + x^4*(b*k^2 - 1) - x^3*(2*b*k - 4) + x^2*(b - 6) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(k*x-1)*(1-2*k*x+(-1+2*k)*x**2)/((1-x)*x*(-k*x+1))**(2/3)/(-1+4
*x+(-6+b)*x**2+(-2*b*k+4)*x**3+(b*k**2-1)*x**4),x)
```

```
[Out] Timed out
```

3.2097 $\int \frac{-3aq+4bpx^3+apx^4}{\sqrt[3]{q+px^4}\left(b^3d+cq+3ab^2dx+3a^2bdx^2+a^3dx^3+cpx^4\right)}dx$

Optimal. Leaf size=228

$$\frac{\log\left(a^2d^{2/3}x^2+\sqrt[3]{px^4+q}\left(b\left(-\sqrt[3]{c}\right)\sqrt[3]{d}-a\sqrt[3]{c}\sqrt[3]{d}\,x\right)+2abd^{2/3}x+b^2d^{2/3}+c^{2/3}\left(px^4+q\right)^{2/3}\right)}{2c^{2/3}\sqrt[3]{d}}-\frac{\log\left(a\sqrt[3]{d}\,x+b\right.}{c}$$

Rubi [F] time = 4.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3aq+4bpx^3+apx^4}{\sqrt[3]{q+px^4}\left(b^3d+cq+3ab^2dx+3a^2bdx^2+a^3dx^3+cpx^4\right)}dx$$

Verification is not applicable to the result.

```
[In] Int[(-3*a*q + 4*b*p*x^3 + a*p*x^4)/((q + p*x^4)^(1/3)*(b^3*d + c*q + 3*a*b^
2*d*x + 3*a^2*b*d*x^2 + a^3*d*x^3 + c*p*x^4)),x]
```

```
[Out] (a*x*(1 + (p*x^4)/q)^(1/3)*Hypergeometric2F1[1/4, 1/3, 5/4, -((p*x^4)/q)])/
(c*(q + p*x^4)^(1/3)) - (a*(b^3*d + 4*c*q)*Defer[Int][1/((q + p*x^4)^(1/3)*
(b^3*d + c*q + 3*a*b^2*d*x + 3*a^2*b*d*x^2 + a^3*d*x^3 + c*p*x^4)), x])/c -
 (3*a^2*b^2*d*Defer[Int][x/((q + p*x^4)^(1/3)*(b^3*d + c*q + 3*a*b^2*d*x +
3*a^2*b*d*x^2 + a^3*d*x^3 + c*p*x^4)), x])/c - (3*a^3*b*d*Defer[Int][x^2/((
q + p*x^4)^(1/3)*(b^3*d + c*q + 3*a*b^2*d*x + 3*a^2*b*d*x^2 + a^3*d*x^3 + c
*p*x^4)), x])/c - ((a^4*d - 4*b*c*p)*Defer[Int][x^3/((q + p*x^4)^(1/3)*(b^3
*d + c*q + 3*a*b^2*d*x + 3*a^2*b*d*x^2 + a^3*d*x^3 + c*p*x^4)), x])/c
```

Rubi steps

$$\int \frac{-3aq+4bpx^3+apx^4}{\sqrt[3]{q+px^4}\left(b^3d+cq+3ab^2dx+3a^2bdx^2+a^3dx^3+cpx^4\right)}dx=\int\left(\frac{a}{c\sqrt[3]{q+px^4}}-\frac{a\left(b^3d+4cq\right)+3a^2b^2d}{c\sqrt[3]{q+px^4}\left(b^3d+cq+3a\right.}\right.$$

$$=-\frac{\int\frac{a\left(b^3d+4cq\right)+3a^2b^2dx+3a^3bdx^2+\left(a^4d-4bcp\right)x^3}{\sqrt[3]{q+px^4}\left(b^3d+cq+3ab^2dx+3a^2bdx^2+a^3dx^3+cpx^4\right)}d}{c}$$

$$=-\frac{\int\left(\frac{a\left(b^3d+4cq\right)}{\sqrt[3]{q+px^4}\left(b^3d+cq+3ab^2dx+3a^2bdx^2+a^3dx^3+cpx^4\right)}\right.}{}$$

$$=\frac{ax\sqrt[3]{1+\frac{px^4}{q}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{5}{4};-\frac{px^4}{q}\right)}{c\sqrt[3]{q+px^4}}-\frac{\left(3a^3bd\right)\int}{}$$

Mathematica [F] time = 1.74, size = 0, normalized size = 0.00

$$\int \frac{-3aq+4bpx^3+apx^4}{\sqrt[3]{q+px^4}\left(b^3d+cq+3ab^2dx+3a^2bdx^2+a^3dx^3+cpx^4\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3*a*q + 4*b*p*x^3 + a*p*x^4)/((q + p*x^4)^(1/3)*(b^3*d + c*q +
3*a*b^2*d*x + 3*a^2*b*d*x^2 + a^3*d*x^3 + c*p*x^4)),x]
```

```
[Out] Integrate[(-3*a*q + 4*b*p*x^3 + a*p*x^4)/((q + p*x^4)^(1/3)*(b^3*d + c*q +
3*a*b^2*d*x + 3*a^2*b*d*x^2 + a^3*d*x^3 + c*p*x^4)), x]
```

IntegrateAlgebraic [A] time = 21.05, size = 228, normalized size = 1.00

$$\frac{\log\left(a^2d^{2/3}x^2+\sqrt[3]{px^4+q}\left(b\left(-\sqrt[3]{c}\right)\sqrt[3]{d}-a\sqrt[3]{c}\sqrt[3]{d}x\right)+2abd^{2/3}x+b^2d^{2/3}+c^{2/3}\left(px^4+q\right)^{2/3}\right)}{2c^{2/3}\sqrt[3]{d}}-\frac{\log\left(a\sqrt[3]{d}x+b\sqrt[3]{d}+\sqrt[3]{c}\sqrt[3]{px^4+q}\right)}{c^{2/3}\sqrt[3]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{c}\sqrt[3]{px^4+q}}{-2a\sqrt[3]{d}x-2b\sqrt[3]{d}+\sqrt[3]{c}\sqrt[3]{px^4+q}}\right)}{c^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*a*q + 4*b*p*x^3 + a*p*x^4)/((q + p*x^4)^(1/3)*(b^3*d
 + c*q + 3*a*b^2*d*x + 3*a^2*b*d*x^2 + a^3*d*x^3 + c*p*x^4)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*c^(1/3)*(q + p*x^4)^(1/3))/(-2*b*d^(1/3) - 2*a*d
^(1/3)*x + c^(1/3)*(q + p*x^4)^(1/3))]])/(c^(2/3)*d^(1/3))) - Log[b*d^(1/3)
+ a*d^(1/3)*x + c^(1/3)*(q + p*x^4)^(1/3)]/(c^(2/3)*d^(1/3)) + Log[b^2*d^(2
/3) + 2*a*b*d^(2/3)*x + a^2*d^(2/3)*x^2 + (-(b*c^(1/3)*d^(1/3)) - a*c^(1/3)
*d^(1/3)*x)*(q + p*x^4)^(1/3) + c^(2/3)*(q + p*x^4)^(2/3)]/(2*c^(2/3)*d^(1/
3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*p*x^4+4*b*p*x^3-3*a*q)/(p*x^4+q)^(1/3)/(a^3*d*x^3+3*a^2*b*d*x^
2+c*p*x^4+3*a*b^2*d*x+b^3*d+c*q),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*p*x^4+4*b*p*x^3-3*a*q)/(p*x^4+q)^(1/3)/(a^3*d*x^3+3*a^2*b*d*x^
2+c*p*x^4+3*a*b^2*d*x+b^3*d+c*q),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{pa\,x^{4}+4bp\,x^{3}-3aq}{\left(p\,x^{4}+q\right)^{\frac{1}{3}}\left(a^{3}d\,x^{3}+3a^{2}bd\,x^{2}+cp\,x^{4}+3a\,b^{2}dx+b^{3}d+cq\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*p*x^4+4*b*p*x^3-3*a*q)/(p*x^4+q)^(1/3)/(a^3*d*x^3+3*a^2*b*d*x^2+c*p*
x^4+3*a*b^2*d*x+b^3*d+c*q),x)
```

```
[Out] int((a*p*x^4+4*b*p*x^3-3*a*q)/(p*x^4+q)^(1/3)/(a^3*d*x^3+3*a^2*b*d*x^2+c*p*
x^4+3*a*b^2*d*x+b^3*d+c*q),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{apx^{4}+4\,bpx^{3}-3\,aq}{\left(a^{3}dx^{3}+3\,a^{2}bdx^{2}+cpx^{4}+3\,ab^{2}dx+b^{3}d+cq\right)\left(px^{4}+q\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*p*x^4+4*b*p*x^3-3*a*q)/(p*x^4+q)^(1/3)/(a^3*d*x^3+3*a^2*b*d*x^
2+c*p*x^4+3*a*b^2*d*x+b^3*d+c*q),x, algorithm="maxima")
```

```
[Out] integrate((a*p*x^4 + 4*b*p*x^3 - 3*a*q)/((a^3*d*x^3 + 3*a^2*b*d*x^2 + c*p*x
^4 + 3*a*b^2*d*x + b^3*d + c*q)*(p*x^4 + q)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,p\,x^4+4\,b\,p\,x^3-3\,a\,q}{{\left(p\,x^4+q\right)}^{1/3}\,\left(d\,a^3\,x^3+3\,d\,a^2\,b\,x^2+3\,d\,a\,b^2\,x+d\,b^3+c\,p\,x^4+c\,q\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*p*x^4 - 3*a*q + 4*b*p*x^3)/((q + p*x^4)^(1/3)*(c*q + b^3*d + c*p*x^4
 + a^3*d*x^3 + 3*a*b^2*d*x + 3*a^2*b*d*x^2)),x)
```

```
[Out] int((a*p*x^4 - 3*a*q + 4*b*p*x^3)/((q + p*x^4)^(1/3)*(c*q + b^3*d + c*p*x^4
 + a^3*d*x^3 + 3*a*b^2*d*x + 3*a^2*b*d*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*p*x**4+4*b*p*x**3-3*a*q)/(p*x**4+q)**(1/3)/(a**3*d*x**3+3*a**2
*b*d*x**2+c*p*x**4+3*a*b**2*d*x+b**3*d+c*q),x)
```

```
[Out] Timed out
```

3.2098 $\int \frac{3+(-1+2k^2)x-3k^2x^2-k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(1-d-(1+2d)x-(d+k^2)x^2+k^2x^3)}\,dx$

Optimal. Leaf size=229

$$\frac{\log\left(d^{2/3}x^2+2d^{2/3}x+d^{2/3}+\left(\sqrt[3]{d}\,x+\sqrt[3]{d}\right)\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}\right)}{2\sqrt[3]{d}}-\frac{\log\left(-\sqrt[3]{d}\right.}{}$$

Rubi [F] time = 5.92, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3+\left(-1+2k^2\right)x-3k^2x^2-k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(1-d-(1+2d)x-\left(d+k^2\right)x^2+k^2x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3 + (-1 + 2*k^2)*x - 3*k^2*x^2 - k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(
1/3)*(1 - d - (1 + 2*d)*x - (d + k^2)*x^2 + k^2*x^3)),x]
```

```
[Out] -((x*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*AppellF1[1/2, 1/3, 1/3, 3/2, x^2,
k^2*x^2])/((1 - x^2)*(1 - k^2*x^2))^(1/3)) - ((4 - d)*(1 - x^2)^(1/3)*(1 -
k^2*x^2)^(1/3)*Defer[Int][1/((1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*(-1 + d +
(1 + 2*d)*x + (d + k^2)*x^2 - k^2*x^3)), x])/((1 - x^2)*(1 - k^2*x^2))^(1/3
) - (2*(1 + d - k^2)*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*Defer[Int][x/((1 -
 x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*(1 - d - (1 + 2*d)*x - (d + k^2)*x^2 + k^2*
x^3)), x])/((1 - x^2)*(1 - k^2*x^2))^(1/3) - ((d + 4*k^2)*(1 - x^2)^(1/3)*(
1 - k^2*x^2)^(1/3)*Defer[Int][x^2/((1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*(1 -
 d - (1 + 2*d)*x - (d + k^2)*x^2 + k^2*x^3)), x])/((1 - x^2)*(1 - k^2*x^2))
^(1/3)
```

Rubi steps

$$\int \frac{3+\left(-1+2k^2\right)x-3k^2x^2-k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(1-d-(1+2d)x-\left(d+k^2\right)x^2+k^2x^3\right)}\,dx = \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{3+\left(-1+2\right.}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,(1-}}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2\right.}}$$

$$= \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{3-(1-2}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,(1-}}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2\right.}}$$

$$= \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \left(-\frac{1}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}}\right.}{\sqrt[3]{(1-}}$$

$$= -\frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{1}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}}}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= -\frac{x\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,\right.}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= -\frac{x\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,\right.}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

Mathematica [F] time = 3.74, size = 0, normalized size = 0.00

$$\int \frac{3+\left(-1+2k^2\right)x-3k^2x^2-k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(1-d-(1+2d)x-\left(d+k^2\right)x^2+k^2x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3 + (-1 + 2*k^2)*x - 3*k^2*x^2 - k^2*x^3)/(((1 - x^2)*(1 - k^2*x
^2))^(1/3)*(1 - d - (1 + 2*d)*x - (d + k^2)*x^2 + k^2*x^3)),x]
```

```
[Out] Integrate[(3 + (-1 + 2*k^2)*x - 3*k^2*x^2 - k^2*x^3)/(((1 - x^2)*(1 - k^2*x
^2))^(1/3)*(1 - d - (1 + 2*d)*x - (d + k^2)*x^2 + k^2*x^3)), x]
```

IntegrateAlgebraic [A] time = 5.71, size = 229, normalized size = 1.00

$$\frac{\log\left(d^{2/3}x^2+2d^{2/3}x+d^{2/3}+\left(\sqrt[3]{d}x+\sqrt[3]{d}\right)\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}\right)}{2\sqrt[3]{d}}-\frac{\log\left(-\sqrt[3]{d}x-\sqrt[3]{d}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\right)}{\sqrt[3]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}{2\sqrt[3]{d}x+2\sqrt[3]{d}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + (-1 + 2*k^2)*x - 3*k^2*x^2 - k^2*x^3)/(((1 - x^2)*(
1 - k^2*x^2))^(1/3)*(1 - d - (1 + 2*d)*x - (d + k^2)*x^2 + k^2*x^3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))/(2*d^(1/3)
 + 2*d^(1/3)*x + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))])/d^(1/3)) - Log[-d^
(1/3) - d^(1/3)*x + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3)]/d^(1/3) + Log[d^(
2/3) + 2*d^(2/3)*x + d^(2/3)*x^2 + (d^(1/3) + d^(1/3)*x)*(1 + (-1 - k^2)*x^
2 + k^2*x^4)^(1/3) + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+(2*k^2-1)*x-3*k^2*x^2-k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(
1-d-(1+2*d)*x-(k^2+d)*x^2+k^2*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{k^2x^3 + 3\,k^2x^2 - \left(2\,k^2 - 1\right)x - 3}{\left(k^2x^3 - \left(k^2 + d\right)x^2 - (2\,d + 1)x - d + 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+(2*k^2-1)*x-3*k^2*x^2-k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(
1-d-(1+2*d)*x-(k^2+d)*x^2+k^2*x^3),x, algorithm="giac")
```

```
[Out] integrate(-(k^2*x^3 + 3*k^2*x^2 - (2*k^2 - 1)*x - 3)/((k^2*x^3 - (k^2 + d)*
x^2 - (2*d + 1)*x - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{3 + \left(2k^2 - 1\right)x - 3k^2x^2 - k^2x^3}{\left(\left(-x^2 + 1\right)\left(-k^2x^2 + 1\right)\right)^{\frac{1}{3}}\left(1 - d - (1 + 2d)\,x - \left(k^2 + d\right)x^2 + k^2x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+(2*k^2-1)*x-3*k^2*x^2-k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d-(1
+2*d)*x-(k^2+d)*x^2+k^2*x^3),x)
```

```
[Out] int((3+(2*k^2-1)*x-3*k^2*x^2-k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d-(1
+2*d)*x-(k^2+d)*x^2+k^2*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{k^2x^3 + 3\,k^2x^2 - \left(2\,k^2 - 1\right)x - 3}{\left(k^2x^3 - \left(k^2 + d\right)x^2 - (2\,d + 1)x - d + 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+(2*k^2-1)*x-3*k^2*x^2-k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(
1-d-(1+2*d)*x-(k^2+d)*x^2+k^2*x^3),x, algorithm="maxima")
```

```
[Out] -integrate((k^2*x^3 + 3*k^2*x^2 - (2*k^2 - 1)*x - 3)/((k^2*x^3 - (k^2 + d)*
x^2 - (2*d + 1)*x - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int -\frac{3\,k^2\,x^2 + k^2\,x^3 - x\,\left(2\,k^2 - 1\right) - 3}{\left(\left(x^2 - 1\right)\,\left(k^2\,x^2 - 1\right)\right)^{1/3}\,\left(d - k^2\,x^3 + x^2\,\left(k^2 + d\right) + x\,\left(2\,d + 1\right) - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*k^2*x^2 + k^2*x^3 - x*(2*k^2 - 1) - 3)/(((x^2 - 1)*(k^2*x^2 - 1))^(1
/3)*(d - k^2*x^3 + x^2*(d + k^2) + x*(2*d + 1) - 1)),x)
```

```
[Out] -int(-(3*k^2*x^2 + k^2*x^3 - x*(2*k^2 - 1) - 3)/(((x^2 - 1)*(k^2*x^2 - 1))^
(1/3)*(d - k^2*x^3 + x^2*(d + k^2) + x*(2*d + 1) - 1)), x)
```

sympy [**F**(**-1**)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+(2*k**2-1)*x-3*k**2*x**2-k**2*x**3)/((-x**2+1)*(-k**2*x**2+1))
**(1/3)/(1-d-(1+2*d)*x-(k**2+d)*x**2+k**2*x**3),x)
```

```
[Out] Timed out
```

3.2099 $\int \frac{3+(1-2k^2)x-3k^2x^2+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(-1+d-(1+2d)x+(d+k^2)x^2+k^2x^3)}\,dx$

Optimal. Leaf size=229

$$\frac{\log\left(d^{2/3}x^2-2d^{2/3}x+d^{2/3}+\left(\sqrt[3]{d}-\sqrt[3]{d}\,x\right)\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}\right)}{2\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{d}\,x\right.}{}$$

Rubi [F] time = 5.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3+\left(1-2k^2\right)x-3k^2x^2+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d-(1+2d)x+\left(d+k^2\right)x^2+k^2x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3 + (1 - 2*k^2)*x - 3*k^2*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1
/3)*(-1 + d - (1 + 2*d)*x + (d + k^2)*x^2 + k^2*x^3)),x]
```

```
[Out] (x*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*AppellF1[1/2, 1/3, 1/3, 3/2, x^2, k^
2*x^2])/((1 - x^2)*(1 - k^2*x^2))^(1/3) - ((4 - d)*(1 - x^2)^(1/3)*(1 - k^2
*x^2)^(1/3)*Defer[Int][1/((1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*(1 - d + (1 +
 2*d)*x - (d + k^2)*x^2 - k^2*x^3)), x])/((1 - x^2)*(1 - k^2*x^2))^(1/3) -
(2*(1 + d - k^2)*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*Defer[Int][x/((1 - x^2
)^(1/3)*(1 - k^2*x^2)^(1/3)*(1 - d + (1 + 2*d)*x - (d + k^2)*x^2 - k^2*x^3)
), x])/((1 - x^2)*(1 - k^2*x^2))^(1/3) + ((d + 4*k^2)*(1 - x^2)^(1/3)*(1 -
k^2*x^2)^(1/3)*Defer[Int][x^2/((1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*(1 - d +
 (1 + 2*d)*x - (d + k^2)*x^2 - k^2*x^3)), x])/((1 - x^2)*(1 - k^2*x^2))^(1/
3)
```

Rubi steps

$$\begin{aligned}
\int \frac{3+\left(1-2k^2\right)x-3k^2x^2+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d-(1+2d)x+\left(d+k^2\right)x^2+k^2x^3\right)}\,dx &= \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{3+(1-2k}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,(-1+}}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2\right.}}\\
&= \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \left(\frac{1}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}}+\right.}{\sqrt[3]{\left(1-x\right.}}\\
&= \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{1}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}}\,dx}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}\\
&= \frac{x\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,k^2x\right.}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}\\
&= \frac{x\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,k^2x\right.}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}
\end{aligned}$$

Mathematica [F] time = 3.72, size = 0, normalized size = 0.00

$$\int \frac{3+\left(1-2k^2\right)x-3k^2x^2+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d-(1+2d)x+\left(d+k^2\right)x^2+k^2x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3 + (1 - 2*k^2)*x - 3*k^2*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^
2))^(1/3)*(-1 + d - (1 + 2*d)*x + (d + k^2)*x^2 + k^2*x^3)),x]
```

```
[Out] Integrate[(3 + (1 - 2*k^2)*x - 3*k^2*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^
2))^(1/3)*(-1 + d - (1 + 2*d)*x + (d + k^2)*x^2 + k^2*x^3)), x]
```

IntegrateAlgebraic [A] time = 5.69, size = 229, normalized size = 1.00

$$\frac{\log\left(d^{2/3}x^2-2d^{2/3}x+d^{2/3}+\left(\sqrt[3]{d}-\sqrt[3]{d}x\right)\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}\right)}{2\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{d}x-\sqrt[3]{d}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\right)}{\sqrt[3]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}{-2\sqrt[3]{d}x+2\sqrt[3]{d}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 + (1 - 2*k^2)*x - 3*k^2*x^2 + k^2*x^3)/(((1 - x^2)*(1
 - k^2*x^2))^(1/3)*(-1 + d - (1 + 2*d)*x + (d + k^2)*x^2 + k^2*x^3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))/(2*d^(1/3)
 - 2*d^(1/3)*x + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))])/d^(1/3)) - Log[-d^
(1/3) + d^(1/3)*x + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3)]/d^(1/3) + Log[d^(
2/3) - 2*d^(2/3)*x + d^(2/3)*x^2 + (d^(1/3) - d^(1/3)*x)*(1 + (-1 - k^2)*x^
2 + k^2*x^4)^(1/3) + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+(-2*k^2+1)*x-3*k^2*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/
(-1+d-(1+2*d)*x+(k^2+d)*x^2+k^2*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3-3\,k^2x^2-\left(2\,k^2-1\right)x+3}{\left(k^2x^3+\left(k^2+d\right)x^2-(2\,d+1)x+d-1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+(-2*k^2+1)*x-3*k^2*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/
(-1+d-(1+2*d)*x+(k^2+d)*x^2+k^2*x^3),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^3 - 3*k^2*x^2 - (2*k^2 - 1)*x + 3)/((k^2*x^3 + (k^2 + d)*x
^2 - (2*d + 1)*x + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{3+\left(-2k^2+1\right)x-3k^2x^2+k^2x^3}{\left(\left(-x^2+1\right)\left(-k^2x^2+1\right)\right)^{\frac{1}{3}}\left(-1+d-(1+2d)\,x+\left(k^2+d\right)x^2+k^2x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+(-2*k^2+1)*x-3*k^2*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d-
(1+2*d)*x+(k^2+d)*x^2+k^2*x^3),x)
```

```
[Out] int((3+(-2*k^2+1)*x-3*k^2*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d-
(1+2*d)*x+(k^2+d)*x^2+k^2*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3 - 3\,k^2x^2 - \left(2\,k^2 - 1\right)x + 3}{\left(k^2x^3 + \left(k^2 + d\right)x^2 - (2\,d + 1)x + d - 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+(-2*k^2+1)*x-3*k^2*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/
(-1+d-(1+2*d)*x+(k^2+d)*x^2+k^2*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^3 - 3*k^2*x^2 - (2*k^2 - 1)*x + 3)/((k^2*x^3 + (k^2 + d)*x
^2 - (2*d + 1)*x + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{3\,k^2\,x^2 - k^2\,x^3 + x\,\left(2\,k^2 - 1\right) - 3}{\left(\left(x^2 - 1\right)\,\left(k^2\,x^2 - 1\right)\right)^{1/3}\,\left(d + k^2\,x^3 + x^2\,\left(k^2 + d\right) - x\,\left(2\,d + 1\right) - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*k^2*x^2 - k^2*x^3 + x*(2*k^2 - 1) - 3)/(((x^2 - 1)*(k^2*x^2 - 1))^(
1/3)*(d + k^2*x^3 + x^2*(d + k^2) - x*(2*d + 1) - 1)),x)
```

```
[Out] -int((3*k^2*x^2 - k^2*x^3 + x*(2*k^2 - 1) - 3)/(((x^2 - 1)*(k^2*x^2 - 1))^(
1/3)*(d + k^2*x^3 + x^2*(d + k^2) - x*(2*d + 1) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+(-2*k**2+1)*x-3*k**2*x**2+k**2*x**3)/((-x**2+1)*(-k**2*x**2+1)
)**(1/3)/(-1+d-(1+2*d)*x+(k**2+d)*x**2+k**2*x**3),x)
```

```
[Out] Timed out
```

3.2100 $\int \frac{\sqrt{-b^4+a^4x^4}\,(b^4+a^4x^4)}{b^8-cx^4+a^8x^8}\,dx$

Optimal. Leaf size=229

$$-\frac{\tan^{-1}\left(\frac{-\frac{a^4x^4}{\sqrt{2}\sqrt[4]{2a^4b^4-c}}+\frac{x^2\sqrt[4]{2a^4b^4-c}}{\sqrt{2}}+\frac{b^4}{\sqrt{2}\sqrt[4]{2a^4b^4-c}}}{x\sqrt{a^4x^4-b^4}}\right)}{2\sqrt{2}\sqrt[4]{2a^4b^4-c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{2a^4b^4-c}\sqrt{a^4x^4-b^4}}{x^2\sqrt{2a^4b^4-c}+a^4x^4-b^4}\right)}{2\sqrt{2}\sqrt[4]{2a^4b^4-c}}$$

Rubi [C] time = 1.64, antiderivative size = 508, normalized size of antiderivative = 2.22, number of steps used = 18, number of rules used = 7, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {6728, 406, 224, 221, 409, 1219, 1218}

$$\frac{b\left(1-\frac{2a^4b^4+c}{\sqrt{c^2-4a^8b^8}}\right)\sqrt{1-\frac{a^4x^4}{b^4}}F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}+\frac{b\left(\frac{2a^4b^4+c}{\sqrt{c^2-4a^8b^8}}+1\right)\sqrt{1-\frac{a^4x^4}{b^4}}F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(-\frac{\sqrt{2}a^2b^2}{\sqrt{c-\sqrt{c^2-4a^8b^8}}};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(\frac{\sqrt{2}a^2b^2}{\sqrt{c-\sqrt{c^2-4a^8b^8}}};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(-\frac{\sqrt{2}a^2b^2}{\sqrt{c+\sqrt{c^2-4a^8b^8}}};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(\frac{\sqrt{2}a^2b^2}{\sqrt{c+\sqrt{c^2-4a^8b^8}}};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-b^4 + a^4*x^4]*(b^4 + a^4*x^4))/(b^8 - c*x^4 + a^8*x^8),x]
```

```
[Out] (b*(1 - (2*a^4*b^4 + c)/Sqrt[-4*a^8*b^8 + c^2])*Sqrt[1 - (a^4*x^4)/b^4]*Ell
ipticF[ArcSin[(a*x)/b], -1])/(2*a*Sqrt[-b^4 + a^4*x^4]) + (b*(1 + (2*a^4*b^
4 + c)/Sqrt[-4*a^8*b^8 + c^2])*Sqrt[1 - (a^4*x^4)/b^4]*EllipticF[ArcSin[(a*
x)/b], -1])/(2*a*Sqrt[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/b^4]*Ellipti
cPi[-((Sqrt[2]*a^2*b^2)/Sqrt[c - Sqrt[-4*a^8*b^8 + c^2]]), ArcSin[(a*x)/b],
 -1])/(2*a*Sqrt[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[(S
qrt[2]*a^2*b^2)/Sqrt[c - Sqrt[-4*a^8*b^8 + c^2]], ArcSin[(a*x)/b], -1])/(2*
a*Sqrt[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[-((Sqrt[2]*
a^2*b^2)/Sqrt[c + Sqrt[-4*a^8*b^8 + c^2]]), ArcSin[(a*x)/b], -1])/(2*a*Sqrt
[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[(Sqrt[2]*a^2*b^2)
/Sqrt[c + Sqrt[-4*a^8*b^8 + c^2]], ArcSin[(a*x)/b], -1])/(2*a*Sqrt[-b^4 + a
^4*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{-b^4+a^4x^4}\,\left(b^4+a^4x^4\right)}{b^8-cx^4+a^8x^8}\,dx = \int \left(\frac{\left(a^4+\frac{a^4\left(2a^4b^4+c\right)}{\sqrt{-4a^8b^8+c^2}}\right)\sqrt{-b^4+a^4x^4}}{-c-\sqrt{-4a^8b^8+c^2}+2a^8x^4}+\frac{\left(a^4-\frac{a^4\left(2a^4b^4+c\right)}{\sqrt{-4a^8b^8+c^2}}\right)\sqrt{-b^4+a^4x^4}}{-c+\sqrt{-4a^8b^8+c^2}+2a^8x^4}\right)dx$$

$$= \left(a^4\left(1-\frac{2a^4b^4+c}{\sqrt{-4a^8b^8+c^2}}\right)\right)\int \frac{\sqrt{-b^4+a^4x^4}}{-c+\sqrt{-4a^8b^8+c^2}+2a^8x^4}\,dx+\left(a^4\left(1+\frac{2a^4b^4+c}{\sqrt{-4a^8}}\right.\right.$$

$$= \frac{1}{2}\left(1-\frac{2a^4b^4+c}{\sqrt{-4a^8b^8+c^2}}\right)\int \frac{1}{\sqrt{-b^4+a^4x^4}}\,dx+\frac{1}{2}\left(1+\frac{2a^4b^4+c}{\sqrt{-4a^8b^8+c^2}}\right)\int \frac{}{\sqrt{-b^4}}$$

$$= -\left(\frac{1}{2}\int \frac{1}{\left(1-\frac{\sqrt{2}\,a^4x^2}{\sqrt{c-\sqrt{-4a^8b^8+c^2}}}\right)\sqrt{-b^4+a^4x^4}}\,dx\right)-\frac{1}{2}\int \frac{1}{\left(1+\frac{\sqrt{2}\,a^4x^2}{\sqrt{c-\sqrt{-4a^8b^8+c^2}}}\right)\sqrt{-}}$$

$$= \frac{b\left(1-\frac{2a^4b^4+c}{\sqrt{-4a^8b^8+c^2}}\right)\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{-b^4+a^4x^4}}+\frac{b\left(1+\frac{2a^4b^4+c}{\sqrt{-4a^8b^8+c^2}}\right)\sqrt{1-\frac{a^4x^4}{b^4}}}{2a\sqrt{-b^4+a}}$$

$$= \frac{b\left(1-\frac{2a^4b^4+c}{\sqrt{-4a^8b^8+c^2}}\right)\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{-b^4+a^4x^4}}+\frac{b\left(1+\frac{2a^4b^4+c}{\sqrt{-4a^8b^8+c^2}}\right)\sqrt{1-\frac{a^4x^4}{b^4}}}{2a\sqrt{-b^4+a}}$$

Mathematica [C] time = 1.64, size = 326, normalized size = 1.42

$$-\frac{i\sqrt{1-\frac{a^4x^4}{b^4}}\left(2F\left(i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-\Pi\left(-\frac{\sqrt{2}b^2}{a^2\sqrt{\frac{c-\sqrt{c^2-4a^8b^8}}{a^8}}};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-\Pi\left(\frac{\sqrt{2}b^2}{a^2\sqrt{\frac{c-\sqrt{c^2-4a^8b^8}}{a^8}}};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-\Pi\left(-\frac{\sqrt{2}b^2}{a^2\sqrt{\frac{c+\sqrt{c^2-4a^8b^8}}{a^8}}};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-\Pi\left(\frac{\sqrt{2}b^2}{a^2\sqrt{\frac{c+\sqrt{c^2-4a^8b^8}}{a^8}}};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)\right)}{2\sqrt{-\frac{a^2}{b^2}}\sqrt{a^4x^4-b^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-b^4 + a^4*x^4]*(b^4 + a^4*x^4))/(b^8 - c*x^4 + a^8*x^8),x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (a^4*x^4)/b^4]*(2*EllipticF[I*ArcSinh[Sqrt[-(a^2/b^2)]*x
], -1] - EllipticPi[-((Sqrt[2]*b^2)/(a^2*Sqrt[(c - Sqrt[-4*a^8*b^8 + c^2])/
a^8])), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] - EllipticPi[(Sqrt[2]*b^2)/(a^2*
Sqrt[(c - Sqrt[-4*a^8*b^8 + c^2])/a^8]), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1]
```

```
 - EllipticPi[-((Sqrt[2]*b^2)/(a^2*Sqrt[(c + Sqrt[-4*a^8*b^8 + c^2])/a^8]))
, I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] - EllipticPi[(Sqrt[2]*b^2)/(a^2*Sqrt[(
c + Sqrt[-4*a^8*b^8 + c^2])/a^8]), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1]))/(Sq
rt[-(a^2/b^2)]*Sqrt[-b^4 + a^4*x^4])
```

IntegrateAlgebraic [A] time = 1.22, size = 228, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{2a^4b^4-c}\,\sqrt{a^4x^4-b^4}}{x^2\sqrt{2a^4b^4-c}-a^4x^4+b^4}\right)}{2\sqrt{2}\sqrt[4]{2a^4b^4-c}}-\frac{\tanh^{-1}\left(\frac{\frac{a^4x^4}{\sqrt{2}\sqrt[4]{2a^4b^4-c}}+\frac{x^2\sqrt[4]{2a^4b^4-c}}{\sqrt{2}}-\frac{b^4}{\sqrt{2}\sqrt[4]{2a^4b^4-c}}}{x\sqrt{a^4x^4-b^4}}\right)}{2\sqrt{2}\sqrt[4]{2a^4b^4-c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-b^4 + a^4*x^4]*(b^4 + a^4*x^4))/(b^8 - c*x^4 + a^
8*x^8),x]
```

```
[Out] ArcTan[(Sqrt[2]*(2*a^4*b^4 - c)^(1/4)*x*Sqrt[-b^4 + a^4*x^4])/(b^4 + Sqrt[2
*a^4*b^4 - c]*x^2 - a^4*x^4)]/(2*Sqrt[2]*(2*a^4*b^4 - c)^(1/4)) - ArcTanh[(
-(b^4/(Sqrt[2]*(2*a^4*b^4 - c)^(1/4))) + ((2*a^4*b^4 - c)^(1/4)*x^2)/Sqrt[2
] + (a^4*x^4)/(Sqrt[2]*(2*a^4*b^4 - c)^(1/4)))/(x*Sqrt[-b^4 + a^4*x^4])]/(2
*Sqrt[2]*(2*a^4*b^4 - c)^(1/4))
```

fricas [B] time = 47.63, size = 746, normalized size = 3.26

$$\frac{1}{2}\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{1}{4}}\arctan\left(\frac{2\sqrt{a^4x^4-b^4}\left((2a^4b^4-c)x^3\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{1}{4}}+\left((2a^8b^4-a^4c)x^5-(2a^4b^8-b^4c)x\right)\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{3}{4}}\right)-\left((2a^4b^{12}-b^8c+(2a^{12}b^4-a^8c)x^8-(8a^8b^8-6a^4b^4c+c^2)x^4)\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{3}{4}}+2\left((2a^8b^4-a^4c)x^6-(2a^4b^8-b^4c)x^2\right)\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{1}{4}}\right)\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{1}{4}}}{a^8x^8+b^8-cx^4}\right)+\frac{1}{8}\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{1}{4}}\log\left(\frac{2\left((2a^8b^4-a^4c)x^6-(2a^4b^8-b^4c)x^2\right)\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{3}{4}}+2\left(a^4x^5-b^4x-(2a^4b^4-c)x^3\sqrt{-\frac{1}{2a^4b^4-c}}\right)\sqrt{a^4x^4-b^4}-\left(a^8x^8+b^8-(4a^4b^4-c)x^4\right)\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{1}{4}}}{2(a^8x^8+b^8-cx^4)}\right)-\frac{1}{8}\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{1}{4}}\log\left(-\frac{2\left((2a^8b^4-a^4c)x^6-(2a^4b^8-b^4c)x^2\right)\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{3}{4}}-2\left(a^4x^5-b^4x-(2a^4b^4-c)x^3\sqrt{-\frac{1}{2a^4b^4-c}}\right)\sqrt{a^4x^4-b^4}-\left(a^8x^8+b^8-(4a^4b^4-c)x^4\right)\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{1}{4}}}{2(a^8x^8+b^8-cx^4)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^4)^(1/2)*(a^4*x^4+b^4)/(a^8*x^8+b^8-c*x^4),x, algorith
m="fricas")
```

```
[Out] 1/2*(-1/(2*a^4*b^4 - c))^(1/4)*arctan((2*sqrt(a^4*x^4 - b^4)*((2*a^4*b^4 -
c)*x^3*(-1/(2*a^4*b^4 - c))^(1/4) + ((2*a^8*b^4 - a^4*c)*x^5 - (2*a^4*b^8 -
 b^4*c)*x)*(-1/(2*a^4*b^4 - c))^(3/4)) - ((2*a^4*b^12 - b^8*c + (2*a^12*b^4
 - a^8*c)*x^8 - (8*a^8*b^8 - 6*a^4*b^4*c + c^2)*x^4)*(-1/(2*a^4*b^4 - c))^(
3/4) + 2*((2*a^8*b^4 - a^4*c)*x^6 - (2*a^4*b^8 - b^4*c)*x^2)*(-1/(2*a^4*b^4
 - c))^(1/4))*(-1/(2*a^4*b^4 - c))^(1/4))/(a^8*x^8 + b^8 - c*x^4)) + 1/8*(-
1/(2*a^4*b^4 - c))^(1/4)*log(1/2*(2*((2*a^8*b^4 - a^4*c)*x^6 - (2*a^4*b^8 -
 b^4*c)*x^2)*(-1/(2*a^4*b^4 - c))^(3/4) + 2*(a^4*x^5 - b^4*x - (2*a^4*b^4 -
 c)*x^3*sqrt(-1/(2*a^4*b^4 - c)))*sqrt(a^4*x^4 - b^4) - (a^8*x^8 + b^8 - (4
*a^4*b^4 - c)*x^4)*(-1/(2*a^4*b^4 - c))^(1/4))/(a^8*x^8 + b^8 - c*x^4)) - 1
/8*(-1/(2*a^4*b^4 - c))^(1/4)*log(-1/2*(2*((2*a^8*b^4 - a^4*c)*x^6 - (2*a^4
*b^8 - b^4*c)*x^2)*(-1/(2*a^4*b^4 - c))^(3/4) - 2*(a^4*x^5 - b^4*x - (2*a^4
*b^4 - c)*x^3*sqrt(-1/(2*a^4*b^4 - c)))*sqrt(a^4*x^4 - b^4) - (a^8*x^8 + b^
8 - (4*a^4*b^4 - c)*x^4)*(-1/(2*a^4*b^4 - c))^(1/4))/(a^8*x^8 + b^8 - c*x^4
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^4x^4+b^4\right)\sqrt{a^4x^4-b^4}}{a^8x^8+b^8-cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^4)^(1/2)*(a^4*x^4+b^4)/(a^8*x^8+b^8-c*x^4),x, algorith
m="giac")
```

```
[Out] integrate((a^4*x^4 + b^4)*sqrt(a^4*x^4 - b^4)/(a^8*x^8 + b^8 - c*x^4), x)
```

maple [A] time = 0.06, size = 286, normalized size = 1.25

$$\frac{\sqrt{2}\,\ln\left(\frac{\frac{a^4x^4-b^4}{2x^2}-\frac{\left(2a^4b^4-c\right)^{\frac{1}{4}}\sqrt{a^4x^4-b^4}\,\sqrt{2}}{2x}+\frac{\sqrt{2a^4b^4-c}}{2}}{\frac{a^4x^4-b^4}{2x^2}+\frac{\left(2a^4b^4-c\right)^{\frac{1}{4}}\sqrt{a^4x^4-b^4}\,\sqrt{2}}{2x}+\frac{\sqrt{2a^4b^4-c}}{2}}\right)}{8\left(2a^4b^4-c\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{a^4x^4-b^4}\,\sqrt{2}}{\left(2a^4b^4-c\right)^{\frac{1}{4}}x}+1\right)}{4\left(2a^4b^4-c\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\arctan\left(-\frac{\sqrt{a^4x^4-b^4}\,\sqrt{2}}{\left(2a^4b^4-c\right)^{\frac{1}{4}}x}+1\right)}{4\left(2a^4b^4-c\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^4*x^4-b^4)^(1/2)*(a^4*x^4+b^4)/(a^8*x^8+b^8-c*x^4),x)
```

```
[Out] 1/8*2^(1/2)/(2*a^4*b^4-c)^(1/4)*ln((1/2*(a^4*x^4-b^4)/x^2-1/2*(2*a^4*b^4-c)
^(1/4)*(a^4*x^4-b^4)^(1/2)*2^(1/2)/x+1/2*(2*a^4*b^4-c)^(1/2))/(1/2*(a^4*x^4
-b^4)/x^2+1/2*(2*a^4*b^4-c)^(1/4)*(a^4*x^4-b^4)^(1/2)*2^(1/2)/x+1/2*(2*a^4*
b^4-c)^(1/2)))+1/4*2^(1/2)/(2*a^4*b^4-c)^(1/4)*arctan(1/(2*a^4*b^4-c)^(1/4)
*(a^4*x^4-b^4)^(1/2)*2^(1/2)/x+1)-1/4*2^(1/2)/(2*a^4*b^4-c)^(1/4)*arctan(-1
/(2*a^4*b^4-c)^(1/4)*(a^4*x^4-b^4)^(1/2)*2^(1/2)/x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^4x^4+b^4\right)\sqrt{a^4x^4-b^4}}{a^8x^8+b^8-cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^4)^(1/2)*(a^4*x^4+b^4)/(a^8*x^8+b^8-c*x^4),x, algorith
m="maxima")
```

```
[Out] integrate((a^4*x^4 + b^4)*sqrt(a^4*x^4 - b^4)/(a^8*x^8 + b^8 - c*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^4\,x^4+b^4\right)\,\sqrt{a^4\,x^4-b^4}}{a^8\,x^8+b^8-c\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b^4 + a^4*x^4)*(a^4*x^4 - b^4)^(1/2))/(b^8 - c*x^4 + a^8*x^8),x)
```

```
[Out] int(((b^4 + a^4*x^4)*(a^4*x^4 - b^4)^(1/2))/(b^8 - c*x^4 + a^8*x^8), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**4*x**4-b**4)**(1/2)*(a**4*x**4+b**4)/(a**8*x**8+b**8-c*x**4),
x)
```

```
[Out] Timed out
```

3.2101 $\int \frac{-b^8+a^8x^8}{\sqrt{-b^4+a^4x^4}\left(b^8-cx^4+a^8x^8\right)}\,dx$

Optimal. Leaf size=229

$$-\frac{\tan^{-1}\left(\frac{-\frac{a^4x^4}{\sqrt{2}\sqrt[4]{2a^4b^4-c}}+\frac{x^2\sqrt[4]{2a^4b^4-c}}{\sqrt{2}}+\frac{b^4}{\sqrt{2}\sqrt[4]{2a^4b^4-c}}}{x\sqrt{a^4x^4-b^4}}\right)}{2\sqrt{2}\sqrt[4]{2a^4b^4-c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x\sqrt[4]{2a^4b^4-c}\sqrt{a^4x^4-b^4}}{x^2\sqrt{2a^4b^4-c}+a^4x^4-b^4}\right)}{2\sqrt{2}\sqrt[4]{2a^4b^4-c}}$$

Rubi [C] time = 1.22, antiderivative size = 508, normalized size of antiderivative = 2.22, number of steps used = 19, number of rules used = 8, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1586, 6728, 406, 224, 221, 409, 1219, 1218}

$$\frac{b\left(1-\frac{2a^4b^4+c}{\sqrt{c^2-4a^8b^8}}\right)\sqrt{1-\frac{a^4x^4}{b^4}}F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}+\frac{b\left(\frac{2a^4b^4+c}{\sqrt{c^2-4a^8b^8}}+1\right)\sqrt{1-\frac{a^4x^4}{b^4}}F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(-\frac{\sqrt{2}a^2b^2}{\sqrt{c-\sqrt{c^2-4a^8b^8}}};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(\frac{\sqrt{2}a^2b^2}{\sqrt{c-\sqrt{c^2-4a^8b^8}}};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(-\frac{\sqrt{2}a^2b^2}{\sqrt{c+\sqrt{c^2-4a^8b^8}}};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(\frac{\sqrt{2}a^2b^2}{\sqrt{c+\sqrt{c^2-4a^8b^8}}};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b^8 + a^8*x^8)/(Sqrt[-b^4 + a^4*x^4]*(b^8 - c*x^4 + a^8*x^8)),x]
```

```
[Out] (b*(1 - (2*a^4*b^4 + c)/Sqrt[-4*a^8*b^8 + c^2])*Sqrt[1 - (a^4*x^4)/b^4]*Ell
ipticF[ArcSin[(a*x)/b], -1])/(2*a*Sqrt[-b^4 + a^4*x^4]) + (b*(1 + (2*a^4*b^
4 + c)/Sqrt[-4*a^8*b^8 + c^2])*Sqrt[1 - (a^4*x^4)/b^4]*EllipticF[ArcSin[(a*
x)/b], -1])/(2*a*Sqrt[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/b^4]*Ellipti
cPi[-((Sqrt[2]*a^2*b^2)/Sqrt[c - Sqrt[-4*a^8*b^8 + c^2]]), ArcSin[(a*x)/b],
 -1])/(2*a*Sqrt[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[(S
qrt[2]*a^2*b^2)/Sqrt[c - Sqrt[-4*a^8*b^8 + c^2]], ArcSin[(a*x)/b], -1])/(2*
a*Sqrt[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[-((Sqrt[2]*
a^2*b^2)/Sqrt[c + Sqrt[-4*a^8*b^8 + c^2]]), ArcSin[(a*x)/b], -1])/(2*a*Sqrt
[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[(Sqrt[2]*a^2*b^2)
/Sqrt[c + Sqrt[-4*a^8*b^8 + c^2]], ArcSin[(a*x)/b], -1])/(2*a*Sqrt[-b^4 + a
^4*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-b^8+a^8x^8}{\sqrt{-b^4+a^4x^4}\left(b^8-cx^4+a^8x^8\right)}\,dx=\int \frac{\sqrt{-b^4+a^4x^4}\left(b^4+a^4x^4\right)}{b^8-cx^4+a^8x^8}\,dx$$

$$=\int \left(\frac{\left(a^4+\frac{a^4(2a^4b^4+c)}{\sqrt{-4a^8b^8+c^2}}\right)\sqrt{-b^4+a^4x^4}}{-c-\sqrt{-4a^8b^8+c^2}+2a^8x^4}+\frac{\left(a^4-\frac{a^4(2a^4b^4+c)}{\sqrt{-4a^8b^8+c^2}}\right)\sqrt{-b^4+a^4x^4}}{-c+\sqrt{-4a^8b^8+c^2}+2a^8x^4}\right) \ldots$$

$$=\left(a^4\left(1-\frac{2a^4b^4+c}{\sqrt{-4a^8b^8+c^2}}\right)\right)\int \frac{\sqrt{-b^4+a^4x^4}}{-c+\sqrt{-4a^8b^8+c^2}+2a^8x^4}\,dx+\left(a^4\left(1+\ldots\right.\right.$$

$$=\frac{1}{2}\left(1-\frac{2a^4b^4+c}{\sqrt{-4a^8b^8+c^2}}\right)\int \frac{1}{\sqrt{-b^4+a^4x^4}}\,dx+\frac{1}{2}\left(1+\frac{2a^4b^4+c}{\sqrt{-4a^8b^8+c^2}}\right)\int \ldots$$

$$=-\left(\frac{1}{2}\int \frac{1}{\left(1-\frac{\sqrt{2}\,a^4x^2}{\sqrt{c-\sqrt{-4a^8b^8+c^2}}}\right)\sqrt{-b^4+a^4x^4}}\,dx\right)-\frac{1}{2}\int \frac{1}{\left(1+\frac{\sqrt{2}\,a^4x^2}{\sqrt{c-\sqrt{-4a^8b^8+c^2}}}\right.\ldots}$$

$$=\frac{b\left(1-\frac{2a^4b^4+c}{\sqrt{-4a^8b^8+c^2}}\right)\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{-b^4+a^4x^4}}+\frac{b\left(1+\frac{2a^4b^4+c}{\sqrt{-4a^8b^8+c^2}}\right)\sqrt{1\ldots}}{2a\sqrt{-\ldots}}$$

$$=\frac{b\left(1-\frac{2a^4b^4+c}{\sqrt{-4a^8b^8+c^2}}\right)\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{-b^4+a^4x^4}}+\frac{b\left(1+\frac{2a^4b^4+c}{\sqrt{-4a^8b^8+c^2}}\right)\sqrt{1\ldots}}{2a\sqrt{-\ldots}}$$

Mathematica [C] time = 1.31, size = 326, normalized size = 1.42

$$-\frac{i\sqrt{1-\frac{a^4x^4}{b^4}}\left(2F\left(i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-\Pi\left(-\frac{\sqrt{2}b^2}{a^2\sqrt{\frac{c-\sqrt{c^2-4a^8b^8}}{a^8}}};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-\Pi\left(\frac{\sqrt{2}b^2}{a^2\sqrt{\frac{c-\sqrt{c^2-4a^8b^8}}{a^8}}};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-\Pi\left(-\frac{\sqrt{2}b^2}{a^2\sqrt{\frac{c+\sqrt{c^2-4a^8b^8}}{a^8}}};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-\Pi\left(\frac{\sqrt{2}b^2}{a^2\sqrt{\frac{c+\sqrt{c^2-4a^8b^8}}{a^8}}};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)\right)}{2\sqrt{-\frac{a^2}{b^2}}\sqrt{a^4x^4-b^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b^8 + a^8*x^8)/(Sqrt[-b^4 + a^4*x^4]*(b^8 - c*x^4 + a^8*x^8)),x
]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (a^4*x^4)/b^4]*(2*EllipticF[I*ArcSinh[Sqrt[-(a^2/b^2)]*x
], -1] - EllipticPi[-((Sqrt[2]*b^2)/(a^2*Sqrt[(c - Sqrt[-4*a^8*b^8 + c^2])/
a^8])), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] - EllipticPi[(Sqrt[2]*b^2)/(a^2*
Sqrt[(c - Sqrt[-4*a^8*b^8 + c^2])/a^8]), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1]
 - EllipticPi[-((Sqrt[2]*b^2)/(a^2*Sqrt[(c + Sqrt[-4*a^8*b^8 + c^2])/a^8]))
, I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] - EllipticPi[(Sqrt[2]*b^2)/(a^2*Sqrt[(
c + Sqrt[-4*a^8*b^8 + c^2])/a^8]), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1]))/(Sq
rt[-(a^2/b^2)]*Sqrt[-b^4 + a^4*x^4])
```

IntegrateAlgebraic [A] time = 1.46, size = 228, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{2a^4b^4-c}\,\sqrt{a^4x^4-b^4}}{x^2\sqrt{2a^4b^4-c}-a^4x^4+b^4}\right)}{2\sqrt{2}\sqrt[4]{2a^4b^4-c}}-\frac{\tanh^{-1}\left(\frac{\frac{a^4x^4}{\sqrt{2}\sqrt[4]{2a^4b^4-c}}+\frac{x^2\sqrt[4]{2a^4b^4-c}}{\sqrt{2}}-\frac{b^4}{\sqrt{2}\sqrt[4]{2a^4b^4-c}}}{x\sqrt{a^4x^4-b^4}}\right)}{2\sqrt{2}\sqrt[4]{2a^4b^4-c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b^8 + a^8*x^8)/(Sqrt[-b^4 + a^4*x^4]*(b^8 - c*x^4 + a^
8*x^8)),x]
```

```
[Out] ArcTan[(Sqrt[2]*(2*a^4*b^4 - c)^(1/4)*x*Sqrt[-b^4 + a^4*x^4])/(b^4 + Sqrt[2
*a^4*b^4 - c]*x^2 - a^4*x^4)]/(2*Sqrt[2]*(2*a^4*b^4 - c)^(1/4)) - ArcTanh[(
-(b^4/(Sqrt[2]*(2*a^4*b^4 - c)^(1/4))) + ((2*a^4*b^4 - c)^(1/4)*x^2)/Sqrt[2
] + (a^4*x^4)/(Sqrt[2]*(2*a^4*b^4 - c)^(1/4)))/(x*Sqrt[-b^4 + a^4*x^4])]/(2
*Sqrt[2]*(2*a^4*b^4 - c)^(1/4))
```

fricas [B] time = 40.91, size = 746, normalized size = 3.26

$$\frac{1}{2}\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{1}{4}}\arctan\left(\frac{2\sqrt{a^4x^4-b^4}\left((2a^4b^4-c)x^3\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{1}{4}}+\left((2a^8b^4-a^4c)x^5-(2a^4b^8-b^4c)x\right)\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{3}{4}}\right)-\left((2a^4b^{12}-b^8c+(2a^{12}b^4-a^8c)x^8-(8a^8b^8-6a^4b^4c+c^2)x^4)\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{3}{4}}+2\left((2a^8b^4-a^4c)x^6-(2a^4b^8-b^4c)x^2\right)\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{1}{4}}\right)\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{1}{4}}}{a^8x^8+b^8-cx^4}\right)+\frac{1}{8}\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{1}{4}}\log\left(\frac{2\left((2a^8b^4-a^4c)x^6-(2a^4b^8-b^4c)x^2\right)\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{3}{4}}+2\left(a^4x^5-b^4x-(2a^4b^4-c)x^3\sqrt{-\frac{1}{2a^4b^4-c}}\right)\sqrt{a^4x^4-b^4}-\left(a^8x^8+b^8-(4a^4b^4-c)x^4\right)\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{1}{4}}}{2(a^8x^8+b^8-cx^4)}\right)-\frac{1}{8}\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{1}{4}}\log\left(-\frac{2\left((2a^8b^4-a^4c)x^6-(2a^4b^8-b^4c)x^2\right)\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{3}{4}}-2\left(a^4x^5-b^4x-(2a^4b^4-c)x^3\sqrt{-\frac{1}{2a^4b^4-c}}\right)\sqrt{a^4x^4-b^4}-\left(a^8x^8+b^8-(4a^4b^4-c)x^4\right)\left(-\frac{1}{2a^4b^4-c}\right)^{\frac{1}{4}}}{2(a^8x^8+b^8-cx^4)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^8*x^8-b^8)/(a^4*x^4-b^4)^(1/2)/(a^8*x^8+b^8-c*x^4),x, algorith
m="fricas")
```

```
[Out] 1/2*(-1/(2*a^4*b^4 - c))^(1/4)*arctan((2*sqrt(a^4*x^4 - b^4)*((2*a^4*b^4 -
c)*x^3*(-1/(2*a^4*b^4 - c))^(1/4) + ((2*a^8*b^4 - a^4*c)*x^5 - (2*a^4*b^8 -
 b^4*c)*x)*(-1/(2*a^4*b^4 - c))^(3/4)) - ((2*a^4*b^12 - b^8*c + (2*a^12*b^4
 - a^8*c)*x^8 - (8*a^8*b^8 - 6*a^4*b^4*c + c^2)*x^4)*(-1/(2*a^4*b^4 - c))^(
3/4) + 2*((2*a^8*b^4 - a^4*c)*x^6 - (2*a^4*b^8 - b^4*c)*x^2)*(-1/(2*a^4*b^4
 - c))^(1/4))*(-1/(2*a^4*b^4 - c))^(1/4))/(a^8*x^8 + b^8 - c*x^4)) + 1/8*(-
1/(2*a^4*b^4 - c))^(1/4)*log(1/2*(2*((2*a^8*b^4 - a^4*c)*x^6 - (2*a^4*b^8 -
 b^4*c)*x^2)*(-1/(2*a^4*b^4 - c))^(3/4) + 2*(a^4*x^5 - b^4*x - (2*a^4*b^4 -
 c)*x^3*sqrt(-1/(2*a^4*b^4 - c)))*sqrt(a^4*x^4 - b^4) - (a^8*x^8 + b^8 - (4
*a^4*b^4 - c)*x^4)*(-1/(2*a^4*b^4 - c))^(1/4))/(a^8*x^8 + b^8 - c*x^4)) - 1
/8*(-1/(2*a^4*b^4 - c))^(1/4)*log(-1/2*(2*((2*a^8*b^4 - a^4*c)*x^6 - (2*a^4
*b^8 - b^4*c)*x^2)*(-1/(2*a^4*b^4 - c))^(3/4) - 2*(a^4*x^5 - b^4*x - (2*a^4
*b^4 - c)*x^3*sqrt(-1/(2*a^4*b^4 - c)))*sqrt(a^4*x^4 - b^4) - (a^8*x^8 + b^
8 - (4*a^4*b^4 - c)*x^4)*(-1/(2*a^4*b^4 - c))^(1/4))/(a^8*x^8 + b^8 - c*x^4
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^8x^8-b^8}{\left(a^8x^8+b^8-cx^4\right)\sqrt{a^4x^4-b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^8*x^8-b^8)/(a^4*x^4-b^4)^(1/2)/(a^8*x^8+b^8-c*x^4),x, algorith
m="giac")
```

```
[Out] integrate((a^8*x^8 - b^8)/((a^8*x^8 + b^8 - c*x^4)*sqrt(a^4*x^4 - b^4)), x)
```

maple [C] time = 0.06, size = 332, normalized size = 1.45

$$\frac{\sqrt{\frac{a^2x^2}{b^2}+1}\,\sqrt{1-\frac{a^2x^2}{b^2}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{a^2}{b^2}},i\right)}{\sqrt{-\frac{a^2}{b^2}}\,\sqrt{a^4x^4-b^4}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(a^8\_Z^8+b^8-c\_Z^4\right)}\frac{\left(2b^8-\_\alpha^4c\right)\left(-\frac{\mathrm{arctanh}\left(\frac{\_\alpha^2\left(\_\alpha^6a^8+a^4b^4x^2-\_\alpha^2c\right)}{b^4\sqrt{a^4\_\alpha^4-b^4}\,\sqrt{a^4x^4-b^4}}\right)}{\sqrt{a^4\_\alpha^4-b^4}}+\frac{2\_\alpha^3\left(\_\alpha^4a^8-c\right)\sqrt{\frac{a^2x^2}{b^2}+1}\,\sqrt{1-\frac{a^2x^2}{b^2}}\,\mathrm{EllipticPi}\left(x\sqrt{-\frac{a^2}{b^2}},\frac{\_\alpha^2\left(\_\alpha^4a^8-c\right)}{a^2b^6},\frac{\sqrt{\frac{a^2}{b^2}}}{\sqrt{-\frac{a^2}{b^2}}}\right)}{\sqrt{-\frac{a^2}{b^2}}\,b^8\sqrt{a^4x^4-b^4}}\right)}{\_\alpha^3\left(2\_\alpha^4a^8-c\right)}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^8*x^8-b^8)/(a^4*x^4-b^4)^(1/2)/(a^8*x^8+b^8-c*x^4),x)
```

```
[Out] 1/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4
)^(1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)-1/8*sum((2*b^8-_alpha^4*c)/_alpha^3
/(2*_alpha^4*a^8-c)*(-1/(_alpha^4*a^4-b^4)^(1/2)*arctanh(_alpha^2/b^4*(_alp
ha^6*a^8+a^4*b^4*x^2-_alpha^2*c)/(_alpha^4*a^4-b^4)^(1/2)/(a^4*x^4-b^4)^(1/
2))+2/(-a^2/b^2)^(1/2)*_alpha^3*(_alpha^4*a^8-c)/b^8*(a^2*x^2/b^2+1)^(1/2)*
(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticPi(x*(-a^2/b^2)^(1/2),_al
pha^2*(_alpha^4*a^8-c)/a^2/b^6,(a^2/b^2)^(1/2)/(-a^2/b^2)^(1/2))),_alpha=Ro
otOf(_Z^8*a^8+b^8-_Z^4*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^8x^8-b^8}{\left(a^8x^8+b^8-cx^4\right)\sqrt{a^4x^4-b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^8*x^8-b^8)/(a^4*x^4-b^4)^(1/2)/(a^8*x^8+b^8-c*x^4),x, algorith
m="maxima")
```

```
[Out] integrate((a^8*x^8 - b^8)/((a^8*x^8 + b^8 - c*x^4)*sqrt(a^4*x^4 - b^4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{b^8-a^8\,x^8}{\sqrt{a^4\,x^4-b^4}\,\left(a^8\,x^8+b^8-c\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^8 - a^8*x^8)/((a^4*x^4 - b^4)^(1/2)*(b^8 - c*x^4 + a^8*x^8)),x)
```

```
[Out] int(-(b^8 - a^8*x^8)/((a^4*x^4 - b^4)^(1/2)*(b^8 - c*x^4 + a^8*x^8)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**8*x**8-b**8)/(a**4*x**4-b**4)**(1/2)/(a**8*x**8+b**8-c*x**4),
x)
```

```
[Out] Timed out
```

3.2102 $\int \frac{-1+2kx+(1-2k)x^2}{((1-x)x(1-kx))^{2/3}\left(1-(2+b)x+(1+bk)x^2\right)} dx$

Optimal. Leaf size=230

$$-\frac{\log\left(\left(kx^3+(-k-1)x^2+x\right)^{2/3}\left(b^{2/3}x-b^{2/3}kx^2\right)+bk^2x^4-2bkx^3+\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{4/3}+bx^2\right)}{2\sqrt[3]{b}}+\frac{\log}{}$$

Rubi [F] time = 4.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+2kx+(1-2k)x^2}{((1-x)x(1-kx))^{2/3}\left(1-(2+b)x+(1+bk)x^2\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + 2*k*x + (1 - 2*k)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(1 - (2 + b)*
x + (1 + b*k)*x^2)),x]
```

```
[Out] -(((1 + Sqrt[4 + b - 4*k]/Sqrt[b] - 2*k)*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2
/3)*Defer[Int][(1 - x)^(1/3)/(x^(2/3)*(1 - k*x)^(2/3)*(-2 - b - Sqrt[b]*Sqr
t[4 + b - 4*k] + 2*(1 + b*k)*x)), x])/((1 - x)*x*(1 - k*x))^(2/3)) - ((1 -
Sqrt[4 + b - 4*k]/Sqrt[b] - 2*k)*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defe
r[Int][(1 - x)^(1/3)/(x^(2/3)*(1 - k*x)^(2/3)*(-2 - b + Sqrt[b]*Sqrt[4 + b
- 4*k] + 2*(1 + b*k)*x)), x])/((1 - x)*x*(1 - k*x))^(2/3)
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+2kx+(1-2k)x^2}{((1-x)x(1-kx))^{2/3}\left(1-(2+b)x+(1+bk)x^2\right)} dx &= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{-1+2kx+(1-2k)x^2}{(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\left(1-(2+b)x+(\right.}}{((1-x)x(1-kx))^{2/3}}\\
&= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{\sqrt[3]{1-x}\,(-1+(-1+2k)x)}{x^{2/3}(1-kx)^{2/3}\left(1-(2+b)x+(1+bk)x^2\right.}}{((1-x)x(1-kx))^{2/3}}\\
&= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \left(\frac{\left(-1-\frac{\sqrt{4+b-4k}}{\sqrt{b}}+2k\right)\sqrt[3]{1}}{x^{2/3}(1-kx)^{2/3}\left(-2-b-\sqrt{b}\,\sqrt{4+b-4}\right.}\right.}{((1-x)x(}\\
&= \frac{\left(\left(-1-\frac{\sqrt{4+b-4k}}{\sqrt{b}}+2k\right)(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{}{x^{2/3}(1}}{((1-x)x(1-kx))^{2/3}}
\end{aligned}$$

Mathematica [F] time = 8.57, size = 0, normalized size = 0.00

$$\int \frac{-1+2kx+(1-2k)x^2}{((1-x)x(1-kx))^{2/3}\left(1-(2+b)x+(1+bk)x^2\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + 2*k*x + (1 - 2*k)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(1 - (2
 + b)*x + (1 + b*k)*x^2)),x]
```

```
[Out] Integrate[(-1 + 2*k*x + (1 - 2*k)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3)*(1 - (2
 + b)*x + (1 + b*k)*x^2)), x]
```

IntegrateAlgebraic [A] time = 1.06, size = 230, normalized size = 1.00

$$-\frac{\log\left(\left(kx^3+(-k-1)x^2+x\right)^{2/3}\left(b^{2/3}x-b^{2/3}kx^2\right)+bk^2x^4-2bkx^3+\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{4/3}+bx^2\right)}{2\sqrt[3]{b}}+\frac{\log\left(\sqrt{b}\,kx^2+\sqrt[6]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}-\sqrt{b}\,x\right)}{\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}}{-2\sqrt[3]{b}kx^2+2\sqrt[3]{b}x+\left(kx^3+(-k-1)x^2+x\right)^{2/3}}\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*k*x + (1 - 2*k)*x^2)/(((1 - x)*x*(1 - k*x))^(2/3
)*(1 - (2 + b)*x + (1 + b*k)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(x + (-1 - k)*x^2 + k*x^3)^(2/3))/(2*b^(1/3)*x - 2
*b^(1/3)*k*x^2 + (x + (-1 - k)*x^2 + k*x^3)^(2/3))])/b^(1/3) + Log[-(Sqrt[b
]*x) + Sqrt[b]*k*x^2 + b^(1/6)*(x + (-1 - k)*x^2 + k*x^3)^(2/3)]/b^(1/3) -
Log[b*x^2 - 2*b*k*x^3 + b*k^2*x^4 + (b^(2/3)*x - b^(2/3)*k*x^2)*(x + (-1 -
k)*x^2 + k*x^3)^(2/3) + b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(4/3)]/(2*b^(1/3
))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*k*x+(1-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-(2+b)*x+(b*k+1
)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(2\,k-1)x^2-2\,kx+1}{((kx-1)(x-1)x)^{\frac{2}{3}}\left((bk+1)x^2-(b+2)x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*k*x+(1-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-(2+b)*x+(b*k+1
)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-((2*k - 1)*x^2 - 2*k*x + 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((b*k +
 1)*x^2 - (b + 2)*x + 1)), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{-1+2kx+(1-2k)\,x^2}{((1-x)\,x\,(-kx+1))^{\frac{2}{3}}\left(1-(2+b)\,x+(bk+1)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*k*x+(1-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-(2+b)*x+(b*k+1)*x^2)
,x)
```

```
[Out] int((-1+2*k*x+(1-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-(2+b)*x+(b*k+1)*x^2)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(2\,k-1)x^2-2\,kx+1}{((kx-1)(x-1)x)^{\frac{2}{3}}\left((bk+1)x^2-(b+2)x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*k*x+(1-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-(2+b)*x+(b*k+1
)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate(((2*k - 1)*x^2 - 2*k*x + 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((b*k +
 1)*x^2 - (b + 2)*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(2k-1)\,x^2-2\,k\,x+1}{(x\,(k\,x-1)\,(x-1))^{2/3}\,\left((b\,k+1)\,x^2+(-b-2)\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(2*k - 1) - 2*k*x + 1)/((x*(k*x - 1)*(x - 1))^(2/3)*(x^2*(b*k + 1
) - x*(b + 2) + 1)),x)
```

```
[Out] int(-(x^2*(2*k - 1) - 2*k*x + 1)/((x*(k*x - 1)*(x - 1))^(2/3)*(x^2*(b*k + 1
) - x*(b + 2) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*k*x+(1-2*k)*x**2)/((1-x)*x*(-k*x+1))**(2/3)/(1-(2+b)*x+(b*k
+1)*x**2),x)
```

```
[Out] Timed out
```

3.2103 $\int \frac{x^3(3+x^2)}{(1+x^2)\sqrt[3]{1+x^2-x^3}\,(1+x^2+x^3)}\,dx$

Optimal. Leaf size=230

$$\log\left(\sqrt[3]{-x^3+x^2+1}+x\right)-\frac{\log\left(2^{2/3}\sqrt[3]{-x^3+x^2+1}+2x\right)}{\sqrt[3]{2}}-\frac{1}{2}\log\left(x^2-\sqrt[3]{-x^3+x^2+1}\,x+\left(-x^3+x^2+1\right)^{2/3}\right)+\frac{\log \ldots}{}$$

Rubi [F] time = 6.35, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3\left(3+x^2\right)}{\left(1+x^2\right)\sqrt[3]{1+x^2-x^3}\,\left(1+x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*(3 + x^2))/((1 + x^2)*(1 + x^2 - x^3)^(1/3)*(1 + x^2 + x^3)),x]
```

```
[Out] -1/4*((2 + 2^(1/3)*(2/(29 + 3*Sqrt[93])^(1/3) + (58 + 6*Sqrt[93])^(1/3)) -
6*x)^(1/3)*(2 + (2*2^(1/3) + (58 + 6*Sqrt[93])^(2/3))/(29 + 3*Sqrt[93])^(1/
3) - 6*x)^(2/3)*(1 - (2^(2/3)*(2*2^(1/3) + 2*(29 + 3*Sqrt[93])^(1/3) + (58
+ 6*Sqrt[93])^(2/3) - 6*(29 + 3*Sqrt[93])^(1/3)*x))/(6 + 3*2^(1/3)*(29 + 3*
Sqrt[93])^(2/3) + 2^(1/6)*(29 + 3*Sqrt[93])^(1/3)*Sqrt[3*(4 - 2*(2/(29 + 3*
Sqrt[93]))^(2/3) - 2^(1/3)*(29 + 3*Sqrt[93])^(2/3))]))^(1/3)*(1 - (2^(2/3)*
(2*2^(1/3) + 2*(29 + 3*Sqrt[93])^(1/3) + (58 + 6*Sqrt[93])^(2/3) - 6*(29 +
3*Sqrt[93])^(1/3)*x))/(6 + 3*2^(1/3)*(29 + 3*Sqrt[93])^(2/3) - I*2^(1/6)*(2
9 + 3*Sqrt[93])^(1/3)*Sqrt[3*(-4 + 2*(2/(29 + 3*Sqrt[93]))^(2/3) + 2^(1/3)*
(29 + 3*Sqrt[93])^(2/3))]))^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, ((29 + 3*Sqr
t[93])^(1/3)*(2*(2/(29 + 3*Sqrt[93])^(1/3) + (58 + 6*Sqrt[93])^(1/3)) + 2*2
^(2/3)*(1 - 3*x)))/(6 + 3*2^(1/3)*(29 + 3*Sqrt[93])^(2/3) + 2^(1/6)*(29 + 3
*Sqrt[93])^(1/3)*Sqrt[3*(4 - 2*(2/(29 + 3*Sqrt[93]))^(2/3) - 2^(1/3)*(29 +
3*Sqrt[93])^(2/3))]), ((29 + 3*Sqrt[93])^(1/3)*(2*(2/(29 + 3*Sqrt[93])^(1/3
) + (58 + 6*Sqrt[93])^(1/3)) + 2*2^(2/3)*(1 - 3*x)))/(6 + 3*2^(1/3)*(29 + 3
*Sqrt[93])^(2/3) - I*2^(1/6)*(29 + 3*Sqrt[93])^(1/3)*Sqrt[3*(-4 + 2*(2/(29
+ 3*Sqrt[93]))^(2/3) + 2^(1/3)*(29 + 3*Sqrt[93])^(2/3))])])/(1 + x^2 - x^3)
^(1/3) - 3*Defer[Int][1/((1 + x^2 - x^3)^(1/3)*(1 + x^2 + x^3)), x] - Defer
[Int][x^2/((1 + x^2 - x^3)^(1/3)*(1 + x^2 + x^3)), x] + ((I/3)*(-2 + 2*(2/(
29 + 3*Sqrt[93]))^(2/3) + 2^(1/3)*(29 + 3*Sqrt[93])^(2/3) - 2*((2/(29 + 3*S
qrt[93]))^(1/3) + ((29 + 3*Sqrt[93])/2)^(1/3))*(1 - 3*x) + 2*(1 - 3*x)^2)^(
1/3)*(2 + 2^(1/3)*(2/(29 + 3*Sqrt[93])^(1/3) + (58 + 6*Sqrt[93])^(1/3)) - 6
*x)^(1/3)*Defer[Subst][Defer[Int][1/(((-1/3 + I) - x)*((2/(29 + 3*Sqrt[93])
^(1/3) + (58 + 6*Sqrt[93])^(1/3))/(3*2^(2/3)) - x)^(1/3)*((-2 + 2*(2/(29 +
3*Sqrt[93]))^(2/3) + 2^(1/3)*(29 + 3*Sqrt[93])^(2/3))/18 + (((2/(29 + 3*Sqr
t[93]))^(1/3) + ((29 + 3*Sqrt[93])/2)^(1/3))*x)/3 + x^2)^(1/3)), x], x, -1/
3 + x])/(2^(2/3)*(1 + x^2 - x^3)^(1/3)) + ((I/3)*(-2 + 2*(2/(29 + 3*Sqrt[93
]))^(2/3) + 2^(1/3)*(29 + 3*Sqrt[93])^(2/3) - 2*((2/(29 + 3*Sqrt[93]))^(1/3
) + ((29 + 3*Sqrt[93])/2)^(1/3))*(1 - 3*x) + 2*(1 - 3*x)^2)^(1/3)*(2 + 2^(1
/3)*(2/(29 + 3*Sqrt[93])^(1/3) + (58 + 6*Sqrt[93])^(1/3)) - 6*x)^(1/3)*Defe
r[Subst][Defer[Int][1/((((2/(29 + 3*Sqrt[93])^(1/3) + (58 + 6*Sqrt[93])^(1/3
))/(3*2^(2/3)) - x)^(1/3)*((1/3 + I) + x)*((-2 + 2*(2/(29 + 3*Sqrt[93]))^(2
/3) + 2^(1/3)*(29 + 3*Sqrt[93])^(2/3))/18 + (((2/(29 + 3*Sqrt[93]))^(1/3) +
 ((29 + 3*Sqrt[93])/2)^(1/3))*x)/3 + x^2)^(1/3)), x], x, -1/3 + x])/(2^(2/3
)*(1 + x^2 - x^3)^(1/3))
```

Rubi steps

$$\int \frac{x^3\left(3+x^2\right)}{\left(1+x^2\right)\sqrt[3]{1+x^2-x^3}\left(1+x^2+x^3\right)}\,dx = \int\left(\frac{1}{\sqrt[3]{1+x^2-x^3}}+\frac{2}{\left(1+x^2\right)\sqrt[3]{1+x^2-x^3}}+\frac{-3-x^2}{\sqrt[3]{1+x^2-x^3}\,(1+\ldots}\right.$$

$$= 2\int \frac{1}{\left(1+x^2\right)\sqrt[3]{1+x^2-x^3}}\,dx+\int \frac{1}{\sqrt[3]{1+x^2-x^3}}\,dx+\int \frac{\ldots}{\sqrt[3]{1+x\ldots}}$$

$$= 2\int\left(\frac{i}{2(i-x)\sqrt[3]{1+x^2-x^3}}+\frac{i}{2(i+x)\sqrt[3]{1+x^2-x^3}}\right)dx+\int\left(-\ldots\right.$$

$$= i\int \frac{1}{(i-x)\sqrt[3]{1+x^2-x^3}}\,dx+i\int \frac{1}{(i+x)\sqrt[3]{1+x^2-x^3}}\,dx-3\int \ldots$$

$$= i\,\text{Subst}\left(\int \frac{1}{\left(\left(-\frac{1}{3}+i\right)-x\right)\sqrt[3]{\frac{29}{27}+\frac{x}{3}-x^3}}\,dx,x,-\frac{1}{3}+x\right)+i\,\text{Subs}\ldots$$

$$= -\frac{\left(2+2\sqrt[3]{\frac{2}{29+3\sqrt{93}}}+2^{2/3}\sqrt[3]{29+3\sqrt{93}}-6x\right)\sqrt[3]{1-\frac{2\left(2+2\ldots\right.}{6+3\sqrt[3]{2}\left(29+3\sqrt{\ldots}\right.}}}{\ldots}$$

Mathematica [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(3+x^2\right)}{\left(1+x^2\right)\sqrt[3]{1+x^2-x^3}\left(1+x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(3 + x^2))/((1 + x^2)*(1 + x^2 - x^3)^(1/3)*(1 + x^2 + x^3))
,x]
```

```
[Out] Integrate[(x^3*(3 + x^2))/((1 + x^2)*(1 + x^2 - x^3)^(1/3)*(1 + x^2 + x^3))
, x]
```

IntegrateAlgebraic [A] time = 1.01, size = 230, normalized size = 1.00

$$\log\left(\sqrt[3]{-x^3+x^2+1}+x\right)-\frac{\log\left(2^{2/3}\sqrt[3]{-x^3+x^2+1}+2x\right)}{\sqrt[3]{2}}-\frac{1}{2}\log\left(x^2-\sqrt[3]{-x^3+x^2+1}\,x+\left(-x^3+x^2+1\right)^{2/3}\right)+\frac{\log\left(-2x^2+2^{2/3}\sqrt[3]{-x^3+x^2+1}\,x-\sqrt[3]{2}\left(-x^3+x^2+1\right)^{2/3}\right)}{2\sqrt[3]{2}}+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{-x^3+x^2+1}-x}\right)-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{-x^3+x^2+1}-x}\right)}{\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(3 + x^2))/((1 + x^2)*(1 + x^2 - x^3)^(1/3)*(1 + x^
2 + x^3)),x]
```

```
[Out] Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(1 + x^2 - x^3)^(1/3))] - (Sqrt[3]*ArcTa
n[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 + x^2 - x^3)^(1/3))])/2^(1/3) + Log[x + (1 +
```

```
 x^2 - x^3)^(1/3)] - Log[2*x + 2^(2/3)*(1 + x^2 - x^3)^(1/3)]/2^(1/3) - Log
[x^2 - x*(1 + x^2 - x^3)^(1/3) + (1 + x^2 - x^3)^(2/3)]/2 + Log[-2*x^2 + 2^
(2/3)*x*(1 + x^2 - x^3)^(1/3) - 2^(1/3)*(1 + x^2 - x^3)^(2/3)]/(2*2^(1/3))
```

fricas [C] time = 2.64, size = 471, normalized size = 2.05

$$\frac{1}{4}\cdot 2^{\frac{2}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)\log\left(-\frac{x\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{3}-6\cdot 2^{\frac{1}{3}}x\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}+8x-24\left(-x^{3}+x^{2}+1\right)^{\frac{1}{3}}}{8x}\right)-\frac{1}{8}\left(2^{\frac{2}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)-2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}}\right)\log\left(-\frac{3\left(2^{\frac{2}{3}}\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}}\,x\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)+2^{\frac{1}{3}}x\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}-8\left(-x^{3}+x^{2}+1\right)^{\frac{1}{3}}\right)}{8x}\right)-\frac{1}{8}\left(2^{\frac{2}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)+2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}}\right)\log\left(\frac{3\left(2^{\frac{2}{3}}\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}}\,x\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)-2^{\frac{1}{3}}x\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}+8\left(-x^{3}+x^{2}+1\right)^{\frac{1}{3}}\right)}{8x}\right)-\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^{3}+x^{2}+1\right)^{\frac{1}{3}}}{3x}\right)+\log\left(\frac{x\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{3}+32x+24\left(-x^{3}+x^{2}+1\right)^{\frac{1}{3}}}{8x}\right)-\frac{1}{2}\log\left(\frac{x^{2}-\left(-x^{3}+x^{2}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+x^{2}+1\right)^{\frac{2}{3}}}{x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+3)/(x^2+1)/(-x^3+x^2+1)^(1/3)/(x^3+x^2+1),x, algorithm="
fricas")
```

```
[Out] 1/4*2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))*log(-1/8*(x*(I*sqrt(3)*(-1)
^(1/3) - (-1)^(1/3))^3 - 6*2^(1/3)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2
+ 8*x - 24*(-x^3 + x^2 + 1)^(1/3))/x) - 1/8*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3)
- (-1)^(1/3)) - 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3
))^2))*log(-3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-
1)^(1/3))^2)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 2^(1/3)*x*(I*sqrt(3)*(
-1)^(1/3) - (-1)^(1/3))^2 - 8*(-x^3 + x^2 + 1)^(1/3))/x) - 1/8*(2^(2/3)*(I*
sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1
)^(1/3) - (-1)^(1/3))^2))*log(3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(
3)*(-1)^(1/3) - (-1)^(1/3))^2)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) - 2^(1
/3)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 + 8*(-x^3 + x^2 + 1)^(1/3))/x)
- sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + x^2 + 1)^(1/3))/x) + l
og(1/8*(x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^3 + 32*x + 24*(-x^3 + x^2 + 1
)^(1/3))/x) - 1/2*log((x^2 - (-x^3 + x^2 + 1)^(1/3)*x + (-x^3 + x^2 + 1)^(2
/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}+3\right)x^{3}}{\left(x^{3}+x^{2}+1\right)\left(-x^{3}+x^{2}+1\right)^{\frac{1}{3}}\left(x^{2}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+3)/(x^2+1)/(-x^3+x^2+1)^(1/3)/(x^3+x^2+1),x, algorithm="
giac")
```

```
[Out] integrate((x^2 + 3)*x^3/((x^3 + x^2 + 1)*(-x^3 + x^2 + 1)^(1/3)*(x^2 + 1)),
 x)
```

maple [F] time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(x^{2}+3\right)}{\left(x^{2}+1\right)\left(-x^{3}+x^{2}+1\right)^{\frac{1}{3}}\left(x^{3}+x^{2}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+3)/(x^2+1)/(-x^3+x^2+1)^(1/3)/(x^3+x^2+1),x)
```

```
[Out] int(x^3*(x^2+3)/(x^2+1)/(-x^3+x^2+1)^(1/3)/(x^3+x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}+3\right)x^{3}}{\left(x^{3}+x^{2}+1\right)\left(-x^{3}+x^{2}+1\right)^{\frac{1}{3}}\left(x^{2}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+3)/(x^2+1)/(-x^3+x^2+1)^(1/3)/(x^3+x^2+1),x, algorithm="
maxima")
```

```
[Out] integrate((x^2 + 3)*x^3/((x^3 + x^2 + 1)*(-x^3 + x^2 + 1)^(1/3)*(x^2 + 1)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \left(x^2+3\right)}{\left(x^2+1\right) \left(x^3+x^2+1\right) \left(-x^3+x^2+1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x^2 + 3))/((x^2 + 1)*(x^2 + x^3 + 1)*(x^2 - x^3 + 1)^(1/3)),x)
```

```
[Out] int((x^3*(x^2 + 3))/((x^2 + 1)*(x^2 + x^3 + 1)*(x^2 - x^3 + 1)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(x^2+3\right)}{\left(x^2+1\right) \sqrt[3]{-x^3+x^2+1} \left(x^3+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+3)/(x**2+1)/(-x**3+x**2+1)**(1/3)/(x**3+x**2+1),x)
```

```
[Out] Integral(x**3*(x**2 + 3)/((x**2 + 1)*(-x**3 + x**2 + 1)**(1/3)*(x**3 + x**2
 + 1)), x)
```

3.2104 $\int \frac{(1+x^2)\sqrt[4]{-x^3+x^4}}{-1+x+2x^2}\,dx$

Optimal. Leaf size=230

$$\frac{1}{16}\sqrt[4]{x^4-x^3}\,(4x-5)-\frac{57}{32}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)+\frac{4}{3}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-x^3}}\right)+\frac{57}{32}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)-\frac{4}{3}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{}}{\sqrt[4]{x^4}}\right.$$

Rubi [B] time = 1.05, antiderivative size = 480, normalized size of antiderivative = 2.09, number of steps used = 40, number of rules used = 18, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.621, Rules used = {2056, 6728, 50, 63, 240, 212, 206, 203, 101, 157, 93, 297, 1162, 617, 204, 1165, 628, 298}

$$-\frac{1}{4}\sqrt[4]{x^4-x^3}\,(1-x)-\frac{1}{16}\sqrt[4]{x^4-x^3}+\frac{5\sqrt[4]{x^4-x^3}\log\left(\frac{\sqrt{x}}{\sqrt{x-1}}-\frac{\sqrt{2}\sqrt[4]{x}}{\sqrt[4]{x-1}}+1\right)}{24\sqrt{2}\sqrt[4]{x-1}\,x^{3/4}}-\frac{5\sqrt[4]{x^4-x^3}\log\left(\frac{\sqrt{x}}{\sqrt{x-1}}+\frac{\sqrt{2}\sqrt[4]{x}}{\sqrt[4]{x-1}}+1\right)}{24\sqrt{2}\sqrt[4]{x-1}\,x^{3/4}}-\frac{5\sqrt[4]{x^4-x^3}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{12\sqrt{2}\sqrt[4]{x-1}\,x^{3/4}}+\frac{5\sqrt[4]{x^4-x^3}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{x}}{\sqrt[4]{x-1}}+1\right)}{12\sqrt{2}\sqrt[4]{x-1}\,x^{3/4}}+\frac{57\sqrt[4]{x^4-x^3}\tan^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{32\sqrt[4]{x-1}\,x^{3/4}}+\frac{4\sqrt[4]{2}\sqrt[4]{x^4-x^3}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{3\sqrt[4]{x-1}\,x^{3/4}}+\frac{57\sqrt[4]{x^4-x^3}\tanh^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{32\sqrt[4]{x-1}\,x^{3/4}}-\frac{4\sqrt[4]{2}\sqrt[4]{x^4-x^3}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{3\sqrt[4]{x-1}\,x^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(-x^3 + x^4)^(1/4))/(-1 + x + 2*x^2),x]
```

```
[Out] -1/16*(-x^3 + x^4)^(1/4) - ((1 - x)*(-x^3 + x^4)^(1/4))/4 - (5*(-x^3 + x^4)
^(1/4)*ArcTan[1 - (Sqrt[2]*x^(1/4))/(-1 + x)^(1/4)])/(12*Sqrt[2]*(-1 + x)^(
1/4)*x^(3/4)) + (5*(-x^3 + x^4)^(1/4)*ArcTan[1 + (Sqrt[2]*x^(1/4))/(-1 + x)
^(1/4)])/(12*Sqrt[2]*(-1 + x)^(1/4)*x^(3/4)) + (57*(-x^3 + x^4)^(1/4)*ArcTa
n[(-1 + x)^(1/4)/x^(1/4)])/(32*(-1 + x)^(1/4)*x^(3/4)) + (4*2^(1/4)*(-x^3 +
 x^4)^(1/4)*ArcTan[(2^(1/4)*x^(1/4))/(-1 + x)^(1/4)])/(3*(-1 + x)^(1/4)*x^(
3/4)) + (57*(-x^3 + x^4)^(1/4)*ArcTanh[(-1 + x)^(1/4)/x^(1/4)])/(32*(-1 + x
)^(1/4)*x^(3/4)) - (4*2^(1/4)*(-x^3 + x^4)^(1/4)*ArcTanh[(2^(1/4)*x^(1/4))/
(-1 + x)^(1/4)])/(3*(-1 + x)^(1/4)*x^(3/4)) + (5*(-x^3 + x^4)^(1/4)*Log[1 -
 (Sqrt[2]*x^(1/4))/(-1 + x)^(1/4) + Sqrt[x]/Sqrt[-1 + x]])/(24*Sqrt[2]*(-1
+ x)^(1/4)*x^(3/4)) - (5*(-x^3 + x^4)^(1/4)*Log[1 + (Sqrt[2]*x^(1/4))/(-1 +
 x)^(1/4) + Sqrt[x]/Sqrt[-1 + x]])/(24*Sqrt[2]*(-1 + x)^(1/4)*x^(3/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(1+x^2\right)\sqrt[4]{-x^3+x^4}}{-1+x+2x^2}\,dx = \frac{\sqrt[4]{-x^3+x^4}\int \frac{\sqrt[4]{-1+x}\,x^{3/4}\left(1+x^2\right)}{-1+x+2x^2}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{\sqrt[4]{-x^3+x^4}\int \left(\frac{1}{2}\sqrt[4]{-1+x}\,x^{3/4}+\frac{(3-x)\sqrt[4]{-1+x}\,x^{3/4}}{2\left(-1+x+2x^2\right)}\right)dx}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{\sqrt[4]{-x^3+x^4}\int \sqrt[4]{-1+x}\,x^{3/4}\,dx}{2\sqrt[4]{-1+x}\,x^{3/4}} + \frac{\sqrt[4]{-x^3+x^4}\int \frac{(3-x)\sqrt[4]{-1+x}\,x^{3/4}}{-1+x+2x^2}\,dx}{2\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= -\frac{1}{4}(1-x)\sqrt[4]{-x^3+x^4} + \frac{\left(3\sqrt[4]{-x^3+x^4}\right)\int \frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\,dx}{16\sqrt[4]{-1+x}\,x^{3/4}} + \frac{\sqrt[4]{-x^3+x^4}\int \left(\frac{10\sqrt[4]{-1+x}\,x^{3/4}}{3(-2+4x)}\right.}{2\sqrt[4]{-1+x}\,x}$$

$$= \frac{3}{16}\sqrt[4]{-x^3+x^4} - \frac{1}{4}(1-x)\sqrt[4]{-x^3+x^4} - \frac{\left(3\sqrt[4]{-x^3+x^4}\right)\int \frac{1}{(-1+x)^{3/4}\sqrt[4]{x}}\,dx}{64\sqrt[4]{-1+x}\,x^{3/4}} + \frac{\left(5\sqrt[4]{-x^3\right.}{}$$

$$= -\frac{1}{16}\sqrt[4]{-x^3+x^4} - \frac{1}{4}(1-x)\sqrt[4]{-x^3+x^4} - \frac{\left(3\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^4}}\,dx,x,\sqrt[4]{-\right.}{16\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= -\frac{1}{16}\sqrt[4]{-x^3+x^4} - \frac{1}{4}(1-x)\sqrt[4]{-x^3+x^4} + \frac{\left(5\sqrt[4]{-x^3+x^4}\right)\int \frac{1}{(-1+x)^{3/4}\sqrt[4]{x}}\,dx}{48\sqrt[4]{-1+x}\,x^{3/4}} - \frac{\left(3\sqrt[4]{-\right.}{}$$

$$= -\frac{1}{16}\sqrt[4]{-x^3+x^4} - \frac{1}{4}(1-x)\sqrt[4]{-x^3+x^4} - \frac{\left(3\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{-1+}}{\sqrt[4]{x}}\right.}{32\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= -\frac{1}{16}\sqrt[4]{-x^3+x^4} - \frac{1}{4}(1-x)\sqrt[4]{-x^3+x^4} - \frac{3\sqrt[4]{-x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{32\sqrt[4]{-1+x}\,x^{3/4}} - \frac{3\sqrt[4]{-x^3+}}{32}$$

$$= -\frac{1}{16}\sqrt[4]{-x^3+x^4} - \frac{1}{4}(1-x)\sqrt[4]{-x^3+x^4} - \frac{3\sqrt[4]{-x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{32\sqrt[4]{-1+x}\,x^{3/4}} + \frac{4\sqrt[4]{2}\sqrt[4]{-x}}{3}$$

$$= -\frac{1}{16}\sqrt[4]{-x^3+x^4} - \frac{1}{4}(1-x)\sqrt[4]{-x^3+x^4} + \frac{57\sqrt[4]{-x^3+x^4}\tan^{-1}\left(\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{32\sqrt[4]{-1+x}\,x^{3/4}} + \frac{4\sqrt[4]{2}\sqrt[4]{-}}{}$$

$$= -\frac{1}{16}\sqrt[4]{-x^3+x^4} - \frac{1}{4}(1-x)\sqrt[4]{-x^3+x^4} - \frac{5\sqrt[4]{-x^3+x^4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{x}}{\sqrt[4]{-1+x}}\right)}{12\sqrt{2}\sqrt[4]{-1+x}\,x^{3/4}} + \frac{5\sqrt[4]{-x}}{}$$

Mathematica [C] time = 0.18, size = 126, normalized size = 0.55

$$\frac{\sqrt[4]{(x-1)x^3}\left(-30\sqrt[4]{x}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};1-x\right)+12(x-1)\sqrt[4]{x}\,{}_2F_1\left(-\frac{3}{4},\frac{5}{4};\frac{9}{4};1-x\right)+5\left(27\sqrt[4]{x}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};1-x\right)-5\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{1}{x}-1\right)-16\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{x-1}{2x}\right)\right)\right)}{30x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(-x^3 + x^4)^(1/4))/(-1 + x + 2*x^2),x]
```

```
[Out] (((-1 + x)*x^3)^(1/4)*(-30*x^(1/4)*Hypergeometric2F1[-3/4, 1/4, 5/4, 1 - x]
 + 12*(-1 + x)*x^(1/4)*Hypergeometric2F1[-3/4, 5/4, 9/4, 1 - x] + 5*(27*x^(
1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, 1 - x] - 5*Hypergeometric2F1[1/4, 1,
5/4, -1 + x^(-1)] - 16*Hypergeometric2F1[1/4, 1, 5/4, (-1 + x)/(2*x)])))/(3
0*x)
```

IntegrateAlgebraic [A] time = 0.81, size = 230, normalized size = 1.00

$$\frac{1}{16}\sqrt[4]{x^4-x^3}\,(4x-5)-\frac{57}{32}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)+\frac{4}{3}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-x^3}}\right)+\frac{57}{32}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)-\frac{4}{3}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt[4]{x^4-x^3}}\right)+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt[4]{x^4-x^3}}{\sqrt{x^4-x^3}-x^2}\right)}{12\sqrt{2}}-\frac{5\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt{2}}+\frac{\sqrt{x^4-x^3}}{\sqrt{2}}}{x\sqrt[4]{x^4-x^3}}\right)}{12\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^2)*(-x^3 + x^4)^(1/4))/(-1 + x + 2*x^2),x]
```

```
[Out] ((-5 + 4*x)*(-x^3 + x^4)^(1/4))/16 - (57*ArcTan[x/(-x^3 + x^4)^(1/4)])/32 +
 (4*2^(1/4)*ArcTan[(2^(1/4)*x)/(-x^3 + x^4)^(1/4)])/3 + (5*ArcTan[(Sqrt[2]*
x*(-x^3 + x^4)^(1/4))/(-x^2 + Sqrt[-x^3 + x^4])])/(12*Sqrt[2]) + (57*ArcTan
h[x/(-x^3 + x^4)^(1/4)])/32 - (4*2^(1/4)*ArcTanh[(2^(1/4)*x)/(-x^3 + x^4)^(
1/4)])/3 - (5*ArcTanh[(x^2/Sqrt[2] + Sqrt[-x^3 + x^4]/Sqrt[2])/(x*(-x^3 + x
^4)^(1/4))])/(12*Sqrt[2])
```

fricas [B] time = 0.54, size = 423, normalized size = 1.84

$$\frac{5}{12}\sqrt{2}\arctan\left(\frac{\sqrt{2}x\sqrt{\frac{x^2+\sqrt{2}(x^4-x^3)^{\frac{1}{4}}x+\sqrt{x^4-x^3}}{x^2}}-x-\sqrt{2}(x^4-x^3)^{\frac{1}{4}}}{x}\right)+\frac{5}{12}\sqrt{2}\arctan\left(\frac{\sqrt{2}x\sqrt{\frac{x^2-\sqrt{2}(x^4-x^3)^{\frac{1}{4}}x+\sqrt{x^4-x^3}}{x^2}}+x-\sqrt{2}(x^4-x^3)^{\frac{1}{4}}}{x}\right)-\frac{5}{48}\sqrt{2}\log\left(\frac{4\left(x^2+\sqrt{2}(x^4-x^3)^{\frac{1}{4}}x+\sqrt{x^4-x^3}\right)}{x^2}\right)+\frac{5}{48}\sqrt{2}\log\left(\frac{4\left(x^2-\sqrt{2}(x^4-x^3)^{\frac{1}{4}}x+\sqrt{x^4-x^3}\right)}{x^2}\right)+\frac{1}{16}(x^4-x^3)^{\frac{1}{4}}(4x-5)+\frac{8}{3}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{2^{\frac{3}{4}}x\sqrt{\frac{\sqrt{2}x^2+\sqrt{x^4-x^3}}{x^2}}-2^{\frac{3}{4}}(x^4-x^3)^{\frac{1}{4}}}{2x}\right)-\frac{2}{3}\cdot 2^{\frac{1}{4}}\log\left(\frac{2^{\frac{1}{4}}x+(x^4-x^3)^{\frac{1}{4}}}{x}\right)+\frac{2}{3}\cdot 2^{\frac{1}{4}}\log\left(-\frac{2^{\frac{1}{4}}x-(x^4-x^3)^{\frac{1}{4}}}{x}\right)+\frac{57}{32}\arctan\left(\frac{(x^4-x^3)^{\frac{1}{4}}}{x}\right)+\frac{57}{64}\log\left(\frac{x+(x^4-x^3)^{\frac{1}{4}}}{x}\right)-\frac{57}{64}\log\left(-\frac{x-(x^4-x^3)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4-x^3)^(1/4)/(2*x^2+x-1),x, algorithm="fricas")
```

```
[Out] 5/12*sqrt(2)*arctan((sqrt(2)*x*sqrt((x^2 + sqrt(2)*(x^4 - x^3)^(1/4)*x + sq
rt(x^4 - x^3))/x^2) - x - sqrt(2)*(x^4 - x^3)^(1/4))/x) + 5/12*sqrt(2)*arct
an((sqrt(2)*x*sqrt((x^2 - sqrt(2)*(x^4 - x^3)^(1/4)*x + sqrt(x^4 - x^3))/x^
2) + x - sqrt(2)*(x^4 - x^3)^(1/4))/x) - 5/48*sqrt(2)*log(4*(x^2 + sqrt(2)*
(x^4 - x^3)^(1/4)*x + sqrt(x^4 - x^3))/x^2) + 5/48*sqrt(2)*log(4*(x^2 - sqr
t(2)*(x^4 - x^3)^(1/4)*x + sqrt(x^4 - x^3))/x^2) + 1/16*(x^4 - x^3)^(1/4)*(
4*x - 5) + 8/3*2^(1/4)*arctan(1/2*(2^(3/4)*x*sqrt((sqrt(2)*x^2 + sqrt(x^4 -
 x^3))/x^2) - 2^(3/4)*(x^4 - x^3)^(1/4))/x) - 2/3*2^(1/4)*log((2^(1/4)*x +
(x^4 - x^3)^(1/4))/x) + 2/3*2^(1/4)*log(-(2^(1/4)*x - (x^4 - x^3)^(1/4))/x)
 + 57/32*arctan((x^4 - x^3)^(1/4)/x) + 57/64*log((x + (x^4 - x^3)^(1/4))/x)
 - 57/64*log(-(x - (x^4 - x^3)^(1/4))/x)
```

giac [A] time = 0.48, size = 244, normalized size = 1.06

$$-\frac{1}{16}\left(5\left(-\frac{1}{x}+1\right)^{\frac{5}{4}}-\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)x^2+\frac{1}{3}\cdot 8^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)+\frac{5}{24}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)\right)+\frac{5}{24}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)\right)+\frac{5}{48}\sqrt{2}\log\left(\sqrt{2}\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+\sqrt{-\frac{1}{x}+1}+1\right)-\frac{5}{48}\sqrt{2}\log\left(-\sqrt{2}\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+\sqrt{-\frac{1}{x}+1}+1\right)+\frac{2}{3}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)-\frac{2}{3}\cdot 2^{\frac{1}{4}}\log\left(\left|-2^{\frac{1}{4}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right|\right)-\frac{57}{32}\arctan\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)-\frac{57}{64}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right)+\frac{57}{64}\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4-x^3)^(1/4)/(2*x^2+x-1),x, algorithm="giac")
```

```
[Out] -1/16*(5*(-1/x + 1)^(5/4) - (-1/x + 1)^(1/4))*x^2 + 1/3*8^(3/4)*arctan(1/2*
2^(3/4)*(-1/x + 1)^(1/4)) + 5/24*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(-
1/x + 1)^(1/4))) + 5/24*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(-1/x + 1)
^(1/4))) + 5/48*sqrt(2)*log(sqrt(2)*(-1/x + 1)^(1/4) + sqrt(-1/x + 1) + 1)
- 5/48*sqrt(2)*log(-sqrt(2)*(-1/x + 1)^(1/4) + sqrt(-1/x + 1) + 1) + 2/3*2^
(1/4)*log(2^(1/4) + (-1/x + 1)^(1/4)) - 2/3*2^(1/4)*log(abs(-2^(1/4) + (-1/
x + 1)^(1/4))) - 57/32*arctan((-1/x + 1)^(1/4)) - 57/64*log((-1/x + 1)^(1/4
) + 1) + 57/64*log(abs((-1/x + 1)^(1/4) - 1))
```

maple [C] time = 6.60, size = 2431, normalized size = 10.57

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4-x^3)^(1/4)/(2*x^2+x-1),x)
```

[Out] 1/16*(-5+4*x)*(x^3*(-1+x))^(1/4)+(57/64*ln((2*(x^4-3*x^3+3*x^2-x)^(3/4)+2*(x^4-3*x^3+3*x^2-x)^(1/2)*x+2*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+2*x^3-2*(x^4-3*x^3+3*x^2-x)^(1/2)-4*(x^4-3*x^3+3*x^2-x)^(1/4)*x-5*x^2+2*(x^4-3*x^3+3*x^2-x)^(1/4)+4*x-1)/(-1+x)^2)+57/64*RootOf(_Z^2+1)*ln((-2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+1)*x+2*RootOf(_Z^2+1)*x^3+2*(x^4-3*x^3+3*x^2-x)^(3/4)+2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+1)-2*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2-5*RootOf(_Z^2+1)*x^2+4*(x^4-3*x^3+3*x^2-x)^(1/4)*x+4*RootOf(_Z^2+1)*x-2*(x^4-3*x^3+3*x^2-x)^(1/4)-RootOf(_Z^2+1))/(-1+x)^2)-5/24*RootOf(_Z^2+RootOf(_Z^2+1))*ln((2*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2+1)*(x^4-3*x^3+3*x^2-x)^(1/2)*x-2*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2+1)*(x^4-3*x^3+3*x^2-x)^(1/2)+2*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^2+1)*x^2+2*(x^4-3*x^3+3*x^2-x)^(3/4)-4*RootOf(_Z^2+1)*(x^4-3*x^3+3*x^2-x)^(1/4)*x+RootOf(_Z^2+RootOf(_Z^2+1))*x^2+2*RootOf(_Z^2+1)*(x^4-3*x^3+3*x^2-x)^(1/4)-2*RootOf(_Z^2+RootOf(_Z^2+1))*x+RootOf(_Z^2+RootOf(_Z^2+1)))/(-1+x)^2/(-1+2*x))+5/24*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))*ln((-2*RootOf(_Z^2+RootOf(_Z^2+1))*(x^4-3*x^3+3*x^2-x)^(1/2)*x-2*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^2+1)*x^2-RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2+1)*x^2+2*(x^4-3*x^3+3*x^2-x)^(3/4)+2*RootOf(_Z^2+RootOf(_Z^2+1))*(x^4-3*x^3+3*x^2-x)^(1/2)+4*RootOf(_Z^2+1)*(x^4-3*x^3+3*x^2-x)^(1/4)*x+2*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2+1)*x-2*RootOf(_Z^2+1)*(x^4-3*x^3+3*x^2-x)^(1/4)-RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1)))/(-1+x)^2/(-1+2*x))+1/192*RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))-16384*RootOf(_Z^2+RootOf(_Z^2+1)))*ln((-2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))-16384*RootOf(_Z^2+RootOf(_Z^2+1)))*RootOf(_Z^2+1)*x+2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))-16384*RootOf(_Z^2+RootOf(_Z^2+1)))*RootOf(_Z^2+1)-2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))-16384*RootOf(_Z^2+RootOf(_Z^2+1)))*x+256*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))*x^2+2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))-16384*RootOf(_Z^2+RootOf(_Z^2+1)))-512*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2+1)*(x^4-3*x^3+3*x^2-x)^(1/4)*x+256*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^2+RootOf(_Z^2+1))*x^2-3*RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))-16384*RootOf(_Z^2+RootOf(_Z^2+1)))*x^3+512*(x^4-3*x^3+3*x^2-x)^(3/4)+256*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2+1)*(x^4-3*x^3+3*x^2-x)^(1/4)-512*RootOf(_Z^2+RootOf(_Z^2+1))*(x^4-3*x^3+3*x^2-x)^(1/4)*x+7*RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))-16384*RootOf(_Z^2+RootOf(_Z^2+1)))*x^2+256*RootOf(_Z^2+RootOf(_Z^2+1))*(x^4-3*x^3+3*x^2-x)^(1/4)-5*RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))-16384*RootOf(_Z^2+RootOf(_Z^2+1)))*x+RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))-16384*RootOf(_Z^2+RootOf(_Z^2+1))))/(-1+x)^2/(1+x))+1/192*RootOf(_Z^2+1)*RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))-16384*RootOf(_Z^2+RootOf(_Z^2+1)))*ln((2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))-16384*RootOf(_Z^2+RootOf(_Z^2+1)))*RootOf(_Z^2+1)*x-2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))-16384*RootOf(_Z^2+RootOf(_Z^2+1)))*RootOf(_Z^2+1)-2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))-16384*RootOf(_Z^2+RootOf(_Z^2+1)))*x-256*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))*x^2-3*RootOf(_Z^2+1)*RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))-16384*RootOf(_Z^2+RootOf(_Z^2+1)))*x^3+2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))-16384*RootOf(_Z^2+RootOf(_Z^2+1)))+512*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2+1)*(x^4-3*x^3+3*x^2-x)^(1/4)*x-256*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^2+RootOf(_Z^2+1))*x^2+7*RootOf(_Z^2+1)*RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))-16384*RootOf(_Z^2+RootOf(_Z^2+1)))*x^2+512*(x^4-3*x^3+3*x^2-x)^(3/4)-256*RootOf(_Z^2+RootOf(_Z^2+1))*RootOf(_Z^2+1)*(x^4-3*x^3+3*x^2-x)^(1/4)

```
+512*RootOf(_Z^2+RootOf(_Z^2+1))*(x^4-3*x^3+3*x^2-x)^(1/4)*x-5*RootOf(_Z^2+
1)*RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf(_Z^2+1))-16384*RootO
f(_Z^2+RootOf(_Z^2+1)))*x-256*RootOf(_Z^2+RootOf(_Z^2+1))*(x^4-3*x^3+3*x^2-
x)^(1/4)+RootOf(_Z^2+1)*RootOf(_Z^2-16384*RootOf(_Z^2+1)*RootOf(_Z^2+RootOf
(_Z^2+1))-16384*RootOf(_Z^2+RootOf(_Z^2+1))))/(-1+x)^2/(1+x)))*(x^3*(-1+x))
^(1/4)/(-1+x)/x*(x*(-1+x)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^3\right)^{\frac{1}{4}}\left(x^2+1\right)}{2\,x^2+x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4-x^3)^(1/4)/(2*x^2+x-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^3)^(1/4)*(x^2 + 1)/(2*x^2 + x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^2+1\right)\,\left(x^4-x^3\right)^{1/4}}{2\,x^2+x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*(x^4 - x^3)^(1/4))/(x + 2*x^2 - 1),x)
```

```
[Out] int(((x^2 + 1)*(x^4 - x^3)^(1/4))/(x + 2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\,(x-1)}\,\left(x^2+1\right)}{(x+1)\,(2x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4-x**3)**(1/4)/(2*x**2+x-1),x)
```

```
[Out] Integral((x**3*(x - 1))**(1/4)*(x**2 + 1)/((x + 1)*(2*x - 1)), x)
```

3.2105 $\int \frac{1}{\sqrt[3]{-1+x^2}\left(3+x^2\right)}\,dx$

Optimal. Leaf size=231

$$\frac{\tan^{-1}\left(\frac{2^{2/3}x}{2\sqrt{3}\sqrt[3]{x^2-1}+2^{2/3}\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{3\ 2^{2/3}x\sqrt[3]{x^2-1}}{-\sqrt[3]{2}x^2-6\left(x^2-1\right)^{2/3}+3\ 2^{2/3}\sqrt[3]{x^2-1}-3\sqrt[3]{2}}\right)}{6\ 2^{2/3}}-\frac{i\tanh^{-1}\left(\frac{2i\sqrt[3]{2}\sqrt{3}x-i2^{2/3}\sqrt{3}x\sqrt[3]{x^2-1}}{\sqrt[3]{2}x^2-6\left(x^2-1\right)^{2/3}+3\ 2^{2/3}\sqrt[3]{x^2-1}-3\sqrt[3]{2}}\right.}{6\ 2^{2/3}\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 136, normalized size of antiderivative = 0.59, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {393}

$$-\frac{(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left((-1)^{2/3}\sqrt[3]{2}\sqrt[3]{x^2-1}+1\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{1}{2}\left(-\frac{1}{2}\right)^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{-1}x}{\sqrt[3]{2}\sqrt[3]{x^2-1}+\sqrt[3]{-1}}\right)-\frac{(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{1}{6}\left(-\frac{1}{2}\right)^{2/3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -1/2*((-1)^(2/3)*ArcTan[Sqrt[3]/x])/(2^(2/3)*Sqrt[3]) - ((-1)^(2/3)*ArcTan[
(Sqrt[3]*(1 + (-1)^(2/3)*2^(1/3)*(-1 + x^2)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3])
 + ((-1/2)^(2/3)*ArcTanh[x])/6 - ((-1/2)^(2/3)*ArcTanh[((-1)^(1/3)*x)/((-1)
^(1/3) + 2^(1/3)*(-1 + x^2)^(1/3))])/2
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-1+x^2}\left(3+x^2\right)}\,dx=-\frac{(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1+(-1)^{2/3}\sqrt[3]{2}\sqrt[3]{-1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{1}{6}\left(-\frac{1}{2}\right)^{2/3}\tanh^{-1}($$

Mathematica [C] time = 0.16, size = 116, normalized size = 0.50

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{x^2-1}\left(x^2+3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-1 + x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((-1 + x^2)^(1/3)*(3 + x^2
)*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3,
 2, 5/2, x^2, -1/3*x^2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

IntegrateAlgebraic [F] time = 4.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-1+x^2}\,(3+x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((-1 + x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((-1 + x^2)^(1/3)*(3 + x^2)), x]
```

fricas [B] time = 1.49, size = 1896, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] 1/20736*432^(5/6)*sqrt(3)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 2
7) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) +
 216*2^(1/3)*(x^4 + 3*x^2))*(x^2 - 1)^(2/3) + 72*(x^5 + 18*x^4 + 24*x^3 - 1
8*x^2 - 9*x)*(x^2 - 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5
/6)*sqrt(3)*log(2592*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) + 144*432^(1
/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) + 216*2^(1/3)*(x
^4 + 3*x^2))*(x^2 - 1)^(2/3) + 72*(x^5 + 18*x^4 + 24*x^3 - 18*x^2 - 9*x)*(x
^2 - 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/20736*432^(5/6)*sqrt(3)*log
(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x
^5 - x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(
x^2 - 1)^(2/3) - 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(x^2 - 1)^(1/3))
/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/20736*432^(5/6)*sqrt(3)*log(2592*(6*2^(2/
3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 - x^3) - (43
2^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(x^2 - 1)^(2/3)
 - 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(x^2 - 1)^(1/3))/(x^6 + 9*x^4 +
 27*x^2 + 27)) + 1/1296*432^(5/6)*arctan(-1/36*(432^(5/6)*(x^5 - 18*x^3 + 9
*x)*(x^2 - 1)^(1/3) - sqrt(3)*2^(1/3)*(432^(5/6)*(x^4 + 9*x^2)*(x^2 - 1)^(2
/3) + 288*sqrt(3)*(2*x^4 - 3*x^2)*(x^2 - 1)^(1/3) + 6*432^(1/6)*(x^6 + 141*
x^4 - 153*x^2 + 27)) + 648*432^(1/6)*(3*x^3 - x)*(x^2 - 1)^(2/3) + 72*sqrt(
3)*(7*x^5 + 6*x^3 - 9*x))/(x^6 - 225*x^4 + 243*x^2 - 27)) + 1/2592*432^(5/6
)*arctan(-1/18*(sqrt(2)*(18*sqrt(3)*2^(2/3)*(29*x^11 + 879*x^9 - 12078*x^7
+ 10638*x^5 - 3807*x^3 + 243*x) - 2*(x^2 - 1)^(2/3)*(432^(5/6)*(x^10 + 153*
x^8 - 1701*x^6 + 459*x^4) - 216*sqrt(3)*2^(1/3)*(31*x^9 - 297*x^7 - 27*x^5
- 27*x^3)) + 36*(x^2 - 1)^(1/3)*(sqrt(3)*(x^11 + 1167*x^9 - 13158*x^7 + 175
50*x^5 - 4779*x^3 + 243*x) - 8*sqrt(3)*(13*x^10 - 6*x^8 - 1404*x^6 + 1350*x
^4 - 81*x^2)) - 3*432^(1/6)*(x^12 + 7620*x^10 - 92115*x^8 + 169776*x^6 - 10
9269*x^4 + 16524*x^2 - 729))*sqrt((6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27)
 + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) + 2
16*2^(1/3)*(x^4 + 3*x^2))*(x^2 - 1)^(2/3) + 72*(x^5 + 18*x^4 + 24*x^3 - 18*
x^2 - 9*x)*(x^2 - 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 216*(sqrt(3)*2^(
2/3)*(x^10 + 144*x^8 - 918*x^6 + 2808*x^4 - 243*x^2) - 3*432^(1/6)*(31*x^9
- 568*x^7 + 1710*x^5 - 432*x^3 + 27*x))*(x^2 - 1)^(2/3) - 18*sqrt(3)*(x^12
- 366*x^10 + 14535*x^8 - 42660*x^6 + 58239*x^4 - 14094*x^2 + 729) + 144*sqr
t(3)*(11*x^11 - 807*x^9 + 4518*x^7 - 5238*x^5 + 3807*x^3 - 243*x) + (x^2 -
1)^(1/3)*(432^(5/6)*(x^11 - 1215*x^9 + 11754*x^7 - 21006*x^5 + 5589*x^3 - 2
43*x) - 432*sqrt(3)*2^(1/3)*(13*x^10 - 120*x^8 + 1242*x^6 - 1728*x^4 + 81*x
^2)))/(x^12 - 8334*x^10 + 110727*x^8 - 301860*x^6 + 187839*x^4 - 21870*x^2
+ 729)) + 1/2592*432^(5/6)*arctan(1/18*(sqrt(2)*(18*sqrt(3)*2^(2/3)*(29*x^1
1 + 879*x^9 - 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) + 2*(x^2 - 1)^(2/3)
*(432^(5/6)*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) + 216*sqrt(3)*2^(1/3)*(31
*x^9 - 297*x^7 - 27*x^5 - 27*x^3)) + 36*(x^2 - 1)^(1/3)*(sqrt(3)*(x^11 + 11
67*x^9 - 13158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) + 8*sqrt(3)*(13*x^10 - 6
*x^8 - 1404*x^6 + 1350*x^4 - 81*x^2)) + 3*432^(1/6)*(x^12 + 7620*x^10 - 921
```

```
15*x^8 + 169776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sqrt((6*2^(2/3)*(x^6 +
 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 - x^3) - (432^(5/6)*s
qrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(x^2 - 1)^(2/3) - 72*(x^5
 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(x^2 - 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 +
 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^6 + 2808*x^4 - 243*x^2
) + 3*432^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^3 + 27*x))*(x^2 - 1)^(
2/3) - 18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 42660*x^6 + 58239*x^4 - 14
094*x^2 + 729) - 144*sqrt(3)*(11*x^11 - 807*x^9 + 4518*x^7 - 5238*x^5 + 380
7*x^3 - 243*x) - (x^2 - 1)^(1/3)*(432^(5/6)*(x^11 - 1215*x^9 + 11754*x^7 -
21006*x^5 + 5589*x^3 - 243*x) + 432*sqrt(3)*2^(1/3)*(13*x^10 - 120*x^8 + 12
42*x^6 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 110727*x^8 - 301860*x^6 +
 187839*x^4 - 21870*x^2 + 729))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3\right)\left(x^2-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)*(x^2 - 1)^(1/3)), x)
```

maple [C] time = 18.42, size = 874, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/3)/(x^2+3),x)
```

```
[Out] -1/432*ln((RootOf(_Z^6+108)^4*x^6-72*x^5*RootOf(_Z^6+108)^4+225*RootOf(_Z^6
+108)^4*x^4+36*RootOf(_Z^6+108)^2*(x^2-1)^(1/3)*x^5+72*RootOf(_Z^6+108)^4*x
^3-648*RootOf(_Z^6+108)^2*(x^2-1)^(1/3)*x^4-189*RootOf(_Z^6+108)^4*x^2+864*
RootOf(_Z^6+108)^2*(x^2-1)^(1/3)*x^3+648*(x^2-1)^(2/3)*x^4+648*RootOf(_Z^6+
108)^2*(x^2-1)^(1/3)*x^2-4536*x^3*(x^2-1)^(2/3)+27*RootOf(_Z^6+108)^4-324*R
ootOf(_Z^6+108)^2*(x^2-1)^(1/3)*x+1944*(x^2-1)^(2/3)*x^2+1944*x*(x^2-1)^(2/
3))/(x^2+3)^3)*RootOf(_Z^6+108)^4-1/72*ln((RootOf(_Z^6+108)^4*x^6-72*x^5*Ro
otOf(_Z^6+108)^4+225*RootOf(_Z^6+108)^4*x^4+36*RootOf(_Z^6+108)^2*(x^2-1)^(
1/3)*x^5+72*RootOf(_Z^6+108)^4*x^3-648*RootOf(_Z^6+108)^2*(x^2-1)^(1/3)*x^4
-189*RootOf(_Z^6+108)^4*x^2+864*RootOf(_Z^6+108)^2*(x^2-1)^(1/3)*x^3+648*(x
^2-1)^(2/3)*x^4+648*RootOf(_Z^6+108)^2*(x^2-1)^(1/3)*x^2-4536*x^3*(x^2-1)^(
2/3)+27*RootOf(_Z^6+108)^4-324*RootOf(_Z^6+108)^2*(x^2-1)^(1/3)*x+1944*(x^2
-1)^(2/3)*x^2+1944*x*(x^2-1)^(2/3))/(x^2+3)^3)*RootOf(_Z^6+108)-1/36*RootOf
(_Z^6+108)*ln((1296*(x^2-1)^(2/3)*x^4+3888*(x^2-1)^(2/3)*x^2-9072*x^3*(x^2-
1)^(2/3)+3888*x*(x^2-1)^(2/3)-RootOf(_Z^6+108)^4*x^6-225*RootOf(_Z^6+108)^4
*x^4+72*x^5*RootOf(_Z^6+108)^4-1296*x^5*RootOf(_Z^6+108)+4050*RootOf(_Z^6+1
08)*x^4+486*RootOf(_Z^6+108)-27*RootOf(_Z^6+108)^4+189*RootOf(_Z^6+108)^4*x
^2-3402*RootOf(_Z^6+108)*x^2-72*RootOf(_Z^6+108)^4*x^3+1296*RootOf(_Z^6+108
)*x^3+18*RootOf(_Z^6+108)*x^6-36*RootOf(_Z^6+108)^2*(x^2-1)^(1/3)*x^5+648*R
ootOf(_Z^6+108)^2*(x^2-1)^(1/3)*x^4-864*RootOf(_Z^6+108)^2*(x^2-1)^(1/3)*x^
3-648*RootOf(_Z^6+108)^2*(x^2-1)^(1/3)*x^2+324*RootOf(_Z^6+108)^2*(x^2-1)^(
1/3)*x+6*RootOf(_Z^6+108)^5*(x^2-1)^(1/3)*x^5-108*RootOf(_Z^6+108)^5*(x^2-1
)^(1/3)*x^4+144*RootOf(_Z^6+108)^5*(x^2-1)^(1/3)*x^3+108*RootOf(_Z^6+108)^5
*(x^2-1)^(1/3)*x^2-54*RootOf(_Z^6+108)^5*(x^2-1)^(1/3)*x)/(x^2+3)^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3\right)\left(x^2-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/3)/(x^2+3),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 3)*(x^2 - 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(x^2-1\right)^{1/3}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/3)*(x^2 + 3)),x)
[Out] int(1/((x^2 - 1)^(1/3)*(x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/3)/(x**2+3),x)
[Out] Integral(1/(((x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.2106 $\int \frac{1+x}{\left(1+3x+x^2\right)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=231

$$\frac{\log\left(5\sqrt[3]{1-x^3}+\sqrt[3]{2}\,5^{2/3}x-\sqrt[3]{2}\,5^{2/3}\right)}{\sqrt[3]{2}\,5^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{5\sqrt{3}\sqrt[3]{1-x^3}}{5\sqrt[3]{1-x^3}-2\sqrt[3]{2}\,5^{2/3}x+2\sqrt[3]{2}\,5^{2/3}}\right)}{\sqrt[3]{2}\,5^{2/3}}-\frac{\log\left(5\left(1-x^3\right)^{2/3}+\left(\sqrt[3]{2}\,5^{2/3}-\sqrt[3]{\ }\right.\right.}{}$$

Rubi [F] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x}{\left(1+3x+x^2\right)\sqrt[3]{1-x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x)/((1 + 3*x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] ((5 - Sqrt[5])*Defer[Int][1/((3 - Sqrt[5] + 2*x)*(1 - x^3)^(1/3)), x])/5 +
((5 + Sqrt[5])*Defer[Int][1/((3 + Sqrt[5] + 2*x)*(1 - x^3)^(1/3)), x])/5
```

Rubi steps

$$\int \frac{1+x}{\left(1+3x+x^2\right)\sqrt[3]{1-x^3}}\,dx=\int\left(\frac{1-\frac{1}{\sqrt{5}}}{\left(3-\sqrt{5}+2x\right)\sqrt[3]{1-x^3}}+\frac{1+\frac{1}{\sqrt{5}}}{\left(3+\sqrt{5}+2x\right)\sqrt[3]{1-x^3}}\right)dx$$
$$=\frac{1}{5}\left(5-\sqrt{5}\right)\int\frac{1}{\left(3-\sqrt{5}+2x\right)\sqrt[3]{1-x^3}}\,dx+\frac{1}{5}\left(5+\sqrt{5}\right)\int\frac{1}{\left(3+\sqrt{5}+2x\right)\sqrt[3]{1}}$$

Mathematica [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1+x}{\left(1+3x+x^2\right)\sqrt[3]{1-x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x)/((1 + 3*x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] Integrate[(1 + x)/((1 + 3*x + x^2)*(1 - x^3)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 1.26, size = 231, normalized size = 1.00

$$\frac{\log\left(5\sqrt[3]{1-x^3}+\sqrt[3]{2}\,5^{2/3}x-\sqrt[3]{2}\,5^{2/3}\right)}{\sqrt[3]{2}\,5^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{5\sqrt{3}\sqrt[3]{1-x^3}}{5\sqrt[3]{1-x^3}-2\sqrt[3]{2}\,5^{2/3}x+2\sqrt[3]{2}\,5^{2/3}}\right)}{\sqrt[3]{2}\,5^{2/3}}-\frac{\log\left(5\left(1-x^3\right)^{2/3}+\left(\sqrt[3]{2}\,5^{2/3}-\sqrt[3]{2}\,5^{2/3}x\right)\sqrt[3]{1-x^3}+2^{2/3}\sqrt[3]{5}x^2-2\,2^{2/3}\sqrt[3]{5}x+2^{2/3}\sqrt[3]{5}\right)}{2\sqrt[3]{2}\,5^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((1 + 3*x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(5*Sqrt[3]*(1 - x^3)^(1/3))/(2*2^(1/3)*5^(2/3) - 2*2^(1/3)*
5^(2/3)*x + 5*(1 - x^3)^(1/3))])/(2^(1/3)*5^(2/3)) + Log[-(2^(1/3)*5^(2/3))
 + 2^(1/3)*5^(2/3)*x + 5*(1 - x^3)^(1/3)]/(2^(1/3)*5^(2/3)) - Log[2^(2/3)*5
^(1/3) - 2*2^(2/3)*5^(1/3)*x + 2^(2/3)*5^(1/3)*x^2 + (2^(1/3)*5^(2/3) - 2^(
1/3)*5^(2/3)*x)*(1 - x^3)^(1/3) + 5*(1 - x^3)^(2/3)]/(2*2^(1/3)*5^(2/3))
```

fricas [A] time = 10.16, size = 324, normalized size = 1.40

$$\frac{1}{30}\cdot 50^{\frac{1}{6}}\sqrt{3}\sqrt{2}\arctan\left(\frac{50^{\frac{1}{6}}\sqrt{3}\left(2\cdot 50^{\frac{2}{3}}\sqrt{2}\left(3x^4+8x^3+3x^2+8x+3\right)\left(-x^3+1\right)^{\frac{2}{3}}+50^{\frac{1}{3}}\sqrt{2}\left(41x^6-11x^5+50x^4-35x^3+50x^2-11x+41\right)-20\sqrt{2}\left(11x^5-15x^4+15x^3-15x^2+15x-11\right)\left(-x^3+1\right)^{\frac{1}{3}}\right)}{30\left(19x^6-69x^5+30x^4-85x^3+30x^2-69x+19\right)}\right)-\frac{1}{300}\cdot 50^{\frac{2}{3}}\log\left(\frac{50^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{2}{3}}\left(3x^2-x+3\right)+50^{\frac{1}{3}}\left(11x^4-4x^3+11x^2-4x+11\right)-20\left(2x^3-x^2+x-2\right)\left(-x^3+1\right)^{\frac{1}{3}}}{x^4+6x^3+11x^2+6x+1}\right)+\frac{1}{150}\cdot 50^{\frac{2}{3}}\log\left(\frac{50^{\frac{2}{3}}\left(x^2+3x+1\right)-10\cdot 50^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}(x-1)-50\left(-x^3+1\right)^{\frac{2}{3}}}{x^2+3x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+3*x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/30*50^(1/6)*sqrt(3)*sqrt(2)*arctan(1/30*50^(1/6)*sqrt(3)*(2*50^(2/3)*sqrt
(2)*(3*x^4 + 8*x^3 + 3*x^2 + 8*x + 3)*(-x^3 + 1)^(2/3) + 50^(1/3)*sqrt(2)*(
41*x^6 - 11*x^5 + 50*x^4 - 35*x^3 + 50*x^2 - 11*x + 41) - 20*sqrt(2)*(11*x^
5 - 15*x^4 + 15*x^3 - 15*x^2 + 15*x - 11)*(-x^3 + 1)^(1/3))/(19*x^6 - 69*x^
5 + 30*x^4 - 85*x^3 + 30*x^2 - 69*x + 19)) - 1/300*50^(2/3)*log((50^(2/3)*(
-x^3 + 1)^(2/3)*(3*x^2 - x + 3) + 50^(1/3)*(11*x^4 - 4*x^3 + 11*x^2 - 4*x +
 11) - 20*(2*x^3 - x^2 + x - 2)*(-x^3 + 1)^(1/3))/(x^4 + 6*x^3 + 11*x^2 + 6
*x + 1)) + 1/150*50^(2/3)*log((50^(2/3)*(x^2 + 3*x + 1) - 10*50^(1/3)*(-x^3
 + 1)^(1/3)*(x - 1) - 50*(-x^3 + 1)^(2/3))/(x^2 + 3*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^2+3x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+3*x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/((-x^3 + 1)^(1/3)*(x^2 + 3*x + 1)), x)
```

maple [C] time = 9.19, size = 1175, normalized size = 5.09

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2+3*x+1)/(-x^3+1)^(1/3),x)
```

```
[Out] -1/10*ln(-(15*RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(_Z^3-20)+100*_Z^2)*Root
Of(_Z^3-20)^3*x+100*RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(_Z^3-20)+100*_Z^2
)^2*RootOf(_Z^3-20)^2*x+25*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-20)^2+10*_Z*Ro
otOf(_Z^3-20)+100*_Z^2)*RootOf(_Z^3-20)^2-11*(-x^3+1)^(1/3)*RootOf(_Z^3-20)
^2*x-50*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(_Z^3-20)+100*_
Z^2)*RootOf(_Z^3-20)*x+11*(-x^3+1)^(1/3)*RootOf(_Z^3-20)^2+50*(-x^3+1)^(1/3
)*RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(_Z^3-20)+100*_Z^2)*RootOf(_Z^3-20)+
39*RootOf(_Z^3-20)*x^2+260*RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(_Z^3-20)+1
00*_Z^2)*x^2+27*RootOf(_Z^3-20)*x+180*RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf
(_Z^3-20)+100*_Z^2)*x+110*(-x^3+1)^(2/3)+39*RootOf(_Z^3-20)+260*RootOf(Root
Of(_Z^3-20)^2+10*_Z*RootOf(_Z^3-20)+100*_Z^2))/(x^2+3*x+1))*RootOf(_Z^3-20)
-ln(-(15*RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(_Z^3-20)+100*_Z^2)*RootOf(_Z
^3-20)^3*x+100*RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(_Z^3-20)+100*_Z^2)^2*R
ootOf(_Z^3-20)^2*x+25*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(
_Z^3-20)+100*_Z^2)*RootOf(_Z^3-20)^2-11*(-x^3+1)^(1/3)*RootOf(_Z^3-20)^2*x-
50*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(_Z^3-20)+100*_Z^2)*
RootOf(_Z^3-20)*x+11*(-x^3+1)^(1/3)*RootOf(_Z^3-20)^2+50*(-x^3+1)^(1/3)*Roo
tOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(_Z^3-20)+100*_Z^2)*RootOf(_Z^3-20)+39*Ro
otOf(_Z^3-20)*x^2+260*RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(_Z^3-20)+100*_Z
^2)*x^2+27*RootOf(_Z^3-20)*x+180*RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(_Z^3
-20)+100*_Z^2)*x+110*(-x^3+1)^(2/3)+39*RootOf(_Z^3-20)+260*RootOf(RootOf(_Z
^3-20)^2+10*_Z*RootOf(_Z^3-20)+100*_Z^2))/(x^2+3*x+1))*RootOf(RootOf(_Z^3-2
0)^2+10*_Z*RootOf(_Z^3-20)+100*_Z^2)+RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(
_Z^3-20)+100*_Z^2)*ln((5*RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(_Z^3-20)+100
```

```
*_Z^2)*RootOf(_Z^3-20)^3*x-100*RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(_Z^3-2
0)+100*_Z^2)^2*RootOf(_Z^3-20)^2*x+25*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-20)
^2+10*_Z*RootOf(_Z^3-20)+100*_Z^2)*RootOf(_Z^3-20)^2+6*(-x^3+1)^(1/3)*RootO
f(_Z^3-20)^2*x-50*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(_Z^3
-20)+100*_Z^2)*RootOf(_Z^3-20)*x-6*(-x^3+1)^(1/3)*RootOf(_Z^3-20)^2+50*(-x^
3+1)^(1/3)*RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(_Z^3-20)+100*_Z^2)*RootOf(
_Z^3-20)-13*RootOf(_Z^3-20)*x^2+260*RootOf(RootOf(_Z^3-20)^2+10*_Z*RootOf(_
Z^3-20)+100*_Z^2)*x^2+RootOf(_Z^3-20)*x-20*RootOf(RootOf(_Z^3-20)^2+10*_Z*R
ootOf(_Z^3-20)+100*_Z^2)*x-60*(-x^3+1)^(2/3)-13*RootOf(_Z^3-20)+260*RootOf(
RootOf(_Z^3-20)^2+10*_Z*RootOf(_Z^3-20)+100*_Z^2))/(x^2+3*x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^2+3\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+3*x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
[Out] integrate((x + 1)/((-x^3 + 1)^(1/3)*(x^2 + 3*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x+1}{\left(1-x^3\right)^{1/3}\,\left(x^2+3\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((1 - x^3)^(1/3)*(3*x + x^2 + 1)),x)
[Out] int((x + 1)/((1 - x^3)^(1/3)*(3*x + x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt[3]{-\left(x-1\right)\left(x^2+x+1\right)}\,\left(x^2+3x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+3*x+1)/(-x**3+1)**(1/3),x)
[Out] Integral((x + 1)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 + 3*x + 1)), x)
```

3.2107 $$\int \frac{3k+2(1+k^2)x-k(1+k^2)x^2-4k^2x^3-k^3x^4}{((1-x^2)(1-k^2x^2))^{2/3}(1-d-(1+2d)kx-(1+dk^2)x^2+kx^3)}\,dx$$

Optimal. Leaf size=231

$$-\frac{\log\left(-\sqrt[3]{d}\,kx-\sqrt[3]{d}+\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}\right)}{d^{2/3}}+\frac{\log\left(d^{2/3}k^2x^2+2d^{2/3}kx+d^{2/3}+\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}\,(\cdot\right.}{2d^{2/3}}$$

Rubi [F] time = 9.49, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3k+2(1+k^2)x-k(1+k^2)x^2-4k^2x^3-k^3x^4}{((1-x^2)(1-k^2x^2))^{2/3}(1-d-(1+2d)kx-(1+dk^2)x^2+kx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3*k + 2*(1 + k^2)*x - k*(1 + k^2)*x^2 - 4*k^2*x^3 - k^3*x^4)/(((1 - x^
2)*(1 - k^2*x^2))^(2/3)*(1 - d - (1 + 2*d)*k*x - (1 + d*k^2)*x^2 + k*x^3)),
x]
```

```
[Out] -((k*(5 + d*k^2)*x*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*AppellF1[1/2, 2/3, 2
/3, 3/2, x^2, k^2*x^2])/((1 - x^2)*(1 - k^2*x^2))^(2/3)) + (3^(3/4)*Sqrt[2
+ Sqrt[3]]*k^(4/3)*Sqrt[(-1 - k^2 + 2*k^2*x^2)^2]*((-1 + k^2)^(2/3) + 2^(2/
3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))*Sqrt[((-1 + k^2)^(4/3) - 2^(2/3
)*k^(2/3)*(-1 + k^2)^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3) + 2*2^(1/3)*k^(4
/3)*((1 - x^2)*(1 - k^2*x^2))^(2/3))/((1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2
/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))/((
1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(
1/3))], -7 - 4*Sqrt[3]])/(2^(2/3)*(1 + k^2 - 2*k^2*x^2)*Sqrt[(-1 - k^2*(1 -
 2*x^2))^2]*Sqrt[((-1 + k^2)^(2/3)*((-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1
- x^2)*(1 - k^2*x^2))^(1/3)))/((1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(
2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))^2]) + (k*(8 - d^2*k^2 - d*(5 - k^2))*
(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][1/((1 - x^2)^(2/3)*(1 - k^2*
x^2)^(2/3)*(1 - d - (1 + 2*d)*k*x - (1 + d*k^2)*x^2 + k*x^3)), x])/((1 - x^
2)*(1 - k^2*x^2))^(2/3) + ((2 - (2 + 11*d)*k^2 - d*(1 + 2*d)*k^4)*(1 - x^2)
^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][x/((1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3
)*(1 - d - (1 + 2*d)*k*x - (1 + d*k^2)*x^2 + k*x^3)), x])/((1 - x^2)*(1 - k
^2*x^2))^(2/3) - (k*(6 + (2 + 8*d)*k^2 + d^2*k^4)*(1 - x^2)^(2/3)*(1 - k^2*
x^2)^(2/3)*Defer[Int][x^2/((1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*(1 - d - (1
+ 2*d)*k*x - (1 + d*k^2)*x^2 + k*x^3)), x])/((1 - x^2)*(1 - k^2*x^2))^(2/3)
```

Rubi steps

$$\int \frac{3k+2\left(1+k^2\right)x-k\left(1+k^2\right)x^2-4k^2x^3-k^3x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d-(1+2d)kx-\left(1+dk^2\right)x^2+kx^3\right)}\,dx = \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int \frac{3k\cdots}{\left(1-x^2\right)^{2/3}\cdots}}{\left(\left(1-x^2\right)\left(1\cdots\right.\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int\left(-\frac{k\cdots}{\left(1-x^2\right)\cdots}\right.}{\cdots}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int \frac{k\left(8-d^2k^2\cdots\right.}{\cdots}}{\left(\left(1\cdots\right.\right.}$$

$$= -\frac{k\left(5+dk^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{\cdots}}{\left(\left(1-x^2\right)\left(1-k^2\cdots\right.\right.}$$

$$= -\frac{k\left(5+dk^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{\cdots}}{\left(\left(1-x^2\right)\left(1-k^2\cdots\right.\right.}$$

$$= -\frac{k\left(5+dk^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{\cdots}}{\left(\left(1-x^2\right)\left(1-k^2\cdots\right.\right.}$$

$$= -\frac{k\left(5+dk^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{\cdots}}{\left(\left(1-x^2\right)\left(1-k^2\cdots\right.\right.}$$

Mathematica [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{3k+2\left(1+k^2\right)x-k\left(1+k^2\right)x^2-4k^2x^3-k^3x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d-(1+2d)kx-\left(1+dk^2\right)x^2+kx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3*k + 2*(1 + k^2)*x - k*(1 + k^2)*x^2 - 4*k^2*x^3 - k^3*x^4)/(((
1 - x^2)*(1 - k^2*x^2))^(2/3)*(1 - d - (1 + 2*d)*k*x - (1 + d*k^2)*x^2 + k*
x^3)),x]
```

```
[Out] Integrate[(3*k + 2*(1 + k^2)*x - k*(1 + k^2)*x^2 - 4*k^2*x^3 - k^3*x^4)/(((
1 - x^2)*(1 - k^2*x^2))^(2/3)*(1 - d - (1 + 2*d)*k*x - (1 + d*k^2)*x^2 + k*
x^3)), x]
```

IntegrateAlgebraic [A] time = 8.18, size = 231, normalized size = 1.00

$$-\frac{\log\left(-\sqrt[3]{d}\,kx-\sqrt[3]{d}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\right)}{d^{2/3}}+\frac{\log\left(d^{2/3}k^2x^2+2d^{2/3}kx+d^{2/3}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\left(\sqrt[3]{d}\,kx+\sqrt[3]{d}\right)+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}\right)}{2d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\,kx+\sqrt{3}\sqrt[3]{d}}{\sqrt[3]{d}\,kx+\sqrt[3]{d}+2\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*k + 2*(1 + k^2)*x - k*(1 + k^2)*x^2 - 4*k^2*x^3 - k^3
*x^4)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)*(1 - d - (1 + 2*d)*k*x - (1 + d*k^2)
*x^2 + k*x^3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3) + Sqrt[3]*d^(1/3)*k*x)/(d^(1/3) + d^(1/3
)*k*x + 2*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))])/d^(2/3)) - Log[-d^(1/3) -
 d^(1/3)*k*x + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3)]/d^(2/3) + Log[d^(2/3)
+ 2*d^(2/3)*k*x + d^(2/3)*k^2*x^2 + (d^(1/3) + d^(1/3)*k*x)*(1 + (-1 - k^2)
*x^2 + k^2*x^4)^(1/3) + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*k+2*(k^2+1)*x-k*(k^2+1)*x^2-4*k^2*x^3-k^3*x^4)/((-x^2+1)*(-k^2
*x^2+1))^(2/3)/(1-d-(1+2*d)*k*x-(d*k^2+1)*x^2+k*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{k^3x^4 + 4\,k^2x^3 + \left(k^2+1\right)kx^2 - 2\left(k^2+1\right)x - 3\,k}{\left(kx^3 - (2\,d+1)kx - \left(dk^2+1\right)x^2 - d + 1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*k+2*(k^2+1)*x-k*(k^2+1)*x^2-4*k^2*x^3-k^3*x^4)/((-x^2+1)*(-k^2
*x^2+1))^(2/3)/(1-d-(1+2*d)*k*x-(d*k^2+1)*x^2+k*x^3),x, algorithm="giac")
```

```
[Out] integrate(-(k^3*x^4 + 4*k^2*x^3 + (k^2 + 1)*k*x^2 - 2*(k^2 + 1)*x - 3*k)/((
k*x^3 - (2*d + 1)*k*x - (d*k^2 + 1)*x^2 - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^
(2/3)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{3k + 2\left(k^2+1\right)x - k\left(k^2+1\right)x^2 - 4k^2x^3 - k^3x^4}{\left(\left(-x^2+1\right)\left(-k^2x^2+1\right)\right)^{\frac{2}{3}}\left(1 - d - (1+2d)\,kx - \left(d\,k^2+1\right)x^2 + k\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*k+2*(k^2+1)*x-k*(k^2+1)*x^2-4*k^2*x^3-k^3*x^4)/((-x^2+1)*(-k^2*x^2+1
))^(2/3)/(1-d-(1+2*d)*k*x-(d*k^2+1)*x^2+k*x^3),x)
```

```
[Out] int((3*k+2*(k^2+1)*x-k*(k^2+1)*x^2-4*k^2*x^3-k^3*x^4)/((-x^2+1)*(-k^2*x^2+1
))^(2/3)/(1-d-(1+2*d)*k*x-(d*k^2+1)*x^2+k*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{k^3x^4 + 4\,k^2x^3 + \left(k^2+1\right)kx^2 - 2\left(k^2+1\right)x - 3\,k}{\left(kx^3 - (2\,d+1)kx - \left(dk^2+1\right)x^2 - d + 1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*k+2*(k^2+1)*x-k*(k^2+1)*x^2-4*k^2*x^3-k^3*x^4)/((-x^2+1)*(-k^2
*x^2+1))^(2/3)/(1-d-(1+2*d)*k*x-(d*k^2+1)*x^2+k*x^3),x, algorithm="maxima")
```

```
[Out] -integrate((k^3*x^4 + 4*k^2*x^3 + (k^2 + 1)*k*x^2 - 2*(k^2 + 1)*x - 3*k)/((
k*x^3 - (2*d + 1)*k*x - (d*k^2 + 1)*x^2 - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^
(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{4\,k^2\,x^3-2\,x\,\left(k^2+1\right)-3\,k+k^3\,x^4+k\,x^2\,\left(k^2+1\right)}{{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)}^{2/3}\,\left(-k\,x^3+\left(d\,k^2+1\right)\,x^2+k\,\left(2\,d+1\right)\,x+d-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*k^2*x^3 - 2*x*(k^2 + 1) - 3*k + k^3*x^4 + k*x^2*(k^2 + 1))/(((x^2 -
1)*(k^2*x^2 - 1))^(2/3)*(d + x^2*(d*k^2 + 1) - k*x^3 + k*x*(2*d + 1) - 1)),
x)
```

```
[Out] int((4*k^2*x^3 - 2*x*(k^2 + 1) - 3*k + k^3*x^4 + k*x^2*(k^2 + 1))/(((x^2 -
1)*(k^2*x^2 - 1))^(2/3)*(d + x^2*(d*k^2 + 1) - k*x^3 + k*x*(2*d + 1) - 1)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*k+2*(k**2+1)*x-k*(k**2+1)*x**2-4*k**2*x**3-k**3*x**4)/((-x**2+
1)*(-k**2*x**2+1))**(2/3)/(1-d-(1+2*d)*k*x-(d*k**2+1)*x**2+k*x**3),x)
```

```
[Out] Timed out
```

3.2108 $\int \frac{\sqrt{-x+x^4}\left(b+ax^6\right)}{-d+cx^6}\,dx$

Optimal. Leaf size=231

$$-\frac{\sqrt{-\left(\sqrt{d}\left(\sqrt{c}+\sqrt{d}\right)\right)}\,(ad+bc)\tan^{-1}\left(\frac{x\sqrt{x^4-x}\sqrt{-\sqrt{c}\sqrt{d}-d}}{\sqrt{d}\,(x-1)\left(x^2+x+1\right)}\right)}{3c^{3/2}d}+\frac{\sqrt{\sqrt{d}\left(\sqrt{c}-\sqrt{d}\right)}\,(ad+bc)\tan^{-1}\left(\frac{x\sqrt{x^4-x}\sqrt{\sqrt{c}\sqrt{d}-d}}{\sqrt{d}\,(x-1)\left(x^2+x+1\right)}\right)}{3c^{3/2}d}$$

Rubi [A] time = 0.85, antiderivative size = 268, normalized size of antiderivative = 1.16, number of steps used = 18, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {2056, 6715, 1693, 195, 217, 206, 1175, 402, 377, 205, 208}

$$\frac{\sqrt{x^4-x}\sqrt{\sqrt{c}-\sqrt{d}}\,(ad+bc)\tan^{-1}\left(\frac{x^{3/2}\sqrt{\sqrt{c}-\sqrt{d}}}{\sqrt[4]{d}\sqrt{x^3-1}}\right)}{3c^{3/2}d^{3/4}\sqrt{x^3-1}\sqrt{x}}+\frac{\sqrt{x^4-x}\sqrt{\sqrt{c}+\sqrt{d}}\,(ad+bc)\tanh^{-1}\left(\frac{x^{3/2}\sqrt{\sqrt{c}+\sqrt{d}}}{\sqrt[4]{d}\sqrt{x^3-1}}\right)}{3c^{3/2}d^{3/4}\sqrt{x^3-1}\sqrt{x}}+\frac{a\sqrt{x^4-x}\,x}{3c}-\frac{a\sqrt{x^4-x}\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{x^3-1}}\right)}{3c\sqrt{x^3-1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-x + x^4]*(b + a*x^6))/(-d + c*x^6),x]
```

```
[Out] (a*x*Sqrt[-x + x^4])/(3*c) + (Sqrt[Sqrt[c] - Sqrt[d]]*(b*c + a*d)*Sqrt[-x +
 x^4]*ArcTan[(Sqrt[Sqrt[c] - Sqrt[d]]*x^(3/2))/(d^(1/4)*Sqrt[-1 + x^3])])/(
3*c^(3/2)*d^(3/4)*Sqrt[x]*Sqrt[-1 + x^3]) - (a*Sqrt[-x + x^4]*ArcTanh[x^(3/
2)/Sqrt[-1 + x^3]])/(3*c*Sqrt[x]*Sqrt[-1 + x^3]) + (Sqrt[Sqrt[c] + Sqrt[d]]
*(b*c + a*d)*Sqrt[-x + x^4]*ArcTanh[(Sqrt[Sqrt[c] + Sqrt[d]]*x^(3/2))/(d^(1
/4)*Sqrt[-1 + x^3])])/(3*c^(3/2)*d^(3/4)*Sqrt[x]*Sqrt[-1 + x^3])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[-(a*c), 2]}, -Dist[c/(2*r), Int[(d + e*x^2)^q/(r - c*x^2), x], x] - Di
st[c/(2*r), Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e, q}
, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1693

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\sqrt{-x+x^4}\left(b+ax^6\right)}{-d+cx^6}\,dx = \frac{\sqrt{-x+x^4}\int \frac{\sqrt{x}\sqrt{-1+x^3}\left(b+ax^6\right)}{-d+cx^6}\,dx}{\sqrt{x}\sqrt{-1+x^3}}$$

$$= \frac{\left(2\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{\sqrt{-1+x^2}\left(b+ax^4\right)}{-d+cx^4}\,dx, x, x^{3/2}\right)}{3\sqrt{x}\sqrt{-1+x^3}}$$

$$= \frac{\left(2\sqrt{-x+x^4}\right)\text{Subst}\left(\int \left(\frac{a\sqrt{-1+x^2}}{c}+\frac{(bc+ad)\sqrt{-1+x^2}}{c\left(-d+cx^4\right)}\right)dx, x, x^{3/2}\right)}{3\sqrt{x}\sqrt{-1+x^3}}$$

$$= \frac{\left(2a\sqrt{-x+x^4}\right)\text{Subst}\left(\int \sqrt{-1+x^2}\,dx, x, x^{3/2}\right)}{3c\sqrt{x}\sqrt{-1+x^3}} + \frac{\left(2(bc+ad)\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{\sqrt{}}{-}\right.}{3c\sqrt{x}\sqrt{-1+x^3}}$$

$$= \frac{ax\sqrt{-x+x^4}}{3c} - \frac{\left(a\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, x^{3/2}\right)}{3c\sqrt{x}\sqrt{-1+x^3}} - \frac{\left((bc+ad)\sqrt{-x+x^4}\right)\text{S}}{3\sqrt{c}\sqrt{d}}$$

$$= \frac{ax\sqrt{-x+x^4}}{3c} - \frac{\left(a\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^{3/2}}{\sqrt{-1+x^3}}\right)}{3c\sqrt{x}\sqrt{-1+x^3}} - \frac{\left(\left(-1+\frac{\sqrt{d}}{\sqrt{c}}\right)(bc+ad)\right.}{}$$

$$= \frac{ax\sqrt{-x+x^4}}{3c} - \frac{a\sqrt{-x+x^4}\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{-1+x^3}}\right)}{3c\sqrt{x}\sqrt{-1+x^3}} - \frac{\left(\left(-1+\frac{\sqrt{d}}{\sqrt{c}}\right)(bc+ad)\sqrt{-x+x^4}\right)\text{Sub}}{3\sqrt{c}\sqrt{d}}$$

$$= \frac{ax\sqrt{-x+x^4}}{3c} + \frac{\sqrt{\sqrt{c}-\sqrt{d}}\,(bc+ad)\sqrt{-x+x^4}\tan^{-1}\left(\frac{\sqrt{\sqrt{c}-\sqrt{d}}\,x^{3/2}}{\sqrt[4]{d}\sqrt{-1+x^3}}\right)}{3c^{3/2}d^{3/4}\sqrt{x}\sqrt{-1+x^3}} - \frac{a\sqrt{-x+x^4}\,\text{t}}{3c\sqrt{x}}$$

Mathematica [C] time = 1.60, size = 487, normalized size = 2.11

$$\frac{\sqrt{x}\sqrt{x^3-1}\left(-\frac{(ad+bc)\left(\sqrt[4]{d}\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{x^3-1}}\right)+\sqrt{\sqrt{c}+\sqrt{d}}\tanh^{-1}\left(\frac{-\sqrt[4]{d}x^{3/2}+i\sqrt[4]{c}}{\sqrt{x^3-1}\sqrt{\sqrt{c}+\sqrt{d}}}\right)\right)}{d^{3/4}}-\frac{(ad+bc)\left(\sqrt[4]{d}\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{x^3-1}}\right)-\sqrt{\sqrt{c}+\sqrt{d}}\tanh^{-1}\left(\frac{\sqrt[4]{d}x^{3/2}+i\sqrt[4]{c}}{\sqrt{x^3-1}\sqrt{\sqrt{c}+\sqrt{d}}}\right)\right)}{d^{3/4}}+\frac{(ad+bc)\left(\sqrt{\sqrt{c}-\sqrt{d}}\tan^{-1}\left(\frac{\sqrt[4]{c}-\sqrt[4]{d}x^{3/2}}{\sqrt{x^3-1}\sqrt{\sqrt{c}-\sqrt{d}}}\right)+\sqrt[4]{d}\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{x^3-1}}\right)\right)}{d^{3/4}}+\frac{(ad+bc)\left(\sqrt[4]{d}\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{x^3-1}}\right)-\sqrt{\sqrt{c}-\sqrt{d}}\tan^{-1}\left(\frac{\sqrt[4]{c}+\sqrt[4]{d}x^{3/2}}{\sqrt{x^3-1}\sqrt{\sqrt{c}-\sqrt{d}}}\right)\right)}{d^{3/4}}+\frac{2a\sqrt{c}\left(x^3-1\right)\left(\sin^{-1}\left(x^{3/2}\right)+\sqrt{1-x^3}\,x^{3/2}\right)}{\sqrt{-\left(x^3-1\right)^2}}\right)}{6c^{3/2}\sqrt{x\left(x^3-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-x + x^4]*(b + a*x^6))/(-d + c*x^6),x]
```

```
[Out] (Sqrt[x]*Sqrt[-1 + x^3]*((2*a*Sqrt[c]*(-1 + x^3)*(x^(3/2)*Sqrt[1 - x^3] + A
rcSin[x^(3/2)]))/Sqrt[-(-1 + x^3)^2] + ((b*c + a*d)*(Sqrt[Sqrt[c] - Sqrt[d]
]*ArcTan[(c^(1/4) - d^(1/4)*x^(3/2))/(Sqrt[Sqrt[c] - Sqrt[d]]*Sqrt[-1 + x^3
])] + d^(1/4)*ArcTanh[x^(3/2)/Sqrt[-1 + x^3]]))/d^(3/4) + ((b*c + a*d)*(-(S
qrt[Sqrt[c] - Sqrt[d]]*ArcTan[(c^(1/4) + d^(1/4)*x^(3/2))/(Sqrt[Sqrt[c] - S
qrt[d]]*Sqrt[-1 + x^3])]) + d^(1/4)*ArcTanh[x^(3/2)/Sqrt[-1 + x^3]]))/d^(3/
4) - ((b*c + a*d)*(d^(1/4)*ArcTanh[x^(3/2)/Sqrt[-1 + x^3]] + Sqrt[Sqrt[c] +
 Sqrt[d]]*ArcTanh[(I*c^(1/4) - d^(1/4)*x^(3/2))/(Sqrt[Sqrt[c] + Sqrt[d]]*Sq
rt[-1 + x^3])]))/d^(3/4) - ((b*c + a*d)*(d^(1/4)*ArcTanh[x^(3/2)/Sqrt[-1 +
x^3]] - Sqrt[Sqrt[c] + Sqrt[d]]*ArcTanh[(I*c^(1/4) + d^(1/4)*x^(3/2))/(Sqrt
[Sqrt[c] + Sqrt[d]]*Sqrt[-1 + x^3])]))/d^(3/4)))/(6*c^(3/2)*Sqrt[x*(-1 + x^
3)])
```

IntegrateAlgebraic [A] time = 5.77, size = 261, normalized size = 1.13

$$-\frac{\sqrt{-\left(\sqrt{d}\left(\sqrt{c}+\sqrt{d}\right)\right)}(ad+bc)\tan^{-1}\left(\frac{x\sqrt{x^4-x}\sqrt{-\sqrt{c}\sqrt{d}-d}}{\sqrt{d}(x-1)\left(x^2+x+1\right)}\right)}{3c^{3/2}d}+\frac{\sqrt{\sqrt{d}\left(\sqrt{c}-\sqrt{d}\right)}(ad+bc)\tan^{-1}\left(\frac{x\sqrt{x^4-x}\sqrt{\sqrt{c}\sqrt{d}-d}}{\sqrt{d}(x-1)\left(x^2+x+1\right)}\right)}{3c^{3/2}d}+\frac{a\sqrt{x^4-x}\,x}{3c}+\frac{a\log\left(\sqrt{x^4-x}-x^2\right)}{6c}-\frac{a\log\left(c\sqrt{x^4-x}+cx^2\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-x + x^4]*(b + a*x^6))/(-d + c*x^6),x]
```

```
[Out] (a*x*Sqrt[-x + x^4])/(3*c) - (Sqrt[-((Sqrt[c] + Sqrt[d])*Sqrt[d])]*(b*c + a
*d)*ArcTan[(Sqrt[-(Sqrt[c]*Sqrt[d]) - d]*x*Sqrt[-x + x^4])/(Sqrt[d]*(-1 + x
)*(1 + x + x^2))]])/(3*c^(3/2)*d) + (Sqrt[(Sqrt[c] - Sqrt[d])*Sqrt[d]]*(b*c
+ a*d)*ArcTan[(Sqrt[Sqrt[c]*Sqrt[d] - d]*x*Sqrt[-x + x^4])/(Sqrt[d]*(-1 + x
)*(1 + x + x^2))]])/(3*c^(3/2)*d) + (a*Log[-x^2 + Sqrt[-x + x^4]])/(6*c) - (
a*Log[c*x^2 + c*Sqrt[-x + x^4]])/(6*c)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)*(a*x^6+b)/(c*x^6-d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)*(a*x^6+b)/(c*x^6-d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [c,d]=[35,45]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [c,d]=[-45,5]Warning, need to choose
 a branch for the root of a polynomial with parameters. This might be wrong
.The choice was done assuming [c,d]=[-13,69]Warning, need to choose a branc
h for the root of a polynomial with parameters. This might be wrong.The cho
ice was done assuming [c,d]=[-66,-2]Warning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming [c,d]=[-9,-88]a/6/c*ln(abs(sqrt(-(1/x)^3+1)-1))-a/6/c*ln(sqrt
(-(1/x)^3+1)+1)+((-2*c*d^3+4*c*d*sqrt(-d^2-d*sqrt(c*d))*sqrt(c*d)+5*d^2*sqr
t(-d^2-d*sqrt(c*d))*sqrt(c*d)+2*d^2*c*d)*a*c^2*abs(d)+(2*c^2*d^4-4*c^2*d^2*
sqrt(-d^2-d*sqrt(c*d))*sqrt(c*d)-5*c*d^3*sqrt(-d^2-d*sqrt(c*d))*sqrt(c*d)-2
*c*d^3*c*d)*a*abs(d)+(-2*c^2*d^2+4*c^2*sqrt(-d^2-d*sqrt(c*d))*sqrt(c*d)+5*c
*d*sqrt(-d^2-d*sqrt(c*d))*sqrt(c*d)+2*c*d*c*d)*b*c^2*abs(d)+(2*c^3*d^3-4*c^
3*d*sqrt(-d^2-d*sqrt(c*d))*sqrt(c*d)-5*c^2*d^2*sqrt(-d^2-d*sqrt(c*d))*sqrt(
c*d)-2*c^2*d^2*c*d)*b*abs(d))/(12*c^4*d^3+3*c^3*d^4-15*c^2*d^5)/abs(c)*atan
(sqrt(-(1/x)^3+1)/sqrt(-(6*c*d+sqrt(6*c*d*6*c*d-12*c*d*(-3*c^2+3*c*d)))/2/3
/c/d))-((-2*c*d^3+4*c*d*sqrt(-d^2+d*sqrt(c*d))*sqrt(c*d)+5*d^2*sqrt(-d^2+d*
sqrt(c*d))*sqrt(c*d)+2*d^2*c*d)*a*c^2*abs(d)+(2*c^2*d^4-4*c^2*d^2*sqrt(-d^2
+d*sqrt(c*d))*sqrt(c*d)-5*c*d^3*sqrt(-d^2+d*sqrt(c*d))*sqrt(c*d)-2*c*d^3*c*
d)*a*abs(d)+(-2*c^2*d^2+4*c^2*sqrt(-d^2+d*sqrt(c*d))*sqrt(c*d)+5*c*d*sqrt(-
d^2+d*sqrt(c*d))*sqrt(c*d)+2*c*d*c*d)*b*c^2*abs(d)+(2*c^3*d^3-4*c^3*d*sqrt(
-d^2+d*sqrt(c*d))*sqrt(c*d)-5*c^2*d^2*sqrt(-d^2+d*sqrt(c*d))*sqrt(c*d)-2*c^
2*d^2*c*d)*b*abs(d))/(12*c^4*d^3+3*c^3*d^4-15*c^2*d^5)/abs(c)*atan(sqrt(-(1
/x)^3+1)/sqrt(-(6*c*d-sqrt(6*c*d*6*c*d-12*c*d*(-3*c^2+3*c*d)))/2/3/c/d))+8*
a*c*1/24/c^2*x*sqrt(x^4-x)
```

maple [C] time = 0.58, size = 681, normalized size = 2.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x)^(1/2)*(a*x^6+b)/(c*x^6-d),x)
```

```
[Out] a/c*(1/3*x*(x^4-x)^(1/2)-(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+
1/2*I*3^(1/2))/(-1+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^
(1/2))/(-1+x))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1
/2)/(-3/2+1/2*I*3^(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/
2)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^
(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2
*I*3^(1/2)))^(1/2))-EllipticPi(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))
/(-1+x))^(1/2),(-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)),((3/2+1/2*I*3^(1/2
))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))))-1/
3*(a*d+b*c)/c*4^(1/2)*sum((-_alpha^3+1)/_alpha^4*(-1+x)^2*(_alpha^5+_alpha^
4+_alpha^3+_alpha^2+_alpha+1)/(c-d)*(1-I*3^(1/2))*(x/(-1+x)*(-3+I*3^(1/2))/
(I*3^(1/2)-1))^(1/2)*(1/(-1+x)*(I*3^(1/2)+2*x+1)/(-1-I*3^(1/2)))^(1/2)*(1/(
-1+x)*(1+2*x-I*3^(1/2))/(I*3^(1/2)-1))^(1/2)/(-3+I*3^(1/2))/(x*(-1+x)*(I*3^
(1/2)+2*x+1)*(1+2*x-I*3^(1/2)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-
1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(
1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-_alpha^5*c/d*EllipticPi(((-3
/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),1/6*(I*_alpha^5*3^(1
/2)*c-3*_alpha^5*c-I*3^(1/2)*d+3*d)/d,((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/
2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))),_alpha=RootOf(_Z^6*c-d
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^6+b\right)\sqrt{x^4-x}}{cx^6-d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x)^(1/2)*(a*x^6+b)/(c*x^6-d),x, algorithm="maxima")
```

```
[Out] integrate((a*x^6 + b)*sqrt(x^4 - x)/(c*x^6 - d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\sqrt{x^4-x}\,\left(a\,x^6+b\right)}{d-c\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^4 - x)^(1/2)*(b + a*x^6))/(d - c*x^6),x)
```

```
[Out] int(-((x^4 - x)^(1/2)*(b + a*x^6))/(d - c*x^6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(x-1\right)\left(x^2+x+1\right)}\,\left(ax^6+b\right)}{cx^6-d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x)**(1/2)*(a*x**6+b)/(c*x**6-d),x)
```

```
[Out] Integral(sqrt(x*(x - 1)*(x**2 + x + 1))*(a*x**6 + b)/(c*x**6 - d), x)
```

3.2109 $\int \frac{a^2b-2a^2x+(2a-b)x^2}{(x(-a+x)(-b+x))^{2/3}\left(a^2d+(b-2ad)x+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=232

$$-\frac{\log\left(d^{2/3}\left(x^2(-a-b)+abx+x^3\right)^{4/3}+\left(x^2(-a-b)+abx+x^3\right)^{2/3}\left(\sqrt[3]{d}\,x^2-b\sqrt[3]{d}\,x\right)+b^2x^2-2bx^3+x^4\right)}{2d^{2/3}}+\frac{\log}{}$$

Rubi [F] time = 4.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a^2b-2a^2x+(2a-b)x^2}{(x(-a+x)(-b+x))^{2/3}\left(a^2d+(b-2ad)x+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a^2*b - 2*a^2*x + (2*a - b)*x^2)/((x*(-a + x)*(-b + x))^(2/3)*(a^2*d +
 (b - 2*a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] ((2*a - b + Sqrt[b^2 + 4*a^2*d - 4*a*b*d])*x^(2/3)*(-a + x)^(2/3)*(-b + x)^
(2/3)*Defer[Int][(-a + x)^(1/3)/(x^(2/3)*(-b + x)^(2/3)*(b - 2*a*d - Sqrt[b
^2 + 4*a^2*d - 4*a*b*d] + 2*(-1 + d)*x)), x])/((a - x)*(b - x)*x)^(2/3) + (
(2*a - b - Sqrt[b^2 + 4*a^2*d - 4*a*b*d])*x^(2/3)*(-a + x)^(2/3)*(-b + x)^(
2/3)*Defer[Int][(-a + x)^(1/3)/(x^(2/3)*(-b + x)^(2/3)*(b - 2*a*d + Sqrt[b^
2 + 4*a^2*d - 4*a*b*d] + 2*(-1 + d)*x)), x])/((a - x)*(b - x)*x)^(2/3)
```

Rubi steps

$$\begin{aligned}\int \frac{a^2b-2a^2x+(2a-b)x^2}{(x(-a+x)(-b+x))^{2/3}\left(a^2d+(b-2ad)x+(-1+d)x^2\right)}\,dx &= \frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{a^2b-2}{x^{2/3}(-a+x)^{2/3}(-b+x)}}{(x(-a+x)(-b+x))^{2/}}\\ &= \frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{\sqrt[3]{-a+x}\,(-ab}{x^{2/3}(-b+x)^{2/3}\left(a^2d+(}}{(x(-a+x)(-b+x))^{2/3}}\\ &= \frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int\left(\frac{\left(2a-b+\sqrt{b}\right.}{x^{2/3}(-b+x)^{2/3}\left(b-2a\right.}\right.}{}\\ &= \frac{\left(\left(2a-b-\sqrt{b^2+4a^2d-4abd}\right)x^{2/3}(-a+x)^{2/3}\right.}{(x(-a}\end{aligned}$$

Mathematica [F] time = 9.04, size = 0, normalized size = 0.00

$$\int \frac{a^2b-2a^2x+(2a-b)x^2}{(x(-a+x)(-b+x))^{2/3}\left(a^2d+(b-2ad)x+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a^2*b - 2*a^2*x + (2*a - b)*x^2)/((x*(-a + x)*(-b + x))^(2/3)*(a
^2*d + (b - 2*a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] Integrate[(a^2*b - 2*a^2*x + (2*a - b)*x^2)/((x*(-a + x)*(-b + x))^(2/3)*(a
^2*d + (b - 2*a*d)*x + (-1 + d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 3.18, size = 232, normalized size = 1.00

$$-\frac{\log\left(d^{2/3}\left(x^2(-a-b)+abx+x^3\right)^{4/3}+\left(x^2(-a-b)+abx+x^3\right)^{2/3}\left(\sqrt[3]{d}\,x^2-b\sqrt[3]{d}\,x\right)+b^2x^2-2bx^3+x^4\right)}{2d^{2/3}}+\frac{\log\left(\sqrt[3]{d}\left(x^2(-a-b)+abx+x^3\right)^{2/3}+bx-x^2\right)}{d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\left(x^2(-a-b)+abx+x^3\right)^{2/3}}{\sqrt[3]{d}\left(x^2(-a-b)+abx+x^3\right)^{2/3}-2bx+2x^2}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2*b - 2*a^2*x + (2*a - b)*x^2)/((x*(-a + x)*(-b + x))
^(2/3)*(a^2*d + (b - 2*a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(a*b*x + (-a - b)*x^2 + x^3)^(2/3))/(-2*b*
x + 2*x^2 + d^(1/3)*(a*b*x + (-a - b)*x^2 + x^3)^(2/3))])/d^(2/3) + Log[b*x
 - x^2 + d^(1/3)*(a*b*x + (-a - b)*x^2 + x^3)^(2/3)]/d^(2/3) - Log[b^2*x^2
- 2*b*x^3 + x^4 + (-(b*d^(1/3)*x) + d^(1/3)*x^2)*(a*b*x + (-a - b)*x^2 + x^
3)^(2/3) + d^(2/3)*(a*b*x + (-a - b)*x^2 + x^3)^(4/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*b-2*a^2*x+(2*a-b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a^2*d+(-2*a*
d+b)*x+(-1+d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2b - 2\,a^2x + (2\,a - b)x^2}{\left(a^2d + (d-1)x^2 - (2\,ad - b)x\right)\left((a-x)(b-x)x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*b-2*a^2*x+(2*a-b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a^2*d+(-2*a*
d+b)*x+(-1+d)*x^2),x, algorithm="giac")
```

```
[Out] integrate((a^2*b - 2*a^2*x + (2*a - b)*x^2)/((a^2*d + (d - 1)*x^2 - (2*a*d
- b)*x)*((a - x)*(b - x)*x)^(2/3)), x)
```

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{a^2b - 2a^2x + (2a-b)\,x^2}{\left(x\,(-a+x)\,(-b+x)\right)^{\frac{2}{3}}\left(a^2d + (-2ad+b)\,x + (-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*b-2*a^2*x+(2*a-b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a^2*d+(-2*a*d+b)*x
+(-1+d)*x^2),x)
```

```
[Out] int((a^2*b-2*a^2*x+(2*a-b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a^2*d+(-2*a*d+b)*x
+(-1+d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2b - 2\,a^2x + (2\,a - b)x^2}{\left(a^2d + (d-1)x^2 - (2\,ad - b)x\right)\left((a-x)(b-x)x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*b-2*a^2*x+(2*a-b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a^2*d+(-2*a*
d+b)*x+(-1+d)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*b - 2*a^2*x + (2*a - b)*x^2)/((a^2*d + (d - 1)*x^2 - (2*a*d
- b)*x)*((a - x)*(b - x)*x)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(2\,a-b\right)+a^2\,b-2\,a^2\,x}{{\left(x\,\left(a-x\right)\,\left(b-x\right)\right)}^{2/3}\,\left(a^2\,d+x\,\left(b-2\,a\,d\right)+x^2\,\left(d-1\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(2*a - b) + a^2*b - 2*a^2*x)/((x*(a - x)*(b - x))^(2/3)*(a^2*d + x
*(b - 2*a*d) + x^2*(d - 1))),x)
```

```
[Out] int((x^2*(2*a - b) + a^2*b - 2*a^2*x)/((x*(a - x)*(b - x))^(2/3)*(a^2*d + x
*(b - 2*a*d) + x^2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*b-2*a**2*x+(2*a-b)*x**2)/(x*(-a+x)*(-b+x))**(2/3)/(a**2*d+(
-2*a*d+b)*x+(-1+d)*x**2),x)
```

```
[Out] Timed out
```

3.2110 $\int \frac{2ab^2x-b(2a+b)x^2+x^4}{\left(x(-a+x)(-b+x)^2\right)^{2/3}\left(-ab^2+b(2a+b)x-(a+2b+d)x^2+x^3\right)}\,dx$

Optimal. Leaf size=232

$$\frac{\log\left(\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}-\sqrt[3]{d}\,x\right)}{d^{2/3}}-\frac{\log\left(\sqrt[3]{d}\,x\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}+\right.}{2d}$$

Rubi [F] time = 15.94, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2ab^2x-b(2a+b)x^2+x^4}{\left(x(-a+x)(-b+x)^2\right)^{2/3}\left(-ab^2+b(2a+b)x-(a+2b+d)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2*a*b^2*x - b*(2*a + b)*x^2 + x^4)/((x*(-a + x)*(-b + x)^2)^(2/3)*(-(a
*b^2) + b*(2*a + b)*x - (a + 2*b + d)*x^2 + x^3)),x]
```

```
[Out] (-3*(b - x)*x*(1 - x/a)^(2/3)*(1 - x/b)^(1/3)*AppellF1[1/3, 2/3, 1/3, 4/3,
x/a, x/b])/(-((a - x)*(b - x)^2*x))^(2/3) + (3*b*(4*a + b)*x^(2/3)*(-a + x)
^(2/3)*(-b + x)^(4/3)*Defer[Subst][Defer[Int][x^3/((-a + x^3)^(2/3)*(-b + x
^3)^(1/3)*(a*b^2 - 2*a*b*(1 + b/(2*a))*x^3 + a*(1 + (2*b + d)/a)*x^6 - x^9)
), x], x, x^(1/3)])/(-((a - x)*(b - x)^2*x))^(2/3) - (3*(a + 3*b + d)*x^(2/
3)*(-a + x)^(2/3)*(-b + x)^(4/3)*Defer[Subst][Defer[Int][x^6/((-a + x^3)^(2
/3)*(-b + x^3)^(1/3)*(a*b^2 - 2*a*b*(1 + b/(2*a))*x^3 + a*(1 + (2*b + d)/a)
*x^6 - x^9)), x], x, x^(1/3)])/(-((a - x)*(b - x)^2*x))^(2/3) + (3*a*b^2*x^
(2/3)*(-a + x)^(2/3)*(-b + x)^(4/3)*Defer[Subst][Defer[Int][1/((-a + x^3)^(
2/3)*(-b + x^3)^(1/3)*(-(a*b^2) + 2*a*b*(1 + b/(2*a))*x^3 - a*(1 + (2*b + d
)/a)*x^6 + x^9)), x], x, x^(1/3)])/(-((a - x)*(b - x)^2*x))^(2/3)
```

Rubi steps

$$\int \frac{2ab^2x-b(2a+b)x^2+x^4}{\left(x(-a+x)(-b+x)^2\right)^{2/3}\left(-ab^2+b(2a+b)x-(a+2b+d)x^2+x^3\right)}\,dx = \int \frac{x\left(2ab^2-b(\cdots\right.}{\left(x(-a+x)(-b+x)^2\right)^{2/3}\left(-ab^2+\cdots\right.}$$

$$= \frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{\cdots}{(-a+\cdots}}{\left(x(-a+\cdots\right.}$$

$$= \frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{\cdots}{(-a+\cdots}}{\left(x(-a+x\cdots\right.}$$

$$= \frac{\left(3x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\text{Subs}\cdots}{\cdots}$$

$$= \frac{\left(3x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\text{Subs}\cdots}{\cdots}$$

$$= \frac{\left(3x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\text{Subs}\cdots}{\left(x(-a+x)(-\cdots\right.}$$

$$= \frac{\left(3x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\text{Subs}\cdots}{\cdots}$$

$$= \frac{\left(3ab^2x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\text{Su}\cdots}{\cdots}$$

$$= -\frac{3(b-x)x\left(1-\frac{x}{a}\right)^{2/3}\sqrt[3]{1-\frac{x}{b}}\,F_1\left(\frac{1}{3}\cdots\right.}{\left(-\left((a-x)(b-x)^2x\right)\right)^{\cdots}}$$

Mathematica [F] time = 5.29, size = 0, normalized size = 0.00

$$\int \frac{2ab^2x-b(2a+b)x^2+x^4}{\left(x(-a+x)(-b+x)^2\right)^{2/3}\left(-ab^2+b(2a+b)x-(a+2b+d)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2*a*b^2*x - b*(2*a + b)*x^2 + x^4)/((x*(-a + x)*(-b + x)^2)^(2/3
)*(-(a*b^2) + b*(2*a + b)*x - (a + 2*b + d)*x^2 + x^3)),x]
```

```
[Out] Integrate[(2*a*b^2*x - b*(2*a + b)*x^2 + x^4)/((x*(-a + x)*(-b + x)^2)^(2/3
)*(-(a*b^2) + b*(2*a + b)*x - (a + 2*b + d)*x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 3.54, size = 232, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}-\sqrt[3]{d}\,x\right)}{d^{2/3}}-\frac{\log\left(\sqrt[3]{d}\,x\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}+\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}+d^{2/3}x^2\right)}{2d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\,x}{2\sqrt[3]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}+\sqrt[3]{d}\,x}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*a*b^2*x - b*(2*a + b)*x^2 + x^4)/((x*(-a + x)*(-b + x
)^2)^(2/3)*(-(a*b^2) + b*(2*a + b)*x - (a + 2*b + d)*x^2 + x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*x)/(d^(1/3)*x + 2*(-(a*b^2*x) + (2*a*b + b
^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3))])/d^(2/3) + Log[-(d^(1/3)*x) + (-(a*
b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3)]/d^(2/3) - Log[d^(
2/3)*x^2 + d^(1/3)*x*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4
)^(1/3) + (-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(2/3)]/(2
*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b^2*x-b*(2*a+b)*x^2+x^4)/(x*(-a+x)*(-b+x)^2)^(2/3)/(-a*b^2+b
*(2*a+b)*x-(a+2*b+d)*x^2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,ab^2x-(2\,a+b)bx^2+x^4}{\left(-(a-x)(b-x)^2x\right)^{\frac{2}{3}}\left(ab^2-(2\,a+b)bx+(a+2\,b+d)x^2-x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b^2*x-b*(2*a+b)*x^2+x^4)/(x*(-a+x)*(-b+x)^2)^(2/3)/(-a*b^2+b
*(2*a+b)*x-(a+2*b+d)*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate(-(2*a*b^2*x - (2*a + b)*b*x^2 + x^4)/((-(a - x)*(b - x)^2*x)^(2/3
)*(a*b^2 - (2*a + b)*b*x + (a + 2*b + d)*x^2 - x^3)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{2a\,b^2x-b\,(2a+b)\,x^2+x^4}{\left(x\,(-a+x)\,(-b+x)^2\right)^{\frac{2}{3}}\left(-a\,b^2+b\,(2a+b)\,x-(a+2b+d)\,x^2+x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*b^2*x-b*(2*a+b)*x^2+x^4)/(x*(-a+x)*(-b+x)^2)^(2/3)/(-a*b^2+b*(2*a+
b)*x-(a+2*b+d)*x^2+x^3),x)
```

```
[Out] int((2*a*b^2*x-b*(2*a+b)*x^2+x^4)/(x*(-a+x)*(-b+x)^2)^(2/3)/(-a*b^2+b*(2*a+
b)*x-(a+2*b+d)*x^2+x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,ab^2x-(2\,a+b)bx^2+x^4}{\left(-(a-x)(b-x)^2x\right)^{\frac{2}{3}}\left(ab^2-(2\,a+b)bx+(a+2\,b+d)x^2-x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b^2*x-b*(2*a+b)*x^2+x^4)/(x*(-a+x)*(-b+x)^2)^(2/3)/(-a*b^2+b
*(2*a+b)*x-(a+2*b+d)*x^2+x^3),x, algorithm="maxima")
```

```
[Out] -integrate((2*a*b^2*x - (2*a + b)*b*x^2 + x^4)/((-(a - x)*(b - x)^2*x)^(2/3
)*(a*b^2 - (2*a + b)*b*x + (a + 2*b + d)*x^2 - x^3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x^4 - b\,x^2\,\left(2\,a+b\right)+2\,a\,b^2\,x}{{\left(-x\,\left(a-x\right)\,{\left(b-x\right)}^2\right)}^{2/3}\,\left(x^2\,\left(a+2\,b+d\right)+a\,b^2-x^3-b\,x\,\left(2\,a+b\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - b*x^2*(2*a + b) + 2*a*b^2*x)/((-x*(a - x)*(b - x)^2)^(2/3)*(x^2
*(a + 2*b + d) + a*b^2 - x^3 - b*x*(2*a + b))),x)
```

```
[Out] int(-(x^4 - b*x^2*(2*a + b) + 2*a*b^2*x)/((-x*(a - x)*(b - x)^2)^(2/3)*(x^2
*(a + 2*b + d) + a*b^2 - x^3 - b*x*(2*a + b))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b**2*x-b*(2*a+b)*x**2+x**4)/(x*(-a+x)*(-b+x)**2)**(2/3)/(-a*
b**2+b*(2*a+b)*x-(a+2*b+d)*x**2+x**3),x)
```

```
[Out] Timed out
```

3.2111 $\int \frac{-3k+2(1+k^2)x+k(1+k^2)x^2-4k^2x^3+k^3x^4}{((1-x^2)(1-k^2x^2))^{2/3}(-1+d-(1+2d)kx+(1+dk^2)x^2+kx^3)}\,dx$

Optimal. Leaf size=232

$$\frac{\log\left(\sqrt[3]{d}\,kx-\sqrt[3]{d}+\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}\right)}{d^{2/3}}-\frac{\log\left(d^{2/3}k^2x^2-2d^{2/3}kx+d^{2/3}+\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}\,(\sqrt[3]{d}\right.}{2d^{2/3}}$$

Rubi [F] time = 9.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-3k+2(1+k^2)x+k(1+k^2)x^2-4k^2x^3+k^3x^4}{((1-x^2)(1-k^2x^2))^{2/3}(-1+d-(1+2d)kx+(1+dk^2)x^2+kx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-3*k + 2*(1 + k^2)*x + k*(1 + k^2)*x^2 - 4*k^2*x^3 + k^3*x^4)/(((1 - x
^2)*(1 - k^2*x^2))^(2/3)*(-1 + d - (1 + 2*d)*k*x + (1 + d*k^2)*x^2 + k*x^3)
),x]
```

```
[Out] -((k*(5 + d*k^2)*x*(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*AppellF1[1/2, 2/3, 2
/3, 3/2, x^2, k^2*x^2])/((1 - x^2)*(1 - k^2*x^2))^(2/3)) - (3^(3/4)*Sqrt[2
+ Sqrt[3]]*k^(4/3)*Sqrt[(-1 - k^2 + 2*k^2*x^2)^2]*((-1 + k^2)^(2/3) + 2^(2/
3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))*Sqrt[((-1 + k^2)^(4/3) - 2^(2/3
)*k^(2/3)*(-1 + k^2)^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3) + 2*2^(1/3)*k^(4
/3)*((1 - x^2)*(1 - k^2*x^2))^(2/3))/((1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2
/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))/((
1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1 - x^2)*(1 - k^2*x^2))^(
1/3))], -7 - 4*Sqrt[3]])/(2^(2/3)*(1 + k^2 - 2*k^2*x^2)*Sqrt[(-1 - k^2*(1 -
 2*x^2))^2]*Sqrt[((-1 + k^2)^(2/3)*((-1 + k^2)^(2/3) + 2^(2/3)*k^(2/3)*((1
- x^2)*(1 - k^2*x^2))^(1/3)))/((1 + Sqrt[3])*(-1 + k^2)^(2/3) + 2^(2/3)*k^(
2/3)*((1 - x^2)*(1 - k^2*x^2))^(1/3))^2]) + (k*(8 - d^2*k^2 - d*(5 - k^2))*
(1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][1/((1 - x^2)^(2/3)*(1 - k^2*
x^2)^(2/3)*(1 - d + (1 + 2*d)*k*x - (1 + d*k^2)*x^2 - k*x^3)), x])/((1 - x^
2)*(1 - k^2*x^2))^(2/3) - ((2 - (2 + 11*d)*k^2 - d*(1 + 2*d)*k^4)*(1 - x^2)
^(2/3)*(1 - k^2*x^2)^(2/3)*Defer[Int][x/((1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3
)*(1 - d + (1 + 2*d)*k*x - (1 + d*k^2)*x^2 - k*x^3)), x])/((1 - x^2)*(1 - k
^2*x^2))^(2/3) - (k*(6 + (2 + 8*d)*k^2 + d^2*k^4)*(1 - x^2)^(2/3)*(1 - k^2*
x^2)^(2/3)*Defer[Int][x^2/((1 - x^2)^(2/3)*(1 - k^2*x^2)^(2/3)*(1 - d + (1
+ 2*d)*k*x - (1 + d*k^2)*x^2 - k*x^3)), x])/((1 - x^2)*(1 - k^2*x^2))^(2/3)
```

Rubi steps

$$\int \frac{-3k+2\left(1+k^2\right)x+k\left(1+k^2\right)x^2-4k^2x^3+k^3x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(-1+d-(1+2d)kx+\left(1+dk^2\right)x^2+kx^3\right)}dx = \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int \frac{\cdots}{\left(1-x^2\right)^{2}\cdots}}{\left(\left(1-x^2\right)\cdots\right.}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int\left(-\frac{\cdots}{\left(1-\cdots\right.}\right.}{\cdots}$$

$$= -\frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\int \frac{k(8-\cdots}{\cdots}}{\left(\cdots\right.}$$

$$= -\frac{k\left(5+dk^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\cdots\right.}{\left(\left(1-x^2\right)\left(1-\cdots\right.\right.}$$

$$= -\frac{k\left(5+dk^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\cdots\right.}{\left(\left(1-x^2\right)\left(1-\cdots\right.\right.}$$

$$= -\frac{k\left(5+dk^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\cdots\right.}{\left(\left(1-x^2\right)\left(1-\cdots\right.\right.}$$

$$= -\frac{k\left(5+dk^2\right)x\left(1-x^2\right)^{2/3}\left(1-k^2x^2\cdots\right.}{\left(\left(1-x^2\right)\left(1-\cdots\right.\right.}$$

Mathematica [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{-3k+2\left(1+k^2\right)x+k\left(1+k^2\right)x^2-4k^2x^3+k^3x^4}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(-1+d-(1+2d)kx+\left(1+dk^2\right)x^2+kx^3\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-3*k + 2*(1 + k^2)*x + k*(1 + k^2)*x^2 - 4*k^2*x^3 + k^3*x^4)/((
(1 - x^2)*(1 - k^2*x^2))^(2/3)*(-1 + d - (1 + 2*d)*k*x + (1 + d*k^2)*x^2 +
k*x^3)),x]
```

```
[Out] Integrate[(-3*k + 2*(1 + k^2)*x + k*(1 + k^2)*x^2 - 4*k^2*x^3 + k^3*x^4)/((
(1 - x^2)*(1 - k^2*x^2))^(2/3)*(-1 + d - (1 + 2*d)*k*x + (1 + d*k^2)*x^2 +
k*x^3)), x]
```

IntegrateAlgebraic [A] time = 8.14, size = 232, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{d}\,kx-\sqrt[3]{d}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\right)}{d^{2/3}}-\frac{\log\left(d^{2/3}k^2x^2-2d^{2/3}kx+d^{2/3}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\left(\sqrt[3]{d}-\sqrt[3]{d}\,kx\right)+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}\right)}{2d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\,kx-\sqrt{3}\sqrt[3]{d}}{\sqrt[3]{d}\,kx-\sqrt[3]{d}-2\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*k + 2*(1 + k^2)*x + k*(1 + k^2)*x^2 - 4*k^2*x^3 + k^
3*x^4)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)*(-1 + d - (1 + 2*d)*k*x + (1 + d*k^
2)*x^2 + k*x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(-(Sqrt[3]*d^(1/3)) + Sqrt[3]*d^(1/3)*k*x)/(-d^(1/3) + d^(1
/3)*k*x - 2*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))])/d^(2/3) + Log[-d^(1/3)
+ d^(1/3)*k*x + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3)]/d^(2/3) - Log[d^(2/3)
 - 2*d^(2/3)*k*x + d^(2/3)*k^2*x^2 + (d^(1/3) - d^(1/3)*k*x)*(1 + (-1 - k^2
)*x^2 + k^2*x^4)^(1/3) + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*k+2*(k^2+1)*x+k*(k^2+1)*x^2-4*k^2*x^3+k^3*x^4)/((-x^2+1)*(-k^
2*x^2+1))^(2/3)/(-1+d-(1+2*d)*k*x+(d*k^2+1)*x^2+k*x^3),x, algorithm="fricas
")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^3x^4 - 4\,k^2x^3 + \left(k^2+1\right)kx^2 + 2\left(k^2+1\right)x - 3\,k}{\left(kx^3 - (2\,d+1)kx + \left(dk^2+1\right)x^2 + d - 1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*k+2*(k^2+1)*x+k*(k^2+1)*x^2-4*k^2*x^3+k^3*x^4)/((-x^2+1)*(-k^
2*x^2+1))^(2/3)/(-1+d-(1+2*d)*k*x+(d*k^2+1)*x^2+k*x^3),x, algorithm="giac")
```

```
[Out] integrate((k^3*x^4 - 4*k^2*x^3 + (k^2 + 1)*k*x^2 + 2*(k^2 + 1)*x - 3*k)/((k
*x^3 - (2*d + 1)*k*x + (d*k^2 + 1)*x^2 + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(
2/3)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{-3k + 2\left(k^2+1\right)x + k\left(k^2+1\right)x^2 - 4k^2x^3 + k^3x^4}{\left(\left(-x^2+1\right)\left(-k^2x^2+1\right)\right)^{\frac{2}{3}}\left(-1 + d - (1+2d)\,kx + \left(d\,k^2+1\right)x^2 + k\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*k+2*(k^2+1)*x+k*(k^2+1)*x^2-4*k^2*x^3+k^3*x^4)/((-x^2+1)*(-k^2*x^2+
1))^(2/3)/(-1+d-(1+2*d)*k*x+(d*k^2+1)*x^2+k*x^3),x)
```

```
[Out] int((-3*k+2*(k^2+1)*x+k*(k^2+1)*x^2-4*k^2*x^3+k^3*x^4)/((-x^2+1)*(-k^2*x^2+
1))^(2/3)/(-1+d-(1+2*d)*k*x+(d*k^2+1)*x^2+k*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^3x^4 - 4\,k^2x^3 + \left(k^2+1\right)kx^2 + 2\left(k^2+1\right)x - 3\,k}{\left(kx^3 - (2\,d+1)kx + \left(dk^2+1\right)x^2 + d - 1\right)\left(\left(k^2x^2-1\right)\left(x^2-1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*k+2*(k^2+1)*x+k*(k^2+1)*x^2-4*k^2*x^3+k^3*x^4)/((-x^2+1)*(-k^
2*x^2+1))^(2/3)/(-1+d-(1+2*d)*k*x+(d*k^2+1)*x^2+k*x^3),x, algorithm="maxima
")
```

```
[Out] integrate((k^3*x^4 - 4*k^2*x^3 + (k^2 + 1)*k*x^2 + 2*(k^2 + 1)*x - 3*k)/((k
*x^3 - (2*d + 1)*k*x + (d*k^2 + 1)*x^2 + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(
2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{2\,x\,\left(k^2+1\right)-3\,k-4\,k^2\,x^3+k^3\,x^4+k\,x^2\,\left(k^2+1\right)}{{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)}^{2/3}\,\left(k\,x^3+\left(d\,k^2+1\right)\,x^2-k\,\left(2\,d+1\right)\,x+d-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x*(k^2 + 1) - 3*k - 4*k^2*x^3 + k^3*x^4 + k*x^2*(k^2 + 1))/(((x^2 -
1)*(k^2*x^2 - 1))^(2/3)*(d + x^2*(d*k^2 + 1) + k*x^3 - k*x*(2*d + 1) - 1)),
x)
```

```
[Out] int((2*x*(k^2 + 1) - 3*k - 4*k^2*x^3 + k^3*x^4 + k*x^2*(k^2 + 1))/(((x^2 -
1)*(k^2*x^2 - 1))^(2/3)*(d + x^2*(d*k^2 + 1) + k*x^3 - k*x*(2*d + 1) - 1)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*k+2*(k**2+1)*x+k*(k**2+1)*x**2-4*k**2*x**3+k**3*x**4)/((-x**2
+1)*(-k**2*x**2+1))**(2/3)/(-1+d-(1+2*d)*k*x+(d*k**2+1)*x**2+k*x**3),x)
```

```
[Out] Timed out
```

3.2112 $\int \frac{1}{\left(-bx+a^2x^2\right)^{3/2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=232

$$\frac{2\sqrt{a^2x^2-bx}\,\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(1601a^6x^3-456a^4bx^2-200a^2b^2x+210b^3\right)}{1155b^5x^4\left(b-a^2x\right)}+\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(-\frac{6a^{1}\cdots}{\cdots}\right.$$

Rubi [F] time = 4.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(-bx+a^2x^2\right)^{3/2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((-(b*x) + a^2*x^2)^(3/2)*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2)),x
]
```

```
[Out] (2*Sqrt[x]*Sqrt[-b + a^2*x]*Defer[Subst][Defer[Int][1/(x^2*(-b + a^2*x^2)^(
3/2)*(a*x^4 + x^2*Sqrt[-(b*x^2) + a^2*x^4])^(3/2)), x], x, Sqrt[x]])/Sqrt[-
(b*x) + a^2*x^2]
```

Rubi steps

$$\int \frac{1}{\left(-bx+a^2x^2\right)^{3/2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx=\frac{\left(\sqrt{x}\,\sqrt{-b+a^2x}\right)\int \frac{1}{x^{3/2}\left(-b+a^2x\right)^{3/2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx}{\sqrt{-bx+a^2x^2}}$$

$$=\frac{\left(2\sqrt{x}\,\sqrt{-b+a^2x}\right)\text{Subst}\left(\int \frac{1}{x^2\left(-b+a^2x^2\right)^{3/2}\left(ax^4+x^2\sqrt{-bx^2+a^2x^4}\right)^{3/2}}\,d\cdots\right.}{\sqrt{-bx+a^2x^2}}$$

Mathematica [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx+a^2x^2\right)^{3/2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((-(b*x) + a^2*x^2)^(3/2)*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3
/2)),x]
```

```
[Out] Integrate[1/((-(b*x) + a^2*x^2)^(3/2)*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3
/2)), x]
```

IntegrateAlgebraic [A] time = 7.39, size = 232, normalized size = 1.00

$$\frac{2\sqrt{a^2x^2-bx}\,\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(1601a^6x^3-456a^4bx^2-200a^2b^2x+210b^3\right)}{1155b^5x^4\left(b-a^2x\right)}+\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(-\frac{6a^{11/2}\sqrt{\sqrt{a^2x^2-bx}-ax}\,\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\sqrt{a^2x^2-bx}-ax}}{\sqrt{b}}\right)}{b^{11/2}x}-\frac{4\left(2533a^5x^2+461a^3bx+245ab^2\right)}{1155b^5x^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-(b*x) + a^2*x^2)^(3/2)*(a*x^2 + x*Sqrt[-(b*x) + a^2
*x^2])^(3/2)),x]
```

```
[Out] (2*Sqrt[-(b*x) + a^2*x^2]*(210*b^3 - 200*a^2*b^2*x - 456*a^4*b*x^2 + 1601*a
^6*x^3)*Sqrt[x*(a*x + Sqrt[-(b*x) + a^2*x^2])])/(1155*b^5*x^4*(b - a^2*x))
+ Sqrt[x*(a*x + Sqrt[-(b*x) + a^2*x^2])]*((-4*(245*a*b^2 + 461*a^3*b*x + 25
33*a^5*x^2))/(1155*b^5*x^3) - (6*a^(11/2)*Sqrt[-(a*x) + Sqrt[-(b*x) + a^2*x
^2]]*ArcTan[(Sqrt[a]*Sqrt[-(a*x) + Sqrt[-(b*x) + a^2*x^2]])/Sqrt[b]])/(b^(1
1/2)*x))
```

fricas [A] time = 0.63, size = 438, normalized size = 1.89

$$\left[\frac{3465\left(a^7x^5-a^5bx^4\right)\sqrt{a}\,\log\left(\frac{a^2x^2+2\sqrt{a^2x^2-bx}\,ax-bx+2\sqrt{a^2x^2-bx}\,\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}\,\sqrt{a}}{a^2x^2-bx}\right)-2\left(5066\,a^7x^4-4144\,a^5bx^3-432\,a^3b^2x^2-490\,ab^3x+\left(1601\,a^6x^3-456\,a^4bx^2-200\,a^2b^2x+210\,b^3\right)\sqrt{a^2x^2-bx}\right)\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}}{1155\left(a^2b^5x^5-b^6x^4\right)},\ -\frac{2\left(3465\left(a^7x^5-a^5bx^4\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}\,\sqrt{-a}}{ax}\right)+\left(5066\,a^7x^4-4144\,a^5bx^3-432\,a^3b^2x^2-490\,ab^3x+\left(1601\,a^6x^3-456\,a^4bx^2-200\,a^2b^2x+210\,b^3\right)\sqrt{a^2x^2-bx}\right)\sqrt{ax^2+\sqrt{a^2x^2-bx}\,x}\right)}{1155\left(a^2b^5x^5-b^6x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b*x)^(3/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algo
rithm="fricas")
```

```
[Out] [1/1155*(3465*(a^7*x^5 - a^5*b*x^4)*sqrt(a)*log((a^2*x^2 + 2*sqrt(a^2*x^2 -
 b*x)*a*x - b*x + 2*sqrt(a^2*x^2 - b*x)*sqrt(a*x^2 + sqrt(a^2*x^2 - b*x)*x)
*sqrt(a))/(a^2*x^2 - b*x)) - 2*(5066*a^7*x^4 - 4144*a^5*b*x^3 - 432*a^3*b^2
*x^2 - 490*a*b^3*x + (1601*a^6*x^3 - 456*a^4*b*x^2 - 200*a^2*b^2*x + 210*b^
3)*sqrt(a^2*x^2 - b*x))*sqrt(a*x^2 + sqrt(a^2*x^2 - b*x)*x))/(a^2*b^5*x^5 -
 b^6*x^4), -2/1155*(3465*(a^7*x^5 - a^5*b*x^4)*sqrt(-a)*arctan(sqrt(a*x^2 +
 sqrt(a^2*x^2 - b*x)*x)*sqrt(-a)/(a*x)) + (5066*a^7*x^4 - 4144*a^5*b*x^3 -
432*a^3*b^2*x^2 - 490*a*b^3*x + (1601*a^6*x^3 - 456*a^4*b*x^2 - 200*a^2*b^2
*x + 210*b^3)*sqrt(a^2*x^2 - b*x))*sqrt(a*x^2 + sqrt(a^2*x^2 - b*x)*x))/(a^
2*b^5*x^5 - b^6*x^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2-bx\right)^{\frac{3}{2}}\left(ax^2+\sqrt{a^2x^2-bx}\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b*x)^(3/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algo
rithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - b*x)^(3/2)*(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2)),
x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2-bx\right)^{\frac{3}{2}}\left(a\,x^2+x\sqrt{a^2x^2-bx}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*x^2-b*x)^(3/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

```
[Out] int(1/(a^2*x^2-b*x)^(3/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2-bx\right)^{\frac{3}{2}}\left(ax^2+\sqrt{a^2x^2-bx}\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b*x)^(3/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algo
rithm="maxima")
```

```
[Out] integrate(1/((a^2*x^2 - b*x)^(3/2)*(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a^2\,x^2-b\,x\right)^{3/2}\,\left(a\,x^2+x\,\sqrt{a^2\,x^2-b\,x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2*x^2 - b*x)^(3/2)*(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2)),x)
```

```
[Out] int(1/((a^2*x^2 - b*x)^(3/2)*(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x \left(ax + \sqrt{a^{2} x^{2} - bx}\right)\right)^{\frac{3}{2}} \left(x \left(a^{2} x - b\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*x**2-b*x)**(3/2)/(a*x**2+x*(a**2*x**2-b*x)**(1/2))**(3/2)
,x)
```

```
[Out] Integral(1/((x*(a*x + sqrt(a**2*x**2 - b*x)))**(3/2)*(x*(a**2*x - b))**(3/2
)), x)
```

3.2113 $\int \frac{1}{x\sqrt[3]{(-1+x)(q-2qx+x^2)}}\,dx$

Optimal. Leaf size=233

$$\frac{\log\left(q^{2/3}x^2-2q^{2/3}x+q^{2/3}+\left((-2q-1)x^2+3qx-q+x^3\right)^{2/3}+\left(\sqrt[3]{q}\,x-\sqrt[3]{q}\right)\sqrt[3]{(-2q-1)x^2+3qx-q+x^3}\right)}{4\sqrt[3]{q}}$$ lo

Rubi [F] time = 22.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

```
[Out] ((-1 - 2*q - (1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[
-((-1 + q)^3*q)])^(2/3))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[-((-1 +
 q)^3*q)])^(1/3) + 3*x)^(1/3)*(-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(
1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q -
15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (3*(1 - 5*q + 4*q^2 +
 (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*((-1 - 2*q
)/3 + x))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3) +
9*((-1 - 2*q)/3 + x)^2)^(1/3)*Defer[Subst][Defer[Int][1/(((1 + 2*q)/3 + x)*
(-1/3*(1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)
^3*q])^(2/3))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3
) + x)^(1/3)*((-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(1 + 6*q - 15*q^2
 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q - 15*q^2 + 8*q^3 +
 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))/9 + ((1 - 5*q + 4*q^2 + (1 + 6*q - 15*
q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*x)/(3*(1 + 6*q - 15*q^2 +
 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3)) + x^2)^(1/3)), x], x, (-1 - 2*
q)/3 + x])/(3*(-q + 3*q*x + (-1 - 2*q)*x^2 + x^3)^(1/3))
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{3}(1+2q)+x\right)\sqrt[3]{-\frac{2}{27}(1-q)^2(1+8q)-\frac{1}{3}(1-4q)(1-q)x+x^3}}\right.$$

$$= \frac{\left(\sqrt[3]{-1-2q-\frac{1-5q+4q^2+\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q}\right)^{2/3}}{\sqrt[3]{1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q}}}+3x}\sqrt[3]{-1+5q-4\cdots}\right.}{}$$

Mathematica [C] time = 0.19, size = 55, normalized size = 0.24

$$\frac{3\left((x-1)\left(-2qx+q+x^2\right)\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x^2-2qx+q}{q(x-1)^2}\right)}{4q(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

```
[Out] (3*((-1 + x)*(q - 2*q*x + x^2))^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (q - 2
*q*x + x^2)/(q*(-1 + x)^2)])/(4*q*(-1 + x)^2)
```

IntegrateAlgebraic [A] time = 0.54, size = 233, normalized size = 1.00

$$\frac{\log\left(q^{2/3}x^2-2q^{2/3}x+q^{2/3}+\left((-2q-1)x^2+3qx-q+x^3\right)^{2/3}+\left(\sqrt[3]{q}\,x-\sqrt[3]{q}\right)\sqrt[3]{(-2q-1)x^2+3qx-q+x^3}\right)}{4\sqrt[3]{q}}-\frac{\log\left(\sqrt[3]{(-2q-1)x^2+3qx-q+x^3}-\sqrt[3]{q}\,x+\sqrt[3]{q}\right)}{2\sqrt[3]{q}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{(-2q-1)x^2+3qx-q+x^3}}{\sqrt[3]{(-2q-1)x^2+3qx-q+x^3}+2\sqrt[3]{q}\,x-2\sqrt[3]{q}}\right)}{2\sqrt[3]{q}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*(-q + 3*q*x + (-1 - 2*q)*x^2 + x^3)^(1/3))/(-
2*q^(1/3) + 2*q^(1/3)*x + (-q + 3*q*x + (-1 - 2*q)*x^2 + x^3)^(1/3))])/q^(1
/3) - Log[q^(1/3) - q^(1/3)*x + (-q + 3*q*x + (-1 - 2*q)*x^2 + x^3)^(1/3)]/
(2*q^(1/3)) + Log[q^(2/3) - 2*q^(2/3)*x + q^(2/3)*x^2 + (-q^(1/3) + q^(1/3)
*x)*(-q + 3*q*x + (-1 - 2*q)*x^2 + x^3)^(1/3) + (-q + 3*q*x + (-1 - 2*q)*x^
2 + x^3)^(2/3)]/(4*q^(1/3))
```

fricas [B] time = 24.48, size = 1496, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="fricas")
```

```
[Out] [1/12*(sqrt(3)*q*sqrt((-q)^(1/3)/q)*log(-((q^3 - 30*q^2 - 51*q - 1)*x^6 + 5
4*(q^3 + 6*q^2 + 2*q)*x^5 - 27*(17*q^3 + 26*q^2 + 2*q)*x^4 + 486*q^3*x + 54
0*(2*q^3 + q^2)*x^3 - 81*q^3 - 135*(8*q^3 + q^2)*x^2 + 9*((2*q^2 - q - 1)*x
^4 - 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^
(2/3)*(-q)^(1/3) + 9*((q^2 + 7*q + 1)*x^5 - (19*q^2 + 25*q + 1)*x^4 + 9*(7*
q^2 + 3*q)*x^3 + 45*q^2*x - 9*(9*q^2 + q)*x^2 - 9*q^2)*(-(2*q + 1)*x^2 + x^
3 + 3*q*x - q)^(1/3)*(-q)^(2/3) + sqrt(3)*(3*((4*q^2 + 13*q + 1)*x^4 - 6*(7
*q^2 + 5*q)*x^3 - 72*q^2*x + 3*(31*q^2 + 5*q)*x^2 + 18*q^2)*(-(2*q + 1)*x^2
 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^3 - 5*q^2 - 5*q)*x^5 + 5*(q^3
+ 7*q^2 + q)*x^4 - 45*q^3*x - 45*(q^3 + q^2)*x^3 + 9*q^3 + 15*(5*q^3 + q^2)
*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) + ((q^3 + 24*q^2 + 3*q - 1)*
x^6 - 54*(q^3 + 2*q^2)*x^5 + 81*(3*q^3 + 2*q^2)*x^4 - 162*q^3*x - 108*(4*q^
3 + q^2)*x^3 + 27*q^3 + 27*(14*q^3 + q^2)*x^2)*(-q)^(1/3))*sqrt((-q)^(1/3)/
q))/x^6) - 2*(-q)^(2/3)*log(((-q)^(2/3)*(q - 1)*x^2 + 3*(-(2*q + 1)*x^2 + x
^3 + 3*q*x - q)^(1/3)*(q*x - q)*(-q)^(1/3) + 3*(-(2*q + 1)*x^2 + x^3 + 3*q*
x - q)^(2/3)*q)/x^2) + (-q)^(2/3)*log((3*((2*q + 1)*x^2 - 6*q*x + 3*q)*(-(2
*q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^2 + 2*q)*x^3 + 9*q^
2*x - (7*q^2 + 2*q)*x^2 - 3*q^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) -
 ((q^2 + 7*q + 1)*x^4 - 18*(q^2 + q)*x^3 - 36*q^2*x + 9*(5*q^2 + q)*x^2 + 9
*q^2)*(-q)^(1/3))/x^4))/q, 1/12*(2*sqrt(3)*q*sqrt(-(-q)^(1/3)/q)*arctan(1/3
*sqrt(3)*(6*((2*q^2 - q - 1)*x^4 - 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*(-(2*
q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) - 6*((q^3 + 7*q^2 + q)*x^5 -
 (19*q^3 + 25*q^2 + q)*x^4 + 45*q^3*x + 9*(7*q^3 + 3*q^2)*x^3 - 9*q^3 - 9*(
9*q^3 + q^2)*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) - ((q^3 - 12*q^2
 - 15*q - 1)*x^6 + 18*(q^3 + 6*q^2 + 2*q)*x^5 - 9*(17*q^3 + 26*q^2 + 2*q)*x
^4 + 162*q^3*x + 180*(2*q^3 + q^2)*x^3 - 27*q^3 - 45*(8*q^3 + q^2)*x^2)*(-q
)^(1/3))*sqrt(-(-q)^(1/3)/q)/((q^3 + 24*q^2 + 3*q - 1)*x^6 - 54*(q^3 + 2*q^
2)*x^5 + 81*(3*q^3 + 2*q^2)*x^4 - 162*q^3*x - 108*(4*q^3 + q^2)*x^3 + 27*q^
3 + 27*(14*q^3 + q^2)*x^2)) - 2*(-q)^(2/3)*log(((-q)^(2/3)*(q - 1)*x^2 + 3*
(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3)*(q*x - q)*(-q)^(1/3) + 3*(-(2*q +
1)*x^2 + x^3 + 3*q*x - q)^(2/3)*q)/x^2) + (-q)^(2/3)*log((3*((2*q + 1)*x^2
 - 6*q*x + 3*q)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^
```

```
2 + 2*q)*x^3 + 9*q^2*x - (7*q^2 + 2*q)*x^2 - 3*q^2)*(-(2*q + 1)*x^2 + x^3 +
 3*q*x - q)^(1/3) - ((q^2 + 7*q + 1)*x^4 - 18*(q^2 + q)*x^3 - 36*q^2*x + 9*
(5*q^2 + q)*x^2 + 9*q^2)*(-q)^(1/3))/x^4))/q]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(2\,qx - x^2 - q\right)(x - 1)\right)^{\frac{1}{3}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="giac")
[Out] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left((-1 + x)\left(-2qx + x^2 + q\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x)
[Out] int(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(2\,qx - x^2 - q\right)(x - 1)\right)^{\frac{1}{3}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="maxima")
[Out] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left((x - 1)\left(x^2 - 2\,q\,x + q\right)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((x - 1)*(q - 2*q*x + x^2))^(1/3)),x)
[Out] int(1/(x*((x - 1)*(q - 2*q*x + x^2))^(1/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x**2+q))**(1/3),x)
[Out] Timed out
```

3.2114 $\int \frac{ab+(-2a+b)x}{\sqrt[3]{x(-a+x)(-b+x)}\left(a^2d+(b-2ad)x+(-1+d)x^2\right)}dx$

Optimal. Leaf size=233

$$\frac{\log\left(a^2d^{2/3}+\sqrt[3]{x^2(-a-b)+abx+x^3}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)+\left(x^2(-a-b)+abx+x^3\right)^{2/3}-2ad^{2/3}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{x^2\right.}}{}$$

Rubi [F] time = 3.69, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ab+(-2a+b)x}{\sqrt[3]{x(-a+x)(-b+x)}\left(a^2d+(b-2ad)x+(-1+d)x^2\right)}dx$$

Verification is not applicable to the result.

```
[In] Int[(a*b + (-2*a + b)*x)/((x*(-a + x)*(-b + x))^(1/3)*(a^2*d + (b - 2*a*d)*
x + (-1 + d)*x^2)),x]
```

```
[Out] -(((2*a - b + Sqrt[b^2 + 4*a^2*d - 4*a*b*d])*x^(1/3)*(-a + x)^(1/3)*(-b + x
)^(1/3)*Defer[Int][1/(x^(1/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*(b - 2*a*d - Sq
rt[b^2 + 4*a^2*d - 4*a*b*d] + 2*(-1 + d)*x)), x])/((a - x)*(b - x)*x)^(1/3)
) - ((2*a - b - Sqrt[b^2 + 4*a^2*d - 4*a*b*d])*x^(1/3)*(-a + x)^(1/3)*(-b +
 x)^(1/3)*Defer[Int][1/(x^(1/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*(b - 2*a*d +
Sqrt[b^2 + 4*a^2*d - 4*a*b*d] + 2*(-1 + d)*x)), x])/((a - x)*(b - x)*x)^(1/
3)
```

Rubi steps

$$\int \frac{ab+(-2a+b)x}{\sqrt[3]{x(-a+x)(-b+x)}\left(a^2d+(b-2ad)x+(-1+d)x^2\right)}dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\int \frac{ab+(-2a+b)x}{\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\left(a^2d+(b-2a\right.}}{\sqrt[3]{x(-a+x)(-b+x)}}$$

$$= \frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\int\left(\frac{-2a+b-\sqrt{b^2+}}{\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\left(b-2ad-\sqrt{}\right.}\right.}{}$$

$$= \frac{\left(\left(-2a+b-\sqrt{b^2+4a^2d-4abd}\right)\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b}\right.}{\sqrt[3]{x(-a}}$$

Mathematica [F] time = 4.91, size = 0, normalized size = 0.00

$$\int \frac{ab+(-2a+b)x}{\sqrt[3]{x(-a+x)(-b+x)}\left(a^2d+(b-2ad)x+(-1+d)x^2\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*b + (-2*a + b)*x)/((x*(-a + x)*(-b + x))^(1/3)*(a^2*d + (b - 2
*a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] Integrate[(a*b + (-2*a + b)*x)/((x*(-a + x)*(-b + x))^(1/3)*(a^2*d + (b - 2
*a*d)*x + (-1 + d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 2.94, size = 233, normalized size = 1.00

$$\frac{\log\left(a^2d^{2/3}+\sqrt[3]{x^2(-a-b)+abx+x^3}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)+\left(x^2(-a-b)+abx+x^3\right)^{2/3}-2ad^{2/3}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{x^2(-a-b)+abx+x^3}+a\sqrt[3]{d}-\sqrt[3]{d}\,x\right)}{\sqrt[3]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^2(-a-b)+abx+x^3}}{\sqrt[3]{x^2(-a-b)+abx+x^3}-2a\sqrt[3]{d}+2\sqrt[3]{d}\,x}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b + (-2*a + b)*x)/((x*(-a + x)*(-b + x))^(1/3)*(a^2*d
 + (b - 2*a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*(a*b*x + (-a - b)*x^2 + x^3)^(1/3))/(-2*a*d^(1/3
) + 2*d^(1/3)*x + (a*b*x + (-a - b)*x^2 + x^3)^(1/3))])/d^(1/3)) - Log[a*d^
(1/3) - d^(1/3)*x + (a*b*x + (-a - b)*x^2 + x^3)^(1/3)]/d^(1/3) + Log[a^2*d
^(2/3) - 2*a*d^(2/3)*x + d^(2/3)*x^2 + (-(a*d^(1/3)) + d^(1/3)*x)*(a*b*x +
(-a - b)*x^2 + x^3)^(1/3) + (a*b*x + (-a - b)*x^2 + x^3)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b+(-2*a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(a^2*d+(-2*a*d+b)*x+(-1+
d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab-(2\,a-b)x}{\left(a^2d+(d-1)x^2-(2\,ad-b)x\right)\left((a-x)(b-x)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b+(-2*a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(a^2*d+(-2*a*d+b)*x+(-1+
d)*x^2),x, algorithm="giac")
```

```
[Out] integrate((a*b - (2*a - b)*x)/((a^2*d + (d - 1)*x^2 - (2*a*d - b)*x)*((a -
x)*(b - x)*x)^(1/3)), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{ab+(-2a+b)\,x}{\left(x\,(-a+x)\,(-b+x)\right)^{\frac{1}{3}}\left(a^2d+(-2ad+b)\,x+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b+(-2*a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(a^2*d+(-2*a*d+b)*x+(-1+d)*x^2
),x)
```

```
[Out] int((a*b+(-2*a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(a^2*d+(-2*a*d+b)*x+(-1+d)*x^2
),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab-(2\,a-b)x}{\left(a^2d+(d-1)x^2-(2\,ad-b)x\right)\left((a-x)(b-x)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b+(-2*a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(a^2*d+(-2*a*d+b)*x+(-1+
d)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((a*b - (2*a - b)*x)/((a^2*d + (d - 1)*x^2 - (2*a*d - b)*x)*((a -
x)*(b - x)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,b-x\,\left(2\,a-b\right)}{{\left(x\,\left(a-x\right)\,\left(b-x\right)\right)}^{1/3}\,\left(a^2\,d+x\,\left(b-2\,a\,d\right)+x^2\,\left(d-1\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b - x*(2*a - b))/((x*(a - x)*(b - x))^(1/3)*(a^2*d + x*(b - 2*a*d) +
 x^2*(d - 1))),x)
```

```
[Out] int((a*b - x*(2*a - b))/((x*(a - x)*(b - x))^(1/3)*(a^2*d + x*(b - 2*a*d) +
 x^2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b+(-2*a+b)*x)/(x*(-a+x)*(-b+x))**(1/3)/(a**2*d+(-2*a*d+b)*x+(-
1+d)*x**2),x)
```

```
[Out] Timed out
```

3.2115 $\int \frac{1+x^4}{\left(-1-x^2+x^4\right)\sqrt[4]{-x^2+x^4}}\,dx$

Optimal. Leaf size=233

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-x^2}}\right)-\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4-x^2}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x}}\right.$$

Rubi [A] time = 1.28, antiderivative size = 393, normalized size of antiderivative = 1.69, number of steps used = 18, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2056, 6715, 6728, 240, 212, 206, 203, 377}

$$\frac{\sqrt{x}\sqrt[4]{x^2-1}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{\sqrt[4]{x^4-x^2}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\sqrt{x}\sqrt[4]{x^2-1}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{\sqrt[4]{x^4-x^2}}-\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\sqrt{x}\sqrt[4]{x^2-1}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{\sqrt[4]{x^4-x^2}}+\frac{\sqrt{x}\sqrt[4]{x^2-1}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{\sqrt[4]{x^4-x^2}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\sqrt{x}\sqrt[4]{x^2-1}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{\sqrt[4]{x^4-x^2}}-\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\sqrt{x}\sqrt[4]{x^2-1}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{\sqrt[4]{x^4-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/((-1 - x^2 + x^4)*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] (Sqrt[x]*(-1 + x^2)^(1/4)*ArcTan[Sqrt[x]/(-1 + x^2)^(1/4)])/(-x^2 + x^4)^(1
/4) - (((3 + Sqrt[5])/2)^(1/4)*Sqrt[x]*(-1 + x^2)^(1/4)*ArcTan[((2/(3 + Sqr
t[5]))^(1/4)*Sqrt[x])/(-1 + x^2)^(1/4)])/(-x^2 + x^4)^(1/4) - (((3 - Sqrt[5
])/2)^(1/4)*Sqrt[x]*(-1 + x^2)^(1/4)*ArcTan[(((3 + Sqrt[5])/2)^(1/4)*Sqrt[x
])/(-1 + x^2)^(1/4)])/(-x^2 + x^4)^(1/4) + (Sqrt[x]*(-1 + x^2)^(1/4)*ArcTan
h[Sqrt[x]/(-1 + x^2)^(1/4)])/(-x^2 + x^4)^(1/4) - (((3 + Sqrt[5])/2)^(1/4)*
Sqrt[x]*(-1 + x^2)^(1/4)*ArcTanh[((2/(3 + Sqrt[5]))^(1/4)*Sqrt[x])/(-1 + x^
2)^(1/4)])/(-x^2 + x^4)^(1/4) - (((3 - Sqrt[5])/2)^(1/4)*Sqrt[x]*(-1 + x^2)
^(1/4)*ArcTanh[(((3 + Sqrt[5])/2)^(1/4)*Sqrt[x])/(-1 + x^2)^(1/4)])/(-x^2 +
 x^4)^(1/4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1+x^4}{\left(-1-x^2+x^4\right)\sqrt[4]{-x^2+x^4}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{-1+x^2}\right)\int \frac{1+x^4}{\sqrt{x}\sqrt[4]{-1+x^2}\left(-1-x^2+x^4\right)}\,dx}{\sqrt[4]{-x^2+x^4}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{1+x^8}{\sqrt[4]{-1+x^4}\left(-1-x^4+x^8\right)}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^4}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[4]{-1+x^4}} + \frac{2+x^4}{\sqrt[4]{-1+x^4}\left(-1-x^4+x^8\right)}\right)dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^4}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-1+x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^4}} + \frac{\left(2\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \cdots\right.}{\sqrt[4]{-\cdots}}$$

$$= \frac{\left(2\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}} + \frac{\left(2\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{\left(\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}} + \frac{\left(\sqrt{x}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \cdots\right.}{\sqrt[4]{-x^2+x^{\cdots}}}$$

$$= \frac{\sqrt{x}\sqrt[4]{-1+x^2}\,\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}} + \frac{\sqrt{x}\sqrt[4]{-1+x^2}\,\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}} + \frac{\left(2\left(1-\cdots\right.\right.}{\cdots}$$

$$= \frac{\sqrt{x}\sqrt[4]{-1+x^2}\,\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}} + \frac{\sqrt{x}\sqrt[4]{-1+x^2}\,\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}} + \frac{\left(\left(1-\cdots\right.\right.}{\cdots}$$

$$= \frac{\sqrt{x}\sqrt[4]{-1+x^2}\,\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{\sqrt[4]{-x^2+x^4}} - \frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\sqrt{x}\sqrt[4]{-1+x^2}\,\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\cdots}{\sqrt[4]{-1+\cdots}}\right.}{\sqrt[4]{-x^2+x^4}}$$

Mathematica [A] time = 0.29, size = 268, normalized size = 1.15

$$\frac{\sqrt{x}\sqrt[4]{x^2-1}\left(2\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)-2^{3/4}\sqrt[4]{3+\sqrt{5}}\,\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\sqrt{x}}{\sqrt[4]{x^2-1}}\right)-2^{3/4}\sqrt[4]{3-\sqrt{5}}\,\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\sqrt{x}}{\sqrt[4]{x^2-1}}\right)+2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)-2^{3/4}\sqrt[4]{3+\sqrt{5}}\,\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\sqrt{x}}{\sqrt[4]{x^2-1}}\right)-2^{3/4}\sqrt[4]{3-\sqrt{5}}\,\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\sqrt{x}}{\sqrt[4]{x^2-1}}\right)\right)}{2\sqrt[4]{x^2\left(x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/((-1 - x^2 + x^4)*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] (Sqrt[x]*(-1 + x^2)^(1/4)*(2*ArcTan[Sqrt[x]/(-1 + x^2)^(1/4)] - 2^(3/4)*(3
+ Sqrt[5])^(1/4)*ArcTan[((2/(3 + Sqrt[5]))^(1/4)*Sqrt[x])/(-1 + x^2)^(1/4)]
 - 2^(3/4)*(3 - Sqrt[5])^(1/4)*ArcTan[(((3 + Sqrt[5])/2)^(1/4)*Sqrt[x])/(-1
 + x^2)^(1/4)] + 2*ArcTanh[Sqrt[x]/(-1 + x^2)^(1/4)] - 2^(3/4)*(3 + Sqrt[5]
)^(1/4)*ArcTanh[((2/(3 + Sqrt[5]))^(1/4)*Sqrt[x])/(-1 + x^2)^(1/4)] - 2^(3/
4)*(3 - Sqrt[5])^(1/4)*ArcTanh[(((3 + Sqrt[5])/2)^(1/4)*Sqrt[x])/(-1 + x^2)
^(1/4)]))/(2*(x^2*(-1 + x^2))^(1/4))
```

IntegrateAlgebraic [A] time = 0.77, size = 233, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-x^2}}\right)-\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4-x^2}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-x^2}}\right)-\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\,\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)/((-1 - x^2 + x^4)*(-x^2 + x^4)^(1/4)),x]
```

```
[Out] ArcTan[x/(-x^2 + x^4)^(1/4)] - Sqrt[(1 + Sqrt[5])/2]*ArcTan[(Sqrt[-1/2 + Sq
rt[5]/2]*x)/(-x^2 + x^4)^(1/4)] - Sqrt[(-1 + Sqrt[5])/2]*ArcTan[(Sqrt[1/2 +
 Sqrt[5]/2]*x)/(-x^2 + x^4)^(1/4)] + ArcTanh[x/(-x^2 + x^4)^(1/4)] - Sqrt[(
1 + Sqrt[5])/2]*ArcTanh[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(-x^2 + x^4)^(1/4)] - Sq
rt[(-1 + Sqrt[5])/2]*ArcTanh[(Sqrt[1/2 + Sqrt[5]/2]*x)/(-x^2 + x^4)^(1/4)]
```

fricas [B] time = 106.33, size = 1300, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-x^2-1)/(x^4-x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(sqrt(5) + 1)*arctan(-1/163724*(sqrt(2)*(sqrt(5)*sqrt(2)*(
224*x^5 - 310*x^3 + 43*x) - sqrt(2)*(215*x^5 - 663*x^3 + 224*x) - sqrt(x^4
- x^2)*(sqrt(5)*sqrt(2)*(86*x^3 - 267*x) - sqrt(2)*(448*x^3 - 439*x)))*sqrt
(40157*sqrt(5) + 36899)*sqrt(sqrt(5) + 1) - 81862*((x^4 - x^2)^(3/4)*(sqrt(
2)*(2*x^2 - 1) + sqrt(5)*sqrt(2)) + (x^4 - x^2)^(1/4)*(sqrt(5)*sqrt(2)*(x^4
 - x^2) - sqrt(2)*(x^4 - 3*x^2)))*sqrt(sqrt(5) + 1))/(x^5 - x^3 - x)) + 1/2
*sqrt(2)*sqrt(sqrt(5) - 1)*arctan(1/163724*(sqrt(2)*(sqrt(5)*sqrt(2)*(224*x
^5 - 310*x^3 + 43*x) + sqrt(2)*(215*x^5 - 663*x^3 + 224*x) + sqrt(x^4 - x^2
)*(sqrt(5)*sqrt(2)*(86*x^3 - 267*x) + sqrt(2)*(448*x^3 - 439*x)))*sqrt(4015
7*sqrt(5) - 36899)*sqrt(sqrt(5) - 1) + 81862*((x^4 - x^2)^(3/4)*(sqrt(2)*(2
*x^2 - 1) - sqrt(5)*sqrt(2)) + (x^4 - x^2)^(1/4)*(sqrt(5)*sqrt(2)*(x^4 - x^
2) + sqrt(2)*(x^4 - 3*x^2)))*sqrt(sqrt(5) - 1))/(x^5 - x^3 - x)) + 1/8*sqrt
(2)*sqrt(sqrt(5) + 1)*log((4*(x^4 - x^2)^(3/4)*(448*x^2 + sqrt(5)*(86*x^2 +
 181) - 9) + (sqrt(5)*sqrt(2)*(9*x^5 - 371*x^3 + 181*x) - sqrt(2)*(905*x^5
- 923*x^3 + 9*x) - 2*sqrt(x^4 - x^2)*(sqrt(5)*sqrt(2)*(181*x^3 - 95*x) - sq
rt(2)*(9*x^3 - 457*x)))*sqrt(sqrt(5) + 1) - 4*(9*x^4 - 457*x^2 - sqrt(5)*(1
81*x^4 - 95*x^2))*(x^4 - x^2)^(1/4))/(x^5 - x^3 - x)) - 1/8*sqrt(2)*sqrt(sq
rt(5) + 1)*log((4*(x^4 - x^2)^(3/4)*(448*x^2 + sqrt(5)*(86*x^2 + 181) - 9)
- (sqrt(5)*sqrt(2)*(9*x^5 - 371*x^3 + 181*x) - sqrt(2)*(905*x^5 - 923*x^3 +
 9*x) - 2*sqrt(x^4 - x^2)*(sqrt(5)*sqrt(2)*(181*x^3 - 95*x) - sqrt(2)*(9*x^
3 - 457*x)))*sqrt(sqrt(5) + 1) - 4*(9*x^4 - 457*x^2 - sqrt(5)*(181*x^4 - 95
*x^2))*(x^4 - x^2)^(1/4))/(x^5 - x^3 - x)) - 1/8*sqrt(2)*sqrt(sqrt(5) - 1)*
log((4*(x^4 - x^2)^(3/4)*(448*x^2 - sqrt(5)*(86*x^2 + 181) - 9) + (sqrt(5)*
sqrt(2)*(9*x^5 - 371*x^3 + 181*x) + sqrt(2)*(905*x^5 - 923*x^3 + 9*x) + 2*s
qrt(x^4 - x^2)*(sqrt(5)*sqrt(2)*(181*x^3 - 95*x) + sqrt(2)*(9*x^3 - 457*x))
)*sqrt(sqrt(5) - 1) + 4*(9*x^4 - 457*x^2 + sqrt(5)*(181*x^4 - 95*x^2))*(x^4
 - x^2)^(1/4))/(x^5 - x^3 - x)) + 1/8*sqrt(2)*sqrt(sqrt(5) - 1)*log((4*(x^4
 - x^2)^(3/4)*(448*x^2 - sqrt(5)*(86*x^2 + 181) - 9) - (sqrt(5)*sqrt(2)*(9*
x^5 - 371*x^3 + 181*x) + sqrt(2)*(905*x^5 - 923*x^3 + 9*x) + 2*sqrt(x^4 - x
^2)*(sqrt(5)*sqrt(2)*(181*x^3 - 95*x) + sqrt(2)*(9*x^3 - 457*x)))*sqrt(sqrt
(5) - 1) + 4*(9*x^4 - 457*x^2 + sqrt(5)*(181*x^4 - 95*x^2))*(x^4 - x^2)^(1/
4))/(x^5 - x^3 - x)) - 1/2*arctan(2*((x^4 - x^2)^(1/4)*x^2 + (x^4 - x^2)^(3
/4))/x) + 1/2*log((2*x^3 + 2*(x^4 - x^2)^(1/4)*x^2 + 2*sqrt(x^4 - x^2)*x -
x + 2*(x^4 - x^2)^(3/4))/x)
```

giac [A] time = 0.48, size = 232, normalized size = 1.00

$$-\frac{1}{2}\sqrt{2\sqrt{5}-2}\arctan\left(\frac{\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)+\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)-\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}-\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)+\frac{1}{4}\sqrt{2\sqrt{5}+2}\log\left(\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)-\frac{1}{4}\sqrt{2\sqrt{5}+2}\log\left(\left|-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right|\right)+\arctan\left(\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}\right)-\frac{1}{2}\log\left(\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}+1\right)+\frac{1}{2}\log\left(-\left(-\frac{1}{x^2}+1\right)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-x^2-1)/(x^4-x^2)^(1/4),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2*sqrt(5) - 2)*arctan((-1/x^2 + 1)^(1/4)/sqrt(1/2*sqrt(5) + 1/2))
 - 1/2*sqrt(2*sqrt(5) + 2)*arctan((-1/x^2 + 1)^(1/4)/sqrt(1/2*sqrt(5) - 1/2
```

```
)) + 1/4*sqrt(2*sqrt(5) - 2)*log(sqrt(1/2*sqrt(5) + 1/2) + (-1/x^2 + 1)^(1/
4)) - 1/4*sqrt(2*sqrt(5) - 2)*log(sqrt(1/2*sqrt(5) + 1/2) - (-1/x^2 + 1)^(1
/4)) + 1/4*sqrt(2*sqrt(5) + 2)*log(sqrt(1/2*sqrt(5) - 1/2) + (-1/x^2 + 1)^(
1/4)) - 1/4*sqrt(2*sqrt(5) + 2)*log(abs(-sqrt(1/2*sqrt(5) - 1/2) + (-1/x^2
+ 1)^(1/4))) + arctan((-1/x^2 + 1)^(1/4)) - 1/2*log((-1/x^2 + 1)^(1/4) + 1)
 + 1/2*log(-(-1/x^2 + 1)^(1/4) + 1)
```

maple [F] time = 10.01, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^4-x^2-1\right)\left(x^4-x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^4-x^2-1)/(x^4-x^2)^(1/4),x)
```

```
[Out] int((x^4+1)/(x^4-x^2-1)/(x^4-x^2)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^4-x^2\right)^{\frac{1}{4}}\left(x^4-x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-x^2-1)/(x^4-x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/((x^4 - x^2)^(1/4)*(x^4 - x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x^4+1}{\left(x^4-x^2\right)^{1/4}\left(-x^4+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 + 1)/((x^4 - x^2)^(1/4)*(x^2 - x^4 + 1)),x)
```

```
[Out] int(-(x^4 + 1)/((x^4 - x^2)^(1/4)*(x^2 - x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\sqrt[4]{x^2\left(x-1\right)\left(x+1\right)}\left(x^4-x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**4-x**2-1)/(x**4-x**2)**(1/4),x)
```

```
[Out] Integral((x**4 + 1)/((x**2*(x - 1)*(x + 1))**(1/4)*(x**4 - x**2 - 1)), x)
```

3.2116 $\displaystyle \int \frac{x^3\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$

Optimal. Leaf size=233

$$\frac{\left(384a^2x^4-136ax^2-255\right)\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{1920a^2}+\frac{\left(-384a^2x^5+568ax^3+85x\right)\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}}{1920ab}$$

Rubi [F] time = 2.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (
a^2*x^2)/b^2]],x]
```

```
[Out] Defer[Int][(x^3*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a*x^2 + b*x*Sqrt[-(a/b
^2) + (a^2*x^2)/b^2]], x]
```

Rubi steps

$$\int \frac{x^3\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx=\int \frac{x^3\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}\,dx$$

Mathematica [C] time = 12.15, size = 733, normalized size = 3.15

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a*x^2 + b*x*Sqrt[-(a/b^
2) + (a^2*x^2)/b^2]],x]
```

```
[Out] (b*x^2*Sqrt[(a*(-1 + a*x^2))/b^2]*((2*x^2*Sqrt[x*(a*x + b*Sqrt[(a*(-1 + a*x
^2))/b^2])]*(-3*(a + (a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])^2)^6 - Sqrt[2]*a^
5*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])^2*Sqrt[x*(a*x + b*Sqrt[(a*(-1 + a*x^
2))/b^2])]*Hypergeometric2F1[-9/2, -3/2, -1/2, 1 - 2*a*x^2 - 2*b*x*Sqrt[(a*
(-1 + a*x^2))/b^2]]))/b - (2*Sqrt[2]*a^5*x^3*Sqrt[(a*(-1 + a*x^2))/b^2]*(a*
x + b*Sqrt[(a*(-1 + a*x^2))/b^2])^2*(3*Hypergeometric2F1[-7/2, -5/2, -3/2,
1 - 2*a*x^2 - 2*b*x*Sqrt[(a*(-1 + a*x^2))/b^2]] - 10*(-1 + 2*a*x^2 + 2*b*x*
Sqrt[(a*(-1 + a*x^2))/b^2])*Hypergeometric2F1[-7/2, -3/2, -1/2, 1 - 2*a*x^2
 - 2*b*x*Sqrt[(a*(-1 + a*x^2))/b^2]] + 15*(1 + 8*a^2*x^4 - 4*b*x*Sqrt[(a*(-
1 + a*x^2))/b^2] + 8*a*x^2*(-1 + b*x*Sqrt[(a*(-1 + a*x^2))/b^2]))*Hypergeom
etric2F1[-7/2, -1/2, 1/2, 1 - 2*a*x^2 - 2*b*x*Sqrt[(a*(-1 + a*x^2))/b^2]]))
/(-1 + a*x^2 + b*x*Sqrt[(a*(-1 + a*x^2))/b^2]) - (10*Sqrt[x*(a*x + b*Sqrt[(
```

```
a*(-1 + a*x^2))/b^2])]*(a + (a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])^2)^2*(-(a
+ (a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])^2)^4 - Sqrt[2]*a^3*(a*x + b*Sqrt[(a*
(-1 + a*x^2))/b^2])^2*Sqrt[x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*Hypergeo
metric2F1[-5/2, -1/2, 1/2, 1 - 2*a*x^2 - 2*b*x*Sqrt[(a*(-1 + a*x^2))/b^2]])
)/(a*b)))/(60*a*(-1 + a*x^2 + b*x*Sqrt[(a*(-1 + a*x^2))/b^2])*(a + (a*x + b
*Sqrt[(a*(-1 + a*x^2))/b^2])^2)^5)
```

IntegrateAlgebraic [A] time = 3.53, size = 233, normalized size = 1.00

$$\frac{\left(384a^2x^4-136ax^2-255\right)\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{1920a^2}+\frac{\left(-384a^2x^5+568ax^3+85x\right)\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{1920ab}+\frac{17\log\left(b\left(-\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)+\sqrt{2}\sqrt{a}\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}-ax\right)}{128\sqrt{2}a^{3/2}b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a*x^2 + b*x*Sq
rt[-(a/b^2) + (a^2*x^2)/b^2]],x]
```

```
[Out] (Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*(-255 - 136*a*x^2 + 384*a^2*x^4)*Sqrt[a*x^2
 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]])/(1920*a^2) + ((85*x + 568*a*x^3 - 3
84*a^2*x^5)*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]])/(1920*a*b) +
(17*Log[-(a*x) - b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2] + Sqrt[2]*Sqrt[a]*Sqrt[a*
x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]])/(128*Sqrt[2]*a^(3/2)*b)
```

fricas [A] time = 37.47, size = 359, normalized size = 1.54

$$\left[\frac{255\sqrt{2}\sqrt{a}\log\left(-4a^2x^2-4abx\sqrt{\frac{a^2x^2-a}{b^2}}+2\left(\sqrt{2}a^{\frac{3}{2}}x+\sqrt{2}\sqrt{a}b\sqrt{\frac{a^2x^2-a}{b^2}}\right)\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}+a\right)-4\left(384a^3x^5-568a^2x^3-85ax-\left(384a^2bx^4-136abx^2-255b\right)\sqrt{\frac{a^2x^2-a}{b^2}}\right)\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}}{7680a^2b}, \frac{255\sqrt{2}\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\sqrt{-a}}{2ax}\right)-2\left(384a^3x^5-568a^2x^3-85ax-\left(384a^2bx^4-136abx^2-255b\right)\sqrt{\frac{a^2x^2-a}{b^2}}\right)\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}}{3840a^2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a/b^2+a^2*x^2/b^2)^(1/2)/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1
/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(255*sqrt(2)*sqrt(a)*log(-4*a^2*x^2 - 4*a*b*x*sqrt((a^2*x^2 - a)/b^
2) + 2*(sqrt(2)*a^(3/2)*x + sqrt(2)*sqrt(a)*b*sqrt((a^2*x^2 - a)/b^2))*sqrt
(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2)) + a) - 4*(384*a^3*x^5 - 568*a^2*x^3 -
 85*a*x - (384*a^2*b*x^4 - 136*a*b*x^2 - 255*b)*sqrt((a^2*x^2 - a)/b^2))*sq
rt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2)))/(a^2*b), 1/3840*(255*sqrt(2)*sqrt(
-a)*arctan(1/2*sqrt(2)*sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*sqrt(-a)/(
a*x)) - 2*(384*a^3*x^5 - 568*a^2*x^3 - 85*a*x - (384*a^2*b*x^4 - 136*a*b*x^
2 - 255*b)*sqrt((a^2*x^2 - a)/b^2))*sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2
)))/(a^2*b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a/b^2+a^2*x^2/b^2)^(1/2)/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1
/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \sqrt{-\frac{a}{b^{2}}+\frac{a^{2} x^{2}}{b^{2}}}}{\sqrt{a \,x^{2}+b x \sqrt{-\frac{a}{b^{2}}+\frac{a^{2} x^{2}}{b^{2}}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a/b^2+a^2*x^2/b^2)^(1/2)/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(
1/2),x)
```

```
[Out] int(x^3*(-a/b^2+a^2*x^2/b^2)^(1/2)/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(
1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\,x^3}{\sqrt{ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\,bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a/b^2+a^2*x^2/b^2)^(1/2)/(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1
/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2/b^2 - a/b^2)*x^3/sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b
^2)*b*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}}{\sqrt{a\,x^2+b\,x\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*((a^2*x^2)/b^2 - a/b^2)^(1/2))/(a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2
)^(1/2))^(1/2),x)
```

```
[Out] int((x^3*((a^2*x^2)/b^2 - a/b^2)^(1/2))/(a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2
)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{\frac{a(ax^2-1)}{b^2}}}{\sqrt{x\left(ax + b\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a/b**2+a**2*x**2/b**2)**(1/2)/(a*x**2+b*x*(-a/b**2+a**2*x*
*2/b**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*(a*x**2 - 1)/b**2)/sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2
 - a/b**2))), x)
```

3.2117 $\int \frac{\sqrt[3]{-x+x^3}\left(-2+x^4\right)}{x^4\left(1+x^2\right)}\,dx$

Optimal. Leaf size=234

$$-\frac{1}{2}\log\left(\sqrt[3]{x^3-x}-x\right)-\frac{\log\left(2^{2/3}\sqrt[3]{x^3-x}-2x\right)}{2^{2/3}}-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x}+x}\right)-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3-x}+x}\right)}{2^{2/3}}-\frac{3\sqrt[3]{x^3}}{}$$

Rubi [C] time = 0.80, antiderivative size = 316, normalized size of antiderivative = 1.35, number of steps used = 18, number of rules used = 17, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.630, Rules used = {2056, 6725, 264, 277, 329, 275, 331, 292, 31, 634, 618, 204, 628, 466, 465, 511, 510}

$$\frac{3\sqrt[3]{x^3-x}\left(\left(1-3x^2\right)x^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{2x^2}{1-x^2}\right)-3\left(x^4+x^2\right)\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};-\frac{2x^2}{1-x^2}\right)+3x^4-4x^2+1\right)}{8x^3\left(1-x^2\right)}-\frac{3\sqrt[3]{x^3-x}}{2x}+\frac{3\sqrt[3]{x^3-x}\left(1-x^2\right)}{8x^3}-\frac{\sqrt[3]{x^3-x}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2-1}}\right)}{2\sqrt[3]{x}\sqrt[3]{x^2-1}}+\frac{\sqrt[3]{x^3-x}\log\left(\frac{x^{4/3}}{\left(x^2-1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2-1}}+1\right)}{4\sqrt[3]{x}\sqrt[3]{x^2-1}}-\frac{\sqrt{3}\sqrt[3]{x^3-x}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{x}\sqrt[3]{x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-x + x^3)^(1/3)*(-2 + x^4))/(x^4*(1 + x^2)),x]
```

```
[Out] (-3*(-x + x^3)^(1/3))/(2*x) + (3*(1 - x^2)*(-x + x^3)^(1/3))/(8*x^3) - (Sqr
t[3]*(-x + x^3)^(1/3)*ArcTan[(1 + (2*x^(2/3))/(-1 + x^2)^(1/3))/Sqrt[3]])/(
2*x^(1/3)*(-1 + x^2)^(1/3)) + (3*(-x + x^3)^(1/3)*(1 - 4*x^2 + 3*x^4 + x^2*
(1 - 3*x^2)*Hypergeometric2F1[2/3, 1, 5/3, (-2*x^2)/(1 - x^2)] - 3*(x^2 + x
^4)*Hypergeometric2F1[2/3, 2, 5/3, (-2*x^2)/(1 - x^2)]))/(8*x^3*(1 - x^2))
- ((-x + x^3)^(1/3)*Log[1 - x^(2/3)/(-1 + x^2)^(1/3)])/(2*x^(1/3)*(-1 + x^2
)^(1/3)) + ((-x + x^3)^(1/3)*Log[1 + x^(4/3)/(-1 + x^2)^(2/3) + x^(2/3)/(-1
 + x^2)^(1/3)])/(4*x^(1/3)*(-1 + x^2)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{-x+x^3}\left(-2+x^4\right)}{x^4\left(1+x^2\right)}\,dx = \frac{\sqrt[3]{-x+x^3}\int \frac{\sqrt[3]{-1+x^2}\left(-2+x^4\right)}{x^{11/3}\left(1+x^2\right)}\,dx}{\sqrt[3]{x}\sqrt[3]{-1+x^2}}$$

$$= \frac{\sqrt[3]{-x+x^3}\int \left(-\frac{\sqrt[3]{-1+x^2}}{x^{11/3}}+\frac{\sqrt[3]{-1+x^2}}{x^{5/3}}-\frac{\sqrt[3]{-1+x^2}}{x^{11/3}\left(1+x^2\right)}\right)dx}{\sqrt[3]{x}\sqrt[3]{-1+x^2}}$$

$$= -\frac{\sqrt[3]{-x+x^3}\int \frac{\sqrt[3]{-1+x^2}}{x^{11/3}}\,dx}{\sqrt[3]{x}\sqrt[3]{-1+x^2}} + \frac{\sqrt[3]{-x+x^3}\int \frac{\sqrt[3]{-1+x^2}}{x^{5/3}}\,dx}{\sqrt[3]{x}\sqrt[3]{-1+x^2}} - \frac{\sqrt[3]{-x+x^3}\int \frac{\sqrt[3]{-1+x^2}}{x^{11/3}\left(1+x^2\right)}\,dx}{\sqrt[3]{x}\sqrt[3]{-1+x^2}}$$

$$= -\frac{3\sqrt[3]{-x+x^3}}{2x} + \frac{3\left(1-x^2\right)\sqrt[3]{-x+x^3}}{8x^3} + \frac{\sqrt[3]{-x+x^3}\int \frac{\sqrt[3]{x}}{\left(-1+x^2\right)^{2/3}}\,dx}{\sqrt[3]{x}\sqrt[3]{-1+x^2}} - \frac{\left(3\sqrt[3]{-x+x^3}\right)\mathrm{S}}{\sqrt[3]{}}$$

$$= -\frac{3\sqrt[3]{-x+x^3}}{2x} + \frac{3\left(1-x^2\right)\sqrt[3]{-x+x^3}}{8x^3} - \frac{\left(3\sqrt[3]{-x+x^3}\right)\mathrm{Subst}\left(\int \frac{\sqrt[3]{-1+x^3}}{x^5\left(1+x^3\right)}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{x}\sqrt[3]{-1+x^2}}$$

$$= -\frac{3\sqrt[3]{-x+x^3}}{2x} + \frac{3\left(1-x^2\right)\sqrt[3]{-x+x^3}}{8x^3} - \frac{\left(3\sqrt[3]{-x+x^3}\right)\mathrm{Subst}\left(\int \frac{\sqrt[3]{1-x^3}}{x^5\left(1+x^3\right)}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{x}\sqrt[3]{1-x^2}}$$

$$= -\frac{3\sqrt[3]{-x+x^3}}{2x} + \frac{3\left(1-x^2\right)\sqrt[3]{-x+x^3}}{8x^3} + \frac{3\sqrt[3]{-x+x^3}\left(1-4x^2+3x^4+x^2\left(1-3x^2\right)\,{}_2\right.}{8}$$

$$= -\frac{3\sqrt[3]{-x+x^3}}{2x} + \frac{3\left(1-x^2\right)\sqrt[3]{-x+x^3}}{8x^3} + \frac{3\sqrt[3]{-x+x^3}\left(1-4x^2+3x^4+x^2\left(1-3x^2\right)\,{}_2\right.}{8}$$

$$= -\frac{3\sqrt[3]{-x+x^3}}{2x} + \frac{3\left(1-x^2\right)\sqrt[3]{-x+x^3}}{8x^3} + \frac{3\sqrt[3]{-x+x^3}\left(1-4x^2+3x^4+x^2\left(1-3x^2\right)\,{}_2\right.}{8}$$

$$= -\frac{3\sqrt[3]{-x+x^3}}{2x} + \frac{3\left(1-x^2\right)\sqrt[3]{-x+x^3}}{8x^3} + \frac{3\sqrt[3]{-x+x^3}\left(1-4x^2+3x^4+x^2\left(1-3x^2\right)\,{}_2\right.}{8}$$

$$= -\frac{3\sqrt[3]{-x+x^3}}{2x} + \frac{3\left(1-x^2\right)\sqrt[3]{-x+x^3}}{8x^3} - \frac{\sqrt{3}\sqrt[3]{-x+x^3}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{x}\sqrt[3]{-1+x^2}} + 3\sqrt[3]{-x+}$$

Mathematica [C] time = 0.12, size = 137, normalized size = 0.59

$$-\frac{3\sqrt[3]{x\left(x^2-1\right)}\left(\sqrt[3]{1-x^2}\left(\left(3x^2-1\right)x^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2x^2}{x^2-1}\right)+3\left(x^4+x^2\right)\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{2x^2}{x^2-1}\right)-4x^4+6x^2-2\right)-4x^2\left(x^2-1\right)\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};x^2\right)\right)}{8x^3\left(1-x^2\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-x + x^3)^(1/3)*(-2 + x^4))/(x^4*(1 + x^2)),x]
```

```
[Out] (-3*(x*(-1 + x^2))^(1/3)*(-4*x^2*(-1 + x^2)*Hypergeometric2F1[-1/3, -1/3, 2
/3, x^2] + (1 - x^2)^(1/3)*(-2 + 6*x^2 - 4*x^4 + x^2*(-1 + 3*x^2)*Hypergeom
etric2F1[2/3, 1, 5/3, (2*x^2)/(-1 + x^2)] + 3*(x^2 + x^4)*Hypergeometric2F1
[2/3, 2, 5/3, (2*x^2)/(-1 + x^2)])))/(8*x^3*(1 - x^2)^(4/3))
```

IntegrateAlgebraic [A] time = 0.58, size = 234, normalized size = 1.00

$$-\frac{1}{2}\log\left(\sqrt[3]{x^3-x}-x\right)-\frac{\log\left(2^{2/3}\sqrt[3]{x^3-x}-2x\right)}{2^{2/3}}-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x}+x}\right)-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3-x}+x}\right)}{2^{2/3}}-\frac{3\sqrt[3]{x^3-x}\left(5x^2-1\right)}{4x^3}+\frac{1}{4}\log\left(\sqrt[3]{x^3-x}x+\left(x^3-x\right)^{2/3}+x^2\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^3-x}x+\sqrt[3]{2}\left(x^3-x\right)^{2/3}+2x^2\right)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-x + x^3)^(1/3)*(-2 + x^4))/(x^4*(1 + x^2)),x]
```

```
[Out] (-3*(-1 + 5*x^2)*(-x + x^3)^(1/3))/(4*x^3) - (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x
 + 2*(-x + x^3)^(1/3))])/2 - (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(-x +
 x^3)^(1/3))])/2^(2/3) - Log[-x + (-x + x^3)^(1/3)]/2 - Log[-2*x + 2^(2/3)*
(-x + x^3)^(1/3)]/2^(2/3) + Log[x^2 + x*(-x + x^3)^(1/3) + (-x + x^3)^(2/3)
]/4 + Log[2*x^2 + 2^(2/3)*x*(-x + x^3)^(1/3) + 2^(1/3)*(-x + x^3)^(2/3)]/(2
*2^(2/3))
```

fricas [B] time = 7.76, size = 387, normalized size = 1.65

$$-\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}x^3\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(6\cdot 4^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(19x^5-16x^3+x\right)\left(x^3-x\right)^{\frac{1}{3}}+12(-1)^{\frac{1}{3}}\left(5x^4+4x^2-1\right)\left(x^3-x\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(71x^6-111x^4+33x^2-1\right)\right)}{6\left(109x^6-105x^4+3x^2+1\right)}\right)+4^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^3\log\left(\frac{6\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^3-x\right)^{\frac{2}{3}}\left(5x^2-1\right)-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(19x^4-16x^2+1\right)+24\left(2x^3-x\right)\left(x^3-x\right)^{\frac{1}{3}}}{x^4+2x^2+1}\right)-2\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^3\log\left(-\frac{3\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3-x\right)^{\frac{1}{3}}x+4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^2+1\right)+6\left(x^3-x\right)^{\frac{2}{3}}}{x^2+1}\right)+12\sqrt{3}x^3\arctan\left(-\frac{44032959556\sqrt{3}\left(x^3-x\right)^{\frac{1}{3}}x+\sqrt{3}\left(16754327161x^2-2707204793\right)-10524305234\sqrt{3}\left(x^3-x\right)^{\frac{2}{3}}}{81835897185x^2-1102302937}\right)+6x^3\log\left(-3\left(x^3-x\right)^{\frac{1}{3}}x+3\left(x^3-x\right)^{\frac{2}{3}}+1\right)+18\left(x^3-x\right)^{\frac{1}{3}}\left(5x^2-1\right)}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)^(1/3)*(x^4-2)/x^4/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/24*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^3*arctan(1/6*4^(1/6)*sqrt(3)*(6*4^(2/
3)*(-1)^(2/3)*(19*x^5 - 16*x^3 + x)*(x^3 - x)^(1/3) + 12*(-1)^(1/3)*(5*x^4
+ 4*x^2 - 1)*(x^3 - x)^(2/3) + 4^(1/3)*(71*x^6 - 111*x^4 + 33*x^2 - 1))/(10
9*x^6 - 105*x^4 + 3*x^2 + 1)) + 4^(2/3)*(-1)^(1/3)*x^3*log((6*4^(1/3)*(-1)^
(2/3)*(x^3 - x)^(2/3)*(5*x^2 - 1) - 4^(2/3)*(-1)^(1/3)*(19*x^4 - 16*x^2 + 1
) + 24*(2*x^3 - x)*(x^3 - x)^(1/3))/(x^4 + 2*x^2 + 1)) - 2*4^(2/3)*(-1)^(1/
3)*x^3*log(-(3*4^(2/3)*(-1)^(1/3)*(x^3 - x)^(1/3)*x + 4^(1/3)*(-1)^(2/3)*(x
^2 + 1) + 6*(x^3 - x)^(2/3))/(x^2 + 1)) + 12*sqrt(3)*x^3*arctan(-(440329595
56*sqrt(3)*(x^3 - x)^(1/3)*x + sqrt(3)*(16754327161*x^2 - 2707204793) - 105
24305234*sqrt(3)*(x^3 - x)^(2/3))/(81835897185*x^2 - 1102302937)) + 6*x^3*l
og(-3*(x^3 - x)^(1/3)*x + 3*(x^3 - x)^(2/3) + 1) + 18*(x^3 - x)^(1/3)*(5*x^
2 - 1))/x^3
```

giac [A] time = 0.50, size = 171, normalized size = 0.73

$$\frac{1}{2}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{3}{4}\left(-\frac{1}{x^2}+1\right)^{\frac{4}{3}}+\frac{1}{4}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}\right)-\frac{1}{2}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right|\right)-3\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+\frac{1}{4}\log\left(\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{2}\log\left(\left|\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)^(1/3)*(x^4-2)/x^4/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-1/x^2 + 1)^(1
/3))) + 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-1/x^2 + 1)^(1/3) + 1)) - 3/4*(-
1/x^2 + 1)^(4/3) + 1/4*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-1/x^2 + 1)^(1/3) + (
-1/x^2 + 1)^(2/3)) - 1/2*2^(1/3)*log(abs(-2^(1/3) + (-1/x^2 + 1)^(1/3))) -
3*(-1/x^2 + 1)^(1/3) + 1/4*log((-1/x^2 + 1)^(2/3) + (-1/x^2 + 1)^(1/3) + 1)
 - 1/2*log(abs((-1/x^2 + 1)^(1/3) - 1))
```

maple [F] time = 1.57, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-x\right)^{\frac{1}{3}}\left(x^4-2\right)}{x^4\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x)^(1/3)*(x^4-2)/x^4/(x^2+1),x)
```

```
[Out] int((x^3-x)^(1/3)*(x^4-2)/x^4/(x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-2\right)\left(x^3-x\right)^{\frac{1}{3}}}{\left(x^2+1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)^(1/3)*(x^4-2)/x^4/(x^2+1),x, algorithm="maxima")
[Out] integrate((x^4 - 2)*(x^3 - x)^(1/3)/((x^2 + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^3-x\right)^{1/3}\,\left(x^4-2\right)}{x^4\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - x)^(1/3)*(x^4 - 2))/(x^4*(x^2 + 1)),x)
[Out] int(((x^3 - x)^(1/3)*(x^4 - 2))/(x^4*(x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x\left(x-1\right)\left(x+1\right)}\,\left(x^4-2\right)}{x^4\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x)**(1/3)*(x**4-2)/x**4/(x**2+1),x)
[Out] Integral((x*(x - 1)*(x + 1))**(1/3)*(x**4 - 2)/(x**4*(x**2 + 1)), x)
```

3.2118 $\int \frac{2(3aqx-2bpx^3+apx^5)}{\sqrt[3]{q+px^4}\left(b^3d+cq+3ab^2dx^2+(3a^2bd+cp)x^4+a^3dx^6\right)}\,dx$

Optimal. Leaf size=234

$$-\frac{\log\left(a^2d^{2/3}x^4+\sqrt[3]{px^4+q}\left(-a\sqrt[3]{c}\sqrt[3]{d}\,x^2-b\sqrt[3]{c}\sqrt[3]{d}\right)+2abd^{2/3}x^2+b^2d^{2/3}+c^{2/3}\left(px^4+q\right)^{2/3}\right)}{2c^{2/3}\sqrt[3]{d}}+\frac{\log\left(a\sqrt[3]{d}\,x^2+\right.}{}$$

Rubi [F] time = 3.80, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2\left(3aqx-2bpx^3+apx^5\right)}{\sqrt[3]{q+px^4}\left(b^3d+cq+3ab^2dx^2+\left(3a^2bd+cp\right)x^4+a^3dx^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2*(3*a*q*x - 2*b*p*x^3 + a*p*x^5))/((q + p*x^4)^(1/3)*(b^3*d + c*q + 3
*a*b^2*d*x^2 + (3*a^2*b*d + c*p)*x^4 + a^3*d*x^6)),x]
```

```
[Out] 2*b*p*Defer[Subst][Defer[Int][x/((q + p*x^2)^(1/3)*(-(b^3*d) - c*q - 3*a*b^
2*d*x - (3*a^2*b*d + c*p)*x^2 - a^3*d*x^3)), x], x, x^2] + 3*a*q*Defer[Subs
t][Defer[Int][1/((q + p*x^2)^(1/3)*(b^3*d + c*q + 3*a*b^2*d*x + (3*a^2*b*d
+ c*p)*x^2 + a^3*d*x^3)), x], x, x^2] + a*p*Defer[Subst][Defer[Int][x^2/((q
 + p*x^2)^(1/3)*(b^3*d + c*q + 3*a*b^2*d*x + (3*a^2*b*d + c*p)*x^2 + a^3*d*
x^3)), x], x, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2\left(3aqx-2bpx^3+apx^5\right)}{\sqrt[3]{q+px^4}\left(b^3d+cq+3ab^2dx^2+\left(3a^2bd+cp\right)x^4+a^3dx^6\right)}\,dx &= 2\int \frac{3aqx-2bpx^3+ap}{\sqrt[3]{q+px^4}\left(b^3d+cq+3ab^2dx^2+\left(3a^2\right.\right.}\\
&= 2\int \frac{x\left(3aq-2bpx^2+ap\right.}{\sqrt[3]{q+px^4}\left(b^3d+cq+3ab^2dx^2+\left(3a^2\right.\right.}\\
&= \text{Subst}\left(\int \frac{3aq-2bpx}{\sqrt[3]{q+px^2}\left(b^3d+cq+3ab^2dx+\right.}\right.\\
&= \text{Subst}\left(\int\left(\frac{2bp}{\sqrt[3]{q+px^2}\left(-b^3d-cq-3ab^2dx\right.}\right.\right.\\
&= (ap)\,\text{Subst}\left(\int \frac{}{\sqrt[3]{q+px^2}\left(b^3d+cq+3ab^2\right.}\right.
\end{aligned}$$

Mathematica [F] time = 2.20, size = 79, normalized size = 0.34

$$2\int \frac{apx^5+3aqx-2bpx^3}{\sqrt[3]{px^4+q}\left(a^3dx^6+x^4\left(3a^2bd+cp\right)+3ab^2dx^2+b^3d+cq\right)}\,dx$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*(3*a*q*x - 2*b*p*x^3 + a*p*x^5))/((q + p*x^4)^(1/3)*(b^3*d + c
*q + 3*a*b^2*d*x^2 + (3*a^2*b*d + c*p)*x^4 + a^3*d*x^6)),x]
```

[Out] 2*Integrate[(3*a*q*x - 2*b*p*x^3 + a*p*x^5)/((q + p*x^4)^(1/3)*(b^3*d + c*q + 3*a*b^2*d*x^2 + (3*a^2*b*d + c*p)*x^4 + a^3*d*x^6)), x]

IntegrateAlgebraic [A] time = 15.33, size = 234, normalized size = 1.00

$$-\frac{\log\left(a^2d^{2/3}x^4+\sqrt[3]{px^4+q}\left(-a\sqrt[3]{c}\sqrt[3]{d}x^2-b\sqrt[3]{c}\sqrt[3]{d}\right)+2abd^{2/3}x^2+b^2d^{2/3}+c^{2/3}\left(px^4+q\right)^{2/3}\right)}{2c^{2/3}\sqrt[3]{d}}+\frac{\log\left(a\sqrt[3]{d}x^2+b\sqrt[3]{d}+\sqrt[3]{c}\sqrt[3]{px^4+q}\right)}{c^{2/3}\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{c}\sqrt[3]{px^4+q}}{-2a\sqrt[3]{d}x^2-2b\sqrt[3]{d}+\sqrt[3]{c}\sqrt[3]{px^4+q}}\right)}{c^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2*(3*a*q*x - 2*b*p*x^3 + a*p*x^5))/((q + p*x^4)^(1/3)*(b^3*d + c*q + 3*a*b^2*d*x^2 + (3*a^2*b*d + c*p)*x^4 + a^3*d*x^6)),x]

[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*c^(1/3)*(q + p*x^4)^(1/3))/(-2*b*d^(1/3) - 2*a*d^(1/3)*x^2 + c^(1/3)*(q + p*x^4)^(1/3))]])/(c^(2/3)*d^(1/3)) + Log[b*d^(1/3) + a*d^(1/3)*x^2 + c^(1/3)*(q + p*x^4)^(1/3)]/(c^(2/3)*d^(1/3)) - Log[b^2*d^(2/3) + 2*a*b*d^(2/3)*x^2 + a^2*d^(2/3)*x^4 + (-(b*c^(1/3)*d^(1/3)) - a*c^(1/3)*d^(1/3)*x^2)*(q + p*x^4)^(1/3) + c^(2/3)*(q + p*x^4)^(2/3)]/(2*c^(2/3)*d^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*(a*p*x^5-2*b*p*x^3+3*a*q*x)/(p*x^4+q)^(1/3)/(b^3*d+c*q+3*a*b^2*d*x^2+(3*a^2*b*d+c*p)*x^4+a^3*d*x^6),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\left(apx^5-2\,bpx^3+3\,aqx\right)}{\left(a^3dx^6+3\,ab^2dx^2+\left(3\,a^2bd+cp\right)x^4+b^3d+cq\right)\left(px^4+q\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*(a*p*x^5-2*b*p*x^3+3*a*q*x)/(p*x^4+q)^(1/3)/(b^3*d+c*q+3*a*b^2*d*x^2+(3*a^2*b*d+c*p)*x^4+a^3*d*x^6),x, algorithm="giac")

[Out] integrate(2*(a*p*x^5 - 2*b*p*x^3 + 3*a*q*x)/((a^3*d*x^6 + 3*a*b^2*d*x^2 + (3*a^2*b*d + c*p)*x^4 + b^3*d + c*q)*(p*x^4 + q)^(1/3)), x)

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{2ap\,x^5-4bp\,x^3+6aqx}{\left(p\,x^4+q\right)^{\frac{1}{3}}\left(b^3d+cq+3a\,b^2d\,x^2+\left(3a^2bd+cp\right)x^4+a^3d\,x^6\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2*(a*p*x^5-2*b*p*x^3+3*a*q*x)/(p*x^4+q)^(1/3)/(b^3*d+c*q+3*a*b^2*d*x^2+(3*a^2*b*d+c*p)*x^4+a^3*d*x^6),x)

[Out] int(2*(a*p*x^5-2*b*p*x^3+3*a*q*x)/(p*x^4+q)^(1/3)/(b^3*d+c*q+3*a*b^2*d*x^2+(3*a^2*b*d+c*p)*x^4+a^3*d*x^6),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\int \frac{apx^5-2\,bpx^3+3\,aqx}{\left(a^3dx^6+3\,ab^2dx^2+\left(3\,a^2bd+cp\right)x^4+b^3d+cq\right)\left(px^4+q\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*(a*p*x^5-2*b*p*x^3+3*a*q*x)/(p*x^4+q)^(1/3)/(b^3*d+c*q+3*a*b^2*
d*x^2+(3*a^2*b*d+c*p)*x^4+a^3*d*x^6),x, algorithm="maxima")
```

```
[Out] 2*integrate((a*p*x^5 - 2*b*p*x^3 + 3*a*q*x)/((a^3*d*x^6 + 3*a*b^2*d*x^2 + (
3*a^2*b*d + c*p)*x^4 + b^3*d + c*q)*(p*x^4 + q)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{2\,a\,p\,x^5-4\,b\,p\,x^3+6\,a\,q\,x}{{\left(p\,x^4+q\right)}^{1/3}\,\left(c\,q+b^3\,d+x^4\,\left(3\,b\,d\,a^2+c\,p\right)+a^3\,d\,x^6+3\,a\,b^2\,d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*p*x^5 - 4*b*p*x^3 + 6*a*q*x)/((q + p*x^4)^(1/3)*(c*q + b^3*d + x^4
*(c*p + 3*a^2*b*d) + a^3*d*x^6 + 3*a*b^2*d*x^2)),x)
```

```
[Out] int((2*a*p*x^5 - 4*b*p*x^3 + 6*a*q*x)/((q + p*x^4)^(1/3)*(c*q + b^3*d + x^4
*(c*p + 3*a^2*b*d) + a^3*d*x^6 + 3*a*b^2*d*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*(a*p*x**5-2*b*p*x**3+3*a*q*x)/(p*x**4+q)**(1/3)/(b**3*d+c*q+3*a
*b**2*d*x**2+(3*a**2*b*d+c*p)*x**4+a**3*d*x**6),x)
```

```
[Out] Timed out
```

3.2119 $\int \frac{(b^2-2bx+x^2)(-a^2b+4abx-(2a+3b)x^2+2x^3)}{(x(-a+x)^2(-b+x)^3)^{3/4}(bd+(a^2-d)x-2ax^2+x^3)}\,dx$

Optimal. Leaf size=235

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(-a^2b^3x+x^4(a^2+6ab+3b^2)+x^3(-3a^2b-6ab^2-b^3)+x^2(3a^2b^2+2ab^3)+x^5(-2a-3b)+x^6\right)^{3/4}}{x(x-a)^2(b-x)^2}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(-a^2b^3x+x^4(a^2+6ab+3b^2\right.}{}\right.}{}$$

Rubi [F] time = 10.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(b^2-2bx+x^2)(-a^2b+4abx-(2a+3b)x^2+2x^3)}{(x(-a+x)^2(-b+x)^3)^{3/4}(bd+(a^2-d)x-2ax^2+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((b^2 - 2*b*x + x^2)*(-(a^2*b) + 4*a*b*x - (2*a + 3*b)*x^2 + 2*x^3))/((
x*(-a + x)^2*(-b + x)^3)^(3/4)*(b*d + (a^2 - d)*x - 2*a*x^2 + x^3)),x]
```

```
[Out] (12*b*x^(3/4)*(-a + x)^(3/2)*(-b + x)^(9/4)*Defer[Subst][Defer[Int][x^4/(Sq
rt[-a + x^4]*(-b + x^4)^(1/4)*(-(b*d) - a^2*(1 - d/a^2)*x^4 + 2*a*x^8 - x^1
2)), x], x, x^(1/4)])/(-((a - x)^2*(b - x)^3*x))^(3/4) + (4*a*b*x^(3/4)*(-a
 + x)^(3/2)*(-b + x)^(9/4)*Defer[Subst][Defer[Int][1/(Sqrt[-a + x^4]*(-b +
x^4)^(1/4)*(b*d + a^2*(1 - d/a^2)*x^4 - 2*a*x^8 + x^12)), x], x, x^(1/4)])/
(-((a - x)^2*(b - x)^3*x))^(3/4) + (8*x^(3/4)*(-a + x)^(3/2)*(-b + x)^(9/4)
*Defer[Subst][Defer[Int][x^8/(Sqrt[-a + x^4]*(-b + x^4)^(1/4)*(b*d + a^2*(1
 - d/a^2)*x^4 - 2*a*x^8 + x^12)), x], x, x^(1/4)])/(-((a - x)^2*(b - x)^3*x
))^(3/4)
```

Rubi steps

$$\begin{aligned}
\int \frac{(b^2-2bx+x^2)(-a^2b+4abx-(2a+3b)x^2+2x^3)}{(x(-a+x)^2(-b+x)^3)^{3/4}(bd+(a^2-d)x-2ax^2+x^3)}\,dx &= \int \frac{(-b+x)^2(-a^2b+4abx-(2a+3b)x^2+2}{(x(-a+x)^2(-b+x)^3)^{3/4}(bd+(a^2-d)x-2a} \\
&= \frac{\left(x^{3/4}(-a+x)^{3/2}(-b+x)^{9/4}\right)\int \frac{-a^2b+4abx-(2a}{x^{3/4}(-a+x)^{3/2}\sqrt[4]{-b+x}(bd}}{(x(-a+x)^2(-b+x)^3)^{3/4}} \\
&= \frac{\left(x^{3/4}(-a+x)^{3/2}(-b+x)^{9/4}\right)\int \frac{ab-3bx+2}{x^{3/4}\sqrt{-a+x}\sqrt[4]{-b+x}(bd+(}}{(x(-a+x)^2(-b+x)^3)^{3/4}} \\
&= \frac{\left(4x^{3/4}(-a+x)^{3/2}(-b+x)^{9/4}\right)\text{Subst}\left(\int \frac{}{\sqrt{-a+x^4}\sqrt[4]{-b}}\right.}{(x(-a+x)^2(-b+x} \\
&= \frac{\left(4x^{3/4}(-a+x)^{3/2}(-b+x)^{9/4}\right)\text{Subst}\left(\int\left(\frac{}{\sqrt{-a+x^4}\sqrt[4]{-}}\right.\right.}{} \\
&= \frac{\left(8x^{3/4}(-a+x)^{3/2}(-b+x)^{9/4}\right)\text{Subst}\left(\int \frac{}{\sqrt{-a+x^4}\sqrt[4]{-b}}\right.}{(x(-a+x)^2(-b+x}
\end{aligned}$$

Mathematica [F] time = 4.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2-2bx+x^2\right)\left(-a^2b+4abx-(2a+3b)x^2+2x^3\right)}{\left(x(-a+x)^2(-b+x)^3\right)^{3/4}\left(bd+\left(a^2-d\right)x-2ax^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((b^2 - 2*b*x + x^2)*(-(a^2*b) + 4*a*b*x - (2*a + 3*b)*x^2 + 2*x^
3))/((x*(-a + x)^2*(-b + x)^3)^(3/4)*(b*d + (a^2 - d)*x - 2*a*x^2 + x^3)),x
]
```

```
[Out] Integrate[((b^2 - 2*b*x + x^2)*(-(a^2*b) + 4*a*b*x - (2*a + 3*b)*x^2 + 2*x^
3))/((x*(-a + x)^2*(-b + x)^3)^(3/4)*(b*d + (a^2 - d)*x - 2*a*x^2 + x^3)),
x]
```

IntegrateAlgebraic [A] time = 4.29, size = 235, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\left(-a^2b^3x+x^4\left(a^2+6ab+3b^2\right)+x^3\left(-3a^2b-6ab^2-b^3\right)+x^2\left(3a^2b^2+2ab^3\right)+x^5(-2a-3b)+x^6\right)^{3/4}}{x(x-a)^2(b-x)^2}\right)}{d^{3/4}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\left(-a^2b^3x+x^4\left(a^2+6ab+3b^2\right)+x^3\left(-3a^2b-6ab^2-b^3\right)+x^2\left(3a^2b^2+2ab^3\right)+x^5(-2a-3b)+x^6\right)^{3/4}}{x(x-a)^2(b-x)^2}\right)}{d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b^2 - 2*b*x + x^2)*(-(a^2*b) + 4*a*b*x - (2*a + 3*b)*x
^2 + 2*x^3))/((x*(-a + x)^2*(-b + x)^3)^(3/4)*(b*d + (a^2 - d)*x - 2*a*x^2
+ x^3)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(-(a^2*b^3*x) + (3*a^2*b^2 + 2*a*b^3)*x^2 + (-3*a^2*b -
6*a*b^2 - b^3)*x^3 + (a^2 + 6*a*b + 3*b^2)*x^4 + (-2*a - 3*b)*x^5 + x^6)^(3
/4))/((b - x)^2*x*(-a + x)^2)])/d^(3/4) - (2*ArcTanh[(d^(1/4)*(-(a^2*b^3*x)
 + (3*a^2*b^2 + 2*a*b^3)*x^2 + (-3*a^2*b - 6*a*b^2 - b^3)*x^3 + (a^2 + 6*a*
b + 3*b^2)*x^4 + (-2*a - 3*b)*x^5 + x^6)^(3/4))/((b - x)^2*x*(-a + x)^2)])/
d^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2-2*b*x+x^2)*(-a^2*b+4*a*b*x-(2*a+3*b)*x^2+2*x^3)/(x*(-a+x)^2*
(-b+x)^3)^(3/4)/(b*d+(a^2-d)*x-2*a*x^2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2b-4\,abx+(2\,a+3\,b)x^2-2\,x^3\right)\left(b^2-2\,bx+x^2\right)}{\left(-(a-x)^2(b-x)^3x\right)^{\frac{3}{4}}\left(2\,ax^2-x^3-bd-\left(a^2-d\right)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2-2*b*x+x^2)*(-a^2*b+4*a*b*x-(2*a+3*b)*x^2+2*x^3)/(x*(-a+x)^2*
(-b+x)^3)^(3/4)/(b*d+(a^2-d)*x-2*a*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate((a^2*b - 4*a*b*x + (2*a + 3*b)*x^2 - 2*x^3)*(b^2 - 2*b*x + x^2)/(
(-(a - x)^2*(b - x)^3*x)^(3/4)*(2*a*x^2 - x^3 - b*d - (a^2 - d)*x)), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2-2bx+x^2\right)\left(-a^2b+4abx-(2a+3b)\,x^2+2x^3\right)}{\left(x\,(-a+x)^2\,(-b+x)^3\right)^{\frac{3}{4}}\left(bd+\left(a^2-d\right)x-2a\,x^2+x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2-2*b*x+x^2)*(-a^2*b+4*a*b*x-(2*a+3*b)*x^2+2*x^3)/(x*(-a+x)^2*(-b+x)
^3)^(3/4)/(b*d+(a^2-d)*x-2*a*x^2+x^3),x)
```

```
[Out] int((b^2-2*b*x+x^2)*(-a^2*b+4*a*b*x-(2*a+3*b)*x^2+2*x^3)/(x*(-a+x)^2*(-b+x)
^3)^(3/4)/(b*d+(a^2-d)*x-2*a*x^2+x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2b - 4\,abx + (2\,a + 3\,b)x^2 - 2\,x^3\right)\left(b^2 - 2\,bx + x^2\right)}{\left(-(a-x)^2(b-x)^3x\right)^{\frac{3}{4}}\left(2\,ax^2 - x^3 - bd - \left(a^2 - d\right)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2-2*b*x+x^2)*(-a^2*b+4*a*b*x-(2*a+3*b)*x^2+2*x^3)/(x*(-a+x)^2*
(-b+x)^3)^(3/4)/(b*d+(a^2-d)*x-2*a*x^2+x^3),x, algorithm="maxima")
```

```
[Out] integrate((a^2*b - 4*a*b*x + (2*a + 3*b)*x^2 - 2*x^3)*(b^2 - 2*b*x + x^2)/(
(-(a - x)^2*(b - x)^3*x)^(3/4)*(2*a*x^2 - x^3 - b*d - (a^2 - d)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(b^2 - 2\,b\,x + x^2\right)\,\left(x^2\,\left(2\,a + 3\,b\right) + a^2\,b - 2\,x^3 - 4\,a\,b\,x\right)}{\left(-x\,\left(a-x\right)^2\,\left(b-x\right)^3\right)^{3/4}\,\left(x^3 - 2\,a\,x^2 + \left(a^2 - d\right)\,x + b\,d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b^2 - 2*b*x + x^2)*(x^2*(2*a + 3*b) + a^2*b - 2*x^3 - 4*a*b*x))/((-x
*(a - x)^2*(b - x)^3)^(3/4)*(b*d - 2*a*x^2 - x*(d - a^2) + x^3)),x)
```

```
[Out] int(-((b^2 - 2*b*x + x^2)*(x^2*(2*a + 3*b) + a^2*b - 2*x^3 - 4*a*b*x))/((-x
*(a - x)^2*(b - x)^3)^(3/4)*(b*d - 2*a*x^2 - x*(d - a^2) + x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2-2*b*x+x**2)*(-a**2*b+4*a*b*x-(2*a+3*b)*x**2+2*x**3)/(x*(-a+
x)**2*(-b+x)**3)**(3/4)/(b*d+(a**2-d)*x-2*a*x**2+x**3),x)
```

```
[Out] Timed out
```

3.2120 $\int \frac{(-2+(1+k)x)\left(1-2(1+k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(a+k^2\right)x^4\right)}{x^3((1-x)x(1-kx))^{2/3}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$

Optimal. Leaf size=235

$$\frac{\left(a+b^2\right)\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{\left(-a-b^2\right)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-\right.\right.}{2b^{2/3}}$$

Rubi [F] time = 38.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+(1+k)x)\left(1-2(1+k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(a+k^2\right)x^4\right)}{x^3((1-x)x(1-kx))^{2/3}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + (1 + k)*x)*(1 - 2*(1 + k)*x + (1 + 4*k + k^2)*x^2 - 2*(k + k^2)*
x^3 + (a + k^2)*x^4))/(x^3*((1 - x)*x*(1 - k*x))^(2/3)*(1 - (1 + k)*x + (-b
 + k)*x^2)),x]
```

```
[Out] (-3*(a*(1 + 2*b + k^2) + 2*b*k*(1 + 3*k + k^2) - k^2*(1 + 4*k + k^2))*(1 -
x)*(1 - k*x))/(2*(b - k)^2*((1 - x)*x*(1 - k*x))^(2/3)) - (6*(1 + k)^2*(9*b
*k^2 - 3*k^3 - a*(1 + 3*b - k + k^2) - b^2*(1 + 8*k + k^2))*(1 - x)*(1 - k*
x))/(5*(b - k)^3*((1 - x)*x*(1 - k*x))^(2/3)) - (3*(14 + 13*k + 14*k^2)*(8*
b*k^3 - 2*k^4 + 4*b^3*(1 + 3*k + k^2) + b^2*(1 - 12*k^2 + k^4) + a*(1 + 2*b
^2 + k^4 + 4*b*(1 + k + k^2)))*(1 - x)*(1 - k*x))/(40*(b - k)^4*((1 - x)*x*
(1 - k*x))^(2/3)) - (3*(8*b*k^3 - 2*k^4 + 4*b^3*(1 + 3*k + k^2) + b^2*(1 -
12*k^2 + k^4) + a*(1 + 2*b^2 + k^4 + 4*b*(1 + k + k^2)))*(1 - x)*(1 - k*x))
/(8*(b - k)^4*x^2*((1 - x)*x*(1 - k*x))^(2/3)) - (3*(1 + k)*(9*b*k^2 - 3*k^
3 - a*(1 + 3*b - k + k^2) - b^2*(1 + 8*k + k^2))*(1 - x)*(1 - k*x))/(5*(b -
 k)^3*x*((1 - x)*x*(1 - k*x))^(2/3)) - (21*(1 + k)*(8*b*k^3 - 2*k^4 + 4*b^3
*(1 + 3*k + k^2) + b^2*(1 - 12*k^2 + k^4) + a*(1 + 2*b^2 + k^4 + 4*b*(1 + k
 + k^2)))*(1 - x)*(1 - k*x))/(40*(b - k)^4*x*((1 - x)*x*(1 - k*x))^(2/3)) +
 (3*(1 + k)*(a + k^2)*(1 - x)*(((1 - k)*x)/(1 - k*x))^(2/3)*(1 - k*x)*Hyper
geometric2F1[1/3, 2/3, 4/3, (1 - x)/(1 - k*x)])/((1 - k)*(b - k)*((1 - x)*x
*(1 - k*x))^(2/3)) - (3*(1 + k)*(2 + k + 2*k^2)*(9*b*k^2 - 3*k^3 - a*(1 + 3
*b - k + k^2) - b^2*(1 + 8*k + k^2))*(1 - x)*(((1 - k)*x)/(1 - k*x))^(2/3)*
(1 - k*x)*Hypergeometric2F1[1/3, 2/3, 4/3, (1 - x)/(1 - k*x)])/(5*(1 - k)*(
b - k)^3*((1 - x)*x*(1 - k*x))^(2/3)) - (3*(1 + k)*(7 - 4*k + 7*k^2)*(8*b*k
^3 - 2*k^4 + 4*b^3*(1 + 3*k + k^2) + b^2*(1 - 12*k^2 + k^4) + a*(1 + 2*b^2
+ k^4 + 4*b*(1 + k + k^2)))*(1 - x)*(((1 - k)*x)/(1 - k*x))^(2/3)*(1 - k*x)
*Hypergeometric2F1[1/3, 2/3, 4/3, (1 - x)/(1 - k*x)])/(20*(1 - k)*(b - k)^4
*((1 - x)*x*(1 - k*x))^(2/3)) - (3*(1 + k)*(a*(1 + 2*b + k^2) + k*(2*b*(1 +
 3*k + k^2) - k*(1 + 4*k + k^2)))*(1 - x)*(((1 - k)*x)/(1 - k*x))^(2/3)*(1
- k*x)*Hypergeometric2F1[1/3, 2/3, 4/3, (1 - x)/(1 - k*x)])/(2*(1 - k)*(b -
 k)^2*((1 - x)*x*(1 - k*x))^(2/3)) + ((a + b^2)*(1 + 5*b^2 + 5*b^2*k + k^5
+ 5*b*(1 + k^2) + 5*b*k*(1 + k^2) + (1 + 4*b^3 - k - k^5 + k^6 + b^2*(13 +
14*k + 13*k^2) + b*(7 + 2*k + 2*k^2 + 2*k^3 + 7*k^4))/Sqrt[4*b + (-1 + k)^2
])*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Int][1/((1 - x)^(2/3)*x^(11/
3)*(1 - k*x)^(2/3)*(-1 - k - Sqrt[1 + 4*b - 2*k + k^2] + 2*(-b + k)*x)), x]
)/((b - k)^4*((1 - x)*x*(1 - k*x))^(2/3)) + ((a + b^2)*(1 + k^5 + 5*b^2*(1
+ k) + 5*b*(1 + k + k^2 + k^3) - (1 + 4*b^3 - k - k^5 + k^6 + b^2*(13 + 14*
k + 13*k^2) + b*(7 + 2*k + 2*k^2 + 2*k^3 + 7*k^4))/Sqrt[4*b + (-1 + k)^2])*
(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Int][1/((1 - x)^(2/3)*x^(11/3)*
(1 - k*x)^(2/3)*(-1 - k + Sqrt[1 + 4*b - 2*k + k^2] + 2*(-b + k)*x)), x])/(
(b - k)^4*((1 - x)*x*(1 - k*x))^(2/3))
```

Rubi steps

$$\int \frac{(-2+(1+k)x)\left(1-2(1+k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(a+k^2\right)x^4\right)}{x^3((1-x)x(1-kx))^{2/3}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx = \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)}{}$$

$$= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)}{}$$

$$= -\frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right.}{}$$

$$= -\frac{3\left(a\left(1+2b+k^2\right)+2bk\right.}{2(}$$

$$= -\frac{3\left(a\left(1+2b+k^2\right)+2bk\right.}{2(}$$

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$$= -\frac{3\left(a\left(1+2b+k^2\right)+2bk\right.}{2(}$$

$$= -\frac{3\left(a\left(1+2b+k^2\right)+2bk\right.}{2(}$$

Mathematica [F] time = 8.23, size = 0, normalized size = 0.00

$$\int \frac{(-2+(1+k)x)\left(1-2(1+k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(a+k^2\right)x^4\right)}{x^3((1-x)x(1-kx))^{2/3}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + (1 + k)*x)*(1 - 2*(1 + k)*x + (1 + 4*k + k^2)*x^2 - 2*(k +
 k^2)*x^3 + (a + k^2)*x^4))/(x^3*((1 - x)*x*(1 - k*x))^(2/3)*(1 - (1 + k)*x
 + (-b + k)*x^2)),x]
```

```
[Out] Integrate[((-2 + (1 + k)*x)*(1 - 2*(1 + k)*x + (1 + 4*k + k^2)*x^2 - 2*(k +
 k^2)*x^3 + (a + k^2)*x^4))/(x^3*((1 - x)*x*(1 - k*x))^(2/3)*(1 - (1 + k)*x
 + (-b + k)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.55, size = 235, normalized size = 1.00

$$\frac{\left(a+b^2\right)\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{\left(-a-b^2\right)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2b^{2/3}}+\frac{\left(\sqrt{3}\,a+\sqrt{3}\,b^2\right)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{b^{2/3}}+\frac{3\sqrt[3]{kx^3-kx^2-x^2+x}\left(4bx^2+kx^2-kx-x+1\right)}{4x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + (1 + k)*x)*(1 - 2*(1 + k)*x + (1 + 4*k + k^2)*x^2
 - 2*(k + k^2)*x^3 + (a + k^2)*x^4))/(x^3*((1 - x)*x*(1 - k*x))^(2/3)*(1 -
(1 + k)*x + (-b + k)*x^2)),x]
```

```
[Out] (3*(1 - x - k*x + 4*b*x^2 + k*x^2)*(x - x^2 - k*x^2 + k*x^3)^(1/3))/(4*x^3)
 + ((Sqrt[3]*a + Sqrt[3]*b^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(x
+ (-1 - k)*x^2 + k*x^3)^(1/3))])/b^(2/3) + ((a + b^2)*Log[-(b^(1/3)*x) + (x
 + (-1 - k)*x^2 + k*x^3)^(1/3)])/b^(2/3) + ((-a - b^2)*Log[b^(2/3)*x^2 + b^
(1/3)*x*(x + (-1 - k)*x^2 + k*x^3)^(1/3) + (x + (-1 - k)*x^2 + k*x^3)^(2/3)
])/(2*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-2*(1+k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2+a)*x
^4)/x^3/((1-x)*x*(-k*x+1))^(2/3)/(1-(1+k)*x+(-b+k)*x^2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(\left(k^2+a\right)x^4 - 2\left(k^2+k\right)x^3 + \left(k^2+4k+1\right)x^2 - 2(k+1)x + 1\right)((k+1)x-2)}{((kx-1)(x-1)x)^{\frac{2}{3}}\left((b-k)x^2+(k+1)x-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-2*(1+k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2+a)*x
^4)/x^3/((1-x)*x*(-k*x+1))^(2/3)/(1-(1+k)*x+(-b+k)*x^2),x, algorithm="giac"
)
```

```
[Out] integrate(-((k^2 + a)*x^4 - 2*(k^2 + k)*x^3 + (k^2 + 4*k + 1)*x^2 - 2*(k +
1)*x + 1)*((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(2/3)*((b - k)*x^2 + (k +
1)*x - 1)*x^3), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(-2+(1+k)x\right)\left(1-2(1+k)x+\left(k^2+4k+1\right)x^2-2\left(k^2+k\right)x^3+\left(k^2+a\right)x^4\right)}{x^3\left((1-x)x(-kx+1)\right)^{\frac{2}{3}}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+(1+k)*x)*(1-2*(1+k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2+a)*x^4)/x^
3/((1-x)*x*(-k*x+1))^(2/3)/(1-(1+k)*x+(-b+k)*x^2),x)
```

```
[Out] int((-2+(1+k)*x)*(1-2*(1+k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2+a)*x^4)/x^
3/((1-x)*x*(-k*x+1))^(2/3)/(1-(1+k)*x+(-b+k)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(\left(k^2+a\right)x^4 - 2\left(k^2+k\right)x^3 + \left(k^2+4k+1\right)x^2 - 2(k+1)x + 1\right)((k+1)x-2)}{((kx-1)(x-1)x)^{\frac{2}{3}}\left((b-k)x^2+(k+1)x-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-2*(1+k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2+a)*x
^4)/x^3/((1-x)*x*(-k*x+1))^(2/3)/(1-(1+k)*x+(-b+k)*x^2),x, algorithm="maxim
a")
```

```
[Out] -integrate(((k^2 + a)*x^4 - 2*(k^2 + k)*x^3 + (k^2 + 4*k + 1)*x^2 - 2*(k +
1)*x + 1)*((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(2/3)*((b - k)*x^2 + (k +
1)*x - 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(x\,(k+1)-2)\,\left(x^2\,\left(k^2+4\,k+1\right)-2\,x\,(k+1)+x^4\,\left(k^2+a\right)-2\,x^3\,\left(k^2+k\right)+1\right)}{x^3\,(x\,(k\,x-1)\,(x-1))^{2/3}\,\left((b-k)\,x^2+(k+1)\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x*(k + 1) - 2)*(x^2*(4*k + k^2 + 1) - 2*x*(k + 1) + x^4*(a + k^2) -
2*x^3*(k + k^2) + 1))/(x^3*(x*(k*x - 1)*(x - 1))^(2/3)*(x*(k + 1) + x^2*(b
- k) - 1)),x)
```

```
[Out] int(-((x*(k + 1) - 2)*(x^2*(4*k + k^2 + 1) - 2*x*(k + 1) + x^4*(a + k^2) -
2*x^3*(k + k^2) + 1))/(x^3*(x*(k*x - 1)*(x - 1))^(2/3)*(x*(k + 1) + x^2*(b
- k) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-2*(1+k)*x+(k**2+4*k+1)*x**2-2*(k**2+k)*x**3+(k**2
+a)*x**4)/x**3/((1-x)*x*(-k*x+1))**(2/3)/(1-(1+k)*x+(-b+k)*x**2),x)
```

```
[Out] Timed out
```

3.2121 $\int \frac{1+akx+kx^2}{\left(-1+kx^2\right)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}\,dx$

Optimal. Leaf size=236

$$\frac{a\sqrt{k}\tan^{-1}\left(\frac{\left(2\sqrt{k}-2k^{3/2}\right)x^2}{k^2x^4+\left(kx^2+1\right)\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}-2kx^2+1}\right)}{2\left(\sqrt{k}-1\right)\left(\sqrt{k}+1\right)}+\frac{\tan^{-1}\left(\frac{\left(-k-2i\sqrt{k}+1\right)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2+1}\right)}{\left(\sqrt{k}-1\right)\left(\sqrt{k}+1\right)}+\frac{\tan^{-1}\left(\frac{\left(-k+2i\sqrt{k}+1\right)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+k\ldots}\right)}{\left(\sqrt{k}-1\right)\left(\sqrt{k}+1\right)}$$

Rubi [C] time = 2.38, antiderivative size = 389, normalized size of antiderivative = 1.65, number of steps used = 16, number of rules used = 8, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {6719, 6725, 419, 2113, 537, 571, 93, 205}

$$-\frac{\sqrt{1-x^2}\left(2-a\sqrt{k}\right)\sqrt{1-k^2x^2}\tan^{-1}\left(\frac{\sqrt{k}\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{2(1-k)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}+\frac{\sqrt{1-x^2}\left(a\sqrt{k}+2\right)\sqrt{1-k^2x^2}\tan^{-1}\left(\frac{\sqrt{k}\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{2(1-k)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}-\frac{\sqrt{1-x^2}\left(2-a\sqrt{k}\right)\sqrt{1-k^2x^2}\,\Pi\left(k;\sin^{-1}(x)|k^2\right)}{2\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}-\frac{\sqrt{1-x^2}\left(a\sqrt{k}+2\right)\sqrt{1-k^2x^2}\,\Pi\left(k;\sin^{-1}(x)|k^2\right)}{2\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}+\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*k*x + k*x^2)/((-1 + k*x^2)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),x]
```

```
[Out] -1/2*((2 - a*Sqrt[k])*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*ArcTan[(Sqrt[k]*Sqrt[
1 - x^2])/Sqrt[1 - k^2*x^2]])/((1 - k)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) + ((2
 + a*Sqrt[k])*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*ArcTan[(Sqrt[k]*Sqrt[1 - x^2]
)/Sqrt[1 - k^2*x^2]])/(2*(1 - k)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) + (Sqrt[1 -
 x^2]*Sqrt[1 - k^2*x^2]*EllipticF[ArcSin[x], k^2])/Sqrt[(1 - x^2)*(1 - k^2*
x^2)] - ((2 - a*Sqrt[k])*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[k, ArcS
in[x], k^2])/(2*Sqrt[(1 - x^2)*(1 - k^2*x^2)]) - ((2 + a*Sqrt[k])*Sqrt[1 -
x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[k, ArcSin[x], k^2])/(2*Sqrt[(1 - x^2)*(1
- k^2*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2113

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+akx+kx^2}{\left(-1+kx^2\right)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}\,dx = \frac{\left(\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \frac{1+akx+kx^2}{\sqrt{1-x^2}\left(-1+kx^2\right)\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \left(\frac{1}{\sqrt{1-x^2}\sqrt{1-k^2x^2}}+\frac{2+akx}{\sqrt{1-x^2}\left(-1+kx^2\right)\sqrt{1-k^2x^2}}\right)dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{\left(\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \frac{1}{\sqrt{1-x^2}\sqrt{1-k^2x^2}}\,dx}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \frac{}{\sqrt{}}}{\sqrt{\left(1-x^2\right)\left(\right.}}$$

$$= \frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(\sqrt{1-x^2}\sqrt{1-k^2x^2}\right)\int \left(-\frac{}{2(1-\sqrt{}}\right.}{\sqrt{(}}$$

$$= \frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(\left(-2-a\sqrt{k}\right)\sqrt{1-x^2}\sqrt{1-k^2x^2}\right.}{2\sqrt{\left(1-x^2\right)\left(\right.}}$$

$$= \frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(\left(-2-a\sqrt{k}\right)\sqrt{1-x^2}\sqrt{1-k^2x^2}\right.}{2\sqrt{\left(1-x^2\right)\left(\right.}}$$

$$= \frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(2-a\sqrt{k}\right)\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi}{2\sqrt{\left(1-x^2\right)\left(1-k^2\right.}}$$

$$= \frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)}{\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} - \frac{\left(2-a\sqrt{k}\right)\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi}{2\sqrt{\left(1-x^2\right)\left(1-k^2\right.}}$$

$$= -\frac{\left(2-a\sqrt{k}\right)\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\tan^{-1}\left(\frac{\sqrt{k}\sqrt{1-x^2}}{\sqrt{1-k^2x^2}}\right)}{2(1-k)\sqrt{\left(1-x^2\right)\left(1-k^2x^2\right)}} + \frac{\left(2+a\sqrt{k}\right)\sqrt{1-}}{2(1-k}$$

Mathematica [C] time = 0.46, size = 204, normalized size = 0.86

$$\frac{ak\sqrt{x^2-1}\sqrt{k^2x^2-1}\tanh^{-1}\left(\frac{\sqrt{-((k-1)k)}\sqrt{x^2-1}}{\sqrt{k-1}\sqrt{k^2x^2-1}}\right)+\sqrt{k-1}\sqrt{-((k-1)k)}\sqrt{1-x^2}\sqrt{1-k^2x^2}\,F\left(\sin^{-1}(x)|k^2\right)-2\sqrt{k-1}\sqrt{-((k-1)k)}\sqrt{1-x^2}\sqrt{1-k^2x^2}\,\Pi\left(k;\sin^{-1}(x)|k^2\right)}{\sqrt{k-1}\sqrt{-((k-1)k)}\sqrt{\left(x^2-1\right)\left(k^2x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*k*x + k*x^2)/((-1 + k*x^2)*Sqrt[(1 - x^2)*(1 - k^2*x^2)]),
x]
```

```
[Out] (a*k*Sqrt[-1 + x^2]*Sqrt[-1 + k^2*x^2]*ArcTanh[(Sqrt[-((-1 + k)*k)]*Sqrt[-1
 + x^2])/(Sqrt[-1 + k]*Sqrt[-1 + k^2*x^2])] + Sqrt[-1 + k]*Sqrt[-((-1 + k)*
k)]*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticF[ArcSin[x], k^2] - 2*Sqrt[-1 +
 k]*Sqrt[-((-1 + k)*k)]*Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*EllipticPi[k, ArcSi
n[x], k^2])/(Sqrt[-1 + k]*Sqrt[-((-1 + k)*k)]*Sqrt[(-1 + x^2)*(-1 + k^2*x^2
)])
```

IntegrateAlgebraic [A] time = 3.67, size = 236, normalized size = 1.00

$$\frac{a\sqrt{k}\tan^{-1}\left(\frac{\left(2\sqrt{k}-2k^{3/2}\right)x^2}{k^2x^4+\left(kx^2+1\right)\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}-2kx^2+1}\right)}{2\left(\sqrt{k}-1\right)\left(\sqrt{k}+1\right)}+\frac{\tan^{-1}\left(\frac{\left(-k-2i\sqrt{k}+1\right)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2+1}\right)}{\left(\sqrt{k}-1\right)\left(\sqrt{k}+1\right)}+\frac{\tan^{-1}\left(\frac{\left(-k+2i\sqrt{k}+1\right)x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2+1}\right)}{\left(\sqrt{k}-1\right)\left(\sqrt{k}+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + a*k*x + k*x^2)/((-1 + k*x^2)*Sqrt[(1 - x^2)*(1 - k^
2*x^2)]),x]
```

```
[Out] ArcTan[((1 - (2*I)*Sqrt[k] - k)*x)/(1 + k*x^2 + Sqrt[1 + (-1 - k^2)*x^2 + k
^2*x^4])]/((-1 + Sqrt[k])*(1 + Sqrt[k])) + ArcTan[((1 + (2*I)*Sqrt[k] - k)*
x)/(1 + k*x^2 + Sqrt[1 + (-1 - k^2)*x^2 + k^2*x^4])]/((-1 + Sqrt[k])*(1 + S
qrt[k])) + (a*Sqrt[k]*ArcTan[((2*Sqrt[k] - 2*k^(3/2))*x^2)/(1 - 2*k*x^2 + k
^2*x^4 + (1 + k*x^2)*Sqrt[1 + (-1 - k^2)*x^2 + k^2*x^4])])/(2*(-1 + Sqrt[k]
)*(1 + Sqrt[k]))
```

fricas [B] time = 1.83, size = 1793, normalized size = 7.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*k*x+k*x^2+1)/(k*x^2-1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algori
thm="fricas")
```

```
[Out] -1/8*sqrt(-(a^2*k + 4*sqrt(a^2*k/(k^4 - 4*k^3 + 6*k^2 - 4*k + 1))*(k^2 - 2*
k + 1) + 4)/(k^2 - 2*k + 1))*log(-2*(sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)*(a^3
*k + (a^3*k^2 - 4*a*k)*x^2 + 2*(a^2*k^3 - 2*(a^2 + 2)*k^2 + (a^2 + 8)*k - 4
)*sqrt(a^2*k/(k^4 - 4*k^3 + 6*k^2 - 4*k + 1))*x - 4*a) + (2*a^2*k^3*x^4 + 2
*(a*k^3 - 2*a*k^2 + a*k)*x^3 + 2*a^2*k - 2*(a^2*k^3 + a^2*k)*x^2 + 2*(a*k^2
 - 2*a*k + a)*x - (4*(k^4 - 2*k^3 + k^2)*x^4 + (a*k^5 - 4*a*k^4 + 6*a*k^3 -
 4*a*k^2 + a*k)*x^3 - 4*(k^4 - 2*k^3 + 2*k^2 - 2*k + 1)*x^2 + 4*k^2 + (a*k^
4 - 4*a*k^3 + 6*a*k^2 - 4*a*k + a)*x - 8*k + 4)*sqrt(a^2*k/(k^4 - 4*k^3 + 6
*k^2 - 4*k + 1)))*sqrt(-(a^2*k + 4*sqrt(a^2*k/(k^4 - 4*k^3 + 6*k^2 - 4*k +
1))*(k^2 - 2*k + 1) + 4)/(k^2 - 2*k + 1)))/(k^2*x^4 - 2*k*x^2 + 1)) + 1/8*s
qrt(-(a^2*k + 4*sqrt(a^2*k/(k^4 - 4*k^3 + 6*k^2 - 4*k + 1))*(k^2 - 2*k + 1)
 + 4)/(k^2 - 2*k + 1))*log(-2*(sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)*(a^3*k + (
a^3*k^2 - 4*a*k)*x^2 + 2*(a^2*k^3 - 2*(a^2 + 2)*k^2 + (a^2 + 8)*k - 4)*sqrt
(a^2*k/(k^4 - 4*k^3 + 6*k^2 - 4*k + 1))*x - 4*a) - (2*a^2*k^3*x^4 + 2*(a*k^
3 - 2*a*k^2 + a*k)*x^3 + 2*a^2*k - 2*(a^2*k^3 + a^2*k)*x^2 + 2*(a*k^2 - 2*a
*k + a)*x - (4*(k^4 - 2*k^3 + k^2)*x^4 + (a*k^5 - 4*a*k^4 + 6*a*k^3 - 4*a*k
^2 + a*k)*x^3 - 4*(k^4 - 2*k^3 + 2*k^2 - 2*k + 1)*x^2 + 4*k^2 + (a*k^4 - 4*
a*k^3 + 6*a*k^2 - 4*a*k + a)*x - 8*k + 4)*sqrt(a^2*k/(k^4 - 4*k^3 + 6*k^2 -
 4*k + 1)))*sqrt(-(a^2*k + 4*sqrt(a^2*k/(k^4 - 4*k^3 + 6*k^2 - 4*k + 1))*(k
^2 - 2*k + 1) + 4)/(k^2 - 2*k + 1)))/(k^2*x^4 - 2*k*x^2 + 1)) - 1/8*sqrt(-(
a^2*k - 4*sqrt(a^2*k/(k^4 - 4*k^3 + 6*k^2 - 4*k + 1))*(k^2 - 2*k + 1) + 4)/
(k^2 - 2*k + 1))*log(-2*(sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)*(a^3*k + (a^3*k^
2 - 4*a*k)*x^2 - 2*(a^2*k^3 - 2*(a^2 + 2)*k^2 + (a^2 + 8)*k - 4)*sqrt(a^2*k
/(k^4 - 4*k^3 + 6*k^2 - 4*k + 1))*x - 4*a) + (2*a^2*k^3*x^4 + 2*(a*k^3 - 2*
a*k^2 + a*k)*x^3 + 2*a^2*k - 2*(a^2*k^3 + a^2*k)*x^2 + 2*(a*k^2 - 2*a*k + a
)*x + (4*(k^4 - 2*k^3 + k^2)*x^4 + (a*k^5 - 4*a*k^4 + 6*a*k^3 - 4*a*k^2 + a
*k)*x^3 - 4*(k^4 - 2*k^3 + 2*k^2 - 2*k + 1)*x^2 + 4*k^2 + (a*k^4 - 4*a*k^3
+ 6*a*k^2 - 4*a*k + a)*x - 8*k + 4)*sqrt(a^2*k/(k^4 - 4*k^3 + 6*k^2 - 4*k +
 1)))*sqrt(-(a^2*k - 4*sqrt(a^2*k/(k^4 - 4*k^3 + 6*k^2 - 4*k + 1))*(k^2 - 2
*k + 1) + 4)/(k^2 - 2*k + 1)))/(k^2*x^4 - 2*k*x^2 + 1)) + 1/8*sqrt(-(a^2*k
- 4*sqrt(a^2*k/(k^4 - 4*k^3 + 6*k^2 - 4*k + 1))*(k^2 - 2*k + 1) + 4)/(k^2 -
 2*k + 1))*log(-2*(sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)*(a^3*k + (a^3*k^2 - 4*
a*k)*x^2 - 2*(a^2*k^3 - 2*(a^2 + 2)*k^2 + (a^2 + 8)*k - 4)*sqrt(a^2*k/(k^4
- 4*k^3 + 6*k^2 - 4*k + 1))*x - 4*a) - (2*a^2*k^3*x^4 + 2*(a*k^3 - 2*a*k^2
+ a*k)*x^3 + 2*a^2*k - 2*(a^2*k^3 + a^2*k)*x^2 + 2*(a*k^2 - 2*a*k + a)*x +
```

```
(4*(k^4 - 2*k^3 + k^2)*x^4 + (a*k^5 - 4*a*k^4 + 6*a*k^3 - 4*a*k^2 + a*k)*x^
3 - 4*(k^4 - 2*k^3 + 2*k^2 - 2*k + 1)*x^2 + 4*k^2 + (a*k^4 - 4*a*k^3 + 6*a*
k^2 - 4*a*k + a)*x - 8*k + 4)*sqrt(a^2*k/(k^4 - 4*k^3 + 6*k^2 - 4*k + 1)))*
sqrt(-(a^2*k - 4*sqrt(a^2*k/(k^4 - 4*k^3 + 6*k^2 - 4*k + 1))*(k^2 - 2*k + 1
) + 4)/(k^2 - 2*k + 1)))/(k^2*x^4 - 2*k*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{akx + kx^2 + 1}{\left(kx^2 - 1\right)\sqrt{\left(k^2x^2 - 1\right)\left(x^2 - 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*k*x+k*x^2+1)/(k*x^2-1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((a*k*x + k*x^2 + 1)/((k*x^2 - 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))),
x)
```

maple [C] time = 0.05, size = 364, normalized size = 1.54

$$\frac{\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\operatorname{EllipticF}(x,k)}{\sqrt{k^2x^4-k^2x^2-x^2+1}} + \frac{a\operatorname{arctanh}\left(-\frac{x^2k}{\sqrt{-\frac{1}{k}+2-k}\,\sqrt{k^2x^4-k^2x^2-x^2+1}} + \frac{x^2k^2}{2\sqrt{-\frac{1}{k}+2-k}\,\sqrt{k^2x^4-k^2x^2-x^2+1}} + \frac{k}{2\sqrt{-\frac{1}{k}+2-k}\,\sqrt{k^2x^4-k^2x^2-x^2+1}} + \frac{x^2}{2\sqrt{-\frac{1}{k}+2-k}\,\sqrt{k^2x^4-k^2x^2-x^2+1}} + \frac{1}{2\sqrt{-\frac{1}{k}+2-k}\,\sqrt{k^2x^4-k^2x^2-x^2+1}\,k} - \frac{1}{\sqrt{-\frac{1}{k}+2-k}\,\sqrt{k^2x^4-k^2x^2-x^2+1}}\right)}{2\sqrt{-\frac{1}{k}+2-k}} - \frac{2\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\operatorname{EllipticPi}(x,k,k)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*k*x+k*x^2+1)/(k*x^2-1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x)
```

```
[Out] (-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*EllipticF(x
,k)+1/2*a/(-1/k+2-k)^(1/2)*arctanh(-1/(-1/k+2-k)^(1/2)/(k^2*x^4-k^2*x^2-x^2
+1)^(1/2)*x^2*k+1/2/(-1/k+2-k)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*x^2*k^2+
1/2/(-1/k+2-k)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*k+1/2/(-1/k+2-k)^(1/2)/(
k^2*x^4-k^2*x^2-x^2+1)^(1/2)*x^2+1/2/(-1/k+2-k)^(1/2)/(k^2*x^4-k^2*x^2-x^2+
1)^(1/2)/k-1/(-1/k+2-k)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2))-2*(-x^2+1)^(1/
2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*EllipticPi(x,k,k)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{akx + kx^2 + 1}{\left(kx^2 - 1\right)\sqrt{\left(k^2x^2 - 1\right)\left(x^2 - 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*k*x+k*x^2+1)/(k*x^2-1)/((-x^2+1)*(-k^2*x^2+1))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((a*k*x + k*x^2 + 1)/((k*x^2 - 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{k\,x^2 + a\,k\,x + 1}{\left(k\,x^2 - 1\right)\,\sqrt{\left(x^2 - 1\right)\,\left(k^2\,x^2 - 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^2 + a*k*x + 1)/((k*x^2 - 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)),x)
```

```
[Out] int((k*x^2 + a*k*x + 1)/((k*x^2 - 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{akx + kx^2 + 1}{\sqrt{(x-1)(x+1)(kx-1)(kx+1)}\left(kx^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*k*x+k*x**2+1)/(k*x**2-1)/((-x**2+1)*(-k**2*x**2+1))**(1/2),x)
```

```
[Out] Integral((a*k*x + k*x**2 + 1)/(sqrt((x - 1)*(x + 1)*(k*x - 1)*(k*x + 1))*(k
*x**2 - 1)), x)
```

3.2122 $\int \frac{3k+(2-k^2)x-3kx^2-k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(1-d-(1+2d)kx+(-1-dk^2)x^2+kx^3)}\,dx$

Optimal. Leaf size=236

$$\frac{\log\left(d^{2/3}k^2x^2+2d^{2/3}kx+d^{2/3}+\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}\left(\sqrt[3]{d}\,kx+\sqrt[3]{d}\right)+\left(k^2x^4+(-k^2-1)x^2+1\right)^{2/3}\right)}{2\sqrt[3]{d}}-\log$$

Rubi [F] time = 6.51, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3k+(2-k^2)x-3kx^2-k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(1-d-(1+2d)kx+(-1-dk^2)x^2+kx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3*k + (2 - k^2)*x - 3*k*x^2 - k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3
)*(1 - d - (1 + 2*d)*k*x + (-1 - d*k^2)*x^2 + k*x^3)),x]
```

```
[Out] -((k*x*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*AppellF1[1/2, 1/3, 1/3, 3/2, x^2
, k^2*x^2])/((1 - x^2)*(1 - k^2*x^2))^(1/3)) + ((4 - d)*k*(1 - x^2)^(1/3)*(
1 - k^2*x^2)^(1/3)*Defer[Int][1/((1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*(1 - d
 - (1 + 2*d)*k*x - (1 + d*k^2)*x^2 + k*x^3)), x])/((1 - x^2)*(1 - k^2*x^2))
^(1/3) + (2*(1 - (1 + d)*k^2)*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*Defer[Int
][x/((1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*(1 - d - (1 + 2*d)*k*x - (1 + d*k^
2)*x^2 + k*x^3)), x])/((1 - x^2)*(1 - k^2*x^2))^(1/3) - (k*(4 + d*k^2)*(1 -
 x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*Defer[Int][x^2/((1 - x^2)^(1/3)*(1 - k^2*x^
2)^(1/3)*(1 - d - (1 + 2*d)*k*x - (1 + d*k^2)*x^2 + k*x^3)), x])/((1 - x^2)
*(1 - k^2*x^2))^(1/3)
```

Rubi steps

$$\int \frac{3k+\left(2-k^2\right)x-3kx^2-k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(1-d-(1+2d)kx+\left(-1-dk^2\right)x^2+kx^3\right)}\,dx = \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{3k+(2\cdots}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,(1-\cdots}}{\sqrt[3]{\left(1-x^2\right)\left(1-k\cdots\right.}}$$

$$= \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{3k+(2-\cdots}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,(1-\cdots}}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2\cdots\right.}}$$

$$= \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \left(-\frac{k}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\cdots}\right.}{\sqrt[3]{(1-\cdots}}$$

$$= \frac{\left(\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\right)\int \frac{(4-d)k+2(1-\cdots}{\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,(1-\cdots}}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2\cdots\right.}}$$

$$= -\frac{kx\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,\cdots\right.}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= -\frac{kx\sqrt[3]{1-x^2}\,\sqrt[3]{1-k^2x^2}\,F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,\cdots\right.}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

Mathematica [F] time = 1.33, size = 0, normalized size = 0.00

$$\int \frac{3k+\left(2-k^2\right)x-3kx^2-k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(1-d-(1+2d)kx+\left(-1-dk^2\right)x^2+kx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3*k + (2 - k^2)*x - 3*k*x^2 - k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2)
)^(1/3)*(1 - d - (1 + 2*d)*k*x + (-1 - d*k^2)*x^2 + k*x^3)),x]
```

```
[Out] Integrate[(3*k + (2 - k^2)*x - 3*k*x^2 - k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2)
)^(1/3)*(1 - d - (1 + 2*d)*k*x + (-1 - d*k^2)*x^2 + k*x^3)), x]
```

IntegrateAlgebraic [A] time = 5.64, size = 236, normalized size = 1.00

$$\frac{\log\left(d^{2/3}k^2x^2+2d^{2/3}kx+d^{2/3}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\left(\sqrt[3]{d}\,kx+\sqrt[3]{d}\right)+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}\right)}{2\sqrt[3]{d}}-\frac{\log\left(-\sqrt[3]{d}\,kx-\sqrt[3]{d}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\right)}{\sqrt[3]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}{2\sqrt[3]{d}\,kx+2\sqrt[3]{d}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*k + (2 - k^2)*x - 3*k*x^2 - k^2*x^3)/(((1 - x^2)*(1 -
 k^2*x^2))^(1/3)*(1 - d - (1 + 2*d)*k*x + (-1 - d*k^2)*x^2 + k*x^3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))/(2*d^(1/3)
 + 2*d^(1/3)*k*x + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))])/d^(1/3)) - Log[-
d^(1/3) - d^(1/3)*k*x + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3)]/d^(1/3) + Log
[d^(2/3) + 2*d^(2/3)*k*x + d^(2/3)*k^2*x^2 + (d^(1/3) + d^(1/3)*k*x)*(1 + (
-1 - k^2)*x^2 + k^2*x^4)^(1/3) + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(2*d
^(1/3))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*k+(-k^2+2)*x-3*k*x^2-k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1
-d-(1+2*d)*k*x+(-d*k^2-1)*x^2+k*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{k^2x^3 + 3\,kx^2 + (k^2 - 2)x - 3\,k}{(kx^3 - (2\,d + 1)kx - (dk^2 + 1)x^2 - d + 1)\left((k^2x^2 - 1)(x^2 - 1)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*k+(-k^2+2)*x-3*k*x^2-k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1
-d-(1+2*d)*k*x+(-d*k^2-1)*x^2+k*x^3),x, algorithm="giac")
```

```
[Out] integrate(-(k^2*x^3 + 3*k*x^2 + (k^2 - 2)*x - 3*k)/((k*x^3 - (2*d + 1)*k*x
- (d*k^2 + 1)*x^2 - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

maple [**F**] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{3k + (-k^2 + 2)\,x - 3k\,x^2 - k^2x^3}{\left((-x^2 + 1)\,(-k^2x^2 + 1)\right)^{\frac{1}{3}}\left(1 - d - (1 + 2d)\,kx + (-d\,k^2 - 1)\,x^2 + k\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*k+(-k^2+2)*x-3*k*x^2-k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d-(1+
2*d)*k*x+(-d*k^2-1)*x^2+k*x^3),x)
```

```
[Out] int((3*k+(-k^2+2)*x-3*k*x^2-k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d-(1+
2*d)*k*x+(-d*k^2-1)*x^2+k*x^3),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{k^2x^3 + 3\,kx^2 + (k^2 - 2)x - 3\,k}{(kx^3 - (2\,d + 1)kx - (dk^2 + 1)x^2 - d + 1)\left((k^2x^2 - 1)(x^2 - 1)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*k+(-k^2+2)*x-3*k*x^2-k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1
-d-(1+2*d)*k*x+(-d*k^2-1)*x^2+k*x^3),x, algorithm="maxima")
```

```
[Out] -integrate((k^2*x^3 + 3*k*x^2 + (k^2 - 2)*x - 3*k)/((k*x^3 - (2*d + 1)*k*x
- (d*k^2 + 1)*x^2 - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(k^2 - 2) - 3\,k + k^2\,x^3 + 3\,k\,x^2}{\left((x^2 - 1)\,(k^2\,x^2 - 1)\right)^{1/3}\,\left(-k\,x^3 + (d\,k^2 + 1)\,x^2 + k\,(2\,d + 1)\,x + d - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(k^2 - 2) - 3*k + k^2*x^3 + 3*k*x^2)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/3
)*(d + x^2*(d*k^2 + 1) - k*x^3 + k*x*(2*d + 1) - 1)),x)
```

```
[Out] int((x*(k^2 - 2) - 3*k + k^2*x^3 + 3*k*x^2)/((((x^2 - 1)*(k^2*x^2 - 1))^(1/3
)*(d + x^2*(d*k^2 + 1) - k*x^3 + k*x*(2*d + 1) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*k+(-k**2+2)*x-3*k*x**2-k**2*x**3)/((-x**2+1)*(-k**2*x**2+1))**
(1/3)/(1-d-(1+2*d)*k*x+(-d*k**2-1)*x**2+k*x**3),x)
```

```
[Out] Timed out
```

3.2123 $\int \frac{3k+(-2+k^2)x-3kx^2+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(-1+d-(1+2d)kx+(1+dk^2)x^2+kx^3)}\,dx$

Optimal. Leaf size=236

$$\frac{\log\left(d^{2/3}k^2x^2-2d^{2/3}kx+d^{2/3}+\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}\left(\sqrt[3]{d}-\sqrt[3]{d}\,kx\right)+\left(k^2x^4+(-k^2-1)x^2+1\right)^{2/3}\right)}{2\sqrt[3]{d}}-\frac{\log}{}$$

Rubi [F] time = 6.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{3k+(-2+k^2)x-3kx^2+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(-1+d-(1+2d)kx+(1+dk^2)x^2+kx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3*k + (-2 + k^2)*x - 3*k*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/
3)*(-1 + d - (1 + 2*d)*k*x + (1 + d*k^2)*x^2 + k*x^3)),x]
```

```
[Out] (k*x*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*AppellF1[1/2, 1/3, 1/3, 3/2, x^2,
k^2*x^2])/((1 - x^2)*(1 - k^2*x^2))^(1/3) - ((4 - d)*k*(1 - x^2)^(1/3)*(1 -
 k^2*x^2)^(1/3)*Defer[Int][1/((1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*(1 - d +
(1 + 2*d)*k*x - (1 + d*k^2)*x^2 - k*x^3)), x])/((1 - x^2)*(1 - k^2*x^2))^(1
/3) + (2*(1 - (1 + d)*k^2)*(1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*Defer[Int][x
/((1 - x^2)^(1/3)*(1 - k^2*x^2)^(1/3)*(1 - d + (1 + 2*d)*k*x - (1 + d*k^2)*
x^2 - k*x^3)), x])/((1 - x^2)*(1 - k^2*x^2))^(1/3) + (k*(4 + d*k^2)*(1 - x^
2)^(1/3)*(1 - k^2*x^2)^(1/3)*Defer[Int][x^2/((1 - x^2)^(1/3)*(1 - k^2*x^2)^
(1/3)*(1 - d + (1 + 2*d)*k*x - (1 + d*k^2)*x^2 - k*x^3)), x])/((1 - x^2)*(1
 - k^2*x^2))^(1/3)
```

Rubi steps

$$\int \frac{3k+\left(-2+k^2\right)x-3kx^2+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d-(1+2d)kx+\left(1+dk^2\right)x^2+kx^3\right)}dx = \frac{\left(\sqrt[3]{1-x^2}\sqrt[3]{1-k^2x^2}\right)\int \frac{3k+(-\cdots}{\sqrt[3]{1-x^2}\sqrt[3]{1-k^2x^2}(-\cdots}}{\sqrt[3]{\left(1-x^2\right)\left(1-k\cdots\right.}}$$

$$= \frac{\left(\sqrt[3]{1-x^2}\sqrt[3]{1-k^2x^2}\right)\int \frac{-3k+(2\cdots}{\sqrt[3]{1-x^2}\sqrt[3]{1-k^2x^2}(1\cdots}}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2\cdots\right.}}$$

$$= \frac{\left(\sqrt[3]{1-x^2}\sqrt[3]{1-k^2x^2}\right)\int \left(\frac{k}{\sqrt[3]{1-x^2}\sqrt[3]{1-k^2x^2}}\cdots\right.}{\sqrt[3]{(1-x\cdots}}$$

$$= -\frac{\left(\sqrt[3]{1-x^2}\sqrt[3]{1-k^2x^2}\right)\int \frac{(4-d)k-2(\cdots}{\sqrt[3]{1-x^2}\sqrt[3]{1-k^2x^2}(\cdots}}{\sqrt[3]{\left(1-x^2\right)\left(1-\cdots\right.}}$$

$$= \frac{kx\sqrt[3]{1-x^2}\sqrt[3]{1-k^2x^2}F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,k\cdots\right.}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

$$= \frac{kx\sqrt[3]{1-x^2}\sqrt[3]{1-k^2x^2}F_1\left(\frac{1}{2};\frac{1}{3},\frac{1}{3};\frac{3}{2};x^2,k\cdots\right.}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}}$$

Mathematica [F] time = 1.35, size = 0, normalized size = 0.00

$$\int \frac{3k+\left(-2+k^2\right)x-3kx^2+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(-1+d-(1+2d)kx+\left(1+dk^2\right)x^2+kx^3\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(3*k + (-2 + k^2)*x - 3*k*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2
))^(1/3)*(-1 + d - (1 + 2*d)*k*x + (1 + d*k^2)*x^2 + k*x^3)),x]
```

```
[Out] Integrate[(3*k + (-2 + k^2)*x - 3*k*x^2 + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2
))^(1/3)*(-1 + d - (1 + 2*d)*k*x + (1 + d*k^2)*x^2 + k*x^3)), x]
```

IntegrateAlgebraic [A] time = 5.64, size = 236, normalized size = 1.00

$$\frac{\log\left(d^{2/3}k^2x^2-2d^{2/3}kx+d^{2/3}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\left(\sqrt[3]{d}-\sqrt[3]{d}kx\right)+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}\right)}{2\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{d}kx-\sqrt[3]{d}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\right)}{\sqrt[3]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}{-2\sqrt[3]{d}kx+2\sqrt[3]{d}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3*k + (-2 + k^2)*x - 3*k*x^2 + k^2*x^3)/(((1 - x^2)*(1
- k^2*x^2))^(1/3)*(-1 + d - (1 + 2*d)*k*x + (1 + d*k^2)*x^2 + k*x^3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))/(2*d^(1/3)
 - 2*d^(1/3)*k*x + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))])/d^(1/3)) - Log[-
d^(1/3) + d^(1/3)*k*x + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3)]/d^(1/3) + Log
[d^(2/3) - 2*d^(2/3)*k*x + d^(2/3)*k^2*x^2 + (d^(1/3) - d^(1/3)*k*x)*(1 + (
-1 - k^2)*x^2 + k^2*x^4)^(1/3) + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(2*d
^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*k+(k^2-2)*x-3*k*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1
+d-(1+2*d)*k*x+(d*k^2+1)*x^2+k*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^{2}x^{3}-3\,kx^{2}+\left(k^{2}-2\right)x+3\,k}{\left(kx^{3}-(2\,d+1)kx+\left(dk^{2}+1\right)x^{2}+d-1\right)\left(\left(k^{2}x^{2}-1\right)\left(x^{2}-1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*k+(k^2-2)*x-3*k*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1
+d-(1+2*d)*k*x+(d*k^2+1)*x^2+k*x^3),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^3 - 3*k*x^2 + (k^2 - 2)*x + 3*k)/((k*x^3 - (2*d + 1)*k*x +
 (d*k^2 + 1)*x^2 + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{3k+\left(k^{2}-2\right)x-3k\,x^{2}+k^{2}x^{3}}{\left(\left(-x^{2}+1\right)\left(-k^{2}x^{2}+1\right)\right)^{\frac{1}{3}}\left(-1+d-(1+2d)\,kx+\left(d\,k^{2}+1\right)x^{2}+k\,x^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*k+(k^2-2)*x-3*k*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d-(1+
2*d)*k*x+(d*k^2+1)*x^2+k*x^3),x)
```

```
[Out] int((3*k+(k^2-2)*x-3*k*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d-(1+
2*d)*k*x+(d*k^2+1)*x^2+k*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^{2}x^{3}-3\,kx^{2}+\left(k^{2}-2\right)x+3\,k}{\left(kx^{3}-(2\,d+1)kx+\left(dk^{2}+1\right)x^{2}+d-1\right)\left(\left(k^{2}x^{2}-1\right)\left(x^{2}-1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*k+(k^2-2)*x-3*k*x^2+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1
+d-(1+2*d)*k*x+(d*k^2+1)*x^2+k*x^3),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^3 - 3*k*x^2 + (k^2 - 2)*x + 3*k)/((k*x^3 - (2*d + 1)*k*x +
 (d*k^2 + 1)*x^2 + d - 1)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{3\,k+x\,\left(k^2-2\right)+k^2\,x^3-3\,k\,x^2}{{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)}^{1/3}\,\left(k\,x^3+\left(d\,k^2+1\right)\,x^2-k\,\left(2\,d+1\right)\,x+d-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*k + x*(k^2 - 2) + k^2*x^3 - 3*k*x^2)/((((x^2 - 1)*(k^2*x^2 - 1))^(1/3
)*(d + x^2*(d*k^2 + 1) + k*x^3 - k*x*(2*d + 1) - 1)),x)
```

```
[Out] int((3*k + x*(k^2 - 2) + k^2*x^3 - 3*k*x^2)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/3
)*(d + x^2*(d*k^2 + 1) + k*x^3 - k*x*(2*d + 1) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*k+(k**2-2)*x-3*k*x**2+k**2*x**3)/((-x**2+1)*(-k**2*x**2+1))**(
1/3)/(-1+d-(1+2*d)*k*x+(d*k**2+1)*x**2+k*x**3),x)
```

```
[Out] Timed out
```

3.2124 $\int \frac{d+cx^4}{x\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$

Optimal. Leaf size=236

$$\frac{2d}{\sqrt{\sqrt{a^2x^2+b^2}+ax}}+\frac{2d\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{b^2c\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}{12a^4}+c\left(\sqrt{a\right.\ldots$$

Rubi [A] time = 0.73, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6742, 2119, 453, 329, 298, 203, 206, 448}

$$\frac{2d}{\sqrt{\sqrt{a^2x^2+b^2}+ax}}+\frac{2d\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{b^2c\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}{12a^4}+\frac{c\left(\sqrt{a^2x^2+b^2}+ax\right)^{7/2}}{56a^4}+\frac{b^8c}{72a^4\left(\sqrt{a^2x^2+b^2}+ax\right)^{9/2}}-\frac{b^6c}{20a^4\left(\sqrt{a^2x^2+b^2}+ax\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*x^4)/(x*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

```
[Out] (b^8*c)/(72*a^4*(a*x + Sqrt[b^2 + a^2*x^2])^(9/2)) - (b^6*c)/(20*a^4*(a*x +
 Sqrt[b^2 + a^2*x^2])^(5/2)) + (2*d)/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]] - (b^2
*c*(a*x + Sqrt[b^2 + a^2*x^2])^(3/2))/(12*a^4) + (c*(a*x + Sqrt[b^2 + a^2*x
^2])^(7/2))/(56*a^4) + (2*d*ArcTan[Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]/Sqrt[b]]
)/Sqrt[b] - (2*d*ArcTanh[Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]/Sqrt[b]])/Sqrt[b]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeqQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{d+cx^4}{x\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx = \int \left(\frac{d}{x\sqrt{ax+\sqrt{b^2+a^2x^2}}}+\frac{cx^3}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}\right)dx$$

$$= c\int \frac{x^3}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx + d\int \frac{1}{x\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$$

$$= \frac{c\,\text{Subst}\left(\int \frac{\left(-b^2+x^2\right)^3\left(b^2+x^2\right)}{x^{11/2}}\,dx, x, ax+\sqrt{b^2+a^2x^2}\right)}{16a^4} + d\,\text{Subst}\left(\int \frac{b^2+x^2}{x^{3/2}\left(-b^2+x^2\right)}\,d\right.$$

$$= \frac{2d}{\sqrt{ax+\sqrt{b^2+a^2x^2}}} + \frac{c\,\text{Subst}\left(\int\left(-\frac{b^8}{x^{11/2}}+\frac{2b^6}{x^{7/2}}-2b^2\sqrt{x}+x^{5/2}\right)dx, x, ax+\sqrt{b^2+\right.}{16a^4}$$

$$= \frac{b^8c}{72a^4\left(ax+\sqrt{b^2+a^2x^2}\right)^{9/2}} - \frac{b^6c}{20a^4\left(ax+\sqrt{b^2+a^2x^2}\right)^{5/2}} + \frac{2d}{\sqrt{ax+\sqrt{b^2+a^2x^2}}} -$$

$$= \frac{b^8c}{72a^4\left(ax+\sqrt{b^2+a^2x^2}\right)^{9/2}} - \frac{b^6c}{20a^4\left(ax+\sqrt{b^2+a^2x^2}\right)^{5/2}} + \frac{2d}{\sqrt{ax+\sqrt{b^2+a^2x^2}}} -$$

$$= \frac{b^8c}{72a^4\left(ax+\sqrt{b^2+a^2x^2}\right)^{9/2}} - \frac{b^6c}{20a^4\left(ax+\sqrt{b^2+a^2x^2}\right)^{5/2}} + \frac{2d}{\sqrt{ax+\sqrt{b^2+a^2x^2}}} -$$

Mathematica [C] time = 8.04, size = 915, normalized size = 3.88

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*x^4)/(x*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

```
[Out] (2*(-1155*a^4*d*(2*a*x - Sqrt[b^2 + a^2*x^2])*(a*x + Sqrt[b^2 + a^2*x^2]) +
 (c*Sqrt[b^2 + a^2*x^2]*(128*b^10 - 5040*a^9*x^9*(a*x + Sqrt[b^2 + a^2*x^2]
) - 15*a^5*b^4*x^5*(97*a*x + Sqrt[b^2 + a^2*x^2]) + 144*a*b^8*x*(9*a*x + 4*
Sqrt[b^2 + a^2*x^2]) + 195*a^3*b^6*x^3*(9*a*x + 8*Sqrt[b^2 + a^2*x^2]) - 18
0*a^7*b^2*x^7*(37*a*x + 23*Sqrt[b^2 + a^2*x^2])))/((a*x + Sqrt[b^2 + a^2*x^
2])^3*(b^2 + a*x*(a*x + Sqrt[b^2 + a^2*x^2]))) + (c*Sqrt[b^2 + a^2*x^2]*(b^
2 + 2*a*x*(a*x + Sqrt[b^2 + a^2*x^2]))^4*(-304*b^10 + 5040*a^9*x^9*(a*x + S
qrt[b^2 + a^2*x^2]) - 342*a*b^8*x*(9*a*x + 4*Sqrt[b^2 + a^2*x^2]) + 180*a^7
*b^2*x^7*(59*a*x + 45*Sqrt[b^2 + a^2*x^2]) + 15*a^5*b^4*x^5*(317*a*x + 89*S
qrt[b^2 + a^2*x^2]) - 15*a^3*b^6*x^3*(249*a*x + 247*Sqrt[b^2 + a^2*x^2])))/
(12*a*b^12*x + 292*a^3*b^10*x^3 + 2072*a^5*b^8*x^5 + 6400*a^7*b^6*x^7 + 972
8*a^9*b^4*x^9 + 7168*a^11*b^2*x^11 + 2048*a^13*x^13 + b^12*Sqrt[b^2 + a^2*x
^2] + 72*a^2*b^10*x^2*Sqrt[b^2 + a^2*x^2] + 840*a^4*b^8*x^4*Sqrt[b^2 + a^2*
x^2] + 3584*a^6*b^6*x^6*Sqrt[b^2 + a^2*x^2] + 6912*a^8*b^4*x^8*Sqrt[b^2 + a
^2*x^2] + 6144*a^10*b^2*x^10*Sqrt[b^2 + a^2*x^2] + 2048*a^12*x^12*Sqrt[b^2
+ a^2*x^2]) - (1155*a^4*d*(b^2 + a^2*x^2)*(a*x + Sqrt[b^2 + a^2*x^2])^2*(-2
*b^2 - a*x*(a*x + Sqrt[b^2 + a^2*x^2]) + 2*(b^2 + 2*a*x*(a*x + Sqrt[b^2 + a
^2*x^2]))*Hypergeometric2F1[3/4, 1, 7/4, (a*x + Sqrt[b^2 + a^2*x^2])^2/b^2]
))/(b^2 + a*x*(a*x + Sqrt[b^2 + a^2*x^2]))^2))/(3465*a^4*b^2*Sqrt[a*x + Sqr
t[b^2 + a^2*x^2]])
```

IntegrateAlgebraic [A] time = 0.47, size = 236, normalized size = 1.00

$$\frac{2d}{\sqrt{\sqrt{a^2x^2+b^2}+ax}}+\frac{2d\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{b^2c\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}{12a^4}+\frac{c\left(\sqrt{a^2x^2+b^2}+ax\right)^{7/2}}{56a^4}+\frac{b^8c}{72a^4\left(\sqrt{a^2x^2+b^2}+ax\right)^{9/2}}-\frac{b^6c}{20a^4\left(\sqrt{a^2x^2+b^2}+ax\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + c*x^4)/(x*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

```
[Out] (b^8*c)/(72*a^4*(a*x + Sqrt[b^2 + a^2*x^2])^(9/2)) - (b^6*c)/(20*a^4*(a*x +
 Sqrt[b^2 + a^2*x^2])^(5/2)) + (2*d)/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]] - (b^2
*c*(a*x + Sqrt[b^2 + a^2*x^2])^(3/2))/(12*a^4) + (c*(a*x + Sqrt[b^2 + a^2*x
^2])^(7/2))/(56*a^4) + (2*d*ArcTan[Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]/Sqrt[b]]
)/Sqrt[b] - (2*d*ArcTanh[Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]/Sqrt[b]])/Sqrt[b]
```

fricas [A] time = 0.60, size = 482, normalized size = 2.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+d)/x/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/315*(630*a^4*b^(3/2)*d*arctan(sqrt(a*x + sqrt(a^2*x^2 + b^2))/sqrt(b)) +
 315*a^4*b^(3/2)*d*log((b^2 + sqrt(a*x + sqrt(a^2*x^2 + b^2))*((a*x - b)*sq
rt(b) - sqrt(a^2*x^2 + b^2)*sqrt(b)) + sqrt(a^2*x^2 + b^2)*b)/x) - 2*(35*a^
5*c*x^5 + a^3*b^2*c*x^3 - (8*a*b^4*c - 315*a^5*d)*x - (35*a^4*c*x^4 + 6*a^2
*b^2*c*x^2 - 16*b^4*c + 315*a^4*d)*sqrt(a^2*x^2 + b^2))*sqrt(a*x + sqrt(a^2
*x^2 + b^2)))/(a^4*b^2), 1/315*(630*a^4*sqrt(-b)*b*d*arctan(sqrt(a*x + sqrt
(a^2*x^2 + b^2))*sqrt(-b)/b) - 315*a^4*sqrt(-b)*b*d*log(-(b^2 + sqrt(a*x +
sqrt(a^2*x^2 + b^2))*((a*x + b)*sqrt(-b) - sqrt(a^2*x^2 + b^2)*sqrt(-b)) -
```

```
sqrt(a^2*x^2 + b^2)*b)/x) - 2*(35*a^5*c*x^5 + a^3*b^2*c*x^3 - (8*a*b^4*c -
315*a^5*d)*x - (35*a^4*c*x^4 + 6*a^2*b^2*c*x^2 - 16*b^4*c + 315*a^4*d)*sqrt
(a^2*x^2 + b^2))*sqrt(a*x + sqrt(a^2*x^2 + b^2)))/(a^4*b^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^4 + d}{\sqrt{ax + \sqrt{a^2x^2 + b^2}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+d)/x/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate((c*x^4 + d)/(sqrt(a*x + sqrt(a^2*x^2 + b^2))*x), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{c\,x^4 + d}{x\sqrt{ax + \sqrt{a^2x^2 + b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+d)/x/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
[Out] int((c*x^4+d)/x/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^4 + d}{\sqrt{ax + \sqrt{a^2x^2 + b^2}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+d)/x/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxima"
)
[Out] integrate((c*x^4 + d)/(sqrt(a*x + sqrt(a^2*x^2 + b^2))*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,x^4 + d}{x\,\sqrt{a\,x + \sqrt{a^2\,x^2 + b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + c*x^4)/(x*(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2)),x)
[Out] int((d + c*x^4)/(x*(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^4 + d}{x\sqrt{ax + \sqrt{a^2x^2 + b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+d)/x/(a*x+(a**2*x**2+b**2)**(1/2))**(1/2),x)
[Out] Integral((c*x**4 + d)/(x*sqrt(a*x + sqrt(a**2*x**2 + b**2))), x)
```

3.2125 $\int \frac{\sqrt{1+x^2}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=236

$$\frac{\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\left(3072x^3+4096x^2+1814x+1712\right)+\sqrt{x^2+1}\left(\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2}}}\right.}{}$$

Rubi [F] time = 0.47, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+x^2}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/Sqrt[x + Sqrt[1 + x^2
]],x]
```

```
[Out] Defer[Int][(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/Sqrt[x + Sqrt[
1 + x^2]], x]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{x+\sqrt{1+x^2}}}\,dx=\int \frac{\sqrt{1+x^2}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Mathematica [F] time = 4.51, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+x^2}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}}{\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/Sqrt[x + Sqrt[1
 + x^2]],x]
```

```
[Out] Integrate[(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/Sqrt[x + Sqrt[1
 + x^2]], x]
```

IntegrateAlgebraic [A] time = 0.35, size = 236, normalized size = 1.00

$$\frac{\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\left(3072x^3+4096x^2+1814x+1712\right)+\sqrt{x^2+1}\left(\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\left(3072x^2+4096x+278\right)+\left(30720x^3-4096x^2-36930x-632\right)\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)+\left(30720x^4-4096x^3-21570x^2-2680x-24993\right)\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}}{26880\left(\sqrt{x^2+1}+x\right)^{5/2}}-\frac{263}{256}\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]])/Sqrt[x
 + Sqrt[1 + x^2]],x]
```

```
[Out] ((-24993 - 2680*x - 21570*x^2 - 4096*x^3 + 30720*x^4)*Sqrt[1 + Sqrt[x + Sqr
t[1 + x^2]]] + (1712 + 1814*x + 4096*x^2 + 3072*x^3)*Sqrt[x + Sqrt[1 + x^2]
]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] + Sqrt[1 + x^2]*((-632 - 36930*x - 4096
*x^2 + 30720*x^3)*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] + (278 + 4096*x + 3072*
x^2)*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]))/(26880*(x
+ Sqrt[1 + x^2])^(5/2)) - (263*ArcTanh[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]])/
256
```

fricas [A] time = 0.47, size = 129, normalized size = 0.55

$$-\frac{1}{26880}\left(672\,x^2-2\,\sqrt{x^2+1}\,(336\,x+139)-\left(10752\,x^3+784\,x^2-\left(10752\,x^2+784\,x+24993\right)\sqrt{x^2+1}+38049\,x-632\right)\sqrt{x+\sqrt{x^2+1}}-1258\,x-1712\right)\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}-\frac{263}{512}\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}+1\right)+\frac{263}{512}\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x+(x^2+1)^(1/2))
^(1/2),x, algorithm="fricas")
```

```
[Out] -1/26880*(672*x^2 - 2*sqrt(x^2 + 1)*(336*x + 139) - (10752*x^3 + 784*x^2 -
(10752*x^2 + 784*x + 24993)*sqrt(x^2 + 1) + 38049*x - 632)*sqrt(x + sqrt(x^
2 + 1)) - 1258*x - 1712)*sqrt(sqrt(x + sqrt(x^2 + 1)) + 1) - 263/512*log(sq
rt(sqrt(x + sqrt(x^2 + 1)) + 1) + 1) + 263/512*log(sqrt(sqrt(x + sqrt(x^2 +
 1)) + 1) - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x+(x^2+1)^(1/2))
^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}\,\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x+(x^2+1)^(1/2))^(1/2)
,x)
```

```
[Out] int((x^2+1)^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x+(x^2+1)^(1/2))^(1/2)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}\,\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)/(x+(x^2+1)^(1/2))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 1)*sqrt(sqrt(x + sqrt(x^2 + 1)) + 1)/sqrt(x + sqrt(x^2
 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}\,\sqrt{x^2+1}}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2)*(x^2 + 1)^(1/2))/(x + (x^2 + 1
)^(1/2))^(1/2),x)
```

```
[Out] int((((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2)*(x^2 + 1)^(1/2))/(x + (x^2 + 1
)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}\,\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}}{\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)*(1+(x+(x**2+1)**(1/2))**(1/2))**(1/2)/(x+(x**2+1)
**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)*sqrt(sqrt(x + sqrt(x**2 + 1)) + 1)/sqrt(x + sqrt(x*
*2 + 1)), x)
```

3.2126 $\int \sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=236

$$\frac{\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\left(2240x^3+1536x^2+40688x-1542\right)+\sqrt{x^2+1}\left(\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+}}}\right.}{}$$

Rubi [F] time = 0.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + x^2]*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]
,x]
```

```
[Out] Defer[Int][Sqrt[1 + x^2]*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 +
 x^2]]], x]
```

Rubi steps

$$\int \sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx = \int \sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Mathematica [F] time = 2.74, size = 0, normalized size = 0.00

$$\int \sqrt{1+x^2}\sqrt{x+\sqrt{1+x^2}}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 + x^2]*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 +
x^2]]],x]
```

```
[Out] Integrate[Sqrt[1 + x^2]*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 +
x^2]]], x]
```

IntegrateAlgebraic [A] time = 0.32, size = 236, normalized size = 1.00

$$\frac{\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\left(2240x^3+1536x^2+40688x-1542\right)+\sqrt{x^2+1}\left(\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\left(2240x^2+1536x+39568\right)+\left(40320x^3-2560x^2+92032x+2825\right)\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)+\left(40320x^4-2560x^3+112192x^2+1545x+31736\right)\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}}{55440\left(\sqrt{x^2+1}+x\right)^{3/2}}-\frac{1}{16}\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^2]*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x +
Sqrt[1 + x^2]]],x]
```

```
[Out] ((31736 + 1545*x + 112192*x^2 - 2560*x^3 + 40320*x^4)*Sqrt[1 + Sqrt[x + Sqr
t[1 + x^2]]] + (-1542 + 40688*x + 1536*x^2 + 2240*x^3)*Sqrt[x + Sqrt[1 + x^
2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] + Sqrt[1 + x^2]*((2825 + 92032*x - 25
60*x^2 + 40320*x^3)*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] + (39568 + 1536*x + 2
```

```
240*x^2)*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]))/(55440
*(x + Sqrt[1 + x^2])^(3/2)) - ArcTanh[Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]]/16
```

fricas [A] time = 0.40, size = 119, normalized size = 0.50

$$\frac{1}{55440}\left(1120\,x^2+2\,\sqrt{x^2+1}\,(560\,x-771)-\left(8400\,x^2-5\,\sqrt{x^2+1}\,(5712\,x+565)+4105\,x-31736\right)\sqrt{x+\sqrt{x^2+1}}+3078\,x+39568\right)\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}-\frac{1}{32}\,\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}+1\right)+\frac{1}{32}\,\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)*(x+(x^2+1)^(1/2))^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))
^(1/2),x, algorithm="fricas")
```

```
[Out] 1/55440*(1120*x^2 + 2*sqrt(x^2 + 1)*(560*x - 771) - (8400*x^2 - 5*sqrt(x^2
+ 1)*(5712*x + 565) + 4105*x - 31736)*sqrt(x + sqrt(x^2 + 1)) + 3078*x + 39
568)*sqrt(sqrt(x + sqrt(x^2 + 1)) + 1) - 1/32*log(sqrt(sqrt(x + sqrt(x^2 +
1)) + 1) + 1) + 1/32*log(sqrt(sqrt(x + sqrt(x^2 + 1)) + 1) - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)*(x+(x^2+1)^(1/2))^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))
^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+1}\,\sqrt{x+\sqrt{x^2+1}}\,\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)*(x+(x^2+1)^(1/2))^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)
,x)
```

```
[Out] int((x^2+1)^(1/2)*(x+(x^2+1)^(1/2))^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+1}\,\sqrt{x+\sqrt{x^2+1}}\,\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)*(x+(x^2+1)^(1/2))^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 1)*sqrt(x + sqrt(x^2 + 1))*sqrt(sqrt(x + sqrt(x^2 + 1)
) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\sqrt{x+\sqrt{x^2+1}}+1}\,\sqrt{x^2+1}\,\sqrt{x+\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2)*(x^2 + 1)^(1/2)*(x + (x^2 + 1)^
(1/2))^(1/2),x)
```

```
[Out] int(((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2)*(x^2 + 1)^(1/2)*(x + (x^2 + 1)^
(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x + \sqrt{x^2 + 1}}\,\sqrt{x^2 + 1}\,\sqrt{\sqrt{x + \sqrt{x^2 + 1}} + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)*(x+(x**2+1)**(1/2))**(1/2)*(1+(x+(x**2+1)**(1/2))
**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x + sqrt(x**2 + 1))*sqrt(x**2 + 1)*sqrt(sqrt(x + sqrt(x**2 +
1)) + 1), x)
```

3.2127 $\int \frac{\sqrt[4]{-x^3+x^4}}{x(-b+ax)}\,dx$

Optimal. Leaf size=237

$$-\frac{\sqrt{2}\sqrt[4]{a-b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x\sqrt[4]{x^4-x^3}\sqrt[4]{a-b}}{x^2\sqrt{a-b}-\sqrt{b}\sqrt{x^4-x^3}}\right)}{a\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt[4]{a-b}\tanh^{-1}\left(\frac{\frac{x^2\sqrt[4]{a-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}\sqrt{x^4-x^3}}{\sqrt{2}\sqrt[4]{a-b}}}{x\sqrt[4]{x^4-x^3}}\right)}{a\sqrt[4]{b}}-\frac{2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)}{a}+\frac{2\tanh^{-1}\left(\right.}{a}$$

Rubi [A] time = 0.25, antiderivative size = 233, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {2042, 105, 63, 240, 212, 206, 203, 93, 298, 205, 208}

$$\frac{2\sqrt[4]{x^4-x^3}\sqrt[4]{b-a}\tan^{-1}\left(\frac{\sqrt[4]{x}\sqrt[4]{b-a}}{\sqrt[4]{b}\sqrt[4]{x-1}}\right)}{a\sqrt[4]{b}\sqrt[4]{x-1}\,x^{3/4}}-\frac{2\sqrt[4]{x^4-x^3}\sqrt[4]{b-a}\tanh^{-1}\left(\frac{\sqrt[4]{x}\sqrt[4]{b-a}}{\sqrt[4]{b}\sqrt[4]{x-1}}\right)}{a\sqrt[4]{b}\sqrt[4]{x-1}\,x^{3/4}}+\frac{2\sqrt[4]{x^4-x^3}\tan^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{a\sqrt[4]{x-1}\,x^{3/4}}+\frac{2\sqrt[4]{x^4-x^3}\tanh^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{a\sqrt[4]{x-1}\,x^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + x^4)^(1/4)/(x*(-b + a*x)),x]
```

```
[Out] (2*(-x^3 + x^4)^(1/4)*ArcTan[(-1 + x)^(1/4)/x^(1/4)])/(a*(-1 + x)^(1/4)*x^(
3/4)) + (2*(-a + b)^(1/4)*(-x^3 + x^4)^(1/4)*ArcTan[((-a + b)^(1/4)*x^(1/4)
)/(b^(1/4)*(-1 + x)^(1/4))])/(a*b^(1/4)*(-1 + x)^(1/4)*x^(3/4)) + (2*(-x^3
+ x^4)^(1/4)*ArcTanh[(-1 + x)^(1/4)/x^(1/4)])/(a*(-1 + x)^(1/4)*x^(3/4)) -
(2*(-a + b)^(1/4)*(-x^3 + x^4)^(1/4)*ArcTanh[((-a + b)^(1/4)*x^(1/4))/(b^(1
/4)*(-1 + x)^(1/4))])/(a*b^(1/4)*(-1 + x)^(1/4)*x^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{-x^3+x^4}}{x(-b+ax)}\,dx &= \frac{\sqrt[4]{-x^3+x^4}\int \frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}\,(-b+ax)}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}} \\
&= \frac{\sqrt[4]{-x^3+x^4}\int \frac{1}{(-1+x)^{3/4}\sqrt[4]{x}}\,dx}{a\sqrt[4]{-1+x}\,x^{3/4}} - \frac{\left((a-b)\sqrt[4]{-x^3+x^4}\right)\int \frac{1}{(-1+x)^{3/4}\sqrt[4]{x}\,(-b+ax)}\,dx}{a\sqrt[4]{-1+x}\,x^{3/4}} \\
&= \frac{\left(4\sqrt[4]{-x^3+x^4}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^4}}\,dx, x, \sqrt[4]{-1+x}\right)}{a\sqrt[4]{-1+x}\,x^{3/4}} - \frac{\left(4(a-b)\sqrt[4]{-x^3+x^4}\right)\mathrm{Subst}\left(\int \frac{x^2}{-b-(a-b)\cdots}\right.}{a\sqrt[4]{-1+x}\,x^{3/4}} \\
&= \frac{\left(4\sqrt[4]{-x^3+x^4}\right)\mathrm{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{a\sqrt[4]{-1+x}\,x^{3/4}} + \frac{\left(2(a-b)\sqrt[4]{-x^3+x^4}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{-a+b}\,x^2}\cdots\right.}{a\sqrt{-a+b}\,\sqrt[4]{-1+x}\,x^{3/4}} \\
&= \frac{2\sqrt[4]{-a+b}\,\sqrt[4]{-x^3+x^4}\,\tan^{-1}\left(\frac{\sqrt[4]{-a+b}\,\sqrt[4]{x}}{\sqrt[4]{b}\,\sqrt[4]{-1+x}}\right)}{a\sqrt[4]{b}\,\sqrt[4]{-1+x}\,x^{3/4}} - \frac{2\sqrt[4]{-a+b}\,\sqrt[4]{-x^3+x^4}\,\tanh^{-1}\left(\frac{\sqrt[4]{-a+b}\,\sqrt[4]{x}}{\sqrt[4]{b}\,\sqrt[4]{-1+x}}\right)}{a\sqrt[4]{b}\,\sqrt[4]{-1+x}\,x^{3/4}} + \cdots \\
&= \frac{2\sqrt[4]{-x^3+x^4}\,\tan^{-1}\left(\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{a\sqrt[4]{-1+x}\,x^{3/4}} + \frac{2\sqrt[4]{-a+b}\,\sqrt[4]{-x^3+x^4}\,\tan^{-1}\left(\frac{\sqrt[4]{-a+b}\,\sqrt[4]{x}}{\sqrt[4]{b}\,\sqrt[4]{-1+x}}\right)}{a\sqrt[4]{b}\,\sqrt[4]{-1+x}\,x^{3/4}} + \frac{2\sqrt[4]{-x^3+x^4}\,\tan\cdots}{a\sqrt[4]{-1+\cdots}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 68, normalized size = 0.29

$$
\frac{4\sqrt[4]{(x-1)x^3}\left(\sqrt[4]{x}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};1-x\right)-{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{b-bx}{ax-bx}\right)\right)}{ax}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + x^4)^(1/4)/(x*(-b + a*x)),x]
```

```
[Out] (4*((-1 + x)*x^3)^(1/4)*(x^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, 1 - x] -
Hypergeometric2F1[1/4, 1, 5/4, (b - b*x)/(a*x - b*x)]))/(a*x)
```

IntegrateAlgebraic [A] time = 1.06, size = 237, normalized size = 1.00

$$
-\frac{\sqrt{2}\sqrt[4]{a-b}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x\sqrt[4]{x^4-x^3}\sqrt[4]{a-b}}{x^2\sqrt{a-b}-\sqrt{b}\sqrt{x^4-x^3}}\right)}{a\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt[4]{a-b}\,\tanh^{-1}\left(\frac{\frac{x^2\sqrt[4]{a-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}\sqrt{x^4-x^3}}{\sqrt{2}\sqrt[4]{a-b}}}{x\sqrt[4]{x^4-x^3}}\right)}{a\sqrt[4]{b}}-\frac{2\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)}{a}+\frac{2\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-x^3 + x^4)^(1/4)/(x*(-b + a*x)),x]
```

```
[Out] (-2*ArcTan[x/(-x^3 + x^4)^(1/4)])/a - (Sqrt[2]*(a - b)^(1/4)*ArcTan[(Sqrt[2
]*(a - b)^(1/4)*b^(1/4)*x*(-x^3 + x^4)^(1/4))/(Sqrt[a - b]*x^2 - Sqrt[b]*Sq
rt[-x^3 + x^4])])/(a*b^(1/4)) + (2*ArcTanh[x/(-x^3 + x^4)^(1/4)])/a - (Sqrt
[2]*(a - b)^(1/4)*ArcTanh[(((a - b)^(1/4)*x^2)/(Sqrt[2]*b^(1/4)) + (b^(1/4)
*Sqrt[-x^3 + x^4])/(Sqrt[2]*(a - b)^(1/4)))/(x*(-x^3 + x^4)^(1/4))])/(a*b^(
1/4))
```

fricas [A] time = 0.45, size = 297, normalized size = 1.25

$$
\frac{4\,a\left(-\frac{a-b}{a^4b}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^3bx\sqrt{\frac{a^2x^2\sqrt{-\frac{a-b}{a^4b}}+\sqrt{x^4-x^3}}{x^2}}\left(-\frac{a-b}{a^4b}\right)^{\frac{3}{4}}-\left(x^4-x^3\right)^{\frac{1}{4}}a^3b\left(-\frac{a-b}{a^4b}\right)^{\frac{3}{4}}}{(a-b)x}\right)-a\left(-\frac{a-b}{a^4b}\right)^{\frac{1}{4}}\log\left(\frac{ax\left(-\frac{a-b}{a^4b}\right)^{\frac{1}{4}}+\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+a\left(-\frac{a-b}{a^4b}\right)^{\frac{1}{4}}\log\left(-\frac{ax\left(-\frac{a-b}{a^4b}\right)^{\frac{1}{4}}-\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+2\arctan\left(\frac{\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+\log\left(\frac{x+\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)-\log\left(-\frac{x-\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x/(a*x-b),x, algorithm="fricas")
```

```
[Out] (4*a*(-(a - b)/(a^4*b))^(1/4)*arctan(-(a^3*b*x*sqrt((a^2*x^2*sqrt(-(a - b)/
(a^4*b)) + sqrt(x^4 - x^3))/x^2)*(-(a - b)/(a^4*b))^(3/4) - (x^4 - x^3)^(1/
4)*a^3*b*(-(a - b)/(a^4*b))^(3/4))/((a - b)*x)) - a*(-(a - b)/(a^4*b))^(1/4
)*log((a*x*(-(a - b)/(a^4*b))^(1/4) + (x^4 - x^3)^(1/4))/x) + a*(-(a - b)/(
a^4*b))^(1/4)*log(-(a*x*(-(a - b)/(a^4*b))^(1/4) - (x^4 - x^3)^(1/4))/x) +
2*arctan((x^4 - x^3)^(1/4)/x) + log((x + (x^4 - x^3)^(1/4))/x) - log(-(x -
(x^4 - x^3)^(1/4))/x))/a
```

giac [A] time = 3.00, size = 325, normalized size = 1.37

$$-\frac{2\arctan\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)}{a}-\frac{\log\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right)}{a}+\frac{\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)}{a}+\frac{\sqrt{2}\left(ab^{3}-b^{4}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a-b}{b}\right)^{\frac{1}{4}}+2\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)}{2\left(\frac{a-b}{b}\right)^{\frac{1}{4}}}\right)}{ab}+\frac{\sqrt{2}\left(ab^{3}-b^{4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a-b}{b}\right)^{\frac{1}{4}}-2\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)}{2\left(\frac{a-b}{b}\right)^{\frac{1}{4}}}\right)}{ab}+\frac{\sqrt{2}\left(ab^{3}-b^{4}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\left(\frac{a-b}{b}\right)^{\frac{1}{4}}\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+\sqrt{\frac{a-b}{b}}+\sqrt{-\frac{1}{x}+1}\right)}{2\,ab}-\frac{\sqrt{2}\left(ab^{3}-b^{4}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\left(\frac{a-b}{b}\right)^{\frac{1}{4}}\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+\sqrt{\frac{a-b}{b}}+\sqrt{-\frac{1}{x}+1}\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x/(a*x-b),x, algorithm="giac")
```

```
[Out] -2*arctan((-1/x + 1)^(1/4))/a - log((-1/x + 1)^(1/4) + 1)/a + log(abs((-1/x
 + 1)^(1/4) - 1))/a + sqrt(2)*(a*b^3 - b^4)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(
2)*((a - b)/b)^(1/4) + 2*(-1/x + 1)^(1/4))/((a - b)/b)^(1/4))/(a*b) + sqrt(
2)*(a*b^3 - b^4)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*((a - b)/b)^(1/4) - 2*(
-1/x + 1)^(1/4))/((a - b)/b)^(1/4))/(a*b) + 1/2*sqrt(2)*(a*b^3 - b^4)^(1/4)
*log(sqrt(2)*((a - b)/b)^(1/4)*(-1/x + 1)^(1/4) + sqrt((a - b)/b) + sqrt(-1
/x + 1))/(a*b) - 1/2*sqrt(2)*(a*b^3 - b^4)^(1/4)*log(-sqrt(2)*((a - b)/b)^(
1/4)*(-1/x + 1)^(1/4) + sqrt((a - b)/b) + sqrt(-1/x + 1))/(a*b)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}-x^{3}\right)^{\frac{1}{4}}}{x\left(ax-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3)^(1/4)/x/(a*x-b),x)
```

```
[Out] int((x^4-x^3)^(1/4)/x/(a*x-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}-x^{3}\right)^{\frac{1}{4}}}{(ax-b)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3)^(1/4)/x/(a*x-b),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^3)^(1/4)/((a*x - b)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(x^4-x^3\right)^{1/4}}{x\,\left(b-a\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - x^3)^(1/4)/(x*(b - a*x)),x)
```

```
[Out] int(-(x^4 - x^3)^(1/4)/(x*(b - a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3 (x-1)}}{x (ax-b)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3)**(1/4)/x/(a*x-b),x)
```

```
[Out] Integral((x**3*(x - 1))**(1/4)/(x*(a*x - b)), x)
```

3.2128

$$\int \frac{x^3(-2ab+(a+b)x)}{(x(-a+x)(-b+x))^{2/3}\left(-a^2b^2+2ab(a+b)x-\left(a^2+4ab+b^2\right)x^2+2(a+b)x^3+(-1+d)x^4\right)}\,dx$$

Optimal. Leaf size=237

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}x}{\sqrt[6]{d}x-2\sqrt[3]{x^2(-a-b)+abx+x^3}}\right)}{2d^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}x}{2\sqrt[3]{x^2(-a-b)+abx+x^3}+\sqrt[6]{d}x}\right)}{2d^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{d}x}{\sqrt[3]{x^2(-a-b)+abx+x^3}}\right)}{d^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{}}{(x^2(\cdot}\right.}{}$$

Rubi [F] time = 31.66, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3(-2ab+(a+b)x)}{(x(-a+x)(-b+x))^{2/3}\left(-a^2b^2+2ab(a+b)x-\left(a^2+4ab+b^2\right)x^2+2(a+b)x^3+(-1+d)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*(-2*a*b + (a + b)*x))/((x*(-a + x)*(-b + x))^(2/3)*(-(a^2*b^2) + 2
*a*b*(a + b)*x - (a^2 + 4*a*b + b^2)*x^2 + 2*(a + b)*x^3 + (-1 + d)*x^4)),x
]
```

```
[Out] (-3*(a + b)*((b*(a - x))/(a*(b - x)))^(2/3)*(b - x)*x*Hypergeometric2F1[1/3
, 2/3, 4/3, -(((a - b)*x)/(a*(b - x)))])/(b*(1 - d)*((a - x)*(b - x)*x)^(2/
3)) + (3*a^2*b^2*(a + b)*x^(2/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Subst]
[Defer[Int][1/((-a + x^3)^(2/3)*(-b + x^3)^(2/3)*(a^2*b^2 - 2*a^2*b*(1 + b/
a)*x^3 + a^2*(1 + (b*(4*a + b))/a^2)*x^6 - 2*a*(1 + b/a)*x^9 + (1 - d)*x^12
)), x], x, x^(1/3)])/((1 - d)*((a - x)*(b - x)*x)^(2/3)) - (6*a*b*(a + b)^2
*x^(2/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][x^3/((-a + x
^3)^(2/3)*(-b + x^3)^(2/3)*(a^2*b^2 - 2*a^2*b*(1 + b/a)*x^3 + a^2*(1 + (b*(
4*a + b))/a^2)*x^6 - 2*a*(1 + b/a)*x^9 + (1 - d)*x^12)), x], x, x^(1/3)])/(
(1 - d)*((a - x)*(b - x)*x)^(2/3)) + (3*(a + b)*(a^2 + 4*a*b + b^2)*x^(2/3)
*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][x^6/((-a + x^3)^(2/3
)*(-b + x^3)^(2/3)*(a^2*b^2 - 2*a^2*b*(1 + b/a)*x^3 + a^2*(1 + (b*(4*a + b)
)/a^2)*x^6 - 2*a*(1 + b/a)*x^9 + (1 - d)*x^12)), x], x, x^(1/3)])/((1 - d)*
((a - x)*(b - x)*x)^(2/3)) - (6*(a^2 + b^2 + a*b*(1 + d))*x^(2/3)*(-a + x)^
(2/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][x^9/((-a + x^3)^(2/3)*(-b + x^
3)^(2/3)*(a^2*b^2 - 2*a^2*b*(1 + b/a)*x^3 + a^2*(1 + (b*(4*a + b))/a^2)*x^6
 - 2*a*(1 + b/a)*x^9 + (1 - d)*x^12)), x], x, x^(1/3)])/((1 - d)*((a - x)*(
b - x)*x)^(2/3))
```

Rubi steps

$$\int \frac{x^3(-2ab+(a+b)x)}{(x(-a+x)(-b+x))^{2/3}\left(-a^2b^2+2ab(a+b)x-\left(a^2+4ab+b^2\right)x^2+2(a+b)x^3+(-1+d)x^4\right)}\,dx = \frac{\left(x^{2/3}(-a\right.}{}$$

$$= \frac{\left(3x^{2/3}(-\right.}{}$$

$$= \frac{\left(3x^{2/3}(-\right.}{}$$

$$= -\frac{\left(3(a+\right.}{}$$

$$= -\frac{3(a+b}{}$$

$$= -\frac{3(a+b}{}$$

Mathematica [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x^3(-2ab+(a+b)x)}{(x(-a+x)(-b+x))^{2/3}\left(-a^2b^2+2ab(a+b)x-\left(a^2+4ab+b^2\right)x^2+2(a+b)x^3+(-1+d)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(-2*a*b + (a + b)*x))/((x*(-a + x)*(-b + x))^(2/3)*(-(a^2*b^
2) + 2*a*b*(a + b)*x - (a^2 + 4*a*b + b^2)*x^2 + 2*(a + b)*x^3 + (-1 + d)*x
^4)),x]
```

```
[Out] Integrate[(x^3*(-2*a*b + (a + b)*x))/((x*(-a + x)*(-b + x))^(2/3)*(-(a^2*b^
2) + 2*a*b*(a + b)*x - (a^2 + 4*a*b + b^2)*x^2 + 2*(a + b)*x^3 + (-1 + d)*x
^4)), x]
```

IntegrateAlgebraic [A] time = 6.88, size = 237, normalized size = 1.00

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\,x}{\sqrt[6]{d}\,x-2\sqrt[3]{x^2(-a-b)+abx+x^3}}\right)}{2d^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\,x}{2\sqrt[3]{x^2(-a-b)+abx+x^3}+\sqrt[6]{d}\,x}\right)}{2d^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{d}\,x}{\sqrt[3]{x^2(-a-b)+abx+x^3}}\right)}{d^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{d}\,x\sqrt[3]{x^2(-a-b)+abx+x^3}}{\left(x^2(-a-b)+abx+x^3\right)^{2/3}+\sqrt[3]{d}\,x^2}\right)}{2d^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(-2*a*b + (a + b)*x))/((x*(-a + x)*(-b + x))^(2/3)*
(-(a^2*b^2) + 2*a*b*(a + b)*x - (a^2 + 4*a*b + b^2)*x^2 + 2*(a + b)*x^3 + (
-1 + d)*x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/6)*x)/(d^(1/6)*x - 2*(a*b*x + (-a - b)*x^2 +
x^3)^(1/3))])/(2*d^(5/6)) - (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/6)*x)/(d^(1/6)*x
+ 2*(a*b*x + (-a - b)*x^2 + x^3)^(1/3))])/(2*d^(5/6)) + ArcTanh[(d^(1/6)*x)
/(a*b*x + (-a - b)*x^2 + x^3)^(1/3)]/d^(5/6) + ArcTanh[(d^(1/6)*x*(a*b*x +
(-a - b)*x^2 + x^3)^(1/3))/(d^(1/3)*x^2 + (a*b*x + (-a - b)*x^2 + x^3)^(2/3
))]/(2*d^(5/6))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(2/3)/(-a^2*b^2+2*a*b*(a+b
)*x-(a^2+4*a*b+b^2)*x^2+2*(a+b)*x^3+(-1+d)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(2\,ab-(a+b)x)x^3}{\left((d-1)x^4-a^2b^2+2\,(a+b)abx+2\,(a+b)x^3-\left(a^2+4\,ab+b^2\right)x^2\right)((a-x)(b-x)x)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(2/3)/(-a^2*b^2+2*a*b*(a+b
)*x-(a^2+4*a*b+b^2)*x^2+2*(a+b)*x^3+(-1+d)*x^4),x, algorithm="giac")
```

```
[Out] integrate(-(2*a*b - (a + b)*x)*x^3/(((d - 1)*x^4 - a^2*b^2 + 2*(a + b)*a*b*
x + 2*(a + b)*x^3 - (a^2 + 4*a*b + b^2)*x^2)*((a - x)*(b - x)*x)^(2/3)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(-2ab+\left(a+b\right)x\right)}{\left(x\left(-a+x\right)\left(-b+x\right)\right)^{\frac{2}{3}}\left(-a^{2}b^{2}+2ab\left(a+b\right)x-\left(a^{2}+4ab+b^{2}\right)x^{2}+2\left(a+b\right)x^{3}+\left(-1+d\right)x^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(2/3)/(-a^2*b^2+2*a*b*(a+b)*x-(a
^2+4*a*b+b^2)*x^2+2*(a+b)*x^3+(-1+d)*x^4),x)
```

```
[Out] int(x^3*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(2/3)/(-a^2*b^2+2*a*b*(a+b)*x-(a
^2+4*a*b+b^2)*x^2+2*(a+b)*x^3+(-1+d)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(2\,ab-(a+b)x)x^3}{\left((d-1)x^4-a^2b^2+2\,(a+b)abx+2\,(a+b)x^3-\left(a^2+4\,ab+b^2\right)x^2\right)((a-x)(b-x)x)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(2/3)/(-a^2*b^2+2*a*b*(a+b
)*x-(a^2+4*a*b+b^2)*x^2+2*(a+b)*x^3+(-1+d)*x^4),x, algorithm="maxima")
```

```
[Out] -integrate((2*a*b - (a + b)*x)*x^3/(((d - 1)*x^4 - a^2*b^2 + 2*(a + b)*a*b*
x + 2*(a + b)*x^3 - (a^2 + 4*a*b + b^2)*x^2)*((a - x)*(b - x)*x)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^3\,\left(2\,a\,b-x\,\left(a+b\right)\right)}{{\left(x\,\left(a-x\right)\,\left(b-x\right)\right)}^{2/3}\,\left(2\,x^3\,\left(a+b\right)-x^2\,\left(a^2+4\,a\,b+b^2\right)-a^2\,b^2+x^4\,\left(d-1\right)+2\,a\,b\,x\,\left(a+b\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(2*a*b - x*(a + b)))/((x*(a - x)*(b - x))^(2/3)*(2*x^3*(a + b) -
x^2*(4*a*b + a^2 + b^2) - a^2*b^2 + x^4*(d - 1) + 2*a*b*x*(a + b))),x)
```

```
[Out] -int((x^3*(2*a*b - x*(a + b)))/((x*(a - x)*(b - x))^(2/3)*(2*x^3*(a + b) -
x^2*(4*a*b + a^2 + b^2) - a^2*b^2 + x^4*(d - 1) + 2*a*b*x*(a + b))), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))**(2/3)/(-a**2*b**2+2*a*b*
(a+b)*x-(a**2+4*a*b+b**2)*x**2+2*(a+b)*x**3+(-1+d)*x**4),x)
```

```
[Out] Timed out
```

3.2129 $\int \frac{(-4b+ax^3)\sqrt[3]{b+ax^3}}{x^5(-2b+ax^3)}\,dx$

Optimal. Leaf size=238

$$-\frac{a^{4/3}\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{ax^3+b}-3\sqrt[3]{a}\,x\right)}{2\sqrt[3]{2}\,3^{2/3}b}-\frac{a^{4/3}\tan^{-1}\left(\frac{3^{5/6}\sqrt[3]{a}\,x}{2\sqrt[3]{2}\sqrt[3]{ax^3+b}+\sqrt[3]{3}\sqrt[3]{a}\,x}\right)}{2\sqrt[3]{2}\sqrt[6]{3}\,b}+\frac{a^{4/3}\log\left(3a^{2/3}x^2+\sqrt[3]{2}\,3^{2/3}\sqrt[3]{a}\,x\sqrt[3]{ax^3+b}\right.}{4\sqrt[3]{2}\,3^{2/3}b}$$

Rubi [A] time = 0.35, antiderivative size = 226, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {580, 583, 12, 494, 292, 31, 634, 617, 204, 628}

$$-\frac{a^{4/3}\log\left(\sqrt[3]{2}-\frac{\sqrt[3]{3}\sqrt[3]{a}\,x}{\sqrt[3]{ax^3+b}}\right)}{2\sqrt[3]{2}\,3^{2/3}b}-\frac{a^{4/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a}\,x}{\sqrt[6]{3}\sqrt[3]{ax^3+b}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt[6]{3}\,b}+\frac{a^{4/3}\log\left(\frac{3^{2/3}a^{2/3}x^2}{(ax^3+b)^{2/3}}+\frac{\sqrt[3]{6}\sqrt[3]{a}\,x}{\sqrt[3]{ax^3+b}}+2^{2/3}\right)}{4\sqrt[3]{2}\,3^{2/3}b}-\frac{a\sqrt[3]{ax^3+b}}{bx}-\frac{\sqrt[3]{ax^3+b}}{2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((-4*b + a*x^3)*(b + a*x^3)^(1/3))/(x^5*(-2*b + a*x^3)),x]
```

```
[Out] -1/2*(b + a*x^3)^(1/3)/x^4 - (a*(b + a*x^3)^(1/3))/(b*x) - (a^(4/3)*ArcTan[
1/Sqrt[3] + (2^(2/3)*a^(1/3)*x)/(3^(1/6)*(b + a*x^3)^(1/3))])/(2*2^(1/3)*3^
(1/6)*b) - (a^(4/3)*Log[2^(1/3) - (3^(1/3)*a^(1/3)*x)/(b + a*x^3)^(1/3)])/(
2*2^(1/3)*3^(2/3)*b) + (a^(4/3)*Log[2^(2/3) + (3^(2/3)*a^(2/3)*x^2)/(b + a*
x^3)^(2/3) + (6^(1/3)*a^(1/3)*x)/(b + a*x^3)^(1/3)])/(4*2^(1/3)*3^(2/3)*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(-4b+ax^3\right)\sqrt[3]{b+ax^3}}{x^5\left(-2b+ax^3\right)}\,dx = -\frac{\sqrt[3]{b+ax^3}}{2x^4} - \frac{\int \frac{16ab^2+4a^2bx^3}{x^2\left(-2b+ax^3\right)\left(b+ax^3\right)^{2/3}}\,dx}{8b}$$

$$= -\frac{\sqrt[3]{b+ax^3}}{2x^4} - \frac{a\sqrt[3]{b+ax^3}}{bx} - \frac{\int \frac{24a^2b^3x}{\left(-2b+ax^3\right)\left(b+ax^3\right)^{2/3}}\,dx}{16b^3}$$

$$= -\frac{\sqrt[3]{b+ax^3}}{2x^4} - \frac{a\sqrt[3]{b+ax^3}}{bx} - \frac{1}{2}\left(3a^2\right)\int \frac{x}{\left(-2b+ax^3\right)\left(b+ax^3\right)^{2/3}}\,dx$$

$$= -\frac{\sqrt[3]{b+ax^3}}{2x^4} - \frac{a\sqrt[3]{b+ax^3}}{bx} - \frac{1}{2}\left(3a^2\right)\text{Subst}\left(\int \frac{x}{-2b+3abx^3}\,dx, x, \frac{x}{\sqrt[3]{b+ax^3}}\right)$$

$$= -\frac{\sqrt[3]{b+ax^3}}{2x^4} - \frac{a\sqrt[3]{b+ax^3}}{bx} - \frac{a^{5/3}\,\text{Subst}\left(\int \frac{1}{-\sqrt[3]{2}\sqrt[3]{b}+\sqrt[3]{3}\sqrt[3]{a}\sqrt[3]{b}x}\,dx, x, \frac{x}{\sqrt[3]{b+ax^3}}\right)}{2\sqrt[3]{6}\,b^{2/3}} + \frac{a^{5/3}}{\cdots}$$

$$= -\frac{\sqrt[3]{b+ax^3}}{2x^4} - \frac{a\sqrt[3]{b+ax^3}}{bx} - \frac{a^{4/3}\log\left(\sqrt[3]{2}-\frac{\sqrt[3]{3}\sqrt[3]{a}x}{\sqrt[3]{b+ax^3}}\right)}{2\sqrt[3]{2}\,3^{2/3}b} + \frac{a^{4/3}\,\text{Subst}\left(\int \frac{\sqrt[3]{6}\sqrt[3]{a}\,b^{2/3}\cdots}{2^{2/3}b^{2/3}+\sqrt[3]{6}\sqrt[3]{\cdots}}\cdots\right)}{4\sqrt[3]{\cdots}}$$

$$= -\frac{\sqrt[3]{b+ax^3}}{2x^4} - \frac{a\sqrt[3]{b+ax^3}}{bx} - \frac{a^{4/3}\log\left(\sqrt[3]{2}-\frac{\sqrt[3]{3}\sqrt[3]{a}x}{\sqrt[3]{b+ax^3}}\right)}{2\sqrt[3]{2}\,3^{2/3}b} + \frac{a^{4/3}\log\left(2^{2/3}+\frac{3^{2/3}a^{2/3}x^2}{\left(b+ax^3\right)^{2/3}}+\cdots\right)}{4\sqrt[3]{2}\,3^{2/3}b}$$

$$= -\frac{\sqrt[3]{b+ax^3}}{2x^4} - \frac{a\sqrt[3]{b+ax^3}}{bx} - \frac{a^{4/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{3}\sqrt[3]{a}x}{\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt[6]{3}\,b} - \frac{a^{4/3}\log\left(\sqrt[3]{2}-\frac{\sqrt[3]{3}\sqrt[3]{a}x}{\sqrt[3]{b+ax^3}}\right)}{2\sqrt[3]{2}\,3^{2/3}b} + \cdots$$

Mathematica [C] time = 0.24, size = 108, normalized size = 0.45

$$\frac{\frac{3a^2x^6\left(\frac{2ax^3}{b}+2\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{3ax^3}{ax^3-2b}\right)}{\left(2-\frac{ax^3}{b}\right)^{2/3}}-4\left(2a^2x^6+3abx^3+b^2\right)}{8bx^4\left(ax^3+b\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-4*b + a*x^3)*(b + a*x^3)^(1/3))/(x^5*(-2*b + a*x^3)),x]
```

```
[Out] (-4*(b^2 + 3*a*b*x^3 + 2*a^2*x^6) + (3*a^2*x^6*(2 + (2*a*x^3)/b)^(2/3)*Hype
rgeometric2F1[2/3, 2/3, 5/3, (3*a*x^3)/(-2*b + a*x^3)])/(2 - (a*x^3)/b)^(2/
3))/(8*b*x^4*(b + a*x^3)^(2/3))
```

IntegrateAlgebraic [A] time = 0.58, size = 238, normalized size = 1.00

$$-\frac{a^{4/3}\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{ax^3+b}-3\sqrt[3]{a}x\right)}{2\sqrt[3]{2}\,3^{2/3}b}-\frac{a^{4/3}\tan^{-1}\left(\frac{3^{5/6}\sqrt[3]{a}x}{2\sqrt[3]{2}\sqrt[3]{ax^3+b}+\sqrt[3]{3}\sqrt[3]{a}x}\right)}{2\sqrt[3]{2}\sqrt[6]{3}\,b}+\frac{a^{4/3}\log\left(3a^{2/3}x^2+\sqrt[3]{2}\,3^{2/3}\sqrt[3]{a}x\sqrt[3]{ax^3+b}+2^{2/3}\sqrt[3]{3}\left(ax^3+b\right)^{2/3}\right)}{4\sqrt[3]{2}\,3^{2/3}b}+\frac{\left(-2ax^3-b\right)\sqrt[3]{ax^3+b}}{2bx^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4*b + a*x^3)*(b + a*x^3)^(1/3))/(x^5*(-2*b + a*x^3)),
x]
```

```
[Out] ((-b - 2*a*x^3)*(b + a*x^3)^(1/3))/(2*b*x^4) - (a^(4/3)*ArcTan[(3^(5/6)*a^(
1/3)*x)/(3^(1/3)*a^(1/3)*x + 2*2^(1/3)*(b + a*x^3)^(1/3))])/(2*2^(1/3)*3^(1
```

```
/6)*b) - (a^(4/3)*Log[-3*a^(1/3)*x + 2^(1/3)*3^(2/3)*(b + a*x^3)^(1/3)])/(2
*2^(1/3)*3^(2/3)*b) + (a^(4/3)*Log[3*a^(2/3)*x^2 + 2^(1/3)*3^(2/3)*a^(1/3)*
x*(b + a*x^3)^(1/3) + 2^(2/3)*3^(1/3)*(b + a*x^3)^(2/3)])/(4*2^(1/3)*3^(2/3
)*b)
```

fricas [B] time = 21.86, size = 418, normalized size = 1.76

$$-\frac{2\cdot 18^{\frac{2}{3}}\sqrt{3}\left(-a\right)^{\frac{1}{3}}ax^{4}\arctan\left(\frac{4\cdot 18^{\frac{2}{3}}\sqrt{3}\left(4a^{2}x^{7}-7abx^{4}-2b^{2}x\right)\left(ax^{3}+b\right)^{\frac{2}{3}}\left(-a\right)^{\frac{1}{3}}+6\cdot 18^{\frac{1}{3}}\sqrt{3}\left(55a^{2}x^{8}+50abx^{5}+4b^{2}x^{2}\right)\left(ax^{3}+b\right)^{\frac{1}{3}}\left(-a\right)^{\frac{2}{3}}+\sqrt{3}\left(377a^{3}x^{9}+600a^{2}bx^{6}+204ab^{2}x^{3}+8b^{3}\right)}{3\left(487a^{3}x^{9}+480a^{2}bx^{6}+12ab^{2}x^{3}-8b^{3}\right)}\right)-2\cdot 18^{\frac{2}{3}}\left(-a\right)^{\frac{1}{3}}ax^{4}\log\left(-\frac{3\cdot 18^{\frac{2}{3}}\left(ax^{3}+b\right)^{\frac{1}{3}}\left(-a\right)^{\frac{1}{3}}ax^{2}+18\left(ax^{3}+b\right)^{\frac{2}{3}}ax+18^{\frac{1}{3}}\left(ax^{3}-2b\right)\left(-a\right)^{\frac{2}{3}}}{18\left(ax^{3}-2b\right)}\right)+18^{\frac{2}{3}}\left(-a\right)^{\frac{1}{3}}ax^{4}\log\left(\frac{36\cdot 18^{\frac{1}{3}}\left(4ax^{4}+bx\right)\left(ax^{3}+b\right)^{\frac{2}{3}}\left(-a\right)^{\frac{2}{3}}-18^{\frac{2}{3}}\left(55a^{2}x^{6}+50abx^{3}+4b^{2}\right)\left(-a\right)^{\frac{1}{3}}+54\left(7a^{2}x^{5}+4abx^{2}\right)\left(ax^{3}+b\right)^{\frac{1}{3}}}{18\left(a^{2}x^{6}-4abx^{3}+4b^{2}\right)}\right)+108\left(2ax^{3}+b\right)\left(ax^{3}+b\right)^{\frac{1}{3}}}{216bx^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-4*b)*(a*x^3+b)^(1/3)/x^5/(a*x^3-2*b),x, algorithm="fricas"
)
```

```
[Out] -1/216*(2*18^(2/3)*sqrt(3)*(-a)^(1/3)*a*x^4*arctan(1/3*(4*18^(2/3)*sqrt(3)*
(4*a^2*x^7 - 7*a*b*x^4 - 2*b^2*x)*(a*x^3 + b)^(2/3)*(-a)^(1/3) + 6*18^(1/3)
*sqrt(3)*(55*a^2*x^8 + 50*a*b*x^5 + 4*b^2*x^2)*(a*x^3 + b)^(1/3)*(-a)^(2/3)
 + sqrt(3)*(377*a^3*x^9 + 600*a^2*b*x^6 + 204*a*b^2*x^3 + 8*b^3))/(487*a^3*
x^9 + 480*a^2*b*x^6 + 12*a*b^2*x^3 - 8*b^3)) - 2*18^(2/3)*(-a)^(1/3)*a*x^4*
log(-1/18*(3*18^(2/3)*(a*x^3 + b)^(1/3)*(-a)^(1/3)*a*x^2 + 18*(a*x^3 + b)^(
2/3)*a*x + 18^(1/3)*(a*x^3 - 2*b)*(-a)^(2/3))/(a*x^3 - 2*b)) + 18^(2/3)*(-a
)^(1/3)*a*x^4*log(1/18*(36*18^(1/3)*(4*a*x^4 + b*x)*(a*x^3 + b)^(2/3)*(-a)^
(2/3) - 18^(2/3)*(55*a^2*x^6 + 50*a*b*x^3 + 4*b^2)*(-a)^(1/3) + 54*(7*a^2*x
^5 + 4*a*b*x^2)*(a*x^3 + b)^(1/3))/(a^2*x^6 - 4*a*b*x^3 + 4*b^2)) + 108*(2*
a*x^3 + b)*(a*x^3 + b)^(1/3))/(b*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{3}+b\right)^{\frac{1}{3}}\left(ax^{3}-4\,b\right)}{\left(ax^{3}-2\,b\right)x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-4*b)*(a*x^3+b)^(1/3)/x^5/(a*x^3-2*b),x, algorithm="giac")
```

```
[Out] integrate((a*x^3 + b)^(1/3)*(a*x^3 - 4*b)/((a*x^3 - 2*b)*x^5), x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{3}-4b\right)\left(a\,x^{3}+b\right)^{\frac{1}{3}}}{x^{5}\left(a\,x^{3}-2b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3-4*b)*(a*x^3+b)^(1/3)/x^5/(a*x^3-2*b),x)
```

```
[Out] int((a*x^3-4*b)*(a*x^3+b)^(1/3)/x^5/(a*x^3-2*b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{3}+b\right)^{\frac{1}{3}}\left(ax^{3}-4\,b\right)}{\left(ax^{3}-2\,b\right)x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3-4*b)*(a*x^3+b)^(1/3)/x^5/(a*x^3-2*b),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^3 + b)^(1/3)*(a*x^3 - 4*b)/((a*x^3 - 2*b)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x^3+b\right)^{1/3}\,\left(4\,b-a\,x^3\right)}{x^5\,\left(2\,b-a\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a*x^3)^(1/3)*(4*b - a*x^3))/(x^5*(2*b - a*x^3)),x)
[Out] int(((b + a*x^3)^(1/3)*(4*b - a*x^3))/(x^5*(2*b - a*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^3-4b\right)\sqrt[3]{ax^3+b}}{x^5\left(ax^3-2b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3-4*b)*(a*x**3+b)**(1/3)/x**5/(a*x**3-2*b),x)
[Out] Integral((a*x**3 - 4*b)*(a*x**3 + b)**(1/3)/(x**5*(a*x**3 - 2*b)), x)
```

3.2130 $\int \frac{-2a^2bx+a(3a+2b)x^2-4ax^3+x^4}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(-a^2d+2adx-(b+d)x^2+x^3\right)}\,dx$

Optimal. Leaf size=238

$$-\frac{\log\left(a^2d^{2/3}+\sqrt[3]{x^3(-a-b)+abx^2+x^4}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)+\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}-2ad^{2/3}x+d^{2/3}x^2\right)}{2d^{2/3}}+\cdots$$

Rubi [F] time = 13.76, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-2a^2bx+a(3a+2b)x^2-4ax^3+x^4}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(-a^2d+2adx-(b+d)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-2*a^2*b*x + a*(3*a + 2*b)*x^2 - 4*a*x^3 + x^4)/((x^2*(-a + x)*(-b + x
))^(2/3)*(-(a^2*d) + 2*a*d*x - (b + d)*x^2 + x^3)),x]
```

```
[Out] (9*a*x^(4/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x^4*(-a
 + x^3)^(1/3))/((-b + x^3)^(2/3)*(a^2*d - 2*a*d*x^3 + b*(1 + d/b)*x^6 - x^9
)), x], x, x^(1/3)])/((a - x)*(b - x)*x^2)^(2/3) + (6*a*b*x^(4/3)*(-a + x)^
(2/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x*(-a + x^3)^(1/3))/((-b + x^
3)^(2/3)*(-(a^2*d) + 2*a*d*x^3 - b*(1 + d/b)*x^6 + x^9)), x], x, x^(1/3)])/
((a - x)*(b - x)*x^2)^(2/3) + (3*x^(4/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defe
r[Subst][Defer[Int][(x^7*(-a + x^3)^(1/3))/((-b + x^3)^(2/3)*(-(a^2*d) + 2*
a*d*x^3 - b*(1 + d/b)*x^6 + x^9)), x], x, x^(1/3)])/((a - x)*(b - x)*x^2)^(
2/3)
```

Rubi steps

$$\int \frac{-2a^2bx+a(3a+2b)x^2-4ax^3+x^4}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(-a^2d+2adx-(b+d)x^2+x^3\right)}\,dx = \frac{\left(x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{-2a^2bx+a\cdots}{x^{4/3}(-a+x)^{2/3}(-b+x\cdots}}{\left(x^2(-a+x)(-b+x)\right)^{\cdots}}$$

$$= \frac{\left(x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{-2a^2b+(3\cdots}{\sqrt[3]{x}(-a+x)^{2/3}(-b+x\cdots}}{\left(x^2(-a+x)(-b+x)\right)^{\cdots}}$$

$$= \frac{\left(x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{\sqrt[3]{-a+x}\,(2a\cdots}{\sqrt[3]{x}(-b+x)^{2/3}(-a^2d\cdots}}{\left(x^2(-a+x)(-b+x)\right)^{2/3}}$$

$$= \frac{\left(3x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\text{Subst}\left(\int \frac{x\cdots}{(-b+x^3)\cdots}\right.}{\left(x^2(-a+x)(-b+\cdots\right.}$$

$$= \frac{\left(3x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\text{Subst}\left(\int\left(\frac{\cdots}{(-b+x^{\cdots}}\right.\right.}{\cdots}$$

$$= \frac{\left(3x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\text{Subst}\left(\int \frac{\cdots}{(-b+x^3)\cdots}\right.}{\left(x^2(-a+x)(-b\cdots\right.}$$

Mathematica [F] time = 4.16, size = 0, normalized size = 0.00

$$\int \frac{-2a^2bx+a(3a+2b)x^2-4ax^3+x^4}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(-a^2d+2adx-(b+d)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-2*a^2*b*x + a*(3*a + 2*b)*x^2 - 4*a*x^3 + x^4)/((x^2*(-a + x)*(
-b + x))^(2/3)*(-(a^2*d) + 2*a*d*x - (b + d)*x^2 + x^3)),x]
```

```
[Out] Integrate[(-2*a^2*b*x + a*(3*a + 2*b)*x^2 - 4*a*x^3 + x^4)/((x^2*(-a + x)*(
-b + x))^(2/3)*(-(a^2*d) + 2*a*d*x - (b + d)*x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 4.11, size = 238, normalized size = 1.00

$$-\frac{\log\left(a^2d^{2/3}+\sqrt[3]{x^3(-a-b)+abx^2+x^4}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)+\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}-2ad^{2/3}x+d^{2/3}x^2\right)}{2d^{2/3}}+\frac{\log\left(\sqrt[3]{x^3(-a-b)+abx^2+x^4}+a\sqrt[3]{d}-\sqrt[3]{d}\,x\right)}{d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,a\sqrt[3]{d}-\sqrt{3}\sqrt[3]{d}\,x}{-2\sqrt[3]{x^3(-a-b)+abx^2+x^4}+a\sqrt[3]{d}-\sqrt[3]{d}\,x}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*a^2*b*x + a*(3*a + 2*b)*x^2 - 4*a*x^3 + x^4)/((x^2*(
-a + x)*(-b + x))^(2/3)*(-(a^2*d) + 2*a*d*x - (b + d)*x^2 + x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*a*d^(1/3) - Sqrt[3]*d^(1/3)*x)/(a*d^(1/3) - d^(1/3
)*x - 2*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3))])/d^(2/3) + Log[a*d^(1/3) - d
^(1/3)*x + (a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3)]/d^(2/3) - Log[a^2*d^(2/3)
- 2*a*d^(2/3)*x + d^(2/3)*x^2 + (-(a*d^(1/3)) + d^(1/3)*x)*(a*b*x^2 + (-a -
 b)*x^3 + x^4)^(1/3) + (a*b*x^2 + (-a - b)*x^3 + x^4)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a^2*b*x+a*(3*a+2*b)*x^2-4*a*x^3+x^4)/(x^2*(-a+x)*(-b+x))^(2/3
)/(-a^2*d+2*a*d*x-(b+d)*x^2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,a^2bx-(3\,a+2\,b)ax^2+4\,ax^3-x^4}{\left((a-x)(b-x)x^2\right)^{\frac{2}{3}}\left(a^2d-2\,adx+(b+d)x^2-x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a^2*b*x+a*(3*a+2*b)*x^2-4*a*x^3+x^4)/(x^2*(-a+x)*(-b+x))^(2/3
)/(-a^2*d+2*a*d*x-(b+d)*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate((2*a^2*b*x - (3*a + 2*b)*a*x^2 + 4*a*x^3 - x^4)/(((a - x)*(b - x)
*x^2)^(2/3)*(a^2*d - 2*a*d*x + (b + d)*x^2 - x^3)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{-2a^2bx+a\,(3a+2b)\,x^2-4a\,x^3+x^4}{\left(x^2\,(-a+x)\,(-b+x)\right)^{\frac{2}{3}}\left(-a^2d+2adx-(b+d)\,x^2+x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*a^2*b*x+a*(3*a+2*b)*x^2-4*a*x^3+x^4)/(x^2*(-a+x)*(-b+x))^(2/3)/(-a^
2*d+2*a*d*x-(b+d)*x^2+x^3),x)
```

```
[Out] int((-2*a^2*b*x+a*(3*a+2*b)*x^2-4*a*x^3+x^4)/(x^2*(-a+x)*(-b+x))^(2/3)/(-a^
2*d+2*a*d*x-(b+d)*x^2+x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,a^2bx - (3\,a + 2\,b)ax^2 + 4\,ax^3 - x^4}{\left((a - x)(b - x)x^2\right)^{\frac{2}{3}}\left(a^2d - 2\,adx + (b + d)x^2 - x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a^2*b*x+a*(3*a+2*b)*x^2-4*a*x^3+x^4)/(x^2*(-a+x)*(-b+x))^(2/3
)/(-a^2*d+2*a*d*x-(b+d)*x^2+x^3),x, algorithm="maxima")
```

```
[Out] integrate((2*a^2*b*x - (3*a + 2*b)*a*x^2 + 4*a*x^3 - x^4)/(((a - x)*(b - x)
*x^2)^(2/3)*(a^2*d - 2*a*d*x + (b + d)*x^2 - x^3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{4\,a\,x^3 - x^4 - a\,x^2\,\left(3\,a + 2\,b\right) + 2\,a^2\,b\,x}{{\left(x^2\,\left(a - x\right)\,\left(b - x\right)\right)}^{2/3}\,\left(d\,a^2 - 2\,d\,a\,x - x^3 + \left(b + d\right)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*a*x^3 - x^4 - a*x^2*(3*a + 2*b) + 2*a^2*b*x)/((x^2*(a - x)*(b - x))^
(2/3)*(x^2*(b + d) + a^2*d - x^3 - 2*a*d*x)),x)
```

```
[Out] int((4*a*x^3 - x^4 - a*x^2*(3*a + 2*b) + 2*a^2*b*x)/((x^2*(a - x)*(b - x))^
(2/3)*(x^2*(b + d) + a^2*d - x^3 - 2*a*d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*a**2*b*x+a*(3*a+2*b)*x**2-4*a*x**3+x**4)/(x**2*(-a+x)*(-b+x))
**(2/3)/(-a**2*d+2*a*d*x-(b+d)*x**2+x**3),x)
```

```
[Out] Timed out
```

3.2131 $\int \frac{(-2+(1+k)x)\left(1-2(1+k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(a+k^2\right)x^4\right)}{x^4\sqrt[3]{(1-x)x(1-kx)}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$

Optimal. Leaf size=238

$$\frac{\left(a+b^2\right)\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}+\frac{\left(-\sqrt{3}\,a-\sqrt{3}\,b^2\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{\sqrt[3]{b}}+\frac{\left(-a-b^2\right)\log\left(b^{2/}\right.}{}$$

Rubi [F] time = 32.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+(1+k)x)\left(1-2(1+k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(a+k^2\right)x^4\right)}{x^4\sqrt[3]{(1-x)x(1-kx)}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + (1 + k)*x)*(1 - 2*(1 + k)*x + (1 + 4*k + k^2)*x^2 - 2*(k + k^2)*
x^3 + (a + k^2)*x^4))/(x^4*((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x + (-b
 + k)*x^2)),x]
```

```
[Out] (-3*(8*b*k^3 - 2*k^4 + 4*b^3*(1 + 3*k + k^2) + b^2*(1 - 12*k^2 + k^4) + a*(
1 + 2*b^2 + k^4 + 4*b*(1 + k + k^2)))*(1 - x)*(1 - k*x))/(10*(b - k)^4*x^3*
((1 - x)*x*(1 - k*x))^(1/3)) - (3*(1 + k)*(9*b*k^2 - 3*k^3 - a*(1 + 3*b - k
 + k^2) - b^2*(1 + 8*k + k^2))*(1 - x)*(1 - k*x))/(7*(b - k)^3*x^2*((1 - x)
*x*(1 - k*x))^(1/3)) - (12*(1 + k)*(8*b*k^3 - 2*k^4 + 4*b^3*(1 + 3*k + k^2)
 + b^2*(1 - 12*k^2 + k^4) + a*(1 + 2*b^2 + k^4 + 4*b*(1 + k + k^2)))*(1 - x
)*(1 - k*x))/(35*(b - k)^4*x^2*((1 - x)*x*(1 - k*x))^(1/3)) - (3*(a*(1 + 2*
b + k^2) + 2*b*k*(1 + 3*k + k^2) - k^2*(1 + 4*k + k^2))*(1 - x)*(1 - k*x))/
(4*(b - k)^2*x*((1 - x)*x*(1 - k*x))^(1/3)) - (15*(1 + k)^2*(9*b*k^2 - 3*k^
3 - a*(1 + 3*b - k + k^2) - b^2*(1 + 8*k + k^2))*(1 - x)*(1 - k*x))/(28*(b
- k)^3*x*((1 - x)*x*(1 - k*x))^(1/3)) - (3*(20 + 19*k + 20*k^2)*(8*b*k^3 -
2*k^4 + 4*b^3*(1 + 3*k + k^2) + b^2*(1 - 12*k^2 + k^4) + a*(1 + 2*b^2 + k^4
 + 4*b*(1 + k + k^2)))*(1 - x)*(1 - k*x))/(140*(b - k)^4*x*((1 - x)*x*(1 -
k*x))^(1/3)) + (3*(1 + k)*(a + k^2)*(1 - x)*(((1 - k)*x)/(1 - k*x))^(4/3)*(
1 - k*x)*Hypergeometric2F1[2/3, 4/3, 5/3, (1 - x)/(1 - k*x)])/(2*(1 - k)*(b
 - k)*x*((1 - x)*x*(1 - k*x))^(1/3)) - (3*(1 + k)*(5 + 4*k + 5*k^2)*(9*b*k^
2 - 3*k^3 - a*(1 + 3*b - k + k^2) - b^2*(1 + 8*k + k^2))*(1 - x)*(((1 - k)*
x)/(1 - k*x))^(4/3)*(1 - k*x)*Hypergeometric2F1[2/3, 4/3, 5/3, (1 - x)/(1 -
 k*x)])/(28*(1 - k)*(b - k)^3*x*((1 - x)*x*(1 - k*x))^(1/3)) - (3*(1 + k)*(
4 - k + 4*k^2)*(8*b*k^3 - 2*k^4 + 4*b^3*(1 + 3*k + k^2) + b^2*(1 - 12*k^2 +
 k^4) + a*(1 + 2*b^2 + k^4 + 4*b*(1 + k + k^2)))*(1 - x)*(((1 - k)*x)/(1 -
k*x))^(4/3)*(1 - k*x)*Hypergeometric2F1[2/3, 4/3, 5/3, (1 - x)/(1 - k*x)])/
(28*(1 - k)*(b - k)^4*x*((1 - x)*x*(1 - k*x))^(1/3)) - (3*(1 + k)*(a*(1 + 2
*b + k^2) + k*(2*b*(1 + 3*k + k^2) - k*(1 + 4*k + k^2)))*(1 - x)*(((1 - k)*
x)/(1 - k*x))^(4/3)*(1 - k*x)*Hypergeometric2F1[2/3, 4/3, 5/3, (1 - x)/(1 -
 k*x)])/(4*(1 - k)*(b - k)^2*x*((1 - x)*x*(1 - k*x))^(1/3)) + ((a + b^2)*(1
 + 5*b^2 + 5*b^2*k + k^5 + 5*b*(1 + k^2) + 5*b*k*(1 + k^2) + (1 + 4*b^3 - k
 - k^5 + k^6 + b^2*(13 + 14*k + 13*k^2) + b*(7 + 2*k + 2*k^2 + 2*k^3 + 7*k^
4))/Sqrt[4*b + (-1 + k)^2])*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Int
][1/((1 - x)^(1/3)*x^(13/3)*(1 - k*x)^(1/3)*(-1 - k - Sqrt[1 + 4*b - 2*k +
k^2] + 2*(-b + k)*x)), x])/((b - k)^4*((1 - x)*x*(1 - k*x))^(1/3)) + ((a +
b^2)*(1 + k^5 + 5*b^2*(1 + k) + 5*b*(1 + k + k^2 + k^3) - (1 + 4*b^3 - k -
k^5 + k^6 + b^2*(13 + 14*k + 13*k^2) + b*(7 + 2*k + 2*k^2 + 2*k^3 + 7*k^4))
/Sqrt[4*b + (-1 + k)^2])*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Int][1
/((1 - x)^(1/3)*x^(13/3)*(1 - k*x)^(1/3)*(-1 - k + Sqrt[1 + 4*b - 2*k + k^2
] + 2*(-b + k)*x)), x])/((b - k)^4*((1 - x)*x*(1 - k*x))^(1/3))
```

Rubi steps

$$\int \frac{(-2+(1+k)x)\left(1-2(1+k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(a+k^2\right)x^4\right)}{x^4\sqrt[3]{(1-x)x(1-kx)}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx = \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)}{}$$

$$= \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)}{}$$

$$= -\frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right.}{}$$

$$= -\frac{3\left(8bk^3-2k^4+4b^3\right.}{}$$

$$= -\frac{3\left(8bk^3-2k^4+4b^3\right.}{}$$

$$= -\frac{3\left(8bk^3-2k^4+4b^3\right.}{}$$

$$= -\frac{3\left(8bk^3-2k^4+4b^3\right.}{}$$

$$= -\frac{3\left(8bk^3-2k^4+4b^3\right.}{}$$

Mathematica [F] time = 6.61, size = 0, normalized size = 0.00

$$\int \frac{(-2+(1+k)x)\left(1-2(1+k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(a+k^2\right)x^4\right)}{x^4\sqrt[3]{(1-x)x(1-kx)}\left(1-(1+k)x+(-b+k)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + (1 + k)*x)*(1 - 2*(1 + k)*x + (1 + 4*k + k^2)*x^2 - 2*(k +
 k^2)*x^3 + (a + k^2)*x^4))/(x^4*((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x
 + (-b + k)*x^2)),x]
```

```
[Out] Integrate[((-2 + (1 + k)*x)*(1 - 2*(1 + k)*x + (1 + 4*k + k^2)*x^2 - 2*(k +
 k^2)*x^3 + (a + k^2)*x^4))/(x^4*((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x
 + (-b + k)*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.51, size = 238, normalized size = 1.00

$$\frac{\left(a+b^2\right)\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}+\frac{\left(-\sqrt{3}\,a-\sqrt{3}\,b^2\right)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{\sqrt[3]{b}\,x+2\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{\sqrt[3]{b}}+\frac{\left(-a-b^2\right)\log\left(b^{2/3}x^2+\sqrt[3]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2\sqrt[3]{b}}+\frac{3\left(kx^3-kx^2-x^2+x\right)^{2/3}\left(5bx^2+2kx^2-2kx-2x+2\right)}{10x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + (1 + k)*x)*(1 - 2*(1 + k)*x + (1 + 4*k + k^2)*x^2
 - 2*(k + k^2)*x^3 + (a + k^2)*x^4))/(x^4*((1 - x)*x*(1 - k*x))^(1/3)*(1 -
(1 + k)*x + (-b + k)*x^2)),x]
```

```
[Out] (3*(2 - 2*x - 2*k*x + 5*b*x^2 + 2*k*x^2)*(x - x^2 - k*x^2 + k*x^3)^(2/3))/(
10*x^4) + ((-(Sqrt[3]*a) - Sqrt[3]*b^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)
*x + 2*(x + (-1 - k)*x^2 + k*x^3)^(1/3))])/b^(1/3) + ((a + b^2)*Log[-(b^(1/
3)*x) + (x + (-1 - k)*x^2 + k*x^3)^(1/3)])/b^(1/3) + ((-a - b^2)*Log[b^(2/3
)*x^2 + b^(1/3)*x*(x + (-1 - k)*x^2 + k*x^3)^(1/3) + (x + (-1 - k)*x^2 + k*
x^3)^(2/3)])/(2*b^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-2*(1+k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2+a)*x
^4)/x^4/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x+(-b+k)*x^2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(\left(k^2+a\right)x^4-2\left(k^2+k\right)x^3+\left(k^2+4\,k+1\right)x^2-2\,(k+1)x+1\right)((k+1)x-2)}{((kx-1)(x-1)x)^{\frac{1}{3}}\left((b-k)x^2+(k+1)x-1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-2*(1+k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2+a)*x
^4)/x^4/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x+(-b+k)*x^2),x, algorithm="giac"
)
```

```
[Out] integrate(-((k^2 + a)*x^4 - 2*(k^2 + k)*x^3 + (k^2 + 4*k + 1)*x^2 - 2*(k +
1)*x + 1)*((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((b - k)*x^2 + (k +
1)*x - 1)*x^4), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(-2+(1+k)\,x)\left(1-2\,(1+k)\,x+\left(k^2+4k+1\right)x^2-2\left(k^2+k\right)x^3+\left(k^2+a\right)x^4\right)}{x^4\,((1-x)\,x\,(-kx+1))^{\frac{1}{3}}\left(1-(1+k)\,x+(-b+k)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+(1+k)*x)*(1-2*(1+k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2+a)*x^4)/x^
4/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x+(-b+k)*x^2),x)
```

```
[Out] int((-2+(1+k)*x)*(1-2*(1+k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2+a)*x^4)/x^
4/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x+(-b+k)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(\left(k^2+a\right)x^4-2\left(k^2+k\right)x^3+\left(k^2+4\,k+1\right)x^2-2\,(k+1)x+1\right)((k+1)x-2)}{((kx-1)(x-1)x)^{\frac{1}{3}}\left((b-k)x^2+(k+1)x-1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-2*(1+k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2+a)*x
^4)/x^4/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x+(-b+k)*x^2),x, algorithm="maxim
a")
```

```
[Out] -integrate(((k^2 + a)*x^4 - 2*(k^2 + k)*x^3 + (k^2 + 4*k + 1)*x^2 - 2*(k +
1)*x + 1)*((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((b - k)*x^2 + (k +
1)*x - 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(x\,(k+1)-2)\,\left(x^2\,\left(k^2+4\,k+1\right)-2\,x\,(k+1)+x^4\,\left(k^2+a\right)-2\,x^3\,\left(k^2+k\right)+1\right)}{x^4\,(x\,(k\,x-1)\,(x-1))^{1/3}\,\left((b-k)\,x^2+(k+1)\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x*(k + 1) - 2)*(x^2*(4*k + k^2 + 1) - 2*x*(k + 1) + x^4*(a + k^2) -
2*x^3*(k + k^2) + 1))/(x^4*(x*(k*x - 1)*(x - 1))^(1/3)*(x*(k + 1) + x^2*(b
- k) - 1)),x)
```

```
[Out] int(-((x*(k + 1) - 2)*(x^2*(4*k + k^2 + 1) - 2*x*(k + 1) + x^4*(a + k^2) -
2*x^3*(k + k^2) + 1))/(x^4*(x*(k*x - 1)*(x - 1))^(1/3)*(x*(k + 1) + x^2*(b
- k) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-2*(1+k)*x+(k**2+4*k+1)*x**2-2*(k**2+k)*x**3+(k**2
+a)*x**4)/x**4/((1-x)*x*(-k*x+1))**(1/3)/(1-(1+k)*x+(-b+k)*x**2),x)
```

```
[Out] Timed out
```

3.2132 $\int \frac{x^6\sqrt{x+x^4}}{b+ax^6}\,dx$

Optimal. Leaf size=238

$$\frac{\sqrt[4]{-1}\sqrt{\sqrt{b}\left(\sqrt{a}-i\sqrt{b}\right)}\tan^{-1}\left(\frac{(1+i)\sqrt{x^4+x}\sqrt{\sqrt{a}\sqrt{b}-ib}}{\sqrt{2}x^2\left(\sqrt{a}-i\sqrt{b}\right)}\right)}{3a^{3/2}}+\frac{(-1)^{3/4}\sqrt{\sqrt{b}\left(\sqrt{a}+i\sqrt{b}\right)}\tan^{-1}\left(\frac{(1+i)x\sqrt{x^4+x}\sqrt{\sqrt{a}\sqrt{b}+ib}}{\sqrt{2}\sqrt{b}(x+1)\left(x^2-x+1\right)}\right)}{3a^{3/2}}+$$

Rubi [A] time = 0.63, antiderivative size = 253, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2056, 1493, 1491, 1292, 195, 215, 1175, 402, 377, 208}

$$\frac{\sqrt[4]{b}\sqrt{x^4+x}\sqrt{\sqrt{-a}+\sqrt{b}}\tanh^{-1}\left(\frac{x^{3/2}\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}\sqrt{x^3+1}}\right)}{3(-a)^{3/2}\sqrt{x^3+1}\sqrt{x}}-\frac{\sqrt[4]{b}\sqrt{x^4+x}\sqrt{\sqrt{-a}\sqrt{b}+a}\tanh^{-1}\left(\frac{x^{3/2}\sqrt{\sqrt{-a}\sqrt{b}+a}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{x^3+1}}\right)}{3(-a)^{7/4}\sqrt{x^3+1}\sqrt{x}}+\frac{\sqrt{x^4+x}\,x}{3a}+\frac{\sqrt{x^4+x}\sinh^{-1}\left(x^{3/2}\right)}{3a\sqrt{x^3+1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*Sqrt[x + x^4])/(b + a*x^6),x]
```

```
[Out] (x*Sqrt[x + x^4])/(3*a) + (Sqrt[x + x^4]*ArcSinh[x^(3/2)])/(3*a*Sqrt[x]*Sqr
t[1 + x^3]) + (Sqrt[Sqrt[-a] + Sqrt[b]]*b^(1/4)*Sqrt[x + x^4]*ArcTanh[(Sqrt
[Sqrt[-a] + Sqrt[b]]*x^(3/2))/(b^(1/4)*Sqrt[1 + x^3])])/(3*(-a)^(3/2)*Sqrt[
x]*Sqrt[1 + x^3]) - (Sqrt[a + Sqrt[-a]*Sqrt[b]]*b^(1/4)*Sqrt[x + x^4]*ArcTa
nh[(Sqrt[a + Sqrt[-a]*Sqrt[b]]*x^(3/2))/((-a)^(1/4)*b^(1/4)*Sqrt[1 + x^3])]
)/(3*(-a)^(7/4)*Sqrt[x]*Sqrt[1 + x^3])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[-(a*c), 2]}, -Dist[c/(2*r), Int[(d + e*x^2)^q/(r - c*x^2), x], x] - Di
st[c/(2*r), Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e, q}
, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1292

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Dist[f^4/c, Int[(f*x)^(m - 4)*(d + e*x^2)^q, x], x] - Dist[(a
*f^4)/c, Int[((f*x)^(m - 4)*(d + e*x^2)^q)/(a + c*x^4), x], x] /; FreeQ[{a,
 c, d, e, f, q}, x] &&  !IntegerQ[q] && GtQ[m, 3]
```

Rule 1491

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k
 - 1)*(d + e*x^(n/k))^q*(a + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /
; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1493

```
Int[((f_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n
_))^(q_.), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/f, Subst[Int[x^(k
*(m + 1) - 1)*(d + (e*x^(k*n))/f)^q*(a + (c*x^(2*k*n))/f)^p, x], x, (f*x)^(
1/k)], x]] /; FreeQ[{a, c, d, e, f, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
 && FractionQ[m] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{x^6\sqrt{x+x^4}}{b+ax^6}\,dx = \frac{\sqrt{x+x^4}\int \frac{x^{13/2}\sqrt{1+x^3}}{b+ax^6}\,dx}{\sqrt{x}\sqrt{1+x^3}}$$

$$= \frac{\left(2\sqrt{x+x^4}\right)\text{Subst}\left(\int \frac{x^{14}\sqrt{1+x^6}}{b+ax^{12}}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt{1+x^3}}$$

$$= \frac{\left(2\sqrt{x+x^4}\right)\text{Subst}\left(\int \frac{x^4\sqrt{1+x^2}}{b+ax^4}\,dx, x, x^{3/2}\right)}{3\sqrt{x}\sqrt{1+x^3}}$$

$$= \frac{\left(2\sqrt{x+x^4}\right)\text{Subst}\left(\int \sqrt{1+x^2}\,dx, x, x^{3/2}\right)}{3a\sqrt{x}\sqrt{1+x^3}} - \frac{\left(2b\sqrt{x+x^4}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{b+ax^4}\,dx, x, x^{3/2}\right)}{3a\sqrt{x}\sqrt{1+x^3}}$$

$$= \frac{x\sqrt{x+x^4}}{3a} + \frac{\sqrt{x+x^4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^{3/2}\right)}{3a\sqrt{x}\sqrt{1+x^3}} - \frac{\left(\sqrt{-a}\sqrt{b}\sqrt{x+x^4}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{-a}\sqrt{b}-ax^2}\right.}{3a\sqrt{x}\sqrt{1+x^3}}$$

$$= \frac{x\sqrt{x+x^4}}{3a} + \frac{\sqrt{x+x^4}\sinh^{-1}\left(x^{3/2}\right)}{3a\sqrt{x}\sqrt{1+x^3}} + \frac{\left(\sqrt{-a}\left(-a+\sqrt{-a}\sqrt{b}\right)\sqrt{b}\sqrt{x+x^4}\right)\text{Subst}\left(\int \frac{}{\sqrt{1+x^2}\,(\sqrt{}\right.}{3a^2\sqrt{x}\sqrt{1+x^3}}$$

$$= \frac{x\sqrt{x+x^4}}{3a} + \frac{\sqrt{x+x^4}\sinh^{-1}\left(x^{3/2}\right)}{3a\sqrt{x}\sqrt{1+x^3}} + \frac{\left(\sqrt{-a}\left(-a+\sqrt{-a}\sqrt{b}\right)\sqrt{b}\sqrt{x+x^4}\right)\text{Subst}\left(\int \frac{}{\sqrt{-a}\sqrt{b}-}\right.}{3a^2\sqrt{x}\sqrt{1+x^3}}$$

$$= \frac{x\sqrt{x+x^4}}{3a} + \frac{\sqrt{x+x^4}\sinh^{-1}\left(x^{3/2}\right)}{3a\sqrt{x}\sqrt{1+x^3}} + \frac{\sqrt{\sqrt{-a}+\sqrt{b}}\,\sqrt[4]{b}\sqrt{x+x^4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{-a}+\sqrt{b}}\,x^{3/2}}{\sqrt[4]{b}\sqrt{1+x^3}}\right)}{3(-a)^{3/2}\sqrt{x}\sqrt{1+x^3}} - \ldots$$

Mathematica [A] time = 2.79, size = 176, normalized size = 0.74

$$\frac{x\sqrt{x^4+x}\left(\frac{\sqrt{\frac{1}{x^3}+1}\left(\sqrt[4]{b}\left(\sqrt{\sqrt{-a}-\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{\frac{1}{x^3}+1}}{\sqrt{\sqrt{-a}-\sqrt{b}}}\right)-\sqrt{\sqrt{-a}+\sqrt{b}}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{\frac{1}{x^3}+1}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right)\right)+\sqrt{-a}\tanh^{-1}\left(\sqrt{\frac{1}{x^3}+1}\right)\right)}{\sqrt{-a}\left(x^3+1\right)}+1\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*Sqrt[x + x^4])/(b + a*x^6),x]
```

```
[Out] (x*Sqrt[x + x^4]*(1 + (Sqrt[1 + x^(-3)]*(Sqrt[-a]*ArcTanh[Sqrt[1 + x^(-3)]]
 + b^(1/4)*(Sqrt[Sqrt[-a] - Sqrt[b]]*ArcTan[(b^(1/4)*Sqrt[1 + x^(-3)])/Sqrt
[Sqrt[-a] - Sqrt[b]]] - Sqrt[Sqrt[-a] + Sqrt[b]]*ArcTanh[(b^(1/4)*Sqrt[1 +
x^(-3)])/Sqrt[Sqrt[-a] + Sqrt[b]]])))/(Sqrt[-a]*(1 + x^3))))/(3*a)
```

IntegrateAlgebraic [A] time = 1.35, size = 233, normalized size = 0.98

$$\frac{\sqrt[4]{-1}\sqrt{\sqrt{b}\left(\sqrt{a}-i\sqrt{b}\right)}\tan^{-1}\left(\frac{(-1)^{3/4}x\sqrt{x^4+x}\sqrt{\sqrt{a}\sqrt{b}-ib}}{\sqrt{b}(x+1)\left(x^2-x+1\right)}\right)}{3a^{3/2}}+\frac{(-1)^{3/4}\sqrt{\sqrt{b}\left(\sqrt{a}+i\sqrt{b}\right)}\tan^{-1}\left(\frac{\sqrt[4]{-1}x\sqrt{x^4+x}\sqrt{\sqrt{a}\sqrt{b}+ib}}{\sqrt{b}(x+1)\left(x^2-x+1\right)}\right)}{3a^{3/2}}+\frac{\sqrt{x^4+x}\,x}{3a}+\frac{\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+x}}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^6*Sqrt[x + x^4])/(b + a*x^6),x]
```

```
[Out] (x*Sqrt[x + x^4])/(3*a) + ((-1)^(1/4)*Sqrt[(Sqrt[a] - I*Sqrt[b])*Sqrt[b]]*A
rcTan[((-1)^(3/4)*Sqrt[Sqrt[a]*Sqrt[b] - I*b]*x*Sqrt[x + x^4])/(Sqrt[b]*(1
+ x)*(1 - x + x^2))]])/(3*a^(3/2)) + ((-1)^(3/4)*Sqrt[(Sqrt[a] + I*Sqrt[b])*
Sqrt[b]]*ArcTan[((-1)^(1/4)*Sqrt[Sqrt[a]*Sqrt[b] + I*b]*x*Sqrt[x + x^4])/(S
qrt[b]*(1 + x)*(1 - x + x^2))]])/(3*a^(3/2)) + ArcTanh[x^2/Sqrt[x + x^4]]/(3
*a)
```

fricas [B] time = 173.97, size = 1784, normalized size = 7.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^4+x)^(1/2)/(a*x^6+b),x, algorithm="fricas")
```

```
[Out] -1/12*(a*sqrt((a^3*sqrt(-b/a^5) - b)/a^3)*log((2*((9*a^4*b + 73*a^3*b^2 + 2
79*a^2*b^3 + 567*a*b^4)*x^4 - (a^4*b - 12*a^3*b^2 - 120*a^2*b^3 - 216*a*b^4
 + 243*b^5)*x + ((a^7 - 12*a^6*b - 120*a^5*b^2 - 216*a^4*b^3 + 243*a^3*b^4)
*x^4 + (9*a^6*b + 73*a^5*b^2 + 279*a^4*b^3 + 567*a^3*b^4)*x)*sqrt(-b/a^5))*
sqrt(x^4 + x) + ((a^6 - 2*a^5*b - 69*a^4*b^2 - 108*a^3*b^3 + 486*a^2*b^4)*x
^6 - a^5*b + 22*a^4*b^2 + 171*a^3*b^3 + 324*a^2*b^4 + 2*(9*a^5*b + 73*a^4*b
^2 + 279*a^3*b^3 + 567*a^2*b^4)*x^3 + (10*a^7*b + 51*a^6*b^2 + 108*a^5*b^3
+ 243*a^4*b^4 - (8*a^8 + 95*a^7*b + 450*a^6*b^2 + 891*a^5*b^3)*x^6 + 2*(a^8
 - 12*a^7*b - 120*a^6*b^2 - 216*a^5*b^3 + 243*a^4*b^4)*x^3)*sqrt(-b/a^5))*s
qrt((a^3*sqrt(-b/a^5) - b)/a^3))/(a*x^6 + b)) - a*sqrt((a^3*sqrt(-b/a^5) -
b)/a^3)*log((2*((9*a^4*b + 73*a^3*b^2 + 279*a^2*b^3 + 567*a*b^4)*x^4 - (a^4
*b - 12*a^3*b^2 - 120*a^2*b^3 - 216*a*b^4 + 243*b^5)*x + ((a^7 - 12*a^6*b -
 120*a^5*b^2 - 216*a^4*b^3 + 243*a^3*b^4)*x^4 + (9*a^6*b + 73*a^5*b^2 + 279
*a^4*b^3 + 567*a^3*b^4)*x)*sqrt(-b/a^5))*sqrt(x^4 + x) - ((a^6 - 2*a^5*b -
69*a^4*b^2 - 108*a^3*b^3 + 486*a^2*b^4)*x^6 - a^5*b + 22*a^4*b^2 + 171*a^3*
b^3 + 324*a^2*b^4 + 2*(9*a^5*b + 73*a^4*b^2 + 279*a^3*b^3 + 567*a^2*b^4)*x^
3 + (10*a^7*b + 51*a^6*b^2 + 108*a^5*b^3 + 243*a^4*b^4 - (8*a^8 + 95*a^7*b
+ 450*a^6*b^2 + 891*a^5*b^3)*x^6 + 2*(a^8 - 12*a^7*b - 120*a^6*b^2 - 216*a^
5*b^3 + 243*a^4*b^4)*x^3)*sqrt(-b/a^5))*sqrt((a^3*sqrt(-b/a^5) - b)/a^3))/(
a*x^6 + b)) + a*sqrt(-(a^3*sqrt(-b/a^5) + b)/a^3)*log((2*((9*a^4*b + 73*a^3
*b^2 + 279*a^2*b^3 + 567*a*b^4)*x^4 - (a^4*b - 12*a^3*b^2 - 120*a^2*b^3 - 2
16*a*b^4 + 243*b^5)*x - ((a^7 - 12*a^6*b - 120*a^5*b^2 - 216*a^4*b^3 + 243*
a^3*b^4)*x^4 + (9*a^6*b + 73*a^5*b^2 + 279*a^4*b^3 + 567*a^3*b^4)*x)*sqrt(-
b/a^5))*sqrt(x^4 + x) + ((a^6 - 2*a^5*b - 69*a^4*b^2 - 108*a^3*b^3 + 486*a^
2*b^4)*x^6 - a^5*b + 22*a^4*b^2 + 171*a^3*b^3 + 324*a^2*b^4 + 2*(9*a^5*b +
73*a^4*b^2 + 279*a^3*b^3 + 567*a^2*b^4)*x^3 - (10*a^7*b + 51*a^6*b^2 + 108*
a^5*b^3 + 243*a^4*b^4 - (8*a^8 + 95*a^7*b + 450*a^6*b^2 + 891*a^5*b^3)*x^6
+ 2*(a^8 - 12*a^7*b - 120*a^6*b^2 - 216*a^5*b^3 + 243*a^4*b^4)*x^3)*sqrt(-b
/a^5))*sqrt(-(a^3*sqrt(-b/a^5) + b)/a^3))/(a*x^6 + b)) - a*sqrt(-(a^3*sqrt(
-b/a^5) + b)/a^3)*log((2*((9*a^4*b + 73*a^3*b^2 + 279*a^2*b^3 + 567*a*b^4)*
x^4 - (a^4*b - 12*a^3*b^2 - 120*a^2*b^3 - 216*a*b^4 + 243*b^5)*x - ((a^7 -
12*a^6*b - 120*a^5*b^2 - 216*a^4*b^3 + 243*a^3*b^4)*x^4 + (9*a^6*b + 73*a^5
*b^2 + 279*a^4*b^3 + 567*a^3*b^4)*x)*sqrt(-b/a^5))*sqrt(x^4 + x) - ((a^6 -
2*a^5*b - 69*a^4*b^2 - 108*a^3*b^3 + 486*a^2*b^4)*x^6 - a^5*b + 22*a^4*b^2
+ 171*a^3*b^3 + 324*a^2*b^4 + 2*(9*a^5*b + 73*a^4*b^2 + 279*a^3*b^3 + 567*a
^2*b^4)*x^3 - (10*a^7*b + 51*a^6*b^2 + 108*a^5*b^3 + 243*a^4*b^4 - (8*a^8 +
 95*a^7*b + 450*a^6*b^2 + 891*a^5*b^3)*x^6 + 2*(a^8 - 12*a^7*b - 120*a^6*b^
2 - 216*a^5*b^3 + 243*a^4*b^4)*x^3)*sqrt(-b/a^5))*sqrt(-(a^3*sqrt(-b/a^5) +
 b)/a^3))/(a*x^6 + b)) - 4*sqrt(x^4 + x)*x - 2*log(-2*x^3 - 2*sqrt(x^4 + x)
*x - 1))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^4+x)^(1/2)/(a*x^6+b),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[-34,84]Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [a,b]=[-24,-10]Warning, need to cho
ose a branch for the root of a polynomial with parameters. This might be wr
ong.The choice was done assuming [a,b]=[80,82]Warning, need to choose a bra
nch for the root of a polynomial with parameters. This might be wrong.The c
hoice was done assuming [a,b]=[57,-56]Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done assuming [a,b]=[45,-8]-1/6/a*ln(abs(sqrt((1/x)^3+1)-1))+1/6/a*ln(sqr
t((1/x)^3+1)+1)+((-2*a*b^2+4*a*sqrt(-a*b)*sqrt(-b^2-b*sqrt(-a*b))+2*b*a*b-5
*b*sqrt(-a*b)*sqrt(-b^2-b*sqrt(-a*b)))*a^2*abs(b)+(-2*a^2*b^3+4*a^2*b*sqrt(
-a*b)*sqrt(-b^2-b*sqrt(-a*b))+2*a*b^2*a*b-5*a*b^2*sqrt(-a*b)*sqrt(-b^2-b*sq
rt(-a*b)))*abs(b))/(12*a^4*b^2-3*a^3*b^3-15*a^2*b^4)/abs(a)*atan(sqrt((1/x)
^3+1)/sqrt(-(6*a*b+sqrt(6*a*b*6*a*b-12*a*b*(3*a^2+3*a*b)))/2/3/a/b))-((-2*a
*b^2+4*a*sqrt(-a*b)*sqrt(-b^2+b*sqrt(-a*b))+2*b*a*b-5*b*sqrt(-a*b)*sqrt(-b^
2+b*sqrt(-a*b)))*a^2*abs(b)+(-2*a^2*b^3+4*a^2*b*sqrt(-a*b)*sqrt(-b^2+b*sqrt
(-a*b))+2*a*b^2*a*b-5*a*b^2*sqrt(-a*b)*sqrt(-b^2+b*sqrt(-a*b)))*abs(b))/(12
*a^4*b^2-3*a^3*b^3-15*a^2*b^4)/abs(a)*atan(sqrt((1/x)^3+1)/sqrt(-(6*a*b-sqr
t(6*a*b*6*a*b-12*a*b*(3*a^2+3*a*b)))/2/3/a/b))+8*a*1/24/a^2*x*sqrt(x^4+x)
```

maple [C] time = 0.58, size = 676, normalized size = 2.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^4+x)^(1/2)/(a*x^6+b),x)
```

```
[Out] 1/a*(1/3*x*(x^4+x)^(1/2)-(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x/(1/2+1
/2*I*3^(1/2))/(1+x))^(1/2)*(1+x)^2*(-(x-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/
2))/(1+x))^(1/2)*(-(x-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2)/(
3/2+1/2*I*3^(1/2))/(x*(1+x)*(x-1/2+1/2*I*3^(1/2))*(x-1/2-1/2*I*3^(1/2)))^(1
/2)*(-EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),((-
3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2))+EllipticPi(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2))/(1+x))
^(1/2),(1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-
1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))))-1/3*b/a*
4^(1/2)*sum((-_alpha^3-1)/_alpha^4*(1+x)^2*(_alpha^5-_alpha^4+_alpha^3-_alp
ha^2+_alpha-1)/(a+b)*(-1-I*3^(1/2))*(x/(1+x)*(I*3^(1/2)+3)/(1+I*3^(1/2)))^(
1/2)*(-1/(1+x)*(-1+2*x+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)*(-1/(1+x)*(-1+2*x-I*
3^(1/2))/(1+I*3^(1/2)))^(1/2)/(I*3^(1/2)+3)/(x*(1+x)*(-1+2*x+I*3^(1/2))*(-1
+2*x-I*3^(1/2)))^(1/2)*(EllipticF(((3/2+1/2*I*3^(1/2))*x/(1/2+1/2*I*3^(1/2)
)/(1+x))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-_alpha^5*a/b*EllipticPi(((3/2+1/2*I*3^(1/2
))*x/(1/2+1/2*I*3^(1/2))/(1+x))^(1/2),-1/6*(I*_alpha^5*3^(1/2)*a+3*_alpha^5
*a-I*3^(1/2)*b-3*b)/b,((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*
I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))),_alpha=RootOf(_Z^6*a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4 + x}\,x^6}{ax^6 + b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^4+x)^(1/2)/(a*x^6+b),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x)*x^6/(a*x^6 + b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,\sqrt{x^4+x}}{a\,x^6+b} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(x + x^4)^(1/2))/(b + a*x^6),x)
```

```
[Out] int((x^6*(x + x^4)^(1/2))/(b + a*x^6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\sqrt{x\,(x+1)\left(x^2-x+1\right)}}{ax^6+b} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(x**4+x)**(1/2)/(a*x**6+b),x)
```

```
[Out] Integral(x**6*sqrt(x*(x + 1)*(x**2 - x + 1))/(a*x**6 + b), x)
```

3.2133 $\int \frac{(-2+2x^4+5x^7)\sqrt[3]{x-x^3+x^5+x^8}}{(2+x^2+2x^4+2x^7)^2}\,dx$

Optimal. Leaf size=238

$$\frac{\log\left(2\sqrt[3]{x^8+x^5-x^3+x}+2^{2/3}\sqrt[3]{3}\,x\right)}{6\sqrt[3]{2}\,3^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^8+x^5-x^3+x}}{2^{2/3}\sqrt[3]{3}\,x-\sqrt[3]{x^8+x^5-x^3+x}}\right)}{6\sqrt[3]{2}\sqrt[6]{3}}-\frac{\log\left(\sqrt[3]{2}\,3^{2/3}x^2-2^{2/3}\sqrt[3]{3}\sqrt[3]{x^8+x^5-x^3+x}\right.}{12\sqrt[3]{2}\,3^{2/3}}$$

Rubi [F] time = 2.92, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+2x^4+5x^7)\sqrt[3]{x-x^3+x^5+x^8}}{(2+x^2+2x^4+2x^7)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + 2*x^4 + 5*x^7)*(x - x^3 + x^5 + x^8)^(1/3))/(2 + x^2 + 2*x^4 + 2
*x^7)^2,x]
```

```
[Out] (-21*(x - x^3 + x^5 + x^8)^(1/3)*Defer[Subst][Defer[Int][(x^3*(1 - x^6 + x^
12 + x^21)^(1/3))/(2 + x^6 + 2*x^12 + 2*x^21)^2, x], x, x^(1/3)])/(x^(1/3)*
(1 - x^2 + x^4 + x^7)^(1/3)) - (15*(x - x^3 + x^5 + x^8)^(1/3)*Defer[Subst]
[Defer[Int][(x^9*(1 - x^6 + x^12 + x^21)^(1/3))/(2 + x^6 + 2*x^12 + 2*x^21)
^2, x], x, x^(1/3)])/(2*x^(1/3)*(1 - x^2 + x^4 + x^7)^(1/3)) - (9*(x - x^3
+ x^5 + x^8)^(1/3)*Defer[Subst][Defer[Int][(x^15*(1 - x^6 + x^12 + x^21)^(1
/3))/(2 + x^6 + 2*x^12 + 2*x^21)^2, x], x, x^(1/3)])/(x^(1/3)*(1 - x^2 + x^
4 + x^7)^(1/3)) + (15*(x - x^3 + x^5 + x^8)^(1/3)*Defer[Subst][Defer[Int][(
x^3*(1 - x^6 + x^12 + x^21)^(1/3))/(2 + x^6 + 2*x^12 + 2*x^21), x], x, x^(1
/3)])/(2*x^(1/3)*(1 - x^2 + x^4 + x^7)^(1/3))
```

Rubi steps

$$\int \frac{\left(-2+2x^4+5x^7\right)\sqrt[3]{x-x^3+x^5+x^8}}{\left(2+x^2+2x^4+2x^7\right)^2}\,dx = \frac{\sqrt[3]{x-x^3+x^5+x^8}\int \frac{\sqrt[3]{x}\sqrt[3]{1-x^2+x^4+x^7}\left(-2+2x^4+5x^7\right)}{\left(2+x^2+2x^4+2x^7\right)^2}dx}{\sqrt[3]{x}\sqrt[3]{1-x^2+x^4+x^7}}$$

$$= \frac{\left(3\sqrt[3]{x-x^3+x^5+x^8}\right)\text{Subst}\left(\int \frac{x^3\sqrt[3]{1-x^6+x^{12}+x^{21}}\left(-2+2x^{12}+5x^{21}\right)}{\left(2+x^6+2x^{12}+2x^{21}\right)^2}dx,\right.}{\sqrt[3]{x}\sqrt[3]{1-x^2+x^4+x^7}}$$

$$= \frac{\left(3\sqrt[3]{x-x^3+x^5+x^8}\right)\text{Subst}\left(\int\left(-\frac{x^3\left(14+5x^6+6x^{12}\right)\sqrt[3]{1-x^6+x^{12}+x^{21}}}{2\left(2+x^6+2x^{12}+2x^{21}\right)^2}+\right.\right.}{\sqrt[3]{x}\sqrt[3]{1-x^2+x^4+x^7}}$$

$$= -\frac{\left(3\sqrt[3]{x-x^3+x^5+x^8}\right)\text{Subst}\left(\int \frac{x^3\left(14+5x^6+6x^{12}\right)\sqrt[3]{1-x^6+x^{12}+x^{21}}}{\left(2+x^6+2x^{12}+2x^{21}\right)^2}dx,\right.}{2\sqrt[3]{x}\sqrt[3]{1-x^2+x^4+x^7}}$$

$$= -\frac{\left(3\sqrt[3]{x-x^3+x^5+x^8}\right)\text{Subst}\left(\int\left(\frac{14x^3\sqrt[3]{1-x^6+x^{12}+x^{21}}}{\left(2+x^6+2x^{12}+2x^{21}\right)^2}+\frac{5x^9\sqrt[3]{1-x^6+}}{\left(2+x^6+2x^{12}\right.}\right.\right.}{2\sqrt[3]{x}\sqrt[3]{1-x^2+x^4+x^7}}$$

$$= -\frac{\left(15\sqrt[3]{x-x^3+x^5+x^8}\right)\text{Subst}\left(\int \frac{x^9\sqrt[3]{1-x^6+x^{12}+x^{21}}}{\left(2+x^6+2x^{12}+2x^{21}\right)^2}dx,x,\sqrt[3]{x}\right)}{2\sqrt[3]{x}\sqrt[3]{1-x^2+x^4+x^7}}+$$

Mathematica [**F**] time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{\left(-2+2x^4+5x^7\right)\sqrt[3]{x-x^3+x^5+x^8}}{\left(2+x^2+2x^4+2x^7\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + 2*x^4 + 5*x^7)*(x - x^3 + x^5 + x^8)^(1/3))/(2 + x^2 + 2*x
^4 + 2*x^7)^2,x]
```

```
[Out] Integrate[((-2 + 2*x^4 + 5*x^7)*(x - x^3 + x^5 + x^8)^(1/3))/(2 + x^2 + 2*x
^4 + 2*x^7)^2, x]
```

IntegrateAlgebraic [**A**] time = 5.34, size = 238, normalized size = 1.00

$$\frac{\log\left(2\sqrt[3]{x^8+x^5-x^3+x}+2^{2/3}\sqrt[3]{3}x\right)}{6\sqrt[3]{2}\,3^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^8+x^5-x^3+x}}{2^{2/3}\sqrt[3]{3}x-\sqrt[3]{x^8+x^5-x^3+x}}\right)}{6\sqrt[3]{2}\sqrt[6]{3}}-\frac{\log\left(\sqrt[3]{2}\,3^{2/3}x^2-2^{2/3}\sqrt[3]{3}\sqrt[3]{x^8+x^5-x^3+x}\,x+2\left(x^8+x^5-x^3+x\right)^{2/3}\right)}{12\sqrt[3]{2}\,3^{2/3}}-\frac{\sqrt[3]{x^8+x^5-x^3+x}\,x}{2\left(2x^7+2x^4+x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + 2*x^4 + 5*x^7)*(x - x^3 + x^5 + x^8)^(1/3))/(2 +
x^2 + 2*x^4 + 2*x^7)^2,x]
```

```
[Out] -1/2*(x*(x - x^3 + x^5 + x^8)^(1/3))/(2 + x^2 + 2*x^4 + 2*x^7) + ArcTan[(Sq
rt[3]*(x - x^3 + x^5 + x^8)^(1/3))/(2^(2/3)*3^(1/3)*x - (x - x^3 + x^5 + x^
8)^(1/3))]/(6*2^(1/3)*3^(1/6)) + Log[2^(2/3)*3^(1/3)*x + 2*(x - x^3 + x^5 +
 x^8)^(1/3)]/(6*2^(1/3)*3^(2/3)) - Log[2^(1/3)*3^(2/3)*x^2 - 2^(2/3)*3^(1/3
)*x*(x - x^3 + x^5 + x^8)^(1/3) + 2*(x - x^3 + x^5 + x^8)^(2/3)]/(12*2^(1/3
)*3^(2/3))
```

fricas [**B**] time = 65.92, size = 635, normalized size = 2.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^7+2*x^4-2)*(x^8+x^5-x^3+x)^(1/3)/(2*x^7+2*x^4+x^2+2)^2,x, al
gorithm="fricas")
```

```
[Out] -1/648*(6*18^(1/6)*sqrt(6)*(2*x^7 + 2*x^4 + x^2 + 2)*arctan(-1/18*18^(1/6)*
(6*18^(2/3)*sqrt(6)*(4*x^15 + 8*x^12 - 50*x^10 + 4*x^9 + 8*x^8 - 50*x^7 + 6
3*x^5 - 50*x^3 + 4*x)*(x^8 + x^5 - x^3 + x)^(1/3) - 72*sqrt(6)*(2*x^14 + 4*
x^11 - 7*x^9 + 2*x^8 + 4*x^7 - 7*x^6 - 7*x^2 + 2)*(x^8 + x^5 - x^3 + x)^(2/
3) + 18^(1/3)*sqrt(6)*(8*x^21 + 24*x^18 - 204*x^16 + 24*x^15 + 24*x^14 - 40
8*x^13 + 8*x^12 + 648*x^11 - 204*x^10 - 408*x^9 + 624*x^8 + 24*x^7 - 785*x^
6 + 624*x^4 - 204*x^2 + 8))/(8*x^21 + 24*x^18 + 12*x^16 + 24*x^15 + 24*x^14
 + 24*x^13 + 8*x^12 - 432*x^11 + 12*x^10 + 24*x^9 - 456*x^8 + 24*x^7 + 511*
x^6 - 456*x^4 + 12*x^2 + 8)) + 18^(2/3)*(2*x^7 + 2*x^4 + x^2 + 2)*log(-(36*
18^(1/3)*(x^8 + x^5 - x^3 + x)^(2/3)*(x^7 + x^4 - 4*x^2 + 1) - 18^(2/3)*(4*
x^14 + 8*x^11 - 50*x^9 + 4*x^8 + 8*x^7 - 50*x^6 + 63*x^4 - 50*x^2 + 4) - 54
*(4*x^8 + 4*x^5 - 7*x^3 + 4*x)*(x^8 + x^5 - x^3 + x)^(1/3))/(4*x^14 + 8*x^1
1 + 4*x^9 + 4*x^8 + 8*x^7 + 4*x^6 + 9*x^4 + 4*x^2 + 4)) - 2*18^(2/3)*(2*x^7
 + 2*x^4 + x^2 + 2)*log((3*18^(2/3)*(x^8 + x^5 - x^3 + x)^(1/3)*x + 18^(1/3
)*(2*x^7 + 2*x^4 + x^2 + 2) + 18*(x^8 + x^5 - x^3 + x)^(2/3))/(2*x^7 + 2*x^
4 + x^2 + 2)) + 324*(x^8 + x^5 - x^3 + x)^(1/3)*x)/(2*x^7 + 2*x^4 + x^2 + 2
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+x^5-x^3+x\right)^{\frac{1}{3}}\left(5\,x^7+2\,x^4-2\right)}{\left(2\,x^7+2\,x^4+x^2+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^7+2*x^4-2)*(x^8+x^5-x^3+x)^(1/3)/(2*x^7+2*x^4+x^2+2)^2,x, al
gorithm="giac")
```

```
[Out] integrate((x^8 + x^5 - x^3 + x)^(1/3)*(5*x^7 + 2*x^4 - 2)/(2*x^7 + 2*x^4 +
x^2 + 2)^2, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^7+2x^4-2\right)\left(x^8+x^5-x^3+x\right)^{\frac{1}{3}}}{\left(2x^7+2x^4+x^2+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^7+2*x^4-2)*(x^8+x^5-x^3+x)^(1/3)/(2*x^7+2*x^4+x^2+2)^2,x)
```

```
[Out] int((5*x^7+2*x^4-2)*(x^8+x^5-x^3+x)^(1/3)/(2*x^7+2*x^4+x^2+2)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+x^5-x^3+x\right)^{\frac{1}{3}}\left(5\,x^7+2\,x^4-2\right)}{\left(2\,x^7+2\,x^4+x^2+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^7+2*x^4-2)*(x^8+x^5-x^3+x)^(1/3)/(2*x^7+2*x^4+x^2+2)^2,x, al
gorithm="maxima")
```

```
[Out] integrate((x^8 + x^5 - x^3 + x)^(1/3)*(5*x^7 + 2*x^4 - 2)/(2*x^7 + 2*x^4 +
x^2 + 2)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(5\,x^7+2\,x^4-2\right)\,{\left(x^8+x^5-x^3+x\right)}^{1/3}}{{\left(2\,x^7+2\,x^4+x^2+2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^4 + 5*x^7 - 2)*(x - x^3 + x^5 + x^8)^(1/3))/(x^2 + 2*x^4 + 2*x^7
+ 2)^2,x)
```

```
[Out] int(((2*x^4 + 5*x^7 - 2)*(x - x^3 + x^5 + x^8)^(1/3))/(x^2 + 2*x^4 + 2*x^7
+ 2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x\left(x+1\right)\left(x^6-x^5+x^4-x+1\right)}\left(5x^7+2x^4-2\right)}{\left(2x^7+2x^4+x^2+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**7+2*x**4-2)*(x**8+x**5-x**3+x)**(1/3)/(2*x**7+2*x**4+x**2+2
)**2,x)
```

```
[Out] Integral((x*(x + 1)*(x**6 - x**5 + x**4 - x + 1))**(1/3)*(5*x**7 + 2*x**4 -
 2)/(2*x**7 + 2*x**4 + x**2 + 2)**2, x)
```

3.2134 $\int \frac{-b^5+a^5x^5}{\sqrt{b^2x+a^2x^3}\left(b^5+a^5x^5\right)}\,dx$

Optimal. Leaf size=239

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{5\sqrt{a}\sqrt{b}}-\frac{2\sqrt{2\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{5\sqrt{a}\sqrt{b}}-\frac{2\sqrt{2\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\right.}{5\sqrt{a}\sqrt{b}}$$

Rubi [F] time = 3.67, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-b^5+a^5x^5}{\sqrt{b^2x+a^2x^3}\left(b^5+a^5x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-b^5 + a^5*x^5)/(Sqrt[b^2*x + a^2*x^3]*(b^5 + a^5*x^5)),x]
```

```
[Out] -1/5*(Sqrt[2]*Sqrt[x]*Sqrt[b^2 + a^2*x^2]*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[b]*S
qrt[x])/Sqrt[b^2 + a^2*x^2]])/(Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) + (4*
Sqrt[x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2]*EllipticF[2*ArcTan[(Sqr
t[a]*Sqrt[x])/Sqrt[b]], 1/2])/(5*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) - (
16*b^4*Sqrt[x]*Sqrt[b^2 + a^2*x^2]*Defer[Subst][Defer[Int][1/(Sqrt[b^2 + a^
2*x^4]*(b^4 - a*b^3*x^2 + a^2*b^2*x^4 - a^3*b*x^6 + a^4*x^8)), x], x, Sqrt[
x]])/(5*Sqrt[b^2*x + a^2*x^3]) + (12*a*b^3*Sqrt[x]*Sqrt[b^2 + a^2*x^2]*Defe
r[Subst][Defer[Int][x^2/(Sqrt[b^2 + a^2*x^4]*(b^4 - a*b^3*x^2 + a^2*b^2*x^4
 - a^3*b*x^6 + a^4*x^8)), x], x, Sqrt[x]])/(5*Sqrt[b^2*x + a^2*x^3]) - (8*a
^2*b^2*Sqrt[x]*Sqrt[b^2 + a^2*x^2]*Defer[Subst][Defer[Int][x^4/(Sqrt[b^2 +
a^2*x^4]*(b^4 - a*b^3*x^2 + a^2*b^2*x^4 - a^3*b*x^6 + a^4*x^8)), x], x, Sqr
t[x]])/(5*Sqrt[b^2*x + a^2*x^3]) + (4*a^3*b*Sqrt[x]*Sqrt[b^2 + a^2*x^2]*Def
er[Subst][Defer[Int][x^6/(Sqrt[b^2 + a^2*x^4]*(b^4 - a*b^3*x^2 + a^2*b^2*x^
4 - a^3*b*x^6 + a^4*x^8)), x], x, Sqrt[x]])/(5*Sqrt[b^2*x + a^2*x^3])
```

Rubi steps

$$\int \frac{-b^5+a^5x^5}{\sqrt{b^2x+a^2x^3}\left(b^5+a^5x^5\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{b^2+a^2x^2}\right)\int \frac{-b^5+a^5x^5}{\sqrt{x}\sqrt{b^2+a^2x^2}\left(b^5+a^5x^5\right)}\,dx}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{-b^5+a^5x^{10}}{\sqrt{b^2+a^2x^4}\left(b^5+a^5x^{10}\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt{b^2+a^2x^4}} - \frac{2b^5}{\sqrt{b^2+a^2x^4}\left(b^5+a^5x^{10}\right)}\right)dx, x, \sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2+a^2x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}} - \frac{\left(4b^5\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Su}}{}$$

$$= \frac{\sqrt{x}\,(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\left(4b^5\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\right.}{}$$

$$= \frac{\sqrt{x}\,(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\left(4b\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int\right.}{5\sqrt{b^2x+}}$$

$$= \frac{\sqrt{x}\,(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int\right.}{5\sqrt{b^2x+a^2}}$$

$$= \frac{4\sqrt{x}\,(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{5\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\left(2b\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\right.}{5\sqrt{b^2x+}}$$

$$= -\frac{\sqrt{2}\sqrt{x}\sqrt{b^2+a^2x^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{x}}{\sqrt{b^2+a^2x^2}}\right)}{5\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} + \frac{4\sqrt{x}\,(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}\,F\left(2\tan^{-1}\right.}{5\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}}$$

Mathematica [C] time = 2.18, size = 574, normalized size = 2.40

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-b^5 + a^5*x^5)/(Sqrt[b^2*x + a^2*x^3]*(b^5 + a^5*x^5)),x]
```

```
[Out] (2*(-1)^(1/10)*Sqrt[1 + b^2/(a^2*x^2)]*x^(3/2)*((1 + (-1)^(1/5))^2*(1 - 3*(
-1)^(1/5) + (-1)^(2/5))*EllipticF[I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] + 2
*((-1)^(2/5)*EllipticPi[-I, I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] + (1 - (-
1)^(1/5) + 2*(-1)^(2/5) - (-1)^(3/5) + (-1)^(4/5))*EllipticPi[-(-1)^(1/10),
 I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] - EllipticPi[(-1)^(3/10), I*ArcSinh[
Sqrt[(I*b)/a]/Sqrt[x]], -1] + (-1)^(1/5)*EllipticPi[(-1)^(3/10), I*ArcSinh[
Sqrt[(I*b)/a]/Sqrt[x]], -1] + (-1)^(3/5)*EllipticPi[(-1)^(3/10), I*ArcSinh[
Sqrt[(I*b)/a]/Sqrt[x]], -1] - (-1)^(4/5)*EllipticPi[(-1)^(3/10), I*ArcSinh[
Sqrt[(I*b)/a]/Sqrt[x]], -1] + (-1)^(2/5)*EllipticPi[(-1)^(7/10), I*ArcSinh[
Sqrt[(I*b)/a]/Sqrt[x]], -1] - EllipticPi[-(-1)^(9/10), I*ArcSinh[Sqrt[(I*b)
/a]/Sqrt[x]], -1] + (-1)^(1/5)*EllipticPi[-(-1)^(9/10), I*ArcSinh[Sqrt[(I*b
)/a]/Sqrt[x]], -1] + (-1)^(3/5)*EllipticPi[-(-1)^(9/10), I*ArcSinh[Sqrt[(I*
b)/a]/Sqrt[x]], -1] - (-1)^(4/5)*EllipticPi[-(-1)^(9/10), I*ArcSinh[Sqrt[(I
*b)/a]/Sqrt[x]], -1])))/((-1 + (-1)^(1/5))^2*(1 + (-1)^(1/5))^4*(1 + (-1)^(
2/5))*(1 - (-1)^(1/5) + (-1)^(2/5))*Sqrt[(I*b)/a]*Sqrt[x*(b^2 + a^2*x^2)])
```

IntegrateAlgebraic [A] time = 1.28, size = 239, normalized size = 1.00

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{5\sqrt{a}\sqrt{b}}-\frac{2\sqrt{2\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{5\sqrt{a}\sqrt{b}}-\frac{2\sqrt{2\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{5\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b^5 + a^5*x^5)/(Sqrt[b^2*x + a^2*x^3]*(b^5 + a^5*x^5))
,x]
```

```
[Out] -1/5*(Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])/(b^2 +
 a^2*x^2)])/(Sqrt[a]*Sqrt[b]) - (2*Sqrt[2*(1 + Sqrt[5])]*ArcTan[(Sqrt[-1/2
+ Sqrt[5]/2]*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])/(b^2 + a^2*x^2)])/(5*Sq
rt[a]*Sqrt[b]) - (2*Sqrt[2*(-1 + Sqrt[5])]*ArcTanh[(Sqrt[1/2 + Sqrt[5]/2]*S
qrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])/(b^2 + a^2*x^2)])/(5*Sqrt[a]*Sqrt[b])
```

fricas [B] time = 0.81, size = 2091, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^5*x^5-b^5)/(a^2*x^3+b^2*x)^(1/2)/(a^5*x^5+b^5),x, algorithm="f
ricas")
```

```
[Out] [-1/10*sqrt(2)*sqrt(-(5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) + 1)/(a*b))*log(2*(
2*a^4*x^4 + 6*a^2*b^2*x^2 + 2*b^4 + sqrt(2)*(a^3*b*x^2 + 2*a^2*b^2*x + a*b^
3 - 5*sqrt(1/5)*(a^4*b^2*x^2 + a^2*b^4)*sqrt(1/(a^2*b^2)))*sqrt(a^2*x^3 + b
^2*x)*sqrt(-(5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) + 1)/(a*b)) - 10*sqrt(1/5)*(
a^4*b^2*x^3 + a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 - a^3*b*x^3 + a^2*b^2*
x^2 - a*b^3*x + b^4)) + 1/10*sqrt(2)*sqrt(-(5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2
)) + 1)/(a*b))*log(2*(2*a^4*x^4 + 6*a^2*b^2*x^2 + 2*b^4 - sqrt(2)*(a^3*b*x^
2 + 2*a^2*b^2*x + a*b^3 - 5*sqrt(1/5)*(a^4*b^2*x^2 + a^2*b^4)*sqrt(1/(a^2*b
^2)))*sqrt(a^2*x^3 + b^2*x)*sqrt(-(5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) + 1)/(
a*b)) - 10*sqrt(1/5)*(a^4*b^2*x^3 + a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4
- a^3*b*x^3 + a^2*b^2*x^2 - a*b^3*x + b^4)) - 1/10*sqrt(2)*sqrt((5*sqrt(1/5
)*a*b*sqrt(1/(a^2*b^2)) - 1)/(a*b))*log(2*(2*a^4*x^4 + 6*a^2*b^2*x^2 + 2*b^
4 + sqrt(2)*(a^3*b*x^2 + 2*a^2*b^2*x + a*b^3 + 5*sqrt(1/5)*(a^4*b^2*x^2 + a
^2*b^4)*sqrt(1/(a^2*b^2)))*sqrt(a^2*x^3 + b^2*x)*sqrt((5*sqrt(1/5)*a*b*sqrt
(1/(a^2*b^2)) - 1)/(a*b)) + 10*sqrt(1/5)*(a^4*b^2*x^3 + a^2*b^4*x)*sqrt(1/(
a^2*b^2)))/(a^4*x^4 - a^3*b*x^3 + a^2*b^2*x^2 - a*b^3*x + b^4)) + 1/10*sqrt
(2)*sqrt((5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) - 1)/(a*b))*log(2*(2*a^4*x^4 +
6*a^2*b^2*x^2 + 2*b^4 - sqrt(2)*(a^3*b*x^2 + 2*a^2*b^2*x + a*b^3 + 5*sqrt(1
/5)*(a^4*b^2*x^2 + a^2*b^4)*sqrt(1/(a^2*b^2)))*sqrt(a^2*x^3 + b^2*x)*sqrt((
5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) - 1)/(a*b)) + 10*sqrt(1/5)*(a^4*b^2*x^3 +
 a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 - a^3*b*x^3 + a^2*b^2*x^2 - a*b^3*x
 + b^4)) + 1/20*sqrt(2)*sqrt(-1/(a*b))*log((a^4*x^4 - 12*a^3*b*x^3 + 6*a^2*
b^2*x^2 - 12*a*b^3*x + b^4 + 4*sqrt(2)*(a^3*b*x^2 - 2*a^2*b^2*x + a*b^3)*sq
rt(a^2*x^3 + b^2*x)*sqrt(-1/(a*b)))/(a^4*x^4 + 4*a^3*b*x^3 + 6*a^2*b^2*x^2
+ 4*a*b^3*x + b^4)), -1/10*sqrt(2)*sqrt(1/(a*b))*arctan(2*sqrt(2)*sqrt(a^2*
x^3 + b^2*x)*a*b*sqrt(1/(a*b))/(a^2*x^2 - 2*a*b*x + b^2)) - 1/10*sqrt(2)*sq
rt(-(5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) + 1)/(a*b))*log(2*(2*a^4*x^4 + 6*a^2
*b^2*x^2 + 2*b^4 + sqrt(2)*(a^3*b*x^2 + 2*a^2*b^2*x + a*b^3 - 5*sqrt(1/5)*(
a^4*b^2*x^2 + a^2*b^4)*sqrt(1/(a^2*b^2)))*sqrt(a^2*x^3 + b^2*x)*sqrt(-(5*sq
rt(1/5)*a*b*sqrt(1/(a^2*b^2)) + 1)/(a*b)) - 10*sqrt(1/5)*(a^4*b^2*x^3 + a^2
*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 - a^3*b*x^3 + a^2*b^2*x^2 - a*b^3*x + b
^4)) + 1/10*sqrt(2)*sqrt(-(5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) + 1)/(a*b))*lo
g(2*(2*a^4*x^4 + 6*a^2*b^2*x^2 + 2*b^4 - sqrt(2)*(a^3*b*x^2 + 2*a^2*b^2*x +
 a*b^3 - 5*sqrt(1/5)*(a^4*b^2*x^2 + a^2*b^4)*sqrt(1/(a^2*b^2)))*sqrt(a^2*x^
3 + b^2*x)*sqrt(-(5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) + 1)/(a*b)) - 10*sqrt(1
/5)*(a^4*b^2*x^3 + a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 - a^3*b*x^3 + a^2
```

```
*b^2*x^2 - a*b^3*x + b^4)) - 1/10*sqrt(2)*sqrt((5*sqrt(1/5)*a*b*sqrt(1/(a^2
*b^2)) - 1)/(a*b))*log(2*(2*a^4*x^4 + 6*a^2*b^2*x^2 + 2*b^4 + sqrt(2)*(a^3*
b*x^2 + 2*a^2*b^2*x + a*b^3 + 5*sqrt(1/5)*(a^4*b^2*x^2 + a^2*b^4)*sqrt(1/(a
^2*b^2)))*sqrt(a^2*x^3 + b^2*x)*sqrt((5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) - 1
)/(a*b)) + 10*sqrt(1/5)*(a^4*b^2*x^3 + a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x
^4 - a^3*b*x^3 + a^2*b^2*x^2 - a*b^3*x + b^4)) + 1/10*sqrt(2)*sqrt((5*sqrt(
1/5)*a*b*sqrt(1/(a^2*b^2)) - 1)/(a*b))*log(2*(2*a^4*x^4 + 6*a^2*b^2*x^2 + 2
*b^4 - sqrt(2)*(a^3*b*x^2 + 2*a^2*b^2*x + a*b^3 + 5*sqrt(1/5)*(a^4*b^2*x^2
+ a^2*b^4)*sqrt(1/(a^2*b^2)))*sqrt(a^2*x^3 + b^2*x)*sqrt((5*sqrt(1/5)*a*b*s
qrt(1/(a^2*b^2)) - 1)/(a*b)) + 10*sqrt(1/5)*(a^4*b^2*x^3 + a^2*b^4*x)*sqrt(
1/(a^2*b^2)))/(a^4*x^4 - a^3*b*x^3 + a^2*b^2*x^2 - a*b^3*x + b^4))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^5x^5 - b^5}{\left(a^5x^5 + b^5\right)\sqrt{a^2x^3 + b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^5*x^5-b^5)/(a^2*x^3+b^2*x)^(1/2)/(a^5*x^5+b^5),x, algorithm="g
iac")
```

```
[Out] integrate((a^5*x^5 - b^5)/((a^5*x^5 + b^5)*sqrt(a^2*x^3 + b^2*x)), x)
```

maple [C] time = 0.12, size = 487, normalized size = 2.04

$$\frac{ib\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{2}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}, \frac{\sqrt{2}}{2}\right)}{a\sqrt{a^2x^3+b^2x}} - \frac{2i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(a^4\_Z^4-a^3b\_Z^3+a^2b^2\_Z^2-ab^3\_Z+b^4\right)}\frac{\left(-a^3\_\alpha^3+2\_\alpha^2a^2b-3\_\alpha ab^2+4b^3\right)\_\alpha\left(\_\alpha^2a^2-i\_\alpha ab-\_\alpha ab+ib^2\right)\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}, -\frac{\_\alpha\left(i\_\alpha^2a^2-i\_\alpha ab+\_\alpha ab-b^2\right)a}{b^3}, \frac{\sqrt{2}}{2}\right)}{\left(4a^3\_\alpha^3-3\_\alpha^2a^2b+2\_\alpha ab^2-b^3\right)\sqrt{x\left(a^2x^2+b^2\right)}}\right)}{5b^2} - \frac{2ib^2\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{2}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}, -\frac{ib}{a\left(-\frac{ib}{a}+\frac{b}{a}\right)}, \frac{\sqrt{2}}{2}\right)}{5a^2\sqrt{a^2x^3+b^2x}\,\left(-\frac{ib}{a}+\frac{b}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^5*x^5-b^5)/(a^2*x^3+b^2*x)^(1/2)/(a^5*x^5+b^5),x)
```

```
[Out] I*b/a*(-I*(x+I*b/a)/b*a)^(1/2)*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1
/2)/(a^2*x^3+b^2*x)^(1/2)*EllipticF((-I*(x+I*b/a)/b*a)^(1/2),1/2*2^(1/2))-2
/5*I/b^2*2^(1/2)*sum((-_alpha^3*a^3+2*_alpha^2*a^2*b-3*_alpha*a*b^2+4*b^3)/
(4*_alpha^3*a^3-3*_alpha^2*a^2*b+2*_alpha*a*b^2-b^3)*_alpha*(-I*_alpha*a*b+
_alpha^2*a^2+I*b^2-_alpha*a*b)*(-I*(x+I*b/a)/b*a)^(1/2)*(I*(x-I*b/a)/b*a)^(
1/2)*(I*x/b*a)^(1/2)/(x*(a^2*x^2+b^2))^(1/2)*EllipticPi((-I*(x+I*b/a)/b*a)^
(1/2),-_alpha*(I*_alpha^2*a^2-I*_alpha*a*b+_alpha*a*b-b^2)*a/b^3,1/2*2^(1/2
)),_alpha=RootOf(_Z^4*a^4-_Z^3*a^3*b+_Z^2*a^2*b^2-_Z*a*b^3+b^4))-2/5*I*b^2/
a^2*(-I*(x+I*b/a)/b*a)^(1/2)*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1/2
)/(a^2*x^3+b^2*x)^(1/2)/(-I*b/a+b/a)*EllipticPi((-I*(x+I*b/a)/b*a)^(1/2),-I
*b/a/(-I*b/a+b/a),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^5x^5 - b^5}{\left(a^5x^5 + b^5\right)\sqrt{a^2x^3 + b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^5*x^5-b^5)/(a^2*x^3+b^2*x)^(1/2)/(a^5*x^5+b^5),x, algorithm="m
axima")
```

```
[Out] integrate((a^5*x^5 - b^5)/((a^5*x^5 + b^5)*sqrt(a^2*x^3 + b^2*x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^5 - a^5*x^5)/((b^5 + a^5*x^5)*(b^2*x + a^2*x^3)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-b)\left(a^4x^4+a^3bx^3+a^2b^2x^2+ab^3x+b^4\right)}{\sqrt{x\left(a^2x^2+b^2\right)}\,(ax+b)\left(a^4x^4-a^3bx^3+a^2b^2x^2-ab^3x+b^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**5*x**5-b**5)/(a**2*x**3+b**2*x)**(1/2)/(a**5*x**5+b**5),x)
```

```
[Out] Integral((a*x - b)*(a**4*x**4 + a**3*b*x**3 + a**2*b**2*x**2 + a*b**3*x + b
**4)/(sqrt(x*(a**2*x**2 + b**2))*(a*x + b)*(a**4*x**4 - a**3*b*x**3 + a**2*
b**2*x**2 - a*b**3*x + b**4)), x)
```

3.2135 $\int \frac{b^5+a^5x^5}{\sqrt{b^2x+a^2x^3}\left(-b^5+a^5x^5\right)}\,dx$

Optimal. Leaf size=239

$$-\frac{2\sqrt{2\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{5\sqrt{a}\sqrt{b}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{5\sqrt{a}\sqrt{b}}-\frac{2\sqrt{2\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\cdots}}{\cdots}\right)}{5\sqrt{a}\sqrt{b}}$$

Rubi [F] time = 3.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{b^5+a^5x^5}{\sqrt{b^2x+a^2x^3}\left(-b^5+a^5x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(b^5 + a^5*x^5)/(Sqrt[b^2*x + a^2*x^3]*(-b^5 + a^5*x^5)),x]
```

```
[Out] -1/5*(Sqrt[2]*Sqrt[x]*Sqrt[b^2 + a^2*x^2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[b]*
Sqrt[x])/Sqrt[b^2 + a^2*x^2]])/(Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) + (4
*Sqrt[x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2]*EllipticF[2*ArcTan[(Sq
rt[a]*Sqrt[x])/Sqrt[b]], 1/2])/(5*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) -
(16*b^4*Sqrt[x]*Sqrt[b^2 + a^2*x^2]*Defer[Subst][Defer[Int][1/(Sqrt[b^2 + a
^2*x^4]*(b^4 + a*b^3*x^2 + a^2*b^2*x^4 + a^3*b*x^6 + a^4*x^8)), x], x, Sqrt
[x]])/(5*Sqrt[b^2*x + a^2*x^3]) - (12*a*b^3*Sqrt[x]*Sqrt[b^2 + a^2*x^2]*Def
er[Subst][Defer[Int][x^2/(Sqrt[b^2 + a^2*x^4]*(b^4 + a*b^3*x^2 + a^2*b^2*x^
4 + a^3*b*x^6 + a^4*x^8)), x], x, Sqrt[x]])/(5*Sqrt[b^2*x + a^2*x^3]) - (8*
a^2*b^2*Sqrt[x]*Sqrt[b^2 + a^2*x^2]*Defer[Subst][Defer[Int][x^4/(Sqrt[b^2 +
 a^2*x^4]*(b^4 + a*b^3*x^2 + a^2*b^2*x^4 + a^3*b*x^6 + a^4*x^8)), x], x, Sq
rt[x]])/(5*Sqrt[b^2*x + a^2*x^3]) - (4*a^3*b*Sqrt[x]*Sqrt[b^2 + a^2*x^2]*De
fer[Subst][Defer[Int][x^6/(Sqrt[b^2 + a^2*x^4]*(b^4 + a*b^3*x^2 + a^2*b^2*x
^4 + a^3*b*x^6 + a^4*x^8)), x], x, Sqrt[x]])/(5*Sqrt[b^2*x + a^2*x^3])
```

Rubi steps

$$\int \frac{b^5+a^5x^5}{\sqrt{b^2x+a^2x^3}\left(-b^5+a^5x^5\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{b^2+a^2x^2}\right)\int \frac{b^5+a^5x^5}{\sqrt{x}\sqrt{b^2+a^2x^2}\left(-b^5+a^5x^5\right)}\,dx}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{b^5+a^5x^{10}}{\sqrt{b^2+a^2x^4}\left(-b^5+a^5x^{10}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt{b^2+a^2x^4}}+\frac{2b^5}{\sqrt{b^2+a^2x^4}\left(-b^5+a^5x^{10}\right)}\right)dx,x,\sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b^2+a^2x^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}} + \frac{\left(4b^5\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Sub}}{\sqrt{}}$$

$$= \frac{\sqrt{x}(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} + \frac{\left(4b^5\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{\sqrt{x}(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\left(4b\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int\right.}{5\sqrt{b^2x+}}$$

$$= \frac{\sqrt{x}(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int\right.}{5\sqrt{b^2x+a^2x^3}}$$

$$= \frac{4\sqrt{x}(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{5\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} - \frac{\left(2b\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int\right.}{5\sqrt{b^2x+}}$$

$$= -\frac{\sqrt{2}\sqrt{x}\sqrt{b^2+a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{x}}{\sqrt{b^2+a^2x^2}}\right)}{5\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}} + \frac{4\sqrt{x}(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}\,F\left(2\tan^{-1}\right.}{5\sqrt{a}\sqrt{b}\sqrt{b^2x+a^2x^3}}$$

Mathematica [C] time = 1.92, size = 574, normalized size = 2.40

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b^5 + a^5*x^5)/(Sqrt[b^2*x + a^2*x^3]*(-b^5 + a^5*x^5)),x]
```

```
[Out] (2*(-1)^(1/10)*Sqrt[1 + b^2/(a^2*x^2)]*x^(3/2)*((1 + (-1)^(1/5))^2*(1 - 3*(
-1)^(1/5) + (-1)^(2/5))*EllipticF[I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] + 2
*((-1)^(2/5)*EllipticPi[I, I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] + (1 - (-1
)^(1/5) + 2*(-1)^(2/5) - (-1)^(3/5) + (-1)^(4/5))*EllipticPi[(-1)^(1/10), I
*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] - EllipticPi[-(-1)^(3/10), I*ArcSinh[S
qrt[(I*b)/a]/Sqrt[x]], -1] + (-1)^(1/5)*EllipticPi[-(-1)^(3/10), I*ArcSinh[
Sqrt[(I*b)/a]/Sqrt[x]], -1] + (-1)^(3/5)*EllipticPi[-(-1)^(3/10), I*ArcSinh
[Sqrt[(I*b)/a]/Sqrt[x]], -1] - (-1)^(4/5)*EllipticPi[-(-1)^(3/10), I*ArcSin
h[Sqrt[(I*b)/a]/Sqrt[x]], -1] + (-1)^(2/5)*EllipticPi[-(-1)^(7/10), I*ArcSi
nh[Sqrt[(I*b)/a]/Sqrt[x]], -1] - EllipticPi[(-1)^(9/10), I*ArcSinh[Sqrt[(I*
b)/a]/Sqrt[x]], -1] + (-1)^(1/5)*EllipticPi[(-1)^(9/10), I*ArcSinh[Sqrt[(I*
b)/a]/Sqrt[x]], -1] + (-1)^(3/5)*EllipticPi[(-1)^(9/10), I*ArcSinh[Sqrt[(I*
b)/a]/Sqrt[x]], -1] - (-1)^(4/5)*EllipticPi[(-1)^(9/10), I*ArcSinh[Sqrt[(I*
b)/a]/Sqrt[x]], -1])))/((-1 + (-1)^(1/5))^2*(1 + (-1)^(1/5))^4*(1 + (-1)^(2
/5))*(1 - (-1)^(1/5) + (-1)^(2/5))*Sqrt[(I*b)/a]*Sqrt[x*(b^2 + a^2*x^2)])
```

IntegrateAlgebraic [A] time = 1.21, size = 239, normalized size = 1.00

$$-\frac{2\sqrt{2\left(\sqrt{5}-1\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,\sqrt{a}\,\sqrt{b}\,\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{5\sqrt{a}\,\sqrt{b}}-\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{5\sqrt{a}\,\sqrt{b}}-\frac{2\sqrt{2\left(1+\sqrt{5}\right)}\,\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,\sqrt{a}\,\sqrt{b}\,\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{5\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^5 + a^5*x^5)/(Sqrt[b^2*x + a^2*x^3]*(-b^5 + a^5*x^5))
,x]
```

```
[Out] (-2*Sqrt[2*(-1 + Sqrt[5])]*ArcTan[(Sqrt[1/2 + Sqrt[5]/2]*Sqrt[a]*Sqrt[b]*Sq
rt[b^2*x + a^2*x^3])/(b^2 + a^2*x^2)])/(5*Sqrt[a]*Sqrt[b]) - (Sqrt[2]*ArcTa
nh[(Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])/(b^2 + a^2*x^2)])/(5*Sqr
t[a]*Sqrt[b]) - (2*Sqrt[2*(1 + Sqrt[5])]*ArcTanh[(Sqrt[-1/2 + Sqrt[5]/2]*Sq
rt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])/(b^2 + a^2*x^2)])/(5*Sqrt[a]*Sqrt[b])
```

fricas [B] time = 0.84, size = 2075, normalized size = 8.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^5*x^5+b^5)/(a^2*x^3+b^2*x)^(1/2)/(a^5*x^5-b^5),x, algorithm="f
ricas")
```

```
[Out] [1/10*sqrt(2)*sqrt((5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) + 1)/(a*b))*log(2*(2*
a^4*x^4 + 6*a^2*b^2*x^2 + 2*b^4 + sqrt(2)*(a^3*b*x^2 - 2*a^2*b^2*x + a*b^3
- 5*sqrt(1/5)*(a^4*b^2*x^2 + a^2*b^4)*sqrt(1/(a^2*b^2)))*sqrt(a^2*x^3 + b^2
*x)*sqrt((5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) + 1)/(a*b)) + 10*sqrt(1/5)*(a^4
*b^2*x^3 + a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 + a^3*b*x^3 + a^2*b^2*x^2
 + a*b^3*x + b^4)) - 1/10*sqrt(2)*sqrt((5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) +
 1)/(a*b))*log(2*(2*a^4*x^4 + 6*a^2*b^2*x^2 + 2*b^4 - sqrt(2)*(a^3*b*x^2 -
2*a^2*b^2*x + a*b^3 - 5*sqrt(1/5)*(a^4*b^2*x^2 + a^2*b^4)*sqrt(1/(a^2*b^2))
)*sqrt(a^2*x^3 + b^2*x)*sqrt((5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) + 1)/(a*b))
 + 10*sqrt(1/5)*(a^4*b^2*x^3 + a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 + a^3
*b*x^3 + a^2*b^2*x^2 + a*b^3*x + b^4)) + 1/10*sqrt(2)*sqrt(-(5*sqrt(1/5)*a*
b*sqrt(1/(a^2*b^2)) - 1)/(a*b))*log(2*(2*a^4*x^4 + 6*a^2*b^2*x^2 + 2*b^4 +
sqrt(2)*(a^3*b*x^2 - 2*a^2*b^2*x + a*b^3 + 5*sqrt(1/5)*(a^4*b^2*x^2 + a^2*b
^4)*sqrt(1/(a^2*b^2)))*sqrt(a^2*x^3 + b^2*x)*sqrt(-(5*sqrt(1/5)*a*b*sqrt(1/
(a^2*b^2)) - 1)/(a*b)) - 10*sqrt(1/5)*(a^4*b^2*x^3 + a^2*b^4*x)*sqrt(1/(a^2
*b^2)))/(a^4*x^4 + a^3*b*x^3 + a^2*b^2*x^2 + a*b^3*x + b^4)) - 1/10*sqrt(2)
*sqrt(-(5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) - 1)/(a*b))*log(2*(2*a^4*x^4 + 6*
a^2*b^2*x^2 + 2*b^4 - sqrt(2)*(a^3*b*x^2 - 2*a^2*b^2*x + a*b^3 + 5*sqrt(1/5
)*(a^4*b^2*x^2 + a^2*b^4)*sqrt(1/(a^2*b^2)))*sqrt(a^2*x^3 + b^2*x)*sqrt(-(5
*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) - 1)/(a*b)) - 10*sqrt(1/5)*(a^4*b^2*x^3 +
a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 + a^3*b*x^3 + a^2*b^2*x^2 + a*b^3*x
+ b^4)) + 1/20*sqrt(2)*sqrt(1/(a*b))*log((a^4*x^4 + 12*a^3*b*x^3 + 6*a^2*b^
2*x^2 + 12*a*b^3*x + b^4 - 4*sqrt(2)*(a^3*b*x^2 + 2*a^2*b^2*x + a*b^3)*sqrt
(a^2*x^3 + b^2*x)*sqrt(1/(a*b)))/(a^4*x^4 - 4*a^3*b*x^3 + 6*a^2*b^2*x^2 - 4
*a*b^3*x + b^4)), 1/10*sqrt(2)*sqrt(-1/(a*b))*arctan(2*sqrt(2)*sqrt(a^2*x^3
 + b^2*x)*a*b*sqrt(-1/(a*b))/(a^2*x^2 + 2*a*b*x + b^2)) + 1/10*sqrt(2)*sqrt
((5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) + 1)/(a*b))*log(2*(2*a^4*x^4 + 6*a^2*b^
2*x^2 + 2*b^4 + sqrt(2)*(a^3*b*x^2 - 2*a^2*b^2*x + a*b^3 - 5*sqrt(1/5)*(a^4
*b^2*x^2 + a^2*b^4)*sqrt(1/(a^2*b^2)))*sqrt(a^2*x^3 + b^2*x)*sqrt((5*sqrt(1
/5)*a*b*sqrt(1/(a^2*b^2)) + 1)/(a*b)) + 10*sqrt(1/5)*(a^4*b^2*x^3 + a^2*b^4
*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 + a^3*b*x^3 + a^2*b^2*x^2 + a*b^3*x + b^4))
 - 1/10*sqrt(2)*sqrt((5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) + 1)/(a*b))*log(2*(
2*a^4*x^4 + 6*a^2*b^2*x^2 + 2*b^4 - sqrt(2)*(a^3*b*x^2 - 2*a^2*b^2*x + a*b^
3 - 5*sqrt(1/5)*(a^4*b^2*x^2 + a^2*b^4)*sqrt(1/(a^2*b^2)))*sqrt(a^2*x^3 + b
^2*x)*sqrt((5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) + 1)/(a*b)) + 10*sqrt(1/5)*(a
^4*b^2*x^3 + a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 + a^3*b*x^3 + a^2*b^2*x
```

```
^2 + a*b^3*x + b^4)) + 1/10*sqrt(2)*sqrt(-(5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)
) - 1)/(a*b))*log(2*(2*a^4*x^4 + 6*a^2*b^2*x^2 + 2*b^4 + sqrt(2)*(a^3*b*x^2
 - 2*a^2*b^2*x + a*b^3 + 5*sqrt(1/5)*(a^4*b^2*x^2 + a^2*b^4)*sqrt(1/(a^2*b^
2)))*sqrt(a^2*x^3 + b^2*x)*sqrt(-(5*sqrt(1/5)*a*b*sqrt(1/(a^2*b^2)) - 1)/(a
*b)) - 10*sqrt(1/5)*(a^4*b^2*x^3 + a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 +
 a^3*b*x^3 + a^2*b^2*x^2 + a*b^3*x + b^4)) - 1/10*sqrt(2)*sqrt(-(5*sqrt(1/5
)*a*b*sqrt(1/(a^2*b^2)) - 1)/(a*b))*log(2*(2*a^4*x^4 + 6*a^2*b^2*x^2 + 2*b^
4 - sqrt(2)*(a^3*b*x^2 - 2*a^2*b^2*x + a*b^3 + 5*sqrt(1/5)*(a^4*b^2*x^2 + a
^2*b^4)*sqrt(1/(a^2*b^2)))*sqrt(a^2*x^3 + b^2*x)*sqrt(-(5*sqrt(1/5)*a*b*sqr
t(1/(a^2*b^2)) - 1)/(a*b)) - 10*sqrt(1/5)*(a^4*b^2*x^3 + a^2*b^4*x)*sqrt(1/
(a^2*b^2)))/(a^4*x^4 + a^3*b*x^3 + a^2*b^2*x^2 + a*b^3*x + b^4))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^5x^5+b^5}{\left(a^5x^5-b^5\right)\sqrt{a^2x^3+b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^5*x^5+b^5)/(a^2*x^3+b^2*x)^(1/2)/(a^5*x^5-b^5),x, algorithm="g
iac")
```

```
[Out] integrate((a^5*x^5 + b^5)/((a^5*x^5 - b^5)*sqrt(a^2*x^3 + b^2*x)), x)
```

maple [C] time = 0.12, size = 482, normalized size = 2.02

$$\frac{ib\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{2}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a^2x^3+b^2x}}+\frac{2ib^2\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{2}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},-\frac{ib}{a\left(-\frac{ib}{a}-\frac{b}{a}\right)},\frac{\sqrt{2}}{2}\right)}{5a^2\sqrt{a^2x^3+b^2x}\,\left(-\frac{ib}{a}-\frac{b}{a}\right)}+\frac{2i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(a^4\_Z^4+a^3b\_Z^3+a^2b^2\_Z^2+ab^3\_Z+b^4\right)}\frac{\left(-a^3\_\alpha^3-2\_\alpha^2a^2b-3\_\alpha ab^2-4b^3\right)\_\alpha\left(\_\alpha^2a^2-i\_\alpha ab+\_\alpha ab-ib^2\right)\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},-\frac{\_\alpha\left(i\_\alpha^2a^2+i\_\alpha ab+\_\alpha ab+b^2\right)a}{b^3},\frac{\sqrt{2}}{2}\right)}{\left(4a^3\_\alpha^3+3\_\alpha^2a^2b+2\_\alpha ab^2+b^3\right)\sqrt{x\left(a^2x^2+b^2\right)}}\right)}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^5*x^5+b^5)/(a^2*x^3+b^2*x)^(1/2)/(a^5*x^5-b^5),x)
```

```
[Out] I*b/a*(-I*(x+I*b/a)/b*a)^(1/2)*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1
/2)/(a^2*x^3+b^2*x)^(1/2)*EllipticF((-I*(x+I*b/a)/b*a)^(1/2),1/2*2^(1/2))+2
/5*I*b^2/a^2*(-I*(x+I*b/a)/b*a)^(1/2)*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/
b*a)^(1/2)/(a^2*x^3+b^2*x)^(1/2)/(-I*b/a-b/a)*EllipticPi((-I*(x+I*b/a)/b*a)
^(1/2),-I*b/a/(-I*b/a-b/a),1/2*2^(1/2))+2/5*I/b^2*2^(1/2)*sum((-_alpha^3*a^
3-2*_alpha^2*a^2*b-3*_alpha*a*b^2-4*b^3)/(4*_alpha^3*a^3+3*_alpha^2*a^2*b+2
*_alpha*a*b^2+b^3)*_alpha*(-I*_alpha*a*b+_alpha^2*a^2-I*b^2+_alpha*a*b)*(-I
*(x+I*b/a)/b*a)^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1/2)/(x*(a^2*x^2+b
^2))^(1/2)*EllipticPi((-I*(x+I*b/a)/b*a)^(1/2),-_alpha*(I*_alpha^2*a^2+I*_a
lpha*a*b+_alpha*a*b+b^2)*a/b^3,1/2*2^(1/2)),_alpha=RootOf(_Z^4*a^4+_Z^3*a^3
*b+_Z^2*a^2*b^2+_Z*a*b^3+b^4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^5x^5+b^5}{\left(a^5x^5-b^5\right)\sqrt{a^2x^3+b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^5*x^5+b^5)/(a^2*x^3+b^2*x)^(1/2)/(a^5*x^5-b^5),x, algorithm="m
axima")
```

```
[Out] integrate((a^5*x^5 + b^5)/((a^5*x^5 - b^5)*sqrt(a^2*x^3 + b^2*x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^5 + a^5*x^5)/((b^5 - a^5*x^5)*(b^2*x + a^2*x^3)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+b)\left(a^4x^4-a^3bx^3+a^2b^2x^2-ab^3x+b^4\right)}{\sqrt{x\left(a^2x^2+b^2\right)}\,(ax-b)\left(a^4x^4+a^3bx^3+a^2b^2x^2+ab^3x+b^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**5*x**5+b**5)/(a**2*x**3+b**2*x)**(1/2)/(a**5*x**5-b**5),x)
```

```
[Out] Integral((a*x + b)*(a**4*x**4 - a**3*b*x**3 + a**2*b**2*x**2 - a*b**3*x + b
**4)/(sqrt(x*(a**2*x**2 + b**2))*(a*x - b)*(a**4*x**4 + a**3*b*x**3 + a**2*
b**2*x**2 + a*b**3*x + b**4)), x)
```

3.2136 $\int \frac{-2b-ax^4+2x^8}{x^4\sqrt[4]{-b+ax^4}\left(-b+2ax^4\right)}\,dx$

Optimal. Leaf size=239

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2a^{5/4}}+\frac{\left(5a^2-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x\sqrt[4]{ax^4-b}}{\sqrt{ax^4-b}-\sqrt{a}\,x^2}\right)}{2\sqrt{2}\,a^{5/4}b}+\frac{\left(5a^2-b\right)\tanh^{-1}\left(\frac{\sqrt{ax^4-b}+\sqrt{a}\,x^2}{\sqrt{2}\sqrt[4]{a}\,x\sqrt[4]{ax^4-b}}\right)}{2\sqrt{2}\,a^{5/4}b}+\frac{2\left(ax^4-\right.}{3bx}$$

Rubi [A] time = 0.89, antiderivative size = 346, normalized size of antiderivative = 1.45, number of steps used = 17, number of rules used = 13, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6725, 240, 212, 206, 203, 264, 377, 211, 1165, 628, 1162, 617, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2a^{5/4}}-\frac{\left(5a^2-b\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}\right)}{2\sqrt{2}\,a^{5/4}b}+\frac{\left(5a^2-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}+1\right)}{2\sqrt{2}\,a^{5/4}b}-\frac{\left(5a^2-b\right)\log\left(-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}+\frac{\sqrt{a}\,x^2}{\sqrt{ax^4-b}}+1\right)}{4\sqrt{2}\,a^{5/4}b}+\frac{\left(5a^2-b\right)\log\left(\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4-b}}+\frac{\sqrt{a}\,x^2}{\sqrt{ax^4-b}}+1\right)}{4\sqrt{2}\,a^{5/4}b}+\frac{2\left(ax^4-b\right)^{3/4}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*b - a*x^4 + 2*x^8)/(x^4*(-b + a*x^4)^(1/4)*(-b + 2*a*x^4)),x]
```

```
[Out] (2*(-b + a*x^4)^(3/4))/(3*b*x^3) + ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)]/(
2*a^(5/4)) - ((5*a^2 - b)*ArcTan[1 - (Sqrt[2]*a^(1/4)*x)/(-b + a*x^4)^(1/4)
])/(2*Sqrt[2]*a^(5/4)*b) + ((5*a^2 - b)*ArcTan[1 + (Sqrt[2]*a^(1/4)*x)/(-b
+ a*x^4)^(1/4)])/(2*Sqrt[2]*a^(5/4)*b) + ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(
1/4)]/(2*a^(5/4)) - ((5*a^2 - b)*Log[1 + (Sqrt[a]*x^2)/Sqrt[-b + a*x^4] - (
Sqrt[2]*a^(1/4)*x)/(-b + a*x^4)^(1/4)])/(4*Sqrt[2]*a^(5/4)*b) + ((5*a^2 - b
)*Log[1 + (Sqrt[a]*x^2)/Sqrt[-b + a*x^4] + (Sqrt[2]*a^(1/4)*x)/(-b + a*x^4)
^(1/4)])/(4*Sqrt[2]*a^(5/4)*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-2b-ax^4+2x^8}{x^4\sqrt[4]{-b+ax^4}\left(-b+2ax^4\right)}\,dx=\int\left(\frac{1}{a\sqrt[4]{-b+ax^4}}+\frac{2}{x^4\sqrt[4]{-b+ax^4}}+\frac{-5a^2+b}{a\sqrt[4]{-b+ax^4}\left(-b+2ax^4\right)}\right)dx$$

$$=2\int\frac{1}{x^4\sqrt[4]{-b+ax^4}}\,dx+\frac{\int\frac{1}{\sqrt[4]{-b+ax^4}}\,dx}{a}+\frac{\left(-5a^2+b\right)\int\frac{1}{\sqrt[4]{-b+ax^4}\left(-b+2ax^4\right)}\,dx}{a}$$

$$=\frac{2\left(-b+ax^4\right)^{3/4}}{3bx^3}+\frac{\text{Subst}\left(\int\frac{1}{1-ax^4}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)}{a}+\frac{\left(-5a^2+b\right)\text{Subst}\left(\int\frac{1}{-b-a\ldots}\right.}{a}$$

$$=\frac{2\left(-b+ax^4\right)^{3/4}}{3bx^3}+\frac{\text{Subst}\left(\int\frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)}{2a}+\frac{\text{Subst}\left(\int\frac{1}{1+\sqrt{a}\,x^2}\,dx,x,\ldots\right.}{2a}$$

$$=\frac{2\left(-b+ax^4\right)^{3/4}}{3bx^3}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{2a^{5/4}}+\frac{\left(5a^2-b\right)\text{Subst}\left(\int\ldots\right.}{\ldots}$$

$$=\frac{2\left(-b+ax^4\right)^{3/4}}{3bx^3}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{2a^{5/4}}-\frac{\left(5a^2-b\right)\log\left(1+\ldots\right.}{4\sqrt{2}\ldots}$$

$$=\frac{2\left(-b+ax^4\right)^{3/4}}{3bx^3}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{2a^{5/4}}-\frac{\left(5a^2-b\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{-b+ax^4}}\right)}{2\sqrt{2}\,a^{5/4}b}+\frac{\left(5a^2-\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.31, size = 292, normalized size = 1.22

$$\frac{16a^{5/4}\left(ax^4-b\right)^{3/4}-6\sqrt{2}x^3\left(5a^2-b\right)\left(\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)-\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}+1\right)\right)-3\sqrt{2}x^3\left(5a^2-b\right)\left(\log\left(-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}+\frac{\sqrt{a}x^2}{\sqrt{ax^4-b}}+1\right)-\log\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}+\frac{\sqrt{a}x^2}{\sqrt{ax^4-b}}+1\right)\right)+12bx^3\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)+12bx^3\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{24a^{5/4}bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*b - a*x^4 + 2*x^8)/(x^4*(-b + a*x^4)^(1/4)*(-b + 2*a*x^4)),x]
```

```
[Out] (16*a^(5/4)*(-b + a*x^4)^(3/4) + 12*b*x^3*ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(
1/4)] - 6*Sqrt[2]*(5*a^2 - b)*x^3*(ArcTan[1 - (Sqrt[2]*a^(1/4)*x)/(-b + a*x
^4)^(1/4)] - ArcTan[1 + (Sqrt[2]*a^(1/4)*x)/(-b + a*x^4)^(1/4)]) + 12*b*x^3
*ArcTanh[(a^(1/4)*x)/(-b + a*x^4)^(1/4)] - 3*Sqrt[2]*(5*a^2 - b)*x^3*(Log[1
 + (Sqrt[a]*x^2)/Sqrt[-b + a*x^4] - (Sqrt[2]*a^(1/4)*x)/(-b + a*x^4)^(1/4)]
 - Log[1 + (Sqrt[a]*x^2)/Sqrt[-b + a*x^4] + (Sqrt[2]*a^(1/4)*x)/(-b + a*x^4
)^(1/4)]))/(24*a^(5/4)*b*x^3)
```

IntegrateAlgebraic [A] time = 1.01, size = 239, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-b}}\right)}{2a^{5/4}}+\frac{\left(5a^2-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}x\sqrt[4]{ax^4-b}}{\sqrt{ax^4-b}-\sqrt{a}x^2}\right)}{2\sqrt{2}\,a^{5/4}b}+\frac{\left(5a^2-b\right)\tanh^{-1}\left(\frac{\sqrt{ax^4-b}+\sqrt{a}x^2}{\sqrt{2}\sqrt[4]{a}x\sqrt[4]{ax^4-b}}\right)}{2\sqrt{2}\,a^{5/4}b}+\frac{2\left(ax^4-b\right)^{3/4}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-2*b - a*x^4 + 2*x^8)/(x^4*(-b + a*x^4)^(1/4)*(-b + 2*a
*x^4)),x]
```

```
[Out] (2*(-b + a*x^4)^(3/4))/(3*b*x^3) + ArcTan[(a^(1/4)*x)/(-b + a*x^4)^(1/4)]/(
2*a^(5/4)) + ((5*a^2 - b)*ArcTan[(Sqrt[2]*a^(1/4)*x*(-b + a*x^4)^(1/4))/(-(
Sqrt[a]*x^2) + Sqrt[-b + a*x^4])])/(2*Sqrt[2]*a^(5/4)*b) + ArcTanh[(a^(1/4)
*x)/(-b + a*x^4)^(1/4)]/(2*a^(5/4)) + ((5*a^2 - b)*ArcTanh[(Sqrt[a]*x^2 + S
qrt[-b + a*x^4])/(Sqrt[2]*a^(1/4)*x*(-b + a*x^4)^(1/4))])/(2*Sqrt[2]*a^(5/4
)*b)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-a*x^4-2*b)/x^4/(a*x^4-b)^(1/4)/(2*a*x^4-b),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^8 - ax^4 - 2\,b}{\left(2\,ax^4 - b\right)\left(ax^4 - b\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-a*x^4-2*b)/x^4/(a*x^4-b)^(1/4)/(2*a*x^4-b),x, algorithm="g
iac")
```

```
[Out] integrate((2*x^8 - a*x^4 - 2*b)/((2*a*x^4 - b)*(a*x^4 - b)^(1/4)*x^4), x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{2x^8 - a\,x^4 - 2b}{x^4\left(a\,x^4 - b\right)^{\frac{1}{4}}\left(2a\,x^4 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8-a*x^4-2*b)/x^4/(a*x^4-b)^(1/4)/(2*a*x^4-b),x)
```

```
[Out] int((2*x^8-a*x^4-2*b)/x^4/(a*x^4-b)^(1/4)/(2*a*x^4-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^8 - ax^4 - 2\,b}{\left(2\,ax^4 - b\right)\left(ax^4 - b\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-a*x^4-2*b)/x^4/(a*x^4-b)^(1/4)/(2*a*x^4-b),x, algorithm="m
axima")
```

```
[Out] integrate((2*x^8 - a*x^4 - 2*b)/((2*a*x^4 - b)*(a*x^4 - b)^(1/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{-2\,x^8 + a\,x^4 + 2\,b}{x^4\left(a\,x^4 - b\right)^{1/4}\left(b - 2\,a\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b + a*x^4 - 2*x^8)/(x^4*(a*x^4 - b)^(1/4)*(b - 2*a*x^4)),x)
```

```
[Out] int((2*b + a*x^4 - 2*x^8)/(x^4*(a*x^4 - b)^(1/4)*(b - 2*a*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-ax^4 - 2b + 2x^8}{x^4\sqrt[4]{ax^4 - b}\left(2ax^4 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8-a*x**4-2*b)/x**4/(a*x**4-b)**(1/4)/(2*a*x**4-b),x)
```

```
[Out] Integral((-a*x**4 - 2*b + 2*x**8)/(x**4*(a*x**4 - b)**(1/4)*(2*a*x**4 - b))
, x)
```

3.2137 $\int \frac{-1+x^{16}}{\sqrt{-1+x^4}\left(1+x^8+x^{16}\right)}\,dx$

Optimal. Leaf size=239

$$\frac{\tan^{-1}\left(\frac{\frac{x^4}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{x^4-1}}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\frac{x^4}{\sqrt{2}\sqrt[4]{3}}-\frac{\sqrt[4]{3}x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}\sqrt[4]{3}}}{x\sqrt{x^4-1}}\right)}{4\sqrt{2}\sqrt[4]{3}}-\frac{\tanh^{-1}\left(\frac{\frac{x^4}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{x^4-1}}\right)}{4\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^4}{\sqrt{2}\sqrt[4]{3}}+\frac{\sqrt[4]{3}x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}\sqrt[4]{3}}}{x\sqrt{x^4-1}}\right)}{4\sqrt{2}\sqrt[4]{3}}$$

Rubi [C] time = 4.44, antiderivative size = 853, normalized size of antiderivative = 3.57, number of steps used = 153, number of rules used = 22, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.880, Rules used = {1586, 6728, 1729, 1209, 1188, 222, 1185, 1215, 1457, 540, 253, 538, 537, 1248, 735, 844, 217, 206, 725, 204, 406, 409}

$$-\frac{1}{32}\sqrt{\frac{1}{6}\left(3+i\sqrt{3}\right)}\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-i\sqrt{3}\right)x^2+2}{\sqrt{2\left(3+i\sqrt{3}\right)}\sqrt{x^4-1}}\right)+\frac{1}{32}\sqrt{\frac{1}{6}\left(3+i\sqrt{3}\right)}\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{4-\left(1+i\sqrt{3}\right)^2x^2}{2\sqrt{2\left(3+i\sqrt{3}\right)}\sqrt{x^4-1}}\right)+\frac{\left(i+\sqrt{3}\right)\sqrt{x^2-1}\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{8\sqrt{6}\sqrt{x^4-1}}+\frac{3\left(1+i\sqrt{3}\right)\sqrt{x^2-1}\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{8\sqrt{2}\sqrt{x^4-1}}+\frac{3\left(1-i\sqrt{3}\right)\sqrt{x^2-1}\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{8\sqrt{2}\sqrt{x^4-1}}-\frac{\left(i-\sqrt{3}\right)\sqrt{x^2-1}\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{8\sqrt{6}\sqrt{x^4-1}}-\frac{\sqrt{1-x^2}\sqrt{x^2+1}\Pi\left(\frac{1}{2}\left(-i-\sqrt{3}\right);\sin^{-1}(x)|-1\right)}{4\sqrt{x^4-1}}-\frac{\sqrt{1-x^2}\sqrt{x^2+1}\Pi\left(-\frac{4}{\left(i-\sqrt{3}\right)^2};\sin^{-1}(x)|-1\right)}{4\sqrt{x^4-1}}-\frac{\sqrt{1-x^2}\sqrt{x^2+1}\Pi\left(\frac{1}{2}\left(i-\sqrt{3}\right);\sin^{-1}(x)|-1\right)}{4\sqrt{x^4-1}}-\frac{\sqrt{1-x^2}\sqrt{x^2+1}\Pi\left(\frac{1}{\sqrt{\frac{1}{2}\left(1-i\sqrt{3}\right)}};\sin^{-1}(x)|-1\right)}{4\sqrt{x^4-1}}-\frac{\sqrt{1-x^2}\sqrt{x^2+1}\Pi\left(\frac{2}{1-i\sqrt{3}};\sin^{-1}(x)|-1\right)}{4\sqrt{x^4-1}}-\frac{\sqrt{1-x^2}\sqrt{x^2+1}\Pi\left(\frac{1}{\sqrt{\frac{1}{2}\left(1+i\sqrt{3}\right)}};\sin^{-1}(x)|-1\right)}{4\sqrt{x^4-1}}-\frac{\sqrt{1-x^2}\sqrt{x^2+1}\Pi\left(\frac{2}{1+i\sqrt{3}};\sin^{-1}(x)|-1\right)}{4\sqrt{x^4-1}}-\frac{\sqrt{1-x^2}\sqrt{x^2+1}\Pi\left(-\frac{4}{\left(i+\sqrt{3}\right)^2};\sin^{-1}(x)|-1\right)}{4\sqrt{x^4-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^16)/(Sqrt[-1 + x^4]*(1 + x^8 + x^16)),x]
```

```
[Out] -1/32*(Sqrt[(3 + I*Sqrt[3])/6]*(I + Sqrt[3])*ArcTan[(2 + (1 - I*Sqrt[3])*x^
2)/(Sqrt[2*(3 + I*Sqrt[3])]*Sqrt[-1 + x^4])]) + (Sqrt[(3 + I*Sqrt[3])/6]*(I
 + Sqrt[3])*ArcTan[(4 - (1 + I*Sqrt[3])^2*x^2)/(2*Sqrt[2*(3 + I*Sqrt[3])]*S
qrt[-1 + x^4])])/32 - ((I - Sqrt[3])*Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF
[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(8*Sqrt[6]*Sqrt[-1 + x^4]) + (3*
(1 - I*Sqrt[3])*Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/S
qrt[-1 + x^2]], 1/2])/(8*Sqrt[2]*Sqrt[-1 + x^4]) + (3*(1 + I*Sqrt[3])*Sqrt[
-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])
/(8*Sqrt[2]*Sqrt[-1 + x^4]) + ((I + Sqrt[3])*Sqrt[-1 + x^2]*Sqrt[1 + x^2]*E
llipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(8*Sqrt[6]*Sqrt[-1 + x^4
]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticPi[(-I - Sqrt[3])/2, ArcSin[x], -
1])/(4*Sqrt[-1 + x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticPi[-4/(I - Sq
rt[3])^2, ArcSin[x], -1])/(4*Sqrt[-1 + x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]
*EllipticPi[(I - Sqrt[3])/2, ArcSin[x], -1])/(4*Sqrt[-1 + x^4]) - (Sqrt[1 -
 x^2]*Sqrt[1 + x^2]*EllipticPi[1/Sqrt[(1 - I*Sqrt[3])/2], ArcSin[x], -1])/(
4*Sqrt[-1 + x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticPi[2/(1 - I*Sqrt[3
]), ArcSin[x], -1])/(4*Sqrt[-1 + x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*Ellip
ticPi[1/Sqrt[(1 + I*Sqrt[3])/2], ArcSin[x], -1])/(4*Sqrt[-1 + x^4]) - (Sqrt
[1 - x^2]*Sqrt[1 + x^2]*EllipticPi[2/(1 + I*Sqrt[3]), ArcSin[x], -1])/(4*Sq
rt[-1 + x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticPi[-4/(I + Sqrt[3])^2,
 ArcSin[x], -1])/(4*Sqrt[-1 + x^4])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 540

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x
] + Dist[(b*c - a*d)/b, Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
```

```
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Simp[(e*x*(q + c*x^2))/(c*Sqrt[a + c*x^4]), x] - Simp[(Sqr
t[2]*e*q*Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticE[ArcSin[x/Sqrt[(a +
q*x^2)/(2*q)]], 1/2])/(Sqrt[-a]*c*Sqrt[a + c*x^4]), x] /; EqQ[c*d + e*q, 0]
 && IntegerQ[q]] /; FreeQ[{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[(c*d + e*q)/c, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e
/c, Int[(q - c*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[c*d + e*q, 0]] /; FreeQ[
{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1215

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[c/(c*d + e*q), Int[1/Sqrt[a + c*x^4], x], x] + Di
st[e/(c*d + e*q), Int[(q - c*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
 FreeQ[{a, c, d, e}, x] && GtQ[-(a*c), 0] &&  !LtQ[c, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1457

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + c*x^(2*n))^FracPart[p]/((d +
e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p]), Int[(d + e*x^n)^(p + q)*
(f + g*x^n)^r*(a/d + (c*x^n)/e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, n, p,
 q, r}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1729

```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Dist[d, I
nt[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, Int[(x*(a + c*x^4)^p)/(d
^2 - e^2*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^{16}}{\sqrt{-1+x^4}\left(1+x^8+x^{16}\right)}\,dx &= \int \frac{\sqrt{-1+x^4}\left(1+x^4+x^8+x^{12}\right)}{1+x^8+x^{16}}\,dx\\
&= \int \left(\frac{(1-x)\sqrt{-1+x^4}}{8\left(1-x+x^2\right)}+\frac{(1+x)\sqrt{-1+x^4}}{8\left(1+x+x^2\right)}+\frac{\sqrt{-1+x^4}}{4\left(1-x^2+x^4\right)}+\frac{\sqrt{-1+x^4}\left(1+x^4\right.}{2\left(1-x^4+x^8\right)}\right.\\
&= \frac{1}{8}\int \frac{(1-x)\sqrt{-1+x^4}}{1-x+x^2}\,dx+\frac{1}{8}\int \frac{(1+x)\sqrt{-1+x^4}}{1+x+x^2}\,dx+\frac{1}{4}\int \frac{\sqrt{-1+x^4}}{1-x^2+x^4}\,dx+\frac{1}{2}\\
&= \text{rest of steps removed due to Latex formating problem}
\end{aligned}$$

Mathematica [C] time = 2.75, size = 195, normalized size = 0.82

$$\frac{\sqrt{1-x^4}\left(4F\left(\sin^{-1}(x)|-1\right)-\Pi\left(-\frac{i}{2}-\frac{\sqrt{3}}{2};\sin^{-1}(x)|-1\right)-\Pi\left(\frac{i}{2}-\frac{\sqrt{3}}{2};\sin^{-1}(x)|-1\right)-\Pi\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2};\sin^{-1}(x)|-1\right)-\Pi\left(\frac{1}{2}-\frac{i\sqrt{3}}{2};\sin^{-1}(x)|-1\right)-\Pi\left(\frac{1}{2}+\frac{i\sqrt{3}}{2};\sin^{-1}(x)|-1\right)-\Pi\left(\frac{1}{2}\left(-i+\sqrt{3}\right);\sin^{-1}(x)|-1\right)-\Pi\left(\frac{1}{2}i\left(i+\sqrt{3}\right);\sin^{-1}(x)|-1\right)-\Pi\left(\frac{1}{2}\left(i+\sqrt{3}\right);\sin^{-1}(x)|-1\right)\right)}{4\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^16)/(Sqrt[-1 + x^4]*(1 + x^8 + x^16)),x]
```

```
[Out] (Sqrt[1 - x^4]*(4*EllipticF[ArcSin[x], -1] - EllipticPi[-1/2*I - Sqrt[3]/2,
 ArcSin[x], -1] - EllipticPi[I/2 - Sqrt[3]/2, ArcSin[x], -1] - EllipticPi[-
1/2 - (I/2)*Sqrt[3], ArcSin[x], -1] - EllipticPi[1/2 - (I/2)*Sqrt[3], ArcSi
n[x], -1] - EllipticPi[1/2 + (I/2)*Sqrt[3], ArcSin[x], -1] - EllipticPi[(-I
 + Sqrt[3])/2, ArcSin[x], -1] - EllipticPi[(I/2)*(I + Sqrt[3]), ArcSin[x],
-1] - EllipticPi[(I + Sqrt[3])/2, ArcSin[x], -1]))/(4*Sqrt[-1 + x^4])
```

IntegrateAlgebraic [C] time = 2.28, size = 278, normalized size = 1.16

$$\frac{\tan^{-1}\left(\frac{\frac{x^4}{\sqrt{2}}-\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{x^4-1}}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\frac{x^4}{\sqrt{2}\sqrt[4]{3}}+\frac{\left(2i-\sqrt{-1+4i\sqrt{3}}\right)x^2}{\sqrt{2}\sqrt[4]{3}}-\frac{1}{\sqrt{2}\sqrt[4]{3}}}{x\sqrt{x^4-1}}\right)}{4\sqrt{2}\sqrt[4]{3}}-\frac{\tanh^{-1}\left(\frac{\frac{x^4}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{x^4-1}}\right)}{4\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^4}{\sqrt{2}\sqrt[4]{3}}+\frac{\left(2i+\sqrt{-1-4i\sqrt{3}}\right)x^2}{\sqrt{2}\sqrt[4]{3}}-\frac{1}{\sqrt{2}\sqrt[4]{3}}}{x\sqrt{x^4-1}}\right)}{4\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^16)/(Sqrt[-1 + x^4]*(1 + x^8 + x^16)),x]
```

```
[Out] ArcTan[(-(1/Sqrt[2]) - x^2/Sqrt[2] + x^4/Sqrt[2])/(x*Sqrt[-1 + x^4])]/(4*Sq
rt[2]) + ArcTan[(-(1/(Sqrt[2]*3^(1/4))) + ((2*I - Sqrt[-1 + (4*I)*Sqrt[3]])
*x^2)/(Sqrt[2]*3^(1/4)) + x^4/(Sqrt[2]*3^(1/4)))/(x*Sqrt[-1 + x^4])]/(4*Sqr
t[2]*3^(1/4)) - ArcTanh[(-(1/Sqrt[2]) + x^2/Sqrt[2] + x^4/Sqrt[2])/(x*Sqrt[
-1 + x^4])]/(4*Sqrt[2]) - ArcTanh[(-(1/(Sqrt[2]*3^(1/4))) + ((2*I + Sqrt[-1
```

```
 - (4*I)*Sqrt[3]])*x^2)/(Sqrt[2]*3^(1/4)) + x^4/(Sqrt[2]*3^(1/4)))/(x*Sqrt[
-1 + x^4])]/(4*Sqrt[2]*3^(1/4))
```

fricas [B] time = 1.04, size = 1173, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^16-1)/(x^4-1)^(1/2)/(x^16+x^8+1),x, algorithm="fricas")
```

```
[Out] -1/24*3^(3/4)*sqrt(2)*arctan(1/3*(3*x^16 + 6*x^12 + 9*x^8 + 6*x^4 + sqrt(3)
*(2*3^(3/4)*sqrt(2)*(x^14 - 12*x^10 + 12*x^6 - x^2) + 3^(1/4)*sqrt(2)*(x^16
 - 28*x^12 + 45*x^8 - 28*x^4 + 1) + 12*(x^11 + x^7 + x^3 + 4*sqrt(3)*(x^9 -
 x^5))*sqrt(x^4 - 1))*sqrt((12*x^6 - 12*x^2 + sqrt(3)*(x^8 + x^4 + 1) + 2*s
qrt(x^4 - 1)*(3*3^(1/4)*sqrt(2)*x^3 + 3^(3/4)*sqrt(2)*(x^5 - x)))/(x^8 + x^
4 + 1)) + 12*sqrt(3)*(x^14 - x^2) + 6*sqrt(x^4 - 1)*(3*3^(3/4)*sqrt(2)*(x^1
1 - 3*x^7 + x^3) + 3^(1/4)*sqrt(2)*(x^13 - 12*x^9 + 12*x^5 - x)) + 3)/(x^16
 - 46*x^12 + 99*x^8 - 46*x^4 + 1)) + 1/24*3^(3/4)*sqrt(2)*arctan(1/3*(3*x^1
6 + 6*x^12 + 9*x^8 + 6*x^4 - sqrt(3)*(2*3^(3/4)*sqrt(2)*(x^14 - 12*x^10 + 1
2*x^6 - x^2) + 3^(1/4)*sqrt(2)*(x^16 - 28*x^12 + 45*x^8 - 28*x^4 + 1) - 12*
(x^11 + x^7 + x^3 + 4*sqrt(3)*(x^9 - x^5))*sqrt(x^4 - 1))*sqrt((12*x^6 - 12
*x^2 + sqrt(3)*(x^8 + x^4 + 1) - 2*sqrt(x^4 - 1)*(3*3^(1/4)*sqrt(2)*x^3 + 3
^(3/4)*sqrt(2)*(x^5 - x)))/(x^8 + x^4 + 1)) + 12*sqrt(3)*(x^14 - x^2) - 6*s
qrt(x^4 - 1)*(3*3^(3/4)*sqrt(2)*(x^11 - 3*x^7 + x^3) + 3^(1/4)*sqrt(2)*(x^1
3 - 12*x^9 + 12*x^5 - x)) + 3)/(x^16 - 46*x^12 + 99*x^8 - 46*x^4 + 1)) - 1/
96*3^(3/4)*sqrt(2)*log(12*(12*x^6 - 12*x^2 + sqrt(3)*(x^8 + x^4 + 1) + 2*sq
rt(x^4 - 1)*(3*3^(1/4)*sqrt(2)*x^3 + 3^(3/4)*sqrt(2)*(x^5 - x)))/(x^8 + x^4
 + 1)) + 1/96*3^(3/4)*sqrt(2)*log(12*(12*x^6 - 12*x^2 + sqrt(3)*(x^8 + x^4
+ 1) - 2*sqrt(x^4 - 1)*(3*3^(1/4)*sqrt(2)*x^3 + 3^(3/4)*sqrt(2)*(x^5 - x)))
/(x^8 + x^4 + 1)) - 1/8*sqrt(2)*arctan((x^8 - x^4 + 2*sqrt(2)*(x^5 - x^3 -
x)*sqrt(x^4 - 1) + (4*sqrt(x^4 - 1)*x^3 + sqrt(2)*(x^8 - 2*x^6 - 3*x^4 + 2*
x^2 + 1))*sqrt((x^8 + 4*x^6 - x^4 + 2*sqrt(2)*(x^5 + x^3 - x)*sqrt(x^4 - 1)
 - 4*x^2 + 1)/(x^8 - x^4 + 1)) + 1)/(x^8 - 4*x^6 - x^4 + 4*x^2 + 1)) + 1/8*
sqrt(2)*arctan((x^8 - x^4 - 2*sqrt(2)*(x^5 - x^3 - x)*sqrt(x^4 - 1) + (4*sq
rt(x^4 - 1)*x^3 - sqrt(2)*(x^8 - 2*x^6 - 3*x^4 + 2*x^2 + 1))*sqrt((x^8 + 4*
x^6 - x^4 - 2*sqrt(2)*(x^5 + x^3 - x)*sqrt(x^4 - 1) - 4*x^2 + 1)/(x^8 - x^4
 + 1)) + 1)/(x^8 - 4*x^6 - x^4 + 4*x^2 + 1)) - 1/32*sqrt(2)*log(4*(x^8 + 4*
x^6 - x^4 + 2*sqrt(2)*(x^5 + x^3 - x)*sqrt(x^4 - 1) - 4*x^2 + 1)/(x^8 - x^4
 + 1)) + 1/32*sqrt(2)*log(4*(x^8 + 4*x^6 - x^4 - 2*sqrt(2)*(x^5 + x^3 - x)*
sqrt(x^4 - 1) - 4*x^2 + 1)/(x^8 - x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{16}-1}{\left(x^{16}+x^8+1\right)\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^16-1)/(x^4-1)^(1/2)/(x^16+x^8+1),x, algorithm="giac")
```

```
[Out] integrate((x^16 - 1)/((x^16 + x^8 + 1)*sqrt(x^4 - 1)), x)
```

maple [C] time = 0.11, size = 685, normalized size = 2.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^16-1)/(x^4-1)^(1/2)/(x^16+x^8+1),x)
```

```
[Out] -I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*EllipticF(I*x,I)+1/8*(-1/2+1/
2*I*3^(1/2))*(1/2/(-3/2+1/2*I*3^(1/2))^(1/2)*arctanh((1/2+1/2*I*3^(1/2))*(x
^2+1/2-1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))^(1/2)/(x^4-1)^(1/2))+I*(-1/2-1/2
```

```
*I*3^(1/2))*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*EllipticPi(I*x,1/2-1
/2*I*3^(1/2),I))+1/8*(-1/2-1/2*I*3^(1/2))*(1/2/(-3/2-1/2*I*3^(1/2))^(1/2)*a
rctanh((1/2-1/2*I*3^(1/2))*(x^2+1/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))^(1/
2)/(x^4-1)^(1/2))+I*(-1/2+1/2*I*3^(1/2))*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-
1)^(1/2)*EllipticPi(I*x,1/2+1/2*I*3^(1/2),I))+1/16*sum(_alpha*(-1/(_alpha^2
-2)^(1/2)*arctanh(_alpha^2*(_alpha^2+x^2-1)/(_alpha^2-2)^(1/2)/(x^4-1)^(1/2
))+2*I*(-_alpha^3+_alpha)*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*Ellipt
icPi(I*x,_alpha^2-1,I)),_alpha=RootOf(_Z^4-_Z^2+1))+1/8*(1/2+1/2*I*3^(1/2))
*(-1/2/(-3/2-1/2*I*3^(1/2))^(1/2)*arctanh((-1/2+1/2*I*3^(1/2))*(x^2+1/2+1/2
*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))^(1/2)/(x^4-1)^(1/2))+I*(1/2-1/2*I*3^(1/2))
*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*EllipticPi(I*x,1/2+1/2*I*3^(1/2
),I))+1/8*(1/2-1/2*I*3^(1/2))*(-1/2/(-3/2+1/2*I*3^(1/2))^(1/2)*arctanh((-1/
2-1/2*I*3^(1/2))*(x^2+1/2-1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))^(1/2)/(x^4-1)
^(1/2))+I*(1/2+1/2*I*3^(1/2))*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*El
lipticPi(I*x,1/2-1/2*I*3^(1/2),I))+1/16*sum(_alpha*(-1/(_alpha^4-1)^(1/2)*a
rctanh(_alpha^2*(_alpha^6-_alpha^2+x^2)/(_alpha^4-1)^(1/2)/(x^4-1)^(1/2))+2
*I*(-_alpha^7+_alpha^3)*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*Elliptic
Pi(I*x,_alpha^6-_alpha^2,I)),_alpha=RootOf(_Z^8-_Z^4+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{16}-1}{\left(x^{16}+x^8+1\right)\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^16-1)/(x^4-1)^(1/2)/(x^16+x^8+1),x, algorithm="maxima")
```

```
[Out] integrate((x^16 - 1)/((x^16 + x^8 + 1)*sqrt(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{16}-1}{\sqrt{x^4-1}\,\left(x^{16}+x^8+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^16 - 1)/((x^4 - 1)^(1/2)*(x^8 + x^16 + 1)),x)
```

```
[Out] int((x^16 - 1)/((x^4 - 1)^(1/2)*(x^8 + x^16 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2+1\right)\left(x^4+1\right)\left(x^8+1\right)}{\sqrt{(x-1)(x+1)\left(x^2+1\right)}\,\left(x^2-x+1\right)\left(x^2+x+1\right)\left(x^4-x^2+1\right)\left(x^8-x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**16-1)/(x**4-1)**(1/2)/(x**16+x**8+1),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)*(x**4 + 1)*(x**8 + 1)/(sqrt((x - 1)*(x
+ 1)*(x**2 + 1))*(x**2 - x + 1)*(x**2 + x + 1)*(x**4 - x**2 + 1)*(x**8 - x*
*4 + 1)), x)
```

3.2138 $\int \frac{2abx-3ax^2+x^3}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-a^2d+2adx-(b+d)x^2+x^3\right)}\,dx$

Optimal. Leaf size=241

$$-\frac{\log\left(a^2d^{2/3}+\sqrt[3]{x^3(-a-b)+abx^2+x^4}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)+\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}-2ad^{2/3}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}+\frac{\log}{}$$

Rubi [F] time = 15.69, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2abx-3ax^2+x^3}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-a^2d+2adx-(b+d)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2*a*b*x - 3*a*x^2 + x^3)/((x^2*(-a + x)*(-b + x))^(1/3)*(-(a^2*d) + 2*
a*d*x - (b + d)*x^2 + x^3)),x]
```

```
[Out] (3*x*(1 - x/a)^(1/3)*(1 - x/b)^(1/3)*AppellF1[1/3, 1/3, 1/3, 4/3, x/a, x/b]
)/((a - x)*(b - x)*x^2)^(1/3) - (6*a*(b - d)*x^(2/3)*(-a + x)^(1/3)*(-b + x
)^(1/3)*Defer[Subst][Defer[Int][x^3/((-a + x^3)^(1/3)*(-b + x^3)^(1/3)*(a^2
*d - 2*a*d*x^3 + b*(1 + d/b)*x^6 - x^9)), x], x, x^(1/3)])/((a - x)*(b - x)
*x^2)^(1/3) + (3*(3*a - b - d)*x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[
Subst][Defer[Int][x^6/((-a + x^3)^(1/3)*(-b + x^3)^(1/3)*(a^2*d - 2*a*d*x^3
 + b*(1 + d/b)*x^6 - x^9)), x], x, x^(1/3)])/((a - x)*(b - x)*x^2)^(1/3) +
(3*a^2*d*x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][Defer[Int][1/((
-a + x^3)^(1/3)*(-b + x^3)^(1/3)*(-(a^2*d) + 2*a*d*x^3 - b*(1 + d/b)*x^6 +
x^9)), x], x, x^(1/3)])/((a - x)*(b - x)*x^2)^(1/3)
```

Rubi steps

$$\int \frac{2abx - 3ax^2 + x^3}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-a^2d + 2adx - (b+d)x^2 + x^3\right)}\,dx = \int \frac{x\left(2ab - 3ax + x^2\right)}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-a^2d + 2adx - (b+d)x^2 \cdots\right.}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\int \frac{\sqrt[3]{x}\left(2ab-3ax+x^2\right)}{\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\left(-a^2d+2adx-\cdots\right.}}{\sqrt[3]{x^2(-a+x)(-b+x)}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\text{Subst}\left(\int \frac{x^3(2a\cdots}{\sqrt[3]{-a+x^3}\,\sqrt[3]{-b+x^3}\,(\cdots}\right.}{\sqrt[3]{x^2(-a+x)(-b+x\cdots}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\text{Subst}\left(\int\left(\frac{1}{\sqrt[3]{-a+x^3}\,\sqrt[3]{-b+x^3}}\cdots\right.\right.}{\sqrt[3]{x^2(-a+\cdots}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-a+x^3}\,\sqrt[3]{-b+x^3}}\cdots\right.}{\sqrt[3]{x^2(-a+x)(-b+x)}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\text{Subst}\left(\int\left(\frac{2\cdots}{\sqrt[3]{-a+x^3}\,\sqrt[3]{-b+x^3}\cdots}\right.\right.}{\cdots}$$

$$= \frac{\left(3(3a-b-d)x^{2/3}\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\text{Subst}\left(\int \frac{\cdots}{\sqrt[3]{\cdots}}\right.}{\sqrt[3]{x^2(-a+x)(\cdots}}$$

$$= \frac{3x\sqrt[3]{1-\frac{x}{a}}\,\sqrt[3]{1-\frac{x}{b}}\,F_1\left(\frac{1}{3};\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{x}{a},\frac{x}{b}\right)}{\sqrt[3]{(a-x)(b-x)x^2}} + \frac{\left(3(3a-b\cdots\right.}{\cdots}$$

Mathematica [F] time = 3.29, size = 0, normalized size = 0.00

$$\int \frac{2abx - 3ax^2 + x^3}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-a^2d + 2adx - (b+d)x^2 + x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2*a*b*x - 3*a*x^2 + x^3)/((x^2*(-a + x)*(-b + x))^(1/3)*(-(a^2*d
) + 2*a*d*x - (b + d)*x^2 + x^3)),x]
```

```
[Out] Integrate[(2*a*b*x - 3*a*x^2 + x^3)/((x^2*(-a + x)*(-b + x))^(1/3)*(-(a^2*d
) + 2*a*d*x - (b + d)*x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 1.12, size = 241, normalized size = 1.00

$$-\frac{\log\left(a^2d^{2/3} + \sqrt[3]{x^3(-a-b)+abx^2+x^4}\left(\sqrt[3]{d}\,x - a\sqrt[3]{d}\right) + \left(x^3(-a-b)+abx^2+x^4\right)^{2/3} - 2ad^{2/3}x + d^{2/3}x^2\right)}{2\sqrt[3]{d}} + \frac{\log\left(\sqrt[3]{x^3(-a-b)+abx^2+x^4} + a\sqrt[3]{d} - \sqrt[3]{d}\,x\right)}{\sqrt[3]{d}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^3(-a-b)+abx^2+x^4}}{\sqrt[3]{x^3(-a-b)+abx^2+x^4}-2a\sqrt[3]{d}+2\sqrt[3]{d}\,x}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*a*b*x - 3*a*x^2 + x^3)/((x^2*(-a + x)*(-b + x))^(1/3)
*(-(a^2*d) + 2*a*d*x - (b + d)*x^2 + x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3))/(-2*a*d^(1/3
) + 2*d^(1/3)*x + (a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3))])/d^(1/3) + Log[a*d
```

```
^(1/3) - d^(1/3)*x + (a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3)]/d^(1/3) - Log[a^
2*d^(2/3) - 2*a*d^(2/3)*x + d^(2/3)*x^2 + (-(a*d^(1/3)) + d^(1/3)*x)*(a*b*x
^2 + (-a - b)*x^3 + x^4)^(1/3) + (a*b*x^2 + (-a - b)*x^3 + x^4)^(2/3)]/(2*d
^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b*x-3*a*x^2+x^3)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a^2*d+2*a*d*x-(
b+d)*x^2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,abx - 3\,ax^2 + x^3}{\left((a-x)(b-x)x^2\right)^{\frac{1}{3}}\left(a^2d - 2\,adx + (b+d)x^2 - x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b*x-3*a*x^2+x^3)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a^2*d+2*a*d*x-(
b+d)*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate(-(2*a*b*x - 3*a*x^2 + x^3)/(((a - x)*(b - x)*x^2)^(1/3)*(a^2*d -
2*a*d*x + (b + d)*x^2 - x^3)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{2abx - 3a\,x^2 + x^3}{\left(x^2\left(-a+x\right)\left(-b+x\right)\right)^{\frac{1}{3}}\left(-a^2d + 2adx - \left(b+d\right)x^2 + x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*b*x-3*a*x^2+x^3)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a^2*d+2*a*d*x-(b+d)*x
^2+x^3),x)
```

```
[Out] int((2*a*b*x-3*a*x^2+x^3)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a^2*d+2*a*d*x-(b+d)*x
^2+x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,abx - 3\,ax^2 + x^3}{\left((a-x)(b-x)x^2\right)^{\frac{1}{3}}\left(a^2d - 2\,adx + (b+d)x^2 - x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b*x-3*a*x^2+x^3)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a^2*d+2*a*d*x-(
b+d)*x^2+x^3),x, algorithm="maxima")
```

```
[Out] -integrate((2*a*b*x - 3*a*x^2 + x^3)/(((a - x)*(b - x)*x^2)^(1/3)*(a^2*d -
2*a*d*x + (b + d)*x^2 - x^3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x^3 - 3\,a\,x^2 + 2\,a\,b\,x}{\left(x^2\,\left(a-x\right)\,\left(b-x\right)\right)^{1/3}\,\left(d\,a^2 - 2\,d\,a\,x - x^3 + \left(b+d\right)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 3*a*x^2 + 2*a*b*x)/((x^2*(a - x)*(b - x))^(1/3)*(x^2*(b + d) +
a^2*d - x^3 - 2*a*d*x)),x)
```

```
[Out] int(-(x^3 - 3*a*x^2 + 2*a*b*x)/((x^2*(a - x)*(b - x))^(1/3)*(x^2*(b + d) +
a^2*d - x^3 - 2*a*d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b*x-3*a*x**2+x**3)/(x**2*(-a+x)*(-b+x))**(1/3)/(-a**2*d+2*a*
d*x-(b+d)*x**2+x**3),x)
```

```
[Out] Timed out
```

3.2139 $\int \frac{(-2+k^2)x+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\left(-1+d+(-2d+k^2)x^2+dx^4\right)}\,dx$

Optimal. Leaf size=241

$$-\frac{\log\left(d^{2/3}x^4-2d^{2/3}x^2+d^{2/3}+\left(\sqrt[3]{d}-\sqrt[3]{d}\,x^2\right)\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}\right)}{4\sqrt[3]{d}}+\frac{\log$$

Rubi [F] time = 0.82, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+k^2)\,x+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\left(-1+d+(-2d+k^2)\,x^2+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3)*(-1 + d + (-2
*d + k^2)*x^2 + d*x^4)),x]
```

```
[Out] Defer[Subst][Defer[Int][(-2 + k^2 + k^2*x)/((-1 + d + (-2*d + k^2)*x + d*x^
2)*(1 + (-1 - k^2)*x + k^2*x^2)^(1/3)), x], x, x^2]/2
```

Rubi steps

$$\int \frac{(-2+k^2)\,x+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\left(-1+d+(-2d+k^2)\,x^2+dx^4\right)}\,dx=\int \frac{x\left(-2+k^2+k^2x^2\right)}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\left(-1+d+(-2d+k^2)\,x\right.}$$

$$=\frac{1}{2}\,\text{Subst}\left(\int \frac{-2+k^2+k^2x}{\sqrt[3]{(1-x)(1-k^2x)}\left(-1+d+(-2d\right.}\right.$$

$$=\frac{1}{2}\,\text{Subst}\left(\int \frac{-2+k^2+k^2x}{\left(-1+d+(-2d+k^2)\,x+dx^2\right)\sqrt[3]{1}}\right.$$

Mathematica [F] time = 11.80, size = 0, normalized size = 0.00

$$\int \frac{(-2+k^2)\,x+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\left(-1+d+(-2d+k^2)\,x^2+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3)*(-1 + d
 + (-2*d + k^2)*x^2 + d*x^4)),x]
```

```
[Out] Integrate[((-2 + k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3)*(-1 + d
 + (-2*d + k^2)*x^2 + d*x^4)), x]
```

IntegrateAlgebraic [A] time = 5.45, size = 241, normalized size = 1.00

$$-\frac{\log\left(d^{2/3}x^4-2d^{2/3}x^2+d^{2/3}+\left(\sqrt[3]{d}-\sqrt[3]{d}\,x^2\right)\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}\right)}{4\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{d}\,x^2-\sqrt[3]{d}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\right)}{2\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}{-2\sqrt[3]{d}\,x^2+2\sqrt[3]{d}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}\right)}{2\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3
)*(-1 + d + (-2*d + k^2)*x^2 + d*x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))/(2*d^(1/3) -
 2*d^(1/3)*x^2 + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))])/(2*d^(1/3)) + Log[
-d^(1/3) + d^(1/3)*x^2 + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3)]/(2*d^(1/3))
- Log[d^(2/3) - 2*d^(2/3)*x^2 + d^(2/3)*x^4 + (d^(1/3) - d^(1/3)*x^2)*(1 +
(-1 - k^2)*x^2 + k^2*x^4)^(1/3) + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(4*
d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((k^2-2)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d+(k^2-2*d)
*x^2+d*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3 + \left(k^2 - 2\right)x}{\left(dx^4 + \left(k^2 - 2\,d\right)x^2 + d - 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((k^2-2)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d+(k^2-2*d)
*x^2+d*x^4),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^3 + (k^2 - 2)*x)/((d*x^4 + (k^2 - 2*d)*x^2 + d - 1)*((k^2*
x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(k^2 - 2\right)x + k^2x^3}{\left(\left(-x^2 + 1\right)\left(-k^2x^2 + 1\right)\right)^{\frac{1}{3}}\left(-1 + d + \left(k^2 - 2d\right)x^2 + d\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((k^2-2)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d+(k^2-2*d)*x^2+d
*x^4),x)
```

```
[Out] int(((k^2-2)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d+(k^2-2*d)*x^2+d
*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3 + \left(k^2 - 2\right)x}{\left(dx^4 + \left(k^2 - 2\,d\right)x^2 + d - 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((k^2-2)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d+(k^2-2*d)
*x^2+d*x^4),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^3 + (k^2 - 2)*x)/((d*x^4 + (k^2 - 2*d)*x^2 + d - 1)*((k^2*
x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(k^2-2\right)+k^2\,x^3}{\left(\left(x^2-1\right)\left(k^2\,x^2-1\right)\right)^{1/3}\left(d\,x^4+\left(k^2-2\,d\right)\,x^2+d-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(k^2 - 2) + k^2*x^3)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/3)*(d - x^2*(2*d
- k^2) + d*x^4 - 1)),x)
```

```
[Out] int((x*(k^2 - 2) + k^2*x^3)/(((x^2 - 1)*(k^2*x^2 - 1))^(1/3)*(d - x^2*(2*d
- k^2) + d*x^4 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((k**2-2)*x+k**2*x**3)/((-x**2+1)*(-k**2*x**2+1))**(1/3)/(-1+d+(k
**2-2*d)*x**2+d*x**4),x)
```

```
[Out] Timed out
```

3.2140 $$\int \frac{(-1+2k^2)x-2k^4x^3+k^4x^5}{\left((1-x^2)(1-k^2x^2)\right)^{2/3}\left(1-d+(d-2k^2)x^2+k^4x^4\right)}\,dx$$

Optimal. Leaf size=241

$$\frac{\log\left(d^{2/3}x^4-2d^{2/3}x^2+d^{2/3}+\left(\sqrt[3]{d}-\sqrt[3]{d}\,x^2\right)\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{4/3}\right)}{4\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{\right.}}{}$$

Rubi [C] time = 2.87, antiderivative size = 451, normalized size of antiderivative = 1.87, number of steps used = 10, number of rules used = 7, integrand size = 73, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.096, Rules used = {1594, 6715, 6719, 1586, 6728, 137, 136}

$$\frac{3k^2\left(1-x^2\right)\left(\sqrt{d}-\sqrt{d-4k^2\left(1-k^2\right)}\right)\left(1-k^2x^2\right)F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{k^2(1-x^2)}{1-k^2},\frac{2k^4(1-x^2)}{-2(1-k^2)k^2+d-\sqrt{d}\sqrt{d-4k^2(1-k^2)}}\right)}{2\sqrt{d}\left(-\sqrt{d}\sqrt{d-4k^2\left(1-k^2\right)}+d-2\left(1-k^2\right)k^2\right)\sqrt[3]{\frac{1-k^2x^2}{1-k^2}}\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}}+\frac{3k^2\left(1-x^2\right)\left(\sqrt{d-4k^2\left(1-k^2\right)}+\sqrt{d}\right)\left(1-k^2x^2\right)F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{k^2(1-x^2)}{1-k^2},\frac{2k^4(1-x^2)}{-2(1-k^2)k^2+d+\sqrt{d}\sqrt{d-4k^2(1-k^2)}}\right)}{2\sqrt{d}\left(\sqrt{d}\sqrt{d-4k^2\left(1-k^2\right)}+d-2\left(1-k^2\right)k^2\right)\sqrt[3]{\frac{1-k^2x^2}{1-k^2}}\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + 2*k^2)*x - 2*k^4*x^3 + k^4*x^5)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)
*(1 - d + (d - 2*k^2)*x^2 + k^4*x^4)),x]
```

```
[Out] (3*k^2*(Sqrt[d] - Sqrt[d - 4*k^2*(1 - k^2)])*(1 - x^2)*(1 - k^2*x^2)*Appell
F1[1/3, -1/3, 1, 4/3, -((k^2*(1 - x^2))/(1 - k^2)), (2*k^4*(1 - x^2))/(d -
2*k^2*(1 - k^2) - Sqrt[d]*Sqrt[d - 4*k^2*(1 - k^2)])])/(2*Sqrt[d]*(d - 2*k^
2*(1 - k^2) - Sqrt[d]*Sqrt[d - 4*k^2*(1 - k^2)])*((1 - k^2*x^2)/(1 - k^2))^
(1/3)*((1 - x^2)*(1 - k^2*x^2))^(2/3)) + (3*k^2*(Sqrt[d] + Sqrt[d - 4*k^2*(
1 - k^2)])*(1 - x^2)*(1 - k^2*x^2)*AppellF1[1/3, -1/3, 1, 4/3, -((k^2*(1 -
x^2))/(1 - k^2)), (2*k^4*(1 - x^2))/(d - 2*k^2*(1 - k^2) + Sqrt[d]*Sqrt[d -
 4*k^2*(1 - k^2)])])/(2*Sqrt[d]*(d - 2*k^2*(1 - k^2) + Sqrt[d]*Sqrt[d - 4*k
^2*(1 - k^2)])*((1 - k^2*x^2)/(1 - k^2))^(1/3)*((1 - x^2)*(1 - k^2*x^2))^(2
/3))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
```

```
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(-1+2k^2\right)x-2k^4x^3+k^4x^5}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d+\left(d-2k^2\right)x^2+k^4x^4\right)}\,dx = \int \frac{x\left(-1+2k^2-2k^4x^2+k^4x^4\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d+\left(d-2k^2\right)x^2+k\right.}$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{-1+2k^2-2k^4x+k^4x^2}{\left((1-x)\left(1-k^2x\right)\right)^{2/3}\left(1-d+\left(d-2k^2\right.\right.}\right.$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\operatorname{Subst}\left(\int \frac{-1+2k^2-}{(1-x)^{2/3}\left(1-k^2x\right)^{2/3}(}\right.}{2\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\operatorname{Subst}\left(\int \frac{\sqrt[3]{1-k^2x}\left(-1+2k\right.}{(1-x)^{2/3}\left(1-d+\left(d-2\right.\right.}\right.}{2\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}}$$

$$= \frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\operatorname{Subst}\left(\int\left(\frac{\left(-k^2+\frac{k^2\sqrt{d-}}{\sqrt{}}\right.}{(1-x)^{2/3}\left(d-2k^2-\sqrt{}\right.}\right.\right.}{2\left(\left(1-\right.\right.}$$

$$= -\frac{\left(k^2\left(1-\frac{\sqrt{d-4k^2+4k^4}}{\sqrt{d}}\right)\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\operatorname{Sub}}{2\left(\left(1-x^2\right)\left(1\right.\right.}$$

$$= -\frac{\left(k^2\left(1-\frac{\sqrt{d-4k^2+4k^4}}{\sqrt{d}}\right)\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)\right)\operatorname{Subst}}{2\left(\left(1-x^2\right)\left(1-k^2x^2\right.\right.}$$

$$= \frac{3k^2\left(1-\frac{\sqrt{d-4k^2+4k^4}}{\sqrt{d}}\right)\left(1-x^2\right)\left(1-k^2x^2\right)F_1\left(\frac{1}{3};-\frac{1}{3},1\right.}{2\left(d-2k^2\left(1-k^2\right)-\sqrt{d}\sqrt{d-4k^2+4k^4}\right)}$$

Mathematica [F] time = 1.89, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+2k^2\right)x-2k^4x^3+k^4x^5}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(1-d+\left(d-2k^2\right)x^2+k^4x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + 2*k^2)*x - 2*k^4*x^3 + k^4*x^5)/(((1 - x^2)*(1 - k^2*x^2))
^(2/3)*(1 - d + (d - 2*k^2)*x^2 + k^4*x^4)),x]
```

```
[Out] Integrate[((-1 + 2*k^2)*x - 2*k^4*x^3 + k^4*x^5)/(((1 - x^2)*(1 - k^2*x^2))
^(2/3)*(1 - d + (d - 2*k^2)*x^2 + k^4*x^4)), x]
```

IntegrateAlgebraic [A] time = 5.53, size = 241, normalized size = 1.00

$$\frac{\log\left(d^{2/3}x^4-2d^{2/3}x^2+d^{2/3}+\left(\sqrt[3]{d}-\sqrt[3]{d}\,x^2\right)\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{4/3}\right)}{4\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{d}\,x^2-\sqrt[3]{d}+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}\right)}{2\sqrt[3]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}}{-2\sqrt[3]{d}\,x^2+2\sqrt[3]{d}+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}}\right)}{2\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + 2*k^2)*x - 2*k^4*x^3 + k^4*x^5)/(((1 - x^2)*(1 -
k^2*x^2))^(2/3)*(1 - d + (d - 2*k^2)*x^2 + k^4*x^4)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3))/(2*d^(1
/3) - 2*d^(1/3)*x^2 + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3))]])/d^(1/3) - Log
[-d^(1/3) + d^(1/3)*x^2 + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(2*d^(1/3))
 + Log[d^(2/3) - 2*d^(2/3)*x^2 + d^(2/3)*x^4 + (d^(1/3) - d^(1/3)*x^2)*(1 +
 (-1 - k^2)*x^2 + k^2*x^4)^(2/3) + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(4/3)]/(4
*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((2*k^2-1)*x-2*k^4*x^3+k^4*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(1-
d+(-2*k^2+d)*x^2+k^4*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^4x^5 - 2\,k^4x^3 + \left(2\,k^2 - 1\right)x}{\left(k^4x^4 - \left(2\,k^2 - d\right)x^2 - d + 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((2*k^2-1)*x-2*k^4*x^3+k^4*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(1-
d+(-2*k^2+d)*x^2+k^4*x^4),x, algorithm="giac")
```

```
[Out] integrate((k^4*x^5 - 2*k^4*x^3 + (2*k^2 - 1)*x)/((k^4*x^4 - (2*k^2 - d)*x^2
 - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(2/3)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(2k^2 - 1\right)x - 2k^4x^3 + k^4x^5}{\left(\left(-x^2 + 1\right)\left(-k^2x^2 + 1\right)\right)^{\frac{2}{3}}\left(1 - d + \left(-2k^2 + d\right)x^2 + k^4x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*k^2-1)*x-2*k^4*x^3+k^4*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(1-d+(-2*
k^2+d)*x^2+k^4*x^4),x)
```

```
[Out] int(((2*k^2-1)*x-2*k^4*x^3+k^4*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(1-d+(-2*
k^2+d)*x^2+k^4*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^4x^5 - 2\,k^4x^3 + \left(2\,k^2 - 1\right)x}{\left(k^4x^4 - \left(2\,k^2 - d\right)x^2 - d + 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((2*k^2-1)*x-2*k^4*x^3+k^4*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(1-
d+(-2*k^2+d)*x^2+k^4*x^4),x, algorithm="maxima")
```

```
[Out] integrate((k^4*x^5 - 2*k^4*x^3 + (2*k^2 - 1)*x)/((k^4*x^4 - (2*k^2 - d)*x^2
 - d + 1)*((k^2*x^2 - 1)*(x^2 - 1))^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{k^4\,x^5 - 2\,k^4\,x^3 + x\,\left(2\,k^2 - 1\right)}{\left(\left(x^2 - 1\right)\,\left(k^2\,x^2 - 1\right)\right)^{2/3}\,\left(k^4\,x^4 - d + x^2\,\left(d - 2\,k^2\right) + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^4*x^5 - 2*k^4*x^3 + x*(2*k^2 - 1))/(((x^2 - 1)*(k^2*x^2 - 1))^(2/3)*
(k^4*x^4 - d + x^2*(d - 2*k^2) + 1)),x)
```

```
[Out] int((k^4*x^5 - 2*k^4*x^3 + x*(2*k^2 - 1))/(((x^2 - 1)*(k^2*x^2 - 1))^(2/3)*
(k^4*x^4 - d + x^2*(d - 2*k^2) + 1)), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2*k**2-1)*x-2*k**4*x**3+k**4*x**5)/((-x**2+1)*(-k**2*x**2+1))**
(2/3)/(1-d+(-2*k**2+d)*x**2+k**4*x**4),x)
```

```
[Out] Timed out
```

3.2141 $\int \frac{(-2+x^6)(1-x^4+x^6)}{\sqrt[4]{1+x^6}\,(1+2x^6+x^8+x^{12})}\,dx$

Optimal. Leaf size=241

$$\frac{1}{2}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^6+1}}{\sqrt{x^6+1}-x^2}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^6+1}}{\sqrt{x^6+1}-x^2}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}$$

Rubi [F] time = 1.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+x^6)(1-x^4+x^6)}{\sqrt[4]{1+x^6}\,(1+2x^6+x^8+x^{12})}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + x^6)*(1 - x^4 + x^6))/((1 + x^6)^(1/4)*(1 + 2*x^6 + x^8 + x^12))
,x]
```

```
[Out] x*Hypergeometric2F1[1/6, 1/4, 7/6, -x^6] - 3*Defer[Int][1/((1 + x^6)^(1/4)*
(1 + 2*x^6 + x^8 + x^12)), x] + 2*Defer[Int][x^4/((1 + x^6)^(1/4)*(1 + 2*x^
6 + x^8 + x^12)), x] - 3*Defer[Int][x^6/((1 + x^6)^(1/4)*(1 + 2*x^6 + x^8 +
 x^12)), x] - Defer[Int][x^8/((1 + x^6)^(1/4)*(1 + 2*x^6 + x^8 + x^12)), x]
 - Defer[Int][x^10/((1 + x^6)^(1/4)*(1 + 2*x^6 + x^8 + x^12)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-2+x^6)(1-x^4+x^6)}{\sqrt[4]{1+x^6}\,(1+2x^6+x^8+x^{12})}\,dx &= \int \left(\frac{1}{\sqrt[4]{1+x^6}}-\frac{3-2x^4+3x^6+x^8+x^{10}}{\sqrt[4]{1+x^6}\,(1+2x^6+x^8+x^{12})}\right)dx \\
&= \int \frac{1}{\sqrt[4]{1+x^6}}\,dx-\int \frac{3-2x^4+3x^6+x^8+x^{10}}{\sqrt[4]{1+x^6}\,(1+2x^6+x^8+x^{12})}\,dx \\
&= x\,{}_2F_1\left(\frac{1}{6},\frac{1}{4};\frac{7}{6};-x^6\right)-\int \left(\frac{3}{\sqrt[4]{1+x^6}\,(1+2x^6+x^8+x^{12})}-\frac{}{\sqrt[4]{1+x^6}\,(1-}\right. \\
&= x\,{}_2F_1\left(\frac{1}{6},\frac{1}{4};\frac{7}{6};-x^6\right)+2\int \frac{x^4}{\sqrt[4]{1+x^6}\,(1+2x^6+x^8+x^{12})}\,dx-3\int \frac{}{\sqrt[4]{1}}
\end{aligned}$$

Mathematica [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(-2+x^6)(1-x^4+x^6)}{\sqrt[4]{1+x^6}\,(1+2x^6+x^8+x^{12})}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + x^6)*(1 - x^4 + x^6))/((1 + x^6)^(1/4)*(1 + 2*x^6 + x^8 +
x^12)),x]
```

```
[Out] Integrate[((-2 + x^6)*(1 - x^4 + x^6))/((1 + x^6)^(1/4)*(1 + 2*x^6 + x^8 +
x^12)), x]
```

IntegrateAlgebraic [A] time = 16.06, size = 241, normalized size = 1.00

$$\frac{1}{2}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^6+1}}{\sqrt{x^6+1}-x^2}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^6+1}}{\sqrt{x^6+1}-x^2}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^6+1}}{\sqrt{x^6+1}+x^2}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^6+1}}{\sqrt{x^6+1}+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^6)*(1 - x^4 + x^6))/((1 + x^6)^(1/4)*(1 + 2*x^6
 + x^8 + x^12)),x]
```

```
[Out] (Sqrt[(2 - Sqrt[2])/2]*ArcTan[(Sqrt[2 - Sqrt[2]]*x*(1 + x^6)^(1/4))/(-x^2 +
 Sqrt[1 + x^6])])/2 - (Sqrt[(2 + Sqrt[2])/2]*ArcTan[(Sqrt[2 + Sqrt[2]]*x*(1
 + x^6)^(1/4))/(-x^2 + Sqrt[1 + x^6])])/2 + (Sqrt[(2 - Sqrt[2])/2]*ArcTanh[
(Sqrt[2 - Sqrt[2]]*x*(1 + x^6)^(1/4))/(x^2 + Sqrt[1 + x^6])])/2 - (Sqrt[(2
+ Sqrt[2])/2]*ArcTanh[(Sqrt[2 + Sqrt[2]]*x*(1 + x^6)^(1/4))/(x^2 + Sqrt[1 +
 x^6])])/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-x^4+1)/(x^6+1)^(1/4)/(x^12+x^8+2*x^6+1),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}-x^{4}+1\right)\left(x^{6}-2\right)}{\left(x^{12}+x^{8}+2\,x^{6}+1\right)\left(x^{6}+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-x^4+1)/(x^6+1)^(1/4)/(x^12+x^8+2*x^6+1),x, algorithm
="giac")
```

```
[Out] integrate((x^6 - x^4 + 1)*(x^6 - 2)/((x^12 + x^8 + 2*x^6 + 1)*(x^6 + 1)^(1/
4)), x)
```

maple [C] time = 26.68, size = 682, normalized size = 2.83

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-2)*(x^6-x^4+1)/(x^6+1)^(1/4)/(x^12+x^8+2*x^6+1),x)
```

```
[Out] -1/16*RootOf(_Z^8+16)^5*ln((RootOf(_Z^8+16)^8*x^4-4*(x^6+1)^(1/2)*RootOf(_Z
^8+16)^6*x^2+4*RootOf(_Z^8+16)^4*x^6-8*(x^6+1)^(1/4)*RootOf(_Z^8+16)^5*x^3-
4*RootOf(_Z^8+16)^4*x^4+16*RootOf(_Z^8+16)^3*(x^6+1)^(3/4)*x+16*RootOf(_Z^8
+16)^2*(x^6+1)^(1/2)*x^2-16*x^6+4*RootOf(_Z^8+16)^4-16)/(RootOf(_Z^8+16)^4*
x^4-4*x^6-4))-1/4*RootOf(_Z^8+16)*ln((RootOf(_Z^8+16)^8*x^4+4*(x^6+1)^(1/2)
*RootOf(_Z^8+16)^6*x^2+4*RootOf(_Z^8+16)^4*x^6+8*(x^6+1)^(1/4)*RootOf(_Z^8+
16)^5*x^3+4*RootOf(_Z^8+16)^4*x^4+16*RootOf(_Z^8+16)^3*(x^6+1)^(3/4)*x+16*R
ootOf(_Z^8+16)^2*(x^6+1)^(1/2)*x^2+16*x^6+4*RootOf(_Z^8+16)^4+16)/(RootOf(_
Z^8+16)^4*x^4-4*x^6-4))-1/8*RootOf(_Z^8+16)^3*ln(-(-RootOf(_Z^8+16)^10*x^4+
4*RootOf(_Z^8+16)^6*x^6+4*x^4*RootOf(_Z^8+16)^6-16*RootOf(_Z^8+16)^4*(x^6+1
)^(1/2)*x^2-16*RootOf(_Z^8+16)^2*x^6+32*RootOf(_Z^8+16)^3*(x^6+1)^(3/4)*x+4
*RootOf(_Z^8+16)^6-64*RootOf(_Z^8+16)*(x^6+1)^(1/4)*x^3+64*x^2*(x^6+1)^(1/2
)-16*RootOf(_Z^8+16)^2)/(RootOf(_Z^8+16)^4*x^4+4*x^6+4))+1/32*RootOf(_Z^8+1
6)^7*ln(-(-RootOf(_Z^8+16)^10*x^4+4*RootOf(_Z^8+16)^6*x^6-4*x^4*RootOf(_Z^8
+16)^6+16*RootOf(_Z^8+16)^4*(x^6+1)^(1/2)*x^2+16*RootOf(_Z^8+16)^2*x^6+32*R
ootOf(_Z^8+16)^3*(x^6+1)^(3/4)*x+4*RootOf(_Z^8+16)^6+64*RootOf(_Z^8+16)*(x^
6+1)^(1/4)*x^3+64*x^2*(x^6+1)^(1/2)+16*RootOf(_Z^8+16)^2)/(RootOf(_Z^8+16)^
4*x^4+4*x^6+4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}-x^{4}+1\right)\left(x^{6}-2\right)}{\left(x^{12}+x^{8}+2\,x^{6}+1\right)\left(x^{6}+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-2)*(x^6-x^4+1)/(x^6+1)^(1/4)/(x^12+x^8+2*x^6+1),x, algorithm
="maxima")
```

```
[Out] integrate((x^6 - x^4 + 1)*(x^6 - 2)/((x^12 + x^8 + 2*x^6 + 1)*(x^6 + 1)^(1/
4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^6-2\right)\,\left(x^6-x^4+1\right)}{{\left(x^6+1\right)}^{1/4}\,\left(x^{12}+x^8+2\,x^6+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - 2)*(x^6 - x^4 + 1))/((x^6 + 1)^(1/4)*(2*x^6 + x^8 + x^12 + 1)),
x)
```

```
[Out] int(((x^6 - 2)*(x^6 - x^4 + 1))/((x^6 + 1)^(1/4)*(2*x^6 + x^8 + x^12 + 1)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6} - 2\right) \left(x^{6} - x^{4} + 1\right)}{\sqrt[4]{\left(x^{2} + 1\right) \left(x^{4} - x^{2} + 1\right)} \left(x^{12} + x^{8} + 2 x^{6} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-2)*(x**6-x**4+1)/(x**6+1)**(1/4)/(x**12+x**8+2*x**6+1),x)
```

```
[Out] Integral((x**6 - 2)*(x**6 - x**4 + 1)/(((x**2 + 1)*(x**4 - x**2 + 1))**(1/4
)*(x**12 + x**8 + 2*x**6 + 1)), x)
```

3.2142 $\int \frac{(-q+2px^3)(aq+bx+apx^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^3(cq+dx+cpx^3)}\,dx$

Optimal. Leaf size=242

$$\frac{\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}\,(acpx^3+acq-2adx+2bcx)}{2c^2x^2}+\frac{\log\left(\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+px^3+q\right)(-ac^2p}{c^3}$$

Rubi [F] time = 18.81, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-q+2px^3)(aq+bx+apx^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^3(cq+dx+cpx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-q + 2*p*x^3)*(a*q + b*x + a*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3
+ p^2*x^6])/(x^3*(c*q + d*x + c*p*x^3)),x]
```

```
[Out] (2*a*p*Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6], x])/c - (a*q
*Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x^3, x])/c - ((b*c
- a*d)*Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x^2, x])/c^2
+ (d*(b*c - a*d)*Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x,
x])/(c^3*q) - (d^2*(b*c - a*d)*Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3
+ p^2*x^6]/(c*q + d*x + c*p*x^3), x])/(c^3*q) + (3*(b*c - a*d)*p*Defer[Int]
[(x*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6])/(c*q + d*x + c*p*x^3), x])
/c - (d*(b*c - a*d)*p*Defer[Int][(x^2*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^
2*x^6])/(c*q + d*x + c*p*x^3), x])/(c^2*q)
```

Rubi steps

$$\int \frac{(-q+2px^3)(aq+bx+apx^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^3(cq+dx+cpx^3)}\,dx=\int\left(\frac{2ap\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{c}-\frac{aq\sqrt{}}{}\right.$$

$$=-\frac{(bc-ad)\int\frac{\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^2}\,dx}{c^2}+\frac{(2ap)}{}$$

$$=-\frac{(bc-ad)\int\frac{\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^2}\,dx}{c^2}+\frac{(2ap)}{}$$

$$=-\frac{(bc-ad)\int\frac{\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^2}\,dx}{c^2}+\frac{(2ap)}{}$$

Mathematica [F] time = 2.32, size = 0, normalized size = 0.00

$$\int \frac{(-q+2px^3)(aq+bx+apx^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}}{x^3(cq+dx+cpx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-q + 2*p*x^3)*(a*q + b*x + a*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*
q*x^3 + p^2*x^6])/(x^3*(c*q + d*x + c*p*x^3)),x]
```

```
[Out] Integrate[((-q + 2*p*x^3)*(a*q + b*x + a*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*
q*x^3 + p^2*x^6])/(x^3*(c*q + d*x + c*p*x^3)), x]
```

IntegrateAlgebraic [A] time = 1.16, size = 242, normalized size = 1.00

$$\frac{\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}\left(acpx^3+acq-2adx+2bcx\right)}{2c^2x^2}+\frac{\log\left(\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+px^3+q\right)\left(-ac^2pq+ad^2-bcd\right)}{c^3}-\frac{2(ad-bc)\sqrt{2c^2pq-d^2}\tan^{-1}\left(\frac{x\sqrt{2c^2pq-d^2}}{c\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+cpx^3+cq+dx}\right)}{c^3}+\frac{\log(x)\left(ac^2pq-ad^2+bcd\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-q + 2*p*x^3)*(a*q + b*x + a*p*x^3)*Sqrt[q^2 - 2*p*q*x
^2 + 2*p*q*x^3 + p^2*x^6])/(x^3*(c*q + d*x + c*p*x^3)),x]
```

```
[Out] ((a*c*q + 2*b*c*x - 2*a*d*x + a*c*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 +
 p^2*x^6])/(2*c^2*x^2) - (2*(-(b*c) + a*d)*Sqrt[-d^2 + 2*c^2*p*q]*ArcTan[(S
qrt[-d^2 + 2*c^2*p*q]*x)/(c*q + d*x + c*p*x^3 + c*Sqrt[q^2 - 2*p*q*x^2 + 2*
p*q*x^3 + p^2*x^6])])/c^3 + ((b*c*d - a*d^2 + a*c^2*p*q)*Log[x])/c^3 + ((-(
b*c*d) + a*d^2 - a*c^2*p*q)*Log[q + p*x^3 + Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^
3 + p^2*x^6]])/c^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(a*p*x^3+a*q+b*x)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(
1/2)/x^3/(c*p*x^3+c*q+d*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(a*p*x^3+a*q+b*x)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(
1/2)/x^3/(c*p*x^3+c*q+d*x),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError >> type
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\left(2p\,x^3-q\right)\left(ap\,x^3+aq+bx\right)\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}}{x^3\left(cp\,x^3+cq+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*p*x^3-q)*(a*p*x^3+a*q+b*x)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)/x
^3/(c*p*x^3+c*q+d*x),x)
```

```
[Out] int((2*p*x^3-q)*(a*p*x^3+a*q+b*x)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)/x
^3/(c*p*x^3+c*q+d*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6+2\,pqx^3-2\,pqx^2+q^2}\left(apx^3+aq+bx\right)\left(2\,px^3-q\right)}{\left(cpx^3+cq+dx\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(a*p*x^3+a*q+b*x)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(
1/2)/x^3/(c*p*x^3+c*q+d*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)*(a*p*x^3 + a*q + b*x)
*(2*p*x^3 - q)/((c*p*x^3 + c*q + d*x)*x^3), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(q-2\,p\,x^3\right)\,\left(a\,p\,x^3+b\,x+a\,q\right)\,\sqrt{p^2\,x^6+2\,p\,q\,x^3-2\,p\,q\,x^2+q^2}}{x^3\,\left(c\,p\,x^3+d\,x+c\,q\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q - 2*p*x^3)*(a*q + b*x + a*p*x^3)*(p^2*x^6 + q^2 - 2*p*q*x^2 + 2*p*
q*x^3)^(1/2))/(x^3*(c*q + d*x + c*p*x^3)),x)
```

```
[Out] -int(((q - 2*p*x^3)*(a*q + b*x + a*p*x^3)*(p^2*x^6 + q^2 - 2*p*q*x^2 + 2*p*
q*x^3)^(1/2))/(x^3*(c*q + d*x + c*p*x^3)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x**3-q)*(a*p*x**3+a*q+b*x)*(p**2*x**6+2*p*q*x**3-2*p*q*x**2+
q**2)**(1/2)/x**3/(c*p*x**3+c*q+d*x),x)
```

```
[Out] Timed out
```

3.2143 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal. Leaf size=242

$$\frac{x}{2\sqrt{\sqrt{x^4+1}+x^2}}+2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-2\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)+\text{t}$$

Rubi [F] time = 0.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] Defer[Int][1/Sqrt[x^2 + Sqrt[1 + x^4]], x] - I*Defer[Int][1/((I - x)*Sqrt[x
^2 + Sqrt[1 + x^4]]), x] - I*Defer[Int][1/((I + x)*Sqrt[x^2 + Sqrt[1 + x^4]
]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^2}{(1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}\,dx &= \int \left(\frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}-\frac{2}{(1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}\right)dx\\
&= -\left(2\int \frac{1}{(1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}\,dx\right)+\int \frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx\\
&= -\left(2\int \left(\frac{i}{2(i-x)\sqrt{x^2+\sqrt{1+x^4}}}+\frac{i}{2(i+x)\sqrt{x^2+\sqrt{1+x^4}}}\right)dx\right)+\int \frac{}{\sqrt{x^2}}\\
&= -\left(i\int \frac{1}{(i-x)\sqrt{x^2+\sqrt{1+x^4}}}\,dx\right)-i\int \frac{1}{(i+x)\sqrt{x^2+\sqrt{1+x^4}}}\,dx+\int \frac{}{\sqrt{x^2}}
\end{aligned}$$

Mathematica [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]]), x]
```

IntegrateAlgebraic [A] time = 0.96, size = 326, normalized size = 1.35

$$\frac{x}{2\sqrt{\sqrt{x^4+1}+x^2}}+2\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-2\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)+\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}-2\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] x/(2*Sqrt[x^2 + Sqrt[1 + x^4]]) + 2*Sqrt[2]*ArcTan[(-(1/Sqrt[2]) + x^2/Sqrt
[2] + Sqrt[1 + x^4]/Sqrt[2])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])] - 2*Sqrt[1 + Sq
rt[2]]*ArcTan[(-Sqrt[1/2 + 1/Sqrt[2]] + Sqrt[1/2 + 1/Sqrt[2]]*x^2 + Sqrt[1/
2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])] + ArcTanh[(-(1
/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2])/(x*Sqrt[x^2 + Sqrt[1 + x^4
]])]/Sqrt[2] - 2*Sqrt[-1 + Sqrt[2]]*ArcTanh[(-Sqrt[-1/2 + 1/Sqrt[2]] + Sqrt
[-1/2 + 1/Sqrt[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2
+ Sqrt[1 + x^4]])]
```

fricas [B] time = 7.80, size = 420, normalized size = 1.74

$$-\frac{1}{2}\left(x^{3}-\sqrt{x^{4}+1}\,x\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}-2\sqrt{\sqrt{2}+1}\arctan\left(\frac{\left(2x^{2}-\left(x^{2}+\sqrt{2}+1\right)\sqrt{-8\sqrt{2}+12}+\sqrt{x^{4}+1}\left(\sqrt{-8\sqrt{2}+12}-2\right)+2\sqrt{2}-2\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}\sqrt{\sqrt{2}+1}}{4x}\right)-\sqrt{2}\arctan\left(-\frac{\left(\sqrt{2}x^{2}-\sqrt{2}\sqrt{x^{4}+1}\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}}{2x}\right)+\frac{1}{8}\sqrt{2}\log\left(4x^{4}+4\sqrt{x^{4}+1}\,x^{2}+2\left(\sqrt{2}x^{3}+\sqrt{2}\sqrt{x^{4}+1}\,x\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}+1\right)-\frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(\frac{2\left(\sqrt{2}x^{2}+2x^{2}+\left(x^{3}+\sqrt{2}\left(x^{3}+2x\right)-\sqrt{x^{4}+1}\left(\sqrt{2}x+x\right)+3x\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}\sqrt{\sqrt{2}-1}+\sqrt{x^{4}+1}\left(\sqrt{2}+1\right)\right)}{x^{2}+1}\right)+\frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(\frac{2\left(\sqrt{2}x^{2}+2x^{2}-\left(x^{3}+\sqrt{2}\left(x^{3}+2x\right)-\sqrt{x^{4}+1}\left(\sqrt{2}x+x\right)+3x\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}\sqrt{\sqrt{2}-1}+\sqrt{x^{4}+1}\left(\sqrt{2}+1\right)\right)}{x^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x^3 - sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) - 2*sqrt(sqrt(2) + 1
)*arctan(1/4*(2*x^2 - (x^2 + sqrt(2) + 1)*sqrt(-8*sqrt(2) + 12) + sqrt(x^4
+ 1)*(sqrt(-8*sqrt(2) + 12) - 2) + 2*sqrt(2) - 2)*sqrt(x^2 + sqrt(x^4 + 1))
*sqrt(sqrt(2) + 1)/x) - sqrt(2)*arctan(-1/2*(sqrt(2)*x^2 - sqrt(2)*sqrt(x^4
 + 1))*sqrt(x^2 + sqrt(x^4 + 1))/x) + 1/8*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 +
1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)
) + 1) - 1/2*sqrt(sqrt(2) - 1)*log(2*(sqrt(2)*x^2 + 2*x^2 + (x^3 + sqrt(2)*
(x^3 + 2*x) - sqrt(x^4 + 1)*(sqrt(2)*x + x) + 3*x)*sqrt(x^2 + sqrt(x^4 + 1)
)*sqrt(sqrt(2) - 1) + sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 + 1)) + 1/2*sqrt(sq
rt(2) - 1)*log(2*(sqrt(2)*x^2 + 2*x^2 - (x^3 + sqrt(2)*(x^3 + 2*x) - sqrt(x
^4 + 1)*(sqrt(2)*x + x) + 3*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(sqrt(2) - 1)
+ sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^2+\sqrt{x^4+1}}\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^2 + sqrt(x^4 + 1))*(x^2 + 1)), x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(x^2+1\right)\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int((x^2-1)/(x^2+1)/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^2+\sqrt{x^4+1}}\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 - 1)/(sqrt(x^2 + sqrt(x^4 + 1))*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2-1}{\left(x^2+1\right)\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + 1)*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
[Out] int((x^2 - 1)/((x^2 + 1)*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{\left(x^2+1\right)\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**2+(x**4+1)**(1/2))**(1/2),x)
[Out] Integral((x - 1)*(x + 1)/((x**2 + 1)*sqrt(x**2 + sqrt(x**4 + 1))), x)
```

3.2144 $\int \frac{(-1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx$

Optimal. Leaf size=242

$$\frac{1}{2}\sqrt{\sqrt{x^4+1}+x^2}\,x+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)}{\sqrt{2}}-2\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-2\sqrt{2}\,\tanh^{-1}\left(\right.$$

Rubi [F] time = 0.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2),x]
```

```
[Out] Defer[Int][Sqrt[x^2 + Sqrt[1 + x^4]], x] - I*Defer[Int][Sqrt[x^2 + Sqrt[1 +
 x^4]]/(I - x), x] - I*Defer[Int][Sqrt[x^2 + Sqrt[1 + x^4]]/(I + x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx &= \int \left(\sqrt{x^2+\sqrt{1+x^4}}-\frac{2\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\right)dx\\
&= -\left(2\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx\right)+\int \sqrt{x^2+\sqrt{1+x^4}}\,dx\\
&= -\left(2\int \left(\frac{i\sqrt{x^2+\sqrt{1+x^4}}}{2(i-x)}+\frac{i\sqrt{x^2+\sqrt{1+x^4}}}{2(i+x)}\right)dx\right)+\int \sqrt{x^2+\sqrt{1+x^4}}\,dx\\
&= -\left(i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{i-x}\,dx\right)-i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{i+x}\,dx+\int \sqrt{x^2+\sqrt{1+x^4}}\,d
\end{aligned}$$

Mathematica [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2),x]
```

```
[Out] Integrate[((-1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2), x]
```

IntegrateAlgebraic [A] time = 1.07, size = 326, normalized size = 1.35

$$\frac{1}{2}\sqrt{\sqrt{x^4+1}+x^2}\,x+\frac{\tan^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}-2\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\,x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-2\sqrt{2}\,\tanh^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)+2\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\,x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2),x]
```

```
[Out] (x*Sqrt[x^2 + Sqrt[1 + x^4]])/2 + ArcTan[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt
[1 + x^4]/Sqrt[2])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])]/Sqrt[2] - 2*Sqrt[-1 + Sqr
t[2]]*ArcTan[(-Sqrt[1/2 + 1/Sqrt[2]] + Sqrt[1/2 + 1/Sqrt[2]]*x^2 + Sqrt[1/2
 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])] - 2*Sqrt[2]*Arc
Tanh[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2])/(x*Sqrt[x^2 + Sqr
t[1 + x^4]])] + 2*Sqrt[1 + Sqrt[2]]*ArcTanh[(-Sqrt[-1/2 + 1/Sqrt[2]] + Sqrt
[-1/2 + 1/Sqrt[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2
+ Sqrt[1 + x^4]])]
```

fricas [B] time = 3.78, size = 399, normalized size = 1.65

$$\frac{1}{2}\sqrt{x^2+\sqrt{x^4+1}}\,x-2\sqrt{\sqrt{2}-1}\arctan\left(\frac{\left(2\sqrt{2}x^2+2x^2+\sqrt{x^4+1}\left(\left(\sqrt{2}+1\right)\sqrt{-8\sqrt{2}+12}-2\sqrt{2}-2\right)-\left(x^2+\sqrt{2}\left(x^2+2\right)+3\right)\sqrt{-8\sqrt{2}+12}+2\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}-1}}{4x}\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\left(\sqrt{2}x^2-\sqrt{2}\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{2x}\right)+\frac{1}{2}\sqrt{2}\log\left(4x^4+4\sqrt{x^4+1}\,x^2-2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)+\frac{1}{2}\sqrt{\sqrt{2}+1}\log\left(\frac{2\left(\sqrt{2}x^2+2x^2+\left(x^3+\sqrt{2}x-\sqrt{x^4+1}\,x+x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}+1}+\sqrt{x^4+1}\left(\sqrt{2}+1\right)\right)}{x^2+1}\right)-\frac{1}{2}\sqrt{\sqrt{2}+1}\log\left(\frac{2\left(\sqrt{2}x^2+2x^2-\left(x^3+\sqrt{2}x-\sqrt{x^4+1}\,x+x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}+1}+\sqrt{x^4+1}\left(\sqrt{2}+1\right)\right)}{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 + sqrt(x^4 + 1))*x - 2*sqrt(sqrt(2) - 1)*arctan(1/4*(2*sqrt(2)
*x^2 + 2*x^2 + sqrt(x^4 + 1)*((sqrt(2) + 1)*sqrt(-8*sqrt(2) + 12) - 2*sqrt(
2) - 2) - (x^2 + sqrt(2)*(x^2 + 2) + 3)*sqrt(-8*sqrt(2) + 12) + 2)*sqrt(x^2
 + sqrt(x^4 + 1))*sqrt(sqrt(2) - 1)/x) - 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x
^2 - sqrt(2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))/x) + 1/2*sqrt(2)*log(
4*x^4 + 4*sqrt(x^4 + 1)*x^2 - 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*sqr
t(x^2 + sqrt(x^4 + 1)) + 1) + 1/2*sqrt(sqrt(2) + 1)*log(2*(sqrt(2)*x^2 + 2*
x^2 + (x^3 + sqrt(2)*x - sqrt(x^4 + 1)*x + x)*sqrt(x^2 + sqrt(x^4 + 1))*sqr
t(sqrt(2) + 1) + sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 + 1)) - 1/2*sqrt(sqrt(2)
 + 1)*log(2*(sqrt(2)*x^2 + 2*x^2 - (x^3 + sqrt(2)*x - sqrt(x^4 + 1)*x + x)*
sqrt(x^2 + sqrt(x^4 + 1))*sqrt(sqrt(2) + 1) + sqrt(x^4 + 1)*(sqrt(2) + 1))/
(x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-1\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - 1)/(x^2 + 1), x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-1\right)\sqrt{x^2+\sqrt{x^4+1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1),x)
```

```
[Out] int((x^2-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-1\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - 1)/(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^2-1\right)\sqrt{\sqrt{x^4+1}+x^2}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^2 + 1),x)
[Out] int(((x^2 - 1)*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\sqrt{x^2+\sqrt{x^4+1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(x**2+(x**4+1)**(1/2))**(1/2)/(x**2+1),x)
[Out] Integral((x - 1)*(x + 1)*sqrt(x**2 + sqrt(x**4 + 1))/(x**2 + 1), x)
```

3.2145 $\int \sqrt{1+x^2}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=242

$$\frac{\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\left(2560x^3+2048x^2+345x+184\right)+\sqrt{x^2+1}\left(\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+}}}\right.}{}$$

Rubi [F] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{1+x^2}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + x^2]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]],x]
```

```
[Out] Defer[Int][Sqrt[1 + x^2]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]], x]
```

Rubi steps

$$\int \sqrt{1+x^2}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx=\int \sqrt{1+x^2}\sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$$

Mathematica [A] time = 0.65, size = 373, normalized size = 1.54

$$\frac{\left(x^2+\sqrt{x^2+1}x+1\right)\left(79065\left(2x^2+2\sqrt{x^2+1}x+1\right)\log\left(1-\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)-79065\left(2x^2+2\sqrt{x^2+1}x+1\right)\log\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}+1\right)+2\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\left(35840x^4+512\left(5\sqrt{\sqrt{x^2+1}+x}\sqrt{x^2+1}-6\sqrt{x^2+1}+4\sqrt{\sqrt{x^2+1}+x}+377\right)x^2+\left(2048\sqrt{\sqrt{x^2+1}+x}\sqrt{x^2+1}+175104\sqrt{x^2+1}+345\sqrt{\sqrt{x^2+1}+x}-1254\right)x+282\sqrt{x^2+1}-935\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+x}+184\sqrt{\sqrt{x^2+1}+x}+512\left(70\sqrt{x^2+1}+5\sqrt{\sqrt{x^2+1}+x}-6\right)x^3+78032\right)\right)}{80640\sqrt{x^2+1}\left(\sqrt{x^2+1}+x\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]],x]
```

```
[Out] ((1 + x^2 + x*Sqrt[1 + x^2])*(2*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]*(78032 +
35840*x^4 + 282*Sqrt[1 + x^2] + 184*Sqrt[x + Sqrt[1 + x^2]] - 935*Sqrt[1 +
x^2]*Sqrt[x + Sqrt[1 + x^2]] + 512*x^3*(-6 + 70*Sqrt[1 + x^2] + 5*Sqrt[x +
Sqrt[1 + x^2]]) + 512*x^2*(377 - 6*Sqrt[1 + x^2] + 4*Sqrt[x + Sqrt[1 + x^2]
] + 5*Sqrt[1 + x^2]*Sqrt[x + Sqrt[1 + x^2]]) + x*(-1254 + 175104*Sqrt[1 + x
^2] + 345*Sqrt[x + Sqrt[1 + x^2]] + 2048*Sqrt[1 + x^2]*Sqrt[x + Sqrt[1 + x^
2]])) + 79065*(1 + 2*x^2 + 2*x*Sqrt[1 + x^2])*Log[1 - Sqrt[1 + Sqrt[x + Sqr
t[1 + x^2]]]] - 79065*(1 + 2*x^2 + 2*x*Sqrt[1 + x^2])*Log[1 + Sqrt[1 + Sqrt
[x + Sqrt[1 + x^2]]]]))/(80640*Sqrt[1 + x^2]*(x + Sqrt[1 + x^2])^3)
```

IntegrateAlgebraic [A] time = 0.31, size = 242, normalized size = 1.00

$$\frac{\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\left(2560x^3+2048x^2+345x+184\right)+\sqrt{x^2+1}\left(\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\left(2560x^2+2048x-935\right)+\left(35840x^3-3072x^2+175104x+282\right)\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)+\left(35840x^4-3072x^3+193024x^2-1254x+78032\right)\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}}{80640\sqrt{x^2+1}x+40320\left(2x^2+1\right)}-\frac{251}{128}\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^2]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]],x]
```

```
[Out] ((78032 - 1254*x + 193024*x^2 - 3072*x^3 + 35840*x^4)*Sqrt[1 + Sqrt[x + Sqr
t[1 + x^2]]] + (184 + 345*x + 2048*x^2 + 2560*x^3)*Sqrt[x + Sqrt[1 + x^2]]*
Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] + Sqrt[1 + x^2]*((282 + 175104*x - 3072*x
```

```
^2 + 35840*x^3)*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] + (-935 + 2048*x + 2560*x
^2)*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]))/(80640*x*Sq
rt[1 + x^2] + 40320*(1 + 2*x^2)) - (251*ArcTanh[Sqrt[1 + Sqrt[x + Sqrt[1 +
x^2]]]])/128
```

fricas [A] time = 0.46, size = 118, normalized size = 0.49

$$-\frac{1}{40320}\left(1120\,x^2 - 2\,\sqrt{x^2+1}\,(9520\,x+141) + \left(1680\,x^2 - 5\,\sqrt{x^2+1}\,(336\,x-187) - 2215\,x - 184\right)\sqrt{x+\sqrt{x^2+1}} + 1818\,x - 78032\right)\sqrt{\sqrt{x+\sqrt{x^2+1}}+1} - \frac{251}{256}\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}+1\right) + \frac{251}{256}\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/40320*(1120*x^2 - 2*sqrt(x^2 + 1)*(9520*x + 141) + (1680*x^2 - 5*sqrt(x^
2 + 1)*(336*x - 187) - 2215*x - 184)*sqrt(x + sqrt(x^2 + 1)) + 1818*x - 780
32)*sqrt(sqrt(x + sqrt(x^2 + 1)) + 1) - 251/256*log(sqrt(sqrt(x + sqrt(x^2
+ 1)) + 1) + 1) + 251/256*log(sqrt(sqrt(x + sqrt(x^2 + 1)) + 1) - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2),x, algorithm="gia
c")
```

```
[Out] Timed out
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+1}\,\sqrt{1+\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] int((x^2+1)^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+1}\,\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x^2 + 1)*sqrt(sqrt(x + sqrt(x^2 + 1)) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\sqrt{x+\sqrt{x^2+1}}+1}\,\sqrt{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2)*(x^2 + 1)^(1/2),x)
```

```
[Out] int(((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2)*(x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+1}\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)*(1+(x+(x**2+1)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)*sqrt(sqrt(x + sqrt(x**2 + 1)) + 1), x)
```

3.2146 $\int \frac{x(-a+x)}{\left(x^2(-a+x)\right)^{2/3}\left(a^2d-2adx+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=243

$$\frac{\log\left(x-\sqrt[6]{d}\,\sqrt[3]{x^3-ax^2}\right)}{2ad^{2/3}}+\frac{\log\left(\sqrt[6]{d}\,\sqrt[3]{x^3-ax^2}+x\right)}{2ad^{2/3}}-\frac{\log\left(-\sqrt[6]{d}\,x\sqrt[3]{x^3-ax^2}+\sqrt[3]{d}\,\left(x^3-ax^2\right)^{2/3}+x^2\right)}{4ad^{2/3}}-\frac{\log\left(\sqrt[6]{d}\,x\right.}{}$$

Rubi [A] time = 0.84, antiderivative size = 418, normalized size of antiderivative = 1.72, number of steps used = 9, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6719, 911, 105, 59, 91}

$$-\frac{x^{4/3}(x-a)^{2/3}\log\left(-2a\sqrt{d}\left(\sqrt{d}+1\right)-2(1-d)x\right)}{4ad^{2/3}\left(-\left(x^2(a-x)\right)\right)^{2/3}}-\frac{x^{4/3}(x-a)^{2/3}\log\left(2(1-d)x-2a\left(1-\sqrt{d}\right)\sqrt{d}\right)}{4ad^{2/3}\left(-\left(x^2(a-x)\right)\right)^{2/3}}+\frac{3x^{4/3}(x-a)^{2/3}\log\left(-\sqrt[3]{x-a}-\frac{\sqrt[3]{x}}{\sqrt[6]{d}}\right)}{4ad^{2/3}\left(-\left(x^2(a-x)\right)\right)^{2/3}}+\frac{3x^{4/3}(x-a)^{2/3}\log\left(\frac{\sqrt[3]{x}}{\sqrt[6]{d}}-\sqrt[3]{x-a}\right)}{4ad^{2/3}\left(-\left(x^2(a-x)\right)\right)^{2/3}}+\frac{\sqrt{3}\,x^{4/3}(x-a)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-a}}\right)}{2ad^{2/3}\left(-\left(x^2(a-x)\right)\right)^{2/3}}+\frac{\sqrt{3}\,x^{4/3}(x-a)^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-a}}+\frac{1}{\sqrt{3}}\right)}{2ad^{2/3}\left(-\left(x^2(a-x)\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-a + x))/((x^2*(-a + x))^(2/3)*(a^2*d - 2*a*d*x + (-1 + d)*x^2)),x]
```

```
[Out] (Sqrt[3]*x^(4/3)*(-a + x)^(2/3)*ArcTan[1/Sqrt[3] - (2*x^(1/3))/(Sqrt[3]*d^(
1/6)*(-a + x)^(1/3))])/(2*a*d^(2/3)*(-((a - x)*x^2))^(2/3)) + (Sqrt[3]*x^(4
/3)*(-a + x)^(2/3)*ArcTan[1/Sqrt[3] + (2*x^(1/3))/(Sqrt[3]*d^(1/6)*(-a + x)
^(1/3))])/(2*a*d^(2/3)*(-((a - x)*x^2))^(2/3)) - (x^(4/3)*(-a + x)^(2/3)*Lo
g[-2*a*(1 + Sqrt[d])*Sqrt[d] - 2*(1 - d)*x])/(4*a*d^(2/3)*(-((a - x)*x^2))^
(2/3)) - (x^(4/3)*(-a + x)^(2/3)*Log[-2*a*(1 - Sqrt[d])*Sqrt[d] + 2*(1 - d)
*x])/(4*a*d^(2/3)*(-((a - x)*x^2))^(2/3)) + (3*x^(4/3)*(-a + x)^(2/3)*Log[-
(x^(1/3)/d^(1/6)) - (-a + x)^(1/3)])/(4*a*d^(2/3)*(-((a - x)*x^2))^(2/3)) +
 (3*x^(4/3)*(-a + x)^(2/3)*Log[x^(1/3)/d^(1/6) - (-a + x)^(1/3)])/(4*a*d^(2
/3)*(-((a - x)*x^2))^(2/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
```

```
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{x(-a+x)}{\left(x^2(-a+x)\right)^{2/3}\left(a^2d-2adx+(-1+d)x^2\right)}\,dx = \frac{\left(x^{4/3}(-a+x)^{2/3}\right)\int \frac{\sqrt[3]{-a+x}}{\sqrt[3]{x}\left(a^2d-2adx+(-1+d)x^2\right)}\,dx}{\left(x^2(-a+x)\right)^{2/3}}$$

$$= \frac{\left(x^{4/3}(-a+x)^{2/3}\right)\int \left(\frac{(-1+d)\sqrt[3]{-a+x}}{a\sqrt{d}\sqrt[3]{x}\left(-2a\sqrt{d}-2ad-2(1-d)x\right)} + \frac{(-1+}{a\sqrt{d}\sqrt[3]{x}\left(-2a\right.}\right.}{\left(x^2(-a+x)\right)^{2/3}}$$

$$= -\frac{\left((1-d)x^{4/3}(-a+x)^{2/3}\right)\int \frac{\sqrt[3]{-a+x}}{\sqrt[3]{x}\left(-2a\sqrt{d}-2ad-2(1-d)x\right)}\,dx}{a\sqrt{d}\left(x^2(-a+x)\right)^{2/3}} - \frac{\left((1\right.}{}$$

$$= \frac{\left((1-d)x^{4/3}(-a+x)^{2/3}\right)\int \frac{1}{\sqrt[3]{x}(-a+x)^{2/3}\left(-2a\sqrt{d}-2ad-2(1-d)x\right)}\,dx}{\left(1-\sqrt{d}\right)\sqrt{d}\left(x^2(-a+x)\right)^{2/3}}$$

$$= \frac{\sqrt{3}\,x^{4/3}(-a+x)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{-a+x}}\right)}{2ad^{2/3}\left(-\left((a-x)x^2\right)\right)^{2/3}} + \frac{\sqrt{3}\,x^{4/3}(-a}{}$$

Mathematica [C] time = 0.18, size = 71, normalized size = 0.29

$$-\frac{3x^2\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x}{\sqrt{d}(a-x)}\right)+{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x}{\sqrt{d}(x-a)}\right)\right)}{4ad\left(x^2(x-a)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-a + x))/((x^2*(-a + x))^(2/3)*(a^2*d - 2*a*d*x + (-1 + d)*x^
2)),x]
```

```
[Out] (-3*x^2*(Hypergeometric2F1[2/3, 1, 5/3, x/(Sqrt[d]*(a - x))] + Hypergeometr
ic2F1[2/3, 1, 5/3, x/(Sqrt[d]*(-a + x))]))/(4*a*d*(x^2*(-a + x))^(2/3))
```

IntegrateAlgebraic [A] time = 0.55, size = 243, normalized size = 1.00

$$\frac{\log\left(x-\sqrt[6]{d}\sqrt[3]{x^3-ax^2}\right)}{2ad^{2/3}}+\frac{\log\left(\sqrt[6]{d}\sqrt[3]{x^3-ax^2}+x\right)}{2ad^{2/3}}-\frac{\log\left(-\sqrt[6]{d}\,x\sqrt[3]{x^3-ax^2}+\sqrt[3]{d}\left(x^3-ax^2\right)^{2/3}+x^2\right)}{4ad^{2/3}}-\frac{\log\left(\sqrt[6]{d}\,x\sqrt[3]{x^3-ax^2}+\sqrt[3]{d}\left(x^3-ax^2\right)^{2/3}+x^2\right)}{4ad^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{d}\left(x^3-ax^2\right)^{2/3}+x^2}\right)}{2ad^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-a + x))/((x^2*(-a + x))^(2/3)*(a^2*d - 2*a*d*x + (-
1 + d)*x^2)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*d^(1/3)*(-(a*x^2) + x^3)^(2/3))
])/(a*d^(2/3)) + Log[x - d^(1/6)*(-(a*x^2) + x^3)^(1/3)]/(2*a*d^(2/3)) + Lo
```

```
g[x + d^(1/6)*(-(a*x^2) + x^3)^(1/3)]/(2*a*d^(2/3)) - Log[x^2 - d^(1/6)*x*(
-(a*x^2) + x^3)^(1/3) + d^(1/3)*(-(a*x^2) + x^3)^(2/3)]/(4*a*d^(2/3)) - Log
[x^2 + d^(1/6)*x*(-(a*x^2) + x^3)^(1/3) + d^(1/3)*(-(a*x^2) + x^3)^(2/3)]/(
4*a*d^(2/3))
```

fricas [A] time = 0.44, size = 167, normalized size = 0.69

$$\frac{2\sqrt{3}\left(d^{2}\right)^{\frac{1}{6}}d\arctan\left(\frac{\sqrt{3}\left(\left(d^{2}\right)^{\frac{1}{3}}x^{2}+2\left(-ax^{2}+x^{3}\right)^{\frac{2}{3}}d\right)\left(d^{2}\right)^{\frac{1}{6}}}{3\,dx^{2}}\right)-\left(d^{2}\right)^{\frac{2}{3}}\log\left(\frac{\left(d^{2}\right)^{\frac{2}{3}}x^{2}+\left(-ax^{2}+x^{3}\right)^{\frac{2}{3}}\left(d^{2}\right)^{\frac{1}{3}}d-\left(ad^{2}-d^{2}x\right)\left(-ax^{2}+x^{3}\right)^{\frac{1}{3}}}{x^{2}}\right)+2\left(d^{2}\right)^{\frac{2}{3}}\log\left(-\frac{\left(d^{2}\right)^{\frac{1}{3}}x^{2}-\left(-ax^{2}+x^{3}\right)^{\frac{2}{3}}d}{x^{2}}\right)}{4\,ad^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+x)/(x^2*(-a+x))^(2/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x, algorit
hm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(1/3*sqrt(3)*((d^2)^(1/3)*x^2 + 2*(-a*x^
2 + x^3)^(2/3)*d)*(d^2)^(1/6)/(d*x^2)) - (d^2)^(2/3)*log(((d^2)^(2/3)*x^2 +
 (-a*x^2 + x^3)^(2/3)*(d^2)^(1/3)*d - (a*d^2 - d^2*x)*(-a*x^2 + x^3)^(1/3))
/x^2) + 2*(d^2)^(2/3)*log(-((d^2)^(1/3)*x^2 - (-a*x^2 + x^3)^(2/3)*d)/x^2))
/(a*d^2)
```

giac [A] time = 2.64, size = 108, normalized size = 0.44

$$\frac{\sqrt{3}\,|d|^{\frac{4}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\,d^{\frac{1}{3}}\left(2\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}+\frac{1}{d^{\frac{1}{3}}}\right)\right)}{2\,ad^{2}}-\frac{|d|^{\frac{4}{3}}\log\left(\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}+\frac{\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}}{d^{\frac{1}{3}}}+\frac{1}{d^{\frac{2}{3}}}\right)}{4\,ad^{2}}+\frac{\log\left(\left|\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}-\frac{1}{d^{\frac{1}{3}}}\right|\right)}{2\,ad^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+x)/(x^2*(-a+x))^(2/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x, algorit
hm="giac")
```

```
[Out] 1/2*sqrt(3)*abs(d)^(4/3)*arctan(1/3*sqrt(3)*d^(1/3)*(2*(-a/x + 1)^(2/3) + 1
/d^(1/3)))/(a*d^2) - 1/4*abs(d)^(4/3)*log((-a/x + 1)^(4/3) + (-a/x + 1)^(2/
3)/d^(1/3) + 1/d^(2/3))/(a*d^2) + 1/2*log(abs((-a/x + 1)^(2/3) - 1/d^(1/3))
)/(a*d^(2/3))
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x\left(-a+x\right)}{\left(x^{2}\left(-a+x\right)\right)^{\frac{2}{3}}\left(a^{2}d-2adx+\left(-1+d\right)x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a+x)/(x^2*(-a+x))^(2/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x)
```

```
[Out] int(x*(-a+x)/(x^2*(-a+x))^(2/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(a-x)x}{\left(a^{2}d-2\,adx+(d-1)x^{2}\right)\left(-(a-x)x^{2}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+x)/(x^2*(-a+x))^(2/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x, algorit
hm="maxima")
```

```
[Out] -integrate((a - x)*x/((a^2*d - 2*a*d*x + (d - 1)*x^2)*(-(a - x)*x^2)^(2/3))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x\,(a-x)}{\left(-x^2\,(a-x)\right)^{2/3}\,\left(d\,a^2-2\,d\,a\,x+(d-1)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a - x))/((-x^2*(a - x))^(2/3)*(a^2*d + x^2*(d - 1) - 2*a*d*x)),x)
[Out] int(-(x*(a - x))/((-x^2*(a - x))^(2/3)*(a^2*d + x^2*(d - 1) - 2*a*d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+x)/(x**2*(-a+x))**(2/3)/(a**2*d-2*a*d*x+(-1+d)*x**2),x)
[Out] Timed out
```

3.2147 $\int \frac{x}{\sqrt[3]{x^2(-a+x)}\left(a^2d-2adx+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=243

$$\frac{\log\left(x-\sqrt[6]{d}\sqrt[3]{x^3-ax^2}\right)}{2a\sqrt[3]{d}}+\frac{\log\left(\sqrt[6]{d}\sqrt[3]{x^3-ax^2}+x\right)}{2a\sqrt[3]{d}}-\frac{\log\left(-\sqrt[6]{d}\,x\sqrt[3]{x^3-ax^2}+\sqrt[3]{d}\left(x^3-ax^2\right)^{2/3}+x^2\right)}{4a\sqrt[3]{d}}-\frac{\log\left(\sqrt[6]{d}\,x\right.}{}$$

Rubi [A] time = 0.61, antiderivative size = 418, normalized size of antiderivative = 1.72, number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6719, 911, 105, 59, 91}

$$-\frac{x^{2/3}\sqrt[3]{x-a}\log\left(-2a\sqrt{d}\left(\sqrt{d}+1\right)-2(1-d)x\right)}{4a\sqrt[3]{d}\sqrt[3]{-\left(x^2(a-x)\right)}}-\frac{x^{2/3}\sqrt[3]{x-a}\log\left(2(1-d)x-2a\left(1-\sqrt{d}\right)\sqrt{d}\right)}{4a\sqrt[3]{d}\sqrt[3]{-\left(x^2(a-x)\right)}}+\frac{3x^{2/3}\sqrt[3]{x-a}\log\left(-\sqrt[6]{d}\sqrt[3]{x-a}-\sqrt[3]{x}\right)}{4a\sqrt[3]{d}\sqrt[3]{-\left(x^2(a-x)\right)}}+\frac{3x^{2/3}\sqrt[3]{x-a}\log\left(\sqrt[6]{d}\sqrt[3]{x-a}-\sqrt[3]{x}\right)}{4a\sqrt[3]{d}\sqrt[3]{-\left(x^2(a-x)\right)}}+\frac{\sqrt{3}x^{2/3}\sqrt[3]{x-a}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{x}}\right)}{2a\sqrt[3]{d}\sqrt[3]{-\left(x^2(a-x)\right)}}+\frac{\sqrt{3}x^{2/3}\sqrt[3]{x-a}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{2a\sqrt[3]{d}\sqrt[3]{-\left(x^2(a-x)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((x^2*(-a + x))^(1/3)*(a^2*d - 2*a*d*x + (-1 + d)*x^2)),x]
```

```
[Out] (Sqrt[3]*x^(2/3)*(-a + x)^(1/3)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(-a + x)^(1/3
))/(Sqrt[3]*x^(1/3))])/(2*a*d^(1/3)*(-((a - x)*x^2))^(1/3)) + (Sqrt[3]*x^(2
/3)*(-a + x)^(1/3)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(-a + x)^(1/3))/(Sqrt[3]*x
^(1/3))])/(2*a*d^(1/3)*(-((a - x)*x^2))^(1/3)) - (x^(2/3)*(-a + x)^(1/3)*Lo
g[-2*a*(1 + Sqrt[d])*Sqrt[d] - 2*(1 - d)*x])/(4*a*d^(1/3)*(-((a - x)*x^2))^
(1/3)) - (x^(2/3)*(-a + x)^(1/3)*Log[-2*a*(1 - Sqrt[d])*Sqrt[d] + 2*(1 - d)
*x])/(4*a*d^(1/3)*(-((a - x)*x^2))^(1/3)) + (3*x^(2/3)*(-a + x)^(1/3)*Log[-
x^(1/3) - d^(1/6)*(-a + x)^(1/3)])/(4*a*d^(1/3)*(-((a - x)*x^2))^(1/3)) + (
3*x^(2/3)*(-a + x)^(1/3)*Log[-x^(1/3) + d^(1/6)*(-a + x)^(1/3)])/(4*a*d^(1/
3)*(-((a - x)*x^2))^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
```

```
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{x^2(-a+x)}\left(a^2d-2adx+(-1+d)x^2\right)}\,dx = \frac{\left(x^{2/3}\sqrt[3]{-a+x}\right)\int \frac{\sqrt[3]{x}}{\sqrt[3]{-a+x}\left(a^2d-2adx+(-1+d)x^2\right)}\,dx}{\sqrt[3]{x^2(-a+x)}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{-a+x}\right)\int \left(\frac{(-1+d)\sqrt[3]{x}}{a\sqrt{d}\sqrt[3]{-a+x}\left(-2a\sqrt{d}-2ad-2(1-d)x\right)} + \frac{(-}{a\sqrt{d}\sqrt[3]{-a+x}\,(-}\right.}{\sqrt[3]{x^2(-a+x)}}$$

$$= -\frac{\left((1-d)x^{2/3}\sqrt[3]{-a+x}\right)\int \frac{\sqrt[3]{x}}{\sqrt[3]{-a+x}\left(-2a\sqrt{d}-2ad-2(1-d)x\right)}\,dx}{a\sqrt{d}\sqrt[3]{x^2(-a+x)}} - \frac{\left((1-\right.}{}$$

$$= \frac{\left((1-d)x^{2/3}\sqrt[3]{-a+x}\right)\int \frac{1}{x^{2/3}\sqrt[3]{-a+x}\left(-2a\sqrt{d}-2ad-2(1-d)x\right)}\,dx}{\left(1-\sqrt{d}\right)\sqrt[3]{x^2(-a+x)}} - \frac{\left((1\right.}{}$$

$$= \frac{\sqrt{3}\,x^{2/3}\sqrt[3]{-a+x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[3]{-a+x}}{\sqrt{3}\sqrt[3]{x}}\right)}{2a\sqrt[3]{d}\sqrt[3]{-\left((a-x)x^2\right)}} + \frac{\sqrt{3}\,x^{2/3}\sqrt[3]{-a+x}}{2a\sqrt[3]{d}}$$

Mathematica [C] time = 0.13, size = 73, normalized size = 0.30

$$\frac{3x\left({}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{x}{\sqrt{d}(a-x)}\right)-{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{x}{\sqrt{d}(x-a)}\right)\right)}{2a\sqrt{d}\sqrt[3]{x^2(x-a)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((x^2*(-a + x))^(1/3)*(a^2*d - 2*a*d*x + (-1 + d)*x^2)),x]
```

```
[Out] (3*x*(Hypergeometric2F1[1/3, 1, 4/3, x/(Sqrt[d]*(a - x))] - Hypergeometric2
F1[1/3, 1, 4/3, x/(Sqrt[d]*(-a + x))]))/(2*a*Sqrt[d]*(x^2*(-a + x))^(1/3))
```

IntegrateAlgebraic [A] time = 0.50, size = 243, normalized size = 1.00

$$\frac{\log\left(x-\sqrt[6]{d}\sqrt[3]{x^3-ax^2}\right)}{2a\sqrt[3]{d}}+\frac{\log\left(\sqrt[6]{d}\sqrt[3]{x^3-ax^2}+x\right)}{2a\sqrt[3]{d}}-\frac{\log\left(-\sqrt[6]{d}\,x\sqrt[3]{x^3-ax^2}+\sqrt[3]{d}\left(x^3-ax^2\right)^{2/3}+x^2\right)}{4a\sqrt[3]{d}}-\frac{\log\left(\sqrt[6]{d}\,x\sqrt[3]{x^3-ax^2}+\sqrt[3]{d}\left(x^3-ax^2\right)^{2/3}+x^2\right)}{4a\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{d}\left(x^3-ax^2\right)^{2/3}+x^2}\right)}{2a\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((x^2*(-a + x))^(1/3)*(a^2*d - 2*a*d*x + (-1 + d)*x^2)
),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*d^(1/3)*(-(a*x^2) + x^3)^(2/3))])/(2
*a*d^(1/3)) + Log[x - d^(1/6)*(-(a*x^2) + x^3)^(1/3)]/(2*a*d^(1/3)) + Log[x
 + d^(1/6)*(-(a*x^2) + x^3)^(1/3)]/(2*a*d^(1/3)) - Log[x^2 - d^(1/6)*x*(-(a
```

```
*x^2) + x^3)^(1/3) + d^(1/3)*(-(a*x^2) + x^3)^(2/3)]/(4*a*d^(1/3)) - Log[x^
2 + d^(1/6)*x*(-(a*x^2) + x^3)^(1/3) + d^(1/3)*(-(a*x^2) + x^3)^(2/3)]/(4*a
*d^(1/3))
```

fricas [A] time = 0.51, size = 372, normalized size = 1.53

$$\left[\frac{\sqrt{3}\,d\sqrt{-\frac{1}{d^{\frac{2}{3}}}}\log\left(\frac{2\,a^{2}d-4\,adx+(2\,d+1)x^{2}+\sqrt{3}\left(d^{\frac{1}{3}}x^{2}+2\left(-ax^{2}+x^{3}\right)^{\frac{1}{3}}(ad-dx)+\left(-ax^{2}+x^{3}\right)^{\frac{2}{3}}d^{\frac{2}{3}}\right)\sqrt{-\frac{1}{d^{\frac{2}{3}}}}-3\left(-ax^{2}+x^{3}\right)^{\frac{2}{3}}d^{\frac{1}{3}}}{a^{2}d-2\,adx+(d-1)x^{2}}\right)+2\,d^{\frac{2}{3}}\log\left(-\frac{d^{\frac{2}{3}}x^{2}-\left(-ax^{2}+x^{3}\right)^{\frac{2}{3}}d}{x^{2}}\right)-d^{\frac{2}{3}}\log\left(\frac{d^{\frac{1}{3}}x^{2}-\left(-ax^{2}+x^{3}\right)^{\frac{1}{3}}(ad-dx)+\left(-ax^{2}+x^{3}\right)^{\frac{2}{3}}d^{\frac{2}{3}}}{x^{2}}\right)}{4\,ad}, -\frac{2\sqrt{3}\,d^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}x^{2}+2\left(-ax^{2}+x^{3}\right)^{\frac{2}{3}}d^{\frac{2}{3}}\right)}{3\,d^{\frac{1}{3}}x^{2}}\right)-2\,d^{\frac{2}{3}}\log\left(-\frac{d^{\frac{2}{3}}x^{2}-\left(-ax^{2}+x^{3}\right)^{\frac{2}{3}}d}{x^{2}}\right)+d^{\frac{2}{3}}\log\left(\frac{d^{\frac{1}{3}}x^{2}-\left(-ax^{2}+x^{3}\right)^{\frac{1}{3}}(ad-dx)+\left(-ax^{2}+x^{3}\right)^{\frac{2}{3}}d^{\frac{2}{3}}}{x^{2}}\right)}{4\,ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2*(-a+x))^(1/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x, algorithm="fri
cas")
```

```
[Out] [1/4*(sqrt(3)*d*sqrt(-1/d^(2/3))*log((2*a^2*d - 4*a*d*x + (2*d + 1)*x^2 + s
qrt(3)*(d^(1/3)*x^2 + 2*(-a*x^2 + x^3)^(1/3)*(a*d - d*x) + (-a*x^2 + x^3)^(
2/3)*d^(2/3))*sqrt(-1/d^(2/3)) - 3*(-a*x^2 + x^3)^(2/3)*d^(1/3))/(a^2*d - 2
*a*d*x + (d - 1)*x^2)) + 2*d^(2/3)*log(-(d^(2/3)*x^2 - (-a*x^2 + x^3)^(2/3)
*d)/x^2) - d^(2/3)*log((d^(1/3)*x^2 - (-a*x^2 + x^3)^(1/3)*(a*d - d*x) + (-
a*x^2 + x^3)^(2/3)*d^(2/3))/x^2))/(a*d), -1/4*(2*sqrt(3)*d^(2/3)*arctan(1/3
*sqrt(3)*(d^(1/3)*x^2 + 2*(-a*x^2 + x^3)^(2/3)*d^(2/3))/(d^(1/3)*x^2)) - 2*
d^(2/3)*log(-(d^(2/3)*x^2 - (-a*x^2 + x^3)^(2/3)*d)/x^2) + d^(2/3)*log((d^(
1/3)*x^2 - (-a*x^2 + x^3)^(1/3)*(a*d - d*x) + (-a*x^2 + x^3)^(2/3)*d^(2/3))
/x^2))/(a*d)]
```

giac [A] time = 0.50, size = 108, normalized size = 0.44

$$-\frac{\sqrt{3}\,|d|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\,\sqrt{3}\,d^{\frac{1}{3}}\left(2\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}+\frac{1}{d^{\frac{1}{3}}}\right)\right)}{2\,ad}-\frac{|d|^{\frac{2}{3}}\log\left(\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}+\frac{\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}}{d^{\frac{1}{3}}}+\frac{1}{d^{\frac{2}{3}}}\right)}{4\,ad}+\frac{\log\left(\left|\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}-\frac{1}{d^{\frac{1}{3}}}\right|\right)}{2\,ad^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2*(-a+x))^(1/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x, algorithm="gia
c")
```

```
[Out] -1/2*sqrt(3)*abs(d)^(2/3)*arctan(1/3*sqrt(3)*d^(1/3)*(2*(-a/x + 1)^(2/3) +
1/d^(1/3)))/(a*d) - 1/4*abs(d)^(2/3)*log((-a/x + 1)^(4/3) + (-a/x + 1)^(2/3
)/d^(1/3) + 1/d^(2/3))/(a*d) + 1/2*log(abs((-a/x + 1)^(2/3) - 1/d^(1/3)))/(
a*d^(1/3))
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^{2}\left(-a+x\right)\right)^{\frac{1}{3}}\left(a^{2}d-2adx+\left(-1+d\right)x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2*(-a+x))^(1/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x)
```

```
[Out] int(x/(x^2*(-a+x))^(1/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a^{2}d-2\,adx+(d-1)x^{2}\right)\left(-(a-x)x^{2}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2*(-a+x))^(1/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x, algorithm="max
ima")
```

```
[Out] integrate(x/((a^2*d - 2*a*d*x + (d - 1)*x^2)*(-(a - x)*x^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(-x^2\,\left(a-x\right)\right)^{1/3}\,\left(d\,a^2-2\,d\,a\,x+\left(d-1\right)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((-x^2*(a - x))^(1/3)*(a^2*d + x^2*(d - 1) - 2*a*d*x)),x)
[Out] int(x/((-x^2*(a - x))^(1/3)*(a^2*d + x^2*(d - 1) - 2*a*d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{x^{2}\left(-a+x\right)}\left(a^{2}d-2adx+dx^{2}-x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2*(-a+x))**(1/3)/(a**2*d-2*a*d*x+(-1+d)*x**2),x)
[Out] Integral(x/((x**2*(-a + x))**(1/3)*(a**2*d - 2*a*d*x + d*x**2 - x**2)), x)
```

3.2148 $\int \frac{-ax+x^2}{\left(x^2(-a+x)\right)^{2/3}\left(a^2d-2adx+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=243

$$\frac{\log\left(x-\sqrt[6]{d}\sqrt[3]{x^3-ax^2}\right)}{2ad^{2/3}}+\frac{\log\left(\sqrt[6]{d}\sqrt[3]{x^3-ax^2}+x\right)}{2ad^{2/3}}-\frac{\log\left(-\sqrt[6]{d}\,x\sqrt[3]{x^3-ax^2}+\sqrt[3]{d}\left(x^3-ax^2\right)^{2/3}+x^2\right)}{4ad^{2/3}}-\frac{\log\left(\sqrt[6]{d}\,x\right.}{}$$

Rubi [A] time = 0.78, antiderivative size = 418, normalized size of antiderivative = 1.72, number of steps used = 10, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1593, 6719, 911, 105, 59, 91}

$$-\frac{x^{4/3}(x-a)^{2/3}\log\left(-2a\sqrt{d}\left(\sqrt{d}+1\right)-2(1-d)x\right)}{4ad^{2/3}\left(-\left(x^2(a-x)\right)\right)^{2/3}}-\frac{x^{4/3}(x-a)^{2/3}\log\left(2(1-d)x-2a\left(1-\sqrt{d}\right)\sqrt{d}\right)}{4ad^{2/3}\left(-\left(x^2(a-x)\right)\right)^{2/3}}+\frac{3x^{4/3}(x-a)^{2/3}\log\left(-\sqrt[3]{x-a}-\frac{\sqrt[3]{x}}{\sqrt[6]{d}}\right)}{4ad^{2/3}\left(-\left(x^2(a-x)\right)\right)^{2/3}}+\frac{3x^{4/3}(x-a)^{2/3}\log\left(\frac{\sqrt[3]{x}}{\sqrt[6]{d}}-\sqrt[3]{x-a}\right)}{4ad^{2/3}\left(-\left(x^2(a-x)\right)\right)^{2/3}}+\frac{\sqrt{3}\,x^{4/3}(x-a)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-a}}\right)}{2ad^{2/3}\left(-\left(x^2(a-x)\right)\right)^{2/3}}+\frac{\sqrt{3}\,x^{4/3}(x-a)^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-a}}+\frac{1}{\sqrt{3}}\right)}{2ad^{2/3}\left(-\left(x^2(a-x)\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-(a*x) + x^2)/((x^2*(-a + x))^(2/3)*(a^2*d - 2*a*d*x + (-1 + d)*x^2)),
x]
```

```
[Out] (Sqrt[3]*x^(4/3)*(-a + x)^(2/3)*ArcTan[1/Sqrt[3] - (2*x^(1/3))/(Sqrt[3]*d^(
1/6)*(-a + x)^(1/3))])/(2*a*d^(2/3)*(-((a - x)*x^2))^(2/3)) + (Sqrt[3]*x^(4
/3)*(-a + x)^(2/3)*ArcTan[1/Sqrt[3] + (2*x^(1/3))/(Sqrt[3]*d^(1/6)*(-a + x)
^(1/3))])/(2*a*d^(2/3)*(-((a - x)*x^2))^(2/3)) - (x^(4/3)*(-a + x)^(2/3)*Lo
g[-2*a*(1 + Sqrt[d])*Sqrt[d] - 2*(1 - d)*x])/(4*a*d^(2/3)*(-((a - x)*x^2))^
(2/3)) - (x^(4/3)*(-a + x)^(2/3)*Log[-2*a*(1 - Sqrt[d])*Sqrt[d] + 2*(1 - d)
*x])/(4*a*d^(2/3)*(-((a - x)*x^2))^(2/3)) + (3*x^(4/3)*(-a + x)^(2/3)*Log[-
(x^(1/3)/d^(1/6)) - (-a + x)^(1/3)])/(4*a*d^(2/3)*(-((a - x)*x^2))^(2/3)) +
 (3*x^(4/3)*(-a + x)^(2/3)*Log[x^(1/3)/d^(1/6) - (-a + x)^(1/3)])/(4*a*d^(2
/3)*(-((a - x)*x^2))^(2/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
```

```
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{-ax+x^2}{\left(x^2(-a+x)\right)^{2/3}\left(a^2d-2adx+(-1+d)x^2\right)}\,dx = \int \frac{x(-a+x)}{\left(x^2(-a+x)\right)^{2/3}\left(a^2d-2adx+(-1+d)x^2\right)}\,dx$$

$$= \frac{\left(x^{4/3}(-a+x)^{2/3}\right)\int \frac{\sqrt[3]{-a+x}}{\sqrt[3]{x}\left(a^2d-2adx+(-1+d)x^2\right)}\,dx}{\left(x^2(-a+x)\right)^{2/3}}$$

$$= \frac{\left(x^{4/3}(-a+x)^{2/3}\right)\int \left(\frac{(-1+d)\sqrt[3]{-a+x}}{a\sqrt{d}\sqrt[3]{x}\left(-2a\sqrt{d}-2ad-2(1-d)x\right)} + \frac{(-1+}{a\sqrt{d}\sqrt[3]{x}\left(-2a\right.}\right.}{\left(x^2(-a+x)\right)^{2/3}}$$

$$= -\frac{\left((1-d)x^{4/3}(-a+x)^{2/3}\right)\int \frac{\sqrt[3]{-a+x}}{\sqrt[3]{x}\left(-2a\sqrt{d}-2ad-2(1-d)x\right)}\,dx}{a\sqrt{d}\left(x^2(-a+x)\right)^{2/3}} - \frac{\left((1\right.}{}$$

$$= \frac{\left((1-d)x^{4/3}(-a+x)^{2/3}\right)\int \frac{1}{\sqrt[3]{x}(-a+x)^{2/3}\left(-2a\sqrt{d}-2ad-2(1-d)x\right)}\,dx}{\left(1-\sqrt{d}\right)\sqrt{d}\left(x^2(-a+x)\right)^{2/3}}$$

$$= \frac{\sqrt{3}\,x^{4/3}(-a+x)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{-a+x}}\right)}{2ad^{2/3}\left(-\left((a-x)x^2\right)\right)^{2/3}} + \frac{\sqrt{3}\,x^{4/3}(-a}{}$$

Mathematica [C] time = 0.12, size = 71, normalized size = 0.29

$$-\frac{3x^2\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x}{\sqrt{d}(a-x)}\right)+{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x}{\sqrt{d}(x-a)}\right)\right)}{4ad\left(x^2(x-a)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-(a*x) + x^2)/((x^2*(-a + x))^(2/3)*(a^2*d - 2*a*d*x + (-1 + d)*
x^2)),x]
```

```
[Out] (-3*x^2*(Hypergeometric2F1[2/3, 1, 5/3, x/(Sqrt[d]*(a - x))] + Hypergeometr
ic2F1[2/3, 1, 5/3, x/(Sqrt[d]*(-a + x))]))/(4*a*d*(x^2*(-a + x))^(2/3))
```

IntegrateAlgebraic [A] time = 0.55, size = 243, normalized size = 1.00

$$\frac{\log\left(x-\sqrt[6]{d}\sqrt[3]{x^3-ax^2}\right)}{2ad^{2/3}}+\frac{\log\left(\sqrt[6]{d}\sqrt[3]{x^3-ax^2}+x\right)}{2ad^{2/3}}-\frac{\log\left(-\sqrt[6]{d}\,x\sqrt[3]{x^3-ax^2}+\sqrt[3]{d}\left(x^3-ax^2\right)^{2/3}+x^2\right)}{4ad^{2/3}}-\frac{\log\left(\sqrt[6]{d}\,x\sqrt[3]{x^3-ax^2}+\sqrt[3]{d}\left(x^3-ax^2\right)^{2/3}+x^2\right)}{4ad^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{d}\left(x^3-ax^2\right)^{2/3}+x^2}\right)}{2ad^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*x) + x^2)/((x^2*(-a + x))^(2/3)*(a^2*d - 2*a*d*x +
(-1 + d)*x^2)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*d^(1/3)*(-(a*x^2) + x^3)^(2/3))
])/(a*d^(2/3)) + Log[x - d^(1/6)*(-(a*x^2) + x^3)^(1/3)]/(2*a*d^(2/3)) + Lo
g[x + d^(1/6)*(-(a*x^2) + x^3)^(1/3)]/(2*a*d^(2/3)) - Log[x^2 - d^(1/6)*x*(
-(a*x^2) + x^3)^(1/3) + d^(1/3)*(-(a*x^2) + x^3)^(2/3)]/(4*a*d^(2/3)) - Log
[x^2 + d^(1/6)*x*(-(a*x^2) + x^3)^(1/3) + d^(1/3)*(-(a*x^2) + x^3)^(2/3)]/(
4*a*d^(2/3))
```

fricas [A] time = 0.42, size = 167, normalized size = 0.69

$$\frac{2\sqrt{3}\left(d^2\right)^{\frac{1}{6}}d\arctan\left(\frac{\sqrt{3}\left(\left(d^2\right)^{\frac{1}{3}}x^2+2\left(-ax^2+x^3\right)^{\frac{2}{3}}d\right)\left(d^2\right)^{\frac{1}{6}}}{3\,dx^2}\right)-\left(d^2\right)^{\frac{2}{3}}\log\left(\frac{\left(d^2\right)^{\frac{2}{3}}x^2+\left(-ax^2+x^3\right)^{\frac{2}{3}}\left(d^2\right)^{\frac{1}{3}}d-\left(ad^2-d^2x\right)\left(-ax^2+x^3\right)^{\frac{1}{3}}}{x^2}\right)+2\left(d^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(d^2\right)^{\frac{1}{3}}x^2-\left(-ax^2+x^3\right)^{\frac{2}{3}}d}{x^2}\right)}{4\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+x^2)/(x^2*(-a+x))^(2/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(1/3*sqrt(3)*((d^2)^(1/3)*x^2 + 2*(-a*x^
2 + x^3)^(2/3)*d)*(d^2)^(1/6)/(d*x^2)) - (d^2)^(2/3)*log(((d^2)^(2/3)*x^2 +
 (-a*x^2 + x^3)^(2/3)*(d^2)^(1/3)*d - (a*d^2 - d^2*x)*(-a*x^2 + x^3)^(1/3))
/x^2) + 2*(d^2)^(2/3)*log(-((d^2)^(1/3)*x^2 - (-a*x^2 + x^3)^(2/3)*d)/x^2))
/(a*d^2)
```

giac [A] time = 0.49, size = 108, normalized size = 0.44

$$\frac{\sqrt{3}|d|^{\frac{4}{3}}\arctan\left(\frac{1}{3}\sqrt{3}d^{\frac{1}{3}}\left(2\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}+\frac{1}{d^{\frac{1}{3}}}\right)\right)}{2\,ad^2}-\frac{|d|^{\frac{4}{3}}\log\left(\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}+\frac{\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}}{d^{\frac{1}{3}}}+\frac{1}{d^{\frac{2}{3}}}\right)}{4\,ad^2}+\frac{\log\left(\left|\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}-\frac{1}{d^{\frac{1}{3}}}\right|\right)}{2\,ad^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+x^2)/(x^2*(-a+x))^(2/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x, algor
ithm="giac")
```

```
[Out] 1/2*sqrt(3)*abs(d)^(4/3)*arctan(1/3*sqrt(3)*d^(1/3)*(2*(-a/x + 1)^(2/3) + 1
/d^(1/3)))/(a*d^2) - 1/4*abs(d)^(4/3)*log((-a/x + 1)^(4/3) + (-a/x + 1)^(2/
3)/d^(1/3) + 1/d^(2/3))/(a*d^2) + 1/2*log(abs((-a/x + 1)^(2/3) - 1/d^(1/3))
)/(a*d^(2/3))
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{-ax+x^2}{\left(x^2\left(-a+x\right)\right)^{\frac{2}{3}}\left(a^2d-2adx+\left(-1+d\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x+x^2)/(x^2*(-a+x))^(2/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x)
```

```
[Out] int((-a*x+x^2)/(x^2*(-a+x))^(2/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax - x^2}{\left(a^2d - 2\,adx + (d-1)x^2\right)\left(-(a-x)x^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+x^2)/(x^2*(-a+x))^(2/3)/(a^2*d-2*a*d*x+(-1+d)*x^2),x, algor
ithm="maxima")
```

```
[Out] -integrate((a*x - x^2)/((a^2*d - 2*a*d*x + (d - 1)*x^2)*(-(a - x)*x^2)^(2/3
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{a\,x - x^2}{\left(-x^2\,\left(a-x\right)\right)^{2/3}\,\left(d\,a^2 - 2\,d\,a\,x + \left(d-1\right)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x - x^2)/((-x^2*(a - x))^(2/3)*(a^2*d + x^2*(d - 1) - 2*a*d*x)),x)
```

```
[Out] -int((a*x - x^2)/((-x^2*(a - x))^(2/3)*(a^2*d + x^2*(d - 1) - 2*a*d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+x**2)/(x**2*(-a+x))**(2/3)/(a**2*d-2*a*d*x+(-1+d)*x**2),x)
```

```
[Out] Timed out
```

3.2149 $\int \frac{-1+x}{(1+x)\sqrt[3]{-1+x^3}}\,dx$

Optimal. Leaf size=243

$$\frac{\log\left(-2\sqrt[3]{x^3-1}+\sqrt[3]{2}x-\sqrt[3]{2}\right)}{\sqrt[3]{2}}-\frac{1}{3}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^3-1}}{\sqrt{3}}+\frac{\sqrt[3]{2}x}{\sqrt{3}}-\frac{\sqrt[3]{2}}{\sqrt{3}}}{\sqrt[3]{x^3-1}}\right)}{\sqrt[3]{2}}+\frac{1}{6}\log\left(\sqrt[3]{\right.$$

Rubi [A] time = 0.10, antiderivative size = 139, normalized size of antiderivative = 0.57, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2152, 239, 2148}

$$-\frac{1}{2}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{3\log\left(2^{2/3}\sqrt[3]{x^3-1}-x+1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{x^3-1}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log\left((1-x)(x+1)^2\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/((1 + x)*(-1 + x^3)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(1 - (2^(1/3)*(1 - x))/(-1 + x^3)^(1/3))/Sqrt[3]])/2^(1/3
)) + ArcTan[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[(1 - x)*(1
+ x)^2]/(2*2^(1/3)) - Log[-x + (-1 + x^3)^(1/3)]/2 + (3*Log[1 - x + 2^(2/3)
*(-1 + x^3)^(1/3)])/(2*2^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2152

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rubi steps

$$\begin{aligned}\int\frac{-1+x}{(1+x)\sqrt[3]{-1+x^3}}\,dx&=-\left(2\int\frac{1}{(1+x)\sqrt[3]{-1+x^3}}\,dx\right)+\int\frac{1}{\sqrt[3]{-1+x^3}}\,dx\\&=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{-1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log\left((1-x)(1+x)^2\right)}{2\sqrt[3]{2}}-\frac{1}{2}\log\left(-x+\sqrt[3]{-1\right.\end{aligned}$$

Mathematica [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{-1+x}{(1+x)\sqrt[3]{-1+x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x)/((1 + x)*(-1 + x^3)^(1/3)),x]
```

```
[Out] Integrate[(-1 + x)/((1 + x)*(-1 + x^3)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 4.98, size = 243, normalized size = 1.00

$$\frac{\log\left(-2\sqrt[3]{x^3-1}+\sqrt[3]{2}x-\sqrt[3]{2}\right)}{\sqrt[3]{2}}-\frac{1}{3}\log\left(\sqrt[3]{x^3-1}-x\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^3-1}}{\sqrt{3}}+\frac{\sqrt[3]{2}x}{\sqrt{3}}-\frac{\sqrt[3]{2}}{\sqrt{3}}}{\sqrt[3]{x^3-1}}\right)}{\sqrt[3]{2}}+\frac{1}{6}\log\left(\sqrt[3]{x^3-1}x+\left(x^3-1\right)^{2/3}+x^2\right)-\frac{\log\left(4\left(x^3-1\right)^{2/3}+\left(2\sqrt[3]{2}x-2\sqrt[3]{2}\right)\sqrt[3]{x^3-1}+2^{2/3}x^2-2\,2^{2/3}x+2^{2/3}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)/((1 + x)*(-1 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*x)/(x + 2*(-1 + x^3)^(1/3))]/Sqrt[3] - (Sqrt[3]*ArcTan[(-(2
^(1/3)/Sqrt[3]) + (2^(1/3)*x)/Sqrt[3] + (-1 + x^3)^(1/3)/Sqrt[3])/(-1 + x^3
)^(1/3)])/2^(1/3) + Log[-2^(1/3) + 2^(1/3)*x - 2*(-1 + x^3)^(1/3)]/2^(1/3)
- Log[-x + (-1 + x^3)^(1/3)]/3 + Log[x^2 + x*(-1 + x^3)^(1/3) + (-1 + x^3)^
(2/3)]/6 - Log[2^(2/3) - 2*2^(2/3)*x + 2^(2/3)*x^2 + (-2*2^(1/3) + 2*2^(1/3
)*x)*(-1 + x^3)^(1/3) + 4*(-1 + x^3)^(2/3)]/(2*2^(1/3))
```

fricas [A] time = 2.45, size = 370, normalized size = 1.52

$$\frac{1}{6}\cdot4^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{4\cdot4^{\frac{2}{3}}\sqrt{3}\left(x^4+2x^3+2x^2+2x+1\right)\left(x^3-1\right)^{\frac{2}{3}}+2\cdot4^{\frac{1}{3}}\sqrt{3}\left(5x^5-5x^4+6x^3-6x^2+5x-5\right)\left(x^3-1\right)^{\frac{1}{3}}+\sqrt{3}\left(13x^6+2x^5+19x^4-4x^3+19x^2+2x+13\right)}{3\left(3x^6-18x^5-3x^4-28x^3-3x^2-18x+3\right)}\right)+\frac{1}{3}\sqrt{3}\arctan\left(-\frac{4\sqrt{3}\left(x^3-1\right)^{\frac{1}{3}}x^2-2\sqrt{3}\left(x^3-1\right)^{\frac{2}{3}}x+\sqrt{3}\left(x^3-1\right)}{9x^3-1}\right)-\frac{1}{12}\cdot4^{\frac{1}{3}}\log\left(\frac{8\cdot4^{\frac{1}{3}}\left(x^3-1\right)^{\frac{2}{3}}\left(x^2+1\right)+4^{\frac{2}{3}}\left(5x^4+6x^2+5\right)+4\left(3x^3-x^2+x-3\right)\left(x^3-1\right)^{\frac{1}{3}}}{x^4+4x^3+6x^2+4x+1}\right)+\frac{1}{6}\cdot4^{\frac{1}{3}}\log\left(\frac{4^{\frac{2}{3}}\left(x^3-1\right)^{\frac{1}{3}}(x-1)+4^{\frac{1}{3}}\left(x^2+2x+1\right)-4\left(x^3-1\right)^{\frac{2}{3}}}{x^2+2x+1}\right)-\frac{1}{6}\log\left(-3\left(x^3-1\right)^{\frac{1}{3}}x^2+3\left(x^3-1\right)^{\frac{2}{3}}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3-1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*4^(1/3)*sqrt(3)*arctan(1/3*(4*4^(2/3)*sqrt(3)*(x^4 + 2*x^3 + 2*x^2 + 2*
x + 1)*(x^3 - 1)^(2/3) + 2*4^(1/3)*sqrt(3)*(5*x^5 - 5*x^4 + 6*x^3 - 6*x^2 +
 5*x - 5)*(x^3 - 1)^(1/3) + sqrt(3)*(13*x^6 + 2*x^5 + 19*x^4 - 4*x^3 + 19*x
^2 + 2*x + 13))/(3*x^6 - 18*x^5 - 3*x^4 - 28*x^3 - 3*x^2 - 18*x + 3)) + 1/3
*sqrt(3)*arctan(-(4*sqrt(3)*(x^3 - 1)^(1/3)*x^2 - 2*sqrt(3)*(x^3 - 1)^(2/3)
*x + sqrt(3)*(x^3 - 1))/(9*x^3 - 1)) - 1/12*4^(1/3)*log((8*4^(1/3)*(x^3 - 1
)^(2/3)*(x^2 + 1) + 4^(2/3)*(5*x^4 + 6*x^2 + 5) + 4*(3*x^3 - x^2 + x - 3)*(
x^3 - 1)^(1/3))/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1)) + 1/6*4^(1/3)*log((4^(2/3)
*(x^3 - 1)^(1/3)*(x - 1) + 4^(1/3)*(x^2 + 2*x + 1) - 4*(x^3 - 1)^(2/3))/(x^
2 + 2*x + 1)) - 1/6*log(-3*(x^3 - 1)^(1/3)*x^2 + 3*(x^3 - 1)^(2/3)*x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^3-1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3-1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/((x^3 - 1)^(1/3)*(x + 1)), x)
```

maple [F] time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{-1+x}{(1+x)\left(x^3-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(1+x)/(x^3-1)^(1/3),x)
```

```
[Out] int((-1+x)/(1+x)/(x^3-1)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^3-1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3-1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/((x^3 - 1)^(1/3)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x-1}{\left(x^3-1\right)^{1/3}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x^3 - 1)^(1/3)*(x + 1)),x)
```

```
[Out] int((x - 1)/((x^3 - 1)^(1/3)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt[3]{(x-1)\left(x^2+x+1\right)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x**3-1)**(1/3),x)
```

```
[Out] Integral((x - 1)/(((x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

3.2150 $\int \frac{\left(1+x^2\right)\sqrt[3]{-x^2+x^3}}{-1+x^2}\,dx$

Optimal. Leaf size=243

$$\frac{1}{6}\sqrt[3]{x^3-x^2}\,(3x-1)-\frac{17}{9}\log\left(\sqrt[3]{x^3-x^2}-x\right)+\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^3-x^2}-2x\right)+\frac{17}{18}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)\right.$$

Rubi [A] time = 1.01, antiderivative size = 385, normalized size of antiderivative = 1.58, number of steps used = 36, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2056, 1586, 6733, 6725, 331, 292, 31, 634, 618, 204, 628, 321, 494, 617}

$$\frac{1}{2}\sqrt[3]{x^3-x^2}\,x-\frac{1}{6}\sqrt[3]{x^3-x^2}-\frac{17\sqrt[3]{x^3-x^2}\log\left(1-\frac{\sqrt[3]{x}}{\sqrt[3]{x-1}}\right)}{9\sqrt[3]{x-1}\,x^{2/3}}+\frac{\sqrt[3]{2}\sqrt[3]{x^3-x^2}\log\left(1-\frac{\sqrt[3]{2}\sqrt[3]{x}}{\sqrt[3]{x-1}}\right)}{\sqrt[3]{x-1}\,x^{2/3}}+\frac{17\sqrt[3]{x^3-x^2}\log\left(\frac{x^{2/3}}{(x-1)^{2/3}}+\frac{\sqrt[3]{x}}{\sqrt[3]{x-1}}+1\right)}{18\sqrt[3]{x-1}\,x^{2/3}}-\frac{\sqrt[3]{x^3-x^2}\log\left(\frac{2^{2/3}x^{2/3}}{(x-1)^{2/3}}+\frac{\sqrt[3]{2}\sqrt[3]{x}}{\sqrt[3]{x-1}}+1\right)}{2^{2/3}\sqrt[3]{x-1}\,x^{2/3}}-\frac{17\sqrt[3]{x^3-x^2}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{x}}{\sqrt[3]{x-1}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{x-1}\,x^{2/3}}+\frac{\sqrt[3]{2}\sqrt{3}\sqrt[3]{x^3-x^2}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{x}}{\sqrt[3]{x-1}}+1}{\sqrt{3}}\right)}{\sqrt[3]{x-1}\,x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(-x^2 + x^3)^(1/3))/(-1 + x^2),x]
```

```
[Out] -1/6*(-x^2 + x^3)^(1/3) + (x*(-x^2 + x^3)^(1/3))/2 - (17*(-x^2 + x^3)^(1/3)
*ArcTan[(1 + (2*x^(1/3))/(-1 + x)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*(-1 + x)^(1/3
)*x^(2/3)) + (2^(1/3)*Sqrt[3]*(-x^2 + x^3)^(1/3)*ArcTan[(1 + (2*2^(1/3)*x^(
1/3))/(-1 + x)^(1/3))/Sqrt[3]])/((-1 + x)^(1/3)*x^(2/3)) - (17*(-x^2 + x^3)
^(1/3)*Log[1 - x^(1/3)/(-1 + x)^(1/3)])/(9*(-1 + x)^(1/3)*x^(2/3)) + (2^(1/
3)*(-x^2 + x^3)^(1/3)*Log[1 - (2^(1/3)*x^(1/3))/(-1 + x)^(1/3)])/((-1 + x)^
(1/3)*x^(2/3)) + (17*(-x^2 + x^3)^(1/3)*Log[1 + x^(1/3)/(-1 + x)^(1/3) + x^
(2/3)/(-1 + x)^(2/3)])/(18*(-1 + x)^(1/3)*x^(2/3)) - ((-x^2 + x^3)^(1/3)*Lo
g[1 + (2^(1/3)*x^(1/3))/(-1 + x)^(1/3) + (2^(2/3)*x^(2/3))/(-1 + x)^(2/3)])
/(2^(2/3)*(-1 + x)^(1/3)*x^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{\left(1+x^2\right)\sqrt[3]{-x^2+x^3}}{-1+x^2}\,dx = \frac{\sqrt[3]{-x^2+x^3}\int \frac{\sqrt[3]{-1+x}\,x^{2/3}\left(1+x^2\right)}{-1+x^2}\,dx}{\sqrt[3]{-1+x}\,x^{2/3}}$$

$$= \frac{\sqrt[3]{-x^2+x^3}\int \frac{x^{2/3}\left(1+x^2\right)}{(-1+x)^{2/3}(1+x)}\,dx}{\sqrt[3]{-1+x}\,x^{2/3}}$$

$$= \frac{\left(3\sqrt[3]{-x^2+x^3}\right)\text{Subst}\left(\int \frac{x^4\left(1+x^6\right)}{\left(-1+x^3\right)^{2/3}\left(1+x^3\right)}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-1+x}\,x^{2/3}}$$

$$= \frac{\left(3\sqrt[3]{-x^2+x^3}\right)\text{Subst}\left(\int \left(\frac{2x}{\left(-1+x^3\right)^{2/3}} - \frac{x^4}{\left(-1+x^3\right)^{2/3}} + \frac{x^7}{\left(-1+x^3\right)^{2/3}} - \frac{2x}{\left(-1+x^3\right)^{2/3}\left(1+x^3\right)}\right) dx, x\right.}{\sqrt[3]{-1+x}\,x^{2/3}}$$

$$= -\frac{\left(3\sqrt[3]{-x^2+x^3}\right)\text{Subst}\left(\int \frac{x^4}{\left(-1+x^3\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-1+x}\,x^{2/3}} + \frac{\left(3\sqrt[3]{-x^2+x^3}\right)\text{Subst}\left(\int \frac{x^7}{\left(-1+x^3\right)}\right.}{\sqrt[3]{-1+x}\,x^{2/3}}$$

$$= -\sqrt[3]{-x^2+x^3} + \frac{1}{2}x\sqrt[3]{-x^2+x^3} - \frac{\left(2\sqrt[3]{-x^2+x^3}\right)\text{Subst}\left(\int \frac{x}{\left(-1+x^3\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-1+x}\,x^{2/3}} + \cdots$$

$$= -\frac{1}{6}\sqrt[3]{-x^2+x^3} + \frac{1}{2}x\sqrt[3]{-x^2+x^3} + \frac{\left(5\sqrt[3]{-x^2+x^3}\right)\text{Subst}\left(\int \frac{x}{\left(-1+x^3\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{3\sqrt[3]{-1+x}\,x^{2/3}} + \cdots$$

$$= -\frac{1}{6}\sqrt[3]{-x^2+x^3} + \frac{1}{2}x\sqrt[3]{-x^2+x^3} - \frac{2\sqrt[3]{-x^2+x^3}\log\left(1-\frac{\sqrt[3]{x}}{\sqrt[3]{-1+x}}\right)}{\sqrt[3]{-1+x}\,x^{2/3}} + \frac{\sqrt[3]{2}\sqrt[3]{-x^2+x^3}\cdots}{\sqrt[3]{-1}\cdots}$$

$$= -\frac{1}{6}\sqrt[3]{-x^2+x^3} + \frac{1}{2}x\sqrt[3]{-x^2+x^3} - \frac{4\sqrt[3]{-x^2+x^3}\log\left(1-\frac{\sqrt[3]{x}}{\sqrt[3]{-1+x}}\right)}{3\sqrt[3]{-1+x}\,x^{2/3}} + \frac{\sqrt[3]{2}\sqrt[3]{-x^2+x^3}\cdots}{\sqrt[3]{-1}\cdots}$$

$$= -\frac{1}{6}\sqrt[3]{-x^2+x^3} + \frac{1}{2}x\sqrt[3]{-x^2+x^3} - \frac{2\sqrt{3}\sqrt[3]{-x^2+x^3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{x}}{\sqrt[3]{-1+x}}}{\sqrt{3}}\right)}{\sqrt[3]{-1+x}\,x^{2/3}} + \frac{\sqrt[3]{2}\sqrt{3}\sqrt[3]{-}\cdots}{\cdots}$$

$$= -\frac{1}{6}\sqrt[3]{-x^2+x^3} + \frac{1}{2}x\sqrt[3]{-x^2+x^3} + \frac{2\sqrt[3]{-x^2+x^3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{x}}{\sqrt[3]{-1+x}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{-1+x}\,x^{2/3}} - \frac{2\sqrt{3}\sqrt[3]{-x^2+x}\cdots}{\sqrt[3]{-1}\cdots}$$

$$= -\frac{1}{6}\sqrt[3]{-x^2+x^3} + \frac{1}{2}x\sqrt[3]{-x^2+x^3} + \frac{\sqrt[3]{-x^2+x^3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{x}}{\sqrt[3]{-1+x}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{-1+x}\,x^{2/3}} - \frac{2\sqrt{3}\sqrt[3]{-x^2+x}\cdots}{\sqrt[3]{-1}\cdots}$$

Mathematica [C] time = 0.08, size = 85, normalized size = 0.35

$$\frac{3\sqrt[3]{(x-1)x^2}\left((x-1)\sqrt[3]{x}\,{}_2F_1\left(-\frac{2}{3},\frac{4}{3};\frac{7}{3};1-x\right)+8\sqrt[3]{x}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};1-x\right)-4\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{x-1}{2x}\right)\right)}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(-x^2 + x^3)^(1/3))/(-1 + x^2),x]
```

```
[Out] (3*((-1 + x)*x^2)^(1/3)*((-1 + x)*x^(1/3)*Hypergeometric2F1[-2/3, 4/3, 7/3,
 1 - x] + 8*x^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, 1 - x] - 4*Hypergeomet
ric2F1[1/3, 1, 4/3, (-1 + x)/(2*x)]))/(4*x)
```

IntegrateAlgebraic [A] time = 0.65, size = 243, normalized size = 1.00

$$\frac{1}{6}\sqrt[3]{x^3-x^2}\,(3x-1)-\frac{17}{9}\log\left(\sqrt[3]{x^3-x^2}-x\right)+\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^3-x^2}-2x\right)+\frac{17}{18}\log\left(x^2+\sqrt[3]{x^3-x^2}\,x+\left(x^3-x^2\right)^{2/3}\right)-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^3-x^2}\,x+\sqrt[3]{2}\left(x^3-x^2\right)^{2/3}\right)}{2^{2/3}}-\frac{17\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x^2}+x}\right)}{3\sqrt{3}}+\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3-x^2}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^2)*(-x^2 + x^3)^(1/3))/(-1 + x^2),x]
```

```
[Out] ((-1 + 3*x)*(-x^2 + x^3)^(1/3))/6 - (17*ArcTan[(Sqrt[3]*x)/(x + 2*(-x^2 + x
^3)^(1/3))])/(3*Sqrt[3]) + 2^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*
(-x^2 + x^3)^(1/3))] - (17*Log[-x + (-x^2 + x^3)^(1/3)])/9 + 2^(1/3)*Log[-2
*x + 2^(2/3)*(-x^2 + x^3)^(1/3)] + (17*Log[x^2 + x*(-x^2 + x^3)^(1/3) + (-x
^2 + x^3)^(2/3)])/18 - Log[2*x^2 + 2^(2/3)*x*(-x^2 + x^3)^(1/3) + 2^(1/3)*(
-x^2 + x^3)^(2/3)]/2^(2/3)
```

fricas [A] time = 0.43, size = 222, normalized size = 0.91

$$-\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(x^3-x^2\right)^{\frac{1}{3}}+\sqrt{3}x}{3x}\right)+\frac{17}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3-x^2\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{6}\left(x^3-x^2\right)^{\frac{1}{3}}(3x-1)+2^{\frac{1}{3}}\log\left(-\frac{2^{\frac{1}{3}}x-\left(x^3-x^2\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{2}\cdot2^{\frac{1}{3}}\log\left(\frac{2^{\frac{2}{3}}x^2+2^{\frac{1}{3}}\left(x^3-x^2\right)^{\frac{1}{3}}x+\left(x^3-x^2\right)^{\frac{2}{3}}}{x^2}\right)-\frac{17}{9}\log\left(-\frac{x-\left(x^3-x^2\right)^{\frac{1}{3}}}{x}\right)+\frac{17}{18}\log\left(\frac{x^2+\left(x^3-x^2\right)^{\frac{1}{3}}x+\left(x^3-x^2\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^3-x^2)^(1/3)/(x^2-1),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*2^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(x^3 - x^2)^(1/3) + sqrt(3)*x)
/x) + 17/9*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 - x^2)^(1/3))/x)
+ 1/6*(x^3 - x^2)^(1/3)*(3*x - 1) + 2^(1/3)*log(-(2^(1/3)*x - (x^3 - x^2)^(
1/3))/x) - 1/2*2^(1/3)*log((2^(2/3)*x^2 + 2^(1/3)*(x^3 - x^2)^(1/3)*x + (x^
3 - x^2)^(2/3))/x^2) - 17/9*log(-(x - (x^3 - x^2)^(1/3))/x) + 17/18*log((x^
2 + (x^3 - x^2)^(1/3)*x + (x^3 - x^2)^(2/3))/x^2)
```

giac [A] time = 0.86, size = 174, normalized size = 0.72

$$\frac{1}{6}\left(\left(-\frac{1}{x}+1\right)^{\frac{4}{3}}+2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}\right)x^2-\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}\right)\right)+\frac{17}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{2}\cdot2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}\right)+2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}\right|\right)+\frac{17}{18}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}+1\right)-\frac{17}{9}\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^3-x^2)^(1/3)/(x^2-1),x, algorithm="giac")
```

```
[Out] 1/6*((-1/x + 1)^(4/3) + 2*(-1/x + 1)^(1/3))*x^2 - sqrt(3)*2^(1/3)*arctan(1/
6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-1/x + 1)^(1/3))) + 17/9*sqrt(3)*arctan(1/3
*sqrt(3)*(2*(-1/x + 1)^(1/3) + 1)) - 1/2*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-1/
x + 1)^(1/3) + (-1/x + 1)^(2/3)) + 2^(1/3)*log(abs(-2^(1/3) + (-1/x + 1)^(1
/3))) + 17/18*log((-1/x + 1)^(2/3) + (-1/x + 1)^(1/3) + 1) - 17/9*log(abs((
-1/x + 1)^(1/3) - 1))
```

maple [C] time = 5.51, size = 2018, normalized size = 8.30

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^3-x^2)^(1/3)/(x^2-1),x)
```

```
[Out] 1/6*(-1+3*x)*((-1+x)*x^2)^(1/3)+(-17/9*ln(-(RootOf(81*RootOf(_Z^3-2)^2+9*_Z
*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^2-3*RootOf(81*RootOf(_Z^3-2)^2+9
*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x+2*RootOf(81*RootOf(_Z^3-2)^2+
9*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4+270*(x^3-2*x^2+x)^(2/3)*RootOf
(_Z^3-2)^2*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)-432*RootOf(
81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*(x^3-2*x^2+x
)^(1/3)*x+180*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_
Z^3-2)^2*x^2+432*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*RootO
f(_Z^3-2)^2*(x^3-2*x^2+x)^(1/3)-414*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(
_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x+234*RootOf(_Z^3-2)^2*RootOf(81*RootOf(_Z^3
-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)-2916*(x^3-2*x^2+x)^(2/3)-4860*(x^3-2*x^2+x)
^(1/3)*x+8100*x^2+4860*(x^3-2*x^2+x)^(1/3)-12960*x+4860)/(-1+x))+1/9*RootOf
(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*ln((RootOf(_Z^3-2)^2*RootOf(
81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)^2*x^2+3*RootOf(_Z^3-2)^3*Root
Of(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*x^2-3*RootOf(81*RootOf(_Z^
3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x-9*RootOf(81*RootOf(_Z
^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x+27*(x^3-2*x^2+x)^(2/3)
*RootOf(_Z^3-2)^2*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)+2*Ro
otOf(_Z^3-2)^2*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)^2+6*Roo
tOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3+45*(x^3-
2*x^2+x)^(1/3)*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(
_Z^3-2)*x-243*RootOf(_Z^3-2)^2*(x^3-2*x^2+x)^(1/3)*x-45*(x^3-2*x^2+x)^(1/3)
*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)-99*Roo
tOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*x^2+243*RootOf(_Z^3-2)^2*
(x^3-2*x^2+x)^(1/3)-297*RootOf(_Z^3-2)*x^2+108*RootOf(81*RootOf(_Z^3-2)^2+9
*_Z*RootOf(_Z^3-2)+_Z^2)*x+324*RootOf(_Z^3-2)*x+1296*(x^3-2*x^2+x)^(2/3)-9*
RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)-27*RootOf(_Z^3-2))/(-1
+x)/(1+x))+RootOf(_Z^3-2)*ln(-(RootOf(_Z^3-2)^2*RootOf(81*RootOf(_Z^3-2)^2+
9*_Z*RootOf(_Z^3-2)+_Z^2)^2*x^2+27*RootOf(_Z^3-2)^3*RootOf(81*RootOf(_Z^3-2
)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*x^2-3*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(
_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x-81*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootO
f(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x+81*(x^3-2*x^2+x)^(2/3)*RootOf(_Z^3-2)^2*
RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)+2*RootOf(_Z^3-2)^2*Roo
tOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)^2+54*RootOf(81*RootOf(_Z^
3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3-216*(x^3-2*x^2+x)^(1/3)*R
ootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x-729*Ro
otOf(_Z^3-2)^2*(x^3-2*x^2+x)^(1/3)*x+216*(x^3-2*x^2+x)^(1/3)*RootOf(81*Root
Of(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)+117*RootOf(81*RootOf(
_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*x^2+729*RootOf(_Z^3-2)^2*(x^3-2*x^2+x)^
(1/3)+3159*RootOf(_Z^3-2)*x^2-162*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z
^3-2)+_Z^2)*x-4374*RootOf(_Z^3-2)*x-2430*(x^3-2*x^2+x)^(2/3)+45*RootOf(81*R
ootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)+1215*RootOf(_Z^3-2))/(-1+x)/(1+x)
)-17/162*RootOf(_Z^3-2)^2*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z
^2)*ln((5*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^
3-2)^4*x^2-15*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf
(_Z^3-2)^4*x+10*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)^2*Root
Of(_Z^3-2)^4+270*(x^3-2*x^2+x)^(2/3)*RootOf(_Z^3-2)^2*RootOf(81*RootOf(_Z^3
-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)+162*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(
_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*(x^3-2*x^2+x)^(1/3)*x-342*RootOf(81*RootOf(_
Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^2-162*RootOf(81*RootO
f(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*(x^3-2*x^2+x)^(1/3)+
396*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x
-54*RootOf(_Z^3-2)^2*RootOf(81*RootOf(_Z^3-2)^2+9*_Z*RootOf(_Z^3-2)+_Z^2)+7
776*(x^3-2*x^2+x)^(2/3)-4860*(x^3-2*x^2+x)^(1/3)*x-1296*x^2+4860*(x^3-2*x^2
+x)^(1/3)+1620*x-324)/(-1+x)))*((-1+x)*x^2)^(1/3)*((-1+x)^2*x)^(1/3)/(-1+x)
/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-x^2\right)^{\frac{1}{3}}\left(x^2+1\right)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^3-x^2)^(1/3)/(x^2-1),x, algorithm="maxima")
[Out] integrate((x^3 - x^2)^(1/3)*(x^2 + 1)/(x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^2+1\right)\left(x^3-x^2\right)^{1/3}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*(x^3 - x^2)^(1/3))/(x^2 - 1),x)
[Out] int(((x^2 + 1)*(x^3 - x^2)^(1/3))/(x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x^2\left(x-1\right)}\left(x^2+1\right)}{\left(x-1\right)\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**3-x**2)**(1/3)/(x**2-1),x)
[Out] Integral((x**2*(x - 1))**(1/3)*(x**2 + 1)/((x - 1)*(x + 1)), x)
```

3.2151 $\int \frac{\left(1-2k^2\right)x+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(1-d+\left(d-2k^2\right)x^2+k^4x^4\right)}\,dx$

Optimal. Leaf size=243

$$\frac{\log\left(\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+k^2x^2-1\right)}{2d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}{\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}-2k^2x^2+2}\right)}{2d^{2/3}}-\frac{\log\left(d^{2/3}\left(k^2x^4+\left(-\right.\right.\right.}{}$$

Rubi [F] time = 0.85, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-2k^2\right)x+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(1-d+\left(d-2k^2\right)x^2+k^4x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 - 2*k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3)*(1 - d + (d
- 2*k^2)*x^2 + k^4*x^4)),x]
```

```
[Out] Defer[Subst][Defer[Int][(1 - 2*k^2 + k^2*x)/((1 + (-1 - k^2)*x + k^2*x^2)^(
1/3)*(1 - d + (d - 2*k^2)*x + k^4*x^2)), x], x, x^2]/2
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-2k^2\right)x+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(1-d+\left(d-2k^2\right)x^2+k^4x^4\right)}\,dx &= \int \frac{x\left(1-2k^2+k^2x^2\right)}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(1-d+\left(d-2k^2\right)x^2+k^4\right.} \\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1-2k^2+k^2x}{\sqrt[3]{(1-x)\left(1-k^2x\right)}\left(1-d+\left(d-2k^2\right)\right.}\right. \\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1-2k^2+k^2x}{\sqrt[3]{1+\left(-1-k^2\right)x+k^2x^2}\left(1-d+\left(d\right.\right.}\right.\end{aligned}$$

Mathematica [F] time = 13.14, size = 0, normalized size = 0.00

$$\int \frac{\left(1-2k^2\right)x+k^2x^3}{\sqrt[3]{\left(1-x^2\right)\left(1-k^2x^2\right)}\left(1-d+\left(d-2k^2\right)x^2+k^4x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 - 2*k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3)*(1 - d
 + (d - 2*k^2)*x^2 + k^4*x^4)),x]
```

```
[Out] Integrate[((1 - 2*k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3)*(1 - d
 + (d - 2*k^2)*x^2 + k^4*x^4)), x]
```

IntegrateAlgebraic [A] time = 3.86, size = 243, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}+k^2x^2-1\right)}{2d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}}{\sqrt[3]{d}\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}-2k^2x^2+2}\right)}{2d^{2/3}}-\frac{\log\left(d^{2/3}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+\sqrt[3]{k^2x^4+\left(-k^2-1\right)x^2+1}\left(\sqrt[3]{d}-\sqrt[3]{d}k^2x^2\right)+k^4x^4-2k^2x^2+1\right)}{4d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 - 2*k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/
3)*(1 - d + (d - 2*k^2)*x^2 + k^4*x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))/(2 -
 2*k^2*x^2 + d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))]])/(2*d^(2/3)) +
Log[-1 + k^2*x^2 + d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3)]/(2*d^(2/3)
) - Log[1 - 2*k^2*x^2 + k^4*x^4 + (d^(1/3) - d^(1/3)*k^2*x^2)*(1 + (-1 - k^
2)*x^2 + k^2*x^4)^(1/3) + d^(2/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(4*
d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2*k^2+1)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d+(-2*k^2
+d)*x^2+k^4*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3 - \left(2\,k^2 - 1\right)x}{\left(k^4x^4 - \left(2\,k^2 - d\right)x^2 - d + 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2*k^2+1)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d+(-2*k^2
+d)*x^2+k^4*x^4),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^3 - (2*k^2 - 1)*x)/((k^4*x^4 - (2*k^2 - d)*x^2 - d + 1)*((
k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(-2k^2 + 1\right)x + k^2x^3}{\left(\left(-x^2 + 1\right)\left(-k^2x^2 + 1\right)\right)^{\frac{1}{3}}\left(1 - d + \left(-2k^2 + d\right)x^2 + k^4x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-2*k^2+1)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d+(-2*k^2+d)*x^
2+k^4*x^4),x)
```

```
[Out] int(((-2*k^2+1)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d+(-2*k^2+d)*x^
2+k^4*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3 - \left(2\,k^2 - 1\right)x}{\left(k^4x^4 - \left(2\,k^2 - d\right)x^2 - d + 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2*k^2+1)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(1-d+(-2*k^2
+d)*x^2+k^4*x^4),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^3 - (2*k^2 - 1)*x)/((k^4*x^4 - (2*k^2 - d)*x^2 - d + 1)*((
k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int -\frac{k^2 x^3 - x\,\left(2\,k^2 - 1\right)}{\left(\left(x^2 - 1\right)\,\left(k^2\,x^2 - 1\right)\right)^{1/3}\,\left(k^4\,x^4 - d + x^2\,\left(d - 2\,k^2\right) + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^3 - x*(2*k^2 - 1))/(((x^2 - 1)*(k^2*x^2 - 1))^(1/3)*(k^4*x^4 - d
 + x^2*(d - 2*k^2) + 1)),x)
```

```
[Out] -int(-(k^2*x^3 - x*(2*k^2 - 1))/(((x^2 - 1)*(k^2*x^2 - 1))^(1/3)*(k^4*x^4 -
 d + x^2*(d - 2*k^2) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2*k**2+1)*x+k**2*x**3)/((-x**2+1)*(-k**2*x**2+1))**(1/3)/(1-d+
(-2*k**2+d)*x**2+k**4*x**4),x)
```

```
[Out] Timed out
```

3.2152 $\int \frac{(2-k^2)x-2x^3+k^2x^5}{((1-x^2)(1-k^2x^2))^{2/3}(-1+d+(-2d+k^2)x^2+dx^4)}\,dx$

Optimal. Leaf size=243

$$-\frac{\log\left(\sqrt[3]{d}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}+k^2x^2-1\right)}{2d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}}{\sqrt[3]{d}\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}-2k^2x^2+2}\right)}{2d^{2/3}}+\frac{\log\left(d^{2/3}\left(k^2x^4+\right.\right.}{}$$

Rubi [C] time = 2.44, antiderivative size = 251, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 7, integrand size = 66, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.106, Rules used = {1594, 6715, 6719, 1586, 6728, 137, 136}

$$\frac{3\left(1-x^2\right)^2\left(\frac{1-k^2x^2}{1-k^2}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{k^2(1-x^2)}{1-k^2},\frac{2d(1-x^2)}{k^2-\sqrt{k^4-4dk^2+4d}}\right)}{8\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}}+\frac{3\left(1-x^2\right)^2\left(\frac{1-k^2x^2}{1-k^2}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{k^2(1-x^2)}{1-k^2},\frac{2d(1-x^2)}{k^2+\sqrt{k^4-4dk^2+4d}}\right)}{8\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((2 - k^2)*x - 2*x^3 + k^2*x^5)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)*(-1 + 
d + (-2*d + k^2)*x^2 + d*x^4)),x]
```

```
[Out] (3*(1 - x^2)^2*((1 - k^2*x^2)/(1 - k^2))^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -
((k^2*(1 - x^2))/(1 - k^2)), (2*d*(1 - x^2))/(k^2 - Sqrt[4*d - 4*d*k^2 + k^
4])])/(8*((1 - x^2)*(1 - k^2*x^2))^(2/3)) + (3*(1 - x^2)^2*((1 - k^2*x^2)/(
1 - k^2))^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((k^2*(1 - x^2))/(1 - k^2)), (2
*d*(1 - x^2))/(k^2 + Sqrt[4*d - 4*d*k^2 + k^4])])/(8*((1 - x^2)*(1 - k^2*x^
2))^(2/3))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(2-k^2\right)x-2x^3+k^2x^5}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(-1+d+\left(-2d+k^2\right)x^2+dx^4\right)}\,dx=\int \frac{x\left(2-k^2-2x^2+k^2x^4\right)}{\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}\left(-1+d+\left(-2d+k\right.\right.}$$

$$=\frac{1}{2}\,\text{Subst}\left(\int \frac{2-k^2-2x+k^2}{\left((1-x)\left(1-k^2x\right)\right)^{2/3}\left(-1+d+\left(\right.\right.}\right.$$

$$=\frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{}{(1-x)^{2/3}\left(1-k\right.}\right.}{2\left(\left(1-x^2\right)\left(1-k^2\right.\right.}$$

$$=\frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{\sqrt[3]{1-}}{\left(1-k^2x\right)^{2/3}\left(-\right.}\right.}{2\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right.}$$

$$=\frac{\left(\left(1-x^2\right)^{2/3}\left(1-k^2x^2\right)^{2/3}\right)\text{Subst}\left(\int\left(\frac{\left(-k^2\right.}{\left(-2d+k^2-\right.}\right.\right.}{2}$$

$$=\frac{\left(\left(-k^2-\sqrt{4d-4dk^2+k^4}\right)\left(1-x^2\right)^{2/3}\left(1-k^2x\right.\right.}{2\left(\left(1-\right.\right.}$$

$$=\frac{\left(\left(-k^2-\sqrt{4d-4dk^2+k^4}\right)\left(1-x^2\right)^{2/3}\left(\frac{-1+k^2x^2}{-1+k^2}\right.\right.}{2\left(\right.}$$

$$=\frac{3\left(1-x^2\right)^2\left(\frac{1-k^2x^2}{1-k^2}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{k^2\left(1-x^2\right)}{1-k^2},\right.}{8\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}}$$

Mathematica [F] time = 1.88, size = 0, normalized size = 0.00

$$\int \frac{(2-k^2)x-2x^3+k^2x^5}{\left((1-x^2)(1-k^2x^2)\right)^{2/3}\left(-1+d+(-2d+k^2)x^2+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((2 - k^2)*x - 2*x^3 + k^2*x^5)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)*
(-1 + d + (-2*d + k^2)*x^2 + d*x^4)),x]
```

```
[Out] Integrate[((2 - k^2)*x - 2*x^3 + k^2*x^5)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)*
(-1 + d + (-2*d + k^2)*x^2 + d*x^4)), x]
```

IntegrateAlgebraic [A] time = 5.55, size = 243, normalized size = 1.00

$$-\frac{\log\left(\sqrt[3]{d}\left(k^2x^4+(-k^2-1)x^2+1\right)^{2/3}+k^2x^2-1\right)}{2d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\left(k^2x^4+(-k^2-1)x^2+1\right)^{2/3}}{\sqrt[3]{d}\left(k^2x^4+(-k^2-1)x^2+1\right)^{2/3}-2k^2x^2+2}\right)}{2d^{2/3}}+\frac{\log\left(d^{2/3}\left(k^2x^4+(-k^2-1)x^2+1\right)^{4/3}+\left(k^2x^4+(-k^2-1)x^2+1\right)^{2/3}\left(\sqrt[3]{d}-\sqrt[3]{d}k^2x^2\right)+k^4x^4-2k^2x^2+1\right)}{4d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 - k^2)*x - 2*x^3 + k^2*x^5)/(((1 - x^2)*(1 - k^2*x^2
))^(2/3)*(-1 + d + (-2*d + k^2)*x^2 + d*x^4)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3))
/(2 - 2*k^2*x^2 + d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3))])/d^(2/3) -
 Log[-1 + k^2*x^2 + d^(1/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(2*d^(2/3
)) + Log[1 - 2*k^2*x^2 + k^4*x^4 + (d^(1/3) - d^(1/3)*k^2*x^2)*(1 + (-1 - k
^2)*x^2 + k^2*x^4)^(2/3) + d^(2/3)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(4/3)]/(4
*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-k^2+2)*x-2*x^3+k^2*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(-1+d+(k
^2-2*d)*x^2+d*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^5-2x^3-(k^2-2)x}{\left(dx^4+(k^2-2d)x^2+d-1\right)\left((k^2x^2-1)(x^2-1)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-k^2+2)*x-2*x^3+k^2*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(-1+d+(k
^2-2*d)*x^2+d*x^4),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^5 - 2*x^3 - (k^2 - 2)*x)/((d*x^4 + (k^2 - 2*d)*x^2 + d - 1
)*((k^2*x^2 - 1)*(x^2 - 1))^(2/3)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(-k^2+2)x-2x^3+k^2x^5}{\left((-x^2+1)(-k^2x^2+1)\right)^{\frac{2}{3}}\left(-1+d+(k^2-2d)x^2+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-k^2+2)*x-2*x^3+k^2*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(-1+d+(k^2-2*d
)*x^2+d*x^4),x)
```

```
[Out] int(((-k^2+2)*x-2*x^3+k^2*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(-1+d+(k^2-2*d
)*x^2+d*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^5 - 2\,x^3 - \left(k^2 - 2\right)x}{\left(dx^4 + \left(k^2 - 2\,d\right)x^2 + d - 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-k^2+2)*x-2*x^3+k^2*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(-1+d+(k
^2-2*d)*x^2+d*x^4),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^5 - 2*x^3 - (k^2 - 2)*x)/((d*x^4 + (k^2 - 2*d)*x^2 + d - 1
)*((k^2*x^2 - 1)*(x^2 - 1))^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x\,\left(k^2-2\right) - k^2\,x^5 + 2\,x^3}{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)^{2/3}\,\left(d\,x^4 + \left(k^2-2\,d\right)\,x^2 + d - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(k^2 - 2) - k^2*x^5 + 2*x^3)/(((x^2 - 1)*(k^2*x^2 - 1))^(2/3)*(d -
x^2*(2*d - k^2) + d*x^4 - 1)),x)
```

```
[Out] -int((x*(k^2 - 2) - k^2*x^5 + 2*x^3)/(((x^2 - 1)*(k^2*x^2 - 1))^(2/3)*(d -
x^2*(2*d - k^2) + d*x^4 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-k**2+2)*x-2*x**3+k**2*x**5)/((-x**2+1)*(-k**2*x**2+1))**(2/3)/
(-1+d+(k**2-2*d)*x**2+d*x**4),x)
```

```
[Out] Timed out
```

3.2153 $\int \frac{1}{x^4\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$

Optimal. Leaf size=243

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\,\sqrt{b}\,\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{\sqrt{2}\,\sqrt{b}}\right)}{32\sqrt{2}\,b^{7/2}}+\frac{a^{5/2}\left(15x^2\sqrt{ax^2+b^2}+5bx^2\right)+a^{3/2}\left(-8b^2\sqrt{ax^2+b^2}-56b^3\right)}{\frac{384a^{3/2}b^4x^3\sqrt{ax^2+b^2}}{\left(\sqrt{ax^2+b^2}+b\right)^{3/2}}+\frac{192a^{3/2}b^3x^3\left(ax^2+2b^2\right)}{\left(\sqrt{ax^2+b^2}+b\right)^{3/2}}}$$

Rubi [F] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^4\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

```
[Out] Defer[Int][1/(x^4*Sqrt[b + Sqrt[b^2 + a*x^2]]), x]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{b+\sqrt{b^2+ax^2}}}\,dx=\int \frac{1}{x^4\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Mathematica [C] time = 0.61, size = 196, normalized size = 0.81

$$\frac{a^2x\left(17\left(2b\sqrt{ax^2+b^2}+ax^2+2b^2\right){}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{b-\sqrt{b^2+ax^2}}{2b}\right)-2\left(\left(2b\sqrt{ax^2+b^2}+ax^2+2b^2\right){}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};\frac{b-\sqrt{b^2+ax^2}}{2b}\right)+5b\left(7\sqrt{ax^2+b^2}-b\right)\right)\right)}{160b\left(\sqrt{ax^2+b^2}-b\right)^3\left(\sqrt{ax^2+b^2}+b\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

```
[Out] (a^2*x*(17*(2*b^2 + a*x^2 + 2*b*Sqrt[b^2 + a*x^2])*Hypergeometric2F1[-5/2,
1, -3/2, (b - Sqrt[b^2 + a*x^2])/(2*b)] - 2*(5*b*(-b + 7*Sqrt[b^2 + a*x^2])
 + (2*b^2 + a*x^2 + 2*b*Sqrt[b^2 + a*x^2])*Hypergeometric2F1[-5/2, 2, -3/2,
 (b - Sqrt[b^2 + a*x^2])/(2*b)])))/(160*b*(-b + Sqrt[b^2 + a*x^2])^3*(b + S
qrt[b^2 + a*x^2])^(5/2))
```

IntegrateAlgebraic [A] time = 0.39, size = 211, normalized size = 0.87

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\,\sqrt{b}\,\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{64\sqrt{2}\,b^{7/2}}+\frac{a^{5/2}\left(15x^2\sqrt{ax^2+b^2}+5bx^2\right)+a^{3/2}\left(-8b^2\sqrt{ax^2+b^2}-56b^3\right)}{\frac{384a^{3/2}b^4x^3\sqrt{ax^2+b^2}}{\left(\sqrt{ax^2+b^2}+b\right)^{3/2}}+\frac{192a^{3/2}b^3x^3\left(ax^2+2b^2\right)}{\left(\sqrt{ax^2+b^2}+b\right)^{3/2}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

```
[Out] (a^(3/2)*(-56*b^3 - 8*b^2*Sqrt[b^2 + a*x^2]) + a^(5/2)*(5*b*x^2 + 15*x^2*Sq
rt[b^2 + a*x^2]))/((384*a^(3/2)*b^4*x^3*Sqrt[b^2 + a*x^2])/(b + Sqrt[b^2 +
a*x^2])^(3/2) + (192*a^(3/2)*b^3*x^3*(2*b^2 + a*x^2))/(b + Sqrt[b^2 + a*x^2
])^(3/2)) + (5*a^(3/2)*ArcTan[(Sqrt[a]*x)/(Sqrt[2]*Sqrt[b]*Sqrt[b + Sqrt[b^
2 + a*x^2]])])/(64*Sqrt[2]*b^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b+\sqrt{ax^2+b^2}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b + sqrt(a*x^2 + b^2))*x^4), x)
```

maple [C] time = 0.04, size = 31, normalized size = 0.13

$$-\frac{\sqrt{2}\ \mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{1}{4},\frac{3}{4}\right],\left[-\frac{1}{2},\frac{3}{2}\right],-\frac{x^2a}{b^2}\right)}{6\left(b^2\right)^{\frac{1}{4}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
[Out] -1/6/(b^2)^(1/4)*2^(1/2)/x^3*hypergeom([-3/2,1/4,3/4],[-1/2,3/2],-x^2*a/b^2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b+\sqrt{ax^2+b^2}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b + sqrt(a*x^2 + b^2))*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{b+\sqrt{b^2+a\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(b + (a*x^2 + b^2)^(1/2))^(1/2)),x)
[Out] int(1/(x^4*(b + (a*x^2 + b^2)^(1/2))^(1/2)), x)
```

sympy [C] time = 1.28, size = 49, normalized size = 0.20

$$-\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_{3}F_{2}\left(\begin{matrix}-\frac{3}{2},\frac{1}{4},\frac{3}{4}\\-\frac{1}{2},\frac{3}{2}\end{matrix}\middle|\frac{ax^{2}e^{i\pi}}{b^{2}}\right)}{6\pi\sqrt{b}\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b+(a*x**2+b**2)**(1/2))**(1/2),x)
```

```
[Out] -gamma(1/4)*gamma(3/4)*hyper((-3/2, 1/4, 3/4), (-1/2, 3/2), a*x**2*exp_pola
r(I*pi)/b**2)/(6*pi*sqrt(b)*x**3)
```

3.2154 $$\int \frac{-a(a-5b)-(3a+5b)x+4x^2}{\sqrt[3]{(-a+x)(-b+x)}\left(-a^5+bd-\left(-5a^4+d\right)x-10a^3x^2+10a^2x^3-5ax^4+x^5\right)}\,dx$$

Optimal. Leaf size=244

$$\frac{\log\left(a^2-\sqrt[3]{d}\sqrt[3]{x(-a-b)+ab+x^2}-2ax+x^2\right)}{d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\sqrt[3]{x(-a-b)+ab+x^2}}{2a^2+\sqrt[3]{d}\sqrt[3]{x(-a-b)+ab+x^2}-4ax+2x^2}\right)}{d^{2/3}}-\frac{\log\left(a^4-4a^3x+\right.}{}$$

Rubi [F] time = 7.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-a(a-5b)-(3a+5b)x+4x^2}{\sqrt[3]{(-a+x)(-b+x)}\left(-a^5+bd-\left(-5a^4+d\right)x-10a^3x^2+10a^2x^3-5ax^4+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a*(a - 5*b)) - (3*a + 5*b)*x + 4*x^2)/(((-a + x)*(-b + x))^(1/3)*(-a
^5 + b*d - (-5*a^4 + d)*x - 10*a^3*x^2 + 10*a^2*x^3 - 5*a*x^4 + x^5)),x]
```

```
[Out] (-3*(a - 5*b)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][Defer[Int][x^4/((a
 - b + x^3)^(1/3)*(a*(1 - b/a)*d + d*x^3 - x^15)), x], x, (-a + x)^(1/3)])/
((a - x)*(b - x))^(1/3) + (12*a*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][
Defer[Int][x^4/((a - b + x^3)^(1/3)*(-(a*(1 - b/a)*d) - d*x^3 + x^15)), x],
 x, (-a + x)^(1/3)])/((a - x)*(b - x))^(1/3) + (12*(-a + x)^(1/3)*(-b + x)^
(1/3)*Defer[Subst][Defer[Int][x^7/((a - b + x^3)^(1/3)*(-(a*(1 - b/a)*d) -
d*x^3 + x^15)), x], x, (-a + x)^(1/3)])/((a - x)*(b - x))^(1/3)
```

Rubi steps

$$\int \frac{-a(a-5b)-(3a+5b)x+4x^2}{\sqrt[3]{(-a+x)(-b+x)}\left(-a^5+bd-\left(-5a^4+d\right)x-10a^3x^2+10a^2x^3-5ax^4+x^5\right)}\,dx = \frac{\left(\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\int \cdots}{\cdots}$$

$$= \frac{\left(\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\int \cdots}{\cdots}$$

$$= \frac{\left(\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\int \cdots}{\cdots}$$

$$= \frac{\left(\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\int \left(\cdots\right.}{\cdots}$$

$$= \frac{\left(4\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\int \cdots}{\cdots}$$

$$= \frac{\left(12\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\text{Sul}\cdots}{\cdots}$$

$$= \frac{\left(12\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\text{Sul}\cdots}{\cdots}$$

$$= \frac{\left(12\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\text{Sul}\cdots}{\cdots}$$

$$= \frac{\left(12\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\text{Sul}\cdots}{\cdots}$$

Mathematica [F] time = 3.87, size = 0, normalized size = 0.00

$$\int \frac{-a(a-5b)-(3a+5b)x+4x^2}{\sqrt[3]{(-a+x)(-b+x)}\left(-a^5+bd-\left(-5a^4+d\right)x-10a^3x^2+10a^2x^3-5ax^4+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-(a*(a - 5*b)) - (3*a + 5*b)*x + 4*x^2)/(((-a + x)*(-b + x))^(1/
3)*(-a^5 + b*d - (-5*a^4 + d)*x - 10*a^3*x^2 + 10*a^2*x^3 - 5*a*x^4 + x^5))
,x]
```

```
[Out] Integrate[(-(a*(a - 5*b)) - (3*a + 5*b)*x + 4*x^2)/(((-a + x)*(-b + x))^(1/
3)*(-a^5 + b*d - (-5*a^4 + d)*x - 10*a^3*x^2 + 10*a^2*x^3 - 5*a*x^4 + x^5))
, x]
```

IntegrateAlgebraic [A] time = 0.66, size = 244, normalized size = 1.00

$$\frac{\log\left(a^2-\sqrt[3]{d}\sqrt[3]{x(-a-b)+ab+x^2}-2ax+x^2\right)}{d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\sqrt[3]{x(-a-b)+ab+x^2}}{2a^2+\sqrt[3]{d}\sqrt[3]{x(-a-b)+ab+x^2}-4ax+2x^2}\right)}{d^{2/3}}-\frac{\log\left(a^4-4a^3x+\sqrt[3]{x(-a-b)+ab+x^2}\left(a^2\sqrt[3]{d}-2a\sqrt[3]{d}x+\sqrt[3]{d}x^2\right)+6a^2x^2+d^{2/3}\left(x(-a-b)+ab+x^2\right)^{2/3}-4ax^3+x^4\right)}{2d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*(a - 5*b)) - (3*a + 5*b)*x + 4*x^2)/(((-a + x)*(-b
+ x))^(1/3)*(-a^5 + b*d - (-5*a^4 + d)*x - 10*a^3*x^2 + 10*a^2*x^3 - 5*a*x^
4 + x^5)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(a*b + (-a - b)*x + x^2)^(1/3))/(2*a^2 - 4
*a*x + 2*x^2 + d^(1/3)*(a*b + (-a - b)*x + x^2)^(1/3))])/d^(2/3) + Log[a^2
- 2*a*x + x^2 - d^(1/3)*(a*b + (-a - b)*x + x^2)^(1/3)]/d^(2/3) - Log[a^4 -
 4*a^3*x + 6*a^2*x^2 - 4*a*x^3 + x^4 + d^(2/3)*(a*b + (-a - b)*x + x^2)^(2/
3) + (a*b + (-a - b)*x + x^2)^(1/3)*(a^2*d^(1/3) - 2*a*d^(1/3)*x + d^(1/3)*
x^2)]/(2*d^(2/3))
```

fricas **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a-5*b)-(3*a+5*b)*x+4*x^2)/((-a+x)*(-b+x))^(1/3)/(-a^5+b*d-(-
5*a^4+d)*x-10*a^3*x^2+10*a^2*x^3-5*a*x^4+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a-5\,b)a+(3\,a+5\,b)x-4\,x^2}{\left(a^5+10\,a^3x^2-10\,a^2x^3+5\,ax^4-x^5-bd-\left(5\,a^4-d\right)x\right)((a-x)(b-x))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a-5*b)-(3*a+5*b)*x+4*x^2)/((-a+x)*(-b+x))^(1/3)/(-a^5+b*d-(-
5*a^4+d)*x-10*a^3*x^2+10*a^2*x^3-5*a*x^4+x^5),x, algorithm="giac")
```

```
[Out] integrate(((a - 5*b)*a + (3*a + 5*b)*x - 4*x^2)/((a^5 + 10*a^3*x^2 - 10*a^2
*x^3 + 5*a*x^4 - x^5 - b*d - (5*a^4 - d)*x)*((a - x)*(b - x))^(1/3)), x)
```

maple **[F]** time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{-a\,(a-5b)-(3a+5b)\,x+4x^2}{((-a+x)\,(-b+x))^{\frac{1}{3}}\left(-a^5+bd-\left(-5a^4+d\right)x-10a^3x^2+10a^2x^3-5a\,x^4+x^5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*(a-5*b)-(3*a+5*b)*x+4*x^2)/((-a+x)*(-b+x))^(1/3)/(-a^5+b*d-(-5*a^4+
d)*x-10*a^3*x^2+10*a^2*x^3-5*a*x^4+x^5),x)
```

```
[Out] int((-a*(a-5*b)-(3*a+5*b)*x+4*x^2)/((-a+x)*(-b+x))^(1/3)/(-a^5+b*d-(-5*a^4+
d)*x-10*a^3*x^2+10*a^2*x^3-5*a*x^4+x^5),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a-5\,b)a+(3\,a+5\,b)x-4\,x^2}{\left(a^5+10\,a^3x^2-10\,a^2x^3+5\,ax^4-x^5-bd-\left(5\,a^4-d\right)x\right)((a-x)(b-x))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a-5*b)-(3*a+5*b)*x+4*x^2)/((-a+x)*(-b+x))^(1/3)/(-a^5+b*d-(-
5*a^4+d)*x-10*a^3*x^2+10*a^2*x^3-5*a*x^4+x^5),x, algorithm="maxima")
```

```
[Out] integrate(((a - 5*b)*a + (3*a + 5*b)*x - 4*x^2)/((a^5 + 10*a^3*x^2 - 10*a^2
*x^3 + 5*a*x^4 - x^5 - b*d - (5*a^4 - d)*x)*((a - x)*(b - x))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{-4\,x^2+\left(3\,a+5\,b\right)\,x+a\,\left(a-5\,b\right)}{{\left(\left(a-x\right)\,\left(b-x\right)\right)}^{1/3}\,\left(5\,a\,x^4-b\,d+x\,\left(d-5\,a^4\right)+a^5-x^5-10\,a^2\,x^3+10\,a^3\,x^2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(a - 5*b) + x*(3*a + 5*b) - 4*x^2)/(((a - x)*(b - x))^(1/3)*(5*a*x^4
 - b*d + x*(d - 5*a^4) + a^5 - x^5 - 10*a^2*x^3 + 10*a^3*x^2)),x)
```

```
[Out] int((a*(a - 5*b) + x*(3*a + 5*b) - 4*x^2)/(((a - x)*(b - x))^(1/3)*(5*a*x^4
 - b*d + x*(d - 5*a^4) + a^5 - x^5 - 10*a^2*x^3 + 10*a^3*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-a+x)(a-5b+4x)}{\sqrt[3]{(-a+x)(-b+x)}\left(-a^5+5a^4x-10a^3x^2+10a^2x^3-5ax^4+bd-dx+x^5\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a-5*b)-(3*a+5*b)*x+4*x**2)/((-a+x)*(-b+x))**(1/3)/(-a**5+b*d
-(-5*a**4+d)*x-10*a**3*x**2+10*a**2*x**3-5*a*x**4+x**5),x)
```

```
[Out] Integral((-a + x)*(a - 5*b + 4*x)/(((-a + x)*(-b + x))**(1/3)*(-a**5 + 5*a*
*4*x - 10*a**3*x**2 + 10*a**2*x**3 - 5*a*x**4 + b*d - d*x + x**5)), x)
```

3.2155 $\int \frac{x^6\sqrt{-x+x^4}}{b+ax^6}\,dx$

Optimal. Leaf size=244

$$-\frac{\sqrt[4]{-1}\sqrt{\sqrt{b}\left(\sqrt{a}-i\sqrt{b}\right)}\tan^{-1}\left(\frac{(1+i)\sqrt{x^4-x}\sqrt{\sqrt{a}\sqrt{b}-ib}}{\sqrt{2}x^2\left(\sqrt{a}-i\sqrt{b}\right)}\right)}{3a^{3/2}}-\frac{(-1)^{3/4}\sqrt{\sqrt{b}\left(\sqrt{a}+i\sqrt{b}\right)}\tan^{-1}\left(\frac{(1+i)x\sqrt{x^4-x}\sqrt{\sqrt{a}\sqrt{b}+ib}}{\sqrt{2}\sqrt{b}(x-1)\left(x^2+x+1\right)}\right)}{3a^{3/2}}$$

Rubi [A] time = 0.64, antiderivative size = 271, normalized size of antiderivative = 1.11, number of steps used = 18, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {2056, 1493, 1491, 1292, 195, 217, 206, 1175, 402, 377, 208}

$$-\frac{\sqrt[4]{b}\sqrt{x^4-x}\sqrt{\sqrt{-a}+\sqrt{b}}\tanh^{-1}\left(\frac{x^{3/2}\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}\sqrt{x^3-1}}\right)}{3(-a)^{3/2}\sqrt{x^3-1}\sqrt{x}}+\frac{\sqrt[4]{b}\sqrt{x^4-x}\sqrt{\sqrt{-a}\sqrt{b}+a}\tanh^{-1}\left(\frac{x^{3/2}\sqrt{\sqrt{-a}\sqrt{b}+a}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{x^3-1}}\right)}{3(-a)^{7/4}\sqrt{x^3-1}\sqrt{x}}+\frac{\sqrt{x^4-x}\,x}{3a}-\frac{\sqrt{x^4-x}\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{x^3-1}}\right)}{3a\sqrt{x^3-1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*Sqrt[-x + x^4])/(b + a*x^6),x]
```

```
[Out] (x*Sqrt[-x + x^4])/(3*a) - (Sqrt[-x + x^4]*ArcTanh[x^(3/2)/Sqrt[-1 + x^3]])
/(3*a*Sqrt[x]*Sqrt[-1 + x^3]) - (Sqrt[Sqrt[-a] + Sqrt[b]]*b^(1/4)*Sqrt[-x +
 x^4]*ArcTanh[(Sqrt[Sqrt[-a] + Sqrt[b]]*x^(3/2))/(b^(1/4)*Sqrt[-1 + x^3])])
/(3*(-a)^(3/2)*Sqrt[x]*Sqrt[-1 + x^3]) + (Sqrt[a + Sqrt[-a]*Sqrt[b]]*b^(1/4
)*Sqrt[-x + x^4]*ArcTanh[(Sqrt[a + Sqrt[-a]*Sqrt[b]]*x^(3/2))/((-a)^(1/4)*b
^(1/4)*Sqrt[-1 + x^3])])/(3*(-a)^(7/4)*Sqrt[x]*Sqrt[-1 + x^3])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[-(a*c), 2]}, -Dist[c/(2*r), Int[(d + e*x^2)^q/(r - c*x^2), x], x] - Di
st[c/(2*r), Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e, q}
, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1292

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Dist[f^4/c, Int[(f*x)^(m - 4)*(d + e*x^2)^q, x], x] - Dist[(a
*f^4)/c, Int[((f*x)^(m - 4)*(d + e*x^2)^q)/(a + c*x^4), x], x] /; FreeQ[{a,
 c, d, e, f, q}, x] &&  !IntegerQ[q] && GtQ[m, 3]
```

Rule 1491

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_
.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k
 - 1)*(d + e*x^(n/k))^q*(a + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /
; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1493

```
Int[((f_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n
_))^(q_.), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/f, Subst[Int[x^(k
*(m + 1) - 1)*(d + (e*x^(k*n))/f)^q*(a + (c*x^(2*k*n))/f)^p, x], x, (f*x)^(
1/k)], x]] /; FreeQ[{a, c, d, e, f, p, q}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
 && FractionQ[m] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6\sqrt{-x+x^4}}{b+ax^6}\,dx &= \frac{\sqrt{-x+x^4}\int \frac{x^{13/2}\sqrt{-1+x^3}}{b+ax^6}\,dx}{\sqrt{x}\sqrt{-1+x^3}}\\
&= \frac{\left(2\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{x^{14}\sqrt{-1+x^6}}{b+ax^{12}}\,dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt{-1+x^3}}\\
&= \frac{\left(2\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{x^{4}\sqrt{-1+x^2}}{b+ax^{4}}\,dx,x,x^{3/2}\right)}{3\sqrt{x}\sqrt{-1+x^3}}\\
&= \frac{\left(2\sqrt{-x+x^4}\right)\text{Subst}\left(\int \sqrt{-1+x^2}\,dx,x,x^{3/2}\right)}{3a\sqrt{x}\sqrt{-1+x^3}} - \frac{\left(2b\sqrt{-x+x^4}\right)\text{Subst}\left(\int \frac{\sqrt{-1+x^2}}{b+ax^4}\,dx,x,x^3 \ldots\right.}{3a\sqrt{x}\sqrt{-1+x^3}}\\
&= \frac{x\sqrt{-x+x^4}}{3a} - \frac{\sqrt{-x+x^4}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx,x,x^{3/2}\right)}{3a\sqrt{x}\sqrt{-1+x^3}} - \frac{\left(\sqrt{-a}\sqrt{b}\sqrt{-x+x^4}\right)\text{Subst}\left(\int \ldots\right.}{3a\sqrt{x}\sqrt{-1+\ldots}}\\
&= \frac{x\sqrt{-x+x^4}}{3a} - \frac{\sqrt{-x+x^4}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x^{3/2}}{\sqrt{-1+x^3}}\right)}{3a\sqrt{x}\sqrt{-1+x^3}} - \frac{\left(\sqrt{-a}\left(-a+\sqrt{-a}\sqrt{b}\right)\sqrt{b}\sqrt{-x \ldots}\right.}{3\ldots}\\
&= \frac{x\sqrt{-x+x^4}}{3a} - \frac{\sqrt{-x+x^4}\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{-1+x^3}}\right)}{3a\sqrt{x}\sqrt{-1+x^3}} - \frac{\left(\sqrt{-a}\left(-a+\sqrt{-a}\sqrt{b}\right)\sqrt{b}\sqrt{-x+x^4}\right)\text{Subs}\ldots}{3a^2\sqrt{x}\sqrt{-\ldots}}\\
&= \frac{x\sqrt{-x+x^4}}{3a} - \frac{\sqrt{-x+x^4}\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{-1+x^3}}\right)}{3a\sqrt{x}\sqrt{-1+x^3}} - \frac{\sqrt{\sqrt{-a}+\sqrt{b}}\,\sqrt[4]{b}\sqrt{-x+x^4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{-a}+\ldots}}{\sqrt[4]{b}\sqrt{\ldots}}\right.}{3(-a)^{3/2}\sqrt{x}\sqrt{-1+x^3}}
\end{aligned}
$$

Mathematica [A] time = 3.50, size = 181, normalized size = 0.74

$$
\frac{x\sqrt{x\left(x^3-1\right)}\left(\frac{\sqrt{\frac{1}{x^3}-1}\left(-\sqrt[4]{b}\sqrt{\sqrt{-a}+\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{\frac{1}{x^3}-1}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right)+\sqrt[4]{b}\sqrt{\sqrt{-a}-\sqrt{b}}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{\frac{1}{x^3}-1}}{\sqrt{\sqrt{-a}-\sqrt{b}}}\right)+\sqrt{-a}\tan^{-1}\left(\sqrt{\frac{1}{x^3}-1}\right)\right)}{\sqrt{-a}\left(x^3-1\right)}+1\right)}{3a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*Sqrt[-x + x^4])/(b + a*x^6),x]
```

```
[Out] (x*Sqrt[x*(-1 + x^3)]*(1 + (Sqrt[-1 + x^(-3)]*(Sqrt[-a]*ArcTan[Sqrt[-1 + x^
(-3)]] - Sqrt[Sqrt[-a] + Sqrt[b]]*b^(1/4)*ArcTan[(b^(1/4)*Sqrt[-1 + x^(-3)]
)/Sqrt[Sqrt[-a] + Sqrt[b]]] + Sqrt[Sqrt[-a] - Sqrt[b]]*b^(1/4)*ArcTanh[(b^(
1/4)*Sqrt[-1 + x^(-3)])/Sqrt[Sqrt[-a] - Sqrt[b]]]))/(Sqrt[-a]*(-1 + x^3))))
/(3*a)
```

IntegrateAlgebraic [A] time = 1.30, size = 266, normalized size = 1.09

$$
-\frac{\sqrt[4]{-1}\sqrt{\sqrt{b}\left(\sqrt{a}-i\sqrt{b}\right)}\tan^{-1}\left(\frac{(-1)^{3/4}x\sqrt{x^4-x}\sqrt{\sqrt{a}\sqrt{b}-ib}}{\sqrt{b}(x-1)\left(x^2+x+1\right)}\right)}{3a^{3/2}}-\frac{(-1)^{3/4}\sqrt{\sqrt{b}\left(\sqrt{a}+i\sqrt{b}\right)}\tan^{-1}\left(\frac{\sqrt[4]{-1}x\sqrt{x^4-x}\sqrt{\sqrt{a}\sqrt{b}+ib}}{\sqrt{b}(x-1)\left(x^2+x+1\right)}\right)}{3a^{3/2}}+\frac{\sqrt{x^4-x}\,x}{3a}+\frac{\log\left(\sqrt{x^4-x}-x^2\right)}{6a}-\frac{\log\left(a\sqrt{x^4-x}+ax^2\right)}{6a}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^6*Sqrt[-x + x^4])/(b + a*x^6),x]
```

```
[Out] (x*Sqrt[-x + x^4])/(3*a) - ((-1)^(1/4)*Sqrt[(Sqrt[a] - I*Sqrt[b])*Sqrt[b]]*
ArcTan[((-1)^(3/4)*Sqrt[Sqrt[a]*Sqrt[b] - I*b]*x*Sqrt[-x + x^4])/(Sqrt[b]*(
-1 + x)*(1 + x + x^2))]])/(3*a^(3/2)) - ((-1)^(3/4)*Sqrt[(Sqrt[a] + I*Sqrt[b
])*Sqrt[b]]*ArcTan[((-1)^(1/4)*Sqrt[Sqrt[a]*Sqrt[b] + I*b]*x*Sqrt[-x + x^4]
)/(Sqrt[b]*(-1 + x)*(1 + x + x^2))]])/(3*a^(3/2)) + Log[-x^2 + Sqrt[-x + x^4
]]/(6*a) - Log[a*x^2 + a*Sqrt[-x + x^4]]/(6*a)
```

fricas [B] time = 112.40, size = 1800, normalized size = 7.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^4-x)^(1/2)/(a*x^6+b),x, algorithm="fricas")
```

```
[Out] 1/12*(a*sqrt((a^3*sqrt(-b/a^5) - b)/a^3)*log(-(2*((9*a^4*b + 73*a^3*b^2 + 2
79*a^2*b^3 + 567*a*b^4)*x^4 + (a^4*b - 12*a^3*b^2 - 120*a^2*b^3 - 216*a*b^4
 + 243*b^5)*x + ((a^7 - 12*a^6*b - 120*a^5*b^2 - 216*a^4*b^3 + 243*a^3*b^4)
*x^4 - (9*a^6*b + 73*a^5*b^2 + 279*a^4*b^3 + 567*a^3*b^4)*x)*sqrt(-b/a^5))*
sqrt(x^4 - x) + ((a^6 - 2*a^5*b - 69*a^4*b^2 - 108*a^3*b^3 + 486*a^2*b^4)*x
^6 - a^5*b + 22*a^4*b^2 + 171*a^3*b^3 + 324*a^2*b^4 - 2*(9*a^5*b + 73*a^4*b
^2 + 279*a^3*b^3 + 567*a^2*b^4)*x^3 + (10*a^7*b + 51*a^6*b^2 + 108*a^5*b^3
+ 243*a^4*b^4 - (8*a^8 + 95*a^7*b + 450*a^6*b^2 + 891*a^5*b^3)*x^6 - 2*(a^8
 - 12*a^7*b - 120*a^6*b^2 - 216*a^5*b^3 + 243*a^4*b^4)*x^3)*sqrt(-b/a^5))*s
qrt((a^3*sqrt(-b/a^5) - b)/a^3))/(a*x^6 + b)) - a*sqrt((a^3*sqrt(-b/a^5) -
b)/a^3)*log(-(2*((9*a^4*b + 73*a^3*b^2 + 279*a^2*b^3 + 567*a*b^4)*x^4 + (a^
4*b - 12*a^3*b^2 - 120*a^2*b^3 - 216*a*b^4 + 243*b^5)*x + ((a^7 - 12*a^6*b
- 120*a^5*b^2 - 216*a^4*b^3 + 243*a^3*b^4)*x^4 - (9*a^6*b + 73*a^5*b^2 + 27
9*a^4*b^3 + 567*a^3*b^4)*x)*sqrt(-b/a^5))*sqrt(x^4 - x) - ((a^6 - 2*a^5*b -
 69*a^4*b^2 - 108*a^3*b^3 + 486*a^2*b^4)*x^6 - a^5*b + 22*a^4*b^2 + 171*a^3
*b^3 + 324*a^2*b^4 - 2*(9*a^5*b + 73*a^4*b^2 + 279*a^3*b^3 + 567*a^2*b^4)*x
^3 + (10*a^7*b + 51*a^6*b^2 + 108*a^5*b^3 + 243*a^4*b^4 - (8*a^8 + 95*a^7*b
 + 450*a^6*b^2 + 891*a^5*b^3)*x^6 - 2*(a^8 - 12*a^7*b - 120*a^6*b^2 - 216*a
^5*b^3 + 243*a^4*b^4)*x^3)*sqrt(-b/a^5))*sqrt((a^3*sqrt(-b/a^5) - b)/a^3))/
(a*x^6 + b)) + a*sqrt(-(a^3*sqrt(-b/a^5) + b)/a^3)*log(-(2*((9*a^4*b + 73*a
^3*b^2 + 279*a^2*b^3 + 567*a*b^4)*x^4 + (a^4*b - 12*a^3*b^2 - 120*a^2*b^3 -
 216*a*b^4 + 243*b^5)*x - ((a^7 - 12*a^6*b - 120*a^5*b^2 - 216*a^4*b^3 + 24
3*a^3*b^4)*x^4 - (9*a^6*b + 73*a^5*b^2 + 279*a^4*b^3 + 567*a^3*b^4)*x)*sqrt
(-b/a^5))*sqrt(x^4 - x) + ((a^6 - 2*a^5*b - 69*a^4*b^2 - 108*a^3*b^3 + 486*
a^2*b^4)*x^6 - a^5*b + 22*a^4*b^2 + 171*a^3*b^3 + 324*a^2*b^4 - 2*(9*a^5*b
+ 73*a^4*b^2 + 279*a^3*b^3 + 567*a^2*b^4)*x^3 - (10*a^7*b + 51*a^6*b^2 + 10
8*a^5*b^3 + 243*a^4*b^4 - (8*a^8 + 95*a^7*b + 450*a^6*b^2 + 891*a^5*b^3)*x^
6 - 2*(a^8 - 12*a^7*b - 120*a^6*b^2 - 216*a^5*b^3 + 243*a^4*b^4)*x^3)*sqrt(
-b/a^5))*sqrt(-(a^3*sqrt(-b/a^5) + b)/a^3))/(a*x^6 + b)) - a*sqrt(-(a^3*sqr
t(-b/a^5) + b)/a^3)*log(-(2*((9*a^4*b + 73*a^3*b^2 + 279*a^2*b^3 + 567*a*b^
4)*x^4 + (a^4*b - 12*a^3*b^2 - 120*a^2*b^3 - 216*a*b^4 + 243*b^5)*x - ((a^7
 - 12*a^6*b - 120*a^5*b^2 - 216*a^4*b^3 + 243*a^3*b^4)*x^4 - (9*a^6*b + 73*
a^5*b^2 + 279*a^4*b^3 + 567*a^3*b^4)*x)*sqrt(-b/a^5))*sqrt(x^4 - x) - ((a^6
 - 2*a^5*b - 69*a^4*b^2 - 108*a^3*b^3 + 486*a^2*b^4)*x^6 - a^5*b + 22*a^4*b
^2 + 171*a^3*b^3 + 324*a^2*b^4 - 2*(9*a^5*b + 73*a^4*b^2 + 279*a^3*b^3 + 56
7*a^2*b^4)*x^3 - (10*a^7*b + 51*a^6*b^2 + 108*a^5*b^3 + 243*a^4*b^4 - (8*a^
8 + 95*a^7*b + 450*a^6*b^2 + 891*a^5*b^3)*x^6 - 2*(a^8 - 12*a^7*b - 120*a^6
*b^2 - 216*a^5*b^3 + 243*a^4*b^4)*x^3)*sqrt(-b/a^5))*sqrt(-(a^3*sqrt(-b/a^5
) + b)/a^3))/(a*x^6 + b)) + 4*sqrt(x^4 - x)*x + 2*log(2*x^3 - 2*sqrt(x^4 -
x)*x - 1))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^4-x)^(1/2)/(a*x^6+b),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[-11,-20]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [a,b]=[-92,-22]Warning, need to ch
oose a branch for the root of a polynomial with parameters. This might be w
rong.The choice was done assuming [a,b]=[23,98]Warning, need to choose a br
anch for the root of a polynomial with parameters. This might be wrong.The
choice was done assuming [a,b]=[-99,-57]Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
was done assuming [a,b]=[-53,-45]1/6/a*ln(abs(sqrt(-(1/x)^3+1)-1))-1/6/a*ln
(sqrt(-(1/x)^3+1)+1)+((2*a*b^2-4*a*sqrt(-a*b)*sqrt(-b^2-b*sqrt(-a*b))-2*b*a
*b+5*b*sqrt(-a*b)*sqrt(-b^2-b*sqrt(-a*b)))*a^2*abs(b)+(2*a^2*b^3-4*a^2*b*sq
rt(-a*b)*sqrt(-b^2-b*sqrt(-a*b))-2*a*b^2*a*b+5*a*b^2*sqrt(-a*b)*sqrt(-b^2-b
*sqrt(-a*b)))*abs(b))/(12*a^4*b^2-3*a^3*b^3-15*a^2*b^4)/abs(a)*atan(sqrt(-(
1/x)^3+1)/sqrt(-(6*a*b+sqrt(6*a*b*6*a*b-12*a*b*(3*a^2+3*a*b)))/2/3/a/b))-((
-2*a*b^2+4*a*sqrt(-a*b)*sqrt(-b^2+b*sqrt(-a*b))+2*b*a*b-5*b*sqrt(-a*b)*sqrt
(-b^2+b*sqrt(-a*b)))*a^2*abs(b)+(-2*a^2*b^3+4*a^2*b*sqrt(-a*b)*sqrt(-b^2+b*
sqrt(-a*b))+2*a*b^2*a*b-5*a*b^2*sqrt(-a*b)*sqrt(-b^2+b*sqrt(-a*b)))*abs(b))
/(12*a^4*b^2-3*a^3*b^3-15*a^2*b^4)/abs(a)*atan(sqrt(-(1/x)^3+1)/sqrt(-(6*a*
b-sqrt(6*a*b*6*a*b-12*a*b*(3*a^2+3*a*b)))/2/3/a/b))+8*a*1/24/a^2*x*sqrt(x^4
-x)
```

maple [C] time = 0.56, size = 669, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^4-x)^(1/2)/(a*x^6+b),x)
```

```
[Out] 1/a*(1/3*x*(x^4-x)^(1/2)-(1/2-1/2*I*3^(1/2))*((-3/2+1/2*I*3^(1/2))*x/(-1/2+
1/2*I*3^(1/2))/(-1+x))^(1/2)*(-1+x)^2*((x+1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^
(1/2))/(-1+x))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1
/2)/(-3/2+1/2*I*3^(1/2))/(x*(-1+x)*(x+1/2+1/2*I*3^(1/2))*(x+1/2-1/2*I*3^(1/
2)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^
(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2
*I*3^(1/2)))^(1/2))-EllipticPi(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))
/(-1+x))^(1/2),(-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)),((3/2+1/2*I*3^(1/2
))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))))+1/
3*b/a*4^(1/2)*sum((-_alpha^3+1)/_alpha^4*(-1+x)^2*(_alpha^5+_alpha^4+_alpha
^3+_alpha^2+_alpha+1)/(a+b)*(1-I*3^(1/2))*(x/(-1+x)*(-3+I*3^(1/2))/(I*3^(1/
2)-1))^(1/2)*(1/(-1+x)*(I*3^(1/2)+2*x+1)/(-1-I*3^(1/2)))^(1/2)*(1/(-1+x)*(1
+2*x-I*3^(1/2))/(I*3^(1/2)-1))^(1/2)/(-3+I*3^(1/2))/(x*(-1+x)*(I*3^(1/2)+2*
x+1)*(1+2*x-I*3^(1/2)))^(1/2)*(EllipticF(((-3/2+1/2*I*3^(1/2))*x/(-1/2+1/2*
I*3^(1/2))/(-1+x))^(1/2),((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/2+1/2*
I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+_alpha^5*a/b*EllipticPi(((-3/2+1/2*I
*3^(1/2))*x/(-1/2+1/2*I*3^(1/2))/(-1+x))^(1/2),-1/6*(I*_alpha^5*3^(1/2)*a-3
*_alpha^5*a+I*3^(1/2)*b-3*b)/b,((3/2+1/2*I*3^(1/2))*(1/2-1/2*I*3^(1/2))/(1/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))),_alpha=RootOf(_Z^6*a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4 - x}\, x^6}{a x^6 + b}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^4-x)^(1/2)/(a*x^6+b),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 - x)*x^6/(a*x^6 + b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,\sqrt{x^4-x}}{a\,x^6+b} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(x^4 - x)^(1/2))/(b + a*x^6),x)
[Out] int((x^6*(x^4 - x)^(1/2))/(b + a*x^6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6} \sqrt{x \left(x - 1\right) \left(x^{2} + x + 1\right)}}{a x^{6} + b}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(x**4-x)**(1/2)/(a*x**6+b),x)
[Out] Integral(x**6*sqrt(x*(x - 1)*(x**2 + x + 1))/(a*x**6 + b), x)
```

3.2156 $\int \frac{b^6+a^6x^6}{\sqrt{-b^2x+a^2x^3}\left(-b^6+a^6x^6\right)}\,dx$

Optimal. Leaf size=244

$$\frac{2\sqrt{a^2x^3-b^2x}}{3\left(b^2-a^2x^2\right)}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,3^{3/4}\sqrt{a}\sqrt{b}\sqrt{a^2x^3-b^2x}}{\sqrt{3}\,a^2x^2-3abx-\sqrt{3}\,b^2}\right)}{3\sqrt[4]{3}\sqrt{a}\sqrt{b}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\frac{a^{3/2}x^2}{\sqrt{2}\sqrt[4]{3}\sqrt{b}}-\frac{b^{3/2}}{\sqrt{2}\sqrt[4]{3}\sqrt{a}}+\frac{\sqrt[4]{3}\sqrt{a}\sqrt{b}\,x}{\sqrt{2}}}{\sqrt{a^2x^3-b^2x}}\right)}{3\sqrt[4]{3}\sqrt{a}\sqrt{b}}$$

Rubi [C] time = 3.10, antiderivative size = 519, normalized size of antiderivative = 2.13, number of steps used = 36, number of rules used = 17, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.378, Rules used = {2056, 6715, 6725, 224, 221, 2073, 1152, 414, 21, 423, 427, 426, 424, 253, 6728, 1219, 1218}

$$-\frac{x(b-ax)}{3b\sqrt{a^2x^3-b^2x}}-\frac{x(ax+b)}{3b\sqrt{a^2x^3-b^2x}}+\frac{4\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{3\sqrt{a}\sqrt{a^2x^3-b^2x}}-\frac{2\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\Pi\left(-\frac{2a}{a-\sqrt{3}\sqrt{-a^2}};\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{3\sqrt{a}\sqrt{a^2x^3-b^2x}}-\frac{2\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\Pi\left(\frac{2a}{a-\sqrt{3}\sqrt{-a^2}};\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{3\sqrt{a}\sqrt{a^2x^3-b^2x}}-\frac{2\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\Pi\left(-\frac{2a}{a+\sqrt{3}\sqrt{-a^2}};\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{3\sqrt{a}\sqrt{a^2x^3-b^2x}}-\frac{2\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\Pi\left(\frac{2a}{a+\sqrt{3}\sqrt{-a^2}};\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{3\sqrt{a}\sqrt{a^2x^3-b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b^6 + a^6*x^6)/(Sqrt[-(b^2*x) + a^2*x^3]*(-b^6 + a^6*x^6)),x]
```

```
[Out] -1/3*(x*(b - a*x))/(b*Sqrt[-(b^2*x) + a^2*x^3]) - (x*(b + a*x))/(3*b*Sqrt[-
(b^2*x) + a^2*x^3]) + (4*Sqrt[b]*Sqrt[x]*Sqrt[1 - (a^2*x^2)/b^2]*EllipticF[
ArcSin[(Sqrt[a]*Sqrt[x])/Sqrt[b]], -1])/(3*Sqrt[a]*Sqrt[-(b^2*x) + a^2*x^3]
) - (2*Sqrt[b]*Sqrt[x]*Sqrt[1 - (a^2*x^2)/b^2]*EllipticPi[(-2*a)/(a - Sqrt[
3]*Sqrt[-a^2]), ArcSin[(Sqrt[a]*Sqrt[x])/Sqrt[b]], -1])/(3*Sqrt[a]*Sqrt[-(b
^2*x) + a^2*x^3]) - (2*Sqrt[b]*Sqrt[x]*Sqrt[1 - (a^2*x^2)/b^2]*EllipticPi[(
2*a)/(a - Sqrt[3]*Sqrt[-a^2]), ArcSin[(Sqrt[a]*Sqrt[x])/Sqrt[b]], -1])/(3*S
qrt[a]*Sqrt[-(b^2*x) + a^2*x^3]) - (2*Sqrt[b]*Sqrt[x]*Sqrt[1 - (a^2*x^2)/b^
2]*EllipticPi[(-2*a)/(a + Sqrt[3]*Sqrt[-a^2]), ArcSin[(Sqrt[a]*Sqrt[x])/Sqr
t[b]], -1])/(3*Sqrt[a]*Sqrt[-(b^2*x) + a^2*x^3]) - (2*Sqrt[b]*Sqrt[x]*Sqrt[
1 - (a^2*x^2)/b^2]*EllipticPi[(2*a)/(a + Sqrt[3]*Sqrt[-a^2]), ArcSin[(Sqrt[
a]*Sqrt[x])/Sqrt[b]], -1])/(3*Sqrt[a]*Sqrt[-(b^2*x) + a^2*x^3])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
```

```
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{b^6+a^6x^6}{\sqrt{-b^2x+a^2x^3}\left(-b^6+a^6x^6\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\int \frac{b^6+a^6x^6}{\sqrt{x}\sqrt{-b^2+a^2x^2}\left(-b^6+a^6x^6\right)}\,dx}{\sqrt{-b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{b^6+a^6x^{12}}{\sqrt{-b^2+a^2x^4}\left(-b^6+a^6x^{12}\right)}\,dx,x,\sqrt{x}\right)}{\sqrt{-b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\text{Subst}\left(\int\left(\frac{1}{\sqrt{-b^2+a^2x^4}}+\frac{2b^6}{\sqrt{-b^2+a^2x^4}\left(-b^6+a^6x^{12}\right)}\right)dx,x,\sqrt{x}\right)}{\sqrt{-b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b^2+a^2x^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{-b^2x+a^2x^3}} + \frac{\left(4b^6\sqrt{x}\sqrt{-b^2+a^2x^2}\right.}{}$$

$$= \frac{\left(4b^6\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\text{Subst}\left(\int\left(-\frac{1}{6b^5\left(b-ax^2\right)\sqrt{-b^2+a^2x^4}}-\frac{1}{6b^5\left(b+ax^2\right)\sqrt{-b^2+a^2x^4}}\right.\right.}{\sqrt{-b^2x+a^2}}$$

$$= \frac{2\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{-b^2x+a^2x^3}} - \frac{\left(2b\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\text{Subst}\left(\int\right.}{3\sqrt{-b^2x+}}$$

$$= \frac{2\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)\middle|-1\right)}{\sqrt{a}\sqrt{-b^2x+a^2x^3}} - \frac{\left(2b\sqrt{x}\sqrt{-b-ax}\sqrt{b-ax}\right)\text{Subs}}{3\sqrt{-b^2x}}$$

$$= -\frac{x(b-ax)}{3b\sqrt{-b^2x+a^2x^3}} - \frac{x(b+ax)}{3b\sqrt{-b^2x+a^2x^3}} + \frac{2\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right.\right.}{\sqrt{a}\sqrt{-b^2x+a^2x^3}}$$

$$= -\frac{x(b-ax)}{3b\sqrt{-b^2x+a^2x^3}} - \frac{x(b+ax)}{3b\sqrt{-b^2x+a^2x^3}} + \frac{2\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right.\right.}{\sqrt{a}\sqrt{-b^2x+a^2x^3}}$$

$$= -\frac{x(b-ax)}{3b\sqrt{-b^2x+a^2x^3}} - \frac{x(b+ax)}{3b\sqrt{-b^2x+a^2x^3}} + \frac{2\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right.\right.}{\sqrt{a}\sqrt{-b^2x+a^2x^3}}$$

$$= -\frac{x(b-ax)}{3b\sqrt{-b^2x+a^2x^3}} - \frac{x(b+ax)}{3b\sqrt{-b^2x+a^2x^3}} + \frac{2\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right.\right.}{\sqrt{a}\sqrt{-b^2x+a^2x^3}}$$

$$= -\frac{x(b-ax)}{3b\sqrt{-b^2x+a^2x^3}} - \frac{x(b+ax)}{3b\sqrt{-b^2x+a^2x^3}} - \frac{\sqrt{x}\,(b+ax)\sqrt{1-\frac{ax}{b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{b}}\right.\right.}{3\sqrt{a}\sqrt{b}\sqrt{1+\frac{ax}{b}}\sqrt{-b^2x+a^2}}$$

$$= -\frac{x(b-ax)}{3b\sqrt{-b^2x+a^2x^3}} - \frac{x(b+ax)}{3b\sqrt{-b^2x+a^2x^3}} + \frac{4\sqrt{b}\sqrt{x}\sqrt{1-\frac{a^2x^2}{b^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right.\right.}{3\sqrt{a}\sqrt{-b^2x+a^2x^3}}$$

Mathematica [C] time = 2.11, size = 255, normalized size = 1.05

$$\frac{2\left(-x^{3/2}-\frac{i x^2\sqrt{1-\frac{b^2}{a^2x^2}}\left(2F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{b}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(-\frac{2i}{-i+\sqrt{3}};i\sinh^{-1}\left(\frac{\sqrt{-\frac{b}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(\frac{2i}{-i+\sqrt{3}};i\sinh^{-1}\left(\frac{\sqrt{-\frac{b}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(-\frac{2i}{i+\sqrt{3}};i\sinh^{-1}\left(\frac{\sqrt{-\frac{b}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(\frac{2i}{i+\sqrt{3}};i\sinh^{-1}\left(\frac{\sqrt{-\frac{b}{a}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{\sqrt{-\frac{b}{a}}}\right)}{3\sqrt{x}\sqrt{a^2x^3-b^2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^6 + a^6*x^6)/(Sqrt[-(b^2*x) + a^2*x^3]*(-b^6 + a^6*x^6)),x]
```

```
[Out] (2*(-x^(3/2) - (I*Sqrt[1 - b^2/(a^2*x^2)]*x^2*(2*EllipticF[I*ArcSinh[Sqrt[-
(b/a)]/Sqrt[x]], -1] - EllipticPi[(-2*I)/(-I + Sqrt[3]), I*ArcSinh[Sqrt[-(b
/a)]/Sqrt[x]], -1] - EllipticPi[(2*I)/(-I + Sqrt[3]), I*ArcSinh[Sqrt[-(b/a)
]/Sqrt[x]], -1] - EllipticPi[(-2*I)/(I + Sqrt[3]), I*ArcSinh[Sqrt[-(b/a)]/S
qrt[x]], -1] - EllipticPi[(2*I)/(I + Sqrt[3]), I*ArcSinh[Sqrt[-(b/a)]/Sqrt[
x]], -1]))/Sqrt[-(b/a)]))/(3*Sqrt[x]*Sqrt[-(b^2*x) + a^2*x^3])
```

IntegrateAlgebraic [A] time = 0.94, size = 244, normalized size = 1.00

$$\frac{2\sqrt{a^2x^3-b^2x}}{3\left(b^2-a^2x^2\right)}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,3^{3/4}\sqrt{a}\sqrt{b}\sqrt{a^2x^3-b^2x}}{\sqrt{3}a^2x^2-3abx-\sqrt{3}b^2}\right)}{3\sqrt[4]{3}\sqrt{a}\sqrt{b}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\frac{a^{3/2}x^2}{\sqrt{2}\sqrt[4]{3}\sqrt{b}}-\frac{b^{3/2}}{\sqrt{2}\sqrt[4]{3}\sqrt{a}}+\frac{\sqrt[4]{3}\sqrt{a}\sqrt{b}x}{\sqrt{2}}}{\sqrt{a^2x^3-b^2x}}\right)}{3\sqrt[4]{3}\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^6 + a^6*x^6)/(Sqrt[-(b^2*x) + a^2*x^3]*(-b^6 + a^6*x^
6)),x]
```

```
[Out] (2*Sqrt[-(b^2*x) + a^2*x^3])/(3*(b^2 - a^2*x^2)) - (Sqrt[2]*ArcTan[(Sqrt[2]
*3^(3/4)*Sqrt[a]*Sqrt[b]*Sqrt[-(b^2*x) + a^2*x^3])/(-(Sqrt[3]*b^2) - 3*a*b*
x + Sqrt[3]*a^2*x^2)])/(3*3^(1/4)*Sqrt[a]*Sqrt[b]) - (Sqrt[2]*ArcTanh[(-(b^
(3/2)/(Sqrt[2]*3^(1/4)*Sqrt[a])) + (3^(1/4)*Sqrt[a]*Sqrt[b]*x)/Sqrt[2] + (a
^(3/2)*x^2)/(Sqrt[2]*3^(1/4)*Sqrt[b]))/Sqrt[-(b^2*x) + a^2*x^3]])/(3*3^(1/4
)*Sqrt[a]*Sqrt[b])
```

fricas [B] time = 0.71, size = 1145, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^6*x^6+b^6)/(a^2*x^3-b^2*x)^(1/2)/(a^6*x^6-b^6),x, algorithm="f
ricas")
```

```
[Out] -1/12*(4*sqrt(2)*(1/3)^(1/4)*(a^2*x^2 - b^2)*(1/(a^2*b^2))^(1/4)*arctan(1/2
*((3*sqrt(2)*(1/3)^(3/4)*a^2*b^2*x*(1/(a^2*b^2))^(3/4) - sqrt(2)*(1/3)^(1/4
)*(a^2*x^2 - b^2)*(1/(a^2*b^2))^(1/4))*sqrt(a^2*x^3 - b^2*x) - (2*a^2*x^3 -
 2*b^2*x - (3*sqrt(2)*(1/3)^(3/4)*a^2*b^2*x*(1/(a^2*b^2))^(3/4) + sqrt(2)*(
1/3)^(1/4)*(a^2*x^2 - b^2)*(1/(a^2*b^2))^(1/4))*sqrt(a^2*x^3 - b^2*x))*sqrt
((a^4*x^4 + a^2*b^2*x^2 + b^4 + 12*sqrt(1/3)*(a^4*b^2*x^3 - a^2*b^4*x)*sqrt
(1/(a^2*b^2)) + 6*(sqrt(2)*(1/3)^(1/4)*a^2*b^2*x*(1/(a^2*b^2))^(1/4) + sqrt
(2)*(1/3)^(3/4)*(a^4*b^2*x^2 - a^2*b^4)*(1/(a^2*b^2))^(3/4))*sqrt(a^2*x^3 -
 b^2*x))/(a^4*x^4 + a^2*b^2*x^2 + b^4)))/(a^2*x^3 - b^2*x)) + 4*sqrt(2)*(1/
3)^(1/4)*(a^2*x^2 - b^2)*(1/(a^2*b^2))^(1/4)*arctan(1/2*((3*sqrt(2)*(1/3)^(
3/4)*a^2*b^2*x*(1/(a^2*b^2))^(3/4) - sqrt(2)*(1/3)^(1/4)*(a^2*x^2 - b^2)*(1
/(a^2*b^2))^(1/4))*sqrt(a^2*x^3 - b^2*x) + (2*a^2*x^3 - 2*b^2*x + (3*sqrt(2
)*(1/3)^(3/4)*a^2*b^2*x*(1/(a^2*b^2))^(3/4) + sqrt(2)*(1/3)^(1/4)*(a^2*x^2
- b^2)*(1/(a^2*b^2))^(1/4))*sqrt(a^2*x^3 - b^2*x))*sqrt((a^4*x^4 + a^2*b^2*
x^2 + b^4 + 12*sqrt(1/3)*(a^4*b^2*x^3 - a^2*b^4*x)*sqrt(1/(a^2*b^2)) - 6*(s
```

```
qrt(2)*(1/3)^(1/4)*a^2*b^2*x*(1/(a^2*b^2))^(1/4) + sqrt(2)*(1/3)^(3/4)*(a^4
*b^2*x^2 - a^2*b^4)*(1/(a^2*b^2))^(3/4))*sqrt(a^2*x^3 - b^2*x))/(a^4*x^4 +
a^2*b^2*x^2 + b^4)))/(a^2*x^3 - b^2*x)) + sqrt(2)*(1/3)^(1/4)*(a^2*x^2 - b^
2)*(1/(a^2*b^2))^(1/4)*log((a^4*x^4 + a^2*b^2*x^2 + b^4 + 12*sqrt(1/3)*(a^4
*b^2*x^3 - a^2*b^4*x)*sqrt(1/(a^2*b^2)) + 6*(sqrt(2)*(1/3)^(1/4)*a^2*b^2*x*
(1/(a^2*b^2))^(1/4) + sqrt(2)*(1/3)^(3/4)*(a^4*b^2*x^2 - a^2*b^4)*(1/(a^2*b
^2))^(3/4))*sqrt(a^2*x^3 - b^2*x))/(a^4*x^4 + a^2*b^2*x^2 + b^4)) - sqrt(2)
*(1/3)^(1/4)*(a^2*x^2 - b^2)*(1/(a^2*b^2))^(1/4)*log((a^4*x^4 + a^2*b^2*x^2
 + b^4 + 12*sqrt(1/3)*(a^4*b^2*x^3 - a^2*b^4*x)*sqrt(1/(a^2*b^2)) - 6*(sqrt
(2)*(1/3)^(1/4)*a^2*b^2*x*(1/(a^2*b^2))^(1/4) + sqrt(2)*(1/3)^(3/4)*(a^4*b^
2*x^2 - a^2*b^4)*(1/(a^2*b^2))^(3/4))*sqrt(a^2*x^3 - b^2*x))/(a^4*x^4 + a^2
*b^2*x^2 + b^4)) + 8*sqrt(a^2*x^3 - b^2*x))/(a^2*x^2 - b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^6x^6 + b^6}{\left(a^6x^6 - b^6\right)\sqrt{a^2x^3 - b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^6*x^6+b^6)/(a^2*x^3-b^2*x)^(1/2)/(a^6*x^6-b^6),x, algorithm="g
iac")
```

```
[Out] integrate((a^6*x^6 + b^6)/((a^6*x^6 - b^6)*sqrt(a^2*x^3 - b^2*x)), x)
```

maple [C] time = 0.11, size = 828, normalized size = 3.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^6*x^6+b^6)/(a^2*x^3-b^2*x)^(1/2)/(a^6*x^6-b^6),x)
```

```
[Out] b/a*((x+b/a)/b*a)^(1/2)*(-2*(x-b/a)/b*a)^(1/2)*(-a*x/b)^(1/2)/(a^2*x^3-b^2*
x)^(1/2)*EllipticF(((x+b/a)/b*a)^(1/2),1/2*2^(1/2))+1/3*2^(1/2)*sum((-_alph
a*a-2*b)/(2*_alpha*a+b)*_alpha*((x+b/a)/b*a)^(1/2)*(-(x-b/a)/b*a)^(1/2)*(-a
*x/b)^(1/2)/(x*(a^2*x^2-b^2))^(1/2)*EllipticPi(((x+b/a)/b*a)^(1/2),-_alpha/
b*a,1/2*2^(1/2)),_alpha=RootOf(_Z^2*a^2+_Z*a*b+b^2))+1/3*b*(-(a^2*x^2+a*b*x
)/b^2/a/((x-b/a)*(a^2*x^2+a*b*x))^(1/2)-1/2/a*((x+b/a)/b*a)^(1/2)*(-2*(x-b/
a)/b*a)^(1/2)*(-a*x/b)^(1/2)/(a^2*x^3-b^2*x)^(1/2)*EllipticF(((x+b/a)/b*a)^
(1/2),1/2*2^(1/2))+1/2/b*((x+b/a)/b*a)^(1/2)*(-2*(x-b/a)/b*a)^(1/2)*(-a*x/b
)^(1/2)/(a^2*x^3-b^2*x)^(1/2)*(-2*b/a*EllipticE(((x+b/a)/b*a)^(1/2),1/2*2^(
1/2))+b/a*EllipticF(((x+b/a)/b*a)^(1/2),1/2*2^(1/2))))-1/9/a*2^(1/2)*sum((-
_alpha*a+2*b)/(2*_alpha*a-b)*(_alpha*a-2*b)*((x+b/a)/b*a)^(1/2)*(-(x-b/a)/b
*a)^(1/2)*(-a*x/b)^(1/2)/(x*(a^2*x^2-b^2))^(1/2)*EllipticPi(((x+b/a)/b*a)^(
1/2),-1/3*(_alpha*a-2*b)/b,1/2*2^(1/2)),_alpha=RootOf(_Z^2*a^2-_Z*a*b+b^2))
-1/3*b*(-(a^2*x^2-a*b*x)/b^2/a/((x+b/a)*(a^2*x^2-a*b*x))^(1/2)+1/2/a*((x+b/
a)/b*a)^(1/2)*(-2*(x-b/a)/b*a)^(1/2)*(-a*x/b)^(1/2)/(a^2*x^3-b^2*x)^(1/2)*E
llipticF(((x+b/a)/b*a)^(1/2),1/2*2^(1/2))+1/2/b*((x+b/a)/b*a)^(1/2)*(-2*(x-
b/a)/b*a)^(1/2)*(-a*x/b)^(1/2)/(a^2*x^3-b^2*x)^(1/2)*(-2*b/a*EllipticE(((x+
b/a)/b*a)^(1/2),1/2*2^(1/2))+b/a*EllipticF(((x+b/a)/b*a)^(1/2),1/2*2^(1/2))
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^6x^6 + b^6}{\left(a^6x^6 - b^6\right)\sqrt{a^2x^3 - b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^6*x^6+b^6)/(a^2*x^3-b^2*x)^(1/2)/(a^6*x^6-b^6),x, algorithm="m
axima")
```

```
[Out] integrate((a^6*x^6 + b^6)/((a^6*x^6 - b^6)*sqrt(a^2*x^3 - b^2*x)), x)
```

mupad [B] time = 8.57, size = 236, normalized size = 0.97

$$\frac{2\sqrt{a^2x^3-b^2x}}{3\left(b^2-a^2x^2\right)}+\frac{3^{1/4}\sqrt{-\frac{1}{27}\mathrm{i}}\,\ln\left(\frac{(-1)^{1/4}3^{3/4}b^2-(-1)^{1/4}3^{3/4}a^2x^2-3(-1)^{3/4}3^{1/4}a\,b\,x+\sqrt{a}\,\sqrt{b}\,\sqrt{a^2x^3-b^2x}\,6\mathrm{i}}{-a^2x^2+1\mathrm{i}\,\sqrt{3}\,a\,b\,x+b^2}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{3^{1/4}\sqrt{\frac{1}{27}\mathrm{i}}\,\ln\left(\frac{(-1)^{3/4}3^{3/4}b^2-(-1)^{3/4}3^{3/4}a^2x^2-3(-1)^{1/4}3^{1/4}a\,b\,x+\sqrt{a}\,\sqrt{b}\,\sqrt{a^2x^3-b^2x}\,6\mathrm{i}}{a^2x^2+1\mathrm{i}\,\sqrt{3}\,a\,b\,x-b^2}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^6 + a^6*x^6)/((b^6 - a^6*x^6)*(a^2*x^3 - b^2*x)^(1/2)),x)
```

```
[Out] (2*(a^2*x^3 - b^2*x)^(1/2))/(3*(b^2 - a^2*x^2)) + (3^(1/4)*(-1i/27)^(1/2)*l
og((a^(1/2)*b^(1/2)*(a^2*x^3 - b^2*x)^(1/2)*6i + (-1)^(1/4)*3^(3/4)*b^2 - (
-1)^(1/4)*3^(3/4)*a^2*x^2 - 3*(-1)^(3/4)*3^(1/4)*a*b*x)/(b^2 - a^2*x^2 + 3^
(1/2)*a*b*x*1i)))/(a^(1/2)*b^(1/2)) + (3^(1/4)*(1i/27)^(1/2)*log((a^(1/2)*b
^(1/2)*(a^2*x^3 - b^2*x)^(1/2)*6i + (-1)^(3/4)*3^(3/4)*b^2 - (-1)^(3/4)*3^(
3/4)*a^2*x^2 - 3*(-1)^(1/4)*3^(1/4)*a*b*x)/(a^2*x^2 - b^2 + 3^(1/2)*a*b*x*1
i)))/(a^(1/2)*b^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{2}x^{2}+b^{2}\right)\left(a^{4}x^{4}-a^{2}b^{2}x^{2}+b^{4}\right)}{\sqrt{x\left(ax-b\right)\left(ax+b\right)}\,\left(ax-b\right)\left(ax+b\right)\left(a^{2}x^{2}-abx+b^{2}\right)\left(a^{2}x^{2}+abx+b^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**6*x**6+b**6)/(a**2*x**3-b**2*x)**(1/2)/(a**6*x**6-b**6),x)
```

```
[Out] Integral((a**2*x**2 + b**2)*(a**4*x**4 - a**2*b**2*x**2 + b**4)/(sqrt(x*(a*
x - b)*(a*x + b))*(a*x - b)*(a*x + b)*(a**2*x**2 - a*b*x + b**2)*(a**2*x**2
 + a*b*x + b**2)), x)
```

3.2157 $\int \frac{a^2b-2a^2x+(2a-b)x^2}{(x(-a+x)(-b+x))^{2/3}\left(a^2+(-2a+bd)x+(1-d)x^2\right)}\,dx$

Optimal. Leaf size=245

$$-\frac{\log\left(\left(x^2(-a-b)+abx+x^3\right)^{2/3}\left(d^{2/3}x^2-bd^{2/3}x\right)+\sqrt[3]{d}\left(x^2(-a-b)+abx+x^3\right)^{4/3}+b^2dx^2-2bdx^3+dx^4\right)}{2\sqrt[3]{d}}+\frac{\log\cdots}{\cdots}$$

Rubi [F] time = 3.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a^2b-2a^2x+(2a-b)x^2}{(x(-a+x)(-b+x))^{2/3}\left(a^2+(-2a+bd)x+(1-d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a^2*b - 2*a^2*x + (2*a - b)*x^2)/((x*(-a + x)*(-b + x))^(2/3)*(a^2 + (
-2*a + b*d)*x + (1 - d)*x^2)),x]
```

```
[Out] ((2*a - b + Sqrt[4*a^2 - 4*a*b + b^2*d]/Sqrt[d])*x^(2/3)*(-a + x)^(2/3)*(-b
 + x)^(2/3)*Defer[Int][(-a + x)^(1/3)/(x^(2/3)*(-b + x)^(2/3)*(-2*a + b*d -
 Sqrt[d]*Sqrt[4*a^2 - 4*a*b + b^2*d] + 2*(1 - d)*x)), x])/((a - x)*(b - x)*
x)^(2/3) + ((2*a - b - Sqrt[4*a^2 - 4*a*b + b^2*d]/Sqrt[d])*x^(2/3)*(-a + x
)^(2/3)*(-b + x)^(2/3)*Defer[Int][(-a + x)^(1/3)/(x^(2/3)*(-b + x)^(2/3)*(-
2*a + b*d + Sqrt[d]*Sqrt[4*a^2 - 4*a*b + b^2*d] + 2*(1 - d)*x)), x])/((a -
x)*(b - x)*x)^(2/3)
```

Rubi steps

$$\begin{aligned}\int \frac{a^2b-2a^2x+(2a-b)x^2}{(x(-a+x)(-b+x))^{2/3}\left(a^2+(-2a+bd)x+(1-d)x^2\right)}\,dx &= \frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{a^2b-2a^2x+(\cdots}{x^{2/3}(-a+x)^{2/3}(-b+x)^{2/3}\left(a^2\cdots\right.}}{(x(-a+x)(-b+x))^{2/3}}\\ &= \frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{\sqrt[3]{-a+x}\,(-ab+(2a-\cdots}{x^{2/3}(-b+x)^{2/3}\left(a^2+(-2a+bd\cdots\right.}}{(x(-a+x)(-b+x))^{2/3}}\\ &= \frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int\left(\frac{\left(2a-b+\frac{\sqrt{4a^2-\cdots}}{\cdots}\right.}{x^{2/3}(-b+x)^{2/3}\left(-2a+bd-\sqrt{\cdots}\right.}\cdots\right.}{(\cdots}\\ &= \frac{\left(\left(2a-b-\frac{\sqrt{4a^2-4ab+b^2d}}{\sqrt{d}}\right)x^{2/3}(-a+x)^{2/3}(-b+x)^{2/3}\cdots\right.}{(x(-a+x)(\cdots}\end{aligned}$$

Mathematica [F] time = 9.84, size = 0, normalized size = 0.00

$$\int \frac{a^2b-2a^2x+(2a-b)x^2}{(x(-a+x)(-b+x))^{2/3}\left(a^2+(-2a+bd)x+(1-d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a^2*b - 2*a^2*x + (2*a - b)*x^2)/((x*(-a + x)*(-b + x))^(2/3)*(a
^2 + (-2*a + b*d)*x + (1 - d)*x^2)),x]
```

```
[Out] Integrate[(a^2*b - 2*a^2*x + (2*a - b)*x^2)/((x*(-a + x)*(-b + x))^(2/3)*(a
^2 + (-2*a + b*d)*x + (1 - d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 3.22, size = 245, normalized size = 1.00

$$-\frac{\log\left(\left(x^2(-a-b)+abx+x^3\right)^{2/3}\left(d^{2/3}x^2-bd^{2/3}x\right)+\sqrt[3]{d}\left(x^2(-a-b)+abx+x^3\right)^{4/3}+b^2dx^2-2bdx^3+dx^4\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[6]{d}\left(x^2(-a-b)+abx+x^3\right)^{2/3}+b\sqrt{d}\,x-\sqrt{d}\,x^2\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(x^2(-a-b)+abx+x^3\right)^{2/3}}{\left(x^2(-a-b)+abx+x^3\right)^{2/3}-2b\sqrt[3]{d}\,x+2\sqrt[3]{d}\,x^2}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2*b - 2*a^2*x + (2*a - b)*x^2)/((x*(-a + x)*(-b + x))
^(2/3)*(a^2 + (-2*a + b*d)*x + (1 - d)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(a*b*x + (-a - b)*x^2 + x^3)^(2/3))/(-2*b*d^(1/3)*
x + 2*d^(1/3)*x^2 + (a*b*x + (-a - b)*x^2 + x^3)^(2/3))])/d^(1/3) + Log[b*S
qrt[d]*x - Sqrt[d]*x^2 + d^(1/6)*(a*b*x + (-a - b)*x^2 + x^3)^(2/3)]/d^(1/3
) - Log[b^2*d*x^2 - 2*b*d*x^3 + d*x^4 + (-(b*d^(2/3)*x) + d^(2/3)*x^2)*(a*b
*x + (-a - b)*x^2 + x^3)^(2/3) + d^(1/3)*(a*b*x + (-a - b)*x^2 + x^3)^(4/3)
]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*b-2*a^2*x+(2*a-b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a^2+(b*d-2*a
)*x+(1-d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{a^2b-2\,a^2x+(2\,a-b)x^2}{((a-x)(b-x)x)^{\frac{2}{3}}\left((d-1)x^2-a^2-(bd-2\,a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*b-2*a^2*x+(2*a-b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a^2+(b*d-2*a
)*x+(1-d)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*b - 2*a^2*x + (2*a - b)*x^2)/(((a - x)*(b - x)*x)^(2/3)*((d
 - 1)*x^2 - a^2 - (b*d - 2*a)*x)), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{a^2b-2a^2x+(2a-b)\,x^2}{\left(x\,(-a+x)\,(-b+x)\right)^{\frac{2}{3}}\left(a^2+(bd-2a)\,x+(1-d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*b-2*a^2*x+(2*a-b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a^2+(b*d-2*a)*x+(1
-d)*x^2),x)
```

```
[Out] int((a^2*b-2*a^2*x+(2*a-b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a^2+(b*d-2*a)*x+(1
-d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2b-2\,a^2x+(2\,a-b)x^2}{((a-x)(b-x)x)^{\frac{2}{3}}\left((d-1)x^2-a^2-(bd-2\,a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*b-2*a^2*x+(2*a-b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a^2+(b*d-2*a
)*x+(1-d)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*b - 2*a^2*x + (2*a - b)*x^2)/(((a - x)*(b - x)*x)^(2/3)*((d
 - 1)*x^2 - a^2 - (b*d - 2*a)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x^2\,\left(2\,a-b\right)+a^2\,b-2\,a^2\,x}{{\left(x\,\left(a-x\right)\,\left(b-x\right)\right)}^{2/3}\,\left(x\,\left(2\,a-b\,d\right)-a^2+x^2\,\left(d-1\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(2*a - b) + a^2*b - 2*a^2*x)/((x*(a - x)*(b - x))^(2/3)*(x*(2*a -
 b*d) - a^2 + x^2*(d - 1))),x)
```

```
[Out] int(-(x^2*(2*a - b) + a^2*b - 2*a^2*x)/((x*(a - x)*(b - x))^(2/3)*(x*(2*a -
 b*d) - a^2 + x^2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*b-2*a**2*x+(2*a-b)*x**2)/(x*(-a+x)*(-b+x))**(2/3)/(a**2+(b*
d-2*a)*x+(1-d)*x**2),x)
```

```
[Out] Timed out
```

3.2158 $\int \frac{b^6+a^6x^6}{\sqrt{b^2x+a^2x^3}\left(-b^6+a^6x^6\right)}\,dx$

Optimal. Leaf size=245

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt{a}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt{a}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt{2}\sqrt{a}\sqrt{b}}$$

Rubi [C] time = 8.42, antiderivative size = 956, normalized size of antiderivative = 3.90, number of steps used = 185, number of rules used = 21, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.477, Rules used = {2056, 1586, 6715, 6725, 1729, 1209, 1198, 220, 1196, 1211, 1699, 208, 1248, 735, 844, 217, 206, 725, 205, 1217, 1707}

$$-\frac{2\sqrt{x}\sqrt{b^2+a^2x^2}\tan^{-1}\left(\frac{\sqrt{-a}\sqrt{b}\sqrt{x}}{\sqrt{b^2+a^2x^2}}\right)}{3\sqrt{-a}\sqrt{b}\sqrt{a^2x^3+b^2x}}-\frac{2\sqrt{x}\sqrt{b^2+a^2x^2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{b}\sqrt{x}}{\sqrt{b^2+a^2x^2}}\right)}{3\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}-\frac{\sqrt{x}\sqrt{b^2+a^2x^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{x}}{\sqrt{b^2+a^2x^2}}\right)}{3\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}-\frac{\sqrt{x}\sqrt{b^2+a^2x^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{x}}{\sqrt{b^2+a^2x^2}}\right)}{3\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}+\frac{\left(3+i\sqrt{3}\right)\sqrt{x}(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{6\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}-\frac{\left(1+i\sqrt{3}\right)\sqrt{x}(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{3\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}+\frac{\left(3-i\sqrt{3}\right)\sqrt{x}(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{6\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}-\frac{\left(1-i\sqrt{3}\right)\sqrt{x}(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{3\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}-\frac{\left(i-\sqrt{3}\right)\sqrt{x}(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{3\left(3i-\sqrt{3}\right)\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}-\frac{\sqrt{x}(b+ax)\sqrt{\frac{b^2+a^2x^2}{(b+ax)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)|\frac{1}{2}\right)}{3\left(1+\sqrt[3]{-1}\right)\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b^6 + a^6*x^6)/(Sqrt[b^2*x + a^2*x^3]*(-b^6 + a^6*x^6)),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[b^2 + a^2*x^2]*ArcTan[(Sqrt[-a]*Sqrt[b]*Sqrt[x])/Sqrt[b^2
+ a^2*x^2]])/(3*Sqrt[-a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) - (2*Sqrt[x]*Sqrt[b
^2 + a^2*x^2]*ArcTan[(Sqrt[a]*Sqrt[b]*Sqrt[x])/Sqrt[b^2 + a^2*x^2]])/(3*Sqr
t[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) - (Sqrt[x]*Sqrt[b^2 + a^2*x^2]*ArcTan[(
Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[x])/Sqrt[b^2 + a^2*x^2]])/(3*Sqrt[2]*Sqrt[a]*S
qrt[b]*Sqrt[b^2*x + a^2*x^3]) - (Sqrt[x]*Sqrt[b^2 + a^2*x^2]*ArcTanh[(Sqrt[
2]*Sqrt[a]*Sqrt[b]*Sqrt[x])/Sqrt[b^2 + a^2*x^2]])/(3*Sqrt[2]*Sqrt[a]*Sqrt[b
]*Sqrt[b^2*x + a^2*x^3]) - (Sqrt[x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x
)^2]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]], 1/2])/(3*(1 + (-1)^(1/3
))*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) - ((I - Sqrt[3])*Sqrt[x]*(b + a*x
)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt[x])/Sq
rt[b]], 1/2])/(3*(3*I - Sqrt[3])*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) - (
(1 - I*Sqrt[3])*Sqrt[x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2]*Ellipti
cF[2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]], 1/2])/(3*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x
 + a^2*x^3]) + ((3 - I*Sqrt[3])*Sqrt[x]*(b + a*x)*Sqrt[(b^2 + a^2*x^2)/(b +
 a*x)^2]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]], 1/2])/(6*Sqrt[a]*Sq
rt[b]*Sqrt[b^2*x + a^2*x^3]) - ((1 + I*Sqrt[3])*Sqrt[x]*(b + a*x)*Sqrt[(b^2
 + a^2*x^2)/(b + a*x)^2]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]], 1/2
])/(3*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3]) + ((3 + I*Sqrt[3])*Sqrt[x]*(b
+ a*x)*Sqrt[(b^2 + a^2*x^2)/(b + a*x)^2]*EllipticF[2*ArcTan[(Sqrt[a]*Sqrt[x
])/Sqrt[b]], 1/2])/(6*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
```

```
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Dist[d, I
nt[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, Int[(x*(a + c*x^4)^p)/(d
^2 - e^2*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{b^6+a^6x^6}{\sqrt{b^2x+a^2x^3}\left(-b^6+a^6x^6\right)}\,dx = \frac{\left(\sqrt{x}\sqrt{b^2+a^2x^2}\right)\int \frac{b^6+a^6x^6}{\sqrt{x}\sqrt{b^2+a^2x^2}\left(-b^6+a^6x^6\right)}\,dx}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(\sqrt{x}\sqrt{b^2+a^2x^2}\right)\int \frac{\sqrt{b^2+a^2x^2}\left(b^4-a^2b^2x^2+a^4x^4\right)}{\sqrt{x}\left(-b^6+a^6x^6\right)}\,dx}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{b^2+a^2x^4}\left(b^4-a^2b^2x^4+a^4x^8\right)}{-b^6+a^6x^{12}}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2x+a^2x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{b^2+a^2x^2}\right)\text{Subst}\left(\int\left(-\frac{\sqrt{b^2+a^2x^4}}{12b^{3/2}\left(\sqrt{b}-\sqrt{a}\,x\right)}-\frac{\sqrt{b^2+a^2x^4}}{12b^{3/2}\left(\sqrt{b}-i\sqrt{a}\,x\right)}-\frac{\sqrt{b^2+a^2x^4}}{12b^{3/2}\left(\sqrt{b}+i\sqrt{a}\right.}\right.\right.}{}$$

= rest of steps removed due to Latex formating problem

Mathematica [C] time = 2.62, size = 296, normalized size = 1.21

$$\frac{2ix^{3/2}\sqrt{\frac{b^2}{a^2x^2}+1}\left(-3F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)+\Pi\left(-i;i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)+\Pi\left(i;i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)+\Pi\left(-\frac{i}{2}-\frac{\sqrt{3}}{2};i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)+\Pi\left(\frac{i}{2}-\frac{\sqrt{3}}{2};i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)+\Pi\left(\frac{1}{2}\left(-i+\sqrt{3}\right);i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)+\Pi\left(\frac{1}{2}\left(i+\sqrt{3}\right);i\sinh^{-1}\left(\frac{\sqrt{\frac{ib}{a}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{3\sqrt{\frac{ib}{a}}\sqrt{x\left(a^2x^2+b^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^6 + a^6*x^6)/(Sqrt[b^2*x + a^2*x^3]*(-b^6 + a^6*x^6)),x]
```

```
[Out] (((2*I)/3)*Sqrt[1 + b^2/(a^2*x^2)]*x^(3/2)*(-3*EllipticF[I*ArcSinh[Sqrt[(I*
b)/a]/Sqrt[x]], -1] + EllipticPi[-I, I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1]
+ EllipticPi[I, I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] + EllipticPi[-1/2*I -
 Sqrt[3]/2, I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] + EllipticPi[I/2 - Sqrt[3
]/2, I*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] + EllipticPi[(-I + Sqrt[3])/2, I
*ArcSinh[Sqrt[(I*b)/a]/Sqrt[x]], -1] + EllipticPi[(I + Sqrt[3])/2, I*ArcSin
h[Sqrt[(I*b)/a]/Sqrt[x]], -1]))/(Sqrt[(I*b)/a]*Sqrt[x*(b^2 + a^2*x^2)])
```

IntegrateAlgebraic [A] time = 0.61, size = 245, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt{a}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt{a}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}}{a^2x^2+b^2}\right)}{3\sqrt{2}\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^6 + a^6*x^6)/(Sqrt[b^2*x + a^2*x^3]*(-b^6 + a^6*x^6))
,x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])/(b^2 + a^2*x^2)])/(3*Sqr
t[a]*Sqrt[b]) - ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])/(b^2
 + a^2*x^2)]/(3*Sqrt[2]*Sqrt[a]*Sqrt[b]) - (2*ArcTanh[(Sqrt[a]*Sqrt[b]*Sqrt
[b^2*x + a^2*x^3])/(b^2 + a^2*x^2)])/(3*Sqrt[a]*Sqrt[b]) - ArcTanh[(Sqrt[2]
*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])/(b^2 + a^2*x^2)]/(3*Sqrt[2]*Sqrt[a]
*Sqrt[b])
```

fricas [B] time = 0.83, size = 828, normalized size = 3.38

$$\left[-\frac{2\sqrt{2}ab\sqrt{\frac{1}{ab}}\arctan\left(\frac{2\sqrt{2}\sqrt{a^2x^3+b^2x}\,ab\sqrt{\frac{1}{ab}}}{a^2x^2-2abx+b^2}\right)-\sqrt{2}ab\sqrt{\frac{1}{ab}}\log\left(\frac{a^4x^4+12a^3bx^3+6a^2b^2x^2+12ab^3x+b^4-4\sqrt{2}\left(a^3bx^2+2a^2b^2x+ab^3\right)\sqrt{a^2x^3+b^2x}\sqrt{\frac{1}{ab}}}{a^4x^4-4a^3bx^3+6a^2b^2x^2-4ab^3x+b^4}\right)-8\sqrt{ab}\arctan\left(\frac{\sqrt{a^2x^3+b^2x}\left(a^2x^2-abx+b^2\right)\sqrt{ab}}{2\left(a^3bx^3+ab^3x\right)}\right)-4\sqrt{ab}\log\left(\frac{a^4x^4+6a^3bx^3+3a^2b^2x^2+6ab^3x+b^4-4\sqrt{a^2x^3+b^2x}\left(a^2x^2+abx+b^2\right)\sqrt{ab}}{a^4x^4-2a^3bx^3+3a^2b^2x^2-2ab^3x+b^4}\right)}{24ab}, \frac{2\sqrt{2}ab\sqrt{-\frac{1}{ab}}\arctan\left(\frac{2\sqrt{2}\sqrt{a^2x^3+b^2x}\,ab\sqrt{-\frac{1}{ab}}}{a^2x^2+2abx+b^2}\right)+\sqrt{2}ab\sqrt{-\frac{1}{ab}}\log\left(\frac{a^4x^4-12a^3bx^3+6a^2b^2x^2-12ab^3x+b^4+4\sqrt{2}\left(a^3bx^2-2a^2b^2x+ab^3\right)\sqrt{a^2x^3+b^2x}\sqrt{-\frac{1}{ab}}}{a^4x^4+4a^3bx^3+6a^2b^2x^2+4ab^3x+b^4}\right)+8\sqrt{-ab}\arctan\left(\frac{\sqrt{a^2x^3+b^2x}\left(a^2x^2+abx+b^2\right)\sqrt{-ab}}{2\left(a^3bx^3+ab^3x\right)}\right)-4\sqrt{-ab}\log\left(\frac{a^4x^4-6a^3bx^3+3a^2b^2x^2-6ab^3x+b^4-4\sqrt{a^2x^3+b^2x}\left(a^2x^2-abx+b^2\right)\sqrt{-ab}}{a^4x^4+2a^3bx^3+3a^2b^2x^2+2ab^3x+b^4}\right)}{24ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^6*x^6+b^6)/(a^2*x^3+b^2*x)^(1/2)/(a^6*x^6-b^6),x, algorithm="f
ricas")
```

```
[Out] [-1/24*(2*sqrt(2)*a*b*sqrt(1/(a*b))*arctan(2*sqrt(2)*sqrt(a^2*x^3 + b^2*x)*
a*b*sqrt(1/(a*b))/(a^2*x^2 - 2*a*b*x + b^2)) - sqrt(2)*a*b*sqrt(1/(a*b))*lo
g((a^4*x^4 + 12*a^3*b*x^3 + 6*a^2*b^2*x^2 + 12*a*b^3*x + b^4 - 4*sqrt(2)*(a
^3*b*x^2 + 2*a^2*b^2*x + a*b^3)*sqrt(a^2*x^3 + b^2*x)*sqrt(1/(a*b)))/(a^4*x
^4 - 4*a^3*b*x^3 + 6*a^2*b^2*x^2 - 4*a*b^3*x + b^4)) - 8*sqrt(a*b)*arctan(1
/2*sqrt(a^2*x^3 + b^2*x)*(a^2*x^2 - a*b*x + b^2)*sqrt(a*b)/(a^3*b*x^3 + a*b
^3*x)) - 4*sqrt(a*b)*log((a^4*x^4 + 6*a^3*b*x^3 + 3*a^2*b^2*x^2 + 6*a*b^3*x
 + b^4 - 4*sqrt(a^2*x^3 + b^2*x)*(a^2*x^2 + a*b*x + b^2)*sqrt(a*b))/(a^4*x^
4 - 2*a^3*b*x^3 + 3*a^2*b^2*x^2 - 2*a*b^3*x + b^4)))/(a*b), 1/24*(2*sqrt(2)
*a*b*sqrt(-1/(a*b))*arctan(2*sqrt(2)*sqrt(a^2*x^3 + b^2*x)*a*b*sqrt(-1/(a*b
))/(a^2*x^2 + 2*a*b*x + b^2)) + sqrt(2)*a*b*sqrt(-1/(a*b))*log((a^4*x^4 - 1
2*a^3*b*x^3 + 6*a^2*b^2*x^2 - 12*a*b^3*x + b^4 + 4*sqrt(2)*(a^3*b*x^2 - 2*a
^2*b^2*x + a*b^3)*sqrt(a^2*x^3 + b^2*x)*sqrt(-1/(a*b)))/(a^4*x^4 + 4*a^3*b*
x^3 + 6*a^2*b^2*x^2 + 4*a*b^3*x + b^4)) + 8*sqrt(-a*b)*arctan(1/2*sqrt(a^2*
x^3 + b^2*x)*(a^2*x^2 + a*b*x + b^2)*sqrt(-a*b)/(a^3*b*x^3 + a*b^3*x)) - 4*
sqrt(-a*b)*log((a^4*x^4 - 6*a^3*b*x^3 + 3*a^2*b^2*x^2 - 6*a*b^3*x + b^4 - 4
*sqrt(a^2*x^3 + b^2*x)*(a^2*x^2 - a*b*x + b^2)*sqrt(-a*b))/(a^4*x^4 + 2*a^3
*b*x^3 + 3*a^2*b^2*x^2 + 2*a*b^3*x + b^4)))/(a*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^6x^6+b^6}{\left(a^6x^6-b^6\right)\sqrt{a^2x^3+b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^6*x^6+b^6)/(a^2*x^3+b^2*x)^(1/2)/(a^6*x^6-b^6),x, algorithm="g
iac")
```

```
[Out] integrate((a^6*x^6 + b^6)/((a^6*x^6 - b^6)*sqrt(a^2*x^3 + b^2*x)), x)
```

maple [C] time = 0.05, size = 681, normalized size = 2.78

$$\frac{ib\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{2}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a^2x^3+b^2x}}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^2a^2+\_Zab+b^2\right)}\frac{\left(-\_\alpha a-2b\right)\left(i\_\alpha a+ib+b\right)\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},\frac{\_\alpha a-ib+b}{b},\frac{\sqrt{2}}{2}\right)}{\left(2\_\alpha a+b\right)\sqrt{x\left(a^2x^2+b^2\right)}}\right)}{3a}+\frac{ib^2\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{2}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},-\frac{ib}{a\left(-\frac{ib}{a}-\frac{b}{a}\right)},\frac{\sqrt{2}}{2}\right)}{3a^2\sqrt{a^2x^3+b^2x}\,\left(-\frac{ib}{a}-\frac{b}{a}\right)}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^2a^2-\_Zab+b^2\right)}\frac{\left(-\_\alpha a+2b\right)\left(i\_\alpha a-ib+b\right)\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},-\frac{\_\alpha a-ib-b}{b},\frac{\sqrt{2}}{2}\right)}{\left(2\_\alpha a-b\right)\sqrt{x\left(a^2x^2+b^2\right)}}\right)}{3a}-\frac{ib^2\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}}\,\sqrt{2}\,\sqrt{\frac{i\left(x-\frac{ib}{a}\right)a}{b}}\,\sqrt{\frac{ixa}{b}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{i\left(x+\frac{ib}{a}\right)a}{b}},-\frac{ib}{a\left(-\frac{ib}{a}+\frac{b}{a}\right)},\frac{\sqrt{2}}{2}\right)}{3a^2\sqrt{a^2x^3+b^2x}\,\left(-\frac{ib}{a}+\frac{b}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^6*x^6+b^6)/(a^2*x^3+b^2*x)^(1/2)/(a^6*x^6-b^6),x)
```

```
[Out] I*b/a*(-I*(x+I*b/a)/b*a)^(1/2)*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1
/2)/(a^2*x^3+b^2*x)^(1/2)*EllipticF((-I*(x+I*b/a)/b*a)^(1/2),1/2*2^(1/2))+1
/3*I/a*2^(1/2)*sum((-_alpha*a-2*b)/(2*_alpha*a+b)*(I*_alpha*a+I*b+b)*(-I*(x
+I*b/a)/b*a)^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1/2)/(x*(a^2*x^2+b^2)
)^(1/2)*EllipticPi((-I*(x+I*b/a)/b*a)^(1/2),(_alpha*a+b-I*b)/b,1/2*2^(1/2))
,_alpha=RootOf(_Z^2*a^2+_Z*a*b+b^2))+1/3*I*b^2/a^2*(-I*(x+I*b/a)/b*a)^(1/2)
*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1/2)/(a^2*x^3+b^2*x)^(1/2)/(-I*
b/a-b/a)*EllipticPi((-I*(x+I*b/a)/b*a)^(1/2),-I*b/a/(-I*b/a-b/a),1/2*2^(1/2
))+1/3*I/a*2^(1/2)*sum((-_alpha*a+2*b)/(2*_alpha*a-b)*(I*_alpha*a-I*b+b)*(-
I*(x+I*b/a)/b*a)^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1/2)/(x*(a^2*x^2+
b^2))^(1/2)*EllipticPi((-I*(x+I*b/a)/b*a)^(1/2),-(_alpha*a-b-I*b)/b,1/2*2^(
1/2)),_alpha=RootOf(_Z^2*a^2-_Z*a*b+b^2))-1/3*I*b^2/a^2*(-I*(x+I*b/a)/b*a)^
(1/2)*2^(1/2)*(I*(x-I*b/a)/b*a)^(1/2)*(I*x/b*a)^(1/2)/(a^2*x^3+b^2*x)^(1/2)
/(-I*b/a+b/a)*EllipticPi((-I*(x+I*b/a)/b*a)^(1/2),-I*b/a/(-I*b/a+b/a),1/2*2
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^6x^6 + b^6}{(a^6x^6 - b^6)\sqrt{a^2x^3 + b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^6*x^6+b^6)/(a^2*x^3+b^2*x)^(1/2)/(a^6*x^6-b^6),x, algorithm="m
axima")
```

```
[Out] integrate((a^6*x^6 + b^6)/((a^6*x^6 - b^6)*sqrt(a^2*x^3 + b^2*x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^6 + a^6*x^6)/((b^6 - a^6*x^6)*(b^2*x + a^2*x^3)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2x^2 + b^2)(a^4x^4 - a^2b^2x^2 + b^4)}{\sqrt{x(a^2x^2 + b^2)}\,(ax - b)(ax + b)(a^2x^2 - abx + b^2)(a^2x^2 + abx + b^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**6*x**6+b**6)/(a**2*x**3+b**2*x)**(1/2)/(a**6*x**6-b**6),x)
```

```
[Out] Integral((a**2*x**2 + b**2)*(a**4*x**4 - a**2*b**2*x**2 + b**4)/(sqrt(x*(a*
*2*x**2 + b**2))*(a*x - b)*(a*x + b)*(a**2*x**2 - a*b*x + b**2)*(a**2*x**2
+ a*b*x + b**2)), x)
```

3.2159 $\int \frac{(2+5x^7)\sqrt[3]{-x-x^3+x^8}}{(-1+x^7)(-1+x^2+x^7)}\,dx$

Optimal. Leaf size=245

$$-\log\left(\sqrt[3]{x^8-x^3-x}+x\right)+\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^8-x^3-x}+2x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{2\sqrt[3]{x^8-x^3-x}-x}\right)-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\ }{2}\right.$$

Rubi [F] time = 9.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(2+5x^7)\sqrt[3]{-x-x^3+x^8}}{(-1+x^7)(-1+x^2+x^7)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((2 + 5*x^7)*(-x - x^3 + x^8)^(1/3))/((-1 + x^7)*(-1 + x^2 + x^7)),x]

[Out] ((-x - x^3 + x^8)^(1/3)*Defer[Subst][Defer[Int][(-1 - x^6 + x^21)^(1/3)/(-1
 + x), x], x, x^(1/3)])/(x^(1/3)*(-1 - x^2 + x^7)^(1/3)) - ((1 - I*Sqrt[3])
*(-x - x^3 + x^8)^(1/3)*Defer[Subst][Defer[Int][(-1 - x^6 + x^21)^(1/3)/(1
- I*Sqrt[3] + 2*x), x], x, x^(1/3)])/(x^(1/3)*(-1 - x^2 + x^7)^(1/3)) - ((1
 + I*Sqrt[3])*(-x - x^3 + x^8)^(1/3)*Defer[Subst][Defer[Int][(-1 - x^6 + x^
21)^(1/3)/(1 + I*Sqrt[3] + 2*x), x], x, x^(1/3)])/(x^(1/3)*(-1 - x^2 + x^7)
^(1/3)) + ((-x - x^3 + x^8)^(1/3)*Defer[Subst][Defer[Int][(-1 - x^6 + x^21)
^(1/3)/(1 + x + x^2 + x^3 + x^4 + x^5 + x^6), x], x, x^(1/3)])/(x^(1/3)*(-1
 - x^2 + x^7)^(1/3)) + (2*(-x - x^3 + x^8)^(1/3)*Defer[Subst][Defer[Int][(x
*(-1 - x^6 + x^21)^(1/3))/(1 + x + x^2 + x^3 + x^4 + x^5 + x^6), x], x, x^(
1/3)])/(x^(1/3)*(-1 - x^2 + x^7)^(1/3)) + (3*(-x - x^3 + x^8)^(1/3)*Defer[S
ubst][Defer[Int][(x^2*(-1 - x^6 + x^21)^(1/3))/(1 + x + x^2 + x^3 + x^4 + x
^5 + x^6), x], x, x^(1/3)])/(x^(1/3)*(-1 - x^2 + x^7)^(1/3)) + (4*(-x - x^3
 + x^8)^(1/3)*Defer[Subst][Defer[Int][(x^3*(-1 - x^6 + x^21)^(1/3))/(1 + x
+ x^2 + x^3 + x^4 + x^5 + x^6), x], x, x^(1/3)])/(x^(1/3)*(-1 - x^2 + x^7)^
(1/3)) - (2*(-x - x^3 + x^8)^(1/3)*Defer[Subst][Defer[Int][(x^4*(-1 - x^6 +
 x^21)^(1/3))/(1 + x + x^2 + x^3 + x^4 + x^5 + x^6), x], x, x^(1/3)])/(x^(1
/3)*(-1 - x^2 + x^7)^(1/3)) - ((-x - x^3 + x^8)^(1/3)*Defer[Subst][Defer[In
t][(x^5*(-1 - x^6 + x^21)^(1/3))/(1 + x + x^2 + x^3 + x^4 + x^5 + x^6), x],
 x, x^(1/3)])/(x^(1/3)*(-1 - x^2 + x^7)^(1/3)) + (2*(-x - x^3 + x^8)^(1/3)*
Defer[Subst][Defer[Int][(-1 - x^6 + x^21)^(1/3)/(1 - x + x^3 - x^4 + x^6 -
x^8 + x^9 - x^11 + x^12), x], x, x^(1/3)])/(x^(1/3)*(-1 - x^2 + x^7)^(1/3))
 - (3*(-x - x^3 + x^8)^(1/3)*Defer[Subst][Defer[Int][(x*(-1 - x^6 + x^21)^(
1/3))/(1 - x + x^3 - x^4 + x^6 - x^8 + x^9 - x^11 + x^12), x], x, x^(1/3)])
/(x^(1/3)*(-1 - x^2 + x^7)^(1/3)) + (5*(-x - x^3 + x^8)^(1/3)*Defer[Subst][
Defer[Int][(x^3*(-1 - x^6 + x^21)^(1/3))/(1 - x + x^3 - x^4 + x^6 - x^8 + x
^9 - x^11 + x^12), x], x, x^(1/3)])/(x^(1/3)*(-1 - x^2 + x^7)^(1/3)) + ((-x
 - x^3 + x^8)^(1/3)*Defer[Subst][Defer[Int][(x^4*(-1 - x^6 + x^21)^(1/3))/(
1 - x + x^3 - x^4 + x^6 - x^8 + x^9 - x^11 + x^12), x], x, x^(1/3)])/(x^(1/
3)*(-1 - x^2 + x^7)^(1/3)) - (7*(-x - x^3 + x^8)^(1/3)*Defer[Subst][Defer[I
nt][(x^5*(-1 - x^6 + x^21)^(1/3))/(1 - x + x^3 - x^4 + x^6 - x^8 + x^9 - x^
11 + x^12), x], x, x^(1/3)])/(x^(1/3)*(-1 - x^2 + x^7)^(1/3)) + (8*(-x - x^
3 + x^8)^(1/3)*Defer[Subst][Defer[Int][(x^6*(-1 - x^6 + x^21)^(1/3))/(1 - x
 + x^3 - x^4 + x^6 - x^8 + x^9 - x^11 + x^12), x], x, x^(1/3)])/(x^(1/3)*(-
1 - x^2 + x^7)^(1/3)) - (10*(-x - x^3 + x^8)^(1/3)*Defer[Subst][Defer[Int][
(x^8*(-1 - x^6 + x^21)^(1/3))/(1 - x + x^3 - x^4 + x^6 - x^8 + x^9 - x^11 +
 x^12), x], x, x^(1/3)])/(x^(1/3)*(-1 - x^2 + x^7)^(1/3)) + (11*(-x - x^3 +
 x^8)^(1/3)*Defer[Subst][Defer[Int][(x^9*(-1 - x^6 + x^21)^(1/3))/(1 - x +
x^3 - x^4 + x^6 - x^8 + x^9 - x^11 + x^12), x], x, x^(1/3)])/(x^(1/3)*(-1 -
```

```
x^2 + x^7)^(1/3)) + ((-x - x^3 + x^8)^(1/3)*Defer[Subst][Defer[Int][(x^11*
(-1 - x^6 + x^21)^(1/3))/(1 - x + x^3 - x^4 + x^6 - x^8 + x^9 - x^11 + x^12
), x], x, x^(1/3)])/(x^(1/3)*(-1 - x^2 + x^7)^(1/3)) - (6*(-x - x^3 + x^8)^
(1/3)*Defer[Subst][Defer[Int][(x^3*(-1 - x^6 + x^21)^(1/3))/(-1 + x^6 + x^2
1), x], x, x^(1/3)])/(x^(1/3)*(-1 - x^2 + x^7)^(1/3)) - (21*(-x - x^3 + x^8
)^(1/3)*Defer[Subst][Defer[Int][(x^18*(-1 - x^6 + x^21)^(1/3))/(-1 + x^6 +
x^21), x], x, x^(1/3)])/(x^(1/3)*(-1 - x^2 + x^7)^(1/3))
```

Rubi steps

$$\int \frac{(2+5x^7)\sqrt[3]{-x-x^3+x^8}}{(-1+x^7)(-1+x^2+x^7)}\,dx = \frac{\sqrt[3]{-x-x^3+x^8}\int \frac{\sqrt[3]{x}\sqrt[3]{-1-x^2+x^7}(2+5x^7)}{(-1+x^7)(-1+x^2+x^7)}\,dx}{\sqrt[3]{x}\sqrt[3]{-1-x^2+x^7}}$$

$$= \frac{\sqrt[3]{-x-x^3+x^8}\int \left(\frac{\sqrt[3]{x}\sqrt[3]{-1-x^2+x^7}}{-1+x} + \frac{\sqrt[3]{x}(1+2x+3x^2+4x^3+5x^4-x^5)\sqrt[3]{-1-x^2+x^7}}{1+x+x^2+x^3+x^4+x^5+x^6} + \frac{\sqrt[3]{x}(-2-7\ldots}{-1\ldots}\right.}{\sqrt[3]{x}\sqrt[3]{-1-x^2+x^7}}$$

$$= \frac{\sqrt[3]{-x-x^3+x^8}\int \frac{\sqrt[3]{x}\sqrt[3]{-1-x^2+x^7}}{-1+x}\,dx}{\sqrt[3]{x}\sqrt[3]{-1-x^2+x^7}} + \frac{\sqrt[3]{-x-x^3+x^8}\int \frac{\sqrt[3]{x}(1+2x+3x^2+4x^3+5x^4-x^5)\ldots}{1+x+x^2+x^3+x^4+x^5+\ldots}}{\sqrt[3]{x}\sqrt[3]{-1-x^2+x^7}}$$

$$= \frac{\left(3\sqrt[3]{-x-x^3+x^8}\right)\text{Subst}\left(\int \frac{x^3\sqrt[3]{-1-x^6+x^{21}}}{-1+x^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{-1-x^2+x^7}} + \frac{\left(3\sqrt[3]{-x-x^3+x^8}\right)\text{Sub}\ldots}{}$$

$$= \frac{\left(3\sqrt[3]{-x-x^3+x^8}\right)\text{Subst}\left(\int \left(\sqrt[3]{-1-x^6+x^{21}} + \frac{\sqrt[3]{-1-x^6+x^{21}}}{-1+x^3}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{-1-x^2+x^7}} + \frac{(3\ldots}{}$$

$$= \frac{\left(3\sqrt[3]{-x-x^3+x^8}\right)\text{Subst}\left(\int \frac{\sqrt[3]{-1-x^6+x^{21}}}{-1+x^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{-1-x^2+x^7}} + \frac{\left(3\sqrt[3]{-x-x^3+x^8}\right)\text{Subst}\ldots}{}$$

$$= \frac{\left(3\sqrt[3]{-x-x^3+x^8}\right)\text{Subst}\left(\int \left(\frac{\sqrt[3]{-1-x^6+x^{21}}}{3(-1+x)} + \frac{(-2-x)\sqrt[3]{-1-x^6+x^{21}}}{3(1+x+x^2)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{-1-x^2+x^7}} + \frac{(3\ldots}{}$$

$$= \frac{\sqrt[3]{-x-x^3+x^8}\,\text{Subst}\left(\int \frac{\sqrt[3]{-1-x^6+x^{21}}}{-1+x}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{-1-x^2+x^7}} + \frac{\sqrt[3]{-x-x^3+x^8}\,\text{Subst}\left(\int \frac{(-2\ldots}{}\right.}{\sqrt[3]{x}\sqrt[3]{-1-\ldots}}$$

$$= \frac{\sqrt[3]{-x-x^3+x^8}\,\text{Subst}\left(\int \frac{\sqrt[3]{-1-x^6+x^{21}}}{-1+x}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{-1-x^2+x^7}} + \frac{\sqrt[3]{-x-x^3+x^8}\,\text{Subst}\left(\int\left(\frac{(\ldots}{}\right.\right.}{}$$

$$= \frac{\sqrt[3]{-x-x^3+x^8}\,\text{Subst}\left(\int \frac{\sqrt[3]{-1-x^6+x^{21}}}{-1+x}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x}\sqrt[3]{-1-x^2+x^7}} + \frac{\sqrt[3]{-x-x^3+x^8}\,\text{Subst}\left(\int \frac{}{1+\ldots}\right.}{\sqrt[3]{x}\sqrt[3]{-1-\ldots}}$$

Mathematica [F] time = 180.09, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[((2 + 5*x^7)*(-x - x^3 + x^8)^(1/3))/((-1 + x^7)*(-1 + x^2 + x^7)
),x]

[Out] $Aborted
```

IntegrateAlgebraic [A] time = 0.72, size = 245, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^8-x^3-x}+x\right)+\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{x^8-x^3-x}+2x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^8-x^3-x}-x}\right)-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^8-x^3-x}-x}\right)+\frac{1}{2}\log\left(x^2-\sqrt[3]{x^8-x^3-x}\,x+\left(x^8-x^3-x\right)^{2/3}\right)-\frac{\log\left(-2x^2+2^{2/3}\sqrt[3]{x^8-x^3-x}\,x-\sqrt[3]{2}\left(x^8-x^3-x\right)^{2/3}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 5*x^7)*(-x - x^3 + x^8)^(1/3))/((-1 + x^7)*(-1 + x
^2 + x^7)),x]
```

```
[Out] Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(-x - x^3 + x^8)^(1/3))] - 2^(1/3)*Sqrt[
3]*ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(-x - x^3 + x^8)^(1/3))] - Log[x + (-x
- x^3 + x^8)^(1/3)] + 2^(1/3)*Log[2*x + 2^(2/3)*(-x - x^3 + x^8)^(1/3)] + L
og[x^2 - x*(-x - x^3 + x^8)^(1/3) + (-x - x^3 + x^8)^(2/3)]/2 - Log[-2*x^2
+ 2^(2/3)*x*(-x - x^3 + x^8)^(1/3) - 2^(1/3)*(-x - x^3 + x^8)^(2/3)]/2^(2/3
)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^7+2)*(x^8-x^3-x)^(1/3)/(x^7-1)/(x^7+x^2-1),x, algorithm="fri
cas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8-x^3-x\right)^{\frac{1}{3}}\left(5x^7+2\right)}{\left(x^7+x^2-1\right)\left(x^7-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^7+2)*(x^8-x^3-x)^(1/3)/(x^7-1)/(x^7+x^2-1),x, algorithm="gia
c")
```

```
[Out] integrate((x^8 - x^3 - x)^(1/3)*(5*x^7 + 2)/((x^7 + x^2 - 1)*(x^7 - 1)), x)
```

maple [F] time = 2.59, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^7+2\right)\left(x^8-x^3-x\right)^{\frac{1}{3}}}{\left(x^7-1\right)\left(x^7+x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^7+2)*(x^8-x^3-x)^(1/3)/(x^7-1)/(x^7+x^2-1),x)
```

```
[Out] int((5*x^7+2)*(x^8-x^3-x)^(1/3)/(x^7-1)/(x^7+x^2-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8-x^3-x\right)^{\frac{1}{3}}\left(5x^7+2\right)}{\left(x^7+x^2-1\right)\left(x^7-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^7+2)*(x^8-x^3-x)^(1/3)/(x^7-1)/(x^7+x^2-1),x, algorithm="max
ima")
```

```
[Out] integrate((x^8 - x^3 - x)^(1/3)*(5*x^7 + 2)/((x^7 + x^2 - 1)*(x^7 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(5\,x^7+2\right)\,{\left(x^8-x^3-x\right)}^{1/3}}{\left(x^7-1\right)\,\left(x^7+x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x^7 + 2)*(x^8 - x^3 - x)^(1/3))/((x^7 - 1)*(x^2 + x^7 - 1)),x)
```

```
[Out] int(((5*x^7 + 2)*(x^8 - x^3 - x)^(1/3))/((x^7 - 1)*(x^2 + x^7 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**7+2)*(x**8-x**3-x)**(1/3)/(x**7-1)/(x**7+x**2-1),x)
```

```
[Out] Timed out
```

3.2160 $\int \frac{x^6(-4+x^3)}{(-1+x^3)^{3/4}(1-2x^3+x^6+x^8)}\,dx$

Optimal. Leaf size=245

$$\frac{1}{2}\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^3-1}}{\sqrt{x^3-1}-x^2}\right)-\frac{1}{2}\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\left(\sqrt{\frac{2}{2-\sqrt{2}}}-\frac{2}{\sqrt{2-\sqrt{2}}}\right)x\sqrt[4]{x^3-1}}{\sqrt{x^3-1}-x^2}\right)-\frac{1}{2}\sqrt{2-\sqrt{2}}\,\tan$$

Rubi [F] time = 1.42, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^6\left(-4+x^3\right)}{\left(-1+x^3\right)^{3/4}\left(1-2x^3+x^6+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^6*(-4 + x^3))/((-1 + x^3)^(3/4)*(1 - 2*x^3 + x^6 + x^8)),x]
```

```
[Out] (x^2*(1 - x^3)^(3/4)*Hypergeometric2F1[2/3, 3/4, 5/3, x^3])/(2*(-1 + x^3)^(
3/4)) - Defer[Int][x/((-1 + x^3)^(3/4)*(1 - 2*x^3 + x^6 + x^8)), x] + 2*Def
er[Int][x^4/((-1 + x^3)^(3/4)*(1 - 2*x^3 + x^6 + x^8)), x] - 4*Defer[Int][x
^6/((-1 + x^3)^(3/4)*(1 - 2*x^3 + x^6 + x^8)), x] - Defer[Int][x^7/((-1 + x
^3)^(3/4)*(1 - 2*x^3 + x^6 + x^8)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6\left(-4+x^3\right)}{\left(-1+x^3\right)^{3/4}\left(1-2x^3+x^6+x^8\right)}\,dx &= \int \left(\frac{x}{\left(-1+x^3\right)^{3/4}}+\frac{x\left(-1+2x^3-4x^5-x^6\right)}{\left(-1+x^3\right)^{3/4}\left(1-2x^3+x^6+x^8\right)}\right)dx\\ &= \int \frac{x}{\left(-1+x^3\right)^{3/4}}\,dx+\int \frac{x\left(-1+2x^3-4x^5-x^6\right)}{\left(-1+x^3\right)^{3/4}\left(1-2x^3+x^6+x^8\right)}\,dx\\ &= \frac{\left(1-x^3\right)^{3/4}\int \frac{x}{\left(1-x^3\right)^{3/4}}\,dx}{\left(-1+x^3\right)^{3/4}}+\int \left(-\frac{x}{\left(-1+x^3\right)^{3/4}\left(1-2x^3+x^6+x^8\right)}+\ldots\right.\\ &= \frac{x^2\left(1-x^3\right)^{3/4}\,{}_2F_1\left(\frac{2}{3},\frac{3}{4};\frac{5}{3};x^3\right)}{2\left(-1+x^3\right)^{3/4}}+2\int \frac{x^4}{\left(-1+x^3\right)^{3/4}\left(1-2x^3+x^6+x^8\right)}\ldots\end{aligned}$$

Mathematica [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(-4+x^3\right)}{\left(-1+x^3\right)^{3/4}\left(1-2x^3+x^6+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^6*(-4 + x^3))/((-1 + x^3)^(3/4)*(1 - 2*x^3 + x^6 + x^8)),x]
```

```
[Out] Integrate[(x^6*(-4 + x^3))/((-1 + x^3)^(3/4)*(1 - 2*x^3 + x^6 + x^8)), x]
```

IntegrateAlgebraic [A] time = 4.08, size = 225, normalized size = 0.92

$$\frac{1}{2}\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^3-1}}{\sqrt{x^3-1}-x^2}\right)+\frac{1}{2}\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^3-1}}{\sqrt{x^3-1}-x^2}\right)-\frac{1}{2}\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^3-1}}{\sqrt{x^3-1}+x^2}\right)-\frac{1}{2}\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^3-1}}{\sqrt{x^3-1}+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^6*(-4 + x^3))/((-1 + x^3)^(3/4)*(1 - 2*x^3 + x^6 + x^
8)),x]
```

```
[Out] (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]]*x*(-1 + x^3)^(1/4))/(-x^2 + Sq
rt[-1 + x^3])])/2 + (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]]*x*(-1 + x^
3)^(1/4))/(-x^2 + Sqrt[-1 + x^3])])/2 - (Sqrt[2 - Sqrt[2]]*ArcTanh[(Sqrt[2
- Sqrt[2]]*x*(-1 + x^3)^(1/4))/(x^2 + Sqrt[-1 + x^3])])/2 - (Sqrt[2 + Sqrt[
2]]*ArcTanh[(Sqrt[2 + Sqrt[2]]*x*(-1 + x^3)^(1/4))/(x^2 + Sqrt[-1 + x^3])])
/2
```

fricas [B] time = 0.86, size = 1425, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^3-4)/(x^3-1)^(3/4)/(x^8+x^6-2*x^3+1),x, algorithm="fricas"
)
```

```
[Out] 1/4*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(x*sqr
t(sqrt(2) + 2) - x*sqrt(-sqrt(2) + 2) - 2*x*sqrt((2*x^2 + (x^3 - 1)^(1/4)*(
sqrt(2)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) + 2)) + 2*sqrt(x^3 -
1))/x^2) + 2*sqrt(2)*(x^3 - 1)^(1/4))/(x*sqrt(sqrt(2) + 2) + x*sqrt(-sqrt(2
) + 2))) + 1/4*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arc
tan((x*sqrt(sqrt(2) + 2) - x*sqrt(-sqrt(2) + 2) + 2*x*sqrt((2*x^2 - (x^3 -
1)^(1/4)*(sqrt(2)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) + 2)) + 2*s
qrt(x^3 - 1))/x^2) - 2*sqrt(2)*(x^3 - 1)^(1/4))/(x*sqrt(sqrt(2) + 2) + x*sq
rt(-sqrt(2) + 2))) - 1/4*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2)
 + 2))*arctan((x*sqrt(sqrt(2) + 2) + x*sqrt(-sqrt(2) + 2) - 2*x*sqrt((2*x^2
 + (x^3 - 1)^(1/4)*(sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2) +
 2)) + 2*sqrt(x^3 - 1))/x^2) + 2*sqrt(2)*(x^3 - 1)^(1/4))/(x*sqrt(sqrt(2) +
 2) - x*sqrt(-sqrt(2) + 2))) - 1/4*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqr
t(-sqrt(2) + 2))*arctan(-(x*sqrt(sqrt(2) + 2) + x*sqrt(-sqrt(2) + 2) + 2*x*
sqrt((2*x^2 - (x^3 - 1)^(1/4)*(sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt
(-sqrt(2) + 2)) + 2*sqrt(x^3 - 1))/x^2) - 2*sqrt(2)*(x^3 - 1)^(1/4))/(x*sqr
t(sqrt(2) + 2) - x*sqrt(-sqrt(2) + 2))) - 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) +
 sqrt(2)*sqrt(-sqrt(2) + 2))*log(2*(2*x^2 + (x^3 - 1)^(1/4)*(sqrt(2)*x*sqrt
(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2) + 2)) + 2*sqrt(x^3 - 1))/x^2) + 1/1
6*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(2*(2*x^2 - (
x^3 - 1)^(1/4)*(sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2) + 2))
 + 2*sqrt(x^3 - 1))/x^2) - 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-
sqrt(2) + 2))*log(2*(2*x^2 + (x^3 - 1)^(1/4)*(sqrt(2)*x*sqrt(sqrt(2) + 2) -
 sqrt(2)*x*sqrt(-sqrt(2) + 2)) + 2*sqrt(x^3 - 1))/x^2) + 1/16*(sqrt(2)*sqrt
(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(2*(2*x^2 - (x^3 - 1)^(1/4)*
(sqrt(2)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) + 2)) + 2*sqrt(x^3 -
 1))/x^2) + 1/2*sqrt(sqrt(2) + 2)*arctan(-(x*sqrt(-sqrt(2) + 2) - 2*x*sqrt(
(x^2 + (x^3 - 1)^(1/4)*x*sqrt(-sqrt(2) + 2) + sqrt(x^3 - 1))/x^2) + 2*(x^3
- 1)^(1/4))/(x*sqrt(sqrt(2) + 2))) + 1/2*sqrt(sqrt(2) + 2)*arctan((x*sqrt(-
sqrt(2) + 2) + 2*x*sqrt((x^2 - (x^3 - 1)^(1/4)*x*sqrt(-sqrt(2) + 2) + sqrt(
x^3 - 1))/x^2) - 2*(x^3 - 1)^(1/4))/(x*sqrt(sqrt(2) + 2))) + 1/2*sqrt(-sqrt
(2) + 2)*arctan(-(x*sqrt(sqrt(2) + 2) - 2*x*sqrt((x^2 + (x^3 - 1)^(1/4)*x*s
qrt(sqrt(2) + 2) + sqrt(x^3 - 1))/x^2) + 2*(x^3 - 1)^(1/4))/(x*sqrt(-sqrt(2
) + 2))) + 1/2*sqrt(-sqrt(2) + 2)*arctan((x*sqrt(sqrt(2) + 2) + 2*x*sqrt((x
^2 - (x^3 - 1)^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(x^3 - 1))/x^2) - 2*(x^3 - 1
)^(1/4))/(x*sqrt(-sqrt(2) + 2))) - 1/8*sqrt(sqrt(2) + 2)*log((x^2 + (x^3 -
```

```
1)^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(x^3 - 1))/x^2) + 1/8*sqrt(sqrt(2) + 2)*
log((x^2 - (x^3 - 1)^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(x^3 - 1))/x^2) - 1/8*
sqrt(-sqrt(2) + 2)*log((x^2 + (x^3 - 1)^(1/4)*x*sqrt(-sqrt(2) + 2) + sqrt(x
^3 - 1))/x^2) + 1/8*sqrt(-sqrt(2) + 2)*log((x^2 - (x^3 - 1)^(1/4)*x*sqrt(-s
qrt(2) + 2) + sqrt(x^3 - 1))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-4\right)x^6}{\left(x^8+x^6-2\,x^3+1\right)\left(x^3-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^3-4)/(x^3-1)^(3/4)/(x^8+x^6-2*x^3+1),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 4)*x^6/((x^8 + x^6 - 2*x^3 + 1)*(x^3 - 1)^(3/4)), x)
```

maple [C] time = 6.78, size = 461, normalized size = 1.88

$$-\frac{\mathrm{RootOf}\left(\_Z^8+1\right)^3\ln\left(-\frac{\mathrm{RootOf}(\_Z^8+1)^9x^4+2(x^3-1)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^8+1)^6x^3-\mathrm{RootOf}(\_Z^8+1)^5x^3+2\sqrt{x^3-1}\,\mathrm{RootOf}(\_Z^8+1)^3x^2+\mathrm{RootOf}(\_Z^8+1)^5+2(x^3-1)^{\frac{3}{4}}x}{\mathrm{RootOf}(\_Z^8+1)^4x^4+x^3-1}\right)}{2}-\frac{\mathrm{RootOf}\left(\_Z^8+1\right)^5\ln\left(\frac{\mathrm{RootOf}(\_Z^8+1)^7x^4-2\sqrt{x^3-1}\,\mathrm{RootOf}(\_Z^8+1)^5x^2+\mathrm{RootOf}(\_Z^8+1)^3x^3+2(x^3-1)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^8+1)^2x^3-2(x^3-1)^{\frac{3}{4}}x-\mathrm{RootOf}(\_Z^8+1)^3}{\mathrm{RootOf}(\_Z^8+1)^4x^4-x^3+1}\right)}{2}-\frac{\mathrm{RootOf}\left(\_Z^8+1\right)\ln\left(\frac{\mathrm{RootOf}(\_Z^8+1)^{11}x^4+\mathrm{RootOf}(\_Z^8+1)^7x^3-\mathrm{RootOf}(\_Z^8+1)^7-2(x^3-1)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^8+1)^2x^3-2\sqrt{x^3-1}\,\mathrm{RootOf}(\_Z^8+1)x^2-2(x^3-1)^{\frac{3}{4}}x}{\mathrm{RootOf}(\_Z^8+1)^4x^4-x^3+1}\right)}{2}+\frac{\mathrm{RootOf}\left(\_Z^8+1\right)^7\ln\left(\frac{2\sqrt{x^3-1}\,\mathrm{RootOf}(\_Z^8+1)^7x^2+2(x^3-1)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^8+1)^6x^3+\mathrm{RootOf}(\_Z^8+1)^5x^4-\mathrm{RootOf}(\_Z^8+1)x^3-2(x^3-1)^{\frac{3}{4}}x+\mathrm{RootOf}(\_Z^8+1)}{\mathrm{RootOf}(\_Z^8+1)^4x^4+x^3-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^3-4)/(x^3-1)^(3/4)/(x^8+x^6-2*x^3+1),x)
```

```
[Out] -1/2*RootOf(_Z^8+1)^3*ln(-(RootOf(_Z^8+1)^9*x^4+2*(x^3-1)^(1/4)*RootOf(_Z^8
+1)^6*x^3-RootOf(_Z^8+1)^5*x^3+2*(x^3-1)^(1/2)*RootOf(_Z^8+1)^3*x^2+RootOf(
_Z^8+1)^5+2*(x^3-1)^(3/4)*x)/(RootOf(_Z^8+1)^4*x^4+x^3-1))-1/2*RootOf(_Z^8+
1)^5*ln((RootOf(_Z^8+1)^7*x^4-2*(x^3-1)^(1/2)*RootOf(_Z^8+1)^5*x^2+RootOf(_
Z^8+1)^3*x^3+2*(x^3-1)^(1/4)*RootOf(_Z^8+1)^2*x^3-2*(x^3-1)^(3/4)*x-RootOf(
_Z^8+1)^3)/(RootOf(_Z^8+1)^4*x^4-x^3+1))-1/2*RootOf(_Z^8+1)*ln((RootOf(_Z^8
+1)^11*x^4+RootOf(_Z^8+1)^7*x^3-RootOf(_Z^8+1)^7-2*(x^3-1)^(1/4)*RootOf(_Z^
8+1)^2*x^3-2*(x^3-1)^(1/2)*RootOf(_Z^8+1)*x^2-2*(x^3-1)^(3/4)*x)/(RootOf(_Z
^8+1)^4*x^4-x^3+1))+1/2*RootOf(_Z^8+1)^7*ln((2*(x^3-1)^(1/2)*RootOf(_Z^8+1)
^7*x^2+2*(x^3-1)^(1/4)*RootOf(_Z^8+1)^6*x^3+RootOf(_Z^8+1)^5*x^4-RootOf(_Z^
8+1)*x^3-2*(x^3-1)^(3/4)*x+RootOf(_Z^8+1))/(RootOf(_Z^8+1)^4*x^4+x^3-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3-4\right)x^6}{\left(x^8+x^6-2\,x^3+1\right)\left(x^3-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^3-4)/(x^3-1)^(3/4)/(x^8+x^6-2*x^3+1),x, algorithm="maxima"
)
```

```
[Out] integrate((x^3 - 4)*x^6/((x^8 + x^6 - 2*x^3 + 1)*(x^3 - 1)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,\left(x^3-4\right)}{{\left(x^3-1\right)}^{3/4}\,\left(x^8+x^6-2\,x^3+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(x^3 - 4))/((x^3 - 1)^(3/4)*(x^6 - 2*x^3 + x^8 + 1)),x)
```

```
[Out] int((x^6*(x^3 - 4))/((x^3 - 1)^(3/4)*(x^6 - 2*x^3 + x^8 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}\left(x^{3}-4\right)}{\left(\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{3}{4}}\left(x^{8}+x^{6}-2x^{3}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(x**3-4)/(x**3-1)**(3/4)/(x**8+x**6-2*x**3+1),x)
```

```
[Out] Integral(x**6*(x**3 - 4)/(((x - 1)*(x**2 + x + 1))**(3/4)*(x**8 + x**6 - 2*
x**3 + 1)), x)
```

3.2161 $\int \frac{x^6(4+x^5)}{(-1+x^5)^{3/4}(1-2x^5+x^8+x^{10})}\,dx$

Optimal. Leaf size=245

$$-\frac{1}{2}\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^5-1}}{\sqrt{x^5-1}-x^2}\right)+\frac{1}{2}\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\left(\sqrt{\frac{2}{2-\sqrt{2}}}-\frac{2}{\sqrt{2-\sqrt{2}}}\right)x\sqrt[4]{x^5-1}}{\sqrt{x^5-1}-x^2}\right)+\frac{1}{2}\sqrt{2-\sqrt{2}}\,\text{t}$$

Rubi [F] time = 1.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^6\left(4+x^5\right)}{\left(-1+x^5\right)^{3/4}\left(1-2x^5+x^8+x^{10}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^6*(4 + x^5))/((-1 + x^5)^(3/4)*(1 - 2*x^5 + x^8 + x^10)),x]
```

```
[Out] (x^2*(1 - x^5)^(3/4)*Hypergeometric2F1[2/5, 3/4, 7/5, x^5])/(2*(-1 + x^5)^(
3/4)) - Defer[Int][x/((-1 + x^5)^(3/4)*(1 - 2*x^5 + x^8 + x^10)), x] + 6*De
fer[Int][x^6/((-1 + x^5)^(3/4)*(1 - 2*x^5 + x^8 + x^10)), x] - Defer[Int][x
^9/((-1 + x^5)^(3/4)*(1 - 2*x^5 + x^8 + x^10)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6\left(4+x^5\right)}{\left(-1+x^5\right)^{3/4}\left(1-2x^5+x^8+x^{10}\right)}\,dx &= \int \left(\frac{x}{\left(-1+x^5\right)^{3/4}}+\frac{x\left(-1+6x^5-x^8\right)}{\left(-1+x^5\right)^{3/4}\left(1-2x^5+x^8+x^{10}\right)}\right)dx\\
&= \int \frac{x}{\left(-1+x^5\right)^{3/4}}\,dx+\int \frac{x\left(-1+6x^5-x^8\right)}{\left(-1+x^5\right)^{3/4}\left(1-2x^5+x^8+x^{10}\right)}\,dx\\
&= \frac{\left(1-x^5\right)^{3/4}\int \frac{x}{\left(1-x^5\right)^{3/4}}\,dx}{\left(-1+x^5\right)^{3/4}}+\int \left(-\frac{x}{\left(-1+x^5\right)^{3/4}\left(1-2x^5+x^8+x^{10}\right)}+\right.\\
&= \frac{x^2\left(1-x^5\right)^{3/4}\,{}_2F_1\left(\frac{2}{5},\frac{3}{4};\frac{7}{5};x^5\right)}{2\left(-1+x^5\right)^{3/4}}+6\int \frac{x^6}{\left(-1+x^5\right)^{3/4}\left(1-2x^5+x^8+x\right.}
\end{aligned}$$

Mathematica [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(4+x^5\right)}{\left(-1+x^5\right)^{3/4}\left(1-2x^5+x^8+x^{10}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^6*(4 + x^5))/((-1 + x^5)^(3/4)*(1 - 2*x^5 + x^8 + x^10)),x]
```

```
[Out] Integrate[(x^6*(4 + x^5))/((-1 + x^5)^(3/4)*(1 - 2*x^5 + x^8 + x^10)), x]
```

IntegrateAlgebraic [A] time = 13.34, size = 225, normalized size = 0.92

$$-\frac{1}{2}\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^5-1}}{\sqrt{x^5-1}-x^2}\right)-\frac{1}{2}\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^5-1}}{\sqrt{x^5-1}-x^2}\right)+\frac{1}{2}\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^5-1}}{\sqrt{x^5-1}+x^2}\right)+\frac{1}{2}\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^5-1}}{\sqrt{x^5-1}+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^6*(4 + x^5))/((-1 + x^5)^(3/4)*(1 - 2*x^5 + x^8 + x^1
0)),x]
```

```
[Out] -1/2*(Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]]*x*(-1 + x^5)^(1/4))/(-x^2
 + Sqrt[-1 + x^5])]) - (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]]*x*(-1 +
 x^5)^(1/4))/(-x^2 + Sqrt[-1 + x^5])])/2 + (Sqrt[2 - Sqrt[2]]*ArcTanh[(Sqrt
[2 - Sqrt[2]]*x*(-1 + x^5)^(1/4))/(x^2 + Sqrt[-1 + x^5])])/2 + (Sqrt[2 + Sq
rt[2]]*ArcTanh[(Sqrt[2 + Sqrt[2]]*x*(-1 + x^5)^(1/4))/(x^2 + Sqrt[-1 + x^5]
)])/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^5+4)/(x^5-1)^(3/4)/(x^10+x^8-2*x^5+1),x, algorithm="fricas
")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^5+4)x^6}{(x^{10}+x^8-2\,x^5+1)(x^5-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^5+4)/(x^5-1)^(3/4)/(x^10+x^8-2*x^5+1),x, algorithm="giac")
```

```
[Out] integrate((x^5 + 4)*x^6/((x^10 + x^8 - 2*x^5 + 1)*(x^5 - 1)^(3/4)), x)
```

maple [C] time = 11.33, size = 462, normalized size = 1.89

$$-\frac{\mathrm{RootOf}(\_Z^8+1)^3\ln\left(-\frac{\mathrm{RootOf}(\_Z^8+1)^9x^4-2\,\mathrm{RootOf}(\_Z^8+1)^6(x^5-1)^{\frac{1}{4}}x^3-\mathrm{RootOf}(\_Z^8+1)^5x^5+2\,\mathrm{RootOf}(\_Z^8+1)^3\sqrt{x^5-1}\,x^2+\mathrm{RootOf}(\_Z^8+1)^5-2(x^5-1)^{\frac{3}{4}}x}{\mathrm{RootOf}(\_Z^8+1)^4x^4+x^5-1}\right)}{2}-\frac{\mathrm{RootOf}(\_Z^8+1)\ln\left(\frac{\mathrm{RootOf}(\_Z^8+1)^{11}x^4+\mathrm{RootOf}(\_Z^8+1)^7x^5-\mathrm{RootOf}(\_Z^8+1)^7+2(x^5-1)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^8+1)^2x^3-2\,\mathrm{RootOf}(\_Z^8+1)\sqrt{x^5-1}\,x^2+2(x^5-1)^{\frac{3}{4}}x}{\mathrm{RootOf}(\_Z^8+1)^4x^4-x^5+1}\right)}{2}+\frac{\mathrm{RootOf}(\_Z^8+1)^7\ln\left(\frac{2\sqrt{x^5-1}\,\mathrm{RootOf}(\_Z^8+1)^7x^2-2\,\mathrm{RootOf}(\_Z^8+1)^6(x^5-1)^{\frac{1}{4}}x^3+\mathrm{RootOf}(\_Z^8+1)^5x^4-\mathrm{RootOf}(\_Z^8+1)x^5+2(x^5-1)^{\frac{3}{4}}x+\mathrm{RootOf}(\_Z^8+1)}{\mathrm{RootOf}(\_Z^8+1)^4x^4+x^5-1}\right)}{2}+\frac{\mathrm{RootOf}(\_Z^8+1)^5\ln\left(-\frac{\mathrm{RootOf}(\_Z^8+1)^7x^4-2\sqrt{x^5-1}\,\mathrm{RootOf}(\_Z^8+1)^5x^2+\mathrm{RootOf}(\_Z^8+1)^3x^5+2(x^5-1)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^8+1)^2x^3-2(x^5-1)^{\frac{3}{4}}x-\mathrm{RootOf}(\_Z^8+1)^3}{\mathrm{RootOf}(\_Z^8+1)^4x^4-x^5+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^5+4)/(x^5-1)^(3/4)/(x^10+x^8-2*x^5+1),x)
```

```
[Out] -1/2*RootOf(_Z^8+1)^3*ln(-(RootOf(_Z^8+1)^9*x^4-2*RootOf(_Z^8+1)^6*(x^5-1)^
(1/4)*x^3-RootOf(_Z^8+1)^5*x^5+2*RootOf(_Z^8+1)^3*(x^5-1)^(1/2)*x^2+RootOf(
_Z^8+1)^5-2*(x^5-1)^(3/4)*x)/(RootOf(_Z^8+1)^4*x^4+x^5-1))-1/2*RootOf(_Z^8+
1)*ln((RootOf(_Z^8+1)^11*x^4+RootOf(_Z^8+1)^7*x^5-RootOf(_Z^8+1)^7+2*(x^5-1
)^(1/4)*RootOf(_Z^8+1)^2*x^3-2*RootOf(_Z^8+1)*(x^5-1)^(1/2)*x^2+2*(x^5-1)^(
3/4)*x)/(RootOf(_Z^8+1)^4*x^4-x^5+1))+1/2*RootOf(_Z^8+1)^7*ln((2*(x^5-1)^(1
/2)*RootOf(_Z^8+1)^7*x^2-2*RootOf(_Z^8+1)^6*(x^5-1)^(1/4)*x^3+RootOf(_Z^8+1
)^5*x^4-RootOf(_Z^8+1)*x^5+2*(x^5-1)^(3/4)*x+RootOf(_Z^8+1))/(RootOf(_Z^8+1
)^4*x^4+x^5-1))+1/2*RootOf(_Z^8+1)^5*ln(-(RootOf(_Z^8+1)^7*x^4-2*(x^5-1)^(1
/2)*RootOf(_Z^8+1)^5*x^2+RootOf(_Z^8+1)^3*x^5+2*(x^5-1)^(1/4)*RootOf(_Z^8+1
)^2*x^3-2*(x^5-1)^(3/4)*x-RootOf(_Z^8+1)^3)/(RootOf(_Z^8+1)^4*x^4-x^5+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^5+4)x^6}{(x^{10}+x^8-2\,x^5+1)(x^5-1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^5+4)/(x^5-1)^(3/4)/(x^10+x^8-2*x^5+1),x, algorithm="maxima
")
```

```
[Out] integrate((x^5 + 4)*x^6/((x^10 + x^8 - 2*x^5 + 1)*(x^5 - 1)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,\left(x^5+4\right)}{{\left(x^5-1\right)}^{3/4}\,\left(x^{10}+x^8-2\,x^5+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(x^5 + 4))/((x^5 - 1)^(3/4)*(x^8 - 2*x^5 + x^10 + 1)),x)
```

```
[Out] int((x^6*(x^5 + 4))/((x^5 - 1)^(3/4)*(x^8 - 2*x^5 + x^10 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6} \left(x^{5} + 4\right)}{\left(\left(x - 1\right) \left(x^{4} + x^{3} + x^{2} + x + 1\right)\right)^{\frac{3}{4}} \left(x^{10} + x^{8} - 2 x^{5} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(x**5+4)/(x**5-1)**(3/4)/(x**10+x**8-2*x**5+1),x)
```

```
[Out] Integral(x**6*(x**5 + 4)/(((x - 1)*(x**4 + x**3 + x**2 + x + 1))**(3/4)*(x*
*10 + x**8 - 2*x**5 + 1)), x)
```

3.2162 $\int \frac{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx$

Optimal. Leaf size=245

$$\frac{1}{2}\sqrt{\sqrt{x^4+1}+x^2}\,x-\frac{\tan^{-1}\left(\frac{\sqrt{2}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)}{\sqrt{2}}+\sqrt{2\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\sqrt{2}\tanh^{-1}$$

Rubi [F] time = 0.83, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2),x]
```

```
[Out] (I/2)*Defer[Int][(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]])/(I - x), x] + (I
/2)*Defer[Int][(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]])/(I + x), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx &= \int \left(\frac{i\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{2(i-x)}+\frac{i\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{2(i+x)}\right)dx\\ &= \frac{1}{2}i\int \frac{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{i-x}\,dx+\frac{1}{2}i\int \frac{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{i+x}\,dx\end{aligned}$$

Mathematica [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2),x]
```

```
[Out] Integrate[(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2), x]
```

IntegrateAlgebraic [A] time = 1.38, size = 329, normalized size = 1.34

$$\frac{1}{2}\sqrt{\sqrt{x^4+1}+x^2}\,x-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}+\sqrt{2\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)+\sqrt{2\left(1+\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2),x]
```

```
[Out] (x*Sqrt[x^2 + Sqrt[1 + x^4]])/2 - ArcTan[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt
[1 + x^4]/Sqrt[2])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])]/Sqrt[2] + Sqrt[2*(-1 + Sq
rt[2])]*ArcTan[(-Sqrt[1/2 + 1/Sqrt[2]] + Sqrt[1/2 + 1/Sqrt[2]]*x^2 + Sqrt[1
```

```
/2 + 1/Sqrt[2]]*Sqrt[1 + x^4]])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])] - Sqrt[2]*Arc
Tanh[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2])/(x*Sqrt[x^2 + Sqr
t[1 + x^4]])] + Sqrt[2*(1 + Sqrt[2])]*ArcTanh[(-Sqrt[-1/2 + 1/Sqrt[2]] + Sq
rt[-1/2 + 1/Sqrt[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^
2 + Sqrt[1 + x^4]])]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))/(x^2 + 1), x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1),x)
```

```
[Out] int((x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^2+1),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))/(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^2 + 1),x)
```

```
[Out] int(((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}\,\sqrt{x^4+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(1/2)*(x**2+(x**4+1)**(1/2))**(1/2)/(x**2+1),x)
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))*sqrt(x**4 + 1)/(x**2 + 1), x)
```

3.2163 $\int \frac{(b+ax)\left(-3aq+4bpx^3+apx^4\right)}{\left(q+px^4\right)^{2/3}\left(b^3c+dq+3ab^2cx+3a^2bcx^2+a^3cx^3+dpx^4\right)}\,dx$

Optimal. Leaf size=247

$$\frac{\log\left(a^2\sqrt[3]{c}\,x+ab\sqrt[3]{c}\;+a\sqrt[3]{d}\,\sqrt[3]{px^4+q}\right)}{c^{2/3}\sqrt[3]{d}}-\frac{\log\left(a^4c^{2/3}x^2+2a^3bc^{2/3}x+a^2b^2c^{2/3}+a^2d^{2/3}\left(px^4+q\right)^{2/3}+\sqrt[3]{px^4+q}\right.}{2c^{2/3}\sqrt[3]{d}}$$

Rubi [F] time = 6.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(b+ax)\left(-3aq+4bpx^3+apx^4\right)}{\left(q+px^4\right)^{2/3}\left(b^3c+dq+3ab^2cx+3a^2bcx^2+a^3cx^3+dpx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((b + a*x)*(-3*a*q + 4*b*p*x^3 + a*p*x^4))/((q + p*x^4)^(2/3)*(b^3*c +
d*q + 3*a*b^2*c*x + 3*a^2*b*c*x^2 + a^3*c*x^3 + d*p*x^4)),x]
```

```
[Out] -1/2*(3^(3/4)*Sqrt[2 - Sqrt[3]]*a^2*(q^(1/3) - (q + p*x^4)^(1/3))*Sqrt[(q^(
2/3) + q^(1/3)*(q + p*x^4)^(1/3) + (q + p*x^4)^(2/3))/((1 - Sqrt[3])*q^(1/3
) - (q + p*x^4)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*q^(1/3) - (q + p*
x^4)^(1/3))/((1 - Sqrt[3])*q^(1/3) - (q + p*x^4)^(1/3))], -7 + 4*Sqrt[3]])/
(d*p*x^2*Sqrt[-((q^(1/3)*(q^(1/3) - (q + p*x^4)^(1/3)))/((1 - Sqrt[3])*q^(1
/3) - (q + p*x^4)^(1/3))^2)]) - (a*(a^4*c - 5*b*d*p)*x*(1 + (p*x^4)/q)^(2/3
)*Hypergeometric2F1[1/4, 2/3, 5/4, -((p*x^4)/q)])/(d^2*p*(q + p*x^4)^(2/3))
 + (a*(a^4*c*(b^3*c + d*q) - b*d*p*(5*b^3*c + 8*d*q))*Defer[Int][1/((q + p*
x^4)^(2/3)*(b^3*c + d*q + 3*a*b^2*c*x + 3*a^2*b*c*x^2 + a^3*c*x^3 + d*p*x^4
)), x])/(d^2*p) + (a^2*(3*a^4*b^2*c^2 - 4*d*p*(4*b^3*c + d*q))*Defer[Int][x
/((q + p*x^4)^(2/3)*(b^3*c + d*q + 3*a*b^2*c*x + 3*a^2*b*c*x^2 + a^3*c*x^3
+ d*p*x^4)), x])/(d^2*p) + (3*a^3*b*c*(a^4*c - 6*b*d*p)*Defer[Int][x^2/((q
+ p*x^4)^(2/3)*(b^3*c + d*q + 3*a*b^2*c*x + 3*a^2*b*c*x^2 + a^3*c*x^3 + d*p
*x^4)), x])/(d^2*p) + ((a^8*c^2 - 8*a^4*b*c*d*p + 4*b^2*d^2*p^2)*Defer[Int]
[x^3/((q + p*x^4)^(2/3)*(b^3*c + d*q + 3*a*b^2*c*x + 3*a^2*b*c*x^2 + a^3*c*
x^3 + d*p*x^4)), x])/(d^2*p)
```

Rubi steps

$$\int \frac{(b+ax)\left(-3aq+4bpx^3+apx^4\right)}{\left(q+px^4\right)^{2/3}\left(b^3c+dq+3ab^2cx+3a^2bcx^2+a^3cx^3+dpx^4\right)}\,dx = \int \left(-\frac{a\left(a^4c-5bdp\right)}{d^2p\left(q+px^4\right)^{2/3}}+\frac{a^2x}{d\left(q+px^4\right)^{2/3}}+\frac{a\left(\right.}{}\right.$$

$$= \frac{a^2\int \frac{x}{\left(q+px^4\right)^{2/3}}\,dx}{d} + \frac{\int \frac{a\left(a^4c\left(b^3c+dq\right)-bdp\left(5b^3c+8dq\right.\right.}{}}{}$$

$$= \frac{a^2\,\text{Subst}\left(\int \frac{1}{\left(q+px^2\right)^{2/3}}\,dx,x,x^2\right)}{2d} + \frac{\int \left(\frac{}{\left(q+px^4\right)^{2/}}\right.}{}$$

$$= -\frac{a\left(a^4c-5bdp\right)x\left(1+\frac{px^4}{q}\right)^{2/3}\,{}_2F_1\left(\frac{1}{4},\frac{2}{3};\frac{5}{4};-\right.}{d^2p\left(q+px^4\right)^{2/3}}$$

$$= -\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\sqrt[3]{q}-\sqrt[3]{q+px^4}\right)\sqrt{\frac{q^{2/3}+\sqrt[3]{}}{\left(\left(1-\right.\right.}}}{2dpx^2}$$

Mathematica [F] time = 2.13, size = 0, normalized size = 0.00

$$\int \frac{(b+ax)\left(-3aq+4bpx^3+apx^4\right)}{\left(q+px^4\right)^{2/3}\left(b^3c+dq+3ab^2cx+3a^2bcx^2+a^3cx^3+dpx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((b + a*x)*(-3*a*q + 4*b*p*x^3 + a*p*x^4))/((q + p*x^4)^(2/3)*(b^
3*c + d*q + 3*a*b^2*c*x + 3*a^2*b*c*x^2 + a^3*c*x^3 + d*p*x^4)),x]
```

```
[Out] Integrate[((b + a*x)*(-3*a*q + 4*b*p*x^3 + a*p*x^4))/((q + p*x^4)^(2/3)*(b^
3*c + d*q + 3*a*b^2*c*x + 3*a^2*b*c*x^2 + a^3*c*x^3 + d*p*x^4)), x]
```

IntegrateAlgebraic [A] time = 23.14, size = 247, normalized size = 1.00

$$\frac{\log\left(a^2\sqrt[3]{c}\,x+ab\sqrt[3]{c}+a\sqrt[3]{d}\sqrt[3]{px^4+q}\right)}{c^{2/3}\sqrt[3]{d}}-\frac{\log\left(a^4c^{2/3}x^2+2a^3bc^{2/3}x+a^2b^2c^{2/3}+a^2d^{2/3}\left(px^4+q\right)^{2/3}+\sqrt[3]{px^4+q}\left(a^3\left(-\sqrt[3]{c}\right)\sqrt[3]{d}\,x-a^2b\sqrt[3]{c}\sqrt[3]{d}\right)\right)}{2c^{2/3}\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}a\sqrt[3]{c}x+\sqrt{3}b\sqrt[3]{c}}{a\sqrt[3]{c}x+b\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{px^4+q}}\right)}{c^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b + a*x)*(-3*a*q + 4*b*p*x^3 + a*p*x^4))/((q + p*x^4)^
(2/3)*(b^3*c + d*q + 3*a*b^2*c*x + 3*a^2*b*c*x^2 + a^3*c*x^3 + d*p*x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*b*c^(1/3) + Sqrt[3]*a*c^(1/3)*x)/(b*c^(1/3) + a*c^
(1/3)*x - 2*d^(1/3)*(q + p*x^4)^(1/3))])/(c^(2/3)*d^(1/3)) + Log[a*b*c^(1/3
) + a^2*c^(1/3)*x + a*d^(1/3)*(q + p*x^4)^(1/3)]/(c^(2/3)*d^(1/3)) - Log[a^
2*b^2*c^(2/3) + 2*a^3*b*c^(2/3)*x + a^4*c^(2/3)*x^2 + (-(a^2*b*c^(1/3)*d^(1
/3)) - a^3*c^(1/3)*d^(1/3)*x)*(q + p*x^4)^(1/3) + a^2*d^(2/3)*(q + p*x^4)^(
2/3)]/(2*c^(2/3)*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*(a*p*x^4+4*b*p*x^3-3*a*q)/(p*x^4+q)^(2/3)/(a^3*c*x^3+3*a^
2*b*c*x^2+d*p*x^4+3*a*b^2*c*x+b^3*c+d*q),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*(a*p*x^4+4*b*p*x^3-3*a*q)/(p*x^4+q)^(2/3)/(a^3*c*x^3+3*a^
2*b*c*x^2+d*p*x^4+3*a*b^2*c*x+b^3*c+d*q),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{(ax+b)\left(pa\,x^{4}+4bp\,x^{3}-3aq\right)}{\left(p\,x^{4}+q\right)^{\frac{2}{3}}\left(a^{3}c\,x^{3}+3a^{2}bc\,x^{2}+dp\,x^{4}+3a\,b^{2}cx+b^{3}c+dq\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)*(a*p*x^4+4*b*p*x^3-3*a*q)/(p*x^4+q)^(2/3)/(a^3*c*x^3+3*a^2*b*c*
x^2+d*p*x^4+3*a*b^2*c*x+b^3*c+d*q),x)
```

```
[Out] int((a*x+b)*(a*p*x^4+4*b*p*x^3-3*a*q)/(p*x^4+q)^(2/3)/(a^3*c*x^3+3*a^2*b*c*
x^2+d*p*x^4+3*a*b^2*c*x+b^3*c+d*q),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(apx^{4}+4\,bpx^{3}-3\,aq\right)(ax+b)}{\left(a^{3}cx^{3}+3\,a^{2}bcx^{2}+dpx^{4}+3\,ab^{2}cx+b^{3}c+dq\right)\left(px^{4}+q\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*(a*p*x^4+4*b*p*x^3-3*a*q)/(p*x^4+q)^(2/3)/(a^3*c*x^3+3*a^
2*b*c*x^2+d*p*x^4+3*a*b^2*c*x+b^3*c+d*q),x, algorithm="maxima")
```

```
[Out] integrate((a*p*x^4 + 4*b*p*x^3 - 3*a*q)*(a*x + b)/((a^3*c*x^3 + 3*a^2*b*c*x
^2 + d*p*x^4 + 3*a*b^2*c*x + b^3*c + d*q)*(p*x^4 + q)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b+a\,x)\,\left(a\,p\,x^4+4\,b\,p\,x^3-3\,a\,q\right)}{{\left(p\,x^4+q\right)}^{2/3}\,\left(c\,a^3\,x^3+3\,c\,a^2\,b\,x^2+3\,c\,a\,b^2\,x+c\,b^3+d\,p\,x^4+d\,q\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a*x)*(a*p*x^4 - 3*a*q + 4*b*p*x^3))/((q + p*x^4)^(2/3)*(d*q + b^3
*c + d*p*x^4 + a^3*c*x^3 + 3*a*b^2*c*x + 3*a^2*b*c*x^2)),x)
```

```
[Out] int(((b + a*x)*(a*p*x^4 - 3*a*q + 4*b*p*x^3))/((q + p*x^4)^(2/3)*(d*q + b^3
*c + d*p*x^4 + a^3*c*x^3 + 3*a*b^2*c*x + 3*a^2*b*c*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*(a*p*x**4+4*b*p*x**3-3*a*q)/(p*x**4+q)**(2/3)/(a**3*c*x**
3+3*a**2*b*c*x**2+d*p*x**4+3*a*b**2*c*x+b**3*c+d*q),x)
```

```
[Out] Timed out
```

3.2164 $\int \frac{\sqrt{b^2+a^2x^3}\left(2b^2+cx^3+a^2x^6\right)}{x^7\left(b^2+a^2x^6\right)}\,dx$

Optimal. Leaf size=247

$$\frac{\sqrt{a^2x^3+b^2}\left(-a^2x^3-2b^2-2cx^3\right)}{6b^2x^6}+\frac{\sqrt{a-ib}\left((-1)^{3/4}a^2b-\sqrt[4]{-1}\,ac\right)\tan^{-1}\left(\frac{\sqrt[4]{-1}\,\sqrt{a^2x^3+b^2}}{\sqrt{b}\,\sqrt{a-ib}}\right)}{3b^{5/2}}-\frac{\sqrt{a+ib}\left((-1)^{3/4}ac-\sqrt[4]{-}\right.}{}$$

Rubi [B] time = 3.60, antiderivative size = 744, normalized size of antiderivative = 3.01, number of steps used = 27, number of rules used = 15, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6725, 266, 47, 51, 63, 208, 50, 6715, 825, 827, 1169, 634, 618, 206, 628}

$$-\frac{c\sqrt{a^2x^3+b^2}}{3b^2x^3}-\frac{a^2\sqrt{a^2x^3+b^2}}{6b^2x^3}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{b}\right)}{3b}-\frac{\sqrt{a^2x^3+b^2}}{3x^6}-\frac{a^2c\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{b}\right)}{3b^3}+\frac{a^2\left(\sqrt{a^2+b^2}\left(b^2-c\right)+a^2b+b^3\right)\log\left(-\sqrt{2}\sqrt{b}\sqrt{\sqrt{a^2+b^2}+b}\sqrt{a^2x^3+b^2}+b\left(\sqrt{a^2+b^2}+b\right)+a^2x^3\right)}{6\sqrt{2}b^{5/2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+b}}-\frac{a^2\left(\sqrt{a^2+b^2}\left(b^2-c\right)+a^2b+b^3\right)\log\left(\sqrt{2}\sqrt{b}\sqrt{\sqrt{a^2+b^2}+b}\sqrt{a^2x^3+b^2}+b\left(\sqrt{a^2+b^2}+b\right)+a^2x^3\right)}{6\sqrt{2}b^{5/2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+b}}-\frac{a^2\left(-\sqrt{a^2+b^2}\left(b^2-c\right)+a^2b+b^3\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\sqrt{a^2+b^2}+b}-\sqrt{2}\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{b-\sqrt{a^2+b^2}}}\right)}{3\sqrt{2}b^{5/2}\sqrt{a^2+b^2}\sqrt{b-\sqrt{a^2+b^2}}}+\frac{a^2\left(-\sqrt{a^2+b^2}\left(b^2-c\right)+a^2b+b^3\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a^2x^3+b^2}+\sqrt{b}\sqrt{\sqrt{a^2+b^2}+b}}{\sqrt{b}\sqrt{b-\sqrt{a^2+b^2}}}\right)}{3\sqrt{2}b^{5/2}\sqrt{a^2+b^2}\sqrt{b-\sqrt{a^2+b^2}}}+\frac{a^4\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{b}\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + a^2*x^3]*(2*b^2 + c*x^3 + a^2*x^6))/(x^7*(b^2 + a^2*x^6)),x
]
```

```
[Out] -1/3*Sqrt[b^2 + a^2*x^3]/x^6 - (a^2*Sqrt[b^2 + a^2*x^3])/(6*b^2*x^3) - (c*S
qrt[b^2 + a^2*x^3])/(3*b^2*x^3) + (a^4*ArcTanh[Sqrt[b^2 + a^2*x^3]/b])/(6*b
^3) + (2*a^2*ArcTanh[Sqrt[b^2 + a^2*x^3]/b])/(3*b) - (a^2*c*ArcTanh[Sqrt[b^
2 + a^2*x^3]/b])/(3*b^3) - (a^2*(a^2*b + b^3 - Sqrt[a^2 + b^2]*(b^2 - c))*A
rcTanh[(Sqrt[b]*Sqrt[b + Sqrt[a^2 + b^2]] - Sqrt[2]*Sqrt[b^2 + a^2*x^3])/(S
qrt[b]*Sqrt[b - Sqrt[a^2 + b^2]])])/(3*Sqrt[2]*b^(5/2)*Sqrt[a^2 + b^2]*Sqrt
[b - Sqrt[a^2 + b^2]]) + (a^2*(a^2*b + b^3 - Sqrt[a^2 + b^2]*(b^2 - c))*Arc
Tanh[(Sqrt[b]*Sqrt[b + Sqrt[a^2 + b^2]] + Sqrt[2]*Sqrt[b^2 + a^2*x^3])/(Sqr
t[b]*Sqrt[b - Sqrt[a^2 + b^2]])])/(3*Sqrt[2]*b^(5/2)*Sqrt[a^2 + b^2]*Sqrt[b
 - Sqrt[a^2 + b^2]]) + (a^2*(a^2*b + b^3 + Sqrt[a^2 + b^2]*(b^2 - c))*Log[b
*(b + Sqrt[a^2 + b^2]) + a^2*x^3 - Sqrt[2]*Sqrt[b]*Sqrt[b + Sqrt[a^2 + b^2]
]*Sqrt[b^2 + a^2*x^3]])/(6*Sqrt[2]*b^(5/2)*Sqrt[a^2 + b^2]*Sqrt[b + Sqrt[a^
2 + b^2]]) - (a^2*(a^2*b + b^3 + Sqrt[a^2 + b^2]*(b^2 - c))*Log[b*(b + Sqrt
[a^2 + b^2]) + a^2*x^3 + Sqrt[2]*Sqrt[b]*Sqrt[b + Sqrt[a^2 + b^2]]*Sqrt[b^2
 + a^2*x^3]])/(6*Sqrt[2]*b^(5/2)*Sqrt[a^2 + b^2]*Sqrt[b + Sqrt[a^2 + b^2]])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 825

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m
- 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{b^2+a^2x^3}\left(2b^2+cx^3+a^2x^6\right)}{x^7\left(b^2+a^2x^6\right)}\,dx = \int\left(\frac{2\sqrt{b^2+a^2x^3}}{x^7}+\frac{c\sqrt{b^2+a^2x^3}}{b^2x^4}-\frac{a^2\sqrt{b^2+a^2x^3}}{b^2x}+\frac{a^2x^2\sqrt{b^2+a^2x^3}}{b^2\left(b^2+\right.}\right.$$

$$= 2\int\frac{\sqrt{b^2+a^2x^3}}{x^7}\,dx-\frac{a^2\int\frac{\sqrt{b^2+a^2x^3}}{x}\,dx}{b^2}+\frac{a^2\int\frac{x^2\sqrt{b^2+a^2x^3}\left(-c+a^2x^3\right)}{b^2+a^2x^6}\,dx}{b^2}$$

$$= \frac{2}{3}\,\text{Subst}\left(\int\frac{\sqrt{b^2+a^2x}}{x^3}\,dx,x,x^3\right)-\frac{a^2\,\text{Subst}\left(\int\frac{\sqrt{b^2+a^2x}}{x}\,dx,x,x^3\right)}{3b^2}+$$

$$= -\frac{\sqrt{b^2+a^2x^3}}{3x^6}-\frac{c\sqrt{b^2+a^2x^3}}{3b^2x^3}+\frac{1}{6}a^2\,\text{Subst}\left(\int\frac{1}{x^2\sqrt{b^2+a^2x}}\,dx,x,x^3\right.$$

$$= -\frac{\sqrt{b^2+a^2x^3}}{3x^6}-\frac{a^2\sqrt{b^2+a^2x^3}}{6b^2x^3}-\frac{c\sqrt{b^2+a^2x^3}}{3b^2x^3}-\frac{2}{3}\,\text{Subst}\left(\int\frac{1}{-\frac{b^2}{a^2}+\frac{x}{a}}\right.$$

$$= -\frac{\sqrt{b^2+a^2x^3}}{3x^6}-\frac{a^2\sqrt{b^2+a^2x^3}}{6b^2x^3}-\frac{c\sqrt{b^2+a^2x^3}}{3b^2x^3}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b^2+a^2x^3}}{b}\right.}{3b}$$

$$= -\frac{\sqrt{b^2+a^2x^3}}{3x^6}-\frac{a^2\sqrt{b^2+a^2x^3}}{6b^2x^3}-\frac{c\sqrt{b^2+a^2x^3}}{3b^2x^3}+\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b^2+a^2x^3}}{b}\right.}{6b^3}$$

$$= -\frac{\sqrt{b^2+a^2x^3}}{3x^6}-\frac{a^2\sqrt{b^2+a^2x^3}}{6b^2x^3}-\frac{c\sqrt{b^2+a^2x^3}}{3b^2x^3}+\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b^2+a^2x^3}}{b}\right.}{6b^3}$$

$$= -\frac{\sqrt{b^2+a^2x^3}}{3x^6}-\frac{a^2\sqrt{b^2+a^2x^3}}{6b^2x^3}-\frac{c\sqrt{b^2+a^2x^3}}{3b^2x^3}+\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b^2+a^2x^3}}{b}\right.}{6b^3}$$

Mathematica [C] time = 0.81, size = 442, normalized size = 1.79

$$\frac{-3b^{7/2}\left(-\sqrt{-a^2}\,cx^3\sqrt{\sqrt{-a^2}-b}\,\sqrt{a^2x^3+b^2}\,\tan^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{\sqrt{-a^2}-b}}\right)+x^3\sqrt{\sqrt{-a^2}+b}\left(a^2b+\sqrt{-a^2}\,c\right)\sqrt{a^2x^3+b^2}\,\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{\sqrt{-a^2}+b}}\right)+a^2\sqrt{b}\,cx^3\sqrt{\frac{a^2x^3}{b^2}+1}\,\tanh^{-1}\left(\sqrt{\frac{a^2x^3}{b^2}+1}\right)+a^2bx^3\sqrt{\sqrt{-a^2}-b}\,\sqrt{a^2x^3+b^2}\,\tan^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{\sqrt{-a^2}-b}}\right)-2a^2b^{3/2}x^3\sqrt{a^2x^3+b^2}\,\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{b}\right)+a^2\sqrt{b}\,cx^3+b^{5/2}c\right)-4a^4x^3\left(a^2x^3+b^2\right)^2\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{a^2x^3}{b^2}+1\right)}{9b^6x^3\sqrt{a^2x^3+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + a^2*x^3]*(2*b^2 + c*x^3 + a^2*x^6))/(x^7*(b^2 + a^2*x
^6)),x]
```

```
[Out] (-3*b^(7/2)*(b^(5/2)*c + a^2*Sqrt[b]*c*x^3 + a^2*Sqrt[Sqrt[-a^2] - b]*b*x^3
*Sqrt[b^2 + a^2*x^3]*ArcTan[Sqrt[b^2 + a^2*x^3]/(Sqrt[Sqrt[-a^2] - b]*Sqrt[
b])] - Sqrt[-a^2]*Sqrt[Sqrt[-a^2] - b]*c*x^3*Sqrt[b^2 + a^2*x^3]*ArcTan[Sqr
t[b^2 + a^2*x^3]/(Sqrt[Sqrt[-a^2] - b]*Sqrt[b])] - 2*a^2*b^(3/2)*x^3*Sqrt[b
^2 + a^2*x^3]*ArcTanh[Sqrt[b^2 + a^2*x^3]/b] + Sqrt[Sqrt[-a^2] + b]*(a^2*b
+ Sqrt[-a^2]*c)*x^3*Sqrt[b^2 + a^2*x^3]*ArcTanh[Sqrt[b^2 + a^2*x^3]/(Sqrt[b
]*Sqrt[Sqrt[-a^2] + b])] + a^2*Sqrt[b]*c*x^3*Sqrt[1 + (a^2*x^3)/b^2]*ArcTan
h[Sqrt[1 + (a^2*x^3)/b^2]]) - 4*a^4*x^3*(b^2 + a^2*x^3)^2*Hypergeometric2F1
[3/2, 3, 5/2, 1 + (a^2*x^3)/b^2])/(9*b^6*x^3*Sqrt[b^2 + a^2*x^3])
```

IntegrateAlgebraic [A] time = 0.89, size = 247, normalized size = 1.00

$$\frac{\sqrt{a^2x^3+b^2}\left(-a^2x^3-2b^2-2cx^3\right)}{6b^2x^6}+\frac{\sqrt{a-ib}\left((-1)^{3/4}a^2b-\sqrt[4]{-1}\,ac\right)\tan^{-1}\left(\frac{\sqrt[4]{-1}\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{a-ib}}\right)}{3b^{5/2}}-\frac{\sqrt{a+ib}\left((-1)^{3/4}ac-\sqrt[4]{-1}\,a^2b\right)\tan^{-1}\left(\frac{(-1)^{3/4}\sqrt{a^2x^3+b^2}}{\sqrt{b}\sqrt{a+ib}}\right)}{3b^{5/2}}+\frac{\left(a^4+4a^2b^2-2a^2c\right)\tanh^{-1}\left(\frac{\sqrt{a^2x^3+b^2}}{b}\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[b^2 + a^2*x^3]*(2*b^2 + c*x^3 + a^2*x^6))/(x^7*(b^
2 + a^2*x^6)),x]
```

```
[Out] (Sqrt[b^2 + a^2*x^3]*(-2*b^2 - a^2*x^3 - 2*c*x^3))/(6*b^2*x^6) + (Sqrt[a -
I*b]*((-1)^(3/4)*a^2*b - (-1)^(1/4)*a*c)*ArcTan[((-1)^(1/4)*Sqrt[b^2 + a^2*
x^3])/(Sqrt[a - I*b]*Sqrt[b])])/(3*b^(5/2)) - (Sqrt[a + I*b]*(-((-1)^(1/4)*
a^2*b) + (-1)^(3/4)*a*c)*ArcTan[((-1)^(3/4)*Sqrt[b^2 + a^2*x^3])/(Sqrt[a +
I*b]*Sqrt[b])])/(3*b^(5/2)) + ((a^4 + 4*a^2*b^2 - 2*a^2*c)*ArcTanh[Sqrt[b^2
 + a^2*x^3]/b])/(6*b^3)
```

fricas [B] time = 15.87, size = 10278, normalized size = 41.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^3+b^2)^(1/2)*(a^2*x^6+c*x^3+2*b^2)/x^7/(a^2*x^6+b^2),x, al
gorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(2)*b^13*x^6*sqrt((a^8*b^4 + a^6*b^6 + (a^4 + a^2*b^2)*c^4 + 2
*(a^6*b^2 + a^4*b^4)*c^2 - (a^2*b^8 - 2*a^2*b^6*c - b^6*c^2)*sqrt((a^10*b^4
 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10))/(a^8*b
^4 + 4*a^6*b^4*c - 4*a^4*b^2*c^3 + a^4*c^4 - 2*(a^6*b^2 - 2*a^4*b^4)*c^2))*
((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^1
0)^(3/4)*sqrt((a^10*b^4 + 4*a^8*b^4*c - 4*a^6*b^2*c^3 + a^6*c^4 - 2*(a^8*b^
2 - 2*a^6*b^4)*c^2)/b^10)*arctan((sqrt(2)*sqrt(a^2*x^3 + b^2)*((a^8*b^22 +
2*a^6*b^22*c + 2*a^4*b^20*c^3 - a^4*b^18*c^4)*sqrt((a^10*b^4 + a^8*b^6 + (a
^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10)*sqrt((a^10*b^4 + 4*a^8
*b^4*c - 4*a^6*b^2*c^3 + a^6*c^4 - 2*(a^8*b^2 - 2*a^6*b^4)*c^2)/b^10) + (a^
12*b^20 - a^10*b^18*c^2 - 5*a^8*b^16*c^4 - 3*a^6*b^14*c^6 + a^6*b^12*c^7 +
(a^8*b^14 + 2*a^6*b^16)*c^5 - (a^10*b^16 - 4*a^8*b^18)*c^3 - (a^12*b^18 - 2
*a^10*b^20)*c)*sqrt((a^10*b^4 + 4*a^8*b^4*c - 4*a^6*b^2*c^3 + a^6*c^4 - 2*(
a^8*b^2 - 2*a^6*b^4)*c^2)/b^10))*sqrt((a^8*b^4 + a^6*b^6 + (a^4 + a^2*b^2)*
c^4 + 2*(a^6*b^2 + a^4*b^4)*c^2 - (a^2*b^8 - 2*a^2*b^6*c - b^6*c^2)*sqrt((a
^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10))
/(a^8*b^4 + 4*a^6*b^4*c - 4*a^4*b^2*c^3 + a^4*c^4 - 2*(a^6*b^2 - 2*a^4*b^4)
*c^2))*((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c
^2)/b^10)^(3/4) + sqrt(2)*(b^18*sqrt((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*
c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10)*sqrt((a^10*b^4 + 4*a^8*b^4*c - 4*a^6
*b^2*c^3 + a^6*c^4 - 2*(a^8*b^2 - 2*a^6*b^4)*c^2)/b^10) + (a^4*b^16 - a^4*b
^14*c + a^2*b^14*c^2 - a^2*b^12*c^3)*sqrt((a^10*b^4 + 4*a^8*b^4*c - 4*a^6*b
^2*c^3 + a^6*c^4 - 2*(a^8*b^2 - 2*a^6*b^4)*c^2)/b^10))*sqrt((a^8*b^4 + a^6*
b^6 + (a^4 + a^2*b^2)*c^4 + 2*(a^6*b^2 + a^4*b^4)*c^2 - (a^2*b^8 - 2*a^2*b^
6*c - b^6*c^2)*sqrt((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2
+ a^6*b^4)*c^2)/b^10))/(a^8*b^4 + 4*a^6*b^4*c - 4*a^4*b^2*c^3 + a^4*c^4 - 2
*(a^6*b^2 - 2*a^4*b^4)*c^2))*sqrt((a^18*b^10 + a^16*b^12 + (a^10*b^2 + a^8*
b^4)*c^8 - 4*(a^10*b^4 + a^8*b^6)*c^7 + 4*(a^10*b^6 + a^8*b^8)*c^6 - 4*(a^1
2*b^6 + a^10*b^8)*c^5 - 2*(a^14*b^6 - 3*a^12*b^8 - 4*a^10*b^10)*c^4 + 4*(a^
14*b^8 + a^12*b^10)*c^3 + (a^20*b^8 + a^18*b^10 + (a^12 + a^10*b^2)*c^8 - 4
*(a^12*b^2 + a^10*b^4)*c^7 + 4*(a^12*b^4 + a^10*b^6)*c^6 - 4*(a^14*b^4 + a^
12*b^6)*c^5 - 2*(a^16*b^4 - 3*a^14*b^6 - 4*a^12*b^8)*c^4 + 4*(a^16*b^6 + a^
14*b^8)*c^3 + 4*(a^16*b^8 + a^14*b^10)*c^2 + 4*(a^18*b^8 + a^16*b^10)*c)*x^
3 + 4*(a^14*b^10 + a^12*b^12)*c^2 + sqrt(2)*(a^16*b^10 + a^14*b^12 + (a^10*
b^4 + a^8*b^6)*c^6 - 4*(a^10*b^6 + a^8*b^8)*c^5 - (a^12*b^6 - 3*a^10*b^8 -
4*a^8*b^10)*c^4 - (a^14*b^8 - 3*a^12*b^10 - 4*a^10*b^12)*c^2 + 4*(a^14*b^10
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+ a^12*b^12)*c + (a^10*b^14 + 5*a^6*b^10*c^4 - a^6*b^8*c^5 + 2*(a^8*b^10 - 4*a^6*b^12)*c^3 - 2*(3*a^8*b^12 - 2*a^6*b^14)*c^2 - (a^10*b^12 - 4*a^8*b^1 4)*c)*sqrt((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10))*sqrt(a^2*x^3 + b^2)*sqrt((a^8*b^4 + a^6*b^6 + (a^4 + a^2*b^2) *c^4 + 2*(a^6*b^2 + a^4*b^4)*c^2 - (a^2*b^8 - 2*a^2*b^6*c - b^6*c^2)*sqrt((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10))/(a^8*b^4 + 4*a^6*b^4*c - 4*a^4*b^2*c^3 + a^4*c^4 - 2*(a^6*b^2 - 2*a^4*b^4)*c^2))*((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)* c^2)/b^10)^(1/4) + 4*(a^16*b^10 + a^14*b^12)*c + (a^14*b^12 + a^12*b^14 + (a^8*b^6 + a^6*b^8)*c^6 - 4*(a^8*b^8 + a^6*b^10)*c^5 - (a^10*b^8 - 3*a^8*b^1 0 - 4*a^6*b^12)*c^4 - (a^12*b^10 - 3*a^10*b^12 - 4*a^8*b^14)*c^2 + 4*(a^12* b^12 + a^10*b^14)*c)*sqrt((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^ 8*b^2 + a^6*b^4)*c^2)/b^10))/(a^2 + b^2))*((a^10*b^4 + a^8*b^6 + (a^6 + a^4 *b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10)^(3/4) + (a^16*b^18 + a^14*b^20 - (a^8*b^10 + a^6*b^12)*c^8 + 2*(a^8*b^12 + a^6*b^14)*c^7 - 2*(a^10*b^12 + a^8*b^14)*c^6 + 6*(a^10*b^14 + a^8*b^16)*c^5 + 6*(a^12*b^16 + a^10*b^18)*c ^3 + 2*(a^14*b^16 + a^12*b^18)*c^2 + 2*(a^14*b^18 + a^12*b^20)*c)*sqrt((a^1 0*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10)*sq rt((a^10*b^4 + 4*a^8*b^4*c - 4*a^6*b^2*c^3 + a^6*c^4 - 2*(a^8*b^2 - 2*a^6*b ^4)*c^2)/b^10) + (a^20*b^16 + a^18*b^18 - (a^10*b^6 + a^8*b^8)*c^10 + 2*(a^ 10*b^8 + a^8*b^10)*c^9 - 3*(a^12*b^8 + a^10*b^10)*c^8 + 8*(a^12*b^10 + a^10 *b^12)*c^7 - 2*(a^14*b^10 + a^12*b^12)*c^6 + 12*(a^14*b^12 + a^12*b^14)*c^5 + 2*(a^16*b^12 + a^14*b^14)*c^4 + 8*(a^16*b^14 + a^14*b^16)*c^3 + 3*(a^18* b^14 + a^16*b^16)*c^2 + 2*(a^18*b^16 + a^16*b^18)*c)*sqrt((a^10*b^4 + 4*a^8 *b^4*c - 4*a^6*b^2*c^3 + a^6*c^4 - 2*(a^8*b^2 - 2*a^6*b^4)*c^2)/b^10))/(a^2 6*b^12 + a^24*b^14 + (a^14 + a^12*b^2)*c^12 - 4*(a^14*b^2 + a^12*b^4)*c^11 + 2*(a^16*b^2 + 3*a^14*b^4 + 2*a^12*b^6)*c^10 - 12*(a^16*b^4 + a^14*b^6)*c^ 9 - (a^18*b^4 - 15*a^16*b^6 - 16*a^14*b^8)*c^8 - 8*(a^18*b^6 + a^16*b^8)*c^ 7 - 4*(a^20*b^6 - 5*a^18*b^8 - 6*a^16*b^10)*c^6 + 8*(a^20*b^8 + a^18*b^10)* c^5 - (a^22*b^8 - 15*a^20*b^10 - 16*a^18*b^12)*c^4 + 12*(a^22*b^10 + a^20*b ^12)*c^3 + 2*(a^24*b^10 + 3*a^22*b^12 + 2*a^20*b^14)*c^2 + 4*(a^24*b^12 + a ^22*b^14)*c)) + 4*sqrt(2)*b^13*x^6*sqrt((a^8*b^4 + a^6*b^6 + (a^4 + a^2*b^2)*c^4 + 2*(a^6*b^2 + a^4*b^4)*c^2 - (a^2*b^8 - 2*a^2*b^6*c - b^6*c^2)*sqrt((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10))/(a^8*b^4 + 4*a^6*b^4*c - 4*a^4*b^2*c^3 + a^4*c^4 - 2*(a^6*b^2 - 2*a^4*b^ 4)*c^2))*((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4) *c^2)/b^10)^(3/4)*sqrt((a^10*b^4 + 4*a^8*b^4*c - 4*a^6*b^2*c^3 + a^6*c^4 - 2*(a^8*b^2 - 2*a^6*b^4)*c^2)/b^10)*arctan((sqrt(2)*sqrt(a^2*x^3 + b^2)*((a^ 8*b^22 + 2*a^6*b^22*c + 2*a^4*b^20*c^3 - a^4*b^18*c^4)*sqrt((a^10*b^4 + a^8 *b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10)*sqrt((a^10*b^ 4 + 4*a^8*b^4*c - 4*a^6*b^2*c^3 + a^6*c^4 - 2*(a^8*b^2 - 2*a^6*b^4)*c^2)/b^ 10) + (a^12*b^20 - a^10*b^18*c^2 - 5*a^8*b^16*c^4 - 3*a^6*b^14*c^6 + a^6*b^ 12*c^7 + (a^8*b^14 + 2*a^6*b^16)*c^5 - (a^10*b^16 - 4*a^8*b^18)*c^3 - (a^12 *b^18 - 2*a^10*b^20)*c)*sqrt((a^10*b^4 + 4*a^8*b^4*c - 4*a^6*b^2*c^3 + a^6* c^4 - 2*(a^8*b^2 - 2*a^6*b^4)*c^2)/b^10))*sqrt((a^8*b^4 + a^6*b^6 + (a^4 + a^2*b^2)*c^4 + 2*(a^6*b^2 + a^4*b^4)*c^2 - (a^2*b^8 - 2*a^2*b^6*c - b^6*c^2)*sqrt((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^ 2)/b^10))/(a^8*b^4 + 4*a^6*b^4*c - 4*a^4*b^2*c^3 + a^4*c^4 - 2*(a^6*b^2 - 2 *a^4*b^4)*c^2))*((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a ^6*b^4)*c^2)/b^10)^(3/4) + sqrt(2)*(b^18*sqrt((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10)*sqrt((a^10*b^4 + 4*a^8*b^4* c - 4*a^6*b^2*c^3 + a^6*c^4 - 2*(a^8*b^2 - 2*a^6*b^4)*c^2)/b^10) + (a^4*b^1 6 - a^4*b^14*c + a^2*b^14*c^2 - a^2*b^12*c^3)*sqrt((a^10*b^4 + 4*a^8*b^4*c - 4*a^6*b^2*c^3 + a^6*c^4 - 2*(a^8*b^2 - 2*a^6*b^4)*c^2)/b^10))*sqrt((a^8*b ^4 + a^6*b^6 + (a^4 + a^2*b^2)*c^4 + 2*(a^6*b^2 + a^4*b^4)*c^2 - (a^2*b^8 - 2*a^2*b^6*c - b^6*c^2)*sqrt((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2* (a^8*b^2 + a^6*b^4)*c^2)/b^10))/(a^8*b^4 + 4*a^6*b^4*c - 4*a^4*b^2*c^3 + a^ 4*c^4 - 2*(a^6*b^2 - 2*a^4*b^4)*c^2))*sqrt((a^18*b^10 + a^16*b^12 + (a^10*b ^2 + a^8*b^4)*c^8 - 4*(a^10*b^4 + a^8*b^6)*c^7 + 4*(a^10*b^6 + a^8*b^8)*c^6

- 4*(a^12*b^6 + a^10*b^8)*c^5 - 2*(a^14*b^6 - 3*a^12*b^8 - 4*a^10*b^10)*c^4 + 4*(a^14*b^8 + a^12*b^10)*c^3 + (a^20*b^8 + a^18*b^10 + (a^12 + a^10*b^2)*c^8 - 4*(a^12*b^2 + a^10*b^4)*c^7 + 4*(a^12*b^4 + a^10*b^6)*c^6 - 4*(a^14*b^4 + a^12*b^6)*c^5 - 2*(a^16*b^4 - 3*a^14*b^6 - 4*a^12*b^8)*c^4 + 4*(a^16*b^6 + a^14*b^8)*c^3 + 4*(a^16*b^8 + a^14*b^10)*c^2 + 4*(a^18*b^8 + a^16*b^10)*c)*x^3 + 4*(a^14*b^10 + a^12*b^12)*c^2 - sqrt(2)*(a^16*b^10 + a^14*b^12 + (a^10*b^4 + a^8*b^6)*c^6 - 4*(a^10*b^6 + a^8*b^8)*c^5 - (a^12*b^6 - 3*a^10*b^8 - 4*a^8*b^10)*c^4 - (a^14*b^8 - 3*a^12*b^10 - 4*a^10*b^12)*c^2 + 4*(a^14*b^10 + a^12*b^12)*c + (a^10*b^14 + 5*a^6*b^10*c^4 - a^6*b^8*c^5 + 2*(a^8*b^10 - 4*a^6*b^12)*c^3 - 2*(3*a^8*b^12 - 2*a^6*b^14)*c^2 - (a^10*b^12 - 4*a^8*b^14)*c)*sqrt((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10))*sqrt(a^2*x^3 + b^2)*sqrt((a^8*b^4 + a^6*b^6 + (a^4 + a^2*b^2)*c^4 + 2*(a^6*b^2 + a^4*b^4)*c^2 - (a^2*b^8 - 2*a^2*b^6*c - b^6*c^2)*sqrt((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10))/(a^8*b^4 + 4*a^6*b^4*c - 4*a^4*b^2*c^3 + a^4*c^4 - 2*(a^6*b^2 - 2*a^4*b^4)*c^2))*((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10)^(1/4) + 4*(a^16*b^10 + a^14*b^12)*c + (a^14*b^12 + a^12*b^14 + (a^8*b^6 + a^6*b^8)*c^6 - 4*(a^8*b^8 + a^6*b^10)*c^5 - (a^10*b^8 - 3*a^8*b^10 - 4*a^6*b^12)*c^4 - (a^12*b^10 - 3*a^10*b^12 - 4*a^8*b^14)*c^2 + 4*(a^12*b^12 + a^10*b^14)*c)*sqrt((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10))/(a^2 + b^2))*((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10)^(3/4) - (a^16*b^18 + a^14*b^20 - (a^8*b^10 + a^6*b^12)*c^8 + 2*(a^8*b^12 + a^6*b^14)*c^7 - 2*(a^10*b^12 + a^8*b^14)*c^6 + 6*(a^10*b^14 + a^8*b^16)*c^5 + 6*(a^12*b^16 + a^10*b^18)*c^3 + 2*(a^14*b^16 + a^12*b^18)*c^2 + 2*(a^14*b^18 + a^12*b^20)*c)*sqrt((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10)*sqrt((a^10*b^4 + 4*a^8*b^4*c - 4*a^6*b^2*c^3 + a^6*c^4 - 2*(a^8*b^2 - 2*a^6*b^4)*c^2)/b^10) - (a^20*b^16 + a^18*b^18 - (a^10*b^6 + a^8*b^8)*c^10 + 2*(a^10*b^8 + a^8*b^10)*c^9 - 3*(a^12*b^8 + a^10*b^10)*c^8 + 8*(a^12*b^10 + a^10*b^12)*c^7 - 2*(a^14*b^10 + a^12*b^12)*c^6 + 12*(a^14*b^12 + a^12*b^14)*c^5 + 2*(a^16*b^12 + a^14*b^14)*c^4 + 8*(a^16*b^14 + a^14*b^16)*c^3 + 3*(a^18*b^14 + a^16*b^16)*c^2 + 2*(a^18*b^16 + a^16*b^18)*c)*sqrt((a^10*b^4 + 4*a^8*b^4*c - 4*a^6*b^2*c^3 + a^6*c^4 - 2*(a^8*b^2 - 2*a^6*b^4)*c^2)/b^10))/(a^26*b^12 + a^24*b^14 + (a^14 + a^12*b^2)*c^12 - 4*(a^14*b^2 + a^12*b^4)*c^11 + 2*(a^16*b^2 + 3*a^14*b^4 + 2*a^12*b^6)*c^10 - 12*(a^16*b^4 + a^14*b^6)*c^9 - (a^18*b^4 - 15*a^16*b^6 - 16*a^14*b^8)*c^8 - 8*(a^18*b^6 + a^16*b^8)*c^7 - 4*(a^20*b^6 - 5*a^18*b^8 - 6*a^16*b^10)*c^6 + 8*(a^20*b^8 + a^18*b^10)*c^5 - (a^22*b^8 - 15*a^20*b^10 - 16*a^18*b^12)*c^4 + 12*(a^22*b^10 + a^20*b^12)*c^3 + 2*(a^24*b^10 + 3*a^22*b^12 + 2*a^20*b^14)*c^2 + 4*(a^24*b^12 + a^22*b^14)*c)) - (a^14*b^4 + 5*a^12*b^6 + 4*a^10*b^8 - 2*(a^8 + a^6*b^2)*c^5 + (a^10 + 5*a^8*b^2 + 4*a^6*b^4)*c^4 - 4*(a^10*b^2 + a^8*b^4)*c^3 + 2*(a^12*b^2 + 5*a^10*b^4 + 4*a^8*b^6)*c^2 - 2*(a^12*b^4 + a^10*b^6)*c)*x^6*log(b + sqrt(a^2*x^3 + b^2)) + (a^14*b^4 + 5*a^12*b^6 + 4*a^10*b^8 - 2*(a^8 + a^6*b^2)*c^5 + (a^10 + 5*a^8*b^2 + 4*a^6*b^4)*c^4 - 4*(a^10*b^2 + a^8*b^4)*c^3 + 2*(a^12*b^2 + 5*a^10*b^4 + 4*a^8*b^6)*c^2 - 2*(a^12*b^4 + a^10*b^6)*c)*x^6*log(-b + sqrt(a^2*x^3 + b^2)) + sqrt(2)*((a^4*b^11 - 2*a^4*b^9*c - a^2*b^9*c^2)*x^6*sqrt((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10) + (a^10*b^7 + a^8*b^9 + (a^6*b^3 + a^4*b^5)*c^4 + 2*(a^8*b^5 + a^6*b^7)*c^2)*x^6)*sqrt((a^8*b^4 + a^6*b^6 + (a^4 + a^2*b^2)*c^4 + 2*(a^6*b^2 + a^4*b^4)*c^2 - (a^2*b^8 - 2*a^2*b^6*c - b^6*c^2)*sqrt((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10))/(a^8*b^4 + 4*a^6*b^4*c - 4*a^4*b^2*c^3 + a^4*c^4 - 2*(a^6*b^2 - 2*a^4*b^4)*c^2))*((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10)^(1/4)*log((a^18*b^10 + a^16*b^12 + (a^10*b^2 + a^8*b^4)*c^8 - 4*(a^10*b^4 + a^8*b^6)*c^7 + 4*(a^10*b^6 + a^8*b^8)*c^6 - 4*(a^12*b^6 + a^10*b^8)*c^5 - 2*(a^14*b^6 - 3*a^12*b^8 - 4*a^10*b^10)*c^4 + 4*(a^14*b^8 + a^12*b^10)*c^3 + (a^20*b^8 + a^18*b^10 + (a^12 + a^10*b^2)*c^8 - 4*(a^12*b^2 + a^10*b^4)*c^7 + 4*(a^12*b^4 + a^10*b^6)*c^6 - 4*(a^14*b^4 + a^12*b^6)*c^5 - 2*(a^16*b^4 - 3*a^14*b^6 - 4*a^12*b^8)*c^4 + 4*(a^16*b^6 + a^14*b^8)*c^3

```
+ 4*(a^16*b^8 + a^14*b^10)*c^2 + 4*(a^18*b^8 + a^16*b^10)*c)*x^3 + 4*(a^14*
b^10 + a^12*b^12)*c^2 + sqrt(2)*(a^16*b^10 + a^14*b^12 + (a^10*b^4 + a^8*b^
6)*c^6 - 4*(a^10*b^6 + a^8*b^8)*c^5 - (a^12*b^6 - 3*a^10*b^8 - 4*a^8*b^10)*
c^4 - (a^14*b^8 - 3*a^12*b^10 - 4*a^10*b^12)*c^2 + 4*(a^14*b^10 + a^12*b^12
)*c + (a^10*b^14 + 5*a^6*b^10*c^4 - a^6*b^8*c^5 + 2*(a^8*b^10 - 4*a^6*b^12)
*c^3 - 2*(3*a^8*b^12 - 2*a^6*b^14)*c^2 - (a^10*b^12 - 4*a^8*b^14)*c)*sqrt((
a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10)
)*sqrt(a^2*x^3 + b^2)*sqrt((a^8*b^4 + a^6*b^6 + (a^4 + a^2*b^2)*c^4 + 2*(a^
6*b^2 + a^4*b^4)*c^2 - (a^2*b^8 - 2*a^2*b^6*c - b^6*c^2)*sqrt((a^10*b^4 + a
^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10))/(a^8*b^4 +
 4*a^6*b^4*c - 4*a^4*b^2*c^3 + a^4*c^4 - 2*(a^6*b^2 - 2*a^4*b^4)*c^2))*((a^
10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10)^(
1/4) + 4*(a^16*b^10 + a^14*b^12)*c + (a^14*b^12 + a^12*b^14 + (a^8*b^6 + a^
6*b^8)*c^6 - 4*(a^8*b^8 + a^6*b^10)*c^5 - (a^10*b^8 - 3*a^8*b^10 - 4*a^6*b^
12)*c^4 - (a^12*b^10 - 3*a^10*b^12 - 4*a^8*b^14)*c^2 + 4*(a^12*b^12 + a^10*
b^14)*c)*sqrt((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*
b^4)*c^2)/b^10))/(a^2 + b^2)) - sqrt(2)*((a^4*b^11 - 2*a^4*b^9*c - a^2*b^9*
c^2)*x^6*sqrt((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*
b^4)*c^2)/b^10) + (a^10*b^7 + a^8*b^9 + (a^6*b^3 + a^4*b^5)*c^4 + 2*(a^8*b^
5 + a^6*b^7)*c^2)*x^6)*sqrt((a^8*b^4 + a^6*b^6 + (a^4 + a^2*b^2)*c^4 + 2*(a
^6*b^2 + a^4*b^4)*c^2 - (a^2*b^8 - 2*a^2*b^6*c - b^6*c^2)*sqrt((a^10*b^4 +
a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10))/(a^8*b^4
+ 4*a^6*b^4*c - 4*a^4*b^2*c^3 + a^4*c^4 - 2*(a^6*b^2 - 2*a^4*b^4)*c^2))*((a
^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10)^
(1/4)*log((a^18*b^10 + a^16*b^12 + (a^10*b^2 + a^8*b^4)*c^8 - 4*(a^10*b^4 +
 a^8*b^6)*c^7 + 4*(a^10*b^6 + a^8*b^8)*c^6 - 4*(a^12*b^6 + a^10*b^8)*c^5 -
2*(a^14*b^6 - 3*a^12*b^8 - 4*a^10*b^10)*c^4 + 4*(a^14*b^8 + a^12*b^10)*c^3
+ (a^20*b^8 + a^18*b^10 + (a^12 + a^10*b^2)*c^8 - 4*(a^12*b^2 + a^10*b^4)*c
^7 + 4*(a^12*b^4 + a^10*b^6)*c^6 - 4*(a^14*b^4 + a^12*b^6)*c^5 - 2*(a^16*b^
4 - 3*a^14*b^6 - 4*a^12*b^8)*c^4 + 4*(a^16*b^6 + a^14*b^8)*c^3 + 4*(a^16*b^
8 + a^14*b^10)*c^2 + 4*(a^18*b^8 + a^16*b^10)*c)*x^3 + 4*(a^14*b^10 + a^12*
b^12)*c^2 - sqrt(2)*(a^16*b^10 + a^14*b^12 + (a^10*b^4 + a^8*b^6)*c^6 - 4*(
a^10*b^6 + a^8*b^8)*c^5 - (a^12*b^6 - 3*a^10*b^8 - 4*a^8*b^10)*c^4 - (a^14*
b^8 - 3*a^12*b^10 - 4*a^10*b^12)*c^2 + 4*(a^14*b^10 + a^12*b^12)*c + (a^10*
b^14 + 5*a^6*b^10*c^4 - a^6*b^8*c^5 + 2*(a^8*b^10 - 4*a^6*b^12)*c^3 - 2*(3*
a^8*b^12 - 2*a^6*b^14)*c^2 - (a^10*b^12 - 4*a^8*b^14)*c)*sqrt((a^10*b^4 + a
^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10))*sqrt(a^2*x
^3 + b^2)*sqrt((a^8*b^4 + a^6*b^6 + (a^4 + a^2*b^2)*c^4 + 2*(a^6*b^2 + a^4*
b^4)*c^2 - (a^2*b^8 - 2*a^2*b^6*c - b^6*c^2)*sqrt((a^10*b^4 + a^8*b^6 + (a^
6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10))/(a^8*b^4 + 4*a^6*b^4*c
 - 4*a^4*b^2*c^3 + a^4*c^4 - 2*(a^6*b^2 - 2*a^4*b^4)*c^2))*((a^10*b^4 + a^8
*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^10)^(1/4) + 4*(a^
16*b^10 + a^14*b^12)*c + (a^14*b^12 + a^12*b^14 + (a^8*b^6 + a^6*b^8)*c^6 -
 4*(a^8*b^8 + a^6*b^10)*c^5 - (a^10*b^8 - 3*a^8*b^10 - 4*a^6*b^12)*c^4 - (a
^12*b^10 - 3*a^10*b^12 - 4*a^8*b^14)*c^2 + 4*(a^12*b^12 + a^10*b^14)*c)*sqr
t((a^10*b^4 + a^8*b^6 + (a^6 + a^4*b^2)*c^4 + 2*(a^8*b^2 + a^6*b^4)*c^2)/b^
10))/(a^2 + b^2)) + 2*(2*a^10*b^7 + 2*a^8*b^9 + 2*(a^6*b^3 + a^4*b^5)*c^4 +
 (a^12*b^5 + a^10*b^7 + 2*(a^6*b + a^4*b^3)*c^5 + (a^8*b + a^6*b^3)*c^4 + 4
*(a^8*b^3 + a^6*b^5)*c^3 + 2*(a^10*b^3 + a^8*b^5)*c^2 + 2*(a^10*b^5 + a^8*b
^7)*c)*x^3 + 4*(a^8*b^5 + a^6*b^7)*c^2)*sqrt(a^2*x^3 + b^2))/((a^10*b^7 + a
^8*b^9 + (a^6*b^3 + a^4*b^5)*c^4 + 2*(a^8*b^5 + a^6*b^7)*c^2)*x^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^3+b^2)^(1/2)*(a^2*x^6+c*x^3+2*b^2)/x^7/(a^2*x^6+b^2),x, al
gorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueWarning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [abs(b)]=[-4,-35]sym2poly/r2sym(co
nst gen & e,const index_m & i,const vecteur & l) Error: Bad Argument ValueW
arning, need to choose a branch for the root of a polynomial with parameter
s. This might be wrong.The choice was done assuming [abs(b)]=[12,21]sym2pol
y/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argum
ent ValueWarning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [abs(b)]=[24,
16]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error:
 Bad Argument ValueWarning, need to choose a branch for the root of a polyn
omial with parameters. This might be wrong.The choice was done assuming [ab
s(b)]=[-71,69]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur
& l) Error: Bad Argument ValueWarning, need to choose a branch for the root
 of a polynomial with parameters. This might be wrong.The choice was done a
ssuming [abs(b)]=[-22,57]sym2poly/r2sym(const gen & e,const index_m & i,con
st vecteur & l) Error: Bad Argument ValueWarning, need to choose a branch f
or the root of a polynomial with parameters. This might be wrong.The choice
 was done assuming [abs(b)]=[84,29]sym2poly/r2sym(const gen & e,const index
_m & i,const vecteur & l) Error: Bad Argument ValueWarning, need to choose
a branch for the root of a polynomial with parameters. This might be wrong.
The choice was done assuming [abs(b)]=[-27,-21]sym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueWarning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [abs(b)]=[-43,4]sym2poly/r2sym(con
st gen & e,const index_m & i,const vecteur & l) Error: Bad Argument ValueWa
rning, need to choose a branch for the root of a polynomial with parameters
. This might be wrong.The choice was done assuming [abs(b)]=[-60,90]sym2pol
y/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argum
ent ValueWarning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [abs(b)]=[-10
,65]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument ValueWarning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming [a
bs(b)]=[-4,-25]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur
 & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2s
ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument V
aluesym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument ValueEvaluation time: 7.12Done
```

maple [C] time = 0.04, size = 864, normalized size = 3.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^3+b^2)^(1/2)*(a^2*x^6+c*x^3+2*b^2)/x^7/(a^2*x^6+b^2),x)
```

```
[Out] c/b^2*(-1/3*(a^2*x^3+b^2)^(1/2)/x^3-1/3*a^2*arctanh((a^2*x^3+b^2)^(1/2)/(b^
2)^(1/2))/(b^2)^(1/2))-1/3*(a^2*x^3+b^2)^(1/2)/x^6-1/6*a^2/b^2*(a^2*x^3+b^2
)^(1/2)/x^3+1/6*a^4/b^2*arctanh((a^2*x^3+b^2)^(1/2)/(b^2)^(1/2))/(b^2)^(1/2
)-a^2/b^2*(2/3*(a^2*x^3+b^2)^(1/2)-2/3*b^2*arctanh((a^2*x^3+b^2)^(1/2)/(b^2
)^(1/2))/(b^2)^(1/2))+a^2/b^2*(2/3*(a^2*x^3+b^2)^(1/2)+1/6*I/a^3/b^2*2^(1/2
)*sum((-_alpha^3*a^2*b^2+_alpha^3*a^2*c+a^2*b^2+b^2*c)/_alpha^3/(a^2+b^2)*(
-a*b^2)^(1/3)*(1/2*I*a*(2*x+1/a*(-I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)))
/(-a*b^2)^(1/3))^(1/2)*(a*(x-1/a*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/
```

```
2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*a*(2*x+1/a*(I*3^(1/2)*(-a*b^2)^(1/3)+(-a*
b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(a^2*x^3+b^2)^(1/2)*(2*a^2*(_alpha^5*a^2
-_alpha^2*b^2)+I*a^3*(-a*b^2)^(1/3)*_alpha^4*3^(1/2)-I*a^2*_alpha^3*3^(1/2)
*(-a*b^2)^(2/3)-a^3*(-a*b^2)^(1/3)*_alpha^4-a^2*_alpha^3*(-a*b^2)^(2/3)-I*(
-a*b^2)^(1/3)*_alpha*3^(1/2)*b^2*a+I*(-a*b^2)^(2/3)*3^(1/2)*b^2+(-a*b^2)^(1
/3)*_alpha*b^2*a+(-a*b^2)^(2/3)*b^2)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/a*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/a*(-a*b^2)^(1/3))*3^(1/2)*a/(-a*b^2)^(1/3))^(1/2)
,1/2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^5*a^3-I*(-a*b^2)^(2/3)*3^(1/2)*_alp
ha^4*a^2+I*3^(1/2)*_alpha^3*a^2*b^2-3*(-a*b^2)^(2/3)*_alpha^4*a^2-2*I*(-a*b
^2)^(1/3)*3^(1/2)*_alpha^2*a*b^2+I*(-a*b^2)^(2/3)*3^(1/2)*_alpha*b^2-3*_alp
ha^3*a^2*b^2-I*3^(1/2)*b^4+3*(-a*b^2)^(2/3)*_alpha*b^2+3*b^4)/b^2/(a^2+b^2)
,(I*3^(1/2)/a*(-a*b^2)^(1/3)/(-3/2/a*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/a*(-a*b^2
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^6*a^2+b^2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^6 + cx^3 + 2\,b^2\right)\sqrt{a^2x^3 + b^2}}{\left(a^2x^6 + b^2\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^3+b^2)^(1/2)*(a^2*x^6+c*x^3+2*b^2)/x^7/(a^2*x^6+b^2),x, al
gorithm="maxima")
```

```
[Out] integrate((a^2*x^6 + c*x^3 + 2*b^2)*sqrt(a^2*x^3 + b^2)/((a^2*x^6 + b^2)*x^
7), x)
```

mupad [B] time = 14.50, size = 274, normalized size = 1.11

$$\frac{a^2\,\ln\left(\frac{\left(b+\sqrt{a^2\,x^3+b^2}\right)^3\,\left(b-\sqrt{a^2\,x^3+b^2}\right)}{x^6}\right)\,\left(a^2+4\,b^2-2\,c\right)}{12\,b^3}-\frac{\sqrt{a^2\,x^3+b^2}\,\left(a^2+2\,c\right)}{6\,b^2\,x^3}-\frac{\sqrt{a^2\,x^3+b^2}}{3\,x^6}+\frac{a\,\ln\left(\frac{2\,b^2-a\,b\,1{}\mathrm{i}+a^2\,x^3+\sqrt{b}\,\sqrt{a^2\,x^3+b^2}\,\sqrt{-b+a\,1{}\mathrm{i}}\,2{}\mathrm{i}}{a\,x^3+b\,1{}\mathrm{i}}\right)\,\left(-a\,b+c\,1{}\mathrm{i}\right)\,\sqrt{-b+a\,1{}\mathrm{i}}\,1{}\mathrm{i}}{6\,b^{5/2}}+\frac{a\,\ln\left(\frac{a\,b\,1{}\mathrm{i}+2\,b^2+a^2\,x^3-2\,\sqrt{b}\,\sqrt{a^2\,x^3+b^2}\,\sqrt{b+a\,1{}\mathrm{i}}}{-a\,x^3+b\,1{}\mathrm{i}}\right)\,\left(a\,b+c\,1{}\mathrm{i}\right)\,\sqrt{b+a\,1{}\mathrm{i}}}{6\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b^2 + a^2*x^3)^(1/2)*(c*x^3 + 2*b^2 + a^2*x^6))/(x^7*(b^2 + a^2*x^6))
,x)
```

```
[Out] (a^2*log(((b + (b^2 + a^2*x^3)^(1/2))^3*(b - (b^2 + a^2*x^3)^(1/2)))/x^6)*(
a^2 - 2*c + 4*b^2))/(12*b^3) - ((b^2 + a^2*x^3)^(1/2)*(2*c + a^2))/(6*b^2*x
^3) - (b^2 + a^2*x^3)^(1/2)/(3*x^6) + (a*log((2*b^2 - a*b*1i + a^2*x^3 + b^
(1/2)*(b^2 + a^2*x^3)^(1/2)*(a*1i - b)^(1/2)*2i)/(b*1i + a*x^3))*(c*1i - a*
b)*(a*1i - b)^(1/2)*1i)/(6*b^(5/2)) + (a*log((a*b*1i + 2*b^2 + a^2*x^3 - 2*
b^(1/2)*(b^2 + a^2*x^3)^(1/2)*(a*1i + b)^(1/2))/(b*1i - a*x^3))*(c*1i + a*b
)*(a*1i + b)^(1/2))/(6*b^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**3+b**2)**(1/2)*(a**2*x**6+c*x**3+2*b**2)/x**7/(a**2*x**6
+b**2),x)
```

```
[Out] Timed out
```

3.2165 $\int \frac{x^2\left(-2+x^8\right)\sqrt[4]{2-2x^4+x^8}}{\left(2+x^8\right)\left(4-x^4+2x^8\right)}\,dx$

Optimal. Leaf size=248

$$\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^8-2x^4+2}}{\sqrt{2}\,x^2-\sqrt{x^8-2x^4+2}}\right)}{2\sqrt[4]{2}}+\frac{\sqrt[4]{3}\,\tan^{-1}\left(\frac{6^{3/4}x\sqrt[4]{x^8-2x^4+2}}{\sqrt{6}\,\sqrt{x^8-2x^4+2}-3x^2}\right)}{2\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}\,x\sqrt[4]{x^8-2x^4+2}}{2x^2+\sqrt{2}\,\sqrt{x^8-2x^4+2}}\right)}{2\sqrt[4]{2}}-\frac{\sqrt[4]{3}\,\tanh^{-1}\left(\frac{6^{3/4}x\sqrt[4]{x^8-2x^4+2}}{3x^2+\sqrt{6}\,\sqrt{x^8-2x^4+}}\right)}{2\,2^{3/4}}$$

Rubi [F] time = 8.72, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(-2+x^8\right)\sqrt[4]{2-2x^4+x^8}}{\left(2+x^8\right)\left(4-x^4+2x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(-2 + x^8)*(2 - 2*x^4 + x^8)^(1/4))/((2 + x^8)*(4 - x^4 + 2*x^8)),
x]

[Out] -1/4*((-1)^(7/8)*Defer[Int][(2 - 2*x^4 + x^8)^(1/4)/((-2)^(1/8) - x), x])/2
^(1/8) + ((-1)^(3/8)*Defer[Int][(2 - 2*x^4 + x^8)^(1/4)/(-((-1)^(5/8)*2^(1/
8)) - x), x])/(4*2^(1/8)) - ((-1)^(7/8)*Defer[Int][(2 - 2*x^4 + x^8)^(1/4)/
((-2)^(1/8) + x), x])/(4*2^(1/8)) + ((-1)^(3/8)*Defer[Int][(2 - 2*x^4 + x^8
)^(1/4)/(-((-1)^(5/8)*2^(1/8)) + x), x])/(4*2^(1/8)) + Defer[Int][(2 - 2*x^
4 + x^8)^(1/4)/(Sqrt[-1 + I] - 2^(1/8)*x), x]/(2*Sqrt[-1 + I]*2^(3/4)) + De
fer[Int][(2 - 2*x^4 + x^8)^(1/4)/(Sqrt[1 - I] - 2^(1/8)*x), x]/(2*Sqrt[1 -
I]*2^(3/4)) + Defer[Int][(2 - 2*x^4 + x^8)^(1/4)/(Sqrt[-1 + I] + 2^(1/8)*x)
, x]/(2*Sqrt[-1 + I]*2^(3/4)) + Defer[Int][(2 - 2*x^4 + x^8)^(1/4)/(Sqrt[1
- I] + 2^(1/8)*x), x]/(2*Sqrt[1 - I]*2^(3/4)) + ((I/2)*Defer[Int][(2 - 2*x^
4 + x^8)^(1/4)/(Sqrt[-Sqrt[1 - I*Sqrt[31]]] - Sqrt[2]*x), x])/Sqrt[-31*Sqrt
[1 - I*Sqrt[31]]] - ((I + Sqrt[31])*Defer[Int][(2 - 2*x^4 + x^8)^(1/4)/(Sqr
t[-Sqrt[1 - I*Sqrt[31]]] - Sqrt[2]*x), x])/(2*Sqrt[-31*Sqrt[1 - I*Sqrt[31]]
]) + ((I/2)*Defer[Int][(2 - 2*x^4 + x^8)^(1/4)/((1 - I*Sqrt[31])^(1/4) - Sq
rt[2]*x), x])/(Sqrt[31]*(1 - I*Sqrt[31])^(1/4)) - ((I/2)*(1 - I*Sqrt[31])^(
3/4)*Defer[Int][(2 - 2*x^4 + x^8)^(1/4)/((1 - I*Sqrt[31])^(1/4) - Sqrt[2]*x
), x])/Sqrt[31] - ((I/2)*Defer[Int][(2 - 2*x^4 + x^8)^(1/4)/(Sqrt[-Sqrt[1 +
 I*Sqrt[31]]] - Sqrt[2]*x), x])/Sqrt[-31*Sqrt[1 + I*Sqrt[31]]] + ((I - Sqrt
[31])*Defer[Int][(2 - 2*x^4 + x^8)^(1/4)/(Sqrt[-Sqrt[1 + I*Sqrt[31]]] - Sqr
t[2]*x), x])/(2*Sqrt[-31*Sqrt[1 + I*Sqrt[31]]]) - ((I/2)*Defer[Int][(2 - 2*
x^4 + x^8)^(1/4)/((1 + I*Sqrt[31])^(1/4) - Sqrt[2]*x), x])/(Sqrt[31]*(1 + I
*Sqrt[31])^(1/4)) + ((I/2)*(1 + I*Sqrt[31])^(3/4)*Defer[Int][(2 - 2*x^4 + x
^8)^(1/4)/((1 + I*Sqrt[31])^(1/4) - Sqrt[2]*x), x])/Sqrt[31] + ((I/2)*Defer
[Int][(2 - 2*x^4 + x^8)^(1/4)/(Sqrt[-Sqrt[1 - I*Sqrt[31]]] + Sqrt[2]*x), x]
)/Sqrt[-31*Sqrt[1 - I*Sqrt[31]]] - ((I + Sqrt[31])*Defer[Int][(2 - 2*x^4 +
x^8)^(1/4)/(Sqrt[-Sqrt[1 - I*Sqrt[31]]] + Sqrt[2]*x), x])/(2*Sqrt[-31*Sqrt[
1 - I*Sqrt[31]]]) + ((I/2)*Defer[Int][(2 - 2*x^4 + x^8)^(1/4)/((1 - I*Sqrt[
31])^(1/4) + Sqrt[2]*x), x])/(Sqrt[31]*(1 - I*Sqrt[31])^(1/4)) - ((I/2)*(1
- I*Sqrt[31])^(3/4)*Defer[Int][(2 - 2*x^4 + x^8)^(1/4)/((1 - I*Sqrt[31])^(1
/4) + Sqrt[2]*x), x])/Sqrt[31] - ((I/2)*Defer[Int][(2 - 2*x^4 + x^8)^(1/4)/
(Sqrt[-Sqrt[1 + I*Sqrt[31]]] + Sqrt[2]*x), x])/Sqrt[-31*Sqrt[1 + I*Sqrt[31]
]] + ((I - Sqrt[31])*Defer[Int][(2 - 2*x^4 + x^8)^(1/4)/(Sqrt[-Sqrt[1 + I*S
qrt[31]]] + Sqrt[2]*x), x])/(2*Sqrt[-31*Sqrt[1 + I*Sqrt[31]]]) - ((I/2)*Def
er[Int][(2 - 2*x^4 + x^8)^(1/4)/((1 + I*Sqrt[31])^(1/4) + Sqrt[2]*x), x])/(
Sqrt[31]*(1 + I*Sqrt[31])^(1/4)) + ((I/2)*(1 + I*Sqrt[31])^(3/4)*Defer[Int]
[(2 - 2*x^4 + x^8)^(1/4)/((1 + I*Sqrt[31])^(1/4) + Sqrt[2]*x), x])/Sqrt[31]
```

Rubi steps

$$\int \frac{x^2\left(-2+x^8\right)\sqrt[4]{2-2x^4+x^8}}{\left(2+x^8\right)\left(4-x^4+2x^8\right)}\,dx = \int\left(-\frac{2x^6\sqrt[4]{2-2x^4+x^8}}{2+x^8}+\frac{x^2\left(-1+4x^4\right)\sqrt[4]{2-2x^4+x^8}}{4-x^4+2x^8}\right)dx$$

$$= -\left(2\int\frac{x^6\sqrt[4]{2-2x^4+x^8}}{2+x^8}\,dx\right)+\int\frac{x^2\left(-1+4x^4\right)\sqrt[4]{2-2x^4+x^8}}{4-x^4+2x^8}\,dx$$

$$= -\left(2\int\left(\frac{x^2\sqrt[4]{2-2x^4+x^8}}{2\left(-i\sqrt{2}+x^4\right)}+\frac{x^2\sqrt[4]{2-2x^4+x^8}}{2\left(i\sqrt{2}+x^4\right)}\right)dx\right)+\int\left(-\frac{x^2\sqrt[4]{2-2x^4}\cdots}{4-x^4+2\cdots}\right.$$

$$= 4\int\frac{x^6\sqrt[4]{2-2x^4+x^8}}{4-x^4+2x^8}\,dx-\int\frac{x^2\sqrt[4]{2-2x^4+x^8}}{-i\sqrt{2}+x^4}\,dx-\int\frac{x^2\sqrt[4]{2-2x^4+x^8}}{i\sqrt{2}+x^4}\cdots$$

$$= 4\int\left(\frac{i\left(1+i\sqrt{31}\right)x^2\sqrt[4]{2-2x^4+x^8}}{\sqrt{31}\left(1+i\sqrt{31}-4x^4\right)}-\frac{i\left(-1+i\sqrt{31}\right)x^2\sqrt[4]{2-2x^4+x^8}}{\sqrt{31}\left(-1+i\sqrt{31}+4x^4\right)}\right)d\cdots$$

$$= \frac{1}{2}\int\frac{\sqrt[4]{2-2x^4+x^8}}{\sqrt[4]{-2}-x^2}\,dx+\frac{1}{2}\int\frac{\sqrt[4]{2-2x^4+x^8}}{-(-1)^{3/4}\sqrt[4]{2}-x^2}\,dx-\frac{1}{2}\int\frac{\sqrt[4]{2-2x^4+x^8}}{\sqrt[4]{-2}+x^2}\cdots$$

$$= \frac{1}{2}\int\left(-\frac{(-1)^{7/8}\sqrt[4]{2-2x^4+x^8}}{2\sqrt[8]{2}\left(\sqrt[8]{-2}-x\right)}-\frac{(-1)^{7/8}\sqrt[4]{2-2x^4+x^8}}{2\sqrt[8]{2}\left(\sqrt[8]{-2}+x\right)}\right)dx-\frac{1}{2}\int\left(-\frac{(-1\cdots}{2\sqrt[8]{\cdots}}\right.$$

$$= \frac{\int\frac{\sqrt[4]{2-2x^4+x^8}}{\sqrt{-1+i}-\sqrt[8]{2}\,x}\,dx}{2\sqrt{-1+i}\,2^{3/4}}+\frac{\int\frac{\sqrt[4]{2-2x^4+x^8}}{\sqrt{-1+i}+\sqrt[8]{2}\,x}\,dx}{2\sqrt{-1+i}\,2^{3/4}}+\frac{\int\frac{\sqrt[4]{2-2x^4+x^8}}{\sqrt{1-i}-\sqrt[8]{2}\,x}\,dx}{2\sqrt{1-i}\,2^{3/4}}+\frac{\int\frac{\sqrt[4]{2-2x^4+x^8}}{\sqrt{1-i}+\sqrt[8]{2}\,x}\,dx}{2\sqrt{1-i}\,2^{3/4}}$$

$$= \frac{\int\frac{\sqrt[4]{2-2x^4+x^8}}{\sqrt{-1+i}-\sqrt[8]{2}\,x}\,dx}{2\sqrt{-1+i}\,2^{3/4}}+\frac{\int\frac{\sqrt[4]{2-2x^4+x^8}}{\sqrt{-1+i}+\sqrt[8]{2}\,x}\,dx}{2\sqrt{-1+i}\,2^{3/4}}+\frac{\int\frac{\sqrt[4]{2-2x^4+x^8}}{\sqrt{1-i}-\sqrt[8]{2}\,x}\,dx}{2\sqrt{1-i}\,2^{3/4}}+\frac{\int\frac{\sqrt[4]{2-2x^4+x^8}}{\sqrt{1-i}+\sqrt[8]{2}\,x}\,dx}{2\sqrt{1-i}\,2^{3/4}}$$

$$= \frac{\int\frac{\sqrt[4]{2-2x^4+x^8}}{\sqrt{-1+i}-\sqrt[8]{2}\,x}\,dx}{2\sqrt{-1+i}\,2^{3/4}}+\frac{\int\frac{\sqrt[4]{2-2x^4+x^8}}{\sqrt{-1+i}+\sqrt[8]{2}\,x}\,dx}{2\sqrt{-1+i}\,2^{3/4}}+\frac{\int\frac{\sqrt[4]{2-2x^4+x^8}}{\sqrt{1-i}-\sqrt[8]{2}\,x}\,dx}{2\sqrt{1-i}\,2^{3/4}}+\frac{\int\frac{\sqrt[4]{2-2x^4+x^8}}{\sqrt{1-i}+\sqrt[8]{2}\,x}\,dx}{2\sqrt{1-i}\,2^{3/4}}$$

Mathematica [F] time = 0.31, size = 0, normalized size = 0.00

$$\int\frac{x^2\left(-2+x^8\right)\sqrt[4]{2-2x^4+x^8}}{\left(2+x^8\right)\left(4-x^4+2x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(-2 + x^8)*(2 - 2*x^4 + x^8)^(1/4))/((2 + x^8)*(4 - x^4 + 2*
x^8)),x]
```

```
[Out] Integrate[(x^2*(-2 + x^8)*(2 - 2*x^4 + x^8)^(1/4))/((2 + x^8)*(4 - x^4 + 2*
x^8)), x]
```

IntegrateAlgebraic [A] time = 1.87, size = 253, normalized size = 1.02

$$\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^8-2x^4+2}}{\sqrt{2}\,x^2-\sqrt{x^8-2x^4+2}}\right)}{2\sqrt[4]{2}}-\frac{\sqrt[4]{3}\,\tan^{-1}\left(\frac{\frac{\sqrt{x^8-2x^4+2}}{\sqrt[4]{6}}-\frac{\sqrt[4]{3}\,x^2}{2^{3/4}}}{x\sqrt[4]{x^8-2x^4+2}}\right)}{2\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}\,x\sqrt[4]{x^8-2x^4+2}}{2x^2+\sqrt{2}\,\sqrt{x^8-2x^4+2}}\right)}{2\sqrt[4]{2}}-\frac{\sqrt[4]{3}\,\tanh^{-1}\left(\frac{6^{3/4}x\sqrt[4]{x^8-2x^4+2}}{3x^2+\sqrt{6}\,\sqrt{x^8-2x^4+2}}\right)}{2\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(-2 + x^8)*(2 - 2*x^4 + x^8)^(1/4))/((2 + x^8)*(4 -
 x^4 + 2*x^8)),x]
```

```
[Out] ArcTan[(2^(3/4)*x*(2 - 2*x^4 + x^8)^(1/4))/(Sqrt[2]*x^2 - Sqrt[2 - 2*x^4 +
x^8])]/(2*2^(1/4)) - (3^(1/4)*ArcTan[(-((3^(1/4)*x^2)/2^(3/4)) + Sqrt[2 - 2
*x^4 + x^8]/6^(1/4))/(x*(2 - 2*x^4 + x^8)^(1/4))])/(2*2^(3/4)) + ArcTanh[(2
*2^(1/4)*x*(2 - 2*x^4 + x^8)^(1/4))/(2*x^2 + Sqrt[2]*Sqrt[2 - 2*x^4 + x^8])
]/(2*2^(1/4)) - (3^(1/4)*ArcTanh[(6^(3/4)*x*(2 - 2*x^4 + x^8)^(1/4))/(3*x^2
 + Sqrt[6]*Sqrt[2 - 2*x^4 + x^8])])/(2*2^(3/4))
```

fricas [B] time = 26.59, size = 1328, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^8-2)*(x^8-2*x^4+2)^(1/4)/(x^8+2)/(2*x^8-x^4+4),x, algorith
m="fricas")
```

```
[Out] -1/16*8^(3/4)*sqrt(2)*arctan(1/4*(4*8^(1/4)*sqrt(2)*(x^8 - 2*x^4 + 2)^(1/4)
*x^3 + 8^(3/4)*sqrt(2)*(x^8 - 2*x^4 + 2)^(3/4)*x + (8^(3/4)*sqrt(2)*(x^8 -
2*x^4 + 2)^(1/4)*x^3 + 2*8^(1/4)*sqrt(2)*(x^8 - 2*x^4 + 2)^(3/4)*x - 8*sqrt
(x^8 - 2*x^4 + 2)*x^2 - 2*sqrt(2)*(x^8 + 2))*sqrt((2*x^8 + 4*8^(1/4)*sqrt(2
)*(x^8 - 2*x^4 + 2)^(1/4)*x^3 + 8^(3/4)*sqrt(2)*(x^8 - 2*x^4 + 2)^(3/4)*x +
 8*sqrt(2)*sqrt(x^8 - 2*x^4 + 2)*x^2 + 4)/(x^8 + 2)))/(x^8 - 4*x^4 + 2)) -
1/16*8^(3/4)*sqrt(2)*arctan(1/4*(4*8^(1/4)*sqrt(2)*(x^8 - 2*x^4 + 2)^(1/4)*
x^3 + 8^(3/4)*sqrt(2)*(x^8 - 2*x^4 + 2)^(3/4)*x + (8^(3/4)*sqrt(2)*(x^8 - 2
*x^4 + 2)^(1/4)*x^3 + 2*8^(1/4)*sqrt(2)*(x^8 - 2*x^4 + 2)^(3/4)*x + 8*sqrt(
x^8 - 2*x^4 + 2)*x^2 + 2*sqrt(2)*(x^8 + 2))*sqrt((2*x^8 - 4*8^(1/4)*sqrt(2)
*(x^8 - 2*x^4 + 2)^(1/4)*x^3 - 8^(3/4)*sqrt(2)*(x^8 - 2*x^4 + 2)^(3/4)*x +
8*sqrt(2)*sqrt(x^8 - 2*x^4 + 2)*x^2 + 4)/(x^8 + 2)))/(x^8 - 4*x^4 + 2)) + 1
/64*8^(3/4)*sqrt(2)*log(4*(2*x^8 + 4*8^(1/4)*sqrt(2)*(x^8 - 2*x^4 + 2)^(1/4
)*x^3 + 8^(3/4)*sqrt(2)*(x^8 - 2*x^4 + 2)^(3/4)*x + 8*sqrt(2)*sqrt(x^8 - 2*
x^4 + 2)*x^2 + 4)/(x^8 + 2)) - 1/64*8^(3/4)*sqrt(2)*log(4*(2*x^8 - 4*8^(1/4
)*sqrt(2)*(x^8 - 2*x^4 + 2)^(1/4)*x^3 - 8^(3/4)*sqrt(2)*(x^8 - 2*x^4 + 2)^(
3/4)*x + 8*sqrt(2)*sqrt(x^8 - 2*x^4 + 2)*x^2 + 4)/(x^8 + 2)) + 1/4*3^(1/4)*
2^(1/4)*arctan(1/6*(6*3^(3/4)*2^(3/4)*(x^8 - 2*x^4 + 2)^(1/4)*x^3 + 12*3^(1
/4)*2^(1/4)*(x^8 - 2*x^4 + 2)^(3/4)*x + sqrt(6)*(6*3^(1/4)*2^(1/4)*(x^8 - 2
*x^4 + 2)^(1/4)*x^3 + 2*3^(3/4)*2^(3/4)*(x^8 - 2*x^4 + 2)^(3/4)*x - 12*sqrt
(x^8 - 2*x^4 + 2)*x^2 - sqrt(3)*sqrt(2)*(2*x^8 - x^4 + 4))*sqrt((2*x^8 + 2*
3^(3/4)*2^(3/4)*(x^8 - 2*x^4 + 2)^(1/4)*x^3 - x^4 + 4*sqrt(3)*sqrt(2)*sqrt(
x^8 - 2*x^4 + 2)*x^2 + 4*3^(1/4)*2^(1/4)*(x^8 - 2*x^4 + 2)^(3/4)*x + 4)/(2*
x^8 - x^4 + 4)))/(2*x^8 - 7*x^4 + 4)) + 1/4*3^(1/4)*2^(1/4)*arctan(1/6*(6*3
^(3/4)*2^(3/4)*(x^8 - 2*x^4 + 2)^(1/4)*x^3 + 12*3^(1/4)*2^(1/4)*(x^8 - 2*x^
4 + 2)^(3/4)*x + sqrt(6)*(6*3^(1/4)*2^(1/4)*(x^8 - 2*x^4 + 2)^(1/4)*x^3 + 2
*3^(3/4)*2^(3/4)*(x^8 - 2*x^4 + 2)^(3/4)*x + 12*sqrt(x^8 - 2*x^4 + 2)*x^2 +
 sqrt(3)*sqrt(2)*(2*x^8 - x^4 + 4))*sqrt((2*x^8 - 2*3^(3/4)*2^(3/4)*(x^8 -
2*x^4 + 2)^(1/4)*x^3 - x^4 + 4*sqrt(3)*sqrt(2)*sqrt(x^8 - 2*x^4 + 2)*x^2 -
4*3^(1/4)*2^(1/4)*(x^8 - 2*x^4 + 2)^(3/4)*x + 4)/(2*x^8 - x^4 + 4)))/(2*x^8
 - 7*x^4 + 4)) - 1/16*3^(1/4)*2^(1/4)*log(6*(2*x^8 + 2*3^(3/4)*2^(3/4)*(x^8
 - 2*x^4 + 2)^(1/4)*x^3 - x^4 + 4*sqrt(3)*sqrt(2)*sqrt(x^8 - 2*x^4 + 2)*x^2
 + 4*3^(1/4)*2^(1/4)*(x^8 - 2*x^4 + 2)^(3/4)*x + 4)/(2*x^8 - x^4 + 4)) + 1/
16*3^(1/4)*2^(1/4)*log(6*(2*x^8 - 2*3^(3/4)*2^(3/4)*(x^8 - 2*x^4 + 2)^(1/4)
```

```
*x^3 - x^4 + 4*sqrt(3)*sqrt(2)*sqrt(x^8 - 2*x^4 + 2)*x^2 - 4*3^(1/4)*2^(1/4
)*(x^8 - 2*x^4 + 2)^(3/4)*x + 4)/(2*x^8 - x^4 + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{8}-2\,x^{4}+2\right)^{\frac{1}{4}}\left(x^{8}-2\right)x^{2}}{\left(2\,x^{8}-x^{4}+4\right)\left(x^{8}+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^8-2)*(x^8-2*x^4+2)^(1/4)/(x^8+2)/(2*x^8-x^4+4),x, algorith
m="giac")
```

```
[Out] integrate((x^8 - 2*x^4 + 2)^(1/4)*(x^8 - 2)*x^2/((2*x^8 - x^4 + 4)*(x^8 + 2
)), x)
```

maple [F] time = 1.52, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\left(x^{8}-2\right)\left(x^{8}-2x^{4}+2\right)^{\frac{1}{4}}}{\left(x^{8}+2\right)\left(2x^{8}-x^{4}+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^8-2)*(x^8-2*x^4+2)^(1/4)/(x^8+2)/(2*x^8-x^4+4),x)
```

```
[Out] int(x^2*(x^8-2)*(x^8-2*x^4+2)^(1/4)/(x^8+2)/(2*x^8-x^4+4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{8}-2\,x^{4}+2\right)^{\frac{1}{4}}\left(x^{8}-2\right)x^{2}}{\left(2\,x^{8}-x^{4}+4\right)\left(x^{8}+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^8-2)*(x^8-2*x^4+2)^(1/4)/(x^8+2)/(2*x^8-x^4+4),x, algorith
m="maxima")
```

```
[Out] integrate((x^8 - 2*x^4 + 2)^(1/4)*(x^8 - 2)*x^2/((2*x^8 - x^4 + 4)*(x^8 + 2
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(x^8-2\right)\,{\left(x^8-2\,x^4+2\right)}^{1/4}}{\left(x^8+2\right)\,\left(2\,x^8-x^4+4\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^8 - 2)*(x^8 - 2*x^4 + 2)^(1/4))/((x^8 + 2)*(2*x^8 - x^4 + 4)),x
)
```

```
[Out] int((x^2*(x^8 - 2)*(x^8 - 2*x^4 + 2)^(1/4))/((x^8 + 2)*(2*x^8 - x^4 + 4)),
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**8-2)*(x**8-2*x**4+2)**(1/4)/(x**8+2)/(2*x**8-x**4+4),x)
```

```
[Out] Timed out
```

3.2166 $\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[3]{ax^2+x\sqrt{-b+a^2x^2}}}\,dx$

Optimal. Leaf size=248

$$-\frac{\log\left(\frac{\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{x\sqrt{a^2x^2-b}+ax^2}}{\sqrt[3]{b}}+\frac{2^{2/3}a^{2/3}\left(x\sqrt{a^2x^2-b}+ax^2\right)^{2/3}}{b^{2/3}}+1\right)}{2\,2^{2/3}a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\frac{\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{x\sqrt{a^2x^2-b}+ax^2}}{\sqrt[3]{b}}-1\right)}{2^{2/3}a^{2/3}\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{x\sqrt{\ldots}}}{\sqrt{3}\ldots}\right.}{2^{2/3}a^{2/3}\ldots}$$

Rubi [F] time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[3]{ax^2+x\sqrt{-b+a^2x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[-b + a^2*x^2]*(a*x^2 + x*Sqrt[-b + a^2*x^2])^(1/3)),x]
[Out] Defer[Int][1/(Sqrt[-b + a^2*x^2]*(a*x^2 + x*Sqrt[-b + a^2*x^2])^(1/3)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[3]{ax^2+x\sqrt{-b+a^2x^2}}}\,dx=\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[3]{ax^2+x\sqrt{-b+a^2x^2}}}\,dx$$

Mathematica [A] time = 13.65, size = 225, normalized size = 0.91

$$\frac{\sqrt{a^2x^2-b}\left(ax\left(\sqrt{a^2x^2-b}+ax\right)\right)^{4/3}\left(-2\log\left(\sqrt{a^2x^2-b}+ax\right)+3\log\left(\sqrt[3]{b}-\sqrt[3]{\left(\sqrt{a^2x^2-b}+ax\right)^2+b}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{\left(\sqrt{a^2x^2-b}+ax\right)^2+b}}{\sqrt[3]{b}}+1}{\sqrt{3}}\right)\right)}{2\,2^{2/3}a^2\sqrt[3]{b}\,x\sqrt[3]{x\left(\sqrt{a^2x^2-b}+ax\right)}\left(ax\left(\sqrt{a^2x^2-b}+ax\right)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-b + a^2*x^2]*(a*x^2 + x*Sqrt[-b + a^2*x^2])^(1/3)),x]
[Out] (Sqrt[-b + a^2*x^2]*(a*x*(a*x + Sqrt[-b + a^2*x^2]))^(4/3)*(2*Sqrt[3]*ArcTa
n[(1 + (2*(b + (a*x + Sqrt[-b + a^2*x^2])^2)^(1/3))/b^(1/3))/Sqrt[3]] - 2*L
og[a*x + Sqrt[-b + a^2*x^2]] + 3*Log[b^(1/3) - (b + (a*x + Sqrt[-b + a^2*x^
2])^2)^(1/3)]))/(2*2^(2/3)*a^2*b^(1/3)*x*(x*(a*x + Sqrt[-b + a^2*x^2]))^(1/
3)*(-b + a*x*(a*x + Sqrt[-b + a^2*x^2])))
```

IntegrateAlgebraic [A] time = 2.47, size = 248, normalized size = 1.00

$$-\frac{\log\left(\frac{\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{x\sqrt{a^2x^2-b}+ax^2}}{\sqrt[3]{b}}+\frac{2^{2/3}a^{2/3}\left(x\sqrt{a^2x^2-b}+ax^2\right)^{2/3}}{b^{2/3}}+1\right)}{2\,2^{2/3}a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\frac{\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{x\sqrt{a^2x^2-b}+ax^2}}{\sqrt[3]{b}}-1\right)}{2^{2/3}a^{2/3}\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{x\sqrt{a^2x^2-b}+ax^2}}{\sqrt{3}\sqrt[3]{b}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-b + a^2*x^2]*(a*x^2 + x*Sqrt[-b + a^2*x^2])^(1/
3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*2^(1/3)*a^(1/3)*(a*x^2 + x*Sqrt[-b + a^2*x^2
])^(1/3))/(Sqrt[3]*b^(1/3))])/(2^(2/3)*a^(2/3)*b^(1/3)) + Log[-1 + (2^(1/3)
*a^(1/3)*(a*x^2 + x*Sqrt[-b + a^2*x^2])^(1/3))/b^(1/3)]/(2^(2/3)*a^(2/3)*b^
(1/3)) - Log[1 + (2^(1/3)*a^(1/3)*(a*x^2 + x*Sqrt[-b + a^2*x^2])^(1/3))/b^(
1/3) + (2^(2/3)*a^(2/3)*(a*x^2 + x*Sqrt[-b + a^2*x^2])^(2/3))/b^(2/3)]/(2*2
^(2/3)*a^(2/3)*b^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(a*x^2+x*(a^2*x^2-b)^(1/2))^(1/3),x, algorith
m="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2-b}\left(ax^2+\sqrt{a^2x^2-b}\,x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(a*x^2+x*(a^2*x^2-b)^(1/2))^(1/3),x, algorith
m="giac")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 - b)*(a*x^2 + sqrt(a^2*x^2 - b)*x)^(1/3)), x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2-b}\,\left(a\,x^2+x\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*x^2-b)^(1/2)/(a*x^2+x*(a^2*x^2-b)^(1/2))^(1/3),x)
```

```
[Out] int(1/(a^2*x^2-b)^(1/2)/(a*x^2+x*(a^2*x^2-b)^(1/2))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2-b}\left(ax^2+\sqrt{a^2x^2-b}\,x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(a*x^2+x*(a^2*x^2-b)^(1/2))^(1/3),x, algorith
m="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 - b)*(a*x^2 + sqrt(a^2*x^2 - b)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(x\,\sqrt{a^2\,x^2-b}+a\,x^2\right)^{1/3}\,\sqrt{a^2\,x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x*(a^2*x^2 - b)^(1/2) + a*x^2)^(1/3)*(a^2*x^2 - b)^(1/2)),x)
[Out] int(1/((x*(a^2*x^2 - b)^(1/2) + a*x^2)^(1/3)*(a^2*x^2 - b)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x\left(ax+\sqrt{a^2x^2-b}\right)}\sqrt{a^2x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*x**2-b)**(1/2)/(a*x**2+x*(a**2*x**2-b)**(1/2))**(1/3),x)
[Out] Integral(1/((x*(a*x + sqrt(a**2*x**2 - b)))**(1/3)*sqrt(a**2*x**2 - b)), x)
```

3.2167 $\int \frac{\sqrt{-b+ax}}{1+\sqrt{ax+\sqrt{-b+ax}}}\,dx$

Optimal. Leaf size=248

$$\sqrt{ax-b}\left(\frac{\sqrt{\sqrt{ax-b}+ax}}{a}-\frac{2}{a}\right)-\frac{3\sqrt{\sqrt{ax-b}+ax}}{2a}+\frac{(4b-19)\log\left(2\sqrt{ax-b}-2\sqrt{\sqrt{ax-b}+ax}+1\right)}{4a}+\frac{2\log}{}$$

Rubi [A] time = 0.71, antiderivative size = 442, normalized size of antiderivative = 1.78, number of steps used = 25, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6742, 634, 618, 206, 628, 612, 621, 989, 982, 1019, 1076, 1, 1025, 1024}

$$\frac{\sqrt{\sqrt{ax-b}+ax}\left(2\sqrt{ax-b}+1\right)}{2a}-\frac{2\sqrt{ax-b}}{a}-\frac{2\sqrt{\sqrt{ax-b}+ax}}{a}+\frac{\log\left(-\sqrt{ax-b}-ax+1\right)}{a}+\frac{2\tanh^{-1}\left(\sqrt{\sqrt{ax-b}+ax}\right)}{a}+\frac{2(3-2b)\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{\sqrt{5-4b}}\right)}{a\sqrt{5-4b}}-\frac{(1-4b)\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{2\sqrt{\sqrt{ax-b}+ax}}\right)}{4a}+\frac{2(1-b)\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{2\sqrt{\sqrt{ax-b}+ax}}\right)}{a}+\frac{\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{2\sqrt{\sqrt{ax-b}+ax}}\right)}{a}-\frac{4(1-b)\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{\sqrt{5-4b}\sqrt{\sqrt{ax-b}+ax}}\right)}{a\sqrt{5-4b}}-\frac{2\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{\sqrt{5-4b}\sqrt{\sqrt{ax-b}+ax}}\right)}{a\sqrt{5-4b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-b + a*x]/(1 + Sqrt[a*x + Sqrt[-b + a*x]]),x]
```

```
[Out] (-2*Sqrt[-b + a*x])/a - (2*Sqrt[a*x + Sqrt[-b + a*x]])/a + (Sqrt[a*x + Sqrt
[-b + a*x]]*(1 + 2*Sqrt[-b + a*x]))/(2*a) + (2*ArcTanh[Sqrt[a*x + Sqrt[-b +
 a*x]]])/a + (2*(3 - 2*b)*ArcTanh[(1 + 2*Sqrt[-b + a*x])/Sqrt[5 - 4*b]])/(a
*Sqrt[5 - 4*b]) + ArcTanh[(1 + 2*Sqrt[-b + a*x])/(2*Sqrt[a*x + Sqrt[-b + a*
x]])]/a - ((1 - 4*b)*ArcTanh[(1 + 2*Sqrt[-b + a*x])/(2*Sqrt[a*x + Sqrt[-b +
 a*x]])])/(4*a) + (2*(1 - b)*ArcTanh[(1 + 2*Sqrt[-b + a*x])/(2*Sqrt[a*x + S
qrt[-b + a*x]])])/a - (2*ArcTanh[(1 + 2*Sqrt[-b + a*x])/(Sqrt[5 - 4*b]*Sqrt
[a*x + Sqrt[-b + a*x]])])/(a*Sqrt[5 - 4*b]) - (4*(1 - b)*ArcTanh[(1 + 2*Sqr
t[-b + a*x])/(Sqrt[5 - 4*b]*Sqrt[a*x + Sqrt[-b + a*x]])])/(a*Sqrt[5 - 4*b])
 + Log[1 - a*x - Sqrt[-b + a*x]]/a
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 989

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
4*d*f, 0]
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{-b+ax}}{1+\sqrt{ax+\sqrt{-b+ax}}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2}{1+\sqrt{b+x+x^2}}\,dx, x, \sqrt{-b+ax}\right)}{a}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-1+\frac{-1+b+x}{-1+b+x+x^2}+\sqrt{b+x+x^2}+\frac{(1-b)\sqrt{b+x+x^2}}{-1+b+x+x^2}-\frac{x\sqrt{b+x+x^2}}{-1+b+x+x^2}\right)dx, x, \cdots\right.}{a}$$

$$= -\frac{2\sqrt{-b+ax}}{a}+\frac{2\,\text{Subst}\left(\int \frac{-1+b+x}{-1+b+x+x^2}\,dx, x, \sqrt{-b+ax}\right)}{a}+\frac{2\,\text{Subst}\left(\int \sqrt{b+x+\cdots}\right.}{\cdots}$$

$$= -\frac{2\sqrt{-b+ax}}{a}-\frac{2\sqrt{ax+\sqrt{-b+ax}}}{a}+\frac{\sqrt{ax+\sqrt{-b+ax}}\left(1+2\sqrt{-b+ax}\right)}{2a}+\cdots$$

$$= -\frac{2\sqrt{-b+ax}}{a}-\frac{2\sqrt{ax+\sqrt{-b+ax}}}{a}+\frac{\sqrt{ax+\sqrt{-b+ax}}\left(1+2\sqrt{-b+ax}\right)}{2a}+\cdots$$

$$= -\frac{2\sqrt{-b+ax}}{a}-\frac{2\sqrt{ax+\sqrt{-b+ax}}}{a}+\frac{\sqrt{ax+\sqrt{-b+ax}}\left(1+2\sqrt{-b+ax}\right)}{2a}+\cdots$$

$$= -\frac{2\sqrt{-b+ax}}{a}-\frac{2\sqrt{ax+\sqrt{-b+ax}}}{a}+\frac{\sqrt{ax+\sqrt{-b+ax}}\left(1+2\sqrt{-b+ax}\right)}{2a}+\cdots$$

$$= -\frac{2\sqrt{-b+ax}}{a}-\frac{2\sqrt{ax+\sqrt{-b+ax}}}{a}+\frac{\sqrt{ax+\sqrt{-b+ax}}\left(1+2\sqrt{-b+ax}\right)}{2a}+\cdots$$

$$= -\frac{2\sqrt{-b+ax}}{a}-\frac{2\sqrt{ax+\sqrt{-b+ax}}}{a}+\frac{\sqrt{ax+\sqrt{-b+ax}}\left(1+2\sqrt{-b+ax}\right)}{2a}+\cdots$$

Mathematica [B] time = 0.79, size = 503, normalized size = 2.03

$$\frac{-8\sqrt{5-4b}\sqrt{ax-b}-6\sqrt{5-4b}\sqrt{\sqrt{ax-b}+ax}+4\sqrt{5-4b}\sqrt{ax-b}\sqrt{\sqrt{ax-b}+ax}+4\sqrt{5-4b}\log\left(-\sqrt{ax-b}-ax+1\right)-8(2b-3)\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{\sqrt{5-4b}}\right)-\sqrt{5-4b}(4b-11)\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{2\sqrt{\sqrt{ax-b}+ax}}\right)-4\left(2b+\sqrt{5-4b}-3\right)\tanh^{-1}\left(\frac{-2\sqrt{5-4b}\sqrt{ax-b}-4b-\sqrt{5-4b}+1}{4\sqrt{\sqrt{ax-b}+ax}}\right)+8b\tanh^{-1}\left(\frac{2\sqrt{5-4b}\sqrt{ax-b}-4b+\sqrt{5-4b}+1}{4\sqrt{\sqrt{ax-b}+ax}}\right)-4\sqrt{5-4b}\tanh^{-1}\left(\frac{2\sqrt{5-4b}\sqrt{ax-b}-4b+\sqrt{5-4b}+1}{4\sqrt{\sqrt{ax-b}+ax}}\right)-12\tanh^{-1}\left(\frac{2\sqrt{5-4b}\sqrt{ax-b}-4b+\sqrt{5-4b}+1}{4\sqrt{\sqrt{ax-b}+ax}}\right)}{4a\sqrt{5-4b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-b + a*x]/(1 + Sqrt[a*x + Sqrt[-b + a*x]]),x]
```

```
[Out] (-8*Sqrt[5 - 4*b]*Sqrt[-b + a*x] - 6*Sqrt[5 - 4*b]*Sqrt[a*x + Sqrt[-b + a*x
]] + 4*Sqrt[5 - 4*b]*Sqrt[-b + a*x]*Sqrt[a*x + Sqrt[-b + a*x]] - 8*(-3 + 2*
b)*ArcTanh[(1 + 2*Sqrt[-b + a*x])/Sqrt[5 - 4*b]] - Sqrt[5 - 4*b]*(-11 + 4*b
)*ArcTanh[(1 + 2*Sqrt[-b + a*x])/(2*Sqrt[a*x + Sqrt[-b + a*x]])] - 4*(-3 +
Sqrt[5 - 4*b] + 2*b)*ArcTanh[(1 - Sqrt[5 - 4*b] - 4*b - 2*Sqrt[5 - 4*b]*Sqr
t[-b + a*x])/(4*Sqrt[a*x + Sqrt[-b + a*x]])] - 12*ArcTanh[(1 + Sqrt[5 - 4*b
] - 4*b + 2*Sqrt[5 - 4*b]*Sqrt[-b + a*x])/(4*Sqrt[a*x + Sqrt[-b + a*x]])] -
 4*Sqrt[5 - 4*b]*ArcTanh[(1 + Sqrt[5 - 4*b] - 4*b + 2*Sqrt[5 - 4*b]*Sqrt[-b
 + a*x])/(4*Sqrt[a*x + Sqrt[-b + a*x]])] + 8*b*ArcTanh[(1 + Sqrt[5 - 4*b] -
 4*b + 2*Sqrt[5 - 4*b]*Sqrt[-b + a*x])/(4*Sqrt[a*x + Sqrt[-b + a*x]])] + 4*
Sqrt[5 - 4*b]*Log[1 - a*x - Sqrt[-b + a*x]])/(4*a*Sqrt[5 - 4*b])
```

IntegrateAlgebraic [A] time = 0.58, size = 232, normalized size = 0.94

$$\frac{\sqrt{\sqrt{ax-b}+ax}\left(2\sqrt{ax-b}-3\right)}{2a}-\frac{2\sqrt{ax-b}}{a}+\frac{(4b-19)\log\left(a\left(-2\sqrt{ax-b}-1\right)+2a\sqrt{\sqrt{ax-b}+ax}\right)}{4a}+\frac{2\log\left(-2(ax-b)+\sqrt{\sqrt{ax-b}+ax}\left(2\sqrt{ax-b}-1\right)-2b+1\right)}{a}-\frac{4(2b-3)\tan^{-1}\left(\frac{-2\sqrt{ax-b}+2\sqrt{\sqrt{ax-b}+ax}+1}{\sqrt{4b-5}}\right)}{a\sqrt{4b-5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-b + a*x]/(1 + Sqrt[a*x + Sqrt[-b + a*x]]),x]
```

```
[Out] (-2*Sqrt[-b + a*x])/a + (Sqrt[a*x + Sqrt[-b + a*x]]*(-3 + 2*Sqrt[-b + a*x])
)/(2*a) - (4*(-3 + 2*b)*ArcTan[(1 - 2*Sqrt[-b + a*x] + 2*Sqrt[a*x + Sqrt[-b
 + a*x]])/Sqrt[-5 + 4*b]])/(a*Sqrt[-5 + 4*b]) + ((-19 + 4*b)*Log[a*(-1 - 2*
Sqrt[-b + a*x]) + 2*a*Sqrt[a*x + Sqrt[-b + a*x]]])/(4*a) + (2*Log[1 - 2*b -
 2*(-b + a*x) + Sqrt[a*x + Sqrt[-b + a*x]]*(-1 + 2*Sqrt[-b + a*x])])/a
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)^(1/2)/(1+(a*x+(a*x-b)^(1/2))^(1/2)),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (irrational residues)
```

giac [A] time = 1.07, size = 395, normalized size = 1.59

$$\frac{1}{2}\sqrt{ax+\sqrt{ax-b}}\left(\frac{2\sqrt{ax-b}}{a}-\frac{3}{a}\right)+\frac{(4b-11)\log\left(\left|-2\sqrt{ax-b}+2\sqrt{ax+\sqrt{ax-b}}-1\right|\right)}{4a}+\frac{2(2b-3)\arctan\left(-\frac{2\sqrt{ax-b}-2\sqrt{ax+\sqrt{ax-b}}+3}{\sqrt{4b-5}}\right)}{a\sqrt{4b-5}}-\frac{2(2b-3)\arctan\left(-\frac{2\sqrt{ax-b}-2\sqrt{ax+\sqrt{ax-b}}-1}{\sqrt{4b-5}}\right)}{a\sqrt{4b-5}}+\frac{2(2b-3)\arctan\left(\frac{2\sqrt{ax-b}+1}{\sqrt{4b-5}}\right)}{a\sqrt{4b-5}}+\frac{\log\left(ax+\sqrt{ax-b}-1\right)}{a}-\frac{\log\left(\left(\sqrt{ax-b}-\sqrt{ax+\sqrt{ax-b}}\right)^2+b+3\sqrt{ax-b}-3\sqrt{ax+\sqrt{ax-b}}+1\right)}{a}+\frac{\log\left(\left(\sqrt{ax-b}-\sqrt{ax+\sqrt{ax-b}}\right)^2+b-\sqrt{ax-b}+\sqrt{ax+\sqrt{ax-b}}-1\right)}{a}-\frac{2\sqrt{ax-b}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)^(1/2)/(1+(a*x+(a*x-b)^(1/2))^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a*x + sqrt(a*x - b))*(2*sqrt(a*x - b)/a - 3/a) + 1/4*(4*b - 11)*lo
g(abs(-2*sqrt(a*x - b) + 2*sqrt(a*x + sqrt(a*x - b)) - 1))/a + 2*(2*b - 3)*
arctan(-(2*sqrt(a*x - b) - 2*sqrt(a*x + sqrt(a*x - b)) + 3)/sqrt(4*b - 5))/
(a*sqrt(4*b - 5)) - 2*(2*b - 3)*arctan(-(2*sqrt(a*x - b) - 2*sqrt(a*x + sqr
t(a*x - b)) - 1)/sqrt(4*b - 5))/(a*sqrt(4*b - 5)) + 2*(2*b - 3)*arctan((2*s
qrt(a*x - b) + 1)/sqrt(4*b - 5))/(a*sqrt(4*b - 5)) + log(a*x + sqrt(a*x - b
) - 1)/a - log((sqrt(a*x - b) - sqrt(a*x + sqrt(a*x - b)))^2 + b + 3*sqrt(a
*x - b) - 3*sqrt(a*x + sqrt(a*x - b)) + 1)/a + log((sqrt(a*x - b) - sqrt(a*
x + sqrt(a*x - b)))^2 + b - sqrt(a*x - b) + sqrt(a*x + sqrt(a*x - b)) - 1)/
a - 2*sqrt(a*x - b)/a
```

maple [B] time = 0.05, size = 1634, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-b)^(1/2)/(1+(a*x+(a*x-b)^(1/2))^(1/2)),x)
```

```
[Out] 1/2*(a*x+(a*x-b)^(1/2))^(1/2)/a+1/a*arctanh(1/2*(2-(5-4*b)^(1/2)*((a*x-b)^(
1/2)+1/2+1/2*(5-4*b)^(1/2)))/(((a*x-b)^(1/2)+1/2+1/2*(5-4*b)^(1/2))^2-(5-4*
b)^(1/2)*((a*x-b)^(1/2)+1/2+1/2*(5-4*b)^(1/2))+1)^(1/2))-2/a*(a*x-b)^(1/2)-
1/a*(((a*x-b)^(1/2)+1/2+1/2*(5-4*b)^(1/2))^2-(5-4*b)^(1/2)*((a*x-b)^(1/2)+1
/2+1/2*(5-4*b)^(1/2))+1)^(1/2)+1/a*ln(a*x+(a*x-b)^(1/2)-1)+3/2/a*ln((a*x-b)
^(1/2)+1/2+(((a*x-b)^(1/2)+1/2-1/2*(5-4*b)^(1/2))^2+(5-4*b)^(1/2)*((a*x-b)^
(1/2)+1/2-1/2*(5-4*b)^(1/2))+1)^(1/2))-1/a*(((a*x-b)^(1/2)+1/2-1/2*(5-4*b)^
(1/2))^2+(5-4*b)^(1/2)*((a*x-b)^(1/2)+1/2-1/2*(5-4*b)^(1/2))+1)^(1/2)+1/a*a
rctanh(1/2*(2+(5-4*b)^(1/2)*((a*x-b)^(1/2)+1/2-1/2*(5-4*b)^(1/2)))/(((a*x-b
)^(1/2)+1/2-1/2*(5-4*b)^(1/2))^2+(5-4*b)^(1/2)*((a*x-b)^(1/2)+1/2-1/2*(5-4*
b)^(1/2))+1)^(1/2))+3/2/a*ln((a*x-b)^(1/2)+1/2+(((a*x-b)^(1/2)+1/2+1/2*(5-4
*b)^(1/2))^2-(5-4*b)^(1/2)*((a*x-b)^(1/2)+1/2+1/2*(5-4*b)^(1/2))+1)^(1/2))-
1/4/a*ln(1/2+(a*x-b)^(1/2)+(a*x+(a*x-b)^(1/2))^(1/2))+(a*x-b)^(1/2)*(a*x+(a
*x-b)^(1/2))^(1/2)/a+1/a*ln(1/2+(a*x-b)^(1/2)+(a*x+(a*x-b)^(1/2))^(1/2))*b+
1/2/a*(5-4*b)^(1/2)*ln((a*x-b)^(1/2)+1/2+(((a*x-b)^(1/2)+1/2+1/2*(5-4*b)^(1
/2))^2-(5-4*b)^(1/2)*((a*x-b)^(1/2)+1/2+1/2*(5-4*b)^(1/2))+1)^(1/2))+3/a/(5
-4*b)^(1/2)*(((a*x-b)^(1/2)+1/2-1/2*(5-4*b)^(1/2))^2+(5-4*b)^(1/2)*((a*x-b)
^(1/2)+1/2-1/2*(5-4*b)^(1/2))+1)^(1/2)-3/a/(5-4*b)^(1/2)*arctanh(1/2*(2+(5-
4*b)^(1/2)*((a*x-b)^(1/2)+1/2-1/2*(5-4*b)^(1/2)))/(((a*x-b)^(1/2)+1/2-1/2*(
5-4*b)^(1/2))^2+(5-4*b)^(1/2)*((a*x-b)^(1/2)+1/2-1/2*(5-4*b)^(1/2))+1)^(1/2
))-1/a*b*ln((a*x-b)^(1/2)+1/2+(((a*x-b)^(1/2)+1/2-1/2*(5-4*b)^(1/2))^2+(5-4
*b)^(1/2)*((a*x-b)^(1/2)+1/2-1/2*(5-4*b)^(1/2))+1)^(1/2))+3/a/(5-4*b)^(1/2)
*arctanh(1/2*(2-(5-4*b)^(1/2)*((a*x-b)^(1/2)+1/2+1/2*(5-4*b)^(1/2)))/(((a*x
-b)^(1/2)+1/2+1/2*(5-4*b)^(1/2))^2-(5-4*b)^(1/2)*((a*x-b)^(1/2)+1/2+1/2*(5-
4*b)^(1/2))+1)^(1/2))-1/a*b*ln((a*x-b)^(1/2)+1/2+(((a*x-b)^(1/2)+1/2+1/2*(5
-4*b)^(1/2))^2-(5-4*b)^(1/2)*((a*x-b)^(1/2)+1/2+1/2*(5-4*b)^(1/2))+1)^(1/2)
)-6/a/(-5+4*b)^(1/2)*arctan((1+2*(a*x-b)^(1/2))/(-5+4*b)^(1/2))-3/a/(5-4*b)
^(1/2)*(((a*x-b)^(1/2)+1/2+1/2*(5-4*b)^(1/2))^2-(5-4*b)^(1/2)*((a*x-b)^(1/2
)+1/2+1/2*(5-4*b)^(1/2))+1)^(1/2)-1/2/a*(5-4*b)^(1/2)*ln((a*x-b)^(1/2)+1/2+
(((a*x-b)^(1/2)+1/2-1/2*(5-4*b)^(1/2))^2+(5-4*b)^(1/2)*((a*x-b)^(1/2)+1/2-1
/2*(5-4*b)^(1/2))+1)^(1/2))+2/a/(5-4*b)^(1/2)*b*(((a*x-b)^(1/2)+1/2+1/2*(5-
4*b)^(1/2))^2-(5-4*b)^(1/2)*((a*x-b)^(1/2)+1/2+1/2*(5-4*b)^(1/2))+1)^(1/2)-
2/a/(5-4*b)^(1/2)*b*arctanh(1/2*(2-(5-4*b)^(1/2)*((a*x-b)^(1/2)+1/2+1/2*(5-
4*b)^(1/2)))/(((a*x-b)^(1/2)+1/2+1/2*(5-4*b)^(1/2))^2-(5-4*b)^(1/2)*((a*x-b
)^(1/2)+1/2+1/2*(5-4*b)^(1/2))+1)^(1/2))+2/a/(5-4*b)^(1/2)*b*arctanh(1/2*(2
+(5-4*b)^(1/2)*((a*x-b)^(1/2)+1/2-1/2*(5-4*b)^(1/2)))/(((a*x-b)^(1/2)+1/2-1
/2*(5-4*b)^(1/2))^2+(5-4*b)^(1/2)*((a*x-b)^(1/2)+1/2-1/2*(5-4*b)^(1/2))+1)^
(1/2))-2/a/(5-4*b)^(1/2)*b*(((a*x-b)^(1/2)+1/2-1/2*(5-4*b)^(1/2))^2+(5-4*b)
^(1/2)*((a*x-b)^(1/2)+1/2-1/2*(5-4*b)^(1/2))+1)^(1/2)+4/a/(-5+4*b)^(1/2)*ar
ctan((1+2*(a*x-b)^(1/2))/(-5+4*b)^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax-b}}{\sqrt{ax+\sqrt{ax-b}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)^(1/2)/(1+(a*x+(a*x-b)^(1/2))^(1/2)),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*x - b)/(sqrt(a*x + sqrt(a*x - b)) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x-b}}{\sqrt{a\,x+\sqrt{a\,x-b}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - b)^(1/2)/((a*x + (a*x - b)^(1/2))^(1/2) + 1),x)
[Out] int((a*x - b)^(1/2)/((a*x + (a*x - b)^(1/2))^(1/2) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax-b}}{\sqrt{ax+\sqrt{ax-b}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)**(1/2)/(1+(a*x+(a*x-b)**(1/2))**(1/2)),x)
[Out] Integral(sqrt(a*x - b)/(sqrt(a*x + sqrt(a*x - b)) + 1), x)
```

3.2168 $\int x\sqrt{1+\sqrt{2}+\sqrt{2}x+x^2}\,dx$

Optimal. Leaf size=249

$$\left(\frac{1}{2}+\frac{1}{4\sqrt{2}}\right)\log\left(-2\sqrt{x^2+\sqrt{2}x+\sqrt{2}+1}+2x+\sqrt{2}\right)+\frac{415040037844153019277323111 0852x^{10}+2494559}{}$$

Rubi [A] time = 0.11, antiderivative size = 99, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {640, 612, 619, 215}

$$\frac{1}{3}\left(x^2+\sqrt{2}x+\sqrt{2}+1\right)^{3/2}-\frac{1}{4}\left(\sqrt{2}x+1\right)\sqrt{x^2+\sqrt{2}x+\sqrt{2}+1}-\frac{1}{8}\left(4+\sqrt{2}\right)\sinh^{-1}\left(\sqrt{\frac{1}{7}\left(2\sqrt{2}-1\right)}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + Sqrt[2] + Sqrt[2]*x + x^2],x]
```

```
[Out] -1/4*((1 + Sqrt[2]*x)*Sqrt[1 + Sqrt[2] + Sqrt[2]*x + x^2]) + (1 + Sqrt[2] +
 Sqrt[2]*x + x^2)^(3/2)/3 - ((4 + Sqrt[2])*ArcSinh[Sqrt[(-1 + 2*Sqrt[2])/7]
*(1 + Sqrt[2]*x)])/8
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{1+\sqrt{2}+\sqrt{2}x+x^2}\,dx = \frac{1}{3}\left(1+\sqrt{2}+\sqrt{2}x+x^2\right)^{3/2} - \frac{\int \sqrt{1+\sqrt{2}+\sqrt{2}x+x^2}\,dx}{\sqrt{2}}$$
$$= -\frac{1}{4}\left(1+\sqrt{2}x\right)\sqrt{1+\sqrt{2}+\sqrt{2}x+x^2} + \frac{1}{3}\left(1+\sqrt{2}+\sqrt{2}x+x^2\right)^{3/2} - \frac{1}{8}\left(4+\sqrt{2}\right.$$
$$= -\frac{1}{4}\left(1+\sqrt{2}x\right)\sqrt{1+\sqrt{2}+\sqrt{2}x+x^2} + \frac{1}{3}\left(1+\sqrt{2}+\sqrt{2}x+x^2\right)^{3/2} - \frac{1}{8}\sqrt{1+2\cdots}$$
$$= -\frac{1}{4}\left(1+\sqrt{2}x\right)\sqrt{1+\sqrt{2}+\sqrt{2}x+x^2} + \frac{1}{3}\left(1+\sqrt{2}+\sqrt{2}x+x^2\right)^{3/2} - \frac{1}{8}\left(4+\sqrt{2}\right.$$

Mathematica [A] time = 0.10, size = 86, normalized size = 0.35

$$\frac{1}{24}\left(2\sqrt{x^2+\sqrt{2}x+\sqrt{2}+1}\left(4x^2+\sqrt{2}x+4\sqrt{2}+1\right)-3\left(4+\sqrt{2}\right)\sinh^{-1}\left(\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(2x+\sqrt{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + Sqrt[2] + Sqrt[2]*x + x^2],x]
```

```
[Out] (2*Sqrt[1 + Sqrt[2] + Sqrt[2]*x + x^2]*(1 + 4*Sqrt[2] + Sqrt[2]*x + 4*x^2)
- 3*(4 + Sqrt[2])*ArcSinh[Sqrt[(-1 + 2*Sqrt[2])/14]*(Sqrt[2] + 2*x)])/24
```

IntegrateAlgebraic [A] time = 0.36, size = 91, normalized size = 0.37

$$\frac{1}{12}\sqrt{x^2+\sqrt{2}x+\sqrt{2}+1}\left(4x^2+\sqrt{2}x+4\sqrt{2}+1\right)+\frac{1}{8}\left(4+\sqrt{2}\right)\log\left(-2\sqrt{x^2+\sqrt{2}x+\sqrt{2}+1}+2x+\sqrt{2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[1 + Sqrt[2] + Sqrt[2]*x + x^2],x]
```

```
[Out] (Sqrt[1 + Sqrt[2] + Sqrt[2]*x + x^2]*(1 + 4*Sqrt[2] + Sqrt[2]*x + 4*x^2))/1
2 + ((4 + Sqrt[2])*Log[Sqrt[2] + 2*x - 2*Sqrt[1 + Sqrt[2] + Sqrt[2]*x + x^2
]])/8
```

fricas [A] time = 0.52, size = 64, normalized size = 0.26

$$\frac{1}{8}\left(\sqrt{2}+4\right)\log\left(-2x-\sqrt{2}+2\sqrt{x^2+\sqrt{2}(x+1)+1}\right)+\frac{1}{12}\left(4x^2+\sqrt{2}(x+4)+1\right)\sqrt{x^2+\sqrt{2}(x+1)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2^(1/2)+2^(1/2)*x+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2) + 4)*log(-2*x - sqrt(2) + 2*sqrt(x^2 + sqrt(2)*(x + 1) + 1)) +
 1/12*(4*x^2 + sqrt(2)*(x + 4) + 1)*sqrt(x^2 + sqrt(2)*(x + 1) + 1)
```

giac [A] time = 0.16, size = 77, normalized size = 0.31

$$\frac{1}{8}\sqrt{2}\left(2\sqrt{2}+1\right)\log\left(-\sqrt{2}\left(x-\sqrt{x^2+\sqrt{2}x+\sqrt{2}+1}\right)-1\right)+\frac{1}{24}\left(2\left(4x+\sqrt{2}\right)x+\sqrt{2}\left(\sqrt{2}+8\right)\right)\sqrt{x^2+\sqrt{2}x+\sqrt{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2^(1/2)+2^(1/2)*x+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(2*sqrt(2) + 1)*log(-sqrt(2)*(x - sqrt(x^2 + sqrt(2)*x + sqrt(2
) + 1)) - 1) + 1/24*(2*(4*x + sqrt(2))*x + sqrt(2)*(sqrt(2) + 8))*sqrt(x^2
+ sqrt(2)*x + sqrt(2) + 1)
```

maple [A] time = 0.05, size = 74, normalized size = 0.30

$$\frac{\left(1+\sqrt{2}+\sqrt{2}\,x+x^{2}\right)^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(\frac{\left(\sqrt{2}+2x\right)\sqrt{1+\sqrt{2}+\sqrt{2}\,x+x^{2}}}{4}+\frac{\left(2+4\sqrt{2}\right)\operatorname{arcsinh}\left(\frac{x+\frac{\sqrt{2}}{2}}{\sqrt{\frac{1}{2}+\sqrt{2}}}\right)}{8}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+2^(1/2)+2^(1/2)*x+x^2)^(1/2),x)
```

```
[Out] 1/3*(1+2^(1/2)+2^(1/2)*x+x^2)^(3/2)-1/2*2^(1/2)*(1/4*(2^(1/2)+2*x)*(1+2^(1/
2)+2^(1/2)*x+x^2)^(1/2)+1/8*(2+4*2^(1/2))*arcsinh(1/(1/2+2^(1/2))^(1/2)*(x+
1/2*2^(1/2))))
```

maxima [A] time = 0.52, size = 107, normalized size = 0.43

$$-\frac{1}{4}\sqrt{2}\left(\sqrt{2}+1\right)\operatorname{arsinh}\left(\frac{2x+\sqrt{2}}{\sqrt{4\sqrt{2}+2}}\right)-\frac{1}{4}\sqrt{2}\sqrt{x^{2}+\sqrt{2}x+\sqrt{2}+1}\,x+\frac{1}{3}\left(x^{2}+\sqrt{2}x+\sqrt{2}+1\right)^{\frac{3}{2}}+\frac{1}{8}\sqrt{2}\operatorname{arsinh}\left(\frac{2x+\sqrt{2}}{\sqrt{4\sqrt{2}+2}}\right)-\frac{1}{4}\sqrt{x^{2}+\sqrt{2}x+\sqrt{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2^(1/2)+2^(1/2)*x+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2) + 1)*arcsinh((2*x + sqrt(2))/sqrt(4*sqrt(2) + 2)) - 1
/4*sqrt(2)*sqrt(x^2 + sqrt(2)*x + sqrt(2) + 1)*x + 1/3*(x^2 + sqrt(2)*x + s
qrt(2) + 1)^(3/2) + 1/8*sqrt(2)*arcsinh((2*x + sqrt(2))/sqrt(4*sqrt(2) + 2)
) - 1/4*sqrt(x^2 + sqrt(2)*x + sqrt(2) + 1)
```

mupad [B] time = 1.69, size = 76, normalized size = 0.31

$$\frac{\left(8\,x^{2}+2\sqrt{2}\,x+8\sqrt{2}+2\right)\sqrt{x^{2}+\sqrt{2}\,x+\sqrt{2}+1}}{24}+\ln\left(x+\sqrt{x^{2}+\sqrt{2}\,x+\sqrt{2}+1}+\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{8}-\frac{\sqrt{2}\,\left(\sqrt{2}+1\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2^(1/2)*x + 2^(1/2) + x^2 + 1)^(1/2),x)
```

```
[Out] ((2*2^(1/2)*x + 8*2^(1/2) + 8*x^2 + 2)*(2^(1/2)*x + 2^(1/2) + x^2 + 1)^(1/2
))/24 + log(x + (2^(1/2)*x + 2^(1/2) + x^2 + 1)^(1/2) + 2^(1/2)/2)*(2^(1/2)
/8 - (2^(1/2)*(2^(1/2) + 1))/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^{2}+\sqrt{2}x+1+\sqrt{2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2**(1/2)+2**(1/2)*x+x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2 + sqrt(2)*x + 1 + sqrt(2)), x)
```

3.2169 $\int \frac{-ax+x^2}{\left(x^2(-a+x)\right)^{2/3}\left(a^2-2ax+(1-d)x^2\right)}\,dx$

Optimal. Leaf size=249

$$\frac{\log\left(\sqrt[3]{x^3-ax^2}-\sqrt[6]{d}\,x\right)}{2a\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{x^3-ax^2}+\sqrt[6]{d}\,x\right)}{2a\sqrt[3]{d}}-\frac{\log\left(-\sqrt[6]{d}\,x\sqrt[3]{x^3-ax^2}+\left(x^3-ax^2\right)^{2/3}+\sqrt[3]{d}\,x^2\right)}{4a\sqrt[3]{d}}-\frac{\log\left(\sqrt[6]{d}\,x\right.}{}$$

Rubi [A] time = 0.90, antiderivative size = 408, normalized size of antiderivative = 1.64, number of steps used = 10, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {1593, 6719, 911, 105, 59, 91}

$$-\frac{x^{4/3}(x-a)^{2/3}\log\left(2a\left(1-\sqrt{d}\right)-2(1-d)x\right)}{4a\sqrt[3]{d}\left(-\left(x^2(a-x)\right)\right)^{2/3}}-\frac{x^{4/3}(x-a)^{2/3}\log\left(2(1-d)x-2a\left(\sqrt{d}+1\right)\right)}{4a\sqrt[3]{d}\left(-\left(x^2(a-x)\right)\right)^{2/3}}+\frac{3x^{4/3}(x-a)^{2/3}\log\left(-\sqrt[3]{x-a}-\sqrt[6]{d}\sqrt[3]{x}\right)}{4a\sqrt[3]{d}\left(-\left(x^2(a-x)\right)\right)^{2/3}}+\frac{3x^{4/3}(x-a)^{2/3}\log\left(\sqrt[6]{d}\sqrt[3]{x}-\sqrt[3]{x-a}\right)}{4a\sqrt[3]{d}\left(-\left(x^2(a-x)\right)\right)^{2/3}}+\frac{\sqrt{3}x^{4/3}(x-a)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{x-a}}\right)}{2a\sqrt[3]{d}\left(-\left(x^2(a-x)\right)\right)^{2/3}}+\frac{\sqrt{3}x^{4/3}(x-a)^{2/3}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{x-a}}+\frac{1}{\sqrt{3}}\right)}{2a\sqrt[3]{d}\left(-\left(x^2(a-x)\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-(a*x) + x^2)/((x^2*(-a + x))^(2/3)*(a^2 - 2*a*x + (1 - d)*x^2)),x]
```

```
[Out] (Sqrt[3]*x^(4/3)*(-a + x)^(2/3)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*x^(1/3))/(Sqr
t[3]*(-a + x)^(1/3))])/(2*a*d^(1/3)*(-((a - x)*x^2))^(2/3)) + (Sqrt[3]*x^(4
/3)*(-a + x)^(2/3)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*x^(1/3))/(Sqrt[3]*(-a + x)
^(1/3))])/(2*a*d^(1/3)*(-((a - x)*x^2))^(2/3)) - (x^(4/3)*(-a + x)^(2/3)*Lo
g[2*a*(1 - Sqrt[d]) - 2*(1 - d)*x])/(4*a*d^(1/3)*(-((a - x)*x^2))^(2/3)) -
(x^(4/3)*(-a + x)^(2/3)*Log[-2*a*(1 + Sqrt[d]) + 2*(1 - d)*x])/(4*a*d^(1/3)
*(-((a - x)*x^2))^(2/3)) + (3*x^(4/3)*(-a + x)^(2/3)*Log[-(d^(1/6)*x^(1/3))
 - (-a + x)^(1/3)])/(4*a*d^(1/3)*(-((a - x)*x^2))^(2/3)) + (3*x^(4/3)*(-a +
 x)^(2/3)*Log[d^(1/6)*x^(1/3) - (-a + x)^(1/3)])/(4*a*d^(1/3)*(-((a - x)*x^
2))^(2/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
```

```
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{-ax+x^2}{\left(x^2(-a+x)\right)^{2/3}\left(a^2-2ax+(1-d)x^2\right)}\,dx = \int \frac{x(-a+x)}{\left(x^2(-a+x)\right)^{2/3}\left(a^2-2ax+(1-d)x^2\right)}\,dx$$

$$= \frac{\left(x^{4/3}(-a+x)^{2/3}\right)\int \frac{\sqrt[3]{-a+x}}{\sqrt[3]{x}\left(a^2-2ax+(1-d)x^2\right)}\,dx}{\left(x^2(-a+x)\right)^{2/3}}$$

$$= \frac{\left(x^{4/3}(-a+x)^{2/3}\right)\int \left(\frac{(1-d)\sqrt[3]{-a+x}}{a\sqrt{d}\sqrt[3]{x}\left(2a-2a\sqrt{d}-2(1-d)x\right)}+\frac{(1-d)\sqrt[3]{-a+x}}{a\sqrt{d}\sqrt[3]{x}\left(-2a-2a\sqrt{d}+\cdots\right.}\right.}{\left(x^2(-a+x)\right)^{2/3}}$$

$$= \frac{\left((1-d)x^{4/3}(-a+x)^{2/3}\right)\int \frac{\sqrt[3]{-a+x}}{\sqrt[3]{x}\left(2a-2a\sqrt{d}-2(1-d)x\right)}\,dx}{a\sqrt{d}\left(x^2(-a+x)\right)^{2/3}}+\frac{\left((1-d)x^{4/3}\cdots\right.}{\cdots}$$

$$= \frac{\left((1-d)x^{4/3}(-a+x)^{2/3}\right)\int \frac{1}{\sqrt[3]{x}(-a+x)^{2/3}\left(-2a-2a\sqrt{d}+2(1-d)x\right)}\,dx}{\left(1-\sqrt{d}\right)\left(x^2(-a+x)\right)^{2/3}}-\frac{\left((1\cdots\right.}{\cdots}$$

$$= \frac{\sqrt{3}\,x^{4/3}(-a+x)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{-a+x}}\right)}{2a\sqrt[3]{d}\left(-\left((a-x)x^2\right)\right)^{2/3}}+\frac{\sqrt{3}\,x^{4/3}(-a+x)^{2/3}\cdots}{2a\sqrt[3]{d}\left(-\cdots\right.}$$

Mathematica [C] time = 0.17, size = 68, normalized size = 0.27

$$-\frac{3x^2\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{\sqrt{d}\,x}{a-x}\right)+{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{\sqrt{d}\,x}{x-a}\right)\right)}{4a\left(x^2(x-a)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-(a*x) + x^2)/((x^2*(-a + x))^(2/3)*(a^2 - 2*a*x + (1 - d)*x^2))
,x]
```

```
[Out] (-3*x^2*(Hypergeometric2F1[2/3, 1, 5/3, (Sqrt[d]*x)/(a - x)] + Hypergeometr
ic2F1[2/3, 1, 5/3, (Sqrt[d]*x)/(-a + x)]))/(4*a*(x^2*(-a + x))^(2/3))
```

IntegrateAlgebraic [A] time = 0.63, size = 247, normalized size = 0.99

$$\frac{\log\left(\sqrt[3]{x^3-ax^2}-\sqrt[6]{d}\,x\right)}{2a\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{x^3-ax^2}+\sqrt[6]{d}\,x\right)}{2a\sqrt[3]{d}}-\frac{\log\left(-\sqrt[6]{d}\,x\sqrt[3]{x^3-ax^2}+\left(x^3-ax^2\right)^{2/3}+\sqrt[3]{d}\,x^2\right)}{4a\sqrt[3]{d}}-\frac{\log\left(\sqrt[6]{d}\,x\sqrt[3]{x^3-ax^2}+\left(x^3-ax^2\right)^{2/3}+\sqrt[3]{d}\,x^2\right)}{4a\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\left(x^3-ax^2\right)^{2/3}}{\sqrt{3}\sqrt[3]{d}}+\frac{x^2}{\sqrt{3}}}{x^2}\right)}{2a\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*x) + x^2)/((x^2*(-a + x))^(2/3)*(a^2 - 2*a*x + (1 -
 d)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(x^2/Sqrt[3] + (2*(-(a*x^2) + x^3)^(2/3))/(Sqrt[3]*d^(1/3))
)/x^2]])/(2*a*d^(1/3)) + Log[-(d^(1/6)*x) + (-(a*x^2) + x^3)^(1/3)]/(2*a*d^(
1/3)) + Log[d^(1/6)*x + (-(a*x^2) + x^3)^(1/3)]/(2*a*d^(1/3)) - Log[d^(1/3)
*x^2 - d^(1/6)*x*(-(a*x^2) + x^3)^(1/3) + (-(a*x^2) + x^3)^(2/3)]/(4*a*d^(1
/3)) - Log[d^(1/3)*x^2 + d^(1/6)*x*(-(a*x^2) + x^3)^(1/3) + (-(a*x^2) + x^3
)^(2/3)]/(4*a*d^(1/3))
```

fricas [A] time = 0.57, size = 357, normalized size = 1.43

$$\left[\frac{\sqrt{3}\,d\sqrt{-\frac{1}{d^{\frac{2}{3}}}}\log\left(-\frac{(d+2)x^2+2a^2-4ax-\sqrt{3}\left(d^{\frac{4}{3}}x^2+2\left(-ax^2+x^3\right)^{\frac{1}{3}}(a-x)d^{\frac{2}{3}}+\left(-ax^2+x^3\right)^{\frac{2}{3}}d\right)\sqrt{-\frac{1}{d^{\frac{2}{3}}}}-3\left(-ax^2+x^3\right)^{\frac{2}{3}}d^{\frac{2}{3}}}{(d-1)x^2-a^2+2ax}\right)-d^{\frac{2}{3}}\log\left(\frac{d^{\frac{2}{3}}x^2-\left(-ax^2+x^3\right)^{\frac{1}{3}}(a-x)+\left(-ax^2+x^3\right)^{\frac{2}{3}}d^{\frac{1}{3}}}{x^2}\right)+2d^{\frac{2}{3}}\log\left(-\frac{d^{\frac{1}{3}}x^2-\left(-ax^2+x^3\right)^{\frac{2}{3}}}{x^2}\right)}{4ad},\ \frac{2\sqrt{3}\,d^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(d^{\frac{1}{3}}x^2+2\left(-ax^2+x^3\right)^{\frac{2}{3}}\right)}{3d^{\frac{1}{3}}x^2}\right)-d^{\frac{2}{3}}\log\left(\frac{d^{\frac{2}{3}}x^2-\left(-ax^2+x^3\right)^{\frac{1}{3}}(a-x)+\left(-ax^2+x^3\right)^{\frac{2}{3}}d^{\frac{1}{3}}}{x^2}\right)+2d^{\frac{2}{3}}\log\left(-\frac{d^{\frac{1}{3}}x^2-\left(-ax^2+x^3\right)^{\frac{2}{3}}}{x^2}\right)}{4ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+x^2)/(x^2*(-a+x))^(2/3)/(a^2-2*a*x+(1-d)*x^2),x, algorithm=
"fricas")
```

```
[Out] [1/4*(sqrt(3)*d*sqrt(-1/d^(2/3))*log(-((d + 2)*x^2 + 2*a^2 - 4*a*x - sqrt(3
)*(d^(4/3)*x^2 + 2*(-a*x^2 + x^3)^(1/3)*(a - x)*d^(2/3) + (-a*x^2 + x^3)^(2
/3)*d)*sqrt(-1/d^(2/3)) - 3*(-a*x^2 + x^3)^(2/3)*d^(2/3))/((d - 1)*x^2 - a^
2 + 2*a*x)) - d^(2/3)*log((d^(2/3)*x^2 - (-a*x^2 + x^3)^(1/3)*(a - x) + (-a
*x^2 + x^3)^(2/3)*d^(1/3))/x^2) + 2*d^(2/3)*log(-(d^(1/3)*x^2 - (-a*x^2 + x
^3)^(2/3))/x^2))/(a*d), 1/4*(2*sqrt(3)*d^(2/3)*arctan(1/3*sqrt(3)*(d^(1/3)*
x^2 + 2*(-a*x^2 + x^3)^(2/3))/(d^(1/3)*x^2)) - d^(2/3)*log((d^(2/3)*x^2 - (
-a*x^2 + x^3)^(1/3)*(a - x) + (-a*x^2 + x^3)^(2/3)*d^(1/3))/x^2) + 2*d^(2/3
)*log(-(d^(1/3)*x^2 - (-a*x^2 + x^3)^(2/3))/x^2))/(a*d)]
```

giac [A] time = 0.25, size = 100, normalized size = 0.40

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)}{2\,ad^{\frac{1}{3}}}-\frac{\log\left(\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}+d^{\frac{1}{3}}\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}+d^{\frac{2}{3}}\right)}{4\,ad^{\frac{1}{3}}}+\frac{\log\left(\left|\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}-d^{\frac{1}{3}}\right|\right)}{2\,ad^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+x^2)/(x^2*(-a+x))^(2/3)/(a^2-2*a*x+(1-d)*x^2),x, algorithm=
"giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-a/x + 1)^(2/3) + d^(1/3))/d^(1/3))/(a*d
^(1/3)) - 1/4*log((-a/x + 1)^(4/3) + d^(1/3)*(-a/x + 1)^(2/3) + d^(2/3))/(a
*d^(1/3)) + 1/2*log(abs((-a/x + 1)^(2/3) - d^(1/3)))/(a*d^(1/3))
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{-ax+x^2}{\left(x^2\left(-a+x\right)\right)^{\frac{2}{3}}\left(a^2-2ax+(1-d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x+x^2)/(x^2*(-a+x))^(2/3)/(a^2-2*a*x+(1-d)*x^2),x)
```

```
[Out] int((-a*x+x^2)/(x^2*(-a+x))^(2/3)/(a^2-2*a*x+(1-d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax - x^2}{\left(-(a-x)x^2\right)^{\frac{2}{3}}\left((d-1)x^2 - a^2 + 2\,ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+x^2)/(x^2*(-a+x))^(2/3)/(a^2-2*a*x+(1-d)*x^2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x - x^2)/((-(a - x)*x^2)^(2/3)*((d - 1)*x^2 - a^2 + 2*a*x)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x - x^2}{\left(-x^2\,\left(a-x\right)\right)^{2/3}\,\left(-a^2 + 2\,a\,x + \left(d-1\right)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - x^2)/((-x^2*(a - x))^(2/3)*(2*a*x - a^2 + x^2*(d - 1))),x)
```

```
[Out] int((a*x - x^2)/((-x^2*(a - x))^(2/3)*(2*a*x - a^2 + x^2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+x**2)/(x**2*(-a+x))**(2/3)/(a**2-2*a*x+(1-d)*x**2),x)
```

```
[Out] Timed out
```

3.2170 $\int \frac{x}{\sqrt[3]{x^2(-a+x)}\left(-a^2+2ax+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=249

$$-\frac{\log\left(\sqrt[3]{x^3-ax^2}-\sqrt[6]{d}\,x\right)}{2ad^{2/3}}-\frac{\log\left(\sqrt[3]{x^3-ax^2}+\sqrt[6]{d}\,x\right)}{2ad^{2/3}}+\frac{\log\left(-\sqrt[6]{d}\,x\sqrt[3]{x^3-ax^2}+\left(x^3-ax^2\right)^{2/3}+\sqrt[3]{d}\,x^2\right)}{4ad^{2/3}}+\frac{\log\left(\sqrt[6]{d}\right.}{}$$

Rubi [A] time = 0.62, antiderivative size = 408, normalized size of antiderivative = 1.64, number of steps used = 9, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {6719, 911, 105, 59, 91}

$$\frac{x^{2/3}\sqrt[3]{x-a}\log\left(2a\left(1-\sqrt{d}\right)-2(1-d)x\right)}{4ad^{2/3}\sqrt[3]{-\left(x^2(a-x)\right)}}+\frac{x^{2/3}\sqrt[3]{x-a}\log\left(2(1-d)x-2a\left(\sqrt{d}+1\right)\right)}{4ad^{2/3}\sqrt[3]{-\left(x^2(a-x)\right)}}-\frac{3x^{2/3}\sqrt[3]{x-a}\log\left(-\frac{\sqrt[3]{x-a}}{\sqrt[6]{d}}-\sqrt[3]{x}\right)}{4ad^{2/3}\sqrt[3]{-\left(x^2(a-x)\right)}}-\frac{3x^{2/3}\sqrt[3]{x-a}\log\left(\frac{\sqrt[3]{x-a}}{\sqrt[6]{d}}-\sqrt[3]{x}\right)}{4ad^{2/3}\sqrt[3]{-\left(x^2(a-x)\right)}}-\frac{\sqrt{3}x^{2/3}\sqrt[3]{x-a}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x}}\right)}{2ad^{2/3}\sqrt[3]{-\left(x^2(a-x)\right)}}-\frac{\sqrt{3}x^{2/3}\sqrt[3]{x-a}\tan^{-1}\left(\frac{2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{2ad^{2/3}\sqrt[3]{-\left(x^2(a-x)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((x^2*(-a + x))^(1/3)*(-a^2 + 2*a*x + (-1 + d)*x^2)),x]
```

```
[Out] -1/2*(Sqrt[3]*x^(2/3)*(-a + x)^(1/3)*ArcTan[1/Sqrt[3] - (2*(-a + x)^(1/3))/
(Sqrt[3]*d^(1/6)*x^(1/3))])/(a*d^(2/3)*(-((a - x)*x^2))^(1/3)) - (Sqrt[3]*x
^(2/3)*(-a + x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(-a + x)^(1/3))/(Sqrt[3]*d^(1/6
)*x^(1/3))])/(2*a*d^(2/3)*(-((a - x)*x^2))^(1/3)) + (x^(2/3)*(-a + x)^(1/3)
*Log[2*a*(1 - Sqrt[d]) - 2*(1 - d)*x])/(4*a*d^(2/3)*(-((a - x)*x^2))^(1/3))
 + (x^(2/3)*(-a + x)^(1/3)*Log[-2*a*(1 + Sqrt[d]) + 2*(1 - d)*x])/(4*a*d^(2
/3)*(-((a - x)*x^2))^(1/3)) - (3*x^(2/3)*(-a + x)^(1/3)*Log[-x^(1/3) - (-a
+ x)^(1/3)/d^(1/6)])/(4*a*d^(2/3)*(-((a - x)*x^2))^(1/3)) - (3*x^(2/3)*(-a
+ x)^(1/3)*Log[-x^(1/3) + (-a + x)^(1/3)/d^(1/6)])/(4*a*d^(2/3)*(-((a - x)*
x^2))^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
```

```
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{x^2(-a+x)}\left(-a^2+2ax+(-1+d)x^2\right)}dx = \frac{\left(x^{2/3}\sqrt[3]{-a+x}\right)\int \frac{\sqrt[3]{x}}{\sqrt[3]{-a+x}\left(-a^2+2ax+(-1+d)x^2\right)}dx}{\sqrt[3]{x^2(-a+x)}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{-a+x}\right)\int\left(\frac{(-1+d)\sqrt[3]{x}}{a\sqrt{d}\sqrt[3]{-a+x}\left(2a-2a\sqrt{d}-2(1-d)x\right)} + \frac{(-1+d}{a\sqrt{d}\sqrt[3]{-a+x}\left(-2a\right.}\right.}{\sqrt[3]{x^2(-a+x)}}$$

$$= -\frac{\left((1-d)x^{2/3}\sqrt[3]{-a+x}\right)\int \frac{\sqrt[3]{x}}{\sqrt[3]{-a+x}\left(2a-2a\sqrt{d}-2(1-d)x\right)}dx}{a\sqrt{d}\sqrt[3]{x^2(-a+x)}} - \frac{\left((1-d\right.}{}$$

$$= -\frac{\left((1-d)x^{2/3}\sqrt[3]{-a+x}\right)\int \frac{1}{x^{2/3}\sqrt[3]{-a+x}\left(-2a-2a\sqrt{d}+2(1-d)x\right)}dx}{\left(1-\sqrt{d}\right)\sqrt{d}\sqrt[3]{x^2(-a+x)}} - \frac{\left((1\right.}{}$$

$$= -\frac{\sqrt{3}x^{2/3}\sqrt[3]{-a+x}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{-a+x}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x}}\right)}{2ad^{2/3}\sqrt[3]{-\left((a-x)x^2\right)}} - \frac{\sqrt{3}x^{2/3}\sqrt[3]{-a+x}}{2ad^{2/3}\sqrt[3]{}}$$

Mathematica [C] time = 0.11, size = 73, normalized size = 0.29

$$\frac{3x\left({}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{\sqrt{d}x}{x-a}\right)-{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{\sqrt{d}x}{a-x}\right)\right)}{2a\sqrt{d}\sqrt[3]{x^2(x-a)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((x^2*(-a + x))^(1/3)*(-a^2 + 2*a*x + (-1 + d)*x^2)),x]
```

```
[Out] (3*x*(-Hypergeometric2F1[1/3, 1, 4/3, (Sqrt[d]*x)/(a - x)] + Hypergeometric
2F1[1/3, 1, 4/3, (Sqrt[d]*x)/(-a + x)]))/(2*a*Sqrt[d]*(x^2*(-a + x))^(1/3))
```

IntegrateAlgebraic [A] time = 0.53, size = 247, normalized size = 0.99

$$-\frac{\log\left(\sqrt[3]{x^3-ax^2}-\sqrt[6]{d}x\right)}{2ad^{2/3}}-\frac{\log\left(\sqrt[3]{x^3-ax^2}+\sqrt[6]{d}x\right)}{2ad^{2/3}}+\frac{\log\left(-\sqrt[6]{d}x\sqrt[3]{x^3-ax^2}+\left(x^3-ax^2\right)^{2/3}+\sqrt[3]{d}x^2\right)}{4ad^{2/3}}+\frac{\log\left(\sqrt[6]{d}x\sqrt[3]{x^3-ax^2}+\left(x^3-ax^2\right)^{2/3}+\sqrt[3]{d}x^2\right)}{4ad^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\left(x^3-ax^2\right)^{2/3}}{\sqrt{3}\sqrt[3]{d}}+\frac{x^2}{\sqrt{3}}}{x^2}\right)}{2ad^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((x^2*(-a + x))^(1/3)*(-a^2 + 2*a*x + (-1 + d)*x^2)),x
]
```

```
[Out] (Sqrt[3]*ArcTan[(x^2/Sqrt[3] + (2*(-(a*x^2) + x^3)^(2/3))/(Sqrt[3]*d^(1/3))
)/x^2])/(2*a*d^(2/3)) - Log[-(d^(1/6)*x) + (-(a*x^2) + x^3)^(1/3)]/(2*a*d^(
```

```
2/3)) - Log[d^(1/6)*x + (-(a*x^2) + x^3)^(1/3)]/(2*a*d^(2/3)) + Log[d^(1/3)
*x^2 - d^(1/6)*x*(-(a*x^2) + x^3)^(1/3) + (-(a*x^2) + x^3)^(2/3)]/(4*a*d^(2
/3)) + Log[d^(1/3)*x^2 + d^(1/6)*x*(-(a*x^2) + x^3)^(1/3) + (-(a*x^2) + x^3
)^(2/3)]/(4*a*d^(2/3))
```

fricas [A] time = 1.14, size = 194, normalized size = 0.78

$$\frac{2\,\sqrt{3}\,d\sqrt{-\left(-d^2\right)^{\frac{1}{3}}}\,\arctan\left(-\frac{\sqrt{3}\left(\left(-d^2\right)^{\frac{1}{3}}dx^2-2\left(-ax^2+x^3\right)^{\frac{2}{3}}\left(-d^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-d^2\right)^{\frac{1}{3}}}}{3\,d^2x^2}\right)+\left(-d^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(-d^2\right)^{\frac{1}{3}}dx^2+\left(-ax^2+x^3\right)^{\frac{1}{3}}(ad-dx)-\left(-ax^2+x^3\right)^{\frac{2}{3}}\left(-d^2\right)^{\frac{2}{3}}}{x^2}\right)-2\,\left(-d^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(-d^2\right)^{\frac{2}{3}}x^2-\left(-ax^2+x^3\right)^{\frac{2}{3}}d}{x^2}\right)}{4\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2*(-a+x))^(1/3)/(-a^2+2*a*x+(-1+d)*x^2),x, algorithm="fricas
")
```

```
[Out] 1/4*(2*sqrt(3)*d*sqrt(-(-d^2)^(1/3))*arctan(-1/3*sqrt(3)*((-d^2)^(1/3)*d*x^
2 - 2*(-a*x^2 + x^3)^(2/3)*(-d^2)^(2/3))*sqrt(-(-d^2)^(1/3))/(d^2*x^2)) + (
-d^2)^(2/3)*log(-((-d^2)^(1/3)*d*x^2 + (-a*x^2 + x^3)^(1/3)*(a*d - d*x) - (
-a*x^2 + x^3)^(2/3)*(-d^2)^(2/3))/x^2) - 2*(-d^2)^(2/3)*log(-((-d^2)^(2/3)*
x^2 - (-a*x^2 + x^3)^(2/3)*d)/x^2))/(a*d^2)
```

giac [A] time = 0.25, size = 100, normalized size = 0.40

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)}{2\,ad^{\frac{2}{3}}}+\frac{\log\left(\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}+d^{\frac{1}{3}}\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}+d^{\frac{2}{3}}\right)}{4\,ad^{\frac{2}{3}}}-\frac{\log\left(\left|\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}-d^{\frac{1}{3}}\right|\right)}{2\,ad^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2*(-a+x))^(1/3)/(-a^2+2*a*x+(-1+d)*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-a/x + 1)^(2/3) + d^(1/3))/d^(1/3))/(a*d
^(2/3)) + 1/4*log((-a/x + 1)^(4/3) + d^(1/3)*(-a/x + 1)^(2/3) + d^(2/3))/(a
*d^(2/3)) - 1/2*log(abs((-a/x + 1)^(2/3) - d^(1/3)))/(a*d^(2/3))
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^2\left(-a+x\right)\right)^{\frac{1}{3}}\left(-a^2+2ax+\left(-1+d\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2*(-a+x))^(1/3)/(-a^2+2*a*x+(-1+d)*x^2),x)
```

```
[Out] int(x/(x^2*(-a+x))^(1/3)/(-a^2+2*a*x+(-1+d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-(a-x)x^2\right)^{\frac{1}{3}}\left((d-1)x^2-a^2+2\,ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2*(-a+x))^(1/3)/(-a^2+2*a*x+(-1+d)*x^2),x, algorithm="maxima
")
```

```
[Out] integrate(x/((-(a - x)*x^2)^(1/3)*((d - 1)*x^2 - a^2 + 2*a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(-x^2\,\left(a-x\right)\right)^{1/3}\,\left(-a^2+2\,a\,x+\left(d-1\right)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((-x^2*(a - x))^(1/3)*(2*a*x - a^2 + x^2*(d - 1))),x)
```

```
[Out] int(x/((-x^2*(a - x))^(1/3)*(2*a*x - a^2 + x^2*(d - 1))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{x^2\,(-a+x)}\,\left(-a^2+2ax+dx^2-x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2*(-a+x))**(1/3)/(-a**2+2*a*x+(-1+d)*x**2),x)
```

```
[Out] Integral(x/((x**2*(-a + x))**(1/3)*(-a**2 + 2*a*x + d*x**2 - x**2)), x)
```

3.2171

$$\int \frac{-abx^2+x^4}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(a^2b^2-2ab(a+b)x+\left(a^2+4ab+b^2-d\right)x^2-2(a+b)x^3+x^4\right)}\,dx$$

Optimal. Leaf size=249

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\,x}{\sqrt[6]{d}\,x-2\sqrt[3]{x^3(-a-b)+abx^2+x^4}}\right)}{2d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\,x}{2\sqrt[3]{x^3(-a-b)+abx^2+x^4}+\sqrt[6]{d}\,x}\right)}{2d^{5/6}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{d}\,x}{\sqrt[3]{x^3(-a-b)+abx^2+x^4}}\right)}{d^{5/6}}-\frac{\tanh^{-1}}{}$$

Rubi [F] time = 19.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-abx^2+x^4}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(a^2b^2-2ab(a+b)x+\left(a^2+4ab+b^2-d\right)x^2-2(a+b)x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a*b*x^2) + x^4)/((x^2*(-a + x)*(-b + x))^(2/3)*(a^2*b^2 - 2*a*b*(a +
 b)*x + (a^2 + 4*a*b + b^2 - d)*x^2 - 2*(a + b)*x^3 + x^4)),x]
```

```
[Out] (3*a*b*x^(4/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][x^4/((
-a + x^3)^(2/3)*(-b + x^3)^(2/3)*(-(a^2*b^2) + 2*a^2*b*(1 + b/a)*x^3 - a^2*
(1 + (4*a*b + b^2 - d)/a^2)*x^6 + 2*a*(1 + b/a)*x^9 - x^12)), x], x, x^(1/3
)])/((a - x)*(b - x)*x^2)^(2/3) + (3*x^(4/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*
Defer[Subst][Defer[Int][x^10/((-a + x^3)^(2/3)*(-b + x^3)^(2/3)*(a^2*b^2 -
2*a^2*b*(1 + b/a)*x^3 + a^2*(1 + (4*a*b + b^2 - d)/a^2)*x^6 - 2*a*(1 + b/a)
*x^9 + x^12)), x], x, x^(1/3)])/((a - x)*(b - x)*x^2)^(2/3)
```

Rubi steps

$$\int \frac{-abx^2+x^4}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(a^2b^2-2ab(a+b)x+\left(a^2+4ab+b^2-d\right)x^2-2(a+b)x^3+x^4\right)}\,dx=\int \frac{}{\left(x^2(-a+x)(\right.}$$

$$=\frac{\left(x^{4/3}(-a+x)^{2/3}\right.}{}$$

$$=\frac{\left(3x^{4/3}(-a+x)^{2/3}\right.}{}$$

$$=\frac{\left(3x^{4/3}(-a+x)^{2/3}\right.}{}$$

$$=\frac{\left(3x^{4/3}(-a+x)^{2/3}\right.}{}$$

Mathematica [F] time = 3.25, size = 0, normalized size = 0.00

$$\int \frac{-abx^2+x^4}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(a^2b^2-2ab(a+b)x+\left(a^2+4ab+b^2-d\right)x^2-2(a+b)x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-(a*b*x^2) + x^4)/((x^2*(-a + x)*(-b + x))^(2/3)*(a^2*b^2 - 2*a*
b*(a + b)*x + (a^2 + 4*a*b + b^2 - d)*x^2 - 2*(a + b)*x^3 + x^4)),x]
```

```
[Out] Integrate[(-(a*b*x^2) + x^4)/((x^2*(-a + x)*(-b + x))^(2/3)*(a^2*b^2 - 2*a*
b*(a + b)*x + (a^2 + 4*a*b + b^2 - d)*x^2 - 2*(a + b)*x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 1.45, size = 249, normalized size = 1.00

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}x}{\sqrt[6]{d}x-2\sqrt[3]{x^3(-a-b)+abx^2+x^4}}\right)}{2d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}x}{2\sqrt[3]{x^3(-a-b)+abx^2+x^4}+\sqrt[6]{d}x}\right)}{2d^{5/6}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{d}x}{\sqrt[3]{x^3(-a-b)+abx^2+x^4}}\right)}{d^{5/6}}-\frac{\tanh^{-1}\left(\frac{\frac{\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}}{\sqrt[6]{d}}+\sqrt[6]{d}x^2}{x\sqrt[3]{x^3(-a-b)+abx^2+x^4}}\right)}{2d^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*b*x^2) + x^4)/((x^2*(-a + x)*(-b + x))^(2/3)*(a^2*b
^2 - 2*a*b*(a + b)*x + (a^2 + 4*a*b + b^2 - d)*x^2 - 2*(a + b)*x^3 + x^4)),
x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/6)*x)/(d^(1/6)*x - 2*(a*b*x^2 + (-a - b)
*x^3 + x^4)^(1/3))]])/d^(5/6) + (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/6)*x)/(d^(1/6)
*x + 2*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3))]])/(2*d^(5/6)) - ArcTanh[(d^(1/
6)*x)/(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3)]/d^(5/6) - ArcTanh[(d^(1/6)*x^2
+ (a*b*x^2 + (-a - b)*x^3 + x^4)^(2/3)/d^(1/6))/(x*(a*b*x^2 + (-a - b)*x^3
+ x^4)^(1/3))]/(2*d^(5/6))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*x^2+x^4)/(x^2*(-a+x)*(-b+x))^(2/3)/(a^2*b^2-2*a*b*(a+b)*x+(
a^2+4*a*b+b^2-d)*x^2-2*(a+b)*x^3+x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{abx^2-x^4}{\left(a^2b^2-2\,(a+b)abx-2\,(a+b)x^3+x^4+\left(a^2+4\,ab+b^2-d\right)x^2\right)\left((a-x)(b-x)x^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*x^2+x^4)/(x^2*(-a+x)*(-b+x))^(2/3)/(a^2*b^2-2*a*b*(a+b)*x+(
a^2+4*a*b+b^2-d)*x^2-2*(a+b)*x^3+x^4),x, algorithm="giac")
```

```
[Out] integrate(-(a*b*x^2 - x^4)/((a^2*b^2 - 2*(a + b)*a*b*x - 2*(a + b)*x^3 + x^
4 + (a^2 + 4*a*b + b^2 - d)*x^2)*((a - x)*(b - x)*x^2)^(2/3)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{-ab\,x^2+x^4}{\left(x^2\,(-a+x)\,(-b+x)\right)^{\frac{2}{3}}\left(a^2b^2-2ab\,(a+b)\,x+\left(a^2+4ab+b^2-d\right)x^2-2\,(a+b)\,x^3+x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*b*x^2+x^4)/(x^2*(-a+x)*(-b+x))^(2/3)/(a^2*b^2-2*a*b*(a+b)*x+(a^2+4*
a*b+b^2-d)*x^2-2*(a+b)*x^3+x^4),x)
```

```
[Out] int((-a*b*x^2+x^4)/(x^2*(-a+x)*(-b+x))^(2/3)/(a^2*b^2-2*a*b*(a+b)*x+(a^2+4*
a*b+b^2-d)*x^2-2*(a+b)*x^3+x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{abx^2 - x^4}{\left(a^2b^2 - 2\,(a+b)abx - 2\,(a+b)x^3 + x^4 + \left(a^2 + 4\,ab + b^2 - d\right)x^2\right)\left((a-x)(b-x)x^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*x^2+x^4)/(x^2*(-a+x)*(-b+x))^(2/3)/(a^2*b^2-2*a*b*(a+b)*x+(
a^2+4*a*b+b^2-d)*x^2-2*(a+b)*x^3+x^4),x, algorithm="maxima")
```

```
[Out] -integrate((a*b*x^2 - x^4)/((a^2*b^2 - 2*(a + b)*a*b*x - 2*(a + b)*x^3 + x^
4 + (a^2 + 4*a*b + b^2 - d)*x^2)*((a - x)*(b - x)*x^2)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 - a\,b\,x^2}{{\left(x^2\,\left(a-x\right)\,\left(b-x\right)\right)}^{2/3}\,\left(x^4 - 2\,x^3\,\left(a+b\right) + a^2\,b^2 + x^2\,\left(a^2 + 4\,a\,b + b^2 - d\right) - 2\,a\,b\,x\,\left(a+b\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - a*b*x^2)/((x^2*(a - x)*(b - x))^(2/3)*(x^4 - 2*x^3*(a + b) + a^2
*b^2 + x^2*(4*a*b - d + a^2 + b^2) - 2*a*b*x*(a + b))),x)
```

```
[Out] int((x^4 - a*b*x^2)/((x^2*(a - x)*(b - x))^(2/3)*(x^4 - 2*x^3*(a + b) + a^2
*b^2 + x^2*(4*a*b - d + a^2 + b^2) - 2*a*b*x*(a + b))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b*x**2+x**4)/(x**2*(-a+x)*(-b+x))**(2/3)/(a**2*b**2-2*a*b*(a+
b)*x+(a**2+4*a*b+b**2-d)*x**2-2*(a+b)*x**3+x**4),x)
```

```
[Out] Timed out
```

3.2172 $\int \frac{\sqrt[4]{\frac{1+4x^2+ax^2+6x^4+4ax^4+4x^6+6ax^6+x^8+4ax^8+ax^{10}}{x^2}}}{x}\,dx$

Optimal. Leaf size=249

$$\frac{\sqrt[4]{\frac{ax^{10}+4ax^8+6ax^6+4ax^4+ax^2+x^8+4x^6+6x^4+4x^2+1}{x^2}}\left(\frac{\sqrt{x}\tan^{-1}\left(\frac{\sqrt[4]{ax^2+1}}{\sqrt[4]{a}\sqrt{x}}\right)}{4a^{3/4}}+\frac{\sqrt{x}\tanh^{-1}\left(\frac{\sqrt[4]{ax^2+1}}{\sqrt[4]{a}\sqrt{x}}\right)}{4a^{3/4}}+\frac{1}{2}x^2\sqrt[4]{ax^2+1}-2\sqrt[4]{ax^2+1}+\right.}{\left(x^2+1\right)\sqrt[4]{ax^2+1}}$$

Rubi [A] time = 0.41, antiderivative size = 244, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 9, integrand size = 60, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6688, 6718, 453, 279, 329, 331, 298, 203, 206}

$$-\frac{(4a+1)\sqrt{x}\sqrt[4]{\frac{\left(x^2+1\right)^4\left(ax^2+1\right)}{x^2}}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+1}}\right)}{4a^{3/4}\left(x^2+1\right)\sqrt[4]{ax^2+1}}+\frac{(4a+1)\sqrt{x}\sqrt[4]{\frac{\left(x^2+1\right)^4\left(ax^2+1\right)}{x^2}}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{ax^2+1}}\right)}{4a^{3/4}\left(x^2+1\right)\sqrt[4]{ax^2+1}}+\frac{(4a+1)x^2\sqrt[4]{\frac{\left(x^2+1\right)^4\left(ax^2+1\right)}{x^2}}}{2\left(x^2+1\right)}-\frac{2\left(ax^2+1\right)\sqrt[4]{\frac{\left(x^2+1\right)^4\left(ax^2+1\right)}{x^2}}}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x^2 + a*x^2 + 6*x^4 + 4*a*x^4 + 4*x^6 + 6*a*x^6 + x^8 + 4*a*x^8
 + a*x^10)/x^2)^(1/4)/x,x]
```

```
[Out] ((1 + 4*a)*x^2*(((1 + x^2)^4*(1 + a*x^2))/x^2)^(1/4))/(2*(1 + x^2)) - (2*(1
 + a*x^2)*(((1 + x^2)^4*(1 + a*x^2))/x^2)^(1/4))/(1 + x^2) - ((1 + 4*a)*Sqr
t[x]*(((1 + x^2)^4*(1 + a*x^2))/x^2)^(1/4)*ArcTan[(a^(1/4)*Sqrt[x])/(1 + a*
x^2)^(1/4)])/(4*a^(3/4)*(1 + x^2)*(1 + a*x^2)^(1/4)) + ((1 + 4*a)*Sqrt[x]*(
((1 + x^2)^4*(1 + a*x^2))/x^2)^(1/4)*ArcTanh[(a^(1/4)*Sqrt[x])/(1 + a*x^2)^
(1/4)])/(4*a^(3/4)*(1 + x^2)*(1 + a*x^2)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !FreeQ[w, x] &&  !Fr
eeQ[z, x]
```

Rubi steps

$$\int \frac{\sqrt[4]{\frac{1+4x^2+ax^2+6x^4+4ax^4+4x^6+6ax^6+x^8+4ax^8+ax^{10}}{x^2}}}{x}\,dx = \int \frac{\sqrt[4]{\frac{\left(1+x^2\right)^4\left(1+ax^2\right)}{x^2}}}{x}\,dx$$

$$= \frac{\left(\sqrt{x}\sqrt[4]{\frac{\left(1+x^2\right)^4\left(1+ax^2\right)}{x^2}}\right)\int \frac{\left(1+x^2\right)\sqrt[4]{1+ax^2}}{x^{3/2}}\,dx}{\left(1+x^2\right)\sqrt[4]{1+ax^2}}$$

$$= -\frac{2\left(1+ax^2\right)\sqrt[4]{\frac{\left(1+x^2\right)^4\left(1+ax^2\right)}{x^2}}}{1+x^2} - \frac{\left((-1-4a)\sqrt{x}\sqrt[4]{\frac{\left(1+x^2\right)^4\left(1+ax \ldots\right.}{x^2}}\right.}{\left(1+x^2\right)\sqrt[4]{1 \ldots}}$$

$$= \frac{(1+4a)x^2\sqrt[4]{\frac{\left(1+x^2\right)^4\left(1+ax^2\right)}{x^2}}}{2\left(1+x^2\right)} - \frac{2\left(1+ax^2\right)\sqrt[4]{\frac{\left(1+x^2\right)^4\left(1+ax^2\right)}{x^2}}}{1+x^2} - \ldots$$

$$= \frac{(1+4a)x^2\sqrt[4]{\frac{\left(1+x^2\right)^4\left(1+ax^2\right)}{x^2}}}{2\left(1+x^2\right)} - \frac{2\left(1+ax^2\right)\sqrt[4]{\frac{\left(1+x^2\right)^4\left(1+ax^2\right)}{x^2}}}{1+x^2} - \ldots$$

$$= \frac{(1+4a)x^2\sqrt[4]{\frac{\left(1+x^2\right)^4\left(1+ax^2\right)}{x^2}}}{2\left(1+x^2\right)} - \frac{2\left(1+ax^2\right)\sqrt[4]{\frac{\left(1+x^2\right)^4\left(1+ax^2\right)}{x^2}}}{1+x^2} - \ldots$$

$$= \frac{(1+4a)x^2\sqrt[4]{\frac{\left(1+x^2\right)^4\left(1+ax^2\right)}{x^2}}}{2\left(1+x^2\right)} - \frac{2\left(1+ax^2\right)\sqrt[4]{\frac{\left(1+x^2\right)^4\left(1+ax^2\right)}{x^2}}}{1+x^2} - \ldots$$

$$= \frac{(1+4a)x^2\sqrt[4]{\frac{\left(1+x^2\right)^4\left(1+ax^2\right)}{x^2}}}{2\left(1+x^2\right)} - \frac{2\left(1+ax^2\right)\sqrt[4]{\frac{\left(1+x^2\right)^4\left(1+ax^2\right)}{x^2}}}{1+x^2} - \ldots$$

Mathematica [C] time = 0.05, size = 84, normalized size = 0.34

$$-\frac{2\sqrt[4]{\frac{\left(x^2+1\right)^4\left(ax^2+1\right)}{x^2}}\left(3\left(ax^2+1\right)^{5/4}-(4a+1)x^2\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-ax^2\right)\right)}{3\left(x^2+1\right)\sqrt[4]{ax^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x^2 + a*x^2 + 6*x^4 + 4*a*x^4 + 4*x^6 + 6*a*x^6 + x^8 + 4
*a*x^8 + a*x^10)/x^2)^(1/4)/x,x]
```

```
[Out] (-2*(((1 + x^2)^4*(1 + a*x^2))/x^2)^(1/4)*(3*(1 + a*x^2)^(5/4) - (1 + 4*a)*
x^2*Hypergeometric2F1[-1/4, 3/4, 7/4, -(a*x^2)]))/(3*(1 + x^2)*(1 + a*x^2)^
(1/4))
```

IntegrateAlgebraic [A] time = 0.30, size = 237, normalized size = 0.95

$$\frac{(-4a-1)\tan^{-1}\left(\frac{\sqrt[4]{a}\left(x^2+1\right)}{\sqrt[4]{\frac{ax^{10}+4ax^8+6ax^6+4ax^4+ax^2+x^8+4x^6+6x^4+4x^2+1}{x^2}}}\right)}{4a^{3/4}}+\frac{(4a+1)\tanh^{-1}\left(\frac{\sqrt[4]{a}\left(x^2+1\right)}{\sqrt[4]{\frac{ax^{10}+4ax^8+6ax^6+4ax^4+ax^2+x^8+4x^6+6x^4+4x^2+1}{x^2}}}\right)}{4a^{3/4}}+\frac{\left(x^2-4\right)\sqrt[4]{\frac{ax^{10}+4ax^8+6ax^6+4ax^4+ax^2+x^8+4x^6+6x^4+4x^2+1}{x^2}}}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + 4*x^2 + a*x^2 + 6*x^4 + 4*a*x^4 + 4*x^6 + 6*a*x^6
+ x^8 + 4*a*x^8 + a*x^10)/x^2)^(1/4)/x,x]
```

```
[Out] ((-4 + x^2)*((1 + 4*x^2 + a*x^2 + 6*x^4 + 4*a*x^4 + 4*x^6 + 6*a*x^6 + x^8 +
 4*a*x^8 + a*x^10)/x^2)^(1/4))/(2*(1 + x^2)) + ((-1 - 4*a)*ArcTan[(a^(1/4)*
(1 + x^2))/((1 + 4*x^2 + a*x^2 + 6*x^4 + 4*a*x^4 + 4*x^6 + 6*a*x^6 + x^8 +
4*a*x^8 + a*x^10)/x^2)^(1/4)])/(4*a^(3/4)) + ((1 + 4*a)*ArcTanh[(a^(1/4)*(1
 + x^2))/((1 + 4*x^2 + a*x^2 + 6*x^4 + 4*a*x^4 + 4*x^6 + 6*a*x^6 + x^8 + 4*
a*x^8 + a*x^10)/x^2)^(1/4)])/(4*a^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x^10+4*a*x^8+x^8+6*a*x^6+4*x^6+4*a*x^4+6*x^4+a*x^2+4*x^2+1)/x
^2)^(1/4)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax^{10}+4\,ax^{8}+x^{8}+6\,ax^{6}+4\,x^{6}+4\,ax^{4}+6\,x^{4}+ax^{2}+4\,x^{2}+1}{x^{2}}\right)^{\frac{1}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x^10+4*a*x^8+x^8+6*a*x^6+4*x^6+4*a*x^4+6*x^4+a*x^2+4*x^2+1)/x
^2)^(1/4)/x,x, algorithm="giac")
```

```
[Out] integrate(((a*x^10 + 4*a*x^8 + x^8 + 6*a*x^6 + 4*x^6 + 4*a*x^4 + 6*x^4 + a*
x^2 + 4*x^2 + 1)/x^2)^(1/4)/x, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{a\,x^{10}+4a\,x^{8}+x^{8}+6a\,x^{6}+4x^{6}+4a\,x^{4}+6x^{4}+a\,x^{2}+4x^{2}+1}{x^{2}}\right)^{\frac{1}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^10+4*a*x^8+x^8+6*a*x^6+4*x^6+4*a*x^4+6*x^4+a*x^2+4*x^2+1)/x^2)^(1
/4)/x,x)
```

```
[Out] int(((a*x^10+4*a*x^8+x^8+6*a*x^6+4*x^6+4*a*x^4+6*x^4+a*x^2+4*x^2+1)/x^2)^(1
/4)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax^{10}+4\,ax^{8}+x^{8}+6\,ax^{6}+4\,x^{6}+4\,ax^{4}+6\,x^{4}+ax^{2}+4\,x^{2}+1}{x^{2}}\right)^{\frac{1}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x^10+4*a*x^8+x^8+6*a*x^6+4*x^6+4*a*x^4+6*x^4+a*x^2+4*x^2+1)/x
^2)^(1/4)/x,x, algorithm="maxima")
```

```
[Out] integrate(((a*x^10 + 4*a*x^8 + x^8 + 6*a*x^6 + 4*x^6 + 4*a*x^4 + 6*x^4 + a*
x^2 + 4*x^2 + 1)/x^2)^(1/4)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{a\,x^2+4\,a\,x^4+6\,a\,x^6+4\,a\,x^8+a\,x^{10}+4\,x^2+6\,x^4+4\,x^6+x^8+1}{x^2}\right)^{1/4}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^2 + 4*a*x^4 + 6*a*x^6 + 4*a*x^8 + a*x^10 + 4*x^2 + 6*x^4 + 4*x^6
+ x^8 + 1)/x^2)^(1/4)/x,x)
```

```
[Out] int(((a*x^2 + 4*a*x^4 + 6*a*x^6 + 4*a*x^8 + a*x^10 + 4*x^2 + 6*x^4 + 4*x^6
+ x^8 + 1)/x^2)^(1/4)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x**10+4*a*x**8+x**8+6*a*x**6+4*x**6+4*a*x**4+6*x**4+a*x**2+4*
x**2+1)/x**2)**(1/4)/x,x)
```

```
[Out] Timed out
```

3.2173 $\int \frac{\sqrt[3]{2-8x+8x^2}}{3+x} dx$

Optimal. Leaf size=250

$$-\frac{\sqrt[3]{(1-2x)^2}\left(7^{2/3}\sqrt[3]{2x-1}+7\right)\left(\sqrt[3]{7}(2x-1)^{2/3}-7^{2/3}\sqrt[3]{2x-1}+7\right)^2\left(\frac{3(2x-1)^{2/3}}{2^{2/3}}+\sqrt[3]{2}\,7^{2/3}\log\left(7^{2/3}\sqrt[3]{2x-1}+7\right)-\cdots\right.}{14(x+3)\sqrt[3]{2x-1}\left(-2\sqrt[3]{7}\,x+(14x-7)^{2/3}-\cdots\right.}$$

Rubi [A] time = 0.09, antiderivative size = 179, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {646, 50, 56, 617, 204, 31}

$$\frac{3\sqrt[3]{4x^2-4x+1}}{2^{2/3}}-\frac{\left(\frac{7}{2}\right)^{2/3}\sqrt[3]{4x^2-4x+1}\log(x+3)}{(2x-1)^{2/3}}+\frac{3\left(\frac{7}{2}\right)^{2/3}\sqrt[3]{4x^2-4x+1}\log\left(\sqrt[3]{8x-4}+2^{2/3}\sqrt[3]{7}\right)}{(2x-1)^{2/3}}+\frac{\sqrt[3]{2}\sqrt{3}\,7^{2/3}\sqrt[3]{4x^2-4x+1}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{2x-1}}{\sqrt{3}\sqrt[3]{7}}\right)}{(2x-1)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 8*x + 8*x^2)^(1/3)/(3 + x),x]
```

```
[Out] (3*(1 - 4*x + 4*x^2)^(1/3))/2^(2/3) + (2^(1/3)*Sqrt[3]*7^(2/3)*(1 - 4*x + 4
*x^2)^(1/3)*ArcTan[1/Sqrt[3] - (2*(-1 + 2*x)^(1/3))/(Sqrt[3]*7^(1/3))])/(-1
 + 2*x)^(2/3) - ((7/2)^(2/3)*(1 - 4*x + 4*x^2)^(1/3)*Log[3 + x])/(-1 + 2*x)
^(2/3) + (3*(7/2)^(2/3)*(1 - 4*x + 4*x^2)^(1/3)*Log[2^(2/3)*7^(1/3) + (-4 +
 8*x)^(1/3)])/(-1 + 2*x)^(2/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rubi steps

$$\int \frac{\sqrt[3]{2-8x+8x^2}}{3+x}\,dx = \frac{\sqrt[3]{2-8x+8x^2}\int \frac{(-4+8x)^{2/3}}{3+x}\,dx}{(-4+8x)^{2/3}}$$

$$= \frac{3\sqrt[3]{1-4x+4x^2}}{2^{2/3}} - \frac{\left(28\sqrt[3]{2-8x+8x^2}\right)\int \frac{1}{(3+x)\sqrt[3]{-4+8x}}\,dx}{(-4+8x)^{2/3}}$$

$$= \frac{3\sqrt[3]{1-4x+4x^2}}{2^{2/3}} - \frac{\left(\frac{7}{2}\right)^{2/3}\sqrt[3]{1-4x+4x^2}\,\log(3+x)}{(-1+2x)^{2/3}} - \frac{\left(42\sqrt[3]{2-8x+8x^2}\right)\text{Subst}\left(\int \ldots\right.}{(-4 \ldots}$$

$$= \frac{3\sqrt[3]{1-4x+4x^2}}{2^{2/3}} - \frac{\left(\frac{7}{2}\right)^{2/3}\sqrt[3]{1-4x+4x^2}\,\log(3+x)}{(-1+2x)^{2/3}} + \frac{3\left(\frac{7}{2}\right)^{2/3}\sqrt[3]{1-4x+4x^2}\,\log\left(2^{2/3}\ldots\right.}{(-1+2x)^{2/3}}$$

$$= \frac{3\sqrt[3]{1-4x+4x^2}}{2^{2/3}} + \frac{\sqrt[3]{2}\sqrt{3}\,7^{2/3}\sqrt[3]{1-4x+4x^2}\,\tan^{-1}\left(\frac{7-2\,7^{2/3}\sqrt[3]{-1+2x}}{7\sqrt{3}}\right)}{(-1+2x)^{2/3}} - \frac{\left(\frac{7}{2}\right)^{2/3}\sqrt[3]{1-4x\ldots}}{(-1\ldots}$$

Mathematica [C] time = 0.04, size = 37, normalized size = 0.15

$$-\frac{3\sqrt[3]{(1-2x)^2}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{1}{7}(1-2x)\right)-1\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 8*x + 8*x^2)^(1/3)/(3 + x),x]
```

```
[Out] (-3*((1 - 2*x)^2)^(1/3)*(-1 + Hypergeometric2F1[2/3, 1, 5/3, (1 - 2*x)/7]))
/2^(2/3)
```

IntegrateAlgebraic [A] time = 5.53, size = 250, normalized size = 1.00

$$-\frac{\sqrt[3]{(1-2x)^2}\left(7^{2/3}\sqrt[3]{2x-1}+7\right)\left(\sqrt[3]{7}(2x-1)^{2/3}-7^{2/3}\sqrt[3]{2x-1}+7\right)^2\left(\frac{3(2x-1)^{2/3}}{2^{2/3}}+\sqrt[3]{2}\,7^{2/3}\log\left(7^{2/3}\sqrt[3]{2x-1}+7\right)-\left(\frac{7}{2}\right)^{2/3}\log\left(-\sqrt[3]{7}(2x-1)^{2/3}+7^{2/3}\sqrt[3]{2x-1}-7\right)+\sqrt[3]{2}\sqrt{3}\,7^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{2x-1}}{\sqrt{3}\sqrt[3]{7}}\right)\right)}{14(x+3)\sqrt[3]{2x-1}\left(-2\sqrt[3]{7}x+(14x-7)^{2/3}-7\sqrt[3]{2x-1}+\sqrt[3]{7}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - 8*x + 8*x^2)^(1/3)/(3 + x),x]
```

```
[Out] -1/14*(((1 - 2*x)^2)^(1/3)*(7 + 7^(2/3)*(-1 + 2*x)^(1/3))*(7 - 7^(2/3)*(-1
+ 2*x)^(1/3) + 7^(1/3)*(-1 + 2*x)^(2/3))^2*((3*(-1 + 2*x)^(2/3))/2^(2/3) +
2^(1/3)*Sqrt[3]*7^(2/3)*ArcTan[1/Sqrt[3] - (2*(-1 + 2*x)^(1/3))/(Sqrt[3]*7^
(1/3))] + 2^(1/3)*7^(2/3)*Log[7 + 7^(2/3)*(-1 + 2*x)^(1/3)] - (7/2)^(2/3)*L
og[-7 + 7^(2/3)*(-1 + 2*x)^(1/3) - 7^(1/3)*(-1 + 2*x)^(2/3)]))/((3 + x)*(-1
 + 2*x)^(1/3)*(7^(1/3) - 2*7^(1/3)*x - 7*(-1 + 2*x)^(1/3) + (-7 + 14*x)^(2/
3)))
```

fricas [A] time = 0.85, size = 169, normalized size = 0.68

$$98^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{98^{\frac{2}{3}}\sqrt{3}\left(8x^2-8x+2\right)^{\frac{1}{3}}-7\sqrt{3}(2x-1)}{21(2x-1)}\right)-\frac{1}{2}\cdot 98^{\frac{1}{3}}\log\left(\frac{98^{\frac{2}{3}}\left(4x^2-4x+1\right)-7\cdot 98^{\frac{1}{3}}\left(8x^2-8x+2\right)^{\frac{1}{3}}(2x-1)+49\left(8x^2-8x+2\right)^{\frac{2}{3}}}{4x^2-4x+1}\right)+98^{\frac{1}{3}}\log\left(\frac{98^{\frac{1}{3}}(2x-1)+7\left(8x^2-8x+2\right)^{\frac{1}{3}}}{2x-1}\right)+\frac{3}{2}\left(8x^2-8x+2\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^2-8*x+2)^(1/3)/(3+x),x, algorithm="fricas")
```

```
[Out] 98^(1/3)*sqrt(3)*arctan(1/21*(98^(2/3)*sqrt(3)*(8*x^2 - 8*x + 2)^(1/3) - 7*
sqrt(3)*(2*x - 1))/(2*x - 1)) - 1/2*98^(1/3)*log((98^(2/3)*(4*x^2 - 4*x + 1
) - 7*98^(1/3)*(8*x^2 - 8*x + 2)^(1/3)*(2*x - 1) + 49*(8*x^2 - 8*x + 2)^(2/
3))/(4*x^2 - 4*x + 1)) + 98^(1/3)*log((98^(1/3)*(2*x - 1) + 7*(8*x^2 - 8*x
+ 2)^(1/3))/(2*x - 1)) + 3/2*(8*x^2 - 8*x + 2)^(1/3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(8\,x^2-8\,x+2\right)^{\frac{1}{3}}}{x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^2-8*x+2)^(1/3)/(3+x),x, algorithm="giac")
```

```
[Out] integrate((8*x^2 - 8*x + 2)^(1/3)/(x + 3), x)
```

maple [A] time = 0.04, size = 110, normalized size = 0.44

$$\frac{3\,2^{\frac{1}{3}}\left(\left(-1+2x\right)^{2}\right)^{\frac{1}{3}}}{2}+\frac{\left(7^{\frac{2}{3}}\ln\left(\left(-1+2x\right)^{\frac{1}{3}}+7^{\frac{1}{3}}\right)-\frac{7^{\frac{2}{3}}\ln\left(\left(-1+2x\right)^{\frac{2}{3}}-7^{\frac{1}{3}}\left(-1+2x\right)^{\frac{1}{3}}+7^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,7^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,7^{\frac{2}{3}}\left(-1+2x\right)^{\frac{1}{3}}}{7}-1\right)}{3}\right)\right)2^{\frac{1}{3}}\left(\left(-1+2x\right)^{2}\right)^{\frac{1}{3}}}{\left(-1+2x\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^2-8*x+2)^(1/3)/(3+x),x)
```

```
[Out] 3/2*2^(1/3)*((-1+2*x)^2)^(1/3)+(7^(2/3)*ln((-1+2*x)^(1/3)+7^(1/3))-1/2*7^(2
/3)*ln((-1+2*x)^(2/3)-7^(1/3)*(-1+2*x)^(1/3)+7^(2/3))-3^(1/2)*7^(2/3)*arcta
n(1/3*3^(1/2)*(2/7*7^(2/3)*(-1+2*x)^(1/3)-1)))*2^(1/3)*((-1+2*x)^2)^(1/3)/(
-1+2*x)^(2/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(8\,x^2-8\,x+2\right)^{\frac{1}{3}}}{x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^2-8*x+2)^(1/3)/(3+x),x, algorithm="maxima")
```

```
[Out] integrate((8*x^2 - 8*x + 2)^(1/3)/(x + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(8\,x^2-8\,x+2\right)}^{1/3}}{x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^2 - 8*x + 2)^(1/3)/(x + 3),x)
```

```
[Out] int((8*x^2 - 8*x + 2)^(1/3)/(x + 3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\sqrt[3]{2}\int \frac{\sqrt[3]{4x^2-4x+1}}{x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**2-8*x+2)**(1/3)/(3+x),x)
```

```
[Out] 2**(1/3)*Integral((4*x**2 - 4*x + 1)**(1/3)/(x + 3), x)
```

3.2174 $\int \frac{(b+ax)(-aq+bpx^2)}{(q+px^3)^{2/3}(b^3c+dq+3ab^2cx+3a^2bcx^2+(a^3c+dp)x^3)}\,dx$

Optimal. Leaf size=250

$$-\frac{\log\left(a^2b^2c^{2/3}x^2+2ab^3c^{2/3}x+\sqrt[3]{px^3+q}\left(b^3\left(-\sqrt[3]{c}\right)\sqrt[3]{d}-ab^2\sqrt[3]{c}\sqrt[3]{d}x\right)+b^4c^{2/3}+b^2d^{2/3}\left(px^3+q\right)^{2/3}\right)}{6c^{2/3}\sqrt[3]{d}}+\frac{\log\left(ab\sqrt[3]{\cdots}\right.}{}$$

Rubi [F] time = 3.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(b+ax)\left(-aq+bpx^2\right)}{\left(q+px^3\right)^{2/3}\left(b^3c+dq+3ab^2cx+3a^2bcx^2+\left(a^3c+dp\right)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((b + a*x)*(-(a*q) + b*p*x^2))/((q + p*x^3)^(2/3)*(b^3*c + d*q + 3*a*b^
2*c*x + 3*a^2*b*c*x^2 + (a^3*c + d*p)*x^3)),x]
```

```
[Out] (a*b*p*x*(1 + (p*x^3)/q)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((p*x^3)/q
)])/((a^3*c + d*p)*(q + p*x^3)^(2/3)) - (a*b*(b^3*c*p + a^3*c*q + 2*d*p*q)*
Defer[Int][1/((q + p*x^3)^(2/3)*(b^3*c + d*q + 3*a*b^2*c*x + 3*a^2*b*c*x^2
+ (a^3*c + d*p)*x^3)), x])/(a^3*c + d*p) - (a^2*(3*b^3*c*p + a^3*c*q + d*p*
q)*Defer[Int][x/((q + p*x^3)^(2/3)*(b^3*c + d*q + 3*a*b^2*c*x + 3*a^2*b*c*x
^2 + (a^3*c + d*p)*x^3)), x])/(a^3*c + d*p) - (b^2*p*(2*a^3*c - d*p)*Defer[
Int][x^2/((q + p*x^3)^(2/3)*(b^3*c + d*q + 3*a*b^2*c*x + 3*a^2*b*c*x^2 + (a
^3*c + d*p)*x^3)), x])/(a^3*c + d*p)
```

Rubi steps

$$\int \frac{(b+ax)\left(-aq+bpx^2\right)}{\left(q+px^3\right)^{2/3}\left(b^3c+dq+3ab^2cx+3a^2bcx^2+\left(a^3c+dp\right)x^3\right)}\,dx=\int\left(\frac{abp}{\left(a^3c+dp\right)\left(q+px^3\right)^{2/3}}-\frac{ab\left(b^3cp+a^3c\cdots\right.}{\left(a^3c+dp\right)\left(q\cdots\right.}\right.$$

$$=-\frac{\int\frac{ab\left(b^3cp+a^3cq+2dpq\right)+a^2\left(3b^3cp+a^3cq+dpq\right)x+b^2p\left(2a^3\cdots\right.}{\left(q+px^3\right)^{2/3}\left(b^3c+dq+3ab^2cx+3a^2bcx^2+\left(a^3c+dp\right)\cdots\right.}}{a^3c+dp}$$

$$=-\frac{\int\left(\frac{ab\left(b^3cp+a^3cq+2dpq\right)}{\left(q+px^3\right)^{2/3}\left(b^3c+dq+3ab^2cx+3a^2bcx^2+\left(a^3c+dp\right)x^3\right)}\cdots\right.}{\cdots}$$

$$=\frac{abpx\left(1+\frac{px^3}{q}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{px^3}{q}\right)}{\left(a^3c+dp\right)\left(q+px^3\right)^{2/3}}-\frac{\left(b^2p\left(\cdots\right.\right.}{\cdots}$$

Mathematica [F] time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{(b+ax)\left(-aq+bpx^2\right)}{\left(q+px^3\right)^{2/3}\left(b^3c+dq+3ab^2cx+3a^2bcx^2+\left(a^3c+dp\right)x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((b + a*x)*(-(a*q) + b*p*x^2))/((q + p*x^3)^(2/3)*(b^3*c + d*q +
3*a*b^2*c*x + 3*a^2*b*c*x^2 + (a^3*c + d*p)*x^3)),x]
```

```
[Out] Integrate[((b + a*x)*(-(a*q) + b*p*x^2))/((q + p*x^3)^(2/3)*(b^3*c + d*q +
3*a*b^2*c*x + 3*a^2*b*c*x^2 + (a^3*c + d*p)*x^3)), x]
```

IntegrateAlgebraic [A] time = 17.99, size = 250, normalized size = 1.00

$$-\frac{\log\left(a^2b^2c^{2/3}x^2+2ab^3c^{2/3}x+\sqrt[3]{px^3+q}\left(b^3\left(-\sqrt[3]{c}\right)\sqrt[3]{d}-ab^2\sqrt[3]{c}\sqrt[3]{d}x\right)+b^4c^{2/3}+b^2d^{2/3}\left(px^3+q\right)^{2/3}\right)}{6c^{2/3}\sqrt[3]{d}}+\frac{\log\left(ab\sqrt[3]{c}x+b^2\sqrt[3]{c}+b\sqrt[3]{d}\sqrt[3]{px^3+q}\right)}{3c^{2/3}\sqrt[3]{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}a\sqrt[3]{c}x+\sqrt{3}b\sqrt[3]{c}}{a\sqrt[3]{c}x+b\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{px^3+q}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b + a*x)*(-(a*q) + b*p*x^2))/((q + p*x^3)^(2/3)*(b^3*c
 + d*q + 3*a*b^2*c*x + 3*a^2*b*c*x^2 + (a^3*c + d*p)*x^3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*b*c^(1/3) + Sqrt[3]*a*c^(1/3)*x)/(b*c^(1/3) + a*c^(1/3)*x -
 2*d^(1/3)*(q + p*x^3)^(1/3))]/(Sqrt[3]*c^(2/3)*d^(1/3)) + Log[b^2*c^(1/3)
+ a*b*c^(1/3)*x + b*d^(1/3)*(q + p*x^3)^(1/3)]/(3*c^(2/3)*d^(1/3)) - Log[b^
4*c^(2/3) + 2*a*b^3*c^(2/3)*x + a^2*b^2*c^(2/3)*x^2 + (-(b^3*c^(1/3)*d^(1/3
)) - a*b^2*c^(1/3)*d^(1/3)*x)*(q + p*x^3)^(1/3) + b^2*d^(2/3)*(q + p*x^3)^(
2/3)]/(6*c^(2/3)*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*(b*p*x^2-a*q)/(p*x^3+q)^(2/3)/(b^3*c+d*q+3*a*b^2*c*x+3*a^
2*b*c*x^2+(a^3*c+d*p)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bpx^2-aq\right)(ax+b)}{\left(3\,a^2bcx^2+3\,ab^2cx+b^3c+\left(a^3c+dp\right)x^3+dq\right)\left(px^3+q\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*(b*p*x^2-a*q)/(p*x^3+q)^(2/3)/(b^3*c+d*q+3*a*b^2*c*x+3*a^
2*b*c*x^2+(a^3*c+d*p)*x^3),x, algorithm="giac")
```

```
[Out] integrate((b*p*x^2 - a*q)*(a*x + b)/((3*a^2*b*c*x^2 + 3*a*b^2*c*x + b^3*c +
 (a^3*c + d*p)*x^3 + d*q)*(p*x^3 + q)^(2/3)), x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{(ax+b)\left(bp\,x^2-aq\right)}{\left(p\,x^3+q\right)^{\frac{2}{3}}\left(b^3c+dq+3a\,b^2cx+3a^2bc\,x^2+\left(a^3c+dp\right)x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)*(b*p*x^2-a*q)/(p*x^3+q)^(2/3)/(b^3*c+d*q+3*a*b^2*c*x+3*a^2*b*c*
x^2+(a^3*c+d*p)*x^3),x)
```

```
[Out] int((a*x+b)*(b*p*x^2-a*q)/(p*x^3+q)^(2/3)/(b^3*c+d*q+3*a*b^2*c*x+3*a^2*b*c*
x^2+(a^3*c+d*p)*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bpx^2 - aq)(ax + b)}{(3\,a^2bcx^2 + 3\,ab^2cx + b^3c + (a^3c + dp)x^3 + dq)(px^3 + q)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*(b*p*x^2-a*q)/(p*x^3+q)^(2/3)/(b^3*c+d*q+3*a*b^2*c*x+3*a^
2*b*c*x^2+(a^3*c+d*p)*x^3),x, algorithm="maxima")
```

```
[Out] integrate((b*p*x^2 - a*q)*(a*x + b)/((3*a^2*b*c*x^2 + 3*a*b^2*c*x + b^3*c +
 (a^3*c + d*p)*x^3 + d*q)*(p*x^3 + q)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(a\,q - b\,p\,x^2)\,(b + a\,x)}{(p\,x^3 + q)^{2/3}\,\left(d\,q + x^3\,(c\,a^3 + d\,p) + b^3\,c + 3\,a\,b^2\,c\,x + 3\,a^2\,b\,c\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*q - b*p*x^2)*(b + a*x))/((q + p*x^3)^(2/3)*(d*q + x^3*(d*p + a^3*c
) + b^3*c + 3*a*b^2*c*x + 3*a^2*b*c*x^2)),x)
```

```
[Out] int(-((a*q - b*p*x^2)*(b + a*x))/((q + p*x^3)^(2/3)*(d*q + x^3*(d*p + a^3*c
) + b^3*c + 3*a*b^2*c*x + 3*a^2*b*c*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*(b*p*x**2-a*q)/(p*x**3+q)**(2/3)/(b**3*c+d*q+3*a*b**2*c*x
+3*a**2*b*c*x**2+(a**3*c+d*p)*x**3),x)
```

```
[Out] Timed out
```

3.2175 $$\int \frac{(-b+x)\left(-a(a-2b)-2bx+x^2\right)}{((-a+x)(-b+x))^{2/3}\left(a^4-b^2d-2\left(2a^3-bd\right)x+\left(6a^2-d\right)x^2-4ax^3+x^4\right)}\,dx$$

Optimal. Leaf size=250

$$\frac{\log\left(a^2-\sqrt[3]{d}\left(x(-a-b)+ab+x^2\right)^{2/3}-2ax+x^2\right)}{2d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\left(x(-a-b)+ab+x^2\right)^{2/3}}{2a^2+\sqrt[3]{d}\left(x(-a-b)+ab+x^2\right)^{2/3}-4ax+2x^2}\right)}{2d^{2/3}}-\frac{\log\left(a^4-4a^3\right.}{}$$

Rubi [F] time = 7.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-b+x)\left(-a(a-2b)-2bx+x^2\right)}{((-a+x)(-b+x))^{2/3}\left(a^4-b^2d-2\left(2a^3-bd\right)x+\left(6a^2-d\right)x^2-4ax^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-b + x)*(-(a*(a - 2*b)) - 2*b*x + x^2))/(((-a + x)*(-b + x))^(2/3)*(a
^4 - b^2*d - 2*(2*a^3 - b*d)*x + (6*a^2 - d)*x^2 - 4*a*x^3 + x^4)),x]
```

```
[Out] (-3*((a - x)*(b - x))^(1/3)*Defer[Subst][Defer[Int][(x^6*(-a + b + x^3)^(1/
3))/(a^2*(1 + (b*(-2*a + b))/a^2)*d - 2*a*(1 - b/a)*d*x^3 + d*x^6 - x^12),
x], x, (a - x)^(1/3)])/((a - x)^(1/3)*(b - x)^(1/3)) - (3*a*((a - x)*(b - x
))^(1/3)*Defer[Subst][Defer[Int][(x^3*(-a + b + x^3)^(1/3))/(-(a^2*(1 + (b*
(-2*a + b))/a^2)*d) + 2*a*(1 - b/a)*d*x^3 - d*x^6 + x^12), x], x, (a - x)^(
1/3)])/((a - x)^(1/3)*(b - x)^(1/3)) + (3*(a - 2*b)*((a - x)*(b - x))^(1/3)
*Defer[Subst][Defer[Int][(x^3*(-a + b + x^3)^(1/3))/(a^2*(1 + b^2/a^2)*d +
2*b*d*x^3 - 2*a*d*(b + x^3) + x^6*(d - x^6)), x], x, (a - x)^(1/3)])/((a -
x)^(1/3)*(b - x)^(1/3))
```

Rubi steps

$$\int \frac{(-b+x)\left(-a(a-2b)-2bx+x^2\right)}{((-a+x)(-b+x))^{2/3}\left(a^4-b^2d-2\left(2a^3-bd\right)x+\left(6a^2-d\right)x^2-4ax^3+x^4\right)}dx = \int \frac{\sqrt[3]{(a-x)(b-}}{a^4-b^2d-2\left(2a^3-bd\right)x}$$

$$= \frac{\sqrt[3]{(a-x)(b-x)}\int \frac{\sqrt[3]{a}}{a^4-b^2d-2(2a}}{\sqrt[3]{a-x}\sqrt[3]{}}$$

$$= \frac{\sqrt[3]{(a-x)(b-x)}\int \left(\frac{2}{-a^4+b^2d+2}\right.}{}$$

$$= \frac{\sqrt[3]{(a-x)(b-x)}\int \frac{}{a^4-b^2d-2(2a}}{\sqrt[3]{a-x}\sqrt[3]{}}$$

$$= -\frac{\left(3\sqrt[3]{(a-x)(b-x)}\right)\text{Subst}\Big(}{}$$

$$= -\frac{\left(3\sqrt[3]{(a-x)(b-x)}\right)\text{Subst}\Big(}{}$$

$$= -\frac{\left(3\sqrt[3]{(a-x)(b-x)}\right)\text{Subst}\Big(}{}$$

$$= -\frac{\left(3\sqrt[3]{(a-x)(b-x)}\right)\text{Subst}\Big(}{}$$

Mathematica [F] time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)\left(-a(a-2b)-2bx+x^2\right)}{((-a+x)(-b+x))^{2/3}\left(a^4-b^2d-2\left(2a^3-bd\right)x+\left(6a^2-d\right)x^2-4ax^3+x^4\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-b + x)*(-(a*(a - 2*b)) - 2*b*x + x^2))/(((-a + x)*(-b + x))^(2
/3)*(a^4 - b^2*d - 2*(2*a^3 - b*d)*x + (6*a^2 - d)*x^2 - 4*a*x^3 + x^4)),x]
```

```
[Out] Integrate[((-b + x)*(-(a*(a - 2*b)) - 2*b*x + x^2))/(((-a + x)*(-b + x))^(2
/3)*(a^4 - b^2*d - 2*(2*a^3 - b*d)*x + (6*a^2 - d)*x^2 - 4*a*x^3 + x^4)), x
]
```

IntegrateAlgebraic [A] time = 1.16, size = 250, normalized size = 1.00

$$\frac{\log\left(a^2-\sqrt[3]{d}\left(x(-a-b)+ab+x^2\right)^{2/3}-2ax+x^2\right)}{2d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\left(x(-a-b)+ab+x^2\right)^{2/3}}{2a^2+\sqrt[3]{d}\left(x(-a-b)+ab+x^2\right)^{2/3}-4ax+2x^2}\right)}{2d^{2/3}}-\frac{\log\left(a^4-4a^3x+\left(x(-a-b)+ab+x^2\right)^{2/3}\left(a^2\sqrt[3]{d}-2a\sqrt[3]{d}x+\sqrt[3]{d}x^2\right)+6a^2x^2+d^{2/3}\left(x(-a-b)+ab+x^2\right)^{4/3}-4ax^3+x^4\right)}{4d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + x)*(-(a*(a - 2*b)) - 2*b*x + x^2))/(((-a + x)*(-b
 + x))^(2/3)*(a^4 - b^2*d - 2*(2*a^3 - b*d)*x + (6*a^2 - d)*x^2 - 4*a*x^3 +
 x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(a*b + (-a - b)*x + x^2)^(2/3))/(2*a^2 - 4
*a*x + 2*x^2 + d^(1/3)*(a*b + (-a - b)*x + x^2)^(2/3))])/(2*d^(2/3)) + Log[
a^2 - 2*a*x + x^2 - d^(1/3)*(a*b + (-a - b)*x + x^2)^(2/3)]/(2*d^(2/3)) - L
og[a^4 - 4*a^3*x + 6*a^2*x^2 - 4*a*x^3 + x^4 + d^(2/3)*(a*b + (-a - b)*x +
x^2)^(4/3) + (a*b + (-a - b)*x + x^2)^(2/3)*(a^2*d^(1/3) - 2*a*d^(1/3)*x +
d^(1/3)*x^2)]/(4*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-a*(a-2*b)-2*b*x+x^2)/((-a+x)*(-b+x))^(2/3)/(a^4-b^2*d-2*
(2*a^3-b*d)*x+(6*a^2-d)*x^2-4*a*x^3+x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a-2\,b)a+2\,bx-x^2\right)(b-x)}{\left(a^4-4\,ax^3+x^4-b^2d+\left(6\,a^2-d\right)x^2-2\left(2\,a^3-bd\right)x\right)((a-x)(b-x))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-a*(a-2*b)-2*b*x+x^2)/((-a+x)*(-b+x))^(2/3)/(a^4-b^2*d-2*
(2*a^3-b*d)*x+(6*a^2-d)*x^2-4*a*x^3+x^4),x, algorithm="giac")
```

```
[Out] integrate(((a - 2*b)*a + 2*b*x - x^2)*(b - x)/((a^4 - 4*a*x^3 + x^4 - b^2*d
 + (6*a^2 - d)*x^2 - 2*(2*a^3 - b*d)*x)*((a - x)*(b - x))^(2/3)), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)\left(-a\,(a-2b)-2bx+x^2\right)}{((-a+x)\,(-b+x))^{\frac{2}{3}}\left(a^4-b^2d-2\left(2a^3-bd\right)x+\left(6a^2-d\right)x^2-4a\,x^3+x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b+x)*(-a*(a-2*b)-2*b*x+x^2)/((-a+x)*(-b+x))^(2/3)/(a^4-b^2*d-2*(2*a^3
-b*d)*x+(6*a^2-d)*x^2-4*a*x^3+x^4),x)
```

```
[Out] int((-b+x)*(-a*(a-2*b)-2*b*x+x^2)/((-a+x)*(-b+x))^(2/3)/(a^4-b^2*d-2*(2*a^3
-b*d)*x+(6*a^2-d)*x^2-4*a*x^3+x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a-2\,b)a+2\,bx-x^2\right)(b-x)}{\left(a^4-4\,ax^3+x^4-b^2d+\left(6\,a^2-d\right)x^2-2\left(2\,a^3-bd\right)x\right)((a-x)(b-x))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-a*(a-2*b)-2*b*x+x^2)/((-a+x)*(-b+x))^(2/3)/(a^4-b^2*d-2*
(2*a^3-b*d)*x+(6*a^2-d)*x^2-4*a*x^3+x^4),x, algorithm="maxima")
```

```
[Out] integrate(((a - 2*b)*a + 2*b*x - x^2)*(b - x)/((a^4 - 4*a*x^3 + x^4 - b^2*d
 + (6*a^2 - d)*x^2 - 2*(2*a^3 - b*d)*x)*((a - x)*(b - x))^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(b-x)\,\left(-x^2+2\,b\,x+a\,(a-2\,b)\right)}{((a-x)\,(b-x))^{2/3}\,\left(x^2\,\left(d-6\,a^2\right)-2\,x\,\left(b\,d-2\,a^3\right)+b^2\,d+4\,a\,x^3-a^4-x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b - x)*(2*b*x + a*(a - 2*b) - x^2))/(((a - x)*(b - x))^(2/3)*(x^2*(d
 - 6*a^2) - 2*x*(b*d - 2*a^3) + b^2*d + 4*a*x^3 - a^4 - x^4)),x)
```

```
[Out] int(-((b - x)*(2*b*x + a*(a - 2*b) - x^2))/(((a - x)*(b - x))^(2/3)*(x^2*(d
 - 6*a^2) - 2*x*(b*d - 2*a^3) + b^2*d + 4*a*x^3 - a^4 - x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-a*(a-2*b)-2*b*x+x**2)/((-a+x)*(-b+x))**(2/3)/(a**4-b**2*
d-2*(2*a**3-b*d)*x+(6*a**2-d)*x**2-4*a*x**3+x**4),x)
```

```
[Out] Timed out
```

3.2176 $\int \frac{\sqrt{1+x^5}\left(2+x^5\right)}{x^6\left(-1-x^5+ax^{10}\right)}\,dx$

Optimal. Leaf size=250

$$\frac{\left(4\sqrt{2}\,a^{3/2}+\sqrt{2}\,\sqrt{4a+1}\,\sqrt{a}+\sqrt{2}\,\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{x^5+1}}{\sqrt{-2a-\sqrt{4a+1}-1}}\right)}{5\sqrt{4a+1}\,\sqrt{-2a-\sqrt{4a+1}-1}}+\frac{\left(-4\sqrt{2}\,a^{3/2}+\sqrt{2}\,\sqrt{4a+1}\,\sqrt{a}-\sqrt{2}\,\sqrt{a}\right)\tan^{-1}}{5\sqrt{4a+1}\,\sqrt{-2a+\sqrt{4a+1}-}}$$

Rubi [A] time = 0.81, antiderivative size = 89, normalized size of antiderivative = 0.36, number of steps used = 18, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6728, 266, 47, 63, 207, 50, 6715, 824, 826, 1161, 618, 206}

$$\frac{2}{5}\sqrt{a}\tanh^{-1}\left(\sqrt{4a+1}-2\sqrt{a}\sqrt{x^5+1}\right)-\frac{2}{5}\sqrt{a}\tanh^{-1}\left(2\sqrt{a}\sqrt{x^5+1}+\sqrt{4a+1}\right)+\frac{2\sqrt{x^5+1}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x^5]*(2 + x^5))/(x^6*(-1 - x^5 + a*x^10)),x]
```

```
[Out] (2*Sqrt[1 + x^5])/(5*x^5) + (2*Sqrt[a]*ArcTanh[Sqrt[1 + 4*a] - 2*Sqrt[a]*Sq
rt[1 + x^5]])/5 - (2*Sqrt[a]*ArcTanh[Sqrt[1 + 4*a] + 2*Sqrt[a]*Sqrt[1 + x^5
]])/5
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x^5}\left(2+x^5\right)}{x^6\left(-1-x^5+ax^{10}\right)}\,dx=\int\left(-\frac{2\sqrt{1+x^5}}{x^6}+\frac{\sqrt{1+x^5}}{x}-\frac{x^4\sqrt{1+x^5}\left(-1-2a+ax^5\right)}{-1-x^5+ax^{10}}\right)dx$$

$$=-\left(2\int\frac{\sqrt{1+x^5}}{x^6}\,dx\right)+\int\frac{\sqrt{1+x^5}}{x}\,dx-\int\frac{x^4\sqrt{1+x^5}\left(-1-2a+ax^5\right)}{-1-x^5+ax^{10}}\,dx$$

$$=\frac{1}{5}\text{Subst}\left(\int\frac{\sqrt{1+x}}{x}\,dx,x,x^5\right)-\frac{1}{5}\text{Subst}\left(\int\frac{\sqrt{1+x}\,(-1-2a+ax)}{-1-x+ax^2}\,dx,x,x^5\right)-\frac{2}{5}$$

$$=\frac{2\sqrt{1+x^5}}{5x^5}-\frac{\text{Subst}\left(\int\frac{-2a^2-a^2x}{\sqrt{1+x}\left(-1-x+ax^2\right)}\,dx,x,x^5\right)}{5a}$$

$$=\frac{2\sqrt{1+x^5}}{5x^5}-\frac{2\,\text{Subst}\left(\int\frac{-a^2-a^2x^2}{a+(-1-2a)x^2+ax^4}\,dx,x,\sqrt{1+x^5}\right)}{5a}$$

$$=\frac{2\sqrt{1+x^5}}{5x^5}+\frac{1}{5}\text{Subst}\left(\int\frac{1}{1-\frac{\sqrt{1+4a}\,x}{\sqrt{a}}+x^2}\,dx,x,\sqrt{1+x^5}\right)+\frac{1}{5}\text{Subst}\left(\int\frac{1}{1+\frac{\sqrt{1+}}{\sqrt{}}}\right.$$

$$=\frac{2\sqrt{1+x^5}}{5x^5}-\frac{2}{5}\text{Subst}\left(\int\frac{1}{\frac{1}{a}-x^2}\,dx,x,-\frac{\sqrt{1+4a}}{\sqrt{a}}+2\sqrt{1+x^5}\right)-\frac{2}{5}\text{Subst}\left(\int\frac{1}{\frac{1}{a}-}\right.$$

$$=\frac{2\sqrt{1+x^5}}{5x^5}+\frac{2}{5}\sqrt{a}\tanh^{-1}\left(\sqrt{a}\left(\frac{\sqrt{1+4a}}{\sqrt{a}}-2\sqrt{1+x^5}\right)\right)-\frac{2}{5}\sqrt{a}\tanh^{-1}\left(\sqrt{a}\left(\frac{\sqrt{1}}{}\right.\right.$$

Mathematica [A] time = 0.44, size = 204, normalized size = 0.82

$$\frac{\sqrt{4a-2\sqrt{4a+1}+2}\left(4a+\sqrt{4a+1}+1\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{2a-\sqrt{4a+1}+1}}\right)}{10\sqrt{a}\sqrt{4a+1}}+\frac{\left(-4a+\sqrt{4a+1}-1\right)\sqrt{2a+\sqrt{4a+1}+1}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{2a+\sqrt{4a+1}+1}}\right)}{5\sqrt{2}\sqrt{a}\sqrt{4a+1}}+\frac{2\sqrt{x^5+1}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x^5]*(2 + x^5))/(x^6*(-1 - x^5 + a*x^10)),x]
```

```
[Out] (2*Sqrt[1 + x^5])/(5*x^5) + (Sqrt[2 + 4*a - 2*Sqrt[1 + 4*a]]*(1 + 4*a + Sqr
t[1 + 4*a])*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[1 + x^5])/Sqrt[1 + 2*a - Sqrt[1 +
 4*a]]])/(10*Sqrt[a]*Sqrt[1 + 4*a]) + ((-1 - 4*a + Sqrt[1 + 4*a])*Sqrt[1 +
2*a + Sqrt[1 + 4*a]]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sqrt[1 + x^5])/Sqrt[1 + 2*a +
 Sqrt[1 + 4*a]]])/(5*Sqrt[2]*Sqrt[a]*Sqrt[1 + 4*a])
```

IntegrateAlgebraic [A] time = 0.64, size = 250, normalized size = 1.00

$$\frac{\left(4\sqrt{2}a^{3/2}+\sqrt{2}\sqrt{4a+1}\sqrt{a}+\sqrt{2}\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{-2a-\sqrt{4a+1}-1}}\right)}{5\sqrt{4a+1}\sqrt{-2a-\sqrt{4a+1}-1}}+\frac{\left(-4\sqrt{2}a^{3/2}+\sqrt{2}\sqrt{4a+1}\sqrt{a}-\sqrt{2}\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{-2a+\sqrt{4a+1}-1}}\right)}{5\sqrt{4a+1}\sqrt{-2a+\sqrt{4a+1}-1}}+\frac{2\sqrt{x^5+1}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x^5]*(2 + x^5))/(x^6*(-1 - x^5 + a*x^10)),x]
```

```
[Out] (2*Sqrt[1 + x^5])/(5*x^5) + ((Sqrt[2]*Sqrt[a] + 4*Sqrt[2]*a^(3/2) + Sqrt[2]
*Sqrt[a]*Sqrt[1 + 4*a])*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[1 + x^5])/Sqrt[-1 - 2*
a - Sqrt[1 + 4*a]]])/(5*Sqrt[1 + 4*a]*Sqrt[-1 - 2*a - Sqrt[1 + 4*a]]) + ((-
(Sqrt[2]*Sqrt[a]) - 4*Sqrt[2]*a^(3/2) + Sqrt[2]*Sqrt[a]*Sqrt[1 + 4*a])*ArcT
an[(Sqrt[2]*Sqrt[a]*Sqrt[1 + x^5])/Sqrt[-1 - 2*a + Sqrt[1 + 4*a]]])/(5*Sqrt
[1 + 4*a]*Sqrt[-1 - 2*a + Sqrt[1 + 4*a]])
```

fricas [A] time = 0.81, size = 103, normalized size = 0.41

$$\left[\frac{\sqrt{a}\,x^{5}\log\left(\frac{ax^{10}-2\,\sqrt{x^{5}+1}\,\sqrt{a}\,x^{5}+x^{5}+1}{ax^{10}-x^{5}-1}\right)+2\,\sqrt{x^{5}+1}}{5\,x^{5}},\ \frac{2\left(\sqrt{-a}\,x^{5}\arctan\left(\frac{\sqrt{-a}\,x^{5}}{\sqrt{x^{5}+1}}\right)+\sqrt{x^{5}+1}\right)}{5\,x^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(1/2)*(x^5+2)/x^6/(a*x^10-x^5-1),x, algorithm="fricas")
```

```
[Out] [1/5*(sqrt(a)*x^5*log((a*x^10 - 2*sqrt(x^5 + 1)*sqrt(a)*x^5 + x^5 + 1)/(a*x
^10 - x^5 - 1)) + 2*sqrt(x^5 + 1))/x^5, 2/5*(sqrt(-a)*x^5*arctan(sqrt(-a)*x
^5/sqrt(x^5 + 1)) + sqrt(x^5 + 1))/x^5]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(1/2)*(x^5+2)/x^6/(a*x^10-x^5-1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:a: recursive definition (in sto)  Error: Bad
 Argument ValueWarning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a]=[-1
6]a: recursive definition (in sto)  Error: Bad Argument ValueWarning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a]=[-4]Done
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{5}+1}\,\left(x^{5}+2\right)}{x^{6}\left(a\,x^{10}-x^{5}-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)^(1/2)*(x^5+2)/x^6/(a*x^10-x^5-1),x)
```

```
[Out] int((x^5+1)^(1/2)*(x^5+2)/x^6/(a*x^10-x^5-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{5}+2\right)\sqrt{x^{5}+1}}{\left(ax^{10}-x^{5}-1\right)x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)^(1/2)*(x^5+2)/x^6/(a*x^10-x^5-1),x, algorithm="maxima")
```

```
[Out] integrate((x^5 + 2)*sqrt(x^5 + 1)/((a*x^10 - x^5 - 1)*x^6), x)
```

mupad [B] time = 2.56, size = 60, normalized size = 0.24

$$\frac{2\,\sqrt{x^{5}+1}}{5\,x^{5}}+\frac{\sqrt{a}\,\ln\left(\frac{a\,x^{10}+x^{5}-2\,\sqrt{a}\,x^{5}\,\sqrt{x^{5}+1}+1}{-4\,a\,x^{10}+4\,x^{5}+4}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^5 + 1)^(1/2)*(x^5 + 2))/(x^6*(x^5 - a*x^10 + 1)),x)
```

```
[Out] (2*(x^5 + 1)^(1/2))/(5*x^5) + (a^(1/2)*log((a*x^10 + x^5 - 2*a^(1/2)*x^5*(x
^5 + 1)^(1/2) + 1)/(4*x^5 - 4*a*x^10 + 4)))/5
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+1)**(1/2)*(x**5+2)/x**6/(a*x**10-x**5-1),x)
```

```
[Out] Timed out
```

3.2177 $\int \frac{(-2+x^5)\sqrt{-1+x^5}}{x^6(1-x^5+ax^{10})}\,dx$

Optimal. Leaf size=250

$$\frac{\left(-4\sqrt{2}\,a^{3/2}+\sqrt{2}\,\sqrt{1-4a}\,\sqrt{a}+\sqrt{2}\,\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{x^5-1}}{\sqrt{2a-\sqrt{1-4a}-1}}\right)}{5\sqrt{1-4a}\,\sqrt{2a-\sqrt{1-4a}-1}}+\frac{\left(4\sqrt{2}\,a^{3/2}+\sqrt{2}\,\sqrt{1-4a}\,\sqrt{a}-\sqrt{2}\,\sqrt{a}\right)\tan^{-1}\left(\cdots\right)}{5\sqrt{1-4a}\,\sqrt{2a+\sqrt{1-4a}-1}}$$

Rubi [A] time = 0.73, antiderivative size = 97, normalized size of antiderivative = 0.39, number of steps used = 16, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {6728, 266, 47, 63, 203, 50, 6715, 824, 826, 1164, 628}

$$\frac{1}{5}\sqrt{a}\,\log\left(-\sqrt{a}\,\left(1-x^5\right)+\sqrt{a}-\sqrt{x^5-1}\right)-\frac{1}{5}\sqrt{a}\,\log\left(-\sqrt{a}\,\left(1-x^5\right)+\sqrt{a}+\sqrt{x^5-1}\right)+\frac{2\sqrt{x^5-1}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((-2 + x^5)*Sqrt[-1 + x^5])/(x^6*(1 - x^5 + a*x^10)),x]
```

```
[Out] (2*Sqrt[-1 + x^5])/(5*x^5) + (Sqrt[a]*Log[Sqrt[a] - Sqrt[a]*(1 - x^5) - Sqr
t[-1 + x^5]])/5 - (Sqrt[a]*Log[Sqrt[a] - Sqrt[a]*(1 - x^5) + Sqrt[-1 + x^5]
])/5
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-2+x^5\right)\sqrt{-1+x^5}}{x^6\left(1-x^5+ax^{10}\right)}\,dx &= \int \left(-\frac{2\sqrt{-1+x^5}}{x^6}-\frac{\sqrt{-1+x^5}}{x}+\frac{x^4\sqrt{-1+x^5}\left(-1+2a+ax^5\right)}{1-x^5+ax^{10}}\right)dx \\
&= -\left(2\int \frac{\sqrt{-1+x^5}}{x^6}\,dx\right)-\int \frac{\sqrt{-1+x^5}}{x}\,dx+\int \frac{x^4\sqrt{-1+x^5}\left(-1+2a+ax^5\right)}{1-x^5+ax^{10}}\,dx \\
&= -\left(\frac{1}{5}\operatorname{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx,x,x^5\right)\right)+\frac{1}{5}\operatorname{Subst}\left(\int \frac{\sqrt{-1+x}\,(-1+2a+ax)}{1-x+ax^2}\,dx,x,x^5 \right. \\
&= \frac{2\sqrt{-1+x^5}}{5x^5}+\frac{\operatorname{Subst}\left(\int \frac{-2a^2+a^2x}{\sqrt{-1+x}\left(1-x+ax^2\right)}\,dx,x,x^5\right)}{5a} \\
&= \frac{2\sqrt{-1+x^5}}{5x^5}+\frac{2\operatorname{Subst}\left(\int \frac{-a^2+a^2x^2}{a+(-1+2a)x^2+ax^4}\,dx,x,\sqrt{-1+x^5}\right)}{5a} \\
&= \frac{2\sqrt{-1+x^5}}{5x^5}+\frac{1}{5}\sqrt{a}\operatorname{Subst}\left(\int \frac{\frac{1}{\sqrt{a}}+2x}{-1-\frac{x}{\sqrt{a}}-x^2}\,dx,x,\sqrt{-1+x^5}\right)+\frac{1}{5}\sqrt{a}\operatorname{Subst}\left(\int \frac{}{-1} \right. \\
&= \frac{2\sqrt{-1+x^5}}{5x^5}+\frac{1}{5}\sqrt{a}\log\left(\sqrt{a}-\sqrt{a}\left(1-x^5\right)-\sqrt{-1+x^5}\right)-\frac{1}{5}\sqrt{a}\log\left(\sqrt{a}-\sqrt{a}\,(1 \right.
\end{aligned}
$$

Mathematica [A] time = 0.75, size = 238, normalized size = 0.95

$$
\frac{2}{5}\left(-\frac{\sqrt{a}\left(4a+\sqrt{1-4a}-1\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5-1}}{\sqrt{-2a-\sqrt{1-4a}+1}}\right)}{\sqrt{2-8a}\sqrt{-2a-\sqrt{1-4a}+1}}-\frac{\left(-4a+\sqrt{1-4a}+1\right)\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5-1}}{\sqrt{-2a+\sqrt{1-4a}+1}}\right)}{\sqrt{2-8a}\sqrt{-2a+\sqrt{1-4a}+1}}+\tan^{-1}\left(\sqrt{x^5-1}\right)+\frac{x^5+\sqrt{1-x^5}\,x^5\tanh^{-1}\left(\sqrt{1-x^5}\right)-1}{x^5\sqrt{x^5-1}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((-2 + x^5)*Sqrt[-1 + x^5])/(x^6*(1 - x^5 + a*x^10)),x]
```

```
[Out] (2*(ArcTan[Sqrt[-1 + x^5]] + (-1 + x^5 + x^5*Sqrt[1 - x^5]*ArcTanh[Sqrt[1 -
 x^5]])/(x^5*Sqrt[-1 + x^5]) - (Sqrt[a]*(-1 + Sqrt[1 - 4*a] + 4*a)*ArcTanh[
(Sqrt[2]*Sqrt[a]*Sqrt[-1 + x^5])/Sqrt[1 - Sqrt[1 - 4*a] - 2*a]])/(Sqrt[2 -
8*a]*Sqrt[1 - Sqrt[1 - 4*a] - 2*a]) - ((1 + Sqrt[1 - 4*a] - 4*a)*Sqrt[a]*Ar
cTanh[(Sqrt[2]*Sqrt[a]*Sqrt[-1 + x^5])/Sqrt[1 + Sqrt[1 - 4*a] - 2*a]])/(Sqr
t[2 - 8*a]*Sqrt[1 + Sqrt[1 - 4*a] - 2*a])))/5
```

IntegrateAlgebraic [A] time = 0.42, size = 250, normalized size = 1.00

$$
\frac{\left(-4\sqrt{2}a^{3/2}+\sqrt{2}\sqrt{1-4a}\sqrt{a}+\sqrt{2}\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5-1}}{\sqrt{2a-\sqrt{1-4a}-1}}\right)}{5\sqrt{1-4a}\sqrt{2a-\sqrt{1-4a}-1}}+\frac{\left(4\sqrt{2}a^{3/2}+\sqrt{2}\sqrt{1-4a}\sqrt{a}-\sqrt{2}\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5-1}}{\sqrt{2a+\sqrt{1-4a}-1}}\right)}{5\sqrt{1-4a}\sqrt{2a+\sqrt{1-4a}-1}}+\frac{2\sqrt{x^5-1}}{5x^5}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x^5)*Sqrt[-1 + x^5])/(x^6*(1 - x^5 + a*x^10)),x]
```

```
[Out] (2*Sqrt[-1 + x^5])/(5*x^5) + ((Sqrt[2]*Sqrt[a] + Sqrt[2]*Sqrt[1 - 4*a]*Sqrt
[a] - 4*Sqrt[2]*a^(3/2))*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[-1 + x^5])/Sqrt[-1 -
Sqrt[1 - 4*a] + 2*a]])/(5*Sqrt[1 - 4*a]*Sqrt[-1 - Sqrt[1 - 4*a] + 2*a]) + (
(-(Sqrt[2]*Sqrt[a]) + Sqrt[2]*Sqrt[1 - 4*a]*Sqrt[a] + 4*Sqrt[2]*a^(3/2))*Ar
cTan[(Sqrt[2]*Sqrt[a]*Sqrt[-1 + x^5])/Sqrt[-1 + Sqrt[1 - 4*a] + 2*a]])/(5*S
qrt[1 - 4*a]*Sqrt[-1 + Sqrt[1 - 4*a] + 2*a])
```

fricas [A] time = 1.87, size = 103, normalized size = 0.41

$$\left[\frac{\sqrt{a}\,x^5\log\left(\frac{ax^{10}-2\sqrt{x^5-1}\,\sqrt{a}\,x^5+x^5-1}{ax^{10}-x^5+1}\right)+2\sqrt{x^5-1}}{5\,x^5},\ \frac{2\left(\sqrt{-a}\,x^5\arctan\left(\frac{\sqrt{-a}\,x^5}{\sqrt{x^5-1}}\right)+\sqrt{x^5-1}\right)}{5\,x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-2)*(x^5-1)^(1/2)/x^6/(a*x^10-x^5+1),x, algorithm="fricas")
```

```
[Out] [1/5*(sqrt(a)*x^5*log((a*x^10 - 2*sqrt(x^5 - 1)*sqrt(a)*x^5 + x^5 - 1)/(a*x
^10 - x^5 + 1)) + 2*sqrt(x^5 - 1))/x^5, 2/5*(sqrt(-a)*x^5*arctan(sqrt(-a)*x
^5/sqrt(x^5 - 1)) + sqrt(x^5 - 1))/x^5]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-2)*(x^5-1)^(1/2)/x^6/(a*x^10-x^5+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:a: recursive definition (in sto)  Error: Bad
 Argument ValueWarning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a]=[6]
a: recursive definition (in sto)  Error: Bad Argument ValueWarning, need to
 choose a branch for the root of a polynomial with parameters. This might b
e wrong.The choice was done assuming [a]=[39]Precision problem choosing roo
t in common_EXT, current precision 14Done
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{\left(x^5-2\right)\sqrt{x^5-1}}{x^6\left(a\,x^{10}-x^5+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-2)*(x^5-1)^(1/2)/x^6/(a*x^10-x^5+1),x)
```

```
[Out] int((x^5-2)*(x^5-1)^(1/2)/x^6/(a*x^10-x^5+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^5-1}\left(x^5-2\right)}{\left(ax^{10}-x^5+1\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-2)*(x^5-1)^(1/2)/x^6/(a*x^10-x^5+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^5 - 1)*(x^5 - 2)/((a*x^10 - x^5 + 1)*x^6), x)
```

mupad [B] time = 2.54, size = 60, normalized size = 0.24

$$\frac{2\sqrt{x^5-1}}{5\,x^5}+\frac{\sqrt{a}\ \ln\left(\frac{a\,x^{10}+x^5-2\,\sqrt{a}\,x^5\,\sqrt{x^5-1}-1}{4\,a\,x^{10}-4\,x^5+4}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^5 - 1)^(1/2)*(x^5 - 2))/(x^6*(a*x^10 - x^5 + 1)),x)
```

```
[Out] (2*(x^5 - 1)^(1/2))/(5*x^5) + (a^(1/2)*log((a*x^10 + x^5 - 2*a^(1/2)*x^5*(x
^5 - 1)^(1/2) - 1)/(4*a*x^10 - 4*x^5 + 4)))/5
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-2)*(x**5-1)**(1/2)/x**6/(a*x**10-x**5+1),x)
```

```
[Out] Timed out
```

3.2178 $\int \frac{(-2q+px^3)(aq+bx^2+apx^3)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^5(cq+dx^2+cpx^3)}\,dx$

Optimal. Leaf size=251

$$\frac{\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}\,\left(acpx^3+acq-2adx^2+2bcx^2\right)}{2c^2x^4}+\frac{\log\left(\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}+px^3+q\right)(-}{c^3}$$

Rubi [F] time = 13.94, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2q+px^3)(aq+bx^2+apx^3)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^5(cq+dx^2+cpx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*q + p*x^3)*(a*q + b*x^2 + a*p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^
4 + p^2*x^6])/(x^5*(c*q + d*x^2 + c*p*x^3)),x]
```

```
[Out] (-2*a*q*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^5, x])/c -
 (2*(b*c - a*d)*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^3,
 x])/c^2 + (a*p*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^2,
 x])/c + (2*d*(b*c - a*d)*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2
*x^6]/x, x])/(c^3*q) + (3*(b*c - a*d)*p*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2
*p*q*x^4 + p^2*x^6]/(c*q + d*x^2 + c*p*x^3), x])/c - (2*d^2*(b*c - a*d)*Def
er[Int][(x*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6])/(c*q + d*x^2 + c*p*
x^3), x])/(c^3*q) - (2*d*(b*c - a*d)*p*Defer[Int][(x^2*Sqrt[q^2 + 2*p*q*x^3
 - 2*p*q*x^4 + p^2*x^6])/(c*q + d*x^2 + c*p*x^3), x])/(c^2*q)
```

Rubi steps

$$\int \frac{(-2q+px^3)(aq+bx^2+apx^3)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^5(cq+dx^2+cpx^3)}\,dx=\int\left(-\frac{2aq\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{cx^5}-\right.$$

$$=-\frac{(2(bc-ad))\int\frac{\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^3}\,dx}{c^2}-$$

$$=-\frac{(2(bc-ad))\int\frac{\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^3}\,dx}{c^2}-$$

$$=-\frac{(2(bc-ad))\int\frac{\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^3}\,dx}{c^2}-$$

Mathematica [F] time = 4.42, size = 0, normalized size = 0.00

$$\int \frac{(-2q+px^3)(aq+bx^2+apx^3)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^5(cq+dx^2+cpx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2*q + p*x^3)*(a*q + b*x^2 + a*p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*
p*q*x^4 + p^2*x^6])/(x^5*(c*q + d*x^2 + c*p*x^3)),x]
```

```
[Out] Integrate[((-2*q + p*x^3)*(a*q + b*x^2 + a*p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*
p*q*x^4 + p^2*x^6])/(x^5*(c*q + d*x^2 + c*p*x^3)), x]
```

IntegrateAlgebraic [A] time = 1.35, size = 251, normalized size = 1.00

$$\frac{\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}\left(acpx^3+acq-2adx^2+2bcx^2\right)}{2c^2x^4}+\frac{\log\left(\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}+px^3+q\right)\left(-ac^2pq+ad^2-bcd\right)}{c^3}-\frac{2(ad-bc)\sqrt{2c^2pq-d^2}\tan^{-1}\left(\frac{x^2\sqrt{2c^2pq-d^2}}{c\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}+cpx^3+cq+dx^2}\right)}{c^3}+\frac{2\log(x)\left(ac^2pq-ad^2+bcd\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*q + p*x^3)*(a*q + b*x^2 + a*p*x^3)*Sqrt[q^2 + 2*p*q
*x^3 - 2*p*q*x^4 + p^2*x^6])/(x^5*(c*q + d*x^2 + c*p*x^3)),x]
```

```
[Out] ((a*c*q + 2*b*c*x^2 - 2*a*d*x^2 + a*c*p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x
^4 + p^2*x^6])/(2*c^2*x^4) - (2*(-(b*c) + a*d)*Sqrt[-d^2 + 2*c^2*p*q]*ArcTa
n[(Sqrt[-d^2 + 2*c^2*p*q]*x^2)/(c*q + d*x^2 + c*p*x^3 + c*Sqrt[q^2 + 2*p*q*
x^3 - 2*p*q*x^4 + p^2*x^6])])/c^3 + (2*(b*c*d - a*d^2 + a*c^2*p*q)*Log[x])/
c^3 + ((-(b*c*d) + a*d^2 - a*c^2*p*q)*Log[q + p*x^3 + Sqrt[q^2 + 2*p*q*x^3
- 2*p*q*x^4 + p^2*x^6]])/c^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(a*p*x^3+b*x^2+a*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)
^(1/2)/x^5/(c*p*x^3+d*x^2+c*q),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(a*p*x^3+b*x^2+a*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)
^(1/2)/x^5/(c*p*x^3+d*x^2+c*q),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\left(p\,x^{3}-2q\right)\left(ap\,x^{3}+b\,x^{2}+aq\right)\sqrt{p^{2}x^{6}-2pq\,x^{4}+2pq\,x^{3}+q^{2}}}{x^{5}\left(cp\,x^{3}+d\,x^{2}+cq\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^3-2*q)*(a*p*x^3+b*x^2+a*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)
/x^5/(c*p*x^3+d*x^2+c*q),x)
```

```
[Out] int((p*x^3-2*q)*(a*p*x^3+b*x^2+a*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)
/x^5/(c*p*x^3+d*x^2+c*q),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^{2}x^{6}-2\,pqx^{4}+2\,pqx^{3}+q^{2}}\left(apx^{3}+bx^{2}+aq\right)\left(px^{3}-2\,q\right)}{\left(cpx^{3}+dx^{2}+cq\right)x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(a*p*x^3+b*x^2+a*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)
^(1/2)/x^5/(c*p*x^3+d*x^2+c*q),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(p^2*x^6 - 2*p*q*x^4 + 2*p*q*x^3 + q^2)*(a*p*x^3 + b*x^2 + a*
q)*(p*x^3 - 2*q)/((c*p*x^3 + d*x^2 + c*q)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(2\,q-p\,x^3\right)\,\left(a\,p\,x^3+b\,x^2+a\,q\right)\,\sqrt{p^2\,x^6-2\,p\,q\,x^4+2\,p\,q\,x^3+q^2}}{x^5\,\left(c\,p\,x^3+d\,x^2+c\,q\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*q - p*x^3)*(a*q + b*x^2 + a*p*x^3)*(p^2*x^6 + q^2 + 2*p*q*x^3 - 2*
p*q*x^4)^(1/2))/(x^5*(c*q + d*x^2 + c*p*x^3)),x)
```

```
[Out] int(-((2*q - p*x^3)*(a*q + b*x^2 + a*p*x^3)*(p^2*x^6 + q^2 + 2*p*q*x^3 - 2*
p*q*x^4)^(1/2))/(x^5*(c*q + d*x^2 + c*p*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**3-2*q)*(a*p*x**3+b*x**2+a*q)*(p**2*x**6-2*p*q*x**4+2*p*q*x*
*3+q**2)**(1/2)/x**5/(c*p*x**3+d*x**2+c*q),x)
```

```
[Out] Timed out
```

3.2179 $\int -\frac{(-1+x)x\left(-1+2x+\left(-2k+k^2\right)x^2\right)}{((1-x)x(1-kx))^{2/3}\left(1-4kx+\left(-b+6k^2\right)x^2+\left(2b-4k^3\right)x^3+\left(-b+k^4\right)x^4\right)}\,dx$

Optimal. Leaf size=252

$$\frac{\log\left(-\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+k^2x^2-2kx+1\right)}{2b^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}}{\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+2k^2x^2-4kx+2}\right)}{2b^{2/3}}-\frac{\log\left(b^{2/3}\left(k\right.\right.}{}$$

Rubi [F] time = 10.81, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int -\frac{(-1+x)x\left(-1+2x+\left(-2k+k^2\right)x^2\right)}{((1-x)x(1-kx))^{2/3}\left(1-4kx+\left(-b+6k^2\right)x^2+\left(2b-4k^3\right)x^3+\left(-b+k^4\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[-(((-1 + x)*x*(-1 + 2*x + (-2*k + k^2)*x^2))/(((1 - x)*x*(1 - k*x))^(2/
3)*(1 - 4*k*x + (-b + 6*k^2)*x^2 + (2*b - 4*k^3)*x^3 + (-b + k^4)*x^4))),x]
```

```
[Out] (3*(2 - k)*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst][Defer[Int][(x
^6*(1 - x^3)^(1/3)*(1 - k*x^3)^(1/3))/(1 - 4*k*x^3 - b*(1 - (6*k^2)/b)*x^6
+ 2*b*(1 - (2*k^3)/b)*x^9 - b*(1 - k^4/b)*x^12), x], x, x^(1/3)])/((1 - x)*
x*(1 - k*x))^(2/3) + (3*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst][
Defer[Int][(x^3*(1 - x^3)^(1/3)*(1 - k*x^3)^(1/3))/(-1 + 4*k*x^3 + b*(1 - (
6*k^2)/b)*x^6 - 2*b*(1 - (2*k^3)/b)*x^9 + b*(1 - k^4/b)*x^12), x], x, x^(1/
3)])/((1 - x)*x*(1 - k*x))^(2/3)
```

Rubi steps

$$\int -\frac{(-1+x)x\left(-1+2x+\left(-2k+k^2\right)x^2\right)}{((1-x)x(1-kx))^{2/3}\left(1-4kx+\left(-b+6k^2\right)x^2+\left(2b-4k^3\right)x^3+\left(-b+k^4\right)x^4\right)}\,dx = -\frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right.}{}$$

$$= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)}{}$$

$$= \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)}{((1}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right.}{}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right.}{}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right.}{}$$

Mathematica [F] time = 3.62, size = 87, normalized size = 0.35

$$-\int \frac{(x-1)x\left(\left(k^2-2k\right)x^2+2x-1\right)}{((1-x)x(1-kx))^{2/3}\left(x^4\left(k^4-b\right)+x^3\left(2b-4k^3\right)+x^2\left(6k^2-b\right)-4kx+1\right)}\,dx$$

Antiderivative was successfully verified.

```
[In] Integrate[-(((-1 + x)*x*(-1 + 2*x + (-2*k + k^2)*x^2))/(((1 - x)*x*(1 - k*x
))^(2/3)*(1 - 4*k*x + (-b + 6*k^2)*x^2 + (2*b - 4*k^3)*x^3 + (-b + k^4)*x^4
))),x]
```

```
[Out] -Integrate[((-1 + x)*x*(-1 + 2*x + (-2*k + k^2)*x^2))/(((1 - x)*x*(1 - k*x)
)^(2/3)*(1 - 4*k*x + (-b + 6*k^2)*x^2 + (2*b - 4*k^3)*x^3 + (-b + k^4)*x^4)
), x]
```

IntegrateAlgebraic [A] time = 3.23, size = 252, normalized size = 1.00

$$\frac{\log\left(-\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+k^2x^2-2kx+1\right)}{2b^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}}{\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+2k^2x^2-4kx+2}\right)}{2b^{2/3}}-\frac{\log\left(b^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{4/3}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\left(\sqrt[3]{b}k^2x^2-2\sqrt[3]{b}kx+\sqrt[3]{b}\right)+k^4x^4-4k^3x^3+6k^2x^2-4kx+1\right)}{4b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[-(((-1 + x)*x*(-1 + 2*x + (-2*k + k^2)*x^2))/(((1 - x)*x
*(1 - k*x))^(2/3)*(1 - 4*k*x + (-b + 6*k^2)*x^2 + (2*b - 4*k^3)*x^3 + (-b +
 k^4)*x^4))),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3))/(2 - 4*k
*x + 2*k^2*x^2 + b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3))])/(2*b^(2/3)) +
Log[1 - 2*k*x + k^2*x^2 - b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(2
/3)) - Log[1 - 4*k*x + 6*k^2*x^2 - 4*k^3*x^3 + k^4*x^4 + (b^(1/3) - 2*b^(1/
3)*k*x + b^(1/3)*k^2*x^2)*(x + (-1 - k)*x^2 + k*x^3)^(2/3) + b^(2/3)*(x + (
-1 - k)*x^2 + k*x^3)^(4/3)]/(4*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-1+x)*x*(-1+2*x+(k^2-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-4*k*
x+(6*k^2-b)*x^2+(-4*k^3+2*b)*x^3+(k^4-b)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(\left(k^2-2\,k\right)x^2+2\,x-1\right)(x-1)x}{\left(\left(k^4-b\right)x^4-2\left(2\,k^3-b\right)x^3+\left(6\,k^2-b\right)x^2-4\,kx+1\right)((kx-1)(x-1)x)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-1+x)*x*(-1+2*x+(k^2-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-4*k*
x+(6*k^2-b)*x^2+(-4*k^3+2*b)*x^3+(k^4-b)*x^4),x, algorithm="giac")
```

```
[Out] integrate(-((k^2 - 2*k)*x^2 + 2*x - 1)*(x - 1)*x/(((k^4 - b)*x^4 - 2*(2*k^3
 - b)*x^3 + (6*k^2 - b)*x^2 - 4*k*x + 1)*((k*x - 1)*(x - 1)*x)^(2/3)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int -\frac{(-1+x)\,x\left(-1+2x+\left(k^2-2k\right)x^2\right)}{((1-x)\,x\,(-kx+1))^{\frac{2}{3}}\left(1-4kx+\left(6k^2-b\right)x^2+\left(-4k^3+2b\right)x^3+\left(k^4-b\right)x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(-1+x)*x*(-1+2*x+(k^2-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-4*k*x+(6*k
^2-b)*x^2+(-4*k^3+2*b)*x^3+(k^4-b)*x^4),x)
```

```
[Out] int(-(-1+x)*x*(-1+2*x+(k^2-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-4*k*x+(6*k
^2-b)*x^2+(-4*k^3+2*b)*x^3+(k^4-b)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(\left(k^2-2\,k\right)x^2+2\,x-1\right)(x-1)x}{\left(\left(k^4-b\right)x^4-2\left(2\,k^3-b\right)x^3+\left(6\,k^2-b\right)x^2-4\,kx+1\right)\left((kx-1)(x-1)x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-1+x)*x*(-1+2*x+(k^2-2*k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-4*k*
x+(6*k^2-b)*x^2+(-4*k^3+2*b)*x^3+(k^4-b)*x^4),x, algorithm="maxima")
```

```
[Out] -integrate(((k^2 - 2*k)*x^2 + 2*x - 1)*(x - 1)*x/(((k^4 - b)*x^4 - 2*(2*k^3
 - b)*x^3 + (6*k^2 - b)*x^2 - 4*k*x + 1)*((k*x - 1)*(x - 1)*x)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x\,\left(x-1\right)\,\left(\left(2\,k-k^2\right)\,x^2-2\,x+1\right)}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{2/3}\,\left(\left(b-k^4\right)\,x^4+\left(4\,k^3-2\,b\right)\,x^3+\left(b-6\,k^2\right)\,x^2+4\,k\,x-1\right)} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(x - 1)*(x^2*(2*k - k^2) - 2*x + 1))/((x*(k*x - 1)*(x - 1))^(2/3)*(
x^4*(b - k^4) + x^2*(b - 6*k^2) + 4*k*x - x^3*(2*b - 4*k^3) - 1)),x)
```

```
[Out] -int((x*(x - 1)*(x^2*(2*k - k^2) - 2*x + 1))/((x*(k*x - 1)*(x - 1))^(2/3)*(
x^4*(b - k^4) + x^2*(b - 6*k^2) + 4*k*x - x^3*(2*b - 4*k^3) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-1+x)*x*(-1+2*x+(k**2-2*k)*x**2)/((1-x)*x*(-k*x+1))**(2/3)/(1-4
*k*x+(6*k**2-b)*x**2+(-4*k**3+2*b)*x**3+(k**4-b)*x**4),x)
```

```
[Out] Timed out
```

3.2180 $\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^8+a\left(q+px^3\right)^4\right)}{x^{13}}\,dx$

Optimal. Leaf size=252

$$\log(x)\left(ap^3q^3+2bpq\right)+\frac{1}{2}\left(-ap^3q^3-2bpq\right)\log\left(\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}+px^3+q\right)+\frac{\sqrt{p^2x^6-2pqx^4+2p}}{}$$

Rubi [F] time = 1.77, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^8+a\left(q+px^3\right)^4\right)}{x^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(b*x^8 + a*
(q + p*x^3)^4))/x^13,x]
```

```
[Out] -2*a*q^5*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^13, x] -
7*a*p*q^4*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^10, x] -
 8*a*p^2*q^3*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^7, x]
 - 2*b*q*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^5, x] - 2
*a*p^3*q^2*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^4, x] +
 b*p*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^2, x] + 2*a*p
^4*q*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x, x] + a*p^5*D
efer[Int][x^2*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6], x]
```

Rubi steps

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^8+a\left(q+px^3\right)^4\right)}{x^{13}}\,dx=\int\left(-\frac{2aq^5\sqrt{q^2+2pqx^3-2pqx^4+p^2x^{}}}{x^{13}}\right.$$

$$=(bp)\int\frac{\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}}{x^2}\,d$$

Mathematica [F] time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^8+a\left(q+px^3\right)^4\right)}{x^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(b*x^
8 + a*(q + p*x^3)^4))/x^13,x]
```

```
[Out] Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(b*x^
8 + a*(q + p*x^3)^4))/x^13, x]
```

IntegrateAlgebraic [A] time = 0.51, size = 252, normalized size = 1.00

$$\log(x)\left(ap^3q^3+2bpq\right)+\frac{1}{2}\left(-ap^3q^3-2bpq\right)\log\left(\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}+px^3+q\right)+\frac{\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}\left(2ap^5x^{15}-ap^4qx^{13}+10ap^4qx^{12}-3ap^3q^2x^{11}-3ap^3q^2x^{10}+20ap^3q^2x^9-3ap^2q^3x^8-3ap^2q^3x^7+20ap^2q^3x^6-apq^4x^4+10apq^4x^3+2aq^5+6bpx^{11}+6bqx^8\right)}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x
^6]*(b*x^8 + a*(q + p*x^3)^4))/x^13,x]
```

```
[Out] (Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(2*a*q^5 + 10*a*p*q^4*x^3 - a*
p*q^4*x^4 + 20*a*p^2*q^3*x^6 - 3*a*p^2*q^3*x^7 + 6*b*q*x^8 - 3*a*p^2*q^3*x^
8 + 20*a*p^3*q^2*x^9 - 3*a*p^3*q^2*x^10 + 6*b*p*x^11 - 3*a*p^3*q^2*x^11 + 1
0*a*p^4*q*x^12 - a*p^4*q*x^13 + 2*a*p^5*x^15))/(12*x^12) + (2*b*p*q + a*p^3
*q^3)*Log[x] + ((-2*b*p*q - a*p^3*q^3)*Log[q + p*x^3 + Sqrt[q^2 + 2*p*q*x^3
 - 2*p*q*x^4 + p^2*x^6]])/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(b*x^8+a*(p*x
^3+q)^4)/x^13,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^8 + \left(px^3 + q\right)^4 a\right)\sqrt{p^2x^6 - 2\,pqx^4 + 2\,pqx^3 + q^2}\,\left(px^3 - 2\,q\right)}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(b*x^8+a*(p*x
^3+q)^4)/x^13,x, algorithm="giac")
```

```
[Out] integrate((b*x^8 + (p*x^3 + q)^4*a)*sqrt(p^2*x^6 - 2*p*q*x^4 + 2*p*q*x^3 +
q^2)*(p*x^3 - 2*q)/x^13, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\left(p\,x^3 - 2q\right)\sqrt{p^2x^6 - 2pq\,x^4 + 2pq\,x^3 + q^2}\,\left(b\,x^8 + a\left(p\,x^3 + q\right)^4\right)}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(b*x^8+a*(p*x^3+q)^
4)/x^13,x)
```

```
[Out] int((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(b*x^8+a*(p*x^3+q)^
4)/x^13,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^8 + \left(px^3 + q\right)^4 a\right)\sqrt{p^2x^6 - 2\,pqx^4 + 2\,pqx^3 + q^2}\,\left(px^3 - 2\,q\right)}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(b*x^8+a*(p*x
^3+q)^4)/x^13,x, algorithm="maxima")
```

```
[Out] integrate((b*x^8 + (p*x^3 + q)^4*a)*sqrt(p^2*x^6 - 2*p*q*x^4 + 2*p*q*x^3 +
q^2)*(p*x^3 - 2*q)/x^13, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a\left(p\,x^3+q\right)^4+b\,x^8\right)\left(2\,q-p\,x^3\right)\sqrt{p^2\,x^6-2\,p\,q\,x^4+2\,p\,q\,x^3+q^2}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*(q + p*x^3)^4 + b*x^8)*(2*q - p*x^3)*(p^2*x^6 + q^2 + 2*p*q*x^3 -
2*p*q*x^4)^(1/2))/x^13,x)
```

```
[Out] int(-((a*(q + p*x^3)^4 + b*x^8)*(2*q - p*x^3)*(p^2*x^6 + q^2 + 2*p*q*x^3 -
2*p*q*x^4)^(1/2))/x^13, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(px^3-2q\right)\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}\,\left(ap^4x^{12}+4ap^3qx^9+6ap^2q^2x^6+4apq^3x^3+aq^4+bx^8\right)}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**3-2*q)*(p**2*x**6-2*p*q*x**4+2*p*q*x**3+q**2)**(1/2)*(b*x**
8+a*(p*x**3+q)**4)/x**13,x)
```

```
[Out] Integral((p*x**3 - 2*q)*sqrt(p**2*x**6 - 2*p*q*x**4 + 2*p*q*x**3 + q**2)*(a
*p**4*x**12 + 4*a*p**3*q*x**9 + 6*a*p**2*q**2*x**6 + 4*a*p*q**3*x**3 + a*q*
*4 + b*x**8)/x**13, x)
```

3.2181 $\int \frac{1}{x\sqrt[5]{1-4x+6x^2-4x^3+x^4}}\,dx$

Optimal. Leaf size=253

$$(x-1)^{4/5}\left(\log\left(\sqrt[5]{x-1}+1\right)+\frac{1}{4}\left(-1-\sqrt{5}\right)\log\left(2(x-1)^{2/5}+\left(-1-\sqrt{5}\right)\sqrt[5]{x-1}+2\right)+\frac{1}{4}\left(\sqrt{5}-1\right)\log\left(2(x-1)^{2}\right.\right.$$

Rubi [F] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt[5]{1-4x+6x^2-4x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(1 - 4*x + 6*x^2 - 4*x^3 + x^4)^(1/5)),x]
[Out] Defer[Int][1/(x*(1 - 4*x + 6*x^2 - 4*x^3 + x^4)^(1/5)), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt[5]{1-4x+6x^2-4x^3+x^4}}\,dx=\int \frac{1}{x\sqrt[5]{1-4x+6x^2-4x^3+x^4}}\,dx$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.11

$$\frac{5(x-1)\,{}_2F_1\left(\frac{1}{5},1;\frac{6}{5};1-x\right)}{\sqrt[5]{(x-1)^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - 4*x + 6*x^2 - 4*x^3 + x^4)^(1/5)),x]
[Out] (5*(-1 + x)*Hypergeometric2F1[1/5, 1, 6/5, 1 - x])/((-1 + x)^4)^(1/5)
```

IntegrateAlgebraic [A] time = 1.21, size = 411, normalized size = 1.62

$$\frac{1}{5}\log\left(x^4-4x^3+6x^2-4x+1\right)+\log\left(\sqrt[5]{x^4-4x^3+6x^2-4x+1}+x-1\right)+\frac{1}{4}\left(-1-\sqrt{5}\right)\log\left(2x^2+2\left(x^4-4x^3+6x^2-4x+1\right)^{2/5}+\left(\sqrt{5}(1-x)-x+1\right)\sqrt[5]{x^4-4x^3+6x^2-4x+1}-4x+2\right)+\frac{1}{4}\left(\sqrt{5}-1\right)\log\left(2x^2+2\left(x^4-4x^3+6x^2-4x+1\right)^{2/5}+\left(\sqrt{5}(x-1)-x+1\right)\sqrt[5]{x^4-4x^3+6x^2-4x+1}-4x+2\right)-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{10+2\sqrt{5}}\sqrt[5]{x^4-4x^3+6x^2-4x+1}}{\left(\sqrt{5}-1\right)\sqrt[5]{x^4-4x^3+6x^2-4x+1}+4x-4}\right)+\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{10-2\sqrt{5}}\sqrt[5]{x^4-4x^3+6x^2-4x+1}}{\left(1+\sqrt{5}\right)\sqrt[5]{x^4-4x^3+6x^2-4x+1}-4x+4}\right)-\frac{4}{5}\log(x-1)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(1 - 4*x + 6*x^2 - 4*x^3 + x^4)^(1/5)),x]
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[10 + 2*Sqrt[5]]*(1 - 4*x + 6*x^2 - 4*x
^3 + x^4)^(1/5))/(-4 + 4*x + (-1 + Sqrt[5])*(1 - 4*x + 6*x^2 - 4*x^3 + x^4)
^(1/5))]) + Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[10 - 2*Sqrt[5]]*(1 - 4*x + 6
*x^2 - 4*x^3 + x^4)^(1/5))/(4 - 4*x + (1 + Sqrt[5])*(1 - 4*x + 6*x^2 - 4*x^
3 + x^4)^(1/5))] - (4*Log[-1 + x])/5 + Log[1 - 4*x + 6*x^2 - 4*x^3 + x^4]/5
 + Log[-1 + x + (1 - 4*x + 6*x^2 - 4*x^3 + x^4)^(1/5)] + ((-1 - Sqrt[5])*Lo
g[2 - 4*x + 2*x^2 + (1 + Sqrt[5]*(1 - x) - x)*(1 - 4*x + 6*x^2 - 4*x^3 + x^
4)^(1/5) + 2*(1 - 4*x + 6*x^2 - 4*x^3 + x^4)^(2/5)])/4 + ((-1 + Sqrt[5])*Lo
g[2 - 4*x + 2*x^2 + (1 + Sqrt[5]*(-1 + x) - x)*(1 - 4*x + 6*x^2 - 4*x^3 + x
^4)^(1/5) + 2*(1 - 4*x + 6*x^2 - 4*x^3 + x^4)^(2/5)])/4
```

fricas [B] time = 1.81, size = 1084, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-4*x^3+6*x^2-4*x+1)^(1/5),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(5) - 2*sqrt(1/2*sqrt(5) - 5/2) + 1)*log(-1/64*((x - 1)*(sqrt(5)
+ 2*sqrt(1/2*sqrt(5) - 5/2) + 1)^3 + (x - 1)*(sqrt(5) + 2*sqrt(1/2*sqrt(5)
- 5/2) + 1)*(sqrt(5) - 2*sqrt(1/2*sqrt(5) - 5/2) + 1)^2 - 4*(x - 1)*(sqrt(5
) + 2*sqrt(1/2*sqrt(5) - 5/2) + 1)^2 + 16*(x - 1)*(sqrt(5) + 2*sqrt(1/2*sqr
t(5) - 5/2) + 1) + ((x - 1)*(sqrt(5) + 2*sqrt(1/2*sqrt(5) - 5/2) + 1)^2 - 4
*(x - 1)*(sqrt(5) + 2*sqrt(1/2*sqrt(5) - 5/2) + 1))*(sqrt(5) - 2*sqrt(1/2*s
qrt(5) - 5/2) + 1) - 64*(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)^(1/5))/(x - 1)) - 1
/4*(sqrt(5) + 2*sqrt(1/2*sqrt(5) - 5/2) + 1)*log(1/64*((x - 1)*(sqrt(5) + 2
*sqrt(1/2*sqrt(5) - 5/2) + 1)^3 - 4*(x - 1)*(sqrt(5) + 2*sqrt(1/2*sqrt(5) -
 5/2) + 1)^2 + 16*(x - 1)*(sqrt(5) + 2*sqrt(1/2*sqrt(5) - 5/2) + 1) - 64*x
+ 64*(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)^(1/5) + 64)/(x - 1)) + 1/4*(sqrt(5) -
2*sqrt(-3/16*(sqrt(5) + 2*sqrt(1/2*sqrt(5) - 5/2) + 1)^2 - 1/8*(sqrt(5) + 2
*sqrt(1/2*sqrt(5) - 5/2) - 3)*(sqrt(5) - 2*sqrt(1/2*sqrt(5) - 5/2) + 1) - 3
/16*(sqrt(5) - 2*sqrt(1/2*sqrt(5) - 5/2) + 1)^2 + 1/2*sqrt(5) + sqrt(1/2*sq
rt(5) - 5/2) - 5/2) - 1)*log(1/64*((x - 1)*(sqrt(5) + 2*sqrt(1/2*sqrt(5) -
5/2) + 1)*(sqrt(5) - 2*sqrt(1/2*sqrt(5) - 5/2) + 1)^2 + 4*sqrt(-3/16*(sqrt(
5) + 2*sqrt(1/2*sqrt(5) - 5/2) + 1)^2 - 1/8*(sqrt(5) + 2*sqrt(1/2*sqrt(5) -
 5/2) - 3)*(sqrt(5) - 2*sqrt(1/2*sqrt(5) - 5/2) + 1) - 3/16*(sqrt(5) - 2*sq
rt(1/2*sqrt(5) - 5/2) + 1)^2 + 1/2*sqrt(5) + sqrt(1/2*sqrt(5) - 5/2) - 5/2)
*(x - 1)*(sqrt(5) + 2*sqrt(1/2*sqrt(5) - 5/2) + 1)*(sqrt(5) - 2*sqrt(1/2*sq
rt(5) - 5/2) + 1) + ((x - 1)*(sqrt(5) + 2*sqrt(1/2*sqrt(5) - 5/2) + 1)^2 -
4*(x - 1)*(sqrt(5) + 2*sqrt(1/2*sqrt(5) - 5/2) + 1))*(sqrt(5) - 2*sqrt(1/2*
sqrt(5) - 5/2) + 1) + 128*(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)^(1/5))/(x - 1)) +
 1/4*(sqrt(5) + 2*sqrt(-3/16*(sqrt(5) + 2*sqrt(1/2*sqrt(5) - 5/2) + 1)^2 -
1/8*(sqrt(5) + 2*sqrt(1/2*sqrt(5) - 5/2) - 3)*(sqrt(5) - 2*sqrt(1/2*sqrt(5)
 - 5/2) + 1) - 3/16*(sqrt(5) - 2*sqrt(1/2*sqrt(5) - 5/2) + 1)^2 + 1/2*sqrt(
5) + sqrt(1/2*sqrt(5) - 5/2) - 5/2) - 1)*log(1/64*((x - 1)*(sqrt(5) + 2*sqr
t(1/2*sqrt(5) - 5/2) + 1)*(sqrt(5) - 2*sqrt(1/2*sqrt(5) - 5/2) + 1)^2 - 4*s
qrt(-3/16*(sqrt(5) + 2*sqrt(1/2*sqrt(5) - 5/2) + 1)^2 - 1/8*(sqrt(5) + 2*sq
rt(1/2*sqrt(5) - 5/2) - 3)*(sqrt(5) - 2*sqrt(1/2*sqrt(5) - 5/2) + 1) - 3/16
*(sqrt(5) - 2*sqrt(1/2*sqrt(5) - 5/2) + 1)^2 + 1/2*sqrt(5) + sqrt(1/2*sqrt(
5) - 5/2) - 5/2)*(x - 1)*(sqrt(5) + 2*sqrt(1/2*sqrt(5) - 5/2) + 1)*(sqrt(5)
 - 2*sqrt(1/2*sqrt(5) - 5/2) + 1) + ((x - 1)*(sqrt(5) + 2*sqrt(1/2*sqrt(5)
- 5/2) + 1)^2 - 4*(x - 1)*(sqrt(5) + 2*sqrt(1/2*sqrt(5) - 5/2) + 1))*(sqrt(
5) - 2*sqrt(1/2*sqrt(5) - 5/2) + 1) + 128*(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)^(
1/5))/(x - 1)) + log((x + (x^4 - 4*x^3 + 6*x^2 - 4*x + 1)^(1/5) - 1)/(x - 1
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4 - 4\,x^3 + 6\,x^2 - 4\,x + 1\right)^{\frac{1}{5}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-4*x^3+6*x^2-4*x+1)^(1/5),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 - 4*x^3 + 6*x^2 - 4*x + 1)^(1/5)*x), x)
```

maple [C] time = 11.89, size = 11302, normalized size = 44.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4-4*x^3+6*x^2-4*x+1)^(1/5),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4 - 4x^3 + 6x^2 - 4x + 1\right)^{\frac{1}{5}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-4*x^3+6*x^2-4*x+1)^(1/5),x, algorithm="maxima")
[Out] integrate(1/((x^4 - 4*x^3 + 6*x^2 - 4*x + 1)^(1/5)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(x^4 - 4x^3 + 6x^2 - 4x + 1\right)^{1/5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(6*x^2 - 4*x - 4*x^3 + x^4 + 1)^(1/5)),x)
[Out] int(1/(x*(6*x^2 - 4*x - 4*x^3 + x^4 + 1)^(1/5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[5]{(x-1)^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4-4*x**3+6*x**2-4*x+1)**(1/5),x)
[Out] Integral(1/(x*((x - 1)**4)**(1/5)), x)
```

3.2182 $\int \frac{1-x+x^2}{(-1+x^2)\sqrt[3]{x^2+x^4}}\,dx$

Optimal. Leaf size=254

$$\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}-2x\right)}{4\sqrt[3]{2}}-\frac{3\log\left(2^{2/3}\sqrt[3]{x^4+x^2}+2x\right)}{4\sqrt[3]{2}}+\frac{3\log\left(-2x^2+2^{2/3}\sqrt[3]{x^4+x^2}\,x-\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{8\sqrt[3]{2}}-\frac{\log}{}$$

Rubi [C] time = 0.87, antiderivative size = 127, normalized size of antiderivative = 0.50, number of steps used = 18, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2056, 6725, 364, 959, 466, 429, 465, 510}

$$\frac{3\sqrt[3]{x^2+1}\,x^2F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};x^2,-x^2\right)}{4\sqrt[3]{x^4+x^2}}-\frac{6\sqrt[3]{x^2+1}\,xF_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};x^2,-x^2\right)}{\sqrt[3]{x^4+x^2}}+\frac{3\sqrt[3]{x^2+1}\,x\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^4+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x + x^2)/((-1 + x^2)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] (-6*x*(1 + x^2)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6, x^2, -x^2])/(x^2 + x^4)^(1
/3) + (3*x^2*(1 + x^2)^(1/3)*AppellF1[2/3, 1, 1/3, 5/3, x^2, -x^2])/(4*(x^2
 + x^4)^(1/3)) + (3*x*(1 + x^2)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -x^2
])/(x^2 + x^4)^(1/3)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1-x+x^2}{\left(-1+x^2\right)\sqrt[3]{x^2+x^4}}\,dx = \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{1-x+x^2}{x^{2/3}\left(-1+x^2\right)\sqrt[3]{1+x^2}}\,dx}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \left(\frac{1}{x^{2/3}\sqrt[3]{1+x^2}}+\frac{2-x}{x^{2/3}\left(-1+x^2\right)\sqrt[3]{1+x^2}}\right)dx}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{1}{x^{2/3}\sqrt[3]{1+x^2}}\,dx}{\sqrt[3]{x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{2-x}{x^{2/3}\left(-1+x^2\right)\sqrt[3]{1+x^2}}\,dx}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\;{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \left(-\frac{1}{2(1-x)x^{2/3}\sqrt[3]{1+x^2}}-\frac{3}{2x^{2/3}(1+x)\sqrt[3]{1+x^2}}\right)}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\;{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{1}{(1-x)x^{2/3}\sqrt[3]{1+x^2}}\,dx}{2\sqrt[3]{x^2+x^4}} - \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)}{2\sqrt[3]{x}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\;{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{1}{x^{2/3}\left(1-x^2\right)\sqrt[3]{1+x^2}}\,dx}{2\sqrt[3]{x^2+x^4}} - \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)}{2\sqrt[3]{x}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\;{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\left(1-x^6\right)\sqrt[3]{1+x^6}}\,dx,x,\sqrt[3]{x}\right)}{2\sqrt[3]{x^2+x^4}} - $$

$$= -\frac{6x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};x^2,-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{3x\sqrt[3]{1+x^2}\;{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right.}{}$$

$$= -\frac{6x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};x^2,-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{3x^2\sqrt[3]{1+x^2}\,F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};x^2,-x^2\right)}{4\sqrt[3]{x^2+x^4}} + \frac{3x\sqrt[3]{1+x}}{}$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1-x+x^2}{\left(-1+x^2\right)\sqrt[3]{x^2+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x + x^2)/((-1 + x^2)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] Integrate[(1 - x + x^2)/((-1 + x^2)*(x^2 + x^4)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.73, size = 254, normalized size = 1.00

$$\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}-2x\right)}{4\sqrt[3]{2}}-\frac{3\log\left(2^{2/3}\sqrt[3]{x^4+x^2}+2x\right)}{4\sqrt[3]{2}}+\frac{3\log\left(-2x^2+2^{2/3}\sqrt[3]{x^4+x^2}\,x-\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{8\sqrt[3]{2}}-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^4+x^2}\,x+\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{8\sqrt[3]{2}}-\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}-x}\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}+x}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x + x^2)/((-1 + x^2)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] (-3*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(x^2 + x^4)^(1/3))])/(4*2^(1/3
)) - (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(x^2 + x^4)^(1/3))])/(4*2^(1/
3)) + Log[-2*x + 2^(2/3)*(x^2 + x^4)^(1/3)]/(4*2^(1/3)) - (3*Log[2*x + 2^(2
/3)*(x^2 + x^4)^(1/3)])/(4*2^(1/3)) + (3*Log[-2*x^2 + 2^(2/3)*x*(x^2 + x^4)
^(1/3) - 2^(1/3)*(x^2 + x^4)^(2/3)])/(8*2^(1/3)) - Log[2*x^2 + 2^(2/3)*x*(x
^2 + x^4)^(1/3) + 2^(1/3)*(x^2 + x^4)^(2/3)]/(8*2^(1/3))
```

fricas [F] time = 7.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(x^4+x^2\right)^{\frac{2}{3}}\left(x^2-x+1\right)}{x^6-x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2-1)/(x^4+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((x^4 + x^2)^(2/3)*(x^2 - x + 1)/(x^6 - x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-x+1}{\left(x^4+x^2\right)^{\frac{1}{3}}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2-1)/(x^4+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)/((x^4 + x^2)^(1/3)*(x^2 - 1)), x)
```

maple [F] time = 6.85, size = 0, normalized size = 0.00

$$\int \frac{x^2-x+1}{\left(x^2-1\right)\left(x^4+x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+1)/(x^2-1)/(x^4+x^2)^(1/3),x)
```

```
[Out] int((x^2-x+1)/(x^2-1)/(x^4+x^2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - x + 1}{\left(x^4 + x^2\right)^{\frac{1}{3}}\left(x^2 - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2-1)/(x^4+x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)/((x^4 + x^2)^(1/3)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 - x + 1}{\left(x^4 + x^2\right)^{1/3}\left(x^2 - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x + 1)/((x^2 + x^4)^(1/3)*(x^2 - 1)),x)
```

```
[Out] int((x^2 - x + 1)/((x^2 + x^4)^(1/3)*(x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - x + 1}{\sqrt[3]{x^2\left(x^2 + 1\right)}\,(x - 1)\,(x + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+1)/(x**2-1)/(x**4+x**2)**(1/3),x)
```

```
[Out] Integral((x**2 - x + 1)/((x**2*(x**2 + 1))**(1/3)*(x - 1)*(x + 1)), x)
```

3.2183 $\int \frac{1+x+x^2}{(-1+x^2)\sqrt[3]{x^2+x^4}}\,dx$

Optimal. Leaf size=254

$$\frac{3\log\left(2^{2/3}\sqrt[3]{x^4+x^2}-2x\right)}{4\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}+2x\right)}{4\sqrt[3]{2}}+\frac{\log\left(-2x^2+2^{2/3}\sqrt[3]{x^4+x^2}\,x-\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{8\sqrt[3]{2}}-\frac{3\log}{}$$

Rubi [C] time = 0.76, antiderivative size = 127, normalized size of antiderivative = 0.50, number of steps used = 18, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2056, 6725, 364, 959, 466, 429, 465, 510}

$$-\frac{3\sqrt[3]{x^2+1}\,x^2F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};x^2,-x^2\right)}{4\sqrt[3]{x^4+x^2}}-\frac{6\sqrt[3]{x^2+1}\,xF_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};x^2,-x^2\right)}{\sqrt[3]{x^4+x^2}}+\frac{3\sqrt[3]{x^2+1}\,x\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^4+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x + x^2)/((-1 + x^2)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] (-6*x*(1 + x^2)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6, x^2, -x^2])/(x^2 + x^4)^(1
/3) - (3*x^2*(1 + x^2)^(1/3)*AppellF1[2/3, 1, 1/3, 5/3, x^2, -x^2])/(4*(x^2
 + x^4)^(1/3)) + (3*x*(1 + x^2)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -x^2
])/(x^2 + x^4)^(1/3)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x+x^2}{\left(-1+x^2\right)\sqrt[3]{x^2+x^4}}\,dx = \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{1+x+x^2}{x^{2/3}\left(-1+x^2\right)\sqrt[3]{1+x^2}}\,dx}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \left(\frac{1}{x^{2/3}\sqrt[3]{1+x^2}}+\frac{2+x}{x^{2/3}\left(-1+x^2\right)\sqrt[3]{1+x^2}}\right)dx}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{1}{x^{2/3}\sqrt[3]{1+x^2}}\,dx}{\sqrt[3]{x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{2+x}{x^{2/3}\left(-1+x^2\right)\sqrt[3]{1+x^2}}\,dx}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\;{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \left(-\frac{3}{2(1-x)x^{2/3}\sqrt[3]{1+x^2}}-\frac{1}{2x^{2/3}(1+x)\sqrt[3]{1+x^2}}\right)}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\;{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{1}{x^{2/3}(1+x)\sqrt[3]{1+x^2}}\,dx}{2\sqrt[3]{x^2+x^4}} - \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)}{2\sqrt[3]{}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\;{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{1}{x^{2/3}\left(1-x^2\right)\sqrt[3]{1+x^2}}\,dx}{2\sqrt[3]{x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)}{2\sqrt[3]{x}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\;{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\left(1-x^6\right)\sqrt[3]{1+x^6}}\,dx,x,\sqrt[3]{x}\right)}{2\sqrt[3]{x^2+x^4}} + $$

$$= -\frac{6x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};x^2,-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{3x\sqrt[3]{1+x^2}\;{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right.}{}$$

$$= -\frac{6x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};x^2,-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{3x^2\sqrt[3]{1+x^2}\,F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};x^2,-x^2\right)}{4\sqrt[3]{x^2+x^4}} + \frac{3x\sqrt[3]{1+x}}{}$$

Mathematica [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^2}{\left(-1+x^2\right)\sqrt[3]{x^2+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x + x^2)/((-1 + x^2)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] Integrate[(1 + x + x^2)/((-1 + x^2)*(x^2 + x^4)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 0.68, size = 254, normalized size = 1.00

$$\frac{3\log\left(2^{2/3}\sqrt[3]{x^4+x^2}-2x\right)}{4\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}+2x\right)}{4\sqrt[3]{2}}+\frac{\log\left(-2x^2+2^{2/3}\sqrt[3]{x^4+x^2}\,x-\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{8\sqrt[3]{2}}-\frac{3\log\left(2x^2+2^{2/3}\sqrt[3]{x^4+x^2}\,x+\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{8\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}-x}\right)}{4\sqrt[3]{2}}-\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}+x}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x + x^2)/((-1 + x^2)*(x^2 + x^4)^(1/3)),x]
```

```
[Out] -1/4*(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(x^2 + x^4)^(1/3))])/2^(1/3)
 - (3*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(x^2 + x^4)^(1/3))])/(4*2^(1/
3)) + (3*Log[-2*x + 2^(2/3)*(x^2 + x^4)^(1/3)])/(4*2^(1/3)) - Log[2*x + 2^(
2/3)*(x^2 + x^4)^(1/3)]/(4*2^(1/3)) + Log[-2*x^2 + 2^(2/3)*x*(x^2 + x^4)^(1
/3) - 2^(1/3)*(x^2 + x^4)^(2/3)]/(8*2^(1/3)) - (3*Log[2*x^2 + 2^(2/3)*x*(x^
2 + x^4)^(1/3) + 2^(1/3)*(x^2 + x^4)^(2/3)])/(8*2^(1/3))
```

fricas [F] time = 7.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(x^4+x^2\right)^{\frac{2}{3}}\left(x^2+x+1\right)}{x^6-x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2-1)/(x^4+x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((x^4 + x^2)^(2/3)*(x^2 + x + 1)/(x^6 - x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+x+1}{\left(x^4+x^2\right)^{\frac{1}{3}}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2-1)/(x^4+x^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^2 + x + 1)/((x^4 + x^2)^(1/3)*(x^2 - 1)), x)
```

maple [F] time = 6.91, size = 0, normalized size = 0.00

$$\int \frac{x^2+x+1}{\left(x^2-1\right)\left(x^4+x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/(x^2-1)/(x^4+x^2)^(1/3),x)
```

```
[Out] int((x^2+x+1)/(x^2-1)/(x^4+x^2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + x + 1}{\left(x^4 + x^2\right)^{\frac{1}{3}}\left(x^2 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2-1)/(x^4+x^2)^(1/3),x, algorithm="maxima")
[Out] integrate((x^2 + x + 1)/((x^4 + x^2)^(1/3)*(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 + x + 1}{\left(x^4 + x^2\right)^{1/3}\left(x^2 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/((x^2 + x^4)^(1/3)*(x^2 - 1)),x)
[Out] int((x + x^2 + 1)/((x^2 + x^4)^(1/3)*(x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + x + 1}{\sqrt[3]{x^2\left(x^2 + 1\right)}\,(x - 1)(x + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(x**2-1)/(x**4+x**2)**(1/3),x)
[Out] Integral((x**2 + x + 1)/((x**2*(x**2 + 1))**(1/3)*(x - 1)*(x + 1)), x)
```

3.2184 $\int \frac{\left(-1+x^3\right)^{2/3}\left(4+x^3\right)}{x^6\left(-2-x^3+x^6\right)}\,dx$

Optimal. Leaf size=254

$$\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}-x\right)}{6\ 2^{2/3}}-\frac{1}{3}2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^3-1}-2x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}+x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}+\frac{\left(x^3-1\right.}{}$$

Rubi [F] time = 0.74, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(4+x^3\right)}{x^6\left(-2-x^3+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x^3)^(2/3)*(4 + x^3))/(x^6*(-2 - x^3 + x^6)),x]
```

```
[Out] -1/4*(-1 + x^3)^(2/3)/x^2 - (2*(-1 + x^3)^(5/3))/(5*x^5) - (x*(-1 + x^3)^(2
/3)*AppellF1[1/3, -2/3, 1, 4/3, x^3, x^3/2])/(4*(1 - x^3)^(2/3)) + ArcTan[(
1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - Log[-x + (-1 + x^3)^(1/3
)]/4 - Defer[Int][(-1 + x^3)^(2/3)/(1 + x), x]/3 + ((1 + I*Sqrt[3])*Defer[I
nt][(-1 + x^3)^(2/3)/(-1 - I*Sqrt[3] + 2*x), x])/3 + ((1 - I*Sqrt[3])*Defer
[Int][(-1 + x^3)^(2/3)/(-1 + I*Sqrt[3] + 2*x), x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-1+x^3\right)^{2/3}\left(4+x^3\right)}{x^6\left(-2-x^3+x^6\right)}\,dx &= \int \left(-\frac{2\left(-1+x^3\right)^{2/3}}{x^6}+\frac{\left(-1+x^3\right)^{2/3}}{2x^3}-\frac{\left(-1+x^3\right)^{2/3}}{3(1+x)}+\frac{(-2+x)\left(-1+x^3\right)^{2/3}}{3\left(1-x+x^2\right)}+\frac{(-}{2}\right. \\
&= -\left(\frac{1}{3}\int \frac{\left(-1+x^3\right)^{2/3}}{1+x}\,dx\right)+\frac{1}{3}\int \frac{(-2+x)\left(-1+x^3\right)^{2/3}}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{\left(-1+x^3\right)^{2/3}}{x^3}\,dx \\
&= -\frac{\left(-1+x^3\right)^{2/3}}{4x^2}-\frac{2\left(-1+x^3\right)^{5/3}}{5x^5}-\frac{1}{3}\int \frac{\left(-1+x^3\right)^{2/3}}{1+x}\,dx+\frac{1}{3}\int \left(\frac{\left(1+i\sqrt{3}\right)\left(-1+\right.}{-1-i\sqrt{3}+}\right. \\
&= -\frac{\left(-1+x^3\right)^{2/3}}{4x^2}-\frac{2\left(-1+x^3\right)^{5/3}}{5x^5}-\frac{x\left(-1+x^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,\frac{x^3}{2}\right)}{4\left(1-x^3\right)^{2/3}}+\frac{\tan^{-1}\left(\right.}{2}
\end{aligned}$$

Mathematica [A] time = 0.50, size = 236, normalized size = 0.93

$$\frac{2\log\left(\sqrt[3]{2}-\frac{x}{\sqrt[3]{x^3-1}}\right)-8\sqrt[3]{2}\log\left(1-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}\right)+8\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{\frac{2^{2/3}x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)-\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+\frac{x^2}{\left(x^3-1\right)^{2/3}}+2^{2/3}\right)+4\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+\frac{2^{2/3}x^2}{\left(x^3-1\right)^{2/3}}+1\right)}{12\ 2^{2/3}}+\left(x^3-1\right)^{2/3}\left(\frac{2}{5x^5}-\frac{13}{20x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)^(2/3)*(4 + x^3))/(x^6*(-2 - x^3 + x^6)),x]
```

```
[Out] (2/(5*x^5) - 13/(20*x^2))*(-1 + x^3)^(2/3) + (8*2^(1/3)*Sqrt[3]*ArcTan[(1 +
 (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + (2^(2/3)*
```

```
x)/(-1 + x^3)^(1/3))/Sqrt[3]] + 2*Log[2^(1/3) - x/(-1 + x^3)^(1/3)] - 8*2^(
1/3)*Log[1 - (2^(1/3)*x)/(-1 + x^3)^(1/3)] - Log[2^(2/3) + x^2/(-1 + x^3)^(
2/3) + (2^(1/3)*x)/(-1 + x^3)^(1/3)] + 4*2^(1/3)*Log[1 + (2^(2/3)*x^2)/(-1
+ x^3)^(2/3) + (2^(1/3)*x)/(-1 + x^3)^(1/3)])/(12*2^(2/3))
```

IntegrateAlgebraic [A] time = 0.66, size = 254, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}-x\right)}{6\,2^{2/3}}-\frac{1}{3}2^{2/3}\log\left(2^{2/3}\sqrt[3]{x^3-1}-2x\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}+x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^3-1}+x}\right)}{\sqrt{3}}+\frac{\left(x^3-1\right)^{2/3}\left(8-13x^3\right)}{20x^5}+\frac{\log\left(2^{2/3}\sqrt[3]{x^3-1}x+\sqrt[3]{2}\left(x^3-1\right)^{2/3}+2x^2\right)}{3\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}x+2^{2/3}\left(x^3-1\right)^{2/3}+x^2\right)}{12\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(2/3)*(4 + x^3))/(x^6*(-2 - x^3 + x^6)),x]
```

```
[Out] ((8 - 13*x^3)*(-1 + x^3)^(2/3))/(20*x^5) - ArcTan[(Sqrt[3]*x)/(x + 2*2^(1/3
)*(-1 + x^3)^(1/3))]/(2*2^(2/3)*Sqrt[3]) + (2^(2/3)*ArcTan[(Sqrt[3]*x)/(x +
 2^(2/3)*(-1 + x^3)^(1/3))])/Sqrt[3] + Log[-x + 2^(1/3)*(-1 + x^3)^(1/3)]/(
6*2^(2/3)) - (2^(2/3)*Log[-2*x + 2^(2/3)*(-1 + x^3)^(1/3)])/3 + Log[2*x^2 +
 2^(2/3)*x*(-1 + x^3)^(1/3) + 2^(1/3)*(-1 + x^3)^(2/3)]/(3*2^(1/3)) - Log[x
^2 + 2^(1/3)*x*(-1 + x^3)^(1/3) + 2^(2/3)*(-1 + x^3)^(2/3)]/(12*2^(2/3))
```

fricas [B] time = 5.97, size = 521, normalized size = 2.05

$$-\frac{80\sqrt{3}\left(-4\right)^{\frac{1}{3}}x^5\arctan\left(\frac{3\sqrt{3}\left(-4\right)^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(x^3-1\right)^{\frac{2}{3}}+6\sqrt{3}\left(-4\right)^{\frac{1}{3}}\left(19x^8-16x^5+x^2\right)\left(x^3-1\right)^{\frac{1}{3}}-\sqrt{3}\left(71x^9-111x^6+33x^3-1\right)}{3\left(109x^9-105x^6+3x^3+1\right)}\right)-20\cdot 4^{\frac{1}{6}}\sqrt{3}x^5\arctan\left(\frac{4^{\frac{1}{6}}\left(12\cdot 4^{\frac{2}{3}}\sqrt{3}\left(2x^7-5x^4+2x\right)\left(x^3-1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\sqrt{3}\left(91x^9-168x^6+84x^3-8\right)+12\sqrt{3}\left(19x^8-22x^5+4x^2\right)\left(x^3-1\right)^{\frac{1}{3}}\right)}{6\left(53x^9-48x^6-12x^3+8\right)}\right)-10\cdot 4^{\frac{2}{3}}x^5\log\left(\frac{6\cdot 4^{\frac{1}{3}}\left(x^3-1\right)^{\frac{1}{3}}x^2+4^{\frac{2}{3}}\left(x^3-2\right)-12\left(x^3-1\right)^{\frac{2}{3}}x}{x^3-2}\right)+5\cdot 4^{\frac{2}{3}}x^5\log\left(\frac{6\cdot 4^{\frac{2}{3}}\left(2x^4-x\right)\left(x^3-1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(19x^6-22x^3+4\right)+6\left(5x^5-4x^2\right)\left(x^3-1\right)^{\frac{1}{3}}}{x^6-4x^3+4}\right)-80\left(-4\right)^{\frac{1}{3}}x^5\log\left(-\frac{3\left(-4\right)^{\frac{2}{3}}\left(x^3-1\right)^{\frac{1}{3}}x^2-6\left(x^3-1\right)^{\frac{2}{3}}x+\left(-4\right)^{\frac{1}{3}}\left(x^3+1\right)}{x^3+1}\right)+40\left(-4\right)^{\frac{1}{3}}x^5\log\left(-\frac{6\left(-4\right)^{\frac{1}{3}}\left(5x^4-x\right)\left(x^3-1\right)^{\frac{2}{3}}-\left(-4\right)^{\frac{2}{3}}\left(19x^6-16x^3+1\right)-24\left(2x^5-x^2\right)\left(x^3-1\right)^{\frac{1}{3}}}{x^6+2x^3+1}\right)+36\left(13x^3-8\right)\left(x^3-1\right)^{\frac{2}{3}}}{720x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^3+4)/x^6/(x^6-x^3-2),x, algorithm="fricas")
```

```
[Out] -1/720*(80*sqrt(3)*(-4)^(1/3)*x^5*arctan(1/3*(3*sqrt(3)*(-4)^(2/3)*(5*x^7 +
 4*x^4 - x)*(x^3 - 1)^(2/3) + 6*sqrt(3)*(-4)^(1/3)*(19*x^8 - 16*x^5 + x^2)*
(x^3 - 1)^(1/3) - sqrt(3)*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x
^6 + 3*x^3 + 1)) - 20*4^(1/6)*sqrt(3)*x^5*arctan(1/6*4^(1/6)*(12*4^(2/3)*sq
rt(3)*(2*x^7 - 5*x^4 + 2*x)*(x^3 - 1)^(2/3) + 4^(1/3)*sqrt(3)*(91*x^9 - 168
*x^6 + 84*x^3 - 8) + 12*sqrt(3)*(19*x^8 - 22*x^5 + 4*x^2)*(x^3 - 1)^(1/3))/
(53*x^9 - 48*x^6 - 12*x^3 + 8)) - 10*4^(2/3)*x^5*log((6*4^(1/3)*(x^3 - 1)^(
1/3)*x^2 + 4^(2/3)*(x^3 - 2) - 12*(x^3 - 1)^(2/3)*x)/(x^3 - 2)) + 5*4^(2/3)
*x^5*log((6*4^(2/3)*(2*x^4 - x)*(x^3 - 1)^(2/3) + 4^(1/3)*(19*x^6 - 22*x^3
+ 4) + 6*(5*x^5 - 4*x^2)*(x^3 - 1)^(1/3))/(x^6 - 4*x^3 + 4)) - 80*(-4)^(1/3
)*x^5*log(-(3*(-4)^(2/3)*(x^3 - 1)^(1/3)*x^2 - 6*(x^3 - 1)^(2/3)*x + (-4)^(
1/3)*(x^3 + 1))/(x^3 + 1)) + 40*(-4)^(1/3)*x^5*log(-(6*(-4)^(1/3)*(5*x^4 -
x)*(x^3 - 1)^(2/3) - (-4)^(2/3)*(19*x^6 - 16*x^3 + 1) - 24*(2*x^5 - x^2)*(x
^3 - 1)^(1/3))/(x^6 + 2*x^3 + 1)) + 36*(13*x^3 - 8)*(x^3 - 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+4\right)\left(x^3-1\right)^{\frac{2}{3}}}{\left(x^6-x^3-2\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^3+4)/x^6/(x^6-x^3-2),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 4)*(x^3 - 1)^(2/3)/((x^6 - x^3 - 2)*x^6), x)
```

maple [C] time = 4.95, size = 1771, normalized size = 6.97

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)*(x^3+4)/x^6/(x^6-x^3-2),x)
```

```
[Out] -1/20*(13*x^6-21*x^3+8)/x^5/(x^3-1)^(1/3)-1/12*ln((-36*RootOf(RootOf(_Z^3-2
)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^3+3*RootOf(RootOf(_Z^
3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^3+30*(x^3-1)^(2/3)*R
ootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x+30*(
x^3-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z
^3-2)*x^2+RootOf(_Z^3-2)^2*(x^3-1)^(1/3)*x^2+24*RootOf(RootOf(_Z^3-2)^2+6*_
Z*RootOf(_Z^3-2)+36*_Z^2)*x^3-2*RootOf(_Z^3-2)*x^3+2*x*(x^3-1)^(2/3)-24*Roo
tOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)+2*RootOf(_Z^3-2))/(x^3-2)
)*RootOf(_Z^3-2)-1/2*ln((-36*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36
*_Z^2)^2*RootOf(_Z^3-2)^2*x^3+3*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)
+36*_Z^2)*RootOf(_Z^3-2)^3*x^3+30*(x^3-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+6*_
Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x+30*(x^3-1)^(1/3)*RootOf(RootOf
(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^2+RootOf(_Z^3-2)^2
*(x^3-1)^(1/3)*x^2+24*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*
x^3-2*RootOf(_Z^3-2)*x^3+2*x*(x^3-1)^(2/3)-24*RootOf(RootOf(_Z^3-2)^2+6*_Z*
RootOf(_Z^3-2)+36*_Z^2)+2*RootOf(_Z^3-2))/(x^3-2))*RootOf(RootOf(_Z^3-2)^2+
6*_Z*RootOf(_Z^3-2)+36*_Z^2)+1/2*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2
)+36*_Z^2)*ln(-(36*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*R
ootOf(_Z^3-2)^2*x^3+9*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*
RootOf(_Z^3-2)^3*x^3+30*(x^3-1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_
Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x+30*(x^3-1)^(1/3)*RootOf(RootOf(_Z^3-2)^2
+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^2+4*RootOf(_Z^3-2)^2*(x^3-1)
^(1/3)*x^2+36*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+9*Ro
otOf(_Z^3-2)*x^3+8*x*(x^3-1)^(2/3)-24*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_
Z^3-2)+36*_Z^2)-6*RootOf(_Z^3-2))/(x^3-2))+1/3*ln(-(-18*RootOf(RootOf(_Z^3-
2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^3*x^3+6*RootOf(RootOf(_Z
^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^4*x^3+12*(x^3-1)^(2/3)*
RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x+3*R
ootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^3-Root
Of(_Z^3-2)^2*x^3+24*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*(x
^3-1)^(1/3)*x^2-RootOf(_Z^3-2)*(x^3-1)^(1/3)*x^2-x*(x^3-1)^(2/3)-3*RootOf(R
ootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)+RootOf(_Z^3-2)^
2)/(1+x)/(x^2-x+1))*RootOf(_Z^3-2)^2+2*ln(-(-18*RootOf(RootOf(_Z^3-2)^2+6*_
Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^3*x^3+6*RootOf(RootOf(_Z^3-2)^2+
6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^4*x^3+12*(x^3-1)^(2/3)*RootOf(R
ootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x+3*RootOf(Ro
otOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^3-RootOf(_Z^3-
2)^2*x^3+24*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*(x^3-1)^(1
/3)*x^2-RootOf(_Z^3-2)*(x^3-1)^(1/3)*x^2-x*(x^3-1)^(2/3)-3*RootOf(RootOf(_Z
^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)+RootOf(_Z^3-2)^2)/(1+x)
/(x^2-x+1))*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^
3-2)-2*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*
ln((-36*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-
2)^3*x^3+6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3
-2)^4*x^3-18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z
^3-2)*x^3+3*RootOf(_Z^3-2)^2*x^3+6*RootOf(_Z^3-2)*(x^3-1)^(1/3)*x^2+6*x*(x^
3-1)^(2/3)+6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z
^3-2)-RootOf(_Z^3-2)^2)/(1+x)/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^3+4\right)\left(x^3-1\right)^{\frac{2}{3}}}{\left(x^6-x^3-2\right)x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^3+4)/x^6/(x^6-x^3-2),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 4)*(x^3 - 1)^(2/3)/((x^6 - x^3 - 2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(x^3-1\right)^{2/3}\,\left(x^3+4\right)}{x^6\,\left(-x^6+x^3+2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3 - 1)^(2/3)*(x^3 + 4))/(x^6*(x^3 - x^6 + 2)),x)
```

```
[Out] int(-((x^3 - 1)^(2/3)*(x^3 + 4))/(x^6*(x^3 - x^6 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}\left(x^{3}+4\right)}{x^{6}\left(x+1\right)\left(x^{3}-2\right)\left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)*(x**3+4)/x**6/(x**6-x**3-2),x)
```

```
[Out] Integral(((x - 1)*(x**2 + x + 1))**(2/3)*(x**3 + 4)/(x**6*(x + 1)*(x**3 - 2
)*(x**2 - x + 1)), x)
```

3.2185 $\int \frac{\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}{\sqrt{-b+a^2x^2}\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx$

Optimal. Leaf size=254

$$\frac{4\log\left(\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}-\sqrt[3]{c}\right)}{3ac^{2/3}}-\frac{2\log\left(\sqrt[3]{c}\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}+c\right.}{3ac^{2/3}}$$

Rubi [F] time = 1.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}{\sqrt{-b+a^2x^2}\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)/(Sqrt[-b + a^2*x^2]*(a*x +
 Sqrt[-b + a^2*x^2])^(1/4)),x]
```

```
[Out] Defer[Int][(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)/(Sqrt[-b + a^2*x^2]
*(a*x + Sqrt[-b + a^2*x^2])^(1/4)), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}{\sqrt{-b+a^2x^2}\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx=\int \frac{\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}{\sqrt{-b+a^2x^2}\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx$$

Mathematica [C] time = 0.34, size = 74, normalized size = 0.29

$$\frac{3\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}{c}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)/(Sqrt[-b + a^2*x^2]*
(a*x + Sqrt[-b + a^2*x^2])^(1/4)),x]
```

```
[Out] (3*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(4/3)*Hypergeometric2F1[4/3, 2, 7
/3, (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))/c])/(a*c^2)
```

IntegrateAlgebraic [A] time = 0.96, size = 254, normalized size = 1.00

$$\frac{4\log\left(\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}-\sqrt[3]{c}\right)}{3ac^{2/3}}-\frac{2\log\left(\sqrt[3]{c}\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}+c^{2/3}\right)}{3ac^{2/3}}-\frac{4\tan^{-1}\left(\frac{2\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{3}\sqrt[3]{c}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,ac^{2/3}}-\frac{4\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{a\sqrt[4]{\sqrt{a^2x^2-b}+ax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)/(Sqrt[-b +
a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4)),x]
```

```
[Out] (-4*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3))/(a*(a*x + Sqrt[-b + a^2*x
^2])^(1/4)) - (4*ArcTan[1/Sqrt[3] + (2*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4
))^(1/3))/(Sqrt[3]*c^(1/3))]])/(Sqrt[3]*a*c^(2/3)) + (4*Log[-c^(1/3) + (c +
(a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)])/(3*a*c^(2/3)) - (2*Log[c^(2/3) +
c^(1/3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + (c + (a*x + Sqrt[-b
+ a^2*x^2])^(1/4))^(2/3)])/(3*a*c^(2/3))
```

fricas [A] time = 0.69, size = 255, normalized size = 1.00

$$-\frac{2\left(2\sqrt{3}b\left(c^2\right)^{\frac{1}{6}}c\arctan\left(\frac{\sqrt{3}\sqrt{c^2}c+2\sqrt{3}\left(c^2\right)^{\frac{5}{6}}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}{3c^2}\right)+b\left(c^2\right)^{\frac{2}{3}}\log\left(\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{2}{3}}c+\left(c^2\right)^{\frac{1}{3}}c+\left(c^2\right)^{\frac{2}{3}}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}\right)-2b\left(c^2\right)^{\frac{2}{3}}\log\left(\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}c-\left(c^2\right)^{\frac{2}{3}}\right)+6\left(ac^2x-\sqrt{a^2x^2-b}\,c^2\right)\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{3}{4}}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}\right)}{3abc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3)/(a^2*x^2-b)^(1/2)/(a*x+(a
^2*x^2-b)^(1/2))^(1/4),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(3)*b*(c^2)^(1/6)*c*arctan(1/3*(sqrt(3)*sqrt(c^2)*c + 2*sqrt(3)
*(c^2)^(5/6)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3))/c^2) + b*(c^2)^(2
/3)*log((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3)*c + (c^2)^(1/3)*c + (c^
2)^(2/3)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)) - 2*b*(c^2)^(2/3)*log
((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)*c - (c^2)^(2/3)) + 6*(a*c^2*x
- sqrt(a^2*x^2 - b)*c^2)*(a*x + sqrt(a^2*x^2 - b))^(3/4)*(c + (a*x + sqrt(a
^2*x^2 - b))^(1/4))^(1/3))/(a*b*c^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3)/(a^2*x^2-b)^(1/2)/(a*x+(a
^2*x^2-b)^(1/2))^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}{\sqrt{a^2x^2-b}\,\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3)/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2
-b)^(1/2))^(1/4),x)
```

```
[Out] int((c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3)/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2
-b)^(1/2))^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}{\sqrt{a^2x^2-b}\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3)/(a^2*x^2-b)^(1/2)/(a*x+(a
^2*x^2-b)^(1/2))^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)/(sqrt(a^2*x^2 - b)*(a
*x + sqrt(a^2*x^2 - b))^(1/4)), x)
```

mupad [B] time = 2.19, size = 99, normalized size = 0.39

$$-\frac{6\left(c+\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/4}\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{2}{3};\ \frac{5}{3};\ -\frac{c}{\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/4}}\right)}{a\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/4}\left(\frac{c}{\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/4}}+1\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(1/3)/((a*x + (a^2*x^2 - b)^(1/
2))^(1/4)*(a^2*x^2 - b)^(1/2)),x)
```

```
[Out] -(6*(c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(1/3)*hypergeom([-1/3, 2/3], 5/
3, -c/(a*x + (a^2*x^2 - b)^(1/2))^(1/4)))/(a*(a*x + (a^2*x^2 - b)^(1/2))^(1
/4)*(c/(a*x + (a^2*x^2 - b)^(1/2))^(1/4) + 1)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}}{\sqrt[4]{ax+\sqrt{a^2x^2-b}}\,\sqrt{a^2x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a**2*x**2-b)**(1/2))**(1/4))**(1/3)/(a**2*x**2-b)**(1/2)
/(a*x+(a**2*x**2-b)**(1/2))**(1/4),x)
```

```
[Out] Integral((c + (a*x + sqrt(a**2*x**2 - b))**(1/4))**(1/3)/((a*x + sqrt(a**2*
x**2 - b))**(1/4)*sqrt(a**2*x**2 - b)), x)
```

3.2186 $\int \frac{(x+x^2)\sqrt[4]{x^3+x^4}}{-1+x+x^2}\,dx$

Optimal. Leaf size=255

$$\frac{1}{8}\sqrt[4]{x^4+x^3}\,(4x+1)-\frac{29}{16}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)+\sqrt{\frac{1}{5}\left(2+2\sqrt{5}\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4+x^3}}\right)+\sqrt{\frac{1}{5}\left(2\sqrt{5}-2\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\cdots}}{\sqrt[4]{x^4+\cdots}}\right.$$

Rubi [A] time = 0.45, antiderivative size = 416, normalized size of antiderivative = 1.63, number of steps used = 26, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {1593, 2056, 903, 50, 63, 331, 298, 203, 206, 905, 911, 93}

$$\frac{1}{2}\sqrt[4]{x^4+x^3}\,x+\frac{1}{8}\sqrt[4]{x^4+x^3}-\frac{29\sqrt[4]{x^4+x^3}\,\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{16\sqrt[4]{x+1}\,x^{3/4}}+\frac{2^{3/4}\sqrt[4]{3+\sqrt{5}}\,\sqrt[4]{x^4+x^3}\,\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{\sqrt{5}\,\sqrt[4]{x+1}\,x^{3/4}}+\frac{2^{3/4}\sqrt[4]{3-\sqrt{5}}\,\sqrt[4]{x^4+x^3}\,\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}\,\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{\sqrt{5}\,\sqrt[4]{x+1}\,x^{3/4}}+\frac{29\sqrt[4]{x^4+x^3}\,\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{16\sqrt[4]{x+1}\,x^{3/4}}-\frac{2^{3/4}\sqrt[4]{3+\sqrt{5}}\,\sqrt[4]{x^4+x^3}\,\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{\sqrt{5}\,\sqrt[4]{x+1}\,x^{3/4}}-\frac{2^{3/4}\sqrt[4]{3-\sqrt{5}}\,\sqrt[4]{x^4+x^3}\,\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}\,\sqrt[4]{x}}{\sqrt[4]{x+1}}\right)}{\sqrt{5}\,\sqrt[4]{x+1}\,x^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((x + x^2)*(x^3 + x^4)^(1/4))/(-1 + x + x^2),x]
```

```
[Out] (x^3 + x^4)^(1/4)/8 + (x*(x^3 + x^4)^(1/4))/2 - (29*(x^3 + x^4)^(1/4)*ArcTa
n[x^(1/4)/(1 + x)^(1/4)])/(16*x^(3/4)*(1 + x)^(1/4)) + (2^(3/4)*(3 + Sqrt[5
])^(1/4)*(x^3 + x^4)^(1/4)*ArcTan[((2/(3 + Sqrt[5]))^(1/4)*x^(1/4))/(1 + x)
^(1/4)])/(Sqrt[5]*x^(3/4)*(1 + x)^(1/4)) + (2^(3/4)*(3 - Sqrt[5])^(1/4)*(x^
3 + x^4)^(1/4)*ArcTan[(((3 + Sqrt[5])/2)^(1/4)*x^(1/4))/(1 + x)^(1/4)])/(Sq
rt[5]*x^(3/4)*(1 + x)^(1/4)) + (29*(x^3 + x^4)^(1/4)*ArcTanh[x^(1/4)/(1 + x
)^(1/4)])/(16*x^(3/4)*(1 + x)^(1/4)) - (2^(3/4)*(3 + Sqrt[5])^(1/4)*(x^3 +
x^4)^(1/4)*ArcTanh[((2/(3 + Sqrt[5]))^(1/4)*x^(1/4))/(1 + x)^(1/4)])/(Sqrt[
5]*x^(3/4)*(1 + x)^(1/4)) - (2^(3/4)*(3 - Sqrt[5])^(1/4)*(x^3 + x^4)^(1/4)*
ArcTanh[(((3 + Sqrt[5])/2)^(1/4)*x^(1/4))/(1 + x)^(1/4)])/(Sqrt[5]*x^(3/4)*
(1 + x)^(1/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 903

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Dist[g/c^2, Int[Simp[2*c*e*f + c*d*g - b*e
*g + c*e*g*x, x]*(d + e*x)^(m - 1)*(f + g*x)^(n - 2), x], x] + Dist[1/c^2,
Int[(Simp[c^2*d*f^2 - 2*a*c*e*f*g - a*c*d*g^2 + a*b*e*g^2 + (c^2*e*f^2 + 2*
c^2*d*f*g - 2*b*c*e*f*g - b*c*d*g^2 + b^2*e*g^2 - a*c*e*g^2)*x, x]*(d + e*x
)^(m - 1)*(f + g*x)^(n - 2))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
!IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0] && GtQ[n, 1]
```

Rule 905

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(e*g)/c, Int[(d + e*x)^(m - 1)*(f + g
*x)^(n - 1), x], x] + Dist[1/c, Int[(Simp[c*d*f - a*e*g + (c*e*f + c*d*g -
b*e*g)*x, x]*(d + e*x)^(m - 1)*(f + g*x)^(n - 1))/(a + b*x + c*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b
*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0] && GtQ[n, 0
]
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{(x+x^2)\sqrt[4]{x^3+x^4}}{-1+x+x^2}\,dx = \int \frac{x(1+x)\sqrt[4]{x^3+x^4}}{-1+x+x^2}\,dx$$

$$= \frac{\sqrt[4]{x^3+x^4}\int \frac{x^{7/4}(1+x)^{5/4}}{-1+x+x^2}\,dx}{x^{3/4}\sqrt[4]{1+x}}$$

$$= \frac{\sqrt[4]{x^3+x^4}\int x^{3/4}\sqrt[4]{1+x}\,dx}{x^{3/4}\sqrt[4]{1+x}} + \frac{\sqrt[4]{x^3+x^4}\int \frac{x^{3/4}\sqrt[4]{1+x}}{-1+x+x^2}\,dx}{x^{3/4}\sqrt[4]{1+x}}$$

$$= \frac{1}{2}x\sqrt[4]{x^3+x^4} + \frac{\sqrt[4]{x^3+x^4}\int \frac{x^{3/4}}{(1+x)^{3/4}}\,dx}{8x^{3/4}\sqrt[4]{1+x}} + \frac{\sqrt[4]{x^3+x^4}\int \frac{1}{\sqrt[4]{x}(1+x)^{3/4}}\,dx}{x^{3/4}\sqrt[4]{1+x}} + \frac{\sqrt[4]{x^3+x^4}\int \frac{}{\sqrt[4]{x}\,(\cdots}}{x^{3/4}\sqrt[4]{\cdots}}$$

$$= \frac{1}{8}\sqrt[4]{x^3+x^4} + \frac{1}{2}x\sqrt[4]{x^3+x^4} - \frac{\left(3\sqrt[4]{x^3+x^4}\right)\int \frac{1}{\sqrt[4]{x}(1+x)^{3/4}}\,dx}{32x^{3/4}\sqrt[4]{1+x}} + \frac{\sqrt[4]{x^3+x^4}\int\left(-\frac{}{\sqrt{5}\left(-1+\sqrt{5}\cdots\right.}\right.}{\cdots}$$

$$= \frac{1}{8}\sqrt[4]{x^3+x^4} + \frac{1}{2}x\sqrt[4]{x^3+x^4} - \frac{\left(3\sqrt[4]{x^3+x^4}\right)\text{Subst}\left(\int \frac{x^2}{(1+x^4)^{3/4}}\,dx, x, \sqrt[4]{x}\right)}{8x^{3/4}\sqrt[4]{1+x}} + \frac{\left(4\sqrt[4]{x^3+x\cdots}\right.}{\cdots}$$

$$= \frac{1}{8}\sqrt[4]{x^3+x^4} + \frac{1}{2}x\sqrt[4]{x^3+x^4} - \frac{\left(3\sqrt[4]{x^3+x^4}\right)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{8x^{3/4}\sqrt[4]{1+x}} + \frac{\left(2\sqrt[4]{x^3+x^4}\cdots\right.}{\cdots}$$

$$= \frac{1}{8}\sqrt[4]{x^3+x^4} + \frac{1}{2}x\sqrt[4]{x^3+x^4} - \frac{2\sqrt[4]{x^3+x^4}\,\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}} + \frac{2\sqrt[4]{x^3+x^4}\,\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{x^{3/4}\sqrt[4]{1+x}}$$

$$= \frac{1}{8}\sqrt[4]{x^3+x^4} + \frac{1}{2}x\sqrt[4]{x^3+x^4} - \frac{29\sqrt[4]{x^3+x^4}\,\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{1+x}}\right)}{16x^{3/4}\sqrt[4]{1+x}} + \frac{2^{3/4}\sqrt[4]{3+\sqrt{5}}\,\sqrt[4]{x^3+x^4}\,\text{ta}\cdots}{\sqrt{5}\,x^{3/4}\sqrt[4]{1+\cdots}}$$

Mathematica [C] time = 0.43, size = 186, normalized size = 0.73

$$\frac{4}{15}\sqrt[4]{x^3(x+1)}\left(\frac{5\,_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};-x\right)}{\sqrt[4]{x+1}} - \frac{5\,_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-x\right)}{\sqrt[4]{x+1}} + \frac{5\,_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-x\right)}{\sqrt[4]{x+1}} - \frac{2\sqrt{5}\,_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{(-1+\sqrt{5})x}{(1+\sqrt{5})(x+1)}\right)}{(1+\sqrt{5})(x+1)} - \frac{2\sqrt{5}\,_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{(1+\sqrt{5})x}{(-1+\sqrt{5})(x+1)}\right)}{(\sqrt{5}-1)(x+1)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((x + x^2)*(x^3 + x^4)^(1/4))/(-1 + x + x^2),x]
```

```
[Out] (4*(x^3*(1 + x))^(1/4)*((5*Hypergeometric2F1[-5/4, 3/4, 7/4, -x])/(1 + x)^(
1/4) - (5*Hypergeometric2F1[-1/4, 3/4, 7/4, -x])/(1 + x)^(1/4) + (5*Hyperge
ometric2F1[3/4, 3/4, 7/4, -x])/(1 + x)^(1/4) - (2*Sqrt[5]*Hypergeometric2F1
[3/4, 1, 7/4, ((-1 + Sqrt[5])*x)/((1 + Sqrt[5])*(1 + x))])/((1 + Sqrt[5])*(
1 + x)) - (2*Sqrt[5]*Hypergeometric2F1[3/4, 1, 7/4, ((1 + Sqrt[5])*x)/((-1
+ Sqrt[5])*(1 + x))])/((-1 + Sqrt[5])*(1 + x))))/15
```

IntegrateAlgebraic [A] time = 1.04, size = 255, normalized size = 1.00

$$\frac{1}{8}\sqrt[4]{x^4+x^3}(4x+1)-\frac{29}{16}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)+\sqrt{\frac{1}{5}\left(2+2\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}x}{\sqrt[4]{x^4+x^3}}\right)+\sqrt{\frac{1}{5}\left(2\sqrt{5}-2\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}x}{\sqrt[4]{x^4+x^3}}\right)+\frac{29}{16}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4+x^3}}\right)-\sqrt{\frac{1}{5}\left(2+2\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}x}{\sqrt[4]{x^4+x^3}}\right)-\sqrt{\frac{1}{5}\left(2\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}x}{\sqrt[4]{x^4+x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((x + x^2)*(x^3 + x^4)^(1/4))/(-1 + x + x^2),x]
```

```
[Out] ((1 + 4*x)*(x^3 + x^4)^(1/4))/8 - (29*ArcTan[x/(x^3 + x^4)^(1/4)])/16 + Sqr
t[(2 + 2*Sqrt[5])/5]*ArcTan[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(x^3 + x^4)^(1/4)] +
 Sqrt[(-2 + 2*Sqrt[5])/5]*ArcTan[(Sqrt[1/2 + Sqrt[5]/2]*x)/(x^3 + x^4)^(1/4
)] + (29*ArcTanh[x/(x^3 + x^4)^(1/4)])/16 - Sqrt[(2 + 2*Sqrt[5])/5]*ArcTanh
[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(x^3 + x^4)^(1/4)] - Sqrt[(-2 + 2*Sqrt[5])/5]*A
rcTanh[(Sqrt[1/2 + Sqrt[5]/2]*x)/(x^3 + x^4)^(1/4)]
```

fricas [B] time = 0.60, size = 438, normalized size = 1.72

$$\frac{2}{5}\sqrt{5}\sqrt{2\sqrt{5}-2}\arctan\left(\frac{\sqrt{2}x\sqrt{2\sqrt{5}-2}\sqrt{\frac{\sqrt{5}x^2+x^2+2\sqrt{x^4+x^3}}{x^2}}-2\left(x^4+x^3\right)^{\frac{1}{4}}\sqrt{2\sqrt{5}-2}}{4x}\right)+\frac{2}{5}\sqrt{5}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{\sqrt{2}x\sqrt{2\sqrt{5}+2}\sqrt{\frac{\sqrt{5}x^2-x^2+2\sqrt{x^4+x^3}}{x^2}}-2\left(x^4+x^3\right)^{\frac{1}{4}}\sqrt{2\sqrt{5}+2}}{4x}\right)-\frac{1}{10}\sqrt{5}\sqrt{2\sqrt{5}+2}\log\left(\frac{\left(\sqrt{5}x-x\right)\sqrt{2\sqrt{5}+2}+4\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{10}\sqrt{5}\sqrt{2\sqrt{5}+2}\log\left(-\frac{\left(\sqrt{5}x-x\right)\sqrt{2\sqrt{5}+2}-4\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{10}\sqrt{5}\sqrt{2\sqrt{5}-2}\log\left(\frac{\left(\sqrt{5}x+x\right)\sqrt{2\sqrt{5}-2}+4\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{10}\sqrt{5}\sqrt{2\sqrt{5}-2}\log\left(-\frac{\left(\sqrt{5}x+x\right)\sqrt{2\sqrt{5}-2}-4\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{8}\left(x^4+x^3\right)^{\frac{1}{4}}(4x+1)+\frac{29}{16}\arctan\left(\frac{\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{29}{32}\log\left(\frac{x+\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)-\frac{29}{32}\log\left(-\frac{x-\left(x^4+x^3\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)*(x^4+x^3)^(1/4)/(x^2+x-1),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*sqrt(2*sqrt(5) - 2)*arctan(1/4*(sqrt(2)*x*sqrt(2*sqrt(5) - 2)*s
qrt((sqrt(5)*x^2 + x^2 + 2*sqrt(x^4 + x^3))/x^2) - 2*(x^4 + x^3)^(1/4)*sqrt
(2*sqrt(5) - 2))/x) + 2/5*sqrt(5)*sqrt(2*sqrt(5) + 2)*arctan(1/4*(sqrt(2)*x
*sqrt(2*sqrt(5) + 2)*sqrt((sqrt(5)*x^2 - x^2 + 2*sqrt(x^4 + x^3))/x^2) - 2*
(x^4 + x^3)^(1/4)*sqrt(2*sqrt(5) + 2))/x) - 1/10*sqrt(5)*sqrt(2*sqrt(5) + 2
)*log(((sqrt(5)*x - x)*sqrt(2*sqrt(5) + 2) + 4*(x^4 + x^3)^(1/4))/x) + 1/10
*sqrt(5)*sqrt(2*sqrt(5) + 2)*log(-((sqrt(5)*x - x)*sqrt(2*sqrt(5) + 2) - 4*
(x^4 + x^3)^(1/4))/x) - 1/10*sqrt(5)*sqrt(2*sqrt(5) - 2)*log(((sqrt(5)*x +
x)*sqrt(2*sqrt(5) - 2) + 4*(x^4 + x^3)^(1/4))/x) + 1/10*sqrt(5)*sqrt(2*sqrt
(5) - 2)*log(-((sqrt(5)*x + x)*sqrt(2*sqrt(5) - 2) - 4*(x^4 + x^3)^(1/4))/x
) + 1/8*(x^4 + x^3)^(1/4)*(4*x + 1) + 29/16*arctan((x^4 + x^3)^(1/4)/x) + 2
9/32*log((x + (x^4 + x^3)^(1/4))/x) - 29/32*log(-(x - (x^4 + x^3)^(1/4))/x)
```

giac [A] time = 1.12, size = 238, normalized size = 0.93

$$\frac{1}{8}\left(\left(\frac{1}{x}+1\right)^{\frac{5}{4}}+3\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right)x^2-\frac{1}{5}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{\left(\frac{1}{x}+1\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{5}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{\left(\frac{1}{x}+1\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{10}\sqrt{10\sqrt{5}-10}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}+\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right)-\frac{1}{10}\sqrt{10\sqrt{5}+10}\log\left(\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}+\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right)+\frac{1}{10}\sqrt{10\sqrt{5}-10}\log\left(\left|-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}+\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right|\right)+\frac{1}{10}\sqrt{10\sqrt{5}+10}\log\left(\left|-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}+\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right|\right)+\frac{29}{16}\arctan\left(\left(\frac{1}{x}+1\right)^{\frac{1}{4}}\right)+\frac{29}{32}\log\left(\left(\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right)-\frac{29}{32}\log\left(\left|\left(\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)*(x^4+x^3)^(1/4)/(x^2+x-1),x, algorithm="giac")
```

```
[Out] 1/8*((1/x + 1)^(5/4) + 3*(1/x + 1)^(1/4))*x^2 - 1/5*sqrt(10*sqrt(5) - 10)*a
rctan((1/x + 1)^(1/4)/sqrt(1/2*sqrt(5) + 1/2)) - 1/5*sqrt(10*sqrt(5) + 10)*
arctan((1/x + 1)^(1/4)/sqrt(1/2*sqrt(5) - 1/2)) - 1/10*sqrt(10*sqrt(5) - 10
)*log(sqrt(1/2*sqrt(5) + 1/2) + (1/x + 1)^(1/4)) - 1/10*sqrt(10*sqrt(5) + 1
0)*log(sqrt(1/2*sqrt(5) - 1/2) + (1/x + 1)^(1/4)) + 1/10*sqrt(10*sqrt(5) -
10)*log(abs(-sqrt(1/2*sqrt(5) + 1/2) + (1/x + 1)^(1/4))) + 1/10*sqrt(10*sqr
t(5) + 10)*log(abs(-sqrt(1/2*sqrt(5) - 1/2) + (1/x + 1)^(1/4))) + 29/16*arc
tan((1/x + 1)^(1/4)) + 29/32*log((1/x + 1)^(1/4) + 1) - 29/32*log(abs((1/x
+ 1)^(1/4) - 1))
```

maple [C] time = 11.67, size = 4380, normalized size = 17.18

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)*(x^4+x^3)^(1/4)/(x^2+x-1),x)
```

[Out] 1/8*(1+4*x)*(x^3*(1+x))^(1/4)+(-29/32*ln((2*(x^4+3*x^3+3*x^2+x)^(3/4)-2*(x^4+3*x^3+3*x^2+x)^(1/2)*x+2*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2-2*x^3-2*(x^4+3*x^3+3*x^2+x)^(1/2)+4*(x^4+3*x^3+3*x^2+x)^(1/4)*x-5*x^2+2*(x^4+3*x^3+3*x^2+x)^(1/4)-4*x-1)/(1+x)^2)-1/640*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*ln((325*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*RootOf(25*_Z^4-81920*_Z^2-268435456)^4*x^3+325*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*RootOf(25*_Z^4-81920*_Z^2-268435456)^4*x^2-8437760*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(25*_Z^4-81920*_Z^2-268435456)^2*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*x-325*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*RootOf(25*_Z^4-81920*_Z^2-268435456)^4*x+8151040*RootOf(25*_Z^4-81920*_Z^2-268435456)^2*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*x^3+2359296000*(x^4+3*x^3+3*x^2+x)^(3/4)*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-8437760*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(25*_Z^4-81920*_Z^2-268435456)^2*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)-1520435200*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(25*_Z^4-81920*_Z^2-268435456)^2*x^2-325*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*RootOf(25*_Z^4-81920*_Z^2-268435456)^4+19865600*RootOf(25*_Z^4-81920*_Z^2-268435456)^2*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*x^2-16374562816*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*x-3040870400*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(25*_Z^4-81920*_Z^2-268435456)^2*x+15278080*RootOf(25*_Z^4-81920*_Z^2-268435456)^2*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*x+15971909632*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*x^3+4982162063360*(x^4+3*x^3+3*x^2+x)^(3/4)-16374562816*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)-1520435200*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-2748779069440*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2+3563520*RootOf(25*_Z^4-81920*_Z^2-268435456)^2*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)+41070624768*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*x^2-5497558138880*(x^4+3*x^3+3*x^2+x)^(1/4)*x+34225520640*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*x-2748779069440*(x^4+3*x^3+3*x^2+x)^(1/4)+9126805504*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920))/(5*x*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-5*RootOf(25*_Z^4-81920*_Z^2-268435456)^2+16384*x)/(1+x)^2)+1/128*RootOf(25*_Z^4-81920*_Z^2-268435456)*ln(-(325*x^3*RootOf(25*_Z^4-81920*_Z^2-268435456)^5+325*x^2*RootOf(25*_Z^4-81920*_Z^2-268435456)^5+8437760*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(25*_Z^4-81920*_Z^2-268435456)^3*x-325*x*RootOf(25*_Z^4-81920*_Z^2-268435456)^5-10280960*RootOf(25*_Z^4-81920*_Z^2-268435456)^3*x^3+471859200*(x^4+3*x^3+3*x^2+x)^(3/4)*RootOf(25*_Z^4-81920*_Z^2-268435456)^2+8437760*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(25*_Z^4-81920*_Z^2-268435456)^3-304087040*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(25*_Z^4-81920*_Z^2-268435456)^2*x^2-325*RootOf(25*_Z^4-81920*_Z^2-268435456)^5-21995520*RootOf(25*_Z^4-81920*_Z^2-268435456)^3*x^2-44023414784*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(25*_Z^4-81920*_Z^2-268435456)*x-608174080*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(25*_Z^4-81920*_Z^2-268435456)^2*x-13148160*RootOf(25*_Z^4-81920*_Z^2-268435456)^3*x+46170898432*RootOf(25*_Z^4-81920*_Z^2-268435456)*x^3-2542620639232*(x^4+3*x^3+3*x^2+x)^(3/4)-44023414784*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(25*_Z^4-81920*_Z^2-268435456)-304087040*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(25*_Z^4-81920*_Z^2-268435456)^2+1546188226560*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2-1433600*RootOf(25*_Z^4-81920*_Z^2-268435456)^3+109655883776*RootOf(25*_Z^4-81920*_Z^2-268435456)*x^2+3092376453120*(x^4+3*x^3+3*x^2+x)^(1/4)*x+80799072256*RootOf(25*_Z^4-81920*_Z^2-268435456)*x+1546188226560*(x^4+3*x^3+3*x^2+x)^(1/4)+17314086912*RootOf(25*_Z^4-81920*_Z^2-268435456))/(5*x*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-5*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-32768*x+16384)/(1+x)^2)+1/2097152*RootOf(25*_Z^4-81920*_Z^2-268435456)^2*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*ln(-(-425*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*RootOf(25*_Z^4-81920*_Z^2-268435456)^4*x^3-425*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*RootOf(25*_Z^4-81920*_Z^2-268435456)^4*x^2+9994240*(x^4+3*x^3+3*x^2+x)^(1/2)*R

```
ootOf(25*_Z^4-81920*_Z^2-268435456)^2*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_
Z^2-268435456)^2-81920)*x+425*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-2684
35456)^2-81920)*RootOf(25*_Z^4-81920*_Z^2-268435456)^4*x+13271040*RootOf(25
*_Z^4-81920*_Z^2-268435456)^2*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-2684
35456)^2-81920)*x^3+4718592000*(x^4+3*x^3+3*x^2+x)^(3/4)*RootOf(25*_Z^4-819
20*_Z^2-268435456)^2+9994240*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(25*_Z^4-81920
*_Z^2-268435456)^2*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81
920)+3040870400*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(25*_Z^4-81920*_Z^2-2684354
56)^2*x^2+425*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*
RootOf(25*_Z^4-81920*_Z^2-268435456)^4+28590080*RootOf(25*_Z^4-81920*_Z^2-2
68435456)^2*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*x^
2-55297703936*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(_Z^2+25*RootOf(25*_Z^4-81920
*_Z^2-268435456)^2-81920)*x+6081740800*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(25*
_Z^4-81920*_Z^2-268435456)^2*x+17367040*RootOf(25*_Z^4-81920*_Z^2-268435456
)^2*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*x-55834574
848*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*x^3-254262
06392320*(x^4+3*x^3+3*x^2+x)^(3/4)-55297703936*(x^4+3*x^3+3*x^2+x)^(1/2)*Ro
otOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)+3040870400*(x^4+
3*x^3+3*x^2+x)^(1/4)*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-15461882265600*
(x^4+3*x^3+3*x^2+x)^(1/4)*x^2+2048000*RootOf(25*_Z^4-81920*_Z^2-268435456)^
2*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)-132607115264
*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*x^2-309237645
31200*(x^4+3*x^3+3*x^2+x)^(1/4)*x-97710505984*RootOf(_Z^2+25*RootOf(25*_Z^4
-81920*_Z^2-268435456)^2-81920)*x-15461882265600*(x^4+3*x^3+3*x^2+x)^(1/4)-
20937965568*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920))/(
5*x*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-5*RootOf(25*_Z^4-81920*_Z^2-2684
35456)^2-32768*x+16384)/(1+x)^2)+5/2097152*ln((-425*x^3*RootOf(25*_Z^4-8192
0*_Z^2-268435456)^5-425*x^2*RootOf(25*_Z^4-81920*_Z^2-268435456)^5-9994240*
(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(25*_Z^4-81920*_Z^2-268435456)^3*x+425*x*Ro
otOf(25*_Z^4-81920*_Z^2-268435456)^5-10485760*RootOf(25*_Z^4-81920*_Z^2-268
435456)^3*x^3+943718400*(x^4+3*x^3+3*x^2+x)^(3/4)*RootOf(25*_Z^4-81920*_Z^2
-268435456)^2-9994240*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(25*_Z^4-81920*_Z^2-2
68435456)^3+608174080*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(25*_Z^4-81920*_Z^2-2
68435456)^2*x^2+425*RootOf(25*_Z^4-81920*_Z^2-268435456)^5-25804800*RootOf(
25*_Z^4-81920*_Z^2-268435456)^3*x^2-22548578304*(x^4+3*x^3+3*x^2+x)^(1/2)*R
ootOf(25*_Z^4-81920*_Z^2-268435456)*x+1216348160*(x^4+3*x^3+3*x^2+x)^(1/4)*
RootOf(25*_Z^4-81920*_Z^2-268435456)^2*x-20152320*RootOf(25*_Z^4-81920*_Z^2
-268435456)^3*x-16911433728*RootOf(25*_Z^4-81920*_Z^2-268435456)*x^3+199286
4825344*(x^4+3*x^3+3*x^2+x)^(3/4)-22548578304*(x^4+3*x^3+3*x^2+x)^(1/2)*Roo
tOf(25*_Z^4-81920*_Z^2-268435456)+608174080*(x^4+3*x^3+3*x^2+x)^(1/4)*RootO
f(25*_Z^4-81920*_Z^2-268435456)^2+1099511627776*(x^4+3*x^3+3*x^2+x)^(1/4)*x
^2-4833280*RootOf(25*_Z^4-81920*_Z^2-268435456)^3-43486543872*RootOf(25*_Z^
4-81920*_Z^2-268435456)*x^2+2199023255552*(x^4+3*x^3+3*x^2+x)^(1/4)*x-36238
786560*RootOf(25*_Z^4-81920*_Z^2-268435456)*x+1099511627776*(x^4+3*x^3+3*x^
2+x)^(1/4)-9663676416*RootOf(25*_Z^4-81920*_Z^2-268435456))/(5*x*RootOf(25*
_Z^4-81920*_Z^2-268435456)^2-5*RootOf(25*_Z^4-81920*_Z^2-268435456)^2+16384
*x)/(1+x)^2)*RootOf(25*_Z^4-81920*_Z^2-268435456)^3-1/128*ln((-425*x^3*Root
Of(25*_Z^4-81920*_Z^2-268435456)^5-425*x^2*RootOf(25*_Z^4-81920*_Z^2-268435
456)^5-9994240*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(25*_Z^4-81920*_Z^2-26843545
6)^3*x+425*x*RootOf(25*_Z^4-81920*_Z^2-268435456)^5-10485760*RootOf(25*_Z^4
-81920*_Z^2-268435456)^3*x^3+943718400*(x^4+3*x^3+3*x^2+x)^(3/4)*RootOf(25*
_Z^4-81920*_Z^2-268435456)^2-9994240*(x^4+3*x^3+3*x^2+x)^(1/2)*RootOf(25*_Z
^4-81920*_Z^2-268435456)^3+608174080*(x^4+3*x^3+3*x^2+x)^(1/4)*RootOf(25*_Z
^4-81920*_Z^2-268435456)^2*x^2+425*RootOf(25*_Z^4-81920*_Z^2-268435456)^5-2
5804800*RootOf(25*_Z^4-81920*_Z^2-268435456)^3*x^2-22548578304*(x^4+3*x^3+3
*x^2+x)^(1/2)*RootOf(25*_Z^4-81920*_Z^2-268435456)*x+1216348160*(x^4+3*x^3+
3*x^2+x)^(1/4)*RootOf(25*_Z^4-81920*_Z^2-268435456)^2*x-20152320*RootOf(25*
_Z^4-81920*_Z^2-268435456)^3*x-16911433728*RootOf(25*_Z^4-81920*_Z^2-268435
456)*x^3+1992864825344*(x^4+3*x^3+3*x^2+x)^(3/4)-22548578304*(x^4+3*x^3+3*x
```

```
^2+x)^(1/2)*RootOf(25*_Z^4-81920*_Z^2-268435456)+608174080*(x^4+3*x^3+3*x^2
+x)^(1/4)*RootOf(25*_Z^4-81920*_Z^2-268435456)^2+1099511627776*(x^4+3*x^3+3
*x^2+x)^(1/4)*x^2-4833280*RootOf(25*_Z^4-81920*_Z^2-268435456)^3-4348654387
2*RootOf(25*_Z^4-81920*_Z^2-268435456)*x^2+2199023255552*(x^4+3*x^3+3*x^2+x
)^(1/4)*x-36238786560*RootOf(25*_Z^4-81920*_Z^2-268435456)*x+1099511627776*
(x^4+3*x^3+3*x^2+x)^(1/4)-9663676416*RootOf(25*_Z^4-81920*_Z^2-268435456))/
(5*x*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-5*RootOf(25*_Z^4-81920*_Z^2-268
435456)^2+16384*x)/(1+x)^2)*RootOf(25*_Z^4-81920*_Z^2-268435456)+29/524288*
RootOf(25*_Z^4-81920*_Z^2-268435456)*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z
^2-268435456)^2-81920)*ln((-2*RootOf(25*_Z^4-81920*_Z^2-268435456)*RootOf(_
Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*(x^4+3*x^3+3*x^2+x)^(1
/2)*x+2*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*RootOf
(25*_Z^4-81920*_Z^2-268435456)*x^3-2*RootOf(25*_Z^4-81920*_Z^2-268435456)*R
ootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*(x^4+3*x^3+3*x^
2+x)^(1/2)+5*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2-268435456)^2-81920)*R
ootOf(25*_Z^4-81920*_Z^2-268435456)*x^2+32768*(x^4+3*x^3+3*x^2+x)^(3/4)-327
68*(x^4+3*x^3+3*x^2+x)^(1/4)*x^2+4*RootOf(_Z^2+25*RootOf(25*_Z^4-81920*_Z^2
-268435456)^2-81920)*RootOf(25*_Z^4-81920*_Z^2-268435456)*x-65536*(x^4+3*x^
3+3*x^2+x)^(1/4)*x+RootOf(25*_Z^4-81920*_Z^2-268435456)*RootOf(_Z^2+25*Root
Of(25*_Z^4-81920*_Z^2-268435456)^2-81920)-32768*(x^4+3*x^3+3*x^2+x)^(1/4))/
(1+x)^2))/x/(1+x)*(x^3*(1+x))^(1/4)*(x*(1+x)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3\right)^{\frac{1}{4}}\left(x^2+x\right)}{x^2+x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)*(x^4+x^3)^(1/4)/(x^2+x-1),x, algorithm="maxima")
[Out] integrate((x^4 + x^3)^(1/4)*(x^2 + x)/(x^2 + x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^4+x^3\right)^{1/4}\,\left(x^2+x\right)}{x^2+x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + x^4)^(1/4)*(x + x^2))/(x + x^2 - 1),x)
[Out] int(((x^3 + x^4)^(1/4)*(x + x^2))/(x + x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[4]{x^3\,(x+1)}\,(x+1)}{x^2+x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)*(x**4+x**3)**(1/4)/(x**2+x-1),x)
[Out] Integral(x*(x**3*(x + 1))**(1/4)*(x + 1)/(x**2 + x - 1), x)
```

3.2187

$$\int \frac{(b+x^4)^2}{\sqrt[4]{ab^4+4ab^3x^4+b^4x^4+6ab^2x^8+4b^3x^8+4abx^{12}+6b^2x^{12}+ax^{16}+4bx^{16}+x^{20}}}\,dx$$

Optimal. Leaf size=255

$$\frac{x\left(ab^4+x^{12}\left(4ab+6b^2\right)+x^4\left(4ab^3+b^4\right)+x^8\left(6ab^2+4b^3\right)+x^{16}(a+4b)+x^{20}\right)^{3/4}}{4\left(b+x^4\right)^3}+\frac{1}{8}(a-4b)\tan^{-1}\left(\sqrt[4]{ab^4+\ldots}\right.$$

Rubi [A] time = 0.52, antiderivative size = 137, normalized size of antiderivative = 0.54, number of steps used = 11, number of rules used = 8, integrand size = 80, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6, 6688, 6719, 388, 240, 212, 206, 203}

$$\frac{x\left(a+x^4\right)\left(b+x^4\right)}{4\sqrt[4]{\left(a+x^4\right)\left(b+x^4\right)^4}}-\frac{(a-4b)\sqrt[4]{a+x^4}\left(b+x^4\right)\tan^{-1}\left(\frac{x}{\sqrt[4]{a+x^4}}\right)}{8\sqrt[4]{\left(a+x^4\right)\left(b+x^4\right)^4}}-\frac{(a-4b)\sqrt[4]{a+x^4}\left(b+x^4\right)\tanh^{-1}\left(\frac{x}{\sqrt[4]{a+x^4}}\right)}{8\sqrt[4]{\left(a+x^4\right)\left(b+x^4\right)^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + x^4)^2/(a*b^4 + 4*a*b^3*x^4 + b^4*x^4 + 6*a*b^2*x^8 + 4*b^3*x^8 +
4*a*b*x^12 + 6*b^2*x^12 + a*x^16 + 4*b*x^16 + x^20)^(1/4),x]
```

```
[Out] (x*(a + x^4)*(b + x^4))/(4*((a + x^4)*(b + x^4)^4)^(1/4)) - ((a - 4*b)*(a +
 x^4)^(1/4)*(b + x^4)*ArcTan[x/(a + x^4)^(1/4)])/(8*((a + x^4)*(b + x^4)^4)
^(1/4)) - ((a - 4*b)*(a + x^4)^(1/4)*(b + x^4)*ArcTanh[x/(a + x^4)^(1/4)])/
(8*((a + x^4)*(b + x^4)^4)^(1/4))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\left(b+x^4\right)^2}{\sqrt[4]{ab^4+4ab^3x^4+b^4x^4+6ab^2x^8+4b^3x^8+4abx^{12}+6b^2x^{12}+ax^{16}+4bx^{16}+x^{20}}}\,dx = \int \frac{}{\sqrt[4]{ab^4+\left(4ab^3+\right.}}$$

$$= \int \frac{}{\sqrt[4]{ab^4+\left(4ab^3+\right.}}$$

$$= \int \frac{}{\sqrt[4]{ab^4+\left(4ab^3+\right.}}$$

$$= \int \frac{}{\sqrt[4]{ab^4+\left(4ab^3+\right.}}$$

$$= \int \frac{\left(b+x^4\right)^2}{\sqrt[4]{\left(a+x^4\right)\left(b+\right.}}$$

$$= \frac{\left(\sqrt[4]{a+x^4}\,\left(b+x^4\right)\right)}{\sqrt[4]{\left(a+x^4\right)\left(b\right.}}$$

$$= \frac{x\left(a+x^4\right)\left(b+x^4\right.}{4\sqrt[4]{\left(a+x^4\right)\left(b+x^4\right.}}$$

$$= \frac{x\left(a+x^4\right)\left(b+x^4\right.}{4\sqrt[4]{\left(a+x^4\right)\left(b+x^4\right.}}$$

$$= \frac{x\left(a+x^4\right)\left(b+x^4\right.}{4\sqrt[4]{\left(a+x^4\right)\left(b+x^4\right.}}$$

$$= \frac{x\left(a+x^4\right)\left(b+x^4\right.}{4\sqrt[4]{\left(a+x^4\right)\left(b+x^4\right.}}$$

Mathematica [A] time = 0.14, size = 77, normalized size = 0.30

$$\frac{\left(b+x^4\right)\left(x\left(a+x^4\right)-\frac{1}{2}(a-4b)\sqrt[4]{a+x^4}\left(\tan^{-1}\left(\frac{x}{\sqrt[4]{a+x^4}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{a+x^4}}\right)\right)\right)}{4\sqrt[4]{\left(a+x^4\right)\left(b+x^4\right)^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + x^4)^2/(a*b^4 + 4*a*b^3*x^4 + b^4*x^4 + 6*a*b^2*x^8 + 4*b^3*
x^8 + 4*a*b*x^12 + 6*b^2*x^12 + a*x^16 + 4*b*x^16 + x^20)^(1/4),x]
```

```
[Out] ((b + x^4)*(x*(a + x^4) - ((a - 4*b)*(a + x^4)^(1/4)*(ArcTan[x/(a + x^4)^(1
/4)] + ArcTanh[x/(a + x^4)^(1/4)]))/2))/(4*((a + x^4)*(b + x^4)^4)^(1/4))
```

IntegrateAlgebraic [A] time = 0.47, size = 255, normalized size = 1.00

$$\frac{x\left(ab^4+x^{12}\left(4ab+6b^2\right)+x^4\left(4ab^3+b^4\right)+x^8\left(6ab^2+4b^3\right)+x^{16}(a+4b)+x^{20}\right)^{3/4}}{4\left(b+x^4\right)^3}+\frac{1}{8}(a-4b)\tan^{-1}\left(\frac{\sqrt[4]{ab^4+x^{12}\left(4ab+6b^2\right)+x^4\left(4ab^3+b^4\right)+x^8\left(6ab^2+4b^3\right)+x^{16}(a+4b)+x^{20}}}{x\left(b+x^4\right)}\right)+\frac{1}{8}(4b-a)\tanh^{-1}\left(\frac{\sqrt[4]{ab^4+x^{12}\left(4ab+6b^2\right)+x^4\left(4ab^3+b^4\right)+x^8\left(6ab^2+4b^3\right)+x^{16}(a+4b)+x^{20}}}{x\left(b+x^4\right)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + x^4)^2/(a*b^4 + 4*a*b^3*x^4 + b^4*x^4 + 6*a*b^2*x^8
 + 4*b^3*x^8 + 4*a*b*x^12 + 6*b^2*x^12 + a*x^16 + 4*b*x^16 + x^20)^(1/4),x]
```

```
[Out] (x*(a*b^4 + (4*a*b^3 + b^4)*x^4 + (6*a*b^2 + 4*b^3)*x^8 + (4*a*b + 6*b^2)*x
^12 + (a + 4*b)*x^16 + x^20)^(3/4))/(4*(b + x^4)^3) + ((a - 4*b)*ArcTan[(a*
b^4 + (4*a*b^3 + b^4)*x^4 + (6*a*b^2 + 4*b^3)*x^8 + (4*a*b + 6*b^2)*x^12 +
(a + 4*b)*x^16 + x^20)^(1/4)/(x*(b + x^4))])/8 + ((-a + 4*b)*ArcTanh[(a*b^4
 + (4*a*b^3 + b^4)*x^4 + (6*a*b^2 + 4*b^3)*x^8 + (4*a*b + 6*b^2)*x^12 + (a
+ 4*b)*x^16 + x^20)^(1/4)/(x*(b + x^4))])/8
```

fricas [B] time = 0.59, size = 498, normalized size = 1.95

$$\frac{2\left((a-4b)x^{12}+3\left(ab-4b^2\right)x^8+3\left(ab^2-4b^3\right)x^4+ab^3-4b^4\right)\arctan\left(\frac{\left(x^{20}+(a+4b)x^{16}+2\left(2ab+3b^2\right)x^{12}+2\left(3ab^2+2b^3\right)x^8+ab^4+\left(4ab^3+b^4\right)x^4\right)^{\frac{1}{4}}}{x^5+bx}\right)-\left((a-4b)x^{12}+3\left(ab-4b^2\right)x^8+3\left(ab^2-4b^3\right)x^4+ab^3-4b^4\right)\log\left(\frac{x^5+bx+\left(x^{20}+(a+4b)x^{16}+2\left(2ab+3b^2\right)x^{12}+2\left(3ab^2+2b^3\right)x^8+ab^4+\left(4ab^3+b^4\right)x^4\right)^{\frac{1}{4}}}{x^5+bx}\right)+\left((a-4b)x^{12}+3\left(ab-4b^2\right)x^8+3\left(ab^2-4b^3\right)x^4+ab^3-4b^4\right)\log\left(-\frac{x^5+bx-\left(x^{20}+(a+4b)x^{16}+2\left(2ab+3b^2\right)x^{12}+2\left(3ab^2+2b^3\right)x^8+ab^4+\left(4ab^3+b^4\right)x^4\right)^{\frac{1}{4}}}{x^5+bx}\right)+4\left(x^{20}+(a+4b)x^{16}+2\left(2ab+3b^2\right)x^{12}+2\left(3ab^2+2b^3\right)x^8+ab^4+\left(4ab^3+b^4\right)x^4\right)^{\frac{3}{4}}x}{16\left(x^{12}+3bx^8+3b^2x^4+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+b)^2/(x^20+a*x^16+4*b*x^16+4*a*b*x^12+6*b^2*x^12+6*a*b^2*x^8
+4*b^3*x^8+4*a*b^3*x^4+b^4*x^4+a*b^4)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/16*(2*((a - 4*b)*x^12 + 3*(a*b - 4*b^2)*x^8 + 3*(a*b^2 - 4*b^3)*x^4 + a*b
^3 - 4*b^4)*arctan((x^20 + (a + 4*b)*x^16 + 2*(2*a*b + 3*b^2)*x^12 + 2*(3*a
*b^2 + 2*b^3)*x^8 + a*b^4 + (4*a*b^3 + b^4)*x^4)^(1/4)/(x^5 + b*x)) - ((a -
 4*b)*x^12 + 3*(a*b - 4*b^2)*x^8 + 3*(a*b^2 - 4*b^3)*x^4 + a*b^3 - 4*b^4)*l
og((x^5 + b*x + (x^20 + (a + 4*b)*x^16 + 2*(2*a*b + 3*b^2)*x^12 + 2*(3*a*b^
2 + 2*b^3)*x^8 + a*b^4 + (4*a*b^3 + b^4)*x^4)^(1/4))/(x^5 + b*x)) + ((a - 4
*b)*x^12 + 3*(a*b - 4*b^2)*x^8 + 3*(a*b^2 - 4*b^3)*x^4 + a*b^3 - 4*b^4)*log
(-(x^5 + b*x - (x^20 + (a + 4*b)*x^16 + 2*(2*a*b + 3*b^2)*x^12 + 2*(3*a*b^2
 + 2*b^3)*x^8 + a*b^4 + (4*a*b^3 + b^4)*x^4)^(1/4))/(x^5 + b*x)) + 4*(x^20
+ (a + 4*b)*x^16 + 2*(2*a*b + 3*b^2)*x^12 + 2*(3*a*b^2 + 2*b^3)*x^8 + a*b^4
 + (4*a*b^3 + b^4)*x^4)^(3/4)*x)/(x^12 + 3*b*x^8 + 3*b^2*x^4 + b^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+b\right)^2}{\left(x^{20}+ax^{16}+4\,bx^{16}+4\,abx^{12}+6\,b^2x^{12}+6\,ab^2x^8+4\,b^3x^8+4\,ab^3x^4+b^4x^4+ab^4\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+b)^2/(x^20+a*x^16+4*b*x^16+4*a*b*x^12+6*b^2*x^12+6*a*b^2*x^8
+4*b^3*x^8+4*a*b^3*x^4+b^4*x^4+a*b^4)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^4 + b)^2/(x^20 + a*x^16 + 4*b*x^16 + 4*a*b*x^12 + 6*b^2*x^12 +
 6*a*b^2*x^8 + 4*b^3*x^8 + 4*a*b^3*x^4 + b^4*x^4 + a*b^4)^(1/4), x)
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+b\right)^2}{\left(x^{20}+a\,x^{16}+4b\,x^{16}+4ab\,x^{12}+6b^2x^{12}+6a\,b^2x^8+4b^3x^8+4a\,b^3x^4+b^4x^4+a\,b^4\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+b)^2/(x^20+a*x^16+4*b*x^16+4*a*b*x^12+6*b^2*x^12+6*a*b^2*x^8+4*b^3
*x^8+4*a*b^3*x^4+b^4*x^4+a*b^4)^(1/4),x)
```

```
[Out] int((x^4+b)^2/(x^20+a*x^16+4*b*x^16+4*a*b*x^12+6*b^2*x^12+6*a*b^2*x^8+4*b^3
*x^8+4*a*b^3*x^4+b^4*x^4+a*b^4)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+b\right)^2}{\left(x^{20}+ax^{16}+4\,bx^{16}+4\,abx^{12}+6\,b^2x^{12}+6\,ab^2x^8+4\,b^3x^8+4\,ab^3x^4+b^4x^4+ab^4\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+b)^2/(x^20+a*x^16+4*b*x^16+4*a*b*x^12+6*b^2*x^12+6*a*b^2*x^8
+4*b^3*x^8+4*a*b^3*x^4+b^4*x^4+a*b^4)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + b)^2/(x^20 + a*x^16 + 4*b*x^16 + 4*a*b*x^12 + 6*b^2*x^12 +
 6*a*b^2*x^8 + 4*b^3*x^8 + 4*a*b^3*x^4 + b^4*x^4 + a*b^4)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^4+b\right)^2}{\left(b^4\,x^4+a\,b^4+4\,b^3\,x^8+4\,a\,b^3\,x^4+6\,b^2\,x^{12}+6\,a\,b^2\,x^8+4\,b\,x^{16}+4\,a\,b\,x^{12}+x^{20}+a\,x^{16}\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + x^4)^2/(a*b^4 + a*x^16 + 4*b*x^16 + x^20 + b^4*x^4 + 4*b^3*x^8 + 6
*b^2*x^12 + 4*a*b^3*x^4 + 6*a*b^2*x^8 + 4*a*b*x^12)^(1/4),x)
```

```
[Out] int((b + x^4)^2/(a*b^4 + a*x^16 + 4*b*x^16 + x^20 + b^4*x^4 + 4*b^3*x^8 + 6
*b^2*x^12 + 4*a*b^3*x^4 + 6*a*b^2*x^8 + 4*a*b*x^12)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b+x^4\right)^2}{\sqrt[4]{\left(a+x^4\right)\left(b+x^4\right)^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+b)**2/(x**20+a*x**16+4*b*x**16+4*a*b*x**12+6*b**2*x**12+6*a
*b**2*x**8+4*b**3*x**8+4*a*b**3*x**4+b**4*x**4+a*b**4)**(1/4),x)
```

```
[Out] Integral((b + x**4)**2/((a + x**4)*(b + x**4)**4)**(1/4), x)
```

3.2188 $\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^4+a(q+px^3)^4\right)}{x^7}\,dx$

Optimal. Leaf size=255

$$\frac{1}{2}\log(x)\left(ap^3q^3+2bpq\right)+\frac{1}{2}\left(-ap^3q^3-2bpq\right)\log\left(\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+px^3+q\right)+\frac{\sqrt{p^2x^6+2pqx^3-2p}}{}$$

Rubi [F] time = 2.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^4+a(q+px^3)^4\right)}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*x^4 + a*
(q + p*x^3)^4))/x^7,x]
```

```
[Out] (7*a*p^2*q*(q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6)^(3/2))/9 + 2*b*p*Defer[I
nt][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6], x] - a*q^5*Defer[Int][Sqrt
[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x^7, x] - 2*a*p*q^4*Defer[Int][Sqrt
[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x^4, x] - b*q*Defer[Int][Sqrt[q^2 -
 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x^3, x] + 2*a*p^2*q^3*Defer[Int][Sqrt[q^2
 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x, x] + (14*a*p^3*q^2*Defer[Int][x*Sqrt
[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6], x])/3 + a*p^3*q^2*Defer[Int][x^2*S
qrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6], x] + 2*a*p^5*Defer[Int][x^8*Sqr
t[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6], x]
```

Rubi steps

$$\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^4+a(q+px^3)^4\right)}{x^7}\,dx = \int \left(2bp\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6} - \frac{a}{}\right.$$

$$= (2bp)\int \sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\,dx$$

$$= \frac{7}{9}ap^2q\left(q^2-2pqx^2+2pqx^3+p^2x^6\right)^{3/2}+($$

$$= \frac{7}{9}ap^2q\left(q^2-2pqx^2+2pqx^3+p^2x^6\right)^{3/2}+($$

$$= \frac{7}{9}ap^2q\left(q^2-2pqx^2+2pqx^3+p^2x^6\right)^{3/2}+($$

$$= \frac{7}{9}ap^2q\left(q^2-2pqx^2+2pqx^3+p^2x^6\right)^{3/2}+($$

Mathematica [F] time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^4+a(q+px^3)^4\right)}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*x^
4 + a*(q + p*x^3)^4))/x^7,x]
```

[Out] Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*x^4 + a*(q + p*x^3)^4))/x^7, x]

IntegrateAlgebraic [A] time = 0.45, size = 255, normalized size = 1.00

$$\frac{1}{2}\log(x)\left(ap^3q^3+2bpq\right)+\frac{1}{2}\left(-ap^3q^3-2bpq\right)\log\left(\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+px^3+q\right)+\frac{\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}\left(2ap^5x^{15}+10ap^4qx^{12}-ap^4qx^{11}+20ap^3q^2x^9-3ap^3q^2x^8-3ap^3q^2x^7+20ap^2q^3x^6-3ap^2q^3x^5-3ap^2q^3x^4+10apq^4x^3-apq^4x^2+2aq^5+6bpx^7+6bqx^4\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*x^4 + a*(q + p*x^3)^4))/x^7,x]
```

```
[Out] (Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(2*a*q^5 - a*p*q^4*x^2 + 10*a*p*q^4*x^3 + 6*b*q*x^4 - 3*a*p^2*q^3*x^4 - 3*a*p^2*q^3*x^5 + 20*a*p^2*q^3*x^6 + 6*b*p*x^7 - 3*a*p^3*q^2*x^7 - 3*a*p^3*q^2*x^8 + 20*a*p^3*q^2*x^9 - a*p^4*q*x^11 + 10*a*p^4*q*x^12 + 2*a*p^5*x^15))/(12*x^6) + ((2*b*p*q + a*p^3*q^3)*Log[x])/2 + ((-2*b*p*q - a*p^3*q^3)*Log[q + p*x^3 + Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]])/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^4+a*(p*x^3+q)^4)/x^7,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6+2\,pqx^3-2\,pqx^2+q^2}\left(\left(px^3+q\right)^4a+bx^4\right)\left(2\,px^3-q\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^4+a*(p*x^3+q)^4)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)*((p*x^3 + q)^4*a + b*x^4)*(2*p*x^3 - q)/x^7, x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\left(2p\,x^3-q\right)\sqrt{p^2x^6+2pq\,x^3-2pq\,x^2+q^2}\,\left(b\,x^4+a\left(p\,x^3+q\right)^4\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^4+a*(p*x^3+q)^4)/x^7,x)
```

```
[Out] int((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^4+a*(p*x^3+q)^4)/x^7,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6+2\,pqx^3-2\,pqx^2+q^2}\left(\left(px^3+q\right)^4a+bx^4\right)\left(2\,px^3-q\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^4+a*(p*x
^3+q)^4)/x^7,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)*((p*x^3 + q)^4*a + b*
x^4)*(2*p*x^3 - q)/x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(q-2\,p\,x^3\right)\,\left(a\,{\left(p\,x^3+q\right)}^4+b\,x^4\right)\,\sqrt{p^2\,x^6+2\,p\,q\,x^3-2\,p\,q\,x^2+q^2}}{x^7} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q - 2*p*x^3)*(a*(q + p*x^3)^4 + b*x^4)*(p^2*x^6 + q^2 - 2*p*q*x^2 +
2*p*q*x^3)^(1/2))/x^7,x)
```

```
[Out] -int(((q - 2*p*x^3)*(a*(q + p*x^3)^4 + b*x^4)*(p^2*x^6 + q^2 - 2*p*q*x^2 +
2*p*q*x^3)^(1/2))/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2 p x^{3} - q\right) \sqrt{p^{2} x^{6} + 2 p q x^{3} - 2 p q x^{2} + q^{2}} \left(a p^{4} x^{12} + 4 a p^{3} q x^{9} + 6 a p^{2} q^{2} x^{6} + 4 a p q^{3} x^{3} + a q^{4} + b x^{4}\right)}{x^{7}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x**3-q)*(p**2*x**6+2*p*q*x**3-2*p*q*x**2+q**2)**(1/2)*(b*x**
4+a*(p*x**3+q)**4)/x**7,x)
```

```
[Out] Integral((2*p*x**3 - q)*sqrt(p**2*x**6 + 2*p*q*x**3 - 2*p*q*x**2 + q**2)*(a
*p**4*x**12 + 4*a*p**3*q*x**9 + 6*a*p**2*q**2*x**6 + 4*a*p*q**3*x**3 + a*q*
*4 + b*x**4)/x**7, x)
```

3.2189 $\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[6]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=255

$$-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{3}\sqrt[6]{c}}\right)}{a\sqrt[6]{c}}+\frac{4\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{3}\sqrt[6]{c}}+\frac{1}{\sqrt{3}}\right)}{a\sqrt[6]{c}}-\frac{8\tanh^{-1}\left(\frac{\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[6]{c}}\right)}{a\sqrt[6]{c}}-$$

Rubi [F] time = 0.35, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[6]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[-b + a^2*x^2]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6)),x]
```

```
[Out] Defer[Int][1/(Sqrt[-b + a^2*x^2]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/
6)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[6]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[6]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Mathematica [A] time = 0.52, size = 298, normalized size = 1.17

$$-\frac{2\left(-\log\left(-\sqrt[6]{c}\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\sqrt[3]{c}\right)+\log\left(\sqrt[6]{c}\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\sqrt[3]{c}\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[6]{c}}}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[6]{c}}+1}{\sqrt{3}}\right)+4\tanh^{-1}\left(\frac{\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[6]{c}}\right)\right)}{a\sqrt[6]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-b + a^2*x^2]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6
)),x]
```

```
[Out] (-2*(2*Sqrt[3]*ArcTan[(1 - (2*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6))
/c^(1/6))/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + (2*(c + (a*x + Sqrt[-b + a^2*x^2
])^(1/4))^(1/6))/c^(1/6))/Sqrt[3]] + 4*ArcTanh[(c + (a*x + Sqrt[-b + a^2*x^
2])^(1/4))^(1/6)/c^(1/6)] - Log[c^(1/3) - c^(1/6)*(c + (a*x + Sqrt[-b + a^2
*x^2])^(1/4))^(1/6) + (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)] + Log[c
^(1/3) + c^(1/6)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6) + (c + (a*x +
 Sqrt[-b + a^2*x^2])^(1/4))^(1/3)]))/(a*c^(1/6))
```

IntegrateAlgebraic [A] time = 1.26, size = 255, normalized size = 1.00

$$-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{3}\sqrt[6]{c}}\right)}{a\sqrt[6]{c}}+\frac{4\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{3}\sqrt[6]{c}}+\frac{1}{\sqrt{3}}\right)}{a\sqrt[6]{c}}-\frac{8\tanh^{-1}\left(\frac{\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[6]{c}}\right)}{a\sqrt[6]{c}}-\frac{4\tanh^{-1}\left(\frac{\frac{\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[6]{c}}+\sqrt[6]{c}}{\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}\right)}{a\sqrt[6]{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-b + a^2*x^2]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1
/4))^(1/6)),x]
```

```
[Out] (-4*Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1
/6))/(Sqrt[3]*c^(1/6))]])/(a*c^(1/6)) + (4*Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(c
+ (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6))/(Sqrt[3]*c^(1/6))]])/(a*c^(1/6))
- (8*ArcTanh[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6)/c^(1/6)]])/(a*c^(1
/6)) - (4*ArcTanh[(c^(1/6) + (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)/c
^(1/6))/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6)]])/(a*c^(1/6))
```

fricas [B] time = 0.85, size = 590, normalized size = 2.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/6),x, al
gorithm="fricas")
```

```
[Out] -8*sqrt(3)*(1/(a^6*c))^(1/6)*arctan(2/3*sqrt(3)*sqrt(a^5*(c + (a*x + sqrt(a
^2*x^2 - b))^(1/4))^(1/6)*c*(1/(a^6*c))^(5/6) + a^4*c*(1/(a^6*c))^(2/3) + (
c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3))*a*(1/(a^6*c))^(1/6) - 2/3*sqrt(
3)*a*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/6)*(1/(a^6*c))^(1/6) - 1/3*sq
rt(3)) - 8*sqrt(3)*(1/(a^6*c))^(1/6)*arctan(1/3*sqrt(3)*sqrt(-4*a^5*(c + (a
*x + sqrt(a^2*x^2 - b))^(1/4))^(1/6)*c*(1/(a^6*c))^(5/6) + 4*a^4*c*(1/(a^6*
c))^(2/3) + 4*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3))*a*(1/(a^6*c))^(1
/6) - 2/3*sqrt(3)*a*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/6)*(1/(a^6*c))
^(1/6) + 1/3*sqrt(3)) - 2*(1/(a^6*c))^(1/6)*log(4*a^5*(c + (a*x + sqrt(a^2*
x^2 - b))^(1/4))^(1/6)*c*(1/(a^6*c))^(5/6) + 4*a^4*c*(1/(a^6*c))^(2/3) + 4*
(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)) + 2*(1/(a^6*c))^(1/6)*log(-4*a
^5*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/6)*c*(1/(a^6*c))^(5/6) + 4*a^4*
c*(1/(a^6*c))^(2/3) + 4*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)) - 4*(1
/(a^6*c))^(1/6)*log(a^5*c*(1/(a^6*c))^(5/6) + (c + (a*x + sqrt(a^2*x^2 - b)
)^(1/4))^(1/6)) + 4*(1/(a^6*c))^(1/6)*log(-a^5*c*(1/(a^6*c))^(5/6) + (c + (
a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/6))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/6),x, al
gorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^{2} x^{2}-b}\, \left(c +\left(a x +\sqrt{a^{2} x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/6),x)
```

```
[Out] int(1/(a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2-b}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/6),x, al
gorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 - b)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/6))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c+\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/4}\right)^{1/6}\,\sqrt{a^2\,x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(1/6)*(a^2*x^2 - b)^(1/2)),x
)
```

```
[Out] int(1/((c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(1/6)*(a^2*x^2 - b)^(1/2)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}\,\sqrt{a^2x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*x**2-b)**(1/2)/(c+(a*x+(a**2*x**2-b)**(1/2))**(1/4))**(1/
6),x)
```

```
[Out] Integral(1/((c + (a*x + sqrt(a**2*x**2 - b))**(1/4))**(1/6)*sqrt(a**2*x**2
- b)), x)
```

3.2190 $\int \frac{(1-2k^2)x+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(-1+d+(1-2dk^2)x^2+dk^4x^4)}\,dx$

Optimal. Leaf size=256

$$-\frac{\log\left(d^{2/3}k^4x^4-2d^{2/3}k^2x^2+d^{2/3}+\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}\left(\sqrt[3]{d}-\sqrt[3]{d}\,k^2x^2\right)+\left(k^2x^4+(-k^2-1)x^2+1\right)^{2/3}\right)}{4\sqrt[3]{d}}+\cdots$$

Rubi [F] time = 0.87, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-2k^2)x+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(-1+d+(1-2dk^2)x^2+dk^4x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 - 2*k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3)*(-1 + d + (1
 - 2*d*k^2)*x^2 + d*k^4*x^4)),x]
```

```
[Out] Defer[Subst][Defer[Int][(1 - 2*k^2 + k^2*x)/((1 + (-1 - k^2)*x + k^2*x^2)^(
1/3)*(-1 + d + (1 - 2*d*k^2)*x + d*k^4*x^2)), x], x, x^2]/2
```

Rubi steps

$$\int \frac{(1-2k^2)x+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(-1+d+(1-2dk^2)x^2+dk^4x^4)}\,dx=\int \frac{x(1-2k^2+k^2x^2)}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(-1+d+(1-2dk^2)x^2\cdots}$$

$$=\frac{1}{2}\text{Subst}\left(\int \frac{1-2k^2+k^2x}{\sqrt[3]{(1-x)(1-k^2x)}\,(-1+d+(1-2d\cdots}\right.$$

$$=\frac{1}{2}\text{Subst}\left(\int \frac{1-2k^2+k^2x}{\sqrt[3]{1+(-1-k^2)x+k^2x^2}\,(-1+d+\cdots}\right.$$

Mathematica [F] time = 14.27, size = 0, normalized size = 0.00

$$\int \frac{(1-2k^2)x+k^2x^3}{\sqrt[3]{(1-x^2)(1-k^2x^2)}\,(-1+d+(1-2dk^2)x^2+dk^4x^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 - 2*k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3)*(-1 +
d + (1 - 2*d*k^2)*x^2 + d*k^4*x^4)),x]
```

```
[Out] Integrate[((1 - 2*k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/3)*(-1 +
d + (1 - 2*d*k^2)*x^2 + d*k^4*x^4)), x]
```

IntegrateAlgebraic [A] time = 4.97, size = 256, normalized size = 1.00

$$-\frac{\log\left(d^{2/3}k^4x^4-2d^{2/3}k^2x^2+d^{2/3}+\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}\left(\sqrt[3]{d}-\sqrt[3]{d}\,k^2x^2\right)+\left(k^2x^4+(-k^2-1)x^2+1\right)^{2/3}\right)}{4\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{d}\,k^2x^2-\sqrt[3]{d}+\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}\right)}{2\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}}{-2\sqrt[3]{d}\,k^2x^2+2\sqrt[3]{d}+\sqrt[3]{k^2x^4+(-k^2-1)x^2+1}}\right)}{2\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 - 2*k^2)*x + k^2*x^3)/(((1 - x^2)*(1 - k^2*x^2))^(1/
3)*(-1 + d + (1 - 2*d*k^2)*x^2 + d*k^4*x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))/(2*d^(1/3) -
 2*d^(1/3)*k^2*x^2 + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3))])/(2*d^(1/3)) +
Log[-d^(1/3) + d^(1/3)*k^2*x^2 + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3)]/(2*d
^(1/3)) - Log[d^(2/3) - 2*d^(2/3)*k^2*x^2 + d^(2/3)*k^4*x^4 + (d^(1/3) - d^
(1/3)*k^2*x^2)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(1/3) + (1 + (-1 - k^2)*x^2 +
 k^2*x^4)^(2/3)]/(4*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2*k^2+1)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d+(-2*d*
k^2+1)*x^2+d*k^4*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3 - \left(2\,k^2 - 1\right)x}{\left(dk^4x^4 - \left(2\,dk^2 - 1\right)x^2 + d - 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2*k^2+1)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d+(-2*d*
k^2+1)*x^2+d*k^4*x^4),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^3 - (2*k^2 - 1)*x)/((d*k^4*x^4 - (2*d*k^2 - 1)*x^2 + d - 1
)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(-2k^2+1\right)x + k^2x^3}{\left(\left(-x^2+1\right)\left(-k^2x^2+1\right)\right)^{\frac{1}{3}}\left(-1+d+\left(-2d\,k^2+1\right)x^2+d\,k^4x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-2*k^2+1)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d+(-2*d*k^2+1)
*x^2+d*k^4*x^4),x)
```

```
[Out] int(((-2*k^2+1)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d+(-2*d*k^2+1)
*x^2+d*k^4*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^3 - \left(2\,k^2 - 1\right)x}{\left(dk^4x^4 - \left(2\,dk^2 - 1\right)x^2 + d - 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2*k^2+1)*x+k^2*x^3)/((-x^2+1)*(-k^2*x^2+1))^(1/3)/(-1+d+(-2*d*
k^2+1)*x^2+d*k^4*x^4),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^3 - (2*k^2 - 1)*x)/((d*k^4*x^4 - (2*d*k^2 - 1)*x^2 + d - 1
)*((k^2*x^2 - 1)*(x^2 - 1))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int -\frac{k^2 x^3 - x \left(2 k^2 - 1\right)}{\left(\left(x^2 - 1\right) \left(k^2 x^2 - 1\right)\right)^{1/3} \left(d - x^2 \left(2 d k^2 - 1\right) + d k^4 x^4 - 1\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^3 - x*(2*k^2 - 1))/(((x^2 - 1)*(k^2*x^2 - 1))^(1/3)*(d - x^2*(2*
d*k^2 - 1) + d*k^4*x^4 - 1)),x)
```

```
[Out] -int(-(k^2*x^3 - x*(2*k^2 - 1))/(((x^2 - 1)*(k^2*x^2 - 1))^(1/3)*(d - x^2*(
2*d*k^2 - 1) + d*k^4*x^4 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-2*k**2+1)*x+k**2*x**3)/((-x**2+1)*(-k**2*x**2+1))**(1/3)/(-1+d
+(-2*d*k**2+1)*x**2+d*k**4*x**4),x)
```

```
[Out] Timed out
```

3.2191 $$\int \frac{(2-k^2)x-2x^3+k^2x^5}{((1-x^2)(1-k^2x^2))^{2/3}(1-d+(-2+dk^2)x^2+x^4)}\,dx$$

Optimal. Leaf size=256

$$\frac{\log\left(d^{2/3}k^4x^4-2d^{2/3}k^2x^2+d^{2/3}+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{2/3}\left(\sqrt[3]{d}-\sqrt[3]{d}\,k^2x^2\right)+\left(k^2x^4+\left(-k^2-1\right)x^2+1\right)^{4/3}\right.}{4\sqrt[3]{d}}$$

Rubi [C] time = 2.57, antiderivative size = 275, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 7, integrand size = 66, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.106, Rules used = {1594, 6715, 6719, 1586, 6728, 137, 136}

$$\frac{3\left(1-x^2\right)^2\left(\frac{1-k^2x^2}{1-k^2}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{k^2(1-x^2)}{1-k^2},\frac{2(1-x^2)}{\sqrt{d}\left(\sqrt{d}\,k^2-\sqrt{dk^4-4k^2+4}\right)}\right)}{8d\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}}+\frac{3\left(1-x^2\right)^2\left(\frac{1-k^2x^2}{1-k^2}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{k^2(1-x^2)}{1-k^2},\frac{2(1-x^2)}{\sqrt{d}\left(\sqrt{d}\,k^2+\sqrt{dk^4-4k^2+4}\right)}\right)}{8d\left(\left(1-x^2\right)\left(1-k^2x^2\right)\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((2 - k^2)*x - 2*x^3 + k^2*x^5)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)*(1 - d
 + (-2 + d*k^2)*x^2 + x^4)),x]
```

```
[Out] (3*(1 - x^2)^2*((1 - k^2*x^2)/(1 - k^2))^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -
((k^2*(1 - x^2))/(1 - k^2)), (2*(1 - x^2))/(Sqrt[d]*(Sqrt[d]*k^2 - Sqrt[4 -
 4*k^2 + d*k^4]))])/(8*d*((1 - x^2)*(1 - k^2*x^2))^(2/3)) + (3*(1 - x^2)^2*
((1 - k^2*x^2)/(1 - k^2))^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((k^2*(1 - x^2)
)/(1 - k^2)), (2*(1 - x^2))/(Sqrt[d]*(Sqrt[d]*k^2 + Sqrt[4 - 4*k^2 + d*k^4]
))])/(8*d*((1 - x^2)*(1 - k^2*x^2))^(2/3))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(2-k^2)x-2x^3+k^2x^5}{\left((1-x^2)(1-k^2x^2)\right)^{2/3}\left(1-d+(-2+dk^2)x^2+x^4\right)}\,dx = \int \frac{x(2-k^2-2x^2+k^2x^4)}{\left((1-x^2)(1-k^2x^2)\right)^{2/3}\left(1-d+(-2+dk^2)x^2+\right.}$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{2-k^2-2x+k^2x^2}{\left((1-x)(1-k^2x)\right)^{2/3}\left(1-d+(-2+dk\right.}\right.$$

$$= \frac{\left((1-x^2)^{2/3}(1-k^2x^2)^{2/3}\right)\text{Subst}\left(\int \frac{2-k^2-2}{(1-x)^{2/3}(1-k^2x)^{2/3}(1}\right.}{2\left((1-x^2)(1-k^2x^2)\right)^{2/3}}$$

$$= \frac{\left((1-x^2)^{2/3}(1-k^2x^2)^{2/3}\right)\text{Subst}\left(\int \frac{\sqrt[3]{1-x}\,(2-k^2-}{(1-k^2x)^{2/3}(1-d+(-2}\right.}{2\left((1-x^2)(1-k^2x^2)\right)^{2/3}}$$

$$= \frac{\left((1-x^2)^{2/3}(1-k^2x^2)^{2/3}\right)\text{Subst}\left(\int\left(\frac{\left(-k^2+\frac{\sqrt{4-}}{}\right.}{\left(-2+dk^2-\sqrt{d}\sqrt{4-}\right.}\right.\right.}{2\left((1\right.}$$

$$= \frac{\left(\left(-k^2-\frac{\sqrt{4-4k^2+dk^4}}{\sqrt{d}}\right)(1-x^2)^{2/3}(1-k^2x^2)^{2/3}\right)\text{Subst}}{2\left((1-x^2)(1-\right.}$$

$$= \frac{\left(\left(-k^2-\frac{\sqrt{4-4k^2+dk^4}}{\sqrt{d}}\right)(1-x^2)^{2/3}\left(\frac{-1+k^2x^2}{-1+k^2}\right)^{2/3}\right)\text{Subst}}{2\left((1-x^2)\right.}$$

$$= \frac{3(1-x^2)^2\left(\frac{1-k^2x^2}{1-k^2}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{k^2(1-x^2)}{1-k^2},\frac{}{\sqrt{d}\left(\sqrt{d}\right.}\right.}{8d\left((1-x^2)(1-k^2x^2)\right)^{2/3}}$$

Mathematica [F] time = 2.50, size = 0, normalized size = 0.00

$$\int \frac{(2-k^2)\,x-2x^3+k^2x^5}{\left((1-x^2)\,(1-k^2x^2)\right)^{2/3}\left(1-d+(-2+dk^2)\,x^2+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((2 - k^2)*x - 2*x^3 + k^2*x^5)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)*
(1 - d + (-2 + d*k^2)*x^2 + x^4)),x]
```

```
[Out] Integrate[((2 - k^2)*x - 2*x^3 + k^2*x^5)/(((1 - x^2)*(1 - k^2*x^2))^(2/3)*
(1 - d + (-2 + d*k^2)*x^2 + x^4)), x]
```

IntegrateAlgebraic [A] time = 5.02, size = 256, normalized size = 1.00

$$\frac{\log\left(d^{2/3}k^4x^4-2d^{2/3}k^2x^2+d^{2/3}+\left(k^2x^4+(-k^2-1)\,x^2+1\right)^{2/3}\left(\sqrt[3]{d}-\sqrt[3]{d}\,k^2x^2\right)+\left(k^2x^4+(-k^2-1)\,x^2+1\right)^{4/3}\right)}{4\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{d}\,k^2x^2-\sqrt[3]{d}+\left(k^2x^4+(-k^2-1)\,x^2+1\right)^{2/3}\right)}{2\sqrt[3]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(k^2x^4+(-k^2-1)x^2+1\right)^{2/3}}{-2\sqrt[3]{d}\,k^2x^2+2\sqrt[3]{d}+\left(k^2x^4+(-k^2-1)x^2+1\right)^{2/3}}\right)}{2\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 - k^2)*x - 2*x^3 + k^2*x^5)/(((1 - x^2)*(1 - k^2*x^2
))^(2/3)*(1 - d + (-2 + d*k^2)*x^2 + x^4)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3))/(2*d^(1
/3) - 2*d^(1/3)*k^2*x^2 + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3))])/d^(1/3) -
 Log[-d^(1/3) + d^(1/3)*k^2*x^2 + (1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3)]/(2*
d^(1/3)) + Log[d^(2/3) - 2*d^(2/3)*k^2*x^2 + d^(2/3)*k^4*x^4 + (d^(1/3) - d
^(1/3)*k^2*x^2)*(1 + (-1 - k^2)*x^2 + k^2*x^4)^(2/3) + (1 + (-1 - k^2)*x^2
+ k^2*x^4)^(4/3)]/(4*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-k^2+2)*x-2*x^3+k^2*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(1-d+(d*
k^2-2)*x^2+x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^5-2\,x^3-(k^2-2)x}{\left(x^4+(dk^2-2)x^2-d+1\right)\left((k^2x^2-1)(x^2-1)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-k^2+2)*x-2*x^3+k^2*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(1-d+(d*
k^2-2)*x^2+x^4),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^5 - 2*x^3 - (k^2 - 2)*x)/((x^4 + (d*k^2 - 2)*x^2 - d + 1)*
((k^2*x^2 - 1)*(x^2 - 1))^(2/3)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(-k^2+2)\,x-2x^3+k^2x^5}{\left((-x^2+1)\,(-k^2x^2+1)\right)^{\frac{2}{3}}\left(1-d+(d\,k^2-2)\,x^2+x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-k^2+2)*x-2*x^3+k^2*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(1-d+(d*k^2-2)
*x^2+x^4),x)
```

```
[Out] int(((-k^2+2)*x-2*x^3+k^2*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(1-d+(d*k^2-2)
*x^2+x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^5 - 2\,x^3 - \left(k^2 - 2\right)x}{\left(x^4 + \left(dk^2 - 2\right)x^2 - d + 1\right)\left(\left(k^2x^2 - 1\right)\left(x^2 - 1\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-k^2+2)*x-2*x^3+k^2*x^5)/((-x^2+1)*(-k^2*x^2+1))^(2/3)/(1-d+(d*
k^2-2)*x^2+x^4),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^5 - 2*x^3 - (k^2 - 2)*x)/((x^4 + (d*k^2 - 2)*x^2 - d + 1)*
((k^2*x^2 - 1)*(x^2 - 1))^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x\,\left(k^2-2\right)-k^2\,x^5+2\,x^3}{{\left(\left(x^2-1\right)\,\left(k^2\,x^2-1\right)\right)}^{2/3}\,\left(x^4+\left(d\,k^2-2\right)\,x^2-d+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(k^2 - 2) - k^2*x^5 + 2*x^3)/(((x^2 - 1)*(k^2*x^2 - 1))^(2/3)*(x^2*
(d*k^2 - 2) - d + x^4 + 1)),x)
```

```
[Out] -int((x*(k^2 - 2) - k^2*x^5 + 2*x^3)/(((x^2 - 1)*(k^2*x^2 - 1))^(2/3)*(x^2*
(d*k^2 - 2) - d + x^4 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-k**2+2)*x-2*x**3+k**2*x**5)/((-x**2+1)*(-k**2*x**2+1))**(2/3)/
(1-d+(d*k**2-2)*x**2+x**4),x)
```

```
[Out] Timed out
```

3.2192 $\int \frac{\left(-1+x^4\right) \sqrt[4]{-x^2+x^4}}{-1-x^2+x^4} \, dx$

Optimal. Leaf size=257

$$\frac{1}{2} \sqrt[4]{x^4-x^2}\, x-\frac{3}{4} \tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)+\sqrt{\frac{1}{10} \left(1+\sqrt{5}\right)} \tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\, x}{\sqrt[4]{x^4-x^2}}\right)+\sqrt{\frac{1}{10} \left(\sqrt{5}-1\right)} \tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\, x}{\sqrt[4]{x^4-x^2}}\right)$$

Rubi [A] time = 0.81, antiderivative size = 435, normalized size of antiderivative = 1.69, number of steps used = 25, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {2056, 6728, 279, 329, 331, 298, 203, 206, 1269, 1518, 1528, 494}

$$\frac{1}{2} \sqrt[4]{x^4-x^2}\, x-\frac{3 \sqrt[4]{x^4-x^2} \tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{4 \sqrt[4]{x^2-1} \sqrt{x}}+\frac{\sqrt[4]{\frac{1}{2} \left(3+\sqrt{5}\right)} \sqrt[4]{x^4-x^2} \tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}} \sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{\sqrt{5} \sqrt[4]{x^2-1} \sqrt{x}}+\frac{\sqrt[4]{\frac{1}{2} \left(3-\sqrt{5}\right)} \sqrt[4]{x^4-x^2} \tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)} \sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{\sqrt{5} \sqrt[4]{x^2-1} \sqrt{x}}+\frac{3 \sqrt[4]{x^4-x^2} \tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{4 \sqrt[4]{x^2-1} \sqrt{x}}-\frac{\sqrt[4]{\frac{1}{2} \left(3+\sqrt{5}\right)} \sqrt[4]{x^4-x^2} \tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}} \sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{\sqrt{5} \sqrt[4]{x^2-1} \sqrt{x}}-\frac{\sqrt[4]{\frac{1}{2} \left(3-\sqrt{5}\right)} \sqrt[4]{x^4-x^2} \tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)} \sqrt{x}}{\sqrt[4]{x^2-1}}\right)}{\sqrt{5} \sqrt[4]{x^2-1} \sqrt{x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^4)*(-x^2 + x^4)^(1/4))/(-1 - x^2 + x^4),x]
```

```
[Out] (x*(-x^2 + x^4)^(1/4))/2 - (3*(-x^2 + x^4)^(1/4)*ArcTan[Sqrt[x]/(-1 + x^2)^
(1/4)])/(4*Sqrt[x]*(-1 + x^2)^(1/4)) + (((3 + Sqrt[5])/2)^(1/4)*(-x^2 + x^4
)^(1/4)*ArcTan[((2/(3 + Sqrt[5]))^(1/4)*Sqrt[x])/(-1 + x^2)^(1/4)])/(Sqrt[5
]*Sqrt[x]*(-1 + x^2)^(1/4)) + (((3 - Sqrt[5])/2)^(1/4)*(-x^2 + x^4)^(1/4)*A
rcTan[(((3 + Sqrt[5])/2)^(1/4)*Sqrt[x])/(-1 + x^2)^(1/4)])/(Sqrt[5]*Sqrt[x]
*(-1 + x^2)^(1/4)) + (3*(-x^2 + x^4)^(1/4)*ArcTanh[Sqrt[x]/(-1 + x^2)^(1/4)
])/(4*Sqrt[x]*(-1 + x^2)^(1/4)) - (((3 + Sqrt[5])/2)^(1/4)*(-x^2 + x^4)^(1/
4)*ArcTanh[((2/(3 + Sqrt[5]))^(1/4)*Sqrt[x])/(-1 + x^2)^(1/4)])/(Sqrt[5]*Sq
rt[x]*(-1 + x^2)^(1/4)) - (((3 - Sqrt[5])/2)^(1/4)*(-x^2 + x^4)^(1/4)*ArcTa
nh[(((3 + Sqrt[5])/2)^(1/4)*Sqrt[x])/(-1 + x^2)^(1/4)])/(Sqrt[5]*Sqrt[x]*(-
1 + x^2)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 1269

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/f, Subst[I
nt[x^(k*(m + 1) - 1)*(d + (e*x^(2*k))/f^2)^q*(a + (b*x^(2*k))/f^k + (c*x^(4
*k))/f^4)^p, x], x, (f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, p, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1518

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^
(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Dist[(e*f^n)/c, Int[(f*x)^(m - n)*(
d + e*x^n)^(q - 1), x], x] - Dist[f^n/c, Int[((f*x)^(m - n)*(d + e*x^n)^(q
- 1)*Simp[a*e - (c*d - b*e)*x^n, x])/(a + b*x^n + c*x^(2*n)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n,
 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, n - 1] && LeQ[m, 2*n - 1]
```

Rule 1528

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q, (
f*x)^m/(a + b*x^n + c*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, e, f, q, n}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&  !IntegerQ[q] &&
IntegerQ[m]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(-1+x^4\right)\sqrt[4]{-x^2+x^4}}{-1-x^2+x^4}\,dx = \frac{\sqrt[4]{-x^2+x^4}\int \frac{\sqrt{x}\sqrt[4]{-1+x^2}\left(-1+x^4\right)}{-1-x^2+x^4}\,dx}{\sqrt{x}\sqrt[4]{-1+x^2}}$$

$$= \frac{\sqrt[4]{-x^2+x^4}\int \left(\sqrt{x}\sqrt[4]{-1+x^2}+\frac{x^{5/2}\sqrt[4]{-1+x^2}}{-1-x^2+x^4}\right)dx}{\sqrt{x}\sqrt[4]{-1+x^2}}$$

$$= \frac{\sqrt[4]{-x^2+x^4}\int \sqrt{x}\sqrt[4]{-1+x^2}\,dx}{\sqrt{x}\sqrt[4]{-1+x^2}} + \frac{\sqrt[4]{-x^2+x^4}\int \frac{x^{5/2}\sqrt[4]{-1+x^2}}{-1-x^2+x^4}\,dx}{\sqrt{x}\sqrt[4]{-1+x^2}}$$

$$= \frac{1}{2}x\sqrt[4]{-x^2+x^4} - \frac{\sqrt[4]{-x^2+x^4}\int \frac{\sqrt{x}}{\left(-1+x^2\right)^{3/4}}\,dx}{4\sqrt{x}\sqrt[4]{-1+x^2}} + \frac{\left(2\sqrt[4]{-x^2+x^4}\right)\text{Subst}\left(\int \frac{x^6\sqrt[4]{-1+x^4}}{-1-x^4+x^8}\right.}{\sqrt{x}\sqrt[4]{-1+x^2}}$$

$$= \frac{1}{2}x\sqrt[4]{-x^2+x^4} - \frac{\sqrt[4]{-x^2+x^4}\,\text{Subst}\left(\int \frac{x^2}{\left(-1+x^4\right)^{3/4}}\,dx, x, \sqrt{x}\right)}{2\sqrt{x}\sqrt[4]{-1+x^2}} + \frac{\left(2\sqrt[4]{-x^2+x^4}\right)\text{Su}}{\sqrt{x}}$$

$$= \frac{1}{2}x\sqrt[4]{-x^2+x^4} - \frac{\sqrt[4]{-x^2+x^4}\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{2\sqrt{x}\sqrt[4]{-1+x^2}} + \frac{\left(2\sqrt[4]{-x^2+x^4}\right)\text{Sub}}{\sqrt{x}}$$

$$= \frac{1}{2}x\sqrt[4]{-x^2+x^4} - \frac{\sqrt[4]{-x^2+x^4}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{4\sqrt{x}\sqrt[4]{-1+x^2}} + \frac{\sqrt[4]{-x^2+x^4}\,\text{Subst}\left(\right.}{4\sqrt{x}\sqrt[4]{}}$$

$$= \frac{1}{2}x\sqrt[4]{-x^2+x^4} - \frac{3\sqrt[4]{-x^2+x^4}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{4\sqrt{x}\sqrt[4]{-1+x^2}} + \frac{3\sqrt[4]{-x^2+x^4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{4\sqrt{x}\sqrt[4]{-1+x^2}}$$

$$= \frac{1}{2}x\sqrt[4]{-x^2+x^4} - \frac{3\sqrt[4]{-x^2+x^4}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{4\sqrt{x}\sqrt[4]{-1+x^2}} + \frac{3\sqrt[4]{-x^2+x^4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{4\sqrt{x}\sqrt[4]{-1+x^2}}$$

$$= \frac{1}{2}x\sqrt[4]{-x^2+x^4} - \frac{3\sqrt[4]{-x^2+x^4}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{-1+x^2}}\right)}{4\sqrt{x}\sqrt[4]{-1+x^2}} + \frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\sqrt[4]{-x^2+x^4}\tan^{-1}}{\sqrt{5}\sqrt{x}\sqrt[4]{-1+x^2}}$$

Mathematica [C] time = 2.07, size = 231, normalized size = 0.90

$$\frac{\sqrt[4]{x^2\left(x^2-1\right)}\left(\frac{4\left(\sqrt{5}-1\right)^{3/4}\left(5+\sqrt{5}\right)x^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{\left(1+\sqrt{5}\right)x^2}{2x^2+\sqrt{5}-1}\right)}{\sqrt[4]{1-x^2}\left(2x^2+\sqrt{5}-1\right)^{3/4}}+\frac{4\left(\sqrt{5}-5\right)x^{3/2}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{\left(-1+\sqrt{5}\right)x^2}{\left(1+\sqrt{5}\right)\left(x^2-1\right)}\right)}{x^2-1}-\frac{45\left(\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)-\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt[4]{x^2-1}}\right)\right)}{\sqrt[4]{x^2-1}}\right)}{60\sqrt{x}}+\frac{1}{2}\sqrt[4]{x^2\left(x^2-1\right)}\,x$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x^4)*(-x^2 + x^4)^(1/4))/(-1 - x^2 + x^4),x]
```

```
[Out] (x*(x^2*(-1 + x^2))^(1/4))/2 + ((x^2*(-1 + x^2))^(1/4)*((-45*(ArcTan[Sqrt[x
]/(-1 + x^2)^(1/4)] - ArcTanh[Sqrt[x]/(-1 + x^2)^(1/4)]))/(-1 + x^2)^(1/4)
+ (4*(-1 + Sqrt[5])^(3/4)*(5 + Sqrt[5])*x^(3/2)*Hypergeometric2F1[3/4, 3/4,
```

```
7/4, ((1 + Sqrt[5])*x^2)/(-1 + Sqrt[5] + 2*x^2)])/((1 - x^2)^(1/4)*(-1 + S
qrt[5] + 2*x^2)^(3/4)) + (4*(-5 + Sqrt[5])*x^(3/2)*Hypergeometric2F1[3/4, 1
, 7/4, ((-1 + Sqrt[5])*x^2)/((1 + Sqrt[5])*(-1 + x^2))])/(-1 + x^2)))/(60*S
qrt[x])
```

IntegrateAlgebraic [**A**] time = 0.96, size = 257, normalized size = 1.00

$$\frac{1}{2}\sqrt[4]{x^4-x^2}\,x-\frac{3}{4}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)+\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-x^2}}\right)+\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4-x^2}}\right)+\frac{3}{4}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^2}}\right)-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-x^2}}\right)-\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^4)*(-x^2 + x^4)^(1/4))/(-1 - x^2 + x^4),x]
```

```
[Out] (x*(-x^2 + x^4)^(1/4))/2 - (3*ArcTan[x/(-x^2 + x^4)^(1/4)])/4 + Sqrt[(1 + S
qrt[5])/10]*ArcTan[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(-x^2 + x^4)^(1/4)] + Sqrt[(-
1 + Sqrt[5])/10]*ArcTan[(Sqrt[1/2 + Sqrt[5]/2]*x)/(-x^2 + x^4)^(1/4)] + (3*
ArcTanh[x/(-x^2 + x^4)^(1/4)])/4 - Sqrt[(1 + Sqrt[5])/10]*ArcTanh[(Sqrt[-1/
2 + Sqrt[5]/2]*x)/(-x^2 + x^4)^(1/4)] - Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[(Sq
rt[1/2 + Sqrt[5]/2]*x)/(-x^2 + x^4)^(1/4)]
```

fricas [**B**] time = 70.42, size = 1255, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4-x^2)^(1/4)/(x^4-x^2-1),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(10)*sqrt(sqrt(5) + 1)*arctan(-1/818620*(sqrt(2)*(sqrt(10)*sqrt(x
^4 - x^2)*(430*x^3 - sqrt(5)*(448*x^3 - 439*x) - 1335*x) - sqrt(10)*(1120*x
^5 - 1550*x^3 - sqrt(5)*(215*x^5 - 663*x^3 + 224*x) + 215*x))*sqrt(40157*sq
rt(5) + 36899)*sqrt(sqrt(5) + 1) + 81862*(sqrt(10)*(x^4 - x^2)^(3/4)*(sqrt(
5)*(2*x^2 - 1) + 5) + sqrt(10)*(5*x^4 - 5*x^2 - sqrt(5)*(x^4 - 3*x^2))*(x^4
 - x^2)^(1/4))*sqrt(sqrt(5) + 1))/(x^5 - x^3 - x)) - 1/10*sqrt(10)*sqrt(sqr
t(5) - 1)*arctan(1/818620*(sqrt(2)*(sqrt(10)*sqrt(x^4 - x^2)*(430*x^3 + sqr
t(5)*(448*x^3 - 439*x) - 1335*x) + sqrt(10)*(1120*x^5 - 1550*x^3 + sqrt(5)*
(215*x^5 - 663*x^3 + 224*x) + 215*x))*sqrt(40157*sqrt(5) - 36899)*sqrt(sqrt
(5) - 1) + 81862*(sqrt(10)*(x^4 - x^2)^(3/4)*(sqrt(5)*(2*x^2 - 1) - 5) + sq
rt(10)*(5*x^4 - 5*x^2 + sqrt(5)*(x^4 - 3*x^2))*(x^4 - x^2)^(1/4))*sqrt(sqrt
(5) - 1))/(x^5 - x^3 - x)) - 1/40*sqrt(10)*sqrt(sqrt(5) + 1)*log((20*(x^4 -
 x^2)^(3/4)*(448*x^2 + sqrt(5)*(86*x^2 + 181) - 9) + (2*sqrt(10)*sqrt(x^4 -
 x^2)*(905*x^3 - sqrt(5)*(9*x^3 - 457*x) - 475*x) - sqrt(10)*(45*x^5 - 1855
*x^3 - sqrt(5)*(905*x^5 - 923*x^3 + 9*x) + 905*x))*sqrt(sqrt(5) + 1) - 20*(
9*x^4 - 457*x^2 - sqrt(5)*(181*x^4 - 95*x^2))*(x^4 - x^2)^(1/4))/(x^5 - x^3
 - x)) + 1/40*sqrt(10)*sqrt(sqrt(5) + 1)*log((20*(x^4 - x^2)^(3/4)*(448*x^2
 + sqrt(5)*(86*x^2 + 181) - 9) - (2*sqrt(10)*sqrt(x^4 - x^2)*(905*x^3 - sqr
t(5)*(9*x^3 - 457*x) - 475*x) - sqrt(10)*(45*x^5 - 1855*x^3 - sqrt(5)*(905*
x^5 - 923*x^3 + 9*x) + 905*x))*sqrt(sqrt(5) + 1) - 20*(9*x^4 - 457*x^2 - sq
rt(5)*(181*x^4 - 95*x^2))*(x^4 - x^2)^(1/4))/(x^5 - x^3 - x)) - 1/40*sqrt(1
0)*sqrt(sqrt(5) - 1)*log((20*(x^4 - x^2)^(3/4)*(448*x^2 - sqrt(5)*(86*x^2 +
 181) - 9) + (2*sqrt(10)*sqrt(x^4 - x^2)*(905*x^3 + sqrt(5)*(9*x^3 - 457*x)
 - 475*x) + sqrt(10)*(45*x^5 - 1855*x^3 + sqrt(5)*(905*x^5 - 923*x^3 + 9*x)
 + 905*x))*sqrt(sqrt(5) - 1) + 20*(9*x^4 - 457*x^2 + sqrt(5)*(181*x^4 - 95*
x^2))*(x^4 - x^2)^(1/4))/(x^5 - x^3 - x)) + 1/40*sqrt(10)*sqrt(sqrt(5) - 1)
*log((20*(x^4 - x^2)^(3/4)*(448*x^2 - sqrt(5)*(86*x^2 + 181) - 9) - (2*sqrt
(10)*sqrt(x^4 - x^2)*(905*x^3 + sqrt(5)*(9*x^3 - 457*x) - 475*x) + sqrt(10)
*(45*x^5 - 1855*x^3 + sqrt(5)*(905*x^5 - 923*x^3 + 9*x) + 905*x))*sqrt(sqrt
(5) - 1) + 20*(9*x^4 - 457*x^2 + sqrt(5)*(181*x^4 - 95*x^2))*(x^4 - x^2)^(1
/4))/(x^5 - x^3 - x)) + 1/2*(x^4 - x^2)^(1/4)*x + 3/8*arctan(2*((x^4 - x^2)
^(1/4)*x^2 + (x^4 - x^2)^(3/4))/x) + 3/8*log((2*x^3 + 2*(x^4 - x^2)^(1/4)*x
^2 + 2*sqrt(x^4 - x^2)*x - x + 2*(x^4 - x^2)^(3/4))/x)
```

giac [A] time = 1.92, size = 248, normalized size = 0.96

$$-\frac{1}{2}x^{2}\left(-\frac{1}{x^{2}}+1\right)^{\frac{1}{4}}+\frac{1}{10}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{\left(-\frac{1}{x^{2}}+1\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{10}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{\left(-\frac{1}{x^{2}}+1\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{10\sqrt{5}-10}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}+\left(-\frac{1}{x^{2}}+1\right)^{\frac{1}{4}}\right)-\frac{1}{20}\sqrt{10\sqrt{5}-10}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}-\left(-\frac{1}{x^{2}}+1\right)^{\frac{1}{4}}\right)+\frac{1}{20}\sqrt{10\sqrt{5}+10}\log\left(\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}+\left(-\frac{1}{x^{2}}+1\right)^{\frac{1}{4}}\right)-\frac{1}{20}\sqrt{10\sqrt{5}+10}\log\left(\left|-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}+\left(-\frac{1}{x^{2}}+1\right)^{\frac{1}{4}}\right|\right)-\frac{3}{4}\arctan\left(\left(-\frac{1}{x^{2}}+1\right)^{\frac{1}{4}}\right)-\frac{3}{8}\log\left(\left(-\frac{1}{x^{2}}+1\right)^{\frac{1}{4}}+1\right)+\frac{3}{8}\log\left(-\left(-\frac{1}{x^{2}}+1\right)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4-x^2)^(1/4)/(x^4-x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*x^2*(-1/x^2 + 1)^(1/4) + 1/10*sqrt(10*sqrt(5) - 10)*arctan((-1/x^2 + 1
)^(1/4)/sqrt(1/2*sqrt(5) + 1/2)) + 1/10*sqrt(10*sqrt(5) + 10)*arctan((-1/x^
2 + 1)^(1/4)/sqrt(1/2*sqrt(5) - 1/2)) + 1/20*sqrt(10*sqrt(5) - 10)*log(sqrt
(1/2*sqrt(5) + 1/2) + (-1/x^2 + 1)^(1/4)) - 1/20*sqrt(10*sqrt(5) - 10)*log(
sqrt(1/2*sqrt(5) + 1/2) - (-1/x^2 + 1)^(1/4)) + 1/20*sqrt(10*sqrt(5) + 10)*
log(sqrt(1/2*sqrt(5) - 1/2) + (-1/x^2 + 1)^(1/4)) - 1/20*sqrt(10*sqrt(5) +
10)*log(abs(-sqrt(1/2*sqrt(5) - 1/2) + (-1/x^2 + 1)^(1/4))) - 3/4*arctan((-
1/x^2 + 1)^(1/4)) - 3/8*log((-1/x^2 + 1)^(1/4) + 1) + 3/8*log(-(-1/x^2 + 1)
^(1/4) + 1)
```

maple [F] time = 12.33, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}-1\right)\left(x^{4}-x^{2}\right)^{\frac{1}{4}}}{x^{4}-x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)*(x^4-x^2)^(1/4)/(x^4-x^2-1),x)
```

```
[Out] int((x^4-1)*(x^4-x^2)^(1/4)/(x^4-x^2-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}-x^{2}\right)^{\frac{1}{4}}\left(x^{4}-1\right)}{x^{4}-x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)*(x^4-x^2)^(1/4)/(x^4-x^2-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^2)^(1/4)*(x^4 - 1)/(x^4 - x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(x^{4}-1\right)\left(x^{4}-x^{2}\right)^{1/4}}{-x^{4}+x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^4 - 1)*(x^4 - x^2)^(1/4))/(x^2 - x^4 + 1),x)
```

```
[Out] -int(((x^4 - 1)*(x^4 - x^2)^(1/4))/(x^2 - x^4 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^{2}\left(x-1\right)\left(x+1\right)}\,\left(x-1\right)\left(x+1\right)\left(x^{2}+1\right)}{x^{4}-x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)*(x**4-x**2)**(1/4)/(x**4-x**2-1),x)
```

```
[Out] Integral((x**2*(x - 1)*(x + 1))**(1/4)*(x - 1)*(x + 1)*(x**2 + 1)/(x**4 - x
**2 - 1), x)
```

3.2193 $\int \frac{1+(3-2k)x-(4+k)x^2+3kx^3}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+(1+5b)x-(10b+k)x^2+10bx^3-5bx^4+bx^5\right)}\,dx$

Optimal. Leaf size=257

$$-\frac{\log\left(b^{2/3}x^4-4b^{2/3}x^3+6b^{2/3}x^2-4b^{2/3}x+b^{2/3}+\left(\sqrt[3]{b}\,x^2-2\sqrt[3]{b}\,x+\sqrt[3]{b}\right)\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1\right.\right.}{2\sqrt[3]{b}}$$

Rubi [F] time = 11.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+(3-2k)x-(4+k)x^2+3kx^3}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+(1+5b)x-(10b+k)x^2+10bx^3-5bx^4+bx^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + (3 - 2*k)*x - (4 + k)*x^2 + 3*k*x^3)/(((1 - x)*x*(1 - k*x))^(1/3)*
(-b + (1 + 5*b)*x - (10*b + k)*x^2 + 10*b*x^3 - 5*b*x^4 + b*x^5)),x]
```

```
[Out] (6*(2 - k)*(-1 + x)^(1/3)*x^(1/3)*(-1 + k*x)^(1/3)*Defer[Subst][Defer[Int][
(x^4*(-1 + x^3)^(2/3))/((-1 + k*x^3)^(1/3)*(-x^3 + k*x^6 - b*(-1 + x^3)^5))
, x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3) - (9*k*(-1 + x)^(1/3)*x^(1/3
)*(-1 + k*x)^(1/3)*Defer[Subst][Defer[Int][(x^7*(-1 + x^3)^(2/3))/((-1 + k*
x^3)^(1/3)*(-x^3 + k*x^6 - b*(-1 + x^3)^5)), x], x, x^(1/3)])/((1 - x)*x*(1
 - k*x))^(1/3) - (3*(-1 + x)^(1/3)*x^(1/3)*(-1 + k*x)^(1/3)*Defer[Subst][De
fer[Int][(x*(-1 + x^3)^(2/3))/((-1 + k*x^3)^(1/3)*(x^3 - k*x^6 + b*(-1 + x^
3)^5)), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\int \frac{1+(3-2k)x-(4+k)x^2+3kx^3}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+(1+5b)x-(10b+k)x^2+10bx^3-5bx^4+bx^5\right)}\,dx=\int \frac{(1-x)\left(1+2(2-k)x\right.}{\sqrt[3]{(-1+x)x(-1+kx)}\left(b(-1+\right.}$$

$$=\frac{\left(\sqrt[3]{-1+x}\sqrt[3]{x}\sqrt[3]{-1+kx}\right)\int \frac{}{\sqrt[3]{-1+}}}{\sqrt[3]{(-1+x)x(}}$$

$$=-\frac{\left(\sqrt[3]{-1+x}\sqrt[3]{x}\sqrt[3]{-1+kx}\right)\int \frac{(-}{\sqrt[3]{x}}}{\sqrt[3]{(-1+x)x(-}}$$

$$=-\frac{\left(3\sqrt[3]{-1+x}\sqrt[3]{x}\sqrt[3]{-1+kx}\right)\text{Sub}}{\sqrt[3]{(-1}}$$

$$=-\frac{\left(3\sqrt[3]{-1+x}\sqrt[3]{x}\sqrt[3]{-1+kx}\right)\text{Sub}}{}$$

$$=-\frac{\left(3\sqrt[3]{-1+x}\sqrt[3]{x}\sqrt[3]{-1+kx}\right)\text{Sub}}{}$$

$$=-\frac{\left(3\sqrt[3]{-1+x}\sqrt[3]{x}\sqrt[3]{-1+kx}\right)\text{Sub}}{\sqrt[3]{(-1}}$$

Mathematica [F] time = 8.80, size = 0, normalized size = 0.00

$$\int \frac{1+(3-2k)x-(4+k)x^2+3kx^3}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+(1+5b)x-(10b+k)x^2+10bx^3-5bx^4+bx^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + (3 - 2*k)*x - (4 + k)*x^2 + 3*k*x^3)/(((1 - x)*x*(1 - k*x))^
(1/3)*(-b + (1 + 5*b)*x - (10*b + k)*x^2 + 10*b*x^3 - 5*b*x^4 + b*x^5)),x]
```

```
[Out] Integrate[(1 + (3 - 2*k)*x - (4 + k)*x^2 + 3*k*x^3)/(((1 - x)*x*(1 - k*x))^
(1/3)*(-b + (1 + 5*b)*x - (10*b + k)*x^2 + 10*b*x^3 - 5*b*x^4 + b*x^5)), x]
```

IntegrateAlgebraic [A] time = 3.03, size = 257, normalized size = 1.00

$$-\frac{\log\left(b^{2/3}x^4-4b^{2/3}x^3+6b^{2/3}x^2-4b^{2/3}x+b^{2/3}+\left(\sqrt[3]{b}x^2-2\sqrt[3]{b}x+\sqrt[3]{b}\right)\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2\sqrt[3]{b}}+\frac{\log\left(-\sqrt[3]{b}x^2+2\sqrt[3]{b}x-\sqrt[3]{b}+\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{kx^3+(-k-1)x^2+x}}{2\sqrt[3]{b}x^2-4\sqrt[3]{b}x+2\sqrt[3]{b}+\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + (3 - 2*k)*x - (4 + k)*x^2 + 3*k*x^3)/(((1 - x)*x*(1
 - k*x))^(1/3)*(-b + (1 + 5*b)*x - (10*b + k)*x^2 + 10*b*x^3 - 5*b*x^4 + b*
x^5)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(2*b^(1/3) - 4*b
^(1/3)*x + 2*b^(1/3)*x^2 + (x + (-1 - k)*x^2 + k*x^3)^(1/3))])/b^(1/3) + Lo
g[-b^(1/3) + 2*b^(1/3)*x - b^(1/3)*x^2 + (x + (-1 - k)*x^2 + k*x^3)^(1/3)]/
b^(1/3) - Log[b^(2/3) - 4*b^(2/3)*x + 6*b^(2/3)*x^2 - 4*b^(2/3)*x^3 + b^(2/
3)*x^4 + (b^(1/3) - 2*b^(1/3)*x + b^(1/3)*x^2)*(x + (-1 - k)*x^2 + k*x^3)^(
1/3) + (x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(3-2*k)*x-(4+k)*x^2+3*k*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(-b+(1+5
*b)*x-(10*b+k)*x^2+10*b*x^3-5*b*x^4+b*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,kx^3-(k+4)x^2-(2\,k-3)x+1}{\left(bx^5-5\,bx^4+10\,bx^3-(10\,b+k)x^2+(5\,b+1)x-b\right)\left((kx-1)(x-1)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(3-2*k)*x-(4+k)*x^2+3*k*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(-b+(1+5
*b)*x-(10*b+k)*x^2+10*b*x^3-5*b*x^4+b*x^5),x, algorithm="giac")
```

```
[Out] integrate((3*k*x^3 - (k + 4)*x^2 - (2*k - 3)*x + 1)/((b*x^5 - 5*b*x^4 + 10*
b*x^3 - (10*b + k)*x^2 + (5*b + 1)*x - b)*((k*x - 1)*(x - 1)*x)^(1/3)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1+(3-2k)\,x-(4+k)\,x^2+3k\,x^3}{\left((1-x)\,x\,(-kx+1)\right)^{\frac{1}{3}}\left(-b+(1+5b)\,x-(10b+k)\,x^2+10b\,x^3-5b\,x^4+b\,x^5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(3-2*k)*x-(4+k)*x^2+3*k*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(-b+(1+5*b)*x-
(10*b+k)*x^2+10*b*x^3-5*b*x^4+b*x^5),x)
```

```
[Out] int((1+(3-2*k)*x-(4+k)*x^2+3*k*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(-b+(1+5*b)*x-
(10*b+k)*x^2+10*b*x^3-5*b*x^4+b*x^5),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,kx^3-(k+4)x^2-(2\,k-3)x+1}{\left(bx^5-5\,bx^4+10\,bx^3-(10\,b+k)x^2+(5\,b+1)x-b\right)((kx-1)(x-1)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(3-2*k)*x-(4+k)*x^2+3*k*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(-b+(1+5
*b)*x-(10*b+k)*x^2+10*b*x^3-5*b*x^4+b*x^5),x, algorithm="maxima")
```

```
[Out] integrate((3*k*x^3 - (k + 4)*x^2 - (2*k - 3)*x + 1)/((b*x^5 - 5*b*x^4 + 10*
b*x^3 - (10*b + k)*x^2 + (5*b + 1)*x - b)*((k*x - 1)*(x - 1)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int -\frac{-3\,k\,x^3+(k+4)\,x^2+(2\,k-3)\,x-1}{(x\,(k\,x-1)\,(x-1))^{1/3}\,\left(-b\,x^5+5\,b\,x^4-10\,b\,x^3+(10\,b+k)\,x^2+(-5\,b-1)\,x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*k - 3) + x^2*(k + 4) - 3*k*x^3 - 1)/((x*(k*x - 1)*(x - 1))^(1/3)*
(b - 10*b*x^3 + 5*b*x^4 - b*x^5 + x^2*(10*b + k) - x*(5*b + 1))),x)
```

```
[Out] -int(-(x*(2*k - 3) + x^2*(k + 4) - 3*k*x^3 - 1)/((x*(k*x - 1)*(x - 1))^(1/3
)*(b - 10*b*x^3 + 5*b*x^4 - b*x^5 + x^2*(10*b + k) - x*(5*b + 1))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)\left(3kx^2+2kx-4x-1\right)}{\sqrt[3]{x\,(x-1)\,(kx-1)}\,\left(bx^5-5bx^4+10bx^3-10bx^2+5bx-b-kx^2+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(3-2*k)*x-(4+k)*x**2+3*k*x**3)/((1-x)*x*(-k*x+1))**(1/3)/(-b+(
1+5*b)*x-(10*b+k)*x**2+10*b*x**3-5*b*x**4+b*x**5),x)
```

```
[Out] Integral((x - 1)*(3*k*x**2 + 2*k*x - 4*x - 1)/((x*(x - 1)*(k*x - 1))**(1/3)
*(b*x**5 - 5*b*x**4 + 10*b*x**3 - 10*b*x**2 + 5*b*x - b - k*x**2 + x)), x)
```

3.2194 $\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=257

$$-\frac{4\log\left(\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}-\sqrt[3]{c}\right)}{3ac^{4/3}}+\frac{2\log\left(\sqrt[3]{c}\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}+\right.}{3ac^{4/3}}$$

Rubi [F] time = 1.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*(c + (a*x + Sqrt
[-b + a^2*x^2])^(1/4))^(1/3)),x]
```

```
[Out] Defer[Int][1/(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*(c + (a*x
 + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\sqrt[3]{c+}}$$

Mathematica [C] time = 0.30, size = 74, normalized size = 0.29

$$\frac{6\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}{c}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*(c + (a*x
+ Sqrt[-b + a^2*x^2])^(1/4))^(1/3)),x]
```

```
[Out] (6*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3)*Hypergeometric2F1[2/3, 2, 5
/3, (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))/c])/(a*c^2)
```

IntegrateAlgebraic [A] time = 0.57, size = 257, normalized size = 1.00

$$-\frac{4\log\left(\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}-\sqrt[3]{c}\right)}{3ac^{4/3}}+\frac{2\log\left(\sqrt[3]{c}\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}+c^{2/3}\right)}{3ac^{4/3}}-\frac{4\tan^{-1}\left(\frac{2\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{3}\sqrt[3]{c}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}ac^{4/3}}-\frac{4\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}}{ac\sqrt[4]{\sqrt{a^2x^2-b}+ax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*(
c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)),x]
```

```
[Out] (-4*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3))/(a*c*(a*x + Sqrt[-b + a^2
*x^2])^(1/4)) - (4*ArcTan[1/Sqrt[3] + (2*(c + (a*x + Sqrt[-b + a^2*x^2])^(1
/4))^(1/3))/(Sqrt[3]*c^(1/3))]])/(Sqrt[3]*a*c^(4/3)) - (4*Log[-c^(1/3) + (c
+ (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)]])/(3*a*c^(4/3)) + (2*Log[c^(2/3)
+ c^(1/3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + (c + (a*x + Sqrt[-
b + a^2*x^2])^(1/4))^(2/3)]])/(3*a*c^(4/3))
```

fricas [A] time = 0.91, size = 758, normalized size = 2.95

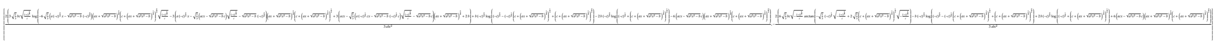

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4)/(c+(a*x+(a^2*x^
2-b)^(1/2))^(1/4))^(1/3),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(1/3)*b*c*sqrt((-c)^(1/3)/c)*log(-6*sqrt(1/3)*(a*(-c)^(2/3)*x -
 sqrt(a^2*x^2 - b)*(-c)^(2/3))*(a*x + sqrt(a^2*x^2 - b))^(3/4)*(c + (a*x +
sqrt(a^2*x^2 - b))^(1/4))^(2/3)*sqrt((-c)^(1/3)/c) - 3*(a*(-c)^(2/3)*x - sq
rt(1/3)*(a*c*x - sqrt(a^2*x^2 - b)*c)*sqrt((-c)^(1/3)/c) - sqrt(a^2*x^2 - b
)*(-c)^(2/3))*(a*x + sqrt(a^2*x^2 - b))^(3/4)*(c + (a*x + sqrt(a^2*x^2 - b)
)^(1/4))^(1/3) + 3*(a*c*x - sqrt(1/3)*(a*(-c)^(1/3)*c*x - sqrt(a^2*x^2 - b)
*(-c)^(1/3)*c)*sqrt((-c)^(1/3)/c) - sqrt(a^2*x^2 - b)*c)*(a*x + sqrt(a^2*x^
2 - b))^(3/4) + 2*b) + b*(-c)^(2/3)*log((-c)^(2/3) - (-c)^(1/3)*(c + (a*x +
 sqrt(a^2*x^2 - b))^(1/4))^(1/3) + (c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2
/3)) - 2*b*(-c)^(2/3)*log((-c)^(1/3) + (c + (a*x + sqrt(a^2*x^2 - b))^(1/4)
)^(1/3)) - 6*(a*c*x - sqrt(a^2*x^2 - b)*c)*(a*x + sqrt(a^2*x^2 - b))^(3/4)*
(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3))/(a*b*c^2), -2/3*(6*sqrt(1/3)*b
*c*sqrt(-(-c)^(1/3)/c)*arctan(-sqrt(1/3)*(-c)^(1/3)*sqrt(-(-c)^(1/3)/c) + 2
*sqrt(1/3)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)*sqrt(-(-c)^(1/3)/c))
 - b*(-c)^(2/3)*log((-c)^(2/3) - (-c)^(1/3)*(c + (a*x + sqrt(a^2*x^2 - b))^
(1/4))^(1/3) + (c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3)) + 2*b*(-c)^(2/3
)*log((-c)^(1/3) + (c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)) + 6*(a*c*x
- sqrt(a^2*x^2 - b)*c)*(a*x + sqrt(a^2*x^2 - b))^(3/4)*(c + (a*x + sqrt(a^2
*x^2 - b))^(1/4))^(2/3))/(a*b*c^2)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4)/(c+(a*x+(a^2*x^
2-b)^(1/2))^(1/4))^(1/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^{2}x^{2}-b}\,\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{4}}\left(c+\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4)/(c+(a*x+(a^2*x^2-b)^(
1/2))^(1/4))^(1/3),x)
```

```
[Out] int(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4)/(c+(a*x+(a^2*x^2-b)^(
1/2))^(1/4))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2-b}\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4)/(c+(a*x+(a^2*x^
2-b)^(1/2))^(1/4))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 - b)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*(c + (a*x +
sqrt(a^2*x^2 - b))^(1/4))^(1/3)), x)
```

mupad [B] time = 2.60, size = 99, normalized size = 0.39

$$-\frac{3\left(\frac{c}{\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/4}}+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{4}{3};\ \frac{7}{3};\ -\frac{c}{\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/4}}\right)}{a\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/4}\left(c+\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/4}\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x + (a^2*x^2 - b)^(1/2))^(1/4)*(c + (a*x + (a^2*x^2 - b)^(1/2))^(
1/4))^(1/3)*(a^2*x^2 - b)^(1/2)),x)
```

```
[Out] -(3*(c/(a*x + (a^2*x^2 - b)^(1/2))^(1/4) + 1)^(1/3)*hypergeom([1/3, 4/3], 7
/3, -c/(a*x + (a^2*x^2 - b)^(1/2))^(1/4)))/(a*(a*x + (a^2*x^2 - b)^(1/2))^(
1/4)*(c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}\,\sqrt[4]{ax+\sqrt{a^2x^2-b}}\,\sqrt{a^2x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*x**2-b)**(1/2)/(a*x+(a**2*x**2-b)**(1/2))**(1/4)/(c+(a*x+
(a**2*x**2-b)**(1/2))**(1/4))**(1/3),x)
```

```
[Out] Integral(1/((c + (a*x + sqrt(a**2*x**2 - b))**(1/4))**(1/3)*(a*x + sqrt(a**
2*x**2 - b))**(1/4)*sqrt(a**2*x**2 - b)), x)
```

3.2195 $\int \frac{(-b+x)\left(ab-2bx+x^2\right)}{\left(x(-a+x)(-b+x)^2\right)^{2/3}\left(b-(1+ad)x+dx^2\right)}\,dx$

Optimal. Leaf size=258

$$-\frac{\log\left(d^{2/3}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}+\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}\left(\sqrt[3]{d}\,x-b\sqrt[3]{d}\right.\right.}{2\sqrt[3]{d}}$$

Rubi [F] time = 8.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-b+x)\left(ab-2bx+x^2\right)}{\left(x(-a+x)(-b+x)^2\right)^{2/3}\left(b-(1+ad)x+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-b + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2)^(2/3)*(b - (1 +
 a*d)*x + d*x^2)),x]
```

```
[Out] (-3*(b - x)*x*(1 - x/a)^(2/3)*(1 - x/b)^(1/3)*AppellF1[1/3, 2/3, 1/3, 4/3,
x/a, x/b])/(d*(-((a - x)*(b - x)^2*x))^(2/3)) + ((1 + a*d - 2*b*d + Sqrt[1
+ 2*a*d - 4*b*d + a^2*d^2])*x^(2/3)*(-a + x)^(2/3)*(-b + x)^(4/3)*Defer[Int
][1/(x^(2/3)*(-a + x)^(2/3)*(-b + x)^(1/3)*(-1 - a*d - Sqrt[1 + 2*a*d - 4*b
*d + a^2*d^2] + 2*d*x)), x])/(d*(-((a - x)*(b - x)^2*x))^(2/3)) + ((1 + a*d
 - 2*b*d - Sqrt[1 + 2*a*d - 4*b*d + a^2*d^2])*x^(2/3)*(-a + x)^(2/3)*(-b +
x)^(4/3)*Defer[Int][1/(x^(2/3)*(-a + x)^(2/3)*(-b + x)^(1/3)*(-1 - a*d + Sq
rt[1 + 2*a*d - 4*b*d + a^2*d^2] + 2*d*x)), x])/(d*(-((a - x)*(b - x)^2*x))^
(2/3))
```

Rubi steps

$$\int \frac{(-b+x)\left(ab-2bx+x^2\right)}{\left(x(-a+x)(-b+x)^2\right)^{2/3}\left(b-(1+ad)x+dx^2\right)}\,dx=\frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{ab-2bx+x^2}{x^{2/3}(-a+x)^{2/3}\sqrt[3]{-b+x}\left(b-(1+ad)x+dx\right.}}{\left(x(-a+x)(-b+x)^2\right)^{2/3}}$$

$$=\frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\int\left(\frac{1}{dx^{2/3}(-a+x)^{2/3}\sqrt[3]{-b+x}}-\frac{}{dx^{2/3}(-a}\right.}{\left(x(-a+x)(-b+x)^2\right)^{2/3}}$$

$$=\frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\int\frac{1}{x^{2/3}(-a+x)^{2/3}\sqrt[3]{-b+x}}\,dx}{d\left(x(-a+x)(-b+x)^2\right)^{2/3}}-\frac{\left(x^{2/3}\right.}{}$$

$$=-\frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\int\left(\frac{-1-ad+2bd-\sqrt{1+2ad}}{x^{2/3}(-a+x)^{2/3}\sqrt[3]{-b+x}\left(-1-ad-\sqrt{1}\right.}\right.}{d\left(x(-\right.}$$

$$=-\frac{\left(\left(-1-ad+2bd-\sqrt{1+2ad-4bd+a^2d^2}\right)x^{2/3}(-a+x)^{2}\right.}{d\left(x(-a+\right.}$$

$$=-\frac{3(b-x)x\left(1-\frac{x}{a}\right)^{2/3}\sqrt[3]{1-\frac{x}{b}}\,F_1\left(\frac{1}{3};\frac{2}{3},\frac{1}{3};\frac{4}{3};\frac{x}{a},\frac{x}{b}\right)}{d\left(-\left((a-x)(b-x)^2x\right)\right)^{2/3}}-\frac{\left(\left(-1-\right.\right.}{}$$

Mathematica [F] time = 7.30, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)\left(ab-2bx+x^2\right)}{\left(x(-a+x)(-b+x)^2\right)^{2/3}\left(b-(1+ad)x+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-b + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2)^(2/3)*(b
- (1 + a*d)*x + d*x^2)),x]
```

```
[Out] Integrate[((-b + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2)^(2/3)*(b
- (1 + a*d)*x + d*x^2)), x]
```

IntegrateAlgebraic [A] time = 3.46, size = 258, normalized size = 1.00

$$-\frac{\log\left(d^{2/3}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}+\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}\left(\sqrt[3]{d}\,x-b\sqrt[3]{d}\right)+b^2-2bx+x^2\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{d}\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}+b-x\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,b-\sqrt{3}\,x}{-2\sqrt[3]{d}\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}+b-x}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2)^
(2/3)*(b - (1 + a*d)*x + d*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*b - Sqrt[3]*x)/(b - x - 2*d^(1/3)*(-(a*b^2*x) + (2
*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3))])/d^(1/3) + Log[b - x + d^(1
/3)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3)]/d^(1/3)
- Log[b^2 - 2*b*x + x^2 + (-(b*d^(1/3)) + d^(1/3)*x)*(-(a*b^2*x) + (2*a*b +
 b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3) + d^(2/3)*(-(a*b^2*x) + (2*a*b + b^
2)*x^2 + (-a - 2*b)*x^3 + x^4)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(2/3)/(b-(a*d+1)*x+d*x
^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(ab-2\,bx+x^2\right)(b-x)}{\left(-(a-x)(b-x)^2x\right)^{\frac{2}{3}}\left(dx^2-(ad+1)x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(2/3)/(b-(a*d+1)*x+d*x
^2),x, algorithm="giac")
```

```
[Out] integrate(-(a*b - 2*b*x + x^2)*(b - x)/((-(a - x)*(b - x)^2*x)^(2/3)*(d*x^2
 - (a*d + 1)*x + b)), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)\left(ab-2bx+x^2\right)}{\left(x\,(-a+x)\,(-b+x)^2\right)^{\frac{2}{3}}\left(b-(ad+1)\,x+d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(2/3)/(b-(a*d+1)*x+d*x^2),x)
```

```
[Out] int((-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(2/3)/(b-(a*d+1)*x+d*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(ab-2\,bx+x^2\right)(b-x)}{\left(-(a-x)(b-x)^2x\right)^{\frac{2}{3}}\left(dx^2-(ad+1)x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(2/3)/(b-(a*d+1)*x+d*x
^2),x, algorithm="maxima")
```

```
[Out] -integrate((a*b - 2*b*x + x^2)*(b - x)/((-(a - x)*(b - x)^2*x)^(2/3)*(d*x^2
 - (a*d + 1)*x + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(b-x)\,\left(x^2-2\,b\,x+a\,b\right)}{{\left(-x\,(a-x)\,{(b-x)}^2\right)}^{2/3}\,\left(d\,x^2+(-a\,d-1)\,x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b - x)*(a*b - 2*b*x + x^2))/((-x*(a - x)*(b - x)^2)^(2/3)*(b - x*(a*
d + 1) + d*x^2)),x)
```

```
[Out] int(-((b - x)*(a*b - 2*b*x + x^2))/((-x*(a - x)*(b - x)^2)^(2/3)*(b - x*(a*
d + 1) + d*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(a*b-2*b*x+x**2)/(x*(-a+x)*(-b+x)**2)**(2/3)/(b-(a*d+1)*x+
d*x**2),x)
```

```
[Out] Timed out
```

3.2196 $\int \frac{(a-5b+4x)(-a^3+3a^2x-3ax^2+x^3)}{((-a+x)(-b+x))^{2/3}(-a^5+bd-(-5a^4+d)x-10a^3x^2+10a^2x^3-5ax^4+x^5)}\,dx$

Optimal. Leaf size=258

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}a^2-2\sqrt{3}ax+\sqrt{3}x^2}{a^2+2\sqrt[3]{d}\sqrt[3]{x(-a-b)+ab+x^2}-2ax+x^2}\right)}{\sqrt[3]{d}}+\frac{\log\left(a^3-2a^2x-a\sqrt[3]{d}\sqrt[3]{x(-a-b)+ab+x^2}+ax^2\right)}{\sqrt[3]{d}}-\frac{\log\left(a^6-4a^5x\right.}{}$$

Rubi [F] time = 9.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a-5b+4x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{((-a+x)(-b+x))^{2/3}\left(-a^5+bd-\left(-5a^4+d\right)x-10a^3x^2+10a^2x^3-5ax^4+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a - 5*b + 4*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/((((-a + x)*(-b + x))
^(2/3)*(-a^5 + b*d - (-5*a^4 + d)*x - 10*a^3*x^2 + 10*a^2*x^3 - 5*a*x^4 + x
^5)),x]
```

```
[Out] (-3*(a - 5*b)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][x^9/((a
 - b + x^3)^(2/3)*(a*(1 - b/a)*d + d*x^3 - x^15)), x], x, (-a + x)^(1/3)])/
((a - x)*(b - x))^(2/3) + (12*a*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Subst][
Defer[Int][x^9/((a - b + x^3)^(2/3)*(-(a*(1 - b/a)*d) - d*x^3 + x^15)), x],
 x, (-a + x)^(1/3)])/((a - x)*(b - x))^(2/3) + (12*(-a + x)^(2/3)*(-b + x)^
(2/3)*Defer[Subst][Defer[Int][x^12/((a - b + x^3)^(2/3)*(-(a*(1 - b/a)*d) -
 d*x^3 + x^15)), x], x, (-a + x)^(1/3)])/((a - x)*(b - x))^(2/3)
```

Rubi steps

$$\int \frac{(a-5b+4x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{((-a+x)(-b+x))^{2/3}\left(-a^5+bd-\left(-5a^4+d\right)x-10a^3x^2+10a^2x^3-5ax^4+x^5\right)}\,dx = \frac{\left((-a+x)^{2/3}(-b+x)^{2/3}\right)\ldots}{\ldots}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{2/3}\right)\ldots}{\ldots}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{2/3}\right)\ldots}{\ldots}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{2/3}\right)\ldots}{\ldots}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{2/3}\right)\ldots}{\ldots}$$

$$= \frac{\left(4(-a+x)^{2/3}(-b+x)^{2/3}\ldots\right.}{\ldots}$$

$$= \frac{\left(12(-a+x)^{2/3}(-b+x)^{2/3}\ldots\right.}{\ldots}$$

$$= \frac{\left(12(-a+x)^{2/3}(-b+x)^{2/3}\ldots\right.}{\ldots}$$

$$= \frac{\left(12(-a+x)^{2/3}(-b+x)^{2/3}\ldots\right.}{\ldots}$$

$$= \frac{\left(12(-a+x)^{2/3}(-b+x)^{2/3}\ldots\right.}{\ldots}$$

Mathematica [F] time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{(a-5b+4x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{((-a+x)(-b+x))^{2/3}\left(-a^5+bd-\left(-5a^4+d\right)x-10a^3x^2+10a^2x^3-5ax^4+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a - 5*b + 4*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/(((-a + x)*(-b
 + x))^(2/3)*(-a^5 + b*d - (-5*a^4 + d)*x - 10*a^3*x^2 + 10*a^2*x^3 - 5*a*x
^4 + x^5)),x]
```

```
[Out] Integrate[((a - 5*b + 4*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/(((-a + x)*(-b
 + x))^(2/3)*(-a^5 + b*d - (-5*a^4 + d)*x - 10*a^3*x^2 + 10*a^2*x^3 - 5*a*x
^4 + x^5)), x]
```

IntegrateAlgebraic [A] time = 5.43, size = 258, normalized size = 1.00

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}a^2-2\sqrt{3}ax+\sqrt{3}x^2}{a^2+2\sqrt[3]{d}\sqrt[3]{x(-a-b)+ab+x^2}-2ax+x^2}\right)}{\sqrt[3]{d}}+\frac{\log\left(a^3-2a^2x-a\sqrt[3]{d}\sqrt[3]{x(-a-b)+ab+x^2}+ax^2\right)}{\sqrt[3]{d}}-\frac{\log\left(a^6-4a^5x+6a^4x^2-4a^3x^3+a^2d^{2/3}\left(x(-a-b)+ab+x^2\right)^{2/3}+a^2x^4+\sqrt[3]{x(-a-b)+ab+x^2}\left(a^4\sqrt[3]{d}-2a^3\sqrt[3]{d}x+a^2\sqrt[3]{d}x^2\right)\right)}{2\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a - 5*b + 4*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/(((-a
 + x)*(-b + x))^(2/3)*(-a^5 + b*d - (-5*a^4 + d)*x - 10*a^3*x^2 + 10*a^2*x^
3 - 5*a*x^4 + x^5)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*a^2 - 2*Sqrt[3]*a*x + Sqrt[3]*x^2)/(a^2 - 2*a*x +
x^2 + 2*d^(1/3)*(a*b + (-a - b)*x + x^2)^(1/3))])/d^(1/3) + Log[a^3 - 2*a^2
*x + a*x^2 - a*d^(1/3)*(a*b + (-a - b)*x + x^2)^(1/3)]/d^(1/3) - Log[a^6 -
4*a^5*x + 6*a^4*x^2 - 4*a^3*x^3 + a^2*x^4 + a^2*d^(2/3)*(a*b + (-a - b)*x +
 x^2)^(2/3) + (a*b + (-a - b)*x + x^2)^(1/3)*(a^4*d^(1/3) - 2*a^3*d^(1/3)*x
 + a^2*d^(1/3)*x^2)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-5*b+4*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/((-a+x)*(-b+x))^(2/3)/(-a^
5+b*d-(-5*a^4+d)*x-10*a^3*x^2+10*a^2*x^3-5*a*x^4+x^5),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{3}-3\,a^{2}x+3\,ax^{2}-x^{3}\right)(a-5\,b+4\,x)}{\left(a^{5}+10\,a^{3}x^{2}-10\,a^{2}x^{3}+5\,ax^{4}-x^{5}-bd-\left(5\,a^{4}-d\right)x\right)\left((a-x)(b-x)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-5*b+4*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/((-a+x)*(-b+x))^(2/3)/(-a^
5+b*d-(-5*a^4+d)*x-10*a^3*x^2+10*a^2*x^3-5*a*x^4+x^5),x, algorithm="giac")
```

```
[Out] integrate((a^3 - 3*a^2*x + 3*a*x^2 - x^3)*(a - 5*b + 4*x)/((a^5 + 10*a^3*x^
2 - 10*a^2*x^3 + 5*a*x^4 - x^5 - b*d - (5*a^4 - d)*x)*((a - x)*(b - x))^(2/
3)), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{(a-5b+4x)\left(-a^{3}+3a^{2}x-3a\,x^{2}+x^{3}\right)}{\left((-a+x)\,(-b+x)\right)^{\frac{2}{3}}\left(-a^{5}+bd-\left(-5a^{4}+d\right)x-10a^{3}x^{2}+10a^{2}x^{3}-5a\,x^{4}+x^{5}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-5*b+4*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/((-a+x)*(-b+x))^(2/3)/(-a^5+b*d-
(-5*a^4+d)*x-10*a^3*x^2+10*a^2*x^3-5*a*x^4+x^5),x)
```

```
[Out] int((a-5*b+4*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/((-a+x)*(-b+x))^(2/3)/(-a^5+b*d-
(-5*a^4+d)*x-10*a^3*x^2+10*a^2*x^3-5*a*x^4+x^5),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{3}-3\,a^{2}x+3\,ax^{2}-x^{3}\right)(a-5\,b+4\,x)}{\left(a^{5}+10\,a^{3}x^{2}-10\,a^{2}x^{3}+5\,ax^{4}-x^{5}-bd-\left(5\,a^{4}-d\right)x\right)\left((a-x)(b-x)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-5*b+4*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/((-a+x)*(-b+x))^(2/3)/(-a^
5+b*d-(-5*a^4+d)*x-10*a^3*x^2+10*a^2*x^3-5*a*x^4+x^5),x, algorithm="maxima"
)
```

```
[Out] integrate((a^3 - 3*a^2*x + 3*a*x^2 - x^3)*(a - 5*b + 4*x)/((a^5 + 10*a^3*x^
2 - 10*a^2*x^3 + 5*a*x^4 - x^5 - b*d - (5*a^4 - d)*x)*((a - x)*(b - x))^(2/
3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int -\frac{(a-5\,b+4\,x)\,\left(a^3-3\,a^2\,x+3\,a\,x^2-x^3\right)}{((a-x)\,(b-x))^{2/3}\,\left(5\,a\,x^4-b\,d+x\,\left(d-5\,a^4\right)+a^5-x^5-10\,a^2\,x^3+10\,a^3\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a - 5*b + 4*x)*(3*a*x^2 - 3*a^2*x + a^3 - x^3))/(((a - x)*(b - x))^(2
/3)*(5*a*x^4 - b*d + x*(d - 5*a^4) + a^5 - x^5 - 10*a^2*x^3 + 10*a^3*x^2)),
x)
```

```
[Out] -int(-((a - 5*b + 4*x)*(3*a*x^2 - 3*a^2*x + a^3 - x^3))/(((a - x)*(b - x))^
(2/3)*(5*a*x^4 - b*d + x*(d - 5*a^4) + a^5 - x^5 - 10*a^2*x^3 + 10*a^3*x^2)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-a+x)^3\,(a-5b+4x)}{((-a+x)\,(-b+x))^{\frac{2}{3}}\left(-a^5+5a^4x-10a^3x^2+10a^2x^3-5ax^4+bd-dx+x^5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-5*b+4*x)*(-a**3+3*a**2*x-3*a*x**2+x**3)/((-a+x)*(-b+x))**(2/3)
/(-a**5+b*d-(-5*a**4+d)*x-10*a**3*x**2+10*a**2*x**3-5*a*x**4+x**5),x)
```

```
[Out] Integral((-a + x)**3*(a - 5*b + 4*x)/(((-a + x)*(-b + x))**(2/3)*(-a**5 + 5
*a**4*x - 10*a**3*x**2 + 10*a**2*x**3 - 5*a*x**4 + b*d - d*x + x**5)), x)
```

3.2197 $\int \frac{(-1+x+2x^2)\sqrt[4]{-x^3+x^4}}{-1-x+x^2}\,dx$

Optimal. Leaf size=261

$$\frac{1}{4}\sqrt[4]{x^4-x^3}\,(4x+11)-\frac{49}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)+\sqrt{2\left(11+5\sqrt{5}\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-x^3}}\right)-\sqrt{2\left(5\sqrt{5}-11\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\cdots}}{\sqrt[4]{x\cdots}}\right.$$

Rubi [B] time = 1.15, antiderivative size = 672, normalized size of antiderivative = 2.57, number of steps used = 34, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2056, 6728, 50, 63, 240, 212, 206, 203, 101, 157, 93, 298}

$$-\sqrt[4]{x^4-x^3}\,(1-x)+\frac{1}{2}\left(3+\sqrt{5}\right)\sqrt[4]{x^4-x^3}+\frac{1}{2}\left(3-\sqrt{5}\right)\sqrt[4]{x^4-x^3}+\frac{3}{4}\sqrt[4]{x^4-x^3}+\frac{\left(13+7\sqrt{5}\right)\sqrt[4]{x^4-x^3}\tan^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{4\sqrt[4]{x-1}\,x^{3/4}}+\frac{\left(13-7\sqrt{5}\right)\sqrt[4]{x^4-x^3}\tan^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{4\sqrt[4]{x-1}\,x^{3/4}}-\frac{3\sqrt[4]{x^4-x^3}\tan^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{8\sqrt[4]{x-1}\,x^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\sqrt[4]{x^4-x^3}\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{\sqrt[4]{2}\sqrt[4]{x-1}\,x^{3/4}}-\frac{\left(3-\sqrt{5}\right)^{5/4}\sqrt[4]{x^4-x^3}\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{\sqrt[4]{2}\sqrt[4]{x-1}\,x^{3/4}}+\frac{\left(13+7\sqrt{5}\right)\sqrt[4]{x^4-x^3}\tanh^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{4\sqrt[4]{x-1}\,x^{3/4}}+\frac{\left(13-7\sqrt{5}\right)\sqrt[4]{x^4-x^3}\tanh^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{4\sqrt[4]{x-1}\,x^{3/4}}-\frac{3\sqrt[4]{x^4-x^3}\tanh^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{8\sqrt[4]{x-1}\,x^{3/4}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\sqrt[4]{x^4-x^3}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{\sqrt[4]{2}\sqrt[4]{x-1}\,x^{3/4}}+\frac{\left(3-\sqrt{5}\right)^{5/4}\sqrt[4]{x^4-x^3}\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{\sqrt[4]{2}\sqrt[4]{x-1}\,x^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x + 2*x^2)*(-x^3 + x^4)^(1/4))/(-1 - x + x^2),x]
```

```
[Out] (3*(-x^3 + x^4)^(1/4))/4 + ((3 - Sqrt[5])*(-x^3 + x^4)^(1/4))/2 + ((3 + Sqr
t[5])*(-x^3 + x^4)^(1/4))/2 - (1 - x)*(-x^3 + x^4)^(1/4) - (3*(-x^3 + x^4)^
(1/4)*ArcTan[(-1 + x)^(1/4)/x^(1/4)])/(8*(-1 + x)^(1/4)*x^(3/4)) + ((13 - 7
*Sqrt[5])*(-x^3 + x^4)^(1/4)*ArcTan[(-1 + x)^(1/4)/x^(1/4)])/(4*(-1 + x)^(1
/4)*x^(3/4)) + ((13 + 7*Sqrt[5])*(-x^3 + x^4)^(1/4)*ArcTan[(-1 + x)^(1/4)/x
^(1/4)])/(4*(-1 + x)^(1/4)*x^(3/4)) + ((3 + Sqrt[5])^(5/4)*(-x^3 + x^4)^(1/
4)*ArcTan[((2/(3 + Sqrt[5]))^(1/4)*x^(1/4))/(-1 + x)^(1/4)])/(2^(1/4)*(-1 +
 x)^(1/4)*x^(3/4)) - ((3 - Sqrt[5])^(5/4)*(-x^3 + x^4)^(1/4)*ArcTan[(((3 +
Sqrt[5])/2)^(1/4)*x^(1/4))/(-1 + x)^(1/4)])/(2^(1/4)*(-1 + x)^(1/4)*x^(3/4)
) - (3*(-x^3 + x^4)^(1/4)*ArcTanh[(-1 + x)^(1/4)/x^(1/4)])/(8*(-1 + x)^(1/4
)*x^(3/4)) + ((13 - 7*Sqrt[5])*(-x^3 + x^4)^(1/4)*ArcTanh[(-1 + x)^(1/4)/x^
(1/4)])/(4*(-1 + x)^(1/4)*x^(3/4)) + ((13 + 7*Sqrt[5])*(-x^3 + x^4)^(1/4)*A
rcTanh[(-1 + x)^(1/4)/x^(1/4)])/(4*(-1 + x)^(1/4)*x^(3/4)) - ((3 + Sqrt[5])
^(5/4)*(-x^3 + x^4)^(1/4)*ArcTanh[((2/(3 + Sqrt[5]))^(1/4)*x^(1/4))/(-1 + x
)^(1/4)])/(2^(1/4)*(-1 + x)^(1/4)*x^(3/4)) + ((3 - Sqrt[5])^(5/4)*(-x^3 + x
^4)^(1/4)*ArcTanh[(((3 + Sqrt[5])/2)^(1/4)*x^(1/4))/(-1 + x)^(1/4)])/(2^(1/
4)*(-1 + x)^(1/4)*x^(3/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
```

```
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(-1+x+2x^2\right)\sqrt[4]{-x^3+x^4}}{-1-x+x^2}\,dx = \frac{\sqrt[4]{-x^3+x^4}\int \frac{\sqrt[4]{-1+x}\,x^{3/4}\left(-1+x+2x^2\right)}{-1-x+x^2}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{\sqrt[4]{-x^3+x^4}\int \left(2\sqrt[4]{-1+x}\,x^{3/4}+\frac{\sqrt[4]{-1+x}\,x^{3/4}(1+3x)}{-1-x+x^2}\right)dx}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{\sqrt[4]{-x^3+x^4}\int \frac{\sqrt[4]{-1+x}\,x^{3/4}(1+3x)}{-1-x+x^2}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}} + \frac{\left(2\sqrt[4]{-x^3+x^4}\right)\int \sqrt[4]{-1+x}\,x^{3/4}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= -\left((1-x)\sqrt[4]{-x^3+x^4}\right) + \frac{\left(3\sqrt[4]{-x^3+x^4}\right)\int \frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\,dx}{4\sqrt[4]{-1+x}\,x^{3/4}} + \frac{\sqrt[4]{-x^3+x^4}\int \left(\frac{(3+\sqrt{\ldots}}{-\ldots}\right.}{\ldots}$$

$$= \frac{3}{4}\sqrt[4]{-x^3+x^4} - (1-x)\sqrt[4]{-x^3+x^4} - \frac{\left(3\sqrt[4]{-x^3+x^4}\right)\int \frac{1}{(-1+x)^{3/4}\sqrt[4]{x}}\,dx}{16\sqrt[4]{-1+x}\,x^{3/4}} + \frac{\left(\left(3\ldots\right.\right.}{\ldots}$$

$$= \frac{3}{4}\sqrt[4]{-x^3+x^4} + \frac{1}{2}\left(3-\sqrt{5}\right)\sqrt[4]{-x^3+x^4} + \frac{1}{2}\left(3+\sqrt{5}\right)\sqrt[4]{-x^3+x^4} - (1-x)\ldots$$

$$= \frac{3}{4}\sqrt[4]{-x^3+x^4} + \frac{1}{2}\left(3-\sqrt{5}\right)\sqrt[4]{-x^3+x^4} + \frac{1}{2}\left(3+\sqrt{5}\right)\sqrt[4]{-x^3+x^4} - (1-x)\ldots$$

$$= \frac{3}{4}\sqrt[4]{-x^3+x^4} + \frac{1}{2}\left(3-\sqrt{5}\right)\sqrt[4]{-x^3+x^4} + \frac{1}{2}\left(3+\sqrt{5}\right)\sqrt[4]{-x^3+x^4} - (1-x)\ldots$$

$$= \frac{3}{4}\sqrt[4]{-x^3+x^4} + \frac{1}{2}\left(3-\sqrt{5}\right)\sqrt[4]{-x^3+x^4} + \frac{1}{2}\left(3+\sqrt{5}\right)\sqrt[4]{-x^3+x^4} - (1-x)\ldots$$

$$= \frac{3}{4}\sqrt[4]{-x^3+x^4} + \frac{1}{2}\left(3-\sqrt{5}\right)\sqrt[4]{-x^3+x^4} + \frac{1}{2}\left(3+\sqrt{5}\right)\sqrt[4]{-x^3+x^4} - (1-x)\ldots$$

$$= \frac{3}{4}\sqrt[4]{-x^3+x^4} + \frac{1}{2}\left(3-\sqrt{5}\right)\sqrt[4]{-x^3+x^4} + \frac{1}{2}\left(3+\sqrt{5}\right)\sqrt[4]{-x^3+x^4} - (1-x)\ldots$$

Mathematica [C] time = 0.26, size = 168, normalized size = 0.64

$$\frac{4\sqrt[4]{(x-1)x^3}\left(15\sqrt[4]{x}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};1-x\right)+2(x-1)\sqrt[4]{x}\,{}_2F_1\left(-\frac{3}{4},\frac{5}{4};\frac{9}{4};1-x\right)+5\sqrt[4]{x}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};1-x\right)+5\left(\sqrt{5}-2\right){}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{\left(-1+\sqrt{5}\right)(x-1)}{\left(1+\sqrt{5}\right)x}\right)-5\left(2+\sqrt{5}\right){}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{\left(1+\sqrt{5}\right)(x-1)}{\left(-1+\sqrt{5}\right)x}\right)\right)}{5x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x + 2*x^2)*(-x^3 + x^4)^(1/4))/(-1 - x + x^2),x]
```

```
[Out] (4*((-1 + x)*x^3)^(1/4)*(15*x^(1/4)*Hypergeometric2F1[-3/4, 1/4, 5/4, 1 - x
] + 2*(-1 + x)*x^(1/4)*Hypergeometric2F1[-3/4, 5/4, 9/4, 1 - x] + 5*x^(1/4)
*Hypergeometric2F1[1/4, 1/4, 5/4, 1 - x] + 5*(-2 + Sqrt[5])*Hypergeometric2
F1[1/4, 1, 5/4, ((-1 + Sqrt[5])*(-1 + x))/((1 + Sqrt[5])*x)] - 5*(2 + Sqrt[
5])*Hypergeometric2F1[1/4, 1, 5/4, ((1 + Sqrt[5])*(-1 + x))/((-1 + Sqrt[5])
*x)]))/(5*x)
```

IntegrateAlgebraic [A] time = 1.59, size = 261, normalized size = 1.00

$$\frac{1}{4}\sqrt[4]{x^4-x^3}(4x+11)-\frac{49}{8}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)+\sqrt{2\left(11+5\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}x}{\sqrt[4]{x^4-x^3}}\right)-\sqrt{2\left(5\sqrt{5}-11\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}x}{\sqrt[4]{x^4-x^3}}\right)+\frac{49}{8}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)-\sqrt{2\left(11+5\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}x}{\sqrt[4]{x^4-x^3}}\right)+\sqrt{2\left(5\sqrt{5}-11\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}x}{\sqrt[4]{x^4-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x + 2*x^2)*(-x^3 + x^4)^(1/4))/(-1 - x + x^2),x]
```

```
[Out] ((11 + 4*x)*(-x^3 + x^4)^(1/4))/4 - (49*ArcTan[x/(-x^3 + x^4)^(1/4)])/8 + S
qrt[2*(11 + 5*Sqrt[5])]*ArcTan[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(-x^3 + x^4)^(1/4
)] - Sqrt[2*(-11 + 5*Sqrt[5])]*ArcTan[(Sqrt[1/2 + Sqrt[5]/2]*x)/(-x^3 + x^4
)^(1/4)] + (49*ArcTanh[x/(-x^3 + x^4)^(1/4)])/8 - Sqrt[2*(11 + 5*Sqrt[5])]*
ArcTanh[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(-x^3 + x^4)^(1/4)] + Sqrt[2*(-11 + 5*Sq
rt[5])]*ArcTanh[(Sqrt[1/2 + Sqrt[5]/2]*x)/(-x^3 + x^4)^(1/4)]
```

fricas [B] time = 0.91, size = 474, normalized size = 1.82

$$-2\sqrt{10\sqrt{5}-22}\arctan\left(\frac{\sqrt{2}\left(\sqrt{5}x+3x\right)\sqrt{10\sqrt{5}-22}\sqrt{\frac{\sqrt{5}x^2+x^2+2\sqrt{x^4-x^3}}{x^2}}-2\left(x^4-x^3\right)^{\frac{1}{4}}\sqrt{10\sqrt{5}-22}\left(\sqrt{5}+3\right)}{8x}\right)-2\sqrt{10\sqrt{5}+22}\arctan\left(\frac{\sqrt{2}\left(\sqrt{5}x-3x\right)\sqrt{10\sqrt{5}+22}\sqrt{\frac{\sqrt{5}x^2-x^2+2\sqrt{x^4-x^3}}{x^2}}-2\left(x^4-x^3\right)^{\frac{1}{4}}\sqrt{10\sqrt{5}+22}\left(\sqrt{5}-3\right)}{8x}\right)-\frac{1}{2}\sqrt{10\sqrt{5}+22}\log\left(\frac{\left(\sqrt{5}x-2x\right)\sqrt{10\sqrt{5}+22}+2\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\sqrt{10\sqrt{5}+22}\log\left(-\frac{\left(\sqrt{5}x-2x\right)\sqrt{10\sqrt{5}+22}-2\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\sqrt{10\sqrt{5}-22}\log\left(\frac{\left(\sqrt{5}x+2x\right)\sqrt{10\sqrt{5}-22}+2\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{2}\sqrt{10\sqrt{5}-22}\log\left(-\frac{\left(\sqrt{5}x+2x\right)\sqrt{10\sqrt{5}-22}-2\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{4}\left(x^4-x^3\right)^{\frac{1}{4}}(4x+11)+\frac{49}{8}\arctan\left(\frac{\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{49}{16}\log\left(\frac{x+\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)-\frac{49}{16}\log\left(-\frac{x-\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+x-1)*(x^4-x^3)^(1/4)/(x^2-x-1),x, algorithm="fricas")
```

```
[Out] -2*sqrt(10*sqrt(5) - 22)*arctan(1/8*(sqrt(2)*(sqrt(5)*x + 3*x)*sqrt(10*sqrt
(5) - 22)*sqrt((sqrt(5)*x^2 + x^2 + 2*sqrt(x^4 - x^3))/x^2) - 2*(x^4 - x^3)
^(1/4)*sqrt(10*sqrt(5) - 22)*(sqrt(5) + 3))/x) - 2*sqrt(10*sqrt(5) + 22)*ar
ctan(1/8*(sqrt(2)*(sqrt(5)*x - 3*x)*sqrt(10*sqrt(5) + 22)*sqrt((sqrt(5)*x^2
 - x^2 + 2*sqrt(x^4 - x^3))/x^2) - 2*(x^4 - x^3)^(1/4)*sqrt(10*sqrt(5) + 22
)*(sqrt(5) - 3))/x) - 1/2*sqrt(10*sqrt(5) + 22)*log(((sqrt(5)*x - 2*x)*sqrt
(10*sqrt(5) + 22) + 2*(x^4 - x^3)^(1/4))/x) + 1/2*sqrt(10*sqrt(5) + 22)*log
(-((sqrt(5)*x - 2*x)*sqrt(10*sqrt(5) + 22) - 2*(x^4 - x^3)^(1/4))/x) + 1/2*
sqrt(10*sqrt(5) - 22)*log(((sqrt(5)*x + 2*x)*sqrt(10*sqrt(5) - 22) + 2*(x^4
 - x^3)^(1/4))/x) - 1/2*sqrt(10*sqrt(5) - 22)*log(-((sqrt(5)*x + 2*x)*sqrt(
10*sqrt(5) - 22) - 2*(x^4 - x^3)^(1/4))/x) + 1/4*(x^4 - x^3)^(1/4)*(4*x + 1
1) + 49/8*arctan((x^4 - x^3)^(1/4)/x) + 49/16*log((x + (x^4 - x^3)^(1/4))/x
) - 49/16*log(-(x - (x^4 - x^3)^(1/4))/x)
```

giac [A] time = 1.10, size = 261, normalized size = 1.00

$$\frac{1}{4}\left(11\left(-\frac{1}{x}+1\right)^{\frac{5}{4}}-15\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)x^2-\sqrt{10\sqrt{5}-22}\arctan\left(\frac{\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\sqrt{10\sqrt{5}+22}\arctan\left(\frac{\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{2}\sqrt{10\sqrt{5}-22}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)+\frac{1}{2}\sqrt{10\sqrt{5}+22}\log\left(\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)+\frac{1}{2}\sqrt{10\sqrt{5}-22}\log\left(\left|-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right|\right)-\frac{1}{2}\sqrt{10\sqrt{5}+22}\log\left(\left|-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right|\right)-\frac{49}{8}\arctan\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)-\frac{49}{16}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right)+\frac{49}{16}\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+x-1)*(x^4-x^3)^(1/4)/(x^2-x-1),x, algorithm="giac")
```

```
[Out] 1/4*(11*(-1/x + 1)^(5/4) - 15*(-1/x + 1)^(1/4))*x^2 - sqrt(10*sqrt(5) - 22)
*arctan((-1/x + 1)^(1/4)/sqrt(1/2*sqrt(5) + 1/2)) + sqrt(10*sqrt(5) + 22)*a
rctan((-1/x + 1)^(1/4)/sqrt(1/2*sqrt(5) - 1/2)) - 1/2*sqrt(10*sqrt(5) - 22)
*log(sqrt(1/2*sqrt(5) + 1/2) + (-1/x + 1)^(1/4)) + 1/2*sqrt(10*sqrt(5) + 22
)*log(sqrt(1/2*sqrt(5) - 1/2) + (-1/x + 1)^(1/4)) + 1/2*sqrt(10*sqrt(5) - 2
2)*log(abs(-sqrt(1/2*sqrt(5) + 1/2) + (-1/x + 1)^(1/4))) - 1/2*sqrt(10*sqrt
```

```
(5) + 22)*log(abs(-sqrt(1/2*sqrt(5) - 1/2) + (-1/x + 1)^(1/4))) - 49/8*arct
an((-1/x + 1)^(1/4)) - 49/16*log((-1/x + 1)^(1/4) + 1) + 49/16*log(abs((-1/
x + 1)^(1/4) - 1))
```

maple [C] time = 10.51, size = 4137, normalized size = 15.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+x-1)*(x^4-x^3)^(1/4)/(x^2-x-1),x)
```

```
[Out] 1/4*(11+4*x)*(x^3*(-1+x))^(1/4)+(-1/16*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^
2+_Z^2+2816)*ln((RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*RootOf(_Z
^4+2816*_Z^2-65536)^4*x^3-4*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816
)*RootOf(_Z^4+2816*_Z^2-65536)^4*x^2+5*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^
2+_Z^2+2816)*RootOf(_Z^4+2816*_Z^2-65536)^4*x-8192*(x^4-3*x^3+3*x^2-x)^(1/2
)*RootOf(_Z^4+2816*_Z^2-65536)^2*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2
+2816)*x+13568*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*RootOf(_Z^4
+2816*_Z^2-65536)^2*x^3-2*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*
RootOf(_Z^4+2816*_Z^2-65536)^4+8192*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4+2
816*_Z^2-65536)^2*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)-40192*Ro
otOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*RootOf(_Z^4+2816*_Z^2-65536)
^2*x^2+368640*RootOf(_Z^4+2816*_Z^2-65536)^2*(x^4-3*x^3+3*x^2-x)^(3/4)-6062
08*RootOf(_Z^4+2816*_Z^2-65536)^2*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+39680*RootO
f(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*RootOf(_Z^4+2816*_Z^2-65536)^2*
x-25296896*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+
_Z^2+2816)*x+28180480*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*x^3+
1212416*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4+2816*_Z^2-65536)^2*x-13056*Ro
otOf(_Z^4+2816*_Z^2-65536)^2*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+281
6)+25296896*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2
+_Z^2+2816)-76677120*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*x^2-6
06208*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4+2816*_Z^2-65536)^2+1059061760*(
x^4-3*x^3+3*x^2-x)^(3/4)-1713373184*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+68812800*
RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*x+3426746368*(x^4-3*x^3+3*
x^2-x)^(1/4)*x-20316160*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)-17
13373184*(x^4-3*x^3+3*x^2-x)^(1/4))/(x*RootOf(_Z^4+2816*_Z^2-65536)^2-2*Roo
tOf(_Z^4+2816*_Z^2-65536)^2+3328*x-5376)/(-1+x)^2)+1/16*RootOf(_Z^4+2816*_Z
^2-65536)*ln(-(RootOf(_Z^4+2816*_Z^2-65536)^5*x^3-4*RootOf(_Z^4+2816*_Z^2-6
5536)^5*x^2+5*RootOf(_Z^4+2816*_Z^2-65536)^5*x+8192*(x^4-3*x^3+3*x^2-x)^(1/
2)*RootOf(_Z^4+2816*_Z^2-65536)^3*x-7936*RootOf(_Z^4+2816*_Z^2-65536)^3*x^3
-2*RootOf(_Z^4+2816*_Z^2-65536)^5-8192*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^
4+2816*_Z^2-65536)^3+17664*RootOf(_Z^4+2816*_Z^2-65536)^3*x^2+368640*RootOf
(_Z^4+2816*_Z^2-65536)^2*(x^4-3*x^3+3*x^2-x)^(3/4)-606208*RootOf(_Z^4+2816*
_Z^2-65536)^2*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2-11520*RootOf(_Z^4+2816*_Z^2-655
36)^3*x+1212416*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4+2816*_Z^2-65536)^2*x-
2228224*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4+2816*_Z^2-65536)*x-2097152*Ro
otOf(_Z^4+2816*_Z^2-65536)*x^3+1792*RootOf(_Z^4+2816*_Z^2-65536)^3-606208*(
x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4+2816*_Z^2-65536)^2+2228224*(x^4-3*x^3+
3*x^2-x)^(1/2)*RootOf(_Z^4+2816*_Z^2-65536)+4784128*RootOf(_Z^4+2816*_Z^2-6
5536)*x^2-20971520*(x^4-3*x^3+3*x^2-x)^(3/4)+6291456*(x^4-3*x^3+3*x^2-x)^(1
/4)*x^2-3276800*RootOf(_Z^4+2816*_Z^2-65536)*x-12582912*(x^4-3*x^3+3*x^2-x)
^(1/4)*x+589824*RootOf(_Z^4+2816*_Z^2-65536)+6291456*(x^4-3*x^3+3*x^2-x)^(1
/4))/(x*RootOf(_Z^4+2816*_Z^2-65536)^2-2*RootOf(_Z^4+2816*_Z^2-65536)^2-512
*x-256)/(-1+x)^2)+1/4096*RootOf(_Z^4+2816*_Z^2-65536)^2*RootOf(RootOf(_Z^4+
2816*_Z^2-65536)^2+_Z^2+2816)*ln((5*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_
Z^2+2816)*RootOf(_Z^4+2816*_Z^2-65536)^4*x^3-20*RootOf(RootOf(_Z^4+2816*_Z^
2-65536)^2+_Z^2+2816)*RootOf(_Z^4+2816*_Z^2-65536)^4*x^2+25*RootOf(RootOf(_
Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*RootOf(_Z^4+2816*_Z^2-65536)^4*x-49408*(x
^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4+2816*_Z^2-65536)^2*RootOf(RootOf(_Z^4+2
816*_Z^2-65536)^2+_Z^2+2816)*x-41088*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+
```

_Z^2+2816)*RootOf(_Z^4+2816*_Z^2-65536)^2*x^3-10*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*RootOf(_Z^4+2816*_Z^2-65536)^4+49408*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4+2816*_Z^2-65536)^2*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)+93952*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*RootOf(_Z^4+2816*_Z^2-65536)^2*x^2-184320*RootOf(_Z^4+2816*_Z^2-65536)^2*(x^4-3*x^3+3*x^2-x)^(3/4)-303104*RootOf(_Z^4+2816*_Z^2-65536)^2*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2-64640*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*RootOf(_Z^4+2816*_Z^2-65536)^2*x+1048576*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*x+1048576*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*x^3+606208*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4+2816*_Z^2-65536)^2*x+11776*RootOf(_Z^4+2816*_Z^2-65536)^2*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)-1048576*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)-2392064*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*x^2-303104*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4+2816*_Z^2-65536)^2+10485760*(x^4-3*x^3+3*x^2-x)^(3/4)+3145728*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+1638400*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*x-6291456*(x^4-3*x^3+3*x^2-x)^(1/4)*x-294912*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)+3145728*(x^4-3*x^3+3*x^2-x)^(1/4))/(x*RootOf(_Z^4+2816*_Z^2-65536)^2-2*RootOf(_Z^4+2816*_Z^2-65536)^2-512*x-256)/(-1+x)^2)-1/4096*ln((5*RootOf(_Z^4+2816*_Z^2-65536)^5*x^3-20*RootOf(_Z^4+2816*_Z^2-65536)^5*x^2+25*RootOf(_Z^4+2816*_Z^2-65536)^5*x+49408*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4+2816*_Z^2-65536)^3*x+69248*RootOf(_Z^4+2816*_Z^2-65536)^3*x^3-10*RootOf(_Z^4+2816*_Z^2-65536)^5-49408*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4+2816*_Z^2-65536)^3-206592*RootOf(_Z^4+2816*_Z^2-65536)^3*x^2+184320*RootOf(_Z^4+2816*_Z^2-65536)^2*(x^4-3*x^3+3*x^2-x)^(3/4)+303104*RootOf(_Z^4+2816*_Z^2-65536)^2*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+205440*RootOf(_Z^4+2816*_Z^2-65536)^3*x-606208*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4+2816*_Z^2-65536)^2*x+140181504*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4+2816*_Z^2-65536)*x+156401664*RootOf(_Z^4+2816*_Z^2-65536)*x^3-68096*RootOf(_Z^4+2816*_Z^2-65536)^3+303104*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4+2816*_Z^2-65536)^2-140181504*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4+2816*_Z^2-65536)-425558016*RootOf(_Z^4+2816*_Z^2-65536)*x^2+529530880*(x^4-3*x^3+3*x^2-x)^(3/4)+856686592*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+381911040*RootOf(_Z^4+2816*_Z^2-65536)*x-1713373184*(x^4-3*x^3+3*x^2-x)^(1/4)*x-112754688*RootOf(_Z^4+2816*_Z^2-65536)+856686592*(x^4-3*x^3+3*x^2-x)^(1/4))/(x*RootOf(_Z^4+2816*_Z^2-65536)^2-2*RootOf(_Z^4+2816*_Z^2-65536)^2+3328*x-5376)/(-1+x)^2)*RootOf(_Z^4+2816*_Z^2-65536)^3-11/16*ln((5*RootOf(_Z^4+2816*_Z^2-65536)^5*x^3-20*RootOf(_Z^4+2816*_Z^2-65536)^5*x^2+25*RootOf(_Z^4+2816*_Z^2-65536)^5*x+49408*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4+2816*_Z^2-65536)^3*x+69248*RootOf(_Z^4+2816*_Z^2-65536)^3*x^3-10*RootOf(_Z^4+2816*_Z^2-65536)^5-49408*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4+2816*_Z^2-65536)^3-206592*RootOf(_Z^4+2816*_Z^2-65536)^3*x^2+184320*RootOf(_Z^4+2816*_Z^2-65536)^2*(x^4-3*x^3+3*x^2-x)^(3/4)+303104*RootOf(_Z^4+2816*_Z^2-65536)^2*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+205440*RootOf(_Z^4+2816*_Z^2-65536)^3*x-606208*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4+2816*_Z^2-65536)^2*x+140181504*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4+2816*_Z^2-65536)*x+156401664*RootOf(_Z^4+2816*_Z^2-65536)*x^3-68096*RootOf(_Z^4+2816*_Z^2-65536)^3+303104*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(_Z^4+2816*_Z^2-65536)^2-140181504*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^4+2816*_Z^2-65536)-425558016*RootOf(_Z^4+2816*_Z^2-65536)*x^2+529530880*(x^4-3*x^3+3*x^2-x)^(3/4)+856686592*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+381911040*RootOf(_Z^4+2816*_Z^2-65536)*x-1713373184*(x^4-3*x^3+3*x^2-x)^(1/4)*x-112754688*RootOf(_Z^4+2816*_Z^2-65536)+856686592*(x^4-3*x^3+3*x^2-x)^(1/4))/(x*RootOf(_Z^4+2816*_Z^2-65536)^2-2*RootOf(_Z^4+2816*_Z^2-65536)^2+3328*x-5376)/(-1+x)^2)*RootOf(_Z^4+2816*_Z^2-65536)-49/4096*RootOf(_Z^4+2816*_Z^2-65536)*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*ln((2*RootOf(_Z^4+2816*_Z^2-65536)*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*(x^4-3*x^3+3*x^2-x)^(1/2)*x-2*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*RootOf(_Z^4+2816*_Z^2-65536)*x^3-2*RootOf(_Z^4+2816*_Z^2-65536)*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*(x^4-3*x^3+3*x^2-x)^(1/2)+5*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*RootOf(_Z^4+2816*_Z^2-65536)*x^

```
2-4*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)*RootOf(_Z^4+2816*_Z^2-
65536)*x+512*(x^4-3*x^3+3*x^2-x)^(3/4)-512*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+Ro
otOf(_Z^4+2816*_Z^2-65536)*RootOf(RootOf(_Z^4+2816*_Z^2-65536)^2+_Z^2+2816)
+1024*(x^4-3*x^3+3*x^2-x)^(1/4)*x-512*(x^4-3*x^3+3*x^2-x)^(1/4))/(-1+x)^2)+
49/16*ln((2*(x^4-3*x^3+3*x^2-x)^(3/4)+2*(x^4-3*x^3+3*x^2-x)^(1/2)*x+2*(x^4-
3*x^3+3*x^2-x)^(1/4)*x^2+2*x^3-2*(x^4-3*x^3+3*x^2-x)^(1/2)-4*(x^4-3*x^3+3*x
^2-x)^(1/4)*x-5*x^2+2*(x^4-3*x^3+3*x^2-x)^(1/4)+4*x-1)/(-1+x)^2))*(x^3*(-1+
x))^(1/4)/(-1+x)/x*(x*(-1+x)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4 - x^3\right)^{\frac{1}{4}} \left(2\,x^2 + x - 1\right)}{x^2 - x - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+x-1)*(x^4-x^3)^(1/4)/(x^2-x-1),x, algorithm="maxima")
[Out] integrate((x^4 - x^3)^(1/4)*(2*x^2 + x - 1)/(x^2 - x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(x^4 - x^3\right)^{1/4}\,\left(2\,x^2 + x - 1\right)}{-x^2 + x + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^4 - x^3)^(1/4)*(x + 2*x^2 - 1))/(x - x^2 + 1),x)
[Out] int(-((x^4 - x^3)^(1/4)*(x + 2*x^2 - 1))/(x - x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\,(x-1)}\;(x+1)\,(2x-1)}{x^2 - x - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+x-1)*(x**4-x**3)**(1/4)/(x**2-x-1),x)
[Out] Integral((x**3*(x - 1))**(1/4)*(x + 1)*(2*x - 1)/(x**2 - x - 1), x)
```

3.2198 $\int \frac{(2+x^6)(-1+x^4+x^6)}{\sqrt[4]{-1+x^6}\,(1-2x^6+x^8+x^{12})}\,dx$

Optimal. Leaf size=261

$$-\frac{1}{2}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^6-1}}{\sqrt{x^6-1}-x^2}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\left(\sqrt{\frac{2}{2-\sqrt{2}}}-\frac{2}{\sqrt{2-\sqrt{2}}}\right)x\sqrt[4]{x^6-1}}{\sqrt{x^6-1}-x^2}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(\right.}$$

Rubi [F] time = 1.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(2+x^6)(-1+x^4+x^6)}{\sqrt[4]{-1+x^6}\,(1-2x^6+x^8+x^{12})}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((2 + x^6)*(-1 + x^4 + x^6))/((-1 + x^6)^(1/4)*(1 - 2*x^6 + x^8 + x^12)
),x]
```

```
[Out] (x*(1 - x^6)^(1/4)*Hypergeometric2F1[1/6, 1/4, 7/6, x^6])/(-1 + x^6)^(1/4)
- 3*Defer[Int][1/((-1 + x^6)^(1/4)*(1 - 2*x^6 + x^8 + x^12)), x] + 2*Defer[
Int][x^4/((-1 + x^6)^(1/4)*(1 - 2*x^6 + x^8 + x^12)), x] + 3*Defer[Int][x^6
/((-1 + x^6)^(1/4)*(1 - 2*x^6 + x^8 + x^12)), x] - Defer[Int][x^8/((-1 + x^
6)^(1/4)*(1 - 2*x^6 + x^8 + x^12)), x] + Defer[Int][x^10/((-1 + x^6)^(1/4)*
(1 - 2*x^6 + x^8 + x^12)), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+x^6)(-1+x^4+x^6)}{\sqrt[4]{-1+x^6}\,(1-2x^6+x^8+x^{12})}\,dx &= \int \left(\frac{1}{\sqrt[4]{-1+x^6}}-\frac{3-2x^4-3x^6+x^8-x^{10}}{\sqrt[4]{-1+x^6}\,(1-2x^6+x^8+x^{12})}\right)dx\\
&= \int \frac{1}{\sqrt[4]{-1+x^6}}\,dx-\int \frac{3-2x^4-3x^6+x^8-x^{10}}{\sqrt[4]{-1+x^6}\,(1-2x^6+x^8+x^{12})}\,dx\\
&= \frac{\sqrt[4]{1-x^6}\int \frac{1}{\sqrt[4]{1-x^6}}\,dx}{\sqrt[4]{-1+x^6}}-\int \left(\frac{3}{\sqrt[4]{-1+x^6}\,(1-2x^6+x^8+x^{12})}-\frac{}{\sqrt[4]{-1+x^6}\,(}\right.\\
&= \frac{x\sqrt[4]{1-x^6}\,{}_2F_1\left(\frac{1}{6},\frac{1}{4};\frac{7}{6};x^6\right)}{\sqrt[4]{-1+x^6}}+2\int \frac{x^4}{\sqrt[4]{-1+x^6}\,(1-2x^6+x^8+x^{12})}\,dx-3
\end{aligned}$$

Mathematica [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{(2+x^6)(-1+x^4+x^6)}{\sqrt[4]{-1+x^6}\,(1-2x^6+x^8+x^{12})}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((2 + x^6)*(-1 + x^4 + x^6))/((-1 + x^6)^(1/4)*(1 - 2*x^6 + x^8 +
 x^12)),x]
```

[Out] Integrate[((2 + x^6)*(-1 + x^4 + x^6))/((-1 + x^6)^(1/4)*(1 - 2*x^6 + x^8 + x^12)), x]

IntegrateAlgebraic [A] time = 16.14, size = 241, normalized size = 0.92

$$-\frac{1}{2}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^6-1}}{\sqrt{x^6-1}-x^2}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^6-1}}{\sqrt{x^6-1}-x^2}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^6-1}}{\sqrt{x^6-1}+x^2}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^6-1}}{\sqrt{x^6-1}+x^2}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + x^6)*(-1 + x^4 + x^6))/((-1 + x^6)^(1/4)*(1 - 2*x^6 + x^8 + x^12)),x]

[Out] -1/2*(Sqrt[(2 + Sqrt[2])/2]*ArcTan[(Sqrt[2 - Sqrt[2]]*x*(-1 + x^6)^(1/4))/(-x^2 + Sqrt[-1 + x^6])]) - (Sqrt[(2 - Sqrt[2])/2]*ArcTan[(Sqrt[2 + Sqrt[2]]*x*(-1 + x^6)^(1/4))/(-x^2 + Sqrt[-1 + x^6])])/2 - (Sqrt[(2 + Sqrt[2])/2]*ArcTanh[(Sqrt[2 - Sqrt[2]]*x*(-1 + x^6)^(1/4))/(x^2 + Sqrt[-1 + x^6])])/2 - (Sqrt[(2 - Sqrt[2])/2]*ArcTanh[(Sqrt[2 + Sqrt[2]]*x*(-1 + x^6)^(1/4))/(x^2 + Sqrt[-1 + x^6])])/2

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+2)*(x^6+x^4-1)/(x^6-1)^(1/4)/(x^12+x^8-2*x^6+1),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^4-1\right)\left(x^6+2\right)}{\left(x^{12}+x^8-2\,x^6+1\right)\left(x^6-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+2)*(x^6+x^4-1)/(x^6-1)^(1/4)/(x^12+x^8-2*x^6+1),x, algorithm="giac")

[Out] integrate((x^6 + x^4 - 1)*(x^6 + 2)/((x^12 + x^8 - 2*x^6 + 1)*(x^6 - 1)^(1/4)), x)

maple [C] time = 26.27, size = 684, normalized size = 2.62

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6+2)*(x^6+x^4-1)/(x^6-1)^(1/4)/(x^12+x^8-2*x^6+1),x)

[Out] -1/8*RootOf(_Z^8+16)^3*ln(-(-RootOf(_Z^8+16)^10*x^4+4*RootOf(_Z^8+16)^6*x^6-4*x^4*RootOf(_Z^8+16)^6+16*RootOf(_Z^8+16)^4*(x^6-1)^(1/2)*x^2+16*RootOf(_Z^8+16)^2*x^6+32*RootOf(_Z^8+16)^3*(x^6-1)^(3/4)*x-4*RootOf(_Z^8+16)^6+64*RootOf(_Z^8+16)*(x^6-1)^(1/4)*x^3+64*x^2*(x^6-1)^(1/2)-16*RootOf(_Z^8+16)^2)/(RootOf(_Z^8+16)^4*x^4+4*x^6-4))-1/32*RootOf(_Z^8+16)^7*ln(-(-RootOf(_Z^8+16)^10*x^4+4*RootOf(_Z^8+16)^6*x^6+4*x^4*RootOf(_Z^8+16)^6-16*RootOf(_Z^8+16)^4*(x^6-1)^(1/2)*x^2-16*RootOf(_Z^8+16)^2*x^6+32*RootOf(_Z^8+16)^3*(x^6-1)^(3/4)*x-4*RootOf(_Z^8+16)^6-64*RootOf(_Z^8+16)*(x^6-1)^(1/4)*x^3+64*x^2*(x^6-1)^(1/2)+16*RootOf(_Z^8+16)^2)/(RootOf(_Z^8+16)^4*x^4+4*x^6-4))+1/4*RootOf(_Z^8+16)*ln((-RootOf(_Z^8+16)^8*x^4+4*(x^6-1)^(1/2)*RootOf(_Z^8+16)^6*x

```
^2-4*RootOf(_Z^8+16)^4*x^6-8*(x^6-1)^(1/4)*RootOf(_Z^8+16)^5*x^3+4*RootOf(_
Z^8+16)^4*x^4+16*RootOf(_Z^8+16)^3*(x^6-1)^(3/4)*x-16*RootOf(_Z^8+16)^2*(x^
6-1)^(1/2)*x^2+16*x^6+4*RootOf(_Z^8+16)^4-16)/(RootOf(_Z^8+16)^4*x^4-4*x^6+
4))-1/16*RootOf(_Z^8+16)^5*ln((-RootOf(_Z^8+16)^8*x^4-4*(x^6-1)^(1/2)*RootO
f(_Z^8+16)^6*x^2-4*RootOf(_Z^8+16)^4*x^6+8*(x^6-1)^(1/4)*RootOf(_Z^8+16)^5*
x^3-4*RootOf(_Z^8+16)^4*x^4+16*RootOf(_Z^8+16)^3*(x^6-1)^(3/4)*x-16*RootOf(
_Z^8+16)^2*(x^6-1)^(1/2)*x^2-16*x^6+4*RootOf(_Z^8+16)^4+16)/(RootOf(_Z^8+16
)^4*x^4-4*x^6+4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}+x^{4}-1\right)\left(x^{6}+2\right)}{\left(x^{12}+x^{8}-2\,x^{6}+1\right)\left(x^{6}-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+2)*(x^6+x^4-1)/(x^6-1)^(1/4)/(x^12+x^8-2*x^6+1),x, algorithm
="maxima")
```

```
[Out] integrate((x^6 + x^4 - 1)*(x^6 + 2)/((x^12 + x^8 - 2*x^6 + 1)*(x^6 - 1)^(1/
4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^6+2\right)\,\left(x^6+x^4-1\right)}{{\left(x^6-1\right)}^{1/4}\,\left(x^{12}+x^8-2\,x^6+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 + 2)*(x^4 + x^6 - 1))/((x^6 - 1)^(1/4)*(x^8 - 2*x^6 + x^12 + 1)),
x)
```

```
[Out] int(((x^6 + 2)*(x^4 + x^6 - 1))/((x^6 - 1)^(1/4)*(x^8 - 2*x^6 + x^12 + 1)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}+2\right)\left(x^{6}+x^{4}-1\right)}{\sqrt[4]{\left(x-1\right)\left(x+1\right)\left(x^{2}-x+1\right)\left(x^{2}+x+1\right)}\left(x^{12}+x^{8}-2x^{6}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+2)*(x**6+x**4-1)/(x**6-1)**(1/4)/(x**12+x**8-2*x**6+1),x)
```

```
[Out] Integral((x**6 + 2)*(x**6 + x**4 - 1)/(((x - 1)*(x + 1)*(x**2 - x + 1)*(x**
2 + x + 1))**(1/4)*(x**12 + x**8 - 2*x**6 + 1)), x)
```

3.2199 $\int \frac{1}{(d+cx)^2\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$

Optimal. Leaf size=261

$$-\frac{a\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}\right)}{\sqrt{c}\sqrt{a^2d^2+b^2c^2}\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}+\frac{a\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}\right)}{\sqrt{c}\sqrt{a^2d^2+b^2c^2}\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}-\frac{1}{c\sqrt{\sqrt{a^2x^2+b^2}+ax}\,(cx+}$$

Rubi [A] time = 0.61, antiderivative size = 310, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2119, 1648, 12, 707, 1093, 205}

$$-\frac{a\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}\right)}{\sqrt{c}\sqrt{a^2d^2+b^2c^2}\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}+\frac{a\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}\right)}{\sqrt{c}\sqrt{a^2d^2+b^2c^2}\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}+\frac{2a\sqrt{\sqrt{a^2x^2+b^2}+ax}}{c\left(-c\left(\sqrt{a^2x^2+b^2}+ax\right)^2-2ad\left(\sqrt{a^2x^2+b^2}+ax\right)+b^2c\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + c*x)^2*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

```
[Out] (2*a*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/(c*(b^2*c - 2*a*d*(a*x + Sqrt[b^2 + a
^2*x^2]) - c*(a*x + Sqrt[b^2 + a^2*x^2])^2)) - (a*ArcTan[(Sqrt[c]*Sqrt[a*x
+ Sqrt[b^2 + a^2*x^2]])/Sqrt[a*d - Sqrt[b^2*c^2 + a^2*d^2]]])/(Sqrt[c]*Sqrt
[b^2*c^2 + a^2*d^2]*Sqrt[a*d - Sqrt[b^2*c^2 + a^2*d^2]]) + (a*ArcTan[(Sqrt[
c]*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/Sqrt[a*d + Sqrt[b^2*c^2 + a^2*d^2]]])/(
Sqrt[c]*Sqrt[b^2*c^2 + a^2*d^2]*Sqrt[a*d + Sqrt[b^2*c^2 + a^2*d^2]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 707

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Sym
bol] :> Dist[2*e, Subst[Int[1/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1648

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
```

```
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*
e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$\int \frac{1}{(d+cx)^2\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx = (2a)\,\text{Subst}\left(\int \frac{b^2+x^2}{\sqrt{x}\,\left(-b^2c+2adx+cx^2\right)^2}\,dx, x, ax+\sqrt{b^2+a^2x^2}\right)$$

$$= \frac{2a\sqrt{ax+\sqrt{b^2+a^2x^2}}}{c\left(b^2c-2ad\left(ax+\sqrt{b^2+a^2x^2}\right)-c\left(ax+\sqrt{b^2+a^2x^2}\right)^2\right)} + \frac{a\,\text{Subst}\left(\int -\cdots\right.}{\cdots}$$

$$= \frac{2a\sqrt{ax+\sqrt{b^2+a^2x^2}}}{c\left(b^2c-2ad\left(ax+\sqrt{b^2+a^2x^2}\right)-c\left(ax+\sqrt{b^2+a^2x^2}\right)^2\right)} - \frac{a\,\text{Subst}\left(\int \frac{\cdots}{\sqrt{x}(\cdots}\right.}{\cdots}$$

$$= \frac{2a\sqrt{ax+\sqrt{b^2+a^2x^2}}}{c\left(b^2c-2ad\left(ax+\sqrt{b^2+a^2x^2}\right)-c\left(ax+\sqrt{b^2+a^2x^2}\right)^2\right)} - \frac{(2a)\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{2a\sqrt{ax+\sqrt{b^2+a^2x^2}}}{c\left(b^2c-2ad\left(ax+\sqrt{b^2+a^2x^2}\right)-c\left(ax+\sqrt{b^2+a^2x^2}\right)^2\right)} - \frac{a\,\text{Subst}\left(\int \frac{\cdots}{ad-\cdots}\right.}{\cdots}$$

$$= \frac{2a\sqrt{ax+\sqrt{b^2+a^2x^2}}}{c\left(b^2c-2ad\left(ax+\sqrt{b^2+a^2x^2}\right)-c\left(ax+\sqrt{b^2+a^2x^2}\right)^2\right)} - \frac{a\tan^{-1}\left(\cdots\right.}{\sqrt{c}\,\sqrt{b^2c^2+a^2d\cdots}}$$

Mathematica [A] time = 0.89, size = 273, normalized size = 1.05

$$-\frac{a\left(-\frac{\sqrt{-\sqrt{a^2d^2+b^2c^2}-ad}\,\tan^{-1}\left(\frac{b\sqrt{c}}{\sqrt{\sqrt{a^2x^2+b^2}+ax}\,\sqrt{-\sqrt{a^2d^2+b^2c^2}-ad}}\right)}{b\sqrt{a^2d^2+b^2c^2}}+\frac{\sqrt{\sqrt{a^2d^2+b^2c^2}-ad}\,\tan^{-1}\left(\frac{b\sqrt{c}}{\sqrt{\sqrt{a^2x^2+b^2}+ax}\,\sqrt{\sqrt{a^2d^2+b^2c^2}-ad}}\right)}{b\sqrt{a^2d^2+b^2c^2}}+\frac{\sqrt{c}}{\sqrt{\sqrt{a^2x^2+b^2}+ax}\,(acx+ad)}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + c*x)^2*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

```
[Out] -((a*(Sqrt[c]/((a*d + a*c*x)*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]) - (Sqrt[-(a*d
) - Sqrt[b^2*c^2 + a^2*d^2]]*ArcTan[(b*Sqrt[c])/(Sqrt[-(a*d) - Sqrt[b^2*c^2
 + a^2*d^2]]*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])]])/(b*Sqrt[b^2*c^2 + a^2*d^2])
 + (Sqrt[-(a*d) + Sqrt[b^2*c^2 + a^2*d^2]]*ArcTan[(b*Sqrt[c])/(Sqrt[-(a*d)
+ Sqrt[b^2*c^2 + a^2*d^2]]*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])]])/(b*Sqrt[b^2*c
^2 + a^2*d^2])))/c^(3/2))
```

IntegrateAlgebraic [A] time = 1.13, size = 261, normalized size = 1.00

$$-\frac{a\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}\right)}{\sqrt{c}\sqrt{a^2d^2+b^2c^2}\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}+\frac{a\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}\right)}{\sqrt{c}\sqrt{a^2d^2+b^2c^2}\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}-\frac{1}{c\sqrt{\sqrt{a^2x^2+b^2}+ax}\,(cx+d)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((d + c*x)^2*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

```
[Out] -(1/(c*(d + c*x)*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])) - (a*ArcTan[(Sqrt[c]*Sqr
t[a*x + Sqrt[b^2 + a^2*x^2]])/Sqrt[a*d - Sqrt[b^2*c^2 + a^2*d^2]]])/(Sqrt[c
]*Sqrt[b^2*c^2 + a^2*d^2]*Sqrt[a*d - Sqrt[b^2*c^2 + a^2*d^2]]) + (a*ArcTan[
(Sqrt[c]*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/Sqrt[a*d + Sqrt[b^2*c^2 + a^2*d^2
]]])/(Sqrt[c]*Sqrt[b^2*c^2 + a^2*d^2]*Sqrt[a*d + Sqrt[b^2*c^2 + a^2*d^2]])
```

fricas [B] time = 0.71, size = 1174, normalized size = 4.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x+d)^2/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*((b^2*c^2*x + b^2*c*d)*sqrt((a^3*d + (b^4*c^5 + a^2*b^2*c^3*d^2)*sqrt(a
^4/(b^6*c^8 + a^2*b^4*c^6*d^2)))/(b^4*c^5 + a^2*b^2*c^3*d^2))*log(sqrt(a*x
+ sqrt(a^2*x^2 + b^2))*a^2 + (a*b^2*c^2 + a^3*d^2 - (b^4*c^5*d + a^2*b^2*c^
3*d^3)*sqrt(a^4/(b^6*c^8 + a^2*b^4*c^6*d^2)))*sqrt((a^3*d + (b^4*c^5 + a^2*
b^2*c^3*d^2)*sqrt(a^4/(b^6*c^8 + a^2*b^4*c^6*d^2)))/(b^4*c^5 + a^2*b^2*c^3*
d^2))) - (b^2*c^2*x + b^2*c*d)*sqrt((a^3*d + (b^4*c^5 + a^2*b^2*c^3*d^2)*sq
rt(a^4/(b^6*c^8 + a^2*b^4*c^6*d^2)))/(b^4*c^5 + a^2*b^2*c^3*d^2))*log(sqrt(
a*x + sqrt(a^2*x^2 + b^2))*a^2 - (a*b^2*c^2 + a^3*d^2 - (b^4*c^5*d + a^2*b^
2*c^3*d^3)*sqrt(a^4/(b^6*c^8 + a^2*b^4*c^6*d^2)))*sqrt((a^3*d + (b^4*c^5 +
a^2*b^2*c^3*d^2)*sqrt(a^4/(b^6*c^8 + a^2*b^4*c^6*d^2)))/(b^4*c^5 + a^2*b^2*
c^3*d^2))) + (b^2*c^2*x + b^2*c*d)*sqrt((a^3*d - (b^4*c^5 + a^2*b^2*c^3*d^2
)*sqrt(a^4/(b^6*c^8 + a^2*b^4*c^6*d^2)))/(b^4*c^5 + a^2*b^2*c^3*d^2))*log(s
qrt(a*x + sqrt(a^2*x^2 + b^2))*a^2 + (a*b^2*c^2 + a^3*d^2 + (b^4*c^5*d + a^
2*b^2*c^3*d^3)*sqrt(a^4/(b^6*c^8 + a^2*b^4*c^6*d^2)))*sqrt((a^3*d - (b^4*c^
5 + a^2*b^2*c^3*d^2)*sqrt(a^4/(b^6*c^8 + a^2*b^4*c^6*d^2)))/(b^4*c^5 + a^2*
b^2*c^3*d^2))) - (b^2*c^2*x + b^2*c*d)*sqrt((a^3*d - (b^4*c^5 + a^2*b^2*c^3
*d^2)*sqrt(a^4/(b^6*c^8 + a^2*b^4*c^6*d^2)))/(b^4*c^5 + a^2*b^2*c^3*d^2))*l
og(sqrt(a*x + sqrt(a^2*x^2 + b^2))*a^2 - (a*b^2*c^2 + a^3*d^2 + (b^4*c^5*d
+ a^2*b^2*c^3*d^3)*sqrt(a^4/(b^6*c^8 + a^2*b^4*c^6*d^2)))*sqrt((a^3*d - (b^
4*c^5 + a^2*b^2*c^3*d^2)*sqrt(a^4/(b^6*c^8 + a^2*b^4*c^6*d^2)))/(b^4*c^5 +
a^2*b^2*c^3*d^2))) + 2*sqrt(a*x + sqrt(a^2*x^2 + b^2))*(a*x - sqrt(a^2*x^2
+ b^2)))/(b^2*c^2*x + b^2*c*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+\sqrt{a^2x^2+b^2}}\,(cx+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x+d)^2/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(a*x + sqrt(a^2*x^2 + b^2))*(c*x + d)^2), x)

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx+d\right)^{2} \sqrt{ax+\sqrt{a^{2}x^{2}+b^{2}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x+d)^2/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)

[Out] int(1/(c*x+d)^2/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+\sqrt{a^{2}x^{2}+b^{2}}}\, \left(cx+d\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x+d)^2/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxima"
)

[Out] integrate(1/(sqrt(a*x + sqrt(a^2*x^2 + b^2))*(c*x + d)^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a\,x+\sqrt{a^2\,x^2+b^2}}\,{\left(d+c\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x + (b^2 + a^2*x^2)^(1/2))^(1/2)*(d + c*x)^2),x)

[Out] int(1/((a*x + (b^2 + a^2*x^2)^(1/2))^(1/2)*(d + c*x)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + \sqrt{a^{2}x^{2} + b^{2}}} \left(cx + d\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x+d)**2/(a*x+(a**2*x**2+b**2)**(1/2))**(1/2),x)

[Out] Integral(1/(sqrt(a*x + sqrt(a**2*x**2 + b**2))*(c*x + d)**2), x)

3.2200 $\int \frac{(1+x^2)^2}{(-1+x^2)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal. Leaf size=261

$$\frac{x(x^2-5)}{2(x^2-1)\sqrt{\sqrt{x^4+1}+x^2}}-4\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)+\sqrt{2\left(7+5\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{2}-1\right)}\,x\sqrt{\sqrt{x^4}}}{\sqrt{x^4+1}+x^2+}\right.$$

Rubi [F] time = 0.72, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+x^2)^2}{(-1+x^2)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^2)^2/((-1 + x^2)^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] Defer[Int][1/Sqrt[x^2 + Sqrt[1 + x^4]], x] + Defer[Int][1/((-1 - x)*Sqrt[x^
2 + Sqrt[1 + x^4]]), x] + Defer[Int][1/((-1 + x)^2*Sqrt[x^2 + Sqrt[1 + x^4]
]), x] + Defer[Int][1/((-1 + x)*Sqrt[x^2 + Sqrt[1 + x^4]]), x] + Defer[Int]
[1/((1 + x)^2*Sqrt[x^2 + Sqrt[1 + x^4]]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x^2)^2}{(-1+x^2)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx &= \int \left(\frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}+\frac{1}{(-1-x)\sqrt{x^2+\sqrt{1+x^4}}}+\frac{1}{(-1+x)^2\sqrt{x^2+\sqrt{1+}}}\right.\\ &= \int \frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}\,dx+\int \frac{1}{(-1-x)\sqrt{x^2+\sqrt{1+x^4}}}\,dx+\int \frac{}{(-1+x)^2\sqrt{}}\end{aligned}$$

Mathematica [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(1+x^2)^2}{(-1+x^2)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^2)^2/((-1 + x^2)^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] Integrate[(1 + x^2)^2/((-1 + x^2)^2*Sqrt[x^2 + Sqrt[1 + x^4]]), x]
```

IntegrateAlgebraic [A] time = 2.40, size = 341, normalized size = 1.31

$$\frac{x(x^2-5)}{2(x^2-1)\sqrt{\sqrt{x^4+1}+x^2}}-4\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)+\sqrt{14+10\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)+\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}-\sqrt{10\sqrt{2}-14}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)^2/((-1 + x^2)^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] (x*(-5 + x^2))/(2*(-1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]]) - 4*Sqrt[2]*ArcTan[
(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2])/(x*Sqrt[x^2 + Sqrt[1 +
 x^4]])] + Sqrt[14 + 10*Sqrt[2]]*ArcTan[(-Sqrt[-1/2 + 1/Sqrt[2]] + Sqrt[-1/
2 + 1/Sqrt[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sq
rt[1 + x^4]])] + ArcTanh[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2
])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])]/Sqrt[2] - Sqrt[-14 + 10*Sqrt[2]]*ArcTanh[
(-Sqrt[1/2 + 1/Sqrt[2]] + Sqrt[1/2 + 1/Sqrt[2]]*x^2 + Sqrt[1/2 + 1/Sqrt[2]]
*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])]
```

fricas [B] time = 8.83, size = 517, normalized size = 1.98

$$\frac{8\left(x^2-1\right)\sqrt{10\sqrt{2}+14}\arctan\left(\frac{\left(4x^2-2\sqrt{2}\left(x^2+3\right)+\sqrt{x^4+1}\left(\left(\sqrt{2}-2\right)\sqrt{-8\sqrt{2}+12}+2\sqrt{2}-4\right)+\left(2x^2-\sqrt{2}\left(x^2+1\right)\right)\sqrt{-8\sqrt{2}+12}+8\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{10\sqrt{2}+14}}{8x}\right)+16\sqrt{2}\left(x^2-1\right)\arctan\left(-\frac{\left(\sqrt{2}x^2-\sqrt{2}\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{2x}\right)+\sqrt{2}\left(x^2-1\right)\log\left(4x^4+4\sqrt{x^4+1}x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)+2\left(x^2-1\right)\sqrt{10\sqrt{2}-14}\log\left(-\frac{2\sqrt{2}x^2+4x^2+\left(4x^3+\sqrt{2}\left(3x^3-7x\right)-\sqrt{x^4+1}\left(3\sqrt{2}x+4x\right)-10x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{10\sqrt{2}-14}+2\sqrt{x^4+1}\left(\sqrt{2}+1\right)}{x^2-1}\right)-2\left(x^2-1\right)\sqrt{10\sqrt{2}-14}\log\left(-\frac{2\sqrt{2}x^2+4x^2-\left(4x^3+\sqrt{2}\left(3x^3-7x\right)-\sqrt{x^4+1}\left(3\sqrt{2}x+4x\right)-10x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{10\sqrt{2}-14}+2\sqrt{x^4+1}\left(\sqrt{2}+1\right)}{x^2-1}\right)-4\left(x^5-5x^3-\sqrt{x^4+1}\left(x^3-5x\right)\right)\sqrt{x^2+\sqrt{x^4+1}}}{8\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^2-1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/8*(8*(x^2 - 1)*sqrt(10*sqrt(2) + 14)*arctan(1/8*(4*x^2 - 2*sqrt(2)*(x^2 +
 3) + sqrt(x^4 + 1)*((sqrt(2) - 2)*sqrt(-8*sqrt(2) + 12) + 2*sqrt(2) - 4) +
 (2*x^2 - sqrt(2)*(x^2 + 1))*sqrt(-8*sqrt(2) + 12) + 8)*sqrt(x^2 + sqrt(x^4
 + 1))*sqrt(10*sqrt(2) + 14)/x) + 16*sqrt(2)*(x^2 - 1)*arctan(-1/2*(sqrt(2)
*x^2 - sqrt(2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))/x) + sqrt(2)*(x^2 -
 1)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1
)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1) + 2*(x^2 - 1)*sqrt(10*sqrt(2) - 14)*log
(-(2*sqrt(2)*x^2 + 4*x^2 + (4*x^3 + sqrt(2)*(3*x^3 - 7*x) - sqrt(x^4 + 1)*(
3*sqrt(2)*x + 4*x) - 10*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(10*sqrt(2) - 14)
+ 2*sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 - 1)) - 2*(x^2 - 1)*sqrt(10*sqrt(2) -
 14)*log(-(2*sqrt(2)*x^2 + 4*x^2 - (4*x^3 + sqrt(2)*(3*x^3 - 7*x) - sqrt(x^
4 + 1)*(3*sqrt(2)*x + 4*x) - 10*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(10*sqrt(2
) - 14) + 2*sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 - 1)) - 4*(x^5 - 5*x^3 - sqrt
(x^4 + 1)*(x^3 - 5*x))*sqrt(x^2 + sqrt(x^4 + 1)))/(x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2}{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^2-1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((x^2 + 1)^2/(sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - 1)^2), x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2}{\left(x^2-1\right)^2\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2/(x^2-1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int((x^2+1)^2/(x^2-1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2}{\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^2-1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((x^2 + 1)^2/(sqrt(x^2 + sqrt(x^4 + 1))*(x^2 - 1)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^2+1\right)^2}{\left(x^2-1\right)^2\sqrt{\sqrt{x^4+1}+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^2/((x^2 - 1)^2*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

```
[Out] int((x^2 + 1)^2/((x^2 - 1)^2*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2}{\left(x-1\right)^2\left(x+1\right)^2\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2/(x**2-1)**2/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] Integral((x**2 + 1)**2/((x - 1)**2*(x + 1)**2*sqrt(x**2 + sqrt(x**4 + 1))),
 x)
```

3.2201 $\int \frac{a(ab+ac-2bc)-2(a^2-bc)x+(2a-b-c)x^2}{((-a+x)(-b+x)(-c+x))^{2/3}(a^2-bcd+(-2a+bd+cd)x+(1-d)x^2)}\,dx$

Optimal. Leaf size=262

$$-\frac{\log\left(a^2+d^{2/3}\left(x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3\right)^{2/3}+\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)\sqrt[3]{x^2(-a-b-c)+x(ab+ac+b}\right.}{2\sqrt[3]{d}}$$

Rubi [F] time = 12.90, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a(ab+ac-2bc)-2(a^2-bc)x+(2a-b-c)x^2}{((-a+x)(-b+x)(-c+x))^{2/3}(a^2-bcd+(-2a+bd+cd)x+(1-d)x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*(a*b + a*c - 2*b*c) - 2*(a^2 - b*c)*x + (2*a - b - c)*x^2)/(((-a + x
)*(-b + x)*(-c + x))^(2/3)*(a^2 - b*c*d + (-2*a + b*d + c*d)*x + (1 - d)*x^
2)),x]
```

```
[Out] ((2*a - b - c + Sqrt[4*a^2 - 4*a*(b + c) + 2*b*c*(2 - d) + b^2*d + c^2*d]/S
qrt[d])*(-a + x)^(2/3)*(-b + x)^(2/3)*(-c + x)^(2/3)*Defer[Int][(-a + x)^(1
/3)/((-b + x)^(2/3)*(-c + x)^(2/3)*(-2*a + b*d + c*d - Sqrt[d]*Sqrt[4*a^2 -
 4*a*b - 4*a*c + 4*b*c + b^2*d - 2*b*c*d + c^2*d] + 2*(1 - d)*x)), x])/(-((
a - x)*(b - x)*(c - x)))^(2/3) + ((2*a - b - c - Sqrt[4*a^2 - 4*a*(b + c) +
 2*b*c*(2 - d) + b^2*d + c^2*d]/Sqrt[d])*(-a + x)^(2/3)*(-b + x)^(2/3)*(-c
+ x)^(2/3)*Defer[Int][(-a + x)^(1/3)/((-b + x)^(2/3)*(-c + x)^(2/3)*(-2*a +
 b*d + c*d + Sqrt[d]*Sqrt[4*a^2 - 4*a*b - 4*a*c + 4*b*c + b^2*d - 2*b*c*d +
 c^2*d] + 2*(1 - d)*x)), x])/(-((a - x)*(b - x)*(c - x)))^(2/3)
```

Rubi steps

$$\int \frac{a(ab+ac-2bc)-2(a^2-bc)x+(2a-b-c)x^2}{((-a+x)(-b+x)(-c+x))^{2/3}(a^2-bcd+(-2a+bd+cd)x+(1-d)x^2)}\,dx = \frac{\left((-a+x)^{2/3}(-b+x)^{2/3}(-c+x)^{2/}\right.}{((-}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{2/3}(-c+x)^{2/}\right.}{((-a+}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{2/3}(-c+x)^{2/}\right.}{}$$

$$= \frac{\left(\left(2a-b-c-\frac{\sqrt{4a^2-4a(b+c)+2bc(2-}}{\sqrt{d}}\right.\right.}{}$$

Mathematica [F] time = 3.50, size = 0, normalized size = 0.00

$$\int \frac{a(ab+ac-2bc)-2(a^2-bc)x+(2a-b-c)x^2}{((-a+x)(-b+x)(-c+x))^{2/3}(a^2-bcd+(-2a+bd+cd)x+(1-d)x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*(a*b + a*c - 2*b*c) - 2*(a^2 - b*c)*x + (2*a - b - c)*x^2)/(((
-a + x)*(-b + x)*(-c + x))^(2/3)*(a^2 - b*c*d + (-2*a + b*d + c*d)*x + (1 -
 d)*x^2)),x]
```

```
[Out] Integrate[(a*(a*b + a*c - 2*b*c) - 2*(a^2 - b*c)*x + (2*a - b - c)*x^2)/(((
-a + x)*(-b + x)*(-c + x))^(2/3)*(a^2 - b*c*d + (-2*a + b*d + c*d)*x + (1 -
 d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 7.20, size = 262, normalized size = 1.00

$$-\frac{\log \left(a^2+d^{2/3} \left(x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3\right)^{2/3}+\left(\sqrt[3]{d}\, x-a\sqrt[3]{d}\right) \sqrt[3]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}-2ax+x^2\right)}{2\sqrt[3]{d}}+\frac{\log \left(\sqrt[3]{d}\, \sqrt[3]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}+a-x\right)}{\sqrt[3]{d}}+\frac{\sqrt{3} \tan ^{-1}\left(\frac{\sqrt{3}\, a-\sqrt{3}\, x}{-2 \sqrt[3]{d}\, \sqrt[3]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}+a-x}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*(a*b + a*c - 2*b*c) - 2*(a^2 - b*c)*x + (2*a - b - c)
*x^2)/(((-a + x)*(-b + x)*(-c + x))^(2/3)*(a^2 - b*c*d + (-2*a + b*d + c*d)
*x + (1 - d)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*a - Sqrt[3]*x)/(a - x - 2*d^(1/3)*(-(a*b*c) + (a*b
 + a*c + b*c)*x + (-a - b - c)*x^2 + x^3)^(1/3))])/d^(1/3) + Log[a - x + d^
(1/3)*(-(a*b*c) + (a*b + a*c + b*c)*x + (-a - b - c)*x^2 + x^3)^(1/3)]/d^(1
/3) - Log[a^2 - 2*a*x + x^2 + (-(a*d^(1/3)) + d^(1/3)*x)*(-(a*b*c) + (a*b +
 a*c + b*c)*x + (-a - b - c)*x^2 + x^3)^(1/3) + d^(2/3)*(-(a*b*c) + (a*b +
a*c + b*c)*x + (-a - b - c)*x^2 + x^3)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(a*b+a*c-2*b*c)-2*(a^2-b*c)*x+(2*a-b-c)*x^2)/((-a+x)*(-b+x)*(-
c+x))^(2/3)/(a^2-b*c*d+(b*d+c*d-2*a)*x+(1-d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(2\,a-b-c)x^2+(ab+ac-2\,bc)a-2\left(a^2-bc\right)x}{(-(a-x)(b-x)(c-x))^{\frac{2}{3}}\left(bcd+(d-1)x^2-a^2-(bd+cd-2\,a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(a*b+a*c-2*b*c)-2*(a^2-b*c)*x+(2*a-b-c)*x^2)/((-a+x)*(-b+x)*(-
c+x))^(2/3)/(a^2-b*c*d+(b*d+c*d-2*a)*x+(1-d)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-((2*a - b - c)*x^2 + (a*b + a*c - 2*b*c)*a - 2*(a^2 - b*c)*x)/((
-(a - x)*(b - x)*(c - x))^(2/3)*(b*c*d + (d - 1)*x^2 - a^2 - (b*d + c*d - 2
*a)*x)), x)
```

maple [F] time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{a\left(ab+ac-2bc\right)-2\left(a^2-bc\right)x+\left(2a-b-c\right)x^2}{\left(\left(-a+x\right)\left(-b+x\right)\left(-c+x\right)\right)^{\frac{2}{3}}\left(a^2-bcd+\left(bd+cd-2a\right)x+\left(1-d\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(a*b+a*c-2*b*c)-2*(a^2-b*c)*x+(2*a-b-c)*x^2)/((-a+x)*(-b+x)*(-c+x))^
(2/3)/(a^2-b*c*d+(b*d+c*d-2*a)*x+(1-d)*x^2),x)
```

```
[Out] int((a*(a*b+a*c-2*b*c)-2*(a^2-b*c)*x+(2*a-b-c)*x^2)/((-a+x)*(-b+x)*(-c+x))^
(2/3)/(a^2-b*c*d+(b*d+c*d-2*a)*x+(1-d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(2\,a-b-c)x^2+(ab+ac-2\,bc)a-2\left(a^2-bc\right)x}{(-(a-x)(b-x)(c-x))^{\frac{2}{3}}\left(bcd+(d-1)x^2-a^2-(bd+cd-2\,a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(a*b+a*c-2*b*c)-2*(a^2-b*c)*x+(2*a-b-c)*x^2)/((-a+x)*(-b+x)*(-
c+x))^(2/3)/(a^2-b*c*d+(b*d+c*d-2*a)*x+(1-d)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate(((2*a - b - c)*x^2 + (a*b + a*c - 2*b*c)*a - 2*(a^2 - b*c)*x)/((
-(a - x)*(b - x)*(c - x))^(2/3)*(b*c*d + (d - 1)*x^2 - a^2 - (b*d + c*d - 2
*a)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{2\,x\,\left(b\,c-a^2\right)-x^2\,\left(b-2\,a+c\right)+a\,\left(a\,b+a\,c-2\,b\,c\right)}{{\left(-\left(a-x\right)\,\left(b-x\right)\,\left(c-x\right)\right)}^{2/3}\,\left(x\,\left(b\,d-2\,a+c\,d\right)+a^2-x^2\,\left(d-1\right)-b\,c\,d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x*(b*c - a^2) - x^2*(b - 2*a + c) + a*(a*b + a*c - 2*b*c))/((-(a - x
)*(b - x)*(c - x))^(2/3)*(x*(b*d - 2*a + c*d) + a^2 - x^2*(d - 1) - b*c*d))
,x)
```

```
[Out] int((2*x*(b*c - a^2) - x^2*(b - 2*a + c) + a*(a*b + a*c - 2*b*c))/((-(a - x
)*(b - x)*(c - x))^(2/3)*(x*(b*d - 2*a + c*d) + a^2 - x^2*(d - 1) - b*c*d))
, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(a*b+a*c-2*b*c)-2*(a**2-b*c)*x+(2*a-b-c)*x**2)/((-a+x)*(-b+x)*
(-c+x))**(2/3)/(a**2-b*c*d+(b*d+c*d-2*a)*x+(1-d)*x**2),x)
```

```
[Out] Timed out
```

3.2202 $\int \frac{1+x^4}{(-1+x^4)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

Optimal. Leaf size=262

$$\frac{\tan^{-1}\left(\frac{x\sqrt{2a-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2+\sqrt{a}}\right)}{\sqrt{2a-c}}-\frac{\sqrt{-2a-2b-c}\,\tan^{-1}\left(\frac{x\sqrt{-2a-2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2-2\sqrt{a}\,x+\sqrt{a}}\right)}{2(2a+2b+c)}-\frac{\sqrt{-2a+2b\cdots}}{}$$

Rubi [F] time = 1.36, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x^4}{(-1+x^4)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^4)/((-1 + x^4)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] Defer[Int][1/Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4], x] - (I/2)*Defer[Int][1
/((I - x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x] + Defer[Int][1/((-1 +
x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x]/2 - (I/2)*Defer[Int][1/((I +
x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x] - Defer[Int][1/((1 + x)*Sqrt[
a + b*x + c*x^2 + b*x^3 + a*x^4]), x]/2
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{(-1+x^4)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx &= \int \left(\frac{1}{\sqrt{a+bx+cx^2+bx^3+ax^4}}+\frac{2}{(-1+x^4)\sqrt{a+bx+cx^2+b\cdots}}\right.\\
&= 2\int \frac{1}{(-1+x^4)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx+\int \frac{}{\sqrt{a+bx+c\cdots}}\\
&= 2\int \left(\frac{1}{2(-1+x^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}-\frac{}{2(1+x^2)\sqrt{a+\cdots}}\right.\\
&= \int \frac{1}{\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx+\int \frac{1}{(-1+x^2)\sqrt{a+bx+cx^2\cdots}}\\
&= \int \frac{1}{\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx-\int \left(\frac{i}{2(i-x)\sqrt{a+bx+cx^2\cdots}}\right.\\
&= -\left(\frac{1}{2}i\int \frac{1}{(i-x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx\right)-\frac{1}{2}i\int \frac{}{(i+x)\sqrt{\cdots}}
\end{aligned}$$

Mathematica [C] time = 3.14, size = 6061, normalized size = 23.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^4)/((-1 + x^4)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 2.33, size = 262, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x\sqrt{2a-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2+\sqrt{a}}\right)}{\sqrt{2a-c}}-\frac{\sqrt{-2a-2b-c}\,\tan^{-1}\left(\frac{x\sqrt{-2a-2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2-2\sqrt{a}\,x+\sqrt{a}}\right)}{2(2a+2b+c)}-\frac{\sqrt{-2a+2b-c}\,\tan^{-1}\left(\frac{x\sqrt{-2a+2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2+2\sqrt{a}\,x+\sqrt{a}}\right)}{2(2a-2b+c)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)/((-1 + x^4)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x
^4]),x]
```

```
[Out] ArcTan[(Sqrt[2*a - c]*x)/(Sqrt[a] + Sqrt[a]*x^2 - Sqrt[a + b*x + c*x^2 + b*
x^3 + a*x^4])]/Sqrt[2*a - c] - (Sqrt[-2*a - 2*b - c]*ArcTan[(Sqrt[-2*a - 2*
b - c]*x)/(Sqrt[a] - 2*Sqrt[a]*x + Sqrt[a]*x^2 - Sqrt[a + b*x + c*x^2 + b*x
^3 + a*x^4])])/(2*(2*a + 2*b + c)) - (Sqrt[-2*a + 2*b - c]*ArcTan[(Sqrt[-2*
a + 2*b - c]*x)/(Sqrt[a] + 2*Sqrt[a]*x + Sqrt[a]*x^2 - Sqrt[a + b*x + c*x^2
 + b*x^3 + a*x^4])])/(2*(2*a - 2*b + c))
```

fricas [B] time = 7.11, size = 4887, normalized size = 18.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] [1/8*((4*a^2 + 4*a*b - 2*b*c - c^2)*sqrt(2*a - 2*b + c)*log(((24*a^2 - 16*a
*b + b^2 + 4*a*c)*x^4 + 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x^3 + 2*(
24*a^2 + 3*b^2 - 4*(a + 2*b)*c + 4*c^2)*x^2 + 4*sqrt(a*x^4 + b*x^3 + c*x^2
+ b*x + a)*((4*a - b)*x^2 + 2*(2*a + b - c)*x + 4*a - b)*sqrt(2*a - 2*b + c
) + 24*a^2 - 16*a*b + b^2 + 4*a*c + 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*
c)*x)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1)) + (4*a^2 - 4*a*b + 2*b*c - c^2)*sqrt
(2*a + 2*b + c)*log(((24*a^2 + 16*a*b + b^2 + 4*a*c)*x^4 - 4*(8*a^2 - 4*a*b
 - 3*b^2 - 2*(2*a + b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a - 2*b)*c + 4*c^2)*
x^2 - 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a + b)*x^2 - 2*(2*a - b -
 c)*x + 4*a + b)*sqrt(2*a + 2*b + c) + 24*a^2 + 16*a*b + b^2 + 4*a*c - 4*(8
*a^2 - 4*a*b - 3*b^2 - 2*(2*a + b)*c)*x)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)) -
 2*(4*a^2 - 4*b^2 + 4*a*c + c^2)*sqrt(-2*a + c)*log(-((8*a^2 - b^2 - 4*a*c)
*x^4 + 8*(2*a*b - b*c)*x^3 - 2*(8*a^2 + b^2 - 12*a*c + 4*c^2)*x^2 + 8*a^2 +
 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(b*x^2 - 2*(2*a - c)*x + b)*sqrt(-
2*a + c) - b^2 - 4*a*c + 8*(2*a*b - b*c)*x)/(x^4 + 2*x^2 + 1)))/(8*a^3 - 8*
a*b^2 - 2*a*c^2 - c^3 + 4*(a^2 + b^2)*c), -1/8*(4*(4*a^2 - 4*b^2 + 4*a*c +
c^2)*sqrt(2*a - c)*arctan(-1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(b*x^2
 - 2*(2*a - c)*x + b)*sqrt(2*a - c)/((2*a^2 - a*c)*x^4 + (2*a*b - b*c)*x^3
+ (2*a*c - c^2)*x^2 + 2*a^2 - a*c + (2*a*b - b*c)*x)) - (4*a^2 + 4*a*b - 2*
b*c - c^2)*sqrt(2*a - 2*b + c)*log(((24*a^2 - 16*a*b + b^2 + 4*a*c)*x^4 + 4
*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a + 2
*b)*c + 4*c^2)*x^2 + 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x^2
 + 2*(2*a + b - c)*x + 4*a - b)*sqrt(2*a - 2*b + c) + 24*a^2 - 16*a*b + b^2
 + 4*a*c + 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x)/(x^4 + 4*x^3 + 6*x^
2 + 4*x + 1)) - (4*a^2 - 4*a*b + 2*b*c - c^2)*sqrt(2*a + 2*b + c)*log(((24*
a^2 + 16*a*b + b^2 + 4*a*c)*x^4 - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(2*a + b)*c)
*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a - 2*b)*c + 4*c^2)*x^2 - 4*sqrt(a*x^4 + b*x^
3 + c*x^2 + b*x + a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(2*a
 + 2*b + c) + 24*a^2 + 16*a*b + b^2 + 4*a*c - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*
(2*a + b)*c)*x)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)))/(8*a^3 - 8*a*b^2 - 2*a*c^
2 - c^3 + 4*(a^2 + b^2)*c), 1/8*(2*(4*a^2 - 4*a*b + 2*b*c - c^2)*sqrt(-2*a
- 2*b - c)*arctan(1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a + b)*x^2
- 2*(2*a - b - c)*x + 4*a + b)*sqrt(-2*a - 2*b - c)/((2*a^2 + 2*a*b + a*c)*
x^4 + (2*a*b + 2*b^2 + b*c)*x^3 + (2*(a + b)*c + c^2)*x^2 + 2*a^2 + 2*a*b +
 a*c + (2*a*b + 2*b^2 + b*c)*x)) + (4*a^2 + 4*a*b - 2*b*c - c^2)*sqrt(2*a -
 2*b + c)*log(((24*a^2 - 16*a*b + b^2 + 4*a*c)*x^4 + 4*(8*a^2 + 4*a*b - 3*b
^2 - 2*(2*a - b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a + 2*b)*c + 4*c^2)*x^2 +
4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x^2 + 2*(2*a + b - c)*x
+ 4*a - b)*sqrt(2*a - 2*b + c) + 24*a^2 - 16*a*b + b^2 + 4*a*c + 4*(8*a^2 +
 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1)) - 2*(4*
a^2 - 4*b^2 + 4*a*c + c^2)*sqrt(-2*a + c)*log(-((8*a^2 - b^2 - 4*a*c)*x^4 +
```

8*(2*a*b - b*c)*x^3 - 2*(8*a^2 + b^2 - 12*a*c + 4*c^2)*x^2 + 8*a^2 + 4*sqr
t(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(b*x^2 - 2*(2*a - c)*x + b)*sqrt(-2*a +
c) - b^2 - 4*a*c + 8*(2*a*b - b*c)*x)/(x^4 + 2*x^2 + 1)))/(8*a^3 - 8*a*b^2
- 2*a*c^2 - c^3 + 4*(a^2 + b^2)*c), -1/8*(4*(4*a^2 - 4*b^2 + 4*a*c + c^2)*s
qrt(2*a - c)*arctan(-1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(b*x^2 - 2*(
2*a - c)*x + b)*sqrt(2*a - c)/((2*a^2 - a*c)*x^4 + (2*a*b - b*c)*x^3 + (2*a
*c - c^2)*x^2 + 2*a^2 - a*c + (2*a*b - b*c)*x)) - 2*(4*a^2 - 4*a*b + 2*b*c
- c^2)*sqrt(-2*a - 2*b - c)*arctan(1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a
)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(-2*a - 2*b - c)/((2*a^
2 + 2*a*b + a*c)*x^4 + (2*a*b + 2*b^2 + b*c)*x^3 + (2*(a + b)*c + c^2)*x^2
+ 2*a^2 + 2*a*b + a*c + (2*a*b + 2*b^2 + b*c)*x)) - (4*a^2 + 4*a*b - 2*b*c
- c^2)*sqrt(2*a - 2*b + c)*log(((24*a^2 - 16*a*b + b^2 + 4*a*c)*x^4 + 4*(8*
a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a + 2*b)*
c + 4*c^2)*x^2 + 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x^2 + 2
*(2*a + b - c)*x + 4*a - b)*sqrt(2*a - 2*b + c) + 24*a^2 - 16*a*b + b^2 + 4
*a*c + 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x)/(x^4 + 4*x^3 + 6*x^2 +
4*x + 1)))/(8*a^3 - 8*a*b^2 - 2*a*c^2 - c^3 + 4*(a^2 + b^2)*c), 1/8*(2*(4*a
^2 + 4*a*b - 2*b*c - c^2)*sqrt(-2*a + 2*b - c)*arctan(-1/2*sqrt(a*x^4 + b*x
^3 + c*x^2 + b*x + a)*((4*a - b)*x^2 + 2*(2*a + b - c)*x + 4*a - b)*sqrt(-2
*a + 2*b - c)/((2*a^2 - 2*a*b + a*c)*x^4 + (2*a*b - 2*b^2 + b*c)*x^3 + (2*(
a - b)*c + c^2)*x^2 + 2*a^2 - 2*a*b + a*c + (2*a*b - 2*b^2 + b*c)*x)) + (4*
a^2 - 4*a*b + 2*b*c - c^2)*sqrt(2*a + 2*b + c)*log(((24*a^2 + 16*a*b + b^2
+ 4*a*c)*x^4 - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(2*a + b)*c)*x^3 + 2*(24*a^2 +
3*b^2 - 4*(a - 2*b)*c + 4*c^2)*x^2 - 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a
)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(2*a + 2*b + c) + 24*a^
2 + 16*a*b + b^2 + 4*a*c - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(2*a + b)*c)*x)/(x^
4 - 4*x^3 + 6*x^2 - 4*x + 1)) - 2*(4*a^2 - 4*b^2 + 4*a*c + c^2)*sqrt(-2*a +
c)*log(-((8*a^2 - b^2 - 4*a*c)*x^4 + 8*(2*a*b - b*c)*x^3 - 2*(8*a^2 + b^2
- 12*a*c + 4*c^2)*x^2 + 8*a^2 + 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(b*
x^2 - 2*(2*a - c)*x + b)*sqrt(-2*a + c) - b^2 - 4*a*c + 8*(2*a*b - b*c)*x)/
(x^4 + 2*x^2 + 1)))/(8*a^3 - 8*a*b^2 - 2*a*c^2 - c^3 + 4*(a^2 + b^2)*c), -1
/8*(4*(4*a^2 - 4*b^2 + 4*a*c + c^2)*sqrt(2*a - c)*arctan(-1/2*sqrt(a*x^4 +
b*x^3 + c*x^2 + b*x + a)*(b*x^2 - 2*(2*a - c)*x + b)*sqrt(2*a - c)/((2*a^2
- a*c)*x^4 + (2*a*b - b*c)*x^3 + (2*a*c - c^2)*x^2 + 2*a^2 - a*c + (2*a*b -
b*c)*x)) - 2*(4*a^2 + 4*a*b - 2*b*c - c^2)*sqrt(-2*a + 2*b - c)*arctan(-1/
2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x^2 + 2*(2*a + b - c)*x
+ 4*a - b)*sqrt(-2*a + 2*b - c)/((2*a^2 - 2*a*b + a*c)*x^4 + (2*a*b - 2*b^2
+ b*c)*x^3 + (2*(a - b)*c + c^2)*x^2 + 2*a^2 - 2*a*b + a*c + (2*a*b - 2*b^
2 + b*c)*x)) - (4*a^2 - 4*a*b + 2*b*c - c^2)*sqrt(2*a + 2*b + c)*log(((24*a
^2 + 16*a*b + b^2 + 4*a*c)*x^4 - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(2*a + b)*c)*
x^3 + 2*(24*a^2 + 3*b^2 - 4*(a - 2*b)*c + 4*c^2)*x^2 - 4*sqrt(a*x^4 + b*x^3
+ c*x^2 + b*x + a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(2*a
+ 2*b + c) + 24*a^2 + 16*a*b + b^2 + 4*a*c - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(
2*a + b)*c)*x)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)))/(8*a^3 - 8*a*b^2 - 2*a*c^2
- c^3 + 4*(a^2 + b^2)*c), 1/4*((4*a^2 + 4*a*b - 2*b*c - c^2)*sqrt(-2*a + 2
*b - c)*arctan(-1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x^2 +
2*(2*a + b - c)*x + 4*a - b)*sqrt(-2*a + 2*b - c)/((2*a^2 - 2*a*b + a*c)*x^
4 + (2*a*b - 2*b^2 + b*c)*x^3 + (2*(a - b)*c + c^2)*x^2 + 2*a^2 - 2*a*b + a
*c + (2*a*b - 2*b^2 + b*c)*x)) + (4*a^2 - 4*a*b + 2*b*c - c^2)*sqrt(-2*a -
2*b - c)*arctan(1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a + b)*x^2 -
2*(2*a - b - c)*x + 4*a + b)*sqrt(-2*a - 2*b - c)/((2*a^2 + 2*a*b + a*c)*x^
4 + (2*a*b + 2*b^2 + b*c)*x^3 + (2*(a + b)*c + c^2)*x^2 + 2*a^2 + 2*a*b + a
*c + (2*a*b + 2*b^2 + b*c)*x)) - (4*a^2 - 4*b^2 + 4*a*c + c^2)*sqrt(-2*a +
c)*log(-((8*a^2 - b^2 - 4*a*c)*x^4 + 8*(2*a*b - b*c)*x^3 - 2*(8*a^2 + b^2 -
12*a*c + 4*c^2)*x^2 + 8*a^2 + 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(b*x
^2 - 2*(2*a - c)*x + b)*sqrt(-2*a + c) - b^2 - 4*a*c + 8*(2*a*b - b*c)*x)/(
x^4 + 2*x^2 + 1)))/(8*a^3 - 8*a*b^2 - 2*a*c^2 - c^3 + 4*(a^2 + b^2)*c), -1/
4*(2*(4*a^2 - 4*b^2 + 4*a*c + c^2)*sqrt(2*a - c)*arctan(-1/2*sqrt(a*x^4 + b
*x^3 + c*x^2 + b*x + a)*(b*x^2 - 2*(2*a - c)*x + b)*sqrt(2*a - c)/((2*a^2 -

```
a*c)*x^4 + (2*a*b - b*c)*x^3 + (2*a*c - c^2)*x^2 + 2*a^2 - a*c + (2*a*b -
b*c)*x)) - (4*a^2 + 4*a*b - 2*b*c - c^2)*sqrt(-2*a + 2*b - c)*arctan(-1/2*s
qrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x^2 + 2*(2*a + b - c)*x + 4
*a - b)*sqrt(-2*a + 2*b - c)/((2*a^2 - 2*a*b + a*c)*x^4 + (2*a*b - 2*b^2 +
b*c)*x^3 + (2*(a - b)*c + c^2)*x^2 + 2*a^2 - 2*a*b + a*c + (2*a*b - 2*b^2 +
b*c)*x)) - (4*a^2 - 4*a*b + 2*b*c - c^2)*sqrt(-2*a - 2*b - c)*arctan(1/2*s
qrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4
*a + b)*sqrt(-2*a - 2*b - c)/((2*a^2 + 2*a*b + a*c)*x^4 + (2*a*b + 2*b^2 +
b*c)*x^3 + (2*(a + b)*c + c^2)*x^2 + 2*a^2 + 2*a*b + a*c + (2*a*b + 2*b^2 +
b*c)*x)))/(8*a^3 - 8*a*b^2 - 2*a*c^2 - c^3 + 4*(a^2 + b^2)*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\sqrt{ax^4+bx^3+cx^2+bx+a}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((x^4 + 1)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(x^4 - 1)), x)
```

maple [C] time = 0.03, size = 82800, normalized size = 316.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^4-1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\sqrt{ax^4+bx^3+cx^2+bx+a}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((x^4 + 1)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4+1}{\left(x^4-1\right)\sqrt{a\,x^4+b\,x^3+c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/((x^4 - 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)),x)
```

```
[Out] int((x^4 + 1)/((x^4 - 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{(x-1)(x+1)\left(x^2+1\right)\sqrt{ax^4+a+bx^3+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**4-1)/(a*x**4+b*x**3+c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((x**4 + 1)/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(a*x**4 + a + b*x**3 +
b*x + c*x**2)), x)
```

3.2203 $\int \frac{(-1+x^2)^2}{(1+x^2)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal. Leaf size=262

$$\frac{x(x^2+5)}{2(x^2+1)\sqrt{\sqrt{x^4+1}+x^2}}+4\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\sqrt{2\left(7+5\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}x\sqrt{\sqrt{x^4+1}}}{\sqrt{x^4+1}+x^2+1}\right.$$

Rubi [F] time = 1.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x^2)^2}{(1+x^2)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x^2)^2/((1 + x^2)^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] Defer[Int][1/Sqrt[x^2 + Sqrt[1 + x^4]], x] - Defer[Int][1/((I - x)^2*Sqrt[x
^2 + Sqrt[1 + x^4]]), x] - I*Defer[Int][1/((I - x)*Sqrt[x^2 + Sqrt[1 + x^4]
]), x] - Defer[Int][1/((I + x)^2*Sqrt[x^2 + Sqrt[1 + x^4]]), x] - I*Defer[I
nt][1/((I + x)*Sqrt[x^2 + Sqrt[1 + x^4]]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x^2)^2}{(1+x^2)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx &= \int \left(\frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}+\frac{4}{(1+x^2)^2\sqrt{x^2+\sqrt{1+x^4}}}-\frac{4}{(1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}\right.\\
&= 4\int \frac{1}{(1+x^2)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx-4\int \frac{1}{(1+x^2)\sqrt{x^2+\sqrt{1+x^4}}}\,dx+\int \\
&= -\left(4\int \left(\frac{i}{2(i-x)\sqrt{x^2+\sqrt{1+x^4}}}+\frac{i}{2(i+x)\sqrt{x^2+\sqrt{1+x^4}}}\right)dx\right)+4\int \left(-\frac{}{4(i}\right.\\
&= -\left(2i\int \frac{1}{(i-x)\sqrt{x^2+\sqrt{1+x^4}}}\,dx\right)-2i\int \frac{1}{(i+x)\sqrt{x^2+\sqrt{1+x^4}}}\,dx-2\int \\
&= -\left(2i\int \frac{1}{(i-x)\sqrt{x^2+\sqrt{1+x^4}}}\,dx\right)-2i\int \frac{1}{(i+x)\sqrt{x^2+\sqrt{1+x^4}}}\,dx-2\int \\
&= i\int \frac{1}{(i-x)\sqrt{x^2+\sqrt{1+x^4}}}\,dx+i\int \frac{1}{(i+x)\sqrt{x^2+\sqrt{1+x^4}}}\,dx-2i\int \frac{}{(i-x)}
\end{aligned}$$

Mathematica [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(-1+x^2)^2}{(1+x^2)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^2)^2/((1 + x^2)^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] Integrate[(-1 + x^2)^2/((1 + x^2)^2*Sqrt[x^2 + Sqrt[1 + x^4]]), x]
```

IntegrateAlgebraic [A] time = 1.99, size = 342, normalized size = 1.31

$$\frac{x\left(x^2+5\right)}{2\left(x^2+1\right)\sqrt{\sqrt{x^4+1}+x^2}}+4\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-\sqrt{14+10\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)+\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}-\sqrt{10\sqrt{2}-14}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)^2/((1 + x^2)^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] (x*(5 + x^2))/(2*(1 + x^2)*Sqrt[x^2 + Sqrt[1 + x^4]]) + 4*Sqrt[2]*ArcTan[(-
(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2])/(x*Sqrt[x^2 + Sqrt[1 + x
^4]])] - Sqrt[14 + 10*Sqrt[2]]*ArcTan[(-Sqrt[1/2 + 1/Sqrt[2]] + Sqrt[1/2 +
1/Sqrt[2]]*x^2 + Sqrt[1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1
+ x^4]])] + ArcTanh[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2])/(x
*Sqrt[x^2 + Sqrt[1 + x^4]])]/Sqrt[2] - Sqrt[-14 + 10*Sqrt[2]]*ArcTanh[(-Sqr
t[-1/2 + 1/Sqrt[2]] + Sqrt[-1/2 + 1/Sqrt[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*S
qrt[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])]
```

fricas [B] time = 11.46, size = 517, normalized size = 1.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2/(x^2+1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/8*(8*(x^2 + 1)*sqrt(10*sqrt(2) + 14)*arctan(-1/8*(4*x^2 - 2*sqrt(2)*(x^2
- 3) - sqrt(x^4 + 1)*((sqrt(2) - 2)*sqrt(-8*sqrt(2) + 12) - 2*sqrt(2) + 4)
- (2*x^2 - sqrt(2)*(x^2 - 1))*sqrt(-8*sqrt(2) + 12) - 8)*sqrt(x^2 + sqrt(x^
4 + 1))*sqrt(10*sqrt(2) + 14)/x) - 16*sqrt(2)*(x^2 + 1)*arctan(-1/2*(sqrt(2
)*x^2 - sqrt(2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))/x) + sqrt(2)*(x^2
+ 1)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 +
1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1) - 2*(x^2 + 1)*sqrt(10*sqrt(2) - 14)*lo
g((2*sqrt(2)*x^2 + 4*x^2 + (4*x^3 + sqrt(2)*(3*x^3 + 7*x) - sqrt(x^4 + 1)*(
3*sqrt(2)*x + 4*x) + 10*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(10*sqrt(2) - 14)
+ 2*sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 + 1)) + 2*(x^2 + 1)*sqrt(10*sqrt(2) -
 14)*log((2*sqrt(2)*x^2 + 4*x^2 - (4*x^3 + sqrt(2)*(3*x^3 + 7*x) - sqrt(x^4
 + 1)*(3*sqrt(2)*x + 4*x) + 10*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(10*sqrt(2)
 - 14) + 2*sqrt(x^4 + 1)*(sqrt(2) + 1))/(x^2 + 1)) - 4*(x^5 + 5*x^3 - sqrt(
x^4 + 1)*(x^3 + 5*x))*sqrt(x^2 + sqrt(x^4 + 1)))/(x^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2-1\right)^2}{\sqrt{x^2+\sqrt{x^4+1}}\,\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2/(x^2+1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((x^2 - 1)^2/(sqrt(x^2 + sqrt(x^4 + 1))*(x^2 + 1)^2), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}-1\right)^{2}}{\left(x^{2}+1\right)^{2} \sqrt{x^{2}+\sqrt{x^{4}+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^2/(x^2+1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int((x^2-1)^2/(x^2+1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}-1\right)^{2}}{\sqrt{x^{2}+\sqrt{x^{4}+1}} \left(x^{2}+1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2/(x^2+1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((x^2 - 1)^2/(sqrt(x^2 + sqrt(x^4 + 1))*(x^2 + 1)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^{2}-1\right)^{2}}{\left(x^{2}+1\right)^{2} \sqrt{\sqrt{x^{4}+1}+x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^2/((x^2 + 1)^2*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

```
[Out] int((x^2 - 1)^2/((x^2 + 1)^2*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)^{2} (x+1)^{2}}{\left(x^{2}+1\right)^{2} \sqrt{x^{2}+\sqrt{x^{4}+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**2/(x**2+1)**2/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] Integral((x - 1)**2*(x + 1)**2/((x**2 + 1)**2*sqrt(x**2 + sqrt(x**4 + 1))),
 x)
```

3.2204 $\int \frac{2ab^2-b(2a+b)x+x^3}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-ab^2+b(2a+b)x-(a+2b+d)x^2+x^3\right)}dx$

Optimal. Leaf size=263

$$-\frac{\log\left(\sqrt[3]{d}\,x\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}+\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}+d^{2/3}x^2\right.}{2\sqrt[3]{d}}$$

Rubi [F] time = 11.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2ab^2-b(2a+b)x+x^3}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-ab^2+b(2a+b)x-(a+2b+d)x^2+x^3\right)}dx$$

Verification is not applicable to the result.

```
[In] Int[(2*a*b^2 - b*(2*a + b)*x + x^3)/((x*(-a + x)*(-b + x)^2)^(1/3)*(-(a*b^2
) + b*(2*a + b)*x - (a + 2*b + d)*x^2 + x^3)),x]
```

```
[Out] (6*a*b*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x*(-b
 + x^3)^(1/3))/((-a + x^3)^(1/3)*(a*b^2 - 2*a*b*(1 + b/(2*a))*x^3 + a*(1 +
(2*b + d)/a)*x^6 - x^9)), x], x, x^(1/3)]])/(-((a - x)*(b - x)^2*x))^(1/3) +
 (3*b*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x^4*(-
b + x^3)^(1/3))/((-a + x^3)^(1/3)*(-(a*b^2) + 2*a*b*(1 + b/(2*a))*x^3 - a*(
1 + (2*b + d)/a)*x^6 + x^9)), x], x, x^(1/3)]])/(-((a - x)*(b - x)^2*x))^(1/
3) + (3*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x^7*
(-b + x^3)^(1/3))/((-a + x^3)^(1/3)*(-(a*b^2) + 2*a*b*(1 + b/(2*a))*x^3 - a
*(1 + (2*b + d)/a)*x^6 + x^9)), x], x, x^(1/3)]])/(-((a - x)*(b - x)^2*x))^(
1/3)
```

Rubi steps

$$\int \frac{2ab^2-b(2a+b)x+x^3}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-ab^2+b(2a+b)x-(a+2b+d)x^2+x^3\right)}dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\int \frac{}{\sqrt[3]{x}\sqrt[3]{-a+}}}{\sqrt[3]{x(-a+}}$$

$$= \frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\int \frac{}{\sqrt[3]{x}\sqrt[3]{-a+}}}{\sqrt[3]{x(-a+x)(-}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\text{Subst}\left(\int\right.}{\sqrt[3]{x(-}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\text{Subst}\left(\int\right.}{\sqrt[3]{x(}}$$

Mathematica [F] time = 3.60, size = 0, normalized size = 0.00

$$\int \frac{2ab^2-b(2a+b)x+x^3}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-ab^2+b(2a+b)x-(a+2b+d)x^2+x^3\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2*a*b^2 - b*(2*a + b)*x + x^3)/((x*(-a + x)*(-b + x)^2)^(1/3)*(-
(a*b^2) + b*(2*a + b)*x - (a + 2*b + d)*x^2 + x^3)),x]
```

```
[Out] Integrate[(2*a*b^2 - b*(2*a + b)*x + x^3)/((x*(-a + x)*(-b + x)^2)^(1/3)*(-
(a*b^2) + b*(2*a + b)*x - (a + 2*b + d)*x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.67, size = 263, normalized size = 1.00

$$-\frac{\log \left(\sqrt[3]{d}\, x \sqrt[3]{x^2 \left(2 a b+b^2\right)-a b^2 x+x^3 (-a-2 b)+x^4}+\left(x^2 \left(2 a b+b^2\right)-a b^2 x+x^3 (-a-2 b)+x^4\right)^{2/3}+d^{2/3} x^2\right)}{2 \sqrt[3]{d}}+\frac{\log \left(\sqrt[3]{x^2 \left(2 a b+b^2\right)-a b^2 x+x^3 (-a-2 b)+x^4}-\sqrt[3]{d}\, x\right)}{\sqrt[3]{d}}+\frac{\sqrt{3} \tan ^{-1}\left(\frac{\sqrt{3} \sqrt[3]{x^2 \left(2 a b+b^2\right)-a b^2 x+x^3 (-a-2 b)+x^4}}{\sqrt[3]{x^2 \left(2 a b+b^2\right)-a b^2 x+x^3 (-a-2 b)+x^4}+2 \sqrt[3]{d}\, x}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*a*b^2 - b*(2*a + b)*x + x^3)/((x*(-a + x)*(-b + x)^2)
^(1/3)*(-(a*b^2) + b*(2*a + b)*x - (a + 2*b + d)*x^2 + x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 +
 x^4)^(1/3))/(2*d^(1/3)*x + (-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^
3 + x^4)^(1/3))])/d^(1/3) + Log[-(d^(1/3)*x) + (-(a*b^2*x) + (2*a*b + b^2)*
x^2 + (-a - 2*b)*x^3 + x^4)^(1/3)]/d^(1/3) - Log[d^(2/3)*x^2 + d^(1/3)*x*(-
(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3) + (-(a*b^2*x) +
 (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b^2-b*(2*a+b)*x+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a*b^2+b*(2*
a+b)*x-(a+2*b+d)*x^2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2 a b^2 - (2 a + b) b x + x^3}{\left(-(a - x)(b - x)^2 x\right)^{\frac{1}{3}} \left(a b^2 - (2 a + b) b x + (a + 2 b + d) x^2 - x^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b^2-b*(2*a+b)*x+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a*b^2+b*(2*
a+b)*x-(a+2*b+d)*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate(-(2*a*b^2 - (2*a + b)*b*x + x^3)/((-(a - x)*(b - x)^2*x)^(1/3)*(a
*b^2 - (2*a + b)*b*x + (a + 2*b + d)*x^2 - x^3)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{2 a b^2 - b (2 a + b) x + x^3}{\left(x (-a + x) (-b + x)^2\right)^{\frac{1}{3}} \left(-a b^2 + b (2 a + b) x - (a + 2 b + d) x^2 + x^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*b^2-b*(2*a+b)*x+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a*b^2+b*(2*a+b)*x
-(a+2*b+d)*x^2+x^3),x)
```

```
[Out] int((2*a*b^2-b*(2*a+b)*x+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a*b^2+b*(2*a+b)*x
-(a+2*b+d)*x^2+x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,ab^2 - (2\,a+b)bx + x^3}{\left(-(a-x)(b-x)^2x\right)^{\frac{1}{3}}\left(ab^2 - (2\,a+b)bx + (a+2\,b+d)x^2 - x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b^2-b*(2*a+b)*x+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a*b^2+b*(2*
a+b)*x-(a+2*b+d)*x^2+x^3),x, algorithm="maxima")
```

```
[Out] -integrate((2*a*b^2 - (2*a + b)*b*x + x^3)/((-(a - x)*(b - x)^2*x)^(1/3)*(a
*b^2 - (2*a + b)*b*x + (a + 2*b + d)*x^2 - x^3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{2\,a\,b^2 + x^3 - b\,x\,(2\,a+b)}{\left(-x\,(a-x)\,(b-x)^2\right)^{1/3}\left(x^2\,(a+2\,b+d) + a\,b^2 - x^3 - b\,x\,(2\,a+b)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*a*b^2 + x^3 - b*x*(2*a + b))/((-x*(a - x)*(b - x)^2)^(1/3)*(x^2*(a
+ 2*b + d) + a*b^2 - x^3 - b*x*(2*a + b))),x)
```

```
[Out] int(-(2*a*b^2 + x^3 - b*x*(2*a + b))/((-x*(a - x)*(b - x)^2)^(1/3)*(x^2*(a
+ 2*b + d) + a*b^2 - x^3 - b*x*(2*a + b))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b**2-b*(2*a+b)*x+x**3)/(x*(-a+x)*(-b+x)**2)**(1/3)/(-a*b**2+
b*(2*a+b)*x-(a+2*b+d)*x**2+x**3),x)
```

```
[Out] Timed out
```

3.2205 $\int \frac{-d+cx}{x\sqrt[3]{-b+ax^3}}\,dx$

Optimal. Leaf size=263

$$\frac{c\log\left(a^{2/3}x^2+\sqrt[3]{a}\,x\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}\right)}{6\sqrt[3]{a}}-\frac{d\log\left(-\sqrt[3]{b}\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}+b^{2/3}\right)}{6\sqrt[3]{b}}-\frac{c\log\left(\sqrt[3]{ax^3-b}-\sqrt[3]{\right.}}{3\sqrt[3]{a}}$$

Rubi [A] time = 0.15, antiderivative size = 163, normalized size of antiderivative = 0.62, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1844, 239, 266, 56, 617, 204, 31}

$$-\frac{c\log\left(\sqrt[3]{ax^3-b}-\sqrt[3]{a}\,x\right)}{2\sqrt[3]{a}}+\frac{c\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a}x}{\sqrt[3]{ax^3-b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}+\frac{d\log\left(\sqrt[3]{ax^3-b}+\sqrt[3]{b}\right)}{2\sqrt[3]{b}}+\frac{d\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{d\log(x)}{2\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-d + c*x)/(x*(-b + a*x^3)^(1/3)),x]
```

```
[Out] (c*ArcTan[(1 + (2*a^(1/3)*x)/(-b + a*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(1/3)
) + (d*ArcTan[(b^(1/3) - 2*(-b + a*x^3)^(1/3))/(Sqrt[3]*b^(1/3))])/(Sqrt[3]
*b^(1/3)) - (d*Log[x])/(2*b^(1/3)) + (d*Log[b^(1/3) + (-b + a*x^3)^(1/3)])/
(2*b^(1/3)) - (c*Log[-(a^(1/3)*x) + (-b + a*x^3)^(1/3)])/(2*a^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-d+cx}{x\sqrt[3]{-b+ax^3}}\,dx &= \int \left(\frac{c}{\sqrt[3]{-b+ax^3}}-\frac{d}{x\sqrt[3]{-b+ax^3}}\right)dx\\
&= c\int \frac{1}{\sqrt[3]{-b+ax^3}}\,dx-d\int \frac{1}{x\sqrt[3]{-b+ax^3}}\,dx\\
&= \frac{c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a}x}{\sqrt[3]{-b+ax^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{c\log\left(-\sqrt[3]{a}x+\sqrt[3]{-b+ax^3}\right)}{2\sqrt[3]{a}}-\frac{1}{3}d\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{-b+ax}}\,dx,x,x^3\right.\\
&= \frac{c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a}x}{\sqrt[3]{-b+ax^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{d\log(x)}{2\sqrt[3]{b}}-\frac{c\log\left(-\sqrt[3]{a}x+\sqrt[3]{-b+ax^3}\right)}{2\sqrt[3]{a}}-\frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{b^{2/3}-\sqrt[3]{b}\right.\\
&= \frac{c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a}x}{\sqrt[3]{-b+ax^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{d\log(x)}{2\sqrt[3]{b}}+\frac{d\log\left(\sqrt[3]{b}+\sqrt[3]{-b+ax^3}\right)}{2\sqrt[3]{b}}-\frac{c\log\left(-\sqrt[3]{a}x+\sqrt[3]{-b+ax\right.}{2\sqrt[3]{a}}\\
&= \frac{c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a}x}{\sqrt[3]{-b+ax^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}+\frac{d\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-b+ax^3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{d\log(x)}{2\sqrt[3]{b}}+\frac{d\log\left(\sqrt[3]{b}+\sqrt[3]{-b+ax^3}\right)}{2\sqrt[3]{b}}-\frac{c\,l}{}
\end{aligned}$$

Mathematica [C] time = 0.18, size = 159, normalized size = 0.60

$$\frac{1}{6}\left(\frac{c\left(\log\left(\frac{a^{2/3}x^2}{\left(ax^3-b\right)^{2/3}}+\frac{\sqrt[3]{a}x}{\sqrt[3]{ax^3-b}}+1\right)-2\log\left(1-\frac{\sqrt[3]{a}x}{\sqrt[3]{ax^3-b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a}x}{\sqrt[3]{ax^3-b}}+1}{\sqrt{3}}\right)\right)}{\sqrt[3]{a}}-\frac{3d\left(ax^3-b\right)^{2/3}{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-\frac{ax^3}{b}\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-d + c*x)/(x*(-b + a*x^3)^(1/3)),x]
```

```
[Out] ((-3*d*(-b + a*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, 1 - (a*x^3)/b])/b
+ (c*(2*Sqrt[3]*ArcTan[(1 + (2*a^(1/3)*x)/(-b + a*x^3)^(1/3))/Sqrt[3]] - 2*
Log[1 - (a^(1/3)*x)/(-b + a*x^3)^(1/3)] + Log[1 + (a^(2/3)*x^2)/(-b + a*x^3
)^(2/3) + (a^(1/3)*x)/(-b + a*x^3)^(1/3)]))/a^(1/3))/6
```

IntegrateAlgebraic [A] time = 4.65, size = 263, normalized size = 1.00

$$\frac{c\log\left(a^{2/3}x^2+\sqrt[3]{a}\,x\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}\right)}{6\sqrt[3]{a}}-\frac{d\log\left(-\sqrt[3]{b}\,\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}+b^{2/3}\right)}{6\sqrt[3]{b}}-\frac{c\log\left(\sqrt[3]{ax^3-b}-\sqrt[3]{a}\,x\right)}{3\sqrt[3]{a}}-\frac{c\tan^{-1}\left(\frac{\frac{2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{a}}+\frac{x}{\sqrt{3}}}{x}\right)}{\sqrt{3}\sqrt[3]{a}}+\frac{d\log\left(\sqrt[3]{ax^3-b}+\sqrt[3]{b}\right)}{3\sqrt[3]{b}}+\frac{d\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-d + c*x)/(x*(-b + a*x^3)^(1/3)),x]
```

```
[Out] -((c*ArcTan[(x/Sqrt[3] + (2*(-b + a*x^3)^(1/3))/(Sqrt[3]*a^(1/3)))/x])/(Sqr
t[3]*a^(1/3))) + (d*ArcTan[1/Sqrt[3] - (2*(-b + a*x^3)^(1/3))/(Sqrt[3]*b^(1
/3))])/(Sqrt[3]*b^(1/3)) + (d*Log[b^(1/3) + (-b + a*x^3)^(1/3)])/(3*b^(1/3)
) - (c*Log[-(a^(1/3)*x) + (-b + a*x^3)^(1/3)])/(3*a^(1/3)) - (d*Log[b^(2/3)
 - b^(1/3)*(-b + a*x^3)^(1/3) + (-b + a*x^3)^(2/3)])/(6*b^(1/3)) + (c*Log[a
^(2/3)*x^2 + a^(1/3)*x*(-b + a*x^3)^(1/3) + (-b + a*x^3)^(2/3)])/(6*a^(1/3)
)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x-d)/x/(a*x^3-b)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx-d}{\left(ax^3-b\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x-d)/x/(a*x^3-b)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*x - d)/((a*x^3 - b)^(1/3)*x), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{cx-d}{x\left(a\,x^3-b\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x-d)/x/(a*x^3-b)^(1/3),x)
```

```
[Out] int((c*x-d)/x/(a*x^3-b)^(1/3),x)
```

maxima [A] time = 0.44, size = 209, normalized size = 0.79

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a^{\frac{1}{3}}+\frac{2\left(ax^3-b\right)^{\frac{1}{3}}}{x}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{\log\left(a^{\frac{2}{3}}+\frac{\left(ax^3-b\right)^{\frac{1}{3}}a^{\frac{1}{3}}}{x}+\frac{\left(ax^3-b\right)^{\frac{2}{3}}}{x^2}\right)}{a^{\frac{1}{3}}}+\frac{2\log\left(-a^{\frac{1}{3}}+\frac{\left(ax^3-b\right)^{\frac{1}{3}}}{x}\right)}{a^{\frac{1}{3}}}\right)c-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(ax^3-b\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}+\frac{\log\left(\left(ax^3-b\right)^{\frac{2}{3}}-\left(ax^3-b\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}-\frac{2\log\left(\left(ax^3-b\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x-d)/x/(a*x^3-b)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(a^(1/3) + 2*(a*x^3 - b)^(1/3)/x)/a^(1/3
))/a^(1/3) - log(a^(2/3) + (a*x^3 - b)^(1/3)*a^(1/3)/x + (a*x^3 - b)^(2/3)/
x^2)/a^(1/3) + 2*log(-a^(1/3) + (a*x^3 - b)^(1/3)/x)/a^(1/3))*c - 1/6*(2*sq
rt(3)*arctan(1/3*sqrt(3)*(2*(a*x^3 - b)^(1/3) - b^(1/3))/b^(1/3))/b^(1/3) +
 log((a*x^3 - b)^(2/3) - (a*x^3 - b)^(1/3)*b^(1/3) + b^(2/3))/b^(1/3) - 2*l
og((a*x^3 - b)^(1/3) + b^(1/3))/b^(1/3))*d
```

mupad [B] time = 2.13, size = 165, normalized size = 0.63

$$\frac{d\,\ln\left(d^2\,{\left(a\,x^3-b\right)}^{1/3}+b^{1/3}\,d^2\right)}{3\,b^{1/3}}-\frac{\ln\left(d^2\,{\left(a\,x^3-b\right)}^{1/3}+\frac{b^{1/3}\,{\left(d-\sqrt{3}\,d\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(d-\sqrt{3}\,d\,1{}\mathrm{i}\right)}{6\,b^{1/3}}-\frac{\ln\left(d^2\,{\left(a\,x^3-b\right)}^{1/3}+\frac{b^{1/3}\,{\left(d+\sqrt{3}\,d\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(d+\sqrt{3}\,d\,1{}\mathrm{i}\right)}{6\,b^{1/3}}+\frac{c\,x\,{\left(1-\frac{a\,x^3}{b}\right)}^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ \frac{a\,x^3}{b}\right)}{{\left(a\,x^3-b\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(d - c*x)/(x*(a*x^3 - b)^(1/3)),x)
```

```
[Out] (d*log(d^2*(a*x^3 - b)^(1/3) + b^(1/3)*d^2))/(3*b^(1/3)) - (log(d^2*(a*x^3
- b)^(1/3) + (b^(1/3)*(d - 3^(1/2)*d*1i)^2)/4)*(d - 3^(1/2)*d*1i))/(6*b^(1/
3)) - (log(d^2*(a*x^3 - b)^(1/3) + (b^(1/3)*(d + 3^(1/2)*d*1i)^2)/4)*(d + 3
^(1/2)*d*1i))/(6*b^(1/3)) + (c*x*(1 - (a*x^3)/b)^(1/3)*hypergeom([1/3, 1/3]
, 4/3, (a*x^3)/b))/(a*x^3 - b)^(1/3)
```

sympy [C] time = 3.11, size = 80, normalized size = 0.30

$$\frac{cxe^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{ax^3}{b}\right)}{3\sqrt[3]{b}\,\Gamma\left(\frac{4}{3}\right)}+\frac{d\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^3}\right)}{3\sqrt[3]{a}\,x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x-d)/x/(a*x**3-b)**(1/3),x)
```

```
[Out] c*x*exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), a*x**3/b)/(3*b**(1/3)
*gamma(4/3)) + d*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*exp_polar(2*I*pi)/(
a*x**3))/(3*a**(1/3)*x*gamma(4/3))
```

3.2206 $\int \frac{-b^4+c^2x^2+a^4x^4}{\sqrt{-b^4+a^4x^4}\left(b^4+a^4x^4\right)}\,dx$

Optimal. Leaf size=263

$$-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\left(2a^2b^2+ic^2\right)\tan^{-1}\left(\frac{(1+i)abx}{\sqrt{a^4x^4-b^4}+a^2x^2+ib^2}\right)}{a^3b^3}+\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\left(2a^2b^2-ic^2\right)\tanh^{-1}\left(\frac{(1-i)\sqrt{a^4x^4-b^4}+(1-i)a^2x^2+(1+i)b^2}{2\sqrt{3-2\sqrt{2}}\,abx}\right)}{a^3b^3}+\left(\frac{1}{8}\right.$$

Rubi [C] time = 1.08, antiderivative size = 350, normalized size of antiderivative = 1.33, number of steps used = 16, number of rules used = 7, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.137, Rules used = {6725, 224, 221, 1211, 1699, 205, 208}

$$\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{a\sqrt{a^4x^4-b^4}}-\frac{\left(2\sqrt{-a^4}\,b^2-c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}\,bx}{\sqrt{a^4x^4-b^4}}\right)}{4\sqrt{2}\left(-a^4\right)^{3/4}b^3}-\frac{\left(2\sqrt{-a^4}\,b^2+c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}\,bx}{\sqrt{a^4x^4-b^4}}\right)}{4\sqrt{2}\left(-a^4\right)^{3/4}b^3}-\frac{\left(2a^4b^2-\sqrt{-a^4}\,c^2\right)\sqrt{1-\frac{a^4x^4}{b^4}}F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{4a^5b\sqrt{a^4x^4-b^4}}-\frac{\left(2a^4b^2+\sqrt{-a^4}\,c^2\right)\sqrt{1-\frac{a^4x^4}{b^4}}F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{4a^5b\sqrt{a^4x^4-b^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b^4 + c^2*x^2 + a^4*x^4)/(Sqrt[-b^4 + a^4*x^4]*(b^4 + a^4*x^4)),x]
```

```
[Out] -1/4*((2*Sqrt[-a^4]*b^2 - c^2)*ArcTan[(Sqrt[2]*(-a^4)^(1/4)*b*x)/Sqrt[-b^4
+ a^4*x^4]])/(Sqrt[2]*(-a^4)^(3/4)*b^3) - ((2*Sqrt[-a^4]*b^2 + c^2)*ArcTanh
[(Sqrt[2]*(-a^4)^(1/4)*b*x)/Sqrt[-b^4 + a^4*x^4]])/(4*Sqrt[2]*(-a^4)^(3/4)*
b^3) + (b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticF[ArcSin[(a*x)/b], -1])/(a*Sqrt[-
b^4 + a^4*x^4]) - ((2*a^4*b^2 - Sqrt[-a^4]*c^2)*Sqrt[1 - (a^4*x^4)/b^4]*Ell
ipticF[ArcSin[(a*x)/b], -1])/(4*a^5*b*Sqrt[-b^4 + a^4*x^4]) - ((2*a^4*b^2 +
 Sqrt[-a^4]*c^2)*Sqrt[1 - (a^4*x^4)/b^4]*EllipticF[ArcSin[(a*x)/b], -1])/(4
*a^5*b*Sqrt[-b^4 + a^4*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-b^4+c^2x^2+a^4x^4}{\sqrt{-b^4+a^4x^4}\left(b^4+a^4x^4\right)}\,dx &= \int \left(\frac{1}{\sqrt{-b^4+a^4x^4}}-\frac{2b^4-c^2x^2}{\sqrt{-b^4+a^4x^4}\left(b^4+a^4x^4\right)}\right)dx\\
&= \int \frac{1}{\sqrt{-b^4+a^4x^4}}\,dx-\int \frac{2b^4-c^2x^2}{\sqrt{-b^4+a^4x^4}\left(b^4+a^4x^4\right)}\,dx\\
&= \frac{\sqrt{1-\frac{a^4x^4}{b^4}}\int \frac{1}{\sqrt{1-\frac{a^4x^4}{b^4}}}\,dx}{\sqrt{-b^4+a^4x^4}}-\int \left(-\frac{\sqrt{-a^4}\left(2\sqrt{-a^4}\,b^4-b^2c^2\right)}{2a^4b^2\left(b^2-\sqrt{-a^4}\,x^2\right)\sqrt{-b^4+a^4x^4}}+\frac{}{2a^4b}\right.\\
&= \frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{a\sqrt{-b^4+a^4x^4}}+\frac{1}{2}\left(-2b^2-\frac{c^2}{\sqrt{-a^4}}\right)\int \frac{1}{\left(b^2+\sqrt{-a^4}\,x^2\right)\sqrt{-b}}\\
&= \frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{a\sqrt{-b^4+a^4x^4}}-\frac{\left(2b^2+\frac{c^2}{\sqrt{-a^4}}\right)\int \frac{1}{\sqrt{-b^4+a^4x^4}}\,dx}{4b^2}-\frac{\left(2b^2+\frac{c^2}{\sqrt{-}}\right.}{}\\
&= \frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{a\sqrt{-b^4+a^4x^4}}-\frac{1}{4}\left(2b^2+\frac{c^2}{\sqrt{-a^4}}\right)\text{Subst}\left(\int \frac{1}{b^2-2\sqrt{-a^4}\,b^4}\right.\\
&= -\frac{\left(2\sqrt{-a^4}\,b^2-c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}\,bx}{\sqrt{-b^4+a^4x^4}}\right)}{4\sqrt{2}\left(-a^4\right)^{3/4}b^3}-\frac{\left(2\sqrt{-a^4}\,b^2+c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}\,bx}{\sqrt{-b^4+a^4x^4}}\right)}{4\sqrt{2}\left(-a^4\right)^{3/4}b^3}
\end{aligned}
$$

Mathematica [C] time = 0.65, size = 305, normalized size = 1.16

$$
\frac{x\left(5c^2x^2\sqrt{1-\frac{a^4x^4}{b^4}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)+3a^4x^4\sqrt{1-\frac{a^4x^4}{b^4}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)-\frac{75b^{12}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)}{\left(a^4x^4+b^4\right)\left(5b^4F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)+2a^4x^4\left(F_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)-2F_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)\right)\right)}\right)}{15b^4\sqrt{a^4x^4-b^4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-b^4 + c^2*x^2 + a^4*x^4)/(Sqrt[-b^4 + a^4*x^4]*(b^4 + a^4*x^4))
,x]
```

```
[Out] (x*(5*c^2*x^2*Sqrt[1 - (a^4*x^4)/b^4]*AppellF1[3/4, 1/2, 1, 7/4, (a^4*x^4)/
b^4, -((a^4*x^4)/b^4)] + 3*a^4*x^4*Sqrt[1 - (a^4*x^4)/b^4]*AppellF1[5/4, 1/
2, 1, 9/4, (a^4*x^4)/b^4, -((a^4*x^4)/b^4)] - (75*b^12*AppellF1[1/4, 1/2, 1
, 5/4, (a^4*x^4)/b^4, -((a^4*x^4)/b^4)])/((b^4 + a^4*x^4)*(5*b^4*AppellF1[1
/4, 1/2, 1, 5/4, (a^4*x^4)/b^4, -((a^4*x^4)/b^4)] + 2*a^4*x^4*(-2*AppellF1[
```

```
5/4, 1/2, 2, 9/4, (a^4*x^4)/b^4, -((a^4*x^4)/b^4)] + AppellF1[5/4, 3/2, 1,
9/4, (a^4*x^4)/b^4, -((a^4*x^4)/b^4)]))))/(15*b^4*Sqrt[-b^4 + a^4*x^4])
```

IntegrateAlgebraic [A] time = 1.55, size = 227, normalized size = 0.86

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\left(2a^2b^2-ic^2\right)\tan^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a^4x^4-b^4}}{abx}\right)}{a^3b^3}+\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\left(2a^2b^2+ic^2\right)\log\left(\sqrt{a^4x^4-b^4}+a^2x^2-(1-i)abx+ib^2\right)}{a^3b^3}-\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\left(2a^2b^2+ic^2\right)\log\left(ia^5b^3x^2+(1+i)a^4b^4x-a^3b^5+ia^3b^3\sqrt{a^4x^4-b^4}\right)}{a^3b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b^4 + c^2*x^2 + a^4*x^4)/(Sqrt[-b^4 + a^4*x^4]*(b^4 +
a^4*x^4)),x]
```

```
[Out] ((1/8 + I/8)*(2*a^2*b^2 - I*c^2)*ArcTan[((1/2 + I/2)*Sqrt[-b^4 + a^4*x^4])/
(a*b*x)])/(a^3*b^3) + ((1/8 + I/8)*(2*a^2*b^2 + I*c^2)*Log[I*b^2 - (1 - I)*
a*b*x + a^2*x^2 + Sqrt[-b^4 + a^4*x^4]])/(a^3*b^3) - ((1/8 + I/8)*(2*a^2*b^
2 + I*c^2)*Log[-(a^3*b^5) + (1 + I)*a^4*b^4*x + I*a^5*b^3*x^2 + I*a^3*b^3*S
qrt[-b^4 + a^4*x^4]])/(a^3*b^3)
```

fricas [A] time = 5.94, size = 131, normalized size = 0.50

$$\frac{2\left(2\,a^2b^2+c^2\right)\arctan\left(\frac{\sqrt{a^4x^4-b^4}\,ax}{a^2bx^2+b^3}\right)-\left(2\,a^2b^2-c^2\right)\log\left(\frac{a^4x^4+2\,a^2b^2x^2-b^4+2\,\sqrt{a^4x^4-b^4}\,abx}{a^4x^4+b^4}\right)}{8\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^4+c^2*x^2)/(a^4*x^4-b^4)^(1/2)/(a^4*x^4+b^4),x, algori
thm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*b^2 + c^2)*arctan(sqrt(a^4*x^4 - b^4)*a*x/(a^2*b*x^2 + b^3))
- (2*a^2*b^2 - c^2)*log((a^4*x^4 + 2*a^2*b^2*x^2 - b^4 + 2*sqrt(a^4*x^4 - b
^4)*a*b*x)/(a^4*x^4 + b^4)))/(a^3*b^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^4x^4-b^4+c^2x^2}{\left(a^4x^4+b^4\right)\sqrt{a^4x^4-b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^4+c^2*x^2)/(a^4*x^4-b^4)^(1/2)/(a^4*x^4+b^4),x, algori
thm="giac")
```

```
[Out] integrate((a^4*x^4 - b^4 + c^2*x^2)/((a^4*x^4 + b^4)*sqrt(a^4*x^4 - b^4)),
x)
```

maple [C] time = 0.06, size = 268, normalized size = 1.02

$$\frac{\sqrt{\frac{a^2x^2}{b^2}+1}\,\sqrt{1-\frac{a^2x^2}{b^2}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{a^2}{b^2}},i\right)}{\sqrt{-\frac{a^2}{b^2}}\,\sqrt{a^4x^4-b^4}}-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4a^4+b^4\right)}\frac{\left(-\_\alpha^2c^2+2b^4\right)\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\_\alpha^2\left(\_\alpha^2+x^2\right)a^4}{\sqrt{-2b^4}\,\sqrt{a^4x^4-b^4}}\right)}{\sqrt{-b^4}}+\frac{4\_\alpha^3a^4\sqrt{\frac{a^2x^2}{b^2}+1}\,\sqrt{1-\frac{a^2x^2}{b^2}}\,\operatorname{EllipticPi}\left(x\sqrt{-\frac{a^2}{b^2}},\frac{\_\alpha^2a^2}{b^2},\frac{\sqrt{\frac{a^2}{b^2}}}{\sqrt{-\frac{a^2}{b^2}}}\right)}{\sqrt{-\frac{a^2}{b^2}}\,b^4\sqrt{a^4x^4-b^4}}\right)}{\_\alpha^3}}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^4*x^4-b^4+c^2*x^2)/(a^4*x^4-b^4)^(1/2)/(a^4*x^4+b^4),x)
```

```
[Out] 1/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4
)^(1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)-1/16/a^4*sum((-_alpha^2*c^2+2*b^4)/
```

```
_alpha^3*(-2^(1/2)/(-b^4)^(1/2)*arctanh(_alpha^2*(_alpha^2+x^2)*a^4/(-2*b^4
)^(1/2)/(a^4*x^4-b^4)^(1/2))+4/(-a^2/b^2)^(1/2)*_alpha^3*a^4/b^4*(a^2*x^2/b
^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticPi(x*(-a^2/b^
2)^(1/2),_alpha^2*a^2/b^2,(a^2/b^2)^(1/2)/(-a^2/b^2)^(1/2))),_alpha=RootOf(
_Z^4*a^4+b^4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^4x^4 - b^4 + c^2x^2}{(a^4x^4 + b^4)\sqrt{a^4x^4 - b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^4*x^4-b^4+c^2*x^2)/(a^4*x^4-b^4)^(1/2)/(a^4*x^4+b^4),x, algori
thm="maxima")
```

```
[Out] integrate((a^4*x^4 - b^4 + c^2*x^2)/((a^4*x^4 + b^4)*sqrt(a^4*x^4 - b^4)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a^4\,x^4 - b^4 + c^2\,x^2}{\left(a^4\,x^4 + b^4\right)\,\sqrt{a^4\,x^4 - b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^4*x^4 - b^4 + c^2*x^2)/((b^4 + a^4*x^4)*(a^4*x^4 - b^4)^(1/2)),x)
```

```
[Out] int((a^4*x^4 - b^4 + c^2*x^2)/((b^4 + a^4*x^4)*(a^4*x^4 - b^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^4x^4 - b^4 + c^2x^2}{\sqrt{(ax - b)(ax + b)\left(a^2x^2 + b^2\right)}\left(a^4x^4 + b^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**4*x**4-b**4+c**2*x**2)/(a**4*x**4-b**4)**(1/2)/(a**4*x**4+b**
4),x)
```

```
[Out] Integral((a**4*x**4 - b**4 + c**2*x**2)/(sqrt((a*x - b)*(a*x + b)*(a**2*x**
2 + b**2))*(a**4*x**4 + b**4)), x)
```

3.2207 $\int \frac{x^2}{(1-x^4)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

Optimal. Leaf size=265

$$\frac{\tan^{-1}\left(\frac{x\sqrt{2a-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2+\sqrt{a}}\right)}{2\sqrt{2a-c}}+\frac{\sqrt{-2a-2b-c}\,\tan^{-1}\left(\frac{x\sqrt{-2a-2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2-2\sqrt{a}\,x+\sqrt{a}}\right)}{4(2a+2b+c)}+\frac{\sqrt{-2a+2b-c}}{}$$

Rubi [F] time = 1.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{(1-x^4)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/((1 - x^4)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] (-1/4*I)*Defer[Int][1/((I - x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x] -
 Defer[Int][1/((-1 + x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x]/4 - (I/4
)*Defer[Int][1/((I + x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x] + Defer[
Int][1/((1 + x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x]/4
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(1-x^4)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx &= \int \left(-\frac{1}{2(-1+x^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}-\frac{1}{2(1+x^2)\sqrt{a+bx+}}\right.\\ &= -\left(\frac{1}{2}\int \frac{1}{(-1+x^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx\right)-\frac{1}{2}\int \frac{}{(1+x^2)\sqrt{}}\\ &= -\left(\frac{1}{2}\int \left(\frac{i}{2(i-x)\sqrt{a+bx+cx^2+bx^3+ax^4}}+\frac{i}{2(i+x)\sqrt{a+bx+cx}}\right.\right.\\ &= -\left(\frac{1}{4}i\int \frac{1}{(i-x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx\right)-\frac{1}{4}i\int \frac{}{(i+x)\sqrt{a+b}}\end{aligned}$$

Mathematica [C] time = 2.69, size = 5410, normalized size = 20.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((1 - x^4)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 2.16, size = 265, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x\sqrt{2a-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2+\sqrt{a}}\right)}{2\sqrt{2a-c}}+\frac{\sqrt{-2a-2b-c}\,\tan^{-1}\left(\frac{x\sqrt{-2a-2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2-2\sqrt{a}\,x+\sqrt{a}}\right)}{4(2a+2b+c)}+\frac{\sqrt{-2a+2b-c}\,\tan^{-1}\left(\frac{x\sqrt{-2a+2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2+2\sqrt{a}\,x+\sqrt{a}}\right)}{4(2a-2b+c)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((1 - x^4)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[2*a - c]*x)/(Sqrt[a] + Sqrt[a]*x^2 - Sqrt[a + b*x + c*x^2 + b*
x^3 + a*x^4])]/(2*Sqrt[2*a - c]) + (Sqrt[-2*a - 2*b - c]*ArcTan[(Sqrt[-2*a
- 2*b - c]*x)/(Sqrt[a] - 2*Sqrt[a]*x + Sqrt[a]*x^2 - Sqrt[a + b*x + c*x^2 +
 b*x^3 + a*x^4])])/(4*(2*a + 2*b + c)) + (Sqrt[-2*a + 2*b - c]*ArcTan[(Sqrt
[-2*a + 2*b - c]*x)/(Sqrt[a] + 2*Sqrt[a]*x + Sqrt[a]*x^2 - Sqrt[a + b*x + c
*x^2 + b*x^3 + a*x^4])])/(4*(2*a - 2*b + c))
```

fricas [B] time = 3.71, size = 4886, normalized size = 18.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

```
[Out] [1/16*((4*a^2 + 4*a*b - 2*b*c - c^2)*sqrt(2*a - 2*b + c)*log(((24*a^2 - 16*
a*b + b^2 + 4*a*c)*x^4 + 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x^3 + 2*
(24*a^2 + 3*b^2 - 4*(a + 2*b)*c + 4*c^2)*x^2 - 4*sqrt(a*x^4 + b*x^3 + c*x^2
 + b*x + a)*((4*a - b)*x^2 + 2*(2*a + b - c)*x + 4*a - b)*sqrt(2*a - 2*b +
c) + 24*a^2 - 16*a*b + b^2 + 4*a*c + 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)
*c)*x)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1)) + (4*a^2 - 4*a*b + 2*b*c - c^2)*sqr
t(2*a + 2*b + c)*log(((24*a^2 + 16*a*b + b^2 + 4*a*c)*x^4 - 4*(8*a^2 - 4*a*
b - 3*b^2 - 2*(2*a + b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a - 2*b)*c + 4*c^2)
*x^2 + 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a + b)*x^2 - 2*(2*a - b
- c)*x + 4*a + b)*sqrt(2*a + 2*b + c) + 24*a^2 + 16*a*b + b^2 + 4*a*c - 4*(
8*a^2 - 4*a*b - 3*b^2 - 2*(2*a + b)*c)*x)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1))
- 2*(4*a^2 - 4*b^2 + 4*a*c + c^2)*sqrt(-2*a + c)*log(-((8*a^2 - b^2 - 4*a*c
)*x^4 + 8*(2*a*b - b*c)*x^3 - 2*(8*a^2 + b^2 - 12*a*c + 4*c^2)*x^2 + 8*a^2
+ 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(b*x^2 - 2*(2*a - c)*x + b)*sqrt(
-2*a + c) - b^2 - 4*a*c + 8*(2*a*b - b*c)*x)/(x^4 + 2*x^2 + 1)))/(8*a^3 - 8
*a*b^2 - 2*a*c^2 - c^3 + 4*(a^2 + b^2)*c), -1/16*(4*(4*a^2 - 4*b^2 + 4*a*c
+ c^2)*sqrt(2*a - c)*arctan(-1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(b*x
^2 - 2*(2*a - c)*x + b)*sqrt(2*a - c)/((2*a^2 - a*c)*x^4 + (2*a*b - b*c)*x^
3 + (2*a*c - c^2)*x^2 + 2*a^2 - a*c + (2*a*b - b*c)*x)) - (4*a^2 + 4*a*b -
2*b*c - c^2)*sqrt(2*a - 2*b + c)*log(((24*a^2 - 16*a*b + b^2 + 4*a*c)*x^4 +
 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a +
 2*b)*c + 4*c^2)*x^2 - 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x
^2 + 2*(2*a + b - c)*x + 4*a - b)*sqrt(2*a - 2*b + c) + 24*a^2 - 16*a*b + b
^2 + 4*a*c + 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x)/(x^4 + 4*x^3 + 6*
x^2 + 4*x + 1)) - (4*a^2 - 4*a*b + 2*b*c - c^2)*sqrt(2*a + 2*b + c)*log(((2
4*a^2 + 16*a*b + b^2 + 4*a*c)*x^4 - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(2*a + b)*
c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a - 2*b)*c + 4*c^2)*x^2 + 4*sqrt(a*x^4 + b*
x^3 + c*x^2 + b*x + a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(2
*a + 2*b + c) + 24*a^2 + 16*a*b + b^2 + 4*a*c - 4*(8*a^2 - 4*a*b - 3*b^2 -
2*(2*a + b)*c)*x)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)))/(8*a^3 - 8*a*b^2 - 2*a*
c^2 - c^3 + 4*(a^2 + b^2)*c), -1/16*(2*(4*a^2 - 4*a*b + 2*b*c - c^2)*sqrt(-
2*a - 2*b - c)*arctan(1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a + b)*
x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(-2*a - 2*b - c)/((2*a^2 + 2*a*b + a
*c)*x^4 + (2*a*b + 2*b^2 + b*c)*x^3 + (2*(a + b)*c + c^2)*x^2 + 2*a^2 + 2*a
*b + a*c + (2*a*b + 2*b^2 + b*c)*x)) - (4*a^2 + 4*a*b - 2*b*c - c^2)*sqrt(2
*a - 2*b + c)*log(((24*a^2 - 16*a*b + b^2 + 4*a*c)*x^4 + 4*(8*a^2 + 4*a*b -
 3*b^2 - 2*(2*a - b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a + 2*b)*c + 4*c^2)*x^
2 - 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x^2 + 2*(2*a + b - c
)*x + 4*a - b)*sqrt(2*a - 2*b + c) + 24*a^2 - 16*a*b + b^2 + 4*a*c + 4*(8*a
^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1)) + 2
*(4*a^2 - 4*b^2 + 4*a*c + c^2)*sqrt(-2*a + c)*log(-((8*a^2 - b^2 - 4*a*c)*x
^4 + 8*(2*a*b - b*c)*x^3 - 2*(8*a^2 + b^2 - 12*a*c + 4*c^2)*x^2 + 8*a^2 + 4
*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(b*x^2 - 2*(2*a - c)*x + b)*sqrt(-2*
a + c) - b^2 - 4*a*c + 8*(2*a*b - b*c)*x)/(x^4 + 2*x^2 + 1)))/(8*a^3 - 8*a*
b^2 - 2*a*c^2 - c^3 + 4*(a^2 + b^2)*c), -1/16*(4*(4*a^2 - 4*b^2 + 4*a*c + c
```

```
^2)*sqrt(2*a - c)*arctan(-1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(b*x^2
 - 2*(2*a - c)*x + b)*sqrt(2*a - c)/((2*a^2 - a*c)*x^4 + (2*a*b - b*c)*x^3 +
 (2*a*c - c^2)*x^2 + 2*a^2 - a*c + (2*a*b - b*c)*x)) + 2*(4*a^2 - 4*a*b + 2
*b*c - c^2)*sqrt(-2*a - 2*b - c)*arctan(1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*
x + a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(-2*a - 2*b - c)/(
(2*a^2 + 2*a*b + a*c)*x^4 + (2*a*b + 2*b^2 + b*c)*x^3 + (2*(a + b)*c + c^2)
*x^2 + 2*a^2 + 2*a*b + a*c + (2*a*b + 2*b^2 + b*c)*x)) - (4*a^2 + 4*a*b - 2
*b*c - c^2)*sqrt(2*a - 2*b + c)*log(((24*a^2 - 16*a*b + b^2 + 4*a*c)*x^4 +
4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a +
2*b)*c + 4*c^2)*x^2 - 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x^
2 + 2*(2*a + b - c)*x + 4*a - b)*sqrt(2*a - 2*b + c) + 24*a^2 - 16*a*b + b^
2 + 4*a*c + 4*(8*a^2 + 4*a*b - 3*b^2 - 2*(2*a - b)*c)*x)/(x^4 + 4*x^3 + 6*x
^2 + 4*x + 1)))/(8*a^3 - 8*a*b^2 - 2*a*c^2 - c^3 + 4*(a^2 + b^2)*c), -1/16*
(2*(4*a^2 + 4*a*b - 2*b*c - c^2)*sqrt(-2*a + 2*b - c)*arctan(-1/2*sqrt(a*x^
4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x^2 + 2*(2*a + b - c)*x + 4*a - b)*
sqrt(-2*a + 2*b - c)/((2*a^2 - 2*a*b + a*c)*x^4 + (2*a*b - 2*b^2 + b*c)*x^3
 + (2*(a - b)*c + c^2)*x^2 + 2*a^2 - 2*a*b + a*c + (2*a*b - 2*b^2 + b*c)*x)
) - (4*a^2 - 4*a*b + 2*b*c - c^2)*sqrt(2*a + 2*b + c)*log(((24*a^2 + 16*a*b
 + b^2 + 4*a*c)*x^4 - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(2*a + b)*c)*x^3 + 2*(24
*a^2 + 3*b^2 - 4*(a - 2*b)*c + 4*c^2)*x^2 + 4*sqrt(a*x^4 + b*x^3 + c*x^2 +
b*x + a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(2*a + 2*b + c)
+ 24*a^2 + 16*a*b + b^2 + 4*a*c - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(2*a + b)*c)
*x)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)) + 2*(4*a^2 - 4*b^2 + 4*a*c + c^2)*sqrt
(-2*a + c)*log(-((8*a^2 - b^2 - 4*a*c)*x^4 + 8*(2*a*b - b*c)*x^3 - 2*(8*a^2
 + b^2 - 12*a*c + 4*c^2)*x^2 + 8*a^2 + 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x +
 a)*(b*x^2 - 2*(2*a - c)*x + b)*sqrt(-2*a + c) - b^2 - 4*a*c + 8*(2*a*b - b
*c)*x)/(x^4 + 2*x^2 + 1)))/(8*a^3 - 8*a*b^2 - 2*a*c^2 - c^3 + 4*(a^2 + b^2)
*c), -1/16*(4*(4*a^2 - 4*b^2 + 4*a*c + c^2)*sqrt(2*a - c)*arctan(-1/2*sqrt(
a*x^4 + b*x^3 + c*x^2 + b*x + a)*(b*x^2 - 2*(2*a - c)*x + b)*sqrt(2*a - c)/
((2*a^2 - a*c)*x^4 + (2*a*b - b*c)*x^3 + (2*a*c - c^2)*x^2 + 2*a^2 - a*c +
(2*a*b - b*c)*x)) + 2*(4*a^2 + 4*a*b - 2*b*c - c^2)*sqrt(-2*a + 2*b - c)*ar
ctan(-1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x^2 + 2*(2*a + b
 - c)*x + 4*a - b)*sqrt(-2*a + 2*b - c)/((2*a^2 - 2*a*b + a*c)*x^4 + (2*a*b
 - 2*b^2 + b*c)*x^3 + (2*(a - b)*c + c^2)*x^2 + 2*a^2 - 2*a*b + a*c + (2*a*
b - 2*b^2 + b*c)*x)) - (4*a^2 - 4*a*b + 2*b*c - c^2)*sqrt(2*a + 2*b + c)*lo
g(((24*a^2 + 16*a*b + b^2 + 4*a*c)*x^4 - 4*(8*a^2 - 4*a*b - 3*b^2 - 2*(2*a
+ b)*c)*x^3 + 2*(24*a^2 + 3*b^2 - 4*(a - 2*b)*c + 4*c^2)*x^2 + 4*sqrt(a*x^4
 + b*x^3 + c*x^2 + b*x + a)*((4*a + b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*s
qrt(2*a + 2*b + c) + 24*a^2 + 16*a*b + b^2 + 4*a*c - 4*(8*a^2 - 4*a*b - 3*b
^2 - 2*(2*a + b)*c)*x)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)))/(8*a^3 - 8*a*b^2 -
 2*a*c^2 - c^3 + 4*(a^2 + b^2)*c), -1/8*((4*a^2 + 4*a*b - 2*b*c - c^2)*sqrt
(-2*a + 2*b - c)*arctan(-1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a -
b)*x^2 + 2*(2*a + b - c)*x + 4*a - b)*sqrt(-2*a + 2*b - c)/((2*a^2 - 2*a*b
+ a*c)*x^4 + (2*a*b - 2*b^2 + b*c)*x^3 + (2*(a - b)*c + c^2)*x^2 + 2*a^2 -
2*a*b + a*c + (2*a*b - 2*b^2 + b*c)*x)) + (4*a^2 - 4*a*b + 2*b*c - c^2)*sqr
t(-2*a - 2*b - c)*arctan(1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a +
b)*x^2 - 2*(2*a - b - c)*x + 4*a + b)*sqrt(-2*a - 2*b - c)/((2*a^2 + 2*a*b
+ a*c)*x^4 + (2*a*b + 2*b^2 + b*c)*x^3 + (2*(a + b)*c + c^2)*x^2 + 2*a^2 +
2*a*b + a*c + (2*a*b + 2*b^2 + b*c)*x)) + (4*a^2 - 4*b^2 + 4*a*c + c^2)*sqr
t(-2*a + c)*log(-((8*a^2 - b^2 - 4*a*c)*x^4 + 8*(2*a*b - b*c)*x^3 - 2*(8*a^
2 + b^2 - 12*a*c + 4*c^2)*x^2 + 8*a^2 + 4*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x
+ a)*(b*x^2 - 2*(2*a - c)*x + b)*sqrt(-2*a + c) - b^2 - 4*a*c + 8*(2*a*b -
b*c)*x)/(x^4 + 2*x^2 + 1)))/(8*a^3 - 8*a*b^2 - 2*a*c^2 - c^3 + 4*(a^2 + b^2
)*c), -1/8*(2*(4*a^2 - 4*b^2 + 4*a*c + c^2)*sqrt(2*a - c)*arctan(-1/2*sqrt(
a*x^4 + b*x^3 + c*x^2 + b*x + a)*(b*x^2 - 2*(2*a - c)*x + b)*sqrt(2*a - c)/
((2*a^2 - a*c)*x^4 + (2*a*b - b*c)*x^3 + (2*a*c - c^2)*x^2 + 2*a^2 - a*c +
(2*a*b - b*c)*x)) + (4*a^2 + 4*a*b - 2*b*c - c^2)*sqrt(-2*a + 2*b - c)*arct
an(-1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a - b)*x^2 + 2*(2*a + b -
 c)*x + 4*a - b)*sqrt(-2*a + 2*b - c)/((2*a^2 - 2*a*b + a*c)*x^4 + (2*a*b -
```

```
 2*b^2 + b*c)*x^3 + (2*(a - b)*c + c^2)*x^2 + 2*a^2 - 2*a*b + a*c + (2*a*b
- 2*b^2 + b*c)*x)) + (4*a^2 - 4*a*b + 2*b*c - c^2)*sqrt(-2*a - 2*b - c)*arc
tan(1/2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*((4*a + b)*x^2 - 2*(2*a - b -
 c)*x + 4*a + b)*sqrt(-2*a - 2*b - c)/((2*a^2 + 2*a*b + a*c)*x^4 + (2*a*b +
 2*b^2 + b*c)*x^3 + (2*(a + b)*c + c^2)*x^2 + 2*a^2 + 2*a*b + a*c + (2*a*b
+ 2*b^2 + b*c)*x)))/(8*a^3 - 8*a*b^2 - 2*a*c^2 - c^3 + 4*(a^2 + b^2)*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\sqrt{ax^4+bx^3+cx^2+bx+a}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(-x^2/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(x^4 - 1)), x)
```

maple [C] time = 0.03, size = 81718, normalized size = 308.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\sqrt{ax^4+bx^3+cx^2+bx+a}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+1)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
[Out] -integrate(x^2/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^2}{\left(x^4-1\right)\,\sqrt{a\,x^4+b\,x^3+c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^4 - 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)),x)
[Out] -int(x^2/((x^4 - 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^4\sqrt{ax^4+a+bx^3+bx+cx^2}-\sqrt{ax^4+a+bx^3+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+1)/(a*x**4+b*x**3+c*x**2+b*x+a)**(1/2),x)
[Out] -Integral(x**2/(x**4*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) - sqrt(a*x**4
 + a + b*x**3 + b*x + c*x**2)), x)
```

3.2208

$$\int \frac{x(-b+x)\left(-a^2b+2a^2x+(-2a+b)x^2\right)}{(x(-a+x)(-b+x))^{2/3}\left(-a^4+4a^3x+\left(-6a^2+b^2d\right)x^2+2(2a-bd)x^3+(-1+d)x^4\right)}\,dx$$

Optimal. Leaf size=265

$$\frac{\log\left(a^2-\sqrt[3]{d}\left(x^2(-a-b)+abx+x^3\right)^{2/3}-2ax+x^2\right)}{2d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\left(x^2(-a-b)+abx+x^3\right)^{2/3}}{2a^2+\sqrt[3]{d}\left(x^2(-a-b)+abx+x^3\right)^{2/3}-4ax+2x^2}\right)}{2d^{2/3}}-\frac{\log\left(a^4-4a\right.}{}$$

Rubi [F] time = 14.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(-b+x)\left(-a^2b+2a^2x+(-2a+b)x^2\right)}{(x(-a+x)(-b+x))^{2/3}\left(-a^4+4a^3x+\left(-6a^2+b^2d\right)x^2+2(2a-bd)x^3+(-1+d)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-b + x)*(-(a^2*b) + 2*a^2*x + (-2*a + b)*x^2))/((x*(-a + x)*(-b + x
))^(2/3)*(-a^4 + 4*a^3*x + (-6*a^2 + b^2*d)*x^2 + 2*(2*a - b*d)*x^3 + (-1 +
 d)*x^4)),x]
```

```
[Out] (3*a*b*x^(2/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x^3*(
-a + x^3)^(1/3)*(-b + x^3)^(1/3))/(-a^4 + 4*a^3*x^3 - 6*a^2*(1 - (b^2*d)/(6
*a^2))*x^6 + 4*a*(1 - (b*d)/(2*a))*x^9 - (1 - d)*x^12), x], x, x^(1/3)])/((
a - x)*(b - x)*x)^(2/3) + (3*(2*a - b)*x^(2/3)*(-a + x)^(2/3)*(-b + x)^(2/3
)*Defer[Subst][Defer[Int][(x^6*(-a + x^3)^(1/3)*(-b + x^3)^(1/3))/(a^4 - 4*
a^3*x^3 + 6*a^2*(1 - (b^2*d)/(6*a^2))*x^6 - 4*a*(1 - (b*d)/(2*a))*x^9 + (1
- d)*x^12), x], x, x^(1/3)])/((a - x)*(b - x)*x)^(2/3)
```

Rubi steps

$$\int \frac{x(-b+x)\left(-a^2b+2a^2x+(-2a+b)x^2\right)}{(x(-a+x)(-b+x))^{2/3}\left(-a^4+4a^3x+\left(-6a^2+b^2d\right)x^2+2(2a-bd)x^3+(-1+d)x^4\right)}\,dx=\frac{\left(x^{2/3}(-a+x)^{2/3}(-b\right.}{}$$

$$=\frac{\left(x^{2/3}(-a+x)^{2/3}(-b\right.}{}$$

$$=\frac{\left(3x^{2/3}(-a+x)^{2/3}(-b\right.}{}$$

$$=\frac{\left(3x^{2/3}(-a+x)^{2/3}(-b\right.}{}$$

$$=\frac{\left(3(2a-b)x^{2/3}(-a+\right.}{}$$

Mathematica [F] time = 1.93, size = 0, normalized size = 0.00

$$\int \frac{x(-b+x)\left(-a^2b+2a^2x+(-2a+b)x^2\right)}{(x(-a+x)(-b+x))^{2/3}\left(-a^4+4a^3x+\left(-6a^2+b^2d\right)x^2+2(2a-bd)x^3+(-1+d)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(-b + x)*(-(a^2*b) + 2*a^2*x + (-2*a + b)*x^2))/((x*(-a + x)*(
-b + x))^(2/3)*(-a^4 + 4*a^3*x + (-6*a^2 + b^2*d)*x^2 + 2*(2*a - b*d)*x^3 +
 (-1 + d)*x^4)),x]
```

```
[Out] Integrate[(x*(-b + x)*(-(a^2*b) + 2*a^2*x + (-2*a + b)*x^2))/((x*(-a + x)*(
-b + x))^(2/3)*(-a^4 + 4*a^3*x + (-6*a^2 + b^2*d)*x^2 + 2*(2*a - b*d)*x^3 +
 (-1 + d)*x^4)), x]
```

IntegrateAlgebraic [A] time = 3.41, size = 265, normalized size = 1.00

$$\frac{\log \left(a^2-\sqrt[3]{d} \left(x^2(-a-b)+abx+x^3\right)^{2/3}-2ax+x^2\right)}{2d^{2/3}}+\frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt{3} \sqrt[3]{d} \left(x^2(-a-b)+abx+x^3\right)^{2/3}}{2a^2+\sqrt[3]{d} \left(x^2(-a-b)+abx+x^3\right)^{2/3}-4ax+2x^2}\right)}{2d^{2/3}}-\frac{\log \left(a^4-4a^3x+\left(x^2(-a-b)+abx+x^3\right)^{2/3} \left(a^2 \sqrt[3]{d}-2a \sqrt[3]{d} x+\sqrt[3]{d} x^2\right)+6a^2x^2+d^{2/3} \left(x^2(-a-b)+abx+x^3\right)^{4/3}-4ax^3+x^4\right)}{4d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-b + x)*(-(a^2*b) + 2*a^2*x + (-2*a + b)*x^2))/((x*(
-a + x)*(-b + x))^(2/3)*(-a^4 + 4*a^3*x + (-6*a^2 + b^2*d)*x^2 + 2*(2*a - b
*d)*x^3 + (-1 + d)*x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(a*b*x + (-a - b)*x^2 + x^3)^(2/3))/(2*a^2
 - 4*a*x + 2*x^2 + d^(1/3)*(a*b*x + (-a - b)*x^2 + x^3)^(2/3))])/(2*d^(2/3)
) + Log[a^2 - 2*a*x + x^2 - d^(1/3)*(a*b*x + (-a - b)*x^2 + x^3)^(2/3)]/(2*
d^(2/3)) - Log[a^4 - 4*a^3*x + 6*a^2*x^2 - 4*a*x^3 + x^4 + (a^2*d^(1/3) - 2
*a*d^(1/3)*x + d^(1/3)*x^2)*(a*b*x + (-a - b)*x^2 + x^3)^(2/3) + d^(2/3)*(a
*b*x + (-a - b)*x^2 + x^3)^(4/3)]/(4*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(-a^2*b+2*a^2*x+(-2*a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(-
a^4+4*a^3*x+(b^2*d-6*a^2)*x^2+2*(-b*d+2*a)*x^3+(-1+d)*x^4),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2 b-2\, a^2 x+(2\, a-b)x^2\right)(b-x)x}{\left((d-1)x^4-a^4+4\, a^3 x-2\,(bd-2\, a)x^3+\left(b^2 d-6\, a^2\right)x^2\right)((a-x)(b-x)x)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(-a^2*b+2*a^2*x+(-2*a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(-
a^4+4*a^3*x+(b^2*d-6*a^2)*x^2+2*(-b*d+2*a)*x^3+(-1+d)*x^4),x, algorithm="gi
ac")
```

```
[Out] integrate((a^2*b - 2*a^2*x + (2*a - b)*x^2)*(b - x)*x/(((d - 1)*x^4 - a^4 +
 4*a^3*x - 2*(b*d - 2*a)*x^3 + (b^2*d - 6*a^2)*x^2)*((a - x)*(b - x)*x)^(2/
3)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x\,(-b+x)\left(-a^2 b+2a^2 x+(-2a+b)\,x^2\right)}{\left(x\,(-a+x)\,(-b+x)\right)^{\frac{2}{3}}\left(-a^4+4a^3 x+\left(b^2 d-6a^2\right)x^2+2\,(-bd+2a)\,x^3+(-1+d)\,x^4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b+x)*(-a^2*b+2*a^2*x+(-2*a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(-a^4+4*
a^3*x+(b^2*d-6*a^2)*x^2+2*(-b*d+2*a)*x^3+(-1+d)*x^4),x)
```

```
[Out] int(x*(-b+x)*(-a^2*b+2*a^2*x+(-2*a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(-a^4+4*
a^3*x+(b^2*d-6*a^2)*x^2+2*(-b*d+2*a)*x^3+(-1+d)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2b - 2\,a^2x + (2\,a - b)x^2\right)(b - x)x}{\left((d - 1)x^4 - a^4 + 4\,a^3x - 2\,(bd - 2\,a)x^3 + \left(b^2d - 6\,a^2\right)x^2\right)\left((a - x)(b - x)x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(-a^2*b+2*a^2*x+(-2*a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(-
a^4+4*a^3*x+(b^2*d-6*a^2)*x^2+2*(-b*d+2*a)*x^3+(-1+d)*x^4),x, algorithm="ma
xima")
```

```
[Out] integrate((a^2*b - 2*a^2*x + (2*a - b)*x^2)*(b - x)*x/(((d - 1)*x^4 - a^4 +
 4*a^3*x - 2*(b*d - 2*a)*x^3 + (b^2*d - 6*a^2)*x^2)*((a - x)*(b - x)*x)^(2/
3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int -\frac{x\,(b - x)\,\left(x^2\,(2\,a - b) + a^2\,b - 2\,a^2\,x\right)}{\left(x\,(a - x)\,(b - x)\right)^{2/3}\,\left(x^2\,\left(b^2\,d - 6\,a^2\right) + 2\,x^3\,(2\,a - b\,d) + 4\,a^3\,x - a^4 + x^4\,(d - 1)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b - x)*(x^2*(2*a - b) + a^2*b - 2*a^2*x))/((x*(a - x)*(b - x))^(2/3
)*(x^2*(b^2*d - 6*a^2) + 2*x^3*(2*a - b*d) + 4*a^3*x - a^4 + x^4*(d - 1))),
x)
```

```
[Out] -int(-(x*(b - x)*(x^2*(2*a - b) + a^2*b - 2*a^2*x))/((x*(a - x)*(b - x))^(2
/3)*(x^2*(b^2*d - 6*a^2) + 2*x^3*(2*a - b*d) + 4*a^3*x - a^4 + x^4*(d - 1))
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(-a**2*b+2*a**2*x+(-2*a+b)*x**2)/(x*(-a+x)*(-b+x))**(2/3
)/(-a**4+4*a**3*x+(b**2*d-6*a**2)*x**2+2*(-b*d+2*a)*x**3+(-1+d)*x**4),x)
```

```
[Out] Timed out
```

3.2209 $\int \frac{\left(-1+x^3\right)^{2/3}\left(8-8x^3+x^6\right)}{x^6\left(-4+x^3\right)\left(-2+x^3\right)}\,dx$

Optimal. Leaf size=265

$$\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}-x\right)}{6\,2^{2/3}}-\frac{\log\left(6^{2/3}\sqrt[3]{x^3-1}-3x\right)}{8\sqrt[3]{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}+x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\,2^{2/3}\sqrt[3]{x^3-1}+\sqrt[3]{3}x}\right)}{8\sqrt[3]{2}}+\frac{\left(x^3-1\right)}{}$$

Rubi [C] time = 0.85, antiderivative size = 173, normalized size of antiderivative = 0.65, number of steps used = 9, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {6725, 264, 277, 239, 430, 429}

$$\frac{x\left(x^3-1\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,\frac{x^3}{4}\right)}{16\left(1-x^3\right)^{2/3}}-\frac{x\left(x^3-1\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,\frac{x^3}{2}\right)}{4\left(1-x^3\right)^{2/3}}+\frac{1}{8}\log\left(\sqrt[3]{x^3-1}-x\right)-\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\left(x^3-1\right)^{5/3}}{5x^5}+\frac{\left(x^3-1\right)^{2/3}}{8x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^3)^(2/3)*(8 - 8*x^3 + x^6))/(x^6*(-4 + x^3)*(-2 + x^3)),x]
```

```
[Out] (-1 + x^3)^(2/3)/(8*x^2) + (-1 + x^3)^(5/3)/(5*x^5) + (x*(-1 + x^3)^(2/3)*A
ppellF1[1/3, -2/3, 1, 4/3, x^3, x^3/4])/(16*(1 - x^3)^(2/3)) - (x*(-1 + x^3
)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, x^3, x^3/2])/(4*(1 - x^3)^(2/3)) - ArcT
an[(1 + (2*x)/(-1 + x^3)^(1/3))/Sqrt[3]]/(4*Sqrt[3]) + Log[-x + (-1 + x^3)^
(1/3)]/8
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
```

```
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(8-8x^3+x^6\right)}{x^6\left(-4+x^3\right)\left(-2+x^3\right)}\,dx = \int \left(\frac{\left(-1+x^3\right)^{2/3}}{x^6} - \frac{\left(-1+x^3\right)^{2/3}}{4x^3} - \frac{\left(-1+x^3\right)^{2/3}}{4\left(-4+x^3\right)} + \frac{\left(-1+x^3\right)^{2/3}}{2\left(-2+x^3\right)}\right) dx$$

$$= -\left(\frac{1}{4}\int \frac{\left(-1+x^3\right)^{2/3}}{x^3}\,dx\right) - \frac{1}{4}\int \frac{\left(-1+x^3\right)^{2/3}}{-4+x^3}\,dx + \frac{1}{2}\int \frac{\left(-1+x^3\right)^{2/3}}{-2+x^3}\,dx + \int$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{8x^2} + \frac{\left(-1+x^3\right)^{5/3}}{5x^5} - \frac{1}{4}\int \frac{1}{\sqrt[3]{-1+x^3}}\,dx - \frac{\left(-1+x^3\right)^{2/3}\int \frac{\left(1-x^3\right)^{2/3}}{-4+x^3}\,dx}{4\left(1-x^3\right)^{2/3}}$$

$$= \frac{\left(-1+x^3\right)^{2/3}}{8x^2} + \frac{\left(-1+x^3\right)^{5/3}}{5x^5} + \frac{x\left(-1+x^3\right)^{2/3} F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,\frac{x^3}{4}\right)}{16\left(1-x^3\right)^{2/3}} - \frac{x\left(-1+\right.}{}$$

Mathematica [A] time = 0.41, size = 259, normalized size = 0.98

$$\frac{1}{96}\left(8\sqrt[3]{2}\log\left(\sqrt[3]{2}-\frac{x}{\sqrt[3]{x^3-1}}\right)-2\,6^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3-1}}\right)-8\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\frac{2^{2/3}x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)+6\,2^{2/3}\sqrt[6]{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{6}x}{\sqrt[3]{x^3-1}}+1}{\sqrt{3}}\right)-4\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+\frac{x^2}{\left(x^3-1\right)^{2/3}}+2^{2/3}\right)+6^{2/3}\log\left(\frac{2^{2/3}\sqrt[3]{3}x}{\sqrt[3]{x^3-1}}+\frac{3^{2/3}x^2}{\left(x^3-1\right)^{2/3}}+2\sqrt[3]{2}\right)\right)+\left(x^3-1\right)^{2/3}\left(\frac{13}{40x^2}-\frac{1}{5x^5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^3)^(2/3)*(8 - 8*x^3 + x^6))/(x^6*(-4 + x^3)*(-2 + x^3)),
x]
```

```
[Out] (-1/5*1/x^5 + 13/(40*x^2))*(-1 + x^3)^(2/3) + (-8*2^(1/3)*Sqrt[3]*ArcTan[(1
 + (2^(2/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] + 6*2^(2/3)*3^(1/6)*ArcTan[(1 + (
6^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] + 8*2^(1/3)*Log[2^(1/3) - x/(-1 + x^3
)^(1/3)] - 4*2^(1/3)*Log[2^(2/3) + x^2/(-1 + x^3)^(2/3) + (2^(1/3)*x)/(-1 +
 x^3)^(1/3)] - 2*6^(2/3)*Log[2^(2/3) - (3^(1/3)*x)/(-1 + x^3)^(1/3)] + 6^(2
/3)*Log[2*2^(1/3) + (3^(2/3)*x^2)/(-1 + x^3)^(2/3) + (2^(2/3)*3^(1/3)*x)/(-
1 + x^3)^(1/3)])/96
```

IntegrateAlgebraic [A] time = 0.77, size = 265, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}-x\right)}{6\,2^{2/3}}-\frac{\log\left(6^{2/3}\sqrt[3]{x^3-1}-3x\right)}{8\sqrt[3]{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}+x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\,2^{2/3}\sqrt[3]{x^3-1}+\sqrt[3]{3}x}\right)}{8\sqrt[3]{2}}+\frac{\left(x^3-1\right)^{2/3}\left(13x^3-8\right)}{40x^5}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}x+2^{2/3}\left(x^3-1\right)^{2/3}+x^2\right)}{12\,2^{2/3}}+\frac{\log\left(6^{2/3}\sqrt[3]{x^3-1}x+2\sqrt[3]{6}\left(x^3-1\right)^{2/3}+3x^2\right)}{16\sqrt[3]{6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(2/3)*(8 - 8*x^3 + x^6))/(x^6*(-4 + x^3)*(-2
 + x^3)),x]
```

```
[Out] ((-1 + x^3)^(2/3)*(-8 + 13*x^3))/(40*x^5) - ArcTan[(Sqrt[3]*x)/(x + 2*2^(1/
3)*(-1 + x^3)^(1/3))]/(2*2^(2/3)*Sqrt[3]) + (3^(1/6)*ArcTan[(3^(5/6)*x)/(3^
(1/3)*x + 2*2^(2/3)*(-1 + x^3)^(1/3))])/(8*2^(1/3)) + Log[-x + 2^(1/3)*(-1
+ x^3)^(1/3)]/(6*2^(2/3)) - Log[-3*x + 6^(2/3)*(-1 + x^3)^(1/3)]/(8*6^(1/3)
) - Log[x^2 + 2^(1/3)*x*(-1 + x^3)^(1/3) + 2^(2/3)*(-1 + x^3)^(2/3)]/(12*2^
```

```
(2/3)) + Log[3*x^2 + 6^(2/3)*x*(-1 + x^3)^(1/3) + 2*6^(1/3)*(-1 + x^3)^(2/3
)]/(16*6^(1/3))
```

fricas [B] time = 11.22, size = 552, normalized size = 2.08

$$\frac{30 \cdot 6^{\frac{1}{6}} \sqrt{2} \left(-1\right)^{\frac{1}{3}} x^{5} \arctan\left(\frac{6^{\frac{1}{6}} \left(24 \cdot 6^{\frac{2}{3}} \sqrt{2} \left(-1\right)^{\frac{2}{3}} \left(5 x^{7} - 22 x^{4} + 8 x\right) \left(x^{3} - 1\right)^{\frac{2}{3}} - 36 \sqrt{2} \left(-1\right)^{\frac{1}{3}} \left(109 x^{8} - 116 x^{5} + 16 x^{2}\right) \left(x^{3} - 1\right)^{\frac{1}{3}} + 6^{\frac{1}{3}} \sqrt{2} \left(1189 x^{9} - 2064 x^{6} + 912 x^{3} - 64\right)\right)}{6 \left(971 x^{9} - 960 x^{6} - 48 x^{3} + 64\right)}\right) + 10 \cdot 6^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} x^{5} \log\left(\frac{18 \cdot 6^{\frac{1}{3}} \left(-1\right)^{\frac{2}{3}} \left(x^{3} - 1\right)^{\frac{1}{3}} x^{2} - 6^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} \left(x^{3} - 4\right) - 36 \left(x^{3} - 1\right)^{\frac{2}{3}} x}{x^{3} - 4}\right) - 5 \cdot 6^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} x^{5} \log\left(-\frac{12 \cdot 6^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} \left(5 x^{4} - 2 x\right) \left(x^{3} - 1\right)^{\frac{2}{3}} - 6^{\frac{1}{3}} \left(-1\right)^{\frac{2}{3}} \left(109 x^{6} - 116 x^{3} + 16\right) - 18 \left(11 x^{5} - 8 x^{2}\right) \left(x^{3} - 1\right)^{\frac{1}{3}}}{x^{6} - 8 x^{3} + 16}\right) + 40 \cdot 4^{\frac{1}{6}} \sqrt{3} x^{5} \arctan\left(\frac{4^{\frac{1}{6}} \left(12 \cdot 4^{\frac{2}{3}} \sqrt{3} \left(2 x^{7} - 5 x^{4} + 2 x\right) \left(x^{3} - 1\right)^{\frac{2}{3}} + 4^{\frac{1}{3}} \sqrt{3} \left(91 x^{9} - 168 x^{6} + 84 x^{3} - 8\right) + 12 \sqrt{3} \left(19 x^{8} - 22 x^{5} + 4 x^{2}\right) \left(x^{3} - 1\right)^{\frac{1}{3}}\right)}{6 \left(53 x^{9} - 48 x^{6} - 12 x^{3} + 8\right)}\right) + 20 \cdot 4^{\frac{2}{3}} x^{5} \log\left(\frac{6 \cdot 4^{\frac{1}{3}} \left(x^{3} - 1\right)^{\frac{1}{3}} x^{2} + 4^{\frac{2}{3}} \left(x^{3} - 2\right) - 12 \left(x^{3} - 1\right)^{\frac{2}{3}} x}{x^{3} - 2}\right) - 10 \cdot 4^{\frac{2}{3}} x^{5} \log\left(\frac{6 \cdot 4^{\frac{2}{3}} \left(2 x^{4} - x\right) \left(x^{3} - 1\right)^{\frac{2}{3}} + 4^{\frac{1}{3}} \left(19 x^{6} - 22 x^{3} + 4\right) + 6 \left(5 x^{5} - 4 x^{2}\right) \left(x^{3} - 1\right)^{\frac{1}{3}}}{x^{6} - 4 x^{3} + 4}\right) + 36 \left(13 x^{3} - 8\right) \left(x^{3} - 1\right)^{\frac{2}{3}}}{1440 x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6-8*x^3+8)/x^6/(x^3-4)/(x^3-2),x, algorithm="fri
cas")
```

```
[Out] 1/1440*(30*6^(1/6)*sqrt(2)*(-1)^(1/3)*x^5*arctan(1/6*6^(1/6)*(24*6^(2/3)*sq
rt(2)*(-1)^(2/3)*(5*x^7 - 22*x^4 + 8*x)*(x^3 - 1)^(2/3) - 36*sqrt(2)*(-1)^(
1/3)*(109*x^8 - 116*x^5 + 16*x^2)*(x^3 - 1)^(1/3) + 6^(1/3)*sqrt(2)*(1189*x
^9 - 2064*x^6 + 912*x^3 - 64))/(971*x^9 - 960*x^6 - 48*x^3 + 64)) + 10*6^(2
/3)*(-1)^(1/3)*x^5*log((18*6^(1/3)*(-1)^(2/3)*(x^3 - 1)^(1/3)*x^2 - 6^(2/3)
*(-1)^(1/3)*(x^3 - 4) - 36*(x^3 - 1)^(2/3)*x)/(x^3 - 4)) - 5*6^(2/3)*(-1)^(
1/3)*x^5*log(-(12*6^(2/3)*(-1)^(1/3)*(5*x^4 - 2*x)*(x^3 - 1)^(2/3) - 6^(1/3
)*(-1)^(2/3)*(109*x^6 - 116*x^3 + 16) - 18*(11*x^5 - 8*x^2)*(x^3 - 1)^(1/3)
)/(x^6 - 8*x^3 + 16)) + 40*4^(1/6)*sqrt(3)*x^5*arctan(1/6*4^(1/6)*(12*4^(2/
3)*sqrt(3)*(2*x^7 - 5*x^4 + 2*x)*(x^3 - 1)^(2/3) + 4^(1/3)*sqrt(3)*(91*x^9
- 168*x^6 + 84*x^3 - 8) + 12*sqrt(3)*(19*x^8 - 22*x^5 + 4*x^2)*(x^3 - 1)^(1
/3))/(53*x^9 - 48*x^6 - 12*x^3 + 8)) + 20*4^(2/3)*x^5*log((6*4^(1/3)*(x^3 -
 1)^(1/3)*x^2 + 4^(2/3)*(x^3 - 2) - 12*(x^3 - 1)^(2/3)*x)/(x^3 - 2)) - 10*4
^(2/3)*x^5*log((6*4^(2/3)*(2*x^4 - x)*(x^3 - 1)^(2/3) + 4^(1/3)*(19*x^6 - 2
2*x^3 + 4) + 6*(5*x^5 - 4*x^2)*(x^3 - 1)^(1/3))/(x^6 - 4*x^3 + 4)) + 36*(13
*x^3 - 8)*(x^3 - 1)^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6} - 8 \, x^{3} + 8\right)\left(x^{3} - 1\right)^{\frac{2}{3}}}{\left(x^{3} - 2\right)\left(x^{3} - 4\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6-8*x^3+8)/x^6/(x^3-4)/(x^3-2),x, algorithm="gia
c")
```

```
[Out] integrate((x^6 - 8*x^3 + 8)*(x^3 - 1)^(2/3)/((x^3 - 2)*(x^3 - 4)*x^6), x)
```

maple [F] time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}-1\right)^{\frac{2}{3}} \left(x^{6}-8 x^{3}+8\right)}{x^{6} \left(x^{3}-4\right) \left(x^{3}-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)*(x^6-8*x^3+8)/x^6/(x^3-4)/(x^3-2),x)
```

```
[Out] int((x^3-1)^(2/3)*(x^6-8*x^3+8)/x^6/(x^3-4)/(x^3-2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6} - 8 \, x^{3} + 8\right)\left(x^{3} - 1\right)^{\frac{2}{3}}}{\left(x^{3} - 2\right)\left(x^{3} - 4\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6-8*x^3+8)/x^6/(x^3-4)/(x^3-2),x, algorithm="max
ima")
```

```
[Out] integrate((x^6 - 8*x^3 + 8)*(x^3 - 1)^(2/3)/((x^3 - 2)*(x^3 - 4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^3-1\right)^{2/3}\,\left(x^6-8\,x^3+8\right)}{x^6\,\left(x^3-2\right)\,\left(x^3-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(x^6 - 8*x^3 + 8))/(x^6*(x^3 - 2)*(x^3 - 4)),x)
```

```
[Out] int(((x^3 - 1)^(2/3)*(x^6 - 8*x^3 + 8))/(x^6*(x^3 - 2)*(x^3 - 4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)*(x**6-8*x**3+8)/x**6/(x**3-4)/(x**3-2),x)
```

```
[Out] Timed out
```

3.2210 $\int \frac{\left(c+\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}\right)^{4/3}}{\sqrt{-b+a^2x^2}}\,dx$

Optimal. Leaf size=265

$$\frac{4c^{4/3}\log\left(\sqrt[3]{\left(\sqrt{a^2x^2-b}+ax\right)^{3/4}+c}-\sqrt[3]{c}\right)}{3a}-\frac{2c^{4/3}\log\left(\sqrt[3]{c}\sqrt[3]{\left(\sqrt{a^2x^2-b}+ax\right)^{3/4}+c}+\left(\left(\sqrt{a^2x^2-b}+ax\right)^{3,\ldots}\right.\right.}{3a}$$

Rubi [F] time = 0.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(c+\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}\right)^{4/3}}{\sqrt{-b+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + (a*x + Sqrt[-b + a^2*x^2])^(3/4))^(4/3)/Sqrt[-b + a^2*x^2],x]
```

```
[Out] Defer[Int][(c + (a*x + Sqrt[-b + a^2*x^2])^(3/4))^(4/3)/Sqrt[-b + a^2*x^2],
 x]
```

Rubi steps

$$\int \frac{\left(c+\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}\right)^{4/3}}{\sqrt{-b+a^2x^2}}\,dx=\int \frac{\left(c+\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}\right)^{4/3}}{\sqrt{-b+a^2x^2}}\,dx$$

Mathematica [C] time = 0.36, size = 199, normalized size = 0.75

$$\frac{-2c^2\left(\frac{c\sqrt[4]{\sqrt{a^2x^2-b}+ax}+\sqrt{a^2x^2-b}+ax}{\sqrt{a^2x^2-b}+ax}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{c}{\left(ax+\sqrt{a^2x^2-b}\right)^{3/4}}\right)+6c\left(\sqrt{a^2x^2-b}+ax\right)^{3/4}+\left(\sqrt{a^2x^2-b}+ax\right)^{3/2}+5c^2}{a\left(\left(\sqrt{a^2x^2-b}+ax\right)^{3/4}+c\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + (a*x + Sqrt[-b + a^2*x^2])^(3/4))^(4/3)/Sqrt[-b + a^2*x^2],x
]
```

```
[Out] (5*c^2 + 6*c*(a*x + Sqrt[-b + a^2*x^2])^(3/4) + (a*x + Sqrt[-b + a^2*x^2])^
(3/2) - 2*c^2*((a*x + Sqrt[-b + a^2*x^2] + c*(a*x + Sqrt[-b + a^2*x^2])^(1/
4))/(a*x + Sqrt[-b + a^2*x^2]))^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, -(c/
(a*x + Sqrt[-b + a^2*x^2])^(3/4))])/(a*(c + (a*x + Sqrt[-b + a^2*x^2])^(3/4
))^(2/3))
```

IntegrateAlgebraic [A] time = 0.95, size = 288, normalized size = 1.09

$$\frac{4c^{4/3}\log\left(\sqrt[3]{\left(\sqrt{a^2x^2-b}+ax\right)^{3/4}+c}-\sqrt[3]{c}\right)}{3a}-\frac{2c^{4/3}\log\left(\sqrt[3]{c}\sqrt[3]{\left(\sqrt{a^2x^2-b}+ax\right)^{3/4}+c}+\left(\left(\sqrt{a^2x^2-b}+ax\right)^{3/4}+c\right)^{2/3}+c^{2/3}\right)}{3a}-\frac{4c^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{\left(\sqrt{a^2x^2-b}+ax\right)^{3/4}+c}}{\sqrt{3}\sqrt[3]{c}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}a}+\frac{5c\sqrt[3]{\left(\sqrt{a^2x^2-b}+ax\right)^{3/4}+c}}{a}+\frac{\left(\sqrt{a^2x^2-b}+ax\right)^{3/4}\sqrt[3]{\left(\sqrt{a^2x^2-b}+ax\right)^{3/4}+c}}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + (a*x + Sqrt[-b + a^2*x^2])^(3/4))^(4/3)/Sqrt[-b + a
^2*x^2],x]
```

```
[Out] (5*c*(c + (a*x + Sqrt[-b + a^2*x^2])^(3/4))^(1/3))/a + ((a*x + Sqrt[-b + a^
2*x^2])^(3/4)*(c + (a*x + Sqrt[-b + a^2*x^2])^(3/4))^(1/3))/a - (4*c^(4/3)*
ArcTan[1/Sqrt[3] + (2*(c + (a*x + Sqrt[-b + a^2*x^2])^(3/4))^(1/3))/(Sqrt[3
]*c^(1/3))])/(Sqrt[3]*a) + (4*c^(4/3)*Log[-c^(1/3) + (c + (a*x + Sqrt[-b +
a^2*x^2])^(3/4))^(1/3)])/(3*a) - (2*c^(4/3)*Log[c^(2/3) + c^(1/3)*(c + (a*x
 + Sqrt[-b + a^2*x^2])^(3/4))^(1/3) + (c + (a*x + Sqrt[-b + a^2*x^2])^(3/4)
)^(2/3)])/(3*a)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a^2*x^2-b)^(1/2))^(3/4))^(4/3)/(a^2*x^2-b)^(1/2),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a^2*x^2-b)^(1/2))^(3/4))^(4/3)/(a^2*x^2-b)^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\left(c+\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{3}{4}}\right)^{\frac{4}{3}}}{\sqrt{a^{2}x^{2}-b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+(a*x+(a^2*x^2-b)^(1/2))^(3/4))^(4/3)/(a^2*x^2-b)^(1/2),x)
```

```
[Out] int((c+(a*x+(a^2*x^2-b)^(1/2))^(3/4))^(4/3)/(a^2*x^2-b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{3}{4}}\right)^{\frac{4}{3}}}{\sqrt{a^{2}x^{2}-b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a^2*x^2-b)^(1/2))^(3/4))^(4/3)/(a^2*x^2-b)^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate((c + (a*x + sqrt(a^2*x^2 - b))^(3/4))^(4/3)/sqrt(a^2*x^2 - b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{3/4}\right)}^{4/3}}{\sqrt{a^2\,x^2-b}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + (a*x + (a^2*x^2 - b)^(1/2))^(3/4))^(4/3)/(a^2*x^2 - b)^(1/2),x)
```

```
[Out] int((c + (a*x + (a^2*x^2 - b)^(1/2))^(3/4))^(4/3)/(a^2*x^2 - b)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + \left(ax + \sqrt{a^2x^2 - b}\right)^{\frac{3}{4}}\right)^{\frac{4}{3}}}{\sqrt{a^2x^2 - b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a**2*x**2-b)**(1/2))**(3/4))**(4/3)/(a**2*x**2-b)**(1/2)
,x)
```

```
[Out] Integral((c + (a*x + sqrt(a**2*x**2 - b))**(3/4))**(4/3)/sqrt(a**2*x**2 - b
), x)
```

3.2211 $\int \frac{1}{(-b+ax)\sqrt[3]{b^3+a^3x^3}}\,dx$

Optimal. Leaf size=266

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a^3x^3+b^3}}{\sqrt[3]{a^3x^3+b^3}+\sqrt[3]{2}ax+\sqrt[3]{2}b}\right)}{2\sqrt[3]{2}ab}+\frac{\log\left(\sqrt[3]{2}a^{3/2}\sqrt{b}x-2\sqrt{a}\sqrt{b}\sqrt[3]{a^3x^3+b^3}+\sqrt[3]{2}\sqrt{a}b^{3/2}\right)}{2\sqrt[3]{2}ab}-\frac{\log\left(4ab\left(a^3x^3+b^3\right)^{2/3}\right.}{}$$

Rubi [A] time = 0.07, antiderivative size = 134, normalized size of antiderivative = 0.50, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2148}

$$\frac{3\log\left(2^{2/3}a\sqrt[3]{a^3x^3+b^3}-a(ax+b)\right)}{4\sqrt[3]{2}ab}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(ax+b)}{\sqrt[3]{a^3x^3+b^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}ab}-\frac{\log\left(-(b-ax)^2(ax+b)\right)}{4\sqrt[3]{2}ab}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-b + a*x)*(b^3 + a^3*x^3)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(b + a*x))/(b^3 + a^3*x^3)^(1/3))/Sqrt[3
]])/(2^(1/3)*a*b) - Log[-((b - a*x)^2*(b + a*x))]/(4*2^(1/3)*a*b) + (3*Log[
-(a*(b + a*x)) + 2^(2/3)*a*(b^3 + a^3*x^3)^(1/3)])/(4*2^(1/3)*a*b)
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(-b+ax)\sqrt[3]{b^3+a^3x^3}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(b+ax)}{\sqrt[3]{b^3+a^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}ab}-\frac{\log\left(-(b-ax)^2(b+ax)\right)}{4\sqrt[3]{2}ab}+\frac{3\log\left(-a(b+ax)+2^{2/3}a\sqrt[3]{\right.}}{4\sqrt[3]{2}ab}$$

Mathematica [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(-b+ax)\sqrt[3]{b^3+a^3x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((-b + a*x)*(b^3 + a^3*x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((-b + a*x)*(b^3 + a^3*x^3)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 1.42, size = 266, normalized size = 1.00

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a^3x^3+b^3}}{\sqrt[3]{a^3x^3+b^3}+\sqrt[3]{2}ax+\sqrt[3]{2}b}\right)}{2\sqrt[3]{2}ab}+\frac{\log\left(\sqrt[3]{2}a^{3/2}\sqrt{b}x-2\sqrt{a}\sqrt{b}\sqrt[3]{a^3x^3+b^3}+\sqrt[3]{2}\sqrt{a}b^{3/2}\right)}{2\sqrt[3]{2}ab}-\frac{\log\left(4ab\left(a^3x^3+b^3\right)^{2/3}+2^{2/3}a^3bx^2+2\,2^{2/3}a^2b^2x+\left(2\sqrt[3]{2}a^2bx+2\sqrt[3]{2}ab^2\right)\sqrt[3]{a^3x^3+b^3}+2^{2/3}ab^3\right)}{4\sqrt[3]{2}ab}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-b + a*x)*(b^3 + a^3*x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(b^3 + a^3*x^3)^(1/3))/(2^(1/3)*b + 2^(1/3)*a*x +
(b^3 + a^3*x^3)^(1/3))])/(2*2^(1/3)*a*b) + Log[2^(1/3)*Sqrt[a]*b^(3/2) + 2^
(1/3)*a^(3/2)*Sqrt[b]*x - 2*Sqrt[a]*Sqrt[b]*(b^3 + a^3*x^3)^(1/3)]/(2*2^(1/
3)*a*b) - Log[2^(2/3)*a*b^3 + 2*2^(2/3)*a^2*b^2*x + 2^(2/3)*a^3*b*x^2 + (2*
2^(1/3)*a*b^2 + 2*2^(1/3)*a^2*b*x)*(b^3 + a^3*x^3)^(1/3) + 4*a*b*(b^3 + a^3
*x^3)^(2/3)]/(4*2^(1/3)*a*b)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-b)/(a^3*x^3+b^3)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^3x^3+b^3\right)^{\frac{1}{3}}(ax-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-b)/(a^3*x^3+b^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((a^3*x^3 + b^3)^(1/3)*(a*x - b)), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax-b)\left(a^3x^3+b^3\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-b)/(a^3*x^3+b^3)^(1/3),x)
```

```
[Out] int(1/(a*x-b)/(a^3*x^3+b^3)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^3x^3+b^3\right)^{\frac{1}{3}}(ax-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-b)/(a^3*x^3+b^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^3*x^3 + b^3)^(1/3)*(a*x - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{1}{\left(a^3\,x^3+b^3\right)^{1/3}\,\left(b-a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((b^3 + a^3*x^3)^(1/3)*(b - a*x)),x)
```

```
[Out] -int(1/((b^3 + a^3*x^3)^(1/3)*(b - a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(ax+b)\left(a^2x^2-abx+b^2\right)}\,(ax-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-b)/(a**3*x**3+b**3)**(1/3),x)
```

```
[Out] Integral(1/(((a*x + b)*(a**2*x**2 - a*b*x + b**2))**(1/3)*(a*x - b)), x)
```

3.2212 $\int \frac{\sqrt[3]{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=266

$$\frac{3b\sqrt[3]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{2a}+\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}-1\right)}{2\sqrt[3]{2}\,a}-\frac{b\log\left(2^{2/3}\left(bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2\right)^{2/3}+\sqrt[3]{2}\right.}{4\sqrt[3]{2}\,a}$$

Rubi [F] time = 0.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])^(1/3)/Sqrt[-(a/b^2) + (a^2
*x^2)/b^2],x]
```

```
[Out] Defer[Int][(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])^(1/3)/Sqrt[-(a/b^2)
 + (a^2*x^2)/b^2], x]
```

Rubi steps

$$\int \frac{\sqrt[3]{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt[3]{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Mathematica [A] time = 1.44, size = 331, normalized size = 1.24

$$\frac{ax\sqrt[3]{x\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}\left(bx\sqrt{\frac{a(ax^2-1)}{b^2}}+ax^2-1\right)\left(-\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)^2+a}+\left(\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)^2+a\right)^{2/3}\right)+6\sqrt[3]{2}\sqrt[3]{ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)}+2\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)^2+a}\right)-2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)^2+a}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)}{4\sqrt[3]{2}\sqrt{\frac{a(ax^2-1)}{b^2}}\left(ax\left(b\sqrt{\frac{a(ax^2-1)}{b^2}}+ax\right)\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])^(1/3)/Sqrt[-(a/b^2)
+ (a^2*x^2)/b^2],x]
```

```
[Out] (a*x*(x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2]))^(1/3)*(-1 + a*x^2 + b*x*Sqrt[
(a*(-1 + a*x^2))/b^2])*(6*2^(1/3)*(a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])
)^(1/3) - 2*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2*(a + (a*x + b*Sqrt[(a*(-1 + a*x^
2))/b^2])^2)^(1/3))/a^(1/3))/Sqrt[3]] + 2*a^(1/3)*Log[a^(1/3) - (a + (a*x +
 b*Sqrt[(a*(-1 + a*x^2))/b^2])^2)^(1/3)] - a^(1/3)*Log[a^(2/3) + a^(1/3)*(a
 + (a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2])^2)^(1/3) + (a + (a*x + b*Sqrt[(a*(-
1 + a*x^2))/b^2])^2)^(2/3)]))/(4*2^(1/3)*Sqrt[(a*(-1 + a*x^2))/b^2]*(a*x*(a
*x + b*Sqrt[(a*(-1 + a*x^2))/b^2]))^(4/3))
```

IntegrateAlgebraic [A] time = 3.85, size = 266, normalized size = 1.00

$$\frac{3b\sqrt[3]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{2a}+\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}-1\right)}{2\sqrt[3]{2}a}-\frac{b\log\left(2^{2/3}\left(bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}+1\right)}{4\sqrt[3]{2}a}-\frac{\sqrt{3}b\tan^{-1}\left(\frac{2\sqrt[3]{2}\sqrt[3]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])^(1/3)/Sqrt[
-(a/b^2) + (a^2*x^2)/b^2],x]
```

```
[Out] (3*b*(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])^(1/3))/(2*a) - (Sqrt[3]*b
*ArcTan[1/Sqrt[3] + (2*2^(1/3)*(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])
^(1/3))/Sqrt[3]])/(2*2^(1/3)*a) + (b*Log[-1 + 2^(1/3)*(a*x^2 + b*x*Sqrt[-(a
/b^2) + (a^2*x^2)/b^2])^(1/3)])/(2*2^(1/3)*a) - (b*Log[1 + 2^(1/3)*(a*x^2 +
 b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])^(1/3) + 2^(2/3)*(a*x^2 + b*x*Sqrt[-(a/
b^2) + (a^2*x^2)/b^2])^(2/3)])/(4*2^(1/3)*a)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/3)/(-a/b^2+a^2*x^2/b^2)
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,bx\right)^{\frac{1}{3}}}{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/3)/(-a/b^2+a^2*x^2/b^2)
^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)^(1/3)/sqrt(a^2*x^2/b^2 -
a/b^2), x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)^{\frac{1}{3}}}{\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/3)/(-a/b^2+a^2*x^2/b^2)^(1/2)
,x)
```

```
[Out] int((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/3)/(-a/b^2+a^2*x^2/b^2)^(1/2)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\,bx\right)^{\frac{1}{3}}}{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/3)/(-a/b^2+a^2*x^2/b^2)
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)^(1/3)/sqrt(a^2*x^2/b^2 -
a/b^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x^2+b\,x\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}\right)^{1/3}}{\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/3)/((a^2*x^2)/b^2 - a/b^
2)^(1/2),x)
```

```
[Out] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/3)/((a^2*x^2)/b^2 - a/b^
2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x\left(ax+b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)}}{\sqrt{\frac{a\left(ax^2-1\right)}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x*(-a/b**2+a**2*x**2/b**2)**(1/2))**(1/3)/(-a/b**2+a**2
*x**2/b**2)**(1/2),x)
```

```
[Out] Integral((x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))**(1/3)/sqrt(a*(a*x**2
- 1)/b**2), x)
```

3.2213 $\int \frac{-b+x}{\left((-a+x)(-b+x)^2\right)^{2/3}(b-ad+(-1+d)x)}\,dx$

Optimal. Leaf size=267

$$-\frac{\log\left(d^{2/3}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}+\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}\left(\sqrt[3]{d}\,x-b\sqrt[3]{d}\right)+\right.}{2\sqrt[3]{d}\,(a-b)}$$

Rubi [A] time = 0.63, antiderivative size = 239, normalized size of antiderivative = 0.90, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {6719, 91}

$$-\frac{(x-a)^{2/3}(x-b)^{4/3}\log(-ad+b-(1-d)x)}{2\sqrt[3]{d}\,(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{3(x-a)^{2/3}(x-b)^{4/3}\log\left(\frac{\sqrt[3]{x-b}}{\sqrt[3]{d}}-\sqrt[3]{x-a}\right)}{2\sqrt[3]{d}\,(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{\sqrt{3}\,(x-a)^{2/3}(x-b)^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{x-b}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{x-a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{d}\,(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + x)/(((-a + x)*(-b + x)^2)^(2/3)*(b - a*d + (-1 + d)*x)),x]
```

```
[Out] (Sqrt[3]*(-a + x)^(2/3)*(-b + x)^(4/3)*ArcTan[1/Sqrt[3] + (2*(-b + x)^(1/3)
)/(Sqrt[3]*d^(1/3)*(-a + x)^(1/3))]])/((a - b)*d^(1/3)*(-((a - x)*(b - x)^2)
)^(2/3)) - ((-a + x)^(2/3)*(-b + x)^(4/3)*Log[b - a*d - (1 - d)*x])/(2*(a -
 b)*d^(1/3)*(-((a - x)*(b - x)^2))^(2/3)) + (3*(-a + x)^(2/3)*(-b + x)^(4/3
)*Log[-(-a + x)^(1/3) + (-b + x)^(1/3)/d^(1/3)])/(2*(a - b)*d^(1/3)*(-((a -
 x)*(b - x)^2))^(2/3))
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}\int \frac{-b+x}{\left((-a+x)(-b+x)^2\right)^{2/3}(b-ad+(-1+d)x)}\,dx &= \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{1}{(-a+x)^{2/3}\sqrt[3]{-b+x}\,(b-ad+(-1+d)x)}\,dx}{\left((-a+x)(-b+x)^2\right)^{2/3}}\\ &= \frac{\sqrt{3}\,(-a+x)^{2/3}(-b+x)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-b+x}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{-a+x}}\right)}{(a-b)\sqrt[3]{d}\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}-\frac{(-a+}{2}\end{aligned}$$

Mathematica [C] time = 0.06, size = 62, normalized size = 0.23

$$-\frac{3(b-x)^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{b-x}{ad-dx}\right)}{2d(a-b)\left((x-a)(b-x)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + x)/(((-a + x)*(-b + x)^2)^(2/3)*(b - a*d + (-1 + d)*x)),x]
```

```
[Out] (-3*(b - x)^2*Hypergeometric2F1[2/3, 1, 5/3, (b - x)/(a*d - d*x)])/(2*(a -
b)*d*((b - x)^2*(-a + x))^(2/3))
```

IntegrateAlgebraic [A] time = 5.87, size = 267, normalized size = 1.00

$$-\frac{\log \left(d^{2/3} \left(x \left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}+\sqrt[3]{x \left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3} \left(\sqrt[3]{d}\, x-b\sqrt[3]{d}\right)+b^2-2bx+x^2\right)}{2\sqrt[3]{d}\,(a-b)}+\frac{\log \left(\sqrt[3]{d}\sqrt[3]{x \left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}+b-x\right)}{\sqrt[3]{d}\,(a-b)}+\frac{\sqrt{3}\tan ^{-1}\left(\frac{\sqrt{3}b-\sqrt{3}x}{-2\sqrt[3]{d}\sqrt[3]{x(2ab+b^2)-ab^2+x^2(-a-2b)+x^3}+b-x}\right)}{\sqrt[3]{d}\,(a-b)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + x)/(((-a + x)*(-b + x)^2)^(2/3)*(b - a*d + (-1 + d
)*x)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*b - Sqrt[3]*x)/(b - x - 2*d^(1/3)*(-(a*b^2) + (2*a
*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3))])/((a - b)*d^(1/3)) + Log[b - x
+ d^(1/3)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3)]/((a -
b)*d^(1/3)) - Log[b^2 - 2*b*x + x^2 + (-(b*d^(1/3)) + d^(1/3)*x)*(-(a*b^2)
+ (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3) + d^(2/3)*(-(a*b^2) + (2*a*
b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(2/3)]/(2*(a - b)*d^(1/3))
```

fricas [A] time = 0.90, size = 715, normalized size = 2.68

$$\left[-\frac{\sqrt{3} d \sqrt{\frac{(-d)^{\frac{1}{3}}}{d}} \log\left(\frac{2abd+(2d+1)x^2+b^2-2((a+b)d+b)x+\sqrt{3}\left(\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{1}{3}}(b-x)(-d)^{\frac{2}{3}}+2\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{2}{3}}d+(b^2-2bx+x^2)(-d)^{\frac{1}{3}}\right)\sqrt{\frac{(-d)^{\frac{1}{3}}}{d}}-3\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{1}{3}}(b-x)(-d)^{\frac{1}{3}}}{abd+(d-1)x^2-b^2-((a+b)d-2b)x}\right)+(-d)^{\frac{2}{3}}\log\left(-\frac{\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{1}{3}}(b-x)(-d)^{\frac{2}{3}}-\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{2}{3}}d+(b^2-2bx+x^2)(-d)^{\frac{1}{3}}}{b^2-2bx+x^2}\right)-2(-d)^{\frac{2}{3}}\log\left(-\frac{(b-x)(-d)^{\frac{2}{3}}+\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{1}{3}}d}{b-x}\right)}{2(a-b)d}, -\frac{2\sqrt{3} d \sqrt{-\frac{(-d)^{\frac{1}{3}}}{d}} \arctan\left(-\frac{\sqrt{3}\left((b-x)(-d)^{\frac{1}{3}}+2\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{1}{3}}(-d)^{\frac{2}{3}}\right)\sqrt{-\frac{(-d)^{\frac{1}{3}}}{d}}}{3(b-x)}\right)+(-d)^{\frac{2}{3}}\log\left(-\frac{\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{1}{3}}(b-x)(-d)^{\frac{2}{3}}-\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{2}{3}}d+(b^2-2bx+x^2)(-d)^{\frac{1}{3}}}{b^2-2bx+x^2}\right)-2(-d)^{\frac{2}{3}}\log\left(-\frac{(b-x)(-d)^{\frac{2}{3}}+\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{1}{3}}d}{b-x}\right)}{2(a-b)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)/((-a+x)*(-b+x)^2)^(2/3)/(b-a*d+(-1+d)*x),x, algorithm="fri
cas")
```

```
[Out] [-1/2*(sqrt(3)*d*sqrt((-d)^(1/3)/d)*log((2*a*b*d + (2*d + 1)*x^2 + b^2 - 2*
((a + b)*d + b)*x + sqrt(3)*((-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*
x)^(1/3)*(b - x)*(-d)^(2/3) + 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^
2)*x)^(2/3)*d + (b^2 - 2*b*x + x^2)*(-d)^(1/3))*sqrt((-d)^(1/3)/d) - 3*(-a*
b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(b - x)*(-d)^(1/3))/(a*b
*d + (d - 1)*x^2 - b^2 - ((a + b)*d - 2*b)*x)) + (-d)^(2/3)*log(-((-a*b^2 -
 (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(b - x)*(-d)^(2/3) - (-a*b^2
- (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*d + (b^2 - 2*b*x + x^2)*(-d)
^(1/3))/(b^2 - 2*b*x + x^2)) - 2*(-d)^(2/3)*log(-((b - x)*(-d)^(2/3) + (-a*
b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*d)/(b - x)))/((a - b)*d)
, -1/2*(2*sqrt(3)*d*sqrt(-(-d)^(1/3)/d)*arctan(-1/3*sqrt(3)*((b - x)*(-d)^(
1/3) + 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(-d)^(2/3))
*sqrt(-(-d)^(1/3)/d)/(b - x)) + (-d)^(2/3)*log(-((-a*b^2 - (a + 2*b)*x^2 +
x^3 + (2*a*b + b^2)*x)^(1/3)*(b - x)*(-d)^(2/3) - (-a*b^2 - (a + 2*b)*x^2 +
 x^3 + (2*a*b + b^2)*x)^(2/3)*d + (b^2 - 2*b*x + x^2)*(-d)^(1/3))/(b^2 - 2*
b*x + x^2)) - 2*(-d)^(2/3)*log(-((b - x)*(-d)^(2/3) + (-a*b^2 - (a + 2*b)*x
^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*d)/(b - x)))/((a - b)*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b-x}{\left(-(a-x)(b-x)^2\right)^{\frac{2}{3}}\,(ad-(d-1)x-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)/((-a+x)*(-b+x)^2)^(2/3)/(b-a*d+(-1+d)*x),x, algorithm="gia
c")
```

```
[Out] integrate((b - x)/((-(a - x)*(b - x)^2)^(2/3)*(a*d - (d - 1)*x - b)), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{-b+x}{\left((-a+x)(-b+x)^{2}\right)^{\frac{2}{3}}(b-ad+(-1+d)x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b+x)/((-a+x)*(-b+x)^2)^(2/3)/(b-a*d+(-1+d)*x),x)
```

```
[Out] int((-b+x)/((-a+x)*(-b+x)^2)^(2/3)/(b-a*d+(-1+d)*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b-x}{\left(-(a-x)(b-x)^{2}\right)^{\frac{2}{3}}(ad-(d-1)x-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)/((-a+x)*(-b+x)^2)^(2/3)/(b-a*d+(-1+d)*x),x, algorithm="max
ima")
```

```
[Out] integrate((b - x)/((-(a - x)*(b - x)^2)^(2/3)*(a*d - (d - 1)*x - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{b-x}{\left(-(a-x)\,(b-x)^{2}\right)^{2/3}\,(b-a\,d+x\,(d-1))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - x)/((-(a - x)*(b - x)^2)^(2/3)*(b - a*d + x*(d - 1))),x)
```

```
[Out] int(-(b - x)/((-(a - x)*(b - x)^2)^(2/3)*(b - a*d + x*(d - 1))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-b+x}{\left((-a+x)(-b+x)^{2}\right)^{\frac{2}{3}}(-ad+b+dx-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)/((-a+x)*(-b+x)**2)**(2/3)/(b-a*d+(-1+d)*x),x)
```

```
[Out] Integral((-b + x)/(((-a + x)*(-b + x)**2)**(2/3)*(-a*d + b + d*x - x)), x)
```

3.2214

$$\int \frac{(b+ax)^2\left(-2aq+3bpx^2+apx^3\right)}{\sqrt{q+px^3}\left(b^4c+dq^2+4ab^3cx+6a^2b^2cx^2+\left(4a^3bc+2dpq\right)x^3+a^4cx^4+dp^2x^6\right)}\,dx$$

Optimal. Leaf size=267

$$\frac{\tanh^{-1}\left(\frac{\sqrt{px^3+q}\left(\sqrt{2}\,a\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{2}\,b\sqrt[4]{c}\sqrt[4]{d}\right)}{a^2\sqrt{c}\,x^2+2ab\sqrt{c}\,x+b^2\sqrt{c}+\sqrt{d}\,px^3+\sqrt{d}\,q}\right)}{\sqrt{2}\,c^{3/4}\sqrt[4]{d}}+\frac{\tan^{-1}\left(\frac{a\sqrt[4]{c}\,x+b\sqrt[4]{c}}{a\sqrt[4]{c}\,x+b\sqrt[4]{c}-\sqrt{2}\sqrt[4]{d}\sqrt{px^3+q}}\right)}{\sqrt{2}\,c^{3/4}\sqrt[4]{d}}-\frac{\tan^{-1}\left(\frac{a\sqrt[4]{c}\,x+b\sqrt[4]{c}}{a\sqrt[4]{c}\,x+b\sqrt[4]{c}+\sqrt{2}\sqrt[4]{d}\sqrt{px^3+q}}\right)}{\sqrt{2}\,c^{3/4}\sqrt[4]{d}}$$

Rubi [F] time = 13.39, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(b+ax)^2\left(-2aq+3bpx^2+apx^3\right)}{\sqrt{q+px^3}\left(b^4c+dq^2+4ab^3cx+6a^2b^2cx^2+\left(4a^3bc+2dpq\right)x^3+a^4cx^4+dp^2x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((b + a*x)^2*(-2*a*q + 3*b*p*x^2 + a*p*x^3))/(Sqrt[q + p*x^3]*(b^4*c +
d*q^2 + 4*a*b^3*c*x + 6*a^2*b^2*c*x^2 + (4*a^3*b*c + 2*d*p*q)*x^3 + a^4*c*x
^4 + d*p^2*x^6)),x]
```

```
[Out] 2*a*b^2*q*Defer[Int][1/(Sqrt[q + p*x^3]*(-(b^4*c) - 4*a*b^3*c*x - 6*a^2*b^2
*c*x^2 - 4*a^3*b*c*x^3 - a^4*c*x^4 - d*(q + p*x^3)^2)), x] + 4*a^2*b*q*Defe
r[Int][x/(Sqrt[q + p*x^3]*(-(b^4*c) - 4*a*b^3*c*x - 6*a^2*b^2*c*x^2 - 4*a^3
*b*c*x^3 - a^4*c*x^4 - d*(q + p*x^3)^2)), x] - (3*b^3*p - 2*a^3*q)*Defer[In
t][x^2/(Sqrt[q + p*x^3]*(-(b^4*c) - 4*a*b^3*c*x - 6*a^2*b^2*c*x^2 - 4*a^3*b
*c*x^3 - a^4*c*x^4 - d*(q + p*x^3)^2)), x] + 7*a*b^2*p*Defer[Int][x^3/(Sqrt
[q + p*x^3]*(b^4*c + 4*a*b^3*c*x + 6*a^2*b^2*c*x^2 + 4*a^3*b*c*x^3 + a^4*c*
x^4 + d*(q + p*x^3)^2)), x] + 5*a^2*b*p*Defer[Int][x^4/(Sqrt[q + p*x^3]*(b^
4*c + 4*a*b^3*c*x + 6*a^2*b^2*c*x^2 + 4*a^3*b*c*x^3 + a^4*c*x^4 + d*(q + p*
x^3)^2)), x] + a^3*p*Defer[Int][x^5/(Sqrt[q + p*x^3]*(b^4*c + 4*a*b^3*c*x +
 6*a^2*b^2*c*x^2 + 4*a^3*b*c*x^3 + a^4*c*x^4 + d*(q + p*x^3)^2)), x]
```

Rubi steps

$$\int \frac{(b+ax)^2\left(-2aq+3bpx^2+apx^3\right)}{\sqrt{q+px^3}\left(b^4c+dq^2+4ab^3cx+6a^2b^2cx^2+\left(4a^3bc+2dpq\right)x^3+a^4cx^4+dp^2x^6\right)}\,dx = \int \frac{}{\sqrt{q+px^3}\left(b^4c\left(1\right.\right.}$$

$$= \int \left(\frac{}{\sqrt{q+px^3}\left(-b^4c\right.} \right.$$

$$= \left(a^3p\right)\int \frac{}{\sqrt{q+px^3}\left(\right.}$$

$$= \left(a^3p\right)\int \frac{}{\sqrt{q+px^3}\left(\right.}$$

Mathematica [C] time = 7.40, size = 52549, normalized size = 196.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((b + a*x)^2*(-2*a*q + 3*b*p*x^2 + a*p*x^3))/(Sqrt[q + p*x^3]*(b^
4*c + d*q^2 + 4*a*b^3*c*x + 6*a^2*b^2*c*x^2 + (4*a^3*b*c + 2*d*p*q)*x^3 + a
^4*c*x^4 + d*p^2*x^6)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((b + a*x)^2*(-2*a*q + 3*b*p*x^2 + a*p*x^3))/(Sqrt[q + p
*x^3]*(b^4*c + d*q^2 + 4*a*b^3*c*x + 6*a^2*b^2*c*x^2 + (4*a^3*b*c + 2*d*p*q
)*x^3 + a^4*c*x^4 + d*p^2*x^6)),x]
```

```
[Out] $Aborted
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^2*(a*p*x^3+3*b*p*x^2-2*a*q)/(p*x^3+q)^(1/2)/(b^4*c+d*q^2+
4*a*b^3*c*x+6*a^2*b^2*c*x^2+(4*a^3*b*c+2*d*p*q)*x^3+a^4*c*x^4+d*p^2*x^6),x,
 algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^2*(a*p*x^3+3*b*p*x^2-2*a*q)/(p*x^3+q)^(1/2)/(b^4*c+d*q^2+
4*a*b^3*c*x+6*a^2*b^2*c*x^2+(4*a^3*b*c+2*d*p*q)*x^3+a^4*c*x^4+d*p^2*x^6),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 2.39, size = 7274, normalized size = 27.24

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)^2*(a*p*x^3+3*b*p*x^2-2*a*q)/(p*x^3+q)^(1/2)/(b^4*c+d*q^2+4*a*b^
3*c*x+6*a^2*b^2*c*x^2+(4*a^3*b*c+2*d*p*q)*x^3+a^4*c*x^4+d*p^2*x^6),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(apx^{3} + 3\,bpx^{2} - 2\,aq\right)(ax + b)^{2}}{\left(a^{4}cx^{4} + dp^{2}x^{6} + 6\,a^{2}b^{2}cx^{2} + 4\,ab^{3}cx + b^{4}c + 2\left(2\,a^{3}bc + dpq\right)x^{3} + dq^{2}\right)\sqrt{px^{3} + q}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^2*(a*p*x^3+3*b*p*x^2-2*a*q)/(p*x^3+q)^(1/2)/(b^4*c+d*q^2+
4*a*b^3*c*x+6*a^2*b^2*c*x^2+(4*a^3*b*c+2*d*p*q)*x^3+a^4*c*x^4+d*p^2*x^6),x,
 algorithm="maxima")
```

```
[Out] integrate((a*p*x^3 + 3*b*p*x^2 - 2*a*q)*(a*x + b)^2/((a^4*c*x^4 + d*p^2*x^6
 + 6*a^2*b^2*c*x^2 + 4*a*b^3*c*x + b^4*c + 2*(2*a^3*b*c + d*p*q)*x^3 + d*q^
2)*sqrt(p*x^3 + q)), x)
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a*x)^2*(a*p*x^3 - 2*a*q + 3*b*p*x^2))/((q + p*x^3)^(1/2)*(x^3*(2*
d*p*q + 4*a^3*b*c) + b^4*c + d*q^2 + a^4*c*x^4 + d*p^2*x^6 + 6*a^2*b^2*c*x^
2 + 4*a*b^3*c*x)),x)
```

```
[Out] \text{Hanged}
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+b)^2 \left(apx^3-2aq+3bpx^2\right)}{\sqrt{px^3+q} \left(a^4cx^4+4a^3bcx^3+6a^2b^2cx^2+4ab^3cx+b^4c+dp^2x^6+2dpqx^3+dq^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)**2*(a*p*x**3+3*b*p*x**2-2*a*q)/(p*x**3+q)**(1/2)/(b**4*c+
d*q**2+4*a*b**3*c*x+6*a**2*b**2*c*x**2+(4*a**3*b*c+2*d*p*q)*x**3+a**4*c*x**
4+d*p**2*x**6),x)
```

```
[Out] Integral((a*x + b)**2*(a*p*x**3 - 2*a*q + 3*b*p*x**2)/(sqrt(p*x**3 + q)*(a*
*4*c*x**4 + 4*a**3*b*c*x**3 + 6*a**2*b**2*c*x**2 + 4*a*b**3*c*x + b**4*c +
d*p**2*x**6 + 2*d*p*q*x**3 + d*q**2)), x)
```

3.2215 $\int \frac{-1+k^4x^4}{\sqrt{(1-x)x(1-k^2x)}\,(a+bx^2+ak^4x^4)}\,dx$

Optimal. Leaf size=268

$$-\frac{\tan^{-1}\left(\frac{\sqrt{k^2x^3+(-k^2-1)x^2+x}\,\sqrt{-\sqrt{a}\,\sqrt{2ak^2-b}+ak^2+a}}{\sqrt{a}\,(x-1)(k^2x-1)}\right)}{\sqrt{a}\,\sqrt{-\sqrt{a}\,\sqrt{2ak^2-b}+ak^2+a}}-\frac{\sqrt{\sqrt{a}\left(\sqrt{2ak^2-b}+\sqrt{a}\,k^2+\sqrt{a}\right)}\tan^{-1}\left(\frac{\sqrt{k^2x^3+(-k^2-1)x^2+x}\,\sqrt{\sqrt{a}\,\sqrt{}}}{\sqrt{a}\,(x-1)(k^2x-1}\right.}{a\left(\sqrt{2ak^2-b}+\sqrt{a}\,k^2+\sqrt{a}\right)}$$

Rubi [C] time = 5.96, antiderivative size = 583, normalized size of antiderivative = 2.18, number of steps used = 28, number of rules used = 10, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6718, 6688, 6728, 714, 115, 6725, 934, 12, 168, 537}

$$\frac{(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,\Pi\left(-\frac{\sqrt{2}\,\sqrt{a}}{\sqrt{-b-\sqrt{b^2-4a^2k^4}}};\sin^{-1}\left(\sqrt{-k^2}\,\sqrt{-x}\right)|\frac{1}{k^2}\right)}{a\sqrt{-k^2}\,\sqrt{x-x^2}\,\sqrt{(1-x)x(1-k^2x)}}+\frac{(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,\Pi\left(\frac{\sqrt{2}\,\sqrt{a}}{\sqrt{-b-\sqrt{b^2-4a^2k^4}}};\sin^{-1}\left(\sqrt{-k^2}\,\sqrt{-x}\right)|\frac{1}{k^2}\right)}{a\sqrt{-k^2}\,\sqrt{x-x^2}\,\sqrt{(1-x)x(1-k^2x)}}+\frac{(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,\Pi\left(-\frac{\sqrt{2}\,\sqrt{a}}{\sqrt{\sqrt{b^2-4a^2k^4}-b}};\sin^{-1}\left(\sqrt{-k^2}\,\sqrt{-x}\right)|\frac{1}{k^2}\right)}{a\sqrt{-k^2}\,\sqrt{x-x^2}\,\sqrt{(1-x)x(1-k^2x)}}+\frac{(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,\Pi\left(\frac{\sqrt{2}\,\sqrt{a}}{\sqrt{\sqrt{b^2-4a^2k^4}-b}};\sin^{-1}\left(\sqrt{-k^2}\,\sqrt{-x}\right)|\frac{1}{k^2}\right)}{a\sqrt{-k^2}\,\sqrt{x-x^2}\,\sqrt{(1-x)x(1-k^2x)}}+\frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{a\sqrt{(1-x)x(1-k^2x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + k^4*x^4)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(a + b*x^2 + a*k^4*x^4)),x]
```

```
[Out] (2*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticF[ArcSin[Sqrt[x]], k^2])/(a*
Sqrt[(1 - x)*x*(1 - k^2*x)]) + ((1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^2*x]*El
lipticPi[-((Sqrt[2]*Sqrt[a])/Sqrt[-b - Sqrt[b^2 - 4*a^2*k^4]]), ArcSin[Sqrt
[-k^2]*Sqrt[-x]], k^(-2)])/(a*Sqrt[-k^2]*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x
 - x^2]) + ((1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticPi[(Sqrt[2]*Sq
rt[a])/Sqrt[-b - Sqrt[b^2 - 4*a^2*k^4]], ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2
)])/(a*Sqrt[-k^2]*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2]) + ((1 - x)*Sqr
t[-x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticPi[-((Sqrt[2]*Sqrt[a])/Sqrt[-b + Sqrt
[b^2 - 4*a^2*k^4]]), ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/(a*Sqrt[-k^2]*Sq
rt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2]) + ((1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1
 - k^2*x]*EllipticPi[(Sqrt[2]*Sqrt[a])/Sqrt[-b + Sqrt[b^2 - 4*a^2*k^4]], Ar
cSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/(a*Sqrt[-k^2]*Sqrt[(1 - x)*x*(1 - k^2*x
)]*Sqrt[x - x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 714

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c*x)/b]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0]
 && RationalQ[b]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-1+k^4x^4}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(a+bx^2+ak^4x^4\right)}\,dx = \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{-1+k^4x^4}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\left(a+bx^2+ak^4x^4\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{-1+k^4x^4}{\sqrt{1-k^2x}\,\sqrt{x-x^2}\left(a+bx^2+ak^4x^4\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \left(\frac{1}{a\sqrt{1-k^2x}\,\sqrt{x-x^2}} - \frac{2a+bx^2}{a\sqrt{1-k^2x}\,\sqrt{x-x^2}\left(a+bx^2+ak^4x \cdots\right.}\cdots\right.}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-k^2x}\,\sqrt{x-x^2}}\,dx}{a\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\cdots\right.}{a\sqrt{(1\cdots}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}}\,dx}{a\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2\cdots}\right.}{\cdots}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{a\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\left(b-\sqrt{b^2-4a^2k^4}\right)\sqrt{\cdots}\right.}{\cdots}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{a\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\left(b-\sqrt{b^2-4a^2k^4}\right)\sqrt{\cdots}\right.}{\cdots}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{a\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(\left(b-\sqrt{b^2-4a^2k^4}\right)\sqrt{\cdots}\right.}{2\cdots}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{a\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(\left(b-\sqrt{b^2-4a^2k^4}\right)\sqrt{\cdots}\right.}{\cdots}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{a\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(\left(b-\sqrt{b^2-4a^2k^4}\right)\sqrt{\cdots}\right.}{\cdots}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{a\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\left(b-\sqrt{b^2-4a^2k^4}\right)\sqrt{\cdots}\right.}{\cdots}$$

$$= \frac{2\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{a\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{(1-x)\sqrt{-x}\,\sqrt{x}\,\sqrt{1-k\cdots}}{a\sqrt{-k\cdots}}$$

Mathematica [C] time = 18.67, size = 13957, normalized size = 52.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + k^4*x^4)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(a + b*x^2 + a*k^4*x^
4)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 1.09, size = 268, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{k^2x^3+(-k^2-1)x^2+x}\sqrt{-\sqrt{a}\sqrt{2ak^2-b}+ak^2+a}}{\sqrt{a}(x-1)(k^2x-1)}\right)}{\sqrt{a}\sqrt{-\sqrt{a}\sqrt{2ak^2-b}+ak^2+a}}-\frac{\sqrt{\sqrt{a}\left(\sqrt{2ak^2-b}+\sqrt{a}k^2+\sqrt{a}\right)}\tan^{-1}\left(\frac{\sqrt{k^2x^3+(-k^2-1)x^2+x}\sqrt{\sqrt{a}\sqrt{2ak^2-b}+ak^2+a}}{\sqrt{a}(x-1)(k^2x-1)}\right)}{a\left(\sqrt{2ak^2-b}+\sqrt{a}k^2+\sqrt{a}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + k^4*x^4)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(a + b*x^2 +
 a*k^4*x^4)),x]
```

```
[Out] -(ArcTan[(Sqrt[a + a*k^2 - Sqrt[a]*Sqrt[-b + 2*a*k^2]]*Sqrt[x + (-1 - k^2)*
x^2 + k^2*x^3])/(Sqrt[a]*(-1 + x)*(-1 + k^2*x))]]/(Sqrt[a]*Sqrt[a + a*k^2 -
Sqrt[a]*Sqrt[-b + 2*a*k^2]])) - (Sqrt[Sqrt[a]*(Sqrt[a] + Sqrt[a]*k^2 + Sqrt
[-b + 2*a*k^2])]*ArcTan[(Sqrt[a + a*k^2 + Sqrt[a]*Sqrt[-b + 2*a*k^2]]*Sqrt[
x + (-1 - k^2)*x^2 + k^2*x^3])/(Sqrt[a]*(-1 + x)*(-1 + k^2*x))]])/(a*(Sqrt[a
] + Sqrt[a]*k^2 + Sqrt[-b + 2*a*k^2]))
```

fricas [B] time = 7.00, size = 2407, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^4*x^4-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(a*k^4*x^4+b*x^2+a),x, alg
orithm="fricas")
```

```
[Out] 1/4*sqrt(-(k^2 + (a^2*k^4 + a^2 + a*b)*sqrt((2*a*k^2 - b)/(a^5*k^8 + a^5 +
2*a^4*b + a^3*b^2 + 2*(a^5 + a^4*b)*k^4)) + 1)/(a^2*k^4 + a^2 + a*b))*log((
a*k^4*x^4 - 2*(a*k^4 + a*k^2)*x^3 + (4*a*k^2 - b)*x^2 - 2*(a*k^2 + a)*x + 2
*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(a^2*k^2 + (a^2*k^4 + a^2*k^2)*x^2 + a^2
 - (2*a^2*k^2 - a*b)*x - (a^4*k^4 + a^4 + a^3*b + (a^4*k^6 + (a^4 + a^3*b)*
k^2)*x^2 - (a^4*k^6 + a^4*k^4 + a^4 + a^3*b + (a^4 + a^3*b)*k^2)*x)*sqrt((2
*a*k^2 - b)/(a^5*k^8 + a^5 + 2*a^4*b + a^3*b^2 + 2*(a^5 + a^4*b)*k^4)))*sqr
t(-(k^2 + (a^2*k^4 + a^2 + a*b)*sqrt((2*a*k^2 - b)/(a^5*k^8 + a^5 + 2*a^4*b
 + a^3*b^2 + 2*(a^5 + a^4*b)*k^4)) + 1)/(a^2*k^4 + a^2 + a*b)) + 2*((a^3*k^
6 + (a^3 + a^2*b)*k^2)*x^3 - (a^3*k^6 + a^3*k^4 + a^3 + a^2*b + (a^3 + a^2*
b)*k^2)*x^2 + (a^3*k^4 + a^3 + a^2*b)*x)*sqrt((2*a*k^2 - b)/(a^5*k^8 + a^5
+ 2*a^4*b + a^3*b^2 + 2*(a^5 + a^4*b)*k^4)) + a)/(a*k^4*x^4 + b*x^2 + a)) -
 1/4*sqrt(-(k^2 + (a^2*k^4 + a^2 + a*b)*sqrt((2*a*k^2 - b)/(a^5*k^8 + a^5 +
 2*a^4*b + a^3*b^2 + 2*(a^5 + a^4*b)*k^4)) + 1)/(a^2*k^4 + a^2 + a*b))*log(
(a*k^4*x^4 - 2*(a*k^4 + a*k^2)*x^3 + (4*a*k^2 - b)*x^2 - 2*(a*k^2 + a)*x -
2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(a^2*k^2 + (a^2*k^4 + a^2*k^2)*x^2 + a^
2 - (2*a^2*k^2 - a*b)*x - (a^4*k^4 + a^4 + a^3*b + (a^4*k^6 + (a^4 + a^3*b)
*k^2)*x^2 - (a^4*k^6 + a^4*k^4 + a^4 + a^3*b + (a^4 + a^3*b)*k^2)*x)*sqrt((
2*a*k^2 - b)/(a^5*k^8 + a^5 + 2*a^4*b + a^3*b^2 + 2*(a^5 + a^4*b)*k^4)))*sq
rt(-(k^2 + (a^2*k^4 + a^2 + a*b)*sqrt((2*a*k^2 - b)/(a^5*k^8 + a^5 + 2*a^4*
b + a^3*b^2 + 2*(a^5 + a^4*b)*k^4)) + 1)/(a^2*k^4 + a^2 + a*b)) + 2*((a^3*k
^6 + (a^3 + a^2*b)*k^2)*x^3 - (a^3*k^6 + a^3*k^4 + a^3 + a^2*b + (a^3 + a^2
*b)*k^2)*x^2 + (a^3*k^4 + a^3 + a^2*b)*x)*sqrt((2*a*k^2 - b)/(a^5*k^8 + a^5
 + 2*a^4*b + a^3*b^2 + 2*(a^5 + a^4*b)*k^4)) + a)/(a*k^4*x^4 + b*x^2 + a))
```

```
+ 1/4*sqrt(-(k^2 - (a^2*k^4 + a^2 + a*b)*sqrt((2*a*k^2 - b)/(a^5*k^8 + a^5
+ 2*a^4*b + a^3*b^2 + 2*(a^5 + a^4*b)*k^4)) + 1)/(a^2*k^4 + a^2 + a*b))*log
((a*k^4*x^4 - 2*(a*k^4 + a*k^2)*x^3 + (4*a*k^2 - b)*x^2 - 2*(a*k^2 + a)*x +
 2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(a^2*k^2 + (a^2*k^4 + a^2*k^2)*x^2 + a
^2 - (2*a^2*k^2 - a*b)*x + (a^4*k^4 + a^4 + a^3*b + (a^4*k^6 + (a^4 + a^3*b
)*k^2)*x^2 - (a^4*k^6 + a^4*k^4 + a^4 + a^3*b + (a^4 + a^3*b)*k^2)*x)*sqrt(
(2*a*k^2 - b)/(a^5*k^8 + a^5 + 2*a^4*b + a^3*b^2 + 2*(a^5 + a^4*b)*k^4)))*s
qrt(-(k^2 - (a^2*k^4 + a^2 + a*b)*sqrt((2*a*k^2 - b)/(a^5*k^8 + a^5 + 2*a^4
*b + a^3*b^2 + 2*(a^5 + a^4*b)*k^4)) + 1)/(a^2*k^4 + a^2 + a*b)) - 2*((a^3*
k^6 + (a^3 + a^2*b)*k^2)*x^3 - (a^3*k^6 + a^3*k^4 + a^3 + a^2*b + (a^3 + a^
2*b)*k^2)*x^2 + (a^3*k^4 + a^3 + a^2*b)*x)*sqrt((2*a*k^2 - b)/(a^5*k^8 + a^
5 + 2*a^4*b + a^3*b^2 + 2*(a^5 + a^4*b)*k^4)) + a)/(a*k^4*x^4 + b*x^2 + a))
 - 1/4*sqrt(-(k^2 - (a^2*k^4 + a^2 + a*b)*sqrt((2*a*k^2 - b)/(a^5*k^8 + a^5
 + 2*a^4*b + a^3*b^2 + 2*(a^5 + a^4*b)*k^4)) + 1)/(a^2*k^4 + a^2 + a*b))*lo
g((a*k^4*x^4 - 2*(a*k^4 + a*k^2)*x^3 + (4*a*k^2 - b)*x^2 - 2*(a*k^2 + a)*x
- 2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(a^2*k^2 + (a^2*k^4 + a^2*k^2)*x^2 +
a^2 - (2*a^2*k^2 - a*b)*x + (a^4*k^4 + a^4 + a^3*b + (a^4*k^6 + (a^4 + a^3*
b)*k^2)*x^2 - (a^4*k^6 + a^4*k^4 + a^4 + a^3*b + (a^4 + a^3*b)*k^2)*x)*sqrt
((2*a*k^2 - b)/(a^5*k^8 + a^5 + 2*a^4*b + a^3*b^2 + 2*(a^5 + a^4*b)*k^4)))*
sqrt(-(k^2 - (a^2*k^4 + a^2 + a*b)*sqrt((2*a*k^2 - b)/(a^5*k^8 + a^5 + 2*a^
4*b + a^3*b^2 + 2*(a^5 + a^4*b)*k^4)) + 1)/(a^2*k^4 + a^2 + a*b)) - 2*((a^3
*k^6 + (a^3 + a^2*b)*k^2)*x^3 - (a^3*k^6 + a^3*k^4 + a^3 + a^2*b + (a^3 + a
^2*b)*k^2)*x^2 + (a^3*k^4 + a^3 + a^2*b)*x)*sqrt((2*a*k^2 - b)/(a^5*k^8 + a
^5 + 2*a^4*b + a^3*b^2 + 2*(a^5 + a^4*b)*k^4)) + a)/(a*k^4*x^4 + b*x^2 + a)
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^4x^4-1}{\left(ak^4x^4+bx^2+a\right)\sqrt{\left(k^2x-1\right)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^4*x^4-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(a*k^4*x^4+b*x^2+a),x, alg
orithm="giac")
```

```
[Out] integrate((k^4*x^4 - 1)/((a*k^4*x^4 + b*x^2 + a)*sqrt((k^2*x - 1)*(x - 1)*x
)), x)
```

maple [C] time = 0.07, size = 314, normalized size = 1.17

$$-\frac{2\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\mathrm{EllipticF}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{a\,k^2\sqrt{k^2x^3-k^2x^2-x^2+x}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(a\,k^4\_Z^4+b\_Z^2+a\right)}\frac{\left(-\_\alpha^2b-2a\right)\left(a\,k^6\_\alpha^3+a\,k^4\_\alpha^2+k^2\_\alpha a+k^2\_\alpha b+a+b\right)\sqrt{-\left(x-\frac{1}{k^2}\right)k^2}\,\sqrt{\frac{-1+x}{\frac{1}{k^2}-1}}\,\sqrt{k^2x}\,\mathrm{EllipticPi}\left(\sqrt{-\left(x-\frac{1}{k^2}\right)k^2},\frac{a\,k^6\_\alpha^3+a\,k^4\_\alpha^2+k^2\_\alpha a+k^2\_\alpha b+a+b}{a\,k^4+a+b},\sqrt{\frac{1}{k^2\left(\frac{1}{k^2}-1\right)}}\right)}{\_\alpha\left(2a\,k^4\_\alpha^2+b\right)\left(a\,k^4+a+b\right)\sqrt{x\left(k^2x^2-k^2x-x+1\right)}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^4*x^4-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(a*k^4*x^4+b*x^2+a),x)
```

```
[Out] -2/a/k^2*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2
*x^3-k^2*x^2-x^2+x)^(1/2)*EllipticF((-(x-1/k^2)*k^2)^(1/2),(1/k^2/(1/k^2-1)
)^(1/2))-1/a*sum((-_alpha^2*b-2*a)/_alpha/(2*_alpha^2*a*k^4+b)*(_alpha^3*a*
k^6+_alpha^2*a*k^4+_alpha*a*k^2+_alpha*b*k^2+a+b)/(a*k^4+a+b)*(-(x-1/k^2)*k
^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(x*(k^2*x^2-k^2*x-x+1))^(1
/2)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),(_alpha^3*a*k^6+_alpha^2*a*k^4+_alpha
*a*k^2+_alpha*b*k^2+a+b)/(a*k^4+a+b),(1/k^2/(1/k^2-1))^(1/2)),_alpha=RootOf
(_Z^4*a*k^4+_Z^2*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^4x^4-1}{\left(ak^4x^4+bx^2+a\right)\sqrt{\left(k^2x-1\right)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^4*x^4-1)/((1-x)*x*(-k^2*x+1))^(1/2)/(a*k^4*x^4+b*x^2+a),x, alg
orithm="maxima")
```

```
[Out] integrate((k^4*x^4 - 1)/((a*k^4*x^4 + b*x^2 + a)*sqrt((k^2*x - 1)*(x - 1)*x
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{k^4 x^4 - 1}{\sqrt{x\left(k^2 x - 1\right)(x - 1)}\left(a k^4 x^4 + b x^2 + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^4*x^4 - 1)/((x*(k^2*x - 1)*(x - 1))^(1/2)*(a + b*x^2 + a*k^4*x^4)),x
)
```

```
[Out] int((k^4*x^4 - 1)/((x*(k^2*x - 1)*(x - 1))^(1/2)*(a + b*x^2 + a*k^4*x^4)),
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k**4*x**4-1)/((1-x)*x*(-k**2*x+1))**(1/2)/(a*k**4*x**4+b*x**2+a)
,x)
```

```
[Out] Timed out
```

3.2216 $\int \frac{x^5(-4a+3x)}{\sqrt[3]{x^2(-a+x)}\left(-a^2+2ax-x^2+dx^8\right)}\,dx$

Optimal. Leaf size=268

$$-\frac{\log\left(a^2\sqrt[3]{d}\,x^4-a^2\sqrt[6]{d}\,x^2\sqrt[3]{x^3-ax^2}+a^2\left(x^3-ax^2\right)^{2/3}\right)}{4d^{2/3}}-\frac{\log\left(a^2\sqrt[3]{d}\,x^4+a^2\sqrt[6]{d}\,x^2\sqrt[3]{x^3-ax^2}+a^2\left(x^3-ax^2\right)^{2/3}\right)}{4d^{2/3}}+$$

Rubi [F] time = 2.97, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^5(-4a+3x)}{\sqrt[3]{x^2(-a+x)}\left(-a^2+2ax-x^2+dx^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^5*(-4*a + 3*x))/((x^2*(-a + x))^(1/3)*(-a^2 + 2*a*x - x^2 + d*x^8)),
x]
```

```
[Out] (12*a*x^(2/3)*(-a + x)^(1/3)*Defer[Subst][Defer[Int][x^15/((-a + x^3)^(1/3)
*(a^2 - 2*a*x^3 + x^6 - d*x^24)), x], x, x^(1/3)])/(-((a - x)*x^2))^(1/3) +
 (9*x^(2/3)*(-a + x)^(1/3)*Defer[Subst][Defer[Int][x^18/((-a + x^3)^(1/3)*(
-a^2 + 2*a*x^3 - x^6 + d*x^24)), x], x, x^(1/3)])/(-((a - x)*x^2))^(1/3)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5(-4a+3x)}{\sqrt[3]{x^2(-a+x)}\left(-a^2+2ax-x^2+dx^8\right)}\,dx &= \frac{\left(x^{2/3}\sqrt[3]{-a+x}\right)\int \frac{x^{13/3}(-4a+3x)}{\sqrt[3]{-a+x}\left(-a^2+2ax-x^2+dx^8\right)}\,dx}{\sqrt[3]{x^2(-a+x)}}\\
&= \frac{\left(3x^{2/3}\sqrt[3]{-a+x}\right)\text{Subst}\left(\int \frac{x^{15}\left(-4a+3x^3\right)}{\sqrt[3]{-a+x^3}\left(-a^2+2ax^3-x^6+dx^{24}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x^2(-a+x)}}\\
&= \frac{\left(3x^{2/3}\sqrt[3]{-a+x}\right)\text{Subst}\left(\int\left(\frac{4ax^{15}}{\sqrt[3]{-a+x^3}\left(a^2-2ax^3+x^6-dx^{24}\right)}+\frac{3x^{18}}{\sqrt[3]{-a+x^3}\left(-a^2+2a\right.}\right.\right.}{\sqrt[3]{x^2(-a+x)}}\\
&= \frac{\left(9x^{2/3}\sqrt[3]{-a+x}\right)\text{Subst}\left(\int \frac{x^{18}}{\sqrt[3]{-a+x^3}\left(-a^2+2ax^3-x^6+dx^{24}\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x^2(-a+x)}}+\frac{(1}{}
\end{aligned}$$

Mathematica [C] time = 1.88, size = 738, normalized size = 2.75

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(-4*a + 3*x))/((x^2*(-a + x))^(1/3)*(-a^2 + 2*a*x - x^2 + d*
x^8)),x]
```

```
[Out] -1/4*(a^4*x*(4*RootSum[-1 + 6*#1 - 15*#1^2 + 20*#1^3 - 15*#1^4 + 6*#1^5 - #
1^6 + a^6*d*#1^8 & , (-6*(x/(-a + x))^(1/3)*#1^4 + 2*Sqrt[3]*ArcTan[(1 + (2
*(x/(-a + x))^(1/3))/#1^(1/3))/Sqrt[3]]*#1^(13/3) - 2*Log[-(x/(-a + x))^(1/
```

```
3) + #1^(1/3)]*#1^(13/3) + Log[(x/(-a + x))^(2/3) + (x/(-a + x))^(1/3)*#1^(
1/3) + #1^(2/3)]*#1^(13/3))/(3 - 15*#1 + 30*#1^2 - 30*#1^3 + 15*#1^4 - 3*#1
^5 + 4*a^6*d*#1^7) & ] - 5*RootSum[-1 + 6*#1 - 15*#1^2 + 20*#1^3 - 15*#1^4
+ 6*#1^5 - #1^6 + a^6*d*#1^8 & , (-6*(x/(-a + x))^(1/3)*#1^5 + 2*Sqrt[3]*Ar
cTan[(1 + (2*(x/(-a + x))^(1/3))/#1^(1/3))/Sqrt[3]]*#1^(16/3) - 2*Log[-(x/(
-a + x))^(1/3) + #1^(1/3)]*#1^(16/3) + Log[(x/(-a + x))^(2/3) + (x/(-a + x)
)^(1/3)*#1^(1/3) + #1^(2/3)]*#1^(16/3))/(3 - 15*#1 + 30*#1^2 - 30*#1^3 + 15
*#1^4 - 3*#1^5 + 4*a^6*d*#1^7) & ] + RootSum[-1 + 6*#1 - 15*#1^2 + 20*#1^3
- 15*#1^4 + 6*#1^5 - #1^6 + a^6*d*#1^8 & , (-6*(x/(-a + x))^(1/3)*#1^6 + 2*
Sqrt[3]*ArcTan[(1 + (2*(x/(-a + x))^(1/3))/#1^(1/3))/Sqrt[3]]*#1^(19/3) - 2
*Log[-(x/(-a + x))^(1/3) + #1^(1/3)]*#1^(19/3) + Log[(x/(-a + x))^(2/3) + (
x/(-a + x))^(1/3)*#1^(1/3) + #1^(2/3)]*#1^(19/3))/(3 - 15*#1 + 30*#1^2 - 30
*#1^3 + 15*#1^4 - 3*#1^5 + 4*a^6*d*#1^7) & ]))/((x/(-a + x))^(1/3)*(x^2*(-a
 + x))^(1/3))
```

IntegrateAlgebraic [F] time = 180.00, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(-4*a + 3*x))/((x^2*(-a + x))^(1/3)*(-a^2 + 2*a*x -
 x^2 + d*x^8)),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.53, size = 167, normalized size = 0.62

$$-\frac{2\,\sqrt{3}\left(d^{2}\right)^{\frac{1}{6}} d \arctan\left(\frac{\sqrt{3}\left(\left(d^{2}\right)^{\frac{1}{3}} d x^{4}+2\left(-a x^{2}+x^{3}\right)^{\frac{2}{3}}\left(d^{2}\right)^{\frac{2}{3}}\right)\left(d^{2}\right)^{\frac{1}{6}}}{3\,d^{2} x^{4}}\right)-2\left(d^{2}\right)^{\frac{2}{3}} \log\left(-\frac{\left(d^{2}\right)^{\frac{2}{3}} x^{4}-\left(-a x^{2}+x^{3}\right)^{\frac{2}{3}} d}{x^{4}}\right)+\left(d^{2}\right)^{\frac{2}{3}} \log\left(\frac{\left(d^{2}\right)^{\frac{1}{3}} d x^{6}+\left(-a x^{2}+x^{3}\right)^{\frac{2}{3}}\left(d^{2}\right)^{\frac{2}{3}} x^{2}-\left(-a x^{2}+x^{3}\right)^{\frac{1}{3}}\left(a d-d x\right)}{x^{6}}\right)}{4\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*a+3*x)/(x^2*(-a+x))^(1/3)/(d*x^8-a^2+2*a*x-x^2),x, algori
thm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(1/3*sqrt(3)*((d^2)^(1/3)*d*x^4 + 2*(-a
*x^2 + x^3)^(2/3)*(d^2)^(2/3))*(d^2)^(1/6)/(d^2*x^4)) - 2*(d^2)^(2/3)*log(-
((d^2)^(2/3)*x^4 - (-a*x^2 + x^3)^(2/3)*d)/x^4) + (d^2)^(2/3)*log(((d^2)^(1
/3)*d*x^6 + (-a*x^2 + x^3)^(2/3)*(d^2)^(2/3)*x^2 - (-a*x^2 + x^3)^(1/3)*(a*
d - d*x))/x^6))/d^2
```

giac [A] time = 1.14, size = 1, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*a+3*x)/(x^2*(-a+x))^(1/3)/(d*x^8-a^2+2*a*x-x^2),x, algori
thm="giac")
```

```
[Out] 0
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \left(-4 a +3 x \right)}{\left(x^{2} \left(-a +x \right)\right)^{\frac{1}{3}} \left(d \,x^{8}-a^{2}+2 a x -x^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-4*a+3*x)/(x^2*(-a+x))^(1/3)/(d*x^8-a^2+2*a*x-x^2),x)
```

```
[Out] int(x^5*(-4*a+3*x)/(x^2*(-a+x))^(1/3)/(d*x^8-a^2+2*a*x-x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(4\,a-3\,x)x^5}{\left(dx^8-a^2+2\,ax-x^2\right)\left(-(a-x)x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-4*a+3*x)/(x^2*(-a+x))^(1/3)/(d*x^8-a^2+2*a*x-x^2),x, algori
thm="maxima")
```

```
[Out] -integrate((4*a - 3*x)*x^5/((d*x^8 - a^2 + 2*a*x - x^2)*(-(a - x)*x^2)^(1/3
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x^5\,\left(4\,a-3\,x\right)}{{\left(-x^2\,\left(a-x\right)\right)}^{1/3}\,\left(-a^2+2\,a\,x+d\,x^8-x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^5*(4*a - 3*x))/((-x^2*(a - x))^(1/3)*(2*a*x + d*x^8 - a^2 - x^2)),x
)
```

```
[Out] int(-(x^5*(4*a - 3*x))/((-x^2*(a - x))^(1/3)*(2*a*x + d*x^8 - a^2 - x^2)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\,(-4a+3x)}{\sqrt[3]{x^2\,(-a+x)}\,\left(-a^2+2ax+dx^8-x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-4*a+3*x)/(x**2*(-a+x))**(1/3)/(d*x**8-a**2+2*a*x-x**2),x)
```

```
[Out] Integral(x**5*(-4*a + 3*x)/((x**2*(-a + x))**(1/3)*(-a**2 + 2*a*x + d*x**8
 - x**2)), x)
```

3.2217 $\int \frac{\sqrt{1-x}}{8(1+x)^{7/2}}\,dx$

Optimal. Leaf size=269

$$\frac{\left(\frac{1}{960}-\frac{\sqrt{1-x}}{960}\right)\left(\sqrt{x+1}-1\right)^9\left(\frac{15\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}-\frac{30\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}+\frac{40\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{50\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}-\frac{46\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}-\frac{50\left(\sqrt{1-x}-1\right)}{\left(\sqrt{x+1}-1\right)^3}\right.}{\left(x+\sqrt{1-x}\sqrt{x+1}-2\sqrt{x+1}+1\right)^5}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 0.15, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {12, 45, 37}

$$-\frac{(1-x)^{3/2}}{120(x+1)^{3/2}}-\frac{(1-x)^{3/2}}{40(x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(8*(1 + x)^(7/2)),x]
```

```
[Out] -1/40*(1 - x)^(3/2)/(1 + x)^(5/2) - (1 - x)^(3/2)/(120*(1 + x)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x}}{8(1+x)^{7/2}}\,dx &= \frac{1}{8}\int \frac{\sqrt{1-x}}{(1+x)^{7/2}}\,dx\\ &= -\frac{(1-x)^{3/2}}{40(1+x)^{5/2}}+\frac{1}{40}\int \frac{\sqrt{1-x}}{(1+x)^{5/2}}\,dx\\ &= -\frac{(1-x)^{3/2}}{40(1+x)^{5/2}}-\frac{(1-x)^{3/2}}{120(1+x)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.09

$$-\frac{(1-x)^{3/2}(x+4)}{120(x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(8*(1 + x)^(7/2)),x]
```

```
[Out] -1/120*((1 - x)^(3/2)*(4 + x))/(1 + x)^(5/2)
```

IntegrateAlgebraic [A] time = 0.06, size = 34, normalized size = 0.13

$$-\frac{(1-x)^{3/2}\left(\frac{3(1-x)}{x+1}+5\right)}{240(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x]/(8*(1 + x)^(7/2)),x]
```

```
[Out] -1/240*((1 - x)^(3/2)*(5 + (3*(1 - x))/(1 + x)))/(1 + x)^(3/2)
```

fricas [A] time = 0.65, size = 54, normalized size = 0.20

$$-\frac{4\,x^{3}+12\,x^{2}-\left(x^{2}+3\,x-4\right)\sqrt{x+1}\,\sqrt{-x+1}+12\,x+4}{120\,\left(x^{3}+3\,x^{2}+3\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/8*(1-x)^(1/2)/(1+x)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/120*(4*x^3 + 12*x^2 - (x^2 + 3*x - 4)*sqrt(x + 1)*sqrt(-x + 1) + 12*x +
4)/(x^3 + 3*x^2 + 3*x + 1)
```

giac [A] time = 0.19, size = 133, normalized size = 0.49

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{5}}{2560\,(x+1)^{\frac{5}{2}}}+\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{1536\,(x+1)^{\frac{3}{2}}}-\frac{\sqrt{2}-\sqrt{-x+1}}{256\,\sqrt{x+1}}+\frac{\left(\frac{30\left(\sqrt{2}-\sqrt{-x+1}\right)^{4}}{(x+1)^{2}}-\frac{5\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}-3\right)(x+1)^{\frac{5}{2}}}{7680\left(\sqrt{2}-\sqrt{-x+1}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/8*(1-x)^(1/2)/(1+x)^(7/2),x, algorithm="giac")
```

```
[Out] 1/2560*(sqrt(2) - sqrt(-x + 1))^5/(x + 1)^(5/2) + 1/1536*(sqrt(2) - sqrt(-x
 + 1))^3/(x + 1)^(3/2) - 1/256*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + 1/768
0*(30*(sqrt(2) - sqrt(-x + 1))^4/(x + 1)^2 - 5*(sqrt(2) - sqrt(-x + 1))^2/(
x + 1) - 3)*(x + 1)^(5/2)/(sqrt(2) - sqrt(-x + 1))^5
```

maple [A] time = 0.00, size = 18, normalized size = 0.07

$$-\frac{(4+x)\,(1-x)^{\frac{3}{2}}}{120\,(1+x)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/8*(1-x)^(1/2)/(1+x)^(7/2),x)
```

```
[Out] -1/120*(4+x)*(1-x)^(3/2)/(1+x)^(5/2)
```

maxima [A] time = 0.31, size = 64, normalized size = 0.24

$$-\frac{\sqrt{-x^{2}+1}}{20\,\left(x^{3}+3\,x^{2}+3\,x+1\right)}+\frac{\sqrt{-x^{2}+1}}{120\,\left(x^{2}+2\,x+1\right)}+\frac{\sqrt{-x^{2}+1}}{120\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/8*(1-x)^(1/2)/(1+x)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(-x^2 + 1)/(x^3 + 3*x^2 + 3*x + 1) + 1/120*sqrt(-x^2 + 1)/(x^2 +
2*x + 1) + 1/120*sqrt(-x^2 + 1)/(x + 1)
```

mupad [B] time = 2.13, size = 49, normalized size = 0.18

$$\frac{3\,x\,\sqrt{1-x}-4\,\sqrt{1-x}+x^2\,\sqrt{1-x}}{\sqrt{x+1}\,\left(120\,x^2+240\,x+120\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(8*(x + 1)^(7/2)),x)
```

```
[Out] (3*x*(1 - x)^(1/2) - 4*(1 - x)^(1/2) + x^2*(1 - x)^(1/2))/((x + 1)^(1/2)*(2
40*x + 120*x^2 + 120))
```

sympy [A] time = 5.16, size = 162, normalized size = 0.60

$$\frac{\begin{cases} \frac{\sqrt{-1+\frac{2}{x+1}}}{15}+\frac{\sqrt{-1+\frac{2}{x+1}}}{15(x+1)}-\frac{2\sqrt{-1+\frac{2}{x+1}}}{5(x+1)^2} & \text{for } \frac{2}{|x+1|}>1 \\ \frac{i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{-30x+15(x+1)^2-30}-\frac{i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{-30x+15(x+1)^2-30}-\frac{8i\sqrt{1-\frac{2}{x+1}}}{-30x+15(x+1)^2-30}+\frac{12i\sqrt{1-\frac{2}{x+1}}}{(x+1)\left(-30x+15(x+1)^2-30\right)} & \text{otherwise} \end{cases}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/8*(1-x)**(1/2)/(1+x)**(7/2),x)
```

```
[Out] Piecewise((sqrt(-1 + 2/(x + 1))/15 + sqrt(-1 + 2/(x + 1))/(15*(x + 1)) - 2*
sqrt(-1 + 2/(x + 1))/(5*(x + 1)**2), 2/Abs(x + 1) > 1), (I*sqrt(1 - 2/(x +
1))*(x + 1)**2/(-30*x + 15*(x + 1)**2 - 30) - I*sqrt(1 - 2/(x + 1))*(x + 1)
/(-30*x + 15*(x + 1)**2 - 30) - 8*I*sqrt(1 - 2/(x + 1))/(-30*x + 15*(x + 1)
**2 - 30) + 12*I*sqrt(1 - 2/(x + 1))/((x + 1)*(-30*x + 15*(x + 1)**2 - 30))
, True))/8
```

3.2218 $\int \frac{2ab^2-b(2a+b)x+x^3}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-ab^2d+b(2a+b)dx-(1+ad+2bd)x^2+dx^3\right)}\,dx$

Optimal. Leaf size=269

$$\frac{\log\left(x-\sqrt[3]{d}\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}\right)}{d^{2/3}}-\frac{\log\left(d^{2/3}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}\right.}{2d}$$

Rubi [F] time = 15.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2ab^2-b(2a+b)x+x^3}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-ab^2d+b(2a+b)dx-(1+ad+2bd)x^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2*a*b^2 - b*(2*a + b)*x + x^3)/((x*(-a + x)*(-b + x)^2)^(1/3)*(-(a*b^2
*d) + b*(2*a + b)*d*x - (1 + a*d + 2*b*d)*x^2 + d*x^3)),x]
```

```
[Out] (6*a*b*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x*(-b
 + x^3)^(1/3))/((-a + x^3)^(1/3)*(a*b^2*d - 2*a*b*(1 + b/(2*a))*d*x^3 + (1
+ (a + 2*b)*d)*x^6 - d*x^9)), x], x, x^(1/3)])/(-((a - x)*(b - x)^2*x))^(1/
3) + (3*b*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x^
4*(-b + x^3)^(1/3))/((-a + x^3)^(1/3)*(-(a*b^2*d) + 2*a*b*(1 + b/(2*a))*d*x
^3 - (1 + (a + 2*b)*d)*x^6 + d*x^9)), x], x, x^(1/3)])/(-((a - x)*(b - x)^2
*x))^(1/3) + (3*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[In
t][(x^7*(-b + x^3)^(1/3))/((-a + x^3)^(1/3)*(-(a*b^2*d) + 2*a*b*(1 + b/(2*a
))*d*x^3 - (1 + (a + 2*b)*d)*x^6 + d*x^9)), x], x, x^(1/3)])/(-((a - x)*(b
- x)^2*x))^(1/3)
```

Rubi steps

$$\int \frac{2ab^2-b(2a+b)x+x^3}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-ab^2d+b(2a+b)dx-(1+ad+2bd)x^2+dx^3\right)}\,dx=\frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\int \frac{}{\sqrt[3]{x}\sqrt[3]{-}}}{\sqrt[3]{x(-}}$$

$$=\frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\int \frac{}{\sqrt[3]{x}\sqrt[3]{-}}}{\sqrt[3]{x(-a+}}$$

$$=\frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\text{Subst}}{}$$

$$=\frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\text{Subst}}{}$$

$$=\frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\text{Subst}}{}$$

Mathematica [F] time = 3.39, size = 0, normalized size = 0.00

$$\int \frac{2ab^2-b(2a+b)x+x^3}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-ab^2d+b(2a+b)dx-(1+ad+2bd)x^2+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2*a*b^2 - b*(2*a + b)*x + x^3)/((x*(-a + x)*(-b + x)^2)^(1/3)*(-
(a*b^2*d) + b*(2*a + b)*d*x - (1 + a*d + 2*b*d)*x^2 + d*x^3)),x]
```

```
[Out] Integrate[(2*a*b^2 - b*(2*a + b)*x + x^3)/((x*(-a + x)*(-b + x)^2)^(1/3)*(-
(a*b^2*d) + b*(2*a + b)*d*x - (1 + a*d + 2*b*d)*x^2 + d*x^3)), x]
```

IntegrateAlgebraic [A] time = 0.74, size = 269, normalized size = 1.00

$$\frac{\log\left(x-\sqrt[3]{d}\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}\right)}{d^{2/3}}-\frac{\log\left(d^{2/3}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}+\sqrt[3]{d}\,x\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}+x^2\right)}{2d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\sqrt[3]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}}{\sqrt[3]{d}\sqrt[3]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}+2x}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2*a*b^2 - b*(2*a + b)*x + x^3)/((x*(-a + x)*(-b + x)^2)
^(1/3)*(-(a*b^2*d) + b*(2*a + b)*d*x - (1 + a*d + 2*b*d)*x^2 + d*x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*
b)*x^3 + x^4)^(1/3))/(2*x + d^(1/3)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a -
 2*b)*x^3 + x^4)^(1/3))])/d^(2/3) + Log[x - d^(1/3)*(-(a*b^2*x) + (2*a*b +
b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3)]/d^(2/3) - Log[x^2 + d^(1/3)*x*(-(a*
b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3) + d^(2/3)*(-(a*b^2
*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b^2-b*(2*a+b)*x+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a*b^2*d+b*(
2*a+b)*d*x-(a*d+2*b*d+1)*x^2+d*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2\,ab^2-(2\,a+b)bx+x^3}{\left(ab^2d-(2\,a+b)bdx-dx^3+(ad+2\,bd+1)x^2\right)\left(-(a-x)(b-x)^2x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b^2-b*(2*a+b)*x+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a*b^2*d+b*(
2*a+b)*d*x-(a*d+2*b*d+1)*x^2+d*x^3),x, algorithm="giac")
```

```
[Out] integrate(-(2*a*b^2 - (2*a + b)*b*x + x^3)/((a*b^2*d - (2*a + b)*b*d*x - d*
x^3 + (a*d + 2*b*d + 1)*x^2)*(-(a - x)*(b - x)^2*x)^(1/3)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{2a\,b^2-b\,(2a+b)\,x+x^3}{\left(x\,(-a+x)\,(-b+x)^2\right)^{\frac{1}{3}}\left(-a\,b^2d+b\,(2a+b)\,dx-(ad+2bd+1)\,x^2+d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*b^2-b*(2*a+b)*x+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a*b^2*d+b*(2*a+b)
*d*x-(a*d+2*b*d+1)*x^2+d*x^3),x)
```

```
[Out] int((2*a*b^2-b*(2*a+b)*x+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a*b^2*d+b*(2*a+b)
*d*x-(a*d+2*b*d+1)*x^2+d*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\,ab^2-(2\,a+b)bx+x^3}{\left(ab^2d-(2\,a+b)bdx-dx^3+(ad+2\,bd+1)x^2\right)\left(-(a-x)(b-x)^2x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b^2-b*(2*a+b)*x+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a*b^2*d+b*(
2*a+b)*d*x-(a*d+2*b*d+1)*x^2+d*x^3),x, algorithm="maxima")
```

```
[Out] -integrate((2*a*b^2 - (2*a + b)*b*x + x^3)/((a*b^2*d - (2*a + b)*b*d*x - d*
x^3 + (a*d + 2*b*d + 1)*x^2)*(-(a - x)*(b - x)^2*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{2\,a\,b^2+x^3-b\,x\,(2\,a+b)}{\left(-x\,(a-x)\,(b-x)^2\right)^{1/3}\,\left(d\,x^3-x^2\,(a\,d+2\,b\,d+1)-a\,b^2\,d+b\,d\,x\,(2\,a+b)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*b^2 + x^3 - b*x*(2*a + b))/((-x*(a - x)*(b - x)^2)^(1/3)*(d*x^3 -
x^2*(a*d + 2*b*d + 1) - a*b^2*d + b*d*x*(2*a + b))),x)
```

```
[Out] int((2*a*b^2 + x^3 - b*x*(2*a + b))/((-x*(a - x)*(b - x)^2)^(1/3)*(d*x^3 -
x^2*(a*d + 2*b*d + 1) - a*b^2*d + b*d*x*(2*a + b))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*b**2-b*(2*a+b)*x+x**3)/(x*(-a+x)*(-b+x)**2)**(1/3)/(-a*b**2*
d+b*(2*a+b)*d*x-(a*d+2*b*d+1)*x**2+d*x**3),x)
```

```
[Out] Timed out
```

3.2219 $\int \frac{(-x+x^2)\sqrt[4]{-x^3+x^4}}{-1-x+x^2}\,dx$

Optimal. Leaf size=269

$$\frac{1}{8}\sqrt[4]{x^4-x^3}\,(4x-1)-\frac{29}{16}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)+\sqrt{\frac{1}{5}\left(2+2\sqrt{5}\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-x^3}}\right)+\sqrt{\frac{1}{5}\left(2\sqrt{5}-2\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\cdots}}{\sqrt[4]{x^4\cdots}}\right.$$

Rubi [A] time = 0.45, antiderivative size = 436, normalized size of antiderivative = 1.62, number of steps used = 26, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {1593, 2056, 903, 50, 63, 240, 212, 206, 203, 905, 911, 93, 298}

$$-\frac{1}{2}\sqrt[4]{x^4-x^3}\,(1-x)+\frac{3}{8}\sqrt[4]{x^4-x^3}+\frac{29\sqrt[4]{x^4-x^3}\,\tan^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{16\sqrt[4]{x-1}\,x^{3/4}}+\frac{2^{3/4}\sqrt[4]{3+\sqrt{5}}\,\sqrt[4]{x^4-x^3}\,\tan^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{\sqrt{5}\,\sqrt[4]{x-1}\,x^{3/4}}+\frac{2^{3/4}\sqrt[4]{3-\sqrt{5}}\,\sqrt[4]{x^4-x^3}\,\tan^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}\,\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{\sqrt{5}\,\sqrt[4]{x-1}\,x^{3/4}}+\frac{29\sqrt[4]{x^4-x^3}\,\tanh^{-1}\left(\frac{\sqrt[4]{x-1}}{\sqrt[4]{x}}\right)}{16\sqrt[4]{x-1}\,x^{3/4}}-\frac{2^{3/4}\sqrt[4]{3+\sqrt{5}}\,\sqrt[4]{x^4-x^3}\,\tanh^{-1}\left(\frac{\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{\sqrt{5}\,\sqrt[4]{x-1}\,x^{3/4}}-\frac{2^{3/4}\sqrt[4]{3-\sqrt{5}}\,\sqrt[4]{x^4-x^3}\,\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{2}(3+\sqrt{5})}\,\sqrt[4]{x}}{\sqrt[4]{x-1}}\right)}{\sqrt{5}\,\sqrt[4]{x-1}\,x^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-x + x^2)*(-x^3 + x^4)^(1/4))/(-1 - x + x^2),x]
```

```
[Out] (3*(-x^3 + x^4)^(1/4))/8 - ((1 - x)*(-x^3 + x^4)^(1/4))/2 + (29*(-x^3 + x^4
)^(1/4)*ArcTan[(-1 + x)^(1/4)/x^(1/4)])/(16*(-1 + x)^(1/4)*x^(3/4)) + (2^(3
/4)*(3 + Sqrt[5])^(1/4)*(-x^3 + x^4)^(1/4)*ArcTan[((2/(3 + Sqrt[5]))^(1/4)*
x^(1/4))/(-1 + x)^(1/4)])/(Sqrt[5]*(-1 + x)^(1/4)*x^(3/4)) + (2^(3/4)*(3 -
Sqrt[5])^(1/4)*(-x^3 + x^4)^(1/4)*ArcTan[(((3 + Sqrt[5])/2)^(1/4)*x^(1/4))/
(-1 + x)^(1/4)])/(Sqrt[5]*(-1 + x)^(1/4)*x^(3/4)) + (29*(-x^3 + x^4)^(1/4)*
ArcTanh[(-1 + x)^(1/4)/x^(1/4)])/(16*(-1 + x)^(1/4)*x^(3/4)) - (2^(3/4)*(3
+ Sqrt[5])^(1/4)*(-x^3 + x^4)^(1/4)*ArcTanh[((2/(3 + Sqrt[5]))^(1/4)*x^(1/4
))/(-1 + x)^(1/4)])/(Sqrt[5]*(-1 + x)^(1/4)*x^(3/4)) - (2^(3/4)*(3 - Sqrt[5
])^(1/4)*(-x^3 + x^4)^(1/4)*ArcTanh[(((3 + Sqrt[5])/2)^(1/4)*x^(1/4))/(-1 +
 x)^(1/4)])/(Sqrt[5]*(-1 + x)^(1/4)*x^(3/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 903

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Dist[g/c^2, Int[Simp[2*c*e*f + c*d*g - b*e
*g + c*e*g*x, x]*(d + e*x)^(m - 1)*(f + g*x)^(n - 2), x], x] + Dist[1/c^2,
Int[(Simp[c^2*d*f^2 - 2*a*c*e*f*g - a*c*d*g^2 + a*b*e*g^2 + (c^2*e*f^2 + 2*
c^2*d*f*g - 2*b*c*e*f*g - b*c*d*g^2 + b^2*e*g^2 - a*c*e*g^2)*x, x]*(d + e*x
)^(m - 1)*(f + g*x)^(n - 2))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
!IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0] && GtQ[n, 1]
```

Rule 905

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(e*g)/c, Int[(d + e*x)^(m - 1)*(f + g
*x)^(n - 1), x], x] + Dist[1/c, Int[(Simp[c*d*f - a*e*g + (c*e*f + c*d*g -
b*e*g)*x, x]*(d + e*x)^(m - 1)*(f + g*x)^(n - 1))/(a + b*x + c*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b
*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0] && GtQ[n, 0
]
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
```

```
IntegerQ[n]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\left(-x+x^2\right)\sqrt[4]{-x^3+x^4}}{-1-x+x^2}\,dx = \int \frac{(-1+x)x\sqrt[4]{-x^3+x^4}}{-1-x+x^2}\,dx$$

$$= \frac{\sqrt[4]{-x^3+x^4}\int \frac{(-1+x)^{5/4}x^{7/4}}{-1-x+x^2}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{\sqrt[4]{-x^3+x^4}\int \sqrt[4]{-1+x}\,x^{3/4}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}} + \frac{\sqrt[4]{-x^3+x^4}\int \frac{\sqrt[4]{-1+x}\,x^{3/4}}{-1-x+x^2}\,dx}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= -\frac{1}{2}(1-x)\sqrt[4]{-x^3+x^4} + \frac{\left(3\sqrt[4]{-x^3+x^4}\right)\int \frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\,dx}{8\sqrt[4]{-1+x}\,x^{3/4}} + \frac{\sqrt[4]{-x^3+x^4}\int \frac{1}{(-1+x)^{3/4}\sqrt[4]{x}}\ldots}{\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{3}{8}\sqrt[4]{-x^3+x^4} - \frac{1}{2}(1-x)\sqrt[4]{-x^3+x^4} - \frac{\left(3\sqrt[4]{-x^3+x^4}\right)\int \frac{1}{(-1+x)^{3/4}\sqrt[4]{x}}\,dx}{32\sqrt[4]{-1+x}\,x^{3/4}} + \frac{\sqrt[4]{-x^3 \ldots}}{\ldots}$$

$$= \frac{3}{8}\sqrt[4]{-x^3+x^4} - \frac{1}{2}(1-x)\sqrt[4]{-x^3+x^4} - \frac{\left(3\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+x^4}}\,dx, x, \sqrt[4]{-1 \ldots}\right.}{8\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{3}{8}\sqrt[4]{-x^3+x^4} - \frac{1}{2}(1-x)\sqrt[4]{-x^3+x^4} - \frac{\left(3\sqrt[4]{-x^3+x^4}\right)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\ldots\right.}{8\sqrt[4]{-1+x}\,x^{3/4}}$$

$$= \frac{3}{8}\sqrt[4]{-x^3+x^4} - \frac{1}{2}(1-x)\sqrt[4]{-x^3+x^4} + \frac{2\sqrt[4]{-x^3+x^4}\,\tan^{-1}\left(\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{\sqrt[4]{-1+x}\,x^{3/4}} + \frac{2\sqrt[4]{-x^3+\ldots}}{\ldots}$$

$$= \frac{3}{8}\sqrt[4]{-x^3+x^4} - \frac{1}{2}(1-x)\sqrt[4]{-x^3+x^4} + \frac{29\sqrt[4]{-x^3+x^4}\,\tan^{-1}\left(\frac{\sqrt[4]{-1+x}}{\sqrt[4]{x}}\right)}{16\sqrt[4]{-1+x}\,x^{3/4}} + \frac{2^{3/4}\sqrt[4]{3+\ldots}}{\ldots}$$

Mathematica [C] time = 0.19, size = 226, normalized size = 0.84

$$\frac{2\sqrt[4]{(x-1)x^3}\left(10\sqrt[4]{x}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{4};\frac{5}{4};1-x\right)-10\sqrt[4]{x}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};1-x\right)+10\sqrt[4]{x}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};1-x\right)+\sqrt{5}\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(-1+\sqrt{5})(x-1)}{(1+\sqrt{5})x}\right)-5\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(-1+\sqrt{5})(x-1)}{(1+\sqrt{5})x}\right)-\sqrt{5}\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(1+\sqrt{5})(x-1)}{(-1+\sqrt{5})x}\right)-5\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(1+\sqrt{5})(x-1)}{(-1+\sqrt{5})x}\right)\right)}{5x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-x + x^2)*(-x^3 + x^4)^(1/4))/(-1 - x + x^2),x]
```

```
[Out] (2*((-1 + x)*x^3)^(1/4)*(10*x^(1/4)*Hypergeometric2F1[-7/4, 1/4, 5/4, 1 - x
] - 10*x^(1/4)*Hypergeometric2F1[-3/4, 1/4, 5/4, 1 - x] + 10*x^(1/4)*Hyperg
eometric2F1[1/4, 1/4, 5/4, 1 - x] - 5*Hypergeometric2F1[1/4, 1, 5/4, ((-1 +
 Sqrt[5])*(-1 + x))/((1 + Sqrt[5])*x)] + Sqrt[5]*Hypergeometric2F1[1/4, 1,
5/4, ((-1 + Sqrt[5])*(-1 + x))/((1 + Sqrt[5])*x)] - 5*Hypergeometric2F1[1/4
, 1, 5/4, ((1 + Sqrt[5])*(-1 + x))/((-1 + Sqrt[5])*x)] - Sqrt[5]*Hypergeome
tric2F1[1/4, 1, 5/4, ((1 + Sqrt[5])*(-1 + x))/((-1 + Sqrt[5])*x)]))/(5*x)
```

IntegrateAlgebraic [A] time = 1.06, size = 269, normalized size = 1.00

$$\frac{1}{8}\sqrt[4]{x^4-x^3}\,(4x-1)-\frac{29}{16}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)+\sqrt{\frac{1}{5}\left(2+2\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-x^3}}\right)+\sqrt{\frac{1}{5}\left(2\sqrt{5}-2\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4-x^3}}\right)+\frac{29}{16}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-x^3}}\right)-\sqrt{\frac{1}{5}\left(2+2\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\,x}{\sqrt[4]{x^4-x^3}}\right)-\sqrt{\frac{1}{5}\left(2\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\,x}{\sqrt[4]{x^4-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-x + x^2)*(-x^3 + x^4)^(1/4))/(-1 - x + x^2),x]
```

```
[Out] ((-1 + 4*x)*(-x^3 + x^4)^(1/4))/8 - (29*ArcTan[x/(-x^3 + x^4)^(1/4)])/16 +
Sqrt[(2 + 2*Sqrt[5])/5]*ArcTan[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(-x^3 + x^4)^(1/4
)] + Sqrt[(-2 + 2*Sqrt[5])/5]*ArcTan[(Sqrt[1/2 + Sqrt[5]/2]*x)/(-x^3 + x^4)
^(1/4)] + (29*ArcTanh[x/(-x^3 + x^4)^(1/4)])/16 - Sqrt[(2 + 2*Sqrt[5])/5]*A
rcTanh[(Sqrt[-1/2 + Sqrt[5]/2]*x)/(-x^3 + x^4)^(1/4)] - Sqrt[(-2 + 2*Sqrt[5
])/5]*ArcTanh[(Sqrt[1/2 + Sqrt[5]/2]*x)/(-x^3 + x^4)^(1/4)]
```

fricas [B] time = 0.49, size = 462, normalized size = 1.72

$$\frac{2}{5}\sqrt{5}\sqrt{2\sqrt{5}-2}\arctan\left(\frac{\sqrt{2}x\sqrt{2\sqrt{5}-2}\sqrt{\frac{\sqrt{5}x^2+x^2+2\sqrt{x^4-x^3}}{x^2}}-2\left(x^4-x^3\right)^{\frac{1}{4}}\sqrt{2\sqrt{5}-2}}{4x}\right)+\frac{2}{5}\sqrt{5}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{\sqrt{2}x\sqrt{2\sqrt{5}+2}\sqrt{\frac{\sqrt{5}x^2-x^2+2\sqrt{x^4-x^3}}{x^2}}-2\left(x^4-x^3\right)^{\frac{1}{4}}\sqrt{2\sqrt{5}+2}}{4x}\right)-\frac{1}{10}\sqrt{5}\sqrt{2\sqrt{5}+2}\log\left(\frac{\left(\sqrt{5}x-x\right)\sqrt{2\sqrt{5}+2}+4\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{10}\sqrt{5}\sqrt{2\sqrt{5}+2}\log\left(-\frac{\left(\sqrt{5}x-x\right)\sqrt{2\sqrt{5}+2}-4\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{10}\sqrt{5}\sqrt{2\sqrt{5}-2}\log\left(\frac{\left(\sqrt{5}x+x\right)\sqrt{2\sqrt{5}-2}+4\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{10}\sqrt{5}\sqrt{2\sqrt{5}-2}\log\left(-\frac{\left(\sqrt{5}x+x\right)\sqrt{2\sqrt{5}-2}-4\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{8}\left(x^4-x^3\right)^{\frac{1}{4}}(4x-1)+\frac{29}{16}\arctan\left(\frac{\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)+\frac{29}{32}\log\left(\frac{x+\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)-\frac{29}{32}\log\left(-\frac{x-\left(x^4-x^3\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)*(x^4-x^3)^(1/4)/(x^2-x-1),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*sqrt(2*sqrt(5) - 2)*arctan(1/4*(sqrt(2)*x*sqrt(2*sqrt(5) - 2)*s
qrt((sqrt(5)*x^2 + x^2 + 2*sqrt(x^4 - x^3))/x^2) - 2*(x^4 - x^3)^(1/4)*sqrt
(2*sqrt(5) - 2))/x) + 2/5*sqrt(5)*sqrt(2*sqrt(5) + 2)*arctan(1/4*(sqrt(2)*x
*sqrt(2*sqrt(5) + 2)*sqrt((sqrt(5)*x^2 - x^2 + 2*sqrt(x^4 - x^3))/x^2) - 2*
(x^4 - x^3)^(1/4)*sqrt(2*sqrt(5) + 2))/x) - 1/10*sqrt(5)*sqrt(2*sqrt(5) + 2
)*log(((sqrt(5)*x - x)*sqrt(2*sqrt(5) + 2) + 4*(x^4 - x^3)^(1/4))/x) + 1/10
*sqrt(5)*sqrt(2*sqrt(5) + 2)*log(-((sqrt(5)*x - x)*sqrt(2*sqrt(5) + 2) - 4*
(x^4 - x^3)^(1/4))/x) - 1/10*sqrt(5)*sqrt(2*sqrt(5) - 2)*log(((sqrt(5)*x +
x)*sqrt(2*sqrt(5) - 2) + 4*(x^4 - x^3)^(1/4))/x) + 1/10*sqrt(5)*sqrt(2*sqrt
(5) - 2)*log(-((sqrt(5)*x + x)*sqrt(2*sqrt(5) - 2) - 4*(x^4 - x^3)^(1/4))/x
) + 1/8*(x^4 - x^3)^(1/4)*(4*x - 1) + 29/16*arctan((x^4 - x^3)^(1/4)/x) + 2
9/32*log((x + (x^4 - x^3)^(1/4))/x) - 29/32*log(-(x - (x^4 - x^3)^(1/4))/x)
```

giac [A] time = 0.70, size = 260, normalized size = 0.97

$$-\frac{1}{8}\left(\left(-\frac{1}{x}+1\right)^{\frac{5}{4}}+3\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)x^2+\frac{1}{5}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{5}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)+\frac{1}{10}\sqrt{10\sqrt{5}-10}\log\left(\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)+\frac{1}{10}\sqrt{10\sqrt{5}+10}\log\left(\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)-\frac{1}{10}\sqrt{10\sqrt{5}-10}\log\left(\left|-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right|\right)-\frac{1}{10}\sqrt{10\sqrt{5}+10}\log\left(\left|-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}+\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right|\right)-\frac{29}{16}\arctan\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}\right)-\frac{29}{32}\log\left(\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}+1\right)+\frac{29}{32}\log\left(\left|\left(-\frac{1}{x}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)*(x^4-x^3)^(1/4)/(x^2-x-1),x, algorithm="giac")
```

```
[Out] -1/8*((-1/x + 1)^(5/4) + 3*(-1/x + 1)^(1/4))*x^2 + 1/5*sqrt(10*sqrt(5) - 10
)*arctan((-1/x + 1)^(1/4)/sqrt(1/2*sqrt(5) + 1/2)) + 1/5*sqrt(10*sqrt(5) +
10)*arctan((-1/x + 1)^(1/4)/sqrt(1/2*sqrt(5) - 1/2)) + 1/10*sqrt(10*sqrt(5)
 - 10)*log(sqrt(1/2*sqrt(5) + 1/2) + (-1/x + 1)^(1/4)) + 1/10*sqrt(10*sqrt(
5) + 10)*log(sqrt(1/2*sqrt(5) - 1/2) + (-1/x + 1)^(1/4)) - 1/10*sqrt(10*sqr
t(5) - 10)*log(abs(-sqrt(1/2*sqrt(5) + 1/2) + (-1/x + 1)^(1/4))) - 1/10*sqr
t(10*sqrt(5) + 10)*log(abs(-sqrt(1/2*sqrt(5) - 1/2) + (-1/x + 1)^(1/4))) -
29/16*arctan((-1/x + 1)^(1/4)) - 29/32*log((-1/x + 1)^(1/4) + 1) + 29/32*lo
g(abs((-1/x + 1)^(1/4) - 1))
```

maple [C] time = 11.62, size = 4526, normalized size = 16.83

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x)*(x^4-x^3)^(1/4)/(x^2-x-1),x)
```

```
[Out] 1/8*(-1+4*x)*(x^3*(-1+x))^(1/4)+(-1/160*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*
_Z^2-1048576)^2+5120)*ln((325*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-10485
76)^2+5120)*RootOf(25*_Z^4+5120*_Z^2-1048576)^4*x^3-1300*RootOf(_Z^2+25*Roo
tOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)*RootOf(25*_Z^4+5120*_Z^2-1048576)^4*
x^2+1625*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)*RootOf(25
*_Z^4+5120*_Z^2-1048576)^4*x-527360*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(25*_Z^
4+5120*_Z^2-1048576)^2*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5
120)*x+642560*RootOf(25*_Z^4+5120*_Z^2-1048576)^2*RootOf(_Z^2+25*RootOf(25*
_Z^4+5120*_Z^2-1048576)^2+5120)*x^3-650*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*
_Z^2-1048576)^2+5120)*RootOf(25*_Z^4+5120*_Z^2-1048576)^4+527360*(x^4-3*x^3
+3*x^2-x)^(1/2)*RootOf(25*_Z^4+5120*_Z^2-1048576)^2*RootOf(_Z^2+25*RootOf(2
5*_Z^4+5120*_Z^2-1048576)^2+5120)-1838080*RootOf(25*_Z^4+5120*_Z^2-1048576)
^2*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)*x^2+36864000*(x
^4-3*x^3+3*x^2-x)^(3/4)*RootOf(25*_Z^4+5120*_Z^2-1048576)^2-60620800*RootOf
(25*_Z^4+5120*_Z^2-1048576)^2*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+1748480*RootOf(
25*_Z^4+5120*_Z^2-1048576)^2*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-104857
6)^2+5120)*x-171966464*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+25*RootOf(25*_
Z^4+5120*_Z^2-1048576)^2+5120)*x+180355072*RootOf(_Z^2+25*RootOf(25*_Z^4+51
20*_Z^2-1048576)^2+5120)*x^3+121241600*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(25*
_Z^4+5120*_Z^2-1048576)^2*x-552960*RootOf(25*_Z^4+5120*_Z^2-1048576)^2*Root
Of(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)+171966464*(x^4-3*x^3+3
*x^2-x)^(1/2)*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)-4734
32064*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)*x^2-60620800
*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+12415139840*
(x^4-3*x^3+3*x^2-x)^(3/4)-19964887040*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+4057989
12*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)*x+39929774080*(
x^4-3*x^3+3*x^2-x)^(1/4)*x-112721920*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^
2-1048576)^2+5120)-19964887040*(x^4-3*x^3+3*x^2-x)^(1/4))/(5*x*RootOf(25*_Z
^4+5120*_Z^2-1048576)^2-10*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+2048*x-3072)
/(-1+x)^2)-1/32*RootOf(25*_Z^4+5120*_Z^2-1048576)*ln((325*x^3*RootOf(25*_Z^
4+5120*_Z^2-1048576)^5-1300*x^2*RootOf(25*_Z^4+5120*_Z^2-1048576)^5+1625*x*
RootOf(25*_Z^4+5120*_Z^2-1048576)^5+527360*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf
(25*_Z^4+5120*_Z^2-1048576)^3*x-509440*RootOf(25*_Z^4+5120*_Z^2-1048576)^3*
x^3-650*RootOf(25*_Z^4+5120*_Z^2-1048576)^5-527360*(x^4-3*x^3+3*x^2-x)^(1/2
)*RootOf(25*_Z^4+5120*_Z^2-1048576)^3+1305600*RootOf(25*_Z^4+5120*_Z^2-1048
576)^3*x^2-7372800*(x^4-3*x^3+3*x^2-x)^(3/4)*RootOf(25*_Z^4+5120*_Z^2-10485
76)^2+12124160*RootOf(25*_Z^4+5120*_Z^2-1048576)^2*(x^4-3*x^3+3*x^2-x)^(1/4
)*x^2-1082880*RootOf(25*_Z^4+5120*_Z^2-1048576)^3*x-24248320*(x^4-3*x^3+3*x
^2-x)^(1/4)*RootOf(25*_Z^4+5120*_Z^2-1048576)^2*x-63963136*(x^4-3*x^3+3*x^2
-x)^(1/2)*RootOf(25*_Z^4+5120*_Z^2-1048576)*x+62390272*RootOf(25*_Z^4+5120*
_Z^2-1048576)*x^3+286720*RootOf(25*_Z^4+5120*_Z^2-1048576)^3+12124160*(x^4-
3*x^3+3*x^2-x)^(1/4)*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+63963136*(x^4-3*x^
3+3*x^2-x)^(1/2)*RootOf(25*_Z^4+5120*_Z^2-1048576)-151519232*x^2*RootOf(25*
_Z^4+5120*_Z^2-1048576)+973078528*(x^4-3*x^3+3*x^2-x)^(3/4)-1509949440*(x^4
-3*x^3+3*x^2-x)^(1/4)*x^2+115867648*RootOf(25*_Z^4+5120*_Z^2-1048576)*x+301
9898880*(x^4-3*x^3+3*x^2-x)^(1/4)*x-26738688*RootOf(25*_Z^4+5120*_Z^2-10485
76)-1509949440*(x^4-3*x^3+3*x^2-x)^(1/4))/(5*x*RootOf(25*_Z^4+5120*_Z^2-104
8576)^2-10*RootOf(25*_Z^4+5120*_Z^2-1048576)^2-1024*x+1024)/(-1+x)^2)-29/32
768*RootOf(25*_Z^4+5120*_Z^2-1048576)*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z
^2-1048576)^2+5120)*ln((2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(25*_Z^4+5120*_Z^
2-1048576)*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)*x-2*Roo
tOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)*RootOf(25*_Z^4+5120*_
```

```
Z^2-1048576)*x^3-2*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(25*_Z^4+5120*_Z^2-10485
76)*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)+5*RootOf(_Z^2+
25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)*RootOf(25*_Z^4+5120*_Z^2-10485
76)*x^2-4*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)*RootOf(2
5*_Z^4+5120*_Z^2-1048576)*x+2048*(x^4-3*x^3+3*x^2-x)^(3/4)-2048*(x^4-3*x^3+
3*x^2-x)^(1/4)*x^2+RootOf(25*_Z^4+5120*_Z^2-1048576)*RootOf(_Z^2+25*RootOf(
25*_Z^4+5120*_Z^2-1048576)^2+5120)+4096*(x^4-3*x^3+3*x^2-x)^(1/4)*x-2048*(x
^4-3*x^3+3*x^2-x)^(1/4))/(-1+x)^2)-29/32*ln((2*(x^4-3*x^3+3*x^2-x)^(3/4)-2*
(x^4-3*x^3+3*x^2-x)^(1/2)*x+2*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2-2*x^3+2*(x^4-3*
x^3+3*x^2-x)^(1/2)-4*(x^4-3*x^3+3*x^2-x)^(1/4)*x+5*x^2+2*(x^4-3*x^3+3*x^2-x
)^(1/4)-4*x+1)/(-1+x)^2)-1/32768*RootOf(25*_Z^4+5120*_Z^2-1048576)^2*RootOf
(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)*ln(-(1075*RootOf(_Z^2+25
*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)*RootOf(25*_Z^4+5120*_Z^2-1048576
)^4*x^3-4300*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)*RootO
f(25*_Z^4+5120*_Z^2-1048576)^4*x^2+5375*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*
_Z^2-1048576)^2+5120)*RootOf(25*_Z^4+5120*_Z^2-1048576)^4*x-1679360*(x^4-3*
x^3+3*x^2-x)^(1/2)*RootOf(25*_Z^4+5120*_Z^2-1048576)^2*RootOf(_Z^2+25*RootO
f(25*_Z^4+5120*_Z^2-1048576)^2+5120)*x-1674240*RootOf(25*_Z^4+5120*_Z^2-104
8576)^2*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)*x^3-2150*R
ootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)*RootOf(25*_Z^4+5120
*_Z^2-1048576)^4+1679360*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(25*_Z^4+5120*_Z^2
-1048576)^2*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)+427520
0*RootOf(25*_Z^4+5120*_Z^2-1048576)^2*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z
^2-1048576)^2+5120)*x^2+73728000*(x^4-3*x^3+3*x^2-x)^(3/4)*RootOf(25*_Z^4+5
120*_Z^2-1048576)^2+121241600*RootOf(25*_Z^4+5120*_Z^2-1048576)^2*(x^4-3*x^
3+3*x^2-x)^(1/4)*x^2-3527680*RootOf(25*_Z^4+5120*_Z^2-1048576)^2*RootOf(_Z^
2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)*x+216006656*(x^4-3*x^3+3*x^2
-x)^(1/2)*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)*x+190840
832*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)*x^3-242483200*
(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(25*_Z^4+5120*_Z^2-1048576)^2*x+926720*Root
Of(25*_Z^4+5120*_Z^2-1048576)^2*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-104
8576)^2+5120)-216006656*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(_Z^2+25*RootOf(25*
_Z^4+5120*_Z^2-1048576)^2+5120)-463470592*RootOf(_Z^2+25*RootOf(25*_Z^4+512
0*_Z^2-1048576)^2+5120)*x^2+121241600*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(25*_
Z^4+5120*_Z^2-1048576)^2-9730785280*(x^4-3*x^3+3*x^2-x)^(3/4)-15099494400*(
x^4-3*x^3+3*x^2-x)^(1/4)*x^2+354418688*RootOf(_Z^2+25*RootOf(25*_Z^4+5120*_
Z^2-1048576)^2+5120)*x+30198988800*(x^4-3*x^3+3*x^2-x)^(1/4)*x-81788928*Roo
tOf(_Z^2+25*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+5120)-15099494400*(x^4-3*x^
3+3*x^2-x)^(1/4))/(5*x*RootOf(25*_Z^4+5120*_Z^2-1048576)^2-10*RootOf(25*_Z^
4+5120*_Z^2-1048576)^2-1024*x+1024)/(-1+x)^2)-5/32768*ln((1075*x^3*RootOf(2
5*_Z^4+5120*_Z^2-1048576)^5-4300*x^2*RootOf(25*_Z^4+5120*_Z^2-1048576)^5+53
75*x*RootOf(25*_Z^4+5120*_Z^2-1048576)^5+1679360*(x^4-3*x^3+3*x^2-x)^(1/2)*
RootOf(25*_Z^4+5120*_Z^2-1048576)^3*x+2114560*RootOf(25*_Z^4+5120*_Z^2-1048
576)^3*x^3-2150*RootOf(25*_Z^4+5120*_Z^2-1048576)^5-1679360*(x^4-3*x^3+3*x^
2-x)^(1/2)*RootOf(25*_Z^4+5120*_Z^2-1048576)^3-6036480*RootOf(25*_Z^4+5120*
_Z^2-1048576)^3*x^2+14745600*(x^4-3*x^3+3*x^2-x)^(3/4)*RootOf(25*_Z^4+5120*
_Z^2-1048576)^2+24248320*RootOf(25*_Z^4+5120*_Z^2-1048576)^2*(x^4-3*x^3+3*x
^2-x)^(1/4)*x^2+5729280*RootOf(25*_Z^4+5120*_Z^2-1048576)^3*x-48496640*(x^4
-3*x^3+3*x^2-x)^(1/4)*RootOf(25*_Z^4+5120*_Z^2-1048576)^2*x+559939584*(x^4-
3*x^3+3*x^2-x)^(1/2)*RootOf(25*_Z^4+5120*_Z^2-1048576)*x+578813952*RootOf(2
5*_Z^4+5120*_Z^2-1048576)*x^3-1807360*RootOf(25*_Z^4+5120*_Z^2-1048576)^3+2
4248320*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(25*_Z^4+5120*_Z^2-1048576)^2-55993
9584*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(25*_Z^4+5120*_Z^2-1048576)-1519386624
*x^2*RootOf(25*_Z^4+5120*_Z^2-1048576)+4966055936*(x^4-3*x^3+3*x^2-x)^(3/4)
+7985954816*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+1302331392*RootOf(25*_Z^4+5120*_Z
^2-1048576)*x-15971909632*(x^4-3*x^3+3*x^2-x)^(1/4)*x-361758720*RootOf(25*_
Z^4+5120*_Z^2-1048576)+7985954816*(x^4-3*x^3+3*x^2-x)^(1/4))/(5*x*RootOf(25
*_Z^4+5120*_Z^2-1048576)^2-10*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+2048*x-30
72)/(-1+x)^2)*RootOf(25*_Z^4+5120*_Z^2-1048576)^3-1/32*ln((1075*x^3*RootOf(
```

```
25*_Z^4+5120*_Z^2-1048576)^5-4300*x^2*RootOf(25*_Z^4+5120*_Z^2-1048576)^5+5
375*x*RootOf(25*_Z^4+5120*_Z^2-1048576)^5+1679360*(x^4-3*x^3+3*x^2-x)^(1/2)
*RootOf(25*_Z^4+5120*_Z^2-1048576)^3*x+2114560*RootOf(25*_Z^4+5120*_Z^2-104
8576)^3*x^3-2150*RootOf(25*_Z^4+5120*_Z^2-1048576)^5-1679360*(x^4-3*x^3+3*x
^2-x)^(1/2)*RootOf(25*_Z^4+5120*_Z^2-1048576)^3-6036480*RootOf(25*_Z^4+5120
*_Z^2-1048576)^3*x^2+14745600*(x^4-3*x^3+3*x^2-x)^(3/4)*RootOf(25*_Z^4+5120
*_Z^2-1048576)^2+24248320*RootOf(25*_Z^4+5120*_Z^2-1048576)^2*(x^4-3*x^3+3*
x^2-x)^(1/4)*x^2+5729280*RootOf(25*_Z^4+5120*_Z^2-1048576)^3*x-48496640*(x^
4-3*x^3+3*x^2-x)^(1/4)*RootOf(25*_Z^4+5120*_Z^2-1048576)^2*x+559939584*(x^4
-3*x^3+3*x^2-x)^(1/2)*RootOf(25*_Z^4+5120*_Z^2-1048576)*x+578813952*RootOf(
25*_Z^4+5120*_Z^2-1048576)*x^3-1807360*RootOf(25*_Z^4+5120*_Z^2-1048576)^3+
24248320*(x^4-3*x^3+3*x^2-x)^(1/4)*RootOf(25*_Z^4+5120*_Z^2-1048576)^2-5599
39584*(x^4-3*x^3+3*x^2-x)^(1/2)*RootOf(25*_Z^4+5120*_Z^2-1048576)-151938662
4*x^2*RootOf(25*_Z^4+5120*_Z^2-1048576)+4966055936*(x^4-3*x^3+3*x^2-x)^(3/4
)+7985954816*(x^4-3*x^3+3*x^2-x)^(1/4)*x^2+1302331392*RootOf(25*_Z^4+5120*_
Z^2-1048576)*x-15971909632*(x^4-3*x^3+3*x^2-x)^(1/4)*x-361758720*RootOf(25*
_Z^4+5120*_Z^2-1048576)+7985954816*(x^4-3*x^3+3*x^2-x)^(1/4))/(5*x*RootOf(2
5*_Z^4+5120*_Z^2-1048576)^2-10*RootOf(25*_Z^4+5120*_Z^2-1048576)^2+2048*x-3
072)/(-1+x)^2)*RootOf(25*_Z^4+5120*_Z^2-1048576))*(x^3*(-1+x))^(1/4)/(-1+x)
/x*(x*(-1+x)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4-x^3\right)^{\frac{1}{4}}\left(x^2-x\right)}{x^2-x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x)*(x^4-x^3)^(1/4)/(x^2-x-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^3)^(1/4)*(x^2 - x)/(x^2 - x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x-x^2\right)\left(x^4-x^3\right)^{1/4}}{-x^2+x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - x^2)*(x^4 - x^3)^(1/4))/(x - x^2 + 1),x)
```

```
[Out] int(((x - x^2)*(x^4 - x^3)^(1/4))/(x - x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[4]{x^3\left(x-1\right)}\left(x-1\right)}{x^2-x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x)*(x**4-x**3)**(1/4)/(x**2-x-1),x)
```

```
[Out] Integral(x*(x**3*(x - 1))**(1/4)*(x - 1)/(x**2 - x - 1), x)
```

3.2220 $\int \frac{1}{\sqrt{1+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=269

$$\frac{\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}\,(6b-16ax)+\sqrt{a^2x^2-b}\left(8\sqrt{\sqrt{a^2x^2-b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}-16\sqrt{\sqrt{\sqrt{a^2x^2}}}\right.}{12a\sqrt{a^2x^2-b}+12a^2x}$$

Rubi [F] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{1+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]],x]
```

```
[Out] Defer[Int][1/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]], x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{1}{\sqrt{1+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Mathematica [A] time = 1.23, size = 188, normalized size = 0.70

$$\frac{2\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}\left(-\frac{9b}{\sqrt{\sqrt{a^2x^2-b}+ax}}+\frac{6b}{\sqrt{a^2x^2-b}+ax}+8\sqrt{\sqrt{a^2x^2-b}+ax}-16\right)-9b\log\left(1-\frac{1}{\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}}\right)+9b\log\left(\frac{1}{\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}}+1\right)}{24a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]],x]
```

```
[Out] (2*Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]*(-16 + (6*b)/(a*x + Sqrt[-b + a
^2*x^2]) - (9*b)/Sqrt[a*x + Sqrt[-b + a^2*x^2]] + 8*Sqrt[a*x + Sqrt[-b + a^
2*x^2]]) - 9*b*Log[1 - 1/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]] + 9*b*Lo
g[1 + 1/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]])/(24*a)
```

IntegrateAlgebraic [A] time = 0.46, size = 269, normalized size = 1.00

$$\frac{\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}\,(6b-16ax)+\sqrt{a^2x^2-b}\left(8\sqrt{\sqrt{a^2x^2-b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}-16\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}\right)+(8ax-9b)\sqrt{\sqrt{a^2x^2-b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}}{12a\sqrt{a^2x^2-b}+12a^2x}+\frac{3b\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]],x]
```

```
[Out] ((6*b - 16*a*x)*Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]] + (-9*b + 8*a*x)*S
qrt[a*x + Sqrt[-b + a^2*x^2]]*Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]] + Sq
rt[-b + a^2*x^2]*(-16*Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]] + 8*Sqrt[a*x
 + Sqrt[-b + a^2*x^2]]*Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]))/(12*a^2*x
```

```
 + 12*a*Sqrt[-b + a^2*x^2]) + (3*b*ArcTanh[Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^
2*x^2]]]])/(4*a)
```

fricas [A] time = 0.45, size = 152, normalized size = 0.57

$$\frac{9\,b\log\left(\sqrt{\sqrt{ax+\sqrt{a^2x^2-b}}+1}+1\right)-9\,b\log\left(\sqrt{\sqrt{ax+\sqrt{a^2x^2-b}}+1}-1\right)+2\left(6\,ax-\left(9\,ax-9\,\sqrt{a^2x^2-b}-8\right)\sqrt{ax+\sqrt{a^2x^2-b}}-6\,\sqrt{a^2x^2-b}-16\right)\sqrt{\sqrt{ax+\sqrt{a^2x^2-b}}+1}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(9*b*log(sqrt(sqrt(a*x + sqrt(a^2*x^2 - b)) + 1) + 1) - 9*b*log(sqrt(s
qrt(a*x + sqrt(a^2*x^2 - b)) + 1) - 1) + 2*(6*a*x - (9*a*x - 9*sqrt(a^2*x^2
 - b) - 8)*sqrt(a*x + sqrt(a^2*x^2 - b)) - 6*sqrt(a^2*x^2 - b) - 16)*sqrt(s
qrt(a*x + sqrt(a^2*x^2 - b)) + 1))/a
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1+\sqrt{ax+\sqrt{a^2x^2-b}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] int(1/(1+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{ax+\sqrt{a^2x^2-b}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(sqrt(a*x + sqrt(a^2*x^2 - b)) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\sqrt{a\,x+\sqrt{a^2\,x^2-b}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x + (a^2*x^2 - b)^(1/2))^(1/2) + 1)^(1/2),x)
```

```
[Out] int(1/((a*x + (a^2*x^2 - b)^(1/2))^(1/2) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{ax+\sqrt{a^2x^2-b}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(a*x+(a**2*x**2-b)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sqrt(a*x + sqrt(a**2*x**2 - b)) + 1), x)
```

3.2221

$$\int \frac{(-1+x)(-1+kx)\left(-2x+(1+k)x^2\right)}{((1-x)x(1-kx))^{2/3}\left(b-2b(1+k)x+\left(b+4bk+bk^2\right)x^2-2bk(1+k)x^3+\left(-1+bk^2\right)x^4\right)}\,dx$$

Optimal. Leaf size=270

$$\frac{\log\left(x-\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{2b^{2/3}}+\frac{\log\left(\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+x\right)}{2b^{2/3}}-\frac{\log\left(-\sqrt[6]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+\right.}{}$$

Rubi [F] time = 21.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x)(-1+kx)\left(-2x+(1+k)x^2\right)}{((1-x)x(1-kx))^{2/3}\left(b-2b(1+k)x+\left(b+4bk+bk^2\right)x^2-2bk(1+k)x^3+\left(-1+bk^2\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x)*(-1 + k*x)*(-2*x + (1 + k)*x^2))/(((1 - x)*x*(1 - k*x))^(2/3)
*(b - 2*b*(1 + k)*x + (b + 4*b*k + b*k^2)*x^2 - 2*b*k*(1 + k)*x^3 + (-1 + b
*k^2)*x^4)),x]
```

```
[Out] (6*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst][Defer[Int][(x^3*(1 -
x^3)^(1/3)*(1 - k*x^3)^(1/3))/(x^12 - b*(-1 + x^3)^2*(-1 + k*x^3)^2), x], x
, x^(1/3)]])/((1 - x)*x*(1 - k*x))^(2/3) + (3*(1 + k)*(1 - x)^(2/3)*x^(2/3)*
(1 - k*x)^(2/3)*Defer[Subst][Defer[Int][(x^6*(1 - x^3)^(1/3)*(1 - k*x^3)^(1
/3))/(-x^12 + b*(-1 + x^3)^2*(-1 + k*x^3)^2), x], x, x^(1/3)]])/((1 - x)*x*(
1 - k*x))^(2/3)
```

Rubi steps

$$\int \frac{(-1+x)(-1+kx)\left(-2x+(1+k)x^2\right)}{((1-x)x(1-kx))^{2/3}\left(b-2b(1+k)x+\left(b+4bk+bk^2\right)x^2-2bk(1+k)x^3+\left(-1+bk^2\right)x^4\right)}\,dx = \int \frac{}{((1-x)x(1 \cdots}$$

$$= \frac{\left((1-x)^{2/3}x^{2/3}\right. \cdots}{}$$

$$= -\frac{\left((1-x)^{2/3}x^{2/3}\right. \cdots}{}$$

$$= \frac{\left((1-x)^{2/3}x^{2/3}\right. \cdots}{}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}\right. \cdots}{}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}\right. \cdots}{}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}\right. \cdots}{}$$

$$= \frac{\left(6(1-x)^{2/3}x^{2/3}\right. \cdots}{}$$

$$= \frac{\left(6(1-x)^{2/3}x^{2/3}\right. \cdots}{}$$

Mathematica [F] time = 3.90, size = 0, normalized size = 0.00

$$\int \frac{(-1+x)(-1+kx)\left(-2x+(1+k)x^2\right)}{((1-x)x(1-kx))^{2/3}\left(b-2b(1+k)x+\left(b+4bk+bk^2\right)x^2-2bk(1+k)x^3+\left(-1+bk^2\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x)*(-1 + k*x)*(-2*x + (1 + k)*x^2))/(((1 - x)*x*(1 - k*x))
^(2/3)*(b - 2*b*(1 + k)*x + (b + 4*b*k + b*k^2)*x^2 - 2*b*k*(1 + k)*x^3 + (
-1 + b*k^2)*x^4)),x]
```

```
[Out] Integrate[((-1 + x)*(-1 + k*x)*(-2*x + (1 + k)*x^2))/(((1 - x)*x*(1 - k*x))
^(2/3)*(b - 2*b*(1 + k)*x + (b + 4*b*k + b*k^2)*x^2 - 2*b*k*(1 + k)*x^3 + (
-1 + b*k^2)*x^4)), x]
```

IntegrateAlgebraic [A] time = 0.63, size = 270, normalized size = 1.00

$$\frac{\log\left(x-\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{2b^{2/3}}+\frac{\log\left(\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+x\right)}{2b^{2/3}}-\frac{\log\left(-\sqrt[6]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+x^2\right)}{4b^{2/3}}-\frac{\log\left(\sqrt[6]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+x^2\right)}{4b^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+x^2}\right)}{2b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)*(-1 + k*x)*(-2*x + (1 + k)*x^2))/(((1 - x)*x*(
1 - k*x))^(2/3)*(b - 2*b*(1 + k)*x + (b + 4*b*k + b*k^2)*x^2 - 2*b*k*(1 + k
)*x^3 + (-1 + b*k^2)*x^4)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*b^(1/3)*(x + (-1 - k)*x^2 + k*x
^3)^(2/3))])/b^(2/3) + Log[x - b^(1/6)*(x + (-1 - k)*x^2 + k*x^3)^(1/3)]/(2
*b^(2/3)) + Log[x + b^(1/6)*(x + (-1 - k)*x^2 + k*x^3)^(1/3)]/(2*b^(2/3)) -
 Log[x^2 - b^(1/6)*x*(x + (-1 - k)*x^2 + k*x^3)^(1/3) + b^(1/3)*(x + (-1 -
k)*x^2 + k*x^3)^(2/3)]/(4*b^(2/3)) - Log[x^2 + b^(1/6)*x*(x + (-1 - k)*x^2
+ k*x^3)^(1/3) + b^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(4*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(k*x-1)*(-2*x+(1+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-2*b*(
1+k)*x+(b*k^2+4*b*k+b)*x^2-2*b*k*(1+k)*x^3+(b*k^2-1)*x^4),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [A] time = 2.63, size = 289, normalized size = 1.07

$$-\frac{|b|\log\left(\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{2}{3}}+\left(-\frac{1}{b}\right)^{\frac{1}{3}}\right)}{2\left(-b^5\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\left(-b^5\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(-\frac{1}{b}\right)^{\frac{1}{6}}+2\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{1}{3}}}{\left(-\frac{1}{b}\right)^{\frac{1}{6}}}\right)}{2b^4}-\frac{\sqrt{3}\left(-b^5\right)^{\frac{2}{3}}\arctan\left(-\frac{\sqrt{3}\left(-\frac{1}{b}\right)^{\frac{1}{6}}-2\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{1}{3}}}{\left(-\frac{1}{b}\right)^{\frac{1}{6}}}\right)}{2b^4}-\frac{\left(-b^5\right)^{\frac{2}{3}}\log\left(\sqrt{3}\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{6}}+\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{2}{3}}+\left(-\frac{1}{b}\right)^{\frac{1}{3}}\right)}{4b^4}-\frac{\left(-b^5\right)^{\frac{2}{3}}\log\left(-\sqrt{3}\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{6}}+\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{2}{3}}+\left(-\frac{1}{b}\right)^{\frac{1}{3}}\right)}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(k*x-1)*(-2*x+(1+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-2*b*(
1+k)*x+(b*k^2+4*b*k+b)*x^2-2*b*k*(1+k)*x^3+(b*k^2-1)*x^4),x, algorithm="gia
c")
```

```
[Out] -1/2*abs(b)*log((k - k/x - 1/x + 1/x^2)^(2/3) + (-1/b)^(1/3))/(-b^5)^(1/3)
+ 1/2*sqrt(3)*(-b^5)^(2/3)*arctan((sqrt(3)*(-1/b)^(1/6) + 2*(k - k/x - 1/x
+ 1/x^2)^(1/3))/(-1/b)^(1/6))/b^4 - 1/2*sqrt(3)*(-b^5)^(2/3)*arctan(-(sqrt(
3)*(-1/b)^(1/6) - 2*(k - k/x - 1/x + 1/x^2)^(1/3))/(-1/b)^(1/6))/b^4 - 1/4*
(-b^5)^(2/3)*log(sqrt(3)*(k - k/x - 1/x + 1/x^2)^(1/3)*(-1/b)^(1/6) + (k -
k/x - 1/x + 1/x^2)^(2/3) + (-1/b)^(1/3))/b^4 - 1/4*(-b^5)^(2/3)*log(-sqrt(3
)*(k - k/x - 1/x + 1/x^2)^(1/3)*(-1/b)^(1/6) + (k - k/x - 1/x + 1/x^2)^(2/3
) + (-1/b)^(1/3))/b^4
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(-1+x)\,(kx-1)\left(-2x+(1+k)\,x^2\right)}{((1-x)\,x\,(-kx+1))^{\frac{2}{3}}\left(b-2b\,(1+k)\,x+\left(b\,k^2+4bk+b\right)x^2-2bk\,(1+k)\,x^3+\left(b\,k^2-1\right)x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)*(k*x-1)*(-2*x+(1+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-2*b*(1+k)*x
+(b*k^2+4*b*k+b)*x^2-2*b*k*(1+k)*x^3+(b*k^2-1)*x^4),x)
```

```
[Out] int((-1+x)*(k*x-1)*(-2*x+(1+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-2*b*(1+k)*x
+(b*k^2+4*b*k+b)*x^2-2*b*k*(1+k)*x^3+(b*k^2-1)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left((k+1)x^2-2\,x\right)(kx-1)(x-1)}{\left(2\,b(k+1)kx^3-\left(bk^2-1\right)x^4+2\,b(k+1)x-\left(bk^2+4\,bk+b\right)x^2-b\right)((kx-1)(x-1)x)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(k*x-1)*(-2*x+(1+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(b-2*b*(
1+k)*x+(b*k^2+4*b*k+b)*x^2-2*b*k*(1+k)*x^3+(b*k^2-1)*x^4),x, algorithm="max
ima")
```

```
[Out] -integrate(((k + 1)*x^2 - 2*x)*(k*x - 1)*(x - 1)/((2*b*(k + 1)*k*x^3 - (b*k
^2 - 1)*x^4 + 2*b*(k + 1)*x - (b*k^2 + 4*b*k + b)*x^2 - b)*((k*x - 1)*(x -
1)*x)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(2\,x-x^2\,\left(k+1\right)\right)\,\left(k\,x-1\right)\,\left(x-1\right)}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{2/3}\,\left(\left(b\,k^2-1\right)\,x^4-2\,b\,k\,\left(k+1\right)\,x^3+\left(b\,k^2+4\,b\,k+b\right)\,x^2-2\,b\,\left(k+1\right)\,x+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*x - x^2*(k + 1))*(k*x - 1)*(x - 1))/((x*(k*x - 1)*(x - 1))^(2/3)*(
b + x^4*(b*k^2 - 1) + x^2*(b + 4*b*k + b*k^2) - 2*b*x*(k + 1) - 2*b*k*x^3*(
k + 1))),x)
```

```
[Out] int(-((2*x - x^2*(k + 1))*(k*x - 1)*(x - 1))/((x*(k*x - 1)*(x - 1))^(2/3)*(
b + x^4*(b*k^2 - 1) + x^2*(b + 4*b*k + b*k^2) - 2*b*x*(k + 1) - 2*b*k*x^3*(
k + 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(k*x-1)*(-2*x+(1+k)*x**2)/((1-x)*x*(-k*x+1))**(2/3)/(b-2*b
*(1+k)*x+(b*k**2+4*b*k+b)*x**2-2*b*k*(1+k)*x**3+(b*k**2-1)*x**4),x)
```

```
[Out] Timed out
```

3.2222 $$\int \frac{\left(-1+2(-2+k)x+3kx^2\right)\left(-1+3x-3x^2+x^3\right)}{((1-x)x(1-kx))^{2/3}\left(-b+(1+5b)x-(10b+k)x^2+10bx^3-5bx^4+bx^5\right)}\,dx$$

Optimal. Leaf size=270

$$\frac{\log\left(-\sqrt[3]{b}\,x^2+2\sqrt[3]{b}\,x-\sqrt[3]{b}+\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{b^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x^2-2\sqrt{3}\sqrt[3]{b}\,x+\sqrt{3}\sqrt[3]{b}}{\sqrt[3]{b}\,x^2-2\sqrt[3]{b}\,x+\sqrt[3]{b}+2\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{b^{2/3}}-\frac{\log\left(b^{2/}\right.}{}$$

Rubi [F] time = 10.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-1+2(-2+k)x+3kx^2\right)\left(-1+3x-3x^2+x^3\right)}{((1-x)x(1-kx))^{2/3}\left(-b+(1+5b)x-(10b+k)x^2+10bx^3-5bx^4+bx^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + 2*(-2 + k)*x + 3*k*x^2)*(-1 + 3*x - 3*x^2 + x^3))/(((1 - x)*x*(1
 - k*x))^(2/3)*(-b + (1 + 5*b)*x - (10*b + k)*x^2 + 10*b*x^3 - 5*b*x^4 + b*
x^5)),x]
```

```
[Out] (6*(2 - k)*(-1 + x)^(2/3)*x^(2/3)*(-1 + k*x)^(2/3)*Defer[Subst][Defer[Int][
(x^3*(-1 + x^3)^(7/3))/((-1 + k*x^3)^(2/3)*(-x^3 + k*x^6 - b*(-1 + x^3)^5))
, x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(2/3) - (9*k*(-1 + x)^(2/3)*x^(2/3
)*(-1 + k*x)^(2/3)*Defer[Subst][Defer[Int][(x^6*(-1 + x^3)^(7/3))/((-1 + k*
x^3)^(2/3)*(-x^3 + k*x^6 - b*(-1 + x^3)^5)), x], x, x^(1/3)])/((1 - x)*x*(1
 - k*x))^(2/3) - (3*(-1 + x)^(2/3)*x^(2/3)*(-1 + k*x)^(2/3)*Defer[Subst][De
fer[Int][(-1 + x^3)^(7/3)/((-1 + k*x^3)^(2/3)*(x^3 - k*x^6 + b*(-1 + x^3)^5
)), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(2/3)
```

Rubi steps

$$\int \frac{\left(-1+2(-2+k)x+3kx^2\right)\left(-1+3x-3x^2+x^3\right)}{((1-x)x(1-kx))^{2/3}\left(-b+(1+5b)x-(10b+k)x^2+10bx^3-5bx^4+bx^5\right)}\,dx=\int \frac{(1-x)^3\left(1+2(2\right.}{((-1+x)x(-1+kx))^{2/3}\left(\right.}$$

$$=\frac{\left((-1+x)^{2/3}x^{2/3}(-1+kx)^{2/3}\right.}{((-1}$$

$$=-\frac{\left((-1+x)^{2/3}x^{2/3}(-1+kx)^{2}\right.}{((-1+}$$

$$=-\frac{\left(3(-1+x)^{2/3}x^{2/3}(-1+kx)\right.}{}$$

$$=-\frac{\left(3(-1+x)^{2/3}x^{2/3}(-1+kx)\right.}{}$$

$$=-\frac{\left(3(-1+x)^{2/3}x^{2/3}(-1+kx)\right.}{}$$

$$=-\frac{\left(3(-1+x)^{2/3}x^{2/3}(-1+kx)\right.}{}$$

Mathematica [F] time = 1.55, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+2(-2+k)x+3kx^2\right)\left(-1+3x-3x^2+x^3\right)}{((1-x)x(1-kx))^{2/3}\left(-b+(1+5b)x-(10b+k)x^2+10bx^3-5bx^4+bx^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + 2*(-2 + k)*x + 3*k*x^2)*(-1 + 3*x - 3*x^2 + x^3))/(((1 - x
)*x*(1 - k*x))^(2/3)*(-b + (1 + 5*b)*x - (10*b + k)*x^2 + 10*b*x^3 - 5*b*x^
4 + b*x^5)),x]
```

```
[Out] Integrate[((-1 + 2*(-2 + k)*x + 3*k*x^2)*(-1 + 3*x - 3*x^2 + x^3))/(((1 - x
)*x*(1 - k*x))^(2/3)*(-b + (1 + 5*b)*x - (10*b + k)*x^2 + 10*b*x^3 - 5*b*x^
4 + b*x^5)), x]
```

IntegrateAlgebraic [A] time = 6.99, size = 270, normalized size = 1.00

$$\frac{\log\left(-\sqrt[3]{b}x^2+2\sqrt[3]{b}x-\sqrt[3]{b}+\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{b^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x^2-2\sqrt{3}\sqrt[3]{b}x+\sqrt{3}\sqrt[3]{b}}{\sqrt[3]{b}x^2-2\sqrt[3]{b}x+\sqrt[3]{b}+2\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{b^{2/3}}-\frac{\log\left(b^{2/3}x^4-4b^{2/3}x^3+6b^{2/3}x^2-4b^{2/3}x+b^{2/3}+\left(\sqrt[3]{b}x^2-2\sqrt[3]{b}x+\sqrt[3]{b}\right)\sqrt[3]{kx^3+(-k-1)x^2+x}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + 2*(-2 + k)*x + 3*k*x^2)*(-1 + 3*x - 3*x^2 + x^3))
/(((1 - x)*x*(1 - k*x))^(2/3)*(-b + (1 + 5*b)*x - (10*b + k)*x^2 + 10*b*x^3
 - 5*b*x^4 + b*x^5)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3) - 2*Sqrt[3]*b^(1/3)*x + Sqrt[3]*b^(1/3)*x^
2)/(b^(1/3) - 2*b^(1/3)*x + b^(1/3)*x^2 + 2*(x + (-1 - k)*x^2 + k*x^3)^(1/3
))])/b^(2/3) + Log[-b^(1/3) + 2*b^(1/3)*x - b^(1/3)*x^2 + (x + (-1 - k)*x^2
 + k*x^3)^(1/3)]/b^(2/3) - Log[b^(2/3) - 4*b^(2/3)*x + 6*b^(2/3)*x^2 - 4*b^
(2/3)*x^3 + b^(2/3)*x^4 + (b^(1/3) - 2*b^(1/3)*x + b^(1/3)*x^2)*(x + (-1 -
k)*x^2 + k*x^3)^(1/3) + (x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*(-2+k)*x+3*k*x^2)*(x^3-3*x^2+3*x-1)/((1-x)*x*(-k*x+1))^(2/3
)/(-b+(1+5*b)*x-(10*b+k)*x^2+10*b*x^3-5*b*x^4+b*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,kx^2+2\,(k-2)x-1\right)\left(x^3-3\,x^2+3\,x-1\right)}{\left(bx^5-5\,bx^4+10\,bx^3-(10\,b+k)x^2+(5\,b+1)x-b\right)((kx-1)(x-1)x)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*(-2+k)*x+3*k*x^2)*(x^3-3*x^2+3*x-1)/((1-x)*x*(-k*x+1))^(2/3
)/(-b+(1+5*b)*x-(10*b+k)*x^2+10*b*x^3-5*b*x^4+b*x^5),x, algorithm="giac")
```

```
[Out] integrate((3*k*x^2 + 2*(k - 2)*x - 1)*(x^3 - 3*x^2 + 3*x - 1)/((b*x^5 - 5*b
*x^4 + 10*b*x^3 - (10*b + k)*x^2 + (5*b + 1)*x - b)*((k*x - 1)*(x - 1)*x)^(
2/3)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+2\,(-2+k)\,x+3k\,x^2\right)\left(x^3-3x^2+3x-1\right)}{((1-x)\,x\,(-kx+1))^{\frac{2}{3}}\left(-b+(1+5b)\,x-(10b+k)\,x^2+10b\,x^3-5b\,x^4+b\,x^5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*(-2+k)*x+3*k*x^2)*(x^3-3*x^2+3*x-1)/((1-x)*x*(-k*x+1))^(2/3)/(-b+
(1+5*b)*x-(10*b+k)*x^2+10*b*x^3-5*b*x^4+b*x^5),x)
```

```
[Out] int((-1+2*(-2+k)*x+3*k*x^2)*(x^3-3*x^2+3*x-1)/((1-x)*x*(-k*x+1))^(2/3)/(-b+
(1+5*b)*x-(10*b+k)*x^2+10*b*x^3-5*b*x^4+b*x^5),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,kx^2+2\,(k-2)x-1\right)\left(x^3-3\,x^2+3\,x-1\right)}{\left(bx^5-5\,bx^4+10\,bx^3-(10\,b+k)x^2+(5\,b+1)x-b\right)\left((kx-1)(x-1)x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*(-2+k)*x+3*k*x^2)*(x^3-3*x^2+3*x-1)/((1-x)*x*(-k*x+1))^(2/3
)/(-b+(1+5*b)*x-(10*b+k)*x^2+10*b*x^3-5*b*x^4+b*x^5),x, algorithm="maxima")
```

```
[Out] integrate((3*k*x^2 + 2*(k - 2)*x - 1)*(x^3 - 3*x^2 + 3*x - 1)/((b*x^5 - 5*b
*x^4 + 10*b*x^3 - (10*b + k)*x^2 + (5*b + 1)*x - b)*((k*x - 1)*(x - 1)*x)^(
2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(2\,x\,\left(k-2\right)+3\,k\,x^2-1\right)\,\left(x^3-3\,x^2+3\,x-1\right)}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{2/3}\,\left(-b\,x^5+5\,b\,x^4-10\,b\,x^3+\left(10\,b+k\right)\,x^2+\left(-5\,b-1\right)\,x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*x*(k - 2) + 3*k*x^2 - 1)*(3*x - 3*x^2 + x^3 - 1))/((x*(k*x - 1)*(x
 - 1))^(2/3)*(b - 10*b*x^3 + 5*b*x^4 - b*x^5 + x^2*(10*b + k) - x*(5*b + 1)
)),x)
```

```
[Out] -int(((2*x*(k - 2) + 3*k*x^2 - 1)*(3*x - 3*x^2 + x^3 - 1))/((x*(k*x - 1)*(x
 - 1))^(2/3)*(b - 10*b*x^3 + 5*b*x^4 - b*x^5 + x^2*(10*b + k) - x*(5*b + 1)
)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-1\right)^{3}\left(3kx^2+2kx-4x-1\right)}{\left(x\left(x-1\right)\left(kx-1\right)\right)^{\frac{2}{3}}\left(bx^5-5bx^4+10bx^3-10bx^2+5bx-b-kx^2+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*(-2+k)*x+3*k*x**2)*(x**3-3*x**2+3*x-1)/((1-x)*x*(-k*x+1))**
(2/3)/(-b+(1+5*b)*x-(10*b+k)*x**2+10*b*x**3-5*b*x**4+b*x**5),x)
```

```
[Out] Integral((x - 1)**3*(3*k*x**2 + 2*k*x - 4*x - 1)/((x*(x - 1)*(k*x - 1))**(2
/3)*(b*x**5 - 5*b*x**4 + 10*b*x**3 - 10*b*x**2 + 5*b*x - b - k*x**2 + x)),
x)
```

3.2223 $\int \frac{-b^{12}+a^{12}x^{12}}{\sqrt{-b^4+a^4x^4}\left(b^{12}+a^{12}x^{12}\right)}\,dx$

Optimal. Leaf size=270

$$-\frac{\tan^{-1}\left(\frac{-\frac{a^3x^4}{2b}+\frac{b^3}{2a}+abx^2}{x\sqrt{a^4x^4-b^4}}\right)}{12ab}+\frac{\tanh^{-1}\left(\frac{-\frac{a^3x^4}{2b}+\frac{b^3}{2a}-abx^2}{x\sqrt{a^4x^4-b^4}}\right)}{12ab}+\frac{\tanh^{-1}\left(\frac{-\frac{a^3x^4}{\sqrt{2}b}+\frac{b^3}{\sqrt{2}a}-\frac{abx^2}{\sqrt{2}}}{x\sqrt{a^4x^4-b^4}}\right)}{3\sqrt{2}\,ab}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,abx\sqrt{a^4x^4-b^4}}{-a^4x^4+a^2b^2x^2+b^4}\right)}{3\sqrt{2}\,ab}$$

Rubi [C] time = 9.29, antiderivative size = 1043, normalized size of antiderivative = 3.86, number of steps used = 207, number of rules used = 18, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1586, 6725, 1729, 1209, 1201, 224, 221, 1200, 1199, 424, 1219, 1218, 1248, 735, 844, 217, 206, 725}

Antiderivative was successfully verified.

```
[In] Int[(-b^12 + a^12*x^12)/(Sqrt[-b^4 + a^4*x^4]*(b^12 + a^12*x^12)),x]
```

```
[Out] -1/6*((-1)^(2/3)*(a^2 - (-1)^(2/3)*(-a^12)^(1/6))*((-1)^(1/3)*a^8 - (-1)^(2
/3)*a^4*(-a^12)^(1/3) - (-a^12)^(2/3))*b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticF[
ArcSin[(a*x)/b], -1])/(a^11*Sqrt[-b^4 + a^4*x^4]) - ((-1)^(2/3)*(a^2 + (-1)
^(2/3)*(-a^12)^(1/6))*((-1)^(1/3)*a^8 - (-1)^(2/3)*a^4*(-a^12)^(1/3) - (-a^
12)^(2/3))*b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticF[ArcSin[(a*x)/b], -1])/(6*a^1
1*Sqrt[-b^4 + a^4*x^4]) + ((a^2 - (-a^12)^(1/6))*(a^8 + a^4*(-a^12)^(1/3) +
 (-a^12)^(2/3))*b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticF[ArcSin[(a*x)/b], -1])/(
6*a^11*Sqrt[-b^4 + a^4*x^4]) + ((a^2 + (-a^12)^(1/6))*(a^8 + a^4*(-a^12)^(1
/3) + (-a^12)^(2/3))*b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticF[ArcSin[(a*x)/b], -
1])/(6*a^11*Sqrt[-b^4 + a^4*x^4]) - ((-1)^(1/3)*(a^2 - (-1)^(1/3)*(-a^12)^(
1/6))*((-1)^(2/3)*a^8 + (-a^12)^(2/3) + ((-1)^(1/3)*(-a^12)^(4/3))/a^8)*b*S
qrt[1 - (a^4*x^4)/b^4]*EllipticF[ArcSin[(a*x)/b], -1])/(6*a^11*Sqrt[-b^4 +
a^4*x^4]) - ((-1)^(1/3)*(a^2 + (-1)^(1/3)*(-a^12)^(1/6))*((-1)^(2/3)*a^8 +
(-a^12)^(2/3) + ((-1)^(1/3)*(-a^12)^(4/3))/a^8)*b*Sqrt[1 - (a^4*x^4)/b^4]*E
llipticF[ArcSin[(a*x)/b], -1])/(6*a^11*Sqrt[-b^4 + a^4*x^4]) - (b*Sqrt[1 -
(a^4*x^4)/b^4]*EllipticPi[a^10/(-a^12)^(5/6), ArcSin[(a*x)/b], -1])/(3*a*Sq
rt[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[((-1)^(1/3)*a^1
0)/(-a^12)^(5/6), ArcSin[(a*x)/b], -1])/(3*a*Sqrt[-b^4 + a^4*x^4]) - (b*Sqr
t[1 - (a^4*x^4)/b^4]*EllipticPi[((-1)^(2/3)*a^10)/(-a^12)^(5/6), ArcSin[(a*
x)/b], -1])/(3*a*Sqrt[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/b^4]*Ellipti
cPi[(-a^12)^(1/6)/a^2, ArcSin[(a*x)/b], -1])/(3*a*Sqrt[-b^4 + a^4*x^4]) - (
b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[((-1)^(1/3)*(-a^12)^(1/6))/a^2, ArcSin
[(a*x)/b], -1])/(3*a*Sqrt[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/b^4]*Ell
ipticPi[((-1)^(2/3)*(-a^12)^(1/6))/a^2, ArcSin[(a*x)/b], -1])/(3*a*Sqrt[-b^
4 + a^4*x^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1201

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1729

```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Dist[d, I
nt[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, Int[(x*(a + c*x^4)^p)/(d
^2 - e^2*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-b^{12}+a^{12}x^{12}}{\sqrt{-b^4+a^4x^4}\,\left(b^{12}+a^{12}x^{12}\right)}\,dx = \int \frac{\sqrt{-b^4+a^4x^4}\,\left(b^8+a^4b^4x^4+a^8x^8\right)}{b^{12}+a^{12}x^{12}}\,dx$$

$$= \int \left(\frac{\left(b^9+\frac{a^8b^9}{\left(-a^{12}\right)^{2/3}}+\frac{a^4b^9}{\sqrt[3]{-a^{12}}}\right)\sqrt{-b^4+a^4x^4}}{12b^{12}\left(b-\sqrt[12]{-a^{12}}\,x\right)}+\frac{\left(b^9+\frac{a^8b^9}{\left(-a^{12}\right)^{2/3}}+\frac{a^4b^9}{\sqrt[3]{-a^{12}}}\right)\sqrt{-b^4+\right.}{12b^{12}\left(b-i\sqrt[12]{-a^{12}}\,x\right)}\right.$$

$$= \frac{\left(1+\frac{a^8}{\left(-a^{12}\right)^{2/3}}+\frac{a^4}{\sqrt[3]{-a^{12}}}\right)\int\frac{\sqrt{-b^4+a^4x^4}}{b-\sqrt[12]{-a^{12}}\,x}\,dx}{12b^3}+\frac{\left(1+\frac{a^8}{\left(-a^{12}\right)^{2/3}}+\frac{a^4}{\sqrt[3]{-a^{12}}}\right)\int\frac{\sqrt{-b^4+a^4x^4}}{b-i\sqrt[12]{-a^{12}}\,x}}{12b^3}$$

= rest of steps removed due to Latex formating problem

Mathematica [C] time = 1.80, size = 276, normalized size = 1.02

$$-\frac{i\sqrt{1-\frac{a^4x^4}{b^4}}\left(3F\left(i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-\Pi\left(-i;i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-\Pi\left(i;i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-\Pi\left(-\frac{i}{2}-\frac{\sqrt{3}}{2};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-\Pi\left(\frac{i}{2}-\frac{\sqrt{3}}{2};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-\Pi\left(\frac{1}{2}\left(-i+\sqrt{3}\right);i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-\Pi\left(\frac{1}{2}\left(i+\sqrt{3}\right);i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)\right)}{3\sqrt{-\frac{a^2}{b^2}}\sqrt{a^4x^4-b^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b^12 + a^12*x^12)/(Sqrt[-b^4 + a^4*x^4]*(b^12 + a^12*x^12)),x]
```

```
[Out] ((-1/3*I)*Sqrt[1 - (a^4*x^4)/b^4]*(3*EllipticF[I*ArcSinh[Sqrt[-(a^2/b^2)]*x
], -1] - EllipticPi[-I, I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] - EllipticPi[I,
I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] - EllipticPi[-1/2*I - Sqrt[3]/2, I*ArcSi
nh[Sqrt[-(a^2/b^2)]*x], -1] - EllipticPi[I/2 - Sqrt[3]/2, I*ArcSinh[Sqrt[-(
a^2/b^2)]*x], -1] - EllipticPi[(-I + Sqrt[3])/2, I*ArcSinh[Sqrt[-(a^2/b^2)]
*x], -1] - EllipticPi[(I + Sqrt[3])/2, I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1]))
/(Sqrt[-(a^2/b^2)]*Sqrt[-b^4 + a^4*x^4])
```

IntegrateAlgebraic [C] time = 26.42, size = 598, normalized size = 2.21

$$\frac{\left(\frac{i}{6}-\frac{1}{6}\right)\tan^{-1}\left(\frac{(1+i)abx}{\sqrt{a^4x^4-b^4}+a^2x^2+ib^2}\right)}{ab}+\frac{(-1)^{3/4}\tan^{-1}\left(\frac{\sqrt{2}abx}{\sqrt{a^4x^4-b^4}+a^2x^2+ib^2}\right)}{3ab}-\frac{i\left(\sqrt{6}\sqrt[4]{-7-4\sqrt{3}}-\sqrt{2}\sqrt[4]{-7-4\sqrt{3}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{-7-4\sqrt{3}}\left(-i\sqrt{a^4x^4-b^4}-ia^2x^2+b^2\right)}{\sqrt{2}abx}\right)}{6ab}+\frac{i\left(\sqrt{2}\sqrt[4]{4\sqrt{3}-7}+\sqrt{6}\sqrt[4]{4\sqrt{3}-7}\right)\tanh^{-1}\left(\frac{\sqrt[4]{4\sqrt{3}-7}\left(-i\sqrt{a^4x^4-b^4}-ia^2x^2+b^2\right)}{\sqrt{2}abx}\right)}{6ab}-\frac{\sqrt[4]{-1}\tanh^{-1}\left(\frac{\sqrt{2}abx}{\sqrt{a^4x^4-b^4}+a^2x^2+ib^2}\right)}{3ab}+\frac{\left(\frac{1}{24}-\frac{i}{24}\right)\log\left(-ia^4x^4-(1-i)a^3bx^3+\left(-ia^2x^2-(1-i)abx+b^2\right)\sqrt{a^4x^4-b^4}-(1+i)ab^3x+ib^4\right)}{ab}-\frac{\left(\frac{1}{24}-\frac{i}{24}\right)\log\left(a^5bx^4+(1+i)a^4b^2x^3-(1-i)a^2b^4x+\left(a^3bx^2+(1+i)a^2b^2x+iab^3\right)\sqrt{a^4x^4-b^4}-ab^5\right)}{ab}$$

Warning: Unable to verify antiderivative.

```
[In] IntegrateAlgebraic[(-b^12 + a^12*x^12)/(Sqrt[-b^4 + a^4*x^4]*(b^12 + a^12*x
^12)),x]
```

```
[Out] ((-1/6 + I/6)*ArcTan[((1 + I)*a*b*x)/(I*b^2 + a^2*x^2 + Sqrt[-b^4 + a^4*x^4
])])/(a*b) + ((-1)^(3/4)*ArcTan[(Sqrt[2]*a*b*x)/(I*b^2 + a^2*x^2 + Sqrt[-b^
4 + a^4*x^4])])/(3*a*b) - ((I/6)*(-(Sqrt[2]*(-7 - 4*Sqrt[3])^(1/4)) + Sqrt[
6]*(-7 - 4*Sqrt[3])^(1/4))*ArcTanh[((-7 - 4*Sqrt[3])^(1/4)*(b^2 - I*a^2*x^2
 - I*Sqrt[-b^4 + a^4*x^4]))/(Sqrt[2]*a*b*x)])/(a*b) + ((I/6)*(Sqrt[2]*(-7 +
 4*Sqrt[3])^(1/4) + Sqrt[6]*(-7 + 4*Sqrt[3])^(1/4))*ArcTanh[((-7 + 4*Sqrt[3
])^(1/4)*(b^2 - I*a^2*x^2 - I*Sqrt[-b^4 + a^4*x^4]))/(Sqrt[2]*a*b*x)])/(a*b
) - ((-1)^(1/4)*ArcTanh[(Sqrt[2]*a*b*x)/(I*b^2 + a^2*x^2 + Sqrt[-b^4 + a^4*
x^4])])/(3*a*b) + ((1/24 - I/24)*Log[I*b^4 - (1 + I)*a*b^3*x - (1 - I)*a^3*
b*x^3 - I*a^4*x^4 + (b^2 - (1 - I)*a*b*x - I*a^2*x^2)*Sqrt[-b^4 + a^4*x^4]]
)/(a*b) - ((1/24 - I/24)*Log[-(a*b^5) - (1 - I)*a^2*b^4*x + (1 + I)*a^4*b^2
*x^3 + a^5*b*x^4 + (I*a*b^3 + (1 + I)*a^2*b^2*x + a^3*b*x^2)*Sqrt[-b^4 + a^
4*x^4]])/(a*b)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^12*x^12-b^12)/(a^4*x^4-b^4)^(1/2)/(a^12*x^12+b^12),x, algorith
m="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^{12}x^{12}-b^{12}}{\left(a^{12}x^{12}+b^{12}\right)\sqrt{a^4x^4-b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^12*x^12-b^12)/(a^4*x^4-b^4)^(1/2)/(a^12*x^12+b^12),x, algorith
m="giac")
```

```
[Out] integrate((a^12*x^12 - b^12)/((a^12*x^12 + b^12)*sqrt(a^4*x^4 - b^4)), x)
```

maple [C] time = 0.10, size = 541, normalized size = 2.00

$$\frac{\sqrt{\frac{a^2x^2}{b^2}+1}\,\sqrt{1-\frac{a^2x^2}{b^2}}\;\mathrm{EllipticF}\left(x\sqrt{-\frac{a^2}{b^2}},i\right)}{\sqrt{-\frac{a^2}{b^2}}\,\sqrt{a^4x^4-b^4}}-\frac{b^4\left(\sum_{\_\alpha=\mathrm{RootOf}\left(a^8\_Z^8-a^4b^4\_Z^4+b^8\right)}\frac{\left(-\_\alpha^4a^4+2b^4\right)\left(-\frac{\mathrm{arctanh}\left(\frac{\_\alpha^2\left(\_\alpha^6a^4-\_\alpha^2b^4+b^4x^2\right)a^4}{b^4\sqrt{\_\alpha^4a^4-b^4}\,\sqrt{a^4x^4-b^4}}\right)}{\sqrt{\_\alpha^4a^4-b^4}}+\frac{2a^4\_\alpha^3\left(\_\alpha^4a^4-b^4\right)\sqrt{\frac{a^2x^2}{b^2}+1}\,\sqrt{1-\frac{a^2x^2}{b^2}}\;\mathrm{EllipticPi}\left(x\sqrt{-\frac{a^2}{b^2}},\frac{\_\alpha^2\left(\_\alpha^4a^4-b^4\right)a^2}{b^6},\frac{\sqrt{\frac{a^2}{b^2}}}{\sqrt{-\frac{a^2}{b^2}}}\right)}{\sqrt{-\frac{a^2}{b^2}}\,b^8\sqrt{a^4x^4-b^4}}\right)}{\_\alpha^3\left(2\_\alpha^4a^4-b^4\right)}\right)}{12a^4}-\frac{b^4\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4a^4+b^4\right)}\frac{-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\_\alpha^2\left(\_\alpha^2+x^2\right)a^4}{\sqrt{-2b^4}\,\sqrt{a^4x^4-b^4}}\right)}{\sqrt{-b^4}}+\frac{4\_\alpha^3a^4\sqrt{\frac{a^2x^2}{b^2}+1}\,\sqrt{1-\frac{a^2x^2}{b^2}}\;\mathrm{EllipticPi}\left(x\sqrt{-\frac{a^2}{b^2}},\frac{\_\alpha^2a^2}{b^2},\frac{\sqrt{\frac{a^2}{b^2}}}{\sqrt{-\frac{a^2}{b^2}}}\right)}{\sqrt{-\frac{a^2}{b^2}}\,b^4\sqrt{a^4x^4-b^4}}}{\_\alpha^3}\right)}{24a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^12*x^12-b^12)/(a^4*x^4-b^4)^(1/2)/(a^12*x^12+b^12),x)
```

```
[Out] 1/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4
)^(1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)-1/12*b^4/a^4*sum((-_alpha^4*a^4+2*b
^4)/_alpha^3/(2*_alpha^4*a^4-b^4)*(-1/(_alpha^4*a^4-b^4)^(1/2)*arctanh(_alp
ha^2/b^4*(_alpha^6*a^4-_alpha^2*b^4+b^4*x^2)*a^4/(_alpha^4*a^4-b^4)^(1/2)/(
a^4*x^4-b^4)^(1/2))+2/(-a^2/b^2)^(1/2)*a^4*_alpha^3*(_alpha^4*a^4-b^4)/b^8*
(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticPi(
x*(-a^2/b^2)^(1/2),_alpha^2*(_alpha^4*a^4-b^4)*a^2/b^6,(a^2/b^2)^(1/2)/(-a^
2/b^2)^(1/2))),_alpha=RootOf(_Z^8*a^8-_Z^4*a^4*b^4+b^8))-1/24*b^4/a^4*sum(1
/_alpha^3*(-2^(1/2)/(-b^4)^(1/2)*arctanh(_alpha^2*(_alpha^2+x^2)*a^4/(-2*b^
4)^(1/2)/(a^4*x^4-b^4)^(1/2))+4/(-a^2/b^2)^(1/2)*_alpha^3*a^4/b^4*(a^2*x^2/
b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticPi(x*(-a^2/b
^2)^(1/2),_alpha^2*a^2/b^2,(a^2/b^2)^(1/2)/(-a^2/b^2)^(1/2))),_alpha=RootOf
(_Z^4*a^4+b^4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^{12}x^{12}-b^{12}}{\left(a^{12}x^{12}+b^{12}\right)\sqrt{a^4x^4-b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^12*x^12-b^12)/(a^4*x^4-b^4)^(1/2)/(a^12*x^12+b^12),x, algorith
m="maxima")
```

```
[Out] integrate((a^12*x^12 - b^12)/((a^12*x^12 + b^12)*sqrt(a^4*x^4 - b^4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{b^{12}-a^{12}\,x^{12}}{\sqrt{a^4\,x^4-b^4}\,\left(a^{12}\,x^{12}+b^{12}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^12 - a^12*x^12)/((a^4*x^4 - b^4)^(1/2)*(b^12 + a^12*x^12)),x)
```

```
[Out] int(-(b^12 - a^12*x^12)/((a^4*x^4 - b^4)^(1/2)*(b^12 + a^12*x^12)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**12*x**12-b**12)/(a**4*x**4-b**4)**(1/2)/(a**12*x**12+b**12),x
)
```

```
[Out] Timed out
```

3.2224

$$\int \frac{-a(a-5b)-(3a+5b)x+4x^2}{\sqrt[3]{(-a+x)(-b+x)}\left(b-a^5d-\left(1-5a^4d\right)x-10a^3dx^2+10a^2dx^3-5adx^4+dx^5\right)}\,dx$$

Optimal. Leaf size=272

$$\frac{\log\left(a^2\sqrt{d}-\sqrt[6]{d}\sqrt[3]{x(-a-b)+ab+x^2}-2a\sqrt{d}\,x+\sqrt{d}\,x^2\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x(-a-b)+ab+x^2}}{2a^2\sqrt[3]{d}+\sqrt[3]{x(-a-b)+ab+x^2}-4a\sqrt[3]{d}\,x+2\sqrt[3]{d}\,x^2}\right)}{\sqrt[3]{d}}-\cdots$$

Rubi [F] time = 7.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-a(a-5b)-(3a+5b)x+4x^2}{\sqrt[3]{(-a+x)(-b+x)}\left(b-a^5d-\left(1-5a^4d\right)x-10a^3dx^2+10a^2dx^3-5adx^4+dx^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a*(a - 5*b)) - (3*a + 5*b)*x + 4*x^2)/(((-a + x)*(-b + x))^(1/3)*(b
- a^5*d - (1 - 5*a^4*d)*x - 10*a^3*d*x^2 + 10*a^2*d*x^3 - 5*a*d*x^4 + d*x^5
)),x]
```

```
[Out] (-3*(a - 5*b)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][Defer[Int][x^4/((a
 - b + x^3)^(1/3)*(a*(1 - b/a) + x^3 - d*x^15)), x], x, (-a + x)^(1/3)])/((
a - x)*(b - x))^(1/3) + (12*a*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][De
fer[Int][x^4/((a - b + x^3)^(1/3)*(-(a*(1 - b/a)) - x^3 + d*x^15)), x], x,
(-a + x)^(1/3)])/((a - x)*(b - x))^(1/3) + (12*(-a + x)^(1/3)*(-b + x)^(1/3
)*Defer[Subst][Defer[Int][x^7/((a - b + x^3)^(1/3)*(-(a*(1 - b/a)) - x^3 +
d*x^15)), x], x, (-a + x)^(1/3)])/((a - x)*(b - x))^(1/3)
```

Rubi steps

$$\int \frac{-a(a-5b)-(3a+5b)x+4x^2}{\sqrt[3]{(-a+x)(-b+x)}\left(b-a^5d-\left(1-5a^4d\right)x-10a^3dx^2+10a^2dx^3-5adx^4+dx^5\right)}\,dx = \frac{\left(\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\int}{}$$

$$= \frac{\left(\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\int}{}$$

$$= \frac{\left(\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\int}{}$$

$$= \frac{\left(\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\int}{}$$

$$= \frac{\left(4\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)\int}{}$$

$$= \frac{\left(12\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)}{}$$

$$= \frac{\left(12\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)}{}$$

$$= \frac{\left(12\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)}{}$$

$$= \frac{\left(12\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)}{}$$

Mathematica [F] time = 4.35, size = 0, normalized size = 0.00

$$\int \frac{-a(a-5b)-(3a+5b)x+4x^2}{\sqrt[3]{(-a+x)(-b+x)}\left(b-a^5d-\left(1-5a^4d\right)x-10a^3dx^2+10a^2dx^3-5adx^4+dx^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-(a*(a - 5*b)) - (3*a + 5*b)*x + 4*x^2)/(((-a + x)*(-b + x))^(1/
3)*(b - a^5*d - (1 - 5*a^4*d)*x - 10*a^3*d*x^2 + 10*a^2*d*x^3 - 5*a*d*x^4 +
 d*x^5)),x]
```

```
[Out] Integrate[(-(a*(a - 5*b)) - (3*a + 5*b)*x + 4*x^2)/(((-a + x)*(-b + x))^(1/
3)*(b - a^5*d - (1 - 5*a^4*d)*x - 10*a^3*d*x^2 + 10*a^2*d*x^3 - 5*a*d*x^4 +
 d*x^5)), x]
```

IntegrateAlgebraic [A] time = 0.79, size = 272, normalized size = 1.00

$$\frac{\log\left(a^2\sqrt{d}-\sqrt[6]{d}\,\sqrt[3]{x(-a-b)+ab+x^2}-2a\sqrt{d}\,x+\sqrt{d}\,x^2\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{x(-a-b)+ab+x^2}}{2a^2\sqrt[3]{d}+\sqrt[3]{x(-a-b)+ab+x^2}-4a\sqrt[3]{d}\,x+2\sqrt[3]{d}\,x^2}\right)}{\sqrt[3]{d}}-\frac{\log\left(a^4d-4a^3dx+\sqrt[3]{x(-a-b)+ab+x^2}\left(a^2d^{2/3}-2ad^{2/3}x+d^{2/3}x^2\right)+6a^2dx^2+\sqrt[3]{d}\left(x(-a-b)+ab+x^2\right)^{2/3}-4adx^3+dx^4\right)}{2\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*(a - 5*b)) - (3*a + 5*b)*x + 4*x^2)/(((-a + x)*(-b
+ x))^(1/3)*(b - a^5*d - (1 - 5*a^4*d)*x - 10*a^3*d*x^2 + 10*a^2*d*x^3 - 5*
a*d*x^4 + d*x^5)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(a*b + (-a - b)*x + x^2)^(1/3))/(2*a^2*d^(1/3) - 4
*a*d^(1/3)*x + 2*d^(1/3)*x^2 + (a*b + (-a - b)*x + x^2)^(1/3))])/d^(1/3) +
Log[a^2*Sqrt[d] - 2*a*Sqrt[d]*x + Sqrt[d]*x^2 - d^(1/6)*(a*b + (-a - b)*x +
 x^2)^(1/3)]/d^(1/3) - Log[a^4*d - 4*a^3*d*x + 6*a^2*d*x^2 - 4*a*d*x^3 + d*
x^4 + d^(1/3)*(a*b + (-a - b)*x + x^2)^(2/3) + (a*b + (-a - b)*x + x^2)^(1/
3)*(a^2*d^(2/3) - 2*a*d^(2/3)*x + d^(2/3)*x^2)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a-5*b)-(3*a+5*b)*x+4*x^2)/((-a+x)*(-b+x))^(1/3)/(b-a^5*d-(-5
*a^4*d+1)*x-10*a^3*d*x^2+10*a^2*d*x^3-5*a*d*x^4+d*x^5),x, algorithm="fricas
")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a-5\,b)a+(3\,a+5\,b)x-4\,x^2}{\left(a^5d+10\,a^3dx^2-10\,a^2dx^3+5\,adx^4-dx^5-\left(5\,a^4d-1\right)x-b\right)((a-x)(b-x))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a-5*b)-(3*a+5*b)*x+4*x^2)/((-a+x)*(-b+x))^(1/3)/(b-a^5*d-(-5
*a^4*d+1)*x-10*a^3*d*x^2+10*a^2*d*x^3-5*a*d*x^4+d*x^5),x, algorithm="giac")
```

```
[Out] integrate(((a - 5*b)*a + (3*a + 5*b)*x - 4*x^2)/((a^5*d + 10*a^3*d*x^2 - 10
*a^2*d*x^3 + 5*a*d*x^4 - d*x^5 - (5*a^4*d - 1)*x - b)*((a - x)*(b - x))^(1/
3)), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{-a\,(a-5b)-(3a+5b)\,x+4x^2}{\left((-a+x)\,(-b+x)\right)^{\frac{1}{3}}\left(b-a^5d-\left(-5a^4d+1\right)x-10a^3d\,x^2+10a^2d\,x^3-5ad\,x^4+d\,x^5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*(a-5*b)-(3*a+5*b)*x+4*x^2)/((-a+x)*(-b+x))^(1/3)/(b-a^5*d-(-5*a^4*d
+1)*x-10*a^3*d*x^2+10*a^2*d*x^3-5*a*d*x^4+d*x^5),x)
```

```
[Out] int((-a*(a-5*b)-(3*a+5*b)*x+4*x^2)/((-a+x)*(-b+x))^(1/3)/(b-a^5*d-(-5*a^4*d
+1)*x-10*a^3*d*x^2+10*a^2*d*x^3-5*a*d*x^4+d*x^5),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a-5\,b)a+(3\,a+5\,b)x-4\,x^2}{\left(a^5d+10\,a^3dx^2-10\,a^2dx^3+5\,adx^4-dx^5-\left(5\,a^4d-1\right)x-b\right)((a-x)(b-x))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a-5*b)-(3*a+5*b)*x+4*x^2)/((-a+x)*(-b+x))^(1/3)/(b-a^5*d-(-5
*a^4*d+1)*x-10*a^3*d*x^2+10*a^2*d*x^3-5*a*d*x^4+d*x^5),x, algorithm="maxima
")
```

```
[Out] integrate(((a - 5*b)*a + (3*a + 5*b)*x - 4*x^2)/((a^5*d + 10*a^3*d*x^2 - 10
*a^2*d*x^3 + 5*a*d*x^4 - d*x^5 - (5*a^4*d - 1)*x - b)*((a - x)*(b - x))^(1/
3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{-4x^2 + (3a+5b)\,x + a\,(a-5b)}{((a-x)\,(b-x))^{1/3}\,\left(b - a^5\,d + d\,x^5 + x\,\left(5\,a^4\,d - 1\right) + 10\,a^2\,d\,x^3 - 10\,a^3\,d\,x^2 - 5\,a\,d\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*(a - 5*b) + x*(3*a + 5*b) - 4*x^2)/(((a - x)*(b - x))^(1/3)*(b - a^
5*d + d*x^5 + x*(5*a^4*d - 1) + 10*a^2*d*x^3 - 10*a^3*d*x^2 - 5*a*d*x^4)),x
)
```

```
[Out] -int((a*(a - 5*b) + x*(3*a + 5*b) - 4*x^2)/(((a - x)*(b - x))^(1/3)*(b - a^
5*d + d*x^5 + x*(5*a^4*d - 1) + 10*a^2*d*x^3 - 10*a^3*d*x^2 - 5*a*d*x^4)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-a+x)\,(a-5b+4x)}{\sqrt[3]{(-a+x)\,(-b+x)}\,\left(-a^5d + 5a^4dx - 10a^3dx^2 + 10a^2dx^3 - 5adx^4 + b + dx^5 - x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a-5*b)-(3*a+5*b)*x+4*x**2)/((-a+x)*(-b+x))**(1/3)/(b-a**5*d-
(-5*a**4*d+1)*x-10*a**3*d*x**2+10*a**2*d*x**3-5*a*d*x**4+d*x**5),x)
```

```
[Out] Integral((-a + x)*(a - 5*b + 4*x)/(((-a + x)*(-b + x))**(1/3)*(-a**5*d + 5*
a**4*d*x - 10*a**3*d*x**2 + 10*a**2*d*x**3 - 5*a*d*x**4 + b + d*x**5 - x)),
 x)
```

3.2225 $\int \frac{ab-(a+b)x+x^2}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(a^2-b^2d-2(a-bd)x+(1-d)x^2\right)}\,dx$

Optimal. Leaf size=273

$$-\frac{\log\left(a^2+d^{2/3}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}+\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}\left(\sqrt[3]{d}\,x-\right.\right.}{4\sqrt[3]{d}\,(a-b)}$$

Rubi [A] time = 1.13, antiderivative size = 513, normalized size of antiderivative = 1.88, number of steps used = 10, number of rules used = 6, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {6719, 24, 911, 105, 59, 91}

$$-\frac{(x-a)^{2/3}(x-b)^{4/3}\log\left(2\left(1-\sqrt{d}\right)\left(a+b\sqrt{d}\right)-2(1-d)x\right)}{4\sqrt[3]{d}\,(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}-\frac{(x-a)^{2/3}(x-b)^{4/3}\log\left(2(1-d)x-2\left(\sqrt{d}+1\right)\left(a-b\sqrt{d}\right)\right)}{4\sqrt[3]{d}\,(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{3(x-a)^{2/3}(x-b)^{4/3}\log\left(-\sqrt[3]{x-a}-\sqrt[6]{d}\sqrt[3]{x-b}\right)}{4\sqrt[3]{d}\,(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{3(x-a)^{2/3}(x-b)^{4/3}\log\left(\sqrt[6]{d}\sqrt[3]{x-b}-\sqrt[3]{x-a}\right)}{4\sqrt[3]{d}\,(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{\sqrt{3}(x-a)^{2/3}(x-b)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[3]{x-b}}{\sqrt{3}\sqrt[3]{x-a}}\right)}{2\sqrt[3]{d}\,(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{\sqrt{3}(x-a)^{2/3}(x-b)^{4/3}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[3]{x-b}}{\sqrt{3}\sqrt[3]{x-a}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{d}\,(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b - (a + b)*x + x^2)/(((-a + x)*(-b + x)^2)^(2/3)*(a^2 - b^2*d - 2*(
a - b*d)*x + (1 - d)*x^2)),x]
```

```
[Out] (Sqrt[3]*(-a + x)^(2/3)*(-b + x)^(4/3)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(-b +
x)^(1/3))/(Sqrt[3]*(-a + x)^(1/3))])/(2*(a - b)*d^(1/3)*(-((a - x)*(b - x)^
2))^(2/3)) + (Sqrt[3]*(-a + x)^(2/3)*(-b + x)^(4/3)*ArcTan[1/Sqrt[3] + (2*d
^(1/6)*(-b + x)^(1/3))/(Sqrt[3]*(-a + x)^(1/3))])/(2*(a - b)*d^(1/3)*(-((a
- x)*(b - x)^2))^(2/3)) - ((-a + x)^(2/3)*(-b + x)^(4/3)*Log[2*(1 - Sqrt[d]
)*(a + b*Sqrt[d]) - 2*(1 - d)*x])/(4*(a - b)*d^(1/3)*(-((a - x)*(b - x)^2))
^(2/3)) - ((-a + x)^(2/3)*(-b + x)^(4/3)*Log[-2*(1 + Sqrt[d])*(a - b*Sqrt[d
]) + 2*(1 - d)*x])/(4*(a - b)*d^(1/3)*(-((a - x)*(b - x)^2))^(2/3)) + (3*(-
a + x)^(2/3)*(-b + x)^(4/3)*Log[-(-a + x)^(1/3) - d^(1/6)*(-b + x)^(1/3)])/
(4*(a - b)*d^(1/3)*(-((a - x)*(b - x)^2))^(2/3)) + (3*(-a + x)^(2/3)*(-b +
x)^(4/3)*Log[-(-a + x)^(1/3) + d^(1/6)*(-b + x)^(1/3)])/(4*(a - b)*d^(1/3)*
(-((a - x)*(b - x)^2))^(2/3))
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{ab-(a+b)x+x^2}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(a^2-b^2d-2(a-bd)x+(1-d)x^2\right)}\,dx = \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{ab-(a+b}{(-a+x)^{2/3}(-b+x)^{4/3}\left(a^2-b^2\right.}}{\left((-a+x)(-b+x)^2\right)^{2/3}}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{\sqrt[3]{-a+x}}{\sqrt[3]{-b+x}\left(a^2-b^2d-2(a-bd)\right.}}{\left((-a+x)(-b+x)^2\right)^{2/3}}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \left(\frac{(1-d)\sqrt[3]{}}{(a-b)\sqrt{d}\sqrt[3]{-b+x}\left(2a-2(\right.}\right.}{\left((-\right.}$$

$$= \frac{\left((1-d)(-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{\sqrt[3]{}}{\sqrt[3]{-b+x}\left(2a-2(a-\right.}}{(a-b)\sqrt{d}\left((-a+x)(-b+x)^2\right.}$$

$$= \frac{\left((1-d)(-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{}{(-a+x)^{2/3}\sqrt[3]{-b+x}}}{\left(1-\sqrt{d}\right)\left((-a+x)(-b+\right.}$$

$$= \frac{\sqrt{3}(-a+x)^{2/3}(-b+x)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[3]{-b}}{\sqrt{3}\sqrt[3]{-a}}\right.}{2(a-b)\sqrt[3]{d}\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}$$

Mathematica [C] time = 0.28, size = 88, normalized size = 0.32

$$-\frac{3\sqrt[3]{(x-a)(b-x)^2}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{\sqrt{d}(b-x)}{x-a}\right)+{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{\sqrt{d}(x-b)}{x-a}\right)\right)}{4(a-b)(x-a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b - (a + b)*x + x^2)/(((-a + x)*(-b + x)^2)^(2/3)*(a^2 - b^2*d
 - 2*(a - b*d)*x + (1 - d)*x^2)),x]
```

[Out] (-3*((b - x)^2*(-a + x))^(1/3)*(Hypergeometric2F1[2/3, 1, 5/3, (Sqrt[d]*(b - x))/(-a + x)] + Hypergeometric2F1[2/3, 1, 5/3, (Sqrt[d]*(-b + x))/(-a + x)]))/(4*(a - b)*(-a + x))

IntegrateAlgebraic [A] time = 6.54, size = 273, normalized size = 1.00

$$-\frac{\log\left(a^2+d^{2/3}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}+\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}\left(\sqrt[3]{d}x-a\sqrt[3]{d}\right)-2ax+x^2\right)}{4\sqrt[3]{d}(a-b)}+\frac{\log\left(\sqrt[3]{d}\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}+a-x\right)}{2\sqrt[3]{d}(a-b)}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}a-\sqrt{3}x}{-2\sqrt[3]{d}\sqrt[3]{x(2ab+b^2)-ab^2+x^2(-a-2b)+x^3}+a-x}\right)}{2\sqrt[3]{d}(a-b)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b - (a + b)*x + x^2)/(((-a + x)*(-b + x)^2)^(2/3)*(a^
2 - b^2*d - 2*(a - b*d)*x + (1 - d)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*a - Sqrt[3]*x)/(a - x - 2*d^(1/3)*(-(a*b^2) + (2*a
*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3))])/(2*(a - b)*d^(1/3)) + Log[a -
x + d^(1/3)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3)]/(2*(
a - b)*d^(1/3)) - Log[a^2 - 2*a*x + x^2 + (-(a*d^(1/3)) + d^(1/3)*x)*(-(a*b
^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3) + d^(2/3)*(-(a*b^2) + (
2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(2/3)]/(4*(a - b)*d^(1/3))
```

fricas [A] time = 0.65, size = 740, normalized size = 2.71

$$\left[-\frac{\sqrt{3}d\sqrt{\frac{(-d)^{\frac{1}{3}}}{d}}\log\left(-\frac{b^2d+(d+2)x^2+2a^2-2(bd+2a)x+\sqrt{3}\left(2\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{1}{3}}(a-x)(-d)^{\frac{2}{3}}+\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{2}{3}}d-\left(b^2d-2bdx+dx^2\right)(-d)^{\frac{1}{3}}\right)\sqrt{\frac{(-d)^{\frac{1}{3}}}{d}}-3\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{2}{3}}(-d)^{\frac{2}{3}}}{b^2d+(d-1)x^2-a^2-2(bd-a)x}\right)+(-d)^{\frac{2}{3}}\log\left(\frac{\left(b^2-2bx+x^2\right)(-d)^{\frac{2}{3}}-\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{1}{3}}(a-x)-\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{2}{3}}(-d)^{\frac{1}{3}}}{b^2-2bx+x^2}\right)-2(-d)^{\frac{2}{3}}\log\left(\frac{\left(b^2-2bx+x^2\right)(-d)^{\frac{1}{3}}+\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{2}{3}}}{b^2-2bx+x^2}\right)}{4(a-b)d}, \frac{2\sqrt{3}d\sqrt{-\frac{(-d)^{\frac{1}{3}}}{d}}\arctan\left(-\frac{\sqrt{3}\left(\left(b^2-2bx+x^2\right)(-d)^{\frac{1}{3}}-2\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{2}{3}}\right)\sqrt{-\frac{(-d)^{\frac{1}{3}}}{d}}}{3\left(b^2-2bx+x^2\right)}\right)-(-d)^{\frac{2}{3}}\log\left(\frac{\left(b^2-2bx+x^2\right)(-d)^{\frac{2}{3}}-\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{1}{3}}(a-x)-\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{2}{3}}(-d)^{\frac{1}{3}}}{b^2-2bx+x^2}\right)+2(-d)^{\frac{2}{3}}\log\left(\frac{\left(b^2-2bx+x^2\right)(-d)^{\frac{1}{3}}+\left(-ab^2-(a+2b)x^2+x^3+(2ab+b^2)x\right)^{\frac{2}{3}}}{b^2-2bx+x^2}\right)}{4(a-b)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-(a+b)*x+x^2)/((-a+x)*(-b+x)^2)^(2/3)/(a^2-b^2*d-2*(-b*d+a)*x
+(1-d)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(3)*d*sqrt((-d)^(1/3)/d)*log(-(b^2*d + (d + 2)*x^2 + 2*a^2 - 2*(
b*d + 2*a)*x + sqrt(3)*(2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^
(1/3)*(a - x)*(-d)^(2/3) + (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)
^(2/3)*d - (b^2*d - 2*b*d*x + d*x^2)*(-d)^(1/3))*sqrt((-d)^(1/3)/d) - 3*(-a
*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*(-d)^(2/3))/(b^2*d + (d
 - 1)*x^2 - a^2 - 2*(b*d - a)*x)) + (-d)^(2/3)*log(((b^2 - 2*b*x + x^2)*(-d
)^(2/3) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(a - x) -
(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*(-d)^(1/3))/(b^2 - 2
*b*x + x^2)) - 2*(-d)^(2/3)*log(((b^2 - 2*b*x + x^2)*(-d)^(1/3) + (-a*b^2 -
 (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3))/(b^2 - 2*b*x + x^2)))/((a -
b)*d), 1/4*(2*sqrt(3)*d*sqrt(-(-d)^(1/3)/d)*arctan(-1/3*sqrt(3)*((b^2 - 2*b
*x + x^2)*(-d)^(1/3) - 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(
2/3))*sqrt(-(-d)^(1/3)/d)/(b^2 - 2*b*x + x^2)) - (-d)^(2/3)*log(((b^2 - 2*b
*x + x^2)*(-d)^(2/3) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/
3)*(a - x) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*(-d)^(1
/3))/(b^2 - 2*b*x + x^2)) + 2*(-d)^(2/3)*log(((b^2 - 2*b*x + x^2)*(-d)^(1/3
) + (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3))/(b^2 - 2*b*x +
x^2)))/((a - b)*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ab-(a+b)x+x^2}{\left(-(a-x)(b-x)^2\right)^{\frac{2}{3}}\left(b^2d+(d-1)x^2-a^2-2\,(bd-a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-(a+b)*x+x^2)/((-a+x)*(-b+x)^2)^(2/3)/(a^2-b^2*d-2*(-b*d+a)*x
+(1-d)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-(a*b - (a + b)*x + x^2)/((-(a - x)*(b - x)^2)^(2/3)*(b^2*d + (d
- 1)*x^2 - a^2 - 2*(b*d - a)*x)), x)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{ab-(a+b)\,x+x^{2}}{\left((-a+x)\,(-b+x)^{2}\right)^{\frac{2}{3}}\left(a^{2}-b^{2}d-2\,(-bd+a)\,x+(1-d)\,x^{2}\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b-(a+b)*x+x^2)/((-a+x)*(-b+x)^2)^(2/3)/(a^2-b^2*d-2*(-b*d+a)*x+(1-d)
*x^2),x)
```

```
[Out] int((a*b-(a+b)*x+x^2)/((-a+x)*(-b+x)^2)^(2/3)/(a^2-b^2*d-2*(-b*d+a)*x+(1-d)
*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ab-(a+b)x+x^{2}}{\left(-(a-x)(b-x)^{2}\right)^{\frac{2}{3}}\left(b^{2}d+(d-1)x^{2}-a^{2}-2\,(bd-a)x\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-(a+b)*x+x^2)/((-a+x)*(-b+x)^2)^(2/3)/(a^2-b^2*d-2*(-b*d+a)*x
+(1-d)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((a*b - (a + b)*x + x^2)/((-(a - x)*(b - x)^2)^(2/3)*(b^2*d + (d
- 1)*x^2 - a^2 - 2*(b*d - a)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x^{2}+(-a-b)\,x+a\,b}{\left(-(a-x)\,(b-x)^{2}\right)^{2/3}\left(b^{2}\,d+2\,x\,(a-b\,d)-a^{2}+x^{2}\,(d-1)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*b + x^2 - x*(a + b))/((-(a - x)*(b - x)^2)^(2/3)*(b^2*d + 2*x*(a -
b*d) - a^2 + x^2*(d - 1))),x)
```

```
[Out] int(-(a*b + x^2 - x*(a + b))/((-(a - x)*(b - x)^2)^(2/3)*(b^2*d + 2*x*(a -
b*d) - a^2 + x^2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-(a+b)*x+x**2)/((-a+x)*(-b+x)**2)**(2/3)/(a**2-b**2*d-2*(-b*d
+a)*x+(1-d)*x**2),x)
```

```
[Out] Timed out
```

3.2226 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(-1+x^4\right)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=273

$$\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{2}-1\right)}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{\sqrt{x^4+1}+}}{\sqrt{x^4+1}+x^2+1}\right.$$

Rubi [C] time = 1.69, antiderivative size = 309, normalized size of antiderivative = 1.13, number of steps used = 26, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {6725, 2133, 725, 206}

$$\frac{1}{8}\sqrt{1+i}\tanh^{-1}\left(\frac{1-x}{\sqrt{1+i}\sqrt{1-ix^2}}\right)-\frac{1}{8}\sqrt{1-i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)+\frac{1}{8}\sqrt{1-i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{8}\sqrt{1+i}\tanh^{-1}\left(\frac{x+1}{\sqrt{1+i}\sqrt{1-ix^2}}\right)+\frac{1}{8}\sqrt{1-i}\tanh^{-1}\left(\frac{1-x}{\sqrt{1-i}\sqrt{1+ix^2}}\right)+\frac{1}{8}\sqrt{1+i}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)-\frac{1}{8}\sqrt{1+i}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)-\frac{1}{8}\sqrt{1-i}\tanh^{-1}\left(\frac{x+1}{\sqrt{1-i}\sqrt{1+ix^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((-1 + x^4)*Sqrt[1 + x^4]),x]
```

```
[Out] (Sqrt[1 + I]*ArcTanh[(1 - x)/(Sqrt[1 + I]*Sqrt[1 - I*x^2])])/8 - (Sqrt[1 -
I]*ArcTanh[(1 - I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/8 + (Sqrt[1 - I]*ArcTa
nh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/8 - (Sqrt[1 + I]*ArcTanh[(1 +
x)/(Sqrt[1 + I]*Sqrt[1 - I*x^2])])/8 + (Sqrt[1 - I]*ArcTanh[(1 - x)/(Sqrt[1
 - I]*Sqrt[1 + I*x^2])])/8 + (Sqrt[1 + I]*ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sq
rt[1 + I*x^2])])/8 - (Sqrt[1 + I]*ArcTanh[(1 + I*x)/(Sqrt[1 + I]*Sqrt[1 + I
*x^2])])/8 - (Sqrt[1 - I]*ArcTanh[(1 + x)/(Sqrt[1 - I]*Sqrt[1 + I*x^2])])/8
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(-1+x^4\right)\sqrt{1+x^4}}\,dx = \int \left(-\frac{\sqrt{x^2+\sqrt{1+x^4}}}{2\left(1-x^2\right)\sqrt{1+x^4}} - \frac{\sqrt{x^2+\sqrt{1+x^4}}}{2\left(1+x^2\right)\sqrt{1+x^4}}\right)dx$$
$$= -\left(\frac{1}{2}\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx\right) - \frac{1}{2}\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(1+x^2\right)\sqrt{1+x^4}}\,dx$$
$$= -\left(\frac{1}{2}\int \left(\frac{i\sqrt{x^2+\sqrt{1+x^4}}}{2(i-x)\sqrt{1+x^4}} + \frac{i\sqrt{x^2+\sqrt{1+x^4}}}{2(i+x)\sqrt{1+x^4}}\right)dx\right) - \frac{1}{2}\int \left(\frac{\sqrt{x^2+\sqrt{1+x^4}}}{2(1-x)\sqrt{1+x^4}} + \frac{\sqrt{x^2}}{2(1-}\right.$$
$$= -\left(\frac{1}{4}i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(i-x)\sqrt{1+x^4}}\,dx\right) - \frac{1}{4}i\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(i+x)\sqrt{1+x^4}}\,dx - \frac{1}{4}\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1-x)\sqrt{1+x^4}}\,dx$$
$$= -\left(\left(-\frac{1}{8}+\frac{i}{8}\right)\int \frac{1}{(i-x)\sqrt{1+ix^2}}\,dx\right) - \left(-\frac{1}{8}+\frac{i}{8}\right)\int \frac{1}{(i+x)\sqrt{1+ix^2}}\,dx - \left(\frac{1}{8}-\frac{i}{8}\right)\int$$
$$= -\left(\left(-\frac{1}{8}-\frac{i}{8}\right)\text{Subst}\left(\int \frac{1}{(1+i)-x^2}\,dx, x, \frac{-1-x}{\sqrt{1-ix^2}}\right)\right) - \left(-\frac{1}{8}-\frac{i}{8}\right)\text{Subst}\left(\int \frac{1}{(1+i)-}\right.$$
$$= \frac{1}{8}\sqrt{1+i}\,\tanh^{-1}\left(\frac{1-x}{\sqrt{1+i}\sqrt{1-ix^2}}\right) - \frac{1}{8}\sqrt{1-i}\,\tanh^{-1}\left(\frac{1-ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) + \frac{1}{8}\sqrt{1-i}\,\text{t}$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\left(-1+x^4\right)\sqrt{1+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((-1 + x^4)*Sqrt[1 + x^4]),x]
```

```
[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((-1 + x^4)*Sqrt[1 + x^4]), x]
```

IntegrateAlgebraic [A] time = 1.82, size = 389, normalized size = 1.42

$$\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2 + Sqrt[1 + x^4]]/((-1 + x^4)*Sqrt[1 + x^4]),x]
```

```
[Out] (Sqrt[(-1 + Sqrt[2])/2]*ArcTan[(-Sqrt[-1/2 + 1/Sqrt[2]] + Sqrt[-1/2 + 1/Sqr
t[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 + x^
4]])])/2 - (Sqrt[(-1 + Sqrt[2])/2]*ArcTan[(-Sqrt[1/2 + 1/Sqrt[2]] + Sqrt[1/
2 + 1/Sqrt[2]]*x^2 + Sqrt[1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sqr
t[1 + x^4]])])/2 - (Sqrt[(1 + Sqrt[2])/2]*ArcTanh[(-Sqrt[-1/2 + 1/Sqrt[2]]
+ Sqrt[-1/2 + 1/Sqrt[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqr
t[x^2 + Sqrt[1 + x^4]])])/2 - (Sqrt[(1 + Sqrt[2])/2]*ArcTanh[(-Sqrt[1/2 + 1
/Sqrt[2]] + Sqrt[1/2 + 1/Sqrt[2]]*x^2 + Sqrt[1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4]
)/(x*Sqrt[x^2 + Sqrt[1 + x^4]])])/2
```

fricas [A] time = 14.71, size = 373, normalized size = 1.37

$$-\frac{1}{4}\sqrt{2}\sqrt{\sqrt{2}-1}\arctan\left(\frac{\left(x^8-2x^4-2\left(2x^7-2x^3+\sqrt{2}\left(3x^7+x^3\right)-\left(4\sqrt{2}x^5+5x^5-x\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}\right)\sqrt{\sqrt{2}-1}-2\sqrt{2}\left(x^8+3x^4\right)-2\left(3x^6+x^2+\sqrt{2}\left(x^6-x^2\right)\right)\sqrt{x^4+1}+1}{7x^8+10x^4-1}\right)-\frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}+1}\log\left(\frac{2\left(\sqrt{2}x^3+2x^3+\sqrt{x^4+1}\left(\sqrt{2}x+x\right)\right)\sqrt{x^2+\sqrt{x^4+1}}+\left(2\sqrt{2}x^4+3x^4+2\sqrt{x^4+1}\left(\sqrt{2}x^2+x^2\right)+1\right)\sqrt{\sqrt{2}+1}}{x^4-1}\right)+\frac{1}{16}\sqrt{2}\sqrt{\sqrt{2}+1}\log\left(\frac{2\left(\sqrt{2}x^3+2x^3+\sqrt{x^4+1}\left(\sqrt{2}x+x\right)\right)\sqrt{x^2+\sqrt{x^4+1}}-\left(2\sqrt{2}x^4+3x^4+2\sqrt{x^4+1}\left(\sqrt{2}x^2+x^2\right)+1\right)\sqrt{\sqrt{2}+1}}{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4-1)/(x^4+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/4*sqrt(2)*sqrt(sqrt(2) - 1)*arctan((x^8 - 2*x^4 - 2*(2*x^7 - 2*x^3 + sqr
t(2)*(3*x^7 + x^3) - (4*sqrt(2)*x^5 + 5*x^5 - x)*sqrt(x^4 + 1))*sqrt(x^2 +
sqrt(x^4 + 1))*sqrt(sqrt(2) - 1) - 2*sqrt(2)*(x^8 + 3*x^4) - 2*(3*x^6 + x^2
 + sqrt(2)*(x^6 - x^2))*sqrt(x^4 + 1) + 1)/(7*x^8 + 10*x^4 - 1)) - 1/16*sqr
t(2)*sqrt(sqrt(2) + 1)*log((2*(sqrt(2)*x^3 + 2*x^3 + sqrt(x^4 + 1)*(sqrt(2)
*x + x))*sqrt(x^2 + sqrt(x^4 + 1)) + (2*sqrt(2)*x^4 + 3*x^4 + 2*sqrt(x^4 +
1)*(sqrt(2)*x^2 + x^2) + 1)*sqrt(sqrt(2) + 1))/(x^4 - 1)) + 1/16*sqrt(2)*sq
rt(sqrt(2) + 1)*log((2*(sqrt(2)*x^3 + 2*x^3 + sqrt(x^4 + 1)*(sqrt(2)*x + x)
)*sqrt(x^2 + sqrt(x^4 + 1)) - (2*sqrt(2)*x^4 + 3*x^4 + 2*sqrt(x^4 + 1)*(sqr
t(2)*x^2 + x^2) + 1)*sqrt(sqrt(2) + 1))/(x^4 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4-1)/(x^4+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x^4 - 1)), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\left(x^4-1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4-1)/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4-1)/(x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4-1)/(x^4+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x^4 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\left(x^4-1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 - 1)*(x^4 + 1)^(1/2)),x)
```

```
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 - 1)*(x^4 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x-1)(x+1)\left(x^2+1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**4-1)/(x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(x**4
+ 1)), x)
```

3.2227 $\int \frac{-1+x^6}{\sqrt[3]{-x^2+x^4}\left(1+x^6\right)} \, dx$

Optimal. Leaf size=274

$$-\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt[3]{x^4-x^2}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^4-x^2}}\right)}{3\sqrt[3]{2}}-\frac{1}{3}\tan^{-1}\left(\frac{x\sqrt[3]{x^4-x^2}}{\left(x^4-x^2\right)^{2/3}-x^2}\right)-\frac{\tan^{-1}\left(\frac{2^{2/3}x\sqrt[3]{x^4-x^2}}{\sqrt[3]{2}\left(x^4-x^2\right)^{2/3}-2x^2}\right)}{6\sqrt[3]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt{3}}+\frac{(x^4}{}}{x\sqrt[3]{x^4}}\right)}{\sqrt{3}}$$

Rubi [F] time = 2.73, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-1+x^6}{\sqrt[3]{-x^2+x^4}\left(1+x^6\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(-1 + x^6)/((-x^2 + x^4)^(1/3)*(1 + x^6)),x]
```

```
[Out] ((-1)^(1/18)*(1 + I*Sqrt[3])*x^(2/3)*(-1 + x^2)^(1/3)*Defer[Subst][Defer[In
t][(-1 + x^6)^(2/3)/((-1)^(1/18) - x), x], x, x^(1/3)])/(6*(-x^2 + x^4)^(1/
3)) + ((-1)^(1/18)*(1 + I*Sqrt[3])*x^(2/3)*(-1 + x^2)^(1/3)*Defer[Subst][De
fer[Int][(-1 + x^6)^(2/3)/((-1)^(1/18) + x), x], x, x^(1/3)])/(6*(-x^2 + x^
4)^(1/3)) + ((-1)^(1/18)*(1 - I*Sqrt[3])*x^(2/3)*(-1 + x^2)^(1/3)*Defer[Sub
st][Defer[Int][(-1 + x^6)^(2/3)/((-1)^(1/18) - (-1)^(1/9)*x), x], x, x^(1/3
)])/(6*(-x^2 + x^4)^(1/3)) + ((-1)^(1/18)*(1 - I*Sqrt[3])*x^(2/3)*(-1 + x^2
)^(1/3)*Defer[Subst][Defer[Int][(-1 + x^6)^(2/3)/((-1)^(1/18) + (-1)^(1/9)*
x), x], x, x^(1/3)])/(6*(-x^2 + x^4)^(1/3)) + ((-1)^(1/18)*x^(2/3)*(-1 + x^
2)^(1/3)*Defer[Subst][Defer[Int][(-1 + x^6)^(2/3)/((-1)^(1/18) - (-1)^(2/9)
*x), x], x, x^(1/3)])/(6*(-x^2 + x^4)^(1/3)) + ((-1)^(1/18)*x^(2/3)*(-1 + x
^2)^(1/3)*Defer[Subst][Defer[Int][(-1 + x^6)^(2/3)/((-1)^(1/18) + (-1)^(2/9
)*x), x], x, x^(1/3)])/(6*(-x^2 + x^4)^(1/3)) + ((-1)^(1/18)*(1 + I*Sqrt[3]
)*x^(2/3)*(-1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(-1 + x^6)^(2/3)/((-1)^(
1/18) - (-1)^(1/3)*x), x], x, x^(1/3)])/(6*(-x^2 + x^4)^(1/3)) + ((-1)^(1/1
8)*(1 + I*Sqrt[3])*x^(2/3)*(-1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(-1 + x
^6)^(2/3)/((-1)^(1/18) + (-1)^(1/3)*x), x], x, x^(1/3)])/(6*(-x^2 + x^4)^(1
/3)) + ((-1)^(1/18)*(1 - I*Sqrt[3])*x^(2/3)*(-1 + x^2)^(1/3)*Defer[Subst][D
efer[Int][(-1 + x^6)^(2/3)/((-1)^(1/18) - (-1)^(4/9)*x), x], x, x^(1/3)])/(
6*(-x^2 + x^4)^(1/3)) + ((-1)^(1/18)*(1 - I*Sqrt[3])*x^(2/3)*(-1 + x^2)^(1/
3)*Defer[Subst][Defer[Int][(-1 + x^6)^(2/3)/((-1)^(1/18) + (-1)^(4/9)*x), x
], x, x^(1/3)])/(6*(-x^2 + x^4)^(1/3)) + ((-1)^(1/18)*x^(2/3)*(-1 + x^2)^(1
/3)*Defer[Subst][Defer[Int][(-1 + x^6)^(2/3)/((-1)^(1/18) - (-1)^(5/9)*x),
x], x, x^(1/3)])/(6*(-x^2 + x^4)^(1/3)) + ((-1)^(1/18)*x^(2/3)*(-1 + x^2)^(
1/3)*Defer[Subst][Defer[Int][(-1 + x^6)^(2/3)/((-1)^(1/18) + (-1)^(5/9)*x),
 x], x, x^(1/3)])/(6*(-x^2 + x^4)^(1/3)) + ((-1)^(1/18)*(1 + I*Sqrt[3])*x^(
2/3)*(-1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(-1 + x^6)^(2/3)/((-1)^(1/18)
 - (-1)^(2/3)*x), x], x, x^(1/3)])/(6*(-x^2 + x^4)^(1/3)) + ((-1)^(1/18)*(1
 + I*Sqrt[3])*x^(2/3)*(-1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(-1 + x^6)^(
2/3)/((-1)^(1/18) + (-1)^(2/3)*x), x], x, x^(1/3)])/(6*(-x^2 + x^4)^(1/3))
+ ((-1)^(1/18)*(1 - I*Sqrt[3])*x^(2/3)*(-1 + x^2)^(1/3)*Defer[Subst][Defer[
Int][(-1 + x^6)^(2/3)/((-1)^(1/18) - (-1)^(7/9)*x), x], x, x^(1/3)])/(6*(-x
^2 + x^4)^(1/3)) + ((-1)^(1/18)*(1 - I*Sqrt[3])*x^(2/3)*(-1 + x^2)^(1/3)*De
fer[Subst][Defer[Int][(-1 + x^6)^(2/3)/((-1)^(1/18) + (-1)^(7/9)*x), x], x,
 x^(1/3)])/(6*(-x^2 + x^4)^(1/3)) + ((-1)^(1/18)*x^(2/3)*(-1 + x^2)^(1/3)*D
efer[Subst][Defer[Int][(-1 + x^6)^(2/3)/((-1)^(1/18) - (-1)^(8/9)*x), x], x
```

```
, x^(1/3)])/(6*(-x^2 + x^4)^(1/3)) + ((-1)^(1/18)*x^(2/3)*(-1 + x^2)^(1/3)*
Defer[Subst][Defer[Int][(-1 + x^6)^(2/3)/((-1)^(1/18) + (-1)^(8/9)*x), x],
x, x^(1/3)])/(6*(-x^2 + x^4)^(1/3))
```

Rubi steps

$$\int \frac{-1+x^6}{\sqrt[3]{-x^2+x^4}\left(1+x^6\right)}\,dx = \frac{\left(x^{2/3}\sqrt[3]{-1+x^2}\right)\int \frac{-1+x^6}{x^{2/3}\sqrt[3]{-1+x^2}\left(1+x^6\right)}\,dx}{\sqrt[3]{-x^2+x^4}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{-1+x^2}\right)\int \frac{\left(-1+x^2\right)^{2/3}\left(1+x^2+x^4\right)}{x^{2/3}\left(1+x^6\right)}\,dx}{\sqrt[3]{-x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-1+x^2}\right)\mathrm{Subst}\left(\int \frac{\left(-1+x^6\right)^{2/3}\left(1+x^6+x^{12}\right)}{1+x^{18}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-1+x^2}\right)\mathrm{Subst}\left(\int\left(\frac{\left(\sqrt[18]{-1}+(-1)^{7/18}+(-1)^{13/18}\right)\left(-1+x^6\right)^{2/3}}{18\left(\sqrt[18]{-1}-x\right)}+\frac{\left(\sqrt[18]{-1}+(-1)^{7/18}+(-1)^{13/18}\right)\cdots}{18\left(\sqrt[18]{-1}+x\right)}\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(\sqrt[18]{-1}\,x^{2/3}\sqrt[3]{-1+x^2}\right)\mathrm{Subst}\left(\int \frac{\left(-1+x^6\right)^{2/3}}{\sqrt[18]{-1}-(-1)^{2/9}x}\,dx, x, \sqrt[3]{x}\right)}{6\sqrt[3]{-x^2+x^4}} + \frac{\left(\sqrt[18]{-1}\,x^{2/3}\sqrt[3]{-1+x^2}\right)\mathrm{Su}\cdots}{6\sqrt[3]{-\cdots}}$$

Mathematica [F] time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{-1+x^6}{\sqrt[3]{-x^2+x^4}\left(1+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^6)/((-x^2 + x^4)^(1/3)*(1 + x^6)),x]
```

```
[Out] Integrate[(-1 + x^6)/((-x^2 + x^4)^(1/3)*(1 + x^6)), x]
```

IntegrateAlgebraic [C] time = 1.02, size = 269, normalized size = 0.98

$$-\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt[3]{x^4-x^2}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^4-x^2}}\right)}{3\sqrt[3]{2}}-\frac{1}{3}i\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{\left(1-i\sqrt{3}\right)x}{2\sqrt[3]{x^4-x^2}}\right)+\frac{1}{3}i\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{\left(1+i\sqrt{3}\right)x}{2\sqrt[3]{x^4-x^2}}\right)-\frac{\tan^{-1}\left(\frac{2^{2/3}x\sqrt[3]{x^4-x^2}}{\sqrt[3]{2}\left(x^4-x^2\right)^{2/3}-2x^2}\right)}{6\sqrt[3]{2}}-\frac{\tanh^{-1}\left(\frac{\frac{\sqrt[3]{2}x^2}{\sqrt{3}}+\frac{\left(x^4-x^2\right)^{2/3}}{\sqrt[3]{2}\sqrt{3}}}{x\sqrt[3]{x^4-x^2}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^6)/((-x^2 + x^4)^(1/3)*(1 + x^6)),x]
```

```
[Out] (-2*ArcTan[x/(-x^2 + x^4)^(1/3)])/3 - ArcTan[(2^(1/3)*x)/(-x^2 + x^4)^(1/3)
]/(3*2^(1/3)) - (I/3)*(-I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x)/(2*(-x^2 +
x^4)^(1/3))] + (I/3)*(I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x)/(2*(-x^2 + x^
4)^(1/3))] - ArcTan[(2^(2/3)*x*(-x^2 + x^4)^(1/3))/(-2*x^2 + 2^(1/3)*(-x^2
+ x^4)^(2/3))]/(6*2^(1/3)) - ArcTanh[((2^(1/3)*x^2)/Sqrt[3] + (-x^2 + x^4)^
(2/3)/(2^(1/3)*Sqrt[3]))/(x*(-x^2 + x^4)^(1/3))]/(2*2^(1/3)*Sqrt[3])
```

fricas [B] time = 16.94, size = 3724, normalized size = 13.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^4-x^2)^(1/3)/(x^6+1),x, algorithm="fricas")
```

```
[Out] -1/96*sqrt(3)*2^(2/3)*log(8500000*(8*sqrt(3)*2^(1/3)*(x^4 - x^2) + 2*(x^4 -
 x^2)^(2/3)*(sqrt(3)*2^(2/3)*(x^2 - 1) + 6*2^(2/3)*x) + 2^(1/3)*(x^5 + 2*x^
3 + x) + 4*(x^4 - x^2)^(1/3)*(3*x^3 + 2*sqrt(3)*x^2 - 3*x))/(x^5 + 2*x^3 +
x)) - 1/96*sqrt(3)*2^(2/3)*log(2125000*(8*sqrt(3)*2^(1/3)*(x^4 - x^2) + 2*(
x^4 - x^2)^(2/3)*(sqrt(3)*2^(2/3)*(x^2 - 1) + 6*2^(2/3)*x) + 2^(1/3)*(x^5 +
 2*x^3 + x) + 4*(x^4 - x^2)^(1/3)*(3*x^3 + 2*sqrt(3)*x^2 - 3*x))/(x^5 + 2*x
^3 + x)) + 1/96*sqrt(3)*2^(2/3)*log(-2125000*(8*sqrt(3)*2^(1/3)*(x^4 - x^2)
 + 2*(x^4 - x^2)^(2/3)*(sqrt(3)*2^(2/3)*(x^2 - 1) - 6*2^(2/3)*x) - 2^(1/3)*
(x^5 + 2*x^3 + x) - 4*(x^4 - x^2)^(1/3)*(3*x^3 - 2*sqrt(3)*x^2 - 3*x))/(x^5
 + 2*x^3 + x)) + 1/96*sqrt(3)*2^(2/3)*log(-8500000*(8*sqrt(3)*2^(1/3)*(x^4
- x^2) + 2*(x^4 - x^2)^(2/3)*(sqrt(3)*2^(2/3)*(x^2 - 1) - 6*2^(2/3)*x) - 2^
(1/3)*(x^5 + 2*x^3 + x) - 4*(x^4 - x^2)^(1/3)*(3*x^3 - 2*sqrt(3)*x^2 - 3*x)
)/(x^5 + 2*x^3 + x)) + 1/12*2^(2/3)*arctan(-(74071498415429632*x^9 + 164527
9755446275808*x^8 - 2346817955632029696*x^7 - 11516958288123930656*x^6 + 57
30636889080074240*x^5 + 11516958288123930656*x^4 - 2346817955632029696*x^3
- 1645279755446275808*x^2 - 125*sqrt(34)*(4*sqrt(3)*2^(1/3)*(78465570355328
*x^9 - 3301419835659*x^8 + 1100839094578688*x^7 - 595767752585659*x^6 - 361
4058455553280*x^5 + 595767752585659*x^4 + 1100839094578688*x^3 + 3301419835
659*x^2 + 78465570355328*x) + 16*(x^4 - x^2)^(2/3)*(4*sqrt(3)*2^(2/3)*(1513
688563712*x^6 + 57183135266496*x^5 - 26977277846305*x^4 - 167158338888320*x
^3 + 26977277846305*x^2 + 57183135266496*x - 1513688563712) - 2^(2/3)*(7916
3286177664*x^6 - 56815411732213*x^5 - 187311276664960*x^4 + 112551186315710
*x^3 + 187311276664960*x^2 - 56815411732213*x - 79163286177664)) - 2^(1/3)*
(36167723835659*x^9 + 4738598437685248*x^8 - 1343569332842636*x^7 - 1606940
1562314752*x^6 + 2036119636643410*x^5 + 16069401562314752*x^4 - 13435693328
42636*x^3 - 4738598437685248*x^2 + 36167723835659*x) - 4*(183204669874443*x
^7 + 4116235393055744*x^6 - 2225700627116645*x^5 - 10698715224852480*x^4 +
2225700627116645*x^3 + 4116235393055744*x^2 - 531250*sqrt(3)*(1009306368*x^
7 - 511421263*x^6 - 4316628224*x^5 + 1207618962*x^4 + 4316628224*x^3 - 5114
21263*x^2 - 1009306368*x) - 183204669874443*x)*(x^4 - x^2)^(1/3))*sqrt((8*s
qrt(3)*2^(1/3)*(x^4 - x^2) + 2*(x^4 - x^2)^(2/3)*(sqrt(3)*2^(2/3)*(x^2 - 1)
 + 6*2^(2/3)*x) + 2^(1/3)*(x^5 + 2*x^3 + x) + 4*(x^4 - x^2)^(1/3)*(3*x^3 +
2*sqrt(3)*x^2 - 3*x))/(x^5 + 2*x^3 + x)) - 1062500*(x^4 - x^2)^(2/3)*(2*sqr
t(3)*2^(1/3)*(23651383808*x^6 + 470146644789*x^5 - 226386757120*x^4 - 71809
982630*x^3 + 226386757120*x^2 + 470146644789*x - 23651383808) - 2^(1/3)*(61
8463173263*x^6 - 733160605696*x^5 - 6989546598945*x^4 + 2615047352320*x^3 +
 6989546598945*x^2 - 733160605696*x - 618463173263)) - 265625*sqrt(3)*(6130
12268401*x^9 - 500076281856*x^8 - 1596364015228*x^7 + 3500533972992*x^6 + 1
1774899788070*x^5 - 3500533972992*x^4 - 1596364015228*x^3 + 500076281856*x^
2 + 613012268401*x) - 1062500*(x^4 - x^2)^(1/3)*(sqrt(3)*2^(2/3)*(217120826
737*x^7 + 155432605696*x^6 + 1229224098945*x^5 - 689287352320*x^4 - 1229224
098945*x^3 + 155432605696*x^2 - 217120826737*x) - 2*2^(2/3)*(71795383808*x^
7 + 1283539269789*x^6 - 948546757120*x^5 - 5040931232630*x^4 + 948546757120
*x^3 + 1283539269789*x^2 - 71795383808*x)) + 74071498415429632*x)/(47995856
8556831351*x^9 - 1202832749691437056*x^8 - 12744795130528777828*x^7 + 84198
29247840059392*x^6 + 32209010220853194570*x^5 - 8419829247840059392*x^4 - 1
2744795130528777828*x^3 + 1202832749691437056*x^2 + 479958568556831351*x))
- 1/12*2^(2/3)*arctan((74071498415429632*x^9 + 1645279755446275808*x^8 - 23
46817955632029696*x^7 - 11516958288123930656*x^6 + 5730636889080074240*x^5
+ 11516958288123930656*x^4 - 2346817955632029696*x^3 - 1645279755446275808*
x^2 + 125*sqrt(34)*(4*sqrt(3)*2^(1/3)*(78465570355328*x^9 - 3301419835659*x
^8 + 1100839094578688*x^7 - 595767752585659*x^6 - 3614058455553280*x^5 + 59
5767752585659*x^4 + 1100839094578688*x^3 + 3301419835659*x^2 + 784655703553
28*x) + 16*(x^4 - x^2)^(2/3)*(4*sqrt(3)*2^(2/3)*(1513688563712*x^6 + 571831
35266496*x^5 - 26977277846305*x^4 - 167158338888320*x^3 + 26977277846305*x^
2 + 57183135266496*x - 1513688563712) + 2^(2/3)*(79163286177664*x^6 - 56815
411732213*x^5 - 187311276664960*x^4 + 112551186315710*x^3 + 187311276664960
*x^2 - 56815411732213*x - 79163286177664)) + 2^(1/3)*(36167723835659*x^9 +
```

4738598437685248*x^8 - 1343569332842636*x^7 - 16069401562314752*x^6 + 20361
19636643410*x^5 + 16069401562314752*x^4 - 1343569332842636*x^3 - 4738598437
685248*x^2 + 36167723835659*x) + 4*(183204669874443*x^7 + 4116235393055744*
x^6 - 2225700627116645*x^5 - 10698715224852480*x^4 + 2225700627116645*x^3 +
4116235393055744*x^2 + 531250*sqrt(3)*(1009306368*x^7 - 511421263*x^6 - 43
16628224*x^5 + 1207618962*x^4 + 4316628224*x^3 - 511421263*x^2 - 1009306368
*x) - 183204669874443*x)*(x^4 - x^2)^(1/3))*sqrt(-(8*sqrt(3)*2^(1/3)*(x^4 -
x^2) + 2*(x^4 - x^2)^(2/3)*(sqrt(3)*2^(2/3)*(x^2 - 1) - 6*2^(2/3)*x) - 2^(
1/3)*(x^5 + 2*x^3 + x) - 4*(x^4 - x^2)^(1/3)*(3*x^3 - 2*sqrt(3)*x^2 - 3*x))
/(x^5 + 2*x^3 + x)) + 1062500*(x^4 - x^2)^(2/3)*(2*sqrt(3)*2^(1/3)*(2365138
3808*x^6 + 470146644789*x^5 - 226386757120*x^4 - 71809982630*x^3 + 22638675
7120*x^2 + 470146644789*x - 23651383808) + 2^(1/3)*(618463173263*x^6 - 7331
60605696*x^5 - 6989546598945*x^4 + 2615047352320*x^3 + 6989546598945*x^2 -
733160605696*x - 618463173263)) + 265625*sqrt(3)*(613012268401*x^9 - 500076
281856*x^8 - 1596364015228*x^7 + 3500533972992*x^6 + 11774899788070*x^5 - 3
500533972992*x^4 - 1596364015228*x^3 + 500076281856*x^2 + 613012268401*x) +
1062500*(x^4 - x^2)^(1/3)*(sqrt(3)*2^(2/3)*(217120826737*x^7 + 15543260569
6*x^6 + 1229224098945*x^5 - 689287352320*x^4 - 1229224098945*x^3 + 15543260
5696*x^2 - 217120826737*x) + 2*2^(2/3)*(71795383808*x^7 + 1283539269789*x^6
- 948546757120*x^5 - 5040931232630*x^4 + 948546757120*x^3 + 1283539269789*
x^2 - 71795383808*x)) + 74071498415429632*x)/(479958568556831351*x^9 - 1202
832749691437056*x^8 - 12744795130528777828*x^7 + 8419829247840059392*x^6 +
32209010220853194570*x^5 - 8419829247840059392*x^4 - 12744795130528777828*x
^3 + 1202832749691437056*x^2 + 479958568556831351*x)) + 1/6*2^(2/3)*arctan(
-1/2*(3564544*x^5 + 249106968*x^4 - 21387264*x^3 + 2125000*2^(2/3)*(x^4 - x
^2)^(1/3)*(512*x^3 + 59*x^2 - 512*x) + 1062500*2^(1/3)*(x^4 - x^2)^(2/3)*(5
9*x^2 - 2048*x - 59) - 249106968*x^2 - 125*sqrt(34)*2^(1/6)*(4*2^(2/3)*(x^4
- x^2)^(2/3)*(15104*x^2 + 527769*x - 15104) + 3481*2^(1/3)*(x^5 + 2*x^3 +
x) + 4*(x^4 - x^2)^(1/3)*(527769*x^3 - 60416*x^2 - 527769*x)) + 3564544*x)/
(205379*x^5 - 2168870912*x^4 - 1232274*x^3 + 2168870912*x^2 + 205379*x)) -
1/12*sqrt(3)*log(26618852*(x^5 - x^3 + 2*(x^4 - x^2)^(2/3)*(sqrt(3)*(x^2 -
1) + 3*x) + 4*sqrt(3)*(x^4 - x^2) + 2*(x^4 - x^2)^(1/3)*(3*x^3 + sqrt(3)*x^
2 - 3*x) + x)/(x^5 - x^3 + x)) + 1/12*sqrt(3)*log(26618852*(x^5 - x^3 - 2*(
x^4 - x^2)^(2/3)*(sqrt(3)*(x^2 - 1) - 3*x) - 4*sqrt(3)*(x^4 - x^2) + 2*(x^4
- x^2)^(1/3)*(3*x^3 - sqrt(3)*x^2 - 3*x) + x)/(x^5 - x^3 + x)) + 1/3*arcta
n(-(163348821309602766976*x^9 + 388743240267983775195 2*x^8 - 37935518804163
19588608*x^7 - 15549729610719351007808*x^6 + 74237549395230364102 40*x^5 + 1
5549729610719351007808*x^4 - 3793551880416319588608*x^3 - 38874324026798377
51952*x^2 - 338*sqrt(233)*(67166456130593243*x^9 - 1731873489534746816*x^8
- 8262322488125426948*x^7 + 5402376118068558976*x^6 + 16323145607859074167*
x^5 - 5402376118068558976*x^4 - 8262322488125426948*x^3 + 17318734895347468
16*x^2 + 8*(11634681213606448*x^6 + 88410267443510747*x^5 - 141607113799927
264*x^4 - 304974921996124561*x^3 + 141607113799927264*x^2 + 2*sqrt(3)*(2398
449325331968*x^6 + 66317101349416968*x^5 + 306343852456405393*x^4 - 1627179
14879099272*x^3 - 306343852456405393*x^2 + 66317101349416968*x - 2398449325
331968) + 88410267443510747*x - 11634681213606448)*(x^4 - x^2)^(2/3) + 2*sq
rt(3)*(8144636084443696*x^9 + 17828539163673051*x^8 - 307821488566558016*x^
7 + 605786626407170998*x^6 + 591209068879784944*x^5 - 605786626407170998*x^
4 - 307821488566558016*x^3 - 17828539163673051*x^2 + 8144636084443696*x) +
2*(195992612428698075*x^7 + 72793982843270240*x^6 - 2656296320238012626*x^5
- 5018142432545251 2*x^4 + 2656296320238012626*x^3 + 72793982843270240*x^2
+ 6654713*sqrt(3)*(14835265056*x^7 + 109365761599*x^6 + 11690073152*x^5 - 2
05755148299*x^4 - 11690073152*x^3 + 109365761599*x^2 - 14835265056*x) - 195
992612428698075*x)*(x^4 - x^2)^(1/3) + 67166456130593243*x)*sqrt((x^5 - x^3
+ 2*(x^4 - x^2)^(2/3)*(sqrt(3)*(x^2 - 1) + 3*x) + 4*sqrt(3)*(x^4 - x^2) +
2*(x^4 - x^2)^(1/3)*(3*x^3 + sqrt(3)*x^2 - 3*x) + x)/(x^5 - x^3 + x)) + 266
18852*(38271977676337*x^6 - 236679798487232*x^5 - 918515085244470*x^4 + 579
302159719616*x^3 + 918515085244470*x^2 + sqrt(3)*(15779271877184*x^6 + 1705
85488729845*x^5 - 175823521011328*x^4 - 273378905866169*x^3 + 1758235210113

28*x^2 + 170585488729845*x - 15779271877184) - 236679798487232*x - 38271977
676337)*(x^4 - x^2)^(2/3) + 6654713*sqrt(3)*(26791566067249*x^9 - 288529954
513920*x^8 - 464756800679794*x^7 + 1154119818055680*x^6 + 902722035292339*x
^5 - 1154119818055680*x^4 - 464756800679794*x^3 + 288529954513920*x^2 + 267
91566067249*x) + 26618852*(105942562745152*x^7 + 803699152215459*x^6 - 5545
07486722688*x^5 - 1645670282107255*x^4 + 554507486722688*x^3 + 803699152215
459*x^2 + sqrt(3)*(67792071593521*x^7 + 128485705379776*x^6 - 3279072605071
8*x^5 - 272750682636736*x^4 + 32790726050718*x^3 + 128485705379776*x^2 - 67
792071593521*x) - 105942562745152*x)*(x^4 - x^2)^(1/3) + 163348821309602766
976*x)/(138674673083346657193*x^9 - 629353408166624067891 2*x^8 - 3046451904
6388862556482*x^7 + 2517413632666496271564 8*x^6 + 607903634196943784557 71*x
^5 - 2517413632666496271564 8*x^4 - 30464519046388862556482*x^3 + 6293534081
666240678912*x^2 + 138674673083346657193*x)) - 1/3*arctan((163348821309602 7
66976*x^9 + 388743240267983775195 2*x^8 - 37935518804163195886 08*x^7 - 15549
7296107193510078 08*x^6 + 742375493952303641024 0*x^5 + 155497296107193510078
08*x^4 - 37935518804163195886 08*x^3 - 388743240267983775195 2*x^2 - 338*sqrt
(233)*(67166456130593243*x^9 - 173187348953474681 6*x^8 - 826232248812542694
8*x^7 + 540237611806855897 6*x^6 + 16323145607859074167*x^5 - 54023761180685
58976*x^4 - 8262322488125426948*x^3 + 1731873489534746816*x^2 + 8*(11634681
213606448*x^6 + 88410267443510747*x^5 - 141607113799927264*x^4 - 3049749219
96124561*x^3 + 141607113799927264*x^2 - 2*sqrt(3)*(2398449325331968*x^6 + 6
6317101349416968*x^5 + 306343852456405393*x^4 - 162717914879099272*x^3 - 30
6343852456405393*x^2 + 66317101349416968*x - 2398449325331968) + 8841026744
3510747*x - 11634681213606448)*(x^4 - x^2)^(2/3) - 2*sqrt(3)*(8144636084443
696*x^9 + 17828539163673051*x^8 - 307821488566558016*x^7 + 6057866264071709
98*x^6 + 591209068879784944*x^5 - 605786626407170998*x^4 - 3078214885665580
16*x^3 - 17828539163673051*x^2 + 8144636084443696*x) + 2*(19599261242869807
5*x^7 + 72793982843270240*x^6 - 2656296320238012626*x^5 - 50181424325452512
*x^4 + 2656296320238012626*x^3 + 72793982843270240*x^2 - 6654713*sqrt(3)*(1
4835265056*x^7 + 109365761599*x^6 + 11690073152*x^5 - 205755148299*x^4 - 11
690073152*x^3 + 109365761599*x^2 - 14835265056*x) - 195992612428698075*x)*(
x^4 - x^2)^(1/3) + 67166456130593243*x)*sqrt((x^5 - x^3 - 2*(x^4 - x^2)^(2/
3)*(sqrt(3)*(x^2 - 1) - 3*x) - 4*sqrt(3)*(x^4 - x^2) + 2*(x^4 - x^2)^(1/3)*
(3*x^3 - sqrt(3)*x^2 - 3*x) + x)/(x^5 - x^3 + x)) + 26618852*(3827197767633
7*x^6 - 236679798487232*x^5 - 918515085244470*x^4 + 579302159719616*x^3 + 9
18515085244470*x^2 - sqrt(3)*(15779271877184*x^6 + 170585488729845*x^5 - 17
5823521011328*x^4 - 273378905866169*x^3 + 175823521011328*x^2 + 17058548872
9845*x - 15779271877184) - 236679798487232*x - 38271977676337)*(x^4 - x^2)^
(2/3) - 6654713*sqrt(3)*(26791566067249*x^9 - 288529954513920*x^8 - 4647568
00679794*x^7 + 1154119818055680*x^6 + 902722035292339*x^5 - 115411981805568
0*x^4 - 464756800679794*x^3 + 288529954513920*x^2 + 26791566067249*x) + 266
18852*(105942562745152*x^7 + 803699152215459*x^6 - 554507486722688*x^5 - 16
45670282107255*x^4 + 554507486722688*x^3 + 803699152215459*x^2 - sqrt(3)*(6
7792071593521*x^7 + 128485705379776*x^6 - 32790726050718*x^5 - 272750682636
736*x^4 + 32790726050718*x^3 + 128485705379776*x^2 - 67792071593521*x) - 10
5942562745152*x)*(x^4 - x^2)^(1/3) + 163348821309602766976*x)/(138674673083
346657193*x^9 - 6293534081666240678912*x^8 - 30464519046388862556482*x^7 +
25174136326664962715648*x^6 + 60790363419694378455771*x^5 - 251741363266649
62715648*x^4 - 30464519046388862556482*x^3 + 6293534081666240678912*x^2 + 1
38674673083346657193*x)) - 1/3*arctan(-2*(1910654896*x^5 - 17610113139*x^4
- 5731964688*x^3 + 17610113139*x^2 - 6654713*(x^4 - x^2)^(2/3)*(2507*x^2 +
1216*x - 2507) + 6654713*(x^4 - x^2)^(1/3)*(1216*x^3 - 2507*x^2 - 1216*x) +
 1910654896*x)/(15756617843*x^5 + 24725904448*x^4 - 47269853529*x^3 - 24725
904448*x^2 + 15756617843*x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6-1}{\left(x^6+1\right)\left(x^4-x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^4-x^2)^(1/3)/(x^6+1),x, algorithm="giac")
[Out] integrate((x^6 - 1)/((x^6 + 1)*(x^4 - x^2)^(1/3)), x)
```

maple [F] time = 98.64, size = 0, normalized size = 0.00

$$\int \frac{x^{6}-1}{\left(x^{4}-x^{2}\right)^{\frac{1}{3}} \left(x^{6}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)/(x^4-x^2)^(1/3)/(x^6+1),x)
[Out] int((x^6-1)/(x^4-x^2)^(1/3)/(x^6+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}-1}{\left(x^{6}+1\right)\left(x^{4}-x^{2}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^4-x^2)^(1/3)/(x^6+1),x, algorithm="maxima")
[Out] integrate((x^6 - 1)/((x^6 + 1)*(x^4 - x^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6-1}{\left(x^6+1\right)\,{\left(x^4-x^2\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - 1)/((x^6 + 1)*(x^4 - x^2)^(1/3)),x)
[Out] int((x^6 - 1)/((x^6 + 1)*(x^4 - x^2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x - 1\right) \left(x + 1\right) \left(x^{2} - x + 1\right) \left(x^{2} + x + 1\right)}{\sqrt[3]{x^{2} \left(x - 1\right) \left(x + 1\right)} \left(x^{2} + 1\right) \left(x^{4} - x^{2} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)/(x**4-x**2)**(1/3)/(x**6+1),x)
[Out] Integral((x - 1)*(x + 1)*(x**2 - x + 1)*(x**2 + x + 1)/((x**2*(x - 1)*(x +
1))**(1/3)*(x**2 + 1)*(x**4 - x**2 + 1)), x)
```

3.2228 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+x}\,dx$

Optimal. Leaf size=274

$$\sqrt{\sqrt{x^4+1}+x^2}-\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{\sqrt{2}-1}}\right)+\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{2}-1\right)}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\sqrt{1$$

Rubi [F] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/(1 + x),x]
[Out] Defer[Int][Sqrt[x^2 + Sqrt[1 + x^4]]/(1 + x), x]
```

Rubi steps

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+x}\,dx=\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+x}\,dx$$

Mathematica [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/(1 + x),x]
[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/(1 + x), x]
```

IntegrateAlgebraic [A] time = 1.51, size = 274, normalized size = 1.00

$$\sqrt{\sqrt{x^4+1}+x^2}-\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\sqrt{1+\sqrt{2}}\,\sqrt{\sqrt{x^4+1}+x^2}\right)+\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{2\sqrt{2}-2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\,\sqrt{\sqrt{x^4+1}+x^2}\right)-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)+\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2+2\sqrt{2}}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2 + Sqrt[1 + x^4]]/(1 + x),x]
[Out] Sqrt[x^2 + Sqrt[1 + x^4]] - Sqrt[-1 + Sqrt[2]]*ArcTan[Sqrt[1 + Sqrt[2]]*Sqr
t[x^2 + Sqrt[1 + x^4]]] + Sqrt[-1 + Sqrt[2]]*ArcTan[(Sqrt[-2 + 2*Sqrt[2]]*x
*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])] - Sqrt[1 + Sqrt[2]]*
ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]] - Sqrt[2]*ArcTanh[(Sq
rt[2]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])] + Sqrt[1 + Sq
rt[2]]*ArcTanh[(Sqrt[2 + 2*Sqrt[2]]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 +
 Sqrt[1 + x^4])]
```

fricas [B] time = 2.26, size = 436, normalized size = 1.59

$$\sqrt{\sqrt{2}-1}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(x^2+1\right)+2\sqrt{x^4+1}\right)\sqrt{\sqrt{2}+1}\,\sqrt{\sqrt{2}-1}-2\left(x^3-x^2-\sqrt{2}\left(x^2-x\right)-\sqrt{x^4+1}\left(x-\sqrt{2}-1\right)+x+1\right)\sqrt{x^2+\sqrt{x^4+1}}\,\sqrt{\sqrt{2}-1}}{2\left(x^2-2x+1\right)}\right)+\frac{1}{4}\sqrt{2}\log\left(4x^4+4\sqrt{x^4+1}\,x^2-2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)+\frac{1}{4}\sqrt{\sqrt{2}+1}\log\left(-\frac{2\left(\left(2x^3-\sqrt{2}\left(x^3-x^2-x-1\right)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2x\right)-2\right)\sqrt{x^2+\sqrt{x^4+1}}+\left(2x^2-\sqrt{2}\left(x^2+1\right)+2\sqrt{x^4+1}\left(\sqrt{2}-1\right)+2\right)\sqrt{\sqrt{2}+1}\right)}{x^2+2x+1}\right)-\frac{1}{4}\sqrt{\sqrt{2}+1}\log\left(-\frac{2\left(\left(2x^3-\sqrt{2}\left(x^3-x^2-x-1\right)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2x\right)-2\right)\sqrt{x^2+\sqrt{x^4+1}}-\left(2x^2-\sqrt{2}\left(x^2+1\right)+2\sqrt{x^4+1}\left(\sqrt{2}-1\right)+2\right)\sqrt{\sqrt{2}+1}\right)}{x^2+2x+1}\right)+\sqrt{x^2+\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x),x, algorithm="fricas")
```

```
[Out] sqrt(sqrt(2) - 1)*arctan(-1/2*(sqrt(2)*(sqrt(2)*(x^2 + 1) + 2*sqrt(x^4 + 1)
)*sqrt(sqrt(2) + 1)*sqrt(sqrt(2) - 1) - 2*(x^3 - x^2 - sqrt(2)*(x^2 - x) -
sqrt(x^4 + 1)*(x - sqrt(2) - 1) + x + 1)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(sqr
t(2) - 1))/(x^2 - 2*x + 1)) + 1/4*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 -
 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1) +
 1/4*sqrt(sqrt(2) + 1)*log(-2*((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(
x^4 + 1)*(sqrt(2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) + (2*x^2 -
sqrt(2)*(x^2 + 1) + 2*sqrt(x^4 + 1)*(sqrt(2) - 1) + 2)*sqrt(sqrt(2) + 1))/(
x^2 + 2*x + 1)) - 1/4*sqrt(sqrt(2) + 1)*log(-2*((2*x^3 - sqrt(2)*(x^3 - x^2
 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4
+ 1)) - (2*x^2 - sqrt(2)*(x^2 + 1) + 2*sqrt(x^4 + 1)*(sqrt(2) - 1) + 2)*sqr
t(sqrt(2) + 1))/(x^2 + 2*x + 1)) + sqrt(x^2 + sqrt(x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(x + 1), x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{1 + x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\sqrt{x^4 + 1} + x^2}}{x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x + 1),x)
```

```
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/(x + 1), x)
```

3.2229 $\int \frac{b+ax^2}{\left(b-ax^2\right)\sqrt[4]{bx^3+ax^5}}\,dx$

Optimal. Leaf size=275

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[8]{a}\sqrt[8]{b}\,x}{\sqrt[4]{ax^5+bx^3}}\right)}{\sqrt[4]{2}\sqrt[8]{a}\sqrt[8]{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[8]{a}\sqrt[8]{b}\,x}{\sqrt[4]{ax^5+bx^3}}\right)}{\sqrt[4]{2}\sqrt[8]{a}\sqrt[8]{b}}-\frac{\tan^{-1}\left(\frac{2^{3/4}\sqrt[8]{a}\sqrt[8]{b}\,x\sqrt[4]{ax^5+bx^3}}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x^2-\sqrt{ax^5+bx^3}}\right)}{2^{3/4}\sqrt[8]{a}\sqrt[8]{b}}+\frac{\tanh^{-1}\left(\frac{\frac{\sqrt[8]{a}\sqrt[8]{b}\,x^2}{\sqrt[4]{2}}+\frac{\sqrt{ax^5+bx^3}}{2^{3/4}\sqrt[8]{a}\sqrt[8]{b}}}{x\sqrt[4]{ax^5+bx^3}}\right)}{2^{3/4}\sqrt[8]{a}\sqrt[8]{b}}$$

Rubi [C] time = 0.23, antiderivative size = 70, normalized size of antiderivative = 0.25, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2056, 466, 430, 429}

$$\frac{4x\left(ax^2+b\right)F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};\frac{ax^2}{b},-\frac{ax^2}{b}\right)}{b\left(\frac{ax^2}{b}+1\right)^{3/4}\sqrt[4]{ax^5+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(b + a*x^2)/((b - a*x^2)*(b*x^3 + a*x^5)^(1/4)),x]
```

```
[Out] (4*x*(b + a*x^2)*AppellF1[1/8, 1, -3/4, 9/8, (a*x^2)/b, -((a*x^2)/b)])/(b*(
1 + (a*x^2)/b)^(3/4)*(b*x^3 + a*x^5)^(1/4))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{b+ax^2}{\left(b-ax^2\right)\sqrt[4]{bx^3+ax^5}}\,dx = \frac{\left(x^{3/4}\sqrt[4]{b+ax^2}\right)\int \frac{\left(b+ax^2\right)^{3/4}}{x^{3/4}\left(b-ax^2\right)}\,dx}{\sqrt[4]{bx^3+ax^5}}$$
$$= \frac{\left(4x^{3/4}\sqrt[4]{b+ax^2}\right)\text{Subst}\left(\int \frac{\left(b+ax^8\right)^{3/4}}{b-ax^8}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{bx^3+ax^5}}$$
$$= \frac{\left(4x^{3/4}\left(b+ax^2\right)\right)\text{Subst}\left(\int \frac{\left(1+\frac{ax^8}{b}\right)^{3/4}}{b-ax^8}\,dx, x, \sqrt[4]{x}\right)}{\left(1+\frac{ax^2}{b}\right)^{3/4}\sqrt[4]{bx^3+ax^5}}$$
$$= \frac{4x\left(b+ax^2\right)F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};\frac{ax^2}{b},-\frac{ax^2}{b}\right)}{b\left(1+\frac{ax^2}{b}\right)^{3/4}\sqrt[4]{bx^3+ax^5}}$$

Mathematica [C] time = 0.05, size = 65, normalized size = 0.24

$$\frac{4\left(x^3\left(ax^2+b\right)\right)^{3/4}F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};-\frac{ax^2}{b},\frac{ax^2}{b}\right)}{bx^2\left(\frac{ax^2}{b}+1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b + a*x^2)/((b - a*x^2)*(b*x^3 + a*x^5)^(1/4)),x]
```

```
[Out] (4*(x^3*(b + a*x^2))^(3/4)*AppellF1[1/8, -3/4, 1, 9/8, -((a*x^2)/b), (a*x^2
)/b])/(b*x^2*(1 + (a*x^2)/b)^(3/4))
```

IntegrateAlgebraic [A] time = 0.99, size = 318, normalized size = 1.16

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[8]{a}\sqrt[8]{b}x}{\sqrt[4]{ax^5+bx^3}}\right)}{\sqrt[4]{2}\sqrt[8]{a}\sqrt[8]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt[8]{a}\sqrt[8]{b}x}{\sqrt[8]{a}\sqrt[8]{b}x-\sqrt[4]{2}\sqrt[4]{ax^5+bx^3}}\right)}{2^{3/4}\sqrt[8]{a}\sqrt[8]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[8]{a}\sqrt[8]{b}x}{\sqrt[4]{2}\sqrt[4]{ax^5+bx^3}+\sqrt[8]{a}\sqrt[8]{b}x}\right)}{2^{3/4}\sqrt[8]{a}\sqrt[8]{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[8]{a}\sqrt[8]{b}x}{\sqrt[4]{ax^5+bx^3}}\right)}{\sqrt[4]{2}\sqrt[8]{a}\sqrt[8]{b}}+\frac{\tanh^{-1}\left(\frac{\frac{\sqrt[8]{a}\sqrt[8]{b}x^2}{\sqrt[4]{2}}+\frac{\sqrt{ax^5+bx^3}}{2^{3/4}\sqrt[8]{a}\sqrt[8]{b}}}{x\sqrt[4]{ax^5+bx^3}}\right)}{2^{3/4}\sqrt[8]{a}\sqrt[8]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^2)/((b - a*x^2)*(b*x^3 + a*x^5)^(1/4)),x]
```

```
[Out] ArcTan[(2^(1/4)*a^(1/8)*b^(1/8)*x)/(b*x^3 + a*x^5)^(1/4)]/(2^(1/4)*a^(1/8)*
b^(1/8)) - ArcTan[(a^(1/8)*b^(1/8)*x)/(a^(1/8)*b^(1/8)*x - 2^(1/4)*(b*x^3 +
 a*x^5)^(1/4))]/(2^(3/4)*a^(1/8)*b^(1/8)) + ArcTan[(a^(1/8)*b^(1/8)*x)/(a^(
1/8)*b^(1/8)*x + 2^(1/4)*(b*x^3 + a*x^5)^(1/4))]/(2^(3/4)*a^(1/8)*b^(1/8))
+ ArcTanh[(2^(1/4)*a^(1/8)*b^(1/8)*x)/(b*x^3 + a*x^5)^(1/4)]/(2^(1/4)*a^(1/
8)*b^(1/8)) + ArcTanh[((a^(1/8)*b^(1/8)*x^2)/2^(1/4) + Sqrt[b*x^3 + a*x^5]/
(2^(3/4)*a^(1/8)*b^(1/8)))/(x*(b*x^3 + a*x^5)^(1/4))]/(2^(3/4)*a^(1/8)*b^(1
/8))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(-a*x^2+b)/(a*x^5+b*x^3)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ax^2+b}{\left(ax^5+bx^3\right)^{\frac{1}{4}}\left(ax^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(-a*x^2+b)/(a*x^5+b*x^3)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-(a*x^2 + b)/((a*x^5 + b*x^3)^(1/4)*(a*x^2 - b)), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2+b}{\left(-a\,x^2+b\right)\left(a\,x^5+b\,x^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)/(-a*x^2+b)/(a*x^5+b*x^3)^(1/4),x)
```

```
[Out] int((a*x^2+b)/(-a*x^2+b)/(a*x^5+b*x^3)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax^2+b}{\left(ax^5+bx^3\right)^{\frac{1}{4}}\left(ax^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(-a*x^2+b)/(a*x^5+b*x^3)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate((a*x^2 + b)/((a*x^5 + b*x^3)^(1/4)*(a*x^2 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x^2+b}{\left(b-a\,x^2\right)\,{\left(a\,x^5+b\,x^3\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^2)/((b - a*x^2)*(a*x^5 + b*x^3)^(1/4)),x)
```

```
[Out] int((b + a*x^2)/((b - a*x^2)*(a*x^5 + b*x^3)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{b}{ax^2\sqrt[4]{ax^5+bx^3}-b\sqrt[4]{ax^5+bx^3}}\,dx-\int \frac{ax^2}{ax^2\sqrt[4]{ax^5+bx^3}-b\sqrt[4]{ax^5+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b)/(-a*x**2+b)/(a*x**5+b*x**3)**(1/4),x)
```

```
[Out] -Integral(b/(a*x**2*(a*x**5 + b*x**3)**(1/4) - b*(a*x**5 + b*x**3)**(1/4)),
 x) - Integral(a*x**2/(a*x**2*(a*x**5 + b*x**3)**(1/4) - b*(a*x**5 + b*x**3
)**(1/4)), x)
```

3.2230 $\int \frac{(4+x^3)(1+x^3+x^4)}{\sqrt[4]{1+x^3}\,(1+2x^3+x^6+x^8)}\,dx$

Optimal. Leaf size=275

$$-\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\frac{\sqrt{2-\sqrt{2}}\,x^2}{\sqrt{2}-2}-\frac{\sqrt{2-\sqrt{2}}\,\sqrt{x^3+1}}{\sqrt{2}-2}}{x\sqrt[4]{x^3+1}}\right)-\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\frac{\sqrt{x^3+1}}{\sqrt{2+\sqrt{2}}}-\frac{x^2}{\sqrt{2+\sqrt{2}}}}{x\sqrt[4]{x^3+1}}\right)+\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\ \mathrm{t}$$

Rubi [F] time = 1.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(4+x^3)(1+x^3+x^4)}{\sqrt[4]{1+x^3}\,(1+2x^3+x^6+x^8)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((4 + x^3)*(1 + x^3 + x^4))/((1 + x^3)^(1/4)*(1 + 2*x^3 + x^6 + x^8)),x
]
```

```
[Out] 4*Defer[Int][1/((1 + x^3)^(1/4)*(1 + 2*x^3 + x^6 + x^8)), x] + 5*Defer[Int]
[x^3/((1 + x^3)^(1/4)*(1 + 2*x^3 + x^6 + x^8)), x] + 4*Defer[Int][x^4/((1 +
 x^3)^(1/4)*(1 + 2*x^3 + x^6 + x^8)), x] + Defer[Int][x^6/((1 + x^3)^(1/4)*
(1 + 2*x^3 + x^6 + x^8)), x] + Defer[Int][x^7/((1 + x^3)^(1/4)*(1 + 2*x^3 +
 x^6 + x^8)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{(4+x^3)(1+x^3+x^4)}{\sqrt[4]{1+x^3}\,(1+2x^3+x^6+x^8)}\,dx &= \int \left(\frac{4}{\sqrt[4]{1+x^3}\,(1+2x^3+x^6+x^8)}+\frac{5x^3}{\sqrt[4]{1+x^3}\,(1+2x^3+x^6+x^8)}+\frac{}{\sqrt[4]{1+}}\right.\\ &= 4\int \frac{1}{\sqrt[4]{1+x^3}\,(1+2x^3+x^6+x^8)}\,dx+4\int \frac{x^4}{\sqrt[4]{1+x^3}\,(1+2x^3+x^6+x^8)}\end{aligned}$$

Mathematica [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(4+x^3)(1+x^3+x^4)}{\sqrt[4]{1+x^3}\,(1+2x^3+x^6+x^8)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((4 + x^3)*(1 + x^3 + x^4))/((1 + x^3)^(1/4)*(1 + 2*x^3 + x^6 + x
^8)),x]
```

```
[Out] Integrate[((4 + x^3)*(1 + x^3 + x^4))/((1 + x^3)^(1/4)*(1 + 2*x^3 + x^6 + x
^8)), x]
```

IntegrateAlgebraic [A] time = 2.56, size = 257, normalized size = 0.93

$$-\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\,\sqrt{x^3+1}-\sqrt{1-\frac{1}{\sqrt{2}}}\,x^2}{x\sqrt[4]{x^3+1}}\right)-\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\,\sqrt{x^3+1}-\sqrt{1+\frac{1}{\sqrt{2}}}\,x^2}{x\sqrt[4]{x^3+1}}\right)+\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^3+1}}{\sqrt{x^3+1}+x^2}\right)+\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^3+1}}{\sqrt{x^3+1}+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((4 + x^3)*(1 + x^3 + x^4))/((1 + x^3)^(1/4)*(1 + 2*x^3
+ x^6 + x^8)),x]
```

```
[Out] -(Sqrt[(2 - Sqrt[2])/2]*ArcTan[(-(Sqrt[1 - 1/Sqrt[2]]*x^2) + Sqrt[1 - 1/Sqr
t[2]]*Sqrt[1 + x^3])/(x*(1 + x^3)^(1/4))]) - Sqrt[(2 + Sqrt[2])/2]*ArcTan[(
-(Sqrt[1 + 1/Sqrt[2]]*x^2) + Sqrt[1 + 1/Sqrt[2]]*Sqrt[1 + x^3])/(x*(1 + x^3
)^(1/4))] + Sqrt[(2 + Sqrt[2])/2]*ArcTanh[(Sqrt[2 - Sqrt[2]]*x*(1 + x^3)^(1
/4))/(x^2 + Sqrt[1 + x^3])] + Sqrt[(2 - Sqrt[2])/2]*ArcTanh[(Sqrt[2 + Sqrt[
2]]*x*(1 + x^3)^(1/4))/(x^2 + Sqrt[1 + x^3])]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)*(x^4+x^3+1)/(x^3+1)^(1/4)/(x^8+x^6+2*x^3+1),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4 + x^3 + 1\right)\left(x^3 + 4\right)}{\left(x^8 + x^6 + 2\,x^3 + 1\right)\left(x^3 + 1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)*(x^4+x^3+1)/(x^3+1)^(1/4)/(x^8+x^6+2*x^3+1),x, algorithm=
"giac")
```

```
[Out] integrate((x^4 + x^3 + 1)*(x^3 + 4)/((x^8 + x^6 + 2*x^3 + 1)*(x^3 + 1)^(1/4
)), x)
```

maple [C] time = 12.63, size = 694, normalized size = 2.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+4)*(x^4+x^3+1)/(x^3+1)^(1/4)/(x^8+x^6+2*x^3+1),x)
```

```
[Out] 1/16*RootOf(_Z^8+16)^7*ln(-(-RootOf(_Z^8+16)^11*x^4-4*x^4*RootOf(_Z^8+16)^7
+16*RootOf(_Z^8+16)^6*(x^3+1)^(1/4)*x^3+4*RootOf(_Z^8+16)^7*x^3-16*(x^3+1)^
(1/2)*RootOf(_Z^8+16)^5*x^2+4*RootOf(_Z^8+16)^7+16*RootOf(_Z^8+16)^3*x^3-64
*(x^3+1)^(1/2)*RootOf(_Z^8+16)*x^2+128*(x^3+1)^(3/4)*x+16*RootOf(_Z^8+16)^3
)/(RootOf(_Z^8+16)^4*x^4+4*x^3+4))+1/2*RootOf(_Z^8+16)*ln(-(RootOf(_Z^8+16)
^9*x^4-4*(x^3+1)^(1/2)*RootOf(_Z^8+16)^7*x^2+4*RootOf(_Z^8+16)^5*x^4+4*x^3*
RootOf(_Z^8+16)^5-16*(x^3+1)^(1/2)*RootOf(_Z^8+16)^3*x^2-32*(x^3+1)^(1/4)*R
ootOf(_Z^8+16)^2*x^3+4*RootOf(_Z^8+16)^5+64*(x^3+1)^(3/4)*x+16*RootOf(_Z^8+
16)*x^3+16*RootOf(_Z^8+16))/(RootOf(_Z^8+16)^4*x^4-4*x^3-4))-1/4*RootOf(_Z^
8+16)^3*ln(-(-RootOf(_Z^8+16)^11*x^4+4*x^4*RootOf(_Z^8+16)^7-16*RootOf(_Z^8
+16)^6*(x^3+1)^(1/4)*x^3+4*RootOf(_Z^8+16)^7*x^3+16*(x^3+1)^(1/2)*RootOf(_Z
^8+16)^5*x^2+4*RootOf(_Z^8+16)^7-16*RootOf(_Z^8+16)^3*x^3-64*(x^3+1)^(1/2)*
RootOf(_Z^8+16)*x^2+128*(x^3+1)^(3/4)*x-16*RootOf(_Z^8+16)^3)/(RootOf(_Z^8+
16)^4*x^4+4*x^3+4))+1/8*RootOf(_Z^8+16)^5*ln(-(RootOf(_Z^8+16)^9*x^4+4*(x^3
+1)^(1/2)*RootOf(_Z^8+16)^7*x^2-4*RootOf(_Z^8+16)^5*x^4+4*x^3*RootOf(_Z^8+1
6)^5-16*(x^3+1)^(1/2)*RootOf(_Z^8+16)^3*x^2+32*(x^3+1)^(1/4)*RootOf(_Z^8+16
)^2*x^3+4*RootOf(_Z^8+16)^5+64*(x^3+1)^(3/4)*x-16*RootOf(_Z^8+16)*x^3-16*Ro
otOf(_Z^8+16))/(RootOf(_Z^8+16)^4*x^4-4*x^3-4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+x^3+1\right)\left(x^3+4\right)}{\left(x^8+x^6+2\,x^3+1\right)\left(x^3+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4)*(x^4+x^3+1)/(x^3+1)^(1/4)/(x^8+x^6+2*x^3+1),x, algorithm=
"maxima")
```

```
[Out] integrate((x^4 + x^3 + 1)*(x^3 + 4)/((x^8 + x^6 + 2*x^3 + 1)*(x^3 + 1)^(1/4
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^3+4\right)\,\left(x^4+x^3+1\right)}{{\left(x^3+1\right)}^{1/4}\,\left(x^8+x^6+2\,x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 4)*(x^3 + x^4 + 1))/((x^3 + 1)^(1/4)*(2*x^3 + x^6 + x^8 + 1)),x
)
```

```
[Out] int(((x^3 + 4)*(x^3 + x^4 + 1))/((x^3 + 1)^(1/4)*(2*x^3 + x^6 + x^8 + 1)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}+4\right)\left(x^{4}+x^{3}+1\right)}{\sqrt[4]{\left(x+1\right)\left(x^{2}-x+1\right)}\left(x^{8}+x^{6}+2x^{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+4)*(x**4+x**3+1)/(x**3+1)**(1/4)/(x**8+x**6+2*x**3+1),x)
```

```
[Out] Integral((x**3 + 4)*(x**4 + x**3 + 1)/(((x + 1)*(x**2 - x + 1))**(1/4)*(x**
8 + x**6 + 2*x**3 + 1)), x)
```

3.2231 $\int \frac{(-b-ax+x^4)\sqrt[4]{bx^3+ax^4}}{-b+ax}\,dx$

Optimal. Leaf size=276

$$\frac{(19712a^4b^2-9843b^5)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{4096a^{23/4}}-\frac{2\sqrt[4]{2}\,(2a^4b^2-b^5)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{a^{23/4}}+\frac{(9843b^5-19712a^4b^2)\tanh^{-1}\left(\frac{}{\sqrt[4]{a}}\right.}{4096a^{23/4}}$$

Rubi [C] time = 1.76, antiderivative size = 707, normalized size of antiderivative = 2.56, number of steps used = 41, number of rules used = 11, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.306, Rules used = {2056, 6733, 6725, 279, 331, 298, 203, 206, 321, 511, 510}

$$-\frac{371b^5\sqrt[4]{ax^4+bx^3}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{4096a^{23/4}x^{3/4}\sqrt[4]{ax+b}}+\frac{371b^5\sqrt[4]{ax^4+bx^3}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{4096a^{23/4}x^{3/4}\sqrt[4]{ax+b}}-\frac{371b^4\sqrt[4]{ax^4+bx^3}}{6144a^5}+\frac{53b^3x\sqrt[4]{ax^4+bx^3}}{1536a^4}-\frac{1}{2}x\left(1-\frac{b^3}{a^4}\right)\sqrt[4]{ax^4+bx^3}+\frac{65b^2x^2\sqrt[4]{ax^4+bx^3}}{192a^3}+\frac{21bx^3\sqrt[4]{ax^4+bx^3}}{80a^2}-\frac{3b^2(a^4-b^3)\sqrt[4]{ax^4+bx^3}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{16a^{23/4}x^{3/4}\sqrt[4]{ax+b}}+\frac{b^2(2a^4-b^3)\sqrt[4]{ax^4+bx^3}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{2a^{23/4}x^{3/4}\sqrt[4]{ax+b}}+\frac{3b^2(a^4-b^3)\sqrt[4]{ax^4+bx^3}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{16a^{23/4}x^{3/4}\sqrt[4]{ax+b}}-\frac{b^2(2a^4-b^3)\sqrt[4]{ax^4+bx^3}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt[4]{x}}{\sqrt[4]{ax+b}}\right)}{2a^{23/4}x^{3/4}\sqrt[4]{ax+b}}+\frac{4b(2a^4-b^3)\sqrt[4]{ax^4+bx^3}F_1\left(\frac{3}{4};1,-\frac{1}{4};\frac{7}{4};\frac{ax}{b},-\frac{ax}{b}\right)}{3a^5\sqrt[4]{\frac{ax}{b}+1}}-\frac{b(a^4-b^3)\sqrt[4]{ax^4+bx^3}}{8a^5}-\frac{b(2a^4-b^3)\sqrt[4]{ax^4+bx^3}}{a^5}+\frac{x^4\sqrt[4]{ax^4+bx^3}}{5a}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-b - a*x + x^4)*(b*x^3 + a*x^4)^(1/4))/(-b + a*x),x]
```

```
[Out] (-371*b^4*(b*x^3 + a*x^4)^(1/4))/(6144*a^5) - (b*(a^4 - b^3)*(b*x^3 + a*x^4
)^(1/4))/(8*a^5) - (b*(2*a^4 - b^3)*(b*x^3 + a*x^4)^(1/4))/a^5 + (53*b^3*x*
(b*x^3 + a*x^4)^(1/4))/(1536*a^4) - ((1 - b^3/a^4)*x*(b*x^3 + a*x^4)^(1/4))
/2 + (65*b^2*x^2*(b*x^3 + a*x^4)^(1/4))/(192*a^3) + (21*b*x^3*(b*x^3 + a*x^
4)^(1/4))/(80*a^2) + (x^4*(b*x^3 + a*x^4)^(1/4))/(5*a) + (4*b*(2*a^4 - b^3)
*(b*x^3 + a*x^4)^(1/4)*AppellF1[3/4, 1, -1/4, 7/4, (a*x)/b, -((a*x)/b)])/(3
*a^5*(1 + (a*x)/b)^(1/4)) - (371*b^5*(b*x^3 + a*x^4)^(1/4)*ArcTan[(a^(1/4)*
x^(1/4))/(b + a*x)^(1/4)])/(4096*a^(23/4)*x^(3/4)*(b + a*x)^(1/4)) - (3*b^2
*(a^4 - b^3)*(b*x^3 + a*x^4)^(1/4)*ArcTan[(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)
])/(16*a^(23/4)*x^(3/4)*(b + a*x)^(1/4)) + (b^2*(2*a^4 - b^3)*(b*x^3 + a*x^
4)^(1/4)*ArcTan[(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(2*a^(23/4)*x^(3/4)*(b
+ a*x)^(1/4)) + (371*b^5*(b*x^3 + a*x^4)^(1/4)*ArcTanh[(a^(1/4)*x^(1/4))/(b
 + a*x)^(1/4)])/(4096*a^(23/4)*x^(3/4)*(b + a*x)^(1/4)) + (3*b^2*(a^4 - b^3
)*(b*x^3 + a*x^4)^(1/4)*ArcTanh[(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(16*a^(
23/4)*x^(3/4)*(b + a*x)^(1/4)) - (b^2*(2*a^4 - b^3)*(b*x^3 + a*x^4)^(1/4)*A
rcTanh[(a^(1/4)*x^(1/4))/(b + a*x)^(1/4)])/(2*a^(23/4)*x^(3/4)*(b + a*x)^(1
/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{\left(-b-ax+x^4\right)\sqrt[4]{bx^3+ax^4}}{-b+ax}\,dx = \frac{\sqrt[4]{bx^3+ax^4}\int \frac{x^{3/4}\sqrt[4]{b+ax}\left(-b-ax+x^4\right)}{-b+ax}\,dx}{x^{3/4}\sqrt[4]{b+ax}}$$

$$= \frac{\left(4\sqrt[4]{bx^3+ax^4}\right)\text{Subst}\left(\int \frac{x^6\sqrt[4]{b+ax^4}\left(-b-ax^4+x^{16}\right)}{-b+ax^4}\,dx, x, \sqrt[4]{x}\right)}{x^{3/4}\sqrt[4]{b+ax}}$$

$$= \frac{\left(4\sqrt[4]{bx^3+ax^4}\right)\text{Subst}\left(\int \left(-\frac{b\left(2a^4-b^3\right)x^2\sqrt[4]{b+ax^4}}{a^5} - \left(1-\frac{b^3}{a^4}\right)x^6\sqrt[4]{b+ax^4} + \frac{b^2x^{10}\sqrt[4]{\cdots}}{a\cdots}\right.\right.}{x^{3/4}\sqrt[4]{b+ax}\cdots}$$

$$= \frac{\left(4\sqrt[4]{bx^3+ax^4}\right)\text{Subst}\left(\int x^{18}\sqrt[4]{b+ax^4}\,dx, x, \sqrt[4]{x}\right)}{ax^{3/4}\sqrt[4]{b+ax}} + \frac{\left(4b\sqrt[4]{bx^3+ax^4}\right)\text{Subst}\cdots}{a^2x^{3/4}\cdots}$$

$$= -\frac{b\left(2a^4-b^3\right)\sqrt[4]{bx^3+ax^4}}{a^5} - \frac{1}{2}\left(1-\frac{b^3}{a^4}\right)x\sqrt[4]{bx^3+ax^4} + \frac{b^2x^2\sqrt[4]{bx^3+ax^4}}{3a^3} + \cdots$$

$$= -\frac{b\left(a^4-b^3\right)\sqrt[4]{bx^3+ax^4}}{8a^5} - \frac{b\left(2a^4-b^3\right)\sqrt[4]{bx^3+ax^4}}{a^5} + \frac{b^3x\sqrt[4]{bx^3+ax^4}}{24a^4} - \frac{1}{2}\cdots$$

$$= -\frac{7b^4\sqrt[4]{bx^3+ax^4}}{96a^5} - \frac{b\left(a^4-b^3\right)\sqrt[4]{bx^3+ax^4}}{8a^5} - \frac{b\left(2a^4-b^3\right)\sqrt[4]{bx^3+ax^4}}{a^5} + \frac{5b\cdots}{\cdots}$$

$$= -\frac{35b^4\sqrt[4]{bx^3+ax^4}}{1536a^5} - \frac{b\left(a^4-b^3\right)\sqrt[4]{bx^3+ax^4}}{8a^5} - \frac{b\left(2a^4-b^3\right)\sqrt[4]{bx^3+ax^4}}{a^5} + \frac{5\cdots}{\cdots}$$

$$= -\frac{371b^4\sqrt[4]{bx^3+ax^4}}{6144a^5} - \frac{b\left(a^4-b^3\right)\sqrt[4]{bx^3+ax^4}}{8a^5} - \frac{b\left(2a^4-b^3\right)\sqrt[4]{bx^3+ax^4}}{a^5} + \cdots$$

$$= -\frac{371b^4\sqrt[4]{bx^3+ax^4}}{6144a^5} - \frac{b\left(a^4-b^3\right)\sqrt[4]{bx^3+ax^4}}{8a^5} - \frac{b\left(2a^4-b^3\right)\sqrt[4]{bx^3+ax^4}}{a^5} + \cdots$$

$$= -\frac{371b^4\sqrt[4]{bx^3+ax^4}}{6144a^5} - \frac{b\left(a^4-b^3\right)\sqrt[4]{bx^3+ax^4}}{8a^5} - \frac{b\left(2a^4-b^3\right)\sqrt[4]{bx^3+ax^4}}{a^5} + \cdots$$

$$= -\frac{371b^4\sqrt[4]{bx^3+ax^4}}{6144a^5} - \frac{b\left(a^4-b^3\right)\sqrt[4]{bx^3+ax^4}}{8a^5} - \frac{b\left(2a^4-b^3\right)\sqrt[4]{bx^3+ax^4}}{a^5} + \cdots$$

Mathematica [C] time = 0.75, size = 232, normalized size = 0.84

$$\frac{x^3\left(35b^2\left(13056a^4-6541b^3\right)\left(\frac{ax}{b}+1\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{2ax}{b-ax}\right)+15abx\left(19712a^4-9843b^3\right)\left(1-\frac{a^2x^2}{b^2}\right)^{3/4}F_1\left(\frac{7}{4};\frac{3}{4},1;\frac{11}{4};-\frac{ax}{b},\frac{ax}{b}\right)+7\left(1-\frac{ax}{b}\right)^{3/4}\left(-15360a^6x^2+768a^5\left(8x^5-105bx\right)-384a^4b\left(170b-37x^4\right)+18464a^3b^2x^3+26820a^2b^3x^2+49125ab^4x+32705b^5\right)\right)}{215040a^5\left(x^3(ax+b)\right)^{3/4}\left(1-\frac{ax}{b}\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-b - a*x + x^4)*(b*x^3 + a*x^4)^(1/4))/(-b + a*x),x]
```

```
[Out] (x^3*(7*(1 - (a*x)/b)^(3/4)*(32705*b^5 + 49125*a*b^4*x - 15360*a^6*x^2 + 26
820*a^2*b^3*x^2 + 18464*a^3*b^2*x^3 - 384*a^4*b*(170*b - 37*x^4) + 768*a^5*
(-105*b*x + 8*x^5)) + 15*a*b*(19712*a^4 - 9843*b^3)*x*(1 - (a^2*x^2)/b^2)^(
3/4)*AppellF1[7/4, 3/4, 1, 11/4, -((a*x)/b), (a*x)/b] + 35*b^2*(13056*a^4 -
 6541*b^3)*(1 + (a*x)/b)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (-2*a*x)/(b
 - a*x)]))/(215040*a^5*(x^3*(b + a*x))^(3/4)*(1 - (a*x)/b)^(3/4))
```

IntegrateAlgebraic [A] time = 1.54, size = 276, normalized size = 1.00

$$\frac{\left(19712a^4b^2-9843b^5\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{4096a^{23/4}}-\frac{2\sqrt[4]{2}\left(2a^4b^2-b^5\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{a^{23/4}}+\frac{\left(9843b^5-19712a^4b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{4096a^{23/4}}+\frac{2\sqrt[4]{2}\left(2a^4b^2-b^5\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+bx^3}}\right)}{a^{23/4}}+\frac{\sqrt[4]{ax^4+bx^3}\left(-15360a^5x-65280a^4b+6144a^4x^4+8064a^3bx^3+10400a^2b^2x^2+16420ab^3x+32705b^4\right)}{30720a^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b - a*x + x^4)*(b*x^3 + a*x^4)^(1/4))/(-b + a*x),x]
```

```
[Out] ((b*x^3 + a*x^4)^(1/4)*(-65280*a^4*b + 32705*b^4 - 15360*a^5*x + 16420*a*b^
3*x + 10400*a^2*b^2*x^2 + 8064*a^3*b*x^3 + 6144*a^4*x^4))/(30720*a^5) + ((1
9712*a^4*b^2 - 9843*b^5)*ArcTan[(a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/(4096*a
^(23/4)) - (2*2^(1/4)*(2*a^4*b^2 - b^5)*ArcTan[(2^(1/4)*a^(1/4)*x)/(b*x^3 +
 a*x^4)^(1/4)])/a^(23/4) + ((-19712*a^4*b^2 + 9843*b^5)*ArcTanh[(a^(1/4)*x)
/(b*x^3 + a*x^4)^(1/4)])/(4096*a^(23/4)) + (2*2^(1/4)*(2*a^4*b^2 - b^5)*Arc
Tanh[(2^(1/4)*a^(1/4)*x)/(b*x^3 + a*x^4)^(1/4)])/a^(23/4)
```

fricas [B] time = 0.64, size = 1239, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-a*x-b)*(a*x^4+b*x^3)^(1/4)/(a*x-b),x, algorithm="fricas")
```

```
[Out] 1/122880*(491520*2^(1/4)*a^5*((16*a^16*b^8 - 32*a^12*b^11 + 24*a^8*b^14 - 8
*a^4*b^17 + b^20)/a^23)^(1/4)*arctan(1/2*(2^(3/4)*a^17*x*sqrt((sqrt(2)*a^12
*x^2*sqrt((16*a^16*b^8 - 32*a^12*b^11 + 24*a^8*b^14 - 8*a^4*b^17 + b^20)/a^
23) + (4*a^8*b^4 - 4*a^4*b^7 + b^10)*sqrt(a*x^4 + b*x^3))/x^2)*((16*a^16*b^
8 - 32*a^12*b^11 + 24*a^8*b^14 - 8*a^4*b^17 + b^20)/a^23)^(3/4) + 2^(3/4)*(
2*a^21*b^2 - a^17*b^5)*(a*x^4 + b*x^3)^(1/4)*((16*a^16*b^8 - 32*a^12*b^11 +
 24*a^8*b^14 - 8*a^4*b^17 + b^20)/a^23)^(3/4))/((16*a^16*b^8 - 32*a^12*b^11
 + 24*a^8*b^14 - 8*a^4*b^17 + b^20)*x)) + 122880*2^(1/4)*a^5*((16*a^16*b^8
- 32*a^12*b^11 + 24*a^8*b^14 - 8*a^4*b^17 + b^20)/a^23)^(1/4)*log(-(2^(1/4)
*a^6*x*((16*a^16*b^8 - 32*a^12*b^11 + 24*a^8*b^14 - 8*a^4*b^17 + b^20)/a^23
)^(1/4) + (2*a^4*b^2 - b^5)*(a*x^4 + b*x^3)^(1/4))/x) - 122880*2^(1/4)*a^5*
((16*a^16*b^8 - 32*a^12*b^11 + 24*a^8*b^14 - 8*a^4*b^17 + b^20)/a^23)^(1/4)
*log((2^(1/4)*a^6*x*((16*a^16*b^8 - 32*a^12*b^11 + 24*a^8*b^14 - 8*a^4*b^17
 + b^20)/a^23)^(1/4) - (2*a^4*b^2 - b^5)*(a*x^4 + b*x^3)^(1/4))/x) - 60*a^5
*((150981161449947136*a^16*b^8 - 301564036556783616*a^12*b^11 + 22587470666
3079936*a^8*b^14 - 75192259797236736*a^4*b^17 + 9386635211853201*b^20)/a^23
)^(1/4)*arctan((a^17*x*sqrt((a^12*x^2*sqrt((150981161449947136*a^16*b^8 - 3
01564036556783616*a^12*b^11 + 225874706663079936*a^8*b^14 - 751922597972367
36*a^4*b^17 + 9386635211853201*b^20)/a^23) + (388562944*a^8*b^4 - 388050432
*a^4*b^7 + 96884649*b^10)*sqrt(a*x^4 + b*x^3))/x^2)*((150981161449947136*a^
16*b^8 - 301564036556783616*a^12*b^11 + 225874706663079936*a^8*b^14 - 75192
259797236736*a^4*b^17 + 9386635211853201*b^20)/a^23)^(3/4) + (19712*a^21*b^
2 - 9843*a^17*b^5)*(a*x^4 + b*x^3)^(1/4)*((150981161449947136*a^16*b^8 - 30
1564036556783616*a^12*b^11 + 225874706663079936*a^8*b^14 - 7519225979723673
6*a^4*b^17 + 9386635211853201*b^20)/a^23)^(3/4))/((150981161449947136*a^16*
b^8 - 301564036556783616*a^12*b^11 + 225874706663079936*a^8*b^14 - 75192259
797236736*a^4*b^17 + 9386635211853201*b^20)*x)) - 15*a^5*((150981161449947136*a^16*b^8 - 301564036556783616*a^12*b^11 + 225874706663079936*a^8*b^14 -
7519225979723673 6*a^4*b^17 + 9386635211853201*b^20)/a^23)^(1/4)*log(-(a^6*x
*((150981161449947136*a^16*b^8 - 301564036556783616*a^12*b^11 + 22587470666
```

```
3079936*a^8*b^14 - 75192259797236736*a^4*b^17 + 9386635211853201*b^20)/a^23
)^(1/4) + (19712*a^4*b^2 - 9843*b^5)*(a*x^4 + b*x^3)^(1/4))/x) + 15*a^5*((1
50981161449947136*a^16*b^8 - 301564036556783616*a^12*b^11 + 225874706663079
936*a^8*b^14 - 75192259797236736*a^4*b^17 + 9386635211853201*b^20)/a^23)^(1
/4)*log((a^6*x*((150981161449947136*a^16*b^8 - 301564036556783616*a^12*b^11
 + 225874706663079936*a^8*b^14 - 75192259797236736*a^4*b^17 + 9386635211853
201*b^20)/a^23)^(1/4) - (19712*a^4*b^2 - 9843*b^5)*(a*x^4 + b*x^3)^(1/4))/x
) + 4*(6144*a^4*x^4 + 8064*a^3*b*x^3 + 10400*a^2*b^2*x^2 - 65280*a^4*b + 32
705*b^4 - 20*(768*a^5 - 821*a*b^3)*x)*(a*x^4 + b*x^3)^(1/4))/a^5
```

giac [B] time = 0.33, size = 704, normalized size = 2.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-a*x-b)*(a*x^4+b*x^3)^(1/4)/(a*x-b),x, algorithm="giac")
```

```
[Out] 1/8192*sqrt(2)*(19712*a^4*b^2 - 9843*b^5)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^
(1/4) + 2*(a + b/x)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a^5) + 1/8192*sqrt(2)*(1
9712*a^4*b^2 - 9843*b^5)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(a + b
/x)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a^5) + 1/16384*sqrt(2)*(19712*a^4*b^2 -
9843*b^5)*log(sqrt(2)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a + b/x)
)/((-a)^(3/4)*a^5) - 1/16384*sqrt(2)*(19712*a^4*b^2 - 9843*b^5)*log(-sqrt(2
)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(-a) + sqrt(a + b/x))/((-a)^(3/4)*a^5) +
 1/2*sqrt(2)*(2*2^(1/4)*(-a)^(1/4)*a^4*b^2 - 2^(1/4)*(-a)^(1/4)*b^5)*log(2^
(3/4)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(2)*sqrt(-a) + sqrt(a + b/x))/a^6 -
1/2*sqrt(2)*(2*2^(1/4)*(-a)^(1/4)*a^4*b^2 - 2^(1/4)*(-a)^(1/4)*b^5)*log(-2^
(3/4)*(-a)^(1/4)*(a + b/x)^(1/4) + sqrt(2)*sqrt(-a) + sqrt(a + b/x))/a^6 -
1/30720*(65280*(a + b/x)^(17/4)*a^4*b^2 - 245760*(a + b/x)^(13/4)*a^5*b^2 +
 345600*(a + b/x)^(9/4)*a^6*b^2 - 215040*(a + b/x)^(5/4)*a^7*b^2 + 49920*(a
 + b/x)^(1/4)*a^8*b^2 - 32705*(a + b/x)^(17/4)*b^5 + 114400*(a + b/x)^(13/4
)*a*b^5 - 157370*(a + b/x)^(9/4)*a^2*b^5 + 94296*(a + b/x)^(5/4)*a^3*b^5 -
24765*(a + b/x)^(1/4)*a^4*b^5)*x^5/(a^5*b^5) + (2*2^(3/4)*(-a)^(1/4)*a^4*b^
2 - 2^(3/4)*(-a)^(1/4)*b^5)*arctan(1/2*2^(1/4)*(2^(3/4)*(-a)^(1/4) + 2*(a +
 b/x)^(1/4))/(-a)^(1/4))/a^6 + (2*2^(3/4)*(-a)^(1/4)*a^4*b^2 - 2^(3/4)*(-a)
^(1/4)*b^5)*arctan(-1/2*2^(1/4)*(2^(3/4)*(-a)^(1/4) - 2*(a + b/x)^(1/4))/(-
a)^(1/4))/a^6
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}-a x -b \right) \left(a \,x^{4}+b \,x^{3}\right)^{\frac{1}{4}}}{a x -b}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-a*x-b)*(a*x^4+b*x^3)^(1/4)/(a*x-b),x)
```

```
[Out] int((x^4-a*x-b)*(a*x^4+b*x^3)^(1/4)/(a*x-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(a x^{4} + b x^{3}\right)}^{\frac{1}{4}} {\left(x^{4} - a x - b\right)}}{a x - b}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-a*x-b)*(a*x^4+b*x^3)^(1/4)/(a*x-b),x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + b*x^3)^(1/4)*(x^4 - a*x - b)/(a*x - b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x^4+b\,x^3\right)^{1/4}\,\left(-x^4+a\,x+b\right)}{b-a\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^4 + b*x^3)^(1/4)*(b + a*x - x^4))/(b - a*x),x)
[Out] int(((a*x^4 + b*x^3)^(1/4)*(b + a*x - x^4))/(b - a*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\,(ax+b)}\,\left(-ax-b+x^4\right)}{ax-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-a*x-b)*(a*x**4+b*x**3)**(1/4)/(a*x-b),x)
[Out] Integral((x**3*(a*x + b))**(1/4)*(-a*x - b + x**4)/(a*x - b), x)
```

3.2232 $\int \frac{(-4a+b+3x)\left(-b^3+3b^2x-3bx^2+x^3\right)}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(a+b^4d-\left(1+4b^3d\right)x+6b^2dx^2-4bdx^3+dx^4\right)}\,dx$

Optimal. Leaf size=276

$$-\frac{\log\left(a^2+d^{2/3}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{4/3}+\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}\left(\sqrt[3]{d}\,x-a\right.\right.}{2d^{2/3}}$$

Rubi [F] time = 8.63, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-4a+b+3x)\left(-b^3+3b^2x-3bx^2+x^3\right)}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(a+b^4d-\left(1+4b^3d\right)x+6b^2dx^2-4bdx^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-4*a + b + 3*x)*(-b^3 + 3*b^2*x - 3*b*x^2 + x^3))/(((-a + x)*(-b + x)
^2)^(2/3)*(a + b^4*d - (1 + 4*b^3*d)*x + 6*b^2*d*x^2 - 4*b*d*x^3 + d*x^4)),
x]
```

```
[Out] (9*a*(-a + x)^(2/3)*(-b + x)^(4/3)*Defer[Subst][Defer[Int][(a - b + x^3)^(5
/3)/(a^4*(1 + (b*(-4*a^3 + 6*a^2*b - 4*a*b^2 + b^3))/a^4)*d - (1 - 4*(a - b
)^3*d)*x^3 + 6*a^2*(1 + (b*(-2*a + b))/a^2)*d*x^6 + 4*a*(1 - b/a)*d*x^9 + d
*x^12), x], x, (-a + x)^(1/3)])/(-((a - x)*(b - x)^2))^(2/3) + (9*(-a + x)^
(2/3)*(-b + x)^(4/3)*Defer[Subst][Defer[Int][(x^3*(a - b + x^3)^(5/3))/(a^4
*(1 + (b*(-4*a^3 + 6*a^2*b - 4*a*b^2 + b^3))/a^4)*d - (1 - 4*(a - b)^3*d)*x
^3 + 6*a^2*(1 + (b*(-2*a + b))/a^2)*d*x^6 + 4*a*(1 - b/a)*d*x^9 + d*x^12),
x], x, (-a + x)^(1/3)])/(-((a - x)*(b - x)^2))^(2/3) - (3*(4*a - b)*(-a + x
)^(2/3)*(-b + x)^(4/3)*Defer[Subst][Defer[Int][(a - b + x^3)^(5/3)/(a*(1 +
(b^4*d)/a) - (1 + 4*b^3*d)*(a + x^3) + 6*b^2*d*(a + x^3)^2 - 4*b*d*(a + x^3
)^3 + d*(a + x^3)^4), x], x, (-a + x)^(1/3)])/(-((a - x)*(b - x)^2))^(2/3)
```

Rubi steps

$$\int \frac{(-4a+b+3x)\left(-b^3+3b^2x-3bx^2+x^3\right)}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(a+b^4d-\left(1+4b^3d\right)x+6b^2dx^2-4bdx^3+dx^4\right)}dx = \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \cdots}{\cdots}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \cdots}{\cdots}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \cdots}{\cdots}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \left(\cdots \right.}{\cdots}$$

$$= \frac{\left(3(-a+x)^{2/3}(-b+x)^{4/3}\right)\int \cdots}{\cdots}$$

$$= \frac{\left(9(-a+x)^{2/3}(-b+x)^{4/3}\right)\text{Sub}\cdots}{\cdots}$$

$$= \frac{\left(9(-a+x)^{2/3}(-b+x)^{4/3}\right)\text{Sub}\cdots}{\cdots}$$

$$= \frac{\left(9(-a+x)^{2/3}(-b+x)^{4/3}\right)\text{Sub}\cdots}{\cdots}$$

$$= \frac{\left(9(-a+x)^{2/3}(-b+x)^{4/3}\right)\text{Sub}\cdots}{\cdots}$$

Mathematica [F] time = 2.84, size = 0, normalized size = 0.00

$$\int \frac{(-4a+b+3x)\left(-b^3+3b^2x-3bx^2+x^3\right)}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(a+b^4d-\left(1+4b^3d\right)x+6b^2dx^2-4bdx^3+dx^4\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-4*a + b + 3*x)*(-b^3 + 3*b^2*x - 3*b*x^2 + x^3))/(((-a + x)*(-
b + x)^2)^(2/3)*(a + b^4*d - (1 + 4*b^3*d)*x + 6*b^2*d*x^2 - 4*b*d*x^3 + d*
x^4)),x]
```

```
[Out] Integrate[((-4*a + b + 3*x)*(-b^3 + 3*b^2*x - 3*b*x^2 + x^3))/(((-a + x)*(-
b + x)^2)^(2/3)*(a + b^4*d - (1 + 4*b^3*d)*x + 6*b^2*d*x^2 - 4*b*d*x^3 + d*
x^4)), x]
```

IntegrateAlgebraic [A] time = 3.24, size = 276, normalized size = 1.00

$$-\frac{\log\left(a^2+d^{2/3}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{4/3}+\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)-2ax+x^2\right)}{2d^{2/3}}+\frac{\log\left(\sqrt[3]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}+a-x\right)}{d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}}{\sqrt[3]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}-2a+2x}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4*a + b + 3*x)*(-b^3 + 3*b^2*x - 3*b*x^2 + x^3))/(((-
a + x)*(-b + x)^2)^(2/3)*(a + b^4*d - (1 + 4*b^3*d)*x + 6*b^2*d*x^2 - 4*b*d
*x^3 + d*x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x
^2 + x^3)^(2/3))/(-2*a + 2*x + d^(1/3)*(-(a*b^2) + (2*a*b + b^2)*x + (-a -
2*b)*x^2 + x^3)^(2/3))]])/d^(2/3) + Log[a - x + d^(1/3)*(-(a*b^2) + (2*a*b +
 b^2)*x + (-a - 2*b)*x^2 + x^3)^(2/3)]/d^(2/3) - Log[a^2 - 2*a*x + x^2 + (-
(a*d^(1/3)) + d^(1/3)*x)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3
)^(2/3) + d^(2/3)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(4/3)
]/(2*d^(2/3))
```

fricas [A] time = 0.77, size = 351, normalized size = 1.27

$$-\frac{2\sqrt{3}\left(d^2\right)^{\frac{1}{6}}d\arctan\left(\frac{\sqrt{3}\left(d^2\right)^{\frac{1}{6}}\left(\left(b^2d-2bdx+dx^2\right)\left(d^2\right)^{\frac{1}{3}}+2\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}\left(d^2\right)^{\frac{2}{3}}\right)}{3\left(b^2d^2-2bd^2x+d^2x^2\right)}\right)+\left(d^2\right)^{\frac{2}{3}}\log\left(\frac{\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}\left(b^2-2bx+x^2\right)\left(d^2\right)^{\frac{2}{3}}+\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{2}{3}}d+\left(b^4d-4b^3dx+6b^2dx^2-4bdx^3+dx^4\right)\left(d^2\right)^{\frac{1}{3}}}{b^4-4b^3x+6b^2x^2-4bx^3+x^4}\right)-2\left(d^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(b^2-2bx+x^2\right)\left(d^2\right)^{\frac{2}{3}}-\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}d}{b^2-2bx+x^2}\right)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*a+b+3*x)*(-b^3+3*b^2*x-3*b*x^2+x^3)/((-a+x)*(-b+x)^2)^(2/3)/(
a+b^4*d-(4*b^3*d+1)*x+6*b^2*d*x^2-4*b*d*x^3+d*x^4),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(1/3*sqrt(3)*(d^2)^(1/6)*((b^2*d - 2*b*
d*x + d*x^2)*(d^2)^(1/3) + 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*
x)^(1/3)*(d^2)^(2/3))/(b^2*d^2 - 2*b*d^2*x + d^2*x^2)) + (d^2)^(2/3)*log(((
-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(b^2 - 2*b*x + x^2)*(
d^2)^(2/3) + (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*d + (b^
4*d - 4*b^3*d*x + 6*b^2*d*x^2 - 4*b*d*x^3 + d*x^4)*(d^2)^(1/3))/(b^4 - 4*b^
3*x + 6*b^2*x^2 - 4*b*x^3 + x^4)) - 2*(d^2)^(2/3)*log(-((b^2 - 2*b*x + x^2)
*(d^2)^(2/3) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*d)/(b
^2 - 2*b*x + x^2)))/d^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^3-3\,b^2x+3\,bx^2-x^3\right)(4\,a-b-3\,x)}{\left(b^4d+6\,b^2dx^2-4\,bdx^3+dx^4-\left(4\,b^3d+1\right)x+a\right)\left(-(a-x)(b-x)^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*a+b+3*x)*(-b^3+3*b^2*x-3*b*x^2+x^3)/((-a+x)*(-b+x)^2)^(2/3)/(
a+b^4*d-(4*b^3*d+1)*x+6*b^2*d*x^2-4*b*d*x^3+d*x^4),x, algorithm="giac")
```

```
[Out] integrate((b^3 - 3*b^2*x + 3*b*x^2 - x^3)*(4*a - b - 3*x)/((b^4*d + 6*b^2*d
*x^2 - 4*b*d*x^3 + d*x^4 - (4*b^3*d + 1)*x + a)*(-(a - x)*(b - x)^2)^(2/3))
, x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(-4a+b+3x)\left(-b^3+3b^2x-3b\,x^2+x^3\right)}{\left((-a+x)\,(-b+x)^2\right)^{\frac{2}{3}}\left(a+b^4d-\left(4b^3d+1\right)x+6b^2d\,x^2-4bd\,x^3+d\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*a+b+3*x)*(-b^3+3*b^2*x-3*b*x^2+x^3)/((-a+x)*(-b+x)^2)^(2/3)/(a+b^4*
d-(4*b^3*d+1)*x+6*b^2*d*x^2-4*b*d*x^3+d*x^4),x)
```

```
[Out] int((-4*a+b+3*x)*(-b^3+3*b^2*x-3*b*x^2+x^3)/((-a+x)*(-b+x)^2)^(2/3)/(a+b^4*
d-(4*b^3*d+1)*x+6*b^2*d*x^2-4*b*d*x^3+d*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^3 - 3\,b^2x + 3\,bx^2 - x^3\right)(4\,a - b - 3\,x)}{\left(b^4d + 6\,b^2dx^2 - 4\,bdx^3 + dx^4 - \left(4\,b^3d + 1\right)x + a\right)\left(-(a-x)(b-x)^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*a+b+3*x)*(-b^3+3*b^2*x-3*b*x^2+x^3)/((-a+x)*(-b+x)^2)^(2/3)/(
a+b^4*d-(4*b^3*d+1)*x+6*b^2*d*x^2-4*b*d*x^3+d*x^4),x, algorithm="maxima")
```

```
[Out] integrate((b^3 - 3*b^2*x + 3*b*x^2 - x^3)*(4*a - b - 3*x)/((b^4*d + 6*b^2*d
*x^2 - 4*b*d*x^3 + d*x^4 - (4*b^3*d + 1)*x + a)*(-(a - x)*(b - x)^2)^(2/3))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(b-4\,a+3\,x)\,\left(b^3-3\,b^2\,x+3\,b\,x^2-x^3\right)}{\left(-(a-x)\,(b-x)^2\right)^{2/3}\,\left(a+b^4\,d+d\,x^4-x\,\left(4\,d\,b^3+1\right)+6\,b^2\,d\,x^2-4\,b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b - 4*a + 3*x)*(3*b*x^2 - 3*b^2*x + b^3 - x^3))/((-(a - x)*(b - x)^2
)^(2/3)*(a + b^4*d + d*x^4 - x*(4*b^3*d + 1) + 6*b^2*d*x^2 - 4*b*d*x^3)),x)
```

```
[Out] int(-((b - 4*a + 3*x)*(3*b*x^2 - 3*b^2*x + b^3 - x^3))/((-(a - x)*(b - x)^2
)^(2/3)*(a + b^4*d + d*x^4 - x*(4*b^3*d + 1) + 6*b^2*d*x^2 - 4*b*d*x^3)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*a+b+3*x)*(-b**3+3*b**2*x-3*b*x**2+x**3)/((-a+x)*(-b+x)**2)**(
2/3)/(a+b**4*d-(4*b**3*d+1)*x+6*b**2*d*x**2-4*b*d*x**3+d*x**4),x)
```

```
[Out] Timed out
```

3.2233

$$\int \frac{x^2(-2+(1+k)x)}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(2+2k)x+\left(1+4k+k^2\right)x^2-\left(2k+2k^2\right)x^3+\left(-b+k^2\right)x^4\right)}\,dx$$

Optimal. Leaf size=276

$$\frac{\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[6]{b}\,x\right)}{2b^{2/3}}+\frac{\log\left(\sqrt[6]{b}\,x+\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{2b^{2/3}}-\frac{\log\left(-\sqrt[6]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\right.}{4b}$$

Rubi [F] time = 15.64, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2(-2+(1+k)x)}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(2+2k)x+\left(1+4k+k^2\right)x^2-\left(2k+2k^2\right)x^3+\left(-b+k^2\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(-2 + (1 + k)*x))/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (2 + 2*k)*x +
(1 + 4*k + k^2)*x^2 - (2*k + 2*k^2)*x^3 + (-b + k^2)*x^4)),x]
```

```
[Out] (3*(1 + k)*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][x^
10/((1 - x^3)^(1/3)*(1 - k*x^3)^(1/3)*(1 - 2*(1 + k)*x^3 + (1 + k*(4 + k))*
x^6 - 2*k*(1 + k)*x^9 - b*(1 - k^2/b)*x^12)), x], x, x^(1/3)])/((1 - x)*x*(
1 - k*x))^(1/3) + (6*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Def
er[Int][x^7/((1 - x^3)^(1/3)*(1 - k*x^3)^(1/3)*(-1 + 2*(1 + k)*x^3 - (1 + k
*(4 + k))*x^6 + 2*k*(1 + k)*x^9 + b*(1 - k^2/b)*x^12)), x], x, x^(1/3)])/((
1 - x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\int \frac{x^2(-2+(1+k)x)}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(2+2k)x+\left(1+4k+k^2\right)x^2-\left(2k+2k^2\right)x^3+\left(-b+k^2\right)x^4\right)}\,dx=\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right.}{}$$

$$=\frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-k}\right.}{}$$

$$=\frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-k}\right.}{}$$

$$=\frac{\left(6\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-k}\right.}{}$$

Mathematica [F] time = 5.42, size = 0, normalized size = 0.00

$$\int \frac{x^2(-2+(1+k)x)}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(2+2k)x+\left(1+4k+k^2\right)x^2-\left(2k+2k^2\right)x^3+\left(-b+k^2\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(-2 + (1 + k)*x))/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (2 + 2*k
)*x + (1 + 4*k + k^2)*x^2 - (2*k + 2*k^2)*x^3 + (-b + k^2)*x^4)),x]
```

```
[Out] Integrate[(x^2*(-2 + (1 + k)*x))/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (2 + 2*k
)*x + (1 + 4*k + k^2)*x^2 - (2*k + 2*k^2)*x^3 + (-b + k^2)*x^4)), x]
```

IntegrateAlgebraic [A] time = 0.68, size = 274, normalized size = 0.99

$$\frac{\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[6]{b}\,x\right)}{2b^{2/3}}+\frac{\log\left(\sqrt[6]{b}\,x+\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{2b^{2/3}}-\frac{\log\left(-\sqrt[6]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\sqrt[3]{b}\,x^2+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{4b^{2/3}}-\frac{\log\left(\sqrt[6]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\sqrt[3]{b}\,x^2+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{4b^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\left(kx^3+(-k-1)x^2+x\right)^{2/3}}{\sqrt{3}\sqrt[3]{b}}+\frac{x^2}{\sqrt{3}}}{x^2}\right)}{2b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(-2 + (1 + k)*x))/(((1 - x)*x*(1 - k*x))^(1/3)*(1 -
 (2 + 2*k)*x + (1 + 4*k + k^2)*x^2 - (2*k + 2*k^2)*x^3 + (-b + k^2)*x^4)),x
]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(x^2/Sqrt[3] + (2*(x + (-1 - k)*x^2 + k*x^3)^(2/3))/(S
qrt[3]*b^(1/3)))/x^2])/b^(2/3) + Log[-(b^(1/6)*x) + (x + (-1 - k)*x^2 + k*x
^3)^(1/3)]/(2*b^(2/3)) + Log[b^(1/6)*x + (x + (-1 - k)*x^2 + k*x^3)^(1/3)]/
(2*b^(2/3)) - Log[b^(1/3)*x^2 - b^(1/6)*x*(x + (-1 - k)*x^2 + k*x^3)^(1/3)
+ (x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(4*b^(2/3)) - Log[b^(1/3)*x^2 + b^(1/6)
*x*(x + (-1 - k)*x^2 + k*x^3)^(1/3) + (x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(4*
b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(2+2*k)*x+(k^2+4*k+1
)*x^2-(2*k^2+2*k)*x^3+(k^2-b)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.37, size = 123, normalized size = 0.45

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{2}{3}}+b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{2b^{\frac{2}{3}}}-\frac{\log\left(\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{4}{3}}+b^{\frac{1}{3}}\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{4b^{\frac{2}{3}}}+\frac{\log\left(\left|\left(k-\frac{k}{x}-\frac{1}{x}+\frac{1}{x^2}\right)^{\frac{2}{3}}-b^{\frac{1}{3}}\right|\right)}{2b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(2+2*k)*x+(k^2+4*k+1
)*x^2-(2*k^2+2*k)*x^3+(k^2-b)*x^4),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(k - k/x - 1/x + 1/x^2)^(2/3) + b^(1/3))
/b^(1/3))/b^(2/3) - 1/4*log((k - k/x - 1/x + 1/x^2)^(4/3) + b^(1/3)*(k - k/
x - 1/x + 1/x^2)^(2/3) + b^(2/3))/b^(2/3) + 1/2*log(abs((k - k/x - 1/x + 1/
x^2)^(2/3) - b^(1/3)))/b^(2/3)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(-2+\left(1+k\right) x\right)}{\left(\left(1-x\right) x \left(-k x+1\right)\right)^{\frac{1}{3}} \left(1-\left(2+2 k\right) x+\left(k^{2}+4 k+1\right) x^{2}-\left(2 k^{2}+2 k\right) x^{3}+\left(k^{2}-b\right) x^{4}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(2+2*k)*x+(k^2+4*k+1)*x^2-
(2*k^2+2*k)*x^3+(k^2-b)*x^4),x)
```

```
[Out] int(x^2*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(2+2*k)*x+(k^2+4*k+1)*x^2-
(2*k^2+2*k)*x^3+(k^2-b)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((k+1)x-2)x^2}{\left(\left(k^2-b\right)x^4-2\left(k^2+k\right)x^3+\left(k^2+4k+1\right)x^2-2(k+1)x+1\right)((kx-1)(x-1)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(2+2*k)*x+(k^2+4*k+1
)*x^2-(2*k^2+2*k)*x^3+(k^2-b)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(((k + 1)*x - 2)*x^2/(((k^2 - b)*x^4 - 2*(k^2 + k)*x^3 + (k^2 + 4*
k + 1)*x^2 - 2*(k + 1)*x + 1)*((k*x - 1)*(x - 1)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x^2\,\left(x\,\left(k+1\right)-2\right)}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/3}\,\left(\left(b-k^2\right)\,x^4+\left(2\,k^2+2\,k\right)\,x^3+\left(-k^2-4\,k-1\right)\,x^2+\left(2\,k+2\right)\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(x*(k + 1) - 2))/((x*(k*x - 1)*(x - 1))^(1/3)*(x*(2*k + 2) + x^4*
(b - k^2) - x^2*(4*k + k^2 + 1) + x^3*(2*k + 2*k^2) - 1)),x)
```

```
[Out] int(-(x^2*(x*(k + 1) - 2))/((x*(k*x - 1)*(x - 1))^(1/3)*(x*(2*k + 2) + x^4*
(b - k^2) - x^2*(4*k + k^2 + 1) + x^3*(2*k + 2*k^2) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-2+(1+k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(1-(2+2*k)*x+(k**2+4*
k+1)*x**2-(2*k**2+2*k)*x**3+(k**2-b)*x**4),x)
```

```
[Out] Timed out
```

3.2234 $\int \frac{1}{c+\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$

Optimal. Leaf size=276

$$\frac{b^2\log\left(\sqrt{a^2x^2+b^2}+ax\right)}{2ac^3}-\frac{b^2\log\left(\sqrt{\sqrt{a^2x^2+b^2}+ax}+c\right)}{ac^3}+\frac{\sqrt{a^2x^2+b^2}\left(\left(3b^2+2c^4\right)\sqrt{\sqrt{a^2x^2+b^2}+ax}+\cdots\right.}{\cdots}$$

Rubi [A] time = 0.13, antiderivative size = 169, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2117, 1821, 1620}

$$\frac{b^2\log\left(\sqrt{a^2x^2+b^2}+ax\right)}{2ac^3}+\frac{b^2}{ac^2\sqrt{\sqrt{a^2x^2+b^2}+ax}}-\frac{\left(b^2+c^4\right)\log\left(\sqrt{\sqrt{a^2x^2+b^2}+ax}+c\right)}{ac^3}-\frac{b^2}{2ac\left(\sqrt{a^2x^2+b^2}+ax\right)}+\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])^(-1),x]
```

```
[Out] -1/2*b^2/(a*c*(a*x + Sqrt[b^2 + a^2*x^2])) + b^2/(a*c^2*Sqrt[a*x + Sqrt[b^2
 + a^2*x^2]]) + Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]/a + (b^2*Log[a*x + Sqrt[b^2
 + a^2*x^2]])/(2*a*c^3) - ((b^2 + c^4)*Log[c + Sqrt[a*x + Sqrt[b^2 + a^2*x^
2]]])/(a*c^3)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{c+\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx = \frac{\text{Subst}\left(\int \frac{b^2+x^2}{(c+\sqrt{x})x^2}\,dx, x, ax+\sqrt{b^2+a^2x^2}\right)}{2a}$$

$$= \frac{\text{Subst}\left(\int \frac{b^2+x^4}{x^3(c+x)}\,dx, x, \sqrt{ax+\sqrt{b^2+a^2x^2}}\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{b^2}{cx^3}-\frac{b^2}{c^2x^2}+\frac{b^2}{c^3x}+\frac{-b^2-c^4}{c^3(c+x)}\right)dx, x, \sqrt{ax+\sqrt{b^2+a^2x^2}}\right)}{a}$$

$$= -\frac{b^2}{2ac\left(ax+\sqrt{b^2+a^2x^2}\right)} + \frac{b^2}{ac^2\sqrt{ax+\sqrt{b^2+a^2x^2}}} + \frac{\sqrt{ax+\sqrt{b^2+a^2x^2}}}{a} + \frac{b^2 1}{}$$

Mathematica [A] time = 0.19, size = 157, normalized size = 0.57

$$\frac{\frac{b^2\log\left(\sqrt{a^2x^2+b^2}+ax\right)}{2c^3}+\frac{b^2}{c^2\sqrt{\sqrt{a^2x^2+b^2}+ax}}-\frac{\left(b^2+c^4\right)\log\left(\sqrt{\sqrt{a^2x^2+b^2}+ax}+c\right)}{c^3}-\frac{b^2}{2c\left(\sqrt{a^2x^2+b^2}+ax\right)}+\sqrt{\sqrt{a^2x^2+b^2}+ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])^(-1),x]
```

```
[Out] (-1/2*b^2/(c*(a*x + Sqrt[b^2 + a^2*x^2])) + b^2/(c^2*Sqrt[a*x + Sqrt[b^2 +
a^2*x^2]]) + Sqrt[a*x + Sqrt[b^2 + a^2*x^2]] + (b^2*Log[a*x + Sqrt[b^2 + a^
2*x^2]])/(2*c^3) - ((b^2 + c^4)*Log[c + Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]])/c
^3)/a
```

IntegrateAlgebraic [A] time = 0.48, size = 276, normalized size = 1.00

$$\frac{b^2\log\left(\sqrt{a^2x^2+b^2}+ax\right)}{2ac^3}-\frac{b^2\log\left(\sqrt{\sqrt{a^2x^2+b^2}+ax}+c\right)}{ac^3}+\frac{\sqrt{a^2x^2+b^2}\left(\left(3b^2+2c^4\right)\sqrt{\sqrt{a^2x^2+b^2}+ax}+4ac^3x+2b^2c\right)+\sqrt{\sqrt{a^2x^2+b^2}+ax}\left(3ab^2x+2ac^4x-b^2c^2\right)+4a^2c^3x^2+2ab^2cx+2b^2c^3}{2ac^3\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}-\frac{c\log\left(\sqrt{\sqrt{a^2x^2+b^2}+ax}+c\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])^(-1),x]
```

```
[Out] (2*b^2*c^3 + 2*a*b^2*c*x + 4*a^2*c^3*x^2 + (-(b^2*c^2) + 3*a*b^2*x + 2*a*c^
4*x)*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]] + Sqrt[b^2 + a^2*x^2]*(2*b^2*c + 4*a*c
^3*x + (3*b^2 + 2*c^4)*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]))/(2*a*c^3*(a*x + Sq
rt[b^2 + a^2*x^2])^(3/2)) + (b^2*Log[a*x + Sqrt[b^2 + a^2*x^2]])/(2*a*c^3)
- (b^2*Log[c + Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]])/(a*c^3) - (c*Log[c + Sqrt[
a*x + Sqrt[b^2 + a^2*x^2]]])/a
```

fricas [A] time = 0.81, size = 134, normalized size = 0.49

$$\frac{ac^2x+2b^2\log\left(\sqrt{ax+\sqrt{a^2x^2+b^2}}\right)-\sqrt{a^2x^2+b^2}\,c^2-2\left(c^4+b^2\right)\log\left(c+\sqrt{ax+\sqrt{a^2x^2+b^2}}\right)+2\left(c^3-acx+\sqrt{a^2x^2+b^2}\,c\right)\sqrt{ax+\sqrt{a^2x^2+b^2}}}{2\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c^2*x + 2*b^2*log(sqrt(a*x + sqrt(a^2*x^2 + b^2))) - sqrt(a^2*x^2 +
b^2)*c^2 - 2*(c^4 + b^2)*log(c + sqrt(a*x + sqrt(a^2*x^2 + b^2))) + 2*(c^3
- a*c*x + sqrt(a^2*x^2 + b^2)*c)*sqrt(a*x + sqrt(a^2*x^2 + b^2)))/(a*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c+\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)),x, algorithm="giac")
[Out] integrate(1/(c + sqrt(a*x + sqrt(a^2*x^2 + b^2))), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{c+\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)),x)
[Out] int(1/(c+(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c+\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(c + sqrt(a*x + sqrt(a^2*x^2 + b^2))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{c+\sqrt{a\,x+\sqrt{a^2\,x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + (a*x + (b^2 + a^2*x^2)^(1/2))^(1/2)),x)
[Out] int(1/(c + (a*x + (b^2 + a^2*x^2)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c+\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(a*x+(a**2*x**2+b**2)**(1/2))**(1/2)),x)
[Out] Integral(1/(c + sqrt(a*x + sqrt(a**2*x**2 + b**2))), x)
```

3.2235 $\int \frac{(-7+x)\sqrt[3]{1+x-5x^2+3x^3}}{(-5+x)(-1+x)^3}\,dx$

Optimal. Leaf size=278

$$\frac{(x-1)^{4/3}(3x+1)^{2/3}\left(-\frac{21\sqrt[3]{3}\left(7(3x+1)^{4/3}-16\sqrt[3]{3x+1}\right)}{64(3x-3)^{4/3}}+\frac{3\sqrt[3]{3}\left(23(3x+1)^{4/3}-80\sqrt[3]{3x+1}\right)}{64(3x-3)^{4/3}}-\frac{\log\left(6^{2/3}\sqrt[3]{3x-3}-3\sqrt[3]{3x+1}\right)}{4\sqrt[3]{2}}+\frac{\log\left(2\sqrt[3]{6}(3x-3)^{2/}\right.}{}\right.}{\left((x-1)^2(3x+1)\right)^{2/3}}$$

Rubi [A] time = 1.45, antiderivative size = 273, normalized size of antiderivative = 0.98, number of steps used = 22, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {6742, 2081, 2077, 101, 157, 60, 91, 21, 37, 47, 50}

$$-\frac{9\sqrt[3]{3x^3-5x^2+x+1}\,(3x+1)}{32(1-x)^2}+\frac{3\sqrt[3]{3x^3-5x^2+x+1}}{8(1-x)}+\frac{\sqrt[3]{3x^3-5x^2+x+1}\,\log(x-5)}{8\sqrt[3]{2}\,(1-x)^{2/3}\sqrt[3]{3x+1}}-\frac{3\sqrt[3]{3x^3-5x^2+x+1}\,\log\left(-\frac{4}{3}\sqrt[3]{1-x}-\frac{2}{3}\sqrt[3]{2}\sqrt[3]{3x+1}\right)}{8\sqrt[3]{2}\,(1-x)^{2/3}\sqrt[3]{3x+1}}-\frac{\sqrt{3}\sqrt[3]{3x^3-5x^2+x+1}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\,2^{2/3}\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{3x+1}}\right)}{4\sqrt[3]{2}\,(1-x)^{2/3}\sqrt[3]{3x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((-7 + x)*(1 + x - 5*x^2 + 3*x^3)^(1/3))/((-5 + x)*(-1 + x)^3),x]
```

```
[Out] (3*(1 + x - 5*x^2 + 3*x^3)^(1/3))/(8*(1 - x)) - (9*(1 + 3*x)*(1 + x - 5*x^2
 + 3*x^3)^(1/3))/(32*(1 - x)^2) - (Sqrt[3]*(1 + x - 5*x^2 + 3*x^3)^(1/3)*Ar
cTan[1/Sqrt[3] - (2*2^(2/3)*(1 - x)^(1/3))/(Sqrt[3]*(1 + 3*x)^(1/3))])/(4*2
^(1/3)*(1 - x)^(2/3)*(1 + 3*x)^(1/3)) + ((1 + x - 5*x^2 + 3*x^3)^(1/3)*Log[
-5 + x])/(8*2^(1/3)*(1 - x)^(2/3)*(1 + 3*x)^(1/3)) - (3*(1 + x - 5*x^2 + 3*
x^3)^(1/3)*Log[(-4*(1 - x)^(1/3))/3 - (2*2^(1/3)*(1 + 3*x)^(1/3))/3])/(8*2^
(1/3)*(1 - x)^(2/3)*(1 + 3*x)^(1/3))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 2077

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Dist[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int
[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d, e
, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 2081

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(-7+x)\sqrt[3]{1+x-5x^2+3x^3}}{(-5+x)(-1+x)^3}\,dx = \int \left(-\frac{\sqrt[3]{1+x-5x^2+3x^3}}{32(-5+x)} + \frac{3\sqrt[3]{1+x-5x^2+3x^3}}{2(-1+x)^3} + \frac{\sqrt[3]{1+x-5x^2+3x^3}}{8(-1+x)^2} + \right.$$

$$= -\left(\frac{1}{32}\int \frac{\sqrt[3]{1+x-5x^2+3x^3}}{-5+x}\,dx\right) + \frac{1}{32}\int \frac{\sqrt[3]{1+x-5x^2+3x^3}}{-1+x}\,dx + \frac{1}{8}\int$$

$$= -\left(\frac{1}{32}\,\text{Subst}\left(\int \frac{\sqrt[3]{\frac{128}{243}-\frac{16x}{9}+3x^3}}{-\frac{40}{9}+x}\,dx, x, -\frac{5}{9}+x\right)\right) + \frac{1}{32}\,\text{Subst}\left(\int \frac{\sqrt[3]{\frac{128}{243}}}{}\right.$$

$$= -\frac{\left(9\sqrt[3]{3}\sqrt[3]{1+x-5x^2+3x^3}\right)\text{Subst}\left(\int \frac{\left(\frac{128}{81}-\frac{32x}{9}\right)^{2/3}\sqrt[3]{\frac{128}{81}+\frac{16x}{9}}}{-\frac{40}{9}+x}\,dx, x, -\frac{5}{9}+x\right)}{512\;2^{2/3}(1-x)^{2/3}\sqrt[3]{1+3x}} +$$

$$= -\frac{1}{32}\sqrt[3]{1+x-5x^2+3x^3} + \frac{\left(4\sqrt[3]{2}\sqrt[3]{1+x-5x^2+3x^3}\right)\text{Subst}\left(\int \frac{\sqrt[3]{\frac{128}{81}+\frac{16x}{9}}}{\left(\frac{128}{81}-\frac{32x}{9}\right)^{4/3}}\right.}{3\;3^{2/3}(1-x)^{2/3}\sqrt[3]{1+3x}}$$

$$= \frac{3\sqrt[3]{1+x-5x^2+3x^3}}{8(1-x)} - \frac{9(1+3x)\sqrt[3]{1+x-5x^2+3x^3}}{32(1-x)^2} - \frac{\left(2\sqrt[3]{2}\sqrt[3]{1+x-5x^2}\right.}{}$$

$$= \frac{3\sqrt[3]{1+x-5x^2+3x^3}}{8(1-x)} - \frac{9(1+3x)\sqrt[3]{1+x-5x^2+3x^3}}{32(1-x)^2} - \frac{\sqrt{3}\sqrt[3]{1+x-5x^2+}}{4\sqrt[3]{2}(}$$

Mathematica [C] time = 0.03, size = 56, normalized size = 0.20

$$\frac{3\left(8(x-1)^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{4(x-1)}{3x+1}\right)-39x^2-10x+1\right)}{32\left((x-1)^2(3x+1)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-7 + x)*(1 + x - 5*x^2 + 3*x^3)^(1/3))/((-5 + x)*(-1 + x)^3),x]
```

```
[Out] (3*(1 - 10*x - 39*x^2 + 8*(-1 + x)^2*Hypergeometric2F1[2/3, 1, 5/3, (4*(-1
+ x))/(1 + 3*x)]))/(32*((-1 + x)^2*(1 + 3*x))^(2/3))
```

IntegrateAlgebraic [A] time = 17.53, size = 218, normalized size = 0.78

$$\frac{\sqrt[3]{x-1}\,(3x+1)^{2/3}\sqrt[3]{(x-1)^2(3x+1)}\left(-\frac{3\sqrt[3]{3x+1}\left(\frac{3(3x+1)}{x-1}+4\right)}{32\sqrt[3]{x-1}}-\frac{\log\left(\frac{\sqrt[3]{2}\sqrt[3]{3x+1}}{\sqrt[3]{x-1}}-2\right)}{4\sqrt[3]{2}}+\frac{\log\left(\frac{2^{2/3}(3x+1)^{2/3}}{(x-1)^{2/3}}+\frac{2\sqrt[3]{2}\sqrt[3]{3x+1}}{\sqrt[3]{x-1}}+4\right)}{8\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{3x+1}}{\sqrt{3}\sqrt[3]{x-1}}+\frac{1}{\sqrt{3}}\right)}{4\sqrt[3]{2}}\right)}{3x^2-2x-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-7 + x)*(1 + x - 5*x^2 + 3*x^3)^(1/3))/((-5 + x)*(-1 +
 x)^3),x]
```

```
[Out] ((-1 + x)^(1/3)*(1 + 3*x)^(2/3)*((-1 + x)^2*(1 + 3*x))^(1/3)*((-3*(1 + 3*x)
^(1/3)*(4 + (3*(1 + 3*x))/(-1 + x)))/(32*(-1 + x)^(1/3)) + (Sqrt[3]*ArcTan[
1/Sqrt[3] + (2^(1/3)*(1 + 3*x)^(1/3))/(Sqrt[3]*(-1 + x)^(1/3))])/(4*2^(1/3)
) - Log[-2 + (2^(1/3)*(1 + 3*x)^(1/3))/(-1 + x)^(1/3)]/(4*2^(1/3)) + Log[4
+ (2*2^(1/3)*(1 + 3*x)^(1/3))/(-1 + x)^(1/3) + (2^(2/3)*(1 + 3*x)^(2/3))/(-
1 + x)^(2/3)]/(8*2^(1/3))))/(-1 - 2*x + 3*x^2)
```

fricas [A] time = 0.64, size = 236, normalized size = 0.85

$$-\frac{4\sqrt{3}\,2^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(x^{2}-2x+1\right)\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}(x-1)+2\cdot 2^{\frac{1}{6}}(-1)^{\frac{2}{3}}\left(3x^{3}-5x^{2}+x+1\right)^{\frac{1}{3}}\right)}{6(x-1)}\right)+2\cdot 2^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(x^{2}-2x+1\right)\log\left(-\frac{2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(3x^{3}-5x^{2}+x+1\right)^{\frac{1}{3}}(x-1)-2\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^{2}-2x+1\right)-\left(3x^{3}-5x^{2}+x+1\right)^{\frac{2}{3}}}{x^{2}-2x+1}\right)-4\cdot 2^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(x^{2}-2x+1\right)\log\left(\frac{2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x-1)+\left(3x^{3}-5x^{2}+x+1\right)^{\frac{1}{3}}}{x-1}\right)+3\left(3x^{3}-5x^{2}+x+1\right)^{\frac{1}{3}}(13x-1)}{32\left(x^{2}-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7+x)*(3*x^3-5*x^2+x+1)^(1/3)/(-5+x)/(-1+x)^3,x, algorithm="fric
as")
```

```
[Out] -1/32*(4*sqrt(3)*2^(2/3)*(-1)^(1/3)*(x^2 - 2*x + 1)*arctan(1/6*sqrt(3)*2^(1
/6)*(2^(5/6)*(x - 1) + 2*2^(1/6)*(-1)^(2/3)*(3*x^3 - 5*x^2 + x + 1)^(1/3))/
(x - 1)) + 2*2^(2/3)*(-1)^(1/3)*(x^2 - 2*x + 1)*log(-(2^(2/3)*(-1)^(1/3)*(3
*x^3 - 5*x^2 + x + 1)^(1/3)*(x - 1) - 2*2^(1/3)*(-1)^(2/3)*(x^2 - 2*x + 1)
- (3*x^3 - 5*x^2 + x + 1)^(2/3))/(x^2 - 2*x + 1)) - 4*2^(2/3)*(-1)^(1/3)*(x
^2 - 2*x + 1)*log((2^(2/3)*(-1)^(1/3)*(x - 1) + (3*x^3 - 5*x^2 + x + 1)^(1/
3))/(x - 1)) + 3*(3*x^3 - 5*x^2 + x + 1)^(1/3)*(13*x - 1))/(x^2 - 2*x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,x^{3}-5\,x^{2}+x+1\right)^{\frac{1}{3}}(x-7)}{(x-1)^{3}(x-5)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7+x)*(3*x^3-5*x^2+x+1)^(1/3)/(-5+x)/(-1+x)^3,x, algorithm="giac
")
```

```
[Out] integrate((3*x^3 - 5*x^2 + x + 1)^(1/3)*(x - 7)/((x - 1)^3*(x - 5)), x)
```

maple [C] time = 2.96, size = 1046, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7+x)*(3*x^3-5*x^2+x+1)^(1/3)/(-5+x)/(-1+x)^3,x)
```

```
[Out] -3/32*(13*x-1)/(-1+x)^2*(3*x^3-5*x^2+x+1)^(1/3)+1/8*RootOf(_Z^3+4)*ln(-(-46
080*RootOf(RootOf(_Z^3+4)^2+48*_Z*RootOf(_Z^3+4)+2304*_Z^2)^2*RootOf(_Z^3+4
)^3*x^2-1344*RootOf(RootOf(_Z^3+4)^2+48*_Z*RootOf(_Z^3+4)+2304*_Z^2)*RootOf
(_Z^3+4)^4*x^2+46080*RootOf(RootOf(_Z^3+4)^2+48*_Z*RootOf(_Z^3+4)+2304*_Z^2
)^2*RootOf(_Z^3+4)^3*x+1344*RootOf(RootOf(_Z^3+4)^2+48*_Z*RootOf(_Z^3+4)+23
04*_Z^2)*RootOf(_Z^3+4)^4*x+1152*(3*x^3-5*x^2+x+1)^(2/3)*RootOf(RootOf(_Z^3
+4)^2+48*_Z*RootOf(_Z^3+4)+2304*_Z^2)*RootOf(_Z^3+4)^2+6000*RootOf(RootOf(_
Z^3+4)^2+48*_Z*RootOf(_Z^3+4)+2304*_Z^2)*RootOf(_Z^3+4)*x^2+175*RootOf(_Z^3
+4)^2*x^2+4032*(3*x^3-5*x^2+x+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+48*_Z*RootOf
(_Z^3+4)+2304*_Z^2)*x+180*(3*x^3-5*x^2+x+1)^(1/3)*RootOf(_Z^3+4)*x-9120*Roo
tOf(RootOf(_Z^3+4)^2+48*_Z*RootOf(_Z^3+4)+2304*_Z^2)*RootOf(_Z^3+4)*x-266*R
ootOf(_Z^3+4)^2*x+84*(3*x^3-5*x^2+x+1)^(2/3)-4032*(3*x^3-5*x^2+x+1)^(1/3)*R
ootOf(RootOf(_Z^3+4)^2+48*_Z*RootOf(_Z^3+4)+2304*_Z^2)-180*(3*x^3-5*x^2+x+1
)^(1/3)*RootOf(_Z^3+4)+3120*RootOf(RootOf(_Z^3+4)^2+48*_Z*RootOf(_Z^3+4)+23
04*_Z^2)*RootOf(_Z^3+4)+91*RootOf(_Z^3+4)^2)/(-5+x)/(-1+x))+6*RootOf(RootOf
(_Z^3+4)^2+48*_Z*RootOf(_Z^3+4)+2304*_Z^2)*ln((18432*RootOf(RootOf(_Z^3+4)^
2+48*_Z*RootOf(_Z^3+4)+2304*_Z^2)^2*RootOf(_Z^3+4)^3*x^2+1344*RootOf(RootOf
(_Z^3+4)^2+48*_Z*RootOf(_Z^3+4)+2304*_Z^2)*RootOf(_Z^3+4)^4*x^2-18432*RootO
f(RootOf(_Z^3+4)^2+48*_Z*RootOf(_Z^3+4)+2304*_Z^2)^2*RootOf(_Z^3+4)^3*x-134
4*RootOf(RootOf(_Z^3+4)^2+48*_Z*RootOf(_Z^3+4)+2304*_Z^2)*RootOf(_Z^3+4)^4*
x+1152*(3*x^3-5*x^2+x+1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+48*_Z*RootOf(_Z^3+4)
+2304*_Z^2)*RootOf(_Z^3+4)^2+864*RootOf(RootOf(_Z^3+4)^2+48*_Z*RootOf(_Z^3+
4)+2304*_Z^2)*RootOf(_Z^3+4)*x^2+63*RootOf(_Z^3+4)^2*x^2-8640*(3*x^3-5*x^2+
x+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+48*_Z*RootOf(_Z^3+4)+2304*_Z^2)*x-84*(3*
```

```
x^3-5*x^2+x+1)^(1/3)*RootOf(_Z^3+4)*x-2112*RootOf(RootOf(_Z^3+4)^2+48*_Z*Ro
otOf(_Z^3+4)+2304*_Z^2)*RootOf(_Z^3+4)*x-154*RootOf(_Z^3+4)^2*x-180*(3*x^3-
5*x^2+x+1)^(2/3)+8640*(3*x^3-5*x^2+x+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+48*_Z
*RootOf(_Z^3+4)+2304*_Z^2)+84*(3*x^3-5*x^2+x+1)^(1/3)*RootOf(_Z^3+4)+1248*R
ootOf(RootOf(_Z^3+4)^2+48*_Z*RootOf(_Z^3+4)+2304*_Z^2)*RootOf(_Z^3+4)+91*Ro
otOf(_Z^3+4)^2)/(-5+x)/(-1+x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3 x^3-5 x^2+x+1\right)^{\frac{1}{3}}(x-7)}{(x-1)^3(x-5)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7+x)*(3*x^3-5*x^2+x+1)^(1/3)/(-5+x)/(-1+x)^3,x, algorithm="maxi
ma")
```

```
[Out] integrate((3*x^3 - 5*x^2 + x + 1)^(1/3)*(x - 7)/((x - 1)^3*(x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x-7)\,\left(3 x^3-5 x^2+x+1\right)^{1/3}}{(x-1)^3\,(x-5)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 7)*(x - 5*x^2 + 3*x^3 + 1)^(1/3))/((x - 1)^3*(x - 5)),x)
```

```
[Out] int(((x - 7)*(x - 5*x^2 + 3*x^3 + 1)^(1/3))/((x - 1)^3*(x - 5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{(x-1)^2\,(3x+1)}\,\,(x-7)}{(x-5)\,(x-1)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7+x)*(3*x**3-5*x**2+x+1)**(1/3)/(-5+x)/(-1+x)**3,x)
```

```
[Out] Integral(((x - 1)**2*(3*x + 1))**(1/3)*(x - 7)/((x - 5)*(x - 1)**3), x)
```

3.2236 $\int \frac{b^2+a^2x^2}{\left(-b^2+a^2x^2\right)^3\sqrt[3]{-bx^2+ax^3}}\,dx$

Optimal. Leaf size=278

$$-\frac{59\log\left(2a^{2/3}x^2+2^{2/3}\sqrt[3]{a}\,x\sqrt[3]{ax^3-bx^2}+\sqrt[3]{2}\left(ax^3-bx^2\right)^{2/3}\right)}{288\sqrt[3]{2}\sqrt[3]{a}\,b^4}-\frac{\left(ax^3-bx^2\right)^{2/3}\left(-625a^4x^4-67a^3bx^3+1503a^2b\right.}{672b^4x(b-ax)^3(ax+b}$$

Rubi [C] time = 147.51, antiderivative size = 1306, normalized size of antiderivative = 4.70, number of steps used = 10, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {2056, 6733, 6742, 1404, 430, 429}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(b^2 + a^2*x^2)/((-b^2 + a^2*x^2)^3*(-(b*x^2) + a*x^3)^(1/3)),x]
```

```
[Out] (9*x*Gamma[4/3]*(1820*b^4*Hypergeometric2F1[1, 2, 13/3, (2*a*x)/(b + a*x)]
- 2275*a*b^3*x*Hypergeometric2F1[1, 2, 13/3, (2*a*x)/(b + a*x)] - 585*a^2*b
^2*x^2*Hypergeometric2F1[1, 2, 13/3, (2*a*x)/(b + a*x)] + 2457*a^3*b*x^3*Hy
pergeometric2F1[1, 2, 13/3, (2*a*x)/(b + a*x)] - 1053*a^4*x^4*Hypergeometri
c2F1[1, 2, 13/3, (2*a*x)/(b + a*x)] + 2484*a*b^3*x*Hypergeometric2F1[2, 3,
16/3, (2*a*x)/(b + a*x)] - 6534*a^2*b^2*x^2*Hypergeometric2F1[2, 3, 16/3, (
2*a*x)/(b + a*x)] + 5832*a^3*b*x^3*Hypergeometric2F1[2, 3, 16/3, (2*a*x)/(b
 + a*x)] - 1782*a^4*x^4*Hypergeometric2F1[2, 3, 16/3, (2*a*x)/(b + a*x)] +
162*a*x*(7*b - 6*a*x)*(b - a*x)^2*HypergeometricPFQ[{2, 2, 3}, {1, 16/3}, (
2*a*x)/(b + a*x)] + 162*a*(b*x^(1/3) - a*x^(4/3))^3*HypergeometricPFQ[{2, 2
, 2, 3}, {1, 1, 16/3}, (2*a*x)/(b + a*x)]))/(1820*b^4*(b - a*x)*(b + a*x)^3
*(-(b*x^2) + a*x^3)^(1/3)*Gamma[1/3]) - (9*x*Gamma[4/3]*(553280*b^6*Hyperge
ometric2F1[1, 3, 19/3, (2*a*x)/(b + a*x)] - 1521520*a*b^5*x*Hypergeometric2
F1[1, 3, 19/3, (2*a*x)/(b + a*x)] + 1482000*a^2*b^4*x^2*Hypergeometric2F1[1
, 3, 19/3, (2*a*x)/(b + a*x)] + 355680*a^3*b^3*x^3*Hypergeometric2F1[1, 3,
19/3, (2*a*x)/(b + a*x)] - 1723680*a^4*b^2*x^4*Hypergeometric2F1[1, 3, 19/3
, (2*a*x)/(b + a*x)] + 1200420*a^5*b*x^5*Hypergeometric2F1[1, 3, 19/3, (2*a
*x)/(b + a*x)] - 277020*a^6*x^6*Hypergeometric2F1[1, 3, 19/3, (2*a*x)/(b +
a*x)] + 994248*a*b^5*x*Hypergeometric2F1[2, 4, 22/3, (2*a*x)/(b + a*x)] - 4
219830*a^2*b^4*x^2*Hypergeometric2F1[2, 4, 22/3, (2*a*x)/(b + a*x)] + 74746
80*a^3*b^3*x^3*Hypergeometric2F1[2, 4, 22/3, (2*a*x)/(b + a*x)] - 6828300*a
^4*b^2*x^4*Hypergeometric2F1[2, 4, 22/3, (2*a*x)/(b + a*x)] + 3178440*a^5*b
*x^5*Hypergeometric2F1[2, 4, 22/3, (2*a*x)/(b + a*x)] - 599238*a^6*x^6*Hype
rgeometric2F1[2, 4, 22/3, (2*a*x)/(b + a*x)] + 405*a*x*(b - a*x)^2*(1730*b^
3 - 4419*a*b^2*x + 3960*a^2*b*x^2 - 1215*a^3*x^3)*HypergeometricPFQ[{2, 2,
4}, {1, 22/3}, (2*a*x)/(b + a*x)] + 1215*a*(b*x^(1/3) - a*x^(4/3))^3*(191*b
^2 - 336*a*b*x + 153*a^2*x^2)*HypergeometricPFQ[{2, 2, 2, 4}, {1, 1, 22/3},
 (2*a*x)/(b + a*x)] + 36450*a*b^5*x*HypergeometricPFQ[{2, 2, 2, 2, 4}, {1,
1, 1, 22/3}, (2*a*x)/(b + a*x)] - 178605*a^2*b^4*x^2*HypergeometricPFQ[{2,
2, 2, 2, 4}, {1, 1, 1, 22/3}, (2*a*x)/(b + a*x)] + 349920*a^3*b^3*x^3*Hyper
geometricPFQ[{2, 2, 2, 2, 4}, {1, 1, 1, 22/3}, (2*a*x)/(b + a*x)] - 342630*
a^4*b^2*x^4*HypergeometricPFQ[{2, 2, 2, 2, 4}, {1, 1, 1, 22/3}, (2*a*x)/(b
+ a*x)] + 167670*a^5*b*x^5*HypergeometricPFQ[{2, 2, 2, 2, 4}, {1, 1, 1, 22/
3}, (2*a*x)/(b + a*x)] - 32805*a^6*x^6*HypergeometricPFQ[{2, 2, 2, 2, 4}, {
1, 1, 1, 22/3}, (2*a*x)/(b + a*x)] + 2187*a*b^5*x*HypergeometricPFQ[{2, 2,
2, 2, 2, 4}, {1, 1, 1, 1, 22/3}, (2*a*x)/(b + a*x)] - 10935*a^2*b^4*x^2*Hyp
ergeometricPFQ[{2, 2, 2, 2, 2, 4}, {1, 1, 1, 1, 22/3}, (2*a*x)/(b + a*x)] +
 21870*a^3*b^3*x^3*HypergeometricPFQ[{2, 2, 2, 2, 2, 4}, {1, 1, 1, 1, 22/3}
, (2*a*x)/(b + a*x)] - 21870*a^4*b^2*x^4*HypergeometricPFQ[{2, 2, 2, 2, 2,
4}, {1, 1, 1, 1, 22/3}, (2*a*x)/(b + a*x)] + 10935*a^5*b*x^5*Hypergeometric
PFQ[{2, 2, 2, 2, 2, 4}, {1, 1, 1, 1, 22/3}, (2*a*x)/(b + a*x)] - 2187*a^6*x
```

```
^6*HypergeometricPFQ[{2, 2, 2, 2, 2, 4}, {1, 1, 1, 1, 22/3}, (2*a*x)/(b + a
*x)]))/(276640*b^4*(b - a*x)^2*(b + a*x)^4*(-(b*x^2) + a*x^3)^(1/3)*Gamma[1
/3])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1404

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{b^2+a^2x^2}{\left(-b^2+a^2x^2\right)^3\sqrt[3]{-bx^2+ax^3}}\,dx = \frac{\left(x^{2/3}\sqrt[3]{-b+ax}\right)\int \frac{b^2+a^2x^2}{x^{2/3}\sqrt[3]{-b+ax}\left(-b^2+a^2x^2\right)^3}\,dx}{\sqrt[3]{-bx^2+ax^3}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-b+ax}\right)\text{Subst}\left(\int \frac{b^2+a^2x^6}{\sqrt[3]{-b+ax^3}\left(-b^2+a^2x^6\right)^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-bx^2+ax^3}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-b+ax}\right)\text{Subst}\left(\int \left(\frac{2b^2}{\sqrt[3]{-b+ax^3}\left(-b^2+a^2x^6\right)^3}+\frac{1}{\sqrt[3]{-b+ax^3}\left(-b^2+a^2x^6\right)^2}\right)dx,\right.}{\sqrt[3]{-bx^2+ax^3}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-b+ax}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-b+ax^3}\left(-b^2+a^2x^6\right)^2}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-bx^2+ax^3}} + \frac{\left(6b^2x^{2/3}\sqrt[3]{-b\right.}{}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{-b+ax}\right)\text{Subst}\left(\int \frac{1}{\left(-b+ax^3\right)^{7/3}\left(b+ax^3\right)^2}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-bx^2+ax^3}} + \frac{\left(6b^2x^{2/3}\sqrt[3]{-b+\right.}{}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{1-\frac{ax}{b}}\right)\text{Subst}\left(\int \frac{1}{\left(b+ax^3\right)^2\left(1-\frac{ax^3}{b}\right)^{7/3}}\,dx, x, \sqrt[3]{x}\right)}{b^2\sqrt[3]{-bx^2+ax^3}} - \frac{\left(6x^{2/3}\sqrt[3]{1-\frac{ax}{b}}\right)\text{S}}{}$$

$$= \frac{9x\Gamma\left(\frac{4}{3}\right)\left(1820b^4\,{}_2F_1\left(1,2;\frac{13}{3};\frac{2ax}{b+ax}\right)-2275ab^3x\,{}_2F_1\left(1,2;\frac{13}{3};\frac{2ax}{b+ax}\right)-585a\right.}{}$$

Mathematica [C] time = 0.88, size = 156, normalized size = 0.56

$$\frac{x\sqrt[3]{\frac{ax}{b}+1}\left(-625a^4x^4-67a^3bx^3+1503a^2b^2x^2+91ab^3x-1190b^4\right)-826x\sqrt[3]{1-\frac{ax}{b}}\left(b^2-a^2x^2\right)^2\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{2ax}{b+ax}\right)}{672b^4(b-ax)^2\sqrt[3]{x^2(ax-b)}\,(ax+b)^2\sqrt[3]{\frac{ax}{b}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b^2 + a^2*x^2)/((-b^2 + a^2*x^2)^3*(-(b*x^2) + a*x^3)^(1/3)),x]
```

```
[Out] (x*(1 + (a*x)/b)^(1/3)*(-1190*b^4 + 91*a*b^3*x + 1503*a^2*b^2*x^2 - 67*a^3*
b*x^3 - 625*a^4*x^4) - 826*x*(1 - (a*x)/b)^(1/3)*(b^2 - a^2*x^2)^2*Hypergeo
metric2F1[1/3, 1/3, 4/3, (2*a*x)/(b + a*x)])/(672*b^4*(b - a*x)^2*(x^2*(-b
+ a*x))^(1/3)*(b + a*x)^2*(1 + (a*x)/b)^(1/3))
```

IntegrateAlgebraic [A] time = 1.24, size = 278, normalized size = 1.00

$$-\frac{59\log\left(2a^{2/3}x^2+2^{2/3}\sqrt[3]{a}\,x\sqrt[3]{ax^3-bx^2}+\sqrt[3]{2}\left(ax^3-bx^2\right)^{2/3}\right)}{288\sqrt[3]{2}\sqrt[3]{a}\,b^4}-\frac{\left(ax^3-bx^2\right)^{2/3}\left(-625a^4x^4-67a^3bx^3+1503a^2b^2x^2+91ab^3x-1190b^4\right)}{672b^4x(b-ax)^3(ax+b)^2}+\frac{59\log\left(2^{2/3}\sqrt[3]{ax^3-bx^2}-2\sqrt[3]{a}\,x\right)}{144\sqrt[3]{2}\sqrt[3]{a}\,b^4}-\frac{59\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a}\,x}{2^{2/3}\sqrt[3]{ax^3-bx^2}+\sqrt[3]{a}\,x}\right)}{48\sqrt[3]{2}\sqrt{3}\sqrt[3]{a}\,b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^2 + a^2*x^2)/((-b^2 + a^2*x^2)^3*(-(b*x^2) + a*x^3)^(
1/3)),x]
```

```
[Out] -1/672*((-(b*x^2) + a*x^3)^(2/3)*(-1190*b^4 + 91*a*b^3*x + 1503*a^2*b^2*x^2
 - 67*a^3*b*x^3 - 625*a^4*x^4))/(b^4*x*(b - a*x)^3*(b + a*x)^2) - (59*ArcTa
n[(Sqrt[3]*a^(1/3)*x)/(a^(1/3)*x + 2^(2/3)*(-(b*x^2) + a*x^3)^(1/3))])/(48*
2^(1/3)*Sqrt[3]*a^(1/3)*b^4) + (59*Log[-2*a^(1/3)*x + 2^(2/3)*(-(b*x^2) + a
```

```
*x^3)^(1/3)])/(144*2^(1/3)*a^(1/3)*b^4) - (59*Log[2*a^(2/3)*x^2 + 2^(2/3)*a
^(1/3)*x*(-(b*x^2) + a*x^3)^(1/3) + 2^(1/3)*(-(b*x^2) + a*x^3)^(2/3)])/(288
*2^(1/3)*a^(1/3)*b^4)
```

fricas [A] time = 1.43, size = 959, normalized size = 3.45

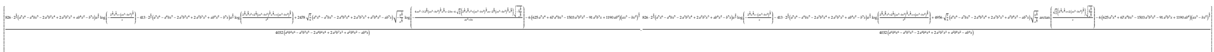

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b^2)/(a^2*x^2-b^2)^3/(a*x^3-b*x^2)^(1/3),x, algorithm="f
ricas")
```

```
[Out] [1/4032*(826*2^(2/3)*(a^5*x^6 - a^4*b*x^5 - 2*a^3*b^2*x^4 + 2*a^2*b^3*x^3 +
 a*b^4*x^2 - b^5*x)*a^(2/3)*log(-(2^(1/3)*a^(1/3)*x - (a*x^3 - b*x^2)^(1/3)
)/x) - 413*2^(2/3)*(a^5*x^6 - a^4*b*x^5 - 2*a^3*b^2*x^4 + 2*a^2*b^3*x^3 + a
*b^4*x^2 - b^5*x)*a^(2/3)*log((2^(2/3)*a^(2/3)*x^2 + 2^(1/3)*(a*x^3 - b*x^2
)^(1/3)*a^(1/3)*x + (a*x^3 - b*x^2)^(2/3))/x^2) + 2478*sqrt(1/6)*(a^6*x^6 -
 a^5*b*x^5 - 2*a^4*b^2*x^4 + 2*a^3*b^3*x^3 + a^2*b^4*x^2 - a*b^5*x)*sqrt(-2
^(1/3)/a^(2/3))*log(-(4*a*x^2 - 3*2^(2/3)*(a*x^3 - b*x^2)^(1/3)*a^(2/3)*x -
 2*b*x - 6*sqrt(1/6)*(2^(1/3)*a^(4/3)*x^2 + (a*x^3 - b*x^2)^(1/3)*a*x - 2^(
2/3)*(a*x^3 - b*x^2)^(2/3)*a^(2/3))*sqrt(-2^(1/3)/a^(2/3)))/(a*x^2 + b*x))
- 6*(625*a^5*x^4 + 67*a^4*b*x^3 - 1503*a^3*b^2*x^2 - 91*a^2*b^3*x + 1190*a*
b^4)*(a*x^3 - b*x^2)^(2/3))/(a^6*b^4*x^6 - a^5*b^5*x^5 - 2*a^4*b^6*x^4 + 2*
a^3*b^7*x^3 + a^2*b^8*x^2 - a*b^9*x), 1/4032*(826*2^(2/3)*(a^5*x^6 - a^4*b*
x^5 - 2*a^3*b^2*x^4 + 2*a^2*b^3*x^3 + a*b^4*x^2 - b^5*x)*a^(2/3)*log(-(2^(1
/3)*a^(1/3)*x - (a*x^3 - b*x^2)^(1/3))/x) - 413*2^(2/3)*(a^5*x^6 - a^4*b*x^
5 - 2*a^3*b^2*x^4 + 2*a^2*b^3*x^3 + a*b^4*x^2 - b^5*x)*a^(2/3)*log((2^(2/3)
*a^(2/3)*x^2 + 2^(1/3)*(a*x^3 - b*x^2)^(1/3)*a^(1/3)*x + (a*x^3 - b*x^2)^(2
/3))/x^2) + 4956*sqrt(1/6)*(a^6*x^6 - a^5*b*x^5 - 2*a^4*b^2*x^4 + 2*a^3*b^3
*x^3 + a^2*b^4*x^2 - a*b^5*x)*sqrt(2^(1/3)/a^(2/3))*arctan(sqrt(1/6)*(2^(1/
3)*a^(1/3)*x + 2*(a*x^3 - b*x^2)^(1/3))*sqrt(2^(1/3)/a^(2/3))/x) - 6*(625*a
^5*x^4 + 67*a^4*b*x^3 - 1503*a^3*b^2*x^2 - 91*a^2*b^3*x + 1190*a*b^4)*(a*x^
3 - b*x^2)^(2/3))/(a^6*b^4*x^6 - a^5*b^5*x^5 - 2*a^4*b^6*x^4 + 2*a^3*b^7*x^
3 + a^2*b^8*x^2 - a*b^9*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b^2)/(a^2*x^2-b^2)^3/(a*x^3-b*x^2)^(1/3),x, algorithm="g
iac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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aic extensions not allowed in a rootofAlgebraic extensions not allowed in a
 rootof-(13*((a-b/x)^(1/3))^2*(a-b/x)-23*((a-b/x)^(1/3))^2*a)/48/b^4/(a-b/x
-2*a)^2+(-336*(a-b/x)^2+105*(a-b/x)*a-24*a^2)/224/b^4/(a-b/x)^(1/3)/(a-b/x)
^2-59/288/(2*a)^(1/3)/b^4*ln(((a-b/x)^(1/3))^2+(2*a)^(1/3)*(a-b/x)^(1/3)+(2
*a)^(1/3)*(2*a)^(1/3))+59*((2*a)^(1/3))^2/96/sqrt(3)/a/b^4*atan(((a-b/x)^(1
/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))+59*(2*a)^(1/3)*(2*a)^(1/3)/288/
b^4/a*ln(abs((a-b/x)^(1/3)-(2*a)^(1/3)))
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{a^{2} x^{2}+b^{2}}{\left(a^{2} x^{2}-b^{2}\right)^{3} \left(a \,x^{3}-b \,x^{2}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2+b^2)/(a^2*x^2-b^2)^3/(a*x^3-b*x^2)^(1/3),x)
```

```
[Out] int((a^2*x^2+b^2)/(a^2*x^2-b^2)^3/(a*x^3-b*x^2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^{2} x^{2} + b^{2}}{{\left(a^{2} x^{2} - b^{2}\right)}^{3} {\left(a x^{3} - b x^{2}\right)}^{\frac{1}{3}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b^2)/(a^2*x^2-b^2)^3/(a*x^3-b*x^2)^(1/3),x, algorithm="m
axima")
```

```
[Out] integrate((a^2*x^2 + b^2)/((a^2*x^2 - b^2)^3*(a*x^3 - b*x^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{a^2\,x^2+b^2}{{\left(b^2-a^2\,x^2\right)}^3\,{\left(a\,x^3-b\,x^2\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^2 + a^2*x^2)/((b^2 - a^2*x^2)^3*(a*x^3 - b*x^2)^(1/3)),x)
```

```
[Out] int(-(b^2 + a^2*x^2)/((b^2 - a^2*x^2)^3*(a*x^3 - b*x^2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^{2} x^{2} + b^{2}}{\sqrt[3]{x^{2} \left(a x - b\right)} \left(a x - b\right)^{3} \left(a x + b\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2+b**2)/(a**2*x**2-b**2)**3/(a*x**3-b*x**2)**(1/3),x)
```

```
[Out] Integral((a**2*x**2 + b**2)/((x**2*(a*x - b))**(1/3)*(a*x - b)**3*(a*x + b)
**3), x)
```

3.2237 $\int \frac{d+cx}{(-d+cx)\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$

Optimal. Leaf size=278

$$\frac{4d\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{-\sqrt{a^2d^2+b^2c^2}-ad}}\right)}{\sqrt{c}\sqrt{-\sqrt{a^2d^2+b^2c^2}-ad}}+\frac{4d\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{\sqrt{a^2d^2+b^2c^2}-ad}}\right)}{\sqrt{c}\sqrt{\sqrt{a^2d^2+b^2c^2}-ad}}+\frac{2\sqrt{a^2x^2+b^2}\,(3acx+6ad)+2\left(3a^2cx^2+6a^2dx+b\right.}{3ac\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}$$

Rubi [A] time = 1.09, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {6742, 2117, 14, 2119, 1628, 828, 826, 1166, 208}

$$-\frac{4d\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}\right)}{\sqrt{c}\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}-\frac{4d\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}\right)}{\sqrt{c}\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}+\frac{4d}{c\sqrt{\sqrt{a^2x^2+b^2}+ax}}-\frac{b^2}{3a\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}+\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*x)/((-d + c*x)*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

```
[Out] -1/3*b^2/(a*(a*x + Sqrt[b^2 + a^2*x^2])^(3/2)) + (4*d)/(c*Sqrt[a*x + Sqrt[b
^2 + a^2*x^2]]) + Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]/a - (4*d*ArcTanh[(Sqrt[c]
*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/Sqrt[a*d - Sqrt[b^2*c^2 + a^2*d^2]]])/(Sq
rt[c]*Sqrt[a*d - Sqrt[b^2*c^2 + a^2*d^2]]) - (4*d*ArcTanh[(Sqrt[c]*Sqrt[a*x
 + Sqrt[b^2 + a^2*x^2]])/Sqrt[a*d + Sqrt[b^2*c^2 + a^2*d^2]]])/(Sqrt[c]*Sqr
t[a*d + Sqrt[b^2*c^2 + a^2*d^2]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{d+cx}{(-d+cx)\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx = \int \left(\frac{1}{\sqrt{ax+\sqrt{b^2+a^2x^2}}} - \frac{2d}{(d-cx)\sqrt{ax+\sqrt{b^2+a^2x^2}}}\right)dx$$

$$= -\left((2d)\int \frac{1}{(d-cx)\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx\right) + \int \frac{1}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{b^2+x^2}{x^{5/2}}\,dx, x, ax+\sqrt{b^2+a^2x^2}\right)}{2a} - (2d)\,\text{Subst}\left(\int \frac{b^2+x}{x^{3/2}\left(b^2c+2ac\right.}\right.$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{x^{5/2}}+\frac{1}{\sqrt{x}}\right)dx, x, ax+\sqrt{b^2+a^2x^2}\right)}{2a} - (2d)\,\text{Subst}\left(\int\left(-\frac{1}{cx^{3/2}}\right.\right.$$

$$= -\frac{b^2}{3a\left(ax+\sqrt{b^2+a^2x^2}\right)^{3/2}} - \frac{4d}{c\sqrt{ax+\sqrt{b^2+a^2x^2}}} + \frac{\sqrt{ax+\sqrt{b^2+a^2x^2}}}{a}$$

$$= -\frac{b^2}{3a\left(ax+\sqrt{b^2+a^2x^2}\right)^{3/2}} + \frac{4d}{c\sqrt{ax+\sqrt{b^2+a^2x^2}}} + \frac{\sqrt{ax+\sqrt{b^2+a^2x^2}}}{a}$$

$$= -\frac{b^2}{3a\left(ax+\sqrt{b^2+a^2x^2}\right)^{3/2}} + \frac{4d}{c\sqrt{ax+\sqrt{b^2+a^2x^2}}} + \frac{\sqrt{ax+\sqrt{b^2+a^2x^2}}}{a}$$

$$= -\frac{b^2}{3a\left(ax+\sqrt{b^2+a^2x^2}\right)^{3/2}} + \frac{4d}{c\sqrt{ax+\sqrt{b^2+a^2x^2}}} + \frac{\sqrt{ax+\sqrt{b^2+a^2x^2}}}{a}$$

$$= -\frac{b^2}{3a\left(ax+\sqrt{b^2+a^2x^2}\right)^{3/2}} + \frac{4d}{c\sqrt{ax+\sqrt{b^2+a^2x^2}}} + \frac{\sqrt{ax+\sqrt{b^2+a^2x^2}}}{a}$$

Mathematica [A] time = 1.28, size = 374, normalized size = 1.35

$$\frac{\frac{12ad\left(ad\left(ad-\sqrt{a^2d^2+b^2c^2}\right)+b^2c^2\right)\tan^{-1}\left(\frac{b\sqrt{c}}{\sqrt{\sqrt{a^2x^2+b^2}+ax}\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}\right)}{b\sqrt{a^2d^2+b^2c^2}\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}} - \frac{12ad\left(ad\left(\sqrt{a^2d^2+b^2c^2}+ad\right)+b^2c^2\right)\tan^{-1}\left(\frac{b\sqrt{c}}{\sqrt{\sqrt{a^2x^2+b^2}+ax}\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}\right)}{b\sqrt{a^2d^2+b^2c^2}\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}} + \frac{\sqrt{c}\left(6a\left(\sqrt{a^2x^2+b^2}+ax\right)(cx+2d)+2b^2c\right)}{\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}}{3ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*x)/((-d + c*x)*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x]
```

```
[Out] ((Sqrt[c]*(2*b^2*c + 6*a*(2*d + c*x)*(a*x + Sqrt[b^2 + a^2*x^2])))/(a*x + S
qrt[b^2 + a^2*x^2])^(3/2) + (12*a*d*(b^2*c^2 + a*d*(a*d - Sqrt[b^2*c^2 + a^
2*d^2]))*ArcTan[(b*Sqrt[c])/(Sqrt[a*d - Sqrt[b^2*c^2 + a^2*d^2]]*Sqrt[a*x +
 Sqrt[b^2 + a^2*x^2]])])/(b*Sqrt[b^2*c^2 + a^2*d^2]*Sqrt[a*d - Sqrt[b^2*c^2
 + a^2*d^2]]) - (12*a*d*(b^2*c^2 + a*d*(a*d + Sqrt[b^2*c^2 + a^2*d^2]))*Arc
Tan[(b*Sqrt[c])/(Sqrt[a*d + Sqrt[b^2*c^2 + a^2*d^2]]*Sqrt[a*x + Sqrt[b^2 +
a^2*x^2]])])/(b*Sqrt[b^2*c^2 + a^2*d^2]*Sqrt[a*d + Sqrt[b^2*c^2 + a^2*d^2]]
))/(3*a*c^(3/2))
```

IntegrateAlgebraic [A] time = 0.74, size = 278, normalized size = 1.00

$$\frac{4d\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{-\sqrt{a^2d^2+b^2c^2}-ad}}\right)}{\sqrt{c}\sqrt{-\sqrt{a^2d^2+b^2c^2}-ad}}+\frac{4d\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{\sqrt{a^2d^2+b^2c^2}-ad}}\right)}{\sqrt{c}\sqrt{\sqrt{a^2d^2+b^2c^2}-ad}}+\frac{2\sqrt{a^2x^2+b^2}\,(3acx+6ad)+2\left(3a^2cx^2+6a^2dx+b^2c\right)}{3ac\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + c*x)/((-d + c*x)*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]),x
]
```

```
[Out] (2*(6*a*d + 3*a*c*x)*Sqrt[b^2 + a^2*x^2] + 2*(b^2*c + 6*a^2*d*x + 3*a^2*c*x
^2))/(3*a*c*(a*x + Sqrt[b^2 + a^2*x^2])^(3/2)) + (4*d*ArcTan[(Sqrt[c]*Sqrt[
a*x + Sqrt[b^2 + a^2*x^2]])/Sqrt[-(a*d) - Sqrt[b^2*c^2 + a^2*d^2]]])/(Sqrt[
c]*Sqrt[-(a*d) - Sqrt[b^2*c^2 + a^2*d^2]]) + (4*d*ArcTan[(Sqrt[c]*Sqrt[a*x
+ Sqrt[b^2 + a^2*x^2]])/Sqrt[-(a*d) + Sqrt[b^2*c^2 + a^2*d^2]]])/(Sqrt[c]*S
qrt[-(a*d) + Sqrt[b^2*c^2 + a^2*d^2]])
```

fricas [B] time = 1.00, size = 781, normalized size = 2.81

$$\frac{2\left(3ab^2c\sqrt{-\frac{b^2c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{b^4c^6}}+ad^3}{b^2c^3}}\log\left(32\sqrt{ax+\sqrt{a^2x^2+b^2}}\,d^3+32\left(b^2c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{b^4c^6}}-ad^3\right)\sqrt{-\frac{b^2c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{b^4c^6}}+ad^3}{b^2c^3}}\right)-3ab^2c\sqrt{-\frac{b^2c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{b^4c^6}}+ad^3}{b^2c^3}}\log\left(32\sqrt{ax+\sqrt{a^2x^2+b^2}}\,d^3-32\left(b^2c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{b^4c^6}}-ad^3\right)\sqrt{-\frac{b^2c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{b^4c^6}}+ad^3}{b^2c^3}}\right)-3ab^2c\sqrt{\frac{b^2c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{b^4c^6}}-ad^3}{b^2c^3}}\log\left(32\sqrt{ax+\sqrt{a^2x^2+b^2}}\,d^3+32\left(b^2c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{b^4c^6}}+ad^3\right)\sqrt{\frac{b^2c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{b^4c^6}}-ad^3}{b^2c^3}}\right)+3ab^2c\sqrt{\frac{b^2c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{b^4c^6}}-ad^3}{b^2c^3}}\log\left(32\sqrt{ax+\sqrt{a^2x^2+b^2}}\,d^3-32\left(b^2c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{b^4c^6}}+ad^3\right)\sqrt{\frac{b^2c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{b^4c^6}}-ad^3}{b^2c^3}}\right)-\left(a^2cx^2+6a^2dx-b^2c-\sqrt{a^2x^2+b^2}\,(acx+6ad)\right)\sqrt{ax+\sqrt{a^2x^2+b^2}}\right)}{3ab^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+d)/(c*x-d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/3*(3*a*b^2*c*sqrt(-(b^2*c^3*sqrt((b^2*c^2*d^4 + a^2*d^6)/(b^4*c^6)) + a*d
^3)/(b^2*c^3))*log(32*sqrt(a*x + sqrt(a^2*x^2 + b^2))*d^3 + 32*(b^2*c^3*sqr
t((b^2*c^2*d^4 + a^2*d^6)/(b^4*c^6)) - a*d^3)*sqrt(-(b^2*c^3*sqrt((b^2*c^2*
d^4 + a^2*d^6)/(b^4*c^6)) + a*d^3)/(b^2*c^3))) - 3*a*b^2*c*sqrt(-(b^2*c^3*s
qrt((b^2*c^2*d^4 + a^2*d^6)/(b^4*c^6)) + a*d^3)/(b^2*c^3))*log(32*sqrt(a*x
+ sqrt(a^2*x^2 + b^2))*d^3 - 32*(b^2*c^3*sqrt((b^2*c^2*d^4 + a^2*d^6)/(b^4*
c^6)) - a*d^3)*sqrt(-(b^2*c^3*sqrt((b^2*c^2*d^4 + a^2*d^6)/(b^4*c^6)) + a*d
^3)/(b^2*c^3))) - 3*a*b^2*c*sqrt((b^2*c^3*sqrt((b^2*c^2*d^4 + a^2*d^6)/(b^4
*c^6)) - a*d^3)/(b^2*c^3))*log(32*sqrt(a*x + sqrt(a^2*x^2 + b^2))*d^3 + 32*
(b^2*c^3*sqrt((b^2*c^2*d^4 + a^2*d^6)/(b^4*c^6)) + a*d^3)*sqrt((b^2*c^3*sqr
t((b^2*c^2*d^4 + a^2*d^6)/(b^4*c^6)) - a*d^3)/(b^2*c^3))) + 3*a*b^2*c*sqrt(
(b^2*c^3*sqrt((b^2*c^2*d^4 + a^2*d^6)/(b^4*c^6)) - a*d^3)/(b^2*c^3))*log(32
*sqrt(a*x + sqrt(a^2*x^2 + b^2))*d^3 - 32*(b^2*c^3*sqrt((b^2*c^2*d^4 + a^2*
d^6)/(b^4*c^6)) + a*d^3)*sqrt((b^2*c^3*sqrt((b^2*c^2*d^4 + a^2*d^6)/(b^4*c^
6)) - a*d^3)/(b^2*c^3))) - (a^2*c*x^2 + 6*a^2*d*x - b^2*c - sqrt(a^2*x^2 +
b^2)*(a*c*x + 6*a*d))*sqrt(a*x + sqrt(a^2*x^2 + b^2)))/(a*b^2*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx+d}{\sqrt{ax+\sqrt{a^2x^2+b^2}}\,(cx-d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+d)/(c*x-d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((c*x + d)/(sqrt(a*x + sqrt(a^2*x^2 + b^2))*(c*x - d)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{cx+d}{(cx-d)\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x+d)/(c*x-d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] int((c*x+d)/(c*x-d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx+d}{\sqrt{ax+\sqrt{a^2x^2+b^2}}\,(cx-d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+d)/(c*x-d)/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((c*x + d)/(sqrt(a*x + sqrt(a^2*x^2 + b^2))*(c*x - d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{d+c\,x}{\sqrt{a\,x+\sqrt{a^2\,x^2+b^2}}\,\left(d-c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(d + c*x)/((a*x + (b^2 + a^2*x^2)^(1/2))^(1/2)*(d - c*x)),x)
```

```
[Out] int(-(d + c*x)/((a*x + (b^2 + a^2*x^2)^(1/2))^(1/2)*(d - c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx+d}{\sqrt{ax+\sqrt{a^2x^2+b^2}}\,\left(cx-d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+d)/(c*x-d)/(a*x+(a**2*x**2+b**2)**(1/2))**(1/2),x)
```

```
[Out] Integral((c*x + d)/(sqrt(a*x + sqrt(a**2*x**2 + b**2))*(c*x - d)), x)
```

3.2238 $$\int \frac{x(-1+kx)(-1+(-1+2k)x)}{\sqrt[3]{(1-x)x(1-kx)}\left(-1+4x+(-6+b)x^2+(4-2bk)x^3+\left(-1+bk^2\right)x^4\right)}\,dx$$

Optimal. Leaf size=279

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}-2x+2}\right)}{2b^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+2x-2}\right)}{2b^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{x-1}\right)}{b^{5/6}}+\tanh^{-1}$$

Rubi [F] time = 11.77, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(-1+kx)(-1+(-1+2k)x)}{\sqrt[3]{(1-x)x(1-kx)}\left(-1+4x+(-6+b)x^2+(4-2bk)x^3+\left(-1+bk^2\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-1 + k*x)*(-1 + (-1 + 2*k)*x))/(((1 - x)*x*(1 - k*x))^(1/3)*(-1 + 4
*x + (-6 + b)*x^2 + (4 - 2*b*k)*x^3 + (-1 + b*k^2)*x^4)),x]
```

```
[Out] (-3*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][(x^4*(1 -
 k*x^3)^(2/3))/((1 - x^3)^(1/3)*(1 - 4*x^3 + 6*(1 - b/6)*x^6 - 4*(1 - (b*k)
/2)*x^9 + (1 - b*k^2)*x^12)), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3)
 - (3*(1 - 2*k)*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int
][(x^7*(1 - k*x^3)^(2/3))/((1 - x^3)^(1/3)*(1 - 4*x^3 + 6*(1 - b/6)*x^6 - 4
*(1 - (b*k)/2)*x^9 + (1 - b*k^2)*x^12)), x], x, x^(1/3)])/((1 - x)*x*(1 - k
*x))^(1/3)
```

Rubi steps

$$\int \frac{x(-1+kx)(-1+(-1+2k)x)}{\sqrt[3]{(1-x)x(1-kx)}\left(-1+4x+(-6+b)x^2+(4-2bk)x^3+\left(-1+bk^2\right)x^4\right)}\,dx = \frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int}{\sqrt[3]{(1}}$$

$$= -\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int}{\sqrt[3]{(1-}}$$

$$= -\frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}}{}$$

$$= -\frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}}{}$$

$$= -\frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subst}}{}$$

Mathematica [F] time = 4.41, size = 0, normalized size = 0.00

$$\int \frac{x(-1+kx)(-1+(-1+2k)x)}{\sqrt[3]{(1-x)x(1-kx)}\left(-1+4x+(-6+b)x^2+(4-2bk)x^3+\left(-1+bk^2\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(-1 + k*x)*(-1 + (-1 + 2*k)*x))/(((1 - x)*x*(1 - k*x))^(1/3)*(
-1 + 4*x + (-6 + b)*x^2 + (4 - 2*b*k)*x^3 + (-1 + b*k^2)*x^4)),x]
```

```
[Out] Integrate[(x*(-1 + k*x)*(-1 + (-1 + 2*k)*x))/(((1 - x)*x*(1 - k*x))^(1/3)*(
-1 + 4*x + (-6 + b)*x^2 + (4 - 2*b*k)*x^3 + (-1 + b*k^2)*x^4)), x]
```

IntegrateAlgebraic [A] time = 3.91, size = 279, normalized size = 1.00

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}-2x+2}\right)}{2b^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+2x-2}\right)}{2b^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{x-1}\right)}{b^{5/6}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[6]{b}x-\sqrt[6]{b}\right)\sqrt[3]{kx^3+(-k-1)x^2+x}}{\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+x^2-2x+1}\right)}{2b^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-1 + k*x)*(-1 + (-1 + 2*k)*x))/(((1 - x)*x*(1 - k*x)
)^(1/3)*(-1 + 4*x + (-6 + b)*x^2 + (4 - 2*b*k)*x^3 + (-1 + b*k^2)*x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/6)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(2 - 2*x
 + b^(1/6)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))])/(2*b^(5/6)) - (Sqrt[3]*ArcTa
n[(Sqrt[3]*b^(1/6)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(-2 + 2*x + b^(1/6)*(x
 + (-1 - k)*x^2 + k*x^3)^(1/3))])/(2*b^(5/6)) + ArcTanh[(b^(1/6)*(x + (-1 -
 k)*x^2 + k*x^3)^(1/3))/(-1 + x)]/b^(5/6) + ArcTanh[((-b^(1/6) + b^(1/6)*x)
*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(1 - 2*x + x^2 + b^(1/3)*(x + (-1 - k)*x
^2 + k*x^3)^(2/3))]/(2*b^(5/6))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(k*x-1)*(-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(-1+4*x+(-6+b)
*x^2+(-2*b*k+4)*x^3+(b*k^2-1)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((2k-1)x-1)(kx-1)x}{\left(\left(bk^2-1\right)x^4-2(bk-2)x^3+(b-6)x^2+4x-1\right)((kx-1)(x-1)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(k*x-1)*(-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(-1+4*x+(-6+b)
*x^2+(-2*b*k+4)*x^3+(b*k^2-1)*x^4),x, algorithm="giac")
```

```
[Out] integrate(((2*k - 1)*x - 1)*(k*x - 1)*x/(((b*k^2 - 1)*x^4 - 2*(b*k - 2)*x^3
 + (b - 6)*x^2 + 4*x - 1)*((k*x - 1)*(x - 1)*x)^(1/3)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x(kx-1)(-1+(-1+2k)x)}{((1-x)x(-kx+1))^{\frac{1}{3}}\left(-1+4x+(-6+b)x^2+(-2bk+4)x^3+\left(bk^2-1\right)x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(k*x-1)*(-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(-1+4*x+(-6+b)*x^2+(
-2*b*k+4)*x^3+(b*k^2-1)*x^4),x)
```

```
[Out] int(x*(k*x-1)*(-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(-1+4*x+(-6+b)*x^2+(
-2*b*k+4)*x^3+(b*k^2-1)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((2k-1)x-1)(kx-1)x}{\left((bk^2-1)x^4-2(bk-2)x^3+(b-6)x^2+4x-1\right)((kx-1)(x-1)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(k*x-1)*(-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(-1+4*x+(-6+b)
*x^2+(-2*b*k+4)*x^3+(b*k^2-1)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(((2*k - 1)*x - 1)*(k*x - 1)*x/(((b*k^2 - 1)*x^4 - 2*(b*k - 2)*x^3
 + (b - 6)*x^2 + 4*x - 1)*((k*x - 1)*(x - 1)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(x\,(2\,k-1)-1)\,(k\,x-1)}{(x\,(k\,x-1)\,(x-1))^{1/3}\,\left((b\,k^2-1)\,x^4+(4-2\,b\,k)\,x^3+(b-6)\,x^2+4\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x*(2*k - 1) - 1)*(k*x - 1))/((x*(k*x - 1)*(x - 1))^(1/3)*(4*x + x^4
*(b*k^2 - 1) - x^3*(2*b*k - 4) + x^2*(b - 6) - 1)),x)
```

```
[Out] int((x*(x*(2*k - 1) - 1)*(k*x - 1))/((x*(k*x - 1)*(x - 1))^(1/3)*(4*x + x^4
*(b*k^2 - 1) - x^3*(2*b*k - 4) + x^2*(b - 6) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(k*x-1)*(-1+(-1+2*k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(-1+4*x+(-6+b
)*x**2+(-2*b*k+4)*x**3+(b*k**2-1)*x**4),x)
```

```
[Out] Timed out
```

3.2239

$$\int \frac{1+(-2+3k)x-(k+4k^2)x^2+3k^2x^3}{\sqrt[3]{(1-x)x(1-kx)}\,(-b+(1+5bk)x-(1+10bk^2)x^2+10bk^3x^3-5bk^4x^4+bk^5x^5)}\,dx$$

Optimal. Leaf size=279

$$-\frac{\log\left(b^{2/3}k^4x^4-4b^{2/3}k^3x^3+6b^{2/3}k^2x^2-4b^{2/3}kx+b^{2/3}+\sqrt[3]{kx^3+(-k-1)x^2+x}\left(\sqrt[3]{b}\,k^2x^2-2\sqrt[3]{b}\,kx+\sqrt[3]{b}\right)+\right.}{2\sqrt[3]{b}}$$

Rubi [F] time = 11.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+(-2+3k)x-(k+4k^2)x^2+3k^2x^3}{\sqrt[3]{(1-x)x(1-kx)}\,(-b+(1+5bk)x-(1+10bk^2)x^2+10bk^3x^3-5bk^4x^4+bk^5x^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + (-2 + 3*k)*x - (k + 4*k^2)*x^2 + 3*k^2*x^3)/(((1 - x)*x*(1 - k*x))
^(1/3)*(-b + (1 + 5*b*k)*x - (1 + 10*b*k^2)*x^2 + 10*b*k^3*x^3 - 5*b*k^4*x^
4 + b*k^5*x^5)),x]
```

```
[Out] (3*(-1 + x)^(1/3)*x^(1/3)*(-1 + k*x)^(1/3)*Defer[Subst][Defer[Int][(x*(-1 +
 k*x^3)^(2/3))/((-1 + x^3)^(1/3)*(-x^3 + x^6 - b*(-1 + k*x^3)^5)), x], x, x
^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3) - (6*(1 - 2*k)*(-1 + x)^(1/3)*x^(1/3)*
(-1 + k*x)^(1/3)*Defer[Subst][Defer[Int][(x^4*(-1 + k*x^3)^(2/3))/((-1 + x^
3)^(1/3)*(-x^3 + x^6 - b*(-1 + k*x^3)^5)), x], x, x^(1/3)])/((1 - x)*x*(1 -
 k*x))^(1/3) + (9*k*(-1 + x)^(1/3)*x^(1/3)*(-1 + k*x)^(1/3)*Defer[Subst][De
fer[Int][(x^7*(-1 + k*x^3)^(2/3))/((-1 + x^3)^(1/3)*(x^3 - x^6 + b*(-1 + k*
x^3)^5)), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\int \frac{1+(-2+3k)x-\left(k+4k^2\right)x^2+3k^2x^3}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+(1+5bk)x-\left(1+10bk^2\right)x^2+10bk^3x^3-5bk^4x^4+bk^5x^5\right)}dx = \int \frac{-1+(2-3k)}{\sqrt[3]{(-1+x)x(-1+k}}$$

$$= \frac{\left(\sqrt[3]{-1+x}\sqrt[3]{x}\sqrt[3]{-1+k}\right.}{}$$

$$= \frac{\left(\sqrt[3]{-1+x}\sqrt[3]{x}\sqrt[3]{-1+k}\right.}{\sqrt[3]{(-}}$$

$$= \frac{\left(3\sqrt[3]{-1+x}\sqrt[3]{x}\sqrt[3]{-1+}\right.}{}$$

$$= \frac{\left(3\sqrt[3]{-1+x}\sqrt[3]{x}\sqrt[3]{-1+}\right.}{}$$

$$= \frac{\left(3\sqrt[3]{-1+x}\sqrt[3]{x}\sqrt[3]{-1+}\right.}{}$$

$$= \frac{\left(3\sqrt[3]{-1+x}\sqrt[3]{x}\sqrt[3]{-1+}\right.}{}$$

Mathematica [F] time = 5.50, size = 0, normalized size = 0.00

$$\int \frac{1+(-2+3k)x-\left(k+4k^2\right)x^2+3k^2x^3}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+(1+5bk)x-\left(1+10bk^2\right)x^2+10bk^3x^3-5bk^4x^4+bk^5x^5\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + (-2 + 3*k)*x - (k + 4*k^2)*x^2 + 3*k^2*x^3)/(((1 - x)*x*(1 -
 k*x))^(1/3)*(-b + (1 + 5*b*k)*x - (1 + 10*b*k^2)*x^2 + 10*b*k^3*x^3 - 5*b*
k^4*x^4 + b*k^5*x^5)),x]
```

```
[Out] Integrate[(1 + (-2 + 3*k)*x - (k + 4*k^2)*x^2 + 3*k^2*x^3)/(((1 - x)*x*(1 -
 k*x))^(1/3)*(-b + (1 + 5*b*k)*x - (1 + 10*b*k^2)*x^2 + 10*b*k^3*x^3 - 5*b*
k^4*x^4 + b*k^5*x^5)), x]
```

IntegrateAlgebraic [A] time = 3.08, size = 279, normalized size = 1.00

$$-\frac{\log\left(b^{2/3}k^4x^4-4b^{2/3}k^3x^3+6b^{2/3}k^2x^2-4b^{2/3}kx+b^{2/3}+\sqrt[3]{kx^3+(-k-1)x^2+x}\left(\sqrt[3]{b}k^2x^2-2\sqrt[3]{b}kx+\sqrt[3]{b}\right)+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2\sqrt[3]{b}}+\frac{\log\left(-\sqrt[3]{b}k^2x^2+2\sqrt[3]{b}kx-\sqrt[3]{b}+\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{kx^3+(-k-1)x^2+x}}{2\sqrt[3]{b}k^2x^2-4\sqrt[3]{b}kx+2\sqrt[3]{b}+\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + (-2 + 3*k)*x - (k + 4*k^2)*x^2 + 3*k^2*x^3)/(((1 -
x)*x*(1 - k*x))^(1/3)*(-b + (1 + 5*b*k)*x - (1 + 10*b*k^2)*x^2 + 10*b*k^3*x
^3 - 5*b*k^4*x^4 + b*k^5*x^5)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(2*b^(1/3) - 4*b
^(1/3)*k*x + 2*b^(1/3)*k^2*x^2 + (x + (-1 - k)*x^2 + k*x^3)^(1/3))])/b^(1/3
) + Log[-b^(1/3) + 2*b^(1/3)*k*x - b^(1/3)*k^2*x^2 + (x + (-1 - k)*x^2 + k*
x^3)^(1/3)]/b^(1/3) - Log[b^(2/3) - 4*b^(2/3)*k*x + 6*b^(2/3)*k^2*x^2 - 4*b
^(2/3)*k^3*x^3 + b^(2/3)*k^4*x^4 + (b^(1/3) - 2*b^(1/3)*k*x + b^(1/3)*k^2*x
```

```
^2)*(x + (-1 - k)*x^2 + k*x^3)^(1/3) + (x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(2
*b^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-2+3*k)*x-(4*k^2+k)*x^2+3*k^2*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(
-b+(5*b*k+1)*x-(10*b*k^2+1)*x^2+10*b*k^3*x^3-5*b*k^4*x^4+b*k^5*x^5),x, algo
rithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-2+3*k)*x-(4*k^2+k)*x^2+3*k^2*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(
-b+(5*b*k+1)*x-(10*b*k^2+1)*x^2+10*b*k^3*x^3-5*b*k^4*x^4+b*k^5*x^5),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1+(-2+3k)\,x-\left(4k^{2}+k\right)x^{2}+3k^{2}x^{3}}{\left((1-x)\,x\,(-kx+1)\right)^{\frac{1}{3}}\left(-b+(5bk+1)\,x-\left(10b\,k^{2}+1\right)x^{2}+10b\,k^{3}x^{3}-5b\,k^{4}x^{4}+b\,k^{5}x^{5}\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(-2+3*k)*x-(4*k^2+k)*x^2+3*k^2*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(-b+(5*
b*k+1)*x-(10*b*k^2+1)*x^2+10*b*k^3*x^3-5*b*k^4*x^4+b*k^5*x^5),x)
```

```
[Out] int((1+(-2+3*k)*x-(4*k^2+k)*x^2+3*k^2*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(-b+(5*
b*k+1)*x-(10*b*k^2+1)*x^2+10*b*k^3*x^3-5*b*k^4*x^4+b*k^5*x^5),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,k^{2}x^{3}-\left(4\,k^{2}+k\right)x^{2}+(3\,k-2)x+1}{\left(bk^{5}x^{5}-5\,bk^{4}x^{4}+10\,bk^{3}x^{3}-\left(10\,bk^{2}+1\right)x^{2}+(5\,bk+1)x-b\right)\left((kx-1)(x-1)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-2+3*k)*x-(4*k^2+k)*x^2+3*k^2*x^3)/((1-x)*x*(-k*x+1))^(1/3)/(
-b+(5*b*k+1)*x-(10*b*k^2+1)*x^2+10*b*k^3*x^3-5*b*k^4*x^4+b*k^5*x^5),x, algo
rithm="maxima")
```

```
[Out] integrate((3*k^2*x^3 - (4*k^2 + k)*x^2 + (3*k - 2)*x + 1)/((b*k^5*x^5 - 5*b
*k^4*x^4 + 10*b*k^3*x^3 - (10*b*k^2 + 1)*x^2 + (5*b*k + 1)*x - b)*((k*x - 1
)*(x - 1)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x\,\left(3\,k-2\right)+3\,k^{2}\,x^{3}-x^{2}\,\left(4\,k^{2}+k\right)+1}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/3}\,\left(b+x^{2}\,\left(10\,b\,k^{2}+1\right)-x\,\left(5\,b\,k+1\right)-10\,b\,k^{3}\,x^{3}+5\,b\,k^{4}\,x^{4}-b\,k^{5}\,x^{5}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(3*k - 2) + 3*k^2*x^3 - x^2*(k + 4*k^2) + 1)/((x*(k*x - 1)*(x - 1))
^(1/3)*(b + x^2*(10*b*k^2 + 1) - x*(5*b*k + 1) - 10*b*k^3*x^3 + 5*b*k^4*x^4
 - b*k^5*x^5)),x)
```

```
[Out] -int((x*(3*k - 2) + 3*k^2*x^3 - x^2*(k + 4*k^2) + 1)/((x*(k*x - 1)*(x - 1))
^(1/3)*(b + x^2*(10*b*k^2 + 1) - x*(5*b*k + 1) - 10*b*k^3*x^3 + 5*b*k^4*x^4
 - b*k^5*x^5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(kx-1)\left(3kx^2-4kx+2x-1\right)}{\sqrt[3]{x(x-1)(kx-1)}\left(bk^5x^5-5bk^4x^4+10bk^3x^3-10bk^2x^2+5bkx-b-x^2+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-2+3*k)*x-(4*k**2+k)*x**2+3*k**2*x**3)/((1-x)*x*(-k*x+1))**(1
/3)/(-b+(5*b*k+1)*x-(10*b*k**2+1)*x**2+10*b*k**3*x**3-5*b*k**4*x**4+b*k**5*
x**5),x)
```

```
[Out] Integral((k*x - 1)*(3*k*x**2 - 4*k*x + 2*x - 1)/((x*(x - 1)*(k*x - 1))**(1/
3)*(b*k**5*x**5 - 5*b*k**4*x**4 + 10*b*k**3*x**3 - 10*b*k**2*x**2 + 5*b*k*x
 - b - x**2 + x)), x)
```

3.2240 $\int \frac{(d+cx)^2}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx$

Optimal. Leaf size=282

$$\frac{2\sqrt{a^2x^2+b^2}\,\left(12a^4c^2x^5+40a^4cdx^4+60a^4d^2x^3-3a^2b^2c^2x^3+30a^2b^2cdx^2+25a^2b^2d^2x-4b^4c^2x+4b^4cd\right)}{15a^2\left(\sqrt{a^2x^2+b^2}+ax\right)^{7/2}}+\frac{2}{}$$

Rubi [A] time = 0.23, antiderivative size = 232, normalized size of antiderivative = 0.82, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2119, 1628}

$$\frac{cd\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}{3a^2}+\frac{b^4cd}{5a^2\left(\sqrt{a^2x^2+b^2}+ax\right)^{5/2}}+\frac{b^2\left(b^2c^2-4a^2d^2\right)}{12a^3\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}-\frac{\sqrt{\sqrt{a^2x^2+b^2}+ax}\,\left(b^2c^2-4a^2d^2\right)}{4a^3}+\frac{c^2\left(\sqrt{a^2x^2+b^2}+ax\right)^{5/2}}{20a^3}-\frac{b^6c^2}{28a^3\left(\sqrt{a^2x^2+b^2}+ax\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*x)^2/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] -1/28*(b^6*c^2)/(a^3*(a*x + Sqrt[b^2 + a^2*x^2])^(7/2)) + (b^4*c*d)/(5*a^2*
(a*x + Sqrt[b^2 + a^2*x^2])^(5/2)) + (b^2*(b^2*c^2 - 4*a^2*d^2))/(12*a^3*(a
*x + Sqrt[b^2 + a^2*x^2])^(3/2)) - ((b^2*c^2 - 4*a^2*d^2)*Sqrt[a*x + Sqrt[b
^2 + a^2*x^2]])/(4*a^3) + (c*d*(a*x + Sqrt[b^2 + a^2*x^2])^(3/2))/(3*a^2) +
 (c^2*(a*x + Sqrt[b^2 + a^2*x^2])^(5/2))/(20*a^3)
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+cx)^2}{\sqrt{ax+\sqrt{b^2+a^2x^2}}}\,dx &= \frac{\text{Subst}\left(\int \frac{(b^2+x^2)\left(-b^2c+2adx+cx^2\right)^2}{x^{9/2}}\,dx, x, ax+\sqrt{b^2+a^2x^2}\right)}{8a^3}\\ &= \frac{\text{Subst}\left(\int \left(\frac{b^6c^2}{x^{9/2}}-\frac{4ab^4cd}{x^{7/2}}-\frac{b^2\left(b^2c^2-4a^2d^2\right)}{x^{5/2}}+\frac{-b^2c^2+4a^2d^2}{\sqrt{x}}+4acd\sqrt{x}+c^2x^{3/2}\right)dx, x, ax\right.}{8a^3}\\ &= -\frac{b^6c^2}{28a^3\left(ax+\sqrt{b^2+a^2x^2}\right)^{7/2}}+\frac{b^4cd}{5a^2\left(ax+\sqrt{b^2+a^2x^2}\right)^{5/2}}+\frac{b^2\left(b^2c^2-4a^2d^2\right.}{12a^3\left(ax+\sqrt{b^2+a^2\right.}}\end{aligned}$$

Mathematica [B] time = 7.65, size = 1949, normalized size = 6.91

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*x)^2/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] (2*d^2*Sqrt[b^2 + a^2*x^2]*(2*a^2*x*Sqrt[b^2 + a^2*x^2] + a*(b^2 + 2*a^2*x^
2))*(6*a^2*x*Sqrt[b^2 + a^2*x^2]*(3*b^2 + a^2*x^2) + a*(7*b^4 + 21*a^2*b^2*
x^2 + 6*a^4*x^4))*(Sqrt[a^2]*b^2 + 2*Sqrt[a^2]*x*(a^2*x + a*Sqrt[b^2 + a^2*
x^2]))^3)/(15*a^4*Sqrt[a^2]*b^2*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]*(a^2*x*Sqrt
[b^2 + a^2*x^2]*(9*b^8 + 120*a^2*b^6*x^2 + 432*a^4*b^4*x^4 + 576*a^6*b^2*x^
6 + 256*a^8*x^8) + a*(b^10 + 41*a^2*b^8*x^2 + 280*a^4*b^6*x^4 + 688*a^6*b^4
*x^6 + 704*a^8*b^2*x^8 + 256*a^10*x^10))) + (4*c*d*Sqrt[b^2 + a^2*x^2]*(2*a
^2*x*Sqrt[b^2 + a^2*x^2] + a*(b^2 + 2*a^2*x^2))*(5*a^2*x*Sqrt[b^2 + a^2*x^2
]*(11*b^4 + 19*a^2*b^2*x^2 + 12*a^4*x^4) + a*(22*b^6 + 95*a^2*b^4*x^2 + 125
*a^4*b^2*x^4 + 60*a^6*x^6))*(Sqrt[a^2]*b^2 + 2*Sqrt[a^2]*x*(a^2*x + a*Sqrt[
b^2 + a^2*x^2]))^3)/(105*a^6*b^2*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]*(Sqrt[a^2]
*Sqrt[b^2 + a^2*x^2]*(b^10 + 50*a^2*b^8*x^2 + 400*a^4*b^6*x^4 + 1120*a^6*b^
4*x^6 + 1280*a^8*b^2*x^8 + 512*a^10*x^10) + 2*a*(5*Sqrt[a^2]*b^10*x + 85*(a
^2)^(3/2)*b^8*x^3 + 416*(a^2)^(5/2)*b^6*x^5 + 848*(a^2)^(7/2)*b^4*x^7 + 768
*(a^2)^(9/2)*b^2*x^9 + 256*(a^2)^(11/2)*x^11))) + (2*c^2*Sqrt[b^2 + a^2*x^2
]*(2*a^2*x*Sqrt[b^2 + a^2*x^2] + a*(b^2 + 2*a^2*x^2))*(28*a^2*x*Sqrt[b^2 +
a^2*x^2]*(-b^6 - a^2*b^4*x^2 + 3*a^4*b^2*x^4 + 2*a^6*x^6) + a*(-8*b^8 - 49*
a^2*b^6*x^2 + 7*a^4*b^4*x^4 + 112*a^6*b^2*x^6 + 56*a^8*x^8))*(Sqrt[a^2]*b^2
 + 2*Sqrt[a^2]*x*(a^2*x + a*Sqrt[b^2 + a^2*x^2]))^3)/(63*a^6*Sqrt[a^2]*b^2*
Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]*(a^2*x*Sqrt[b^2 + a^2*x^2]*(11*b^10 + 220*a
^2*b^8*x^2 + 1232*a^4*b^6*x^4 + 2816*a^6*b^4*x^6 + 2816*a^8*b^2*x^8 + 1024*
a^10*x^10) + a*(b^12 + 61*a^2*b^10*x^2 + 620*a^4*b^8*x^4 + 2352*a^6*b^6*x^6
 + 4096*a^8*b^4*x^8 + 3328*a^10*b^2*x^10 + 1024*a^12*x^12))) - (Sqrt[a^2]*d
^2*x*(a*x + Sqrt[b^2 + a^2*x^2])*((2*b^4)/(3*(a*x + Sqrt[b^2 + a^2*x^2])^(3
/2)) + (2*(a*x + Sqrt[b^2 + a^2*x^2])^(5/2))/5))/(2*b^2*(a/Sqrt[a^2] + (Sqr
t[a^2]*x)/Sqrt[b^2 + a^2*x^2])*(-(Sqrt[a^2]*b^2) + Sqrt[a^2]*(a*x + Sqrt[b^
2 + a^2*x^2])^2)*(Sqrt[a^2]/a - (-(Sqrt[a^2]*b^2) + Sqrt[a^2]*(a*x + Sqrt[b
^2 + a^2*x^2])^2)/(2*a*(a*x + Sqrt[b^2 + a^2*x^2])^2))) - (2*a^2*c*d*x^2*(-
21*b^6 + 105*b^4*(a*x + Sqrt[b^2 + a^2*x^2])^2 - 35*b^2*(a*x + Sqrt[b^2 + a
^2*x^2])^4 + 15*(a*x + Sqrt[b^2 + a^2*x^2])^6))/(105*b^2*(a/Sqrt[a^2] + (Sq
rt[a^2]*x)/Sqrt[b^2 + a^2*x^2])*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]*(-(Sqrt[a^2
]*b^2) + Sqrt[a^2]*(a*x + Sqrt[b^2 + a^2*x^2])^2)^2*(Sqrt[a^2]/a - (-(Sqrt[
a^2]*b^2) + Sqrt[a^2]*(a*x + Sqrt[b^2 + a^2*x^2])^2)/(2*a*(a*x + Sqrt[b^2 +
 a^2*x^2])^2))) - ((a^2)^(3/2)*c^2*x^3*(45*b^8 - 210*b^6*(a*x + Sqrt[b^2 +
a^2*x^2])^2 - 126*b^2*(a*x + Sqrt[b^2 + a^2*x^2])^6 + 35*(a*x + Sqrt[b^2 +
a^2*x^2])^8))/(315*b^2*(a/Sqrt[a^2] + (Sqrt[a^2]*x)/Sqrt[b^2 + a^2*x^2])*Sq
rt[a*x + Sqrt[b^2 + a^2*x^2]]*(-(Sqrt[a^2]*b^2) + Sqrt[a^2]*(a*x + Sqrt[b^2
 + a^2*x^2])^2)^3*(Sqrt[a^2]/a - (-(Sqrt[a^2]*b^2) + Sqrt[a^2]*(a*x + Sqrt[
b^2 + a^2*x^2])^2)/(2*a*(a*x + Sqrt[b^2 + a^2*x^2])^2)))
```

IntegrateAlgebraic [A] time = 0.36, size = 282, normalized size = 1.00

$$\frac{2\sqrt{a^2x^2+b^2}\left(12a^4c^2x^5+40a^4cdx^4+60a^4d^2x^3-3a^2b^2c^2x^3+30a^2b^2cdx^2+25a^2b^2d^2x-4b^4c^2x+4b^4cd\right)}{15a^2\left(\sqrt{a^2x^2+b^2}+ax\right)^{7/2}}+\frac{2\left(84a^6c^2x^6+280a^6cdx^5+420a^6d^2x^4+21a^4b^2c^2x^4+350a^4b^2cdx^3+385a^4b^2d^2x^2-49a^2b^4c^2x^2+98a^2b^4cdx+35a^2b^4d^2-8b^6c^2\right)}{105a^3\left(\sqrt{a^2x^2+b^2}+ax\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + c*x)^2/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]],x]
```

```
[Out] (2*Sqrt[b^2 + a^2*x^2]*(4*b^4*c*d - 4*b^4*c^2*x + 25*a^2*b^2*d^2*x + 30*a^2
*b^2*c*d*x^2 - 3*a^2*b^2*c^2*x^3 + 60*a^4*d^2*x^3 + 40*a^4*c*d*x^4 + 12*a^4
*c^2*x^5))/(15*a^2*(a*x + Sqrt[b^2 + a^2*x^2])^(7/2)) + (2*(-8*b^6*c^2 + 35
*a^2*b^4*d^2 + 98*a^2*b^4*c*d*x - 49*a^2*b^4*c^2*x^2 + 385*a^4*b^2*d^2*x^2
+ 350*a^4*b^2*c*d*x^3 + 21*a^4*b^2*c^2*x^4 + 420*a^6*d^2*x^4 + 280*a^6*c*d*
x^5 + 84*a^6*c^2*x^6))/(105*a^3*(a*x + Sqrt[b^2 + a^2*x^2])^(7/2))
```

fricas [A] time = 1.47, size = 167, normalized size = 0.59

$$-\frac{2\left(15\,a^4c^2x^4+42\,a^4cdx^3+14\,a^2b^2cdx+8\,b^4c^2-35\,a^2b^2d^2+\left(a^2b^2c^2+35\,a^4d^2\right)x^2-\left(15\,a^3c^2x^3+42\,a^3cdx^2+28\,ab^2cd+\left(4\,ab^2c^2+35\,a^3d^2\right)x\right)\sqrt{a^2x^2+b^2}\right)\sqrt{ax+\sqrt{a^2x^2+b^2}}}{105\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+d)^2/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/105*(15*a^4*c^2*x^4 + 42*a^4*c*d*x^3 + 14*a^2*b^2*c*d*x + 8*b^4*c^2 - 35
*a^2*b^2*d^2 + (a^2*b^2*c^2 + 35*a^4*d^2)*x^2 - (15*a^3*c^2*x^3 + 42*a^3*c*
d*x^2 + 28*a*b^2*c*d + (4*a*b^2*c^2 + 35*a^3*d^2)*x)*sqrt(a^2*x^2 + b^2))*s
qrt(a*x + sqrt(a^2*x^2 + b^2))/(a^3*b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx+d)^2}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+d)^2/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x + d)^2/sqrt(a*x + sqrt(a^2*x^2 + b^2)), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(cx+d)^2}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x+d)^2/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] int((c*x+d)^2/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx+d)^2}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+d)^2/(a*x+(a^2*x^2+b^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x + d)^2/sqrt(a*x + sqrt(a^2*x^2 + b^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+c\,x)^2}{\sqrt{a\,x+\sqrt{a^2\,x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + c*x)^2/(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int((d + c*x)^2/(a*x + (b^2 + a^2*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx+d)^2}{\sqrt{ax+\sqrt{a^2x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+d)**2/(a*x+(a**2*x**2+b**2)**(1/2))**(1/2),x)
```

```
[Out] Integral((c*x + d)**2/sqrt(a*x + sqrt(a**2*x**2 + b**2)), x)
```

3.2241 $\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$

Optimal. Leaf size=285

$$-\frac{(-1)^{2/3}\log\left(k^2x^2+2(-1)^{2/3}\sqrt[3]{2}\,(k-1)^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\left(\sqrt[3]{-1}\,2^{2/3}\sqrt[3]{k-1}-\sqrt[3]{-1}\,2^{2/3}\sqrt[3]{k-1}\,kx\right)\cdots\right.}{2\,2^{2/3}\sqrt[3]{k-1}}$$

Rubi [F] time = 0.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
```

```
[Out] ((1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Int][(1 - k*x)^(1/3)/((1 - x)^
(2/3)*x^(2/3)*(1 + (-2 + k)*x)), x])/((1 - x)*x*(1 - k*x))^(2/3)
```

Rubi steps

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx = \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{\sqrt[3]{1-kx}}{(1-x)^{2/3}x^{2/3}(1+(-2+k)x)}\,dx}{((1-x)x(1-kx))^{2/3}}$$

Mathematica [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
```

```
[Out] Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)), x]
```

IntegrateAlgebraic [A] time = 3.21, size = 285, normalized size = 1.00

$$-\frac{(-1)^{2/3}\log\left(k^2x^2+2(-1)^{2/3}\sqrt[3]{2}\,(k-1)^{2/3}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\left(\sqrt[3]{-1}\,2^{2/3}\sqrt[3]{k-1}-\sqrt[3]{-1}\,2^{2/3}\sqrt[3]{k-1}\,kx\right)\sqrt[3]{kx^3+(-k-1)x^2+x}-2kx+1\right)}{2\,2^{2/3}\sqrt[3]{k-1}}+\frac{(-1)^{2/3}\log\left(\sqrt[3]{-1}\,2^{2/3}\sqrt[3]{k-1}\,\sqrt[3]{kx^3+(-k-1)x^2+x}+kx-1\right)}{2^{2/3}\sqrt[3]{k-1}}+\frac{(-1)^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}kx-\sqrt{3}}{-2\sqrt[3]{-1}\,2^{2/3}\sqrt[3]{k-1}\,\sqrt[3]{kx^3+(-k-1)x^2+x}+kx-1}\right)}{2^{2/3}\sqrt[3]{k-1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3))
,x]
```

```
[Out] ((-1)^(2/3)*Sqrt[3]*ArcTan[(-Sqrt[3] + Sqrt[3]*k*x)/(-1 + k*x - 2*(-1)^(1/3
)*2^(2/3)*(-1 + k)^(1/3)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))]])/(2^(2/3)*(-1 +
 k)^(1/3)) + ((-1)^(2/3)*Log[-1 + k*x + (-1)^(1/3)*2^(2/3)*(-1 + k)^(1/3)*(
x + (-1 - k)*x^2 + k*x^3)^(1/3)])/(2^(2/3)*(-1 + k)^(1/3)) - ((-1)^(2/3)*Lo
g[1 - 2*k*x + k^2*x^2 + ((-1)^(1/3)*2^(2/3)*(-1 + k)^(1/3) - (-1)^(1/3)*2^(
2/3)*(-1 + k)^(1/3)*k*x)*(x + (-1 - k)*x^2 + k*x^3)^(1/3) + 2*(-1)^(2/3)*2^
(1/3)*(-1 + k)^(2/3)*(x + (-1 - k)*x^2 + k*x^3)^(2/3)])/(2*2^(2/3)*(-1 + k)
^(1/3))
```

fricas [B] time = 94.95, size = 932, normalized size = 3.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="fric
as")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/3*(24*sqrt(3)*2^(1/3)*((k^5 - 3*k^4 - 4*k^3 +
22*k^2 - 24*k + 8)*x^4 - 2*(k^4 - 10*k^3 + 27*k^2 - 26*k + 8)*x^3 - 6*(k^3
- 4*k^2 + 4*k - 1)*x^2 - 2*(k^2 - 1)*x + k - 1)*(k*x^3 - (k + 1)*x^2 + x)^(
2/3)/(k - 1)^(1/3) - 6*sqrt(3)*2^(2/3)*((k^6 + 27*k^5 - 40*k^4 - 20*k^3 + 4
8*k^2 - 16*k)*x^5 - (33*k^5 + 55*k^4 - 220*k^3 + 132*k^2 + 16*k - 16)*x^4 +
 2*(55*k^4 - 55*k^3 - 66*k^2 + 82*k - 16)*x^3 - 2*(55*k^3 - 99*k^2 + 38*k +
 6)*x^2 + (33*k^2 - 61*k + 28)*x - k + 1)*(k*x^3 - (k + 1)*x^2 + x)^(1/3)/(
k - 1)^(2/3) + sqrt(3)*((k^6 - 48*k^5 - 192*k^4 + 416*k^3 - 48*k^2 - 192*k
+ 64)*x^6 + 6*(7*k^5 + 104*k^4 - 80*k^3 - 176*k^2 + 176*k - 32)*x^5 - 3*(13
9*k^4 + 256*k^3 - 768*k^2 + 352*k + 16)*x^4 + 4*(203*k^3 - 192*k^2 - 120*k
+ 104)*x^3 - 3*(139*k^2 - 208*k + 64)*x^2 + 6*(7*k - 8)*x + 1))/((k^6 + 96*
k^5 - 48*k^4 - 160*k^3 + 240*k^2 - 192*k + 64)*x^6 - 6*(17*k^5 + 64*k^4 - 1
12*k^3 + 80*k^2 - 80*k + 32)*x^5 + 3*(149*k^4 + 32*k^3 - 96*k^2 - 160*k + 8
0)*x^4 - 4*(157*k^3 - 24*k^2 - 168*k + 40)*x^3 + 3*(149*k^2 - 128*k - 16)*x
^2 - 6*(17*k - 16)*x + 1))/(k - 1)^(1/3) - 1/12*2^(1/3)*log((12*2^(2/3)*(k*
x^3 - (k + 1)*x^2 + x)^(2/3)*((k^3 + k^2 - 4*k + 2)*x^2 - 2*(2*k^2 - 3*k +
1)*x + k - 1)/(k - 1)^(2/3) + 6*((k^3 + 8*k^2 - 8*k)*x^3 - (11*k^2 - 8)*x^2
 + (11*k - 8)*x - 1)*(k*x^3 - (k + 1)*x^2 + x)^(1/3) + 2^(1/3)*((k^4 + 28*k
^3 - 12*k^2 - 32*k + 16)*x^4 - 4*(8*k^3 + 15*k^2 - 30*k + 8)*x^3 + 6*(13*k^
2 - 10*k - 2)*x^2 - 4*(8*k - 7)*x + 1)/(k - 1)^(1/3))/((k^4 - 8*k^3 + 24*k^
2 - 32*k + 16)*x^4 + 4*(k^3 - 6*k^2 + 12*k - 8)*x^3 + 6*(k^2 - 4*k + 4)*x^2
 + 4*(k - 2)*x + 1))/(k - 1)^(1/3) + 1/6*2^(1/3)*log((6*2^(1/3)*(k*x^3 - (k
 + 1)*x^2 + x)^(1/3)*(k*x - 1)/(k - 1)^(1/3) - 2^(2/3)*((k^2 - 4*k + 4)*x^2
 + 2*(k - 2)*x + 1)/(k - 1)^(2/3) - 12*(k*x^3 - (k + 1)*x^2 + x)^(2/3))/((k
^2 - 4*k + 4)*x^2 + 2*(k - 2)*x + 1))/(k - 1)^(1/3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{kx-1}{\left((kx-1)(x-1)x\right)^{\frac{2}{3}}\left((k-2)x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="giac
")
```

```
[Out] integrate(-(k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{-kx+1}{\left(1+(-2+k)\,x\right)\left((1-x)\,x\,(-kx+1)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x)
```

```
[Out] int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{kx-1}{\left((kx-1)(x-1)x\right)^{\frac{2}{3}}\left((k-2)x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="maxi
ma")
```

```
[Out] -integrate((k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{k\,x-1}{\left(x\,\left(k-2\right)+1\right)\,{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(k*x - 1)/((x*(k - 2) + 1)*(x*(k*x - 1)*(x - 1))^(2/3)),x)
```

```
[Out] -int((k*x - 1)/((x*(k - 2) + 1)*(x*(k*x - 1)*(x - 1))^(2/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{kx}{kx\left(kx^{3}-kx^{2}-x^{2}+x\right)^{\frac{2}{3}}-2x\left(kx^{3}-kx^{2}-x^{2}+x\right)^{\frac{2}{3}}+\left(kx^{3}-kx^{2}-x^{2}+x\right)^{\frac{2}{3}}}\, dx-\int \left(-\frac{1}{kx\left(kx^{3}-kx^{2}-x^{2}+x\right)^{\frac{2}{3}}-2x\left(kx^{3}-kx^{2}-x^{2}+x\right)^{\frac{2}{3}}+\left(kx^{3}-kx^{2}-x^{2}+x\right)^{\frac{2}{3}}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))**(2/3),x)
```

```
[Out] -Integral(k*x/(k*x*(k*x**3 - k*x**2 - x**2 + x)**(2/3) - 2*x*(k*x**3 - k*x*
*2 - x**2 + x)**(2/3) + (k*x**3 - k*x**2 - x**2 + x)**(2/3)), x) - Integral
(-1/(k*x*(k*x**3 - k*x**2 - x**2 + x)**(2/3) - 2*x*(k*x**3 - k*x**2 - x**2
+ x)**(2/3) + (k*x**3 - k*x**2 - x**2 + x)**(2/3)), x)
```

3.2242 $\int \sqrt{ax+\sqrt{b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$

Optimal. Leaf size=286

$$\frac{\sqrt{\sqrt{a^2x^2+b}+ax}\left(6acx+16c^3\right)\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}+\sqrt{a^2x^2+b}\left(\left(60ax-8c^2\right)\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}\right.}{105a\sqrt{\sqrt{a^2x^2+b}}}$$

Rubi [F] time = 0.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{ax+\sqrt{b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]],x
]
```

```
[Out] Defer[Int][Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x
^2]]], x]
```

Rubi steps

$$\int \sqrt{ax+\sqrt{b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx=\int \sqrt{ax+\sqrt{b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Mathematica [F] time = 9.27, size = 0, normalized size = 0.00

$$\int \sqrt{ax+\sqrt{b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^
2]]],x]
```

```
[Out] Integrate[Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^
2]]], x]
```

IntegrateAlgebraic [A] time = 0.72, size = 286, normalized size = 1.00

$$\frac{\sqrt{\sqrt{a^2x^2+b}+ax}\left(6acx+16c^3\right)\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}+\sqrt{a^2x^2+b}\left(\left(60ax-8c^2\right)\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}+6c\sqrt{\sqrt{a^2x^2+b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}\right)+\left(60a^2x^2-8ac^2x-75b\right)\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{105a\sqrt{\sqrt{a^2x^2+b}+ax}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[b
 + a^2*x^2]]],x]
```

```
[Out] ((-75*b - 8*a*c^2*x + 60*a^2*x^2)*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]] +
 (16*c^3 + 6*a*c*x)*Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[
b + a^2*x^2]]] + Sqrt[b + a^2*x^2]*((-8*c^2 + 60*a*x)*Sqrt[c + Sqrt[a*x + S
```

```
qrt[b + a^2*x^2]]] + 6*c*Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x +
Sqrt[b + a^2*x^2]]]))/(105*a*Sqrt[a*x + Sqrt[b + a^2*x^2]]) - (b*ArcTanh[Sq
rt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]]/Sqrt[c]])/(a*Sqrt[c])
```

fricas [A] time = 2.97, size = 359, normalized size = 1.26

$$\left[\frac{105\,b\sqrt{c}\,\log\left(2\left(a\sqrt{c}\,x-\sqrt{a^2x^2+b}\,\sqrt{c}\right)\sqrt{ax+\sqrt{a^2x^2+b}}\,\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}-2\left(acx-\sqrt{a^2x^2+b}\,c\right)\sqrt{ax+\sqrt{a^2x^2+b}}+b\right)+2\left(16\,c^4+6\,ac^2x+6\,\sqrt{a^2x^2+b}\,c^2-\left(8\,c^3-135\,acx+75\,\sqrt{a^2x^2+b}\,c\right)\sqrt{ax+\sqrt{a^2x^2+b}}\right)\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{210\,ac},\ \frac{105\,b\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{c}\right)+\left(16\,c^4+6\,ac^2x+6\,\sqrt{a^2x^2+b}\,c^2-\left(8\,c^3-135\,acx+75\,\sqrt{a^2x^2+b}\,c\right)\sqrt{ax+\sqrt{a^2x^2+b}}\right)\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{105\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2+b)^(1/2))^(1/2)*(c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*b*sqrt(c)*log(2*(a*sqrt(c)*x - sqrt(a^2*x^2 + b)*sqrt(c))*sqrt(
a*x + sqrt(a^2*x^2 + b))*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b))) - 2*(a*c*x
 - sqrt(a^2*x^2 + b)*c)*sqrt(a*x + sqrt(a^2*x^2 + b)) + b) + 2*(16*c^4 + 6*
a*c^2*x + 6*sqrt(a^2*x^2 + b)*c^2 - (8*c^3 - 135*a*c*x + 75*sqrt(a^2*x^2 +
b)*c)*sqrt(a*x + sqrt(a^2*x^2 + b)))*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b))
))/(a*c), 1/105*(105*b*sqrt(-c)*arctan(sqrt(-c)*sqrt(c + sqrt(a*x + sqrt(a^
2*x^2 + b)))/c) + (16*c^4 + 6*a*c^2*x + 6*sqrt(a^2*x^2 + b)*c^2 - (8*c^3 -
135*a*c*x + 75*sqrt(a^2*x^2 + b)*c)*sqrt(a*x + sqrt(a^2*x^2 + b)))*sqrt(c +
 sqrt(a*x + sqrt(a^2*x^2 + b))))/(a*c)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2+b)^(1/2))^(1/2)*(c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(
1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax+\sqrt{a^2x^2+b}}\,\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+(a^2*x^2+b)^(1/2))^(1/2)*(c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2),x
)
```

```
[Out] int((a*x+(a^2*x^2+b)^(1/2))^(1/2)*(c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2),x
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax+\sqrt{a^2x^2+b}}\,\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2+b)^(1/2))^(1/2)*(c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + sqrt(a^2*x^2 + b))*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 +
b))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\sqrt{a^2\,x^2+b}+a\,x}\,\sqrt{c+\sqrt{\sqrt{a^2\,x^2+b}+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a^2*x^2)^(1/2) + a*x)^(1/2)*(c + ((b + a^2*x^2)^(1/2) + a*x)^(1/2
))^(1/2),x)
```

```
[Out] int(((b + a^2*x^2)^(1/2) + a*x)^(1/2)*(c + ((b + a^2*x^2)^(1/2) + a*x)^(1/2
))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + \sqrt{ax + \sqrt{a^2x^2 + b}}}\,\sqrt{ax + \sqrt{a^2x^2 + b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a**2*x**2+b)**(1/2))**(1/2)*(c+(a*x+(a**2*x**2+b)**(1/2))**
(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(c + sqrt(a*x + sqrt(a**2*x**2 + b)))*sqrt(a*x + sqrt(a**2*x**
2 + b)), x)
```

3.2243 $\int x^4\sqrt{b+a^2x^4}\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$

Optimal. Leaf size=287

$$\frac{\sqrt{a}\,x\left(192a^6x^{12}+360a^4bx^8+212a^2b^2x^4+39b^3\right)\sqrt{\sqrt{a^2x^4+b}+ax^2}+\sqrt{a}\,x\sqrt{a^2x^4+b}\sqrt{\sqrt{a^2x^4+b}+ax^2}\,(19}{1152a^{7/2}bx^2+1536a^{11/2}x^6+1536a^{9/2}x^4\sqrt{a^2x^4+b}+384a^{5/2}b\sqrt{a^2x^4+b}}$$

Rubi [F] time = 0.70, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^4\sqrt{b+a^2x^4}\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^4*Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] Defer[Int][x^4*Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]], x]
```

Rubi steps

$$\int x^4\sqrt{b+a^2x^4}\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx=\int x^4\sqrt{b+a^2x^4}\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int x^4\sqrt{b+a^2x^4}\sqrt{ax^2+\sqrt{b+a^2x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^4*Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] Integrate[x^4*Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]], x]
```

IntegrateAlgebraic [A] time = 1.10, size = 287, normalized size = 1.00

$$\frac{\sqrt{a}\,x\left(192a^6x^{12}+360a^4bx^8+212a^2b^2x^4+39b^3\right)\sqrt{\sqrt{a^2x^4+b}+ax^2}+\sqrt{a}\,x\sqrt{a^2x^4+b}\sqrt{\sqrt{a^2x^4+b}+ax^2}\left(192a^5x^{10}+264a^3bx^6+104ab^2x^2\right)}{1152a^{7/2}bx^2+1536a^{11/2}x^6+1536a^{9/2}x^4\sqrt{a^2x^4+b}+384a^{5/2}b\sqrt{a^2x^4+b}}-\frac{13b^2\log\left(i\sqrt{a^2x^4+b}+i\sqrt{2}\sqrt{a}\,x\sqrt{\sqrt{a^2x^4+b}+ax^2}+iax^2\right)}{128\sqrt{2}\,a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*Sqrt[b + a^2*x^4]*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]],x]
[Out] (Sqrt[a]*x*Sqrt[b + a^2*x^4]*(104*a*b^2*x^2 + 264*a^3*b*x^6 + 192*a^5*x^10)
*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]] + Sqrt[a]*x*(39*b^3 + 212*a^2*b^2*x^4 + 36
0*a^4*b*x^8 + 192*a^6*x^12)*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]])/(1152*a^(7/2)*
b*x^2 + 1536*a^(11/2)*x^6 + 384*a^(5/2)*b*Sqrt[b + a^2*x^4] + 1536*a^(9/2)*
x^4*Sqrt[b + a^2*x^4]) - (13*b^2*Log[I*a*x^2 + I*Sqrt[b + a^2*x^4] + I*Sqrt
[2]*Sqrt[a]*x*Sqrt[a*x^2 + Sqrt[b + a^2*x^4]]])/(128*Sqrt[2]*a^(5/2))
```

fricas [A] time = 3.25, size = 284, normalized size = 0.99

$$\left[\frac{39\sqrt{2}\sqrt{a}\,b^2\log\left(4a^2x^4+4\sqrt{a^2x^4+b}\,ax^2-2\left(\sqrt{2}a^{\frac{3}{2}}x^3+\sqrt{2}\sqrt{a^2x^4+b}\sqrt{a}\,x\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}+b\right)-4\left(8a^4x^7+13a^2bx^3-\left(56a^3x^5+39abx\right)\sqrt{a^2x^4+b}\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{1536a^3},\ \frac{39\sqrt{2}\sqrt{-a}\,b^2\arctan\left(\frac{\sqrt{2}\sqrt{ax^2+\sqrt{a^2x^4+b}}\sqrt{-a}}{2ax}\right)-2\left(8a^4x^7+13a^2bx^3-\left(56a^3x^5+39abx\right)\sqrt{a^2x^4+b}\right)\sqrt{ax^2+\sqrt{a^2x^4+b}}}{768a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a^2*x^4+b)^(1/2)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorith
m="fricas")
```

```
[Out] [1/1536*(39*sqrt(2)*sqrt(a)*b^2*log(4*a^2*x^4 + 4*sqrt(a^2*x^4 + b)*a*x^2 -
 2*(sqrt(2)*a^(3/2)*x^3 + sqrt(2)*sqrt(a^2*x^4 + b)*sqrt(a)*x)*sqrt(a*x^2 +
 sqrt(a^2*x^4 + b)) + b) - 4*(8*a^4*x^7 + 13*a^2*b*x^3 - (56*a^3*x^5 + 39*a
*b*x)*sqrt(a^2*x^4 + b))*sqrt(a*x^2 + sqrt(a^2*x^4 + b)))/a^3, 1/768*(39*sq
rt(2)*sqrt(-a)*b^2*arctan(1/2*sqrt(2)*sqrt(a*x^2 + sqrt(a^2*x^4 + b))*sqrt(
-a)/(a*x)) - 2*(8*a^4*x^7 + 13*a^2*b*x^3 - (56*a^3*x^5 + 39*a*b*x)*sqrt(a^2
*x^4 + b))*sqrt(a*x^2 + sqrt(a^2*x^4 + b)))/a^3]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a^2*x^4+b)^(1/2)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorith
m="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, need to choose a branch for the root of a polyno
mial with parameters. This might be wrong.The choice was done assuming [a,b
,x]=[0,-68,-97]Precision problem choosing root in common_EXT, current preci
sion 14Warning, need to choose a branch for the root of a polynomial with p
arameters. This might be wrong.The choice was done assuming [a,b,x]=[-67,8,
-61]schur row 3 -1.00112e-08Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b,t_nostep]=[24,49,-35]Warning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.The choice was done
 assuming [a,b,t_nostep]=[20,8,5]Warning, need to choose a branch for the r
oot of a polynomial with parameters. This might be wrong.The choice was don
e assuming [a,b,t_nostep]=[3,-23,44]Warning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming [a,b,t_nostep]=[-92,-93,-41]schur row 3 -1.33253e-09Warning,
need to choose a branch for the root of a polynomial with parameters. This
might be wrong.The choice was done assuming [a,b,t_nostep]=[93,-2,-73]Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b,t_nostep]=[40,96,-96]W
arning, need to choose a branch for the root of a polynomial with parameter
s. This might be wrong.The choice was done assuming [a,b,t_nostep]=[-24,-84
,-66]schur row 3 -9.75014e-10Warning, need to choose a branch for the root
of a polynomial with parameters. This might be wrong.The choice was done as
suming [a,b,t_nostep]=[66,44,-64]schur row 3 3.13659e-08Warning, need to ch
oose a branch for the root of a polynomial with parameters. This might be w
rong.The choice was done assuming [a,b,t_nostep]=[-95,-41,-48]Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [a,b,t_nostep]=[1,69,4]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [a,b,t_nostep]=[99,-63,-95]Warning
, need to choose a branch for the root of a polynomial with parameters. Thi
s might be wrong.The choice was done assuming [a,b,t_nostep]=[81,83,-9]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming [a,b,t_nostep]=[-89,27,4]W
arning, need to choose a branch for the root of a polynomial with parameter
s. This might be wrong.The choice was done assuming [a,b,t_nostep]=[56,75,-
77]Warning, need to choose a branch for the root of a polynomial with param
eters. This might be wrong.The choice was done assuming [a,b,t_nostep]=[37,
-64,-25]Warning, need to choose a branch for the root of a polynomial with
parameters. This might be wrong.The choice was done assuming [a,b,t_nostep]
=[-98,-33,59]Warning, need to choose a branch for the root of a polynomial
```

```
with parameters. This might be wrong.The choice was done assuming [a,b,t_no
step]=[92,64,-69]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [a,b,
t_nostep]=[21,-17,-9]Warning, need to choose a branch for the root of a pol
ynomial with parameters. This might be wrong.The choice was done assuming [
a,b,t_nostep]=[-4,96,-65]Warning, need to choose a branch for the root of a
 polynomial with parameters. This might be wrong.The choice was done assumi
ng [a,b,t_nostep]=[-85,-1,-30]schur row 3 -5.59763e-10Warning, need to choo
se a branch for the root of a polynomial with parameters. This might be wro
ng.The choice was done assuming [a,b,t_nostep]=[-75,7,-7]Warning, need to c
hoose a branch for the root of a polynomial with parameters. This might be
wrong.The choice was done assuming [a,b,t_nostep]=[-4,-6,-56]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b,t_nostep]=[-5,-69,82]schur row
3 1.56409e-09Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b,t_no
step]=[-71,32,48]Warning, need to choose a branch for the root of a polynom
ial with parameters. This might be wrong.The choice was done assuming [a,b,
t_nostep]=[73,53,-20]Warning, need to choose a branch for the root of a pol
ynomial with parameters. This might be wrong.The choice was done assuming [
a,b,t_nostep]=[16,51,70]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.The choice was done assumin
g [a,b,t_nostep]=[-53,-39,82]schur row 3 -6.64448e-11Warning, need to choos
e a branch for the root of a polynomial with parameters. This might be wron
g.The choice was done assuming [a,b,t_nostep]=[-15,91,-72]Warning, need to
choose a branch for the root of a polynomial with parameters. This might be
 wrong.The choice was done assuming [a,b,t_nostep]=[74,86,-86]schur row 1 6
.32223e-07Francis algorithm not precise enough for[1.0,0.0,-599084431612,-1
.19816667401e+12,8.97248832883e+22]Warning, need to choose a branch for the
 root of a polynomial with parameters. This might be wrong.The choice was d
one assuming [a,b,t_nostep]=[-28,72,-21]Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
was done assuming [a,b,t_nostep]=[64,-73,13]Warning, need to choose a branc
h for the root of a polynomial with parameters. This might be wrong.The cho
ice was done assuming [a,b,t_nostep]=[-74,31,29]Warning, need to choose a b
ranch for the root of a polynomial with parameters. This might be wrong.The
 choice was done assuming [a,b,t_nostep]=[95,-80,-59]schur row 3 7.64423e-0
9Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.The choice was done assuming [a,b,t_nostep]=[-12,5
3,99]Warning, need to choose a branch for the root of a polynomial with par
ameters. This might be wrong.The choice was done assuming [a,b,t_nostep]=[6
9,-96,-89]Warning, need to choose a branch for the root of a polynomial wit
h parameters. This might be wrong.The choice was done assuming [a,b,t_noste
p]=[26,-78,-6]Unable to divide, perhaps due to rounding error%%%{%%%{%%{[-2
,%%%{4,[1]%%%}]:[1,0,%%%{1,[1]%%%}]%%},[0]%%%}/%%%{%%{[2,1]:[1,0,%%%{1,[1]%
%%}]%%},[0]%%%},[1]%%%}+%%%{%%%{1,[1]%%%},[0]%%%} / %%%{%%%{1/%%%{%%{[2,1]:
[1,0,%%%{1,[1]%%%}]%%},[0]%%%},[0]%%%},[0]%%%} Error: Bad Argument ValueUna
ble to divide, perhaps due to rounding error%%%{%%%{%%{[2,%%%{4,[1]%%%}]:[1
,0,%%%{1,[1]%%%}]%%},[0]%%%}/%%%{%%{[-2,1]:[1,0,%%%{1,[1]%%%}]%%},[0]%%%},[
1]%%%}+%%%{%%%{1,[1]%%%},[0]%%%} / %%%{%%%{1/%%%{%%{[-2,1]:[1,0,%%%{1,[1]%%
%}]%%},[0]%%%},[0]%%%},[0]%%%} Error: Bad Argument ValueEvaluation time: 10
.42integrate((4*a^2*x^8*sqrt(sqrt(a^2*x^4+b)+a*x^2)*sqrt(a^2*x^4+b)-2*a^2*x
^8-4*a*x^6*sqrt(sqrt(a^2*x^4+b)+a*x^2)*sqrt(a^2*x^4+b)+4*b*x^4*sqrt(sqrt(a^
2*x^4+b)+a*x^2)*sqrt(a^2*x^4+b)-2*b*x^4+x^4*sqrt(sqrt(a^2*x^4+b)+a*x^2)*sqr
t(a^2*x^4+b)+2*x^4*(a^2*x^4+b))/(4*a^2*x^4-4*a*x^2+4*b+1),x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^{4}\sqrt{a^{2}x^{4}+b}\,\sqrt{a\,x^{2}+\sqrt{a^{2}x^{4}+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2*x^4+b)^(1/2)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x)
```

```
[Out] int(x^4*(a^2*x^4+b)^(1/2)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2x^4+b}\sqrt{ax^2+\sqrt{a^2x^4+b}}\,x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a^2*x^4+b)^(1/2)*(a*x^2+(a^2*x^4+b)^(1/2))^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(a^2*x^4 + b)*sqrt(a*x^2 + sqrt(a^2*x^4 + b))*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\sqrt{\sqrt{a^2\,x^4+b}+a\,x^2}\,\sqrt{a^2\,x^4+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((b + a^2*x^4)^(1/2) + a*x^2)^(1/2)*(b + a^2*x^4)^(1/2),x)
```

```
[Out] int(x^4*((b + a^2*x^4)^(1/2) + a*x^2)^(1/2)*(b + a^2*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\sqrt{ax^2+\sqrt{a^2x^4+b}}\,\sqrt{a^2x^4+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a**2*x**4+b)**(1/2)*(a*x**2+(a**2*x**4+b)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(a*x**2 + sqrt(a**2*x**4 + b))*sqrt(a**2*x**4 + b), x)
```

3.2244 $\int \frac{x^2}{(b+ax^2)\sqrt[3]{x+x^3}}\,dx$

Optimal. Leaf size=288

$$\frac{\sqrt[3]{b}\log\left(-\sqrt[3]{b}\sqrt[3]{x^3+x}\,x\sqrt[3]{a-b}+x^2(a-b)^{2/3}+b^{2/3}\left(x^3+x\right)^{2/3}\right)}{4a\sqrt[3]{a-b}}-\frac{\sqrt[3]{b}\log\left(x\sqrt[3]{a-b}+\sqrt[3]{b}\sqrt[3]{x^3+x}\right)}{2a\sqrt[3]{a-b}}+\frac{\sqrt{3}\sqrt[3]{b}\ \text{t}}{}$$

Rubi [A] time = 0.58, antiderivative size = 445, normalized size of antiderivative = 1.55, number of steps used = 17, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2042, 466, 465, 494, 481, 200, 31, 634, 618, 204, 628, 617}

$$\frac{\sqrt[3]{b}\sqrt[3]{x}\sqrt[3]{x^2+1}\log\left(\frac{x^{4/3}(a-b)^{2/3}}{\left(x^2+1\right)^{2/3}}-\frac{\sqrt[3]{b}x^{2/3}\sqrt[3]{a-b}}{\sqrt[3]{x^2+1}}+b^{2/3}\right)}{4a\sqrt[3]{x^3+x}\sqrt[3]{a-b}}-\frac{\sqrt[3]{b}\sqrt[3]{x}\sqrt[3]{x^2+1}\log\left(\frac{x^{2/3}\sqrt[3]{a-b}}{\sqrt[3]{x^2+1}}+\sqrt[3]{b}\right)}{2a\sqrt[3]{x^3+x}\sqrt[3]{a-b}}+\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}\sqrt[3]{x^2+1}\tan^{-1}\left(\frac{\sqrt[3]{b}-\frac{2x^{2/3}\sqrt[3]{a-b}}{\sqrt[3]{x^2+1}}}{\sqrt{3}\sqrt[3]{b}}\right)}{2a\sqrt[3]{x^3+x}\sqrt[3]{a-b}}-\frac{\sqrt[3]{x}\sqrt[3]{x^2+1}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2+1}}\right)}{2a\sqrt[3]{x^3+x}}+\frac{\sqrt[3]{x}\sqrt[3]{x^2+1}\log\left(\frac{x^{4/3}}{\left(x^2+1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2+1}}+1\right)}{4a\sqrt[3]{x^3+x}}+\frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{x^2+1}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)}{2a\sqrt[3]{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((b + a*x^2)*(x + x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*x^(1/3)*(1 + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3))/(1 + x^2)^(1/3))/S
qrt[3]])/(2*a*(x + x^3)^(1/3)) + (Sqrt[3]*b^(1/3)*x^(1/3)*(1 + x^2)^(1/3)*A
rcTan[(b^(1/3) - (2*(a - b)^(1/3)*x^(2/3))/(1 + x^2)^(1/3))/(Sqrt[3]*b^(1/3
))])/(2*a*(a - b)^(1/3)*(x + x^3)^(1/3)) - (x^(1/3)*(1 + x^2)^(1/3)*Log[1 -
 x^(2/3)/(1 + x^2)^(1/3)])/(2*a*(x + x^3)^(1/3)) + (x^(1/3)*(1 + x^2)^(1/3)
*Log[1 + x^(4/3)/(1 + x^2)^(2/3) + x^(2/3)/(1 + x^2)^(1/3)])/(4*a*(x + x^3)
^(1/3)) - (b^(1/3)*x^(1/3)*(1 + x^2)^(1/3)*Log[b^(1/3) + ((a - b)^(1/3)*x^(
2/3))/(1 + x^2)^(1/3)])/(2*a*(a - b)^(1/3)*(x + x^3)^(1/3)) + (b^(1/3)*x^(1
/3)*(1 + x^2)^(1/3)*Log[b^(2/3) + ((a - b)^(2/3)*x^(4/3))/(1 + x^2)^(2/3) -
 ((a - b)^(1/3)*b^(1/3)*x^(2/3))/(1 + x^2)^(1/3)])/(4*a*(a - b)^(1/3)*(x +
x^3)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(b+ax^2\right)\sqrt[3]{x+x^3}}\,dx &= \frac{\left(\sqrt[3]{x}\,\sqrt[3]{1+x^2}\right)\int \frac{x^{5/3}}{\sqrt[3]{1+x^2}\left(b+ax^2\right)}\,dx}{\sqrt[3]{x+x^3}} \\
&= \frac{\left(3\sqrt[3]{x}\,\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{x^7}{\sqrt[3]{1+x^6}\left(b+ax^6\right)}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{x+x^3}} \\
&= \frac{\left(3\sqrt[3]{x}\,\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{1+x^3}\left(b+ax^3\right)}\,dx,x,x^{2/3}\right)}{2\sqrt[3]{x+x^3}} \\
&= \frac{\left(3\sqrt[3]{x}\,\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{x^3}{\left(1-x^3\right)\left(b-(-a+b)x^3\right)}\,dx,x,\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2\sqrt[3]{x+x^3}} \\
&= \frac{\left(3\sqrt[3]{x}\,\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^3}\,dx,x,\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2a\sqrt[3]{x+x^3}} - \frac{\left(3b\sqrt[3]{x}\,\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{b+(a-b)x^3}\right.}{2a\sqrt[3]{x+x^3}} \\
&= \frac{\left(\sqrt[3]{x}\,\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2a\sqrt[3]{x+x^3}} + \frac{\left(\sqrt[3]{x}\,\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx,x,\right.}{2a\sqrt[3]{x+x^3}} \\
&= -\frac{\sqrt[3]{x}\,\sqrt[3]{1+x^2}\,\log\left(1-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2a\sqrt[3]{x+x^3}} - \frac{\sqrt[3]{b}\,\sqrt[3]{x}\,\sqrt[3]{1+x^2}\,\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{a-b}\,x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2a\sqrt[3]{a-b}\,\sqrt[3]{x+x^3}} + \frac{\left(\sqrt[3]{x}\,\sqrt[3]{1\right.}{} \\
&= -\frac{\sqrt[3]{x}\,\sqrt[3]{1+x^2}\,\log\left(1-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2a\sqrt[3]{x+x^3}} + \frac{\sqrt[3]{x}\,\sqrt[3]{1+x^2}\,\log\left(1+\frac{x^{4/3}}{\left(1+x^2\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{4a\sqrt[3]{x+x^3}} - \frac{\sqrt[3]{b}\,\sqrt[3]{\cdots}}{} \\
&= \frac{\sqrt{3}\,\sqrt[3]{x}\,\sqrt[3]{1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2a\sqrt[3]{x+x^3}} + \frac{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{x}\,\sqrt[3]{1+x^2}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a-b}\,x^{2/3}}{\sqrt[3]{b}\,\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2a\sqrt[3]{a-b}\,\sqrt[3]{x+x^3}} - \frac{\sqrt[3]{x}\,\sqrt[3]{\cdots}}{}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 53, normalized size = 0.18

$$\frac{3x^3\sqrt[3]{x^2+1}\,F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-x^2,-\frac{ax^2}{b}\right)}{8b\sqrt[3]{x^3+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((b + a*x^2)*(x + x^3)^(1/3)),x]
```

```
[Out] (3*x^3*(1 + x^2)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -x^2, -((a*x^2)/b)])/(8*b
*(x + x^3)^(1/3))
```

IntegrateAlgebraic [A] time = 0.80, size = 291, normalized size = 1.01

$$\frac{\sqrt[3]{b}\log\left(-\sqrt[3]{b}\,\sqrt[3]{x^3+x}\,x\sqrt[3]{a-b}+x^2(a-b)^{2/3}+b^{2/3}\left(x^3+x\right)^{2/3}\right)}{4a\sqrt[3]{a-b}}-\frac{\sqrt[3]{b}\log\left(x\sqrt[3]{a-b}+\sqrt[3]{b}\,\sqrt[3]{x^3+x}\right)}{2a\sqrt[3]{a-b}}+\frac{\sqrt{3}\,\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt{3}x\sqrt[3]{a-b}}{x\sqrt[3]{a-b}-2\sqrt[3]{b}\,\sqrt[3]{x^3+x}}\right)}{2a\sqrt[3]{a-b}}-\frac{\log\left(a\sqrt[3]{x^3+x}-ax\right)}{2a}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x}+x}\right)}{2a}+\frac{\log\left(\sqrt[3]{x^3+x}\,x+\left(x^3+x\right)^{2/3}+x^2\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((b + a*x^2)*(x + x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(x + x^3)^(1/3))])/(2*a) + (Sqrt[3]*b^(1
/3)*ArcTan[(Sqrt[3]*(a - b)^(1/3)*x)/((a - b)^(1/3)*x - 2*b^(1/3)*(x + x^3)
```

```
^(1/3))])/(2*a*(a - b)^(1/3)) - Log[-(a*x) + a*(x + x^3)^(1/3)]/(2*a) - (b^
(1/3)*Log[(a - b)^(1/3)*x + b^(1/3)*(x + x^3)^(1/3)])/(2*a*(a - b)^(1/3)) +
 Log[x^2 + x*(x + x^3)^(1/3) + (x + x^3)^(2/3)]/(4*a) + (b^(1/3)*Log[(a - b
)^(2/3)*x^2 - (a - b)^(1/3)*b^(1/3)*x*(x + x^3)^(1/3) + b^(2/3)*(x + x^3)^(
2/3)])/(4*a*(a - b)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2+b)/(x^3+x)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.41, size = 256, normalized size = 0.89

$$-\frac{b\left(-\frac{a-b}{b}\right)^{\frac{2}{3}}\log\left(\left|-\left(-\frac{a-b}{b}\right)^{\frac{1}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right|\right)}{2\left(a^2-ab\right)}-\frac{3\left(-ab^2+b^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a-b}{b}\right)^{\frac{1}{3}}+2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a-b}{b}\right)^{\frac{1}{3}}}\right)}{2\left(\sqrt{3}a^2b-\sqrt{3}ab^2\right)}+\frac{\left(-ab^2+b^3\right)^{\frac{2}{3}}\log\left(\left(-\frac{a-b}{b}\right)^{\frac{2}{3}}+\left(-\frac{a-b}{b}\right)^{\frac{1}{3}}\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}\right)}{4\left(a^2b-ab^2\right)}-\frac{\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)}{2a}+\frac{\log\left(\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)}{4a}-\frac{\log\left(\left|\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2+b)/(x^3+x)^(1/3),x, algorithm="giac")
```

```
[Out] -1/2*b*(-(a - b)/b)^(2/3)*log(abs(-(-(a - b)/b)^(1/3) + (1/x^2 + 1)^(1/3)))
/(a^2 - a*b) - 3/2*(-a*b^2 + b^3)^(2/3)*arctan(1/3*sqrt(3)*((-(a - b)/b)^(1
/3) + 2*(1/x^2 + 1)^(1/3))/(-(a - b)/b)^(1/3))/(sqrt(3)*a^2*b - sqrt(3)*a*b
^2) + 1/4*(-a*b^2 + b^3)^(2/3)*log((-(a - b)/b)^(2/3) + (-(a - b)/b)^(1/3)*
(1/x^2 + 1)^(1/3) + (1/x^2 + 1)^(2/3))/(a^2*b - a*b^2) - 1/2*sqrt(3)*arctan
(1/3*sqrt(3)*(2*(1/x^2 + 1)^(1/3) + 1))/a + 1/4*log((1/x^2 + 1)^(2/3) + (1/
x^2 + 1)^(1/3) + 1)/a - 1/2*log(abs((1/x^2 + 1)^(1/3) - 1))/a
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a\,x^2+b\right)\left(x^3+x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2+b)/(x^3+x)^(1/3),x)
```

```
[Out] int(x^2/(a*x^2+b)/(x^3+x)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax^2+b\right)\left(x^3+x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2+b)/(x^3+x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a*x^2 + b)*(x^3 + x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(a\,x^2+b\right)\,{\left(x^3+x\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b + a*x^2)*(x + x^3)^(1/3)),x)
```

```
[Out] int(x^2/((b + a*x^2)*(x + x^3)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[3]{x\left(x^2+1\right)}\left(ax^2+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**2+b)/(x**3+x)**(1/3),x)
```

```
[Out] Integral(x**2/((x*(x**2 + 1))**(1/3)*(a*x**2 + b)), x)
```

3.2245 $\int \frac{1+x^6}{\sqrt[3]{x^2+x^4}\left(-1+x^6\right)} dx$

Optimal. Leaf size=288

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+x^2}-x}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+x^2}+x}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}-x}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}+x}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{2}{3}\tanh^{-1}\left(\frac{x}{\sqrt[3]{x^4+x^2}}\right)-$$

Rubi [F] time = 2.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x^6}{\sqrt[3]{x^2+x^4}\left(-1+x^6\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^6)/((x^2 + x^4)^(1/3)*(-1 + x^6)),x]
```

```
[Out] -1/6*(x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6)^(2/3)/(1 -
x), x], x, x^(1/3)])/(x^2 + x^4)^(1/3) - (x^(2/3)*(1 + x^2)^(1/3)*Defer[Sub
st][Defer[Int][(1 + x^6)^(2/3)/(1 + x), x], x, x^(1/3)])/(6*(x^2 + x^4)^(1/
3)) - ((1 + I*Sqrt[3])*x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(1 +
 x^6)^(2/3)/(1 - (-1)^(1/9)*x), x], x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) - ((
1 + I*Sqrt[3])*x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6)^(2
/3)/(1 + (-1)^(1/9)*x), x], x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) - ((1 - I*Sq
rt[3])*x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6)^(2/3)/(1 -
 (-1)^(2/9)*x), x], x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) - ((1 - I*Sqrt[3])*x
^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6)^(2/3)/(1 + (-1)^(2
/9)*x), x], x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) - (x^(2/3)*(1 + x^2)^(1/3)*D
efer[Subst][Defer[Int][(1 + x^6)^(2/3)/(1 - (-1)^(1/3)*x), x], x, x^(1/3)])
/(6*(x^2 + x^4)^(1/3)) - (x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(
1 + x^6)^(2/3)/(1 + (-1)^(1/3)*x), x], x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) -
 ((1 + I*Sqrt[3])*x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6)
^(2/3)/(1 - (-1)^(4/9)*x), x], x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) - ((1 + I
*Sqrt[3])*x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6)^(2/3)/(
1 + (-1)^(4/9)*x), x], x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) - ((1 - I*Sqrt[3]
)*x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6)^(2/3)/(1 - (-1)
^(5/9)*x), x], x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) - ((1 - I*Sqrt[3])*x^(2/3
)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6)^(2/3)/(1 + (-1)^(5/9)*x
), x], x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) - (x^(2/3)*(1 + x^2)^(1/3)*Defer[
Subst][Defer[Int][(1 + x^6)^(2/3)/(1 - (-1)^(2/3)*x), x], x, x^(1/3)])/(6*(
x^2 + x^4)^(1/3)) - (x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(1 + x
^6)^(2/3)/(1 + (-1)^(2/3)*x), x], x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) - ((1
+ I*Sqrt[3])*x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6)^(2/3
)/(1 - (-1)^(7/9)*x), x], x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) - ((1 + I*Sqrt
[3])*x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6)^(2/3)/(1 + (
-1)^(7/9)*x), x], x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) - ((1 - I*Sqrt[3])*x^(
2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6)^(2/3)/(1 - (-1)^(8/9
)*x), x], x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) - ((1 - I*Sqrt[3])*x^(2/3)*(1
+ x^2)^(1/3)*Defer[Subst][Defer[Int][(1 + x^6)^(2/3)/(1 + (-1)^(8/9)*x), x]
, x, x^(1/3)])/(6*(x^2 + x^4)^(1/3))
```

Rubi steps

$$\int \frac{1+x^6}{\sqrt[3]{x^2+x^4}\left(-1+x^6\right)}\,dx = \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{1+x^6}{x^{2/3}\sqrt[3]{1+x^2}\left(-1+x^6\right)}\,dx}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{\left(1+x^2\right)^{2/3}\left(1-x^2+x^4\right)}{x^{2/3}\left(-1+x^6\right)}\,dx}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{\left(1+x^6\right)^{2/3}\left(1-x^6+x^{12}\right)}{-1+x^{18}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int\left(-\frac{\left(1+x^6\right)^{2/3}}{18(1-x)} - \frac{\left(1+x^6\right)^{2/3}}{18(1+x)} - \frac{\left(1+\sqrt[3]{-1}+(-1)^{2/3}\right)\left(1+x^6\right)^{2/3}}{18\left(1-\sqrt[9]{-1}\,x\right)} - \frac{\left(1+\sqrt[3]{-1}\right.}{1\ldots}\right.\right.}{}$$

$$= -\frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{\left(1+x^6\right)^{2/3}}{1-x}\,dx, x, \sqrt[3]{x}\right)}{6\sqrt[3]{x^2+x^4}} - \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{\left(1+x^6\right)^{2/3}}{1+x}\right.}{6\sqrt[3]{x^2+x^4}}$$

Mathematica [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1+x^6}{\sqrt[3]{x^2+x^4}\left(-1+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^6)/((x^2 + x^4)^(1/3)*(-1 + x^6)),x]
[Out] Integrate[(1 + x^6)/((x^2 + x^4)^(1/3)*(-1 + x^6)), x]
```

IntegrateAlgebraic [A] time = 0.90, size = 288, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+x^2}-x}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+x^2}+x}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}-x}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}+x}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{2}{3}\tanh^{-1}\left(\frac{x}{\sqrt[3]{x^4+x^2}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^4+x^2}}\right)}{3\sqrt[3]{2}} - \frac{1}{3}\tanh^{-1}\left(\frac{x^2+\left(x^4+x^2\right)^{2/3}}{x\sqrt[3]{x^4+x^2}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x^2+\frac{\left(x^4+x^2\right)^{2/3}}{\sqrt[3]{2}}}{x\sqrt[3]{x^4+x^2}}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^6)/((x^2 + x^4)^(1/3)*(-1 + x^6)),x]
[Out] -(ArcTan[(Sqrt[3]*x)/(-x + 2*(x^2 + x^4)^(1/3))]/Sqrt[3]) - ArcTan[(Sqrt[3]
*x)/(x + 2*(x^2 + x^4)^(1/3))]/Sqrt[3] - ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(
x^2 + x^4)^(1/3))]/(2*2^(1/3)*Sqrt[3]) - ArcTan[(Sqrt[3]*x)/(x + 2^(2/3)*(x
^2 + x^4)^(1/3))]/(2*2^(1/3)*Sqrt[3]) - (2*ArcTanh[x/(x^2 + x^4)^(1/3)])/3
- ArcTanh[(2^(1/3)*x)/(x^2 + x^4)^(1/3)]/(3*2^(1/3)) - ArcTanh[(x^2 + (x^2
+ x^4)^(2/3))/(x*(x^2 + x^4)^(1/3))]/3 - ArcTanh[(2^(1/3)*x^2 + (x^2 + x^4)
^(2/3)/2^(1/3))/(x*(x^2 + x^4)^(1/3))]/(6*2^(1/3))
```

fricas [B] time = 7.01, size = 456, normalized size = 1.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^4+x^2)^(1/3)/(x^6-1),x, algorithm="fricas")
[Out] 1/12*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(4*sqrt(6)*2^(2/3)*(-1)^
(2/3)*(x^4 + x^2)^(2/3)*(x^2 + 2*x + 1) + 8*sqrt(6)*(-1)^(1/3)*(x^4 + x^2)^
```

```
(1/3)*(x^3 - 2*x^2 + x) - sqrt(6)*2^(1/3)*(x^5 - 8*x^4 - 2*x^3 - 8*x^2 + x)
)/(x^5 + 8*x^4 - 2*x^3 + 8*x^2 + x)) + 1/12*2^(2/3)*(-1)^(1/3)*log(-(4*2^(1
/3)*(-1)^(2/3)*(x^4 + x^2)^(1/3)*x - 2^(2/3)*(-1)^(1/3)*(x^3 + 2*x^2 + x) +
 4*(x^4 + x^2)^(2/3))/(x^3 - 2*x^2 + x)) - 1/24*2^(2/3)*(-1)^(1/3)*log((2^(
1/3)*(-1)^(2/3)*(x^3 - 2*x^2 + x) + 2*2^(2/3)*(-1)^(1/3)*(x^4 + x^2)^(2/3)
+ 4*(x^4 + x^2)^(1/3)*x)/(x^3 - 2*x^2 + x)) - 1/3*sqrt(3)*arctan(1/3*(4*sqr
t(3)*(x^4 + x^2)^(2/3)*(x^2 + x + 1) - 4*sqrt(3)*(x^4 + x^2)^(1/3)*(x^3 - x
^2 + x) - sqrt(3)*(x^5 - 4*x^4 + x^3 - 4*x^2 + x))/(x^5 + 4*x^4 + x^3 + 4*x
^2 + x)) + 1/3*log((x^3 - x^2 + 2*(x^4 + x^2)^(1/3)*x + x - 2*(x^4 + x^2)^(
2/3))/(x^3 + x^2 + x)) + 1/6*log((x^3 - x^2 + 2*(x^4 + x^2)^(1/3)*x + x - 2
*(x^4 + x^2)^(2/3))/(x^3 - x^2 + x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+1}{\left(x^6-1\right)\left(x^4+x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^4+x^2)^(1/3)/(x^6-1),x, algorithm="giac")
[Out] integrate((x^6 + 1)/((x^6 - 1)*(x^4 + x^2)^(1/3)), x)
```

maple [C] time = 55.67, size = 7357, normalized size = 25.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/(x^4+x^2)^(1/3)/(x^6-1),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+1}{\left(x^6-1\right)\left(x^4+x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^4+x^2)^(1/3)/(x^6-1),x, algorithm="maxima")
[Out] integrate((x^6 + 1)/((x^6 - 1)*(x^4 + x^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6+1}{\left(x^4+x^2\right)^{1/3}\,\left(x^6-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)/((x^2 + x^4)^(1/3)*(x^6 - 1)),x)
[Out] int((x^6 + 1)/((x^2 + x^4)^(1/3)*(x^6 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)\left(x^4-x^2+1\right)}{\sqrt[3]{x^2\left(x^2+1\right)}\,(x-1)\,(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/(x**4+x**2)**(1/3)/(x**6-1),x)
```

```
[Out] Integral((x**2 + 1)*(x**4 - x**2 + 1)/((x**2*(x**2 + 1))**(1/3)*(x - 1)*(x
+ 1)*(x**2 - x + 1)*(x**2 + x + 1)), x)
```

3.2246 $\int \frac{1}{\left(-bx+a^2x^2\right)^{5/2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=288

$$\frac{\sqrt{a^2x^2-bx}\,\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(121339a^{10}x^5-148243a^8bx^4+12416a^6b^2x^3+5248a^4b^3x^2+2688a^2b^4x-43\right.}{16380b^7x^5\left(b-a^2x\right)^2}$$

Rubi [F] time = 4.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(-bx+a^2x^2\right)^{5/2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((-(b*x) + a^2*x^2)^(5/2)*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3/2)),x
]
```

```
[Out] (2*Sqrt[x]*Sqrt[-b + a^2*x]*Defer[Subst][Defer[Int][1/(x^4*(-b + a^2*x^2)^(
5/2)*(a*x^4 + x^2*Sqrt[-(b*x^2) + a^2*x^4])^(3/2)), x], x, Sqrt[x]])/Sqrt[-
(b*x) + a^2*x^2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-bx+a^2x^2\right)^{5/2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx &= \frac{\left(\sqrt{x}\sqrt{-b+a^2x}\right)\int \frac{1}{x^{5/2}\left(-b+a^2x\right)^{5/2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx}{\sqrt{-bx+a^2x^2}}\\ &= \frac{\left(2\sqrt{x}\sqrt{-b+a^2x}\right)\text{Subst}\left(\int \frac{1}{x^4\left(-b+a^2x^2\right)^{5/2}\left(ax^4+x^2\sqrt{-bx^2+a^2x^4}\right)^{3/2}}\,d\right.}{\sqrt{-bx+a^2x^2}}\end{aligned}$$

Mathematica [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx+a^2x^2\right)^{5/2}\left(ax^2+x\sqrt{-bx+a^2x^2}\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((-(b*x) + a^2*x^2)^(5/2)*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3
/2)),x]
```

```
[Out] Integrate[1/((-(b*x) + a^2*x^2)^(5/2)*(a*x^2 + x*Sqrt[-(b*x) + a^2*x^2])^(3
/2)), x]
```

IntegrateAlgebraic [A] time = 8.73, size = 288, normalized size = 1.00

$$\frac{\sqrt{a^2x^2-bx}\,\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(121339a^{10}x^5-148243a^8bx^4+12416a^6b^2x^3+5248a^4b^3x^2+2688a^2b^4x-4368b^5\right)}{16380b^7x^5\left(b-a^2x\right)^2}+\sqrt{x\left(\sqrt{a^2x^2-bx}+ax\right)}\left(\frac{109a^{15/2}\sqrt{\sqrt{a^2x^2-bx}-ax}\,\tan^{-1}\left(\frac{\sqrt{a}\sqrt{\sqrt{a^2x^2-bx}-ax}}{\sqrt{b}}\right)}{4b^{15/2}x}-\frac{283847a^9x^4-229768a^7bx^3-24840a^5b^2x^2-9352a^3b^3x-4872ab^4}{8190b^7x^4\left(b-a^2x\right)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-(b*x) + a^2*x^2)^(5/2)*(a*x^2 + x*Sqrt[-(b*x) + a^2
*x^2])^(3/2)),x]
```

```
[Out] (Sqrt[-(b*x) + a^2*x^2]*(-4368*b^5 + 2688*a^2*b^4*x + 5248*a^4*b^3*x^2 + 12
416*a^6*b^2*x^3 - 148243*a^8*b*x^4 + 121339*a^10*x^5)*Sqrt[x*(a*x + Sqrt[-(
b*x) + a^2*x^2])])/(16380*b^7*x^5*(b - a^2*x)^2) + Sqrt[x*(a*x + Sqrt[-(b*x
) + a^2*x^2])]*(-1/8190*(-4872*a*b^4 - 9352*a^3*b^3*x - 24840*a^5*b^2*x^2 -
 229768*a^7*b*x^3 + 283847*a^9*x^4)/(b^7*x^4*(b - a^2*x)) + (109*a^(15/2)*S
qrt[-(a*x) + Sqrt[-(b*x) + a^2*x^2]]*ArcTan[(Sqrt[a]*Sqrt[-(a*x) + Sqrt[-(b
*x) + a^2*x^2]])/Sqrt[b]])/(4*b^(15/2)*x))
```

fricas [A] time = 0.78, size = 566, normalized size = 1.97

$$\left[\frac{446355\left(a^{11}x^{7}-2a^{9}bx^{6}+a^{7}b^{2}x^{5}\right)\sqrt{a}\log\left(\frac{a^{2}x^{2}+2\sqrt{a^{2}x^{2}-bx}\,ax-bx-2\sqrt{a^{2}x^{2}-bx}\sqrt{ax^{2}+\sqrt{a^{2}x^{2}-bx}\,x}\sqrt{a}}{a^{2}x^{2}-bx}\right)+2\left(567694a^{11}x^{6}-1027230a^{9}bx^{5}+409856a^{7}b^{2}x^{4}+30976a^{5}b^{3}x^{3}+8960a^{3}b^{4}x^{2}+9744ab^{5}x+\left(121339a^{10}x^{5}-148243a^{8}bx^{4}+12416a^{6}b^{2}x^{3}+5248a^{4}b^{3}x^{2}+2688a^{2}b^{4}x-4368b^{5}\right)\sqrt{a^{2}x^{2}-bx}\right)\sqrt{ax^{2}+\sqrt{a^{2}x^{2}-bx}\,x}}{32760\left(a^{4}b^{7}x^{7}-2a^{2}b^{8}x^{6}+b^{9}x^{5}\right)},\ \frac{446355\left(a^{11}x^{7}-2a^{9}bx^{6}+a^{7}b^{2}x^{5}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{ax^{2}+\sqrt{a^{2}x^{2}-bx}\,x}\sqrt{-a}}{ax}\right)+\left(567694a^{11}x^{6}-1027230a^{9}bx^{5}+409856a^{7}b^{2}x^{4}+30976a^{5}b^{3}x^{3}+8960a^{3}b^{4}x^{2}+9744ab^{5}x+\left(121339a^{10}x^{5}-148243a^{8}bx^{4}+12416a^{6}b^{2}x^{3}+5248a^{4}b^{3}x^{2}+2688a^{2}b^{4}x-4368b^{5}\right)\sqrt{a^{2}x^{2}-bx}\right)\sqrt{ax^{2}+\sqrt{a^{2}x^{2}-bx}\,x}}{16380\left(a^{4}b^{7}x^{7}-2a^{2}b^{8}x^{6}+b^{9}x^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b*x)^(5/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algo
rithm="fricas")
```

```
[Out] [1/32760*(446355*(a^11*x^7 - 2*a^9*b*x^6 + a^7*b^2*x^5)*sqrt(a)*log((a^2*x^
2 + 2*sqrt(a^2*x^2 - b*x)*a*x - b*x - 2*sqrt(a^2*x^2 - b*x)*sqrt(a*x^2 + sq
rt(a^2*x^2 - b*x)*x)*sqrt(a))/(a^2*x^2 - b*x)) + 2*(567694*a^11*x^6 - 10272
30*a^9*b*x^5 + 409856*a^7*b^2*x^4 + 30976*a^5*b^3*x^3 + 8960*a^3*b^4*x^2 +
9744*a*b^5*x + (121339*a^10*x^5 - 148243*a^8*b*x^4 + 12416*a^6*b^2*x^3 + 52
48*a^4*b^3*x^2 + 2688*a^2*b^4*x - 4368*b^5)*sqrt(a^2*x^2 - b*x))*sqrt(a*x^2
 + sqrt(a^2*x^2 - b*x)*x))/(a^4*b^7*x^7 - 2*a^2*b^8*x^6 + b^9*x^5), 1/16380
*(446355*(a^11*x^7 - 2*a^9*b*x^6 + a^7*b^2*x^5)*sqrt(-a)*arctan(sqrt(a*x^2
+ sqrt(a^2*x^2 - b*x)*x)*sqrt(-a)/(a*x)) + (567694*a^11*x^6 - 1027230*a^9*b
*x^5 + 409856*a^7*b^2*x^4 + 30976*a^5*b^3*x^3 + 8960*a^3*b^4*x^2 + 9744*a*b
^5*x + (121339*a^10*x^5 - 148243*a^8*b*x^4 + 12416*a^6*b^2*x^3 + 5248*a^4*b
^3*x^2 + 2688*a^2*b^4*x - 4368*b^5)*sqrt(a^2*x^2 - b*x))*sqrt(a*x^2 + sqrt(
a^2*x^2 - b*x)*x))/(a^4*b^7*x^7 - 2*a^2*b^8*x^6 + b^9*x^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}x^{2}-bx\right)^{\frac{5}{2}}\left(ax^{2}+\sqrt{a^{2}x^{2}-bx}\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b*x)^(5/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algo
rithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - b*x)^(5/2)*(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2)),
x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}x^{2}-bx\right)^{\frac{5}{2}}\left(a\,x^{2}+x\sqrt{a^{2}x^{2}-bx}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*x^2-b*x)^(5/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

```
[Out] int(1/(a^2*x^2-b*x)^(5/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^2-bx\right)^{\frac{5}{2}}\left(ax^2+\sqrt{a^2x^2-bx}\,x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b*x)^(5/2)/(a*x^2+x*(a^2*x^2-b*x)^(1/2))^(3/2),x, algo
rithm="maxima")
```

```
[Out] integrate(1/((a^2*x^2 - b*x)^(5/2)*(a*x^2 + sqrt(a^2*x^2 - b*x)*x)^(3/2)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a^2\,x^2-b\,x\right)^{5/2}\,\left(a\,x^2+x\,\sqrt{a^2\,x^2-b\,x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2*x^2 - b*x)^(5/2)*(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2)),x)
```

```
[Out] int(1/((a^2*x^2 - b*x)^(5/2)*(a*x^2 + x*(a^2*x^2 - b*x)^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x\left(ax+\sqrt{a^2x^2-bx}\right)\right)^{\frac{3}{2}}\left(x\left(a^2x-b\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*x**2-b*x)**(5/2)/(a*x**2+x*(a**2*x**2-b*x)**(1/2))**(3/2)
,x)
```

```
[Out] Integral(1/((x*(a*x + sqrt(a**2*x**2 - b*x)))**(3/2)*(x*(a**2*x - b))**(5/2
)), x)
```

3.2247 $\int \frac{(a-2b+x)(-b+x)}{\sqrt[3]{(-a+x)(-b+x)}\left(a^4-b^2d-2\left(2a^3-bd\right)x+\left(6a^2-d\right)x^2-4ax^3+x^4\right)}\,dx$

Optimal. Leaf size=289

$$\frac{\tanh^{-1}\left(\frac{\sqrt[3]{x(-a-b)+ab+x^2}\left(a\sqrt[6]{d}-\sqrt[6]{d}\,x\right)}{a^2+\sqrt[3]{d}\left(x(-a-b)+ab+x^2\right)^{2/3}-2ax+x^2}\right)}{2d^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x(-a-b)+ab+x^2}}{\sqrt[6]{d}\sqrt[3]{x(-a-b)+ab+x^2}+2a-2x}\right)}{2d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x(-a-b)+ab+x^2}}{\sqrt[6]{d}\sqrt[3]{x(-a-b)+ab+x^2}-2a+\ldots}\right)}{2d^{5/6}}$$

Rubi [F] time = 6.62, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a-2b+x)(-b+x)}{\sqrt[3]{(-a+x)(-b+x)}\left(a^4-b^2d-2\left(2a^3-bd\right)x+\left(6a^2-d\right)x^2-4ax^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a - 2*b + x)*(-b + x))/(((-a + x)*(-b + x))^(1/3)*(a^4 - b^2*d - 2*(2
*a^3 - b*d)*x + (6*a^2 - d)*x^2 - 4*a*x^3 + x^4)),x]
```

```
[Out] (3*a*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][Defer[Int][(x*(a - b + x^3)
^(2/3))/(-(a^2*(1 + (b*(-2*a + b))/a^2)*d) - 2*a*(1 - b/a)*d*x^3 - d*x^6 +
x^12), x], x, (-a + x)^(1/3)])/((a - x)*(b - x))^(1/3) + (3*(-a + x)^(1/3)*
(-b + x)^(1/3)*Defer[Subst][Defer[Int][(x^4*(a - b + x^3)^(2/3))/(-(a^2*(1
+ (b*(-2*a + b))/a^2)*d) - 2*a*(1 - b/a)*d*x^3 - d*x^6 + x^12), x], x, (-a
+ x)^(1/3)])/((a - x)*(b - x))^(1/3) - (3*(a - 2*b)*(-a + x)^(1/3)*(-b + x)
^(1/3)*Defer[Subst][Defer[Int][(x*(a - b + x^3)^(2/3))/(a^2*(1 + b^2/a^2)*d
 - 2*b*d*x^3 + 2*a*d*(-b + x^3) + x^6*(d - x^6)), x], x, (-a + x)^(1/3)])/(
(a - x)*(b - x))^(1/3)
```

Rubi steps

$$\int \frac{(a-2b+x)(-b+x)}{\sqrt[3]{(-a+x)(-b+x)}\left(a^4-b^2d-2\left(2a^3-bd\right)x+\left(6a^2-d\right)x^2-4ax^3+x^4\right)}dx = \frac{\left(\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\int \frac{}{\sqrt[3]{-a+x}(\ldots}}{\sqrt[3]{(-a\ldots}}$$

$$= \frac{\left(\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\int\left(\frac{}{\sqrt[3]{-a+x}}\ldots\right.}{}$$

$$= \frac{\left(\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\int \frac{}{\sqrt[3]{-a+x}(\ldots}}{\sqrt[3]{(-a\ldots}}$$

$$= \frac{\left(3\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\mathrm{Subst}\left(\ldots\right.}{}$$

$$= \frac{\left(3\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\mathrm{Subst}\left(\ldots\right.}{}$$

$$= \frac{\left(3\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\mathrm{Subst}\left(\ldots\right.}{}$$

$$= \frac{\left(3\sqrt[3]{-a+x}\sqrt[3]{-b+x}\right)\mathrm{Subst}\left(\ldots\right.}{}$$

Mathematica [F] time = 2.58, size = 0, normalized size = 0.00

$$\int \frac{(a-2b+x)(-b+x)}{\sqrt[3]{(-a+x)(-b+x)}\left(a^4-b^2d-2\left(2a^3-bd\right)x+\left(6a^2-d\right)x^2-4ax^3+x^4\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a - 2*b + x)*(-b + x))/(((-a + x)*(-b + x))^(1/3)*(a^4 - b^2*d
 - 2*(2*a^3 - b*d)*x + (6*a^2 - d)*x^2 - 4*a*x^3 + x^4)),x]
```

```
[Out] Integrate[((a - 2*b + x)*(-b + x))/(((-a + x)*(-b + x))^(1/3)*(a^4 - b^2*d
 - 2*(2*a^3 - b*d)*x + (6*a^2 - d)*x^2 - 4*a*x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 1.24, size = 289, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt[3]{x(-a-b)+ab+x^2}\left(a\sqrt[6]{d}-\sqrt[6]{d}x\right)}{a^2+\sqrt[3]{d}\left(x(-a-b)+ab+x^2\right)^{2/3}-2ax+x^2}\right)}{2d^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x(-a-b)+ab+x^2}}{\sqrt[6]{d}\sqrt[3]{x(-a-b)+ab+x^2}+2a-2x}\right)}{2d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x(-a-b)+ab+x^2}}{\sqrt[6]{d}\sqrt[3]{x(-a-b)+ab+x^2}-2a+2x}\right)}{2d^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[3]{x(-a-b)+ab+x^2}}{a-x}\right)}{d^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a - 2*b + x)*(-b + x))/(((-a + x)*(-b + x))^(1/3)*(a^4
 - b^2*d - 2*(2*a^3 - b*d)*x + (6*a^2 - d)*x^2 - 4*a*x^3 + x^4)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/6)*(a*b + (-a - b)*x + x^2)^(1/3))/(2*a
- 2*x + d^(1/6)*(a*b + (-a - b)*x + x^2)^(1/3))])/d^(5/6) + (Sqrt[3]*ArcTan
[(Sqrt[3]*d^(1/6)*(a*b + (-a - b)*x + x^2)^(1/3))/(-2*a + 2*x + d^(1/6)*(a*
b + (-a - b)*x + x^2)^(1/3))])/(2*d^(5/6)) + ArcTanh[(d^(1/6)*(a*b + (-a -
b)*x + x^2)^(1/3))/(a - x)]/d^(5/6) + ArcTanh[((a*d^(1/6) - d^(1/6)*x)*(a*b
```

```
 + (-a - b)*x + x^2)^(1/3))/(a^2 - 2*a*x + x^2 + d^(1/3)*(a*b + (-a - b)*x
+ x^2)^(2/3))]/(2*d^(5/6))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)*(-b+x)/((-a+x)*(-b+x))^(1/3)/(a^4-b^2*d-2*(2*a^3-b*d)*x
+(6*a^2-d)*x^2-4*a*x^3+x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(a-2\,b+x)(b-x)}{\left(a^4-4\,ax^3+x^4-b^2d+\left(6\,a^2-d\right)x^2-2\left(2\,a^3-bd\right)x\right)((a-x)(b-x))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)*(-b+x)/((-a+x)*(-b+x))^(1/3)/(a^4-b^2*d-2*(2*a^3-b*d)*x
+(6*a^2-d)*x^2-4*a*x^3+x^4),x, algorithm="giac")
```

```
[Out] integrate(-(a - 2*b + x)*(b - x)/((a^4 - 4*a*x^3 + x^4 - b^2*d + (6*a^2 - d
)*x^2 - 2*(2*a^3 - b*d)*x)*((a - x)*(b - x))^(1/3)), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(a-2b+x)\,(-b+x)}{((-a+x)\,(-b+x))^{\frac{1}{3}}\left(a^4-b^2d-2\left(2a^3-bd\right)x+\left(6a^2-d\right)x^2-4a\,x^3+x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-2*b+x)*(-b+x)/((-a+x)*(-b+x))^(1/3)/(a^4-b^2*d-2*(2*a^3-b*d)*x+(6*a^
2-d)*x^2-4*a*x^3+x^4),x)
```

```
[Out] int((a-2*b+x)*(-b+x)/((-a+x)*(-b+x))^(1/3)/(a^4-b^2*d-2*(2*a^3-b*d)*x+(6*a^
2-d)*x^2-4*a*x^3+x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(a-2\,b+x)(b-x)}{\left(a^4-4\,ax^3+x^4-b^2d+\left(6\,a^2-d\right)x^2-2\left(2\,a^3-bd\right)x\right)((a-x)(b-x))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)*(-b+x)/((-a+x)*(-b+x))^(1/3)/(a^4-b^2*d-2*(2*a^3-b*d)*x
+(6*a^2-d)*x^2-4*a*x^3+x^4),x, algorithm="maxima")
```

```
[Out] -integrate((a - 2*b + x)*(b - x)/((a^4 - 4*a*x^3 + x^4 - b^2*d + (6*a^2 - d
)*x^2 - 2*(2*a^3 - b*d)*x)*((a - x)*(b - x))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b-x)\,(a-2\,b+x)}{((a-x)\,(b-x))^{1/3}\,\left(x^2\,\left(d-6\,a^2\right)-2\,x\,\left(b\,d-2\,a^3\right)+b^2\,d+4\,a\,x^3-a^4-x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b - x)*(a - 2*b + x))/(((a - x)*(b - x))^(1/3)*(x^2*(d - 6*a^2) - 2*x
*(b*d - 2*a^3) + b^2*d + 4*a*x^3 - a^4 - x^4)),x)
```

```
[Out] int(((b - x)*(a - 2*b + x))/(((a - x)*(b - x))^(1/3)*(x^2*(d - 6*a^2) - 2*x
*(b*d - 2*a^3) + b^2*d + 4*a*x^3 - a^4 - x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)*(-b+x)/((-a+x)*(-b+x))**(1/3)/(a**4-b**2*d-2*(2*a**3-b*
d)*x+(6*a**2-d)*x**2-4*a*x**3+x**4),x)
```

```
[Out] Timed out
```

3.2248 $\int \frac{(-1+x)x(1+(-2+k)x)}{\sqrt[3]{(1-x)x(1-kx)}\left(1-4kx+\left(-b+6k^2\right)x^2+\left(2b-4k^3\right)x^3+\left(-b+k^4\right)x^4\right)}\,dx$

Optimal. Leaf size=289

$$\frac{\tanh^{-1}\left(\frac{\sqrt[3]{kx^3+(-k-1)x^2+x}\left(\sqrt[6]{b}\,kx-\sqrt[6]{b}\right)}{\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+k^2x^2-2kx+1}\right)}{2b^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}-2kx+2}\right)}{2b^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}-\cdots}\right)}{2b^{5/6}}$$

Rubi [F] time = 12.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+x)x(1+(-2+k)x)}{\sqrt[3]{(1-x)x(1-kx)}\left(1-4kx+\left(-b+6k^2\right)x^2+\left(2b-4k^3\right)x^3+\left(-b+k^4\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + x)*x*(1 + (-2 + k)*x))/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - 4*k*x +
 (-b + 6*k^2)*x^2 + (2*b - 4*k^3)*x^3 + (-b + k^4)*x^4)),x]
```

```
[Out] (-3*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][(x^4*(1 -
 x^3)^(2/3))/((1 - k*x^3)^(1/3)*(1 - 4*k*x^3 - b*(1 - (6*k^2)/b)*x^6 + 2*b*
(1 - (2*k^3)/b)*x^9 - b*(1 - k^4/b)*x^12)), x], x, x^(1/3)])/((1 - x)*x*(1
- k*x))^(1/3) + (3*(2 - k)*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subs
t][Defer[Int][(x^7*(1 - x^3)^(2/3))/((1 - k*x^3)^(1/3)*(1 - 4*k*x^3 - b*(1
- (6*k^2)/b)*x^6 + 2*b*(1 - (2*k^3)/b)*x^9 - b*(1 - k^4/b)*x^12)), x], x, x
^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\int \frac{(-1+x)x(1+(-2+k)x)}{\sqrt[3]{(1-x)x(1-kx)}\left(1-4kx+\left(-b+6k^2\right)x^2+\left(2b-4k^3\right)x^3+\left(-b+k^4\right)x^4\right)}\,dx=\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \cdots}{\cdots}$$

$$=-\frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \cdots}{\cdots}$$

$$=-\frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\cdots}{\cdots}$$

$$=-\frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\cdots}{\cdots}$$

$$=-\frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\cdots}{\cdots}$$

Mathematica [F] time = 4.86, size = 0, normalized size = 0.00

$$\int \frac{(-1+x)x(1+(-2+k)x)}{\sqrt[3]{(1-x)x(1-kx)}\left(1-4kx+\left(-b+6k^2\right)x^2+\left(2b-4k^3\right)x^3+\left(-b+k^4\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x)*x*(1 + (-2 + k)*x))/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - 4
*k*x + (-b + 6*k^2)*x^2 + (2*b - 4*k^3)*x^3 + (-b + k^4)*x^4)),x]
```

```
[Out] Integrate[((-1 + x)*x*(1 + (-2 + k)*x))/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - 4
*k*x + (-b + 6*k^2)*x^2 + (2*b - 4*k^3)*x^3 + (-b + k^4)*x^4)), x]
```

IntegrateAlgebraic [A] time = 3.87, size = 289, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt[3]{kx^3+(-k-1)x^2+x}\left(\sqrt[6]{b}\,kx-\sqrt[6]{b}\right)}{\sqrt[3]{b}\left(kx^3+(-k-1)x^2+x\right)^{2/3}+k^2x^2-2kx+1}\right)}{2b^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}-2kx+2}\right)}{2b^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}+2kx-2}\right)}{2b^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[3]{kx^3+(-k-1)x^2+x}}{kx-1}\right)}{b^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x)*x*(1 + (-2 + k)*x))/(((1 - x)*x*(1 - k*x))^(1/
3)*(1 - 4*k*x + (-b + 6*k^2)*x^2 + (2*b - 4*k^3)*x^3 + (-b + k^4)*x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/6)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(2 - 2*k
*x + b^(1/6)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))]])/(2*b^(5/6)) - (Sqrt[3]*Arc
Tan[(Sqrt[3]*b^(1/6)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(-2 + 2*k*x + b^(1/6
)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))]])/(2*b^(5/6)) + ArcTanh[(b^(1/6)*(x + (
-1 - k)*x^2 + k*x^3)^(1/3))/(-1 + k*x)]/b^(5/6) + ArcTanh[((-b^(1/6) + b^(1
/6)*k*x)*(x + (-1 - k)*x^2 + k*x^3)^(1/3))/(1 - 2*k*x + k^2*x^2 + b^(1/3)*(
x + (-1 - k)*x^2 + k*x^3)^(2/3))]/(2*b^(5/6))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*x*(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-4*k*x+(6*k^2-b)
*x^2+(-4*k^3+2*b)*x^3+(k^4-b)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((k-2)x+1)(x-1)x}{\left((k^4-b)x^4-2\left(2k^3-b\right)x^3+\left(6k^2-b\right)x^2-4kx+1\right)((kx-1)(x-1)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*x*(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-4*k*x+(6*k^2-b)
*x^2+(-4*k^3+2*b)*x^3+(k^4-b)*x^4),x, algorithm="giac")
```

```
[Out] integrate(((k - 2)*x + 1)*(x - 1)*x/(((k^4 - b)*x^4 - 2*(2*k^3 - b)*x^3 + (
6*k^2 - b)*x^2 - 4*k*x + 1)*((k*x - 1)*(x - 1)*x)^(1/3)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(-1+x)\,x\,(1+(-2+k)\,x)}{((1-x)\,x\,(-kx+1))^{\frac{1}{3}}\left(1-4kx+\left(6k^2-b\right)x^2+\left(-4k^3+2b\right)x^3+\left(k^4-b\right)x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)*x*(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-4*k*x+(6*k^2-b)*x^2+(
-4*k^3+2*b)*x^3+(k^4-b)*x^4),x)
```

```
[Out] int((-1+x)*x*(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-4*k*x+(6*k^2-b)*x^2+(
-4*k^3+2*b)*x^3+(k^4-b)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((k-2)x+1)(x-1)x}{\left(\left(k^4-b\right)x^4-2\left(2\,k^3-b\right)x^3+\left(6\,k^2-b\right)x^2-4\,kx+1\right)((kx-1)(x-1)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*x*(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-4*k*x+(6*k^2-b)
*x^2+(-4*k^3+2*b)*x^3+(k^4-b)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(((k - 2)*x + 1)*(x - 1)*x/(((k^4 - b)*x^4 - 2*(2*k^3 - b)*x^3 + (
6*k^2 - b)*x^2 - 4*k*x + 1)*((k*x - 1)*(x - 1)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x\,\left(x\,\left(k-2\right)+1\right)\,\left(x-1\right)}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/3}\,\left(\left(b-k^4\right)\,x^4+\left(4\,k^3-2\,b\right)\,x^3+\left(b-6\,k^2\right)\,x^2+4\,k\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(x*(k - 2) + 1)*(x - 1))/((x*(k*x - 1)*(x - 1))^(1/3)*(x^4*(b - k^4
) + x^2*(b - 6*k^2) + 4*k*x - x^3*(2*b - 4*k^3) - 1)),x)
```

```
[Out] -int((x*(x*(k - 2) + 1)*(x - 1))/((x*(k*x - 1)*(x - 1))^(1/3)*(x^4*(b - k^4
) + x^2*(b - 6*k^2) + 4*k*x - x^3*(2*b - 4*k^3) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*x*(1+(-2+k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(1-4*k*x+(6*k**2-
b)*x**2+(-4*k**3+2*b)*x**3+(k**4-b)*x**4),x)
```

```
[Out] Timed out
```

3.2249 $\int \frac{\left(-1+x^3\right)^{2/3}\left(4+x^6\right)}{x^6\left(-4+x^6\right)}\,dx$

Optimal. Leaf size=289

$$\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}-x\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3-1}-3x\right)}{2\ 2^{2/3}\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}+x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}+\sqrt[3]{3}x}\right)}{2\ 2^{2/3}}+\frac{\left(x^3-1\right)}{5}$$

Rubi [C] time = 0.34, antiderivative size = 109, normalized size of antiderivative = 0.38, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6725, 264, 430, 429}

$$-\frac{x\left(x^3-1\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-\frac{x^3}{2}\right)}{4\left(1-x^3\right)^{2/3}}-\frac{x\left(x^3-1\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,\frac{x^3}{2}\right)}{4\left(1-x^3\right)^{2/3}}-\frac{\left(x^3-1\right)^{5/3}}{5x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + x^3)^(2/3)*(4 + x^6))/(x^6*(-4 + x^6)),x]
```

```
[Out] -1/5*(-1 + x^3)^(5/3)/x^5 - (x*(-1 + x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3,
 x^3, -1/2*x^3])/(4*(1 - x^3)^(2/3)) - (x*(-1 + x^3)^(2/3)*AppellF1[1/3, -2
/3, 1, 4/3, x^3, x^3/2])/(4*(1 - x^3)^(2/3))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-1+x^3\right)^{2/3}\left(4+x^6\right)}{x^6\left(-4+x^6\right)}\,dx &= \int \left(-\frac{\left(-1+x^3\right)^{2/3}}{x^6}+\frac{\left(-1+x^3\right)^{2/3}}{2\left(-2+x^3\right)}-\frac{\left(-1+x^3\right)^{2/3}}{2\left(2+x^3\right)}\right)dx\\ &= \frac{1}{2}\int \frac{\left(-1+x^3\right)^{2/3}}{-2+x^3}\,dx-\frac{1}{2}\int \frac{\left(-1+x^3\right)^{2/3}}{2+x^3}\,dx-\int \frac{\left(-1+x^3\right)^{2/3}}{x^6}\,dx\\ &= -\frac{\left(-1+x^3\right)^{5/3}}{5x^5}+\frac{\left(-1+x^3\right)^{2/3}\int \frac{\left(1-x^3\right)^{2/3}}{-2+x^3}\,dx}{2\left(1-x^3\right)^{2/3}}-\frac{\left(-1+x^3\right)^{2/3}\int \frac{\left(1-x^3\right)^{2/3}}{2+x^3}\,dx}{2\left(1-x^3\right)^{2/3}}\\ &= -\frac{\left(-1+x^3\right)^{5/3}}{5x^5}-\frac{x\left(-1+x^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-\frac{x^3}{2}\right)}{4\left(1-x^3\right)^{2/3}}-\frac{x\left(-1+x^3\right)^{2/3}F_1\left(\frac{1}{3};-\right.}{4\left(1-x^3\right)^{2}}\end{aligned}$$

Mathematica [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+x^3\right)^{2/3}\left(4+x^6\right)}{x^6\left(-4+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + x^3)^(2/3)*(4 + x^6))/(x^6*(-4 + x^6)),x]
```

```
[Out] Integrate[((-1 + x^3)^(2/3)*(4 + x^6))/(x^6*(-4 + x^6)), x]
```

IntegrateAlgebraic [A] time = 0.74, size = 289, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}-x\right)}{6\,2^{2/3}}-\frac{\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3-1}-3x\right)}{2\,2^{2/3}\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}+x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\sqrt[6]{3}\tan^{-1}\left(\frac{3^{5/6}x}{2\sqrt[3]{2}\sqrt[3]{x^3-1}+\sqrt[3]{3}x}\right)}{2\,2^{2/3}}+\frac{\left(x^3-1\right)^{2/3}\left(1-x^3\right)}{5x^5}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{x^3-1}x+2^{2/3}\left(x^3-1\right)^{2/3}+x^2\right)}{12\,2^{2/3}}+\frac{\log\left(\sqrt[3]{2}\,3^{2/3}\sqrt[3]{x^3-1}x+2^{2/3}\sqrt[3]{3}\left(x^3-1\right)^{2/3}+3x^2\right)}{4\,2^{2/3}\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + x^3)^(2/3)*(4 + x^6))/(x^6*(-4 + x^6)),x]
```

```
[Out] ((1 - x^3)*(-1 + x^3)^(2/3))/(5*x^5) - ArcTan[(Sqrt[3]*x)/(x + 2*2^(1/3)*(-
1 + x^3)^(1/3))]/(2*2^(2/3)*Sqrt[3]) + (3^(1/6)*ArcTan[(3^(5/6)*x)/(3^(1/3)
*x + 2*2^(1/3)*(-1 + x^3)^(1/3))])/(2*2^(2/3)) + Log[-x + 2^(1/3)*(-1 + x^3
)^(1/3)]/(6*2^(2/3)) - Log[-3*x + 2^(1/3)*3^(2/3)*(-1 + x^3)^(1/3)]/(2*2^(2
/3)*3^(1/3)) - Log[x^2 + 2^(1/3)*x*(-1 + x^3)^(1/3) + 2^(2/3)*(-1 + x^3)^(2
/3)]/(12*2^(2/3)) + Log[3*x^2 + 2^(1/3)*3^(2/3)*x*(-1 + x^3)^(1/3) + 2^(2/3
)*3^(1/3)*(-1 + x^3)^(2/3)]/(4*2^(2/3)*3^(1/3))
```

fricas [B] time = 13.89, size = 534, normalized size = 1.85

$$\frac{10\cdot 12^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^5\log\left(-\frac{18\cdot 12^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^3-1\right)^{\frac{1}{3}}x^2+12^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3+2\right)-36\left(x^3-1\right)^{\frac{2}{3}}x}{x^3+2}\right)-5\cdot 12^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^5\log\left(-\frac{6\cdot 12^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(4x^4-x\right)\left(x^3-1\right)^{\frac{2}{3}}-12^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(55x^6-50x^3+4\right)-18\left(7x^5-4x^2\right)\left(x^3-1\right)^{\frac{1}{3}}}{x^6+4x^3+4}\right)+20\cdot 4^{\frac{1}{6}}\sqrt{3}x^5\arctan\left(\frac{4^{\frac{1}{6}}\left(12\cdot 4^{\frac{2}{3}}\sqrt{3}\left(2x^7-5x^4+2x\right)\left(x^3-1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\sqrt{3}\left(91x^9-168x^6+84x^3-8\right)+12\sqrt{3}\left(19x^8-22x^5+4x^2\right)\left(x^3-1\right)^{\frac{1}{3}}\right)}{6\left(53x^9-48x^6-12x^3+8\right)}\right)+10\cdot 4^{\frac{2}{3}}x^5\log\left(\frac{6\cdot 4^{\frac{1}{3}}\left(x^3-1\right)^{\frac{1}{3}}x^2+4^{\frac{2}{3}}\left(x^3-2\right)-12\left(x^3-1\right)^{\frac{2}{3}}x}{x^3-2}\right)-5\cdot 4^{\frac{2}{3}}x^5\log\left(\frac{6\cdot 4^{\frac{2}{3}}\left(2x^4-x\right)\left(x^3-1\right)^{\frac{2}{3}}+4^{\frac{1}{3}}\left(19x^6-22x^3+4\right)+6\left(5x^5-4x^2\right)\left(x^3-1\right)^{\frac{1}{3}}}{x^6-4x^3+4}\right)-60\cdot 12^{\frac{1}{6}}(-1)^{\frac{1}{3}}x^5\arctan\left(\frac{12^{\frac{1}{6}}\left(12\cdot 12^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(4x^7+7x^4-2x\right)\left(x^3-1\right)^{\frac{2}{3}}+36(-1)^{\frac{1}{3}}\left(55x^8-50x^5+4x^2\right)\left(x^3-1\right)^{\frac{1}{3}}-12^{\frac{1}{3}}\left(377x^9-600x^6+204x^3-8\right)\right)}{6\left(487x^9-480x^6+12x^3+8\right)}\right)-144\left(x^3-1\right)^{\frac{5}{3}}}{720x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+4)/x^6/(x^6-4),x, algorithm="fricas")
```

```
[Out] 1/720*(10*12^(2/3)*(-1)^(1/3)*x^5*log(-(18*12^(1/3)*(-1)^(2/3)*(x^3 - 1)^(1
/3)*x^2 + 12^(2/3)*(-1)^(1/3)*(x^3 + 2) - 36*(x^3 - 1)^(2/3)*x)/(x^3 + 2))
- 5*12^(2/3)*(-1)^(1/3)*x^5*log(-(6*12^(2/3)*(-1)^(1/3)*(4*x^4 - x)*(x^3 -
1)^(2/3) - 12^(1/3)*(-1)^(2/3)*(55*x^6 - 50*x^3 + 4) - 18*(7*x^5 - 4*x^2)*(
x^3 - 1)^(1/3))/(x^6 + 4*x^3 + 4)) + 20*4^(1/6)*sqrt(3)*x^5*arctan(1/6*4^(1
/6)*(12*4^(2/3)*sqrt(3)*(2*x^7 - 5*x^4 + 2*x)*(x^3 - 1)^(2/3) + 4^(1/3)*sqr
t(3)*(91*x^9 - 168*x^6 + 84*x^3 - 8) + 12*sqrt(3)*(19*x^8 - 22*x^5 + 4*x^2)
*(x^3 - 1)^(1/3))/(53*x^9 - 48*x^6 - 12*x^3 + 8)) + 10*4^(2/3)*x^5*log((6*4
```

```
^(1/3)*(x^3 - 1)^(1/3)*x^2 + 4^(2/3)*(x^3 - 2) - 12*(x^3 - 1)^(2/3)*x)/(x^3
 - 2)) - 5*4^(2/3)*x^5*log((6*4^(2/3)*(2*x^4 - x)*(x^3 - 1)^(2/3) + 4^(1/3)
*(19*x^6 - 22*x^3 + 4) + 6*(5*x^5 - 4*x^2)*(x^3 - 1)^(1/3))/(x^6 - 4*x^3 +
4)) - 60*12^(1/6)*(-1)^(1/3)*x^5*arctan(1/6*12^(1/6)*(12*12^(2/3)*(-1)^(2/3
)*(4*x^7 + 7*x^4 - 2*x)*(x^3 - 1)^(2/3) + 36*(-1)^(1/3)*(55*x^8 - 50*x^5 +
4*x^2)*(x^3 - 1)^(1/3) - 12^(1/3)*(377*x^9 - 600*x^6 + 204*x^3 - 8))/(487*x
^9 - 480*x^6 + 12*x^3 + 8)) - 144*(x^3 - 1)^(5/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^6+4)(x^3-1)^{\frac{2}{3}}}{(x^6-4)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+4)/x^6/(x^6-4),x, algorithm="giac")
[Out] integrate((x^6 + 4)*(x^3 - 1)^(2/3)/((x^6 - 4)*x^6), x)
```

maple [F] time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{(x^3-1)^{\frac{2}{3}}(x^6+4)}{x^6(x^6-4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)^(2/3)*(x^6+4)/x^6/(x^6-4),x)
[Out] int((x^3-1)^(2/3)*(x^6+4)/x^6/(x^6-4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^6+4)(x^3-1)^{\frac{2}{3}}}{(x^6-4)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)^(2/3)*(x^6+4)/x^6/(x^6-4),x, algorithm="maxima")
[Out] integrate((x^6 + 4)*(x^3 - 1)^(2/3)/((x^6 - 4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x^3-1)^{2/3}(x^6+4)}{x^6(x^6-4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - 1)^(2/3)*(x^6 + 4))/(x^6*(x^6 - 4)),x)
[Out] int(((x^3 - 1)^(2/3)*(x^6 + 4))/(x^6*(x^6 - 4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((x-1)(x^2+x+1))^{\frac{2}{3}}(x^6+4)}{x^6(x^3-2)(x^3+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)**(2/3)*(x**6+4)/x**6/(x**6-4),x)
```

```
[Out] Integral(((x - 1)*(x**2 + x + 1))**(2/3)*(x**6 + 4)/(x**6*(x**3 - 2)*(x**3
+ 2)), x)
```

3.2250 $\int \frac{\sqrt[6]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}{\sqrt{-b+a^2x^2}}\,dx$

Optimal. Leaf size=289

$$\frac{24\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{a}+\frac{4\sqrt{3}\sqrt[6]{c}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{3}\sqrt[6]{c}}\right)}{a}-\frac{4\sqrt{3}\sqrt[6]{c}\tan^{-1}\left(\frac{2\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{3}\sqrt[6]{c}}+\frac{1}{\sqrt{3}}\right)}{a}$$

Rubi [F] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[6]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}{\sqrt{-b+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6)/Sqrt[-b + a^2*x^2],x]
```

```
[Out] Defer[Int][(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6)/Sqrt[-b + a^2*x^2],
 x]
```

Rubi steps

$$\int \frac{\sqrt[6]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}{\sqrt{-b+a^2x^2}}\,dx=\int \frac{\sqrt[6]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}{\sqrt{-b+a^2x^2}}\,dx$$

Mathematica [C] time = 0.18, size = 73, normalized size = 0.25

$$-\frac{24\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}\left({}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\frac{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}{c}\right)-1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6)/Sqrt[-b + a^2*x^2],x
]
```

```
[Out] (-24*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6)*(-1 + Hypergeometric2F1[1
/6, 1, 7/6, (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))/c]))/a
```

IntegrateAlgebraic [A] time = 1.30, size = 289, normalized size = 1.00

$$\frac{24\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{a}+\frac{4\sqrt{3}\sqrt[6]{c}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{3}\sqrt[6]{c}}\right)}{a}-\frac{4\sqrt{3}\sqrt[6]{c}\tan^{-1}\left(\frac{2\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{3}\sqrt[6]{c}}+\frac{1}{\sqrt{3}}\right)}{a}-\frac{8\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[6]{c}}\right)}{a}-\frac{4\sqrt[6]{c}\tanh^{-1}\left(\frac{\frac{\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[6]{c}}+\sqrt[6]{c}}{\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6)/Sqrt[-b + a
^2*x^2],x]
```

```
[Out] (24*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6))/a + (4*Sqrt[3]*c^(1/6)*Ar
cTan[1/Sqrt[3] - (2*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6))/(Sqrt[3]*
c^(1/6))]])/a - (4*Sqrt[3]*c^(1/6)*ArcTan[1/Sqrt[3] + (2*(c + (a*x + Sqrt[-b
 + a^2*x^2])^(1/4))^(1/6))/(Sqrt[3]*c^(1/6))]])/a - (8*c^(1/6)*ArcTanh[(c +
(a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6)/c^(1/6)]])/a - (4*c^(1/6)*ArcTanh[(c
^(1/6) + (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)/c^(1/6))/(c + (a*x +
Sqrt[-b + a^2*x^2])^(1/4))^(1/6)]])/a
```

fricas [B] time = 2.04, size = 584, normalized size = 2.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/6)/(a^2*x^2-b)^(1/2),x, algo
rithm="fricas")
```

```
[Out] 2*(4*sqrt(3)*a*(c/a^6)^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^2*(c/a^6)^(1/3) +
 a*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/6)*(c/a^6)^(1/6) + (c + (a*x +
sqrt(a^2*x^2 - b))^(1/4))^(1/3))*a^5*(c/a^6)^(5/6) - 2*sqrt(3)*a^5*(c + (a*
x + sqrt(a^2*x^2 - b))^(1/4))^(1/6)*(c/a^6)^(5/6) - sqrt(3)*c)/c) + 4*sqrt(
3)*a*(c/a^6)^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^2*(c/a^6)^(1/3) - a*(c + (a
*x + sqrt(a^2*x^2 - b))^(1/4))^(1/6)*(c/a^6)^(1/6) + (c + (a*x + sqrt(a^2*x
^2 - b))^(1/4))^(1/3))*a^5*(c/a^6)^(5/6) - 2*sqrt(3)*a^5*(c + (a*x + sqrt(a
^2*x^2 - b))^(1/4))^(1/6)*(c/a^6)^(5/6) + sqrt(3)*c)/c) - a*(c/a^6)^(1/6)*l
og(64*a^2*(c/a^6)^(1/3) + 64*a*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/6)*
(c/a^6)^(1/6) + 64*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)) + a*(c/a^6)
^(1/6)*log(64*a^2*(c/a^6)^(1/3) - 64*a*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4)
)^(1/6)*(c/a^6)^(1/6) + 64*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)) - 2
*a*(c/a^6)^(1/6)*log(4*a*(c/a^6)^(1/6) + 4*(c + (a*x + sqrt(a^2*x^2 - b))^(
1/4))^(1/6)) + 2*a*(c/a^6)^(1/6)*log(-4*a*(c/a^6)^(1/6) + 4*(c + (a*x + sqr
t(a^2*x^2 - b))^(1/4))^(1/6)) + 12*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1
/6))/a
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/6)/(a^2*x^2-b)^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\left(c+\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{6}}}{\sqrt{a^{2}x^{2}-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/6)/(a^2*x^2-b)^(1/2),x)
```

```
[Out] int((c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/6)/(a^2*x^2-b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{6}}}{\sqrt{a^{2}x^{2}-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/6)/(a^2*x^2-b)^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/6)/sqrt(a^2*x^2 - b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c+\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/4}\right)^{1/6}}{\sqrt{a^2\,x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(1/6)/(a^2*x^2 - b)^(1/2),x)
```

```
[Out] int((c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(1/6)/(a^2*x^2 - b)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}}{\sqrt{a^2x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a**2*x**2-b)**(1/2))**(1/4))**(1/6)/(a**2*x**2-b)**(1/2)
,x)
```

```
[Out] Integral((c + (a*x + sqrt(a**2*x**2 - b))**(1/4))**(1/6)/sqrt(a**2*x**2 - b
), x)
```

3.2251 $$\int \frac{(-b+x)\left(ab-2bx+x^2\right)}{\left(x(-a+x)(-b+x)^2\right)^{2/3}\left(bd-(a+d)x+x^2\right)}\,dx$$

Optimal. Leaf size=290

$$\frac{\log\left(\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}+b\sqrt[3]{d}-\sqrt[3]{d}\,x\right)}{d^{2/3}}-\frac{\log\left(\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+\right.}{}$$

Rubi [F] time = 6.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-b+x)\left(ab-2bx+x^2\right)}{\left(x(-a+x)(-b+x)^2\right)^{2/3}\left(bd-(a+d)x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-b + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2)^(2/3)*(b*d - (a
 + d)*x + x^2)),x]
```

```
[Out] (-3*(b - x)*x*(1 - x/a)^(2/3)*(1 - x/b)^(1/3)*AppellF1[1/3, 2/3, 1/3, 4/3,
x/a, x/b])/(-((a - x)*(b - x)^2*x))^(2/3) + ((a - 2*b + d + Sqrt[a^2 + 2*a*
d - 4*b*d + d^2])*x^(2/3)*(-a + x)^(2/3)*(-b + x)^(4/3)*Defer[Int][1/(x^(2/
3)*(-a + x)^(2/3)*(-b + x)^(1/3)*(-a - d - Sqrt[a^2 + 2*a*d - 4*b*d + d^2]
+ 2*x)), x])/(-((a - x)*(b - x)^2*x))^(2/3) + ((a - 2*b + d - Sqrt[a^2 + 2*
a*d - 4*b*d + d^2])*x^(2/3)*(-a + x)^(2/3)*(-b + x)^(4/3)*Defer[Int][1/(x^(
2/3)*(-a + x)^(2/3)*(-b + x)^(1/3)*(-a - d + Sqrt[a^2 + 2*a*d - 4*b*d + d^2
] + 2*x)), x])/(-((a - x)*(b - x)^2*x))^(2/3)
```

Rubi steps

$$\int \frac{(-b+x)\left(ab-2bx+x^2\right)}{\left(x(-a+x)(-b+x)^2\right)^{2/3}\left(bd-(a+d)x+x^2\right)}\,dx=\frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\int\frac{ab-2bx+x^2}{x^{2/3}(-a+x)^{2/3}\sqrt[3]{-b+x}\left(bd-(a+d)x+\right.}}{\left(x(-a+x)(-b+x)^2\right)^{2/3}}$$

$$=\frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\int\left(\frac{1}{x^{2/3}(-a+x)^{2/3}\sqrt[3]{-b+x}}+\frac{}{x^{2/3}(-a}\right.}{\left(x(-a+x)(-b+x)^2\right)^{2/3}}$$

$$=\frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\int\frac{1}{x^{2/3}(-a+x)^{2/3}\sqrt[3]{-b+x}}\,dx}{\left(x(-a+x)(-b+x)^2\right)^{2/3}}+\frac{\left(x^{2/}\right.}{}$$

$$=\frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\int\left(\frac{a-2b+d+\sqrt{a^2+2ad-}}{x^{2/3}(-a+x)^{2/3}\sqrt[3]{-b+x}\left(-a-d-\sqrt{a^2}\right.}\right.}{\left(x(-a+\right.}$$

$$=\frac{\left(\left(a-2b+d-\sqrt{a^2+2ad-4bd+d^2}\right)x^{2/3}(-a+x)^{2/3}(-\right.}{\left(x(-a+x)(-\right.}$$

$$=-\frac{3(b-x)x\left(1-\frac{x}{a}\right)^{2/3}\sqrt[3]{1-\frac{x}{b}}\,F_1\left(\frac{1}{3};\frac{2}{3},\frac{1}{3};\frac{4}{3};\frac{x}{a},\frac{x}{b}\right)}{\left(-\left((a-x)(b-x)^2x\right)\right)^{2/3}}+\frac{\left(\left(a-\right.\right.}{}$$

Mathematica [F] time = 7.30, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)\left(ab-2bx+x^2\right)}{\left(x(-a+x)(-b+x)^2\right)^{2/3}\left(bd-(a+d)x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-b + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2)^(2/3)*(b*
d - (a + d)*x + x^2)),x]
```

```
[Out] Integrate[((-b + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2)^(2/3)*(b*
d - (a + d)*x + x^2)), x]
```

IntegrateAlgebraic [A] time = 3.50, size = 290, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}+b\sqrt[3]{d}-\sqrt[3]{d}\,x\right)}{d^{2/3}}-\frac{\log\left(\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}\left(\sqrt[3]{d}\,x-b\sqrt[3]{d}\right)+\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}+b^2d^{2/3}-2bd^{2/3}x+d^{2/3}x^2\right)}{2d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}b\sqrt[3]{d}-\sqrt{3}\sqrt[3]{d}\,x}{-2\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}+b\sqrt[3]{d}-\sqrt[3]{d}\,x}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2)^
(2/3)*(b*d - (a + d)*x + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*b*d^(1/3) - Sqrt[3]*d^(1/3)*x)/(b*d^(1/3) - d^(1/3
)*x - 2*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3))]])/d^
(2/3) + Log[b*d^(1/3) - d^(1/3)*x + (-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a -
 2*b)*x^3 + x^4)^(1/3)]/d^(2/3) - Log[b^2*d^(2/3) - 2*b*d^(2/3)*x + d^(2/3)
*x^2 + (-(b*d^(1/3)) + d^(1/3)*x)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2
*b)*x^3 + x^4)^(1/3) + (-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x
^4)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(2/3)/(b*d-(a+d)*x+x^2
),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(ab-2\,bx+x^2\right)(b-x)}{\left(-(a-x)(b-x)^2x\right)^{\frac{2}{3}}\left(bd-(a+d)x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(2/3)/(b*d-(a+d)*x+x^2
),x, algorithm="giac")
```

```
[Out] integrate(-(a*b - 2*b*x + x^2)*(b - x)/((-(a - x)*(b - x)^2*x)^(2/3)*(b*d -
 (a + d)*x + x^2)), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)\left(ab-2bx+x^2\right)}{\left(x\left(-a+x\right)\left(-b+x\right)^2\right)^{\frac{2}{3}}\left(bd-\left(a+d\right)x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(2/3)/(b*d-(a+d)*x+x^2),x)
```

```
[Out] int((-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(2/3)/(b*d-(a+d)*x+x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ab - 2\,bx + x^2)(b - x)}{\left(-(a - x)(b - x)^2 x\right)^{\frac{2}{3}} \left(bd - (a + d)x + x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(2/3)/(b*d-(a+d)*x+x^2
),x, algorithm="maxima")
```

```
[Out] -integrate((a*b - 2*b*x + x^2)*(b - x)/((-(a - x)*(b - x)^2*x)^(2/3)*(b*d -
 (a + d)*x + x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(b - x)\,\left(x^2 - 2\,b\,x + a\,b\right)}{\left(x^2 + (-a - d)\,x + b\,d\right)\,\left(-x\,(a - x)\,(b - x)^2\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b - x)*(a*b - 2*b*x + x^2))/((b*d + x^2 - x*(a + d))*(-x*(a - x)*(b
- x)^2)^(2/3)),x)
```

```
[Out] int(-((b - x)*(a*b - 2*b*x + x^2))/((b*d + x^2 - x*(a + d))*(-x*(a - x)*(b
- x)^2)^(2/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(a*b-2*b*x+x**2)/(x*(-a+x)*(-b+x)**2)**(2/3)/(b*d-(a+d)*x+
x**2),x)
```

```
[Out] Timed out
```

3.2252 $\int \frac{-d+cx^4}{x\sqrt[3]{-b+ax^3}}\,dx$

Optimal. Leaf size=290

$$-\frac{bc\log\left(\sqrt[3]{ax^3-b}-\sqrt[3]{a}\,x\right)}{9a^{4/3}}-\frac{bc\tan^{-1}\left(\frac{\frac{2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{a}}+\frac{x}{\sqrt{3}}}{x}\right)}{3\sqrt{3}\,a^{4/3}}+\frac{bc\log\left(a^{2/3}x^2+\sqrt[3]{a}\,x\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}\right)}{18a^{4/3}}-\frac{d\log\left(-\sqrt[3]{b}\right.}{}$$

Rubi [A] time = 0.17, antiderivative size = 190, normalized size of antiderivative = 0.66, number of steps used = 9, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1844, 266, 56, 617, 204, 31, 321, 239}

$$-\frac{bc\log\left(\sqrt[3]{ax^3-b}-\sqrt[3]{a}\,x\right)}{6a^{4/3}}+\frac{bc\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{ax^3-b}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}}+\frac{cx\left(ax^3-b\right)^{2/3}}{3a}+\frac{d\log\left(\sqrt[3]{ax^3-b}+\sqrt[3]{b}\right)}{2\sqrt[3]{b}}+\frac{d\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{d\log(x)}{2\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-d + c*x^4)/(x*(-b + a*x^3)^(1/3)),x]
```

```
[Out] (c*x*(-b + a*x^3)^(2/3))/(3*a) + (b*c*ArcTan[(1 + (2*a^(1/3)*x)/(-b + a*x^3
)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*a^(4/3)) + (d*ArcTan[(b^(1/3) - 2*(-b + a*x^3
)^(1/3))/(Sqrt[3]*b^(1/3))])/(Sqrt[3]*b^(1/3)) - (d*Log[x])/(2*b^(1/3)) + (
d*Log[b^(1/3) + (-b + a*x^3)^(1/3)])/(2*b^(1/3)) - (b*c*Log[-(a^(1/3)*x) +
(-b + a*x^3)^(1/3)])/(6*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-d+cx^4}{x\sqrt[3]{-b+ax^3}}\,dx &= \int \left(-\frac{d}{x\sqrt[3]{-b+ax^3}}+\frac{cx^3}{\sqrt[3]{-b+ax^3}}\right)dx \\
&= c\int \frac{x^3}{\sqrt[3]{-b+ax^3}}\,dx - d\int \frac{1}{x\sqrt[3]{-b+ax^3}}\,dx \\
&= \frac{cx\left(-b+ax^3\right)^{2/3}}{3a} + \frac{(bc)\int \frac{1}{\sqrt[3]{-b+ax^3}}\,dx}{3a} - \frac{1}{3}d\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{-b+ax}}\,dx, x, x^3\right) \\
&= \frac{cx\left(-b+ax^3\right)^{2/3}}{3a} + \frac{bc\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{-b+ax^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}} - \frac{d\log(x)}{2\sqrt[3]{b}} - \frac{bc\log\left(-\sqrt[3]{a}\,x+\sqrt[3]{-b+ax^3}\right)}{6a^{4/3}} - \frac{1}{2} \\
&= \frac{cx\left(-b+ax^3\right)^{2/3}}{3a} + \frac{bc\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{-b+ax^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}} - \frac{d\log(x)}{2\sqrt[3]{b}} + \frac{d\log\left(\sqrt[3]{b}+\sqrt[3]{-b+ax^3}\right)}{2\sqrt[3]{b}} - \frac{bc\log}{} \\
&= \frac{cx\left(-b+ax^3\right)^{2/3}}{3a} + \frac{bc\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{-b+ax^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}} + \frac{d\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-b+ax^3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}} - \frac{d\log(x)}{2\sqrt[3]{b}} + \frac{d\log\left(\sqrt[3]{b}\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.26, size = 184, normalized size = 0.63

$$\frac{1}{18}\left(\frac{c\left(b\log\left(\frac{a^{2/3}x^2}{\left(ax^3-b\right)^{2/3}}+\frac{\sqrt[3]{a}\,x}{\sqrt[3]{ax^3-b}}+1\right)+6\sqrt[3]{a}\,x\left(ax^3-b\right)^{2/3}-2b\log\left(1-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{ax^3-b}}\right)+2\sqrt{3}\,b\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{ax^3-b}}+1}{\sqrt{3}}\right)\right)}{a^{4/3}}-\frac{9d\left(ax^3-b\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-\frac{ax^3}{b}\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-d + c*x^4)/(x*(-b + a*x^3)^(1/3)),x]
```

```
[Out] ((-9*d*(-b + a*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, 1 - (a*x^3)/b])/b
+ (c*(6*a^(1/3)*x*(-b + a*x^3)^(2/3) + 2*Sqrt[3]*b*ArcTan[(1 + (2*a^(1/3)*x
)/(-b + a*x^3)^(1/3))/Sqrt[3]] - 2*b*Log[1 - (a^(1/3)*x)/(-b + a*x^3)^(1/3)
] + b*Log[1 + (a^(2/3)*x^2)/(-b + a*x^3)^(2/3) + (a^(1/3)*x)/(-b + a*x^3)^(
1/3)]))/a^(4/3))/18
```

IntegrateAlgebraic [A] time = 4.88, size = 290, normalized size = 1.00

$$-\frac{bc\log\left(\sqrt[3]{ax^3-b}-\sqrt[3]{a}\,x\right)}{9a^{4/3}}-\frac{bc\tan^{-1}\left(\frac{\frac{2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{a}}+\frac{x}{\sqrt{3}}}{x}\right)}{3\sqrt{3}\,a^{4/3}}+\frac{bc\log\left(a^{2/3}x^2+\sqrt[3]{a}\,x\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}\right)}{18a^{4/3}}-\frac{d\log\left(-\sqrt[3]{b}\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}+b^{2/3}\right)}{6\sqrt[3]{b}}+\frac{cx\left(ax^3-b\right)^{2/3}}{3a}+\frac{d\log\left(\sqrt[3]{ax^3-b}+\sqrt[3]{b}\right)}{3\sqrt[3]{b}}+\frac{d\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-d + c*x^4)/(x*(-b + a*x^3)^(1/3)),x]
```

```
[Out] (c*x*(-b + a*x^3)^(2/3))/(3*a) - (b*c*ArcTan[(x/Sqrt[3] + (2*(-b + a*x^3)^(
1/3))/(Sqrt[3]*a^(1/3)))/x])/(3*Sqrt[3]*a^(4/3)) + (d*ArcTan[1/Sqrt[3] - (2
*(-b + a*x^3)^(1/3))/(Sqrt[3]*b^(1/3))])/(Sqrt[3]*b^(1/3)) + (d*Log[b^(1/3)
 + (-b + a*x^3)^(1/3)])/(3*b^(1/3)) - (b*c*Log[-(a^(1/3)*x) + (-b + a*x^3)^
(1/3)])/(9*a^(4/3)) - (d*Log[b^(2/3) - b^(1/3)*(-b + a*x^3)^(1/3) + (-b + a
*x^3)^(2/3)])/(6*b^(1/3)) + (b*c*Log[a^(2/3)*x^2 + a^(1/3)*x*(-b + a*x^3)^(
1/3) + (-b + a*x^3)^(2/3)])/(18*a^(4/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4-d)/x/(a*x^3-b)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^4-d}{\left(ax^3-b\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4-d)/x/(a*x^3-b)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 - d)/((a*x^3 - b)^(1/3)*x), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{c\,x^4-d}{x\left(a\,x^3-b\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4-d)/x/(a*x^3-b)^(1/3),x)
```

```
[Out] int((c*x^4-d)/x/(a*x^3-b)^(1/3),x)
```

maxima [A] time = 0.88, size = 250, normalized size = 0.86

$$-\frac{1}{18}\left(\frac{2\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(a^{\frac{1}{3}}+\frac{2\left(ax^3-b\right)^{\frac{1}{3}}}{x}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b\log\left(a^{\frac{2}{3}}+\frac{\left(ax^3-b\right)^{\frac{1}{3}}a^{\frac{1}{3}}}{x}+\frac{\left(ax^3-b\right)^{\frac{2}{3}}}{x^2}\right)}{a^{\frac{4}{3}}}+\frac{2b\log\left(-a^{\frac{1}{3}}+\frac{\left(ax^3-b\right)^{\frac{1}{3}}}{x}\right)}{a^{\frac{4}{3}}}-\frac{6\left(ax^3-b\right)^{\frac{2}{3}}b}{\left(a^2-\frac{\left(ax^3-b\right)a}{x^3}\right)x^2}\right)c-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(ax^3-b\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}+\frac{\log\left(\left(ax^3-b\right)^{\frac{2}{3}}-\left(ax^3-b\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}-\frac{2\log\left(\left(ax^3-b\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4-d)/x/(a*x^3-b)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/18*(2*sqrt(3)*b*arctan(1/3*sqrt(3)*(a^(1/3) + 2*(a*x^3 - b)^(1/3)/x)/a^(
1/3))/a^(4/3) - b*log(a^(2/3) + (a*x^3 - b)^(1/3)*a^(1/3)/x + (a*x^3 - b)^(
2/3)/x^2)/a^(4/3) + 2*b*log(-a^(1/3) + (a*x^3 - b)^(1/3)/x)/a^(4/3) - 6*(a*
x^3 - b)^(2/3)*b/((a^2 - (a*x^3 - b)*a/x^3)*x^2))*c - 1/6*(2*sqrt(3)*arctan
(1/3*sqrt(3)*(2*(a*x^3 - b)^(1/3) - b^(1/3))/b^(1/3))/b^(1/3) + log((a*x^3
- b)^(2/3) - (a*x^3 - b)^(1/3)*b^(1/3) + b^(2/3))/b^(1/3) - 2*log((a*x^3 -
b)^(1/3) + b^(1/3))/b^(1/3))*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{d-c\,x^4}{x\,{\left(a\,x^3-b\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(d - c*x^4)/(x*(a*x^3 - b)^(1/3)),x)
```

```
[Out] -int((d - c*x^4)/(x*(a*x^3 - b)^(1/3)), x)
```

sympy [C] time = 3.09, size = 83, normalized size = 0.29

$$-\frac{cx^4e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ax^3}{b}\right)}{3\sqrt[3]{b}\,\Gamma\left(\frac{7}{3}\right)}+\frac{d\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{be^{2i\pi}}{ax^3}\right)}{3\sqrt[3]{a}\,x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4-d)/x/(a*x**3-b)**(1/3),x)
```

```
[Out] -c*x**4*exp(2*I*pi/3)*gamma(4/3)*hyper((1/3, 4/3), (7/3,), a*x**3/b)/(3*b**
(1/3)*gamma(7/3)) + d*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*exp_polar(2*I*
pi)/(a*x**3))/(3*a**(1/3)*x*gamma(4/3))
```

3.2253 $\int \frac{ab-2bx+x^2}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(b-(1+ad)x+dx^2\right)}\,dx$

Optimal. Leaf size=291

$$\frac{\log\left(\sqrt[3]{d}\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}+b-x\right)}{d^{2/3}}-\frac{\log\left(d^{2/3}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)\right.}{}$$

Rubi [F] time = 7.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ab-2bx+x^2}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(b-(1+ad)x+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*b - 2*b*x + x^2)/((x*(-a + x)*(-b + x)^2)^(1/3)*(b - (1 + a*d)*x + d
*x^2)),x]
```

```
[Out] (3*x*(1 - x/a)^(1/3)*(1 - x/b)^(2/3)*AppellF1[2/3, 1/3, 2/3, 5/3, x/a, x/b]
)/(2*d*(-((a - x)*(b - x)^2*x))^(1/3)) + ((1 + a*d - 2*b*d + Sqrt[1 + 2*a*d
 - 4*b*d + a^2*d^2])*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Int][1/(x^
(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*(-1 - a*d - Sqrt[1 + 2*a*d - 4*b*d + a^
2*d^2] + 2*d*x)), x])/(d*(-((a - x)*(b - x)^2*x))^(1/3)) + ((1 + a*d - 2*b*
d - Sqrt[1 + 2*a*d - 4*b*d + a^2*d^2])*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3
)*Defer[Int][1/(x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*(-1 - a*d + Sqrt[1 +
2*a*d - 4*b*d + a^2*d^2] + 2*d*x)), x])/(d*(-((a - x)*(b - x)^2*x))^(1/3))
```

Rubi steps

$$\int \frac{ab-2bx+x^2}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(b-(1+ad)x+dx^2\right)}\,dx=\frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\int\frac{ab-2bx+x^2}{\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\left(b-(1+ad)x+dx^2\right)}\,d}{\sqrt[3]{x(-a+x)(-b+x)^2}}$$

$$=\frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\int\left(\frac{1}{d\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}}-\frac{b-}{d\sqrt[3]{x}\sqrt[3]{-a+x}(}\right.}{\sqrt[3]{x(-a+x)(-b+x)^2}}$$

$$=\frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\int\frac{1}{\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}}\,dx}{d\sqrt[3]{x(-a+x)(-b+x)^2}}-\frac{\left(\sqrt[3]{x}\sqrt[3]{-a}\right.}{}$$

$$=-\frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\int\left(\frac{-1-ad+2bd-\sqrt{1+2ad-4bd}}{\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\left(-1-ad-\sqrt{1+2ad}\right.}\right.}{d\sqrt[3]{x(-a}}$$

$$=-\frac{\left(\left(-1-ad+2bd-\sqrt{1+2ad-4bd+a^2d^2}\right)\sqrt[3]{x}\sqrt[3]{-a+x}(-\right.}{d\sqrt[3]{x(-a+}}$$

$$=\frac{3x\sqrt[3]{1-\frac{x}{a}}\left(1-\frac{x}{b}\right)^{2/3}F_1\left(\frac{2}{3};\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{x}{a},\frac{x}{b}\right)}{2d\sqrt[3]{-\left((a-x)(b-x)^2x\right)}}-\frac{\left(\left(-1-ad+2bd-\right.\right.}{}$$

Mathematica [F] time = 5.77, size = 0, normalized size = 0.00

$$\int \frac{ab - 2bx + x^2}{\sqrt[3]{x(-a + x)(-b + x)^2} \left(b - (1 + ad)x + dx^2\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*b - 2*b*x + x^2)/((x*(-a + x)*(-b + x)^2)^(1/3)*(b - (1 + a*d)
*x + d*x^2)),x]
```

```
[Out] Integrate[(a*b - 2*b*x + x^2)/((x*(-a + x)*(-b + x)^2)^(1/3)*(b - (1 + a*d)
*x + d*x^2)), x]
```

IntegrateAlgebraic [A] time = 0.62, size = 291, normalized size = 1.00

$$\frac{\log \left(\sqrt[3]{d} \sqrt[3]{x^2 \left(2ab + b^2\right) - ab^2x + x^3(-a - 2b) + x^4} + b - x\right)}{d^{2/3}} - \frac{\log \left(d^{2/3} \left(x^2 \left(2ab + b^2\right) - ab^2x + x^3(-a - 2b) + x^4\right)^{2/3} + \sqrt[3]{x^2 \left(2ab + b^2\right) - ab^2x + x^3(-a - 2b) + x^4} \left(\sqrt[3]{d}\, x - b\sqrt[3]{d}\right) + b^2 - 2bx + x^2\right)}{2d^{2/3}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt{3} \sqrt[3]{d} \sqrt[3]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}}{\sqrt[3]{d} \sqrt[3]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4} - 2b + 2x}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b - 2*b*x + x^2)/((x*(-a + x)*(-b + x)^2)^(1/3)*(b -
(1 + a*d)*x + d*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*
b)*x^3 + x^4)^(1/3))/(-2*b + 2*x + d^(1/3)*(-(a*b^2*x) + (2*a*b + b^2)*x^2
+ (-a - 2*b)*x^3 + x^4)^(1/3))])/d^(2/3) + Log[b - x + d^(1/3)*(-(a*b^2*x)
+ (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3)]/d^(2/3) - Log[b^2 - 2*b*
x + x^2 + (-(b*d^(1/3)) + d^(1/3)*x)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a
- 2*b)*x^3 + x^4)^(1/3) + d^(2/3)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2
*b)*x^3 + x^4)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(b-(a*d+1)*x+d*x^2),x,
algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab - 2\,bx + x^2}{\left(-(a - x)(b - x)^2 x\right)^{\frac{1}{3}} \left(dx^2 - (ad + 1)x + b\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(b-(a*d+1)*x+d*x^2),x,
algorithm="giac")
```

```
[Out] integrate((a*b - 2*b*x + x^2)/((-(a - x)*(b - x)^2*x)^(1/3)*(d*x^2 - (a*d +
 1)*x + b)), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{ab - 2bx + x^2}{\left(x\,(-a + x)\,(-b + x)^2\right)^{\frac{1}{3}} \left(b - (ad + 1)\,x + d\,x^2\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(b-(a*d+1)*x+d*x^2),x)
```

```
[Out] int((a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(b-(a*d+1)*x+d*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab - 2\,bx + x^2}{\left(-(a-x)(b-x)^2 x\right)^{\frac{1}{3}} \left(dx^2 - (ad+1)x + b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(b-(a*d+1)*x+d*x^2),x,
algorithm="maxima")
```

```
[Out] integrate((a*b - 2*b*x + x^2)/((-(a - x)*(b - x)^2*x)^(1/3)*(d*x^2 - (a*d +
 1)*x + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 - 2\,b\,x + a\,b}{\left(-x\,\left(a-x\right)\,{\left(b-x\right)}^2\right)^{1/3}\,\left(d\,x^2 + \left(-a\,d-1\right)\,x + b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b - 2*b*x + x^2)/((-x*(a - x)*(b - x)^2)^(1/3)*(b - x*(a*d + 1) + d*
x^2)),x)
```

```
[Out] int((a*b - 2*b*x + x^2)/((-x*(a - x)*(b - x)^2)^(1/3)*(b - x*(a*d + 1) + d*
x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*b*x+x**2)/(x*(-a+x)*(-b+x)**2)**(1/3)/(b-(a*d+1)*x+d*x**2)
,x)
```

```
[Out] Timed out
```

3.2254 $\int \frac{-ab^2+(4a-b)bx-3ax^2+x^3}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-a^2+\left(2a+b^2d\right)x-(1+2bd)x^2+dx^3\right)}dx$

Optimal. Leaf size=291

$$-\frac{\log\left(a^2+d^{2/3}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}+\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}\left(\sqrt[3]{d}\right.}{2d^{2/3}}$$

Rubi [F] time = 8.96, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-ab^2+(4a-b)bx-3ax^2+x^3}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-a^2+\left(2a+b^2d\right)x-(1+2bd)x^2+dx^3\right)}dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a*b^2) + (4*a - b)*b*x - 3*a*x^2 + x^3)/((x*(-a + x)*(-b + x)^2)^(1/
3)*(-a^2 + (2*a + b^2*d)*x - (1 + 2*b*d)*x^2 + d*x^3)),x]
```

```
[Out] (3*(3*a - b)*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][
(x^4*(-b + x^3)^(1/3))/((-a + x^3)^(1/3)*(a^2 - 2*a*(1 + (b^2*d)/(2*a))*x^3
 + (1 + 2*b*d)*x^6 - d*x^9)), x], x, x^(1/3)])/(-((a - x)*(b - x)^2*x))^(1/
3) + (3*a*b*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(
x*(-b + x^3)^(1/3))/((-a + x^3)^(1/3)*(-a^2 + 2*a*(1 + (b^2*d)/(2*a))*x^3 -
 (1 + 2*b*d)*x^6 + d*x^9)), x], x, x^(1/3)])/(-((a - x)*(b - x)^2*x))^(1/3)
 + (3*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x^7*(-
b + x^3)^(1/3))/((-a + x^3)^(1/3)*(-a^2 + 2*a*(1 + (b^2*d)/(2*a))*x^3 - (1
+ 2*b*d)*x^6 + d*x^9)), x], x, x^(1/3)])/(-((a - x)*(b - x)^2*x))^(1/3)
```

Rubi steps

$$\int \frac{-ab^2+(4a-b)bx-3ax^2+x^3}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-a^2+\left(2a+b^2d\right)x-(1+2bd)x^2+dx^3\right)}dx=\frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\int \frac{}{\sqrt[3]{x}\sqrt[3]{-a+x}}}{\sqrt[3]{x(-a+x}}$$

$$=\frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\int \frac{\sqrt[3]{}}{\sqrt[3]{x}\sqrt[3]{-a+x}}}{\sqrt[3]{x(-a+x)(-b}}$$

$$=\frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\text{Subst}\left(\int\right.}{\sqrt[3]{x(-a}}$$

$$=\frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\text{Subst}\left(\int\right.}{}$$

$$=\frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\text{Subst}\left(\int\right.}{\sqrt[3]{x(-a}}$$

Mathematica [F] time = 3.63, size = 0, normalized size = 0.00

$$\int \frac{-ab^2+(4a-b)bx-3ax^2+x^3}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-a^2+\left(2a+b^2d\right)x-(1+2bd)x^2+dx^3\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-(a*b^2) + (4*a - b)*b*x - 3*a*x^2 + x^3)/((x*(-a + x)*(-b + x)^
2)^(1/3)*(-a^2 + (2*a + b^2*d)*x - (1 + 2*b*d)*x^2 + d*x^3)),x]
```

```
[Out] Integrate[(-(a*b^2) + (4*a - b)*b*x - 3*a*x^2 + x^3)/((x*(-a + x)*(-b + x)^
2)^(1/3)*(-a^2 + (2*a + b^2*d)*x - (1 + 2*b*d)*x^2 + d*x^3)), x]
```

IntegrateAlgebraic [A] time = 0.72, size = 291, normalized size = 1.00

$$-\frac{\log \left(a^2+d^{2/3} \left(x^2 \left(2 a b+b^2\right)-a b^2 x+x^3 (-a-2 b)+x^4\right)^{2/3}+\sqrt[3]{x^2 \left(2 a b+b^2\right)-a b^2 x+x^3 (-a-2 b)+x^4} \left(\sqrt[3]{d} x-a \sqrt[3]{d}\right)-2 a x+x^2\right)}{2 d^{2/3}}+\frac{\log \left(\sqrt[3]{d} \sqrt[3]{x^2 \left(2 a b+b^2\right)-a b^2 x+x^3 (-a-2 b)+x^4}+a-x\right)}{d^{2/3}}+\frac{\sqrt{3} \tan ^{-1}\left(\frac{\sqrt{3} \sqrt[3]{d} \sqrt[3]{x^2 \left(2 a b+b^2\right)-a b^2 x+x^3 (-a-2 b)+x^4}}{\sqrt[3]{d} \sqrt[3]{x^2 \left(2 a b+b^2\right)-a b^2 x+x^3 (-a-2 b)+x^4}-2 a+2 x}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*b^2) + (4*a - b)*b*x - 3*a*x^2 + x^3)/((x*(-a + x)*
(-b + x)^2)^(1/3)*(-a^2 + (2*a + b^2*d)*x - (1 + 2*b*d)*x^2 + d*x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*
b)*x^3 + x^4)^(1/3))/(-2*a + 2*x + d^(1/3)*(-(a*b^2*x) + (2*a*b + b^2)*x^2
+ (-a - 2*b)*x^3 + x^4)^(1/3))])/d^(2/3) + Log[a - x + d^(1/3)*(-(a*b^2*x)
+ (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3)]/d^(2/3) - Log[a^2 - 2*a*
x + x^2 + (-(a*d^(1/3)) + d^(1/3)*x)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a
- 2*b)*x^3 + x^4)^(1/3) + d^(2/3)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2
*b)*x^3 + x^4)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b^2+(4*a-b)*b*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a^2+
(b^2*d+2*a)*x-(2*b*d+1)*x^2+d*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ab^2 - (4\,a - b)bx + 3\,ax^2 - x^3}{\left(-(a - x)(b - x)^2 x\right)^{\frac{1}{3}} \left(dx^3 - (2\,bd + 1)x^2 - a^2 + \left(b^2 d + 2\,a\right)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b^2+(4*a-b)*b*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a^2+
(b^2*d+2*a)*x-(2*b*d+1)*x^2+d*x^3),x, algorithm="giac")
```

```
[Out] integrate(-(a*b^2 - (4*a - b)*b*x + 3*a*x^2 - x^3)/((-(a - x)*(b - x)^2*x)^
(1/3)*(d*x^3 - (2*b*d + 1)*x^2 - a^2 + (b^2*d + 2*a)*x)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{-a\,b^2 + (4a - b)\,bx - 3a\,x^2 + x^3}{\left(x\,(-a + x)\,(-b + x)^2\right)^{\frac{1}{3}} \left(-a^2 + \left(b^2 d + 2a\right)x - (2bd + 1)\,x^2 + d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*b^2+(4*a-b)*b*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a^2+(b^2*d
+2*a)*x-(2*b*d+1)*x^2+d*x^3),x)
```

```
[Out] int((-a*b^2+(4*a-b)*b*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a^2+(b^2*d
+2*a)*x-(2*b*d+1)*x^2+d*x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ab^2 - (4\,a - b)bx + 3\,ax^2 - x^3}{\left(-(a-x)(b-x)^2x\right)^{\frac{1}{3}}\left(dx^3 - (2\,bd + 1)x^2 - a^2 + \left(b^2d + 2\,a\right)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b^2+(4*a-b)*b*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a^2+
(b^2*d+2*a)*x-(2*b*d+1)*x^2+d*x^3),x, algorithm="maxima")
```

```
[Out] -integrate((a*b^2 - (4*a - b)*b*x + 3*a*x^2 - x^3)/((-(a - x)*(b - x)^2*x)^
(1/3)*(d*x^3 - (2*b*d + 1)*x^2 - a^2 + (b^2*d + 2*a)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{a\,b^2 + 3\,a\,x^2 - x^3 - b\,x\,\left(4\,a - b\right)}{\left(-x\,\left(a-x\right)\,\left(b-x\right)^2\right)^{1/3}\,\left(x\,\left(d\,b^2 + 2\,a\right) + d\,x^3 - x^2\,\left(2\,b\,d + 1\right) - a^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*b^2 + 3*a*x^2 - x^3 - b*x*(4*a - b))/((-x*(a - x)*(b - x)^2)^(1/3)*
(x*(2*a + b^2*d) + d*x^3 - x^2*(2*b*d + 1) - a^2)),x)
```

```
[Out] int(-(a*b^2 + 3*a*x^2 - x^3 - b*x*(4*a - b))/((-x*(a - x)*(b - x)^2)^(1/3)*
(x*(2*a + b^2*d) + d*x^3 - x^2*(2*b*d + 1) - a^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b**2+(4*a-b)*b*x-3*a*x**2+x**3)/(x*(-a+x)*(-b+x)**2)**(1/3)/(
-a**2+(b**2*d+2*a)*x-(2*b*d+1)*x**2+d*x**3),x)
```

```
[Out] Timed out
```

3.2255 $$\int \frac{(-4a+b+3x)\left(-b^3+3b^2x-3bx^2+x^3\right)}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(b^4+ad-\left(4b^3+d\right)x+6b^2x^2-4bx^3+x^4\right)}\,dx$$

Optimal. Leaf size=291

$$-\frac{\log\left(a^2d+\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}\left(d^{2/3}x-ad^{2/3}\right)+\sqrt[3]{d}\,\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+\right.\right.}{2\sqrt[3]{d}}$$

Rubi [F] time = 7.49, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-4a+b+3x)\left(-b^3+3b^2x-3bx^2+x^3\right)}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(b^4+ad-\left(4b^3+d\right)x+6b^2x^2-4bx^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-4*a + b + 3*x)*(-b^3 + 3*b^2*x - 3*b*x^2 + x^3))/(((-a + x)*(-b + x)
^2)^(2/3)*(b^4 + a*d - (4*b^3 + d)*x + 6*b^2*x^2 - 4*b*x^3 + x^4)),x]
```

```
[Out] (9*a*(-a + x)^(2/3)*(-b + x)^(4/3)*Defer[Subst][Defer[Int][(a - b + x^3)^(5
/3)/(a^4*(1 + (b*(-4*a^3 + 6*a^2*b - 4*a*b^2 + b^3))/a^4) + 4*a^3*(1 - (12*
a^2*b - 12*a*b^2 + 4*b^3 + d)/(4*a^3))*x^3 + 6*a^2*(1 + (b*(-2*a + b))/a^2)
*x^6 + 4*a*(1 - b/a)*x^9 + x^12), x], x, (-a + x)^(1/3)])/(-((a - x)*(b - x
)^2))^(2/3) + (9*(-a + x)^(2/3)*(-b + x)^(4/3)*Defer[Subst][Defer[Int][(x^3
*(a - b + x^3)^(5/3))/(a^4*(1 + (b*(-4*a^3 + 6*a^2*b - 4*a*b^2 + b^3))/a^4)
 + 4*a^3*(1 - (12*a^2*b - 12*a*b^2 + 4*b^3 + d)/(4*a^3))*x^3 + 6*a^2*(1 + (
b*(-2*a + b))/a^2)*x^6 + 4*a*(1 - b/a)*x^9 + x^12), x], x, (-a + x)^(1/3)])
/(-((a - x)*(b - x)^2))^(2/3) - (3*(4*a - b)*(-a + x)^(2/3)*(-b + x)^(4/3)*
Defer[Subst][Defer[Int][(a - b + x^3)^(5/3)/(b^4*(1 + (a*d)/b^4) - (4*b^3 +
 d)*(a + x^3) + 6*b^2*(a + x^3)^2 - 4*b*(a + x^3)^3 + (a + x^3)^4), x], x,
(-a + x)^(1/3)])/(-((a - x)*(b - x)^2))^(2/3)
```

Rubi steps

$$\int \frac{(-4a+b+3x)\left(-b^3+3b^2x-3bx^2+x^3\right)}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(b^4+ad-\left(4b^3+d\right)x+6b^2x^2-4bx^3+x^4\right)}\,dx = \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{}{(-a+x)^{2/3}}}{\left((-a+x)\right.}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{}{(-a+x)^{2/3}}}{\left((-a+x)\right.}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{}{(-a+x)^{2/3}}}{\left((-a+x)(-b\right.}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \left(\frac{}{(-a+x)^{2}}\right.}{}$$

$$= \frac{\left(3(-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{}{(-a+x)^{2}}}{\left((-a+x)(-\right.}$$

$$= \frac{\left(9(-a+x)^{2/3}(-b+x)^{4/3}\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{\left(9(-a+x)^{2/3}(-b+x)^{4/3}\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{\left(9(-a+x)^{2/3}(-b+x)^{4/3}\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{\left(9(-a+x)^{2/3}(-b+x)^{4/3}\right)\text{Subst}\left(\int\right.}{}$$

Mathematica [F] time = 2.88, size = 0, normalized size = 0.00

$$\int \frac{(-4a+b+3x)\left(-b^3+3b^2x-3bx^2+x^3\right)}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(b^4+ad-\left(4b^3+d\right)x+6b^2x^2-4bx^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-4*a + b + 3*x)*(-b^3 + 3*b^2*x - 3*b*x^2 + x^3))/(((-a + x)*(-
b + x)^2)^(2/3)*(b^4 + a*d - (4*b^3 + d)*x + 6*b^2*x^2 - 4*b*x^3 + x^4)),x]
```

```
[Out] Integrate[((-4*a + b + 3*x)*(-b^3 + 3*b^2*x - 3*b*x^2 + x^3))/(((-a + x)*(-
b + x)^2)^(2/3)*(b^4 + a*d - (4*b^3 + d)*x + 6*b^2*x^2 - 4*b*x^3 + x^4)), x
]
```

IntegrateAlgebraic [A] time = 3.17, size = 291, normalized size = 1.00

$$-\frac{\log\left(a^2d+\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}\left(d^{2/3}x-ad^{2/3}\right)+\sqrt[3]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{4/3}-2adx+dx^2\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[6]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}+a\sqrt{d}-\sqrt{d}\,x\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}}{\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}-2a\sqrt[3]{d}+2\sqrt[3]{d}\,x}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4*a + b + 3*x)*(-b^3 + 3*b^2*x - 3*b*x^2 + x^3))/(((-
a + x)*(-b + x)^2)^(2/3)*(b^4 + a*d - (4*b^3 + d)*x + 6*b^2*x^2 - 4*b*x^3 +
 x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3
)^(2/3))/(-2*a*d^(1/3) + 2*d^(1/3)*x + (-(a*b^2) + (2*a*b + b^2)*x + (-a -
2*b)*x^2 + x^3)^(2/3))])/d^(1/3) + Log[a*Sqrt[d] - Sqrt[d]*x + d^(1/6)*(-(a
*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(2/3)]/d^(1/3) - Log[a^2*d
- 2*a*d*x + d*x^2 + (-(a*d^(2/3)) + d^(2/3)*x)*(-(a*b^2) + (2*a*b + b^2)*x
+ (-a - 2*b)*x^2 + x^3)^(2/3) + d^(1/3)*(-(a*b^2) + (2*a*b + b^2)*x + (-a -
 2*b)*x^2 + x^3)^(4/3)]/(2*d^(1/3))
```

fricas [A] time = 1.22, size = 798, normalized size = 2.74

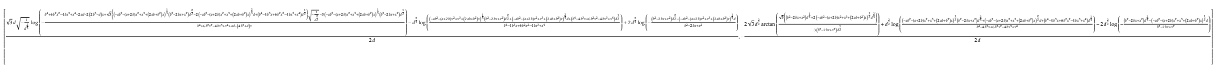

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*a+b+3*x)*(-b^3+3*b^2*x-3*b*x^2+x^3)/((-a+x)*(-b+x)^2)^(2/3)/(
b^4+a*d-(4*b^3+d)*x+6*b^2*x^2-4*b*x^3+x^4),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*d*sqrt(-1/d^(2/3))*log(-(b^4 + 6*b^2*x^2 - 4*b*x^3 + x^4 - 2*
a*d - 2*(2*b^3 - d)*x + sqrt(3)*((-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b
^2)*x)^(1/3)*(b^2 - 2*b*x + x^2)*d^(2/3) - 2*(-a*b^2 - (a + 2*b)*x^2 + x^3
+ (2*a*b + b^2)*x)^(2/3)*d + (b^4 - 4*b^3*x + 6*b^2*x^2 - 4*b*x^3 + x^4)*d^
(1/3))*sqrt(-1/d^(2/3)) - 3*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x
)^(1/3)*(b^2 - 2*b*x + x^2)*d^(1/3))/(b^4 + 6*b^2*x^2 - 4*b*x^3 + x^4 + a*d
 - (4*b^3 + d)*x)) - d^(2/3)*log(((-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b +
b^2)*x)^(1/3)*(b^2 - 2*b*x + x^2)*d^(2/3) + (-a*b^2 - (a + 2*b)*x^2 + x^3 +
 (2*a*b + b^2)*x)^(2/3)*d + (b^4 - 4*b^3*x + 6*b^2*x^2 - 4*b*x^3 + x^4)*d^(
1/3))/(b^4 - 4*b^3*x + 6*b^2*x^2 - 4*b*x^3 + x^4)) + 2*d^(2/3)*log(-((b^2 -
 2*b*x + x^2)*d^(2/3) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1
/3)*d)/(b^2 - 2*b*x + x^2)))/d, -1/2*(2*sqrt(3)*d^(2/3)*arctan(1/3*sqrt(3)*
((b^2 - 2*b*x + x^2)*d^(1/3) + 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b
^2)*x)^(1/3)*d^(2/3))/((b^2 - 2*b*x + x^2)*d^(1/3))) + d^(2/3)*log(((-a*b^2
 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(b^2 - 2*b*x + x^2)*d^(2/3)
 + (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*d + (b^4 - 4*b^3*
x + 6*b^2*x^2 - 4*b*x^3 + x^4)*d^(1/3))/(b^4 - 4*b^3*x + 6*b^2*x^2 - 4*b*x^
3 + x^4)) - 2*d^(2/3)*log(-((b^2 - 2*b*x + x^2)*d^(2/3) - (-a*b^2 - (a + 2*
b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*d)/(b^2 - 2*b*x + x^2)))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^{3} - 3\,b^{2} x + 3\,b x^{2} - x^{3}\right)\left(4\,a - b - 3\,x\right)}{\left(b^{4} + 6\,b^{2} x^{2} - 4\,b x^{3} + x^{4} + a d - \left(4\,b^{3} + d\right)x\right)\left(-\left(a - x\right)\left(b - x\right)^{2}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*a+b+3*x)*(-b^3+3*b^2*x-3*b*x^2+x^3)/((-a+x)*(-b+x)^2)^(2/3)/(
b^4+a*d-(4*b^3+d)*x+6*b^2*x^2-4*b*x^3+x^4),x, algorithm="giac")
```

```
[Out] integrate((b^3 - 3*b^2*x + 3*b*x^2 - x^3)*(4*a - b - 3*x)/((b^4 + 6*b^2*x^2
 - 4*b*x^3 + x^4 + a*d - (4*b^3 + d)*x)*(-(a - x)*(b - x)^2)^(2/3)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(-4a + b + 3x\right)\left(-b^{3} + 3b^{2} x - 3b\,x^{2} + x^{3}\right)}{\left(\left(-a + x\right)\left(-b + x\right)^{2}\right)^{\frac{2}{3}}\left(b^{4} + ad - \left(4b^{3} + d\right)x + 6b^{2} x^{2} - 4b\,x^{3} + x^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*a+b+3*x)*(-b^3+3*b^2*x-3*b*x^2+x^3)/((-a+x)*(-b+x)^2)^(2/3)/(b^4+a*
d-(4*b^3+d)*x+6*b^2*x^2-4*b*x^3+x^4),x)
```

```
[Out] int((-4*a+b+3*x)*(-b^3+3*b^2*x-3*b*x^2+x^3)/((-a+x)*(-b+x)^2)^(2/3)/(b^4+a*
d-(4*b^3+d)*x+6*b^2*x^2-4*b*x^3+x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b^3 - 3\,b^2x + 3\,bx^2 - x^3\right)(4\,a - b - 3\,x)}{\left(b^4 + 6\,b^2x^2 - 4\,bx^3 + x^4 + ad - \left(4\,b^3 + d\right)x\right)\left(-(a - x)(b - x)^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*a+b+3*x)*(-b^3+3*b^2*x-3*b*x^2+x^3)/((-a+x)*(-b+x)^2)^(2/3)/(
b^4+a*d-(4*b^3+d)*x+6*b^2*x^2-4*b*x^3+x^4),x, algorithm="maxima")
```

```
[Out] integrate((b^3 - 3*b^2*x + 3*b*x^2 - x^3)*(4*a - b - 3*x)/((b^4 + 6*b^2*x^2
 - 4*b*x^3 + x^4 + a*d - (4*b^3 + d)*x)*(-(a - x)*(b - x)^2)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(b - 4\,a + 3\,x)\,\left(b^3 - 3\,b^2\,x + 3\,b\,x^2 - x^3\right)}{\left(-(a - x)\,(b - x)^2\right)^{2/3}\,\left(a\,d - 4\,b\,x^3 - x\,\left(4\,b^3 + d\right) + b^4 + x^4 + 6\,b^2\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b - 4*a + 3*x)*(3*b*x^2 - 3*b^2*x + b^3 - x^3))/((-(a - x)*(b - x)^2
)^(2/3)*(a*d - 4*b*x^3 - x*(d + 4*b^3) + b^4 + x^4 + 6*b^2*x^2)),x)
```

```
[Out] int(-((b - 4*a + 3*x)*(3*b*x^2 - 3*b^2*x + b^3 - x^3))/((-(a - x)*(b - x)^2
)^(2/3)*(a*d - 4*b*x^3 - x*(d + 4*b^3) + b^4 + x^4 + 6*b^2*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*a+b+3*x)*(-b**3+3*b**2*x-3*b*x**2+x**3)/((-a+x)*(-b+x)**2)**(
2/3)/(b**4+a*d-(4*b**3+d)*x+6*b**2*x**2-4*b*x**3+x**4),x)
```

```
[Out] Timed out
```

3.2256 $\int \frac{\sqrt[4]{-bx^3+ax^4}}{x(-d+cx)}\,dx$

Optimal. Leaf size=291

$$-\frac{(1-i)\sqrt[4]{ad-bc}\,\tan^{-1}\left(\frac{(1+i)\sqrt[4]{d}\,x\sqrt[4]{ax^4-bx^3}\,\sqrt[4]{ad-bc}}{x^2\sqrt{ad-bc}-i\sqrt{d}\,\sqrt{ax^4-bx^3}}\right)}{c\sqrt[4]{d}}-\frac{(1-i)\sqrt[4]{ad-bc}\,\tanh^{-1}\left(\frac{\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{ad-bc}}{\sqrt[4]{d}}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{d}\,\sqrt{ax^4-bx^3}}{\sqrt[4]{ad-bc}}}{x\sqrt[4]{ax^4-bx^3}}\right)}{c\sqrt[4]{d}}-\frac{2\sqrt[4]{a}\ \mathrm{t}}{}$$

Rubi [A] time = 0.40, antiderivative size = 309, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2042, 105, 63, 331, 298, 203, 206, 93, 205, 208}

$$\frac{2\sqrt[4]{ax^4-bx^3}\,\sqrt[4]{ad-bc}\,\tan^{-1}\left(\frac{\sqrt[4]{x}\,\sqrt[4]{ad-bc}}{\sqrt[4]{d}\,\sqrt[4]{ax-b}}\right)}{c\sqrt[4]{d}\,x^{3/4}\sqrt[4]{ax-b}}-\frac{2\sqrt[4]{ax^4-bx^3}\,\sqrt[4]{ad-bc}\,\tanh^{-1}\left(\frac{\sqrt[4]{x}\,\sqrt[4]{ad-bc}}{\sqrt[4]{d}\,\sqrt[4]{ax-b}}\right)}{c\sqrt[4]{d}\,x^{3/4}\sqrt[4]{ax-b}}-\frac{2\sqrt[4]{a}\,\sqrt[4]{ax^4-bx^3}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt[4]{x}}{\sqrt[4]{ax-b}}\right)}{cx^{3/4}\sqrt[4]{ax-b}}+\frac{2\sqrt[4]{a}\,\sqrt[4]{ax^4-bx^3}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt[4]{x}}{\sqrt[4]{ax-b}}\right)}{cx^{3/4}\sqrt[4]{ax-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-(b*x^3) + a*x^4)^(1/4)/(x*(-d + c*x)),x]
```

```
[Out] (-2*a^(1/4)*(-(b*x^3) + a*x^4)^(1/4)*ArcTan[(a^(1/4)*x^(1/4))/(-b + a*x)^(1
/4)])/(c*x^(3/4)*(-b + a*x)^(1/4)) + (2*(-(b*c) + a*d)^(1/4)*(-(b*x^3) + a*
x^4)^(1/4)*ArcTan[((-(b*c) + a*d)^(1/4)*x^(1/4))/(d^(1/4)*(-b + a*x)^(1/4))
])/(c*d^(1/4)*x^(3/4)*(-b + a*x)^(1/4)) + (2*a^(1/4)*(-(b*x^3) + a*x^4)^(1/
4)*ArcTanh[(a^(1/4)*x^(1/4))/(-b + a*x)^(1/4)])/(c*x^(3/4)*(-b + a*x)^(1/4)
) - (2*(-(b*c) + a*d)^(1/4)*(-(b*x^3) + a*x^4)^(1/4)*ArcTanh[((-(b*c) + a*d
)^(1/4)*x^(1/4))/(d^(1/4)*(-b + a*x)^(1/4))])/(c*d^(1/4)*x^(3/4)*(-b + a*x)
^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\int \frac{\sqrt[4]{-bx^3+ax^4}}{x(-d+cx)}\,dx = \frac{\sqrt[4]{-bx^3+ax^4}\int \frac{\sqrt[4]{-b+ax}}{\sqrt[4]{x}\,(-d+cx)}\,dx}{x^{3/4}\sqrt[4]{-b+ax}}$$

$$= \frac{\left(a\sqrt[4]{-bx^3+ax^4}\right)\int \frac{1}{\sqrt[4]{x}\,(-b+ax)^{3/4}}\,dx}{cx^{3/4}\sqrt[4]{-b+ax}} - \frac{\left((bc-ad)\sqrt[4]{-bx^3+ax^4}\right)\int \frac{1}{\sqrt[4]{x}\,(-b+ax)^{3/4}(-d+cx)}\,dx}{cx^{3/4}\sqrt[4]{-b+ax}}$$

$$= \frac{\left(4a\sqrt[4]{-bx^3+ax^4}\right)\text{Subst}\left(\int \frac{x^2}{\left(-b+ax^4\right)^{3/4}}\,dx, x, \sqrt[4]{x}\right)}{cx^{3/4}\sqrt[4]{-b+ax}} - \frac{\left(4(bc-ad)\sqrt[4]{-bx^3+ax^4}\right)\text{Subst}\left(\int \cdots\right.}{cx^{3/4}\sqrt[4]{-b+ax}}$$

$$= \frac{\left(4a\sqrt[4]{-bx^3+ax^4}\right)\text{Subst}\left(\int \frac{x^2}{1-ax^4}\,dx, x, \frac{\sqrt[4]{x}}{\sqrt[4]{-b+ax}}\right)}{cx^{3/4}\sqrt[4]{-b+ax}} + \frac{\left(2(bc-ad)\sqrt[4]{-bx^3+ax^4}\right)\text{Subst}\left(\int \cdots\right.}{c\sqrt{-bc+ad}\,x^{3/4}\sqrt[4]{\cdots}}$$

$$= \frac{2\sqrt[4]{-bc+ad}\,\sqrt[4]{-bx^3+ax^4}\,\tan^{-1}\left(\frac{\sqrt[4]{-bc+ad}\,\sqrt[4]{x}}{\sqrt[4]{d}\,\sqrt[4]{-b+ax}}\right)}{c\sqrt[4]{d}\,x^{3/4}\sqrt[4]{-b+ax}} - \frac{2\sqrt[4]{-bc+ad}\,\sqrt[4]{-bx^3+ax^4}\,\tanh^{-1}\left(\frac{\sqrt[4]{-bc+\cdots}}{\sqrt[4]{d}\,\sqrt[4]{\cdots}}\right.}{c\sqrt[4]{d}\,x^{3/4}\sqrt[4]{-b+ax}}$$

$$= -\frac{2\sqrt[4]{a}\,\sqrt[4]{-bx^3+ax^4}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt[4]{x}}{\sqrt[4]{-b+ax}}\right)}{cx^{3/4}\sqrt[4]{-b+ax}} + \frac{2\sqrt[4]{-bc+ad}\,\sqrt[4]{-bx^3+ax^4}\,\tan^{-1}\left(\frac{\sqrt[4]{-bc+ad}\,\sqrt[4]{x}}{\sqrt[4]{d}\,\sqrt[4]{-b+ax}}\right)}{c\sqrt[4]{d}\,x^{3/4}\sqrt[4]{-b+ax}} + \frac{2\cdots}{\cdots}$$

Mathematica [C] time = 0.07, size = 105, normalized size = 0.36

$$-\frac{4\sqrt[4]{x^3(ax-b)}\left((bc-ad)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{bcx-adx}{bd-adx}\right)+ad\left(1-\frac{ax}{b}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{ax}{b}\right)\right)}{3cd(b-ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-(b*x^3) + a*x^4)^(1/4)/(x*(-d + c*x)),x]
```

```
[Out] (-4*(x^3*(-b + a*x))^(1/4)*(a*d*(1 - (a*x)/b)^(3/4)*Hypergeometric2F1[3/4,
3/4, 7/4, (a*x)/b] + (b*c - a*d)*Hypergeometric2F1[3/4, 1, 7/4, (b*c*x - a*
d*x)/(b*d - a*d*x)]))/(3*c*d*(b - a*x))
```

IntegrateAlgebraic [A] time = 1.46, size = 293, normalized size = 1.01

$$-\frac{\sqrt{2}\sqrt[4]{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x\sqrt[4]{ax^4-bx^3}\sqrt[4]{bc-ad}}{x^2\sqrt{bc-ad}-\sqrt{d}\sqrt{ax^4-bx^3}}\right)}{c\sqrt[4]{d}}-\frac{\sqrt{2}\sqrt[4]{bc-ad}\,\tanh^{-1}\left(\frac{\frac{x^2\sqrt[4]{bc-ad}}{\sqrt{2}\sqrt[4]{d}}+\frac{\sqrt[4]{d}\sqrt{ax^4-bx^3}}{\sqrt{2}\sqrt[4]{bc-ad}}}{x\sqrt[4]{ax^4-bx^3}}\right)}{c\sqrt[4]{d}}-\frac{2\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-bx^3}}\right)}{c}+\frac{2\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4-bx^3}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(b*x^3) + a*x^4)^(1/4)/(x*(-d + c*x)),x]
```

```
[Out] (-2*a^(1/4)*ArcTan[(a^(1/4)*x)/(-(b*x^3) + a*x^4)^(1/4)])/c - (Sqrt[2]*(b*c
 - a*d)^(1/4)*ArcTan[(Sqrt[2]*d^(1/4)*(b*c - a*d)^(1/4)*x*(-(b*x^3) + a*x^4
)^(1/4))/(Sqrt[b*c - a*d]*x^2 - Sqrt[d]*Sqrt[-(b*x^3) + a*x^4])])/(c*d^(1/4
)) + (2*a^(1/4)*ArcTanh[(a^(1/4)*x)/(-(b*x^3) + a*x^4)^(1/4)])/c - (Sqrt[2]
*(b*c - a*d)^(1/4)*ArcTanh[(((b*c - a*d)^(1/4)*x^2)/(Sqrt[2]*d^(1/4)) + (d^
(1/4)*Sqrt[-(b*x^3) + a*x^4])/(Sqrt[2]*(b*c - a*d)^(1/4)))/(x*(-(b*x^3) + a
*x^4)^(1/4))])/(c*d^(1/4))
```

fricas [A] time = 0.68, size = 441, normalized size = 1.52

$$4\left(-\frac{bc-ad}{c^4d}\right)^{\frac{1}{4}}\arctan\left(-\frac{c^3dx\sqrt{\frac{c^2x^2\sqrt{-\frac{bc-ad}{c^4d}}+\sqrt{ax^4-bx^3}}{x^2}}\left(-\frac{bc-ad}{c^4d}\right)^{\frac{3}{4}}-\left(ax^4-bx^3\right)^{\frac{1}{4}}c^3d\left(-\frac{bc-ad}{c^4d}\right)^{\frac{3}{4}}}{(bc-ad)x}\right)-4\left(\frac{a}{c^4}\right)^{\frac{1}{4}}\arctan\left(\frac{c^3x\sqrt{\frac{c^2x^2\sqrt{\frac{a}{c^4}}+\sqrt{ax^4-bx^3}}{x^2}}\left(\frac{a}{c^4}\right)^{\frac{3}{4}}-\left(ax^4-bx^3\right)^{\frac{1}{4}}c^3\left(\frac{a}{c^4}\right)^{\frac{3}{4}}}{ax}\right)+\left(\frac{a}{c^4}\right)^{\frac{1}{4}}\log\left(\frac{cx\left(\frac{a}{c^4}\right)^{\frac{1}{4}}+\left(ax^4-bx^3\right)^{\frac{1}{4}}}{x}\right)-\left(\frac{a}{c^4}\right)^{\frac{1}{4}}\log\left(-\frac{cx\left(\frac{a}{c^4}\right)^{\frac{1}{4}}-\left(ax^4-bx^3\right)^{\frac{1}{4}}}{x}\right)-\left(-\frac{bc-ad}{c^4d}\right)^{\frac{1}{4}}\log\left(\frac{cx\left(-\frac{bc-ad}{c^4d}\right)^{\frac{1}{4}}+\left(ax^4-bx^3\right)^{\frac{1}{4}}}{x}\right)+\left(-\frac{bc-ad}{c^4d}\right)^{\frac{1}{4}}\log\left(-\frac{cx\left(-\frac{bc-ad}{c^4d}\right)^{\frac{1}{4}}-\left(ax^4-bx^3\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x^3)^(1/4)/x/(c*x-d),x, algorithm="fricas")
```

```
[Out] 4*(-(b*c - a*d)/(c^4*d))^(1/4)*arctan(-(c^3*d*x*sqrt((c^2*x^2*sqrt(-(b*c -
a*d)/(c^4*d)) + sqrt(a*x^4 - b*x^3))/x^2)*(-(b*c - a*d)/(c^4*d))^(3/4) - (a
*x^4 - b*x^3)^(1/4)*c^3*d*(-(b*c - a*d)/(c^4*d))^(3/4))/((b*c - a*d)*x)) -
4*(a/c^4)^(1/4)*arctan((c^3*x*sqrt((c^2*x^2*sqrt(a/c^4) + sqrt(a*x^4 - b*x^
3))/x^2)*(a/c^4)^(3/4) - (a*x^4 - b*x^3)^(1/4)*c^3*(a/c^4)^(3/4))/(a*x)) +
(a/c^4)^(1/4)*log((c*x*(a/c^4)^(1/4) + (a*x^4 - b*x^3)^(1/4))/x) - (a/c^4)^
(1/4)*log(-(c*x*(a/c^4)^(1/4) - (a*x^4 - b*x^3)^(1/4))/x) - (-(b*c - a*d)/(
c^4*d))^(1/4)*log((c*x*(-(b*c - a*d)/(c^4*d))^(1/4) + (a*x^4 - b*x^3)^(1/4)
)/x) + (-(b*c - a*d)/(c^4*d))^(1/4)*log(-(c*x*(-(b*c - a*d)/(c^4*d))^(1/4)
- (a*x^4 - b*x^3)^(1/4))/x)
```

giac [B] time = 0.51, size = 505, normalized size = 1.74

$$\frac{\sqrt{2}\,(-a)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{c}+\frac{\sqrt{2}\,(-a)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{c}+\frac{\sqrt{2}\,(-a)^{\frac{1}{4}}\log\left(\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a-\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x}}\right)}{2\,c}-\frac{\sqrt{2}\,(-a)^{\frac{1}{4}}\log\left(-\sqrt{2}\,(-a)^{\frac{1}{4}}\left(a-\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x}}\right)}{2\,c}-\frac{\sqrt{2}\left(bcd^3-ad^4\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{bc-ad}{d}\right)^{\frac{1}{4}}+2\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2\left(\frac{bc-ad}{d}\right)^{\frac{1}{4}}}\right)}{cd}-\frac{\sqrt{2}\left(bcd^3-ad^4\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{bc-ad}{d}\right)^{\frac{1}{4}}-2\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2\left(\frac{bc-ad}{d}\right)^{\frac{1}{4}}}\right)}{cd}-\frac{\sqrt{2}\left(bcd^3-ad^4\right)^{\frac{1}{4}}\log\left(\sqrt{2}\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{4}}+\sqrt{a-\frac{b}{x}}+\sqrt{\frac{bc-ad}{d}}\right)}{2\,cd}+\frac{\sqrt{2}\left(bcd^3-ad^4\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{4}}+\sqrt{a-\frac{b}{x}}+\sqrt{\frac{bc-ad}{d}}\right)}{2\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x^3)^(1/4)/x/(c*x-d),x, algorithm="giac")
```

```
[Out] sqrt(2)*(-a)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(a - b/x)^(1/
4))/(-a)^(1/4))/c + sqrt(2)*(-a)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1
/4) - 2*(a - b/x)^(1/4))/(-a)^(1/4))/c + 1/2*sqrt(2)*(-a)^(1/4)*log(sqrt(2)
*(-a)^(1/4)*(a - b/x)^(1/4) + sqrt(-a) + sqrt(a - b/x))/c - 1/2*sqrt(2)*(-a
)^(1/4)*log(-sqrt(2)*(-a)^(1/4)*(a - b/x)^(1/4) + sqrt(-a) + sqrt(a - b/x))
/c - sqrt(2)*(b*c*d^3 - a*d^4)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*((b*c - a*
d)/d)^(1/4) + 2*(a - b/x)^(1/4))/((b*c - a*d)/d)^(1/4))/(c*d) - sqrt(2)*(b*
c*d^3 - a*d^4)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*((b*c - a*d)/d)^(1/4) - 2
*(a - b/x)^(1/4))/((b*c - a*d)/d)^(1/4))/(c*d) - 1/2*sqrt(2)*(b*c*d^3 - a*d
^4)^(1/4)*log(sqrt(2)*(a - b/x)^(1/4)*((b*c - a*d)/d)^(1/4) + sqrt(a - b/x)
 + sqrt((b*c - a*d)/d))/(c*d) + 1/2*sqrt(2)*(b*c*d^3 - a*d^4)^(1/4)*log(-sq
rt(2)*(a - b/x)^(1/4)*((b*c - a*d)/d)^(1/4) + sqrt(a - b/x) + sqrt((b*c - a
*d)/d))/(c*d)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{4}-b\,x^{3}\right)^{\frac{1}{4}}}{x\,(cx-d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b*x^3)^(1/4)/x/(c*x-d),x)
```

```
[Out] int((a*x^4-b*x^3)^(1/4)/x/(c*x-d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{4}-bx^{3}\right)^{\frac{1}{4}}}{(cx-d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b*x^3)^(1/4)/x/(c*x-d),x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 - b*x^3)^(1/4)/((c*x - d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a\,x^4-b\,x^3\right)^{1/4}}{x\,\left(d-c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x^4 - b*x^3)^(1/4)/(x*(d - c*x)),x)
[Out] int(-(a*x^4 - b*x^3)^(1/4)/(x*(d - c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^3\,(ax-b)}}{x\,(cx-d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b*x**3)**(1/4)/x/(c*x-d),x)
[Out] Integral((x**3*(a*x - b))**(1/4)/(x*(c*x - d)), x)
```

3.2257

$$\int \frac{(-a+x)(-b+x)\left(-2abx+(a+b)x^2\right)}{(x(-a+x)(-b+x))^{2/3}\left(a^2b^2d-2ab(a+b)dx+\left(a^2+4ab+b^2\right)dx^2-2(a+b)dx^3+(-1+d)x^4\right)}\,dx$$

Optimal. Leaf size=291

$$\frac{\log\left(x-\sqrt[6]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}\right)}{2d^{2/3}}+\frac{\log\left(\sqrt[6]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}+x\right)}{2d^{2/3}}-\frac{\log\left(-\sqrt[6]{d}\,x\sqrt[3]{x^2(-a-b)+abx\right.}{}$$

Rubi [F] time = 48.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-a+x)(-b+x)\left(-2abx+(a+b)x^2\right)}{(x(-a+x)(-b+x))^{2/3}\left(a^2b^2d-2ab(a+b)dx+\left(a^2+4ab+b^2\right)dx^2-2(a+b)dx^3+(-1+d)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-a + x)*(-b + x)*(-2*a*b*x + (a + b)*x^2))/((x*(-a + x)*(-b + x))^(2/
3)*(a^2*b^2*d - 2*a*b*(a + b)*d*x + (a^2 + 4*a*b + b^2)*d*x^2 - 2*(a + b)*d
*x^3 + (-1 + d)*x^4)),x]
```

```
[Out] (3*(a + b)*x^(2/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x
^6*(-a + x^3)^(1/3)*(-b + x^3)^(1/3))/(a^2*b^2*d - 2*a^2*b*(1 + b/a)*d*x^3
+ a^2*(1 + (b*(4*a + b))/a^2)*d*x^6 - 2*a*(1 + b/a)*d*x^9 - (1 - d)*x^12),
x], x, x^(1/3)])/((a - x)*(b - x)*x)^(2/3) + (6*a*b*x^(2/3)*(-a + x)^(2/3)*
(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x^3*(-a + x^3)^(1/3)*(-b + x^3)^(1/
3))/(-(a^2*b^2*d) + 2*a^2*b*(1 + b/a)*d*x^3 - a^2*(1 + (b*(4*a + b))/a^2)*d
*x^6 + 2*a*(1 + b/a)*d*x^9 + (1 - d)*x^12), x], x, x^(1/3)])/((a - x)*(b -
x)*x)^(2/3)
```

Rubi steps

$$\int \frac{(-a+x)(-b+x)\left(-2abx+(a+b)x^2\right)}{(x(-a+x)(-b+x))^{2/3}\left(a^2b^2d-2ab(a+b)dx+\left(a^2+4ab+b^2\right)dx^2-2(a+b)dx^3+(-1+d)x^4\right)}\,dx=\int \frac{}{(x}$$

$$=\frac{\left(x^{2/3}\right.}{}$$

$$=\frac{\left(3x^{2/}\right.}{}$$

$$=\frac{\left(3x^{2/}\right.}{}$$

$$=\frac{\left(6ab\right.}{}$$

Mathematica [F] time = 1.86, size = 0, normalized size = 0.00

$$\int \frac{(-a+x)(-b+x)\left(-2abx+(a+b)x^2\right)}{(x(-a+x)(-b+x))^{2/3}\left(a^2b^2d-2ab(a+b)dx+\left(a^2+4ab+b^2\right)dx^2-2(a+b)dx^3+(-1+d)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-a + x)*(-b + x)*(-2*a*b*x + (a + b)*x^2))/((x*(-a + x)*(-b + x
))^(2/3)*(a^2*b^2*d - 2*a*b*(a + b)*d*x + (a^2 + 4*a*b + b^2)*d*x^2 - 2*(a
+ b)*d*x^3 + (-1 + d)*x^4)),x]
```

```
[Out] Integrate[((-a + x)*(-b + x)*(-2*a*b*x + (a + b)*x^2))/((x*(-a + x)*(-b + x
))^(2/3)*(a^2*b^2*d - 2*a*b*(a + b)*d*x + (a^2 + 4*a*b + b^2)*d*x^2 - 2*(a
+ b)*d*x^3 + (-1 + d)*x^4)), x]
```

IntegrateAlgebraic [A] time = 1.28, size = 291, normalized size = 1.00

$$\frac{\log\left(x-\sqrt[6]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}\right)}{2d^{2/3}}+\frac{\log\left(\sqrt[6]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}+x\right)}{2d^{2/3}}-\frac{\log\left(-\sqrt[6]{d}\,x\sqrt[3]{x^2(-a-b)+abx+x^3}+\sqrt[3]{d}\left(x^2(-a-b)+abx+x^3\right)^{2/3}+x^2\right)}{4d^{2/3}}-\frac{\log\left(\sqrt[6]{d}\,x\sqrt[3]{x^2(-a-b)+abx+x^3}+\sqrt[3]{d}\left(x^2(-a-b)+abx+x^3\right)^{2/3}+x^2\right)}{4d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\sqrt[3]{d}\left(x^2(-a-b)+abx+x^3\right)^{2/3}+x^2}\right)}{2d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-a + x)*(-b + x)*(-2*a*b*x + (a + b)*x^2))/((x*(-a + x
)*(-b + x))^(2/3)*(a^2*b^2*d - 2*a*b*(a + b)*d*x + (a^2 + 4*a*b + b^2)*d*x^
2 - 2*(a + b)*d*x^3 + (-1 + d)*x^4)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*x^2)/(x^2 + 2*d^(1/3)*(a*b*x + (-a - b)*x^2 +
 x^3)^(2/3))]])/d^(2/3) + Log[x - d^(1/6)*(a*b*x + (-a - b)*x^2 + x^3)^(1/3)
]/(2*d^(2/3)) + Log[x + d^(1/6)*(a*b*x + (-a - b)*x^2 + x^3)^(1/3)]/(2*d^(2
/3)) - Log[x^2 - d^(1/6)*x*(a*b*x + (-a - b)*x^2 + x^3)^(1/3) + d^(1/3)*(a*
b*x + (-a - b)*x^2 + x^3)^(2/3)]/(4*d^(2/3)) - Log[x^2 + d^(1/6)*x*(a*b*x +
 (-a - b)*x^2 + x^3)^(1/3) + d^(1/3)*(a*b*x + (-a - b)*x^2 + x^3)^(2/3)]/(4
*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-2*a*b*x+(a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a^2*b
^2*d-2*a*b*(a+b)*d*x+(a^2+4*a*b+b^2)*d*x^2-2*(a+b)*d*x^3+(-1+d)*x^4),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.25, size = 317, normalized size = 1.09

$$-\frac{|d|\log\left(\left(\frac{ab}{x^2}-\frac{a}{x}-\frac{b}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{d}\right)^{\frac{1}{3}}\right)}{2\left(-d^5\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\left(-d^5\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(-\frac{1}{d}\right)^{\frac{1}{6}}+2\left(\frac{ab}{x^2}-\frac{a}{x}-\frac{b}{x}+1\right)^{\frac{1}{3}}}{\left(-\frac{1}{d}\right)^{\frac{1}{6}}}\right)}{2d^4}-\frac{\sqrt{3}\left(-d^5\right)^{\frac{2}{3}}\arctan\left(-\frac{\sqrt{3}\left(-\frac{1}{d}\right)^{\frac{1}{6}}-2\left(\frac{ab}{x^2}-\frac{a}{x}-\frac{b}{x}+1\right)^{\frac{1}{3}}}{\left(-\frac{1}{d}\right)^{\frac{1}{6}}}\right)}{2d^4}-\frac{\left(-d^5\right)^{\frac{2}{3}}\log\left(\sqrt{3}\left(\frac{ab}{x^2}-\frac{a}{x}-\frac{b}{x}+1\right)^{\frac{1}{3}}\left(-\frac{1}{d}\right)^{\frac{1}{6}}+\left(\frac{ab}{x^2}-\frac{a}{x}-\frac{b}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{d}\right)^{\frac{1}{3}}\right)}{4d^4}-\frac{\left(-d^5\right)^{\frac{2}{3}}\log\left(-\sqrt{3}\left(\frac{ab}{x^2}-\frac{a}{x}-\frac{b}{x}+1\right)^{\frac{1}{3}}\left(-\frac{1}{d}\right)^{\frac{1}{6}}+\left(\frac{ab}{x^2}-\frac{a}{x}-\frac{b}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{d}\right)^{\frac{1}{3}}\right)}{4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-2*a*b*x+(a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a^2*b
^2*d-2*a*b*(a+b)*d*x+(a^2+4*a*b+b^2)*d*x^2-2*(a+b)*d*x^3+(-1+d)*x^4),x, alg
orithm="giac")
```

```
[Out] -1/2*abs(d)*log((a*b/x^2 - a/x - b/x + 1)^(2/3) + (-1/d)^(1/3))/(-d^5)^(1/3
) + 1/2*sqrt(3)*(-d^5)^(2/3)*arctan((sqrt(3)*(-1/d)^(1/6) + 2*(a*b/x^2 - a/
x - b/x + 1)^(1/3))/(-1/d)^(1/6))/d^4 - 1/2*sqrt(3)*(-d^5)^(2/3)*arctan(-(s
qrt(3)*(-1/d)^(1/6) - 2*(a*b/x^2 - a/x - b/x + 1)^(1/3))/(-1/d)^(1/6))/d^4
- 1/4*(-d^5)^(2/3)*log(sqrt(3)*(a*b/x^2 - a/x - b/x + 1)^(1/3)*(-1/d)^(1/6)
 + (a*b/x^2 - a/x - b/x + 1)^(2/3) + (-1/d)^(1/3))/d^4 - 1/4*(-d^5)^(2/3)*l
og(-sqrt(3)*(a*b/x^2 - a/x - b/x + 1)^(1/3)*(-1/d)^(1/6) + (a*b/x^2 - a/x -
 b/x + 1)^(2/3) + (-1/d)^(1/3))/d^4
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(-a+x)(-b+x)\left(-2abx+(a+b)x^2\right)}{\left(x(-a+x)(-b+x)\right)^{\frac{2}{3}}\left(a^2b^2d-2ab(a+b)dx+\left(a^2+4ab+b^2\right)dx^2-2(a+b)dx^3+(-1+d)x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x)*(-b+x)*(-2*a*b*x+(a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a^2*b^2*d-2
*a*b*(a+b)*d*x+(a^2+4*a*b+b^2)*d*x^2-2*(a+b)*d*x^3+(-1+d)*x^4),x)
```

```
[Out] int((-a+x)*(-b+x)*(-2*a*b*x+(a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a^2*b^2*d-2
*a*b*(a+b)*d*x+(a^2+4*a*b+b^2)*d*x^2-2*(a+b)*d*x^3+(-1+d)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(2\,abx-(a+b)x^2\right)(a-x)(b-x)}{\left(a^2b^2d-2\,(a+b)abdx-2\,(a+b)dx^3+(d-1)x^4+\left(a^2+4\,ab+b^2\right)dx^2\right)((a-x)(b-x)x)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-2*a*b*x+(a+b)*x^2)/(x*(-a+x)*(-b+x))^(2/3)/(a^2*b
^2*d-2*a*b*(a+b)*d*x+(a^2+4*a*b+b^2)*d*x^2-2*(a+b)*d*x^3+(-1+d)*x^4),x, alg
orithm="maxima")
```

```
[Out] -integrate((2*a*b*x - (a + b)*x^2)*(a - x)*(b - x)/((a^2*b^2*d - 2*(a + b)*
a*b*d*x - 2*(a + b)*d*x^3 + (d - 1)*x^4 + (a^2 + 4*a*b + b^2)*d*x^2)*((a -
x)*(b - x)*x)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^2\,\left(a+b\right)-2\,a\,b\,x\right)\,\left(a-x\right)\,\left(b-x\right)}{{\left(x\,\left(a-x\right)\,\left(b-x\right)\right)}^{2/3}\,\left(x^4\,\left(d-1\right)+a^2\,b^2\,d+d\,x^2\,\left(a^2+4\,a\,b+b^2\right)-2\,d\,x^3\,\left(a+b\right)-2\,a\,b\,d\,x\,\left(a+b\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2*(a + b) - 2*a*b*x)*(a - x)*(b - x))/((x*(a - x)*(b - x))^(2/3)*(x
^4*(d - 1) + a^2*b^2*d + d*x^2*(4*a*b + a^2 + b^2) - 2*d*x^3*(a + b) - 2*a*
b*d*x*(a + b))),x)
```

```
[Out] int(((x^2*(a + b) - 2*a*b*x)*(a - x)*(b - x))/((x*(a - x)*(b - x))^(2/3)*(x
^4*(d - 1) + a^2*b^2*d + d*x^2*(4*a*b + a^2 + b^2) - 2*d*x^3*(a + b) - 2*a*
b*d*x*(a + b))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)*(-2*a*b*x+(a+b)*x**2)/(x*(-a+x)*(-b+x))**(2/3)/(a**
2*b**2*d-2*a*b*(a+b)*d*x+(a**2+4*a*b+b**2)*d*x**2-2*(a+b)*d*x**3+(-1+d)*x**
4),x)
```

```
[Out] Timed out
```

3.2258 $\int \frac{1+x^3+x^6}{\sqrt[4]{x^3+x^5}\left(1-x^6\right)} \, dx$

Optimal. Leaf size=291

$$\frac{1}{3}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^5+x^3}}\right)+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{2\sqrt[4]{2}}+\frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^5+x^3}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{2\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^5+x^3}}{\sqrt{2}x^2-\sqrt{x^5+x^3}}\right)}{6\,2^{3/4}}+\frac{\tan^{-1}\left(\right.}{}$$

Rubi [F] time = 180.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\text{\$Aborted}$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^3 + x^6)/((x^3 + x^5)^(1/4)*(1 - x^6)),x]
```

```
[Out] $Aborted
```

Rubi steps

Aborted

Mathematica [F] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^3+x^6}{\sqrt[4]{x^3+x^5}\left(1-x^6\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^3 + x^6)/((x^3 + x^5)^(1/4)*(1 - x^6)),x]
```

```
[Out] Integrate[(1 + x^3 + x^6)/((x^3 + x^5)^(1/4)*(1 - x^6)), x]
```

IntegrateAlgebraic [A] time = 3.16, size = 291, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^5+x^3}}\right)+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{2\sqrt[4]{2}}+\frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^5+x^3}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{2\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^5+x^3}}{\sqrt{2}x^2-\sqrt{x^5+x^3}}\right)}{6\,2^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x\sqrt[4]{x^5+x^3}}{\sqrt{x^5+x^3}-x^2}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^5+x^3}}{2^{3/4}}}{x\sqrt[4]{x^5+x^3}}\right)}{6\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt{2}}+\frac{\sqrt{x^5+x^3}}{\sqrt{2}}}{x\sqrt[4]{x^5+x^3}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3 + x^6)/((x^3 + x^5)^(1/4)*(1 - x^6)),x]
```

```
[Out] ArcTan[x/(x^3 + x^5)^(1/4)]/3 + ArcTan[(2^(1/4)*x)/(x^3 + x^5)^(1/4)]/(2*2^
(1/4)) - ArcTan[(2^(3/4)*x*(x^3 + x^5)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^3 + x^5
])]/(6*2^(3/4)) + ArcTan[(Sqrt[2]*x*(x^3 + x^5)^(1/4))/(-x^2 + Sqrt[x^3 + x
^5])]/Sqrt[2] + ArcTanh[x/(x^3 + x^5)^(1/4)]/3 + ArcTanh[(2^(1/4)*x)/(x^3 +
 x^5)^(1/4)]/(2*2^(1/4)) + ArcTanh[(x^2/2^(1/4) + Sqrt[x^3 + x^5]/2^(3/4))/
(x*(x^3 + x^5)^(1/4))]/(6*2^(3/4)) + ArcTanh[(x^2/Sqrt[2] + Sqrt[x^3 + x^5]
/Sqrt[2])/(x*(x^3 + x^5)^(1/4))]/Sqrt[2]
```

fricas [B] time = 78.74, size = 1837, normalized size = 6.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^5+x^3)^(1/4)/(-x^6+1),x, algorithm="fricas")
```

```
[Out] -1/4*2^(3/4)*arctan(-1/2*(4*2^(3/4)*(x^5 + x^3)^(1/4)*x^2 - 2^(3/4)*(2*2^(3
/4)*sqrt(x^5 + x^3)*x + 2^(1/4)*(x^4 + 2*x^3 + x^2)) + 4*2^(1/4)*(x^5 + x^3
)^(3/4))/(x^4 - 2*x^3 + x^2)) + 1/16*2^(3/4)*log((4*sqrt(2)*(x^5 + x^3)^(1/
4)*x^2 + 2^(3/4)*(x^4 + 2*x^3 + x^2) + 4*2^(1/4)*sqrt(x^5 + x^3)*x + 4*(x^5
 + x^3)^(3/4))/(x^4 - 2*x^3 + x^2)) - 1/16*2^(3/4)*log((4*sqrt(2)*(x^5 + x^
3)^(1/4)*x^2 - 2^(3/4)*(x^4 + 2*x^3 + x^2) - 4*2^(1/4)*sqrt(x^5 + x^3)*x +
4*(x^5 + x^3)^(3/4))/(x^4 - 2*x^3 + x^2)) - 1/2*sqrt(2)*arctan(-(x^6 + 2*x^
5 + 3*x^4 + 2*x^3 + 2*sqrt(2)*(x^5 + x^3)^(3/4)*(x^2 - 3*x + 1) + x^2 + 2*s
qrt(2)*(x^5 + x^3)^(1/4)*(3*x^4 - x^3 + 3*x^2) + 4*sqrt(x^5 + x^3)*(x^3 + x
^2 + x) - (2*sqrt(2)*sqrt(x^5 + x^3)*(x^3 - 3*x^2 + x) + 16*(x^5 + x^3)^(3/
4)*x + sqrt(2)*(x^6 - 8*x^5 + x^4 - 8*x^3 + x^2) + 4*(x^5 + x^3)^(1/4)*(x^4
 + x^3 + x^2))*sqrt((x^4 + x^3 + 2*sqrt(2)*(x^5 + x^3)^(1/4)*x^2 + x^2 + 4*
sqrt(x^5 + x^3)*x + 2*sqrt(2)*(x^5 + x^3)^(3/4))/(x^4 + x^3 + x^2)))/(x^6 -
 14*x^5 + 3*x^4 - 14*x^3 + x^2)) + 1/2*sqrt(2)*arctan(-(x^6 + 2*x^5 + 3*x^4
 + 2*x^3 - 2*sqrt(2)*(x^5 + x^3)^(3/4)*(x^2 - 3*x + 1) + x^2 - 2*sqrt(2)*(x
^5 + x^3)^(1/4)*(3*x^4 - x^3 + 3*x^2) + 4*sqrt(x^5 + x^3)*(x^3 + x^2 + x) +
 (2*sqrt(2)*sqrt(x^5 + x^3)*(x^3 - 3*x^2 + x) - 16*(x^5 + x^3)^(3/4)*x + sq
rt(2)*(x^6 - 8*x^5 + x^4 - 8*x^3 + x^2) - 4*(x^5 + x^3)^(1/4)*(x^4 + x^3 +
x^2))*sqrt((x^4 + x^3 - 2*sqrt(2)*(x^5 + x^3)^(1/4)*x^2 + x^2 + 4*sqrt(x^5
+ x^3)*x - 2*sqrt(2)*(x^5 + x^3)^(3/4))/(x^4 + x^3 + x^2)))/(x^6 - 14*x^5 +
 3*x^4 - 14*x^3 + x^2)) + 1/8*sqrt(2)*log(4*(x^4 + x^3 + 2*sqrt(2)*(x^5 + x
^3)^(1/4)*x^2 + x^2 + 4*sqrt(x^5 + x^3)*x + 2*sqrt(2)*(x^5 + x^3)^(3/4))/(x
^4 + x^3 + x^2)) - 1/8*sqrt(2)*log(4*(x^4 + x^3 - 2*sqrt(2)*(x^5 + x^3)^(1/
4)*x^2 + x^2 + 4*sqrt(x^5 + x^3)*x - 2*sqrt(2)*(x^5 + x^3)^(3/4))/(x^4 + x^
3 + x^2)) + 1/12*2^(1/4)*arctan(1/2*(2*x^6 + 8*x^5 + 12*x^4 + 8*x^3 + 4*2^(
3/4)*(x^5 + x^3)^(3/4)*(x^2 - 6*x + 1) + 8*sqrt(2)*sqrt(x^5 + x^3)*(x^3 + 2
*x^2 + x) + 2*x^2 + sqrt(2)*(32*sqrt(2)*(x^5 + x^3)^(3/4)*x + 2^(3/4)*(x^6
- 16*x^5 - 2*x^4 - 16*x^3 + x^2) + 4*2^(1/4)*sqrt(x^5 + x^3)*(x^3 - 6*x^2 +
 x) + 8*(x^5 + x^3)^(1/4)*(x^4 + 2*x^3 + x^2))*sqrt((4*2^(3/4)*(x^5 + x^3)^
(1/4)*x^2 + sqrt(2)*(x^4 + 2*x^3 + x^2) + 8*sqrt(x^5 + x^3)*x + 4*2^(1/4)*(
x^5 + x^3)^(3/4))/(x^4 + 2*x^3 + x^2)) + 8*2^(1/4)*(x^5 + x^3)^(1/4)*(3*x^4
 - 2*x^3 + 3*x^2))/(x^6 - 28*x^5 + 6*x^4 - 28*x^3 + x^2)) - 1/12*2^(1/4)*ar
ctan(1/2*(2*x^6 + 8*x^5 + 12*x^4 + 8*x^3 - 4*2^(3/4)*(x^5 + x^3)^(3/4)*(x^2
 - 6*x + 1) + 8*sqrt(2)*sqrt(x^5 + x^3)*(x^3 + 2*x^2 + x) + 2*x^2 + sqrt(2)
*(32*sqrt(2)*(x^5 + x^3)^(3/4)*x - 2^(3/4)*(x^6 - 16*x^5 - 2*x^4 - 16*x^3 +
 x^2) - 4*2^(1/4)*sqrt(x^5 + x^3)*(x^3 - 6*x^2 + x) + 8*(x^5 + x^3)^(1/4)*(
x^4 + 2*x^3 + x^2))*sqrt(-(4*2^(3/4)*(x^5 + x^3)^(1/4)*x^2 - sqrt(2)*(x^4 +
 2*x^3 + x^2) - 8*sqrt(x^5 + x^3)*x + 4*2^(1/4)*(x^5 + x^3)^(3/4))/(x^4 + 2
*x^3 + x^2)) - 8*2^(1/4)*(x^5 + x^3)^(1/4)*(3*x^4 - 2*x^3 + 3*x^2))/(x^6 -
28*x^5 + 6*x^4 - 28*x^3 + x^2)) + 1/48*2^(1/4)*log(8*(4*2^(3/4)*(x^5 + x^3)
^(1/4)*x^2 + sqrt(2)*(x^4 + 2*x^3 + x^2) + 8*sqrt(x^5 + x^3)*x + 4*2^(1/4)*
(x^5 + x^3)^(3/4))/(x^4 + 2*x^3 + x^2)) - 1/48*2^(1/4)*log(-8*(4*2^(3/4)*(x
^5 + x^3)^(1/4)*x^2 - sqrt(2)*(x^4 + 2*x^3 + x^2) - 8*sqrt(x^5 + x^3)*x + 4
*2^(1/4)*(x^5 + x^3)^(3/4))/(x^4 + 2*x^3 + x^2)) + 1/6*arctan(2*((x^5 + x^3
)^(1/4)*x^2 + (x^5 + x^3)^(3/4))/(x^4 - x^3 + x^2)) + 1/6*log((x^4 + x^3 +
2*(x^5 + x^3)^(1/4)*x^2 + x^2 + 2*sqrt(x^5 + x^3)*x + 2*(x^5 + x^3)^(3/4))/
(x^4 - x^3 + x^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^6 + x^3 + 1}{\left(x^6 - 1\right)\left(x^5 + x^3\right)^{\frac{1}{4}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^5+x^3)^(1/4)/(-x^6+1),x, algorithm="giac")
```

```
[Out] integrate(-(x^6 + x^3 + 1)/((x^6 - 1)*(x^5 + x^3)^(1/4)), x)
```

maple [C] time = 28.14, size = 1434, normalized size = 4.93

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+x^3+1)/(x^5+x^3)^(1/4)/(-x^6+1),x)
```

```
[Out] -1/8*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln((RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf
(_Z^4-8)^2*(x^5+x^3)^(1/2)*x-2*(x^5+x^3)^(1/4)*RootOf(_Z^4-8)^2*x^2-RootOf(
_Z^2+RootOf(_Z^4-8)^2)*x^4-2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^3+4*(x^5+x^3)^
(3/4)-RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^2)/(-1+x)^2/x^2)-1/8*RootOf(_Z^4-8)*l
n((-RootOf(_Z^4-8)^3*(x^5+x^3)^(1/2)*x+2*(x^5+x^3)^(1/4)*RootOf(_Z^4-8)^2*x
^2-RootOf(_Z^4-8)*x^4-2*RootOf(_Z^4-8)*x^3+4*(x^5+x^3)^(3/4)-RootOf(_Z^4-8)
*x^2)/(-1+x)^2/x^2)-1/96*ln((-RootOf(_Z^4-8)^3*x^4-2*RootOf(_Z^4-8)^3*x^3+8
*(x^5+x^3)^(1/4)*RootOf(_Z^4-8)^2*x^2-RootOf(_Z^4-8)^3*x^2-16*(x^5+x^3)^(1/
2)*RootOf(_Z^4-8)*x+16*(x^5+x^3)^(3/4))/(1+x)^2/x^2)*RootOf(_Z^4-8)^3-1/96*
ln((-RootOf(_Z^4-8)^3*x^4-2*RootOf(_Z^4-8)^3*x^3+8*(x^5+x^3)^(1/4)*RootOf(_
Z^4-8)^2*x^2-RootOf(_Z^4-8)^3*x^2-16*(x^5+x^3)^(1/2)*RootOf(_Z^4-8)*x+16*(x
^5+x^3)^(3/4))/(1+x)^2/x^2)*RootOf(_Z^4-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)+
1/48*RootOf(_Z^4-8)^2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln(-(-RootOf(_Z^4-8)^3*
x^4+RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x^4+2*RootOf(_Z^4-8)^3*x
^3-2*RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x^3+8*(x^5+x^3)^(1/4)*R
ootOf(_Z^4-8)*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^2-RootOf(_Z^4-8)^3*x^2+RootOf
(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^2*x^2-8*(x^5+x^3)^(1/2)*RootOf(_Z^4-
8)*x-8*RootOf(_Z^2+RootOf(_Z^4-8)^2)*(x^5+x^3)^(1/2)*x+16*(x^5+x^3)^(3/4))/
(1+x)^2/x^2)+1/48*RootOf(_Z^4-8)^3*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln((RootOf
(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^3*x^4-2*RootOf(_Z^4-8)^3*(x^5+x^3)^(
1/2)*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x+RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_
Z^4-8)^3*x^3+RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)^3*x^2+16*(x^5+x^3
)^(3/4)-16*(x^5+x^3)^(1/4)*x^2)/x^2/(x^2-x+1))-1/8*ln((-RootOf(_Z^4-8)^2*x^
4-4*RootOf(_Z^4-8)^2*(x^5+x^3)^(1/2)*x-RootOf(_Z^4-8)^2*x^3-RootOf(_Z^4-8)^
2*x^2+8*(x^5+x^3)^(3/4)+8*(x^5+x^3)^(1/4)*x^2)/x^2/(x^2+x+1))*RootOf(_Z^4-8
)^2+1/8*ln((-RootOf(_Z^4-8)^2*x^4-4*RootOf(_Z^4-8)^2*(x^5+x^3)^(1/2)*x-Root
Of(_Z^4-8)^2*x^3-RootOf(_Z^4-8)^2*x^2+8*(x^5+x^3)^(3/4)+8*(x^5+x^3)^(1/4)*x
^2)/x^2/(x^2+x+1))*RootOf(_Z^4-8)*RootOf(_Z^2+RootOf(_Z^4-8)^2)-1/4*RootOf(
_Z^4-8)*RootOf(_Z^2+RootOf(_Z^4-8)^2)*ln((-(x^5+x^3)^(1/4)*RootOf(_Z^4-8)^3
*RootOf(_Z^2+RootOf(_Z^4-8)^2)*x^2-RootOf(_Z^4-8)^2*x^4-RootOf(_Z^2+RootOf(
_Z^4-8)^2)*RootOf(_Z^4-8)*x^4-2*RootOf(_Z^4-8)^2*(x^5+x^3)^(1/2)*x+2*RootOf
(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)*(x^5+x^3)^(1/2)*x+RootOf(_Z^4-8)^2*x
^3+RootOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)*x^3-RootOf(_Z^4-8)^2*x^2-Ro
otOf(_Z^2+RootOf(_Z^4-8)^2)*RootOf(_Z^4-8)*x^2+8*(x^5+x^3)^(3/4))/x^2/(x^2+
x+1))-1/6*ln((-x^4+2*(x^5+x^3)^(3/4)-2*(x^5+x^3)^(1/2)*x+2*(x^5+x^3)^(1/4)*
x^2-x^3-x^2)/x^2/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^6+x^3+1}{\left(x^6-1\right)\left(x^5+x^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^5+x^3)^(1/4)/(-x^6+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^6 + x^3 + 1)/((x^6 - 1)*(x^5 + x^3)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x^6+x^3+1}{\left(x^5+x^3\right)^{1/4}\,\left(x^6-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 + x^6 + 1)/((x^3 + x^5)^(1/4)*(x^6 - 1)),x)
```

```
[Out] int(-(x^3 + x^6 + 1)/((x^3 + x^5)^(1/4)*(x^6 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{x^6\sqrt[4]{x^5+x^3}-\sqrt[4]{x^5+x^3}}\,dx-\int \frac{x^6}{x^6\sqrt[4]{x^5+x^3}-\sqrt[4]{x^5+x^3}}\,dx-\int \frac{1}{x^6\sqrt[4]{x^5+x^3}-\sqrt[4]{x^5+x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+x**3+1)/(x**5+x**3)**(1/4)/(-x**6+1),x)
```

```
[Out] -Integral(x**3/(x**6*(x**5 + x**3)**(1/4) - (x**5 + x**3)**(1/4)), x) - Int
egral(x**6/(x**6*(x**5 + x**3)**(1/4) - (x**5 + x**3)**(1/4)), x) - Integra
l(1/(x**6*(x**5 + x**3)**(1/4) - (x**5 + x**3)**(1/4)), x)
```

3.2259 $\int \frac{-a-bx+\left(b+ak^2\right)x^2}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(1-2x+k^2x^2\right)}\,dx$

Optimal. Leaf size=293

$$\frac{\left(2iak^2-2a\sqrt{k^2-1}\,k^2-ibk^2-2b\sqrt{k^2-1}\,+2ib\right)\tan^{-1}\left(\frac{\sqrt{k^2-2i\sqrt{k^2-1}-2}\,\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{k^2(x-1)x}\right)}{2k^2\sqrt{k^2-1}\,\sqrt{k^2-2i\sqrt{k^2-1}\,-2}}+\frac{\left(-2iak^2-2a\sqrt{k^2-1}\,k\right.}{}$$

Rubi [C] time = 3.76, antiderivative size = 385, normalized size of antiderivative = 1.31, number of steps used = 17, number of rules used = 10, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {6718, 6688, 6728, 6, 714, 115, 934, 12, 168, 537}

$$-\frac{(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\left(-2\sqrt{1-k^2}\left(ak^2+b\right)+2ak^2+b\left(2-k^2\right)\right)\Pi\left(\frac{1}{1-\sqrt{1-k^2}};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{\left(-k^2\right)^{3/2}\left(1-\sqrt{1-k^2}\right)\sqrt{x-x^2}\sqrt{(1-x)x\left(1-k^2x\right)}}-\frac{(1-x)\sqrt{-x}\sqrt{x}\sqrt{1-k^2x}\left(2\sqrt{1-k^2}\left(ak^2+b\right)+2ak^2+b\left(2-k^2\right)\right)\Pi\left(\frac{1}{\sqrt{1-k^2}+1};\sin^{-1}\left(\sqrt{-k^2}\sqrt{-x}\right)|\frac{1}{k^2}\right)}{\left(-k^2\right)^{3/2}\left(\sqrt{1-k^2}+1\right)\sqrt{x-x^2}\sqrt{(1-x)x\left(1-k^2x\right)}}+\frac{2\sqrt{1-x}\sqrt{x}\sqrt{1-k^2x}\left(a+\frac{b}{k^2}\right)F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-a - b*x + (b + a*k^2)*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(1 - 2*x + k^
2*x^2)),x]
```

```
[Out] (2*(a + b/k^2)*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticF[ArcSin[Sqrt[x]
], k^2])/Sqrt[(1 - x)*x*(1 - k^2*x)] - ((2*a*k^2 + b*(2 - k^2) - 2*Sqrt[1 -
 k^2]*(b + a*k^2))*(1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1 - k^2*x]*EllipticPi[(1 -
 Sqrt[1 - k^2])^(-1), ArcSin[Sqrt[-k^2]*Sqrt[-x]], k^(-2)])/((-k^2)^(3/2)*(
1 - Sqrt[1 - k^2])*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqrt[x - x^2]) - ((2*a*k^2 +
 b*(2 - k^2) + 2*Sqrt[1 - k^2]*(b + a*k^2))*(1 - x)*Sqrt[-x]*Sqrt[x]*Sqrt[1
 - k^2*x]*EllipticPi[(1 + Sqrt[1 - k^2])^(-1), ArcSin[Sqrt[-k^2]*Sqrt[-x]],
 k^(-2)])/((-k^2)^(3/2)*(1 + Sqrt[1 - k^2])*Sqrt[(1 - x)*x*(1 - k^2*x)]*Sqr
t[x - x^2])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 115

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (GtQ[-(b/d), 0] || LtQ[-(b/f), 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
```

```
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 714

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c*x)/b]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0]
 && RationalQ[b]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !FreeQ[w, x] &&  !Fr
eeQ[z, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-a-bx+\left(b+ak^2\right)x^2}{\sqrt{(1-x)x\left(1-k^2x\right)}\left(1-2x+k^2x^2\right)}\,dx = \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{-a-bx+\left(b+ak^2\right)x^2}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\left(1-2x+k^2x^2\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{-a-bx+\left(b+ak^2\right)x^2}{\sqrt{1-k^2x}\,\sqrt{x-x^2}\left(1-2x+k^2x^2\right)}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \left(\frac{a}{\sqrt{1-k^2x}\,\sqrt{x-x^2}} + \frac{b}{k^2\sqrt{1-k^2x}\,\sqrt{x-x^2}} - \frac{b+2ak^2-(\cdots}{k^2\sqrt{1-k^2x}\,\cdots}\right.}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \left(\frac{a+\frac{b}{k^2}}{\sqrt{1-k^2x}\,\sqrt{x-x^2}} - \frac{b+2ak^2-\left(2ak^2+b\left(2-k^2\right)\right)x}{k^2\sqrt{1-k^2x}\,\sqrt{x-x^2}\left(1-2x+k^2x^2\right)}\right)\cdots}{\sqrt{(1-x)x\left(1-k^2x\right)}}$$

$$= \frac{\left(\left(a+\frac{b}{k^2}\right)\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-k^2x}\,\sqrt{x-x^2}}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\sqrt{1}\cdots\right.}{k\cdots}$$

$$= \frac{\left(\left(a+\frac{b}{k^2}\right)\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\right)\int \frac{1}{\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}}\,dx}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{1-x}\,\sqrt{x}\,\cdots\right.}{\cdots}$$

$$= \frac{2\left(a+\frac{b}{k^2}\right)\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\left(-2ak^2-b\left(2\cdots\right.\right.\right.}{\cdots}$$

$$= \frac{2\left(a+\frac{b}{k^2}\right)\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\sqrt{2}\left(-2ak^2-\cdots\right.\right.}{\cdots}$$

$$= \frac{2\left(a+\frac{b}{k^2}\right)\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(\left(-2ak^2-b\left(2\cdots\right.\right.\right.}{\cdots}$$

$$= \frac{2\left(a+\frac{b}{k^2}\right)\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} + \frac{\left(2\left(-2ak^2-b\left(\cdots\right.\right.\right.}{\cdots}$$

$$= \frac{2\left(a+\frac{b}{k^2}\right)\sqrt{1-x}\,\sqrt{x}\,\sqrt{1-k^2x}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|k^2\right)}{\sqrt{(1-x)x\left(1-k^2x\right)}} - \frac{\left(b\left(2-k^2-2\sqrt{\cdots}\right.\right.}{\cdots}$$

Mathematica [C] time = 2.81, size = 244, normalized size = 0.83

$$\frac{i\sqrt{\frac{1}{x-1}+1}\,(x-1)^{3/2}\sqrt{\frac{1-\frac{1}{k^2}}{x-1}+1}\left(\left(b\left(\sqrt{1-k^2}-1\right)-2ak^2\right)\Pi\left(\frac{k^2-1}{k^2-\sqrt{1-k^2}-1};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+\left(2ak^2+b\sqrt{1-k^2}+b\right)\Pi\left(\frac{k^2-1}{k^2+\sqrt{1-k^2}-1};i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)+2a\sqrt{1-k^2}\,k^2F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|1-\frac{1}{k^2}\right)\right)}{k^2\sqrt{1-k^2}\,\sqrt{(x-1)x\left(k^2x-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a - b*x + (b + a*k^2)*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)]*(1 - 2*
x + k^2*x^2)),x]
```

```
[Out] (I*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (1 - k^(-2))/(-1 + x)]*(-1 + x)^(3/2)*(
2*a*k^2*Sqrt[1 - k^2]*EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] + (-
2*a*k^2 + b*(-1 + Sqrt[1 - k^2]))*EllipticPi[(-1 + k^2)/(-1 + k^2 - Sqrt[1
- k^2]), I*ArcSinh[1/Sqrt[-1 + x]], 1 - k^(-2)] + (b + 2*a*k^2 + b*Sqrt[1 -
 k^2])*EllipticPi[(-1 + k^2)/(-1 + k^2 + Sqrt[1 - k^2]), I*ArcSinh[1/Sqrt[-
1 + x]], 1 - k^(-2)]))/(k^2*Sqrt[1 - k^2]*Sqrt[(-1 + x)*x*(-1 + k^2*x)])
```

IntegrateAlgebraic [A] time = 2.73, size = 293, normalized size = 1.00

$$\frac{\left(2iak^2-2a\sqrt{k^2-1}\,k^2-ibk^2-2b\sqrt{k^2-1}+2ib\right)\tan^{-1}\left(\frac{\sqrt{k^2-2i\sqrt{k^2-1}-2}\,\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{k^2(x-1)x}\right)}{2k^2\sqrt{k^2-1}\,\sqrt{k^2-2i\sqrt{k^2-1}-2}}+\frac{\left(-2iak^2-2a\sqrt{k^2-1}\,k^2+ibk^2-2b\sqrt{k^2-1}-2ib\right)\tan^{-1}\left(\frac{\sqrt{k^2+2i\sqrt{k^2-1}-2}\,\sqrt{k^2x^3+\left(-k^2-1\right)x^2+x}}{k^2(x-1)x}\right)}{2k^2\sqrt{k^2-1}\,\sqrt{k^2+2i\sqrt{k^2-1}-2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a - b*x + (b + a*k^2)*x^2)/(Sqrt[(1 - x)*x*(1 - k^2*x)
]*(1 - 2*x + k^2*x^2)),x]
```

```
[Out] (((2*I)*b + (2*I)*a*k^2 - I*b*k^2 - 2*b*Sqrt[-1 + k^2] - 2*a*k^2*Sqrt[-1 +
k^2])*ArcTan[(Sqrt[-2 + k^2 - (2*I)*Sqrt[-1 + k^2]]*Sqrt[x + (-1 - k^2)*x^2
 + k^2*x^3])/(k^2*(-1 + x)*x)])/(2*k^2*Sqrt[-1 + k^2]*Sqrt[-2 + k^2 - (2*I)
*Sqrt[-1 + k^2]]) + (((-2*I)*b - (2*I)*a*k^2 + I*b*k^2 - 2*b*Sqrt[-1 + k^2]
 - 2*a*k^2*Sqrt[-1 + k^2])*ArcTan[(Sqrt[-2 + k^2 + (2*I)*Sqrt[-1 + k^2]]*Sq
rt[x + (-1 - k^2)*x^2 + k^2*x^3])/(k^2*(-1 + x)*x)])/(2*k^2*Sqrt[-1 + k^2]*
Sqrt[-2 + k^2 + (2*I)*Sqrt[-1 + k^2]])
```

fricas [A] time = 1.69, size = 491, normalized size = 1.68

$$\left[-\frac{\left(2\,ak^2+b\right)\sqrt{-k^2+1}\,\log\left(\frac{k^4x^4-4\left(2\,k^4-k^2\right)x^3+2\left(4\,k^4+k^2-2\right)x^2-4\,\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\left(k^2x^2-2\,k^2x+1\right)\sqrt{-k^2+1}-4\left(2\,k^2-1\right)x+1}{k^4x^4-4\,k^2x^3+2\left(k^2+2\right)x^2-4\,x+1}\right)-\left(bk^2-b\right)\log\left(\frac{k^4x^4+4\,k^2x^3-2\left(3\,k^2+2\right)x^2-4\,\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\left(k^2x^2-1\right)+4\,x+1}{k^4x^4-4\,k^2x^3+2\left(k^2+2\right)x^2-4\,x+1}\right)}{4\left(k^4-k^2\right)},\ \frac{2\left(2\,ak^2+b\right)\sqrt{k^2-1}\,\arctan\left(\frac{\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\left(k^2x^2-2\,k^2x+1\right)\sqrt{k^2-1}}{2\left(\left(k^4-k^2\right)x^3-\left(k^4-1\right)x^2+\left(k^2-1\right)x\right)}\right)+\left(bk^2-b\right)\log\left(\frac{k^4x^4+4\,k^2x^3-2\left(3\,k^2+2\right)x^2-4\,\sqrt{k^2x^3-\left(k^2+1\right)x^2+x}\left(k^2x^2-1\right)+4\,x+1}{k^4x^4-4\,k^2x^3+2\left(k^2+2\right)x^2-4\,x+1}\right)}{4\left(k^4-k^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a-b*x+(a*k^2+b)*x^2)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-2*x+1)
,x, algorithm="fricas")
```

```
[Out] [-1/4*((2*a*k^2 + b)*sqrt(-k^2 + 1)*log((k^4*x^4 - 4*(2*k^4 - k^2)*x^3 + 2*
(4*k^4 + k^2 - 2)*x^2 - 4*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2 - 2*k^
2*x + 1)*sqrt(-k^2 + 1) - 4*(2*k^2 - 1)*x + 1)/(k^4*x^4 - 4*k^2*x^3 + 2*(k^
2 + 2)*x^2 - 4*x + 1)) - (b*k^2 - b)*log((k^4*x^4 + 4*k^2*x^3 - 2*(3*k^2 +
2)*x^2 - 4*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2 - 1) + 4*x + 1)/(k^4*
x^4 - 4*k^2*x^3 + 2*(k^2 + 2)*x^2 - 4*x + 1)))/(k^4 - k^2), 1/4*(2*(2*a*k^2
 + b)*sqrt(k^2 - 1)*arctan(1/2*sqrt(k^2*x^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2 -
 2*k^2*x + 1)*sqrt(k^2 - 1)/((k^4 - k^2)*x^3 - (k^4 - 1)*x^2 + (k^2 - 1)*x)
) + (b*k^2 - b)*log((k^4*x^4 + 4*k^2*x^3 - 2*(3*k^2 + 2)*x^2 - 4*sqrt(k^2*x
^3 - (k^2 + 1)*x^2 + x)*(k^2*x^2 - 1) + 4*x + 1)/(k^4*x^4 - 4*k^2*x^3 + 2*(
k^2 + 2)*x^2 - 4*x + 1)))/(k^4 - k^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ak^2+b\right)x^2-bx-a}{\left(k^2x^2-2\,x+1\right)\sqrt{\left(k^2x-1\right)(x-1)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a-b*x+(a*k^2+b)*x^2)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-2*x+1)
,x, algorithm="giac")
```

```
[Out] integrate(((a*k^2 + b)*x^2 - b*x - a)/((k^2*x^2 - 2*x + 1)*sqrt((k^2*x - 1)
*(x - 1)*x)), x)
```

maple [C] time = 0.06, size = 1907, normalized size = 6.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a-b*x+(a*k^2+b)*x^2)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-2*x+1),x)
```

```
[Out] -2*(a*k^2+b)/k^4*(-(x-1/k^2)*k^2)^(1/2)*((-1+x)/(1/k^2-1))^(1/2)*(k^2*x)^(1
/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)*EllipticF((-(x-1/k^2)*k^2)^(1/2),(1/k^2/(
1/k^2-1))^(1/2))+1/k^2*(2/(-k^2+1)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2
-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)*EllipticPi((-(x-1/k^
2)*k^2)^(1/2),1/k^2/(1/k^2-(1+(-k^2+1)^(1/2))/k^2),(1/k^2/(1/k^2-1))^(1/2))
*a-2/(-k^2+1)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1
/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(
1/k^2-(1+(-k^2+1)^(1/2))/k^2),(1/k^2/(1/k^2-1))^(1/2))*b+2/(-k^2+1)*(-k^2*x
+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-
x^2+x)^(1/2)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-(1+(-k^2+1)^(1/
2))/k^2),(1/k^2/(1/k^2-1))^(1/2))/k^2*b+2/(-k^2+1)^(1/2)*(-k^2*x+1)^(1/2)*(
1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2
)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-(1+(-k^2+1)^(1/2))/k^2),(1
/k^2/(1/k^2-1))^(1/2))*a-1/(-k^2+1)^(1/2)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1
/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)*EllipticPi((-
(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2-(1+(-k^2+1)^(1/2))/k^2),(1/k^2/(1/k^2-1))
^(1/2))*b+2/(-k^2+1)^(1/2)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/
2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)*EllipticPi((-(x-1/k^2)*k^2)^
(1/2),1/k^2/(1/k^2-(1+(-k^2+1)^(1/2))/k^2),(1/k^2/(1/k^2-1))^(1/2))/k^2*b-2
/(-k^2+1)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/
(k^2*x^3-k^2*x^2-x^2+x)^(1/2)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^
2-(1+(-k^2+1)^(1/2))/k^2),(1/k^2/(1/k^2-1))^(1/2))*a*k^2+2/(-k^2+1)*(-k^2*x
+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-
x^2+x)^(1/2)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+(-1+(-k^2+1)^(1
/2))/k^2),(1/k^2/(1/k^2-1))^(1/2))*a-2/(-k^2+1)*(-k^2*x+1)^(1/2)*(1/(1/k^2-
1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)*Ellipti
cPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+(-1+(-k^2+1)^(1/2))/k^2),(1/k^2/(1/
k^2-1))^(1/2))*b+2/(-k^2+1)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1
/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)*EllipticPi((-(x-1/k^2)*k^2)
^(1/2),1/k^2/(1/k^2+(-1+(-k^2+1)^(1/2))/k^2),(1/k^2/(1/k^2-1))^(1/2))/k^2*b
-2/(-k^2+1)^(1/2)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x
)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k
^2/(1/k^2+(-1+(-k^2+1)^(1/2))/k^2),(1/k^2/(1/k^2-1))^(1/2))*a+1/(-k^2+1)^(1
/2)*(-k^2*x+1)^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x
^3-k^2*x^2-x^2+x)^(1/2)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+(-1+
(-k^2+1)^(1/2))/k^2),(1/k^2/(1/k^2-1))^(1/2))*b-2/(-k^2+1)^(1/2)*(-k^2*x+1)
^(1/2)*(1/(1/k^2-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2
+x)^(1/2)*EllipticPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+(-1+(-k^2+1)^(1/2)
)/k^2),(1/k^2/(1/k^2-1))^(1/2))/k^2*b-2/(-k^2+1)*(-k^2*x+1)^(1/2)*(1/(1/k^2
-1)*x-1/(1/k^2-1))^(1/2)*(k^2*x)^(1/2)/(k^2*x^3-k^2*x^2-x^2+x)^(1/2)*Ellipt
icPi((-(x-1/k^2)*k^2)^(1/2),1/k^2/(1/k^2+(-1+(-k^2+1)^(1/2))/k^2),(1/k^2/(1
/k^2-1))^(1/2))*a*k^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a-b*x+(a*k^2+b)*x^2)/((1-x)*x*(-k^2*x+1))^(1/2)/(k^2*x^2-2*x+1)
,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(k-1>0)', see `assume?` for more det
ails)Is k-1 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(-a\,k^2-b\right)\,x^2+b\,x+a}{\left(k^2\,x^2-2\,x+1\right)\,\sqrt{x\,\left(k^2\,x-1\right)\,\left(x-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x - x^2*(b + a*k^2))/((k^2*x^2 - 2*x + 1)*(x*(k^2*x - 1)*(x - 1
))^(1/2)),x)
```

```
[Out] int(-(a + b*x - x^2*(b + a*k^2))/((k^2*x^2 - 2*x + 1)*(x*(k^2*x - 1)*(x - 1
))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a-b*x+(a*k**2+b)*x**2)/((1-x)*x*(-k**2*x+1))**(1/2)/(k**2*x**2-
2*x+1),x)
```

```
[Out] Timed out
```

3.2260 $\int \frac{1-2x^4+x^8}{\sqrt[4]{-1+x^4}\left(1-2x^4+2x^8\right)}\,dx$

Optimal. Leaf size=293

$$\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}-x^2}\right)-\frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}$$

Rubi [C] time = 0.28, antiderivative size = 298, normalized size of antiderivative = 1.02, number of steps used = 28, number of rules used = 15, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.441, Rules used = {28, 1428, 416, 530, 240, 212, 206, 203, 377, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\left(\frac{1}{8}+\frac{i}{8}\right)\sqrt[8]{-1}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{x^4-1}}\right)-\frac{\left(\frac{1}{8}+\frac{i}{8}\right)(-1)^{5/8}\tan^{-1}\left(1-\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4-1}}\right)}{\sqrt{2}}+\frac{\left(\frac{1}{8}+\frac{i}{8}\right)(-1)^{5/8}\tan^{-1}\left(\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4-1}}+1\right)}{\sqrt{2}}+\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)-\left(\frac{1}{8}+\frac{i}{8}\right)\sqrt[8]{-1}\tanh^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{x^4-1}}\right)-\frac{\left(\frac{1}{16}+\frac{i}{16}\right)(-1)^{5/8}\log\left(\frac{\sqrt[8]{-1}\sqrt{2}x}{\sqrt[4]{x^4-1}}+\frac{x^2}{\sqrt{x^4-1}}+\sqrt[4]{-1}\right)}{\sqrt{2}}+\frac{\left(\frac{1}{16}+\frac{i}{16}\right)(-1)^{5/8}\log\left(\frac{(-1)^{7/8}\sqrt{2}x}{\sqrt[4]{x^4-1}}-\frac{(-1)^{3/4}x^2}{\sqrt{x^4-1}}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^4 + x^8)/((-1 + x^4)^(1/4)*(1 - 2*x^4 + 2*x^8)),x]
```

```
[Out] ArcTan[x/(-1 + x^4)^(1/4)]/4 - (1/8 + I/8)*(-1)^(1/8)*ArcTan[((-1)^(7/8)*x)
/(-1 + x^4)^(1/4)] - ((1/8 + I/8)*(-1)^(5/8)*ArcTan[1 - ((-1)^(7/8)*Sqrt[2]
*x)/(-1 + x^4)^(1/4)])/Sqrt[2] + ((1/8 + I/8)*(-1)^(5/8)*ArcTan[1 + ((-1)^(
7/8)*Sqrt[2]*x)/(-1 + x^4)^(1/4)])/Sqrt[2] + ArcTanh[x/(-1 + x^4)^(1/4)]/4
- (1/8 + I/8)*(-1)^(1/8)*ArcTanh[((-1)^(7/8)*x)/(-1 + x^4)^(1/4)] - ((1/16
+ I/16)*(-1)^(5/8)*Log[(-1)^(1/4) + x^2/Sqrt[-1 + x^4] + ((-1)^(1/8)*Sqrt[2
]*x)/(-1 + x^4)^(1/4)])/Sqrt[2] + ((1/16 + I/16)*(-1)^(5/8)*Log[1 - ((-1)^(
3/4)*x^2)/Sqrt[-1 + x^4] + ((-1)^(7/8)*Sqrt[2]*x)/(-1 + x^4)^(1/4)])/Sqrt[2
]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1428

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^
n)^q/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[(d + e*x^n)^q/(b + r + 2
*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[q]
```

Rubi steps

$$\int \frac{1-2x^4+x^8}{\sqrt[4]{-1+x^4}\left(1-2x^4+2x^8\right)}\,dx = \int \frac{\left(-1+x^4\right)^{7/4}}{1-2x^4+2x^8}\,dx$$

$$= -\left(2i\int \frac{\left(-1+x^4\right)^{7/4}}{(-2-2i)+4x^4}\,dx\right)+2i\int \frac{\left(-1+x^4\right)^{7/4}}{(-2+2i)+4x^4}\,dx$$

$$= -\left(\frac{1}{8}i\int \frac{(14-2i)-(20-8i)x^4}{\sqrt[4]{-1+x^4}\left((-2-2i)+4x^4\right)}\,dx\right)+\frac{1}{8}i\int \frac{(14+2i)-(20+8i)x^4}{\sqrt[4]{-1+x^4}\left((-2+2i)+4x^4\right)}$$

$$= -\left(\left(-\frac{1}{4}-\frac{5i}{8}\right)\int \frac{1}{\sqrt[4]{-1+x^4}}\,dx\right)+\left(\frac{1}{4}-\frac{5i}{8}\right)\int \frac{1}{\sqrt[4]{-1+x^4}}\,dx-\int \frac{}{\sqrt[4]{-1+x^4}\,(}$$

$$= -\left(\left(-\frac{1}{4}-\frac{5i}{8}\right)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)\right)+\left(\frac{1}{4}-\frac{5i}{8}\right)\text{Subst}\left(\int \frac{1}{1-x}\right.$$

$$= -\left(\left(-\frac{1}{8}-\frac{5i}{16}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt[4]{-1+x^4}}\right)\right)-\left(-\frac{1}{8}-\frac{5i}{16}\right)\text{Subst}\left(\int \frac{}{1+}\right.$$

$$= \frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)-\frac{(-1)^{3/8}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{-1+x^4}}\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)-\frac{(-}{}$$

$$= \frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)-\frac{(-1)^{3/8}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{-1+x^4}}\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)-\frac{(-}{}$$

$$= \frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt[4]{-1+x^4}}\right)-\frac{(-1)^{3/8}\tan^{-1}\left(\frac{(-1)^{7/8}x}{\sqrt[4]{-1+x^4}}\right)}{4\sqrt{2}}-\frac{1}{8}(-1)^{7/8}\tan^{-1}\left(1-\frac{(-1)^{7/8}\sqrt{}}{\sqrt[4]{-1+}}\right.$$

Mathematica [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1-2x^4+x^8}{\sqrt[4]{-1+x^4}\left(1-2x^4+2x^8\right)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(1 - 2*x^4 + x^8)/((-1 + x^4)^(1/4)*(1 - 2*x^4 + 2*x^8)),x]

[Out] Integrate[(1 - 2*x^4 + x^8)/((-1 + x^4)^(1/4)*(1 - 2*x^4 + 2*x^8)), x]

IntegrateAlgebraic [A] time = 1.27, size = 273, normalized size = 0.93

$$\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^4-1}}\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}-x^2}\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}-x^2}\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+x^2}\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^4-1}}{\sqrt{x^4-1}+x^2}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - 2*x^4 + x^8)/((-1 + x^4)^(1/4)*(1 - 2*x^4 + 2*x^8)),x]

[Out] ArcTan[x/(-1 + x^4)^(1/4)]/4 + (Sqrt[(2 + Sqrt[2])/2]*ArcTan[(Sqrt[2 - Sqrt[2]]*x*(-1 + x^4)^(1/4))/(-x^2 + Sqrt[-1 + x^4])])/8 + (Sqrt[(2 - Sqrt[2])/2]*ArcTan[(Sqrt[2 + Sqrt[2]]*x*(-1 + x^4)^(1/4))/(-x^2 + Sqrt[-1 + x^4])])/8 + ArcTanh[x/(-1 + x^4)^(1/4)]/4 + (Sqrt[(2 + Sqrt[2])/2]*ArcTanh[(Sqrt[2 - Sqrt[2]]*x*(-1 + x^4)^(1/4))/(x^2 + Sqrt[-1 + x^4])])/8 + (Sqrt[(2 - Sqrt[2])/2]*ArcTanh[(Sqrt[2 + Sqrt[2]]*x*(-1 + x^4)^(1/4))/(x^2 + Sqrt[-1 + x^4])])/8

fricas [B] time = 0.80, size = 1982, normalized size = 6.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^8-2*x^4+1)/(x^4-1)^(1/4)/(2*x^8-2*x^4+1),x, algorithm="fricas")

[Out] 1/16*sqrt(2)*sqrt(-sqrt(2) + 2)*arctan(-(x*(sqrt(2) + 2)^(3/2) - 3*x*sqrt(sqrt(2) + 2) - (x*(sqrt(2) + 2) - x)*sqrt(-sqrt(2) + 2) - 2*x*sqrt((2*x^2 + (x^4 - 1)^(1/4)*(sqrt(2)*x*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*x*sqrt(sqrt(2) + 2) - (sqrt(2)*x*(sqrt(2) + 2) - sqrt(2)*x)*sqrt(-sqrt(2) + 2)) + 2*sqrt(x^4 - 1))/x^2) + 2*sqrt(2)*(x^4 - 1)^(1/4))/(x*(sqrt(2) + 2)^(3/2) - 3*x*sqrt(sqrt(2) + 2) + (x*(sqrt(2) + 2) - x)*sqrt(-sqrt(2) + 2))) + 1/16*sqrt(2)*sqrt(-sqrt(2) + 2)*arctan((x*(sqrt(2) + 2)^(3/2) - 3*x*sqrt(sqrt(2) + 2) - (x*(sqrt(2) + 2) - x)*sqrt(-sqrt(2) + 2) + 2*x*sqrt((2*x^2 - (x^4 - 1)^(1/4)*(sqrt(2)*x*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*x*sqrt(sqrt(2) + 2) - (sqrt(2)*x*(sqrt(2) + 2) - sqrt(2)*x)*sqrt(-sqrt(2) + 2)) + 2*sqrt(x^4 - 1))/x^2) - 2*sqrt(2)*(x^4 - 1)^(1/4))/(x*(sqrt(2) + 2)^(3/2) - 3*x*sqrt(sqrt(2) + 2) + (x*(sqrt(2) + 2) - x)*sqrt(-sqrt(2) + 2))) + 1/16*sqrt(2)*sqrt(sqrt(2) + 2)*arctan((x*(sqrt(2) + 2)^(3/2) - 3*x*sqrt(sqrt(2) + 2) + (x*(sqrt(2) + 2) - x)*sqrt(-sqrt(2) + 2) - 2*x*sqrt((2*x^2 + (x^4 - 1)^(1/4)*(sqrt(2)*x*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*x*sqrt(sqrt(2) + 2) + (sqrt(2)*x*(sqrt(2) + 2) - sqrt(2)*x)*sqrt(-sqrt(2) + 2)) + 2*sqrt(x^4 - 1))/x^2) + 2*sqrt(2)*(x^4 - 1)^(1/4))/(x*(sqrt(2) + 2)^(3/2) - 3*x*sqrt(sqrt(2) + 2) - (x*(sqrt(2) + 2) - x)*sqrt(-sqrt(2) + 2))) + 1/16*sqrt(2)*sqrt(sqrt(2) + 2)*arctan(-(x*(sqrt(2) + 2)^(3/2) - 3*x*sqrt(sqrt(2) + 2) + (x*(sqrt(2) + 2) - x)*sqrt(-sqrt(2) + 2) + 2*x*sqrt((2*x^2 - (x^4 - 1)^(1/4)*(sqrt(2)*x*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*x*sqrt(sqrt(2) + 2) + (sqrt(2)*x*(sqrt(2) + 2) - sqrt(2)*x)*sqrt(-sqrt(2) + 2)) + 2*sqrt(x^4 - 1))/x^2) - 2*sqrt(2)*(x^4 - 1)^(1/4))/(x*(sqrt(2) + 2)^(3/2) - 3*x*sqrt(sqrt(2) + 2) - (x*(sqrt(2) + 2) - x)*sqrt(-sqrt(2) + 2))) + 1/16*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*arctan(-((x*(sqrt(2) + 2) - x)*sqrt(-sqrt(2) + 2) - 2*x*sqrt((x^2 + (x^4 - 1)^(1/4)*(x*(sqrt(2) + 2) - x)*sqrt(-sqrt(2) + 2) + sqrt(x^4 - 1))/x^2) + 2*(x^4 - 1)^(1/4))/(x*(sqrt(2) + 2)^(3/2) - 3*x*sqrt(sqrt(2) + 2))) + 1/16*(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))*arctan(((x*(sqrt(2) + 2) - x)*sqrt(-sqrt(2) + 2) + 2*x*sqrt((x^2 - (x^4 - 1)^(1/4)*(x*(sqrt(2) + 2) - x)*sqrt(-sqrt(2) + 2) + sqrt(x^4 - 1))/x^2) - 2*(x^4 - 1)^(1/4))/(x*(sqrt(2) + 2)^(3/2) - 3*x*sqrt(sqrt(2) + 2))) + 1/16*(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))*arctan(-(x*(sqrt(2) + 2)^(3/2) - 3*x*sqrt(sqrt(2) + 2) - 2*x*sqrt((x^2 + (x^4 - 1)^(1/4)*(x*(sqrt(2) + 2)^(3/2) - 3*x*sqrt(sqrt(2) + 2)) + sqrt(x^4 - 1))/x^2) +

```
 2*(x^4 - 1)^(1/4))/((x*(sqrt(2) + 2) - x)*sqrt(-sqrt(2) + 2))) + 1/16*(sqr
t(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))*arctan((x*(sqrt(2) + 2)^(3/2) - 3*x*sq
rt(sqrt(2) + 2) + 2*x*sqrt((x^2 - (x^4 - 1)^(1/4)*(x*(sqrt(2) + 2)^(3/2) -
3*x*sqrt(sqrt(2) + 2)) + sqrt(x^4 - 1))/x^2) - 2*(x^4 - 1)^(1/4))/((x*(sqrt
(2) + 2) - x)*sqrt(-sqrt(2) + 2))) + 1/64*sqrt(2)*sqrt(-sqrt(2) + 2)*log(32
*(2*x^2 + (x^4 - 1)^(1/4)*(sqrt(2)*x*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*x*sqrt
(sqrt(2) + 2) + (sqrt(2)*x*(sqrt(2) + 2) - sqrt(2)*x)*sqrt(-sqrt(2) + 2)) +
 2*sqrt(x^4 - 1))/x^2) - 1/64*sqrt(2)*sqrt(-sqrt(2) + 2)*log(32*(2*x^2 - (x
^4 - 1)^(1/4)*(sqrt(2)*x*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*x*sqrt(sqrt(2) + 2
) + (sqrt(2)*x*(sqrt(2) + 2) - sqrt(2)*x)*sqrt(-sqrt(2) + 2)) + 2*sqrt(x^4
- 1))/x^2) - 1/64*sqrt(2)*sqrt(sqrt(2) + 2)*log(32*(2*x^2 + (x^4 - 1)^(1/4)
*(sqrt(2)*x*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*x*sqrt(sqrt(2) + 2) - (sqrt(2)*
x*(sqrt(2) + 2) - sqrt(2)*x)*sqrt(-sqrt(2) + 2)) + 2*sqrt(x^4 - 1))/x^2) +
1/64*sqrt(2)*sqrt(sqrt(2) + 2)*log(32*(2*x^2 - (x^4 - 1)^(1/4)*(sqrt(2)*x*(
sqrt(2) + 2)^(3/2) - 3*sqrt(2)*x*sqrt(sqrt(2) + 2) - (sqrt(2)*x*(sqrt(2) +
2) - sqrt(2)*x)*sqrt(-sqrt(2) + 2)) + 2*sqrt(x^4 - 1))/x^2) + 1/64*(sqrt(sq
rt(2) + 2) - sqrt(-sqrt(2) + 2))*log(256*(x^2 + (x^4 - 1)^(1/4)*(x*(sqrt(2)
 + 2) - x)*sqrt(-sqrt(2) + 2) + sqrt(x^4 - 1))/x^2) - 1/64*(sqrt(sqrt(2) +
2) - sqrt(-sqrt(2) + 2))*log(256*(x^2 - (x^4 - 1)^(1/4)*(x*(sqrt(2) + 2) -
x)*sqrt(-sqrt(2) + 2) + sqrt(x^4 - 1))/x^2) + 1/64*(sqrt(sqrt(2) + 2) + sqr
t(-sqrt(2) + 2))*log(256*(x^2 + (x^4 - 1)^(1/4)*(x*(sqrt(2) + 2)^(3/2) - 3*
x*sqrt(sqrt(2) + 2)) + sqrt(x^4 - 1))/x^2) - 1/64*(sqrt(sqrt(2) + 2) + sqrt
(-sqrt(2) + 2))*log(256*(x^2 - (x^4 - 1)^(1/4)*(x*(sqrt(2) + 2)^(3/2) - 3*x
*sqrt(sqrt(2) + 2)) + sqrt(x^4 - 1))/x^2) - 1/4*arctan((x^4 - 1)^(1/4)/x) +
 1/8*log((x + (x^4 - 1)^(1/4))/x) - 1/8*log(-(x - (x^4 - 1)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 - 2\,x^4 + 1}{\left(2\,x^8 - 2\,x^4 + 1\right)\left(x^4 - 1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-2*x^4+1)/(x^4-1)^(1/4)/(2*x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] integrate((x^8 - 2*x^4 + 1)/((2*x^8 - 2*x^4 + 1)*(x^4 - 1)^(1/4)), x)
```

maple [C] time = 11.17, size = 1017, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-2*x^4+1)/(x^4-1)^(1/4)/(2*x^8-2*x^4+1),x)
```

```
[Out] -1/8*ln(2*(x^4-1)^(3/4)*x-2*x^2*(x^4-1)^(1/2)+2*x^3*(x^4-1)^(1/4)-2*x^4+1)-
1/8*RootOf(_Z^2+1)*ln(2*RootOf(_Z^2+1)*(x^4-1)^(1/2)*x^2-2*RootOf(_Z^2+1)*x
^4+2*(x^4-1)^(3/4)*x-2*x^3*(x^4-1)^(1/4)+RootOf(_Z^2+1))-1/16*RootOf(_Z^2+1
)*RootOf(_Z^4-4*RootOf(_Z^2+1))*ln(((x^4-1)^(1/2)*RootOf(_Z^2+1)*RootOf(_Z^
4-4*RootOf(_Z^2+1))^3*x^2+RootOf(_Z^4-4*RootOf(_Z^2+1))*RootOf(_Z^2+1)^2*x^
4-RootOf(_Z^4-4*RootOf(_Z^2+1))^3*(x^4-1)^(1/2)*x^2-2*(x^4-1)^(1/4)*RootOf(
_Z^4-4*RootOf(_Z^2+1))^2*x^3-RootOf(_Z^4-4*RootOf(_Z^2+1))*x^4-4*(x^4-1)^(3
/4)*x-RootOf(_Z^2+1)*RootOf(_Z^4-4*RootOf(_Z^2+1))+RootOf(_Z^4-4*RootOf(_Z^
2+1)))/(RootOf(_Z^2+1)*x^4-x^4+1))-1/16*RootOf(_Z^4-4*RootOf(_Z^2+1))*ln(-(
(x^4-1)^(1/4)*RootOf(_Z^2+1)*RootOf(_Z^4-4*RootOf(_Z^2+1))^2*x^3+RootOf(_Z^
4-4*RootOf(_Z^2+1))^3*(x^4-1)^(1/2)*x^2-RootOf(_Z^4-4*RootOf(_Z^2+1))*RootO
f(_Z^2+1)*x^4-(x^4-1)^(1/4)*RootOf(_Z^4-4*RootOf(_Z^2+1))^2*x^3-2*RootOf(_Z
^2+1)*(x^4-1)^(3/4)*x-RootOf(_Z^4-4*RootOf(_Z^2+1))*x^4+2*(x^4-1)^(3/4)*x+R
ootOf(_Z^4-4*RootOf(_Z^2+1)))/(RootOf(_Z^2+1)*x^4-x^4+1))-1/32*RootOf(_Z^4-
4*RootOf(_Z^2+1))^3*RootOf(_Z^2+1)*ln(-(RootOf(_Z^4-4*RootOf(_Z^2+1))^3*Roo
tOf(_Z^2+1)^2*x^4-RootOf(_Z^4-4*RootOf(_Z^2+1))^3*RootOf(_Z^2+1)*x^4+2*(x^4
```

```
-1)^(1/4)*RootOf(_Z^2+1)*RootOf(_Z^4-4*RootOf(_Z^2+1))^2*x^3+4*(x^4-1)^(1/2
)*RootOf(_Z^2+1)*RootOf(_Z^4-4*RootOf(_Z^2+1))*x^2-2*(x^4-1)^(1/4)*RootOf(_
Z^4-4*RootOf(_Z^2+1))^2*x^3+4*RootOf(_Z^2+1)*(x^4-1)^(3/4)*x+RootOf(_Z^4-4*
RootOf(_Z^2+1))^3*RootOf(_Z^2+1)+4*(x^4-1)^(3/4)*x)/(RootOf(_Z^2+1)*x^4+x^4
-1))+1/32*RootOf(_Z^4-4*RootOf(_Z^2+1))^3*ln(-(RootOf(_Z^4-4*RootOf(_Z^2+1)
)^3*RootOf(_Z^2+1)^2*x^4-2*RootOf(_Z^4-4*RootOf(_Z^2+1))^3*RootOf(_Z^2+1)*x
^4-4*(x^4-1)^(1/4)*RootOf(_Z^2+1)*RootOf(_Z^4-4*RootOf(_Z^2+1))^2*x^3+RootO
f(_Z^4-4*RootOf(_Z^2+1))^3*x^4-4*(x^4-1)^(1/2)*RootOf(_Z^2+1)*RootOf(_Z^4-4
*RootOf(_Z^2+1))*x^2+4*RootOf(_Z^4-4*RootOf(_Z^2+1))*(x^4-1)^(1/2)*x^2+Root
Of(_Z^4-4*RootOf(_Z^2+1))^3*RootOf(_Z^2+1)+8*(x^4-1)^(3/4)*x-RootOf(_Z^4-4*
RootOf(_Z^2+1))^3)/(RootOf(_Z^2+1)*x^4+x^4-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 - 2\,x^4 + 1}{\left(2\,x^8 - 2\,x^4 + 1\right)\left(x^4 - 1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-2*x^4+1)/(x^4-1)^(1/4)/(2*x^8-2*x^4+1),x, algorithm="maxima"
)
[Out] integrate((x^8 - 2*x^4 + 1)/((2*x^8 - 2*x^4 + 1)*(x^4 - 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^8 - 2\,x^4 + 1}{{\left(x^4 - 1\right)}^{1/4}\,\left(2\,x^8 - 2\,x^4 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 - 2*x^4 + 1)/((x^4 - 1)^(1/4)*(2*x^8 - 2*x^4 + 1)),x)
[Out] int((x^8 - 2*x^4 + 1)/((x^4 - 1)^(1/4)*(2*x^8 - 2*x^4 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-2*x**4+1)/(x**4-1)**(1/4)/(2*x**8-2*x**4+1),x)
[Out] Timed out
```

3.2261 $\int \sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$

Optimal. Leaf size=293

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{\sqrt{c}}\right)}{4ac^{3/2}}+\frac{\left(48a^2cx^2-16ac^3x-6bc\right)\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}+\sqrt{\sqrt{a^2x^2+b}+ax}\sqrt{\sqrt{\sqrt{a^2x}}}\cdots}{\cdots}$$

Rubi [F] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]],x]
```

```
[Out] Defer[Int][Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]], x]
```

Rubi steps

$$\int \sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx=\int \sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Mathematica [A] time = 0.59, size = 220, normalized size = 0.75

$$-\frac{2\left(-\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{\sqrt{c}}\right)}{8c^{3/2}}-\frac{1}{5}\left(\sqrt{\sqrt{a^2x^2+b}+ax}+c\right)^{5/2}+\frac{1}{3}c\left(\sqrt{\sqrt{a^2x^2+b}+ax}+c\right)^{3/2}+\frac{b\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{8c\sqrt{\sqrt{a^2x^2+b}+ax}}+\frac{b\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{4\left(\sqrt{a^2x^2+b}+ax\right)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]],x]
```

```
[Out] (-2*((b*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]])/(4*(a*x + Sqrt[b + a^2*x^2
])) + (b*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]])/(8*c*Sqrt[a*x + Sqrt[b +
a^2*x^2]]) + (c*(c + Sqrt[a*x + Sqrt[b + a^2*x^2]])^(3/2))/3 - (c + Sqrt[a*
x + Sqrt[b + a^2*x^2]])^(5/2)/5 - (b*ArcTanh[Sqrt[c + Sqrt[a*x + Sqrt[b + a
^2*x^2]]]/Sqrt[c]])/(8*c^(3/2))))/a
```

IntegrateAlgebraic [A] time = 0.59, size = 293, normalized size = 1.00

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{\sqrt{c}}\right)}{4ac^{3/2}}+\frac{\left(48a^2cx^2-16ac^3x-6bc\right)\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}+\sqrt{\sqrt{a^2x^2+b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}\left(8ac^2x-15b\right)+\sqrt{a^2x^2+b}\left(\left(48acx-16c^3\right)\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}+8c^2\sqrt{\sqrt{a^2x^2+b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}\right)}{60ac\sqrt{a^2x^2+b}+60a^2cx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]],x]
```

```
[Out] ((-6*b*c - 16*a*c^3*x + 48*a^2*c*x^2)*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]
]] + (-15*b + 8*a*c^2*x)*Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x +
Sqrt[b + a^2*x^2]]] + Sqrt[b + a^2*x^2]*((-16*c^3 + 48*a*c*x)*Sqrt[c + Sqrt
```

```
[a*x + Sqrt[b + a^2*x^2]]] + 8*c^2*Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + S
qrt[a*x + Sqrt[b + a^2*x^2]]]))/(60*a^2*c*x + 60*a*c*Sqrt[b + a^2*x^2]) + (
b*ArcTanh[Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]]/Sqrt[c]])/(4*a*c^(3/2))
```

fricas [A] time = 0.78, size = 359, normalized size = 1.23

$$\left[\frac{15\,b\sqrt{c}\log\left(-2\left(a\sqrt{c}x-\sqrt{a^2x^2+b}\sqrt{c}\right)\sqrt{ax+\sqrt{a^2x^2+b}}\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}-2\left(acx-\sqrt{a^2x^2+b}\,c\right)\sqrt{ax+\sqrt{a^2x^2+b}}+b\right)-2\left(16c^4-54ac^2x+6\sqrt{a^2x^2+b}\,c^2-\left(8c^3+15acx-15\sqrt{a^2x^2+b}\,c\right)\sqrt{ax+\sqrt{a^2x^2+b}}\right)\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{120\,ac^2},-\frac{15\,b\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{c}\right)+\left(16c^4-54ac^2x+6\sqrt{a^2x^2+b}\,c^2-\left(8c^3+15acx-15\sqrt{a^2x^2+b}\,c\right)\sqrt{ax+\sqrt{a^2x^2+b}}\right)\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{60\,ac^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/120*(15*b*sqrt(c)*log(-2*(a*sqrt(c)*x - sqrt(a^2*x^2 + b)*sqrt(c))*sqrt(
a*x + sqrt(a^2*x^2 + b))*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b))) - 2*(a*c*x
 - sqrt(a^2*x^2 + b)*c)*sqrt(a*x + sqrt(a^2*x^2 + b)) + b) - 2*(16*c^4 - 54
*a*c^2*x + 6*sqrt(a^2*x^2 + b)*c^2 - (8*c^3 + 15*a*c*x - 15*sqrt(a^2*x^2 +
b)*c)*sqrt(a*x + sqrt(a^2*x^2 + b)))*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b))
))/(a*c^2), -1/60*(15*b*sqrt(-c)*arctan(sqrt(-c)*sqrt(c + sqrt(a*x + sqrt(a
^2*x^2 + b)))/c) + (16*c^4 - 54*a*c^2*x + 6*sqrt(a^2*x^2 + b)*c^2 - (8*c^3
+ 15*a*c*x - 15*sqrt(a^2*x^2 + b)*c)*sqrt(a*x + sqrt(a^2*x^2 + b)))*sqrt(c
+ sqrt(a*x + sqrt(a^2*x^2 + b))))/(a*c^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b))), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] int((c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\sqrt{c+\sqrt{\sqrt{a^2\,x^2+b}+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + ((b + a^2*x^2)^(1/2) + a*x)^(1/2))^(1/2),x)
```

```
[Out] int((c + ((b + a^2*x^2)^(1/2) + a*x)^(1/2))^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + \sqrt{ax + \sqrt{a^2x^2 + b}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a**2*x**2+b)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(c + sqrt(a*x + sqrt(a**2*x**2 + b))), x)
```

3.2262 $\int \frac{x^3}{\sqrt[3]{x^2(-a+x)}\left(-a^4+4a^3x-6a^2x^2+4ax^3+(-1+d)x^4\right)}\,dx$

Optimal. Leaf size=294

$$-\frac{\log\left(\sqrt[3]{x^3-ax^2}-\sqrt[12]{d}\,x\right)}{4ad^{5/6}}-\frac{\log\left(\sqrt[3]{x^3-ax^2}+\sqrt[12]{d}\,x\right)}{4ad^{5/6}}+\frac{\log\left(\left(x^3-ax^2\right)^{2/3}+\sqrt[6]{d}\,x^2\right)}{4ad^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\,x^2}{\sqrt[6]{d}\,x^2-2\left(x^3-ax^2\right.}\right.}{4ad^{5/6}}$$

Rubi [F] time = 1.79, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3}{\sqrt[3]{x^2(-a+x)}\left(-a^4+4a^3x-6a^2x^2+4ax^3+(-1+d)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/((x^2*(-a + x))^(1/3)*(-a^4 + 4*a^3*x - 6*a^2*x^2 + 4*a*x^3 + (-1 +
 d)*x^4)),x]
```

```
[Out] (3*x^(2/3)*(-a + x)^(1/3)*Defer[Subst][Defer[Int][x^9/((-a + x^3)^(1/3)*(-a
^4 + 4*a^3*x^3 - 6*a^2*x^6 + 4*a*x^9 + (-1 + d)*x^12)), x], x, x^(1/3)])/(-
((a - x)*x^2))^(1/3)
```

Rubi steps

$$\int \frac{x^3}{\sqrt[3]{x^2(-a+x)}\left(-a^4+4a^3x-6a^2x^2+4ax^3+(-1+d)x^4\right)}\,dx=\frac{\left(x^{2/3}\sqrt[3]{-a+x}\right)\int \frac{x^{7/3}}{\sqrt[3]{-a+x}\left(-a^4+4a^3x-6a^2x^2+4ax^3+(-\right.}}{\sqrt[3]{x^2(-a+x)}}$$

$$=\frac{\left(3x^{2/3}\sqrt[3]{-a+x}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{-a+x^3}\left(-a^4+4a^3x^3-6\right.}\right.}{\sqrt[3]{x^2(-a+x)}}$$

Mathematica [A] time = 0.70, size = 219, normalized size = 0.74

$$\frac{x\left(-\log\left(\sqrt[3]{d}\left(\frac{x}{x-a}\right)^{4/3}-\sqrt[6]{d}\left(\frac{x}{x-a}\right)^{2/3}+1\right)+\log\left(\sqrt[3]{d}\left(\frac{x}{x-a}\right)^{4/3}+\sqrt[6]{d}\left(\frac{x}{x-a}\right)^{2/3}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{d}\left(\frac{x}{x-a}\right)^{2/3}}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[6]{d}\left(\frac{x}{x-a}\right)^{2/3}+1}{\sqrt{3}}\right)+4\tanh^{-1}\left(\sqrt[6]{d}\left(\frac{x}{x-a}\right)^{2/3}\right)\right)}{8ad^{5/6}\sqrt[3]{\frac{x}{x-a}}\sqrt[3]{x^2(x-a)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((x^2*(-a + x))^(1/3)*(-a^4 + 4*a^3*x - 6*a^2*x^2 + 4*a*x^3 +
 (-1 + d)*x^4)),x]
```

```
[Out] (x*(2*Sqrt[3]*ArcTan[(1 - 2*d^(1/6)*(x/(-a + x))^(2/3))/Sqrt[3]] - 2*Sqrt[3
]*ArcTan[(1 + 2*d^(1/6)*(x/(-a + x))^(2/3))/Sqrt[3]] + 4*ArcTanh[d^(1/6)*(x
/(-a + x))^(2/3)] - Log[1 - d^(1/6)*(x/(-a + x))^(2/3) + d^(1/3)*(x/(-a + x
))^(4/3)] + Log[1 + d^(1/6)*(x/(-a + x))^(2/3) + d^(1/3)*(x/(-a + x))^(4/3)
]))/(8*a*d^(5/6)*(x/(-a + x))^(1/3)*(x^2*(-a + x))^(1/3))
```

IntegrateAlgebraic [A] time = 1.31, size = 426, normalized size = 1.45

$$-\frac{\log\left(\sqrt[3]{x^3-ax^2}-\sqrt[12]{d}\,x\right)}{4ad^{5/6}}-\frac{\log\left(\sqrt[3]{x^3-ax^2}+\sqrt[12]{d}\,x\right)}{4ad^{5/6}}+\frac{\log\left(\left(x^3-ax^2\right)^{2/3}+\sqrt[6]{d}\,x^2\right)}{4ad^{5/6}}+\frac{\log\left(-\sqrt[12]{d}\,x\sqrt[3]{x^3-ax^2}+\left(x^3-ax^2\right)^{2/3}+\sqrt[6]{d}\,x^2\right)}{8ad^{5/6}}+\frac{\log\left(\sqrt[12]{d}\,x\sqrt[3]{x^3-ax^2}+\left(x^3-ax^2\right)^{2/3}+\sqrt[6]{d}\,x^2\right)}{8ad^{5/6}}-\frac{\log\left(-\sqrt{3}\sqrt[12]{d}\,x\sqrt[3]{x^3-ax^2}+\left(x^3-ax^2\right)^{2/3}+\sqrt[6]{d}\,x^2\right)}{8ad^{5/6}}-\frac{\log\left(\sqrt{3}\sqrt[12]{d}\,x\sqrt[3]{x^3-ax^2}+\left(x^3-ax^2\right)^{2/3}+\sqrt[6]{d}\,x^2\right)}{8ad^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\,x^2\left(x^3-ax^2\right)^{2/3}}{\left(x^3-ax^2\right)^{4/3}-\sqrt[3]{d}\,x^4}\right)}{4ad^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((x^2*(-a + x))^(1/3)*(-a^4 + 4*a^3*x - 6*a^2*x^2 +
4*a*x^3 + (-1 + d)*x^4)),x]
```

```
[Out] -1/4*(Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/6)*x^2*(-(a*x^2) + x^3)^(2/3))/(-(d^(1/3
)*x^4) + (-(a*x^2) + x^3)^(4/3))])/(a*d^(5/6)) - Log[-(d^(1/12)*x) + (-(a*x
^2) + x^3)^(1/3)]/(4*a*d^(5/6)) - Log[d^(1/12)*x + (-(a*x^2) + x^3)^(1/3)]/
(4*a*d^(5/6)) + Log[d^(1/6)*x^2 + (-(a*x^2) + x^3)^(2/3)]/(4*a*d^(5/6)) + L
og[d^(1/6)*x^2 - d^(1/12)*x*(-(a*x^2) + x^3)^(1/3) + (-(a*x^2) + x^3)^(2/3)
]/(8*a*d^(5/6)) + Log[d^(1/6)*x^2 + d^(1/12)*x*(-(a*x^2) + x^3)^(1/3) + (-(
a*x^2) + x^3)^(2/3)]/(8*a*d^(5/6)) - Log[d^(1/6)*x^2 - Sqrt[3]*d^(1/12)*x*(
-(a*x^2) + x^3)^(1/3) + (-(a*x^2) + x^3)^(2/3)]/(8*a*d^(5/6)) - Log[d^(1/6)
*x^2 + Sqrt[3]*d^(1/12)*x*(-(a*x^2) + x^3)^(1/3) + (-(a*x^2) + x^3)^(2/3)]/
(8*a*d^(5/6))
```

fricas [B] time = 0.76, size = 557, normalized size = 1.89

$$-\frac{1}{2}\sqrt{3}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{3}a^5d^4x^2\sqrt{\frac{a^2d^2x^2\left(\frac{1}{a^6d^5}\right)^{\frac{1}{3}}+\left(-ax^2+x^3\right)^{\frac{2}{3}}ad\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}-\left(-ax^2+x^3\right)^{\frac{1}{3}}(a-x)}{x^2}}\left(\frac{1}{a^6d^5}\right)^{\frac{5}{6}}-2\sqrt{3}\left(-ax^2+x^3\right)^{\frac{2}{3}}a^5d^4\left(\frac{1}{a^6d^5}\right)^{\frac{5}{6}}-\sqrt{3}x^2}{3x^2}\right)-\frac{1}{2}\sqrt{3}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{3}a^5d^4x^2\sqrt{\frac{a^2d^2x^2\left(\frac{1}{a^6d^5}\right)^{\frac{1}{3}}-\left(-ax^2+x^3\right)^{\frac{2}{3}}ad\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}-\left(-ax^2+x^3\right)^{\frac{1}{3}}(a-x)}{x^2}}\left(\frac{1}{a^6d^5}\right)^{\frac{5}{6}}-2\sqrt{3}\left(-ax^2+x^3\right)^{\frac{2}{3}}a^5d^4\left(\frac{1}{a^6d^5}\right)^{\frac{5}{6}}+\sqrt{3}x^2}{3x^2}\right)+\frac{1}{8}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\log\left(\frac{a^2d^2x^2\left(\frac{1}{a^6d^5}\right)^{\frac{1}{3}}+\left(-ax^2+x^3\right)^{\frac{2}{3}}ad\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}-\left(-ax^2+x^3\right)^{\frac{1}{3}}(a-x)}{x^2}\right)-\frac{1}{8}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\log\left(\frac{a^2d^2x^2\left(\frac{1}{a^6d^5}\right)^{\frac{1}{3}}-\left(-ax^2+x^3\right)^{\frac{2}{3}}ad\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}-\left(-ax^2+x^3\right)^{\frac{1}{3}}(a-x)}{x^2}\right)+\frac{1}{4}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\log\left(\frac{adx^2\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}+\left(-ax^2+x^3\right)^{\frac{2}{3}}}{x^2}\right)-\frac{1}{4}\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}\log\left(-\frac{adx^2\left(\frac{1}{a^6d^5}\right)^{\frac{1}{6}}-\left(-ax^2+x^3\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2*(-a+x))^(1/3)/(-a^4+4*a^3*x-6*a^2*x^2+4*a*x^3+(-1+d)*x^4
),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*(1/(a^6*d^5))^(1/6)*arctan(1/3*(2*sqrt(3)*a^5*d^4*x^2*sqrt((a^
2*d^2*x^2*(1/(a^6*d^5))^(1/3) + (-a*x^2 + x^3)^(2/3)*a*d*(1/(a^6*d^5))^(1/6
) - (-a*x^2 + x^3)^(1/3)*(a - x))/x^2)*(1/(a^6*d^5))^(5/6) - 2*sqrt(3)*(-a*
x^2 + x^3)^(2/3)*a^5*d^4*(1/(a^6*d^5))^(5/6) - sqrt(3)*x^2)/x^2) - 1/2*sqrt
(3)*(1/(a^6*d^5))^(1/6)*arctan(1/3*(2*sqrt(3)*a^5*d^4*x^2*sqrt((a^2*d^2*x^2
*(1/(a^6*d^5))^(1/3) - (-a*x^2 + x^3)^(2/3)*a*d*(1/(a^6*d^5))^(1/6) - (-a*x
^2 + x^3)^(1/3)*(a - x))/x^2)*(1/(a^6*d^5))^(5/6) - 2*sqrt(3)*(-a*x^2 + x^3
)^(2/3)*a^5*d^4*(1/(a^6*d^5))^(5/6) + sqrt(3)*x^2)/x^2) + 1/8*(1/(a^6*d^5))
^(1/6)*log((a^2*d^2*x^2*(1/(a^6*d^5))^(1/3) + (-a*x^2 + x^3)^(2/3)*a*d*(1/(
a^6*d^5))^(1/6) - (-a*x^2 + x^3)^(1/3)*(a - x))/x^2) - 1/8*(1/(a^6*d^5))^(1
/6)*log((a^2*d^2*x^2*(1/(a^6*d^5))^(1/3) - (-a*x^2 + x^3)^(2/3)*a*d*(1/(a^6
*d^5))^(1/6) - (-a*x^2 + x^3)^(1/3)*(a - x))/x^2) + 1/4*(1/(a^6*d^5))^(1/6)
*log((a*d*x^2*(1/(a^6*d^5))^(1/6) + (-a*x^2 + x^3)^(2/3))/x^2) - 1/4*(1/(a^
6*d^5))^(1/6)*log(-(a*d*x^2*(1/(a^6*d^5))^(1/6) - (-a*x^2 + x^3)^(2/3))/x^2
)
```

giac [A] time = 0.29, size = 209, normalized size = 0.71

$$-\frac{\sqrt{3}\log\left(\sqrt{3}(-d)^{\frac{1}{6}}\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}+(-d)^{\frac{1}{3}}\right)}{8a(-d)^{\frac{5}{6}}}-\frac{\sqrt{3}(-d)^{\frac{1}{6}}\log\left(-\sqrt{3}(-d)^{\frac{1}{6}}\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}+\left(-\frac{a}{x}+1\right)^{\frac{4}{3}}+(-d)^{\frac{1}{3}}\right)}{8ad}-\frac{\arctan\left(\frac{\sqrt{3}(-d)^{\frac{1}{6}}+2\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}}{(-d)^{\frac{1}{6}}}\right)}{4a(-d)^{\frac{5}{6}}}-\frac{\arctan\left(-\frac{\sqrt{3}(-d)^{\frac{1}{6}}-2\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}}{(-d)^{\frac{1}{6}}}\right)}{4a(-d)^{\frac{5}{6}}}-\frac{\arctan\left(\frac{\left(-\frac{a}{x}+1\right)^{\frac{2}{3}}}{(-d)^{\frac{1}{6}}}\right)}{2a(-d)^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2*(-a+x))^(1/3)/(-a^4+4*a^3*x-6*a^2*x^2+4*a*x^3+(-1+d)*x^4
),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(3)*log(sqrt(3)*(-d)^(1/6)*(-a/x + 1)^(2/3) + (-a/x + 1)^(4/3) + (
-d)^(1/3))/(a*(-d)^(5/6)) - 1/8*sqrt(3)*(-d)^(1/6)*log(-sqrt(3)*(-d)^(1/6)*
(-a/x + 1)^(2/3) + (-a/x + 1)^(4/3) + (-d)^(1/3))/(a*d) - 1/4*arctan((sqrt(
3)*(-d)^(1/6) + 2*(-a/x + 1)^(2/3))/(-d)^(1/6))/(a*(-d)^(5/6)) - 1/4*arctan
(-(sqrt(3)*(-d)^(1/6) - 2*(-a/x + 1)^(2/3))/(-d)^(1/6))/(a*(-d)^(5/6)) - 1/
2*arctan((-a/x + 1)^(2/3)/(-d)^(1/6))/(a*(-d)^(5/6))
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(x^{2}\left(-a+x\right)\right)^{\frac{1}{3}}\left(-a^{4}+4a^{3}x-6a^{2}x^{2}+4a\,x^{3}+\left(-1+d\right)x^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2*(-a+x))^(1/3)/(-a^4+4*a^3*x-6*a^2*x^2+4*a*x^3+(-1+d)*x^4),x)
```

```
[Out] int(x^3/(x^2*(-a+x))^(1/3)/(-a^4+4*a^3*x-6*a^2*x^2+4*a*x^3+(-1+d)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left((d-1)x^4 - a^4 + 4\,a^3x - 6\,a^2x^2 + 4\,ax^3\right)\left(-(a-x)x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2*(-a+x))^(1/3)/(-a^4+4*a^3*x-6*a^2*x^2+4*a*x^3+(-1+d)*x^4
),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(((d - 1)*x^4 - a^4 + 4*a^3*x - 6*a^2*x^2 + 4*a*x^3)*(-(a - x
)*x^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(-x^2\,\left(a-x\right)\right)^{1/3}\,\left(-a^4+4\,a^3\,x-6\,a^2\,x^2+4\,a\,x^3+\left(d-1\right)\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((-x^2*(a - x))^(1/3)*(4*a*x^3 + 4*a^3*x - a^4 - 6*a^2*x^2 + x^4*(d
 - 1))),x)
```

```
[Out] int(x^3/((-x^2*(a - x))^(1/3)*(4*a*x^3 + 4*a^3*x - a^4 - 6*a^2*x^2 + x^4*(d
 - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**2*(-a+x))**(1/3)/(-a**4+4*a**3*x-6*a**2*x**2+4*a*x**3+(-
1+d)*x**4),x)
```

```
[Out] Timed out
```

3.2263 $\int \frac{\sqrt{c+\sqrt{ax^2+x\sqrt{-b+a^2x^2}}}}{\sqrt{-b+a^2x^2}}\,dx$

Optimal. Leaf size=294

$$\frac{2\sqrt{\sqrt{x\left(\sqrt{a^2x^2-b}+ax\right)}+c}}{a}-\frac{\sqrt{\sqrt{2}\sqrt{b}-2\sqrt{a}\,c}\left(\sqrt{2}\sqrt{a}\,c-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{\sqrt{x\left(\sqrt{a^2x^2-b}+ax\right)}+c}}{\sqrt{\sqrt{2}\sqrt{b}-2\sqrt{a}\,c}}\right)}{a^{5/4}\left(2\sqrt{a}\,c-\sqrt{2}\sqrt{b}\right)}-\frac{\left(\sqrt{2}\sqrt{a}\,c\right.\cdots}{\cdots}$$

Rubi [F] time = 0.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{c+\sqrt{ax^2+x\sqrt{-b+a^2x^2}}}}{\sqrt{-b+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + Sqrt[a*x^2 + x*Sqrt[-b + a^2*x^2]]]/Sqrt[-b + a^2*x^2],x]
```

```
[Out] Defer[Int][Sqrt[c + Sqrt[a*x^2 + x*Sqrt[-b + a^2*x^2]]]/Sqrt[-b + a^2*x^2],
 x]
```

Rubi steps

$$\int \frac{\sqrt{c+\sqrt{ax^2+x\sqrt{-b+a^2x^2}}}}{\sqrt{-b+a^2x^2}}\,dx=\int \frac{\sqrt{c+\sqrt{ax^2+x\sqrt{-b+a^2x^2}}}}{\sqrt{-b+a^2x^2}}\,dx$$

Mathematica [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+\sqrt{ax^2+x\sqrt{-b+a^2x^2}}}}{\sqrt{-b+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + Sqrt[a*x^2 + x*Sqrt[-b + a^2*x^2]]]/Sqrt[-b + a^2*x^2],x
]
```

```
[Out] Integrate[Sqrt[c + Sqrt[a*x^2 + x*Sqrt[-b + a^2*x^2]]]/Sqrt[-b + a^2*x^2],
x]
```

IntegrateAlgebraic [F] time = 4.31, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+\sqrt{ax^2+x\sqrt{-b+a^2x^2}}}}{\sqrt{-b+a^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[c + Sqrt[a*x^2 + x*Sqrt[-b + a^2*x^2]]]/Sqrt[-b + a
^2*x^2],x]
```

[Out] Defer[IntegrateAlgebraic][Sqrt[c + Sqrt[a*x^2 + x*Sqrt[-b + a^2*x^2]]]/Sqrt
[-b + a^2*x^2], x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x^2+x*(a^2*x^2-b)^(1/2))^(1/2))^(1/2)/(a^2*x^2-b)^(1/2),x,
algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x^2+x*(a^2*x^2-b)^(1/2))^(1/2))^(1/2)/(a^2*x^2-b)^(1/2),x,
algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(4*a
^2*x^2-4*a*x^2+x^2-4*b)]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 1.
4gen.cc:simplify/tmp.type!=_EXT Error: Bad Argument Value
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+\sqrt{a\,x^{2}+x\sqrt{a^{2}x^{2}-b}}}}{\sqrt{a^{2}x^{2}-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+(a*x^2+x*(a^2*x^2-b)^(1/2))^(1/2))^(1/2)/(a^2*x^2-b)^(1/2),x)
```

```
[Out] int((c+(a*x^2+x*(a^2*x^2-b)^(1/2))^(1/2))^(1/2)/(a^2*x^2-b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+\sqrt{ax^{2}+\sqrt{a^{2}x^{2}-b}\,x}}}{\sqrt{a^{2}x^{2}-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x^2+x*(a^2*x^2-b)^(1/2))^(1/2))^(1/2)/(a^2*x^2-b)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate(sqrt(c + sqrt(a*x^2 + sqrt(a^2*x^2 - b)*x))/sqrt(a^2*x^2 - b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+\sqrt{x\,\sqrt{a^2\,x^2-b}+a\,x^2}}}{\sqrt{a^2\,x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + (x*(a^2*x^2 - b)^(1/2) + a*x^2)^(1/2))^(1/2)/(a^2*x^2 - b)^(1/2),x
)
```

```
[Out] int((c + (x*(a^2*x^2 - b)^(1/2) + a*x^2)^(1/2))^(1/2)/(a^2*x^2 - b)^(1/2),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + \sqrt{ax^2 + x\sqrt{a^2x^2 - b}}}}{\sqrt{a^2x^2 - b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x**2+x*(a**2*x**2-b)**(1/2))**(1/2))**(1/2)/(a**2*x**2-b)**
(1/2),x)
```

```
[Out] Integral(sqrt(c + sqrt(a*x**2 + x*sqrt(a**2*x**2 - b)))/sqrt(a**2*x**2 - b)
, x)
```

3.2264 $\int \frac{-a(ab+ac-2bc)+2(a^2-bc)x+(-2a+b+c)x^2}{((-a+x)(-b+x)(-c+x))^{2/3}(-bc+a^2d+(b+c-2ad)x+(-1+d)x^2)}\,dx$

Optimal. Leaf size=296

$$\frac{\log\left(a^2d^{2/3}+\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)\sqrt[3]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}+\left(x^2(-a-b-c)+x(ab+ac+bc)\right.}{2d^{2/3}}$$

Rubi [F] time = 8.97, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-a(ab+ac-2bc)+2\left(a^2-bc\right)x+(-2a+b+c)x^2}{((-a+x)(-b+x)(-c+x))^{2/3}\left(-bc+a^2d+(b+c-2ad)x+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a*(a*b + a*c - 2*b*c)) + 2*(a^2 - b*c)*x + (-2*a + b + c)*x^2)/(((-a
 + x)*(-b + x)*(-c + x))^(2/3)*(-(b*c) + a^2*d + (b + c - 2*a*d)*x + (-1 +
d)*x^2)),x]
```

```
[Out] -(((2*a - b - c + Sqrt[b^2 + c^2 + 4*a^2*d - 4*a*c*d - 2*b*(c + 2*a*d - 2*c
*d)])*(-a + x)^(2/3)*(-b + x)^(2/3)*(-c + x)^(2/3)*Defer[Int][(-a + x)^(1/3
)/((-b + x)^(2/3)*(-c + x)^(2/3)*(b + c - 2*a*d - Sqrt[b^2 - 2*b*c + c^2 +
4*a^2*d - 4*a*b*d - 4*a*c*d + 4*b*c*d] + 2*(-1 + d)*x)), x])/(-((a - x)*(b
- x)*(c - x)))^(2/3)) - ((2*a - b - c - Sqrt[b^2 + c^2 + 4*a^2*d - 4*a*c*d
- 2*b*(c + 2*a*d - 2*c*d)])*(-a + x)^(2/3)*(-b + x)^(2/3)*(-c + x)^(2/3)*De
fer[Int][(-a + x)^(1/3)/((-b + x)^(2/3)*(-c + x)^(2/3)*(b + c - 2*a*d + Sqr
t[b^2 - 2*b*c + c^2 + 4*a^2*d - 4*a*b*d - 4*a*c*d + 4*b*c*d] + 2*(-1 + d)*x
)), x])/(-((a - x)*(b - x)*(c - x)))^(2/3)
```

Rubi steps

$$\int \frac{-a(ab+ac-2bc)+2\left(a^2-bc\right)x+(-2a+b+c)x^2}{((-a+x)(-b+x)(-c+x))^{2/3}\left(-bc+a^2d+(b+c-2ad)x+(-1+d)x^2\right)}\,dx=\frac{\left((-a+x)^{2/3}(-b+x)^{2/3}(-c+\right.}{}$$

$$=\frac{\left((-a+x)^{2/3}(-b+x)^{2/3}(-c+\right.}{((-}$$

$$=\frac{\left((-a+x)^{2/3}(-b+x)^{2/3}(-c+\right.}{}$$

$$=\frac{\left(\left(-2a+b+c-\sqrt{b^2+c^2+4\right.\right.}{}$$

Mathematica [F] time = 3.29, size = 0, normalized size = 0.00

$$\int \frac{-a(ab+ac-2bc)+2\left(a^2-bc\right)x+(-2a+b+c)x^2}{((-a+x)(-b+x)(-c+x))^{2/3}\left(-bc+a^2d+(b+c-2ad)x+(-1+d)x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-(a*(a*b + a*c - 2*b*c)) + 2*(a^2 - b*c)*x + (-2*a + b + c)*x^2)
/(((-a + x)*(-b + x)*(-c + x))^(2/3)*(-(b*c) + a^2*d + (b + c - 2*a*d)*x +
(-1 + d)*x^2)),x]
```

```
[Out] Integrate[(-(a*(a*b + a*c - 2*b*c)) + 2*(a^2 - b*c)*x + (-2*a + b + c)*x^2)
/(((-a + x)*(-b + x)*(-c + x))^(2/3)*(-(b*c) + a^2*d + (b + c - 2*a*d)*x +
(-1 + d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 7.27, size = 296, normalized size = 1.00

$$\frac{\log\left(a^2d^{2/3}+\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)\sqrt[3]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}+\left(x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3\right)^{2/3}-2ad^{2/3}x+d^{2/3}x^2\right)}{2d^{2/3}}-\frac{\log\left(\sqrt[3]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}+a\sqrt[3]{d}-\sqrt[3]{d}\,x\right)}{d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}a\sqrt[3]{d}-\sqrt{3}\sqrt[3]{d}\,x}{-2\sqrt[3]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}+a\sqrt[3]{d}-\sqrt[3]{d}\,x}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*(a*b + a*c - 2*b*c)) + 2*(a^2 - b*c)*x + (-2*a + b
+ c)*x^2)/(((-a + x)*(-b + x)*(-c + x))^(2/3)*(-(b*c) + a^2*d + (b + c - 2*
a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*a*d^(1/3) - Sqrt[3]*d^(1/3)*x)/(a*d^(1/3) - d^(1
/3)*x - 2*(-(a*b*c) + (a*b + a*c + b*c)*x + (-a - b - c)*x^2 + x^3)^(1/3))]
)/d^(2/3)) - Log[a*d^(1/3) - d^(1/3)*x + (-(a*b*c) + (a*b + a*c + b*c)*x +
(-a - b - c)*x^2 + x^3)^(1/3)]/d^(2/3) + Log[a^2*d^(2/3) - 2*a*d^(2/3)*x +
d^(2/3)*x^2 + (-(a*d^(1/3)) + d^(1/3)*x)*(-(a*b*c) + (a*b + a*c + b*c)*x +
(-a - b - c)*x^2 + x^3)^(1/3) + (-(a*b*c) + (a*b + a*c + b*c)*x + (-a - b -
 c)*x^2 + x^3)^(2/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a*b+a*c-2*b*c)+2*(a^2-b*c)*x+(-2*a+b+c)*x^2)/((-a+x)*(-b+x)*
(-c+x))^(2/3)/(-b*c+a^2*d+(-2*a*d+b+c)*x+(-1+d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(2\,a-b-c)x^2+(ab+ac-2\,bc)a-2\left(a^2-bc\right)x}{(-(a-x)(b-x)(c-x))^{\frac{2}{3}}\left(a^2d+(d-1)x^2-bc-(2\,ad-b-c)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a*b+a*c-2*b*c)+2*(a^2-b*c)*x+(-2*a+b+c)*x^2)/((-a+x)*(-b+x)*
(-c+x))^(2/3)/(-b*c+a^2*d+(-2*a*d+b+c)*x+(-1+d)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-((2*a - b - c)*x^2 + (a*b + a*c - 2*b*c)*a - 2*(a^2 - b*c)*x)/((
-(a - x)*(b - x)*(c - x))^(2/3)*(a^2*d + (d - 1)*x^2 - b*c - (2*a*d - b - c
)*x)), x)
```

maple [F] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{-a\left(ab+ac-2bc\right)+2\left(a^2-bc\right)x+\left(-2a+b+c\right)x^2}{\left(\left(-a+x\right)\left(-b+x\right)\left(-c+x\right)\right)^{\frac{2}{3}}\left(-bc+a^2d+\left(-2ad+b+c\right)x+\left(-1+d\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*(a*b+a*c-2*b*c)+2*(a^2-b*c)*x+(-2*a+b+c)*x^2)/((-a+x)*(-b+x)*(-c+x)
)^(2/3)/(-b*c+a^2*d+(-2*a*d+b+c)*x+(-1+d)*x^2),x)
```

```
[Out] int((-a*(a*b+a*c-2*b*c)+2*(a^2-b*c)*x+(-2*a+b+c)*x^2)/((-a+x)*(-b+x)*(-c+x)
)^(2/3)/(-b*c+a^2*d+(-2*a*d+b+c)*x+(-1+d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(2\,a - b - c)x^2 + (ab + ac - 2\,bc)a - 2\left(a^2 - bc\right)x}{(-(a - x)(b - x)(c - x))^{\frac{2}{3}}\left(a^2 d + (d - 1)x^2 - bc - (2\,ad - b - c)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a*b+a*c-2*b*c)+2*(a^2-b*c)*x+(-2*a+b+c)*x^2)/((-a+x)*(-b+x)*
(-c+x))^(2/3)/(-b*c+a^2*d+(-2*a*d+b+c)*x+(-1+d)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate(((2*a - b - c)*x^2 + (a*b + a*c - 2*b*c)*a - 2*(a^2 - b*c)*x)/((
-(a - x)*(b - x)*(c - x))^(2/3)*(a^2*d + (d - 1)*x^2 - b*c - (2*a*d - b - c
)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{2\,x\,\left(b\,c - a^2\right) - x^2\,\left(b - 2\,a + c\right) + a\,\left(a\,b + a\,c - 2\,b\,c\right)}{\left(-\left(a - x\right)\,\left(b - x\right)\,\left(c - x\right)\right)^{2/3}\,\left(x\,\left(b + c - 2\,a\,d\right) - b\,c + a^2\,d + x^2\,\left(d - 1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x*(b*c - a^2) - x^2*(b - 2*a + c) + a*(a*b + a*c - 2*b*c))/((-(a -
x)*(b - x)*(c - x))^(2/3)*(x*(b + c - 2*a*d) - b*c + a^2*d + x^2*(d - 1))),
x)
```

```
[Out] -int((2*x*(b*c - a^2) - x^2*(b - 2*a + c) + a*(a*b + a*c - 2*b*c))/((-(a -
x)*(b - x)*(c - x))^(2/3)*(x*(b + c - 2*a*d) - b*c + a^2*d + x^2*(d - 1))),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*(a*b+a*c-2*b*c)+2*(a**2-b*c)*x+(-2*a+b+c)*x**2)/((-a+x)*(-b+x
)*(-c+x))**(2/3)/(-b*c+a**2*d+(-2*a*d+b+c)*x+(-1+d)*x**2),x)
```

```
[Out] Timed out
```

3.2265

$$\int \frac{x^2(-2ab+(a+b)x)}{\sqrt[3]{x(-a+x)(-b+x)}\left(-a^2b^2+2ab(a+b)x-\left(a^2+4ab+b^2\right)x^2+2(a+b)x^3+(-1+d)x^4\right)}\,dx$$

Optimal. Leaf size=297

$$-\frac{\log\left(\sqrt[3]{x^2(-a-b)+abx+x^3}-\sqrt[6]{d}\,x\right)}{2d^{2/3}}-\frac{\log\left(\sqrt[3]{x^2(-a-b)+abx+x^3}+\sqrt[6]{d}\,x\right)}{2d^{2/3}}+\frac{\log\left(-\sqrt[6]{d}\,x\sqrt[3]{x^2(-a-b)+abx+\right.}{}$$

Rubi [F] time = 24.68, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2(-2ab+(a+b)x)}{\sqrt[3]{x(-a+x)(-b+x)}\left(-a^2b^2+2ab(a+b)x-\left(a^2+4ab+b^2\right)x^2+2(a+b)x^3+(-1+d)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(-2*a*b + (a + b)*x))/((x*(-a + x)*(-b + x))^(1/3)*(-(a^2*b^2) + 2
*a*b*(a + b)*x - (a^2 + 4*a*b + b^2)*x^2 + 2*(a + b)*x^3 + (-1 + d)*x^4)),x
]
```

```
[Out] (6*a*b*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][Defer[Int][x^7/((
-a + x^3)^(1/3)*(-b + x^3)^(1/3)*(a^2*b^2 - 2*a^2*b*(1 + b/a)*x^3 + a^2*(1
+ (b*(4*a + b))/a^2)*x^6 - 2*a*(1 + b/a)*x^9 + (1 - d)*x^12)), x], x, x^(1/
3)])/((a - x)*(b - x)*x)^(1/3) - (3*(a + b)*x^(1/3)*(-a + x)^(1/3)*(-b + x)
^(1/3)*Defer[Subst][Defer[Int][x^10/((-a + x^3)^(1/3)*(-b + x^3)^(1/3)*(a^2
*b^2 - 2*a^2*b*(1 + b/a)*x^3 + a^2*(1 + (b*(4*a + b))/a^2)*x^6 - 2*a*(1 + b
/a)*x^9 + (1 - d)*x^12)), x], x, x^(1/3)])/((a - x)*(b - x)*x)^(1/3)
```

Rubi steps

$$\int \frac{x^2(-2ab+(a+b)x)}{\sqrt[3]{x(-a+x)(-b+x)}\left(-a^2b^2+2ab(a+b)x-\left(a^2+4ab+b^2\right)x^2+2(a+b)x^3+(-1+d)x^4\right)}\,dx=\frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}\right.}{}$$

$$=\frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+}\right.}{}$$

$$=\frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+}\right.}{}$$

$$=\frac{\left(3(-a-b)\sqrt[3]{}\right.}{}$$

Mathematica [F] time = 2.66, size = 0, normalized size = 0.00

$$\int \frac{x^2(-2ab+(a+b)x)}{\sqrt[3]{x(-a+x)(-b+x)}\left(-a^2b^2+2ab(a+b)x-\left(a^2+4ab+b^2\right)x^2+2(a+b)x^3+(-1+d)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(-2*a*b + (a + b)*x))/((x*(-a + x)*(-b + x))^(1/3)*(-(a^2*b^
2) + 2*a*b*(a + b)*x - (a^2 + 4*a*b + b^2)*x^2 + 2*(a + b)*x^3 + (-1 + d)*x
^4)),x]
```

```
[Out] Integrate[(x^2*(-2*a*b + (a + b)*x))/((x*(-a + x)*(-b + x))^(1/3)*(-(a^2*b^
2) + 2*a*b*(a + b)*x - (a^2 + 4*a*b + b^2)*x^2 + 2*(a + b)*x^3 + (-1 + d)*x
^4)), x]
```

IntegrateAlgebraic [A] time = 1.17, size = 295, normalized size = 0.99

$$-\frac{\log \left(\sqrt[3]{x^2(-a-b)+abx+x^3}-\sqrt[6]{d}\,x\right)}{2d^{2/3}}-\frac{\log \left(\sqrt[3]{x^2(-a-b)+abx+x^3}+\sqrt[6]{d}\,x\right)}{2d^{2/3}}+\frac{\log \left(-\sqrt[6]{d}\,x\sqrt[3]{x^2(-a-b)+abx+x^3}+\left(x^2(-a-b)+abx+x^3\right)^{2/3}+\sqrt[3]{d}\,x^2\right)}{4d^{2/3}}+\frac{\log \left(\sqrt[6]{d}\,x\sqrt[3]{x^2(-a-b)+abx+x^3}+\left(x^2(-a-b)+abx+x^3\right)^{2/3}+\sqrt[3]{d}\,x^2\right)}{4d^{2/3}}+\frac{\sqrt{3}\tan ^{-1}\left(\frac{\frac{2\left(x^2(-a-b)+abx+x^3\right)^{2/3}}{\sqrt{3}\sqrt[3]{d}}+\frac{x^2}{\sqrt{3}}}{x^2}\right)}{2d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(-2*a*b + (a + b)*x))/((x*(-a + x)*(-b + x))^(1/3)*
(-(a^2*b^2) + 2*a*b*(a + b)*x - (a^2 + 4*a*b + b^2)*x^2 + 2*(a + b)*x^3 + (
-1 + d)*x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(x^2/Sqrt[3] + (2*(a*b*x + (-a - b)*x^2 + x^3)^(2/3))/(Sqrt
[3]*d^(1/3)))/x^2])/(2*d^(2/3)) - Log[-(d^(1/6)*x) + (a*b*x + (-a - b)*x^2
+ x^3)^(1/3)]/(2*d^(2/3)) - Log[d^(1/6)*x + (a*b*x + (-a - b)*x^2 + x^3)^(1
/3)]/(2*d^(2/3)) + Log[d^(1/3)*x^2 - d^(1/6)*x*(a*b*x + (-a - b)*x^2 + x^3)
^(1/3) + (a*b*x + (-a - b)*x^2 + x^3)^(2/3)]/(4*d^(2/3)) + Log[d^(1/3)*x^2
+ d^(1/6)*x*(a*b*x + (-a - b)*x^2 + x^3)^(1/3) + (a*b*x + (-a - b)*x^2 + x^
3)^(2/3)]/(4*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a^2*b^2+2*a*b*(a+b
)*x-(a^2+4*a*b+b^2)*x^2+2*(a+b)*x^3+(-1+d)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(2\,ab-(a+b)x)x^2}{\left((d-1)x^4-a^2b^2+2\,(a+b)abx+2\,(a+b)x^3-\left(a^2+4\,ab+b^2\right)x^2\right)\left((a-x)(b-x)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a^2*b^2+2*a*b*(a+b
)*x-(a^2+4*a*b+b^2)*x^2+2*(a+b)*x^3+(-1+d)*x^4),x, algorithm="giac")
```

```
[Out] integrate(-(2*a*b - (a + b)*x)*x^2/(((d - 1)*x^4 - a^2*b^2 + 2*(a + b)*a*b*
x + 2*(a + b)*x^3 - (a^2 + 4*a*b + b^2)*x^2)*((a - x)*(b - x)*x)^(1/3)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-2ab+(a+b)\,x\right)}{\left(x\left(-a+x\right)\left(-b+x\right)\right)^{\frac{1}{3}}\left(-a^2b^2+2ab\left(a+b\right)x-\left(a^2+4ab+b^2\right)x^2+2\left(a+b\right)x^3+\left(-1+d\right)x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a^2*b^2+2*a*b*(a+b)*x-(a
^2+4*a*b+b^2)*x^2+2*(a+b)*x^3+(-1+d)*x^4),x)
```

```
[Out] int(x^2*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a^2*b^2+2*a*b*(a+b)*x-(a
^2+4*a*b+b^2)*x^2+2*(a+b)*x^3+(-1+d)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(2\,ab-(a+b)x)x^2}{\left((d-1)x^4-a^2b^2+2\,(a+b)abx+2\,(a+b)x^3-\left(a^2+4\,ab+b^2\right)x^2\right)((a-x)(b-x)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a^2*b^2+2*a*b*(a+b
)*x-(a^2+4*a*b+b^2)*x^2+2*(a+b)*x^3+(-1+d)*x^4),x, algorithm="maxima")
```

```
[Out] -integrate((2*a*b - (a + b)*x)*x^2/(((d - 1)*x^4 - a^2*b^2 + 2*(a + b)*a*b*
x + 2*(a + b)*x^3 - (a^2 + 4*a*b + b^2)*x^2)*((a - x)*(b - x)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^2\,\left(2\,a\,b-x\,\left(a+b\right)\right)}{{\left(x\,\left(a-x\right)\,\left(b-x\right)\right)}^{1/3}\,\left(2\,x^3\,\left(a+b\right)-x^2\,\left(a^2+4\,a\,b+b^2\right)-a^2\,b^2+x^4\,\left(d-1\right)+2\,a\,b\,x\,\left(a+b\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(2*a*b - x*(a + b)))/((x*(a - x)*(b - x))^(1/3)*(2*x^3*(a + b) -
x^2*(4*a*b + a^2 + b^2) - a^2*b^2 + x^4*(d - 1) + 2*a*b*x*(a + b))),x)
```

```
[Out] -int((x^2*(2*a*b - x*(a + b)))/((x*(a - x)*(b - x))^(1/3)*(2*x^3*(a + b) -
x^2*(4*a*b + a^2 + b^2) - a^2*b^2 + x^4*(d - 1) + 2*a*b*x*(a + b))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-2*a*b+(a+b)*x)/(x*(-a+x)*(-b+x))**(1/3)/(-a**2*b**2+2*a*b*
(a+b)*x-(a**2+4*a*b+b**2)*x**2+2*(a+b)*x**3+(-1+d)*x**4),x)
```

```
[Out] Timed out
```

3.2266 $\int \frac{1+x^6}{\sqrt[4]{x^3+x^5}\left(1-x^6\right)}\,dx$

Optimal. Leaf size=297

$$\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^5+x^3}}\right)+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{3\sqrt[4]{2}}+\frac{2}{3}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^5+x^3}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{3\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^5+x^3}}{\sqrt{2}x^2-\sqrt{x^5+x^3}}\right)}{3\,2^{3/4}}+\frac{1}{3}\sqrt{2}$$

Rubi [F] time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$Aborted

Verification is not applicable to the result.

```
[In] Int[(1 + x^6)/((x^3 + x^5)^(1/4)*(1 - x^6)),x]
[Out] $Aborted
```

Rubi steps

Aborted

Mathematica [F] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{1+x^6}{\sqrt[4]{x^3+x^5}\left(1-x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^6)/((x^3 + x^5)^(1/4)*(1 - x^6)),x]
[Out] Integrate[(1 + x^6)/((x^3 + x^5)^(1/4)*(1 - x^6)), x]
```

IntegrateAlgebraic [A] time = 1.34, size = 297, normalized size = 1.00

$$\frac{2}{3}\tan^{-1}\left(\frac{x}{\sqrt[4]{x^5+x^3}}\right)+\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{3\sqrt[4]{2}}+\frac{2}{3}\tanh^{-1}\left(\frac{x}{\sqrt[4]{x^5+x^3}}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}x}{\sqrt[4]{x^5+x^3}}\right)}{3\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{2^{3/4}x\sqrt[4]{x^5+x^3}}{\sqrt{2}x^2-\sqrt{x^5+x^3}}\right)}{3\,2^{3/4}}+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x\sqrt[4]{x^5+x^3}}{\sqrt{x^5+x^3}-x^2}\right)+\frac{\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^5+x^3}}{2^{3/4}}}{x\sqrt[4]{x^5+x^3}}\right)}{3\,2^{3/4}}+\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{\frac{x^2}{\sqrt{2}}+\frac{\sqrt{x^5+x^3}}{\sqrt{2}}}{x\sqrt[4]{x^5+x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^6)/((x^3 + x^5)^(1/4)*(1 - x^6)),x]
[Out] (2*ArcTan[x/(x^3 + x^5)^(1/4)])/3 + ArcTan[(2^(1/4)*x)/(x^3 + x^5)^(1/4)]/(
3*2^(1/4)) - ArcTan[(2^(3/4)*x*(x^3 + x^5)^(1/4))/(Sqrt[2]*x^2 - Sqrt[x^3 +
 x^5])]/(3*2^(3/4)) + (Sqrt[2]*ArcTan[(Sqrt[2]*x*(x^3 + x^5)^(1/4))/(-x^2 +
 Sqrt[x^3 + x^5])])/3 + (2*ArcTanh[x/(x^3 + x^5)^(1/4)])/3 + ArcTanh[(2^(1/
4)*x)/(x^3 + x^5)^(1/4)]/(3*2^(1/4)) + ArcTanh[(x^2/2^(1/4) + Sqrt[x^3 + x^
5]/2^(3/4))/(x*(x^3 + x^5)^(1/4))]/(3*2^(3/4)) + (Sqrt[2]*ArcTanh[(x^2/Sqrt
[2] + Sqrt[x^3 + x^5]/Sqrt[2])/(x*(x^3 + x^5)^(1/4))])/3
```

fricas [B] time = 59.49, size = 1837, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^5+x^3)^(1/4)/(-x^6+1),x, algorithm="fricas")
```

```
[Out] -1/6*2^(3/4)*arctan(-1/2*(4*2^(3/4)*(x^5 + x^3)^(1/4)*x^2 - 2^(3/4)*(2*2^(3
/4)*sqrt(x^5 + x^3)*x + 2^(1/4)*(x^4 + 2*x^3 + x^2)) + 4*2^(1/4)*(x^5 + x^3
)^(3/4))/(x^4 - 2*x^3 + x^2)) + 1/24*2^(3/4)*log((4*sqrt(2)*(x^5 + x^3)^(1/
4)*x^2 + 2^(3/4)*(x^4 + 2*x^3 + x^2) + 4*2^(1/4)*sqrt(x^5 + x^3)*x + 4*(x^5
 + x^3)^(3/4))/(x^4 - 2*x^3 + x^2)) - 1/24*2^(3/4)*log((4*sqrt(2)*(x^5 + x^
3)^(1/4)*x^2 - 2^(3/4)*(x^4 + 2*x^3 + x^2) - 4*2^(1/4)*sqrt(x^5 + x^3)*x +
4*(x^5 + x^3)^(3/4))/(x^4 - 2*x^3 + x^2)) - 1/3*sqrt(2)*arctan(-(x^6 + 2*x^
5 + 3*x^4 + 2*x^3 + 2*sqrt(2)*(x^5 + x^3)^(3/4)*(x^2 - 3*x + 1) + x^2 + 2*s
qrt(2)*(x^5 + x^3)^(1/4)*(3*x^4 - x^3 + 3*x^2) + 4*sqrt(x^5 + x^3)*(x^3 + x
^2 + x) - (2*sqrt(2)*sqrt(x^5 + x^3)*(x^3 - 3*x^2 + x) + 16*(x^5 + x^3)^(3/
4)*x + sqrt(2)*(x^6 - 8*x^5 + x^4 - 8*x^3 + x^2) + 4*(x^5 + x^3)^(1/4)*(x^4
 + x^3 + x^2))*sqrt((x^4 + x^3 + 2*sqrt(2)*(x^5 + x^3)^(1/4)*x^2 + x^2 + 4*
sqrt(x^5 + x^3)*x + 2*sqrt(2)*(x^5 + x^3)^(3/4))/(x^4 + x^3 + x^2)))/(x^6 -
 14*x^5 + 3*x^4 - 14*x^3 + x^2)) + 1/3*sqrt(2)*arctan(-(x^6 + 2*x^5 + 3*x^4
 + 2*x^3 - 2*sqrt(2)*(x^5 + x^3)^(3/4)*(x^2 - 3*x + 1) + x^2 - 2*sqrt(2)*(x
^5 + x^3)^(1/4)*(3*x^4 - x^3 + 3*x^2) + 4*sqrt(x^5 + x^3)*(x^3 + x^2 + x) +
 (2*sqrt(2)*sqrt(x^5 + x^3)*(x^3 - 3*x^2 + x) - 16*(x^5 + x^3)^(3/4)*x + sq
rt(2)*(x^6 - 8*x^5 + x^4 - 8*x^3 + x^2) - 4*(x^5 + x^3)^(1/4)*(x^4 + x^3 +
x^2))*sqrt((x^4 + x^3 - 2*sqrt(2)*(x^5 + x^3)^(1/4)*x^2 + x^2 + 4*sqrt(x^5
+ x^3)*x - 2*sqrt(2)*(x^5 + x^3)^(3/4))/(x^4 + x^3 + x^2)))/(x^6 - 14*x^5 +
 3*x^4 - 14*x^3 + x^2)) + 1/12*sqrt(2)*log(4*(x^4 + x^3 + 2*sqrt(2)*(x^5 +
x^3)^(1/4)*x^2 + x^2 + 4*sqrt(x^5 + x^3)*x + 2*sqrt(2)*(x^5 + x^3)^(3/4))/(
x^4 + x^3 + x^2)) - 1/12*sqrt(2)*log(4*(x^4 + x^3 - 2*sqrt(2)*(x^5 + x^3)^(
1/4)*x^2 + x^2 + 4*sqrt(x^5 + x^3)*x - 2*sqrt(2)*(x^5 + x^3)^(3/4))/(x^4 +
x^3 + x^2)) + 1/6*2^(1/4)*arctan(1/2*(2*x^6 + 8*x^5 + 12*x^4 + 8*x^3 + 4*2^
(3/4)*(x^5 + x^3)^(3/4)*(x^2 - 6*x + 1) + 8*sqrt(2)*sqrt(x^5 + x^3)*(x^3 +
2*x^2 + x) + 2*x^2 + sqrt(2)*(32*sqrt(2)*(x^5 + x^3)^(3/4)*x + 2^(3/4)*(x^6
 - 16*x^5 - 2*x^4 - 16*x^3 + x^2) + 4*2^(1/4)*sqrt(x^5 + x^3)*(x^3 - 6*x^2
+ x) + 8*(x^5 + x^3)^(1/4)*(x^4 + 2*x^3 + x^2))*sqrt((4*2^(3/4)*(x^5 + x^3)
^(1/4)*x^2 + sqrt(2)*(x^4 + 2*x^3 + x^2) + 8*sqrt(x^5 + x^3)*x + 4*2^(1/4)*
(x^5 + x^3)^(3/4))/(x^4 + 2*x^3 + x^2)) + 8*2^(1/4)*(x^5 + x^3)^(1/4)*(3*x^
4 - 2*x^3 + 3*x^2))/(x^6 - 28*x^5 + 6*x^4 - 28*x^3 + x^2)) - 1/6*2^(1/4)*ar
ctan(1/2*(2*x^6 + 8*x^5 + 12*x^4 + 8*x^3 - 4*2^(3/4)*(x^5 + x^3)^(3/4)*(x^2
 - 6*x + 1) + 8*sqrt(2)*sqrt(x^5 + x^3)*(x^3 + 2*x^2 + x) + 2*x^2 + sqrt(2)
*(32*sqrt(2)*(x^5 + x^3)^(3/4)*x - 2^(3/4)*(x^6 - 16*x^5 - 2*x^4 - 16*x^3 +
 x^2) - 4*2^(1/4)*sqrt(x^5 + x^3)*(x^3 - 6*x^2 + x) + 8*(x^5 + x^3)^(1/4)*(
x^4 + 2*x^3 + x^2))*sqrt(-(4*2^(3/4)*(x^5 + x^3)^(1/4)*x^2 - sqrt(2)*(x^4 +
 2*x^3 + x^2) - 8*sqrt(x^5 + x^3)*x + 4*2^(1/4)*(x^5 + x^3)^(3/4))/(x^4 + 2
*x^3 + x^2)) - 8*2^(1/4)*(x^5 + x^3)^(1/4)*(3*x^4 - 2*x^3 + 3*x^2))/(x^6 -
28*x^5 + 6*x^4 - 28*x^3 + x^2)) + 1/24*2^(1/4)*log(8*(4*2^(3/4)*(x^5 + x^3)
^(1/4)*x^2 + sqrt(2)*(x^4 + 2*x^3 + x^2) + 8*sqrt(x^5 + x^3)*x + 4*2^(1/4)*
(x^5 + x^3)^(3/4))/(x^4 + 2*x^3 + x^2)) - 1/24*2^(1/4)*log(-8*(4*2^(3/4)*(x
^5 + x^3)^(1/4)*x^2 - sqrt(2)*(x^4 + 2*x^3 + x^2) - 8*sqrt(x^5 + x^3)*x + 4
*2^(1/4)*(x^5 + x^3)^(3/4))/(x^4 + 2*x^3 + x^2)) + 1/3*arctan(2*((x^5 + x^3
)^(1/4)*x^2 + (x^5 + x^3)^(3/4))/(x^4 - x^3 + x^2)) + 1/3*log((x^4 + x^3 +
2*(x^5 + x^3)^(1/4)*x^2 + x^2 + 2*sqrt(x^5 + x^3)*x + 2*(x^5 + x^3)^(3/4))/
(x^4 - x^3 + x^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^6+1}{\left(x^6-1\right)\left(x^5+x^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^5+x^3)^(1/4)/(-x^6+1),x, algorithm="giac")
```

```
[Out] integrate(-(x^6 + 1)/((x^6 - 1)*(x^5 + x^3)^(1/4)), x)
```

maple [C] time = 24.18, size = 1422, normalized size = 4.79

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/(x^5+x^3)^(1/4)/(-x^6+1),x)
```

```
[Out] -1/12*RootOf(_Z^4+8)*ln(-(RootOf(_Z^4+8)^3*(x^5+x^3)^(1/2)*x-2*(x^5+x^3)^(1
/4)*RootOf(_Z^4+8)^2*x^2-RootOf(_Z^4+8)*x^4+2*RootOf(_Z^4+8)*x^3+4*(x^5+x^3
)^(3/4)-RootOf(_Z^4+8)*x^2)/(1+x)^2/x^2)-1/12*RootOf(_Z^2+RootOf(_Z^4+8)^2)
*ln(-(-(x^5+x^3)^(1/2)*RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*x+2*(
x^5+x^3)^(1/4)*RootOf(_Z^4+8)^2*x^2-RootOf(_Z^2+RootOf(_Z^4+8)^2)*x^4+2*Roo
tOf(_Z^2+RootOf(_Z^4+8)^2)*x^3+4*(x^5+x^3)^(3/4)-RootOf(_Z^2+RootOf(_Z^4+8)
^2)*x^2)/(1+x)^2/x^2)+1/48*ln(-(RootOf(_Z^4+8)^3*x^4-2*RootOf(_Z^4+8)^3*x^3
+8*(x^5+x^3)^(1/4)*RootOf(_Z^4+8)^2*x^2+RootOf(_Z^4+8)^3*x^2-16*(x^5+x^3)^(
1/2)*RootOf(_Z^4+8)*x+16*(x^5+x^3)^(3/4))/(-1+x)^2/x^2)*RootOf(_Z^4+8)^3+1/
48*ln(-(RootOf(_Z^4+8)^3*x^4-2*RootOf(_Z^4+8)^3*x^3+8*(x^5+x^3)^(1/4)*RootO
f(_Z^4+8)^2*x^2+RootOf(_Z^4+8)^3*x^2-16*(x^5+x^3)^(1/2)*RootOf(_Z^4+8)*x+16
*(x^5+x^3)^(3/4))/(-1+x)^2/x^2)*RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z^4+8)
^2)-1/24*RootOf(_Z^4+8)^2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln((-RootOf(_Z^2+Ro
otOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^4+RootOf(_Z^4+8)^3*x^4-2*RootOf(_Z^2+Roo
tOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*x^3+2*RootOf(_Z^4+8)^3*x^3+8*RootOf(_Z^2+Ro
otOf(_Z^4+8)^2)*(x^5+x^3)^(1/4)*RootOf(_Z^4+8)*x^2-RootOf(_Z^2+RootOf(_Z^4+
8)^2)*RootOf(_Z^4+8)^2*x^2+RootOf(_Z^4+8)^3*x^2-8*RootOf(_Z^2+RootOf(_Z^4+8
)^2)*(x^5+x^3)^(1/2)*x-8*(x^5+x^3)^(1/2)*RootOf(_Z^4+8)*x+16*(x^5+x^3)^(3/4
))/(-1+x)^2/x^2)+1/3*ln((x^4+2*(x^5+x^3)^(3/4)+2*(x^5+x^3)^(1/2)*x+2*(x^5+x
^3)^(1/4)*x^2+x^3+x^2)/x^2/(x^2-x+1))-1/24*RootOf(_Z^4+8)^3*RootOf(_Z^2+Roo
tOf(_Z^4+8)^2)*ln((-RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^3*x^4+2*Ro
otOf(_Z^4+8)^3*(x^5+x^3)^(1/2)*RootOf(_Z^2+RootOf(_Z^4+8)^2)*x-RootOf(_Z^2+
RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^3*x^3-RootOf(_Z^2+RootOf(_Z^4+8)^2)*x^2*Ro
otOf(_Z^4+8)^3+16*(x^5+x^3)^(3/4)-16*(x^5+x^3)^(1/4)*x^2)/x^2/(x^2-x+1))+1/
12*ln((RootOf(_Z^4+8)^2*x^4-4*(x^5+x^3)^(1/2)*RootOf(_Z^4+8)^2*x+x^3*RootOf
(_Z^4+8)^2+RootOf(_Z^4+8)^2*x^2+8*(x^5+x^3)^(3/4)-8*(x^5+x^3)^(1/4)*x^2)/x^
2/(x^2+x+1))*RootOf(_Z^4+8)^2-1/12*ln((RootOf(_Z^4+8)^2*x^4-4*(x^5+x^3)^(1/
2)*RootOf(_Z^4+8)^2*x+x^3*RootOf(_Z^4+8)^2+RootOf(_Z^4+8)^2*x^2+8*(x^5+x^3)
^(3/4)-8*(x^5+x^3)^(1/4)*x^2)/x^2/(x^2+x+1))*RootOf(_Z^4+8)*RootOf(_Z^2+Roo
tOf(_Z^4+8)^2)+1/6*RootOf(_Z^4+8)*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln((-(x^5+x
^3)^(1/4)*RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^3*x^2+RootOf(_Z^4+8)
*RootOf(_Z^2+RootOf(_Z^4+8)^2)*x^4+RootOf(_Z^4+8)^2*x^4+2*RootOf(_Z^2+RootO
f(_Z^4+8)^2)*RootOf(_Z^4+8)*(x^5+x^3)^(1/2)*x-2*(x^5+x^3)^(1/2)*RootOf(_Z^4
+8)^2*x-x^3*RootOf(_Z^4+8)*RootOf(_Z^2+RootOf(_Z^4+8)^2)-x^3*RootOf(_Z^4+8)
^2+x^2*RootOf(_Z^4+8)*RootOf(_Z^2+RootOf(_Z^4+8)^2)+RootOf(_Z^4+8)^2*x^2+8*
(x^5+x^3)^(3/4))/x^2/(x^2+x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^6+1}{\left(x^6-1\right)\left(x^5+x^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^5+x^3)^(1/4)/(-x^6+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^6 + 1)/((x^6 - 1)*(x^5 + x^3)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x^6+1}{\left(x^5+x^3\right)^{1/4}\,\left(x^6-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^6 + 1)/((x^3 + x^5)^(1/4)*(x^6 - 1)),x)
```

```
[Out] int(-(x^6 + 1)/((x^3 + x^5)^(1/4)*(x^6 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^6}{x^6\sqrt[4]{x^5+x^3}-\sqrt[4]{x^5+x^3}}\,dx-\int \frac{1}{x^6\sqrt[4]{x^5+x^3}-\sqrt[4]{x^5+x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/(x**5+x**3)**(1/4)/(-x**6+1),x)
```

```
[Out] -Integral(x**6/(x**6*(x**5 + x**3)**(1/4) - (x**5 + x**3)**(1/4)), x) - Int
egral(1/(x**6*(x**5 + x**3)**(1/4) - (x**5 + x**3)**(1/4)), x)
```

3.2267 $\int \frac{b^{16}+a^{16}x^{16}}{\sqrt{-b^4+a^4x^4}\left(-b^{16}+a^{16}x^{16}\right)}\,dx$

Optimal. Leaf size=297

$$-\frac{x}{4\sqrt{a^4x^4-b^4}}-\frac{\tan^{-1}\left(\frac{-\frac{a^3x^4}{2b}+\frac{b^3}{2a}+abx^2}{x\sqrt{a^4x^4-b^4}}\right)}{16ab}+\frac{\tanh^{-1}\left(\frac{-\frac{a^3x^4}{2b}+\frac{b^3}{2a}-abx^2}{x\sqrt{a^4x^4-b^4}}\right)}{16ab}+\frac{\tanh^{-1}\left(\frac{-\frac{a^3x^4}{2^{3/4}b}+\frac{b^3}{2^{3/4}a}-\frac{abx^2}{\sqrt[4]{2}}}{x\sqrt{a^4x^4-b^4}}\right)}{4\,2^{3/4}ab}+\frac{\tan^{-1}\left(\frac{2^{3/4}abx\sqrt{a^4x}}{-a^4x^4+\sqrt{2}\,a^2b}\right.}{4\,2^{3/4}ab}$$

Rubi [C] time = 1.71, antiderivative size = 506, normalized size of antiderivative = 1.70, number of steps used = 44, number of rules used = 20, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6725, 224, 221, 2073, 1152, 414, 21, 423, 427, 426, 424, 253, 409, 1211, 1699, 203, 206, 1429, 1219, 1218}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}bx}{\sqrt{a^4x^4-b^4}}\right)}{8\sqrt{2}\sqrt[4]{-a^4}b}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}bx}{\sqrt{a^4x^4-b^4}}\right)}{8\sqrt{2}\sqrt[4]{-a^4}b}+\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{a^4x^4-b^4}}-\frac{x\left(b^2-a^2x^2\right)}{8b^2\sqrt{a^4x^4-b^4}}-\frac{x\left(a^2x^2+b^2\right)}{8b^2\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(\frac{a^6}{\left(-a^8\right)^{3/4}};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{4a\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(\frac{\sqrt[4]{-a^8}}{a^2};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{4a\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(-\frac{\sqrt{-\sqrt{-a^8}}}{a^2};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{4a\sqrt{a^4x^4-b^4}}-\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(\frac{\sqrt{-\sqrt{-a^8}}}{a^2};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{4a\sqrt{a^4x^4-b^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b^16 + a^16*x^16)/(Sqrt[-b^4 + a^4*x^4]*(-b^16 + a^16*x^16)),x]
```

```
[Out] -1/8*(x*(b^2 - a^2*x^2))/(b^2*Sqrt[-b^4 + a^4*x^4]) - (x*(b^2 + a^2*x^2))/(
8*b^2*Sqrt[-b^4 + a^4*x^4]) - ArcTan[(Sqrt[2]*(-a^4)^(1/4)*b*x)/Sqrt[-b^4 +
 a^4*x^4]]/(8*Sqrt[2]*(-a^4)^(1/4)*b) - ArcTanh[(Sqrt[2]*(-a^4)^(1/4)*b*x)/
Sqrt[-b^4 + a^4*x^4]]/(8*Sqrt[2]*(-a^4)^(1/4)*b) + (b*Sqrt[1 - (a^4*x^4)/b^
4]*EllipticF[ArcSin[(a*x)/b], -1])/(2*a*Sqrt[-b^4 + a^4*x^4]) - (b*Sqrt[1 -
 (a^4*x^4)/b^4]*EllipticPi[a^6/(-a^8)^(3/4), ArcSin[(a*x)/b], -1])/(4*a*Sqr
t[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[(-a^8)^(1/4)/a^2
, ArcSin[(a*x)/b], -1])/(4*a*Sqrt[-b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/
b^4]*EllipticPi[-(Sqrt[-Sqrt[-a^8]]/a^2), ArcSin[(a*x)/b], -1])/(4*a*Sqrt[-
b^4 + a^4*x^4]) - (b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[Sqrt[-Sqrt[-a^8]]/a
^2, ArcSin[(a*x)/b], -1])/(4*a*Sqrt[-b^4 + a^4*x^4])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
```

```
d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

Rule 1429

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> W
ith[{r = Rt[-(a*c), 2]}, -Dist[c/(2*r), Int[(d + e*x^n)^q/(r - c*x^n), x],
x] - Dist[c/(2*r), Int[(d + e*x^n)^q/(r + c*x^n), x], x]] /; FreeQ[{a, c, d
, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[q]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /; !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{b^{16}+a^{16}x^{16}}{\sqrt{-b^4+a^4x^4}\left(-b^{16}+a^{16}x^{16}\right)}\,dx = \int \left(\frac{1}{\sqrt{-b^4+a^4x^4}}+\frac{2b^{16}}{\sqrt{-b^4+a^4x^4}\left(-b^{16}+a^{16}x^{16}\right)}\right)dx$$

$$= \left(2b^{16}\right)\int \frac{1}{\sqrt{-b^4+a^4x^4}\left(-b^{16}+a^{16}x^{16}\right)}\,dx + \int \frac{1}{\sqrt{-b^4+a^4x^4}}\,dx$$

$$= \left(2b^{16}\right)\int \left(-\frac{1}{8b^{14}\left(b^2-a^2x^2\right)\sqrt{-b^4+a^4x^4}} - \frac{1}{8b^{14}\left(b^2+a^2x^2\right)\sqrt{-b^4+a^4x^4}}\right.$$

$$= \frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{a\sqrt{-b^4+a^4x^4}} - \frac{1}{4}b^2\int \frac{1}{\left(b^2-a^2x^2\right)\sqrt{-b^4+a^4x^4}}\,dx - \frac{1}{4}b$$

$$= \frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{a\sqrt{-b^4+a^4x^4}} - \frac{1}{4}\int \frac{1}{\left(1-\frac{\sqrt{-a^4}\,x^2}{b^2}\right)\sqrt{-b^4+a^4x^4}}\,dx - \frac{1}{4}\int$$

$$= -\frac{x\left(b^2-a^2x^2\right)}{8b^2\sqrt{-b^4+a^4x^4}} - \frac{x\left(b^2+a^2x^2\right)}{8b^2\sqrt{-b^4+a^4x^4}} + \frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{a\sqrt{-b^4+a^4x^4}} -$$

$$= -\frac{x\left(b^2-a^2x^2\right)}{8b^2\sqrt{-b^4+a^4x^4}} - \frac{x\left(b^2+a^2x^2\right)}{8b^2\sqrt{-b^4+a^4x^4}} + \frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{a\sqrt{-b^4+a^4x^4}} -$$

$$= -\frac{x\left(b^2-a^2x^2\right)}{8b^2\sqrt{-b^4+a^4x^4}} - \frac{x\left(b^2+a^2x^2\right)}{8b^2\sqrt{-b^4+a^4x^4}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}\,bx}{\sqrt{-b^4+a^4x^4}}\right)}{8\sqrt{2}\sqrt[4]{-a^4}\,b} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}}{\sqrt{-b}}\right.}{8\sqrt{2}\sqrt[4]{-}}$$

$$= -\frac{x\left(b^2-a^2x^2\right)}{8b^2\sqrt{-b^4+a^4x^4}} - \frac{x\left(b^2+a^2x^2\right)}{8b^2\sqrt{-b^4+a^4x^4}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}\,bx}{\sqrt{-b^4+a^4x^4}}\right)}{8\sqrt{2}\sqrt[4]{-a^4}\,b} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}}{\sqrt{-b}}\right.}{8\sqrt{2}\sqrt[4]{-}}$$

$$= -\frac{x\left(b^2-a^2x^2\right)}{8b^2\sqrt{-b^4+a^4x^4}} - \frac{x\left(b^2+a^2x^2\right)}{8b^2\sqrt{-b^4+a^4x^4}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}\,bx}{\sqrt{-b^4+a^4x^4}}\right)}{8\sqrt{2}\sqrt[4]{-a^4}\,b} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}}{\sqrt{-b}}\right.}{8\sqrt{2}\sqrt[4]{-}}$$

$$= -\frac{x\left(b^2-a^2x^2\right)}{8b^2\sqrt{-b^4+a^4x^4}} - \frac{x\left(b^2+a^2x^2\right)}{8b^2\sqrt{-b^4+a^4x^4}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}\,bx}{\sqrt{-b^4+a^4x^4}}\right)}{8\sqrt{2}\sqrt[4]{-a^4}\,b} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}}{\sqrt{-b}}\right.}{8\sqrt{2}\sqrt[4]{-}}$$

Mathematica [C] time = 1.32, size = 373, normalized size = 1.26

$$\frac{x\left(-\sqrt{-\frac{a^2}{b^2}}\right)-3i\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}\,x\right)\middle|-1\right)+i\sqrt{1-\frac{a^4x^4}{b^4}}\,\Pi\left(-i;i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}\,x\right)\middle|-1\right)+i\sqrt{1-\frac{a^4x^4}{b^4}}\,\Pi\left(i;i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}\,x\right)\middle|-1\right)+i\sqrt{1-\frac{a^4x^4}{b^4}}\,\Pi\left(-\sqrt[4]{-1};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}\,x\right)\middle|-1\right)+i\sqrt{1-\frac{a^4x^4}{b^4}}\,\Pi\left(\sqrt[4]{-1};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}\,x\right)\middle|-1\right)+i\sqrt{1-\frac{a^4x^4}{b^4}}\,\Pi\left(-(-1)^{3/4};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}\,x\right)\middle|-1\right)+i\sqrt{1-\frac{a^4x^4}{b^4}}\,\Pi\left((-1)^{3/4};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}\,x\right)\middle|-1\right)}{4\sqrt{-\frac{a^2}{b^2}}\sqrt{a^4x^4-b^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^16 + a^16*x^16)/(Sqrt[-b^4 + a^4*x^4]*(-b^16 + a^16*x^16)),x]
```

```
[Out] (-(Sqrt[-(a^2/b^2)]*x) - (3*I)*Sqrt[1 - (a^4*x^4)/b^4]*EllipticF[I*ArcSinh[
Sqrt[-(a^2/b^2)]*x], -1] + I*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[-I, I*ArcSi
```

```
nh[Sqrt[-(a^2/b^2)]*x], -1] + I*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[I, I*Arc
Sinh[Sqrt[-(a^2/b^2)]*x], -1] + I*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[-(-1)^
(1/4), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] + I*Sqrt[1 - (a^4*x^4)/b^4]*Ellip
ticPi[(-1)^(1/4), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] + I*Sqrt[1 - (a^4*x^4)
/b^4]*EllipticPi[-(-1)^(3/4), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] + I*Sqrt[1
 - (a^4*x^4)/b^4]*EllipticPi[(-1)^(3/4), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1]
)/(4*Sqrt[-(a^2/b^2)]*Sqrt[-b^4 + a^4*x^4])
```

IntegrateAlgebraic [C] time = 4.62, size = 624, normalized size = 2.10

$$-\frac{1}{8}\text{RootSum}\left[\#1^8-8i\#1^6a^2b^2+8\#1^4a^4b^4+32i\#1^2a^6b^6+16a^8b^8\&,\frac{i\#1^6\log\left(-\#1x+\sqrt{a^4x^4-b^4}+a^2x^2+ib^2\right)-i\#1^6\log(x)-2\#1^4a^2b^2\log(x)+2\#1^4a^2b^2\log\left(-\#1x+\sqrt{a^4x^4-b^4}+a^2x^2+ib^2\right)-4i\#1^2a^4b^4\log(x)+4i\#1^2a^4b^4\log\left(-\#1x+\sqrt{a^4x^4-b^4}+a^2x^2+ib^2\right)+8a^6b^6\log\left(-\#1x+\sqrt{a^4x^4-b^4}+a^2x^2+ib^2\right)-8a^6b^6\log(x)}{-i\#1^7-6\#1^5a^2b^2-4i\#1^3a^4b^4+8\#1a^6b^6}\&\right]-\frac{x}{4\sqrt{a^4x^4-b^4}}-\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tan^{-1}\left(\frac{(1+i)abx}{\sqrt{a^4x^4-b^4}+a^2x^2+ib^2}\right)}{ab}+\frac{\left(\frac{1}{32}-\frac{i}{32}\right)\log\left(-ia^4x^4-(1-i)a^3bx^3+\left(-ia^2x^2-(1-i)abx+b^2\right)\sqrt{a^4x^4-b^4}-(1+i)ab^3x+ib^4\right)}{ab}-\frac{\left(\frac{1}{32}-\frac{i}{32}\right)\log\left(a^5bx^4+(1+i)a^4b^2x^3-(1-i)a^2b^4x+\left(a^3bx^2+(1+i)a^2b^2x+iab^3\right)\sqrt{a^4x^4-b^4}-ab^5\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^16 + a^16*x^16)/(Sqrt[-b^4 + a^4*x^4]*(-b^16 + a^16*x
^16)),x]
```

```
[Out] -1/4*x/Sqrt[-b^4 + a^4*x^4] - ((1/8 - I/8)*ArcTan[((1 + I)*a*b*x)/(I*b^2 +
a^2*x^2 + Sqrt[-b^4 + a^4*x^4])])/(a*b) + ((1/32 - I/32)*Log[I*b^4 - (1 + I
)*a*b^3*x - (1 - I)*a^3*b*x^3 - I*a^4*x^4 + (b^2 - (1 - I)*a*b*x - I*a^2*x^
2)*Sqrt[-b^4 + a^4*x^4]])/(a*b) - ((1/32 - I/32)*Log[-(a*b^5) - (1 - I)*a^2
*b^4*x + (1 + I)*a^4*b^2*x^3 + a^5*b*x^4 + (I*a*b^3 + (1 + I)*a^2*b^2*x + a
^3*b*x^2)*Sqrt[-b^4 + a^4*x^4]])/(a*b) - RootSum[16*a^8*b^8 + (32*I)*a^6*b^
6*#1^2 + 8*a^4*b^4*#1^4 - (8*I)*a^2*b^2*#1^6 + #1^8 & , (-8*a^6*b^6*Log[x]
+ 8*a^6*b^6*Log[I*b^2 + a^2*x^2 + Sqrt[-b^4 + a^4*x^4] - x*#1] - (4*I)*a^4*
b^4*Log[x]*#1^2 + (4*I)*a^4*b^4*Log[I*b^2 + a^2*x^2 + Sqrt[-b^4 + a^4*x^4]
- x*#1]*#1^2 - 2*a^2*b^2*Log[x]*#1^4 + 2*a^2*b^2*Log[I*b^2 + a^2*x^2 + Sqrt
[-b^4 + a^4*x^4] - x*#1]*#1^4 - I*Log[x]*#1^6 + I*Log[I*b^2 + a^2*x^2 + Sqr
t[-b^4 + a^4*x^4] - x*#1]*#1^6)/(8*a^6*b^6*#1 - (4*I)*a^4*b^4*#1^3 - 6*a^2*
b^2*#1^5 - I*#1^7) & ]/8
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^16*x^16+b^16)/(a^4*x^4-b^4)^(1/2)/(a^16*x^16-b^16),x, algorith
m="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^{16}x^{16}+b^{16}}{\left(a^{16}x^{16}-b^{16}\right)\sqrt{a^4x^4-b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^16*x^16+b^16)/(a^4*x^4-b^4)^(1/2)/(a^16*x^16-b^16),x, algorith
m="giac")
```

```
[Out] integrate((a^16*x^16 + b^16)/((a^16*x^16 - b^16)*sqrt(a^4*x^4 - b^4)), x)
```

maple [C] time = 0.12, size = 1190, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^16*x^16+b^16)/(a^4*x^4-b^4)^(1/2)/(a^16*x^16-b^16),x)
```

```
[Out] 1/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4
)^(1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)+1/8*b*(-1/2*(a^4*x^3+a^3*b*x^2+a^2*
```

```
b^2*x+a*b^3)/a^2/b^3/((x-b/a)*(a^4*x^3+a^3*b*x^2+a^2*b^2*x+a*b^3))^(1/2)-1/
2/b/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b
^4)^(1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)+1/2/b/(-a^2/b^2)^(1/2)*(a^2*x^2/b
^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*(EllipticF(x*(-a^2/b^
2)^(1/2),I)-EllipticE(x*(-a^2/b^2)^(1/2),I)))-1/16*b^8/a^8*sum(1/_alpha^7*(
-1/(_alpha^4*a^4-b^4)^(1/2)*arctanh(_alpha^2/b^4*(_alpha^6*a^4+b^4*x^2)*a^4
/(_alpha^4*a^4-b^4)^(1/2)/(a^4*x^4-b^4)^(1/2))+2/(-a^2/b^2)^(1/2)*_alpha^7*
a^8/b^8*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*Ell
ipticPi(x*(-a^2/b^2)^(1/2),_alpha^6*a^6/b^6,(a^2/b^2)^(1/2)/(-a^2/b^2)^(1/2
))),_alpha=RootOf(_Z^8*a^8+b^8))-1/4*b^2*(-1/2*(a^4*x^2-a^2*b^2)/b^4*x/a^2/
((x^2+b^2/a^2)*(a^4*x^2-a^2*b^2))^(1/2)+1/2/b^2/(-a^2/b^2)^(1/2)*(a^2*x^2/b
^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticF(x*(-a^2/b^2
)^(1/2),I)+1/2/b^2/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(
1/2)/(a^4*x^4-b^4)^(1/2)*(EllipticF(x*(-a^2/b^2)^(1/2),I)-EllipticE(x*(-a^2
/b^2)^(1/2),I)))-1/32*b^4/a^4*sum(1/_alpha^3*(-2^(1/2)/(-b^4)^(1/2)*arctanh
(_alpha^2*(_alpha^2+x^2)*a^4/(-2*b^4)^(1/2)/(a^4*x^4-b^4)^(1/2))+4/(-a^2/b^
2)^(1/2)*_alpha^3*a^4/b^4*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*
x^4-b^4)^(1/2)*EllipticPi(x*(-a^2/b^2)^(1/2),_alpha^2*a^2/b^2,(a^2/b^2)^(1/
2)/(-a^2/b^2)^(1/2))),_alpha=RootOf(_Z^4*a^4+b^4))-1/8*b*(1/2*(a^4*x^3-a^3*
b*x^2+a^2*b^2*x-a*b^3)/a^2/b^3/((x+b/a)*(a^4*x^3-a^3*b*x^2+a^2*b^2*x-a*b^3)
)^(1/2)+1/2/b/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/
(a^4*x^4-b^4)^(1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)-1/2/b/(-a^2/b^2)^(1/2)*
(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*(EllipticF(
x*(-a^2/b^2)^(1/2),I)-EllipticE(x*(-a^2/b^2)^(1/2),I)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^{16}x^{16}+b^{16}}{\left(a^{16}x^{16}-b^{16}\right)\sqrt{a^4x^4-b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^16*x^16+b^16)/(a^4*x^4-b^4)^(1/2)/(a^16*x^16-b^16),x, algorith
m="maxima")
```

```
[Out] integrate((a^16*x^16 + b^16)/((a^16*x^16 - b^16)*sqrt(a^4*x^4 - b^4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{a^{16}\,x^{16}+b^{16}}{\sqrt{a^4\,x^4-b^4}\,\left(b^{16}-a^{16}\,x^{16}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^16 + a^16*x^16)/((a^4*x^4 - b^4)^(1/2)*(b^16 - a^16*x^16)),x)
```

```
[Out] int(-(b^16 + a^16*x^16)/((a^4*x^4 - b^4)^(1/2)*(b^16 - a^16*x^16)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**16*x**16+b**16)/(a**4*x**4-b**4)**(1/2)/(a**16*x**16-b**16),x
)
```

```
[Out] Timed out
```

3.2268 $\int \frac{b^8+x^4+a^8x^8}{\sqrt{-b^4+a^4x^4}\left(-b^8+a^8x^8\right)}\,dx$

Optimal. Leaf size=299

$$\frac{x\left(2a^4b^4+1\right)\sqrt{a^4x^4-b^4}}{4a^4b^4\left(b^4-a^4x^4\right)}-\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\left(2a^4b^4-1\right)\tan^{-1}\left(\frac{(1+i)abx}{\sqrt{a^4x^4-b^4}+a^2x^2+ib^2}\right)}{a^5b^5}+\frac{\left(\frac{1}{16}-\frac{i}{16}\right)\left(2a^4b^4-1\right)\tanh^{-1}\left(\frac{(1-i)\sqrt{\ }}{}\right.}{a^5b^5}$$

Rubi [C] time = 0.77, antiderivative size = 338, normalized size of antiderivative = 1.13, number of steps used = 20, number of rules used = 11, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.234, Rules used = {6725, 224, 221, 1455, 527, 523, 409, 1211, 1699, 203, 206}

$$-\frac{x\left(\frac{1}{a^4b^4}+2\right)}{4\sqrt{a^4x^4-b^4}}+\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{a\sqrt{a^4x^4-b^4}}-\frac{\left(1-2a^4b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}bx}{\sqrt{a^4x^4-b^4}}\right)}{8\sqrt{2}\left(-a^4\right)^{5/4}b^5}-\frac{\left(1-2a^4b^4\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}bx}{\sqrt{a^4x^4-b^4}}\right)}{8\sqrt{2}\left(-a^4\right)^{5/4}b^5}+\frac{\left(1-2a^4b^4\right)\sqrt{1-\frac{a^4x^4}{b^4}}F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{4a^5b^3\sqrt{a^4x^4-b^4}}-\frac{\left(2a^4b^4+1\right)\sqrt{1-\frac{a^4x^4}{b^4}}F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{4a^5b^3\sqrt{a^4x^4-b^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b^8 + x^4 + a^8*x^8)/(Sqrt[-b^4 + a^4*x^4]*(-b^8 + a^8*x^8)),x]
```

```
[Out] -1/4*((2 + 1/(a^4*b^4))*x)/Sqrt[-b^4 + a^4*x^4] - ((1 - 2*a^4*b^4)*ArcTan[(
Sqrt[2]*(-a^4)^(1/4)*b*x)/Sqrt[-b^4 + a^4*x^4]])/(8*Sqrt[2]*(-a^4)^(5/4)*b^
5) - ((1 - 2*a^4*b^4)*ArcTanh[(Sqrt[2]*(-a^4)^(1/4)*b*x)/Sqrt[-b^4 + a^4*x^
4]])/(8*Sqrt[2]*(-a^4)^(5/4)*b^5) + (b*Sqrt[1 - (a^4*x^4)/b^4]*EllipticF[Ar
cSin[(a*x)/b], -1])/(a*Sqrt[-b^4 + a^4*x^4]) + ((1 - 2*a^4*b^4)*Sqrt[1 - (a
^4*x^4)/b^4]*EllipticF[ArcSin[(a*x)/b], -1])/(4*a^5*b^3*Sqrt[-b^4 + a^4*x^4
]) - ((1 + 2*a^4*b^4)*Sqrt[1 - (a^4*x^4)/b^4]*EllipticF[ArcSin[(a*x)/b], -1
])/(4*a^5*b^3*Sqrt[-b^4 + a^4*x^4])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1455

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, n, q, r}, x] && EqQ[n2,
 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{b^8+x^4+a^8x^8}{\sqrt{-b^4+a^4x^4}\left(-b^8+a^8x^8\right)}\,dx = \int\left(\frac{1}{\sqrt{-b^4+a^4x^4}}+\frac{2b^8+x^4}{\sqrt{-b^4+a^4x^4}\left(-b^8+a^8x^8\right)}\right)dx$$

$$= \int \frac{1}{\sqrt{-b^4+a^4x^4}}\,dx + \int \frac{2b^8+x^4}{\sqrt{-b^4+a^4x^4}\left(-b^8+a^8x^8\right)}\,dx$$

$$= \frac{\sqrt{1-\frac{a^4x^4}{b^4}}\int \frac{1}{\sqrt{1-\frac{a^4x^4}{b^4}}}\,dx}{\sqrt{-b^4+a^4x^4}} + \int \frac{2b^8+x^4}{\left(-b^4+a^4x^4\right)^{3/2}\left(b^4+a^4x^4\right)}\,dx$$

$$= -\frac{\left(2+\frac{1}{a^4b^4}\right)x}{4\sqrt{-b^4+a^4x^4}} + \frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{a\sqrt{-b^4+a^4x^4}} + \frac{\int \frac{b^8\left(1-6a^4b^4\right)-a^4b^4\left(1+2a^4\cdots\right.}{\sqrt{-b^4+a^4x^4}\left(b^4+a^4x^{\cdots}\right.}}{4a^4b^8}$$

$$= -\frac{\left(2+\frac{1}{a^4b^4}\right)x}{4\sqrt{-b^4+a^4x^4}} + \frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{a\sqrt{-b^4+a^4x^4}} - \frac{1}{4}\left(2+\frac{1}{a^4b^4}\right)\int \frac{\cdots}{\sqrt{-b^{\cdots}}}$$

$$= -\frac{\left(2+\frac{1}{a^4b^4}\right)x}{4\sqrt{-b^4+a^4x^4}} + \frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{a\sqrt{-b^4+a^4x^4}} + \frac{\left(\frac{1}{a^4}-2b^4\right)\int \frac{\cdots}{\left(1-\frac{\sqrt{-a^4}\,x^{\cdots}}{b^2}\right.}}{4b^4}$$

$$= -\frac{\left(2+\frac{1}{a^4b^4}\right)x}{4\sqrt{-b^4+a^4x^4}} + \frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{a\sqrt{-b^4+a^4x^4}} - \frac{\left(2+\frac{1}{a^4b^4}\right)b\sqrt{1-\frac{a^4x^4}{b^4}}\cdots}{4a\sqrt{-b^4\cdots}}$$

$$= -\frac{\left(2+\frac{1}{a^4b^4}\right)x}{4\sqrt{-b^4+a^4x^4}} + \frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{a\sqrt{-b^4+a^4x^4}} - \frac{\left(2+\frac{1}{a^4b^4}\right)b\sqrt{1-\frac{a^4x^4}{b^4}}\cdots}{4a\sqrt{-b^4\cdots}}$$

$$= -\frac{\left(2+\frac{1}{a^4b^4}\right)x}{4\sqrt{-b^4+a^4x^4}} + \frac{\left(\frac{1}{a^4}-2b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}\,bx}{\sqrt{-b^4+a^4x^4}}\right)}{8\sqrt{2}\sqrt[4]{-a^4}\,b^5} + \frac{\left(\frac{1}{a^4}-2b^4\right)\tanh^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{-\cdots}}\right.}{8\sqrt{2}\sqrt[4]{-a^4}\,b^5}$$

Mathematica [C] time = 0.74, size = 221, normalized size = 0.74

$$\frac{x\left(\frac{5\left(2a^4b^4-1\right)\left(a^4x^4-b^4\right)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)}{\left(a^4x^4+b^4\right)\left(5b^4F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)-2a^4x^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)\right)\right)}-2a^4-\frac{1}{b^4}\right)}{4a^4\sqrt{a^4x^4-b^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b^8 + x^4 + a^8*x^8)/(Sqrt[-b^4 + a^4*x^4]*(-b^8 + a^8*x^8)),x]
```

```
[Out] (x*(-2*a^4 - b^(-4) + (5*(-1 + 2*a^4*b^4)*(-b^4 + a^4*x^4)*AppellF1[1/4, -1
/2, 1, 5/4, (a^4*x^4)/b^4, -((a^4*x^4)/b^4)])/((b^4 + a^4*x^4)*(5*b^4*Appel
lF1[1/4, -1/2, 1, 5/4, (a^4*x^4)/b^4, -((a^4*x^4)/b^4)] - 2*a^4*x^4*(2*Appe
llF1[5/4, -1/2, 2, 9/4, (a^4*x^4)/b^4, -((a^4*x^4)/b^4)] + AppellF1[5/4, 1/
2, 1, 9/4, (a^4*x^4)/b^4, -((a^4*x^4)/b^4)])))))/(4*a^4*Sqrt[-b^4 + a^4*x^4
])
```

IntegrateAlgebraic [A] time = 1.38, size = 178, normalized size = 0.60

$$\frac{x\left(2a^4b^4+1\right)\sqrt{a^4x^4-b^4}}{4a^4b^4\left(b^4-a^4x^4\right)}+\frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(2a^4b^4-1\right)\tan^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a^4x^4-b^4}}{abx}\right)}{a^5b^5}-\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\left(2a^4b^4-1\right)\tan^{-1}\left(\frac{(1+i)abx}{\sqrt{a^4x^4-b^4}+a^2x^2+ib^2}\right)}{a^5b^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^8 + x^4 + a^8*x^8)/(Sqrt[-b^4 + a^4*x^4]*(-b^8 + a^8*
x^8)),x]
```

```
[Out] ((1 + 2*a^4*b^4)*x*Sqrt[-b^4 + a^4*x^4])/(4*a^4*b^4*(b^4 - a^4*x^4)) + ((1/
16 + I/16)*(-1 + 2*a^4*b^4)*ArcTan[((1/2 + I/2)*Sqrt[-b^4 + a^4*x^4])/(a*b*
x)])/(a^5*b^5) - ((1/8 - I/8)*(-1 + 2*a^4*b^4)*ArcTan[((1 + I)*a*b*x)/(I*b^
2 + a^2*x^2 + Sqrt[-b^4 + a^4*x^4])])/(a^5*b^5)
```

fricas [A] time = 2.51, size = 215, normalized size = 0.72

$$-\frac{4\left(2\,a^{5}b^{5}+ab\right)\sqrt{a^{4}x^{4}-b^{4}}\,x+2\left(2\,a^{4}b^{8}-\left(2\,a^{8}b^{4}-a^{4}\right)x^{4}-b^{4}\right)\arctan\left(\frac{\sqrt{a^{4}x^{4}-b^{4}}\,ax}{a^{2}bx^{2}+b^{3}}\right)+\left(2\,a^{4}b^{8}-\left(2\,a^{8}b^{4}-a^{4}\right)x^{4}-b^{4}\right)\log\left(\frac{a^{4}x^{4}+2\,a^{2}b^{2}x^{2}-b^{4}-2\,\sqrt{a^{4}x^{4}-b^{4}}\,abx}{a^{4}x^{4}+b^{4}}\right)}{16\left(a^{9}b^{5}x^{4}-a^{5}b^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^8*x^8+b^8+x^4)/(a^4*x^4-b^4)^(1/2)/(a^8*x^8-b^8),x, algorithm=
"fricas")
```

```
[Out] -1/16*(4*(2*a^5*b^5 + a*b)*sqrt(a^4*x^4 - b^4)*x + 2*(2*a^4*b^8 - (2*a^8*b^
4 - a^4)*x^4 - b^4)*arctan(sqrt(a^4*x^4 - b^4)*a*x/(a^2*b*x^2 + b^3)) + (2*
a^4*b^8 - (2*a^8*b^4 - a^4)*x^4 - b^4)*log((a^4*x^4 + 2*a^2*b^2*x^2 - b^4 -
 2*sqrt(a^4*x^4 - b^4)*a*b*x)/(a^4*x^4 + b^4)))/(a^9*b^5*x^4 - a^5*b^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^{8}x^{8}+b^{8}+x^{4}}{\left(a^{8}x^{8}-b^{8}\right)\sqrt{a^{4}x^{4}-b^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^8*x^8+b^8+x^4)/(a^4*x^4-b^4)^(1/2)/(a^8*x^8-b^8),x, algorithm=
"giac")
```

```
[Out] integrate((a^8*x^8 + b^8 + x^4)/((a^8*x^8 - b^8)*sqrt(a^4*x^4 - b^4)), x)
```

maple [C] time = 0.05, size = 1031, normalized size = 3.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^8*x^8+b^8+x^4)/(a^4*x^4-b^4)^(1/2)/(a^8*x^8-b^8),x)
```

```
[Out] 1/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4
)^(1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)+1/8/b^3*(2*a^4*b^4+1)/a^4*(-1/2*(a^
4*x^3+a^3*b*x^2+a^2*b^2*x+a*b^3)/a^2/b^3/((x-b/a)*(a^4*x^3+a^3*b*x^2+a^2*b^
2*x+a*b^3))^(1/2)-1/2/b/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b
^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)+1/2/b/(-a^2/b
^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*(
EllipticF(x*(-a^2/b^2)^(1/2),I)-EllipticE(x*(-a^2/b^2)^(1/2),I)))+1/4/a^4/b
^2*(-2*a^4*b^4-1)*(-1/2*(a^4*x^2-a^2*b^2)/b^4*x/a^2/((x^2+b^2/a^2)*(a^4*x^2
-a^2*b^2))^(1/2)+1/2/b^2/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/
b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)+1/2/b^2/(-a^
2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2
)*(EllipticF(x*(-a^2/b^2)^(1/2),I)-EllipticE(x*(-a^2/b^2)^(1/2),I)))+1/32/a
^8*(-2*a^4*b^4+1)*sum(1/_alpha^3*(-2^(1/2)/(-b^4)^(1/2)*arctanh(_alpha^2*(_
alpha^2+x^2)*a^4/(-2*b^4)^(1/2)/(a^4*x^4-b^4)^(1/2))+4/(-a^2/b^2)^(1/2)*_al
pha^3*a^4/b^4*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/
2)*EllipticPi(x*(-a^2/b^2)^(1/2),_alpha^2*a^2/b^2,(a^2/b^2)^(1/2)/(-a^2/b^2
)^(1/2))),_alpha=RootOf(_Z^4*a^4+b^4))+1/8*(-2*a^4*b^4-1)/a^4/b^3*(1/2*(a^4
*x^3-a^3*b*x^2+a^2*b^2*x-a*b^3)/a^2/b^3/((x+b/a)*(a^4*x^3-a^3*b*x^2+a^2*b^2
```

```
*x-a*b^3))^(1/2)+1/2/b/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^
2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)-1/2/b/(-a^2/b^
2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*(E
llipticF(x*(-a^2/b^2)^(1/2),I)-EllipticE(x*(-a^2/b^2)^(1/2),I)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^8x^8+b^8+x^4}{\left(a^8x^8-b^8\right)\sqrt{a^4x^4-b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^8*x^8+b^8+x^4)/(a^4*x^4-b^4)^(1/2)/(a^8*x^8-b^8),x, algorithm=
"maxima")
```

```
[Out] integrate((a^8*x^8 + b^8 + x^4)/((a^8*x^8 - b^8)*sqrt(a^4*x^4 - b^4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{a^8\,x^8+b^8+x^4}{\sqrt{a^4\,x^4-b^4}\,\left(b^8-a^8\,x^8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^8 + x^4 + a^8*x^8)/((a^4*x^4 - b^4)^(1/2)*(b^8 - a^8*x^8)),x)
```

```
[Out] int(-(b^8 + x^4 + a^8*x^8)/((a^4*x^4 - b^4)^(1/2)*(b^8 - a^8*x^8)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^8x^8+b^8+x^4}{\sqrt{(ax-b)(ax+b)\left(a^2x^2+b^2\right)}\,(ax-b)(ax+b)\left(a^2x^2+b^2\right)\left(a^4x^4+b^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**8*x**8+b**8+x**4)/(a**4*x**4-b**4)**(1/2)/(a**8*x**8-b**8),x)
```

```
[Out] Integral((a**8*x**8 + b**8 + x**4)/(sqrt((a*x - b)*(a*x + b)*(a**2*x**2 + b
**2))*(a*x - b)*(a*x + b)*(a**2*x**2 + b**2)*(a**4*x**4 + b**4)), x)
```

3.2269 $\int \frac{-b+x}{\left((-a+x)(-b+x)^2\right)^{2/3}(a-bd+(-1+d)x)}\,dx$

Optimal. Leaf size=301

$$-\frac{\log\left(\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}+b\sqrt[3]{d}-\sqrt[3]{d}\,x\right)}{d^{2/3}(a-b)}+\frac{\log\left(\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}\ (\sqrt[3]{\cdots}\right.}{\cdots}$$

Rubi [A] time = 0.60, antiderivative size = 240, normalized size of antiderivative = 0.80, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {6719, 91}

$$\frac{(x-a)^{2/3}(x-b)^{4/3}\log(a-bd-(1-d)x)}{2d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}-\frac{3(x-a)^{2/3}(x-b)^{4/3}\log\left(\sqrt[3]{d}\sqrt[3]{x-b}-\sqrt[3]{x-a}\right)}{2d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}-\frac{\sqrt{3}(x-a)^{2/3}(x-b)^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{x-b}}{\sqrt{3}\sqrt[3]{x-a}}+\frac{1}{\sqrt{3}}\right)}{d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + x)/(((-a + x)*(-b + x)^2)^(2/3)*(a - b*d + (-1 + d)*x)),x]
```

```
[Out] -((Sqrt[3]*(-a + x)^(2/3)*(-b + x)^(4/3)*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(-b
+ x)^(1/3))/(Sqrt[3]*(-a + x)^(1/3))])/((a - b)*d^(2/3)*(-((a - x)*(b - x)^
2))^(2/3))) + ((-a + x)^(2/3)*(-b + x)^(4/3)*Log[a - b*d - (1 - d)*x])/(2*(
a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(2/3)) - (3*(-a + x)^(2/3)*(-b + x)^(
4/3)*Log[-(-a + x)^(1/3) + d^(1/3)*(-b + x)^(1/3)])/(2*(a - b)*d^(2/3)*(-((
a - x)*(b - x)^2))^(2/3))
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{-b+x}{\left((-a+x)(-b+x)^2\right)^{2/3}(a-bd+(-1+d)x)}\,dx=\frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{1}{(-a+x)^{2/3}\sqrt[3]{-b+x}\,(a-bd+(-1+d)x)}\,dx}{\left((-a+x)(-b+x)^2\right)^{2/3}}$$

$$=-\frac{\sqrt{3}(-a+x)^{2/3}(-b+x)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{-b+x}}{\sqrt{3}\sqrt[3]{-a+x}}\right)}{(a-b)d^{2/3}\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{(-a+\cdots}{2\cdots}$$

Mathematica [C] time = 0.06, size = 57, normalized size = 0.19

$$\frac{3(b-x)^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d(b-x)}{a-x}\right)}{2(a-b)\left((x-a)(b-x)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + x)/(((-a + x)*(-b + x)^2)^(2/3)*(a - b*d + (-1 + d)*x)),x]
```

```
[Out] (3*(b - x)^2*Hypergeometric2F1[2/3, 1, 5/3, (d*(b - x))/(a - x)])/(2*(a - b
)*((b - x)^2*(-a + x))^(2/3))
```

IntegrateAlgebraic [A] time = 5.89, size = 301, normalized size = 1.00

$$-\frac{\log\left(\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}+b\sqrt[3]{d}-\sqrt[3]{d}\,x\right)}{d^{2/3}(a-b)}+\frac{\log\left(\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}\left(\sqrt[3]{d}\,x-b\sqrt[3]{d}\right)+\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}+b^2d^{2/3}-2bd^{2/3}x+d^{2/3}x^2\right)}{2d^{2/3}(a-b)}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,b\sqrt[3]{d}-\sqrt{3}\,\sqrt[3]{d}\,x}{-2\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}+b\sqrt[3]{d}-\sqrt[3]{d}\,x}\right)}{d^{2/3}(a-b)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + x)/(((-a + x)*(-b + x)^2)^(2/3)*(a - b*d + (-1 + d
)*x)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*b*d^(1/3) - Sqrt[3]*d^(1/3)*x)/(b*d^(1/3) - d^(1
/3)*x - 2*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3))])/((a
- b)*d^(2/3))) - Log[b*d^(1/3) - d^(1/3)*x + (-(a*b^2) + (2*a*b + b^2)*x +
(-a - 2*b)*x^2 + x^3)^(1/3)]/((a - b)*d^(2/3)) + Log[b^2*d^(2/3) - 2*b*d^(2
/3)*x + d^(2/3)*x^2 + (-(b*d^(1/3)) + d^(1/3)*x)*(-(a*b^2) + (2*a*b + b^2)*
x + (-a - 2*b)*x^2 + x^3)^(1/3) + (-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*
x^2 + x^3)^(2/3)]/(2*(a - b)*d^(2/3))
```

fricas [A] time = 0.47, size = 290, normalized size = 0.96

$$\frac{2\sqrt{3}\left(d^2\right)^{\frac{1}{6}}d\arctan\left(\frac{\sqrt{3}\left(d^2\right)^{\frac{1}{6}}\left((bd-dx)\left(d^2\right)^{\frac{1}{3}}-2\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}\left(d^2\right)^{\frac{2}{3}}\right)}{3\left(bd^2-d^2x\right)}\right)+\left(d^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}\left(d^2\right)^{\frac{2}{3}}(b-x)-\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{2}{3}}d-\left(b^2d-2bdx+dx^2\right)\left(d^2\right)^{\frac{1}{3}}}{b^2-2bx+x^2}\right)-2\left(d^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(d^2\right)^{\frac{2}{3}}(b-x)+\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}d}{b-x}\right)}{2(a-b)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)/((-a+x)*(-b+x)^2)^(2/3)/(a-b*d+(-1+d)*x),x, algorithm="fri
cas")
```

```
[Out] 1/2*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(1/3*sqrt(3)*(d^2)^(1/6)*((b*d - d*x)*(d
^2)^(1/3) - 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(d^2)^
(2/3))/(b*d^2 - d^2*x)) + (d^2)^(2/3)*log(-((-a*b^2 - (a + 2*b)*x^2 + x^3 +
 (2*a*b + b^2)*x)^(1/3)*(d^2)^(2/3)*(b - x) - (-a*b^2 - (a + 2*b)*x^2 + x^3
 + (2*a*b + b^2)*x)^(2/3)*d - (b^2*d - 2*b*d*x + d*x^2)*(d^2)^(1/3))/(b^2 -
 2*b*x + x^2)) - 2*(d^2)^(2/3)*log(-((d^2)^(2/3)*(b - x) + (-a*b^2 - (a + 2
*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*d)/(b - x)))/((a - b)*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b-x}{\left(-(a-x)(b-x)^2\right)^{\frac{2}{3}}(bd-(d-1)x-a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)/((-a+x)*(-b+x)^2)^(2/3)/(a-b*d+(-1+d)*x),x, algorithm="gia
c")
```

```
[Out] integrate((b - x)/((-(a - x)*(b - x)^2)^(2/3)*(b*d - (d - 1)*x - a)), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{-b+x}{\left((-a+x)(-b+x)^2\right)^{\frac{2}{3}}(a-bd+(-1+d)x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b+x)/((-a+x)*(-b+x)^2)^(2/3)/(a-b*d+(-1+d)*x),x)
```

```
[Out] int((-b+x)/((-a+x)*(-b+x)^2)^(2/3)/(a-b*d+(-1+d)*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b-x}{\left(-(a-x)(b-x)^2\right)^{\frac{2}{3}}(bd-(d-1)x-a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)/((-a+x)*(-b+x)^2)^(2/3)/(a-b*d+(-1+d)*x),x, algorithm="max
ima")
```

```
[Out] integrate((b - x)/((-(a - x)*(b - x)^2)^(2/3)*(b*d - (d - 1)*x - a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{b-x}{\left(-(a-x)\,(b-x)^2\right)^{2/3}\,(a-b\,d+x\,(d-1))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - x)/((-(a - x)*(b - x)^2)^(2/3)*(a - b*d + x*(d - 1))),x)
```

```
[Out] int(-(b - x)/((-(a - x)*(b - x)^2)^(2/3)*(a - b*d + x*(d - 1))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-b+x}{\left((-a+x)\,(-b+x)^2\right)^{\frac{2}{3}}(a-bd+dx-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)/((-a+x)*(-b+x)**2)**(2/3)/(a-b*d+(-1+d)*x),x)
```

```
[Out] Integral((-b + x)/(((-a + x)*(-b + x)**2)**(2/3)*(a - b*d + d*x - x)), x)
```

3.2270 $\int \frac{\sqrt{-b+x^2}\left(c+x^4\right)\sqrt{x+\sqrt{-b+x^2}}}{x}\,dx$

Optimal. Leaf size=301

$$\sqrt{2}\, b^{3/4} c \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{x^2-b}+x}}{\sqrt{x^2-b}-\sqrt{b}+x}\right)+\sqrt{2}\, b^{3/4} c \tanh^{-1}\left(\frac{\frac{\sqrt{x^2-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{x}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt{\sqrt{x^2-b}+x}}\right)+\frac{2\sqrt{x^2-b}\left(-1368b^4x+370\right.}{}$$

Rubi [A] time = 1.25, antiderivative size = 391, normalized size of antiderivative = 1.30, number of steps used = 18, number of rules used = 12, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {6742, 2120, 462, 459, 329, 297, 1162, 617, 204, 1165, 628, 448}

$$-\frac{b^{3/4}c\log\left(\sqrt{x^2-b}-\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{x^2-b}+x}+\sqrt{b}+x\right)}{\sqrt{2}}+\frac{b^{3/4}c\log\left(\sqrt{x^2-b}+\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{x^2-b}+x}+\sqrt{b}+x\right)}{\sqrt{2}}+\sqrt{2}\,b^{3/4}c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sqrt{x^2-b}+x}}{\sqrt[4]{b}}\right)-\sqrt{2}\,b^{3/4}c\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{x^2-b}+x}}{\sqrt[4]{b}}+1\right)-\frac{b^5}{144\left(\sqrt{x^2-b}+x\right)^{9/2}}-\frac{b^4}{80\left(\sqrt{x^2-b}+x\right)^{5/2}}+\frac{b^3}{8\sqrt{\sqrt{x^2-b}+x}}-\frac{1}{24}b^2\left(\sqrt{x^2-b}+x\right)^{3/2}+\frac{1}{3}c\left(\sqrt{x^2-b}+x\right)^{3/2}-\frac{bc}{\sqrt{\sqrt{x^2-b}+x}}+\frac{1}{176}\left(\sqrt{x^2-b}+x\right)^{11/2}+\frac{1}{112}b\left(\sqrt{x^2-b}+x\right)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-b + x^2]*(c + x^4)*Sqrt[x + Sqrt[-b + x^2]])/x,x]
```

```
[Out] -1/144*b^5/(x + Sqrt[-b + x^2])^(9/2) - b^4/(80*(x + Sqrt[-b + x^2])^(5/2))
 + b^3/(8*Sqrt[x + Sqrt[-b + x^2]]) - (b*c)/Sqrt[x + Sqrt[-b + x^2]] - (b^2
*(x + Sqrt[-b + x^2])^(3/2))/24 + (c*(x + Sqrt[-b + x^2])^(3/2))/3 + (b*(x
+ Sqrt[-b + x^2])^(7/2))/112 + (x + Sqrt[-b + x^2])^(11/2)/176 + Sqrt[2]*b^
(3/4)*c*ArcTan[1 - (Sqrt[2]*Sqrt[x + Sqrt[-b + x^2]])/b^(1/4)] - Sqrt[2]*b^
(3/4)*c*ArcTan[1 + (Sqrt[2]*Sqrt[x + Sqrt[-b + x^2]])/b^(1/4)] - (b^(3/4)*c
*Log[Sqrt[b] + x + Sqrt[-b + x^2] - Sqrt[2]*b^(1/4)*Sqrt[x + Sqrt[-b + x^2]
]])/Sqrt[2] + (b^(3/4)*c*Log[Sqrt[b] + x + Sqrt[-b + x^2] + Sqrt[2]*b^(1/4)
*Sqrt[x + Sqrt[-b + x^2]]])/Sqrt[2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{-b+x^2}\left(c+x^4\right)\sqrt{x+\sqrt{-b+x^2}}}{x}\,dx = \int \left(\frac{c\sqrt{-b+x^2}\sqrt{x+\sqrt{-b+x^2}}}{x} + x^3\sqrt{-b+x^2}\sqrt{x+\sqrt{-b+x^2}}\right)$$

$$= c\int \frac{\sqrt{-b+x^2}\sqrt{x+\sqrt{-b+x^2}}}{x}\,dx + \int x^3\sqrt{-b+x^2}\sqrt{x+\sqrt{-b+}}$$

$$= \frac{1}{32}\text{Subst}\left(\int \frac{\left(-b+x^2\right)^2\left(b+x^2\right)^3}{x^{11/2}}\,dx, x, x+\sqrt{-b+x^2}\right) + \frac{1}{2}c\,\text{Su}$$

$$= -\frac{bc}{\sqrt{x+\sqrt{-b+x^2}}} + \frac{1}{32}\text{Subst}\left(\int\left(\frac{b^5}{x^{11/2}} + \frac{b^4}{x^{7/2}} - \frac{2b^3}{x^{3/2}} - 2b^2\sqrt{x}\right.\right.$$

$$= -\frac{b^5}{144\left(x+\sqrt{-b+x^2}\right)^{9/2}} - \frac{b^4}{80\left(x+\sqrt{-b+x^2}\right)^{5/2}} + \frac{b^3}{8\sqrt{x+\sqrt{-b}}}$$

$$= -\frac{b^5}{144\left(x+\sqrt{-b+x^2}\right)^{9/2}} - \frac{b^4}{80\left(x+\sqrt{-b+x^2}\right)^{5/2}} + \frac{b^3}{8\sqrt{x+\sqrt{-b}}}$$

$$= -\frac{b^5}{144\left(x+\sqrt{-b+x^2}\right)^{9/2}} - \frac{b^4}{80\left(x+\sqrt{-b+x^2}\right)^{5/2}} + \frac{b^3}{8\sqrt{x+\sqrt{-b}}}$$

$$= -\frac{b^5}{144\left(x+\sqrt{-b+x^2}\right)^{9/2}} - \frac{b^4}{80\left(x+\sqrt{-b+x^2}\right)^{5/2}} + \frac{b^3}{8\sqrt{x+\sqrt{-b}}}$$

$$= -\frac{b^5}{144\left(x+\sqrt{-b+x^2}\right)^{9/2}} - \frac{b^4}{80\left(x+\sqrt{-b+x^2}\right)^{5/2}} + \frac{b^3}{8\sqrt{x+\sqrt{-b}}}$$

$$= -\frac{b^5}{144\left(x+\sqrt{-b+x^2}\right)^{9/2}} - \frac{b^4}{80\left(x+\sqrt{-b+x^2}\right)^{5/2}} + \frac{b^3}{8\sqrt{x+\sqrt{-b}}}$$

Mathematica [A] time = 8.63, size = 524, normalized size = 1.74

$$\frac{1}{6}c\left(-3\sqrt{2}b^{3/4}\log\left(\sqrt{x^2-b}-\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{x^2-b}+x}+\sqrt{b}+x\right)+3\sqrt{2}b^{3/4}\log\left(\sqrt{x^2-b}+\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{x^2-b}+x}+\sqrt{b}+x\right)+6\sqrt{2}b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sqrt{x^2-b}+x}}{\sqrt[4]{b}}\right)-6\sqrt{2}b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{x^2-b}+x}}{\sqrt[4]{b}}+1\right)+\frac{4x\left(\sqrt{x^2-b}+x\right)-8b}{\sqrt{\sqrt{x^2-b}+x}}\right)+\frac{2\sqrt{x^2-b}\left(304b^5-342b^4x\left(4\sqrt{x^2-b}+9x\right)+15b^3x^3\left(247\sqrt{x^2-b}+249x\right)+15b^2x^5\left(89\sqrt{x^2-b}+317x\right)+5040x^9\left(\sqrt{x^2-b}+x\right)-180bx^7\left(45\sqrt{x^2-b}+59x\right)\right)\left(\sqrt{x^2-b}+x\right)^{9/2}}{3465\left(-b^5+b^4x\left(9\sqrt{x^2-b}+41x\right)-40b^3x^3\left(3\sqrt{x^2-b}+7x\right)+16b^2x^5\left(27\sqrt{x^2-b}+43x\right)+256x^9\left(\sqrt{x^2-b}+x\right)-64bx^7\left(9\sqrt{x^2-b}+11x\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-b + x^2]*(c + x^4)*Sqrt[x + Sqrt[-b + x^2]])/x,x]
```

```
[Out] (2*Sqrt[-b + x^2]*(x + Sqrt[-b + x^2])^(9/2)*(304*b^5 + 5040*x^9*(x + Sqrt[
-b + x^2]) - 342*b^4*x*(9*x + 4*Sqrt[-b + x^2]) - 180*b*x^7*(59*x + 45*Sqrt
[-b + x^2]) + 15*b^2*x^5*(317*x + 89*Sqrt[-b + x^2]) + 15*b^3*x^3*(249*x +
247*Sqrt[-b + x^2])))/(3465*(-b^5 + 256*x^9*(x + Sqrt[-b + x^2]) - 40*b^3*x
^3*(7*x + 3*Sqrt[-b + x^2]) - 64*b*x^7*(11*x + 9*Sqrt[-b + x^2]) + b^4*x*(4
1*x + 9*Sqrt[-b + x^2]) + 16*b^2*x^5*(43*x + 27*Sqrt[-b + x^2]))) + (c*((-8
*b + 4*x*(x + Sqrt[-b + x^2]))/Sqrt[x + Sqrt[-b + x^2]] + 6*Sqrt[2]*b^(3/4)
*ArcTan[1 - (Sqrt[2]*Sqrt[x + Sqrt[-b + x^2]])/b^(1/4)] - 6*Sqrt[2]*b^(3/4)
*ArcTan[1 + (Sqrt[2]*Sqrt[x + Sqrt[-b + x^2]])/b^(1/4)] - 3*Sqrt[2]*b^(3/4)
```

```
*Log[Sqrt[b] + x + Sqrt[-b + x^2] - Sqrt[2]*b^(1/4)*Sqrt[x + Sqrt[-b + x^2]
]] + 3*Sqrt[2]*b^(3/4)*Log[Sqrt[b] + x + Sqrt[-b + x^2] + Sqrt[2]*b^(1/4)*S
qrt[x + Sqrt[-b + x^2]]]))/6
```

IntegrateAlgebraic [A] time = 0.69, size = 301, normalized size = 1.00

$$\sqrt{2}\, b^{3/4} c \tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{\sqrt{x^2-b}+x}}{\sqrt{x^2-b}-\sqrt{b}+x}\right)+\sqrt{2}\, b^{3/4} c \tanh^{-1}\left(\frac{\frac{\sqrt{x^2-b}}{\sqrt{2}\,\sqrt[4]{b}}+\frac{x}{\sqrt{2}\,\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt{\sqrt{x^2-b}+x}}\right)+\frac{2\sqrt{x^2-b}\left(-1368b^4x+3705b^3x^3+10395b^2cx+1335b^2x^5-32340bcx^3-8100bx^7+18480cx^5+5040x^9\right)+2\left(304b^5-3078b^4x^2-2310b^3c+3735b^3x^4+24255b^2cx^2+4755b^2x^6-41580bcx^4-10620bx^8+18480cx^6+5040x^{10}\right)}{3465\left(\sqrt{x^2-b}+x\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-b + x^2]*(c + x^4)*Sqrt[x + Sqrt[-b + x^2]])/x,x]
```

```
[Out] (2*Sqrt[-b + x^2]*(-1368*b^4*x + 10395*b^2*c*x + 3705*b^3*x^3 - 32340*b*c*x
^3 + 1335*b^2*x^5 + 18480*c*x^5 - 8100*b*x^7 + 5040*x^9) + 2*(304*b^5 - 231
0*b^3*c - 3078*b^4*x^2 + 24255*b^2*c*x^2 + 3735*b^3*x^4 - 41580*b*c*x^4 + 4
755*b^2*x^6 + 18480*c*x^6 - 10620*b*x^8 + 5040*x^10))/(3465*(x + Sqrt[-b +
x^2])^(9/2)) + Sqrt[2]*b^(3/4)*c*ArcTan[(Sqrt[2]*b^(1/4)*Sqrt[x + Sqrt[-b +
 x^2]])/(-Sqrt[b] + x + Sqrt[-b + x^2])] + Sqrt[2]*b^(3/4)*c*ArcTanh[(b^(1/
4)/Sqrt[2] + x/(Sqrt[2]*b^(1/4)) + Sqrt[-b + x^2]/(Sqrt[2]*b^(1/4)))/Sqrt[x
 + Sqrt[-b + x^2]]]
```

fricas [A] time = 0.58, size = 269, normalized size = 0.89

$$-\frac{2}{3465}\left(35x^5-19bx^3-\left(152b^2-1155c\right)x-2\left(175x^4-57bx^2-152b^2+1155c\right)\sqrt{x^2-b}\right)\sqrt{x+\sqrt{x^2-b}}+4\left(-b^3c^4\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-b^3c^4\right)^{\frac{1}{4}}b^2c^3\sqrt{x+\sqrt{x^2-b}}-\sqrt{b^4c^6x+\sqrt{x^2-b}\,b^4c^6-\sqrt{-b^3c^4}\,b^3c^4}\left(-b^3c^4\right)^{\frac{1}{4}}}{b^3c^4}\right)-\left(-b^3c^4\right)^{\frac{1}{4}}\log\left(b^2c^3\sqrt{x+\sqrt{x^2-b}}+\left(-b^3c^4\right)^{\frac{3}{4}}\right)+\left(-b^3c^4\right)^{\frac{1}{4}}\log\left(b^2c^3\sqrt{x+\sqrt{x^2-b}}-\left(-b^3c^4\right)^{\frac{3}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-b)^(1/2)*(x^4+c)*(x+(x^2-b)^(1/2))^(1/2)/x,x, algorithm="fri
cas")
```

```
[Out] -2/3465*(35*x^5 - 19*b*x^3 - (152*b^2 - 1155*c)*x - 2*(175*x^4 - 57*b*x^2 -
 152*b^2 + 1155*c)*sqrt(x^2 - b))*sqrt(x + sqrt(x^2 - b)) + 4*(-b^3*c^4)^(1
/4)*arctan(-((-b^3*c^4)^(1/4)*b^2*c^3*sqrt(x + sqrt(x^2 - b)) - sqrt(b^4*c^
6*x + sqrt(x^2 - b)*b^4*c^6 - sqrt(-b^3*c^4)*b^3*c^4)*(-b^3*c^4)^(1/4))/(b^
3*c^4)) - (-b^3*c^4)^(1/4)*log(b^2*c^3*sqrt(x + sqrt(x^2 - b)) + (-b^3*c^4)
^(3/4)) + (-b^3*c^4)^(1/4)*log(b^2*c^3*sqrt(x + sqrt(x^2 - b)) - (-b^3*c^4)
^(3/4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+c\right)\sqrt{x^2-b}\,\sqrt{x+\sqrt{x^2-b}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-b)^(1/2)*(x^4+c)*(x+(x^2-b)^(1/2))^(1/2)/x,x, algorithm="gia
c")
```

```
[Out] integrate((x^4 + c)*sqrt(x^2 - b)*sqrt(x + sqrt(x^2 - b))/x, x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2-b}\,\left(x^4+c\right)\sqrt{x+\sqrt{x^2-b}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-b)^(1/2)*(x^4+c)*(x+(x^2-b)^(1/2))^(1/2)/x,x)
```

```
[Out] int((x^2-b)^(1/2)*(x^4+c)*(x+(x^2-b)^(1/2))^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+c\right)\sqrt{x^2-b}\,\sqrt{x+\sqrt{x^2-b}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-b)^(1/2)*(x^4+c)*(x+(x^2-b)^(1/2))^(1/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate((x^4 + c)*sqrt(x^2 - b)*sqrt(x + sqrt(x^2 - b))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x+\sqrt{x^2-b}}\,\left(x^4+c\right)\,\sqrt{x^2-b}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + (x^2 - b)^(1/2))^(1/2)*(c + x^4)*(x^2 - b)^(1/2))/x,x)
```

```
[Out] int(((x + (x^2 - b)^(1/2))^(1/2)*(c + x^4)*(x^2 - b)^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-b+x^2}\,\left(c+x^4\right)\sqrt{x+\sqrt{-b+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-b)**(1/2)*(x**4+c)*(x+(x**2-b)**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-b + x**2)*(c + x**4)*sqrt(x + sqrt(-b + x**2))/x, x)
```

3.2271 $\int \frac{x^8}{\sqrt{-b^4+a^4x^4}\left(-b^{16}+a^{16}x^{16}\right)}\,dx$

Optimal. Leaf size=303

$$-\frac{x}{8a^8b^8\sqrt{a^4x^4-b^4}}-\frac{\tan^{-1}\left(\frac{-\frac{a^3x^4}{2b}+\frac{b^3}{2a}+abx^2}{x\sqrt{a^4x^4-b^4}}\right)}{32a^9b^9}+\frac{\tanh^{-1}\left(\frac{-\frac{a^3x^4}{2b}+\frac{b^3}{2a}-abx^2}{x\sqrt{a^4x^4-b^4}}\right)}{32a^9b^9}-\frac{\tanh^{-1}\left(\frac{-\frac{a^3x^4}{2^{3/4}b}+\frac{b^3}{2^{3/4}a}-\frac{abx^2}{\sqrt[4]{2}}}{x\sqrt{a^4x^4-b^4}}\right)}{8\,2^{3/4}a^9b^9}-\frac{\tan^{-1}\left(\frac{2^{3/4}abx\sqrt{a^{\cdots}}}{-a^4x^4+\sqrt{2}\,a^{\cdots}}\right)}{8\,2^{3/4}a^9b^{\cdots}}$$

Rubi [C] time = 1.25, antiderivative size = 522, normalized size of antiderivative = 1.72, number of steps used = 40, number of rules used = 19, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.528, Rules used = {6725, 1152, 414, 21, 423, 427, 426, 424, 253, 224, 221, 409, 1211, 1699, 203, 206, 1429, 1219, 1218}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}bx}{\sqrt{a^4x^4-b^4}}\right)}{16\sqrt{2}\left(-a^4\right)^{9/4}b^9}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}bx}{\sqrt{a^4x^4-b^4}}\right)}{16\sqrt{2}\left(-a^4\right)^{9/4}b^9}-\frac{\sqrt{1-\frac{a^4x^4}{b^4}}F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{4a^9b^7\sqrt{a^4x^4-b^4}}-\frac{x\left(b^2-a^2x^2\right)}{16a^8b^{10}\sqrt{a^4x^4-b^4}}-\frac{x\left(a^2x^2+b^2\right)}{16a^8b^{10}\sqrt{a^4x^4-b^4}}+\frac{\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(\frac{a^6}{\left(-a^8\right)^{3/4}};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{8a^9b^7\sqrt{a^4x^4-b^4}}+\frac{\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(\frac{\sqrt[4]{-a^8}}{a^2};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{8a^9b^7\sqrt{a^4x^4-b^4}}+\frac{\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(-\frac{\sqrt{-\sqrt{-a^8}}}{a^2};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{8a^9b^7\sqrt{a^4x^4-b^4}}+\frac{\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(\frac{\sqrt{-\sqrt{-a^8}}}{a^2};\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{8a^9b^7\sqrt{a^4x^4-b^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(Sqrt[-b^4 + a^4*x^4]*(-b^16 + a^16*x^16)),x]
```

```
[Out] -1/16*(x*(b^2 - a^2*x^2))/(a^8*b^10*Sqrt[-b^4 + a^4*x^4]) - (x*(b^2 + a^2*x
^2))/(16*a^8*b^10*Sqrt[-b^4 + a^4*x^4]) - ArcTan[(Sqrt[2]*(-a^4)^(1/4)*b*x)
/Sqrt[-b^4 + a^4*x^4]]/(16*Sqrt[2]*(-a^4)^(9/4)*b^9) - ArcTanh[(Sqrt[2]*(-a
^4)^(1/4)*b*x)/Sqrt[-b^4 + a^4*x^4]]/(16*Sqrt[2]*(-a^4)^(9/4)*b^9) - (Sqrt[
1 - (a^4*x^4)/b^4]*EllipticF[ArcSin[(a*x)/b], -1])/(4*a^9*b^7*Sqrt[-b^4 + a
^4*x^4]) + (Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[a^6/(-a^8)^(3/4), ArcSin[(a*
x)/b], -1])/(8*a^9*b^7*Sqrt[-b^4 + a^4*x^4]) + (Sqrt[1 - (a^4*x^4)/b^4]*Ell
ipticPi[(-a^8)^(1/4)/a^2, ArcSin[(a*x)/b], -1])/(8*a^9*b^7*Sqrt[-b^4 + a^4*
x^4]) + (Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[-(Sqrt[-Sqrt[-a^8]]/a^2), ArcSi
n[(a*x)/b], -1])/(8*a^9*b^7*Sqrt[-b^4 + a^4*x^4]) + (Sqrt[1 - (a^4*x^4)/b^4
]*EllipticPi[Sqrt[-Sqrt[-a^8]]/a^2, ArcSin[(a*x)/b], -1])/(8*a^9*b^7*Sqrt[-
b^4 + a^4*x^4])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 1152

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + (c*x^2)/e)^FracPa
```

```
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1429

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> W
ith[{r = Rt[-(a*c), 2]}, -Dist[c/(2*r), Int[(d + e*x^n)^q/(r - c*x^n), x],
x] - Dist[c/(2*r), Int[(d + e*x^n)^q/(r + c*x^n), x], x]] /; FreeQ[{a, c, d
, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^8}{\sqrt{-b^4+a^4x^4}\left(-b^{16}+a^{16}x^{16}\right)}\,dx = \int \left(\frac{1}{8a^8b^6\left(-b^2+a^2x^2\right)\sqrt{-b^4+a^4x^4}} - \frac{1}{8a^8b^6\left(b^2+a^2x^2\right)\sqrt{-b^4+a^4x^4}}\right.$$

$$= \frac{\int \frac{1}{\sqrt{-b^4+a^4x^4}\left(b^8+a^8x^8\right)}\,dx}{2a^8} + \frac{\int \frac{1}{\left(-b^2+a^2x^2\right)\sqrt{-b^4+a^4x^4}}\,dx}{8a^8b^6} - \frac{\int \frac{1}{\left(b^2+a^2x^2\right)\sqrt{-b^4+a^4x}}}{8a^8b^6}$$

$$= -\frac{\int \frac{1}{\left(1-\frac{\sqrt{-a^4}\,x^2}{b^2}\right)\sqrt{-b^4+a^4x^4}}\,dx}{8a^8b^8} - \frac{\int \frac{1}{\left(1+\frac{\sqrt{-a^4}\,x^2}{b^2}\right)\sqrt{-b^4+a^4x^4}}\,dx}{8a^8b^8} - \frac{\int \frac{1}{\sqrt{-b^4+a^4x^4}\left(\sqrt{\right.}}}{4\sqrt{-a}}$$

$$= -\frac{x\left(b^2-a^2x^2\right)}{16a^8b^{10}\sqrt{-b^4+a^4x^4}} - \frac{x\left(b^2+a^2x^2\right)}{16a^8b^{10}\sqrt{-b^4+a^4x^4}} - 2\frac{\int \frac{1}{\sqrt{-b^4+a^4x^4}}\,dx}{16a^8b^8} - \int$$

$$= -\frac{x\left(b^2-a^2x^2\right)}{16a^8b^{10}\sqrt{-b^4+a^4x^4}} - \frac{x\left(b^2+a^2x^2\right)}{16a^8b^{10}\sqrt{-b^4+a^4x^4}} - \frac{\text{Subst}\left(\int \frac{1}{1-2\sqrt{-a^4}\,b^2x^2}\right.}{16a^8b}$$

$$= -\frac{x\left(b^2-a^2x^2\right)}{16a^8b^{10}\sqrt{-b^4+a^4x^4}} - \frac{x\left(b^2+a^2x^2\right)}{16a^8b^{10}\sqrt{-b^4+a^4x^4}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}\,bx}{\sqrt{-b^4+a^4x^4}}\right)}{16\sqrt{2}\left(-a^4\right)^{9/4}b^9} -$$

$$= -\frac{x\left(b^2-a^2x^2\right)}{16a^8b^{10}\sqrt{-b^4+a^4x^4}} - \frac{x\left(b^2+a^2x^2\right)}{16a^8b^{10}\sqrt{-b^4+a^4x^4}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}\,bx}{\sqrt{-b^4+a^4x^4}}\right)}{16\sqrt{2}\left(-a^4\right)^{9/4}b^9} -$$

$$= -\frac{x\left(b^2-a^2x^2\right)}{16a^8b^{10}\sqrt{-b^4+a^4x^4}} - \frac{x\left(b^2+a^2x^2\right)}{16a^8b^{10}\sqrt{-b^4+a^4x^4}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}\,bx}{\sqrt{-b^4+a^4x^4}}\right)}{16\sqrt{2}\left(-a^4\right)^{9/4}b^9} -$$

$$= -\frac{x\left(b^2-a^2x^2\right)}{16a^8b^{10}\sqrt{-b^4+a^4x^4}} - \frac{x\left(b^2+a^2x^2\right)}{16a^8b^{10}\sqrt{-b^4+a^4x^4}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}\,bx}{\sqrt{-b^4+a^4x^4}}\right)}{16\sqrt{2}\left(-a^4\right)^{9/4}b^9} -$$

Mathematica [C] time = 1.40, size = 379, normalized size = 1.25

$$\frac{x\left(-\sqrt{-\frac{a^2}{b^2}}\right)+i\sqrt{1-\frac{a^4x^4}{b^4}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)+i\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(-i;i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)+i\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(i;i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-i\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(-\sqrt[4]{-1};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-i\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(\sqrt[4]{-1};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-i\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left(-(-1)^{3/4};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)-i\sqrt{1-\frac{a^4x^4}{b^4}}\Pi\left((-1)^{3/4};i\sinh^{-1}\left(\sqrt{-\frac{a^2}{b^2}}x\right)\middle|-1\right)}{8a^8b^8\sqrt{-\frac{a^2}{b^2}}\sqrt{a^4x^4-b^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(Sqrt[-b^4 + a^4*x^4]*(-b^16 + a^16*x^16)),x]
```

```
[Out] (-(Sqrt[-(a^2/b^2)]*x) + I*Sqrt[1 - (a^4*x^4)/b^4]*EllipticF[I*ArcSinh[Sqrt
[-(a^2/b^2)]*x], -1] + I*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[-I, I*ArcSinh[S
qrt[-(a^2/b^2)]*x], -1] + I*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[I, I*ArcSinh
[Sqrt[-(a^2/b^2)]*x], -1] - I*Sqrt[1 - (a^4*x^4)/b^4]*EllipticPi[-(-1)^(1/4
), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] - I*Sqrt[1 - (a^4*x^4)/b^4]*EllipticP
i[(-1)^(1/4), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] - I*Sqrt[1 - (a^4*x^4)/b^4
]*EllipticPi[-(-1)^(3/4), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1] - I*Sqrt[1 - (
a^4*x^4)/b^4]*EllipticPi[(-1)^(3/4), I*ArcSinh[Sqrt[-(a^2/b^2)]*x], -1])/(8
*a^8*Sqrt[-(a^2/b^2)]*b^8*Sqrt[-b^4 + a^4*x^4])
```

IntegrateAlgebraic [C] time = 2.98, size = 612, normalized size = 2.02

$$\frac{\text{RootSum}\left[\#1^8-8i\#1^6a^2b^2+8\#1^4a^4b^4+32i\#1^2a^6b^6+16a^8b^8\&,\frac{i\#1^6\log\left(-\#1x+\sqrt{a^4x^4-b^4}+a^2x^2+ib^2\right)-i\#1^6\log(x)-2\#1^4a^2b^2\log(x)+2\#1^4a^2b^2\log\left(-\#1x+\sqrt{a^4x^4-b^4}+a^2x^2+ib^2\right)-4i\#1^2a^4b^4\log(x)+4i\#1^2a^4b^4\log\left(-\#1x+\sqrt{a^4x^4-b^4}+a^2x^2+ib^2\right)+8a^6b^6\log\left(-\#1x+\sqrt{a^4x^4-b^4}+a^2x^2+ib^2\right)-8a^6b^6\log(x)}{-i\#1^7-6\#1^5a^2b^2-4i\#1^3a^4b^4+8\#1a^6b^6}\&\right]}{16a^8b^8}-\frac{x}{8a^8b^8\sqrt{a^4x^4-b^4}}-\frac{\left(\frac{1}{16}-\frac{i}{16}\right)\tan^{-1}\left(\frac{(1+i)abx}{\sqrt{a^4x^4-b^4}+a^2x^2+ib^2}\right)}{a^9b^9}+\frac{\left(\frac{1}{32}-\frac{i}{32}\right)\tanh^{-1}\left(\frac{-a^4x^4-(1-i)a^3bx^3+\left(-a^2x^2-(1-i)abx-ib^2\right)\sqrt{a^4x^4-b^4}-(1+i)ab^3x+b^4}{-ia^4x^4+(1+i)a^3bx^3+\left(-ia^2x^2+(1+i)abx+b^2\right)\sqrt{a^4x^4-b^4}-(1-i)ab^3x+ib^4}\right)}{a^9b^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/(Sqrt[-b^4 + a^4*x^4]*(-b^16 + a^16*x^16)),x]
```

```
[Out] -1/8*x/(a^8*b^8*Sqrt[-b^4 + a^4*x^4]) - ((1/16 - I/16)*ArcTan[((1 + I)*a*b*
x)/(I*b^2 + a^2*x^2 + Sqrt[-b^4 + a^4*x^4])])/(a^9*b^9) + ((1/32 - I/32)*Ar
cTanh[(b^4 - (1 + I)*a*b^3*x - (1 - I)*a^3*b*x^3 - a^4*x^4 + ((-I)*b^2 - (1
 - I)*a*b*x - a^2*x^2)*Sqrt[-b^4 + a^4*x^4])/(I*b^4 - (1 - I)*a*b^3*x + (1
+ I)*a^3*b*x^3 - I*a^4*x^4 + (b^2 + (1 + I)*a*b*x - I*a^2*x^2)*Sqrt[-b^4 +
a^4*x^4])])/(a^9*b^9) + RootSum[16*a^8*b^8 + (32*I)*a^6*b^6*#1^2 + 8*a^4*b^
4*#1^4 - (8*I)*a^2*b^2*#1^6 + #1^8 & , (-8*a^6*b^6*Log[x] + 8*a^6*b^6*Log[I
*b^2 + a^2*x^2 + Sqrt[-b^4 + a^4*x^4] - x*#1] - (4*I)*a^4*b^4*Log[x]*#1^2 +
 (4*I)*a^4*b^4*Log[I*b^2 + a^2*x^2 + Sqrt[-b^4 + a^4*x^4] - x*#1]*#1^2 - 2*
a^2*b^2*Log[x]*#1^4 + 2*a^2*b^2*Log[I*b^2 + a^2*x^2 + Sqrt[-b^4 + a^4*x^4]
- x*#1]*#1^4 - I*Log[x]*#1^6 + I*Log[I*b^2 + a^2*x^2 + Sqrt[-b^4 + a^4*x^4]
 - x*#1]*#1^6)/(8*a^6*b^6*#1 - (4*I)*a^4*b^4*#1^3 - 6*a^2*b^2*#1^5 - I*#1^7
) & ]/(16*a^8*b^8)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a^4*x^4-b^4)^(1/2)/(a^16*x^16-b^16),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a^{16}x^{16}-b^{16}\right)\sqrt{a^4x^4-b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a^4*x^4-b^4)^(1/2)/(a^16*x^16-b^16),x, algorithm="giac")
```

```
[Out] integrate(x^8/((a^16*x^16 - b^16)*sqrt(a^4*x^4 - b^4)), x)
```

maple [C] time = 0.06, size = 1130, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a^4*x^4-b^4)^(1/2)/(a^16*x^16-b^16),x)
```

```
[Out] 1/16/b^7/a^8*(-1/2*(a^4*x^3+a^3*b*x^2+a^2*b^2*x+a*b^3)/a^2/b^3/((x-b/a)*(a^
4*x^3+a^3*b*x^2+a^2*b^2*x+a*b^3))^(1/2)-1/2/b/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2
+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticF(x*(-a^2/b^2)^
(1/2),I)+1/2/b/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)
/(a^4*x^4-b^4)^(1/2)*(EllipticF(x*(-a^2/b^2)^(1/2),I)-EllipticE(x*(-a^2/b^2
)^(1/2),I)))+1/32/a^16*sum(1/_alpha^7*(-1/(_alpha^4*a^4-b^4)^(1/2)*arctanh(
_alpha^2/b^4*(_alpha^6*a^4+b^4*x^2)*a^4/(_alpha^4*a^4-b^4)^(1/2)/(a^4*x^4-b
^4)^(1/2))+2/(-a^2/b^2)^(1/2)*_alpha^7*a^8/b^8*(a^2*x^2/b^2+1)^(1/2)*(1-a^2
*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticPi(x*(-a^2/b^2)^(1/2),_alpha^6*
a^6/b^6,(a^2/b^2)^(1/2)/(-a^2/b^2)^(1/2))),_alpha=RootOf(_Z^8*a^8+b^8))-1/8
/a^8/b^6*(-1/2*(a^4*x^2-a^2*b^2)/b^4*x/a^2/((x^2+b^2/a^2)*(a^4*x^2-a^2*b^2)
)^(1/2)+1/2/b^2/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2
```

```
)/(a^4*x^4-b^4)^(1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)+1/2/b^2/(-a^2/b^2)^(1
/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*(Ellipt
icF(x*(-a^2/b^2)^(1/2),I)-EllipticE(x*(-a^2/b^2)^(1/2),I)))-1/64/a^12/b^4*s
um(1/_alpha^3*(-2^(1/2)/(-b^4)^(1/2)*arctanh(_alpha^2*(_alpha^2+x^2)*a^4/(-
2*b^4)^(1/2)/(a^4*x^4-b^4)^(1/2))+4/(-a^2/b^2)^(1/2)*_alpha^3*a^4/b^4*(a^2*
x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticPi(x*(-a
^2/b^2)^(1/2),_alpha^2*a^2/b^2,(a^2/b^2)^(1/2)/(-a^2/b^2)^(1/2))),_alpha=Ro
otOf(_Z^4*a^4+b^4))-1/16/b^7/a^8*(1/2*(a^4*x^3-a^3*b*x^2+a^2*b^2*x-a*b^3)/a
^2/b^3/((x+b/a)*(a^4*x^3-a^3*b*x^2+a^2*b^2*x-a*b^3))^(1/2)+1/2/b/(-a^2/b^2)
^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*Elli
pticF(x*(-a^2/b^2)^(1/2),I)-1/2/b/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1
-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*(EllipticF(x*(-a^2/b^2)^(1/2),I)-El
lipticE(x*(-a^2/b^2)^(1/2),I)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a^{16}x^{16}-b^{16}\right)\sqrt{a^4x^4-b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a^4*x^4-b^4)^(1/2)/(a^16*x^16-b^16),x, algorithm="maxima")
[Out] integrate(x^8/((a^16*x^16 - b^16)*sqrt(a^4*x^4 - b^4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^8}{\sqrt{a^4\,x^4-b^4}\,\left(b^{16}-a^{16}\,x^{16}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^8/((a^4*x^4 - b^4)^(1/2)*(b^16 - a^16*x^16)),x)
[Out] -int(x^8/((a^4*x^4 - b^4)^(1/2)*(b^16 - a^16*x^16)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(a**4*x**4-b**4)**(1/2)/(a**16*x**16-b**16),x)
[Out] Timed out
```

3.2272 $\int \frac{x^2}{\left(-b+ax^2\right)\sqrt[3]{-x+x^3}}\,dx$

Optimal. Leaf size=304

$$\frac{\sqrt[3]{b}\log\left(-\sqrt[3]{b}\sqrt[3]{x^3-x}\,x\sqrt[3]{a-b}+x^2(a-b)^{2/3}+b^{2/3}\left(x^3-x\right)^{2/3}\right)}{4a\sqrt[3]{a-b}}-\frac{\sqrt[3]{b}\log\left(x\sqrt[3]{a-b}+\sqrt[3]{b}\sqrt[3]{x^3-x}\right)}{2a\sqrt[3]{a-b}}+\frac{\sqrt{3}\sqrt[3]{b}\tan}{}$$

Rubi [A] time = 0.53, antiderivative size = 457, normalized size of antiderivative = 1.50, number of steps used = 17, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2042, 466, 465, 494, 481, 200, 31, 634, 618, 204, 628, 617}

$$\frac{\sqrt[3]{b}\sqrt[3]{x}\sqrt[3]{x^2-1}\log\left(\frac{x^{4/3}(a-b)^{2/3}}{\left(x^2-1\right)^{2/3}}-\frac{\sqrt[3]{b}x^{2/3}\sqrt[3]{a-b}}{\sqrt[3]{x^2-1}}+b^{2/3}\right)}{4a\sqrt[3]{x^3-x}\sqrt[3]{a-b}}-\frac{\sqrt[3]{b}\sqrt[3]{x}\sqrt[3]{x^2-1}\log\left(\frac{x^{2/3}\sqrt[3]{a-b}}{\sqrt[3]{x^2-1}}+\sqrt[3]{b}\right)}{2a\sqrt[3]{x^3-x}\sqrt[3]{a-b}}+\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}\sqrt[3]{x^2-1}\tan^{-1}\left(\frac{\sqrt[3]{b}-\frac{2x^{2/3}\sqrt[3]{a-b}}{\sqrt[3]{x^2-1}}}{\sqrt{3}\sqrt[3]{b}}\right)}{2a\sqrt[3]{x^3-x}\sqrt[3]{a-b}}-\frac{\sqrt[3]{x}\sqrt[3]{x^2-1}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2-1}}\right)}{2a\sqrt[3]{x^3-x}}+\frac{\sqrt[3]{x}\sqrt[3]{x^2-1}\log\left(\frac{x^{4/3}}{\left(x^2-1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2-1}}+1\right)}{4a\sqrt[3]{x^3-x}}+\frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{x^2-1}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1}{\sqrt{3}}\right)}{2a\sqrt[3]{x^3-x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-b + a*x^2)*(-x + x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*x^(1/3)*(-1 + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3))/(-1 + x^2)^(1/3))
/Sqrt[3]])/(2*a*(-x + x^3)^(1/3)) + (Sqrt[3]*b^(1/3)*x^(1/3)*(-1 + x^2)^(1/
3)*ArcTan[(b^(1/3) - (2*(a - b)^(1/3)*x^(2/3))/(-1 + x^2)^(1/3))/(Sqrt[3]*b
^(1/3))])/(2*a*(a - b)^(1/3)*(-x + x^3)^(1/3)) - (x^(1/3)*(-1 + x^2)^(1/3)*
Log[1 - x^(2/3)/(-1 + x^2)^(1/3)])/(2*a*(-x + x^3)^(1/3)) + (x^(1/3)*(-1 +
x^2)^(1/3)*Log[1 + x^(4/3)/(-1 + x^2)^(2/3) + x^(2/3)/(-1 + x^2)^(1/3)])/(4
*a*(-x + x^3)^(1/3)) - (b^(1/3)*x^(1/3)*(-1 + x^2)^(1/3)*Log[b^(1/3) + ((a
- b)^(1/3)*x^(2/3))/(-1 + x^2)^(1/3)])/(2*a*(a - b)^(1/3)*(-x + x^3)^(1/3))
 + (b^(1/3)*x^(1/3)*(-1 + x^2)^(1/3)*Log[b^(2/3) + ((a - b)^(2/3)*x^(4/3))/
(-1 + x^2)^(2/3) - ((a - b)^(1/3)*b^(1/3)*x^(2/3))/(-1 + x^2)^(1/3)])/(4*a*
(a - b)^(1/3)*(-x + x^3)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\int \frac{x^2}{\left(-b+ax^2\right)\sqrt[3]{-x+x^3}}\,dx = \frac{\left(\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\right)\int \frac{x^{5/3}}{\sqrt[3]{-1+x^2}\left(-b+ax^2\right)}\,dx}{\sqrt[3]{-x+x^3}}$$

$$= \frac{\left(3\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{x^7}{\sqrt[3]{-1+x^6}\left(-b+ax^6\right)}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-x+x^3}}$$

$$= \frac{\left(3\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{-1+x^3}\left(-b+ax^3\right)}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{-x+x^3}}$$

$$= -\frac{\left(3\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{x^3}{\left(1-x^3\right)\left(-b-(a-b)x^3\right)}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2\sqrt[3]{-x+x^3}}$$

$$= \frac{\left(3\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^3}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2a\sqrt[3]{-x+x^3}} + \frac{\left(3b\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{}{-b+(}\right.}{2a\sqrt[3]{-x+x^3}}$$

$$= \frac{\left(\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2a\sqrt[3]{-x+x^3}} + \frac{\left(\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,d\right.}{2a\sqrt[3]{-x+x^3}}$$

$$= -\frac{\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\,\log\left(1-\frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2a\sqrt[3]{-x+x^3}} - \frac{\sqrt[3]{b}\,\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\,\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{a-b}\,x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2a\sqrt[3]{a-b}\,\sqrt[3]{-x+x^3}} + \frac{\left(\sqrt[3]{x}\right.}{}$$

$$= -\frac{\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\,\log\left(1-\frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2a\sqrt[3]{-x+x^3}} + \frac{\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\,\log\left(1+\frac{x^{4/3}}{\left(-1+x^2\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{4a\sqrt[3]{-x+x^3}} -$$

$$= \frac{\sqrt{3}\,\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}}{\sqrt{3}}\right)}{2a\sqrt[3]{-x+x^3}} + \frac{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{x}\,\sqrt[3]{-1+x^2}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a-b}\,x^{2/3}}{\sqrt[3]{b}\,\sqrt[3]{-1+x^2}}}{\sqrt{3}}\right)}{2a\sqrt[3]{a-b}\,\sqrt[3]{-x+x^3}} -$$

Mathematica [C] time = 0.04, size = 54, normalized size = 0.18

$$-\frac{3x^3\sqrt[3]{1-x^2}\,F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^2,\frac{ax^2}{b}\right)}{8b\sqrt[3]{x\left(x^2-1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-b + a*x^2)*(-x + x^3)^(1/3)),x]
```

```
[Out] (-3*x^3*(1 - x^2)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, x^2, (a*x^2)/b])/(8*b*(x
*(-1 + x^2))^(1/3))
```

IntegrateAlgebraic [A] time = 0.77, size = 307, normalized size = 1.01

$$\frac{\sqrt[3]{b}\log\left(-\sqrt[3]{b}\sqrt[3]{x^3-x}\,x\sqrt[3]{a-b}+x^2(a-b)^{2/3}+b^{2/3}\left(x^3-x\right)^{2/3}\right)}{4a\sqrt[3]{a-b}} - \frac{\sqrt[3]{b}\log\left(x\sqrt[3]{a-b}+\sqrt[3]{b}\sqrt[3]{x^3-x}\right)}{2a\sqrt[3]{a-b}} + \frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt{3}x\sqrt[3]{a-b}}{x\sqrt[3]{a-b}-2\sqrt[3]{b}\sqrt[3]{x^3-x}}\right)}{2a\sqrt[3]{a-b}} - \frac{\log\left(a\sqrt[3]{x^3-x}-ax\right)}{2a} + \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x}+x}\right)}{2a} + \frac{\log\left(\sqrt[3]{x^3-x}\,x+\left(x^3-x\right)^{2/3}+x^2\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-b + a*x^2)*(-x + x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x + 2*(-x + x^3)^(1/3))])/(2*a) + (Sqrt[3]*b^(
1/3)*ArcTan[(Sqrt[3]*(a - b)^(1/3)*x)/((a - b)^(1/3)*x - 2*b^(1/3)*(-x + x^
```

```
3)^(1/3))])/(2*a*(a - b)^(1/3)) - Log[-(a*x) + a*(-x + x^3)^(1/3)]/(2*a) -
(b^(1/3)*Log[(a - b)^(1/3)*x + b^(1/3)*(-x + x^3)^(1/3)])/(2*a*(a - b)^(1/3
)) + Log[x^2 + x*(-x + x^3)^(1/3) + (-x + x^3)^(2/3)]/(4*a) + (b^(1/3)*Log[
(a - b)^(2/3)*x^2 - (a - b)^(1/3)*b^(1/3)*x*(-x + x^3)^(1/3) + b^(2/3)*(-x
+ x^3)^(2/3)])/(4*a*(a - b)^(1/3))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2-b)/(x^3-x)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**A**] time = 0.25, size = 272, normalized size = 0.89

$$-\frac{b\left(-\frac{a-b}{b}\right)^{\frac{2}{3}}\log\left(\left|-\left(-\frac{a-b}{b}\right)^{\frac{1}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right|\right)}{2\left(a^2-ab\right)}-\frac{3\left(-ab^2+b^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a-b}{b}\right)^{\frac{1}{3}}+2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a-b}{b}\right)^{\frac{1}{3}}}\right)}{2\left(\sqrt{3}a^2b-\sqrt{3}ab^2\right)}+\frac{\left(-ab^2+b^3\right)^{\frac{2}{3}}\log\left(\left(-\frac{a-b}{b}\right)^{\frac{2}{3}}+\left(-\frac{a-b}{b}\right)^{\frac{1}{3}}\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}\right)}{4\left(a^2b-ab^2\right)}-\frac{\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)}{2a}+\frac{\log\left(\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)}{4a}-\frac{\log\left(\left|\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2-b)/(x^3-x)^(1/3),x, algorithm="giac")
```

```
[Out] -1/2*b*(-(a - b)/b)^(2/3)*log(abs(-(-(a - b)/b)^(1/3) + (-1/x^2 + 1)^(1/3))
)/(a^2 - a*b) - 3/2*(-a*b^2 + b^3)^(2/3)*arctan(1/3*sqrt(3)*((-(a - b)/b)^(
1/3) + 2*(-1/x^2 + 1)^(1/3))/(-(a - b)/b)^(1/3))/(sqrt(3)*a^2*b - sqrt(3)*a
*b^2) + 1/4*(-a*b^2 + b^3)^(2/3)*log((-(a - b)/b)^(2/3) + (-(a - b)/b)^(1/3
)*(-1/x^2 + 1)^(1/3) + (-1/x^2 + 1)^(2/3))/(a^2*b - a*b^2) - 1/2*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*(-1/x^2 + 1)^(1/3) + 1))/a + 1/4*log((-1/x^2 + 1)^(2/3)
 + (-1/x^2 + 1)^(1/3) + 1)/a - 1/2*log(abs((-1/x^2 + 1)^(1/3) - 1))/a
```

maple [**F**] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a\,x^2-b\right)\left(x^3-x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2-b)/(x^3-x)^(1/3),x)
```

```
[Out] int(x^2/(a*x^2-b)/(x^3-x)^(1/3),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax^2-b\right)\left(x^3-x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^2-b)/(x^3-x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a*x^2 - b)*(x^3 - x)^(1/3)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^2}{\left(x^3-x\right)^{1/3}\,\left(b-a\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^3 - x)^(1/3)*(b - a*x^2)),x)
[Out] -int(x^2/((x^3 - x)^(1/3)*(b - a*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[3]{x\,(x-1)\,(x+1)}\,\left(ax^2-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**2-b)/(x**3-x)**(1/3),x)
[Out] Integral(x**2/((x*(x - 1)*(x + 1))**(1/3)*(a*x**2 - b)), x)
```

3.2273 $\int \frac{(1+x^4)\sqrt{x+\sqrt{1+x^2}}}{-1+x^4}\,dx$

Optimal. Leaf size=304

$$\frac{1}{3}\left(\sqrt{x^2+1}+x\right)^{3/2}-\frac{1}{\sqrt{\sqrt{x^2+1}+x}}-\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{\sqrt{2}-1}}\right)+\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{1+\sqrt{2}}}\right)-$$

Rubi [A] time = 0.76, antiderivative size = 343, normalized size of antiderivative = 1.13, number of steps used = 34, number of rules used = 18, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6725, 2117, 14, 2119, 1628, 826, 1166, 204, 206, 207, 203, 2122, 329, 297, 1162, 617, 1165, 628}

$$\frac{1}{3}\left(\sqrt{x^2+1}+x\right)^{3/2}-\frac{1}{\sqrt{\sqrt{x^2+1}+x}}-\frac{\log\left(\sqrt{x^2+1}-\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right)}{\sqrt{2}}+\frac{\log\left(\sqrt{x^2+1}+\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right)}{\sqrt{2}}+\sqrt{1+\sqrt{2}}\tan^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)-\sqrt{\sqrt{2}-1}\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\sqrt{x^2+1}+x}\right)-\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+1\right)-\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)+\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^4)*Sqrt[x + Sqrt[1 + x^2]])/(-1 + x^4),x]
```

```
[Out] -(1/Sqrt[x + Sqrt[1 + x^2]]) + (x + Sqrt[1 + x^2])^(3/2)/3 + Sqrt[1 + Sqrt[
2]]*ArcTan[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]] - Sqrt[-1 + Sqrt[2]]
*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]] + Sqrt[2]*ArcTan[1 - Sqr
t[2]*Sqrt[x + Sqrt[1 + x^2]]] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[x + Sqrt[1
+ x^2]]] - Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x
^2]]] + Sqrt[-1 + Sqrt[2]]*ArcTanh[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]
]] - Log[1 + x + Sqrt[1 + x^2] - Sqrt[2]*Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[2] +
 Log[1 + x + Sqrt[1 + x^2] + Sqrt[2]*Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(1+x^4)\sqrt{x+\sqrt{1+x^2}}}{-1+x^4}\,dx = \int \left(\sqrt{x+\sqrt{1+x^2}} + \frac{2\sqrt{x+\sqrt{1+x^2}}}{-1+x^4}\right) dx$$

$$= 2\int \frac{\sqrt{x+\sqrt{1+x^2}}}{-1+x^4}\,dx + \int \sqrt{x+\sqrt{1+x^2}}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1+x^2}{x^{3/2}}\,dx, x, x+\sqrt{1+x^2}\right) + 2\int \left(-\frac{\sqrt{x+\sqrt{1+x^2}}}{2(1-x^2)} - \frac{\sqrt{x+\sqrt{1+}}}{2(1+x^2)}\right.$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{x^{3/2}} + \sqrt{x}\right) dx, x, x+\sqrt{1+x^2}\right) - \int \frac{\sqrt{x+\sqrt{1+x^2}}}{1-x^2}\,dx - \int \frac{\sqrt{x}}{}$$

$$= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}} + \frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2} - 2\,\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, x+\sqrt{1+x^2}\right)$$

$$= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}} + \frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2} - \frac{1}{2}\int \frac{\sqrt{x+\sqrt{1+x^2}}}{1-x}\,dx - \frac{1}{2}\int \frac{\sqrt{x+}}{1}$$

$$= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}} + \frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1+x^2}{\sqrt{x}\,(1+2x-x^2)}\,dx, x, x\right.$$

$$= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}} + \frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2} - \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{1}{\sqrt{x}} + \frac{2(1+x)}{\sqrt{x}\,(1+2x-x^2}\right.\right.$$

$$= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}} + \frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2} - \frac{\log\left(1+x+\sqrt{1+x^2} - \sqrt{2}\sqrt{x+\sqrt{1-}}\right.}{\sqrt{2}}$$

$$= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}} + \frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2} + \sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x+\sqrt{1+x^2}}\right) -$$

$$= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}} + \frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2} + \sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x+\sqrt{1+x^2}}\right) -$$

$$= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}} + \frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2} - \sqrt{-1+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{x+\sqrt{1+x^2}}}{\sqrt{-1+\sqrt{2}}}\right) +$$

Mathematica [A] time = 1.08, size = 379, normalized size = 1.25

$$\frac{1}{3}\left(\sqrt{x^2+1}+x\right)^{3/2} - \frac{1}{\sqrt{\sqrt{x^2+1}+x}} - \frac{\log\left(\sqrt{x^2+1}-\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right)}{\sqrt{2}} + \frac{\log\left(\sqrt{x^2+1}+\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right)}{\sqrt{2}} + \frac{\left(\sqrt{2}-2\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{\sqrt{2}-1}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}} + \frac{\left(2+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}} + \sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\sqrt{x^2+1}+x}\right) - \sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+1\right) - \frac{\left(\sqrt{2}-2\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{\sqrt{2}-1}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}} - \frac{\left(2+\sqrt{2}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}+x}}{\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^4)*Sqrt[x + Sqrt[1 + x^2]])/(-1 + x^4),x]
```

```
[Out] -(1/Sqrt[x + Sqrt[1 + x^2]]) + (x + Sqrt[1 + x^2])^(3/2)/3 + ((-2 + Sqrt[2]
)*ArcTan[Sqrt[x + Sqrt[1 + x^2]]/Sqrt[-1 + Sqrt[2]]])/Sqrt[2*(-1 + Sqrt[2])
] + ((2 + Sqrt[2])*ArcTan[Sqrt[x + Sqrt[1 + x^2]]/Sqrt[1 + Sqrt[2]]])/Sqrt[
```

```
2*(1 + Sqrt[2])] + Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x + Sqrt[1 + x^2]]] - Sq
rt[2]*ArcTan[1 + Sqrt[2]*Sqrt[x + Sqrt[1 + x^2]]] - ((-2 + Sqrt[2])*ArcTanh
[Sqrt[x + Sqrt[1 + x^2]]/Sqrt[-1 + Sqrt[2]]])/Sqrt[2*(-1 + Sqrt[2])] - ((2
+ Sqrt[2])*ArcTanh[Sqrt[x + Sqrt[1 + x^2]]/Sqrt[1 + Sqrt[2]]])/Sqrt[2*(1 +
Sqrt[2])] - Log[1 + x + Sqrt[1 + x^2] - Sqrt[2]*Sqrt[x + Sqrt[1 + x^2]]]/Sq
rt[2] + Log[1 + x + Sqrt[1 + x^2] + Sqrt[2]*Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[2
]
```

IntegrateAlgebraic [A] time = 0.69, size = 304, normalized size = 1.00

$$\frac{1}{3}\left(\sqrt{x^2+1}+x\right)^{3/2}-\frac{1}{\sqrt{\sqrt{x^2+1}+x}}+\sqrt{1+\sqrt{2}}\tan^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)-\sqrt{\sqrt{2}-1}\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{x^2+1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2+1}+x}}\right)-\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)+\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{x^2+1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2+1}+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^4)*Sqrt[x + Sqrt[1 + x^2]])/(-1 + x^4),x]
```

```
[Out] -(1/Sqrt[x + Sqrt[1 + x^2]]) + (x + Sqrt[1 + x^2])^(3/2)/3 + Sqrt[1 + Sqrt[
2]]*ArcTan[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]] - Sqrt[-1 + Sqrt[2]]
*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]] - Sqrt[2]*ArcTan[(-(1/Sq
rt[2]) + x/Sqrt[2] + Sqrt[1 + x^2]/Sqrt[2])/Sqrt[x + Sqrt[1 + x^2]]] - Sqrt
[1 + Sqrt[2]]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]] + Sqrt[-1
 + Sqrt[2]]*ArcTanh[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]] + Sqrt[2]*Ar
cTanh[(1/Sqrt[2] + x/Sqrt[2] + Sqrt[1 + x^2]/Sqrt[2])/Sqrt[x + Sqrt[1 + x^2
]]]
```

fricas [B] time = 0.52, size = 463, normalized size = 1.52

$$\frac{2}{3}\left(2x-\sqrt{x^2+1}\right)\sqrt{x+\sqrt{x^2+1}}+2\sqrt{\sqrt{2}-1}\arctan\left(\sqrt{x+\sqrt{2}+\sqrt{x^2+1}-1}\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}-\sqrt{x+\sqrt{x^2+1}}\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}\right)-2\sqrt{\sqrt{2}+1}\arctan\left(\sqrt{x+\sqrt{2}+\sqrt{x^2+1}+1}\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)-\sqrt{x+\sqrt{x^2+1}}\sqrt{\sqrt{2}+1}\left(\sqrt{2}-1\right)\right)+2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+x+\sqrt{x^2+1}+1}-\sqrt{2}\sqrt{x+\sqrt{x^2+1}}-1\right)+2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+4x+4\sqrt{x^2+1}+4}-\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+1\right)+\frac{1}{2}\sqrt{2}\log\left(4\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+4x+4\sqrt{x^2+1}+4\right)-\frac{1}{2}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x+\sqrt{x^2+1}}+4x+4\sqrt{x^2+1}+4\right)-\frac{1}{2}\sqrt{\sqrt{2}+1}\log\left(\sqrt{x+\sqrt{x^2+1}}+\sqrt{\sqrt{2}+1}\right)+\frac{1}{2}\sqrt{\sqrt{2}+1}\log\left(\sqrt{x+\sqrt{x^2+1}}-\sqrt{\sqrt{2}+1}\right)+\frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(\sqrt{x+\sqrt{x^2+1}}+\sqrt{\sqrt{2}-1}\right)-\frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(\sqrt{x+\sqrt{x^2+1}}-\sqrt{\sqrt{2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x+(x^2+1)^(1/2))^(1/2)/(x^4-1),x, algorithm="fricas")
```

```
[Out] 2/3*(2*x - sqrt(x^2 + 1))*sqrt(x + sqrt(x^2 + 1)) + 2*sqrt(sqrt(2) - 1)*arc
tan(sqrt(x + sqrt(2) + sqrt(x^2 + 1) - 1)*(sqrt(2) + 1)*sqrt(sqrt(2) - 1) -
 sqrt(x + sqrt(x^2 + 1))*(sqrt(2) + 1)*sqrt(sqrt(2) - 1)) - 2*sqrt(sqrt(2)
+ 1)*arctan(sqrt(x + sqrt(2) + sqrt(x^2 + 1) + 1)*sqrt(sqrt(2) + 1)*(sqrt(2
) - 1) - sqrt(x + sqrt(x^2 + 1))*sqrt(sqrt(2) + 1)*(sqrt(2) - 1)) + 2*sqrt(
2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqrt(x + sqrt(x^2 + 1)) + x + sqrt(x^2 + 1)
+ 1) - sqrt(2)*sqrt(x + sqrt(x^2 + 1)) - 1) + 2*sqrt(2)*arctan(1/2*sqrt(2)*
sqrt(-4*sqrt(2)*sqrt(x + sqrt(x^2 + 1)) + 4*x + 4*sqrt(x^2 + 1) + 4) - sqrt
(2)*sqrt(x + sqrt(x^2 + 1)) + 1) + 1/2*sqrt(2)*log(4*sqrt(2)*sqrt(x + sqrt(
x^2 + 1)) + 4*x + 4*sqrt(x^2 + 1) + 4) - 1/2*sqrt(2)*log(-4*sqrt(2)*sqrt(x
+ sqrt(x^2 + 1)) + 4*x + 4*sqrt(x^2 + 1) + 4) - 1/2*sqrt(sqrt(2) + 1)*log(s
qrt(x + sqrt(x^2 + 1)) + sqrt(sqrt(2) + 1)) + 1/2*sqrt(sqrt(2) + 1)*log(sqr
t(x + sqrt(x^2 + 1)) - sqrt(sqrt(2) + 1)) + 1/2*sqrt(sqrt(2) - 1)*log(sqrt(
x + sqrt(x^2 + 1)) + sqrt(sqrt(2) - 1)) - 1/2*sqrt(sqrt(2) - 1)*log(sqrt(x
+ sqrt(x^2 + 1)) - sqrt(sqrt(2) - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)\sqrt{x+\sqrt{x^2+1}}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x+(x^2+1)^(1/2))^(1/2)/(x^4-1),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)*sqrt(x + sqrt(x^2 + 1))/(x^4 - 1), x)
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}+1\right) \sqrt{x+\sqrt{x^{2}+1}}}{x^{4}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)*(x+(x^2+1)^(1/2))^(1/2)/(x^4-1),x)
```

```
[Out] int((x^4+1)*(x+(x^2+1)^(1/2))^(1/2)/(x^4-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}+1\right) \sqrt{x+\sqrt{x^{2}+1}}}{x^{4}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x+(x^2+1)^(1/2))^(1/2)/(x^4-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)*sqrt(x + sqrt(x^2 + 1))/(x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^{4}+1\right) \sqrt{x+\sqrt{x^{2}+1}}}{x^{4}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)*(x + (x^2 + 1)^(1/2))^(1/2))/(x^4 - 1),x)
```

```
[Out] int(((x^4 + 1)*(x + (x^2 + 1)^(1/2))^(1/2))/(x^4 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{x^{2}+1}}\, \left(x^{4}+1\right)}{\left(x-1\right) \left(x+1\right) \left(x^{2}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)*(x+(x**2+1)**(1/2))**(1/2)/(x**4-1),x)
```

```
[Out] Integral(sqrt(x + sqrt(x**2 + 1))*(x**4 + 1)/((x - 1)*(x + 1)*(x**2 + 1)),
x)
```

3.2274 $\int \frac{(-a+x)(-b+x)}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(-b^2+a^2d+2(b-ad)x+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=305

$$-\frac{\log\left(a^2d^{2/3}+\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)+\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+\right.\right.}{4d^{2/3}(a-b)}$$

Rubi [A] time = 1.23, antiderivative size = 513, normalized size of antiderivative = 1.68, number of steps used = 9, number of rules used = 5, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {6719, 911, 105, 59, 91}

$$-\frac{(x-a)^{2/3}(x-b)^{4/3}\log\left(2\left(\sqrt{d}+1\right)\left(b-a\sqrt{d}\right)-2(1-d)x\right)}{4d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}-\frac{(x-a)^{2/3}(x-b)^{4/3}\log\left(2(1-d)x-2\left(1-\sqrt{d}\right)\left(a\sqrt{d}+b\right)\right)}{4d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{3(x-a)^{2/3}(x-b)^{4/3}\log\left(-\sqrt[3]{x-a}-\frac{\sqrt[3]{x-b}}{\sqrt[6]{d}}\right)}{4d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{3(x-a)^{2/3}(x-b)^{4/3}\log\left(\frac{\sqrt[3]{x-b}}{\sqrt[6]{d}}-\sqrt[3]{x-a}\right)}{4d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{\sqrt{3}(x-a)^{2/3}(x-b)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x-b}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-a}}\right)}{2d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{\sqrt{3}(x-a)^{2/3}(x-b)^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{x-b}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-a}}+\frac{1}{\sqrt{3}}\right)}{2d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((-a + x)*(-b + x))/(((-a + x)*(-b + x)^2)^(2/3)*(-b^2 + a^2*d + 2*(b -
 a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] (Sqrt[3]*(-a + x)^(2/3)*(-b + x)^(4/3)*ArcTan[1/Sqrt[3] - (2*(-b + x)^(1/3)
)/(Sqrt[3]*d^(1/6)*(-a + x)^(1/3))])/(2*(a - b)*d^(2/3)*(-((a - x)*(b - x)^
2))^(2/3)) + (Sqrt[3]*(-a + x)^(2/3)*(-b + x)^(4/3)*ArcTan[1/Sqrt[3] + (2*(
-b + x)^(1/3))/(Sqrt[3]*d^(1/6)*(-a + x)^(1/3))])/(2*(a - b)*d^(2/3)*(-((a
- x)*(b - x)^2))^(2/3)) - ((-a + x)^(2/3)*(-b + x)^(4/3)*Log[2*(1 + Sqrt[d]
)*(b - a*Sqrt[d]) - 2*(1 - d)*x])/(4*(a - b)*d^(2/3)*(-((a - x)*(b - x)^2))
^(2/3)) - ((-a + x)^(2/3)*(-b + x)^(4/3)*Log[-2*(1 - Sqrt[d])*(b + a*Sqrt[d
]) + 2*(1 - d)*x])/(4*(a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(2/3)) + (3*(-
a + x)^(2/3)*(-b + x)^(4/3)*Log[-(-a + x)^(1/3) - (-b + x)^(1/3)/d^(1/6)])/
(4*(a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(2/3)) + (3*(-a + x)^(2/3)*(-b +
x)^(4/3)*Log[-(-a + x)^(1/3) + (-b + x)^(1/3)/d^(1/6)])/(4*(a - b)*d^(2/3)*
(-((a - x)*(b - x)^2))^(2/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[m] && !
IntegerQ[n]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{(-a+x)(-b+x)}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(-b^2+a^2d+2(b-ad)x+(-1+d)x^2\right)}\,dx = \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{\sqrt[3]{-a}}{\sqrt[3]{-b+x}\left(-b^2+a^2d+2(\right.}}{\left((-a+x)(-b+x)^2\right)^{2/3}}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int\left(\frac{(-1}{(a-b)\sqrt{d}\,\sqrt[3]{-b+x}\,(2b}\right.}{}$$

$$= -\frac{\left((1-d)(-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{}{\sqrt[3]{-b+x}\,(2}}{(a-b)\sqrt{d}\,\left((-a+x)(-b\right.}$$

$$= \frac{\left((1-d)(-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{}{(-a+x)^{2/3}\sqrt[3]{}}}{\left(1-\sqrt{d}\right)\sqrt{d}\,\left((-a+x)(\right.}$$

$$= \frac{\sqrt{3}(-a+x)^{2/3}(-b+x)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2}{\sqrt{3}}\right.}{2(a-b)d^{2/3}\left(-\left((a-x)(b-x)^2\right)\right)^{2/}}$$

Mathematica [C] time = 0.41, size = 91, normalized size = 0.30

$$-\frac{3(b-x)^2\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{b-x}{\sqrt{d}(x-a)}\right)+{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x-b}{\sqrt{d}(x-a)}\right)\right)}{4d(a-b)\left((x-a)(b-x)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-a + x)*(-b + x))/(((-a + x)*(-b + x)^2)^(2/3)*(-b^2 + a^2*d +
2*(b - a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] (-3*(b - x)^2*(Hypergeometric2F1[2/3, 1, 5/3, (b - x)/(Sqrt[d]*(-a + x))] +
 Hypergeometric2F1[2/3, 1, 5/3, (-b + x)/(Sqrt[d]*(-a + x))]))/(4*(a - b)*d
*((b - x)^2*(-a + x))^(2/3))
```

IntegrateAlgebraic [A] time = 6.54, size = 305, normalized size = 1.00

$$-\frac{\log\left(a^2d^{2/3}+\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}\left(\sqrt[3]{d}x-a\sqrt[3]{d}\right)+\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}-2ad^{2/3}x+d^{2/3}x^2\right)}{4d^{2/3}(a-b)}+\frac{\log\left(\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}+a\sqrt[3]{d}-\sqrt[3]{d}x\right)}{2d^{2/3}(a-b)}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}a\sqrt[3]{d}-\sqrt{3}\sqrt[3]{d}x}{-2\sqrt[3]{x(2ab+b^2)-ab^2+x^2(-a-2b)+x^3}+a\sqrt[3]{d}-\sqrt[3]{d}x}\right)}{2d^{2/3}(a-b)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-a + x)*(-b + x))/(((-a + x)*(-b + x)^2)^(2/3)*(-b^2 +
 a^2*d + 2*(b - a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*a*d^(1/3) - Sqrt[3]*d^(1/3)*x)/(a*d^(1/3) - d^(1/3
)*x - 2*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3))]])/(2*(a
- b)*d^(2/3)) + Log[a*d^(1/3) - d^(1/3)*x + (-(a*b^2) + (2*a*b + b^2)*x + (
-a - 2*b)*x^2 + x^3)^(1/3)]/(2*(a - b)*d^(2/3)) - Log[a^2*d^(2/3) - 2*a*d^(
2/3)*x + d^(2/3)*x^2 + (-(a*d^(1/3)) + d^(1/3)*x)*(-(a*b^2) + (2*a*b + b^2)
*x + (-a - 2*b)*x^2 + x^3)^(1/3) + (-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)
*x^2 + x^3)^(2/3)]/(4*(a - b)*d^(2/3))
```

fricas [A] time = 3.40, size = 331, normalized size = 1.09

$$\frac{2\sqrt{3}\,d\sqrt{-\left(-d^{2}\right)^{\frac{1}{3}}}\arctan\left(\frac{\sqrt{3}\left(2\left(-ab^{2}-(a+2\,b)x^{2}+x^{3}+\left(2\,ab+b^{2}\right)x\right)^{\frac{2}{3}}d-\left(b^{2}-2\,bx+x^{2}\right)\left(-d^{2}\right)^{\frac{1}{3}}\right)\sqrt{-\left(-d^{2}\right)^{\frac{1}{3}}}}{3\left(b^{2}d-2\,bdx+dx^{2}\right)}\right)-\left(-d^{2}\right)^{\frac{2}{3}}\log\left(-\frac{\left(-ab^{2}-(a+2\,b)x^{2}+x^{3}+\left(2\,ab+b^{2}\right)x\right)^{\frac{2}{3}}\left(-d^{2}\right)^{\frac{1}{3}}d-\left(b^{2}-2\,bx+x^{2}\right)\left(-d^{2}\right)^{\frac{2}{3}}+\left(-ab^{2}-(a+2\,b)x^{2}+x^{3}+\left(2\,ab+b^{2}\right)x\right)^{\frac{1}{3}}\left(ad^{2}-d^{2}x\right)}{b^{2}-2\,bx+x^{2}}\right)+2\left(-d^{2}\right)^{\frac{2}{3}}\log\left(\frac{\left(-ab^{2}-(a+2\,b)x^{2}+x^{3}+\left(2\,ab+b^{2}\right)x\right)^{\frac{2}{3}}d+\left(b^{2}-2\,bx+x^{2}\right)\left(-d^{2}\right)^{\frac{1}{3}}}{b^{2}-2\,bx+x^{2}}\right)}{4\,(a-b)d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)/((-a+x)*(-b+x)^2)^(2/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-
1+d)*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*d*sqrt(-(-d^2)^(1/3))*arctan(1/3*sqrt(3)*(2*(-a*b^2 - (a + 2
*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*d - (b^2 - 2*b*x + x^2)*(-d^2)^(1/3)
)*sqrt(-(-d^2)^(1/3))/(b^2*d - 2*b*d*x + d*x^2)) - (-d^2)^(2/3)*log(-((-a*b
^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*(-d^2)^(1/3)*d - (b^2 - 2
*b*x + x^2)*(-d^2)^(2/3) + (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)
^(1/3)*(a*d^2 - d^2*x))/(b^2 - 2*b*x + x^2)) + 2*(-d^2)^(2/3)*log(((-a*b^2
- (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*d + (b^2 - 2*b*x + x^2)*(-d^
2)^(1/3))/(b^2 - 2*b*x + x^2)))/((a - b)*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a-x)(b-x)}{\left(-(a-x)(b-x)^{2}\right)^{\frac{2}{3}}\left(a^{2}d+(d-1)x^{2}-b^{2}-2\,(ad-b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)/((-a+x)*(-b+x)^2)^(2/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-
1+d)*x^2),x, algorithm="giac")
```

```
[Out] integrate((a - x)*(b - x)/((-(a - x)*(b - x)^2)^(2/3)*(a^2*d + (d - 1)*x^2
- b^2 - 2*(a*d - b)*x)), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{(-a+x)\,(-b+x)}{\left((-a+x)\,(-b+x)^{2}\right)^{\frac{2}{3}}\left(-b^{2}+a^{2}d+2\,(-ad+b)\,x+(-1+d)\,x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x)*(-b+x)/((-a+x)*(-b+x)^2)^(2/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-1+d)*x
^2),x)
```

```
[Out] int((-a+x)*(-b+x)/((-a+x)*(-b+x)^2)^(2/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-1+d)*x
^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a-x)(b-x)}{\left(-(a-x)(b-x)^{2}\right)^{\frac{2}{3}}\left(a^{2}d+(d-1)x^{2}-b^{2}-2\,(ad-b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)/((-a+x)*(-b+x)^2)^(2/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-
1+d)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((a - x)*(b - x)/((-(a - x)*(b - x)^2)^(2/3)*(a^2*d + (d - 1)*x^2
- b^2 - 2*(a*d - b)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a-x)\,(b-x)}{\left(-(a-x)\,(b-x)^2\right)^{2/3}\,\left(a^2\,d+2\,x\,(b-a\,d)-b^2+x^2\,(d-1)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a - x)*(b - x))/((-(a - x)*(b - x)^2)^(2/3)*(a^2*d + 2*x*(b - a*d) -
b^2 + x^2*(d - 1))),x)
```

```
[Out] int(((a - x)*(b - x))/((-(a - x)*(b - x)^2)^(2/3)*(a^2*d + 2*x*(b - a*d) -
b^2 + x^2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)*(-b+x)/((-a+x)*(-b+x)**2)**(2/3)/(-b**2+a**2*d+2*(-a*d+b)*
x+(-1+d)*x**2),x)
```

```
[Out] Timed out
```

3.2275 $\int \frac{ab-(a+b)x+x^2}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(-b^2+a^2d+2(b-ad)x+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=305

$$-\frac{\log\left(a^2d^{2/3}+\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)+\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+\right.\right.}{4d^{2/3}(a-b)}$$

Rubi [A] time = 0.86, antiderivative size = 513, normalized size of antiderivative = 1.68, number of steps used = 10, number of rules used = 6, integrand size = 60, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6719, 24, 911, 105, 59, 91}

$$-\frac{(x-a)^{2/3}(x-b)^{4/3}\log\left(2\left(\sqrt{d}+1\right)\left(b-a\sqrt{d}\right)-2(1-d)x\right)}{4d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}-\frac{(x-a)^{2/3}(x-b)^{4/3}\log\left(2(1-d)x-2\left(1-\sqrt{d}\right)\left(a\sqrt{d}+b\right)\right)}{4d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{3(x-a)^{2/3}(x-b)^{4/3}\log\left(-\sqrt[3]{x-a}-\frac{\sqrt[3]{x-b}}{\sqrt[6]{d}}\right)}{4d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{3(x-a)^{2/3}(x-b)^{4/3}\log\left(\frac{\sqrt[3]{x-b}}{\sqrt[6]{d}}-\sqrt[3]{x-a}\right)}{4d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{\sqrt{3}(x-a)^{2/3}(x-b)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x-b}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-a}}\right)}{2d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{\sqrt{3}(x-a)^{2/3}(x-b)^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{x-b}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-a}}+\frac{1}{\sqrt{3}}\right)}{2d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*b - (a + b)*x + x^2)/(((-a + x)*(-b + x)^2)^(2/3)*(-b^2 + a^2*d + 2*
(b - a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] (Sqrt[3]*(-a + x)^(2/3)*(-b + x)^(4/3)*ArcTan[1/Sqrt[3] - (2*(-b + x)^(1/3)
)/(Sqrt[3]*d^(1/6)*(-a + x)^(1/3))])/(2*(a - b)*d^(2/3)*(-((a - x)*(b - x)^
2))^(2/3)) + (Sqrt[3]*(-a + x)^(2/3)*(-b + x)^(4/3)*ArcTan[1/Sqrt[3] + (2*(
-b + x)^(1/3))/(Sqrt[3]*d^(1/6)*(-a + x)^(1/3))])/(2*(a - b)*d^(2/3)*(-((a
- x)*(b - x)^2))^(2/3)) - ((-a + x)^(2/3)*(-b + x)^(4/3)*Log[2*(1 + Sqrt[d]
)*(b - a*Sqrt[d]) - 2*(1 - d)*x])/(4*(a - b)*d^(2/3)*(-((a - x)*(b - x)^2))
^(2/3)) - ((-a + x)^(2/3)*(-b + x)^(4/3)*Log[-2*(1 - Sqrt[d])*(b + a*Sqrt[d
]) + 2*(1 - d)*x])/(4*(a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(2/3)) + (3*(-
a + x)^(2/3)*(-b + x)^(4/3)*Log[-(-a + x)^(1/3) - (-b + x)^(1/3)/d^(1/6)])/
(4*(a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(2/3)) + (3*(-a + x)^(2/3)*(-b +
x)^(4/3)*Log[-(-a + x)^(1/3) + (-b + x)^(1/3)/d^(1/6)])/(4*(a - b)*d^(2/3)*
(-((a - x)*(b - x)^2))^(2/3))
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{ab-(a+b)x+x^2}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(-b^2+a^2d+2(b-ad)x+(-1+d)x^2\right)}\,dx = \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{ab}{(-a+x)^{2/3}(-b+x)^{4/3}(\cdots}}{\left((-a+x)(-b+x)^2\cdots\right.}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{\sqrt[3]{-a\cdots}}{\sqrt[3]{-b+x}\left(-b^2+a^2d+2\cdots\right.}}{\left((-a+x)(-b+x)^2\right)^{2/3}}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \left(\frac{(-1\cdots}{(a-b)\sqrt{d}\,\sqrt[3]{-b+x}\,(2\cdots}\right.}{\cdots}$$

$$= -\frac{\left((1-d)(-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{\cdots}{\sqrt[3]{-b+x}\,(2\cdots}}{(a-b)\sqrt{d}\,\left((-a+x)(-b\cdots\right.}$$

$$= \frac{\left((1-d)(-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{\cdots}{(-a+x)^{2/3}\sqrt[3]{\cdots}}}{\left(1-\sqrt{d}\right)\sqrt{d}\,\left((-a+x)(\cdots\right.}$$

$$= \frac{\sqrt{3}\,(-a+x)^{2/3}(-b+x)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\cdots}{\sqrt{3}\cdots}\right.}{2(a-b)d^{2/3}\left(-\left((a-x)(b-x)^2\right)\right)^{2/\cdots}}$$

Mathematica [C] time = 0.27, size = 91, normalized size = 0.30

$$-\frac{3\sqrt[3]{(x-a)(b-x)^2}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{b-x}{\sqrt{d}(x-a)}\right)+{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x-b}{\sqrt{d}(x-a)}\right)\right)}{4d(a-b)(x-a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*b - (a + b)*x + x^2)/(((-a + x)*(-b + x)^2)^(2/3)*(-b^2 + a^2*
d + 2*(b - a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] (-3*((b - x)^2*(-a + x))^(1/3)*(Hypergeometric2F1[2/3, 1, 5/3, (b - x)/(Sqr
t[d]*(-a + x))] + Hypergeometric2F1[2/3, 1, 5/3, (-b + x)/(Sqrt[d]*(-a + x)
)]))/(4*(a - b)*d*(-a + x))
```

IntegrateAlgebraic [A] time = 6.45, size = 305, normalized size = 1.00

$$-\frac{\log \left(a^2d^{2/3}+\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)+\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}-2ad^{2/3}x+d^{2/3}x^2\right)}{4d^{2/3}(a-b)}+\frac{\log \left(\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}+a\sqrt[3]{d}-\sqrt[3]{d}\,x\right)}{2d^{2/3}(a-b)}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}a\sqrt[3]{d}-\sqrt{3}\sqrt[3]{d}\,x}{-2\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}+a\sqrt[3]{d}-\sqrt[3]{d}\,x}\right)}{2d^{2/3}(a-b)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b - (a + b)*x + x^2)/(((-a + x)*(-b + x)^2)^(2/3)*(-b
^2 + a^2*d + 2*(b - a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*a*d^(1/3) - Sqrt[3]*d^(1/3)*x)/(a*d^(1/3) - d^(1/3
)*x - 2*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3))]])/(2*(a
- b)*d^(2/3)) + Log[a*d^(1/3) - d^(1/3)*x + (-(a*b^2) + (2*a*b + b^2)*x + (
-a - 2*b)*x^2 + x^3)^(1/3)]/(2*(a - b)*d^(2/3)) - Log[a^2*d^(2/3) - 2*a*d^(
2/3)*x + d^(2/3)*x^2 + (-(a*d^(1/3)) + d^(1/3)*x)*(-(a*b^2) + (2*a*b + b^2)
*x + (-a - 2*b)*x^2 + x^3)^(1/3) + (-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)
*x^2 + x^3)^(2/3)]/(4*(a - b)*d^(2/3))
```

fricas [A] time = 0.65, size = 331, normalized size = 1.09

$$\frac{2\sqrt{3}d\sqrt{-\left(-d^2\right)^{\frac{1}{3}}}\arctan\left(\frac{\sqrt{3}\left(2\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{2}{3}}d-\left(b^2-2bx+x^2\right)\left(-d^2\right)^{\frac{1}{3}}\right)\sqrt{-\left(-d^2\right)^{\frac{1}{3}}}}{3\left(b^2d-2bdx+dx^2\right)}\right)-\left(-d^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{2}{3}}\left(-d^2\right)^{\frac{1}{3}}d-\left(b^2-2bx+x^2\right)\left(-d^2\right)^{\frac{2}{3}}+\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}\left(ad^2-d^2x\right)}{b^2-2bx+x^2}\right)+2\left(-d^2\right)^{\frac{2}{3}}\log\left(\frac{\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{2}{3}}d+\left(b^2-2bx+x^2\right)\left(-d^2\right)^{\frac{1}{3}}}{b^2-2bx+x^2}\right)}{4(a-b)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-(a+b)*x+x^2)/((-a+x)*(-b+x)^2)^(2/3)/(-b^2+a^2*d+2*(-a*d+b)*
x+(-1+d)*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*d*sqrt(-(-d^2)^(1/3))*arctan(1/3*sqrt(3)*(2*(-a*b^2 - (a + 2
*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*d - (b^2 - 2*b*x + x^2)*(-d^2)^(1/3)
)*sqrt(-(-d^2)^(1/3))/(b^2*d - 2*b*d*x + d*x^2)) - (-d^2)^(2/3)*log(-((-a*b
^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*(-d^2)^(1/3)*d - (b^2 - 2
*b*x + x^2)*(-d^2)^(2/3) + (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)
^(1/3)*(a*d^2 - d^2*x))/(b^2 - 2*b*x + x^2)) + 2*(-d^2)^(2/3)*log(((-a*b^2
- (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*d + (b^2 - 2*b*x + x^2)*(-d^
2)^(1/3))/(b^2 - 2*b*x + x^2)))/((a - b)*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab-(a+b)x+x^2}{\left(-(a-x)(b-x)^2\right)^{\frac{2}{3}}\left(a^2d+(d-1)x^2-b^2-2\,(ad-b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-(a+b)*x+x^2)/((-a+x)*(-b+x)^2)^(2/3)/(-b^2+a^2*d+2*(-a*d+b)*
x+(-1+d)*x^2),x, algorithm="giac")
```

```
[Out] integrate((a*b - (a + b)*x + x^2)/((-(a - x)*(b - x)^2)^(2/3)*(a^2*d + (d -
 1)*x^2 - b^2 - 2*(a*d - b)*x)), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{ab-(a+b)\,x+x^2}{\left((-a+x)\,(-b+x)^2\right)^{\frac{2}{3}}\left(-b^2+a^2d+2\,(-ad+b)\,x+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b-(a+b)*x+x^2)/((-a+x)*(-b+x)^2)^(2/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-1+
d)*x^2),x)
```

```
[Out] int((a*b-(a+b)*x+x^2)/((-a+x)*(-b+x)^2)^(2/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-1+
d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab-(a+b)x+x^2}{\left(-(a-x)(b-x)^2\right)^{\frac{2}{3}}\left(a^2d+(d-1)x^2-b^2-2\,(ad-b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-(a+b)*x+x^2)/((-a+x)*(-b+x)^2)^(2/3)/(-b^2+a^2*d+2*(-a*d+b)*
x+(-1+d)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((a*b - (a + b)*x + x^2)/((-(a - x)*(b - x)^2)^(2/3)*(a^2*d + (d -
 1)*x^2 - b^2 - 2*(a*d - b)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2+(-a-b)\,x+a\,b}{\left(-(a-x)\,(b-x)^2\right)^{2/3}\,\left(a^2\,d+2\,x\,(b-a\,d)-b^2+x^2\,(d-1)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b + x^2 - x*(a + b))/((-(a - x)*(b - x)^2)^(2/3)*(a^2*d + 2*x*(b - a
*d) - b^2 + x^2*(d - 1))),x)
```

```
[Out] int((a*b + x^2 - x*(a + b))/((-(a - x)*(b - x)^2)^(2/3)*(a^2*d + 2*x*(b - a
*d) - b^2 + x^2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-(a+b)*x+x**2)/((-a+x)*(-b+x)**2)**(2/3)/(-b**2+a**2*d+2*(-a*
d+b)*x+(-1+d)*x**2),x)
```

```
[Out] Timed out
```

3.2276 $\int \frac{ab-2bx+x^2}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(bd-(a+d)x+x^2\right)}\,dx$

Optimal. Leaf size=306

$$-\frac{\log\left(\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}\left(d^{2/3}x-bd^{2/3}\right)+\sqrt[3]{d}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+\right.\right.}{2\sqrt[3]{d}}$$

Rubi [F] time = 5.66, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ab-2bx+x^2}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(bd-(a+d)x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*b - 2*b*x + x^2)/((x*(-a + x)*(-b + x)^2)^(1/3)*(b*d - (a + d)*x + x
^2)),x]
```

```
[Out] (3*x*(1 - x/a)^(1/3)*(1 - x/b)^(2/3)*AppellF1[2/3, 1/3, 2/3, 5/3, x/a, x/b]
)/(2*(-((a - x)*(b - x)^2*x))^(1/3)) + ((a - 2*b + d + Sqrt[a^2 + 2*a*d - 4
*b*d + d^2])*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Int][1/(x^(1/3)*(-
a + x)^(1/3)*(-b + x)^(2/3)*(-a - d - Sqrt[a^2 + 2*a*d - 4*b*d + d^2] + 2*x
)), x])/(-((a - x)*(b - x)^2*x))^(1/3) + ((a - 2*b + d - Sqrt[a^2 + 2*a*d -
 4*b*d + d^2])*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Int][1/(x^(1/3)*
(-a + x)^(1/3)*(-b + x)^(2/3)*(-a - d + Sqrt[a^2 + 2*a*d - 4*b*d + d^2] + 2
*x)), x])/(-((a - x)*(b - x)^2*x))^(1/3)
```

Rubi steps

$$\int \frac{ab-2bx+x^2}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(bd-(a+d)x+x^2\right)}\,dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{ab-2bx+x^2}{\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+x)^{2/3}\left(bd-(a+d)x+x^2\right)}\,d}{\sqrt[3]{x(-a+x)(-b+x)^2}}$$

$$= \frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int\left(\frac{1}{\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+x)^{2/3}}+\frac{b(}{\sqrt[3]{x}\sqrt[3]{-a+x}\,(}\right.}{\sqrt[3]{x(-a+x)(-b+x)^2}}$$

$$= \frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{1}{\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+x)^{2/3}}\,dx}{\sqrt[3]{x(-a+x)(-b+x)^2}} + \frac{\left(\sqrt[3]{x}\sqrt[3]{-}\right.}{}$$

$$= \frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int\left(\frac{a-2b+d+\sqrt{a^2+2ad-4bd+}}{\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+x)^{2/3}\left(-a-d-\sqrt{a^2+2ad}\right.}\right.}{\sqrt[3]{x(-a+}}$$

$$= \frac{\left(\left(a-2b+d-\sqrt{a^2+2ad-4bd+d^2}\right)\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+\right.}{\sqrt[3]{x(-a+x)(-}}$$

$$= \frac{3x\sqrt[3]{1-\frac{x}{a}}\left(1-\frac{x}{b}\right)^{2/3}F_1\left(\frac{2}{3};\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{x}{a},\frac{x}{b}\right)}{2\sqrt[3]{-\left((a-x)(b-x)^2x\right)}} + \frac{\left(\left(a-2b+d-\right.\right.}{}$$

Mathematica [F] time = 8.17, size = 0, normalized size = 0.00

$$\int \frac{ab-2bx+x^2}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(bd-(a+d)x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*b - 2*b*x + x^2)/((x*(-a + x)*(-b + x)^2)^(1/3)*(b*d - (a + d)
*x + x^2)),x]
```

```
[Out] Integrate[(a*b - 2*b*x + x^2)/((x*(-a + x)*(-b + x)^2)^(1/3)*(b*d - (a + d)
*x + x^2)), x]
```

IntegrateAlgebraic [A] time = 0.63, size = 306, normalized size = 1.00

$$-\frac{\log\left(\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}\left(d^{2/3}x-bd^{2/3}\right)+\sqrt[3]{d}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}+b^2d-2bdx+dx^2\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[6]{d}\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}+b\sqrt{d}-\sqrt{d}\,x\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}}{\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}-2b\sqrt[3]{d}+2\sqrt[3]{d}\,x}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b - 2*b*x + x^2)/((x*(-a + x)*(-b + x)^2)^(1/3)*(b*d
- (a + d)*x + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 +
 x^4)^(1/3))/(-2*b*d^(1/3) + 2*d^(1/3)*x + (-(a*b^2*x) + (2*a*b + b^2)*x^2
+ (-a - 2*b)*x^3 + x^4)^(1/3))])/d^(1/3) + Log[b*Sqrt[d] - Sqrt[d]*x + d^(1
/6)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3)]/d^(1/3)
- Log[b^2*d - 2*b*d*x + d*x^2 + (-(b*d^(2/3)) + d^(2/3)*x)*(-(a*b^2*x) + (2
*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3) + d^(1/3)*(-(a*b^2*x) + (2*a*
b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(b*d-(a+d)*x+x^2),x, al
gorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab-2\,bx+x^2}{\left(-(a-x)(b-x)^2x\right)^{\frac{1}{3}}\left(bd-(a+d)x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(b*d-(a+d)*x+x^2),x, al
gorithm="giac")
```

```
[Out] integrate((a*b - 2*b*x + x^2)/((-(a - x)*(b - x)^2*x)^(1/3)*(b*d - (a + d)*
x + x^2)), x)
```

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{ab-2bx+x^2}{\left(x\,(-a+x)\,(-b+x)^2\right)^{\frac{1}{3}}\left(bd-(a+d)\,x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(b*d-(a+d)*x+x^2),x)
```

```
[Out] int((a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(b*d-(a+d)*x+x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab-2\,bx+x^2}{\left(-(a-x)(b-x)^2x\right)^{\frac{1}{3}}\left(bd-(a+d)x+x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(b*d-(a+d)*x+x^2),x, al
gorithm="maxima")
```

```
[Out] integrate((a*b - 2*b*x + x^2)/((-(a - x)*(b - x)^2*x)^(1/3)*(b*d - (a + d)*
x + x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2-2\,b\,x+a\,b}{\left(x^2+\left(-a-d\right)\,x+b\,d\right)\,{\left(-x\,\left(a-x\right)\,{\left(b-x\right)}^2\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b - 2*b*x + x^2)/((b*d + x^2 - x*(a + d))*(-x*(a - x)*(b - x)^2)^(1/
3)),x)
```

```
[Out] int((a*b - 2*b*x + x^2)/((b*d + x^2 - x*(a + d))*(-x*(a - x)*(b - x)^2)^(1/
3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b-2*b*x+x**2)/(x*(-a+x)*(-b+x)**2)**(1/3)/(b*d-(a+d)*x+x**2),x
)
```

```
[Out] Timed out
```

3.2277 $\int \frac{-ab^2+(4a-b)bx-3ax^2+x^3}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-a^2d+\left(b^2+2ad\right)x-(2b+d)x^2+x^3\right)}\,dx$

Optimal. Leaf size=306

$$-\frac{\log\left(a^2d^{2/3}+\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)+\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+\right.\right.}{2\sqrt[3]{d}}$$

Rubi [F] time = 8.92, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-ab^2+(4a-b)bx-3ax^2+x^3}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-a^2d+\left(b^2+2ad\right)x-(2b+d)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a*b^2) + (4*a - b)*b*x - 3*a*x^2 + x^3)/((x*(-a + x)*(-b + x)^2)^(1/
3)*(-(a^2*d) + (b^2 + 2*a*d)*x - (2*b + d)*x^2 + x^3)),x]
```

```
[Out] (3*(3*a - b)*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][
(x^4*(-b + x^3)^(1/3))/((-a + x^3)^(1/3)*(a^2*d - b^2*(1 + (2*a*d)/b^2)*x^3
 + 2*b*(1 + d/(2*b))*x^6 - x^9)), x], x, x^(1/3)])/(-((a - x)*(b - x)^2*x))
^(1/3) + (3*a*b*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[In
t][(x*(-b + x^3)^(1/3))/((-a + x^3)^(1/3)*(-(a^2*d) + b^2*(1 + (2*a*d)/b^2)
*x^3 - 2*b*(1 + d/(2*b))*x^6 + x^9)), x], x, x^(1/3)])/(-((a - x)*(b - x)^2
*x))^(1/3) + (3*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[In
t][(x^7*(-b + x^3)^(1/3))/((-a + x^3)^(1/3)*(-(a^2*d) + b^2*(1 + (2*a*d)/b^
2)*x^3 - 2*b*(1 + d/(2*b))*x^6 + x^9)), x], x, x^(1/3)])/(-((a - x)*(b - x)
^2*x))^(1/3)
```

Rubi steps

$$\int \frac{-ab^2+(4a-b)bx-3ax^2+x^3}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-a^2d+\left(b^2+2ad\right)x-(2b+d)x^2+x^3\right)}\,dx=\frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int\frac{-a}{\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+}}{\sqrt[3]{x(-a+x)(-b}}$$

$$=\frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int\frac{\sqrt[3]{-b+x}}{\sqrt[3]{x}\sqrt[3]{-a+x}\left(-a^2d\right.}}{\sqrt[3]{x(-a+x)(-b+x}}$$

$$=\frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\text{Subst}\left(\int\frac{}{\sqrt[3]{-a+}}\right.}{\sqrt[3]{x(-a+x)}}$$

$$=\frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\text{Subst}\left(\int\left(\frac{}{\sqrt[3]{-a}}\right.\right.}{}$$

$$=\frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\text{Subst}\left(\int\frac{}{\sqrt[3]{-a+}}\right.}{\sqrt[3]{x(-a+}}$$

Mathematica [F] time = 3.54, size = 0, normalized size = 0.00

$$\int \frac{-ab^2+(4a-b)bx-3ax^2+x^3}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-a^2d+\left(b^2+2ad\right)x-(2b+d)x^2+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-(a*b^2) + (4*a - b)*b*x - 3*a*x^2 + x^3)/((x*(-a + x)*(-b + x)^
2)^(1/3)*(-(a^2*d) + (b^2 + 2*a*d)*x - (2*b + d)*x^2 + x^3)),x]
```

```
[Out] Integrate[(-(a*b^2) + (4*a - b)*b*x - 3*a*x^2 + x^3)/((x*(-a + x)*(-b + x)^
2)^(1/3)*(-(a^2*d) + (b^2 + 2*a*d)*x - (2*b + d)*x^2 + x^3)), x]
```

IntegrateAlgebraic [A] time = 0.72, size = 306, normalized size = 1.00

$$-\frac{\log\left(a^2d^{2/3}+\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)+\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}-2ad^{2/3}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}+a\sqrt[3]{d}-\sqrt[3]{d}\,x\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}}{\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}-2a\sqrt[3]{d}+2\sqrt[3]{d}\,x}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*b^2) + (4*a - b)*b*x - 3*a*x^2 + x^3)/((x*(-a + x)*
(-b + x)^2)^(1/3)*(-(a^2*d) + (b^2 + 2*a*d)*x - (2*b + d)*x^2 + x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 +
 x^4)^(1/3))/(-2*a*d^(1/3) + 2*d^(1/3)*x + (-(a*b^2*x) + (2*a*b + b^2)*x^2
+ (-a - 2*b)*x^3 + x^4)^(1/3))])/d^(1/3) + Log[a*d^(1/3) - d^(1/3)*x + (-(a
*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3)]/d^(1/3) - Log[a^
2*d^(2/3) - 2*a*d^(2/3)*x + d^(2/3)*x^2 + (-(a*d^(1/3)) + d^(1/3)*x)*(-(a*b
^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3) + (-(a*b^2*x) + (2*
a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b^2+(4*a-b)*b*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a^2*
d+(2*a*d+b^2)*x-(2*b+d)*x^2+x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab^2-(4\,a-b)bx+3\,ax^2-x^3}{\left(-(a-x)(b-x)^2x\right)^{\frac{1}{3}}\left(a^2d+(2\,b+d)x^2-x^3-\left(b^2+2\,ad\right)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b^2+(4*a-b)*b*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a^2*
d+(2*a*d+b^2)*x-(2*b+d)*x^2+x^3),x, algorithm="giac")
```

```
[Out] integrate((a*b^2 - (4*a - b)*b*x + 3*a*x^2 - x^3)/((-(a - x)*(b - x)^2*x)^(
1/3)*(a^2*d + (2*b + d)*x^2 - x^3 - (b^2 + 2*a*d)*x)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{-a\,b^2+(4a-b)\,bx-3a\,x^2+x^3}{\left(x\,(-a+x)\,(-b+x)^2\right)^{\frac{1}{3}}\left(-a^2d+\left(2ad+b^2\right)x-(2b+d)\,x^2+x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*b^2+(4*a-b)*b*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a^2*d+(2*a
*d+b^2)*x-(2*b+d)*x^2+x^3),x)
```

```
[Out] int((-a*b^2+(4*a-b)*b*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a^2*d+(2*a
*d+b^2)*x-(2*b+d)*x^2+x^3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ab^2 - (4\,a - b)bx + 3\,ax^2 - x^3}{\left(-(a-x)(b-x)^2x\right)^{\frac{1}{3}}\left(a^2d + (2\,b + d)x^2 - x^3 - \left(b^2 + 2\,ad\right)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b^2+(4*a-b)*b*x-3*a*x^2+x^3)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-a^2*
d+(2*a*d+b^2)*x-(2*b+d)*x^2+x^3),x, algorithm="maxima")
```

```
[Out] integrate((a*b^2 - (4*a - b)*b*x + 3*a*x^2 - x^3)/((-(a - x)*(b - x)^2*x)^(
1/3)*(a^2*d + (2*b + d)*x^2 - x^3 - (b^2 + 2*a*d)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{a\,b^2 + 3\,a\,x^2 - x^3 - b\,x\,\left(4\,a - b\right)}{\left(-x\,\left(a - x\right)\,\left(b - x\right)^2\right)^{1/3}\,\left(x\,\left(b^2 + 2\,a\,d\right) - a^2\,d - x^2\,\left(2\,b + d\right) + x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*b^2 + 3*a*x^2 - x^3 - b*x*(4*a - b))/((-x*(a - x)*(b - x)^2)^(1/3)*
(x*(2*a*d + b^2) - a^2*d - x^2*(2*b + d) + x^3)),x)
```

```
[Out] int(-(a*b^2 + 3*a*x^2 - x^3 - b*x*(4*a - b))/((-x*(a - x)*(b - x)^2)^(1/3)*
(x*(2*a*d + b^2) - a^2*d - x^2*(2*b + d) + x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b**2+(4*a-b)*b*x-3*a*x**2+x**3)/(x*(-a+x)*(-b+x)**2)**(1/3)/(
-a**2*d+(2*a*d+b**2)*x-(2*b+d)*x**2+x**3),x)
```

```
[Out] Timed out
```

3.2278 $\int \frac{-d+cx^7}{x\sqrt[3]{-b+ax^3}}\,dx$

Optimal. Leaf size=306

$$-\frac{2b^2c\log\left(\sqrt[3]{ax^3-b}-\sqrt[3]{a}\,x\right)}{27a^{7/3}}-\frac{2b^2c\tan^{-1}\left(\frac{\frac{2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{a}}+\frac{x}{\sqrt{3}}}{x}\right)}{9\sqrt{3}\,a^{7/3}}+\frac{b^2c\log\left(a^{2/3}x^2+\sqrt[3]{a}\,x\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}\right)}{27a^{7/3}}+\frac{c\left(\right.}{}$$

Rubi [A] time = 0.24, antiderivative size = 219, normalized size of antiderivative = 0.72, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1836, 1844, 266, 56, 617, 204, 31, 321, 239}

$$-\frac{b^2c\log\left(\sqrt[3]{ax^3-b}-\sqrt[3]{a}\,x\right)}{9a^{7/3}}+\frac{2b^2c\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{ax^3-b}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{7/3}}+\frac{2bcx\left(ax^3-b\right)^{2/3}}{9a^2}+\frac{cx^4\left(ax^3-b\right)^{2/3}}{6a}+\frac{d\log\left(\sqrt[3]{ax^3-b}+\sqrt[3]{b}\right)}{2\sqrt[3]{b}}+\frac{d\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{d\log(x)}{2\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-d + c*x^7)/(x*(-b + a*x^3)^(1/3)),x]
```

```
[Out] (2*b*c*x*(-b + a*x^3)^(2/3))/(9*a^2) + (c*x^4*(-b + a*x^3)^(2/3))/(6*a) + (
2*b^2*c*ArcTan[(1 + (2*a^(1/3)*x)/(-b + a*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*
a^(7/3)) + (d*ArcTan[(b^(1/3) - 2*(-b + a*x^3)^(1/3))/(Sqrt[3]*b^(1/3))])/(
Sqrt[3]*b^(1/3)) - (d*Log[x])/(2*b^(1/3)) + (d*Log[b^(1/3) + (-b + a*x^3)^(
1/3)])/(2*b^(1/3)) - (b^2*c*Log[-(a^(1/3)*x) + (-b + a*x^3)^(1/3)])/(9*a^(7
/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1836

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m +
q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{-d+cx^7}{x\sqrt[3]{-b+ax^3}}\,dx = \frac{cx^4\left(-b+ax^3\right)^{2/3}}{6a} + \frac{\int \frac{-6ad+4bcx^4}{x\sqrt[3]{-b+ax^3}}\,dx}{6a}$$

$$= \frac{cx^4\left(-b+ax^3\right)^{2/3}}{6a} + \frac{\int \left(-\frac{6ad}{x\sqrt[3]{-b+ax^3}} + \frac{4bcx^3}{\sqrt[3]{-b+ax^3}}\right)dx}{6a}$$

$$= \frac{cx^4\left(-b+ax^3\right)^{2/3}}{6a} + \frac{(2bc)\int \frac{x^3}{\sqrt[3]{-b+ax^3}}\,dx}{3a} - d\int \frac{1}{x\sqrt[3]{-b+ax^3}}\,dx$$

$$= \frac{2bcx\left(-b+ax^3\right)^{2/3}}{9a^2} + \frac{cx^4\left(-b+ax^3\right)^{2/3}}{6a} + \frac{\left(2b^2c\right)\int \frac{1}{\sqrt[3]{-b+ax^3}}\,dx}{9a^2} - \frac{1}{3}d\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{-b+\ldots}}\right.$$

$$= \frac{2bcx\left(-b+ax^3\right)^{2/3}}{9a^2} + \frac{cx^4\left(-b+ax^3\right)^{2/3}}{6a} + \frac{2b^2c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a}x}{\sqrt[3]{-b+ax^3}}}{\sqrt{3}}\right)}{9\sqrt{3}a^{7/3}} - \frac{d\log(x)}{2\sqrt[3]{b}} - \frac{b^2c\log\left(\ldots\right.}{\ldots}$$

$$= \frac{2bcx\left(-b+ax^3\right)^{2/3}}{9a^2} + \frac{cx^4\left(-b+ax^3\right)^{2/3}}{6a} + \frac{2b^2c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a}x}{\sqrt[3]{-b+ax^3}}}{\sqrt{3}}\right)}{9\sqrt{3}a^{7/3}} - \frac{d\log(x)}{2\sqrt[3]{b}} + \frac{d\log\left(\sqrt[3]{\ldots}\right.}{\ldots}$$

$$= \frac{2bcx\left(-b+ax^3\right)^{2/3}}{9a^2} + \frac{cx^4\left(-b+ax^3\right)^{2/3}}{6a} + \frac{2b^2c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a}x}{\sqrt[3]{-b+ax^3}}}{\sqrt{3}}\right)}{9\sqrt{3}a^{7/3}} + \frac{d\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-b+ax^3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}\ldots$$

Mathematica [C] time = 0.30, size = 216, normalized size = 0.71

$$\frac{c\left(2b^2\log\left(\frac{a^{2/3}x^2}{\left(ax^3-b\right)^{2/3}}+\frac{\sqrt[3]{a}x}{\sqrt[3]{ax^3-b}}+1\right)+9a^{4/3}x^4\left(ax^3-b\right)^{2/3}-4b^2\log\left(1-\frac{\sqrt[3]{a}x}{\sqrt[3]{ax^3-b}}\right)+4\sqrt{3}b^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a}x}{\sqrt[3]{ax^3-b}}+1}{\sqrt{3}}\right)+12\sqrt[3]{a}bx\left(ax^3-b\right)^{2/3}\right)}{54a^{7/3}} - \frac{d\left(ax^3-b\right)^{2/3}{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-\frac{ax^3}{b}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-d + c*x^7)/(x*(-b + a*x^3)^(1/3)),x]
```

```
[Out] -1/2*(d*(-b + a*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, 1 - (a*x^3)/b])/b
 + (c*(12*a^(1/3)*b*x*(-b + a*x^3)^(2/3) + 9*a^(4/3)*x^4*(-b + a*x^3)^(2/3)
 + 4*Sqrt[3]*b^2*ArcTan[(1 + (2*a^(1/3)*x)/(-b + a*x^3)^(1/3))/Sqrt[3]] - 4
*b^2*Log[1 - (a^(1/3)*x)/(-b + a*x^3)^(1/3)] + 2*b^2*Log[1 + (a^(2/3)*x^2)/
(-b + a*x^3)^(2/3) + (a^(1/3)*x)/(-b + a*x^3)^(1/3)]))/(54*a^(7/3))
```

IntegrateAlgebraic [A] time = 4.91, size = 306, normalized size = 1.00

$$-\frac{2b^2c\log\left(\sqrt[3]{ax^3-b}-\sqrt[3]{a}x\right)}{27a^{7/3}} - \frac{2b^2c\tan^{-1}\left(\frac{\frac{2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{a}}+\frac{x}{\sqrt{3}}}{x}\right)}{9\sqrt{3}a^{7/3}} + \frac{b^2c\log\left(a^{2/3}x^2+\sqrt[3]{a}x\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}\right)}{27a^{7/3}} + \frac{c\left(ax^3-b\right)^{2/3}\left(3ax^4+4bx\right)}{18a^2} - \frac{d\log\left(-\sqrt[3]{b}\sqrt[3]{ax^3-b}+\left(ax^3-b\right)^{2/3}+b^{2/3}\right)}{6\sqrt[3]{b}} + \frac{d\log\left(\sqrt[3]{ax^3-b}+\sqrt[3]{b}\right)}{3\sqrt[3]{b}} + \frac{d\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{ax^3-b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-d + c*x^7)/(x*(-b + a*x^3)^(1/3)),x]
```

```
[Out] (c*(-b + a*x^3)^(2/3)*(4*b*x + 3*a*x^4))/(18*a^2) - (2*b^2*c*ArcTan[(x/Sqrt
[3] + (2*(-b + a*x^3)^(1/3))/(Sqrt[3]*a^(1/3)))/x])/(9*Sqrt[3]*a^(7/3)) + (
d*ArcTan[1/Sqrt[3] - (2*(-b + a*x^3)^(1/3))/(Sqrt[3]*b^(1/3))])/(Sqrt[3]*b^
(1/3)) + (d*Log[b^(1/3) + (-b + a*x^3)^(1/3)])/(3*b^(1/3)) - (2*b^2*c*Log[-
(a^(1/3)*x) + (-b + a*x^3)^(1/3)])/(27*a^(7/3)) - (d*Log[b^(2/3) - b^(1/3)*
```

```
(-b + a*x^3)^(1/3) + (-b + a*x^3)^(2/3)])/(6*b^(1/3)) + (b^2*c*Log[a^(2/3)*
x^2 + a^(1/3)*x*(-b + a*x^3)^(1/3) + (-b + a*x^3)^(2/3)])/(27*a^(7/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^7-d)/x/(a*x^3-b)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^7 - d}{\left(ax^3 - b\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^7-d)/x/(a*x^3-b)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^7 - d)/((a*x^3 - b)^(1/3)*x), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{c\,x^7 - d}{x\left(a\,x^3 - b\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^7-d)/x/(a*x^3-b)^(1/3),x)
```

```
[Out] int((c*x^7-d)/x/(a*x^3-b)^(1/3),x)
```

maxima [A] time = 1.19, size = 301, normalized size = 0.98

$$-\frac{1}{54}\left(\frac{4\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(a^{\frac{1}{3}}+\frac{2\left(ax^3-b\right)^{\frac{1}{3}}}{x}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}}-\frac{2b^2\log\left(a^{\frac{2}{3}}+\frac{\left(ax^3-b\right)^{\frac{1}{3}}a^{\frac{1}{3}}}{x}+\frac{\left(ax^3-b\right)^{\frac{2}{3}}}{x^2}\right)}{a^{\frac{7}{3}}}+\frac{4b^2\log\left(-a^{\frac{1}{3}}+\frac{\left(ax^3-b\right)^{\frac{1}{3}}}{x}\right)}{a^{\frac{7}{3}}}-\frac{3\left(\frac{7\left(ax^3-b\right)^{\frac{2}{3}}ab^2}{x^2}-\frac{4\left(ax^3-b\right)^{\frac{5}{3}}b^2}{x^5}\right)}{a^4-\frac{2\left(ax^3-b\right)a^3}{x^3}+\frac{\left(ax^3-b\right)^2a^2}{x^6}}\right)c-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(ax^3-b\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}+\frac{\log\left(\left(ax^3-b\right)^{\frac{2}{3}}-\left(ax^3-b\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}-\frac{2\log\left(\left(ax^3-b\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^7-d)/x/(a*x^3-b)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/54*(4*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(a^(1/3) + 2*(a*x^3 - b)^(1/3)/x)/a
^(1/3))/a^(7/3) - 2*b^2*log(a^(2/3) + (a*x^3 - b)^(1/3)*a^(1/3)/x + (a*x^3
- b)^(2/3)/x^2)/a^(7/3) + 4*b^2*log(-a^(1/3) + (a*x^3 - b)^(1/3)/x)/a^(7/3)
 - 3*(7*(a*x^3 - b)^(2/3)*a*b^2/x^2 - 4*(a*x^3 - b)^(5/3)*b^2/x^5)/(a^4 - 2
*(a*x^3 - b)*a^3/x^3 + (a*x^3 - b)^2*a^2/x^6))*c - 1/6*(2*sqrt(3)*arctan(1/
3*sqrt(3)*(2*(a*x^3 - b)^(1/3) - b^(1/3))/b^(1/3))/b^(1/3) + log((a*x^3 - b
)^(2/3) - (a*x^3 - b)^(1/3)*b^(1/3) + b^(2/3))/b^(1/3) - 2*log((a*x^3 - b)^
(1/3) + b^(1/3))/b^(1/3))*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{d - c\,x^7}{x\left(a\,x^3 - b\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(d - c*x^7)/(x*(a*x^3 - b)^(1/3)),x)
```

```
[Out] -int((d - c*x^7)/(x*(a*x^3 - b)^(1/3)), x)
```

sympy [C] time = 3.40, size = 82, normalized size = 0.27

$$\frac{cx^7e^{-\frac{i\pi}{3}}\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{ax^3}{b}\right)}{3\sqrt[3]{b}\,\Gamma\left(\frac{10}{3}\right)}+\frac{d\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{be^{2i\pi}}{ax^3}\right)}{3\sqrt[3]{a}\,x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**7-d)/x/(a*x**3-b)**(1/3),x)
```

```
[Out] c*x**7*exp(-I*pi/3)*gamma(7/3)*hyper((1/3, 7/3), (10/3,), a*x**3/b)/(3*b**(
1/3)*gamma(10/3)) + d*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*exp_polar(2*I*
pi)/(a*x**3))/(3*a**(1/3)*x*gamma(4/3))
```

3.2279 $\int \frac{\sqrt{c+\sqrt{b+ax}}}{x-\sqrt{b+ax}}\,dx$

Optimal. Leaf size=306

$$\frac{2\left(\sqrt{2}\,c\sqrt{a^2+4b}+\sqrt{2}\,a\sqrt{a^2+4b}+\sqrt{2}\,a^2+\sqrt{2}\,ac+2\sqrt{2}\,b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{ax+b}+c}}{\sqrt{-\sqrt{a^2+4b}-a-2c}}\right)}{\sqrt{a^2+4b}\sqrt{-\sqrt{a^2+4b}-a-2c}}+\frac{2\left(\sqrt{2}\,c\sqrt{a^2+4b}+\sqrt{2}\,a\sqrt{\right.}}{}$$

Rubi [A] time = 1.74, antiderivative size = 256, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {824, 826, 1166, 206}

$$\frac{2\sqrt{2}\left(-a\left(\sqrt{a^2+4b}-c\right)-c\sqrt{a^2+4b}+a^2+2b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{ax+b}+c}}{\sqrt{-\sqrt{a^2+4b}+a+2c}}\right)}{\sqrt{a^2+4b}\sqrt{-\sqrt{a^2+4b}+a+2c}}-\frac{2\sqrt{2}\left(a\left(\sqrt{a^2+4b}+c\right)+c\sqrt{a^2+4b}+a^2+2b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{ax+b}+c}}{\sqrt{\sqrt{a^2+4b}+a+2c}}\right)}{\sqrt{a^2+4b}\sqrt{\sqrt{a^2+4b}+a+2c}}+4\sqrt{\sqrt{ax+b}+c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + Sqrt[b + a*x]]/(x - Sqrt[b + a*x]),x]
```

```
[Out] 4*Sqrt[c + Sqrt[b + a*x]] + (2*Sqrt[2]*(a^2 + 2*b - a*(Sqrt[a^2 + 4*b] - c)
 - Sqrt[a^2 + 4*b]*c)*ArcTanh[(Sqrt[2]*Sqrt[c + Sqrt[b + a*x]])/Sqrt[a - Sq
rt[a^2 + 4*b] + 2*c]])/(Sqrt[a^2 + 4*b]*Sqrt[a - Sqrt[a^2 + 4*b] + 2*c]) -
(2*Sqrt[2]*(a^2 + 2*b + Sqrt[a^2 + 4*b]*c + a*(Sqrt[a^2 + 4*b] + c))*ArcTan
h[(Sqrt[2]*Sqrt[c + Sqrt[b + a*x]])/Sqrt[a + Sqrt[a^2 + 4*b] + 2*c]])/(Sqrt
[a^2 + 4*b]*Sqrt[a + Sqrt[a^2 + 4*b] + 2*c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{c+\sqrt{b+ax}}}{x-\sqrt{b+ax}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{x\sqrt{c+x}}{b+ax-x^2}\,dx, x, \sqrt{b+ax}\right)\right)$$
$$= 4\sqrt{c+\sqrt{b+ax}} + 2\,\text{Subst}\left(\int \frac{-b-(a+c)x}{\sqrt{c+x}\,(b+ax-x^2)}\,dx, x, \sqrt{b+ax}\right)$$
$$= 4\sqrt{c+\sqrt{b+ax}} + 4\,\text{Subst}\left(\int \frac{-b-(-a-c)c+(-a-c)x^2}{b-ac-c^2+(a+2c)x^2-x^4}\,dx, x, \sqrt{c+\sqrt{b+ax}}\right)$$
$$= 4\sqrt{c+\sqrt{b+ax}} + \frac{\left(2\left(a^2+2b-a\left(\sqrt{a^2+4b}-c\right)-\sqrt{a^2+4b}\,c\right)\right)\text{Subst}\left(\int \frac{}{-\frac{1}{2}\sqrt{a^2+4b}-\cdots}\right.}{\sqrt{a^2+4b}}$$
$$= 4\sqrt{c+\sqrt{b+ax}} + \frac{2\sqrt{2}\left(a^2+2b-a\left(\sqrt{a^2+4b}-c\right)-\sqrt{a^2+4b}\,c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c+\sqrt{b}\cdots}}{\sqrt{a-\sqrt{a^2+4}\cdots}}\right.}{\sqrt{a^2+4b}\sqrt{a-\sqrt{a^2+4b}+2c}}$$

Mathematica [A] time = 0.46, size = 255, normalized size = 0.83

$$\frac{2\sqrt{2}\left(a\left(c-\sqrt{a^2+4b}\right)-c\sqrt{a^2+4b}+a^2+2b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{ax+b}+c}}{\sqrt{-\sqrt{a^2+4b}+a+2c}}\right)}{\sqrt{a^2+4b}\sqrt{-\sqrt{a^2+4b}+a+2c}}-\frac{2\sqrt{2}\left(a\left(\sqrt{a^2+4b}+c\right)+c\sqrt{a^2+4b}+a^2+2b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{ax+b}+c}}{\sqrt{\sqrt{a^2+4b}+a+2c}}\right)}{\sqrt{a^2+4b}\sqrt{\sqrt{a^2+4b}+a+2c}}+4\sqrt{\sqrt{ax+b}+c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + Sqrt[b + a*x]]/(x - Sqrt[b + a*x]),x]
```

```
[Out] 4*Sqrt[c + Sqrt[b + a*x]] + (2*Sqrt[2]*(a^2 + 2*b - Sqrt[a^2 + 4*b]*c + a*(
-Sqrt[a^2 + 4*b] + c))*ArcTanh[(Sqrt[2]*Sqrt[c + Sqrt[b + a*x]])/Sqrt[a - S
qrt[a^2 + 4*b] + 2*c]])/(Sqrt[a^2 + 4*b]*Sqrt[a - Sqrt[a^2 + 4*b] + 2*c]) -
 (2*Sqrt[2]*(a^2 + 2*b + Sqrt[a^2 + 4*b]*c + a*(Sqrt[a^2 + 4*b] + c))*ArcTa
nh[(Sqrt[2]*Sqrt[c + Sqrt[b + a*x]])/Sqrt[a + Sqrt[a^2 + 4*b] + 2*c]])/(Sqr
t[a^2 + 4*b]*Sqrt[a + Sqrt[a^2 + 4*b] + 2*c])
```

IntegrateAlgebraic [A] time = 0.73, size = 306, normalized size = 1.00

$$\frac{2\left(\sqrt{2}c\sqrt{a^2+4b}+\sqrt{2}a\sqrt{a^2+4b}+\sqrt{2}a^2+\sqrt{2}ac+2\sqrt{2}b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{ax+b}+c}}{\sqrt{-\sqrt{a^2+4b}-a-2c}}\right)}{\sqrt{a^2+4b}\sqrt{-\sqrt{a^2+4b}-a-2c}}+\frac{2\left(\sqrt{2}c\sqrt{a^2+4b}+\sqrt{2}a\sqrt{a^2+4b}-\sqrt{2}a^2-\sqrt{2}ac-2\sqrt{2}b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{ax+b}+c}}{\sqrt{\sqrt{a^2+4b}-a-2c}}\right)}{\sqrt{a^2+4b}\sqrt{\sqrt{a^2+4b}-a-2c}}+4\sqrt{\sqrt{ax+b}+c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + Sqrt[b + a*x]]/(x - Sqrt[b + a*x]),x]
```

```
[Out] 4*Sqrt[c + Sqrt[b + a*x]] + (2*(Sqrt[2]*a^2 + 2*Sqrt[2]*b + Sqrt[2]*a*Sqrt[
a^2 + 4*b] + Sqrt[2]*a*c + Sqrt[2]*Sqrt[a^2 + 4*b]*c)*ArcTan[(Sqrt[2]*Sqrt[
c + Sqrt[b + a*x]])/Sqrt[-a - Sqrt[a^2 + 4*b] - 2*c]])/(Sqrt[a^2 + 4*b]*Sqr
t[-a - Sqrt[a^2 + 4*b] - 2*c]) + (2*(-(Sqrt[2]*a^2) - 2*Sqrt[2]*b + Sqrt[2]
*a*Sqrt[a^2 + 4*b] - Sqrt[2]*a*c + Sqrt[2]*Sqrt[a^2 + 4*b]*c)*ArcTan[(Sqrt[
2]*Sqrt[c + Sqrt[b + a*x]])/Sqrt[-a + Sqrt[a^2 + 4*b] - 2*c]])/(Sqrt[a^2 +
4*b]*Sqrt[-a + Sqrt[a^2 + 4*b] - 2*c])
```

fricas [B] time = 0.88, size = 1106, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+b)^(1/2))^(1/2)/(x-(a*x+b)^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*sqrt((a^3 + 3*a*b + (a^2 + 2*b)*c + (a^2 + 4*b)*sqrt((a^4 + a^2*c^
2 + 2*a^2*b + b^2 + 2*(a^3 + a*b)*c)/(a^2 + 4*b)))/(a^2 + 4*b))*log(8*sqrt(
2)*(a^4 + 5*a^2*b + 4*b^2 + (a^3 + 4*a*b)*c - (a^3 + 4*a*b)*sqrt((a^4 + a^2
*c^2 + 2*a^2*b + b^2 + 2*(a^3 + a*b)*c)/(a^2 + 4*b)))*sqrt((a^3 + 3*a*b + (
a^2 + 2*b)*c + (a^2 + 4*b)*sqrt((a^4 + a^2*c^2 + 2*a^2*b + b^2 + 2*(a^3 + a
*b)*c)/(a^2 + 4*b)))/(a^2 + 4*b)) + 32*(a^2*b + a*b*c + b^2)*sqrt(c + sqrt(
a*x + b))) + sqrt(2)*sqrt((a^3 + 3*a*b + (a^2 + 2*b)*c + (a^2 + 4*b)*sqrt((
a^4 + a^2*c^2 + 2*a^2*b + b^2 + 2*(a^3 + a*b)*c)/(a^2 + 4*b)))/(a^2 + 4*b))
*log(-8*sqrt(2)*(a^4 + 5*a^2*b + 4*b^2 + (a^3 + 4*a*b)*c - (a^3 + 4*a*b)*sq
rt((a^4 + a^2*c^2 + 2*a^2*b + b^2 + 2*(a^3 + a*b)*c)/(a^2 + 4*b)))*sqrt((a^
3 + 3*a*b + (a^2 + 2*b)*c + (a^2 + 4*b)*sqrt((a^4 + a^2*c^2 + 2*a^2*b + b^2
 + 2*(a^3 + a*b)*c)/(a^2 + 4*b)))/(a^2 + 4*b)) + 32*(a^2*b + a*b*c + b^2)*s
qrt(c + sqrt(a*x + b))) - sqrt(2)*sqrt((a^3 + 3*a*b + (a^2 + 2*b)*c - (a^2
+ 4*b)*sqrt((a^4 + a^2*c^2 + 2*a^2*b + b^2 + 2*(a^3 + a*b)*c)/(a^2 + 4*b)))
/(a^2 + 4*b))*log(8*sqrt(2)*(a^4 + 5*a^2*b + 4*b^2 + (a^3 + 4*a*b)*c + (a^3
 + 4*a*b)*sqrt((a^4 + a^2*c^2 + 2*a^2*b + b^2 + 2*(a^3 + a*b)*c)/(a^2 + 4*b
)))*sqrt((a^3 + 3*a*b + (a^2 + 2*b)*c - (a^2 + 4*b)*sqrt((a^4 + a^2*c^2 + 2
*a^2*b + b^2 + 2*(a^3 + a*b)*c)/(a^2 + 4*b)))/(a^2 + 4*b)) + 32*(a^2*b + a*
b*c + b^2)*sqrt(c + sqrt(a*x + b))) + sqrt(2)*sqrt((a^3 + 3*a*b + (a^2 + 2*
b)*c - (a^2 + 4*b)*sqrt((a^4 + a^2*c^2 + 2*a^2*b + b^2 + 2*(a^3 + a*b)*c)/(
a^2 + 4*b)))/(a^2 + 4*b))*log(-8*sqrt(2)*(a^4 + 5*a^2*b + 4*b^2 + (a^3 + 4*
a*b)*c + (a^3 + 4*a*b)*sqrt((a^4 + a^2*c^2 + 2*a^2*b + b^2 + 2*(a^3 + a*b)*
c)/(a^2 + 4*b)))*sqrt((a^3 + 3*a*b + (a^2 + 2*b)*c - (a^2 + 4*b)*sqrt((a^4
+ a^2*c^2 + 2*a^2*b + b^2 + 2*(a^3 + a*b)*c)/(a^2 + 4*b)))/(a^2 + 4*b)) + 3
2*(a^2*b + a*b*c + b^2)*sqrt(c + sqrt(a*x + b))) + 4*sqrt(c + sqrt(a*x + b)
)
```

giac [A] time = 0.20, size = 210, normalized size = 0.69

$$-\frac{4\sqrt{a^2+4b}\sqrt{-2a-4c+2\sqrt{a^2+4b}}\,b\arctan\left(\frac{\sqrt{c+\sqrt{ax+b}}}{\sqrt{-\frac{1}{2}a-c+\frac{1}{2}\sqrt{(a+2c)^2-4ac-4c^2+4b}}}\right)}{a^3+4ab+\left(a^2+4b\right)^{\frac{3}{2}}}+\frac{4\sqrt{a^2+4b}\sqrt{-2a-4c-2\sqrt{a^2+4b}}\,b\arctan\left(\frac{\sqrt{c+\sqrt{ax+b}}}{\sqrt{-\frac{1}{2}a-c-\frac{1}{2}\sqrt{(a+2c)^2-4ac-4c^2+4b}}}\right)}{a^3+4ab-\left(a^2+4b\right)^{\frac{3}{2}}}+4\sqrt{c+\sqrt{ax+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+b)^(1/2))^(1/2)/(x-(a*x+b)^(1/2)),x, algorithm="giac")
```

```
[Out] -4*sqrt(a^2 + 4*b)*sqrt(-2*a - 4*c + 2*sqrt(a^2 + 4*b))*b*arctan(sqrt(c + s
qrt(a*x + b))/sqrt(-1/2*a - c + 1/2*sqrt((a + 2*c)^2 - 4*a*c - 4*c^2 + 4*b)
))/(a^3 + 4*a*b + (a^2 + 4*b)^(3/2)) + 4*sqrt(a^2 + 4*b)*sqrt(-2*a - 4*c -
2*sqrt(a^2 + 4*b))*b*arctan(sqrt(c + sqrt(a*x + b))/sqrt(-1/2*a - c - 1/2*s
qrt((a + 2*c)^2 - 4*a*c - 4*c^2 + 4*b)))/(a^3 + 4*a*b - (a^2 + 4*b)^(3/2))
+ 4*sqrt(c + sqrt(a*x + b))
```

maple [B] time = 0.06, size = 645, normalized size = 2.11

$$4\sqrt{c+\sqrt{ax+b}}+\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2\sqrt{a^2+4b}-2a-4c}}\right)a}{\sqrt{-2\sqrt{a^2+4b}-2a-4c}}+\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2\sqrt{a^2+4b}-2a-4c}}\right)c}{\sqrt{-2\sqrt{a^2+4b}-2a-4c}}+\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2\sqrt{a^2+4b}-2a-4c}}\right)a^2}{\sqrt{a^2+4b}\sqrt{-2\sqrt{a^2+4b}-2a-4c}}+\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2\sqrt{a^2+4b}-2a-4c}}\right)ac}{\sqrt{a^2+4b}\sqrt{-2\sqrt{a^2+4b}-2a-4c}}+\frac{8\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2\sqrt{a^2+4b}-2a-4c}}\right)b}{\sqrt{a^2+4b}\sqrt{-2\sqrt{a^2+4b}-2a-4c}}+\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2a+2\sqrt{a^2+4b}-4c}}\right)a}{\sqrt{-2a+2\sqrt{a^2+4b}-4c}}+\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2a+2\sqrt{a^2+4b}-4c}}\right)c}{\sqrt{-2a+2\sqrt{a^2+4b}-4c}}-\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2a+2\sqrt{a^2+4b}-4c}}\right)a^2}{\sqrt{a^2+4b}\sqrt{-2a+2\sqrt{a^2+4b}-4c}}-\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2a+2\sqrt{a^2+4b}-4c}}\right)ac}{\sqrt{a^2+4b}\sqrt{-2a+2\sqrt{a^2+4b}-4c}}-\frac{8\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2a+2\sqrt{a^2+4b}-4c}}\right)b}{\sqrt{a^2+4b}\sqrt{-2a+2\sqrt{a^2+4b}-4c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+(a*x+b)^(1/2))^(1/2)/(x-(a*x+b)^(1/2)),x)
```

```
[Out] 4*(c+(a*x+b)^(1/2))^(1/2)+4/(-2*(a^2+4*b)^(1/2)-2*a-4*c)^(1/2)*arctan(2*(c+
(a*x+b)^(1/2))^(1/2)/(-2*(a^2+4*b)^(1/2)-2*a-4*c)^(1/2))*a+4/(-2*(a^2+4*b)^
(1/2)-2*a-4*c)^(1/2)*arctan(2*(c+(a*x+b)^(1/2))^(1/2)/(-2*(a^2+4*b)^(1/2)-2
*a-4*c)^(1/2))*c+4/(a^2+4*b)^(1/2)/(-2*(a^2+4*b)^(1/2)-2*a-4*c)^(1/2)*arcta
n(2*(c+(a*x+b)^(1/2))^(1/2)/(-2*(a^2+4*b)^(1/2)-2*a-4*c)^(1/2))*a^2+4/(a^2+
4*b)^(1/2)/(-2*(a^2+4*b)^(1/2)-2*a-4*c)^(1/2)*arctan(2*(c+(a*x+b)^(1/2))^(1
/2)/(-2*(a^2+4*b)^(1/2)-2*a-4*c)^(1/2))*a*c+8/(a^2+4*b)^(1/2)/(-2*(a^2+4*b)
^(1/2)-2*a-4*c)^(1/2)*arctan(2*(c+(a*x+b)^(1/2))^(1/2)/(-2*(a^2+4*b)^(1/2)-
2*a-4*c)^(1/2))*b+4/(-2*a+2*(a^2+4*b)^(1/2)-4*c)^(1/2)*arctan(2*(c+(a*x+b)^
```

```
(1/2))^(1/2)/(-2*a+2*(a^2+4*b)^(1/2)-4*c)^(1/2))*a+4/(-2*a+2*(a^2+4*b)^(1/2
)-4*c)^(1/2)*arctan(2*(c+(a*x+b)^(1/2))^(1/2)/(-2*a+2*(a^2+4*b)^(1/2)-4*c)^
(1/2))*c-4/(a^2+4*b)^(1/2)/(-2*a+2*(a^2+4*b)^(1/2)-4*c)^(1/2)*arctan(2*(c+(
a*x+b)^(1/2))^(1/2)/(-2*a+2*(a^2+4*b)^(1/2)-4*c)^(1/2))*a^2-4/(a^2+4*b)^(1/
2)/(-2*a+2*(a^2+4*b)^(1/2)-4*c)^(1/2)*arctan(2*(c+(a*x+b)^(1/2))^(1/2)/(-2*
a+2*(a^2+4*b)^(1/2)-4*c)^(1/2))*a*c-8/(a^2+4*b)^(1/2)/(-2*a+2*(a^2+4*b)^(1/
2)-4*c)^(1/2)*arctan(2*(c+(a*x+b)^(1/2))^(1/2)/(-2*a+2*(a^2+4*b)^(1/2)-4*c)
^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+\sqrt{ax+b}}}{x-\sqrt{ax+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+b)^(1/2))^(1/2)/(x-(a*x+b)^(1/2)),x, algorithm="maxima")
[Out] integrate(sqrt(c + sqrt(a*x + b))/(x - sqrt(a*x + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+\sqrt{b+a\,x}}}{x-\sqrt{b+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + (b + a*x)^(1/2))^(1/2)/(x - (b + a*x)^(1/2)),x)
[Out] int((c + (b + a*x)^(1/2))^(1/2)/(x - (b + a*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+\sqrt{ax+b}}}{x-\sqrt{ax+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+b)**(1/2))**(1/2)/(x-(a*x+b)**(1/2)),x)
[Out] Integral(sqrt(c + sqrt(a*x + b))/(x - sqrt(a*x + b)), x)
```

3.2280 $\int \frac{1+x}{(1+2x)\sqrt[3]{27+27x+36x^2+28x^3+9x^4+x^5}}\,dx$

Optimal. Leaf size=308

$$\frac{\log\left(\sqrt[3]{10}\,x^2+5\sqrt[3]{x^5+9x^4+28x^3+36x^2+27x+27}+\sqrt[3]{10}\,x-6\sqrt[3]{10}\right)}{5\sqrt[3]{10}}-\frac{\log\left(10^{2/3}x^4+2\,10^{2/3}x^3-11\,10^{2/3}x^2+\right.}{}$$

Rubi [C] time = 1.02, antiderivative size = 454, normalized size of antiderivative = 1.47, number of steps used = 20, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6688, 6719, 6742, 757, 429, 444, 55, 617, 204, 31}

$$\frac{2x(x+3)\sqrt[3]{x^2+1}\,F_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};\frac{x^2}{9},-x^2\right)}{15\sqrt[3]{(x+3)^3\left(x^2+1\right)}}+\frac{x(x+3)\sqrt[3]{x^2+1}\,F_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};4x^2,-x^2\right)}{5\sqrt[3]{(x+3)^3\left(x^2+1\right)}}-\frac{(x+3)\sqrt[3]{x^2+1}\log\left(1-4x^2\right)}{20\sqrt[3]{10}\sqrt[3]{(x+3)^3\left(x^2+1\right)}}-\frac{(x+3)\sqrt[3]{x^2+1}\log\left(9-x^2\right)}{10\sqrt[3]{10}\sqrt[3]{(x+3)^3\left(x^2+1\right)}}+\frac{3(x+3)\sqrt[3]{x^2+1}\log\left(\sqrt[3]{10}-2\sqrt[3]{x^2+1}\right)}{20\sqrt[3]{10}\sqrt[3]{(x+3)^3\left(x^2+1\right)}}+\frac{3(x+3)\sqrt[3]{x^2+1}\log\left(\sqrt[3]{10}-\sqrt[3]{x^2+1}\right)}{10\sqrt[3]{10}\sqrt[3]{(x+3)^3\left(x^2+1\right)}}+\frac{\sqrt{3}(x+3)\sqrt[3]{x^2+1}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{x^2+1}+\sqrt[3]{5}}{\sqrt{3}\sqrt[3]{5}}\right)}{5\sqrt[3]{10}\sqrt[3]{(x+3)^3\left(x^2+1\right)}}+\frac{\sqrt{3}(x+3)\sqrt[3]{x^2+1}\tan^{-1}\left(\frac{2\,2^{2/3}\sqrt[3]{x^2+1}+\sqrt[3]{5}}{\sqrt{3}\sqrt[3]{5}}\right)}{10\sqrt[3]{10}\sqrt[3]{(x+3)^3\left(x^2+1\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x)/((1 + 2*x)*(27 + 27*x + 36*x^2 + 28*x^3 + 9*x^4 + x^5)^(1/3)),x
]
```

```
[Out] (2*x*(3 + x)*(1 + x^2)^(1/3)*AppellF1[1/2, 1, 1/3, 3/2, x^2/9, -x^2])/(15*(
(3 + x)^3*(1 + x^2))^(1/3)) + (x*(3 + x)*(1 + x^2)^(1/3)*AppellF1[1/2, 1, 1
/3, 3/2, 4*x^2, -x^2])/(5*((3 + x)^3*(1 + x^2))^(1/3)) + (Sqrt[3]*(3 + x)*(
1 + x^2)^(1/3)*ArcTan[(5^(1/3) + 2^(2/3)*(1 + x^2)^(1/3))/(Sqrt[3]*5^(1/3))
])/(5*10^(1/3)*((3 + x)^3*(1 + x^2))^(1/3)) + (Sqrt[3]*(3 + x)*(1 + x^2)^(1
/3)*ArcTan[(5^(1/3) + 2*2^(2/3)*(1 + x^2)^(1/3))/(Sqrt[3]*5^(1/3))])/(10*10
^(1/3)*((3 + x)^3*(1 + x^2))^(1/3)) - ((3 + x)*(1 + x^2)^(1/3)*Log[1 - 4*x^
2])/(20*10^(1/3)*((3 + x)^3*(1 + x^2))^(1/3)) - ((3 + x)*(1 + x^2)^(1/3)*Lo
g[9 - x^2])/(10*10^(1/3)*((3 + x)^3*(1 + x^2))^(1/3)) + (3*(3 + x)*(1 + x^2
)^(1/3)*Log[10^(1/3) - 2*(1 + x^2)^(1/3)])/(20*10^(1/3)*((3 + x)^3*(1 + x^2
))^(1/3)) + (3*(3 + x)*(1 + x^2)^(1/3)*Log[10^(1/3) - (1 + x^2)^(1/3)])/(10
*10^(1/3)*((3 + x)^3*(1 + x^2))^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1+x}{(1+2x)\sqrt[3]{27+27x+36x^2+28x^3+9x^4+x^5}}\,dx = \int \frac{1+x}{(1+2x)\sqrt[3]{(3+x)^3\left(1+x^2\right)}}\,dx$$

$$= \frac{\left((3+x)\sqrt[3]{1+x^2}\right)\int \frac{1+x}{(3+x)(1+2x)\sqrt[3]{1+x^2}}\,dx}{\sqrt[3]{(3+x)^3\left(1+x^2\right)}}$$

$$= \frac{\left((3+x)\sqrt[3]{1+x^2}\right)\int \left(\frac{2}{5(3+x)\sqrt[3]{1+x^2}}+\frac{1}{5(1+2x)\sqrt[3]{1+x^2}}\right)dx}{\sqrt[3]{(3+x)^3\left(1+x^2\right)}}$$

$$= \frac{\left((3+x)\sqrt[3]{1+x^2}\right)\int \frac{1}{(1+2x)\sqrt[3]{1+x^2}}\,dx}{5\sqrt[3]{(3+x)^3\left(1+x^2\right)}} + \frac{\left(2(3+x)\sqrt[3]{1+x^2}\right)\ldots}{5\sqrt[3]{(3+x)^3\,(\ldots}}$$

$$= \frac{\left((3+x)\sqrt[3]{1+x^2}\right)\int \left(\frac{1}{\left(1-4x^2\right)\sqrt[3]{1+x^2}}+\frac{2x}{\sqrt[3]{1+x^2}\left(-1+4x^2\right)}\right)dx}{5\sqrt[3]{(3+x)^3\left(1+x^2\right)}} + \ldots$$

$$= \frac{\left((3+x)\sqrt[3]{1+x^2}\right)\int \frac{1}{\left(1-4x^2\right)\sqrt[3]{1+x^2}}\,dx}{5\sqrt[3]{(3+x)^3\left(1+x^2\right)}} + \frac{\left(2(3+x)\sqrt[3]{1+x^2}\right)\ldots}{5\sqrt[3]{(3+x)\ldots}}$$

$$= \frac{2x(3+x)\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};\frac{x^2}{9},-x^2\right)}{15\sqrt[3]{(3+x)^3\left(1+x^2\right)}} + \frac{x(3+x)\sqrt[3]{1+x\ldots}}{5\sqrt[3]{(3\ldots}}$$

$$= \frac{2x(3+x)\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};\frac{x^2}{9},-x^2\right)}{15\sqrt[3]{(3+x)^3\left(1+x^2\right)}} + \frac{x(3+x)\sqrt[3]{1+x\ldots}}{5\sqrt[3]{(3\ldots}}$$

$$= \frac{2x(3+x)\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};\frac{x^2}{9},-x^2\right)}{15\sqrt[3]{(3+x)^3\left(1+x^2\right)}} + \frac{x(3+x)\sqrt[3]{1+x\ldots}}{5\sqrt[3]{(3\ldots}}$$

$$= \frac{2x(3+x)\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{2};1,\frac{1}{3};\frac{3}{2};\frac{x^2}{9},-x^2\right)}{15\sqrt[3]{(3+x)^3\left(1+x^2\right)}} + \frac{x(3+x)\sqrt[3]{1+x\ldots}}{5\sqrt[3]{(3\ldots}}$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1+x}{(1+2x)\sqrt[3]{27+27x+36x^2+28x^3+9x^4+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x)/((1 + 2*x)*(27 + 27*x + 36*x^2 + 28*x^3 + 9*x^4 + x^5)^(1
/3)),x]
```

```
[Out] Integrate[(1 + x)/((1 + 2*x)*(27 + 27*x + 36*x^2 + 28*x^3 + 9*x^4 + x^5)^(1
/3)), x]
```

IntegrateAlgebraic [A] time = 0.38, size = 308, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{10}\,x^2+5\sqrt[3]{x^5+9x^4+28x^3+36x^2+27x+27}+\sqrt[3]{10}\,x-6\sqrt[3]{10}\right)}{5\sqrt[3]{10}} - \frac{\log\left(10^{2/3}x^4+2\,10^{2/3}x^3-11\,10^{2/3}x^2+25\left(x^5+9x^4+28x^3+36x^2+27x+27\right)^{2/3}+\left(-5\sqrt[3]{10}\,x^2-5\sqrt[3]{10}\,x+30\sqrt[3]{10}\right)\sqrt[3]{x^5+9x^4+28x^3+36x^2+27x+27}-12\,10^{2/3}x+36\,10^{2/3}\right)}{10\sqrt[3]{10}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{5\sqrt{3}\sqrt[3]{x^5+9x^4+28x^3+36x^2+27x+27}}{-2\sqrt[3]{10}\,x^2+5\sqrt[3]{x^5+9x^4+28x^3+36x^2+27x+27}-2\sqrt[3]{10}\,x+12\sqrt[3]{10}}\right)}{5\sqrt[3]{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((1 + 2*x)*(27 + 27*x + 36*x^2 + 28*x^3 + 9*x^4
+ x^5)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(5*Sqrt[3]*(27 + 27*x + 36*x^2 + 28*x^3 + 9*x^4 + x^5)^(1/3
))/(12*10^(1/3) - 2*10^(1/3)*x - 2*10^(1/3)*x^2 + 5*(27 + 27*x + 36*x^2 + 2
8*x^3 + 9*x^4 + x^5)^(1/3))])/(5*10^(1/3)) + Log[-6*10^(1/3) + 10^(1/3)*x +
 10^(1/3)*x^2 + 5*(27 + 27*x + 36*x^2 + 28*x^3 + 9*x^4 + x^5)^(1/3)]/(5*10^
(1/3)) - Log[36*10^(2/3) - 12*10^(2/3)*x - 11*10^(2/3)*x^2 + 2*10^(2/3)*x^3
 + 10^(2/3)*x^4 + (30*10^(1/3) - 5*10^(1/3)*x - 5*10^(1/3)*x^2)*(27 + 27*x
+ 36*x^2 + 28*x^3 + 9*x^4 + x^5)^(1/3) + 25*(27 + 27*x + 36*x^2 + 28*x^3 +
9*x^4 + x^5)^(2/3)]/(10*10^(1/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+2*x)/(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3),x, algorith
m="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(x^{5}+9\,x^{4}+28\,x^{3}+36\,x^{2}+27\,x+27\right)^{\frac{1}{3}}(2\,x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+2*x)/(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3),x, algorith
m="giac")
```

```
[Out] integrate((x + 1)/((x^5 + 9*x^4 + 28*x^3 + 36*x^2 + 27*x + 27)^(1/3)*(2*x +
 1)), x)
```

maple [C] time = 18.74, size = 2482, normalized size = 8.06

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(1+2*x)/(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3),x)
```

```
[Out] 1/50*RootOf(_Z^3-100)*ln(-(1777529944800*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^
(2/3)*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*RootOf
(_Z^3-100)^2*x-84207657169800*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(2/3)+49597
2988575750*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)-2
2512729770640*RootOf(_Z^3-100)*x^3-24547012581240*RootOf(_Z^3-100)*x^2+5289
135307560*RootOf(_Z^3-100)*x-2825392792500*RootOf(_Z^3-100)*x^4-52118325607
572*RootOf(_Z^3-100)+370691534376*RootOf(_Z^3-100)*x^5-1971731746875*RootOf
(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)^2*RootOf(_Z^3-100)
^2*x^3+207195471450*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+62
5*_Z^2)*RootOf(_Z^3-100)^3*x^3-6452940262500*RootOf(81*RootOf(_Z^3-100)^2+2
25*_Z*RootOf(_Z^3-100)+625*_Z^2)^2*RootOf(_Z^3-100)^2*x+678094270200*RootOf
(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*RootOf(_Z^3-100)^3
*x-3527607343500*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_
Z^2)*x^5+26887251093750*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100
```

)+625*_Z^2)*x^4+214237616715000*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*x^3+233596437502500*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*x^2-50332934047500*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*x-59749446875*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)^2*RootOf(_Z^3-100)^2*x^5+6278650650*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*RootOf(_Z^3-100)^3*x^5-477995575000*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)^2*RootOf(_Z^3-100)^2*x^4+50229205200*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*RootOf(_Z^3-100)^3*x^4-5377450218750*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)^2*RootOf(_Z^3-100)^2*x^2+565078558500*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*RootOf(_Z^3-100)^3*x^2+1215958301550*RootOf(_Z^3-100)*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3)*x^3-1215958301550*RootOf(_Z^3-100)*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3)*x^2-9727666412400*RootOf(_Z^3-100)*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3)*x+15357858723072*RootOf(_Z^3-100)^2*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3)+42103828584900*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(2/3)*x+1279821560256*RootOf(_Z^3-100)^2*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3)*x^3-3555059889600*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(2/3)*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*RootOf(_Z^3-100)^2-1279821560256*RootOf(_Z^3-100)^2*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3)*x^2-10238572482048*RootOf(_Z^3-100)^2*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3)*x+14591499618600*RootOf(_Z^3-100)*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3))/(1+2*x)/(3+x)^4)+1/18*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*ln(-(-1777529944800*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(2/3)*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*RootOf(_Z^3-100)^2*x+43774498855800*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(2/3)-144773126687700*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)+102679185456900*RootOf(_Z^3-100)*x^3+153786472335900*RootOf(_Z^3-100)*x^2+65510249544900*RootOf(_Z^3-100)*x+15874233045750*RootOf(_Z^3-100)*x^4+178550275887270*RootOf(_Z^3-100)-495585812160*RootOf(_Z^3-100)*x^5-575542976250*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)^2*RootOf(_Z^3-100)^2*x^3+709823428875*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*RootOf(_Z^3-100)^3*x^3-1883595195000*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)^2*RootOf(_Z^3-100)^2*x+2323058494500*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*RootOf(_Z^3-100)^3*x+401833641600*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*x^5-12871233832500*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*x^4-83254907619000*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*x^3-124694001909000*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*x^2-53117384499000*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*x-17440696250*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)^2*RootOf(_Z^3-100)^2*x^5+21509800875*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*RootOf(_Z^3-100)^3*x^5-139525570000*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)^2*RootOf(_Z^3-100)^2*x^4+172078407000*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*RootOf(_Z^3-100)^3*x^4-1569662662500*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)^2*RootOf(_Z^3-100)^2*x^2+1935882078750*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*RootOf(_Z^3-100)^3*x^2-2339101588050*RootOf(_Z^3-100)*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3)*x^3+2339101588050*RootOf(_Z^3-100)*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3)*x^2+18712812704400*RootOf(_Z^3-100)*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3)*x-15357858723072*RootOf(_Z^3-100)^2*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3)-21887249427900*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(2/3)*

```
x-1279821560256*RootOf(_Z^3-100)^2*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3)*
x^3+3555059889600*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(2/3)*RootOf(81*RootOf(
_Z^3-100)^2+225*_Z*RootOf(_Z^3-100)+625*_Z^2)*RootOf(_Z^3-100)^2+1279821560
256*RootOf(_Z^3-100)^2*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3)*x^2+10238572
482048*RootOf(_Z^3-100)^2*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3)*x-2806921
9056600*RootOf(_Z^3-100)*RootOf(81*RootOf(_Z^3-100)^2+225*_Z*RootOf(_Z^3-10
0)+625*_Z^2)*(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3))/(1+2*x)/(3+x)^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(x^5+9x^4+28x^3+36x^2+27x+27\right)^{\frac{1}{3}}(2x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+2*x)/(x^5+9*x^4+28*x^3+36*x^2+27*x+27)^(1/3),x, algorith
m="maxima")
```

```
[Out] integrate((x + 1)/((x^5 + 9*x^4 + 28*x^3 + 36*x^2 + 27*x + 27)^(1/3)*(2*x +
 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x+1}{(2x+1)\left(x^5+9x^4+28x^3+36x^2+27x+27\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((2*x + 1)*(27*x + 36*x^2 + 28*x^3 + 9*x^4 + x^5 + 27)^(1/3)),x
)
```

```
[Out] int((x + 1)/((2*x + 1)*(27*x + 36*x^2 + 28*x^3 + 9*x^4 + x^5 + 27)^(1/3)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt[3]{(x+3)^3\left(x^2+1\right)}\,(2x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+2*x)/(x**5+9*x**4+28*x**3+36*x**2+27*x+27)**(1/3),x)
```

```
[Out] Integral((x + 1)/(((x + 3)**3*(x**2 + 1))**(1/3)*(2*x + 1)), x)
```

3.2281

$$\int \frac{x(-b+x)(ab+(-2a+b)x)}{\sqrt[3]{x(-a+x)(-b+x)}\left(-a^4+4a^3x+\left(-6a^2+b^2d\right)x^2+2(2a-bd)x^3+(-1+d)x^4\right)}\,dx$$

Optimal. Leaf size=310

$$\frac{\tanh^{-1}\left(\frac{\sqrt[3]{x^2(-a-b)+abx+x^3}\left(a\sqrt[6]{d}-\sqrt[6]{d}\,x\right)}{a^2+\sqrt[3]{d}\left(x^2(-a-b)+abx+x^3\right)^{2/3}-2ax+x^2}\right)}{2d^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}}{\sqrt[6]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}+2a-2x}\right)}{2d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}}{\sqrt[6]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}-2\ldots}\right)}{2d^{5/6}}$$

Rubi [F] time = 12.79, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(-b+x)(ab+(-2a+b)x)}{\sqrt[3]{x(-a+x)(-b+x)}\left(-a^4+4a^3x+\left(-6a^2+b^2d\right)x^2+2(2a-bd)x^3+(-1+d)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-b + x)*(a*b + (-2*a + b)*x))/((x*(-a + x)*(-b + x))^(1/3)*(-a^4 +
4*a^3*x + (-6*a^2 + b^2*d)*x^2 + 2*(2*a - b*d)*x^3 + (-1 + d)*x^4)),x]
```

```
[Out] (3*a*b*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][Defer[Int][(x^4*(
-b + x^3)^(2/3))/((-a + x^3)^(1/3)*(-a^4 + 4*a^3*x^3 - 6*a^2*(1 - (b^2*d)/(
6*a^2))*x^6 + 4*a*(1 - (b*d)/(2*a))*x^9 - (1 - d)*x^12)), x], x, x^(1/3)])/
((a - x)*(b - x)*x)^(1/3) + (3*(2*a - b)*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(1
/3)*Defer[Subst][Defer[Int][(x^7*(-b + x^3)^(2/3))/((-a + x^3)^(1/3)*(a^4 -
 4*a^3*x^3 + 6*a^2*(1 - (b^2*d)/(6*a^2))*x^6 - 4*a*(1 - (b*d)/(2*a))*x^9 +
(1 - d)*x^12)), x], x, x^(1/3)])/((a - x)*(b - x)*x)^(1/3)
```

Rubi steps

$$\int \frac{x(-b+x)(ab+(-2a+b)x)}{\sqrt[3]{x(-a+x)(-b+x)}\left(-a^4+4a^3x+\left(-6a^2+b^2d\right)x^2+2(2a-bd)x^3+(-1+d)x^4\right)}\,dx=\frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b+x}\ldots\right.}{\ldots}$$

$$=\frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b+\ldots}\right.}{\ldots}$$

$$=\frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}\sqrt[3]{-b+\ldots}\right.}{\ldots}$$

$$=\frac{\left(3(2a-b)\sqrt[3]{x}\sqrt[3]{-a+\ldots}\right.}{\ldots}$$

Mathematica [F] time = 3.36, size = 0, normalized size = 0.00

$$\int \frac{x(-b+x)(ab+(-2a+b)x)}{\sqrt[3]{x(-a+x)(-b+x)}\left(-a^4+4a^3x+\left(-6a^2+b^2d\right)x^2+2(2a-bd)x^3+(-1+d)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(-b + x)*(a*b + (-2*a + b)*x))/((x*(-a + x)*(-b + x))^(1/3)*(-
a^4 + 4*a^3*x + (-6*a^2 + b^2*d)*x^2 + 2*(2*a - b*d)*x^3 + (-1 + d)*x^4)),x
]
```

```
[Out] Integrate[(x*(-b + x)*(a*b + (-2*a + b)*x))/((x*(-a + x)*(-b + x))^(1/3)*(-
a^4 + 4*a^3*x + (-6*a^2 + b^2*d)*x^2 + 2*(2*a - b*d)*x^3 + (-1 + d)*x^4)),
x]
```

IntegrateAlgebraic [A] time = 6.93, size = 274, normalized size = 0.88

$$\frac{\tanh^{-1}\left(\frac{\sqrt[3]{x^2(-a-b)+abx+x^3}\left(a\sqrt[6]{d}-\sqrt[6]{d}\,x\right)}{a^2+\sqrt[3]{d}\left(x^2(-a-b)+abx+x^3\right)^{2/3}-2ax+x^2}\right)}{2d^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,a-\sqrt{3}\,x}{-2\sqrt[6]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}+a-x}\right)}{2d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,a-\sqrt{3}\,x}{2\sqrt[6]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}+a-x}\right)}{2d^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[3]{x^2(-a-b)+abx+x^3}}{a-x}\right)}{d^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-b + x)*(a*b + (-2*a + b)*x))/((x*(-a + x)*(-b + x))
^(1/3)*(-a^4 + 4*a^3*x + (-6*a^2 + b^2*d)*x^2 + 2*(2*a - b*d)*x^3 + (-1 + d
)*x^4)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*a - Sqrt[3]*x)/(a - x - 2*d^(1/6)*(a*b*x + (-
a - b)*x^2 + x^3)^(1/3))]])/d^(5/6) + (Sqrt[3]*ArcTan[(Sqrt[3]*a - Sqrt[3]*x
)/(a - x + 2*d^(1/6)*(a*b*x + (-a - b)*x^2 + x^3)^(1/3))]])/(2*d^(5/6)) + Ar
cTanh[(d^(1/6)*(a*b*x + (-a - b)*x^2 + x^3)^(1/3))/(a - x)]/d^(5/6) + ArcTa
nh[((a*d^(1/6) - d^(1/6)*x)*(a*b*x + (-a - b)*x^2 + x^3)^(1/3))/(a^2 - 2*a*
x + x^2 + d^(1/3)*(a*b*x + (-a - b)*x^2 + x^3)^(2/3))]/(2*d^(5/6))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(a*b+(-2*a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a^4+4*a^3*x+(
b^2*d-6*a^2)*x^2+2*(-b*d+2*a)*x^3+(-1+d)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(ab-(2\,a-b)x)(b-x)x}{\left((d-1)x^4-a^4+4\,a^3x-2\,(bd-2\,a)x^3+\left(b^2d-6\,a^2\right)x^2\right)((a-x)(b-x)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(a*b+(-2*a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a^4+4*a^3*x+(
b^2*d-6*a^2)*x^2+2*(-b*d+2*a)*x^3+(-1+d)*x^4),x, algorithm="giac")
```

```
[Out] integrate(-(a*b - (2*a - b)*x)*(b - x)*x/(((d - 1)*x^4 - a^4 + 4*a^3*x - 2*
(b*d - 2*a)*x^3 + (b^2*d - 6*a^2)*x^2)*((a - x)*(b - x)*x)^(1/3)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x\left(-b+x\right)\left(ab+\left(-2a+b\right)x\right)}{\left(x\left(-a+x\right)\left(-b+x\right)\right)^{\frac{1}{3}}\left(-a^{4}+4a^{3}x+\left(b^{2}d-6a^{2}\right)x^{2}+2\left(-bd+2a\right)x^{3}+\left(-1+d\right)x^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b+x)*(a*b+(-2*a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a^4+4*a^3*x+(b^2*d-
6*a^2)*x^2+2*(-b*d+2*a)*x^3+(-1+d)*x^4),x)
```

```
[Out] int(x*(-b+x)*(a*b+(-2*a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a^4+4*a^3*x+(b^2*d-
6*a^2)*x^2+2*(-b*d+2*a)*x^3+(-1+d)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ab-(2a-b)x)(b-x)x}{\left((d-1)x^4-a^4+4a^3x-2(bd-2a)x^3+\left(b^2d-6a^2\right)x^2\right)((a-x)(b-x)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(a*b+(-2*a+b)*x)/(x*(-a+x)*(-b+x))^(1/3)/(-a^4+4*a^3*x+(
b^2*d-6*a^2)*x^2+2*(-b*d+2*a)*x^3+(-1+d)*x^4),x, algorithm="maxima")
```

```
[Out] -integrate((a*b - (2*a - b)*x)*(b - x)*x/(((d - 1)*x^4 - a^4 + 4*a^3*x - 2*
(b*d - 2*a)*x^3 + (b^2*d - 6*a^2)*x^2)*((a - x)*(b - x)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x\,\left(a\,b-x\,\left(2\,a-b\right)\right)\,\left(b-x\right)}{{\left(x\,\left(a-x\right)\,\left(b-x\right)\right)}^{1/3}\,\left(x^2\,\left(b^2\,d-6\,a^2\right)+2\,x^3\,\left(2\,a-b\,d\right)+4\,a^3\,x-a^4+x^4\,\left(d-1\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a*b - x*(2*a - b))*(b - x))/((x*(a - x)*(b - x))^(1/3)*(x^2*(b^2*d
 - 6*a^2) + 2*x^3*(2*a - b*d) + 4*a^3*x - a^4 + x^4*(d - 1))),x)
```

```
[Out] int(-(x*(a*b - x*(2*a - b))*(b - x))/((x*(a - x)*(b - x))^(1/3)*(x^2*(b^2*d
 - 6*a^2) + 2*x^3*(2*a - b*d) + 4*a^3*x - a^4 + x^4*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b+x)*(a*b+(-2*a+b)*x)/(x*(-a+x)*(-b+x))**(1/3)/(-a**4+4*a**3*
x+(b**2*d-6*a**2)*x**2+2*(-b*d+2*a)*x**3+(-1+d)*x**4),x)
```

```
[Out] Timed out
```

3.2282 $\int \frac{-ab-ac+2bc+(2a-b-c)x}{\sqrt[3]{(-a+x)(-b+x)(-c+x)}\left(-bc+a^2d+(b+c-2ad)x+(-1+d)x^2\right)}dx$

Optimal. Leaf size=311

$$-\frac{\log\left(a^2d^{2/3}+\left(\sqrt[3]{d}x-a\sqrt[3]{d}\right)\sqrt[3]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}+\left(x^2(-a-b-c)+x(ab+ac+b\right.\right.}{2\sqrt[3]{d}}$$

Rubi [F] time = 9.85, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{-ab-ac+2bc+(2a-b-c)x}{\sqrt[3]{(-a+x)(-b+x)(-c+x)}\left(-bc+a^2d+(b+c-2ad)x+(-1+d)x^2\right)}dx$$

Verification is not applicable to the result.

```
[In] Int[(-(a*b) - a*c + 2*b*c + (2*a - b - c)*x)/(((-a + x)*(-b + x)*(-c + x))^
(1/3)*(-(b*c) + a^2*d + (b + c - 2*a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] ((2*a - b - c + Sqrt[b^2 + c^2 + 4*a^2*d - 4*a*c*d - 2*b*(c + 2*a*d - 2*c*d
)])*(-a + x)^(1/3)*(-b + x)^(1/3)*(-c + x)^(1/3)*Defer[Int][1/((-a + x)^(1/
3)*(-b + x)^(1/3)*(-c + x)^(1/3)*(b + c - 2*a*d - Sqrt[b^2 - 2*b*c + c^2 +
4*a^2*d - 4*a*b*d - 4*a*c*d + 4*b*c*d] + 2*(-1 + d)*x)), x])/(-((a - x)*(b
- x)*(c - x)))^(1/3) + ((2*a - b - c - Sqrt[b^2 + c^2 + 4*a^2*d - 4*a*c*d -
 2*b*(c + 2*a*d - 2*c*d)])*(-a + x)^(1/3)*(-b + x)^(1/3)*(-c + x)^(1/3)*Def
er[Int][1/((-a + x)^(1/3)*(-b + x)^(1/3)*(-c + x)^(1/3)*(b + c - 2*a*d + Sq
rt[b^2 - 2*b*c + c^2 + 4*a^2*d - 4*a*b*d - 4*a*c*d + 4*b*c*d] + 2*(-1 + d)*
x)), x])/(-((a - x)*(b - x)*(c - x)))^(1/3)
```

Rubi steps

$$\int \frac{-ab-ac+2bc+(2a-b-c)x}{\sqrt[3]{(-a+x)(-b+x)(-c+x)}\left(-bc+a^2d+(b+c-2ad)x+(-1+d)x^2\right)}dx = \frac{\left(\sqrt[3]{-a+x}\sqrt[3]{-b+x}\sqrt[3]{-c+x}\right)\int}{\sqrt[3]{(-}}$$

$$= \frac{\left(\sqrt[3]{-a+x}\sqrt[3]{-b+x}\sqrt[3]{-c+x}\right)\int}{}$$

$$= \frac{\left(\left(2a-b-c-\sqrt{b^2+c^2+4a^2c}\right.\right.}{}$$

Mathematica [F] time = 9.12, size = 0, normalized size = 0.00

$$\int \frac{-ab-ac+2bc+(2a-b-c)x}{\sqrt[3]{(-a+x)(-b+x)(-c+x)}\left(-bc+a^2d+(b+c-2ad)x+(-1+d)x^2\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-(a*b) - a*c + 2*b*c + (2*a - b - c)*x)/(((-a + x)*(-b + x)*(-c
+ x))^(1/3)*(-(b*c) + a^2*d + (b + c - 2*a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] Integrate[(-(a*b) - a*c + 2*b*c + (2*a - b - c)*x)/(((-a + x)*(-b + x)*(-c
+ x))^(1/3)*(-(b*c) + a^2*d + (b + c - 2*a*d)*x + (-1 + d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 3.17, size = 311, normalized size = 1.00

$$-\frac{\log\left(a^2d^{2/3}+\left(\sqrt[3]{d}\,x-a\sqrt[3]{d}\right)\sqrt[3]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}+\left(x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3\right)^{2/3}-2ad^{2/3}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}+a\sqrt[3]{d}-\sqrt[3]{d}\,x\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}}{\sqrt[3]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}-2a\sqrt[3]{d}+2\sqrt[3]{d}\,x}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-(a*b) - a*c + 2*b*c + (2*a - b - c)*x)/(((-a + x)*(-b
+ x)*(-c + x))^(1/3)*(-(b*c) + a^2*d + (b + c - 2*a*d)*x + (-1 + d)*x^2)),x
]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(-(a*b*c) + (a*b + a*c + b*c)*x + (-a - b - c)*x^2
 + x^3)^(1/3))/(-2*a*d^(1/3) + 2*d^(1/3)*x + (-(a*b*c) + (a*b + a*c + b*c)*
x + (-a - b - c)*x^2 + x^3)^(1/3))])/d^(1/3) + Log[a*d^(1/3) - d^(1/3)*x +
(-(a*b*c) + (a*b + a*c + b*c)*x + (-a - b - c)*x^2 + x^3)^(1/3)]/d^(1/3) -
Log[a^2*d^(2/3) - 2*a*d^(2/3)*x + d^(2/3)*x^2 + (-(a*d^(1/3)) + d^(1/3)*x)*
(-(a*b*c) + (a*b + a*c + b*c)*x + (-a - b - c)*x^2 + x^3)^(1/3) + (-(a*b*c)
 + (a*b + a*c + b*c)*x + (-a - b - c)*x^2 + x^3)^(2/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b-a*c+2*b*c+(2*a-b-c)*x)/((-a+x)*(-b+x)*(-c+x))^(1/3)/(-b*c+a
^2*d+(-2*a*d+b+c)*x+(-1+d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ab+ac-2\,bc-(2\,a-b-c)x}{(-(a-x)(b-x)(c-x))^{\frac{1}{3}}\left(a^2d+(d-1)x^2-bc-(2\,ad-b-c)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b-a*c+2*b*c+(2*a-b-c)*x)/((-a+x)*(-b+x)*(-c+x))^(1/3)/(-b*c+a
^2*d+(-2*a*d+b+c)*x+(-1+d)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-(a*b + a*c - 2*b*c - (2*a - b - c)*x)/((-(a - x)*(b - x)*(c - x)
)^(1/3)*(a^2*d + (d - 1)*x^2 - b*c - (2*a*d - b - c)*x)), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{-ab-ac+2bc+(2a-b-c)\,x}{((-a+x)\,(-b+x)\,(-c+x))^{\frac{1}{3}}\left(-bc+a^2d+(-2ad+b+c)\,x+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*b-a*c+2*b*c+(2*a-b-c)*x)/((-a+x)*(-b+x)*(-c+x))^(1/3)/(-b*c+a^2*d+(
-2*a*d+b+c)*x+(-1+d)*x^2),x)
```

```
[Out] int((-a*b-a*c+2*b*c+(2*a-b-c)*x)/((-a+x)*(-b+x)*(-c+x))^(1/3)/(-b*c+a^2*d+(
-2*a*d+b+c)*x+(-1+d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ab+ac-2\,bc-(2\,a-b-c)x}{(-(a-x)(b-x)(c-x))^{\frac{1}{3}}\left(a^2d+(d-1)x^2-bc-(2\,ad-b-c)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b-a*c+2*b*c+(2*a-b-c)*x)/((-a+x)*(-b+x)*(-c+x))^(1/3)/(-b*c+a
^2*d+(-2*a*d+b+c)*x+(-1+d)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((a*b + a*c - 2*b*c - (2*a - b - c)*x)/((-(a - x)*(b - x)*(c - x)
)^(1/3)*(a^2*d + (d - 1)*x^2 - b*c - (2*a*d - b - c)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{a\,b+a\,c-2\,b\,c+x\,\left(b-2\,a+c\right)}{{\left(-\left(a-x\right)\,\left(b-x\right)\,\left(c-x\right)\right)}^{1/3}\,\left(x\,\left(b+c-2\,a\,d\right)-b\,c+a^2\,d+x^2\,\left(d-1\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*b + a*c - 2*b*c + x*(b - 2*a + c))/((-(a - x)*(b - x)*(c - x))^(1/3
)*(x*(b + c - 2*a*d) - b*c + a^2*d + x^2*(d - 1))),x)
```

```
[Out] int(-(a*b + a*c - 2*b*c + x*(b - 2*a + c))/((-(a - x)*(b - x)*(c - x))^(1/3
)*(x*(b + c - 2*a*d) - b*c + a^2*d + x^2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b-a*c+2*b*c+(2*a-b-c)*x)/((-a+x)*(-b+x)*(-c+x))**(1/3)/(-b*c+
a**2*d+(-2*a*d+b+c)*x+(-1+d)*x**2),x)
```

```
[Out] Timed out
```

3.2283

$$\int \frac{x\left(-ab+x^2\right)}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(a^2b^2-2ab(a+b)x+\left(a^2+4ab+b^2-d\right)x^2-2(a+b)x^3+x^4\right)}\,dx$$

Optimal. Leaf size=311

$$\frac{\log\left(\sqrt[3]{x^3(-a-b)+abx^2+x^4}-\sqrt[6]{d}\,x\right)}{2d^{2/3}}+\frac{\log\left(\sqrt[3]{x^3(-a-b)+abx^2+x^4}+\sqrt[6]{d}\,x\right)}{2d^{2/3}}-\frac{\log\left(-\sqrt[6]{d}\,x\sqrt[3]{x^3(-a-b)+abx^2}\right.}{}$$

Rubi [F] time = 15.35, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\left(-ab+x^2\right)}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(a^2b^2-2ab(a+b)x+\left(a^2+4ab+b^2-d\right)x^2-2(a+b)x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-(a*b) + x^2))/((x^2*(-a + x)*(-b + x))^(1/3)*(a^2*b^2 - 2*a*b*(a +
 b)*x + (a^2 + 4*a*b + b^2 - d)*x^2 - 2*(a + b)*x^3 + x^4)),x]
```

```
[Out] (3*a*b*x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][Defer[Int][x^3/((
-a + x^3)^(1/3)*(-b + x^3)^(1/3)*(-(a^2*b^2) + 2*a^2*b*(1 + b/a)*x^3 - a^2*
(1 + (4*a*b + b^2 - d)/a^2)*x^6 + 2*a*(1 + b/a)*x^9 - x^12)), x], x, x^(1/3
)])/((a - x)*(b - x)*x^2)^(1/3) + (3*x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*
Defer[Subst][Defer[Int][x^9/((-a + x^3)^(1/3)*(-b + x^3)^(1/3)*(a^2*b^2 - 2
*a^2*b*(1 + b/a)*x^3 + a^2*(1 + (4*a*b + b^2 - d)/a^2)*x^6 - 2*a*(1 + b/a)*
x^9 + x^12)), x], x, x^(1/3)])/((a - x)*(b - x)*x^2)^(1/3)
```

Rubi steps

$$\int \frac{x\left(-ab+x^2\right)}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(a^2b^2-2ab(a+b)x+\left(a^2+4ab+b^2-d\right)x^2-2(a+b)x^3+x^4\right)}\,dx=\frac{\left(x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-b\right.}}{}$$

$$=\frac{\left(3x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-\right.}}{}$$

$$=\frac{\left(3x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-\right.}}{}$$

$$=\frac{\left(3x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-\right.}}{}$$

Mathematica [F] time = 3.16, size = 0, normalized size = 0.00

$$\int \frac{x\left(-ab+x^2\right)}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(a^2b^2-2ab(a+b)x+\left(a^2+4ab+b^2-d\right)x^2-2(a+b)x^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(-(a*b) + x^2))/((x^2*(-a + x)*(-b + x))^(1/3)*(a^2*b^2 - 2*a*
b*(a + b)*x + (a^2 + 4*a*b + b^2 - d)*x^2 - 2*(a + b)*x^3 + x^4)),x]
```

```
[Out] Integrate[(x*(-(a*b) + x^2))/((x^2*(-a + x)*(-b + x))^(1/3)*(a^2*b^2 - 2*a*
b*(a + b)*x + (a^2 + 4*a*b + b^2 - d)*x^2 - 2*(a + b)*x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.83, size = 309, normalized size = 0.99

$$\frac{\log\left(\sqrt[3]{x^3(-a-b)+abx^2+x^4}-\sqrt[6]{d}\,x\right)}{2d^{2/3}}+\frac{\log\left(\sqrt[3]{x^3(-a-b)+abx^2+x^4}+\sqrt[6]{d}\,x\right)}{2d^{2/3}}-\frac{\log\left(-\sqrt[6]{d}\,x\sqrt[3]{x^3(-a-b)+abx^2+x^4}+\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+\sqrt[3]{d}\,x^2\right)}{4d^{2/3}}-\frac{\log\left(\sqrt[6]{d}\,x\sqrt[3]{x^3(-a-b)+abx^2+x^4}+\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+\sqrt[3]{d}\,x^2\right)}{4d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}}{\sqrt{3}\sqrt[3]{d}}+\frac{x^2}{\sqrt{3}}}{x^2}\right)}{2d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-(a*b) + x^2))/((x^2*(-a + x)*(-b + x))^(1/3)*(a^2*b
^2 - 2*a*b*(a + b)*x + (a^2 + 4*a*b + b^2 - d)*x^2 - 2*(a + b)*x^3 + x^4)),
x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(x^2/Sqrt[3] + (2*(a*b*x^2 + (-a - b)*x^3 + x^4)^(2/3)
)/(Sqrt[3]*d^(1/3)))/x^2])/d^(2/3) + Log[-(d^(1/6)*x) + (a*b*x^2 + (-a - b)
*x^3 + x^4)^(1/3)]/(2*d^(2/3)) + Log[d^(1/6)*x + (a*b*x^2 + (-a - b)*x^3 +
x^4)^(1/3)]/(2*d^(2/3)) - Log[d^(1/3)*x^2 - d^(1/6)*x*(a*b*x^2 + (-a - b)*x
^3 + x^4)^(1/3) + (a*b*x^2 + (-a - b)*x^3 + x^4)^(2/3)]/(4*d^(2/3)) - Log[d
^(1/3)*x^2 + d^(1/6)*x*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3) + (a*b*x^2 + (-
a - b)*x^3 + x^4)^(2/3)]/(4*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(a^2*b^2-2*a*b*(a+b)*x+(a^
2+4*a*b+b^2-d)*x^2-2*(a+b)*x^3+x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(ab-x^2\right)x}{\left(a^2b^2-2\,(a+b)abx-2\,(a+b)x^3+x^4+\left(a^2+4\,ab+b^2-d\right)x^2\right)\left((a-x)(b-x)x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(a^2*b^2-2*a*b*(a+b)*x+(a^
2+4*a*b+b^2-d)*x^2-2*(a+b)*x^3+x^4),x, algorithm="giac")
```

```
[Out] integrate(-(a*b - x^2)*x/((a^2*b^2 - 2*(a + b)*a*b*x - 2*(a + b)*x^3 + x^4
+ (a^2 + 4*a*b + b^2 - d)*x^2)*((a - x)*(b - x)*x^2)^(1/3)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x\left(-ab+x^2\right)}{\left(x^2\left(-a+x\right)\left(-b+x\right)\right)^{\frac{1}{3}}\left(a^2b^2-2ab\left(a+b\right)x+\left(a^2+4ab+b^2-d\right)x^2-2\left(a+b\right)x^3+x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(a^2*b^2-2*a*b*(a+b)*x+(a^2+4*a*
b+b^2-d)*x^2-2*(a+b)*x^3+x^4),x)
```

```
[Out] int(x*(-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(a^2*b^2-2*a*b*(a+b)*x+(a^2+4*a*
b+b^2-d)*x^2-2*(a+b)*x^3+x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ab-x^2)x}{\left(a^2b^2-2(a+b)abx-2(a+b)x^3+x^4+\left(a^2+4ab+b^2-d\right)x^2\right)\left((a-x)(b-x)x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*b+x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(a^2*b^2-2*a*b*(a+b)*x+(a^
2+4*a*b+b^2-d)*x^2-2*(a+b)*x^3+x^4),x, algorithm="maxima")
```

```
[Out] -integrate((a*b - x^2)*x/((a^2*b^2 - 2*(a + b)*a*b*x - 2*(a + b)*x^3 + x^4
+ (a^2 + 4*a*b + b^2 - d)*x^2)*((a - x)*(b - x)*x^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x\left(a\,b-x^2\right)}{\left(x^2\,(a-x)\,(b-x)\right)^{1/3}\,\left(x^4-2\,x^3\,(a+b)+a^2\,b^2+x^2\,\left(a^2+4\,a\,b+b^2-d\right)-2\,a\,b\,x\,(a+b)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a*b - x^2))/((x^2*(a - x)*(b - x))^(1/3)*(x^4 - 2*x^3*(a + b) + a^
2*b^2 + x^2*(4*a*b - d + a^2 + b^2) - 2*a*b*x*(a + b))),x)
```

```
[Out] int(-(x*(a*b - x^2))/((x^2*(a - x)*(b - x))^(1/3)*(x^4 - 2*x^3*(a + b) + a^
2*b^2 + x^2*(4*a*b - d + a^2 + b^2) - 2*a*b*x*(a + b))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*b+x**2)/(x**2*(-a+x)*(-b+x))**(1/3)/(a**2*b**2-2*a*b*(a+b)*
x+(a**2+4*a*b+b**2-d)*x**2-2*(a+b)*x**3+x**4),x)
```

```
[Out] Timed out
```

3.2284 $\int \frac{-b^6+a^6x^6}{\sqrt{-b^2x+a^2x^3}\left(b^6+a^6x^6\right)}\,dx$

Optimal. Leaf size=311

$$-\frac{\tan^{-1}\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{a^2x^3-b^2x}}{a^2x^2-2abx-b^2}\right)}{6\sqrt{a}\sqrt{b}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3-b^2x}}{a^2x^2-abx-b^2}\right)}{3\sqrt{a}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\frac{a^{3/2}x^2}{2\sqrt{b}}-\frac{b^{3/2}}{2\sqrt{a}}+\sqrt{a}\sqrt{b}x}{\sqrt{a^2x^3-b^2x}}\right)}{6\sqrt{a}\sqrt{b}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\frac{a^{3/2}x^2}{\sqrt{2}\sqrt{b}}-\ldots}{\sqrt{a^2x\ldots}}\right)}{3\sqrt{a}\sqrt{b}}$$

Rubi [C] time = 8.75, antiderivative size = 1295, normalized size of antiderivative = 4.16, number of steps used = 209, number of rules used = 20, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2056, 1586, 6715, 6725, 1729, 1209, 1201, 224, 221, 1200, 1199, 424, 1219, 1218, 1248, 735, 844, 217, 206, 725}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(-b^6 + a^6*x^6)/(Sqrt[-(b^2*x) + a^2*x^3]*(b^6 + a^6*x^6)),x]
```

```
[Out] -1/3*((-1)^(2/3)*(a - (-1)^(2/3)*(-a^6)^(1/6))*((-1)^(1/3)*a^4 - (-1)^(2/3)
*a^2*(-a^6)^(1/3) - (-a^6)^(2/3))*Sqrt[b]*Sqrt[x]*Sqrt[1 - (a^2*x^2)/b^2]*E
llipticF[ArcSin[(Sqrt[a]*Sqrt[x])/Sqrt[b]], -1])/(a^(11/2)*Sqrt[-(b^2*x) +
a^2*x^3]) - ((-1)^(2/3)*(a + (-1)^(2/3)*(-a^6)^(1/6))*((-1)^(1/3)*a^4 - (-1
)^(2/3)*a^2*(-a^6)^(1/3) - (-a^6)^(2/3))*Sqrt[b]*Sqrt[x]*Sqrt[1 - (a^2*x^2)
/b^2]*EllipticF[ArcSin[(Sqrt[a]*Sqrt[x])/Sqrt[b]], -1])/(3*a^(11/2)*Sqrt[-(
b^2*x) + a^2*x^3]) - ((-1)^(1/3)*(a - (-1)^(1/3)*(-a^6)^(1/6))*((-1)^(2/3)*
a^4 + ((-1)^(1/3)*a^8)/(-a^6)^(2/3) + (-a^6)^(2/3))*Sqrt[b]*Sqrt[x]*Sqrt[1
- (a^2*x^2)/b^2]*EllipticF[ArcSin[(Sqrt[a]*Sqrt[x])/Sqrt[b]], -1])/(3*a^(11
/2)*Sqrt[-(b^2*x) + a^2*x^3]) - ((-1)^(1/3)*(a + (-1)^(1/3)*(-a^6)^(1/6))*(
(-1)^(2/3)*a^4 + ((-1)^(1/3)*a^8)/(-a^6)^(2/3) + (-a^6)^(2/3))*Sqrt[b]*Sqrt
[x]*Sqrt[1 - (a^2*x^2)/b^2]*EllipticF[ArcSin[(Sqrt[a]*Sqrt[x])/Sqrt[b]], -1
])/(3*a^(11/2)*Sqrt[-(b^2*x) + a^2*x^3]) + ((a - (-a^6)^(1/6))*(a^4 + a^2*(
-a^6)^(1/3) + (-a^6)^(2/3))*Sqrt[b]*Sqrt[x]*Sqrt[1 - (a^2*x^2)/b^2]*Ellipti
cF[ArcSin[(Sqrt[a]*Sqrt[x])/Sqrt[b]], -1])/(3*a^(11/2)*Sqrt[-(b^2*x) + a^2*
x^3]) + ((a + (-a^6)^(1/6))*(a^4 + a^2*(-a^6)^(1/3) + (-a^6)^(2/3))*Sqrt[b]
*Sqrt[x]*Sqrt[1 - (a^2*x^2)/b^2]*EllipticF[ArcSin[(Sqrt[a]*Sqrt[x])/Sqrt[b]
], -1])/(3*a^(11/2)*Sqrt[-(b^2*x) + a^2*x^3]) - (2*Sqrt[b]*Sqrt[x]*Sqrt[1 -
 (a^2*x^2)/b^2]*EllipticPi[a^5/(-a^6)^(5/6), ArcSin[(Sqrt[a]*Sqrt[x])/Sqrt[
b]], -1])/(3*Sqrt[a]*Sqrt[-(b^2*x) + a^2*x^3]) - (2*Sqrt[b]*Sqrt[x]*Sqrt[1
- (a^2*x^2)/b^2]*EllipticPi[((-1)^(1/3)*a^5)/(-a^6)^(5/6), ArcSin[(Sqrt[a]*
Sqrt[x])/Sqrt[b]], -1])/(3*Sqrt[a]*Sqrt[-(b^2*x) + a^2*x^3]) - (2*Sqrt[b]*S
qrt[x]*Sqrt[1 - (a^2*x^2)/b^2]*EllipticPi[((-1)^(2/3)*a^5)/(-a^6)^(5/6), Ar
cSin[(Sqrt[a]*Sqrt[x])/Sqrt[b]], -1])/(3*Sqrt[a]*Sqrt[-(b^2*x) + a^2*x^3])
- (2*Sqrt[b]*Sqrt[x]*Sqrt[1 - (a^2*x^2)/b^2]*EllipticPi[(-a^6)^(1/6)/a, Arc
Sin[(Sqrt[a]*Sqrt[x])/Sqrt[b]], -1])/(3*Sqrt[a]*Sqrt[-(b^2*x) + a^2*x^3]) -
 (2*Sqrt[b]*Sqrt[x]*Sqrt[1 - (a^2*x^2)/b^2]*EllipticPi[((-1)^(1/3)*(-a^6)^(
1/6))/a, ArcSin[(Sqrt[a]*Sqrt[x])/Sqrt[b]], -1])/(3*Sqrt[a]*Sqrt[-(b^2*x) +
 a^2*x^3]) - (2*Sqrt[b]*Sqrt[x]*Sqrt[1 - (a^2*x^2)/b^2]*EllipticPi[((-1)^(2
/3)*(-a^6)^(1/6))/a, ArcSin[(Sqrt[a]*Sqrt[x])/Sqrt[b]], -1])/(3*Sqrt[a]*Sqr
t[-(b^2*x) + a^2*x^3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1201

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(c/a), 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q
, Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1729

```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Dist[d, I
nt[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, Int[(x*(a + c*x^4)^p)/(d
^2 - e^2*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

```
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-b^6+a^6x^6}{\sqrt{-b^2x+a^2x^3}\left(b^6+a^6x^6\right)}\,dx &= \frac{\left(\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\int \frac{-b^6+a^6x^6}{\sqrt{x}\sqrt{-b^2+a^2x^2}\left(b^6+a^6x^6\right)}\,dx}{\sqrt{-b^2x+a^2x^3}}\\
&= \frac{\left(\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\int \frac{\sqrt{-b^2+a^2x^2}\left(b^4+a^2b^2x^2+a^4x^4\right)}{\sqrt{x}\left(b^6+a^6x^6\right)}\,dx}{\sqrt{-b^2x+a^2x^3}}\\
&= \frac{\left(2\sqrt{x}\sqrt{-b^2+a^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{-b^2+a^2x^4}\left(b^4+a^2b^2x^4+a^4x^8\right)}{b^6+a^6x^{12}}\,dx, x, \sqrt{x}\right)}{\sqrt{-b^2x+a^2x^3}}\\
&= \text{rest of steps removed due to Latex formating problem}
\end{aligned}$$

Mathematica [C] time = 3.06, size = 294, normalized size = 0.95

$$-\frac{2ix^{3/2}\sqrt{1-\frac{b^2}{a^2x^2}}\left(3F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{b}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(-i;i\sinh^{-1}\left(\frac{\sqrt{-\frac{b}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(i;i\sinh^{-1}\left(\frac{\sqrt{-\frac{b}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(-\frac{i}{2}-\frac{\sqrt{3}}{2};i\sinh^{-1}\left(\frac{\sqrt{-\frac{b}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(\frac{i}{2}-\frac{\sqrt{3}}{2};i\sinh^{-1}\left(\frac{\sqrt{-\frac{b}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(\frac{1}{2}\left(-i+\sqrt{3}\right);i\sinh^{-1}\left(\frac{\sqrt{-\frac{b}{a}}}{\sqrt{x}}\right)\middle|-1\right)-\Pi\left(\frac{1}{2}\left(i+\sqrt{3}\right);i\sinh^{-1}\left(\frac{\sqrt{-\frac{b}{a}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{3\sqrt{-\frac{b}{a}}\sqrt{a^2x^3-b^2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b^6 + a^6*x^6)/(Sqrt[-(b^2*x) + a^2*x^3]*(b^6 + a^6*x^6)),x]
```

```
[Out] (((-2*I)/3)*Sqrt[1 - b^2/(a^2*x^2)]*x^(3/2)*(3*EllipticF[I*ArcSinh[Sqrt[-(b
/a)]/Sqrt[x]], -1] - EllipticPi[-I, I*ArcSinh[Sqrt[-(b/a)]/Sqrt[x]], -1] -
EllipticPi[I, I*ArcSinh[Sqrt[-(b/a)]/Sqrt[x]], -1] - EllipticPi[-1/2*I - Sq
rt[3]/2, I*ArcSinh[Sqrt[-(b/a)]/Sqrt[x]], -1] - EllipticPi[I/2 - Sqrt[3]/2,
 I*ArcSinh[Sqrt[-(b/a)]/Sqrt[x]], -1] - EllipticPi[(-I + Sqrt[3])/2, I*ArcS
inh[Sqrt[-(b/a)]/Sqrt[x]], -1] - EllipticPi[(I + Sqrt[3])/2, I*ArcSinh[Sqrt
[-(b/a)]/Sqrt[x]], -1]))/(Sqrt[-(b/a)]*Sqrt[-(b^2*x) + a^2*x^3])
```

IntegrateAlgebraic [A] time = 0.91, size = 311, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{a^2x^3-b^2x}}{a^2x^2-2abx-b^2}\right)}{6\sqrt{a}\sqrt{b}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3-b^2x}}{a^2x^2-abx-b^2}\right)}{3\sqrt{a}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\frac{a^{3/2}x^2}{2\sqrt{b}}-\frac{b^{3/2}}{2\sqrt{a}}+\sqrt{a}\sqrt{b}x}{\sqrt{a^2x^3-b^2x}}\right)}{6\sqrt{a}\sqrt{b}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\frac{a^{3/2}x^2}{\sqrt{2}\sqrt{b}}-\frac{b^{3/2}}{\sqrt{2}\sqrt{a}}+\frac{\sqrt{a}\sqrt{b}x}{\sqrt{2}}}{\sqrt{a^2x^3-b^2x}}\right)}{3\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b^6 + a^6*x^6)/(Sqrt[-(b^2*x) + a^2*x^3]*(b^6 + a^6*x^
6)),x]
```

```
[Out] -1/6*ArcTan[(2*Sqrt[a]*Sqrt[b]*Sqrt[-(b^2*x) + a^2*x^3])/(-b^2 - 2*a*b*x +
a^2*x^2)]/(Sqrt[a]*Sqrt[b]) - (Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt
[-(b^2*x) + a^2*x^3])/(-b^2 - a*b*x + a^2*x^2)])/(3*Sqrt[a]*Sqrt[b]) - ArcT
anh[(-1/2*b^(3/2)/Sqrt[a] + Sqrt[a]*Sqrt[b]*x + (a^(3/2)*x^2)/(2*Sqrt[b]))/
Sqrt[-(b^2*x) + a^2*x^3]]/(6*Sqrt[a]*Sqrt[b]) - (Sqrt[2]*ArcTanh[(-(b^(3/2)
/(Sqrt[2]*Sqrt[a])) + (Sqrt[a]*Sqrt[b]*x)/Sqrt[2] + (a^(3/2)*x^2)/(Sqrt[2]*
Sqrt[b]))/Sqrt[-(b^2*x) + a^2*x^3]])/(3*Sqrt[a]*Sqrt[b])
```

fricas [B] time = 1.78, size = 2045, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^6*x^6-b^6)/(a^2*x^3-b^2*x)^(1/2)/(a^6*x^6+b^6),x, algorithm="f
ricas")
```

```
[Out] -1/6*sqrt(2)*(1/4)^(1/4)*(1/(a^2*b^2))^(1/4)*arctan(1/2*((4*sqrt(2)*(1/4)^(
3/4)*a^2*b^2*x*(1/(a^2*b^2))^(3/4) - sqrt(2)*(1/4)^(1/4)*(a^2*x^2 - b^2)*(1
/(a^2*b^2))^(1/4))*sqrt(a^2*x^3 - b^2*x) - (2*a^2*x^3 - 2*b^2*x - (4*sqrt(2
)*(1/4)^(3/4)*a^2*b^2*x*(1/(a^2*b^2))^(3/4) + sqrt(2)*(1/4)^(1/4)*(a^2*x^2
- b^2)*(1/(a^2*b^2))^(1/4))*sqrt(a^2*x^3 - b^2*x))*sqrt((a^4*x^4 + 2*a^2*b^
2*x^2 + b^4 + 8*(sqrt(2)*(1/4)^(1/4)*a^2*b^2*x*(1/(a^2*b^2))^(1/4) + sqrt(2
)*(1/4)^(3/4)*(a^4*b^2*x^2 - a^2*b^4)*(1/(a^2*b^2))^(3/4))*sqrt(a^2*x^3 - b
^2*x) + 8*(a^4*b^2*x^3 - a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 + 2*a^2*b^2
*x^2 + b^4)))/(a^2*x^3 - b^2*x)) - 1/6*sqrt(2)*(1/4)^(1/4)*(1/(a^2*b^2))^(1
/4)*arctan(1/2*((4*sqrt(2)*(1/4)^(3/4)*a^2*b^2*x*(1/(a^2*b^2))^(3/4) - sqrt
(2)*(1/4)^(1/4)*(a^2*x^2 - b^2)*(1/(a^2*b^2))^(1/4))*sqrt(a^2*x^3 - b^2*x)
+ (2*a^2*x^3 - 2*b^2*x + (4*sqrt(2)*(1/4)^(3/4)*a^2*b^2*x*(1/(a^2*b^2))^(3/
4) + sqrt(2)*(1/4)^(1/4)*(a^2*x^2 - b^2)*(1/(a^2*b^2))^(1/4))*sqrt(a^2*x^3
- b^2*x))*sqrt((a^4*x^4 + 2*a^2*b^2*x^2 + b^4 - 8*(sqrt(2)*(1/4)^(1/4)*a^2*
b^2*x*(1/(a^2*b^2))^(1/4) + sqrt(2)*(1/4)^(3/4)*(a^4*b^2*x^2 - a^2*b^4)*(1/
(a^2*b^2))^(3/4))*sqrt(a^2*x^3 - b^2*x) + 8*(a^4*b^2*x^3 - a^2*b^4*x)*sqrt(
1/(a^2*b^2)))/(a^4*x^4 + 2*a^2*b^2*x^2 + b^4)))/(a^2*x^3 - b^2*x)) - 1/24*s
qrt(2)*(1/4)^(1/4)*(1/(a^2*b^2))^(1/4)*log((a^4*x^4 + 2*a^2*b^2*x^2 + b^4 +
 8*(sqrt(2)*(1/4)^(1/4)*a^2*b^2*x*(1/(a^2*b^2))^(1/4) + sqrt(2)*(1/4)^(3/4)
*(a^4*b^2*x^2 - a^2*b^4)*(1/(a^2*b^2))^(3/4))*sqrt(a^2*x^3 - b^2*x) + 8*(a^
4*b^2*x^3 - a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 + 2*a^2*b^2*x^2 + b^4))
+ 1/24*sqrt(2)*(1/4)^(1/4)*(1/(a^2*b^2))^(1/4)*log((a^4*x^4 + 2*a^2*b^2*x^2
 + b^4 - 8*(sqrt(2)*(1/4)^(1/4)*a^2*b^2*x*(1/(a^2*b^2))^(1/4) + sqrt(2)*(1/
4)^(3/4)*(a^4*b^2*x^2 - a^2*b^4)*(1/(a^2*b^2))^(3/4))*sqrt(a^2*x^3 - b^2*x)
 + 8*(a^4*b^2*x^3 - a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 + 2*a^2*b^2*x^2
+ b^4)) - 1/3*sqrt(2)*(1/(a^2*b^2))^(1/4)*arctan(1/2*((sqrt(2)*a^2*b^2*x*(1
/(a^2*b^2))^(3/4) - sqrt(2)*(a^2*x^2 - b^2)*(1/(a^2*b^2))^(1/4))*sqrt(a^2*x
^3 - b^2*x) - (2*a^2*x^3 - 2*b^2*x - (sqrt(2)*a^2*b^2*x*(1/(a^2*b^2))^(3/4)
 + sqrt(2)*(a^2*x^2 - b^2)*(1/(a^2*b^2))^(1/4))*sqrt(a^2*x^3 - b^2*x))*sqrt
((a^4*x^4 - a^2*b^2*x^2 + b^4 + 2*(sqrt(2)*a^2*b^2*x*(1/(a^2*b^2))^(1/4) +
sqrt(2)*(a^4*b^2*x^2 - a^2*b^4)*(1/(a^2*b^2))^(3/4))*sqrt(a^2*x^3 - b^2*x)
+ 4*(a^4*b^2*x^3 - a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 - a^2*b^2*x^2 + b
^4)))/(a^2*x^3 - b^2*x)) - 1/3*sqrt(2)*(1/(a^2*b^2))^(1/4)*arctan(1/2*((sqr
t(2)*a^2*b^2*x*(1/(a^2*b^2))^(3/4) - sqrt(2)*(a^2*x^2 - b^2)*(1/(a^2*b^2))^
(1/4))*sqrt(a^2*x^3 - b^2*x) + (2*a^2*x^3 - 2*b^2*x + (sqrt(2)*a^2*b^2*x*(1
/(a^2*b^2))^(3/4) + sqrt(2)*(a^2*x^2 - b^2)*(1/(a^2*b^2))^(1/4))*sqrt(a^2*x
^3 - b^2*x))*sqrt((a^4*x^4 - a^2*b^2*x^2 + b^4 - 2*(sqrt(2)*a^2*b^2*x*(1/(a
^2*b^2))^(1/4) + sqrt(2)*(a^4*b^2*x^2 - a^2*b^4)*(1/(a^2*b^2))^(3/4))*sqrt(
a^2*x^3 - b^2*x) + 4*(a^4*b^2*x^3 - a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4
- a^2*b^2*x^2 + b^4)))/(a^2*x^3 - b^2*x)) - 1/12*sqrt(2)*(1/(a^2*b^2))^(1/4
)*log((a^4*x^4 - a^2*b^2*x^2 + b^4 + 2*(sqrt(2)*a^2*b^2*x*(1/(a^2*b^2))^(1/
4) + sqrt(2)*(a^4*b^2*x^2 - a^2*b^4)*(1/(a^2*b^2))^(3/4))*sqrt(a^2*x^3 - b^
2*x) + 4*(a^4*b^2*x^3 - a^2*b^4*x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 - a^2*b^2*x^
2 + b^4)) + 1/12*sqrt(2)*(1/(a^2*b^2))^(1/4)*log((a^4*x^4 - a^2*b^2*x^2 + b
^4 - 2*(sqrt(2)*a^2*b^2*x*(1/(a^2*b^2))^(1/4) + sqrt(2)*(a^4*b^2*x^2 - a^2*
b^4)*(1/(a^2*b^2))^(3/4))*sqrt(a^2*x^3 - b^2*x) + 4*(a^4*b^2*x^3 - a^2*b^4*
x)*sqrt(1/(a^2*b^2)))/(a^4*x^4 - a^2*b^2*x^2 + b^4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^6x^6 - b^6}{\left(a^6x^6 + b^6\right)\sqrt{a^2x^3 - b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^6*x^6-b^6)/(a^2*x^3-b^2*x)^(1/2)/(a^6*x^6+b^6),x, algorithm="g
iac")
```

```
[Out] integrate((a^6*x^6 - b^6)/((a^6*x^6 + b^6)*sqrt(a^2*x^3 - b^2*x)), x)
```

maple [C] time = 0.08, size = 475, normalized size = 1.53

$$\frac{b\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{2\left(x-\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{ax}{b}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{a^2x^3-b^2x}}-\frac{\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4a^4-\_Z^2a^2b^2+b^4\right)}\frac{\left(-\_\alpha^2a^2+2b^2\right)\_\alpha\left(\_\alpha a-b\right)\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{\left(x-\frac{b}{a}\right)a}{b}}\,\sqrt{-\frac{ax}{b}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}},-\frac{a^2\_\alpha^2\left(\_\alpha a-b\right)}{b^3},\frac{\sqrt{2}}{2}\right)}{\left(2\_\alpha^2a^2-b^2\right)\sqrt{x\left(a^2x^2-b^2\right)}}\right)}{3b}-\frac{2b^2\left(-\frac{i\sqrt{1+\frac{ax}{b}}\,\sqrt{-\frac{2ax}{b}+2}\,\sqrt{-\frac{ax}{b}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}},-\frac{b}{a\left(-\frac{ib}{a}-\frac{b}{a}\right)},\frac{\sqrt{2}}{2}\right)}{2a^2\sqrt{a^2x^3-b^2x}\,\left(-\frac{ib}{a}-\frac{b}{a}\right)}+\frac{i\sqrt{1+\frac{ax}{b}}\,\sqrt{-\frac{2ax}{b}+2}\,\sqrt{-\frac{ax}{b}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{\left(x+\frac{b}{a}\right)a}{b}},-\frac{b}{a\left(-\frac{b}{a}+\frac{ib}{a}\right)},\frac{\sqrt{2}}{2}\right)}{2a^2\sqrt{a^2x^3-b^2x}\,\left(-\frac{b}{a}+\frac{ib}{a}\right)}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^6*x^6-b^6)/(a^2*x^3-b^2*x)^(1/2)/(a^6*x^6+b^6),x)
```

```
[Out] b/a*((x+b/a)/b*a)^(1/2)*(-2*(x-b/a)/b*a)^(1/2)*(-a*x/b)^(1/2)/(a^2*x^3-b^2*
x)^(1/2)*EllipticF(((x+b/a)/b*a)^(1/2),1/2*2^(1/2))-1/3/b*2^(1/2)*sum((-_al
pha^2*a^2+2*b^2)*_alpha/(2*_alpha^2*a^2-b^2)*(_alpha*a-b)*((x+b/a)/b*a)^(1/
2)*(-(x-b/a)/b*a)^(1/2)*(-a*x/b)^(1/2)/(x*(a^2*x^2-b^2))^(1/2)*EllipticPi((
(x+b/a)/b*a)^(1/2),-a^2*_alpha^2*(_alpha*a-b)/b^3,1/2*2^(1/2)),_alpha=RootO
f(_Z^4*a^4-_Z^2*a^2*b^2+b^4))-2/3*b^2*(-1/2*I/a^2*(1+a*x/b)^(1/2)*(-2*a*x/b
+2)^(1/2)*(-a*x/b)^(1/2)/(a^2*x^3-b^2*x)^(1/2)/(-I*b/a-b/a)*EllipticPi(((x+
b/a)/b*a)^(1/2),-b/a/(-I*b/a-b/a),1/2*2^(1/2))+1/2*I/a^2*(1+a*x/b)^(1/2)*(-
2*a*x/b+2)^(1/2)*(-a*x/b)^(1/2)/(a^2*x^3-b^2*x)^(1/2)/(-b/a+I*b/a)*Elliptic
Pi(((x+b/a)/b*a)^(1/2),-b/a/(-b/a+I*b/a),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^6x^6-b^6}{\left(a^6x^6+b^6\right)\sqrt{a^2x^3-b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^6*x^6-b^6)/(a^2*x^3-b^2*x)^(1/2)/(a^6*x^6+b^6),x, algorithm="m
axima")
```

```
[Out] integrate((a^6*x^6 - b^6)/((a^6*x^6 + b^6)*sqrt(a^2*x^3 - b^2*x)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^6 - a^6*x^6)/((b^6 + a^6*x^6)*(a^2*x^3 - b^2*x)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax-b)(ax+b)\left(a^2x^2-abx+b^2\right)\left(a^2x^2+abx+b^2\right)}{\sqrt{x(ax-b)(ax+b)}\left(a^2x^2+b^2\right)\left(a^4x^4-a^2b^2x^2+b^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**6*x**6-b**6)/(a**2*x**3-b**2*x)**(1/2)/(a**6*x**6+b**6),x)
```

```
[Out] Integral((a*x - b)*(a*x + b)*(a**2*x**2 - a*b*x + b**2)*(a**2*x**2 + a*b*x
+ b**2)/(sqrt(x*(a*x - b)*(a*x + b))*(a**2*x**2 + b**2)*(a**4*x**4 - a**2*b
**2*x**2 + b**4)), x)
```

3.2285 $\int \frac{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{-1+x^4}\,dx$

Optimal. Leaf size=311

$$\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{2}-1\right)}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right.$$

Rubi [F] time = 1.79, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{-1+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]])/(-1 + x^4),x]
```

```
[Out] (-1/4*I)*Defer[Int][(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]])/(I - x), x] -
 Defer[Int][(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 - x), x]/4 - (I/4)
*Defer[Int][(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]])/(I + x), x] - Defer[I
nt][(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x), x]/4
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{-1+x^4}\,dx &= \int \left(-\frac{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{2\left(1-x^2\right)}-\frac{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{2\left(1+x^2\right)}\right)dx\\
&= -\left(\frac{1}{2}\int \frac{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{1-x^2}\,dx\right)-\frac{1}{2}\int \frac{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2}\,dx\\
&= -\left(\frac{1}{2}\int \left(\frac{i\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{2(i-x)}+\frac{i\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{2(i+x)}\right)dx\right)-\frac{1}{2}\int\\
&= -\left(\frac{1}{4}i\int \frac{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{i-x}\,dx\right)-\frac{1}{4}i\int \frac{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{i+x}\,dx-
\end{aligned}$$

Mathematica [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+x^4}\sqrt{x^2+\sqrt{1+x^4}}}{-1+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]])/(-1 + x^4),x]
```

```
[Out] Integrate[(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]])/(-1 + x^4), x]
```

IntegrateAlgebraic [A] time = 2.01, size = 440, normalized size = 1.41

$$\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[1 + x^4]*Sqrt[x^2 + Sqrt[1 + x^4]])/(-1 + x^4),x]
```

```
[Out] Sqrt[(-1 + Sqrt[2])/2]*ArcTan[(-Sqrt[-1/2 + 1/Sqrt[2]] + Sqrt[-1/2 + 1/Sqrt
[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 + x^4
]])] - Sqrt[(-1 + Sqrt[2])/2]*ArcTan[(-Sqrt[1/2 + 1/Sqrt[2]] + Sqrt[1/2 + 1
/Sqrt[2]]*x^2 + Sqrt[1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 +
 x^4]])] + Sqrt[2]*ArcTanh[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt
[2])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])] - Sqrt[(1 + Sqrt[2])/2]*ArcTanh[(-Sqrt[
-1/2 + 1/Sqrt[2]] + Sqrt[-1/2 + 1/Sqrt[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*Sqr
t[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])] - Sqrt[(1 + Sqrt[2])/2]*ArcTanh[
(-Sqrt[1/2 + 1/Sqrt[2]] + Sqrt[1/2 + 1/Sqrt[2]]*x^2 + Sqrt[1/2 + 1/Sqrt[2]]
*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])]
```

fricas [A] time = 5.15, size = 433, normalized size = 1.39

$$-\frac{1}{2}\sqrt{2}\sqrt{\sqrt{2}-1}\arctan\left(\frac{x^8-2x^4-2\left(2x^7-2x^3+\sqrt{2}\left(3x^7+x^3\right)-\left(4\sqrt{2}x^5+5x^5-x\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}-1}-2\sqrt{2}\left(x^8+3x^4\right)-2\left(3x^6+x^2+\sqrt{2}\left(x^6-x^2\right)\right)\sqrt{x^4+1}+1}{7x^8+10x^4-1}\right)-\frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}+1}\log\left(\frac{2\left(\sqrt{2}x^3+2x^3+\sqrt{x^4+1}\left(\sqrt{2}x+x\right)\right)\sqrt{x^2+\sqrt{x^4+1}}+\left(2\sqrt{2}x^4+3x^4+2\sqrt{x^4+1}\left(\sqrt{2}x^2+x^2\right)+1\right)\sqrt{\sqrt{2}+1}}{x^4-1}\right)+\frac{1}{8}\sqrt{2}\sqrt{\sqrt{2}+1}\log\left(\frac{2\left(\sqrt{2}x^3+2x^3+\sqrt{x^4+1}\left(\sqrt{2}x+x\right)\right)\sqrt{x^2+\sqrt{x^4+1}}-\left(2\sqrt{2}x^4+3x^4+2\sqrt{x^4+1}\left(\sqrt{2}x^2+x^2\right)+1\right)\sqrt{\sqrt{2}+1}}{x^4-1}\right)+\frac{1}{4}\sqrt{2}\log\left(4x^4+4\sqrt{x^4+1}x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4-1),x, algorithm="fri
cas")
```

```
[Out] -1/2*sqrt(2)*sqrt(sqrt(2) - 1)*arctan((x^8 - 2*x^4 - 2*(2*x^7 - 2*x^3 + sqr
t(2)*(3*x^7 + x^3) - (4*sqrt(2)*x^5 + 5*x^5 - x)*sqrt(x^4 + 1))*sqrt(x^2 +
sqrt(x^4 + 1))*sqrt(sqrt(2) - 1) - 2*sqrt(2)*(x^8 + 3*x^4) - 2*(3*x^6 + x^2
 + sqrt(2)*(x^6 - x^2))*sqrt(x^4 + 1) + 1)/(7*x^8 + 10*x^4 - 1)) - 1/8*sqrt
(2)*sqrt(sqrt(2) + 1)*log((2*(sqrt(2)*x^3 + 2*x^3 + sqrt(x^4 + 1)*(sqrt(2)*
x + x))*sqrt(x^2 + sqrt(x^4 + 1)) + (2*sqrt(2)*x^4 + 3*x^4 + 2*sqrt(x^4 + 1
)*(sqrt(2)*x^2 + x^2) + 1)*sqrt(sqrt(2) + 1))/(x^4 - 1)) + 1/8*sqrt(2)*sqrt
(sqrt(2) + 1)*log((2*(sqrt(2)*x^3 + 2*x^3 + sqrt(x^4 + 1)*(sqrt(2)*x + x))*
sqrt(x^2 + sqrt(x^4 + 1)) - (2*sqrt(2)*x^4 + 3*x^4 + 2*sqrt(x^4 + 1)*(sqrt(
2)*x^2 + x^2) + 1)*sqrt(sqrt(2) + 1))/(x^4 - 1)) + 1/4*sqrt(2)*log(4*x^4 +
4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*sqrt(x^2 +
sqrt(x^4 + 1)) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}\sqrt{x^2+\sqrt{x^4+1}}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4-1),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))/(x^4 - 1), x)
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4-1),x)
```

```
[Out] int((x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}\,\sqrt{x^2+\sqrt{x^4+1}}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)*(x^2+(x^4+1)^(1/2))^(1/2)/(x^4-1),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x^4 + 1)*sqrt(x^2 + sqrt(x^4 + 1))/(x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x^4+1}\,\sqrt{\sqrt{x^4+1}+x^2}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 - 1),x)
```

```
[Out] int(((x^4 + 1)^(1/2)*((x^4 + 1)^(1/2) + x^2)^(1/2))/(x^4 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}\,\sqrt{x^4+1}}{(x-1)(x+1)\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(1/2)*(x**2+(x**4+1)**(1/2))**(1/2)/(x**4-1),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))*sqrt(x**4 + 1)/((x - 1)*(x + 1)*(x**2
+ 1)), x)
```

3.2286 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=312

$$\frac{2x^4+x^2+\left(-x^2-1\right)\left(\sqrt{x^4+1}+x^2\right)x+\sqrt{x^4+1}\left(2x^2-\left(\sqrt{x^4+1}+x^2\right)x+1\right)+1}{2\left(x^2-1\right)\left(\sqrt{x^4+1}+x^2\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{\sqrt{2}-1}}\right)}{2\sqrt{\sqrt{2}-1}}-\frac{1}{2}\sqrt{1\cdots}$$

Rubi [C] time = 0.19, antiderivative size = 125, normalized size of antiderivative = 0.40, number of steps used = 7, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2133, 731, 725, 206}

$$-\frac{\sqrt{1-ix^2}}{2(x+1)}-\frac{\sqrt{1+ix^2}}{2(x+1)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

```
[Out] -1/2*Sqrt[1 - I*x^2]/(1 + x) - Sqrt[1 + I*x^2]/(2*(1 + x)) - ((1 - I)^(3/2)
*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/4 - ((1 + I)^(3/2)*ArcTa
nh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx &= \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+x)^2\sqrt{1-ix^2}}\,dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1+x)^2\sqrt{1+ix^2}}\,dx \\
&= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}-\frac{1}{2}i\int \frac{1}{(1+x)\sqrt{1-ix^2}}\,dx+\frac{1}{2}i\int \frac{1}{(1+x)\sqrt{1+ix^2}}\,dx \\
&= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}+\frac{1}{2}i\,\text{Subst}\left(\int \frac{1}{(1-i)-x^2}\,dx,x,\frac{1+ix}{\sqrt{1-ix^2}}\right)-\frac{1}{2}i\,\text{Subst}\Bigg(\int \\
&= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\tanh^{-1}\Bigg(
\end{aligned}
$$

Mathematica [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

```
[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]), x]
```

IntegrateAlgebraic [A] time = 3.22, size = 312, normalized size = 1.00

$$\frac{2x^4+x^2+(-x^2-1)\left(\sqrt{x^4+1}+x^2\right)x+\sqrt{x^4+1}\left(2x^2-\left(\sqrt{x^4+1}+x^2\right)x+1\right)+1}{2\left(x^2-1\right)\left(\sqrt{x^4+1}+x^2\right)^{3/2}}+\frac{\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^4+1}+x^2}\right)}{2\sqrt{\sqrt{2}-1}}-\frac{1}{2}\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2\sqrt{2}-2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\frac{\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^4+1}+x^2}\right)}{2\sqrt{1+\sqrt{2}}}+\frac{1}{2}\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\frac{\sqrt{2+2\sqrt{2}}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

```
[Out] (1 + x^2 + 2*x^4 + x*(-1 - x^2)*(x^2 + Sqrt[1 + x^4]) + Sqrt[1 + x^4]*(1 +
2*x^2 - x*(x^2 + Sqrt[1 + x^4])))/(2*(-1 + x^2)*(x^2 + Sqrt[1 + x^4])^(3/2)
) + ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]]/(2*Sqrt[-1 + Sqrt[2
]]) - (Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[-2 + 2*Sqrt[2]]*x*Sqrt[x^2 + Sqrt[1 +
 x^4]])/(1 + x^2 + Sqrt[1 + x^4])])/2 - ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x^2
 + Sqrt[1 + x^4]]]/(2*Sqrt[1 + Sqrt[2]]) + (Sqrt[-1 + Sqrt[2]]*ArcTanh[(Sqr
t[2 + 2*Sqrt[2]]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])])/2
```

fricas [A] time = 2.07, size = 394, normalized size = 1.26

$$\frac{4(x+1)\sqrt{\sqrt{2}+1}\arctan\left(\frac{2\left(x^3+x^2-\sqrt{2}\left(x^3+1\right)+\sqrt{x^4+1}\left(\sqrt{2}x-x-1\right)-x+1\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{\sqrt{2}+1}+\left(2x^2-\sqrt{2}\left(x^2+1\right)+2\sqrt{x^4+1}\left(\sqrt{2}-1\right)+2\right)\sqrt{2\sqrt{2}+2}\sqrt{\sqrt{2}+1}}{2\left(x^2-2x+1\right)}\right)+(x+1)\sqrt{\sqrt{2}-1}\log\left(-\frac{\left(2x^3-\sqrt{2}\left(x^3-x^2-x-1\right)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2x\right)-2\right)\sqrt{x^2+\sqrt{x^4+1}}+\left(\sqrt{2}\left(x^2+1\right)+2\sqrt{x^4+1}\right)\sqrt{\sqrt{2}-1}}{x^2+2x+1}\right)-(x+1)\sqrt{\sqrt{2}-1}\log\left(-\frac{\left(2x^3-\sqrt{2}\left(x^3-x^2-x-1\right)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2x\right)-2\right)\sqrt{x^2+\sqrt{x^4+1}}-\left(\sqrt{2}\left(x^2+1\right)+2\sqrt{x^4+1}\right)\sqrt{\sqrt{2}-1}}{x^2+2x+1}\right)+4\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)}{8(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/8*(4*(x + 1)*sqrt(sqrt(2) + 1)*arctan(1/2*(2*(x^3 + x^2 - sqrt(2)*(x^3 +
1) + sqrt(x^4 + 1)*(sqrt(2)*x - x - 1) - x + 1)*sqrt(x^2 + sqrt(x^4 + 1))*s
qrt(sqrt(2) + 1) + (2*x^2 - sqrt(2)*(x^2 + 1) + 2*sqrt(x^4 + 1)*(sqrt(2) -
1) + 2)*sqrt(2*sqrt(2) + 2)*sqrt(sqrt(2) + 1))/(x^2 - 2*x + 1)) + (x + 1)*s
qrt(sqrt(2) - 1)*log(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)
*(sqrt(2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) + (sqrt(2)*(x^2 + 1
) + 2*sqrt(x^4 + 1))*sqrt(sqrt(2) - 1))/(x^2 + 2*x + 1)) - (x + 1)*sqrt(sqr
t(2) - 1)*log(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(
2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - (sqrt(2)*(x^2 + 1) + 2*s
```

```
qrt(x^4 + 1))*sqrt(sqrt(2) - 1))/(x^2 + 2*x + 1)) + 4*sqrt(x^2 + sqrt(x^4 +
 1))*(x^2 - sqrt(x^4 + 1) - 1))/(x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}\,(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(1+x)^2\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}\,(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}\,(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)^2),x)
```

```
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)^2\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)**2/(x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)**2*sqrt(x**4 + 1)), x)
```

3.2287 $\int \frac{(-b+x)\left(ab-2bx+x^2\right)}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-b^2d+2bdx-\left(-a^2+d\right)x^2-2ax^3+x^4\right)}\,dx$

Optimal. Leaf size=313

$$-\frac{\log\left(a^2x^2+d^{2/3}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}+\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}\ \left(\right.}{4d^{2/3}}$$

Rubi [F] time = 13.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-b+x)\left(ab-2bx+x^2\right)}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-b^2d+2bdx-\left(-a^2+d\right)x^2-2ax^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-b + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2)^(1/3)*(-(b^2*d)
 + 2*b*d*x - (-a^2 + d)*x^2 - 2*a*x^3 + x^4)),x]
```

```
[Out] (6*b*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x^4*(-b
 + x^3)^(1/3))/((-a + x^3)^(1/3)*(b^2*d - 2*b*d*x^3 - a^2*(1 - d/a^2)*x^6 +
 2*a*x^9 - x^12)), x], x, x^(1/3)]])/(-((a - x)*(b - x)^2*x))^(1/3) + (3*a*b
*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x*(-b + x^3
)^(1/3))/((-a + x^3)^(1/3)*(-(b^2*d) + 2*b*d*x^3 + a^2*(1 - d/a^2)*x^6 - 2*
a*x^9 + x^12)), x], x, x^(1/3)]])/(-((a - x)*(b - x)^2*x))^(1/3) + (3*x^(1/3
)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x^7*(-b + x^3)^(1/
3))/((-a + x^3)^(1/3)*(-(b^2*d) + 2*b*d*x^3 + a^2*(1 - d/a^2)*x^6 - 2*a*x^9
 + x^12)), x], x, x^(1/3)]])/(-((a - x)*(b - x)^2*x))^(1/3)
```

Rubi steps

$$\int \frac{(-b+x)\left(ab-2bx+x^2\right)}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-b^2d+2bdx-\left(-a^2+d\right)x^2-2ax^3+x^4\right)}\,dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{}{\sqrt[3]{x}\sqrt[3]{-a+x}}}{\sqrt[3]{x(-a+x)(-}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\text{Subst}\left(\int\right.}{\sqrt[3]{x(-}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\text{Subst}\left(\int\right.}{\sqrt[3]{x(-}}$$

Mathematica [F] time = 3.57, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)\left(ab-2bx+x^2\right)}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-b^2d+2bdx-\left(-a^2+d\right)x^2-2ax^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-b + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2)^(1/3)*(-(
b^2*d) + 2*b*d*x - (-a^2 + d)*x^2 - 2*a*x^3 + x^4)),x]
```

```
[Out] Integrate[((-b + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2)^(1/3)*(-(
b^2*d) + 2*b*d*x - (-a^2 + d)*x^2 - 2*a*x^3 + x^4)), x]
```

IntegrateAlgebraic [A] time = 0.74, size = 313, normalized size = 1.00

$$-\frac{\log\left(a^2x^2+d^{2/3}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}+\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}\left(\sqrt[3]{d}\,x^2-a\sqrt[3]{d}\,x\right)-2ax^3+x^4\right)}{4d^{2/3}}+\frac{\log\left(\sqrt[3]{d}\,\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}+ax-x^2\right)}{2d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}}{\sqrt[3]{d}\,\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}-2ax+2x^2}\right)}{2d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2)^
(1/3)*(-(b^2*d) + 2*b*d*x - (-a^2 + d)*x^2 - 2*a*x^3 + x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*
b)*x^3 + x^4)^(1/3))/(-2*a*x + 2*x^2 + d^(1/3)*(-(a*b^2*x) + (2*a*b + b^2)*
x^2 + (-a - 2*b)*x^3 + x^4)^(1/3))]])/(2*d^(2/3)) + Log[a*x - x^2 + d^(1/3)*
(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3)]/(2*d^(2/3))
- Log[a^2*x^2 - 2*a*x^3 + x^4 + (-(a*d^(1/3)*x) + d^(1/3)*x^2)*(-(a*b^2*x)
+ (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3) + d^(2/3)*(-(a*b^2*x) + (
2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(2/3)]/(4*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-b^2*d+2*b*d*x-
(-a^2+d)*x^2-2*a*x^3+x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ab-2\,bx+x^2\right)(b-x)}{\left(-(a-x)(b-x)^2x\right)^{\frac{1}{3}}\left(2\,ax^3-x^4+b^2d-2\,bdx-\left(a^2-d\right)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-b^2*d+2*b*d*x-
(-a^2+d)*x^2-2*a*x^3+x^4),x, algorithm="giac")
```

```
[Out] integrate((a*b - 2*b*x + x^2)*(b - x)/((-(a - x)*(b - x)^2*x)^(1/3)*(2*a*x^
3 - x^4 + b^2*d - 2*b*d*x - (a^2 - d)*x^2)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)\left(ab-2bx+x^2\right)}{\left(x\,(-a+x)\,(-b+x)^2\right)^{\frac{1}{3}}\left(-b^2d+2bdx-\left(-a^2+d\right)x^2-2a\,x^3+x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-b^2*d+2*b*d*x-(-a^2+
d)*x^2-2*a*x^3+x^4),x)
```

```
[Out] int((-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-b^2*d+2*b*d*x-(-a^2+
d)*x^2-2*a*x^3+x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ab - 2\,bx + x^2)(b - x)}{\left(-(a - x)(b - x)^2x\right)^{\frac{1}{3}}\left(2\,ax^3 - x^4 + b^2d - 2\,bdx - (a^2 - d)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-b^2*d+2*b*d*x-
(-a^2+d)*x^2-2*a*x^3+x^4),x, algorithm="maxima")
```

```
[Out] integrate((a*b - 2*b*x + x^2)*(b - x)/((-(a - x)*(b - x)^2*x)^(1/3)*(2*a*x^
3 - x^4 + b^2*d - 2*b*d*x - (a^2 - d)*x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b - x)\,(x^2 - 2\,b\,x + a\,b)}{\left(-x\,(a - x)\,(b - x)^2\right)^{1/3}\,\left(d\,b^2 - 2\,d\,b\,x - x^4 + 2\,a\,x^3 + (d - a^2)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b - x)*(a*b - 2*b*x + x^2))/((-x*(a - x)*(b - x)^2)^(1/3)*(x^2*(d - a
^2) + b^2*d + 2*a*x^3 - x^4 - 2*b*d*x)),x)
```

```
[Out] int(((b - x)*(a*b - 2*b*x + x^2))/((-x*(a - x)*(b - x)^2)^(1/3)*(x^2*(d - a
^2) + b^2*d + 2*a*x^3 - x^4 - 2*b*d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(a*b-2*b*x+x**2)/(x*(-a+x)*(-b+x)**2)**(1/3)/(-b**2*d+2*b*
d*x-(-a**2+d)*x**2-2*a*x**3+x**4),x)
```

```
[Out] Timed out
```

3.2288 $\int \frac{b^8+a^8x^8}{\sqrt{-b^4+a^4x^4}\left(-b^8+a^8x^8\right)}\,dx$

Optimal. Leaf size=313

$$-\frac{x}{2\sqrt{a^4x^4-b^4}}+\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{a^4x^4-b^4}}{\sqrt{3-2\sqrt{2}}\,ab}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)ax^2}{\sqrt{3-2\sqrt{2}}\,b}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)b}{\sqrt{3-2\sqrt{2}}\,a}}{x}\right)}{ab}-\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{a^4x^4-b^4}}{\sqrt{3+2\sqrt{2}}\,ab}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)ax^2}{\sqrt{3+2\sqrt{2}}\,b}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)b}{\sqrt{3+2\sqrt{2}}}}{x}\right)}{ab}$$

Rubi [A] time = 0.71, antiderivative size = 135, normalized size of antiderivative = 0.43, number of steps used = 20, number of rules used = 11, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6725, 224, 221, 1404, 414, 523, 409, 1211, 1699, 203, 206}

$$-\frac{x}{2\sqrt{a^4x^4-b^4}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}\,bx}{\sqrt{a^4x^4-b^4}}\right)}{4\sqrt{2}\sqrt[4]{-a^4}\,b}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}\,bx}{\sqrt{a^4x^4-b^4}}\right)}{4\sqrt{2}\sqrt[4]{-a^4}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(b^8 + a^8*x^8)/(Sqrt[-b^4 + a^4*x^4]*(-b^8 + a^8*x^8)),x]
```

```
[Out] -1/2*x/Sqrt[-b^4 + a^4*x^4] - ArcTan[(Sqrt[2]*(-a^4)^(1/4)*b*x)/Sqrt[-b^4 +
 a^4*x^4]]/(4*Sqrt[2]*(-a^4)^(1/4)*b) - ArcTanh[(Sqrt[2]*(-a^4)^(1/4)*b*x)/
Sqrt[-b^4 + a^4*x^4]]/(4*Sqrt[2]*(-a^4)^(1/4)*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1211

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1404

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbo
l] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{b^8+a^8x^8}{\sqrt{-b^4+a^4x^4}\left(-b^8+a^8x^8\right)}\,dx = \int \left(\frac{1}{\sqrt{-b^4+a^4x^4}}+\frac{2b^8}{\sqrt{-b^4+a^4x^4}\left(-b^8+a^8x^8\right)}\right)dx$$
$$= \left(2b^8\right)\int \frac{1}{\sqrt{-b^4+a^4x^4}\left(-b^8+a^8x^8\right)}\,dx+\int \frac{1}{\sqrt{-b^4+a^4x^4}}\,dx$$
$$= \left(2b^8\right)\int \frac{1}{\left(-b^4+a^4x^4\right)^{3/2}\left(b^4+a^4x^4\right)}\,dx+\frac{\sqrt{1-\frac{a^4x^4}{b^4}}\int \frac{1}{\sqrt{1-\frac{a^4x^4}{b^4}}}\,dx}{\sqrt{-b^4+a^4x^4}}$$
$$= -\frac{x}{2\sqrt{-b^4+a^4x^4}}+\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{a\sqrt{-b^4+a^4x^4}}+\frac{\int \frac{-3a^4b^4-a^8x^4}{\sqrt{-b^4+a^4x^4}\left(b^4+a^4x^4\right)}\,dx}{2a^4}$$
$$= -\frac{x}{2\sqrt{-b^4+a^4x^4}}+\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{a\sqrt{-b^4+a^4x^4}}-\frac{1}{2}\int \frac{1}{\sqrt{-b^4+a^4x^4}}\,dx-$$
$$= -\frac{x}{2\sqrt{-b^4+a^4x^4}}+\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{a\sqrt{-b^4+a^4x^4}}-\frac{1}{2}\int \frac{1}{\left(1-\frac{\sqrt{-a^4}\,x^2}{b^2}\right)\sqrt{-b^4}}$$
$$= -\frac{x}{2\sqrt{-b^4+a^4x^4}}+\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{-b^4+a^4x^4}}-2\left(\frac{1}{4}\int \frac{1}{\sqrt{-b^4+a^4x^4}}\,dx\right.$$
$$= -\frac{x}{2\sqrt{-b^4+a^4x^4}}+\frac{b\sqrt{1-\frac{a^4x^4}{b^4}}\,F\left(\sin^{-1}\left(\frac{ax}{b}\right)\middle|-1\right)}{2a\sqrt{-b^4+a^4x^4}}-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-2\sqrt{-a^4}\,b}\right.$$
$$= -\frac{x}{2\sqrt{-b^4+a^4x^4}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}\,bx}{\sqrt{-b^4+a^4x^4}}\right)}{4\sqrt{2}\sqrt[4]{-a^4}\,b}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{-a^4}\,bx}{\sqrt{-b^4+a^4x^4}}\right)}{4\sqrt{2}\sqrt[4]{-a^4}\,b}$$

Mathematica [C] time = 0.63, size = 201, normalized size = 0.64

$$\frac{x\left(-\frac{5\left(b^8-a^4b^4x^4\right)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)}{\left(a^4x^4+b^4\right)\left(5b^4F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)-2a^4x^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)\right)\right)}-1\right)}{2\sqrt{a^4x^4-b^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b^8 + a^8*x^8)/(Sqrt[-b^4 + a^4*x^4]*(-b^8 + a^8*x^8)),x]
```

```
[Out] (x*(-1 - (5*(b^8 - a^4*b^4*x^4)*AppellF1[1/4, -1/2, 1, 5/4, (a^4*x^4)/b^4,
-((a^4*x^4)/b^4)])/((b^4 + a^4*x^4)*(5*b^4*AppellF1[1/4, -1/2, 1, 5/4, (a^4
*x^4)/b^4, -((a^4*x^4)/b^4)] - 2*a^4*x^4*(2*AppellF1[5/4, -1/2, 2, 9/4, (a^
4*x^4)/b^4, -((a^4*x^4)/b^4)] + AppellF1[5/4, 1/2, 1, 9/4, (a^4*x^4)/b^4, -
((a^4*x^4)/b^4)])))))/(2*Sqrt[-b^4 + a^4*x^4])
```

IntegrateAlgebraic [A] time = 1.18, size = 247, normalized size = 0.79

$$-\frac{x}{2\sqrt{a^4x^4-b^4}}-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tan^{-1}\left(\frac{(1+i)abx}{\sqrt{a^4x^4-b^4}+a^2x^2+ib^2}\right)}{ab}+\frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tanh^{-1}\left(\frac{-a^4x^4-(1-i)a^3bx^3+\left(-a^2x^2-(1-i)abx-ib^2\right)\sqrt{a^4x^4-b^4}-(1+i)ab^3x+b^4}{-ia^4x^4+(1+i)a^3bx^3+\left(-ia^2x^2+(1+i)abx+b^2\right)\sqrt{a^4x^4-b^4}-(1-i)ab^3x+ib^4}\right)}{ab}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(b^8 + a^8*x^8)/(Sqrt[-b^4 + a^4*x^4]*(-b^8 + a^8*x^8)), x]

[Out] -1/2*x/Sqrt[-b^4 + a^4*x^4] - ((1/4 - I/4)*ArcTan[((1 + I)*a*b*x)/(I*b^2 + a^2*x^2 + Sqrt[-b^4 + a^4*x^4])])/(a*b) + ((1/8 - I/8)*ArcTanh[(b^4 - (1 + I)*a*b^3*x - (1 - I)*a^3*b*x^3 - a^4*x^4 + ((-I)*b^2 - (1 - I)*a*b*x - a^2*x^2)*Sqrt[-b^4 + a^4*x^4])/(I*b^4 - (1 - I)*a*b^3*x + (1 + I)*a^3*b*x^3 - I*a^4*x^4 + (b^2 + (1 + I)*a*b*x - I*a^2*x^2)*Sqrt[-b^4 + a^4*x^4])])/(a*b)

fricas [A] time = 0.85, size = 162, normalized size = 0.52

$$-\frac{4\sqrt{a^4x^4-b^4}\,abx-2\left(a^4x^4-b^4\right)\arctan\left(\frac{\sqrt{a^4x^4-b^4}\,ax}{a^2bx^2+b^3}\right)-\left(a^4x^4-b^4\right)\log\left(\frac{a^4x^4+2\,a^2b^2x^2-b^4-2\,\sqrt{a^4x^4-b^4}\,abx}{a^4x^4+b^4}\right)}{8\left(a^5bx^4-ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^8*x^8+b^8)/(a^4*x^4-b^4)^(1/2)/(a^8*x^8-b^8),x, algorithm="fricas")

[Out] -1/8*(4*sqrt(a^4*x^4 - b^4)*a*b*x - 2*(a^4*x^4 - b^4)*arctan(sqrt(a^4*x^4 - b^4)*a*x/(a^2*b*x^2 + b^3)) - (a^4*x^4 - b^4)*log((a^4*x^4 + 2*a^2*b^2*x^2 - b^4 - 2*sqrt(a^4*x^4 - b^4)*a*b*x)/(a^4*x^4 + b^4)))/(a^5*b*x^4 - a*b^5)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^8x^8+b^8}{\left(a^8x^8-b^8\right)\sqrt{a^4x^4-b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^8*x^8+b^8)/(a^4*x^4-b^4)^(1/2)/(a^8*x^8-b^8),x, algorithm="giac")

[Out] integrate((a^8*x^8 + b^8)/((a^8*x^8 - b^8)*sqrt(a^4*x^4 - b^4)), x)

maple [C] time = 0.05, size = 981, normalized size = 3.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^8*x^8+b^8)/(a^4*x^4-b^4)^(1/2)/(a^8*x^8-b^8),x)

[Out] 1/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)+1/4*b*(-1/2*(a^4*x^3+a^3*b*x^2+a^2*b^2*x+a*b^3)/a^2/b^3/((x-b/a)*(a^4*x^3+a^3*b*x^2+a^2*b^2*x+a*b^3))^(1/2)-1/2/b/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)+1/2/b/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*(EllipticF(x*(-a^2/b^2)^(1/2),I)-EllipticE(x*(-a^2/b^2)^(1/2),I)))-1/2*b^2*(-1/2*(a^4*x^2-a^2*b^2)/b^4*x/a^2/((x^2+b^2/a^2)*(a^4*x^2-a^2*b^2))^(1/2)+1/2/b^2/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)+1/2/b^2/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*(EllipticF(x*(-a^2/b^2)^(1/2),I)-EllipticE(x*(-a^2/b^2)^(1/2),I)))-1/16*b^4/a^4*sum(1/_alpha^3*(-2^(1/2)/(-b^4)^(1/2)*arctanh(_alpha^2*(_alpha^2+x^2)*a^4/(-2*b^4)^(1/2)/(a^4*x^4-b^4)^(1/2))+4/(-a^2/b^2)^(1/2)*_alpha^3*a^4/b^4*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2

```
)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticPi(x*(-a^2/b^2)^(1/2),_alpha^2*a^2/b^2,
(a^2/b^2)^(1/2)/(-a^2/b^2)^(1/2))),_alpha=RootOf(_Z^4*a^4+b^4))-1/4*b*(1/2*
(a^4*x^3-a^3*b*x^2+a^2*b^2*x-a*b^3)/a^2/b^3/((x+b/a)*(a^4*x^3-a^3*b*x^2+a^2
*b^2*x-a*b^3))^(1/2)+1/2/b/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^
2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)-1/2/b/(-a^
2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2
)*(EllipticF(x*(-a^2/b^2)^(1/2),I)-EllipticE(x*(-a^2/b^2)^(1/2),I)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^8x^8 + b^8}{\left(a^8x^8 - b^8\right)\sqrt{a^4x^4 - b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^8*x^8+b^8)/(a^4*x^4-b^4)^(1/2)/(a^8*x^8-b^8),x, algorithm="max
ima")
```

```
[Out] integrate((a^8*x^8 + b^8)/((a^8*x^8 - b^8)*sqrt(a^4*x^4 - b^4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{a^8\,x^8 + b^8}{\sqrt{a^4\,x^4 - b^4}\,\left(b^8 - a^8\,x^8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^8 + a^8*x^8)/((a^4*x^4 - b^4)^(1/2)*(b^8 - a^8*x^8)),x)
```

```
[Out] int(-(b^8 + a^8*x^8)/((a^4*x^4 - b^4)^(1/2)*(b^8 - a^8*x^8)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^8x^8 + b^8}{\sqrt{(ax-b)(ax+b)\left(a^2x^2+b^2\right)}\,(ax-b)(ax+b)\left(a^2x^2+b^2\right)\left(a^4x^4+b^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**8*x**8+b**8)/(a**4*x**4-b**4)**(1/2)/(a**8*x**8-b**8),x)
```

```
[Out] Integral((a**8*x**8 + b**8)/(sqrt((a*x - b)*(a*x + b)*(a**2*x**2 + b**2))*(
a*x - b)*(a*x + b)*(a**2*x**2 + b**2)*(a**4*x**4 + b**4)), x)
```

3.2289 $\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$

Optimal. Leaf size=313

$$-\frac{1}{5}\sqrt{2\sqrt{5}-\frac{22}{5}}\tan^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{2+\sqrt{5}}\,(x+1)}\right)+\frac{1}{5}\sqrt{\frac{22}{5}+2\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{\sqrt{5}-2}\,(x+1)}\right)+\frac{4x^{3/2}-4\sqrt{x^2-1}\,x+}{5\left(x^2-x\right.}$$

Rubi [A] time = 0.57, antiderivative size = 365, normalized size of antiderivative = 1.17, number of steps used = 18, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6742, 736, 826, 1166, 207, 203, 1018, 1034, 725, 206, 204, 985}

$$-\frac{2\sqrt{x^2-1}(1-2x)}{5(-x^2+x+1)}+\frac{2\sqrt{x}(1-2x)}{5(-x^2+x+1)}-\frac{2}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)+\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)-\frac{2}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)+\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)+\frac{1}{5}\sqrt{\frac{2}{5}\left(5\sqrt{5}-11\right)}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x}\right)-\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

```
[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - (2*(1 - 2*x)*Sqrt[-1 + x^2])/(5*(
1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[Sqrt[2/(-1 + Sqrt[5])
]*Sqrt[x]])/5 + Sqrt[2/(5*(-1 + Sqrt[5]))]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sq
rt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])] - (2*Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTan[
(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(
2*(11 + 5*Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]])/5 + Sqrt[2/(
5*(1 + Sqrt[5]))]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt
[-1 + x^2])] - (2*Sqrt[(2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sq
rt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 985

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((b - q + 2*c*x)
*Sqrt[d + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ
[b^2 - 4*a*c]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx = \int \left(-\frac{2\sqrt{x}}{\left(-1-x+x^2\right)^2}+\frac{2x}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}+\frac{1}{\sqrt{-1+x^2}\left(-1-x\right.}\right.$$

$$= -\left(2\int \frac{\sqrt{x}}{\left(-1-x+x^2\right)^2}\,dx\right)+2\int \frac{x}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}\,dx+\int$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{2}{5}\int \frac{-\frac{1}{2}-x}{\sqrt{x}\left(-1-x+x^2\right)}\,dx+\frac{2}{5}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{4}{5}\text{Subst}\left(\int \frac{-\frac{1}{2}-x^2}{-1-x^2+x^4}\,dx,x,\right.$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\tan^{-1}\left(\frac{2-}{\sqrt{2\left(-1\right.}}\right.$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{1}{5}\sqrt{\frac{2}{5}\left(-11+5\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\right.$$

Mathematica [A] time = 0.81, size = 340, normalized size = 1.09

$$\frac{2}{5}\left(\frac{\sqrt{x}(1-2x)}{-x^2+x+1}+\frac{\sqrt{x^2-1}(1-2x)}{x^2-x-1}-\frac{1}{2}\sqrt{\frac{5}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{-\sqrt{5}x+x-2}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)-\sqrt{\sqrt{5}-\frac{2}{5}}\tan^{-1}\left(\frac{\left(\sqrt{5}-1\right)x+2}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)-\sqrt{\frac{5}{2\left(1+\sqrt{5}\right)}}\tanh^{-1}\left(\frac{\sqrt{5}x+x-2}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)-\sqrt{\frac{2}{5}+\sqrt{5}}\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)+\sqrt{\frac{1}{10}\left(5\sqrt{5}-11\right)}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x}\right)-\sqrt{\frac{1}{10}\left(11+5\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

```
[Out] (2*(((1 - 2*x)*Sqrt[x])/(1 + x - x^2) + ((1 - 2*x)*Sqrt[-1 + x^2])/(-1 - x
+ x^2) + Sqrt[(-11 + 5*Sqrt[5])/10]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[x]]
- (Sqrt[(5*(1 + Sqrt[5]))/2]*ArcTan[(-2 + x - Sqrt[5]*x)/(Sqrt[2*(-1 + Sqrt
[5])]*Sqrt[-1 + x^2])])/2 - Sqrt[-2/5 + Sqrt[5]]*ArcTan[(2 + (-1 + Sqrt[5])
*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])] - Sqrt[(11 + 5*Sqrt[5])/10]*Ar
cTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]] - Sqrt[5/(2*(1 + Sqrt[5]))]*ArcTanh[(-
2 + x + Sqrt[5]*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])] - Sqrt[2/5 + Sqr
t[5]]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])]
))/5
```

IntegrateAlgebraic [A] time = 8.92, size = 231, normalized size = 0.74

$$-\frac{1}{5}\sqrt{2\sqrt{5}-\frac{22}{5}}\tan^{-1}\left(\frac{\sqrt{\sqrt{5}-2}\sqrt{x^2-1}}{x+1}\right)+\frac{1}{5}\sqrt{\frac{22}{5}+2\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{5}}\sqrt{x^2-1}}{x+1}\right)+\frac{4x^{3/2}-4\sqrt{x^2-1}x+2\sqrt{x^2-1}-2\sqrt{x}}{5\left(x^2-x-1\right)}+\frac{1}{5}\sqrt{2\sqrt{5}-\frac{22}{5}}\tan^{-1}\left(\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\sqrt{x}\right)-\frac{1}{5}\sqrt{\frac{22}{5}+2\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

```
[Out] (-2*Sqrt[x] + 4*x^(3/2) + 2*Sqrt[-1 + x^2] - 4*x*Sqrt[-1 + x^2])/(5*(-1 - x
 + x^2)) + (Sqrt[-22/5 + 2*Sqrt[5]]*ArcTan[Sqrt[1/2 + Sqrt[5]/2]*Sqrt[x]])/
```

```
5 - (Sqrt[-22/5 + 2*Sqrt[5]]*ArcTan[(Sqrt[-2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1
+ x)])/5 - (Sqrt[22/5 + 2*Sqrt[5]]*ArcTanh[Sqrt[-1/2 + Sqrt[5]/2]*Sqrt[x]])
/5 + (Sqrt[22/5 + 2*Sqrt[5]]*ArcTanh[(Sqrt[2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1
+ x)])/5
```

fricas [A] time = 0.46, size = 424, normalized size = 1.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/50*(4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22)*arctan(1/2*sqrt(2*x^2 -
 sqrt(x^2 - 1)*(2*x + sqrt(5) - 1) + sqrt(5)*x - x)*sqrt(10*sqrt(5) - 22)*(
sqrt(5) + 2) + 1/4*(sqrt(5)*(2*x + 1) - 2*sqrt(x^2 - 1)*(sqrt(5) + 2) + 4*x
 + 3)*sqrt(10*sqrt(5) - 22)) - 4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22
)*arctan(1/4*(sqrt(2)*sqrt(2*x + sqrt(5) - 1)*(sqrt(5) + 2) - 2*sqrt(x)*(sq
rt(5) + 2))*sqrt(10*sqrt(5) - 22)) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
+ 22)*log(sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^
2 - 1) + 2) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(
5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
 + 22)*log(-sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(
x^2 - 1) + 2) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sq
rt(5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) - 40*x^2 - 20*sqrt(x^2 - 1)*(2*x - 1
) + 20*(2*x - 1)*sqrt(x) + 40*x + 40)/(x^2 - x - 1)
```

giac [A] time = 6.01, size = 367, normalized size = 1.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2/5*sqrt(1/10)*sqrt(5*sqrt(5) - 11)*arctan((2*x + sqrt(5) - 2*sqrt(x^2 - 1)
 - 1)/sqrt(2*sqrt(5) - 2)) + 1/5*sqrt(1/10)*sqrt(5*sqrt(5) + 11)*log(abs(-1
53040*x + 22956*sqrt(5)*sqrt(50*sqrt(5) + 110) + 76520*sqrt(5) + 153040*sqr
t(x^2 - 1) - 38260*sqrt(50*sqrt(5) + 110) + 76520)) - 1/5*sqrt(1/10)*sqrt(5
*sqrt(5) + 11)*log(abs(-153040*x - 22956*sqrt(5)*sqrt(50*sqrt(5) + 110) + 7
6520*sqrt(5) + 153040*sqrt(x^2 - 1) + 38260*sqrt(50*sqrt(5) + 110) + 76520)
) + 1/25*sqrt(50*sqrt(5) - 110)*arctan(sqrt(x)/sqrt(1/2*sqrt(5) - 1/2)) - 1
/50*sqrt(50*sqrt(5) + 110)*log(sqrt(x) + sqrt(1/2*sqrt(5) + 1/2)) + 1/50*sq
rt(50*sqrt(5) + 110)*log(abs(sqrt(x) - sqrt(1/2*sqrt(5) + 1/2))) + 4/5*((x
- sqrt(x^2 - 1))^3 + 2*(x - sqrt(x^2 - 1))^2 + 3*x - 3*sqrt(x^2 - 1) - 2)/(
(x - sqrt(x^2 - 1))^4 - 2*(x - sqrt(x^2 - 1))^3 - 2*(x - sqrt(x^2 - 1))^2 -
 2*x + 2*sqrt(x^2 - 1) + 1) + 2/5*(2*x^(3/2) - sqrt(x))/(x^2 - x - 1)
```

maple [B] time = 0.14, size = 902, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x)
```

```
[Out] -6/25*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2-1
/2*5^(1/2)))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2-1/2*5^(1/2))^2+4*(5^(1/2)+1)*(x-
1/2-1/2*5^(1/2))+2+2*5^(1/2))^(1/2))-1/5/(1/2+1/2*5^(1/2))/(x-1/2-1/2*5^(1/
2))*((x-1/2-1/2*5^(1/2))^2+(5^(1/2)+1)*(x-1/2-1/2*5^(1/2))+1/2+1/2*5^(1/2))
^(1/2)+6/5/(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1
/2)+1)*(x-1/2-1/2*5^(1/2)))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2-1/2*5^(1/2))^2+4*
(5^(1/2)+1)*(x-1/2-1/2*5^(1/2))+2+2*5^(1/2))^(1/2))+2/5/(1/2+1/2*5^(1/2))/(
```

```
2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2-1/2*5^(1/2)))/(2
+2*5^(1/2))^(1/2)/(4*(x-1/2-1/2*5^(1/2))^2+4*(5^(1/2)+1)*(x-1/2-1/2*5^(1/2)
)+2+2*5^(1/2))^(1/2))*5^(1/2)-1/5*5^(1/2)/(1/2+1/2*5^(1/2))/(x-1/2-1/2*5^(1
/2))*((x-1/2-1/2*5^(1/2))^2+(5^(1/2)+1)*(x-1/2-1/2*5^(1/2))+1/2+1/2*5^(1/2)
)^(1/2)-6/25*5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*
(x-1/2+1/2*5^(1/2)))/(-2+2*5^(1/2))^(1/2)/(4*(x-1/2+1/2*5^(1/2))^2+4*(-5^(1
/2)+1)*(x-1/2+1/2*5^(1/2))+2-2*5^(1/2))^(1/2))-1/5/(1/2-1/2*5^(1/2))/(x-1/2
+1/2*5^(1/2))*((x-1/2+1/2*5^(1/2))^2+(-5^(1/2)+1)*(x-1/2+1/2*5^(1/2))+1/2-1
/2*5^(1/2))^(1/2)+2/5/(1/2-1/2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^
(1/2)+(-5^(1/2)+1)*(x-1/2+1/2*5^(1/2)))/(-2+2*5^(1/2))^(1/2)/(4*(x-1/2+1/2*
5^(1/2))^2+4*(-5^(1/2)+1)*(x-1/2+1/2*5^(1/2))+2-2*5^(1/2))^(1/2))*5^(1/2)-6
/5/(1/2-1/2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*
(x-1/2+1/2*5^(1/2)))/(-2+2*5^(1/2))^(1/2)/(4*(x-1/2+1/2*5^(1/2))^2+4*(-5^(1
/2)+1)*(x-1/2+1/2*5^(1/2))+2-2*5^(1/2))^(1/2))+1/5*5^(1/2)/(1/2-1/2*5^(1/2)
)/(x-1/2+1/2*5^(1/2))*((x-1/2+1/2*5^(1/2))^2+(-5^(1/2)+1)*(x-1/2+1/2*5^(1/2
))+1/2-1/2*5^(1/2))^(1/2)+2/5*x^(1/2)/(x-1/2-1/2*5^(1/2))-4/5/(2+2*5^(1/2))
^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))-8/25/(2+2*5^(1/2))^(1/2)*arct
anh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))*5^(1/2)+2/5*x^(1/2)/(x-1/2+1/2*5^(1/2))+
4/5/(-2+2*5^(1/2))^(1/2)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))-8/25/(-2+2*
5^(1/2))^(1/2)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2-1}\left(\sqrt{x^2-1}+\sqrt{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{x^2-1}\,\left(\sqrt{x^2-1}+\sqrt{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*((x^2 - 1)^(1/2) + x^(1/2))^2),x)
[Out] int(1/((x^2 - 1)^(1/2)*((x^2 - 1)^(1/2) + x^(1/2))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\left(\sqrt{x}+\sqrt{x^2-1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(x**(1/2)+(x**2-1)**(1/2))**2,x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(sqrt(x) + sqrt(x**2 - 1))**2), x)
```

3.2290 $\int \frac{b+ax^2}{\left(d+cx^2\right)\sqrt[3]{x+x^3}}\,dx$

Optimal. Leaf size=315

$$\frac{(bc-ad)\log\left(x\sqrt[3]{c-d}+\sqrt[3]{d}\sqrt[3]{x^3+x}\right)}{2cd^{2/3}\sqrt[3]{c-d}}-\frac{\sqrt{3}(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}x\sqrt[3]{c-d}}{x\sqrt[3]{c-d}-2\sqrt[3]{d}\sqrt[3]{x^3+x}}\right)}{2cd^{2/3}\sqrt[3]{c-d}}+\frac{(ad-bc)\log\left(-\sqrt[3]{d}\sqrt[3]{x^3+x}x\sqrt[3]{c\right.}}{4cd}$$

Rubi [A] time = 0.52, antiderivative size = 407, normalized size of antiderivative = 1.29, number of steps used = 15, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2056, 584, 329, 275, 239, 466, 465, 377, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{x}\sqrt[3]{x^2+1}(bc-ad)\log\left(\frac{x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{x^2+1}}+\sqrt[3]{d}\right)}{2cd^{2/3}\sqrt[3]{x^3+x}\sqrt[3]{c-d}}-\frac{\sqrt[3]{x}\sqrt[3]{x^2+1}(bc-ad)\log\left(\frac{x^{4/3}(c-d)^{2/3}}{\left(x^2+1\right)^{2/3}}-\frac{\sqrt[3]{d}x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{x^2+1}}+d^{2/3}\right)}{4cd^{2/3}\sqrt[3]{x^3+x}\sqrt[3]{c-d}}-\frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{x^2+1}(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-\frac{2x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{x^2+1}}}{\sqrt{3}\sqrt[3]{d}}\right)}{2cd^{2/3}\sqrt[3]{x^3+x}\sqrt[3]{c-d}}-\frac{3a\sqrt[3]{x}\sqrt[3]{x^2+1}\log\left(x^{2/3}-\sqrt[3]{x^2+1}\right)}{4c\sqrt[3]{x^3+x}}+\frac{\sqrt{3}a\sqrt[3]{x}\sqrt[3]{x^2+1}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)}{2c\sqrt[3]{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^2)/((d + c*x^2)*(x + x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*a*x^(1/3)*(1 + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3))/(1 + x^2)^(1/3))
/Sqrt[3]])/(2*c*(x + x^3)^(1/3)) - (Sqrt[3]*(b*c - a*d)*x^(1/3)*(1 + x^2)^(
1/3)*ArcTan[(d^(1/3) - (2*(c - d)^(1/3)*x^(2/3))/(1 + x^2)^(1/3))/(Sqrt[3]*
d^(1/3))])/(2*c*(c - d)^(1/3)*d^(2/3)*(x + x^3)^(1/3)) + ((b*c - a*d)*x^(1/
3)*(1 + x^2)^(1/3)*Log[d^(1/3) + ((c - d)^(1/3)*x^(2/3))/(1 + x^2)^(1/3)])/
(2*c*(c - d)^(1/3)*d^(2/3)*(x + x^3)^(1/3)) - ((b*c - a*d)*x^(1/3)*(1 + x^2
)^(1/3)*Log[d^(2/3) + ((c - d)^(2/3)*x^(4/3))/(1 + x^2)^(2/3) - ((c - d)^(1
/3)*d^(1/3)*x^(2/3))/(1 + x^2)^(1/3)])/(4*c*(c - d)^(1/3)*d^(2/3)*(x + x^3)
^(1/3)) - (3*a*x^(1/3)*(1 + x^2)^(1/3)*Log[x^(2/3) - (1 + x^2)^(1/3)])/(4*c
*(x + x^3)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{b+ax^2}{\left(d+cx^2\right)\sqrt[3]{x+x^3}}\,dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\int \frac{b+ax^2}{\sqrt[3]{x}\sqrt[3]{1+x^2}\left(d+cx^2\right)}\,dx}{\sqrt[3]{x+x^3}}$$

$$= \frac{\left(\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\int \left(\frac{a}{c\sqrt[3]{x}\sqrt[3]{1+x^2}}+\frac{bc-ad}{c\sqrt[3]{x}\sqrt[3]{1+x^2}\left(d+cx^2\right)}\right)dx}{\sqrt[3]{x+x^3}}$$

$$= \frac{\left(a\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\int \frac{1}{\sqrt[3]{x}\sqrt[3]{1+x^2}}\,dx}{c\sqrt[3]{x+x^3}} + \frac{\left((bc-ad)\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\int \frac{1}{\sqrt[3]{x}\sqrt[3]{1+x^2}\left(d+cx^2\right)}\,dx}{c\sqrt[3]{x+x^3}}$$

$$= \frac{\left(3a\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{c\sqrt[3]{x+x^3}} + \frac{\left(3(bc-ad)\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \cdots\right.}{c\sqrt[3]{x+x^3}}$$

$$= \frac{\left(3a\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^3}}\,dx, x, x^{2/3}\right)}{2c\sqrt[3]{x+x^3}} + \frac{\left(3(bc-ad)\sqrt[3]{x}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \cdots\right.}{2c\sqrt[3]{x+x^3}}$$

$$= \frac{\sqrt{3}\,a\sqrt[3]{x}\sqrt[3]{1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2c\sqrt[3]{x+x^3}} - \frac{3a\sqrt[3]{x}\sqrt[3]{1+x^2}\,\log\left(x^{2/3}-\sqrt[3]{1+x^2}\right)}{4c\sqrt[3]{x+x^3}} + \frac{\left(3(bc-a \cdots\right.}{\cdots}$$

$$= \frac{\sqrt{3}\,a\sqrt[3]{x}\sqrt[3]{1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2c\sqrt[3]{x+x^3}} - \frac{3a\sqrt[3]{x}\sqrt[3]{1+x^2}\,\log\left(x^{2/3}-\sqrt[3]{1+x^2}\right)}{4c\sqrt[3]{x+x^3}} + \frac{\left((bc-ad \cdots\right.}{\cdots}$$

$$= \frac{\sqrt{3}\,a\sqrt[3]{x}\sqrt[3]{1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2c\sqrt[3]{x+x^3}} + \frac{(bc-ad)\sqrt[3]{x}\sqrt[3]{1+x^2}\,\log\left(\sqrt[3]{d}+\frac{\sqrt[3]{c-d}\,x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2c\sqrt[3]{c-d}\,d^{2/3}\sqrt[3]{x+x^3}} - \frac{3a \cdots}{\cdots}$$

$$= \frac{\sqrt{3}\,a\sqrt[3]{x}\sqrt[3]{1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2c\sqrt[3]{x+x^3}} + \frac{(bc-ad)\sqrt[3]{x}\sqrt[3]{1+x^2}\,\log\left(\sqrt[3]{d}+\frac{\sqrt[3]{c-d}\,x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2c\sqrt[3]{c-d}\,d^{2/3}\sqrt[3]{x+x^3}} - \frac{(bc \cdots}{\cdots}$$

$$= \frac{\sqrt{3}\,a\sqrt[3]{x}\sqrt[3]{1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2c\sqrt[3]{x+x^3}} - \frac{\sqrt{3}\,(bc-ad)\sqrt[3]{x}\sqrt[3]{1+x^2}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{c-d}\,x^{2/3}}{\sqrt[3]{d}\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2c\sqrt[3]{c-d}\,d^{2/3}\sqrt[3]{x+x^3}} + \frac{(b \cdots}{\cdots}$$

Mathematica [A] time = 0.59, size = 308, normalized size = 0.98

$$\frac{\sqrt[3]{x}\sqrt[3]{x^2+1}\left(\frac{2(bc-ad)\log\left(\frac{x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{x^2+1}}+\sqrt[3]{d}\right)}{d^{2/3}\sqrt[3]{c-d}} - \frac{(bc-ad)\left(\log\left(\frac{x^{4/3}(c-d)^{2/3}}{\left(x^2+1\right)^{2/3}}-\frac{\sqrt[3]{d}\,x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{x^2+1}}+d^{2/3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{d}\sqrt[3]{x^2+1}}}{\sqrt{3}}\right)\right)}{d^{2/3}\sqrt[3]{c-d}} - 2a\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2+1}}\right) + a\left(\log\left(\frac{x^{4/3}}{\left(x^2+1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2+1}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)\right)\right)}{4c\sqrt[3]{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^2)/((d + c*x^2)*(x + x^3)^(1/3)),x]
```

```
[Out] (x^(1/3)*(1 + x^2)^(1/3)*(-2*a*Log[1 - x^(2/3)/(1 + x^2)^(1/3)] + a*(2*Sqrt
[3]*ArcTan[(1 + (2*x^(2/3))/(1 + x^2)^(1/3))/Sqrt[3]] + Log[1 + x^(4/3)/(1
+ x^2)^(2/3) + x^(2/3)/(1 + x^2)^(1/3)]) + (2*(b*c - a*d)*Log[d^(1/3) + ((c
 - d)^(1/3)*x^(2/3))/(1 + x^2)^(1/3)])/((c - d)^(1/3)*d^(2/3)) - ((b*c - a*
d)*(2*Sqrt[3]*ArcTan[(1 - (2*(c - d)^(1/3)*x^(2/3))/(d^(1/3)*(1 + x^2)^(1/3
)))/Sqrt[3]] + Log[d^(2/3) + ((c - d)^(2/3)*x^(4/3))/(1 + x^2)^(2/3) - ((c
- d)^(1/3)*d^(1/3)*x^(2/3))/(1 + x^2)^(1/3)]))/((c - d)^(1/3)*d^(2/3))))/(4
*c*(x + x^3)^(1/3))
```

IntegrateAlgebraic [A] time = 2.91, size = 318, normalized size = 1.01

$$\frac{(bc-ad)\log\left(x\sqrt[3]{c-d}+\sqrt[3]{d}\sqrt[3]{x^3+x}\right)}{2cd^{2/3}\sqrt[3]{c-d}}-\frac{\sqrt{3}(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}x\sqrt[3]{c-d}}{x\sqrt[3]{c-d}-2\sqrt[3]{d}\sqrt[3]{x^3+x}}\right)}{2cd^{2/3}\sqrt[3]{c-d}}+\frac{(ad-bc)\log\left(-\sqrt[3]{d}\sqrt[3]{x^3+x}\,x\sqrt[3]{c-d}+x^2(c-d)^{2/3}+d^{2/3}\left(x^3+x\right)^{2/3}\right)}{4cd^{2/3}\sqrt[3]{c-d}}-\frac{a\log\left(c\sqrt[3]{x^3+x}-cx\right)}{2c}+\frac{\sqrt{3}a\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x}+x}\right)}{2c}+\frac{a\log\left(\sqrt[3]{x^3+x}\,x+\left(x^3+x\right)^{2/3}+x^2\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^2)/((d + c*x^2)*(x + x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*a*ArcTan[(Sqrt[3]*x)/(x + 2*(x + x^3)^(1/3))])/(2*c) - (Sqrt[3]*(b
*c - a*d)*ArcTan[(Sqrt[3]*(c - d)^(1/3)*x)/((c - d)^(1/3)*x - 2*d^(1/3)*(x
+ x^3)^(1/3))])/(2*c*(c - d)^(1/3)*d^(2/3)) - (a*Log[-(c*x) + c*(x + x^3)^(
1/3)])/(2*c) + ((b*c - a*d)*Log[(c - d)^(1/3)*x + d^(1/3)*(x + x^3)^(1/3)])
/(2*c*(c - d)^(1/3)*d^(2/3)) + (a*Log[x^2 + x*(x + x^3)^(1/3) + (x + x^3)^(
2/3)])/(4*c) + ((-(b*c) + a*d)*Log[(c - d)^(2/3)*x^2 - (c - d)^(1/3)*d^(1/3
)*x*(x + x^3)^(1/3) + d^(2/3)*(x + x^3)^(2/3)])/(4*c*(c - d)^(1/3)*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(c*x^2+d)/(x^3+x)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.37, size = 284, normalized size = 0.90

$$-\frac{\sqrt{3}a\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)}{2c}+\frac{\left(bc\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}-ad\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right|\right)}{2\left(c^2-cd\right)}+\frac{a\log\left(\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)}{4c}-\frac{a\log\left(\left|\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)}{2c}-\frac{\left(\sqrt{3}bc-\sqrt{3}ad\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}+2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}}\right)}{2\left(-cd^2+d^3\right)^{\frac{1}{3}}c}+\frac{(bc-ad)\log\left(\left(-\frac{c-d}{d}\right)^{\frac{2}{3}}+\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}\right)}{4\left(-cd^2+d^3\right)^{\frac{1}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(c*x^2+d)/(x^3+x)^(1/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*(1/x^2 + 1)^(1/3) + 1))/c + 1/2*(b*c*(
-(c - d)/d)^(1/3) - a*d*(-(c - d)/d)^(1/3))*(-(c - d)/d)^(1/3)*log(abs(-(-(
c - d)/d)^(1/3) + (1/x^2 + 1)^(1/3)))/(c^2 - c*d) + 1/4*a*log((1/x^2 + 1)^(
2/3) + (1/x^2 + 1)^(1/3) + 1)/c - 1/2*a*log(abs((1/x^2 + 1)^(1/3) - 1))/c -
 1/2*(sqrt(3)*b*c - sqrt(3)*a*d)*arctan(1/3*sqrt(3)*((-(c - d)/d)^(1/3) + 2
*(1/x^2 + 1)^(1/3))/(-(c - d)/d)^(1/3))/((-c*d^2 + d^3)^(1/3)*c) + 1/4*(b*c
 - a*d)*log((-(c - d)/d)^(2/3) + (-(c - d)/d)^(1/3)*(1/x^2 + 1)^(1/3) + (1/
x^2 + 1)^(2/3))/((-c*d^2 + d^3)^(1/3)*c)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2+b}{\left(c\,x^2+d\right)\left(x^3+x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)/(c*x^2+d)/(x^3+x)^(1/3),x)
[Out] int((a*x^2+b)/(c*x^2+d)/(x^3+x)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2+b}{(cx^2+d)(x^3+x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)/(c*x^2+d)/(x^3+x)^(1/3),x, algorithm="maxima")
[Out] integrate((a*x^2 + b)/((c*x^2 + d)*(x^3 + x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x^2+b}{\left(c\,x^2+d\right)\,\left(x^3+x\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^2)/((d + c*x^2)*(x + x^3)^(1/3)),x)
[Out] int((b + a*x^2)/((d + c*x^2)*(x + x^3)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2+b}{\sqrt[3]{x\left(x^2+1\right)}\left(cx^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b)/(c*x**2+d)/(x**3+x)**(1/3),x)
[Out] Integral((a*x**2 + b)/((x*(x**2 + 1))**(1/3)*(c*x**2 + d)), x)
```

3.2291 $\int \frac{x^3(4ab-3(a+b)x+2x^2)}{(x^2(-a+x)(-b+x))^{2/3}(-ab+(a+b)x-x^2+dx^4)}\,dx$

Optimal. Leaf size=315

$$-\frac{\log\left(a^2b^2-2a^2bx+a^2x^2-2ab^2x+d^{2/3}\left(x^3(-a-b)+abx^2+x^4\right)^{4/3}+\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}\left(ab\sqrt[3]{d}-\right.\right.}{2d^{2/3}}$$

Rubi [F] time = 19.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3\left(4ab-3(a+b)x+2x^2\right)}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(-ab+(a+b)x-x^2+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*(4*a*b - 3*(a + b)*x + 2*x^2))/((x^2*(-a + x)*(-b + x))^(2/3)*(-(a
*b) + (a + b)*x - x^2 + d*x^4)),x]
```

```
[Out] (3*x^2*(1 - x/a)^(2/3)*(1 - x/b)^(2/3)*AppellF1[2/3, 2/3, 2/3, 5/3, x/a, x/
b])/(d*((a - x)*(b - x)*x^2)^(2/3)) + (6*(a + b)*x^(4/3)*(-a + x)^(2/3)*(-b
 + x)^(2/3)*Defer[Subst][Defer[Int][x^4/((-a + x^3)^(2/3)*(-b + x^3)^(2/3)*
(a*b - a*(1 + b/a)*x^3 + x^6 - d*x^12)), x], x, x^(1/3)])/(d*((a - x)*(b -
x)*x^2)^(2/3)) - (6*(1 + 2*a*b*d)*x^(4/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Def
er[Subst][Defer[Int][x^7/((-a + x^3)^(2/3)*(-b + x^3)^(2/3)*(a*b - a*(1 + b
/a)*x^3 + x^6 - d*x^12)), x], x, x^(1/3)])/(d*((a - x)*(b - x)*x^2)^(2/3))
 + (9*(a + b)*x^(4/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][
x^10/((-a + x^3)^(2/3)*(-b + x^3)^(2/3)*(a*b - a*(1 + b/a)*x^3 + x^6 - d*x^
12)), x], x, x^(1/3)])/((a - x)*(b - x)*x^2)^(2/3) + (6*a*b*x^(4/3)*(-a + x
)^(2/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][x/((-a + x^3)^(2/3)*(-b + x^
3)^(2/3)*(-(a*b) + a*(1 + b/a)*x^3 - x^6 + d*x^12)), x], x, x^(1/3)])/(d*((
a - x)*(b - x)*x^2)^(2/3))
```

Rubi steps

$$\int \frac{x^3\left(4ab-3(a+b)x+2x^2\right)}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(-ab+(a+b)x-x^2+dx^4\right)}dx = \frac{\left(x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\int \frac{x^{5/3}\left(4ab-3(a+b)x+\cdots\right.}{(-a+x)^{2/3}(-b+x)^{2/3}\left(-ab+(a\cdots\right.}}{\left(x^2(-a+x)(-b+x)\right)^{2/3}}$$

$$= \frac{\left(3x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\text{Subst}\left(\int \frac{x^7\left(4\cdots\right.}{\left(-a+x^3\right)^{2/3}\left(-b+\cdots\right.}\right.}{\left(x^2(-a+x)(-b+x)\cdots\right.}$$

$$= \frac{\left(3x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\text{Subst}\left(\int\left(\frac{2x}{d\left(-a+x^3\right)^{2/3}\left(-b\cdots\right.}\right.\right.}{\left(x^2(-a+\cdots\right.}$$

$$= \frac{\left(3x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\text{Subst}\left(\int \frac{x\left(2ab-2(a+b)x\cdots\right.}{\left(-a+x^3\right)^{2/3}\left(-b+\cdots\right.}\right.}{d\left(x^2(-a+x)(-b+x\cdots\right.}$$

$$= \frac{\left(3x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\text{Subst}\left(\int\left(\frac{\cdots}{\left(-a+x^3\right)^{2/3}\left(-b\cdots\right.}\right.\right.}{\cdots}$$

$$= \frac{\left(9(a+b)x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right)\text{Subst}\left(\int \frac{\cdots}{\left(-a+x^3\cdots\right.}\right.}{\left(x^2(-a+x)(-b+\cdots\right.}$$

$$= \frac{3x^2\left(1-\frac{x}{a}\right)^{2/3}\left(1-\frac{x}{b}\right)^{2/3}F_1\left(\frac{2}{3};\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{x}{a},\frac{x}{b}\right)}{d\left((a-x)(b-x)x^2\right)^{2/3}} + \frac{\left(9(a+\cdots\right.}{\cdots}$$

Mathematica [F] time = 3.38, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(4ab-3(a+b)x+2x^2\right)}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(-ab+(a+b)x-x^2+dx^4\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(4*a*b - 3*(a + b)*x + 2*x^2))/((x^2*(-a + x)*(-b + x))^(2/3
)*(-(a*b) + (a + b)*x - x^2 + d*x^4)),x]
```

```
[Out] Integrate[(x^3*(4*a*b - 3*(a + b)*x + 2*x^2))/((x^2*(-a + x)*(-b + x))^(2/3
)*(-(a*b) + (a + b)*x - x^2 + d*x^4)), x]
```

IntegrateAlgebraic [A] time = 2.01, size = 315, normalized size = 1.00

$$-\frac{\log\left(a^2b^2-2a^2bx+a^2x^2-2ab^2x+d^{2/3}\left(x^3(-a-b)+abx^2+x^4\right)^{4/3}+\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}\left(ab\sqrt[3]{d}-a\sqrt[3]{d}x-b\sqrt[3]{d}x+\sqrt[3]{d}x^2\right)+4abx^2-2ax^3+b^2x^2-2bx^3+x^4\right)}{2d^{2/3}}+\frac{\log\left(-\sqrt[3]{d}\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+ab-ax-bx+x^2\right)}{d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}}{\sqrt[3]{d}\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+2ab-2ax-2bx+2x^2}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(4*a*b - 3*(a + b)*x + 2*x^2))/((x^2*(-a + x)*(-b +
 x))^(2/3)*(-(a*b) + (a + b)*x - x^2 + d*x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(2/3))/(2*a
*b - 2*a*x - 2*b*x + 2*x^2 + d^(1/3)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(2/3))]
```

```
)/d^(2/3) + Log[a*b - a*x - b*x + x^2 - d^(1/3)*(a*b*x^2 + (-a - b)*x^3 + x
^4)^(2/3)]/d^(2/3) - Log[a^2*b^2 - 2*a^2*b*x - 2*a*b^2*x + a^2*x^2 + 4*a*b*
x^2 + b^2*x^2 - 2*a*x^3 - 2*b*x^3 + x^4 + (a*b*d^(1/3) - a*d^(1/3)*x - b*d^
(1/3)*x + d^(1/3)*x^2)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(2/3) + d^(2/3)*(a*b*
x^2 + (-a - b)*x^3 + x^4)^(4/3)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(-a*b+(a+b)
*x-x^2+d*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,ab - 3\,(a+b)x + 2\,x^2\right)x^3}{\left(dx^4 - ab + (a+b)x - x^2\right)\left((a-x)(b-x)x^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(-a*b+(a+b)
*x-x^2+d*x^4),x, algorithm="giac")
```

```
[Out] integrate((4*a*b - 3*(a + b)*x + 2*x^2)*x^3/((d*x^4 - a*b + (a + b)*x - x^2
)*((a - x)*(b - x)*x^2)^(2/3)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(4ab - 3\,(a+b)\,x + 2x^2\right)}{\left(x^2\,(-a+x)\,(-b+x)\right)^{\frac{2}{3}}\left(-ab + (a+b)\,x - x^2 + d\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(-a*b+(a+b)*x-x^2
+d*x^4),x)
```

```
[Out] int(x^3*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(-a*b+(a+b)*x-x^2
+d*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,ab - 3\,(a+b)x + 2\,x^2\right)x^3}{\left(dx^4 - ab + (a+b)x - x^2\right)\left((a-x)(b-x)x^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(-a*b+(a+b)
*x-x^2+d*x^4),x, algorithm="maxima")
```

```
[Out] integrate((4*a*b - 3*(a + b)*x + 2*x^2)*x^3/((d*x^4 - a*b + (a + b)*x - x^2
)*((a - x)*(b - x)*x^2)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^3\,\left(4\,a\,b + 2\,x^2 - 3\,x\,\left(a+b\right)\right)}{{\left(x^2\,\left(a-x\right)\,\left(b-x\right)\right)}^{2/3}\,\left(-d\,x^4 + x^2 + \left(-a-b\right)\,x + a\,b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(4*a*b + 2*x^2 - 3*x*(a + b)))/((x^2*(a - x)*(b - x))^(2/3)*(a*b
- d*x^4 + x^2 - x*(a + b))),x)
```

```
[Out] -int((x^3*(4*a*b + 2*x^2 - 3*x*(a + b)))/((x^2*(a - x)*(b - x))^(2/3)*(a*b
- d*x^4 + x^2 - x*(a + b))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(4*a*b-3*(a+b)*x+2*x**2)/(x**2*(-a+x)*(-b+x))**(2/3)/(-a*b+(
a+b)*x-x**2+d*x**4),x)
```

```
[Out] Timed out
```

3.2292 $\int \left(1+x^2\right)^{3/2} \sqrt{x+\sqrt{1+x^2}} \sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\, dx$

Optimal. Leaf size=316

$$\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\,\left(1968046080x^7+1130364928x^6+10550149120x^5+6568280064x^4+9\right.$$

Rubi [F] time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(1+x^2\right)^{3/2} \sqrt{x+\sqrt{1+x^2}} \sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\, dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^2)^(3/2)*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]
]],x]
```

```
[Out] Defer[Int][(1 + x^2)^(3/2)*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1
 + x^2]]], x]
```

Rubi steps

$$\int \left(1+x^2\right)^{3/2} \sqrt{x+\sqrt{1+x^2}} \sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\, dx = \int \left(1+x^2\right)^{3/2} \sqrt{x+\sqrt{1+x^2}} \sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\, dx$$

Mathematica [F] time = 5.19, size = 0, normalized size = 0.00

$$\int \left(1+x^2\right)^{3/2} \sqrt{x+\sqrt{1+x^2}} \sqrt{1+\sqrt{x+\sqrt{1+x^2}}}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^2)^(3/2)*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1
+ x^2]]],x]
```

```
[Out] Integrate[(1 + x^2)^(3/2)*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1
+ x^2]]], x]
```

IntegrateAlgebraic [A] time = 0.65, size = 316, normalized size = 1.00

$$\frac{\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\left(1968046080x^7+1130364928x^6+10550149120x^5+6568280064x^4+96561463296x^3+1984342244x^2+66830366096x-1176816782\right)+\sqrt{x^2+1}\left(\sqrt{\sqrt{x^2+1}+x}\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\left(1968046080x^6+1130364928x^5+95661260800x^4+6003097600x^3+92024406016x^2-875910940x+21890925968\right)+\left(66913566720x^7-2099249152x^6+214715203584x^5-10745282560x^4+308588576768x^3+3694527828x^2+88760534448x+2167822549\right)\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)+\left(66913566720x^8-2099249152x^7+248171986944x^6-11794907136x^5+407581982720x^4-1415707308x^3+220397520304x^2+5227043711x+15903121112\right)\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}}{39729930240\left(\sqrt{x^2+1}+x\right)^{7/2}}-\frac{545\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{x^2+1}+x}+1}\right)}{8192}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^2)^(3/2)*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x
+ Sqrt[1 + x^2]]],x]
```

```
[Out] ((15903121112 + 5227043711*x + 220397520304*x^2 - 1415707308*x^3 + 40758198
2720*x^4 - 11794907136*x^5 + 248171986944*x^6 - 2099249152*x^7 + 6691356672
0*x^8)*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] + (-1176816782 + 66830366096*x + 1
984342244*x^2 + 96561463296*x^3 + 6568280064*x^4 + 10550149120*x^5 + 113036
```

```
4928*x^6 + 1968046080*x^7)*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1
 + x^2]]] + Sqrt[1 + x^2]*((2167822549 + 88760534448*x + 3694527828*x^2 + 3
08588576768*x^3 - 10745282560*x^4 + 214715203584*x^5 - 2099249152*x^6 + 669
13566720*x^7)*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]] + (21890925968 - 875910940*
x + 92024406016*x^2 + 6003097600*x^3 + 9566126080*x^4 + 1130364928*x^5 + 19
68046080*x^6)*Sqrt[x + Sqrt[1 + x^2]]*Sqrt[1 + Sqrt[x + Sqrt[1 + x^2]]]))/(
39729930240*(x + Sqrt[1 + x^2])^(7/2)) - (545*ArcTanh[Sqrt[1 + Sqrt[x + Sqr
t[1 + x^2]]]])/8192
```

fricas [A] time = 0.41, size = 159, normalized size = 0.50

$$\frac{1}{39729930240}\left(246005760\,x^4 + 377783296\,x^3 + 987937568\,x^2 + 2\left(123002880\,x^3 - 47596032\,x^2 + 578794096\,x - 588408391\right)\sqrt{x^2+1} - \left(1493606400\,x^4 + 391339520\,x^3 + 7419648592\,x^2 - \left(9857802240\,x^3 + 128933376\,x^2 + 25148050000\,x + 2167822549\right)\sqrt{x^2+1} + 3444246485\,x - 15903121112\right)\sqrt{x+\sqrt{x^2+1}} + 2654539406\,x + 21890925968\right)\sqrt{\sqrt{x+\sqrt{x^2+1}}+1} - \frac{545}{16384}\,\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}+1\right) + \frac{545}{16384}\,\log\left(\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(3/2)*(x+(x^2+1)^(1/2))^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))
^(1/2),x, algorithm="fricas")
```

```
[Out] 1/39729930240*(246005760*x^4 + 377783296*x^3 + 987937568*x^2 + 2*(123002880
*x^3 - 47596032*x^2 + 578794096*x - 588408391)*sqrt(x^2 + 1) - (1493606400*
x^4 + 391339520*x^3 + 7419648592*x^2 - (9857802240*x^3 + 128933376*x^2 + 25
148050000*x + 2167822549)*sqrt(x^2 + 1) + 3444246485*x - 15903121112)*sqrt(
x + sqrt(x^2 + 1)) + 2654539406*x + 21890925968)*sqrt(sqrt(x + sqrt(x^2 + 1
)) + 1) - 545/16384*log(sqrt(sqrt(x + sqrt(x^2 + 1)) + 1) + 1) + 545/16384*
log(sqrt(sqrt(x + sqrt(x^2 + 1)) + 1) - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(3/2)*(x+(x^2+1)^(1/2))^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))
^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \left(x^{2}+1\right)^{\frac{3}{2}} \sqrt{x+\sqrt{x^{2}+1}}\, \sqrt{1+\sqrt{x+\sqrt{x^{2}+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(3/2)*(x+(x^2+1)^(1/2))^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)
,x)
```

```
[Out] int((x^2+1)^(3/2)*(x+(x^2+1)^(1/2))^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))^(1/2)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{2}+1\right)^{\frac{3}{2}} \sqrt{x+\sqrt{x^{2}+1}}\, \sqrt{\sqrt{x+\sqrt{x^{2}+1}}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(3/2)*(x+(x^2+1)^(1/2))^(1/2)*(1+(x+(x^2+1)^(1/2))^(1/2))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^(3/2)*sqrt(x + sqrt(x^2 + 1))*sqrt(sqrt(x + sqrt(x^2 +
1)) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\sqrt{x+\sqrt{x^2+1}}+1}\,\left(x^2+1\right)^{3/2}\sqrt{x+\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2)*(x^2 + 1)^(3/2)*(x + (x^2 + 1)^
(1/2))^(1/2),x)
```

```
[Out] int(((x + (x^2 + 1)^(1/2))^(1/2) + 1)^(1/2)*(x^2 + 1)^(3/2)*(x + (x^2 + 1)^
(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x+\sqrt{x^2+1}}\,\left(x^2+1\right)^{\frac{3}{2}}\sqrt{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(3/2)*(x+(x**2+1)**(1/2))**(1/2)*(1+(x+(x**2+1)**(1/2))
**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x + sqrt(x**2 + 1))*(x**2 + 1)**(3/2)*sqrt(sqrt(x + sqrt(x**2
 + 1)) + 1), x)
```

3.2293

$$\int \frac{(a-5b+4x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{((-a+x)(-b+x))^{2/3}\left(b-a^5d-\left(1-5a^4d\right)x-10a^3dx^2+10a^2dx^3-5adx^4+dx^5\right)}\,dx$$

Optimal. Leaf size=317

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}a^2\sqrt[3]{d}-2\sqrt{3}a\sqrt[3]{d}x+\sqrt{3}\sqrt[3]{d}x^2}{a^2\sqrt[3]{d}+2\sqrt[3]{x(-a-b)+ab+x^2}-2a\sqrt[3]{d}x+\sqrt[3]{d}x^2}\right)}{d^{2/3}}+\frac{\log\left(a^3\sqrt[3]{d}-2a^2\sqrt[3]{d}x-a\sqrt[3]{x(-a-b)+ab+x^2}+a\sqrt[3]{d}x^2\right)}{d^{2/3}}-\frac{\log\left(\right.}{}$$

Rubi [F] time = 8.67, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a-5b+4x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{((-a+x)(-b+x))^{2/3}\left(b-a^5d-\left(1-5a^4d\right)x-10a^3dx^2+10a^2dx^3-5adx^4+dx^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a - 5*b + 4*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/(((-a + x)*(-b + x))
^(2/3)*(b - a^5*d - (1 - 5*a^4*d)*x - 10*a^3*d*x^2 + 10*a^2*d*x^3 - 5*a*d*x
^4 + d*x^5)),x]
```

```
[Out] (-3*(a - 5*b)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][x^9/((a
 - b + x^3)^(2/3)*(a*(1 - b/a) + x^3 - d*x^15)), x], x, (-a + x)^(1/3)])/((
a - x)*(b - x))^(2/3) + (12*a*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Subst][De
fer[Int][x^9/((a - b + x^3)^(2/3)*(-(a*(1 - b/a)) - x^3 + d*x^15)), x], x,
(-a + x)^(1/3)])/((a - x)*(b - x))^(2/3) + (12*(-a + x)^(2/3)*(-b + x)^(2/3
)*Defer[Subst][Defer[Int][x^12/((a - b + x^3)^(2/3)*(-(a*(1 - b/a)) - x^3 +
 d*x^15)), x], x, (-a + x)^(1/3)])/((a - x)*(b - x))^(2/3)
```

Rubi steps

$$\int \frac{(a-5b+4x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{((-a+x)(-b+x))^{2/3}\left(b-a^5d-\left(1-5a^4d\right)x-10a^3dx^2+10a^2dx^3-5adx^4+dx^5\right)}\,dx = \frac{\left((-a+x)^{2/3}(-b+x\right.}{}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x\right.}{}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x\right.}{}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x\right.}{}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x\right.}{}$$

$$= \frac{\left(4(-a+x)^{2/3}(-b+\right.}{}$$

$$= \frac{\left(12(-a+x)^{2/3}(-b\right.}{}$$

$$= \frac{\left(12(-a+x)^{2/3}(-b\right.}{}$$

$$= \frac{\left(12(-a+x)^{2/3}(-b\right.}{}$$

$$= \frac{\left(12(-a+x)^{2/3}(-b\right.}{}$$

Mathematica [F] time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{(a-5b+4x)\left(-a^3+3a^2x-3ax^2+x^3\right)}{((-a+x)(-b+x))^{2/3}\left(b-a^5d-\left(1-5a^4d\right)x-10a^3dx^2+10a^2dx^3-5adx^4+dx^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a - 5*b + 4*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/(((-a + x)*(-b
 + x))^(2/3)*(b - a^5*d - (1 - 5*a^4*d)*x - 10*a^3*d*x^2 + 10*a^2*d*x^3 - 5
*a*d*x^4 + d*x^5)),x]
```

```
[Out] Integrate[((a - 5*b + 4*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/(((-a + x)*(-b
 + x))^(2/3)*(b - a^5*d - (1 - 5*a^4*d)*x - 10*a^3*d*x^2 + 10*a^2*d*x^3 - 5
*a*d*x^4 + d*x^5)), x]
```

IntegrateAlgebraic [A] time = 5.38, size = 317, normalized size = 1.00

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}a^2\sqrt[3]{d}-2\sqrt{3}a\sqrt[3]{d}x+\sqrt{3}\sqrt[3]{d}x^2}{a^2\sqrt[3]{d}+2\sqrt[3]{x(-a-b)+ab+x^2}-2a\sqrt[3]{d}x+\sqrt[3]{d}x^2}\right)}{d^{2/3}}+\frac{\log\left(a^3\sqrt[3]{d}-2a^2\sqrt[3]{d}x-a\sqrt[3]{x(-a-b)+ab+x^2}+a\sqrt[3]{d}x^2\right)}{d^{2/3}}-\frac{\log\left(a^6d^{2/3}-4a^5d^{2/3}x+6a^4d^{2/3}x^2-4a^3d^{2/3}x^3+a^2\left(x(-a-b)+ab+x^2\right)^{2/3}+a^2d^{2/3}x^4+\sqrt[3]{x(-a-b)+ab+x^2}\left(a^4\sqrt[3]{d}-2a^3\sqrt[3]{d}x+a^2\sqrt[3]{d}x^2\right)\right)}{2d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a - 5*b + 4*x)*(-a^3 + 3*a^2*x - 3*a*x^2 + x^3))/(((-a
 + x)*(-b + x))^(2/3)*(b - a^5*d - (1 - 5*a^4*d)*x - 10*a^3*d*x^2 + 10*a^2*
d*x^3 - 5*a*d*x^4 + d*x^5)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*a^2*d^(1/3) - 2*Sqrt[3]*a*d^(1/3)*x + Sqrt[3]*d^(1
/3)*x^2)/(a^2*d^(1/3) - 2*a*d^(1/3)*x + d^(1/3)*x^2 + 2*(a*b + (-a - b)*x +
 x^2)^(1/3))]])/d^(2/3) + Log[a^3*d^(1/3) - 2*a^2*d^(1/3)*x + a*d^(1/3)*x^2
- a*(a*b + (-a - b)*x + x^2)^(1/3)]/d^(2/3) - Log[a^6*d^(2/3) - 4*a^5*d^(2/
3)*x + 6*a^4*d^(2/3)*x^2 - 4*a^3*d^(2/3)*x^3 + a^2*d^(2/3)*x^4 + a^2*(a*b +
 (-a - b)*x + x^2)^(2/3) + (a*b + (-a - b)*x + x^2)^(1/3)*(a^4*d^(1/3) - 2*
a^3*d^(1/3)*x + a^2*d^(1/3)*x^2)]/(2*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-5*b+4*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/((-a+x)*(-b+x))^(2/3)/(b-a
^5*d-(-5*a^4*d+1)*x-10*a^3*d*x^2+10*a^2*d*x^3-5*a*d*x^4+d*x^5),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{3}-3\,a^{2}x+3\,ax^{2}-x^{3}\right)(a-5\,b+4\,x)}{\left(a^{5}d+10\,a^{3}dx^{2}-10\,a^{2}dx^{3}+5\,adx^{4}-dx^{5}-\left(5\,a^{4}d-1\right)x-b\right)((a-x)(b-x))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-5*b+4*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/((-a+x)*(-b+x))^(2/3)/(b-a
^5*d-(-5*a^4*d+1)*x-10*a^3*d*x^2+10*a^2*d*x^3-5*a*d*x^4+d*x^5),x, algorithm
="giac")
```

```
[Out] integrate((a^3 - 3*a^2*x + 3*a*x^2 - x^3)*(a - 5*b + 4*x)/((a^5*d + 10*a^3*
d*x^2 - 10*a^2*d*x^3 + 5*a*d*x^4 - d*x^5 - (5*a^4*d - 1)*x - b)*((a - x)*(b
 - x))^(2/3)), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(a-5b+4x)\left(-a^{3}+3a^{2}x-3a\,x^{2}+x^{3}\right)}{\left((-a+x)\,(-b+x)\right)^{\frac{2}{3}}\left(b-a^{5}d-\left(-5a^{4}d+1\right)x-10a^{3}d\,x^{2}+10a^{2}d\,x^{3}-5ad\,x^{4}+d\,x^{5}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-5*b+4*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/((-a+x)*(-b+x))^(2/3)/(b-a^5*d-(
-5*a^4*d+1)*x-10*a^3*d*x^2+10*a^2*d*x^3-5*a*d*x^4+d*x^5),x)
```

```
[Out] int((a-5*b+4*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/((-a+x)*(-b+x))^(2/3)/(b-a^5*d-(
-5*a^4*d+1)*x-10*a^3*d*x^2+10*a^2*d*x^3-5*a*d*x^4+d*x^5),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{3}-3\,a^{2}x+3\,ax^{2}-x^{3}\right)(a-5\,b+4\,x)}{\left(a^{5}d+10\,a^{3}dx^{2}-10\,a^{2}dx^{3}+5\,adx^{4}-dx^{5}-\left(5\,a^{4}d-1\right)x-b\right)((a-x)(b-x))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-5*b+4*x)*(-a^3+3*a^2*x-3*a*x^2+x^3)/((-a+x)*(-b+x))^(2/3)/(b-a
^5*d-(-5*a^4*d+1)*x-10*a^3*d*x^2+10*a^2*d*x^3-5*a*d*x^4+d*x^5),x, algorithm
="maxima")
```

```
[Out] integrate((a^3 - 3*a^2*x + 3*a*x^2 - x^3)*(a - 5*b + 4*x)/((a^5*d + 10*a^3*
d*x^2 - 10*a^2*d*x^3 + 5*a*d*x^4 - d*x^5 - (5*a^4*d - 1)*x - b)*((a - x)*(b
 - x))^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(a-5\,b+4\,x)\,\left(a^3-3\,a^2\,x+3\,a\,x^2-x^3\right)}{((a-x)\,(b-x))^{2/3}\,\left(b-a^5\,d+d\,x^5+x\,\left(5\,a^4\,d-1\right)+10\,a^2\,d\,x^3-10\,a^3\,d\,x^2-5\,a\,d\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a - 5*b + 4*x)*(3*a*x^2 - 3*a^2*x + a^3 - x^3))/(((a - x)*(b - x))^(
2/3)*(b - a^5*d + d*x^5 + x*(5*a^4*d - 1) + 10*a^2*d*x^3 - 10*a^3*d*x^2 - 5
*a*d*x^4)),x)
```

```
[Out] int(-((a - 5*b + 4*x)*(3*a*x^2 - 3*a^2*x + a^3 - x^3))/(((a - x)*(b - x))^(
2/3)*(b - a^5*d + d*x^5 + x*(5*a^4*d - 1) + 10*a^2*d*x^3 - 10*a^3*d*x^2 - 5
*a*d*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-a+x)^3\,(a-5b+4x)}{((-a+x)\,(-b+x))^{\frac{2}{3}}\left(-a^5d+5a^4dx-10a^3dx^2+10a^2dx^3-5adx^4+b+dx^5-x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-5*b+4*x)*(-a**3+3*a**2*x-3*a*x**2+x**3)/((-a+x)*(-b+x))**(2/3)
/(b-a**5*d-(-5*a**4*d+1)*x-10*a**3*d*x**2+10*a**2*d*x**3-5*a*d*x**4+d*x**5)
,x)
```

```
[Out] Integral((-a + x)**3*(a - 5*b + 4*x)/(((-a + x)*(-b + x))**(2/3)*(-a**5*d +
 5*a**4*d*x - 10*a**3*d*x**2 + 10*a**2*d*x**3 - 5*a*d*x**4 + b + d*x**5 - x
)), x)
```

3.2294 $\int \frac{(2-2x+2x^2-3x^3+3x^4)\sqrt[3]{-x-x^3-x^4+x^6}}{(1+x)(-1+2x-2x^2+x^3)(-1-x^3+x^5)}\,dx$

Optimal. Leaf size=318

$$-\log\left(\sqrt[3]{x^6-x^4-x^3-x}+x\right)+\sqrt[3]{2}\log\left(\sqrt[3]{x^6-x^4-x^3-x}+\sqrt[3]{2}x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^6-x^4-x^3-x}}{\sqrt[3]{x^6-x^4-x^3-x}-2x}\right)-\sqrt[3]{2}\sqrt{}$$

Rubi [F] time = 15.70, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(2-2x+2x^2-3x^3+3x^4)\sqrt[3]{-x-x^3-x^4+x^6}}{(1+x)(-1+2x-2x^2+x^3)(-1-x^3+x^5)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((2 - 2*x + 2*x^2 - 3*x^3 + 3*x^4)*(-x - x^3 - x^4 + x^6)^(1/3))/((1 +
x)*(-1 + 2*x - 2*x^2 + x^3)*(-1 - x^3 + x^5)),x]
```

```
[Out] -(((-x - x^3 - x^4 + x^6)^(1/3)*Defer[Subst][Defer[Int][(-1 - x^6 - x^9 + x
^15)^(1/3)/(-1 + x), x], x, x^(1/3)])/(x^(1/3)*(-1 - x^2 - x^3 + x^5)^(1/3)
)) - (2*(-x - x^3 - x^4 + x^6)^(1/3)*Defer[Subst][Defer[Int][(-1 - x^6 - x^
9 + x^15)^(1/3)/(1 + x), x], x, x^(1/3)])/(x^(1/3)*(-1 - x^2 - x^3 + x^5)^(
1/3)) + (2*(1 + I*Sqrt[3])*(-x - x^3 - x^4 + x^6)^(1/3)*Defer[Subst][Defer[
Int][(-1 - x^6 - x^9 + x^15)^(1/3)/(-1 - I*Sqrt[3] + 2*x), x], x, x^(1/3)])
/(x^(1/3)*(-1 - x^2 - x^3 + x^5)^(1/3)) + ((1 - I*Sqrt[3])*(-x - x^3 - x^4
+ x^6)^(1/3)*Defer[Subst][Defer[Int][(-1 - x^6 - x^9 + x^15)^(1/3)/(1 - I*S
qrt[3] + 2*x), x], x, x^(1/3)])/(x^(1/3)*(-1 - x^2 - x^3 + x^5)^(1/3)) + (2
*(1 - I*Sqrt[3])*(-x - x^3 - x^4 + x^6)^(1/3)*Defer[Subst][Defer[Int][(-1 -
 x^6 - x^9 + x^15)^(1/3)/(-1 + I*Sqrt[3] + 2*x), x], x, x^(1/3)])/(x^(1/3)*
(-1 - x^2 - x^3 + x^5)^(1/3)) + ((1 + I*Sqrt[3])*(-x - x^3 - x^4 + x^6)^(1/
3)*Defer[Subst][Defer[Int][(-1 - x^6 - x^9 + x^15)^(1/3)/(1 + I*Sqrt[3] + 2
*x), x], x, x^(1/3)])/(x^(1/3)*(-1 - x^2 - x^3 + x^5)^(1/3)) + (2*(-x - x^3
 - x^4 + x^6)^(1/3)*Defer[Subst][Defer[Int][(-1 - x^6 - x^9 + x^15)^(1/3)/(
(1 - I*Sqrt[3])^(1/3) + (-2)^(1/3)*x), x], x, x^(1/3)])/((1 - I*Sqrt[3])^(2
/3)*x^(1/3)*(-1 - x^2 - x^3 + x^5)^(1/3)) + (2*(-x - x^3 - x^4 + x^6)^(1/3)
*Defer[Subst][Defer[Int][(-1 - x^6 - x^9 + x^15)^(1/3)/((1 + I*Sqrt[3])^(1/
3) + (-2)^(1/3)*x), x], x, x^(1/3)])/((1 + I*Sqrt[3])^(2/3)*x^(1/3)*(-1 - x
^2 - x^3 + x^5)^(1/3)) + (2*(-x - x^3 - x^4 + x^6)^(1/3)*Defer[Subst][Defer
[Int][(-1 - x^6 - x^9 + x^15)^(1/3)/((1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x), x]
, x, x^(1/3)])/((1 - I*Sqrt[3])^(2/3)*x^(1/3)*(-1 - x^2 - x^3 + x^5)^(1/3))
 + (2*(-x - x^3 - x^4 + x^6)^(1/3)*Defer[Subst][Defer[Int][(-1 - x^6 - x^9
+ x^15)^(1/3)/((1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x), x], x, x^(1/3)])/((1 + I
*Sqrt[3])^(2/3)*x^(1/3)*(-1 - x^2 - x^3 + x^5)^(1/3)) + (2*(-x - x^3 - x^4
+ x^6)^(1/3)*Defer[Subst][Defer[Int][(-1 - x^6 - x^9 + x^15)^(1/3)/((1 - I*
Sqrt[3])^(1/3) - (-1)^(2/3)*2^(1/3)*x), x], x, x^(1/3)])/((1 - I*Sqrt[3])^(
2/3)*x^(1/3)*(-1 - x^2 - x^3 + x^5)^(1/3)) + (2*(-x - x^3 - x^4 + x^6)^(1/3
)*Defer[Subst][Defer[Int][(-1 - x^6 - x^9 + x^15)^(1/3)/((1 + I*Sqrt[3])^(1
/3) - (-1)^(2/3)*2^(1/3)*x), x], x, x^(1/3)])/((1 + I*Sqrt[3])^(2/3)*x^(1/3
)*(-1 - x^2 - x^3 + x^5)^(1/3)) - (9*(-x - x^3 - x^4 + x^6)^(1/3)*Defer[Sub
st][Defer[Int][(x^6*(-1 - x^6 - x^9 + x^15)^(1/3))/(-1 - x^9 + x^15), x], x
, x^(1/3)])/(x^(1/3)*(-1 - x^2 - x^3 + x^5)^(1/3)) + (15*(-x - x^3 - x^4 +
x^6)^(1/3)*Defer[Subst][Defer[Int][(x^12*(-1 - x^6 - x^9 + x^15)^(1/3))/(-1
 - x^9 + x^15), x], x, x^(1/3)])/(x^(1/3)*(-1 - x^2 - x^3 + x^5)^(1/3))
```

Rubi steps

$$\int \frac{\left(2-2x+2x^2-3x^3+3x^4\right)\sqrt[3]{-x-x^3-x^4+x^6}}{(1+x)\left(-1+2x-2x^2+x^3\right)\left(-1-x^3+x^5\right)}\,dx = \frac{\sqrt[3]{-x-x^3-x^4+x^6}\int \frac{\sqrt[3]{x}\left(2-2x+2x^2-3x^3+3x^4\right)\sqrt[3]{-1-x^2-x^3+}}{(1+x)\left(-1+2x-2x^2+x^3\right)\left(-1-x^3+x^5\right)}}{\sqrt[3]{x}\sqrt[3]{-1-x^2-x^3+x^5}}$$

$$= \frac{\left(3\sqrt[3]{-x-x^3-x^4+x^6}\right)\text{Subst}\left(\int \frac{x^3\left(2-2x^3+2x^6-3x^9+3x^{12}}{\left(1+x^3\right)\left(-1+2x^3-2x^6+\right.}\right.}{\sqrt[3]{x}\sqrt[3]{-1-x^2-x^3+x^5}}$$

$$= \frac{\left(3\sqrt[3]{-x-x^3-x^4+x^6}\right)\text{Subst}\left(\int\left(-\frac{\sqrt[3]{-1-x^6-x^9+x^{15}}}{3(-1+x)}-\right.\right.}{}$$

$$= -\frac{\sqrt[3]{-x-x^3-x^4+x^6}\,\text{Subst}\left(\int \frac{\sqrt[3]{-1-x^6-x^9+x^{15}}}{-1+x}\,dx, x, \right.}{\sqrt[3]{x}\sqrt[3]{-1-x^2-x^3+x^5}}$$

$$= -\frac{\sqrt[3]{-x-x^3-x^4+x^6}\,\text{Subst}\left(\int \frac{\sqrt[3]{-1-x^6-x^9+x^{15}}}{-1+x}\,dx, x, \right.}{\sqrt[3]{x}\sqrt[3]{-1-x^2-x^3+x^5}}$$

$$= -\frac{\sqrt[3]{-x-x^3-x^4+x^6}\,\text{Subst}\left(\int \frac{\sqrt[3]{-1-x^6-x^9+x^{15}}}{-1+x}\,dx, x, \right.}{\sqrt[3]{x}\sqrt[3]{-1-x^2-x^3+x^5}}$$

$$= -\frac{\sqrt[3]{-x-x^3-x^4+x^6}\,\text{Subst}\left(\int \frac{\sqrt[3]{-1-x^6-x^9+x^{15}}}{-1+x}\,dx, x, \right.}{\sqrt[3]{x}\sqrt[3]{-1-x^2-x^3+x^5}}$$

$$= -\frac{\sqrt[3]{-x-x^3-x^4+x^6}\,\text{Subst}\left(\int \frac{\sqrt[3]{-1-x^6-x^9+x^{15}}}{-1+x}\,dx, x, \right.}{\sqrt[3]{x}\sqrt[3]{-1-x^2-x^3+x^5}}$$

Mathematica [F] time = 1.93, size = 0, normalized size = 0.00

$$\int \frac{\left(2-2x+2x^2-3x^3+3x^4\right)\sqrt[3]{-x-x^3-x^4+x^6}}{(1+x)\left(-1+2x-2x^2+x^3\right)\left(-1-x^3+x^5\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((2 - 2*x + 2*x^2 - 3*x^3 + 3*x^4)*(-x - x^3 - x^4 + x^6)^(1/3))/
((1 + x)*(-1 + 2*x - 2*x^2 + x^3)*(-1 - x^3 + x^5)),x]
```

```
[Out] Integrate[((2 - 2*x + 2*x^2 - 3*x^3 + 3*x^4)*(-x - x^3 - x^4 + x^6)^(1/3))/
((1 + x)*(-1 + 2*x - 2*x^2 + x^3)*(-1 - x^3 + x^5)), x]
```

IntegrateAlgebraic [A] time = 5.10, size = 318, normalized size = 1.00

$$-\log\left(\sqrt[3]{x^6-x^4-x^3-x}+x\right)+\sqrt[3]{2}\log\left(\sqrt[3]{x^6-x^4-x^3-x}+\sqrt[3]{2}x\right)+\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^6-x^4-x^3-x}}{\sqrt[3]{x^6-x^4-x^3-x}-2x}\right)-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^6-x^4-x^3-x}}{\sqrt[3]{x^6-x^4-x^3-x}-2\sqrt[3]{2}x}\right)+\frac{1}{2}\log\left(x^2-\sqrt[3]{x^6-x^4-x^3-x}\,x+\left(x^6-x^4-x^3-x\right)^{2/3}\right)-\frac{\log\left(2^{2/3}x^2-\sqrt[3]{2}\sqrt[3]{x^6-x^4-x^3-x}\,x+\left(x^6-x^4-x^3-x\right)^{2/3}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 - 2*x + 2*x^2 - 3*x^3 + 3*x^4)*(-x - x^3 - x^4 + x^6
)^(1/3))/((1 + x)*(-1 + 2*x - 2*x^2 + x^3)*(-1 - x^3 + x^5)),x]
```

```
[Out] Sqrt[3]*ArcTan[(Sqrt[3]*(-x - x^3 - x^4 + x^6)^(1/3))/(-2*x + (-x - x^3 - x
^4 + x^6)^(1/3))] - 2^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*(-x - x^3 - x^4 + x^6)^
```

```
(1/3))/(-2*2^(1/3)*x + (-x - x^3 - x^4 + x^6)^(1/3))] - Log[x + (-x - x^3 -
 x^4 + x^6)^(1/3)] + 2^(1/3)*Log[2^(1/3)*x + (-x - x^3 - x^4 + x^6)^(1/3)]
+ Log[x^2 - x*(-x - x^3 - x^4 + x^6)^(1/3) + (-x - x^3 - x^4 + x^6)^(2/3)]/
2 - Log[2^(2/3)*x^2 - 2^(1/3)*x*(-x - x^3 - x^4 + x^6)^(1/3) + (-x - x^3 -
x^4 + x^6)^(2/3)]/2^(2/3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-3*x^3+2*x^2-2*x+2)*(x^6-x^4-x^3-x)^(1/3)/(1+x)/(x^3-2*x^2+
2*x-1)/(x^5-x^3-1),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6 - x^4 - x^3 - x\right)^{\frac{1}{3}} \left(3\,x^4 - 3\,x^3 + 2\,x^2 - 2\,x + 2\right)}{\left(x^5 - x^3 - 1\right)\left(x^3 - 2\,x^2 + 2\,x - 1\right)(x + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-3*x^3+2*x^2-2*x+2)*(x^6-x^4-x^3-x)^(1/3)/(1+x)/(x^3-2*x^2+
2*x-1)/(x^5-x^3-1),x, algorithm="giac")
```

```
[Out] integrate((x^6 - x^4 - x^3 - x)^(1/3)*(3*x^4 - 3*x^3 + 2*x^2 - 2*x + 2)/((x
^5 - x^3 - 1)*(x^3 - 2*x^2 + 2*x - 1)*(x + 1)), x)
```

maple [C] time = 107.01, size = 3484, normalized size = 10.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4-3*x^3+2*x^2-2*x+2)*(x^6-x^4-x^3-x)^(1/3)/(1+x)/(x^3-2*x^2+2*x-1)
/(x^5-x^3-1),x)
```

```
[Out] -1/2*ln(-(17352778151361340*RootOf(_Z^3-2)^4*RootOf(4*RootOf(_Z^3-2)^2+2*_Z
*RootOf(_Z^3-2)+_Z^2)*x^5+19398769220353248*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*
RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^3*x^5-17352778151361340*RootOf(_Z^3-2
)^4*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*x^3-193987692203532
48*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^3*x
^3-9977847437032705*RootOf(_Z^3-2)^4*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf
(_Z^3-2)+_Z^2)*x^2-11154292301703176*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf
(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^3*x^2-9544027983248737O*RootOf(_Z^3-2)^2*x^
5-106693230711942864*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*Ro
otOf(_Z^3-2)*x^5+101047682304680178*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_
Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*(x^6-x^4-x^3-x)^(2/3)-17352778151361340*RootO
f(_Z^3-2)^4*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)-19398769220
353248*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)
^3+95440279832487370*RootOf(_Z^3-2)^2*x^3+106693230711942864*RootOf(4*RootO
f(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x^3-43381945378403350*
RootOf(_Z^3-2)^2*x^2-48496923050883120*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootO
f(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x^2+404190729218720712*RootOf(_Z^3-2)*(x^6-x
^4-x^3-x)^(1/3)*x+549795936176579706*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(
_Z^3-2)+_Z^2)*(x^6-x^4-x^3-x)^(1/3)*x-695401143134438700*(x^6-x^4-x^3-x)^(2
/3)+95440279832487370*RootOf(_Z^3-2)^2+106693230711942864*RootOf(4*RootOf(_
Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2))/(1+x)^2/(-1+x)/(x^2-x+1)
)*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)-ln(-(1735277815136134
```

0*RootOf(_Z^3-2)^4*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*x^5+
1939876922035324 8*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)^2*Roo
tOf(_Z^3-2)^3*x^5-17352778151361340*RootOf(_Z^3-2)^4*RootOf(4*RootOf(_Z^3-2
)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*x^3-19398769220353248*RootOf(4*RootOf(_Z^3-2)
^2+2*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^3*x^3-99778474370327705*RootO
f(_Z^3-2)^4*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*x^2-1115429
23017031176*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z
^3-2)^3*x^2-95440279832487370*RootOf(_Z^3-2)^2*x^5-106693230711942864*RootO
f(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x^5+101047682
304680178*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2
)^2*(x^6-x^4-x^3-x)^(2/3)-17352778151361340*RootOf(_Z^3-2)^4*RootOf(4*RootO
f(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)-19398769220353248*RootOf(4*RootOf(_Z^
3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^3+95440279832487370*RootO
f(_Z^3-2)^2*x^3+106693230711942864*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z
^3-2)+_Z^2)*RootOf(_Z^3-2)*x^3-43381945378403350*RootOf(_Z^3-2)^2*x^2-48496
923050883120*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^
3-2)*x^2+404190729218720712*RootOf(_Z^3-2)*(x^6-x^4-x^3-x)^(1/3)*x+54979593
6176579706*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*(x^6-x^4-x^3
-x)^(1/3)*x-695401143134438700*(x^6-x^4-x^3-x)^(2/3)+95440279832487370*Root
Of(_Z^3-2)^2+106693230711942864*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-
2)+_Z^2)*RootOf(_Z^3-2))/(1+x)^2/(-1+x)/(x^2-x+1))*RootOf(_Z^3-2)+RootOf(_Z
^3-2)*ln((77595076881412992*RootOf(_Z^3-2)^4*RootOf(4*RootOf(_Z^3-2)^2+2*_Z
*RootOf(_Z^3-2)+_Z^2)*x^5+17352778151361340*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*
RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^3*x^5-77595076881412992*RootOf(_Z^3-2
)^4*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*x^3-173527781513613
40*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^3*x
^3-446171692068124704*RootOf(_Z^3-2)^4*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootO
f(_Z^3-2)+_Z^2)*x^2-99778474370327705*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf
(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^3*x^2+737153230373423424*RootOf(_Z^3-2)^2*x
^5+164851392437932730*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*R
ootOf(_Z^3-2)*x^5+202095364609360356*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(
_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*(x^6-x^4-x^3-x)^(2/3)-77595076881412992*Root
Of(_Z^3-2)^4*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)-1735277815
1361340*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2
)^3-737153230373423424*RootOf(_Z^3-2)^2*x^3-164851392437932730*RootOf(4*Roo
tOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x^3-1590699076068966
336*RootOf(_Z^3-2)^2*x^2-355731952102907470*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*
RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x^2+808381458437441424*RootOf(_Z^3-2)*(
x^6-x^4-x^3-x)^(1/3)*x-695401143134438700*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*Ro
otOf(_Z^3-2)+_Z^2)*(x^6-x^4-x^3-x)^(1/3)*x+2199183744706318824*(x^6-x^4-x^3
-x)^(2/3)-737153230373423424*RootOf(_Z^3-2)^2-164851392437932730*RootOf(4*R
ootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2))/(1+x)^2/(-1+x)/(x
^2-x+1))-ln(-(2378228285152225516*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^
3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^5-2378228285152225516*RootOf(4*RootOf(_Z^3-
2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^3-13674812639625296717*
RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^2-
24953827777837657564*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*Ro
otOf(_Z^3-2)^2*x^5-2378228285152225516*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootO
f(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4+24953827777837657564*RootOf(4*RootOf(_Z^
3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^3+27374521598343300905 6
*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*(x^6-
x^4-x^3-x)^(2/3)+65089065472220569699 6*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootO
f(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*(x^6-x^4-x^3-x)^(1/3)*x+322446188180271501
072*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^
2-13326631387188689049 92*x^5+24953827777837657564*RootOf(_Z^3-2)^2*RootOf(4
*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)+13326631387188689049 92*x^3-1508
581754955090751760*(x^6-x^4-x^3-x)^(2/3)+10949808639337320362 24*x*(x^6-x^4-
x^3-x)^(1/3)+23847656166548180405 12*x^2+13326631387188689049 92)/(x^5-x^3-1)
)+ln((-8767520649466242796*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z

```
^2)^2*RootOf(_Z^3-2)^4*x^5+8767520649466242796*RootOf(4*RootOf(_Z^3-2)^2+2*
_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^3+50413243734430896077*RootOf(
4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^2+1362692
66312298092972*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_
Z^3-2)^2*x^5+8767520649466242796*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3
-2)+_Z^2)^2*RootOf(_Z^3-2)^4-136269266312298092972*RootOf(4*RootOf(_Z^3-2)^
2+2*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^3+136872607991716504528*Root
Of(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*(x^6-x^4-x
^3-x)^(2/3)-188572719369386343970*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^
3-2)+_Z^2)*RootOf(_Z^3-2)^2*(x^6-x^4-x^3-x)^(1/3)*x-123792352423379264190*R
ootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^2-713
46848554566765480*x^5-136269266312298092972*RootOf(_Z^3-2)^2*RootOf(4*RootO
f(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)+71346848554566765480*x^3+130178130944
4411393992*(x^6-x^4-x^3-x)^(2/3)+547490431966866018112*x*(x^6-x^4-x^3-x)^(1
/3)+52321022273348961352*x^2+71346848554566765480)/(x^5-x^3-1))+1/4*ln((-87
67520649466242796*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)^2*Roo
tOf(_Z^3-2)^4*x^5+8767520649466242796*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf
(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^3+50413243734430896077*RootOf(4*RootOf(
_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^2+1362692663122980
92972*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*
x^5+8767520649466242796*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)
^2*RootOf(_Z^3-2)^4-136269266312298092972*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*Ro
otOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^3+136872607991716504528*RootOf(4*Root
Of(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*(x^6-x^4-x^3-x)^(2/
3)-188572719369386343970*RootOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2
)*RootOf(_Z^3-2)^2*(x^6-x^4-x^3-x)^(1/3)*x-123792352423379264190*RootOf(4*R
ootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^2-713468485545
66765480*x^5-136269266312298092972*RootOf(_Z^3-2)^2*RootOf(4*RootOf(_Z^3-2)
^2+2*_Z*RootOf(_Z^3-2)+_Z^2)+71346848554566765480*x^3+130178130944441139399
2*(x^6-x^4-x^3-x)^(2/3)+547490431966866018112*x*(x^6-x^4-x^3-x)^(1/3)+52321
022273348961352*x^2+71346848554566765480)/(x^5-x^3-1))*RootOf(_Z^3-2)^2*Roo
tOf(4*RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6 - x^4 - x^3 - x\right)^{\frac{1}{3}}\left(3\,x^4 - 3\,x^3 + 2\,x^2 - 2\,x + 2\right)}{\left(x^5 - x^3 - 1\right)\left(x^3 - 2\,x^2 + 2\,x - 1\right)(x + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4-3*x^3+2*x^2-2*x+2)*(x^6-x^4-x^3-x)^(1/3)/(1+x)/(x^3-2*x^2+
2*x-1)/(x^5-x^3-1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 - x^4 - x^3 - x)^(1/3)*(3*x^4 - 3*x^3 + 2*x^2 - 2*x + 2)/((x
^5 - x^3 - 1)*(x^3 - 2*x^2 + 2*x - 1)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(x^6 - x^4 - x^3 - x\right)^{1/3}\,\left(3\,x^4 - 3\,x^3 + 2\,x^2 - 2\,x + 2\right)}{(x + 1)\,\left(-x^5 + x^3 + 1\right)\,\left(x^3 - 2\,x^2 + 2\,x - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^6 - x^3 - x^4 - x)^(1/3)*(2*x^2 - 2*x - 3*x^3 + 3*x^4 + 2))/((x +
1)*(x^3 - x^5 + 1)*(2*x - 2*x^2 + x^3 - 1)),x)
```

```
[Out] int(-((x^6 - x^3 - x^4 - x)^(1/3)*(2*x^2 - 2*x - 3*x^3 + 3*x^4 + 2))/((x +
1)*(x^3 - x^5 + 1)*(2*x - 2*x^2 + x^3 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x\left(x^{5}-x^{3}-x^{2}-1\right)}\,\left(3x^{4}-3x^{3}+2x^{2}-2x+2\right)}{(x-1)\,(x+1)\left(x^{2}-x+1\right)\left(x^{5}-x^{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4-3*x**3+2*x**2-2*x+2)*(x**6-x**4-x**3-x)**(1/3)/(1+x)/(x**
3-2*x**2+2*x-1)/(x**5-x**3-1),x)
```

```
[Out] Integral((x*(x**5 - x**3 - x**2 - 1))**(1/3)*(3*x**4 - 3*x**3 + 2*x**2 - 2*
x + 2)/((x - 1)*(x + 1)*(x**2 - x + 1)*(x**5 - x**3 - 1)), x)
```

3.2295 $\int \frac{1}{(-b+ax)\sqrt[3]{-b^2x^2+a^3x^3}}\,dx$

Optimal. Leaf size=319

$$-\frac{i\left(\sqrt{3}-i\right)\log\left(\sqrt[3]{-1}\,\sqrt[3]{a^3x^3-b^2x^2}+\sqrt[3]{a}\,x\sqrt[3]{a^2-b}\right)}{2\sqrt[3]{a}\,b\sqrt[3]{a^2-b}}+\frac{\sqrt{-3+3i\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{a}\,x\sqrt[3]{a^2-b}}{\sqrt[3]{a}\,x\sqrt[3]{a^2-b}-2\sqrt[3]{-1}\,\sqrt[3]{a^3x^3-b^2x^2}}\right)}{\sqrt{2}\,\sqrt[3]{a}\,b\sqrt[3]{a^2-b}}+\frac{\left(1+i\sqrt{\right.}}{}$$

Rubi [A] time = 0.14, antiderivative size = 291, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2056, 91}

$$-\frac{x^{2/3}\sqrt[3]{a^3x-b^2}\,\log(ax-b)}{2\sqrt[3]{a}\,b\sqrt[3]{a^2-b}\,\sqrt[3]{a^3x^3-b^2x^2}}+\frac{3x^{2/3}\sqrt[3]{a^3x-b^2}\,\log\left(\frac{\sqrt[3]{a^3x-b^2}}{\sqrt[3]{a}\,\sqrt[3]{a^2-b}}-\sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b\sqrt[3]{a^2-b}\,\sqrt[3]{a^3x^3-b^2x^2}}+\frac{\sqrt{3}\,x^{2/3}\sqrt[3]{a^3x-b^2}\,\tan^{-1}\left(\frac{2\sqrt[3]{a^3x-b^2}}{\sqrt{3}\,\sqrt[3]{a}\,\sqrt[3]{x}\,\sqrt[3]{a^2-b}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{a}\,b\sqrt[3]{a^2-b}\,\sqrt[3]{a^3x^3-b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-b + a*x)*(-(b^2*x^2) + a^3*x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*x^(2/3)*(-b^2 + a^3*x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(-b^2 + a^3*x)^
(1/3))/(Sqrt[3]*a^(1/3)*(a^2 - b)^(1/3)*x^(1/3))])/(a^(1/3)*(a^2 - b)^(1/3)
*b*(-(b^2*x^2) + a^3*x^3)^(1/3)) - (x^(2/3)*(-b^2 + a^3*x)^(1/3)*Log[-b + a
*x])/(2*a^(1/3)*(a^2 - b)^(1/3)*b*(-(b^2*x^2) + a^3*x^3)^(1/3)) + (3*x^(2/3
)*(-b^2 + a^3*x)^(1/3)*Log[-x^(1/3) + (-b^2 + a^3*x)^(1/3)/(a^(1/3)*(a^2 -
b)^(1/3))])/(2*a^(1/3)*(a^2 - b)^(1/3)*b*(-(b^2*x^2) + a^3*x^3)^(1/3))
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{(-b+ax)\sqrt[3]{-b^2x^2+a^3x^3}}\,dx=\frac{\left(x^{2/3}\sqrt[3]{-b^2+a^3x}\right)\int \frac{1}{x^{2/3}(-b+ax)\sqrt[3]{-b^2+a^3x}}\,dx}{\sqrt[3]{-b^2x^2+a^3x^3}}$$

$$=\frac{\sqrt{3}\,x^{2/3}\sqrt[3]{-b^2+a^3x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-b^2+a^3x}}{\sqrt{3}\,\sqrt[3]{a}\,\sqrt[3]{a^2-b}\,\sqrt[3]{x}}\right)}{\sqrt[3]{a}\,\sqrt[3]{a^2-b}\,b\sqrt[3]{-b^2x^2+a^3x^3}}-\frac{x^{2/3}\sqrt[3]{-b^2+a^3x}\,\log(-b+}{2\sqrt[3]{a}\,\sqrt[3]{a^2-b}\,b\sqrt[3]{-b^2x^2+a^3}}$$

Mathematica [C] time = 0.03, size = 56, normalized size = 0.18

$$-\frac{3x\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{\left(a^3-ab\right)x}{a^3x-b^2}\right)}{b\sqrt[3]{x^2\left(a^3x-b^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-b + a*x)*(-(b^2*x^2) + a^3*x^3)^(1/3)),x]
```

```
[Out] (-3*x*Hypergeometric2F1[1/3, 1, 4/3, ((a^3 - a*b)*x)/(-b^2 + a^3*x)])/(b*(x
^2*(-b^2 + a^3*x))^(1/3))
```

IntegrateAlgebraic [A] time = 2.07, size = 365, normalized size = 1.14

$$-\frac{i\left(\sqrt{3}-i\right)\log\left(2\sqrt[3]{a}\,x\sqrt[3]{a^2-b}+\left(1+i\sqrt{3}\right)\sqrt[3]{a^3x^3-b^2x^2}\right)}{2\sqrt[3]{a}\,b\sqrt[3]{a^2-b}}+\frac{\sqrt{-3+3i\sqrt{3}}\,\tan^{-1}\left(\frac{3\sqrt[3]{a}\,x\sqrt[3]{a^2-b}}{-\sqrt{3}\sqrt[3]{a^3x^3-b^2x^2}-3i\sqrt[3]{a^3x^3-b^2x^2}+\sqrt{3}\sqrt[3]{a}\,x\sqrt[3]{a^2-b}}\right)}{\sqrt{2}\sqrt[3]{a}\,b\sqrt[3]{a^2-b}}+\frac{\left(1+i\sqrt{3}\right)\log\left(\left(\sqrt{3}+i\right)\left(a^3x^3-b^2x^2\right)^{2/3}-2ia^{2/3}x^2\left(a^2-b\right)^{2/3}+\sqrt[3]{a}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a^2-b}\sqrt[3]{a^3x^3-b^2x^2}\right)}{4\sqrt[3]{a}\,b\sqrt[3]{a^2-b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-b + a*x)*(-(b^2*x^2) + a^3*x^3)^(1/3)),x]
```

```
[Out] (Sqrt[-3 + (3*I)*Sqrt[3]]*ArcTan[(3*a^(1/3)*(a^2 - b)^(1/3)*x)/(Sqrt[3]*a^(
1/3)*(a^2 - b)^(1/3)*x - (3*I)*(-(b^2*x^2) + a^3*x^3)^(1/3) - Sqrt[3]*(-(b^
2*x^2) + a^3*x^3)^(1/3))])/(Sqrt[2]*a^(1/3)*(a^2 - b)^(1/3)*b) - ((I/2)*(-I
 + Sqrt[3])*Log[2*a^(1/3)*(a^2 - b)^(1/3)*x + (1 + I*Sqrt[3])*(-(b^2*x^2) +
 a^3*x^3)^(1/3)])/(a^(1/3)*(a^2 - b)^(1/3)*b) + ((1 + I*Sqrt[3])*Log[(-2*I)
*a^(2/3)*(a^2 - b)^(2/3)*x^2 + a^(1/3)*(a^2 - b)^(1/3)*(I*x - Sqrt[3]*x)*(-
(b^2*x^2) + a^3*x^3)^(1/3) + (I + Sqrt[3])*(-(b^2*x^2) + a^3*x^3)^(2/3)])/(
4*a^(1/3)*(a^2 - b)^(1/3)*b)
```

fricas [A] time = 0.87, size = 549, normalized size = 1.72

$$\left[\frac{\sqrt{3}\left(a^3-ab\right)\sqrt{-\frac{1}{\left(a^3-ab\right)^{\frac{2}{3}}}}\log\left(-\frac{2b^2x-\left(3a^3-ab\right)x^2+3\left(a^3x^3-b^2x^2\right)^{\frac{1}{3}}\left(a^3-ab\right)^{\frac{2}{3}}x+\sqrt{3}\left(\left(a^3-ab\right)^{\frac{4}{3}}x^2+\left(a^3x^3-b^2x^2\right)^{\frac{1}{3}}\left(a^3-ab\right)x-2\left(a^3x^3-b^2x^2\right)^{\frac{2}{3}}\left(a^3-ab\right)^{\frac{2}{3}}\right)\sqrt{-\frac{1}{\left(a^3-ab\right)^{\frac{2}{3}}}}}{ax^2-bx}\right)+2\left(a^3-ab\right)^{\frac{2}{3}}\log\left(-\frac{\left(a^3-ab\right)^{\frac{1}{3}}x-\left(a^3x^3-b^2x^2\right)^{\frac{1}{3}}}{x}\right)-\left(a^3-ab\right)^{\frac{2}{3}}\log\left(\frac{\left(a^3-ab\right)^{\frac{2}{3}}x^2+\left(a^3x^3-b^2x^2\right)^{\frac{1}{3}}\left(a^3-ab\right)^{\frac{1}{3}}x+\left(a^3x^3-b^2x^2\right)^{\frac{2}{3}}}{x^2}\right)}{2\left(a^3b-ab^2\right)}, \frac{2\sqrt{3}\left(a^3-ab\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(a^3-ab\right)^{\frac{1}{3}}x+2\left(a^3x^3-b^2x^2\right)^{\frac{1}{3}}\right)}{3\left(a^3-ab\right)^{\frac{1}{3}}x}\right)+2\left(a^3-ab\right)^{\frac{2}{3}}\log\left(-\frac{\left(a^3-ab\right)^{\frac{1}{3}}x-\left(a^3x^3-b^2x^2\right)^{\frac{1}{3}}}{x}\right)-\left(a^3-ab\right)^{\frac{2}{3}}\log\left(\frac{\left(a^3-ab\right)^{\frac{2}{3}}x^2+\left(a^3x^3-b^2x^2\right)^{\frac{1}{3}}\left(a^3-ab\right)^{\frac{1}{3}}x+\left(a^3x^3-b^2x^2\right)^{\frac{2}{3}}}{x^2}\right)}{2\left(a^3b-ab^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-b)/(a^3*x^3-b^2*x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*(a^3 - a*b)*sqrt(-1/(a^3 - a*b)^(2/3))*log(-(2*b^2*x - (3*a^3
 - a*b)*x^2 + 3*(a^3*x^3 - b^2*x^2)^(1/3)*(a^3 - a*b)^(2/3)*x + sqrt(3)*((a
^3 - a*b)^(4/3)*x^2 + (a^3*x^3 - b^2*x^2)^(1/3)*(a^3 - a*b)*x - 2*(a^3*x^3
- b^2*x^2)^(2/3)*(a^3 - a*b)^(2/3))*sqrt(-1/(a^3 - a*b)^(2/3)))/(a*x^2 - b*
x)) + 2*(a^3 - a*b)^(2/3)*log(-((a^3 - a*b)^(1/3)*x - (a^3*x^3 - b^2*x^2)^(
1/3))/x) - (a^3 - a*b)^(2/3)*log(((a^3 - a*b)^(2/3)*x^2 + (a^3*x^3 - b^2*x^
2)^(1/3)*(a^3 - a*b)^(1/3)*x + (a^3*x^3 - b^2*x^2)^(2/3))/x^2))/(a^3*b - a*
b^2), 1/2*(2*sqrt(3)*(a^3 - a*b)^(2/3)*arctan(1/3*sqrt(3)*((a^3 - a*b)^(1/3
)*x + 2*(a^3*x^3 - b^2*x^2)^(1/3))/((a^3 - a*b)^(1/3)*x)) + 2*(a^3 - a*b)^(
2/3)*log(-((a^3 - a*b)^(1/3)*x - (a^3*x^3 - b^2*x^2)^(1/3))/x) - (a^3 - a*b
)^(2/3)*log(((a^3 - a*b)^(2/3)*x^2 + (a^3*x^3 - b^2*x^2)^(1/3)*(a^3 - a*b)^
(1/3)*x + (a^3*x^3 - b^2*x^2)^(2/3))/x^2))/(a^3*b - a*b^2)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-b)/(a^3*x^3-b^2*x^2)^(1/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax-b\right)\left(a^3x^3-b^2x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-b)/(a^3*x^3-b^2*x^2)^(1/3),x)
[Out] int(1/(a*x-b)/(a^3*x^3-b^2*x^2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^3x^3 - b^2x^2\right)^{\frac{1}{3}}(ax - b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-b)/(a^3*x^3-b^2*x^2)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((a^3*x^3 - b^2*x^2)^(1/3)*(a*x - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{1}{\left(a^3\,x^3 - b^2\,x^2\right)^{1/3}\,(b - a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a^3*x^3 - b^2*x^2)^(1/3)*(b - a*x)),x)
[Out] -int(1/((a^3*x^3 - b^2*x^2)^(1/3)*(b - a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x^2\left(a^3x - b^2\right)}\,(ax - b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-b)/(a**3*x**3-b**2*x**2)**(1/3),x)
[Out] Integral(1/((x**2*(a**3*x - b**2))**(1/3)*(a*x - b)), x)
```

3.2296 $\int \frac{(-b+x)(-4a+b+3x)}{\sqrt[3]{(-a+x)(-b+x)^2}\left(b^4+ad-\left(4b^3+d\right)x+6b^2x^2-4bx^3+x^4\right)}\,dx$

Optimal. Leaf size=319

$$\frac{\log\left(-\sqrt[3]{d}\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}+b^2-2bx+x^2\right)}{d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a}}{\sqrt[3]{d}\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}}\right.}{d^{2/3}}$$

Rubi [F] time = 7.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-b+x)(-4a+b+3x)}{\sqrt[3]{(-a+x)(-b+x)^2}\left(b^4+ad-\left(4b^3+d\right)x+6b^2x^2-4bx^3+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-b + x)*(-4*a + b + 3*x))/(((-a + x)*(-b + x)^2)^(1/3)*(b^4 + a*d - (
4*b^3 + d)*x + 6*b^2*x^2 - 4*b*x^3 + x^4)),x]
```

```
[Out] (9*a*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x*(a - b + x^3)
^(1/3))/(a^4*(1 + (b*(-4*a^3 + 6*a^2*b - 4*a*b^2 + b^3))/a^4) + 4*a^3*(1 -
(12*a^2*b - 12*a*b^2 + 4*b^3 + d)/(4*a^3))*x^3 + 6*a^2*(1 + (b*(-2*a + b))/
a^2)*x^6 + 4*a*(1 - b/a)*x^9 + x^12), x], x, (-a + x)^(1/3)])/(-((a - x)*(b
 - x)^2))^(1/3) + (9*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][
(x^4*(a - b + x^3)^(1/3))/(a^4*(1 + (b*(-4*a^3 + 6*a^2*b - 4*a*b^2 + b^3))/
a^4) + 4*a^3*(1 - (12*a^2*b - 12*a*b^2 + 4*b^3 + d)/(4*a^3))*x^3 + 6*a^2*(1
 + (b*(-2*a + b))/a^2)*x^6 + 4*a*(1 - b/a)*x^9 + x^12), x], x, (-a + x)^(1/
3)])/(-((a - x)*(b - x)^2))^(1/3) - (3*(4*a - b)*(-a + x)^(1/3)*(-b + x)^(2
/3)*Defer[Subst][Defer[Int][(x*(a - b + x^3)^(1/3))/(b^4*(1 + (a*d)/b^4) -
(4*b^3 + d)*(a + x^3) + 6*b^2*(a + x^3)^2 - 4*b*(a + x^3)^3 + (a + x^3)^4),
 x], x, (-a + x)^(1/3)])/(-((a - x)*(b - x)^2))^(1/3)
```

Rubi steps

$$\int \frac{(-b+x)(-4a+b+3x)}{\sqrt[3]{(-a+x)(-b+x)^2}\left(b^4+ad-\left(4b^3+d\right)x+6b^2x^2-4bx^3+x^4\right)}dx = \frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{\sqrt[3]{-b+}}{\sqrt[3]{-a+x}\left(b^4+ad-(4\right.}}{\sqrt[3]{(-a+x)(-b+x}}$$

$$= \frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int\left(\frac{4a(}{\sqrt[3]{-a+x}\left(-b^4-ad+\right.}\right.}{}$$

$$= \frac{\left(3\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{}{\sqrt[3]{-a+x}\left(b^4+ad-(\right.}}{\sqrt[3]{(-a+x)(-b+x}}$$

$$= \frac{\left(9\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\text{Subst}\left(\int \frac{}{b^4+ad-}\right.}{\sqrt[3]{}}$$

$$= \frac{\left(9\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\text{Subst}\left(\int \frac{}{b^4\left(1+\frac{a}{b}\right.}\right.}{}$$

$$= \frac{\left(9\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\text{Subst}\left(\int\left(\frac{}{a^4\left(1+\right.}\right.\right.}{}$$

$$= \frac{\left(9\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\text{Subst}\left(\int \frac{}{a^4\left(1+\frac{b}{}\right.}\right.}{}$$

Mathematica [**C**] time = 1.76, size = 893, normalized size = 2.80

Antiderivative was successfully verified.

```
[In] Integrate[((-b + x)*(-4*a + b + 3*x))/(((-a + x)*(-b + x)^2)^(1/3)*(b^4 + a
*d - (4*b^3 + d)*x + 6*b^2*x^2 - 4*b*x^3 + x^4)),x]
```

```
[Out] ((a - b)*((b - x)/(a - x))^(2/3)*(-a + x)*(4*RootSum[-d + 3*d*#1 - 3*d*#1^2
 + d*#1^3 - a^3*#1^4 + 3*a^2*b*#1^4 - 3*a*b^2*#1^4 + b^3*#1^4 & , (6*((-b +
 x)/(-a + x))^(1/3) - 2*Sqrt[3]*ArcTan[(1 + (2*((b - x)/(a - x))^(1/3))/#1^
(1/3))/Sqrt[3]]*#1^(1/3) + 2*Log[-((-b + x)/(-a + x))^(1/3) + #1^(1/3)]*#1^
(1/3) - Log[((-b + x)/(-a + x))^(2/3) + ((-b + x)/(-a + x))^(1/3)*#1^(1/3)
+ #1^(2/3)]*#1^(1/3))/(-3*d + 6*d*#1 - 3*d*#1^2 + 4*a^3*#1^3 - 12*a^2*b*#1^
3 + 12*a*b^2*#1^3 - 4*b^3*#1^3) & ] + 5*RootSum[-d + 3*d*#1 - 3*d*#1^2 + d*
#1^3 - a^3*#1^4 + 3*a^2*b*#1^4 - 3*a*b^2*#1^4 + b^3*#1^4 & , (-6*((-b + x)/
(-a + x))^(1/3)*#1 + 2*Sqrt[3]*ArcTan[(1 + (2*((b - x)/(a - x))^(1/3))/#1^(
1/3))/Sqrt[3]]*#1^(4/3) - 2*Log[-((-b + x)/(-a + x))^(1/3) + #1^(1/3)]*#1^(
4/3) + Log[((-b + x)/(-a + x))^(2/3) + ((-b + x)/(-a + x))^(1/3)*#1^(1/3) +
 #1^(2/3)]*#1^(4/3))/(-3*d + 6*d*#1 - 3*d*#1^2 + 4*a^3*#1^3 - 12*a^2*b*#1^3
 + 12*a*b^2*#1^3 - 4*b^3*#1^3) & ] - RootSum[-d + 3*d*#1 - 3*d*#1^2 + d*#1^
3 - a^3*#1^4 + 3*a^2*b*#1^4 - 3*a*b^2*#1^4 + b^3*#1^4 & , (-6*((-b + x)/(-a
 + x))^(1/3)*#1^2 + 2*Sqrt[3]*ArcTan[(1 + (2*((b - x)/(a - x))^(1/3))/#1^(1
/3))/Sqrt[3]]*#1^(7/3) - 2*Log[-((-b + x)/(-a + x))^(1/3) + #1^(1/3)]*#1^(7
```

```
/3) + Log[((-b + x)/(-a + x))^(2/3) + ((-b + x)/(-a + x))^(1/3)*#1^(1/3) +
#1^(2/3)]*#1^(7/3))/(-3*d + 6*d*#1 - 3*d*#1^2 + 4*a^3*#1^3 - 12*a^2*b*#1^3
+ 12*a*b^2*#1^3 - 4*b^3*#1^3) & ]))/(2*((b - x)^2*(-a + x))^(1/3))
```

IntegrateAlgebraic [A] time = 3.15, size = 319, normalized size = 1.00

$$\frac{\log\left(-\sqrt[3]{d}\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}+b^2-2bx+x^2\right)}{d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}}{\sqrt[3]{d}\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}+2b^2-4bx+2x^2}\right)}{d^{2/3}}-\frac{\log\left(d^{2/3}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}+\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}\left(b^2\sqrt[3]{d}-2b\sqrt[3]{d}x+\sqrt[3]{d}x^2\right)+b^4-4b^3x+6b^2x^2-4bx^3+x^4\right)}{2d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + x)*(-4*a + b + 3*x))/(((-a + x)*(-b + x)^2)^(1/3)
*(b^4 + a*d - (4*b^3 + d)*x + 6*b^2*x^2 - 4*b*x^3 + x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x
^2 + x^3)^(1/3))/(2*b^2 - 4*b*x + 2*x^2 + d^(1/3)*(-(a*b^2) + (2*a*b + b^2)
*x + (-a - 2*b)*x^2 + x^3)^(1/3))])/d^(2/3) + Log[b^2 - 2*b*x + x^2 - d^(1/
3)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3)]/d^(2/3) - Log
[b^4 - 4*b^3*x + 6*b^2*x^2 - 4*b*x^3 + x^4 + (b^2*d^(1/3) - 2*b*d^(1/3)*x +
 d^(1/3)*x^2)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3) + d
^(2/3)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(2/3)]/(2*d^(2/3
))
```

fricas [A] time = 1.21, size = 337, normalized size = 1.06

$$\frac{2\sqrt{3}\left(d^2\right)^{\frac{1}{6}}d\arctan\left(\frac{\sqrt{3}\left(d^2\right)^{\frac{1}{6}}\left(\left(b^2-2bx+x^2\right)\left(d^2\right)^{\frac{1}{3}}+2\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}d\right)}{3\left(b^2d-2bdx+dx^2\right)}\right)-\left(d^2\right)^{\frac{2}{3}}\log\left(\frac{\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{2}{3}}d^2+\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}\left(b^2d-2bdx+dx^2\right)\left(d^2\right)^{\frac{1}{3}}+\left(b^4-4b^3x+6b^2x^2-4bx^3+x^4\right)\left(d^2\right)^{\frac{2}{3}}}{b^4-4b^3x+6b^2x^2-4bx^3+x^4}\right)+2\left(d^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(b^2-2bx+x^2\right)\left(d^2\right)^{\frac{1}{3}}-\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}d}{b^2-2bx+x^2}\right)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-4*a+b+3*x)/((-a+x)*(-b+x)^2)^(1/3)/(b^4+a*d-(4*b^3+d)*x+
6*b^2*x^2-4*b*x^3+x^4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(1/3*sqrt(3)*(d^2)^(1/6)*((b^2 - 2*b*x +
 x^2)*(d^2)^(1/3) + 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3
)*d)/(b^2*d - 2*b*d*x + d*x^2)) - (d^2)^(2/3)*log(((-a*b^2 - (a + 2*b)*x^2
+ x^3 + (2*a*b + b^2)*x)^(2/3)*d^2 + (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b
 + b^2)*x)^(1/3)*(b^2*d - 2*b*d*x + d*x^2)*(d^2)^(1/3) + (b^4 - 4*b^3*x + 6
*b^2*x^2 - 4*b*x^3 + x^4)*(d^2)^(2/3))/(b^4 - 4*b^3*x + 6*b^2*x^2 - 4*b*x^3
 + x^4)) + 2*(d^2)^(2/3)*log(-((b^2 - 2*b*x + x^2)*(d^2)^(1/3) - (-a*b^2 -
(a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*d)/(b^2 - 2*b*x + x^2)))/d^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4a-b-3x)(b-x)}{\left(b^4+6b^2x^2-4bx^3+x^4+ad-\left(4b^3+d\right)x\right)\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-4*a+b+3*x)/((-a+x)*(-b+x)^2)^(1/3)/(b^4+a*d-(4*b^3+d)*x+
6*b^2*x^2-4*b*x^3+x^4),x, algorithm="giac")
```

```
[Out] integrate((4*a - b - 3*x)*(b - x)/((b^4 + 6*b^2*x^2 - 4*b*x^3 + x^4 + a*d -
 (4*b^3 + d)*x)*(-(a - x)*(b - x)^2)^(1/3)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)(-4a+b+3x)}{\left((-a+x)(-b+x)^2\right)^{\frac{1}{3}}\left(b^4+ad-\left(4b^3+d\right)x+6b^2x^2-4b\,x^3+x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b+x)*(-4*a+b+3*x)/((-a+x)*(-b+x)^2)^(1/3)/(b^4+a*d-(4*b^3+d)*x+6*b^2*
x^2-4*b*x^3+x^4),x)
```

```
[Out] int((-b+x)*(-4*a+b+3*x)/((-a+x)*(-b+x)^2)^(1/3)/(b^4+a*d-(4*b^3+d)*x+6*b^2*
x^2-4*b*x^3+x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4a-b-3x)(b-x)}{\left(b^4+6b^2x^2-4bx^3+x^4+ad-\left(4b^3+d\right)x\right)\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-4*a+b+3*x)/((-a+x)*(-b+x)^2)^(1/3)/(b^4+a*d-(4*b^3+d)*x+
6*b^2*x^2-4*b*x^3+x^4),x, algorithm="maxima")
```

```
[Out] integrate((4*a - b - 3*x)*(b - x)/((b^4 + 6*b^2*x^2 - 4*b*x^3 + x^4 + a*d -
 (4*b^3 + d)*x)*(-(a - x)*(b - x)^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(b-x)\,(b-4a+3x)}{\left(-(a-x)\,(b-x)^2\right)^{1/3}\,\left(a\,d-4\,b\,x^3-x\,\left(4\,b^3+d\right)+b^4+x^4+6\,b^2\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b - x)*(b - 4*a + 3*x))/((-(a - x)*(b - x)^2)^(1/3)*(a*d - 4*b*x^3 -
 x*(d + 4*b^3) + b^4 + x^4 + 6*b^2*x^2)),x)
```

```
[Out] int(-((b - x)*(b - 4*a + 3*x))/((-(a - x)*(b - x)^2)^(1/3)*(a*d - 4*b*x^3 -
 x*(d + 4*b^3) + b^4 + x^4 + 6*b^2*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-4*a+b+3*x)/((-a+x)*(-b+x)**2)**(1/3)/(b**4+a*d-(4*b**3+d
)*x+6*b**2*x**2-4*b*x**3+x**4),x)
```

```
[Out] Timed out
```

3.2297 $\int \frac{x^4}{\sqrt{-b^4+a^4x^4}\left(-b^8+a^8x^8\right)}\,dx$

Optimal. Leaf size=319

$$-\frac{x}{4a^4b^4\sqrt{a^4x^4-b^4}}-\frac{\left(\frac{1}{16}-\frac{i}{16}\right)\tanh^{-1}\left(\frac{\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{a^4x^4-b^4}}{\sqrt{3-2\sqrt{2}}\,ab}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)ax^2}{\sqrt{3-2\sqrt{2}}\,b}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)b}{\sqrt{3-2\sqrt{2}}\,a}}{x}\right)}{a^5b^5}+\frac{\left(\frac{1}{16}-\frac{i}{16}\right)\tanh^{-1}\left(\frac{\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{a^4x^4-b^4}}{\sqrt{3+2\sqrt{2}}\,ab}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)}{\sqrt{3+2}}\cdots}{x}\right)}{a^5b^5}$$

Rubi [A] time = 0.08, antiderivative size = 120, normalized size of antiderivative = 0.38, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1479, 471, 21, 405}

$$-\frac{x}{4a^4b^4\sqrt{a^4x^4-b^4}}+\frac{\tan^{-1}\left(\frac{ax\left(b^2-a^2x^2\right)}{b\sqrt{a^4x^4-b^4}}\right)}{8a^5b^5}+\frac{\tanh^{-1}\left(\frac{ax\left(a^2x^2+b^2\right)}{b\sqrt{a^4x^4-b^4}}\right)}{8a^5b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[-b^4 + a^4*x^4]*(-b^8 + a^8*x^8)),x]
```

```
[Out] -1/4*x/(a^4*b^4*Sqrt[-b^4 + a^4*x^4]) + ArcTan[(a*x*(b^2 - a^2*x^2))/(b*Sqr
t[-b^4 + a^4*x^4])]/(8*a^5*b^5) + ArcTanh[(a*x*(b^2 + a^2*x^2))/(b*Sqrt[-b^
4 + a^4*x^4])]/(8*a^5*b^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 405

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[-(a*b), 4]}, Simp[(a*ArcTan[(q*x*(a + q^2*x^2))/(a*Sqrt[a + b*x^4])])/(
2*c*q), x] + Simp[(a*ArcTanh[(q*x*(a - q^2*x^2))/(a*Sqrt[a + b*x^4])])/(2*c
*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && NegQ[a*b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1479

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n
2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*(a/d + (c*x^n)/e)^
p, x] /; FreeQ[{a, c, d, e, f, q, m, n, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2
 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{-b^4+a^4x^4}\left(-b^8+a^8x^8\right)}\,dx = \int \frac{x^4}{\left(-b^4+a^4x^4\right)^{3/2}\left(b^4+a^4x^4\right)}\,dx$$
$$= -\frac{x}{4a^4b^4\sqrt{-b^4+a^4x^4}} + \frac{\int \frac{b^4-a^4x^4}{\sqrt{-b^4+a^4x^4}\left(b^4+a^4x^4\right)}\,dx}{4a^4b^4}$$
$$= -\frac{x}{4a^4b^4\sqrt{-b^4+a^4x^4}} - \frac{\int \frac{\sqrt{-b^4+a^4x^4}}{b^4+a^4x^4}\,dx}{4a^4b^4}$$
$$= -\frac{x}{4a^4b^4\sqrt{-b^4+a^4x^4}} + \frac{\tan^{-1}\left(\frac{ax\left(b^2-a^2x^2\right)}{b\sqrt{-b^4+a^4x^4}}\right)}{8a^5b^5} + \frac{\tanh^{-1}\left(\frac{ax\left(b^2+a^2x^2\right)}{b\sqrt{-b^4+a^4x^4}}\right)}{8a^5b^5}$$

Mathematica [C] time = 0.47, size = 205, normalized size = 0.64

$$\frac{x\left(\frac{5\left(b^4-a^4x^4\right)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)}{\left(a^4x^4+b^4\right)\left(5b^4F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)-2a^4x^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{a^4x^4}{b^4},-\frac{a^4x^4}{b^4}\right)\right)\right)}-\frac{1}{b^4}\right)}{4a^4\sqrt{a^4x^4-b^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/(Sqrt[-b^4 + a^4*x^4]*(-b^8 + a^8*x^8)),x]
```

```
[Out] (x*(-b^(-4) + (5*(b^4 - a^4*x^4)*AppellF1[1/4, -1/2, 1, 5/4, (a^4*x^4)/b^4,
 -((a^4*x^4)/b^4)])/((b^4 + a^4*x^4)*(5*b^4*AppellF1[1/4, -1/2, 1, 5/4, (a^
4*x^4)/b^4, -((a^4*x^4)/b^4)] - 2*a^4*x^4*(2*AppellF1[5/4, -1/2, 2, 9/4, (a
^4*x^4)/b^4, -((a^4*x^4)/b^4)] + AppellF1[5/4, 1/2, 1, 9/4, (a^4*x^4)/b^4,
-((a^4*x^4)/b^4)])))))/(4*a^4*Sqrt[-b^4 + a^4*x^4])
```

IntegrateAlgebraic [A] time = 0.93, size = 253, normalized size = 0.79

$$-\frac{x}{4a^4b^4\sqrt{a^4x^4-b^4}} + \frac{\left(\frac{1}{8}-\frac{i}{8}\right)\tan^{-1}\left(\frac{(1+i)abx}{\sqrt{a^4x^4-b^4}+a^2x^2+ib^2}\right)}{a^5b^5} - \frac{\left(\frac{1}{16}-\frac{i}{16}\right)\tanh^{-1}\left(\frac{-a^4x^4-(1-i)a^3bx^3+\left(-a^2x^2-(1-i)abx-ib^2\right)\sqrt{a^4x^4-b^4}-(1+i)ab^3x+b^4}{-ia^4x^4+(1+i)a^3bx^3+\left(-ia^2x^2+(1+i)abx+b^2\right)\sqrt{a^4x^4-b^4}-(1-i)ab^3x+ib^4}\right)}{a^5b^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(Sqrt[-b^4 + a^4*x^4]*(-b^8 + a^8*x^8)),x]
```

```
[Out] -1/4*x/(a^4*b^4*Sqrt[-b^4 + a^4*x^4]) + ((1/8 - I/8)*ArcTan[((1 + I)*a*b*x)
/(I*b^2 + a^2*x^2 + Sqrt[-b^4 + a^4*x^4])])/(a^5*b^5) - ((1/16 - I/16)*ArcT
anh[(b^4 - (1 + I)*a*b^3*x - (1 - I)*a^3*b*x^3 - a^4*x^4 + ((-I)*b^2 - (1 -
 I)*a*b*x - a^2*x^2)*Sqrt[-b^4 + a^4*x^4])/(I*b^4 - (1 - I)*a*b^3*x + (1 +
I)*a^3*b*x^3 - I*a^4*x^4 + (b^2 + (1 + I)*a*b*x - I*a^2*x^2)*Sqrt[-b^4 + a^
4*x^4])])/(a^5*b^5)
```

fricas [A] time = 2.44, size = 166, normalized size = 0.52

$$-\frac{4\sqrt{a^4x^4-b^4}\,abx + 2\left(a^4x^4-b^4\right)\arctan\left(\frac{\sqrt{a^4x^4-b^4}\,ax}{a^2bx^2+b^3}\right) - \left(a^4x^4-b^4\right)\log\left(\frac{a^4x^4+2\,a^2b^2x^2-b^4+2\,\sqrt{a^4x^4-b^4}\,abx}{a^4x^4+b^4}\right)}{16\left(a^9b^5x^4-a^5b^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^4*x^4-b^4)^(1/2)/(a^8*x^8-b^8),x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(a^4*x^4 - b^4)*a*b*x + 2*(a^4*x^4 - b^4)*arctan(sqrt(a^4*x^4
- b^4)*a*x/(a^2*b*x^2 + b^3)) - (a^4*x^4 - b^4)*log((a^4*x^4 + 2*a^2*b^2*x^
2 - b^4 + 2*sqrt(a^4*x^4 - b^4)*a*b*x)/(a^4*x^4 + b^4)))/(a^9*b^5*x^4 - a^5
*b^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a^8x^8 - b^8\right)\sqrt{a^4x^4 - b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^4*x^4-b^4)^(1/2)/(a^8*x^8-b^8),x, algorithm="giac")
[Out] integrate(x^4/((a^8*x^8 - b^8)*sqrt(a^4*x^4 - b^4)), x)
```

maple [C] time = 0.05, size = 921, normalized size = 2.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^4*x^4-b^4)^(1/2)/(a^8*x^8-b^8),x)
[Out] 1/8/b^3/a^4*(-1/2*(a^4*x^3+a^3*b*x^2+a^2*b^2*x+a*b^3)/a^2/b^3/((x-b/a)*(a^4
*x^3+a^3*b*x^2+a^2*b^2*x+a*b^3))^(1/2)-1/2/b/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+
1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticF(x*(-a^2/b^2)^(
1/2),I)+1/2/b/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/
(a^4*x^4-b^4)^(1/2)*(EllipticF(x*(-a^2/b^2)^(1/2),I)-EllipticE(x*(-a^2/b^2)
^(1/2),I)))-1/4/a^4/b^2*(-1/2*(a^4*x^2-a^2*b^2)/b^4*x/a^2/((x^2+b^2/a^2)*(a
^4*x^2-a^2*b^2))^(1/2)+1/2/b^2/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^
2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)+1/2/b^
2/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4
)^(1/2)*(EllipticF(x*(-a^2/b^2)^(1/2),I)-EllipticE(x*(-a^2/b^2)^(1/2),I)))+
1/32/a^8*sum(1/_alpha^3*(-2^(1/2)/(-b^4)^(1/2)*arctanh(_alpha^2*(_alpha^2+x
^2)*a^4/(-2*b^4)^(1/2)/(a^4*x^4-b^4)^(1/2))+4/(-a^2/b^2)^(1/2)*_alpha^3*a^4
/b^4*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*Ellipt
icPi(x*(-a^2/b^2)^(1/2),_alpha^2*a^2/b^2,(a^2/b^2)^(1/2)/(-a^2/b^2)^(1/2)))
,_alpha=RootOf(_Z^4*a^4+b^4))-1/8/b^3/a^4*(1/2*(a^4*x^3-a^3*b*x^2+a^2*b^2*x
-a*b^3)/a^2/b^3/((x+b/a)*(a^4*x^3-a^3*b*x^2+a^2*b^2*x-a*b^3))^(1/2)+1/2/b/(
-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(
1/2)*EllipticF(x*(-a^2/b^2)^(1/2),I)-1/2/b/(-a^2/b^2)^(1/2)*(a^2*x^2/b^2+1)
^(1/2)*(1-a^2*x^2/b^2)^(1/2)/(a^4*x^4-b^4)^(1/2)*(EllipticF(x*(-a^2/b^2)^(1
/2),I)-EllipticE(x*(-a^2/b^2)^(1/2),I)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a^8x^8 - b^8\right)\sqrt{a^4x^4 - b^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^4*x^4-b^4)^(1/2)/(a^8*x^8-b^8),x, algorithm="maxima")
[Out] integrate(x^4/((a^8*x^8 - b^8)*sqrt(a^4*x^4 - b^4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^4}{\sqrt{a^4\,x^4-b^4}\,\left(b^8-a^8\,x^8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4/((a^4*x^4 - b^4)^(1/2)*(b^8 - a^8*x^8)),x)
```

```
[Out] -int(x^4/((a^4*x^4 - b^4)^(1/2)*(b^8 - a^8*x^8)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{(ax-b)(ax+b)(a^2x^2+b^2)}\,(ax-b)(ax+b)(a^2x^2+b^2)(a^4x^4+b^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**4*x**4-b**4)**(1/2)/(a**8*x**8-b**8),x)
```

```
[Out] Integral(x**4/(sqrt((a*x - b)*(a*x + b)*(a**2*x**2 + b**2))*(a*x - b)*(a*x
+ b)*(a**2*x**2 + b**2)*(a**4*x**4 + b**4)), x)
```

3.2298 $\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt{ax+\sqrt{-b+a^2x^2}}\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=319

$$\frac{8\log\left(\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}-\sqrt[3]{c}\right)}{9ac^{7/3}}-\frac{4\log\left(\sqrt[3]{c}\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}+c\right.}{9ac^{7/3}}$$

Rubi [F] time = 1.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt{ax+\sqrt{-b+a^2x^2}}\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[-b + a^2*x^2]*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*(c + (a*x + Sqrt[-
b + a^2*x^2])^(1/4))^(1/3)),x]
```

```
[Out] Defer[Int][1/(Sqrt[-b + a^2*x^2]*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*(c + (a*x +
 Sqrt[-b + a^2*x^2])^(1/4))^(1/3)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt{ax+\sqrt{-b+a^2x^2}}\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt{ax+\sqrt{-b+a^2x^2}}\sqrt[3]{c+}}$$

Mathematica [C] time = 0.31, size = 74, normalized size = 0.23

$$-\frac{6\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}{}_2F_1\left(\frac{2}{3},3;\frac{5}{3};\frac{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}{c}\right)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-b + a^2*x^2]*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*(c + (a*x +
Sqrt[-b + a^2*x^2])^(1/4))^(1/3)),x]
```

```
[Out] (-6*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3)*Hypergeometric2F1[2/3, 3,
5/3, (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))/c])/(a*c^3)
```

IntegrateAlgebraic [A] time = 0.97, size = 319, normalized size = 1.00

$$\frac{8\log\left(\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}-\sqrt[3]{c}\right)}{9ac^{7/3}}-\frac{4\log\left(\sqrt[3]{c}\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}+c^{2/3}\right)}{9ac^{7/3}}+\frac{8\tan^{-1}\left(\frac{2\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{3}\sqrt[3]{c}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}ac^{7/3}}+\frac{8\sqrt[4]{\sqrt{a^2x^2-b}+ax}\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}-6c\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}}{3ac^2\sqrt{\sqrt{a^2x^2-b}+ax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-b + a^2*x^2]*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*(c
+ (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)),x]
```

```
[Out] (-6*c*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3) + 8*(a*x + Sqrt[-b + a^2
*x^2])^(1/4)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3))/(3*a*c^2*Sqrt[a*
x + Sqrt[-b + a^2*x^2]]) + (8*ArcTan[1/Sqrt[3] + (2*(c + (a*x + Sqrt[-b + a
^2*x^2])^(1/4))^(1/3))/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*a*c^(7/3)) + (8*Log[-
c^(1/3) + (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)])/(9*a*c^(7/3)) - (4
*Log[c^(2/3) + c^(1/3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + (c +
(a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3)])/(9*a*c^(7/3))
```

fricas [A] time = 1.23, size = 770, normalized size = 2.41

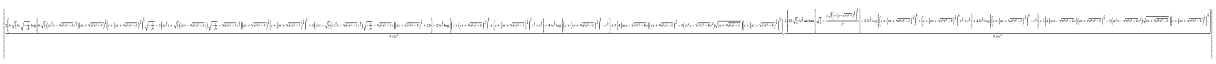

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^
2-b)^(1/2))^(1/4))^(1/3),x, algorithm="fricas")
```

```
[Out] [2/9*(6*sqrt(1/3)*b*c*sqrt(-1/c^(2/3))*log(6*sqrt(1/3)*(a*c^(2/3)*x - sqrt(
a^2*x^2 - b)*c^(2/3))*(a*x + sqrt(a^2*x^2 - b))^(3/4)*(c + (a*x + sqrt(a^2*
x^2 - b))^(1/4))^(2/3)*sqrt(-1/c^(2/3)) - 3*(a*c^(2/3)*x + sqrt(1/3)*(a*c*x
 - sqrt(a^2*x^2 - b)*c)*sqrt(-1/c^(2/3)) - sqrt(a^2*x^2 - b)*c^(2/3))*(a*x
+ sqrt(a^2*x^2 - b))^(3/4)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3) + 3*
(a*c*x - sqrt(1/3)*(a*c^(4/3)*x - sqrt(a^2*x^2 - b)*c^(4/3))*sqrt(-1/c^(2/3
)) - sqrt(a^2*x^2 - b)*c)*(a*x + sqrt(a^2*x^2 - b))^(3/4) + 2*b) - 2*b*c^(2
/3)*log((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3) + (c + (a*x + sqrt(a^2*
x^2 - b))^(1/4))^(1/3)*c^(1/3) + c^(2/3)) + 4*b*c^(2/3)*log((c + (a*x + sqr
t(a^2*x^2 - b))^(1/4))^(1/3) - c^(1/3)) + 3*(4*(a*c*x - sqrt(a^2*x^2 - b)*c
)*(a*x + sqrt(a^2*x^2 - b))^(3/4) - 3*(a*c^2*x - sqrt(a^2*x^2 - b)*c^2)*sqr
t(a*x + sqrt(a^2*x^2 - b)))*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3))/(a
*b*c^3), 2/9*(12*sqrt(1/3)*b*c^(2/3)*arctan(sqrt(1/3) + 2*sqrt(1/3)*(c + (a
*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)/c^(1/3)) - 2*b*c^(2/3)*log((c + (a*x +
 sqrt(a^2*x^2 - b))^(1/4))^(2/3) + (c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1
/3)*c^(1/3) + c^(2/3)) + 4*b*c^(2/3)*log((c + (a*x + sqrt(a^2*x^2 - b))^(1/
4))^(1/3) - c^(1/3)) + 3*(4*(a*c*x - sqrt(a^2*x^2 - b)*c)*(a*x + sqrt(a^2*x
^2 - b))^(3/4) - 3*(a*c^2*x - sqrt(a^2*x^2 - b)*c^2)*sqrt(a*x + sqrt(a^2*x^
2 - b)))*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3))/(a*b*c^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^
2-b)^(1/2))^(1/4))^(1/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^{2} x^{2}-b}\,\sqrt{a x +\sqrt{a^{2} x^{2}-b}}\,\left(c +\left(a x +\sqrt{a^{2} x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^2-b)^(
1/2))^(1/4))^(1/3),x)
```

```
[Out] int(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^2-b)^(
1/2))^(1/4))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2-b}\sqrt{ax+\sqrt{a^2x^2-b}}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^
2-b)^(1/2))^(1/4))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 - b)*sqrt(a*x + sqrt(a^2*x^2 - b))*(c + (a*x + sq
rt(a^2*x^2 - b))^(1/4))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a\,x+\sqrt{a^2\,x^2-b}}\,\left(c+{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{1/4}\right)^{1/3}\,\sqrt{a^2\,x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x + (a^2*x^2 - b)^(1/2))^(1/2)*(c + (a*x + (a^2*x^2 - b)^(1/2))^(
1/4))^(1/3)*(a^2*x^2 - b)^(1/2)),x)
```

```
[Out] int(1/((a*x + (a^2*x^2 - b)^(1/2))^(1/2)*(c + (a*x + (a^2*x^2 - b)^(1/2))^(
1/4))^(1/3)*(a^2*x^2 - b)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}\sqrt{ax+\sqrt{a^2x^2-b}}\sqrt{a^2x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*x**2-b)**(1/2)/(a*x+(a**2*x**2-b)**(1/2))**(1/2)/(c+(a*x+
(a**2*x**2-b)**(1/2))**(1/4))**(1/3),x)
```

```
[Out] Integral(1/((c + (a*x + sqrt(a**2*x**2 - b))**(1/4))**(1/3)*sqrt(a*x + sqrt
(a**2*x**2 - b))*sqrt(a**2*x**2 - b)), x)
```

3.2299

$$\int \frac{x^3(-b+x)\left(2ab-3ax+x^2\right)}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-a^4+4a^3x-6a^2x^2+4ax^3+\left(-1+b^2d\right)x^4-2bdx^5+dx^6\right)}\,dx$$

Optimal. Leaf size=324

$$\frac{\tanh^{-1}\left(\frac{\sqrt[3]{x^3(-a-b)+abx^2+x^4}\left(a\sqrt[6]{d}-\sqrt[6]{d}\,x\right)}{a^2+\sqrt[3]{d}\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}-2ax+x^2}\right)}{2d^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}}{\sqrt[6]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}+2a-2x}\right)}{2d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x^3(-a-b)+ab}}{\sqrt[6]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}}\right)}{2d^{5/6}}$$

Rubi [F] time = 52.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3(-b+x)\left(2ab-3ax+x^2\right)}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-a^4+4a^3x-6a^2x^2+4ax^3+\left(-1+b^2d\right)x^4-2bdx^5+dx^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*(-b + x)*(2*a*b - 3*a*x + x^2))/((x^2*(-a + x)*(-b + x))^(1/3)*(-a
^4 + 4*a^3*x - 6*a^2*x^2 + 4*a*x^3 + (-1 + b^2*d)*x^4 - 2*b*d*x^5 + d*x^6))
,x]
```

```
[Out] (9*a*x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][Defer[Int][(x^12*(-
b + x^3)^(2/3))/((-a + x^3)^(1/3)*(a^4 - 4*a^3*x^3 + 6*a^2*x^6 - 4*a*x^9 +
(1 - b^2*d)*x^12 + 2*b*d*x^15 - d*x^18)), x], x, x^(1/3)])/((a - x)*(b - x)
*x^2)^(1/3) + (6*a*b*x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][Def
er[Int][(x^9*(-b + x^3)^(2/3))/((-a + x^3)^(1/3)*(-a^4 + 4*a^3*x^3 - 6*a^2*
x^6 + 4*a*x^9 - (1 - b^2*d)*x^12 - 2*b*d*x^15 + d*x^18)), x], x, x^(1/3)])/
((a - x)*(b - x)*x^2)^(1/3) + (3*x^(2/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defe
r[Subst][Defer[Int][(x^15*(-b + x^3)^(2/3))/((-a + x^3)^(1/3)*(-a^4 + 4*a^3
*x^3 - 6*a^2*x^6 + 4*a*x^9 - (1 - b^2*d)*x^12 - 2*b*d*x^15 + d*x^18)), x],
x, x^(1/3)])/((a - x)*(b - x)*x^2)^(1/3)
```

Rubi steps

$$\int \frac{x^3(-b+x)\left(2ab-3ax+x^2\right)}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-a^4+4a^3x-6a^2x^2+4ax^3+\left(-1+b^2d\right)x^4-2bdx^5+dx^6\right)}\,dx=\frac{\left(x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-b+\cdots}\right.}{\cdots}$$

$$=\frac{\left(3x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-b+\cdots}\right.}{\cdots}$$

$$=\frac{\left(3x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-b+\cdots}\right.}{\cdots}$$

$$=\frac{\left(3x^{2/3}\sqrt[3]{-a+x}\sqrt[3]{-b+\cdots}\right.}{\cdots}$$

Mathematica [F] time = 3.34, size = 0, normalized size = 0.00

$$\int \frac{x^3(-b+x)\left(2ab-3ax+x^2\right)}{\sqrt[3]{x^2(-a+x)(-b+x)}\left(-a^4+4a^3x-6a^2x^2+4ax^3+\left(-1+b^2d\right)x^4-2bdx^5+dx^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(-b + x)*(2*a*b - 3*a*x + x^2))/((x^2*(-a + x)*(-b + x))^(1/
3)*(-a^4 + 4*a^3*x - 6*a^2*x^2 + 4*a*x^3 + (-1 + b^2*d)*x^4 - 2*b*d*x^5 + d
*x^6)),x]
```

```
[Out] Integrate[(x^3*(-b + x)*(2*a*b - 3*a*x + x^2))/((x^2*(-a + x)*(-b + x))^(1/
3)*(-a^4 + 4*a^3*x - 6*a^2*x^2 + 4*a*x^3 + (-1 + b^2*d)*x^4 - 2*b*d*x^5 + d
*x^6)), x]
```

IntegrateAlgebraic [A] time = 4.39, size = 324, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt[3]{x^3(-a-b)+abx^2+x^4}\left(a\sqrt[6]{d}-\sqrt[6]{d}\,x\right)}{a^2+\sqrt[3]{d}\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}-2ax+x^2}\right)}{2d^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}}{\sqrt[6]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}+2a-2x}\right)}{2d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}}{\sqrt[6]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}-2a+2x}\right)}{2d^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[3]{x^3(-a-b)+abx^2+x^4}}{a-x}\right)}{d^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(-b + x)*(2*a*b - 3*a*x + x^2))/((x^2*(-a + x)*(-b
+ x))^(1/3)*(-a^4 + 4*a^3*x - 6*a^2*x^2 + 4*a*x^3 + (-1 + b^2*d)*x^4 - 2*b*
d*x^5 + d*x^6)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/6)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3))
/(2*a - 2*x + d^(1/6)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3))])/d^(5/6) + (Sq
rt[3]*ArcTan[(Sqrt[3]*d^(1/6)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3))/(-2*a +
 2*x + d^(1/6)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3))])/(2*d^(5/6)) + ArcTan
h[(d^(1/6)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3))/(a - x)]/d^(5/6) + ArcTanh
[((a*d^(1/6) - d^(1/6)*x)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(1/3))/(a^2 - 2*a*
x + x^2 + d^(1/3)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(2/3))]/(2*d^(5/6))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-b+x)*(2*a*b-3*a*x+x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a^4+4*a^
3*x-6*a^2*x^2+4*a*x^3+(b^2*d-1)*x^4-2*b*d*x^5+d*x^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-b+x)*(2*a*b-3*a*x+x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a^4+4*a^
3*x-6*a^2*x^2+4*a*x^3+(b^2*d-1)*x^4-2*b*d*x^5+d*x^6),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^3(-b+x)\left(2ab-3ax+x^2\right)}{\left(x^2(-a+x)(-b+x)\right)^{\frac{1}{3}}\left(-a^4+4a^3x-6a^2x^2+4a\,x^3+\left(b^2d-1\right)x^4-2bd\,x^5+d\,x^6\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-b+x)*(2*a*b-3*a*x+x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a^4+4*a^3*x-6*
a^2*x^2+4*a*x^3+(b^2*d-1)*x^4-2*b*d*x^5+d*x^6),x)
```

```
[Out] int(x^3*(-b+x)*(2*a*b-3*a*x+x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a^4+4*a^3*x-6*
a^2*x^2+4*a*x^3+(b^2*d-1)*x^4-2*b*d*x^5+d*x^6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,ab - 3\,ax + x^2\right)(b-x)x^3}{\left(2\,bdx^5 - dx^6 - \left(b^2d - 1\right)x^4 + a^4 - 4\,a^3x + 6\,a^2x^2 - 4\,ax^3\right)\left((a-x)(b-x)x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-b+x)*(2*a*b-3*a*x+x^2)/(x^2*(-a+x)*(-b+x))^(1/3)/(-a^4+4*a^
3*x-6*a^2*x^2+4*a*x^3+(b^2*d-1)*x^4-2*b*d*x^5+d*x^6),x, algorithm="maxima")
```

```
[Out] integrate((2*a*b - 3*a*x + x^2)*(b - x)*x^3/((2*b*d*x^5 - d*x^6 - (b^2*d -
1)*x^4 + a^4 - 4*a^3*x + 6*a^2*x^2 - 4*a*x^3)*((a - x)*(b - x)*x^2)^(1/3)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x^3\,\left(b-x\right)\,\left(x^2-3\,a\,x+2\,a\,b\right)}{{\left(x^2\,\left(a-x\right)\,\left(b-x\right)\right)}^{1/3}\,\left(-a^4+4\,a^3\,x-6\,a^2\,x^2+4\,a\,x^3+d\,x^6-2\,b\,d\,x^5+\left(b^2\,d-1\right)\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(b - x)*(2*a*b - 3*a*x + x^2))/((x^2*(a - x)*(b - x))^(1/3)*(x^4*
(b^2*d - 1) + 4*a*x^3 + 4*a^3*x + d*x^6 - a^4 - 6*a^2*x^2 - 2*b*d*x^5)),x)
```

```
[Out] int(-(x^3*(b - x)*(2*a*b - 3*a*x + x^2))/((x^2*(a - x)*(b - x))^(1/3)*(x^4*
(b^2*d - 1) + 4*a*x^3 + 4*a^3*x + d*x^6 - a^4 - 6*a^2*x^2 - 2*b*d*x^5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(-b+x\right)\left(2ab - 3ax + x^2\right)}{\sqrt[3]{x^2\left(-a+x\right)\left(-b+x\right)}\left(-a^4 + 4a^3x - 6a^2x^2 + 4ax^3 + b^2dx^4 - 2bdx^5 + dx^6 - x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-b+x)*(2*a*b-3*a*x+x**2)/(x**2*(-a+x)*(-b+x))**(1/3)/(-a**4
+4*a**3*x-6*a**2*x**2+4*a*x**3+(b**2*d-1)*x**4-2*b*d*x**5+d*x**6),x)
```

```
[Out] Integral(x**3*(-b + x)*(2*a*b - 3*a*x + x**2)/((x**2*(-a + x)*(-b + x))**(1
/3)*(-a**4 + 4*a**3*x - 6*a**2*x**2 + 4*a*x**3 + b**2*d*x**4 - 2*b*d*x**5 +
 d*x**6 - x**4)), x)
```

3.2300 $\int \frac{(1+x^3)^{2/3}(-1+x^6)}{x^6(-1-2x^3+2x^6)}\,dx$

Optimal. Leaf size=325

$$\frac{\log\left(3^{5/6}\sqrt[3]{x^3+1}-3x\right)}{2\,3^{2/3}\sqrt[3]{26+15\sqrt{3}}}+\frac{\sqrt[3]{26+15\sqrt{3}}\,\log\left(3^{5/6}\sqrt[3]{x^3+1}+3x\right)}{2\,3^{2/3}}-\frac{1}{2}\sqrt[3]{\frac{1}{3}\left(45+26\sqrt{3}\right)}\tan^{-1}\left(\frac{3^{2/3}x}{\sqrt[6]{3}\,x-2\sqrt[3]{x^3+1}}\right.$$

Rubi [C] time = 0.56, antiderivative size = 176, normalized size of antiderivative = 0.54, number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {6728, 264, 277, 239, 429}

$$-\frac{\left(2+\sqrt{3}\right)xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-x^3,\frac{2x^3}{1-\sqrt{3}}\right)}{1-\sqrt{3}}-\frac{\left(2-\sqrt{3}\right)xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-x^3,\frac{2x^3}{1+\sqrt{3}}\right)}{1+\sqrt{3}}+\log\left(\sqrt[3]{x^3+1}-x\right)-\frac{2\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\left(x^3+1\right)^{5/3}}{5x^5}+\frac{\left(x^3+1\right)^{2/3}}{x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((1 + x^3)^(2/3)*(-1 + x^6))/(x^6*(-1 - 2*x^3 + 2*x^6)),x]
```

```
[Out] (1 + x^3)^(2/3)/x^2 - (1 + x^3)^(5/3)/(5*x^5) - ((2 + Sqrt[3])*x*AppellF1[1
/3, -2/3, 1, 4/3, -x^3, (2*x^3)/(1 - Sqrt[3])])/(1 - Sqrt[3]) - ((2 - Sqrt[
3])*x*AppellF1[1/3, -2/3, 1, 4/3, -x^3, (2*x^3)/(1 + Sqrt[3])])/(1 + Sqrt[3
]) - (2*ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + Log[-x + (1
+ x^3)^(1/3)]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(1+x^3\right)^{2/3}\left(-1+x^6\right)}{x^6\left(-1-2x^3+2x^6\right)}\,dx &= \int \left(\frac{\left(1+x^3\right)^{2/3}}{x^6}-\frac{2\left(1+x^3\right)^{2/3}}{x^3}+\frac{\left(1+x^3\right)^{2/3}\left(-5+4x^3\right)}{-1-2x^3+2x^6}\right) dx\\
&= -\left(2\int \frac{\left(1+x^3\right)^{2/3}}{x^3}\,dx\right)+\int \frac{\left(1+x^3\right)^{2/3}}{x^6}\,dx+\int \frac{\left(1+x^3\right)^{2/3}\left(-5+4x^3\right)}{-1-2x^3+2x^6}\,dx\\
&= \frac{\left(1+x^3\right)^{2/3}}{x^2}-\frac{\left(1+x^3\right)^{5/3}}{5x^5}-2\int \frac{1}{\sqrt[3]{1+x^3}}\,dx+\int \left(\frac{\left(4-2\sqrt{3}\right)\left(1+x^3\right)^{2/3}}{-2-2\sqrt{3}+4x^3}+\frac{\left(4+2\right.}{-2+\ldots}\right.\\
&= \frac{\left(1+x^3\right)^{2/3}}{x^2}-\frac{\left(1+x^3\right)^{5/3}}{5x^5}-\frac{2\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(-x+\sqrt[3]{1+x^3}\right)+\left(2\left(2-\sqrt{3}\right.\right.\\
&= \frac{\left(1+x^3\right)^{2/3}}{x^2}-\frac{\left(1+x^3\right)^{5/3}}{5x^5}-\frac{\left(2+\sqrt{3}\right)xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-x^3,\frac{2x^3}{1-\sqrt{3}}\right)}{1-\sqrt{3}}-\frac{\left(2-\sqrt{3}\right)xF_1\left(\ldots\right.}{\ldots}
\end{aligned}
$$

Mathematica [A] time = 0.75, size = 265, normalized size = 0.82

$$\frac{-12\left(\sqrt{3}-2\right)\log\left(1-\frac{\sqrt[6]{3}x}{\sqrt[3]{x^3+1}}\right)+12\left(2+\sqrt{3}\right)\log\left(\frac{\sqrt[6]{3}x}{\sqrt[3]{x^3+1}}+1\right)-\left(1+\sqrt{3}\right)\left(3+\sqrt{3}\right)\left(6\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2x}{\sqrt[3]{3}\sqrt[3]{x^3+1}}\right)+\sqrt{3}\log\left(-\frac{\sqrt[6]{3}x}{\sqrt[3]{x^3+1}}+\frac{\sqrt[3]{3}x^2}{\left(x^3+1\right)^{2/3}}+1\right)\right)+\left(\sqrt{3}-3\right)\left(\sqrt{3}-1\right)\left(6\tan^{-1}\left(\frac{2x}{\sqrt[3]{3}\sqrt[3]{x^3+1}}+\frac{1}{\sqrt{3}}\right)+\sqrt{3}\log\left(\frac{\sqrt[6]{3}x}{\sqrt[3]{x^3+1}}+\frac{\sqrt[3]{3}x^2}{\left(x^3+1\right)^{2/3}}+1\right)\right)}{24\,3^{2/3}}+\left(x^3+1\right)^{2/3}\left(\frac{4}{5x^2}-\frac{1}{5x^5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)^(2/3)*(-1 + x^6))/(x^6*(-1 - 2*x^3 + 2*x^6)),x]
```

```
[Out] (-1/5*1/x^5 + 4/(5*x^2))*(1 + x^3)^(2/3) + (-12*(-2 + Sqrt[3])*Log[1 - (3^(
1/6)*x)/(1 + x^3)^(1/3)] - (1 + Sqrt[3])*(3 + Sqrt[3])*(6*ArcTan[1/Sqrt[3]
- (2*x)/(3^(1/3)*(1 + x^3)^(1/3))] + Sqrt[3]*Log[1 + (3^(1/3)*x^2)/(1 + x^3
)^(2/3) - (3^(1/6)*x)/(1 + x^3)^(1/3)]) + 12*(2 + Sqrt[3])*Log[1 + (3^(1/6)
*x)/(1 + x^3)^(1/3)] + (-3 + Sqrt[3])*(-1 + Sqrt[3])*(6*ArcTan[1/Sqrt[3] +
(2*x)/(3^(1/3)*(1 + x^3)^(1/3))] + Sqrt[3]*Log[1 + (3^(1/3)*x^2)/(1 + x^3)^
(2/3) + (3^(1/6)*x)/(1 + x^3)^(1/3)]))/(24*3^(2/3))
```

IntegrateAlgebraic [A] time = 2.18, size = 325, normalized size = 1.00

$$\frac{\log\left(3^{5/6}\sqrt[3]{x^3+1}-3x\right)}{2\,3^{2/3}\sqrt[3]{26+15\sqrt{3}}}+\frac{\sqrt[3]{26+15\sqrt{3}}\log\left(3^{5/6}\sqrt[3]{x^3+1}+3x\right)}{2\,3^{2/3}}-\frac{1}{2}\sqrt[3]{\frac{1}{3}\left(45+26\sqrt{3}\right)}\tan^{-1}\left(\frac{3^{2/3}x}{\sqrt[6]{3}x-2\sqrt[3]{x^3+1}}\right)-\frac{\tan^{-1}\left(\frac{3^{2/3}x}{2\sqrt[3]{x^3+1}+\sqrt[6]{3}x}\right)}{2\sqrt[3]{45+26\sqrt{3}}}+\frac{\left(x^3+1\right)^{2/3}\left(4x^3-1\right)}{5x^5}-\frac{\sqrt[3]{26+15\sqrt{3}}\log\left(3^{5/6}\sqrt[3]{x^3+1}x-3^{2/3}\left(x^3+1\right)^{2/3}-3x^2\right)}{4\,3^{2/3}}-\frac{\log\left(3^{5/6}\sqrt[3]{x^3+1}x+3^{2/3}\left(x^3+1\right)^{2/3}+3x^2\right)}{4\,3^{2/3}\sqrt[3]{26+15\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^3)^(2/3)*(-1 + x^6))/(x^6*(-1 - 2*x^3 + 2*x^6)),
x]
```

```
[Out] ((1 + x^3)^(2/3)*(-1 + 4*x^3))/(5*x^5) - (((45 + 26*Sqrt[3])/3)^(1/3)*ArcTa
n[(3^(2/3)*x)/(3^(1/6)*x - 2*(1 + x^3)^(1/3))])/2 - ArcTan[(3^(2/3)*x)/(3^(
1/6)*x + 2*(1 + x^3)^(1/3))]/(2*(45 + 26*Sqrt[3])^(1/3)) + Log[-3*x + 3^(5/
6)*(1 + x^3)^(1/3)]/(2*3^(2/3)*(26 + 15*Sqrt[3])^(1/3)) + ((26 + 15*Sqrt[3]
)^(1/3)*Log[3*x + 3^(5/6)*(1 + x^3)^(1/3)])/(2*3^(2/3)) - ((26 + 15*Sqrt[3]
)^(1/3)*Log[-3*x^2 + 3^(5/6)*x*(1 + x^3)^(1/3) - 3^(2/3)*(1 + x^3)^(2/3)])/
(4*3^(2/3)) - Log[3*x^2 + 3^(5/6)*x*(1 + x^3)^(1/3) + 3^(2/3)*(1 + x^3)^(2/
3)]/(4*3^(2/3)*(26 + 15*Sqrt[3])^(1/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^6-1)/x^6/(2*x^6-2*x^3-1),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}-1\right)\left(x^{3}+1\right)^{\frac{2}{3}}}{\left(2\,x^{6}-2\,x^{3}-1\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^6-1)/x^6/(2*x^6-2*x^3-1),x, algorithm="giac")
```

```
[Out] integrate((x^6 - 1)*(x^3 + 1)^(2/3)/((2*x^6 - 2*x^3 - 1)*x^6), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}+1\right)^{\frac{2}{3}}\left(x^{6}-1\right)}{x^{6}\left(2x^{6}-2x^{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(2/3)*(x^6-1)/x^6/(2*x^6-2*x^3-1),x)
```

```
[Out] int((x^3+1)^(2/3)*(x^6-1)/x^6/(2*x^6-2*x^3-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}-1\right)\left(x^{3}+1\right)^{\frac{2}{3}}}{\left(2\,x^{6}-2\,x^{3}-1\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(2/3)*(x^6-1)/x^6/(2*x^6-2*x^3-1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 - 1)*(x^3 + 1)^(2/3)/((2*x^6 - 2*x^3 - 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(x^{3}+1\right)^{2/3}\,\left(x^{6}-1\right)}{x^{6}\,\left(-2\,x^{6}+2\,x^{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3 + 1)^(2/3)*(x^6 - 1))/(x^6*(2*x^3 - 2*x^6 + 1)),x)
```

```
[Out] -int(((x^3 + 1)^(2/3)*(x^6 - 1))/(x^6*(2*x^3 - 2*x^6 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(2/3)*(x**6-1)/x**6/(2*x**6-2*x**3-1),x)
```

```
[Out] Timed out
```

3.2301 $\int \frac{1+x^4}{\left(-1+x^4\right)\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=326

$$\frac{4}{3}\sqrt{\sqrt{x^2+1}-x}\,x+\frac{2}{3}\sqrt{x^2+1}\,\sqrt{\sqrt{x^2+1}-x}+\sqrt{\sqrt{2}-1}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}-x}}{\sqrt{\sqrt{2}-1}}\right)-\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+1}-}}{\sqrt{1+\sqrt{2}}}\right.$$

Rubi [A] time = 0.82, antiderivative size = 341, normalized size of antiderivative = 1.05, number of steps used = 36, number of rules used = 19, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.679, Rules used = {6725, 2117, 14, 2119, 1628, 828, 826, 1166, 204, 206, 207, 203, 2122, 329, 211, 1165, 628, 1162, 617}

$$\sqrt{\sqrt{x^2+1}+x}-\frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}}+\frac{\log\left(\sqrt{x^2+1}-\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right)}{\sqrt{2}}-\frac{\log\left(\sqrt{x^2+1}+\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+x+1\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{1+\sqrt{2}}}+\frac{\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{\sqrt{2}-1}}+\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\sqrt{x^2+1}+x}\right)-\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\sqrt{x^2+1}+x}+1\right)-\frac{\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{1+\sqrt{2}}}+\frac{\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)}{\sqrt{\sqrt{2}-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^4)/((-1 + x^4)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/3*1/(x + Sqrt[1 + x^2])^(3/2) + Sqrt[x + Sqrt[1 + x^2]] - ArcTan[Sqrt[-1
 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[1 + Sqrt[2]] + ArcTan[Sqrt[1 + Sq
rt[2]]*Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[-1 + Sqrt[2]] + Sqrt[2]*ArcTan[1 - Sqr
t[2]*Sqrt[x + Sqrt[1 + x^2]]] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[x + Sqrt[1
+ x^2]]] - ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[1 + Sqr
t[2]] + ArcTanh[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[-1 + Sqrt[2
]] + Log[1 + x + Sqrt[1 + x^2] - Sqrt[2]*Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[2] -
 Log[1 + x + Sqrt[1 + x^2] + Sqrt[2]*Sqrt[x + Sqrt[1 + x^2]]]/Sqrt[2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^4}{\left(-1+x^4\right)\sqrt{x+\sqrt{1+x^2}}}\,dx = \int \left(\frac{1}{\sqrt{x+\sqrt{1+x^2}}} + \frac{2}{\left(-1+x^4\right)\sqrt{x+\sqrt{1+x^2}}}\right) dx$$

$$= 2\int \frac{1}{\left(-1+x^4\right)\sqrt{x+\sqrt{1+x^2}}}\,dx + \int \frac{1}{\sqrt{x+\sqrt{1+x^2}}}\,dx$$

$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1+x^2}{x^{5/2}}\,dx, x, x+\sqrt{1+x^2}\right) + 2\int \left(-\frac{1}{2\left(1-x^2\right)\sqrt{x+\sqrt{1+x^2}}}\right.$$

$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{1}{x^{5/2}} + \frac{1}{\sqrt{x}}\right) dx, x, x+\sqrt{1+x^2}\right) - \int \frac{1}{\left(1-x^2\right)\sqrt{x+\sqrt{1+x^2}}}$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} - 2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx, x, x+\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} - \frac{1}{2}\int \frac{1}{(1-x)\sqrt{x+\sqrt{1+x^2}}}\,dx - \frac{1}{2}$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} - \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1+x^2}{x^{3/2}\left(1+2x-x^2\right)}\,dx,\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} - \frac{1}{2}\,\mathrm{Subst}\left(\int \left(-\frac{1}{x^{3/2}} + \frac{2(1+x}{x^{3/2}\left(1+2x\right.}\right.\right.$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} + \frac{\log\left(1+x+\sqrt{1+x^2} - \sqrt{2}\sqrt{x+\right.}{\sqrt{2}}$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} + \sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x+\sqrt{1+x^2}}\right)$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} + \sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x+\sqrt{1+x^2}}\right)$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} + \sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x+\sqrt{1+x^2}}\right)$$

$$= -\frac{1}{3\left(x+\sqrt{1+x^2}\right)^{3/2}} + \sqrt{x+\sqrt{1+x^2}} + \frac{\tan^{-1}\left(\frac{\sqrt{x+\sqrt{1+x^2}}}{\sqrt{-1+\sqrt{2}}}\right)}{\sqrt{-1+\sqrt{2}}} - \frac{\tan^{-1}\left(\frac{\sqrt{x+\sqrt{}}}{\sqrt{1+}}\right.}{\sqrt{1+\sqrt{}}}$$

Mathematica [A] time = 0.96, size = 383, normalized size = 1.17

$$\sqrt{\sqrt{x^2+1}+x} - \frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}} + \frac{\log\left(-\frac{\sqrt{2}}{\sqrt{\sqrt{x^2+1}+x}} + \frac{1}{\sqrt{x^2+1}+x} + 1\right)}{\sqrt{2}} - \frac{\log\left(\frac{\sqrt{2}}{\sqrt{\sqrt{x^2+1}+x}} + \frac{1}{\sqrt{x^2+1}+x} + 1\right)}{\sqrt{2}} - \frac{\left(\sqrt{2}-2\right)\tan^{-1}\left(\frac{1}{\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}} - \frac{\left(2+\sqrt{2}\right)\tan^{-1}\left(\frac{1}{\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}} - \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt{\sqrt{x^2+1}+x}}\right) + \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{\sqrt{x^2+1}+x}}+1\right) + \frac{\left(\sqrt{2}-2\right)\tanh^{-1}\left(\frac{1}{\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}} + \frac{\left(2+\sqrt{2}\right)\tanh^{-1}\left(\frac{1}{\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/((-1 + x^4)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/3*1/(x + Sqrt[1 + x^2])^(3/2) + Sqrt[x + Sqrt[1 + x^2]] - ((-2 + Sqrt[2]
)*ArcTan[1/(Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]])])/Sqrt[2*(-1 + Sqrt
[2])] - ((2 + Sqrt[2])*ArcTan[1/(Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]])
])/Sqrt[2*(1 + Sqrt[2])] - Sqrt[2]*ArcTan[1 - Sqrt[2]/Sqrt[x + Sqrt[1 + x^2
]]] + Sqrt[2]*ArcTan[1 + Sqrt[2]/Sqrt[x + Sqrt[1 + x^2]]] + ((-2 + Sqrt[2])
*ArcTanh[1/(Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]])])/Sqrt[2*(-1 + Sqrt
[2])] + ((2 + Sqrt[2])*ArcTanh[1/(Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]
)])/Sqrt[2*(1 + Sqrt[2])] + Log[1 + (x + Sqrt[1 + x^2])^(-1) - Sqrt[2]/Sqrt
[x + Sqrt[1 + x^2]]]/Sqrt[2] - Log[1 + (x + Sqrt[1 + x^2])^(-1) + Sqrt[2]/S
qrt[x + Sqrt[1 + x^2]]]/Sqrt[2]
```

IntegrateAlgebraic [A] time = 0.49, size = 303, normalized size = 0.93

$$\sqrt{\sqrt{x^2+1}+x}-\frac{1}{3\left(\sqrt{x^2+1}+x\right)^{3/2}}-\sqrt{\sqrt{2}-1}\tan^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)+\sqrt{1+\sqrt{2}}\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{\sqrt{x^2+1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2+1}+x}}\right)-\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)+\sqrt{1+\sqrt{2}}\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\frac{\sqrt{x^2+1}}{\sqrt{2}}+\frac{x}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{\sqrt{\sqrt{x^2+1}+x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)/((-1 + x^4)*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -1/3*1/(x + Sqrt[1 + x^2])^(3/2) + Sqrt[x + Sqrt[1 + x^2]] - Sqrt[-1 + Sqrt
[2]]*ArcTan[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]] + Sqrt[1 + Sqrt[2]]
*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]] - Sqrt[2]*ArcTan[(-(1/Sq
rt[2]) + x/Sqrt[2] + Sqrt[1 + x^2]/Sqrt[2])/Sqrt[x + Sqrt[1 + x^2]]] - Sqrt
[-1 + Sqrt[2]]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]] + Sqrt[1
 + Sqrt[2]]*ArcTanh[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]] - Sqrt[2]*Ar
cTanh[(1/Sqrt[2] + x/Sqrt[2] + Sqrt[1 + x^2]/Sqrt[2])/Sqrt[x + Sqrt[1 + x^2
]]]
```

fricas [A] time = 0.50, size = 467, normalized size = 1.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(x^2 - sqrt(x^2 + 1)*x - 1)*sqrt(x + sqrt(x^2 + 1)) + 2*sqrt(2)*arctan
(sqrt(2)*sqrt(sqrt(2)*sqrt(x + sqrt(x^2 + 1)) + x + sqrt(x^2 + 1) + 1) - sq
rt(2)*sqrt(x + sqrt(x^2 + 1)) - 1) + 2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*s
qrt(2)*sqrt(x + sqrt(x^2 + 1)) + 4*x + 4*sqrt(x^2 + 1) + 4) - sqrt(2)*sqrt(
x + sqrt(x^2 + 1)) + 1) - 2*sqrt(sqrt(2) + 1)*arctan(sqrt(x + sqrt(2) + sqr
t(x^2 + 1) - 1)*sqrt(sqrt(2) + 1) - sqrt(x + sqrt(x^2 + 1))*sqrt(sqrt(2) +
1)) + 2*sqrt(sqrt(2) - 1)*arctan(sqrt(x + sqrt(2) + sqrt(x^2 + 1) + 1)*sqrt
(sqrt(2) - 1) - sqrt(x + sqrt(x^2 + 1))*sqrt(sqrt(2) - 1)) - 1/2*sqrt(sqrt(
2) - 1)*log((sqrt(2) + 1)*sqrt(sqrt(2) - 1) + sqrt(x + sqrt(x^2 + 1))) + 1/
2*sqrt(sqrt(2) - 1)*log(-(sqrt(2) + 1)*sqrt(sqrt(2) - 1) + sqrt(x + sqrt(x^
2 + 1))) + 1/2*sqrt(sqrt(2) + 1)*log(sqrt(sqrt(2) + 1)*(sqrt(2) - 1) + sqrt
(x + sqrt(x^2 + 1))) - 1/2*sqrt(sqrt(2) + 1)*log(-sqrt(sqrt(2) + 1)*(sqrt(2
) - 1) + sqrt(x + sqrt(x^2 + 1))) - 1/2*sqrt(2)*log(4*sqrt(2)*sqrt(x + sqrt
(x^2 + 1)) + 4*x + 4*sqrt(x^2 + 1) + 4) + 1/2*sqrt(2)*log(-4*sqrt(2)*sqrt(x
 + sqrt(x^2 + 1)) + 4*x + 4*sqrt(x^2 + 1) + 4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^4-1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate((x^4 + 1)/((x^4 - 1)*sqrt(x + sqrt(x^2 + 1))), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^4-1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^4-1)/(x+(x^2+1)^(1/2))^(1/2),x)
[Out] int((x^4+1)/(x^4-1)/(x+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{\left(x^4-1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^4-1)/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate((x^4 + 1)/((x^4 - 1)*sqrt(x + sqrt(x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4+1}{\left(x^4-1\right)\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/((x^4 - 1)*(x + (x^2 + 1)^(1/2))^(1/2)),x)
[Out] int((x^4 + 1)/((x^4 - 1)*(x + (x^2 + 1)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4+1}{(x-1)(x+1)\sqrt{x+\sqrt{x^2+1}}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**4-1)/(x+(x**2+1)**(1/2))**(1/2),x)
[Out] Integral((x**4 + 1)/((x - 1)*(x + 1)*sqrt(x + sqrt(x**2 + 1))*(x**2 + 1)),
x)
```

3.2302 $\int \frac{x^3}{\sqrt[3]{-x^2+x^4}\left(1+x^6\right)} dx$

Optimal. Leaf size=328

$$\frac{1}{6}\log\left(x^2+\left(x^4-x^2\right)^{2/3}\right)-\frac{1}{12}i\left(\sqrt{3}-i\right)\log\left(-i\sqrt{3}x^2-x^2+2\left(x^4-x^2\right)^{2/3}\right)+\frac{1}{12}i\left(\sqrt{3}+i\right)\log\left(i\sqrt{3}x^2-x^2+2\left(x\right.\right.$$

Rubi [C] time = 2.09, antiderivative size = 160, normalized size of antiderivative = 0.49, number of steps used = 55, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2056, 6725, 959, 466, 465, 511, 510}

$$\frac{\sqrt[3]{1-x^2}\,x^4F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^2,-x^2\right)}{10\sqrt[3]{x^4-x^2}}+\frac{\sqrt[3]{1-x^2}\,x^4F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^2,\sqrt[3]{-1}x^2\right)}{10\sqrt[3]{x^4-x^2}}+\frac{\sqrt[3]{1-x^2}\,x^4F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^2,-(-1)^{2/3}x^2\right)}{10\sqrt[3]{x^4-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3/((-x^2 + x^4)^(1/3)*(1 + x^6)),x]
```

```
[Out] (x^4*(1 - x^2)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, x^2, -x^2])/(10*(-x^2 + x^4
)^(1/3)) + (x^4*(1 - x^2)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, x^2, (-1)^(1/3)*
x^2])/(10*(-x^2 + x^4)^(1/3)) + (x^4*(1 - x^2)^(1/3)*AppellF1[5/3, 1/3, 1,
8/3, x^2, -((-1)^(2/3)*x^2)])/(10*(-x^2 + x^4)^(1/3))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
```

```
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !I
ntegerQ[p] && !IntegersQ[n, 2*p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt[3]{-x^2+x^4}\left(1+x^6\right)}\,dx = \frac{\left(x^{2/3}\sqrt[3]{-1+x^2}\right)\int \frac{x^{7/3}}{\sqrt[3]{-1+x^2}\left(1+x^6\right)}\,dx}{\sqrt[3]{-x^2+x^4}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{-1+x^2}\right)\int \left(\frac{ix^{7/3}}{2\sqrt[3]{-1+x^2}\left(i-x^3\right)}+\frac{ix^{7/3}}{2\sqrt[3]{-1+x^2}\left(i+x^3\right)}\right)dx}{\sqrt[3]{-x^2+x^4}}$$

$$= \frac{\left(ix^{2/3}\sqrt[3]{-1+x^2}\right)\int \frac{x^{7/3}}{\sqrt[3]{-1+x^2}\left(i-x^3\right)}\,dx}{2\sqrt[3]{-x^2+x^4}} + \frac{\left(ix^{2/3}\sqrt[3]{-1+x^2}\right)\int \frac{x^{7/3}}{\sqrt[3]{-1+x^2}\left(i+x^3\right)}\,dx}{2\sqrt[3]{-x^2+x^4}}$$

$$= \frac{\left(ix^{2/3}\sqrt[3]{-1+x^2}\right)\int \left(-\frac{(-1)^{2/3}x^{7/3}}{3\left(\sqrt[6]{-1}-x\right)\sqrt[3]{-1+x^2}}-\frac{(-1)^{2/3}x^{7/3}}{3\left(\sqrt[6]{-1}+\sqrt[3]{-1}\,x\right)\sqrt[3]{-1+x^2}}-\frac{(-1)^{2/3}x^{7/3}}{3\left(\sqrt[6]{-1}-(-1)^{2/3}x\right)\sqrt[3]{-1+\ldots}}\right.}{2\sqrt[3]{-x^2+x^4}}$$

$$= \frac{\left(\sqrt[6]{-1}\,x^{2/3}\sqrt[3]{-1+x^2}\right)\int \frac{x^{7/3}}{\left(\sqrt[6]{-1}-x\right)\sqrt[3]{-1+x^2}}\,dx}{6\sqrt[3]{-x^2+x^4}} + \frac{\left(\sqrt[6]{-1}\,x^{2/3}\sqrt[3]{-1+x^2}\right)\int \frac{x^{7/3}}{\left(\sqrt[6]{-1}+\sqrt[3]{-1}\,x\right)\ldots}}{6\sqrt[3]{-x^2+x^4}}$$

$$= \frac{\left(ix^{2/3}\sqrt[3]{-1+x^2}\right)\int \frac{x^{10/3}}{\sqrt[3]{-1+x^2}\left(-(-1)^{2/3}+\sqrt[3]{-1}\,x^2\right)}\,dx}{6\sqrt[3]{-x^2+x^4}} - \frac{\left(ix^{2/3}\sqrt[3]{-1+x^2}\right)\int \frac{x^{10/3}}{\sqrt[3]{-1+x^2}\left(\sqrt[3]{-1}-\ldots\right.}}{6\sqrt[3]{-x^2+x^4}}$$

$$= \frac{\left(ix^{2/3}\sqrt[3]{-1+x^2}\right)\text{Subst}\left(\int \frac{x^{12}}{\sqrt[3]{-1+x^6}\left(-(-1)^{2/3}+\sqrt[3]{-1}\,x^6\right)}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{-x^2+x^4}} - \frac{\left(ix^{2/3}\sqrt[3]{-1+x^2}\right)\ldots}{}$$

$$= \frac{\left(ix^{2/3}\sqrt[3]{1-x^2}\right)\text{Subst}\left(\int \frac{x^{12}}{\sqrt[3]{1-x^6}\left(-(-1)^{2/3}+\sqrt[3]{-1}\,x^6\right)}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{-x^2+x^4}} - \frac{\left(ix^{2/3}\sqrt[3]{1-x^2}\right)\text{Subs}\ldots}{}$$

$$= \frac{\left(\sqrt[3]{-1}\,x^{2/3}\sqrt[3]{1-x^2}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt[3]{1-x^3}\left(\sqrt[3]{-1}-x^3\right)}\,dx, x, x^{2/3}\right)}{4\sqrt[3]{-x^2+x^4}} + \frac{\left(\sqrt[3]{-1}\,x^{2/3}\sqrt[3]{1-x^2}\right)\text{Su}\ldots}{}$$

$$= \frac{x^4\sqrt[3]{1-x^2}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^2,-x^2\right)}{10\sqrt[3]{-x^2+x^4}} + \frac{x^4\sqrt[3]{1-x^2}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^2,\sqrt[3]{-1}\,x^2\right)}{10\sqrt[3]{-x^2+x^4}} + \frac{x^4\sqrt[3]{1-\ldots}}{}$$

Mathematica [F] time = 1.79, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt[3]{-x^2+x^4}\,(1+x^6)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((-x^2 + x^4)^(1/3)*(1 + x^6)),x]
```

```
[Out] Integrate[x^3/((-x^2 + x^4)^(1/3)*(1 + x^6)), x]
```

IntegrateAlgebraic [A] time = 0.73, size = 312, normalized size = 0.95

$$\frac{1}{6}\log\left(x^2+\left(x^4-x^2\right)^{2/3}\right)+\frac{\log\left(-2x^2+2^{2/3}\sqrt{3}\sqrt[3]{x^4-x^2}\,x-\sqrt[3]{2}\left(x^4-x^2\right)^{2/3}\right)}{24\sqrt[3]{2}}-\frac{\log\left(2x^2+\sqrt[3]{2}\left(x^4-x^2\right)^{2/3}\right)}{12\sqrt[3]{2}}+\frac{\log\left(2x^2+2^{2/3}\sqrt{3}\sqrt[3]{x^4-x^2}\,x+\sqrt[3]{2}\left(x^4-x^2\right)^{2/3}\right)}{24\sqrt[3]{2}}-\frac{1}{12}\log\left(x^4-\left(x^4-x^2\right)^{2/3}x^2+\left(x^4-x^2\right)^{4/3}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{2\left(x^4-x^2\right)^{2/3}-x^2}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{\sqrt[3]{2}\left(x^4-x^2\right)^{2/3}-x^2}\right)}{4\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((-x^2 + x^4)^(1/3)*(1 + x^6)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3]*x^2)/(-x^2 + 2*(-x^2 + x^4)^(2/3))]/Sqrt[3] + ArcTan[(
Sqrt[3]*x^2)/(-x^2 + 2^(1/3)*(-x^2 + x^4)^(2/3))]/(4*2^(1/3)*Sqrt[3]) + Log
[x^2 + (-x^2 + x^4)^(2/3)]/6 + Log[-2*x^2 + 2^(2/3)*Sqrt[3]*x*(-x^2 + x^4)^
(1/3) - 2^(1/3)*(-x^2 + x^4)^(2/3)]/(24*2^(1/3)) - Log[2*x^2 + 2^(1/3)*(-x^
2 + x^4)^(2/3)]/(12*2^(1/3)) + Log[2*x^2 + 2^(2/3)*Sqrt[3]*x*(-x^2 + x^4)^(
1/3) + 2^(1/3)*(-x^2 + x^4)^(2/3)]/(24*2^(1/3)) - Log[x^4 - x^2*(-x^2 + x^4
)^(2/3) + (-x^2 + x^4)^(4/3)]/12
```

fricas [A] time = 2.58, size = 470, normalized size = 1.43

$$-\frac{1}{72}\sqrt{6}\,2^{1/6}(-1)^{1/3}\arctan\left(\frac{2^{1/6}\left(24\sqrt{6}\,2^{2/3}(-1)^{2/3}\left(x^8-2x^6-6x^4-2x^2+1\right)\left(x^4-x^2\right)^{2/3}-12\sqrt{6}(-1)^{1/3}\left(x^{10}-33x^8+110x^6-110x^4+33x^2-1\right)\left(x^4-x^2\right)^{1/3}+\sqrt{6}\,2^{1/3}\left(x^{12}+42x^{10}-417x^8+812x^6-417x^4+42x^2+1\right)\right)}{6\left(x^{12}-102x^{10}+447x^8-628x^6+447x^4-102x^2+1\right)}\right)-\frac{1}{144}2^{2/3}(-1)^{1/3}\log\left(-\frac{12\cdot 2^{2/3}(-1)^{1/3}\left(x^4-x^2\right)^{2/3}\left(x^4-4x^2+1\right)-2^{1/3}(-1)^{2/3}\left(x^8-32x^6+78x^4-32x^2+1\right)+6\left(x^6-11x^4+11x^2-1\right)\left(x^4-x^2\right)^{1/3}}{x^8+4x^6+6x^4+4x^2+1}\right)+\frac{1}{72}2^{2/3}(-1)^{1/3}\log\left(-\frac{6\cdot 2^{1/3}(-1)^{2/3}\left(x^4-x^2\right)^{1/3}\left(x^2-1\right)-2^{2/3}(-1)^{1/3}\left(x^4+2x^2+1\right)+12\left(x^4-x^2\right)^{2/3}}{x^4+2x^2+1}\right)-\frac{1}{6}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(x^2-1\right)-2\sqrt{3}\left(x^4-x^2\right)^{1/3}}{3\left(x^2-1\right)}\right)+\frac{1}{12}\log\left(\frac{x^4-x^2+3\left(x^4-x^2\right)^{1/3}\left(x^2-1\right)+3\left(x^4-x^2\right)^{2/3}+1}{x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-x^2)^(1/3)/(x^6+1),x, algorithm="fricas")
```

```
[Out] -1/72*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(24*sqrt(6)*2^(2/3)*(-1
)^(2/3)*(x^8 - 2*x^6 - 6*x^4 - 2*x^2 + 1)*(x^4 - x^2)^(2/3) - 12*sqrt(6)*(-
1)^(1/3)*(x^10 - 33*x^8 + 110*x^6 - 110*x^4 + 33*x^2 - 1)*(x^4 - x^2)^(1/3)
 + sqrt(6)*2^(1/3)*(x^12 + 42*x^10 - 417*x^8 + 812*x^6 - 417*x^4 + 42*x^2 +
 1))/(x^12 - 102*x^10 + 447*x^8 - 628*x^6 + 447*x^4 - 102*x^2 + 1)) - 1/144
*2^(2/3)*(-1)^(1/3)*log(-(12*2^(2/3)*(-1)^(1/3)*(x^4 - x^2)^(2/3)*(x^4 - 4*
x^2 + 1) - 2^(1/3)*(-1)^(2/3)*(x^8 - 32*x^6 + 78*x^4 - 32*x^2 + 1) + 6*(x^6
 - 11*x^4 + 11*x^2 - 1)*(x^4 - x^2)^(1/3))/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1
)) + 1/72*2^(2/3)*(-1)^(1/3)*log(-(6*2^(1/3)*(-1)^(2/3)*(x^4 - x^2)^(1/3)*(
x^2 - 1) - 2^(2/3)*(-1)^(1/3)*(x^4 + 2*x^2 + 1) + 12*(x^4 - x^2)^(2/3))/(x^
4 + 2*x^2 + 1)) - 1/6*sqrt(3)*arctan(-1/3*(sqrt(3)*(x^2 - 1) - 2*sqrt(3)*(x
^4 - x^2)^(1/3))/(x^2 - 1)) + 1/12*log((x^4 - x^2 + 3*(x^4 - x^2)^(1/3)*(x^
2 - 1) + 3*(x^4 - x^2)^(2/3) + 1)/(x^4 - x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^6+1\right)\left(x^4-x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-x^2)^(1/3)/(x^6+1),x, algorithm="giac")
```

```
[Out] integrate(x^3/((x^6 + 1)*(x^4 - x^2)^(1/3)), x)
```

maple [C] time = 46.88, size = 2437, normalized size = 7.43

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4-x^2)^(1/3)/(x^6+1),x)
```

```
[Out] 1/4*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*ln((347184*RootOf(
RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^4+876300
0*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^4
-1475532*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3
+4)^2*x^2-37242750*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*Roo
tOf(_Z^3+4)^3*x^2+50416182*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_
Z^2)*RootOf(_Z^3+4)^2*(x^4-x^2)^(2/3)+4019850*RootOf(RootOf(_Z^3+4)^2+6*_Z*
RootOf(_Z^3+4)+36*_Z^2)*(x^4-x^2)^(1/3)*RootOf(_Z^3+4)*x^2-130194*RootOf(Ro
otOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^4-8402697*(x^4-x^2)^(1/3)*Roo
tOf(_Z^3+4)^2*x^2-3286125*RootOf(_Z^3+4)*x^4+347184*RootOf(RootOf(_Z^3+4)^2
+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2+8763000*RootOf(_Z^3+4)^3*R
ootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)-4019850*(x^4-x^2)^(1/3)
*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)-144660
*x^2*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)+8402697*RootOf(_Z
^3+4)^2*(x^4-x^2)^(1/3)-3651250*RootOf(_Z^3+4)*x^2-36290688*(x^4-x^2)^(2/3)
-130194*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)-3286125*RootOf
(_Z^3+4))/(x^2+1)^2)+1/24*RootOf(_Z^3+4)*ln(-(52578000*RootOf(RootOf(_Z^3+4
)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^4+57864*RootOf(RootOf
(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^4-223456500*Root
Of(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^2-245
922*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x
^2+50416182*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^
3+4)^2*(x^4-x^2)^(2/3)-54436032*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)
+36*_Z^2)*(x^4-x^2)^(1/3)*RootOf(_Z^3+4)*x^2-15335250*RootOf(RootOf(_Z^3+4)
^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^4-8402697*(x^4-x^2)^(1/3)*RootOf(_Z^3+4)^
2*x^2-16877*RootOf(_Z^3+4)*x^4+52578000*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf
(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2+57864*RootOf(_Z^3+4)^3*RootOf(RootOf(_
Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)+54436032*(x^4-x^2)^(1/3)*RootOf(RootO
f(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)+170878500*x^2*RootO
f(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)+8402697*RootOf(_Z^3+4)^2*(x
^4-x^2)^(1/3)+188058*RootOf(_Z^3+4)*x^2+2679900*(x^4-x^2)^(2/3)-15335250*Ro
otOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)-16877*RootOf(_Z^3+4))/(x
^2+1)^2)+1/6*ln((2521548*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^
2)^2*RootOf(_Z^3+4)^4*x^4-10716579*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3
+4)+36*_Z^2)^2*RootOf(_Z^3+4)^4*x^2-4376418*RootOf(RootOf(_Z^3+4)^2+6*_Z*Ro
otOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x^4+2521548*RootOf(RootOf(_Z^3+4)^2+
6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^4-5735088*RootOf(RootOf(_Z^3+
4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*(x^4-x^2)^(1/3)*RootOf(_Z^3+4)^2*x^2-3384
018*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*(
x^4-x^2)^(2/3)+12190842*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2
)*RootOf(_Z^3+4)^2*x^2+5735088*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+
36*_Z^2)*RootOf(_Z^3+4)^2*(x^4-x^2)^(1/3)+891144*x^4-4376418*RootOf(RootOf(
_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2+1567380*(x^4-x^2)^(
1/3)*x^2+3823392*(x^4-x^2)^(2/3)-2277368*x^2-1567380*(x^4-x^2)^(1/3)+891144
)/(x^4-x^2+1))-1/6*ln(-(1630404*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)
+36*_Z^2)^2*RootOf(_Z^3+4)^4*x^4-6929217*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootO
f(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^4*x^2+1952766*RootOf(RootOf(_Z^3+4)^2+6
*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x^4+1630404*RootOf(RootOf(_Z^3
+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^4+2351070*RootOf(RootOf
(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*(x^4-x^2)^(1/3)*RootOf(_Z^3+4)^2*x^
2-3384018*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+
4)^2*(x^4-x^2)^(2/3)-4155312*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36
```

```
*_Z^2)*RootOf(_Z^3+4)^2*x^2-2351070*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^
3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*(x^4-x^2)^(1/3)+495080*x^4+1952766*RootOf(Ro
otOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2-3823392*(x^4-x
^2)^(1/3)*x^2-1567380*(x^4-x^2)^(2/3)-594096*x^2+3823392*(x^4-x^2)^(1/3)+49
5080)/(x^4-x^2+1))+1/4*ln(-(1630404*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^
3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^4*x^4-6929217*RootOf(RootOf(_Z^3+4)^2+6*_Z*R
ootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^4*x^2+1952766*RootOf(RootOf(_Z^3+4)
^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x^4+1630404*RootOf(RootOf(
_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^4+2351070*RootOf(Ro
otOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*(x^4-x^2)^(1/3)*RootOf(_Z^3+4)^
2*x^2-3384018*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_
Z^3+4)^2*(x^4-x^2)^(2/3)-4155312*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4
)+36*_Z^2)*RootOf(_Z^3+4)^2*x^2-2351070*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf
(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*(x^4-x^2)^(1/3)+495080*x^4+1952766*RootO
f(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2-3823392*(x
^4-x^2)^(1/3)*x^2-1567380*(x^4-x^2)^(2/3)-594096*x^2+3823392*(x^4-x^2)^(1/3
)+495080)/(x^4-x^2+1))*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)
*RootOf(_Z^3+4)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^6+1\right)\left(x^4-x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-x^2)^(1/3)/(x^6+1),x, algorithm="maxima")
[Out] integrate(x^3/((x^6 + 1)*(x^4 - x^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(x^6+1\right)\,\left(x^4-x^2\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((x^6 + 1)*(x^4 - x^2)^(1/3)),x)
[Out] int(x^3/((x^6 + 1)*(x^4 - x^2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt[3]{x^2\,(x-1)\,(x+1)}\,\left(x^2+1\right)\left(x^4-x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4-x**2)**(1/3)/(x**6+1),x)
[Out] Integral(x**3/((x**2*(x - 1)*(x + 1))**(1/3)*(x**2 + 1)*(x**4 - x**2 + 1)),
 x)
```

3.2303 $\int \frac{-1+x^2}{\sqrt[4]{\frac{b+ax}{d+cx}}}\,dx$

Optimal. Leaf size=329

$$\frac{\left(\frac{ax+b}{cx+d}\right)^{3/4}\left(32a^2c^3x^3-96a^2c^3x+36a^2c^2dx^2-96a^2c^2d-3a^2cd^2x-7a^2d^3-36abc^3x^2-42abc^2dx-6abcd^2+4\right.}{96a^3c^2}$$

Rubi [A] time = 0.77, antiderivative size = 624, normalized size of antiderivative = 1.90, number of steps used = 18, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1961, 6742, 290, 298, 205, 208}

$$\frac{15(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt[4]{\frac{ax+b}{cx+d}}}{\sqrt[4]{a}}\right)}{64a^{13/4}c^{11/4}}+\frac{5d(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt[4]{\frac{ax+b}{cx+d}}}{\sqrt[4]{a}}\right)}{8a^{9/4}c^{11/4}}-\frac{15(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt[4]{\frac{ax+b}{cx+d}}}{\sqrt[4]{a}}\right)}{64a^{13/4}c^{11/4}}-\frac{5d(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt[4]{\frac{ax+b}{cx+d}}}{\sqrt[4]{a}}\right)}{8a^{9/4}c^{11/4}}-\frac{\left(c^2-d^2\right)(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt[4]{\frac{ax+b}{cx+d}}}{\sqrt[4]{a}}\right)}{2a^{5/4}c^{11/4}}+\frac{\left(c^2-d^2\right)(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt[4]{\frac{ax+b}{cx+d}}}{\sqrt[4]{a}}\right)}{2a^{5/4}c^{11/4}}+\frac{15(cx+d)(bc-ad)^2\left(\frac{ax+b}{cx+d}\right)^{3/4}}{32a^3c^2}-\frac{3(bc-ad)^3\left(\frac{ax+b}{cx+d}\right)^{3/4}}{8a^2c^2\left(a-\frac{c(ax+b)}{cx+d}\right)^2}+\frac{5d(cx+d)(bc-ad)\left(\frac{ax+b}{cx+d}\right)^{3/4}}{4a^2c^2}-\frac{\left(c^2-d^2\right)(cx+d)\left(\frac{ax+b}{cx+d}\right)^{3/4}}{ac^2}-\frac{(bc-ad)^3\left(\frac{ax+b}{cx+d}\right)^{3/4}}{3ac^2\left(a-\frac{c(ax+b)}{cx+d}\right)^3}-\frac{d(bc-ad)^2\left(\frac{ax+b}{cx+d}\right)^{3/4}}{ac^2\left(a-\frac{c(ax+b)}{cx+d}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/((b + a*x)/(d + c*x))^(1/4),x]
```

```
[Out] (5*d*(b*c - a*d)*((b + a*x)/(d + c*x))^(3/4)*(d + c*x))/(4*a^2*c^2) + (15*(
b*c - a*d)^2*((b + a*x)/(d + c*x))^(3/4)*(d + c*x))/(32*a^3*c^2) - ((c^2 -
d^2)*((b + a*x)/(d + c*x))^(3/4)*(d + c*x))/(a*c^2) - ((b*c - a*d)^3*((b +
a*x)/(d + c*x))^(3/4))/(3*a*c^2*(a - (c*(b + a*x))/(d + c*x))^3) - (d*(b*c
- a*d)^2*((b + a*x)/(d + c*x))^(3/4))/(a*c^2*(a - (c*(b + a*x))/(d + c*x))^
2) - (3*(b*c - a*d)^3*((b + a*x)/(d + c*x))^(3/4))/(8*a^2*c^2*(a - (c*(b +
a*x))/(d + c*x))^2) + (5*d*(b*c - a*d)^2*ArcTan[(c^(1/4)*((b + a*x)/(d + c*
x))^(1/4))/a^(1/4)])/(8*a^(9/4)*c^(11/4)) + (15*(b*c - a*d)^3*ArcTan[(c^(1/
4)*((b + a*x)/(d + c*x))^(1/4))/a^(1/4)])/(64*a^(13/4)*c^(11/4)) - ((b*c -
a*d)*(c^2 - d^2)*ArcTan[(c^(1/4)*((b + a*x)/(d + c*x))^(1/4))/a^(1/4)])/(2*
a^(5/4)*c^(11/4)) - (5*d*(b*c - a*d)^2*ArcTanh[(c^(1/4)*((b + a*x)/(d + c*x
))^(1/4))/a^(1/4)])/(8*a^(9/4)*c^(11/4)) - (15*(b*c - a*d)^3*ArcTanh[(c^(1/
4)*((b + a*x)/(d + c*x))^(1/4))/a^(1/4)])/(64*a^(13/4)*c^(11/4)) + ((b*c -
a*d)*(c^2 - d^2)*ArcTanh[(c^(1/4)*((b + a*x)/(d + c*x))^(1/4))/a^(1/4)])/(2
*a^(5/4)*c^(11/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 1961

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
 + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-1+x^2}{\sqrt[4]{\frac{b+ax}{d+cx}}}\,dx = -\left((4(bc-ad))\,\text{Subst}\left(\int \frac{x^2\left(-1+\frac{\left(b-dx^4\right)^2}{\left(a-cx^4\right)^2}\right)}{\left(a-cx^4\right)^2}\,dx, x, \sqrt[4]{\frac{b+ax}{d+cx}}\right)\right)$$

$$= -\left((4(bc-ad))\,\text{Subst}\left(\int \left(\frac{(bc-ad)^2x^2}{c^2\left(-a+cx^4\right)^4}-\frac{2d(bc-ad)x^2}{c^2\left(-a+cx^4\right)^3}-\frac{\left(c^2-d^2\right)x^2}{c^2\left(-a+cx^4\right)^2}\right)dx, x, \sqrt[4]{\frac{b+ax}{d+cx}}\right)\right)$$

$$= \frac{\left(8d(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{\left(-a+cx^4\right)^3}\,dx, x, \sqrt[4]{\frac{b+ax}{d+cx}}\right)}{c^2} - \frac{\left(4(bc-ad)^3\right)\text{Subst}\left(\int \frac{x^2}{\left(-a+cx^4\right)^4}\,dx, x, \sqrt[4]{\frac{b+ax}{d+cx}}\right.}{c^2}$$

$$= -\frac{\left(c^2-d^2\right)\left(\frac{b+ax}{d+cx}\right)^{3/4}(d+cx)}{ac^2} - \frac{(bc-ad)^3\left(\frac{b+ax}{d+cx}\right)^{3/4}}{3ac^2\left(a-\frac{c(b+ax)}{d+cx}\right)^3} - \frac{d(bc-ad)^2\left(\frac{b+ax}{d+cx}\right)^{3/4}}{ac^2\left(a-\frac{c(b+ax)}{d+cx}\right)^2} - \left(5d(bc-ad)^2\right)\text{Sub}$$

$$= \frac{5d(bc-ad)\left(\frac{b+ax}{d+cx}\right)^{3/4}(d+cx)}{4a^2c^2} - \frac{\left(c^2-d^2\right)\left(\frac{b+ax}{d+cx}\right)^{3/4}(d+cx)}{ac^2} - \frac{(bc-ad)^3\left(\frac{b+ax}{d+cx}\right)^{3/4}}{3ac^2\left(a-\frac{c(b+ax)}{d+cx}\right)^3} - \frac{d(bc-ad)^2}{ac^2\left(a-\right.}$$

$$= \frac{5d(bc-ad)\left(\frac{b+ax}{d+cx}\right)^{3/4}(d+cx)}{4a^2c^2} + \frac{15(bc-ad)^2\left(\frac{b+ax}{d+cx}\right)^{3/4}(d+cx)}{32a^3c^2} - \frac{\left(c^2-d^2\right)\left(\frac{b+ax}{d+cx}\right)^{3/4}(d+cx)}{ac^2} -$$

$$= \frac{5d(bc-ad)\left(\frac{b+ax}{d+cx}\right)^{3/4}(d+cx)}{4a^2c^2} + \frac{15(bc-ad)^2\left(\frac{b+ax}{d+cx}\right)^{3/4}(d+cx)}{32a^3c^2} - \frac{\left(c^2-d^2\right)\left(\frac{b+ax}{d+cx}\right)^{3/4}(d+cx)}{ac^2} -$$

$$= \frac{5d(bc-ad)\left(\frac{b+ax}{d+cx}\right)^{3/4}(d+cx)}{4a^2c^2} + \frac{15(bc-ad)^2\left(\frac{b+ax}{d+cx}\right)^{3/4}(d+cx)}{32a^3c^2} - \frac{\left(c^2-d^2\right)\left(\frac{b+ax}{d+cx}\right)^{3/4}(d+cx)}{ac^2} -$$

Mathematica [C] time = 0.17, size = 171, normalized size = 0.52

$$\frac{4(cx+d)\left(\frac{ax+b}{cx+d}\right)^{3/4}\left(a\left(a\left(d^2-c^2\right){}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{c(b+ax)}{bc-ad}\right)+2d(bc-ad){}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};\frac{c(b+ax)}{bc-ad}\right)\right)+(bc-ad)^2{}_2F_1\left(-\frac{9}{4},\frac{3}{4};\frac{7}{4};\frac{c(b+ax)}{bc-ad}\right)\right)}{3a^3c^2\sqrt[4]{\frac{a(cx+d)}{ad-bc}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((b + a*x)/(d + c*x))^(1/4),x]
```

```
[Out] (4*((b + a*x)/(d + c*x))^(3/4)*(d + c*x)*((b*c - a*d)^2*Hypergeometric2F1[-
9/4, 3/4, 7/4, (c*(b + a*x))/(b*c - a*d)] + a*(2*d*(b*c - a*d)*Hypergeometr
ic2F1[-5/4, 3/4, 7/4, (c*(b + a*x))/(b*c - a*d)] + a*(-c^2 + d^2)*Hypergeom
etric2F1[-1/4, 3/4, 7/4, (c*(b + a*x))/(b*c - a*d)])))/(3*a^3*c^2*((a*(d +
c*x))/(-(b*c) + a*d))^(1/4))
```

IntegrateAlgebraic [B] time = 1.17, size = 711, normalized size = 2.16

$$\frac{\left(32a^3c^2d-7a^3d^3-32a^2bc^3-3a^2bcd^2-5ab^2c^2d+15b^3c^3\right)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt[4]{\frac{ax+b}{cx+d}}}{\sqrt[4]{a}}\right)}{64a^{13/4}c^{11/4}}+\frac{\left(-32a^3c^2d+7a^3d^3+32a^2bc^3+3a^2bcd^2+5ab^2c^2d-15b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt[4]{\frac{ax+b}{cx+d}}}{\sqrt[4]{a}}\right)}{64a^{13/4}c^{11/4}}+\frac{-96a^5c^2d\left(\frac{ax+b}{cx+d}\right)^{3/4}-7a^5d^3\left(\frac{ax+b}{cx+d}\right)^{3/4}+192a^4c^3d\left(\frac{ax+b}{cx+d}\right)^{7/4}+96a^4bc^3\left(\frac{ax+b}{cx+d}\right)^{3/4}+18a^4cd^3\left(\frac{ax+b}{cx+d}\right)^{7/4}-3a^4bcd^2\left(\frac{ax+b}{cx+d}\right)^{3/4}+123a^3b^2c^2d\left(\frac{ax+b}{cx+d}\right)^{3/4}-96a^3c^4d\left(\frac{ax+b}{cx+d}\right)^{11/4}-192a^3bc^4\left(\frac{ax+b}{cx+d}\right)^{7/4}+21a^3c^2d^3\left(\frac{ax+b}{cx+d}\right)^{11/4}-102a^3bc^2d^2\left(\frac{ax+b}{cx+d}\right)^{7/4}-113a^2b^3c^3\left(\frac{ax+b}{cx+d}\right)^{3/4}-42a^2b^2c^3d\left(\frac{ax+b}{cx+d}\right)^{7/4}+96a^2bc^5\left(\frac{ax+b}{cx+d}\right)^{11/4}+9a^2bc^3d^2\left(\frac{ax+b}{cx+d}\right)^{11/4}-45b^3c^5\left(\frac{ax+b}{cx+d}\right)^{11/4}+126ab^3c^4\left(\frac{ax+b}{cx+d}\right)^{7/4}+15ab^2c^4d\left(\frac{ax+b}{cx+d}\right)^{11/4}}{96a^3c^2\left(a-\frac{c(ax+b)}{cx+d}\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^2)/((b + a*x)/(d + c*x))^(1/4),x]
```

```
[Out] (96*a^4*b*c^3*((b + a*x)/(d + c*x))^(3/4) - 113*a^2*b^3*c^3*((b + a*x)/(d +
 c*x))^(3/4) - 96*a^5*c^2*d*((b + a*x)/(d + c*x))^(3/4) + 123*a^3*b^2*c^2*d
*((b + a*x)/(d + c*x))^(3/4) - 3*a^4*b*c*d^2*((b + a*x)/(d + c*x))^(3/4) -
7*a^5*d^3*((b + a*x)/(d + c*x))^(3/4) - 192*a^3*b*c^4*((b + a*x)/(d + c*x))
^(7/4) + 126*a*b^3*c^4*((b + a*x)/(d + c*x))^(7/4) + 192*a^4*c^3*d*((b + a*
x)/(d + c*x))^(7/4) - 42*a^2*b^2*c^3*d*((b + a*x)/(d + c*x))^(7/4) - 102*a^
3*b*c^2*d^2*((b + a*x)/(d + c*x))^(7/4) + 18*a^4*c*d^3*((b + a*x)/(d + c*x)
)^(7/4) + 96*a^2*b*c^5*((b + a*x)/(d + c*x))^(11/4) - 45*b^3*c^5*((b + a*x)
/(d + c*x))^(11/4) - 96*a^3*c^4*d*((b + a*x)/(d + c*x))^(11/4) + 15*a*b^2*c
^4*d*((b + a*x)/(d + c*x))^(11/4) + 9*a^2*b*c^3*d^2*((b + a*x)/(d + c*x))^(
11/4) + 21*a^3*c^2*d^3*((b + a*x)/(d + c*x))^(11/4))/(96*a^3*c^2*(a - (c*(b
 + a*x))/(d + c*x))^3) + ((-32*a^2*b*c^3 + 15*b^3*c^3 + 32*a^3*c^2*d - 5*a*
b^2*c^2*d - 3*a^2*b*c*d^2 - 7*a^3*d^3)*ArcTan[(c^(1/4)*((b + a*x)/(d + c*x)
)^(1/4))/a^(1/4)])/(64*a^(13/4)*c^(11/4)) + ((32*a^2*b*c^3 - 15*b^3*c^3 - 3
2*a^3*c^2*d + 5*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 7*a^3*d^3)*ArcTanh[(c^(1/4)*(
(b + a*x)/(d + c*x))^(1/4))/a^(1/4)])/(64*a^(13/4)*c^(11/4))
```

fricas [B] time = 2.15, size = 6305, normalized size = 19.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/((a*x+b)/(c*x+d))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/384*(12*a^3*c^2*((4116*a^11*b*c*d^11 + 2401*a^12*d^12 + (1048576*a^8*b^4
- 1966080*a^6*b^6 + 1382400*a^4*b^8 - 432000*a^2*b^10 + 50625*b^12)*c^12 -
4*(1048576*a^9*b^3 - 1638400*a^7*b^5 + 921600*a^5*b^7 - 216000*a^3*b^9 + 16
875*a*b^11)*c^11*d + 6*(1048576*a^10*b^2 - 1245184*a^8*b^4 + 471040*a^6*b^6
 - 52800*a^4*b^8 - 1125*a^2*b^10)*c^10*d^2 - 4*(1048576*a^11*b - 917504*a^9
*b^3 + 307200*a^7*b^5 - 83200*a^5*b^7 + 15375*a^3*b^9)*c^9*d^3 + (1048576*a
^12 - 2228224*a^10*b^2 + 2297856*a^8*b^4 - 874240*a^6*b^6 + 93775*a^4*b^8)*
c^8*d^4 + 24*(98304*a^11*b - 75776*a^9*b^3 + 11200*a^7*b^5 + 775*a^5*b^7)*c
^7*d^5 - 4*(229376*a^12 - 67584*a^10*b^2 + 10176*a^8*b^4 - 7895*a^6*b^6)*c^
6*d^6 - 24*(14336*a^11*b - 13184*a^9*b^3 + 2025*a^7*b^5)*c^5*d^7 + 3*(10035
2*a^12 - 20608*a^10*b^2 - 5083*a^8*b^4)*c^4*d^8 - 28*(448*a^11*b + 393*a^9*
b^3)*c^3*d^9 - 98*(448*a^12 - 97*a^10*b^2)*c^2*d^10)/(a^13*c^11))^(1/4)*arc
tan((sqrt((302526*a^17*b*c*d^17 + 117649*a^18*d^18 + (1073741824*a^12*b^6 -
 3019898880*a^10*b^8 + 3538944000*a^8*b^10 - 2211840000*a^6*b^12 + 77760000
```

0*a^4*b^14 - 145800000*a^2*b^16 + 11390625*b^18)*c^18 - 6*(1073741824*a^13*
b^5 - 2684354560*a^11*b^7 + 2752512000*a^9*b^9 - 1474560000*a^7*b^11 + 4320
00000*a^5*b^13 - 64800000*a^3*b^15 + 3796875*a*b^17)*c^17*d + 3*(5368709120
*a^14*b^4 - 11542724608*a^12*b^6 + 9882828800*a^10*b^8 - 4227072000*a^8*b^1
0 + 915840000*a^6*b^12 - 86400000*a^4*b^14 + 1771875*a^2*b^16)*c^16*d^2 - 1
6*(1342177280*a^15*b^3 - 2415919104*a^13*b^5 + 1690828800*a^11*b^7 - 585728
000*a^9*b^9 + 110880000*a^7*b^11 - 13500000*a^5*b^13 + 1096875*a^3*b^15)*c^
15*d^3 + 60*(268435456*a^16*b^2 - 436207616*a^14*b^4 + 323485696*a^12*b^6 -
158433280*a^10*b^8 + 53766400*a^8*b^10 - 10392000*a^6*b^12 + 781875*a^4*b^
14)*c^14*d^4 - 24*(268435456*a^17*b - 671088640*a^15*b^3 + 815267840*a^13*b
^5 - 501350400*a^11*b^7 + 147488000*a^9*b^9 - 17320000*a^7*b^11 + 328125*a^
5*b^13)*c^13*d^5 + 4*(268435456*a^18 - 3019898880*a^16*b^2 + 3971481600*a^1
4*b^4 - 1859256320*a^12*b^6 + 338515200*a^10*b^8 - 23424000*a^8*b^10 + 2100
625*a^6*b^12)*c^12*d^6 + 48*(134217728*a^17*b - 144179200*a^15*b^3 + 711065
60*a^13*b^5 - 29811200*a^11*b^7 + 8516000*a^9*b^9 - 879375*a^7*b^11)*c^11*d
^7 - 6*(234881024*a^18 - 537395200*a^16*b^2 + 627507200*a^14*b^4 - 29063936
0*a^12*b^6 + 44430400*a^10*b^8 - 492125*a^8*b^10)*c^10*d^8 - 20*(121110528*
a^17*b - 109445120*a^15*b^3 + 22353408*a^13*b^5 + 447360*a^11*b^7 + 15975*a
^9*b^9)*c^9*d^9 + 6*(128450560*a^18 - 68812800*a^16*b^2 + 41638400*a^14*b^4
- 23924224*a^12*b^6 + 3566605*a^10*b^8)*c^8*d^10 + 48*(8028160*a^17*b - 82
88000*a^15*b^3 + 1718368*a^13*b^5 + 28305*a^11*b^7)*c^7*d^11 - 4*(56197120*
a^18 - 14112000*a^16*b^2 - 4320960*a^14*b^4 + 103199*a^12*b^6)*c^6*d^12 - 1
68*(62720*a^17*b - 176960*a^15*b^3 + 37083*a^13*b^5)*c^5*d^13 + 2940*(12544
*a^18 - 3584*a^16*b^2 - 461*a^14*b^4)*c^4*d^14 - 16464*(224*a^17*b + 15*a^1
5*b^3)*c^3*d^15 - 7203*(448*a^18 - 115*a^16*b^2)*c^2*d^16)*sqrt((a*x + b)/(
c*x + d)) + (4116*a^18*b*c^6*d^11 + 2401*a^19*c^5*d^12 + (1048576*a^15*b^4
- 1966080*a^13*b^6 + 1382400*a^11*b^8 - 432000*a^9*b^10 + 50625*a^7*b^12)*c
^17 - 4*(1048576*a^16*b^3 - 1638400*a^14*b^5 + 921600*a^12*b^7 - 216000*a^1
0*b^9 + 16875*a^8*b^11)*c^16*d + 6*(1048576*a^17*b^2 - 1245184*a^15*b^4 + 4
71040*a^13*b^6 - 52800*a^11*b^8 - 1125*a^9*b^10)*c^15*d^2 - 4*(1048576*a^18
*b - 917504*a^16*b^3 + 307200*a^14*b^5 - 83200*a^12*b^7 + 15375*a^10*b^9)*c
^14*d^3 + (1048576*a^19 - 2228224*a^17*b^2 + 2297856*a^15*b^4 - 874240*a^13
*b^6 + 93775*a^11*b^8)*c^13*d^4 + 24*(98304*a^18*b - 75776*a^16*b^3 + 11200
*a^14*b^5 + 775*a^12*b^7)*c^12*d^5 - 4*(229376*a^19 - 67584*a^17*b^2 + 1017
6*a^15*b^4 - 7895*a^13*b^6)*c^11*d^6 - 24*(14336*a^18*b - 13184*a^16*b^3 +
2025*a^14*b^5)*c^10*d^7 + 3*(100352*a^19 - 20608*a^17*b^2 - 5083*a^15*b^4)*
c^9*d^8 - 28*(448*a^18*b + 393*a^16*b^3)*c^8*d^9 - 98*(448*a^19 - 97*a^17*b
^2)*c^7*d^10)*sqrt((4116*a^11*b*c*d^11 + 2401*a^12*d^12 + (1048576*a^8*b^4
- 1966080*a^6*b^6 + 1382400*a^4*b^8 - 432000*a^2*b^10 + 50625*b^12)*c^12 -
4*(1048576*a^9*b^3 - 1638400*a^7*b^5 + 921600*a^5*b^7 - 216000*a^3*b^9 + 16
875*a*b^11)*c^11*d + 6*(1048576*a^10*b^2 - 1245184*a^8*b^4 + 471040*a^6*b^6
- 52800*a^4*b^8 - 1125*a^2*b^10)*c^10*d^2 - 4*(1048576*a^11*b - 917504*a^9
*b^3 + 307200*a^7*b^5 - 83200*a^5*b^7 + 15375*a^3*b^9)*c^9*d^3 + (1048576*a
^12 - 2228224*a^10*b^2 + 2297856*a^8*b^4 - 874240*a^6*b^6 + 93775*a^4*b^8)*
c^8*d^4 + 24*(98304*a^11*b - 75776*a^9*b^3 + 11200*a^7*b^5 + 775*a^5*b^7)*c
^7*d^5 - 4*(229376*a^12 - 67584*a^10*b^2 + 10176*a^8*b^4 - 7895*a^6*b^6)*c^
6*d^6 - 24*(14336*a^11*b - 13184*a^9*b^3 + 2025*a^7*b^5)*c^5*d^7 + 3*(10035
2*a^12 - 20608*a^10*b^2 - 5083*a^8*b^4)*c^4*d^8 - 28*(448*a^11*b + 393*a^9*
b^3)*c^3*d^9 - 98*(448*a^12 - 97*a^10*b^2)*c^2*d^10)/(a^13*c^11)))*a^3*c^3*
((4116*a^11*b*c*d^11 + 2401*a^12*d^12 + (1048576*a^8*b^4 - 1966080*a^6*b^6
+ 1382400*a^4*b^8 - 432000*a^2*b^10 + 50625*b^12)*c^12 - 4*(1048576*a^9*b^3
- 1638400*a^7*b^5 + 921600*a^5*b^7 - 216000*a^3*b^9 + 16875*a*b^11)*c^11*d
+ 6*(1048576*a^10*b^2 - 1245184*a^8*b^4 + 471040*a^6*b^6 - 52800*a^4*b^8 -
1125*a^2*b^10)*c^10*d^2 - 4*(1048576*a^11*b - 917504*a^9*b^3 + 307200*a^7*
b^5 - 83200*a^5*b^7 + 15375*a^3*b^9)*c^9*d^3 + (1048576*a^12 - 2228224*a^10
*b^2 + 2297856*a^8*b^4 - 874240*a^6*b^6 + 93775*a^4*b^8)*c^8*d^4 + 24*(9830
4*a^11*b - 75776*a^9*b^3 + 11200*a^7*b^5 + 775*a^5*b^7)*c^7*d^5 - 4*(229376
*a^12 - 67584*a^10*b^2 + 10176*a^8*b^4 - 7895*a^6*b^6)*c^6*d^6 - 24*(14336*
a^11*b - 13184*a^9*b^3 + 2025*a^7*b^5)*c^5*d^7 + 3*(100352*a^12 - 20608*a^1

0*b^2 - 5083*a^8*b^4)*c^4*d^8 - 28*(448*a^11*b + 393*a^9*b^3)*c^3*d^9 - 98*(448*a^12 - 97*a^10*b^2)*c^2*d^10)/(a^13*c^11))^(1/4) - (441*a^11*b*c^4*d^8 + 343*a^12*c^3*d^9 + (32768*a^9*b^3 - 46080*a^7*b^5 + 21600*a^5*b^7 - 3375*a^3*b^9)*c^12 - 3*(32768*a^10*b^2 - 35840*a^8*b^4 + 12000*a^6*b^6 - 1125*a^4*b^8)*c^11*d + 12*(8192*a^11*b - 5632*a^9*b^3 + 680*a^7*b^5 + 75*a^5*b^7)*c^10*d^2 - 4*(8192*a^12 - 4608*a^10*b^2 + 2760*a^8*b^4 - 875*a^6*b^6)*c^9*d^3 - 6*(5632*a^11*b - 4144*a^9*b^3 + 555*a^7*b^5)*c^8*d^4 + 6*(3584*a^12 - 592*a^10*b^2 - 205*a^8*b^4)*c^7*d^5 + 12*(56*a^11*b - 129*a^9*b^3)*c^6*d^6 - 84*(56*a^12 - 11*a^10*b^2)*c^5*d^7)*((a*x + b)/(c*x + d))^(1/4)*((4116*a^11*b*c*d^11 + 2401*a^12*d^12 + (1048576*a^8*b^4 - 1966080*a^6*b^6 + 1382400*a^4*b^8 - 432000*a^2*b^10 + 50625*b^12)*c^12 - 4*(1048576*a^9*b^3 - 1638400*a^7*b^5 + 921600*a^5*b^7 - 216000*a^3*b^9 + 16875*a*b^11)*c^11*d + 6*(1048576*a^10*b^2 - 1245184*a^8*b^4 + 471040*a^6*b^6 - 52800*a^4*b^8 - 1125*a^2*b^10)*c^10*d^2 - 4*(1048576*a^11*b - 917504*a^9*b^3 + 307200*a^7*b^5 - 83200*a^5*b^7 + 15375*a^3*b^9)*c^9*d^3 + (1048576*a^12 - 2228224*a^10*b^2 + 2297856*a^8*b^4 - 874240*a^6*b^6 + 93775*a^4*b^8)*c^8*d^4 + 24*(98304*a^11*b - 75776*a^9*b^3 + 11200*a^7*b^5 + 775*a^5*b^7)*c^7*d^5 - 4*(229376*a^12 - 67584*a^10*b^2 + 10176*a^8*b^4 - 7895*a^6*b^6)*c^6*d^6 - 24*(14336*a^11*b - 13184*a^9*b^3 + 2025*a^7*b^5)*c^5*d^7 + 3*(100352*a^12 - 20608*a^10*b^2 - 5083*a^8*b^4)*c^4*d^8 - 28*(448*a^11*b + 393*a^9*b^3)*c^3*d^9 - 98*(448*a^12 - 97*a^10*b^2)*c^2*d^10)/(a^13*c^11))^(1/4))/(4116*a^11*b*c*d^11 + 2401*a^12*d^12 + (1048576*a^8*b^4 - 1966080*a^6*b^6 + 1382400*a^4*b^8 - 432000*a^2*b^10 + 50625*b^12)*c^12 - 4*(1048576*a^9*b^3 - 1638400*a^7*b^5 + 921600*a^5*b^7 - 216000*a^3*b^9 + 16875*a*b^11)*c^11*d + 6*(1048576*a^10*b^2 - 1245184*a^8*b^4 + 471040*a^6*b^6 - 52800*a^4*b^8 - 1125*a^2*b^10)*c^10*d^2 - 4*(1048576*a^11*b - 917504*a^9*b^3 + 307200*a^7*b^5 - 83200*a^5*b^7 + 15375*a^3*b^9)*c^9*d^3 + (1048576*a^12 - 2228224*a^10*b^2 + 2297856*a^8*b^4 - 874240*a^6*b^6 + 93775*a^4*b^8)*c^8*d^4 + 24*(98304*a^11*b - 75776*a^9*b^3 + 11200*a^7*b^5 + 775*a^5*b^7)*c^7*d^5 - 4*(229376*a^12 - 67584*a^10*b^2 + 10176*a^8*b^4 - 7895*a^6*b^6)*c^6*d^6 - 24*(14336*a^11*b - 13184*a^9*b^3 + 2025*a^7*b^5)*c^5*d^7 + 3*(100352*a^12 - 20608*a^10*b^2 - 5083*a^8*b^4)*c^4*d^8 - 28*(448*a^11*b + 393*a^9*b^3)*c^3*d^9 - 98*(448*a^12 - 97*a^10*b^2)*c^2*d^10)) + 3*a^3*c^2*((4116*a^11*b*c*d^11 + 2401*a^12*d^12 + (1048576*a^8*b^4 - 1966080*a^6*b^6 + 1382400*a^4*b^8 - 432000*a^2*b^10 + 50625*b^12)*c^12 - 4*(1048576*a^9*b^3 - 1638400*a^7*b^5 + 921600*a^5*b^7 - 216000*a^3*b^9 + 16875*a*b^11)*c^11*d + 6*(1048576*a^10*b^2 - 1245184*a^8*b^4 + 471040*a^6*b^6 - 52800*a^4*b^8 - 1125*a^2*b^10)*c^10*d^2 - 4*(1048576*a^11*b - 917504*a^9*b^3 + 307200*a^7*b^5 - 83200*a^5*b^7 + 15375*a^3*b^9)*c^9*d^3 + (1048576*a^12 - 2228224*a^10*b^2 + 2297856*a^8*b^4 - 874240*a^6*b^6 + 93775*a^4*b^8)*c^8*d^4 + 24*(98304*a^11*b - 75776*a^9*b^3 + 11200*a^7*b^5 + 775*a^5*b^7)*c^7*d^5 - 4*(229376*a^12 - 67584*a^10*b^2 + 10176*a^8*b^4 - 7895*a^6*b^6)*c^6*d^6 - 24*(14336*a^11*b - 13184*a^9*b^3 + 2025*a^7*b^5)*c^5*d^7 + 3*(100352*a^12 - 20608*a^10*b^2 - 5083*a^8*b^4)*c^4*d^8 - 28*(448*a^11*b + 393*a^9*b^3)*c^3*d^9 - 98*(448*a^12 - 97*a^10*b^2)*c^2*d^10)/(a^13*c^11))^(1/4)*log(a^10*c^8*((4116*a^11*b*c*d^11 + 2401*a^12*d^12 + (1048576*a^8*b^4 - 1966080*a^6*b^6 + 1382400*a^4*b^8 - 432000*a^2*b^10 + 50625*b^12)*c^12 - 4*(1048576*a^9*b^3 - 1638400*a^7*b^5 + 921600*a^5*b^7 - 216000*a^3*b^9 + 16875*a*b^11)*c^11*d + 6*(1048576*a^10*b^2 - 1245184*a^8*b^4 + 471040*a^6*b^6 - 52800*a^4*b^8 - 1125*a^2*b^10)*c^10*d^2 - 4*(1048576*a^11*b - 917504*a^9*b^3 + 307200*a^7*b^5 - 83200*a^5*b^7 + 15375*a^3*b^9)*c^9*d^3 + (1048576*a^12 - 2228224*a^10*b^2 + 2297856*a^8*b^4 - 874240*a^6*b^6 + 93775*a^4*b^8)*c^8*d^4 + 24*(98304*a^11*b - 75776*a^9*b^3 + 11200*a^7*b^5 + 775*a^5*b^7)*c^7*d^5 - 4*(229376*a^12 - 67584*a^10*b^2 + 10176*a^8*b^4 - 7895*a^6*b^6)*c^6*d^6 - 24*(14336*a^11*b - 13184*a^9*b^3 + 2025*a^7*b^5)*c^5*d^7 + 3*(100352*a^12 - 20608*a^10*b^2 - 5083*a^8*b^4)*c^4*d^8 - 28*(448*a^11*b + 393*a^9*b^3)*c^3*d^9 - 98*(448*a^12 - 97*a^10*b^2)*c^2*d^10)/(a^13*c^11))^(3/4) + (441*a^8*b*c*d^8 + 343*a^9*d^9 + (32768*a^6*b^3 - 46080*a^4*b^5 + 21600*a^2*b^7 - 3375*b^9)*c^9 - 3*(32768*a^7*b^2 - 35840*a^5*b^4 + 12000*a^3*b^6 - 1125*a*b^8)*c^8*d + 12*(8192*a^8*b - 5632*a^6*b^3 + 680*a^4*b^5 + 75*a^2*b^7)*c^7*d^

```
2 - 4*(8192*a^9 - 4608*a^7*b^2 + 2760*a^5*b^4 - 875*a^3*b^6)*c^6*d^3 - 6*(5
632*a^8*b - 4144*a^6*b^3 + 555*a^4*b^5)*c^5*d^4 + 6*(3584*a^9 - 592*a^7*b^2
 - 205*a^5*b^4)*c^4*d^5 + 12*(56*a^8*b - 129*a^6*b^3)*c^3*d^6 - 84*(56*a^9
- 11*a^7*b^2)*c^2*d^7)*((a*x + b)/(c*x + d))^(1/4)) - 3*a^3*c^2*((4116*a^11
*b*c*d^11 + 2401*a^12*d^12 + (1048576*a^8*b^4 - 1966080*a^6*b^6 + 1382400*a
^4*b^8 - 432000*a^2*b^10 + 50625*b^12)*c^12 - 4*(1048576*a^9*b^3 - 1638400*
a^7*b^5 + 921600*a^5*b^7 - 216000*a^3*b^9 + 16875*a*b^11)*c^11*d + 6*(10485
76*a^10*b^2 - 1245184*a^8*b^4 + 471040*a^6*b^6 - 52800*a^4*b^8 - 1125*a^2*b
^10)*c^10*d^2 - 4*(1048576*a^11*b - 917504*a^9*b^3 + 307200*a^7*b^5 - 83200
*a^5*b^7 + 15375*a^3*b^9)*c^9*d^3 + (1048576*a^12 - 2228224*a^10*b^2 + 2297
856*a^8*b^4 - 874240*a^6*b^6 + 93775*a^4*b^8)*c^8*d^4 + 24*(98304*a^11*b -
75776*a^9*b^3 + 11200*a^7*b^5 + 775*a^5*b^7)*c^7*d^5 - 4*(229376*a^12 - 675
84*a^10*b^2 + 10176*a^8*b^4 - 7895*a^6*b^6)*c^6*d^6 - 24*(14336*a^11*b - 13
184*a^9*b^3 + 2025*a^7*b^5)*c^5*d^7 + 3*(100352*a^12 - 20608*a^10*b^2 - 508
3*a^8*b^4)*c^4*d^8 - 28*(448*a^11*b + 393*a^9*b^3)*c^3*d^9 - 98*(448*a^12 -
 97*a^10*b^2)*c^2*d^10)/(a^13*c^11))^(1/4)*log(-a^10*c^8*((4116*a^11*b*c*d^
11 + 2401*a^12*d^12 + (1048576*a^8*b^4 - 1966080*a^6*b^6 + 1382400*a^4*b^8
- 432000*a^2*b^10 + 50625*b^12)*c^12 - 4*(1048576*a^9*b^3 - 1638400*a^7*b^5
 + 921600*a^5*b^7 - 216000*a^3*b^9 + 16875*a*b^11)*c^11*d + 6*(1048576*a^10
*b^2 - 1245184*a^8*b^4 + 471040*a^6*b^6 - 52800*a^4*b^8 - 1125*a^2*b^10)*c^
10*d^2 - 4*(1048576*a^11*b - 917504*a^9*b^3 + 307200*a^7*b^5 - 83200*a^5*b^
7 + 15375*a^3*b^9)*c^9*d^3 + (1048576*a^12 - 2228224*a^10*b^2 + 2297856*a^8
*b^4 - 874240*a^6*b^6 + 93775*a^4*b^8)*c^8*d^4 + 24*(98304*a^11*b - 75776*a
^9*b^3 + 11200*a^7*b^5 + 775*a^5*b^7)*c^7*d^5 - 4*(229376*a^12 - 67584*a^10
*b^2 + 10176*a^8*b^4 - 7895*a^6*b^6)*c^6*d^6 - 24*(14336*a^11*b - 13184*a^9
*b^3 + 2025*a^7*b^5)*c^5*d^7 + 3*(100352*a^12 - 20608*a^10*b^2 - 5083*a^8*b
^4)*c^4*d^8 - 28*(448*a^11*b + 393*a^9*b^3)*c^3*d^9 - 98*(448*a^12 - 97*a^1
0*b^2)*c^2*d^10)/(a^13*c^11))^(3/4) + (441*a^8*b*c*d^8 + 343*a^9*d^9 + (327
68*a^6*b^3 - 46080*a^4*b^5 + 21600*a^2*b^7 - 3375*b^9)*c^9 - 3*(32768*a^7*b
^2 - 35840*a^5*b^4 + 12000*a^3*b^6 - 1125*a*b^8)*c^8*d + 12*(8192*a^8*b - 5
632*a^6*b^3 + 680*a^4*b^5 + 75*a^2*b^7)*c^7*d^2 - 4*(8192*a^9 - 4608*a^7*b^
2 + 2760*a^5*b^4 - 875*a^3*b^6)*c^6*d^3 - 6*(5632*a^8*b - 4144*a^6*b^3 + 55
5*a^4*b^5)*c^5*d^4 + 6*(3584*a^9 - 592*a^7*b^2 - 205*a^5*b^4)*c^4*d^5 + 12*
(56*a^8*b - 129*a^6*b^3)*c^3*d^6 - 84*(56*a^9 - 11*a^7*b^2)*c^2*d^7)*((a*x
+ b)/(c*x + d))^(1/4)) + 4*(32*a^2*c^3*x^3 - 6*a*b*c*d^2 - 7*a^2*d^3 - 3*(3
2*a^2 - 15*b^2)*c^2*d - 36*(a*b*c^3 - a^2*c^2*d)*x^2 - 3*(14*a*b*c^2*d + a^
2*c*d^2 + (32*a^2 - 15*b^2)*c^3)*x)*((a*x + b)/(c*x + d))^(3/4))/(a^3*c^2)
```

giac [B] time = 7.67, size = 1490, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/((a*x+b)/(c*x+d))^(1/4),x, algorithm="giac")
```

```
[Out] -1/768*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)*(6*sqrt(2)*(32*a^2*b^2*c^4 -
 15*b^4*c^4 - 64*a^3*b*c^3*d + 20*a*b^3*c^3*d + 32*a^4*c^2*d^2 - 2*a^2*b^2*
c^2*d^2 + 4*a^3*b*c*d^3 - 7*a^4*d^4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a/c)^(1/
4) + 2*((a*x + b)/(c*x + d))^(1/4))/(-a/c)^(1/4))/((-a*c^3)^(1/4)*a^3*c^2)
+ 6*sqrt(2)*(32*a^2*b^2*c^4 - 15*b^4*c^4 - 64*a^3*b*c^3*d + 20*a*b^3*c^3*d
+ 32*a^4*c^2*d^2 - 2*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - 7*a^4*d^4)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(-a/c)^(1/4) - 2*((a*x + b)/(c*x + d))^(1/4))/(-a/c)^(1
/4))/((-a*c^3)^(1/4)*a^3*c^2) - 3*sqrt(2)*(32*a^2*b^2*c^4 - 15*b^4*c^4 - 64
*a^3*b*c^3*d + 20*a*b^3*c^3*d + 32*a^4*c^2*d^2 - 2*a^2*b^2*c^2*d^2 + 4*a^3*
b*c*d^3 - 7*a^4*d^4)*log(sqrt(2)*((a*x + b)/(c*x + d))^(1/4)*(-a/c)^(1/4) +
 sqrt((a*x + b)/(c*x + d)) + sqrt(-a/c))/((-a*c^3)^(1/4)*a^3*c^2) + 3*sqrt(
2)*(32*a^2*b^2*c^4 - 15*b^4*c^4 - 64*a^3*b*c^3*d + 20*a*b^3*c^3*d + 32*a^4*
c^2*d^2 - 2*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - 7*a^4*d^4)*log(-sqrt(2)*((a*x
 + b)/(c*x + d))^(1/4)*(-a/c)^(1/4) + sqrt((a*x + b)/(c*x + d)) + sqrt(-a/c
))/((-a*c^3)^(1/4)*a^3*c^2) - 8*(96*a^4*b^2*c^4*((a*x + b)/(c*x + d))^(3/4)
```

```
 - 113*a^2*b^4*c^4*((a*x + b)/(c*x + d))^(3/4) - 192*(a*x + b)*a^3*b^2*c^5*
((a*x + b)/(c*x + d))^(3/4)/(c*x + d) + 126*(a*x + b)*a*b^4*c^5*((a*x + b)/
(c*x + d))^(3/4)/(c*x + d) + 96*(a*x + b)^2*a^2*b^2*c^6*((a*x + b)/(c*x + d
))^(3/4)/(c*x + d)^2 - 45*(a*x + b)^2*b^4*c^6*((a*x + b)/(c*x + d))^(3/4)/(
c*x + d)^2 - 192*a^5*b*c^3*d*((a*x + b)/(c*x + d))^(3/4) + 236*a^3*b^3*c^3*
d*((a*x + b)/(c*x + d))^(3/4) + 384*(a*x + b)*a^4*b*c^4*d*((a*x + b)/(c*x +
 d))^(3/4)/(c*x + d) - 168*(a*x + b)*a^2*b^3*c^4*d*((a*x + b)/(c*x + d))^(3
/4)/(c*x + d) - 192*(a*x + b)^2*a^3*b*c^5*d*((a*x + b)/(c*x + d))^(3/4)/(c*
x + d)^2 + 60*(a*x + b)^2*a*b^3*c^5*d*((a*x + b)/(c*x + d))^(3/4)/(c*x + d)
^2 + 96*a^6*c^2*d^2*((a*x + b)/(c*x + d))^(3/4) - 126*a^4*b^2*c^2*d^2*((a*x
 + b)/(c*x + d))^(3/4) - 192*(a*x + b)*a^5*c^3*d^2*((a*x + b)/(c*x + d))^(3
/4)/(c*x + d) - 60*(a*x + b)*a^3*b^2*c^3*d^2*((a*x + b)/(c*x + d))^(3/4)/(c
*x + d) + 96*(a*x + b)^2*a^4*c^4*d^2*((a*x + b)/(c*x + d))^(3/4)/(c*x + d)^
2 - 6*(a*x + b)^2*a^2*b^2*c^4*d^2*((a*x + b)/(c*x + d))^(3/4)/(c*x + d)^2 -
 4*a^5*b*c*d^3*((a*x + b)/(c*x + d))^(3/4) + 120*(a*x + b)*a^4*b*c^2*d^3*((
a*x + b)/(c*x + d))^(3/4)/(c*x + d) + 12*(a*x + b)^2*a^3*b*c^3*d^3*((a*x +
b)/(c*x + d))^(3/4)/(c*x + d)^2 + 7*a^6*d^4*((a*x + b)/(c*x + d))^(3/4) - 1
8*(a*x + b)*a^5*c*d^4*((a*x + b)/(c*x + d))^(3/4)/(c*x + d) - 21*(a*x + b)^
2*a^4*c^2*d^4*((a*x + b)/(c*x + d))^(3/4)/(c*x + d)^2)/((a - (a*x + b)*c/(c
*x + d))^3*a^3*c^2))
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\left(\frac{ax+b}{cx+d}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/((a*x+b)/(c*x+d))^(1/4),x)
```

```
[Out] int((x^2-1)/((a*x+b)/(c*x+d))^(1/4),x)
```

maxima [A] time = 1.65, size = 487, normalized size = 1.48

$$\frac{3\left(3a^2bc^3d^2+7a^3c^2d^3+\left(32a^2b-15b^3\right)c^5-\left(32a^3-5ab^2\right)c^4d\right)\left(\frac{ax+b}{cx+d}\right)^{\frac{11}{4}}-6\left(17a^3bc^2d^2-3a^4cd^3+\left(32a^3b-21ab^3\right)c^4-\left(32a^4-7a^2b^2\right)c^3d\right)\left(\frac{ax+b}{cx+d}\right)^{\frac{7}{4}}-\left(3a^4bcd^2+7a^5d^3-\left(96a^4b-113a^2b^3\right)c^3+3\left(32a^5-41a^3b^2\right)c^2d\right)\left(\frac{ax+b}{cx+d}\right)^{\frac{3}{4}}}{96\left(a^6c^2-\frac{3(ax+b)a^5c^3}{cx+d}+\frac{3(ax+b)^2a^4c^4}{(cx+d)^2}-\frac{(ax+b)^3a^3c^5}{(cx+d)^3}\right)}-\frac{\left(3a^2bcd^2+7a^3d^3+\left(32a^2b-15b^3\right)c^3-\left(32a^3-5ab^2\right)c^2d\right)\left(\frac{2\arctan\left(\frac{\sqrt{c}\left(\frac{ax+b}{cx+d}\right)^{\frac{1}{4}}}{\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\log\left(\frac{\sqrt{c}\left(\frac{ax+b}{cx+d}\right)^{\frac{1}{4}}-\sqrt{\sqrt{a}\sqrt{c}}}{\sqrt{c}\left(\frac{ax+b}{cx+d}\right)^{\frac{1}{4}}+\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}\right)}{128a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/((a*x+b)/(c*x+d))^(1/4),x, algorithm="maxima")
```

```
[Out] 1/96*(3*(3*a^2*b*c^3*d^2 + 7*a^3*c^2*d^3 + (32*a^2*b - 15*b^3)*c^5 - (32*a^
3 - 5*a*b^2)*c^4*d)*((a*x + b)/(c*x + d))^(11/4) - 6*(17*a^3*b*c^2*d^2 - 3*
a^4*c*d^3 + (32*a^3*b - 21*a*b^3)*c^4 - (32*a^4 - 7*a^2*b^2)*c^3*d)*((a*x +
 b)/(c*x + d))^(7/4) - (3*a^4*b*c*d^2 + 7*a^5*d^3 - (96*a^4*b - 113*a^2*b^3
)*c^3 + 3*(32*a^5 - 41*a^3*b^2)*c^2*d)*((a*x + b)/(c*x + d))^(3/4))/(a^6*c^
2 - 3*(a*x + b)*a^5*c^3/(c*x + d) + 3*(a*x + b)^2*a^4*c^4/(c*x + d)^2 - (a*
x + b)^3*a^3*c^5/(c*x + d)^3) - 1/128*(3*a^2*b*c*d^2 + 7*a^3*d^3 + (32*a^2*
b - 15*b^3)*c^3 - (32*a^3 - 5*a*b^2)*c^2*d)*(2*arctan(sqrt(c)*((a*x + b)/(c
*x + d))^(1/4)/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + log
((sqrt(c)*((a*x + b)/(c*x + d))^(1/4) - sqrt(sqrt(a)*sqrt(c)))/(sqrt(c)*((a
*x + b)/(c*x + d))^(1/4) + sqrt(sqrt(a)*sqrt(c))))/(sqrt(sqrt(a)*sqrt(c))*s
qrt(c)))/(a^3*c^2)
```

mupad [B] time = 3.77, size = 1566, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((b + a*x)/(d + c*x))^(1/4),x)
```

```
[Out] ((((b + a*x)/(d + c*x))^(11/4)*((7*a^3*d^3)/32 - (15*b^3*c^3)/32 + a^2*b*c^
3 - a^3*c^2*d + (5*a*b^2*c^2*d)/32 + (3*a^2*b*c*d^2)/32))/a^6 + (((b + a*x)
/(d + c*x))^(7/4)*((3*a^3*d^3)/16 + (21*b^3*c^3)/16 - 2*a^2*b*c^3 + 2*a^3*c
^2*d - (7*a*b^2*c^2*d)/16 - (17*a^2*b*c*d^2)/16))/(a^5*c) - (((b + a*x)/(d
+ c*x))^(3/4)*((7*a^3*d^3)/96 + (113*b^3*c^3)/96 - a^2*b*c^3 + a^3*c^2*d -
(41*a*b^2*c^2*d)/32 + (a^2*b*c*d^2)/32))/(a^4*c^2))/((3*c^2*(b + a*x)^2)/(a
^2*(d + c*x)^2) - (c^3*(b + a*x)^3)/(a^3*(d + c*x)^3) - (3*c*(b + a*x))/(a*
(d + c*x)) + 1) - (atan((c^(1/2)*(a*d - b*c)*((b + a*x)/(d + c*x))^(1/4)*(7
*a^2*d^2 - 32*a^2*c^2 + 15*b^2*c^2 + 10*a*b*c*d)*(225*b^6*c^(13/2) - 960*a^
2*b^4*c^(13/2) + 1024*a^4*b^2*c^(13/2) + 49*a^6*c^(1/2)*d^6 - 448*a^6*c^(5/
2)*d^4 + 1024*a^6*c^(9/2)*d^2 + 42*a^5*b*c^(3/2)*d^5 + 256*a^5*b*c^(7/2)*d^
3 + 1280*a^3*b^3*c^(11/2)*d + 79*a^4*b^2*c^(5/2)*d^4 - 180*a^3*b^3*c^(7/2)*
d^3 - 65*a^2*b^4*c^(9/2)*d^2 - 128*a^4*b^2*c^(9/2)*d^2 - 150*a*b^5*c^(11/2)
*d - 2048*a^5*b*c^(11/2)*d))/(a^(1/4)*(21600*a^2*b^7*c^(39/4) - 3375*b^9*c^
(39/4) - 46080*a^4*b^5*c^(39/4) + 32768*a^6*b^3*c^(39/4) + 343*a^9*c^(3/4)*
d^9 - 4704*a^9*c^(11/4)*d^7 + 21504*a^9*c^(19/4)*d^5 - 32768*a^9*c^(27/4)*d
^3 + 441*a^8*b*c^(7/4)*d^8 + 672*a^8*b*c^(15/4)*d^6 - 33792*a^8*b*c^(23/4)*
d^4 + 98304*a^8*b*c^(31/4)*d^2 - 36000*a^3*b^6*c^(35/4)*d + 107520*a^5*b^4*
c^(35/4)*d - 98304*a^7*b^2*c^(35/4)*d + 924*a^7*b^2*c^(11/4)*d^7 - 1548*a^6
*b^3*c^(15/4)*d^6 - 1230*a^5*b^4*c^(19/4)*d^5 - 3552*a^7*b^2*c^(19/4)*d^5 -
 3330*a^4*b^5*c^(23/4)*d^4 + 24864*a^6*b^3*c^(23/4)*d^4 + 3500*a^3*b^6*c^(2
7/4)*d^3 - 11040*a^5*b^4*c^(27/4)*d^3 + 18432*a^7*b^2*c^(27/4)*d^3 + 900*a^
2*b^7*c^(31/4)*d^2 + 8160*a^4*b^5*c^(31/4)*d^2 - 67584*a^6*b^3*c^(31/4)*d^2
 + 3375*a*b^8*c^(35/4)*d)))*(a*d - b*c)*(7*a^2*d^2 - 32*a^2*c^2 + 15*b^2*c^
2 + 10*a*b*c*d))/(64*a^(13/4)*c^(11/4)) + (atanh((c^(1/2)*(a*d - b*c)*((b +
 a*x)/(d + c*x))^(1/4)*(7*a^2*d^2 - 32*a^2*c^2 + 15*b^2*c^2 + 10*a*b*c*d)*(
225*b^6*c^(13/2) - 960*a^2*b^4*c^(13/2) + 1024*a^4*b^2*c^(13/2) + 49*a^6*c^
(1/2)*d^6 - 448*a^6*c^(5/2)*d^4 + 1024*a^6*c^(9/2)*d^2 + 42*a^5*b*c^(3/2)*d
^5 + 256*a^5*b*c^(7/2)*d^3 + 1280*a^3*b^3*c^(11/2)*d + 79*a^4*b^2*c^(5/2)*d
^4 - 180*a^3*b^3*c^(7/2)*d^3 - 65*a^2*b^4*c^(9/2)*d^2 - 128*a^4*b^2*c^(9/2)
*d^2 - 150*a*b^5*c^(11/2)*d - 2048*a^5*b*c^(11/2)*d))/(a^(1/4)*(21600*a^2*b
^7*c^(39/4) - 3375*b^9*c^(39/4) - 46080*a^4*b^5*c^(39/4) + 32768*a^6*b^3*c^
(39/4) + 343*a^9*c^(3/4)*d^9 - 4704*a^9*c^(11/4)*d^7 + 21504*a^9*c^(19/4)*d
^5 - 32768*a^9*c^(27/4)*d^3 + 441*a^8*b*c^(7/4)*d^8 + 672*a^8*b*c^(15/4)*d^
6 - 33792*a^8*b*c^(23/4)*d^4 + 98304*a^8*b*c^(31/4)*d^2 - 36000*a^3*b^6*c^(
35/4)*d + 107520*a^5*b^4*c^(35/4)*d - 98304*a^7*b^2*c^(35/4)*d + 924*a^7*b^
2*c^(11/4)*d^7 - 1548*a^6*b^3*c^(15/4)*d^6 - 1230*a^5*b^4*c^(19/4)*d^5 - 35
52*a^7*b^2*c^(19/4)*d^5 - 3330*a^4*b^5*c^(23/4)*d^4 + 24864*a^6*b^3*c^(23/4
)*d^4 + 3500*a^3*b^6*c^(27/4)*d^3 - 11040*a^5*b^4*c^(27/4)*d^3 + 18432*a^7*
b^2*c^(27/4)*d^3 + 900*a^2*b^7*c^(31/4)*d^2 + 8160*a^4*b^5*c^(31/4)*d^2 - 6
7584*a^6*b^3*c^(31/4)*d^2 + 3375*a*b^8*c^(35/4)*d)))*(a*d - b*c)*(7*a^2*d^2
 - 32*a^2*c^2 + 15*b^2*c^2 + 10*a*b*c*d))/(64*a^(13/4)*c^(11/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{\sqrt[4]{\frac{ax+b}{cx+d}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/((a*x+b)/(c*x+d))**(1/4),x)
```

```
[Out] Integral((x - 1)*(x + 1)/((a*x + b)/(c*x + d))**(1/4), x)
```

3.2304 $\int \frac{-b+ax^4}{\sqrt{b+ax^4}\left(b-c^2x^2+ax^4\right)}\,dx$

Optimal. Leaf size=329

$$-\frac{\left(\sqrt{2\sqrt{a}\,\sqrt{b}+c^2}+c\right)\sqrt{c\sqrt{2\sqrt{a}\,\sqrt{b}+c^2}-\sqrt{a}\,\sqrt{b}-c^2}\,\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt{c\sqrt{2\sqrt{a}\,\sqrt{b}+c^2}-\sqrt{a}\,\sqrt{b}-c^2}}{\sqrt{ax^4+b}+\sqrt{a}\,x^2+\sqrt{b}}\right)}{\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,c}-\frac{\left(\sqrt{2\sqrt{a}\,\sqrt{b}+\right.}{}$$

Rubi [A] time = 0.15, antiderivative size = 20, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2112, 208}

$$-\frac{\tanh^{-1}\left(\frac{cx}{\sqrt{ax^4+b}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^4)/(Sqrt[b + a*x^4]*(b - c^2*x^2 + a*x^4)),x]
```

```
[Out] -(ArcTanh[(c*x)/Sqrt[b + a*x^4]]/c)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2112

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\int \frac{-b+ax^4}{\sqrt{b+ax^4}\left(b-c^2x^2+ax^4\right)}\,dx=-\left(b\,\text{Subst}\left(\int \frac{1}{b-bc^2x^2}\,dx,x,\frac{x}{\sqrt{b+ax^4}}\right)\right)$$
$$=-\frac{\tanh^{-1}\left(\frac{cx}{\sqrt{b+ax^4}}\right)}{c}$$

Mathematica [C] time = 0.95, size = 199, normalized size = 0.60

$$-\frac{i\sqrt{\frac{ax^4}{b}+1}\left(-\Pi\left(\frac{2i\sqrt{a}\,\sqrt{b}}{c^2-\sqrt{c^4-4ab}};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right)\middle|-1\right)-\Pi\left(\frac{2i\sqrt{a}\,\sqrt{b}}{c^2+\sqrt{c^4-4ab}};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right)\middle|-1\right)+F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right)\middle|-1\right)\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{ax^4+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^4)/(Sqrt[b + a*x^4]*(b - c^2*x^2 + a*x^4)),x]
```

```
[Out] ((-I)*Sqrt[1 + (a*x^4)/b]*(EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]*x]
, -1] - EllipticPi[((2*I)*Sqrt[a]*Sqrt[b])/(c^2 - Sqrt[-4*a*b + c^4]), I*Ar
```

```
cSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]*x], -1] - EllipticPi[((2*I)*Sqrt[a]*Sqrt[b]
)/(c^2 + Sqrt[-4*a*b + c^4]), I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]*x], -1]))
/(Sqrt[(I*Sqrt[a])/Sqrt[b]]*Sqrt[b + a*x^4])
```

IntegrateAlgebraic [A] time = 1.37, size = 52, normalized size = 0.16

$$\frac{\log\left(c^2x - c\sqrt{ax^4+b}\right)}{2c} - \frac{\log\left(\sqrt{ax^4+b} + cx\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^4)/(Sqrt[b + a*x^4]*(b - c^2*x^2 + a*x^4)),x]
[Out] -1/2*Log[c*x + Sqrt[b + a*x^4]]/c + Log[c^2*x - c*Sqrt[b + a*x^4]]/(2*c)
```

fricas [A] time = 0.55, size = 51, normalized size = 0.16

$$\frac{\log\left(\frac{ax^4+c^2x^2-2\sqrt{ax^4+b}\,cx+b}{ax^4-c^2x^2+b}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)/(a*x^4+b)^(1/2)/(a*x^4-c^2*x^2+b),x, algorithm="fricas"
)
[Out] 1/2*log((a*x^4 + c^2*x^2 - 2*sqrt(a*x^4 + b)*c*x + b)/(a*x^4 - c^2*x^2 + b)
)/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4-b}{\left(ax^4-c^2x^2+b\right)\sqrt{ax^4+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)/(a*x^4+b)^(1/2)/(a*x^4-c^2*x^2+b),x, algorithm="giac")
[Out] integrate((a*x^4 - b)/((a*x^4 - c^2*x^2 + b)*sqrt(a*x^4 + b)), x)
```

maple [C] time = 0.12, size = 298, normalized size = 0.91

$$\frac{\sqrt{1-\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{a\,x^4+b}} - \frac{\sum_{\_\alpha=\mathrm{RootOf}\left(a\_Z^4-c^2\_Z^2+b\right)}\frac{\left(-c^2\_\alpha^2+2b\right)\left(-\frac{\operatorname{arctanh}\left(\frac{\_\alpha^2\left(-\_\alpha^2a+a\,x^2+c^2\right)}{\sqrt{c^2\_\alpha^2}\,\sqrt{a\,x^4+b}}\right)}{\sqrt{c^2\_\alpha^2}}+\frac{2\_\alpha\left(\_\alpha^2a-c^2\right)\sqrt{1-\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},\frac{i\left(\_\alpha^2a-c^2\right)}{\sqrt{a}\,\sqrt{b}},\frac{\sqrt{-\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,b\sqrt{a\,x^4+b}}\right)}{\_\alpha\left(2\_\alpha^2a-c^2\right)}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b)/(a*x^4+b)^(1/2)/(a*x^4-c^2*x^2+b),x)
[Out] 1/(I*a^(1/2)/b^(1/2))^(1/2)*(1-I*a^(1/2)/b^(1/2)*x^2)^(1/2)*(1+I*a^(1/2)/b^
(1/2)*x^2)^(1/2)/(a*x^4+b)^(1/2)*EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)-1
/4*sum((-_alpha^2*c^2+2*b)/_alpha/(2*_alpha^2*a-c^2)*(-1/(c^2*_alpha^2)^(1/
2)*arctanh(_alpha^2*(-_alpha^2*a+a*x^2+c^2)/(c^2*_alpha^2)^(1/2)/(a*x^4+b)^
(1/2))+2/(I*a^(1/2)/b^(1/2))^(1/2)*_alpha*(_alpha^2*a-c^2)/b*(1-I*a^(1/2)/b
^(1/2)*x^2)^(1/2)*(1+I*a^(1/2)/b^(1/2)*x^2)^(1/2)/(a*x^4+b)^(1/2)*EllipticP
```

```
i(x*(I*a^(1/2)/b^(1/2))^(1/2),I/a^(1/2)/b^(1/2)*(_alpha^2*a-c^2),(-I*a^(1/2
)/b^(1/2))^(1/2)/(I*a^(1/2)/b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*a-_Z^2*c^2+
b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4 - b}{\left(ax^4 - c^2x^2 + b\right)\sqrt{ax^4 + b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)/(a*x^4+b)^(1/2)/(a*x^4-c^2*x^2+b),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^4 - b)/((a*x^4 - c^2*x^2 + b)*sqrt(a*x^4 + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{b - a\,x^4}{\sqrt{a\,x^4 + b}\,\left(-c^2\,x^2 + a\,x^4 + b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^4)/((b + a*x^4)^(1/2)*(b + a*x^4 - c^2*x^2)),x)
```

```
[Out] int(-(b - a*x^4)/((b + a*x^4)^(1/2)*(b + a*x^4 - c^2*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4 - b}{\sqrt{ax^4 + b}\,\left(ax^4 + b - c^2x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b)/(a*x**4+b)**(1/2)/(a*x**4-c**2*x**2+b),x)
```

```
[Out] Integral((a*x**4 - b)/(sqrt(a*x**4 + b)*(a*x**4 + b - c**2*x**2)), x)
```

3.2305 $\int \frac{(1+x^2)\sqrt[3]{-1-x^2+x^4+x^6}}{x}\,dx$

Optimal. Leaf size=330

$$\left(x^2-1\right)^{2/3}\left(x^2+1\right)^{4/3}\left(\frac{1}{12}\sqrt[3]{x^2-1}\left(3\left(x^2+1\right)^{5/3}-2\left(x^2+1\right)^{2/3}\right)+\frac{1}{2}\sqrt[3]{x^2-1}\left(x^2+1\right)^{2/3}+\frac{1}{18}\log\left(\sqrt[3]{x^2-1}-\sqrt[3]{x^2\right.$$

Rubi [A] time = 0.55, antiderivative size = 488, normalized size of antiderivative = 1.48, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {6719, 101, 157, 50, 59, 105, 91}

$$\frac{1}{4}\sqrt[3]{x^6+x^4-x^2-1}\left(x^2+1\right)+\frac{1}{3}\sqrt[3]{x^6+x^4-x^2-1}+\frac{\sqrt[3]{x^6+x^4-x^2-1}\log\left(x^2\right)}{4\sqrt[3]{x^2-1}\left(x^2+1\right)^{2/3}}+\frac{\sqrt[3]{x^6+x^4-x^2-1}\log\left(x^2-1\right)}{36\sqrt[3]{x^2-1}\left(x^2+1\right)^{2/3}}-\frac{3\sqrt[3]{x^6+x^4-x^2-1}\log\left(-\sqrt[3]{x^2-1}-\sqrt[3]{x^2+1}\right)}{4\sqrt[3]{x^2-1}\left(x^2+1\right)^{2/3}}+\frac{\sqrt[3]{x^6+x^4-x^2-1}\log\left(\frac{\sqrt[3]{x^2+1}}{\sqrt[3]{x^2-1}}-1\right)}{12\sqrt[3]{x^2-1}\left(x^2+1\right)^{2/3}}-\frac{\sqrt{3}\sqrt[3]{x^6+x^4-x^2-1}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x^2+1}}{\sqrt{3}\sqrt[3]{x^2-1}}\right)}{2\sqrt[3]{x^2-1}\left(x^2+1\right)^{2/3}}+\frac{\sqrt{3}\sqrt[3]{x^6+x^4-x^2-1}\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}}{\sqrt{3}\sqrt[3]{x^2-1}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{x^2-1}\left(x^2+1\right)^{2/3}}-\frac{4\sqrt[3]{x^6+x^4-x^2-1}\tan^{-1}\left(\frac{2\sqrt[3]{x^2+1}}{\sqrt{3}\sqrt[3]{x^2-1}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{x^2-1}\left(x^2+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(-1 - x^2 + x^4 + x^6)^(1/3))/x,x]
```

```
[Out] (-1 - x^2 + x^4 + x^6)^(1/3)/3 + ((1 + x^2)*(-1 - x^2 + x^4 + x^6)^(1/3))/4
 - (Sqrt[3]*(-1 - x^2 + x^4 + x^6)^(1/3)*ArcTan[1/Sqrt[3] - (2*(1 + x^2)^(1
/3))/(Sqrt[3]*(-1 + x^2)^(1/3))]])/(2*(-1 + x^2)^(1/3)*(1 + x^2)^(2/3)) - (4
*(-1 - x^2 + x^4 + x^6)^(1/3)*ArcTan[1/Sqrt[3] + (2*(1 + x^2)^(1/3))/(Sqrt[
3]*(-1 + x^2)^(1/3))]])/(3*Sqrt[3]*(-1 + x^2)^(1/3)*(1 + x^2)^(2/3)) + (Sqrt
[3]*(-1 - x^2 + x^4 + x^6)^(1/3)*ArcTan[1/Sqrt[3] + (2*(1 + x^2)^(1/3))/(Sq
rt[3]*(-1 + x^2)^(1/3))]])/(2*(-1 + x^2)^(1/3)*(1 + x^2)^(2/3)) + ((-1 - x^2
 + x^4 + x^6)^(1/3)*Log[x^2])/(4*(-1 + x^2)^(1/3)*(1 + x^2)^(2/3)) + ((-1 -
 x^2 + x^4 + x^6)^(1/3)*Log[-1 + x^2])/(36*(-1 + x^2)^(1/3)*(1 + x^2)^(2/3)
) - (3*(-1 - x^2 + x^4 + x^6)^(1/3)*Log[-(-1 + x^2)^(1/3) - (1 + x^2)^(1/3)
])/(4*(-1 + x^2)^(1/3)*(1 + x^2)^(2/3)) + ((-1 - x^2 + x^4 + x^6)^(1/3)*Log
[-1 + (1 + x^2)^(1/3)/(-1 + x^2)^(1/3)])/(12*(-1 + x^2)^(1/3)*(1 + x^2)^(2/
3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
```

```
 b, c, d, e, f}, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\left(1+x^2\right)\sqrt[3]{-1-x^2+x^4+x^6}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(1+x)\sqrt[3]{(-1+x)(1+x)^2}}{x}\,dx, x, x^2\right)$$

$$= \frac{\sqrt[3]{\left(-1+x^2\right)\left(1+x^2\right)^2}\,\text{Subst}\left(\int \frac{\sqrt[3]{-1+x}\,(1+x)^{5/3}}{x}\,dx, x, x^2\right)}{2\sqrt[3]{-1+x^2}\,\left(1+x^2\right)^{2/3}}$$

$$= \frac{1}{4}\left(1+x^2\right)\sqrt[3]{-\left(\left(1-x^2\right)\left(1+x^2\right)^2\right)} - \frac{\sqrt[3]{\left(-1+x^2\right)\left(1+x^2\right)^2}\,\text{Subst}\left(\int \frac{\left(2-\frac{4x}{3}\right.}{(-1\ldots}\right.}{4\sqrt[3]{-1+x^2}\,\left(1+x^2\right)^{2/\ldots}}$$

$$= \frac{1}{4}\left(1+x^2\right)\sqrt[3]{-\left(\left(1-x^2\right)\left(1+x^2\right)^2\right)} + \frac{\sqrt[3]{\left(-1+x^2\right)\left(1+x^2\right)^2}\,\text{Subst}\left(\int \frac{(1+x\ldots}{(-1+x\ldots}\right.}{3\sqrt[3]{-1+x^2}\,\left(1+x^2\right)^{2/3}}$$

$$= \frac{1}{3}\sqrt[3]{-\left(\left(1-x^2\right)\left(1+x^2\right)^2\right)} + \frac{1}{4}\left(1+x^2\right)\sqrt[3]{-\left(\left(1-x^2\right)\left(1+x^2\right)^2\right)} + \frac{\left(4\sqrt[3]{(-1\ldots}\right.}{\ldots}$$

$$= \frac{1}{3}\sqrt[3]{-\left(\left(1-x^2\right)\left(1+x^2\right)^2\right)} + \frac{1}{4}\left(1+x^2\right)\sqrt[3]{-\left(\left(1-x^2\right)\left(1+x^2\right)^2\right)} - \frac{\sqrt{3}\sqrt[3]{-\ldots}}{\ldots}$$

Mathematica [C] time = 0.11, size = 188, normalized size = 0.57

$$-\frac{3\sqrt[3]{\left(x^2-1\right)\left(x^2+1\right)^2}\left(2^{2/3}x^2\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{1}{2}\left(1-x^2\right)\right)-4\,2^{2/3}\left(x^2+1\right){}_2F_1\left(-\frac{5}{3},\frac{1}{3};\frac{4}{3};\frac{1}{2}\left(1-x^2\right)\right)+2\,2^{2/3}\left(x^2+1\right){}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};\frac{1}{2}\left(1-x^2\right)\right)+2^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{1}{2}\left(1-x^2\right)\right)+2\left(x^2+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{1-x^2}{x^2+1}\right)\right)}{4\left(x^2+1\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(-1 - x^2 + x^4 + x^6)^(1/3))/x,x]
```

```
[Out] (-3*((-1 + x^2)*(1 + x^2)^2)^(1/3)*(-4*2^(2/3)*(1 + x^2)*Hypergeometric2F1[
-5/3, 1/3, 4/3, (1 - x^2)/2] + 2*2^(2/3)*(1 + x^2)*Hypergeometric2F1[-2/3,
1/3, 4/3, (1 - x^2)/2] + 2^(2/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (1 - x^2)
/2] + 2^(2/3)*x^2*Hypergeometric2F1[1/3, 1/3, 4/3, (1 - x^2)/2] + 2*(1 + x^
2)^(2/3)*Hypergeometric2F1[1/3, 1, 4/3, (1 - x^2)/(1 + x^2)]))/(4*(1 + x^2)
^(5/3))
```

IntegrateAlgebraic [A] time = 8.00, size = 308, normalized size = 0.93

$$\frac{\left(x^2-1\right)^{2/3}\left(x^2+1\right)^{4/3}\left(\frac{1}{12}\sqrt[3]{x^2-1}\left(3\left(x^2+1\right)^{5/3}+4\left(x^2+1\right)^{2/3}\right)+\frac{1}{18}\log\left(\sqrt[3]{x^2-1}-\sqrt[3]{x^2+1}\right)-\frac{1}{2}\log\left(\sqrt[3]{x^2-1}+\sqrt[3]{x^2+1}\right)+\frac{1}{4}\log\left(\left(x^2-1\right)^{2/3}-\sqrt[3]{x^2+1}\sqrt[3]{x^2-1}+\left(x^2+1\right)^{2/3}\right)-\frac{1}{36}\log\left(\left(x^2-1\right)^{2/3}+\sqrt[3]{x^2+1}\sqrt[3]{x^2-1}+\left(x^2+1\right)^{2/3}\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^2+1}}{2\sqrt[3]{x^2-1}-\sqrt[3]{x^2+1}}\right)+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^2+1}}{2\sqrt[3]{x^2-1}+\sqrt[3]{x^2+1}}\right)}{6\sqrt{3}}\right)}{\left(\left(x^2-1\right)\left(x^2+1\right)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + x^2)*(-1 - x^2 + x^4 + x^6)^(1/3))/x,x]
```

```
[Out] ((-1 + x^2)^(2/3)*(1 + x^2)^(4/3)*(((-1 + x^2)^(1/3)*(4*(1 + x^2)^(2/3) + 3
*(1 + x^2)^(5/3)))/12 + (Sqrt[3]*ArcTan[(Sqrt[3]*(1 + x^2)^(1/3))/(2*(-1 +
x^2)^(1/3) - (1 + x^2)^(1/3))])/2 + ArcTan[(Sqrt[3]*(1 + x^2)^(1/3))/(2*(-1
 + x^2)^(1/3) + (1 + x^2)^(1/3))]/(6*Sqrt[3]) + Log[(-1 + x^2)^(1/3) - (1 +
 x^2)^(1/3)]/18 - Log[(-1 + x^2)^(1/3) + (1 + x^2)^(1/3)]/2 + Log[(-1 + x^2
)^(2/3) - (-1 + x^2)^(1/3)*(1 + x^2)^(1/3) + (1 + x^2)^(2/3)]/4 - Log[(-1 +
 x^2)^(2/3) + (-1 + x^2)^(1/3)*(1 + x^2)^(1/3) + (1 + x^2)^(2/3)]/36))/((-1
 + x^2)*(1 + x^2)^2)^(2/3)
```

fricas [A] time = 0.44, size = 305, normalized size = 0.92

$$-\frac{1}{18}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x^2+1\right)+2\sqrt{3}\left(x^6+x^4-x^2-1\right)^{\frac{1}{3}}}{3\left(x^2+1\right)}\right)-\frac{1}{2}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(x^2+1\right)-2\sqrt{3}\left(x^6+x^4-x^2-1\right)^{\frac{1}{3}}}{3\left(x^2+1\right)}\right)+\frac{1}{12}\left(x^6+x^4-x^2-1\right)^{\frac{1}{3}}\left(3x^2+7\right)-\frac{1}{36}\log\left(\frac{x^4+2x^2+\left(x^6+x^4-x^2-1\right)^{\frac{1}{3}}\left(x^2+1\right)+\left(x^6+x^4-x^2-1\right)^{\frac{2}{3}}+1}{x^4+2x^2+1}\right)+\frac{1}{4}\log\left(\frac{x^4+2x^2-\left(x^6+x^4-x^2-1\right)^{\frac{1}{3}}\left(x^2+1\right)+\left(x^6+x^4-x^2-1\right)^{\frac{2}{3}}+1}{x^4+2x^2+1}\right)-\frac{1}{2}\log\left(\frac{x^2+\left(x^6+x^4-x^2-1\right)^{\frac{1}{3}}+1}{x^2+1}\right)+\frac{1}{18}\log\left(-\frac{x^2-\left(x^6+x^4-x^2-1\right)^{\frac{1}{3}}+1}{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^6+x^4-x^2-1)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*arctan(1/3*(sqrt(3)*(x^2 + 1) + 2*sqrt(3)*(x^6 + x^4 - x^2 -
1)^(1/3))/(x^2 + 1)) - 1/2*sqrt(3)*arctan(-1/3*(sqrt(3)*(x^2 + 1) - 2*sqrt(
3)*(x^6 + x^4 - x^2 - 1)^(1/3))/(x^2 + 1)) + 1/12*(x^6 + x^4 - x^2 - 1)^(1/
3)*(3*x^2 + 7) - 1/36*log((x^4 + 2*x^2 + (x^6 + x^4 - x^2 - 1)^(1/3)*(x^2 +
 1) + (x^6 + x^4 - x^2 - 1)^(2/3) + 1)/(x^4 + 2*x^2 + 1)) + 1/4*log((x^4 +
2*x^2 - (x^6 + x^4 - x^2 - 1)^(1/3)*(x^2 + 1) + (x^6 + x^4 - x^2 - 1)^(2/3)
 + 1)/(x^4 + 2*x^2 + 1)) - 1/2*log((x^2 + (x^6 + x^4 - x^2 - 1)^(1/3) + 1)/
(x^2 + 1)) + 1/18*log(-(x^2 - (x^6 + x^4 - x^2 - 1)^(1/3) + 1)/(x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^4-x^2-1\right)^{\frac{1}{3}}\left(x^2+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^6+x^4-x^2-1)^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^4 - x^2 - 1)^(1/3)*(x^2 + 1)/x, x)
```

maple [C] time = 49.13, size = 6304, normalized size = 19.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^6+x^4-x^2-1)^(1/3)/x,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6+x^4-x^2-1\right)^{\frac{1}{3}}\left(x^2+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^6+x^4-x^2-1)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] integrate((x^6 + x^4 - x^2 - 1)^(1/3)*(x^2 + 1)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^2+1\right)\,{\left(x^6+x^4-x^2-1\right)}^{1/3}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*(x^4 - x^2 + x^6 - 1)^(1/3))/x,x)
```

```
[Out] int(((x^2 + 1)*(x^4 - x^2 + x^6 - 1)^(1/3))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{(x-1)(x+1)\left(x^2+1\right)^2}\,\left(x^2+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**6+x**4-x**2-1)**(1/3)/x,x)
```

```
[Out] Integral(((x - 1)*(x + 1)*(x**2 + 1)**2)**(1/3)*(x**2 + 1)/x, x)
```

3.2306 $\int \sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\sqrt[3]{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=330

$$\frac{7}{16}x\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\sqrt[3]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}+\frac{\left(-ax^2-10\right)\sqrt[3]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{16b}-\frac{5\log\left(\sqrt[3]{2}\sqrt[3]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+\right.}{24\sqrt[3]{2}\,b}$$

Rubi [F] time = 0.47, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\sqrt[3]{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b
^2])^(1/3),x]
```

```
[Out] Defer[Int][Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2
*x^2)/b^2])^(1/3), x]
```

Rubi steps

$$\int \sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\sqrt[3]{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\sqrt[3]{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Mathematica [C] time = 20.84, size = 10907, normalized size = 33.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*
x^2)/b^2])^(1/3),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 3.72, size = 333, normalized size = 1.01

$$\frac{7}{16}x\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\sqrt[3]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}+\frac{\left(-ax^2-10\right)\sqrt[3]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{16b}-\frac{5\log\left(\sqrt[3]{2}\,b\sqrt[3]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}-b\right)}{24\sqrt[3]{2}\,b}+\frac{5\log\left(2^{2/3}\left(bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}+1\right)}{48\sqrt[3]{2}\,b}+\frac{5\tan^{-1}\left(\frac{2\sqrt[3]{2}\sqrt[3]{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{8\sqrt[3]{2}\sqrt{3}\,b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*(a*x^2 + b*x*Sqrt[-(a/b^2
) + (a^2*x^2)/b^2])^(1/3),x]
```

```
[Out] ((-10 - a*x^2)*(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])^(1/3))/(16*b) +
 (7*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]*(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)
/b^2])^(1/3))/16 + (5*ArcTan[1/Sqrt[3] + (2*2^(1/3)*(a*x^2 + b*x*Sqrt[-(a/b
^2) + (a^2*x^2)/b^2])^(1/3))/Sqrt[3]])/(8*2^(1/3)*Sqrt[3]*b) - (5*Log[-b +
```

```
2^(1/3)*b*(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])^(1/3)])/(24*2^(1/3)*
b) + (5*Log[1 + 2^(1/3)*(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])^(1/3)
+ 2^(2/3)*(a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])^(2/3)])/(48*2^(1/3)*
b)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b^2+a^2*x^2/b^2)^(1/2)*(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))
^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b^2+a^2*x^2/b^2)^(1/2)*(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))
^(1/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \sqrt{-\frac{a}{b^{2}}+\frac{a^{2} x^{2}}{b^{2}}}\, \left(a\,x^{2}+b x \sqrt{-\frac{a}{b^{2}}+\frac{a^{2} x^{2}}{b^{2}}}\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a/b^2+a^2*x^2/b^2)^(1/2)*(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/3)
,x)
```

```
[Out] int((-a/b^2+a^2*x^2/b^2)^(1/2)*(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/3)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^{2}+\sqrt{\frac{a^{2}x^{2}}{b^{2}}-\frac{a}{b^{2}}}\,bx\right)^{\frac{1}{3}} \sqrt{\frac{a^{2}x^{2}}{b^{2}}-\frac{a}{b^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b^2+a^2*x^2/b^2)^(1/2)*(a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))
^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)^(1/3)*sqrt(a^2*x^2/b^2 -
a/b^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a\,x^2+b\,x\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}\right)}^{1/3}\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/3)*((a^2*x^2)/b^2 - a/b^
2)^(1/2),x)
```

```
[Out] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/3)*((a^2*x^2)/b^2 - a/b^
2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{x\left(ax+b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)}\sqrt{\frac{a\left(ax^2-1\right)}{b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a/b**2+a**2*x**2/b**2)**(1/2)*(a*x**2+b*x*(-a/b**2+a**2*x**2/b*
*2)**(1/2))**(1/3),x)
```

```
[Out] Integral((x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))**(1/3)*sqrt(a*(a*x**2
- 1)/b**2), x)
```

3.2307 $\int \frac{-b+ax^2}{\left(-d+cx^2\right)\sqrt[3]{-x+x^3}}\,dx$

Optimal. Leaf size=331

$$\frac{(bc-ad)\log\left(x\sqrt[3]{c-d}+\sqrt[3]{d}\sqrt[3]{x^3-x}\right)}{2cd^{2/3}\sqrt[3]{c-d}}-\frac{\sqrt{3}(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}x\sqrt[3]{c-d}}{x\sqrt[3]{c-d}-2\sqrt[3]{d}\sqrt[3]{x^3-x}}\right)}{2cd^{2/3}\sqrt[3]{c-d}}+\frac{(ad-bc)\log\left(-\sqrt[3]{d}\sqrt[3]{x^3-x}x\sqrt[3]{c}\right.}{4cd^2}$$

Rubi [A] time = 0.55, antiderivative size = 417, normalized size of antiderivative = 1.26, number of steps used = 15, number of rules used = 14, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2056, 584, 329, 275, 239, 466, 465, 377, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{x}\sqrt[3]{x^2-1}(bc-ad)\log\left(\frac{x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{x^2-1}}+\sqrt[3]{d}\right)}{2cd^{2/3}\sqrt[3]{x^3-x}\sqrt[3]{c-d}}-\frac{\sqrt[3]{x}\sqrt[3]{x^2-1}(bc-ad)\log\left(\frac{x^{4/3}(c-d)^{2/3}}{\left(x^2-1\right)^{2/3}}-\frac{\sqrt[3]{d}x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{x^2-1}}+d^{2/3}\right)}{4cd^{2/3}\sqrt[3]{x^3-x}\sqrt[3]{c-d}}-\frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{x^2-1}(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-\frac{2x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{x^2-1}}}{\sqrt{3}\sqrt[3]{d}}\right)}{2cd^{2/3}\sqrt[3]{x^3-x}\sqrt[3]{c-d}}-\frac{3a\sqrt[3]{x}\sqrt[3]{x^2-1}\log\left(x^{2/3}-\sqrt[3]{x^2-1}\right)}{4c\sqrt[3]{x^3-x}}+\frac{\sqrt{3}a\sqrt[3]{x}\sqrt[3]{x^2-1}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1}{\sqrt{3}}\right)}{2c\sqrt[3]{x^3-x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^2)/((-d + c*x^2)*(-x + x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*a*x^(1/3)*(-1 + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3))/(-1 + x^2)^(1/3
))/Sqrt[3]])/(2*c*(-x + x^3)^(1/3)) - (Sqrt[3]*(b*c - a*d)*x^(1/3)*(-1 + x^
2)^(1/3)*ArcTan[(d^(1/3) - (2*(c - d)^(1/3)*x^(2/3))/(-1 + x^2)^(1/3))/(Sqr
t[3]*d^(1/3))])/(2*c*(c - d)^(1/3)*d^(2/3)*(-x + x^3)^(1/3)) + ((b*c - a*d)
*x^(1/3)*(-1 + x^2)^(1/3)*Log[d^(1/3) + ((c - d)^(1/3)*x^(2/3))/(-1 + x^2)^
(1/3)])/(2*c*(c - d)^(1/3)*d^(2/3)*(-x + x^3)^(1/3)) - ((b*c - a*d)*x^(1/3)
*(-1 + x^2)^(1/3)*Log[d^(2/3) + ((c - d)^(2/3)*x^(4/3))/(-1 + x^2)^(2/3) -
((c - d)^(1/3)*d^(1/3)*x^(2/3))/(-1 + x^2)^(1/3)])/(4*c*(c - d)^(1/3)*d^(2/
3)*(-x + x^3)^(1/3)) - (3*a*x^(1/3)*(-1 + x^2)^(1/3)*Log[x^(2/3) - (-1 + x^
2)^(1/3)])/(4*c*(-x + x^3)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{-b+ax^2}{\left(-d+cx^2\right)\sqrt[3]{-x+x^3}}\,dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\int \frac{-b+ax^2}{\sqrt[3]{x}\sqrt[3]{-1+x^2}\left(-d+cx^2\right)}\,dx}{\sqrt[3]{-x+x^3}}$$

$$= \frac{\left(\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\int \left(\frac{a}{c\sqrt[3]{x}\sqrt[3]{-1+x^2}}+\frac{-bc+ad}{c\sqrt[3]{x}\sqrt[3]{-1+x^2}\left(-d+cx^2\right)}\right)dx}{\sqrt[3]{-x+x^3}}$$

$$= \frac{\left(a\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\int \frac{1}{\sqrt[3]{x}\sqrt[3]{-1+x^2}}\,dx}{c\sqrt[3]{-x+x^3}} + \frac{\left((-bc+ad)\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\int \frac{1}{\sqrt[3]{x}\sqrt[3]{-1+x^2}\left(-d+cx^2\right)} \cdots}{c\sqrt[3]{-x+x^3}}$$

$$= \frac{\left(3a\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt[3]{-1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{c\sqrt[3]{-x+x^3}} + \frac{\left(3(-bc+ad)\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\mathrm{Sub}\cdots}{c\sqrt[3]{-x}\cdots}$$

$$= \frac{\left(3a\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{-1+x^3}}\,dx, x, x^{2/3}\right)}{2c\sqrt[3]{-x+x^3}} + \frac{\left(3(-bc+ad)\sqrt[3]{x}\sqrt[3]{-1+x^2}\right)\mathrm{Sub}\cdots}{2c\sqrt[3]{-x}\cdots}$$

$$= \frac{\sqrt{3}\,a\sqrt[3]{x}\sqrt[3]{-1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}}{\sqrt{3}}\right)}{2c\sqrt[3]{-x+x^3}} - \frac{3a\sqrt[3]{x}\sqrt[3]{-1+x^2}\,\log\left(x^{2/3}-\sqrt[3]{-1+x^2}\right)}{4c\sqrt[3]{-x+x^3}} + \cdots$$

$$= \frac{\sqrt{3}\,a\sqrt[3]{x}\sqrt[3]{-1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}}{\sqrt{3}}\right)}{2c\sqrt[3]{-x+x^3}} - \frac{3a\sqrt[3]{x}\sqrt[3]{-1+x^2}\,\log\left(x^{2/3}-\sqrt[3]{-1+x^2}\right)}{4c\sqrt[3]{-x+x^3}} + \cdots$$

$$= \frac{\sqrt{3}\,a\sqrt[3]{x}\sqrt[3]{-1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}}{\sqrt{3}}\right)}{2c\sqrt[3]{-x+x^3}} + \frac{(bc-ad)\sqrt[3]{x}\sqrt[3]{-1+x^2}\,\log\left(\sqrt[3]{d}+\frac{\sqrt[3]{c-d}\,x^{2/3}}{\sqrt[3]{-1+x^2}}\cdots\right.}{2c\sqrt[3]{c-d}\,d^{2/3}\sqrt[3]{-x+x^3}}$$

$$= \frac{\sqrt{3}\,a\sqrt[3]{x}\sqrt[3]{-1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}}{\sqrt{3}}\right)}{2c\sqrt[3]{-x+x^3}} + \frac{(bc-ad)\sqrt[3]{x}\sqrt[3]{-1+x^2}\,\log\left(\sqrt[3]{d}+\frac{\sqrt[3]{c-d}\,x^{2/3}}{\sqrt[3]{-1+x^2}}\cdots\right.}{2c\sqrt[3]{c-d}\,d^{2/3}\sqrt[3]{-x+x^3}}$$

$$= \frac{\sqrt{3}\,a\sqrt[3]{x}\sqrt[3]{-1+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}}{\sqrt{3}}\right)}{2c\sqrt[3]{-x+x^3}} - \frac{\sqrt{3}\,(bc-ad)\sqrt[3]{x}\sqrt[3]{-1+x^2}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{c-d}\,x^{2}\cdots}{\sqrt[3]{d}\sqrt[3]{-1+}\cdots}}{\sqrt{3}}\right.}{2c\sqrt[3]{c-d}\,d^{2/3}\sqrt[3]{-x+x^3}}$$

Mathematica [A] time = 0.60, size = 310, normalized size = 0.94

$$\frac{\sqrt[3]{x}\sqrt[3]{x^2-1}\left(\frac{2(bc-ad)\log\left(\frac{x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{x^2-1}}+\sqrt[3]{d}\right)}{d^{2/3}\sqrt[3]{c-d}}-\frac{(bc-ad)\left(\log\left(\frac{x^{4/3}(c-d)^{2/3}}{\left(x^2-1\right)^{2/3}}-\frac{\sqrt[3]{d}\,x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{x^2-1}}+d^{2/3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{d}\sqrt[3]{x^2-1}}}{\sqrt{3}}\right)\right)}{d^{2/3}\sqrt[3]{c-d}}-2a\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2-1}}\right)+a\left(\log\left(\frac{x^{4/3}}{\left(x^2-1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2-1}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1}{\sqrt{3}}\right)\right)\right)}{4c\sqrt[3]{x\left(x^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^2)/((-d + c*x^2)*(-x + x^3)^(1/3)),x]
```

```
[Out] (x^(1/3)*(-1 + x^2)^(1/3)*(-2*a*Log[1 - x^(2/3)/(-1 + x^2)^(1/3)] + a*(2*Sq
rt[3]*ArcTan[(1 + (2*x^(2/3))/(-1 + x^2)^(1/3))/Sqrt[3]] + Log[1 + x^(4/3)/
(-1 + x^2)^(2/3) + x^(2/3)/(-1 + x^2)^(1/3)]) + (2*(b*c - a*d)*Log[d^(1/3)
+ ((c - d)^(1/3)*x^(2/3))/(-1 + x^2)^(1/3)])/((c - d)^(1/3)*d^(2/3)) - ((b*
c - a*d)*(2*Sqrt[3]*ArcTan[(1 - (2*(c - d)^(1/3)*x^(2/3))/(d^(1/3)*(-1 + x^
2)^(1/3)))/Sqrt[3]] + Log[d^(2/3) + ((c - d)^(2/3)*x^(4/3))/(-1 + x^2)^(2/3
) - ((c - d)^(1/3)*d^(1/3)*x^(2/3))/(-1 + x^2)^(1/3)]))/((c - d)^(1/3)*d^(2
/3))))/(4*c*(x*(-1 + x^2))^(1/3))
```

IntegrateAlgebraic [A] time = 3.19, size = 334, normalized size = 1.01

$$\frac{(bc-ad)\log\left(x\sqrt[3]{c-d}+\sqrt[3]{d}\sqrt[3]{x^3-x}\right)}{2cd^{2/3}\sqrt[3]{c-d}}-\frac{\sqrt{3}(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}x\sqrt[3]{c-d}}{x\sqrt[3]{c-d}-2\sqrt[3]{d}\sqrt[3]{x^3-x}}\right)}{2cd^{2/3}\sqrt[3]{c-d}}+\frac{(ad-bc)\log\left(-\sqrt[3]{d}\sqrt[3]{x^3-x}\,x\sqrt[3]{c-d}+x^2(c-d)^{2/3}+d^{2/3}\left(x^3-x\right)^{2/3}\right)}{4cd^{2/3}\sqrt[3]{c-d}}-\frac{a\log\left(c\sqrt[3]{x^3-x}-cx\right)}{2c}+\frac{\sqrt{3}a\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x}+x}\right)}{2c}+\frac{a\log\left(\sqrt[3]{x^3-x}\,x+\left(x^3-x\right)^{2/3}+x^2\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^2)/((-d + c*x^2)*(-x + x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*a*ArcTan[(Sqrt[3]*x)/(x + 2*(-x + x^3)^(1/3))])/(2*c) - (Sqrt[3]*(
b*c - a*d)*ArcTan[(Sqrt[3]*(c - d)^(1/3)*x)/((c - d)^(1/3)*x - 2*d^(1/3)*(-
x + x^3)^(1/3))])/(2*c*(c - d)^(1/3)*d^(2/3)) - (a*Log[-(c*x) + c*(-x + x^3
)^(1/3)])/(2*c) + ((b*c - a*d)*Log[(c - d)^(1/3)*x + d^(1/3)*(-x + x^3)^(1/
3)])/(2*c*(c - d)^(1/3)*d^(2/3)) + (a*Log[x^2 + x*(-x + x^3)^(1/3) + (-x +
x^3)^(2/3)])/(4*c) + ((-(b*c) + a*d)*Log[(c - d)^(2/3)*x^2 - (c - d)^(1/3)*
d^(1/3)*x*(-x + x^3)^(1/3) + d^(2/3)*(-x + x^3)^(2/3)])/(4*c*(c - d)^(1/3)*
d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(c*x^2-d)/(x^3-x)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.27, size = 300, normalized size = 0.91

$$-\frac{\sqrt{3}a\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)}{2c}+\frac{\left(bc\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}-ad\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right|\right)}{2\left(c^2-cd\right)}+\frac{a\log\left(\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)}{4c}-\frac{a\log\left(\left|\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)}{2c}-\frac{\left(\sqrt{3}bc-\sqrt{3}ad\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}+2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}}\right)}{2\left(-cd^2+d^3\right)^{\frac{1}{3}}c}+\frac{(bc-ad)\log\left(\left(-\frac{c-d}{d}\right)^{\frac{2}{3}}+\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}\right)}{4\left(-cd^2+d^3\right)^{\frac{1}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(c*x^2-d)/(x^3-x)^(1/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*(-1/x^2 + 1)^(1/3) + 1))/c + 1/2*(b*c*
(-(c - d)/d)^(1/3) - a*d*(-(c - d)/d)^(1/3))*(-(c - d)/d)^(1/3)*log(abs(-(-
(c - d)/d)^(1/3) + (-1/x^2 + 1)^(1/3)))/(c^2 - c*d) + 1/4*a*log((-1/x^2 + 1
)^(2/3) + (-1/x^2 + 1)^(1/3) + 1)/c - 1/2*a*log(abs((-1/x^2 + 1)^(1/3) - 1)
)/c - 1/2*(sqrt(3)*b*c - sqrt(3)*a*d)*arctan(1/3*sqrt(3)*((-(c - d)/d)^(1/3
) + 2*(-1/x^2 + 1)^(1/3))/(-(c - d)/d)^(1/3))/((-c*d^2 + d^3)^(1/3)*c) + 1/
4*(b*c - a*d)*log((-(c - d)/d)^(2/3) + (-(c - d)/d)^(1/3)*(-1/x^2 + 1)^(1/3
) + (-1/x^2 + 1)^(2/3))/((-c*d^2 + d^3)^(1/3)*c)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2-b}{\left(c\,x^2-d\right)\left(x^3-x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)/(c*x^2-d)/(x^3-x)^(1/3),x)
[Out] int((a*x^2-b)/(c*x^2-d)/(x^3-x)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - b}{(cx^2 - d)(x^3 - x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)/(c*x^2-d)/(x^3-x)^(1/3),x, algorithm="maxima")
[Out] integrate((a*x^2 - b)/((c*x^2 - d)*(x^3 - x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b - a\,x^2}{(x^3 - x)^{1/3}\,(d - c\,x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a*x^2)/((x^3 - x)^(1/3)*(d - c*x^2)),x)
[Out] int((b - a*x^2)/((x^3 - x)^(1/3)*(d - c*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 - b}{\sqrt[3]{x(x-1)(x+1)}\,(cx^2 - d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)/(c*x**2-d)/(x**3-x)**(1/3),x)
[Out] Integral((a*x**2 - b)/((x*(x - 1)*(x + 1))**(1/3)*(c*x**2 - d)), x)
```

3.2308 $\int \frac{1}{(b+ax)\sqrt[3]{-b^3+a^3x^3}}\,dx$

Optimal. Leaf size=332

$$\frac{(-1)^{5/6}\sqrt{3}\tanh^{-1}\left(\frac{\frac{i\sqrt[3]{a^3x^3-b^3}}{\sqrt{3}}+\frac{\left(-\sqrt{3}a-ia\right)x}{2^{2/3}\sqrt{3}}+\frac{\sqrt{3}b+ib}{2^{2/3}\sqrt{3}}}{\sqrt[3]{a^3x^3-b^3}}\right)}{2\sqrt[3]{2}ab}+\frac{\sqrt[3]{-\frac{1}{2}}\log\left((-1)^{2/3}a^{3/2}\sqrt{b}x-2^{2/3}\sqrt{a}\sqrt{b}\sqrt[3]{a^3x^3-b^3}-(-1)^{2/3}\sqrt{}\right.}{2ab}$$

Rubi [A] time = 0.08, antiderivative size = 139, normalized size of antiderivative = 0.42, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2148}

$$-\frac{3\log\left(2^{2/3}a\sqrt[3]{a^3x^3-b^3}+a(b-ax)\right)}{4\sqrt[3]{2}ab}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(b-ax)}{\sqrt[3]{a^3x^3-b^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}ab}+\frac{\log\left((b-ax)(ax+b)^2\right)}{4\sqrt[3]{2}ab}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b + a*x)*(-b^3 + a^3*x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2^(1/3)*(b - a*x))/(-b^3 + a^3*x^3)^(1/3))/Sqrt[3]])/
(2*2^(1/3)*a*b) + Log[(b - a*x)*(b + a*x)^2]/(4*2^(1/3)*a*b) - (3*Log[a*(b
- a*x) + 2^(2/3)*a*(-b^3 + a^3*x^3)^(1/3)])/(4*2^(1/3)*a*b)
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))]/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(b+ax)\sqrt[3]{-b^3+a^3x^3}}\,dx=\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(b-ax)}{\sqrt[3]{-b^3+a^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}ab}+\frac{\log\left((b-ax)(b+ax)^2\right)}{4\sqrt[3]{2}ab}-\frac{3\log\left(a(b-ax)+2^{2/3}a\sqrt[3]{-}\right.}{4\sqrt[3]{2}ab}$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(b+ax)\sqrt[3]{-b^3+a^3x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((b + a*x)*(-b^3 + a^3*x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((b + a*x)*(-b^3 + a^3*x^3)^(1/3)), x]
```

IntegrateAlgebraic [A] time = 2.41, size = 385, normalized size = 1.16

$$\frac{(-1)^{5/6}\sqrt{3}\tanh^{-1}\left(\frac{\frac{i\sqrt[3]{a^3x^3-b^3}}{\sqrt{3}}+\frac{\left(-\sqrt{3}a-ia\right)x}{2^{2/3}\sqrt{3}}+\frac{\sqrt{3}b+ib}{2^{2/3}\sqrt{3}}}{\sqrt[3]{a^3x^3-b^3}}\right)}{2\sqrt[3]{2}ab}+\frac{\sqrt[3]{-\frac{1}{2}}\log\left(-i\sqrt{3}a^{3/2}\sqrt{b}x+a^{3/2}\sqrt{b}x+2\,2^{2/3}\sqrt{a}\sqrt{b}\sqrt[3]{a^3x^3-b^3}+\left(-1+i\sqrt{3}\right)\sqrt{a}b^{3/2}\right)}{2ab}-\frac{\sqrt[3]{-\frac{1}{2}}\log\left(4\sqrt[3]{2}ab\left(a^3x^3-b^3\right)^{2/3}-i\sqrt{3}a^3bx^2-a^3bx^2+2i\sqrt{3}a^2b^2x+2a^2b^2x+\left(2(-2)^{2/3}a^2bx-2(-2)^{2/3}ab^2\right)\sqrt[3]{a^3x^3-b^3}-i\sqrt{3}ab^3-ab^3\right)}{4ab}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((b + a*x)*(-b^3 + a^3*x^3)^(1/3)),x]
```

```
[Out] ((-1)^(5/6)*Sqrt[3]*ArcTanh[((I*b + Sqrt[3]*b)/(2^(2/3)*Sqrt[3]) + (((-I)*a
 - Sqrt[3]*a)*x)/(2^(2/3)*Sqrt[3]) + (I*(-b^3 + a^3*x^3)^(1/3))/Sqrt[3])/(-
b^3 + a^3*x^3)^(1/3)])/(2*2^(1/3)*a*b) + ((-1/2)^(1/3)*Log[(-1 + I*Sqrt[3])
*Sqrt[a]*b^(3/2) + a^(3/2)*Sqrt[b]*x - I*Sqrt[3]*a^(3/2)*Sqrt[b]*x + 2*2^(2
/3)*Sqrt[a]*Sqrt[b]*(-b^3 + a^3*x^3)^(1/3)])/(2*a*b) - ((-1/2)^(1/3)*Log[-(
a*b^3) - I*Sqrt[3]*a*b^3 + 2*a^2*b^2*x + (2*I)*Sqrt[3]*a^2*b^2*x - a^3*b*x^
2 - I*Sqrt[3]*a^3*b*x^2 + (-2*(-2)^(2/3)*a*b^2 + 2*(-2)^(2/3)*a^2*b*x)*(-b^
3 + a^3*x^3)^(1/3) + 4*2^(1/3)*a*b*(-b^3 + a^3*x^3)^(2/3)])/(4*a*b)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b)/(a^3*x^3-b^3)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^3x^3 - b^3\right)^{\frac{1}{3}}(ax+b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b)/(a^3*x^3-b^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((a^3*x^3 - b^3)^(1/3)*(a*x + b)), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax+b)\left(a^3x^3 - b^3\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b)/(a^3*x^3-b^3)^(1/3),x)
```

```
[Out] int(1/(a*x+b)/(a^3*x^3-b^3)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^3x^3 - b^3\right)^{\frac{1}{3}}(ax+b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b)/(a^3*x^3-b^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^3*x^3 - b^3)^(1/3)*(a*x + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a^3\,x^3 - b^3\right)^{1/3}\,\left(b + a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^3*x^3 - b^3)^(1/3)*(b + a*x)),x)
```

```
[Out] int(1/((a^3*x^3 - b^3)^(1/3)*(b + a*x)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(ax-b)\left(a^2x^2+abx+b^2\right)}\,(ax+b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b)/(a**3*x**3-b**3)**(1/3),x)
```

```
[Out] Integral(1/(((a*x - b)*(a**2*x**2 + a*b*x + b**2))**(1/3)*(a*x + b)), x)
```

3.2309 $\int \frac{(c+bx+ax^2)^{5/2}}{c+bx}\,dx$

Optimal. Leaf size=334

$$-\frac{a^{5/2}c^5\log\left(\sqrt{ax^2+bx+c}-\sqrt{a}\,x\right)}{b^6}+\frac{a^{5/2}c^5\log\left(-b\sqrt{ax^2+bx+c}+\sqrt{a}\,bx+2\sqrt{a}\,c\right)}{b^6}+\frac{\left(256a^5c^5-80a^2b^6c^2+30\right.}{}$$

Rubi [A] time = 0.33, antiderivative size = 309, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {734, 814, 843, 621, 206, 724}

$$-\frac{a^{5/2}c^5\tanh^{-1}\left(\frac{x(b^2-2ac)+bc}{2\sqrt{a}\,c\sqrt{ax^2+bx+c}}\right)}{b^6}+\frac{\left(16a^2c^2+6abx\left(b^2-2ac\right)-6ab^2c+3b^4\right)\left(ax^2+bx+c\right)^{3/2}}{48ab^3}+\frac{\left(-256a^5c^5+80a^2b^6c^2-30ab^8c+3b^{10}\right)\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{ax^2+bx+c}}\right)}{256a^{5/2}b^6}-\frac{\left(-128a^4c^4+32a^3b^2c^3+8a^2b^4c^2+2abx\left(b^2-2ac\right)\left(-16a^2c^2-12ab^2c+3b^4\right)-18ab^6c+3b^8\right)\sqrt{ax^2+bx+c}}{128a^2b^5}+\frac{\left(ax^2+bx+c\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + b*x + a*x^2)^(5/2)/(c + b*x),x]
```

```
[Out] -1/128*((3*b^8 - 18*a*b^6*c + 8*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4
+ 2*a*b*(b^2 - 2*a*c)*(3*b^4 - 12*a*b^2*c - 16*a^2*c^2)*x)*Sqrt[c + b*x + a
*x^2])/(a^2*b^5) + ((3*b^4 - 6*a*b^2*c + 16*a^2*c^2 + 6*a*b*(b^2 - 2*a*c)*x
)*(c + b*x + a*x^2)^(3/2))/(48*a*b^3) + (c + b*x + a*x^2)^(5/2)/(5*b) + ((3
*b^10 - 30*a*b^8*c + 80*a^2*b^6*c^2 - 256*a^5*c^5)*ArcTanh[(b + 2*a*x)/(2*S
qrt[a]*Sqrt[c + b*x + a*x^2])])/(256*a^(5/2)*b^6) - (a^(5/2)*c^5*ArcTanh[(b
*c + (b^2 - 2*a*c)*x)/(2*Sqrt[a]*c*Sqrt[c + b*x + a*x^2])])/b^6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
```

```
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(c+bx+ax^2\right)^{5/2}}{c+bx}\,dx = \frac{\left(c+bx+ax^2\right)^{5/2}}{5b} - \frac{\int \frac{\left(-bc-\left(b^2-2ac\right)x\right)\left(c+bx+ax^2\right)^{3/2}}{c+bx}\,dx}{2b}$$

$$= \frac{\left(3b^4-6ab^2c+16a^2c^2+6ab\left(b^2-2ac\right)x\right)\left(c+bx+ax^2\right)^{3/2}}{48ab^3} + \frac{\left(c+bx+ax^2\right)^{5/2}}{5b} + \int$$

$$= -\frac{\left(3b^8-18ab^6c+8a^2b^4c^2+32a^3b^2c^3-128a^4c^4+2ab\left(b^2-2ac\right)\left(3b^4-12ab^2c-16\right.\right.}{128a^2b^5}$$

$$= -\frac{\left(3b^8-18ab^6c+8a^2b^4c^2+32a^3b^2c^3-128a^4c^4+2ab\left(b^2-2ac\right)\left(3b^4-12ab^2c-16\right.\right.}{128a^2b^5}$$

$$= -\frac{\left(3b^8-18ab^6c+8a^2b^4c^2+32a^3b^2c^3-128a^4c^4+2ab\left(b^2-2ac\right)\left(3b^4-12ab^2c-16\right.\right.}{128a^2b^5}$$

$$= -\frac{\left(3b^8-18ab^6c+8a^2b^4c^2+32a^3b^2c^3-128a^4c^4+2ab\left(b^2-2ac\right)\left(3b^4-12ab^2c-16\right.\right.}{128a^2b^5}$$

Mathematica [A] time = 0.53, size = 275, normalized size = 0.82

$$\frac{-3840a^5c^5\tanh^{-1}\left(\frac{-2acx+b^2x+bc}{2\sqrt{a}\,c\sqrt{x(ax+b)+c}}\right)+15\left(-256a^5c^5+80a^2b^6c^2-30ab^8c+3b^{10}\right)\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\,\sqrt{x(ax+b)+c}}\right)+2\sqrt{a}\,b\sqrt{x(ax+b)+c}\left(-960a^4bc^3x+1920a^4c^4-80a^3b^3cx\left(6ax^2+c\right)+160a^3b^2c^2\left(4ax^2+c\right)+12a^2b^5x\left(84ax^2+109c\right)+24a^2b^4\left(16a^2x^4+2acx^2+c^2\right)+30ab^7x+6ab^6\left(124ax^2+65c\right)-45b^8\right)}{3840a^{5/2}b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + b*x + a*x^2)^(5/2)/(c + b*x),x]
```

```
[Out] (2*Sqrt[a]*b*Sqrt[c + x*(b + a*x)]*(-45*b^8 + 1920*a^4*c^4 + 30*a*b^7*x - 9
60*a^4*b*c^3*x + 160*a^3*b^2*c^2*(c + 4*a*x^2) - 80*a^3*b^3*c*x*(c + 6*a*x^
2) + 12*a^2*b^5*x*(109*c + 84*a*x^2) + 6*a*b^6*(65*c + 124*a*x^2) + 24*a^2*
b^4*(c^2 + 2*a*c*x^2 + 16*a^2*x^4)) + 15*(3*b^10 - 30*a*b^8*c + 80*a^2*b^6*
c^2 - 256*a^5*c^5)*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])] -
 3840*a^5*c^5*ArcTanh[(b*c + b^2*x - 2*a*c*x)/(2*Sqrt[a]*c*Sqrt[c + x*(b +
a*x)])])/(3840*a^(5/2)*b^6)
```

IntegrateAlgebraic [A] time = 2.19, size = 334, normalized size = 1.00

$$-\frac{a^{5/2}c^5\log\left(\sqrt{ax^2+bx+c}-\sqrt{a}\,x\right)}{b^6}+\frac{a^{5/2}c^5\log\left(-b\sqrt{ax^2+bx+c}+\sqrt{a}\,bx+2\sqrt{a}\,c\right)}{b^6}+\frac{\left(256a^5c^5-80a^2b^6c^2+30ab^8c-3b^{10}\right)\log\left(-2\sqrt{a}\,\sqrt{ax^2+bx+c}+2ax+b\right)}{256a^{5/2}b^6}+\frac{\sqrt{ax^2+bx+c}\left(384a^4b^4x^4-480a^4b^3cx^3+640a^4b^2c^2x^2-960a^4bc^3x+1920a^4c^4+1008a^3b^5x^3+48a^3b^4cx^2-80a^3b^3c^2x+160a^3b^2c^3+744a^2b^6x^2+1308a^2b^5cx+24a^2b^4c^2+30ab^7x+390ab^6c-45b^8\right)}{1920a^2b^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + b*x + a*x^2)^(5/2)/(c + b*x),x]
```

```
[Out] (Sqrt[c + b*x + a*x^2]*(-45*b^8 + 390*a*b^6*c + 24*a^2*b^4*c^2 + 160*a^3*b^
2*c^3 + 1920*a^4*c^4 + 30*a*b^7*x + 1308*a^2*b^5*c*x - 80*a^3*b^3*c^2*x - 9
60*a^4*b*c^3*x + 744*a^2*b^6*x^2 + 48*a^3*b^4*c*x^2 + 640*a^4*b^2*c^2*x^2 +
 1008*a^3*b^5*x^3 - 480*a^4*b^3*c*x^3 + 384*a^4*b^4*x^4))/(1920*a^2*b^5) -
(a^(5/2)*c^5*Log[-(Sqrt[a]*x) + Sqrt[c + b*x + a*x^2]])/b^6 + ((-3*b^10 + 3
0*a*b^8*c - 80*a^2*b^6*c^2 + 256*a^5*c^5)*Log[b + 2*a*x - 2*Sqrt[a]*Sqrt[c
+ b*x + a*x^2]])/(256*a^(5/2)*b^6) + (a^(5/2)*c^5*Log[2*Sqrt[a]*c + Sqrt[a]
*b*x - b*Sqrt[c + b*x + a*x^2]])/b^6
```

fricas [A] time = 81.29, size = 707, normalized size = 2.12

$$\left[\frac{3840\,a^{\frac{11}{2}} c^{5} \log\left(-\frac{2\,b^{3} c x + b^{2} c^{2} + 4\,a c^{3} + \left(b^{4} - 4\,a b^{2} c + 8\,a^{2} c^{2}\right) x^{2} - 4\left(b c^{2} + \left(b^{2} c - 2\,a c^{2}\right) x\right) \sqrt{a x^{2} + b x + c}\,\sqrt{a}}{b^{2} x^{2} + 2\,b c x + c^{2}}\right) - 15\left(3\,b^{10} - 30\,a b^{8} c + 80\,a^{2} b^{6} c^{2} - 256\,a^{5} c^{5}\right)\sqrt{a}\,\log\left(-8\,a^{2} x^{2} - 8\,a b x + 4\,\sqrt{a x^{2} + b x + c}\,(2\,a x + b)\sqrt{a} - b^{2} - 4\,a c\right) + 4\left(384\,a^{5} b^{5} x^{4} - 45\,a b^{9} + 390\,a^{2} b^{7} c + 24\,a^{3} b^{5} c^{2} + 160\,a^{4} b^{3} c^{3} + 1920\,a^{5} b c^{4} + 48\left(21\,a^{4} b^{6} - 10\,a^{5} b^{4} c\right) x^{3} + 8\left(93\,a^{3} b^{7} + 6\,a^{4} b^{5} c + 80\,a^{5} b^{3} c^{2}\right) x^{2} + 2\left(15\,a^{2} b^{8} + 654\,a^{3} b^{6} c - 40\,a^{4} b^{4} c^{2} - 480\,a^{5} b^{2} c^{3}\right) x\right)\sqrt{a x^{2} + b x + c}}{7680\,a^{3} b^{6}}, -\frac{3840\,\sqrt{-a}\,a^{5} c^{5} \arctan\left(-\frac{\sqrt{a x^{2} + b x + c}\left(b c + \left(b^{2} - 2\,a c\right) x\right)\sqrt{-a}}{2\left(a^{2} c x^{2} + a b c x + a c^{2}\right)}\right) + 15\left(3\,b^{10} - 30\,a b^{8} c + 80\,a^{2} b^{6} c^{2} - 256\,a^{5} c^{5}\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{a x^{2} + b x + c}\,(2\,a x + b)\sqrt{-a}}{2\left(a^{2} x^{2} + a b x + a c\right)}\right) - 2\left(384\,a^{5} b^{5} x^{4} - 45\,a b^{9} + 390\,a^{2} b^{7} c + 24\,a^{3} b^{5} c^{2} + 160\,a^{4} b^{3} c^{3} + 1920\,a^{5} b c^{4} + 48\left(21\,a^{4} b^{6} - 10\,a^{5} b^{4} c\right) x^{3} + 8\left(93\,a^{3} b^{7} + 6\,a^{4} b^{5} c + 80\,a^{5} b^{3} c^{2}\right) x^{2} + 2\left(15\,a^{2} b^{8} + 654\,a^{3} b^{6} c - 40\,a^{4} b^{4} c^{2} - 480\,a^{5} b^{2} c^{3}\right) x\right)\sqrt{a x^{2} + b x + c}}{3840\,a^{3} b^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x+c)^(5/2)/(b*x+c),x, algorithm="fricas")
```

```
[Out] [1/7680*(3840*a^(11/2)*c^5*log(-(2*b^3*c*x + b^2*c^2 + 4*a*c^3 + (b^4 - 4*a
*b^2*c + 8*a^2*c^2)*x^2 - 4*(b*c^2 + (b^2*c - 2*a*c^2)*x)*sqrt(a*x^2 + b*x
+ c)*sqrt(a))/(b^2*x^2 + 2*b*c*x + c^2)) - 15*(3*b^10 - 30*a*b^8*c + 80*a^2
*b^6*c^2 - 256*a^5*c^5)*sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x + 4*sqrt(a*x^2 + b
*x + c)*(2*a*x + b)*sqrt(a) - b^2 - 4*a*c) + 4*(384*a^5*b^5*x^4 - 45*a*b^9
+ 390*a^2*b^7*c + 24*a^3*b^5*c^2 + 160*a^4*b^3*c^3 + 1920*a^5*b*c^4 + 48*(2
1*a^4*b^6 - 10*a^5*b^4*c)*x^3 + 8*(93*a^3*b^7 + 6*a^4*b^5*c + 80*a^5*b^3*c^
2)*x^2 + 2*(15*a^2*b^8 + 654*a^3*b^6*c - 40*a^4*b^4*c^2 - 480*a^5*b^2*c^3)*
x)*sqrt(a*x^2 + b*x + c))/(a^3*b^6), -1/3840*(3840*sqrt(-a)*a^5*c^5*arctan(
-1/2*sqrt(a*x^2 + b*x + c)*(b*c + (b^2 - 2*a*c)*x)*sqrt(-a)/(a^2*c*x^2 + a*
b*c*x + a*c^2)) + 15*(3*b^10 - 30*a*b^8*c + 80*a^2*b^6*c^2 - 256*a^5*c^5)*s
qrt(-a)*arctan(1/2*sqrt(a*x^2 + b*x + c)*(2*a*x + b)*sqrt(-a)/(a^2*x^2 + a*
b*x + a*c)) - 2*(384*a^5*b^5*x^4 - 45*a*b^9 + 390*a^2*b^7*c + 24*a^3*b^5*c^
2 + 160*a^4*b^3*c^3 + 1920*a^5*b*c^4 + 48*(21*a^4*b^6 - 10*a^5*b^4*c)*x^3 +
 8*(93*a^3*b^7 + 6*a^4*b^5*c + 80*a^5*b^3*c^2)*x^2 + 2*(15*a^2*b^8 + 654*a^
3*b^6*c - 40*a^4*b^4*c^2 - 480*a^5*b^2*c^3)*x)*sqrt(a*x^2 + b*x + c))/(a^3*
b^6)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x+c)^(5/2)/(b*x+c),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 1232, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x+c)^(5/2)/(b*x+c),x)
```

```
[Out] -1/8/b*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(3/2)*c-1/16/b*((x+c/b
)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(1/2)*c^2-3/128/a^2*((x+c/b)^2*a-(2*
a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(1/2)*b^3+3/256/a^(5/2)*ln((-1/2*(2*a*c-b^2)/
b+(x+c/b)*a)/a^(1/2)+((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(1/2))*b
^4+1/8*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(3/2)*x+1/16/a*((x+c/b
)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(3/2)*b+9/32*((x+c/b)^2*a-(2*a*c-b^2
```

```
)/b*(x+c/b)+a*c^2/b^2)^(1/2)*x*c-3/64/a*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+
a*c^2/b^2)^(1/2)*x*b^2+9/64/a*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)
^(1/2)*b*c-1/b^7*a^3*c^6/(a*c^2/b^2)^(1/2)*ln((2*a*c^2/b^2-(2*a*c-b^2)/b*(x
+c/b)+2*(a*c^2/b^2)^(1/2)*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(1/
2))/(x+c/b))+1/5/b*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(5/2)-1/4/
b^3*a*c^3*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(1/2)+1/b^5*a^2*c^4
*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(1/2)+5/16/a^(1/2)*ln((-1/2*
(2*a*c-b^2)/b+(x+c/b)*a)/a^(1/2)+((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b
^2)^(1/2))*c^2-15/128/a^(3/2)*ln((-1/2*(2*a*c-b^2)/b+(x+c/b)*a)/a^(1/2)+((x
+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(1/2))*c*b^2-1/b^6*a^(5/2)*c^5*l
n((-1/2*(2*a*c-b^2)/b+(x+c/b)*a)/a^(1/2)+((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)
+a*c^2/b^2)^(1/2))+1/3/b^3*a*c^2*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b
^2)^(3/2)-1/2/b^4*a^2*c^3*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(1/
2)*x-1/4/b^2*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(3/2)*x*a*c-1/8/
b^2*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(1/2)*x*c^2*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x+c)^(5/2)/(b*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x^2+b\,x+c\right)}^{5/2}}{c+b\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + b*x + a*x^2)^(5/2)/(c + b*x),x)
```

```
[Out] int((c + b*x + a*x^2)^(5/2)/(c + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{2} + bx + c\right)^{\frac{5}{2}}}{bx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x+c)**(5/2)/(b*x+c),x)
```

```
[Out] Integral((a*x**2 + b*x + c)**(5/2)/(b*x + c), x)
```

3.2310 $\int \frac{(-b+x^3)(b+x^3)(-c+x^3)}{\sqrt[3]{ax^2+x^3}}\,dx$

Optimal. Leaf size=339

$$\frac{(135850a^9+176904a^6c-275562a^3b^2-1594323b^2c)\log\left(\sqrt[3]{ax^2+x^3}-x\right)}{1594323}+\frac{(-135850a^9-176904a^6c+275562}{}$$

Rubi [B] time = 1.07, antiderivative size = 993, normalized size of antiderivative = 2.93, number of steps used = 28, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2053, 2011, 59, 2024}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[((-b + x^3)*(b + x^3)*(-c + x^3))/(a*x^2 + x^3)^(1/3),x]
```

```
[Out] (-67925*a^7*(a*x^2 + x^3)^(2/3))/354294 + (7*a*b^2*(a*x^2 + x^3)^(2/3))/18
- (182*a^4*c*(a*x^2 + x^3)^(2/3))/729 + (135850*a^8*(a*x^2 + x^3)^(2/3))/(5
31441*x) - (14*a^2*b^2*(a*x^2 + x^3)^(2/3))/(27*x) + (728*a^5*c*(a*x^2 + x^
3)^(2/3))/(2187*x) + (67925*a^6*x*(a*x^2 + x^3)^(2/3))/413343 - (b^2*x*(a*x
^2 + x^3)^(2/3))/3 + (52*a^3*c*x*(a*x^2 + x^3)^(2/3))/243 - (13585*a^5*x^2*
(a*x^2 + x^3)^(2/3))/91854 - (26*a^2*c*x^2*(a*x^2 + x^3)^(2/3))/135 + (2090
*a^4*x^3*(a*x^2 + x^3)^(2/3))/15309 + (8*a*c*x^3*(a*x^2 + x^3)^(2/3))/45 -
(5225*a^3*x^4*(a*x^2 + x^3)^(2/3))/40824 - (c*x^4*(a*x^2 + x^3)^(2/3))/6 +
(275*a^2*x^5*(a*x^2 + x^3)^(2/3))/2268 - (25*a*x^6*(a*x^2 + x^3)^(2/3))/216
 + (x^7*(a*x^2 + x^3)^(2/3))/9 + (135850*a^9*x^(2/3)*(a + x)^(1/3)*ArcTan[1
/Sqrt[3] + (2*(a + x)^(1/3))/(Sqrt[3]*x^(1/3))])/(531441*Sqrt[3]*(a*x^2 + x
^3)^(1/3)) - (14*a^3*b^2*x^(2/3)*(a + x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(a + x
)^(1/3))/(Sqrt[3]*x^(1/3))])/(27*Sqrt[3]*(a*x^2 + x^3)^(1/3)) + (728*a^6*c*
x^(2/3)*(a + x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(a + x)^(1/3))/(Sqrt[3]*x^(1/3)
)])/(2187*Sqrt[3]*(a*x^2 + x^3)^(1/3)) - (Sqrt[3]*b^2*c*x^(2/3)*(a + x)^(1/
3)*ArcTan[1/Sqrt[3] + (2*(a + x)^(1/3))/(Sqrt[3]*x^(1/3))])/(a*x^2 + x^3)^(
1/3) + (67925*a^9*x^(2/3)*(a + x)^(1/3)*Log[x])/(1594323*(a*x^2 + x^3)^(1/3
)) - (7*a^3*b^2*x^(2/3)*(a + x)^(1/3)*Log[x])/(81*(a*x^2 + x^3)^(1/3)) + (3
64*a^6*c*x^(2/3)*(a + x)^(1/3)*Log[x])/(6561*(a*x^2 + x^3)^(1/3)) - (b^2*c*
x^(2/3)*(a + x)^(1/3)*Log[x])/(2*(a*x^2 + x^3)^(1/3)) + (67925*a^9*x^(2/3)*
(a + x)^(1/3)*Log[-1 + (a + x)^(1/3)/x^(1/3)])/(531441*(a*x^2 + x^3)^(1/3))
 - (7*a^3*b^2*x^(2/3)*(a + x)^(1/3)*Log[-1 + (a + x)^(1/3)/x^(1/3)])/(27*(a
*x^2 + x^3)^(1/3)) + (364*a^6*c*x^(2/3)*(a + x)^(1/3)*Log[-1 + (a + x)^(1/3
)/x^(1/3)])/(2187*(a*x^2 + x^3)^(1/3)) - (3*b^2*c*x^(2/3)*(a + x)^(1/3)*Log
[-1 + (a + x)^(1/3)/x^(1/3)])/(2*(a*x^2 + x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2053

```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Expan
dIntegrand[Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && (Po
lyQ[Pq, x] || PolyQ[Pq, x^n]) && !IntegerQ[p] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\left(-b+x^3\right)\left(b+x^3\right)\left(-c+x^3\right)}{\sqrt[3]{ax^2+x^3}}\,dx = \int \left(\frac{b^2c}{\sqrt[3]{ax^2+x^3}} - \frac{b^2x^3}{\sqrt[3]{ax^2+x^3}} - \frac{cx^6}{\sqrt[3]{ax^2+x^3}} + \frac{x^9}{\sqrt[3]{ax^2+x^3}}\right) dx$$

$$= -\left(b^2\int \frac{x^3}{\sqrt[3]{ax^2+x^3}}\,dx\right) - c\int \frac{x^6}{\sqrt[3]{ax^2+x^3}}\,dx + \left(b^2c\right)\int \frac{1}{\sqrt[3]{ax^2+x^3}}\,dx + \int$$

$$= -\frac{1}{3}b^2x\left(ax^2+x^3\right)^{2/3} - \frac{1}{6}cx^4\left(ax^2+x^3\right)^{2/3} + \frac{1}{9}x^7\left(ax^2+x^3\right)^{2/3} - \frac{1}{27}(25a)\int$$

$$= \frac{7}{18}ab^2\left(ax^2+x^3\right)^{2/3} - \frac{1}{3}b^2x\left(ax^2+x^3\right)^{2/3} + \frac{8}{45}acx^3\left(ax^2+x^3\right)^{2/3} - \frac{1}{6}cx^4\left(ax\right.$$

$$= \frac{7}{18}ab^2\left(ax^2+x^3\right)^{2/3} - \frac{14a^2b^2\left(ax^2+x^3\right)^{2/3}}{27x} - \frac{1}{3}b^2x\left(ax^2+x^3\right)^{2/3} - \frac{26}{135}a^2cx$$

$$= \frac{7}{18}ab^2\left(ax^2+x^3\right)^{2/3} - \frac{14a^2b^2\left(ax^2+x^3\right)^{2/3}}{27x} - \frac{1}{3}b^2x\left(ax^2+x^3\right)^{2/3} + \frac{52}{243}a^3cx$$

$$= \frac{7}{18}ab^2\left(ax^2+x^3\right)^{2/3} - \frac{182}{729}a^4c\left(ax^2+x^3\right)^{2/3} - \frac{14a^2b^2\left(ax^2+x^3\right)^{2/3}}{27x} - \frac{1}{3}b^2x\left(\right.$$

$$= \frac{7}{18}ab^2\left(ax^2+x^3\right)^{2/3} - \frac{182}{729}a^4c\left(ax^2+x^3\right)^{2/3} - \frac{14a^2b^2\left(ax^2+x^3\right)^{2/3}}{27x} + \frac{728a^5}{}$$

$$= \frac{7}{18}ab^2\left(ax^2+x^3\right)^{2/3} - \frac{182}{729}a^4c\left(ax^2+x^3\right)^{2/3} - \frac{14a^2b^2\left(ax^2+x^3\right)^{2/3}}{27x} + \frac{728a^5}{}$$

$$= -\frac{67925a^7\left(ax^2+x^3\right)^{2/3}}{354294} + \frac{7}{18}ab^2\left(ax^2+x^3\right)^{2/3} - \frac{182}{729}a^4c\left(ax^2+x^3\right)^{2/3} - \frac{14a}{}$$

$$= -\frac{67925a^7\left(ax^2+x^3\right)^{2/3}}{354294} + \frac{7}{18}ab^2\left(ax^2+x^3\right)^{2/3} - \frac{182}{729}a^4c\left(ax^2+x^3\right)^{2/3} + \frac{135}{}$$

$$= -\frac{67925a^7\left(ax^2+x^3\right)^{2/3}}{354294} + \frac{7}{18}ab^2\left(ax^2+x^3\right)^{2/3} - \frac{182}{729}a^4c\left(ax^2+x^3\right)^{2/3} + \frac{135}{}$$

$$= -\frac{67925a^7\left(ax^2+x^3\right)^{2/3}}{354294} + \frac{7}{18}ab^2\left(ax^2+x^3\right)^{2/3} - \frac{182}{729}a^4c\left(ax^2+x^3\right)^{2/3} + \frac{135}{}$$

Mathematica [C] time = 0.28, size = 319, normalized size = 0.94

$$\frac{3x\sqrt[3]{\frac{a+x}{a}}\left(a^9\,{}_2F_1\left(-\frac{26}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right) - 9a^9\,{}_2F_1\left(-\frac{23}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right) + 36a^9\,{}_2F_1\left(-\frac{20}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right) + a^3\left(84a^6+20a^3c-b^2\right){}_2F_1\left(-\frac{8}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right) - 3a^3\left(12a^6+5a^3c-b^2\right){}_2F_1\left(-\frac{5}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right) + 3a^3\left(3a^6+2a^3c-b^2\right){}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right) - \left(a^6-b^2\right)\left(a^3+c\right){}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right) - a^6\left(84a^3+c\right){}_2F_1\left(-\frac{17}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right) + 6a^6\left(21a^3+c\right){}_2F_1\left(-\frac{14}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right) - 3a^6\left(42a^3+5c\right){}_2F_1\left(-\frac{11}{3},\frac{1}{3};\frac{4}{3};-\frac{x}{a}\right)\right)}{\sqrt[3]{x^2(a+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + x^3)*(b + x^3)*(-c + x^3))/(a*x^2 + x^3)^(1/3),x]
```

```
[Out] (3*x*((a + x)/a)^(1/3)*(a^9*Hypergeometric2F1[-26/3, 1/3, 4/3, -(x/a)] - 9*
a^9*Hypergeometric2F1[-23/3, 1/3, 4/3, -(x/a)] + 36*a^9*Hypergeometric2F1[-
20/3, 1/3, 4/3, -(x/a)] - a^6*(84*a^3 + c)*Hypergeometric2F1[-17/3, 1/3, 4/
3, -(x/a)] + 6*a^6*(21*a^3 + c)*Hypergeometric2F1[-14/3, 1/3, 4/3, -(x/a)]
- 3*a^6*(42*a^3 + 5*c)*Hypergeometric2F1[-11/3, 1/3, 4/3, -(x/a)] + a^3*(84
*a^6 - b^2 + 20*a^3*c)*Hypergeometric2F1[-8/3, 1/3, 4/3, -(x/a)] - 3*a^3*(1
```

```
2*a^6 - b^2 + 5*a^3*c)*Hypergeometric2F1[-5/3, 1/3, 4/3, -(x/a)] + 3*a^3*(3
*a^6 - b^2 + 2*a^3*c)*Hypergeometric2F1[-2/3, 1/3, 4/3, -(x/a)] - (a^6 - b^
2)*(a^3 + c)*Hypergeometric2F1[1/3, 1/3, 4/3, -(x/a)]))/(x^2*(a + x))^(1/3)
```

IntegrateAlgebraic [A] time = 4.72, size = 339, normalized size = 1.00

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + x^3)*(b + x^3)*(-c + x^3))/(a*x^2 + x^3)^(1/3),x]
```

```
[Out] ((a*x^2 + x^3)^(2/3)*(38038000*a^8 - 77157360*a^2*b^2 + 49533120*a^5*c - 28
528500*a^7*x + 57868020*a*b^2*x - 37149840*a^4*c*x + 24453000*a^6*x^2 - 496
01160*b^2*x^2 + 31842720*a^3*c*x^2 - 22007700*a^5*x^3 - 28658448*a^2*c*x^3
+ 20314800*a^4*x^4 + 26453952*a*c*x^4 - 19045125*a^3*x^5 - 24800580*c*x^5 +
 18042750*a^2*x^6 - 17222625*a*x^7 + 16533720*x^8))/(148803480*x) + ((-1358
50*Sqrt[3]*a^9 + 275562*Sqrt[3]*a^3*b^2 - 176904*Sqrt[3]*a^6*c + 1594323*Sq
rt[3]*b^2*c)*ArcTan[(Sqrt[3]*x)/(x + 2*(a*x^2 + x^3)^(1/3))])/1594323 + ((1
35850*a^9 - 275562*a^3*b^2 + 176904*a^6*c - 1594323*b^2*c)*Log[-x + (a*x^2
+ x^3)^(1/3)])/1594323 + ((-135850*a^9 + 275562*a^3*b^2 - 176904*a^6*c + 15
94323*b^2*c)*Log[x^2 + x*(a*x^2 + x^3)^(1/3) + (a*x^2 + x^3)^(2/3)])/318864
6
```

fricas [A] time = 0.42, size = 325, normalized size = 0.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-b)*(x^3+b)*(x^3-c)/(a*x^2+x^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/446410440*(280*sqrt(3)*(135850*a^9 - 275562*a^3*b^2 + 243*(728*a^6 - 6561
*b^2)*c)*x*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(a*x^2 + x^3)^(1/3))/x) + 280*
(135850*a^9 - 275562*a^3*b^2 + 243*(728*a^6 - 6561*b^2)*c)*x*log(-(x - (a*x
^2 + x^3)^(1/3))/x) - 140*(135850*a^9 - 275562*a^3*b^2 + 243*(728*a^6 - 656
1*b^2)*c)*x*log((x^2 + (a*x^2 + x^3)^(1/3)*x + (a*x^2 + x^3)^(2/3))/x^2) +
3*(38038000*a^8 + 18042750*a^2*x^6 - 17222625*a*x^7 + 16533720*x^8 + 495331
20*a^5*c - 3645*(5225*a^3 + 6804*c)*x^5 + 3888*(5225*a^4 + 6804*a*c)*x^4 -
77157360*a^2*b^2 - 4212*(5225*a^5 + 6804*a^2*c)*x^3 + 360*(67925*a^6 + 8845
2*a^3*c - 137781*b^2)*x^2 - 420*(67925*a^7 + 88452*a^4*c - 137781*a*b^2)*x)
*(a*x^2 + x^3)^(2/3))/x
```

giac [A] time = 0.49, size = 572, normalized size = 1.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-b)*(x^3+b)*(x^3-c)/(a*x^2+x^3)^(1/3),x, algorithm="giac")
```

```
[Out] 1/446410440*(280*sqrt(3)*(135850*a^10 + 176904*a^7*c - 275562*a^4*b^2 - 159
4323*a*b^2*c)*arctan(1/3*sqrt(3)*(2*(a/x + 1)^(1/3) + 1)) - 140*(135850*a^1
0 + 176904*a^7*c - 275562*a^4*b^2 - 1594323*a*b^2*c)*log((a/x + 1)^(2/3) +
(a/x + 1)^(1/3) + 1) + 280*(135850*a^10 + 176904*a^7*c - 275562*a^4*b^2 - 1
594323*a*b^2*c)*log(abs((a/x + 1)^(1/3) - 1)) + 3*(38038000*a^10*(a/x + 1)^
(26/3) - 332832500*a^10*(a/x + 1)^(23/3) + 1289216500*a^10*(a/x + 1)^(20/3)
 + 49533120*a^7*c*(a/x + 1)^(26/3) - 2897952200*a^10*(a/x + 1)^(17/3) - 433
414800*a^7*c*(a/x + 1)^(23/3) + 4158305800*a^10*(a/x + 1)^(14/3) + 16788189
60*a^7*c*(a/x + 1)^(20/3) - 77157360*a^4*b^2*(a/x + 1)^(26/3) - 3938066825*
a^10*(a/x + 1)^(11/3) - 3773716128*a^7*c*(a/x + 1)^(17/3) + 675126900*a^4*b
^2*(a/x + 1)^(23/3) + 2448101425*a^10*(a/x + 1)^(8/3) + 5414949792*a^7*c*(a
```

```
/x + 1)^(14/3) - 2615083380*a^4*b^2*(a/x + 1)^(20/3) - 952462700*a^10*(a/x
+ 1)^(5/3) - 5128154388*a^7*c*(a/x + 1)^(11/3) + 5833647540*a^4*b^2*(a/x +
1)^(17/3) + 204186220*a^10*(a/x + 1)^(2/3) + 3164424732*a^7*c*(a/x + 1)^(8/
3) - 8170413300*a^4*b^2*(a/x + 1)^(14/3) - 1170879948*a^7*c*(a/x + 1)^(5/3)
 + 7338216060*a^4*b^2*(a/x + 1)^(11/3) + 198438660*a^7*c*(a/x + 1)^(2/3) -
4119651900*a^4*b^2*(a/x + 1)^(8/3) + 1319941980*a^4*b^2*(a/x + 1)^(5/3) - 1
84626540*a^4*b^2*(a/x + 1)^(2/3))*x^9/a^9)/a
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}-b\right) \left(x^{3}+b\right) \left(x^{3}-c\right)}{\left(a\,x^{2}+x^{3}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-b)*(x^3+b)*(x^3-c)/(a*x^2+x^3)^(1/3),x)
[Out] int((x^3-b)*(x^3+b)*(x^3-c)/(a*x^2+x^3)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{3}+b\right)\left(x^{3}-b\right)\left(x^{3}-c\right)}{\left(ax^{2}+x^{3}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-b)*(x^3+b)*(x^3-c)/(a*x^2+x^3)^(1/3),x, algorithm="maxima")
[Out] integrate((x^3 + b)*(x^3 - b)*(x^3 - c)/(a*x^2 + x^3)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^3+b\right)\,\left(b-x^3\right)\,\left(c-x^3\right)}{{\left(x^3+a\,x^2\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + x^3)*(b - x^3)*(c - x^3))/(a*x^2 + x^3)^(1/3),x)
[Out] int(((b + x^3)*(b - x^3)*(c - x^3))/(a*x^2 + x^3)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- b + x^{3}\right) \left(b + x^{3}\right) \left(- c + x^{3}\right)}{\sqrt[3]{x^{2} \left(a + x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-b)*(x**3+b)*(x**3-c)/(a*x**2+x**3)**(1/3),x)
[Out] Integral((-b + x**3)*(b + x**3)*(-c + x**3)/(x**2*(a + x))**(1/3), x)
```

3.2311 $\int \frac{x(-a+x)(-b+x)\left(ab-2bx+x^2\right)}{\left(x(-a+x)(-b+x)^2\right)^{2/3}\left(-b^2+2bx-\left(1-a^2d\right)x^2-2adx^3+dx^4\right)}\,dx$

Optimal. Leaf size=340

$$\frac{\log\left(-\sqrt[3]{d}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}+b^2-2bx+x^2\right)}{2d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\left(x^2\left(2ab+b^2\right)-ab^2x\ldots}{\sqrt[3]{d}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2\ldots}\right.}{2d^{2/3}}$$

Rubi [F] time = 23.64, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(-a+x)(-b+x)\left(ab-2bx+x^2\right)}{\left(x(-a+x)(-b+x)^2\right)^{2/3}\left(-b^2+2bx-\left(1-a^2d\right)x^2-2adx^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-a + x)*(-b + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2)^(2/3
)*(-b^2 + 2*b*x - (1 - a^2*d)*x^2 - 2*a*d*x^3 + d*x^4)),x]
```

```
[Out] (6*b*x^(2/3)*(-a + x)^(2/3)*(-b + x)^(4/3)*Defer[Subst][Defer[Int][(x^6*(-a
 + x^3)^(1/3))/((-b + x^3)^(1/3)*(b^2 - 2*b*x^3 + (1 - a^2*d)*x^6 + 2*a*d*x
^9 - d*x^12)), x], x, x^(1/3)])/(-((a - x)*(b - x)^2*x))^(2/3) + (3*a*b*x^(
2/3)*(-a + x)^(2/3)*(-b + x)^(4/3)*Defer[Subst][Defer[Int][(x^3*(-a + x^3)^
(1/3))/((-b + x^3)^(1/3)*(-b^2 + 2*b*x^3 - (1 - a^2*d)*x^6 - 2*a*d*x^9 + d*
x^12)), x], x, x^(1/3)])/(-((a - x)*(b - x)^2*x))^(2/3) + (3*x^(2/3)*(-a +
x)^(2/3)*(-b + x)^(4/3)*Defer[Subst][Defer[Int][(x^9*(-a + x^3)^(1/3))/((-b
 + x^3)^(1/3)*(-b^2 + 2*b*x^3 - (1 - a^2*d)*x^6 - 2*a*d*x^9 + d*x^12)), x],
 x, x^(1/3)])/(-((a - x)*(b - x)^2*x))^(2/3)
```

Rubi steps

$$\int \frac{x(-a+x)(-b+x)\left(ab-2bx+x^2\right)}{\left(x(-a+x)(-b+x)^2\right)^{2/3}\left(-b^2+2bx-\left(1-a^2d\right)x^2-2adx^3+dx^4\right)}\,dx = \frac{\left(x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{\ldots}{\sqrt[3]{-b+\ldots}}}{\left(x(-a+x)(-b\ldots\right.}$$

$$= \frac{\left(3x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\text{Subst}\ldots}{\left(x(-\ldots\right.}$$

$$= \frac{\left(3x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\text{Subst}\ldots}{\ldots}$$

$$= \frac{\left(3x^{2/3}(-a+x)^{2/3}(-b+x)^{4/3}\right)\text{Subst}\ldots}{\left(x(-\ldots\right.}$$

Mathematica [F] time = 3.37, size = 0, normalized size = 0.00

$$\int \frac{x(-a+x)(-b+x)\left(ab-2bx+x^2\right)}{\left(x(-a+x)(-b+x)^2\right)^{2/3}\left(-b^2+2bx-\left(1-a^2d\right)x^2-2adx^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(-a + x)*(-b + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2
)^(2/3)*(-b^2 + 2*b*x - (1 - a^2*d)*x^2 - 2*a*d*x^3 + d*x^4)),x]
```

```
[Out] Integrate[(x*(-a + x)*(-b + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2
)^(2/3)*(-b^2 + 2*b*x - (1 - a^2*d)*x^2 - 2*a*d*x^3 + d*x^4)), x]
```

IntegrateAlgebraic [A] time = 0.84, size = 340, normalized size = 1.00

$$\frac{\log\left(-\sqrt[3]{d}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}+b^2-2bx+x^2\right)}{2d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\left(x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}}{\sqrt[3]{d}\left(x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}+2b^2-4bx+2x^2}\right)}{2d^{2/3}}-\frac{\log\left(d^{2/3}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{4/3}+\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}\left(b^2\sqrt[3]{d}-2b\sqrt[3]{d}\,x+\sqrt[3]{d}\,x^2\right)+b^4-4b^3x+6b^2x^2-4bx^3+x^4\right)}{4d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-a + x)*(-b + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(
-b + x)^2)^(2/3)*(-b^2 + 2*b*x - (1 - a^2*d)*x^2 - 2*a*d*x^3 + d*x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*
b)*x^3 + x^4)^(2/3))/(2*b^2 - 4*b*x + 2*x^2 + d^(1/3)*(-(a*b^2*x) + (2*a*b
+ b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(2/3))]])/(2*d^(2/3)) + Log[b^2 - 2*b*x +
 x^2 - d^(1/3)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(2/3
)]/(2*d^(2/3)) - Log[b^4 - 4*b^3*x + 6*b^2*x^2 - 4*b*x^3 + x^4 + (b^2*d^(1/
3) - 2*b*d^(1/3)*x + d^(1/3)*x^2)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2
*b)*x^3 + x^4)^(2/3) + d^(2/3)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)
*x^3 + x^4)^(4/3)]/(4*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+x)*(-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(2/3)/(-b^2+2
*b*x-(-a^2*d+1)*x^2-2*a*d*x^3+d*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(ab-2\,bx+x^2\right)(a-x)(b-x)x}{\left(2\,adx^3-dx^4-\left(a^2d-1\right)x^2+b^2-2\,bx\right)\left(-(a-x)(b-x)^2x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+x)*(-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(2/3)/(-b^2+2
*b*x-(-a^2*d+1)*x^2-2*a*d*x^3+d*x^4),x, algorithm="giac")
```

```
[Out] integrate(-(a*b - 2*b*x + x^2)*(a - x)*(b - x)*x/((2*a*d*x^3 - d*x^4 - (a^2
*d - 1)*x^2 + b^2 - 2*b*x)*(-(a - x)*(b - x)^2*x)^(2/3)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x\left(-a+x\right)\left(-b+x\right)\left(ab-2bx+x^2\right)}{\left(x\left(-a+x\right)\left(-b+x\right)^2\right)^{\frac{2}{3}}\left(-b^2+2bx-\left(-a^2d+1\right)x^2-2ad\,x^3+d\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a+x)*(-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(2/3)/(-b^2+2*b*x-(
-a^2*d+1)*x^2-2*a*d*x^3+d*x^4),x)
```

```
[Out] int(x*(-a+x)*(-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(2/3)/(-b^2+2*b*x-(
-a^2*d+1)*x^2-2*a*d*x^3+d*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(ab - 2\,bx + x^2\right)(a - x)(b - x)x}{\left(2\,adx^3 - dx^4 - \left(a^2d - 1\right)x^2 + b^2 - 2\,bx\right)\left(-(a - x)(b - x)^2x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+x)*(-b+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(2/3)/(-b^2+2
*b*x-(-a^2*d+1)*x^2-2*a*d*x^3+d*x^4),x, algorithm="maxima")
```

```
[Out] -integrate((a*b - 2*b*x + x^2)*(a - x)*(b - x)*x/((2*a*d*x^3 - d*x^4 - (a^2
*d - 1)*x^2 + b^2 - 2*b*x)*(-(a - x)*(b - x)^2*x)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(a-x\right)\,\left(b-x\right)\,\left(x^2-2\,b\,x+a\,b\right)}{{\left(-x\,\left(a-x\right)\,{\left(b-x\right)}^2\right)}^{2/3}\,\left(-b^2+2\,b\,x+d\,x^4-2\,a\,d\,x^3+\left(a^2\,d-1\right)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a - x)*(b - x)*(a*b - 2*b*x + x^2))/((-x*(a - x)*(b - x)^2)^(2/3)*(
x^2*(a^2*d - 1) + 2*b*x + d*x^4 - b^2 - 2*a*d*x^3)),x)
```

```
[Out] int((x*(a - x)*(b - x)*(a*b - 2*b*x + x^2))/((-x*(a - x)*(b - x)^2)^(2/3)*(
x^2*(a^2*d - 1) + 2*b*x + d*x^4 - b^2 - 2*a*d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+x)*(-b+x)*(a*b-2*b*x+x**2)/(x*(-a+x)*(-b+x)**2)**(2/3)/(-b*
*2+2*b*x-(-a**2*d+1)*x**2-2*a*d*x**3+d*x**4),x)
```

```
[Out] Timed out
```

3.2312 $\int \frac{(-1+(-1+2k)x)(1-2x+x^2)}{\sqrt[3]{(1-x)x(1-kx)}\,(-b+4bx+(1-6b)x^2+(4b-2k)x^3+(-b+k^2)x^4)}\,dx$

Optimal. Leaf size=343

$$-\frac{\log\left(-\sqrt[6]{b}\,x+\sqrt[6]{b}+\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{2b^{2/3}}-\frac{\log\left(\sqrt[6]{b}\,x-\sqrt[6]{b}+\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{2b^{2/3}}+\frac{\log\left(\left(\sqrt[6]{b}-\sqrt[6]{b}\,x\right)\sqrt[3]{\cdots}\right.}{\cdots}$$

Rubi [F] time = 10.66, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-1+(-1+2k)x)\left(1-2x+x^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\,\left(-b+4bx+(1-6b)x^2+(4b-2k)x^3+\left(-b+k^2\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-1 + (-1 + 2*k)*x)*(1 - 2*x + x^2))/(((1 - x)*x*(1 - k*x))^(1/3)*(-b
+ 4*b*x + (1 - 6*b)*x^2 + (4*b - 2*k)*x^3 + (-b + k^2)*x^4)),x]
```

```
[Out] (3*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][(x*(1 - x^
3)^(5/3))/((1 - k*x^3)^(1/3)*(b*(-1 + x^3)^4 - x^6*(-1 + k*x^3)^2)), x], x,
 x^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3) + (3*(1 - 2*k)*(1 - x)^(1/3)*x^(1/3)
*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][(x^4*(1 - x^3)^(5/3))/((1 - k*x^3)
^(1/3)*(b*(-1 + x^3)^4 - x^6*(-1 + k*x^3)^2)), x], x, x^(1/3)])/((1 - x)*x*
(1 - k*x))^(1/3)
```

Rubi steps

$$\int \frac{(-1+(-1+2k)x)\left(1-2x+x^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+4bx+(1-6b)x^2+(4b-2k)x^3+\left(-b+k^2\right)x^4\right)}\,dx = \int \frac{(-1}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+4\right.}$$

$$= \frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{}{\sqrt[3]{1-x}}}{}$$

$$= \frac{\left(\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\int \frac{}{\sqrt[3]{x}\sqrt[3]{}}}{\sqrt[3]{(}}$$

$$= \frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subs}}{}$$

$$= \frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subs}}{\sqrt[3]{}}$$

$$= \frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subs}}{}$$

$$= \frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subs}}{}$$

$$= \frac{\left(3\sqrt[3]{1-x}\sqrt[3]{x}\sqrt[3]{1-kx}\right)\text{Subs}}{\sqrt[3]{}}$$

Mathematica [F] time = 4.59, size = 0, normalized size = 0.00

$$\int \frac{(-1+(-1+2k)x)\left(1-2x+x^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\left(-b+4bx+(1-6b)x^2+(4b-2k)x^3+\left(-b+k^2\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + (-1 + 2*k)*x)*(1 - 2*x + x^2))/(((1 - x)*x*(1 - k*x))^(1/3
)*(-b + 4*b*x + (1 - 6*b)*x^2 + (4*b - 2*k)*x^3 + (-b + k^2)*x^4)),x]
```

```
[Out] Integrate[((-1 + (-1 + 2*k)*x)*(1 - 2*x + x^2))/(((1 - x)*x*(1 - k*x))^(1/3
)*(-b + 4*b*x + (1 - 6*b)*x^2 + (4*b - 2*k)*x^3 + (-b + k^2)*x^4)), x]
```

IntegrateAlgebraic [A] time = 5.34, size = 276, normalized size = 0.80

$$-\frac{\log\left(-\sqrt[3]{b}x^2+2\sqrt[3]{b}x-\sqrt[3]{b}+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2b^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x^2-2\sqrt{3}\sqrt[3]{b}x+\sqrt{3}\sqrt[3]{b}}{\sqrt[3]{b}x^2-2\sqrt[3]{b}x+\sqrt[3]{b}+2\left(kx^3+(-k-1)x^2+x\right)^{2/3}}\right)}{2b^{2/3}}+\frac{\log\left(b^{2/3}x^4-4b^{2/3}x^3+6b^{2/3}x^2-4b^{2/3}x+b^{2/3}+\left(\sqrt[3]{b}x^2-2\sqrt[3]{b}x+\sqrt[3]{b}\right)\left(kx^3+(-k-1)x^2+x\right)^{2/3}+\left(kx^3+(-k-1)x^2+x\right)^{4/3}\right)}{4b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + (-1 + 2*k)*x)*(1 - 2*x + x^2))/(((1 - x)*x*(1 - k
*x))^(1/3)*(-b + 4*b*x + (1 - 6*b)*x^2 + (4*b - 2*k)*x^3 + (-b + k^2)*x^4))
,x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3) - 2*Sqrt[3]*b^(1/3)*x + Sqrt[3]*b^(1/
3)*x^2)/(b^(1/3) - 2*b^(1/3)*x + b^(1/3)*x^2 + 2*(x + (-1 - k)*x^2 + k*x^3)
```

```
^(2/3))])/b^(2/3) - Log[-b^(1/3) + 2*b^(1/3)*x - b^(1/3)*x^2 + (x + (-1 - k
)*x^2 + k*x^3)^(2/3)]/(2*b^(2/3)) + Log[b^(2/3) - 4*b^(2/3)*x + 6*b^(2/3)*x
^2 - 4*b^(2/3)*x^3 + b^(2/3)*x^4 + (b^(1/3) - 2*b^(1/3)*x + b^(1/3)*x^2)*(x
 + (-1 - k)*x^2 + k*x^3)^(2/3) + (x + (-1 - k)*x^2 + k*x^3)^(4/3)]/(4*b^(2/
3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-1+2*k)*x)*(x^2-2*x+1)/((1-x)*x*(-k*x+1))^(1/3)/(-b+4*b*x+(1
-6*b)*x^2+(4*b-2*k)*x^3+(k^2-b)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((2k-1)x-1)\left(x^2-2x+1\right)}{\left(\left(k^2-b\right)x^4+2(2b-k)x^3-(6b-1)x^2+4bx-b\right)((kx-1)(x-1)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-1+2*k)*x)*(x^2-2*x+1)/((1-x)*x*(-k*x+1))^(1/3)/(-b+4*b*x+(1
-6*b)*x^2+(4*b-2*k)*x^3+(k^2-b)*x^4),x, algorithm="giac")
```

```
[Out] integrate(((2*k - 1)*x - 1)*(x^2 - 2*x + 1)/(((k^2 - b)*x^4 + 2*(2*b - k)*x
^3 - (6*b - 1)*x^2 + 4*b*x - b)*((k*x - 1)*(x - 1)*x)^(1/3)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(-1+(-1+2k)x)\left(x^2-2x+1\right)}{((1-x)x(-kx+1))^{\frac{1}{3}}\left(-b+4bx+(1-6b)x^2+(4b-2k)x^3+\left(k^2-b\right)x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+(-1+2*k)*x)*(x^2-2*x+1)/((1-x)*x*(-k*x+1))^(1/3)/(-b+4*b*x+(1-6*b)*
x^2+(4*b-2*k)*x^3+(k^2-b)*x^4),x)
```

```
[Out] int((-1+(-1+2*k)*x)*(x^2-2*x+1)/((1-x)*x*(-k*x+1))^(1/3)/(-b+4*b*x+(1-6*b)*
x^2+(4*b-2*k)*x^3+(k^2-b)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((2k-1)x-1)\left(x^2-2x+1\right)}{\left(\left(k^2-b\right)x^4+2(2b-k)x^3-(6b-1)x^2+4bx-b\right)((kx-1)(x-1)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-1+2*k)*x)*(x^2-2*x+1)/((1-x)*x*(-k*x+1))^(1/3)/(-b+4*b*x+(1
-6*b)*x^2+(4*b-2*k)*x^3+(k^2-b)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(((2*k - 1)*x - 1)*(x^2 - 2*x + 1)/(((k^2 - b)*x^4 + 2*(2*b - k)*x
^3 - (6*b - 1)*x^2 + 4*b*x - b)*((k*x - 1)*(x - 1)*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(x\,(2\,k-1)-1)\,\left(x^2-2\,x+1\right)}{(x\,(k\,x-1)\,(x-1))^{1/3}\,\left(\left(b-k^2\right)\,x^4+(2\,k-4\,b)\,x^3+(6\,b-1)\,x^2-4\,b\,x+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x*(2*k - 1) - 1)*(x^2 - 2*x + 1))/((x*(k*x - 1)*(x - 1))^(1/3)*(b -
x^3*(4*b - 2*k) + x^4*(b - k^2) - 4*b*x + x^2*(6*b - 1))),x)
```

```
[Out] int(-((x*(2*k - 1) - 1)*(x^2 - 2*x + 1))/((x*(k*x - 1)*(x - 1))^(1/3)*(b -
x^3*(4*b - 2*k) + x^4*(b - k^2) - 4*b*x + x^2*(6*b - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-1+2*k)*x)*(x**2-2*x+1)/((1-x)*x*(-k*x+1))**(1/3)/(-b+4*b*x+
(1-6*b)*x**2+(4*b-2*k)*x**3+(k**2-b)*x**4),x)
```

```
[Out] Timed out
```

3.2313 $\int \frac{1}{(b+ax)\sqrt[4]{b^2x+a^2x^3}}\,dx$

Optimal. Leaf size=345

$$\frac{\tan^{-1}\left(\frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{a^2x^3+b^2x}}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{a^2x^3+b^2x}-2^{3/4}ax+2^{3/4}b}\right)}{2\,2^{3/4}a^{3/4}b^{3/4}}-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{a^2x^3+b^2x}}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{a^2x^3+b^2x}+2^{3/4}ax-2^{3/4}b}\right)}{2\,2^{3/4}a^{3/4}b^{3/4}}-\frac{\tanh^{-1}\left(\frac{-2\sqrt[4]{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}-\frac{a^2x^2}{\sqrt[4]{2}}+2^{3/4}ab}{2a^{5/4}\sqrt[4]{b}\,x\sqrt[4]{a^2x^3+b^2x}-2\sqrt[4]{a}\,b^{5/4}\sqrt[4]{a^2x^3}}\right)}{2\,2^{3/4}a^{3/4}b^{3/4}}$$

Rubi [C] time = 0.30, antiderivative size = 150, normalized size of antiderivative = 0.43, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2056, 959, 466, 511, 510}

$$\frac{4x\sqrt[4]{\frac{a^2x^2}{b^2}+1}\,F_1\left(\frac{3}{8};1,\frac{1}{4};\frac{11}{8};\frac{a^2x^2}{b^2},-\frac{a^2x^2}{b^2}\right)}{3b\sqrt[4]{a^2x^3+b^2x}}-\frac{4ax^2\sqrt[4]{\frac{a^2x^2}{b^2}+1}\,F_1\left(\frac{7}{8};1,\frac{1}{4};\frac{15}{8};\frac{a^2x^2}{b^2},-\frac{a^2x^2}{b^2}\right)}{7b^2\sqrt[4]{a^2x^3+b^2x}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((b + a*x)*(b^2*x + a^2*x^3)^(1/4)),x]
```

```
[Out] (4*x*(1 + (a^2*x^2)/b^2)^(1/4)*AppellF1[3/8, 1, 1/4, 11/8, (a^2*x^2)/b^2, -
((a^2*x^2)/b^2)])/(3*b*(b^2*x + a^2*x^3)^(1/4)) - (4*a*x^2*(1 + (a^2*x^2)/b
^2)^(1/4)*AppellF1[7/8, 1, 1/4, 15/8, (a^2*x^2)/b^2, -((a^2*x^2)/b^2)])/(7*
b^2*(b^2*x + a^2*x^3)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{1}{(b+ax)\sqrt[4]{b^2x+a^2x^3}}\,dx = \frac{\left(\sqrt[4]{x}\sqrt[4]{b^2+a^2x^2}\right)\int \frac{1}{\sqrt[4]{x}(b+ax)\sqrt[4]{b^2+a^2x^2}}\,dx}{\sqrt[4]{b^2x+a^2x^3}}$$

$$= -\frac{\left(a\sqrt[4]{x}\sqrt[4]{b^2+a^2x^2}\right)\int \frac{x^{3/4}}{(b^2-a^2x^2)\sqrt[4]{b^2+a^2x^2}}\,dx}{\sqrt[4]{b^2x+a^2x^3}} + \frac{\left(b\sqrt[4]{x}\sqrt[4]{b^2+a^2x^2}\right)\int \frac{1}{\sqrt[4]{x}(b^2-a^2x^2)\sqrt[4]{\ldots}}}{\sqrt[4]{b^2x+a^2x^3}}$$

$$= -\frac{\left(4a\sqrt[4]{x}\sqrt[4]{b^2+a^2x^2}\right)\mathrm{Subst}\left(\int \frac{x^6}{(b^2-a^2x^8)\sqrt[4]{b^2+a^2x^8}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{b^2x+a^2x^3}} + \frac{\left(4b\sqrt[4]{x}\sqrt[4]{b^2+a^2x\ldots}\right.}{\ldots}$$

$$= -\frac{\left(4a\sqrt[4]{x}\sqrt[4]{1+\frac{a^2x^2}{b^2}}\right)\mathrm{Subst}\left(\int \frac{x^6}{(b^2-a^2x^8)\sqrt[4]{1+\frac{a^2x^8}{b^2}}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{b^2x+a^2x^3}} + \frac{\left(4b\sqrt[4]{x}\sqrt[4]{1+\frac{a^2x^2}{b^2}}\right)\ldots}{\ldots}$$

$$= \frac{4x\sqrt[4]{1+\frac{a^2x^2}{b^2}}\,F_1\left(\frac{3}{8};1,\frac{1}{4};\frac{11}{8};\frac{a^2x^2}{b^2},-\frac{a^2x^2}{b^2}\right)}{3b\sqrt[4]{b^2x+a^2x^3}} - \frac{4ax^2\sqrt[4]{1+\frac{a^2x^2}{b^2}}\,F_1\left(\frac{7}{8};1,\frac{1}{4};\frac{15}{8};\frac{a^2x^2}{b^2},-\ldots\right)}{7b^2\sqrt[4]{b^2x+a^2x^3}}$$

Mathematica [F] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{1}{(b+ax)\sqrt[4]{b^2x+a^2x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((b + a*x)*(b^2*x + a^2*x^3)^(1/4)),x]
[Out] Integrate[1/((b + a*x)*(b^2*x + a^2*x^3)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 6.15, size = 345, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{a^2x^3+b^2x}}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{a^2x^3+b^2x}-2^{3/4}ax+2^{3/4}b}\right)}{2\,2^{3/4}a^{3/4}b^{3/4}} - \frac{\tan^{-1}\left(\frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{a^2x^3+b^2x}}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{a^2x^3+b^2x}+2^{3/4}ax-2^{3/4}b}\right)}{2\,2^{3/4}a^{3/4}b^{3/4}} - \frac{\tanh^{-1}\left(\frac{-2\sqrt[4]{2}\sqrt{a}\sqrt{b}\sqrt{a^2x^3+b^2x}-\frac{a^2x^2}{\sqrt[4]{2}}+2^{3/4}abx-\frac{b^2}{\sqrt[4]{2}}}{2a^{5/4}\sqrt[4]{b}\,x\sqrt[4]{a^2x^3+b^2x}-2\sqrt[4]{a}\,b^{5/4}\sqrt[4]{a^2x^3+b^2x}}\right)}{2\,2^{3/4}a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((b + a*x)*(b^2*x + a^2*x^3)^(1/4)),x]
[Out] ArcTan[(2*a^(1/4)*b^(1/4)*(b^2*x + a^2*x^3)^(1/4))/(2^(3/4)*b - 2^(3/4)*a*x
 + 2*a^(1/4)*b^(1/4)*(b^2*x + a^2*x^3)^(1/4))]/(2*2^(3/4)*a^(3/4)*b^(3/4))
- ArcTan[(2*a^(1/4)*b^(1/4)*(b^2*x + a^2*x^3)^(1/4))/(-(2^(3/4)*b) + 2^(3/4
)*a*x + 2*a^(1/4)*b^(1/4)*(b^2*x + a^2*x^3)^(1/4))]/(2*2^(3/4)*a^(3/4)*b^(3
/4)) - ArcTanh[(-(b^2/2^(1/4)) + 2^(3/4)*a*b*x - (a^2*x^2)/2^(1/4) - 2*2^(1
/4)*Sqrt[a]*Sqrt[b]*Sqrt[b^2*x + a^2*x^3])/(-2*a^(1/4)*b^(5/4)*(b^2*x + a^2
*x^3)^(1/4) + 2*a^(5/4)*b^(1/4)*x*(b^2*x + a^2*x^3)^(1/4))]/(2*2^(3/4)*a^(3
/4)*b^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b)/(a^2*x^3+b^2*x)^(1/4),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^3+b^2x\right)^{\frac{1}{4}}(ax+b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b)/(a^2*x^3+b^2*x)^(1/4),x, algorithm="giac")
[Out] integrate(1/((a^2*x^3 + b^2*x)^(1/4)*(a*x + b)), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax+b)\left(a^2x^3+b^2x\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b)/(a^2*x^3+b^2*x)^(1/4),x)
[Out] int(1/(a*x+b)/(a^2*x^3+b^2*x)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2x^3+b^2x\right)^{\frac{1}{4}}(ax+b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b)/(a^2*x^3+b^2*x)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((a^2*x^3 + b^2*x)^(1/4)*(a*x + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a^2\,x^3+b^2\,x\right)^{1/4}\,\left(b+a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b^2*x + a^2*x^3)^(1/4)*(b + a*x)),x)
[Out] int(1/((b^2*x + a^2*x^3)^(1/4)*(b + a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x\left(a^2x^2+b^2\right)}\,(ax+b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b)/(a**2*x**3+b**2*x)**(1/4),x)
[Out] Integral(1/((x*(a**2*x**2 + b**2))**(1/4)*(a*x + b)), x)
```

3.2314 $\int \frac{(-b+x)(-4a+b+3x)}{\sqrt[3]{(-a+x)(-b+x)^2}\left(a+b^4d-\left(1+4b^3d\right)x+6b^2dx^2-4bdx^3+dx^4\right)}dx$

Optimal. Leaf size=347

$$\frac{\log\left(-\sqrt[6]{d}\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}+b^2\sqrt{d}-2b\sqrt{d}\,x+\sqrt{d}\,x^2\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x\left(2ab\cdots\right.}}{\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-\cdots}}\right.}{\sqrt[3]{\cdots}}$$

Rubi [F] time = 7.86, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-b+x)(-4a+b+3x)}{\sqrt[3]{(-a+x)(-b+x)^2}\left(a+b^4d-\left(1+4b^3d\right)x+6b^2dx^2-4bdx^3+dx^4\right)}dx$$

Verification is not applicable to the result.

```
[In] Int[((-b + x)*(-4*a + b + 3*x))/(((-a + x)*(-b + x)^2)^(1/3)*(a + b^4*d - (
1 + 4*b^3*d)*x + 6*b^2*d*x^2 - 4*b*d*x^3 + d*x^4)),x]
```

```
[Out] (9*a*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x*(a - b + x^3)
^(1/3))/(a^4*(1 + (b*(-4*a^3 + 6*a^2*b - 4*a*b^2 + b^3))/a^4)*d - (1 - 4*(a
 - b)^3*d)*x^3 + 6*a^2*(1 + (b*(-2*a + b))/a^2)*d*x^6 + 4*a*(1 - b/a)*d*x^9
 + d*x^12), x], x, (-a + x)^(1/3)]])/(-((a - x)*(b - x)^2))^(1/3) + (9*(-a +
 x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x^4*(a - b + x^3)^(1/3))/
(a^4*(1 + (b*(-4*a^3 + 6*a^2*b - 4*a*b^2 + b^3))/a^4)*d - (1 - 4*(a - b)^3*
d)*x^3 + 6*a^2*(1 + (b*(-2*a + b))/a^2)*d*x^6 + 4*a*(1 - b/a)*d*x^9 + d*x^1
2), x], x, (-a + x)^(1/3)]])/(-((a - x)*(b - x)^2))^(1/3) - (3*(4*a - b)*(-a
 + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x*(a - b + x^3)^(1/3))/
(a*(1 + (b^4*d)/a) - (1 + 4*b^3*d)*(a + x^3) + 6*b^2*d*(a + x^3)^2 - 4*b*d*
(a + x^3)^3 + d*(a + x^3)^4), x], x, (-a + x)^(1/3)]])/(-((a - x)*(b - x)^2)
)^(1/3)
```

Rubi steps

$$\int \frac{(-b+x)(-4a+b+3x)}{\sqrt[3]{(-a+x)(-b+x)^2}\left(a+b^4d-\left(1+4b^3d\right)x+6b^2dx^2-4bdx^3+dx^4\right)}\,dx = \frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{}{\sqrt[3]{-a+x}\,(a+b\cdots}}{\sqrt[3]{(-a+x)(\cdots}}$$

$$= \frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int\Bigg(\frac{}{\sqrt[3]{-a+x}\,(-\cdots}}{}$$

$$= \frac{\left(3\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{}{\sqrt[3]{-a+x}\,(a\cdots}}{\sqrt[3]{(-a+x)(\cdots}}$$

$$= \frac{\left(9\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\text{Subst}\Big(\int \cdots}{}$$

$$= \frac{\left(9\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\text{Subst}\Bigg(\int \cdots}{}$$

$$= \frac{\left(9\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\text{Subst}\Bigg(\int\Bigg(\cdots}{}$$

$$= \frac{\left(9\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\text{Subst}\Bigg(\int \cdots}{}$$

Mathematica [F] time = 2.30, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)(-4a+b+3x)}{\sqrt[3]{(-a+x)(-b+x)^2}\left(a+b^4d-\left(1+4b^3d\right)x+6b^2dx^2-4bdx^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-b + x)*(-4*a + b + 3*x))/(((-a + x)*(-b + x)^2)^(1/3)*(a + b^4
*d - (1 + 4*b^3*d)*x + 6*b^2*d*x^2 - 4*b*d*x^3 + d*x^4)),x]
```

```
[Out] Integrate[((-b + x)*(-4*a + b + 3*x))/(((-a + x)*(-b + x)^2)^(1/3)*(a + b^4
*d - (1 + 4*b^3*d)*x + 6*b^2*d*x^2 - 4*b*d*x^3 + d*x^4)), x]
```

IntegrateAlgebraic [A] time = 3.21, size = 347, normalized size = 1.00

$$\frac{\log\left(-\sqrt[6]{d}\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}+b^2\sqrt{d}-2b\sqrt{d}\,x+\sqrt{d}\,x^2\right)}{\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}}{\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}+2b^2\sqrt[3]{d}-4b\sqrt[3]{d}\,x+2\sqrt[3]{d}\,x^2}\right)}{\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}\left(b^2d^{2/3}-2bd^{2/3}x+d^{2/3}x^2\right)+\sqrt[3]{d}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}+b^4d-4b^3dx+6b^2dx^2-4bdx^3+dx^4\right)}{2\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + x)*(-4*a + b + 3*x))/(((-a + x)*(-b + x)^2)^(1/3)
*(a + b^4*d - (1 + 4*b^3*d)*x + 6*b^2*d*x^2 - 4*b*d*x^3 + d*x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3
)^(1/3))/(2*b^2*d^(1/3) - 4*b*d^(1/3)*x + 2*d^(1/3)*x^2 + (-(a*b^2) + (2*a*
b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3))])/d^(1/3) + Log[b^2*Sqrt[d] - 2*b
*Sqrt[d]*x + Sqrt[d]*x^2 - d^(1/6)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)
```

```
*x^2 + x^3)^(1/3)]/d^(1/3) - Log[b^4*d - 4*b^3*d*x + 6*b^2*d*x^2 - 4*b*d*x^
3 + d*x^4 + (b^2*d^(2/3) - 2*b*d^(2/3)*x + d^(2/3)*x^2)*(-(a*b^2) + (2*a*b
+ b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3) + d^(1/3)*(-(a*b^2) + (2*a*b + b^2)*
x + (-a - 2*b)*x^2 + x^3)^(2/3)]/(2*d^(1/3))
```

fricas [A] time = 1.24, size = 811, normalized size = 2.34

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-4*a+b+3*x)/((-a+x)*(-b+x)^2)^(1/3)/(a+b^4*d-(4*b^3*d+1)*
x+6*b^2*d*x^2-4*b*d*x^3+d*x^4),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*d*sqrt(-1/d^(2/3))*log(-(b^4*d + 6*b^2*d*x^2 - 4*b*d*x^3 + d*
x^4 - 3*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(b^2 - 2*b*x
 + x^2)*d^(2/3) - 2*(2*b^3*d - 1)*x - sqrt(3)*((-a*b^2 - (a + 2*b)*x^2 + x^
3 + (2*a*b + b^2)*x)^(1/3)*(b^2*d - 2*b*d*x + d*x^2) + (b^4*d - 4*b^3*d*x +
 6*b^2*d*x^2 - 4*b*d*x^3 + d*x^4)*d^(1/3) - 2*(-a*b^2 - (a + 2*b)*x^2 + x^3
 + (2*a*b + b^2)*x)^(2/3)*d^(2/3))*sqrt(-1/d^(2/3)) - 2*a)/(b^4*d + 6*b^2*d
*x^2 - 4*b*d*x^3 + d*x^4 - (4*b^3*d + 1)*x + a)) - d^(2/3)*log(((-a*b^2 - (
a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(b^2 - 2*b*x + x^2)*d^(1/3) + (
b^4 - 4*b^3*x + 6*b^2*x^2 - 4*b*x^3 + x^4)*d^(2/3) + (-a*b^2 - (a + 2*b)*x^
2 + x^3 + (2*a*b + b^2)*x)^(2/3))/(b^4 - 4*b^3*x + 6*b^2*x^2 - 4*b*x^3 + x^
4)) + 2*d^(2/3)*log(-((b^2 - 2*b*x + x^2)*d^(1/3) - (-a*b^2 - (a + 2*b)*x^2
 + x^3 + (2*a*b + b^2)*x)^(1/3))/(b^2 - 2*b*x + x^2)))/d, 1/2*(2*sqrt(3)*d^
(2/3)*arctan(1/3*sqrt(3)*((b^2 - 2*b*x + x^2)*d^(1/3) + 2*(-a*b^2 - (a + 2*
b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3))/((b^2 - 2*b*x + x^2)*d^(1/3))) - d^(
2/3)*log(((-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(b^2 - 2*b
*x + x^2)*d^(1/3) + (b^4 - 4*b^3*x + 6*b^2*x^2 - 4*b*x^3 + x^4)*d^(2/3) + (
-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3))/(b^4 - 4*b^3*x + 6*b
^2*x^2 - 4*b*x^3 + x^4)) + 2*d^(2/3)*log(-((b^2 - 2*b*x + x^2)*d^(1/3) - (-
a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3))/(b^2 - 2*b*x + x^2)))
/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,a-b-3\,x)(b-x)}{\left(b^{4}d+6\,b^{2}dx^{2}-4\,bdx^{3}+dx^{4}-\left(4\,b^{3}d+1\right)x+a\right)\left(-(a-x)(b-x)^{2}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-4*a+b+3*x)/((-a+x)*(-b+x)^2)^(1/3)/(a+b^4*d-(4*b^3*d+1)*
x+6*b^2*d*x^2-4*b*d*x^3+d*x^4),x, algorithm="giac")
```

```
[Out] integrate((4*a - b - 3*x)*(b - x)/((b^4*d + 6*b^2*d*x^2 - 4*b*d*x^3 + d*x^4
 - (4*b^3*d + 1)*x + a)*(-(a - x)*(b - x)^2)^(1/3)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)\,(-4a+b+3x)}{\left((-a+x)\,(-b+x)^{2}\right)^{\frac{1}{3}}\left(a+b^{4}d-\left(4b^{3}d+1\right)x+6b^{2}d\,x^{2}-4bd\,x^{3}+d\,x^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b+x)*(-4*a+b+3*x)/((-a+x)*(-b+x)^2)^(1/3)/(a+b^4*d-(4*b^3*d+1)*x+6*b^
2*d*x^2-4*b*d*x^3+d*x^4),x)
```

```
[Out] int((-b+x)*(-4*a+b+3*x)/((-a+x)*(-b+x)^2)^(1/3)/(a+b^4*d-(4*b^3*d+1)*x+6*b^
2*d*x^2-4*b*d*x^3+d*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4\,a-b-3\,x)(b-x)}{\left(b^4d+6\,b^2dx^2-4\,bdx^3+dx^4-\left(4\,b^3d+1\right)x+a\right)\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-4*a+b+3*x)/((-a+x)*(-b+x)^2)^(1/3)/(a+b^4*d-(4*b^3*d+1)*
x+6*b^2*d*x^2-4*b*d*x^3+d*x^4),x, algorithm="maxima")
```

```
[Out] integrate((4*a - b - 3*x)*(b - x)/((b^4*d + 6*b^2*d*x^2 - 4*b*d*x^3 + d*x^4
 - (4*b^3*d + 1)*x + a)*(-(a - x)*(b - x)^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(b-x)\,\,(b-4\,a+3\,x)}{{\left(-\,(a-x)\,\,{(b-x)}^2\right)}^{1/3}\,\left(a+b^4\,d+d\,x^4-x\,\left(4\,d\,b^3+1\right)+6\,b^2\,d\,x^2-4\,b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((b - x)*(b - 4*a + 3*x))/((-(a - x)*(b - x)^2)^(1/3)*(a + b^4*d + d*x
^4 - x*(4*b^3*d + 1) + 6*b^2*d*x^2 - 4*b*d*x^3)),x)
```

```
[Out] int(-((b - x)*(b - 4*a + 3*x))/((-(a - x)*(b - x)^2)^(1/3)*(a + b^4*d + d*x
^4 - x*(4*b^3*d + 1) + 6*b^2*d*x^2 - 4*b*d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-4*a+b+3*x)/((-a+x)*(-b+x)**2)**(1/3)/(a+b**4*d-(4*b**3*d
+1)*x+6*b**2*d*x**2-4*b*d*x**3+d*x**4),x)
```

```
[Out] Timed out
```

3.2315 $\int \frac{1-x^4}{\left(1+x^2+x^4\right)\sqrt[4]{-x^3+x^5}}\,dx$

Optimal. Leaf size=351

$$\frac{\tan^{-1}\left(\frac{3^{7/8}\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^5-x^3}}{3^{3/4}\sqrt{x^5-x^3}-3x^2}\right)}{2^{3/4}\sqrt[8]{3\left(17+12\sqrt{2}\right)}}+\frac{\sqrt[8]{\frac{1}{3}\left(17+12\sqrt{2}\right)}\tan^{-1}\left(\frac{3^{7/8}\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^5-x^3}}{3^{3/4}\sqrt{x^5-x^3}-3x^2}\right)}{2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\frac{\sqrt[8]{3}x^2}{\sqrt{2-\sqrt{2}}}+\frac{\sqrt{x^5-x^3}}{\sqrt[8]{3}\sqrt{2-\sqrt{2}}}}{x\sqrt[4]{x^5-x^3}}\right)}{2^{3/4}\sqrt[8]{3\left(17+12\sqrt{2}\right)}}+\sqrt[8]{\frac{1}{3}}\left(17\right.$$

Rubi [C] time = 0.76, antiderivative size = 167, normalized size of antiderivative = 0.48, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2056, 1586, 6715, 6728, 430, 429}

$$\frac{4\left(\sqrt{3}+3i\right)x\sqrt[4]{1-x^2}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};x^2,-\frac{2x^2}{1-i\sqrt{3}}\right)}{3\left(\sqrt{3}+i\right)\sqrt[4]{x^5-x^3}}+\frac{4\left(-\sqrt{3}+3i\right)x\sqrt[4]{1-x^2}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};x^2,-\frac{2x^2}{1+i\sqrt{3}}\right)}{3\left(-\sqrt{3}+i\right)\sqrt[4]{x^5-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x^4)/((1 + x^2 + x^4)*(-x^3 + x^5)^(1/4)),x]
```

```
[Out] (4*(3*I + Sqrt[3])*x*(1 - x^2)^(1/4)*AppellF1[1/8, -3/4, 1, 9/8, x^2, (-2*x
^2)/(1 - I*Sqrt[3])])/(3*(I + Sqrt[3])*(-x^3 + x^5)^(1/4)) + (4*(3*I - Sqrt
[3])*x*(1 - x^2)^(1/4)*AppellF1[1/8, -3/4, 1, 9/8, x^2, (-2*x^2)/(1 + I*Sqr
t[3])])/(3*(I - Sqrt[3])*(-x^3 + x^5)^(1/4))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
```

```
fQ[x^(m + 1), u, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^4}{\left(1+x^2+x^4\right)\sqrt[4]{-x^3+x^5}}\,dx &= \frac{\left(x^{3/4}\sqrt[4]{-1+x^2}\right)\int \frac{1-x^4}{x^{3/4}\sqrt[4]{-1+x^2}\left(1+x^2+x^4\right)}\,dx}{\sqrt[4]{-x^3+x^5}}\\
&= \frac{\left(x^{3/4}\sqrt[4]{-1+x^2}\right)\int \frac{\left(-1-x^2\right)\left(-1+x^2\right)^{3/4}}{x^{3/4}\left(1+x^2+x^4\right)}\,dx}{\sqrt[4]{-x^3+x^5}}\\
&= \frac{\left(4x^{3/4}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{\left(-1-x^8\right)\left(-1+x^8\right)^{3/4}}{1+x^8+x^{16}}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-x^3+x^5}}\\
&= \frac{\left(4x^{3/4}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \left(\frac{\left(-1+\frac{i}{\sqrt{3}}\right)\left(-1+x^8\right)^{3/4}}{1-i\sqrt{3}+2x^8}+\frac{\left(-1-\frac{i}{\sqrt{3}}\right)\left(-1+x^8\right)^{3/4}}{1+i\sqrt{3}+2x^8}\right)dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-x^3+x^5}}\\
&= \frac{\left(4\left(-3+i\sqrt{3}\right)x^{3/4}\sqrt[4]{-1+x^2}\right)\text{Subst}\left(\int \frac{\left(-1+x^8\right)^{3/4}}{1-i\sqrt{3}+2x^8}\,dx,x,\sqrt[4]{x}\right)}{3\sqrt[4]{-x^3+x^5}} - \frac{\left(4\left(3+i\sqrt{3}\right)x^{3}\right.}{}\\
&= \frac{\left(4\left(-3+i\sqrt{3}\right)x^{3/4}\left(-1+x^2\right)\right)\text{Subst}\left(\int \frac{\left(1-x^8\right)^{3/4}}{1-i\sqrt{3}+2x^8}\,dx,x,\sqrt[4]{x}\right)}{3\left(1-x^2\right)^{3/4}\sqrt[4]{-x^3+x^5}} - \frac{\left(4\left(3+i\sqrt{3}\right)x^{3}\right.}{}\\
&= \frac{4\left(3i+\sqrt{3}\right)x\sqrt[4]{1-x^2}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};x^2,-\frac{2x^2}{1-i\sqrt{3}}\right)}{3\left(i+\sqrt{3}\right)\sqrt[4]{-x^3+x^5}} + \frac{4\left(3i-\sqrt{3}\right)x\sqrt[4]{1-x^2}\,F_1\left(\frac{1}{8}\right.}{3\left(i-\sqrt{3}\right)\sqrt[4]{-}}
\end{aligned}$$

Mathematica [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{\left(1+x^2+x^4\right)\sqrt[4]{-x^3+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^4)/((1 + x^2 + x^4)*(-x^3 + x^5)^(1/4)),x]
[Out] Integrate[(1 - x^4)/((1 + x^2 + x^4)*(-x^3 + x^5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 3.64, size = 377, normalized size = 1.07

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\,3^{7/8}x\sqrt[4]{x^5-x^3}}{\sqrt[8]{17+12\sqrt{2}}\left(3^{3/4}\sqrt{x^5-x^3}-3x^2\right)}\right)}{2^{3/4}\sqrt[8]{3\left(17+12\sqrt{2}\right)}}+\frac{\sqrt[8]{\frac{1}{3}\left(17+12\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt[4]{2}\,3^{7/8}\sqrt[8]{17+12\sqrt{2}}\,x\sqrt[4]{x^5-x^3}}{3^{3/4}\sqrt{x^5-x^3}-3x^2}\right)}{2^{3/4}}+\frac{\tanh^{-1}\left(\frac{3\sqrt[8]{\frac{17}{8748}+\frac{\sqrt{2}}{729}}\,x^2+3^{3/4}\sqrt[8]{\frac{17}{8748}+\frac{\sqrt{2}}{729}}\sqrt{x^5-x^3}}{x\sqrt[4]{x^5-x^3}}\right)}{2^{3/4}\sqrt[8]{3\left(17+12\sqrt{2}\right)}}+\frac{\sqrt[8]{\frac{1}{3}\left(17+12\sqrt{2}\right)}\tanh^{-1}\left(\frac{\frac{\sqrt[8]{\frac{3}{17+12\sqrt{2}}}\,x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^5-x^3}}{\sqrt[4]{2}\sqrt[8]{3\left(17+12\sqrt{2}\right)}}}{x\sqrt[4]{x^5-x^3}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^4)/((1 + x^2 + x^4)*(-x^3 + x^5)^(1/4)),x]
```

```
[Out] ArcTan[(2^(1/4)*3^(7/8)*x*(-x^3 + x^5)^(1/4))/((17 + 12*Sqrt[2])^(1/8)*(-3*
x^2 + 3^(3/4)*Sqrt[-x^3 + x^5]))]/(2^(3/4)*(3*(17 + 12*Sqrt[2]))^(1/8)) + (
((17 + 12*Sqrt[2])/3)^(1/8)*ArcTan[(2^(1/4)*3^(7/8)*(17 + 12*Sqrt[2])^(1/8)
*x*(-x^3 + x^5)^(1/4))/(-3*x^2 + 3^(3/4)*Sqrt[-x^3 + x^5])])/2^(3/4) + ArcT
anh[(3*(17/8748 + Sqrt[2]/729)^(1/8)*x^2 + 3^(3/4)*(17/8748 + Sqrt[2]/729)^
(1/8)*Sqrt[-x^3 + x^5])/(x*(-x^3 + x^5)^(1/4))]/(2^(3/4)*(3*(17 + 12*Sqrt[2
]))^(1/8)) + (((17 + 12*Sqrt[2])/3)^(1/8)*ArcTanh[(((3/(17 + 12*Sqrt[2]))^(
1/8)*x^2)/2^(1/4) + Sqrt[-x^3 + x^5]/(2^(1/4)*(3*(17 + 12*Sqrt[2]))^(1/8)))
/(x*(-x^3 + x^5)^(1/4))])/2^(3/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^4+x^2+1)/(x^5-x^3)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^4-1}{\left(x^5-x^3\right)^{\frac{1}{4}}\left(x^4+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^4+x^2+1)/(x^5-x^3)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-(x^4 - 1)/((x^5 - x^3)^(1/4)*(x^4 + x^2 + 1)), x)
```

maple [C] time = 53.73, size = 2164, normalized size = 6.17

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^4+x^2+1)/(x^5-x^3)^(1/4),x)
```

```
[Out] 1/6*RootOf(_Z^2-RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*ln(-108
*(-16*RootOf(_Z^2-RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*RootO
f(_Z^8+2187)^8*x^4+24*RootOf(_Z^2-RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8
+2187)^2))*RootOf(_Z^8+2187)^8*x^3+16*RootOf(_Z^2-RootOf(_Z^8+2187)*RootOf(
_Z^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+2187)^8*x^2+54*(x^5-x^3)^(1/2)*RootO
f(_Z^8+2187)^5*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^2-RootOf(_Z^8+218
7)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*x+162*(x^5-x^3)^(1/4)*RootOf(_Z^2+Root
Of(_Z^8+2187)^2)*RootOf(_Z^8+2187)^5*x^2+1350*RootOf(_Z^2-RootOf(_Z^8+2187)
*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+2187)^4*x^4+243*RootOf(_Z^8+
2187)^4*RootOf(_Z^2-RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*x^3
+8100*(x^5-x^3)^(3/4)*RootOf(_Z^8+2187)^4-1350*RootOf(_Z^2-RootOf(_Z^8+2187
)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+2187)^4*x^2-24300*(x^5-x^3)
^(1/2)*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^2-RootOf(_Z^8+2187)*RootO
f(_Z^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+2187)*x-72900*(x^5-x^3)^(1/4)*Root
Of(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^8+2187)*x^2-28431*RootOf(_Z^2-RootOf
(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*x^4-56862*RootOf(_Z^2-RootOf(
_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*x^3+39366*(x^5-x^3)^(3/4)+2843
1*RootOf(_Z^2-RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*x^2)/x^2/
(x*RootOf(_Z^8+2187)^4-2*RootOf(_Z^8+2187)^4+81*x)/(x*RootOf(_Z^8+2187)^4-2
*RootOf(_Z^8+2187)^4+135*x+108))-1/6*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*ln(-1
08*(-16*x^4*RootOf(_Z^8+2187)^8*RootOf(_Z^2+RootOf(_Z^8+2187)^2)+24*x^3*Roo
```

```
tOf(_Z^8+2187)^8*RootOf(_Z^2+RootOf(_Z^8+2187)^2)+16*x^2*RootOf(_Z^8+2187)^
8*RootOf(_Z^2+RootOf(_Z^8+2187)^2)+54*(x^5-x^3)^(1/2)*RootOf(_Z^8+2187)^6*R
ootOf(_Z^2+RootOf(_Z^8+2187)^2)*x-162*(x^5-x^3)^(1/4)*RootOf(_Z^8+2187)^6*x
^2-1350*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^8+2187)^4*x^4-243*RootOf
(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^8+2187)^4*x^3+8100*(x^5-x^3)^(3/4)*Roo
tOf(_Z^8+2187)^4+1350*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^8+2187)^4*
x^2+24300*(x^5-x^3)^(1/2)*RootOf(_Z^8+2187)^2*RootOf(_Z^2+RootOf(_Z^8+2187)
^2)*x-72900*(x^5-x^3)^(1/4)*RootOf(_Z^8+2187)^2*x^2-28431*RootOf(_Z^2+RootO
f(_Z^8+2187)^2)*x^4-56862*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*x^3-39366*(x^5-x
^3)^(3/4)+28431*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*x^2)/x^2/(x*RootOf(_Z^8+21
87)^4-2*RootOf(_Z^8+2187)^4-135*x-108)/(x*RootOf(_Z^8+2187)^4-2*RootOf(_Z^8
+2187)^4-81*x))+1/13122*RootOf(_Z^8+2187)^7*RootOf(_Z^2+RootOf(_Z^8+2187)^2
)*RootOf(_Z^2-RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*ln(-108*(
26*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^2-RootOf(_Z^8+2187)*RootOf(_Z
^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+2187)^11*x^4-39*RootOf(_Z^2+RootOf(_Z^
8+2187)^2)*RootOf(_Z^2-RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*
RootOf(_Z^8+2187)^11*x^3-26*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^8+21
87)^11*RootOf(_Z^2-RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*x^2+
243*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^2-RootOf(_Z^8+2187)*RootOf(_
Z^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+2187)^7*x^4-4050*RootOf(_Z^2+RootOf(_
Z^8+2187)^2)*RootOf(_Z^2-RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2)
)*RootOf(_Z^8+2187)^7*x^3-243*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^2-
RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+2187)^7*x^2
-4374*(x^5-x^3)^(1/2)*RootOf(_Z^8+2187)^6*RootOf(_Z^2-RootOf(_Z^8+2187)*Roo
tOf(_Z^2+RootOf(_Z^8+2187)^2))*x-13122*(x^5-x^3)^(1/4)*RootOf(_Z^2+RootOf(_
Z^8+2187)^2)*RootOf(_Z^8+2187)^5*x^2-52488*RootOf(_Z^2+RootOf(_Z^8+2187)^2)
*RootOf(_Z^2-RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^
8+2187)^3*x^4-104976*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^8+2187)^3*R
ootOf(_Z^2-RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*x^3+656100*(
x^5-x^3)^(3/4)*RootOf(_Z^8+2187)^4+52488*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*R
ootOf(_Z^2-RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+
2187)^3*x^2+1968300*(x^5-x^3)^(1/2)*RootOf(_Z^2-RootOf(_Z^8+2187)*RootOf(_Z
^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+2187)^2*x+5904900*(x^5-x^3)^(1/4)*Root
Of(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^8+2187)*x^2+3188646*(x^5-x^3)^(3/4))
/x^2/(x*RootOf(_Z^8+2187)^4-2*RootOf(_Z^8+2187)^4+81*x)/(x*RootOf(_Z^8+2187
)^4-2*RootOf(_Z^8+2187)^4+135*x+108))-1/6*RootOf(_Z^8+2187)*ln(-108*(-16*Ro
otOf(_Z^8+2187)^9*x^4+24*RootOf(_Z^8+2187)^9*x^3+16*RootOf(_Z^8+2187)^9*x^2
-54*(x^5-x^3)^(1/2)*RootOf(_Z^8+2187)^7*x+162*(x^5-x^3)^(1/4)*RootOf(_Z^8+2
187)^6*x^2-1350*RootOf(_Z^8+2187)^5*x^4-243*x^3*RootOf(_Z^8+2187)^5+8100*(x
^5-x^3)^(3/4)*RootOf(_Z^8+2187)^4+1350*RootOf(_Z^8+2187)^5*x^2-24300*(x^5-x
^3)^(1/2)*RootOf(_Z^8+2187)^3*x+72900*(x^5-x^3)^(1/4)*RootOf(_Z^8+2187)^2*x
^2-28431*RootOf(_Z^8+2187)*x^4-56862*RootOf(_Z^8+2187)*x^3-39366*(x^5-x^3)^
(3/4)+28431*RootOf(_Z^8+2187)*x^2)/x^2/(x*RootOf(_Z^8+2187)^4-2*RootOf(_Z^8
+2187)^4-135*x-108)/(x*RootOf(_Z^8+2187)^4-2*RootOf(_Z^8+2187)^4-81*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4-1}{\left(x^5-x^3\right)^{\frac{1}{4}}\left(x^4+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^4+x^2+1)/(x^5-x^3)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/((x^5 - x^3)^(1/4)*(x^4 + x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^4-1}{\left(x^5-x^3\right)^{1/4}\,\left(x^4+x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/((x^5 - x^3)^(1/4)*(x^2 + x^4 + 1)),x)
```

```
[Out] -int((x^4 - 1)/((x^5 - x^3)^(1/4)*(x^2 + x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{x^4\sqrt[4]{x^5-x^3}+x^2\sqrt[4]{x^5-x^3}+\sqrt[4]{x^5-x^3}}\,dx-\int \left(-\frac{1}{x^4\sqrt[4]{x^5-x^3}+x^2\sqrt[4]{x^5-x^3}+\sqrt[4]{x^5-x^3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**4+x**2+1)/(x**5-x**3)**(1/4),x)
```

```
[Out] -Integral(x**4/(x**4*(x**5 - x**3)**(1/4) + x**2*(x**5 - x**3)**(1/4) + (x*
*5 - x**3)**(1/4)), x) - Integral(-1/(x**4*(x**5 - x**3)**(1/4) + x**2*(x**
5 - x**3)**(1/4) + (x**5 - x**3)**(1/4)), x)
```

3.2316 $\int \frac{\sqrt{-b+a^2x^2}}{\sqrt[3]{ax^2+x\sqrt{-b+a^2x^2}}}\,dx$

Optimal. Leaf size=351

$$\frac{ax\sqrt{a^2x^2-b}\left(x\sqrt{a^2x^2-b}+ax^2\right)^{2/3}}{2b}+\frac{\left(5b-4a^2x^2\right)\left(x\sqrt{a^2x^2-b}+ax^2\right)^{2/3}}{8b}-\frac{7b^{2/3}\log\left(\frac{\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{x\sqrt{a^2x^2-b}+ax^2}}{\sqrt[3]{b}}-1\right)}{12\,2^{2/3}a^{2/3}}+$$

Rubi [F] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-b+a^2x^2}}{\sqrt[3]{ax^2+x\sqrt{-b+a^2x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[-b + a^2*x^2]/(a*x^2 + x*Sqrt[-b + a^2*x^2])^(1/3),x]
```

```
[Out] Defer[Int][Sqrt[-b + a^2*x^2]/(a*x^2 + x*Sqrt[-b + a^2*x^2])^(1/3), x]
```

Rubi steps

$$\int \frac{\sqrt{-b+a^2x^2}}{\sqrt[3]{ax^2+x\sqrt{-b+a^2x^2}}}\,dx=\int \frac{\sqrt{-b+a^2x^2}}{\sqrt[3]{ax^2+x\sqrt{-b+a^2x^2}}}\,dx$$

Mathematica [C] time = 24.91, size = 12131, normalized size = 34.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-b + a^2*x^2]/(a*x^2 + x*Sqrt[-b + a^2*x^2])^(1/3),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 3.60, size = 351, normalized size = 1.00

$$\frac{ax\sqrt{a^2x^2-b}\left(x\sqrt{a^2x^2-b}+ax^2\right)^{2/3}}{2b}+\frac{\left(5b-4a^2x^2\right)\left(x\sqrt{a^2x^2-b}+ax^2\right)^{2/3}}{8b}-\frac{7b^{2/3}\log\left(\frac{\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{x\sqrt{a^2x^2-b}+ax^2}}{\sqrt[3]{b}}-1\right)}{12\,2^{2/3}a^{2/3}}+\frac{7b^{2/3}\log\left(\frac{\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{x\sqrt{a^2x^2-b}+ax^2}}{\sqrt[3]{b}}+\frac{2^{2/3}a^{2/3}\left(x\sqrt{a^2x^2-b}+ax^2\right)^{2/3}}{b^{2/3}}+1\right)}{24\,2^{2/3}a^{2/3}}-\frac{7b^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{x\sqrt{a^2x^2-b}+ax^2}}{\sqrt{3}\sqrt[3]{b}}+\frac{1}{\sqrt{3}}\right)}{4\,2^{2/3}\sqrt{3}a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-b + a^2*x^2]/(a*x^2 + x*Sqrt[-b + a^2*x^2])^(1/3),
x]
```

```
[Out] ((5*b - 4*a^2*x^2)*(a*x^2 + x*Sqrt[-b + a^2*x^2])^(2/3))/(8*b) + (a*x*Sqrt[
-b + a^2*x^2]*(a*x^2 + x*Sqrt[-b + a^2*x^2])^(2/3))/(2*b) - (7*b^(2/3)*ArcT
an[1/Sqrt[3] + (2*2^(1/3)*a^(1/3)*(a*x^2 + x*Sqrt[-b + a^2*x^2])^(1/3))/(Sq
rt[3]*b^(1/3))])/(4*2^(2/3)*Sqrt[3]*a^(2/3)) - (7*b^(2/3)*Log[-1 + (2^(1/3)
*a^(1/3)*(a*x^2 + x*Sqrt[-b + a^2*x^2])^(1/3))/b^(1/3)])/(12*2^(2/3)*a^(2/3
)) + (7*b^(2/3)*Log[1 + (2^(1/3)*a^(1/3)*(a*x^2 + x*Sqrt[-b + a^2*x^2])^(1/
3))/b^(1/3) + (2^(2/3)*a^(2/3)*(a*x^2 + x*Sqrt[-b + a^2*x^2])^(2/3))/b^(2/3
)])/(24*2^(2/3)*a^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(a*x^2+x*(a^2*x^2-b)^(1/2))^(1/3),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2 - b}}{\left(ax^2 + \sqrt{a^2x^2 - b}\,x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(a*x^2+x*(a^2*x^2-b)^(1/2))^(1/3),x, algorithm=
"giac")
```

```
[Out] integrate(sqrt(a^2*x^2 - b)/(a*x^2 + sqrt(a^2*x^2 - b)*x)^(1/3), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2 - b}}{\left(a\,x^2 + x\sqrt{a^2x^2 - b}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b)^(1/2)/(a*x^2+x*(a^2*x^2-b)^(1/2))^(1/3),x)
```

```
[Out] int((a^2*x^2-b)^(1/2)/(a*x^2+x*(a^2*x^2-b)^(1/2))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2 - b}}{\left(ax^2 + \sqrt{a^2x^2 - b}\,x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(a*x^2+x*(a^2*x^2-b)^(1/2))^(1/3),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 - b)/(a*x^2 + sqrt(a^2*x^2 - b)*x)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a^2\,x^2-b}}{{\left(x\,\sqrt{a^2\,x^2-b}+a\,x^2\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 - b)^(1/2)/(x*(a^2*x^2 - b)^(1/2) + a*x^2)^(1/3),x)
```

```
[Out] int((a^2*x^2 - b)^(1/2)/(x*(a^2*x^2 - b)^(1/2) + a*x^2)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2 - b}}{\sqrt[3]{x\left(ax + \sqrt{a^2x^2 - b}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b)**(1/2)/(a*x**2+x*(a**2*x**2-b)**(1/2))**(1/3),x)
[Out] Integral(sqrt(a**2*x**2 - b)/(x*(a*x + sqrt(a**2*x**2 - b)))**(1/3), x)
```

3.2317 $\int \frac{x^3}{\sqrt{a+bx+cx^2+bx^3+ax^4}\left(1-x^6\right)}\,dx$

Optimal. Leaf size=352

$$\frac{\sqrt{-2a-2b-c}\tan^{-1}\left(\frac{x\sqrt{-2a-2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2-2\sqrt{a}\,x+\sqrt{a}}\right)}{6(2a+2b+c)}+\frac{\tan^{-1}\left(\frac{x\sqrt{a-b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2-\sqrt{a}\,x+\sqrt{a}}\right)}{3\sqrt{a-b-c}}-\frac{\tan^{-1}\Big(}{}$$

Rubi [F] time = 2.75, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3}{\sqrt{a+bx+cx^2+bx^3+ax^4}\left(1-x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/(Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]*(1 - x^6)),x]
```

```
[Out] -1/6*Defer[Int][1/((-1 + x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x] - De
fer[Int][1/((1 + x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x]/6 + ((1 + I*
Sqrt[3])*Defer[Int][1/((-1 - I*Sqrt[3] + 2*x)*Sqrt[a + b*x + c*x^2 + b*x^3
+ a*x^4]), x])/6 + ((1 - I*Sqrt[3])*Defer[Int][1/((1 - I*Sqrt[3] + 2*x)*Sqr
t[a + b*x + c*x^2 + b*x^3 + a*x^4]), x])/6 + ((1 - I*Sqrt[3])*Defer[Int][1/
((-1 + I*Sqrt[3] + 2*x)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]), x])/6 + ((1
 + I*Sqrt[3])*Defer[Int][1/((1 + I*Sqrt[3] + 2*x)*Sqrt[a + b*x + c*x^2 + b*
x^3 + a*x^4]), x])/6
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{a+bx+cx^2+bx^3+ax^4}\left(1-x^6\right)}\,dx &= \int \left(-\frac{x}{3\left(-1+x^2\right)\sqrt{a+bx+cx^2+bx^3+ax^4}}+\frac{}{6\left(1-x+x^2\right)\sqrt{a}}\right.\\
&= \frac{1}{6}\int \frac{-2+x}{\left(1-x+x^2\right)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx+\frac{1}{6}\int \frac{}{\left(1+x+\right.}\\
&= \frac{1}{6}\int \left(\frac{1+i\sqrt{3}}{\left(-1-i\sqrt{3}+2x\right)\sqrt{a+bx+cx^2+bx^3+ax^4}}+\frac{}{\left(-1+i\sqrt{3}\right.}\right.\\
&= -\left(\frac{1}{6}\int \frac{1}{(-1+x)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx\right)-\frac{1}{6}\int \frac{}{(1+x)\sqrt{}}
\end{aligned}$$

Mathematica [C] time = 7.14, size = 12183, normalized size = 34.61

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/(Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]*(1 - x^6)),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 4.87, size = 352, normalized size = 1.00

$$\frac{\sqrt{-2a-2b-c}\tan^{-1}\left(\frac{x\sqrt{-2a-2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2-2\sqrt{a}\,x+\sqrt{a}}\right)}{6(2a+2b+c)}+\frac{\tan^{-1}\left(\frac{x\sqrt{a-b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2-\sqrt{a}\,x+\sqrt{a}}\right)}{3\sqrt{a-b-c}}-\frac{\tan^{-1}\left(\frac{x\sqrt{a+b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2+\sqrt{a}\,x+\sqrt{a}}\right)}{3\sqrt{a+b-c}}-\frac{\sqrt{-2a+2b-c}\tan^{-1}\left(\frac{x\sqrt{-2a+2b-c}}{-\sqrt{ax^4+a+bx^3+bx+cx^2}+\sqrt{a}\,x^2+2\sqrt{a}\,x+\sqrt{a}}\right)}{6(2a-2b+c)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]*(1 - x^6)),x]
```

```
[Out] (Sqrt[-2*a - 2*b - c]*ArcTan[(Sqrt[-2*a - 2*b - c]*x)/(Sqrt[a] - 2*Sqrt[a]*
x + Sqrt[a]*x^2 - Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(6*(2*a + 2*b +
c)) + ArcTan[(Sqrt[a - b - c]*x)/(Sqrt[a] - Sqrt[a]*x + Sqrt[a]*x^2 - Sqrt[
a + b*x + c*x^2 + b*x^3 + a*x^4])]/(3*Sqrt[a - b - c]) - ArcTan[(Sqrt[a + b
 - c]*x)/(Sqrt[a] + Sqrt[a]*x + Sqrt[a]*x^2 - Sqrt[a + b*x + c*x^2 + b*x^3
+ a*x^4])]/(3*Sqrt[a + b - c]) - (Sqrt[-2*a + 2*b - c]*ArcTan[(Sqrt[-2*a +
2*b - c]*x)/(Sqrt[a] + 2*Sqrt[a]*x + Sqrt[a]*x^2 - Sqrt[a + b*x + c*x^2 + b
*x^3 + a*x^4])])/(6*(2*a - 2*b + c))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2)/(-x^6+1),x, algorithm="fricas
")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2)/(-x^6+1),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a\,x^4+b\,x^3+c\,x^2+bx+a}\,\left(-x^6+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2)/(-x^6+1),x)
```

```
[Out] int(x^3/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2)/(-x^6+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{\left(x^6-1\right)\sqrt{ax^4+bx^3+cx^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2)/(-x^6+1),x, algorithm="maxima
")
```

```
[Out] -integrate(x^3/((x^6 - 1)*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^3}{\left(x^6-1\right)\,\sqrt{a\,x^4+b\,x^3+c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3/((x^6 - 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)),x)
```

```
[Out] -int(x^3/((x^6 - 1)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{x^6\sqrt{ax^4 + a + bx^3 + bx + cx^2} - \sqrt{ax^4 + a + bx^3 + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x**4+b*x**3+c*x**2+b*x+a)**(1/2)/(-x**6+1),x)
```

```
[Out] -Integral(x**3/(x**6*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) - sqrt(a*x**4
 + a + b*x**3 + b*x + c*x**2)), x)
```

3.2318 $\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^6+a(q+px^3)^6\right)}{x^9}\,dx$

Optimal. Leaf size=352

$$\frac{1}{8}\log(x)\left(5ap^4q^4+8bpq\right)+\frac{1}{8}\left(-5ap^4q^4-8bpq\right)\log\left(\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+px^3+q\right)+\frac{\sqrt{p^2x^6+2pqx^3-}}{}$$

Rubi [F] time = 1.76, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^6+a\left(q+px^3\right)^6\right)}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*x^6 + a*
(q + p*x^3)^6))/x^9,x]
```

```
[Out] 2*p*(b + 5*a*p^2*q^4)*Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6
], x] - a*q^7*Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x^9, x
] - 4*a*p*q^6*Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]/x^6, x
] - q*(b + 3*a*p^2*q^4)*Defer[Int][Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x
^6]/x^3, x] + 25*a*p^4*q^3*Defer[Int][x^3*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3
+ p^2*x^6], x] + 24*a*p^5*q^2*Defer[Int][x^6*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x
^3 + p^2*x^6], x] + 11*a*p^6*q*Defer[Int][x^9*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*
x^3 + p^2*x^6], x] + 2*a*p^7*Defer[Int][x^12*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x
^3 + p^2*x^6], x]
```

Rubi steps

$$\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^6+a\left(q+px^3\right)^6\right)}{x^9}\,dx = \int \left(2p\left(b+5ap^2q^4\right)\sqrt{q^2-2pqx^2+2pqx^3}\right.$$

$$= \left(2ap^7\right)\int x^{12}\sqrt{q^2-2pqx^2+2pqx^3+p^2x}$$

Mathematica [F] time = 1.25, size = 0, normalized size = 0.00

$$\int \frac{(-q+2px^3)\sqrt{q^2-2pqx^2+2pqx^3+p^2x^6}\left(bx^6+a\left(q+px^3\right)^6\right)}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*x^
6 + a*(q + p*x^3)^6))/x^9,x]
```

```
[Out] Integrate[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(b*x^
6 + a*(q + p*x^3)^6))/x^9, x]
```

IntegrateAlgebraic [A] time = 0.55, size = 352, normalized size = 1.00

$$\frac{1}{8}\log(x)\left(5ap^4q^4+8bpq\right)+\frac{1}{8}\left(-5ap^4q^4-8bpq\right)\log\left(\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}+px^3+q\right)+\frac{\sqrt{p^2x^6+2pqx^3-2pqx^2+q^2}\left(6ap^7x^{21}+42ap^6qx^{18}-2ap^6qx^{17}+126ap^5q^2x^{15}-10ap^5q^2x^{14}-5ap^5q^2x^{13}+210ap^4q^3x^{12}-20ap^4q^3x^{11}-15ap^4q^3x^{10}-15ap^4q^3x^9+210ap^3q^4x^9-20ap^3q^4x^8-15ap^3q^4x^7-15ap^3q^4x^6+126ap^2q^5x^6-10ap^2q^5x^5-5ap^2q^5x^4+42apq^6x^3-2apq^6x^2+6aq^7+24bpx^9+24bqx^6\right)}{48x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-q + 2*p*x^3)*Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x
^6]*(b*x^6 + a*(q + p*x^3)^6))/x^9,x]
```

```
[Out] (Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]*(6*a*q^7 - 2*a*p*q^6*x^2 + 42*
a*p*q^6*x^3 - 5*a*p^2*q^5*x^4 - 10*a*p^2*q^5*x^5 + 24*b*q*x^6 - 15*a*p^3*q^
4*x^6 + 126*a*p^2*q^5*x^6 - 15*a*p^3*q^4*x^7 - 20*a*p^3*q^4*x^8 + 24*b*p*x^
9 - 15*a*p^4*q^3*x^9 + 210*a*p^3*q^4*x^9 - 15*a*p^4*q^3*x^10 - 20*a*p^4*q^3
*x^11 + 210*a*p^4*q^3*x^12 - 5*a*p^5*q^2*x^13 - 10*a*p^5*q^2*x^14 + 126*a*p
^5*q^2*x^15 - 2*a*p^6*q*x^17 + 42*a*p^6*q*x^18 + 6*a*p^7*x^21))/(48*x^8) +
((8*b*p*q + 5*a*p^4*q^4)*Log[x])/8 + ((-8*b*p*q - 5*a*p^4*q^4)*Log[q + p*x^
3 + Sqrt[q^2 - 2*p*q*x^2 + 2*p*q*x^3 + p^2*x^6]])/8
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^6+a*(p*x
^3+q)^6)/x^9,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6 + 2\,pqx^3 - 2\,pqx^2 + q^2}\left(\left(px^3 + q\right)^6 a + bx^6\right)\left(2\,px^3 - q\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^6+a*(p*x
^3+q)^6)/x^9,x, algorithm="giac")
```

```
[Out] integrate(sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)*((p*x^3 + q)^6*a + b*
x^6)*(2*p*x^3 - q)/x^9, x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\left(2p\,x^3 - q\right)\sqrt{p^2x^6 + 2pq\,x^3 - 2pq\,x^2 + q^2}\,\left(b\,x^6 + a\left(p\,x^3 + q\right)^6\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^6+a*(p*x^3+q)^
6)/x^9,x)
```

```
[Out] int((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^6+a*(p*x^3+q)^
6)/x^9,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{p^2x^6 + 2\,pqx^3 - 2\,pqx^2 + q^2}\left(\left(px^3 + q\right)^6 a + bx^6\right)\left(2\,px^3 - q\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x^3-q)*(p^2*x^6+2*p*q*x^3-2*p*q*x^2+q^2)^(1/2)*(b*x^6+a*(p*x
^3+q)^6)/x^9,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(p^2*x^6 + 2*p*q*x^3 - 2*p*q*x^2 + q^2)*((p*x^3 + q)^6*a + b*
x^6)*(2*p*x^3 - q)/x^9, x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q - 2*p*x^3)*(a*(q + p*x^3)^6 + b*x^6)*(p^2*x^6 + q^2 - 2*p*q*x^2 +
2*p*q*x^3)^(1/2))/x^9,x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2px^3 - q)\sqrt{p^2x^6 + 2pqx^3 - 2pqx^2 + q^2}\,(ap^6x^{18} + 6ap^5qx^{15} + 15ap^4q^2x^{12} + 20ap^3q^3x^9 + 15ap^2q^4x^6 + 6apq^5x^3 + aq^6 + bx^6)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*p*x**3-q)*(p**2*x**6+2*p*q*x**3-2*p*q*x**2+q**2)**(1/2)*(b*x**
6+a*(p*x**3+q)**6)/x**9,x)
```

```
[Out] Integral((2*p*x**3 - q)*sqrt(p**2*x**6 + 2*p*q*x**3 - 2*p*q*x**2 + q**2)*(a
*p**6*x**18 + 6*a*p**5*q*x**15 + 15*a*p**4*q**2*x**12 + 20*a*p**3*q**3*x**9
 + 15*a*p**2*q**4*x**6 + 6*a*p*q**5*x**3 + a*q**6 + b*x**6)/x**9, x)
```

3.2319 $\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^{12}+a\left(q+px^3\right)^6\right)}{x^{17}}\,dx$

Optimal. Leaf size=352

$$\frac{1}{4}\log(x)\left(5ap^4q^4+8bpq\right)+\frac{1}{8}\left(-5ap^4q^4-8bpq\right)\log\left(\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}+px^3+q\right)+\frac{\sqrt{p^2x^6-2pqx^4}}{}$$

Rubi [F] time = 1.82, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^{12}+a\left(q+px^3\right)^6\right)}{x^{17}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(b*x^12 + a
*(q + p*x^3)^6))/x^17,x]
```

```
[Out] -2*a*q^7*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^17, x] -
11*a*p*q^6*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^14, x]
- 24*a*p^2*q^5*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x^11,
 x] - 25*a*p^3*q^4*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]/x
^8, x] - 2*q*(b + 5*a*p^4*q^2)*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4
+ p^2*x^6]/x^5, x] + p*(b + 3*a*p^4*q^2)*Defer[Int][Sqrt[q^2 + 2*p*q*x^3 -
2*p*q*x^4 + p^2*x^6]/x^2, x] + 4*a*p^6*q*Defer[Int][x*Sqrt[q^2 + 2*p*q*x^3
- 2*p*q*x^4 + p^2*x^6], x] + a*p^7*Defer[Int][x^4*Sqrt[q^2 + 2*p*q*x^3 - 2*
p*q*x^4 + p^2*x^6], x]
```

Rubi steps

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^{12}+a\left(q+px^3\right)^6\right)}{x^{17}}\,dx=\int\left(-\frac{2aq^7\sqrt{q^2+2pqx^3-2pqx^4+p^2}}{x^{17}}\right.$$

$$=\left(ap^7\right)\int x^4\sqrt{q^2+2pqx^3-2pqx^4+p^2}$$

Mathematica [F] time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{\left(-2q+px^3\right)\sqrt{q^2+2pqx^3-2pqx^4+p^2x^6}\left(bx^{12}+a\left(q+px^3\right)^6\right)}{x^{17}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(b*x^
12 + a*(q + p*x^3)^6))/x^17,x]
```

```
[Out] Integrate[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(b*x^
12 + a*(q + p*x^3)^6))/x^17, x]
```

IntegrateAlgebraic [A] time = 0.61, size = 352, normalized size = 1.00

$$\frac{1}{4}\log(x)\left(5ap^4q^4+8bpq\right)+\frac{1}{8}\left(-5ap^4q^4-8bpq\right)\log\left(\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}+px^3+q\right)+\frac{\sqrt{p^2x^6-2pqx^4+2pqx^3+q^2}\left(6ap^7x^{21}-2ap^6qx^{19}+42ap^6qx^{18}-5ap^5q^2x^{17}-10ap^5q^2x^{16}+126ap^5q^2x^{15}-15ap^4q^3x^{15}-15ap^4q^3x^{14}-20ap^4q^3x^{13}+210ap^4q^3x^{12}-15ap^3q^4x^{12}-15ap^3q^4x^{11}-20ap^3q^4x^{10}+210ap^3q^4x^9-5ap^2q^5x^8-10ap^2q^5x^7+126ap^2q^5x^6-2apq^6x^4+42apq^6x^3+6aq^7+24bpx^{15}+24bqx^{12}\right)}{48x^{16}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*q + p*x^3)*Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x
^6]*(b*x^12 + a*(q + p*x^3)^6))/x^17,x]
```

```
[Out] (Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]*(6*a*q^7 + 42*a*p*q^6*x^3 - 2*
a*p*q^6*x^4 + 126*a*p^2*q^5*x^6 - 10*a*p^2*q^5*x^7 - 5*a*p^2*q^5*x^8 + 210*
a*p^3*q^4*x^9 - 20*a*p^3*q^4*x^10 - 15*a*p^3*q^4*x^11 + 24*b*q*x^12 + 210*a
*p^4*q^3*x^12 - 15*a*p^3*q^4*x^12 - 20*a*p^4*q^3*x^13 - 15*a*p^4*q^3*x^14 +
 24*b*p*x^15 + 126*a*p^5*q^2*x^15 - 15*a*p^4*q^3*x^15 - 10*a*p^5*q^2*x^16 -
 5*a*p^5*q^2*x^17 + 42*a*p^6*q*x^18 - 2*a*p^6*q*x^19 + 6*a*p^7*x^21))/(48*x
^16) + ((8*b*p*q + 5*a*p^4*q^4)*Log[x])/4 + ((-8*b*p*q - 5*a*p^4*q^4)*Log[q
 + p*x^3 + Sqrt[q^2 + 2*p*q*x^3 - 2*p*q*x^4 + p^2*x^6]])/8
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(b*x^12+a*(p*
x^3+q)^6)/x^17,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{12} + \left(px^3 + q\right)^6 a\right)\sqrt{p^2x^6 - 2\,pqx^4 + 2\,pqx^3 + q^2}\left(px^3 - 2\,q\right)}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(b*x^12+a*(p*
x^3+q)^6)/x^17,x, algorithm="giac")
```

```
[Out] integrate((b*x^12 + (p*x^3 + q)^6*a)*sqrt(p^2*x^6 - 2*p*q*x^4 + 2*p*q*x^3 +
 q^2)*(p*x^3 - 2*q)/x^17, x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(p\,x^{3}-2q\right)\sqrt{p^{2}x^{6}-2x^{4}pq+2pq\,x^{3}+q^{2}}\,\left(b\,x^{12}+a\left(p\,x^{3}+q\right)^{6}\right)}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(b*x^12+a*(p*x^3+q)
^6)/x^17,x)
```

```
[Out] int((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(b*x^12+a*(p*x^3+q)
^6)/x^17,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{12} + \left(px^3 + q\right)^6 a\right)\sqrt{p^2x^6 - 2\,pqx^4 + 2\,pqx^3 + q^2}\left(px^3 - 2\,q\right)}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^3-2*q)*(p^2*x^6-2*p*q*x^4+2*p*q*x^3+q^2)^(1/2)*(b*x^12+a*(p*
x^3+q)^6)/x^17,x, algorithm="maxima")
```

```
[Out] integrate((b*x^12 + (p*x^3 + q)^6*a)*sqrt(p^2*x^6 - 2*p*q*x^4 + 2*p*q*x^3 +
 q^2)*(p*x^3 - 2*q)/x^17, x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*(q + p*x^3)^6 + b*x^12)*(2*q - p*x^3)*(p^2*x^6 + q^2 + 2*p*q*x^3 -
 2*p*q*x^4)^(1/2))/x^17,x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(px^{3} - 2q\right)\sqrt{p^{2}x^{6} - 2pqx^{4} + 2pqx^{3} + q^{2}}\left(ap^{6}x^{18} + 6ap^{5}qx^{15} + 15ap^{4}q^{2}x^{12} + 20ap^{3}q^{3}x^{9} + 15ap^{2}q^{4}x^{6} + 6apq^{5}x^{3} + aq^{6} + bx^{12}\right)}{x^{17}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**3-2*q)*(p**2*x**6-2*p*q*x**4+2*p*q*x**3+q**2)**(1/2)*(b*x**
12+a*(p*x**3+q)**6)/x**17,x)
```

```
[Out] Integral((p*x**3 - 2*q)*sqrt(p**2*x**6 - 2*p*q*x**4 + 2*p*q*x**3 + q**2)*(a
*p**6*x**18 + 6*a*p**5*q*x**15 + 15*a*p**4*q**2*x**12 + 20*a*p**3*q**3*x**9
 + 15*a*p**2*q**4*x**6 + 6*a*p*q**5*x**3 + a*q**6 + b*x**12)/x**17, x)
```

3.2320 $\int \frac{-3b+ax}{\sqrt[3]{b^2-a^2x^2}\left(3b^2+a^2x^2\right)}\,dx$

Optimal. Leaf size=354

$$\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-a^2x^2}\right)}{2^{2/3}ab^{2/3}}-\frac{\log\left(b^{2/3}\left(b^2-a^2x^2\right)^{2/3}\right)}{2\,2^{2/3}ab^{2/3}}-\frac{\log\left(2\sqrt[3]{b}\sqrt[3]{b^2-a^2x^2}+2^{2/3}ax+2^{2/3}b\right)}{2^{2/3}ab^{2/3}}+\frac{\log\left(-2^{2/3}a\sqrt[3]{b}\,x\sqrt[3]{b^2-\right.}{}$$

Rubi [A] time = 0.08, antiderivative size = 229, normalized size of antiderivative = 0.65, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {1009, 1008}

$$\frac{\sqrt[3]{1-\frac{a^2x^2}{b^2}}\log\left(a^2x^2+3b^2\right)}{2\,2^{2/3}a\sqrt[3]{b^2-a^2x^2}}-\frac{3\sqrt[3]{1-\frac{a^2x^2}{b^2}}\log\left(\left(\frac{ax}{b}+1\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{1-\frac{ax}{b}}\right)}{2\,2^{2/3}a\sqrt[3]{b^2-a^2x^2}}+\frac{\sqrt{3}\sqrt[3]{1-\frac{a^2x^2}{b^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(\frac{ax}{b}+1\right)^{2/3}}{\sqrt{3}\sqrt[3]{1-\frac{ax}{b}}}\right)}{2^{2/3}a\sqrt[3]{b^2-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-3*b + a*x)/((b^2 - a^2*x^2)^(1/3)*(3*b^2 + a^2*x^2)),x]
```

```
[Out] (Sqrt[3]*(1 - (a^2*x^2)/b^2)^(1/3)*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 + (a*x)/b
)^(2/3))/(Sqrt[3]*(1 - (a*x)/b)^(1/3))])/(2^(2/3)*a*(b^2 - a^2*x^2)^(1/3))
+ ((1 - (a^2*x^2)/b^2)^(1/3)*Log[3*b^2 + a^2*x^2])/(2*2^(2/3)*a*(b^2 - a^2*
x^2)^(1/3)) - (3*(1 - (a^2*x^2)/b^2)^(1/3)*Log[2^(1/3)*(1 - (a*x)/b)^(1/3)
+ (1 + (a*x)/b)^(2/3)])/(2*2^(2/3)*a*(b^2 - a^2*x^2)^(1/3))
```

Rule 1008

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[(Sqrt[3]*h*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^
(2/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))])/(2^(2/3)*a^(1/3)*f), x] + (-Simp[(
3*h*Log[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2^(5/3)*a^
(1/3)*f), x] + Simp[(h*Log[d + f*x^2])/(2^(5/3)*a^(1/3)*f), x]) /; FreeQ[{a
, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && G
tQ[a, 0]
```

Rule 1009

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Dist[(1 + (c*x^2)/a)^(1/3)/(a + c*x^2)^(1/3), Int[(g + h*x)/
((1 + (c*x^2)/a)^(1/3)*(d + f*x^2)), x], x] /; FreeQ[{a, c, d, f, g, h}, x]
 && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{-3b+ax}{\sqrt[3]{b^2-a^2x^2}\left(3b^2+a^2x^2\right)}\,dx=\frac{\sqrt[3]{1-\frac{a^2x^2}{b^2}}\int \frac{-3b+ax}{\left(3b^2+a^2x^2\right)\sqrt[3]{1-\frac{a^2x^2}{b^2}}}\,dx}{\sqrt[3]{b^2-a^2x^2}}$$

$$=\frac{\sqrt{3}\sqrt[3]{1-\frac{a^2x^2}{b^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1+\frac{ax}{b}\right)^{2/3}}{\sqrt{3}\sqrt[3]{1-\frac{ax}{b}}}\right)}{2^{2/3}a\sqrt[3]{b^2-a^2x^2}}+\frac{\sqrt[3]{1-\frac{a^2x^2}{b^2}}\log\left(3b^2+a^2x^2\right)}{2\,2^{2/3}a\sqrt[3]{b^2-a^2x^2}}-\frac{3\sqrt[3]{1-}}{}$$

Mathematica [C] time = 0.47, size = 250, normalized size = 0.71

$$\frac{x\left(ax\sqrt[3]{1-\frac{a^2x^2}{b^2}}F_1\left(1;\frac{1}{3},1;2;\frac{a^2x^2}{b^2},-\frac{a^2x^2}{3b^2}\right)-\frac{162b^5F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{a^2x^2}{b^2},-\frac{a^2x^2}{3b^2}\right)}{(a^2x^2+3b^2)\left(9b^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{a^2x^2}{b^2},-\frac{a^2x^2}{3b^2}\right)+2a^2x^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{a^2x^2}{b^2},-\frac{a^2x^2}{3b^2}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{a^2x^2}{b^2},-\frac{a^2x^2}{3b^2}\right)\right)\right)}\right)}{6b^2\sqrt[3]{b^2-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-3*b + a*x)/((b^2 - a^2*x^2)^(1/3)*(3*b^2 + a^2*x^2)),x]
```

```
[Out] (x*(a*x*(1 - (a^2*x^2)/b^2)^(1/3)*AppellF1[1, 1/3, 1, 2, (a^2*x^2)/b^2, -1/
3*(a^2*x^2)/b^2] - (162*b^5*AppellF1[1/2, 1/3, 1, 3/2, (a^2*x^2)/b^2, -1/3*
(a^2*x^2)/b^2])/((3*b^2 + a^2*x^2)*(9*b^2*AppellF1[1/2, 1/3, 1, 3/2, (a^2*x
^2)/b^2, -1/3*(a^2*x^2)/b^2] + 2*a^2*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (a^2*
x^2)/b^2, -1/3*(a^2*x^2)/b^2] + AppellF1[3/2, 4/3, 1, 5/2, (a^2*x^2)/b^2, -
1/3*(a^2*x^2)/b^2])))))/(6*b^2*(b^2 - a^2*x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 0.60, size = 354, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-a^2x^2}\right)}{2^{2/3}ab^{2/3}}-\frac{\log\left(b^{2/3}\left(b^2-a^2x^2\right)^{2/3}\right)}{2\,2^{2/3}ab^{2/3}}-\frac{\log\left(2\sqrt[3]{b}\sqrt[3]{b^2-a^2x^2}+2^{2/3}ax+2^{2/3}b\right)}{2^{2/3}ab^{2/3}}+\frac{\log\left(-2^{2/3}a\sqrt[3]{b}x\sqrt[3]{b^2-a^2x^2}-2^{2/3}b^{4/3}\sqrt[3]{b^2-a^2x^2}+2b^{2/3}\left(b^2-a^2x^2\right)^{2/3}+\sqrt[3]{2}a^2x^2+2\sqrt[3]{2}abx+\sqrt[3]{2}b^2\right)}{2\,2^{2/3}ab^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{b^2-a^2x^2}}{\sqrt[3]{b}\sqrt[3]{b^2-a^2x^2}-2^{2/3}ax-2^{2/3}b}\right)}{2^{2/3}ab^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-3*b + a*x)/((b^2 - a^2*x^2)^(1/3)*(3*b^2 + a^2*x^2)),x
]
```

```
[Out] -((Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*(b^2 - a^2*x^2)^(1/3))/(-(2^(2/3)*b) - 2
^(2/3)*a*x + b^(1/3)*(b^2 - a^2*x^2)^(1/3))])/(2^(2/3)*a*b^(2/3))) + Log[b^
(1/3)*(b^2 - a^2*x^2)^(1/3)]/(2^(2/3)*a*b^(2/3)) - Log[b^(2/3)*(b^2 - a^2*x
^2)^(2/3)]/(2*2^(2/3)*a*b^(2/3)) - Log[2^(2/3)*b + 2^(2/3)*a*x + 2*b^(1/3)*
(b^2 - a^2*x^2)^(1/3)]/(2^(2/3)*a*b^(2/3)) + Log[2^(1/3)*b^2 + 2*2^(1/3)*a*
b*x + 2^(1/3)*a^2*x^2 - 2^(2/3)*b^(4/3)*(b^2 - a^2*x^2)^(1/3) - 2^(2/3)*a*b
^(1/3)*x*(b^2 - a^2*x^2)^(1/3) + 2*b^(2/3)*(b^2 - a^2*x^2)^(2/3)]/(2*2^(2/3
)*a*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-3*b)/(-a^2*x^2+b^2)^(1/3)/(a^2*x^2+3*b^2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-3\,b}{\left(a^2x^2+3\,b^2\right)\left(-a^2x^2+b^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-3*b)/(-a^2*x^2+b^2)^(1/3)/(a^2*x^2+3*b^2),x, algorithm="giac
")
```

```
[Out] integrate((a*x - 3*b)/((a^2*x^2 + 3*b^2)*(-a^2*x^2 + b^2)^(1/3)), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{ax-3b}{\left(-a^2x^2+b^2\right)^{\frac{1}{3}}\left(a^2x^2+3b^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-3*b)/(-a^2*x^2+b^2)^(1/3)/(a^2*x^2+3*b^2),x)
```

```
[Out] int((a*x-3*b)/(-a^2*x^2+b^2)^(1/3)/(a^2*x^2+3*b^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-3\,b}{\left(a^2x^2+3\,b^2\right)\left(-a^2x^2+b^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-3*b)/(-a^2*x^2+b^2)^(1/3)/(a^2*x^2+3*b^2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*x - 3*b)/((a^2*x^2 + 3*b^2)*(-a^2*x^2 + b^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{3\,b-a\,x}{\left(b^2-a^2\,x^2\right)^{1/3}\,\left(a^2\,x^2+3\,b^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*b - a*x)/((b^2 - a^2*x^2)^(1/3)*(3*b^2 + a^2*x^2)),x)
```

```
[Out] int(-(3*b - a*x)/((b^2 - a^2*x^2)^(1/3)*(3*b^2 + a^2*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-3b}{\sqrt[3]{-\left(ax-b\right)\left(ax+b\right)}\,\left(a^2x^2+3b^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-3*b)/(-a**2*x**2+b**2)**(1/3)/(a**2*x**2+3*b**2),x)
```

```
[Out] Integral((a*x - 3*b)/((-(a*x - b)*(a*x + b))**(1/3)*(a**2*x**2 + 3*b**2)),
x)
```

3.2321 $\int \frac{x^3(4ab-3(a+b)x+2x^2)}{(x^2(-a+x)(-b+x))^{2/3}(-abd+(a+b)dx-dx^2+x^4)}\,dx$

Optimal. Leaf size=355

$$-\frac{\log\left(a^2b^2d-2a^2bdx+a^2dx^2-2ab^2dx+\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}\left(abd^{2/3}-ad^{2/3}x-bd^{2/3}x+d^{2/3}x^2\right)+4a\right.}{2\sqrt[3]{d}}$$

Rubi [F] time = 27.69, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3\left(4ab-3(a+b)x+2x^2\right)}{\left(x^2(-a+x)(-b+x)\right)^{2/3}\left(-abd+(a+b)dx-dx^2+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*(4*a*b - 3*(a + b)*x + 2*x^2))/((x^2*(-a + x)*(-b + x))^(2/3)*(-(a
*b*d) + (a + b)*d*x - d*x^2 + x^4)),x]
```

```
[Out] (3*x^2*(1 - x/a)^(2/3)*(1 - x/b)^(2/3)*AppellF1[2/3, 2/3, 2/3, 5/3, x/a, x/
b])/((a - x)*(b - x)*x^2)^(2/3) + (6*(a + b)*d*x^(4/3)*(-a + x)^(2/3)*(-b +
 x)^(2/3)*Defer[Subst][Defer[Int][x^4/((-a + x^3)^(2/3)*(-b + x^3)^(2/3)*(a
*b*d - a*(1 + b/a)*d*x^3 + d*x^6 - x^12)), x], x, x^(1/3)])/((a - x)*(b - x
)*x^2)^(2/3) - (6*(2*a*b + d)*x^(4/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[S
ubst][Defer[Int][x^7/((-a + x^3)^(2/3)*(-b + x^3)^(2/3)*(a*b*d - a*(1 + b/a
)*d*x^3 + d*x^6 - x^12)), x], x, x^(1/3)])/((a - x)*(b - x)*x^2)^(2/3) + (9
*(a + b)*x^(4/3)*(-a + x)^(2/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][x^10
/((-a + x^3)^(2/3)*(-b + x^3)^(2/3)*(a*b*d - a*(1 + b/a)*d*x^3 + d*x^6 - x^
12)), x], x, x^(1/3)])/((a - x)*(b - x)*x^2)^(2/3) + (6*a*b*d*x^(4/3)*(-a +
 x)^(2/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][x/((-a + x^3)^(2/3)*(-b +
x^3)^(2/3)*(-(a*b*d) + a*(1 + b/a)*d*x^3 - d*x^6 + x^12)), x], x, x^(1/3)])
/((a - x)*(b - x)*x^2)^(2/3)
```

Rubi steps

$$\int \frac{x^3 \left(4ab - 3(a+b)x + 2x^2\right)}{\left(x^2(-a+x)(-b+x)\right)^{2/3} \left(-abd + (a+b)dx - dx^2 + x^4\right)} dx = \frac{\left(x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right) \int \frac{x^{5/3}\left(4ab-3(a+\cdots\right.}{(-a+x)^{2/3}(-b+x)^{2/3}\left(-abd\cdots\right.}}{\left(x^2(-a+x)(-b+x)\right)^{2/3}}$$

$$= \frac{\left(3x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right) \text{Subst}\left(\int \frac{\cdots}{\left(-a+x^3\right)^{2/3}(-\cdots}\right.}{\left(x^2(-a+x)(-b+\cdots\right.}$$

$$= \frac{\left(3x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right) \text{Subst}\left(\int \left(\frac{2x\cdots}{\left(-a+x^3\right)^{2/3}(\cdots}\right.\right.}{\left(x^2(-\cdots\right.}$$

$$= \frac{\left(3x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right) \text{Subst}\left(\int \frac{x\left(2abd-2(\cdots\right.}{\left(-a+x^3\right)^{2/3}(-\cdots}\right.}{\left(x^2(-a+x)(-b+\cdots\right.}$$

$$= \frac{\left(3x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right) \text{Subst}\left(\int \left(\frac{\cdots}{\left(-a+x^3\right)^{2/3}(\cdots}\right.\right.}{\cdots}$$

$$= \frac{\left(9(a+b)x^{4/3}(-a+x)^{2/3}(-b+x)^{2/3}\right) \text{Subst}\left(\int \frac{\cdots}{(-a+\cdots}\right.}{\left(x^2(-a+x)(-\cdots\right.}$$

$$= \frac{3x^2 \left(1-\frac{x}{a}\right)^{2/3} \left(1-\frac{x}{b}\right)^{2/3} F_1\left(\frac{2}{3}; \frac{2}{3}, \frac{2}{3}; \frac{5}{3}; \frac{x}{a}, \frac{x}{b}\right)}{\left((a-x)(b-x)x^2\right)^{2/3}} + \frac{\left(9(\cdots\right.}{\cdots}$$

Mathematica [F] time = 3.48, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(4ab - 3(a+b)x + 2x^2\right)}{\left(x^2(-a+x)(-b+x)\right)^{2/3} \left(-abd + (a+b)dx - dx^2 + x^4\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(4*a*b - 3*(a + b)*x + 2*x^2))/((x^2*(-a + x)*(-b + x))^(2/3
)*(-(a*b*d) + (a + b)*d*x - d*x^2 + x^4)),x]
```

```
[Out] Integrate[(x^3*(4*a*b - 3*(a + b)*x + 2*x^2))/((x^2*(-a + x)*(-b + x))^(2/3
)*(-(a*b*d) + (a + b)*d*x - d*x^2 + x^4)), x]
```

IntegrateAlgebraic [A] time = 1.28, size = 355, normalized size = 1.00

$$-\frac{\log \left(a^2b^2d - 2a^2bdx + a^2dx^2 - 2ab^2dx + \left(x^3(-a-b) + abx^2 + x^4\right)^{2/3} \left(abd^{2/3} - ad^{2/3}x - bd^{2/3}x + d^{2/3}x^2\right) + 4abdx^2 + \sqrt[3]{d} \left(x^3(-a-b) + abx^2 + x^4\right)^{4/3} - 2adx^3 + b^2dx^2 - 2bdx^3 + dx^4\right)}{2\sqrt[3]{d}} + \frac{\log \left(\sqrt[6]{d} \left(x^3(-a-b) + abx^2 + x^4\right)^{2/3} - ab\sqrt[6]{d} + a\sqrt[6]{d}x + b\sqrt[6]{d}x - \sqrt[6]{d}x^2\right)}{\sqrt[3]{d}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt{3}\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}}{x\left(-2a\sqrt[3]{d}-2b\sqrt[3]{d}\right)+2ab\sqrt[3]{d}+\left(x^3(-a-b)+abx^2+x^4\right)^{2/3}+2\sqrt[3]{d}x^2}\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(4*a*b - 3*(a + b)*x + 2*x^2))/((x^2*(-a + x)*(-b +
 x))^(2/3)*(-(a*b*d) + (a + b)*d*x - d*x^2 + x^4)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(a*b*x^2 + (-a - b)*x^3 + x^4)^(2/3))/(2*a*b*d^(1/
3) + (-2*a*d^(1/3) - 2*b*d^(1/3))*x + 2*d^(1/3)*x^2 + (a*b*x^2 + (-a - b)*x
```

```
^3 + x^4)^(2/3))])/d^(1/3) + Log[-(a*b*Sqrt[d]) + a*Sqrt[d]*x + b*Sqrt[d]*x
 - Sqrt[d]*x^2 + d^(1/6)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(2/3)]/d^(1/3) - Lo
g[a^2*b^2*d - 2*a^2*b*d*x - 2*a*b^2*d*x + a^2*d*x^2 + 4*a*b*d*x^2 + b^2*d*x
^2 - 2*a*d*x^3 - 2*b*d*x^3 + d*x^4 + (a*b*d^(2/3) - a*d^(2/3)*x - b*d^(2/3)
*x + d^(2/3)*x^2)*(a*b*x^2 + (-a - b)*x^3 + x^4)^(2/3) + d^(1/3)*(a*b*x^2 +
 (-a - b)*x^3 + x^4)^(4/3)]/(2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(-a*b*d+(a+
b)*d*x-d*x^2+x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,ab-3\,(a+b)x+2\,x^2\right)x^3}{\left((a-x)(b-x)x^2\right)^{\frac{2}{3}}\left(x^4-abd+(a+b)dx-dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(-a*b*d+(a+
b)*d*x-d*x^2+x^4),x, algorithm="giac")
```

```
[Out] integrate((4*a*b - 3*(a + b)*x + 2*x^2)*x^3/(((a - x)*(b - x)*x^2)^(2/3)*(x
^4 - a*b*d + (a + b)*d*x - d*x^2)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(4ab-3\,(a+b)\,x+2x^2\right)}{\left(x^2\,(-a+x)\,(-b+x)\right)^{\frac{2}{3}}\left(-abd+(a+b)\,dx-d\,x^2+x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(-a*b*d+(a+b)*d*x
-d*x^2+x^4),x)
```

```
[Out] int(x^3*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(-a*b*d+(a+b)*d*x
-d*x^2+x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,ab-3\,(a+b)x+2\,x^2\right)x^3}{\left((a-x)(b-x)x^2\right)^{\frac{2}{3}}\left(x^4-abd+(a+b)dx-dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(4*a*b-3*(a+b)*x+2*x^2)/(x^2*(-a+x)*(-b+x))^(2/3)/(-a*b*d+(a+
b)*d*x-d*x^2+x^4),x, algorithm="maxima")
```

```
[Out] integrate((4*a*b - 3*(a + b)*x + 2*x^2)*x^3/(((a - x)*(b - x)*x^2)^(2/3)*(x
^4 - a*b*d + (a + b)*d*x - d*x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^3\,\left(4\,a\,b+2\,x^2-3\,x\,\left(a+b\right)\right)}{{\left(x^2\,\left(a-x\right)\,\left(b-x\right)\right)}^{2/3}\,\left(-x^4+d\,x^2-d\,\left(a+b\right)\,x+a\,b\,d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(4*a*b + 2*x^2 - 3*x*(a + b)))/((x^2*(a - x)*(b - x))^(2/3)*(d*x^
2 - x^4 - d*x*(a + b) + a*b*d)),x)
```

```
[Out] -int((x^3*(4*a*b + 2*x^2 - 3*x*(a + b)))/((x^2*(a - x)*(b - x))^(2/3)*(d*x^
2 - x^4 - d*x*(a + b) + a*b*d)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(4*a*b-3*(a+b)*x+2*x**2)/(x**2*(-a+x)*(-b+x))**(2/3)/(-a*b*d
+(a+b)*d*x-d*x**2+x**4),x)
```

```
[Out] Timed out
```

3.2322 $\int \frac{x^3}{\sqrt[3]{x^2+x^4}\left(-1+x^6\right)} dx$

Optimal. Leaf size=356

$$-\frac{1}{6}\log\left(\sqrt[3]{x^4+x^2}-x\right)-\frac{1}{6}\log\left(\sqrt[3]{x^4+x^2}+x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}-2x\right)}{12\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}+2x\right)}{12\sqrt[3]{2}}+\frac{1}{12}\log\Big($$

Rubi [C] time = 1.58, antiderivative size = 152, normalized size of antiderivative = 0.43, number of steps used = 43, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2056, 6725, 959, 466, 465, 510}

$$-\frac{\sqrt[3]{x^2+1}\,x^4F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-x^2,-\sqrt[3]{-1}x^2\right)}{10\sqrt[3]{x^4+x^2}}-\frac{\sqrt[3]{x^2+1}\,x^4F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-x^2,(-1)^{2/3}x^2\right)}{10\sqrt[3]{x^4+x^2}}-\frac{\sqrt[3]{x^2+1}\,x^4F_1\left(\frac{5}{3};1,\frac{1}{3};\frac{8}{3};x^2,-x^2\right)}{10\sqrt[3]{x^4+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3/((x^2 + x^4)^(1/3)*(-1 + x^6)),x]
```

```
[Out] -1/10*(x^4*(1 + x^2)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -x^2, -((-1)^(1/3)*x^
2)])/(x^2 + x^4)^(1/3) - (x^4*(1 + x^2)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -x
^2, (-1)^(2/3)*x^2])/(10*(x^2 + x^4)^(1/3)) - (x^4*(1 + x^2)^(1/3)*AppellF1
[5/3, 1, 1/3, 8/3, x^2, -x^2])/(10*(x^2 + x^4)^(1/3))
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 959

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[(d*(g*x)^n)/x^n, Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[(e*(g*x)^n)/x^n, Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
```

```
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt[3]{x^2+x^4}\left(-1+x^6\right)}\,dx = \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{x^{7/3}}{\sqrt[3]{1+x^2}\left(-1+x^6\right)}\,dx}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \left(-\frac{x^{7/3}}{2\sqrt[3]{1+x^2}\left(1-x^3\right)}-\frac{x^{7/3}}{2\sqrt[3]{1+x^2}\left(1+x^3\right)}\right)dx}{\sqrt[3]{x^2+x^4}}$$

$$= -\frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{x^{7/3}}{\sqrt[3]{1+x^2}\left(1-x^3\right)}\,dx}{2\sqrt[3]{x^2+x^4}} - \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{x^{7/3}}{\sqrt[3]{1+x^2}\left(1+x^3\right)}\,dx}{2\sqrt[3]{x^2+x^4}}$$

$$= -\frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \left(-\frac{x^{7/3}}{3(-1-x)\sqrt[3]{1+x^2}}-\frac{x^{7/3}}{3\left(-1+\sqrt[3]{-1}\,x\right)\sqrt[3]{1+x^2}}-\frac{x^{7/3}}{3\left(-1-(-1)^{2/3}x\right)\sqrt[3]{1+x^2}}\right)dx}{2\sqrt[3]{x^2+x^4}} - \cdots$$

$$= \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{x^{7/3}}{(-1-x)\sqrt[3]{1+x^2}}\,dx}{6\sqrt[3]{x^2+x^4}} - \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{x^{7/3}}{(1-x)\sqrt[3]{1+x^2}}\,dx}{6\sqrt[3]{x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\cdots}{6\sqrt[3]{\cdots}}$$

$$= -2\frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{x^{7/3}}{\left(1-x^2\right)\sqrt[3]{1+x^2}}\,dx}{6\sqrt[3]{x^2+x^4}} - 2\frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{x^{7/3}}{\sqrt[3]{1+x^2}\left(1+\sqrt[3]{-1}\,x^2\right)}\,dx}{6\sqrt[3]{x^2+x^4}} - 2\frac{\left(x^{2/3}\cdots\right.}{\cdots}$$

$$= -2\frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\mathrm{Subst}\left(\int \frac{x^9}{\left(1-x^6\right)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{x^2+x^4}} - 2\frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\mathrm{Subst}\left(\int \frac{\cdots}{\sqrt[3]{1+x^{\cdots}}}\cdots\right.}{2\sqrt[3]{x^2+x^{\cdots}}}$$

$$= -2\frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\mathrm{Subst}\left(\int \frac{x^4}{\left(1-x^3\right)\sqrt[3]{1+x^3}}\,dx, x, x^{2/3}\right)}{4\sqrt[3]{x^2+x^4}} - 2\frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\mathrm{Subst}\left(\int \frac{\cdots}{\sqrt[3]{1+x^{\cdots}}}\cdots\right.}{4\sqrt[3]{x^2+x^{\cdots}}}$$

$$= -\frac{x^4\sqrt[3]{1+x^2}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-x^2,-\sqrt[3]{-1}\,x^2\right)}{10\sqrt[3]{x^2+x^4}} - \frac{x^4\sqrt[3]{1+x^2}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-x^2,(-1)^{2/3}x^2\right)}{10\sqrt[3]{x^2+x^4}} \cdots$$

Mathematica [F] time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt[3]{x^2+x^4}\left(-1+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/((x^2 + x^4)^(1/3)*(-1 + x^6)),x]
[Out] Integrate[x^3/((x^2 + x^4)^(1/3)*(-1 + x^6)), x]
```

IntegrateAlgebraic [A] time = 0.95, size = 360, normalized size = 1.01

$$-\frac{1}{6}\log\left(\sqrt[3]{x^4+x^2}-x\right)-\frac{1}{6}\log\left(\sqrt[3]{x^4+x^2}+x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}-2x\right)}{12\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}+2x\right)}{12\sqrt[3]{2}}+\frac{1}{12}\log\left(x^2-\sqrt[3]{x^4+x^2}\,x+\left(x^4+x^2\right)^{2/3}\right)+\frac{1}{12}\log\left(x^2+\sqrt[3]{x^4+x^2}\,x+\left(x^4+x^2\right)^{2/3}\right)-\frac{\log\left(-2x^2+2^{2/3}\sqrt[3]{x^4+x^2}\,x-\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{24\sqrt[3]{2}}-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^4+x^2}\,x+\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{24\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}x^2}{x^2+\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}}\right)}{4\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\frac{x^2}{\sqrt{3}}+\frac{2\left(x^4+x^2\right)^{2/3}}{\sqrt{3}}}{x^2}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((x^2 + x^4)^(1/3)*(-1 + x^6)),x]
```

```
[Out] ArcTan[(Sqrt[3]*x^2)/(x^2 + 2^(1/3)*(x^2 + x^4)^(2/3))]/(4*2^(1/3)*Sqrt[3])
 + ArcTan[(x^2/Sqrt[3] + (2*(x^2 + x^4)^(2/3))/Sqrt[3])/x^2]/(2*Sqrt[3]) -
Log[-x + (x^2 + x^4)^(1/3)]/6 - Log[x + (x^2 + x^4)^(1/3)]/6 + Log[-2*x + 2
^(2/3)*(x^2 + x^4)^(1/3)]/(12*2^(1/3)) + Log[2*x + 2^(2/3)*(x^2 + x^4)^(1/3
)]/(12*2^(1/3)) + Log[x^2 - x*(x^2 + x^4)^(1/3) + (x^2 + x^4)^(2/3)]/12 + L
og[x^2 + x*(x^2 + x^4)^(1/3) + (x^2 + x^4)^(2/3)]/12 - Log[-2*x^2 + 2^(2/3)
*x*(x^2 + x^4)^(1/3) - 2^(1/3)*(x^2 + x^4)^(2/3)]/(24*2^(1/3)) - Log[2*x^2
+ 2^(2/3)*x*(x^2 + x^4)^(1/3) + 2^(1/3)*(x^2 + x^4)^(2/3)]/(24*2^(1/3))
```

fricas [A] time = 2.49, size = 418, normalized size = 1.17

$$-\frac{1}{72}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\left(24\sqrt{6}\,2^{\frac{2}{3}}\left(x^8+2x^6-6x^4+2x^2+1\right)\left(x^4+x^2\right)^{\frac{2}{3}}+\sqrt{6}\,2^{\frac{1}{3}}\left(x^{12}-42x^{10}-417x^8-812x^6-417x^4-42x^2+1\right)-12\sqrt{6}\left(x^{10}+33x^8+110x^6+110x^4+33x^2+1\right)\left(x^4+x^2\right)^{\frac{1}{3}}\right)}{6\left(x^{12}+102x^{10}+447x^8+628x^6+447x^4+102x^2+1\right)}\right)-\frac{1}{144}\cdot2^{\frac{2}{3}}\log\left(\frac{12\cdot2^{\frac{2}{3}}\left(x^4+4x^2+1\right)\left(x^4+x^2\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(x^8+32x^6+78x^4+32x^2+1\right)+6\left(x^6+11x^4+11x^2+1\right)\left(x^4+x^2\right)^{\frac{1}{3}}}{x^8-4x^6+6x^4-4x^2+1}\right)+\frac{1}{72}\cdot2^{\frac{2}{3}}\log\left(-\frac{2^{\frac{2}{3}}\left(x^4-2x^2+1\right)-6\cdot2^{\frac{1}{3}}\left(x^4+x^2\right)^{\frac{1}{3}}\left(x^2+1\right)+12\left(x^4+x^2\right)^{\frac{2}{3}}}{x^4-2x^2+1}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x^2+1\right)+2\sqrt{3}\left(x^4+x^2\right)^{\frac{1}{3}}}{3\left(x^2+1\right)}\right)-\frac{1}{12}\log\left(\frac{x^4+x^2-3\left(x^4+x^2\right)^{\frac{1}{3}}\left(x^2+1\right)+3\left(x^4+x^2\right)^{\frac{2}{3}}+1}{x^4+x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+x^2)^(1/3)/(x^6-1),x, algorithm="fricas")
```

```
[Out] -1/72*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(24*sqrt(6)*2^(2/3)*(x^8 + 2*x^6 -
 6*x^4 + 2*x^2 + 1)*(x^4 + x^2)^(2/3) + sqrt(6)*2^(1/3)*(x^12 - 42*x^10 - 4
17*x^8 - 812*x^6 - 417*x^4 - 42*x^2 + 1) - 12*sqrt(6)*(x^10 + 33*x^8 + 110*
x^6 + 110*x^4 + 33*x^2 + 1)*(x^4 + x^2)^(1/3))/(x^12 + 102*x^10 + 447*x^8 +
 628*x^6 + 447*x^4 + 102*x^2 + 1)) - 1/144*2^(2/3)*log((12*2^(2/3)*(x^4 + 4
*x^2 + 1)*(x^4 + x^2)^(2/3) + 2^(1/3)*(x^8 + 32*x^6 + 78*x^4 + 32*x^2 + 1)
+ 6*(x^6 + 11*x^4 + 11*x^2 + 1)*(x^4 + x^2)^(1/3))/(x^8 - 4*x^6 + 6*x^4 - 4
*x^2 + 1)) + 1/72*2^(2/3)*log(-(2^(2/3)*(x^4 - 2*x^2 + 1) - 6*2^(1/3)*(x^4
+ x^2)^(1/3)*(x^2 + 1) + 12*(x^4 + x^2)^(2/3))/(x^4 - 2*x^2 + 1)) - 1/6*sqr
t(3)*arctan(1/3*(sqrt(3)*(x^2 + 1) + 2*sqrt(3)*(x^4 + x^2)^(1/3))/(x^2 + 1)
) - 1/12*log((x^4 + x^2 - 3*(x^4 + x^2)^(1/3)*(x^2 + 1) + 3*(x^4 + x^2)^(2/
3) + 1)/(x^4 + x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^6-1\right)\left(x^4+x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+x^2)^(1/3)/(x^6-1),x, algorithm="giac")
```

```
[Out] integrate(x^3/((x^6 - 1)*(x^4 + x^2)^(1/3)), x)
```

maple [C] time = 46.14, size = 2987, normalized size = 8.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4+x^2)^(1/3)/(x^6-1),x)
```

```
[Out] -1/24*ln((-3586488*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*Roo
tOf(_Z^3-4)^3*x^4-40450464*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_
Z^2)^2*RootOf(_Z^3-4)^2*x^4+15242574*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z
^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^2+171914472*RootOf(RootOf(_Z^3-4)^2+6*_Z*
```

RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+104139450*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*(x^4+x^2)^(2/3)+7334688*(x^4+x^2)^(1/3)*RootOf(_Z^3-4)^2*x^2+104139450*(x^4+x^2)^(1/3)*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^2+3735925*RootOf(_Z^3-4)*x^4+42135900*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^4-3586488*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3-40450464*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2+7334688*(x^4+x^2)^(1/3)*RootOf(_Z^3-4)^2+104139450*(x^4+x^2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)+31979518*RootOf(_Z^3-4)*x^2+360683304*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^2+29338752*(x^4+x^2)^(2/3)+3735925*RootOf(_Z^3-4)+42135900*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(1+x)^2/(-1+x)^2)*RootOf(_Z^3-4)-1/4*ln((-3586488*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^4-40450464*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^4+15242574*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^2+171914472*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+104139450*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*(x^4+x^2)^(2/3)+7334688*(x^4+x^2)^(1/3)*RootOf(_Z^3-4)^2*x^2+104139450*(x^4+x^2)^(1/3)*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^2+3735925*RootOf(_Z^3-4)*x^4+42135900*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^4-3586488*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3-40450464*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2+7334688*(x^4+x^2)^(1/3)*RootOf(_Z^3-4)^2+104139450*(x^4+x^2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)+31979518*RootOf(_Z^3-4)*x^2+360683304*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^2+29338752*(x^4+x^2)^(2/3)+3735925*RootOf(_Z^3-4)+42135900*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(1+x)^2/(-1+x)^2)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+1/4*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln(-(1051752*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^4+13483488*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^4-4469946*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^2-57304824*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+34713150*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*(x^4+x^2)^(2/3)+3340629*(x^4+x^2)^(1/3)*RootOf(_Z^3-4)^2*x^2+34713150*(x^4+x^2)^(1/3)*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^2+1796743*RootOf(_Z^3-4)*x^4+23034292*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^4+1051752*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3+13483488*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2+3340629*(x^4+x^2)^(1/3)*RootOf(_Z^3-4)^2+34713150*(x^4+x^2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)+6398158*RootOf(_Z^3-4)*x^2+82024552*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^2+13362516*(x^4+x^2)^(2/3)+1796743*RootOf(_Z^3-4)+23034292*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(1+x)^2/(-1+x)^2)+1/6*ln((-1340172*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^4*x^4+5695731*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^4*x^2+5235654*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x^4+9200025*(x^4+x^2)^(1/3)*RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^2-1340172*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^4+9200025*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*(x^4+x^2)^(2/3)+19126281*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x^2+9200025*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*(x^4+x^2)^(1/3)+1451650*x^4+8496*(x^4+x^2)^(1/3)*x^2+5235654*RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+8496*(x^4+x^2)^(2/3)+4587214*x^2+8496*(x^4+x^2)^(1/3)+1451650)/(x^2+x+1)/(x^2-x+1))+1/4*ln((-1340172*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^4*x^4+5695731*RootOf(RootOf(_Z

```
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^4*x^2+5235654*RootOf(
RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x^4+9200025*
(x^4+x^2)^(1/3)*RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4
)+36*_Z^2)*x^2-1340172*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
^2*RootOf(_Z^3-4)^4+9200025*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*
_Z^2)*RootOf(_Z^3-4)^2*(x^4+x^2)^(2/3)+19126281*RootOf(RootOf(_Z^3-4)^2+6*_
Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x^2+9200025*RootOf(RootOf(_Z^3-4
)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*(x^4+x^2)^(1/3)+1451650*x
^4+8496*(x^4+x^2)^(1/3)*x^2+5235654*RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^
2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+8496*(x^4+x^2)^(2/3)+4587214*x^2+8496*(x^4+x
^2)^(1/3)+1451650)/(x^2+x+1)/(x^2-x+1))*RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3
-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)-1/4*RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-
4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln(-(1340172*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^4*x^4-5695731*RootOf(RootOf(_Z^3-
4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^4*x^2+7022550*RootOf(Roo
tOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x^4+9200025*(x^
4+x^2)^(1/3)*RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+3
6*_Z^2)*x^2+1340172*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*
RootOf(_Z^3-4)^4+9200025*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^
2)*RootOf(_Z^3-4)^2*(x^4+x^2)^(2/3)+11531973*RootOf(RootOf(_Z^3-4)^2+6*_Z*R
ootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x^2+9200025*RootOf(RootOf(_Z^3-4)^2
+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*(x^4+x^2)^(1/3)+2634418*x^4+
6124854*(x^4+x^2)^(1/3)*x^2+7022550*RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^
2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+6124854*(x^4+x^2)^(2/3)+5632204*x^2+6124854*
(x^4+x^2)^(1/3)+2634418)/(x^2+x+1)/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^6-1\right)\left(x^4+x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+x^2)^(1/3)/(x^6-1),x, algorithm="maxima")
[Out] integrate(x^3/((x^6 - 1)*(x^4 + x^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(x^4+x^2\right)^{1/3}\,\left(x^6-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((x^2 + x^4)^(1/3)*(x^6 - 1)),x)
[Out] int(x^3/((x^2 + x^4)^(1/3)*(x^6 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt[3]{x^2\left(x^2+1\right)}\,(x-1)\,(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4+x**2)**(1/3)/(x**6-1),x)
[Out] Integral(x**3/((x**2*(x**2 + 1))**(1/3)*(x - 1)*(x + 1)*(x**2 - x + 1)*(x**
2 + x + 1)), x)
```

3.2323 $\int \frac{(c+bx+ax^2)^{5/2}}{(c+bx)^2} dx$

Optimal. Leaf size=362

$$\frac{5\left(2a^{5/2}c^4-a^{3/2}b^2c^3\right)\log\left(\sqrt{ax^2+bx+c}-\sqrt{a}\,x\right)}{2b^6}-\frac{5\left(2a^{5/2}c^4-a^{3/2}b^2c^3\right)\log\left(\sqrt{a}\,(bx+2c)-b\sqrt{ax^2+bx+c}\right)}{2b^6}+\cdots$$

Rubi [A] time = 0.32, antiderivative size = 277, normalized size of antiderivative = 0.77, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {732, 814, 843, 621, 206, 724}

$$-\frac{5a^{3/2}c^3\left(b^2-2ac\right)\tanh^{-1}\left(\frac{x\left(b^2-2ac\right)+bc}{2\sqrt{a}\,c\sqrt{ax^2+bx+c}}\right)}{2b^6}+\frac{5\left(-64a^3c^3+48a^2b^2c^2+2abx\left(16a^2c^2-4ab^2c+b^4\right)-4ab^4c+b^6\right)\sqrt{ax^2+bx+c}}{64ab^5}-\frac{5\left(-128a^4c^4+64a^3b^2c^3-8ab^6c+b^8\right)\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{ax^2+bx+c}}\right)}{128a^{3/2}b^6}+\frac{5\left(6abx-8ac+7b^2\right)\left(ax^2+bx+c\right)^{3/2}}{24b^3}-\frac{\left(ax^2+bx+c\right)^{5/2}}{b(bx+c)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + b*x + a*x^2)^(5/2)/(c + b*x)^2,x]
```

```
[Out] (5*(b^6 - 4*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3 + 2*a*b*(b^4 - 4*a*b^2*c
+ 16*a^2*c^2)*x)*Sqrt[c + b*x + a*x^2])/(64*a*b^5) + (5*(7*b^2 - 8*a*c + 6*
a*b*x)*(c + b*x + a*x^2)^(3/2))/(24*b^3) - (c + b*x + a*x^2)^(5/2)/(b*(c +
b*x)) - (5*(b^8 - 8*a*b^6*c + 64*a^3*b^2*c^3 - 128*a^4*c^4)*ArcTanh[(b + 2*
a*x)/(2*Sqrt[a]*Sqrt[c + b*x + a*x^2])])/(128*a^(3/2)*b^6) - (5*a^(3/2)*c^3
*(b^2 - 2*a*c)*ArcTanh[(b*c + (b^2 - 2*a*c)*x)/(2*Sqrt[a]*c*Sqrt[c + b*x +
a*x^2])])/(2*b^6)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
```

```
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(c+bx+ax^2\right)^{5/2}}{(c+bx)^2}\,dx = -\frac{\left(c+bx+ax^2\right)^{5/2}}{b(c+bx)} + \frac{5\int \frac{(b+2ax)\left(c+bx+ax^2\right)^{3/2}}{c+bx}\,dx}{2b}$$

$$= \frac{5\left(7b^2-8ac+6abx\right)\left(c+bx+ax^2\right)^{3/2}}{24b^3} - \frac{\left(c+bx+ax^2\right)^{5/2}}{b(c+bx)} - \frac{5\int \frac{\left(-abc\left(b^2+4ac\right)-a\left(b^4-4a\right.\right.}{c}}{16}$$

$$= \frac{5\left(b^6-4ab^4c+48a^2b^2c^2-64a^3c^3+2ab\left(b^4-4ab^2c+16a^2c^2\right)x\right)\sqrt{c+bx+ax^2}}{64ab^5} +$$

$$= \frac{5\left(b^6-4ab^4c+48a^2b^2c^2-64a^3c^3+2ab\left(b^4-4ab^2c+16a^2c^2\right)x\right)\sqrt{c+bx+ax^2}}{64ab^5} +$$

$$= \frac{5\left(b^6-4ab^4c+48a^2b^2c^2-64a^3c^3+2ab\left(b^4-4ab^2c+16a^2c^2\right)x\right)\sqrt{c+bx+ax^2}}{64ab^5} +$$

$$= \frac{5\left(b^6-4ab^4c+48a^2b^2c^2-64a^3c^3+2ab\left(b^4-4ab^2c+16a^2c^2\right)x\right)\sqrt{c+bx+ax^2}}{64ab^5} +$$

Mathematica [A] time = 0.63, size = 271, normalized size = 0.75

$$\frac{5\left(64a^3c^3\left(2ac-b^2\right)\tanh^{-1}\left(\frac{-2acx+b^2x+bc}{2\sqrt{a}\,c\sqrt{x(ax+b)+c}}\right)-\left(-128a^4c^4+64a^3b^2c^3-8ab^6c+b^8\right)\tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right)+2\sqrt{a}\,b\left(32a^3bc^2x-64a^3c^3-8a^2b^3cx+48a^2b^2c^2+2ab^5x-4ab^4c+b^6\right)\sqrt{x(ax+b)+c}\right)}{128a^{3/2}b^6}+\frac{5\left(6abx-8ac+7b^2\right)(x(ax+b)+c)^{3/2}}{24b^3}-\frac{(x(ax+b)+c)^{5/2}}{b(bx+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + b*x + a*x^2)^(5/2)/(c + b*x)^2,x]
```

```
[Out] (5*(7*b^2 - 8*a*c + 6*a*b*x)*(c + x*(b + a*x))^(3/2))/(24*b^3) - (c + x*(b
+ a*x))^(5/2)/(b*(c + b*x)) + (5*(2*Sqrt[a]*b*(b^6 - 4*a*b^4*c + 48*a^2*b^2
*c^2 - 64*a^3*c^3 + 2*a*b^5*x - 8*a^2*b^3*c*x + 32*a^3*b*c^2*x)*Sqrt[c + x*
(b + a*x)] - (b^8 - 8*a*b^6*c + 64*a^3*b^2*c^3 - 128*a^4*c^4)*ArcTanh[(b +
2*a*x)/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])] + 64*a^3*c^3*(-b^2 + 2*a*c)*ArcTa
nh[(b*c + b^2*x - 2*a*c*x)/(2*Sqrt[a]*c*Sqrt[c + x*(b + a*x)])]))/(128*a^(3
/2)*b^6)
```

IntegrateAlgebraic [A] time = 1.87, size = 366, normalized size = 1.01

$$\frac{5\left(2a^{5/2}c^4-a^{3/2}b^2c^3\right)\log\left(\sqrt{ax^2+bx+c}-\sqrt{a}\,x\right)}{2b^6}-\frac{5\left(2a^{5/2}c^4-a^{3/2}b^2c^3\right)\log\left(\sqrt{a}\,(bx+2c)-b\sqrt{ax^2+bx+c}\right)}{2b^6}+\frac{\sqrt{ax^2+bx+c}\left(48a^3b^4x^4-80a^3b^3cx^3+160a^3b^2c^2x^2-480a^3bc^3x-960a^3c^4+136a^2b^5x^3-64a^2b^4cx^2+200a^2b^3c^2x+400a^2b^2c^3+118ab^6x^2+146ab^5cx+28ab^4c^2+15b^7x+15b^6c\right)}{192ab^5(bx+c)}-\frac{5\left(128a^4c^4-64a^3b^2c^3+8ab^6c-b^8\right)\log\left(-2a^{3/2}\sqrt{ax^2+bx+c}+2a^2x+ab\right)}{128a^{3/2}b^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + b*x + a*x^2)^(5/2)/(c + b*x)^2,x]
```

```
[Out] (Sqrt[c + b*x + a*x^2]*(15*b^6*c + 28*a*b^4*c^2 + 400*a^2*b^2*c^3 - 960*a^3
*c^4 + 15*b^7*x + 146*a*b^5*c*x + 200*a^2*b^3*c^2*x - 480*a^3*b*c^3*x + 118
*a*b^6*x^2 - 64*a^2*b^4*c*x^2 + 160*a^3*b^2*c^2*x^2 + 136*a^2*b^5*x^3 - 80*
a^3*b^3*c*x^3 + 48*a^3*b^4*x^4))/(192*a*b^5*(c + b*x)) + (5*(-(a^(3/2)*b^2*
c^3) + 2*a^(5/2)*c^4)*Log[-(Sqrt[a]*x) + Sqrt[c + b*x + a*x^2]])/(2*b^6) -
(5*(-b^8 + 8*a*b^6*c - 64*a^3*b^2*c^3 + 128*a^4*c^4)*Log[a*b + 2*a^2*x - 2*
a^(3/2)*Sqrt[c + b*x + a*x^2]])/(128*a^(3/2)*b^6) - (5*(-(a^(3/2)*b^2*c^3)
+ 2*a^(5/2)*c^4)*Log[Sqrt[a]*(2*c + b*x) - b*Sqrt[c + b*x + a*x^2]])/(2*b^6
)
```

fricas [A] time = 27.54, size = 854, normalized size = 2.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x+c)^(5/2)/(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(b^8*c - 8*a*b^6*c^2 + 64*a^3*b^2*c^4 - 128*a^4*c^5 + (b^9 - 8*
a*b^7*c + 64*a^3*b^3*c^3 - 128*a^4*b*c^4)*x)*sqrt(a)*log(-8*a^2*x^2 - 8*a*b
*x - 4*sqrt(a*x^2 + b*x + c)*(2*a*x + b)*sqrt(a) - b^2 - 4*a*c) + 960*(a^3*
b^2*c^4 - 2*a^4*c^5 + (a^3*b^3*c^3 - 2*a^4*b*c^4)*x)*sqrt(a)*log(-(2*b^3*c*
x + b^2*c^2 + 4*a*c^3 + (b^4 - 4*a*b^2*c + 8*a^2*c^2)*x^2 + 4*(b*c^2 + (b^2
*c - 2*a*c^2)*x)*sqrt(a*x^2 + b*x + c)*sqrt(a))/(b^2*x^2 + 2*b*c*x + c^2))
- 4*(48*a^4*b^5*x^4 + 15*a*b^7*c + 28*a^2*b^5*c^2 + 400*a^3*b^3*c^3 - 960*a
^4*b*c^4 + 8*(17*a^3*b^6 - 10*a^4*b^4*c)*x^3 + 2*(59*a^2*b^7 - 32*a^3*b^5*c
 + 80*a^4*b^3*c^2)*x^2 + (15*a*b^8 + 146*a^2*b^6*c + 200*a^3*b^4*c^2 - 480*
a^4*b^2*c^3)*x)*sqrt(a*x^2 + b*x + c))/(a^2*b^7*x + a^2*b^6*c), -1/384*(960
*(a^3*b^2*c^4 - 2*a^4*c^5 + (a^3*b^3*c^3 - 2*a^4*b*c^4)*x)*sqrt(-a)*arctan(
-1/2*sqrt(a*x^2 + b*x + c)*(b*c + (b^2 - 2*a*c)*x)*sqrt(-a)/(a^2*c*x^2 + a*
b*c*x + a*c^2)) - 15*(b^8*c - 8*a*b^6*c^2 + 64*a^3*b^2*c^4 - 128*a^4*c^5 +
(b^9 - 8*a*b^7*c + 64*a^3*b^3*c^3 - 128*a^4*b*c^4)*x)*sqrt(-a)*arctan(1/2*s
qrt(a*x^2 + b*x + c)*(2*a*x + b)*sqrt(-a)/(a^2*x^2 + a*b*x + a*c)) - 2*(48*
a^4*b^5*x^4 + 15*a*b^7*c + 28*a^2*b^5*c^2 + 400*a^3*b^3*c^3 - 960*a^4*b*c^4
 + 8*(17*a^3*b^6 - 10*a^4*b^4*c)*x^3 + 2*(59*a^2*b^7 - 32*a^3*b^5*c + 80*a^
4*b^3*c^2)*x^2 + (15*a*b^8 + 146*a^2*b^6*c + 200*a^3*b^4*c^2 - 480*a^4*b^2*
c^3)*x)*sqrt(a*x^2 + b*x + c))/(a^2*b^7*x + a^2*b^6*c)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x+c)^(5/2)/(b*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 1306, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x+c)^(5/2)/(b*x+c)^2,x)
```

```
[Out] 35/24/b*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(3/2)+1/c^2*((x+c/b)^
2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(5/2)*x-5/3/b^3*a*c*((x+c/b)^2*a-(2*a*
c-b^2)/b*(x+c/b)+a*c^2/b^2)^(3/2)+15/4/b^3*a*c^2*((x+c/b)^2*a-(2*a*c-b^2)/b
*(x+c/b)+a*c^2/b^2)^(1/2)-5/8/b^2*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/
b^2)^(1/2)*x*a*c+5/16/a^(1/2)*ln((-1/2*(2*a*c-b^2)/b+(x+c/b)*a)/a^(1/2)+((x
+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(1/2))*c-5/b^5*a^2*c^3*((x+c/b)^
2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(1/2)+b/a/c^2*((x+c/b)^2*a-(2*a*c-b^2)
/b*(x+c/b)+a*c^2/b^2)^(5/2)+5/b^6*c^4*a^(5/2)*ln((-1/2*(2*a*c-b^2)/b+(x+c/b
)*a)/a^(1/2)+((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(1/2))-5/16/b*((
x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(1/2)*c-1/b/c*((x+c/b)^2*a-(2*a
*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(5/2)+5/32*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+
a*c^2/b^2)^(1/2)*x+5/64/a*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(1/
2)*b-5/128/a^(3/2)*ln((-1/2*(2*a*c-b^2)/b+(x+c/b)*a)/a^(1/2)+((x+c/b)^2*a-(
2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(1/2))*b^2+5/4/b^2*a*((x+c/b)^2*a-(2*a*c-b^
2)/b*(x+c/b)+a*c^2/b^2)^(3/2)*x+5/b^7*a^3*c^5/(a*c^2/b^2)^(1/2)*ln((2*a*c^2
/b^2-(2*a*c-b^2)/b*(x+c/b)+2*(a*c^2/b^2)^(1/2)*((x+c/b)^2*a-(2*a*c-b^2)/b*(
x+c/b)+a*c^2/b^2)^(1/2))/(x+c/b))-5/2/b^5*a^2*c^4/(a*c^2/b^2)^(1/2)*ln((2*a
*c^2/b^2-(2*a*c-b^2)/b*(x+c/b)+2*(a*c^2/b^2)^(1/2)*((x+c/b)^2*a-(2*a*c-b^2)
/b*(x+c/b)+a*c^2/b^2)^(1/2))/(x+c/b))-5/2/b^4*a^(3/2)*c^3*ln((-1/2*(2*a*c-b
^2)/b+(x+c/b)*a)/a^(1/2)+((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(1/2
))+5/2/b^4*a^2*c^2*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(1/2)*x-1/
a/c^2/(x+c/b)*((x+c/b)^2*a-(2*a*c-b^2)/b*(x+c/b)+a*c^2/b^2)^(7/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x+c)^(5/2)/(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x^2+b\,x+c\right)^{5/2}}{{\left(c+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + b*x + a*x^2)^(5/2)/(c + b*x)^2,x)
```

```
[Out] int((c + b*x + a*x^2)^(5/2)/(c + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{2} + bx + c\right)^{\frac{5}{2}}}{\left(bx + c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x+c)**(5/2)/(b*x+c)**2,x)
```

```
[Out] Integral((a*x**2 + b*x + c)**(5/2)/(b*x + c)**2, x)
```

3.2324 $\int \frac{(1+x^4)^2}{(-1+x^4)^2 \sqrt{x^2+\sqrt{1+x^4}}} \, dx$

Optimal. Leaf size=362

$$\frac{x(x^4-3)}{2(x^4-1)\sqrt{\sqrt{x^4+1}+x^2}}+\frac{1}{2}\sqrt{\frac{1}{2}\left(7+5\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{2}-1\right)}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(7+5\sqrt{2}\right)}\tan^{-1}\left(\right.$$

Rubi [F] time = 1.70, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+x^4)^2}{(-1+x^4)^2 \sqrt{x^2+\sqrt{1+x^4}}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^4)^2/((-1 + x^4)^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] Defer[Int][1/Sqrt[x^2 + Sqrt[1 + x^4]], x] - Defer[Int][1/((I - x)^2*Sqrt[x
^2 + Sqrt[1 + x^4]]), x]/4 - (I/4)*Defer[Int][1/((I - x)*Sqrt[x^2 + Sqrt[1
+ x^4]]), x] - Defer[Int][1/((1 - x)*Sqrt[x^2 + Sqrt[1 + x^4]]), x]/4 + Def
er[Int][1/((-1 + x)^2*Sqrt[x^2 + Sqrt[1 + x^4]]), x]/4 - Defer[Int][1/((I +
 x)^2*Sqrt[x^2 + Sqrt[1 + x^4]]), x]/4 - (I/4)*Defer[Int][1/((I + x)*Sqrt[x
^2 + Sqrt[1 + x^4]]), x] + Defer[Int][1/((1 + x)^2*Sqrt[x^2 + Sqrt[1 + x^4]
]), x]/4 - Defer[Int][1/((1 + x)*Sqrt[x^2 + Sqrt[1 + x^4]]), x]/4
```

Rubi steps

$$\int \frac{(1+x^4)^2}{(-1+x^4)^2 \sqrt{x^2+\sqrt{1+x^4}}} \, dx = \int \left(\frac{1}{\sqrt{x^2+\sqrt{1+x^4}}}+\frac{1}{4(-1+x)^2\sqrt{x^2+\sqrt{1+x^4}}}+\frac{1}{4(1+x)^2\sqrt{x^2+\sqrt{1+x^4}}}\right.$$
$$=\frac{1}{4}\int \frac{1}{(-1+x)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx+\frac{1}{4}\int \frac{1}{(1+x)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx+\frac{1}{2}\int$$
$$=\frac{1}{4}\int \frac{1}{(-1+x)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx+\frac{1}{4}\int \frac{1}{(1+x)^2\sqrt{x^2+\sqrt{1+x^4}}}\,dx+\frac{1}{2}\int$$
$$=-\left(\frac{1}{2}i\int \frac{1}{(i-x)\sqrt{x^2+\sqrt{1+x^4}}}\,dx\right)-\frac{1}{2}i\int \frac{1}{(i+x)\sqrt{x^2+\sqrt{1+x^4}}}\,dx-\frac{1}{4}\int$$
$$=-\left(\frac{1}{2}i\int \frac{1}{(i-x)\sqrt{x^2+\sqrt{1+x^4}}}\,dx\right)-\frac{1}{2}i\int \frac{1}{(i+x)\sqrt{x^2+\sqrt{1+x^4}}}\,dx-\frac{1}{4}\int$$
$$=\frac{1}{4}i\int \frac{1}{(i-x)\sqrt{x^2+\sqrt{1+x^4}}}\,dx+\frac{1}{4}i\int \frac{1}{(i+x)\sqrt{x^2+\sqrt{1+x^4}}}\,dx-\frac{1}{2}i\int \frac{}{(i}$$

Mathematica [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\left(1+x^4\right)^2}{\left(-1+x^4\right)^2 \sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^4)^2/((-1 + x^4)^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] Integrate[(1 + x^4)^2/((-1 + x^4)^2*Sqrt[x^2 + Sqrt[1 + x^4]]), x]
```

IntegrateAlgebraic [A] time = 7.32, size = 483, normalized size = 1.33

$$\frac{x\left(x^4-3\right)}{2\left(x^4-1\right)\sqrt{\sqrt{x^4+1}+x^2}}+\frac{1}{2}\sqrt{\frac{7}{2}+\frac{5}{\sqrt{2}}}\tan^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-\frac{1}{2}\sqrt{\frac{7}{2}+\frac{5}{\sqrt{2}}}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)+\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{x^4+1}}{\sqrt{2}}+\frac{x^2}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}-\frac{1}{2}\sqrt{\frac{5}{\sqrt{2}}-\frac{7}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{x^4+1}+\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}x^2-\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)-\frac{1}{2}\sqrt{\frac{5}{\sqrt{2}}-\frac{7}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{x^4+1}+\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}x^2-\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}}{x\sqrt{\sqrt{x^4+1}+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^4)^2/((-1 + x^4)^2*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] (x*(-3 + x^4))/(2*(-1 + x^4)*Sqrt[x^2 + Sqrt[1 + x^4]]) + (Sqrt[7/2 + 5/Sqr
t[2]]*ArcTan[(-Sqrt[-1/2 + 1/Sqrt[2]] + Sqrt[-1/2 + 1/Sqrt[2]]*x^2 + Sqrt[-
1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])])/2 - (Sqrt[7
/2 + 5/Sqrt[2]]*ArcTan[(-Sqrt[1/2 + 1/Sqrt[2]] + Sqrt[1/2 + 1/Sqrt[2]]*x^2
+ Sqrt[1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])])/2 +
ArcTanh[(-(1/Sqrt[2]) + x^2/Sqrt[2] + Sqrt[1 + x^4]/Sqrt[2])/(x*Sqrt[x^2 +
Sqrt[1 + x^4]])]/Sqrt[2] - (Sqrt[-7/2 + 5/Sqrt[2]]*ArcTanh[(-Sqrt[-1/2 + 1/
Sqrt[2]] + Sqrt[-1/2 + 1/Sqrt[2]]*x^2 + Sqrt[-1/2 + 1/Sqrt[2]]*Sqrt[1 + x^4
])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])])/2 - (Sqrt[-7/2 + 5/Sqrt[2]]*ArcTanh[(-Sq
rt[1/2 + 1/Sqrt[2]] + Sqrt[1/2 + 1/Sqrt[2]]*x^2 + Sqrt[1/2 + 1/Sqrt[2]]*Sqr
t[1 + x^4])/(x*Sqrt[x^2 + Sqrt[1 + x^4]])])/2
```

fricas [B] time = 5.68, size = 553, normalized size = 1.53

$$-\frac{4\sqrt{2}\left(x^4-1\right)\sqrt{5\sqrt{2}+7}\arctan\left(\frac{2\left(6x^7+10x^3-\sqrt{2}\left(5x^7+7x^3\right)-\left(x^5-2\sqrt{2}\left(x^5+x\right)+3x\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{5\sqrt{2}+7}-\left(5x^8+10x^4-\sqrt{2}\left(3x^8+4x^4+1\right)-2\left(x^6+3x^2-2\sqrt{2}\left(x^6+x^2\right)\right)\sqrt{x^4+1}+1\right)\sqrt{5\sqrt{2}+7}\sqrt{\sqrt{2}-1}}{7x^8+10x^4-1}\right)+\sqrt{2}\left(x^4-1\right)\sqrt{5\sqrt{2}-7}\log\left(\frac{2\left(\sqrt{2}x^3+2x^3+\sqrt{x^4+1}\left(\sqrt{2}x+x\right)\right)\sqrt{x^2+\sqrt{x^4+1}}+\left(17x^4+2\sqrt{2}\left(6x^4+1\right)+2\sqrt{x^4+1}\left(5\sqrt{2}x^2+7x^2\right)+3\right)\sqrt{5\sqrt{2}-7}}{x^4-1}\right)-\sqrt{2}\left(x^4-1\right)\sqrt{5\sqrt{2}-7}\log\left(\frac{2\left(\sqrt{2}x^3+2x^3+\sqrt{x^4+1}\left(\sqrt{2}x+x\right)\right)\sqrt{x^2+\sqrt{x^4+1}}-\left(17x^4+2\sqrt{2}\left(6x^4+1\right)+2\sqrt{x^4+1}\left(5\sqrt{2}x^2+7x^2\right)+3\right)\sqrt{5\sqrt{2}-7}}{x^4-1}\right)-2\sqrt{2}\left(x^4-1\right)\log\left(4x^4+4\sqrt{x^4+1}x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)+8\left(x^7-3x^3-\left(x^5-3x\right)\sqrt{x^4+1}\right)\sqrt{x^2+\sqrt{x^4+1}}}{16\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^2/(x^4-1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/16*(4*sqrt(2)*(x^4 - 1)*sqrt(5*sqrt(2) + 7)*arctan((2*(6*x^7 + 10*x^3 -
sqrt(2)*(5*x^7 + 7*x^3) - (x^5 - 2*sqrt(2)*(x^5 + x) + 3*x)*sqrt(x^4 + 1))*
sqrt(x^2 + sqrt(x^4 + 1))*sqrt(5*sqrt(2) + 7) - (5*x^8 + 10*x^4 - sqrt(2)*(
3*x^8 + 4*x^4 + 1) - 2*(x^6 + 3*x^2 - 2*sqrt(2)*(x^6 + x^2))*sqrt(x^4 + 1)
+ 1)*sqrt(5*sqrt(2) + 7)*sqrt(sqrt(2) - 1))/(7*x^8 + 10*x^4 - 1)) + sqrt(2)
*(x^4 - 1)*sqrt(5*sqrt(2) - 7)*log((2*(sqrt(2)*x^3 + 2*x^3 + sqrt(x^4 + 1)*
(sqrt(2)*x + x))*sqrt(x^2 + sqrt(x^4 + 1)) + (17*x^4 + 2*sqrt(2)*(6*x^4 + 1
) + 2*sqrt(x^4 + 1)*(5*sqrt(2)*x^2 + 7*x^2) + 3)*sqrt(5*sqrt(2) - 7))/(x^4
- 1)) - sqrt(2)*(x^4 - 1)*sqrt(5*sqrt(2) - 7)*log((2*(sqrt(2)*x^3 + 2*x^3 +
 sqrt(x^4 + 1)*(sqrt(2)*x + x))*sqrt(x^2 + sqrt(x^4 + 1)) - (17*x^4 + 2*sqr
t(2)*(6*x^4 + 1) + 2*sqrt(x^4 + 1)*(5*sqrt(2)*x^2 + 7*x^2) + 3)*sqrt(5*sqrt
(2) - 7))/(x^4 - 1)) - 2*sqrt(2)*(x^4 - 1)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2
+ 2*(sqrt(2)*x^3 + sqrt(2)*sqrt(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1)
+ 8*(x^7 - 3*x^3 - (x^5 - 3*x)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1)))/(x
^4 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4+1\right)^2}{\left(x^4-1\right)^2 \sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^2/(x^4-1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((x^4 + 1)^2/((x^4 - 1)^2*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}+1\right)^{2}}{\left(x^{4}-1\right)^{2} \sqrt{x^{2}+\sqrt{x^{4}+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^2/(x^4-1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int((x^4+1)^2/(x^4-1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}+1\right)^{2}}{\left(x^{4}-1\right)^{2} \sqrt{x^{2}+\sqrt{x^{4}+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^2/(x^4-1)^2/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((x^4 + 1)^2/((x^4 - 1)^2*sqrt(x^2 + sqrt(x^4 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^{4}+1\right)^{2}}{\left(x^{4}-1\right)^{2} \sqrt{\sqrt{x^{4}+1}+x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)^2/((x^4 - 1)^2*((x^4 + 1)^(1/2) + x^2)^(1/2)),x)
```

```
[Out] int((x^4 + 1)^2/((x^4 - 1)^2*((x^4 + 1)^(1/2) + x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{4}+1\right)^{2}}{\left(x-1\right)^{2} \left(x+1\right)^{2} \left(x^{2}+1\right)^{2} \sqrt{x^{2}+\sqrt{x^{4}+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**2/(x**4-1)**2/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] Integral((x**4 + 1)**2/((x - 1)**2*(x + 1)**2*(x**2 + 1)**2*sqrt(x**2 + sqr
t(x**4 + 1))), x)
```

3.2325 $\int \frac{\sqrt{ax+\sqrt{-b+ax}}}{1+\sqrt{-b+ax}}\,dx$

Optimal. Leaf size=363

$$\frac{\sqrt{\frac{\sqrt{ax-b}+ax}{\left(\sqrt{ax-b}+1\right)^2}}\,(2ax-2b-3)\ \ \sqrt{ax-b}\ \sqrt{\frac{\sqrt{ax-b}+ax}{\left(\sqrt{ax-b}+1\right)^2}}}{2a}-\frac{\sqrt{ax-b}\ \sqrt{\frac{\sqrt{ax-b}+ax}{\left(\sqrt{ax-b}+1\right)^2}}}{2a}+\frac{2\sqrt{b}\log\left(\sqrt{ax-b}+1\right)}{a}-\frac{2\sqrt{b}\log\left(2\sqrt{b}\sqrt{\frac{\sqrt{ax-b}+ax}{\left(\sqrt{ax-b}+1\right)^2}}+\right.}{}$$

Rubi [A] time = 0.30, antiderivative size = 148, normalized size of antiderivative = 0.41, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {814, 843, 621, 206, 724}

$$-\frac{\sqrt{\sqrt{ax-b}+ax}\left(3-2\sqrt{ax-b}\right)}{2a}-\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{ax-b}-2b+1}{2\sqrt{b}\sqrt{\sqrt{ax-b}+ax}}\right)}{a}+\frac{(4b+3)\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{2\sqrt{\sqrt{ax-b}+ax}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + Sqrt[-b + a*x]]/(1 + Sqrt[-b + a*x]),x]
```

```
[Out] -1/2*((3 - 2*Sqrt[-b + a*x])*Sqrt[a*x + Sqrt[-b + a*x]])/a - (2*Sqrt[b]*Arc
Tanh[(1 - 2*b + Sqrt[-b + a*x])/(2*Sqrt[b]*Sqrt[a*x + Sqrt[-b + a*x]])])/a
+ ((3 + 4*b)*ArcTanh[(1 + 2*Sqrt[-b + a*x])/(2*Sqrt[a*x + Sqrt[-b + a*x]])]
)/(4*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{ax+\sqrt{-b+ax}}}{1+\sqrt{-b+ax}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x\sqrt{b+x+x^2}}{1+x}\,dx, x, \sqrt{-b+ax}\right)}{a}$$

$$= -\frac{\left(3-2\sqrt{-b+ax}\right)\sqrt{ax+\sqrt{-b+ax}}}{2a} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-3+4b)-\frac{1}{2}(3+4b)x}{(1+x)\sqrt{b+x+x^2}}\,dx, x, \sqrt{-b+ax}\right)}{2a}$$

$$= -\frac{\left(3-2\sqrt{-b+ax}\right)\sqrt{ax+\sqrt{-b+ax}}}{2a} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{b+x+x^2}}\,dx, x, \sqrt{-b+ax}\right)}{a}$$

$$= -\frac{\left(3-2\sqrt{-b+ax}\right)\sqrt{ax+\sqrt{-b+ax}}}{2a} + \frac{(4b)\,\text{Subst}\left(\int \frac{1}{4b-x^2}\,dx, x, \frac{-1+2b-\sqrt{-b+ax}}{\sqrt{ax+\sqrt{-b+ax}}}\right)}{a} + -\ (3$$

$$= -\frac{\left(3-2\sqrt{-b+ax}\right)\sqrt{ax+\sqrt{-b+ax}}}{2a} - \frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{1-2b+\sqrt{-b+ax}}{2\sqrt{b}\sqrt{ax+\sqrt{-b+ax}}}\right)}{a} + \frac{(3+4b)\tan}{}$$

Mathematica [A] time = 0.19, size = 143, normalized size = 0.39

$$\frac{2\sqrt{\sqrt{ax-b}+ax}\left(2\sqrt{ax-b}-3\right)+8\sqrt{b}\,\tanh^{-1}\left(\frac{-\sqrt{ax-b}+2b-1}{2\sqrt{b}\sqrt{\sqrt{ax-b}+ax}}\right)+(4b+3)\tanh^{-1}\left(\frac{2\sqrt{ax-b}+1}{2\sqrt{\sqrt{ax-b}+ax}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + Sqrt[-b + a*x]]/(1 + Sqrt[-b + a*x]),x]
```

```
[Out] (2*Sqrt[a*x + Sqrt[-b + a*x]]*(-3 + 2*Sqrt[-b + a*x]) + 8*Sqrt[b]*ArcTanh[(
-1 + 2*b - Sqrt[-b + a*x])/(2*Sqrt[b]*Sqrt[a*x + Sqrt[-b + a*x]])] + (3 + 4
*b)*ArcTanh[(1 + 2*Sqrt[-b + a*x])/(2*Sqrt[a*x + Sqrt[-b + a*x]])])/(4*a)
```

IntegrateAlgebraic [A] time = 0.40, size = 155, normalized size = 0.43

$$\frac{\sqrt{\sqrt{ax-b}+ax}\left(2\sqrt{ax-b}-3\right)}{2a}+\frac{(-4b-3)\log\left(a\left(-2\sqrt{ax-b}-1\right)+2a\sqrt{\sqrt{ax-b}+ax}\right)}{4a}-\frac{4\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{ax-b}}{\sqrt{b}}-\frac{\sqrt{\sqrt{ax-b}+ax}}{\sqrt{b}}+\frac{1}{\sqrt{b}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x + Sqrt[-b + a*x]]/(1 + Sqrt[-b + a*x]),x]
```

```
[Out] (Sqrt[a*x + Sqrt[-b + a*x]]*(-3 + 2*Sqrt[-b + a*x]))/(2*a) - (4*Sqrt[b]*Arc
Tanh[1/Sqrt[b] + Sqrt[-b + a*x]/Sqrt[b] - Sqrt[a*x + Sqrt[-b + a*x]]/Sqrt[b
]])/a + ((-3 - 4*b)*Log[a*(-1 - 2*Sqrt[-b + a*x]) + 2*a*Sqrt[a*x + Sqrt[-b
+ a*x]]])/(4*a)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a*x-b)^(1/2))^(1/2)/(1+(a*x-b)^(1/2)),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [A] time = 6.04, size = 127, normalized size = 0.35

$$\frac{1}{2}\sqrt{ax+\sqrt{ax-b}}\left(\frac{2\sqrt{ax-b}}{a}-\frac{3}{a}\right)-\frac{(4b+3)\log\left(\left|-2\sqrt{ax-b}+2\sqrt{ax+\sqrt{ax-b}}-1\right|\right)}{4a}-\frac{4b\arctan\left(-\frac{\sqrt{ax-b}-\sqrt{ax+\sqrt{ax-b}}+1}{\sqrt{-b}}\right)}{a\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a*x-b)^(1/2))^(1/2)/(1+(a*x-b)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a*x + sqrt(a*x - b))*(2*sqrt(a*x - b)/a - 3/a) - 1/4*(4*b + 3)*log
(abs(-2*sqrt(a*x - b) + 2*sqrt(a*x + sqrt(a*x - b)) - 1))/a - 4*b*arctan(-(
sqrt(a*x - b) - sqrt(a*x + sqrt(a*x - b)) + 1)/sqrt(-b))/(a*sqrt(-b))
```

maple [A] time = 0.01, size = 266, normalized size = 0.73

$$\frac{\sqrt{ax-b}\,\sqrt{ax+\sqrt{ax-b}}}{a}+\frac{\sqrt{ax+\sqrt{ax-b}}}{2a}+\frac{\ln\left(\frac{1}{2}+\sqrt{ax-b}+\sqrt{ax+\sqrt{ax-b}}\right)b}{a}-\frac{\ln\left(\frac{1}{2}+\sqrt{ax-b}+\sqrt{ax+\sqrt{ax-b}}\right)}{4a}-\frac{2\sqrt{\left(1+\sqrt{ax-b}\right)^2-\sqrt{ax-b}-1+b}}{a}+\frac{\ln\left(\sqrt{ax-b}+\frac{1}{2}+\sqrt{\left(1+\sqrt{ax-b}\right)^2-\sqrt{ax-b}-1+b}\right)}{a}+\frac{2\sqrt{b}\,\ln\left(\frac{2b-\sqrt{ax-b}-1+2\sqrt{b}\,\sqrt{\left(1+\sqrt{ax-b}\right)^2-\sqrt{ax-b}-1+b}}{1+\sqrt{ax-b}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+(a*x-b)^(1/2))^(1/2)/(1+(a*x-b)^(1/2)),x)
```

```
[Out] (a*x-b)^(1/2)*(a*x+(a*x-b)^(1/2))^(1/2)/a+1/2*(a*x+(a*x-b)^(1/2))^(1/2)/a+1
/a*ln(1/2+(a*x-b)^(1/2)+(a*x+(a*x-b)^(1/2))^(1/2))*b-1/4/a*ln(1/2+(a*x-b)^(
1/2)+(a*x+(a*x-b)^(1/2))^(1/2))-2/a*((1+(a*x-b)^(1/2))^2-(a*x-b)^(1/2)-1+b)
^(1/2)+1/a*ln((a*x-b)^(1/2)+1/2+((1+(a*x-b)^(1/2))^2-(a*x-b)^(1/2)-1+b)^(1/
2))+2/a*b^(1/2)*ln((2*b-(a*x-b)^(1/2)-1+2*b^(1/2)*((1+(a*x-b)^(1/2))^2-(a*x
-b)^(1/2)-1+b)^(1/2))/(1+(a*x-b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+\sqrt{ax-b}}}{\sqrt{ax-b}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a*x-b)^(1/2))^(1/2)/(1+(a*x-b)^(1/2)),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*x + sqrt(a*x - b))/(sqrt(a*x - b) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x+\sqrt{a\,x-b}}}{\sqrt{a\,x-b}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + (a*x - b)^(1/2))^(1/2)/((a*x - b)^(1/2) + 1),x)
```

```
[Out] int((a*x + (a*x - b)^(1/2))^(1/2)/((a*x - b)^(1/2) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + \sqrt{ax - b}}}{\sqrt{ax - b} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a*x-b)**(1/2))**(1/2)/(1+(a*x-b)**(1/2)),x)
[Out] Integral(sqrt(a*x + sqrt(a*x - b))/(sqrt(a*x - b) + 1), x)
```

3.2326 $\int \frac{(b+ax^2)\sqrt[3]{x+x^3}}{d+cx^2}\,dx$

Optimal. Leaf size=367

$$\frac{\sqrt[3]{c-d}\,(bc-ad)\log\left(-\sqrt[3]{d}\,\sqrt[3]{x^3+x}\,x\sqrt[3]{c-d}+x^2(c-d)^{2/3}+d^{2/3}\left(x^3+x\right)^{2/3}\right)}{4c^2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{x^3+x}-x\right)(-ac+3ad}{6c^2}$$

Rubi [A] time = 0.86, antiderivative size = 523, normalized size of antiderivative = 1.43, number of steps used = 22, number of rules used = 16, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {2056, 581, 584, 329, 275, 331, 292, 31, 634, 618, 204, 628, 466, 465, 494, 617}

$$\frac{\sqrt[3]{x^3+x}\,\sqrt[3]{c-d}\,(bc-ad)\log\left(\frac{x^{4/3}(c-d)^{2/3}}{\left(x^2+1\right)^{2/3}}-\frac{\sqrt[3]{d}\,x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{x^2+1}}+d^{2/3}\right)}{4c^2\sqrt[3]{d}\,\sqrt[3]{x^2+1}\,\sqrt[3]{x}}-\frac{\sqrt[3]{x^3+x}\,\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2+1}}\right)(a(c-3d)+3bc)}{6c^2\sqrt[3]{x^2+1}\,\sqrt[3]{x}}+\frac{\sqrt[3]{x^3+x}\,\log\left(\frac{x^{4/3}}{\left(x^2+1\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2+1}}+1\right)(a(c-3d)+3bc)}{12c^2\sqrt[3]{x^2+1}\,\sqrt[3]{x}}-\frac{\sqrt[3]{x^3+x}\,\sqrt[3]{c-d}\,(bc-ad)\log\left(\frac{x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{x^2+1}}+\sqrt[3]{d}\right)}{2c^2\sqrt[3]{d}\,\sqrt[3]{x^2+1}\,\sqrt[3]{x}}-\frac{\sqrt[3]{x^3+x}\,\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2+1}}+1}{\sqrt{3}}\right)(a(c-3d)+3bc)}{2\sqrt{3}\,c^2\sqrt[3]{x^2+1}\,\sqrt[3]{x}}-\frac{\sqrt{3}\,\sqrt[3]{x^3+x}\,\sqrt[3]{c-d}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-\frac{2x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{x^2+1}}}{\sqrt{3}\,\sqrt[3]{d}}\right)}{2c^2\sqrt[3]{d}\,\sqrt[3]{x^2+1}\,\sqrt[3]{x}}+\frac{a\sqrt[3]{x^3+x}\,x}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((b + a*x^2)*(x + x^3)^(1/3))/(d + c*x^2),x]
```

```
[Out] (a*x*(x + x^3)^(1/3))/(2*c) - ((3*b*c + a*(c - 3*d))*(x + x^3)^(1/3)*ArcTan
[(1 + (2*x^(2/3))/(1 + x^2)^(1/3))/Sqrt[3]])/(2*Sqrt[3]*c^2*x^(1/3)*(1 + x^
2)^(1/3)) - (Sqrt[3]*(c - d)^(1/3)*(b*c - a*d)*(x + x^3)^(1/3)*ArcTan[(d^(1
/3) - (2*(c - d)^(1/3)*x^(2/3))/(1 + x^2)^(1/3))/(Sqrt[3]*d^(1/3))])/(2*c^2
*d^(1/3)*x^(1/3)*(1 + x^2)^(1/3)) - ((3*b*c + a*(c - 3*d))*(x + x^3)^(1/3)*
Log[1 - x^(2/3)/(1 + x^2)^(1/3)])/(6*c^2*x^(1/3)*(1 + x^2)^(1/3)) + ((3*b*c
 + a*(c - 3*d))*(x + x^3)^(1/3)*Log[1 + x^(4/3)/(1 + x^2)^(2/3) + x^(2/3)/(
1 + x^2)^(1/3)])/(12*c^2*x^(1/3)*(1 + x^2)^(1/3)) - ((c - d)^(1/3)*(b*c - a
*d)*(x + x^3)^(1/3)*Log[d^(1/3) + ((c - d)^(1/3)*x^(2/3))/(1 + x^2)^(1/3)])
/(2*c^2*d^(1/3)*x^(1/3)*(1 + x^2)^(1/3)) + ((c - d)^(1/3)*(b*c - a*d)*(x +
x^3)^(1/3)*Log[d^(2/3) + ((c - d)^(2/3)*x^(4/3))/(1 + x^2)^(2/3) - ((c - d)
^(1/3)*d^(1/3)*x^(2/3))/(1 + x^2)^(1/3)])/(4*c^2*d^(1/3)*x^(1/3)*(1 + x^2)^
(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{(b+ax^2)\sqrt[3]{x+x^3}}{d+cx^2}\,dx = \frac{\sqrt[3]{x+x^3}\int \frac{\sqrt[3]{x}\sqrt[3]{1+x^2}(b+ax^2)}{d+cx^2}\,dx}{\sqrt[3]{x}\sqrt[3]{1+x^2}}$$

$$= \frac{ax\sqrt[3]{x+x^3}}{2c} + \frac{\sqrt[3]{x+x^3}\int \frac{\sqrt[3]{x}\left(\frac{2}{3}(3bc-2ad)+\frac{2}{3}(3bc+a(c-3d))x^2\right)}{(1+x^2)^{2/3}(d+cx^2)}\,dx}{2c\sqrt[3]{x}\sqrt[3]{1+x^2}}$$

$$= \frac{ax\sqrt[3]{x+x^3}}{2c} + \frac{\sqrt[3]{x+x^3}\int \left(\frac{2(3bc+a(c-3d))\sqrt[3]{x}}{3c(1+x^2)^{2/3}} + \frac{\left(-\frac{2}{3}(3bc+a(c-3d))d+\frac{2}{3}c(3bc-2ad)\right)\sqrt[3]{x}}{c(1+x^2)^{2/3}(d+cx^2)}\right)dx}{2c\sqrt[3]{x}\sqrt[3]{1+x^2}}$$

$$= \frac{ax\sqrt[3]{x+x^3}}{2c} + \frac{\left((3bc+a(c-3d))\sqrt[3]{x+x^3}\right)\int \frac{\sqrt[3]{x}}{(1+x^2)^{2/3}}\,dx}{3c^2\sqrt[3]{x}\sqrt[3]{1+x^2}} + \frac{\left((c-d)(bc-ad)\sqrt[3]{x+x^3}\right)}{c^2\sqrt[3]{x}\sqrt[3]{1+}}$$

$$= \frac{ax\sqrt[3]{x+x^3}}{2c} + \frac{\left((3bc+a(c-3d))\sqrt[3]{x+x^3}\right)\text{Subst}\left(\int \frac{x^3}{(1+x^6)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{c^2\sqrt[3]{x}\sqrt[3]{1+x^2}} + \frac{\left(3(c-d)(b\right.}{}$$

$$= \frac{ax\sqrt[3]{x+x^3}}{2c} + \frac{\left((3bc+a(c-3d))\sqrt[3]{x+x^3}\right)\text{Subst}\left(\int \frac{x}{(1+x^3)^{2/3}}\,dx, x, x^{2/3}\right)}{2c^2\sqrt[3]{x}\sqrt[3]{1+x^2}} + \frac{\left(3(c-d)(b\right.}{}$$

$$= \frac{ax\sqrt[3]{x+x^3}}{2c} + \frac{\left((3bc+a(c-3d))\sqrt[3]{x+x^3}\right)\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{2c^2\sqrt[3]{x}\sqrt[3]{1+x^2}} + \frac{\left(3(c-d)(bc\right.}{}$$

$$= \frac{ax\sqrt[3]{x+x^3}}{2c} + \frac{\left((3bc+a(c-3d))\sqrt[3]{x+x^3}\right)\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{6c^2\sqrt[3]{x}\sqrt[3]{1+x^2}} - \frac{\left((3bc+a(c-\right.}{}$$

$$= \frac{ax\sqrt[3]{x+x^3}}{2c} - \frac{(3bc+a(c-3d))\sqrt[3]{x+x^3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{6c^2\sqrt[3]{x}\sqrt[3]{1+x^2}} - \frac{\sqrt[3]{c-d}\,(bc-ad)\sqrt[3]{x+x^3}\ \mathrm{l}}{2c^2\sqrt[3]{d}\sqrt[3]{x}\sqrt[3]{}}$$

$$= \frac{ax\sqrt[3]{x+x^3}}{2c} - \frac{(3bc+a(c-3d))\sqrt[3]{x+x^3}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{1+x^2}}\right)}{6c^2\sqrt[3]{x}\sqrt[3]{1+x^2}} + \frac{(3bc+a(c-3d))\sqrt[3]{x+x^3}}{12c^2\sqrt[3]{x}}$$

$$= \frac{ax\sqrt[3]{x+x^3}}{2c} - \frac{(3bc+a(c-3d))\sqrt[3]{x+x^3}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{1+x^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\,c^2\sqrt[3]{x}\sqrt[3]{1+x^2}} - \frac{\sqrt{3}\sqrt[3]{c-d}\,(bc-ad)\sqrt[3]{x+}}{2c^2\sqrt[3]{d}\sqrt[3]{x}}$$

Mathematica [C] time = 0.26, size = 168, normalized size = 0.46

$$\frac{x\sqrt[3]{x^3+x}\left(2x^2\left(\frac{cx^2}{d}+1\right)^{2/3}(a(c-3d)+3bc)F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-x^2,-\frac{cx^2}{d}\right)+5\left((3bc-2ad)\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(c-d)x^2}{cx^2+d}\right)+2ad\sqrt[3]{x^2+1}\left(\frac{cx^2}{d}+1\right)^{2/3}\right)\right)}{20cd\sqrt[3]{x^2+1}\left(\frac{cx^2}{d}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((b + a*x^2)*(x + x^3)^(1/3))/(d + c*x^2),x]
```

```
[Out] (x*(x + x^3)^(1/3)*(2*(3*b*c + a*(c - 3*d))*x^2*(1 + (c*x^2)/d)^(2/3)*Appel
lF1[5/3, 2/3, 1, 8/3, -x^2, -((c*x^2)/d)] + 5*(2*a*d*(1 + x^2)^(1/3)*(1 + (
```

```
c*x^2)/d)^(2/3) + (3*b*c - 2*a*d)*Hypergeometric2F1[2/3, 2/3, 5/3, ((c - d)
*x^2)/(d + c*x^2)])))/(20*c*d*(1 + x^2)^(1/3)*(1 + (c*x^2)/d)^(2/3))
```

IntegrateAlgebraic [A] time = 1.22, size = 367, normalized size = 1.00

$$\frac{\sqrt[3]{c-d}\,(bc-ad)\log\left(-\sqrt[3]{d}\,\sqrt[3]{x^3+x}\,x\sqrt[3]{c-d}+x^2(c-d)^{2/3}+d^{2/3}\left(x^3+x\right)^{2/3}\right)}{4c^2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{x^3+x}-x\right)(-ac+3ad-3bc)}{6c^2}-\frac{\sqrt[3]{c-d}\,(bc-ad)\log\left(x\sqrt[3]{c-d}+\sqrt[3]{d}\,\sqrt[3]{x^3+x}\right)}{2c^2\sqrt[3]{d}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3+x}+x}\right)(ac-3ad+3bc)}{2\sqrt{3}c^2}-\frac{\sqrt{3}\,\sqrt[3]{c-d}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}x\sqrt[3]{c-d}}{x\sqrt[3]{c-d}-2\sqrt[3]{d}\,\sqrt[3]{x^3+x}}\right)}{2c^2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{x^3+x}\,x+\left(x^3+x\right)^{2/3}+x^2\right)(ac-3ad+3bc)}{12c^2}+\frac{a\sqrt[3]{x^3+x}\,x}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b + a*x^2)*(x + x^3)^(1/3))/(d + c*x^2),x]
```

```
[Out] (a*x*(x + x^3)^(1/3))/(2*c) - ((a*c + 3*b*c - 3*a*d)*ArcTan[(Sqrt[3]*x)/(x
+ 2*(x + x^3)^(1/3))])/(2*Sqrt[3]*c^2) - (Sqrt[3]*(c - d)^(1/3)*(b*c - a*d)
*ArcTan[(Sqrt[3]*(c - d)^(1/3)*x)/((c - d)^(1/3)*x - 2*d^(1/3)*(x + x^3)^(1
/3))])/(2*c^2*d^(1/3)) + ((-(a*c) - 3*b*c + 3*a*d)*Log[-x + (x + x^3)^(1/3)
])/(6*c^2) - ((c - d)^(1/3)*(b*c - a*d)*Log[(c - d)^(1/3)*x + d^(1/3)*(x +
x^3)^(1/3)])/(2*c^2*d^(1/3)) + ((a*c + 3*b*c - 3*a*d)*Log[x^2 + x*(x + x^3)
^(1/3) + (x + x^3)^(2/3)])/(12*c^2) + ((c - d)^(1/3)*(b*c - a*d)*Log[(c - d
)^(2/3)*x^2 - (c - d)^(1/3)*d^(1/3)*x*(x + x^3)^(1/3) + d^(2/3)*(x + x^3)^(
2/3)])/(4*c^2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)*(x^3+x)^(1/3)/(c*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.89, size = 354, normalized size = 0.96

$$\frac{ax^2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}}{2c}+\frac{\left(bc^2-acd-bcd+ad^2\right)\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right|\right)}{2\left(c^3-c^2d\right)}+\frac{\sqrt{3}\,(ac+3bc-3ad)\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)}{6c^2}+\frac{(ac+3bc-3ad)\log\left(\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)}{12c^2}-\frac{(ac+3bc-3ad)\log\left(\left|\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)}{6c^2}-\frac{\left(\sqrt{3}\left(-cd^2+d^3\right)^{\frac{1}{3}}bc-\sqrt{3}\left(-cd^2+d^3\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}+2\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}}\right)}{2c^2d}-\frac{\left(\left(-cd^2+d^3\right)^{\frac{1}{3}}bc-\left(-cd^2+d^3\right)^{\frac{1}{3}}ad\right)\log\left(\left(-\frac{c-d}{d}\right)^{\frac{2}{3}}+\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}+\left(\frac{1}{x^2}+1\right)^{\frac{2}{3}}\right)}{4c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)*(x^3+x)^(1/3)/(c*x^2+d),x, algorithm="giac")
```

```
[Out] 1/2*a*x^2*(1/x^2 + 1)^(1/3)/c + 1/2*(b*c^2 - a*c*d - b*c*d + a*d^2)*(-(c -
d)/d)^(1/3)*log(abs(-(-(c - d)/d)^(1/3) + (1/x^2 + 1)^(1/3)))/(c^3 - c^2*d)
 + 1/6*sqrt(3)*(a*c + 3*b*c - 3*a*d)*arctan(1/3*sqrt(3)*(2*(1/x^2 + 1)^(1/3
) + 1))/c^2 + 1/12*(a*c + 3*b*c - 3*a*d)*log((1/x^2 + 1)^(2/3) + (1/x^2 + 1
)^(1/3) + 1)/c^2 - 1/6*(a*c + 3*b*c - 3*a*d)*log(abs((1/x^2 + 1)^(1/3) - 1)
)/c^2 - 1/2*(sqrt(3)*(-c*d^2 + d^3)^(1/3)*b*c - sqrt(3)*(-c*d^2 + d^3)^(1/3
)*a*d)*arctan(1/3*sqrt(3)*((-(c - d)/d)^(1/3) + 2*(1/x^2 + 1)^(1/3))/(-(c -
 d)/d)^(1/3))/(c^2*d) - 1/4*((-c*d^2 + d^3)^(1/3)*b*c - (-c*d^2 + d^3)^(1/3
)*a*d)*log((-(c - d)/d)^(2/3) + (-(c - d)/d)^(1/3)*(1/x^2 + 1)^(1/3) + (1/x
^2 + 1)^(2/3))/(c^2*d)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^2+b\right)\left(x^3+x\right)^{\frac{1}{3}}}{c\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b)*(x^3+x)^(1/3)/(c*x^2+d),x)
```

```
[Out] int((a*x^2+b)*(x^3+x)^(1/3)/(c*x^2+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^2+b)(x^3+x)^{\frac{1}{3}}}{cx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b)*(x^3+x)^(1/3)/(c*x^2+d),x, algorithm="maxima")
[Out] integrate((a*x^2 + b)*(x^3 + x)^(1/3)/(c*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a\,x^2+b)\,(x^3+x)^{1/3}}{c\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a*x^2)*(x + x^3)^(1/3))/(d + c*x^2),x)
[Out] int(((b + a*x^2)*(x + x^3)^(1/3))/(d + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x(x^2+1)}\,(ax^2+b)}{cx^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b)*(x**3+x)**(1/3)/(c*x**2+d),x)
[Out] Integral((x*(x**2 + 1))**(1/3)*(a*x**2 + b)/(c*x**2 + d), x)
```

3.2327 $\int \frac{ab+ac-2bc+(-2a+b+c)x}{\sqrt[3]{(-a+x)(-b+x)(-c+x)}\left(a^2-bcd+(-2a+bd+cd)x+(1-d)x^2\right)}dx$

Optimal. Leaf size=369

$$-\frac{i\left(\sqrt{3}-i\right)\log\left(\sqrt[3]{-1}\left(a^2-2ax+x^2\right)-d^{2/3}\left(x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3\right)^{2/3}+(-1)^{2/3}\sqrt[3]{d}\,(a-\right.}{4d^{2/3}}$$

Rubi [F] time = 10.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ab+ac-2bc+(-2a+b+c)x}{\sqrt[3]{(-a+x)(-b+x)(-c+x)}\left(a^2-bcd+(-2a+bd+cd)x+(1-d)x^2\right)}dx$$

Verification is not applicable to the result.

```
[In] Int[(a*b + a*c - 2*b*c + (-2*a + b + c)*x)/(((-a + x)*(-b + x)*(-c + x))^(1
/3)*(a^2 - b*c*d + (-2*a + b*d + c*d)*x + (1 - d)*x^2)),x]
```

```
[Out] -(((2*a - b - c + Sqrt[4*a^2 - 4*a*(b + c) + 2*b*c*(2 - d) + b^2*d + c^2*d]
/Sqrt[d])*(-a + x)^(1/3)*(-b + x)^(1/3)*(-c + x)^(1/3)*Defer[Int][1/((-a +
x)^(1/3)*(-b + x)^(1/3)*(-c + x)^(1/3)*(-2*a + b*d + c*d - Sqrt[d]*Sqrt[4*a
^2 - 4*a*b - 4*a*c + 4*b*c + b^2*d - 2*b*c*d + c^2*d] + 2*(1 - d)*x)), x])/
(-((a - x)*(b - x)*(c - x)))^(1/3)) - ((2*a - b - c - Sqrt[4*a^2 - 4*a*(b +
 c) + 2*b*c*(2 - d) + b^2*d + c^2*d]/Sqrt[d])*(-a + x)^(1/3)*(-b + x)^(1/3)
*(-c + x)^(1/3)*Defer[Int][1/((-a + x)^(1/3)*(-b + x)^(1/3)*(-c + x)^(1/3)*
(-2*a + b*d + c*d + Sqrt[d]*Sqrt[4*a^2 - 4*a*b - 4*a*c + 4*b*c + b^2*d - 2*
b*c*d + c^2*d] + 2*(1 - d)*x)), x])/(-((a - x)*(b - x)*(c - x)))^(1/3)
```

Rubi steps

$$\int \frac{ab+ac-2bc+(-2a+b+c)x}{\sqrt[3]{(-a+x)(-b+x)(-c+x)}\left(a^2-bcd+(-2a+bd+cd)x+(1-d)x^2\right)}dx = \frac{\left(\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\,\sqrt[3]{-c+x}\right)\int}{\sqrt[3]{(-a}}$$

$$= \frac{\left(\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\,\sqrt[3]{-c+x}\right)\int}{}$$

$$= \frac{\left(\left(-2a+b+c-\frac{\sqrt{4a^2-4a(b+c)+2b}}{\sqrt{d}}\right.\right.}{}$$

Mathematica [F] time = 9.93, size = 0, normalized size = 0.00

$$\int \frac{ab+ac-2bc+(-2a+b+c)x}{\sqrt[3]{(-a+x)(-b+x)(-c+x)}\left(a^2-bcd+(-2a+bd+cd)x+(1-d)x^2\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*b + a*c - 2*b*c + (-2*a + b + c)*x)/(((-a + x)*(-b + x)*(-c +
x))^(1/3)*(a^2 - b*c*d + (-2*a + b*d + c*d)*x + (1 - d)*x^2)),x]
```

```
[Out] Integrate[(a*b + a*c - 2*b*c + (-2*a + b + c)*x)/(((-a + x)*(-b + x)*(-c +
x))^(1/3)*(a^2 - b*c*d + (-2*a + b*d + c*d)*x + (1 - d)*x^2)), x]
```

IntegrateAlgebraic [A] time = 6.38, size = 455, normalized size = 1.23

$$-\frac{i\left(\sqrt{3}-i\right)\log\left(\sqrt{3}\left(-ia^2+\sqrt[3]{d}\,(ix-ia)\sqrt[3]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}+2iax-ix^2\right)-a^2+2d^{2/3}\left(x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3\right)^{2/3}+\sqrt[3]{d}\,(a-x)\sqrt[3]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}+2ax-x^2\right)}{4d^{2/3}}+\frac{\left(1+i\sqrt{3}\right)\log\left(2\sqrt[3]{d}\,\sqrt[3]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}+\sqrt{3}\,(ia-ix)-a+x\right)}{2d^{2/3}}+\frac{\sqrt{\frac{1}{2}\left(-3+3i\sqrt{3}\right)}\tan^{-1}\left(\frac{3\sqrt[3]{d}\,\sqrt[3]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{x^2(-a-b-c)+x(ab+ac+bc)-abc+x^3}+\sqrt{3}\,a-3ia-\sqrt{3}\,x+3ix}\right)}{d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*b + a*c - 2*b*c + (-2*a + b + c)*x)/(((-a + x)*(-b +
x)*(-c + x))^(1/3)*(a^2 - b*c*d + (-2*a + b*d + c*d)*x + (1 - d)*x^2)),x]
```

```
[Out] (Sqrt[(-3 + (3*I)*Sqrt[3])/2]*ArcTan[(3*d^(1/3)*(-(a*b*c) + (a*b + a*c + b*
c)*x + (-a - b - c)*x^2 + x^3)^(1/3))/((-3*I)*a + Sqrt[3]*a + (3*I)*x - Sqr
t[3]*x + Sqrt[3]*d^(1/3)*(-(a*b*c) + (a*b + a*c + b*c)*x + (-a - b - c)*x^2
 + x^3)^(1/3))])/d^(2/3) + ((1 + I*Sqrt[3])*Log[-a + Sqrt[3]*(I*a - I*x) +
x + 2*d^(1/3)*(-(a*b*c) + (a*b + a*c + b*c)*x + (-a - b - c)*x^2 + x^3)^(1/
3)])/(2*d^(2/3)) - ((I/4)*(-I + Sqrt[3])*Log[-a^2 + 2*a*x - x^2 + d^(1/3)*(
a - x)*(-(a*b*c) + (a*b + a*c + b*c)*x + (-a - b - c)*x^2 + x^3)^(1/3) + 2*
d^(2/3)*(-(a*b*c) + (a*b + a*c + b*c)*x + (-a - b - c)*x^2 + x^3)^(2/3) + S
qrt[3]*((-I)*a^2 + (2*I)*a*x - I*x^2 + d^(1/3)*((-I)*a + I*x)*(-(a*b*c) + (
a*b + a*c + b*c)*x + (-a - b - c)*x^2 + x^3)^(1/3))])/d^(2/3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b+a*c-2*b*c+(-2*a+b+c)*x)/((-a+x)*(-b+x)*(-c+x))^(1/3)/(a^2-b*
c*d+(b*d+c*d-2*a)*x+(1-d)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ab+ac-2\,bc-(2\,a-b-c)x}{(-(a-x)(b-x)(c-x))^{\frac{1}{3}}\left(bcd+(d-1)x^2-a^2-(bd+cd-2\,a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b+a*c-2*b*c+(-2*a+b+c)*x)/((-a+x)*(-b+x)*(-c+x))^(1/3)/(a^2-b*
c*d+(b*d+c*d-2*a)*x+(1-d)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-(a*b + a*c - 2*b*c - (2*a - b - c)*x)/((-(a - x)*(b - x)*(c - x)
)^(1/3)*(b*c*d + (d - 1)*x^2 - a^2 - (b*d + c*d - 2*a)*x)), x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{ab+ac-2bc+(-2a+b+c)\,x}{\left((-a+x)\,(-b+x)\,(-c+x)\right)^{\frac{1}{3}}\left(a^2-bcd+(bd+cd-2a)\,x+(1-d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b+a*c-2*b*c+(-2*a+b+c)*x)/((-a+x)*(-b+x)*(-c+x))^(1/3)/(a^2-b*c*d+(b
*d+c*d-2*a)*x+(1-d)*x^2),x)
```

```
[Out] int((a*b+a*c-2*b*c+(-2*a+b+c)*x)/((-a+x)*(-b+x)*(-c+x))^(1/3)/(a^2-b*c*d+(b
*d+c*d-2*a)*x+(1-d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ab+ac-2\,bc-(2\,a-b-c)x}{(-(a-x)(b-x)(c-x))^{\frac{1}{3}}\left(bcd+(d-1)x^2-a^2-(bd+cd-2\,a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b+a*c-2*b*c+(-2*a+b+c)*x)/((-a+x)*(-b+x)*(-c+x))^(1/3)/(a^2-b*
c*d+(b*d+c*d-2*a)*x+(1-d)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((a*b + a*c - 2*b*c - (2*a - b - c)*x)/((-(a - x)*(b - x)*(c - x)
)^(1/3)*(b*c*d + (d - 1)*x^2 - a^2 - (b*d + c*d - 2*a)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,b+a\,c-2\,b\,c+x\,\left(b-2\,a+c\right)}{{\left(-\left(a-x\right)\,\left(b-x\right)\,\left(c-x\right)\right)}^{1/3}\,\left(x\,\left(b\,d-2\,a+c\,d\right)+a^2-x^2\,\left(d-1\right)-b\,c\,d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*b + a*c - 2*b*c + x*(b - 2*a + c))/((-(a - x)*(b - x)*(c - x))^(1/3)
*(x*(b*d - 2*a + c*d) + a^2 - x^2*(d - 1) - b*c*d)),x)
```

```
[Out] int((a*b + a*c - 2*b*c + x*(b - 2*a + c))/((-(a - x)*(b - x)*(c - x))^(1/3)
*(x*(b*d - 2*a + c*d) + a^2 - x^2*(d - 1) - b*c*d)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*b+a*c-2*b*c+(-2*a+b+c)*x)/((-a+x)*(-b+x)*(-c+x))**(1/3)/(a**2-
b*c*d+(b*d+c*d-2*a)*x+(1-d)*x**2),x)
```

```
[Out] Timed out
```

3.2328 $\int \frac{(-2+x)(1-x+x^2)}{x^3(-1+x+x^2)\sqrt[3]{\frac{1-x+2x^2}{1-x+3x^2}}}\,dx$

Optimal. Leaf size=370

$$\frac{\left(\frac{2x^2-x+1}{3x^2-x+1}\right)^{2/3}(-3x^2+x-1)}{x^2}+\frac{7}{3}\log\left(\sqrt[3]{\frac{2x^2-x+1}{3x^2-x+1}}-1\right)-\frac{2\,2^{2/3}\log\left(6^{2/3}\sqrt[3]{\frac{2x^2-x+1}{3x^2-x+1}}-3\right)}{\sqrt[3]{3}}-\frac{7}{6}\log\left(\left(\frac{2x^2-x+1}{3x^2-x+1}\right)^{2/3}\right.$$

Rubi [F] time = 4.93, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+x)\left(1-x+x^2\right)}{x^3\left(-1+x+x^2\right)\sqrt[3]{\frac{1-x+2x^2}{1-x+3x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + x)*(1 - x + x^2))/(x^3*(-1 + x + x^2)*((1 - x + 2*x^2)/(1 - x +
3*x^2))^(1/3)),x]
```

```
[Out] (2*(1 - x + 2*x^2)^(1/3)*Defer[Int][(1 - x + 3*x^2)^(1/3)/(x^3*(1 - x + 2*x
^2)^(1/3)), x])/(((1 - x + 2*x^2)/(1 - x + 3*x^2))^(1/3)*(1 - x + 3*x^2)^(1
/3)) - ((1 - x + 2*x^2)^(1/3)*Defer[Int][(1 - x + 3*x^2)^(1/3)/(x^2*(1 - x
+ 2*x^2)^(1/3)), x])/(((1 - x + 2*x^2)/(1 - x + 3*x^2))^(1/3)*(1 - x + 3*x^
2)^(1/3)) + (4*(1 - x + 2*x^2)^(1/3)*Defer[Int][(1 - x + 3*x^2)^(1/3)/(x*(1
 - x + 2*x^2)^(1/3)), x])/(((1 - x + 2*x^2)/(1 - x + 3*x^2))^(1/3)*(1 - x +
 3*x^2)^(1/3)) - (4*(1 - x + 2*x^2)^(1/3)*Defer[Int][(1 - x + 3*x^2)^(1/3)/
((1 - Sqrt[5] + 2*x)*(1 - x + 2*x^2)^(1/3)), x])/(((1 - x + 2*x^2)/(1 - x +
 3*x^2))^(1/3)*(1 - x + 3*x^2)^(1/3)) - (4*(1 - x + 2*x^2)^(1/3)*Defer[Int]
[(1 - x + 3*x^2)^(1/3)/((1 + Sqrt[5] + 2*x)*(1 - x + 2*x^2)^(1/3)), x])/(((
1 - x + 2*x^2)/(1 - x + 3*x^2))^(1/3)*(1 - x + 3*x^2)^(1/3))
```

Rubi steps

$$\int \frac{(-2+x)\left(1-x+x^2\right)}{x^3\left(-1+x+x^2\right)\sqrt[3]{\frac{1-x+2x^2}{1-x+3x^2}}}\,dx = \frac{\sqrt[3]{1-x+2x^2}\int \frac{(-2+x)\left(1-x+x^2\right)\sqrt[3]{1-x+3x^2}}{x^3\left(-1+x+x^2\right)\sqrt[3]{1-x+2x^2}}\,dx}{\sqrt[3]{\frac{1-x+2x^2}{1-x+3x^2}}\sqrt[3]{1-x+3x^2}}$$

$$= \frac{\sqrt[3]{1-x+2x^2}\int\left(\frac{2\sqrt[3]{1-x+3x^2}}{x^3\sqrt[3]{1-x+2x^2}} - \frac{\sqrt[3]{1-x+3x^2}}{x^2\sqrt[3]{1-x+2x^2}} + \frac{4\sqrt[3]{1-x+3x^2}}{x\sqrt[3]{1-x+2x^2}} - \frac{2(1+2x)\sqrt[3]{1-x+3x^2}}{\left(-1+x+x^2\right)\sqrt[3]{1-x+2x}}\right.}{\sqrt[3]{\frac{1-x+2x^2}{1-x+3x^2}}\sqrt[3]{1-x+3x^2}}$$

$$= -\frac{\sqrt[3]{1-x+2x^2}\int\frac{\sqrt[3]{1-x+3x^2}}{x^2\sqrt[3]{1-x+2x^2}}\,dx}{\sqrt[3]{\frac{1-x+2x^2}{1-x+3x^2}}\sqrt[3]{1-x+3x^2}} + \frac{\left(2\sqrt[3]{1-x+2x^2}\right)\int\frac{\sqrt[3]{1-x+3x^2}}{x^3\sqrt[3]{1-x+2x^2}}\,dx}{\sqrt[3]{\frac{1-x+2x^2}{1-x+3x^2}}\sqrt[3]{1-x+3x^2}} - \frac{(2}{}$$

$$= -\frac{\sqrt[3]{1-x+2x^2}\int\frac{\sqrt[3]{1-x+3x^2}}{x^2\sqrt[3]{1-x+2x^2}}\,dx}{\sqrt[3]{\frac{1-x+2x^2}{1-x+3x^2}}\sqrt[3]{1-x+3x^2}} + \frac{\left(2\sqrt[3]{1-x+2x^2}\right)\int\frac{\sqrt[3]{1-x+3x^2}}{x^3\sqrt[3]{1-x+2x^2}}\,dx}{\sqrt[3]{\frac{1-x+2x^2}{1-x+3x^2}}\sqrt[3]{1-x+3x^2}} - \frac{(2}{}$$

$$= -\frac{\sqrt[3]{1-x+2x^2}\int\frac{\sqrt[3]{1-x+3x^2}}{x^2\sqrt[3]{1-x+2x^2}}\,dx}{\sqrt[3]{\frac{1-x+2x^2}{1-x+3x^2}}\sqrt[3]{1-x+3x^2}} + \frac{\left(2\sqrt[3]{1-x+2x^2}\right)\int\frac{\sqrt[3]{1-x+3x^2}}{x^3\sqrt[3]{1-x+2x^2}}\,dx}{\sqrt[3]{\frac{1-x+2x^2}{1-x+3x^2}}\sqrt[3]{1-x+3x^2}} + \frac{(4}{}$$

Mathematica [F] time = 1.39, size = 0, normalized size = 0.00

$$\int \frac{(-2+x)\left(1-x+x^2\right)}{x^3\left(-1+x+x^2\right)\sqrt[3]{\frac{1-x+2x^2}{1-x+3x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + x)*(1 - x + x^2))/(x^3*(-1 + x + x^2)*((1 - x + 2*x^2)/(1
- x + 3*x^2))^(1/3)),x]
```

```
[Out] Integrate[((-2 + x)*(1 - x + x^2))/(x^3*(-1 + x + x^2)*((1 - x + 2*x^2)/(1
- x + 3*x^2))^(1/3)), x]
```

IntegrateAlgebraic [A] time = 1.34, size = 370, normalized size = 1.00

$$\frac{\left(\frac{2x^2-x+1}{3x^2-x+1}\right)^{2/3}\left(-3x^2+x-1\right)}{x^2}+\frac{7}{3}\log\left(\sqrt[3]{\frac{2x^2-x+1}{3x^2-x+1}}-1\right)-\frac{2\,2^{2/3}\log\left(6^{2/3}\sqrt[3]{\frac{2x^2-x+1}{3x^2-x+1}}-3\right)}{\sqrt[3]{3}}-\frac{7}{6}\log\left(\left(\frac{2x^2-x+1}{3x^2-x+1}\right)^{2/3}+\sqrt[3]{\frac{2x^2-x+1}{3x^2-x+1}}+1\right)+\frac{2^{2/3}\log\left(2\sqrt[3]{6}\left(\frac{2x^2-x+1}{3x^2-x+1}\right)^{2/3}+6^{2/3}\sqrt[3]{\frac{2x^2-x+1}{3x^2-x+1}}+3\right)}{\sqrt[3]{3}}-2\,2^{2/3}\sqrt[6]{3}\tan^{-1}\left(\frac{2\,2^{2/3}\sqrt[3]{\frac{2x^2-x+1}{3x^2-x+1}}}{3^{5/6}}+\frac{1}{\sqrt{3}}\right)+\frac{7\tan^{-1}\left(\frac{2\sqrt[3]{\frac{2x^2-x+1}{3x^2-x+1}}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + x)*(1 - x + x^2))/(x^3*(-1 + x + x^2)*((1 - x + 2
*x^2)/(1 - x + 3*x^2))^(1/3)),x]
```

```
[Out] ((-1 + x - 3*x^2)*((1 - x + 2*x^2)/(1 - x + 3*x^2))^(2/3))/x^2 - 2*2^(2/3)*
3^(1/6)*ArcTan[1/Sqrt[3] + (2*2^(2/3)*((1 - x + 2*x^2)/(1 - x + 3*x^2))^(1/
3))/3^(5/6)] + (7*ArcTan[1/Sqrt[3] + (2*((1 - x + 2*x^2)/(1 - x + 3*x^2))^(
1/3))/Sqrt[3]])/Sqrt[3] + (7*Log[-1 + ((1 - x + 2*x^2)/(1 - x + 3*x^2))^(1/
3)])/3 - (2*2^(2/3)*Log[-3 + 6^(2/3)*((1 - x + 2*x^2)/(1 - x + 3*x^2))^(1/3
)])/3^(1/3) - (7*Log[1 + ((1 - x + 2*x^2)/(1 - x + 3*x^2))^(1/3) + ((1 - x
+ 2*x^2)/(1 - x + 3*x^2))^(2/3)])/6 + (2^(2/3)*Log[3 + 6^(2/3)*((1 - x + 2*
x^2)/(1 - x + 3*x^2))^(1/3) + 2*6^(1/3)*((1 - x + 2*x^2)/(1 - x + 3*x^2))^(
2/3)])/3^(1/3)
```

fricas [B] time = 18.01, size = 735, normalized size = 1.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)*(x^2-x+1)/x^3/(x^2+x-1)/((2*x^2-x+1)/(3*x^2-x+1))^(1/3),x,
 algorithm="fricas")
```

```
[Out] -1/18*(2*3^(2/3)*(-4)^(1/3)*x^2*log(-(24*3^(2/3)*(-4)^(1/3)*(39*x^4 - 28*x^
3 + 33*x^2 - 10*x + 5)*((2*x^2 - x + 1)/(3*x^2 - x + 1))^(2/3) - 3^(1/3)*(-
4)^(2/3)*(649*x^4 - 538*x^3 + 647*x^2 - 218*x + 109) - 36*(75*x^4 - 58*x^3
+ 69*x^2 - 22*x + 11)*((2*x^2 - x + 1)/(3*x^2 - x + 1))^(1/3))/(x^4 + 2*x^3
 - x^2 - 2*x + 1)) - 4*3^(2/3)*(-4)^(1/3)*x^2*log(-(9*3^(1/3)*(-4)^(2/3)*(3
*x^2 - x + 1)*((2*x^2 - x + 1)/(3*x^2 - x + 1))^(1/3) + 3^(2/3)*(-4)^(1/3)*
(x^2 + x - 1) - 36*(3*x^2 - x + 1)*((2*x^2 - x + 1)/(3*x^2 - x + 1))^(2/3))
/(x^2 + x - 1)) + 12*3^(1/6)*(-4)^(1/3)*x^2*arctan(1/3*3^(1/6)*(12*3^(2/3)*
(-4)^(2/3)*(39*x^6 + 11*x^5 - 34*x^4 + 51*x^3 - 38*x^2 + 15*x - 5)*((2*x^2
- x + 1)/(3*x^2 - x + 1))^(2/3) + 18*(-4)^(1/3)*(1947*x^6 - 2263*x^5 + 3128
*x^4 - 1839*x^3 + 1192*x^2 - 327*x + 109)*((2*x^2 - x + 1)/(3*x^2 - x + 1))
^(1/3) - 3^(1/3)*(16199*x^6 - 20631*x^5 + 29268*x^4 - 18463*x^3 + 12204*x^2
 - 3567*x + 1189))/(17497*x^6 - 20409*x^5 + 28188*x^4 - 16529*x^3 + 10692*x
^2 - 2913*x + 971)) + 42*sqrt(3)*x^2*arctan((26407150*sqrt(3)*(3*x^2 - x +
1)*((2*x^2 - x + 1)/(3*x^2 - x + 1))^(2/3) + 15172108*sqrt(3)*(3*x^2 - x +
1)*((2*x^2 - x + 1)/(3*x^2 - x + 1))^(1/3) + sqrt(3)*(47470762*x^2 - 207896
29*x + 20789629))/(29760814*x^2 - 16852563*x + 16852563)) - 21*x^2*log((x^2
 + 3*(3*x^2 - x + 1)*((2*x^2 - x + 1)/(3*x^2 - x + 1))^(2/3) - 3*(3*x^2 - x
 + 1)*((2*x^2 - x + 1)/(3*x^2 - x + 1))^(1/3))/x^2) + 18*(3*x^2 - x + 1)*((
2*x^2 - x + 1)/(3*x^2 - x + 1))^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2 - x + 1)(x - 2)}{(x^2 + x - 1)x^3 \left(\frac{2x^2 - x + 1}{3x^2 - x + 1}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)*(x^2-x+1)/x^3/(x^2+x-1)/((2*x^2-x+1)/(3*x^2-x+1))^(1/3),x,
 algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)*(x - 2)/((x^2 + x - 1)*x^3*((2*x^2 - x + 1)/(3*x^2
- x + 1))^(1/3)), x)
```

maple [F] time = 1.83, size = 0, normalized size = 0.00

$$\int \frac{(-2 + x)(x^2 - x + 1)}{x^3 (x^2 + x - 1) \left(\frac{2x^2 - x + 1}{3x^2 - x + 1}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+x)*(x^2-x+1)/x^3/(x^2+x-1)/((2*x^2-x+1)/(3*x^2-x+1))^(1/3),x)
```

```
[Out] int((-2+x)*(x^2-x+1)/x^3/(x^2+x-1)/((2*x^2-x+1)/(3*x^2-x+1))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2 - x + 1)(x - 2)}{(x^2 + x - 1)x^3 \left(\frac{2x^2 - x + 1}{3x^2 - x + 1}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)*(x^2-x+1)/x^3/(x^2+x-1)/((2*x^2-x+1)/(3*x^2-x+1))^(1/3),x,
 algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)*(x - 2)/((x^2 + x - 1)*x^3*((2*x^2 - x + 1)/(3*x^2
- x + 1))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x-2)\left(x^2-x+1\right)}{x^3\left(\frac{2x^2-x+1}{3x^2-x+1}\right)^{1/3}\left(x^2+x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 2)*(x^2 - x + 1))/(x^3*((2*x^2 - x + 1)/(3*x^2 - x + 1))^(1/3)*(x
 + x^2 - 1)),x)
```

```
[Out] int(((x - 2)*(x^2 - x + 1))/(x^3*((2*x^2 - x + 1)/(3*x^2 - x + 1))^(1/3)*(x
 + x^2 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)*(x**2-x+1)/x**3/(x**2+x-1)/((2*x**2-x+1)/(3*x**2-x+1))**(1
/3),x)
```

```
[Out] Timed out
```

3.2329 $\int \frac{x^3(5b+9ax^4)}{\sqrt[4]{bx+ax^5}\,(1+bx^5+ax^9)}\,dx$

Optimal. Leaf size=371

$$-\sqrt{2}\tan^{-1}\left(\frac{x^2\sqrt[4]{ax^5+bx}-2^{2/3}x\sqrt[4]{ax^5+bx}}{2^{2/3}x\sqrt[4]{ax^5+bx}+x^2\left(-\sqrt[4]{ax^5+bx}\right)-\sqrt{2}x+2\sqrt[6]{2}}\right)+\sqrt{2}\tan^{-1}\left(\frac{x^2\sqrt[4]{ax^5+bx}-2^{2/3}x\sqrt[4]{ax}}{2^{2/3}x\sqrt[4]{ax^5+bx}+x^2\left(-\sqrt[4]{ax^5+bx}\right.}\right.$$

Rubi [F] time = 2.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3(5b+9ax^4)}{\sqrt[4]{bx+ax^5}\,(1+bx^5+ax^9)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*(5*b + 9*a*x^4))/((b*x + a*x^5)^(1/4)*(1 + b*x^5 + a*x^9)),x]
```

```
[Out] (20*b*x^(1/4)*(b + a*x^4)^(1/4)*Defer[Subst][Defer[Int][x^14/((b + a*x^16)^
(1/4)*(1 + b*x^20 + a*x^36)), x], x, x^(1/4)])/(b*x + a*x^5)^(1/4) + (36*a*
x^(1/4)*(b + a*x^4)^(1/4)*Defer[Subst][Defer[Int][x^30/((b + a*x^16)^(1/4)*
(1 + b*x^20 + a*x^36)), x], x, x^(1/4)])/(b*x + a*x^5)^(1/4)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(5b+9ax^4)}{\sqrt[4]{bx+ax^5}\,(1+bx^5+ax^9)}\,dx &= \frac{\left(\sqrt[4]{x}\sqrt[4]{b+ax^4}\right)\int \frac{x^{11/4}(5b+9ax^4)}{\sqrt[4]{b+ax^4}\,(1+bx^5+ax^9)}\,dx}{\sqrt[4]{bx+ax^5}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^4}\right)\operatorname{Subst}\left(\int \frac{x^{14}(5b+9ax^{16})}{\sqrt[4]{b+ax^{16}}\,(1+bx^{20}+ax^{36})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^5}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{b+ax^4}\right)\operatorname{Subst}\left(\int\left(\frac{5bx^{14}}{\sqrt[4]{b+ax^{16}}\,(1+bx^{20}+ax^{36})}+\frac{9ax^{30}}{\sqrt[4]{b+ax^{16}}\,(1+bx^{20}+ax^{36})}\right)dx,x\right.}{\sqrt[4]{bx+ax^5}}\\
&= \frac{\left(36a\sqrt[4]{x}\sqrt[4]{b+ax^4}\right)\operatorname{Subst}\left(\int \frac{x^{30}}{\sqrt[4]{b+ax^{16}}\,(1+bx^{20}+ax^{36})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx+ax^5}}+\frac{\left(20b\sqrt[4]{x}\sqrt[4]{b+\right.}{}
\end{aligned}$$

Mathematica [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x^3(5b+9ax^4)}{\sqrt[4]{bx+ax^5}\,(1+bx^5+ax^9)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(5*b + 9*a*x^4))/((b*x + a*x^5)^(1/4)*(1 + b*x^5 + a*x^9)),x
]
```

```
[Out] Integrate[(x^3*(5*b + 9*a*x^4))/((b*x + a*x^5)^(1/4)*(1 + b*x^5 + a*x^9)),
x]
```

IntegrateAlgebraic [A] time = 16.74, size = 371, normalized size = 1.00

$$-\sqrt{2}\tan^{-1}\left(\frac{x^2\sqrt[4]{ax^5+bx}-2^{2/3}x\sqrt[4]{ax^5+bx}}{2^{2/3}x\sqrt[4]{ax^5+bx}+x^2\left(-\sqrt[4]{ax^5+bx}\right)-\sqrt{2}x+2\sqrt[6]{2}}\right)+\sqrt{2}\tan^{-1}\left(\frac{x^2\sqrt[4]{ax^5+bx}-2^{2/3}x\sqrt[4]{ax^5+bx}}{2^{2/3}x\sqrt[4]{ax^5+bx}+x^2\left(-\sqrt[4]{ax^5+bx}\right)+\sqrt{2}x-2\sqrt[6]{2}}\right)-\sqrt{2}\tanh^{-1}\left(\frac{-2\,2^{5/6}x\sqrt[4]{ax^5+bx}-\sqrt{2}x^3\sqrt[4]{ax^5+bx}+4\sqrt[6]{2}x^2\sqrt[4]{ax^5+bx}}{x^4\left(-\sqrt{ax^5+bx}\right)+2\,2^{2/3}x^3\sqrt{ax^5+bx}-2\sqrt[3]{2}x^2\sqrt{ax^5+bx}-x^2+2\,2^{2/3}x-2\sqrt[3]{2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(5*b + 9*a*x^4))/((b*x + a*x^5)^(1/4)*(1 + b*x^5 +
a*x^9)),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(-(2^(2/3)*x*(b*x + a*x^5)^(1/4)) + x^2*(b*x + a*x^5)^(1/4
))/(2*2^(1/6) - Sqrt[2]*x + 2^(2/3)*x*(b*x + a*x^5)^(1/4) - x^2*(b*x + a*x^
5)^(1/4))]) + Sqrt[2]*ArcTan[(-(2^(2/3)*x*(b*x + a*x^5)^(1/4)) + x^2*(b*x +
 a*x^5)^(1/4))/(-2*2^(1/6) + Sqrt[2]*x + 2^(2/3)*x*(b*x + a*x^5)^(1/4) - x^
2*(b*x + a*x^5)^(1/4))] - Sqrt[2]*ArcTanh[(-2*2^(5/6)*x*(b*x + a*x^5)^(1/4)
 + 4*2^(1/6)*x^2*(b*x + a*x^5)^(1/4) - Sqrt[2]*x^3*(b*x + a*x^5)^(1/4))/(-2
*2^(1/3) + 2*2^(2/3)*x - x^2 - 2*2^(1/3)*x^2*Sqrt[b*x + a*x^5] + 2*2^(2/3)*
x^3*Sqrt[b*x + a*x^5] - x^4*Sqrt[b*x + a*x^5])]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(9*a*x^4+5*b)/(a*x^5+b*x)^(1/4)/(a*x^9+b*x^5+1),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(9\,ax^4+5\,b\right)x^3}{\left(ax^9+bx^5+1\right)\left(ax^5+bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(9*a*x^4+5*b)/(a*x^5+b*x)^(1/4)/(a*x^9+b*x^5+1),x, algorithm=
"giac")
```

```
[Out] integrate((9*a*x^4 + 5*b)*x^3/((a*x^9 + b*x^5 + 1)*(a*x^5 + b*x)^(1/4)), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(9a\,x^4+5b\right)}{\left(a\,x^5+bx\right)^{\frac{1}{4}}\left(a\,x^9+b\,x^5+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(9*a*x^4+5*b)/(a*x^5+b*x)^(1/4)/(a*x^9+b*x^5+1),x)
```

```
[Out] int(x^3*(9*a*x^4+5*b)/(a*x^5+b*x)^(1/4)/(a*x^9+b*x^5+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(9\,ax^4+5\,b\right)x^3}{\left(ax^9+bx^5+1\right)\left(ax^5+bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(9*a*x^4+5*b)/(a*x^5+b*x)^(1/4)/(a*x^9+b*x^5+1),x, algorithm=
"maxima")
```

```
[Out] integrate((9*a*x^4 + 5*b)*x^3/((a*x^9 + b*x^5 + 1)*(a*x^5 + b*x)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(9\,a\,x^4+5\,b\right)}{{\left(a\,x^5+b\,x\right)}^{1/4}\,\left(a\,x^9+b\,x^5+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(5*b + 9*a*x^4))/((b*x + a*x^5)^(1/4)*(a*x^9 + b*x^5 + 1)),x)
```

```
[Out] int((x^3*(5*b + 9*a*x^4))/((b*x + a*x^5)^(1/4)*(a*x^9 + b*x^5 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(9 a x^{4} + 5 b\right)}{\sqrt[4]{x \left(a x^{4} + b\right)} \left(a x^{9} + b x^{5} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(9*a*x**4+5*b)/(a*x**5+b*x)**(1/4)/(a*x**9+b*x**5+1),x)
```

```
[Out] Integral(x**3*(9*a*x**4 + 5*b)/((x*(a*x**4 + b))**(1/4)*(a*x**9 + b*x**5 +
1)), x)
```

3.2330

$$\int \frac{\left(-2x+(1+k)x^2\right)\left(1-(1+k)x+(a+k)x^2\right)}{((1-x)x(1-kx))^{2/3}\left(1-2(1+k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(-b+k^2\right)x^4\right)}\,dx$$

Optimal. Leaf size=383

$$\frac{\left(a+\sqrt{b}\right)\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[6]{b}\,x\right)}{2b^{5/6}}+\frac{\left(\sqrt{b}-a\right)\log\left(\sqrt[6]{b}\,x+\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{2b^{5/6}}+\frac{\left(a-\sqrt{b}\right)\log}{}$$

Rubi [F] time = 35.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-2x+(1+k)x^2\right)\left(1-(1+k)x+(a+k)x^2\right)}{((1-x)x(1-kx))^{2/3}\left(1-2(1+k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(-b+k^2\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2*x + (1 + k)*x^2)*(1 - (1 + k)*x + (a + k)*x^2))/(((1 - x)*x*(1 - k
*x))^(2/3)*(1 - 2*(1 + k)*x + (1 + 4*k + k^2)*x^2 - 2*(k + k^2)*x^3 + (-b +
 k^2)*x^4)),x]

[Out] (-3*(1 + k)*(a + k)*x*((1 - x)/(1 - k*x))^(2/3)*(1 - k*x)*Hypergeometric2F1
[1/3, 2/3, 4/3, ((1 - k)*x)/(1 - k*x)])/((b - k^2)*((1 - x)*x*(1 - k*x))^(2
/3)) + (3*(1 + k)*(a + k)*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst
][Defer[Int][1/((1 - x^3)^(2/3)*(1 - k*x^3)^(2/3)*(1 - 2*(1 + k)*x^3 + (1 +
 k*(4 + k))*x^6 - 2*k*(1 + k)*x^9 - b*(1 - k^2/b)*x^12)), x], x, x^(1/3)])/
((b - k^2)*((1 - x)*x*(1 - k*x))^(2/3)) - (6*(b + k + k^2 + k^3 + a*(1 + k)
^2)*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst][Defer[Int][x^3/((1 -
 x^3)^(2/3)*(1 - k*x^3)^(2/3)*(1 - 2*(1 + k)*x^3 + (1 + k*(4 + k))*x^6 - 2*
k*(1 + k)*x^9 - b*(1 - k^2/b)*x^12)), x], x, x^(1/3)])/((b - k^2)*((1 - x)*
x*(1 - k*x))^(2/3)) + (3*(1 + k)*(3*b + k + k^2 + k^3 + a*(1 + 4*k + k^2))*
(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst][Defer[Int][x^6/((1 - x^3
)^(2/3)*(1 - k*x^3)^(2/3)*(1 - 2*(1 + k)*x^3 + (1 + k*(4 + k))*x^6 - 2*k*(1
 + k)*x^9 - b*(1 - k^2/b)*x^12)), x], x, x^(1/3)])/((b - k^2)*((1 - x)*x*(1
 - k*x))^(2/3)) - (3*(b*(1 + 2*a + 4*k + k^2) + k*(k + k^3 + 2*a*(1 + k + k
^2)))*(1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Subst][Defer[Int][x^9/((1
 - x^3)^(2/3)*(1 - k*x^3)^(2/3)*(1 - 2*(1 + k)*x^3 + (1 + k*(4 + k))*x^6 -
2*k*(1 + k)*x^9 - b*(1 - k^2/b)*x^12)), x], x, x^(1/3)])/((b - k^2)*((1 - x
)*x*(1 - k*x))^(2/3))
```

Rubi steps

$$\int \frac{\left(-2x+(1+k)x^2\right)\left(1-(1+k)x+(a+k)x^2\right)}{((1-x)x(1-kx))^{2/3}\left(1-2(1+k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(-b+k^2\right)x^4\right)}dx = \int \frac{}{((1-x)x(1-kx)}$$

$$= \frac{\left((1-x)^{2/3}x^{2/3}(1-k\right.}{}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}(1-\right.}{}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}(1-\right.}{}$$

$$= \frac{\left(3(1-x)^{2/3}x^{2/3}(1-\right.}{}$$

$$= -\frac{3(1+k)(a+k)x\left(\right.}{\left(b-k\right.}$$

$$= -\frac{3(1+k)(a+k)x\left(\right.}{\left(b-k\right.}$$

Mathematica [F] time = 1.67, size = 0, normalized size = 0.00

$$\int \frac{\left(-2x+(1+k)x^2\right)\left(1-(1+k)x+(a+k)x^2\right)}{((1-x)x(1-kx))^{2/3}\left(1-2(1+k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(-b+k^2\right)x^4\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2*x + (1 + k)*x^2)*(1 - (1 + k)*x + (a + k)*x^2))/(((1 - x)*x*
(1 - k*x))^(2/3)*(1 - 2*(1 + k)*x + (1 + 4*k + k^2)*x^2 - 2*(k + k^2)*x^3 +
 (-b + k^2)*x^4)),x]
```

```
[Out] Integrate[((-2*x + (1 + k)*x^2)*(1 - (1 + k)*x + (a + k)*x^2))/(((1 - x)*x*
(1 - k*x))^(2/3)*(1 - 2*(1 + k)*x + (1 + 4*k + k^2)*x^2 - 2*(k + k^2)*x^3 +
 (-b + k^2)*x^4)), x]
```

IntegrateAlgebraic [A] time = 1.87, size = 383, normalized size = 1.00

$$\frac{\left(a+\sqrt{b}\right)\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[6]{b}\,x\right)}{2b^{5/6}}+\frac{\left(\sqrt{b}-a\right)\log\left(\sqrt[6]{b}\,x+\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{2b^{5/6}}+\frac{\left(a-\sqrt{b}\right)\log\left(-\sqrt[6]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\sqrt[3]{b}\,x^2+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{4b^{5/6}}+\frac{\left(-a-\sqrt{b}\right)\log\left(\sqrt[6]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\sqrt[3]{b}\,x^2+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{4b^{5/6}}-\frac{\sqrt{3}\left(a-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\,x}{\sqrt[6]{b}\,x-2\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{2b^{5/6}}+\frac{\sqrt{3}\left(a+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\,x}{\sqrt[6]{b}\,x+2\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{2b^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2*x + (1 + k)*x^2)*(1 - (1 + k)*x + (a + k)*x^2))/(((
1 - x)*x*(1 - k*x))^(2/3)*(1 - 2*(1 + k)*x + (1 + 4*k + k^2)*x^2 - 2*(k + k
^2)*x^3 + (-b + k^2)*x^4)),x]
```

```
[Out] -1/2*(Sqrt[3]*(a - Sqrt[b])*ArcTan[(Sqrt[3]*b^(1/6)*x)/(b^(1/6)*x - 2*(x +
(-1 - k)*x^2 + k*x^3)^(1/3))])/b^(5/6) + (Sqrt[3]*(a + Sqrt[b])*ArcTan[(Sqr
t[3]*b^(1/6)*x)/(b^(1/6)*x + 2*(x + (-1 - k)*x^2 + k*x^3)^(1/3))])/(2*b^(5/
```

```
6)) + ((a + Sqrt[b])*Log[-(b^(1/6)*x) + (x + (-1 - k)*x^2 + k*x^3)^(1/3)])/
(2*b^(5/6)) + ((-a + Sqrt[b])*Log[b^(1/6)*x + (x + (-1 - k)*x^2 + k*x^3)^(1
/3)])/(2*b^(5/6)) + ((a - Sqrt[b])*Log[b^(1/3)*x^2 - b^(1/6)*x*(x + (-1 - k
)*x^2 + k*x^3)^(1/3) + (x + (-1 - k)*x^2 + k*x^3)^(2/3)])/(4*b^(5/6)) + ((-
a - Sqrt[b])*Log[b^(1/3)*x^2 + b^(1/6)*x*(x + (-1 - k)*x^2 + k*x^3)^(1/3) +
 (x + (-1 - k)*x^2 + k*x^3)^(2/3)])/(4*b^(5/6))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x+(1+k)*x^2)*(1-(1+k)*x+(a+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(
1-2*(1+k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2-b)*x^4),x, algorithm="fricas
")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+k)x^2-(k+1)x+1\right)\left((k+1)x^2-2\,x\right)}{\left(\left(k^2-b\right)x^4-2\left(k^2+k\right)x^3+\left(k^2+4\,k+1\right)x^2-2\,(k+1)x+1\right)\left((kx-1)(x-1)x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x+(1+k)*x^2)*(1-(1+k)*x+(a+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(
1-2*(1+k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2-b)*x^4),x, algorithm="giac")
```

```
[Out] integrate(((a + k)*x^2 - (k + 1)*x + 1)*((k + 1)*x^2 - 2*x)/(((k^2 - b)*x^4
 - 2*(k^2 + k)*x^3 + (k^2 + 4*k + 1)*x^2 - 2*(k + 1)*x + 1)*((k*x - 1)*(x -
 1)*x)^(2/3)), x)
```

maple [**F**] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(-2x+(1+k)\,x^2\right)\left(1-(1+k)\,x+(a+k)\,x^2\right)}{\left((1-x)\,x\,(-kx+1)\right)^{\frac{2}{3}}\left(1-2\,(1+k)\,x+\left(k^2+4k+1\right)x^2-2\left(k^2+k\right)x^3+\left(k^2-b\right)x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x+(1+k)*x^2)*(1-(1+k)*x+(a+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-2*(1
+k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2-b)*x^4),x)
```

```
[Out] int((-2*x+(1+k)*x^2)*(1-(1+k)*x+(a+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(1-2*(1
+k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2-b)*x^4),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+k)x^2-(k+1)x+1\right)\left((k+1)x^2-2\,x\right)}{\left(\left(k^2-b\right)x^4-2\left(k^2+k\right)x^3+\left(k^2+4\,k+1\right)x^2-2\,(k+1)x+1\right)\left((kx-1)(x-1)x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x+(1+k)*x^2)*(1-(1+k)*x+(a+k)*x^2)/((1-x)*x*(-k*x+1))^(2/3)/(
1-2*(1+k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2-b)*x^4),x, algorithm="maxima
")
```

```
[Out] integrate(((a + k)*x^2 - (k + 1)*x + 1)*((k + 1)*x^2 - 2*x)/(((k^2 - b)*x^4
 - 2*(k^2 + k)*x^3 + (k^2 + 4*k + 1)*x^2 - 2*(k + 1)*x + 1)*((k*x - 1)*(x -
 1)*x)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(2\,x-x^2\,\left(k+1\right)\right)\,\left(\left(a+k\right)\,x^2+\left(-k-1\right)\,x+1\right)}{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)^{2/3}\,\left(x^4\,\left(b-k^2\right)-x^2\,\left(k^2+4\,k+1\right)+2\,x\,\left(k+1\right)+2\,x^3\,\left(k^2+k\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x - x^2*(k + 1))*(x^2*(a + k) - x*(k + 1) + 1))/((x*(k*x - 1)*(x -
1))^(2/3)*(x^4*(b - k^2) - x^2*(4*k + k^2 + 1) + 2*x*(k + 1) + 2*x^3*(k + k
^2) - 1)),x)
```

```
[Out] int(((2*x - x^2*(k + 1))*(x^2*(a + k) - x*(k + 1) + 1))/((x*(k*x - 1)*(x -
1))^(2/3)*(x^4*(b - k^2) - x^2*(4*k + k^2 + 1) + 2*x*(k + 1) + 2*x^3*(k + k
^2) - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x+(1+k)*x**2)*(1-(1+k)*x+(a+k)*x**2)/((1-x)*x*(-k*x+1))**(2/3
)/(1-2*(1+k)*x+(k**2+4*k+1)*x**2-2*(k**2+k)*x**3+(k**2-b)*x**4),x)
```

```
[Out] Timed out
```

3.2331 $$\int \frac{(-2+(1+k)x)\left(1-(1+k)x+(a+k)x^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(2+2k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(-b+k^2\right)x^4\right)}\,dx$$

Optimal. Leaf size=383

$$\frac{\left(a+\sqrt{b}\right)\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[6]{b}\,x\right)}{2b^{2/3}}+\frac{\left(a-\sqrt{b}\right)\log\left(\sqrt[6]{b}\,x+\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{2b^{2/3}}+\frac{\left(\sqrt{b}-a\right)\log}{}$$

Rubi [F] time = 28.93, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-2+(1+k)x)\left(1-(1+k)x+(a+k)x^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(2+2k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(-b+k^2\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-2 + (1 + k)*x)*(1 - (1 + k)*x + (a + k)*x^2))/(((1 - x)*x*(1 - k*x))
^(1/3)*(1 - (2 + 2*k)*x + (1 + 4*k + k^2)*x^2 - 2*(k + k^2)*x^3 + (-b + k^2
)*x^4)),x]
```

```
[Out] (9*(1 + k)*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][x^
4/((1 - x^3)^(1/3)*(1 - k*x^3)^(1/3)*(1 - 2*(1 + k)*x^3 + (1 + k*(4 + k))*x
^6 - 2*k*(1 + k)*x^9 - b*(1 - k^2/b)*x^12)), x], x, x^(1/3)])/((1 - x)*x*(1
 - k*x))^(1/3) - (3*(1 + 2*a + 4*k + k^2)*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(
1/3)*Defer[Subst][Defer[Int][x^7/((1 - x^3)^(1/3)*(1 - k*x^3)^(1/3)*(1 - 2*
(1 + k)*x^3 + (1 + k*(4 + k))*x^6 - 2*k*(1 + k)*x^9 - b*(1 - k^2/b)*x^12)),
 x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3) + (3*(1 + k)*(a + k)*(1 - x)^
(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][x^10/((1 - x^3)^(1/3)
*(1 - k*x^3)^(1/3)*(1 - 2*(1 + k)*x^3 + (1 + k*(4 + k))*x^6 - 2*k*(1 + k)*x
^9 - b*(1 - k^2/b)*x^12)), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3) + (
6*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][x/((1 - x^3
)^(1/3)*(1 - k*x^3)^(1/3)*(-1 + 2*(1 + k)*x^3 - (1 + k*(4 + k))*x^6 + 2*k*(
1 + k)*x^9 + b*(1 - k^2/b)*x^12)), x], x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(
1/3)
```

Rubi steps

$$\int \frac{(-2+(1+k)x)\left(1-(1+k)x+(a+k)x^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(2+2k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(-b+k^2\right)x^4\right)}\,dx=\frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-}\right.}{}$$

$$=\frac{\left(3\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-}\right.}{}$$

$$=\frac{\left(3\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-}\right.}{}$$

$$=\frac{\left(6\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-}\right.}{}$$

Mathematica [F] time = 7.62, size = 0, normalized size = 0.00

$$\int \frac{(-2+(1+k)x)\left(1-(1+k)x+(a+k)x^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\left(1-(2+2k)x+\left(1+4k+k^2\right)x^2-2\left(k+k^2\right)x^3+\left(-b+k^2\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-2 + (1 + k)*x)*(1 - (1 + k)*x + (a + k)*x^2))/(((1 - x)*x*(1 -
 k*x))^(1/3)*(1 - (2 + 2*k)*x + (1 + 4*k + k^2)*x^2 - 2*(k + k^2)*x^3 + (-b
 + k^2)*x^4)),x]
```

```
[Out] Integrate[((-2 + (1 + k)*x)*(1 - (1 + k)*x + (a + k)*x^2))/(((1 - x)*x*(1 -
 k*x))^(1/3)*(1 - (2 + 2*k)*x + (1 + 4*k + k^2)*x^2 - 2*(k + k^2)*x^3 + (-b
 + k^2)*x^4)), x]
```

IntegrateAlgebraic [A] time = 1.42, size = 383, normalized size = 1.00

$$\frac{\left(a+\sqrt{b}\right)\log\left(\sqrt[3]{kx^3+(-k-1)x^2+x}-\sqrt[6]{b}\,x\right)}{2b^{2/3}}+\frac{\left(a-\sqrt{b}\right)\log\left(\sqrt[6]{b}\,x+\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{2b^{2/3}}+\frac{\left(\sqrt{b}-a\right)\log\left(-\sqrt[6]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\sqrt[3]{b}\,x^2+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{4b^{2/3}}+\frac{\left(-a-\sqrt{b}\right)\log\left(\sqrt[6]{b}\,x\sqrt[3]{kx^3+(-k-1)x^2+x}+\sqrt[3]{b}\,x^2+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{4b^{2/3}}-\frac{\sqrt{3}\left(a-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\,x}{\sqrt[6]{b}\,x-2\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{2b^{2/3}}-\frac{\sqrt{3}\left(a+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\,x}{\sqrt[6]{b}\,x+2\sqrt[3]{kx^3+(-k-1)x^2+x}}\right)}{2b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-2 + (1 + k)*x)*(1 - (1 + k)*x + (a + k)*x^2))/(((1 -
x)*x*(1 - k*x))^(1/3)*(1 - (2 + 2*k)*x + (1 + 4*k + k^2)*x^2 - 2*(k + k^2)*
x^3 + (-b + k^2)*x^4)),x]
```

```
[Out] -1/2*(Sqrt[3]*(a - Sqrt[b])*ArcTan[(Sqrt[3]*b^(1/6)*x)/(b^(1/6)*x - 2*(x +
(-1 - k)*x^2 + k*x^3)^(1/3))])/b^(2/3) - (Sqrt[3]*(a + Sqrt[b])*ArcTan[(Sqr
t[3]*b^(1/6)*x)/(b^(1/6)*x + 2*(x + (-1 - k)*x^2 + k*x^3)^(1/3))])/(2*b^(2/
3)) + ((a + Sqrt[b])*Log[-(b^(1/6)*x) + (x + (-1 - k)*x^2 + k*x^3)^(1/3)])/
(2*b^(2/3)) + ((a - Sqrt[b])*Log[b^(1/6)*x + (x + (-1 - k)*x^2 + k*x^3)^(1/
3)])/(2*b^(2/3)) + ((-a + Sqrt[b])*Log[b^(1/3)*x^2 - b^(1/6)*x*(x + (-1 - k
)*x^2 + k*x^3)^(1/3) + (x + (-1 - k)*x^2 + k*x^3)^(2/3)])/(4*b^(2/3)) + ((-
a - Sqrt[b])*Log[b^(1/3)*x^2 + b^(1/6)*x*(x + (-1 - k)*x^2 + k*x^3)^(1/3) +
 (x + (-1 - k)*x^2 + k*x^3)^(2/3)])/(4*b^(2/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(1-(2
+2*k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2-b)*x^4),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+k)x^2-(k+1)x+1\right)((k+1)x-2)}{\left(\left(k^2-b\right)x^4-2\left(k^2+k\right)x^3+\left(k^2+4k+1\right)x^2-2(k+1)x+1\right)\left((kx-1)(x-1)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(1-(2
+2*k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2-b)*x^4),x, algorithm="giac")
```

```
[Out] integrate(((a + k)*x^2 - (k + 1)*x + 1)*((k + 1)*x - 2)/(((k^2 - b)*x^4 - 2
*(k^2 + k)*x^3 + (k^2 + 4*k + 1)*x^2 - 2*(k + 1)*x + 1)*((k*x - 1)*(x - 1)*
x)^(1/3)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(-2+(1+k)x)\left(1-(1+k)x+(a+k)x^{2}\right)}{((1-x)x(-kx+1))^{\frac{1}{3}}\left(1-(2+2k)x+\left(k^{2}+4k+1\right)x^{2}-2\left(k^{2}+k\right)x^{3}+\left(k^{2}-b\right)x^{4}\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(1-(2+2*k)*
x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2-b)*x^4),x)
```

```
[Out] int((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(1-(2+2*k)*
x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2-b)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+k)x^{2}-(k+1)x+1\right)((k+1)x-2)}{\left(\left(k^{2}-b\right)x^{4}-2\left(k^{2}+k\right)x^{3}+\left(k^{2}+4k+1\right)x^{2}-2(k+1)x+1\right)((kx-1)(x-1)x)^{\frac{1}{3}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x^2)/((1-x)*x*(-k*x+1))^(1/3)/(1-(2
+2*k)*x+(k^2+4*k+1)*x^2-2*(k^2+k)*x^3+(k^2-b)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(((a + k)*x^2 - (k + 1)*x + 1)*((k + 1)*x - 2)/(((k^2 - b)*x^4 - 2
*(k^2 + k)*x^3 + (k^2 + 4*k + 1)*x^2 - 2*(k + 1)*x + 1)*((k*x - 1)*(x - 1)*
x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{(x\,(k+1)-2)\,\left((a+k)\,x^{2}+(-k-1)\,x+1\right)}{(x\,(k\,x-1)\,(x-1))^{1/3}\,\left(x\,(2\,k+2)+x^{4}\,\left(b-k^{2}\right)-x^{2}\,\left(k^{2}+4\,k+1\right)+2\,x^{3}\,\left(k^{2}+k\right)-1\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x*(k + 1) - 2)*(x^2*(a + k) - x*(k + 1) + 1))/((x*(k*x - 1)*(x - 1))
^(1/3)*(x*(2*k + 2) + x^4*(b - k^2) - x^2*(4*k + k^2 + 1) + 2*x^3*(k + k^2)
 - 1)),x)
```

```
[Out] int(-((x*(k + 1) - 2)*(x^2*(a + k) - x*(k + 1) + 1))/((x*(k*x - 1)*(x - 1))
^(1/3)*(x*(2*k + 2) + x^4*(b - k^2) - x^2*(4*k + k^2 + 1) + 2*x^3*(k + k^2)
 - 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+(1+k)*x)*(1-(1+k)*x+(a+k)*x**2)/((1-x)*x*(-k*x+1))**(1/3)/(1-
(2+2*k)*x+(k**2+4*k+1)*x**2-2*(k**2+k)*x**3+(k**2-b)*x**4),x)
```

```
[Out] Timed out
```

3.2332 $\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt{ax+\sqrt{-b+a^2x^2}}\sqrt[6]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=383

$$-\frac{7\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{3}\sqrt[6]{c}}\right)}{6\sqrt{3}\,ac^{13/6}}+\frac{7\tan^{-1}\left(\frac{2\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{3}\sqrt[6]{c}}+\frac{1}{\sqrt{3}}\right)}{6\sqrt{3}\,ac^{13/6}}-\frac{7\tanh^{-1}\left(\frac{\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[6]{c}}\right)}{9ac^{13/6}}-\frac{7\tanh^{-1}}{}$$

Rubi [F] time = 1.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt{ax+\sqrt{-b+a^2x^2}}\sqrt[6]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(Sqrt[-b + a^2*x^2]*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*(c + (a*x + Sqrt[-
b + a^2*x^2])^(1/4))^(1/6)),x]
```

```
[Out] Defer[Int][1/(Sqrt[-b + a^2*x^2]*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*(c + (a*x +
 Sqrt[-b + a^2*x^2])^(1/4))^(1/6)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt{ax+\sqrt{-b+a^2x^2}}\sqrt[6]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{1}{\sqrt{-b+a^2x^2}\sqrt{ax+\sqrt{-b+a^2x^2}}\sqrt[6]{c+\sqrt[4]{}}}$$

Mathematica [C] time = 0.36, size = 76, normalized size = 0.20

$$-\frac{24\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{5/6}{}_2F_1\left(\frac{5}{6},3;\frac{11}{6};\frac{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}{c}\right)}{5ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-b + a^2*x^2]*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*(c + (a*x +
Sqrt[-b + a^2*x^2])^(1/4))^(1/6)),x]
```

```
[Out] (-24*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(5/6)*Hypergeometric2F1[5/6, 3,
 11/6, (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))/c])/(5*a*c^3)
```

IntegrateAlgebraic [A] time = 22.29, size = 355, normalized size = 0.93

$$-\frac{7\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{3}\sqrt[6]{c}}\right)}{6\sqrt{3}\,ac^{13/6}}+\frac{7\tan^{-1}\left(\frac{2\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{3}\sqrt[6]{c}}+\frac{1}{\sqrt{3}}\right)}{6\sqrt{3}\,ac^{13/6}}-\frac{7\tanh^{-1}\left(\frac{\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[6]{c}}\right)}{9ac^{13/6}}-\frac{7\tanh^{-1}\left(\frac{\sqrt[6]{c}\sqrt[6]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\sqrt[3]{c}}\right)}{18ac^{13/6}}+\frac{\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{5/6}\left(7\sqrt[4]{\sqrt{a^2x^2-b}+ax}-6c\right)}{3ac^2\sqrt{\sqrt{a^2x^2-b}+ax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-b + a^2*x^2]*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*(c
+ (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6)),x]
```

```
[Out] ((c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(5/6)*(-6*c + 7*(a*x + Sqrt[-b + a^
2*x^2])^(1/4)))/(3*a*c^2*Sqrt[a*x + Sqrt[-b + a^2*x^2]]) - (7*ArcTan[1/Sqrt
[3] - (2*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6))/(Sqrt[3]*c^(1/6))])/
(6*Sqrt[3]*a*c^(13/6)) + (7*ArcTan[1/Sqrt[3] + (2*(c + (a*x + Sqrt[-b + a^2
*x^2])^(1/4))^(1/6))/(Sqrt[3]*c^(1/6))])/(6*Sqrt[3]*a*c^(13/6)) - (7*ArcTan
h[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6)/c^(1/6)])/(9*a*c^(13/6)) - (
7*ArcTanh[(c^(1/6)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/6))/(c^(1/3) +
 (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3))])/(18*a*c^(13/6))
```

fricas [B] time = 0.65, size = 761, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^
2-b)^(1/2))^(1/4))^(1/6),x, algorithm="fricas")
```

```
[Out] -1/36*(28*sqrt(3)*a*b*c^2*(1/(a^6*c^13))^(1/6)*arctan(2/3*sqrt(3)*sqrt(a^5*
(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/6)*c^11*(1/(a^6*c^13))^(5/6) + a^4
*c^9*(1/(a^6*c^13))^(2/3) + (c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3))*a*
c^2*(1/(a^6*c^13))^(1/6) - 2/3*sqrt(3)*a*(c + (a*x + sqrt(a^2*x^2 - b))^(1/
4))^(1/6)*c^2*(1/(a^6*c^13))^(1/6) - 1/3*sqrt(3)) + 28*sqrt(3)*a*b*c^2*(1/(
a^6*c^13))^(1/6)*arctan(2/3*sqrt(3)*sqrt(-a^5*(c + (a*x + sqrt(a^2*x^2 - b)
)^(1/4))^(1/6)*c^11*(1/(a^6*c^13))^(5/6) + a^4*c^9*(1/(a^6*c^13))^(2/3) + (
c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3))*a*c^2*(1/(a^6*c^13))^(1/6) - 2/
3*sqrt(3)*a*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/6)*c^2*(1/(a^6*c^13))^
(1/6) + 1/3*sqrt(3)) + 7*a*b*c^2*(1/(a^6*c^13))^(1/6)*log(a^5*(c + (a*x + s
qrt(a^2*x^2 - b))^(1/4))^(1/6)*c^11*(1/(a^6*c^13))^(5/6) + a^4*c^9*(1/(a^6*
c^13))^(2/3) + (c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)) - 7*a*b*c^2*(1/
(a^6*c^13))^(1/6)*log(-a^5*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/6)*c^11
*(1/(a^6*c^13))^(5/6) + a^4*c^9*(1/(a^6*c^13))^(2/3) + (c + (a*x + sqrt(a^2
*x^2 - b))^(1/4))^(1/3)) + 14*a*b*c^2*(1/(a^6*c^13))^(1/6)*log(a^5*c^11*(1/
(a^6*c^13))^(5/6) + (c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/6)) - 14*a*b*c
^2*(1/(a^6*c^13))^(1/6)*log(-a^5*c^11*(1/(a^6*c^13))^(5/6) + (c + (a*x + sq
rt(a^2*x^2 - b))^(1/4))^(1/6)) - 12*(7*(a*x + sqrt(a^2*x^2 - b))^(3/4)*(a*x
 - sqrt(a^2*x^2 - b)) - 6*(a*c*x - sqrt(a^2*x^2 - b)*c)*sqrt(a*x + sqrt(a^2
*x^2 - b)))*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(5/6))/(a*b*c^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^
2-b)^(1/2))^(1/4))^(1/6),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^{2} x^{2}-b}\, \sqrt{a x+\sqrt{a^{2} x^{2}-b}}\, \left(c+\left(a x+\sqrt{a^{2} x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^2-b)^(
1/2))^(1/4))^(1/6),x)
```

```
[Out] int(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^2-b)^(
1/2))^(1/4))^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2x^2-b}\,\sqrt{ax+\sqrt{a^2x^2-b}}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^
2-b)^(1/2))^(1/4))^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a^2*x^2 - b)*sqrt(a*x + sqrt(a^2*x^2 - b))*(c + (a*x + sq
rt(a^2*x^2 - b))^(1/4))^(1/6)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a\,x+\sqrt{a^2\,x^2-b}}\,\left(c+\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/4}\right)^{1/6}\,\sqrt{a^2\,x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x + (a^2*x^2 - b)^(1/2))^(1/2)*(c + (a*x + (a^2*x^2 - b)^(1/2))^(
1/4))^(1/6)*(a^2*x^2 - b)^(1/2)),x)
```

```
[Out] int(1/((a*x + (a^2*x^2 - b)^(1/2))^(1/2)*(c + (a*x + (a^2*x^2 - b)^(1/2))^(
1/4))^(1/6)*(a^2*x^2 - b)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}\,\sqrt{ax+\sqrt{a^2x^2-b}}\,\sqrt{a^2x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2*x**2-b)**(1/2)/(a*x+(a**2*x**2-b)**(1/2))**(1/2)/(c+(a*x+
(a**2*x**2-b)**(1/2))**(1/4))**(1/6),x)
```

```
[Out] Integral(1/((c + (a*x + sqrt(a**2*x**2 - b))**(1/4))**(1/6)*sqrt(a*x + sqrt
(a**2*x**2 - b))*sqrt(a**2*x**2 - b)), x)
```

3.2333 $\int \frac{x}{\sqrt{1+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=383

$$\frac{35b^2\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}\right)}{128a^2}+\frac{\sqrt{a^2x^2-b}\left(\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}\left(-9216a^2x^2-12288ax+245\right.\right.}{}$$

Rubi [F] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\sqrt{1+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]],x]
```

```
[Out] Defer[Int][x/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]], x]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{x}{\sqrt{1+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Mathematica [A] time = 2.14, size = 444, normalized size = 1.16

$$-\frac{-\frac{b^2\left(\sqrt{\sqrt{a^2x^2-b}+ax}+1\right)^{7/2}}{8\left(\sqrt{a^2x^2-b}+ax\right)^2}+\frac{25b^2\left(\sqrt{\sqrt{a^2x^2-b}+ax}+1\right)^{5/2}}{48\left(\sqrt{a^2x^2-b}+ax\right)^{3/2}}-\frac{163b^2\left(\sqrt{\sqrt{a^2x^2-b}+ax}+1\right)^{3/2}}{192\left(\sqrt{a^2x^2-b}+ax\right)}+\frac{93b^2\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}}{128\sqrt{\sqrt{a^2x^2-b}+ax}}+\frac{35}{256}b^2\log\left(1-\frac{1}{\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}}\right)-\frac{35}{256}b^2\log\left(\frac{1}{\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}}+1\right)-\frac{1}{7}\left(\sqrt{\sqrt{a^2x^2-b}+ax}+1\right)^{7/2}+\frac{3}{5}\left(\sqrt{\sqrt{a^2x^2-b}+ax}+1\right)^{5/2}-\left(\sqrt{\sqrt{a^2x^2-b}+ax}+1\right)^{3/2}+\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]],x]
```

```
[Out] -((Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]] + (93*b^2*Sqrt[1 + Sqrt[a*x + S
qrt[-b + a^2*x^2]]])/(128*Sqrt[a*x + Sqrt[-b + a^2*x^2]]) - (1 + Sqrt[a*x +
 Sqrt[-b + a^2*x^2]])^(3/2) - (163*b^2*(1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]])
^(3/2))/(192*(a*x + Sqrt[-b + a^2*x^2])) + (3*(1 + Sqrt[a*x + Sqrt[-b + a^2
*x^2]])^(5/2))/5 + (25*b^2*(1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]])^(5/2))/(48*
(a*x + Sqrt[-b + a^2*x^2])^(3/2)) - (1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]])^(7
/2)/7 - (b^2*(1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]])^(7/2))/(8*(a*x + Sqrt[-b
+ a^2*x^2])^2) + (35*b^2*Log[1 - 1/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]
])/256 - (35*b^2*Log[1 + 1/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]])/256)/
a^2)
```

IntegrateAlgebraic [A] time = 0.95, size = 383, normalized size = 1.00

$$\frac{35b^2\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}\right)}{128a^2}+\frac{\sqrt{a^2x^2-b}\left(\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}\left(-9216a^2x^2-12288ax+2450b^2+2304b\right)+\left(7680a^2x^2+6144ax-3675b^2-1920b\right)\sqrt{\sqrt{a^2x^2-b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}\right)+\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}\left(-9216a^3x^3-12288a^2x^2+2450ab^2x+6912abx+1680b^2+6144b\right)+\left(7680a^3x^3+6144a^2x^2-3675ab^2x-5760abx-1960b^2-3072b\right)\sqrt{\sqrt{a^2x^2-b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}}{13440a^2\left(2a^2x^2-b\right)+26880a^3x\sqrt{a^2x^2-b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]],x]
```

[Out] ((6144*b + 1680*b^2 + 6912*a*b*x + 2450*a*b^2*x - 12288*a^2*x^2 - 9216*a^3*
x^3)*Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]] + (-3072*b - 1960*b^2 - 5760*
a*b*x - 3675*a*b^2*x + 6144*a^2*x^2 + 7680*a^3*x^3)*Sqrt[a*x + Sqrt[-b + a^
2*x^2]]*Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]] + Sqrt[-b + a^2*x^2]*((230
4*b + 2450*b^2 - 12288*a*x - 9216*a^2*x^2)*Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^
2*x^2]]] + (-1920*b - 3675*b^2 + 6144*a*x + 7680*a^2*x^2)*Sqrt[a*x + Sqrt[-
b + a^2*x^2]]*Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]))/(26880*a^3*x*Sqrt[
-b + a^2*x^2] + 13440*a^2*(-b + 2*a^2*x^2)) + (35*b^2*ArcTanh[Sqrt[1 + Sqrt
[a*x + Sqrt[-b + a^2*x^2]]]])/(128*a^2)

fricas [A] time = 0.45, size = 210, normalized size = 0.55

$$\frac{3675\, b^{2} \log\left(\sqrt{\sqrt{ax + \sqrt{a^{2}x^{2} - b}} + 1} + 1\right) - 3675\, b^{2} \log\left(\sqrt{\sqrt{ax + \sqrt{a^{2}x^{2} - b}} + 1} - 1\right) + 2 \left(3360\, a^{2}x^{2} + 2\,(1225\, ab - 1152\, a)x - 2\, \sqrt{a^{2}x^{2} - b}\,(1680\, ax + 1225\, b + 1152) - \left(3920\, a^{2}x^{2} + 15\,(245\, ab - 128\, a)x - 5\, \sqrt{a^{2}x^{2} - b}\,(784\, ax + 735\, b + 384) - 1960\, b - 3072\right)\sqrt{ax + \sqrt{a^{2}x^{2} - b}} - 1680\, b - 6144\right)\sqrt{\sqrt{ax + \sqrt{a^{2}x^{2} - b}} + 1}}{26880\, a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")

[Out] 1/26880*(3675*b^2*log(sqrt(sqrt(a*x + sqrt(a^2*x^2 - b)) + 1) + 1) - 3675*b
^2*log(sqrt(sqrt(a*x + sqrt(a^2*x^2 - b)) + 1) - 1) + 2*(3360*a^2*x^2 + 2*(
1225*a*b - 1152*a)*x - 2*sqrt(a^2*x^2 - b)*(1680*a*x + 1225*b + 1152) - (39
20*a^2*x^2 + 15*(245*a*b - 128*a)*x - 5*sqrt(a^2*x^2 - b)*(784*a*x + 735*b
+ 384) - 1960*b - 3072)*sqrt(a*x + sqrt(a^2*x^2 - b)) - 1680*b - 6144)*sqrt
(sqrt(a*x + sqrt(a^2*x^2 - b)) + 1))/a^2

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{1 + \sqrt{ax + \sqrt{a^{2}x^{2} - b}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(1+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x)

[Out] int(x/(1+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\sqrt{ax + \sqrt{a^{2}x^{2} - b}} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(x/sqrt(sqrt(a*x + sqrt(a^2*x^2 - b)) + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{\sqrt{a\,x + \sqrt{a^2\,x^2 - b}} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x + (a^2*x^2 - b)^(1/2))^(1/2) + 1)^(1/2),x)
```

```
[Out] int(x/((a*x + (a^2*x^2 - b)^(1/2))^(1/2) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\sqrt{ax + \sqrt{a^2x^2 - b}} + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+(a*x+(a**2*x**2-b)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(x/sqrt(sqrt(a*x + sqrt(a**2*x**2 - b)) + 1), x)
```

3.2334 $\int \frac{\left(d+cx^2\right)\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{\left(-b+a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=384

$$\frac{4\sqrt[8]{b}\,cx\sqrt[4]{\frac{\sqrt{a^2x^2-b}}{\sqrt{b}}+\frac{ax}{\sqrt{b}}}}{5a^2}+\frac{\sqrt{a^2x^2-b}\sqrt[4]{\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}}\left(4a^2\sqrt[8]{b}\,cx^2-5a^2\sqrt[8]{b}\,d-9b^{9/8}c\right)}{5a^3\left(ax-\sqrt{b}\right)\left(ax+\sqrt{b}\right)}-\frac{5\left(a^2d+bc\right)\tan^{-1}\left(\sqrt[4]{\frac{\sqrt{a^2x^2-b}}{\sqrt{b}}\cdots}\right.}{2a^3b^{3/8}}$$

Rubi [B] time = 1.74, antiderivative size = 869, normalized size of antiderivative = 2.26, number of steps used = 35, number of rules used = 17, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {6742, 2122, 288, 329, 301, 211, 1165, 628, 1162, 617, 204, 212, 206, 203, 2120, 463, 459}

Antiderivative was successfully verified.

```
[In] Int[((d + c*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(5/4))/(-b + a^2*x^2)^(3/2),x]
```

```
[Out] (4*c*(a*x + Sqrt[-b + a^2*x^2])^(5/4))/(5*a^3) + (2*b*c*(a*x + Sqrt[-b + a^
2*x^2])^(5/4))/(a^3*(b - (a*x + Sqrt[-b + a^2*x^2])^2)) + (2*d*(a*x + Sqrt[
-b + a^2*x^2])^(5/4))/(a*(b - (a*x + Sqrt[-b + a^2*x^2])^2)) - (5*b^(5/8)*c
*ArcTan[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)])/(2*a^3) - (5*d*ArcTan[(a
*x + Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)])/(2*a*b^(3/8)) - (5*b^(5/8)*c*ArcTa
n[1 - (Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/b^(1/8)])/(2*Sqrt[2]*a^3)
- (5*d*ArcTan[1 - (Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/b^(1/8)])/(2*S
qrt[2]*a*b^(3/8)) + (5*b^(5/8)*c*ArcTan[1 + (Sqrt[2]*(a*x + Sqrt[-b + a^2*x
^2])^(1/4))/b^(1/8)])/(2*Sqrt[2]*a^3) + (5*d*ArcTan[1 + (Sqrt[2]*(a*x + Sqr
t[-b + a^2*x^2])^(1/4))/b^(1/8)])/(2*Sqrt[2]*a*b^(3/8)) - (5*b^(5/8)*c*ArcT
anh[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)])/(2*a^3) - (5*d*ArcTanh[(a*x
+ Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)])/(2*a*b^(3/8)) - (5*b^(5/8)*c*Log[b^(1
/4) - Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[-b
 + a^2*x^2]]])/(4*Sqrt[2]*a^3) - (5*d*Log[b^(1/4) - Sqrt[2]*b^(1/8)*(a*x +
Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]])/(4*Sqrt[2]*a*b
^(3/8)) + (5*b^(5/8)*c*Log[b^(1/4) + Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x
^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]])/(4*Sqrt[2]*a^3) + (5*d*Log[b^
(1/4) + Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[
-b + a^2*x^2]]])/(4*Sqrt[2]*a*b^(3/8))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(d+cx^2\right)\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{\left(-b+a^2x^2\right)^{3/2}}\,dx = \int \left(\frac{d\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{\left(-b+a^2x^2\right)^{3/2}}+\frac{cx^2\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{\left(-b+a^2x^2\right)^{3/2}}\right)dx$$

$$= c\int \frac{x^2\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{\left(-b+a^2x^2\right)^{3/2}}\,dx + d\int \frac{\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{\left(-b+a^2x^2\right)^{3/2}}\,dx$$

$$= \frac{c\,\text{Subst}\left(\int \frac{\sqrt[4]{x}\left(b+x^2\right)^2}{\left(-b+x^2\right)^2}\,dx, x, ax+\sqrt{-b+a^2x^2}\right)}{a^3} + \frac{(4d)\,\text{Subst}\left(\int \frac{x^{9/4}}{\left(-b+x^2\right.}\right.}{}$$

$$= \frac{2bc\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{a^3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} + \frac{2d\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{a\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} + \frac{c\,\text{Sul}}{}$$

$$= \frac{4c\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{5a^3} + \frac{2bc\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{a^3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} + \frac{2d\left(ax\right.}{a\left(b-\left(a\right.\right.}$$

$$= \frac{4c\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{5a^3} + \frac{2bc\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{a^3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} + \frac{2d\left(ax\right.}{a\left(b-\left(a\right.\right.}$$

$$= \frac{4c\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{5a^3} + \frac{2bc\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{a^3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} + \frac{2d\left(ax\right.}{a\left(b-\left(a\right.\right.}$$

$$= \frac{4c\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{5a^3} + \frac{2bc\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{a^3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} + \frac{2d\left(ax\right.}{a\left(b-\left(a\right.\right.}$$

$$= \frac{4c\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{5a^3} + \frac{2bc\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{a^3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} + \frac{2d\left(ax\right.}{a\left(b-\left(a\right.\right.}$$

$$= \frac{4c\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{5a^3} + \frac{2bc\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{a^3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} + \frac{2d\left(ax\right.}{a\left(b-\left(a\right.\right.}$$

$$= \frac{4c\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{5a^3} + \frac{2bc\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{a^3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} + \frac{2d\left(ax\right.}{a\left(b-\left(a\right.\right.}$$

Mathematica [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{(d+cx^2)\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{(-b+a^2x^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + c*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(5/4))/(-b + a^2*x^2)^(3/
2),x]
```

```
[Out] Integrate[((d + c*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(5/4))/(-b + a^2*x^2)^(3/
2), x]
```

IntegrateAlgebraic [A] time = 4.30, size = 446, normalized size = 1.16

$$\frac{4\sqrt[8]{b}\,cx\sqrt[4]{\frac{\sqrt{a^2x^2-b}}{\sqrt{b}}+\frac{ax}{\sqrt{b}}}}{5a^2}+\frac{\sqrt{a^2x^2-b}\,\sqrt[4]{\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}}\left(4a^2\sqrt[8]{b}\,cx^2-5a^2\sqrt[8]{b}\,d-9b^{9/8}c\right)}{5a^3\left(ax-\sqrt{b}\right)\left(ax+\sqrt{b}\right)}-\frac{5\left(a^2d+bc\right)\tan^{-1}\left(\sqrt[4]{\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}}\right)}{2a^3b^{3/8}}+\frac{5\left(a^2d+bc\right)\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}}-1}{\sqrt{2}\sqrt[4]{\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}}}\right)}{2\sqrt{2}\,a^3b^{3/8}}-\frac{5\left(a^2d+bc\right)\tanh^{-1}\left(\sqrt[4]{\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}}\right)}{2a^3b^{3/8}}+\frac{5\left(a^2d+bc\right)\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}}+1}{\sqrt{2}\sqrt[4]{\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}}}\right)}{2\sqrt{2}\,a^3b^{3/8}}$$

Warning: Unable to verify antiderivative.

```
[In] IntegrateAlgebraic[((d + c*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(5/4))/(-b + a^2
*x^2)^(3/2),x]
```

```
[Out] (Sqrt[-b + a^2*x^2]*(-9*b^(9/8)*c - 5*a^2*b^(1/8)*d + 4*a^2*b^(1/8)*c*x^2)*
((a*x + Sqrt[-b + a^2*x^2])/Sqrt[b])^(1/4))/(5*a^3*(-Sqrt[b] + a*x)*(Sqrt[b
] + a*x)) + (4*b^(1/8)*c*x*((a*x)/Sqrt[b] + Sqrt[-b + a^2*x^2]/Sqrt[b])^(1/
4))/(5*a^2) - (5*(b*c + a^2*d)*ArcTan[((a*x + Sqrt[-b + a^2*x^2])/Sqrt[b])^
(1/4)])/(2*a^3*b^(3/8)) + (5*(b*c + a^2*d)*ArcTan[(-1 + Sqrt[(a*x + Sqrt[-b
 + a^2*x^2])/Sqrt[b]])/(Sqrt[2]*((a*x + Sqrt[-b + a^2*x^2])/Sqrt[b])^(1/4))
])/(2*Sqrt[2]*a^3*b^(3/8)) - (5*(b*c + a^2*d)*ArcTanh[((a*x + Sqrt[-b + a^2
*x^2])/Sqrt[b])^(1/4)])/(2*a^3*b^(3/8)) + (5*(b*c + a^2*d)*ArcTanh[(1 + Sqr
t[(a*x + Sqrt[-b + a^2*x^2])/Sqrt[b]])/(Sqrt[2]*((a*x + Sqrt[-b + a^2*x^2])
/Sqrt[b])^(1/4))])/(2*Sqrt[2]*a^3*b^(3/8))
```

fricas [B] time = 0.64, size = 5796, normalized size = 15.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+d)*(a*x+(a^2*x^2-b)^(1/2))^(5/4)/(a^2*x^2-b)^(3/2),x, algo
rithm="fricas")
```

```
[Out] -1/40*(100*sqrt(2)*(a^5*x^2 - a^3*b)*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*
b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3
 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b^3))^(1/8)*arctan
(1/3125*(3125*a^16*d^8 + 25000*a^14*b*c*d^7 + 87500*a^12*b^2*c^2*d^6 + 1750
00*a^10*b^3*c^3*d^5 + 218750*a^8*b^4*c^4*d^4 + 175000*a^6*b^5*c^5*d^3 + 875
00*a^4*b^6*c^6*d^2 + 25000*a^2*b^7*c^7*d + 3125*b^8*c^8 + sqrt(2)*sqrt(-976
5625*sqrt(2)*(a^25*b^2*d^5 + 5*a^23*b^3*c*d^4 + 10*a^21*b^4*c^2*d^3 + 10*a^
19*b^5*c^3*d^2 + 5*a^17*b^6*c^4*d + a^15*b^7*c^5)*(a*x + sqrt(a^2*x^2 - b))
^(1/4)*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*
d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*
b^7*c^7*d + b^8*c^8)/(a^24*b^3))^(5/8) + 9765625*(a^20*d^10 + 10*a^18*b*c*d
^9 + 45*a^16*b^2*c^2*d^8 + 120*a^14*b^3*c^3*d^7 + 210*a^12*b^4*c^4*d^6 + 25
2*a^10*b^5*c^5*d^5 + 210*a^8*b^6*c^6*d^4 + 120*a^6*b^7*c^7*d^3 + 45*a^4*b^8
*c^8*d^2 + 10*a^2*b^9*c^9*d + b^10*c^10)*sqrt(a*x + sqrt(a^2*x^2 - b)) + 97
65625*(a^22*b*d^8 + 8*a^20*b^2*c*d^7 + 28*a^18*b^3*c^2*d^6 + 56*a^16*b^4*c^
3*d^5 + 70*a^14*b^5*c^4*d^4 + 56*a^12*b^6*c^5*d^3 + 28*a^10*b^7*c^6*d^2 + 8
*a^8*b^8*c^7*d + a^6*b^9*c^8)*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2
*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a
```

^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b^3))^(1/4))*a^9*b*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b^3))^(3/8) - 3125*sqrt(2)*(a^19*b*d^5 + 5*a^17*b^2*c*d^4 + 10*a^15*b^3*c^2*d^3 + 10*a^13*b^4*c^3*d^2 + 5*a^11*b^5*c^4*d + a^9*b^6*c^5)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b^3))^(3/8))/(a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)) + 100*sqrt(2)*(a^5*x^2 - a^3*b)*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b^3))^(1/8)*arctan(-(a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8 - sqrt(2)*sqrt(sqrt(2)*(a^25*b^2*d^5 + 5*a^23*b^3*c*d^4 + 10*a^21*b^4*c^2*d^3 + 10*a^19*b^5*c^3*d^2 + 5*a^17*b^6*c^4*d + a^15*b^7*c^5)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b^3))^(5/8) + (a^20*d^10 + 10*a^18*b*c*d^9 + 45*a^16*b^2*c^2*d^8 + 120*a^14*b^3*c^3*d^7 + 210*a^12*b^4*c^4*d^6 + 252*a^10*b^5*c^5*d^5 + 210*a^8*b^6*c^6*d^4 + 120*a^6*b^7*c^7*d^3 + 45*a^4*b^8*c^8*d^2 + 10*a^2*b^9*c^9*d + b^10*c^10)*sqrt(a*x + sqrt(a^2*x^2 - b)) + (a^22*b*d^8 + 8*a^20*b^2*c*d^7 + 28*a^18*b^3*c^2*d^6 + 56*a^16*b^4*c^3*d^5 + 70*a^14*b^5*c^4*d^4 + 56*a^12*b^6*c^5*d^3 + 28*a^10*b^7*c^6*d^2 + 8*a^8*b^8*c^7*d + a^6*b^9*c^8)*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b^3))^(1/4))*a^9*b*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b^3))^(3/8) + sqrt(2)*(a^19*b*d^5 + 5*a^17*b^2*c*d^4 + 10*a^15*b^3*c^2*d^3 + 10*a^13*b^4*c^3*d^2 + 5*a^11*b^5*c^4*d + a^9*b^6*c^5)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b^3))^(3/8))/(a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)) - 25*sqrt(2)*(a^5*x^2 - a^3*b)*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b^3))^(1/8)*log(9765625*sqrt(2)*(a^25*b^2*d^5 + 5*a^23*b^3*c*d^4 + 10*a^21*b^4*c^2*d^3 + 10*a^19*b^5*c^3*d^2 + 5*a^17*b^6*c^4*d + a^15*b^7*c^5)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b^3))^(5/8) + 9765625*(a^20*d^10 + 10*a^18*b*c*d^9 + 45*a^16*b^2*c^2*d^8 + 120*a^14*b^3*c^3*d^7 + 210*a^12*b^4*c^4*d^6 + 252*a^10*b^5*c^5*d^5 + 210*a^8*b^6*c^6*d^4 + 120*a^6*b^7*c^7*d^3 + 45*a^4*b^8*c^8*d^2 + 10*a^2*b^9*c^9*d + b^10*c^10)*sqrt(a*x + sqrt(a^2*x^2 - b)) + 9765625*(a^22*b*d^8 + 8*a^20*b^2*c*d^7 + 28*a^18*b^3*c^2*d^6 + 56*a^16*b^4*c^3*d^5 + 70*a^14*b^5*c^4*d^4 + 56*a^12*b^6*c^5*d^3 + 28*a^10*b^7*c^6*d^2 + 8*a^8*b^8*c^7*d + a^6*b^9*c^8)*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b^3))^(1/4)) + 25*sqrt(2)*(a^5*x^2 - a^3*b)*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b^3))^(1/8)*log(-9765625*sqrt(2)*(a^25*b^2*d^5 + 5*a^23*b^3*c*d^4 + 10*a^21*b^4*c^2*d^3 + 10*a^19*b^5*c^3*d^2 + 5*a^17*b^6*c^4*d + a^15*b^7*c^5)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^

```
4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8
*c^8)/(a^24*b^3))^(5/8) + 9765625*(a^20*d^10 + 10*a^18*b*c*d^9 + 45*a^16*b^
2*c^2*d^8 + 120*a^14*b^3*c^3*d^7 + 210*a^12*b^4*c^4*d^6 + 252*a^10*b^5*c^5*
d^5 + 210*a^8*b^6*c^6*d^4 + 120*a^6*b^7*c^7*d^3 + 45*a^4*b^8*c^8*d^2 + 10*a
^2*b^9*c^9*d + b^10*c^10)*sqrt(a*x + sqrt(a^2*x^2 - b)) + 9765625*(a^22*b*d
^8 + 8*a^20*b^2*c*d^7 + 28*a^18*b^3*c^2*d^6 + 56*a^16*b^4*c^3*d^5 + 70*a^14
*b^5*c^4*d^4 + 56*a^12*b^6*c^5*d^3 + 28*a^10*b^7*c^6*d^2 + 8*a^8*b^8*c^7*d
+ a^6*b^9*c^8)*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*
b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2
+ 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b^3))^(1/4)) - 200*(a^5*x^2 - a^3*b)*((a
^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a
^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d
+ b^8*c^8)/(a^24*b^3))^(1/8)*arctan((sqrt((a^20*d^10 + 10*a^18*b*c*d^9 + 45
*a^16*b^2*c^2*d^8 + 120*a^14*b^3*c^3*d^7 + 210*a^12*b^4*c^4*d^6 + 252*a^10*
b^5*c^5*d^5 + 210*a^8*b^6*c^6*d^4 + 120*a^6*b^7*c^7*d^3 + 45*a^4*b^8*c^8*d^
2 + 10*a^2*b^9*c^9*d + b^10*c^10)*sqrt(a*x + sqrt(a^2*x^2 - b)) + (a^22*b*d
^8 + 8*a^20*b^2*c*d^7 + 28*a^18*b^3*c^2*d^6 + 56*a^16*b^4*c^3*d^5 + 70*a^14
*b^5*c^4*d^4 + 56*a^12*b^6*c^5*d^3 + 28*a^10*b^7*c^6*d^2 + 8*a^8*b^8*c^7*d
+ a^6*b^9*c^8)*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*
b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2
+ 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b^3))^(1/4))*a^9*b*((a^16*d^8 + 8*a^14*b
*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 5
6*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b
^3))^(3/8) - (a^19*b*d^5 + 5*a^17*b^2*c*d^4 + 10*a^15*b^3*c^2*d^3 + 10*a^13
*b^4*c^3*d^2 + 5*a^11*b^5*c^4*d + a^9*b^6*c^5)*(a*x + sqrt(a^2*x^2 - b))^(1
/4)*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5
 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7
*c^7*d + b^8*c^8)/(a^24*b^3))^(3/8))/(a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b
^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3
+ 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)) + 50*(a^5*x^2 - a^3*b)*(
(a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70
*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*
d + b^8*c^8)/(a^24*b^3))^(1/8)*log(3125*a^15*b^2*((a^16*d^8 + 8*a^14*b*c*d^
7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6
*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b^3))^
(5/8) + 3125*(a^10*d^5 + 5*a^8*b*c*d^4 + 10*a^6*b^2*c^2*d^3 + 10*a^4*b^3*c^
3*d^2 + 5*a^2*b^4*c^4*d + b^5*c^5)*(a*x + sqrt(a^2*x^2 - b))^(1/4)) - 50*(a
^5*x^2 - a^3*b)*((a^16*d^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10
*b^3*c^3*d^5 + 70*a^8*b^4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2
 + 8*a^2*b^7*c^7*d + b^8*c^8)/(a^24*b^3))^(1/8)*log(-3125*a^15*b^2*((a^16*d
^8 + 8*a^14*b*c*d^7 + 28*a^12*b^2*c^2*d^6 + 56*a^10*b^3*c^3*d^5 + 70*a^8*b^
4*c^4*d^4 + 56*a^6*b^5*c^5*d^3 + 28*a^4*b^6*c^6*d^2 + 8*a^2*b^7*c^7*d + b^8
*c^8)/(a^24*b^3))^(5/8) + 3125*(a^10*d^5 + 5*a^8*b*c*d^4 + 10*a^6*b^2*c^2*d
^3 + 10*a^4*b^3*c^3*d^2 + 5*a^2*b^4*c^4*d + b^5*c^5)*(a*x + sqrt(a^2*x^2 -
b))^(1/4)) - 8*(4*a^3*c*x^3 - 4*a*b*c*x + (4*a^2*c*x^2 - 5*a^2*d - 9*b*c)*s
qrt(a^2*x^2 - b))*(a*x + sqrt(a^2*x^2 - b))^(1/4))/(a^5*x^2 - a^3*b)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+d)*(a*x+(a^2*x^2-b)^(1/2))^(5/4)/(a^2*x^2-b)^(3/2),x, algo
rithm="giac")

[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^{2}+d\right)\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{5}{4}}}{\left(a^{2}x^{2}-b\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+d)*(a*x+(a^2*x^2-b)^(1/2))^(5/4)/(a^2*x^2-b)^(3/2),x)
[Out] int((c*x^2+d)*(a*x+(a^2*x^2-b)^(1/2))^(5/4)/(a^2*x^2-b)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2}+d\right)\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{5}{4}}}{\left(a^{2}x^{2}-b\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+d)*(a*x+(a^2*x^2-b)^(1/2))^(5/4)/(a^2*x^2-b)^(3/2),x, algo
rithm="maxima")
[Out] integrate((c*x^2 + d)*(a*x + sqrt(a^2*x^2 - b))^(5/4)/(a^2*x^2 - b)^(3/2),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{5/4}\,\left(c\,x^2+d\right)}{{\left(a^2\,x^2-b\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (a^2*x^2 - b)^(1/2))^(5/4)*(d + c*x^2))/(a^2*x^2 - b)^(3/2),x)
[Out] int(((a*x + (a^2*x^2 - b)^(1/2))^(5/4)*(d + c*x^2))/(a^2*x^2 - b)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{5}{4}}\left(cx^{2}+d\right)}{\left(a^{2}x^{2}-b\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+d)*(a*x+(a**2*x**2-b)**(1/2))**(5/4)/(a**2*x**2-b)**(3/2)
,x)
[Out] Integral((a*x + sqrt(a**2*x**2 - b))**(5/4)*(c*x**2 + d)/(a**2*x**2 - b)**(
3/2), x)
```

3.2335 $\int \frac{(-b+ax^2)\sqrt[3]{-x+x^3}}{-d+cx^2}\,dx$

Optimal. Leaf size=385

$$-\frac{\sqrt[3]{c-d}\,(bc-ad)\log\left(-\sqrt[3]{d}\,\sqrt[3]{x^3-x}\,x\sqrt[3]{c-d}+x^2(c-d)^{2/3}+d^{2/3}\left(x^3-x\right)^{2/3}\right)}{4c^2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{x^3-x}-x\right)(ac-3ad+3b}{6c^2}$$

Rubi [A] time = 0.88, antiderivative size = 537, normalized size of antiderivative = 1.39, number of steps used = 22, number of rules used = 16, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2056, 581, 584, 329, 275, 331, 292, 31, 634, 618, 204, 628, 466, 465, 494, 617}

$$-\frac{\sqrt[3]{x^3-x}\,\sqrt[3]{c-d}\,(bc-ad)\log\left(\frac{x^{4/3}(c-d)^{2/3}}{(x^2-1)^{2/3}}-\frac{\sqrt[3]{d}\,x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{x^2-1}}+d^{2/3}\right)}{4c^2\sqrt[3]{d}\,\sqrt[3]{x^2-1}\,\sqrt[3]{x}}+\frac{\sqrt[3]{x^3-x}\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2-1}}\right)(a(c-3d)+3bc)}{6c^2\sqrt[3]{x^2-1}\,\sqrt[3]{x}}-\frac{\sqrt[3]{x^3-x}\log\left(\frac{x^{4/3}}{(x^2-1)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2-1}}+1\right)(a(c-3d)+3bc)}{12c^2\sqrt[3]{x^2-1}\,\sqrt[3]{x}}+\frac{\sqrt[3]{x^3-x}\,\sqrt[3]{c-d}\,(bc-ad)\log\left(\frac{x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{x^2-1}}+\sqrt[3]{d}\right)}{2c^2\sqrt[3]{d}\,\sqrt[3]{x^2-1}\,\sqrt[3]{x}}+\frac{\sqrt[3]{x^3-x}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-1}}+1}{\sqrt{3}}\right)(a(c-3d)+3bc)}{2\sqrt{3}c^2\sqrt[3]{x^2-1}\,\sqrt[3]{x}}+\frac{\sqrt{3}\,\sqrt[3]{x^3-x}\,\sqrt[3]{c-d}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-\frac{2x^{2/3}\sqrt[3]{c-d}}{\sqrt[3]{x^2-1}}}{\sqrt{3}\,\sqrt[3]{d}}\right)}{2c^2\sqrt[3]{d}\,\sqrt[3]{x^2-1}\,\sqrt[3]{x}}+\frac{a\sqrt[3]{x^3-x}\,x}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a*x^2)*(-x + x^3)^(1/3))/(-d + c*x^2),x]
```

```
[Out] (a*x*(-x + x^3)^(1/3))/(2*c) + ((3*b*c + a*(c - 3*d))*(-x + x^3)^(1/3)*ArcT
an[(1 + (2*x^(2/3))/(-1 + x^2)^(1/3))/Sqrt[3]])/(2*Sqrt[3]*c^2*x^(1/3)*(-1
+ x^2)^(1/3)) + (Sqrt[3]*(c - d)^(1/3)*(b*c - a*d)*(-x + x^3)^(1/3)*ArcTan[
(d^(1/3) - (2*(c - d)^(1/3)*x^(2/3))/(-1 + x^2)^(1/3))/(Sqrt[3]*d^(1/3))])/
(2*c^2*d^(1/3)*x^(1/3)*(-1 + x^2)^(1/3)) + ((3*b*c + a*(c - 3*d))*(-x + x^3
)^(1/3)*Log[1 - x^(2/3)/(-1 + x^2)^(1/3)])/(6*c^2*x^(1/3)*(-1 + x^2)^(1/3))
 - ((3*b*c + a*(c - 3*d))*(-x + x^3)^(1/3)*Log[1 + x^(4/3)/(-1 + x^2)^(2/3)
 + x^(2/3)/(-1 + x^2)^(1/3)])/(12*c^2*x^(1/3)*(-1 + x^2)^(1/3)) + ((c - d)^
(1/3)*(b*c - a*d)*(-x + x^3)^(1/3)*Log[d^(1/3) + ((c - d)^(1/3)*x^(2/3))/(-
1 + x^2)^(1/3)])/(2*c^2*d^(1/3)*x^(1/3)*(-1 + x^2)^(1/3)) - ((c - d)^(1/3)*
(b*c - a*d)*(-x + x^3)^(1/3)*Log[d^(2/3) + ((c - d)^(2/3)*x^(4/3))/(-1 + x^
2)^(2/3) - ((c - d)^(1/3)*d^(1/3)*x^(2/3))/(-1 + x^2)^(1/3)])/(4*c^2*d^(1/3
)*x^(1/3)*(-1 + x^2)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[b, 3]*Rt[a, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\left(-b+ax^2\right)\sqrt[3]{-x+x^3}}{-d+cx^2}\,dx = \frac{\sqrt[3]{-x+x^3}\int \frac{\sqrt[3]{x}\sqrt[3]{-1+x^2}\left(-b+ax^2\right)}{-d+cx^2}\,dx}{\sqrt[3]{x}\sqrt[3]{-1+x^2}}$$

$$= \frac{ax\sqrt[3]{-x+x^3}}{2c} + \frac{\sqrt[3]{-x+x^3}\int \frac{\sqrt[3]{x}\left(\frac{2}{3}(3bc-2ad)-\frac{2}{3}(3bc+a(c-3d))x^2\right)}{\left(-1+x^2\right)^{2/3}\left(-d+cx^2\right)}\,dx}{2c\sqrt[3]{x}\sqrt[3]{-1+x^2}}$$

$$= \frac{ax\sqrt[3]{-x+x^3}}{2c} + \frac{\sqrt[3]{-x+x^3}\int \left(-\frac{2(3bc+a(c-3d))\sqrt[3]{x}}{3c\left(-1+x^2\right)^{2/3}} + \frac{\left(-\frac{2}{3}(3bc+a(c-3d))d+\frac{2}{3}c(3bc-2ad)\right)\sqrt[3]{x}}{c\left(-1+x^2\right)^{2/3}\left(-d+cx^2\right)}\right.}{2c\sqrt[3]{x}\sqrt[3]{-1+x^2}}$$

$$= \frac{ax\sqrt[3]{-x+x^3}}{2c} - \frac{\left((3bc+a(c-3d))\sqrt[3]{-x+x^3}\right)\int \frac{\sqrt[3]{x}}{\left(-1+x^2\right)^{2/3}}\,dx}{3c^2\sqrt[3]{x}\sqrt[3]{-1+x^2}} + \frac{\left((c-d)(bc-ad)\right.}{}$$

$$= \frac{ax\sqrt[3]{-x+x^3}}{2c} - \frac{\left((3bc+a(c-3d))\sqrt[3]{-x+x^3}\right)\text{Subst}\left(\int \frac{x^3}{\left(-1+x^6\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{c^2\sqrt[3]{x}\sqrt[3]{-1+x^2}} + \cdots$$

$$= \frac{ax\sqrt[3]{-x+x^3}}{2c} - \frac{\left((3bc+a(c-3d))\sqrt[3]{-x+x^3}\right)\text{Subst}\left(\int \frac{x}{\left(-1+x^3\right)^{2/3}}\,dx, x, x^{2/3}\right)}{2c^2\sqrt[3]{x}\sqrt[3]{-1+x^2}} + \cdots$$

$$= \frac{ax\sqrt[3]{-x+x^3}}{2c} - \frac{\left((3bc+a(c-3d))\sqrt[3]{-x+x^3}\right)\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{2c^2\sqrt[3]{x}\sqrt[3]{-1+x^2}} + \cdots$$

$$= \frac{ax\sqrt[3]{-x+x^3}}{2c} - \frac{\left((3bc+a(c-3d))\sqrt[3]{-x+x^3}\right)\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{6c^2\sqrt[3]{x}\sqrt[3]{-1+x^2}} + \cdots$$

$$= \frac{ax\sqrt[3]{-x+x^3}}{2c} + \frac{(3bc+a(c-3d))\sqrt[3]{-x+x^3}\,\log\left(1-\frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{6c^2\sqrt[3]{x}\sqrt[3]{-1+x^2}} + \frac{\sqrt[3]{c-d}\,(bc-ad\cdots}{2c\cdots}$$

$$= \frac{ax\sqrt[3]{-x+x^3}}{2c} + \frac{(3bc+a(c-3d))\sqrt[3]{-x+x^3}\,\log\left(1-\frac{x^{2/3}}{\sqrt[3]{-1+x^2}}\right)}{6c^2\sqrt[3]{x}\sqrt[3]{-1+x^2}} - \frac{(3bc+a(c-3d\cdots}{}$$

$$= \frac{ax\sqrt[3]{-x+x^3}}{2c} + \frac{(3bc+a(c-3d))\sqrt[3]{-x+x^3}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-1+x^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\,c^2\sqrt[3]{x}\sqrt[3]{-1+x^2}} + \frac{\sqrt{3}\sqrt[3]{c-d}\,(bc-\cdots}{2\cdots}$$

Mathematica [C] time = 0.69, size = 183, normalized size = 0.48

$$\frac{2\left(1-x^2\right)^{2/3}x^4\left(1-\frac{cx^2}{d}\right)^{2/3}(a(c-3d)+3bc)F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};x^2,\frac{cx^2}{d}\right)+5x^2\left(\left(1-x^2\right)^{2/3}(2ad-3bc)\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(c-d)x^2}{cx^2-d}\right)+2ad\left(x^2-1\right)\left(1-\frac{cx^2}{d}\right)^{2/3}\right)}{20cd\left(x\left(x^2-1\right)\right)^{2/3}\left(1-\frac{cx^2}{d}\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-b + a*x^2)*(-x + x^3)^(1/3))/(-d + c*x^2),x]
```

```
[Out] (2*(3*b*c + a*(c - 3*d))*x^4*(1 - x^2)^(2/3)*(1 - (c*x^2)/d)^(2/3)*AppellF1
[5/3, 2/3, 1, 8/3, x^2, (c*x^2)/d] + 5*x^2*(2*a*d*(-1 + x^2)*(1 - (c*x^2)/d
)^(2/3) + (-3*b*c + 2*a*d)*(1 - x^2)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3,
```

```
((c - d)*x^2)/(-d + c*x^2)]))/(20*c*d*(x*(-1 + x^2))^(2/3)*(1 - (c*x^2)/d)
^(2/3))
```

IntegrateAlgebraic [A] time = 1.23, size = 385, normalized size = 1.00

$$-\frac{\sqrt[3]{c-d}\,(bc-ad)\log\left(-\sqrt[3]{d}\,\sqrt[3]{x^3-x}\,x\sqrt[3]{c-d}+x^2(c-d)^{2/3}+d^{2/3}\left(x^3-x\right)^{2/3}\right)}{4c^2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{x^3-x}-x\right)(ac-3ad+3bc)}{6c^2}+\frac{\sqrt[3]{c-d}\,(bc-ad)\log\left(x\sqrt[3]{c-d}+\sqrt[3]{d}\,\sqrt[3]{x^3-x}\right)}{2c^2\sqrt[3]{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^3-x}+x}\right)(ac-3ad+3bc)}{2\sqrt{3}c^2}+\frac{\sqrt{3}\sqrt[3]{c-d}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}x\sqrt[3]{c-d}}{x\sqrt[3]{c-d}-2\sqrt[3]{d}\,\sqrt[3]{x^3-x}}\right)}{2c^2\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{x^3-x}\,x+\left(x^3-x\right)^{2/3}+x^2\right)(-ac+3ad-3bc)}{12c^2}+\frac{a\sqrt[3]{x^3-x}\,x}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a*x^2)*(-x + x^3)^(1/3))/(-d + c*x^2),x]
```

```
[Out] (a*x*(-x + x^3)^(1/3))/(2*c) + ((a*c + 3*b*c - 3*a*d)*ArcTan[(Sqrt[3]*x)/(x
 + 2*(-x + x^3)^(1/3))])/(2*Sqrt[3]*c^2) + (Sqrt[3]*(c - d)^(1/3)*(b*c - a*
d)*ArcTan[(Sqrt[3]*(c - d)^(1/3)*x)/((c - d)^(1/3)*x - 2*d^(1/3)*(-x + x^3)
^(1/3))])/(2*c^2*d^(1/3)) + ((a*c + 3*b*c - 3*a*d)*Log[-x + (-x + x^3)^(1/3
)])/(6*c^2) + ((c - d)^(1/3)*(b*c - a*d)*Log[(c - d)^(1/3)*x + d^(1/3)*(-x
+ x^3)^(1/3)])/(2*c^2*d^(1/3)) + ((-(a*c) - 3*b*c + 3*a*d)*Log[x^2 + x*(-x
+ x^3)^(1/3) + (-x + x^3)^(2/3)])/(12*c^2) - ((c - d)^(1/3)*(b*c - a*d)*Log
[(c - d)^(2/3)*x^2 - (c - d)^(1/3)*d^(1/3)*x*(-x + x^3)^(1/3) + d^(2/3)*(-x
 + x^3)^(2/3)])/(4*c^2*d^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(x^3-x)^(1/3)/(c*x^2-d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 3.07, size = 372, normalized size = 0.97

$$\frac{ax^2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}}{2c}-\frac{\left(bc^2-acd-bcd+ad^2\right)\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right|\right)}{2\left(c^3-c^2d\right)}-\frac{\sqrt{3}(ac+3bc-3ad)\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)}{6c^2}-\frac{(ac+3bc-3ad)\log\left(\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+1\right)}{12c^2}+\frac{(ac+3bc-3ad)\log\left(\left|\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)}{6c^2}+\frac{\left(\sqrt{3}\left(-cd^2+d^3\right)^{\frac{1}{3}}bc-\sqrt{3}\left(-cd^2+d^3\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}+2\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}}\right)}{2c^2d}+\frac{\left(\left(-cd^2+d^3\right)^{\frac{1}{3}}bc-\left(-cd^2+d^3\right)^{\frac{1}{3}}ad\right)\log\left(\left(-\frac{c-d}{d}\right)^{\frac{2}{3}}+\left(-\frac{c-d}{d}\right)^{\frac{1}{3}}\left(-\frac{1}{x^2}+1\right)^{\frac{1}{3}}+\left(-\frac{1}{x^2}+1\right)^{\frac{2}{3}}\right)}{4c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(x^3-x)^(1/3)/(c*x^2-d),x, algorithm="giac")
```

```
[Out] 1/2*a*x^2*(-1/x^2 + 1)^(1/3)/c - 1/2*(b*c^2 - a*c*d - b*c*d + a*d^2)*(-(c -
 d)/d)^(1/3)*log(abs(-(-(c - d)/d)^(1/3) + (-1/x^2 + 1)^(1/3)))/(c^3 - c^2*
d) - 1/6*sqrt(3)*(a*c + 3*b*c - 3*a*d)*arctan(1/3*sqrt(3)*(2*(-1/x^2 + 1)^(
1/3) + 1))/c^2 - 1/12*(a*c + 3*b*c - 3*a*d)*log((-1/x^2 + 1)^(2/3) + (-1/x^
2 + 1)^(1/3) + 1)/c^2 + 1/6*(a*c + 3*b*c - 3*a*d)*log(abs((-1/x^2 + 1)^(1/3
) - 1))/c^2 + 1/2*(sqrt(3)*(-c*d^2 + d^3)^(1/3)*b*c - sqrt(3)*(-c*d^2 + d^3
)^(1/3)*a*d)*arctan(1/3*sqrt(3)*((-(c - d)/d)^(1/3) + 2*(-1/x^2 + 1)^(1/3))
/(-(c - d)/d)^(1/3))/(c^2*d) + 1/4*((-c*d^2 + d^3)^(1/3)*b*c - (-c*d^2 + d^
3)^(1/3)*a*d)*log((-(c - d)/d)^(2/3) + (-(c - d)/d)^(1/3)*(-1/x^2 + 1)^(1/3
) + (-1/x^2 + 1)^(2/3))/(c^2*d)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^{2}-b\right)\left(x^{3}-x\right)^{\frac{1}{3}}}{c\,x^{2}-d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2-b)*(x^3-x)^(1/3)/(c*x^2-d),x)
```

```
[Out] int((a*x^2-b)*(x^3-x)^(1/3)/(c*x^2-d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^2 - b)(x^3 - x)^{\frac{1}{3}}}{cx^2 - d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2-b)*(x^3-x)^(1/3)/(c*x^2-d),x, algorithm="maxima")
[Out] integrate((a*x^2 - b)*(x^3 - x)^(1/3)/(c*x^2 - d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x^3 - x)^{1/3}\,(b - a\,x^2)}{d - c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 - x)^(1/3)*(b - a*x^2))/(d - c*x^2),x)
[Out] int(((x^3 - x)^(1/3)*(b - a*x^2))/(d - c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x\,(x-1)\,(x+1)}\,(ax^2 - b)}{cx^2 - d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2-b)*(x**3-x)**(1/3)/(c*x**2-d),x)
[Out] Integral((x*(x - 1)*(x + 1))**(1/3)*(a*x**2 - b)/(c*x**2 - d), x)
```

3.2336

$$\int \frac{(-ab+(2a-b)x)\left(a^2-2ax+x^2\right)}{\sqrt[3]{x(-a+x)(-b+x)}\left(a^4d-4a^3dx+\left(-b^2+6a^2d\right)x^2+2(b-2ad)x^3+(-1+d)x^4\right)}\,dx$$

Optimal. Leaf size=387

$$-\frac{\log\left(a^2\sqrt[3]{d}+\sqrt[3]{x^2(-a-b)+abx+x^3}\left(a\sqrt[6]{d}-\sqrt[6]{d}\,x\right)+\left(x^2(-a-b)+abx+x^3\right)^{2/3}-2a\sqrt[3]{d}\,x+\sqrt[3]{d}\,x^2\right)}{4d^{2/3}}-\log\left(a^2\cdots\right.$$

Rubi [F] time = 13.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-ab+(2a-b)x)\left(a^2-2ax+x^2\right)}{\sqrt[3]{x(-a+x)(-b+x)}\left(a^4d-4a^3dx+\left(-b^2+6a^2d\right)x^2+2(b-2ad)x^3+(-1+d)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-(a*b) + (2*a - b)*x)*(a^2 - 2*a*x + x^2))/((x*(-a + x)*(-b + x))^(1/
3)*(a^4*d - 4*a^3*d*x + (-b^2 + 6*a^2*d)*x^2 + 2*(b - 2*a*d)*x^3 + (-1 + d)
*x^4)),x]
```

```
[Out] (3*(2*a - b)*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][Defer[Int][
(x^4*(-a + x^3)^(5/3))/((-b + x^3)^(1/3)*(a^4*d - 4*a^3*d*x^3 - b^2*(1 - (6
*a^2*d)/b^2)*x^6 + 2*b*(1 - (2*a*d)/b)*x^9 - (1 - d)*x^12)), x], x, x^(1/3)
])/((a - x)*(b - x)*x)^(1/3) + (3*a*b*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(1/3)
*Defer[Subst][Defer[Int][(x*(-a + x^3)^(5/3))/((-b + x^3)^(1/3)*(-(a^4*d) +
 4*a^3*d*x^3 + b^2*(1 - (6*a^2*d)/b^2)*x^6 - 2*b*(1 - (2*a*d)/b)*x^9 + (1 -
 d)*x^12)), x], x, x^(1/3)])/((a - x)*(b - x)*x)^(1/3)
```

Rubi steps

$$\int \frac{(-ab+(2a-b)x)\left(a^2-2ax+x^2\right)}{\sqrt[3]{x(-a+x)(-b+x)}\left(a^4d-4a^3dx+\left(-b^2+6a^2d\right)x^2+2(b-2ad)x^3+(-1+d)x^4\right)}\,dx=\int \frac{\cdots}{\sqrt[3]{x(-a+x)(-b+\cdots}}$$

$$=\frac{\left(\sqrt[3]{x}\,\sqrt[3]{-a+x}\,\sqrt[3]{-b+\cdots}\right.}{\cdots}$$

$$=\frac{\left(3\sqrt[3]{x}\,\sqrt[3]{-a+x}\,\sqrt[3]{-b+\cdots}\right.}{\cdots}$$

$$=\frac{\left(3\sqrt[3]{x}\,\sqrt[3]{-a+x}\,\sqrt[3]{-b+\cdots}\right.}{\cdots}$$

$$=\frac{\left(3(2a-b)\sqrt[3]{x}\,\sqrt[3]{-a+\cdots}\right.}{\cdots}$$

Mathematica [F] time = 6.93, size = 0, normalized size = 0.00

$$\int \frac{(-ab+(2a-b)x)\left(a^2-2ax+x^2\right)}{\sqrt[3]{x(-a+x)(-b+x)}\left(a^4d-4a^3dx+\left(-b^2+6a^2d\right)x^2+2(b-2ad)x^3+(-1+d)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-(a*b) + (2*a - b)*x)*(a^2 - 2*a*x + x^2))/((x*(-a + x)*(-b + x
))^(1/3)*(a^4*d - 4*a^3*d*x + (-b^2 + 6*a^2*d)*x^2 + 2*(b - 2*a*d)*x^3 + (-
1 + d)*x^4)),x]
```

```
[Out] Integrate[((-(a*b) + (2*a - b)*x)*(a^2 - 2*a*x + x^2))/((x*(-a + x)*(-b + x
))^(1/3)*(a^4*d - 4*a^3*d*x + (-b^2 + 6*a^2*d)*x^2 + 2*(b - 2*a*d)*x^3 + (-
1 + d)*x^4)), x]
```

IntegrateAlgebraic [A] time = 5.95, size = 387, normalized size = 1.00

$$-\frac{\log\left(a^2\sqrt[3]{d}+\sqrt[3]{x^2(-a-b)+abx+x^3}\left(a\sqrt[6]{d}-\sqrt[6]{d}\,x\right)+\left(x^2(-a-b)+abx+x^3\right)^{2/3}-2a\sqrt[3]{d}\,x+\sqrt[3]{d}\,x^2\right)}{4d^{2/3}}-\frac{\log\left(a^2\sqrt[3]{d}+\sqrt[3]{x^2(-a-b)+abx+x^3}\left(\sqrt[6]{d}\,x-a\sqrt[6]{d}\right)+\left(x^2(-a-b)+abx+x^3\right)^{2/3}-2a\sqrt[3]{d}\,x+\sqrt[3]{d}\,x^2\right)}{4d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{a^2}{\sqrt{3}}+\frac{2\left(x^2(-a-b)+abx+x^3\right)^{2/3}}{\sqrt{3}\sqrt[3]{d}}-\frac{2ax}{\sqrt{3}}+\frac{x^2}{\sqrt{3}}}{(a-x)^2}\right)}{2d^{2/3}}+\frac{\log\left(\sqrt[3]{x^2(-a-b)+abx+x^3}+a\sqrt[6]{d}-\sqrt[6]{d}\,x\right)}{2d^{2/3}}+\frac{\log\left(\sqrt[3]{x^2(-a-b)+abx+x^3}+a\left(-\sqrt[6]{d}\right)+\sqrt[6]{d}\,x\right)}{2d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-(a*b) + (2*a - b)*x)*(a^2 - 2*a*x + x^2))/((x*(-a + x
)*(-b + x))^(1/3)*(a^4*d - 4*a^3*d*x + (-b^2 + 6*a^2*d)*x^2 + 2*(b - 2*a*d)
*x^3 + (-1 + d)*x^4)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(a^2/Sqrt[3] - (2*a*x)/Sqrt[3] + x^2/Sqrt[3] + (2*(a*b
*x + (-a - b)*x^2 + x^3)^(2/3))/(Sqrt[3]*d^(1/3)))/(a - x)^2])/d^(2/3) + Lo
g[a*d^(1/6) - d^(1/6)*x + (a*b*x + (-a - b)*x^2 + x^3)^(1/3)]/(2*d^(2/3)) +
 Log[-(a*d^(1/6)) + d^(1/6)*x + (a*b*x + (-a - b)*x^2 + x^3)^(1/3)]/(2*d^(2
/3)) - Log[a^2*d^(1/3) - 2*a*d^(1/3)*x + d^(1/3)*x^2 + (a*d^(1/6) - d^(1/6)
*x)*(a*b*x + (-a - b)*x^2 + x^3)^(1/3) + (a*b*x + (-a - b)*x^2 + x^3)^(2/3)
]/(4*d^(2/3)) - Log[a^2*d^(1/3) - 2*a*d^(1/3)*x + d^(1/3)*x^2 + (-(a*d^(1/6
)) + d^(1/6)*x)*(a*b*x + (-a - b)*x^2 + x^3)^(1/3) + (a*b*x + (-a - b)*x^2
+ x^3)^(2/3)]/(4*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+(2*a-b)*x)*(a^2-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/3)/(a^4*d-4
*a^3*d*x+(6*a^2*d-b^2)*x^2+2*(-2*a*d+b)*x^3+(-1+d)*x^4),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^2-2\,ax+x^2\right)(ab-(2\,a-b)x)}{\left(a^4d-4\,a^3dx+(d-1)x^4-2\,(2\,ad-b)x^3+\left(6\,a^2d-b^2\right)x^2\right)((a-x)(b-x)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+(2*a-b)*x)*(a^2-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/3)/(a^4*d-4
*a^3*d*x+(6*a^2*d-b^2)*x^2+2*(-2*a*d+b)*x^3+(-1+d)*x^4),x, algorithm="giac"
)
```

```
[Out] integrate(-(a^2 - 2*a*x + x^2)*(a*b - (2*a - b)*x)/((a^4*d - 4*a^3*d*x + (d
 - 1)*x^4 - 2*(2*a*d - b)*x^3 + (6*a^2*d - b^2)*x^2)*((a - x)*(b - x)*x)^(1
/3)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(-ab+(2a-b)x)\left(a^2-2ax+x^2\right)}{\left(x(-a+x)(-b+x)\right)^{\frac{1}{3}}\left(a^4d-4a^3dx+\left(6a^2d-b^2\right)x^2+2(-2ad+b)x^3+(-1+d)x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*b+(2*a-b)*x)*(a^2-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/3)/(a^4*d-4*a^3*d
*x+(6*a^2*d-b^2)*x^2+2*(-2*a*d+b)*x^3+(-1+d)*x^4),x)
```

```
[Out] int((-a*b+(2*a-b)*x)*(a^2-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/3)/(a^4*d-4*a^3*d
*x+(6*a^2*d-b^2)*x^2+2*(-2*a*d+b)*x^3+(-1+d)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2-2\,ax+x^2\right)(ab-(2\,a-b)x)}{\left(a^4d-4\,a^3dx+(d-1)x^4-2\,(2\,ad-b)x^3+\left(6\,a^2d-b^2\right)x^2\right)((a-x)(b-x)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+(2*a-b)*x)*(a^2-2*a*x+x^2)/(x*(-a+x)*(-b+x))^(1/3)/(a^4*d-4
*a^3*d*x+(6*a^2*d-b^2)*x^2+2*(-2*a*d+b)*x^3+(-1+d)*x^4),x, algorithm="maxim
a")
```

```
[Out] -integrate((a^2 - 2*a*x + x^2)*(a*b - (2*a - b)*x)/((a^4*d - 4*a^3*d*x + (d
 - 1)*x^4 - 2*(2*a*d - b)*x^3 + (6*a^2*d - b^2)*x^2)*((a - x)*(b - x)*x)^(1
/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{(a\,b-x\,(2\,a-b))\,\left(a^2-2\,a\,x+x^2\right)}{(x\,(a-x)\,(b-x))^{1/3}\,\left(2\,x^3\,(b-2\,a\,d)+x^2\,\left(6\,a^2\,d-b^2\right)+a^4\,d+x^4\,(d-1)-4\,a^3\,d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*b - x*(2*a - b))*(a^2 - 2*a*x + x^2))/((x*(a - x)*(b - x))^(1/3)*(
2*x^3*(b - 2*a*d) + x^2*(6*a^2*d - b^2) + a^4*d + x^4*(d - 1) - 4*a^3*d*x))
,x)
```

```
[Out] -int(((a*b - x*(2*a - b))*(a^2 - 2*a*x + x^2))/((x*(a - x)*(b - x))^(1/3)*(
2*x^3*(b - 2*a*d) + x^2*(6*a^2*d - b^2) + a^4*d + x^4*(d - 1) - 4*a^3*d*x))
, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*b+(2*a-b)*x)*(a**2-2*a*x+x**2)/(x*(-a+x)*(-b+x))**(1/3)/(a**4
*d-4*a**3*d*x+(6*a**2*d-b**2)*x**2+2*(-2*a*d+b)*x**3+(-1+d)*x**4),x)
```

```
[Out] Timed out
```

3.2337 $\int \frac{b+dx}{x^4 \sqrt[4]{\frac{b+ax}{d+cx}}} \, dx$

Optimal. Leaf size=388

$$\frac{\left(\frac{ax+b}{cx+d}\right)^{3/4} \left(-45a^2cd^2x^3 - 45a^2d^3x^2 + 6abc^2dx^3 + 42abcd^2x^2 + 36abd^3x + 60acd^3x^3 + 60ad^4x^2 + 7b^2c^3x^3 + 3b^2\right.}{96b^2d^2x^3}$$

Rubi [A] time = 0.58, antiderivative size = 397, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1962, 577, 457, 290, 298, 205, 208}

$$-\frac{\left(2bcd(5a-6d)+5ad^2(3a-4d)+7b^2c^2\right)(bc-ad)\left(\frac{ax+b}{cx+d}\right)^{3/4}}{32b^2d^2\left(b-\frac{d(ax+b)}{cx+d}\right)}+\frac{\left(2bcd(5a-6d)+5ad^2(3a-4d)+7b^2c^2\right)(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{\frac{ax+b}{cx+d}}}{\sqrt[4]{b}}\right)}{64b^{9/4}d^{11/4}}-\frac{\left(2bcd(5a-6d)+5ad^2(3a-4d)+7b^2c^2\right)(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{\frac{ax+b}{cx+d}}}{\sqrt[4]{b}}\right)}{64b^{9/4}d^{11/4}}+\frac{(d(9a-4d)+7bc)(bc-ad)^2\left(\frac{ax+b}{cx+d}\right)^{3/4}}{24bd^2\left(b-\frac{d(ax+b)}{cx+d}\right)^2}-\frac{(b+dx)(bc-ad)^3\left(\frac{ax+b}{cx+d}\right)^{3/4}}{3bd(cx+d)\left(b-\frac{d(ax+b)}{cx+d}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b + d*x)/(x^4*((b + a*x)/(d + c*x))^(1/4)),x]
```

```
[Out] -1/3*((b*c - a*d)^3*((b + a*x)/(d + c*x))^(3/4)*(b + d*x))/(b*d*(d + c*x)*(
b - (d*(b + a*x))/(d + c*x))^3) + ((b*c - a*d)^2*(7*b*c + (9*a - 4*d)*d)*((
b + a*x)/(d + c*x))^(3/4))/(24*b*d^2*(b - (d*(b + a*x))/(d + c*x))^2) - ((b
*c - a*d)*(7*b^2*c^2 + 2*b*c*(5*a - 6*d)*d + 5*a*(3*a - 4*d)*d^2)*((b + a*x
)/(d + c*x))^(3/4))/(32*b^2*d^2*(b - (d*(b + a*x))/(d + c*x))) + ((b*c - a*
d)*(7*b^2*c^2 + 2*b*c*(5*a - 6*d)*d + 5*a*(3*a - 4*d)*d^2)*ArcTan[(d^(1/4)*
((b + a*x)/(d + c*x))^(1/4))/b^(1/4)])/(64*b^(9/4)*d^(11/4)) - ((b*c - a*d)
*(7*b^2*c^2 + 2*b*c*(5*a - 6*d)*d + 5*a*(3*a - 4*d)*d^2)*ArcTanh[(d^(1/4)*(
(b + a*x)/(d + c*x))^(1/4))/b^(1/4)])/(64*b^(9/4)*d^(11/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
```

```
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 1962

```
Int[(u_)^(r_.)*(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)
*(x_)^(n_.)))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c
- a*d))/n, Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^
((m + 1)/n - 1)*(u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(
b*e - d*x^q)^((m + 1)/n + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q
)], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] &
& IntegerQ[1/n] && IntegersQ[m, r]
```

Rubi steps

$$\int \frac{b+dx}{x^4\sqrt[4]{\frac{b+ax}{d+cx}}}\,dx = -\left((4(bc-ad))\operatorname{Subst}\left(\int \frac{x^2\left(a-cx^4\right)\left(b(a-d)+\left(-bc+d^2\right)x^4\right)}{\left(b-dx^4\right)^4}\,dx, x, \sqrt[4]{\frac{b+ax}{d+cx}}\right)\right)$$

$$= -\frac{(bc-ad)^3\left(\frac{b+ax}{d+cx}\right)^{3/4}(b+dx)}{3bd(d+cx)\left(b-\frac{d(b+ax)}{d+cx}\right)^3} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{x^2\left(3b(a-d)(bc+3ad)-(7bc+5ad)\left(bc-d^2\right)x^4\right)}{\left(b-dx^4\right)^3}\,dx, x, \sqrt[4]{\frac{b\cdots}{d\cdots}}\right.}{3bd}$$

$$= -\frac{(bc-ad)^3\left(\frac{b+ax}{d+cx}\right)^{3/4}(b+dx)}{3bd(d+cx)\left(b-\frac{d(b+ax)}{d+cx}\right)^3} + \frac{(bc-ad)^2(7bc+(9a-4d)d)\left(\frac{b+ax}{d+cx}\right)^{3/4}}{24bd^2\left(b-\frac{d(b+ax)}{d+cx}\right)^2} - \frac{\left((bc-ad)\left(7b^2c^2+2\cdots\right.\right.}{\cdots}$$

$$= -\frac{(bc-ad)^3\left(\frac{b+ax}{d+cx}\right)^{3/4}(b+dx)}{3bd(d+cx)\left(b-\frac{d(b+ax)}{d+cx}\right)^3} + \frac{(bc-ad)^2(7bc+(9a-4d)d)\left(\frac{b+ax}{d+cx}\right)^{3/4}}{24bd^2\left(b-\frac{d(b+ax)}{d+cx}\right)^2} - \frac{(bc-ad)\left(7b^2c^2+2b\cdots\right.}{3\cdots}$$

$$= -\frac{(bc-ad)^3\left(\frac{b+ax}{d+cx}\right)^{3/4}(b+dx)}{3bd(d+cx)\left(b-\frac{d(b+ax)}{d+cx}\right)^3} + \frac{(bc-ad)^2(7bc+(9a-4d)d)\left(\frac{b+ax}{d+cx}\right)^{3/4}}{24bd^2\left(b-\frac{d(b+ax)}{d+cx}\right)^2} - \frac{(bc-ad)\left(7b^2c^2+2b\cdots\right.}{3\cdots}$$

$$= -\frac{(bc-ad)^3\left(\frac{b+ax}{d+cx}\right)^{3/4}(b+dx)}{3bd(d+cx)\left(b-\frac{d(b+ax)}{d+cx}\right)^3} + \frac{(bc-ad)^2(7bc+(9a-4d)d)\left(\frac{b+ax}{d+cx}\right)^{3/4}}{24bd^2\left(b-\frac{d(b+ax)}{d+cx}\right)^2} - \frac{(bc-ad)\left(7b^2c^2+2b\cdots\right.}{3\cdots}$$

Mathematica [C] time = 0.34, size = 155, normalized size = 0.40

$$\frac{\left(\frac{ax+b}{cx+d}\right)^{3/4}\left(x\left(4b^2(cx+d)^2(3d(3a-4d)+7bc)-x\left(2bcd(5a-6d)+5ad^2(3a-4d)+7b^2c^2\right)\left(x(bc-ad)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{d(b+ax)}{b(d+cx)}\right)+3b(cx+d)\right)\right)-32b^3d(cx+d)^2\right)}{96b^3d^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + d*x)/(x^4*((b + a*x)/(d + c*x))^(1/4)),x]
```

```
[Out] (((b + a*x)/(d + c*x))^(3/4)*(-32*b^3*d*(d + c*x)^2 + x*(4*b^2*(7*b*c + 3*(
3*a - 4*d)*d)*(d + c*x)^2 - (7*b^2*c^2 + 2*b*c*(5*a - 6*d)*d + 5*a*(3*a - 4
*d)*d^2)*x*(3*b*(d + c*x) + (b*c - a*d)*x*Hypergeometric2F1[3/4, 1, 7/4, (d
*(b + a*x))/(b*(d + c*x))]))))/(96*b^3*d^2*x^3)
```

IntegrateAlgebraic [B] time = 1.13, size = 815, normalized size = 2.10

$$\frac{-60a^2\left(\frac{b+ax}{d+cx}\right)^{11/4}d^6+45a^3\left(\frac{b+ax}{d+cx}\right)^{11/4}d^5+24abc\left(\frac{b+ax}{d+cx}\right)^{11/4}d^5+168a^2b\left(\frac{b+ax}{d+cx}\right)^{7/4}d^5+36b^2c^2\left(\frac{b+ax}{d+cx}\right)^{11/4}d^4-15a^2bc\left(\frac{b+ax}{d+cx}\right)^{11/4}d^4-126a^3b\left(\frac{b+ax}{d+cx}\right)^{7/4}d^4-144ab^2c\left(\frac{b+ax}{d+cx}\right)^{7/4}d^4-108a^2b^2\left(\frac{b+ax}{d+cx}\right)^{3/4}d^4-9ab^2c^2\left(\frac{b+ax}{d+cx}\right)^{11/4}d^3-24b^3c^2\left(\frac{b+ax}{d+cx}\right)^{7/4}d^3+42a^2b^2c\left(\frac{b+ax}{d+cx}\right)^{7/4}d^3+113a^3b^2\left(\frac{b+ax}{d+cx}\right)^{3/4}d^3+120ab^3c\left(\frac{b+ax}{d+cx}\right)^{3/4}d^3-21b^3c^3\left(\frac{b+ax}{d+cx}\right)^{11/4}d^2+102ab^3c^2\left(\frac{b+ax}{d+cx}\right)^{7/4}d^2-12b^4c^2\left(\frac{b+ax}{d+cx}\right)^{3/4}d^2-123a^2b^3c\left(\frac{b+ax}{d+cx}\right)^{3/4}d^2-18b^4c^3\left(\frac{b+ax}{d+cx}\right)^{7/4}d+3ab^4c^2\left(\frac{b+ax}{d+cx}\right)^{3/4}d+7b^5c^3\left(\frac{b+ax}{d+cx}\right)^{3/4}}{96b^2d^2\left(b-\frac{d(b+ax)}{d+cx}\right)^3}+\frac{\left(20a^2d^4-15a^3d^3-8abcd^3-12b^2c^2d^2+5a^2bcd^2+3ab^2c^2d+7b^3c^3\right)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{\frac{b+ax}{d+cx}}}{\sqrt[4]{b}}\right)}{64b^{9/4}d^{11/4}}+\frac{\left(-20a^2d^4+15a^3d^3+8abcd^3+12b^2c^2d^2-5a^2bcd^2-3ab^2c^2d-7b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{\frac{b+ax}{d+cx}}}{\sqrt[4]{b}}\right)}{64b^{9/4}d^{11/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + d*x)/(x^4*((b + a*x)/(d + c*x))^(1/4)),x]
```

```
[Out] (7*b^5*c^3*((b + a*x)/(d + c*x))^(3/4) + 3*a*b^4*c^2*d*((b + a*x)/(d + c*x)
)^(3/4) - 123*a^2*b^3*c*d^2*((b + a*x)/(d + c*x))^(3/4) - 12*b^4*c^2*d^2*((
b + a*x)/(d + c*x))^(3/4) + 113*a^3*b^2*d^3*((b + a*x)/(d + c*x))^(3/4) + 1
20*a*b^3*c*d^3*((b + a*x)/(d + c*x))^(3/4) - 108*a^2*b^2*d^4*((b + a*x)/(d
+ c*x))^(3/4) - 18*b^4*c^3*d*((b + a*x)/(d + c*x))^(7/4) + 102*a*b^3*c^2*d^
2*((b + a*x)/(d + c*x))^(7/4) + 42*a^2*b^2*c*d^3*((b + a*x)/(d + c*x))^(7/4
) - 24*b^3*c^2*d^3*((b + a*x)/(d + c*x))^(7/4) - 126*a^3*b*d^4*((b + a*x)/(
d + c*x))^(7/4) - 144*a*b^2*c*d^4*((b + a*x)/(d + c*x))^(7/4) + 168*a^2*b*d
^5*((b + a*x)/(d + c*x))^(7/4) - 21*b^3*c^3*d^2*((b + a*x)/(d + c*x))^(11/4
) - 9*a*b^2*c^2*d^3*((b + a*x)/(d + c*x))^(11/4) - 15*a^2*b*c*d^4*((b + a*x
)/(d + c*x))^(11/4) + 36*b^2*c^2*d^4*((b + a*x)/(d + c*x))^(11/4) + 45*a^3*
d^5*((b + a*x)/(d + c*x))^(11/4) + 24*a*b*c*d^5*((b + a*x)/(d + c*x))^(11/4
) - 60*a^2*d^6*((b + a*x)/(d + c*x))^(11/4))/(96*b^2*d^2*(b - (d*(b + a*x))
/(d + c*x))^3) + ((7*b^3*c^3 + 3*a*b^2*c^2*d + 5*a^2*b*c*d^2 - 12*b^2*c^2*d
^2 - 15*a^3*d^3 - 8*a*b*c*d^3 + 20*a^2*d^4)*ArcTan[(d^(1/4)*((b + a*x)/(d +
 c*x))^(1/4))/b^(1/4)])/(64*b^(9/4)*d^(11/4)) + ((-7*b^3*c^3 - 3*a*b^2*c^2*
d - 5*a^2*b*c*d^2 + 12*b^2*c^2*d^2 + 15*a^3*d^3 + 8*a*b*c*d^3 - 20*a^2*d^4)
*ArcTanh[(d^(1/4)*((b + a*x)/(d + c*x))^(1/4))/b^(1/4)])/(64*b^(9/4)*d^(11/
4))
```

fricas [B] time = 4.10, size = 8984, normalized size = 23.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+b)/x^4/((a*x+b)/(c*x+d))^(1/4),x, algorithm="fricas")
```

```
[Out] -1/384*(12*b^2*d^2*x^3*((2401*b^12*c^12 + 4116*a*b^11*c^11*d + 160000*a^8*d
^16 - 32000*(15*a^9 + 8*a^7*b*c)*d^15 + 800*(675*a^10 + 920*a^8*b*c - 288*a
^6*b^2*c^2)*d^14 - 80*(3375*a^11 + 9900*a^9*b*c - 6720*a^7*b^2*c^2 - 5248*a
^5*b^3*c^3)*d^13 + (50625*a^12 + 378000*a^10*b*c - 429600*a^8*b^2*c^2 - 762
880*a^6*b^3*c^3 + 165376*a^4*b^4*c^4)*d^12 - 4*(16875*a^11*b*c - 31500*a^9*
b^2*c^2 - 81600*a^7*b^3*c^3 + 138880*a^5*b^4*c^4 + 62976*a^3*b^5*c^5)*d^11
- 2*(3375*a^10*b^2*c^2 - 44600*a^8*b^3*c^3 - 359520*a^6*b^4*c^4 - 85248*a^4
*b^5*c^5 + 41472*a^2*b^6*c^6)*d^10 - 4*(15375*a^9*b^3*c^3 + 105400*a^7*b^4*
c^4 - 48480*a^5*b^5*c^5 - 104704*a^3*b^6*c^6 - 13824*a*b^7*c^7)*d^9 + (9377
5*a^8*b^4*c^4 - 159840*a^6*b^5*c^5 - 423744*a^4*b^6*c^6 + 101376*a^2*b^7*c^
7 + 20736*b^8*c^8)*d^8 + 24*(775*a^7*b^5*c^5 + 3140*a^5*b^6*c^6 - 10976*a^3
*b^7*c^7 - 4896*a*b^8*c^8)*d^7 + 4*(7895*a^6*b^6*c^6 + 45624*a^4*b^7*c^7 -
1416*a^2*b^8*c^8 - 12096*b^9*c^9)*d^6 - 24*(2025*a^5*b^7*c^7 - 3334*a^3*b^8
```

*c^8 - 3864*a*b^9*c^9)*d^5 - (15249*a^4*b^8*c^8 + 31024*a^2*b^9*c^9 - 42336
*b^10*c^10)*d^4 - 28*(393*a^3*b^9*c^9 + 1148*a*b^10*c^10)*d^3 + 98*(97*a^2*
b^10*c^10 - 168*b^11*c^11)*d^2)/(b^9*d^11))^(1/4)*arctan((sqrt((117649*b^18
*c^18 + 302526*a*b^17*c^17*d + 64000000*a^12*d^24 - 19200000*(15*a^13 + 8*a
^11*b*c)*d^23 + 2400000*(225*a^14 + 280*a^12*b*c - 32*a^10*b^2*c^2)*d^22 -
160000*(3375*a^15 + 7650*a^13*b*c - 1680*a^11*b^2*c^2 - 2368*a^9*b^3*c^3)*d
^21 + 6000*(50625*a^16 + 198000*a^14*b*c - 59600*a^12*b^2*c^2 - 218880*a^10
*b^3*c^3 + 256*a^8*b^4*c^4)*d^20 - 120*(759375*a^17 + 5400000*a^15*b*c - 17
40000*a^13*b^2*c^2 - 14608000*a^11*b^3*c^3 + 1619200*a^9*b^4*c^4 + 3411968*
a^7*b^5*c^5)*d^19 + (11390625*a^18 + 188325000*a^16*b*c - 32400000*a^14*b^2
*c^2 - 1068640000*a^12*b^3*c^3 + 718944000*a^10*b^4*c^4 + 1158789120*a^8*b^
5*c^5 + 26066944*a^6*b^6*c^6)*d^18 - 6*(3796875*a^17*b*c + 2700000*a^15*b^2
*c^2 - 38400000*a^13*b^3*c^3 + 179920000*a^11*b^4*c^4 + 202016000*a^9*b^5*c
^5 - 68751360*a^7*b^6*c^6 - 40943616*a^5*b^7*c^7)*d^17 + 3*(1771875*a^16*b^
2*c^2 + 12600000*a^14*b^3*c^3 + 271640000*a^12*b^4*c^4 + 187488000*a^10*b^5
*c^5 - 441555200*a^8*b^6*c^6 - 190455808*a^6*b^7*c^7 + 184320*a^4*b^8*c^8)*
d^16 - 16*(1096875*a^15*b^3*c^3 + 19293750*a^13*b^4*c^4 + 7500000*a^11*b^5*
c^5 - 91234000*a^9*b^6*c^6 - 21033600*a^7*b^7*c^7 + 28212096*a^5*b^8*c^8 +
5114880*a^3*b^9*c^9)*d^15 + 60*(781875*a^14*b^4*c^4 + 433000*a^12*b^5*c^5 -
11277600*a^10*b^6*c^6 + 3939520*a^8*b^7*c^7 + 16628416*a^6*b^8*c^8 + 20259
84*a^4*b^9*c^9 - 165888*a^2*b^10*c^10)*d^14 - 24*(328125*a^13*b^5*c^5 - 405
5000*a^11*b^6*c^6 + 17584000*a^9*b^7*c^7 + 31071200*a^7*b^8*c^8 - 9901760*a
^5*b^9*c^9 - 9008640*a^3*b^10*c^10 - 497664*a*b^11*c^11)*d^13 + 4*(2100625*
a^12*b^6*c^6 + 55086000*a^10*b^7*c^7 + 50212200*a^8*b^8*c^8 - 153101120*a^6
*b^9*c^9 - 76238400*a^4*b^10*c^10 + 4561920*a^2*b^11*c^11 + 746496*b^12*c^1
2)*d^12 - 48*(879375*a^11*b^7*c^7 + 277625*a^9*b^8*c^8 - 9199200*a^7*b^9*c^
9 - 1007760*a^5*b^10*c^10 + 4747200*a^3*b^11*c^11 + 819072*a*b^12*c^12)*d^1
1 + 6*(492125*a^10*b^8*c^8 - 17593400*a^8*b^9*c^9 + 22208960*a^6*b^10*c^10
+ 48540800*a^4*b^11*c^11 - 259200*a^2*b^12*c^12 - 1741824*b^13*c^13)*d^10 -
20*(15975*a^9*b^9*c^9 + 4653552*a^7*b^10*c^10 + 5058048*a^5*b^11*c^11 - 60
51520*a^3*b^12*c^12 - 2685312*a*b^13*c^13)*d^9 + 6*(3566605*a^8*b^10*c^10 -
379744*a^6*b^11*c^11 - 21549600*a^4*b^12*c^12 - 3158400*a^2*b^13*c^13 + 25
40160*b^14*c^14)*d^8 + 48*(28305*a^7*b^11*c^11 + 869438*a^5*b^12*c^12 - 700
000*a^3*b^13*c^13 - 811440*a*b^14*c^14)*d^7 - 4*(103199*a^6*b^12*c^12 - 626
5560*a^4*b^13*c^13 - 4351200*a^2*b^14*c^14 + 2963520*b^15*c^15)*d^6 - 168*(
37083*a^5*b^13*c^13 - 27160*a^3*b^14*c^14 - 94080*a*b^15*c^15)*d^5 - 2940*(
461*a^4*b^14*c^14 + 2128*a^2*b^15*c^15 - 1764*b^16*c^16)*d^4 - 8232*(30*a^3
*b^15*c^15 + 413*a*b^16*c^16)*d^3 + 7203*(115*a^2*b^16*c^16 - 168*b^17*c^17
)*d^2)*sqrt((a*x + b)/(c*x + d)) + (2401*b^17*c^12*d^5 + 4116*a*b^16*c^11*d
^6 + 160000*a^8*b^5*d^21 - 32000*(15*a^9*b^5 + 8*a^7*b^6*c)*d^20 + 800*(675
*a^10*b^5 + 920*a^8*b^6*c - 288*a^6*b^7*c^2)*d^19 - 80*(3375*a^11*b^5 + 990
0*a^9*b^6*c - 6720*a^7*b^7*c^2 - 5248*a^5*b^8*c^3)*d^18 + (50625*a^12*b^5 +
378000*a^10*b^6*c - 429600*a^8*b^7*c^2 - 762880*a^6*b^8*c^3 + 165376*a^4*b
^9*c^4)*d^17 - 4*(16875*a^11*b^6*c - 31500*a^9*b^7*c^2 - 81600*a^7*b^8*c^3
+ 138880*a^5*b^9*c^4 + 62976*a^3*b^10*c^5)*d^16 - 2*(3375*a^10*b^7*c^2 - 44
600*a^8*b^8*c^3 - 359520*a^6*b^9*c^4 - 85248*a^4*b^10*c^5 + 41472*a^2*b^11*
c^6)*d^15 - 4*(15375*a^9*b^8*c^3 + 105400*a^7*b^9*c^4 - 48480*a^5*b^10*c^5
- 104704*a^3*b^11*c^6 - 13824*a*b^12*c^7)*d^14 + (93775*a^8*b^9*c^4 - 15984
0*a^6*b^10*c^5 - 423744*a^4*b^11*c^6 + 101376*a^2*b^12*c^7 + 20736*b^13*c^8
)*d^13 + 24*(775*a^7*b^10*c^5 + 3140*a^5*b^11*c^6 - 10976*a^3*b^12*c^7 - 48
96*a*b^13*c^8)*d^12 + 4*(7895*a^6*b^11*c^6 + 45624*a^4*b^12*c^7 - 1416*a^2*
b^13*c^8 - 12096*b^14*c^9)*d^11 - 24*(2025*a^5*b^12*c^7 - 3334*a^3*b^13*c^8
- 3864*a*b^14*c^9)*d^10 - (15249*a^4*b^13*c^8 + 31024*a^2*b^14*c^9 - 42336
*b^15*c^10)*d^9 - 28*(393*a^3*b^14*c^9 + 1148*a*b^15*c^10)*d^8 + 98*(97*a^2
*b^15*c^10 - 168*b^16*c^11)*d^7)*sqrt((2401*b^12*c^12 + 4116*a*b^11*c^11*d
+ 160000*a^8*d^16 - 32000*(15*a^9 + 8*a^7*b*c)*d^15 + 800*(675*a^10 + 920*a
^8*b*c - 288*a^6*b^2*c^2)*d^14 - 80*(3375*a^11 + 9900*a^9*b*c - 6720*a^7*b^
2*c^2 - 5248*a^5*b^3*c^3)*d^13 + (50625*a^12 + 378000*a^10*b*c - 429600*a^8
*b^2*c^2 - 762880*a^6*b^3*c^3 + 165376*a^4*b^4*c^4)*d^12 - 4*(16875*a^11*b*

c - 31500*a^9*b^2*c^2 - 81600*a^7*b^3*c^3 + 138880*a^5*b^4*c^4 + 62976*a^3*
b^5*c^5)*d^11 - 2*(3375*a^10*b^2*c^2 - 44600*a^8*b^3*c^3 - 359520*a^6*b^4*c
^4 - 85248*a^4*b^5*c^5 + 41472*a^2*b^6*c^6)*d^10 - 4*(15375*a^9*b^3*c^3 + 1
05400*a^7*b^4*c^4 - 48480*a^5*b^5*c^5 - 104704*a^3*b^6*c^6 - 13824*a*b^7*c^
7)*d^9 + (93775*a^8*b^4*c^4 - 159840*a^6*b^5*c^5 - 423744*a^4*b^6*c^6 + 101
376*a^2*b^7*c^7 + 20736*b^8*c^8)*d^8 + 24*(775*a^7*b^5*c^5 + 3140*a^5*b^6*c
^6 - 10976*a^3*b^7*c^7 - 4896*a*b^8*c^8)*d^7 + 4*(7895*a^6*b^6*c^6 + 45624*
a^4*b^7*c^7 - 1416*a^2*b^8*c^8 - 12096*b^9*c^9)*d^6 - 24*(2025*a^5*b^7*c^7
 - 3334*a^3*b^8*c^8 - 3864*a*b^9*c^9)*d^5 - (15249*a^4*b^8*c^8 + 31024*a^2*b
^9*c^9 - 42336*b^10*c^10)*d^4 - 28*(393*a^3*b^9*c^9 + 1148*a*b^10*c^10)*d^3
 + 98*(97*a^2*b^10*c^10 - 168*b^11*c^11)*d^2)/(b^9*d^11)))*b^2*d^3*((2401*b
^12*c^12 + 4116*a*b^11*c^11*d + 160000*a^8*d^16 - 32000*(15*a^9 + 8*a^7*b*c
)*d^15 + 800*(675*a^10 + 920*a^8*b*c - 288*a^6*b^2*c^2)*d^14 - 80*(3375*a^1
1 + 9900*a^9*b*c - 6720*a^7*b^2*c^2 - 5248*a^5*b^3*c^3)*d^13 + (50625*a^12
 + 378000*a^10*b*c - 429600*a^8*b^2*c^2 - 762880*a^6*b^3*c^3 + 165376*a^4*b^
4*c^4)*d^12 - 4*(16875*a^11*b*c - 31500*a^9*b^2*c^2 - 81600*a^7*b^3*c^3 + 1
38880*a^5*b^4*c^4 + 62976*a^3*b^5*c^5)*d^11 - 2*(3375*a^10*b^2*c^2 - 44600*
a^8*b^3*c^3 - 359520*a^6*b^4*c^4 - 85248*a^4*b^5*c^5 + 41472*a^2*b^6*c^6)*d
^10 - 4*(15375*a^9*b^3*c^3 + 105400*a^7*b^4*c^4 - 48480*a^5*b^5*c^5 - 10470
4*a^3*b^6*c^6 - 13824*a*b^7*c^7)*d^9 + (93775*a^8*b^4*c^4 - 159840*a^6*b^5*
c^5 - 423744*a^4*b^6*c^6 + 101376*a^2*b^7*c^7 + 20736*b^8*c^8)*d^8 + 24*(77
5*a^7*b^5*c^5 + 3140*a^5*b^6*c^6 - 10976*a^3*b^7*c^7 - 4896*a*b^8*c^8)*d^7
 + 4*(7895*a^6*b^6*c^6 + 45624*a^4*b^7*c^7 - 1416*a^2*b^8*c^8 - 12096*b^9*c^
9)*d^6 - 24*(2025*a^5*b^7*c^7 - 3334*a^3*b^8*c^8 - 3864*a*b^9*c^9)*d^5 - (1
5249*a^4*b^8*c^8 + 31024*a^2*b^9*c^9 - 42336*b^10*c^10)*d^4 - 28*(393*a^3*b
^9*c^9 + 1148*a*b^10*c^10)*d^3 + 98*(97*a^2*b^10*c^10 - 168*b^11*c^11)*d^2)
/(b^9*d^11))^(1/4) - (343*b^11*c^9*d^3 + 441*a*b^10*c^8*d^4 + 8000*a^6*b^2*
d^15 - 1200*(15*a^7*b^2 + 8*a^5*b^3*c)*d^14 + 60*(225*a^8*b^2 + 340*a^6*b^3
*c - 176*a^4*b^4*c^2)*d^13 - (3375*a^9*b^2 + 14400*a^7*b^3*c - 17520*a^5*b^
4*c^2 - 11008*a^3*b^5*c^3)*d^12 + 3*(1125*a^8*b^3*c - 2800*a^6*b^4*c^2 - 31
20*a^4*b^5*c^3 + 2112*a^2*b^6*c^4)*d^11 + 12*(75*a^7*b^4*c^2 - 320*a^5*b^5*
c^3 - 1172*a^3*b^6*c^4 - 288*a*b^7*c^5)*d^10 + 4*(875*a^6*b^5*c^3 + 2850*a^
4*b^6*c^4 - 1212*a^2*b^7*c^5 - 432*b^8*c^6)*d^9 - 222*(15*a^5*b^6*c^4 - 32*
a^3*b^7*c^5 - 24*a*b^8*c^6)*d^8 - 6*(205*a^4*b^7*c^5 + 152*a^2*b^8*c^6 - 50
4*b^9*c^7)*d^7 - 12*(129*a^3*b^8*c^6 + 224*a*b^9*c^7)*d^6 + 84*(11*a^2*b^9*
c^7 - 21*b^10*c^8)*d^5)*((a*x + b)/(c*x + d))^(1/4)*((2401*b^12*c^12 + 4116
*a*b^11*c^11*d + 160000*a^8*d^16 - 32000*(15*a^9 + 8*a^7*b*c)*d^15 + 800*(6
75*a^10 + 920*a^8*b*c - 288*a^6*b^2*c^2)*d^14 - 80*(3375*a^11 + 9900*a^9*b*
c - 6720*a^7*b^2*c^2 - 5248*a^5*b^3*c^3)*d^13 + (50625*a^12 + 378000*a^10*b
*c - 429600*a^8*b^2*c^2 - 762880*a^6*b^3*c^3 + 165376*a^4*b^4*c^4)*d^12 - 4
*(16875*a^11*b*c - 31500*a^9*b^2*c^2 - 81600*a^7*b^3*c^3 + 138880*a^5*b^4*c
^4 + 62976*a^3*b^5*c^5)*d^11 - 2*(3375*a^10*b^2*c^2 - 44600*a^8*b^3*c^3 - 3
59520*a^6*b^4*c^4 - 85248*a^4*b^5*c^5 + 41472*a^2*b^6*c^6)*d^10 - 4*(15375*
a^9*b^3*c^3 + 105400*a^7*b^4*c^4 - 48480*a^5*b^5*c^5 - 104704*a^3*b^6*c^6 -
 13824*a*b^7*c^7)*d^9 + (93775*a^8*b^4*c^4 - 159840*a^6*b^5*c^5 - 423744*a^
4*b^6*c^6 + 101376*a^2*b^7*c^7 + 20736*b^8*c^8)*d^8 + 24*(775*a^7*b^5*c^5 +
 3140*a^5*b^6*c^6 - 10976*a^3*b^7*c^7 - 4896*a*b^8*c^8)*d^7 + 4*(7895*a^6*b
^6*c^6 + 45624*a^4*b^7*c^7 - 1416*a^2*b^8*c^8 - 12096*b^9*c^9)*d^6 - 24*(20
25*a^5*b^7*c^7 - 3334*a^3*b^8*c^8 - 3864*a*b^9*c^9)*d^5 - (15249*a^4*b^8*c^
8 + 31024*a^2*b^9*c^9 - 42336*b^10*c^10)*d^4 - 28*(393*a^3*b^9*c^9 + 1148*a
*b^10*c^10)*d^3 + 98*(97*a^2*b^10*c^10 - 168*b^11*c^11)*d^2)/(b^9*d^11))^(1
/4))/(2401*b^12*c^12 + 4116*a*b^11*c^11*d + 160000*a^8*d^16 - 32000*(15*a^9
 + 8*a^7*b*c)*d^15 + 800*(675*a^10 + 920*a^8*b*c - 288*a^6*b^2*c^2)*d^14 -
80*(3375*a^11 + 9900*a^9*b*c - 6720*a^7*b^2*c^2 - 5248*a^5*b^3*c^3)*d^13 +
(50625*a^12 + 378000*a^10*b*c - 429600*a^8*b^2*c^2 - 762880*a^6*b^3*c^3 + 1
65376*a^4*b^4*c^4)*d^12 - 4*(16875*a^11*b*c - 31500*a^9*b^2*c^2 - 81600*a^7
*b^3*c^3 + 138880*a^5*b^4*c^4 + 62976*a^3*b^5*c^5)*d^11 - 2*(3375*a^10*b^2*
c^2 - 44600*a^8*b^3*c^3 - 359520*a^6*b^4*c^4 - 85248*a^4*b^5*c^5 + 41472*a^
2*b^6*c^6)*d^10 - 4*(15375*a^9*b^3*c^3 + 105400*a^7*b^4*c^4 - 48480*a^5*b^5

*c^5 - 104704*a^3*b^6*c^6 - 13824*a*b^7*c^7)*d^9 + (93775*a^8*b^4*c^4 - 159
840*a^6*b^5*c^5 - 423744*a^4*b^6*c^6 + 101376*a^2*b^7*c^7 + 20736*b^8*c^8)*
d^8 + 24*(775*a^7*b^5*c^5 + 3140*a^5*b^6*c^6 - 10976*a^3*b^7*c^7 - 4896*a*b
^8*c^8)*d^7 + 4*(7895*a^6*b^6*c^6 + 45624*a^4*b^7*c^7 - 1416*a^2*b^8*c^8 -
12096*b^9*c^9)*d^6 - 24*(2025*a^5*b^7*c^7 - 3334*a^3*b^8*c^8 - 3864*a*b^9*c
^9)*d^5 - (15249*a^4*b^8*c^8 + 31024*a^2*b^9*c^9 - 42336*b^10*c^10)*d^4 - 2
8*(393*a^3*b^9*c^9 + 1148*a*b^10*c^10)*d^3 + 98*(97*a^2*b^10*c^10 - 168*b^1
1*c^11)*d^2)) + 3*b^2*d^2*x^3*((2401*b^12*c^12 + 4116*a*b^11*c^11*d + 16000
0*a^8*d^16 - 32000*(15*a^9 + 8*a^7*b*c)*d^15 + 800*(675*a^10 + 920*a^8*b*c
- 288*a^6*b^2*c^2)*d^14 - 80*(3375*a^11 + 9900*a^9*b*c - 6720*a^7*b^2*c^2 -
5248*a^5*b^3*c^3)*d^13 + (50625*a^12 + 378000*a^10*b*c - 429600*a^8*b^2*c^
2 - 762880*a^6*b^3*c^3 + 165376*a^4*b^4*c^4)*d^12 - 4*(16875*a^11*b*c - 315
00*a^9*b^2*c^2 - 81600*a^7*b^3*c^3 + 138880*a^5*b^4*c^4 + 62976*a^3*b^5*c^5
)*d^11 - 2*(3375*a^10*b^2*c^2 - 44600*a^8*b^3*c^3 - 359520*a^6*b^4*c^4 - 85
248*a^4*b^5*c^5 + 41472*a^2*b^6*c^6)*d^10 - 4*(15375*a^9*b^3*c^3 + 105400*a
^7*b^4*c^4 - 48480*a^5*b^5*c^5 - 104704*a^3*b^6*c^6 - 13824*a*b^7*c^7)*d^9
+ (93775*a^8*b^4*c^4 - 159840*a^6*b^5*c^5 - 423744*a^4*b^6*c^6 + 101376*a^2
*b^7*c^7 + 20736*b^8*c^8)*d^8 + 24*(775*a^7*b^5*c^5 + 3140*a^5*b^6*c^6 - 10
976*a^3*b^7*c^7 - 4896*a*b^8*c^8)*d^7 + 4*(7895*a^6*b^6*c^6 + 45624*a^4*b^7
*c^7 - 1416*a^2*b^8*c^8 - 12096*b^9*c^9)*d^6 - 24*(2025*a^5*b^7*c^7 - 3334*
a^3*b^8*c^8 - 3864*a*b^9*c^9)*d^5 - (15249*a^4*b^8*c^8 + 31024*a^2*b^9*c^9
- 42336*b^10*c^10)*d^4 - 28*(393*a^3*b^9*c^9 + 1148*a*b^10*c^10)*d^3 + 98*(
97*a^2*b^10*c^10 - 168*b^11*c^11)*d^2)/(b^9*d^11))^(1/4)*log(b^7*d^8*((2401
*b^12*c^12 + 4116*a*b^11*c^11*d + 160000*a^8*d^16 - 32000*(15*a^9 + 8*a^7*b
*c)*d^15 + 800*(675*a^10 + 920*a^8*b*c - 288*a^6*b^2*c^2)*d^14 - 80*(3375*a
^11 + 9900*a^9*b*c - 6720*a^7*b^2*c^2 - 5248*a^5*b^3*c^3)*d^13 + (50625*a^1
2 + 378000*a^10*b*c - 429600*a^8*b^2*c^2 - 762880*a^6*b^3*c^3 + 165376*a^4*
b^4*c^4)*d^12 - 4*(16875*a^11*b*c - 31500*a^9*b^2*c^2 - 81600*a^7*b^3*c^3 +
138880*a^5*b^4*c^4 + 62976*a^3*b^5*c^5)*d^11 - 2*(3375*a^10*b^2*c^2 - 4460
0*a^8*b^3*c^3 - 359520*a^6*b^4*c^4 - 85248*a^4*b^5*c^5 + 41472*a^2*b^6*c^6)
d^10 - 4(15375*a^9*b^3*c^3 + 105400*a^7*b^4*c^4 - 48480*a^5*b^5*c^5 - 104
704*a^3*b^6*c^6 - 13824*a*b^7*c^7)*d^9 + (93775*a^8*b^4*c^4 - 159840*a^6*b^
5*c^5 - 423744*a^4*b^6*c^6 + 101376*a^2*b^7*c^7 + 20736*b^8*c^8)*d^8 + 24*(
775*a^7*b^5*c^5 + 3140*a^5*b^6*c^6 - 10976*a^3*b^7*c^7 - 4896*a*b^8*c^8)*d^
7 + 4*(7895*a^6*b^6*c^6 + 45624*a^4*b^7*c^7 - 1416*a^2*b^8*c^8 - 12096*b^9*
c^9)*d^6 - 24*(2025*a^5*b^7*c^7 - 3334*a^3*b^8*c^8 - 3864*a*b^9*c^9)*d^5 -
(15249*a^4*b^8*c^8 + 31024*a^2*b^9*c^9 - 42336*b^10*c^10)*d^4 - 28*(393*a^3
*b^9*c^9 + 1148*a*b^10*c^10)*d^3 + 98*(97*a^2*b^10*c^10 - 168*b^11*c^11)*d^
2)/(b^9*d^11))^(3/4) + (343*b^9*c^9 + 441*a*b^8*c^8*d + 8000*a^6*d^12 - 120
0*(15*a^7 + 8*a^5*b*c)*d^11 + 60*(225*a^8 + 340*a^6*b*c - 176*a^4*b^2*c^2)*
d^10 - (3375*a^9 + 14400*a^7*b*c - 17520*a^5*b^2*c^2 - 11008*a^3*b^3*c^3)*d
^9 + 3*(1125*a^8*b*c - 2800*a^6*b^2*c^2 - 3120*a^4*b^3*c^3 + 2112*a^2*b^4*c
^4)*d^8 + 12*(75*a^7*b^2*c^2 - 320*a^5*b^3*c^3 - 1172*a^3*b^4*c^4 - 288*a*b
^5*c^5)*d^7 + 4*(875*a^6*b^3*c^3 + 2850*a^4*b^4*c^4 - 1212*a^2*b^5*c^5 - 43
2*b^6*c^6)*d^6 - 222*(15*a^5*b^4*c^4 - 32*a^3*b^5*c^5 - 24*a*b^6*c^6)*d^5 -
6*(205*a^4*b^5*c^5 + 152*a^2*b^6*c^6 - 504*b^7*c^7)*d^4 - 12*(129*a^3*b^6*
c^6 + 224*a*b^7*c^7)*d^3 + 84*(11*a^2*b^7*c^7 - 21*b^8*c^8)*d^2)*((a*x + b)
/(c*x + d))^(1/4)) - 3*b^2*d^2*x^3*((2401*b^12*c^12 + 4116*a*b^11*c^11*d +
160000*a^8*d^16 - 32000*(15*a^9 + 8*a^7*b*c)*d^15 + 800*(675*a^10 + 920*a^8
*b*c - 288*a^6*b^2*c^2)*d^14 - 80*(3375*a^11 + 9900*a^9*b*c - 6720*a^7*b^2*
c^2 - 5248*a^5*b^3*c^3)*d^13 + (50625*a^12 + 378000*a^10*b*c - 429600*a^8*b
^2*c^2 - 762880*a^6*b^3*c^3 + 165376*a^4*b^4*c^4)*d^12 - 4*(16875*a^11*b*c
- 31500*a^9*b^2*c^2 - 81600*a^7*b^3*c^3 + 138880*a^5*b^4*c^4 + 62976*a^3*b^
5*c^5)*d^11 - 2*(3375*a^10*b^2*c^2 - 44600*a^8*b^3*c^3 - 359520*a^6*b^4*c^4
- 85248*a^4*b^5*c^5 + 41472*a^2*b^6*c^6)*d^10 - 4*(15375*a^9*b^3*c^3 + 105
400*a^7*b^4*c^4 - 48480*a^5*b^5*c^5 - 104704*a^3*b^6*c^6 - 13824*a*b^7*c^7)
*d^9 + (93775*a^8*b^4*c^4 - 159840*a^6*b^5*c^5 - 423744*a^4*b^6*c^6 + 10137
6*a^2*b^7*c^7 + 20736*b^8*c^8)*d^8 + 24*(775*a^7*b^5*c^5 + 3140*a^5*b^6*c^6
- 10976*a^3*b^7*c^7 - 4896*a*b^8*c^8)*d^7 + 4*(7895*a^6*b^6*c^6 + 45624*a^

```
4*b^7*c^7 - 1416*a^2*b^8*c^8 - 12096*b^9*c^9)*d^6 - 24*(2025*a^5*b^7*c^7 -
3334*a^3*b^8*c^8 - 3864*a*b^9*c^9)*d^5 - (15249*a^4*b^8*c^8 + 31024*a^2*b^9
*c^9 - 42336*b^10*c^10)*d^4 - 28*(393*a^3*b^9*c^9 + 1148*a*b^10*c^10)*d^3 +
 98*(97*a^2*b^10*c^10 - 168*b^11*c^11)*d^2)/(b^9*d^11))^(1/4)*log(-b^7*d^8*
((2401*b^12*c^12 + 4116*a*b^11*c^11*d + 160000*a^8*d^16 - 32000*(15*a^9 + 8
*a^7*b*c)*d^15 + 800*(675*a^10 + 920*a^8*b*c - 288*a^6*b^2*c^2)*d^14 - 80*(
3375*a^11 + 9900*a^9*b*c - 6720*a^7*b^2*c^2 - 5248*a^5*b^3*c^3)*d^13 + (506
25*a^12 + 378000*a^10*b*c - 429600*a^8*b^2*c^2 - 762880*a^6*b^3*c^3 + 16537
6*a^4*b^4*c^4)*d^12 - 4*(16875*a^11*b*c - 31500*a^9*b^2*c^2 - 81600*a^7*b^3
*c^3 + 138880*a^5*b^4*c^4 + 62976*a^3*b^5*c^5)*d^11 - 2*(3375*a^10*b^2*c^2
- 44600*a^8*b^3*c^3 - 359520*a^6*b^4*c^4 - 85248*a^4*b^5*c^5 + 41472*a^2*b^
6*c^6)*d^10 - 4*(15375*a^9*b^3*c^3 + 105400*a^7*b^4*c^4 - 48480*a^5*b^5*c^5
 - 104704*a^3*b^6*c^6 - 13824*a*b^7*c^7)*d^9 + (93775*a^8*b^4*c^4 - 159840*
a^6*b^5*c^5 - 423744*a^4*b^6*c^6 + 101376*a^2*b^7*c^7 + 20736*b^8*c^8)*d^8
+ 24*(775*a^7*b^5*c^5 + 3140*a^5*b^6*c^6 - 10976*a^3*b^7*c^7 - 4896*a*b^8*c
^8)*d^7 + 4*(7895*a^6*b^6*c^6 + 45624*a^4*b^7*c^7 - 1416*a^2*b^8*c^8 - 1209
6*b^9*c^9)*d^6 - 24*(2025*a^5*b^7*c^7 - 3334*a^3*b^8*c^8 - 3864*a*b^9*c^9)*
d^5 - (15249*a^4*b^8*c^8 + 31024*a^2*b^9*c^9 - 42336*b^10*c^10)*d^4 - 28*(3
93*a^3*b^9*c^9 + 1148*a*b^10*c^10)*d^3 + 98*(97*a^2*b^10*c^10 - 168*b^11*c^
11)*d^2)/(b^9*d^11))^(3/4) + (343*b^9*c^9 + 441*a*b^8*c^8*d + 8000*a^6*d^12
 - 1200*(15*a^7 + 8*a^5*b*c)*d^11 + 60*(225*a^8 + 340*a^6*b*c - 176*a^4*b^2
*c^2)*d^10 - (3375*a^9 + 14400*a^7*b*c - 17520*a^5*b^2*c^2 - 11008*a^3*b^3*
c^3)*d^9 + 3*(1125*a^8*b*c - 2800*a^6*b^2*c^2 - 3120*a^4*b^3*c^3 + 2112*a^2
*b^4*c^4)*d^8 + 12*(75*a^7*b^2*c^2 - 320*a^5*b^3*c^3 - 1172*a^3*b^4*c^4 - 2
88*a*b^5*c^5)*d^7 + 4*(875*a^6*b^3*c^3 + 2850*a^4*b^4*c^4 - 1212*a^2*b^5*c^
5 - 432*b^6*c^6)*d^6 - 222*(15*a^5*b^4*c^4 - 32*a^3*b^5*c^5 - 24*a*b^6*c^6)
*d^5 - 6*(205*a^4*b^5*c^5 + 152*a^2*b^6*c^6 - 504*b^7*c^7)*d^4 - 12*(129*a^
3*b^6*c^6 + 224*a*b^7*c^7)*d^3 + 84*(11*a^2*b^7*c^7 - 21*b^8*c^8)*d^2)*((a*
x + b)/(c*x + d))^(1/4)) + 4*(32*b^2*d^3 - (7*b^2*c^3 + 6*a*b*c^2*d + 60*a*
c*d^3 - 3*(15*a^2*c + 4*b*c^2)*d^2)*x^3 - 3*(b^2*c^2*d + 14*a*b*c*d^2 + 20*
a*d^4 - 5*(3*a^2 + 4*b*c)*d^3)*x^2 + 12*(3*b^2*c*d^2 - 3*a*b*d^3 + 4*b*d^4)
*x)*((a*x + b)/(c*x + d))^(3/4))/(b^2*d^2*x^3)
```

giac [B] time = 6.05, size = 1632, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+b)/x^4/((a*x+b)/(c*x+d))^(1/4),x, algorithm="giac")
```

```
[Out] 1/768*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)*(6*sqrt(2)*(7*b^4*c^4 - 4*a*b
^3*c^3*d + 2*a^2*b^2*c^2*d^2 - 12*b^3*c^3*d^2 - 20*a^3*b*c*d^3 + 4*a*b^2*c^
2*d^3 + 15*a^4*d^4 + 28*a^2*b*c*d^4 - 20*a^3*d^5)*arctan(1/2*sqrt(2)*(sqrt(
2)*(-b/d)^(1/4) + 2*((a*x + b)/(c*x + d))^(1/4))/(-b/d)^(1/4))/((-b*d^3)^(1
/4)*b^2*d^2) + 6*sqrt(2)*(7*b^4*c^4 - 4*a*b^3*c^3*d + 2*a^2*b^2*c^2*d^2 - 1
2*b^3*c^3*d^2 - 20*a^3*b*c*d^3 + 4*a*b^2*c^2*d^3 + 15*a^4*d^4 + 28*a^2*b*c*
d^4 - 20*a^3*d^5)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b/d)^(1/4) - 2*((a*x + b)/
(c*x + d))^(1/4))/(-b/d)^(1/4))/((-b*d^3)^(1/4)*b^2*d^2) - 3*sqrt(2)*(7*b^4
*c^4 - 4*a*b^3*c^3*d + 2*a^2*b^2*c^2*d^2 - 12*b^3*c^3*d^2 - 20*a^3*b*c*d^3
+ 4*a*b^2*c^2*d^3 + 15*a^4*d^4 + 28*a^2*b*c*d^4 - 20*a^3*d^5)*log(sqrt(2)*(
(a*x + b)/(c*x + d))^(1/4)*(-b/d)^(1/4) + sqrt((a*x + b)/(c*x + d)) + sqrt(
-b/d))/((-b*d^3)^(1/4)*b^2*d^2) + 3*sqrt(2)*(7*b^4*c^4 - 4*a*b^3*c^3*d + 2*
a^2*b^2*c^2*d^2 - 12*b^3*c^3*d^2 - 20*a^3*b*c*d^3 + 4*a*b^2*c^2*d^3 + 15*a^
4*d^4 + 28*a^2*b*c*d^4 - 20*a^3*d^5)*log(-sqrt(2)*((a*x + b)/(c*x + d))^(1/
4)*(-b/d)^(1/4) + sqrt((a*x + b)/(c*x + d)) + sqrt(-b/d))/((-b*d^3)^(1/4)*b
^2*d^2) + 8*(7*b^6*c^4*((a*x + b)/(c*x + d))^(3/4) - 4*a*b^5*c^3*d*((a*x +
b)/(c*x + d))^(3/4) - 18*(a*x + b)*b^5*c^4*d*((a*x + b)/(c*x + d))^(3/4)/(c
*x + d) - 126*a^2*b^4*c^2*d^2*((a*x + b)/(c*x + d))^(3/4) + 120*(a*x + b)*a
*b^4*c^3*d^2*((a*x + b)/(c*x + d))^(3/4)/(c*x + d) - 12*b^5*c^3*d^2*((a*x +
 b)/(c*x + d))^(3/4) - 21*(a*x + b)^2*b^4*c^4*d^2*((a*x + b)/(c*x + d))^(3/
```

```
4)/(c*x + d)^2 + 236*a^3*b^3*c*d^3*((a*x + b)/(c*x + d))^(3/4) - 60*(a*x +
b)*a^2*b^3*c^2*d^3*((a*x + b)/(c*x + d))^(3/4)/(c*x + d) + 132*a*b^4*c^2*d^
3*((a*x + b)/(c*x + d))^(3/4) + 12*(a*x + b)^2*a*b^3*c^3*d^3*((a*x + b)/(c*
x + d))^(3/4)/(c*x + d)^2 - 24*(a*x + b)*b^4*c^3*d^3*((a*x + b)/(c*x + d))^
(3/4)/(c*x + d) - 113*a^4*b^2*d^4*((a*x + b)/(c*x + d))^(3/4) - 168*(a*x +
b)*a^3*b^2*c*d^4*((a*x + b)/(c*x + d))^(3/4)/(c*x + d) - 228*a^2*b^3*c*d^4*
((a*x + b)/(c*x + d))^(3/4) - 6*(a*x + b)^2*a^2*b^2*c^2*d^4*((a*x + b)/(c*x
 + d))^(3/4)/(c*x + d)^2 - 120*(a*x + b)*a*b^3*c^2*d^4*((a*x + b)/(c*x + d)
)^(3/4)/(c*x + d) + 36*(a*x + b)^2*b^3*c^3*d^4*((a*x + b)/(c*x + d))^(3/4)/
(c*x + d)^2 + 126*(a*x + b)*a^4*b*d^5*((a*x + b)/(c*x + d))^(3/4)/(c*x + d)
 + 108*a^3*b^2*d^5*((a*x + b)/(c*x + d))^(3/4) + 60*(a*x + b)^2*a^3*b*c*d^5
*((a*x + b)/(c*x + d))^(3/4)/(c*x + d)^2 + 312*(a*x + b)*a^2*b^2*c*d^5*((a*
x + b)/(c*x + d))^(3/4)/(c*x + d) - 12*(a*x + b)^2*a*b^2*c^2*d^5*((a*x + b)
/(c*x + d))^(3/4)/(c*x + d)^2 - 45*(a*x + b)^2*a^4*d^6*((a*x + b)/(c*x + d)
)^(3/4)/(c*x + d)^2 - 168*(a*x + b)*a^3*b*d^6*((a*x + b)/(c*x + d))^(3/4)/(
c*x + d) - 84*(a*x + b)^2*a^2*b*c*d^6*((a*x + b)/(c*x + d))^(3/4)/(c*x + d)
^2 + 60*(a*x + b)^2*a^3*d^7*((a*x + b)/(c*x + d))^(3/4)/(c*x + d)^2)/((b -
(a*x + b)*d/(c*x + d))^3*b^2*d^2))
```

maple [F]   time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{dx+b}{x^4\left(\frac{ax+b}{cx+d}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+b)/x^4/((a*x+b)/(c*x+d))^(1/4),x)
```

```
[Out] int((d*x+b)/x^4/((a*x+b)/(c*x+d))^(1/4),x)
```

maxima [A]   time = 0.43, size = 535, normalized size = 1.38

$$-\frac{3\left(7b^3c^3d^2+3ab^2c^2d^3+20a^2d^6-\left(15a^3+8abc\right)d^5+\left(5a^2bc-12b^2c^2\right)d^4\right)\left(\frac{ax+b}{cx+d}\right)^{\frac{11}{4}}+6\left(3b^4c^3d-17ab^3c^2d^2-28a^2bd^5+3\left(7a^3b+8ab^2c\right)d^4-\left(7a^2b^2c-4b^3c^2\right)d^3\right)\left(\frac{ax+b}{cx+d}\right)^{\frac{7}{4}}-\left(7b^5c^3+3ab^4c^2d-108a^2b^2d^4+\left(113a^3b^2+120ab^3c\right)d^3-3\left(41a^2b^3c+4b^4c^2\right)d^2\right)\left(\frac{ax+b}{cx+d}\right)^{\frac{3}{4}}}{96\left(b^5d^2-\frac{3(ax+b)b^4d^3}{cx+d}+\frac{3(ax+b)^2b^3d^4}{(cx+d)^2}-\frac{(ax+b)^3b^2d^5}{(cx+d)^3}\right)}+\frac{\left(7b^3c^3+3ab^2c^2d+20a^2d^4-\left(15a^3+8abc\right)d^3+\left(5a^2bc-12b^2c^2\right)d^2\right)\left(\frac{2\arctan\left(\frac{\sqrt{d}\left(\frac{ax+b}{cx+d}\right)^{\frac{1}{4}}}{\sqrt{\sqrt{b}\sqrt{d}}}\right)}{\sqrt{\sqrt{b}\sqrt{d}}\sqrt{d}}+\frac{\log\left(\frac{\sqrt{d}\left(\frac{ax+b}{cx+d}\right)^{\frac{1}{4}}-\sqrt{\sqrt{b}\sqrt{d}}}{\sqrt{d}\left(\frac{ax+b}{cx+d}\right)^{\frac{1}{4}}+\sqrt{\sqrt{b}\sqrt{d}}}\right)}{\sqrt{\sqrt{b}\sqrt{d}}\sqrt{d}}\right)}{128b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+b)/x^4/((a*x+b)/(c*x+d))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/96*(3*(7*b^3*c^3*d^2 + 3*a*b^2*c^2*d^3 + 20*a^2*d^6 - (15*a^3 + 8*a*b*c)
*d^5 + (5*a^2*b*c - 12*b^2*c^2)*d^4)*((a*x + b)/(c*x + d))^(11/4) + 6*(3*b^
4*c^3*d - 17*a*b^3*c^2*d^2 - 28*a^2*b*d^5 + 3*(7*a^3*b + 8*a*b^2*c)*d^4 - (
7*a^2*b^2*c - 4*b^3*c^2)*d^3)*((a*x + b)/(c*x + d))^(7/4) - (7*b^5*c^3 + 3*
a*b^4*c^2*d - 108*a^2*b^2*d^4 + (113*a^3*b^2 + 120*a*b^3*c)*d^3 - 3*(41*a^2
*b^3*c + 4*b^4*c^2)*d^2)*((a*x + b)/(c*x + d))^(3/4))/(b^5*d^2 - 3*(a*x + b
)*b^4*d^3/(c*x + d) + 3*(a*x + b)^2*b^3*d^4/(c*x + d)^2 - (a*x + b)^3*b^2*d
^5/(c*x + d)^3) + 1/128*(7*b^3*c^3 + 3*a*b^2*c^2*d + 20*a^2*d^4 - (15*a^3 +
 8*a*b*c)*d^3 + (5*a^2*b*c - 12*b^2*c^2)*d^2)*(2*arctan(sqrt(d)*((a*x + b)/
(c*x + d))^(1/4)/sqrt(sqrt(b)*sqrt(d)))/(sqrt(sqrt(b)*sqrt(d))*sqrt(d)) + l
og((sqrt(d)*((a*x + b)/(c*x + d))^(1/4) - sqrt(sqrt(b)*sqrt(d)))/(sqrt(d)*(
(a*x + b)/(c*x + d))^(1/4) + sqrt(sqrt(b)*sqrt(d))))/(sqrt(sqrt(b)*sqrt(d))
*sqrt(d)))/(b^2*d^2)
```

mupad [B]   time = 4.22, size = 532, normalized size = 1.37

$$\frac{\mathrm{atanh}\left(\frac{d^{1/4}\left(\frac{b+ax}{d+cx}\right)^{1/4}}{b^{1/4}}\right)(ad-bc)\left(15a^2d^2+10abcd-20ad^3+7b^2c^2-12bcd^2\right)}{64b^{9/4}d^{11/4}}-\frac{\mathrm{atan}\left(\frac{d^{1/4}\left(\frac{b+ax}{d+cx}\right)^{1/4}}{b^{1/4}}\right)(ad-bc)\left(15a^2d^2+10abcd-20ad^3+7b^2c^2-12bcd^2\right)}{64b^{9/4}d^{11/4}}-\frac{\frac{c^2\left(\frac{b+ax}{d+cx}\right)^{7/4}\left(\frac{21a^3cd^3}{16}-\frac{7a^2bc^2d^2}{16}-\frac{7a^2cd^4}{4}-\frac{17ab^2c^3d}{16}+\frac{3abc^2d^3}{2}+\frac{3b^3c^4}{16}+\frac{b^2c^3d^2}{4}\right)}{a^3bd^4}-\frac{c\left(\frac{b+ax}{d+cx}\right)^{3/4}\left(\frac{113a^3c^2d^3}{96}-\frac{41a^2bc^3d^2}{32}-\frac{9a^2c^2d^4}{8}+\frac{ab^2c^4d}{32}+\frac{5abc^3d^3}{4}+\frac{7b^3c^5}{96}-\frac{b^2c^4d^2}{8}\right)}{a^3d^5}+\frac{c^3\left(\frac{b+ax}{d+cx}\right)^{11/4}\left(-\frac{15a^3d^3}{32}+\frac{5a^2bcd^2}{32}+\frac{5a^2d^4}{8}+\frac{3ab^2c^2d}{32}-\frac{abcd^3}{4}+\frac{7b^3c^3}{32}-\frac{3b^2c^2d^2}{8}\right)}{a^3b^2d^3}}{\frac{b^3c^3}{a^3d^3}-\frac{c^3(b+ax)^3}{a^3(d+cx)^3}+\frac{3bc^3(b+ax)^2}{a^3d(d+cx)^2}-\frac{3b^2c^3(b+ax)}{a^3d^2(d+cx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + d*x)/(x^4*((b + a*x)/(d + c*x))^(1/4)),x)
```

```
[Out] (atanh((d^(1/4)*((b + a*x)/(d + c*x))^(1/4))/b^(1/4))*(a*d - b*c)*(15*a^2*d
^2 - 20*a*d^3 + 7*b^2*c^2 - 12*b*c*d^2 + 10*a*b*c*d))/(64*b^(9/4)*d^(11/4))
 - (atan((d^(1/4)*((b + a*x)/(d + c*x))^(1/4))/b^(1/4))*(a*d - b*c)*(15*a^2
*d^2 - 20*a*d^3 + 7*b^2*c^2 - 12*b*c*d^2 + 10*a*b*c*d))/(64*b^(9/4)*d^(11/4
)) - ((c^2*((b + a*x)/(d + c*x))^(7/4)*((3*b^3*c^4)/16 - (7*a^2*c*d^4)/4 +
(21*a^3*c*d^3)/16 + (b^2*c^3*d^2)/4 - (7*a^2*b*c^2*d^2)/16 + (3*a*b*c^2*d^3
)/2 - (17*a*b^2*c^3*d)/16))/(a^3*b*d^4) - (c*((b + a*x)/(d + c*x))^(3/4)*((
7*b^3*c^5)/96 - (9*a^2*c^2*d^4)/8 + (113*a^3*c^2*d^3)/96 - (b^2*c^4*d^2)/8
- (41*a^2*b*c^3*d^2)/32 + (5*a*b*c^3*d^3)/4 + (a*b^2*c^4*d)/32))/(a^3*d^5)
+ (c^3*((b + a*x)/(d + c*x))^(11/4)*((5*a^2*d^4)/8 - (15*a^3*d^3)/32 + (7*b
^3*c^3)/32 - (3*b^2*c^2*d^2)/8 - (a*b*c*d^3)/4 + (3*a*b^2*c^2*d)/32 + (5*a^
2*b*c*d^2)/32))/(a^3*b^2*d^3))/((b^3*c^3)/(a^3*d^3) - (c^3*(b + a*x)^3)/(a^
3*(d + c*x)^3) + (3*b*c^3*(b + a*x)^2)/(a^3*d*(d + c*x)^2) - (3*b^2*c^3*(b
+ a*x))/(a^3*d^2*(d + c*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b + dx}{x^4 \sqrt[4]{\frac{ax+b}{cx+d}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+b)/x**4/((a*x+b)/(c*x+d))**(1/4),x)
```

```
[Out] Integral((b + d*x)/(x**4*((a*x + b)/(c*x + d))**(1/4)), x)
```

3.2338 $\int \frac{1+x^6}{\sqrt[4]{-x^3+x^5}\,(1-x^6)}\,dx$

Optimal. Leaf size=390

$$\frac{4\left(x^5-x^3\right)^{3/4}}{3x^2\left(x^2-1\right)}+\frac{\sqrt[4]{2}\tan^{-1}\left(\frac{3^{7/8}\sqrt{2-\sqrt{2}}\,x\sqrt[4]{x^5-x^3}}{3^{3/4}\sqrt{x^5-x^3}-3x^2}\right)}{3\sqrt[8]{3\left(17+12\sqrt{2}\right)}}+\frac{1}{3}\sqrt[4]{2}\sqrt[8]{\frac{1}{3}\left(17+12\sqrt{2}\right)}\tan^{-1}\left(\frac{3^{7/8}\sqrt{2+\sqrt{2}}\,x\sqrt[4]{x^5-x^3}}{3^{3/4}\sqrt{x^5-x^3}-3x^2}\right)+\frac{\sqrt[4]{2}\,\mathrm{ta}\ldots}{\ldots}$$

Rubi [F] time = 180.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$Aborted

Verification is not applicable to the result.

```
[In] Int[(1 + x^6)/((-x^3 + x^5)^(1/4)*(1 - x^6)),x]
```

```
[Out] $Aborted
```

Rubi steps

Aborted

Mathematica [F] time = 1.15, size = 0, normalized size = 0.00

$$\int \frac{1+x^6}{\sqrt[4]{-x^3+x^5}\,\left(1-x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^6)/((-x^3 + x^5)^(1/4)*(1 - x^6)),x]
```

```
[Out] Integrate[(1 + x^6)/((-x^3 + x^5)^(1/4)*(1 - x^6)), x]
```

IntegrateAlgebraic [A] time = 4.70, size = 416, normalized size = 1.07

$$\frac{4\left(x^5-x^3\right)^{3/4}}{3x^2\left(x^2-1\right)}+\frac{\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\,3^{7/8}x\sqrt[4]{x^5-x^3}}{\sqrt[8]{17+12\sqrt{2}}\left(3^{3/4}\sqrt{x^5-x^3}-3x^2\right)}\right)}{3\sqrt[8]{3\left(17+12\sqrt{2}\right)}}+\frac{1}{3}\sqrt[4]{2}\sqrt[8]{\frac{1}{3}\left(17+12\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt[4]{2}\,3^{7/8}\sqrt[8]{17+12\sqrt{2}}\,x\sqrt[4]{x^5-x^3}}{3^{3/4}\sqrt{x^5-x^3}-3x^2}\right)+\frac{\sqrt[4]{2}\tanh^{-1}\left(\frac{3\sqrt[8]{\frac{17}{8748}+\frac{\sqrt{2}}{729}}\,x^2+3^{3/4}\sqrt[8]{\frac{17}{8748}+\frac{\sqrt{2}}{729}}\sqrt{x^5-x^3}}{x\sqrt[4]{x^5-x^3}}\right)}{3\sqrt[8]{3\left(17+12\sqrt{2}\right)}}+\frac{1}{3}\sqrt[4]{2}\sqrt[8]{\frac{1}{3}\left(17+12\sqrt{2}\right)}\tanh^{-1}\left(\frac{\frac{\sqrt[8]{\frac{3}{17+12\sqrt{2}}}\,x^2}{\sqrt[4]{2}}+\frac{\sqrt{x^5-x^3}}{\sqrt[4]{2}\sqrt[8]{3\left(17+12\sqrt{2}\right)}}}{x\sqrt[4]{x^5-x^3}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] IntegrateAlgebraic[(1 + x^6)/((-x^3 + x^5)^(1/4)*(1 - x^6)),x]
```

```
[Out] (4*(-x^3 + x^5)^(3/4))/(3*x^2*(-1 + x^2)) + (2^(1/4)*ArcTan[(2^(1/4)*3^(7/8
)*x*(-x^3 + x^5)^(1/4))/((17 + 12*Sqrt[2])^(1/8)*(-3*x^2 + 3^(3/4)*Sqrt[-x^
3 + x^5]))])/(3*(3*(17 + 12*Sqrt[2]))^(1/8)) + (2^(1/4)*((17 + 12*Sqrt[2])/
3)^(1/8)*ArcTan[(2^(1/4)*3^(7/8)*(17 + 12*Sqrt[2])^(1/8)*x*(-x^3 + x^5)^(1/
4))/(-3*x^2 + 3^(3/4)*Sqrt[-x^3 + x^5])])/3 + (2^(1/4)*ArcTanh[(3*(17/8748
+ Sqrt[2]/729)^(1/8)*x^2 + 3^(3/4)*(17/8748 + Sqrt[2]/729)^(1/8)*Sqrt[-x^3
+ x^5])/(x*(-x^3 + x^5)^(1/4))])/(3*(3*(17 + 12*Sqrt[2]))^(1/8)) + (2^(1/4)
*((17 + 12*Sqrt[2])/3)^(1/8)*ArcTanh[(((3/(17 + 12*Sqrt[2]))^(1/8)*x^2)/2^(
1/4) + Sqrt[-x^3 + x^5]/(2^(1/4)*(3*(17 + 12*Sqrt[2]))^(1/8)))/(x*(-x^3 + x
^5)^(1/4))])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^5-x^3)^(1/4)/(-x^6+1),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^6+1}{\left(x^6-1\right)\left(x^5-x^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^5-x^3)^(1/4)/(-x^6+1),x, algorithm="giac")
```

```
[Out] integrate(-(x^6 + 1)/((x^6 - 1)*(x^5 - x^3)^(1/4)), x)
```

maple [C] time = 48.51, size = 2154, normalized size = 5.52

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/(x^5-x^3)^(1/4)/(-x^6+1),x)
```

```
[Out] 4/3*x/(x^3*(x^2-1))^(1/4)-1/9*RootOf(_Z^2+RootOf(_Z^8+2187)*RootOf(_Z^2+Roo
tOf(_Z^8+2187)^2))*ln(-108*(16*RootOf(_Z^2+RootOf(_Z^8+2187)*RootOf(_Z^2+Ro
otOf(_Z^8+2187)^2))*RootOf(_Z^8+2187)^8*x^4-24*RootOf(_Z^2+RootOf(_Z^8+2187
)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+2187)^8*x^3-16*RootOf(_Z^2+
RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+2187)^8*x^2
+54*(x^5-x^3)^(1/2)*RootOf(_Z^8+2187)^5*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*Ro
otOf(_Z^2+RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*x-162*(x^5-x^
3)^(1/4)*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^8+2187)^5*x^2-1350*Root
Of(_Z^2+RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+218
7)^4*x^4-243*RootOf(_Z^8+2187)^4*RootOf(_Z^2+RootOf(_Z^8+2187)*RootOf(_Z^2+
RootOf(_Z^8+2187)^2))*x^3+8100*(x^5-x^3)^(3/4)*RootOf(_Z^8+2187)^4+1350*Roo
tOf(_Z^2+RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+21
87)^4*x^2-24300*(x^5-x^3)^(1/2)*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^
2+RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+2187)*x+7
2900*(x^5-x^3)^(1/4)*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^8+2187)*x^2
+28431*RootOf(_Z^2+RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*x^4+
56862*RootOf(_Z^2+RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*x^3+3
9366*(x^5-x^3)^(3/4)-28431*RootOf(_Z^2+RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf
(_Z^8+2187)^2))*x^2)/x^2/(x*RootOf(_Z^8+2187)^4-2*RootOf(_Z^8+2187)^4+81*x)
/(x*RootOf(_Z^8+2187)^4-2*RootOf(_Z^8+2187)^4+135*x+108))-1/9*RootOf(_Z^8+2
187)*ln(108*(16*RootOf(_Z^8+2187)^9*x^4-24*RootOf(_Z^8+2187)^9*x^3-16*RootO
f(_Z^8+2187)^9*x^2+54*(x^5-x^3)^(1/2)*RootOf(_Z^8+2187)^7*x-162*(x^5-x^3)^(
1/4)*RootOf(_Z^8+2187)^6*x^2+1350*RootOf(_Z^8+2187)^5*x^4+243*x^3*RootOf(_Z
^8+2187)^5-1350*RootOf(_Z^8+2187)^5*x^2-8100*(x^5-x^3)^(3/4)*RootOf(_Z^8+21
87)^4+24300*(x^5-x^3)^(1/2)*RootOf(_Z^8+2187)^3*x-72900*(x^5-x^3)^(1/4)*Roo
tOf(_Z^8+2187)^2*x^2+28431*RootOf(_Z^8+2187)*x^4+56862*RootOf(_Z^8+2187)*x^
3-28431*RootOf(_Z^8+2187)*x^2+39366*(x^5-x^3)^(3/4))/x^2/(x*RootOf(_Z^8+218
7)^4-2*RootOf(_Z^8+2187)^4-135*x-108)/(x*RootOf(_Z^8+2187)^4-2*RootOf(_Z^8+
2187)^4-81*x))-1/9*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*ln(108*(16*x^4*RootOf(_
Z^8+2187)^8*RootOf(_Z^2+RootOf(_Z^8+2187)^2)-24*x^3*RootOf(_Z^8+2187)^8*Roo
tOf(_Z^2+RootOf(_Z^8+2187)^2)-16*x^2*RootOf(_Z^8+2187)^8*RootOf(_Z^2+RootOf
(_Z^8+2187)^2)-54*(x^5-x^3)^(1/2)*RootOf(_Z^8+2187)^6*RootOf(_Z^2+RootOf(_Z
^8+2187)^2)*x+1350*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^8+2187)^4*x^4
+162*(x^5-x^3)^(1/4)*RootOf(_Z^8+2187)^6*x^2+243*RootOf(_Z^2+RootOf(_Z^8+21
87)^2)*RootOf(_Z^8+2187)^4*x^3-1350*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf
(_Z^8+2187)^4*x^2-8100*(x^5-x^3)^(3/4)*RootOf(_Z^8+2187)^4-24300*(x^5-x^3)^
```

```
(1/2)*RootOf(_Z^8+2187)^2*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*x+28431*RootOf(_
Z^2+RootOf(_Z^8+2187)^2)*x^4+72900*(x^5-x^3)^(1/4)*RootOf(_Z^8+2187)^2*x^2+
56862*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*x^3-28431*RootOf(_Z^2+RootOf(_Z^8+21
87)^2)*x^2+39366*(x^5-x^3)^(3/4))/x^2/(x*RootOf(_Z^8+2187)^4-2*RootOf(_Z^8+
2187)^4-135*x-108)/(x*RootOf(_Z^8+2187)^4-2*RootOf(_Z^8+2187)^4-81*x))+1/19
683*RootOf(_Z^8+2187)^7*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^2+RootOf
(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*ln(-108*(26*RootOf(_Z^2+RootO
f(_Z^8+2187)^2)*RootOf(_Z^2+RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)
^2))*RootOf(_Z^8+2187)^11*x^4-39*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z
^2+RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+2187)^11
*x^3-26*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^8+2187)^11*RootOf(_Z^2+R
ootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*x^2+243*RootOf(_Z^2+Root
Of(_Z^8+2187)^2)*RootOf(_Z^2+RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187
)^2))*RootOf(_Z^8+2187)^7*x^4-4050*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(
_Z^2+RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+2187)^
7*x^3-243*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^2+RootOf(_Z^8+2187)*Ro
otOf(_Z^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+2187)^7*x^2+4374*(x^5-x^3)^(1/2
)*RootOf(_Z^8+2187)^6*RootOf(_Z^2+RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8
+2187)^2))*x+13122*(x^5-x^3)^(1/4)*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(
_Z^8+2187)^5*x^2-52488*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^2+RootOf(
_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*RootOf(_Z^8+2187)^3*x^4-104976
*RootOf(_Z^2+RootOf(_Z^8+2187)^2)*RootOf(_Z^8+2187)^3*RootOf(_Z^2+RootOf(_Z
^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*x^3+52488*RootOf(_Z^2+RootOf(_Z^
8+2187)^2)*RootOf(_Z^2+RootOf(_Z^8+2187)*RootOf(_Z^2+RootOf(_Z^8+2187)^2))*
RootOf(_Z^8+2187)^3*x^2+656100*(x^5-x^3)^(3/4)*RootOf(_Z^8+2187)^4-1968300*
(x^5-x^3)^(1/2)*RootOf(_Z^8+2187)^2*RootOf(_Z^2+RootOf(_Z^8+2187)*RootOf(_Z
^2+RootOf(_Z^8+2187)^2))*x-5904900*(x^5-x^3)^(1/4)*RootOf(_Z^2+RootOf(_Z^8+
2187)^2)*RootOf(_Z^8+2187)*x^2+3188646*(x^5-x^3)^(3/4))/x^2/(x*RootOf(_Z^8+
2187)^4-2*RootOf(_Z^8+2187)^4+81*x)/(x*RootOf(_Z^8+2187)^4-2*RootOf(_Z^8+21
87)^4+135*x+108))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^6+1}{(x^6-1)(x^5-x^3)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^5-x^3)^(1/4)/(-x^6+1),x, algorithm="maxima")
[Out] -integrate((x^6 + 1)/((x^6 - 1)*(x^5 - x^3)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x^6+1}{(x^6-1)\,(x^5-x^3)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^6 + 1)/((x^6 - 1)*(x^5 - x^3)^(1/4)),x)
[Out] int(-(x^6 + 1)/((x^6 - 1)*(x^5 - x^3)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^6}{x^6\sqrt[4]{x^5-x^3}-\sqrt[4]{x^5-x^3}}\,dx-\int \frac{1}{x^6\sqrt[4]{x^5-x^3}-\sqrt[4]{x^5-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/(x**5-x**3)**(1/4)/(-x**6+1),x)
```

```
[Out] -Integral(x**6/(x**6*(x**5 - x**3)**(1/4) - (x**5 - x**3)**(1/4)), x) - Int
egral(1/(x**6*(x**5 - x**3)**(1/4) - (x**5 - x**3)**(1/4)), x)
```

3.2339 $\int \frac{\sqrt{-b+a^2x^2}\left(d+cx^4\right)\sqrt{ax+\sqrt{-b+a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=390

$$\frac{a\sqrt[4]{b}\,d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt{a^2x^2-b}+ax-\sqrt{b}}\right)}{\sqrt{2}}-\frac{a\sqrt[4]{b}\,d\tanh^{-1}\left(\frac{\frac{\sqrt{a^2x^2-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{ax}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt{\sqrt{a^2x^2-b}+ax}}\right)}{\sqrt{2}}+\frac{224a^9cx^{10}+2016a^9dx^6-504a^7bcx^8-20}{}$$

Rubi [A] time = 1.97, antiderivative size = 433, normalized size of antiderivative = 1.11, number of steps used = 20, number of rules used = 14, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6742, 2120, 463, 12, 321, 329, 211, 1165, 628, 1162, 617, 204, 259, 270}

$$2ad\sqrt{\sqrt{a^2x^2-b}+ax}+\frac{2abd\sqrt{\sqrt{a^2x^2-b}+ax}}{\left(\sqrt{a^2x^2-b}+ax\right)^2+b}+\frac{a\sqrt[4]{b}\,d\log\left(\sqrt{a^2x^2-b}-\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{a^2x^2-b}+ax}+ax+\sqrt{b}\right)}{2\sqrt{2}}-\frac{a\sqrt[4]{b}\,d\log\left(\sqrt{a^2x^2-b}+\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{a^2x^2-b}+ax}+ax+\sqrt{b}\right)}{2\sqrt{2}}+\frac{a\sqrt[4]{b}\,d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt[4]{b}}\right)}{\sqrt{2}}-\frac{a\sqrt[4]{b}\,d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}}-\frac{b^4c}{56a^3\left(\sqrt{a^2x^2-b}+ax\right)^{7/2}}-\frac{b^2c\sqrt{\sqrt{a^2x^2-b}+ax}}{4a^3}+\frac{c\left(\sqrt{a^2x^2-b}+ax\right)^{9/2}}{72a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-b + a^2*x^2]*(d + c*x^4)*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/x^2,x]
```

```
[Out] -1/56*(b^4*c)/(a^3*(a*x + Sqrt[-b + a^2*x^2])^(7/2)) - (b^2*c*Sqrt[a*x + Sq
rt[-b + a^2*x^2]])/(4*a^3) + 2*a*d*Sqrt[a*x + Sqrt[-b + a^2*x^2]] + (c*(a*x
 + Sqrt[-b + a^2*x^2])^(9/2))/(72*a^3) + (2*a*b*d*Sqrt[a*x + Sqrt[-b + a^2*
x^2]])/(b + (a*x + Sqrt[-b + a^2*x^2])^2) + (a*b^(1/4)*d*ArcTan[1 - (Sqrt[2
]*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/b^(1/4)])/Sqrt[2] - (a*b^(1/4)*d*ArcTan[1
 + (Sqrt[2]*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/b^(1/4)])/Sqrt[2] + (a*b^(1/4)*
d*Log[Sqrt[b] + a*x + Sqrt[-b + a^2*x^2] - Sqrt[2]*b^(1/4)*Sqrt[a*x + Sqrt[
-b + a^2*x^2]]])/(2*Sqrt[2]) - (a*b^(1/4)*d*Log[Sqrt[b] + a*x + Sqrt[-b + a
^2*x^2] + Sqrt[2]*b^(1/4)*Sqrt[a*x + Sqrt[-b + a^2*x^2]]])/(2*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
```

```
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{-b+a^2x^2}\left(d+cx^4\right)\sqrt{ax+\sqrt{-b+a^2x^2}}}{x^2}\,dx = \int \left(\frac{d\sqrt{-b+a^2x^2}\sqrt{ax+\sqrt{-b+a^2x^2}}}{x^2} + cx^2\sqrt{-b+a^2x^2}\sqrt{\right.$$

$$= c\int x^2\sqrt{-b+a^2x^2}\sqrt{ax+\sqrt{-b+a^2x^2}}\,dx + d\int \frac{\sqrt{-b+a^2}}{}$$

$$= \frac{c\,\mathrm{Subst}\left(\int \frac{(-b+x^2)^2(b+x^2)^2}{x^{9/2}}dx, x, ax+\sqrt{-b+a^2x^2}\right)}{16a^3} + (ad)\,\mathrm{S}$$

$$= \frac{2abd\sqrt{ax+\sqrt{-b+a^2x^2}}}{b+\left(ax+\sqrt{-b+a^2x^2}\right)^2} + \frac{c\,\mathrm{Subst}\left(\int \frac{(-b^2+x^4)^2}{x^{9/2}}dx, x, ax+\right.}{16a^3}$$

$$= \frac{2abd\sqrt{ax+\sqrt{-b+a^2x^2}}}{b+\left(ax+\sqrt{-b+a^2x^2}\right)^2} + \frac{c\,\mathrm{Subst}\left(\int\left(\frac{b^4}{x^{9/2}} - \frac{2b^2}{\sqrt{x}} + x^{7/2}\right)\right.}{16a^3}$$

$$= -\frac{b^4c}{56a^3\left(ax+\sqrt{-b+a^2x^2}\right)^{7/2}} - \frac{b^2c\sqrt{ax+\sqrt{-b+a^2x^2}}}{4a^3} +$$

$$= -\frac{b^4c}{56a^3\left(ax+\sqrt{-b+a^2x^2}\right)^{7/2}} - \frac{b^2c\sqrt{ax+\sqrt{-b+a^2x^2}}}{4a^3} +$$

$$= -\frac{b^4c}{56a^3\left(ax+\sqrt{-b+a^2x^2}\right)^{7/2}} - \frac{b^2c\sqrt{ax+\sqrt{-b+a^2x^2}}}{4a^3} +$$

$$= -\frac{b^4c}{56a^3\left(ax+\sqrt{-b+a^2x^2}\right)^{7/2}} - \frac{b^2c\sqrt{ax+\sqrt{-b+a^2x^2}}}{4a^3} +$$

$$= -\frac{b^4c}{56a^3\left(ax+\sqrt{-b+a^2x^2}\right)^{7/2}} - \frac{b^2c\sqrt{ax+\sqrt{-b+a^2x^2}}}{4a^3} +$$

$$= -\frac{b^4c}{56a^3\left(ax+\sqrt{-b+a^2x^2}\right)^{7/2}} - \frac{b^2c\sqrt{ax+\sqrt{-b+a^2x^2}}}{4a^3} +$$

Mathematica [B] time = 23.41, size = 11755, normalized size = 30.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[-b + a^2*x^2]*(d + c*x^4)*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/x
^2,x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.92, size = 390, normalized size = 1.00

$$\frac{a\sqrt[4]{b}\,d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt{a^2x^2-b}+ax-\sqrt{b}}\right)}{\sqrt{2}}-\frac{a\sqrt[4]{b}\,d\tanh^{-1}\left(\frac{\frac{\sqrt{a^2x^2-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{ax}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt{\sqrt{a^2x^2-b}+ax}}\right)}{\sqrt{2}}+\frac{224a^9cx^{10}+2016a^9dx^6-504a^7bcx^8-2016a^7bdx^4+126a^5b^2cx^6+126a^5b^2dx^2+210a^3b^3cx^4+63a^3b^3d+\sqrt{a^2x^2-b}\left(224a^8cx^9+2016a^8dx^5-392a^6bcx^7-1008a^6bdx^3-42a^4b^2cx^5-126a^4b^2dx+154a^2b^3cx^3-16b^4cx\right)-72ab^4cx^2}{63a^3x\left(\sqrt{a^2x^2-b}+ax\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-b + a^2*x^2]*(d + c*x^4)*Sqrt[a*x + Sqrt[-b + a^2
*x^2]])/x^2,x]
```

```
[Out] (63*a^3*b^3*d - 72*a*b^4*c*x^2 + 126*a^5*b^2*d*x^2 + 210*a^3*b^3*c*x^4 - 20
16*a^7*b*d*x^4 + 126*a^5*b^2*c*x^6 + 2016*a^9*d*x^6 - 504*a^7*b*c*x^8 + 224
*a^9*c*x^10 + Sqrt[-b + a^2*x^2]*(-16*b^4*c*x - 126*a^4*b^2*d*x + 154*a^2*b
^3*c*x^3 - 1008*a^6*b*d*x^3 - 42*a^4*b^2*c*x^5 + 2016*a^8*d*x^5 - 392*a^6*b
*c*x^7 + 224*a^8*c*x^9))/(63*a^3*x*(a*x + Sqrt[-b + a^2*x^2])^(9/2)) + (a*b
^(1/4)*d*ArcTan[(Sqrt[2]*b^(1/4)*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/(-Sqrt[b]
+ a*x + Sqrt[-b + a^2*x^2])])/Sqrt[2] - (a*b^(1/4)*d*ArcTanh[(b^(1/4)/Sqrt[
2] + (a*x)/(Sqrt[2]*b^(1/4)) + Sqrt[-b + a^2*x^2]/(Sqrt[2]*b^(1/4)))/Sqrt[a
*x + Sqrt[-b + a^2*x^2]]])/Sqrt[2]
```

fricas [A] time = 0.84, size = 329, normalized size = 0.84

$$-\frac{252\left(-a^4bd^4\right)^{\frac{1}{4}}a^3x\arctan\left(-\frac{\left(-a^4bd^4\right)^{\frac{3}{4}}\sqrt{ax+\sqrt{a^2x^2-b}}\,ad-\left(-a^4bd^4\right)^{\frac{3}{4}}\sqrt{a^3d^2x+\sqrt{a^2x^2-b}\,a^2d^2+\sqrt{-a^4bd^4}}}{a^4bd^4}\right)+63\left(-a^4bd^4\right)^{\frac{1}{4}}a^3x\log\left(\sqrt{ax+\sqrt{a^2x^2-b}}\,ad+\left(-a^4bd^4\right)^{\frac{1}{4}}\right)-63\left(-a^4bd^4\right)^{\frac{1}{4}}a^3x\log\left(\sqrt{ax+\sqrt{a^2x^2-b}}\,ad-\left(-a^4bd^4\right)^{\frac{1}{4}}\right)+2\left(2a^4cx^5-2a^2bcx^3-\left(189a^4d-16b^2c\right)x-\left(16a^3cx^4-8abcx^2-63a^3d\right)\sqrt{a^2x^2-b}\right)\sqrt{ax+\sqrt{a^2x^2-b}}}{126a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(c*x^4+d)*(a*x+(a^2*x^2-b)^(1/2))^(1/2)/x^2,x,
algorithm="fricas")
```

```
[Out] -1/126*(252*(-a^4*b*d^4)^(1/4)*a^3*x*arctan(-((-a^4*b*d^4)^(3/4)*sqrt(a*x +
 sqrt(a^2*x^2 - b))*a*d - (-a^4*b*d^4)^(3/4)*sqrt(a^3*d^2*x + sqrt(a^2*x^2
- b)*a^2*d^2 + sqrt(-a^4*b*d^4)))/(a^4*b*d^4)) + 63*(-a^4*b*d^4)^(1/4)*a^3*
x*log(sqrt(a*x + sqrt(a^2*x^2 - b))*a*d + (-a^4*b*d^4)^(1/4)) - 63*(-a^4*b*
d^4)^(1/4)*a^3*x*log(sqrt(a*x + sqrt(a^2*x^2 - b))*a*d - (-a^4*b*d^4)^(1/4)
) + 2*(2*a^4*c*x^5 - 2*a^2*b*c*x^3 - (189*a^4*d - 16*b^2*c)*x - (16*a^3*c*x
^4 - 8*a*b*c*x^2 - 63*a^3*d)*sqrt(a^2*x^2 - b))*sqrt(a*x + sqrt(a^2*x^2 - b
)))/(a^3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+d\right)\sqrt{a^2x^2-b}\sqrt{ax+\sqrt{a^2x^2-b}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(c*x^4+d)*(a*x+(a^2*x^2-b)^(1/2))^(1/2)/x^2,x,
algorithm="giac")
```

```
[Out] integrate((c*x^4 + d)*sqrt(a^2*x^2 - b)*sqrt(a*x + sqrt(a^2*x^2 - b))/x^2,
x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2-b}\,\left(c\,x^4+d\right)\sqrt{ax+\sqrt{a^2x^2-b}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b)^(1/2)*(c*x^4+d)*(a*x+(a^2*x^2-b)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int((a^2*x^2-b)^(1/2)*(c*x^4+d)*(a*x+(a^2*x^2-b)^(1/2))^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4+d)\sqrt{a^2x^2-b}\sqrt{ax+\sqrt{a^2x^2-b}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(c*x^4+d)*(a*x+(a^2*x^2-b)^(1/2))^(1/2)/x^2,x,
algorithm="maxima")
```

```
[Out] integrate((c*x^4 + d)*sqrt(a^2*x^2 - b)*sqrt(a*x + sqrt(a^2*x^2 - b))/x^2,
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x+\sqrt{a^2\,x^2-b}}\,\left(c\,x^4+d\right)\,\sqrt{a^2\,x^2-b}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (a^2*x^2 - b)^(1/2))^(1/2)*(d + c*x^4)*(a^2*x^2 - b)^(1/2))/x^2
,x)
```

```
[Out] int(((a*x + (a^2*x^2 - b)^(1/2))^(1/2)*(d + c*x^4)*(a^2*x^2 - b)^(1/2))/x^2
, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+\sqrt{a^2x^2-b}}\sqrt{a^2x^2-b}\left(cx^4+d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b)**(1/2)*(c*x**4+d)*(a*x+(a**2*x**2-b)**(1/2))**(1/2)
/x**2,x)
```

```
[Out] Integral(sqrt(a*x + sqrt(a**2*x**2 - b))*sqrt(a**2*x**2 - b)*(c*x**4 + d)/x
**2, x)
```

3.2340 $\int \frac{\sqrt{-b+a^2x^2}\left(d+cx^4\right)\sqrt{ax+\sqrt{-b+a^2x^2}}}{x}\,dx$

Optimal. Leaf size=397

$$\sqrt{2}\,b^{3/4}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt{a^2x^2-b}+ax-\sqrt{b}}\right)+\sqrt{2}\,b^{3/4}d\tanh^{-1}\left(\frac{\frac{\sqrt{a^2x^2-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{ax}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt{\sqrt{a^2x^2-b}+ax}}\right)+\frac{2\left(5040a^{10}cx^{10}+18480a\right.}{}$$

Rubi [A] time = 1.72, antiderivative size = 499, normalized size of antiderivative = 1.26, number of steps used = 18, number of rules used = 12, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.245, Rules used = {6742, 2120, 462, 459, 329, 297, 1162, 617, 204, 1165, 628, 448}

$$-\frac{b^{3/4}d\log\left(\sqrt{a^2x^2-b}-\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{a^2x^2-b}+ax}+ax+\sqrt{b}\right)}{\sqrt{2}}+\frac{b^{3/4}d\log\left(\sqrt{a^2x^2-b}+\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{a^2x^2-b}+ax}+ax+\sqrt{b}\right)}{\sqrt{2}}+\sqrt{2}\,b^{3/4}d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt[4]{b}}\right)-\sqrt{2}\,b^{3/4}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt[4]{b}}+1\right)+\frac{1}{3}d\left(\sqrt{a^2x^2-b}+ax\right)^{3/2}-\frac{bd}{\sqrt{\sqrt{a^2x^2-b}+ax}}-\frac{b^5c}{144a^4\left(\sqrt{a^2x^2-b}+ax\right)^{9/2}}-\frac{b^4c}{80a^4\left(\sqrt{a^2x^2-b}+ax\right)^{5/2}}+\frac{b^3c}{8a^4\sqrt{\sqrt{a^2x^2-b}+ax}}-\frac{b^2c\left(\sqrt{a^2x^2-b}+ax\right)^{3/2}}{24a^4}+\frac{c\left(\sqrt{a^2x^2-b}+ax\right)^{11/2}}{176a^4}+\frac{bc\left(\sqrt{a^2x^2-b}+ax\right)^{7/2}}{112a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-b + a^2*x^2]*(d + c*x^4)*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/x,x]
```

```
[Out] -1/144*(b^5*c)/(a^4*(a*x + Sqrt[-b + a^2*x^2])^(9/2)) - (b^4*c)/(80*a^4*(a*
x + Sqrt[-b + a^2*x^2])^(5/2)) + (b^3*c)/(8*a^4*Sqrt[a*x + Sqrt[-b + a^2*x^
2]]) - (b*d)/Sqrt[a*x + Sqrt[-b + a^2*x^2]] - (b^2*c*(a*x + Sqrt[-b + a^2*x
^2])^(3/2))/(24*a^4) + (d*(a*x + Sqrt[-b + a^2*x^2])^(3/2))/3 + (b*c*(a*x +
 Sqrt[-b + a^2*x^2])^(7/2))/(112*a^4) + (c*(a*x + Sqrt[-b + a^2*x^2])^(11/2
))/(176*a^4) + Sqrt[2]*b^(3/4)*d*ArcTan[1 - (Sqrt[2]*Sqrt[a*x + Sqrt[-b + a
^2*x^2]])/b^(1/4)] - Sqrt[2]*b^(3/4)*d*ArcTan[1 + (Sqrt[2]*Sqrt[a*x + Sqrt[
-b + a^2*x^2]])/b^(1/4)] - (b^(3/4)*d*Log[Sqrt[b] + a*x + Sqrt[-b + a^2*x^2
] - Sqrt[2]*b^(1/4)*Sqrt[a*x + Sqrt[-b + a^2*x^2]]])/Sqrt[2] + (b^(3/4)*d*L
og[Sqrt[b] + a*x + Sqrt[-b + a^2*x^2] + Sqrt[2]*b^(1/4)*Sqrt[a*x + Sqrt[-b
+ a^2*x^2]]])/Sqrt[2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{-b+a^2x^2}\left(d+cx^4\right)\sqrt{ax+\sqrt{-b+a^2x^2}}}{x}\,dx = \int\left(\frac{d\sqrt{-b+a^2x^2}\sqrt{ax+\sqrt{-b+a^2x^2}}}{x} + cx^3\sqrt{-b+a^2x^2}\sqrt{ax$$

$$= c\int x^3\sqrt{-b+a^2x^2}\sqrt{ax+\sqrt{-b+a^2x^2}}\,dx + d\int \frac{\sqrt{-b+a^2x^2}}{}$$

$$= \frac{c\,\mathrm{Subst}\left(\int \frac{\left(-b+x^2\right)^2\left(b+x^2\right)^3}{x^{11/2}}dx, x, ax+\sqrt{-b+a^2x^2}\right)}{32a^4} + \frac{1}{2}d\,\mathrm{Subs}$$

$$= -\frac{bd}{\sqrt{ax+\sqrt{-b+a^2x^2}}} + \frac{c\,\mathrm{Subst}\left(\int\left(\frac{b^5}{x^{11/2}}+\frac{b^4}{x^{7/2}}-\frac{2b^3}{x^{3/2}}-2b^2\sqrt{}\right.\right.}{}$$

$$= -\frac{b^5c}{144a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{9/2}} - \frac{b^4c}{80a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{5/2}}$$

$$= -\frac{b^5c}{144a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{9/2}} - \frac{b^4c}{80a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{5/2}}$$

$$= -\frac{b^5c}{144a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{9/2}} - \frac{b^4c}{80a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{5/2}}$$

$$= -\frac{b^5c}{144a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{9/2}} - \frac{b^4c}{80a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{5/2}}$$

$$= -\frac{b^5c}{144a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{9/2}} - \frac{b^4c}{80a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{5/2}}$$

$$= -\frac{b^5c}{144a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{9/2}} - \frac{b^4c}{80a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{5/2}}$$

Mathematica [B] time = 24.24, size = 9604, normalized size = 24.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[-b + a^2*x^2]*(d + c*x^4)*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/x
,x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 0.94, size = 397, normalized size = 1.00

$$\sqrt{2}\,b^{3/4}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt{a^2x^2-b}+ax-\sqrt{b}}\right)+\sqrt{2}\,b^{3/4}d\tanh^{-1}\left(\frac{\frac{\sqrt{a^2x^2-b}}{\sqrt{2}\sqrt[4]{b}}+\frac{ax}{\sqrt{2}\sqrt[4]{b}}+\frac{\sqrt[4]{b}}{\sqrt{2}}}{\sqrt{\sqrt{a^2x^2-b}+ax}}\right)+\frac{2\left(5040a^{10}cx^{10}+18480a^{10}dx^6-10620a^8bcx^8-41580a^8bdx^4+4755a^6b^2cx^6+24255a^6b^2dx^2+3735a^4b^3cx^4-2310a^4b^3d-3078a^2b^4cx^2+304b^5c\right)+2\sqrt{a^2x^2-b}\left(5040a^9cx^9+18480a^9dx^5-8100a^7bcx^7-32340a^7bdx^3+1335a^5b^2cx^5+10395a^5b^2dx+3705a^3b^3cx^3-1368ab^4cx\right)}{3465a^4\left(\sqrt{a^2x^2-b}+ax\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-b + a^2*x^2]*(d + c*x^4)*Sqrt[a*x + Sqrt[-b + a^2
*x^2]])/x,x]
```

```
[Out] (2*Sqrt[-b + a^2*x^2]*(-1368*a*b^4*c*x + 10395*a^5*b^2*d*x + 3705*a^3*b^3*c
*x^3 - 32340*a^7*b*d*x^3 + 1335*a^5*b^2*c*x^5 + 18480*a^9*d*x^5 - 8100*a^7*
b*c*x^7 + 5040*a^9*c*x^9) + 2*(304*b^5*c - 2310*a^4*b^3*d - 3078*a^2*b^4*c*
x^2 + 24255*a^6*b^2*d*x^2 + 3735*a^4*b^3*c*x^4 - 41580*a^8*b*d*x^4 + 4755*a
^6*b^2*c*x^6 + 18480*a^10*d*x^6 - 10620*a^8*b*c*x^8 + 5040*a^10*c*x^10))/(3
465*a^4*(a*x + Sqrt[-b + a^2*x^2])^(9/2)) + Sqrt[2]*b^(3/4)*d*ArcTan[(Sqrt[
2]*b^(1/4)*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/(-Sqrt[b] + a*x + Sqrt[-b + a^2*
x^2])] + Sqrt[2]*b^(3/4)*d*ArcTanh[(b^(1/4)/Sqrt[2] + (a*x)/(Sqrt[2]*b^(1/4
)) + Sqrt[-b + a^2*x^2]/(Sqrt[2]*b^(1/4)))/Sqrt[a*x + Sqrt[-b + a^2*x^2]]]
```

fricas [A] time = 0.46, size = 341, normalized size = 0.86

$$\frac{13860 \left(-b^3 d^4\right)^{\frac{1}{4}} a^4 \arctan\left(-\frac{\left(-b^3 d^4\right)^{\frac{1}{4}} \sqrt{ax+\sqrt{a^2x^2-b}}\, b^2 d^3 - \sqrt{a b^4 d^6 x + \sqrt{a^2x^2-b}\, b^4 d^6 - \sqrt{-b^3 d^4}\, b^3 d^4} \left(-b^3 d^4\right)^{\frac{1}{4}}}{b^3 d^4}\right) - 3465 \left(-b^3 d^4\right)^{\frac{1}{4}} a^4 \log\left(\sqrt{ax + \sqrt{a^2x^2-b}}\, b^2 d^3 + \left(-b^3 d^4\right)^{\frac{3}{4}}\right) + 3465 \left(-b^3 d^4\right)^{\frac{1}{4}} a^4 \log\left(\sqrt{ax + \sqrt{a^2x^2-b}}\, b^2 d^3 - \left(-b^3 d^4\right)^{\frac{3}{4}}\right) - 2 \left(35 a^5 c x^5 - 19 a^3 b c x^3 + \left(1155 a^5 d - 152 a b^2 c\right) x - 2 \left(175 a^4 c x^4 - 57 a^2 b c x^2 + 1155 a^4 d - 152 b^2 c\right) \sqrt{a^2x^2-b}\right) \sqrt{ax + \sqrt{a^2x^2-b}}}{3465 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(c*x^4+d)*(a*x+(a^2*x^2-b)^(1/2))^(1/2)/x,x, al
gorithm="fricas")
```

```
[Out] 1/3465*(13860*(-b^3*d^4)^(1/4)*a^4*arctan(-((-b^3*d^4)^(1/4)*sqrt(a*x + sqr
t(a^2*x^2 - b))*b^2*d^3 - sqrt(a*b^4*d^6*x + sqrt(a^2*x^2 - b)*b^4*d^6 - sq
rt(-b^3*d^4)*b^3*d^4)*(-b^3*d^4)^(1/4))/(b^3*d^4)) - 3465*(-b^3*d^4)^(1/4)*
a^4*log(sqrt(a*x + sqrt(a^2*x^2 - b))*b^2*d^3 + (-b^3*d^4)^(3/4)) + 3465*(-
b^3*d^4)^(1/4)*a^4*log(sqrt(a*x + sqrt(a^2*x^2 - b))*b^2*d^3 - (-b^3*d^4)^(
3/4)) - 2*(35*a^5*c*x^5 - 19*a^3*b*c*x^3 + (1155*a^5*d - 152*a*b^2*c)*x - 2
*(175*a^4*c*x^4 - 57*a^2*b*c*x^2 + 1155*a^4*d - 152*b^2*c)*sqrt(a^2*x^2 - b
))*sqrt(a*x + sqrt(a^2*x^2 - b)))/a^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + d\right)\sqrt{a^2x^2 - b}\,\sqrt{ax + \sqrt{a^2x^2 - b}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(c*x^4+d)*(a*x+(a^2*x^2-b)^(1/2))^(1/2)/x,x, al
gorithm="giac")
```

```
[Out] integrate((c*x^4 + d)*sqrt(a^2*x^2 - b)*sqrt(a*x + sqrt(a^2*x^2 - b))/x, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2 - b}\,\left(c\,x^4 + d\right)\sqrt{ax + \sqrt{a^2x^2 - b}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b)^(1/2)*(c*x^4+d)*(a*x+(a^2*x^2-b)^(1/2))^(1/2)/x,x)
```

```
[Out] int((a^2*x^2-b)^(1/2)*(c*x^4+d)*(a*x+(a^2*x^2-b)^(1/2))^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4 + d\right)\sqrt{a^2x^2 - b}\,\sqrt{ax + \sqrt{a^2x^2 - b}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(c*x^4+d)*(a*x+(a^2*x^2-b)^(1/2))^(1/2)/x,x, al
gorithm="maxima")
```

```
[Out] integrate((c*x^4 + d)*sqrt(a^2*x^2 - b)*sqrt(a*x + sqrt(a^2*x^2 - b))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x+\sqrt{a^2\,x^2-b}}\,\left(c\,x^4+d\right)\,\sqrt{a^2\,x^2-b}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (a^2*x^2 - b)^(1/2))^(1/2)*(d + c*x^4)*(a^2*x^2 - b)^(1/2))/x,x
)
```

```
[Out] int(((a*x + (a^2*x^2 - b)^(1/2))^(1/2)*(d + c*x^4)*(a^2*x^2 - b)^(1/2))/x,
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+\sqrt{a^2x^2-b}}\,\sqrt{a^2x^2-b}\,\left(cx^4+d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b)**(1/2)*(c*x**4+d)*(a*x+(a**2*x**2-b)**(1/2))**(1/2)
/x,x)
```

```
[Out] Integral(sqrt(a*x + sqrt(a**2*x**2 - b))*sqrt(a**2*x**2 - b)*(c*x**4 + d)/x
, x)
```

3.2341 $\int \frac{x^5(7b+10ax^3)}{\sqrt[4]{bx^3+ax^6}\,(1+bx^7+ax^{10})}\,dx$

Optimal. Leaf size=399

$$-\sqrt{2}\tan^{-1}\left(\frac{x^2\sqrt[4]{ax^6+bx^3}-2^{2/3}x\sqrt[4]{ax^6+bx^3}}{2^{2/3}x\sqrt[4]{ax^6+bx^3}+x^2\left(-\sqrt[4]{ax^6+bx^3}\right)-\sqrt{2}x+2\sqrt[6]{2}}\right)+\sqrt{2}\tan^{-1}\left(\frac{x^2\sqrt[4]{ax^6+bx^3}-2^{2/3}}{2^{2/3}x\sqrt[4]{ax^6+bx^3}+x^2\left(-\sqrt[4]{ax^6}\right.}\right.$$

Rubi [F] time = 2.42, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^5(7b+10ax^3)}{\sqrt[4]{bx^3+ax^6}\,(1+bx^7+ax^{10})}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^5*(7*b + 10*a*x^3))/((b*x^3 + a*x^6)^(1/4)*(1 + b*x^7 + a*x^10)),x]
```

```
[Out] (28*b*x^(3/4)*(b + a*x^3)^(1/4)*Defer[Subst][Defer[Int][x^20/((b + a*x^12)^
(1/4)*(1 + b*x^28 + a*x^40)), x], x, x^(1/4)])/(b*x^3 + a*x^6)^(1/4) + (40*
a*x^(3/4)*(b + a*x^3)^(1/4)*Defer[Subst][Defer[Int][x^32/((b + a*x^12)^(1/4
)*(1 + b*x^28 + a*x^40)), x], x, x^(1/4)])/(b*x^3 + a*x^6)^(1/4)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5(7b+10ax^3)}{\sqrt[4]{bx^3+ax^6}\,(1+bx^7+ax^{10})}\,dx &= \frac{\left(x^{3/4}\sqrt[4]{b+ax^3}\right)\int \frac{x^{17/4}(7b+10ax^3)}{\sqrt[4]{b+ax^3}\,(1+bx^7+ax^{10})}\,dx}{\sqrt[4]{bx^3+ax^6}}\\
&= \frac{\left(4x^{3/4}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{x^{20}(7b+10ax^{12})}{\sqrt[4]{b+ax^{12}}\,(1+bx^{28}+ax^{40})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx^3+ax^6}}\\
&= \frac{\left(4x^{3/4}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \left(\frac{7bx^{20}}{\sqrt[4]{b+ax^{12}}\,(1+bx^{28}+ax^{40})}+\frac{10ax^{32}}{\sqrt[4]{b+ax^{12}}\,(1+bx^{28}+ax^{40})}\right)\right.}{\sqrt[4]{bx^3+ax^6}}\\
&= \frac{\left(40ax^{3/4}\sqrt[4]{b+ax^3}\right)\text{Subst}\left(\int \frac{x^{32}}{\sqrt[4]{b+ax^{12}}\,(1+bx^{28}+ax^{40})}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{bx^3+ax^6}}+\frac{\left(28bx^{3/4}\right.}{}
\end{aligned}$$

Mathematica [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{x^5(7b+10ax^3)}{\sqrt[4]{bx^3+ax^6}\,(1+bx^7+ax^{10})}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^5*(7*b + 10*a*x^3))/((b*x^3 + a*x^6)^(1/4)*(1 + b*x^7 + a*x^10
)),x]
```

```
[Out] Integrate[(x^5*(7*b + 10*a*x^3))/((b*x^3 + a*x^6)^(1/4)*(1 + b*x^7 + a*x^10
)), x]
```

IntegrateAlgebraic [A] time = 20.12, size = 399, normalized size = 1.00

$$-\sqrt{2}\tan^{-1}\left(\frac{x^2\sqrt[4]{ax^6+bx^3}-2^{2/3}x\sqrt[4]{ax^6+bx^3}}{2^{2/3}x\sqrt[4]{ax^6+bx^3}+x^2\left(-\sqrt[4]{ax^6+bx^3}\right)-\sqrt{2}x+2\sqrt[6]{2}}\right)+\sqrt{2}\tan^{-1}\left(\frac{x^2\sqrt[4]{ax^6+bx^3}-2^{2/3}x\sqrt[4]{ax^6+bx^3}}{2^{2/3}x\sqrt[4]{ax^6+bx^3}+x^2\left(-\sqrt[4]{ax^6+bx^3}\right)+\sqrt{2}x-2\sqrt[6]{2}}\right)-\sqrt{2}\tanh^{-1}\left(\frac{-\sqrt{2}x^3\sqrt[4]{ax^6+bx^3}-2\,2^{5/6}x\sqrt[4]{ax^6+bx^3}+4\sqrt[6]{2}x^2\sqrt[4]{ax^6+bx^3}}{2\,2^{2/3}x^3\sqrt{ax^6+bx^3}+x^4\left(-\sqrt{ax^6+bx^3}\right)-2\sqrt[3]{2}x^2\sqrt{ax^6+bx^3}-x^2+2\,2^{2/3}x-2\sqrt[3]{2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(7*b + 10*a*x^3))/((b*x^3 + a*x^6)^(1/4)*(1 + b*x^7
 + a*x^10)),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(-(2^(2/3)*x*(b*x^3 + a*x^6)^(1/4)) + x^2*(b*x^3 + a*x^6)^
(1/4))/(2*2^(1/6) - Sqrt[2]*x + 2^(2/3)*x*(b*x^3 + a*x^6)^(1/4) - x^2*(b*x^
3 + a*x^6)^(1/4))]) + Sqrt[2]*ArcTan[(-(2^(2/3)*x*(b*x^3 + a*x^6)^(1/4)) +
x^2*(b*x^3 + a*x^6)^(1/4))/(-2*2^(1/6) + Sqrt[2]*x + 2^(2/3)*x*(b*x^3 + a*x
^6)^(1/4) - x^2*(b*x^3 + a*x^6)^(1/4))] - Sqrt[2]*ArcTanh[(-2*2^(5/6)*x*(b*
x^3 + a*x^6)^(1/4) + 4*2^(1/6)*x^2*(b*x^3 + a*x^6)^(1/4) - Sqrt[2]*x^3*(b*x
^3 + a*x^6)^(1/4))/(-2*2^(1/3) + 2*2^(2/3)*x - x^2 - 2*2^(1/3)*x^2*Sqrt[b*x
^3 + a*x^6] + 2*2^(2/3)*x^3*Sqrt[b*x^3 + a*x^6] - x^4*Sqrt[b*x^3 + a*x^6])]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(10*a*x^3+7*b)/(a*x^6+b*x^3)^(1/4)/(a*x^10+b*x^7+1),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(10\,ax^3+7\,b\right)x^5}{\left(ax^{10}+bx^7+1\right)\left(ax^6+bx^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(10*a*x^3+7*b)/(a*x^6+b*x^3)^(1/4)/(a*x^10+b*x^7+1),x, algori
thm="giac")
```

```
[Out] integrate((10*a*x^3 + 7*b)*x^5/((a*x^10 + b*x^7 + 1)*(a*x^6 + b*x^3)^(1/4))
, x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(10a\,x^3+7b\right)}{\left(a\,x^6+b\,x^3\right)^{\frac{1}{4}}\left(a\,x^{10}+b\,x^7+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(10*a*x^3+7*b)/(a*x^6+b*x^3)^(1/4)/(a*x^10+b*x^7+1),x)
```

```
[Out] int(x^5*(10*a*x^3+7*b)/(a*x^6+b*x^3)^(1/4)/(a*x^10+b*x^7+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(10\,ax^3+7\,b\right)x^5}{\left(ax^{10}+bx^7+1\right)\left(ax^6+bx^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(10*a*x^3+7*b)/(a*x^6+b*x^3)^(1/4)/(a*x^10+b*x^7+1),x, algori
thm="maxima")
```

```
[Out] integrate((10*a*x^3 + 7*b)*x^5/((a*x^10 + b*x^7 + 1)*(a*x^6 + b*x^3)^(1/4))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(10\,a\,x^3+7\,b\right)}{{\left(a\,x^6+b\,x^3\right)}^{1/4}\,\left(a\,x^{10}+b\,x^7+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(7*b + 10*a*x^3))/((a*x^6 + b*x^3)^(1/4)*(a*x^10 + b*x^7 + 1)),x)
```

```
[Out] int((x^5*(7*b + 10*a*x^3))/((a*x^6 + b*x^3)^(1/4)*(a*x^10 + b*x^7 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \left(10 a x^{3} + 7 b\right)}{\sqrt[4]{x^{3} \left(a x^{3} + b\right)} \left(a x^{10} + b x^{7} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(10*a*x**3+7*b)/(a*x**6+b*x**3)**(1/4)/(a*x**10+b*x**7+1),x)
```

```
[Out] Integral(x**5*(10*a*x**3 + 7*b)/((x**3*(a*x**3 + b))**(1/4)*(a*x**10 + b*x*
*7 + 1)), x)
```

3.2342 $\int \frac{x^4\left(-2q+px^3\right)\sqrt{q+px^3}}{bx^8+a\left(q+px^3\right)^4}\,dx$

Optimal. Leaf size=399

$$-\frac{\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{x\left(\sqrt{\frac{2}{2-\sqrt{2}}}\,\sqrt[8]{a}\,\sqrt[8]{b}-\frac{2\sqrt[8]{a}\,\sqrt[8]{b}}{\sqrt{2-\sqrt{2}}}\right)\sqrt{px^3+q}}{-\sqrt[4]{a}\,px^3-\sqrt[4]{a}\,q+\sqrt[4]{b}\,x^2}\right)}{4a^{3/8}b^{5/8}}+\frac{\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{b}\,x\sqrt{px^3+q}}{\sqrt[4]{a}\,px^3+\sqrt[4]{a}\,q-\sqrt[4]{b}\,x^2}\right)}{4a^{3/8}b^{5/8}}-\frac{\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\right.}{4a^{3/8}}$$

Rubi [F] time = 2.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4\left(-2q+px^3\right)\sqrt{q+px^3}}{bx^8+a\left(q+px^3\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(-2*q + p*x^3)*Sqrt[q + p*x^3])/(b*x^8 + a*(q + p*x^3)^4),x]
```

```
[Out] -2*q*Defer[Int][(x^4*Sqrt[q + p*x^3])/(b*x^8 + a*(q + p*x^3)^4), x] + p*Def
er[Int][(x^7*Sqrt[q + p*x^3])/(b*x^8 + a*(q + p*x^3)^4), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(-2q+px^3\right)\sqrt{q+px^3}}{bx^8+a\left(q+px^3\right)^4}\,dx &= \int \left(-\frac{2qx^4\sqrt{q+px^3}}{aq^4+4apq^3x^3+6ap^2q^2x^6+bx^8+4ap^3qx^9+ap^4x^{12}}+\frac{}{aq^4+4apq^3x^3+}\right.\\
&= p\int \frac{x^7\sqrt{q+px^3}}{aq^4+4apq^3x^3+6ap^2q^2x^6+bx^8+4ap^3qx^9+ap^4x^{12}}\,dx-(2q)\int \frac{}{aq^4+4}\\
&= p\int \frac{x^7\sqrt{q+px^3}}{bx^8+a\left(q+px^3\right)^4}\,dx-(2q)\int \frac{x^4\sqrt{q+px^3}}{bx^8+a\left(q+px^3\right)^4}\,dx
\end{aligned}$$

Mathematica [C] time = 7.82, size = 65821, normalized size = 164.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(-2*q + p*x^3)*Sqrt[q + p*x^3])/(b*x^8 + a*(q + p*x^3)^4),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 3.48, size = 367, normalized size = 0.92

$$-\frac{\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{b}\,x\sqrt{px^3+q}}{\sqrt[4]{a}\,px^3+\sqrt[4]{a}\,q-\sqrt[4]{b}\,x^2}\right)}{4a^{3/8}b^{5/8}}+\frac{\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{b}\,x\sqrt{px^3+q}}{\sqrt[4]{a}\,px^3+\sqrt[4]{a}\,q-\sqrt[4]{b}\,x^2}\right)}{4a^{3/8}b^{5/8}}-\frac{\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{b}\,x\sqrt{px^3+q}}{\sqrt[4]{a}\,px^3+\sqrt[4]{a}\,q+\sqrt[4]{b}\,x^2}\right)}{4a^{3/8}b^{5/8}}+\frac{\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{b}\,x\sqrt{px^3+q}}{\sqrt[4]{a}\,px^3+\sqrt[4]{a}\,q+\sqrt[4]{b}\,x^2}\right)}{4a^{3/8}b^{5/8}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(-2*q + p*x^3)*Sqrt[q + p*x^3])/(b*x^8 + a*(q + p*x
^3)^4),x]
```

```
[Out] -1/4*(Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]]*a^(1/8)*b^(1/8)*x*Sqrt[q
+ p*x^3])/(a^(1/4)*q - b^(1/4)*x^2 + a^(1/4)*p*x^3)]])/(a^(3/8)*b^(5/8)) + (
Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8)*b^(1/8)*x*Sqrt[q + p*x^
3])/(a^(1/4)*q - b^(1/4)*x^2 + a^(1/4)*p*x^3)]])/(4*a^(3/8)*b^(5/8)) - (Sqrt
[2 + Sqrt[2]]*ArcTanh[(Sqrt[2 - Sqrt[2]]*a^(1/8)*b^(1/8)*x*Sqrt[q + p*x^3])
/(a^(1/4)*q + b^(1/4)*x^2 + a^(1/4)*p*x^3)]])/(4*a^(3/8)*b^(5/8)) + (Sqrt[2
- Sqrt[2]]*ArcTanh[(Sqrt[2 + Sqrt[2]]*a^(1/8)*b^(1/8)*x*Sqrt[q + p*x^3])/(a
^(1/4)*q + b^(1/4)*x^2 + a^(1/4)*p*x^3)]])/(4*a^(3/8)*b^(5/8))
```

fricas [B] time = 17.19, size = 2875, normalized size = 7.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(p*x^3-2*q)*(p*x^3+q)^(1/2)/(b*x^8+a*(p*x^3+q)^4),x, algorith
m="fricas")
```

```
[Out] -1/4*sqrt(2)*(-1/(a^3*b^5))^(1/8)*arctan(-1/2*((sqrt(2)*a*b^2*x^2*(-1/(a^3*
b^5))^(3/8) + sqrt(2)*(a^2*b^3*p*x^3 + a^2*b^3*q)*(-1/(a^3*b^5))^(5/8))*sqr
t(p*x^3 + q) + (2*p*x^4 + 2*q*x + (sqrt(2)*a*b^2*x^2*(-1/(a^3*b^5))^(3/8) -
 sqrt(2)*(a^2*b^3*p*x^3 + a^2*b^3*q)*(-1/(a^3*b^5))^(5/8))*sqrt(p*x^3 + q))
*sqrt((a*p^4*x^12 + 4*a*p^3*q*x^9 + 6*a*p^2*q^2*x^6 + b*x^8 + 4*a*p*q^3*x^3
 + a*q^4 + 4*(a^3*b^4*p^3*x^11 + 3*a^3*b^4*p^2*q*x^8 + 3*a^3*b^4*p*q^2*x^5
+ a^3*b^4*q^3*x^2)*(-1/(a^3*b^5))^(3/4) + 2*(sqrt(2)*a^2*b^4*x^7*(-1/(a^3*b
^5))^(5/8) - sqrt(2)*(a^3*b^5*p*x^8 + a^3*b^5*q*x^5)*(-1/(a^3*b^5))^(7/8) -
 sqrt(2)*(a^2*b^2*p^3*x^10 + 3*a^2*b^2*p^2*q*x^7 + 3*a^2*b^2*p*q^2*x^4 + a^
2*b^2*q^3*x)*(-1/(a^3*b^5))^(3/8) + sqrt(2)*(a*b*p^2*x^9 + 2*a*b*p*q*x^6 +
a*b*q^2*x^3)*(-1/(a^3*b^5))^(1/8))*sqrt(p*x^3 + q) - 4*(a*b^2*p*x^9 + a*b^2
*q*x^6)*(-1/(a^3*b^5))^(1/4))/(a*p^4*x^12 + 4*a*p^3*q*x^9 + 6*a*p^2*q^2*x^6
 + b*x^8 + 4*a*p*q^3*x^3 + a*q^4)))/(p*x^4 + q*x)) - 1/4*sqrt(2)*(-1/(a^3*b
^5))^(1/8)*arctan(-1/2*((sqrt(2)*a*b^2*x^2*(-1/(a^3*b^5))^(3/8) + sqrt(2)*(
a^2*b^3*p*x^3 + a^2*b^3*q)*(-1/(a^3*b^5))^(5/8))*sqrt(p*x^3 + q) - (2*p*x^4
 + 2*q*x - (sqrt(2)*a*b^2*x^2*(-1/(a^3*b^5))^(3/8) - sqrt(2)*(a^2*b^3*p*x^3
 + a^2*b^3*q)*(-1/(a^3*b^5))^(5/8))*sqrt(p*x^3 + q))*sqrt((a*p^4*x^12 + 4*a
*p^3*q*x^9 + 6*a*p^2*q^2*x^6 + b*x^8 + 4*a*p*q^3*x^3 + a*q^4 + 4*(a^3*b^4*p
^3*x^11 + 3*a^3*b^4*p^2*q*x^8 + 3*a^3*b^4*p*q^2*x^5 + a^3*b^4*q^3*x^2)*(-1/
(a^3*b^5))^(3/4) - 2*(sqrt(2)*a^2*b^4*x^7*(-1/(a^3*b^5))^(5/8) - sqrt(2)*(a
^3*b^5*p*x^8 + a^3*b^5*q*x^5)*(-1/(a^3*b^5))^(7/8) - sqrt(2)*(a^2*b^2*p^3*x
^10 + 3*a^2*b^2*p^2*q*x^7 + 3*a^2*b^2*p*q^2*x^4 + a^2*b^2*q^3*x)*(-1/(a^3*b
^5))^(3/8) + sqrt(2)*(a*b*p^2*x^9 + 2*a*b*p*q*x^6 + a*b*q^2*x^3)*(-1/(a^3*b
^5))^(1/8))*sqrt(p*x^3 + q) - 4*(a*b^2*p*x^9 + a*b^2*q*x^6)*(-1/(a^3*b^5))^
(1/4))/(a*p^4*x^12 + 4*a*p^3*q*x^9 + 6*a*p^2*q^2*x^6 + b*x^8 + 4*a*p*q^3*x^
3 + a*q^4)))/(p*x^4 + q*x)) - 1/16*sqrt(2)*(-1/(a^3*b^5))^(1/8)*log((a*p^4*
x^12 + 4*a*p^3*q*x^9 + 6*a*p^2*q^2*x^6 + b*x^8 + 4*a*p*q^3*x^3 + a*q^4 + 4*
(a^3*b^4*p^3*x^11 + 3*a^3*b^4*p^2*q*x^8 + 3*a^3*b^4*p*q^2*x^5 + a^3*b^4*q^3
*x^2)*(-1/(a^3*b^5))^(3/4) + 2*(sqrt(2)*a^2*b^4*x^7*(-1/(a^3*b^5))^(5/8) -
sqrt(2)*(a^3*b^5*p*x^8 + a^3*b^5*q*x^5)*(-1/(a^3*b^5))^(7/8) - sqrt(2)*(a^2
*b^2*p^3*x^10 + 3*a^2*b^2*p^2*q*x^7 + 3*a^2*b^2*p*q^2*x^4 + a^2*b^2*q^3*x)*
(-1/(a^3*b^5))^(3/8) + sqrt(2)*(a*b*p^2*x^9 + 2*a*b*p*q*x^6 + a*b*q^2*x^3)*
(-1/(a^3*b^5))^(1/8))*sqrt(p*x^3 + q) - 4*(a*b^2*p*x^9 + a*b^2*q*x^6)*(-1/(
a^3*b^5))^(1/4))/(a*p^4*x^12 + 4*a*p^3*q*x^9 + 6*a*p^2*q^2*x^6 + b*x^8 + 4*
a*p*q^3*x^3 + a*q^4)) + 1/16*sqrt(2)*(-1/(a^3*b^5))^(1/8)*log((a*p^4*x^12 +
 4*a*p^3*q*x^9 + 6*a*p^2*q^2*x^6 + b*x^8 + 4*a*p*q^3*x^3 + a*q^4 + 4*(a^3*b
^4*p^3*x^11 + 3*a^3*b^4*p^2*q*x^8 + 3*a^3*b^4*p*q^2*x^5 + a^3*b^4*q^3*x^2)*
(-1/(a^3*b^5))^(3/4) - 2*(sqrt(2)*a^2*b^4*x^7*(-1/(a^3*b^5))^(5/8) - sqrt(2
)*(a^3*b^5*p*x^8 + a^3*b^5*q*x^5)*(-1/(a^3*b^5))^(7/8) - sqrt(2)*(a^2*b^2*p
^3*x^10 + 3*a^2*b^2*p^2*q*x^7 + 3*a^2*b^2*p*q^2*x^4 + a^2*b^2*q^3*x)*(-1/(a
^3*b^5))^(3/8) + sqrt(2)*(a*b*p^2*x^9 + 2*a*b*p*q*x^6 + a*b*q^2*x^3)*(-1/(a
^3*b^5))^(1/8))*sqrt(p*x^3 + q) - 4*(a*b^2*p*x^9 + a*b^2*q*x^6)*(-1/(a^3*b^
5))^(1/4))/(a*p^4*x^12 + 4*a*p^3*q*x^9 + 6*a*p^2*q^2*x^6 + b*x^8 + 4*a*p*q^
```

```
3*x^3 + a*q^4)) - 1/2*(-1/(a^3*b^5))^(1/8)*arctan(a*b^2*x*(-1/(a^3*b^5))^(3
/8)/sqrt(p*x^3 + q)) - 1/8*(-1/(a^3*b^5))^(1/8)*log((a*p^4*x^12 + 4*a*p^3*q
*x^9 + 6*a*p^2*q^2*x^6 - b*x^8 + 4*a*p*q^3*x^3 + a*q^4 + 2*(a^3*b^4*p^3*x^1
1 + 3*a^3*b^4*p^2*q*x^8 + 3*a^3*b^4*p*q^2*x^5 + a^3*b^4*q^3*x^2)*(-1/(a^3*b
^5))^(3/4) + 2*(a^2*b^4*x^7*(-1/(a^3*b^5))^(5/8) - (a^3*b^5*p*x^8 + a^3*b^5
*q*x^5)*(-1/(a^3*b^5))^(7/8) + (a^2*b^2*p^3*x^10 + 3*a^2*b^2*p^2*q*x^7 + 3*
a^2*b^2*p*q^2*x^4 + a^2*b^2*q^3*x)*(-1/(a^3*b^5))^(3/8) - (a*b*p^2*x^9 + 2*
a*b*p*q*x^6 + a*b*q^2*x^3)*(-1/(a^3*b^5))^(1/8))*sqrt(p*x^3 + q) - 2*(a^2*b
^3*p^2*x^10 + 2*a^2*b^3*p*q*x^7 + a^2*b^3*q^2*x^4)*sqrt(-1/(a^3*b^5)) + 2*(
a*b^2*p*x^9 + a*b^2*q*x^6)*(-1/(a^3*b^5))^(1/4))/(a*p^4*x^12 + 4*a*p^3*q*x^
9 + 6*a*p^2*q^2*x^6 + b*x^8 + 4*a*p*q^3*x^3 + a*q^4)) + 1/8*(-1/(a^3*b^5))^
(1/8)*log((a*p^4*x^12 + 4*a*p^3*q*x^9 + 6*a*p^2*q^2*x^6 - b*x^8 + 4*a*p*q^3
*x^3 + a*q^4 + 2*(a^3*b^4*p^3*x^11 + 3*a^3*b^4*p^2*q*x^8 + 3*a^3*b^4*p*q^2*
x^5 + a^3*b^4*q^3*x^2)*(-1/(a^3*b^5))^(3/4) - 2*(a^2*b^4*x^7*(-1/(a^3*b^5))
^(5/8) - (a^3*b^5*p*x^8 + a^3*b^5*q*x^5)*(-1/(a^3*b^5))^(7/8) + (a^2*b^2*p^
3*x^10 + 3*a^2*b^2*p^2*q*x^7 + 3*a^2*b^2*p*q^2*x^4 + a^2*b^2*q^3*x)*(-1/(a^
3*b^5))^(3/8) - (a*b*p^2*x^9 + 2*a*b*p*q*x^6 + a*b*q^2*x^3)*(-1/(a^3*b^5))^
(1/8))*sqrt(p*x^3 + q) - 2*(a^2*b^3*p^2*x^10 + 2*a^2*b^3*p*q*x^7 + a^2*b^3*
q^2*x^4)*sqrt(-1/(a^3*b^5)) + 2*(a*b^2*p*x^9 + a*b^2*q*x^6)*(-1/(a^3*b^5))^
(1/4))/(a*p^4*x^12 + 4*a*p^3*q*x^9 + 6*a*p^2*q^2*x^6 + b*x^8 + 4*a*p*q^3*x^
3 + a*q^4))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(p*x^3-2*q)*(p*x^3+q)^(1/2)/(b*x^8+a*(p*x^3+q)^4),x, algorith
m="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.70, size = 1596, normalized size = 4.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(p*x^3-2*q)*(p*x^3+q)^(1/2)/(b*x^8+a*(p*x^3+q)^4),x)
```

```
[Out] 1/4*I/p^2/q^3/b*2^(1/2)*sum((_alpha^6*p^2-_alpha^3*p*q-2*q^2)*_alpha^2/(-3*
_alpha^9*a*p^4-9*_alpha^6*a*p^3*q-9*_alpha^3*a*p^2*q^2-2*_alpha^5*b-3*a*p*q
^3)*(-q*p^2)^(1/3)*(1/2*I*p*(2*x+1/p*(-I*3^(1/2)*(-q*p^2)^(1/3)+(-q*p^2)^(1
/3)))/(-q*p^2)^(1/3))^(1/2)*(p*(x-1/p*(-q*p^2)^(1/3))/(-3*(-q*p^2)^(1/3)+I*
3^(1/2)*(-q*p^2)^(1/3)))^(1/2)*(-1/2*I*p*(2*x+1/p*(I*3^(1/2)*(-q*p^2)^(1/3)
+(-q*p^2)^(1/3)))/(-q*p^2)^(1/3))^(1/2)/(p*x^3+q)^(1/2)*((-q*p^2)^(2/3)*b*q
^2+3*(-q*p^2)^(2/3)*_alpha^4*a*p^4*q^2+(-q*p^2)^(1/3)*_alpha^2*a*p^4*q^3+(-
q*p^2)^(2/3)*_alpha*a*p^3*q^3-(-q*p^2)^(1/3)*_alpha^4*b*p^2*q-(-q*p^2)^(2/3
)*_alpha^3*b*p*q+(-q*p^2)^(1/3)*_alpha*b*p*q^2+I*(-q*p^2)^(2/3)*3^(1/2)*q^2
*b+I*(-q*p^2)^(2/3)*3^(1/2)*_alpha^6*p^2*b+I*(-q*p^2)^(2/3)*p^6*3^(1/2)*_al
pha^10*a-I*(-q*p^2)^(1/3)*p^7*3^(1/2)*_alpha^11*a-I*(-q*p^2)^(1/3)*p^3*3^(1
/2)*_alpha^7*b+2*q*p^2*(_alpha^9*a*p^5+3*_alpha^6*a*p^4*q+3*_alpha^3*a*p^3*
q^2+_alpha^5*b*p+a*p^2*q^3-_alpha^2*b*q)+I*(-q*p^2)^(2/3)*p^3*3^(1/2)*_alph
a*a*q^3+I*(-q*p^2)^(1/3)*3^(1/2)*_alpha^4*q*p^2*b-I*(-q*p^2)^(2/3)*3^(1/2)*
_alpha^3*q*p*b-I*(-q*p^2)^(1/3)*3^(1/2)*_alpha*q^2*p*b-3*I*(-q*p^2)^(1/3)*p
^5*3^(1/2)*_alpha^5*a*q^2+3*I*(-q*p^2)^(2/3)*p^4*3^(1/2)*_alpha^4*a*q^2-I*(
-q*p^2)^(1/3)*p^4*3^(1/2)*_alpha^2*a*q^3-3*I*(-q*p^2)^(1/3)*p^6*3^(1/2)*_al
pha^8*a*q+3*I*(-q*p^2)^(2/3)*p^5*3^(1/2)*_alpha^7*a*q+(-q*p^2)^(1/3)*_alpha
^11*a*p^7+(-q*p^2)^(2/3)*_alpha^10*a*p^6+(-q*p^2)^(1/3)*_alpha^7*b*p^3+(-q*
p^2)^(2/3)*_alpha^6*b*p^2+3*(-q*p^2)^(1/3)*_alpha^8*a*p^6*q+3*(-q*p^2)^(2/3
)*_alpha^7*a*p^5*q+3*(-q*p^2)^(1/3)*_alpha^5*a*p^5*q^2)*EllipticPi(1/3*3^(1
```

```
/2)*(I*(x+1/2/p*(-q*p^2)^(1/3)-1/2*I*3^(1/2)/p*(-q*p^2)^(1/3))*3^(1/2)*p/(-
q*p^2)^(1/3))^(1/2),-1/2/p*(-3*_alpha^2*p^3*(-q*p^2)^(2/3)*a*q^3+3*_alpha^4
*(-q*p^2)^(2/3)*q*p*b-9*_alpha^8*p^5*(-q*p^2)^(2/3)*a*q-9*_alpha^5*p^4*(-q*
p^2)^(2/3)*a*q^2+I*(-q*p^2)^(2/3)*3^(1/2)*_alpha^4*b*p*q-3*I*(-q*p^2)^(2/3)
*3^(1/2)*_alpha^8*a*p^5*q-6*I*(-q*p^2)^(1/3)*3^(1/2)*_alpha^6*a*p^5*q^2-3*I
*(-q*p^2)^(2/3)*3^(1/2)*_alpha^5*a*p^4*q^2-6*I*(-q*p^2)^(1/3)*3^(1/2)*_alph
a^3*a*p^4*q^3-I*(-q*p^2)^(2/3)*3^(1/2)*_alpha^2*a*p^3*q^3-2*I*(-q*p^2)^(1/3
)*3^(1/2)*_alpha^5*b*p^2*q+2*I*(-q*p^2)^(1/3)*3^(1/2)*_alpha^2*b*p*q^2-2*I*
(-q*p^2)^(1/3)*3^(1/2)*_alpha^9*a*p^6*q+I*3^(1/2)*b*p*q^3-3*(-q*p^2)^(2/3)*
_alpha*q^2*b-3*_alpha^11*p^6*(-q*p^2)^(2/3)*a-3*_alpha^7*(-q*p^2)^(2/3)*p^2
*b-3*p^7*_alpha^10*a*q-9*p^6*_alpha^7*a*q^2-9*p^5*_alpha^4*a*q^3-3*p^3*_alp
ha^6*q*b-3*p^4*_alpha*a*q^4+3*_alpha^3*q^2*p^2*b-3*q^3*b*p+I*3^(1/2)*_alpha
*a*p^4*q^4+I*3^(1/2)*_alpha^10*a*p^7*q-I*(-q*p^2)^(2/3)*3^(1/2)*_alpha^7*b*
p^2-I*3^(1/2)*_alpha^3*b*p^2*q^2-I*(-q*p^2)^(2/3)*3^(1/2)*_alpha*b*q^2-I*(-
q*p^2)^(2/3)*3^(1/2)*_alpha^11*a*p^6+3*I*3^(1/2)*_alpha^7*a*p^6*q^2+3*I*3^(
1/2)*_alpha^4*a*p^5*q^3-2*I*(-q*p^2)^(1/3)*3^(1/2)*a*p^3*q^4+I*3^(1/2)*_alp
ha^6*b*p^3*q)/q^3/b,(I*3^(1/2)/p*(-q*p^2)^(1/3)/(-3/2/p*(-q*p^2)^(1/3)+1/2*
I*3^(1/2)/p*(-q*p^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^12*a*p^4+4*_Z^9*a*p^3*
q+6*_Z^6*a*p^2*q^2+_Z^8*b+4*_Z^3*a*p*q^3+a*q^4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^3+q}\left(px^3-2\,q\right)x^4}{bx^8+\left(px^3+q\right)^4a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(p*x^3-2*q)*(p*x^3+q)^(1/2)/(b*x^8+a*(p*x^3+q)^4),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(p*x^3 + q)*(p*x^3 - 2*q)*x^4/(b*x^8 + (p*x^3 + q)^4*a), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(q + p*x^3)^(1/2)*(2*q - p*x^3))/(a*(q + p*x^3)^4 + b*x^8),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(px^3-2q\right)\sqrt{px^3+q}}{ap^4x^{12}+4ap^3qx^9+6ap^2q^2x^6+4apq^3x^3+aq^4+bx^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(p*x**3-2*q)*(p*x**3+q)**(1/2)/(b*x**8+a*(p*x**3+q)**4),x)
```

```
[Out] Integral(x**4*(p*x**3 - 2*q)*sqrt(p*x**3 + q)/(a*p**4*x**12 + 4*a*p**3*q*x*
*9 + 6*a*p**2*q**2*x**6 + 4*a*p*q**3*x**3 + a*q**4 + b*x**8), x)
```

3.2343 $\int \frac{(1+(-2+k)x)(1-2kx+k^2x^2)}{\sqrt[3]{(1-x)x(1-kx)}\left(b-4bkx+(-1+6bk^2)x^2+(2-4bk^3)x^3+(-1+bk^4)x^4\right)}\,dx$

Optimal. Leaf size=404

$$-\frac{\log\left(\sqrt[6]{b}\,k^2x-\sqrt[6]{b}\,k-k\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{2b^{2/3}}-\frac{\log\left(\sqrt[6]{b}\,k^2x-\sqrt[6]{b}\,k+k\sqrt[3]{kx^3+(-k-1)x^2+x}\right)}{2b^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2(\cdots}{\cdots}\right.}{\cdots}$$

Rubi [F] time = 11.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+(-2+k)x)\left(1-2kx+k^2x^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\left(b-4bkx+\left(-1+6bk^2\right)x^2+\left(2-4bk^3\right)x^3+\left(-1+bk^4\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((1 + (-2 + k)*x)*(1 - 2*k*x + k^2*x^2))/(((1 - x)*x*(1 - k*x))^(1/3)*(
b - 4*b*k*x + (-1 + 6*b*k^2)*x^2 + (2 - 4*b*k^3)*x^3 + (-1 + b*k^4)*x^4)),x
]
```

```
[Out] (3*(1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][(x*(1 - k*
x^3)^(5/3))/((1 - x^3)^(1/3)*(-(x^6*(-1 + x^3)^2) + b*(-1 + k*x^3)^4)), x],
 x, x^(1/3)])/((1 - x)*x*(1 - k*x))^(1/3) - (3*(2 - k)*(1 - x)^(1/3)*x^(1/3
)*(1 - k*x)^(1/3)*Defer[Subst][Defer[Int][(x^4*(1 - k*x^3)^(5/3))/((1 - x^3
)^(1/3)*(-(x^6*(-1 + x^3)^2) + b*(-1 + k*x^3)^4)), x], x, x^(1/3)])/((1 - x
)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\int \frac{(1+(-2+k)x)\left(1-2kx+k^2x^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\,\left(b-4bkx+\left(-1+6bk^2\right)x^2+\left(2-4bk^3\right)x^3+\left(-1+bk^4\right)x^4\right)}\,dx = \int \frac{\cdots}{\sqrt[3]{(1-x)x(1-kx)}\ \cdots}$$

$$= \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\cdots}{\cdots}$$

$$= \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\cdots}{\cdots}$$

$$= \frac{\left(3\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\cdots\right.}{\cdots}$$

$$= \frac{\left(3\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\cdots\right.}{\cdots}$$

$$= \frac{\left(3\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\cdots\right.}{\cdots}$$

$$= \frac{\left(3\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\cdots\right.}{\cdots}$$

$$= \frac{\left(3\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\cdots\right.}{\cdots}$$

Mathematica [F] time = 6.20, size = 0, normalized size = 0.00

$$\int \frac{(1+(-2+k)x)\left(1-2kx+k^2x^2\right)}{\sqrt[3]{(1-x)x(1-kx)}\,\left(b-4bkx+\left(-1+6bk^2\right)x^2+\left(2-4bk^3\right)x^3+\left(-1+bk^4\right)x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 + (-2 + k)*x)*(1 - 2*k*x + k^2*x^2))/((((1 - x)*x*(1 - k*x))^(
1/3)*(b - 4*b*k*x + (-1 + 6*b*k^2)*x^2 + (2 - 4*b*k^3)*x^3 + (-1 + b*k^4)*x
^4)),x]
```

```
[Out] Integrate[((1 + (-2 + k)*x)*(1 - 2*k*x + k^2*x^2))/((((1 - x)*x*(1 - k*x))^(
1/3)*(b - 4*b*k*x + (-1 + 6*b*k^2)*x^2 + (2 - 4*b*k^3)*x^3 + (-1 + b*k^4)*x
^4)), x]
```

IntegrateAlgebraic [A] time = 6.51, size = 322, normalized size = 0.80

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}k^2x^2-2\sqrt{3}\sqrt[3]{b}kx+\sqrt{3}\sqrt[3]{b}}{\sqrt[3]{b}k^2x^2-2\sqrt[3]{b}kx+\sqrt[3]{b}+2\left(kx^3+(-k-1)x^2+x\right)^{2/3}}\right)}{2b^{2/3}}-\frac{\log\left(\sqrt[3]{b}k^3x^2-2\sqrt[3]{b}k^2x+\sqrt[3]{b}k-k\left(kx^3+(-k-1)x^2+x\right)^{2/3}\right)}{2b^{2/3}}+\frac{\log\left(b^{2/3}k^6x^4-4b^{2/3}k^5x^3+6b^{2/3}k^4x^2-4b^{2/3}k^3x+b^{2/3}k^2+\left(kx^3+(-k-1)x^2+x\right)^{2/3}\left(\sqrt[3]{b}k^4x^2-2\sqrt[3]{b}k^3x+\sqrt[3]{b}k^2\right)+k^2\left(kx^3+(-k-1)x^2+x\right)^{4/3}\right)}{4b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + (-2 + k)*x)*(1 - 2*k*x + k^2*x^2))/((((1 - x)*x*(1
- k*x))^(1/3)*(b - 4*b*k*x + (-1 + 6*b*k^2)*x^2 + (2 - 4*b*k^3)*x^3 + (-1 +
 b*k^4)*x^4)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3) - 2*Sqrt[3]*b^(1/3)*k*x + Sqrt[3]*b^(
1/3)*k^2*x^2)/(b^(1/3) - 2*b^(1/3)*k*x + b^(1/3)*k^2*x^2 + 2*(x + (-1 - k)*
```

```
x^2 + k*x^3)^(2/3))])/b^(2/3) - Log[b^(1/3)*k - 2*b^(1/3)*k^2*x + b^(1/3)*k
^3*x^2 - k*(x + (-1 - k)*x^2 + k*x^3)^(2/3)]/(2*b^(2/3)) + Log[b^(2/3)*k^2
- 4*b^(2/3)*k^3*x + 6*b^(2/3)*k^4*x^2 - 4*b^(2/3)*k^5*x^3 + b^(2/3)*k^6*x^4
 + (b^(1/3)*k^2 - 2*b^(1/3)*k^3*x + b^(1/3)*k^4*x^2)*(x + (-1 - k)*x^2 + k*
x^3)^(2/3) + k^2*(x + (-1 - k)*x^2 + k*x^3)^(4/3)]/(4*b^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-2+k)*x)*(k^2*x^2-2*k*x+1)/((1-x)*x*(-k*x+1))^(1/3)/(b-4*b*k*
x+(6*b*k^2-1)*x^2+(-4*b*k^3+2)*x^3+(b*k^4-1)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(k^{2} x^{2}-2 \, k x+1\right)\left((k-2) x+1\right)}{\left(\left(b k^{4}-1\right) x^{4}-2 \, \left(2 \, b k^{3}-1\right) x^{3}-4 \, b k x+\left(6 \, b k^{2}-1\right) x^{2}+b\right) \left((k x-1)(x-1) x\right)^{\frac{1}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-2+k)*x)*(k^2*x^2-2*k*x+1)/((1-x)*x*(-k*x+1))^(1/3)/(b-4*b*k*
x+(6*b*k^2-1)*x^2+(-4*b*k^3+2)*x^3+(b*k^4-1)*x^4),x, algorithm="giac")
```

```
[Out] integrate((k^2*x^2 - 2*k*x + 1)*((k - 2)*x + 1)/(((b*k^4 - 1)*x^4 - 2*(2*b*
k^3 - 1)*x^3 - 4*b*k*x + (6*b*k^2 - 1)*x^2 + b)*((k*x - 1)*(x - 1)*x)^(1/3)
), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(1+\left(-2+k\right) x\right) \left(k^{2} x^{2}-2 k x+1\right)}{\left(\left(1-x\right) x \left(-k x+1\right)\right)^{\frac{1}{3}} \left(b-4 b k x+\left(6 b \,k^{2}-1\right) x^{2}+\left(-4 b \,k^{3}+2\right) x^{3}+\left(b \,k^{4}-1\right) x^{4}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(-2+k)*x)*(k^2*x^2-2*k*x+1)/((1-x)*x*(-k*x+1))^(1/3)/(b-4*b*k*x+(6*b
*k^2-1)*x^2+(-4*b*k^3+2)*x^3+(b*k^4-1)*x^4),x)
```

```
[Out] int((1+(-2+k)*x)*(k^2*x^2-2*k*x+1)/((1-x)*x*(-k*x+1))^(1/3)/(b-4*b*k*x+(6*b
*k^2-1)*x^2+(-4*b*k^3+2)*x^3+(b*k^4-1)*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(k^{2} x^{2}-2 \, k x+1\right)\left((k-2) x+1\right)}{\left(\left(b k^{4}-1\right) x^{4}-2 \, \left(2 \, b k^{3}-1\right) x^{3}-4 \, b k x+\left(6 \, b k^{2}-1\right) x^{2}+b\right) \left((k x-1)(x-1) x\right)^{\frac{1}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-2+k)*x)*(k^2*x^2-2*k*x+1)/((1-x)*x*(-k*x+1))^(1/3)/(b-4*b*k*
x+(6*b*k^2-1)*x^2+(-4*b*k^3+2)*x^3+(b*k^4-1)*x^4),x, algorithm="maxima")
```

```
[Out] integrate((k^2*x^2 - 2*k*x + 1)*((k - 2)*x + 1)/(((b*k^4 - 1)*x^4 - 2*(2*b*
k^3 - 1)*x^3 - 4*b*k*x + (6*b*k^2 - 1)*x^2 + b)*((k*x - 1)*(x - 1)*x)^(1/3)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x\,\left(k-2\right)+1\right)\,\left(k^2\,x^2-2\,k\,x+1\right)}{{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/3}\,\left(\left(b\,k^4-1\right)\,x^4+\left(2-4\,b\,k^3\right)\,x^3+\left(6\,b\,k^2-1\right)\,x^2-4\,b\,k\,x+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x*(k - 2) + 1)*(k^2*x^2 - 2*k*x + 1))/((x*(k*x - 1)*(x - 1))^(1/3)*(b
 + x^4*(b*k^4 - 1) + x^2*(6*b*k^2 - 1) - x^3*(4*b*k^3 - 2) - 4*b*k*x)),x)
```

```
[Out] int(((x*(k - 2) + 1)*(k^2*x^2 - 2*k*x + 1))/((x*(k*x - 1)*(x - 1))^(1/3)*(b
 + x^4*(b*k^4 - 1) + x^2*(6*b*k^2 - 1) - x^3*(4*b*k^3 - 2) - 4*b*k*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-2+k)*x)*(k**2*x**2-2*k*x+1)/((1-x)*x*(-k*x+1))**(1/3)/(b-4*b
*k*x+(6*b*k**2-1)*x**2+(-4*b*k**3+2)*x**3+(b*k**4-1)*x**4),x)
```

```
[Out] Timed out
```

3.2344

$$\int \frac{(a-2b+x)\left(a^2-2ax+x^2\right)}{\sqrt[3]{(-a+x)(-b+x)}\left(-b^2+a^4d+2\left(b-2a^3d\right)x+\left(-1+6a^2d\right)x^2-4adx^3+dx^4\right)}\,dx$$

Optimal. Leaf size=405

$$\frac{\log\left(a^2\sqrt[6]{d}-a\sqrt[3]{x(-a-b)+ab+x^2}-a\sqrt[6]{d}\,x\right)}{2d^{2/3}}+\frac{\log\left(a^2\sqrt[6]{d}+a\sqrt[3]{x(-a-b)+ab+x^2}-a\sqrt[6]{d}\,x\right)}{2d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{a^2}{\sqrt{3}}+\ldots}{\ldots}\right)}{\ldots}$$

Rubi [F] time = 9.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a-2b+x)\left(a^2-2ax+x^2\right)}{\sqrt[3]{(-a+x)(-b+x)}\left(-b^2+a^4d+2\left(b-2a^3d\right)x+\left(-1+6a^2d\right)x^2-4adx^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a - 2*b + x)*(a^2 - 2*a*x + x^2))/(((-a + x)*(-b + x))^(1/3)*(-b^2 +
a^4*d + 2*(b - 2*a^3*d)*x + (-1 + 6*a^2*d)*x^2 - 4*a*d*x^3 + d*x^4)),x]
```

```
[Out] (-3*(a - 2*b)*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[Subst][Defer[Int][x^7/((a
 - b + x^3)^(1/3)*(a^2*(1 + (b*(-2*a + b))/a^2) + (2*a - 2*b)*x^3 + x^6 - d
*x^12)), x], x, (-a + x)^(1/3)]])/((a - x)*(b - x))^(1/3) - (3*a*(-a + x)^(1
/3)*(-b + x)^(1/3)*Defer[Subst][Defer[Int][x^7/((a - b + x^3)^(1/3)*(a^2*(1
 + (b*(-2*a + b))/a^2) + 2*a*(1 - b/a)*x^3 + x^6 - d*x^12)), x], x, (-a + x
)^(1/3)]])/((a - x)*(b - x))^(1/3) - (3*(-a + x)^(1/3)*(-b + x)^(1/3)*Defer[
Subst][Defer[Int][x^10/((a - b + x^3)^(1/3)*(a^2*(1 + (b*(-2*a + b))/a^2) +
 2*a*(1 - b/a)*x^3 + x^6 - d*x^12)), x], x, (-a + x)^(1/3)]])/((a - x)*(b -
x))^(1/3)
```

Rubi steps

$$\int \frac{(a-2b+x)\left(a^2-2ax+x^2\right)}{\sqrt[3]{(-a+x)(-b+x)}\,\left(-b^2+a^4d+2\left(b-2a^3d\right)x+\left(-1+6a^2d\right)x^2-4adx^3+dx^4\right)}\,dx = \int \frac{}{\sqrt[3]{(-a+x)(-b+}}$$

$$= \frac{\left(\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)}{}$$

$$= \frac{\left(\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)}{}$$

$$= \frac{\left(\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)}{}$$

$$= \frac{\left(\sqrt[3]{-a+x}\,\sqrt[3]{-b+x}\right)}{}$$

$$= -\frac{\left(3\sqrt[3]{-a+x}\,\sqrt[3]{-b+}\right.}{}$$

$$= -\frac{\left(3\sqrt[3]{-a+x}\,\sqrt[3]{-b+}\right.}{}$$

$$= -\frac{\left(3\sqrt[3]{-a+x}\,\sqrt[3]{-b+}\right.}{}$$

$$= -\frac{\left(3\sqrt[3]{-a+x}\,\sqrt[3]{-b+}\right.}{}$$

$$= -\frac{\left(3\sqrt[3]{-a+x}\,\sqrt[3]{-b+}\right.}{}$$

Mathematica [F] time = 4.63, size = 0, normalized size = 0.00

$$\int \frac{(a-2b+x)\left(a^2-2ax+x^2\right)}{\sqrt[3]{(-a+x)(-b+x)}\,\left(-b^2+a^4d+2\left(b-2a^3d\right)x+\left(-1+6a^2d\right)x^2-4adx^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a - 2*b + x)*(a^2 - 2*a*x + x^2))/(((-a + x)*(-b + x))^(1/3)*(-
b^2 + a^4*d + 2*(b - 2*a^3*d)*x + (-1 + 6*a^2*d)*x^2 - 4*a*d*x^3 + d*x^4)),
x]
```

```
[Out] Integrate[((a - 2*b + x)*(a^2 - 2*a*x + x^2))/(((-a + x)*(-b + x))^(1/3)*(-
b^2 + a^4*d + 2*(b - 2*a^3*d)*x + (-1 + 6*a^2*d)*x^2 - 4*a*d*x^3 + d*x^4)),
 x]
```

IntegrateAlgebraic [A] time = 3.58, size = 405, normalized size = 1.00

$$\frac{\log\left(a^2\sqrt[6]{d}-a\sqrt[3]{x(-a-b)+ab+x^2}-a\sqrt[6]{d}\,x\right)}{2d^{2/3}}+\frac{\log\left(a^2\sqrt[6]{d}+a\sqrt[3]{x(-a-b)+ab+x^2}-a\sqrt[6]{d}\,x\right)}{2d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{a^2}{\sqrt{3}}+\frac{2\left(x(-a-b)+ab+x^2\right)^{2/3}}{\sqrt{3}\sqrt[3]{d}}-\frac{2ax}{\sqrt{3}}+\frac{x^2}{\sqrt{3}}}{(a-x)^2}\right)}{2d^{2/3}}-\frac{\log\left(a^4\sqrt[3]{d}-2a^3\sqrt[3]{d}\,x+a^2\left(x(-a-b)+ab+x^2\right)^{2/3}+a^2\sqrt[3]{d}\,x^2+\sqrt[3]{x(-a-b)+ab+x^2}\left(a^3\sqrt[6]{d}-a^2\sqrt[6]{d}\,x\right)\right)}{4d^{2/3}}-\frac{\log\left(a^4\sqrt[3]{d}-2a^3\sqrt[3]{d}\,x+a^2\left(x(-a-b)+ab+x^2\right)^{2/3}+a^2\sqrt[3]{d}\,x^2+\sqrt[3]{x(-a-b)+ab+x^2}\left(a^2\sqrt[6]{d}\,x-a^3\sqrt[6]{d}\right)\right)}{4d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a - 2*b + x)*(a^2 - 2*a*x + x^2))/(((-a + x)*(-b + x))
^(1/3)*(-b^2 + a^4*d + 2*(b - 2*a^3*d)*x + (-1 + 6*a^2*d)*x^2 - 4*a*d*x^3 +
 d*x^4)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(a^2/Sqrt[3] - (2*a*x)/Sqrt[3] + x^2/Sqrt[3] + (2*(a*b
 + (-a - b)*x + x^2)^(2/3))/(Sqrt[3]*d^(1/3)))/(a - x)^2]])/d^(2/3) + Log[a^
2*d^(1/6) - a*d^(1/6)*x - a*(a*b + (-a - b)*x + x^2)^(1/3)]/(2*d^(2/3)) + L
og[a^2*d^(1/6) - a*d^(1/6)*x + a*(a*b + (-a - b)*x + x^2)^(1/3)]/(2*d^(2/3)
) - Log[a^4*d^(1/3) - 2*a^3*d^(1/3)*x + a^2*d^(1/3)*x^2 + (a^3*d^(1/6) - a^
2*d^(1/6)*x)*(a*b + (-a - b)*x + x^2)^(1/3) + a^2*(a*b + (-a - b)*x + x^2)^
(2/3)]/(4*d^(2/3)) - Log[a^4*d^(1/3) - 2*a^3*d^(1/3)*x + a^2*d^(1/3)*x^2 +
(-(a^3*d^(1/6)) + a^2*d^(1/6)*x)*(a*b + (-a - b)*x + x^2)^(1/3) + a^2*(a*b
+ (-a - b)*x + x^2)^(2/3)]/(4*d^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)*(a^2-2*a*x+x^2)/((-a+x)*(-b+x))^(1/3)/(-b^2+a^4*d+2*(-2
*a^3*d+b)*x+(6*a^2*d-1)*x^2-4*a*d*x^3+d*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{2}-2\,ax+x^{2}\right)\left(a-2\,b+x\right)}{\left(a^{4}d-4\,adx^{3}+dx^{4}+\left(6\,a^{2}d-1\right)x^{2}-b^{2}-2\left(2\,a^{3}d-b\right)x\right)\left(\left(a-x\right)\left(b-x\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)*(a^2-2*a*x+x^2)/((-a+x)*(-b+x))^(1/3)/(-b^2+a^4*d+2*(-2
*a^3*d+b)*x+(6*a^2*d-1)*x^2-4*a*d*x^3+d*x^4),x, algorithm="giac")
```

```
[Out] integrate((a^2 - 2*a*x + x^2)*(a - 2*b + x)/((a^4*d - 4*a*d*x^3 + d*x^4 + (
6*a^2*d - 1)*x^2 - b^2 - 2*(2*a^3*d - b)*x)*((a - x)*(b - x))^(1/3)), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(a-2b+x\right)\left(a^{2}-2ax+x^{2}\right)}{\left(\left(-a+x\right)\left(-b+x\right)\right)^{\frac{1}{3}}\left(-b^{2}+a^{4}d+2\left(-2a^{3}d+b\right)x+\left(6a^{2}d-1\right)x^{2}-4ad\,x^{3}+d\,x^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-2*b+x)*(a^2-2*a*x+x^2)/((-a+x)*(-b+x))^(1/3)/(-b^2+a^4*d+2*(-2*a^3*d
+b)*x+(6*a^2*d-1)*x^2-4*a*d*x^3+d*x^4),x)
```

```
[Out] int((a-2*b+x)*(a^2-2*a*x+x^2)/((-a+x)*(-b+x))^(1/3)/(-b^2+a^4*d+2*(-2*a^3*d
+b)*x+(6*a^2*d-1)*x^2-4*a*d*x^3+d*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{2}-2\,ax+x^{2}\right)\left(a-2\,b+x\right)}{\left(a^{4}d-4\,adx^{3}+dx^{4}+\left(6\,a^{2}d-1\right)x^{2}-b^{2}-2\left(2\,a^{3}d-b\right)x\right)\left(\left(a-x\right)\left(b-x\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)*(a^2-2*a*x+x^2)/((-a+x)*(-b+x))^(1/3)/(-b^2+a^4*d+2*(-2
*a^3*d+b)*x+(6*a^2*d-1)*x^2-4*a*d*x^3+d*x^4),x, algorithm="maxima")
```

```
[Out] integrate((a^2 - 2*a*x + x^2)*(a - 2*b + x)/((a^4*d - 4*a*d*x^3 + d*x^4 + (
6*a^2*d - 1)*x^2 - b^2 - 2*(2*a^3*d - b)*x)*((a - x)*(b - x))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2-2\,a\,x+x^2\right)\,\left(a-2\,b+x\right)}{{\left(\left(a-x\right)\,\left(b-x\right)\right)}^{1/3}\,\left(x^2\,\left(6\,a^2\,d-1\right)+2\,x\,\left(b-2\,a^3\,d\right)+a^4\,d+d\,x^4-b^2-4\,a\,d\,x^3\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a^2 - 2*a*x + x^2)*(a - 2*b + x))/(((a - x)*(b - x))^(1/3)*(x^2*(6*a^
2*d - 1) + 2*x*(b - 2*a^3*d) + a^4*d + d*x^4 - b^2 - 4*a*d*x^3)),x)
```

```
[Out] int(((a^2 - 2*a*x + x^2)*(a - 2*b + x))/(((a - x)*(b - x))^(1/3)*(x^2*(6*a^
2*d - 1) + 2*x*(b - 2*a^3*d) + a^4*d + d*x^4 - b^2 - 4*a*d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-2*b+x)*(a**2-2*a*x+x**2)/((-a+x)*(-b+x))**(1/3)/(-b**2+a**4*d+
2*(-2*a**3*d+b)*x+(6*a**2*d-1)*x**2-4*a*d*x**3+d*x**4),x)
```

```
[Out] Timed out
```

3.2345 $\int \frac{\sqrt{b+ax}\sqrt{c+\sqrt{b+ax}}}{x-\sqrt{b+ax}}\,dx$

Optimal. Leaf size=407

$$\frac{2\left(\sqrt{2}\,a^3+\sqrt{2}\,ac\sqrt{a^2+4b}+\sqrt{2}\,a^2\sqrt{a^2+4b}+\sqrt{2}\,b\sqrt{a^2+4b}+\sqrt{2}\,a^2c+3\sqrt{2}\,ab+2\sqrt{2}\,bc\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{ax+b}+}}{\sqrt{-\sqrt{a^2+4b}-a-}}\right.}{\sqrt{a^2+4b}\sqrt{-\sqrt{a^2+4b}-a-2c}}$$

Rubi [A] time = 4.65, antiderivative size = 255, normalized size of antiderivative = 0.63, number of steps used = 7, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {897, 1287, 1166, 206}

$$-\frac{2\sqrt{2}\left(a^2-\frac{a^3+a^2c+3ab+2bc}{\sqrt{a^2+4b}}+ac+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{ax+b}+c}}{\sqrt{-\sqrt{a^2+4b}+a+2c}}\right)}{\sqrt{-\sqrt{a^2+4b}+a+2c}}-\frac{2\sqrt{2}\left(a^2+\frac{a^3+a^2c+3ab+2bc}{\sqrt{a^2+4b}}+ac+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{ax+b}+c}}{\sqrt{\sqrt{a^2+4b}+a+2c}}\right)}{\sqrt{\sqrt{a^2+4b}+a+2c}}+\frac{4}{3}\left(\sqrt{ax+b}+c\right)^{3/2}+4a\sqrt{\sqrt{ax+b}+c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]])/(x - Sqrt[b + a*x]),x]
```

```
[Out] 4*a*Sqrt[c + Sqrt[b + a*x]] + (4*(c + Sqrt[b + a*x])^(3/2))/3 - (2*Sqrt[2]*
(a^2 + b + a*c - (a^3 + 3*a*b + a^2*c + 2*b*c)/Sqrt[a^2 + 4*b])*ArcTanh[(Sq
rt[2]*Sqrt[c + Sqrt[b + a*x]])/Sqrt[a - Sqrt[a^2 + 4*b] + 2*c]])/Sqrt[a - S
qrt[a^2 + 4*b] + 2*c] - (2*Sqrt[2]*(a^2 + b + a*c + (a^3 + 3*a*b + a^2*c +
2*b*c)/Sqrt[a^2 + 4*b])*ArcTanh[(Sqrt[2]*Sqrt[c + Sqrt[b + a*x]])/Sqrt[a +
Sqrt[a^2 + 4*b] + 2*c]])/Sqrt[a + Sqrt[a^2 + 4*b] + 2*c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{b+ax}\sqrt{c+\sqrt{b+ax}}}{x-\sqrt{b+ax}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{x^2\sqrt{c+x}}{b+ax-x^2}\,dx, x, \sqrt{b+ax}\right)\right)\\
&= -\left(4\,\text{Subst}\left(\int \frac{x^2\left(-c+x^2\right)^2}{b-ac-c^2+(a+2c)x^2-x^4}\,dx, x, \sqrt{c+\sqrt{b+ax}}\right)\right)\\
&= -\left(4\,\text{Subst}\left(\int\left(-a-x^2+\frac{a(b-c(a+c))+\left(a^2+b+ac\right)x^2}{b-ac-c^2+(a+2c)x^2-x^4}\right)dx, x, \sqrt{c+\sqrt{b}}\right.\right.\\
&= 4a\sqrt{c+\sqrt{b+ax}}+\frac{4}{3}\left(c+\sqrt{b+ax}\right)^{3/2}-4\,\text{Subst}\left(\int \frac{a(b-c(a+c))+\left(a^2+\right.}{b-ac-c^2+(a+2c}\right.\\
&= 4a\sqrt{c+\sqrt{b+ax}}+\frac{4}{3}\left(c+\sqrt{b+ax}\right)^{3/2}-\left(2\left(a^2+b+ac-\frac{a^3+3ab+a^2c+}{\sqrt{a^2+4b}}\right.\right.\\
&= 4a\sqrt{c+\sqrt{b+ax}}+\frac{4}{3}\left(c+\sqrt{b+ax}\right)^{3/2}-\frac{2\sqrt{2}\left(a^2+b+ac-\frac{a^3+3ab+a^2c+2bc}{\sqrt{a^2+4b}}\right)}{\sqrt{a-\sqrt{a^2+4b}}}
\end{aligned}
$$

Mathematica [A] time = 4.45, size = 328, normalized size = 0.81

$$
\frac{\sqrt{\sqrt{ax+b}+c}\left(-2a\left(-3\sqrt{a^2+4b}+\sqrt{ax+b}+c\right)+2a\left(3\sqrt{a^2+4b}+\sqrt{ax+b}+c\right)-\frac{3\left(a\sqrt{a^2+4b}-a^2-2b\right)\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{\sqrt{ax+b}+c}{-\sqrt{a^2+4b}+a+2c}}\right)}{\sqrt{\frac{\sqrt{ax+b}+c}{-2\sqrt{a^2+4b}+2a+4c}}}-\frac{3\sqrt{2}\left(a\sqrt{a^2+4b}+a^2+2b\right)\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{\sqrt{ax+b}+c}{\sqrt{a^2+4b}+a+2c}}\right)}{\sqrt{\frac{\sqrt{ax+b}+c}{\sqrt{a^2+4b}+a+2c}}}+4c\sqrt{a^2+4b}+4\sqrt{a^2+4b}\sqrt{ax+b}\right)}{3\sqrt{a^2+4b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]])/(x - Sqrt[b + a*x]),x]
```

```
[Out] (Sqrt[c + Sqrt[b + a*x]]*(4*Sqrt[a^2 + 4*b]*c + 4*Sqrt[a^2 + 4*b]*Sqrt[b +
a*x] - 2*a*(-3*Sqrt[a^2 + 4*b] + c + Sqrt[b + a*x]) + 2*a*(3*Sqrt[a^2 + 4*b
] + c + Sqrt[b + a*x]) - (3*(-a^2 - 2*b + a*Sqrt[a^2 + 4*b])*ArcTanh[Sqrt[2
]*Sqrt[(c + Sqrt[b + a*x])/(a - Sqrt[a^2 + 4*b] + 2*c)]])/Sqrt[(c + Sqrt[b
+ a*x])/(2*a - 2*Sqrt[a^2 + 4*b] + 4*c)] - (3*Sqrt[2]*(a^2 + 2*b + a*Sqrt[a
^2 + 4*b])*ArcTanh[Sqrt[2]*Sqrt[(c + Sqrt[b + a*x])/(a + Sqrt[a^2 + 4*b] +
2*c)]])/Sqrt[(c + Sqrt[b + a*x])/(a + Sqrt[a^2 + 4*b] + 2*c)]))/(3*Sqrt[a^2
 + 4*b])
```

IntegrateAlgebraic [A] time = 0.97, size = 388, normalized size = 0.95

$$
\frac{2\left(\sqrt{2}a^3+\sqrt{2}ac\sqrt{a^2+4b}+\sqrt{2}a^2\sqrt{a^2+4b}+\sqrt{2}b\sqrt{a^2+4b}+\sqrt{2}a^2c+3\sqrt{2}ab+2\sqrt{2}bc\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{ax+b}+c}}{\sqrt{-\sqrt{a^2+4b}-a-2c}}\right)}{\sqrt{a^2+4b}\sqrt{-\sqrt{a^2+4b}-a-2c}}+\frac{2\left(-\sqrt{2}a^3+\sqrt{2}ac\sqrt{a^2+4b}+\sqrt{2}a^2\sqrt{a^2+4b}+\sqrt{2}b\sqrt{a^2+4b}-\sqrt{2}a^2c-3\sqrt{2}ab-2\sqrt{2}bc\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{ax+b}+c}}{\sqrt{\sqrt{a^2+4b}-a-2c}}\right)}{\sqrt{a^2+4b}\sqrt{\sqrt{a^2+4b}-a-2c}}+\frac{4}{3}\sqrt{\sqrt{ax+b}+c}\left(\sqrt{ax+b}+3a+c\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[b + a*x]*Sqrt[c + Sqrt[b + a*x]])/(x - Sqrt[b + a*
x]),x]
```

```
[Out] (4*Sqrt[c + Sqrt[b + a*x]]*(3*a + c + Sqrt[b + a*x]))/3 + (2*(Sqrt[2]*a^3 +
 3*Sqrt[2]*a*b + Sqrt[2]*a^2*Sqrt[a^2 + 4*b] + Sqrt[2]*b*Sqrt[a^2 + 4*b] +
Sqrt[2]*a^2*c + 2*Sqrt[2]*b*c + Sqrt[2]*a*Sqrt[a^2 + 4*b]*c)*ArcTan[(Sqrt[2
]*Sqrt[c + Sqrt[b + a*x]])/Sqrt[-a - Sqrt[a^2 + 4*b] - 2*c]])/(Sqrt[a^2 + 4
*b]*Sqrt[-a - Sqrt[a^2 + 4*b] - 2*c]) + (2*(-(Sqrt[2]*a^3) - 3*Sqrt[2]*a*b
+ Sqrt[2]*a^2*Sqrt[a^2 + 4*b] + Sqrt[2]*b*Sqrt[a^2 + 4*b] - Sqrt[2]*a^2*c -
 2*Sqrt[2]*b*c + Sqrt[2]*a*Sqrt[a^2 + 4*b]*c)*ArcTan[(Sqrt[2]*Sqrt[c + Sqrt
```

```
[b + a*x]])/Sqrt[-a + Sqrt[a^2 + 4*b] - 2*c]])/(Sqrt[a^2 + 4*b]*Sqrt[-a + S
qrt[a^2 + 4*b] - 2*c])
```

fricas [B] time = 0.50, size = 2014, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^(1/2)*(c+(a*x+b)^(1/2))^(1/2)/(x-(a*x+b)^(1/2)),x, algori
thm="fricas")
```

```
[Out] sqrt(2)*sqrt((a^5 + 5*a^3*b + 5*a*b^2 + (a^4 + 4*a^2*b + 2*b^2)*c + (a^2 +
4*b)*sqrt((a^8 + 6*a^6*b + 11*a^4*b^2 + 6*a^2*b^3 + b^4 + (a^6 + 4*a^4*b +
4*a^2*b^2)*c^2 + 2*(a^7 + 5*a^5*b + 7*a^3*b^2 + 2*a*b^3)*c)/(a^2 + 4*b)))/(
a^2 + 4*b))*log(8*sqrt(2)*(a^7 + 7*a^5*b + 13*a^3*b^2 + 4*a*b^3 + (a^6 + 6*
a^4*b + 8*a^2*b^2)*c - (a^4 + 6*a^2*b + 8*b^2)*sqrt((a^8 + 6*a^6*b + 11*a^4
*b^2 + 6*a^2*b^3 + b^4 + (a^6 + 4*a^4*b + 4*a^2*b^2)*c^2 + 2*(a^7 + 5*a^5*b
 + 7*a^3*b^2 + 2*a*b^3)*c)/(a^2 + 4*b)))*sqrt((a^5 + 5*a^3*b + 5*a*b^2 + (a
^4 + 4*a^2*b + 2*b^2)*c + (a^2 + 4*b)*sqrt((a^8 + 6*a^6*b + 11*a^4*b^2 + 6*
a^2*b^3 + b^4 + (a^6 + 4*a^4*b + 4*a^2*b^2)*c^2 + 2*(a^7 + 5*a^5*b + 7*a^3*
b^2 + 2*a*b^3)*c)/(a^2 + 4*b)))/(a^2 + 4*b)) + 32*(a^4*b^2 + 3*a^2*b^3 + b^
4 + (a^3*b^2 + 2*a*b^3)*c)*sqrt(c + sqrt(a*x + b))) - sqrt(2)*sqrt((a^5 + 5
*a^3*b + 5*a*b^2 + (a^4 + 4*a^2*b + 2*b^2)*c + (a^2 + 4*b)*sqrt((a^8 + 6*a^
6*b + 11*a^4*b^2 + 6*a^2*b^3 + b^4 + (a^6 + 4*a^4*b + 4*a^2*b^2)*c^2 + 2*(a
^7 + 5*a^5*b + 7*a^3*b^2 + 2*a*b^3)*c)/(a^2 + 4*b)))/(a^2 + 4*b))*log(-8*sq
rt(2)*(a^7 + 7*a^5*b + 13*a^3*b^2 + 4*a*b^3 + (a^6 + 6*a^4*b + 8*a^2*b^2)*c
 - (a^4 + 6*a^2*b + 8*b^2)*sqrt((a^8 + 6*a^6*b + 11*a^4*b^2 + 6*a^2*b^3 + b
^4 + (a^6 + 4*a^4*b + 4*a^2*b^2)*c^2 + 2*(a^7 + 5*a^5*b + 7*a^3*b^2 + 2*a*b
^3)*c)/(a^2 + 4*b)))*sqrt((a^5 + 5*a^3*b + 5*a*b^2 + (a^4 + 4*a^2*b + 2*b^2
)*c + (a^2 + 4*b)*sqrt((a^8 + 6*a^6*b + 11*a^4*b^2 + 6*a^2*b^3 + b^4 + (a^6
 + 4*a^4*b + 4*a^2*b^2)*c^2 + 2*(a^7 + 5*a^5*b + 7*a^3*b^2 + 2*a*b^3)*c)/(a
^2 + 4*b)))/(a^2 + 4*b)) + 32*(a^4*b^2 + 3*a^2*b^3 + b^4 + (a^3*b^2 + 2*a*b
^3)*c)*sqrt(c + sqrt(a*x + b))) + sqrt(2)*sqrt((a^5 + 5*a^3*b + 5*a*b^2 + (
a^4 + 4*a^2*b + 2*b^2)*c - (a^2 + 4*b)*sqrt((a^8 + 6*a^6*b + 11*a^4*b^2 + 6
*a^2*b^3 + b^4 + (a^6 + 4*a^4*b + 4*a^2*b^2)*c^2 + 2*(a^7 + 5*a^5*b + 7*a^3
*b^2 + 2*a*b^3)*c)/(a^2 + 4*b)))/(a^2 + 4*b))*log(8*sqrt(2)*(a^7 + 7*a^5*b
+ 13*a^3*b^2 + 4*a*b^3 + (a^6 + 6*a^4*b + 8*a^2*b^2)*c + (a^4 + 6*a^2*b + 8
*b^2)*sqrt((a^8 + 6*a^6*b + 11*a^4*b^2 + 6*a^2*b^3 + b^4 + (a^6 + 4*a^4*b +
 4*a^2*b^2)*c^2 + 2*(a^7 + 5*a^5*b + 7*a^3*b^2 + 2*a*b^3)*c)/(a^2 + 4*b)))*
sqrt((a^5 + 5*a^3*b + 5*a*b^2 + (a^4 + 4*a^2*b + 2*b^2)*c - (a^2 + 4*b)*sqr
t((a^8 + 6*a^6*b + 11*a^4*b^2 + 6*a^2*b^3 + b^4 + (a^6 + 4*a^4*b + 4*a^2*b^
2)*c^2 + 2*(a^7 + 5*a^5*b + 7*a^3*b^2 + 2*a*b^3)*c)/(a^2 + 4*b)))/(a^2 + 4*
b)) + 32*(a^4*b^2 + 3*a^2*b^3 + b^4 + (a^3*b^2 + 2*a*b^3)*c)*sqrt(c + sqrt(
a*x + b))) - sqrt(2)*sqrt((a^5 + 5*a^3*b + 5*a*b^2 + (a^4 + 4*a^2*b + 2*b^2
)*c - (a^2 + 4*b)*sqrt((a^8 + 6*a^6*b + 11*a^4*b^2 + 6*a^2*b^3 + b^4 + (a^6
 + 4*a^4*b + 4*a^2*b^2)*c^2 + 2*(a^7 + 5*a^5*b + 7*a^3*b^2 + 2*a*b^3)*c)/(a
^2 + 4*b)))/(a^2 + 4*b))*log(-8*sqrt(2)*(a^7 + 7*a^5*b + 13*a^3*b^2 + 4*a*b
^3 + (a^6 + 6*a^4*b + 8*a^2*b^2)*c + (a^4 + 6*a^2*b + 8*b^2)*sqrt((a^8 + 6*
a^6*b + 11*a^4*b^2 + 6*a^2*b^3 + b^4 + (a^6 + 4*a^4*b + 4*a^2*b^2)*c^2 + 2*
(a^7 + 5*a^5*b + 7*a^3*b^2 + 2*a*b^3)*c)/(a^2 + 4*b)))*sqrt((a^5 + 5*a^3*b
+ 5*a*b^2 + (a^4 + 4*a^2*b + 2*b^2)*c - (a^2 + 4*b)*sqrt((a^8 + 6*a^6*b + 1
1*a^4*b^2 + 6*a^2*b^3 + b^4 + (a^6 + 4*a^4*b + 4*a^2*b^2)*c^2 + 2*(a^7 + 5*
a^5*b + 7*a^3*b^2 + 2*a*b^3)*c)/(a^2 + 4*b)))/(a^2 + 4*b)) + 32*(a^4*b^2 +
3*a^2*b^3 + b^4 + (a^3*b^2 + 2*a*b^3)*c)*sqrt(c + sqrt(a*x + b))) + 4/3*(3*
a + c + sqrt(a*x + b))*sqrt(c + sqrt(a*x + b))
```

giac [A] time = 1.05, size = 259, normalized size = 0.64

$$\frac{4\,\sqrt{a^2+4\,b}\,\sqrt{-2\,a-4\,c+2\,\sqrt{a^2+4\,b}}\,b^2\arctan\left(\frac{\sqrt{c+\sqrt{ax+b}}}{\sqrt{-\frac{1}{2}\,a-c+\frac{1}{2}\,\sqrt{(a+2\,c)^2-4\,ac-4\,c^2+4\,b}}}\right)}{a^4+6\,a^2b+8\,b^2+\left(a^3+4\,ab\right)\sqrt{a^2+4\,b}}-\frac{4\,\sqrt{a^2+4\,b}\,\sqrt{-2\,a-4\,c-2\,\sqrt{a^2+4\,b}}\,b^2\arctan\left(\frac{\sqrt{c+\sqrt{ax+b}}}{\sqrt{-\frac{1}{2}\,a-c-\frac{1}{2}\,\sqrt{(a+2\,c)^2-4\,ac-4\,c^2+4\,b}}}\right)}{a^4+6\,a^2b+8\,b^2-\left(a^3+4\,ab\right)\sqrt{a^2+4\,b}}+4\,a\sqrt{c+\sqrt{ax+b}}+\frac{4}{3}\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^(1/2)*(c+(a*x+b)^(1/2))^(1/2)/(x-(a*x+b)^(1/2)),x, algori
thm="giac")
```

```
[Out] 4*sqrt(a^2 + 4*b)*sqrt(-2*a - 4*c + 2*sqrt(a^2 + 4*b))*b^2*arctan(sqrt(c +
sqrt(a*x + b))/sqrt(-1/2*a - c + 1/2*sqrt((a + 2*c)^2 - 4*a*c - 4*c^2 + 4*b
)))/(a^4 + 6*a^2*b + 8*b^2 + (a^3 + 4*a*b)*sqrt(a^2 + 4*b)) - 4*sqrt(a^2 +
4*b)*sqrt(-2*a - 4*c - 2*sqrt(a^2 + 4*b))*b^2*arctan(sqrt(c + sqrt(a*x + b)
)/sqrt(-1/2*a - c - 1/2*sqrt((a + 2*c)^2 - 4*a*c - 4*c^2 + 4*b)))/(a^4 + 6*
a^2*b + 8*b^2 - (a^3 + 4*a*b)*sqrt(a^2 + 4*b)) + 4*a*sqrt(c + sqrt(a*x + b)
) + 4/3*(c + sqrt(a*x + b))^(3/2)
```

maple [B] time = 0.05, size = 919, normalized size = 2.26

$$\frac{4\left(c+\sqrt{ax+b}\right)^{\frac{3}{2}}}{3}+4a\sqrt{c+\sqrt{ax+b}}+\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2a+2\sqrt{a^2+4b}-4c}}\right)a^2}{\sqrt{-2a+2\sqrt{a^2+4b}-4c}}+\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2a+2\sqrt{a^2+4b}-4c}}\right)ac}{\sqrt{-2a+2\sqrt{a^2+4b}-4c}}-\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2a+2\sqrt{a^2+4b}-4c}}\right)a^3}{\sqrt{a^2+4b}\,\sqrt{-2a+2\sqrt{a^2+4b}-4c}}-\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2a+2\sqrt{a^2+4b}-4c}}\right)a^2c}{\sqrt{a^2+4b}\,\sqrt{-2a+2\sqrt{a^2+4b}-4c}}+\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2a+2\sqrt{a^2+4b}-4c}}\right)b}{\sqrt{-2a+2\sqrt{a^2+4b}-4c}}-\frac{12\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2a+2\sqrt{a^2+4b}-4c}}\right)ab}{\sqrt{a^2+4b}\,\sqrt{-2a+2\sqrt{a^2+4b}-4c}}-\frac{8\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2a+2\sqrt{a^2+4b}-4c}}\right)bc}{\sqrt{a^2+4b}\,\sqrt{-2a+2\sqrt{a^2+4b}-4c}}+\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2\sqrt{a^2+4b}-2a-4c}}\right)a^2}{\sqrt{-2\sqrt{a^2+4b}-2a-4c}}+\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2\sqrt{a^2+4b}-2a-4c}}\right)ac}{\sqrt{-2\sqrt{a^2+4b}-2a-4c}}+\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2\sqrt{a^2+4b}-2a-4c}}\right)a^3}{\sqrt{a^2+4b}\,\sqrt{-2\sqrt{a^2+4b}-2a-4c}}+\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2\sqrt{a^2+4b}-2a-4c}}\right)a^2c}{\sqrt{a^2+4b}\,\sqrt{-2\sqrt{a^2+4b}-2a-4c}}+\frac{4\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2\sqrt{a^2+4b}-2a-4c}}\right)b}{\sqrt{-2\sqrt{a^2+4b}-2a-4c}}+\frac{12\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2\sqrt{a^2+4b}-2a-4c}}\right)ab}{\sqrt{a^2+4b}\,\sqrt{-2\sqrt{a^2+4b}-2a-4c}}+\frac{8\arctan\left(\frac{2\sqrt{c+\sqrt{ax+b}}}{\sqrt{-2\sqrt{a^2+4b}-2a-4c}}\right)bc}{\sqrt{a^2+4b}\,\sqrt{-2\sqrt{a^2+4b}-2a-4c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)^(1/2)*(c+(a*x+b)^(1/2))^(1/2)/(x-(a*x+b)^(1/2)),x)
```

```
[Out] 4/3*(c+(a*x+b)^(1/2))^(3/2)+4*a*(c+(a*x+b)^(1/2))^(1/2)+4/(-2*a+2*(a^2+4*b)
^(1/2)-4*c)^(1/2)*arctan(2*(c+(a*x+b)^(1/2))^(1/2)/(-2*a+2*(a^2+4*b)^(1/2)-
4*c)^(1/2))*a^2+4/(-2*a+2*(a^2+4*b)^(1/2)-4*c)^(1/2)*arctan(2*(c+(a*x+b)^(1
/2))^(1/2)/(-2*a+2*(a^2+4*b)^(1/2)-4*c)^(1/2))*a*c-4/(a^2+4*b)^(1/2)/(-2*a+
2*(a^2+4*b)^(1/2)-4*c)^(1/2)*arctan(2*(c+(a*x+b)^(1/2))^(1/2)/(-2*a+2*(a^2+
4*b)^(1/2)-4*c)^(1/2))*a^3-4/(a^2+4*b)^(1/2)/(-2*a+2*(a^2+4*b)^(1/2)-4*c)^(
1/2)*arctan(2*(c+(a*x+b)^(1/2))^(1/2)/(-2*a+2*(a^2+4*b)^(1/2)-4*c)^(1/2))*a
^2*c+4/(-2*a+2*(a^2+4*b)^(1/2)-4*c)^(1/2)*arctan(2*(c+(a*x+b)^(1/2))^(1/2)/
(-2*a+2*(a^2+4*b)^(1/2)-4*c)^(1/2))*b-12/(a^2+4*b)^(1/2)/(-2*a+2*(a^2+4*b)^
(1/2)-4*c)^(1/2)*arctan(2*(c+(a*x+b)^(1/2))^(1/2)/(-2*a+2*(a^2+4*b)^(1/2)-4
*c)^(1/2))*a*b-8/(a^2+4*b)^(1/2)/(-2*a+2*(a^2+4*b)^(1/2)-4*c)^(1/2)*arctan(
2*(c+(a*x+b)^(1/2))^(1/2)/(-2*a+2*(a^2+4*b)^(1/2)-4*c)^(1/2))*b*c+4/(-2*(a^
2+4*b)^(1/2)-2*a-4*c)^(1/2)*arctan(2*(c+(a*x+b)^(1/2))^(1/2)/(-2*(a^2+4*b)^
(1/2)-2*a-4*c)^(1/2))*a^2+4/(-2*(a^2+4*b)^(1/2)-2*a-4*c)^(1/2)*arctan(2*(c+
(a*x+b)^(1/2))^(1/2)/(-2*(a^2+4*b)^(1/2)-2*a-4*c)^(1/2))*a*c+4/(a^2+4*b)^(1
/2)/(-2*(a^2+4*b)^(1/2)-2*a-4*c)^(1/2)*arctan(2*(c+(a*x+b)^(1/2))^(1/2)/(-2
*(a^2+4*b)^(1/2)-2*a-4*c)^(1/2))*a^3+4/(a^2+4*b)^(1/2)/(-2*(a^2+4*b)^(1/2)-
2*a-4*c)^(1/2)*arctan(2*(c+(a*x+b)^(1/2))^(1/2)/(-2*(a^2+4*b)^(1/2)-2*a-4*c
)^(1/2))*a^2*c+4/(-2*(a^2+4*b)^(1/2)-2*a-4*c)^(1/2)*arctan(2*(c+(a*x+b)^(1/
2))^(1/2)/(-2*(a^2+4*b)^(1/2)-2*a-4*c)^(1/2))*b+12/(a^2+4*b)^(1/2)/(-2*(a^2
+4*b)^(1/2)-2*a-4*c)^(1/2)*arctan(2*(c+(a*x+b)^(1/2))^(1/2)/(-2*(a^2+4*b)^(
1/2)-2*a-4*c)^(1/2))*a*b+8/(a^2+4*b)^(1/2)/(-2*(a^2+4*b)^(1/2)-2*a-4*c)^(1/
2)*arctan(2*(c+(a*x+b)^(1/2))^(1/2)/(-2*(a^2+4*b)^(1/2)-2*a-4*c)^(1/2))*b*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+b}\,\sqrt{c+\sqrt{ax+b}}}{x-\sqrt{ax+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^(1/2)*(c+(a*x+b)^(1/2))^(1/2)/(x-(a*x+b)^(1/2)),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(a*x + b)*sqrt(c + sqrt(a*x + b))/(x - sqrt(a*x + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+\sqrt{b+a\,x}}\,\sqrt{b+a\,x}}{x-\sqrt{b+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + (b + a*x)^(1/2))^(1/2)*(b + a*x)^(1/2))/(x - (b + a*x)^(1/2)),x)
[Out] int(((c + (b + a*x)^(1/2))^(1/2)*(b + a*x)^(1/2))/(x - (b + a*x)^(1/2)), x)
```

sympy [B] time = 123.16, size = 1095, normalized size = 2.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)**(1/2)*(c+(a*x+b)**(1/2))**(1/2)/(x-(a*x+b)**(1/2)),x)
[Out] -4*a**2*c*RootSum(_t**4*(16*a**5*c - 16*a**4*b + 16*a**4*c**2 + 128*a**3*b*
c - 128*a**2*b**2 + 128*a**2*b*c**2 + 256*a*b**2*c - 256*b**3 + 256*b**2*c*
*2) + _t**2*(-4*a**3 - 8*a**2*c - 16*a*b - 32*b*c) + 1, Lambda(_t, _t*log(8
*_t**3*a**4*c - 8*_t**3*a**3*b + 24*_t**3*a**3*c**2 + 16*_t**3*a**2*b*c + 1
6*_t**3*a**2*c**3 - 32*_t**3*a*b**2 + 96*_t**3*a*b*c**2 - 64*_t**3*b**2*c +
 64*_t**3*b*c**3 - 2*_t*a**2 - 4*_t*a*c - 4*_t*b - 4*_t*c**2 + sqrt(c + sqr
t(a*x + b))))) + 4*a**2*RootSum(_t**4*(16*a**4 + 128*a**2*b + 256*b**2) + _
t**2*(-4*a**3 - 8*a**2*c - 16*a*b - 32*b*c) + a*c - b + c**2, Lambda(_t, _t
*log(-16*_t**3*a**2 - 64*_t**3*b + 2*_t*a + 4*_t*c + sqrt(c + sqrt(a*x + b)
)))) + 4*a*b*RootSum(_t**4*(16*a**5*c - 16*a**4*b + 16*a**4*c**2 + 128*a**3
*b*c - 128*a**2*b**2 + 128*a**2*b*c**2 + 256*a*b**2*c - 256*b**3 + 256*b**2
*c**2) + _t**2*(-4*a**3 - 8*a**2*c - 16*a*b - 32*b*c) + 1, Lambda(_t, _t*lo
g(8*_t**3*a**4*c - 8*_t**3*a**3*b + 24*_t**3*a**3*c**2 + 16*_t**3*a**2*b*c
+ 16*_t**3*a**2*c**3 - 32*_t**3*a*b**2 + 96*_t**3*a*b*c**2 - 64*_t**3*b**2*
c + 64*_t**3*b*c**3 - 2*_t*a**2 - 4*_t*a*c - 4*_t*b - 4*_t*c**2 + sqrt(c +
sqrt(a*x + b))))) - 4*a*c**2*RootSum(_t**4*(16*a**5*c - 16*a**4*b + 16*a**4
*c**2 + 128*a**3*b*c - 128*a**2*b**2 + 128*a**2*b*c**2 + 256*a*b**2*c - 256
*b**3 + 256*b**2*c**2) + _t**2*(-4*a**3 - 8*a**2*c - 16*a*b - 32*b*c) + 1,
Lambda(_t, _t*log(8*_t**3*a**4*c - 8*_t**3*a**3*b + 24*_t**3*a**3*c**2 + 16
*_t**3*a**2*b*c + 16*_t**3*a**2*c**3 - 32*_t**3*a*b**2 + 96*_t**3*a*b*c**2
- 64*_t**3*b**2*c + 64*_t**3*b*c**3 - 2*_t*a**2 - 4*_t*a*c - 4*_t*b - 4*_t*
c**2 + sqrt(c + sqrt(a*x + b))))) + 4*a*c*RootSum(_t**4*(16*a**4 + 128*a**2
*b + 256*b**2) + _t**2*(-4*a**3 - 8*a**2*c - 16*a*b - 32*b*c) + a*c - b + c
**2, Lambda(_t, _t*log(-16*_t**3*a**2 - 64*_t**3*b + 2*_t*a + 4*_t*c + sqrt
(c + sqrt(a*x + b))))) + 4*a*sqrt(c + sqrt(a*x + b)) + 4*b*RootSum(_t**4*(1
6*a**4 + 128*a**2*b + 256*b**2) + _t**2*(-4*a**3 - 8*a**2*c - 16*a*b - 32*b
*c) + a*c - b + c**2, Lambda(_t, _t*log(-16*_t**3*a**2 - 64*_t**3*b + 2*_t*
a + 4*_t*c + sqrt(c + sqrt(a*x + b))))) + 4*(c + sqrt(a*x + b))**(3/2)/3
```

3.2346 $\int \frac{1+x}{(1-ax)\sqrt[4]{\frac{1-bx}{c+x}}}\,dx$

Optimal. Leaf size=411

$$-\frac{(abc+4ab+a+4b)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{\frac{1-bx}{c+x}}}{\sqrt{\frac{1-bx}{c+x}}-\sqrt{b}}\right)}{2\sqrt{2}\,a^2b^{5/4}}-\frac{(abc+4ab+a+4b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{\frac{1-bx}{c+x}}}{\sqrt{\frac{1-bx}{c+x}}+\sqrt{b}}\right)}{2\sqrt{2}\,a^2b^{5/4}}-\frac{\sqrt{2}\,(a+1)\sqrt[4]{ac+1}\ \mathrm{ta}}{a^2}$$

Rubi [A] time = 1.18, antiderivative size = 425, normalized size of antiderivative = 1.03, number of steps used = 17, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1977, 579, 584, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{(ab(c+4)+a+4b)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{\frac{1-bx}{c+x}}+\sqrt{\frac{1-bx}{c+x}}+\sqrt{b}\right)}{4\sqrt{2}\,a^2b^{5/4}}-\frac{(ab(c+4)+a+4b)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{\frac{1-bx}{c+x}}+\sqrt{\frac{1-bx}{c+x}}+\sqrt{b}\right)}{4\sqrt{2}\,a^2b^{5/4}}-\frac{(ab(c+4)+a+4b)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\frac{1-bx}{c+x}}}{\sqrt[4]{b}}\right)}{2\sqrt{2}\,a^2b^{5/4}}+\frac{(ab(c+4)+a+4b)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{\frac{1-bx}{c+x}}}{\sqrt[4]{b}}+1\right)}{2\sqrt{2}\,a^2b^{5/4}}-\frac{2(a+1)\sqrt[4]{ac+1}\tan^{-1}\left(\frac{\sqrt[4]{ac+1}\sqrt[4]{\frac{1-bx}{c+x}}}{\sqrt[4]{a-b}}\right)}{a^2\sqrt[4]{a-b}}+\frac{2(a+1)\sqrt[4]{ac+1}\tanh^{-1}\left(\frac{\sqrt[4]{ac+1}\sqrt[4]{\frac{1-bx}{c+x}}}{\sqrt[4]{a-b}}\right)}{a^2\sqrt[4]{a-b}}+\frac{(c+x)\left(\frac{1-bx}{c+x}\right)^{3/4}}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((1 - a*x)*((1 - b*x)/(c + x))^(1/4)),x]
```

```
[Out] ((c + x)*((1 - b*x)/(c + x))^(3/4))/(a*b) - (2*(1 + a)*(1 + a*c)^(1/4)*ArcT
an[((1 + a*c)^(1/4)*((1 - b*x)/(c + x))^(1/4))/(a - b)^(1/4)])/(a^2*(a - b)
^(1/4)) - ((a + 4*b + a*b*(4 + c))*ArcTan[1 - (Sqrt[2]*((1 - b*x)/(c + x))^
(1/4))/b^(1/4)])/(2*Sqrt[2]*a^2*b^(5/4)) + ((a + 4*b + a*b*(4 + c))*ArcTan[
1 + (Sqrt[2]*((1 - b*x)/(c + x))^(1/4))/b^(1/4)])/(2*Sqrt[2]*a^2*b^(5/4)) +
 (2*(1 + a)*(1 + a*c)^(1/4)*ArcTanh[((1 + a*c)^(1/4)*((1 - b*x)/(c + x))^(1
/4))/(a - b)^(1/4)])/(a^2*(a - b)^(1/4)) + ((a + 4*b + a*b*(4 + c))*Log[Sqr
t[b] - Sqrt[2]*b^(1/4)*((1 - b*x)/(c + x))^(1/4) + Sqrt[(1 - b*x)/(c + x)]]
)/(4*Sqrt[2]*a^2*b^(5/4)) - ((a + 4*b + a*b*(4 + c))*Log[Sqrt[b] + Sqrt[2]*
b^(1/4)*((1 - b*x)/(c + x))^(1/4) + Sqrt[(1 - b*x)/(c + x)]])/(4*Sqrt[2]*a^
2*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1977

```
Int[(u_)^(p_.)*(v_)^(q_.)*((g_.)*(x_))^(m_.)*(z_)^(r_.), x_Symbol] :> Int[(
g*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q*ExpandToSum[z, x]^r, x] /; F
reeQ[{g, m, p, q, r}, x] && BinomialQ[{u, v, z}, x] && EqQ[BinomialDegree[u
, x] - BinomialDegree[v, x], 0] && EqQ[BinomialDegree[u, x] - BinomialDegre
e[z, x], 0] &&  !BinomialMatchQ[{u, v, z}, x]
```

Rubi steps

$$\int \frac{1+x}{(1-ax)\sqrt[4]{\frac{1-bx}{c+x}}}\,dx = -\left((4(1+bc))\,\text{Subst}\left(\int \frac{x^2\left(1+b-(-1+c)x^4\right)}{\left(b+x^4\right)^2\left(b+x^4+a\left(-1+cx^4\right)\right)}\,dx, x, \sqrt[4]{\frac{1-bx}{c+x}}\right)\right)$$

$$= -\left((4(1+bc))\,\text{Subst}\left(\int \frac{x^2\left(1+b+(1-c)x^4\right)}{\left(b+x^4\right)^2\left(-a+b+(1+ac)x^4\right)}\,dx, x, \sqrt[4]{\frac{1-bx}{c+x}}\right)\right)$$

$$= \frac{(c+x)\left(\frac{1-bx}{c+x}\right)^{3/4}}{ab} - \frac{\text{Subst}\left(\int \frac{x^2\left((a+3b+4ab)(1+bc)-(1+ac)(1+bc)x^4\right)}{\left(b+x^4\right)\left(-a+b+(1+ac)x^4\right)}\,dx, x, \sqrt[4]{\frac{1-bx}{c+x}}\right)}{ab}$$

$$= \frac{(c+x)\left(\frac{1-bx}{c+x}\right)^{3/4}}{ab} - \frac{\text{Subst}\left(\int \left(\frac{(-a-4b-ab(4+c))x^2}{a\left(b+x^4\right)} + \frac{4(1+a)b(-1-ac)x^2}{a\left(a-b-(1+ac)x^4\right)}\right) dx, x, \sqrt[4]{\frac{1-bx}{c+x}}\right)}{ab}$$

$$= \frac{(c+x)\left(\frac{1-bx}{c+x}\right)^{3/4}}{ab} + \frac{(4(1+a)(1+ac))\,\text{Subst}\left(\int \frac{x^2}{a-b-(1+ac)x^4}\,dx, x, \sqrt[4]{\frac{1-bx}{c+x}}\right)}{a^2} + \frac{(a+4b+\ldots}{}$$

$$= \frac{(c+x)\left(\frac{1-bx}{c+x}\right)^{3/4}}{ab} + \frac{\left(2(1+a)\sqrt{1+ac}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-b}-\sqrt{1+ac}\,x^2}\,dx, x, \sqrt[4]{\frac{1-bx}{c+x}}\right)}{a^2} - \frac{(2(1+\ldots}{}$$

$$= \frac{(c+x)\left(\frac{1-bx}{c+x}\right)^{3/4}}{ab} - \frac{2(1+a)\sqrt[4]{1+ac}\,\tan^{-1}\left(\frac{\sqrt[4]{1+ac}\,\sqrt[4]{\frac{1-bx}{c+x}}}{\sqrt[4]{a-b}}\right)}{a^2\sqrt[4]{a-b}} + \frac{2(1+a)\sqrt[4]{1+ac}\,\tanh^{-1}\left(\ldots\right.}{a^2\sqrt[4]{a-b}}$$

$$= \frac{(c+x)\left(\frac{1-bx}{c+x}\right)^{3/4}}{ab} - \frac{2(1+a)\sqrt[4]{1+ac}\,\tan^{-1}\left(\frac{\sqrt[4]{1+ac}\,\sqrt[4]{\frac{1-bx}{c+x}}}{\sqrt[4]{a-b}}\right)}{a^2\sqrt[4]{a-b}} + \frac{2(1+a)\sqrt[4]{1+ac}\,\tanh^{-1}\left(\ldots\right.}{a^2\sqrt[4]{a-b}}$$

$$= \frac{(c+x)\left(\frac{1-bx}{c+x}\right)^{3/4}}{ab} - \frac{2(1+a)\sqrt[4]{1+ac}\,\tan^{-1}\left(\frac{\sqrt[4]{1+ac}\,\sqrt[4]{\frac{1-bx}{c+x}}}{\sqrt[4]{a-b}}\right)}{a^2\sqrt[4]{a-b}} - \frac{(a+4b+ab(4+c))\tan^{-1}\ldots}{2\sqrt{2}\,a^2b^{5/4}}$$

Mathematica [C] time = 0.19, size = 151, normalized size = 0.37

$$-\frac{4\left(5(a+1)\sqrt[4]{\frac{1-bx}{bc+1}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{b(c+x)}{bc+1}\right) - 5(a+1)\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(a-b)(c+x)}{(ac+1)(1-bx)}\right) + a(c+x)\sqrt[4]{\frac{1-bx}{bc+1}}\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\frac{b(c+x)}{bc+1}\right)\right)}{5a^2\sqrt[4]{\frac{1-bx}{c+x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((1 - a*x)*((1 - b*x)/(c + x))^(1/4)),x]
```

```
[Out] (-4*(5*(1 + a)*((1 - b*x)/(1 + b*c))^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4,
 (b*(c + x))/(1 + b*c)] - 5*(1 + a)*Hypergeometric2F1[1/4, 1, 5/4, ((a - b)
*(c + x))/((1 + a*c)*(1 - b*x))] + a*(c + x)*((1 - b*x)/(1 + b*c))^(1/4)*Hy
pergeometric2F1[1/4, 5/4, 9/4, (b*(c + x))/(1 + b*c)]))/(5*a^2*((1 - b*x)/(
c + x))^(1/4))
```

IntegrateAlgebraic [A] time = 1.28, size = 426, normalized size = 1.04

$$-\frac{(abc+4ab+a+4b)\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{\frac{1-bx}{c+x}}}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1-bx}{c+x}}}\right)}{2\sqrt{2}\,a^2b^{5/4}} - \frac{(abc+4ab+a+4b)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{\frac{1-bx}{c+x}}}{\sqrt{\frac{1-bx}{c+x}}+\sqrt{b}}\right)}{2\sqrt{2}\,a^2b^{5/4}} + \frac{\sqrt{2}(a+1)\sqrt[4]{ac+1}\,\tan^{-1}\left(\frac{\sqrt{b-a}-\sqrt{ac+1}\,\sqrt{\frac{1-bx}{c+x}}}{\sqrt{2}\,\sqrt[4]{b-a}\,\sqrt[4]{ac+1}\,\sqrt[4]{\frac{1-bx}{c+x}}}\right)}{a^2\sqrt[4]{b-a}} + \frac{\sqrt{2}(a+1)\sqrt[4]{ac+1}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b-a}\,\sqrt[4]{ac+1}\,\sqrt[4]{\frac{1-bx}{c+x}}}{\sqrt{ac+1}\,\sqrt{\frac{1-bx}{c+x}}+\sqrt{b-a}}\right)}{a^2\sqrt[4]{b-a}} + \frac{(bc+1)\left(\frac{1-bx}{c+x}\right)^{3/4}}{ab\left(\frac{1-bx}{c+x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((1 - a*x)*((1 - b*x)/(c + x))^(1/4)),x]
```

```
[Out] ((1 + b*c)*((1 - b*x)/(c + x))^(3/4))/(a*b*(b + (1 - b*x)/(c + x))) - ((a +
 4*b + 4*a*b + a*b*c)*ArcTan[(Sqrt[b] - Sqrt[(1 - b*x)/(c + x)])/(Sqrt[2]*b
^(1/4)*((1 - b*x)/(c + x))^(1/4))])/(2*Sqrt[2]*a^2*b^(5/4)) + (Sqrt[2]*(1 +
 a)*(1 + a*c)^(1/4)*ArcTan[(Sqrt[-a + b] - Sqrt[1 + a*c]*Sqrt[(1 - b*x)/(c
+ x)])/(Sqrt[2]*(-a + b)^(1/4)*(1 + a*c)^(1/4)*((1 - b*x)/(c + x))^(1/4))])
/(a^2*(-a + b)^(1/4)) - ((a + 4*b + 4*a*b + a*b*c)*ArcTanh[(Sqrt[2]*b^(1/4)
*((1 - b*x)/(c + x))^(1/4))/(Sqrt[b] + Sqrt[(1 - b*x)/(c + x)])])/(2*Sqrt[2
]*a^2*b^(5/4)) + (Sqrt[2]*(1 + a)*(1 + a*c)^(1/4)*ArcTanh[(Sqrt[2]*(-a + b)
^(1/4)*(1 + a*c)^(1/4)*((1 - b*x)/(c + x))^(1/4))/(Sqrt[-a + b] + Sqrt[1 +
a*c]*Sqrt[(1 - b*x)/(c + x)])])/(a^2*(-a + b)^(1/4))
```

fricas [B] time = 3.61, size = 4062, normalized size = 9.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-a*x+1)/((-b*x+1)/(c+x))^(1/4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*b*(-(a^4*b^4*c^4 + 256*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*b^4 + a^4
+ 256*(a^4 + 3*a^3 + 3*a^2 + a)*b^3 + 4*(a^4*b^3 + 4*(a^4 + a^3)*b^4)*c^3 +
 96*(a^4 + 2*a^3 + a^2)*b^2 + 6*(a^4*b^2 + 16*(a^4 + 2*a^3 + a^2)*b^4 + 8*(
a^4 + a^3)*b^3)*c^2 + 16*(a^4 + a^3)*b + 4*(a^4*b + 64*(a^4 + 3*a^3 + 3*a^2
 + a)*b^4 + 48*(a^4 + 2*a^3 + a^2)*b^3 + 12*(a^4 + a^3)*b^2)*c)/(a^8*b^5))^
(1/4)*arctan((sqrt((a^6*b^6*c^6 + 4096*(a^6 + 6*a^5 + 15*a^4 + 20*a^3 + 15*
a^2 + 6*a + 1)*b^6 + a^6 + 6144*(a^6 + 5*a^5 + 10*a^4 + 10*a^3 + 5*a^2 + a)
*b^5 + 6*(a^6*b^5 + 4*(a^6 + a^5)*b^6)*c^5 + 3840*(a^6 + 4*a^5 + 6*a^4 + 4*
a^3 + a^2)*b^4 + 15*(a^6*b^4 + 16*(a^6 + 2*a^5 + a^4)*b^6 + 8*(a^6 + a^5)*b
^5)*c^4 + 1280*(a^6 + 3*a^5 + 3*a^4 + a^3)*b^3 + 20*(a^6*b^3 + 64*(a^6 + 3*
a^5 + 3*a^4 + a^3)*b^6 + 48*(a^6 + 2*a^5 + a^4)*b^5 + 12*(a^6 + a^5)*b^4)*c
^3 + 240*(a^6 + 2*a^5 + a^4)*b^2 + 15*(a^6*b^2 + 256*(a^6 + 4*a^5 + 6*a^4 +
 4*a^3 + a^2)*b^6 + 256*(a^6 + 3*a^5 + 3*a^4 + a^3)*b^5 + 96*(a^6 + 2*a^5 +
 a^4)*b^4 + 16*(a^6 + a^5)*b^3)*c^2 + 24*(a^6 + a^5)*b + 6*(a^6*b + 1024*(a
^6 + 5*a^5 + 10*a^4 + 10*a^3 + 5*a^2 + a)*b^6 + 1280*(a^6 + 4*a^5 + 6*a^4 +
 4*a^3 + a^2)*b^5 + 640*(a^6 + 3*a^5 + 3*a^4 + a^3)*b^4 + 160*(a^6 + 2*a^5
+ a^4)*b^3 + 20*(a^6 + a^5)*b^2)*c)*sqrt(-(b*x - 1)/(c + x)) - (a^8*b^7*c^4
 + a^8*b^3 + 256*(a^8 + 4*a^7 + 6*a^6 + 4*a^5 + a^4)*b^7 + 256*(a^8 + 3*a^7
 + 3*a^6 + a^5)*b^6 + 96*(a^8 + 2*a^7 + a^6)*b^5 + 16*(a^8 + a^7)*b^4 + 4*(
a^8*b^6 + 4*(a^8 + a^7)*b^7)*c^3 + 6*(a^8*b^5 + 16*(a^8 + 2*a^7 + a^6)*b^7
+ 8*(a^8 + a^7)*b^6)*c^2 + 4*(a^8*b^4 + 64*(a^8 + 3*a^7 + 3*a^6 + a^5)*b^7
+ 48*(a^8 + 2*a^7 + a^6)*b^6 + 12*(a^8 + a^7)*b^5)*c)*sqrt(-(a^4*b^4*c^4 +
256*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*b^4 + a^4 + 256*(a^4 + 3*a^3 + 3*a^2 +
a)*b^3 + 4*(a^4*b^3 + 4*(a^4 + a^3)*b^4)*c^3 + 96*(a^4 + 2*a^3 + a^2)*b^2 +
 6*(a^4*b^2 + 16*(a^4 + 2*a^3 + a^2)*b^4 + 8*(a^4 + a^3)*b^3)*c^2 + 16*(a^4
 + a^3)*b + 4*(a^4*b + 64*(a^4 + 3*a^3 + 3*a^2 + a)*b^4 + 48*(a^4 + 2*a^3 +
 a^2)*b^3 + 12*(a^4 + a^3)*b^2)*c)/(a^8*b^5)))*a^2*b*(-(a^4*b^4*c^4 + 256*(
a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*b^4 + a^4 + 256*(a^4 + 3*a^3 + 3*a^2 + a)*b^
3 + 4*(a^4*b^3 + 4*(a^4 + a^3)*b^4)*c^3 + 96*(a^4 + 2*a^3 + a^2)*b^2 + 6*(a
^4*b^2 + 16*(a^4 + 2*a^3 + a^2)*b^4 + 8*(a^4 + a^3)*b^3)*c^2 + 16*(a^4 + a^
3)*b + 4*(a^4*b + 64*(a^4 + 3*a^3 + 3*a^2 + a)*b^4 + 48*(a^4 + 2*a^3 + a^2)
*b^3 + 12*(a^4 + a^3)*b^2)*c)/(a^8*b^5))^(1/4) - (a^5*b^4*c^3 + a^5*b + 64*
(a^5 + 3*a^4 + 3*a^3 + a^2)*b^4 + 48*(a^5 + 2*a^4 + a^3)*b^3 + 12*(a^5 + a^
4)*b^2 + 3*(a^5*b^3 + 4*(a^5 + a^4)*b^4)*c^2 + 3*(a^5*b^2 + 16*(a^5 + 2*a^4
 + a^3)*b^4 + 8*(a^5 + a^4)*b^3)*c)*(-(b*x - 1)/(c + x))^(1/4)*(-(a^4*b^4*c
^4 + 256*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*b^4 + a^4 + 256*(a^4 + 3*a^3 + 3*a
^2 + a)*b^3 + 4*(a^4*b^3 + 4*(a^4 + a^3)*b^4)*c^3 + 96*(a^4 + 2*a^3 + a^2)*
b^2 + 6*(a^4*b^2 + 16*(a^4 + 2*a^3 + a^2)*b^4 + 8*(a^4 + a^3)*b^3)*c^2 + 16
*(a^4 + a^3)*b + 4*(a^4*b + 64*(a^4 + 3*a^3 + 3*a^2 + a)*b^4 + 48*(a^4 + 2*
a^3 + a^2)*b^3 + 12*(a^4 + a^3)*b^2)*c)/(a^8*b^5))^(1/4))/(a^4*b^4*c^4 + 25
6*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*b^4 + a^4 + 256*(a^4 + 3*a^3 + 3*a^2 + a)
*b^3 + 4*(a^4*b^3 + 4*(a^4 + a^3)*b^4)*c^3 + 96*(a^4 + 2*a^3 + a^2)*b^2 + 6
```

```
*(a^4*b^2 + 16*(a^4 + 2*a^3 + a^2)*b^4 + 8*(a^4 + a^3)*b^3)*c^2 + 16*(a^4 +
 a^3)*b + 4*(a^4*b + 64*(a^4 + 3*a^3 + 3*a^2 + a)*b^4 + 48*(a^4 + 2*a^3 + a
^2)*b^3 + 12*(a^4 + a^3)*b^2)*c)) - 16*a*b*((a^4 + 4*a^3 + 6*a^2 + (a^5 + 4
*a^4 + 6*a^3 + 4*a^2 + a)*c + 4*a + 1)/(a^9 - a^8*b))^(1/4)*arctan((sqrt((a
^9 + 4*a^8 + 6*a^7 + 4*a^6 + a^5 - (a^8 + 4*a^7 + 6*a^6 + 4*a^5 + a^4)*b +
(a^10 + 4*a^9 + 6*a^8 + 4*a^7 + a^6 - (a^9 + 4*a^8 + 6*a^7 + 4*a^6 + a^5)*b
)*c)*sqrt((a^4 + 4*a^3 + 6*a^2 + (a^5 + 4*a^4 + 6*a^3 + 4*a^2 + a)*c + 4*a
+ 1)/(a^9 - a^8*b)) + (a^6 + 6*a^5 + 15*a^4 + 20*a^3 + (a^8 + 6*a^7 + 15*a^
6 + 20*a^5 + 15*a^4 + 6*a^3 + a^2)*c^2 + 15*a^2 + 2*(a^7 + 6*a^6 + 15*a^5 +
 20*a^4 + 15*a^3 + 6*a^2 + a)*c + 6*a + 1)*sqrt(-(b*x - 1)/(c + x)))*a^2*((
a^4 + 4*a^3 + 6*a^2 + (a^5 + 4*a^4 + 6*a^3 + 4*a^2 + a)*c + 4*a + 1)/(a^9 -
 a^8*b))^(1/4) - (a^5 + 3*a^4 + 3*a^3 + a^2 + (a^6 + 3*a^5 + 3*a^4 + a^3)*c
)*((a^4 + 4*a^3 + 6*a^2 + (a^5 + 4*a^4 + 6*a^3 + 4*a^2 + a)*c + 4*a + 1)/(a
^9 - a^8*b))^(1/4)*(-(b*x - 1)/(c + x))^(1/4))/(a^4 + 4*a^3 + 6*a^2 + (a^5
+ 4*a^4 + 6*a^3 + 4*a^2 + a)*c + 4*a + 1)) - a*b*(-(a^4*b^4*c^4 + 256*(a^4
+ 4*a^3 + 6*a^2 + 4*a + 1)*b^4 + a^4 + 256*(a^4 + 3*a^3 + 3*a^2 + a)*b^3 +
4*(a^4*b^3 + 4*(a^4 + a^3)*b^4)*c^3 + 96*(a^4 + 2*a^3 + a^2)*b^2 + 6*(a^4*b
^2 + 16*(a^4 + 2*a^3 + a^2)*b^4 + 8*(a^4 + a^3)*b^3)*c^2 + 16*(a^4 + a^3)*b
 + 4*(a^4*b + 64*(a^4 + 3*a^3 + 3*a^2 + a)*b^4 + 48*(a^4 + 2*a^3 + a^2)*b^3
 + 12*(a^4 + a^3)*b^2)*c)/(a^8*b^5))^(1/4)*log(a^6*b^4*(-(a^4*b^4*c^4 + 256
*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*b^4 + a^4 + 256*(a^4 + 3*a^3 + 3*a^2 + a)*
b^3 + 4*(a^4*b^3 + 4*(a^4 + a^3)*b^4)*c^3 + 96*(a^4 + 2*a^3 + a^2)*b^2 + 6*
(a^4*b^2 + 16*(a^4 + 2*a^3 + a^2)*b^4 + 8*(a^4 + a^3)*b^3)*c^2 + 16*(a^4 +
a^3)*b + 4*(a^4*b + 64*(a^4 + 3*a^3 + 3*a^2 + a)*b^4 + 48*(a^4 + 2*a^3 + a^
2)*b^3 + 12*(a^4 + a^3)*b^2)*c)/(a^8*b^5))^(3/4) + (a^3*b^3*c^3 + 64*(a^3 +
 3*a^2 + 3*a + 1)*b^3 + a^3 + 48*(a^3 + 2*a^2 + a)*b^2 + 3*(a^3*b^2 + 4*(a^
3 + a^2)*b^3)*c^2 + 12*(a^3 + a^2)*b + 3*(a^3*b + 16*(a^3 + 2*a^2 + a)*b^3
+ 8*(a^3 + a^2)*b^2)*c)*(-(b*x - 1)/(c + x))^(1/4)) + a*b*(-(a^4*b^4*c^4 +
256*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*b^4 + a^4 + 256*(a^4 + 3*a^3 + 3*a^2 +
a)*b^3 + 4*(a^4*b^3 + 4*(a^4 + a^3)*b^4)*c^3 + 96*(a^4 + 2*a^3 + a^2)*b^2 +
 6*(a^4*b^2 + 16*(a^4 + 2*a^3 + a^2)*b^4 + 8*(a^4 + a^3)*b^3)*c^2 + 16*(a^4
 + a^3)*b + 4*(a^4*b + 64*(a^4 + 3*a^3 + 3*a^2 + a)*b^4 + 48*(a^4 + 2*a^3 +
 a^2)*b^3 + 12*(a^4 + a^3)*b^2)*c)/(a^8*b^5))^(1/4)*log(-a^6*b^4*(-(a^4*b^4
*c^4 + 256*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*b^4 + a^4 + 256*(a^4 + 3*a^3 + 3
*a^2 + a)*b^3 + 4*(a^4*b^3 + 4*(a^4 + a^3)*b^4)*c^3 + 96*(a^4 + 2*a^3 + a^2
)*b^2 + 6*(a^4*b^2 + 16*(a^4 + 2*a^3 + a^2)*b^4 + 8*(a^4 + a^3)*b^3)*c^2 +
16*(a^4 + a^3)*b + 4*(a^4*b + 64*(a^4 + 3*a^3 + 3*a^2 + a)*b^4 + 48*(a^4 +
2*a^3 + a^2)*b^3 + 12*(a^4 + a^3)*b^2)*c)/(a^8*b^5))^(3/4) + (a^3*b^3*c^3 +
 64*(a^3 + 3*a^2 + 3*a + 1)*b^3 + a^3 + 48*(a^3 + 2*a^2 + a)*b^2 + 3*(a^3*b
^2 + 4*(a^3 + a^2)*b^3)*c^2 + 12*(a^3 + a^2)*b + 3*(a^3*b + 16*(a^3 + 2*a^2
 + a)*b^3 + 8*(a^3 + a^2)*b^2)*c)*(-(b*x - 1)/(c + x))^(1/4)) - 4*a*b*((a^4
 + 4*a^3 + 6*a^2 + (a^5 + 4*a^4 + 6*a^3 + 4*a^2 + a)*c + 4*a + 1)/(a^9 - a^
8*b))^(1/4)*log((a^7 - a^6*b)*((a^4 + 4*a^3 + 6*a^2 + (a^5 + 4*a^4 + 6*a^3
+ 4*a^2 + a)*c + 4*a + 1)/(a^9 - a^8*b))^(3/4) + (a^3 + 3*a^2 + (a^4 + 3*a^
3 + 3*a^2 + a)*c + 3*a + 1)*(-(b*x - 1)/(c + x))^(1/4)) + 4*a*b*((a^4 + 4*a
^3 + 6*a^2 + (a^5 + 4*a^4 + 6*a^3 + 4*a^2 + a)*c + 4*a + 1)/(a^9 - a^8*b))^
(1/4)*log(-(a^7 - a^6*b)*((a^4 + 4*a^3 + 6*a^2 + (a^5 + 4*a^4 + 6*a^3 + 4*a
^2 + a)*c + 4*a + 1)/(a^9 - a^8*b))^(3/4) + (a^3 + 3*a^2 + (a^4 + 3*a^3 + 3
*a^2 + a)*c + 3*a + 1)*(-(b*x - 1)/(c + x))^(1/4)) - 4*(c + x)*(-(b*x - 1)/
(c + x))^(3/4))/(a*b)
```

giac [B] time = 8.01, size = 1194, normalized size = 2.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-a*x+1)/((-b*x+1)/(c+x))^(1/4),x, algorithm="giac")
```

```
[Out] 1/8*(8*(-a^4*c^3 + a^3*b*c^3 - 3*a^3*c^2 + 3*a^2*b*c^2 - 3*a^2*c + 3*a*b*c
- a + b)^(3/4)*(sqrt(2)*a*b*c + sqrt(2)*b*c + sqrt(2)*a + sqrt(2))*arctan(1
```

```
/2*sqrt(2)*(sqrt(2)*(-(a - b)/(a*c + 1))^(1/4) + 2*(-(b*x - 1)/(c + x))^(1/
4))/(-(a - b)/(a*c + 1))^(1/4))/(a^5*c^2 - a^4*b*c^2 + 2*a^4*c - 2*a^3*b*c
+ a^3 - a^2*b) + 8*(-a^4*c^3 + a^3*b*c^3 - 3*a^3*c^2 + 3*a^2*b*c^2 - 3*a^2*
c + 3*a*b*c - a + b)^(3/4)*(sqrt(2)*a*b*c + sqrt(2)*b*c + sqrt(2)*a + sqrt(
2))*arctan(-1/2*sqrt(2)*(sqrt(2)*(-(a - b)/(a*c + 1))^(1/4) - 2*(-(b*x - 1)
/(c + x))^(1/4))/(-(a - b)/(a*c + 1))^(1/4))/(a^5*c^2 - a^4*b*c^2 + 2*a^4*c
 - 2*a^3*b*c + a^3 - a^2*b) - 8*(-a^4*c^3 + a^3*b*c^3 - 3*a^3*c^2 + 3*a^2*b
*c^2 - 3*a^2*c + 3*a*b*c - a + b)^(3/4)*(a*b*c + b*c + a + 1)*log(sqrt(2)*(
-(a - b)/(a*c + 1))^(1/4)*(-(b*x - 1)/(c + x))^(1/4) + sqrt(-(a - b)/(a*c +
 1)) + sqrt(-(b*x - 1)/(c + x)))/(sqrt(2)*a^5*c^2 - sqrt(2)*a^4*b*c^2 + 2*s
qrt(2)*a^4*c - 2*sqrt(2)*a^3*b*c + sqrt(2)*a^3 - sqrt(2)*a^2*b) + 8*(-a^4*c
^3 + a^3*b*c^3 - 3*a^3*c^2 + 3*a^2*b*c^2 - 3*a^2*c + 3*a*b*c - a + b)^(3/4)
*(a*b*c + b*c + a + 1)*log(-sqrt(2)*(-(a - b)/(a*c + 1))^(1/4)*(-(b*x - 1)/
(c + x))^(1/4) + sqrt(-(a - b)/(a*c + 1)) + sqrt(-(b*x - 1)/(c + x)))/(sqrt
(2)*a^5*c^2 - sqrt(2)*a^4*b*c^2 + 2*sqrt(2)*a^4*c - 2*sqrt(2)*a^3*b*c + sqr
t(2)*a^3 - sqrt(2)*a^2*b) + 2*sqrt(2)*(a*b^(11/4)*c^2 + 2*(2*b^2 + b)*a*b^(
3/4)*c + 4*b^(11/4)*c + a*(4*b + 1)*b^(3/4) + 4*b^(7/4))*arctan(1/2*sqrt(2)
*(sqrt(2)*b^(1/4) + 2*(-(b*x - 1)/(c + x))^(1/4))/b^(1/4))/(a^2*b^2) + 2*sq
rt(2)*(a*b^(11/4)*c^2 + 2*(2*b^2 + b)*a*b^(3/4)*c + 4*b^(11/4)*c + a*(4*b +
 1)*b^(3/4) + 4*b^(7/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(-(b*x -
1)/(c + x))^(1/4))/b^(1/4))/(a^2*b^2) - sqrt(2)*(a*b^(11/4)*c^2 + 2*(2*b^2
+ b)*a*b^(3/4)*c + 4*b^(11/4)*c + a*(4*b + 1)*b^(3/4) + 4*b^(7/4))*log(sqrt
(2)*b^(1/4)*(-(b*x - 1)/(c + x))^(1/4) + sqrt(b) + sqrt(-(b*x - 1)/(c + x))
)/(a^2*b^2) + sqrt(2)*(a*b^(11/4)*c^2 + 2*(2*b^2 + b)*a*b^(3/4)*c + 4*b^(11
/4)*c + a*(4*b + 1)*b^(3/4) + 4*b^(7/4))*log(-sqrt(2)*b^(1/4)*(-(b*x - 1)/(
c + x))^(1/4) + sqrt(b) + sqrt(-(b*x - 1)/(c + x)))/(a^2*b^2) + 8*(b^2*c^2*
(-(b*x - 1)/(c + x))^(3/4) + 2*b*c*(-(b*x - 1)/(c + x))^(3/4) + (-(b*x - 1)
/(c + x))^(3/4))/(a*(b - (b*x - 1)/(c + x))*b))*(b*c/(b*c + 1)^2 + 1/(b*c +
 1)^2)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{1+x}{\left(-ax+1\right)\left(\frac{-bx+1}{c+x}\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-a*x+1)/((-b*x+1)/(c+x))^(1/4),x)
```

```
[Out] int((1+x)/(-a*x+1)/((-b*x+1)/(c+x))^(1/4),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-a*x+1)/((-b*x+1)/(c+x))^(1/4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive, negative or zero?
```

mupad [B] time = 44.07, size = 132915, normalized size = 323.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 1)/((-(b*x - 1)/(c + x))^(1/4)*(a*x - 1)),x)
```

```
[Out] log((((256*a*(a - b)*(b*c + 1)^7*(10*a^3*b - 20*a*b^3 - 288*a*b^4 + 12*a^4*
b + a^4 - 80*b^4 + 25*a^2*b^2 + 12*a^2*b^3 + 72*a^3*b^2 - 336*a^2*b^4 + 96*
a^3*b^3 + 48*a^4*b^2 - 128*a^3*b^4 + 64*a^4*b^3 - 22*a^2*b^3*c + 8*a^3*b^2*
c - 180*a^2*b^4*c - 24*a^3*b^3*c + 12*a^4*b^2*c - 192*a^3*b^4*c - 64*a^4*b^
4*c + a^2*b^4*c^2 - 2*a^3*b^3*c^2 + a^4*b^2*c^2 - 52*a*b^4*c + 2*a^4*b*c))/
((-b)^(51/4)*(a*c + 1)^9) + (1024*a^2*(a - b)*(b*c + 1)^6*(-(b*x - 1)/(c +
x))^(1/4)*(((a*c + 1)*(a + 1)^4)/(a^8*(a - b)))^(1/4)*(6*a^3*b - 24*a*b^3 +
 64*a*b^4 + 8*a^4*b + a^4 + 32*b^4 + a^2*b^2 - 56*a^2*b^3 + 16*a^3*b^2 + 32
*a^2*b^4 - 32*a^3*b^3 + 16*a^4*b^2 - 14*a^2*b^3*c + 4*a^3*b^2*c + 72*a^2*b^
4*c - 16*a^3*b^3*c + 8*a^4*b^2*c + 32*a^3*b^4*c + 17*a^2*b^4*c^2 - 2*a^3*b^
3*c^2 + a^4*b^2*c^2 + 32*a^3*b^4*c^2 + 16*a^4*b^4*c^2 + 40*a*b^4*c + 2*a^4*
b*c))/((-b)^(47/4)*(a*c + 1)^9))*(((a*c + 1)*(a + 1)^4)/(a^8*(a - b)))^(3/4
) - (64*(a - b)*(b*c + 1)^6*(-(b*x - 1)/(c + x))^(1/4)*(a + 1)^2*(a + 4*b +
 4*a*b + a*b*c)^2*(64*a*b^2 - 8*a*b - 24*a^2*b - 16*a^3*b + a^3*c + a^2 + 3
2*b^2 + 32*a^2*b^2 + 40*a^2*b^2*c + 2*a^3*b*c^2 + 16*a^3*b^2*c + 9*a^2*b^2*
c^2 + 8*a^3*b^2*c^2 + a^3*b^2*c^3 + 24*a*b^2*c + 10*a^2*b*c + 8*a^3*b*c))/(
a^6*(-b)^(51/4)*(a*c + 1)^8))*(((a*c + 1)*(a + 1)^4)/(a^8*(a - b)))^(1/4) -
 (64*(a - b)*(b*c + 1)^7*(a + 1)^3*(4*a + a*c + 5)*(a + 4*b + 4*a*b + a*b*c
)^3)/(a^7*(-b)^(51/4)*(a*c + 1)^8))*(-(a^2*(4*b^3*c + 6*b^3) + a^3*(6*b^3*c
 + 4*b^3) + b^3 + a*(b^3*c + 4*b^3) + a^4*(4*b^3*c + b^3) + a^5*b^3*c)/(a^8
*b^4 - a^9*b^3))^(1/4) - log((((256*a*(a - b)*(b*c + 1)^7*(10*a^3*b - 20*a*
b^3 - 288*a*b^4 + 12*a^4*b + a^4 - 80*b^4 + 25*a^2*b^2 + 12*a^2*b^3 + 72*a^
3*b^2 - 336*a^2*b^4 + 96*a^3*b^3 + 48*a^4*b^2 - 128*a^3*b^4 + 64*a^4*b^3 -
22*a^2*b^3*c + 8*a^3*b^2*c - 180*a^2*b^4*c - 24*a^3*b^3*c + 12*a^4*b^2*c -
192*a^3*b^4*c - 64*a^4*b^4*c + a^2*b^4*c^2 - 2*a^3*b^3*c^2 + a^4*b^2*c^2 -
52*a*b^4*c + 2*a^4*b*c))/((-b)^(51/4)*(a*c + 1)^9) - (1024*a^2*(a - b)*(b*c
 + 1)^6*(-(b*x - 1)/(c + x))^(1/4)*(((a*c + 1)*(a + 1)^4)/(a^8*(a - b)))^(1
/4)*(6*a^3*b - 24*a*b^3 + 64*a*b^4 + 8*a^4*b + a^4 + 32*b^4 + a^2*b^2 - 56*
a^2*b^3 + 16*a^3*b^2 + 32*a^2*b^4 - 32*a^3*b^3 + 16*a^4*b^2 - 14*a^2*b^3*c
+ 4*a^3*b^2*c + 72*a^2*b^4*c - 16*a^3*b^3*c + 8*a^4*b^2*c + 32*a^3*b^4*c +
17*a^2*b^4*c^2 - 2*a^3*b^3*c^2 + a^4*b^2*c^2 + 32*a^3*b^4*c^2 + 16*a^4*b^4*
c^2 + 40*a*b^4*c + 2*a^4*b*c))/((-b)^(47/4)*(a*c + 1)^9))*(((a*c + 1)*(a +
1)^4)/(a^8*(a - b)))^(3/4) + (64*(a - b)*(b*c + 1)^6*(-(b*x - 1)/(c + x))^(
1/4)*(a + 1)^2*(a + 4*b + 4*a*b + a*b*c)^2*(64*a*b^2 - 8*a*b - 24*a^2*b - 1
6*a^3*b + a^3*c + a^2 + 32*b^2 + 32*a^2*b^2 + 40*a^2*b^2*c + 2*a^3*b*c^2 +
16*a^3*b^2*c + 9*a^2*b^2*c^2 + 8*a^3*b^2*c^2 + a^3*b^2*c^3 + 24*a*b^2*c + 1
0*a^2*b*c + 8*a^3*b*c))/(a^6*(-b)^(51/4)*(a*c + 1)^8))*(((a*c + 1)*(a + 1)^
4)/(a^8*(a - b)))^(1/4) - (64*(a - b)*(b*c + 1)^7*(a + 1)^3*(4*a + a*c + 5)
*(a + 4*b + 4*a*b + a*b*c)^3)/(a^7*(-b)^(51/4)*(a*c + 1)^8))*(-(4*a*b^3 + b
^3 + 6*a^2*b^3 + 4*a^3*b^3 + a^4*b^3 + 4*a^2*b^3*c + 6*a^3*b^3*c + 4*a^4*b^
3*c + a^5*b^3*c + a*b^3*c)/(a^8*b^4 - a^9*b^3))^(1/4) + 2*atan(((-(4*a*b^3
+ b^3 + 6*a^2*b^3 + 4*a^3*b^3 + a^4*b^3 + 4*a^2*b^3*c + 6*a^3*b^3*c + 4*a^4
*b^3*c + a^5*b^3*c + a*b^3*c)/(a^8*b^4 - a^9*b^3))^(1/4)*((-(4*a*b^3 + b^3
+ 6*a^2*b^3 + 4*a^3*b^3 + a^4*b^3 + 4*a^2*b^3*c + 6*a^3*b^3*c + 4*a^4*b^3*c
 + a^5*b^3*c + a*b^3*c)/(a^8*b^4 - a^9*b^3))^(3/4)*((64*(36*a^12*(-b)^(25/4
) - 4*a^13*(-b)^(21/4) + 48*a^13*(-b)^(25/4) - 60*a^11*(-b)^(29/4) - 240*a^
12*(-b)^(29/4) - 192*a^13*(-b)^(29/4) - 180*a^10*(-b)^(33/4) - 240*a^11*(-b
)^(33/4) + 192*a^12*(-b)^(33/4) + 240*a^9*(-b)^(37/4) + 256*a^13*(-b)^(33/4
) + 1200*a^10*(-b)^(37/4) + 1728*a^11*(-b)^(37/4) + 320*a^8*(-b)^(41/4) + 7
68*a^12*(-b)^(37/4) + 1152*a^9*(-b)^(41/4) + 1344*a^10*(-b)^(41/4) + 512*a^
11*(-b)^(41/4) - 144*a^13*(-b)^(29/4)*c^2 + 912*a^12*(-b)^(33/4)*c^2 + 1344
*a^13*(-b)^(33/4)*c^2 + 336*a^13*(-b)^(33/4)*c^3 - 432*a^11*(-b)^(37/4)*c^2
 - 4032*a^12*(-b)^(37/4)*c^2 - 1680*a^12*(-b)^(37/4)*c^3 - 4032*a^13*(-b)^(
37/4)*c^2 - 4624*a^10*(-b)^(41/4)*c^2 - 2688*a^13*(-b)^(37/4)*c^3 - 9408*a^
11*(-b)^(41/4)*c^2 - 504*a^13*(-b)^(37/4)*c^4 - 336*a^11*(-b)^(41/4)*c^3 -
1344*a^12*(-b)^(41/4)*c^2 + 3584*a^9*(-b)^(45/4)*c^2 + 5376*a^12*(-b)^(41/4
)*c^3 + 3584*a^13*(-b)^(41/4)*c^2 + 20160*a^10*(-b)^(45/4)*c^2 + 1848*a^12*
(-b)^(41/4)*c^4 + 6720*a^13*(-b)^(41/4)*c^3 + 8848*a^10*(-b)^(45/4)*c^3 + 3
0912*a^11*(-b)^(45/4)*c^2 + 3360*a^13*(-b)^(41/4)*c^4 + 6720*a^8*(-b)^(49/4
```

)*c^2 + 21504*a^11*(-b)^(45/4)*c^3 + 14336*a^12*(-b)^(45/4)*c^2 + 504*a^13*
(-b)^(41/4)*c^5 + 24192*a^9*(-b)^(49/4)*c^2 + 1848*a^11*(-b)^(45/4)*c^4 + 9
408*a^12*(-b)^(45/4)*c^3 - 4032*a^9*(-b)^(49/4)*c^3 + 28224*a^10*(-b)^(49/4
)*c^2 - 3360*a^12*(-b)^(45/4)*c^4 - 3584*a^13*(-b)^(45/4)*c^3 - 26880*a^10*
(-b)^(49/4)*c^3 + 10752*a^11*(-b)^(49/4)*c^2 - 1176*a^12*(-b)^(45/4)*c^5 -
6720*a^13*(-b)^(45/4)*c^4 - 10584*a^10*(-b)^(49/4)*c^4 - 44352*a^11*(-b)^(4
9/4)*c^3 - 2688*a^13*(-b)^(45/4)*c^5 - 11200*a^8*(-b)^(53/4)*c^3 - 30240*a^
11*(-b)^(49/4)*c^4 - 21504*a^12*(-b)^(49/4)*c^3 - 336*a^13*(-b)^(45/4)*c^6
- 40320*a^9*(-b)^(53/4)*c^3 - 2520*a^11*(-b)^(49/4)*c^5 - 20160*a^12*(-b)^(
49/4)*c^4 + 1120*a^9*(-b)^(53/4)*c^4 - 47040*a^10*(-b)^(53/4)*c^3 + 16800*a
^10*(-b)^(53/4)*c^4 - 17920*a^11*(-b)^(53/4)*c^3 + 336*a^12*(-b)^(49/4)*c^6
 + 4032*a^13*(-b)^(49/4)*c^5 + 8120*a^10*(-b)^(53/4)*c^5 + 33600*a^11*(-b)^
(53/4)*c^4 + 1344*a^13*(-b)^(49/4)*c^6 + 11200*a^8*(-b)^(57/4)*c^4 + 26880*
a^11*(-b)^(53/4)*c^5 + 17920*a^12*(-b)^(53/4)*c^4 + 144*a^13*(-b)^(49/4)*c^
7 + 40320*a^9*(-b)^(57/4)*c^4 + 1680*a^11*(-b)^(53/4)*c^6 + 22848*a^12*(-b)
^(53/4)*c^5 + 2240*a^9*(-b)^(57/4)*c^5 + 47040*a^10*(-b)^(57/4)*c^4 + 1344*
a^12*(-b)^(53/4)*c^6 + 3584*a^13*(-b)^(53/4)*c^5 + 17920*a^11*(-b)^(57/4)*c
^4 + 48*a^12*(-b)^(53/4)*c^7 - 1344*a^13*(-b)^(53/4)*c^6 - 3920*a^10*(-b)^(
57/4)*c^6 - 9408*a^11*(-b)^(57/4)*c^5 - 384*a^13*(-b)^(53/4)*c^7 - 6720*a^8
*(-b)^(61/4)*c^5 - 14784*a^11*(-b)^(57/4)*c^6 - 7168*a^12*(-b)^(57/4)*c^5 -
 36*a^13*(-b)^(53/4)*c^8 - 24192*a^9*(-b)^(61/4)*c^5 - 528*a^11*(-b)^(57/4)
*c^7 - 14784*a^12*(-b)^(57/4)*c^6 - 2688*a^9*(-b)^(61/4)*c^6 - 28224*a^10*(
-b)^(61/4)*c^5 - 768*a^12*(-b)^(57/4)*c^7 - 3584*a^13*(-b)^(57/4)*c^6 - 672
0*a^10*(-b)^(61/4)*c^6 - 10752*a^11*(-b)^(61/4)*c^5 - 60*a^12*(-b)^(57/4)*c
^8 + 192*a^13*(-b)^(57/4)*c^7 + 1104*a^10*(-b)^(61/4)*c^7 - 4032*a^11*(-b)^
(61/4)*c^6 + 48*a^13*(-b)^(57/4)*c^8 + 2240*a^8*(-b)^(65/4)*c^6 + 4608*a^11
*(-b)^(61/4)*c^7 + 4*a^13*(-b)^(57/4)*c^9 + 8064*a^9*(-b)^(65/4)*c^6 + 36*a
^11*(-b)^(61/4)*c^8 + 5184*a^12*(-b)^(61/4)*c^7 + 1216*a^9*(-b)^(65/4)*c^7
 + 9408*a^10*(-b)^(65/4)*c^6 + 144*a^12*(-b)^(61/4)*c^8 + 1536*a^13*(-b)^(61
/4)*c^7 + 3840*a^10*(-b)^(65/4)*c^7 + 3584*a^11*(-b)^(65/4)*c^6 + 12*a^12*(
-b)^(61/4)*c^9 - 148*a^10*(-b)^(65/4)*c^8 + 3648*a^11*(-b)^(65/4)*c^7 - 320
a^8(-b)^(69/4)*c^7 - 624*a^11*(-b)^(65/4)*c^8 + 1024*a^12*(-b)^(65/4)*c^7
 - 1152*a^9*(-b)^(69/4)*c^7 + 12*a^11*(-b)^(65/4)*c^9 - 768*a^12*(-b)^(65/4
)*c^8 - 208*a^9*(-b)^(69/4)*c^8 - 1344*a^10*(-b)^(69/4)*c^7 - 256*a^13*(-b)
^(65/4)*c^8 - 720*a^10*(-b)^(69/4)*c^8 - 512*a^11*(-b)^(69/4)*c^7 + 4*a^10*
(-b)^(69/4)*c^9 - 768*a^11*(-b)^(69/4)*c^8 - 256*a^12*(-b)^(69/4)*c^8 + 36*
a^13*(-b)^(25/4)*c - 276*a^12*(-b)^(29/4)*c - 384*a^13*(-b)^(29/4)*c + 300*
a^11*(-b)^(33/4)*c + 1536*a^12*(-b)^(33/4)*c + 1344*a^13*(-b)^(33/4)*c + 13
80*a^10*(-b)^(37/4)*c + 2304*a^11*(-b)^(37/4)*c - 576*a^12*(-b)^(37/4)*c -
1472*a^9*(-b)^(41/4)*c - 1536*a^13*(-b)^(37/4)*c - 7680*a^10*(-b)^(41/4)*c
- 11328*a^11*(-b)^(41/4)*c - 2240*a^8*(-b)^(45/4)*c - 5120*a^12*(-b)^(41/4)
*c - 8064*a^9*(-b)^(45/4)*c - 9408*a^10*(-b)^(45/4)*c - 3584*a^11*(-b)^(45/
4)*c))/(a^7*b^18 + 9*a^8*b^18*c + 36*a^9*b^18*c^2 + 84*a^10*b^18*c^3 + 126*
a^11*b^18*c^4 + 126*a^12*b^18*c^5 + 84*a^13*b^18*c^6 + 36*a^14*b^18*c^7 + 9
*a^15*b^18*c^8 + a^16*b^18*c^9) - ((-(4*a*b^3 + b^3 + 6*a^2*b^3 + 4*a^3*b^3
 + a^4*b^3 + 4*a^2*b^3*c + 6*a^3*b^3*c + 4*a^4*b^3*c + a^5*b^3*c + a*b^3*c)
/(a^8*b^4 - a^9*b^3))^(1/4)*(-(b*x - 1)/(c + x))^(1/4)*(16*a^13*(-b)^(11/2)
 - 80*a^12*(-b)^(13/2) - 80*a^11*(-b)^(15/2) - 128*a^13*(-b)^(13/2) + 400*a
^10*(-b)^(17/2) + 128*a^12*(-b)^(15/2) + 896*a^9*(-b)^(19/2) + 1152*a^11*(-
b)^(17/2) + 256*a^13*(-b)^(15/2) + 512*a^8*(-b)^(21/2) + 1920*a^10*(-b)^(19
/2) + 768*a^12*(-b)^(17/2) + 1024*a^9*(-b)^(21/2) + 1024*a^11*(-b)^(19/2) +
 512*a^10*(-b)^(21/2) + 448*a^13*(-b)^(15/2)*c^2 - 1344*a^12*(-b)^(17/2)*c^
2 - 2624*a^11*(-b)^(19/2)*c^2 - 2688*a^13*(-b)^(17/2)*c^2 + 1088*a^10*(-b)^
(21/2)*c^2 + 128*a^12*(-b)^(19/2)*c^2 - 896*a^13*(-b)^(17/2)*c^3 + 9600*a^9
*(-b)^(23/2)*c^2 + 9344*a^11*(-b)^(21/2)*c^2 + 1792*a^12*(-b)^(19/2)*c^3 +
4096*a^13*(-b)^(19/2)*c^2 + 7680*a^8*(-b)^(25/2)*c^2 + 21888*a^10*(-b)^(23/
2)*c^2 + 4096*a^11*(-b)^(21/2)*c^3 + 8704*a^12*(-b)^(21/2)*c^2 + 4480*a^13*
(-b)^(19/2)*c^3 + 15360*a^9*(-b)^(25/2)*c^2 + 3328*a^10*(-b)^(23/2)*c^3 + 1
2288*a^11*(-b)^(23/2)*c^2 + 128*a^12*(-b)^(21/2)*c^3 + 1120*a^13*(-b)^(19/2

)*c^4 - 8320*a^9*(-b)^(25/2)*c^3 + 7680*a^10*(-b)^(25/2)*c^2 - 4992*a^11*(-b)^(23/2)*c^3 - 1120*a^12*(-b)^(21/2)*c^4 - 6656*a^13*(-b)^(21/2)*c^3 - 10240*a^8*(-b)^(27/2)*c^3 - 21120*a^10*(-b)^(25/2)*c^3 - 2400*a^11*(-b)^(23/2)*c^4 - 9216*a^12*(-b)^(23/2)*c^3 - 4480*a^13*(-b)^(21/2)*c^4 - 20480*a^9*(-b)^(27/2)*c^3 - 7200*a^10*(-b)^(25/2)*c^4 - 12800*a^11*(-b)^(25/2)*c^3 + 1920*a^12*(-b)^(23/2)*c^4 - 896*a^13*(-b)^(21/2)*c^5 + 640*a^9*(-b)^(27/2)*c^4 - 10240*a^10*(-b)^(27/2)*c^3 - 3200*a^11*(-b)^(25/2)*c^4 + 7680*a^13*(-b)^(23/2)*c^4 + 7680*a^8*(-b)^(29/2)*c^4 + 5760*a^10*(-b)^(27/2)*c^4 - 1280*a^11*(-b)^(25/2)*c^5 + 5120*a^12*(-b)^(25/2)*c^4 + 2688*a^13*(-b)^(23/2)*c^5 + 15360*a^9*(-b)^(29/2)*c^4 + 5120*a^10*(-b)^(27/2)*c^5 + 5120*a^11*(-b)^(27/2)*c^4 - 5248*a^12*(-b)^(25/2)*c^5 + 448*a^13*(-b)^(23/2)*c^6 + 4224*a^9*(-b)^(29/2)*c^5 + 7680*a^10*(-b)^(29/2)*c^4 + 3968*a^11*(-b)^(27/2)*c^5 + 448*a^12*(-b)^(25/2)*c^6 - 6656*a^13*(-b)^(25/2)*c^5 - 3072*a^8*(-b)^(31/2)*c^5 + 5760*a^10*(-b)^(29/2)*c^5 + 2752*a^11*(-b)^(27/2)*c^6 - 2048*a^12*(-b)^(27/2)*c^5 - 896*a^13*(-b)^(25/2)*c^6 - 6144*a^9*(-b)^(31/2)*c^5 - 704*a^10*(-b)^(29/2)*c^6 + 1536*a^11*(-b)^(29/2)*c^5 + 5504*a^12*(-b)^(27/2)*c^6 - 128*a^13*(-b)^(25/2)*c^7 - 2944*a^9*(-b)^(31/2)*c^6 - 3072*a^10*(-b)^(31/2)*c^5 + 384*a^11*(-b)^(29/2)*c^6 - 256*a^12*(-b)^(27/2)*c^7 + 4096*a^13*(-b)^(27/2)*c^6 + 512*a^8*(-b)^(33/2)*c^6 - 4992*a^10*(-b)^(31/2)*c^6 - 1536*a^11*(-b)^(29/2)*c^7 + 1536*a^12*(-b)^(29/2)*c^6 + 128*a^13*(-b)^(27/2)*c^7 + 1024*a^9*(-b)^(33/2)*c^6 - 768*a^10*(-b)^(31/2)*c^7 - 2048*a^11*(-b)^(31/2)*c^6 - 2688*a^12*(-b)^(29/2)*c^7 + 16*a^13*(-b)^(27/2)*c^8 + 640*a^9*(-b)^(33/2)*c^7 + 512*a^10*(-b)^(33/2)*c^6 - 1664*a^11*(-b)^(31/2)*c^7 + 48*a^12*(-b)^(29/2)*c^8 - 1536*a^13*(-b)^(29/2)*c^7 + 1152*a^10*(-b)^(33/2)*c^7 + 304*a^11*(-b)^(31/2)*c^8 - 1024*a^12*(-b)^(31/2)*c^7 + 272*a^10*(-b)^(33/2)*c^8 + 512*a^11*(-b)^(33/2)*c^7 + 512*a^12*(-b)^(31/2)*c^8 + 512*a^11*(-b)^(33/2)*c^8 + 256*a^13*(-b)^(31/2)*c^8 + 256*a^12*(-b)^(33/2)*c^8 - 128*a^13*(-b)^(13/2)*c + 512*a^12*(-b)^(15/2)*c + 768*a^11*(-b)^(17/2)*c + 896*a^13*(-b)^(15/2)*c - 1536*a^10*(-b)^(19/2)*c - 384*a^12*(-b)^(17/2)*c - 4736*a^9*(-b)^(21/2)*c - 5504*a^11*(-b)^(19/2)*c - 1536*a^13*(-b)^(17/2)*c - 3072*a^8*(-b)^(23/2)*c - 10368*a^10*(-b)^(21/2)*c - 4096*a^12*(-b)^(19/2)*c - 6144*a^9*(-b)^(23/2)*c - 5632*a^11*(-b)^(21/2)*c - 3072*a^10*(-b)^(23/2)*c)*64i)/((-b)^(1/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^8*b^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a^12*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9)))*1i - (64*(-(b*x - 1)/(c + x))^(1/4)*(512*(-b)^(19/2) + a^5*(-b)^(9/2) + a^4*(-b)^(11/2) + 2*a^6*(-b)^(9/2) - 16*a^3*(-b)^(13/2) + 18*a^5*(-b)^(11/2) + a^7*(-b)^(9/2) - 144*a^2*(-b)^(15/2) - 176*a^4*(-b)^(13/2) + 49*a^6*(-b)^(11/2) - 576*a^3*(-b)^(15/2) - 560*a^5*(-b)^(13/2) + 48*a^7*(-b)^(11/2) + 2688*a^2*(-b)^(17/2) - 608*a^4*(-b)^(15/2) - 784*a^6*(-b)^(13/2) + 16*a^8*(-b)^(11/2) + 7680*a^3*(-b)^(17/2) + 448*a^5*(-b)^(15/2) - 512*a^7*(-b)^(13/2) + 7680*a^2*(-b)^(19/2) + 11520*a^4*(-b)^(17/2) + 1392*a^6*(-b)^(15/2) - 128*a^8*(-b)^(13/2) + 10240*a^3*(-b)^(19/2) + 9600*a^5*(-b)^(17/2) + 1024*a^7*(-b)^(15/2) + 7680*a^4*(-b)^(19/2) + 4224*a^6*(-b)^(17/2) + 256*a^8*(-b)^(15/2) + 3072*a^5*(-b)^(19/2) + 768*a^7*(-b)^(17/2) + 512*a^6*(-b)^(19/2) + 7680*(-b)^(23/2)*c^2 - 10240*(-b)^(25/2)*c^3 + 7680*(-b)^(27/2)*c^4 - 3072*(-b)^(29/2)*c^5 + 512*(-b)^(31/2)*c^6 + 384*a*(-b)^(17/2) + 3072*a*(-b)^(19/2) - 3072*(-b)^(21/2)*c + a^7*(-b)^(9/2)*c^2 - 35*a^6*(-b)^(11/2)*c^2 + 2*a^8*(-b)^(9/2)*c^2 + 265*a^5*(-b)^(13/2)*c^2 - 86*a^7*(-b)^(11/2)*c^2 + a^9*(-b)^(9/2)*c^2 - 851*a^4*(-b)^(15/2)*c^2 + 738*a^6*(-b)^(13/2)*c^2 - 10*a^7*(-b)^(11/2)*c^3 - 67*a^8*(-b)^(11/2)*c^2 + 2496*a^3*(-b)^(17/2)*c^2 - 2566*a^5*(-b)^(15/2)*c^2 + 224*a^6*(-b)^(13/2)*c^3 + 649*a^7*(-b)^(13/2)*c^2 - 20*a^8*(-b)^(11/2)*c^3 - 16*a^9*(-b)^(11/2)*c^2 - 5184*a^2*(-b)^(19/2)*c^2 + 10432*a^4*(-b)^(17/2)*c^2 - 1358*a^5*(-b)^(15/2)*c^3 - 1907*a^6*(-b)^(15/2)*c^2 + 592*a^7*(-b)^(13/2)*c^3 + 144*a^8*(-b)^(13/2)*c^2 - 10*a^9*(-b)^(11/2)*c^3 - 31104*a^3*(-b)^(19/2)*c^2 + 3784*a^4*(-b)^(17/2)*c^3 + 14912*a^5*(-b)^(17/2)*c^2 - 4364*a^6*(-b)^(15/2)*c^3 + 45*a^7*(-b)^(13/2)*c^4 + 1120*a^7*(-b)^(15/2)*c^2 + 512*a^8*(-b)^(13/2)*c^3 - 32*a^9*(-b)^(13/2)*c^2 + 1152*a^2*(-b)^(21/2)*c^2 - 7552*a^3*(-b)^(19/2)*c^3 - 74624*a^4*(-b)^(19/2)*c^2 + 14288*a^5*(-b)^(17/2)*c

^3 - 771*a^6*(-b)^(15/2)*c^4 + 5824*a^6*(-b)^(17/2)*c^2 - 4974*a^7*(-b)^(15/2)*c^3 + 90*a^8*(-b)^(13/2)*c^4 + 1952*a^8*(-b)^(15/2)*c^2 + 144*a^9*(-b)^(13/2)*c^3 + 6912*a^2*(-b)^(21/2)*c^3 + 23040*a^3*(-b)^(21/2)*c^2 - 36800*a^4*(-b)^(19/2)*c^3 + 3874*a^5*(-b)^(17/2)*c^4 - 91136*a^5*(-b)^(19/2)*c^2 + 19144*a^6*(-b)^(17/2)*c^3 - 2118*a^7*(-b)^(15/2)*c^4 - 5120*a^7*(-b)^(17/2)*c^2 - 2288*a^8*(-b)^(15/2)*c^3 + 45*a^9*(-b)^(13/2)*c^4 + 640*a^9*(-b)^(15/2)*c^2 + 115200*a^2*(-b)^(23/2)*c^2 + 49536*a^3*(-b)^(21/2)*c^3 - 8750*a^4*(-b)^(19/2)*c^4 + 57600*a^4*(-b)^(21/2)*c^2 - 68032*a^5*(-b)^(19/2)*c^3 + 13444*a^6*(-b)^(17/2)*c^4 - 58944*a^6*(-b)^(19/2)*c^2 - 120*a^7*(-b)^(15/2)*c^5 + 9664*a^7*(-b)^(17/2)*c^3 - 1923*a^8*(-b)^(15/2)*c^4 - 5248*a^8*(-b)^(17/2)*c^2 - 320*a^9*(-b)^(15/2)*c^3 + 44160*a^2*(-b)^(23/2)*c^3 + 11040*a^3*(-b)^(21/2)*c^4 + 153600*a^3*(-b)^(23/2)*c^2 + 137728*a^4*(-b)^(21/2)*c^3 - 36988*a^5*(-b)^(19/2)*c^4 + 63360*a^5*(-b)^(21/2)*c^2 + 1644*a^6*(-b)^(17/2)*c^5 - 56512*a^6*(-b)^(19/2)*c^3 + 17058*a^7*(-b)^(17/2)*c^4 - 18560*a^7*(-b)^(19/2)*c^2 - 240*a^8*(-b)^(15/2)*c^5 + 128*a^8*(-b)^(17/2)*c^3 - 576*a^9*(-b)^(15/2)*c^4 - 1280*a^9*(-b)^(17/2)*c^2 - 480*a^2*(-b)^(23/2)*c^4 + 76800*a^3*(-b)^(23/2)*c^3 + 60640*a^4*(-b)^(21/2)*c^4 + 115200*a^4*(-b)^(23/2)*c^2 - 6776*a^5*(-b)^(19/2)*c^5 + 193792*a^5*(-b)^(21/2)*c^3 - 59150*a^6*(-b)^(19/2)*c^4 + 33408*a^6*(-b)^(21/2)*c^2 + 4632*a^7*(-b)^(17/2)*c^5 - 16064*a^7*(-b)^(19/2)*c^3 + 9280*a^8*(-b)^(17/2)*c^4 - 2048*a^8*(-b)^(19/2)*c^2 - 120*a^9*(-b)^(15/2)*c^5 - 896*a^9*(-b)^(17/2)*c^3 - 153600*a^2*(-b)^(25/2)*c^3 - 23040*a^3*(-b)^(23/2)*c^4 + 11620*a^4*(-b)^(21/2)*c^5 + 57600*a^4*(-b)^(23/2)*c^3 + 128864*a^5*(-b)^(21/2)*c^4 + 46080*a^5*(-b)^(23/2)*c^2 - 24752*a^6*(-b)^(19/2)*c^5 + 146688*a^6*(-b)^(21/2)*c^3 + 210*a^7*(-b)^(17/2)*c^6 - 43232*a^7*(-b)^(19/2)*c^4 + 6912*a^7*(-b)^(21/2)*c^2 + 4332*a^8*(-b)^(17/2)*c^5 + 3968*a^8*(-b)^(19/2)*c^3 + 1792*a^9*(-b)^(17/2)*c^4 - 90240*a^2*(-b)^(25/2)*c^4 - 7104*a^3*(-b)^(23/2)*c^5 - 204800*a^3*(-b)^(25/2)*c^3 - 100160*a^4*(-b)^(23/2)*c^4 + 53480*a^5*(-b)^(21/2)*c^5 + 9600*a^5*(-b)^(23/2)*c^3 - 2310*a^6*(-b)^(19/2)*c^6 + 131872*a^6*(-b)^(21/2)*c^4 + 7680*a^6*(-b)^(23/2)*c^2 - 33432*a^7*(-b)^(19/2)*c^5 + 56704*a^7*(-b)^(21/2)*c^3 + 420*a^8*(-b)^(17/2)*c^6 - 13216*a^8*(-b)^(19/2)*c^4 + 1344*a^9*(-b)^(17/2)*c^5 + 2304*a^9*(-b)^(19/2)*c^3 - 9216*a^2*(-b)^(25/2)*c^5 - 192000*a^3*(-b)^(25/2)*c^4 - 48224*a^4*(-b)^(23/2)*c^5 - 153600*a^4*(-b)^(25/2)*c^3 + 7546*a^5*(-b)^(21/2)*c^6 - 179840*a^5*(-b)^(23/2)*c^4 + 94948*a^6*(-b)^(21/2)*c^5 - 9600*a^6*(-b)^(23/2)*c^3 - 6636*a^7*(-b)^(19/2)*c^6 + 63232*a^7*(-b)^(21/2)*c^4 - 19712*a^8*(-b)^(19/2)*c^5 + 8704*a^8*(-b)^(21/2)*c^3 + 210*a^9*(-b)^(17/2)*c^6 - 896*a^9*(-b)^(19/2)*c^4 + 115200*a^2*(-b)^(27/2)*c^4 - 31104*a^3*(-b)^(25/2)*c^5 - 8750*a^4*(-b)^(23/2)*c^6 - 211200*a^4*(-b)^(25/2)*c^4 - 121120*a^5*(-b)^(23/2)*c^5 - 61440*a^5*(-b)^(25/2)*c^3 + 28756*a^6*(-b)^(21/2)*c^6 - 161760*a^6*(-b)^(23/2)*c^4 - 252*a^7*(-b)^(19/2)*c^7 + 80416*a^7*(-b)^(21/2)*c^5 - 3840*a^7*(-b)^(23/2)*c^3 - 6342*a^8*(-b)^(19/2)*c^6 + 9344*a^8*(-b)^(21/2)*c^4 - 4256*a^9*(-b)^(19/2)*c^5 + 81792*a^2*(-b)^(27/2)*c^5 - 832*a^3*(-b)^(25/2)*c^6 + 153600*a^3*(-b)^(27/2)*c^4 - 23552*a^4*(-b)^(25/2)*c^5 - 44380*a^5*(-b)^(23/2)*c^6 - 124800*a^5*(-b)^(25/2)*c^4 + 2184*a^6*(-b)^(21/2)*c^7 - 146336*a^6*(-b)^(23/2)*c^5 - 10240*a^6*(-b)^(25/2)*c^3 + 40698*a^7*(-b)^(21/2)*c^6 - 72320*a^7*(-b)^(23/2)*c^4 - 504*a^8*(-b)^(19/2)*c^7 + 31808*a^8*(-b)^(21/2)*c^5 - 2016*a^9*(-b)^(19/2)*c^6 - 1280*a^9*(-b)^(21/2)*c^4 + 10944*a^2*(-b)^(27/2)*c^6 + 184320*a^3*(-b)^(27/2)*c^5 + 8896*a^4*(-b)^(25/2)*c^6 + 115200*a^4*(-b)^(27/2)*c^4 - 5300*a^5*(-b)^(23/2)*c^7 + 32512*a^5*(-b)^(25/2)*c^5 - 86702*a^6*(-b)^(23/2)*c^6 - 36480*a^6*(-b)^(25/2)*c^4 + 6384*a^7*(-b)^(21/2)*c^7 - 87968*a^7*(-b)^(23/2)*c^5 + 25312*a^8*(-b)^(21/2)*c^6 - 12800*a^8*(-b)^(23/2)*c^4 - 252*a^9*(-b)^(19/2)*c^7 + 4480*a^9*(-b)^(21/2)*c^5 - 46080*a^2*(-b)^(29/2)*c^5 + 49536*a^3*(-b)^(27/2)*c^6 + 3016*a^4*(-b)^(25/2)*c^7 + 218880*a^4*(-b)^(27/2)*c^5 + 44864*a^5*(-b)^(25/2)*c^6 + 46080*a^5*(-b)^(27/2)*c^4 - 21128*a^6*(-b)^(23/2)*c^7 + 66048*a^6*(-b)^(25/2)*c^5 + 210*a^7*(-b)^(21/2)*c^8 - 81536*a^7*(-b)^(23/2)*c^6 - 3840*a^7*(-b)^(25/2)*c^4 + 6216*a^8*(-b)^(21/2)*c^7 - 22912*a^8*(-b)^(23/2)*c^5 + 5824*a^9*(-b)^(21/2)*c^6 - 36480*a^2*(-b)^(29/2)*c^6 + 4224*a^3*(-b)^(27/2)*c^7 - 61440*a^3*(-b)^(29/2)*c^5 + 86656*a^4*(-

b)^(27/2)*c^6 + 18896*a^5*(-b)^(25/2)*c^7 + 144000*a^5*(-b)^(27/2)*c^5 - 13
74*a^6*(-b)^(23/2)*c^8 + 75200*a^6*(-b)^(25/2)*c^6 + 7680*a^6*(-b)^(27/2)*c
^4 - 31284*a^7*(-b)^(23/2)*c^7 + 40576*a^7*(-b)^(25/2)*c^5 + 420*a^8*(-b)^(
21/2)*c^8 - 36736*a^8*(-b)^(23/2)*c^6 + 2016*a^9*(-b)^(21/2)*c^7 - 1280*a^9
*(-b)^(23/2)*c^5 - 5376*a^2*(-b)^(29/2)*c^7 - 84480*a^3*(-b)^(29/2)*c^6 + 1
3888*a^4*(-b)^(27/2)*c^7 - 46080*a^4*(-b)^(29/2)*c^5 + 2173*a^5*(-b)^(25/2)
*c^8 + 70144*a^5*(-b)^(27/2)*c^6 + 42952*a^6*(-b)^(25/2)*c^7 + 49536*a^6*(-
b)^(27/2)*c^5 - 4092*a^7*(-b)^(23/2)*c^8 + 57856*a^7*(-b)^(25/2)*c^6 - 2038
4*a^8*(-b)^(23/2)*c^7 + 8704*a^8*(-b)^(25/2)*c^5 + 210*a^9*(-b)^(21/2)*c^8
 - 6272*a^9*(-b)^(23/2)*c^6 + 7680*a^2*(-b)^(31/2)*c^6 - 26496*a^3*(-b)^(29/
2)*c^7 + 301*a^4*(-b)^(27/2)*c^8 - 103680*a^4*(-b)^(29/2)*c^6 + 12608*a^5*(
-b)^(27/2)*c^7 - 18432*a^5*(-b)^(29/2)*c^5 + 9274*a^6*(-b)^(25/2)*c^8 + 216
96*a^6*(-b)^(27/2)*c^6 - 120*a^7*(-b)^(23/2)*c^9 + 45760*a^7*(-b)^(25/2)*c^
7 + 6912*a^7*(-b)^(27/2)*c^5 - 4062*a^8*(-b)^(23/2)*c^8 + 20096*a^8*(-b)^(2
5/2)*c^6 - 4928*a^9*(-b)^(23/2)*c^7 + 6528*a^2*(-b)^(31/2)*c^7 - 2448*a^3*(
-b)^(29/2)*c^8 + 10240*a^3*(-b)^(31/2)*c^6 - 52736*a^4*(-b)^(29/2)*c^7 - 15
58*a^5*(-b)^(27/2)*c^8 - 71040*a^5*(-b)^(29/2)*c^6 + 546*a^6*(-b)^(25/2)*c^
9 - 4544*a^6*(-b)^(27/2)*c^7 - 3072*a^6*(-b)^(29/2)*c^5 + 14589*a^7*(-b)^(2
5/2)*c^8 - 2432*a^7*(-b)^(27/2)*c^6 - 240*a^8*(-b)^(23/2)*c^9 + 23168*a^8*(
-b)^(25/2)*c^7 - 1344*a^9*(-b)^(23/2)*c^8 + 2304*a^9*(-b)^(25/2)*c^6 + 1008
a^2(-b)^(31/2)*c^8 + 15360*a^3*(-b)^(31/2)*c^7 - 10160*a^4*(-b)^(29/2)*c^
8 + 7680*a^4*(-b)^(31/2)*c^6 - 384*a^5*(-b)^(27/2)*c^9 - 53504*a^5*(-b)^(29
/2)*c^7 - 8099*a^6*(-b)^(27/2)*c^8 - 25728*a^6*(-b)^(29/2)*c^6 + 1668*a^7*(
-b)^(25/2)*c^9 - 13760*a^7*(-b)^(27/2)*c^7 + 10048*a^8*(-b)^(25/2)*c^8 - 20
48*a^8*(-b)^(27/2)*c^6 - 120*a^9*(-b)^(23/2)*c^9 + 4480*a^9*(-b)^(25/2)*c^7
 + 5184*a^3*(-b)^(31/2)*c^8 - 570*a^4*(-b)^(29/2)*c^9 + 19200*a^4*(-b)^(31/
2)*c^7 - 16048*a^5*(-b)^(29/2)*c^8 + 3072*a^5*(-b)^(31/2)*c^6 - 1984*a^6*(-
b)^(27/2)*c^9 - 28416*a^6*(-b)^(29/2)*c^7 + 45*a^7*(-b)^(25/2)*c^10 - 11984
a^7(-b)^(27/2)*c^8 - 3840*a^7*(-b)^(29/2)*c^6 + 1698*a^8*(-b)^(25/2)*c^9
 - 7552*a^8*(-b)^(27/2)*c^7 + 2560*a^9*(-b)^(25/2)*c^8 + 480*a^3*(-b)^(31/2)
*c^9 + 10912*a^4*(-b)^(31/2)*c^8 - 1732*a^5*(-b)^(29/2)*c^9 + 13440*a^5*(-b
)^(31/2)*c^7 - 119*a^6*(-b)^(27/2)*c^10 - 11408*a^6*(-b)^(29/2)*c^8 + 512*a
^6*(-b)^(31/2)*c^6 - 3568*a^7*(-b)^(27/2)*c^9 - 7040*a^7*(-b)^(29/2)*c^7 +
90*a^8*(-b)^(25/2)*c^10 - 7408*a^8*(-b)^(27/2)*c^8 + 576*a^9*(-b)^(25/2)*c^
9 - 1280*a^9*(-b)^(27/2)*c^7 + 2160*a^4*(-b)^(31/2)*c^9 - 35*a^5*(-b)^(29/2
)*c^10 + 11968*a^5*(-b)^(31/2)*c^8 - 1530*a^6*(-b)^(29/2)*c^9 + 4992*a^6*(-
b)^(31/2)*c^7 - 382*a^7*(-b)^(27/2)*c^10 - 2816*a^7*(-b)^(29/2)*c^8 - 2720*
a^8*(-b)^(27/2)*c^9 - 512*a^8*(-b)^(29/2)*c^7 + 45*a^9*(-b)^(25/2)*c^10 - 1
664*a^9*(-b)^(27/2)*c^8 + 129*a^4*(-b)^(31/2)*c^10 + 3856*a^5*(-b)^(31/2)*c
^9 + 10*a^6*(-b)^(29/2)*c^10 + 7152*a^6*(-b)^(31/2)*c^8 - 10*a^7*(-b)^(27/2
)*c^11 + 112*a^7*(-b)^(29/2)*c^9 + 768*a^7*(-b)^(31/2)*c^7 - 407*a^8*(-b)^(
27/2)*c^10 + 512*a^8*(-b)^(29/2)*c^8 - 752*a^9*(-b)^(27/2)*c^9 + 482*a^5*(-
b)^(31/2)*c^10 + 8*a^6*(-b)^(29/2)*c^11 + 3408*a^6*(-b)^(31/2)*c^9 + 221*a^
7*(-b)^(29/2)*c^10 + 2176*a^7*(-b)^(31/2)*c^8 - 20*a^8*(-b)^(27/2)*c^11 + 7
36*a^8*(-b)^(29/2)*c^9 - 144*a^9*(-b)^(27/2)*c^10 + 256*a^9*(-b)^(29/2)*c^8
 + 18*a^5*(-b)^(31/2)*c^11 + 673*a^6*(-b)^(31/2)*c^10 + 32*a^7*(-b)^(29/2)*
c^11 + 1488*a^7*(-b)^(31/2)*c^9 + 272*a^8*(-b)^(29/2)*c^10 + 256*a^8*(-b)^(
31/2)*c^8 - 10*a^9*(-b)^(27/2)*c^11 + 256*a^9*(-b)^(29/2)*c^9 + 52*a^6*(-b)
^(31/2)*c^11 + a^7*(-b)^(29/2)*c^12 + 416*a^7*(-b)^(31/2)*c^10 + 40*a^8*(-b
)^(29/2)*c^11 + 256*a^8*(-b)^(31/2)*c^9 + 96*a^9*(-b)^(29/2)*c^10 + a^6*(-b
)^(31/2)*c^12 + 50*a^7*(-b)^(31/2)*c^11 + 2*a^8*(-b)^(29/2)*c^12 + 96*a^8*(
-b)^(31/2)*c^10 + 16*a^9*(-b)^(29/2)*c^11 + 2*a^7*(-b)^(31/2)*c^12 + 16*a^8
*(-b)^(31/2)*c^11 + a^9*(-b)^(29/2)*c^12 + a^8*(-b)^(31/2)*c^12 - 1152*a*(-
b)^(19/2)*c - 18432*a*(-b)^(21/2)*c + 2*a^6*(-b)^(9/2)*c - 24*a^5*(-b)^(11/
2)*c + 4*a^7*(-b)^(9/2)*c + 70*a^4*(-b)^(13/2)*c - 48*a^6*(-b)^(11/2)*c + 2
a^8(-b)^(9/2)*c - 288*a^3*(-b)^(15/2)*c + 60*a^5*(-b)^(13/2)*c - 8*a^7*(-
b)^(11/2)*c + 1536*a^2*(-b)^(17/2)*c - 656*a^4*(-b)^(15/2)*c - 378*a^6*(-b)
^(13/2)*c + 32*a^8*(-b)^(11/2)*c + 8064*a^3*(-b)^(17/2)*c + 656*a^5*(-b)^(1
5/2)*c - 784*a^7*(-b)^(13/2)*c + 16*a^9*(-b)^(11/2)*c - 9600*a^2*(-b)^(19/2

)*c + 16384*a^4*(-b)^(17/2)*c + 3280*a^6*(-b)^(15/2)*c - 544*a^8*(-b)^(13/2
)*c - 30720*a^3*(-b)^(19/2)*c + 15616*a^5*(-b)^(17/2)*c + 3664*a^7*(-b)^(15
/2)*c - 128*a^9*(-b)^(13/2)*c - 1152*a*(-b)^(21/2)*c^2 - 46080*a^2*(-b)^(21
/2)*c - 49920*a^4*(-b)^(19/2)*c + 6144*a^6*(-b)^(17/2)*c + 1664*a^8*(-b)^(1
5/2)*c - 61440*a^3*(-b)^(21/2)*c - 44160*a^5*(-b)^(19/2)*c - 128*a^7*(-b)^(
17/2)*c + 256*a^9*(-b)^(15/2)*c + 46080*a*(-b)^(23/2)*c^2 - 46080*a^4*(-b)^
(21/2)*c - 20352*a^6*(-b)^(19/2)*c - 512*a^8*(-b)^(17/2)*c + 9600*a*(-b)^(2
3/2)*c^3 - 18432*a^5*(-b)^(21/2)*c - 3840*a^7*(-b)^(19/2)*c - 3072*a^6*(-b)
^(21/2)*c - 61440*a*(-b)^(25/2)*c^3 - 17280*a*(-b)^(25/2)*c^4 + 46080*a*(-b
)^(27/2)*c^4 + 14976*a*(-b)^(27/2)*c^5 - 18432*a*(-b)^(29/2)*c^5 - 6528*a*(
-b)^(29/2)*c^6 + 3072*a*(-b)^(31/2)*c^6 + 1152*a*(-b)^(31/2)*c^7))/((-b)^(1
/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^8*b^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10
*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a^12*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^1
4*b^17*c^8 + a^15*b^17*c^9))) - (-(4*a*b^3 + b^3 + 6*a^2*b^3 + 4*a^3*b^3 +
a^4*b^3 + 4*a^2*b^3*c + 6*a^3*b^3*c + 4*a^4*b^3*c + a^5*b^3*c + a*b^3*c)/(a
^8*b^4 - a^9*b^3))^(1/4)*((-(4*a*b^3 + b^3 + 6*a^2*b^3 + 4*a^3*b^3 + a^4*b^
3 + 4*a^2*b^3*c + 6*a^3*b^3*c + 4*a^4*b^3*c + a^5*b^3*c + a*b^3*c)/(a^8*b^4
 - a^9*b^3))^(3/4)*((64*(36*a^12*(-b)^(25/4) - 4*a^13*(-b)^(21/4) + 48*a^13
*(-b)^(25/4) - 60*a^11*(-b)^(29/4) - 240*a^12*(-b)^(29/4) - 192*a^13*(-b)^(
29/4) - 180*a^10*(-b)^(33/4) - 240*a^11*(-b)^(33/4) + 192*a^12*(-b)^(33/4)
+ 240*a^9*(-b)^(37/4) + 256*a^13*(-b)^(33/4) + 1200*a^10*(-b)^(37/4) + 1728
a^11(-b)^(37/4) + 320*a^8*(-b)^(41/4) + 768*a^12*(-b)^(37/4) + 1152*a^9*(
-b)^(41/4) + 1344*a^10*(-b)^(41/4) + 512*a^11*(-b)^(41/4) - 144*a^13*(-b)^(
29/4)*c^2 + 912*a^12*(-b)^(33/4)*c^2 + 1344*a^13*(-b)^(33/4)*c^2 + 336*a^13
*(-b)^(33/4)*c^3 - 432*a^11*(-b)^(37/4)*c^2 - 4032*a^12*(-b)^(37/4)*c^2 - 1
680*a^12*(-b)^(37/4)*c^3 - 4032*a^13*(-b)^(37/4)*c^2 - 4624*a^10*(-b)^(41/4
)*c^2 - 2688*a^13*(-b)^(37/4)*c^3 - 9408*a^11*(-b)^(41/4)*c^2 - 504*a^13*(-
b)^(37/4)*c^4 - 336*a^11*(-b)^(41/4)*c^3 - 1344*a^12*(-b)^(41/4)*c^2 + 3584
a^9(-b)^(45/4)*c^2 + 5376*a^12*(-b)^(41/4)*c^3 + 3584*a^13*(-b)^(41/4)*c^
2 + 20160*a^10*(-b)^(45/4)*c^2 + 1848*a^12*(-b)^(41/4)*c^4 + 6720*a^13*(-b)
^(41/4)*c^3 + 8848*a^10*(-b)^(45/4)*c^3 + 30912*a^11*(-b)^(45/4)*c^2 + 3360
a^13(-b)^(41/4)*c^4 + 6720*a^8*(-b)^(49/4)*c^2 + 21504*a^11*(-b)^(45/4)*c
^3 + 14336*a^12*(-b)^(45/4)*c^2 + 504*a^13*(-b)^(41/4)*c^5 + 24192*a^9*(-b)
^(49/4)*c^2 + 1848*a^11*(-b)^(45/4)*c^4 + 9408*a^12*(-b)^(45/4)*c^3 - 4032*
a^9*(-b)^(49/4)*c^3 + 28224*a^10*(-b)^(49/4)*c^2 - 3360*a^12*(-b)^(45/4)*c^
4 - 3584*a^13*(-b)^(45/4)*c^3 - 26880*a^10*(-b)^(49/4)*c^3 + 10752*a^11*(-b
)^(49/4)*c^2 - 1176*a^12*(-b)^(45/4)*c^5 - 6720*a^13*(-b)^(45/4)*c^4 - 1058
4*a^10*(-b)^(49/4)*c^4 - 44352*a^11*(-b)^(49/4)*c^3 - 2688*a^13*(-b)^(45/4)
*c^5 - 11200*a^8*(-b)^(53/4)*c^3 - 30240*a^11*(-b)^(49/4)*c^4 - 21504*a^12*
(-b)^(49/4)*c^3 - 336*a^13*(-b)^(45/4)*c^6 - 40320*a^9*(-b)^(53/4)*c^3 - 25
20*a^11*(-b)^(49/4)*c^5 - 20160*a^12*(-b)^(49/4)*c^4 + 1120*a^9*(-b)^(53/4)
*c^4 - 47040*a^10*(-b)^(53/4)*c^3 + 16800*a^10*(-b)^(53/4)*c^4 - 17920*a^11
*(-b)^(53/4)*c^3 + 336*a^12*(-b)^(49/4)*c^6 + 4032*a^13*(-b)^(49/4)*c^5 + 8
120*a^10*(-b)^(53/4)*c^5 + 33600*a^11*(-b)^(53/4)*c^4 + 1344*a^13*(-b)^(49/
4)*c^6 + 11200*a^8*(-b)^(57/4)*c^4 + 26880*a^11*(-b)^(53/4)*c^5 + 17920*a^1
2*(-b)^(53/4)*c^4 + 144*a^13*(-b)^(49/4)*c^7 + 40320*a^9*(-b)^(57/4)*c^4 +
1680*a^11*(-b)^(53/4)*c^6 + 22848*a^12*(-b)^(53/4)*c^5 + 2240*a^9*(-b)^(57/
4)*c^5 + 47040*a^10*(-b)^(57/4)*c^4 + 1344*a^12*(-b)^(53/4)*c^6 + 3584*a^13
*(-b)^(53/4)*c^5 + 17920*a^11*(-b)^(57/4)*c^4 + 48*a^12*(-b)^(53/4)*c^7 - 1
344*a^13*(-b)^(53/4)*c^6 - 3920*a^10*(-b)^(57/4)*c^6 - 9408*a^11*(-b)^(57/4
)*c^5 - 384*a^13*(-b)^(53/4)*c^7 - 6720*a^8*(-b)^(61/4)*c^5 - 14784*a^11*(-
b)^(57/4)*c^6 - 7168*a^12*(-b)^(57/4)*c^5 - 36*a^13*(-b)^(53/4)*c^8 - 24192
a^9(-b)^(61/4)*c^5 - 528*a^11*(-b)^(57/4)*c^7 - 14784*a^12*(-b)^(57/4)*c^
6 - 2688*a^9*(-b)^(61/4)*c^6 - 28224*a^10*(-b)^(61/4)*c^5 - 768*a^12*(-b)^(
57/4)*c^7 - 3584*a^13*(-b)^(57/4)*c^6 - 6720*a^10*(-b)^(61/4)*c^6 - 10752*a
^11*(-b)^(61/4)*c^5 - 60*a^12*(-b)^(57/4)*c^8 + 192*a^13*(-b)^(57/4)*c^7 +
1104*a^10*(-b)^(61/4)*c^7 - 4032*a^11*(-b)^(61/4)*c^6 + 48*a^13*(-b)^(57/4)
*c^8 + 2240*a^8*(-b)^(65/4)*c^6 + 4608*a^11*(-b)^(61/4)*c^7 + 4*a^13*(-b)^(
57/4)*c^9 + 8064*a^9*(-b)^(65/4)*c^6 + 36*a^11*(-b)^(61/4)*c^8 + 5184*a^12*

(-b)^(61/4)*c^7 + 1216*a^9*(-b)^(65/4)*c^7 + 9408*a^10*(-b)^(65/4)*c^6 + 14
4*a^12*(-b)^(61/4)*c^8 + 1536*a^13*(-b)^(61/4)*c^7 + 3840*a^10*(-b)^(65/4)*
c^7 + 3584*a^11*(-b)^(65/4)*c^6 + 12*a^12*(-b)^(61/4)*c^9 - 148*a^10*(-b)^(
65/4)*c^8 + 3648*a^11*(-b)^(65/4)*c^7 - 320*a^8*(-b)^(69/4)*c^7 - 624*a^11*
(-b)^(65/4)*c^8 + 1024*a^12*(-b)^(65/4)*c^7 - 1152*a^9*(-b)^(69/4)*c^7 + 12
a^11(-b)^(65/4)*c^9 - 768*a^12*(-b)^(65/4)*c^8 - 208*a^9*(-b)^(69/4)*c^8
 - 1344*a^10*(-b)^(69/4)*c^7 - 256*a^13*(-b)^(65/4)*c^8 - 720*a^10*(-b)^(69/
4)*c^8 - 512*a^11*(-b)^(69/4)*c^7 + 4*a^10*(-b)^(69/4)*c^9 - 768*a^11*(-b)^
(69/4)*c^8 - 256*a^12*(-b)^(69/4)*c^8 + 36*a^13*(-b)^(25/4)*c - 276*a^12*(-
b)^(29/4)*c - 384*a^13*(-b)^(29/4)*c + 300*a^11*(-b)^(33/4)*c + 1536*a^12*(
-b)^(33/4)*c + 1344*a^13*(-b)^(33/4)*c + 1380*a^10*(-b)^(37/4)*c + 2304*a^1
1*(-b)^(37/4)*c - 576*a^12*(-b)^(37/4)*c - 1472*a^9*(-b)^(41/4)*c - 1536*a^
13*(-b)^(37/4)*c - 7680*a^10*(-b)^(41/4)*c - 11328*a^11*(-b)^(41/4)*c - 224
0*a^8*(-b)^(45/4)*c - 5120*a^12*(-b)^(41/4)*c - 8064*a^9*(-b)^(45/4)*c - 94
08*a^10*(-b)^(45/4)*c - 3584*a^11*(-b)^(45/4)*c))/(a^7*b^18 + 9*a^8*b^18*c
 + 36*a^9*b^18*c^2 + 84*a^10*b^18*c^3 + 126*a^11*b^18*c^4 + 126*a^12*b^18*c^
5 + 84*a^13*b^18*c^6 + 36*a^14*b^18*c^7 + 9*a^15*b^18*c^8 + a^16*b^18*c^9)
 + ((-(4*a*b^3 + b^3 + 6*a^2*b^3 + 4*a^3*b^3 + a^4*b^3 + 4*a^2*b^3*c + 6*a^3
*b^3*c + 4*a^4*b^3*c + a^5*b^3*c + a*b^3*c)/(a^8*b^4 - a^9*b^3))^(1/4)*(-(b
x - 1)/(c + x))^(1/4)(16*a^13*(-b)^(11/2) - 80*a^12*(-b)^(13/2) - 80*a^11
*(-b)^(15/2) - 128*a^13*(-b)^(13/2) + 400*a^10*(-b)^(17/2) + 128*a^12*(-b)^
(15/2) + 896*a^9*(-b)^(19/2) + 1152*a^11*(-b)^(17/2) + 256*a^13*(-b)^(15/2)
 + 512*a^8*(-b)^(21/2) + 1920*a^10*(-b)^(19/2) + 768*a^12*(-b)^(17/2) + 102
4*a^9*(-b)^(21/2) + 1024*a^11*(-b)^(19/2) + 512*a^10*(-b)^(21/2) + 448*a^13
*(-b)^(15/2)*c^2 - 1344*a^12*(-b)^(17/2)*c^2 - 2624*a^11*(-b)^(19/2)*c^2 -
2688*a^13*(-b)^(17/2)*c^2 + 1088*a^10*(-b)^(21/2)*c^2 + 128*a^12*(-b)^(19/2
)*c^2 - 896*a^13*(-b)^(17/2)*c^3 + 9600*a^9*(-b)^(23/2)*c^2 + 9344*a^11*(-b
)^(21/2)*c^2 + 1792*a^12*(-b)^(19/2)*c^3 + 4096*a^13*(-b)^(19/2)*c^2 + 7680
a^8(-b)^(25/2)*c^2 + 21888*a^10*(-b)^(23/2)*c^2 + 4096*a^11*(-b)^(21/2)*c
^3 + 8704*a^12*(-b)^(21/2)*c^2 + 4480*a^13*(-b)^(19/2)*c^3 + 15360*a^9*(-b)
^(25/2)*c^2 + 3328*a^10*(-b)^(23/2)*c^3 + 12288*a^11*(-b)^(23/2)*c^2 + 128*
a^12*(-b)^(21/2)*c^3 + 1120*a^13*(-b)^(19/2)*c^4 - 8320*a^9*(-b)^(25/2)*c^3
 + 7680*a^10*(-b)^(25/2)*c^2 - 4992*a^11*(-b)^(23/2)*c^3 - 1120*a^12*(-b)^(
21/2)*c^4 - 6656*a^13*(-b)^(21/2)*c^3 - 10240*a^8*(-b)^(27/2)*c^3 - 21120*a
^10*(-b)^(25/2)*c^3 - 2400*a^11*(-b)^(23/2)*c^4 - 9216*a^12*(-b)^(23/2)*c^3
 - 4480*a^13*(-b)^(21/2)*c^4 - 20480*a^9*(-b)^(27/2)*c^3 - 7200*a^10*(-b)^(
25/2)*c^4 - 12800*a^11*(-b)^(25/2)*c^3 + 1920*a^12*(-b)^(23/2)*c^4 - 896*a^
13*(-b)^(21/2)*c^5 + 640*a^9*(-b)^(27/2)*c^4 - 10240*a^10*(-b)^(27/2)*c^3 -
 3200*a^11*(-b)^(25/2)*c^4 + 7680*a^13*(-b)^(23/2)*c^4 + 7680*a^8*(-b)^(29/
2)*c^4 + 5760*a^10*(-b)^(27/2)*c^4 - 1280*a^11*(-b)^(25/2)*c^5 + 5120*a^12*
(-b)^(25/2)*c^4 + 2688*a^13*(-b)^(23/2)*c^5 + 15360*a^9*(-b)^(29/2)*c^4 + 5
120*a^10*(-b)^(27/2)*c^5 + 5120*a^11*(-b)^(27/2)*c^4 - 5248*a^12*(-b)^(25/2
)*c^5 + 448*a^13*(-b)^(23/2)*c^6 + 4224*a^9*(-b)^(29/2)*c^5 + 7680*a^10*(-b
)^(29/2)*c^4 + 3968*a^11*(-b)^(27/2)*c^5 + 448*a^12*(-b)^(25/2)*c^6 - 6656*
a^13*(-b)^(25/2)*c^5 - 3072*a^8*(-b)^(31/2)*c^5 + 5760*a^10*(-b)^(29/2)*c^5
 + 2752*a^11*(-b)^(27/2)*c^6 - 2048*a^12*(-b)^(27/2)*c^5 - 896*a^13*(-b)^(2
5/2)*c^6 - 6144*a^9*(-b)^(31/2)*c^5 - 704*a^10*(-b)^(29/2)*c^6 + 1536*a^11*
(-b)^(29/2)*c^5 + 5504*a^12*(-b)^(27/2)*c^6 - 128*a^13*(-b)^(25/2)*c^7 - 29
44*a^9*(-b)^(31/2)*c^6 - 3072*a^10*(-b)^(31/2)*c^5 + 384*a^11*(-b)^(29/2)*c
^6 - 256*a^12*(-b)^(27/2)*c^7 + 4096*a^13*(-b)^(27/2)*c^6 + 512*a^8*(-b)^(3
3/2)*c^6 - 4992*a^10*(-b)^(31/2)*c^6 - 1536*a^11*(-b)^(29/2)*c^7 + 1536*a^1
2*(-b)^(29/2)*c^6 + 128*a^13*(-b)^(27/2)*c^7 + 1024*a^9*(-b)^(33/2)*c^6 - 7
68*a^10*(-b)^(31/2)*c^7 - 2048*a^11*(-b)^(31/2)*c^6 - 2688*a^12*(-b)^(29/2)
*c^7 + 16*a^13*(-b)^(27/2)*c^8 + 640*a^9*(-b)^(33/2)*c^7 + 512*a^10*(-b)^(3
3/2)*c^6 - 1664*a^11*(-b)^(31/2)*c^7 + 48*a^12*(-b)^(29/2)*c^8 - 1536*a^13*
(-b)^(29/2)*c^7 + 1152*a^10*(-b)^(33/2)*c^7 + 304*a^11*(-b)^(31/2)*c^8 - 10
24*a^12*(-b)^(31/2)*c^7 + 272*a^10*(-b)^(33/2)*c^8 + 512*a^11*(-b)^(33/2)*c
^7 + 512*a^12*(-b)^(31/2)*c^8 + 512*a^11*(-b)^(33/2)*c^8 + 256*a^13*(-b)^(3
1/2)*c^8 + 256*a^12*(-b)^(33/2)*c^8 - 128*a^13*(-b)^(13/2)*c + 512*a^12*(-b

)^(15/2)*c + 768*a^11*(-b)^(17/2)*c + 896*a^13*(-b)^(15/2)*c - 1536*a^10*(-
b)^(19/2)*c - 384*a^12*(-b)^(17/2)*c - 4736*a^9*(-b)^(21/2)*c - 5504*a^11*(
-b)^(19/2)*c - 1536*a^13*(-b)^(17/2)*c - 3072*a^8*(-b)^(23/2)*c - 10368*a^1
0*(-b)^(21/2)*c - 4096*a^12*(-b)^(19/2)*c - 6144*a^9*(-b)^(23/2)*c - 5632*a
^11*(-b)^(21/2)*c - 3072*a^10*(-b)^(23/2)*c)*64i)/((-b)^(1/4)*(a^6*b^17 + 9
*a^7*b^17*c + 36*a^8*b^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a
^11*b^17*c^5 + 84*a^12*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15
*b^17*c^9)))*1i + (64*(-(b*x - 1)/(c + x))^(1/4)*(512*(-b)^(19/2) + a^5*(-b
)^(9/2) + a^4*(-b)^(11/2) + 2*a^6*(-b)^(9/2) - 16*a^3*(-b)^(13/2) + 18*a^5*
(-b)^(11/2) + a^7*(-b)^(9/2) - 144*a^2*(-b)^(15/2) - 176*a^4*(-b)^(13/2) +
49*a^6*(-b)^(11/2) - 576*a^3*(-b)^(15/2) - 560*a^5*(-b)^(13/2) + 48*a^7*(-b
)^(11/2) + 2688*a^2*(-b)^(17/2) - 608*a^4*(-b)^(15/2) - 784*a^6*(-b)^(13/2)
 + 16*a^8*(-b)^(11/2) + 7680*a^3*(-b)^(17/2) + 448*a^5*(-b)^(15/2) - 512*a^
7*(-b)^(13/2) + 7680*a^2*(-b)^(19/2) + 11520*a^4*(-b)^(17/2) + 1392*a^6*(-b
)^(15/2) - 128*a^8*(-b)^(13/2) + 10240*a^3*(-b)^(19/2) + 9600*a^5*(-b)^(17/
2) + 1024*a^7*(-b)^(15/2) + 7680*a^4*(-b)^(19/2) + 4224*a^6*(-b)^(17/2) + 2
56*a^8*(-b)^(15/2) + 3072*a^5*(-b)^(19/2) + 768*a^7*(-b)^(17/2) + 512*a^6*(
-b)^(19/2) + 7680*(-b)^(23/2)*c^2 - 10240*(-b)^(25/2)*c^3 + 7680*(-b)^(27/2
)*c^4 - 3072*(-b)^(29/2)*c^5 + 512*(-b)^(31/2)*c^6 + 384*a*(-b)^(17/2) + 30
72*a*(-b)^(19/2) - 3072*(-b)^(21/2)*c + a^7*(-b)^(9/2)*c^2 - 35*a^6*(-b)^(1
1/2)*c^2 + 2*a^8*(-b)^(9/2)*c^2 + 265*a^5*(-b)^(13/2)*c^2 - 86*a^7*(-b)^(11
/2)*c^2 + a^9*(-b)^(9/2)*c^2 - 851*a^4*(-b)^(15/2)*c^2 + 738*a^6*(-b)^(13/2
)*c^2 - 10*a^7*(-b)^(11/2)*c^3 - 67*a^8*(-b)^(11/2)*c^2 + 2496*a^3*(-b)^(17
/2)*c^2 - 2566*a^5*(-b)^(15/2)*c^2 + 224*a^6*(-b)^(13/2)*c^3 + 649*a^7*(-b)
^(13/2)*c^2 - 20*a^8*(-b)^(11/2)*c^3 - 16*a^9*(-b)^(11/2)*c^2 - 5184*a^2*(-
b)^(19/2)*c^2 + 10432*a^4*(-b)^(17/2)*c^2 - 1358*a^5*(-b)^(15/2)*c^3 - 1907
a^6(-b)^(15/2)*c^2 + 592*a^7*(-b)^(13/2)*c^3 + 144*a^8*(-b)^(13/2)*c^2 -
10*a^9*(-b)^(11/2)*c^3 - 31104*a^3*(-b)^(19/2)*c^2 + 3784*a^4*(-b)^(17/2)*c
^3 + 14912*a^5*(-b)^(17/2)*c^2 - 4364*a^6*(-b)^(15/2)*c^3 + 45*a^7*(-b)^(13
/2)*c^4 + 1120*a^7*(-b)^(15/2)*c^2 + 512*a^8*(-b)^(13/2)*c^3 - 32*a^9*(-b)^
(13/2)*c^2 + 1152*a^2*(-b)^(21/2)*c^2 - 7552*a^3*(-b)^(19/2)*c^3 - 74624*a^
4*(-b)^(19/2)*c^2 + 14288*a^5*(-b)^(17/2)*c^3 - 771*a^6*(-b)^(15/2)*c^4 + 5
824*a^6*(-b)^(17/2)*c^2 - 4974*a^7*(-b)^(15/2)*c^3 + 90*a^8*(-b)^(13/2)*c^4
 + 1952*a^8*(-b)^(15/2)*c^2 + 144*a^9*(-b)^(13/2)*c^3 + 6912*a^2*(-b)^(21/2
)*c^3 + 23040*a^3*(-b)^(21/2)*c^2 - 36800*a^4*(-b)^(19/2)*c^3 + 3874*a^5*(-
b)^(17/2)*c^4 - 91136*a^5*(-b)^(19/2)*c^2 + 19144*a^6*(-b)^(17/2)*c^3 - 211
8*a^7*(-b)^(15/2)*c^4 - 5120*a^7*(-b)^(17/2)*c^2 - 2288*a^8*(-b)^(15/2)*c^3
 + 45*a^9*(-b)^(13/2)*c^4 + 640*a^9*(-b)^(15/2)*c^2 + 115200*a^2*(-b)^(23/2
)*c^2 + 49536*a^3*(-b)^(21/2)*c^3 - 8750*a^4*(-b)^(19/2)*c^4 + 57600*a^4*(-
b)^(21/2)*c^2 - 68032*a^5*(-b)^(19/2)*c^3 + 13444*a^6*(-b)^(17/2)*c^4 - 589
44*a^6*(-b)^(19/2)*c^2 - 120*a^7*(-b)^(15/2)*c^5 + 9664*a^7*(-b)^(17/2)*c^3
 - 1923*a^8*(-b)^(15/2)*c^4 - 5248*a^8*(-b)^(17/2)*c^2 - 320*a^9*(-b)^(15/2
)*c^3 + 44160*a^2*(-b)^(23/2)*c^3 + 11040*a^3*(-b)^(21/2)*c^4 + 153600*a^3*
(-b)^(23/2)*c^2 + 137728*a^4*(-b)^(21/2)*c^3 - 36988*a^5*(-b)^(19/2)*c^4 +
63360*a^5*(-b)^(21/2)*c^2 + 1644*a^6*(-b)^(17/2)*c^5 - 56512*a^6*(-b)^(19/2
)*c^3 + 17058*a^7*(-b)^(17/2)*c^4 - 18560*a^7*(-b)^(19/2)*c^2 - 240*a^8*(-b
)^(15/2)*c^5 + 128*a^8*(-b)^(17/2)*c^3 - 576*a^9*(-b)^(15/2)*c^4 - 1280*a^9
*(-b)^(17/2)*c^2 - 480*a^2*(-b)^(23/2)*c^4 + 76800*a^3*(-b)^(23/2)*c^3 + 60
640*a^4*(-b)^(21/2)*c^4 + 115200*a^4*(-b)^(23/2)*c^2 - 6776*a^5*(-b)^(19/2)
*c^5 + 193792*a^5*(-b)^(21/2)*c^3 - 59150*a^6*(-b)^(19/2)*c^4 + 33408*a^6*(
-b)^(21/2)*c^2 + 4632*a^7*(-b)^(17/2)*c^5 - 16064*a^7*(-b)^(19/2)*c^3 + 928
0*a^8*(-b)^(17/2)*c^4 - 2048*a^8*(-b)^(19/2)*c^2 - 120*a^9*(-b)^(15/2)*c^5
 - 896*a^9*(-b)^(17/2)*c^3 - 153600*a^2*(-b)^(25/2)*c^3 - 23040*a^3*(-b)^(23
/2)*c^4 + 11620*a^4*(-b)^(21/2)*c^5 + 57600*a^4*(-b)^(23/2)*c^3 + 128864*a^
5*(-b)^(21/2)*c^4 + 46080*a^5*(-b)^(23/2)*c^2 - 24752*a^6*(-b)^(19/2)*c^5 +
 146688*a^6*(-b)^(21/2)*c^3 + 210*a^7*(-b)^(17/2)*c^6 - 43232*a^7*(-b)^(19/
2)*c^4 + 6912*a^7*(-b)^(21/2)*c^2 + 4332*a^8*(-b)^(17/2)*c^5 + 3968*a^8*(-b
)^(19/2)*c^3 + 1792*a^9*(-b)^(17/2)*c^4 - 90240*a^2*(-b)^(25/2)*c^4 - 7104*
a^3*(-b)^(23/2)*c^5 - 204800*a^3*(-b)^(25/2)*c^3 - 100160*a^4*(-b)^(23/2)*c

^4 + 53480*a^5*(-b)^(21/2)*c^5 + 9600*a^5*(-b)^(23/2)*c^3 - 2310*a^6*(-b)^(
19/2)*c^6 + 131872*a^6*(-b)^(21/2)*c^4 + 7680*a^6*(-b)^(23/2)*c^2 - 33432*a
^7*(-b)^(19/2)*c^5 + 56704*a^7*(-b)^(21/2)*c^3 + 420*a^8*(-b)^(17/2)*c^6 -
13216*a^8*(-b)^(19/2)*c^4 + 1344*a^9*(-b)^(17/2)*c^5 + 2304*a^9*(-b)^(19/2)
*c^3 - 9216*a^2*(-b)^(25/2)*c^5 - 192000*a^3*(-b)^(25/2)*c^4 - 48224*a^4*(-
b)^(23/2)*c^5 - 153600*a^4*(-b)^(25/2)*c^3 + 7546*a^5*(-b)^(21/2)*c^6 - 179
840*a^5*(-b)^(23/2)*c^4 + 94948*a^6*(-b)^(21/2)*c^5 - 9600*a^6*(-b)^(23/2)*
c^3 - 6636*a^7*(-b)^(19/2)*c^6 + 63232*a^7*(-b)^(21/2)*c^4 - 19712*a^8*(-b)
^(19/2)*c^5 + 8704*a^8*(-b)^(21/2)*c^3 + 210*a^9*(-b)^(17/2)*c^6 - 896*a^9*
(-b)^(19/2)*c^4 + 115200*a^2*(-b)^(27/2)*c^4 - 31104*a^3*(-b)^(25/2)*c^5 -
8750*a^4*(-b)^(23/2)*c^6 - 211200*a^4*(-b)^(25/2)*c^4 - 121120*a^5*(-b)^(23
/2)*c^5 - 61440*a^5*(-b)^(25/2)*c^3 + 28756*a^6*(-b)^(21/2)*c^6 - 161760*a^
6*(-b)^(23/2)*c^4 - 252*a^7*(-b)^(19/2)*c^7 + 80416*a^7*(-b)^(21/2)*c^5 - 3
840*a^7*(-b)^(23/2)*c^3 - 6342*a^8*(-b)^(19/2)*c^6 + 9344*a^8*(-b)^(21/2)*c
^4 - 4256*a^9*(-b)^(19/2)*c^5 + 81792*a^2*(-b)^(27/2)*c^5 - 832*a^3*(-b)^(2
5/2)*c^6 + 153600*a^3*(-b)^(27/2)*c^4 - 23552*a^4*(-b)^(25/2)*c^5 - 44380*a
^5*(-b)^(23/2)*c^6 - 124800*a^5*(-b)^(25/2)*c^4 + 2184*a^6*(-b)^(21/2)*c^7
 - 146336*a^6*(-b)^(23/2)*c^5 - 10240*a^6*(-b)^(25/2)*c^3 + 40698*a^7*(-b)^(
21/2)*c^6 - 72320*a^7*(-b)^(23/2)*c^4 - 504*a^8*(-b)^(19/2)*c^7 + 31808*a^8
*(-b)^(21/2)*c^5 - 2016*a^9*(-b)^(19/2)*c^6 - 1280*a^9*(-b)^(21/2)*c^4 + 10
944*a^2*(-b)^(27/2)*c^6 + 184320*a^3*(-b)^(27/2)*c^5 + 8896*a^4*(-b)^(25/2)
*c^6 + 115200*a^4*(-b)^(27/2)*c^4 - 5300*a^5*(-b)^(23/2)*c^7 + 32512*a^5*(-
b)^(25/2)*c^5 - 86702*a^6*(-b)^(23/2)*c^6 - 36480*a^6*(-b)^(25/2)*c^4 + 638
4*a^7*(-b)^(21/2)*c^7 - 87968*a^7*(-b)^(23/2)*c^5 + 25312*a^8*(-b)^(21/2)*c
^6 - 12800*a^8*(-b)^(23/2)*c^4 - 252*a^9*(-b)^(19/2)*c^7 + 4480*a^9*(-b)^(2
1/2)*c^5 - 46080*a^2*(-b)^(29/2)*c^5 + 49536*a^3*(-b)^(27/2)*c^6 + 3016*a^4
*(-b)^(25/2)*c^7 + 218880*a^4*(-b)^(27/2)*c^5 + 44864*a^5*(-b)^(25/2)*c^6 +
 46080*a^5*(-b)^(27/2)*c^4 - 21128*a^6*(-b)^(23/2)*c^7 + 66048*a^6*(-b)^(25
/2)*c^5 + 210*a^7*(-b)^(21/2)*c^8 - 81536*a^7*(-b)^(23/2)*c^6 - 3840*a^7*(-
b)^(25/2)*c^4 + 6216*a^8*(-b)^(21/2)*c^7 - 22912*a^8*(-b)^(23/2)*c^5 + 5824
a^9(-b)^(21/2)*c^6 - 36480*a^2*(-b)^(29/2)*c^6 + 4224*a^3*(-b)^(27/2)*c^7
 - 61440*a^3*(-b)^(29/2)*c^5 + 86656*a^4*(-b)^(27/2)*c^6 + 18896*a^5*(-b)^(
25/2)*c^7 + 144000*a^5*(-b)^(27/2)*c^5 - 1374*a^6*(-b)^(23/2)*c^8 + 75200*a
^6*(-b)^(25/2)*c^6 + 7680*a^6*(-b)^(27/2)*c^4 - 31284*a^7*(-b)^(23/2)*c^7 +
 40576*a^7*(-b)^(25/2)*c^5 + 420*a^8*(-b)^(21/2)*c^8 - 36736*a^8*(-b)^(23/2
)*c^6 + 2016*a^9*(-b)^(21/2)*c^7 - 1280*a^9*(-b)^(23/2)*c^5 - 5376*a^2*(-b)
^(29/2)*c^7 - 84480*a^3*(-b)^(29/2)*c^6 + 13888*a^4*(-b)^(27/2)*c^7 - 46080
a^4(-b)^(29/2)*c^5 + 2173*a^5*(-b)^(25/2)*c^8 + 70144*a^5*(-b)^(27/2)*c^6
 + 42952*a^6*(-b)^(25/2)*c^7 + 49536*a^6*(-b)^(27/2)*c^5 - 4092*a^7*(-b)^(2
3/2)*c^8 + 57856*a^7*(-b)^(25/2)*c^6 - 20384*a^8*(-b)^(23/2)*c^7 + 8704*a^8
*(-b)^(25/2)*c^5 + 210*a^9*(-b)^(21/2)*c^8 - 6272*a^9*(-b)^(23/2)*c^6 + 768
0*a^2*(-b)^(31/2)*c^6 - 26496*a^3*(-b)^(29/2)*c^7 + 301*a^4*(-b)^(27/2)*c^8
 - 103680*a^4*(-b)^(29/2)*c^6 + 12608*a^5*(-b)^(27/2)*c^7 - 18432*a^5*(-b)^
(29/2)*c^5 + 9274*a^6*(-b)^(25/2)*c^8 + 21696*a^6*(-b)^(27/2)*c^6 - 120*a^7
*(-b)^(23/2)*c^9 + 45760*a^7*(-b)^(25/2)*c^7 + 6912*a^7*(-b)^(27/2)*c^5 - 4
062*a^8*(-b)^(23/2)*c^8 + 20096*a^8*(-b)^(25/2)*c^6 - 4928*a^9*(-b)^(23/2)*
c^7 + 6528*a^2*(-b)^(31/2)*c^7 - 2448*a^3*(-b)^(29/2)*c^8 + 10240*a^3*(-b)^
(31/2)*c^6 - 52736*a^4*(-b)^(29/2)*c^7 - 1558*a^5*(-b)^(27/2)*c^8 - 71040*a
^5*(-b)^(29/2)*c^6 + 546*a^6*(-b)^(25/2)*c^9 - 4544*a^6*(-b)^(27/2)*c^7 - 3
072*a^6*(-b)^(29/2)*c^5 + 14589*a^7*(-b)^(25/2)*c^8 - 2432*a^7*(-b)^(27/2)*
c^6 - 240*a^8*(-b)^(23/2)*c^9 + 23168*a^8*(-b)^(25/2)*c^7 - 1344*a^9*(-b)^(
23/2)*c^8 + 2304*a^9*(-b)^(25/2)*c^6 + 1008*a^2*(-b)^(31/2)*c^8 + 15360*a^3
*(-b)^(31/2)*c^7 - 10160*a^4*(-b)^(29/2)*c^8 + 7680*a^4*(-b)^(31/2)*c^6 - 3
84*a^5*(-b)^(27/2)*c^9 - 53504*a^5*(-b)^(29/2)*c^7 - 8099*a^6*(-b)^(27/2)*c
^8 - 25728*a^6*(-b)^(29/2)*c^6 + 1668*a^7*(-b)^(25/2)*c^9 - 13760*a^7*(-b)^
(27/2)*c^7 + 10048*a^8*(-b)^(25/2)*c^8 - 2048*a^8*(-b)^(27/2)*c^6 - 120*a^9
*(-b)^(23/2)*c^9 + 4480*a^9*(-b)^(25/2)*c^7 + 5184*a^3*(-b)^(31/2)*c^8 - 57
0*a^4*(-b)^(29/2)*c^9 + 19200*a^4*(-b)^(31/2)*c^7 - 16048*a^5*(-b)^(29/2)*c
^8 + 3072*a^5*(-b)^(31/2)*c^6 - 1984*a^6*(-b)^(27/2)*c^9 - 28416*a^6*(-b)^(

29/2)*c^7 + 45*a^7*(-b)^(25/2)*c^10 - 11984*a^7*(-b)^(27/2)*c^8 - 3840*a^7*(-b)^(29/2)*c^6 + 1698*a^8*(-b)^(25/2)*c^9 - 7552*a^8*(-b)^(27/2)*c^7 + 2560*a^9*(-b)^(25/2)*c^8 + 480*a^3*(-b)^(31/2)*c^9 + 10912*a^4*(-b)^(31/2)*c^8 - 1732*a^5*(-b)^(29/2)*c^9 + 13440*a^5*(-b)^(31/2)*c^7 - 119*a^6*(-b)^(27/2)*c^10 - 11408*a^6*(-b)^(29/2)*c^8 + 512*a^6*(-b)^(31/2)*c^6 - 3568*a^7*(-b)^(27/2)*c^9 - 7040*a^7*(-b)^(29/2)*c^7 + 90*a^8*(-b)^(25/2)*c^10 - 7408*a^8*(-b)^(27/2)*c^8 + 576*a^9*(-b)^(25/2)*c^9 - 1280*a^9*(-b)^(27/2)*c^7 + 2160*a^4*(-b)^(31/2)*c^9 - 35*a^5*(-b)^(29/2)*c^10 + 11968*a^5*(-b)^(31/2)*c^8 - 1530*a^6*(-b)^(29/2)*c^9 + 4992*a^6*(-b)^(31/2)*c^7 - 382*a^7*(-b)^(27/2)*c^10 - 2816*a^7*(-b)^(29/2)*c^8 - 2720*a^8*(-b)^(27/2)*c^9 - 512*a^8*(-b)^(29/2)*c^7 + 45*a^9*(-b)^(25/2)*c^10 - 1664*a^9*(-b)^(27/2)*c^8 + 129*a^4*(-b)^(31/2)*c^10 + 3856*a^5*(-b)^(31/2)*c^9 + 10*a^6*(-b)^(29/2)*c^10 + 7152*a^6*(-b)^(31/2)*c^8 - 10*a^7*(-b)^(27/2)*c^11 + 112*a^7*(-b)^(29/2)*c^9 + 768*a^7*(-b)^(31/2)*c^7 - 407*a^8*(-b)^(27/2)*c^10 + 512*a^8*(-b)^(29/2)*c^8 - 752*a^9*(-b)^(27/2)*c^9 + 482*a^5*(-b)^(31/2)*c^10 + 8*a^6*(-b)^(29/2)*c^11 + 3408*a^6*(-b)^(31/2)*c^9 + 221*a^7*(-b)^(29/2)*c^10 + 2176*a^7*(-b)^(31/2)*c^8 - 20*a^8*(-b)^(27/2)*c^11 + 736*a^8*(-b)^(29/2)*c^9 - 144*a^9*(-b)^(27/2)*c^10 + 256*a^9*(-b)^(29/2)*c^8 + 18*a^5*(-b)^(31/2)*c^11 + 673*a^6*(-b)^(31/2)*c^10 + 32*a^7*(-b)^(29/2)*c^11 + 1488*a^7*(-b)^(31/2)*c^9 + 272*a^8*(-b)^(29/2)*c^10 + 256*a^8*(-b)^(31/2)*c^8 - 10*a^9*(-b)^(27/2)*c^11 + 256*a^9*(-b)^(29/2)*c^9 + 52*a^6*(-b)^(31/2)*c^11 + a^7*(-b)^(29/2)*c^12 + 416*a^7*(-b)^(31/2)*c^10 + 40*a^8*(-b)^(29/2)*c^11 + 256*a^8*(-b)^(31/2)*c^9 + 96*a^9*(-b)^(29/2)*c^10 + a^6*(-b)^(31/2)*c^12 + 50*a^7*(-b)^(31/2)*c^11 + 2*a^8*(-b)^(29/2)*c^12 + 96*a^8*(-b)^(31/2)*c^10 + 16*a^9*(-b)^(29/2)*c^11 + 2*a^7*(-b)^(31/2)*c^12 + 16*a^8*(-b)^(31/2)*c^11 + a^9*(-b)^(29/2)*c^12 + a^8*(-b)^(31/2)*c^12 - 1152*a*(-b)^(19/2)*c - 18432*a*(-b)^(21/2)*c + 2*a^6*(-b)^(9/2)*c - 24*a^5*(-b)^(11/2)*c + 4*a^7*(-b)^(9/2)*c + 70*a^4*(-b)^(13/2)*c - 48*a^6*(-b)^(11/2)*c + 2*a^8*(-b)^(9/2)*c - 288*a^3*(-b)^(15/2)*c + 60*a^5*(-b)^(13/2)*c - 8*a^7*(-b)^(11/2)*c + 1536*a^2*(-b)^(17/2)*c - 656*a^4*(-b)^(15/2)*c - 378*a^6*(-b)^(13/2)*c + 32*a^8*(-b)^(11/2)*c + 8064*a^3*(-b)^(17/2)*c + 656*a^5*(-b)^(15/2)*c - 784*a^7*(-b)^(13/2)*c + 16*a^9*(-b)^(11/2)*c - 9600*a^2*(-b)^(19/2)*c + 16384*a^4*(-b)^(17/2)*c + 3280*a^6*(-b)^(15/2)*c - 544*a^8*(-b)^(13/2)*c - 30720*a^3*(-b)^(19/2)*c + 15616*a^5*(-b)^(17/2)*c + 3664*a^7*(-b)^(15/2)*c - 128*a^9*(-b)^(13/2)*c - 1152*a*(-b)^(21/2)*c^2 - 46080*a^2*(-b)^(21/2)*c - 49920*a^4*(-b)^(19/2)*c + 6144*a^6*(-b)^(17/2)*c + 1664*a^8*(-b)^(15/2)*c - 61440*a^3*(-b)^(21/2)*c - 44160*a^5*(-b)^(19/2)*c - 128*a^7*(-b)^(17/2)*c + 256*a^9*(-b)^(15/2)*c + 46080*a*(-b)^(23/2)*c^2 - 46080*a^4*(-b)^(21/2)*c - 20352*a^6*(-b)^(19/2)*c - 512*a^8*(-b)^(17/2)*c + 9600*a*(-b)^(23/2)*c^3 - 18432*a^5*(-b)^(21/2)*c - 3840*a^7*(-b)^(19/2)*c - 3072*a^6*(-b)^(21/2)*c - 61440*a*(-b)^(25/2)*c^3 - 17280*a*(-b)^(25/2)*c^4 + 46080*a*(-b)^(27/2)*c^4 + 14976*a*(-b)^(27/2)*c^5 - 18432*a*(-b)^(29/2)*c^5 - 6528*a*(-b)^(29/2)*c^6 + 3072*a*(-b)^(31/2)*c^6 + 1152*a*(-b)^(31/2)*c^7))/((-b)^(1/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^8*b^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a^12*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9))))/((-(4*a*b^3 + b^3 + 6*a^2*b^3 + 4*a^3*b^3 + a^4*b^3 + 4*a^2*b^3*c + 6*a^3*b^3*c + 4*a^4*b^3*c + a^5*b^3*c + a*b^3*c)/(a^8*b^4 - a^9*b^3))^(1/4)*((-(4*a*b^3 + b^3 + 6*a^2*b^3 + 4*a^3*b^3 + a^4*b^3 + 4*a^2*b^3*c + 6*a^3*b^3*c + 4*a^4*b^3*c + a^5*b^3*c + a*b^3*c)/(a^8*b^4 - a^9*b^3))^(3/4)*((64*(36*a^12*(-b)^(25/4) - 4*a^13*(-b)^(21/4) + 48*a^13*(-b)^(25/4) - 60*a^11*(-b)^(29/4) - 240*a^12*(-b)^(29/4) - 192*a^13*(-b)^(29/4) - 180*a^10*(-b)^(33/4) - 240*a^11*(-b)^(33/4) + 192*a^12*(-b)^(33/4) + 240*a^9*(-b)^(37/4) + 256*a^13*(-b)^(33/4) + 1200*a^10*(-b)^(37/4) + 1728*a^11*(-b)^(37/4) + 320*a^8*(-b)^(41/4) + 768*a^12*(-b)^(37/4) + 1152*a^9*(-b)^(41/4) + 1344*a^10*(-b)^(41/4) + 512*a^11*(-b)^(41/4) - 144*a^13*(-b)^(29/4)*c^2 + 912*a^12*(-b)^(33/4)*c^2 + 1344*a^13*(-b)^(33/4)*c^2 + 336*a^13*(-b)^(33/4)*c^3 - 432*a^11*(-b)^(37/4)*c^2 - 4032*a^12*(-b)^(37/4)*c^2 - 1680*a^12*(-b)^(37/4)*c^3 - 4032*a^13*(-b)^(37/4)*c^2 - 4624*a^10*(-b)^(41/4)*c^2 - 2688*a^13*(-b)^(37/4)*c^3 - 9408*a^11*(-b)^(41/4)*c^2 - 504*a^13*(-b)^(37/4)*c^4 - 336*a^11*(-b)^(4

1/4)*c^3 - 1344*a^12*(-b)^(41/4)*c^2 + 3584*a^9*(-b)^(45/4)*c^2 + 5376*a^12
*(-b)^(41/4)*c^3 + 3584*a^13*(-b)^(41/4)*c^2 + 20160*a^10*(-b)^(45/4)*c^2 +
 1848*a^12*(-b)^(41/4)*c^4 + 6720*a^13*(-b)^(41/4)*c^3 + 8848*a^10*(-b)^(45
/4)*c^3 + 30912*a^11*(-b)^(45/4)*c^2 + 3360*a^13*(-b)^(41/4)*c^4 + 6720*a^8
*(-b)^(49/4)*c^2 + 21504*a^11*(-b)^(45/4)*c^3 + 14336*a^12*(-b)^(45/4)*c^2
+ 504*a^13*(-b)^(41/4)*c^5 + 24192*a^9*(-b)^(49/4)*c^2 + 1848*a^11*(-b)^(45
/4)*c^4 + 9408*a^12*(-b)^(45/4)*c^3 - 4032*a^9*(-b)^(49/4)*c^3 + 28224*a^10
*(-b)^(49/4)*c^2 - 3360*a^12*(-b)^(45/4)*c^4 - 3584*a^13*(-b)^(45/4)*c^3 -
26880*a^10*(-b)^(49/4)*c^3 + 10752*a^11*(-b)^(49/4)*c^2 - 1176*a^12*(-b)^(4
5/4)*c^5 - 6720*a^13*(-b)^(45/4)*c^4 - 10584*a^10*(-b)^(49/4)*c^4 - 44352*a
^11*(-b)^(49/4)*c^3 - 2688*a^13*(-b)^(45/4)*c^5 - 11200*a^8*(-b)^(53/4)*c^3
 - 30240*a^11*(-b)^(49/4)*c^4 - 21504*a^12*(-b)^(49/4)*c^3 - 336*a^13*(-b)^
(45/4)*c^6 - 40320*a^9*(-b)^(53/4)*c^3 - 2520*a^11*(-b)^(49/4)*c^5 - 20160*
a^12*(-b)^(49/4)*c^4 + 1120*a^9*(-b)^(53/4)*c^4 - 47040*a^10*(-b)^(53/4)*c^
3 + 16800*a^10*(-b)^(53/4)*c^4 - 17920*a^11*(-b)^(53/4)*c^3 + 336*a^12*(-b)
^(49/4)*c^6 + 4032*a^13*(-b)^(49/4)*c^5 + 8120*a^10*(-b)^(53/4)*c^5 + 33600
a^11(-b)^(53/4)*c^4 + 1344*a^13*(-b)^(49/4)*c^6 + 11200*a^8*(-b)^(57/4)*c
^4 + 26880*a^11*(-b)^(53/4)*c^5 + 17920*a^12*(-b)^(53/4)*c^4 + 144*a^13*(-b
)^(49/4)*c^7 + 40320*a^9*(-b)^(57/4)*c^4 + 1680*a^11*(-b)^(53/4)*c^6 + 2284
8*a^12*(-b)^(53/4)*c^5 + 2240*a^9*(-b)^(57/4)*c^5 + 47040*a^10*(-b)^(57/4)*
c^4 + 1344*a^12*(-b)^(53/4)*c^6 + 3584*a^13*(-b)^(53/4)*c^5 + 17920*a^11*(-
b)^(57/4)*c^4 + 48*a^12*(-b)^(53/4)*c^7 - 1344*a^13*(-b)^(53/4)*c^6 - 3920*
a^10*(-b)^(57/4)*c^6 - 9408*a^11*(-b)^(57/4)*c^5 - 384*a^13*(-b)^(53/4)*c^7
 - 6720*a^8*(-b)^(61/4)*c^5 - 14784*a^11*(-b)^(57/4)*c^6 - 7168*a^12*(-b)^(
57/4)*c^5 - 36*a^13*(-b)^(53/4)*c^8 - 24192*a^9*(-b)^(61/4)*c^5 - 528*a^11*
(-b)^(57/4)*c^7 - 14784*a^12*(-b)^(57/4)*c^6 - 2688*a^9*(-b)^(61/4)*c^6 - 2
8224*a^10*(-b)^(61/4)*c^5 - 768*a^12*(-b)^(57/4)*c^7 - 3584*a^13*(-b)^(57/4
)*c^6 - 6720*a^10*(-b)^(61/4)*c^6 - 10752*a^11*(-b)^(61/4)*c^5 - 60*a^12*(-
b)^(57/4)*c^8 + 192*a^13*(-b)^(57/4)*c^7 + 1104*a^10*(-b)^(61/4)*c^7 - 4032
a^11(-b)^(61/4)*c^6 + 48*a^13*(-b)^(57/4)*c^8 + 2240*a^8*(-b)^(65/4)*c^6
+ 4608*a^11*(-b)^(61/4)*c^7 + 4*a^13*(-b)^(57/4)*c^9 + 8064*a^9*(-b)^(65/4)
*c^6 + 36*a^11*(-b)^(61/4)*c^8 + 5184*a^12*(-b)^(61/4)*c^7 + 1216*a^9*(-b)^
(65/4)*c^7 + 9408*a^10*(-b)^(65/4)*c^6 + 144*a^12*(-b)^(61/4)*c^8 + 1536*a^
13*(-b)^(61/4)*c^7 + 3840*a^10*(-b)^(65/4)*c^7 + 3584*a^11*(-b)^(65/4)*c^6
+ 12*a^12*(-b)^(61/4)*c^9 - 148*a^10*(-b)^(65/4)*c^8 + 3648*a^11*(-b)^(65/4
)*c^7 - 320*a^8*(-b)^(69/4)*c^7 - 624*a^11*(-b)^(65/4)*c^8 + 1024*a^12*(-b)
^(65/4)*c^7 - 1152*a^9*(-b)^(69/4)*c^7 + 12*a^11*(-b)^(65/4)*c^9 - 768*a^12
*(-b)^(65/4)*c^8 - 208*a^9*(-b)^(69/4)*c^8 - 1344*a^10*(-b)^(69/4)*c^7 - 25
6*a^13*(-b)^(65/4)*c^8 - 720*a^10*(-b)^(69/4)*c^8 - 512*a^11*(-b)^(69/4)*c^
7 + 4*a^10*(-b)^(69/4)*c^9 - 768*a^11*(-b)^(69/4)*c^8 - 256*a^12*(-b)^(69/4
)*c^8 + 36*a^13*(-b)^(25/4)*c - 276*a^12*(-b)^(29/4)*c - 384*a^13*(-b)^(29/
4)*c + 300*a^11*(-b)^(33/4)*c + 1536*a^12*(-b)^(33/4)*c + 1344*a^13*(-b)^(3
3/4)*c + 1380*a^10*(-b)^(37/4)*c + 2304*a^11*(-b)^(37/4)*c - 576*a^12*(-b)^
(37/4)*c - 1472*a^9*(-b)^(41/4)*c - 1536*a^13*(-b)^(37/4)*c - 7680*a^10*(-b
)^(41/4)*c - 11328*a^11*(-b)^(41/4)*c - 2240*a^8*(-b)^(45/4)*c - 5120*a^12*
(-b)^(41/4)*c - 8064*a^9*(-b)^(45/4)*c - 9408*a^10*(-b)^(45/4)*c - 3584*a^1
1*(-b)^(45/4)*c))/(a^7*b^18 + 9*a^8*b^18*c + 36*a^9*b^18*c^2 + 84*a^10*b^18
*c^3 + 126*a^11*b^18*c^4 + 126*a^12*b^18*c^5 + 84*a^13*b^18*c^6 + 36*a^14*b
^18*c^7 + 9*a^15*b^18*c^8 + a^16*b^18*c^9) - ((-(4*a*b^3 + b^3 + 6*a^2*b^3
+ 4*a^3*b^3 + a^4*b^3 + 4*a^2*b^3*c + 6*a^3*b^3*c + 4*a^4*b^3*c + a^5*b^3*c
 + a*b^3*c)/(a^8*b^4 - a^9*b^3))^(1/4)*(-(b*x - 1)/(c + x))^(1/4)*(16*a^13*
(-b)^(11/2) - 80*a^12*(-b)^(13/2) - 80*a^11*(-b)^(15/2) - 128*a^13*(-b)^(13
/2) + 400*a^10*(-b)^(17/2) + 128*a^12*(-b)^(15/2) + 896*a^9*(-b)^(19/2) + 1
152*a^11*(-b)^(17/2) + 256*a^13*(-b)^(15/2) + 512*a^8*(-b)^(21/2) + 1920*a^
10*(-b)^(19/2) + 768*a^12*(-b)^(17/2) + 1024*a^9*(-b)^(21/2) + 1024*a^11*(-
b)^(19/2) + 512*a^10*(-b)^(21/2) + 448*a^13*(-b)^(15/2)*c^2 - 1344*a^12*(-b
)^(17/2)*c^2 - 2624*a^11*(-b)^(19/2)*c^2 - 2688*a^13*(-b)^(17/2)*c^2 + 1088
a^10(-b)^(21/2)*c^2 + 128*a^12*(-b)^(19/2)*c^2 - 896*a^13*(-b)^(17/2)*c^3
 + 9600*a^9*(-b)^(23/2)*c^2 + 9344*a^11*(-b)^(21/2)*c^2 + 1792*a^12*(-b)^(1

9/2)*c^3 + 4096*a^13*(-b)^(19/2)*c^2 + 7680*a^8*(-b)^(25/2)*c^2 + 21888*a^1
0*(-b)^(23/2)*c^2 + 4096*a^11*(-b)^(21/2)*c^3 + 8704*a^12*(-b)^(21/2)*c^2 +
 4480*a^13*(-b)^(19/2)*c^3 + 15360*a^9*(-b)^(25/2)*c^2 + 3328*a^10*(-b)^(23
/2)*c^3 + 12288*a^11*(-b)^(23/2)*c^2 + 128*a^12*(-b)^(21/2)*c^3 + 1120*a^13
*(-b)^(19/2)*c^4 - 8320*a^9*(-b)^(25/2)*c^3 + 7680*a^10*(-b)^(25/2)*c^2 - 4
992*a^11*(-b)^(23/2)*c^3 - 1120*a^12*(-b)^(21/2)*c^4 - 6656*a^13*(-b)^(21/2
)*c^3 - 10240*a^8*(-b)^(27/2)*c^3 - 21120*a^10*(-b)^(25/2)*c^3 - 2400*a^11*
(-b)^(23/2)*c^4 - 9216*a^12*(-b)^(23/2)*c^3 - 4480*a^13*(-b)^(21/2)*c^4 - 2
0480*a^9*(-b)^(27/2)*c^3 - 7200*a^10*(-b)^(25/2)*c^4 - 12800*a^11*(-b)^(25/
2)*c^3 + 1920*a^12*(-b)^(23/2)*c^4 - 896*a^13*(-b)^(21/2)*c^5 + 640*a^9*(-b
)^(27/2)*c^4 - 10240*a^10*(-b)^(27/2)*c^3 - 3200*a^11*(-b)^(25/2)*c^4 + 768
0*a^13*(-b)^(23/2)*c^4 + 7680*a^8*(-b)^(29/2)*c^4 + 5760*a^10*(-b)^(27/2)*c
^4 - 1280*a^11*(-b)^(25/2)*c^5 + 5120*a^12*(-b)^(25/2)*c^4 + 2688*a^13*(-b)
^(23/2)*c^5 + 15360*a^9*(-b)^(29/2)*c^4 + 5120*a^10*(-b)^(27/2)*c^5 + 5120*
a^11*(-b)^(27/2)*c^4 - 5248*a^12*(-b)^(25/2)*c^5 + 448*a^13*(-b)^(23/2)*c^6
 + 4224*a^9*(-b)^(29/2)*c^5 + 7680*a^10*(-b)^(29/2)*c^4 + 3968*a^11*(-b)^(2
7/2)*c^5 + 448*a^12*(-b)^(25/2)*c^6 - 6656*a^13*(-b)^(25/2)*c^5 - 3072*a^8*
(-b)^(31/2)*c^5 + 5760*a^10*(-b)^(29/2)*c^5 + 2752*a^11*(-b)^(27/2)*c^6 - 2
048*a^12*(-b)^(27/2)*c^5 - 896*a^13*(-b)^(25/2)*c^6 - 6144*a^9*(-b)^(31/2)*
c^5 - 704*a^10*(-b)^(29/2)*c^6 + 1536*a^11*(-b)^(29/2)*c^5 + 5504*a^12*(-b)
^(27/2)*c^6 - 128*a^13*(-b)^(25/2)*c^7 - 2944*a^9*(-b)^(31/2)*c^6 - 3072*a^
10*(-b)^(31/2)*c^5 + 384*a^11*(-b)^(29/2)*c^6 - 256*a^12*(-b)^(27/2)*c^7 +
4096*a^13*(-b)^(27/2)*c^6 + 512*a^8*(-b)^(33/2)*c^6 - 4992*a^10*(-b)^(31/2)
*c^6 - 1536*a^11*(-b)^(29/2)*c^7 + 1536*a^12*(-b)^(29/2)*c^6 + 128*a^13*(-b
)^(27/2)*c^7 + 1024*a^9*(-b)^(33/2)*c^6 - 768*a^10*(-b)^(31/2)*c^7 - 2048*a
^11*(-b)^(31/2)*c^6 - 2688*a^12*(-b)^(29/2)*c^7 + 16*a^13*(-b)^(27/2)*c^8 +
 640*a^9*(-b)^(33/2)*c^7 + 512*a^10*(-b)^(33/2)*c^6 - 1664*a^11*(-b)^(31/2)
*c^7 + 48*a^12*(-b)^(29/2)*c^8 - 1536*a^13*(-b)^(29/2)*c^7 + 1152*a^10*(-b)
^(33/2)*c^7 + 304*a^11*(-b)^(31/2)*c^8 - 1024*a^12*(-b)^(31/2)*c^7 + 272*a^
10*(-b)^(33/2)*c^8 + 512*a^11*(-b)^(33/2)*c^7 + 512*a^12*(-b)^(31/2)*c^8 +
512*a^11*(-b)^(33/2)*c^8 + 256*a^13*(-b)^(31/2)*c^8 + 256*a^12*(-b)^(33/2)*
c^8 - 128*a^13*(-b)^(13/2)*c + 512*a^12*(-b)^(15/2)*c + 768*a^11*(-b)^(17/2
)*c + 896*a^13*(-b)^(15/2)*c - 1536*a^10*(-b)^(19/2)*c - 384*a^12*(-b)^(17/
2)*c - 4736*a^9*(-b)^(21/2)*c - 5504*a^11*(-b)^(19/2)*c - 1536*a^13*(-b)^(1
7/2)*c - 3072*a^8*(-b)^(23/2)*c - 10368*a^10*(-b)^(21/2)*c - 4096*a^12*(-b)
^(19/2)*c - 6144*a^9*(-b)^(23/2)*c - 5632*a^11*(-b)^(21/2)*c - 3072*a^10*(-
b)^(23/2)*c)*64i)/((-b)^(1/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^8*b^17*c^2 +
84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a^12*b^17*c^6
 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9)))*1i - (64*(-(b*x - 1
)/(c + x))^(1/4)*(512*(-b)^(19/2) + a^5*(-b)^(9/2) + a^4*(-b)^(11/2) + 2*a^
6*(-b)^(9/2) - 16*a^3*(-b)^(13/2) + 18*a^5*(-b)^(11/2) + a^7*(-b)^(9/2) - 1
44*a^2*(-b)^(15/2) - 176*a^4*(-b)^(13/2) + 49*a^6*(-b)^(11/2) - 576*a^3*(-b
)^(15/2) - 560*a^5*(-b)^(13/2) + 48*a^7*(-b)^(11/2) + 2688*a^2*(-b)^(17/2)
 - 608*a^4*(-b)^(15/2) - 784*a^6*(-b)^(13/2) + 16*a^8*(-b)^(11/2) + 7680*a^3
*(-b)^(17/2) + 448*a^5*(-b)^(15/2) - 512*a^7*(-b)^(13/2) + 7680*a^2*(-b)^(1
9/2) + 11520*a^4*(-b)^(17/2) + 1392*a^6*(-b)^(15/2) - 128*a^8*(-b)^(13/2) +
 10240*a^3*(-b)^(19/2) + 9600*a^5*(-b)^(17/2) + 1024*a^7*(-b)^(15/2) + 7680
a^4(-b)^(19/2) + 4224*a^6*(-b)^(17/2) + 256*a^8*(-b)^(15/2) + 3072*a^5*(-
b)^(19/2) + 768*a^7*(-b)^(17/2) + 512*a^6*(-b)^(19/2) + 7680*(-b)^(23/2)*c^
2 - 10240*(-b)^(25/2)*c^3 + 7680*(-b)^(27/2)*c^4 - 3072*(-b)^(29/2)*c^5 + 5
12*(-b)^(31/2)*c^6 + 384*a*(-b)^(17/2) + 3072*a*(-b)^(19/2) - 3072*(-b)^(21
/2)*c + a^7*(-b)^(9/2)*c^2 - 35*a^6*(-b)^(11/2)*c^2 + 2*a^8*(-b)^(9/2)*c^2
 + 265*a^5*(-b)^(13/2)*c^2 - 86*a^7*(-b)^(11/2)*c^2 + a^9*(-b)^(9/2)*c^2 - 8
51*a^4*(-b)^(15/2)*c^2 + 738*a^6*(-b)^(13/2)*c^2 - 10*a^7*(-b)^(11/2)*c^3 -
 67*a^8*(-b)^(11/2)*c^2 + 2496*a^3*(-b)^(17/2)*c^2 - 2566*a^5*(-b)^(15/2)*c
^2 + 224*a^6*(-b)^(13/2)*c^3 + 649*a^7*(-b)^(13/2)*c^2 - 20*a^8*(-b)^(11/2)
*c^3 - 16*a^9*(-b)^(11/2)*c^2 - 5184*a^2*(-b)^(19/2)*c^2 + 10432*a^4*(-b)^(
17/2)*c^2 - 1358*a^5*(-b)^(15/2)*c^3 - 1907*a^6*(-b)^(15/2)*c^2 + 592*a^7*(
-b)^(13/2)*c^3 + 144*a^8*(-b)^(13/2)*c^2 - 10*a^9*(-b)^(11/2)*c^3 - 31104*a

^3*(-b)^(19/2)*c^2 + 3784*a^4*(-b)^(17/2)*c^3 + 14912*a^5*(-b)^(17/2)*c^2 - 4364*a^6*(-b)^(15/2)*c^3 + 45*a^7*(-b)^(13/2)*c^4 + 1120*a^7*(-b)^(15/2)*c^2 + 512*a^8*(-b)^(13/2)*c^3 - 32*a^9*(-b)^(13/2)*c^2 + 1152*a^2*(-b)^(21/2)*c^2 - 7552*a^3*(-b)^(19/2)*c^3 - 74624*a^4*(-b)^(19/2)*c^2 + 14288*a^5*(-b)^(17/2)*c^3 - 771*a^6*(-b)^(15/2)*c^4 + 5824*a^6*(-b)^(17/2)*c^2 - 4974*a^7*(-b)^(15/2)*c^3 + 90*a^8*(-b)^(13/2)*c^4 + 1952*a^8*(-b)^(15/2)*c^2 + 144*a^9*(-b)^(13/2)*c^3 + 6912*a^2*(-b)^(21/2)*c^3 + 23040*a^3*(-b)^(21/2)*c^2 - 36800*a^4*(-b)^(19/2)*c^3 + 3874*a^5*(-b)^(17/2)*c^4 - 91136*a^5*(-b)^(19/2)*c^2 + 19144*a^6*(-b)^(17/2)*c^3 - 2118*a^7*(-b)^(15/2)*c^4 - 5120*a^7*(-b)^(17/2)*c^2 - 2288*a^8*(-b)^(15/2)*c^3 + 45*a^9*(-b)^(13/2)*c^4 + 640*a^9*(-b)^(15/2)*c^2 + 115200*a^2*(-b)^(23/2)*c^2 + 49536*a^3*(-b)^(21/2)*c^3 - 8750*a^4*(-b)^(19/2)*c^4 + 57600*a^4*(-b)^(21/2)*c^2 - 68032*a^5*(-b)^(19/2)*c^3 + 13444*a^6*(-b)^(17/2)*c^4 - 58944*a^6*(-b)^(19/2)*c^2 - 120*a^7*(-b)^(15/2)*c^5 + 9664*a^7*(-b)^(17/2)*c^3 - 1923*a^8*(-b)^(15/2)*c^4 - 5248*a^8*(-b)^(17/2)*c^2 - 320*a^9*(-b)^(15/2)*c^3 + 44160*a^2*(-b)^(23/2)*c^3 + 11040*a^3*(-b)^(21/2)*c^4 + 153600*a^3*(-b)^(23/2)*c^2 + 137728*a^4*(-b)^(21/2)*c^3 - 36988*a^5*(-b)^(19/2)*c^4 + 63360*a^5*(-b)^(21/2)*c^2 + 1644*a^6*(-b)^(17/2)*c^5 - 56512*a^6*(-b)^(19/2)*c^3 + 17058*a^7*(-b)^(17/2)*c^4 - 18560*a^7*(-b)^(19/2)*c^2 - 240*a^8*(-b)^(15/2)*c^5 + 128*a^8*(-b)^(17/2)*c^3 - 576*a^9*(-b)^(15/2)*c^4 - 1280*a^9*(-b)^(17/2)*c^2 - 480*a^2*(-b)^(23/2)*c^4 + 76800*a^3*(-b)^(23/2)*c^3 + 60640*a^4*(-b)^(21/2)*c^4 + 115200*a^4*(-b)^(23/2)*c^2 - 6776*a^5*(-b)^(19/2)*c^5 + 193792*a^5*(-b)^(21/2)*c^3 - 59150*a^6*(-b)^(19/2)*c^4 + 33408*a^6*(-b)^(21/2)*c^2 + 4632*a^7*(-b)^(17/2)*c^5 - 16064*a^7*(-b)^(19/2)*c^3 + 9280*a^8*(-b)^(17/2)*c^4 - 2048*a^8*(-b)^(19/2)*c^2 - 120*a^9*(-b)^(15/2)*c^5 - 896*a^9*(-b)^(17/2)*c^3 - 153600*a^2*(-b)^(25/2)*c^3 - 23040*a^3*(-b)^(23/2)*c^4 + 11620*a^4*(-b)^(21/2)*c^5 + 57600*a^4*(-b)^(23/2)*c^3 + 128864*a^5*(-b)^(21/2)*c^4 + 46080*a^5*(-b)^(23/2)*c^2 - 24752*a^6*(-b)^(19/2)*c^5 + 146688*a^6*(-b)^(21/2)*c^3 + 210*a^7*(-b)^(17/2)*c^6 - 43232*a^7*(-b)^(19/2)*c^4 + 6912*a^7*(-b)^(21/2)*c^2 + 4332*a^8*(-b)^(17/2)*c^5 + 3968*a^8*(-b)^(19/2)*c^3 + 1792*a^9*(-b)^(17/2)*c^4 - 90240*a^2*(-b)^(25/2)*c^4 - 7104*a^3*(-b)^(23/2)*c^5 - 204800*a^3*(-b)^(25/2)*c^3 - 100160*a^4*(-b)^(23/2)*c^4 + 53480*a^5*(-b)^(21/2)*c^5 + 9600*a^5*(-b)^(23/2)*c^3 - 2310*a^6*(-b)^(19/2)*c^6 + 131872*a^6*(-b)^(21/2)*c^4 + 7680*a^6*(-b)^(23/2)*c^2 - 33432*a^7*(-b)^(19/2)*c^5 + 56704*a^7*(-b)^(21/2)*c^3 + 420*a^8*(-b)^(17/2)*c^6 - 13216*a^8*(-b)^(19/2)*c^4 + 1344*a^9*(-b)^(17/2)*c^5 + 2304*a^9*(-b)^(19/2)*c^3 - 9216*a^2*(-b)^(25/2)*c^5 - 192000*a^3*(-b)^(25/2)*c^4 - 48224*a^4*(-b)^(23/2)*c^5 - 153600*a^4*(-b)^(25/2)*c^3 + 7546*a^5*(-b)^(21/2)*c^6 - 179840*a^5*(-b)^(23/2)*c^4 + 94948*a^6*(-b)^(21/2)*c^5 - 9600*a^6*(-b)^(23/2)*c^3 - 6636*a^7*(-b)^(19/2)*c^6 + 63232*a^7*(-b)^(21/2)*c^4 - 19712*a^8*(-b)^(19/2)*c^5 + 8704*a^8*(-b)^(21/2)*c^3 + 210*a^9*(-b)^(17/2)*c^6 - 896*a^9*(-b)^(19/2)*c^4 + 115200*a^2*(-b)^(27/2)*c^4 - 31104*a^3*(-b)^(25/2)*c^5 - 8750*a^4*(-b)^(23/2)*c^6 - 211200*a^4*(-b)^(25/2)*c^4 - 121120*a^5*(-b)^(23/2)*c^5 - 61440*a^5*(-b)^(25/2)*c^3 + 28756*a^6*(-b)^(21/2)*c^6 - 161760*a^6*(-b)^(23/2)*c^4 - 252*a^7*(-b)^(19/2)*c^7 + 80416*a^7*(-b)^(21/2)*c^5 - 3840*a^7*(-b)^(23/2)*c^3 - 6342*a^8*(-b)^(19/2)*c^6 + 9344*a^8*(-b)^(21/2)*c^4 - 4256*a^9*(-b)^(19/2)*c^5 + 81792*a^2*(-b)^(27/2)*c^5 - 832*a^3*(-b)^(25/2)*c^6 + 153600*a^3*(-b)^(27/2)*c^4 - 23552*a^4*(-b)^(25/2)*c^5 - 44380*a^5*(-b)^(23/2)*c^6 - 124800*a^5*(-b)^(25/2)*c^4 + 2184*a^6*(-b)^(21/2)*c^7 - 146336*a^6*(-b)^(23/2)*c^5 - 10240*a^6*(-b)^(25/2)*c^3 + 40698*a^7*(-b)^(21/2)*c^6 - 72320*a^7*(-b)^(23/2)*c^4 - 504*a^8*(-b)^(19/2)*c^7 + 31808*a^8*(-b)^(21/2)*c^5 - 2016*a^9*(-b)^(19/2)*c^6 - 1280*a^9*(-b)^(21/2)*c^4 + 10944*a^2*(-b)^(27/2)*c^6 + 184320*a^3*(-b)^(27/2)*c^5 + 8896*a^4*(-b)^(25/2)*c^6 + 115200*a^4*(-b)^(27/2)*c^4 - 5300*a^5*(-b)^(23/2)*c^7 + 32512*a^5*(-b)^(25/2)*c^5 - 86702*a^6*(-b)^(23/2)*c^6 - 36480*a^6*(-b)^(25/2)*c^4 + 6384*a^7*(-b)^(21/2)*c^7 - 87968*a^7*(-b)^(23/2)*c^5 + 25312*a^8*(-b)^(21/2)*c^6 - 12800*a^8*(-b)^(23/2)*c^4 - 252*a^9*(-b)^(19/2)*c^7 + 4480*a^9*(-b)^(21/2)*c^5 - 46080*a^2*(-b)^(29/2)*c^5 + 49536*a^3*(-b)^(27/2)*c^6 + 3016*a^4*(-b)^(25/2)*c^7 + 218880*a^4*(-b)^(27/2)*c^5 + 44864*a^5*(-b)^(25/2)*c^6 + 46080*a^5*(-b)^(27/2)*c^4 - 211

28*a^6*(-b)^(23/2)*c^7 + 66048*a^6*(-b)^(25/2)*c^5 + 210*a^7*(-b)^(21/2)*c^8 - 81536*a^7*(-b)^(23/2)*c^6 - 3840*a^7*(-b)^(25/2)*c^4 + 6216*a^8*(-b)^(21/2)*c^7 - 22912*a^8*(-b)^(23/2)*c^5 + 5824*a^9*(-b)^(21/2)*c^6 - 36480*a^2*(-b)^(29/2)*c^6 + 4224*a^3*(-b)^(27/2)*c^7 - 61440*a^3*(-b)^(29/2)*c^5 + 86656*a^4*(-b)^(27/2)*c^6 + 18896*a^5*(-b)^(25/2)*c^7 + 144000*a^5*(-b)^(27/2)*c^5 - 1374*a^6*(-b)^(23/2)*c^8 + 75200*a^6*(-b)^(25/2)*c^6 + 7680*a^6*(-b)^(27/2)*c^4 - 31284*a^7*(-b)^(23/2)*c^7 + 40576*a^7*(-b)^(25/2)*c^5 + 420*a^8*(-b)^(21/2)*c^8 - 36736*a^8*(-b)^(23/2)*c^6 + 2016*a^9*(-b)^(21/2)*c^7 - 1280*a^9*(-b)^(23/2)*c^5 - 5376*a^2*(-b)^(29/2)*c^7 - 84480*a^3*(-b)^(29/2)*c^6 + 13888*a^4*(-b)^(27/2)*c^7 - 46080*a^4*(-b)^(29/2)*c^5 + 2173*a^5*(-b)^(25/2)*c^8 + 70144*a^5*(-b)^(27/2)*c^6 + 42952*a^6*(-b)^(25/2)*c^7 + 49536*a^6*(-b)^(27/2)*c^5 - 4092*a^7*(-b)^(23/2)*c^8 + 57856*a^7*(-b)^(25/2)*c^6 - 20384*a^8*(-b)^(23/2)*c^7 + 8704*a^8*(-b)^(25/2)*c^5 + 210*a^9*(-b)^(21/2)*c^8 - 6272*a^9*(-b)^(23/2)*c^6 + 7680*a^2*(-b)^(31/2)*c^6 - 26496*a^3*(-b)^(29/2)*c^7 + 301*a^4*(-b)^(27/2)*c^8 - 103680*a^4*(-b)^(29/2)*c^6 + 12608*a^5*(-b)^(27/2)*c^7 - 18432*a^5*(-b)^(29/2)*c^5 + 9274*a^6*(-b)^(25/2)*c^8 + 21696*a^6*(-b)^(27/2)*c^6 - 120*a^7*(-b)^(23/2)*c^9 + 45760*a^7*(-b)^(25/2)*c^7 + 6912*a^7*(-b)^(27/2)*c^5 - 4062*a^8*(-b)^(23/2)*c^8 + 20096*a^8*(-b)^(25/2)*c^6 - 4928*a^9*(-b)^(23/2)*c^7 + 6528*a^2*(-b)^(31/2)*c^7 - 2448*a^3*(-b)^(29/2)*c^8 + 10240*a^3*(-b)^(31/2)*c^6 - 52736*a^4*(-b)^(29/2)*c^7 - 1558*a^5*(-b)^(27/2)*c^8 - 71040*a^5*(-b)^(29/2)*c^6 + 546*a^6*(-b)^(25/2)*c^9 - 4544*a^6*(-b)^(27/2)*c^7 - 3072*a^6*(-b)^(29/2)*c^5 + 14589*a^7*(-b)^(25/2)*c^8 - 2432*a^7*(-b)^(27/2)*c^6 - 240*a^8*(-b)^(23/2)*c^9 + 23168*a^8*(-b)^(25/2)*c^7 - 1344*a^9*(-b)^(23/2)*c^8 + 2304*a^9*(-b)^(25/2)*c^6 + 1008*a^2*(-b)^(31/2)*c^8 + 15360*a^3*(-b)^(31/2)*c^7 - 10160*a^4*(-b)^(29/2)*c^8 + 7680*a^4*(-b)^(31/2)*c^6 - 384*a^5*(-b)^(27/2)*c^9 - 53504*a^5*(-b)^(29/2)*c^7 - 8099*a^6*(-b)^(27/2)*c^8 - 25728*a^6*(-b)^(29/2)*c^6 + 1668*a^7*(-b)^(25/2)*c^9 - 13760*a^7*(-b)^(27/2)*c^7 + 10048*a^8*(-b)^(25/2)*c^8 - 2048*a^8*(-b)^(27/2)*c^6 - 120*a^9*(-b)^(23/2)*c^9 + 4480*a^9*(-b)^(25/2)*c^7 + 5184*a^3*(-b)^(31/2)*c^8 - 570*a^4*(-b)^(29/2)*c^9 + 19200*a^4*(-b)^(31/2)*c^7 - 16048*a^5*(-b)^(29/2)*c^8 + 3072*a^5*(-b)^(31/2)*c^6 - 1984*a^6*(-b)^(27/2)*c^9 - 28416*a^6*(-b)^(29/2)*c^7 + 45*a^7*(-b)^(25/2)*c^10 - 11984*a^7*(-b)^(27/2)*c^8 - 3840*a^7*(-b)^(29/2)*c^6 + 1698*a^8*(-b)^(25/2)*c^9 - 7552*a^8*(-b)^(27/2)*c^7 + 2560*a^9*(-b)^(25/2)*c^8 + 480*a^3*(-b)^(31/2)*c^9 + 10912*a^4*(-b)^(31/2)*c^8 - 1732*a^5*(-b)^(29/2)*c^9 + 13440*a^5*(-b)^(31/2)*c^7 - 119*a^6*(-b)^(27/2)*c^10 - 11408*a^6*(-b)^(29/2)*c^8 + 512*a^6*(-b)^(31/2)*c^6 - 3568*a^7*(-b)^(27/2)*c^9 - 7040*a^7*(-b)^(29/2)*c^7 + 90*a^8*(-b)^(25/2)*c^10 - 7408*a^8*(-b)^(27/2)*c^8 + 576*a^9*(-b)^(25/2)*c^9 - 1280*a^9*(-b)^(27/2)*c^7 + 2160*a^4*(-b)^(31/2)*c^9 - 35*a^5*(-b)^(29/2)*c^10 + 11968*a^5*(-b)^(31/2)*c^8 - 1530*a^6*(-b)^(29/2)*c^9 + 4992*a^6*(-b)^(31/2)*c^7 - 382*a^7*(-b)^(27/2)*c^10 - 2816*a^7*(-b)^(29/2)*c^8 - 2720*a^8*(-b)^(27/2)*c^9 - 512*a^8*(-b)^(29/2)*c^7 + 45*a^9*(-b)^(25/2)*c^10 - 1664*a^9*(-b)^(27/2)*c^8 + 129*a^4*(-b)^(31/2)*c^10 + 3856*a^5*(-b)^(31/2)*c^9 + 10*a^6*(-b)^(29/2)*c^10 + 7152*a^6*(-b)^(31/2)*c^8 - 10*a^7*(-b)^(27/2)*c^11 + 112*a^7*(-b)^(29/2)*c^9 + 768*a^7*(-b)^(31/2)*c^7 - 407*a^8*(-b)^(27/2)*c^10 + 512*a^8*(-b)^(29/2)*c^8 - 752*a^9*(-b)^(27/2)*c^9 + 482*a^5*(-b)^(31/2)*c^10 + 8*a^6*(-b)^(29/2)*c^11 + 3408*a^6*(-b)^(31/2)*c^9 + 221*a^7*(-b)^(29/2)*c^10 + 2176*a^7*(-b)^(31/2)*c^8 - 20*a^8*(-b)^(27/2)*c^11 + 736*a^8*(-b)^(29/2)*c^9 - 144*a^9*(-b)^(27/2)*c^10 + 256*a^9*(-b)^(29/2)*c^8 + 18*a^5*(-b)^(31/2)*c^11 + 673*a^6*(-b)^(31/2)*c^10 + 32*a^7*(-b)^(29/2)*c^11 + 1488*a^7*(-b)^(31/2)*c^9 + 272*a^8*(-b)^(29/2)*c^10 + 256*a^8*(-b)^(31/2)*c^8 - 10*a^9*(-b)^(27/2)*c^11 + 256*a^9*(-b)^(29/2)*c^9 + 52*a^6*(-b)^(31/2)*c^11 + a^7*(-b)^(29/2)*c^12 + 416*a^7*(-b)^(31/2)*c^10 + 40*a^8*(-b)^(29/2)*c^11 + 256*a^8*(-b)^(31/2)*c^9 + 96*a^9*(-b)^(29/2)*c^10 + a^6*(-b)^(31/2)*c^12 + 50*a^7*(-b)^(31/2)*c^11 + 2*a^8*(-b)^(29/2)*c^12 + 96*a^8*(-b)^(31/2)*c^10 + 16*a^9*(-b)^(29/2)*c^11 + 2*a^7*(-b)^(31/2)*c^12 + 16*a^8*(-b)^(31/2)*c^11 + a^9*(-b)^(29/2)*c^12 + a^8*(-b)^(31/2)*c^12 - 1152*a*(-b)^(19/2)*c - 18432*a*(-b)^(21/2)*c + 2*a^6*(-b)^(9/2)*c - 24*a^5*(-b)^(11/2)*c + 4*a^7*(-b)^(9/2)*c + 70*a^4*(-b)^(13/2)*c - 48*a^6*(-b)^(

11/2)*c + 2*a^8*(-b)^(9/2)*c - 288*a^3*(-b)^(15/2)*c + 60*a^5*(-b)^(13/2)*c - 8*a^7*(-b)^(11/2)*c + 1536*a^2*(-b)^(17/2)*c - 656*a^4*(-b)^(15/2)*c - 378*a^6*(-b)^(13/2)*c + 32*a^8*(-b)^(11/2)*c + 8064*a^3*(-b)^(17/2)*c + 656*a^5*(-b)^(15/2)*c - 784*a^7*(-b)^(13/2)*c + 16*a^9*(-b)^(11/2)*c - 9600*a^2*(-b)^(19/2)*c + 16384*a^4*(-b)^(17/2)*c + 3280*a^6*(-b)^(15/2)*c - 544*a^8*(-b)^(13/2)*c - 30720*a^3*(-b)^(19/2)*c + 15616*a^5*(-b)^(17/2)*c + 3664*a^7*(-b)^(15/2)*c - 128*a^9*(-b)^(13/2)*c - 1152*a*(-b)^(21/2)*c^2 - 46080*a^2*(-b)^(21/2)*c - 49920*a^4*(-b)^(19/2)*c + 6144*a^6*(-b)^(17/2)*c + 1664*a^8*(-b)^(15/2)*c - 61440*a^3*(-b)^(21/2)*c - 44160*a^5*(-b)^(19/2)*c - 128*a^7*(-b)^(17/2)*c + 256*a^9*(-b)^(15/2)*c + 46080*a*(-b)^(23/2)*c^2 - 46080*a^4*(-b)^(21/2)*c - 20352*a^6*(-b)^(19/2)*c - 512*a^8*(-b)^(17/2)*c + 9600*a*(-b)^(23/2)*c^3 - 18432*a^5*(-b)^(21/2)*c - 3840*a^7*(-b)^(19/2)*c - 3072*a^6*(-b)^(21/2)*c - 61440*a*(-b)^(25/2)*c^3 - 17280*a*(-b)^(25/2)*c^4 + 46080*a*(-b)^(27/2)*c^4 + 14976*a*(-b)^(27/2)*c^5 - 18432*a*(-b)^(29/2)*c^5 - 6528*a*(-b)^(29/2)*c^6 + 3072*a*(-b)^(31/2)*c^6 + 1152*a*(-b)^(31/2)*c^7))/((-b)^(1/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^8*b^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a^12*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9)))*1i - (128*(5*a^4*(-b)^(21/4) - 320*(-b)^(37/4) + 19*a^5*(-b)^(21/4) + 27*a^6*(-b)^(21/4) - 55*a^3*(-b)^(25/4) + 17*a^7*(-b)^(21/4) - 269*a^4*(-b)^(25/4) + 4*a^8*(-b)^(21/4) - 525*a^5*(-b)^(25/4) + 180*a^2*(-b)^(29/4) - 511*a^6*(-b)^(25/4) + 1104*a^3*(-b)^(29/4) - 248*a^7*(-b)^(25/4) + 2808*a^4*(-b)^(29/4) - 48*a^8*(-b)^(25/4) + 3792*a^5*(-b)^(29/4) - 784*a^2*(-b)^(33/4) + 2868*a^6*(-b)^(29/4) - 2976*a^3*(-b)^(33/4) + 1152*a^7*(-b)^(29/4) - 5920*a^4*(-b)^(33/4) + 192*a^8*(-b)^(29/4) - 6800*a^5*(-b)^(33/4) - 6336*a^2*(-b)^(37/4) - 4560*a^6*(-b)^(33/4) - 10240*a^3*(-b)^(37/4) - 1664*a^7*(-b)^(33/4) - 9920*a^4*(-b)^(37/4) - 256*a^8*(-b)^(33/4) - 5760*a^5*(-b)^(37/4) - 1856*a^6*(-b)^(37/4) - 256*a^7*(-b)^(37/4) - 6720*(-b)^(45/4)*c^2 + 11200*(-b)^(49/4)*c^3 - 11200*(-b)^(53/4)*c^4 + 6720*(-b)^(57/4)*c^5 - 2240*(-b)^(61/4)*c^6 + 320*(-b)^(65/4)*c^7 - 80*a*(-b)^(33/4) - 2176*a*(-b)^(37/4) + 2240*(-b)^(41/4)*c + a^6*(-b)^(21/4)*c^2 + 3*a^7*(-b)^(21/4)*c^2 + 3*a^8*(-b)^(21/4)*c^2 - 71*a^5*(-b)^(25/4)*c^2 + a^9*(-b)^(21/4)*c^2 - 265*a^6*(-b)^(25/4)*c^2 - 10*a^6*(-b)^(25/4)*c^3 - 369*a^7*(-b)^(25/4)*c^2 + 849*a^4*(-b)^(29/4)*c^2 - 30*a^7*(-b)^(25/4)*c^3 - 227*a^8*(-b)^(25/4)*c^2 + 3851*a^5*(-b)^(29/4)*c^2 - 30*a^8*(-b)^(25/4)*c^3 - 52*a^9*(-b)^(25/4)*c^2 + 368*a^5*(-b)^(29/4)*c^3 + 6939*a^6*(-b)^(29/4)*c^2 - 10*a^9*(-b)^(25/4)*c^3 - 3607*a^3*(-b)^(33/4)*c^2 + 1392*a^6*(-b)^(29/4)*c^3 + 6201*a^7*(-b)^(29/4)*c^2 - 19689*a^4*(-b)^(33/4)*c^2 + 45*a^6*(-b)^(29/4)*c^4 + 1968*a^7*(-b)^(29/4)*c^3 + 2744*a^8*(-b)^(29/4)*c^2 - 3198*a^4*(-b)^(33/4)*c^3 - 44241*a^5*(-b)^(33/4)*c^2 + 135*a^7*(-b)^(29/4)*c^4 + 1232*a^8*(-b)^(29/4)*c^3 + 480*a^9*(-b)^(29/4)*c^2 + 4880*a^2*(-b)^(37/4)*c^2 - 14874*a^5*(-b)^(33/4)*c^3 - 52259*a^6*(-b)^(33/4)*c^2 + 135*a^8*(-b)^(29/4)*c^4 + 288*a^9*(-b)^(29/4)*c^3 + 33088*a^3*(-b)^(37/4)*c^2 - 1107*a^5*(-b)^(33/4)*c^4 - 27546*a^6*(-b)^(33/4)*c^3 - 34116*a^7*(-b)^(33/4)*c^2 + 45*a^9*(-b)^(29/4)*c^4 + 9976*a^3*(-b)^(37/4)*c^3 + 93600*a^4*(-b)^(37/4)*c^2 - 4233*a^6*(-b)^(33/4)*c^4 - 25374*a^7*(-b)^(33/4)*c^3 - 11616*a^8*(-b)^(33/4)*c^2 + 56280*a^4*(-b)^(37/4)*c^3 + 142912*a^5*(-b)^(37/4)*c^2 - 120*a^6*(-b)^(33/4)*c^5 - 6057*a^7*(-b)^(33/4)*c^4 - 11616*a^8*(-b)^(33/4)*c^3 - 1600*a^9*(-b)^(33/4)*c^2 + 11200*a^2*(-b)^(41/4)*c^2 + 7290*a^4*(-b)^(37/4)*c^4 + 131064*a^5*(-b)^(37/4)*c^3 + 126608*a^6*(-b)^(37/4)*c^2 - 360*a^7*(-b)^(33/4)*c^5 - 3843*a^8*(-b)^(33/4)*c^4 - 2112*a^9*(-b)^(33/4)*c^3 - 7952*a^2*(-b)^(41/4)*c^3 + 12096*a^3*(-b)^(41/4)*c^2 + 34566*a^5*(-b)^(37/4)*c^4 + 161096*a^6*(-b)^(37/4)*c^3 + 64512*a^7*(-b)^(37/4)*c^2 - 360*a^8*(-b)^(33/4)*c^5 - 912*a^9*(-b)^(33/4)*c^4 - 59136*a^3*(-b)^(41/4)*c^3 - 13440*a^4*(-b)^(41/4)*c^2 + 2148*a^5*(-b)^(37/4)*c^5 + 65334*a^6*(-b)^(37/4)*c^4 + 110064*a^7*(-b)^(37/4)*c^3 + 17216*a^8*(-b)^(37/4)*c^2 - 120*a^9*(-b)^(33/4)*c^5 - 16590*a^3*(-b)^(41/4)*c^4 - 180768*a^4*(-b)^(41/4)*c^3 - 44800*a^5*(-b)^(41/4)*c^2 + 8292*a^6*(-b)^(37/4)*c^5 + 61506*a^7*(-b)^(37/4)*c^4 + 39552*a^8*(-b)^(37/4)*c^3 + 1792*a^9*(-b)^(37/4)*c^2 - 133056*a^2*(-b)^(45/4)*c^2 - 96810*a^4*(-b)^(41/4)*c^4 - 296128*a^5*(-b)^(41/4)*c^3 + 210*a^6*(

-b)^(37/4)*c^6 - 44352*a^6*(-b)^(41/4)*c^2 + 11988*a^7*(-b)^(37/4)*c^5 + 28
824*a^8*(-b)^(37/4)*c^4 + 5824*a^9*(-b)^(37/4)*c^3 - 55552*a^2*(-b)^(45/4)*
c^3 - 215040*a^3*(-b)^(45/4)*c^2 - 10752*a^4*(-b)^(41/4)*c^5 - 233226*a^5*(
-b)^(41/4)*c^4 - 281232*a^6*(-b)^(41/4)*c^3 + 630*a^7*(-b)^(37/4)*c^6 - 201
60*a^7*(-b)^(41/4)*c^2 + 7692*a^8*(-b)^(37/4)*c^5 + 5376*a^9*(-b)^(37/4)*c^
4 + 5208*a^2*(-b)^(45/4)*c^4 - 119616*a^3*(-b)^(45/4)*c^3 - 208320*a^4*(-b)
^(45/4)*c^2 - 51912*a^5*(-b)^(41/4)*c^5 - 296814*a^6*(-b)^(41/4)*c^4 - 1545
60*a^7*(-b)^(41/4)*c^3 + 630*a^8*(-b)^(37/4)*c^6 - 3584*a^8*(-b)^(41/4)*c^2
 + 1848*a^9*(-b)^(37/4)*c^5 + 51296*a^3*(-b)^(45/4)*c^4 - 125440*a^4*(-b)^(
45/4)*c^3 - 2814*a^5*(-b)^(41/4)*c^6 - 120960*a^5*(-b)^(45/4)*c^2 - 99960*a
^6*(-b)^(41/4)*c^5 - 210336*a^7*(-b)^(41/4)*c^4 - 45248*a^8*(-b)^(41/4)*c^3
 + 210*a^9*(-b)^(37/4)*c^6 + 16940*a^3*(-b)^(45/4)*c^5 + 185808*a^4*(-b)^(4
5/4)*c^4 - 56000*a^5*(-b)^(45/4)*c^3 - 10962*a^6*(-b)^(41/4)*c^6 - 38976*a^
6*(-b)^(45/4)*c^2 - 95928*a^7*(-b)^(41/4)*c^5 - 78624*a^8*(-b)^(41/4)*c^4 -
 5376*a^9*(-b)^(41/4)*c^3 + 221760*a^2*(-b)^(49/4)*c^3 + 103404*a^4*(-b)^(4
5/4)*c^5 + 343392*a^5*(-b)^(45/4)*c^4 - 252*a^6*(-b)^(41/4)*c^7 + 5376*a^6*
(-b)^(45/4)*c^3 - 16002*a^7*(-b)^(41/4)*c^6 - 5376*a^7*(-b)^(45/4)*c^2 - 45
864*a^8*(-b)^(41/4)*c^5 - 12096*a^9*(-b)^(41/4)*c^4 + 110880*a^2*(-b)^(49/4
)*c^4 + 358400*a^3*(-b)^(49/4)*c^3 + 10458*a^4*(-b)^(45/4)*c^6 + 259644*a^5
*(-b)^(45/4)*c^5 + 359128*a^6*(-b)^(45/4)*c^4 - 756*a^7*(-b)^(41/4)*c^7 + 1
3888*a^7*(-b)^(45/4)*c^3 - 10374*a^8*(-b)^(41/4)*c^6 - 8736*a^9*(-b)^(41/4)
*c^5 + 3080*a^2*(-b)^(49/4)*c^5 + 268800*a^3*(-b)^(49/4)*c^4 + 347200*a^4*(
-b)^(49/4)*c^3 + 51534*a^5*(-b)^(45/4)*c^6 + 343588*a^6*(-b)^(45/4)*c^5 + 2
15040*a^7*(-b)^(45/4)*c^4 - 756*a^8*(-b)^(41/4)*c^7 + 3584*a^8*(-b)^(45/4)*
c^3 - 2520*a^9*(-b)^(41/4)*c^6 - 3584*a^3*(-b)^(49/4)*c^5 + 347200*a^4*(-b)
^(49/4)*c^4 + 2520*a^5*(-b)^(45/4)*c^7 + 201600*a^5*(-b)^(49/4)*c^3 + 10126
2*a^6*(-b)^(45/4)*c^6 + 252840*a^7*(-b)^(45/4)*c^5 + 68544*a^8*(-b)^(45/4)*
c^4 - 252*a^9*(-b)^(41/4)*c^7 - 9926*a^3*(-b)^(49/4)*c^6 - 71568*a^4*(-b)^(
49/4)*c^5 + 252000*a^5*(-b)^(49/4)*c^4 + 9912*a^6*(-b)^(45/4)*c^7 + 64960*a
^6*(-b)^(49/4)*c^3 + 99162*a^7*(-b)^(45/4)*c^6 + 98112*a^8*(-b)^(45/4)*c^5
 + 8960*a^9*(-b)^(45/4)*c^4 - 221760*a^2*(-b)^(53/4)*c^4 - 65898*a^4*(-b)^(4
9/4)*c^6 - 197792*a^5*(-b)^(49/4)*c^5 + 210*a^6*(-b)^(45/4)*c^8 + 97440*a^6
*(-b)^(49/4)*c^4 + 14616*a^7*(-b)^(45/4)*c^7 + 8960*a^7*(-b)^(49/4)*c^3 + 4
8384*a^8*(-b)^(45/4)*c^6 + 15680*a^9*(-b)^(45/4)*c^5 - 121856*a^2*(-b)^(53/
4)*c^5 - 358400*a^3*(-b)^(53/4)*c^4 - 6564*a^4*(-b)^(49/4)*c^7 - 177114*a^5
*(-b)^(49/4)*c^6 - 254968*a^6*(-b)^(49/4)*c^5 + 630*a^7*(-b)^(45/4)*c^8 + 1
5680*a^7*(-b)^(49/4)*c^4 + 9576*a^8*(-b)^(45/4)*c^7 + 9408*a^9*(-b)^(45/4)*
c^6 - 8624*a^2*(-b)^(53/4)*c^6 - 310464*a^3*(-b)^(53/4)*c^5 - 347200*a^4*(-
b)^(53/4)*c^4 - 33276*a^5*(-b)^(49/4)*c^7 - 248206*a^6*(-b)^(49/4)*c^6 - 17
5392*a^7*(-b)^(49/4)*c^5 + 630*a^8*(-b)^(45/4)*c^8 + 2352*a^9*(-b)^(45/4)*c
^7 - 36288*a^3*(-b)^(53/4)*c^6 - 430080*a^4*(-b)^(53/4)*c^5 - 1518*a^5*(-b)
^(49/4)*c^8 - 201600*a^5*(-b)^(53/4)*c^4 - 67164*a^6*(-b)^(49/4)*c^7 - 1920
24*a^7*(-b)^(49/4)*c^6 - 62272*a^8*(-b)^(49/4)*c^5 + 210*a^9*(-b)^(45/4)*c^
8 + 2232*a^3*(-b)^(53/4)*c^7 - 47712*a^4*(-b)^(53/4)*c^6 - 347200*a^5*(-b)^
(53/4)*c^5 - 6042*a^6*(-b)^(49/4)*c^8 - 64960*a^6*(-b)^(53/4)*c^4 - 67476*a
^7*(-b)^(49/4)*c^7 - 77952*a^8*(-b)^(49/4)*c^6 - 8960*a^9*(-b)^(49/4)*c^5 +
 133056*a^2*(-b)^(57/4)*c^5 + 20184*a^4*(-b)^(53/4)*c^7 + 4928*a^5*(-b)^(53
/4)*c^6 - 120*a^6*(-b)^(49/4)*c^9 - 161280*a^6*(-b)^(53/4)*c^5 - 9018*a^7*(
-b)^(49/4)*c^8 - 8960*a^7*(-b)^(53/4)*c^4 - 33744*a^8*(-b)^(49/4)*c^7 - 129
92*a^9*(-b)^(49/4)*c^6 + 77504*a^2*(-b)^(57/4)*c^6 + 215040*a^3*(-b)^(57/4)
*c^5 + 2433*a^4*(-b)^(53/4)*c^8 + 65016*a^5*(-b)^(53/4)*c^7 + 72912*a^6*(-b
)^(53/4)*c^6 - 360*a^7*(-b)^(49/4)*c^9 - 38976*a^7*(-b)^(53/4)*c^5 - 5982*a
^8*(-b)^(49/4)*c^8 - 6720*a^9*(-b)^(49/4)*c^7 + 6960*a^2*(-b)^(57/4)*c^7 +
202944*a^3*(-b)^(57/4)*c^6 + 208320*a^4*(-b)^(57/4)*c^5 + 12999*a^5*(-b)^(5
3/4)*c^8 + 102984*a^6*(-b)^(53/4)*c^7 + 75264*a^7*(-b)^(53/4)*c^6 - 360*a^8
*(-b)^(49/4)*c^9 - 3584*a^8*(-b)^(53/4)*c^5 - 1488*a^9*(-b)^(49/4)*c^8 + 35
072*a^3*(-b)^(57/4)*c^7 + 291200*a^4*(-b)^(57/4)*c^6 + 582*a^5*(-b)^(53/4)*
c^9 + 120960*a^5*(-b)^(57/4)*c^5 + 27471*a^6*(-b)^(53/4)*c^8 + 87216*a^7*(-
b)^(53/4)*c^7 + 32704*a^8*(-b)^(53/4)*c^6 - 120*a^9*(-b)^(49/4)*c^9 + 869*a

^3*(-b)^(57/4)*c^8 + 69600*a^4*(-b)^(57/4)*c^7 + 246400*a^5*(-b)^(57/4)*c^6 + 2358*a^6*(-b)^(53/4)*c^9 + 38976*a^6*(-b)^(57/4)*c^5 + 28749*a^7*(-b)^(53/4)*c^8 + 38016*a^8*(-b)^(53/4)*c^7 + 5376*a^9*(-b)^(53/4)*c^6 - 44352*a^2*(-b)^(61/4)*c^6 + 1335*a^4*(-b)^(57/4)*c^8 + 65088*a^5*(-b)^(57/4)*c^7 + 45*a^6*(-b)^(53/4)*c^10 + 122304*a^6*(-b)^(57/4)*c^6 + 3582*a^7*(-b)^(53/4)*c^9 + 5376*a^7*(-b)^(57/4)*c^5 + 14916*a^8*(-b)^(53/4)*c^8 + 6720*a^9*(-b)^(53/4)*c^7 - 26880*a^2*(-b)^(61/4)*c^7 - 71680*a^3*(-b)^(61/4)*c^6 - 380*a^4*(-b)^(57/4)*c^9 - 5289*a^5*(-b)^(57/4)*c^8 + 22192*a^6*(-b)^(57/4)*c^7 + 135*a^7*(-b)^(53/4)*c^10 + 32704*a^7*(-b)^(57/4)*c^6 + 2418*a^8*(-b)^(53/4)*c^9 + 3072*a^9*(-b)^(53/4)*c^8 - 2668*a^2*(-b)^(61/4)*c^8 - 71616*a^3*(-b)^(61/4)*c^7 - 69440*a^4*(-b)^(61/4)*c^6 - 2416*a^5*(-b)^(57/4)*c^9 - 17171*a^6*(-b)^(57/4)*c^8 - 7872*a^7*(-b)^(57/4)*c^7 + 135*a^8*(-b)^(53/4)*c^10 + 3584*a^8*(-b)^(57/4)*c^6 + 612*a^9*(-b)^(53/4)*c^9 - 14384*a^3*(-b)^(61/4)*c^8 - 104960*a^4*(-b)^(61/4)*c^7 - 123*a^5*(-b)^(57/4)*c^10 - 40320*a^5*(-b)^(61/4)*c^6 - 5784*a^6*(-b)^(57/4)*c^9 - 19464*a^7*(-b)^(57/4)*c^8 - 8256*a^8*(-b)^(57/4)*c^7 + 45*a^9*(-b)^(53/4)*c^10 - 670*a^3*(-b)^(61/4)*c^9 - 31752*a^4*(-b)^(61/4)*c^8 - 91200*a^5*(-b)^(61/4)*c^7 - 517*a^6*(-b)^(57/4)*c^10 - 12992*a^6*(-b)^(61/4)*c^6 - 6656*a^7*(-b)^(57/4)*c^9 - 10032*a^8*(-b)^(57/4)*c^8 - 1792*a^9*(-b)^(57/4)*c^7 + 6336*a^2*(-b)^(65/4)*c^7 - 2830*a^4*(-b)^(61/4)*c^9 - 36272*a^5*(-b)^(61/4)*c^8 - 10*a^6*(-b)^(57/4)*c^11 - 46848*a^6*(-b)^(61/4)*c^7 - 813*a^7*(-b)^(57/4)*c^10 - 1792*a^7*(-b)^(61/4)*c^6 - 3724*a^8*(-b)^(57/4)*c^9 - 1984*a^9*(-b)^(57/4)*c^8 + 3952*a^2*(-b)^(65/4)*c^8 + 10240*a^3*(-b)^(65/4)*c^7 - 43*a^4*(-b)^(61/4)*c^10 - 4374*a^5*(-b)^(61/4)*c^9 - 21868*a^6*(-b)^(61/4)*c^8 - 30*a^7*(-b)^(57/4)*c^11 - 13120*a^7*(-b)^(61/4)*c^7 - 567*a^8*(-b)^(57/4)*c^10 - 816*a^9*(-b)^(57/4)*c^9 + 412*a^2*(-b)^(65/4)*c^9 + 10656*a^3*(-b)^(65/4)*c^8 + 9920*a^4*(-b)^(65/4)*c^7 - 57*a^5*(-b)^(61/4)*c^10 - 2586*a^6*(-b)^(61/4)*c^9 - 5760*a^7*(-b)^(61/4)*c^8 - 30*a^8*(-b)^(57/4)*c^11 - 1536*a^8*(-b)^(61/4)*c^7 - 148*a^9*(-b)^(57/4)*c^10 + 2304*a^3*(-b)^(65/4)*c^9 + 15840*a^4*(-b)^(65/4)*c^8 + 8*a^5*(-b)^(61/4)*c^11 + 5760*a^5*(-b)^(65/4)*c^7 + 183*a^6*(-b)^(61/4)*c^10 + 236*a^7*(-b)^(61/4)*c^9 + 128*a^8*(-b)^(61/4)*c^8 - 10*a^9*(-b)^(57/4)*c^11 + 125*a^3*(-b)^(65/4)*c^10 + 5352*a^4*(-b)^(65/4)*c^9 + 14000*a^5*(-b)^(65/4)*c^8 + 40*a^6*(-b)^(61/4)*c^11 + 1856*a^6*(-b)^(65/4)*c^7 + 461*a^7*(-b)^(61/4)*c^10 + 864*a^8*(-b)^(61/4)*c^9 + 256*a^9*(-b)^(61/4)*c^8 + 595*a^4*(-b)^(65/4)*c^10 + 6608*a^5*(-b)^(65/4)*c^9 + a^6*(-b)^(61/4)*c^12 + 7344*a^6*(-b)^(65/4)*c^8 + 72*a^7*(-b)^(61/4)*c^11 + 256*a^7*(-b)^(65/4)*c^7 + 360*a^8*(-b)^(61/4)*c^10 + 256*a^9*(-b)^(61/4)*c^9 + 18*a^4*(-b)^(65/4)*c^11 + 1131*a^5*(-b)^(65/4)*c^10 + 4572*a^6*(-b)^(65/4)*c^9 + 3*a^7*(-b)^(61/4)*c^12 + 2112*a^7*(-b)^(65/4)*c^8 + 56*a^8*(-b)^(61/4)*c^11 + 96*a^9*(-b)^(61/4)*c^10 + 70*a^5*(-b)^(65/4)*c^11 + 1073*a^6*(-b)^(65/4)*c^10 + 1680*a^7*(-b)^(65/4)*c^9 + 3*a^8*(-b)^(61/4)*c^12 + 256*a^8*(-b)^(65/4)*c^8 + 16*a^9*(-b)^(61/4)*c^11 + a^5*(-b)^(65/4)*c^12 + 102*a^6*(-b)^(65/4)*c^11 + 508*a^7*(-b)^(65/4)*c^10 + 256*a^8*(-b)^(65/4)*c^9 + a^9*(-b)^(61/4)*c^12 + 3*a^6*(-b)^(65/4)*c^12 + 66*a^7*(-b)^(65/4)*c^11 + 96*a^8*(-b)^(65/4)*c^10 + 3*a^7*(-b)^(65/4)*c^12 + 16*a^8*(-b)^(65/4)*c^11 + a^8*(-b)^(65/4)*c^12 - 64*a*(-b)^(37/4)*c + 15232*a*(-b)^(41/4)*c + 6*a^5*(-b)^(21/4)*c + 22*a^6*(-b)^(21/4)*c + 30*a^7*(-b)^(21/4)*c - 116*a^4*(-b)^(25/4)*c + 18*a^8*(-b)^(21/4)*c - 504*a^5*(-b)^(25/4)*c + 4*a^9*(-b)^(21/4)*c - 864*a^6*(-b)^(25/4)*c + 706*a^3*(-b)^(29/4)*c - 728*a^7*(-b)^(25/4)*c + 3698*a^4*(-b)^(29/4)*c - 300*a^8*(-b)^(25/4)*c + 7914*a^5*(-b)^(29/4)*c - 48*a^9*(-b)^(25/4)*c - 1476*a^2*(-b)^(33/4)*c + 8806*a^6*(-b)^(29/4)*c - 9472*a^3*(-b)^(33/4)*c + 5324*a^7*(-b)^(29/4)*c - 25368*a^4*(-b)^(33/4)*c + 1632*a^8*(-b)^(29/4)*c - 36528*a^5*(-b)^(33/4)*c + 192*a^9*(-b)^(29/4)*c + 1536*a^2*(-b)^(37/4)*c - 30212*a^6*(-b)^(33/4)*c + 10176*a^3*(-b)^(37/4)*c - 14064*a^7*(-b)^(33/4)*c + 25600*a^4*(-b)^(37/4)*c - 3264*a^8*(-b)^(33/4)*c + 33600*a^5*(-b)^(37/4)*c - 256*a^9*(-b)^(33/4)*c + 2688*a*(-b)^(41/4)*c^2 + 44352*a^2*(-b)^(41/4)*c + 24576*a^6*(-b)^(37/4)*c + 71680*a^3*(-b)^(41/4)*c + 9536*a^7*(-b)^(37/4)*c + 69440*a^4*(-b)^(41/4)*c + 1536*a^8*(-b)^(37/4)*c + 40320*a^5*(-b)^(41/4)*c - 45696*a*(-b)^(45/4)*c^2 + 12992*a^6*(-b)^(41/4)*c - 10304*a*(-b)

^(45/4)*c^3 + 1792*a^7*(-b)^(41/4)*c + 76160*a*(-b)^(49/4)*c^3 + 19040*a*(-b)^(49/4)*c^4 - 76160*a*(-b)^(53/4)*c^4 - 20160*a*(-b)^(53/4)*c^5 + 45696*a*(-b)^(57/4)*c^5 + 12544*a*(-b)^(57/4)*c^6 - 15232*a*(-b)^(61/4)*c^6 - 4288*a*(-b)^(61/4)*c^7 + 2176*a*(-b)^(65/4)*c^7 + 624*a*(-b)^(65/4)*c^8))/(a^7*b^18 + 9*a^8*b^18*c + 36*a^9*b^18*c^2 + 84*a^10*b^18*c^3 + 126*a^11*b^18*c^4 + 126*a^12*b^18*c^5 + 84*a^13*b^18*c^6 + 36*a^14*b^18*c^7 + 9*a^15*b^18*c^8 + a^16*b^18*c^9) + (-(4*a*b^3 + b^3 + 6*a^2*b^3 + 4*a^3*b^3 + a^4*b^3 + 4*a^2*b^3*c + 6*a^3*b^3*c + 4*a^4*b^3*c + a^5*b^3*c + a*b^3*c)/(a^8*b^4 - a^9*b^3))^(1/4)*((-(4*a*b^3 + b^3 + 6*a^2*b^3 + 4*a^3*b^3 + a^4*b^3 + 4*a^2*b^3*c + 6*a^3*b^3*c + 4*a^4*b^3*c + a^5*b^3*c + a*b^3*c)/(a^8*b^4 - a^9*b^3))^(3/4)*((64*(36*a^12*(-b)^(25/4) - 4*a^13*(-b)^(21/4) + 48*a^13*(-b)^(25/4) - 60*a^11*(-b)^(29/4) - 240*a^12*(-b)^(29/4) - 192*a^13*(-b)^(29/4) - 180*a^10*(-b)^(33/4) - 240*a^11*(-b)^(33/4) + 192*a^12*(-b)^(33/4) + 240*a^9*(-b)^(37/4) + 256*a^13*(-b)^(33/4) + 1200*a^10*(-b)^(37/4) + 1728*a^11*(-b)^(37/4) + 320*a^8*(-b)^(41/4) + 768*a^12*(-b)^(37/4) + 1152*a^9*(-b)^(41/4) + 1344*a^10*(-b)^(41/4) + 512*a^11*(-b)^(41/4) - 144*a^13*(-b)^(29/4)*c^2 + 912*a^12*(-b)^(33/4)*c^2 + 1344*a^13*(-b)^(33/4)*c^2 + 336*a^13*(-b)^(33/4)*c^3 - 432*a^11*(-b)^(37/4)*c^2 - 4032*a^12*(-b)^(37/4)*c^2 - 1680*a^12*(-b)^(37/4)*c^3 - 4032*a^13*(-b)^(37/4)*c^2 - 4624*a^10*(-b)^(41/4)*c^2 - 2688*a^13*(-b)^(37/4)*c^3 - 9408*a^11*(-b)^(41/4)*c^2 - 504*a^13*(-b)^(37/4)*c^4 - 336*a^11*(-b)^(41/4)*c^3 - 1344*a^12*(-b)^(41/4)*c^2 + 3584*a^9*(-b)^(45/4)*c^2 + 5376*a^12*(-b)^(41/4)*c^3 + 3584*a^13*(-b)^(41/4)*c^2 + 20160*a^10*(-b)^(45/4)*c^2 + 1848*a^12*(-b)^(41/4)*c^4 + 6720*a^13*(-b)^(41/4)*c^3 + 8848*a^10*(-b)^(45/4)*c^3 + 30912*a^11*(-b)^(45/4)*c^2 + 3360*a^13*(-b)^(41/4)*c^4 + 6720*a^8*(-b)^(49/4)*c^2 + 21504*a^11*(-b)^(45/4)*c^3 + 14336*a^12*(-b)^(45/4)*c^2 + 504*a^13*(-b)^(41/4)*c^5 + 24192*a^9*(-b)^(49/4)*c^2 + 1848*a^11*(-b)^(45/4)*c^4 + 9408*a^12*(-b)^(45/4)*c^3 - 4032*a^9*(-b)^(49/4)*c^3 + 28224*a^10*(-b)^(49/4)*c^2 - 3360*a^12*(-b)^(45/4)*c^4 - 3584*a^13*(-b)^(45/4)*c^3 - 26880*a^10*(-b)^(49/4)*c^3 + 10752*a^11*(-b)^(49/4)*c^2 - 1176*a^12*(-b)^(45/4)*c^5 - 6720*a^13*(-b)^(45/4)*c^4 - 10584*a^10*(-b)^(49/4)*c^4 - 44352*a^11*(-b)^(49/4)*c^3 - 2688*a^13*(-b)^(45/4)*c^5 - 11200*a^8*(-b)^(53/4)*c^3 - 30240*a^11*(-b)^(49/4)*c^4 - 21504*a^12*(-b)^(49/4)*c^3 - 336*a^13*(-b)^(45/4)*c^6 - 40320*a^9*(-b)^(53/4)*c^3 - 2520*a^11*(-b)^(49/4)*c^5 - 20160*a^12*(-b)^(49/4)*c^4 + 1120*a^9*(-b)^(53/4)*c^4 - 47040*a^10*(-b)^(53/4)*c^3 + 16800*a^10*(-b)^(53/4)*c^4 - 17920*a^11*(-b)^(53/4)*c^3 + 336*a^12*(-b)^(49/4)*c^6 + 4032*a^13*(-b)^(49/4)*c^5 + 8120*a^10*(-b)^(53/4)*c^5 + 33600*a^11*(-b)^(53/4)*c^4 + 1344*a^13*(-b)^(49/4)*c^6 + 11200*a^8*(-b)^(57/4)*c^4 + 26880*a^11*(-b)^(53/4)*c^5 + 17920*a^12*(-b)^(53/4)*c^4 + 144*a^13*(-b)^(49/4)*c^7 + 40320*a^9*(-b)^(57/4)*c^4 + 1680*a^11*(-b)^(53/4)*c^6 + 22848*a^12*(-b)^(53/4)*c^5 + 2240*a^9*(-b)^(57/4)*c^5 + 47040*a^10*(-b)^(57/4)*c^4 + 1344*a^12*(-b)^(53/4)*c^6 + 3584*a^13*(-b)^(53/4)*c^5 + 17920*a^11*(-b)^(57/4)*c^4 + 48*a^12*(-b)^(53/4)*c^7 - 1344*a^13*(-b)^(53/4)*c^6 - 3920*a^10*(-b)^(57/4)*c^6 - 9408*a^11*(-b)^(57/4)*c^5 - 384*a^13*(-b)^(53/4)*c^7 - 6720*a^8*(-b)^(61/4)*c^5 - 14784*a^11*(-b)^(57/4)*c^6 - 7168*a^12*(-b)^(57/4)*c^5 - 36*a^13*(-b)^(53/4)*c^8 - 24192*a^9*(-b)^(61/4)*c^5 - 528*a^11*(-b)^(57/4)*c^7 - 14784*a^12*(-b)^(57/4)*c^6 - 2688*a^9*(-b)^(61/4)*c^6 - 28224*a^10*(-b)^(61/4)*c^5 - 768*a^12*(-b)^(57/4)*c^7 - 3584*a^13*(-b)^(57/4)*c^6 - 6720*a^10*(-b)^(61/4)*c^6 - 10752*a^11*(-b)^(61/4)*c^5 - 60*a^12*(-b)^(57/4)*c^8 + 192*a^13*(-b)^(57/4)*c^7 + 1104*a^10*(-b)^(61/4)*c^7 - 4032*a^11*(-b)^(61/4)*c^6 + 48*a^13*(-b)^(57/4)*c^8 + 2240*a^8*(-b)^(65/4)*c^6 + 4608*a^11*(-b)^(61/4)*c^7 + 4*a^13*(-b)^(57/4)*c^9 + 8064*a^9*(-b)^(65/4)*c^6 + 36*a^11*(-b)^(61/4)*c^8 + 5184*a^12*(-b)^(61/4)*c^7 + 1216*a^9*(-b)^(65/4)*c^7 + 9408*a^10*(-b)^(65/4)*c^6 + 144*a^12*(-b)^(61/4)*c^8 + 1536*a^13*(-b)^(61/4)*c^7 + 3840*a^10*(-b)^(65/4)*c^7 + 3584*a^11*(-b)^(65/4)*c^6 + 12*a^12*(-b)^(61/4)*c^9 - 148*a^10*(-b)^(65/4)*c^8 + 3648*a^11*(-b)^(65/4)*c^7 - 320*a^8*(-b)^(69/4)*c^7 - 624*a^11*(-b)^(65/4)*c^8 + 1024*a^12*(-b)^(65/4)*c^7 - 1152*a^9*(-b)^(69/4)*c^7 + 12*a^11*(-b)^(65/4)*c^9 - 768*a^12*(-b)^(65/4)*c^8 - 208*a^9*(-b)^(69/4)*c^8 - 1344*a^10*(-b)^(69/4)*c^7 - 256*a^13*(-b)^(65/4)*c^8 - 720*a^10*(-b)^(69/4)*c^8 - 5

12*a^11*(-b)^(69/4)*c^7 + 4*a^10*(-b)^(69/4)*c^9 - 768*a^11*(-b)^(69/4)*c^8
 - 256*a^12*(-b)^(69/4)*c^8 + 36*a^13*(-b)^(25/4)*c - 276*a^12*(-b)^(29/4)*
c - 384*a^13*(-b)^(29/4)*c + 300*a^11*(-b)^(33/4)*c + 1536*a^12*(-b)^(33/4)
*c + 1344*a^13*(-b)^(33/4)*c + 1380*a^10*(-b)^(37/4)*c + 2304*a^11*(-b)^(37
/4)*c - 576*a^12*(-b)^(37/4)*c - 1472*a^9*(-b)^(41/4)*c - 1536*a^13*(-b)^(3
7/4)*c - 7680*a^10*(-b)^(41/4)*c - 11328*a^11*(-b)^(41/4)*c - 2240*a^8*(-b)
^(45/4)*c - 5120*a^12*(-b)^(41/4)*c - 8064*a^9*(-b)^(45/4)*c - 9408*a^10*(-
b)^(45/4)*c - 3584*a^11*(-b)^(45/4)*c))/(a^7*b^18 + 9*a^8*b^18*c + 36*a^9*b
^18*c^2 + 84*a^10*b^18*c^3 + 126*a^11*b^18*c^4 + 126*a^12*b^18*c^5 + 84*a^1
3*b^18*c^6 + 36*a^14*b^18*c^7 + 9*a^15*b^18*c^8 + a^16*b^18*c^9) + ((-(4*a*
b^3 + b^3 + 6*a^2*b^3 + 4*a^3*b^3 + a^4*b^3 + 4*a^2*b^3*c + 6*a^3*b^3*c + 4
*a^4*b^3*c + a^5*b^3*c + a*b^3*c)/(a^8*b^4 - a^9*b^3))^(1/4)*(-(b*x - 1)/(c
 + x))^(1/4)*(16*a^13*(-b)^(11/2) - 80*a^12*(-b)^(13/2) - 80*a^11*(-b)^(15/
2) - 128*a^13*(-b)^(13/2) + 400*a^10*(-b)^(17/2) + 128*a^12*(-b)^(15/2) + 8
96*a^9*(-b)^(19/2) + 1152*a^11*(-b)^(17/2) + 256*a^13*(-b)^(15/2) + 512*a^8
*(-b)^(21/2) + 1920*a^10*(-b)^(19/2) + 768*a^12*(-b)^(17/2) + 1024*a^9*(-b)
^(21/2) + 1024*a^11*(-b)^(19/2) + 512*a^10*(-b)^(21/2) + 448*a^13*(-b)^(15/
2)*c^2 - 1344*a^12*(-b)^(17/2)*c^2 - 2624*a^11*(-b)^(19/2)*c^2 - 2688*a^13*
(-b)^(17/2)*c^2 + 1088*a^10*(-b)^(21/2)*c^2 + 128*a^12*(-b)^(19/2)*c^2 - 89
6*a^13*(-b)^(17/2)*c^3 + 9600*a^9*(-b)^(23/2)*c^2 + 9344*a^11*(-b)^(21/2)*c
^2 + 1792*a^12*(-b)^(19/2)*c^3 + 4096*a^13*(-b)^(19/2)*c^2 + 7680*a^8*(-b)^
(25/2)*c^2 + 21888*a^10*(-b)^(23/2)*c^2 + 4096*a^11*(-b)^(21/2)*c^3 + 8704*
a^12*(-b)^(21/2)*c^2 + 4480*a^13*(-b)^(19/2)*c^3 + 15360*a^9*(-b)^(25/2)*c^
2 + 3328*a^10*(-b)^(23/2)*c^3 + 12288*a^11*(-b)^(23/2)*c^2 + 128*a^12*(-b)^
(21/2)*c^3 + 1120*a^13*(-b)^(19/2)*c^4 - 8320*a^9*(-b)^(25/2)*c^3 + 7680*a^
10*(-b)^(25/2)*c^2 - 4992*a^11*(-b)^(23/2)*c^3 - 1120*a^12*(-b)^(21/2)*c^4
 - 6656*a^13*(-b)^(21/2)*c^3 - 10240*a^8*(-b)^(27/2)*c^3 - 21120*a^10*(-b)^(
25/2)*c^3 - 2400*a^11*(-b)^(23/2)*c^4 - 9216*a^12*(-b)^(23/2)*c^3 - 4480*a^
13*(-b)^(21/2)*c^4 - 20480*a^9*(-b)^(27/2)*c^3 - 7200*a^10*(-b)^(25/2)*c^4
 - 12800*a^11*(-b)^(25/2)*c^3 + 1920*a^12*(-b)^(23/2)*c^4 - 896*a^13*(-b)^(2
1/2)*c^5 + 640*a^9*(-b)^(27/2)*c^4 - 10240*a^10*(-b)^(27/2)*c^3 - 3200*a^11
*(-b)^(25/2)*c^4 + 7680*a^13*(-b)^(23/2)*c^4 + 7680*a^8*(-b)^(29/2)*c^4 + 5
760*a^10*(-b)^(27/2)*c^4 - 1280*a^11*(-b)^(25/2)*c^5 + 5120*a^12*(-b)^(25/2
)*c^4 + 2688*a^13*(-b)^(23/2)*c^5 + 15360*a^9*(-b)^(29/2)*c^4 + 5120*a^10*(
-b)^(27/2)*c^5 + 5120*a^11*(-b)^(27/2)*c^4 - 5248*a^12*(-b)^(25/2)*c^5 + 44
8*a^13*(-b)^(23/2)*c^6 + 4224*a^9*(-b)^(29/2)*c^5 + 7680*a^10*(-b)^(29/2)*c
^4 + 3968*a^11*(-b)^(27/2)*c^5 + 448*a^12*(-b)^(25/2)*c^6 - 6656*a^13*(-b)^
(25/2)*c^5 - 3072*a^8*(-b)^(31/2)*c^5 + 5760*a^10*(-b)^(29/2)*c^5 + 2752*a^
11*(-b)^(27/2)*c^6 - 2048*a^12*(-b)^(27/2)*c^5 - 896*a^13*(-b)^(25/2)*c^6 -
 6144*a^9*(-b)^(31/2)*c^5 - 704*a^10*(-b)^(29/2)*c^6 + 1536*a^11*(-b)^(29/2
)*c^5 + 5504*a^12*(-b)^(27/2)*c^6 - 128*a^13*(-b)^(25/2)*c^7 - 2944*a^9*(-b
)^(31/2)*c^6 - 3072*a^10*(-b)^(31/2)*c^5 + 384*a^11*(-b)^(29/2)*c^6 - 256*a
^12*(-b)^(27/2)*c^7 + 4096*a^13*(-b)^(27/2)*c^6 + 512*a^8*(-b)^(33/2)*c^6 -
 4992*a^10*(-b)^(31/2)*c^6 - 1536*a^11*(-b)^(29/2)*c^7 + 1536*a^12*(-b)^(29
/2)*c^6 + 128*a^13*(-b)^(27/2)*c^7 + 1024*a^9*(-b)^(33/2)*c^6 - 768*a^10*(-
b)^(31/2)*c^7 - 2048*a^11*(-b)^(31/2)*c^6 - 2688*a^12*(-b)^(29/2)*c^7 + 16*
a^13*(-b)^(27/2)*c^8 + 640*a^9*(-b)^(33/2)*c^7 + 512*a^10*(-b)^(33/2)*c^6 -
 1664*a^11*(-b)^(31/2)*c^7 + 48*a^12*(-b)^(29/2)*c^8 - 1536*a^13*(-b)^(29/2
)*c^7 + 1152*a^10*(-b)^(33/2)*c^7 + 304*a^11*(-b)^(31/2)*c^8 - 1024*a^12*(-
b)^(31/2)*c^7 + 272*a^10*(-b)^(33/2)*c^8 + 512*a^11*(-b)^(33/2)*c^7 + 512*a
^12*(-b)^(31/2)*c^8 + 512*a^11*(-b)^(33/2)*c^8 + 256*a^13*(-b)^(31/2)*c^8 +
 256*a^12*(-b)^(33/2)*c^8 - 128*a^13*(-b)^(13/2)*c + 512*a^12*(-b)^(15/2)*c
 + 768*a^11*(-b)^(17/2)*c + 896*a^13*(-b)^(15/2)*c - 1536*a^10*(-b)^(19/2)*
c - 384*a^12*(-b)^(17/2)*c - 4736*a^9*(-b)^(21/2)*c - 5504*a^11*(-b)^(19/2)
*c - 1536*a^13*(-b)^(17/2)*c - 3072*a^8*(-b)^(23/2)*c - 10368*a^10*(-b)^(21
/2)*c - 4096*a^12*(-b)^(19/2)*c - 6144*a^9*(-b)^(23/2)*c - 5632*a^11*(-b)^(
21/2)*c - 3072*a^10*(-b)^(23/2)*c)*64i)/((-b)^(1/4)*(a^6*b^17 + 9*a^7*b^17*
c + 36*a^8*b^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c
^5 + 84*a^12*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9)

))*1i + (64*(-(b*x - 1)/(c + x))^(1/4)*(512*(-b)^(19/2) + a^5*(-b)^(9/2) + a^4*(-b)^(11/2) + 2*a^6*(-b)^(9/2) - 16*a^3*(-b)^(13/2) + 18*a^5*(-b)^(11/2) + a^7*(-b)^(9/2) - 144*a^2*(-b)^(15/2) - 176*a^4*(-b)^(13/2) + 49*a^6*(-b)^(11/2) - 576*a^3*(-b)^(15/2) - 560*a^5*(-b)^(13/2) + 48*a^7*(-b)^(11/2) + 2688*a^2*(-b)^(17/2) - 608*a^4*(-b)^(15/2) - 784*a^6*(-b)^(13/2) + 16*a^8*(-b)^(11/2) + 7680*a^3*(-b)^(17/2) + 448*a^5*(-b)^(15/2) - 512*a^7*(-b)^(13/2) + 7680*a^2*(-b)^(19/2) + 11520*a^4*(-b)^(17/2) + 1392*a^6*(-b)^(15/2) - 128*a^8*(-b)^(13/2) + 10240*a^3*(-b)^(19/2) + 9600*a^5*(-b)^(17/2) + 1024*a^7*(-b)^(15/2) + 7680*a^4*(-b)^(19/2) + 4224*a^6*(-b)^(17/2) + 256*a^8*(-b)^(15/2) + 3072*a^5*(-b)^(19/2) + 768*a^7*(-b)^(17/2) + 512*a^6*(-b)^(19/2) + 7680*(-b)^(23/2)*c^2 - 10240*(-b)^(25/2)*c^3 + 7680*(-b)^(27/2)*c^4 - 3072*(-b)^(29/2)*c^5 + 512*(-b)^(31/2)*c^6 + 384*a*(-b)^(17/2) + 3072*a*(-b)^(19/2) - 3072*(-b)^(21/2)*c + a^7*(-b)^(9/2)*c^2 - 35*a^6*(-b)^(11/2)*c^2 + 2*a^8*(-b)^(9/2)*c^2 + 265*a^5*(-b)^(13/2)*c^2 - 86*a^7*(-b)^(11/2)*c^2 + a^9*(-b)^(9/2)*c^2 - 851*a^4*(-b)^(15/2)*c^2 + 738*a^6*(-b)^(13/2)*c^2 - 10*a^7*(-b)^(11/2)*c^3 - 67*a^8*(-b)^(11/2)*c^2 + 2496*a^3*(-b)^(17/2)*c^2 - 2566*a^5*(-b)^(15/2)*c^2 + 224*a^6*(-b)^(13/2)*c^3 + 649*a^7*(-b)^(13/2)*c^2 - 20*a^8*(-b)^(11/2)*c^3 - 16*a^9*(-b)^(11/2)*c^2 - 5184*a^2*(-b)^(19/2)*c^2 + 10432*a^4*(-b)^(17/2)*c^2 - 1358*a^5*(-b)^(15/2)*c^3 - 1907*a^6*(-b)^(15/2)*c^2 + 592*a^7*(-b)^(13/2)*c^3 + 144*a^8*(-b)^(13/2)*c^2 - 10*a^9*(-b)^(11/2)*c^3 - 31104*a^3*(-b)^(19/2)*c^2 + 3784*a^4*(-b)^(17/2)*c^3 + 14912*a^5*(-b)^(17/2)*c^2 - 4364*a^6*(-b)^(15/2)*c^3 + 45*a^7*(-b)^(13/2)*c^4 + 1120*a^7*(-b)^(15/2)*c^2 + 512*a^8*(-b)^(13/2)*c^3 - 32*a^9*(-b)^(13/2)*c^2 + 1152*a^2*(-b)^(21/2)*c^2 - 7552*a^3*(-b)^(19/2)*c^3 - 74624*a^4*(-b)^(19/2)*c^2 + 14288*a^5*(-b)^(17/2)*c^3 - 771*a^6*(-b)^(15/2)*c^4 + 5824*a^6*(-b)^(17/2)*c^2 - 4974*a^7*(-b)^(15/2)*c^3 + 90*a^8*(-b)^(13/2)*c^4 + 1952*a^8*(-b)^(15/2)*c^2 + 144*a^9*(-b)^(13/2)*c^3 + 6912*a^2*(-b)^(21/2)*c^3 + 23040*a^3*(-b)^(21/2)*c^2 - 36800*a^4*(-b)^(19/2)*c^3 + 3874*a^5*(-b)^(17/2)*c^4 - 91136*a^5*(-b)^(19/2)*c^2 + 19144*a^6*(-b)^(17/2)*c^3 - 2118*a^7*(-b)^(15/2)*c^4 - 5120*a^7*(-b)^(17/2)*c^2 - 2288*a^8*(-b)^(15/2)*c^3 + 45*a^9*(-b)^(13/2)*c^4 + 640*a^9*(-b)^(15/2)*c^2 + 115200*a^2*(-b)^(23/2)*c^2 + 49536*a^3*(-b)^(21/2)*c^3 - 8750*a^4*(-b)^(19/2)*c^4 + 57600*a^4*(-b)^(21/2)*c^2 - 68032*a^5*(-b)^(19/2)*c^3 + 13444*a^6*(-b)^(17/2)*c^4 - 58944*a^6*(-b)^(19/2)*c^2 - 120*a^7*(-b)^(15/2)*c^5 + 9664*a^7*(-b)^(17/2)*c^3 - 1923*a^8*(-b)^(15/2)*c^4 - 5248*a^8*(-b)^(17/2)*c^2 - 320*a^9*(-b)^(15/2)*c^3 + 44160*a^2*(-b)^(23/2)*c^3 + 11040*a^3*(-b)^(21/2)*c^4 + 153600*a^3*(-b)^(23/2)*c^2 + 137728*a^4*(-b)^(21/2)*c^3 - 36988*a^5*(-b)^(19/2)*c^4 + 63360*a^5*(-b)^(21/2)*c^2 + 1644*a^6*(-b)^(17/2)*c^5 - 56512*a^6*(-b)^(19/2)*c^3 + 17058*a^7*(-b)^(17/2)*c^4 - 18560*a^7*(-b)^(19/2)*c^2 - 240*a^8*(-b)^(15/2)*c^5 + 128*a^8*(-b)^(17/2)*c^3 - 576*a^9*(-b)^(15/2)*c^4 - 1280*a^9*(-b)^(17/2)*c^2 - 480*a^2*(-b)^(23/2)*c^4 + 76800*a^3*(-b)^(23/2)*c^3 + 60640*a^4*(-b)^(21/2)*c^4 + 115200*a^4*(-b)^(23/2)*c^2 - 6776*a^5*(-b)^(19/2)*c^5 + 193792*a^5*(-b)^(21/2)*c^3 - 59150*a^6*(-b)^(19/2)*c^4 + 33408*a^6*(-b)^(21/2)*c^2 + 4632*a^7*(-b)^(17/2)*c^5 - 16064*a^7*(-b)^(19/2)*c^3 + 9280*a^8*(-b)^(17/2)*c^4 - 2048*a^8*(-b)^(19/2)*c^2 - 120*a^9*(-b)^(15/2)*c^5 - 896*a^9*(-b)^(17/2)*c^3 - 153600*a^2*(-b)^(25/2)*c^3 - 23040*a^3*(-b)^(23/2)*c^4 + 11620*a^4*(-b)^(21/2)*c^5 + 57600*a^4*(-b)^(23/2)*c^3 + 128864*a^5*(-b)^(21/2)*c^4 + 46080*a^5*(-b)^(23/2)*c^2 - 24752*a^6*(-b)^(19/2)*c^5 + 146688*a^6*(-b)^(21/2)*c^3 + 210*a^7*(-b)^(17/2)*c^6 - 43232*a^7*(-b)^(19/2)*c^4 + 6912*a^7*(-b)^(21/2)*c^2 + 4332*a^8*(-b)^(17/2)*c^5 + 3968*a^8*(-b)^(19/2)*c^3 + 1792*a^9*(-b)^(17/2)*c^4 - 90240*a^2*(-b)^(25/2)*c^4 - 7104*a^3*(-b)^(23/2)*c^5 - 204800*a^3*(-b)^(25/2)*c^3 - 100160*a^4*(-b)^(23/2)*c^4 + 53480*a^5*(-b)^(21/2)*c^5 + 9600*a^5*(-b)^(23/2)*c^3 - 2310*a^6*(-b)^(19/2)*c^6 + 131872*a^6*(-b)^(21/2)*c^4 + 7680*a^6*(-b)^(23/2)*c^2 - 33432*a^7*(-b)^(19/2)*c^5 + 56704*a^7*(-b)^(21/2)*c^3 + 420*a^8*(-b)^(17/2)*c^6 - 13216*a^8*(-b)^(19/2)*c^4 + 1344*a^9*(-b)^(17/2)*c^5 + 2304*a^9*(-b)^(19/2)*c^3 - 9216*a^2*(-b)^(25/2)*c^5 - 192000*a^3*(-b)^(25/2)*c^4 - 48224*a^4*(-b)^(23/2)*c^5 - 153600*a^4*(-b)^(25/2)*c^3 + 7546*a^5*(-b)^(21/2)*c^6 - 179840*a^5*(-b)^(23/2)*c^4 + 94948*a^6*(-b)^(21/2)*c^5 - 9600*a^6*(-b)^(23/2)*c^3 - 6636 *a^7*(-b)^(19/2)*c^6 + 63232*a^7*(-b)^(21/2)*c^4 - 19712*a^8*(-b)^(19/2)*c^5 + 8704*a^8*(-b)^(21/2)*c^3 + 210*a^9*(-b)^(17/2)*c^6 - 896*a^9*(-b)^(19/2)*c^4 + 115200*a^2*(-b)^(27/2)*c^4 - 31104*a^3*(-b)^(25/2)*c^5 - 8750*a^4*(-b)^(23/2)*c^6 - 211200*a^4*(-b)^(25/2)*c^4 - 121120*a^5*(-b)^(23/2)*c^5 - 61440*a^5*(-b)^(25/2)*c^3 + 28756*a^6*(-b)^(21/2)*c^6 - 161760*a^6*(-b)^(23/2)*c^4 - 252*a^7*(-b)^(19/2)*c^7 + 80416*a^7*(-b)^(21/2)*c^5 - 3840*a^7*(-b)^(23/2)*c^3 - 6342*a^8*(-b)^(19/2)*c^6 + 9344*a^8*(-b)^(21/2)*c^4 - 4256*a^9*(-b)^(19/2)*c^5 + 81792*a^2*(-b)^(27/2)*c^5 - 832*a^3*(-b)^(25/2)*c^6 + 153600*a^3*(-b)^(27/2)*c^4 - 23552*a^4*(-b)^(25/2)*c^5 - 44380*a^5*(-b)^(23/2)*c^6 - 124800*a^5*(-b)^(25/2)*c^4 + 2184*a^6*(-b)^(21/2)*c^7 - 146336*a^6*(-b)^(23/2)*c^5 - 10240*a^6*(-b)^(25/2)*c^3 + 40698*a^7*(-b)^(21/2)*c^6 - 72320*a^7*(-b)^(23/2)*c^4 - 504*a^8*(-b)^(19/2)*c^7 + 31808*a^8*(-b)^(21/2)*c^5 - 2016*a^9*(-b)^(19/2)*c^6 - 1280*a^9*(-b)^(21/2)*c^4 + 10944*a^2*(-b)^(27/2)*c^6 + 184320*a^3*(-b)^(27/2)*c^5 + 8896*a^4*(-b)^(25/2)*c^6 + 115200*a^4*(-b)^(27/2)*c^4 - 5300*a^5*(-b)^(23/2)*c^7 + 32512*a^5*(-b)^(25/2)*c^5 - 86702*a^6*(-b)^(23/2)*c^6 - 36480*a^6*(-b)^(25/2)*c^4 + 6384*a^7*(-b)^(21/2)*c^7 - 87968*a^7*(-b)^(23/2)*c^5 + 25312*a^8*(-b)^(21/2)*c^6 - 12800*a^8*(-b)^(23/2)*c^4 - 252*a^9*(-b)^(19/2)*c^7 + 4480*a^9*(-b)^(21/2)*c^5 - 46080*a^2*(-b)^(29/2)*c^5 + 49536*a^3*(-b)^(27/2)*c^6 + 3016*a^4*(-b)^(25/2)*c^7 + 218880*a^4*(-b)^(27/2)*c^5 + 44864*a^5*(-b)^(25/2)*c^6 + 46080*a^5*(-b)^(27/2)*c^4 - 21128*a^6*(-b)^(23/2)*c^7 + 66048*a^6*(-b)^(25/2)*c^5 + 210*a^7*(-b)^(21/2)*c^8 - 81536*a^7*(-b)^(23/2)*c^6 - 3840*a^7*(-b)^(25/2)*c^4 + 6216*a^8*(-b)^(21/2)*c^7 - 22912*a^8*(-b)^(23/2)*c^5 + 5824*a^9*(-b)^(21/2)*c^6 - 36480*a^2*(-b)^(29/2)*c^6 + 4224*a^3*(-b)^(27/2)*c^7 - 61440*a^3*(-b)^(29/2)*c^5 + 86656*a^4*(-b)^(27/2)*c^6 + 18896*a^5*(-b)^(25/2)*c^7 + 144000*a^5*(-b)^(27/2)*c^5 - 1374*a^6*(-b)^(23/2)*c^8 + 75200*a^6*(-b)^(25/2)*c^6 + 7680*a^6*(-b)^(27/2)*c^4 - 31284*a^7*(-b)^(23/2)*c^7 + 40576*a^7*(-b)^(25/2)*c^5 + 420*a^8*(-b)^(21/2)*c^8 - 36736*a^8*(-b)^(23/2)*c^6 + 2016*a^9*(-b)^(21/2)*c^7 - 1280*a^9*(-b)^(23/2)*c^5 - 5376*a^2*(-b)^(29/2)*c^7 - 84480*a^3*(-b)^(29/2)*c^6 + 13888*a^4*(-b)^(27/2)*c^7 - 46080*a^4*(-b)^(29/2)*c^5 + 2173*a^5*(-b)^(25/2)*c^8 + 70144*a^5*(-b)^(27/2)*c^6 + 42952*a^6*(-b)^(25/2)*c^7 + 49536*a^6*(-b)^(27/2)*c^5 - 4092*a^7*(-b)^(23/2)*c^8 + 57856*a^7*(-b)^(25/2)*c^6 - 20384*a^8*(-b)^(23/2)*c^7 + 8704*a^8*(-b)^(25/2)*c^5 + 210*a^9*(-b)^(21/2)*c^8 - 6272*a^9*(-b)^(23/2)*c^6 + 7680*a^2*(-b)^(31/2)*c^6 - 26496*a^3*(-b)^(29/2)*c^7 + 301*a^4*(-b)^(27/2)*c^8 - 103680*a^4*(-b)^(29/2)*c^6 + 12608*a^5*(-b)^(27/2)*c^7 - 18432*a^5*(-b)^(29/2)*c^5 + 9274*a^6*(-b)^(25/2)*c^8 + 21696*a^6*(-b)^(27/2)*c^6 - 120*a^7*(-b)^(23/2)*c^9 + 45760*a^7*(-b)^(25/2)*c^7 + 6912*a^7*(-b)^(27/2)*c^5 - 4062*a^8*(-b)^(23/2)*c^8 + 20096*a^8*(-b)^(25/2)*c^6 - 4928*a^9*(-b)^(23/2)*c^7 + 6528*a^2*(-b)^(31/2)*c^7 - 2448*a^3*(-b)^(29/2)*c^8 + 10240*a^3*(-b)^(31/2)*c^6 - 52736*a^4*(-b)^(29/2)*c^7 - 1558*a^5*(-b)^(27/2)*c^8 - 71040*a^5*(-b)^(29/2)*c^6 + 546*a^6*(-b)^(25/2)*c^9 - 4544*a^6*(-b)^(27/2)*c^7 - 3072*a^6*(-b)^(29/2)*c^5 + 14589*a^7*(-b)^(25/2)*c^8 - 2432*a^7*(-b)^(27/2)*c^6 - 240*a^8*(-b)^(23/2)*c^9 + 23168*a^8*(-b)^(25/2)*c^7 - 1344*a^9*(-b)^(23/2)*c^8 + 2304*a^9*(-b)^(25/2)*c^6 + 1008*a^2*(-b)^(31/2)*c^8 + 15360*a^3*(-b)^(31/2)*c^7 - 10160*a^4*(-b)^(29/2)*c^8 + 7680*a^4*(-b)^(31/2)*c^6 - 384*a^5*(-b)^(27/2)*c^9 - 53504*a^5*(-b)^(29/2)*c^7 - 8099*a^6*(-b)^(27/2)*c^8 - 25728*a^6*(-b)^(29/2)*c^6 + 1668*a^7*(-b)^(25/2)*c^9 - 13760*a^7*(-b)^(27/2)*c^7 + 10048*a^8*(-b)^(25/2)*c^8 - 2048*a^8*(-b)^(27/2)*c^6 - 120*a^9*(-b)^(23/2)*c^9 + 4480*a^9*(-b)^(25/2)*c^7 + 5184*a^3*(-b)^(31/2)*c^8 - 570*a^4*(-b)^(29/2)*c^9 + 19200*a^4*(-b)^(31/2)*c^7 - 16048*a^5*(-b)^(29/2)*c^8 + 3072*a^5*(-b)^(31/2)*c^6 - 1984*a^6*(-b)^(27/2)*c^9 - 28416*a^6*(-b)^(29/2)*c^7 + 45*a^7*(-b)^(25/2)*c^10 - 11984*a^7*(-b)^(27/2)*c^8 - 3840*a^7*(-b)^(29/2)*c^6 + 1698*a^8*(-b)^(25/2)*c^9 - 7552*a^8*(-b)^(27/2)*c^7 + 2560*a^9*(-b)^(25/2)*c^8 + 480*a^3*(-b)^(31/2)*c^9 + 10912*a^4*(-b)^(31/2)*c^8 - 1732*a^5*(-b)^(29/2)*c^9 + 13440*a^5*(-b)^(31/2)*c^7 - 119*a^6*(-b)^(27/2)*c^10 - 11408*a^6*(-b)^(29/2)*c^8 + 512*a^6*(-b)^(31/2)*c^6 - 3568*a^7*(-b)^(27/2)*c^9 - 7040*a^7*(-b)^(29/2)*c^7 + 90*a^8*(-b)^(25/2)*c^10 - 7408*a^8*(-b)^(27/2)*c^8 + 576*a^9*(-b)^(25/2)*c^9 - 1280*a^9*(-b)^(27/2)*c^7 + 2160*a^4*(-

b)^(31/2)*c^9 - 35*a^5*(-b)^(29/2)*c^10 + 11968*a^5*(-b)^(31/2)*c^8 - 1530*
a^6*(-b)^(29/2)*c^9 + 4992*a^6*(-b)^(31/2)*c^7 - 382*a^7*(-b)^(27/2)*c^10 -
2816*a^7*(-b)^(29/2)*c^8 - 2720*a^8*(-b)^(27/2)*c^9 - 512*a^8*(-b)^(29/2)*
c^7 + 45*a^9*(-b)^(25/2)*c^10 - 1664*a^9*(-b)^(27/2)*c^8 + 129*a^4*(-b)^(31
/2)*c^10 + 3856*a^5*(-b)^(31/2)*c^9 + 10*a^6*(-b)^(29/2)*c^10 + 7152*a^6*(-
b)^(31/2)*c^8 - 10*a^7*(-b)^(27/2)*c^11 + 112*a^7*(-b)^(29/2)*c^9 + 768*a^7
*(-b)^(31/2)*c^7 - 407*a^8*(-b)^(27/2)*c^10 + 512*a^8*(-b)^(29/2)*c^8 - 752
a^9(-b)^(27/2)*c^9 + 482*a^5*(-b)^(31/2)*c^10 + 8*a^6*(-b)^(29/2)*c^11 +
3408*a^6*(-b)^(31/2)*c^9 + 221*a^7*(-b)^(29/2)*c^10 + 2176*a^7*(-b)^(31/2)*
c^8 - 20*a^8*(-b)^(27/2)*c^11 + 736*a^8*(-b)^(29/2)*c^9 - 144*a^9*(-b)^(27/
2)*c^10 + 256*a^9*(-b)^(29/2)*c^8 + 18*a^5*(-b)^(31/2)*c^11 + 673*a^6*(-b)^
(31/2)*c^10 + 32*a^7*(-b)^(29/2)*c^11 + 1488*a^7*(-b)^(31/2)*c^9 + 272*a^8*
(-b)^(29/2)*c^10 + 256*a^8*(-b)^(31/2)*c^8 - 10*a^9*(-b)^(27/2)*c^11 + 256*
a^9*(-b)^(29/2)*c^9 + 52*a^6*(-b)^(31/2)*c^11 + a^7*(-b)^(29/2)*c^12 + 416*
a^7*(-b)^(31/2)*c^10 + 40*a^8*(-b)^(29/2)*c^11 + 256*a^8*(-b)^(31/2)*c^9 +
96*a^9*(-b)^(29/2)*c^10 + a^6*(-b)^(31/2)*c^12 + 50*a^7*(-b)^(31/2)*c^11 +
2*a^8*(-b)^(29/2)*c^12 + 96*a^8*(-b)^(31/2)*c^10 + 16*a^9*(-b)^(29/2)*c^11
+ 2*a^7*(-b)^(31/2)*c^12 + 16*a^8*(-b)^(31/2)*c^11 + a^9*(-b)^(29/2)*c^12 +
a^8*(-b)^(31/2)*c^12 - 1152*a*(-b)^(19/2)*c - 18432*a*(-b)^(21/2)*c + 2*a^
6*(-b)^(9/2)*c - 24*a^5*(-b)^(11/2)*c + 4*a^7*(-b)^(9/2)*c + 70*a^4*(-b)^(1
3/2)*c - 48*a^6*(-b)^(11/2)*c + 2*a^8*(-b)^(9/2)*c - 288*a^3*(-b)^(15/2)*c
+ 60*a^5*(-b)^(13/2)*c - 8*a^7*(-b)^(11/2)*c + 1536*a^2*(-b)^(17/2)*c - 656
a^4(-b)^(15/2)*c - 378*a^6*(-b)^(13/2)*c + 32*a^8*(-b)^(11/2)*c + 8064*a^
3*(-b)^(17/2)*c + 656*a^5*(-b)^(15/2)*c - 784*a^7*(-b)^(13/2)*c + 16*a^9*(-
b)^(11/2)*c - 9600*a^2*(-b)^(19/2)*c + 16384*a^4*(-b)^(17/2)*c + 3280*a^6*(
-b)^(15/2)*c - 544*a^8*(-b)^(13/2)*c - 30720*a^3*(-b)^(19/2)*c + 15616*a^5*
(-b)^(17/2)*c + 3664*a^7*(-b)^(15/2)*c - 128*a^9*(-b)^(13/2)*c - 1152*a*(-b
)^(21/2)*c^2 - 46080*a^2*(-b)^(21/2)*c - 49920*a^4*(-b)^(19/2)*c + 6144*a^6
*(-b)^(17/2)*c + 1664*a^8*(-b)^(15/2)*c - 61440*a^3*(-b)^(21/2)*c - 44160*a
^5*(-b)^(19/2)*c - 128*a^7*(-b)^(17/2)*c + 256*a^9*(-b)^(15/2)*c + 46080*a*
(-b)^(23/2)*c^2 - 46080*a^4*(-b)^(21/2)*c - 20352*a^6*(-b)^(19/2)*c - 512*a
^8*(-b)^(17/2)*c + 9600*a*(-b)^(23/2)*c^3 - 18432*a^5*(-b)^(21/2)*c - 3840*
a^7*(-b)^(19/2)*c - 3072*a^6*(-b)^(21/2)*c - 61440*a*(-b)^(25/2)*c^3 - 1728
0*a*(-b)^(25/2)*c^4 + 46080*a*(-b)^(27/2)*c^4 + 14976*a*(-b)^(27/2)*c^5 - 1
8432*a*(-b)^(29/2)*c^5 - 6528*a*(-b)^(29/2)*c^6 + 3072*a*(-b)^(31/2)*c^6 +
1152*a*(-b)^(31/2)*c^7))/((-b)^(1/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^8*b^17
*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a^12*b^
17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9)))*1i))*(-(4*a*
b^3 + b^3 + 6*a^2*b^3 + 4*a^3*b^3 + a^4*b^3 + 4*a^2*b^3*c + 6*a^3*b^3*c + 4
*a^4*b^3*c + a^5*b^3*c + a*b^3*c)/(a^8*b^4 - a^9*b^3))^(1/4) + (atan(((((-b
)^(7/4)*1i - (a*(-b)^(3/4)*(4*b + b*c + 1)*1i)/4)*((((-b)^(7/4)*1i - (a*(-b
)^(3/4)*(4*b + b*c + 1)*1i)/4)^3*((64*(36*a^12*(-b)^(25/4) - 4*a^13*(-b)^(2
1/4) + 48*a^13*(-b)^(25/4) - 60*a^11*(-b)^(29/4) - 240*a^12*(-b)^(29/4) - 1
92*a^13*(-b)^(29/4) - 180*a^10*(-b)^(33/4) - 240*a^11*(-b)^(33/4) + 192*a^1
2*(-b)^(33/4) + 240*a^9*(-b)^(37/4) + 256*a^13*(-b)^(33/4) + 1200*a^10*(-b)
^(37/4) + 1728*a^11*(-b)^(37/4) + 320*a^8*(-b)^(41/4) + 768*a^12*(-b)^(37/4
) + 1152*a^9*(-b)^(41/4) + 1344*a^10*(-b)^(41/4) + 512*a^11*(-b)^(41/4) - 1
44*a^13*(-b)^(29/4)*c^2 + 912*a^12*(-b)^(33/4)*c^2 + 1344*a^13*(-b)^(33/4)*
c^2 + 336*a^13*(-b)^(33/4)*c^3 - 432*a^11*(-b)^(37/4)*c^2 - 4032*a^12*(-b)^
(37/4)*c^2 - 1680*a^12*(-b)^(37/4)*c^3 - 4032*a^13*(-b)^(37/4)*c^2 - 4624*a
^10*(-b)^(41/4)*c^2 - 2688*a^13*(-b)^(37/4)*c^3 - 9408*a^11*(-b)^(41/4)*c^2
- 504*a^13*(-b)^(37/4)*c^4 - 336*a^11*(-b)^(41/4)*c^3 - 1344*a^12*(-b)^(41
/4)*c^2 + 3584*a^9*(-b)^(45/4)*c^2 + 5376*a^12*(-b)^(41/4)*c^3 + 3584*a^13*
(-b)^(41/4)*c^2 + 20160*a^10*(-b)^(45/4)*c^2 + 1848*a^12*(-b)^(41/4)*c^4 +
6720*a^13*(-b)^(41/4)*c^3 + 8848*a^10*(-b)^(45/4)*c^3 + 30912*a^11*(-b)^(45
/4)*c^2 + 3360*a^13*(-b)^(41/4)*c^4 + 6720*a^8*(-b)^(49/4)*c^2 + 21504*a^11
*(-b)^(45/4)*c^3 + 14336*a^12*(-b)^(45/4)*c^2 + 504*a^13*(-b)^(41/4)*c^5 +
24192*a^9*(-b)^(49/4)*c^2 + 1848*a^11*(-b)^(45/4)*c^4 + 9408*a^12*(-b)^(45/
4)*c^3 - 4032*a^9*(-b)^(49/4)*c^3 + 28224*a^10*(-b)^(49/4)*c^2 - 3360*a^12*

(-b)^(45/4)*c^4 - 3584*a^13*(-b)^(45/4)*c^3 - 26880*a^10*(-b)^(49/4)*c^3 + 10752*a^11*(-b)^(49/4)*c^2 - 1176*a^12*(-b)^(45/4)*c^5 - 6720*a^13*(-b)^(45/4)*c^4 - 10584*a^10*(-b)^(49/4)*c^4 - 44352*a^11*(-b)^(49/4)*c^3 - 2688*a^13*(-b)^(45/4)*c^5 - 11200*a^8*(-b)^(53/4)*c^3 - 30240*a^11*(-b)^(49/4)*c^4 - 21504*a^12*(-b)^(49/4)*c^3 - 336*a^13*(-b)^(45/4)*c^6 - 40320*a^9*(-b)^(53/4)*c^3 - 2520*a^11*(-b)^(49/4)*c^5 - 20160*a^12*(-b)^(49/4)*c^4 + 1120*a^9*(-b)^(53/4)*c^4 - 47040*a^10*(-b)^(53/4)*c^3 + 16800*a^10*(-b)^(53/4)*c^4 - 17920*a^11*(-b)^(53/4)*c^3 + 336*a^12*(-b)^(49/4)*c^6 + 4032*a^13*(-b)^(49/4)*c^5 + 8120*a^10*(-b)^(53/4)*c^5 + 33600*a^11*(-b)^(53/4)*c^4 + 1344*a^13*(-b)^(49/4)*c^6 + 11200*a^8*(-b)^(57/4)*c^4 + 26880*a^11*(-b)^(53/4)*c^5 + 17920*a^12*(-b)^(53/4)*c^4 + 144*a^13*(-b)^(49/4)*c^7 + 40320*a^9*(-b)^(57/4)*c^4 + 1680*a^11*(-b)^(53/4)*c^6 + 22848*a^12*(-b)^(53/4)*c^5 + 2240*a^9*(-b)^(57/4)*c^5 + 47040*a^10*(-b)^(57/4)*c^4 + 1344*a^12*(-b)^(53/4)*c^6 + 3584*a^13*(-b)^(53/4)*c^5 + 17920*a^11*(-b)^(57/4)*c^4 + 48*a^12*(-b)^(53/4)*c^7 - 1344*a^13*(-b)^(53/4)*c^6 - 3920*a^10*(-b)^(57/4)*c^6 - 9408*a^11*(-b)^(57/4)*c^5 - 384*a^13*(-b)^(53/4)*c^7 - 6720*a^8*(-b)^(61/4)*c^5 - 14784*a^11*(-b)^(57/4)*c^6 - 7168*a^12*(-b)^(57/4)*c^5 - 36*a^13*(-b)^(53/4)*c^8 - 24192*a^9*(-b)^(61/4)*c^5 - 528*a^11*(-b)^(57/4)*c^7 - 14784*a^12*(-b)^(57/4)*c^6 - 2688*a^9*(-b)^(61/4)*c^6 - 28224*a^10*(-b)^(61/4)*c^5 - 768*a^12*(-b)^(57/4)*c^7 - 3584*a^13*(-b)^(57/4)*c^6 - 6720*a^10*(-b)^(61/4)*c^6 - 10752*a^11*(-b)^(61/4)*c^5 - 60*a^12*(-b)^(57/4)*c^8 + 192*a^13*(-b)^(57/4)*c^7 + 1104*a^10*(-b)^(61/4)*c^7 - 4032*a^11*(-b)^(61/4)*c^6 + 48*a^13*(-b)^(57/4)*c^8 + 2240*a^8*(-b)^(65/4)*c^6 + 4608*a^11*(-b)^(61/4)*c^7 + 4*a^13*(-b)^(57/4)*c^9 + 8064*a^9*(-b)^(65/4)*c^6 + 36*a^11*(-b)^(61/4)*c^8 + 5184*a^12*(-b)^(61/4)*c^7 + 1216*a^9*(-b)^(65/4)*c^7 + 9408*a^10*(-b)^(65/4)*c^6 + 144*a^12*(-b)^(61/4)*c^8 + 1536*a^13*(-b)^(61/4)*c^7 + 3840*a^10*(-b)^(65/4)*c^7 + 3584*a^11*(-b)^(65/4)*c^6 + 12*a^12*(-b)^(61/4)*c^9 - 148*a^10*(-b)^(65/4)*c^8 + 3648*a^11*(-b)^(65/4)*c^7 - 320*a^8*(-b)^(69/4)*c^7 - 624*a^11*(-b)^(65/4)*c^8 + 1024*a^12*(-b)^(65/4)*c^7 - 1152*a^9*(-b)^(69/4)*c^7 + 12*a^11*(-b)^(65/4)*c^9 - 768*a^12*(-b)^(65/4)*c^8 - 208*a^9*(-b)^(69/4)*c^8 - 1344*a^10*(-b)^(69/4)*c^7 - 256*a^13*(-b)^(65/4)*c^8 - 720*a^10*(-b)^(69/4)*c^8 - 512*a^11*(-b)^(69/4)*c^7 + 4*a^10*(-b)^(69/4)*c^9 - 768*a^11*(-b)^(69/4)*c^8 - 256*a^12*(-b)^(69/4)*c^8 + 36*a^13*(-b)^(25/4)*c - 276*a^12*(-b)^(29/4)*c - 384*a^13*(-b)^(29/4)*c + 300*a^11*(-b)^(33/4)*c + 1536*a^12*(-b)^(33/4)*c + 1344*a^13*(-b)^(33/4)*c + 1380*a^10*(-b)^(37/4)*c + 2304*a^11*(-b)^(37/4)*c - 576*a^12*(-b)^(37/4)*c - 1472*a^9*(-b)^(41/4)*c - 1536*a^13*(-b)^(37/4)*c - 7680*a^10*(-b)^(41/4)*c - 11328*a^11*(-b)^(41/4)*c - 2240*a^8*(-b)^(45/4)*c - 5120*a^12*(-b)^(41/4)*c - 8064*a^9*(-b)^(45/4)*c - 9408*a^10*(-b)^(45/4)*c - 3584*a^11*(-b)^(45/4)*c))/(a^7*b^18 + 9*a^8*b^18*c + 36*a^9*b^18*c^2 + 84*a^10*b^18*c^3 + 126*a^11*b^18*c^4 + 126*a^12*b^18*c^5 + 84*a^13*b^18*c^6 + 36*a^14*b^18*c^7 + 9*a^15*b^18*c^8 + a^16*b^18*c^9) - (64*((-b)^(7/4)*1i - (a*(-b)^(3/4)*(4*b + b*c + 1)*1i)/4)*(-(b*x - 1)/(c + x))^(1/4)*(16*a^13*(-b)^(11/2) - 80*a^12*(-b)^(13/2) - 80*a^11*(-b)^(15/2) - 128*a^13*(-b)^(13/2) + 400*a^10*(-b)^(17/2) + 128*a^12*(-b)^(15/2) + 896*a^9*(-b)^(19/2) + 1152*a^11*(-b)^(17/2) + 256*a^13*(-b)^(15/2) + 512*a^8*(-b)^(21/2) + 1920*a^10*(-b)^(19/2) + 768*a^12*(-b)^(17/2) + 1024*a^9*(-b)^(21/2) + 1024*a^11*(-b)^(19/2) + 512*a^10*(-b)^(21/2) + 448*a^13*(-b)^(15/2)*c^2 - 1344*a^12*(-b)^(17/2)*c^2 - 2624*a^11*(-b)^(19/2)*c^2 - 2688*a^13*(-b)^(17/2)*c^2 + 1088*a^10*(-b)^(21/2)*c^2 + 128*a^12*(-b)^(19/2)*c^2 - 896*a^13*(-b)^(17/2)*c^3 + 9600*a^9*(-b)^(23/2)*c^2 + 9344*a^11*(-b)^(21/2)*c^2 + 1792*a^12*(-b)^(19/2)*c^3 + 4096*a^13*(-b)^(19/2)*c^2 + 7680*a^8*(-b)^(25/2)*c^2 + 21888*a^10*(-b)^(23/2)*c^2 + 4096*a^11*(-b)^(21/2)*c^3 + 8704*a^12*(-b)^(21/2)*c^2 + 4480*a^13*(-b)^(19/2)*c^3 + 15360*a^9*(-b)^(25/2)*c^2 + 3328*a^10*(-b)^(23/2)*c^3 + 12288*a^11*(-b)^(23/2)*c^2 + 128*a^12*(-b)^(21/2)*c^3 + 1120*a^13*(-b)^(19/2)*c^4 - 8320*a^9*(-b)^(25/2)*c^3 + 7680*a^10*(-b)^(25/2)*c^2 - 4992*a^11*(-b)^(23/2)*c^3 - 1120*a^12*(-b)^(21/2)*c^4 - 6656*a^13*(-b)^(21/2)*c^3 - 10240*a^8*(-b)^(27/2)*c^3 - 21120*a^10*(-b)^(25/2)*c^3 - 2400*a^11*(-b)^(23/2)*c^4 - 9216*a^12*(-b)^(23/2)*c^3 - 4480*a^13*(-b)^(21/2)*c^4 - 20480*a^9*(-b)^(27/2)*c^3 - 7200*a^10*(-b

)^(25/2)*c^4 - 12800*a^11*(-b)^(25/2)*c^3 + 1920*a^12*(-b)^(23/2)*c^4 - 896
a^13(-b)^(21/2)*c^5 + 640*a^9*(-b)^(27/2)*c^4 - 10240*a^10*(-b)^(27/2)*c^
3 - 3200*a^11*(-b)^(25/2)*c^4 + 7680*a^13*(-b)^(23/2)*c^4 + 7680*a^8*(-b)^(
29/2)*c^4 + 5760*a^10*(-b)^(27/2)*c^4 - 1280*a^11*(-b)^(25/2)*c^5 + 5120*a^
12*(-b)^(25/2)*c^4 + 2688*a^13*(-b)^(23/2)*c^5 + 15360*a^9*(-b)^(29/2)*c^4
+ 5120*a^10*(-b)^(27/2)*c^5 + 5120*a^11*(-b)^(27/2)*c^4 - 5248*a^12*(-b)^(2
5/2)*c^5 + 448*a^13*(-b)^(23/2)*c^6 + 4224*a^9*(-b)^(29/2)*c^5 + 7680*a^10*
(-b)^(29/2)*c^4 + 3968*a^11*(-b)^(27/2)*c^5 + 448*a^12*(-b)^(25/2)*c^6 - 66
56*a^13*(-b)^(25/2)*c^5 - 3072*a^8*(-b)^(31/2)*c^5 + 5760*a^10*(-b)^(29/2)*
c^5 + 2752*a^11*(-b)^(27/2)*c^6 - 2048*a^12*(-b)^(27/2)*c^5 - 896*a^13*(-b)
^(25/2)*c^6 - 6144*a^9*(-b)^(31/2)*c^5 - 704*a^10*(-b)^(29/2)*c^6 + 1536*a^
11*(-b)^(29/2)*c^5 + 5504*a^12*(-b)^(27/2)*c^6 - 128*a^13*(-b)^(25/2)*c^7 -
 2944*a^9*(-b)^(31/2)*c^6 - 3072*a^10*(-b)^(31/2)*c^5 + 384*a^11*(-b)^(29/2
)*c^6 - 256*a^12*(-b)^(27/2)*c^7 + 4096*a^13*(-b)^(27/2)*c^6 + 512*a^8*(-b)
^(33/2)*c^6 - 4992*a^10*(-b)^(31/2)*c^6 - 1536*a^11*(-b)^(29/2)*c^7 + 1536*
a^12*(-b)^(29/2)*c^6 + 128*a^13*(-b)^(27/2)*c^7 + 1024*a^9*(-b)^(33/2)*c^6
- 768*a^10*(-b)^(31/2)*c^7 - 2048*a^11*(-b)^(31/2)*c^6 - 2688*a^12*(-b)^(29
/2)*c^7 + 16*a^13*(-b)^(27/2)*c^8 + 640*a^9*(-b)^(33/2)*c^7 + 512*a^10*(-b)
^(33/2)*c^6 - 1664*a^11*(-b)^(31/2)*c^7 + 48*a^12*(-b)^(29/2)*c^8 - 1536*a^
13*(-b)^(29/2)*c^7 + 1152*a^10*(-b)^(33/2)*c^7 + 304*a^11*(-b)^(31/2)*c^8 -
 1024*a^12*(-b)^(31/2)*c^7 + 272*a^10*(-b)^(33/2)*c^8 + 512*a^11*(-b)^(33/2
)*c^7 + 512*a^12*(-b)^(31/2)*c^8 + 512*a^11*(-b)^(33/2)*c^8 + 256*a^13*(-b)
^(31/2)*c^8 + 256*a^12*(-b)^(33/2)*c^8 - 128*a^13*(-b)^(13/2)*c + 512*a^12*
(-b)^(15/2)*c + 768*a^11*(-b)^(17/2)*c + 896*a^13*(-b)^(15/2)*c - 1536*a^10
*(-b)^(19/2)*c - 384*a^12*(-b)^(17/2)*c - 4736*a^9*(-b)^(21/2)*c - 5504*a^1
1*(-b)^(19/2)*c - 1536*a^13*(-b)^(17/2)*c - 3072*a^8*(-b)^(23/2)*c - 10368*
a^10*(-b)^(21/2)*c - 4096*a^12*(-b)^(19/2)*c - 6144*a^9*(-b)^(23/2)*c - 563
2*a^11*(-b)^(21/2)*c - 3072*a^10*(-b)^(23/2)*c))/(a^2*(-b)^(9/4)*(a^6*b^17
+ 9*a^7*b^17*c + 36*a^8*b^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 12
6*a^11*b^17*c^5 + 84*a^12*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a
^15*b^17*c^9))))/(a^6*b^6) + (64*(-(b*x - 1)/(c + x))^(1/4)*(512*(-b)^(19/2
) + a^5*(-b)^(9/2) + a^4*(-b)^(11/2) + 2*a^6*(-b)^(9/2) - 16*a^3*(-b)^(13/2
) + 18*a^5*(-b)^(11/2) + a^7*(-b)^(9/2) - 144*a^2*(-b)^(15/2) - 176*a^4*(-b
)^(13/2) + 49*a^6*(-b)^(11/2) - 576*a^3*(-b)^(15/2) - 560*a^5*(-b)^(13/2) +
 48*a^7*(-b)^(11/2) + 2688*a^2*(-b)^(17/2) - 608*a^4*(-b)^(15/2) - 784*a^6*
(-b)^(13/2) + 16*a^8*(-b)^(11/2) + 7680*a^3*(-b)^(17/2) + 448*a^5*(-b)^(15/
2) - 512*a^7*(-b)^(13/2) + 7680*a^2*(-b)^(19/2) + 11520*a^4*(-b)^(17/2) + 1
392*a^6*(-b)^(15/2) - 128*a^8*(-b)^(13/2) + 10240*a^3*(-b)^(19/2) + 9600*a^
5*(-b)^(17/2) + 1024*a^7*(-b)^(15/2) + 7680*a^4*(-b)^(19/2) + 4224*a^6*(-b)
^(17/2) + 256*a^8*(-b)^(15/2) + 3072*a^5*(-b)^(19/2) + 768*a^7*(-b)^(17/2)
+ 512*a^6*(-b)^(19/2) + 7680*(-b)^(23/2)*c^2 - 10240*(-b)^(25/2)*c^3 + 7680
*(-b)^(27/2)*c^4 - 3072*(-b)^(29/2)*c^5 + 512*(-b)^(31/2)*c^6 + 384*a*(-b)^
(17/2) + 3072*a*(-b)^(19/2) - 3072*(-b)^(21/2)*c + a^7*(-b)^(9/2)*c^2 - 35*
a^6*(-b)^(11/2)*c^2 + 2*a^8*(-b)^(9/2)*c^2 + 265*a^5*(-b)^(13/2)*c^2 - 86*a
^7*(-b)^(11/2)*c^2 + a^9*(-b)^(9/2)*c^2 - 851*a^4*(-b)^(15/2)*c^2 + 738*a^6
*(-b)^(13/2)*c^2 - 10*a^7*(-b)^(11/2)*c^3 - 67*a^8*(-b)^(11/2)*c^2 + 2496*a
^3*(-b)^(17/2)*c^2 - 2566*a^5*(-b)^(15/2)*c^2 + 224*a^6*(-b)^(13/2)*c^3 + 6
49*a^7*(-b)^(13/2)*c^2 - 20*a^8*(-b)^(11/2)*c^3 - 16*a^9*(-b)^(11/2)*c^2 -
5184*a^2*(-b)^(19/2)*c^2 + 10432*a^4*(-b)^(17/2)*c^2 - 1358*a^5*(-b)^(15/2)
*c^3 - 1907*a^6*(-b)^(15/2)*c^2 + 592*a^7*(-b)^(13/2)*c^3 + 144*a^8*(-b)^(1
3/2)*c^2 - 10*a^9*(-b)^(11/2)*c^3 - 31104*a^3*(-b)^(19/2)*c^2 + 3784*a^4*(-
b)^(17/2)*c^3 + 14912*a^5*(-b)^(17/2)*c^2 - 4364*a^6*(-b)^(15/2)*c^3 + 45*a
^7*(-b)^(13/2)*c^4 + 1120*a^7*(-b)^(15/2)*c^2 + 512*a^8*(-b)^(13/2)*c^3 - 3
2*a^9*(-b)^(13/2)*c^2 + 1152*a^2*(-b)^(21/2)*c^2 - 7552*a^3*(-b)^(19/2)*c^3
 - 74624*a^4*(-b)^(19/2)*c^2 + 14288*a^5*(-b)^(17/2)*c^3 - 771*a^6*(-b)^(15
/2)*c^4 + 5824*a^6*(-b)^(17/2)*c^2 - 4974*a^7*(-b)^(15/2)*c^3 + 90*a^8*(-b)
^(13/2)*c^4 + 1952*a^8*(-b)^(15/2)*c^2 + 144*a^9*(-b)^(13/2)*c^3 + 6912*a^2
*(-b)^(21/2)*c^3 + 23040*a^3*(-b)^(21/2)*c^2 - 36800*a^4*(-b)^(19/2)*c^3 +
3874*a^5*(-b)^(17/2)*c^4 - 91136*a^5*(-b)^(19/2)*c^2 + 19144*a^6*(-b)^(17/2

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)*c^3 - 2118*a^7*(-b)^(15/2)*c^4 - 5120*a^7*(-b)^(17/2)*c^2 - 2288*a^8*(-b)
^(15/2)*c^3 + 45*a^9*(-b)^(13/2)*c^4 + 640*a^9*(-b)^(15/2)*c^2 + 115200*a^2
*(-b)^(23/2)*c^2 + 49536*a^3*(-b)^(21/2)*c^3 - 8750*a^4*(-b)^(19/2)*c^4 + 5
7600*a^4*(-b)^(21/2)*c^2 - 68032*a^5*(-b)^(19/2)*c^3 + 13444*a^6*(-b)^(17/2
)*c^4 - 58944*a^6*(-b)^(19/2)*c^2 - 120*a^7*(-b)^(15/2)*c^5 + 9664*a^7*(-b)
^(17/2)*c^3 - 1923*a^8*(-b)^(15/2)*c^4 - 5248*a^8*(-b)^(17/2)*c^2 - 320*a^9
*(-b)^(15/2)*c^3 + 44160*a^2*(-b)^(23/2)*c^3 + 11040*a^3*(-b)^(21/2)*c^4 +
153600*a^3*(-b)^(23/2)*c^2 + 137728*a^4*(-b)^(21/2)*c^3 - 36988*a^5*(-b)^(1
9/2)*c^4 + 63360*a^5*(-b)^(21/2)*c^2 + 1644*a^6*(-b)^(17/2)*c^5 - 56512*a^6
*(-b)^(19/2)*c^3 + 17058*a^7*(-b)^(17/2)*c^4 - 18560*a^7*(-b)^(19/2)*c^2 -
240*a^8*(-b)^(15/2)*c^5 + 128*a^8*(-b)^(17/2)*c^3 - 576*a^9*(-b)^(15/2)*c^4
 - 1280*a^9*(-b)^(17/2)*c^2 - 480*a^2*(-b)^(23/2)*c^4 + 76800*a^3*(-b)^(23/
2)*c^3 + 60640*a^4*(-b)^(21/2)*c^4 + 115200*a^4*(-b)^(23/2)*c^2 - 6776*a^5*
(-b)^(19/2)*c^5 + 193792*a^5*(-b)^(21/2)*c^3 - 59150*a^6*(-b)^(19/2)*c^4 +
33408*a^6*(-b)^(21/2)*c^2 + 4632*a^7*(-b)^(17/2)*c^5 - 16064*a^7*(-b)^(19/2
)*c^3 + 9280*a^8*(-b)^(17/2)*c^4 - 2048*a^8*(-b)^(19/2)*c^2 - 120*a^9*(-b)^
(15/2)*c^5 - 896*a^9*(-b)^(17/2)*c^3 - 153600*a^2*(-b)^(25/2)*c^3 - 23040*a
^3*(-b)^(23/2)*c^4 + 11620*a^4*(-b)^(21/2)*c^5 + 57600*a^4*(-b)^(23/2)*c^3
+ 128864*a^5*(-b)^(21/2)*c^4 + 46080*a^5*(-b)^(23/2)*c^2 - 24752*a^6*(-b)^(
19/2)*c^5 + 146688*a^6*(-b)^(21/2)*c^3 + 210*a^7*(-b)^(17/2)*c^6 - 43232*a^
7*(-b)^(19/2)*c^4 + 6912*a^7*(-b)^(21/2)*c^2 + 4332*a^8*(-b)^(17/2)*c^5 + 3
968*a^8*(-b)^(19/2)*c^3 + 1792*a^9*(-b)^(17/2)*c^4 - 90240*a^2*(-b)^(25/2)*
c^4 - 7104*a^3*(-b)^(23/2)*c^5 - 204800*a^3*(-b)^(25/2)*c^3 - 100160*a^4*(-
b)^(23/2)*c^4 + 53480*a^5*(-b)^(21/2)*c^5 + 9600*a^5*(-b)^(23/2)*c^3 - 2310
*a^6*(-b)^(19/2)*c^6 + 131872*a^6*(-b)^(21/2)*c^4 + 7680*a^6*(-b)^(23/2)*c^
2 - 33432*a^7*(-b)^(19/2)*c^5 + 56704*a^7*(-b)^(21/2)*c^3 + 420*a^8*(-b)^(1
7/2)*c^6 - 13216*a^8*(-b)^(19/2)*c^4 + 1344*a^9*(-b)^(17/2)*c^5 + 2304*a^9*
(-b)^(19/2)*c^3 - 9216*a^2*(-b)^(25/2)*c^5 - 192000*a^3*(-b)^(25/2)*c^4 - 4
8224*a^4*(-b)^(23/2)*c^5 - 153600*a^4*(-b)^(25/2)*c^3 + 7546*a^5*(-b)^(21/2
)*c^6 - 179840*a^5*(-b)^(23/2)*c^4 + 94948*a^6*(-b)^(21/2)*c^5 - 9600*a^6*(
-b)^(23/2)*c^3 - 6636*a^7*(-b)^(19/2)*c^6 + 63232*a^7*(-b)^(21/2)*c^4 - 197
12*a^8*(-b)^(19/2)*c^5 + 8704*a^8*(-b)^(21/2)*c^3 + 210*a^9*(-b)^(17/2)*c^6
 - 896*a^9*(-b)^(19/2)*c^4 + 115200*a^2*(-b)^(27/2)*c^4 - 31104*a^3*(-b)^(2
5/2)*c^5 - 8750*a^4*(-b)^(23/2)*c^6 - 211200*a^4*(-b)^(25/2)*c^4 - 121120*a
^5*(-b)^(23/2)*c^5 - 61440*a^5*(-b)^(25/2)*c^3 + 28756*a^6*(-b)^(21/2)*c^6
- 161760*a^6*(-b)^(23/2)*c^4 - 252*a^7*(-b)^(19/2)*c^7 + 80416*a^7*(-b)^(21
/2)*c^5 - 3840*a^7*(-b)^(23/2)*c^3 - 6342*a^8*(-b)^(19/2)*c^6 + 9344*a^8*(-
b)^(21/2)*c^4 - 4256*a^9*(-b)^(19/2)*c^5 + 81792*a^2*(-b)^(27/2)*c^5 - 832*
a^3*(-b)^(25/2)*c^6 + 153600*a^3*(-b)^(27/2)*c^4 - 23552*a^4*(-b)^(25/2)*c^
5 - 44380*a^5*(-b)^(23/2)*c^6 - 124800*a^5*(-b)^(25/2)*c^4 + 2184*a^6*(-b)^
(21/2)*c^7 - 146336*a^6*(-b)^(23/2)*c^5 - 10240*a^6*(-b)^(25/2)*c^3 + 40698
*a^7*(-b)^(21/2)*c^6 - 72320*a^7*(-b)^(23/2)*c^4 - 504*a^8*(-b)^(19/2)*c^7
+ 31808*a^8*(-b)^(21/2)*c^5 - 2016*a^9*(-b)^(19/2)*c^6 - 1280*a^9*(-b)^(21/
2)*c^4 + 10944*a^2*(-b)^(27/2)*c^6 + 184320*a^3*(-b)^(27/2)*c^5 + 8896*a^4*
(-b)^(25/2)*c^6 + 115200*a^4*(-b)^(27/2)*c^4 - 5300*a^5*(-b)^(23/2)*c^7 + 3
2512*a^5*(-b)^(25/2)*c^5 - 86702*a^6*(-b)^(23/2)*c^6 - 36480*a^6*(-b)^(25/2
)*c^4 + 6384*a^7*(-b)^(21/2)*c^7 - 87968*a^7*(-b)^(23/2)*c^5 + 25312*a^8*(-
b)^(21/2)*c^6 - 12800*a^8*(-b)^(23/2)*c^4 - 252*a^9*(-b)^(19/2)*c^7 + 4480*
a^9*(-b)^(21/2)*c^5 - 46080*a^2*(-b)^(29/2)*c^5 + 49536*a^3*(-b)^(27/2)*c^6
 + 3016*a^4*(-b)^(25/2)*c^7 + 218880*a^4*(-b)^(27/2)*c^5 + 44864*a^5*(-b)^(
25/2)*c^6 + 46080*a^5*(-b)^(27/2)*c^4 - 21128*a^6*(-b)^(23/2)*c^7 + 66048*a
^6*(-b)^(25/2)*c^5 + 210*a^7*(-b)^(21/2)*c^8 - 81536*a^7*(-b)^(23/2)*c^6 -
3840*a^7*(-b)^(25/2)*c^4 + 6216*a^8*(-b)^(21/2)*c^7 - 22912*a^8*(-b)^(23/2)
*c^5 + 5824*a^9*(-b)^(21/2)*c^6 - 36480*a^2*(-b)^(29/2)*c^6 + 4224*a^3*(-b)
^(27/2)*c^7 - 61440*a^3*(-b)^(29/2)*c^5 + 86656*a^4*(-b)^(27/2)*c^6 + 18896
*a^5*(-b)^(25/2)*c^7 + 144000*a^5*(-b)^(27/2)*c^5 - 1374*a^6*(-b)^(23/2)*c^
8 + 75200*a^6*(-b)^(25/2)*c^6 + 7680*a^6*(-b)^(27/2)*c^4 - 31284*a^7*(-b)^(
23/2)*c^7 + 40576*a^7*(-b)^(25/2)*c^5 + 420*a^8*(-b)^(21/2)*c^8 - 36736*a^8
*(-b)^(23/2)*c^6 + 2016*a^9*(-b)^(21/2)*c^7 - 1280*a^9*(-b)^(23/2)*c^5 - 53
```

76*a^2*(-b)^(29/2)*c^7 - 84480*a^3*(-b)^(29/2)*c^6 + 13888*a^4*(-b)^(27/2)*
c^7 - 46080*a^4*(-b)^(29/2)*c^5 + 2173*a^5*(-b)^(25/2)*c^8 + 70144*a^5*(-b)
^(27/2)*c^6 + 42952*a^6*(-b)^(25/2)*c^7 + 49536*a^6*(-b)^(27/2)*c^5 - 4092*
a^7*(-b)^(23/2)*c^8 + 57856*a^7*(-b)^(25/2)*c^6 - 20384*a^8*(-b)^(23/2)*c^7
 + 8704*a^8*(-b)^(25/2)*c^5 + 210*a^9*(-b)^(21/2)*c^8 - 6272*a^9*(-b)^(23/2
)*c^6 + 7680*a^2*(-b)^(31/2)*c^6 - 26496*a^3*(-b)^(29/2)*c^7 + 301*a^4*(-b)
^(27/2)*c^8 - 103680*a^4*(-b)^(29/2)*c^6 + 12608*a^5*(-b)^(27/2)*c^7 - 1843
2*a^5*(-b)^(29/2)*c^5 + 9274*a^6*(-b)^(25/2)*c^8 + 21696*a^6*(-b)^(27/2)*c^
6 - 120*a^7*(-b)^(23/2)*c^9 + 45760*a^7*(-b)^(25/2)*c^7 + 6912*a^7*(-b)^(27
/2)*c^5 - 4062*a^8*(-b)^(23/2)*c^8 + 20096*a^8*(-b)^(25/2)*c^6 - 4928*a^9*(
-b)^(23/2)*c^7 + 6528*a^2*(-b)^(31/2)*c^7 - 2448*a^3*(-b)^(29/2)*c^8 + 1024
0*a^3*(-b)^(31/2)*c^6 - 52736*a^4*(-b)^(29/2)*c^7 - 1558*a^5*(-b)^(27/2)*c^
8 - 71040*a^5*(-b)^(29/2)*c^6 + 546*a^6*(-b)^(25/2)*c^9 - 4544*a^6*(-b)^(27
/2)*c^7 - 3072*a^6*(-b)^(29/2)*c^5 + 14589*a^7*(-b)^(25/2)*c^8 - 2432*a^7*(
-b)^(27/2)*c^6 - 240*a^8*(-b)^(23/2)*c^9 + 23168*a^8*(-b)^(25/2)*c^7 - 1344
a^9(-b)^(23/2)*c^8 + 2304*a^9*(-b)^(25/2)*c^6 + 1008*a^2*(-b)^(31/2)*c^8
 + 15360*a^3*(-b)^(31/2)*c^7 - 10160*a^4*(-b)^(29/2)*c^8 + 7680*a^4*(-b)^(31
/2)*c^6 - 384*a^5*(-b)^(27/2)*c^9 - 53504*a^5*(-b)^(29/2)*c^7 - 8099*a^6*(-
b)^(27/2)*c^8 - 25728*a^6*(-b)^(29/2)*c^6 + 1668*a^7*(-b)^(25/2)*c^9 - 1376
0*a^7*(-b)^(27/2)*c^7 + 10048*a^8*(-b)^(25/2)*c^8 - 2048*a^8*(-b)^(27/2)*c^
6 - 120*a^9*(-b)^(23/2)*c^9 + 4480*a^9*(-b)^(25/2)*c^7 + 5184*a^3*(-b)^(31/
2)*c^8 - 570*a^4*(-b)^(29/2)*c^9 + 19200*a^4*(-b)^(31/2)*c^7 - 16048*a^5*(-
b)^(29/2)*c^8 + 3072*a^5*(-b)^(31/2)*c^6 - 1984*a^6*(-b)^(27/2)*c^9 - 28416
a^6(-b)^(29/2)*c^7 + 45*a^7*(-b)^(25/2)*c^10 - 11984*a^7*(-b)^(27/2)*c^8
 - 3840*a^7*(-b)^(29/2)*c^6 + 1698*a^8*(-b)^(25/2)*c^9 - 7552*a^8*(-b)^(27/2
)*c^7 + 2560*a^9*(-b)^(25/2)*c^8 + 480*a^3*(-b)^(31/2)*c^9 + 10912*a^4*(-b)
^(31/2)*c^8 - 1732*a^5*(-b)^(29/2)*c^9 + 13440*a^5*(-b)^(31/2)*c^7 - 119*a^
6*(-b)^(27/2)*c^10 - 11408*a^6*(-b)^(29/2)*c^8 + 512*a^6*(-b)^(31/2)*c^6 -
3568*a^7*(-b)^(27/2)*c^9 - 7040*a^7*(-b)^(29/2)*c^7 + 90*a^8*(-b)^(25/2)*c^
10 - 7408*a^8*(-b)^(27/2)*c^8 + 576*a^9*(-b)^(25/2)*c^9 - 1280*a^9*(-b)^(27
/2)*c^7 + 2160*a^4*(-b)^(31/2)*c^9 - 35*a^5*(-b)^(29/2)*c^10 + 11968*a^5*(-
b)^(31/2)*c^8 - 1530*a^6*(-b)^(29/2)*c^9 + 4992*a^6*(-b)^(31/2)*c^7 - 382*a
^7*(-b)^(27/2)*c^10 - 2816*a^7*(-b)^(29/2)*c^8 - 2720*a^8*(-b)^(27/2)*c^9 -
 512*a^8*(-b)^(29/2)*c^7 + 45*a^9*(-b)^(25/2)*c^10 - 1664*a^9*(-b)^(27/2)*c
^8 + 129*a^4*(-b)^(31/2)*c^10 + 3856*a^5*(-b)^(31/2)*c^9 + 10*a^6*(-b)^(29/
2)*c^10 + 7152*a^6*(-b)^(31/2)*c^8 - 10*a^7*(-b)^(27/2)*c^11 + 112*a^7*(-b)
^(29/2)*c^9 + 768*a^7*(-b)^(31/2)*c^7 - 407*a^8*(-b)^(27/2)*c^10 + 512*a^8*
(-b)^(29/2)*c^8 - 752*a^9*(-b)^(27/2)*c^9 + 482*a^5*(-b)^(31/2)*c^10 + 8*a^
6*(-b)^(29/2)*c^11 + 3408*a^6*(-b)^(31/2)*c^9 + 221*a^7*(-b)^(29/2)*c^10 +
2176*a^7*(-b)^(31/2)*c^8 - 20*a^8*(-b)^(27/2)*c^11 + 736*a^8*(-b)^(29/2)*c^
9 - 144*a^9*(-b)^(27/2)*c^10 + 256*a^9*(-b)^(29/2)*c^8 + 18*a^5*(-b)^(31/2)
*c^11 + 673*a^6*(-b)^(31/2)*c^10 + 32*a^7*(-b)^(29/2)*c^11 + 1488*a^7*(-b)^
(31/2)*c^9 + 272*a^8*(-b)^(29/2)*c^10 + 256*a^8*(-b)^(31/2)*c^8 - 10*a^9*(-
b)^(27/2)*c^11 + 256*a^9*(-b)^(29/2)*c^9 + 52*a^6*(-b)^(31/2)*c^11 + a^7*(-
b)^(29/2)*c^12 + 416*a^7*(-b)^(31/2)*c^10 + 40*a^8*(-b)^(29/2)*c^11 + 256*a
^8*(-b)^(31/2)*c^9 + 96*a^9*(-b)^(29/2)*c^10 + a^6*(-b)^(31/2)*c^12 + 50*a^
7*(-b)^(31/2)*c^11 + 2*a^8*(-b)^(29/2)*c^12 + 96*a^8*(-b)^(31/2)*c^10 + 16*
a^9*(-b)^(29/2)*c^11 + 2*a^7*(-b)^(31/2)*c^12 + 16*a^8*(-b)^(31/2)*c^11 + a
^9*(-b)^(29/2)*c^12 + a^8*(-b)^(31/2)*c^12 - 1152*a*(-b)^(19/2)*c - 18432*a
*(-b)^(21/2)*c + 2*a^6*(-b)^(9/2)*c - 24*a^5*(-b)^(11/2)*c + 4*a^7*(-b)^(9/
2)*c + 70*a^4*(-b)^(13/2)*c - 48*a^6*(-b)^(11/2)*c + 2*a^8*(-b)^(9/2)*c - 2
88*a^3*(-b)^(15/2)*c + 60*a^5*(-b)^(13/2)*c - 8*a^7*(-b)^(11/2)*c + 1536*a^
2*(-b)^(17/2)*c - 656*a^4*(-b)^(15/2)*c - 378*a^6*(-b)^(13/2)*c + 32*a^8*(-
b)^(11/2)*c + 8064*a^3*(-b)^(17/2)*c + 656*a^5*(-b)^(15/2)*c - 784*a^7*(-b)
^(13/2)*c + 16*a^9*(-b)^(11/2)*c - 9600*a^2*(-b)^(19/2)*c + 16384*a^4*(-b)^
(17/2)*c + 3280*a^6*(-b)^(15/2)*c - 544*a^8*(-b)^(13/2)*c - 30720*a^3*(-b)^
(19/2)*c + 15616*a^5*(-b)^(17/2)*c + 3664*a^7*(-b)^(15/2)*c - 128*a^9*(-b)^
(13/2)*c - 1152*a*(-b)^(21/2)*c^2 - 46080*a^2*(-b)^(21/2)*c - 49920*a^4*(-b
)^(19/2)*c + 6144*a^6*(-b)^(17/2)*c + 1664*a^8*(-b)^(15/2)*c - 61440*a^3*(-

b)^(21/2)*c - 44160*a^5*(-b)^(19/2)*c - 128*a^7*(-b)^(17/2)*c + 256*a^9*(-b
)^(15/2)*c + 46080*a*(-b)^(23/2)*c^2 - 46080*a^4*(-b)^(21/2)*c - 20352*a^6*
(-b)^(19/2)*c - 512*a^8*(-b)^(17/2)*c + 9600*a*(-b)^(23/2)*c^3 - 18432*a^5*
(-b)^(21/2)*c - 3840*a^7*(-b)^(19/2)*c - 3072*a^6*(-b)^(21/2)*c - 61440*a*(
-b)^(25/2)*c^3 - 17280*a*(-b)^(25/2)*c^4 + 46080*a*(-b)^(27/2)*c^4 + 14976*
a*(-b)^(27/2)*c^5 - 18432*a*(-b)^(29/2)*c^5 - 6528*a*(-b)^(29/2)*c^6 + 3072
a(-b)^(31/2)*c^6 + 1152*a*(-b)^(31/2)*c^7))/((-b)^(1/4)*(a^6*b^17 + 9*a^7
*b^17*c + 36*a^8*b^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*
b^17*c^5 + 84*a^12*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^1
7*c^9)))*1i)/(a^2*b^2) - (((-b)^(7/4)*1i - (a*(-b)^(3/4)*(4*b + b*c + 1)*1i
)/4)*((((-b)^(7/4)*1i - (a*(-b)^(3/4)*(4*b + b*c + 1)*1i)/4)^3*((64*(36*a^1
2*(-b)^(25/4) - 4*a^13*(-b)^(21/4) + 48*a^13*(-b)^(25/4) - 60*a^11*(-b)^(29
/4) - 240*a^12*(-b)^(29/4) - 192*a^13*(-b)^(29/4) - 180*a^10*(-b)^(33/4) -
240*a^11*(-b)^(33/4) + 192*a^12*(-b)^(33/4) + 240*a^9*(-b)^(37/4) + 256*a^1
3*(-b)^(33/4) + 1200*a^10*(-b)^(37/4) + 1728*a^11*(-b)^(37/4) + 320*a^8*(-b
)^(41/4) + 768*a^12*(-b)^(37/4) + 1152*a^9*(-b)^(41/4) + 1344*a^10*(-b)^(41
/4) + 512*a^11*(-b)^(41/4) - 144*a^13*(-b)^(29/4)*c^2 + 912*a^12*(-b)^(33/4
)*c^2 + 1344*a^13*(-b)^(33/4)*c^2 + 336*a^13*(-b)^(33/4)*c^3 - 432*a^11*(-b
)^(37/4)*c^2 - 4032*a^12*(-b)^(37/4)*c^2 - 1680*a^12*(-b)^(37/4)*c^3 - 4032
a^13(-b)^(37/4)*c^2 - 4624*a^10*(-b)^(41/4)*c^2 - 2688*a^13*(-b)^(37/4)*c
^3 - 9408*a^11*(-b)^(41/4)*c^2 - 504*a^13*(-b)^(37/4)*c^4 - 336*a^11*(-b)^(
41/4)*c^3 - 1344*a^12*(-b)^(41/4)*c^2 + 3584*a^9*(-b)^(45/4)*c^2 + 5376*a^1
2*(-b)^(41/4)*c^3 + 3584*a^13*(-b)^(41/4)*c^2 + 20160*a^10*(-b)^(45/4)*c^2
+ 1848*a^12*(-b)^(41/4)*c^4 + 6720*a^13*(-b)^(41/4)*c^3 + 8848*a^10*(-b)^(4
5/4)*c^3 + 30912*a^11*(-b)^(45/4)*c^2 + 3360*a^13*(-b)^(41/4)*c^4 + 6720*a^
8*(-b)^(49/4)*c^2 + 21504*a^11*(-b)^(45/4)*c^3 + 14336*a^12*(-b)^(45/4)*c^2
 + 504*a^13*(-b)^(41/4)*c^5 + 24192*a^9*(-b)^(49/4)*c^2 + 1848*a^11*(-b)^(4
5/4)*c^4 + 9408*a^12*(-b)^(45/4)*c^3 - 4032*a^9*(-b)^(49/4)*c^3 + 28224*a^1
0*(-b)^(49/4)*c^2 - 3360*a^12*(-b)^(45/4)*c^4 - 3584*a^13*(-b)^(45/4)*c^3 -
 26880*a^10*(-b)^(49/4)*c^3 + 10752*a^11*(-b)^(49/4)*c^2 - 1176*a^12*(-b)^(
45/4)*c^5 - 6720*a^13*(-b)^(45/4)*c^4 - 10584*a^10*(-b)^(49/4)*c^4 - 44352*
a^11*(-b)^(49/4)*c^3 - 2688*a^13*(-b)^(45/4)*c^5 - 11200*a^8*(-b)^(53/4)*c^
3 - 30240*a^11*(-b)^(49/4)*c^4 - 21504*a^12*(-b)^(49/4)*c^3 - 336*a^13*(-b)
^(45/4)*c^6 - 40320*a^9*(-b)^(53/4)*c^3 - 2520*a^11*(-b)^(49/4)*c^5 - 20160
a^12(-b)^(49/4)*c^4 + 1120*a^9*(-b)^(53/4)*c^4 - 47040*a^10*(-b)^(53/4)*c
^3 + 16800*a^10*(-b)^(53/4)*c^4 - 17920*a^11*(-b)^(53/4)*c^3 + 336*a^12*(-b
)^(49/4)*c^6 + 4032*a^13*(-b)^(49/4)*c^5 + 8120*a^10*(-b)^(53/4)*c^5 + 3360
0*a^11*(-b)^(53/4)*c^4 + 1344*a^13*(-b)^(49/4)*c^6 + 11200*a^8*(-b)^(57/4)*
c^4 + 26880*a^11*(-b)^(53/4)*c^5 + 17920*a^12*(-b)^(53/4)*c^4 + 144*a^13*(-
b)^(49/4)*c^7 + 40320*a^9*(-b)^(57/4)*c^4 + 1680*a^11*(-b)^(53/4)*c^6 + 228
48*a^12*(-b)^(53/4)*c^5 + 2240*a^9*(-b)^(57/4)*c^5 + 47040*a^10*(-b)^(57/4)
*c^4 + 1344*a^12*(-b)^(53/4)*c^6 + 3584*a^13*(-b)^(53/4)*c^5 + 17920*a^11*(
-b)^(57/4)*c^4 + 48*a^12*(-b)^(53/4)*c^7 - 1344*a^13*(-b)^(53/4)*c^6 - 3920
a^10(-b)^(57/4)*c^6 - 9408*a^11*(-b)^(57/4)*c^5 - 384*a^13*(-b)^(53/4)*c^
7 - 6720*a^8*(-b)^(61/4)*c^5 - 14784*a^11*(-b)^(57/4)*c^6 - 7168*a^12*(-b)^
(57/4)*c^5 - 36*a^13*(-b)^(53/4)*c^8 - 24192*a^9*(-b)^(61/4)*c^5 - 528*a^11
*(-b)^(57/4)*c^7 - 14784*a^12*(-b)^(57/4)*c^6 - 2688*a^9*(-b)^(61/4)*c^6 -
28224*a^10*(-b)^(61/4)*c^5 - 768*a^12*(-b)^(57/4)*c^7 - 3584*a^13*(-b)^(57/
4)*c^6 - 6720*a^10*(-b)^(61/4)*c^6 - 10752*a^11*(-b)^(61/4)*c^5 - 60*a^12*(
-b)^(57/4)*c^8 + 192*a^13*(-b)^(57/4)*c^7 + 1104*a^10*(-b)^(61/4)*c^7 - 403
2*a^11*(-b)^(61/4)*c^6 + 48*a^13*(-b)^(57/4)*c^8 + 2240*a^8*(-b)^(65/4)*c^6
 + 4608*a^11*(-b)^(61/4)*c^7 + 4*a^13*(-b)^(57/4)*c^9 + 8064*a^9*(-b)^(65/4
)*c^6 + 36*a^11*(-b)^(61/4)*c^8 + 5184*a^12*(-b)^(61/4)*c^7 + 1216*a^9*(-b)
^(65/4)*c^7 + 9408*a^10*(-b)^(65/4)*c^6 + 144*a^12*(-b)^(61/4)*c^8 + 1536*a
^13*(-b)^(61/4)*c^7 + 3840*a^10*(-b)^(65/4)*c^7 + 3584*a^11*(-b)^(65/4)*c^6
 + 12*a^12*(-b)^(61/4)*c^9 - 148*a^10*(-b)^(65/4)*c^8 + 3648*a^11*(-b)^(65/
4)*c^7 - 320*a^8*(-b)^(69/4)*c^7 - 624*a^11*(-b)^(65/4)*c^8 + 1024*a^12*(-b
)^(65/4)*c^7 - 1152*a^9*(-b)^(69/4)*c^7 + 12*a^11*(-b)^(65/4)*c^9 - 768*a^1
2*(-b)^(65/4)*c^8 - 208*a^9*(-b)^(69/4)*c^8 - 1344*a^10*(-b)^(69/4)*c^7 - 2

56*a^13*(-b)^(65/4)*c^8 - 720*a^10*(-b)^(69/4)*c^8 - 512*a^11*(-b)^(69/4)*c
^7 + 4*a^10*(-b)^(69/4)*c^9 - 768*a^11*(-b)^(69/4)*c^8 - 256*a^12*(-b)^(69/
4)*c^8 + 36*a^13*(-b)^(25/4)*c - 276*a^12*(-b)^(29/4)*c - 384*a^13*(-b)^(29
/4)*c + 300*a^11*(-b)^(33/4)*c + 1536*a^12*(-b)^(33/4)*c + 1344*a^13*(-b)^(
33/4)*c + 1380*a^10*(-b)^(37/4)*c + 2304*a^11*(-b)^(37/4)*c - 576*a^12*(-b)
^(37/4)*c - 1472*a^9*(-b)^(41/4)*c - 1536*a^13*(-b)^(37/4)*c - 7680*a^10*(-
b)^(41/4)*c - 11328*a^11*(-b)^(41/4)*c - 2240*a^8*(-b)^(45/4)*c - 5120*a^12
*(-b)^(41/4)*c - 8064*a^9*(-b)^(45/4)*c - 9408*a^10*(-b)^(45/4)*c - 3584*a^
11*(-b)^(45/4)*c))/(a^7*b^18 + 9*a^8*b^18*c + 36*a^9*b^18*c^2 + 84*a^10*b^1
8*c^3 + 126*a^11*b^18*c^4 + 126*a^12*b^18*c^5 + 84*a^13*b^18*c^6 + 36*a^14*
b^18*c^7 + 9*a^15*b^18*c^8 + a^16*b^18*c^9) + (64*((-b)^(7/4)*1i - (a*(-b)^
(3/4)*(4*b + b*c + 1)*1i)/4)*(-(b*x - 1)/(c + x))^(1/4)*(16*a^13*(-b)^(11/2
) - 80*a^12*(-b)^(13/2) - 80*a^11*(-b)^(15/2) - 128*a^13*(-b)^(13/2) + 400*
a^10*(-b)^(17/2) + 128*a^12*(-b)^(15/2) + 896*a^9*(-b)^(19/2) + 1152*a^11*(
-b)^(17/2) + 256*a^13*(-b)^(15/2) + 512*a^8*(-b)^(21/2) + 1920*a^10*(-b)^(1
9/2) + 768*a^12*(-b)^(17/2) + 1024*a^9*(-b)^(21/2) + 1024*a^11*(-b)^(19/2)
+ 512*a^10*(-b)^(21/2) + 448*a^13*(-b)^(15/2)*c^2 - 1344*a^12*(-b)^(17/2)*c
^2 - 2624*a^11*(-b)^(19/2)*c^2 - 2688*a^13*(-b)^(17/2)*c^2 + 1088*a^10*(-b)
^(21/2)*c^2 + 128*a^12*(-b)^(19/2)*c^2 - 896*a^13*(-b)^(17/2)*c^3 + 9600*a^
9*(-b)^(23/2)*c^2 + 9344*a^11*(-b)^(21/2)*c^2 + 1792*a^12*(-b)^(19/2)*c^3 +
4096*a^13*(-b)^(19/2)*c^2 + 7680*a^8*(-b)^(25/2)*c^2 + 21888*a^10*(-b)^(23
/2)*c^2 + 4096*a^11*(-b)^(21/2)*c^3 + 8704*a^12*(-b)^(21/2)*c^2 + 4480*a^13
*(-b)^(19/2)*c^3 + 15360*a^9*(-b)^(25/2)*c^2 + 3328*a^10*(-b)^(23/2)*c^3 +
12288*a^11*(-b)^(23/2)*c^2 + 128*a^12*(-b)^(21/2)*c^3 + 1120*a^13*(-b)^(19/
2)*c^4 - 8320*a^9*(-b)^(25/2)*c^3 + 7680*a^10*(-b)^(25/2)*c^2 - 4992*a^11*(
-b)^(23/2)*c^3 - 1120*a^12*(-b)^(21/2)*c^4 - 6656*a^13*(-b)^(21/2)*c^3 - 10
240*a^8*(-b)^(27/2)*c^3 - 21120*a^10*(-b)^(25/2)*c^3 - 2400*a^11*(-b)^(23/2
)*c^4 - 9216*a^12*(-b)^(23/2)*c^3 - 4480*a^13*(-b)^(21/2)*c^4 - 20480*a^9*(
-b)^(27/2)*c^3 - 7200*a^10*(-b)^(25/2)*c^4 - 12800*a^11*(-b)^(25/2)*c^3 + 1
920*a^12*(-b)^(23/2)*c^4 - 896*a^13*(-b)^(21/2)*c^5 + 640*a^9*(-b)^(27/2)*c
^4 - 10240*a^10*(-b)^(27/2)*c^3 - 3200*a^11*(-b)^(25/2)*c^4 + 7680*a^13*(-b
)^(23/2)*c^4 + 7680*a^8*(-b)^(29/2)*c^4 + 5760*a^10*(-b)^(27/2)*c^4 - 1280*
a^11*(-b)^(25/2)*c^5 + 5120*a^12*(-b)^(25/2)*c^4 + 2688*a^13*(-b)^(23/2)*c^
5 + 15360*a^9*(-b)^(29/2)*c^4 + 5120*a^10*(-b)^(27/2)*c^5 + 5120*a^11*(-b)^
(27/2)*c^4 - 5248*a^12*(-b)^(25/2)*c^5 + 448*a^13*(-b)^(23/2)*c^6 + 4224*a^
9*(-b)^(29/2)*c^5 + 7680*a^10*(-b)^(29/2)*c^4 + 3968*a^11*(-b)^(27/2)*c^5 +
448*a^12*(-b)^(25/2)*c^6 - 6656*a^13*(-b)^(25/2)*c^5 - 3072*a^8*(-b)^(31/2
)*c^5 + 5760*a^10*(-b)^(29/2)*c^5 + 2752*a^11*(-b)^(27/2)*c^6 - 2048*a^12*(
-b)^(27/2)*c^5 - 896*a^13*(-b)^(25/2)*c^6 - 6144*a^9*(-b)^(31/2)*c^5 - 704*
a^10*(-b)^(29/2)*c^6 + 1536*a^11*(-b)^(29/2)*c^5 + 5504*a^12*(-b)^(27/2)*c^
6 - 128*a^13*(-b)^(25/2)*c^7 - 2944*a^9*(-b)^(31/2)*c^6 - 3072*a^10*(-b)^(3
1/2)*c^5 + 384*a^11*(-b)^(29/2)*c^6 - 256*a^12*(-b)^(27/2)*c^7 + 4096*a^13*
(-b)^(27/2)*c^6 + 512*a^8*(-b)^(33/2)*c^6 - 4992*a^10*(-b)^(31/2)*c^6 - 153
6*a^11*(-b)^(29/2)*c^7 + 1536*a^12*(-b)^(29/2)*c^6 + 128*a^13*(-b)^(27/2)*c
^7 + 1024*a^9*(-b)^(33/2)*c^6 - 768*a^10*(-b)^(31/2)*c^7 - 2048*a^11*(-b)^(
31/2)*c^6 - 2688*a^12*(-b)^(29/2)*c^7 + 16*a^13*(-b)^(27/2)*c^8 + 640*a^9*(
-b)^(33/2)*c^7 + 512*a^10*(-b)^(33/2)*c^6 - 1664*a^11*(-b)^(31/2)*c^7 + 48*
a^12*(-b)^(29/2)*c^8 - 1536*a^13*(-b)^(29/2)*c^7 + 1152*a^10*(-b)^(33/2)*c^
7 + 304*a^11*(-b)^(31/2)*c^8 - 1024*a^12*(-b)^(31/2)*c^7 + 272*a^10*(-b)^(3
3/2)*c^8 + 512*a^11*(-b)^(33/2)*c^7 + 512*a^12*(-b)^(31/2)*c^8 + 512*a^11*(
-b)^(33/2)*c^8 + 256*a^13*(-b)^(31/2)*c^8 + 256*a^12*(-b)^(33/2)*c^8 - 128*
a^13*(-b)^(13/2)*c + 512*a^12*(-b)^(15/2)*c + 768*a^11*(-b)^(17/2)*c + 896*
a^13*(-b)^(15/2)*c - 1536*a^10*(-b)^(19/2)*c - 384*a^12*(-b)^(17/2)*c - 473
6*a^9*(-b)^(21/2)*c - 5504*a^11*(-b)^(19/2)*c - 1536*a^13*(-b)^(17/2)*c - 3
072*a^8*(-b)^(23/2)*c - 10368*a^10*(-b)^(21/2)*c - 4096*a^12*(-b)^(19/2)*c
- 6144*a^9*(-b)^(23/2)*c - 5632*a^11*(-b)^(21/2)*c - 3072*a^10*(-b)^(23/2)*
c))/(a^2*(-b)^(9/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^8*b^17*c^2 + 84*a^9*b^1
7*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a^12*b^17*c^6 + 36*a^13*
b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9))))/(a^6*b^6) - (64*(-(b*x - 1)/

(c + x))^(1/4)*(512*(-b)^(19/2) + a^5*(-b)^(9/2) + a^4*(-b)^(11/2) + 2*a^6*(-b)^(9/2) - 16*a^3*(-b)^(13/2) + 18*a^5*(-b)^(11/2) + a^7*(-b)^(9/2) - 144*a^2*(-b)^(15/2) - 176*a^4*(-b)^(13/2) + 49*a^6*(-b)^(11/2) - 576*a^3*(-b)^(15/2) - 560*a^5*(-b)^(13/2) + 48*a^7*(-b)^(11/2) + 2688*a^2*(-b)^(17/2) - 608*a^4*(-b)^(15/2) - 784*a^6*(-b)^(13/2) + 16*a^8*(-b)^(11/2) + 7680*a^3*(-b)^(17/2) + 448*a^5*(-b)^(15/2) - 512*a^7*(-b)^(13/2) + 7680*a^2*(-b)^(19/2) + 11520*a^4*(-b)^(17/2) + 1392*a^6*(-b)^(15/2) - 128*a^8*(-b)^(13/2) + 10240*a^3*(-b)^(19/2) + 9600*a^5*(-b)^(17/2) + 1024*a^7*(-b)^(15/2) + 7680*a^4*(-b)^(19/2) + 4224*a^6*(-b)^(17/2) + 256*a^8*(-b)^(15/2) + 3072*a^5*(-b)^(19/2) + 768*a^7*(-b)^(17/2) + 512*a^6*(-b)^(19/2) + 7680*(-b)^(23/2)*c^2 - 10240*(-b)^(25/2)*c^3 + 7680*(-b)^(27/2)*c^4 - 3072*(-b)^(29/2)*c^5 + 512*(-b)^(31/2)*c^6 + 384*a*(-b)^(17/2) + 3072*a*(-b)^(19/2) - 3072*(-b)^(21/2)*c + a^7*(-b)^(9/2)*c^2 - 35*a^6*(-b)^(11/2)*c^2 + 2*a^8*(-b)^(9/2)*c^2 + 265*a^5*(-b)^(13/2)*c^2 - 86*a^7*(-b)^(11/2)*c^2 + a^9*(-b)^(9/2)*c^2 - 851*a^4*(-b)^(15/2)*c^2 + 738*a^6*(-b)^(13/2)*c^2 - 10*a^7*(-b)^(11/2)*c^3 - 67*a^8*(-b)^(11/2)*c^2 + 2496*a^3*(-b)^(17/2)*c^2 - 2566*a^5*(-b)^(15/2)*c^2 + 224*a^6*(-b)^(13/2)*c^3 + 649*a^7*(-b)^(13/2)*c^2 - 20*a^8*(-b)^(11/2)*c^3 - 16*a^9*(-b)^(11/2)*c^2 - 5184*a^2*(-b)^(19/2)*c^2 + 10432*a^4*(-b)^(17/2)*c^2 - 1358*a^5*(-b)^(15/2)*c^3 - 1907*a^6*(-b)^(15/2)*c^2 + 592*a^7*(-b)^(13/2)*c^3 + 144*a^8*(-b)^(13/2)*c^2 - 10*a^9*(-b)^(11/2)*c^3 - 31104*a^3*(-b)^(19/2)*c^2 + 3784*a^4*(-b)^(17/2)*c^3 + 14912*a^5*(-b)^(17/2)*c^2 - 4364*a^6*(-b)^(15/2)*c^3 + 45*a^7*(-b)^(13/2)*c^4 + 1120*a^7*(-b)^(15/2)*c^2 + 512*a^8*(-b)^(13/2)*c^3 - 32*a^9*(-b)^(13/2)*c^2 + 1152*a^2*(-b)^(21/2)*c^2 - 7552*a^3*(-b)^(19/2)*c^3 - 74624*a^4*(-b)^(19/2)*c^2 + 14288*a^5*(-b)^(17/2)*c^3 - 771*a^6*(-b)^(15/2)*c^4 + 5824*a^6*(-b)^(17/2)*c^2 - 4974*a^7*(-b)^(15/2)*c^3 + 90*a^8*(-b)^(13/2)*c^4 + 1952*a^8*(-b)^(15/2)*c^2 + 144*a^9*(-b)^(13/2)*c^3 + 6912*a^2*(-b)^(21/2)*c^3 + 23040*a^3*(-b)^(21/2)*c^2 - 36800*a^4*(-b)^(19/2)*c^3 + 3874*a^5*(-b)^(17/2)*c^4 - 91136*a^5*(-b)^(19/2)*c^2 + 19144*a^6*(-b)^(17/2)*c^3 - 2118*a^7*(-b)^(15/2)*c^4 - 5120*a^7*(-b)^(17/2)*c^2 - 2288*a^8*(-b)^(15/2)*c^3 + 45*a^9*(-b)^(13/2)*c^4 + 640*a^9*(-b)^(15/2)*c^2 + 115200*a^2*(-b)^(23/2)*c^2 + 49536*a^3*(-b)^(21/2)*c^3 - 8750*a^4*(-b)^(19/2)*c^4 + 57600*a^4*(-b)^(21/2)*c^2 - 68032*a^5*(-b)^(19/2)*c^3 + 13444*a^6*(-b)^(17/2)*c^4 - 58944*a^6*(-b)^(19/2)*c^2 - 120*a^7*(-b)^(15/2)*c^5 + 9664*a^7*(-b)^(17/2)*c^3 - 1923*a^8*(-b)^(15/2)*c^4 - 5248*a^8*(-b)^(17/2)*c^2 - 320*a^9*(-b)^(15/2)*c^3 + 44160*a^2*(-b)^(23/2)*c^3 + 11040*a^3*(-b)^(21/2)*c^4 + 153600*a^3*(-b)^(23/2)*c^2 + 137728*a^4*(-b)^(21/2)*c^3 - 36988*a^5*(-b)^(19/2)*c^4 + 63360*a^5*(-b)^(21/2)*c^2 + 1644*a^6*(-b)^(17/2)*c^5 - 56512*a^6*(-b)^(19/2)*c^3 + 17058*a^7*(-b)^(17/2)*c^4 - 18560*a^7*(-b)^(19/2)*c^2 - 240*a^8*(-b)^(15/2)*c^5 + 128*a^8*(-b)^(17/2)*c^3 - 576*a^9*(-b)^(15/2)*c^4 - 1280*a^9*(-b)^(17/2)*c^2 - 480*a^2*(-b)^(23/2)*c^4 + 76800*a^3*(-b)^(23/2)*c^3 + 60640*a^4*(-b)^(21/2)*c^4 + 115200*a^4*(-b)^(23/2)*c^2 - 6776*a^5*(-b)^(19/2)*c^5 + 193792*a^5*(-b)^(21/2)*c^3 - 59150*a^6*(-b)^(19/2)*c^4 + 33408*a^6*(-b)^(21/2)*c^2 + 4632*a^7*(-b)^(17/2)*c^5 - 16064*a^7*(-b)^(19/2)*c^3 + 9280*a^8*(-b)^(17/2)*c^4 - 2048*a^8*(-b)^(19/2)*c^2 - 120*a^9*(-b)^(15/2)*c^5 - 896*a^9*(-b)^(17/2)*c^3 - 153600*a^2*(-b)^(25/2)*c^3 - 23040*a^3*(-b)^(23/2)*c^4 + 11620*a^4*(-b)^(21/2)*c^5 + 57600*a^4*(-b)^(23/2)*c^3 + 128864*a^5*(-b)^(21/2)*c^4 + 46080*a^5*(-b)^(23/2)*c^2 - 24752*a^6*(-b)^(19/2)*c^5 + 146688*a^6*(-b)^(21/2)*c^3 + 210*a^7*(-b)^(17/2)*c^6 - 43232*a^7*(-b)^(19/2)*c^4 + 6912*a^7*(-b)^(21/2)*c^2 + 4332*a^8*(-b)^(17/2)*c^5 + 3968*a^8*(-b)^(19/2)*c^3 + 1792*a^9*(-b)^(17/2)*c^4 - 90240*a^2*(-b)^(25/2)*c^4 - 7104*a^3*(-b)^(23/2)*c^5 - 204800*a^3*(-b)^(25/2)*c^3 - 100160*a^4*(-b)^(23/2)*c^4 + 53480*a^5*(-b)^(21/2)*c^5 + 9600*a^5*(-b)^(23/2)*c^3 - 2310*a^6*(-b)^(19/2)*c^6 + 131872*a^6*(-b)^(21/2)*c^4 + 7680*a^6*(-b)^(23/2)*c^2 - 33432*a^7*(-b)^(19/2)*c^5 + 56704*a^7*(-b)^(21/2)*c^3 + 420*a^8*(-b)^(17/2)*c^6 - 13216*a^8*(-b)^(19/2)*c^4 + 1344*a^9*(-b)^(17/2)*c^5 + 2304*a^9*(-b)^(19/2)*c^3 - 9216*a^2*(-b)^(25/2)*c^5 - 192000*a^3*(-b)^(25/2)*c^4 - 48224*a^4*(-b)^(23/2)*c^5 - 153600*a^4*(-b)^(25/2)*c^3 + 7546*a^5*(-b)^(21/2)*c^6 - 179840*a^5*(-b)^(23/2)*c^4 + 94948*a^6*(-b)^(21/2)*c^5 - 9600*a^6*(-b)^(23/2)*c^3 - 6636*a^7*(-b)^(19/2)*c^6 + 6

3232*a^7*(-b)^(21/2)*c^4 - 19712*a^8*(-b)^(19/2)*c^5 + 8704*a^8*(-b)^(21/2)
*c^3 + 210*a^9*(-b)^(17/2)*c^6 - 896*a^9*(-b)^(19/2)*c^4 + 115200*a^2*(-b)^
(27/2)*c^4 - 31104*a^3*(-b)^(25/2)*c^5 - 8750*a^4*(-b)^(23/2)*c^6 - 211200*
a^4*(-b)^(25/2)*c^4 - 121120*a^5*(-b)^(23/2)*c^5 - 61440*a^5*(-b)^(25/2)*c^
3 + 28756*a^6*(-b)^(21/2)*c^6 - 161760*a^6*(-b)^(23/2)*c^4 - 252*a^7*(-b)^(
19/2)*c^7 + 80416*a^7*(-b)^(21/2)*c^5 - 3840*a^7*(-b)^(23/2)*c^3 - 6342*a^8
*(-b)^(19/2)*c^6 + 9344*a^8*(-b)^(21/2)*c^4 - 4256*a^9*(-b)^(19/2)*c^5 + 81
792*a^2*(-b)^(27/2)*c^5 - 832*a^3*(-b)^(25/2)*c^6 + 153600*a^3*(-b)^(27/2)*
c^4 - 23552*a^4*(-b)^(25/2)*c^5 - 44380*a^5*(-b)^(23/2)*c^6 - 124800*a^5*(-
b)^(25/2)*c^4 + 2184*a^6*(-b)^(21/2)*c^7 - 146336*a^6*(-b)^(23/2)*c^5 - 102
40*a^6*(-b)^(25/2)*c^3 + 40698*a^7*(-b)^(21/2)*c^6 - 72320*a^7*(-b)^(23/2)*
c^4 - 504*a^8*(-b)^(19/2)*c^7 + 31808*a^8*(-b)^(21/2)*c^5 - 2016*a^9*(-b)^(
19/2)*c^6 - 1280*a^9*(-b)^(21/2)*c^4 + 10944*a^2*(-b)^(27/2)*c^6 + 184320*a
^3*(-b)^(27/2)*c^5 + 8896*a^4*(-b)^(25/2)*c^6 + 115200*a^4*(-b)^(27/2)*c^4
- 5300*a^5*(-b)^(23/2)*c^7 + 32512*a^5*(-b)^(25/2)*c^5 - 86702*a^6*(-b)^(23
/2)*c^6 - 36480*a^6*(-b)^(25/2)*c^4 + 6384*a^7*(-b)^(21/2)*c^7 - 87968*a^7*
(-b)^(23/2)*c^5 + 25312*a^8*(-b)^(21/2)*c^6 - 12800*a^8*(-b)^(23/2)*c^4 - 2
52*a^9*(-b)^(19/2)*c^7 + 4480*a^9*(-b)^(21/2)*c^5 - 46080*a^2*(-b)^(29/2)*c
^5 + 49536*a^3*(-b)^(27/2)*c^6 + 3016*a^4*(-b)^(25/2)*c^7 + 218880*a^4*(-b)
^(27/2)*c^5 + 44864*a^5*(-b)^(25/2)*c^6 + 46080*a^5*(-b)^(27/2)*c^4 - 21128
a^6(-b)^(23/2)*c^7 + 66048*a^6*(-b)^(25/2)*c^5 + 210*a^7*(-b)^(21/2)*c^8
- 81536*a^7*(-b)^(23/2)*c^6 - 3840*a^7*(-b)^(25/2)*c^4 + 6216*a^8*(-b)^(21/
2)*c^7 - 22912*a^8*(-b)^(23/2)*c^5 + 5824*a^9*(-b)^(21/2)*c^6 - 36480*a^2*(
-b)^(29/2)*c^6 + 4224*a^3*(-b)^(27/2)*c^7 - 61440*a^3*(-b)^(29/2)*c^5 + 866
56*a^4*(-b)^(27/2)*c^6 + 18896*a^5*(-b)^(25/2)*c^7 + 144000*a^5*(-b)^(27/2)
*c^5 - 1374*a^6*(-b)^(23/2)*c^8 + 75200*a^6*(-b)^(25/2)*c^6 + 7680*a^6*(-b)
^(27/2)*c^4 - 31284*a^7*(-b)^(23/2)*c^7 + 40576*a^7*(-b)^(25/2)*c^5 + 420*a
^8*(-b)^(21/2)*c^8 - 36736*a^8*(-b)^(23/2)*c^6 + 2016*a^9*(-b)^(21/2)*c^7 -
1280*a^9*(-b)^(23/2)*c^5 - 5376*a^2*(-b)^(29/2)*c^7 - 84480*a^3*(-b)^(29/2
)*c^6 + 13888*a^4*(-b)^(27/2)*c^7 - 46080*a^4*(-b)^(29/2)*c^5 + 2173*a^5*(-
b)^(25/2)*c^8 + 70144*a^5*(-b)^(27/2)*c^6 + 42952*a^6*(-b)^(25/2)*c^7 + 495
36*a^6*(-b)^(27/2)*c^5 - 4092*a^7*(-b)^(23/2)*c^8 + 57856*a^7*(-b)^(25/2)*c
^6 - 20384*a^8*(-b)^(23/2)*c^7 + 8704*a^8*(-b)^(25/2)*c^5 + 210*a^9*(-b)^(2
1/2)*c^8 - 6272*a^9*(-b)^(23/2)*c^6 + 7680*a^2*(-b)^(31/2)*c^6 - 26496*a^3*
(-b)^(29/2)*c^7 + 301*a^4*(-b)^(27/2)*c^8 - 103680*a^4*(-b)^(29/2)*c^6 + 12
608*a^5*(-b)^(27/2)*c^7 - 18432*a^5*(-b)^(29/2)*c^5 + 9274*a^6*(-b)^(25/2)*
c^8 + 21696*a^6*(-b)^(27/2)*c^6 - 120*a^7*(-b)^(23/2)*c^9 + 45760*a^7*(-b)^
(25/2)*c^7 + 6912*a^7*(-b)^(27/2)*c^5 - 4062*a^8*(-b)^(23/2)*c^8 + 20096*a^
8*(-b)^(25/2)*c^6 - 4928*a^9*(-b)^(23/2)*c^7 + 6528*a^2*(-b)^(31/2)*c^7 - 2
448*a^3*(-b)^(29/2)*c^8 + 10240*a^3*(-b)^(31/2)*c^6 - 52736*a^4*(-b)^(29/2)
*c^7 - 1558*a^5*(-b)^(27/2)*c^8 - 71040*a^5*(-b)^(29/2)*c^6 + 546*a^6*(-b)^
(25/2)*c^9 - 4544*a^6*(-b)^(27/2)*c^7 - 3072*a^6*(-b)^(29/2)*c^5 + 14589*a^
7*(-b)^(25/2)*c^8 - 2432*a^7*(-b)^(27/2)*c^6 - 240*a^8*(-b)^(23/2)*c^9 + 23
168*a^8*(-b)^(25/2)*c^7 - 1344*a^9*(-b)^(23/2)*c^8 + 2304*a^9*(-b)^(25/2)*c
^6 + 1008*a^2*(-b)^(31/2)*c^8 + 15360*a^3*(-b)^(31/2)*c^7 - 10160*a^4*(-b)^
(29/2)*c^8 + 7680*a^4*(-b)^(31/2)*c^6 - 384*a^5*(-b)^(27/2)*c^9 - 53504*a^5
*(-b)^(29/2)*c^7 - 8099*a^6*(-b)^(27/2)*c^8 - 25728*a^6*(-b)^(29/2)*c^6 + 1
668*a^7*(-b)^(25/2)*c^9 - 13760*a^7*(-b)^(27/2)*c^7 + 10048*a^8*(-b)^(25/2)
*c^8 - 2048*a^8*(-b)^(27/2)*c^6 - 120*a^9*(-b)^(23/2)*c^9 + 4480*a^9*(-b)^(
25/2)*c^7 + 5184*a^3*(-b)^(31/2)*c^8 - 570*a^4*(-b)^(29/2)*c^9 + 19200*a^4*
(-b)^(31/2)*c^7 - 16048*a^5*(-b)^(29/2)*c^8 + 3072*a^5*(-b)^(31/2)*c^6 - 19
84*a^6*(-b)^(27/2)*c^9 - 28416*a^6*(-b)^(29/2)*c^7 + 45*a^7*(-b)^(25/2)*c^1
0 - 11984*a^7*(-b)^(27/2)*c^8 - 3840*a^7*(-b)^(29/2)*c^6 + 1698*a^8*(-b)^(2
5/2)*c^9 - 7552*a^8*(-b)^(27/2)*c^7 + 2560*a^9*(-b)^(25/2)*c^8 + 480*a^3*(-
b)^(31/2)*c^9 + 10912*a^4*(-b)^(31/2)*c^8 - 1732*a^5*(-b)^(29/2)*c^9 + 1344
0*a^5*(-b)^(31/2)*c^7 - 119*a^6*(-b)^(27/2)*c^10 - 11408*a^6*(-b)^(29/2)*c^
8 + 512*a^6*(-b)^(31/2)*c^6 - 3568*a^7*(-b)^(27/2)*c^9 - 7040*a^7*(-b)^(29/
2)*c^7 + 90*a^8*(-b)^(25/2)*c^10 - 7408*a^8*(-b)^(27/2)*c^8 + 576*a^9*(-b)^
(25/2)*c^9 - 1280*a^9*(-b)^(27/2)*c^7 + 2160*a^4*(-b)^(31/2)*c^9 - 35*a^5*(

-b)^(29/2)*c^10 + 11968*a^5*(-b)^(31/2)*c^8 - 1530*a^6*(-b)^(29/2)*c^9 + 49
92*a^6*(-b)^(31/2)*c^7 - 382*a^7*(-b)^(27/2)*c^10 - 2816*a^7*(-b)^(29/2)*c^
8 - 2720*a^8*(-b)^(27/2)*c^9 - 512*a^8*(-b)^(29/2)*c^7 + 45*a^9*(-b)^(25/2)
*c^10 - 1664*a^9*(-b)^(27/2)*c^8 + 129*a^4*(-b)^(31/2)*c^10 + 3856*a^5*(-b)
^(31/2)*c^9 + 10*a^6*(-b)^(29/2)*c^10 + 7152*a^6*(-b)^(31/2)*c^8 - 10*a^7*(
-b)^(27/2)*c^11 + 112*a^7*(-b)^(29/2)*c^9 + 768*a^7*(-b)^(31/2)*c^7 - 407*a
^8*(-b)^(27/2)*c^10 + 512*a^8*(-b)^(29/2)*c^8 - 752*a^9*(-b)^(27/2)*c^9 + 4
82*a^5*(-b)^(31/2)*c^10 + 8*a^6*(-b)^(29/2)*c^11 + 3408*a^6*(-b)^(31/2)*c^9
 + 221*a^7*(-b)^(29/2)*c^10 + 2176*a^7*(-b)^(31/2)*c^8 - 20*a^8*(-b)^(27/2)
*c^11 + 736*a^8*(-b)^(29/2)*c^9 - 144*a^9*(-b)^(27/2)*c^10 + 256*a^9*(-b)^(
29/2)*c^8 + 18*a^5*(-b)^(31/2)*c^11 + 673*a^6*(-b)^(31/2)*c^10 + 32*a^7*(-b
)^(29/2)*c^11 + 1488*a^7*(-b)^(31/2)*c^9 + 272*a^8*(-b)^(29/2)*c^10 + 256*a
^8*(-b)^(31/2)*c^8 - 10*a^9*(-b)^(27/2)*c^11 + 256*a^9*(-b)^(29/2)*c^9 + 52
a^6(-b)^(31/2)*c^11 + a^7*(-b)^(29/2)*c^12 + 416*a^7*(-b)^(31/2)*c^10 + 4
0*a^8*(-b)^(29/2)*c^11 + 256*a^8*(-b)^(31/2)*c^9 + 96*a^9*(-b)^(29/2)*c^10
 + a^6*(-b)^(31/2)*c^12 + 50*a^7*(-b)^(31/2)*c^11 + 2*a^8*(-b)^(29/2)*c^12 +
 96*a^8*(-b)^(31/2)*c^10 + 16*a^9*(-b)^(29/2)*c^11 + 2*a^7*(-b)^(31/2)*c^12
 + 16*a^8*(-b)^(31/2)*c^11 + a^9*(-b)^(29/2)*c^12 + a^8*(-b)^(31/2)*c^12 -
1152*a*(-b)^(19/2)*c - 18432*a*(-b)^(21/2)*c + 2*a^6*(-b)^(9/2)*c - 24*a^5*
(-b)^(11/2)*c + 4*a^7*(-b)^(9/2)*c + 70*a^4*(-b)^(13/2)*c - 48*a^6*(-b)^(11
/2)*c + 2*a^8*(-b)^(9/2)*c - 288*a^3*(-b)^(15/2)*c + 60*a^5*(-b)^(13/2)*c -
 8*a^7*(-b)^(11/2)*c + 1536*a^2*(-b)^(17/2)*c - 656*a^4*(-b)^(15/2)*c - 378
a^6(-b)^(13/2)*c + 32*a^8*(-b)^(11/2)*c + 8064*a^3*(-b)^(17/2)*c + 656*a^
5*(-b)^(15/2)*c - 784*a^7*(-b)^(13/2)*c + 16*a^9*(-b)^(11/2)*c - 9600*a^2*(
-b)^(19/2)*c + 16384*a^4*(-b)^(17/2)*c + 3280*a^6*(-b)^(15/2)*c - 544*a^8*(
-b)^(13/2)*c - 30720*a^3*(-b)^(19/2)*c + 15616*a^5*(-b)^(17/2)*c + 3664*a^7
*(-b)^(15/2)*c - 128*a^9*(-b)^(13/2)*c - 1152*a*(-b)^(21/2)*c^2 - 46080*a^2
*(-b)^(21/2)*c - 49920*a^4*(-b)^(19/2)*c + 6144*a^6*(-b)^(17/2)*c + 1664*a^
8*(-b)^(15/2)*c - 61440*a^3*(-b)^(21/2)*c - 44160*a^5*(-b)^(19/2)*c - 128*a
^7*(-b)^(17/2)*c + 256*a^9*(-b)^(15/2)*c + 46080*a*(-b)^(23/2)*c^2 - 46080*
a^4*(-b)^(21/2)*c - 20352*a^6*(-b)^(19/2)*c - 512*a^8*(-b)^(17/2)*c + 9600*
a*(-b)^(23/2)*c^3 - 18432*a^5*(-b)^(21/2)*c - 3840*a^7*(-b)^(19/2)*c - 3072
a^6(-b)^(21/2)*c - 61440*a*(-b)^(25/2)*c^3 - 17280*a*(-b)^(25/2)*c^4 + 46
080*a*(-b)^(27/2)*c^4 + 14976*a*(-b)^(27/2)*c^5 - 18432*a*(-b)^(29/2)*c^5 -
 6528*a*(-b)^(29/2)*c^6 + 3072*a*(-b)^(31/2)*c^6 + 1152*a*(-b)^(31/2)*c^7))
/((-b)^(1/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^8*b^17*c^2 + 84*a^9*b^17*c^3 +
 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a^12*b^17*c^6 + 36*a^13*b^17*c^
7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9)))*1i)/(a^2*b^2))/((128*(5*a^4*(-b)^(21
/4) - 320*(-b)^(37/4) + 19*a^5*(-b)^(21/4) + 27*a^6*(-b)^(21/4) - 55*a^3*(-
b)^(25/4) + 17*a^7*(-b)^(21/4) - 269*a^4*(-b)^(25/4) + 4*a^8*(-b)^(21/4) -
525*a^5*(-b)^(25/4) + 180*a^2*(-b)^(29/4) - 511*a^6*(-b)^(25/4) + 1104*a^3*
(-b)^(29/4) - 248*a^7*(-b)^(25/4) + 2808*a^4*(-b)^(29/4) - 48*a^8*(-b)^(25/
4) + 3792*a^5*(-b)^(29/4) - 784*a^2*(-b)^(33/4) + 2868*a^6*(-b)^(29/4) - 29
76*a^3*(-b)^(33/4) + 1152*a^7*(-b)^(29/4) - 5920*a^4*(-b)^(33/4) + 192*a^8*
(-b)^(29/4) - 6800*a^5*(-b)^(33/4) - 6336*a^2*(-b)^(37/4) - 4560*a^6*(-b)^(
33/4) - 10240*a^3*(-b)^(37/4) - 1664*a^7*(-b)^(33/4) - 9920*a^4*(-b)^(37/4)
 - 256*a^8*(-b)^(33/4) - 5760*a^5*(-b)^(37/4) - 1856*a^6*(-b)^(37/4) - 256*
a^7*(-b)^(37/4) - 6720*(-b)^(45/4)*c^2 + 11200*(-b)^(49/4)*c^3 - 11200*(-b)
^(53/4)*c^4 + 6720*(-b)^(57/4)*c^5 - 2240*(-b)^(61/4)*c^6 + 320*(-b)^(65/4)
*c^7 - 80*a*(-b)^(33/4) - 2176*a*(-b)^(37/4) + 2240*(-b)^(41/4)*c + a^6*(-b
)^(21/4)*c^2 + 3*a^7*(-b)^(21/4)*c^2 + 3*a^8*(-b)^(21/4)*c^2 - 71*a^5*(-b)^
(25/4)*c^2 + a^9*(-b)^(21/4)*c^2 - 265*a^6*(-b)^(25/4)*c^2 - 10*a^6*(-b)^(2
5/4)*c^3 - 369*a^7*(-b)^(25/4)*c^2 + 849*a^4*(-b)^(29/4)*c^2 - 30*a^7*(-b)^
(25/4)*c^3 - 227*a^8*(-b)^(25/4)*c^2 + 3851*a^5*(-b)^(29/4)*c^2 - 30*a^8*(-
b)^(25/4)*c^3 - 52*a^9*(-b)^(25/4)*c^2 + 368*a^5*(-b)^(29/4)*c^3 + 6939*a^6
*(-b)^(29/4)*c^2 - 10*a^9*(-b)^(25/4)*c^3 - 3607*a^3*(-b)^(33/4)*c^2 + 1392
a^6(-b)^(29/4)*c^3 + 6201*a^7*(-b)^(29/4)*c^2 - 19689*a^4*(-b)^(33/4)*c^2
 + 45*a^6*(-b)^(29/4)*c^4 + 1968*a^7*(-b)^(29/4)*c^3 + 2744*a^8*(-b)^(29/4)
*c^2 - 3198*a^4*(-b)^(33/4)*c^3 - 44241*a^5*(-b)^(33/4)*c^2 + 135*a^7*(-b)^

(29/4)*c^4 + 1232*a^8*(-b)^(29/4)*c^3 + 480*a^9*(-b)^(29/4)*c^2 + 4880*a^2*
(-b)^(37/4)*c^2 - 14874*a^5*(-b)^(33/4)*c^3 - 52259*a^6*(-b)^(33/4)*c^2 + 1
35*a^8*(-b)^(29/4)*c^4 + 288*a^9*(-b)^(29/4)*c^3 + 33088*a^3*(-b)^(37/4)*c^
2 - 1107*a^5*(-b)^(33/4)*c^4 - 27546*a^6*(-b)^(33/4)*c^3 - 34116*a^7*(-b)^(
33/4)*c^2 + 45*a^9*(-b)^(29/4)*c^4 + 9976*a^3*(-b)^(37/4)*c^3 + 93600*a^4*(
-b)^(37/4)*c^2 - 4233*a^6*(-b)^(33/4)*c^4 - 25374*a^7*(-b)^(33/4)*c^3 - 116
16*a^8*(-b)^(33/4)*c^2 + 56280*a^4*(-b)^(37/4)*c^3 + 142912*a^5*(-b)^(37/4)
*c^2 - 120*a^6*(-b)^(33/4)*c^5 - 6057*a^7*(-b)^(33/4)*c^4 - 11616*a^8*(-b)^
(33/4)*c^3 - 1600*a^9*(-b)^(33/4)*c^2 + 11200*a^2*(-b)^(41/4)*c^2 + 7290*a^
4*(-b)^(37/4)*c^4 + 131064*a^5*(-b)^(37/4)*c^3 + 126608*a^6*(-b)^(37/4)*c^2
 - 360*a^7*(-b)^(33/4)*c^5 - 3843*a^8*(-b)^(33/4)*c^4 - 2112*a^9*(-b)^(33/4
)*c^3 - 7952*a^2*(-b)^(41/4)*c^3 + 12096*a^3*(-b)^(41/4)*c^2 + 34566*a^5*(-
b)^(37/4)*c^4 + 161096*a^6*(-b)^(37/4)*c^3 + 64512*a^7*(-b)^(37/4)*c^2 - 36
0*a^8*(-b)^(33/4)*c^5 - 912*a^9*(-b)^(33/4)*c^4 - 59136*a^3*(-b)^(41/4)*c^3
 - 13440*a^4*(-b)^(41/4)*c^2 + 2148*a^5*(-b)^(37/4)*c^5 + 65334*a^6*(-b)^(3
7/4)*c^4 + 110064*a^7*(-b)^(37/4)*c^3 + 17216*a^8*(-b)^(37/4)*c^2 - 120*a^9
*(-b)^(33/4)*c^5 - 16590*a^3*(-b)^(41/4)*c^4 - 180768*a^4*(-b)^(41/4)*c^3 -
 44800*a^5*(-b)^(41/4)*c^2 + 8292*a^6*(-b)^(37/4)*c^5 + 61506*a^7*(-b)^(37/
4)*c^4 + 39552*a^8*(-b)^(37/4)*c^3 + 1792*a^9*(-b)^(37/4)*c^2 - 133056*a^2*
(-b)^(45/4)*c^2 - 96810*a^4*(-b)^(41/4)*c^4 - 296128*a^5*(-b)^(41/4)*c^3 +
210*a^6*(-b)^(37/4)*c^6 - 44352*a^6*(-b)^(41/4)*c^2 + 11988*a^7*(-b)^(37/4)
*c^5 + 28824*a^8*(-b)^(37/4)*c^4 + 5824*a^9*(-b)^(37/4)*c^3 - 55552*a^2*(-b
)^(45/4)*c^3 - 215040*a^3*(-b)^(45/4)*c^2 - 10752*a^4*(-b)^(41/4)*c^5 - 233
226*a^5*(-b)^(41/4)*c^4 - 281232*a^6*(-b)^(41/4)*c^3 + 630*a^7*(-b)^(37/4)*
c^6 - 20160*a^7*(-b)^(41/4)*c^2 + 7692*a^8*(-b)^(37/4)*c^5 + 5376*a^9*(-b)^
(37/4)*c^4 + 5208*a^2*(-b)^(45/4)*c^4 - 119616*a^3*(-b)^(45/4)*c^3 - 208320
a^4(-b)^(45/4)*c^2 - 51912*a^5*(-b)^(41/4)*c^5 - 296814*a^6*(-b)^(41/4)*c
^4 - 154560*a^7*(-b)^(41/4)*c^3 + 630*a^8*(-b)^(37/4)*c^6 - 3584*a^8*(-b)^(
41/4)*c^2 + 1848*a^9*(-b)^(37/4)*c^5 + 51296*a^3*(-b)^(45/4)*c^4 - 125440*a
^4*(-b)^(45/4)*c^3 - 2814*a^5*(-b)^(41/4)*c^6 - 120960*a^5*(-b)^(45/4)*c^2
 - 99960*a^6*(-b)^(41/4)*c^5 - 210336*a^7*(-b)^(41/4)*c^4 - 45248*a^8*(-b)^(
41/4)*c^3 + 210*a^9*(-b)^(37/4)*c^6 + 16940*a^3*(-b)^(45/4)*c^5 + 185808*a^
4*(-b)^(45/4)*c^4 - 56000*a^5*(-b)^(45/4)*c^3 - 10962*a^6*(-b)^(41/4)*c^6 -
 38976*a^6*(-b)^(45/4)*c^2 - 95928*a^7*(-b)^(41/4)*c^5 - 78624*a^8*(-b)^(41
/4)*c^4 - 5376*a^9*(-b)^(41/4)*c^3 + 221760*a^2*(-b)^(49/4)*c^3 + 103404*a^
4*(-b)^(45/4)*c^5 + 343392*a^5*(-b)^(45/4)*c^4 - 252*a^6*(-b)^(41/4)*c^7 +
5376*a^6*(-b)^(45/4)*c^3 - 16002*a^7*(-b)^(41/4)*c^6 - 5376*a^7*(-b)^(45/4)
*c^2 - 45864*a^8*(-b)^(41/4)*c^5 - 12096*a^9*(-b)^(41/4)*c^4 + 110880*a^2*(
-b)^(49/4)*c^4 + 358400*a^3*(-b)^(49/4)*c^3 + 10458*a^4*(-b)^(45/4)*c^6 + 2
59644*a^5*(-b)^(45/4)*c^5 + 359128*a^6*(-b)^(45/4)*c^4 - 756*a^7*(-b)^(41/4
)*c^7 + 13888*a^7*(-b)^(45/4)*c^3 - 10374*a^8*(-b)^(41/4)*c^6 - 8736*a^9*(-
b)^(41/4)*c^5 + 3080*a^2*(-b)^(49/4)*c^5 + 268800*a^3*(-b)^(49/4)*c^4 + 347
200*a^4*(-b)^(49/4)*c^3 + 51534*a^5*(-b)^(45/4)*c^6 + 343588*a^6*(-b)^(45/4
)*c^5 + 215040*a^7*(-b)^(45/4)*c^4 - 756*a^8*(-b)^(41/4)*c^7 + 3584*a^8*(-b
)^(45/4)*c^3 - 2520*a^9*(-b)^(41/4)*c^6 - 3584*a^3*(-b)^(49/4)*c^5 + 347200
a^4(-b)^(49/4)*c^4 + 2520*a^5*(-b)^(45/4)*c^7 + 201600*a^5*(-b)^(49/4)*c^
3 + 101262*a^6*(-b)^(45/4)*c^6 + 252840*a^7*(-b)^(45/4)*c^5 + 68544*a^8*(-b
)^(45/4)*c^4 - 252*a^9*(-b)^(41/4)*c^7 - 9926*a^3*(-b)^(49/4)*c^6 - 71568*a
^4*(-b)^(49/4)*c^5 + 252000*a^5*(-b)^(49/4)*c^4 + 9912*a^6*(-b)^(45/4)*c^7
 + 64960*a^6*(-b)^(49/4)*c^3 + 99162*a^7*(-b)^(45/4)*c^6 + 98112*a^8*(-b)^(4
5/4)*c^5 + 8960*a^9*(-b)^(45/4)*c^4 - 221760*a^2*(-b)^(53/4)*c^4 - 65898*a^
4*(-b)^(49/4)*c^6 - 197792*a^5*(-b)^(49/4)*c^5 + 210*a^6*(-b)^(45/4)*c^8 +
97440*a^6*(-b)^(49/4)*c^4 + 14616*a^7*(-b)^(45/4)*c^7 + 8960*a^7*(-b)^(49/4
)*c^3 + 48384*a^8*(-b)^(45/4)*c^6 + 15680*a^9*(-b)^(45/4)*c^5 - 121856*a^2*
(-b)^(53/4)*c^5 - 358400*a^3*(-b)^(53/4)*c^4 - 6564*a^4*(-b)^(49/4)*c^7 - 1
77114*a^5*(-b)^(49/4)*c^6 - 254968*a^6*(-b)^(49/4)*c^5 + 630*a^7*(-b)^(45/4
)*c^8 + 15680*a^7*(-b)^(49/4)*c^4 + 9576*a^8*(-b)^(45/4)*c^7 + 9408*a^9*(-b
)^(45/4)*c^6 - 8624*a^2*(-b)^(53/4)*c^6 - 310464*a^3*(-b)^(53/4)*c^5 - 3472
00*a^4*(-b)^(53/4)*c^4 - 33276*a^5*(-b)^(49/4)*c^7 - 248206*a^6*(-b)^(49/4)

*c^6 - 175392*a^7*(-b)^(49/4)*c^5 + 630*a^8*(-b)^(45/4)*c^8 + 2352*a^9*(-b)
^(45/4)*c^7 - 36288*a^3*(-b)^(53/4)*c^6 - 430080*a^4*(-b)^(53/4)*c^5 - 1518
a^5(-b)^(49/4)*c^8 - 201600*a^5*(-b)^(53/4)*c^4 - 67164*a^6*(-b)^(49/4)*c
^7 - 192024*a^7*(-b)^(49/4)*c^6 - 62272*a^8*(-b)^(49/4)*c^5 + 210*a^9*(-b)^
(45/4)*c^8 + 2232*a^3*(-b)^(53/4)*c^7 - 47712*a^4*(-b)^(53/4)*c^6 - 347200*
a^5*(-b)^(53/4)*c^5 - 6042*a^6*(-b)^(49/4)*c^8 - 64960*a^6*(-b)^(53/4)*c^4
- 67476*a^7*(-b)^(49/4)*c^7 - 77952*a^8*(-b)^(49/4)*c^6 - 8960*a^9*(-b)^(49
/4)*c^5 + 133056*a^2*(-b)^(57/4)*c^5 + 20184*a^4*(-b)^(53/4)*c^7 + 4928*a^5
*(-b)^(53/4)*c^6 - 120*a^6*(-b)^(49/4)*c^9 - 161280*a^6*(-b)^(53/4)*c^5 - 9
018*a^7*(-b)^(49/4)*c^8 - 8960*a^7*(-b)^(53/4)*c^4 - 33744*a^8*(-b)^(49/4)*
c^7 - 12992*a^9*(-b)^(49/4)*c^6 + 77504*a^2*(-b)^(57/4)*c^6 + 215040*a^3*(-
b)^(57/4)*c^5 + 2433*a^4*(-b)^(53/4)*c^8 + 65016*a^5*(-b)^(53/4)*c^7 + 7291
2*a^6*(-b)^(53/4)*c^6 - 360*a^7*(-b)^(49/4)*c^9 - 38976*a^7*(-b)^(53/4)*c^5
 - 5982*a^8*(-b)^(49/4)*c^8 - 6720*a^9*(-b)^(49/4)*c^7 + 6960*a^2*(-b)^(57/
4)*c^7 + 202944*a^3*(-b)^(57/4)*c^6 + 208320*a^4*(-b)^(57/4)*c^5 + 12999*a^
5*(-b)^(53/4)*c^8 + 102984*a^6*(-b)^(53/4)*c^7 + 75264*a^7*(-b)^(53/4)*c^6
- 360*a^8*(-b)^(49/4)*c^9 - 3584*a^8*(-b)^(53/4)*c^5 - 1488*a^9*(-b)^(49/4)
*c^8 + 35072*a^3*(-b)^(57/4)*c^7 + 291200*a^4*(-b)^(57/4)*c^6 + 582*a^5*(-b
)^(53/4)*c^9 + 120960*a^5*(-b)^(57/4)*c^5 + 27471*a^6*(-b)^(53/4)*c^8 + 872
16*a^7*(-b)^(53/4)*c^7 + 32704*a^8*(-b)^(53/4)*c^6 - 120*a^9*(-b)^(49/4)*c^
9 + 869*a^3*(-b)^(57/4)*c^8 + 69600*a^4*(-b)^(57/4)*c^7 + 246400*a^5*(-b)^(
57/4)*c^6 + 2358*a^6*(-b)^(53/4)*c^9 + 38976*a^6*(-b)^(57/4)*c^5 + 28749*a^
7*(-b)^(53/4)*c^8 + 38016*a^8*(-b)^(53/4)*c^7 + 5376*a^9*(-b)^(53/4)*c^6 -
44352*a^2*(-b)^(61/4)*c^6 + 1335*a^4*(-b)^(57/4)*c^8 + 65088*a^5*(-b)^(57/4
)*c^7 + 45*a^6*(-b)^(53/4)*c^10 + 122304*a^6*(-b)^(57/4)*c^6 + 3582*a^7*(-b
)^(53/4)*c^9 + 5376*a^7*(-b)^(57/4)*c^5 + 14916*a^8*(-b)^(53/4)*c^8 + 6720*
a^9*(-b)^(53/4)*c^7 - 26880*a^2*(-b)^(61/4)*c^7 - 71680*a^3*(-b)^(61/4)*c^6
 - 380*a^4*(-b)^(57/4)*c^9 - 5289*a^5*(-b)^(57/4)*c^8 + 22192*a^6*(-b)^(57/
4)*c^7 + 135*a^7*(-b)^(53/4)*c^10 + 32704*a^7*(-b)^(57/4)*c^6 + 2418*a^8*(-
b)^(53/4)*c^9 + 3072*a^9*(-b)^(53/4)*c^8 - 2668*a^2*(-b)^(61/4)*c^8 - 71616
a^3(-b)^(61/4)*c^7 - 69440*a^4*(-b)^(61/4)*c^6 - 2416*a^5*(-b)^(57/4)*c^9
 - 17171*a^6*(-b)^(57/4)*c^8 - 7872*a^7*(-b)^(57/4)*c^7 + 135*a^8*(-b)^(53/
4)*c^10 + 3584*a^8*(-b)^(57/4)*c^6 + 612*a^9*(-b)^(53/4)*c^9 - 14384*a^3*(-
b)^(61/4)*c^8 - 104960*a^4*(-b)^(61/4)*c^7 - 123*a^5*(-b)^(57/4)*c^10 - 403
20*a^5*(-b)^(61/4)*c^6 - 5784*a^6*(-b)^(57/4)*c^9 - 19464*a^7*(-b)^(57/4)*c
^8 - 8256*a^8*(-b)^(57/4)*c^7 + 45*a^9*(-b)^(53/4)*c^10 - 670*a^3*(-b)^(61/
4)*c^9 - 31752*a^4*(-b)^(61/4)*c^8 - 91200*a^5*(-b)^(61/4)*c^7 - 517*a^6*(-
b)^(57/4)*c^10 - 12992*a^6*(-b)^(61/4)*c^6 - 6656*a^7*(-b)^(57/4)*c^9 - 100
32*a^8*(-b)^(57/4)*c^8 - 1792*a^9*(-b)^(57/4)*c^7 + 6336*a^2*(-b)^(65/4)*c^
7 - 2830*a^4*(-b)^(61/4)*c^9 - 36272*a^5*(-b)^(61/4)*c^8 - 10*a^6*(-b)^(57/
4)*c^11 - 46848*a^6*(-b)^(61/4)*c^7 - 813*a^7*(-b)^(57/4)*c^10 - 1792*a^7*(
-b)^(61/4)*c^6 - 3724*a^8*(-b)^(57/4)*c^9 - 1984*a^9*(-b)^(57/4)*c^8 + 3952
a^2(-b)^(65/4)*c^8 + 10240*a^3*(-b)^(65/4)*c^7 - 43*a^4*(-b)^(61/4)*c^10
- 4374*a^5*(-b)^(61/4)*c^9 - 21868*a^6*(-b)^(61/4)*c^8 - 30*a^7*(-b)^(57/4)
*c^11 - 13120*a^7*(-b)^(61/4)*c^7 - 567*a^8*(-b)^(57/4)*c^10 - 816*a^9*(-b)
^(57/4)*c^9 + 412*a^2*(-b)^(65/4)*c^9 + 10656*a^3*(-b)^(65/4)*c^8 + 9920*a^
4*(-b)^(65/4)*c^7 - 57*a^5*(-b)^(61/4)*c^10 - 2586*a^6*(-b)^(61/4)*c^9 - 57
60*a^7*(-b)^(61/4)*c^8 - 30*a^8*(-b)^(57/4)*c^11 - 1536*a^8*(-b)^(61/4)*c^7
 - 148*a^9*(-b)^(57/4)*c^10 + 2304*a^3*(-b)^(65/4)*c^9 + 15840*a^4*(-b)^(65
/4)*c^8 + 8*a^5*(-b)^(61/4)*c^11 + 5760*a^5*(-b)^(65/4)*c^7 + 183*a^6*(-b)^
(61/4)*c^10 + 236*a^7*(-b)^(61/4)*c^9 + 128*a^8*(-b)^(61/4)*c^8 - 10*a^9*(-
b)^(57/4)*c^11 + 125*a^3*(-b)^(65/4)*c^10 + 5352*a^4*(-b)^(65/4)*c^9 + 1400
0*a^5*(-b)^(65/4)*c^8 + 40*a^6*(-b)^(61/4)*c^11 + 1856*a^6*(-b)^(65/4)*c^7
+ 461*a^7*(-b)^(61/4)*c^10 + 864*a^8*(-b)^(61/4)*c^9 + 256*a^9*(-b)^(61/4)*
c^8 + 595*a^4*(-b)^(65/4)*c^10 + 6608*a^5*(-b)^(65/4)*c^9 + a^6*(-b)^(61/4)
*c^12 + 7344*a^6*(-b)^(65/4)*c^8 + 72*a^7*(-b)^(61/4)*c^11 + 256*a^7*(-b)^(
65/4)*c^7 + 360*a^8*(-b)^(61/4)*c^10 + 256*a^9*(-b)^(61/4)*c^9 + 18*a^4*(-b
)^(65/4)*c^11 + 1131*a^5*(-b)^(65/4)*c^10 + 4572*a^6*(-b)^(65/4)*c^9 + 3*a^
7*(-b)^(61/4)*c^12 + 2112*a^7*(-b)^(65/4)*c^8 + 56*a^8*(-b)^(61/4)*c^11 + 9

6*a^9*(-b)^(61/4)*c^10 + 70*a^5*(-b)^(65/4)*c^11 + 1073*a^6*(-b)^(65/4)*c^1
0 + 1680*a^7*(-b)^(65/4)*c^9 + 3*a^8*(-b)^(61/4)*c^12 + 256*a^8*(-b)^(65/4)
*c^8 + 16*a^9*(-b)^(61/4)*c^11 + a^5*(-b)^(65/4)*c^12 + 102*a^6*(-b)^(65/4)
*c^11 + 508*a^7*(-b)^(65/4)*c^10 + 256*a^8*(-b)^(65/4)*c^9 + a^9*(-b)^(61/4
)*c^12 + 3*a^6*(-b)^(65/4)*c^12 + 66*a^7*(-b)^(65/4)*c^11 + 96*a^8*(-b)^(65
/4)*c^10 + 3*a^7*(-b)^(65/4)*c^12 + 16*a^8*(-b)^(65/4)*c^11 + a^8*(-b)^(65/
4)*c^12 - 64*a*(-b)^(37/4)*c + 15232*a*(-b)^(41/4)*c + 6*a^5*(-b)^(21/4)*c
 + 22*a^6*(-b)^(21/4)*c + 30*a^7*(-b)^(21/4)*c - 116*a^4*(-b)^(25/4)*c + 18*
a^8*(-b)^(21/4)*c - 504*a^5*(-b)^(25/4)*c + 4*a^9*(-b)^(21/4)*c - 864*a^6*(
-b)^(25/4)*c + 706*a^3*(-b)^(29/4)*c - 728*a^7*(-b)^(25/4)*c + 3698*a^4*(-b
)^(29/4)*c - 300*a^8*(-b)^(25/4)*c + 7914*a^5*(-b)^(29/4)*c - 48*a^9*(-b)^(
25/4)*c - 1476*a^2*(-b)^(33/4)*c + 8806*a^6*(-b)^(29/4)*c - 9472*a^3*(-b)^(
33/4)*c + 5324*a^7*(-b)^(29/4)*c - 25368*a^4*(-b)^(33/4)*c + 1632*a^8*(-b)^
(29/4)*c - 36528*a^5*(-b)^(33/4)*c + 192*a^9*(-b)^(29/4)*c + 1536*a^2*(-b)^
(37/4)*c - 30212*a^6*(-b)^(33/4)*c + 10176*a^3*(-b)^(37/4)*c - 14064*a^7*(-
b)^(33/4)*c + 25600*a^4*(-b)^(37/4)*c - 3264*a^8*(-b)^(33/4)*c + 33600*a^5*
(-b)^(37/4)*c - 256*a^9*(-b)^(33/4)*c + 2688*a*(-b)^(41/4)*c^2 + 44352*a^2*
(-b)^(41/4)*c + 24576*a^6*(-b)^(37/4)*c + 71680*a^3*(-b)^(41/4)*c + 9536*a^
7*(-b)^(37/4)*c + 69440*a^4*(-b)^(41/4)*c + 1536*a^8*(-b)^(37/4)*c + 40320*
a^5*(-b)^(41/4)*c - 45696*a*(-b)^(45/4)*c^2 + 12992*a^6*(-b)^(41/4)*c - 103
04*a*(-b)^(45/4)*c^3 + 1792*a^7*(-b)^(41/4)*c + 76160*a*(-b)^(49/4)*c^3 + 1
9040*a*(-b)^(49/4)*c^4 - 76160*a*(-b)^(53/4)*c^4 - 20160*a*(-b)^(53/4)*c^5
 + 45696*a*(-b)^(57/4)*c^5 + 12544*a*(-b)^(57/4)*c^6 - 15232*a*(-b)^(61/4)*c
^6 - 4288*a*(-b)^(61/4)*c^7 + 2176*a*(-b)^(65/4)*c^7 + 624*a*(-b)^(65/4)*c^
8))/(a^7*b^18 + 9*a^8*b^18*c + 36*a^9*b^18*c^2 + 84*a^10*b^18*c^3 + 126*a^1
1*b^18*c^4 + 126*a^12*b^18*c^5 + 84*a^13*b^18*c^6 + 36*a^14*b^18*c^7 + 9*a^
15*b^18*c^8 + a^16*b^18*c^9) + (((-b)^(7/4)*1i - (a*(-b)^(3/4)*(4*b + b*c +
 1)*1i)/4)*((((-b)^(7/4)*1i - (a*(-b)^(3/4)*(4*b + b*c + 1)*1i)/4)^3*((64*(
36*a^12*(-b)^(25/4) - 4*a^13*(-b)^(21/4) + 48*a^13*(-b)^(25/4) - 60*a^11*(-
b)^(29/4) - 240*a^12*(-b)^(29/4) - 192*a^13*(-b)^(29/4) - 180*a^10*(-b)^(33
/4) - 240*a^11*(-b)^(33/4) + 192*a^12*(-b)^(33/4) + 240*a^9*(-b)^(37/4) + 2
56*a^13*(-b)^(33/4) + 1200*a^10*(-b)^(37/4) + 1728*a^11*(-b)^(37/4) + 320*a
^8*(-b)^(41/4) + 768*a^12*(-b)^(37/4) + 1152*a^9*(-b)^(41/4) + 1344*a^10*(-
b)^(41/4) + 512*a^11*(-b)^(41/4) - 144*a^13*(-b)^(29/4)*c^2 + 912*a^12*(-b)
^(33/4)*c^2 + 1344*a^13*(-b)^(33/4)*c^2 + 336*a^13*(-b)^(33/4)*c^3 - 432*a^
11*(-b)^(37/4)*c^2 - 4032*a^12*(-b)^(37/4)*c^2 - 1680*a^12*(-b)^(37/4)*c^3
 - 4032*a^13*(-b)^(37/4)*c^2 - 4624*a^10*(-b)^(41/4)*c^2 - 2688*a^13*(-b)^(3
7/4)*c^3 - 9408*a^11*(-b)^(41/4)*c^2 - 504*a^13*(-b)^(37/4)*c^4 - 336*a^11*
(-b)^(41/4)*c^3 - 1344*a^12*(-b)^(41/4)*c^2 + 3584*a^9*(-b)^(45/4)*c^2 + 53
76*a^12*(-b)^(41/4)*c^3 + 3584*a^13*(-b)^(41/4)*c^2 + 20160*a^10*(-b)^(45/4
)*c^2 + 1848*a^12*(-b)^(41/4)*c^4 + 6720*a^13*(-b)^(41/4)*c^3 + 8848*a^10*(
-b)^(45/4)*c^3 + 30912*a^11*(-b)^(45/4)*c^2 + 3360*a^13*(-b)^(41/4)*c^4 + 6
720*a^8*(-b)^(49/4)*c^2 + 21504*a^11*(-b)^(45/4)*c^3 + 14336*a^12*(-b)^(45/
4)*c^2 + 504*a^13*(-b)^(41/4)*c^5 + 24192*a^9*(-b)^(49/4)*c^2 + 1848*a^11*(
-b)^(45/4)*c^4 + 9408*a^12*(-b)^(45/4)*c^3 - 4032*a^9*(-b)^(49/4)*c^3 + 282
24*a^10*(-b)^(49/4)*c^2 - 3360*a^12*(-b)^(45/4)*c^4 - 3584*a^13*(-b)^(45/4)
*c^3 - 26880*a^10*(-b)^(49/4)*c^3 + 10752*a^11*(-b)^(49/4)*c^2 - 1176*a^12*
(-b)^(45/4)*c^5 - 6720*a^13*(-b)^(45/4)*c^4 - 10584*a^10*(-b)^(49/4)*c^4 -
44352*a^11*(-b)^(49/4)*c^3 - 2688*a^13*(-b)^(45/4)*c^5 - 11200*a^8*(-b)^(53
/4)*c^3 - 30240*a^11*(-b)^(49/4)*c^4 - 21504*a^12*(-b)^(49/4)*c^3 - 336*a^1
3*(-b)^(45/4)*c^6 - 40320*a^9*(-b)^(53/4)*c^3 - 2520*a^11*(-b)^(49/4)*c^5 -
 20160*a^12*(-b)^(49/4)*c^4 + 1120*a^9*(-b)^(53/4)*c^4 - 47040*a^10*(-b)^(5
3/4)*c^3 + 16800*a^10*(-b)^(53/4)*c^4 - 17920*a^11*(-b)^(53/4)*c^3 + 336*a^
12*(-b)^(49/4)*c^6 + 4032*a^13*(-b)^(49/4)*c^5 + 8120*a^10*(-b)^(53/4)*c^5
 + 33600*a^11*(-b)^(53/4)*c^4 + 1344*a^13*(-b)^(49/4)*c^6 + 11200*a^8*(-b)^(
57/4)*c^4 + 26880*a^11*(-b)^(53/4)*c^5 + 17920*a^12*(-b)^(53/4)*c^4 + 144*a
^13*(-b)^(49/4)*c^7 + 40320*a^9*(-b)^(57/4)*c^4 + 1680*a^11*(-b)^(53/4)*c^6
 + 22848*a^12*(-b)^(53/4)*c^5 + 2240*a^9*(-b)^(57/4)*c^5 + 47040*a^10*(-b)^
(57/4)*c^4 + 1344*a^12*(-b)^(53/4)*c^6 + 3584*a^13*(-b)^(53/4)*c^5 + 17920*

a^11*(-b)^(57/4)*c^4 + 48*a^12*(-b)^(53/4)*c^7 - 1344*a^13*(-b)^(53/4)*c^6 - 3920*a^10*(-b)^(57/4)*c^6 - 9408*a^11*(-b)^(57/4)*c^5 - 384*a^13*(-b)^(53/4)*c^7 - 6720*a^8*(-b)^(61/4)*c^5 - 14784*a^11*(-b)^(57/4)*c^6 - 7168*a^12*(-b)^(57/4)*c^5 - 36*a^13*(-b)^(53/4)*c^8 - 24192*a^9*(-b)^(61/4)*c^5 - 528*a^11*(-b)^(57/4)*c^7 - 14784*a^12*(-b)^(57/4)*c^6 - 2688*a^9*(-b)^(61/4)*c^6 - 28224*a^10*(-b)^(61/4)*c^5 - 768*a^12*(-b)^(57/4)*c^7 - 3584*a^13*(-b)^(57/4)*c^6 - 6720*a^10*(-b)^(61/4)*c^6 - 10752*a^11*(-b)^(61/4)*c^5 - 60*a^12*(-b)^(57/4)*c^8 + 192*a^13*(-b)^(57/4)*c^7 + 1104*a^10*(-b)^(61/4)*c^7 - 4032*a^11*(-b)^(61/4)*c^6 + 48*a^13*(-b)^(57/4)*c^8 + 2240*a^8*(-b)^(65/4)*c^6 + 4608*a^11*(-b)^(61/4)*c^7 + 4*a^13*(-b)^(57/4)*c^9 + 8064*a^9*(-b)^(65/4)*c^6 + 36*a^11*(-b)^(61/4)*c^8 + 5184*a^12*(-b)^(61/4)*c^7 + 1216*a^9*(-b)^(65/4)*c^7 + 9408*a^10*(-b)^(65/4)*c^6 + 144*a^12*(-b)^(61/4)*c^8 + 1536*a^13*(-b)^(61/4)*c^7 + 3840*a^10*(-b)^(65/4)*c^7 + 3584*a^11*(-b)^(65/4)*c^6 + 12*a^12*(-b)^(61/4)*c^9 - 148*a^10*(-b)^(65/4)*c^8 + 3648*a^11*(-b)^(65/4)*c^7 - 320*a^8*(-b)^(69/4)*c^7 - 624*a^11*(-b)^(65/4)*c^8 + 1024*a^12*(-b)^(65/4)*c^7 - 1152*a^9*(-b)^(69/4)*c^7 + 12*a^11*(-b)^(65/4)*c^9 - 768*a^12*(-b)^(65/4)*c^8 - 208*a^9*(-b)^(69/4)*c^8 - 1344*a^10*(-b)^(69/4)*c^7 - 256*a^13*(-b)^(65/4)*c^8 - 720*a^10*(-b)^(69/4)*c^8 - 512*a^11*(-b)^(69/4)*c^7 + 4*a^10*(-b)^(69/4)*c^9 - 768*a^11*(-b)^(69/4)*c^8 - 256*a^12*(-b)^(69/4)*c^8 + 36*a^13*(-b)^(25/4)*c - 276*a^12*(-b)^(29/4)*c - 384*a^13*(-b)^(29/4)*c + 300*a^11*(-b)^(33/4)*c + 1536*a^12*(-b)^(33/4)*c + 1344*a^13*(-b)^(33/4)*c + 1380*a^10*(-b)^(37/4)*c + 2304*a^11*(-b)^(37/4)*c - 576*a^12*(-b)^(37/4)*c - 1472*a^9*(-b)^(41/4)*c - 1536*a^13*(-b)^(37/4)*c - 7680*a^10*(-b)^(41/4)*c - 11328*a^11*(-b)^(41/4)*c - 2240*a^8*(-b)^(45/4)*c - 5120*a^12*(-b)^(41/4)*c - 8064*a^9*(-b)^(45/4)*c - 9408*a^10*(-b)^(45/4)*c - 3584*a^11*(-b)^(45/4)*c))/(a^7*b^18 + 9*a^8*b^18*c + 36*a^9*b^18*c^2 + 84*a^10*b^18*c^3 + 126*a^11*b^18*c^4 + 126*a^12*b^18*c^5 + 84*a^13*b^18*c^6 + 36*a^14*b^18*c^7 + 9*a^15*b^18*c^8 + a^16*b^18*c^9) - (64*((-b)^(7/4)*1i - (a*(-b)^(3/4)*(4*b + b*c + 1)*1i)/4)*(-(b*x - 1)/(c + x))^(1/4)*(16*a^13*(-b)^(11/2) - 80*a^12*(-b)^(13/2) - 80*a^11*(-b)^(15/2) - 128*a^13*(-b)^(13/2) + 400*a^10*(-b)^(17/2) + 128*a^12*(-b)^(15/2) + 896*a^9*(-b)^(19/2) + 1152*a^11*(-b)^(17/2) + 256*a^13*(-b)^(15/2) + 512*a^8*(-b)^(21/2) + 1920*a^10*(-b)^(19/2) + 768*a^12*(-b)^(17/2) + 1024*a^9*(-b)^(21/2) + 1024*a^11*(-b)^(19/2) + 512*a^10*(-b)^(21/2) + 448*a^13*(-b)^(15/2)*c^2 - 1344*a^12*(-b)^(17/2)*c^2 - 2624*a^11*(-b)^(19/2)*c^2 - 2688*a^13*(-b)^(17/2)*c^2 + 1088*a^10*(-b)^(21/2)*c^2 + 128*a^12*(-b)^(19/2)*c^2 - 896*a^13*(-b)^(17/2)*c^3 + 9600*a^9*(-b)^(23/2)*c^2 + 9344*a^11*(-b)^(21/2)*c^2 + 1792*a^12*(-b)^(19/2)*c^3 + 4096*a^13*(-b)^(19/2)*c^2 + 7680*a^8*(-b)^(25/2)*c^2 + 21888*a^10*(-b)^(23/2)*c^2 + 4096*a^11*(-b)^(21/2)*c^3 + 8704*a^12*(-b)^(21/2)*c^2 + 4480*a^13*(-b)^(19/2)*c^3 + 15360*a^9*(-b)^(25/2)*c^2 + 3328*a^10*(-b)^(23/2)*c^3 + 12288*a^11*(-b)^(23/2)*c^2 + 128*a^12*(-b)^(21/2)*c^3 + 1120*a^13*(-b)^(19/2)*c^4 - 8320*a^9*(-b)^(25/2)*c^3 + 7680*a^10*(-b)^(25/2)*c^2 - 4992*a^11*(-b)^(23/2)*c^3 - 1120*a^12*(-b)^(21/2)*c^4 - 6656*a^13*(-b)^(21/2)*c^3 - 10240*a^8*(-b)^(27/2)*c^3 - 21120*a^10*(-b)^(25/2)*c^3 - 2400*a^11*(-b)^(23/2)*c^4 - 9216*a^12*(-b)^(23/2)*c^3 - 4480*a^13*(-b)^(21/2)*c^4 - 20480*a^9*(-b)^(27/2)*c^3 - 7200*a^10*(-b)^(25/2)*c^4 - 12800*a^11*(-b)^(25/2)*c^3 + 1920*a^12*(-b)^(23/2)*c^4 - 896*a^13*(-b)^(21/2)*c^5 + 640*a^9*(-b)^(27/2)*c^4 - 10240*a^10*(-b)^(27/2)*c^3 - 3200*a^11*(-b)^(25/2)*c^4 + 7680*a^13*(-b)^(23/2)*c^4 + 7680*a^8*(-b)^(29/2)*c^4 + 5760*a^10*(-b)^(27/2)*c^4 - 1280*a^11*(-b)^(25/2)*c^5 + 5120*a^12*(-b)^(25/2)*c^4 + 2688*a^13*(-b)^(23/2)*c^5 + 15360*a^9*(-b)^(29/2)*c^4 + 5120*a^10*(-b)^(27/2)*c^5 + 5120*a^11*(-b)^(27/2)*c^4 - 5248*a^12*(-b)^(25/2)*c^5 + 448*a^13*(-b)^(23/2)*c^6 + 4224*a^9*(-b)^(29/2)*c^5 + 7680*a^10*(-b)^(29/2)*c^4 + 3968*a^11*(-b)^(27/2)*c^5 + 448*a^12*(-b)^(25/2)*c^6 - 6656*a^13*(-b)^(25/2)*c^5 - 3072*a^8*(-b)^(31/2)*c^5 + 5760*a^10*(-b)^(29/2)*c^5 + 2752*a^11*(-b)^(27/2)*c^6 - 2048*a^12*(-b)^(27/2)*c^5 - 896*a^13*(-b)^(25/2)*c^6 - 6144*a^9*(-b)^(31/2)*c^5 - 704*a^10*(-b)^(29/2)*c^6 + 1536*a^11*(-b)^(29/2)*c^5 + 5504*a^12*(-b)^(27/2)*c^6 - 128*a^13*(-b)^(25/2)*c^7 - 2944*a^9*(-b)^(31/2)*c^6 - 3072*a^10*(-b)^(31/2)*c^5 + 384*a^11*(-b)^(29/2)*c^6 - 256*a^12*(-b)^(27/2)*c^7 + 4096 *a^13*(-b)^(27/2)*c^6 + 512*a^8*(-b)^(33/2)*c^6 - 4992*a^10*(-b)^(31/2)*c^6 - 1536*a^11*(-b)^(29/2)*c^7 + 1536*a^12*(-b)^(29/2)*c^6 + 128*a^13*(-b)^(27/2)*c^7 + 1024*a^9*(-b)^(33/2)*c^6 - 768*a^10*(-b)^(31/2)*c^7 - 2048*a^11*(-b)^(31/2)*c^6 - 2688*a^12*(-b)^(29/2)*c^7 + 16*a^13*(-b)^(27/2)*c^8 + 640*a^9*(-b)^(33/2)*c^7 + 512*a^10*(-b)^(33/2)*c^6 - 1664*a^11*(-b)^(31/2)*c^7 + 48*a^12*(-b)^(29/2)*c^8 - 1536*a^13*(-b)^(29/2)*c^7 + 1152*a^10*(-b)^(33/2)*c^7 + 304*a^11*(-b)^(31/2)*c^8 - 1024*a^12*(-b)^(31/2)*c^7 + 272*a^10*(-b)^(33/2)*c^8 + 512*a^11*(-b)^(33/2)*c^7 + 512*a^12*(-b)^(31/2)*c^8 + 512*a^11*(-b)^(33/2)*c^8 + 256*a^13*(-b)^(31/2)*c^8 + 256*a^12*(-b)^(33/2)*c^8 - 128*a^13*(-b)^(13/2)*c + 512*a^12*(-b)^(15/2)*c + 768*a^11*(-b)^(17/2)*c + 896*a^13*(-b)^(15/2)*c - 1536*a^10*(-b)^(19/2)*c - 384*a^12*(-b)^(17/2)*c - 4736*a^9*(-b)^(21/2)*c - 5504*a^11*(-b)^(19/2)*c - 1536*a^13*(-b)^(17/2)*c - 3072*a^8*(-b)^(23/2)*c - 10368*a^10*(-b)^(21/2)*c - 4096*a^12*(-b)^(19/2)*c - 6144*a^9*(-b)^(23/2)*c - 5632*a^11*(-b)^(21/2)*c - 3072*a^10*(-b)^(23/2)*c))/(a^2*(-b)^(9/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^8*b^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a^12*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9))))/(a^6*b^6) + (64*(-(b*x - 1)/(c + x))^(1/4)*(512*(-b)^(19/2) + a^5*(-b)^(9/2) + a^4*(-b)^(11/2) + 2*a^6*(-b)^(9/2) - 16*a^3*(-b)^(13/2) + 18*a^5*(-b)^(11/2) + a^7*(-b)^(9/2) - 144*a^2*(-b)^(15/2) - 176*a^4*(-b)^(13/2) + 49*a^6*(-b)^(11/2) - 576*a^3*(-b)^(15/2) - 560*a^5*(-b)^(13/2) + 48*a^7*(-b)^(11/2) + 2688*a^2*(-b)^(17/2) - 608*a^4*(-b)^(15/2) - 784*a^6*(-b)^(13/2) + 16*a^8*(-b)^(11/2) + 7680*a^3*(-b)^(17/2) + 448*a^5*(-b)^(15/2) - 512*a^7*(-b)^(13/2) + 7680*a^2*(-b)^(19/2) + 11520*a^4*(-b)^(17/2) + 1392*a^6*(-b)^(15/2) - 128*a^8*(-b)^(13/2) + 10240*a^3*(-b)^(19/2) + 9600*a^5*(-b)^(17/2) + 1024*a^7*(-b)^(15/2) + 7680*a^4*(-b)^(19/2) + 4224*a^6*(-b)^(17/2) + 256*a^8*(-b)^(15/2) + 3072*a^5*(-b)^(19/2) + 768*a^7*(-b)^(17/2) + 512*a^6*(-b)^(19/2) + 7680*(-b)^(23/2)*c^2 - 10240*(-b)^(25/2)*c^3 + 7680*(-b)^(27/2)*c^4 - 3072*(-b)^(29/2)*c^5 + 512*(-b)^(31/2)*c^6 + 384*a*(-b)^(17/2) + 3072*a*(-b)^(19/2) - 3072*(-b)^(21/2)*c + a^7*(-b)^(9/2)*c^2 - 35*a^6*(-b)^(11/2)*c^2 + 2*a^8*(-b)^(9/2)*c^2 + 265*a^5*(-b)^(13/2)*c^2 - 86*a^7*(-b)^(11/2)*c^2 + a^9*(-b)^(9/2)*c^2 - 851*a^4*(-b)^(15/2)*c^2 + 738*a^6*(-b)^(13/2)*c^2 - 10*a^7*(-b)^(11/2)*c^3 - 67*a^8*(-b)^(11/2)*c^2 + 2496*a^3*(-b)^(17/2)*c^2 - 2566*a^5*(-b)^(15/2)*c^2 + 224*a^6*(-b)^(13/2)*c^3 + 649*a^7*(-b)^(13/2)*c^2 - 20*a^8*(-b)^(11/2)*c^3 - 16*a^9*(-b)^(11/2)*c^2 - 5184*a^2*(-b)^(19/2)*c^2 + 10432*a^4*(-b)^(17/2)*c^2 - 1358*a^5*(-b)^(15/2)*c^3 - 1907*a^6*(-b)^(15/2)*c^2 + 592*a^7*(-b)^(13/2)*c^3 + 144*a^8*(-b)^(13/2)*c^2 - 10*a^9*(-b)^(11/2)*c^3 - 31104*a^3*(-b)^(19/2)*c^2 + 3784*a^4*(-b)^(17/2)*c^3 + 14912*a^5*(-b)^(17/2)*c^2 - 4364*a^6*(-b)^(15/2)*c^3 + 45*a^7*(-b)^(13/2)*c^4 + 1120*a^7*(-b)^(15/2)*c^2 + 512*a^8*(-b)^(13/2)*c^3 - 32*a^9*(-b)^(13/2)*c^2 + 1152*a^2*(-b)^(21/2)*c^2 - 7552*a^3*(-b)^(19/2)*c^3 - 74624*a^4*(-b)^(19/2)*c^2 + 14288*a^5*(-b)^(17/2)*c^3 - 771*a^6*(-b)^(15/2)*c^4 + 5824*a^6*(-b)^(17/2)*c^2 - 4974*a^7*(-b)^(15/2)*c^3 + 90*a^8*(-b)^(13/2)*c^4 + 1952*a^8*(-b)^(15/2)*c^2 + 144*a^9*(-b)^(13/2)*c^3 + 6912*a^2*(-b)^(21/2)*c^3 + 23040*a^3*(-b)^(21/2)*c^2 - 36800*a^4*(-b)^(19/2)*c^3 + 3874*a^5*(-b)^(17/2)*c^4 - 91136*a^5*(-b)^(19/2)*c^2 + 19144*a^6*(-b)^(17/2)*c^3 - 2118*a^7*(-b)^(15/2)*c^4 - 5120*a^7*(-b)^(17/2)*c^2 - 2288*a^8*(-b)^(15/2)*c^3 + 45*a^9*(-b)^(13/2)*c^4 + 640*a^9*(-b)^(15/2)*c^2 + 115200*a^2*(-b)^(23/2)*c^2 + 49536*a^3*(-b)^(21/2)*c^3 - 8750*a^4*(-b)^(19/2)*c^4 + 57600*a^4*(-b)^(21/2)*c^2 - 68032*a^5*(-b)^(19/2)*c^3 + 13444*a^6*(-b)^(17/2)*c^4 - 58944*a^6*(-b)^(19/2)*c^2 - 120*a^7*(-b)^(15/2)*c^5 + 9664*a^7*(-b)^(17/2)*c^3 - 1923*a^8*(-b)^(15/2)*c^4 - 5248*a^8*(-b)^(17/2)*c^2 - 320*a^9*(-b)^(15/2)*c^3 + 44160*a^2*(-b)^(23/2)*c^3 + 11040*a^3*(-b)^(21/2)*c^4 + 153600*a^3*(-b)^(23/2)*c^2 + 137728*a^4*(-b)^(21/2)*c^3 - 36988*a^5*(-b)^(19/2)*c^4 + 63360*a^5*(-b)^(21/2)*c^2 + 1644*a^6*(-b)^(17/2)*c^5 - 56512*a^6*(-b)^(19/2)*c^3 + 17058*a^7*(-b)^(17/2)*c^4 - 18560*a^7*(-b)^(19/2)*c^2 - 240*a^8*(-b)^(15/2)*c^5 + 128*a^8*(-b)^(17/2)*c^3 - 576*a^9*(-b)^(15/2)*c^4 - 1280*a^9*(-b)^(17/2)*c^2 - 480*a^2*(-b)^(23/2)*c^4 + 76800*a^3*(-b)^(23/2)*c^3 + 60640*a^4*(-b)^(21/2)*c^4 + 115200*a^4*(-b)^(23/2)*c^2 - 6776*a^5*(-b)^(19/2)*c^5 + 193792*a^5*(-b)^(21/2

)*c^3 - 59150*a^6*(-b)^(19/2)*c^4 + 33408*a^6*(-b)^(21/2)*c^2 + 4632*a^7*(-b)^(17/2)*c^5 - 16064*a^7*(-b)^(19/2)*c^3 + 9280*a^8*(-b)^(17/2)*c^4 - 2048*a^8*(-b)^(19/2)*c^2 - 120*a^9*(-b)^(15/2)*c^5 - 896*a^9*(-b)^(17/2)*c^3 - 153600*a^2*(-b)^(25/2)*c^3 - 23040*a^3*(-b)^(23/2)*c^4 + 11620*a^4*(-b)^(21/2)*c^5 + 57600*a^4*(-b)^(23/2)*c^3 + 128864*a^5*(-b)^(21/2)*c^4 + 46080*a^5*(-b)^(23/2)*c^2 - 24752*a^6*(-b)^(19/2)*c^5 + 146688*a^6*(-b)^(21/2)*c^3 + 210*a^7*(-b)^(17/2)*c^6 - 43232*a^7*(-b)^(19/2)*c^4 + 6912*a^7*(-b)^(21/2)*c^2 + 4332*a^8*(-b)^(17/2)*c^5 + 3968*a^8*(-b)^(19/2)*c^3 + 1792*a^9*(-b)^(17/2)*c^4 - 90240*a^2*(-b)^(25/2)*c^4 - 7104*a^3*(-b)^(23/2)*c^5 - 204800*a^3*(-b)^(25/2)*c^3 - 100160*a^4*(-b)^(23/2)*c^4 + 53480*a^5*(-b)^(21/2)*c^5 + 9600*a^5*(-b)^(23/2)*c^3 - 2310*a^6*(-b)^(19/2)*c^6 + 131872*a^6*(-b)^(21/2)*c^4 + 7680*a^6*(-b)^(23/2)*c^2 - 33432*a^7*(-b)^(19/2)*c^5 + 56704*a^7*(-b)^(21/2)*c^3 + 420*a^8*(-b)^(17/2)*c^6 - 13216*a^8*(-b)^(19/2)*c^4 + 1344*a^9*(-b)^(17/2)*c^5 + 2304*a^9*(-b)^(19/2)*c^3 - 9216*a^2*(-b)^(25/2)*c^5 - 192000*a^3*(-b)^(25/2)*c^4 - 48224*a^4*(-b)^(23/2)*c^5 - 153600*a^4*(-b)^(25/2)*c^3 + 7546*a^5*(-b)^(21/2)*c^6 - 179840*a^5*(-b)^(23/2)*c^4 + 94948*a^6*(-b)^(21/2)*c^5 - 9600*a^6*(-b)^(23/2)*c^3 - 6636*a^7*(-b)^(19/2)*c^6 + 63232*a^7*(-b)^(21/2)*c^4 - 19712*a^8*(-b)^(19/2)*c^5 + 8704*a^8*(-b)^(21/2)*c^3 + 210*a^9*(-b)^(17/2)*c^6 - 896*a^9*(-b)^(19/2)*c^4 + 115200*a^2*(-b)^(27/2)*c^4 - 31104*a^3*(-b)^(25/2)*c^5 - 8750*a^4*(-b)^(23/2)*c^6 - 211200*a^4*(-b)^(25/2)*c^4 - 121120*a^5*(-b)^(23/2)*c^5 - 61440*a^5*(-b)^(25/2)*c^3 + 28756*a^6*(-b)^(21/2)*c^6 - 161760*a^6*(-b)^(23/2)*c^4 - 252*a^7*(-b)^(19/2)*c^7 + 80416*a^7*(-b)^(21/2)*c^5 - 3840*a^7*(-b)^(23/2)*c^3 - 6342*a^8*(-b)^(19/2)*c^6 + 9344*a^8*(-b)^(21/2)*c^4 - 4256*a^9*(-b)^(19/2)*c^5 + 81792*a^2*(-b)^(27/2)*c^5 - 832*a^3*(-b)^(25/2)*c^6 + 153600*a^3*(-b)^(27/2)*c^4 - 23552*a^4*(-b)^(25/2)*c^5 - 44380*a^5*(-b)^(23/2)*c^6 - 124800*a^5*(-b)^(25/2)*c^4 + 2184*a^6*(-b)^(21/2)*c^7 - 146336*a^6*(-b)^(23/2)*c^5 - 10240*a^6*(-b)^(25/2)*c^3 + 40698*a^7*(-b)^(21/2)*c^6 - 72320*a^7*(-b)^(23/2)*c^4 - 504*a^8*(-b)^(19/2)*c^7 + 31808*a^8*(-b)^(21/2)*c^5 - 2016*a^9*(-b)^(19/2)*c^6 - 1280*a^9*(-b)^(21/2)*c^4 + 10944*a^2*(-b)^(27/2)*c^6 + 184320*a^3*(-b)^(27/2)*c^5 + 8896*a^4*(-b)^(25/2)*c^6 + 115200*a^4*(-b)^(27/2)*c^4 - 5300*a^5*(-b)^(23/2)*c^7 + 32512*a^5*(-b)^(25/2)*c^5 - 86702*a^6*(-b)^(23/2)*c^6 - 36480*a^6*(-b)^(25/2)*c^4 + 6384*a^7*(-b)^(21/2)*c^7 - 87968*a^7*(-b)^(23/2)*c^5 + 25312*a^8*(-b)^(21/2)*c^6 - 12800*a^8*(-b)^(23/2)*c^4 - 252*a^9*(-b)^(19/2)*c^7 + 4480*a^9*(-b)^(21/2)*c^5 - 46080*a^2*(-b)^(29/2)*c^5 + 49536*a^3*(-b)^(27/2)*c^6 + 3016*a^4*(-b)^(25/2)*c^7 + 218880*a^4*(-b)^(27/2)*c^5 + 44864*a^5*(-b)^(25/2)*c^6 + 46080*a^5*(-b)^(27/2)*c^4 - 21128*a^6*(-b)^(23/2)*c^7 + 66048*a^6*(-b)^(25/2)*c^5 + 210*a^7*(-b)^(21/2)*c^8 - 81536*a^7*(-b)^(23/2)*c^6 - 3840*a^7*(-b)^(25/2)*c^4 + 6216*a^8*(-b)^(21/2)*c^7 - 22912*a^8*(-b)^(23/2)*c^5 + 5824*a^9*(-b)^(21/2)*c^6 - 36480*a^2*(-b)^(29/2)*c^6 + 4224*a^3*(-b)^(27/2)*c^7 - 61440*a^3*(-b)^(29/2)*c^5 + 86656*a^4*(-b)^(27/2)*c^6 + 18896*a^5*(-b)^(25/2)*c^7 + 144000*a^5*(-b)^(27/2)*c^5 - 1374*a^6*(-b)^(23/2)*c^8 + 75200*a^6*(-b)^(25/2)*c^6 + 7680*a^6*(-b)^(27/2)*c^4 - 31284*a^7*(-b)^(23/2)*c^7 + 40576*a^7*(-b)^(25/2)*c^5 + 420*a^8*(-b)^(21/2)*c^8 - 36736*a^8*(-b)^(23/2)*c^6 + 2016*a^9*(-b)^(21/2)*c^7 - 1280*a^9*(-b)^(23/2)*c^5 - 5376*a^2*(-b)^(29/2)*c^7 - 84480*a^3*(-b)^(29/2)*c^6 + 13888*a^4*(-b)^(27/2)*c^7 - 46080*a^4*(-b)^(29/2)*c^5 + 2173*a^5*(-b)^(25/2)*c^8 + 70144*a^5*(-b)^(27/2)*c^6 + 42952*a^6*(-b)^(25/2)*c^7 + 49536*a^6*(-b)^(27/2)*c^5 - 4092*a^7*(-b)^(23/2)*c^8 + 57856*a^7*(-b)^(25/2)*c^6 - 20384*a^8*(-b)^(23/2)*c^7 + 8704*a^8*(-b)^(25/2)*c^5 + 210*a^9*(-b)^(21/2)*c^8 - 6272*a^9*(-b)^(23/2)*c^6 + 7680*a^2*(-b)^(31/2)*c^6 - 26496*a^3*(-b)^(29/2)*c^7 + 301*a^4*(-b)^(27/2)*c^8 - 103680*a^4*(-b)^(29/2)*c^6 + 12608*a^5*(-b)^(27/2)*c^7 - 18432*a^5*(-b)^(29/2)*c^5 + 9274*a^6*(-b)^(25/2)*c^8 + 21696*a^6*(-b)^(27/2)*c^6 - 120*a^7*(-b)^(23/2)*c^9 + 45760*a^7*(-b)^(25/2)*c^7 + 6912*a^7*(-b)^(27/2)*c^5 - 4062*a^8*(-b)^(23/2)*c^8 + 20096*a^8*(-b)^(25/2)*c^6 - 4928*a^9*(-b)^(23/2)*c^7 + 6528*a^2*(-b)^(31/2)*c^7 - 2448*a^3*(-b)^(29/2)*c^8 + 10240*a^3*(-b)^(31/2)*c^6 - 52736*a^4*(-b)^(29/2)*c^7 - 1558*a^5*(-b)^(27/2)*c^8 - 71040*a^5*(-b)^(29/2)*c^6 + 546*a^6*(-b)^(25/2)*c^9 - 4544*a^6*(-b)^(27/2)*c^7 - 3072*a^6*(-b)^(29/2)*c^5 + 14

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589*a^7*(-b)^(25/2)*c^8 - 2432*a^7*(-b)^(27/2)*c^6 - 240*a^8*(-b)^(23/2)*c^
9 + 23168*a^8*(-b)^(25/2)*c^7 - 1344*a^9*(-b)^(23/2)*c^8 + 2304*a^9*(-b)^(2
5/2)*c^6 + 1008*a^2*(-b)^(31/2)*c^8 + 15360*a^3*(-b)^(31/2)*c^7 - 10160*a^4
*(-b)^(29/2)*c^8 + 7680*a^4*(-b)^(31/2)*c^6 - 384*a^5*(-b)^(27/2)*c^9 - 535
04*a^5*(-b)^(29/2)*c^7 - 8099*a^6*(-b)^(27/2)*c^8 - 25728*a^6*(-b)^(29/2)*c
^6 + 1668*a^7*(-b)^(25/2)*c^9 - 13760*a^7*(-b)^(27/2)*c^7 + 10048*a^8*(-b)^
(25/2)*c^8 - 2048*a^8*(-b)^(27/2)*c^6 - 120*a^9*(-b)^(23/2)*c^9 + 4480*a^9*
(-b)^(25/2)*c^7 + 5184*a^3*(-b)^(31/2)*c^8 - 570*a^4*(-b)^(29/2)*c^9 + 1920
0*a^4*(-b)^(31/2)*c^7 - 16048*a^5*(-b)^(29/2)*c^8 + 3072*a^5*(-b)^(31/2)*c^
6 - 1984*a^6*(-b)^(27/2)*c^9 - 28416*a^6*(-b)^(29/2)*c^7 + 45*a^7*(-b)^(25/
2)*c^10 - 11984*a^7*(-b)^(27/2)*c^8 - 3840*a^7*(-b)^(29/2)*c^6 + 1698*a^8*(
-b)^(25/2)*c^9 - 7552*a^8*(-b)^(27/2)*c^7 + 2560*a^9*(-b)^(25/2)*c^8 + 480*
a^3*(-b)^(31/2)*c^9 + 10912*a^4*(-b)^(31/2)*c^8 - 1732*a^5*(-b)^(29/2)*c^9
 + 13440*a^5*(-b)^(31/2)*c^7 - 119*a^6*(-b)^(27/2)*c^10 - 11408*a^6*(-b)^(29
/2)*c^8 + 512*a^6*(-b)^(31/2)*c^6 - 3568*a^7*(-b)^(27/2)*c^9 - 7040*a^7*(-b
)^(29/2)*c^7 + 90*a^8*(-b)^(25/2)*c^10 - 7408*a^8*(-b)^(27/2)*c^8 + 576*a^9
*(-b)^(25/2)*c^9 - 1280*a^9*(-b)^(27/2)*c^7 + 2160*a^4*(-b)^(31/2)*c^9 - 35
*a^5*(-b)^(29/2)*c^10 + 11968*a^5*(-b)^(31/2)*c^8 - 1530*a^6*(-b)^(29/2)*c^
9 + 4992*a^6*(-b)^(31/2)*c^7 - 382*a^7*(-b)^(27/2)*c^10 - 2816*a^7*(-b)^(29
/2)*c^8 - 2720*a^8*(-b)^(27/2)*c^9 - 512*a^8*(-b)^(29/2)*c^7 + 45*a^9*(-b)^
(25/2)*c^10 - 1664*a^9*(-b)^(27/2)*c^8 + 129*a^4*(-b)^(31/2)*c^10 + 3856*a^
5*(-b)^(31/2)*c^9 + 10*a^6*(-b)^(29/2)*c^10 + 7152*a^6*(-b)^(31/2)*c^8 - 10
*a^7*(-b)^(27/2)*c^11 + 112*a^7*(-b)^(29/2)*c^9 + 768*a^7*(-b)^(31/2)*c^7 -
 407*a^8*(-b)^(27/2)*c^10 + 512*a^8*(-b)^(29/2)*c^8 - 752*a^9*(-b)^(27/2)*c
^9 + 482*a^5*(-b)^(31/2)*c^10 + 8*a^6*(-b)^(29/2)*c^11 + 3408*a^6*(-b)^(31/
2)*c^9 + 221*a^7*(-b)^(29/2)*c^10 + 2176*a^7*(-b)^(31/2)*c^8 - 20*a^8*(-b)^
(27/2)*c^11 + 736*a^8*(-b)^(29/2)*c^9 - 144*a^9*(-b)^(27/2)*c^10 + 256*a^9*
(-b)^(29/2)*c^8 + 18*a^5*(-b)^(31/2)*c^11 + 673*a^6*(-b)^(31/2)*c^10 + 32*a
^7*(-b)^(29/2)*c^11 + 1488*a^7*(-b)^(31/2)*c^9 + 272*a^8*(-b)^(29/2)*c^10 +
 256*a^8*(-b)^(31/2)*c^8 - 10*a^9*(-b)^(27/2)*c^11 + 256*a^9*(-b)^(29/2)*c^
9 + 52*a^6*(-b)^(31/2)*c^11 + a^7*(-b)^(29/2)*c^12 + 416*a^7*(-b)^(31/2)*c^
10 + 40*a^8*(-b)^(29/2)*c^11 + 256*a^8*(-b)^(31/2)*c^9 + 96*a^9*(-b)^(29/2)
*c^10 + a^6*(-b)^(31/2)*c^12 + 50*a^7*(-b)^(31/2)*c^11 + 2*a^8*(-b)^(29/2)*
c^12 + 96*a^8*(-b)^(31/2)*c^10 + 16*a^9*(-b)^(29/2)*c^11 + 2*a^7*(-b)^(31/2
)*c^12 + 16*a^8*(-b)^(31/2)*c^11 + a^9*(-b)^(29/2)*c^12 + a^8*(-b)^(31/2)*c
^12 - 1152*a*(-b)^(19/2)*c - 18432*a*(-b)^(21/2)*c + 2*a^6*(-b)^(9/2)*c - 2
4*a^5*(-b)^(11/2)*c + 4*a^7*(-b)^(9/2)*c + 70*a^4*(-b)^(13/2)*c - 48*a^6*(-
b)^(11/2)*c + 2*a^8*(-b)^(9/2)*c - 288*a^3*(-b)^(15/2)*c + 60*a^5*(-b)^(13/
2)*c - 8*a^7*(-b)^(11/2)*c + 1536*a^2*(-b)^(17/2)*c - 656*a^4*(-b)^(15/2)*c
 - 378*a^6*(-b)^(13/2)*c + 32*a^8*(-b)^(11/2)*c + 8064*a^3*(-b)^(17/2)*c +
656*a^5*(-b)^(15/2)*c - 784*a^7*(-b)^(13/2)*c + 16*a^9*(-b)^(11/2)*c - 9600
*a^2*(-b)^(19/2)*c + 16384*a^4*(-b)^(17/2)*c + 3280*a^6*(-b)^(15/2)*c - 544
*a^8*(-b)^(13/2)*c - 30720*a^3*(-b)^(19/2)*c + 15616*a^5*(-b)^(17/2)*c + 36
64*a^7*(-b)^(15/2)*c - 128*a^9*(-b)^(13/2)*c - 1152*a*(-b)^(21/2)*c^2 - 460
80*a^2*(-b)^(21/2)*c - 49920*a^4*(-b)^(19/2)*c + 6144*a^6*(-b)^(17/2)*c + 1
664*a^8*(-b)^(15/2)*c - 61440*a^3*(-b)^(21/2)*c - 44160*a^5*(-b)^(19/2)*c -
 128*a^7*(-b)^(17/2)*c + 256*a^9*(-b)^(15/2)*c + 46080*a*(-b)^(23/2)*c^2 -
46080*a^4*(-b)^(21/2)*c - 20352*a^6*(-b)^(19/2)*c - 512*a^8*(-b)^(17/2)*c +
 9600*a*(-b)^(23/2)*c^3 - 18432*a^5*(-b)^(21/2)*c - 3840*a^7*(-b)^(19/2)*c
- 3072*a^6*(-b)^(21/2)*c - 61440*a*(-b)^(25/2)*c^3 - 17280*a*(-b)^(25/2)*c^
4 + 46080*a*(-b)^(27/2)*c^4 + 14976*a*(-b)^(27/2)*c^5 - 18432*a*(-b)^(29/2)
*c^5 - 6528*a*(-b)^(29/2)*c^6 + 3072*a*(-b)^(31/2)*c^6 + 1152*a*(-b)^(31/2)
*c^7))/((-b)^(1/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^8*b^17*c^2 + 84*a^9*b^17
*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a^12*b^17*c^6 + 36*a^13*b
^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9))))/(a^2*b^2) + (((-b)^(7/4)*1i -
 (a*(-b)^(3/4)*(4*b + b*c + 1)*1i)/4)*((((-b)^(7/4)*1i - (a*(-b)^(3/4)*(4*b
 + b*c + 1)*1i)/4)^3*((64*(36*a^12*(-b)^(25/4) - 4*a^13*(-b)^(21/4) + 48*a^
13*(-b)^(25/4) - 60*a^11*(-b)^(29/4) - 240*a^12*(-b)^(29/4) - 192*a^13*(-b)
^(29/4) - 180*a^10*(-b)^(33/4) - 240*a^11*(-b)^(33/4) + 192*a^12*(-b)^(33/4
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) + 240*a^9*(-b)^(37/4) + 256*a^13*(-b)^(33/4) + 1200*a^10*(-b)^(37/4) + 17
28*a^11*(-b)^(37/4) + 320*a^8*(-b)^(41/4) + 768*a^12*(-b)^(37/4) + 1152*a^9
*(-b)^(41/4) + 1344*a^10*(-b)^(41/4) + 512*a^11*(-b)^(41/4) - 144*a^13*(-b)
^(29/4)*c^2 + 912*a^12*(-b)^(33/4)*c^2 + 1344*a^13*(-b)^(33/4)*c^2 + 336*a^
13*(-b)^(33/4)*c^3 - 432*a^11*(-b)^(37/4)*c^2 - 4032*a^12*(-b)^(37/4)*c^2 -
 1680*a^12*(-b)^(37/4)*c^3 - 4032*a^13*(-b)^(37/4)*c^2 - 4624*a^10*(-b)^(41
/4)*c^2 - 2688*a^13*(-b)^(37/4)*c^3 - 9408*a^11*(-b)^(41/4)*c^2 - 504*a^13*
(-b)^(37/4)*c^4 - 336*a^11*(-b)^(41/4)*c^3 - 1344*a^12*(-b)^(41/4)*c^2 + 35
84*a^9*(-b)^(45/4)*c^2 + 5376*a^12*(-b)^(41/4)*c^3 + 3584*a^13*(-b)^(41/4)*
c^2 + 20160*a^10*(-b)^(45/4)*c^2 + 1848*a^12*(-b)^(41/4)*c^4 + 6720*a^13*(-
b)^(41/4)*c^3 + 8848*a^10*(-b)^(45/4)*c^3 + 30912*a^11*(-b)^(45/4)*c^2 + 33
60*a^13*(-b)^(41/4)*c^4 + 6720*a^8*(-b)^(49/4)*c^2 + 21504*a^11*(-b)^(45/4)
*c^3 + 14336*a^12*(-b)^(45/4)*c^2 + 504*a^13*(-b)^(41/4)*c^5 + 24192*a^9*(-
b)^(49/4)*c^2 + 1848*a^11*(-b)^(45/4)*c^4 + 9408*a^12*(-b)^(45/4)*c^3 - 403
2*a^9*(-b)^(49/4)*c^3 + 28224*a^10*(-b)^(49/4)*c^2 - 3360*a^12*(-b)^(45/4)*
c^4 - 3584*a^13*(-b)^(45/4)*c^3 - 26880*a^10*(-b)^(49/4)*c^3 + 10752*a^11*(
-b)^(49/4)*c^2 - 1176*a^12*(-b)^(45/4)*c^5 - 6720*a^13*(-b)^(45/4)*c^4 - 10
584*a^10*(-b)^(49/4)*c^4 - 44352*a^11*(-b)^(49/4)*c^3 - 2688*a^13*(-b)^(45/
4)*c^5 - 11200*a^8*(-b)^(53/4)*c^3 - 30240*a^11*(-b)^(49/4)*c^4 - 21504*a^1
2*(-b)^(49/4)*c^3 - 336*a^13*(-b)^(45/4)*c^6 - 40320*a^9*(-b)^(53/4)*c^3 -
2520*a^11*(-b)^(49/4)*c^5 - 20160*a^12*(-b)^(49/4)*c^4 + 1120*a^9*(-b)^(53/
4)*c^4 - 47040*a^10*(-b)^(53/4)*c^3 + 16800*a^10*(-b)^(53/4)*c^4 - 17920*a^
11*(-b)^(53/4)*c^3 + 336*a^12*(-b)^(49/4)*c^6 + 4032*a^13*(-b)^(49/4)*c^5 +
 8120*a^10*(-b)^(53/4)*c^5 + 33600*a^11*(-b)^(53/4)*c^4 + 1344*a^13*(-b)^(4
9/4)*c^6 + 11200*a^8*(-b)^(57/4)*c^4 + 26880*a^11*(-b)^(53/4)*c^5 + 17920*a
^12*(-b)^(53/4)*c^4 + 144*a^13*(-b)^(49/4)*c^7 + 40320*a^9*(-b)^(57/4)*c^4
+ 1680*a^11*(-b)^(53/4)*c^6 + 22848*a^12*(-b)^(53/4)*c^5 + 2240*a^9*(-b)^(5
7/4)*c^5 + 47040*a^10*(-b)^(57/4)*c^4 + 1344*a^12*(-b)^(53/4)*c^6 + 3584*a^
13*(-b)^(53/4)*c^5 + 17920*a^11*(-b)^(57/4)*c^4 + 48*a^12*(-b)^(53/4)*c^7 -
 1344*a^13*(-b)^(53/4)*c^6 - 3920*a^10*(-b)^(57/4)*c^6 - 9408*a^11*(-b)^(57
/4)*c^5 - 384*a^13*(-b)^(53/4)*c^7 - 6720*a^8*(-b)^(61/4)*c^5 - 14784*a^11*
(-b)^(57/4)*c^6 - 7168*a^12*(-b)^(57/4)*c^5 - 36*a^13*(-b)^(53/4)*c^8 - 241
92*a^9*(-b)^(61/4)*c^5 - 528*a^11*(-b)^(57/4)*c^7 - 14784*a^12*(-b)^(57/4)*
c^6 - 2688*a^9*(-b)^(61/4)*c^6 - 28224*a^10*(-b)^(61/4)*c^5 - 768*a^12*(-b)
^(57/4)*c^7 - 3584*a^13*(-b)^(57/4)*c^6 - 6720*a^10*(-b)^(61/4)*c^6 - 10752
a^11(-b)^(61/4)*c^5 - 60*a^12*(-b)^(57/4)*c^8 + 192*a^13*(-b)^(57/4)*c^7
+ 1104*a^10*(-b)^(61/4)*c^7 - 4032*a^11*(-b)^(61/4)*c^6 + 48*a^13*(-b)^(57/
4)*c^8 + 2240*a^8*(-b)^(65/4)*c^6 + 4608*a^11*(-b)^(61/4)*c^7 + 4*a^13*(-b)
^(57/4)*c^9 + 8064*a^9*(-b)^(65/4)*c^6 + 36*a^11*(-b)^(61/4)*c^8 + 5184*a^1
2*(-b)^(61/4)*c^7 + 1216*a^9*(-b)^(65/4)*c^7 + 9408*a^10*(-b)^(65/4)*c^6 +
144*a^12*(-b)^(61/4)*c^8 + 1536*a^13*(-b)^(61/4)*c^7 + 3840*a^10*(-b)^(65/4
)*c^7 + 3584*a^11*(-b)^(65/4)*c^6 + 12*a^12*(-b)^(61/4)*c^9 - 148*a^10*(-b)
^(65/4)*c^8 + 3648*a^11*(-b)^(65/4)*c^7 - 320*a^8*(-b)^(69/4)*c^7 - 624*a^1
1*(-b)^(65/4)*c^8 + 1024*a^12*(-b)^(65/4)*c^7 - 1152*a^9*(-b)^(69/4)*c^7 +
12*a^11*(-b)^(65/4)*c^9 - 768*a^12*(-b)^(65/4)*c^8 - 208*a^9*(-b)^(69/4)*c^
8 - 1344*a^10*(-b)^(69/4)*c^7 - 256*a^13*(-b)^(65/4)*c^8 - 720*a^10*(-b)^(6
9/4)*c^8 - 512*a^11*(-b)^(69/4)*c^7 + 4*a^10*(-b)^(69/4)*c^9 - 768*a^11*(-b
)^(69/4)*c^8 - 256*a^12*(-b)^(69/4)*c^8 + 36*a^13*(-b)^(25/4)*c - 276*a^12*
(-b)^(29/4)*c - 384*a^13*(-b)^(29/4)*c + 300*a^11*(-b)^(33/4)*c + 1536*a^12
*(-b)^(33/4)*c + 1344*a^13*(-b)^(33/4)*c + 1380*a^10*(-b)^(37/4)*c + 2304*a
^11*(-b)^(37/4)*c - 576*a^12*(-b)^(37/4)*c - 1472*a^9*(-b)^(41/4)*c - 1536*
a^13*(-b)^(37/4)*c - 7680*a^10*(-b)^(41/4)*c - 11328*a^11*(-b)^(41/4)*c - 2
240*a^8*(-b)^(45/4)*c - 5120*a^12*(-b)^(41/4)*c - 8064*a^9*(-b)^(45/4)*c -
9408*a^10*(-b)^(45/4)*c - 3584*a^11*(-b)^(45/4)*c))/(a^7*b^18 + 9*a^8*b^18*
c + 36*a^9*b^18*c^2 + 84*a^10*b^18*c^3 + 126*a^11*b^18*c^4 + 126*a^12*b^18*
c^5 + 84*a^13*b^18*c^6 + 36*a^14*b^18*c^7 + 9*a^15*b^18*c^8 + a^16*b^18*c^9
) + (64*((-b)^(7/4)*1i - (a*(-b)^(3/4)*(4*b + b*c + 1)*1i)/4)*(-(b*x - 1)/(
c + x))^(1/4)*(16*a^13*(-b)^(11/2) - 80*a^12*(-b)^(13/2) - 80*a^11*(-b)^(15
/2) - 128*a^13*(-b)^(13/2) + 400*a^10*(-b)^(17/2) + 128*a^12*(-b)^(15/2) +

896*a^9*(-b)^(19/2) + 1152*a^11*(-b)^(17/2) + 256*a^13*(-b)^(15/2) + 512*a^8*(-b)^(21/2) + 1920*a^10*(-b)^(19/2) + 768*a^12*(-b)^(17/2) + 1024*a^9*(-b)^(21/2) + 1024*a^11*(-b)^(19/2) + 512*a^10*(-b)^(21/2) + 448*a^13*(-b)^(15/2)*c^2 - 1344*a^12*(-b)^(17/2)*c^2 - 2624*a^11*(-b)^(19/2)*c^2 - 2688*a^13*(-b)^(17/2)*c^2 + 1088*a^10*(-b)^(21/2)*c^2 + 128*a^12*(-b)^(19/2)*c^2 - 896*a^13*(-b)^(17/2)*c^3 + 9600*a^9*(-b)^(23/2)*c^2 + 9344*a^11*(-b)^(21/2)*c^2 + 1792*a^12*(-b)^(19/2)*c^3 + 4096*a^13*(-b)^(19/2)*c^2 + 7680*a^8*(-b)^(25/2)*c^2 + 21888*a^10*(-b)^(23/2)*c^2 + 4096*a^11*(-b)^(21/2)*c^3 + 8704*a^12*(-b)^(21/2)*c^2 + 4480*a^13*(-b)^(19/2)*c^3 + 15360*a^9*(-b)^(25/2)*c^2 + 3328*a^10*(-b)^(23/2)*c^3 + 12288*a^11*(-b)^(23/2)*c^2 + 128*a^12*(-b)^(21/2)*c^3 + 1120*a^13*(-b)^(19/2)*c^4 - 8320*a^9*(-b)^(25/2)*c^3 + 7680*a^10*(-b)^(25/2)*c^2 - 4992*a^11*(-b)^(23/2)*c^3 - 1120*a^12*(-b)^(21/2)*c^4 - 6656*a^13*(-b)^(21/2)*c^3 - 10240*a^8*(-b)^(27/2)*c^3 - 21120*a^10*(-b)^(25/2)*c^3 - 2400*a^11*(-b)^(23/2)*c^4 - 9216*a^12*(-b)^(23/2)*c^3 - 4480*a^13*(-b)^(21/2)*c^4 - 20480*a^9*(-b)^(27/2)*c^3 - 7200*a^10*(-b)^(25/2)*c^4 - 12800*a^11*(-b)^(25/2)*c^3 + 1920*a^12*(-b)^(23/2)*c^4 - 896*a^13*(-b)^(21/2)*c^5 + 640*a^9*(-b)^(27/2)*c^4 - 10240*a^10*(-b)^(27/2)*c^3 - 3200*a^11*(-b)^(25/2)*c^4 + 7680*a^13*(-b)^(23/2)*c^4 + 7680*a^8*(-b)^(29/2)*c^4 + 5760*a^10*(-b)^(27/2)*c^4 - 1280*a^11*(-b)^(25/2)*c^5 + 5120*a^12*(-b)^(25/2)*c^4 + 2688*a^13*(-b)^(23/2)*c^5 + 15360*a^9*(-b)^(29/2)*c^4 + 5120*a^10*(-b)^(27/2)*c^5 + 5120*a^11*(-b)^(27/2)*c^4 - 5248*a^12*(-b)^(25/2)*c^5 + 448*a^13*(-b)^(23/2)*c^6 + 4224*a^9*(-b)^(29/2)*c^5 + 7680*a^10*(-b)^(29/2)*c^4 + 3968*a^11*(-b)^(27/2)*c^5 + 448*a^12*(-b)^(25/2)*c^6 - 6656*a^13*(-b)^(25/2)*c^5 - 3072*a^8*(-b)^(31/2)*c^5 + 5760*a^10*(-b)^(29/2)*c^5 + 2752*a^11*(-b)^(27/2)*c^6 - 2048*a^12*(-b)^(27/2)*c^5 - 896*a^13*(-b)^(25/2)*c^6 - 6144*a^9*(-b)^(31/2)*c^5 - 704*a^10*(-b)^(29/2)*c^6 + 1536*a^11*(-b)^(29/2)*c^5 + 5504*a^12*(-b)^(27/2)*c^6 - 128*a^13*(-b)^(25/2)*c^7 - 2944*a^9*(-b)^(31/2)*c^6 - 3072*a^10*(-b)^(31/2)*c^5 + 384*a^11*(-b)^(29/2)*c^6 - 256*a^12*(-b)^(27/2)*c^7 + 4096*a^13*(-b)^(27/2)*c^6 + 512*a^8*(-b)^(33/2)*c^6 - 4992*a^10*(-b)^(31/2)*c^6 - 1536*a^11*(-b)^(29/2)*c^7 + 1536*a^12*(-b)^(29/2)*c^6 + 128*a^13*(-b)^(27/2)*c^7 + 1024*a^9*(-b)^(33/2)*c^6 - 768*a^10*(-b)^(31/2)*c^7 - 2048*a^11*(-b)^(31/2)*c^6 - 2688*a^12*(-b)^(29/2)*c^7 + 16*a^13*(-b)^(27/2)*c^8 + 640*a^9*(-b)^(33/2)*c^7 + 512*a^10*(-b)^(33/2)*c^6 - 1664*a^11*(-b)^(31/2)*c^7 + 48*a^12*(-b)^(29/2)*c^8 - 1536*a^13*(-b)^(29/2)*c^7 + 1152*a^10*(-b)^(33/2)*c^7 + 304*a^11*(-b)^(31/2)*c^8 - 1024*a^12*(-b)^(31/2)*c^7 + 272*a^10*(-b)^(33/2)*c^8 + 512*a^11*(-b)^(33/2)*c^7 + 512*a^12*(-b)^(31/2)*c^8 + 512*a^11*(-b)^(33/2)*c^8 + 256*a^13*(-b)^(31/2)*c^8 + 256*a^12*(-b)^(33/2)*c^8 - 128*a^13*(-b)^(13/2)*c + 512*a^12*(-b)^(15/2)*c + 768*a^11*(-b)^(17/2)*c + 896*a^13*(-b)^(15/2)*c - 1536*a^10*(-b)^(19/2)*c - 384*a^12*(-b)^(17/2)*c - 4736*a^9*(-b)^(21/2)*c - 5504*a^11*(-b)^(19/2)*c - 1536*a^13*(-b)^(17/2)*c - 3072*a^8*(-b)^(23/2)*c - 10368*a^10*(-b)^(21/2)*c - 4096*a^12*(-b)^(19/2)*c - 6144*a^9*(-b)^(23/2)*c - 5632*a^11*(-b)^(21/2)*c - 3072*a^10*(-b)^(23/2)*c))/(a^2*(-b)^(9/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^8*b^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a^12*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9))))/(a^6*b^6) - (64*(-(b*x - 1)/(c + x))^(1/4)*(512*(-b)^(19/2) + a^5*(-b)^(9/2) + a^4*(-b)^(11/2) + 2*a^6*(-b)^(9/2) - 16*a^3*(-b)^(13/2) + 18*a^5*(-b)^(11/2) + a^7*(-b)^(9/2) - 144*a^2*(-b)^(15/2) - 176*a^4*(-b)^(13/2) + 49*a^6*(-b)^(11/2) - 576*a^3*(-b)^(15/2) - 560*a^5*(-b)^(13/2) + 48*a^7*(-b)^(11/2) + 2688*a^2*(-b)^(17/2) - 608*a^4*(-b)^(15/2) - 784*a^6*(-b)^(13/2) + 16*a^8*(-b)^(11/2) + 7680*a^3*(-b)^(17/2) + 448*a^5*(-b)^(15/2) - 512*a^7*(-b)^(13/2) + 7680*a^2*(-b)^(19/2) + 11520*a^4*(-b)^(17/2) + 1392*a^6*(-b)^(15/2) - 128*a^8*(-b)^(13/2) + 10240*a^3*(-b)^(19/2) + 9600*a^5*(-b)^(17/2) + 1024*a^7*(-b)^(15/2) + 7680*a^4*(-b)^(19/2) + 4224*a^6*(-b)^(17/2) + 256*a^8*(-b)^(15/2) + 3072*a^5*(-b)^(19/2) + 768*a^7*(-b)^(17/2) + 512*a^6*(-b)^(19/2) + 7680*(-b)^(23/2)*c^2 - 10240*(-b)^(25/2)*c^3 + 7680*(-b)^(27/2)*c^4 - 3072*(-b)^(29/2)*c^5 + 512*(-b)^(31/2)*c^6 + 384*a*(-b)^(17/2) + 3072*a*(-b)^(19/2) - 3072*(-b)^(21/2)*c + a^7*(-b)^(9/2)*c^2 - 35*a^6*(-b)^(11/2)*c^2 + 2*a^8*(-b)^(9/2)*c^2 + 265*a^5*(-b)^(13/2)*c^2 - 86*a^7*(-b)^(11/

2)*c^2 + a^9*(-b)^(9/2)*c^2 - 851*a^4*(-b)^(15/2)*c^2 + 738*a^6*(-b)^(13/2)
*c^2 - 10*a^7*(-b)^(11/2)*c^3 - 67*a^8*(-b)^(11/2)*c^2 + 2496*a^3*(-b)^(17/
2)*c^2 - 2566*a^5*(-b)^(15/2)*c^2 + 224*a^6*(-b)^(13/2)*c^3 + 649*a^7*(-b)^
(13/2)*c^2 - 20*a^8*(-b)^(11/2)*c^3 - 16*a^9*(-b)^(11/2)*c^2 - 5184*a^2*(-b
)^(19/2)*c^2 + 10432*a^4*(-b)^(17/2)*c^2 - 1358*a^5*(-b)^(15/2)*c^3 - 1907*
a^6*(-b)^(15/2)*c^2 + 592*a^7*(-b)^(13/2)*c^3 + 144*a^8*(-b)^(13/2)*c^2 - 1
0*a^9*(-b)^(11/2)*c^3 - 31104*a^3*(-b)^(19/2)*c^2 + 3784*a^4*(-b)^(17/2)*c^
3 + 14912*a^5*(-b)^(17/2)*c^2 - 4364*a^6*(-b)^(15/2)*c^3 + 45*a^7*(-b)^(13/
2)*c^4 + 1120*a^7*(-b)^(15/2)*c^2 + 512*a^8*(-b)^(13/2)*c^3 - 32*a^9*(-b)^(
13/2)*c^2 + 1152*a^2*(-b)^(21/2)*c^2 - 7552*a^3*(-b)^(19/2)*c^3 - 74624*a^4
*(-b)^(19/2)*c^2 + 14288*a^5*(-b)^(17/2)*c^3 - 771*a^6*(-b)^(15/2)*c^4 + 58
24*a^6*(-b)^(17/2)*c^2 - 4974*a^7*(-b)^(15/2)*c^3 + 90*a^8*(-b)^(13/2)*c^4
+ 1952*a^8*(-b)^(15/2)*c^2 + 144*a^9*(-b)^(13/2)*c^3 + 6912*a^2*(-b)^(21/2)
*c^3 + 23040*a^3*(-b)^(21/2)*c^2 - 36800*a^4*(-b)^(19/2)*c^3 + 3874*a^5*(-b
)^(17/2)*c^4 - 91136*a^5*(-b)^(19/2)*c^2 + 19144*a^6*(-b)^(17/2)*c^3 - 2118
a^7(-b)^(15/2)*c^4 - 5120*a^7*(-b)^(17/2)*c^2 - 2288*a^8*(-b)^(15/2)*c^3
+ 45*a^9*(-b)^(13/2)*c^4 + 640*a^9*(-b)^(15/2)*c^2 + 115200*a^2*(-b)^(23/2)
*c^2 + 49536*a^3*(-b)^(21/2)*c^3 - 8750*a^4*(-b)^(19/2)*c^4 + 57600*a^4*(-b
)^(21/2)*c^2 - 68032*a^5*(-b)^(19/2)*c^3 + 13444*a^6*(-b)^(17/2)*c^4 - 5894
4*a^6*(-b)^(19/2)*c^2 - 120*a^7*(-b)^(15/2)*c^5 + 9664*a^7*(-b)^(17/2)*c^3
- 1923*a^8*(-b)^(15/2)*c^4 - 5248*a^8*(-b)^(17/2)*c^2 - 320*a^9*(-b)^(15/2)
*c^3 + 44160*a^2*(-b)^(23/2)*c^3 + 11040*a^3*(-b)^(21/2)*c^4 + 153600*a^3*(
-b)^(23/2)*c^2 + 137728*a^4*(-b)^(21/2)*c^3 - 36988*a^5*(-b)^(19/2)*c^4 + 6
3360*a^5*(-b)^(21/2)*c^2 + 1644*a^6*(-b)^(17/2)*c^5 - 56512*a^6*(-b)^(19/2)
*c^3 + 17058*a^7*(-b)^(17/2)*c^4 - 18560*a^7*(-b)^(19/2)*c^2 - 240*a^8*(-b)
^(15/2)*c^5 + 128*a^8*(-b)^(17/2)*c^3 - 576*a^9*(-b)^(15/2)*c^4 - 1280*a^9*
(-b)^(17/2)*c^2 - 480*a^2*(-b)^(23/2)*c^4 + 76800*a^3*(-b)^(23/2)*c^3 + 606
40*a^4*(-b)^(21/2)*c^4 + 115200*a^4*(-b)^(23/2)*c^2 - 6776*a^5*(-b)^(19/2)*
c^5 + 193792*a^5*(-b)^(21/2)*c^3 - 59150*a^6*(-b)^(19/2)*c^4 + 33408*a^6*(-
b)^(21/2)*c^2 + 4632*a^7*(-b)^(17/2)*c^5 - 16064*a^7*(-b)^(19/2)*c^3 + 9280
a^8(-b)^(17/2)*c^4 - 2048*a^8*(-b)^(19/2)*c^2 - 120*a^9*(-b)^(15/2)*c^5 -
896*a^9*(-b)^(17/2)*c^3 - 153600*a^2*(-b)^(25/2)*c^3 - 23040*a^3*(-b)^(23/
2)*c^4 + 11620*a^4*(-b)^(21/2)*c^5 + 57600*a^4*(-b)^(23/2)*c^3 + 128864*a^5
*(-b)^(21/2)*c^4 + 46080*a^5*(-b)^(23/2)*c^2 - 24752*a^6*(-b)^(19/2)*c^5 +
146688*a^6*(-b)^(21/2)*c^3 + 210*a^7*(-b)^(17/2)*c^6 - 43232*a^7*(-b)^(19/2
)*c^4 + 6912*a^7*(-b)^(21/2)*c^2 + 4332*a^8*(-b)^(17/2)*c^5 + 3968*a^8*(-b)
^(19/2)*c^3 + 1792*a^9*(-b)^(17/2)*c^4 - 90240*a^2*(-b)^(25/2)*c^4 - 7104*a
^3*(-b)^(23/2)*c^5 - 204800*a^3*(-b)^(25/2)*c^3 - 100160*a^4*(-b)^(23/2)*c^
4 + 53480*a^5*(-b)^(21/2)*c^5 + 9600*a^5*(-b)^(23/2)*c^3 - 2310*a^6*(-b)^(1
9/2)*c^6 + 131872*a^6*(-b)^(21/2)*c^4 + 7680*a^6*(-b)^(23/2)*c^2 - 33432*a^
7*(-b)^(19/2)*c^5 + 56704*a^7*(-b)^(21/2)*c^3 + 420*a^8*(-b)^(17/2)*c^6 - 1
3216*a^8*(-b)^(19/2)*c^4 + 1344*a^9*(-b)^(17/2)*c^5 + 2304*a^9*(-b)^(19/2)*
c^3 - 9216*a^2*(-b)^(25/2)*c^5 - 192000*a^3*(-b)^(25/2)*c^4 - 48224*a^4*(-b
)^(23/2)*c^5 - 153600*a^4*(-b)^(25/2)*c^3 + 7546*a^5*(-b)^(21/2)*c^6 - 1798
40*a^5*(-b)^(23/2)*c^4 + 94948*a^6*(-b)^(21/2)*c^5 - 9600*a^6*(-b)^(23/2)*c
^3 - 6636*a^7*(-b)^(19/2)*c^6 + 63232*a^7*(-b)^(21/2)*c^4 - 19712*a^8*(-b)^
(19/2)*c^5 + 8704*a^8*(-b)^(21/2)*c^3 + 210*a^9*(-b)^(17/2)*c^6 - 896*a^9*(
-b)^(19/2)*c^4 + 115200*a^2*(-b)^(27/2)*c^4 - 31104*a^3*(-b)^(25/2)*c^5 - 8
750*a^4*(-b)^(23/2)*c^6 - 211200*a^4*(-b)^(25/2)*c^4 - 121120*a^5*(-b)^(23/
2)*c^5 - 61440*a^5*(-b)^(25/2)*c^3 + 28756*a^6*(-b)^(21/2)*c^6 - 161760*a^6
*(-b)^(23/2)*c^4 - 252*a^7*(-b)^(19/2)*c^7 + 80416*a^7*(-b)^(21/2)*c^5 - 38
40*a^7*(-b)^(23/2)*c^3 - 6342*a^8*(-b)^(19/2)*c^6 + 9344*a^8*(-b)^(21/2)*c^
4 - 4256*a^9*(-b)^(19/2)*c^5 + 81792*a^2*(-b)^(27/2)*c^5 - 832*a^3*(-b)^(25
/2)*c^6 + 153600*a^3*(-b)^(27/2)*c^4 - 23552*a^4*(-b)^(25/2)*c^5 - 44380*a^
5*(-b)^(23/2)*c^6 - 124800*a^5*(-b)^(25/2)*c^4 + 2184*a^6*(-b)^(21/2)*c^7 -
146336*a^6*(-b)^(23/2)*c^5 - 10240*a^6*(-b)^(25/2)*c^3 + 40698*a^7*(-b)^(2
1/2)*c^6 - 72320*a^7*(-b)^(23/2)*c^4 - 504*a^8*(-b)^(19/2)*c^7 + 31808*a^8*
(-b)^(21/2)*c^5 - 2016*a^9*(-b)^(19/2)*c^6 - 1280*a^9*(-b)^(21/2)*c^4 + 109
44*a^2*(-b)^(27/2)*c^6 + 184320*a^3*(-b)^(27/2)*c^5 + 8896*a^4*(-b)^(25/2)*

c^6 + 115200*a^4*(-b)^(27/2)*c^4 - 5300*a^5*(-b)^(23/2)*c^7 + 32512*a^5*(-b
)^(25/2)*c^5 - 86702*a^6*(-b)^(23/2)*c^6 - 36480*a^6*(-b)^(25/2)*c^4 + 6384
a^7(-b)^(21/2)*c^7 - 87968*a^7*(-b)^(23/2)*c^5 + 25312*a^8*(-b)^(21/2)*c^
6 - 12800*a^8*(-b)^(23/2)*c^4 - 252*a^9*(-b)^(19/2)*c^7 + 4480*a^9*(-b)^(21
/2)*c^5 - 46080*a^2*(-b)^(29/2)*c^5 + 49536*a^3*(-b)^(27/2)*c^6 + 3016*a^4*
(-b)^(25/2)*c^7 + 218880*a^4*(-b)^(27/2)*c^5 + 44864*a^5*(-b)^(25/2)*c^6 +
46080*a^5*(-b)^(27/2)*c^4 - 21128*a^6*(-b)^(23/2)*c^7 + 66048*a^6*(-b)^(25/
2)*c^5 + 210*a^7*(-b)^(21/2)*c^8 - 81536*a^7*(-b)^(23/2)*c^6 - 3840*a^7*(-b
)^(25/2)*c^4 + 6216*a^8*(-b)^(21/2)*c^7 - 22912*a^8*(-b)^(23/2)*c^5 + 5824*
a^9*(-b)^(21/2)*c^6 - 36480*a^2*(-b)^(29/2)*c^6 + 4224*a^3*(-b)^(27/2)*c^7
- 61440*a^3*(-b)^(29/2)*c^5 + 86656*a^4*(-b)^(27/2)*c^6 + 18896*a^5*(-b)^(2
5/2)*c^7 + 144000*a^5*(-b)^(27/2)*c^5 - 1374*a^6*(-b)^(23/2)*c^8 + 75200*a^
6*(-b)^(25/2)*c^6 + 7680*a^6*(-b)^(27/2)*c^4 - 31284*a^7*(-b)^(23/2)*c^7 +
40576*a^7*(-b)^(25/2)*c^5 + 420*a^8*(-b)^(21/2)*c^8 - 36736*a^8*(-b)^(23/2)
*c^6 + 2016*a^9*(-b)^(21/2)*c^7 - 1280*a^9*(-b)^(23/2)*c^5 - 5376*a^2*(-b)^
(29/2)*c^7 - 84480*a^3*(-b)^(29/2)*c^6 + 13888*a^4*(-b)^(27/2)*c^7 - 46080*
a^4*(-b)^(29/2)*c^5 + 2173*a^5*(-b)^(25/2)*c^8 + 70144*a^5*(-b)^(27/2)*c^6
+ 42952*a^6*(-b)^(25/2)*c^7 + 49536*a^6*(-b)^(27/2)*c^5 - 4092*a^7*(-b)^(23
/2)*c^8 + 57856*a^7*(-b)^(25/2)*c^6 - 20384*a^8*(-b)^(23/2)*c^7 + 8704*a^8*
(-b)^(25/2)*c^5 + 210*a^9*(-b)^(21/2)*c^8 - 6272*a^9*(-b)^(23/2)*c^6 + 7680
a^2(-b)^(31/2)*c^6 - 26496*a^3*(-b)^(29/2)*c^7 + 301*a^4*(-b)^(27/2)*c^8
- 103680*a^4*(-b)^(29/2)*c^6 + 12608*a^5*(-b)^(27/2)*c^7 - 18432*a^5*(-b)^(
29/2)*c^5 + 9274*a^6*(-b)^(25/2)*c^8 + 21696*a^6*(-b)^(27/2)*c^6 - 120*a^7*
(-b)^(23/2)*c^9 + 45760*a^7*(-b)^(25/2)*c^7 + 6912*a^7*(-b)^(27/2)*c^5 - 40
62*a^8*(-b)^(23/2)*c^8 + 20096*a^8*(-b)^(25/2)*c^6 - 4928*a^9*(-b)^(23/2)*c
^7 + 6528*a^2*(-b)^(31/2)*c^7 - 2448*a^3*(-b)^(29/2)*c^8 + 10240*a^3*(-b)^(
31/2)*c^6 - 52736*a^4*(-b)^(29/2)*c^7 - 1558*a^5*(-b)^(27/2)*c^8 - 71040*a^
5*(-b)^(29/2)*c^6 + 546*a^6*(-b)^(25/2)*c^9 - 4544*a^6*(-b)^(27/2)*c^7 - 30
72*a^6*(-b)^(29/2)*c^5 + 14589*a^7*(-b)^(25/2)*c^8 - 2432*a^7*(-b)^(27/2)*c
^6 - 240*a^8*(-b)^(23/2)*c^9 + 23168*a^8*(-b)^(25/2)*c^7 - 1344*a^9*(-b)^(2
3/2)*c^8 + 2304*a^9*(-b)^(25/2)*c^6 + 1008*a^2*(-b)^(31/2)*c^8 + 15360*a^3*
(-b)^(31/2)*c^7 - 10160*a^4*(-b)^(29/2)*c^8 + 7680*a^4*(-b)^(31/2)*c^6 - 38
4*a^5*(-b)^(27/2)*c^9 - 53504*a^5*(-b)^(29/2)*c^7 - 8099*a^6*(-b)^(27/2)*c^
8 - 25728*a^6*(-b)^(29/2)*c^6 + 1668*a^7*(-b)^(25/2)*c^9 - 13760*a^7*(-b)^(
27/2)*c^7 + 10048*a^8*(-b)^(25/2)*c^8 - 2048*a^8*(-b)^(27/2)*c^6 - 120*a^9*
(-b)^(23/2)*c^9 + 4480*a^9*(-b)^(25/2)*c^7 + 5184*a^3*(-b)^(31/2)*c^8 - 570
a^4(-b)^(29/2)*c^9 + 19200*a^4*(-b)^(31/2)*c^7 - 16048*a^5*(-b)^(29/2)*c^
8 + 3072*a^5*(-b)^(31/2)*c^6 - 1984*a^6*(-b)^(27/2)*c^9 - 28416*a^6*(-b)^(2
9/2)*c^7 + 45*a^7*(-b)^(25/2)*c^10 - 11984*a^7*(-b)^(27/2)*c^8 - 3840*a^7*(
-b)^(29/2)*c^6 + 1698*a^8*(-b)^(25/2)*c^9 - 7552*a^8*(-b)^(27/2)*c^7 + 2560
a^9(-b)^(25/2)*c^8 + 480*a^3*(-b)^(31/2)*c^9 + 10912*a^4*(-b)^(31/2)*c^8
- 1732*a^5*(-b)^(29/2)*c^9 + 13440*a^5*(-b)^(31/2)*c^7 - 119*a^6*(-b)^(27/2
)*c^10 - 11408*a^6*(-b)^(29/2)*c^8 + 512*a^6*(-b)^(31/2)*c^6 - 3568*a^7*(-b
)^(27/2)*c^9 - 7040*a^7*(-b)^(29/2)*c^7 + 90*a^8*(-b)^(25/2)*c^10 - 7408*a^
8*(-b)^(27/2)*c^8 + 576*a^9*(-b)^(25/2)*c^9 - 1280*a^9*(-b)^(27/2)*c^7 + 21
60*a^4*(-b)^(31/2)*c^9 - 35*a^5*(-b)^(29/2)*c^10 + 11968*a^5*(-b)^(31/2)*c^
8 - 1530*a^6*(-b)^(29/2)*c^9 + 4992*a^6*(-b)^(31/2)*c^7 - 382*a^7*(-b)^(27/
2)*c^10 - 2816*a^7*(-b)^(29/2)*c^8 - 2720*a^8*(-b)^(27/2)*c^9 - 512*a^8*(-b
)^(29/2)*c^7 + 45*a^9*(-b)^(25/2)*c^10 - 1664*a^9*(-b)^(27/2)*c^8 + 129*a^4
*(-b)^(31/2)*c^10 + 3856*a^5*(-b)^(31/2)*c^9 + 10*a^6*(-b)^(29/2)*c^10 + 71
52*a^6*(-b)^(31/2)*c^8 - 10*a^7*(-b)^(27/2)*c^11 + 112*a^7*(-b)^(29/2)*c^9
+ 768*a^7*(-b)^(31/2)*c^7 - 407*a^8*(-b)^(27/2)*c^10 + 512*a^8*(-b)^(29/2)*
c^8 - 752*a^9*(-b)^(27/2)*c^9 + 482*a^5*(-b)^(31/2)*c^10 + 8*a^6*(-b)^(29/2
)*c^11 + 3408*a^6*(-b)^(31/2)*c^9 + 221*a^7*(-b)^(29/2)*c^10 + 2176*a^7*(-b
)^(31/2)*c^8 - 20*a^8*(-b)^(27/2)*c^11 + 736*a^8*(-b)^(29/2)*c^9 - 144*a^9*
(-b)^(27/2)*c^10 + 256*a^9*(-b)^(29/2)*c^8 + 18*a^5*(-b)^(31/2)*c^11 + 673*
a^6*(-b)^(31/2)*c^10 + 32*a^7*(-b)^(29/2)*c^11 + 1488*a^7*(-b)^(31/2)*c^9 +
272*a^8*(-b)^(29/2)*c^10 + 256*a^8*(-b)^(31/2)*c^8 - 10*a^9*(-b)^(27/2)*c^
11 + 256*a^9*(-b)^(29/2)*c^9 + 52*a^6*(-b)^(31/2)*c^11 + a^7*(-b)^(29/2)*c^

12 + 416*a^7*(-b)^(31/2)*c^10 + 40*a^8*(-b)^(29/2)*c^11 + 256*a^8*(-b)^(31/
2)*c^9 + 96*a^9*(-b)^(29/2)*c^10 + a^6*(-b)^(31/2)*c^12 + 50*a^7*(-b)^(31/2
)*c^11 + 2*a^8*(-b)^(29/2)*c^12 + 96*a^8*(-b)^(31/2)*c^10 + 16*a^9*(-b)^(29
/2)*c^11 + 2*a^7*(-b)^(31/2)*c^12 + 16*a^8*(-b)^(31/2)*c^11 + a^9*(-b)^(29/
2)*c^12 + a^8*(-b)^(31/2)*c^12 - 1152*a*(-b)^(19/2)*c - 18432*a*(-b)^(21/2)
*c + 2*a^6*(-b)^(9/2)*c - 24*a^5*(-b)^(11/2)*c + 4*a^7*(-b)^(9/2)*c + 70*a^
4*(-b)^(13/2)*c - 48*a^6*(-b)^(11/2)*c + 2*a^8*(-b)^(9/2)*c - 288*a^3*(-b)^
(15/2)*c + 60*a^5*(-b)^(13/2)*c - 8*a^7*(-b)^(11/2)*c + 1536*a^2*(-b)^(17/2
)*c - 656*a^4*(-b)^(15/2)*c - 378*a^6*(-b)^(13/2)*c + 32*a^8*(-b)^(11/2)*c
+ 8064*a^3*(-b)^(17/2)*c + 656*a^5*(-b)^(15/2)*c - 784*a^7*(-b)^(13/2)*c +
16*a^9*(-b)^(11/2)*c - 9600*a^2*(-b)^(19/2)*c + 16384*a^4*(-b)^(17/2)*c + 3
280*a^6*(-b)^(15/2)*c - 544*a^8*(-b)^(13/2)*c - 30720*a^3*(-b)^(19/2)*c + 1
5616*a^5*(-b)^(17/2)*c + 3664*a^7*(-b)^(15/2)*c - 128*a^9*(-b)^(13/2)*c - 1
152*a*(-b)^(21/2)*c^2 - 46080*a^2*(-b)^(21/2)*c - 49920*a^4*(-b)^(19/2)*c +
 6144*a^6*(-b)^(17/2)*c + 1664*a^8*(-b)^(15/2)*c - 61440*a^3*(-b)^(21/2)*c
- 44160*a^5*(-b)^(19/2)*c - 128*a^7*(-b)^(17/2)*c + 256*a^9*(-b)^(15/2)*c +
 46080*a*(-b)^(23/2)*c^2 - 46080*a^4*(-b)^(21/2)*c - 20352*a^6*(-b)^(19/2)*
c - 512*a^8*(-b)^(17/2)*c + 9600*a*(-b)^(23/2)*c^3 - 18432*a^5*(-b)^(21/2)*
c - 3840*a^7*(-b)^(19/2)*c - 3072*a^6*(-b)^(21/2)*c - 61440*a*(-b)^(25/2)*c
^3 - 17280*a*(-b)^(25/2)*c^4 + 46080*a*(-b)^(27/2)*c^4 + 14976*a*(-b)^(27/2
)*c^5 - 18432*a*(-b)^(29/2)*c^5 - 6528*a*(-b)^(29/2)*c^6 + 3072*a*(-b)^(31/
2)*c^6 + 1152*a*(-b)^(31/2)*c^7))/((-b)^(1/4)*(a^6*b^17 + 9*a^7*b^17*c + 36
*a^8*b^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 8
4*a^12*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9))))/(a
^2*b^2)))*((-b)^(7/4)*1i - (a*(-b)^(3/4)*(4*b + b*c + 1)*1i)/4)*2i)/(a^2*b^
2) + (atan((((b*((-b)^(3/4) + a*((-b)^(3/4) + ((-b)^(3/4)*c)/4)) + (a*(-b)^
(3/4))/4)*((64*(-(b*x - 1)/(c + x))^(1/4)*(512*(-b)^(19/2) + a^5*(-b)^(9/2)
 + a^4*(-b)^(11/2) + 2*a^6*(-b)^(9/2) - 16*a^3*(-b)^(13/2) + 18*a^5*(-b)^(1
1/2) + a^7*(-b)^(9/2) - 144*a^2*(-b)^(15/2) - 176*a^4*(-b)^(13/2) + 49*a^6*
(-b)^(11/2) - 576*a^3*(-b)^(15/2) - 560*a^5*(-b)^(13/2) + 48*a^7*(-b)^(11/2
) + 2688*a^2*(-b)^(17/2) - 608*a^4*(-b)^(15/2) - 784*a^6*(-b)^(13/2) + 16*a
^8*(-b)^(11/2) + 7680*a^3*(-b)^(17/2) + 448*a^5*(-b)^(15/2) - 512*a^7*(-b)^
(13/2) + 7680*a^2*(-b)^(19/2) + 11520*a^4*(-b)^(17/2) + 1392*a^6*(-b)^(15/2
) - 128*a^8*(-b)^(13/2) + 10240*a^3*(-b)^(19/2) + 9600*a^5*(-b)^(17/2) + 10
24*a^7*(-b)^(15/2) + 7680*a^4*(-b)^(19/2) + 4224*a^6*(-b)^(17/2) + 256*a^8*
(-b)^(15/2) + 3072*a^5*(-b)^(19/2) + 768*a^7*(-b)^(17/2) + 512*a^6*(-b)^(19
/2) + 7680*(-b)^(23/2)*c^2 - 10240*(-b)^(25/2)*c^3 + 7680*(-b)^(27/2)*c^4 -
 3072*(-b)^(29/2)*c^5 + 512*(-b)^(31/2)*c^6 + 384*a*(-b)^(17/2) + 3072*a*(-
b)^(19/2) - 3072*(-b)^(21/2)*c + a^7*(-b)^(9/2)*c^2 - 35*a^6*(-b)^(11/2)*c^
2 + 2*a^8*(-b)^(9/2)*c^2 + 265*a^5*(-b)^(13/2)*c^2 - 86*a^7*(-b)^(11/2)*c^2
 + a^9*(-b)^(9/2)*c^2 - 851*a^4*(-b)^(15/2)*c^2 + 738*a^6*(-b)^(13/2)*c^2 -
 10*a^7*(-b)^(11/2)*c^3 - 67*a^8*(-b)^(11/2)*c^2 + 2496*a^3*(-b)^(17/2)*c^2
 - 2566*a^5*(-b)^(15/2)*c^2 + 224*a^6*(-b)^(13/2)*c^3 + 649*a^7*(-b)^(13/2)
*c^2 - 20*a^8*(-b)^(11/2)*c^3 - 16*a^9*(-b)^(11/2)*c^2 - 5184*a^2*(-b)^(19/
2)*c^2 + 10432*a^4*(-b)^(17/2)*c^2 - 1358*a^5*(-b)^(15/2)*c^3 - 1907*a^6*(-
b)^(15/2)*c^2 + 592*a^7*(-b)^(13/2)*c^3 + 144*a^8*(-b)^(13/2)*c^2 - 10*a^9*
(-b)^(11/2)*c^3 - 31104*a^3*(-b)^(19/2)*c^2 + 3784*a^4*(-b)^(17/2)*c^3 + 14
912*a^5*(-b)^(17/2)*c^2 - 4364*a^6*(-b)^(15/2)*c^3 + 45*a^7*(-b)^(13/2)*c^4
 + 1120*a^7*(-b)^(15/2)*c^2 + 512*a^8*(-b)^(13/2)*c^3 - 32*a^9*(-b)^(13/2)*
c^2 + 1152*a^2*(-b)^(21/2)*c^2 - 7552*a^3*(-b)^(19/2)*c^3 - 74624*a^4*(-b)^
(19/2)*c^2 + 14288*a^5*(-b)^(17/2)*c^3 - 771*a^6*(-b)^(15/2)*c^4 + 5824*a^6
*(-b)^(17/2)*c^2 - 4974*a^7*(-b)^(15/2)*c^3 + 90*a^8*(-b)^(13/2)*c^4 + 1952
a^8(-b)^(15/2)*c^2 + 144*a^9*(-b)^(13/2)*c^3 + 6912*a^2*(-b)^(21/2)*c^3 +
 23040*a^3*(-b)^(21/2)*c^2 - 36800*a^4*(-b)^(19/2)*c^3 + 3874*a^5*(-b)^(17/
2)*c^4 - 91136*a^5*(-b)^(19/2)*c^2 + 19144*a^6*(-b)^(17/2)*c^3 - 2118*a^7*(
-b)^(15/2)*c^4 - 5120*a^7*(-b)^(17/2)*c^2 - 2288*a^8*(-b)^(15/2)*c^3 + 45*a
^9*(-b)^(13/2)*c^4 + 640*a^9*(-b)^(15/2)*c^2 + 115200*a^2*(-b)^(23/2)*c^2 +
 49536*a^3*(-b)^(21/2)*c^3 - 8750*a^4*(-b)^(19/2)*c^4 + 57600*a^4*(-b)^(21/
2)*c^2 - 68032*a^5*(-b)^(19/2)*c^3 + 13444*a^6*(-b)^(17/2)*c^4 - 58944*a^6*

(-b)^(19/2)*c^2 - 120*a^7*(-b)^(15/2)*c^5 + 9664*a^7*(-b)^(17/2)*c^3 - 1923
a^8(-b)^(15/2)*c^4 - 5248*a^8*(-b)^(17/2)*c^2 - 320*a^9*(-b)^(15/2)*c^3 +
 44160*a^2*(-b)^(23/2)*c^3 + 11040*a^3*(-b)^(21/2)*c^4 + 153600*a^3*(-b)^(2
3/2)*c^2 + 137728*a^4*(-b)^(21/2)*c^3 - 36988*a^5*(-b)^(19/2)*c^4 + 63360*a
^5*(-b)^(21/2)*c^2 + 1644*a^6*(-b)^(17/2)*c^5 - 56512*a^6*(-b)^(19/2)*c^3 +
 17058*a^7*(-b)^(17/2)*c^4 - 18560*a^7*(-b)^(19/2)*c^2 - 240*a^8*(-b)^(15/2
)*c^5 + 128*a^8*(-b)^(17/2)*c^3 - 576*a^9*(-b)^(15/2)*c^4 - 1280*a^9*(-b)^(
17/2)*c^2 - 480*a^2*(-b)^(23/2)*c^4 + 76800*a^3*(-b)^(23/2)*c^3 + 60640*a^4
*(-b)^(21/2)*c^4 + 115200*a^4*(-b)^(23/2)*c^2 - 6776*a^5*(-b)^(19/2)*c^5 +
193792*a^5*(-b)^(21/2)*c^3 - 59150*a^6*(-b)^(19/2)*c^4 + 33408*a^6*(-b)^(21
/2)*c^2 + 4632*a^7*(-b)^(17/2)*c^5 - 16064*a^7*(-b)^(19/2)*c^3 + 9280*a^8*(
-b)^(17/2)*c^4 - 2048*a^8*(-b)^(19/2)*c^2 - 120*a^9*(-b)^(15/2)*c^5 - 896*a
^9*(-b)^(17/2)*c^3 - 153600*a^2*(-b)^(25/2)*c^3 - 23040*a^3*(-b)^(23/2)*c^4
 + 11620*a^4*(-b)^(21/2)*c^5 + 57600*a^4*(-b)^(23/2)*c^3 + 128864*a^5*(-b)^
(21/2)*c^4 + 46080*a^5*(-b)^(23/2)*c^2 - 24752*a^6*(-b)^(19/2)*c^5 + 146688
a^6(-b)^(21/2)*c^3 + 210*a^7*(-b)^(17/2)*c^6 - 43232*a^7*(-b)^(19/2)*c^4
 + 6912*a^7*(-b)^(21/2)*c^2 + 4332*a^8*(-b)^(17/2)*c^5 + 3968*a^8*(-b)^(19/2
)*c^3 + 1792*a^9*(-b)^(17/2)*c^4 - 90240*a^2*(-b)^(25/2)*c^4 - 7104*a^3*(-b
)^(23/2)*c^5 - 204800*a^3*(-b)^(25/2)*c^3 - 100160*a^4*(-b)^(23/2)*c^4 + 53
480*a^5*(-b)^(21/2)*c^5 + 9600*a^5*(-b)^(23/2)*c^3 - 2310*a^6*(-b)^(19/2)*c
^6 + 131872*a^6*(-b)^(21/2)*c^4 + 7680*a^6*(-b)^(23/2)*c^2 - 33432*a^7*(-b)
^(19/2)*c^5 + 56704*a^7*(-b)^(21/2)*c^3 + 420*a^8*(-b)^(17/2)*c^6 - 13216*a
^8*(-b)^(19/2)*c^4 + 1344*a^9*(-b)^(17/2)*c^5 + 2304*a^9*(-b)^(19/2)*c^3 -
9216*a^2*(-b)^(25/2)*c^5 - 192000*a^3*(-b)^(25/2)*c^4 - 48224*a^4*(-b)^(23/
2)*c^5 - 153600*a^4*(-b)^(25/2)*c^3 + 7546*a^5*(-b)^(21/2)*c^6 - 179840*a^5
*(-b)^(23/2)*c^4 + 94948*a^6*(-b)^(21/2)*c^5 - 9600*a^6*(-b)^(23/2)*c^3 - 6
636*a^7*(-b)^(19/2)*c^6 + 63232*a^7*(-b)^(21/2)*c^4 - 19712*a^8*(-b)^(19/2)
*c^5 + 8704*a^8*(-b)^(21/2)*c^3 + 210*a^9*(-b)^(17/2)*c^6 - 896*a^9*(-b)^(1
9/2)*c^4 + 115200*a^2*(-b)^(27/2)*c^4 - 31104*a^3*(-b)^(25/2)*c^5 - 8750*a^
4*(-b)^(23/2)*c^6 - 211200*a^4*(-b)^(25/2)*c^4 - 121120*a^5*(-b)^(23/2)*c^5
 - 61440*a^5*(-b)^(25/2)*c^3 + 28756*a^6*(-b)^(21/2)*c^6 - 161760*a^6*(-b)^
(23/2)*c^4 - 252*a^7*(-b)^(19/2)*c^7 + 80416*a^7*(-b)^(21/2)*c^5 - 3840*a^7
*(-b)^(23/2)*c^3 - 6342*a^8*(-b)^(19/2)*c^6 + 9344*a^8*(-b)^(21/2)*c^4 - 42
56*a^9*(-b)^(19/2)*c^5 + 81792*a^2*(-b)^(27/2)*c^5 - 832*a^3*(-b)^(25/2)*c^
6 + 153600*a^3*(-b)^(27/2)*c^4 - 23552*a^4*(-b)^(25/2)*c^5 - 44380*a^5*(-b)
^(23/2)*c^6 - 124800*a^5*(-b)^(25/2)*c^4 + 2184*a^6*(-b)^(21/2)*c^7 - 14633
6*a^6*(-b)^(23/2)*c^5 - 10240*a^6*(-b)^(25/2)*c^3 + 40698*a^7*(-b)^(21/2)*c
^6 - 72320*a^7*(-b)^(23/2)*c^4 - 504*a^8*(-b)^(19/2)*c^7 + 31808*a^8*(-b)^(
21/2)*c^5 - 2016*a^9*(-b)^(19/2)*c^6 - 1280*a^9*(-b)^(21/2)*c^4 + 10944*a^2
*(-b)^(27/2)*c^6 + 184320*a^3*(-b)^(27/2)*c^5 + 8896*a^4*(-b)^(25/2)*c^6 +
115200*a^4*(-b)^(27/2)*c^4 - 5300*a^5*(-b)^(23/2)*c^7 + 32512*a^5*(-b)^(25/
2)*c^5 - 86702*a^6*(-b)^(23/2)*c^6 - 36480*a^6*(-b)^(25/2)*c^4 + 6384*a^7*(
-b)^(21/2)*c^7 - 87968*a^7*(-b)^(23/2)*c^5 + 25312*a^8*(-b)^(21/2)*c^6 - 12
800*a^8*(-b)^(23/2)*c^4 - 252*a^9*(-b)^(19/2)*c^7 + 4480*a^9*(-b)^(21/2)*c^
5 - 46080*a^2*(-b)^(29/2)*c^5 + 49536*a^3*(-b)^(27/2)*c^6 + 3016*a^4*(-b)^(
25/2)*c^7 + 218880*a^4*(-b)^(27/2)*c^5 + 44864*a^5*(-b)^(25/2)*c^6 + 46080*
a^5*(-b)^(27/2)*c^4 - 21128*a^6*(-b)^(23/2)*c^7 + 66048*a^6*(-b)^(25/2)*c^5
 + 210*a^7*(-b)^(21/2)*c^8 - 81536*a^7*(-b)^(23/2)*c^6 - 3840*a^7*(-b)^(25/
2)*c^4 + 6216*a^8*(-b)^(21/2)*c^7 - 22912*a^8*(-b)^(23/2)*c^5 + 5824*a^9*(-
b)^(21/2)*c^6 - 36480*a^2*(-b)^(29/2)*c^6 + 4224*a^3*(-b)^(27/2)*c^7 - 6144
0*a^3*(-b)^(29/2)*c^5 + 86656*a^4*(-b)^(27/2)*c^6 + 18896*a^5*(-b)^(25/2)*c
^7 + 144000*a^5*(-b)^(27/2)*c^5 - 1374*a^6*(-b)^(23/2)*c^8 + 75200*a^6*(-b)
^(25/2)*c^6 + 7680*a^6*(-b)^(27/2)*c^4 - 31284*a^7*(-b)^(23/2)*c^7 + 40576*
a^7*(-b)^(25/2)*c^5 + 420*a^8*(-b)^(21/2)*c^8 - 36736*a^8*(-b)^(23/2)*c^6 +
 2016*a^9*(-b)^(21/2)*c^7 - 1280*a^9*(-b)^(23/2)*c^5 - 5376*a^2*(-b)^(29/2)
*c^7 - 84480*a^3*(-b)^(29/2)*c^6 + 13888*a^4*(-b)^(27/2)*c^7 - 46080*a^4*(-
b)^(29/2)*c^5 + 2173*a^5*(-b)^(25/2)*c^8 + 70144*a^5*(-b)^(27/2)*c^6 + 4295
2*a^6*(-b)^(25/2)*c^7 + 49536*a^6*(-b)^(27/2)*c^5 - 4092*a^7*(-b)^(23/2)*c^
8 + 57856*a^7*(-b)^(25/2)*c^6 - 20384*a^8*(-b)^(23/2)*c^7 + 8704*a^8*(-b)^(

25/2)*c^5 + 210*a^9*(-b)^(21/2)*c^8 - 6272*a^9*(-b)^(23/2)*c^6 + 7680*a^2*(
-b)^(31/2)*c^6 - 26496*a^3*(-b)^(29/2)*c^7 + 301*a^4*(-b)^(27/2)*c^8 - 1036
80*a^4*(-b)^(29/2)*c^6 + 12608*a^5*(-b)^(27/2)*c^7 - 18432*a^5*(-b)^(29/2)*
c^5 + 9274*a^6*(-b)^(25/2)*c^8 + 21696*a^6*(-b)^(27/2)*c^6 - 120*a^7*(-b)^(
23/2)*c^9 + 45760*a^7*(-b)^(25/2)*c^7 + 6912*a^7*(-b)^(27/2)*c^5 - 4062*a^8
*(-b)^(23/2)*c^8 + 20096*a^8*(-b)^(25/2)*c^6 - 4928*a^9*(-b)^(23/2)*c^7 + 6
528*a^2*(-b)^(31/2)*c^7 - 2448*a^3*(-b)^(29/2)*c^8 + 10240*a^3*(-b)^(31/2)*
c^6 - 52736*a^4*(-b)^(29/2)*c^7 - 1558*a^5*(-b)^(27/2)*c^8 - 71040*a^5*(-b)
^(29/2)*c^6 + 546*a^6*(-b)^(25/2)*c^9 - 4544*a^6*(-b)^(27/2)*c^7 - 3072*a^6
*(-b)^(29/2)*c^5 + 14589*a^7*(-b)^(25/2)*c^8 - 2432*a^7*(-b)^(27/2)*c^6 - 2
40*a^8*(-b)^(23/2)*c^9 + 23168*a^8*(-b)^(25/2)*c^7 - 1344*a^9*(-b)^(23/2)*c
^8 + 2304*a^9*(-b)^(25/2)*c^6 + 1008*a^2*(-b)^(31/2)*c^8 + 15360*a^3*(-b)^(
31/2)*c^7 - 10160*a^4*(-b)^(29/2)*c^8 + 7680*a^4*(-b)^(31/2)*c^6 - 384*a^5*
(-b)^(27/2)*c^9 - 53504*a^5*(-b)^(29/2)*c^7 - 8099*a^6*(-b)^(27/2)*c^8 - 25
728*a^6*(-b)^(29/2)*c^6 + 1668*a^7*(-b)^(25/2)*c^9 - 13760*a^7*(-b)^(27/2)*
c^7 + 10048*a^8*(-b)^(25/2)*c^8 - 2048*a^8*(-b)^(27/2)*c^6 - 120*a^9*(-b)^(
23/2)*c^9 + 4480*a^9*(-b)^(25/2)*c^7 + 5184*a^3*(-b)^(31/2)*c^8 - 570*a^4*(
-b)^(29/2)*c^9 + 19200*a^4*(-b)^(31/2)*c^7 - 16048*a^5*(-b)^(29/2)*c^8 + 30
72*a^5*(-b)^(31/2)*c^6 - 1984*a^6*(-b)^(27/2)*c^9 - 28416*a^6*(-b)^(29/2)*c
^7 + 45*a^7*(-b)^(25/2)*c^10 - 11984*a^7*(-b)^(27/2)*c^8 - 3840*a^7*(-b)^(2
9/2)*c^6 + 1698*a^8*(-b)^(25/2)*c^9 - 7552*a^8*(-b)^(27/2)*c^7 + 2560*a^9*(
-b)^(25/2)*c^8 + 480*a^3*(-b)^(31/2)*c^9 + 10912*a^4*(-b)^(31/2)*c^8 - 1732
a^5(-b)^(29/2)*c^9 + 13440*a^5*(-b)^(31/2)*c^7 - 119*a^6*(-b)^(27/2)*c^10
 - 11408*a^6*(-b)^(29/2)*c^8 + 512*a^6*(-b)^(31/2)*c^6 - 3568*a^7*(-b)^(27/
2)*c^9 - 7040*a^7*(-b)^(29/2)*c^7 + 90*a^8*(-b)^(25/2)*c^10 - 7408*a^8*(-b)
^(27/2)*c^8 + 576*a^9*(-b)^(25/2)*c^9 - 1280*a^9*(-b)^(27/2)*c^7 + 2160*a^4
*(-b)^(31/2)*c^9 - 35*a^5*(-b)^(29/2)*c^10 + 11968*a^5*(-b)^(31/2)*c^8 - 15
30*a^6*(-b)^(29/2)*c^9 + 4992*a^6*(-b)^(31/2)*c^7 - 382*a^7*(-b)^(27/2)*c^1
0 - 2816*a^7*(-b)^(29/2)*c^8 - 2720*a^8*(-b)^(27/2)*c^9 - 512*a^8*(-b)^(29/
2)*c^7 + 45*a^9*(-b)^(25/2)*c^10 - 1664*a^9*(-b)^(27/2)*c^8 + 129*a^4*(-b)^
(31/2)*c^10 + 3856*a^5*(-b)^(31/2)*c^9 + 10*a^6*(-b)^(29/2)*c^10 + 7152*a^6
*(-b)^(31/2)*c^8 - 10*a^7*(-b)^(27/2)*c^11 + 112*a^7*(-b)^(29/2)*c^9 + 768*
a^7*(-b)^(31/2)*c^7 - 407*a^8*(-b)^(27/2)*c^10 + 512*a^8*(-b)^(29/2)*c^8 -
752*a^9*(-b)^(27/2)*c^9 + 482*a^5*(-b)^(31/2)*c^10 + 8*a^6*(-b)^(29/2)*c^11
 + 3408*a^6*(-b)^(31/2)*c^9 + 221*a^7*(-b)^(29/2)*c^10 + 2176*a^7*(-b)^(31/
2)*c^8 - 20*a^8*(-b)^(27/2)*c^11 + 736*a^8*(-b)^(29/2)*c^9 - 144*a^9*(-b)^(
27/2)*c^10 + 256*a^9*(-b)^(29/2)*c^8 + 18*a^5*(-b)^(31/2)*c^11 + 673*a^6*(-
b)^(31/2)*c^10 + 32*a^7*(-b)^(29/2)*c^11 + 1488*a^7*(-b)^(31/2)*c^9 + 272*a
^8*(-b)^(29/2)*c^10 + 256*a^8*(-b)^(31/2)*c^8 - 10*a^9*(-b)^(27/2)*c^11 + 2
56*a^9*(-b)^(29/2)*c^9 + 52*a^6*(-b)^(31/2)*c^11 + a^7*(-b)^(29/2)*c^12 + 4
16*a^7*(-b)^(31/2)*c^10 + 40*a^8*(-b)^(29/2)*c^11 + 256*a^8*(-b)^(31/2)*c^9
 + 96*a^9*(-b)^(29/2)*c^10 + a^6*(-b)^(31/2)*c^12 + 50*a^7*(-b)^(31/2)*c^11
 + 2*a^8*(-b)^(29/2)*c^12 + 96*a^8*(-b)^(31/2)*c^10 + 16*a^9*(-b)^(29/2)*c^
11 + 2*a^7*(-b)^(31/2)*c^12 + 16*a^8*(-b)^(31/2)*c^11 + a^9*(-b)^(29/2)*c^1
2 + a^8*(-b)^(31/2)*c^12 - 1152*a*(-b)^(19/2)*c - 18432*a*(-b)^(21/2)*c + 2
a^6(-b)^(9/2)*c - 24*a^5*(-b)^(11/2)*c + 4*a^7*(-b)^(9/2)*c + 70*a^4*(-b)
^(13/2)*c - 48*a^6*(-b)^(11/2)*c + 2*a^8*(-b)^(9/2)*c - 288*a^3*(-b)^(15/2)
*c + 60*a^5*(-b)^(13/2)*c - 8*a^7*(-b)^(11/2)*c + 1536*a^2*(-b)^(17/2)*c -
656*a^4*(-b)^(15/2)*c - 378*a^6*(-b)^(13/2)*c + 32*a^8*(-b)^(11/2)*c + 8064
a^3(-b)^(17/2)*c + 656*a^5*(-b)^(15/2)*c - 784*a^7*(-b)^(13/2)*c + 16*a^9
*(-b)^(11/2)*c - 9600*a^2*(-b)^(19/2)*c + 16384*a^4*(-b)^(17/2)*c + 3280*a^
6*(-b)^(15/2)*c - 544*a^8*(-b)^(13/2)*c - 30720*a^3*(-b)^(19/2)*c + 15616*a
^5*(-b)^(17/2)*c + 3664*a^7*(-b)^(15/2)*c - 128*a^9*(-b)^(13/2)*c - 1152*a*
(-b)^(21/2)*c^2 - 46080*a^2*(-b)^(21/2)*c - 49920*a^4*(-b)^(19/2)*c + 6144*
a^6*(-b)^(17/2)*c + 1664*a^8*(-b)^(15/2)*c - 61440*a^3*(-b)^(21/2)*c - 4416
0*a^5*(-b)^(19/2)*c - 128*a^7*(-b)^(17/2)*c + 256*a^9*(-b)^(15/2)*c + 46080
a(-b)^(23/2)*c^2 - 46080*a^4*(-b)^(21/2)*c - 20352*a^6*(-b)^(19/2)*c - 51
2*a^8*(-b)^(17/2)*c + 9600*a*(-b)^(23/2)*c^3 - 18432*a^5*(-b)^(21/2)*c - 38
40*a^7*(-b)^(19/2)*c - 3072*a^6*(-b)^(21/2)*c - 61440*a*(-b)^(25/2)*c^3 - 1

7280*a*(-b)^(25/2)*c^4 + 46080*a*(-b)^(27/2)*c^4 + 14976*a*(-b)^(27/2)*c^5 - 18432*a*(-b)^(29/2)*c^5 - 6528*a*(-b)^(29/2)*c^6 + 3072*a*(-b)^(31/2)*c^6 + 1152*a*(-b)^(31/2)*c^7))/((-b)^(1/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^8*b^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a^12*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9)) + (((64*(36*a^12*(-b)^(25/4) - 4*a^13*(-b)^(21/4) + 48*a^13*(-b)^(25/4) - 60*a^11*(-b)^(29/4) - 240*a^12*(-b)^(29/4) - 192*a^13*(-b)^(29/4) - 180*a^10*(-b)^(33/4) - 240*a^11*(-b)^(33/4) + 192*a^12*(-b)^(33/4) + 240*a^9*(-b)^(37/4) + 256*a^13*(-b)^(33/4) + 1200*a^10*(-b)^(37/4) + 1728*a^11*(-b)^(37/4) + 320*a^8*(-b)^(41/4) + 768*a^12*(-b)^(37/4) + 1152*a^9*(-b)^(41/4) + 1344*a^10*(-b)^(41/4) + 512*a^11*(-b)^(41/4) - 144*a^13*(-b)^(29/4)*c^2 + 912*a^12*(-b)^(33/4)*c^2 + 1344*a^13*(-b)^(33/4)*c^2 + 336*a^13*(-b)^(33/4)*c^3 - 432*a^11*(-b)^(37/4)*c^2 - 4032*a^12*(-b)^(37/4)*c^2 - 1680*a^12*(-b)^(37/4)*c^3 - 4032*a^13*(-b)^(37/4)*c^2 - 4624*a^10*(-b)^(41/4)*c^2 - 2688*a^13*(-b)^(37/4)*c^3 - 9408*a^11*(-b)^(41/4)*c^2 - 504*a^13*(-b)^(37/4)*c^4 - 336*a^11*(-b)^(41/4)*c^3 - 1344*a^12*(-b)^(41/4)*c^2 + 3584*a^9*(-b)^(45/4)*c^2 + 5376*a^12*(-b)^(41/4)*c^3 + 3584*a^13*(-b)^(41/4)*c^2 + 20160*a^10*(-b)^(45/4)*c^2 + 1848*a^12*(-b)^(41/4)*c^4 + 6720*a^13*(-b)^(41/4)*c^3 + 8848*a^10*(-b)^(45/4)*c^3 + 30912*a^11*(-b)^(45/4)*c^2 + 3360*a^13*(-b)^(41/4)*c^4 + 6720*a^8*(-b)^(49/4)*c^2 + 21504*a^11*(-b)^(45/4)*c^3 + 14336*a^12*(-b)^(45/4)*c^2 + 504*a^13*(-b)^(41/4)*c^5 + 24192*a^9*(-b)^(49/4)*c^2 + 1848*a^11*(-b)^(45/4)*c^4 + 9408*a^12*(-b)^(45/4)*c^3 - 4032*a^9*(-b)^(49/4)*c^3 + 28224*a^10*(-b)^(49/4)*c^2 - 3360*a^12*(-b)^(45/4)*c^4 - 3584*a^13*(-b)^(45/4)*c^3 - 26880*a^10*(-b)^(49/4)*c^3 + 10752*a^11*(-b)^(49/4)*c^2 - 1176*a^12*(-b)^(45/4)*c^5 - 6720*a^13*(-b)^(45/4)*c^4 - 10584*a^10*(-b)^(49/4)*c^4 - 44352*a^11*(-b)^(49/4)*c^3 - 2688*a^13*(-b)^(45/4)*c^5 - 11200*a^8*(-b)^(53/4)*c^3 - 30240*a^11*(-b)^(49/4)*c^4 - 21504*a^12*(-b)^(49/4)*c^3 - 336*a^13*(-b)^(45/4)*c^6 - 40320*a^9*(-b)^(53/4)*c^3 - 2520*a^11*(-b)^(49/4)*c^5 - 20160*a^12*(-b)^(49/4)*c^4 + 1120*a^9*(-b)^(53/4)*c^4 - 47040*a^10*(-b)^(53/4)*c^3 + 16800*a^10*(-b)^(53/4)*c^4 - 17920*a^11*(-b)^(53/4)*c^3 + 336*a^12*(-b)^(49/4)*c^6 + 4032*a^13*(-b)^(49/4)*c^5 + 8120*a^10*(-b)^(53/4)*c^5 + 33600*a^11*(-b)^(53/4)*c^4 + 1344*a^13*(-b)^(49/4)*c^6 + 11200*a^8*(-b)^(57/4)*c^4 + 26880*a^11*(-b)^(53/4)*c^5 + 17920*a^12*(-b)^(53/4)*c^4 + 144*a^13*(-b)^(49/4)*c^7 + 40320*a^9*(-b)^(57/4)*c^4 + 1680*a^11*(-b)^(53/4)*c^6 + 22848*a^12*(-b)^(53/4)*c^5 + 2240*a^9*(-b)^(57/4)*c^5 + 47040*a^10*(-b)^(57/4)*c^4 + 1344*a^12*(-b)^(53/4)*c^6 + 3584*a^13*(-b)^(53/4)*c^5 + 17920*a^11*(-b)^(57/4)*c^4 + 48*a^12*(-b)^(53/4)*c^7 - 1344*a^13*(-b)^(53/4)*c^6 - 3920*a^10*(-b)^(57/4)*c^6 - 9408*a^11*(-b)^(57/4)*c^5 - 384*a^13*(-b)^(53/4)*c^7 - 6720*a^8*(-b)^(61/4)*c^5 - 14784*a^11*(-b)^(57/4)*c^6 - 7168*a^12*(-b)^(57/4)*c^5 - 36*a^13*(-b)^(53/4)*c^8 - 24192*a^9*(-b)^(61/4)*c^5 - 528*a^11*(-b)^(57/4)*c^7 - 14784*a^12*(-b)^(57/4)*c^6 - 2688*a^9*(-b)^(61/4)*c^6 - 28224*a^10*(-b)^(61/4)*c^5 - 768*a^12*(-b)^(57/4)*c^7 - 3584*a^13*(-b)^(57/4)*c^6 - 6720*a^10*(-b)^(61/4)*c^6 - 10752*a^11*(-b)^(61/4)*c^5 - 60*a^12*(-b)^(57/4)*c^8 + 192*a^13*(-b)^(57/4)*c^7 + 1104*a^10*(-b)^(61/4)*c^7 - 4032*a^11*(-b)^(61/4)*c^6 + 48*a^13*(-b)^(57/4)*c^8 + 2240*a^8*(-b)^(65/4)*c^6 + 4608*a^11*(-b)^(61/4)*c^7 + 4*a^13*(-b)^(57/4)*c^9 + 8064*a^9*(-b)^(65/4)*c^6 + 36*a^11*(-b)^(61/4)*c^8 + 5184*a^12*(-b)^(61/4)*c^7 + 1216*a^9*(-b)^(65/4)*c^7 + 9408*a^10*(-b)^(65/4)*c^6 + 144*a^12*(-b)^(61/4)*c^8 + 1536*a^13*(-b)^(61/4)*c^7 + 3840*a^10*(-b)^(65/4)*c^7 + 3584*a^11*(-b)^(65/4)*c^6 + 12*a^12*(-b)^(61/4)*c^9 - 148*a^10*(-b)^(65/4)*c^8 + 3648*a^11*(-b)^(65/4)*c^7 - 320*a^8*(-b)^(69/4)*c^7 - 624*a^11*(-b)^(65/4)*c^8 + 1024*a^12*(-b)^(65/4)*c^7 - 1152*a^9*(-b)^(69/4)*c^7 + 12*a^11*(-b)^(65/4)*c^9 - 768*a^12*(-b)^(65/4)*c^8 - 208*a^9*(-b)^(69/4)*c^8 - 1344*a^10*(-b)^(69/4)*c^7 - 256*a^13*(-b)^(65/4)*c^8 - 720*a^10*(-b)^(69/4)*c^8 - 512*a^11*(-b)^(69/4)*c^7 + 4*a^10*(-b)^(69/4)*c^9 - 768*a^11*(-b)^(69/4)*c^8 - 256*a^12*(-b)^(69/4)*c^8 + 36*a^13*(-b)^(25/4)*c - 276*a^12*(-b)^(29/4)*c - 384*a^13*(-b)^(29/4)*c + 300*a^11*(-b)^(33/4)*c + 1536*a^12*(-b)^(33/4)*c + 1344*a^13*(-b)^(33/4)*c + 1380*a^10*(-b)^(37/4)*c + 2304*a^11*(-b)^(37/4)*c - 576*a^12*(-b)^(37/4)*c - 1472*a^9*(-b)^(41/4)*c - 1536*a^13*(-b)^(37/4)*c - 7680*a^

10*(-b)^(41/4)*c - 11328*a^11*(-b)^(41/4)*c - 2240*a^8*(-b)^(45/4)*c - 5120
a^12(-b)^(41/4)*c - 8064*a^9*(-b)^(45/4)*c - 9408*a^10*(-b)^(45/4)*c - 35
84*a^11*(-b)^(45/4)*c))/(a^7*b^18 + 9*a^8*b^18*c + 36*a^9*b^18*c^2 + 84*a^1
0*b^18*c^3 + 126*a^11*b^18*c^4 + 126*a^12*b^18*c^5 + 84*a^13*b^18*c^6 + 36*
a^14*b^18*c^7 + 9*a^15*b^18*c^8 + a^16*b^18*c^9) - (64*(-(b*x - 1)/(c + x))
^(1/4)*(b*((-b)^(3/4) + a*((-b)^(3/4) + ((-b)^(3/4)*c)/4)) + (a*(-b)^(3/4))
/4)*(16*a^13*(-b)^(11/2) - 80*a^12*(-b)^(13/2) - 80*a^11*(-b)^(15/2) - 128*
a^13*(-b)^(13/2) + 400*a^10*(-b)^(17/2) + 128*a^12*(-b)^(15/2) + 896*a^9*(-
b)^(19/2) + 1152*a^11*(-b)^(17/2) + 256*a^13*(-b)^(15/2) + 512*a^8*(-b)^(21
/2) + 1920*a^10*(-b)^(19/2) + 768*a^12*(-b)^(17/2) + 1024*a^9*(-b)^(21/2) +
1024*a^11*(-b)^(19/2) + 512*a^10*(-b)^(21/2) + 448*a^13*(-b)^(15/2)*c^2 -
1344*a^12*(-b)^(17/2)*c^2 - 2624*a^11*(-b)^(19/2)*c^2 - 2688*a^13*(-b)^(17/
2)*c^2 + 1088*a^10*(-b)^(21/2)*c^2 + 128*a^12*(-b)^(19/2)*c^2 - 896*a^13*(-
b)^(17/2)*c^3 + 9600*a^9*(-b)^(23/2)*c^2 + 9344*a^11*(-b)^(21/2)*c^2 + 1792
a^12(-b)^(19/2)*c^3 + 4096*a^13*(-b)^(19/2)*c^2 + 7680*a^8*(-b)^(25/2)*c^
2 + 21888*a^10*(-b)^(23/2)*c^2 + 4096*a^11*(-b)^(21/2)*c^3 + 8704*a^12*(-b)
^(21/2)*c^2 + 4480*a^13*(-b)^(19/2)*c^3 + 15360*a^9*(-b)^(25/2)*c^2 + 3328*
a^10*(-b)^(23/2)*c^3 + 12288*a^11*(-b)^(23/2)*c^2 + 128*a^12*(-b)^(21/2)*c^
3 + 1120*a^13*(-b)^(19/2)*c^4 - 8320*a^9*(-b)^(25/2)*c^3 + 7680*a^10*(-b)^(
25/2)*c^2 - 4992*a^11*(-b)^(23/2)*c^3 - 1120*a^12*(-b)^(21/2)*c^4 - 6656*a^
13*(-b)^(21/2)*c^3 - 10240*a^8*(-b)^(27/2)*c^3 - 21120*a^10*(-b)^(25/2)*c^3
- 2400*a^11*(-b)^(23/2)*c^4 - 9216*a^12*(-b)^(23/2)*c^3 - 4480*a^13*(-b)^(
21/2)*c^4 - 20480*a^9*(-b)^(27/2)*c^3 - 7200*a^10*(-b)^(25/2)*c^4 - 12800*a
^11*(-b)^(25/2)*c^3 + 1920*a^12*(-b)^(23/2)*c^4 - 896*a^13*(-b)^(21/2)*c^5
+ 640*a^9*(-b)^(27/2)*c^4 - 10240*a^10*(-b)^(27/2)*c^3 - 3200*a^11*(-b)^(25
/2)*c^4 + 7680*a^13*(-b)^(23/2)*c^4 + 7680*a^8*(-b)^(29/2)*c^4 + 5760*a^10*
(-b)^(27/2)*c^4 - 1280*a^11*(-b)^(25/2)*c^5 + 5120*a^12*(-b)^(25/2)*c^4 + 2
688*a^13*(-b)^(23/2)*c^5 + 15360*a^9*(-b)^(29/2)*c^4 + 5120*a^10*(-b)^(27/2
)*c^5 + 5120*a^11*(-b)^(27/2)*c^4 - 5248*a^12*(-b)^(25/2)*c^5 + 448*a^13*(-
b)^(23/2)*c^6 + 4224*a^9*(-b)^(29/2)*c^5 + 7680*a^10*(-b)^(29/2)*c^4 + 3968
a^11(-b)^(27/2)*c^5 + 448*a^12*(-b)^(25/2)*c^6 - 6656*a^13*(-b)^(25/2)*c^
5 - 3072*a^8*(-b)^(31/2)*c^5 + 5760*a^10*(-b)^(29/2)*c^5 + 2752*a^11*(-b)^(
27/2)*c^6 - 2048*a^12*(-b)^(27/2)*c^5 - 896*a^13*(-b)^(25/2)*c^6 - 6144*a^9
*(-b)^(31/2)*c^5 - 704*a^10*(-b)^(29/2)*c^6 + 1536*a^11*(-b)^(29/2)*c^5 + 5
504*a^12*(-b)^(27/2)*c^6 - 128*a^13*(-b)^(25/2)*c^7 - 2944*a^9*(-b)^(31/2)*
c^6 - 3072*a^10*(-b)^(31/2)*c^5 + 384*a^11*(-b)^(29/2)*c^6 - 256*a^12*(-b)^
(27/2)*c^7 + 4096*a^13*(-b)^(27/2)*c^6 + 512*a^8*(-b)^(33/2)*c^6 - 4992*a^1
0*(-b)^(31/2)*c^6 - 1536*a^11*(-b)^(29/2)*c^7 + 1536*a^12*(-b)^(29/2)*c^6 +
128*a^13*(-b)^(27/2)*c^7 + 1024*a^9*(-b)^(33/2)*c^6 - 768*a^10*(-b)^(31/2)
*c^7 - 2048*a^11*(-b)^(31/2)*c^6 - 2688*a^12*(-b)^(29/2)*c^7 + 16*a^13*(-b)
^(27/2)*c^8 + 640*a^9*(-b)^(33/2)*c^7 + 512*a^10*(-b)^(33/2)*c^6 - 1664*a^1
1*(-b)^(31/2)*c^7 + 48*a^12*(-b)^(29/2)*c^8 - 1536*a^13*(-b)^(29/2)*c^7 + 1
152*a^10*(-b)^(33/2)*c^7 + 304*a^11*(-b)^(31/2)*c^8 - 1024*a^12*(-b)^(31/2)
*c^7 + 272*a^10*(-b)^(33/2)*c^8 + 512*a^11*(-b)^(33/2)*c^7 + 512*a^12*(-b)^
(31/2)*c^8 + 512*a^11*(-b)^(33/2)*c^8 + 256*a^13*(-b)^(31/2)*c^8 + 256*a^12
*(-b)^(33/2)*c^8 - 128*a^13*(-b)^(13/2)*c + 512*a^12*(-b)^(15/2)*c + 768*a^
11*(-b)^(17/2)*c + 896*a^13*(-b)^(15/2)*c - 1536*a^10*(-b)^(19/2)*c - 384*a
^12*(-b)^(17/2)*c - 4736*a^9*(-b)^(21/2)*c - 5504*a^11*(-b)^(19/2)*c - 1536
a^13(-b)^(17/2)*c - 3072*a^8*(-b)^(23/2)*c - 10368*a^10*(-b)^(21/2)*c - 4
096*a^12*(-b)^(19/2)*c - 6144*a^9*(-b)^(23/2)*c - 5632*a^11*(-b)^(21/2)*c -
3072*a^10*(-b)^(23/2)*c))/(a^2*(-b)^(9/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^
8*b^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a
^12*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9)))*(b*((-
b)^(3/4) + a*((-b)^(3/4) + ((-b)^(3/4)*c)/4)) + (a*(-b)^(3/4))/4)^3)/(a^6*b
^6))*1i)/(a^2*b^2) + ((b*((-b)^(3/4) + a*((-b)^(3/4) + ((-b)^(3/4)*c)/4)) +
(a*(-b)^(3/4))/4)*((64*(-(b*x - 1)/(c + x))^(1/4)*(512*(-b)^(19/2) + a^5*(
-b)^(9/2) + a^4*(-b)^(11/2) + 2*a^6*(-b)^(9/2) - 16*a^3*(-b)^(13/2) + 18*a^
5*(-b)^(11/2) + a^7*(-b)^(9/2) - 144*a^2*(-b)^(15/2) - 176*a^4*(-b)^(13/2)
+ 49*a^6*(-b)^(11/2) - 576*a^3*(-b)^(15/2) - 560*a^5*(-b)^(13/2) + 48*a^7*(

-b)^(11/2) + 2688*a^2*(-b)^(17/2) - 608*a^4*(-b)^(15/2) - 784*a^6*(-b)^(13/2) + 16*a^8*(-b)^(11/2) + 7680*a^3*(-b)^(17/2) + 448*a^5*(-b)^(15/2) - 512*a^7*(-b)^(13/2) + 7680*a^2*(-b)^(19/2) + 11520*a^4*(-b)^(17/2) + 1392*a^6*(-b)^(15/2) - 128*a^8*(-b)^(13/2) + 10240*a^3*(-b)^(19/2) + 9600*a^5*(-b)^(17/2) + 1024*a^7*(-b)^(15/2) + 7680*a^4*(-b)^(19/2) + 4224*a^6*(-b)^(17/2) + 256*a^8*(-b)^(15/2) + 3072*a^5*(-b)^(19/2) + 768*a^7*(-b)^(17/2) + 512*a^6*(-b)^(19/2) + 7680*(-b)^(23/2)*c^2 - 10240*(-b)^(25/2)*c^3 + 7680*(-b)^(27/2)*c^4 - 3072*(-b)^(29/2)*c^5 + 512*(-b)^(31/2)*c^6 + 384*a*(-b)^(17/2) + 3072*a*(-b)^(19/2) - 3072*(-b)^(21/2)*c + a^7*(-b)^(9/2)*c^2 - 35*a^6*(-b)^(11/2)*c^2 + 2*a^8*(-b)^(9/2)*c^2 + 265*a^5*(-b)^(13/2)*c^2 - 86*a^7*(-b)^(11/2)*c^2 + a^9*(-b)^(9/2)*c^2 - 851*a^4*(-b)^(15/2)*c^2 + 738*a^6*(-b)^(13/2)*c^2 - 10*a^7*(-b)^(11/2)*c^3 - 67*a^8*(-b)^(11/2)*c^2 + 2496*a^3*(-b)^(17/2)*c^2 - 2566*a^5*(-b)^(15/2)*c^2 + 224*a^6*(-b)^(13/2)*c^3 + 649*a^7*(-b)^(13/2)*c^2 - 20*a^8*(-b)^(11/2)*c^3 - 16*a^9*(-b)^(11/2)*c^2 - 5184*a^2*(-b)^(19/2)*c^2 + 10432*a^4*(-b)^(17/2)*c^2 - 1358*a^5*(-b)^(15/2)*c^3 - 1907*a^6*(-b)^(15/2)*c^2 + 592*a^7*(-b)^(13/2)*c^3 + 144*a^8*(-b)^(13/2)*c^2 - 10*a^9*(-b)^(11/2)*c^3 - 31104*a^3*(-b)^(19/2)*c^2 + 3784*a^4*(-b)^(17/2)*c^3 + 14912*a^5*(-b)^(17/2)*c^2 - 4364*a^6*(-b)^(15/2)*c^3 + 45*a^7*(-b)^(13/2)*c^4 + 1120*a^7*(-b)^(15/2)*c^2 + 512*a^8*(-b)^(13/2)*c^3 - 32*a^9*(-b)^(13/2)*c^2 + 1152*a^2*(-b)^(21/2)*c^2 - 7552*a^3*(-b)^(19/2)*c^3 - 74624*a^4*(-b)^(19/2)*c^2 + 14288*a^5*(-b)^(17/2)*c^3 - 771*a^6*(-b)^(15/2)*c^4 + 5824*a^6*(-b)^(17/2)*c^2 - 4974*a^7*(-b)^(15/2)*c^3 + 90*a^8*(-b)^(13/2)*c^4 + 1952*a^8*(-b)^(15/2)*c^2 + 144*a^9*(-b)^(13/2)*c^3 + 6912*a^2*(-b)^(21/2)*c^3 + 23040*a^3*(-b)^(21/2)*c^2 - 36800*a^4*(-b)^(19/2)*c^3 + 3874*a^5*(-b)^(17/2)*c^4 - 91136*a^5*(-b)^(19/2)*c^2 + 19144*a^6*(-b)^(17/2)*c^3 - 2118*a^7*(-b)^(15/2)*c^4 - 5120*a^7*(-b)^(17/2)*c^2 - 2288*a^8*(-b)^(15/2)*c^3 + 45*a^9*(-b)^(13/2)*c^4 + 640*a^9*(-b)^(15/2)*c^2 + 115200*a^2*(-b)^(23/2)*c^2 + 49536*a^3*(-b)^(21/2)*c^3 - 8750*a^4*(-b)^(19/2)*c^4 + 57600*a^4*(-b)^(21/2)*c^2 - 68032*a^5*(-b)^(19/2)*c^3 + 13444*a^6*(-b)^(17/2)*c^4 - 58944*a^6*(-b)^(19/2)*c^2 - 120*a^7*(-b)^(15/2)*c^5 + 9664*a^7*(-b)^(17/2)*c^3 - 1923*a^8*(-b)^(15/2)*c^4 - 5248*a^8*(-b)^(17/2)*c^2 - 320*a^9*(-b)^(15/2)*c^3 + 44160*a^2*(-b)^(23/2)*c^3 + 11040*a^3*(-b)^(21/2)*c^4 + 153600*a^3*(-b)^(23/2)*c^2 + 137728*a^4*(-b)^(21/2)*c^3 - 36988*a^5*(-b)^(19/2)*c^4 + 63360*a^5*(-b)^(21/2)*c^2 + 1644*a^6*(-b)^(17/2)*c^5 - 56512*a^6*(-b)^(19/2)*c^3 + 17058*a^7*(-b)^(17/2)*c^4 - 18560*a^7*(-b)^(19/2)*c^2 - 240*a^8*(-b)^(15/2)*c^5 + 128*a^8*(-b)^(17/2)*c^3 - 576*a^9*(-b)^(15/2)*c^4 - 1280*a^9*(-b)^(17/2)*c^2 - 480*a^2*(-b)^(23/2)*c^4 + 76800*a^3*(-b)^(23/2)*c^3 + 60640*a^4*(-b)^(21/2)*c^4 + 115200*a^4*(-b)^(23/2)*c^2 - 6776*a^5*(-b)^(19/2)*c^5 + 193792*a^5*(-b)^(21/2)*c^3 - 59150*a^6*(-b)^(19/2)*c^4 + 33408*a^6*(-b)^(21/2)*c^2 + 4632*a^7*(-b)^(17/2)*c^5 - 16064*a^7*(-b)^(19/2)*c^3 + 9280*a^8*(-b)^(17/2)*c^4 - 2048*a^8*(-b)^(19/2)*c^2 - 120*a^9*(-b)^(15/2)*c^5 - 896*a^9*(-b)^(17/2)*c^3 - 153600*a^2*(-b)^(25/2)*c^3 - 23040*a^3*(-b)^(23/2)*c^4 + 11620*a^4*(-b)^(21/2)*c^5 + 57600*a^4*(-b)^(23/2)*c^3 + 128864*a^5*(-b)^(21/2)*c^4 + 46080*a^5*(-b)^(23/2)*c^2 - 24752*a^6*(-b)^(19/2)*c^5 + 146688*a^6*(-b)^(21/2)*c^3 + 210*a^7*(-b)^(17/2)*c^6 - 43232*a^7*(-b)^(19/2)*c^4 + 6912*a^7*(-b)^(21/2)*c^2 + 4332*a^8*(-b)^(17/2)*c^5 + 3968*a^8*(-b)^(19/2)*c^3 + 1792*a^9*(-b)^(17/2)*c^4 - 90240*a^2*(-b)^(25/2)*c^4 - 7104*a^3*(-b)^(23/2)*c^5 - 204800*a^3*(-b)^(25/2)*c^3 - 100160*a^4*(-b)^(23/2)*c^4 + 53480*a^5*(-b)^(21/2)*c^5 + 9600*a^5*(-b)^(23/2)*c^3 - 2310*a^6*(-b)^(19/2)*c^6 + 131872*a^6*(-b)^(21/2)*c^4 + 7680*a^6*(-b)^(23/2)*c^2 - 33432*a^7*(-b)^(19/2)*c^5 + 56704*a^7*(-b)^(21/2)*c^3 + 420*a^8*(-b)^(17/2)*c^6 - 13216*a^8*(-b)^(19/2)*c^4 + 1344*a^9*(-b)^(17/2)*c^5 + 2304*a^9*(-b)^(19/2)*c^3 - 9216*a^2*(-b)^(25/2)*c^5 - 192000*a^3*(-b)^(25/2)*c^4 - 48224*a^4*(-b)^(23/2)*c^5 - 153600*a^4*(-b)^(25/2)*c^3 + 7546*a^5*(-b)^(21/2)*c^6 - 179840*a^5*(-b)^(23/2)*c^4 + 94948*a^6*(-b)^(21/2)*c^5 - 9600*a^6*(-b)^(23/2)*c^3 - 6636*a^7*(-b)^(19/2)*c^6 + 63232*a^7*(-b)^(21/2)*c^4 - 19712*a^8*(-b)^(19/2)*c^5 + 8704*a^8*(-b)^(21/2)*c^3 + 210*a^9*(-b)^(17/2)*c^6 - 896*a^9*(-b)^(19/2)*c^4 + 115200*a^2*(-b)^(27/2)*c^4 - 31104*a^3*(-b)^(25/2)*c^5 - 8750*a^4*(-b)^(23/2)*c^6 - 211200*a^4*(-b)^(25/2)*c^4 - 121120*a^5*(-b)^(

23/2)*c^5 - 61440*a^5*(-b)^(25/2)*c^3 + 28756*a^6*(-b)^(21/2)*c^6 - 161760*
a^6*(-b)^(23/2)*c^4 - 252*a^7*(-b)^(19/2)*c^7 + 80416*a^7*(-b)^(21/2)*c^5 -
3840*a^7*(-b)^(23/2)*c^3 - 6342*a^8*(-b)^(19/2)*c^6 + 9344*a^8*(-b)^(21/2)
*c^4 - 4256*a^9*(-b)^(19/2)*c^5 + 81792*a^2*(-b)^(27/2)*c^5 - 832*a^3*(-b)^
(25/2)*c^6 + 153600*a^3*(-b)^(27/2)*c^4 - 23552*a^4*(-b)^(25/2)*c^5 - 44380
a^5(-b)^(23/2)*c^6 - 124800*a^5*(-b)^(25/2)*c^4 + 2184*a^6*(-b)^(21/2)*c^
7 - 146336*a^6*(-b)^(23/2)*c^5 - 10240*a^6*(-b)^(25/2)*c^3 + 40698*a^7*(-b)
^(21/2)*c^6 - 72320*a^7*(-b)^(23/2)*c^4 - 504*a^8*(-b)^(19/2)*c^7 + 31808*a
^8*(-b)^(21/2)*c^5 - 2016*a^9*(-b)^(19/2)*c^6 - 1280*a^9*(-b)^(21/2)*c^4 +
10944*a^2*(-b)^(27/2)*c^6 + 184320*a^3*(-b)^(27/2)*c^5 + 8896*a^4*(-b)^(25/
2)*c^6 + 115200*a^4*(-b)^(27/2)*c^4 - 5300*a^5*(-b)^(23/2)*c^7 + 32512*a^5*
(-b)^(25/2)*c^5 - 86702*a^6*(-b)^(23/2)*c^6 - 36480*a^6*(-b)^(25/2)*c^4 + 6
384*a^7*(-b)^(21/2)*c^7 - 87968*a^7*(-b)^(23/2)*c^5 + 25312*a^8*(-b)^(21/2)
*c^6 - 12800*a^8*(-b)^(23/2)*c^4 - 252*a^9*(-b)^(19/2)*c^7 + 4480*a^9*(-b)^
(21/2)*c^5 - 46080*a^2*(-b)^(29/2)*c^5 + 49536*a^3*(-b)^(27/2)*c^6 + 3016*a
^4*(-b)^(25/2)*c^7 + 218880*a^4*(-b)^(27/2)*c^5 + 44864*a^5*(-b)^(25/2)*c^6
+ 46080*a^5*(-b)^(27/2)*c^4 - 21128*a^6*(-b)^(23/2)*c^7 + 66048*a^6*(-b)^(
25/2)*c^5 + 210*a^7*(-b)^(21/2)*c^8 - 81536*a^7*(-b)^(23/2)*c^6 - 3840*a^7*
(-b)^(25/2)*c^4 + 6216*a^8*(-b)^(21/2)*c^7 - 22912*a^8*(-b)^(23/2)*c^5 + 58
24*a^9*(-b)^(21/2)*c^6 - 36480*a^2*(-b)^(29/2)*c^6 + 4224*a^3*(-b)^(27/2)*c
^7 - 61440*a^3*(-b)^(29/2)*c^5 + 86656*a^4*(-b)^(27/2)*c^6 + 18896*a^5*(-b)
^(25/2)*c^7 + 144000*a^5*(-b)^(27/2)*c^5 - 1374*a^6*(-b)^(23/2)*c^8 + 75200
a^6(-b)^(25/2)*c^6 + 7680*a^6*(-b)^(27/2)*c^4 - 31284*a^7*(-b)^(23/2)*c^7
+ 40576*a^7*(-b)^(25/2)*c^5 + 420*a^8*(-b)^(21/2)*c^8 - 36736*a^8*(-b)^(23
/2)*c^6 + 2016*a^9*(-b)^(21/2)*c^7 - 1280*a^9*(-b)^(23/2)*c^5 - 5376*a^2*(-
b)^(29/2)*c^7 - 84480*a^3*(-b)^(29/2)*c^6 + 13888*a^4*(-b)^(27/2)*c^7 - 460
80*a^4*(-b)^(29/2)*c^5 + 2173*a^5*(-b)^(25/2)*c^8 + 70144*a^5*(-b)^(27/2)*c
^6 + 42952*a^6*(-b)^(25/2)*c^7 + 49536*a^6*(-b)^(27/2)*c^5 - 4092*a^7*(-b)^
(23/2)*c^8 + 57856*a^7*(-b)^(25/2)*c^6 - 20384*a^8*(-b)^(23/2)*c^7 + 8704*a
^8*(-b)^(25/2)*c^5 + 210*a^9*(-b)^(21/2)*c^8 - 6272*a^9*(-b)^(23/2)*c^6 + 7
680*a^2*(-b)^(31/2)*c^6 - 26496*a^3*(-b)^(29/2)*c^7 + 301*a^4*(-b)^(27/2)*c
^8 - 103680*a^4*(-b)^(29/2)*c^6 + 12608*a^5*(-b)^(27/2)*c^7 - 18432*a^5*(-b
)^(29/2)*c^5 + 9274*a^6*(-b)^(25/2)*c^8 + 21696*a^6*(-b)^(27/2)*c^6 - 120*a
^7*(-b)^(23/2)*c^9 + 45760*a^7*(-b)^(25/2)*c^7 + 6912*a^7*(-b)^(27/2)*c^5 -
4062*a^8*(-b)^(23/2)*c^8 + 20096*a^8*(-b)^(25/2)*c^6 - 4928*a^9*(-b)^(23/2
)*c^7 + 6528*a^2*(-b)^(31/2)*c^7 - 2448*a^3*(-b)^(29/2)*c^8 + 10240*a^3*(-b
)^(31/2)*c^6 - 52736*a^4*(-b)^(29/2)*c^7 - 1558*a^5*(-b)^(27/2)*c^8 - 71040
a^5(-b)^(29/2)*c^6 + 546*a^6*(-b)^(25/2)*c^9 - 4544*a^6*(-b)^(27/2)*c^7 -
3072*a^6*(-b)^(29/2)*c^5 + 14589*a^7*(-b)^(25/2)*c^8 - 2432*a^7*(-b)^(27/2
)*c^6 - 240*a^8*(-b)^(23/2)*c^9 + 23168*a^8*(-b)^(25/2)*c^7 - 1344*a^9*(-b)
^(23/2)*c^8 + 2304*a^9*(-b)^(25/2)*c^6 + 1008*a^2*(-b)^(31/2)*c^8 + 15360*a
^3*(-b)^(31/2)*c^7 - 10160*a^4*(-b)^(29/2)*c^8 + 7680*a^4*(-b)^(31/2)*c^6 -
384*a^5*(-b)^(27/2)*c^9 - 53504*a^5*(-b)^(29/2)*c^7 - 8099*a^6*(-b)^(27/2)
*c^8 - 25728*a^6*(-b)^(29/2)*c^6 + 1668*a^7*(-b)^(25/2)*c^9 - 13760*a^7*(-b
)^(27/2)*c^7 + 10048*a^8*(-b)^(25/2)*c^8 - 2048*a^8*(-b)^(27/2)*c^6 - 120*a
^9*(-b)^(23/2)*c^9 + 4480*a^9*(-b)^(25/2)*c^7 + 5184*a^3*(-b)^(31/2)*c^8 -
570*a^4*(-b)^(29/2)*c^9 + 19200*a^4*(-b)^(31/2)*c^7 - 16048*a^5*(-b)^(29/2)
*c^8 + 3072*a^5*(-b)^(31/2)*c^6 - 1984*a^6*(-b)^(27/2)*c^9 - 28416*a^6*(-b)
^(29/2)*c^7 + 45*a^7*(-b)^(25/2)*c^10 - 11984*a^7*(-b)^(27/2)*c^8 - 3840*a^
7*(-b)^(29/2)*c^6 + 1698*a^8*(-b)^(25/2)*c^9 - 7552*a^8*(-b)^(27/2)*c^7 + 2
560*a^9*(-b)^(25/2)*c^8 + 480*a^3*(-b)^(31/2)*c^9 + 10912*a^4*(-b)^(31/2)*c
^8 - 1732*a^5*(-b)^(29/2)*c^9 + 13440*a^5*(-b)^(31/2)*c^7 - 119*a^6*(-b)^(2
7/2)*c^10 - 11408*a^6*(-b)^(29/2)*c^8 + 512*a^6*(-b)^(31/2)*c^6 - 3568*a^7*
(-b)^(27/2)*c^9 - 7040*a^7*(-b)^(29/2)*c^7 + 90*a^8*(-b)^(25/2)*c^10 - 7408
a^8(-b)^(27/2)*c^8 + 576*a^9*(-b)^(25/2)*c^9 - 1280*a^9*(-b)^(27/2)*c^7 +
2160*a^4*(-b)^(31/2)*c^9 - 35*a^5*(-b)^(29/2)*c^10 + 11968*a^5*(-b)^(31/2)
*c^8 - 1530*a^6*(-b)^(29/2)*c^9 + 4992*a^6*(-b)^(31/2)*c^7 - 382*a^7*(-b)^(
27/2)*c^10 - 2816*a^7*(-b)^(29/2)*c^8 - 2720*a^8*(-b)^(27/2)*c^9 - 512*a^8*
(-b)^(29/2)*c^7 + 45*a^9*(-b)^(25/2)*c^10 - 1664*a^9*(-b)^(27/2)*c^8 + 129*

a^4*(-b)^(31/2)*c^10 + 3856*a^5*(-b)^(31/2)*c^9 + 10*a^6*(-b)^(29/2)*c^10 + 7152*a^6*(-b)^(31/2)*c^8 - 10*a^7*(-b)^(27/2)*c^11 + 112*a^7*(-b)^(29/2)*c^9 + 768*a^7*(-b)^(31/2)*c^7 - 407*a^8*(-b)^(27/2)*c^10 + 512*a^8*(-b)^(29/2)*c^8 - 752*a^9*(-b)^(27/2)*c^9 + 482*a^5*(-b)^(31/2)*c^10 + 8*a^6*(-b)^(29/2)*c^11 + 3408*a^6*(-b)^(31/2)*c^9 + 221*a^7*(-b)^(29/2)*c^10 + 2176*a^7*(-b)^(31/2)*c^8 - 20*a^8*(-b)^(27/2)*c^11 + 736*a^8*(-b)^(29/2)*c^9 - 144*a^9*(-b)^(27/2)*c^10 + 256*a^9*(-b)^(29/2)*c^8 + 18*a^5*(-b)^(31/2)*c^11 + 673*a^6*(-b)^(31/2)*c^10 + 32*a^7*(-b)^(29/2)*c^11 + 1488*a^7*(-b)^(31/2)*c^9 + 272*a^8*(-b)^(29/2)*c^10 + 256*a^8*(-b)^(31/2)*c^8 - 10*a^9*(-b)^(27/2)*c^11 + 256*a^9*(-b)^(29/2)*c^9 + 52*a^6*(-b)^(31/2)*c^11 + a^7*(-b)^(29/2)*c^12 + 416*a^7*(-b)^(31/2)*c^10 + 40*a^8*(-b)^(29/2)*c^11 + 256*a^8*(-b)^(31/2)*c^9 + 96*a^9*(-b)^(29/2)*c^10 + a^6*(-b)^(31/2)*c^12 + 50*a^7*(-b)^(31/2)*c^11 + 2*a^8*(-b)^(29/2)*c^12 + 96*a^8*(-b)^(31/2)*c^10 + 16*a^9*(-b)^(29/2)*c^11 + 2*a^7*(-b)^(31/2)*c^12 + 16*a^8*(-b)^(31/2)*c^11 + a^9*(-b)^(29/2)*c^12 + a^8*(-b)^(31/2)*c^12 - 1152*a*(-b)^(19/2)*c - 18432*a*(-b)^(21/2)*c + 2*a^6*(-b)^(9/2)*c - 24*a^5*(-b)^(11/2)*c + 4*a^7*(-b)^(9/2)*c + 70*a^4*(-b)^(13/2)*c - 48*a^6*(-b)^(11/2)*c + 2*a^8*(-b)^(9/2)*c - 288*a^3*(-b)^(15/2)*c + 60*a^5*(-b)^(13/2)*c - 8*a^7*(-b)^(11/2)*c + 1536*a^2*(-b)^(17/2)*c - 656*a^4*(-b)^(15/2)*c - 378*a^6*(-b)^(13/2)*c + 32*a^8*(-b)^(11/2)*c + 8064*a^3*(-b)^(17/2)*c + 656*a^5*(-b)^(15/2)*c - 784*a^7*(-b)^(13/2)*c + 16*a^9*(-b)^(11/2)*c - 9600*a^2*(-b)^(19/2)*c + 16384*a^4*(-b)^(17/2)*c + 3280*a^6*(-b)^(15/2)*c - 544*a^8*(-b)^(13/2)*c - 30720*a^3*(-b)^(19/2)*c + 15616*a^5*(-b)^(17/2)*c + 3664*a^7*(-b)^(15/2)*c - 128*a^9*(-b)^(13/2)*c - 1152*a*(-b)^(21/2)*c^2 - 46080*a^2*(-b)^(21/2)*c - 49920*a^4*(-b)^(19/2)*c + 6144*a^6*(-b)^(17/2)*c + 1664*a^8*(-b)^(15/2)*c - 61440*a^3*(-b)^(21/2)*c - 44160*a^5*(-b)^(19/2)*c - 128*a^7*(-b)^(17/2)*c + 256*a^9*(-b)^(15/2)*c + 46080*a*(-b)^(23/2)*c^2 - 46080*a^4*(-b)^(21/2)*c - 20352*a^6*(-b)^(19/2)*c - 512*a^8*(-b)^(17/2)*c + 9600*a*(-b)^(23/2)*c^3 - 18432*a^5*(-b)^(21/2)*c - 3840*a^7*(-b)^(19/2)*c - 3072*a^6*(-b)^(21/2)*c - 61440*a*(-b)^(25/2)*c^3 - 17280*a*(-b)^(25/2)*c^4 + 46080*a*(-b)^(27/2)*c^4 + 14976*a*(-b)^(27/2)*c^5 - 18432*a*(-b)^(29/2)*c^5 - 6528*a*(-b)^(29/2)*c^6 + 3072*a*(-b)^(31/2)*c^6 + 1152*a*(-b)^(31/2)*c^7))/((-b)^(1/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^8*b^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a^12*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9)) - (((64*(36*a^12*(-b)^(25/4) - 4*a^13*(-b)^(21/4) + 48*a^13*(-b)^(25/4) - 60*a^11*(-b)^(29/4) - 240*a^12*(-b)^(29/4) - 192*a^13*(-b)^(29/4) - 180*a^10*(-b)^(33/4) - 240*a^11*(-b)^(33/4) + 192*a^12*(-b)^(33/4) + 240*a^9*(-b)^(37/4) + 256*a^13*(-b)^(33/4) + 1200*a^10*(-b)^(37/4) + 1728*a^11*(-b)^(37/4) + 320*a^8*(-b)^(41/4) + 768*a^12*(-b)^(37/4) + 1152*a^9*(-b)^(41/4) + 1344*a^10*(-b)^(41/4) + 512*a^11*(-b)^(41/4) - 144*a^13*(-b)^(29/4)*c^2 + 912*a^12*(-b)^(33/4)*c^2 + 1344*a^13*(-b)^(33/4)*c^2 + 336*a^13*(-b)^(33/4)*c^3 - 432*a^11*(-b)^(37/4)*c^2 - 4032*a^12*(-b)^(37/4)*c^2 - 1680*a^12*(-b)^(37/4)*c^3 - 4032*a^13*(-b)^(37/4)*c^2 - 4624*a^10*(-b)^(41/4)*c^2 - 2688*a^13*(-b)^(37/4)*c^3 - 9408*a^11*(-b)^(41/4)*c^2 - 504*a^13*(-b)^(37/4)*c^4 - 336*a^11*(-b)^(41/4)*c^3 - 1344*a^12*(-b)^(41/4)*c^2 + 3584*a^9*(-b)^(45/4)*c^2 + 5376*a^12*(-b)^(41/4)*c^3 + 3584*a^13*(-b)^(41/4)*c^2 + 20160*a^10*(-b)^(45/4)*c^2 + 1848*a^12*(-b)^(41/4)*c^4 + 6720*a^13*(-b)^(41/4)*c^3 + 8848*a^10*(-b)^(45/4)*c^3 + 30912*a^11*(-b)^(45/4)*c^2 + 3360*a^13*(-b)^(41/4)*c^4 + 6720*a^8*(-b)^(49/4)*c^2 + 21504*a^11*(-b)^(45/4)*c^3 + 14336*a^12*(-b)^(45/4)*c^2 + 504*a^13*(-b)^(41/4)*c^5 + 24192*a^9*(-b)^(49/4)*c^2 + 1848*a^11*(-b)^(45/4)*c^4 + 9408*a^12*(-b)^(45/4)*c^3 - 4032*a^9*(-b)^(49/4)*c^3 + 28224*a^10*(-b)^(49/4)*c^2 - 3360*a^12*(-b)^(45/4)*c^4 - 3584*a^13*(-b)^(45/4)*c^3 - 26880*a^10*(-b)^(49/4)*c^3 + 10752*a^11*(-b)^(49/4)*c^2 - 1176*a^12*(-b)^(45/4)*c^5 - 6720*a^13*(-b)^(45/4)*c^4 - 10584*a^10*(-b)^(49/4)*c^4 - 44352*a^11*(-b)^(49/4)*c^3 - 2688*a^13*(-b)^(45/4)*c^5 - 11200*a^8*(-b)^(53/4)*c^3 - 30240*a^11*(-b)^(49/4)*c^4 - 21504*a^12*(-b)^(49/4)*c^3 - 336*a^13*(-b)^(45/4)*c^6 - 40320*a^9*(-b)^(53/4)*c^3 - 2520*a^11*(-b)^(49/4)*c^5 - 20160*a^12*(-b)^(49/4)*c^4 + 1120*a^9*(-b)^(53/4)*c^4 - 47040*a^10*(-b)^(53/4)*c^3 + 16800*a^10*(-b)^(53/4)*c^4 - 17920*a^11*(-b)^(53/4)*c^3

+ 336*a^12*(-b)^(49/4)*c^6 + 4032*a^13*(-b)^(49/4)*c^5 + 8120*a^10*(-b)^(53/4)*c^5 + 33600*a^11*(-b)^(53/4)*c^4 + 1344*a^13*(-b)^(49/4)*c^6 + 11200*a^8*(-b)^(57/4)*c^4 + 26880*a^11*(-b)^(53/4)*c^5 + 17920*a^12*(-b)^(53/4)*c^4 + 144*a^13*(-b)^(49/4)*c^7 + 40320*a^9*(-b)^(57/4)*c^4 + 1680*a^11*(-b)^(53/4)*c^6 + 22848*a^12*(-b)^(53/4)*c^5 + 2240*a^9*(-b)^(57/4)*c^5 + 47040*a^10*(-b)^(57/4)*c^4 + 1344*a^12*(-b)^(53/4)*c^6 + 3584*a^13*(-b)^(53/4)*c^5 + 17920*a^11*(-b)^(57/4)*c^4 + 48*a^12*(-b)^(53/4)*c^7 - 1344*a^13*(-b)^(53/4)*c^6 - 3920*a^10*(-b)^(57/4)*c^6 - 9408*a^11*(-b)^(57/4)*c^5 - 384*a^13*(-b)^(53/4)*c^7 - 6720*a^8*(-b)^(61/4)*c^5 - 14784*a^11*(-b)^(57/4)*c^6 - 7168*a^12*(-b)^(57/4)*c^5 - 36*a^13*(-b)^(53/4)*c^8 - 24192*a^9*(-b)^(61/4)*c^5 - 528*a^11*(-b)^(57/4)*c^7 - 14784*a^12*(-b)^(57/4)*c^6 - 2688*a^9*(-b)^(61/4)*c^6 - 28224*a^10*(-b)^(61/4)*c^5 - 768*a^12*(-b)^(57/4)*c^7 - 3584*a^13*(-b)^(57/4)*c^6 - 6720*a^10*(-b)^(61/4)*c^6 - 10752*a^11*(-b)^(61/4)*c^5 - 60*a^12*(-b)^(57/4)*c^8 + 192*a^13*(-b)^(57/4)*c^7 + 1104*a^10*(-b)^(61/4)*c^7 - 4032*a^11*(-b)^(61/4)*c^6 + 48*a^13*(-b)^(57/4)*c^8 + 2240*a^8*(-b)^(65/4)*c^6 + 4608*a^11*(-b)^(61/4)*c^7 + 4*a^13*(-b)^(57/4)*c^9 + 8064*a^9*(-b)^(65/4)*c^6 + 36*a^11*(-b)^(61/4)*c^8 + 5184*a^12*(-b)^(61/4)*c^7 + 1216*a^9*(-b)^(65/4)*c^7 + 9408*a^10*(-b)^(65/4)*c^6 + 144*a^12*(-b)^(61/4)*c^8 + 1536*a^13*(-b)^(61/4)*c^7 + 3840*a^10*(-b)^(65/4)*c^7 + 3584*a^11*(-b)^(65/4)*c^6 + 12*a^12*(-b)^(61/4)*c^9 - 148*a^10*(-b)^(65/4)*c^8 + 3648*a^11*(-b)^(65/4)*c^7 - 320*a^8*(-b)^(69/4)*c^7 - 624*a^11*(-b)^(65/4)*c^8 + 1024*a^12*(-b)^(65/4)*c^7 - 1152*a^9*(-b)^(69/4)*c^7 + 12*a^11*(-b)^(65/4)*c^9 - 768*a^12*(-b)^(65/4)*c^8 - 208*a^9*(-b)^(69/4)*c^8 - 1344*a^10*(-b)^(69/4)*c^7 - 256*a^13*(-b)^(65/4)*c^8 - 720*a^10*(-b)^(69/4)*c^8 - 512*a^11*(-b)^(69/4)*c^7 + 4*a^10*(-b)^(69/4)*c^9 - 768*a^11*(-b)^(69/4)*c^8 - 256*a^12*(-b)^(69/4)*c^8 + 36*a^13*(-b)^(25/4)*c - 276*a^12*(-b)^(29/4)*c - 384*a^13*(-b)^(29/4)*c + 300*a^11*(-b)^(33/4)*c + 1536*a^12*(-b)^(33/4)*c + 1344*a^13*(-b)^(33/4)*c + 1380*a^10*(-b)^(37/4)*c + 2304*a^11*(-b)^(37/4)*c - 576*a^12*(-b)^(37/4)*c - 1472*a^9*(-b)^(41/4)*c - 1536*a^13*(-b)^(37/4)*c - 7680*a^10*(-b)^(41/4)*c - 11328*a^11*(-b)^(41/4)*c - 2240*a^8*(-b)^(45/4)*c - 5120*a^12*(-b)^(41/4)*c - 8064*a^9*(-b)^(45/4)*c - 9408*a^10*(-b)^(45/4)*c - 3584*a^11*(-b)^(45/4)*c))/(a^7*b^18 + 9*a^8*b^18*c + 36*a^9*b^18*c^2 + 84*a^10*b^18*c^3 + 126*a^11*b^18*c^4 + 126*a^12*b^18*c^5 + 84*a^13*b^18*c^6 + 36*a^14*b^18*c^7 + 9*a^15*b^18*c^8 + a^16*b^18*c^9) + (64*(-(b*x - 1)/(c + x))^(1/4)*(b*((-b)^(3/4) + a*((-b)^(3/4) + ((-b)^(3/4)*c)/4)) + (a*(-b)^(3/4))/4)*(16*a^13*(-b)^(11/2) - 80*a^12*(-b)^(13/2) - 80*a^11*(-b)^(15/2) - 128*a^13*(-b)^(13/2) + 400*a^10*(-b)^(17/2) + 128*a^12*(-b)^(15/2) + 896*a^9*(-b)^(19/2) + 1152*a^11*(-b)^(17/2) + 256*a^13*(-b)^(15/2) + 512*a^8*(-b)^(21/2) + 1920*a^10*(-b)^(19/2) + 768*a^12*(-b)^(17/2) + 1024*a^9*(-b)^(21/2) + 1024*a^11*(-b)^(19/2) + 512*a^10*(-b)^(21/2) + 448*a^13*(-b)^(15/2)*c^2 - 1344*a^12*(-b)^(17/2)*c^2 - 2624*a^11*(-b)^(19/2)*c^2 - 2688*a^13*(-b)^(17/2)*c^2 + 1088*a^10*(-b)^(21/2)*c^2 + 128*a^12*(-b)^(19/2)*c^2 - 896*a^13*(-b)^(17/2)*c^3 + 9600*a^9*(-b)^(23/2)*c^2 + 9344*a^11*(-b)^(21/2)*c^2 + 1792*a^12*(-b)^(19/2)*c^3 + 4096*a^13*(-b)^(19/2)*c^2 + 7680*a^8*(-b)^(25/2)*c^2 + 21888*a^10*(-b)^(23/2)*c^2 + 4096*a^11*(-b)^(21/2)*c^3 + 8704*a^12*(-b)^(21/2)*c^2 + 4480*a^13*(-b)^(19/2)*c^3 + 15360*a^9*(-b)^(25/2)*c^2 + 3328*a^10*(-b)^(23/2)*c^3 + 12288*a^11*(-b)^(23/2)*c^2 + 128*a^12*(-b)^(21/2)*c^3 + 1120*a^13*(-b)^(19/2)*c^4 - 8320*a^9*(-b)^(25/2)*c^3 + 7680*a^10*(-b)^(25/2)*c^2 - 4992*a^11*(-b)^(23/2)*c^3 - 1120*a^12*(-b)^(21/2)*c^4 - 6656*a^13*(-b)^(21/2)*c^3 - 10240*a^8*(-b)^(27/2)*c^3 - 21120*a^10*(-b)^(25/2)*c^3 - 2400*a^11*(-b)^(23/2)*c^4 - 9216*a^12*(-b)^(23/2)*c^3 - 4480*a^13*(-b)^(21/2)*c^4 - 20480*a^9*(-b)^(27/2)*c^3 - 7200*a^10*(-b)^(25/2)*c^4 - 12800*a^11*(-b)^(25/2)*c^3 + 1920*a^12*(-b)^(23/2)*c^4 - 896*a^13*(-b)^(21/2)*c^5 + 640*a^9*(-b)^(27/2)*c^4 - 10240*a^10*(-b)^(27/2)*c^3 - 3200*a^11*(-b)^(25/2)*c^4 + 7680*a^13*(-b)^(23/2)*c^4 + 7680*a^8*(-b)^(29/2)*c^4 + 5760*a^10*(-b)^(27/2)*c^4 - 1280*a^11*(-b)^(25/2)*c^5 + 5120*a^12*(-b)^(25/2)*c^4 + 2688*a^13*(-b)^(23/2)*c^5 + 15360*a^9*(-b)^(29/2)*c^4 + 5120*a^10*(-b)^(27/2)*c^5 + 5120*a^11*(-b)^(27/2)*c^4 - 5248*a^12*(-b)^(25/2)*c^5 + 448*a^13*(-b)^(23/2)*c^6 + 4224*a^9*(-b)^(29/2)*c^5 + 7680*a^10*(-b)^(29/2)*c

^4 + 3968*a^11*(-b)^(27/2)*c^5 + 448*a^12*(-b)^(25/2)*c^6 - 6656*a^13*(-b)^(25/2)*c^5 - 3072*a^8*(-b)^(31/2)*c^5 + 5760*a^10*(-b)^(29/2)*c^5 + 2752*a^11*(-b)^(27/2)*c^6 - 2048*a^12*(-b)^(27/2)*c^5 - 896*a^13*(-b)^(25/2)*c^6 - 6144*a^9*(-b)^(31/2)*c^5 - 704*a^10*(-b)^(29/2)*c^6 + 1536*a^11*(-b)^(29/2)*c^5 + 5504*a^12*(-b)^(27/2)*c^6 - 128*a^13*(-b)^(25/2)*c^7 - 2944*a^9*(-b)^(31/2)*c^6 - 3072*a^10*(-b)^(31/2)*c^5 + 384*a^11*(-b)^(29/2)*c^6 - 256*a^12*(-b)^(27/2)*c^7 + 4096*a^13*(-b)^(27/2)*c^6 + 512*a^8*(-b)^(33/2)*c^6 - 4992*a^10*(-b)^(31/2)*c^6 - 1536*a^11*(-b)^(29/2)*c^7 + 1536*a^12*(-b)^(29/2)*c^6 + 128*a^13*(-b)^(27/2)*c^7 + 1024*a^9*(-b)^(33/2)*c^6 - 768*a^10*(-b)^(31/2)*c^7 - 2048*a^11*(-b)^(31/2)*c^6 - 2688*a^12*(-b)^(29/2)*c^7 + 16*a^13*(-b)^(27/2)*c^8 + 640*a^9*(-b)^(33/2)*c^7 + 512*a^10*(-b)^(33/2)*c^6 - 1664*a^11*(-b)^(31/2)*c^7 + 48*a^12*(-b)^(29/2)*c^8 - 1536*a^13*(-b)^(29/2)*c^7 + 1152*a^10*(-b)^(33/2)*c^7 + 304*a^11*(-b)^(31/2)*c^8 - 1024*a^12*(-b)^(31/2)*c^7 + 272*a^10*(-b)^(33/2)*c^8 + 512*a^11*(-b)^(33/2)*c^7 + 512*a^12*(-b)^(31/2)*c^8 + 512*a^11*(-b)^(33/2)*c^8 + 256*a^13*(-b)^(31/2)*c^8 + 256*a^12*(-b)^(33/2)*c^8 - 128*a^13*(-b)^(13/2)*c + 512*a^12*(-b)^(15/2)*c + 768*a^11*(-b)^(17/2)*c + 896*a^13*(-b)^(15/2)*c - 1536*a^10*(-b)^(19/2)*c - 384*a^12*(-b)^(17/2)*c - 4736*a^9*(-b)^(21/2)*c - 5504*a^11*(-b)^(19/2)*c - 1536*a^13*(-b)^(17/2)*c - 3072*a^8*(-b)^(23/2)*c - 10368*a^10*(-b)^(21/2)*c - 4096*a^12*(-b)^(19/2)*c - 6144*a^9*(-b)^(23/2)*c - 5632*a^11*(-b)^(21/2)*c - 3072*a^10*(-b)^(23/2)*c))/(a^2*(-b)^(9/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^8*b^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a^12*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9)))*(b*((-b)^(3/4) + a*((-b)^(3/4) + ((-b)^(3/4)*c)/4)) + (a*(-b)^(3/4))/4)^3)/(a^6*b^6))*1i)/(a^2*b^2))/((128*(5*a^4*(-b)^(21/4) - 320*(-b)^(37/4) + 19*a^5*(-b)^(21/4) + 27*a^6*(-b)^(21/4) - 55*a^3*(-b)^(25/4) + 17*a^7*(-b)^(21/4) - 269*a^4*(-b)^(25/4) + 4*a^8*(-b)^(21/4) - 525*a^5*(-b)^(25/4) + 180*a^2*(-b)^(29/4) - 511*a^6*(-b)^(25/4) + 1104*a^3*(-b)^(29/4) - 248*a^7*(-b)^(25/4) + 2808*a^4*(-b)^(29/4) - 48*a^8*(-b)^(25/4) + 3792*a^5*(-b)^(29/4) - 784*a^2*(-b)^(33/4) + 2868*a^6*(-b)^(29/4) - 2976*a^3*(-b)^(33/4) + 1152*a^7*(-b)^(29/4) - 5920*a^4*(-b)^(33/4) + 192*a^8*(-b)^(29/4) - 6800*a^5*(-b)^(33/4) - 6336*a^2*(-b)^(37/4) - 4560*a^6*(-b)^(33/4) - 10240*a^3*(-b)^(37/4) - 1664*a^7*(-b)^(33/4) - 9920*a^4*(-b)^(37/4) - 256*a^8*(-b)^(33/4) - 5760*a^5*(-b)^(37/4) - 1856*a^6*(-b)^(37/4) - 256*a^7*(-b)^(37/4) - 6720*(-b)^(45/4)*c^2 + 11200*(-b)^(49/4)*c^3 - 11200*(-b)^(53/4)*c^4 + 6720*(-b)^(57/4)*c^5 - 2240*(-b)^(61/4)*c^6 + 320*(-b)^(65/4)*c^7 - 80*a*(-b)^(33/4) - 2176*a*(-b)^(37/4) + 2240*(-b)^(41/4)*c + a^6*(-b)^(21/4)*c^2 + 3*a^7*(-b)^(21/4)*c^2 + 3*a^8*(-b)^(21/4)*c^2 - 71*a^5*(-b)^(25/4)*c^2 + a^9*(-b)^(21/4)*c^2 - 265*a^6*(-b)^(25/4)*c^2 - 10*a^6*(-b)^(25/4)*c^3 - 369*a^7*(-b)^(25/4)*c^2 + 849*a^4*(-b)^(29/4)*c^2 - 30*a^7*(-b)^(25/4)*c^3 - 227*a^8*(-b)^(25/4)*c^2 + 3851*a^5*(-b)^(29/4)*c^2 - 30*a^8*(-b)^(25/4)*c^3 - 52*a^9*(-b)^(25/4)*c^2 + 368*a^5*(-b)^(29/4)*c^3 + 6939*a^6*(-b)^(29/4)*c^2 - 10*a^9*(-b)^(25/4)*c^3 - 3607*a^3*(-b)^(33/4)*c^2 + 1392*a^6*(-b)^(29/4)*c^3 + 6201*a^7*(-b)^(29/4)*c^2 - 19689*a^4*(-b)^(33/4)*c^2 + 45*a^6*(-b)^(29/4)*c^4 + 1968*a^7*(-b)^(29/4)*c^3 + 2744*a^8*(-b)^(29/4)*c^2 - 3198*a^4*(-b)^(33/4)*c^3 - 44241*a^5*(-b)^(33/4)*c^2 + 135*a^7*(-b)^(29/4)*c^4 + 1232*a^8*(-b)^(29/4)*c^3 + 480*a^9*(-b)^(29/4)*c^2 + 4880*a^2*(-b)^(37/4)*c^2 - 14874*a^5*(-b)^(33/4)*c^3 - 52259*a^6*(-b)^(33/4)*c^2 + 135*a^8*(-b)^(29/4)*c^4 + 288*a^9*(-b)^(29/4)*c^3 + 33088*a^3*(-b)^(37/4)*c^2 - 1107*a^5*(-b)^(33/4)*c^4 - 27546*a^6*(-b)^(33/4)*c^3 - 34116*a^7*(-b)^(33/4)*c^2 + 45*a^9*(-b)^(29/4)*c^4 + 9976*a^3*(-b)^(37/4)*c^3 + 93600*a^4*(-b)^(37/4)*c^2 - 4233*a^6*(-b)^(33/4)*c^4 - 25374*a^7*(-b)^(33/4)*c^3 - 11616*a^8*(-b)^(33/4)*c^2 + 56280*a^4*(-b)^(37/4)*c^3 + 142912*a^5*(-b)^(37/4)*c^2 - 120*a^6*(-b)^(33/4)*c^5 - 6057*a^7*(-b)^(33/4)*c^4 - 11616*a^8*(-b)^(33/4)*c^3 - 1600*a^9*(-b)^(33/4)*c^2 + 11200*a^2*(-b)^(41/4)*c^2 + 7290*a^4*(-b)^(37/4)*c^4 + 131064*a^5*(-b)^(37/4)*c^3 + 126608*a^6*(-b)^(37/4)*c^2 - 360*a^7*(-b)^(33/4)*c^5 - 3843*a^8*(-b)^(33/4)*c^4 - 2112*a^9*(-b)^(33/4)*c^3 - 7952*a^2*(-b)^(41/4)*c^3 + 12096*a^3*(-b)^(41/4)*c^2 + 34566*a^5*(-b)^(37/4)*c^4 + 161096*a^6*(-b)^(37/4)*c^3 + 64512*a^7*(-b)^(37/4)*c^2 - 360*a^8*(-b)^(33/4)*c^5 - 9

12*a^9*(-b)^(33/4)*c^4 - 59136*a^3*(-b)^(41/4)*c^3 - 13440*a^4*(-b)^(41/4)*
c^2 + 2148*a^5*(-b)^(37/4)*c^5 + 65334*a^6*(-b)^(37/4)*c^4 + 110064*a^7*(-b
)^(37/4)*c^3 + 17216*a^8*(-b)^(37/4)*c^2 - 120*a^9*(-b)^(33/4)*c^5 - 16590*
a^3*(-b)^(41/4)*c^4 - 180768*a^4*(-b)^(41/4)*c^3 - 44800*a^5*(-b)^(41/4)*c^
2 + 8292*a^6*(-b)^(37/4)*c^5 + 61506*a^7*(-b)^(37/4)*c^4 + 39552*a^8*(-b)^(
37/4)*c^3 + 1792*a^9*(-b)^(37/4)*c^2 - 133056*a^2*(-b)^(45/4)*c^2 - 96810*a
^4*(-b)^(41/4)*c^4 - 296128*a^5*(-b)^(41/4)*c^3 + 210*a^6*(-b)^(37/4)*c^6 -
 44352*a^6*(-b)^(41/4)*c^2 + 11988*a^7*(-b)^(37/4)*c^5 + 28824*a^8*(-b)^(37
/4)*c^4 + 5824*a^9*(-b)^(37/4)*c^3 - 55552*a^2*(-b)^(45/4)*c^3 - 215040*a^3
*(-b)^(45/4)*c^2 - 10752*a^4*(-b)^(41/4)*c^5 - 233226*a^5*(-b)^(41/4)*c^4 -
 281232*a^6*(-b)^(41/4)*c^3 + 630*a^7*(-b)^(37/4)*c^6 - 20160*a^7*(-b)^(41/
4)*c^2 + 7692*a^8*(-b)^(37/4)*c^5 + 5376*a^9*(-b)^(37/4)*c^4 + 5208*a^2*(-b
)^(45/4)*c^4 - 119616*a^3*(-b)^(45/4)*c^3 - 208320*a^4*(-b)^(45/4)*c^2 - 51
912*a^5*(-b)^(41/4)*c^5 - 296814*a^6*(-b)^(41/4)*c^4 - 154560*a^7*(-b)^(41/
4)*c^3 + 630*a^8*(-b)^(37/4)*c^6 - 3584*a^8*(-b)^(41/4)*c^2 + 1848*a^9*(-b)
^(37/4)*c^5 + 51296*a^3*(-b)^(45/4)*c^4 - 125440*a^4*(-b)^(45/4)*c^3 - 2814
a^5(-b)^(41/4)*c^6 - 120960*a^5*(-b)^(45/4)*c^2 - 99960*a^6*(-b)^(41/4)*c
^5 - 210336*a^7*(-b)^(41/4)*c^4 - 45248*a^8*(-b)^(41/4)*c^3 + 210*a^9*(-b)^
(37/4)*c^6 + 16940*a^3*(-b)^(45/4)*c^5 + 185808*a^4*(-b)^(45/4)*c^4 - 56000
a^5(-b)^(45/4)*c^3 - 10962*a^6*(-b)^(41/4)*c^6 - 38976*a^6*(-b)^(45/4)*c^
2 - 95928*a^7*(-b)^(41/4)*c^5 - 78624*a^8*(-b)^(41/4)*c^4 - 5376*a^9*(-b)^(
41/4)*c^3 + 221760*a^2*(-b)^(49/4)*c^3 + 103404*a^4*(-b)^(45/4)*c^5 + 34339
2*a^5*(-b)^(45/4)*c^4 - 252*a^6*(-b)^(41/4)*c^7 + 5376*a^6*(-b)^(45/4)*c^3
 - 16002*a^7*(-b)^(41/4)*c^6 - 5376*a^7*(-b)^(45/4)*c^2 - 45864*a^8*(-b)^(41
/4)*c^5 - 12096*a^9*(-b)^(41/4)*c^4 + 110880*a^2*(-b)^(49/4)*c^4 + 358400*a
^3*(-b)^(49/4)*c^3 + 10458*a^4*(-b)^(45/4)*c^6 + 259644*a^5*(-b)^(45/4)*c^5
 + 359128*a^6*(-b)^(45/4)*c^4 - 756*a^7*(-b)^(41/4)*c^7 + 13888*a^7*(-b)^(4
5/4)*c^3 - 10374*a^8*(-b)^(41/4)*c^6 - 8736*a^9*(-b)^(41/4)*c^5 + 3080*a^2*
(-b)^(49/4)*c^5 + 268800*a^3*(-b)^(49/4)*c^4 + 347200*a^4*(-b)^(49/4)*c^3 +
 51534*a^5*(-b)^(45/4)*c^6 + 343588*a^6*(-b)^(45/4)*c^5 + 215040*a^7*(-b)^(
45/4)*c^4 - 756*a^8*(-b)^(41/4)*c^7 + 3584*a^8*(-b)^(45/4)*c^3 - 2520*a^9*(
-b)^(41/4)*c^6 - 3584*a^3*(-b)^(49/4)*c^5 + 347200*a^4*(-b)^(49/4)*c^4 + 25
20*a^5*(-b)^(45/4)*c^7 + 201600*a^5*(-b)^(49/4)*c^3 + 101262*a^6*(-b)^(45/4
)*c^6 + 252840*a^7*(-b)^(45/4)*c^5 + 68544*a^8*(-b)^(45/4)*c^4 - 252*a^9*(-
b)^(41/4)*c^7 - 9926*a^3*(-b)^(49/4)*c^6 - 71568*a^4*(-b)^(49/4)*c^5 + 2520
00*a^5*(-b)^(49/4)*c^4 + 9912*a^6*(-b)^(45/4)*c^7 + 64960*a^6*(-b)^(49/4)*c
^3 + 99162*a^7*(-b)^(45/4)*c^6 + 98112*a^8*(-b)^(45/4)*c^5 + 8960*a^9*(-b)^
(45/4)*c^4 - 221760*a^2*(-b)^(53/4)*c^4 - 65898*a^4*(-b)^(49/4)*c^6 - 19779
2*a^5*(-b)^(49/4)*c^5 + 210*a^6*(-b)^(45/4)*c^8 + 97440*a^6*(-b)^(49/4)*c^4
 + 14616*a^7*(-b)^(45/4)*c^7 + 8960*a^7*(-b)^(49/4)*c^3 + 48384*a^8*(-b)^(4
5/4)*c^6 + 15680*a^9*(-b)^(45/4)*c^5 - 121856*a^2*(-b)^(53/4)*c^5 - 358400*
a^3*(-b)^(53/4)*c^4 - 6564*a^4*(-b)^(49/4)*c^7 - 177114*a^5*(-b)^(49/4)*c^6
 - 254968*a^6*(-b)^(49/4)*c^5 + 630*a^7*(-b)^(45/4)*c^8 + 15680*a^7*(-b)^(4
9/4)*c^4 + 9576*a^8*(-b)^(45/4)*c^7 + 9408*a^9*(-b)^(45/4)*c^6 - 8624*a^2*(
-b)^(53/4)*c^6 - 310464*a^3*(-b)^(53/4)*c^5 - 347200*a^4*(-b)^(53/4)*c^4 -
33276*a^5*(-b)^(49/4)*c^7 - 248206*a^6*(-b)^(49/4)*c^6 - 175392*a^7*(-b)^(4
9/4)*c^5 + 630*a^8*(-b)^(45/4)*c^8 + 2352*a^9*(-b)^(45/4)*c^7 - 36288*a^3*(
-b)^(53/4)*c^6 - 430080*a^4*(-b)^(53/4)*c^5 - 1518*a^5*(-b)^(49/4)*c^8 - 20
1600*a^5*(-b)^(53/4)*c^4 - 67164*a^6*(-b)^(49/4)*c^7 - 192024*a^7*(-b)^(49/
4)*c^6 - 62272*a^8*(-b)^(49/4)*c^5 + 210*a^9*(-b)^(45/4)*c^8 + 2232*a^3*(-b
)^(53/4)*c^7 - 47712*a^4*(-b)^(53/4)*c^6 - 347200*a^5*(-b)^(53/4)*c^5 - 604
2*a^6*(-b)^(49/4)*c^8 - 64960*a^6*(-b)^(53/4)*c^4 - 67476*a^7*(-b)^(49/4)*c
^7 - 77952*a^8*(-b)^(49/4)*c^6 - 8960*a^9*(-b)^(49/4)*c^5 + 133056*a^2*(-b)
^(57/4)*c^5 + 20184*a^4*(-b)^(53/4)*c^7 + 4928*a^5*(-b)^(53/4)*c^6 - 120*a^
6*(-b)^(49/4)*c^9 - 161280*a^6*(-b)^(53/4)*c^5 - 9018*a^7*(-b)^(49/4)*c^8 -
 8960*a^7*(-b)^(53/4)*c^4 - 33744*a^8*(-b)^(49/4)*c^7 - 12992*a^9*(-b)^(49/
4)*c^6 + 77504*a^2*(-b)^(57/4)*c^6 + 215040*a^3*(-b)^(57/4)*c^5 + 2433*a^4*
(-b)^(53/4)*c^8 + 65016*a^5*(-b)^(53/4)*c^7 + 72912*a^6*(-b)^(53/4)*c^6 - 3
60*a^7*(-b)^(49/4)*c^9 - 38976*a^7*(-b)^(53/4)*c^5 - 5982*a^8*(-b)^(49/4)*c

^8 - 6720*a^9*(-b)^(49/4)*c^7 + 6960*a^2*(-b)^(57/4)*c^7 + 202944*a^3*(-b)^(57/4)*c^6 + 208320*a^4*(-b)^(57/4)*c^5 + 12999*a^5*(-b)^(53/4)*c^8 + 102984*a^6*(-b)^(53/4)*c^7 + 75264*a^7*(-b)^(53/4)*c^6 - 360*a^8*(-b)^(49/4)*c^9 - 3584*a^8*(-b)^(53/4)*c^5 - 1488*a^9*(-b)^(49/4)*c^8 + 35072*a^3*(-b)^(57/4)*c^7 + 291200*a^4*(-b)^(57/4)*c^6 + 582*a^5*(-b)^(53/4)*c^9 + 120960*a^5*(-b)^(57/4)*c^5 + 27471*a^6*(-b)^(53/4)*c^8 + 87216*a^7*(-b)^(53/4)*c^7 + 32704*a^8*(-b)^(53/4)*c^6 - 120*a^9*(-b)^(49/4)*c^9 + 869*a^3*(-b)^(57/4)*c^8 + 69600*a^4*(-b)^(57/4)*c^7 + 246400*a^5*(-b)^(57/4)*c^6 + 2358*a^6*(-b)^(53/4)*c^9 + 38976*a^6*(-b)^(57/4)*c^5 + 28749*a^7*(-b)^(53/4)*c^8 + 38016*a^8*(-b)^(53/4)*c^7 + 5376*a^9*(-b)^(53/4)*c^6 - 44352*a^2*(-b)^(61/4)*c^6 + 1335*a^4*(-b)^(57/4)*c^8 + 65088*a^5*(-b)^(57/4)*c^7 + 45*a^6*(-b)^(53/4)*c^10 + 122304*a^6*(-b)^(57/4)*c^6 + 3582*a^7*(-b)^(53/4)*c^9 + 5376*a^7*(-b)^(57/4)*c^5 + 14916*a^8*(-b)^(53/4)*c^8 + 6720*a^9*(-b)^(53/4)*c^7 - 26880*a^2*(-b)^(61/4)*c^7 - 71680*a^3*(-b)^(61/4)*c^6 - 380*a^4*(-b)^(57/4)*c^9 - 5289*a^5*(-b)^(57/4)*c^8 + 22192*a^6*(-b)^(57/4)*c^7 + 135*a^7*(-b)^(53/4)*c^10 + 32704*a^7*(-b)^(57/4)*c^6 + 2418*a^8*(-b)^(53/4)*c^9 + 3072*a^9*(-b)^(53/4)*c^8 - 2668*a^2*(-b)^(61/4)*c^8 - 71616*a^3*(-b)^(61/4)*c^7 - 69440*a^4*(-b)^(61/4)*c^6 - 2416*a^5*(-b)^(57/4)*c^9 - 17171*a^6*(-b)^(57/4)*c^8 - 7872*a^7*(-b)^(57/4)*c^7 + 135*a^8*(-b)^(53/4)*c^10 + 3584*a^8*(-b)^(57/4)*c^6 + 612*a^9*(-b)^(53/4)*c^9 - 14384*a^3*(-b)^(61/4)*c^8 - 104960*a^4*(-b)^(61/4)*c^7 - 123*a^5*(-b)^(57/4)*c^10 - 40320*a^5*(-b)^(61/4)*c^6 - 5784*a^6*(-b)^(57/4)*c^9 - 19464*a^7*(-b)^(57/4)*c^8 - 8256*a^8*(-b)^(57/4)*c^7 + 45*a^9*(-b)^(53/4)*c^10 - 670*a^3*(-b)^(61/4)*c^9 - 31752*a^4*(-b)^(61/4)*c^8 - 91200*a^5*(-b)^(61/4)*c^7 - 517*a^6*(-b)^(57/4)*c^10 - 12992*a^6*(-b)^(61/4)*c^6 - 6656*a^7*(-b)^(57/4)*c^9 - 10032*a^8*(-b)^(57/4)*c^8 - 1792*a^9*(-b)^(57/4)*c^7 + 6336*a^2*(-b)^(65/4)*c^7 - 2830*a^4*(-b)^(61/4)*c^9 - 36272*a^5*(-b)^(61/4)*c^8 - 10*a^6*(-b)^(57/4)*c^11 - 46848*a^6*(-b)^(61/4)*c^7 - 813*a^7*(-b)^(57/4)*c^10 - 1792*a^7*(-b)^(61/4)*c^6 - 3724*a^8*(-b)^(57/4)*c^9 - 1984*a^9*(-b)^(57/4)*c^8 + 3952*a^2*(-b)^(65/4)*c^8 + 10240*a^3*(-b)^(65/4)*c^7 - 43*a^4*(-b)^(61/4)*c^10 - 4374*a^5*(-b)^(61/4)*c^9 - 21868*a^6*(-b)^(61/4)*c^8 - 30*a^7*(-b)^(57/4)*c^11 - 13120*a^7*(-b)^(61/4)*c^7 - 567*a^8*(-b)^(57/4)*c^10 - 816*a^9*(-b)^(57/4)*c^9 + 412*a^2*(-b)^(65/4)*c^9 + 10656*a^3*(-b)^(65/4)*c^8 + 9920*a^4*(-b)^(65/4)*c^7 - 57*a^5*(-b)^(61/4)*c^10 - 2586*a^6*(-b)^(61/4)*c^9 - 5760*a^7*(-b)^(61/4)*c^8 - 30*a^8*(-b)^(57/4)*c^11 - 1536*a^8*(-b)^(61/4)*c^7 - 148*a^9*(-b)^(57/4)*c^10 + 2304*a^3*(-b)^(65/4)*c^9 + 15840*a^4*(-b)^(65/4)*c^8 + 8*a^5*(-b)^(61/4)*c^11 + 5760*a^5*(-b)^(65/4)*c^7 + 183*a^6*(-b)^(61/4)*c^10 + 236*a^7*(-b)^(61/4)*c^9 + 128*a^8*(-b)^(61/4)*c^8 - 10*a^9*(-b)^(57/4)*c^11 + 125*a^3*(-b)^(65/4)*c^10 + 5352*a^4*(-b)^(65/4)*c^9 + 14000*a^5*(-b)^(65/4)*c^8 + 40*a^6*(-b)^(61/4)*c^11 + 1856*a^6*(-b)^(65/4)*c^7 + 461*a^7*(-b)^(61/4)*c^10 + 864*a^8*(-b)^(61/4)*c^9 + 256*a^9*(-b)^(61/4)*c^8 + 595*a^4*(-b)^(65/4)*c^10 + 6608*a^5*(-b)^(65/4)*c^9 + a^6*(-b)^(61/4)*c^12 + 7344*a^6*(-b)^(65/4)*c^8 + 72*a^7*(-b)^(61/4)*c^11 + 256*a^7*(-b)^(65/4)*c^7 + 360*a^8*(-b)^(61/4)*c^10 + 256*a^9*(-b)^(61/4)*c^9 + 18*a^4*(-b)^(65/4)*c^11 + 1131*a^5*(-b)^(65/4)*c^10 + 4572*a^6*(-b)^(65/4)*c^9 + 3*a^7*(-b)^(61/4)*c^12 + 2112*a^7*(-b)^(65/4)*c^8 + 56*a^8*(-b)^(61/4)*c^11 + 96*a^9*(-b)^(61/4)*c^10 + 70*a^5*(-b)^(65/4)*c^11 + 1073*a^6*(-b)^(65/4)*c^10 + 1680*a^7*(-b)^(65/4)*c^9 + 3*a^8*(-b)^(61/4)*c^12 + 256*a^8*(-b)^(65/4)*c^8 + 16*a^9*(-b)^(61/4)*c^11 + a^5*(-b)^(65/4)*c^12 + 102*a^6*(-b)^(65/4)*c^11 + 508*a^7*(-b)^(65/4)*c^10 + 256*a^8*(-b)^(65/4)*c^9 + a^9*(-b)^(61/4)*c^12 + 3*a^6*(-b)^(65/4)*c^12 + 66*a^7*(-b)^(65/4)*c^11 + 96*a^8*(-b)^(65/4)*c^10 + 3*a^7*(-b)^(65/4)*c^12 + 16*a^8*(-b)^(65/4)*c^11 + a^8*(-b)^(65/4)*c^12 - 64*a*(-b)^(37/4)*c + 15232*a*(-b)^(41/4)*c + 6*a^5*(-b)^(21/4)*c + 22*a^6*(-b)^(21/4)*c + 30*a^7*(-b)^(21/4)*c - 116*a^4*(-b)^(25/4)*c + 18*a^8*(-b)^(21/4)*c - 504*a^5*(-b)^(25/4)*c + 4*a^9*(-b)^(21/4)*c - 864*a^6*(-b)^(25/4)*c + 706*a^3*(-b)^(29/4)*c - 728*a^7*(-b)^(25/4)*c + 3698*a^4*(-b)^(29/4)*c - 300*a^8*(-b)^(25/4)*c + 7914*a^5*(-b)^(29/4)*c - 48*a^9*(-b)^(25/4)*c - 1476*a^2*(-b)^(33/4)*c + 8806*a^6*(-b)^(29/4)*c - 9472*a^3*(-b)^(33/4)*c + 5324*a^7*(-b)^(29/4)*c - 25368*a^4*(-b)^(33/4)*c + 1632*a^8*(-b)^(29/4)*c - 36528*a^5*(-b)

^(33/4)*c + 192*a^9*(-b)^(29/4)*c + 1536*a^2*(-b)^(37/4)*c - 30212*a^6*(-b)
^(33/4)*c + 10176*a^3*(-b)^(37/4)*c - 14064*a^7*(-b)^(33/4)*c + 25600*a^4*(
-b)^(37/4)*c - 3264*a^8*(-b)^(33/4)*c + 33600*a^5*(-b)^(37/4)*c - 256*a^9*(
-b)^(33/4)*c + 2688*a*(-b)^(41/4)*c^2 + 44352*a^2*(-b)^(41/4)*c + 24576*a^6
*(-b)^(37/4)*c + 71680*a^3*(-b)^(41/4)*c + 9536*a^7*(-b)^(37/4)*c + 69440*a
^4*(-b)^(41/4)*c + 1536*a^8*(-b)^(37/4)*c + 40320*a^5*(-b)^(41/4)*c - 45696
a(-b)^(45/4)*c^2 + 12992*a^6*(-b)^(41/4)*c - 10304*a*(-b)^(45/4)*c^3 + 17
92*a^7*(-b)^(41/4)*c + 76160*a*(-b)^(49/4)*c^3 + 19040*a*(-b)^(49/4)*c^4 -
76160*a*(-b)^(53/4)*c^4 - 20160*a*(-b)^(53/4)*c^5 + 45696*a*(-b)^(57/4)*c^5
 + 12544*a*(-b)^(57/4)*c^6 - 15232*a*(-b)^(61/4)*c^6 - 4288*a*(-b)^(61/4)*c
^7 + 2176*a*(-b)^(65/4)*c^7 + 624*a*(-b)^(65/4)*c^8))/(a^7*b^18 + 9*a^8*b^1
8*c + 36*a^9*b^18*c^2 + 84*a^10*b^18*c^3 + 126*a^11*b^18*c^4 + 126*a^12*b^1
8*c^5 + 84*a^13*b^18*c^6 + 36*a^14*b^18*c^7 + 9*a^15*b^18*c^8 + a^16*b^18*c
^9) + ((b*((-b)^(3/4) + a*((-b)^(3/4) + ((-b)^(3/4)*c)/4)) + (a*(-b)^(3/4))
/4)*((64*(-(b*x - 1)/(c + x))^(1/4)*(512*(-b)^(19/2) + a^5*(-b)^(9/2) + a^4
*(-b)^(11/2) + 2*a^6*(-b)^(9/2) - 16*a^3*(-b)^(13/2) + 18*a^5*(-b)^(11/2) +
 a^7*(-b)^(9/2) - 144*a^2*(-b)^(15/2) - 176*a^4*(-b)^(13/2) + 49*a^6*(-b)^(
11/2) - 576*a^3*(-b)^(15/2) - 560*a^5*(-b)^(13/2) + 48*a^7*(-b)^(11/2) + 26
88*a^2*(-b)^(17/2) - 608*a^4*(-b)^(15/2) - 784*a^6*(-b)^(13/2) + 16*a^8*(-b
)^(11/2) + 7680*a^3*(-b)^(17/2) + 448*a^5*(-b)^(15/2) - 512*a^7*(-b)^(13/2)
 + 7680*a^2*(-b)^(19/2) + 11520*a^4*(-b)^(17/2) + 1392*a^6*(-b)^(15/2) - 12
8*a^8*(-b)^(13/2) + 10240*a^3*(-b)^(19/2) + 9600*a^5*(-b)^(17/2) + 1024*a^7
*(-b)^(15/2) + 7680*a^4*(-b)^(19/2) + 4224*a^6*(-b)^(17/2) + 256*a^8*(-b)^(
15/2) + 3072*a^5*(-b)^(19/2) + 768*a^7*(-b)^(17/2) + 512*a^6*(-b)^(19/2) +
7680*(-b)^(23/2)*c^2 - 10240*(-b)^(25/2)*c^3 + 7680*(-b)^(27/2)*c^4 - 3072*
(-b)^(29/2)*c^5 + 512*(-b)^(31/2)*c^6 + 384*a*(-b)^(17/2) + 3072*a*(-b)^(19
/2) - 3072*(-b)^(21/2)*c + a^7*(-b)^(9/2)*c^2 - 35*a^6*(-b)^(11/2)*c^2 + 2*
a^8*(-b)^(9/2)*c^2 + 265*a^5*(-b)^(13/2)*c^2 - 86*a^7*(-b)^(11/2)*c^2 + a^9
*(-b)^(9/2)*c^2 - 851*a^4*(-b)^(15/2)*c^2 + 738*a^6*(-b)^(13/2)*c^2 - 10*a^
7*(-b)^(11/2)*c^3 - 67*a^8*(-b)^(11/2)*c^2 + 2496*a^3*(-b)^(17/2)*c^2 - 256
6*a^5*(-b)^(15/2)*c^2 + 224*a^6*(-b)^(13/2)*c^3 + 649*a^7*(-b)^(13/2)*c^2 -
 20*a^8*(-b)^(11/2)*c^3 - 16*a^9*(-b)^(11/2)*c^2 - 5184*a^2*(-b)^(19/2)*c^2
 + 10432*a^4*(-b)^(17/2)*c^2 - 1358*a^5*(-b)^(15/2)*c^3 - 1907*a^6*(-b)^(15
/2)*c^2 + 592*a^7*(-b)^(13/2)*c^3 + 144*a^8*(-b)^(13/2)*c^2 - 10*a^9*(-b)^(
11/2)*c^3 - 31104*a^3*(-b)^(19/2)*c^2 + 3784*a^4*(-b)^(17/2)*c^3 + 14912*a^
5*(-b)^(17/2)*c^2 - 4364*a^6*(-b)^(15/2)*c^3 + 45*a^7*(-b)^(13/2)*c^4 + 112
0*a^7*(-b)^(15/2)*c^2 + 512*a^8*(-b)^(13/2)*c^3 - 32*a^9*(-b)^(13/2)*c^2 +
1152*a^2*(-b)^(21/2)*c^2 - 7552*a^3*(-b)^(19/2)*c^3 - 74624*a^4*(-b)^(19/2)
*c^2 + 14288*a^5*(-b)^(17/2)*c^3 - 771*a^6*(-b)^(15/2)*c^4 + 5824*a^6*(-b)^
(17/2)*c^2 - 4974*a^7*(-b)^(15/2)*c^3 + 90*a^8*(-b)^(13/2)*c^4 + 1952*a^8*(
-b)^(15/2)*c^2 + 144*a^9*(-b)^(13/2)*c^3 + 6912*a^2*(-b)^(21/2)*c^3 + 23040
a^3(-b)^(21/2)*c^2 - 36800*a^4*(-b)^(19/2)*c^3 + 3874*a^5*(-b)^(17/2)*c^4
 - 91136*a^5*(-b)^(19/2)*c^2 + 19144*a^6*(-b)^(17/2)*c^3 - 2118*a^7*(-b)^(1
5/2)*c^4 - 5120*a^7*(-b)^(17/2)*c^2 - 2288*a^8*(-b)^(15/2)*c^3 + 45*a^9*(-b
)^(13/2)*c^4 + 640*a^9*(-b)^(15/2)*c^2 + 115200*a^2*(-b)^(23/2)*c^2 + 49536
a^3(-b)^(21/2)*c^3 - 8750*a^4*(-b)^(19/2)*c^4 + 57600*a^4*(-b)^(21/2)*c^2
 - 68032*a^5*(-b)^(19/2)*c^3 + 13444*a^6*(-b)^(17/2)*c^4 - 58944*a^6*(-b)^(
19/2)*c^2 - 120*a^7*(-b)^(15/2)*c^5 + 9664*a^7*(-b)^(17/2)*c^3 - 1923*a^8*(
-b)^(15/2)*c^4 - 5248*a^8*(-b)^(17/2)*c^2 - 320*a^9*(-b)^(15/2)*c^3 + 44160
a^2(-b)^(23/2)*c^3 + 11040*a^3*(-b)^(21/2)*c^4 + 153600*a^3*(-b)^(23/2)*c
^2 + 137728*a^4*(-b)^(21/2)*c^3 - 36988*a^5*(-b)^(19/2)*c^4 + 63360*a^5*(-b
)^(21/2)*c^2 + 1644*a^6*(-b)^(17/2)*c^5 - 56512*a^6*(-b)^(19/2)*c^3 + 17058
a^7(-b)^(17/2)*c^4 - 18560*a^7*(-b)^(19/2)*c^2 - 240*a^8*(-b)^(15/2)*c^5
+ 128*a^8*(-b)^(17/2)*c^3 - 576*a^9*(-b)^(15/2)*c^4 - 1280*a^9*(-b)^(17/2)*
c^2 - 480*a^2*(-b)^(23/2)*c^4 + 76800*a^3*(-b)^(23/2)*c^3 + 60640*a^4*(-b)^
(21/2)*c^4 + 115200*a^4*(-b)^(23/2)*c^2 - 6776*a^5*(-b)^(19/2)*c^5 + 193792
a^5(-b)^(21/2)*c^3 - 59150*a^6*(-b)^(19/2)*c^4 + 33408*a^6*(-b)^(21/2)*c^
2 + 4632*a^7*(-b)^(17/2)*c^5 - 16064*a^7*(-b)^(19/2)*c^3 + 9280*a^8*(-b)^(1
7/2)*c^4 - 2048*a^8*(-b)^(19/2)*c^2 - 120*a^9*(-b)^(15/2)*c^5 - 896*a^9*(-b

)^(17/2)*c^3 - 153600*a^2*(-b)^(25/2)*c^3 - 23040*a^3*(-b)^(23/2)*c^4 + 116
20*a^4*(-b)^(21/2)*c^5 + 57600*a^4*(-b)^(23/2)*c^3 + 128864*a^5*(-b)^(21/2)
*c^4 + 46080*a^5*(-b)^(23/2)*c^2 - 24752*a^6*(-b)^(19/2)*c^5 + 146688*a^6*(
-b)^(21/2)*c^3 + 210*a^7*(-b)^(17/2)*c^6 - 43232*a^7*(-b)^(19/2)*c^4 + 6912
a^7(-b)^(21/2)*c^2 + 4332*a^8*(-b)^(17/2)*c^5 + 3968*a^8*(-b)^(19/2)*c^3
+ 1792*a^9*(-b)^(17/2)*c^4 - 90240*a^2*(-b)^(25/2)*c^4 - 7104*a^3*(-b)^(23/
2)*c^5 - 204800*a^3*(-b)^(25/2)*c^3 - 100160*a^4*(-b)^(23/2)*c^4 + 53480*a^
5*(-b)^(21/2)*c^5 + 9600*a^5*(-b)^(23/2)*c^3 - 2310*a^6*(-b)^(19/2)*c^6 + 1
31872*a^6*(-b)^(21/2)*c^4 + 7680*a^6*(-b)^(23/2)*c^2 - 33432*a^7*(-b)^(19/2
)*c^5 + 56704*a^7*(-b)^(21/2)*c^3 + 420*a^8*(-b)^(17/2)*c^6 - 13216*a^8*(-b
)^(19/2)*c^4 + 1344*a^9*(-b)^(17/2)*c^5 + 2304*a^9*(-b)^(19/2)*c^3 - 9216*a
^2*(-b)^(25/2)*c^5 - 192000*a^3*(-b)^(25/2)*c^4 - 48224*a^4*(-b)^(23/2)*c^5
 - 153600*a^4*(-b)^(25/2)*c^3 + 7546*a^5*(-b)^(21/2)*c^6 - 179840*a^5*(-b)^
(23/2)*c^4 + 94948*a^6*(-b)^(21/2)*c^5 - 9600*a^6*(-b)^(23/2)*c^3 - 6636*a^
7*(-b)^(19/2)*c^6 + 63232*a^7*(-b)^(21/2)*c^4 - 19712*a^8*(-b)^(19/2)*c^5 +
 8704*a^8*(-b)^(21/2)*c^3 + 210*a^9*(-b)^(17/2)*c^6 - 896*a^9*(-b)^(19/2)*c
^4 + 115200*a^2*(-b)^(27/2)*c^4 - 31104*a^3*(-b)^(25/2)*c^5 - 8750*a^4*(-b)
^(23/2)*c^6 - 211200*a^4*(-b)^(25/2)*c^4 - 121120*a^5*(-b)^(23/2)*c^5 - 614
40*a^5*(-b)^(25/2)*c^3 + 28756*a^6*(-b)^(21/2)*c^6 - 161760*a^6*(-b)^(23/2)
*c^4 - 252*a^7*(-b)^(19/2)*c^7 + 80416*a^7*(-b)^(21/2)*c^5 - 3840*a^7*(-b)^
(23/2)*c^3 - 6342*a^8*(-b)^(19/2)*c^6 + 9344*a^8*(-b)^(21/2)*c^4 - 4256*a^9
*(-b)^(19/2)*c^5 + 81792*a^2*(-b)^(27/2)*c^5 - 832*a^3*(-b)^(25/2)*c^6 + 15
3600*a^3*(-b)^(27/2)*c^4 - 23552*a^4*(-b)^(25/2)*c^5 - 44380*a^5*(-b)^(23/2
)*c^6 - 124800*a^5*(-b)^(25/2)*c^4 + 2184*a^6*(-b)^(21/2)*c^7 - 146336*a^6*
(-b)^(23/2)*c^5 - 10240*a^6*(-b)^(25/2)*c^3 + 40698*a^7*(-b)^(21/2)*c^6 - 7
2320*a^7*(-b)^(23/2)*c^4 - 504*a^8*(-b)^(19/2)*c^7 + 31808*a^8*(-b)^(21/2)*
c^5 - 2016*a^9*(-b)^(19/2)*c^6 - 1280*a^9*(-b)^(21/2)*c^4 + 10944*a^2*(-b)^
(27/2)*c^6 + 184320*a^3*(-b)^(27/2)*c^5 + 8896*a^4*(-b)^(25/2)*c^6 + 115200
a^4(-b)^(27/2)*c^4 - 5300*a^5*(-b)^(23/2)*c^7 + 32512*a^5*(-b)^(25/2)*c^5
 - 86702*a^6*(-b)^(23/2)*c^6 - 36480*a^6*(-b)^(25/2)*c^4 + 6384*a^7*(-b)^(2
1/2)*c^7 - 87968*a^7*(-b)^(23/2)*c^5 + 25312*a^8*(-b)^(21/2)*c^6 - 12800*a^
8*(-b)^(23/2)*c^4 - 252*a^9*(-b)^(19/2)*c^7 + 4480*a^9*(-b)^(21/2)*c^5 - 46
080*a^2*(-b)^(29/2)*c^5 + 49536*a^3*(-b)^(27/2)*c^6 + 3016*a^4*(-b)^(25/2)*
c^7 + 218880*a^4*(-b)^(27/2)*c^5 + 44864*a^5*(-b)^(25/2)*c^6 + 46080*a^5*(-
b)^(27/2)*c^4 - 21128*a^6*(-b)^(23/2)*c^7 + 66048*a^6*(-b)^(25/2)*c^5 + 210
a^7(-b)^(21/2)*c^8 - 81536*a^7*(-b)^(23/2)*c^6 - 3840*a^7*(-b)^(25/2)*c^4
 + 6216*a^8*(-b)^(21/2)*c^7 - 22912*a^8*(-b)^(23/2)*c^5 + 5824*a^9*(-b)^(21
/2)*c^6 - 36480*a^2*(-b)^(29/2)*c^6 + 4224*a^3*(-b)^(27/2)*c^7 - 61440*a^3*
(-b)^(29/2)*c^5 + 86656*a^4*(-b)^(27/2)*c^6 + 18896*a^5*(-b)^(25/2)*c^7 + 1
44000*a^5*(-b)^(27/2)*c^5 - 1374*a^6*(-b)^(23/2)*c^8 + 75200*a^6*(-b)^(25/2
)*c^6 + 7680*a^6*(-b)^(27/2)*c^4 - 31284*a^7*(-b)^(23/2)*c^7 + 40576*a^7*(-
b)^(25/2)*c^5 + 420*a^8*(-b)^(21/2)*c^8 - 36736*a^8*(-b)^(23/2)*c^6 + 2016*
a^9*(-b)^(21/2)*c^7 - 1280*a^9*(-b)^(23/2)*c^5 - 5376*a^2*(-b)^(29/2)*c^7 -
 84480*a^3*(-b)^(29/2)*c^6 + 13888*a^4*(-b)^(27/2)*c^7 - 46080*a^4*(-b)^(29
/2)*c^5 + 2173*a^5*(-b)^(25/2)*c^8 + 70144*a^5*(-b)^(27/2)*c^6 + 42952*a^6*
(-b)^(25/2)*c^7 + 49536*a^6*(-b)^(27/2)*c^5 - 4092*a^7*(-b)^(23/2)*c^8 + 57
856*a^7*(-b)^(25/2)*c^6 - 20384*a^8*(-b)^(23/2)*c^7 + 8704*a^8*(-b)^(25/2)*
c^5 + 210*a^9*(-b)^(21/2)*c^8 - 6272*a^9*(-b)^(23/2)*c^6 + 7680*a^2*(-b)^(3
1/2)*c^6 - 26496*a^3*(-b)^(29/2)*c^7 + 301*a^4*(-b)^(27/2)*c^8 - 103680*a^4
*(-b)^(29/2)*c^6 + 12608*a^5*(-b)^(27/2)*c^7 - 18432*a^5*(-b)^(29/2)*c^5 +
9274*a^6*(-b)^(25/2)*c^8 + 21696*a^6*(-b)^(27/2)*c^6 - 120*a^7*(-b)^(23/2)*
c^9 + 45760*a^7*(-b)^(25/2)*c^7 + 6912*a^7*(-b)^(27/2)*c^5 - 4062*a^8*(-b)^
(23/2)*c^8 + 20096*a^8*(-b)^(25/2)*c^6 - 4928*a^9*(-b)^(23/2)*c^7 + 6528*a^
2*(-b)^(31/2)*c^7 - 2448*a^3*(-b)^(29/2)*c^8 + 10240*a^3*(-b)^(31/2)*c^6 -
52736*a^4*(-b)^(29/2)*c^7 - 1558*a^5*(-b)^(27/2)*c^8 - 71040*a^5*(-b)^(29/2
)*c^6 + 546*a^6*(-b)^(25/2)*c^9 - 4544*a^6*(-b)^(27/2)*c^7 - 3072*a^6*(-b)^
(29/2)*c^5 + 14589*a^7*(-b)^(25/2)*c^8 - 2432*a^7*(-b)^(27/2)*c^6 - 240*a^8
*(-b)^(23/2)*c^9 + 23168*a^8*(-b)^(25/2)*c^7 - 1344*a^9*(-b)^(23/2)*c^8 + 2
304*a^9*(-b)^(25/2)*c^6 + 1008*a^2*(-b)^(31/2)*c^8 + 15360*a^3*(-b)^(31/2)*

c^7 - 10160*a^4*(-b)^(29/2)*c^8 + 7680*a^4*(-b)^(31/2)*c^6 - 384*a^5*(-b)^(
27/2)*c^9 - 53504*a^5*(-b)^(29/2)*c^7 - 8099*a^6*(-b)^(27/2)*c^8 - 25728*a^
6*(-b)^(29/2)*c^6 + 1668*a^7*(-b)^(25/2)*c^9 - 13760*a^7*(-b)^(27/2)*c^7 +
10048*a^8*(-b)^(25/2)*c^8 - 2048*a^8*(-b)^(27/2)*c^6 - 120*a^9*(-b)^(23/2)*
c^9 + 4480*a^9*(-b)^(25/2)*c^7 + 5184*a^3*(-b)^(31/2)*c^8 - 570*a^4*(-b)^(2
9/2)*c^9 + 19200*a^4*(-b)^(31/2)*c^7 - 16048*a^5*(-b)^(29/2)*c^8 + 3072*a^5
*(-b)^(31/2)*c^6 - 1984*a^6*(-b)^(27/2)*c^9 - 28416*a^6*(-b)^(29/2)*c^7 + 4
5*a^7*(-b)^(25/2)*c^10 - 11984*a^7*(-b)^(27/2)*c^8 - 3840*a^7*(-b)^(29/2)*c
^6 + 1698*a^8*(-b)^(25/2)*c^9 - 7552*a^8*(-b)^(27/2)*c^7 + 2560*a^9*(-b)^(2
5/2)*c^8 + 480*a^3*(-b)^(31/2)*c^9 + 10912*a^4*(-b)^(31/2)*c^8 - 1732*a^5*(
-b)^(29/2)*c^9 + 13440*a^5*(-b)^(31/2)*c^7 - 119*a^6*(-b)^(27/2)*c^10 - 114
08*a^6*(-b)^(29/2)*c^8 + 512*a^6*(-b)^(31/2)*c^6 - 3568*a^7*(-b)^(27/2)*c^9
 - 7040*a^7*(-b)^(29/2)*c^7 + 90*a^8*(-b)^(25/2)*c^10 - 7408*a^8*(-b)^(27/2
)*c^8 + 576*a^9*(-b)^(25/2)*c^9 - 1280*a^9*(-b)^(27/2)*c^7 + 2160*a^4*(-b)^
(31/2)*c^9 - 35*a^5*(-b)^(29/2)*c^10 + 11968*a^5*(-b)^(31/2)*c^8 - 1530*a^6
*(-b)^(29/2)*c^9 + 4992*a^6*(-b)^(31/2)*c^7 - 382*a^7*(-b)^(27/2)*c^10 - 28
16*a^7*(-b)^(29/2)*c^8 - 2720*a^8*(-b)^(27/2)*c^9 - 512*a^8*(-b)^(29/2)*c^7
 + 45*a^9*(-b)^(25/2)*c^10 - 1664*a^9*(-b)^(27/2)*c^8 + 129*a^4*(-b)^(31/2)
*c^10 + 3856*a^5*(-b)^(31/2)*c^9 + 10*a^6*(-b)^(29/2)*c^10 + 7152*a^6*(-b)^
(31/2)*c^8 - 10*a^7*(-b)^(27/2)*c^11 + 112*a^7*(-b)^(29/2)*c^9 + 768*a^7*(-
b)^(31/2)*c^7 - 407*a^8*(-b)^(27/2)*c^10 + 512*a^8*(-b)^(29/2)*c^8 - 752*a^
9*(-b)^(27/2)*c^9 + 482*a^5*(-b)^(31/2)*c^10 + 8*a^6*(-b)^(29/2)*c^11 + 340
8*a^6*(-b)^(31/2)*c^9 + 221*a^7*(-b)^(29/2)*c^10 + 2176*a^7*(-b)^(31/2)*c^8
 - 20*a^8*(-b)^(27/2)*c^11 + 736*a^8*(-b)^(29/2)*c^9 - 144*a^9*(-b)^(27/2)*
c^10 + 256*a^9*(-b)^(29/2)*c^8 + 18*a^5*(-b)^(31/2)*c^11 + 673*a^6*(-b)^(31
/2)*c^10 + 32*a^7*(-b)^(29/2)*c^11 + 1488*a^7*(-b)^(31/2)*c^9 + 272*a^8*(-b
)^(29/2)*c^10 + 256*a^8*(-b)^(31/2)*c^8 - 10*a^9*(-b)^(27/2)*c^11 + 256*a^9
*(-b)^(29/2)*c^9 + 52*a^6*(-b)^(31/2)*c^11 + a^7*(-b)^(29/2)*c^12 + 416*a^7
*(-b)^(31/2)*c^10 + 40*a^8*(-b)^(29/2)*c^11 + 256*a^8*(-b)^(31/2)*c^9 + 96*
a^9*(-b)^(29/2)*c^10 + a^6*(-b)^(31/2)*c^12 + 50*a^7*(-b)^(31/2)*c^11 + 2*a
^8*(-b)^(29/2)*c^12 + 96*a^8*(-b)^(31/2)*c^10 + 16*a^9*(-b)^(29/2)*c^11 + 2
a^7(-b)^(31/2)*c^12 + 16*a^8*(-b)^(31/2)*c^11 + a^9*(-b)^(29/2)*c^12 + a^
8*(-b)^(31/2)*c^12 - 1152*a*(-b)^(19/2)*c - 18432*a*(-b)^(21/2)*c + 2*a^6*(
-b)^(9/2)*c - 24*a^5*(-b)^(11/2)*c + 4*a^7*(-b)^(9/2)*c + 70*a^4*(-b)^(13/2
)*c - 48*a^6*(-b)^(11/2)*c + 2*a^8*(-b)^(9/2)*c - 288*a^3*(-b)^(15/2)*c + 6
0*a^5*(-b)^(13/2)*c - 8*a^7*(-b)^(11/2)*c + 1536*a^2*(-b)^(17/2)*c - 656*a^
4*(-b)^(15/2)*c - 378*a^6*(-b)^(13/2)*c + 32*a^8*(-b)^(11/2)*c + 8064*a^3*(
-b)^(17/2)*c + 656*a^5*(-b)^(15/2)*c - 784*a^7*(-b)^(13/2)*c + 16*a^9*(-b)^
(11/2)*c - 9600*a^2*(-b)^(19/2)*c + 16384*a^4*(-b)^(17/2)*c + 3280*a^6*(-b)
^(15/2)*c - 544*a^8*(-b)^(13/2)*c - 30720*a^3*(-b)^(19/2)*c + 15616*a^5*(-b
)^(17/2)*c + 3664*a^7*(-b)^(15/2)*c - 128*a^9*(-b)^(13/2)*c - 1152*a*(-b)^(
21/2)*c^2 - 46080*a^2*(-b)^(21/2)*c - 49920*a^4*(-b)^(19/2)*c + 6144*a^6*(-
b)^(17/2)*c + 1664*a^8*(-b)^(15/2)*c - 61440*a^3*(-b)^(21/2)*c - 44160*a^5*
(-b)^(19/2)*c - 128*a^7*(-b)^(17/2)*c + 256*a^9*(-b)^(15/2)*c + 46080*a*(-b
)^(23/2)*c^2 - 46080*a^4*(-b)^(21/2)*c - 20352*a^6*(-b)^(19/2)*c - 512*a^8*
(-b)^(17/2)*c + 9600*a*(-b)^(23/2)*c^3 - 18432*a^5*(-b)^(21/2)*c - 3840*a^7
*(-b)^(19/2)*c - 3072*a^6*(-b)^(21/2)*c - 61440*a*(-b)^(25/2)*c^3 - 17280*a
*(-b)^(25/2)*c^4 + 46080*a*(-b)^(27/2)*c^4 + 14976*a*(-b)^(27/2)*c^5 - 1843
2*a*(-b)^(29/2)*c^5 - 6528*a*(-b)^(29/2)*c^6 + 3072*a*(-b)^(31/2)*c^6 + 115
2*a*(-b)^(31/2)*c^7))/((-b)^(1/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^8*b^17*c^
2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a^12*b^17*
c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9)) + (((64*(36*a^12
*(-b)^(25/4) - 4*a^13*(-b)^(21/4) + 48*a^13*(-b)^(25/4) - 60*a^11*(-b)^(29/
4) - 240*a^12*(-b)^(29/4) - 192*a^13*(-b)^(29/4) - 180*a^10*(-b)^(33/4) - 2
40*a^11*(-b)^(33/4) + 192*a^12*(-b)^(33/4) + 240*a^9*(-b)^(37/4) + 256*a^13
*(-b)^(33/4) + 1200*a^10*(-b)^(37/4) + 1728*a^11*(-b)^(37/4) + 320*a^8*(-b)
^(41/4) + 768*a^12*(-b)^(37/4) + 1152*a^9*(-b)^(41/4) + 1344*a^10*(-b)^(41/
4) + 512*a^11*(-b)^(41/4) - 144*a^13*(-b)^(29/4)*c^2 + 912*a^12*(-b)^(33/4)
*c^2 + 1344*a^13*(-b)^(33/4)*c^2 + 336*a^13*(-b)^(33/4)*c^3 - 432*a^11*(-b)

^(37/4)*c^2 - 4032*a^12*(-b)^(37/4)*c^2 - 1680*a^12*(-b)^(37/4)*c^3 - 4032*a^13*(-b)^(37/4)*c^2 - 4624*a^10*(-b)^(41/4)*c^2 - 2688*a^13*(-b)^(37/4)*c^3 - 9408*a^11*(-b)^(41/4)*c^2 - 504*a^13*(-b)^(37/4)*c^4 - 336*a^11*(-b)^(41/4)*c^3 - 1344*a^12*(-b)^(41/4)*c^2 + 3584*a^9*(-b)^(45/4)*c^2 + 5376*a^12*(-b)^(41/4)*c^3 + 3584*a^13*(-b)^(41/4)*c^2 + 20160*a^10*(-b)^(45/4)*c^2 + 1848*a^12*(-b)^(41/4)*c^4 + 6720*a^13*(-b)^(41/4)*c^3 + 8848*a^10*(-b)^(45/4)*c^3 + 30912*a^11*(-b)^(45/4)*c^2 + 3360*a^13*(-b)^(41/4)*c^4 + 6720*a^8*(-b)^(49/4)*c^2 + 21504*a^11*(-b)^(45/4)*c^3 + 14336*a^12*(-b)^(45/4)*c^2 + 504*a^13*(-b)^(41/4)*c^5 + 24192*a^9*(-b)^(49/4)*c^2 + 1848*a^11*(-b)^(45/4)*c^4 + 9408*a^12*(-b)^(45/4)*c^3 - 4032*a^9*(-b)^(49/4)*c^3 + 28224*a^10*(-b)^(49/4)*c^2 - 3360*a^12*(-b)^(45/4)*c^4 - 3584*a^13*(-b)^(45/4)*c^3 - 26880*a^10*(-b)^(49/4)*c^3 + 10752*a^11*(-b)^(49/4)*c^2 - 1176*a^12*(-b)^(45/4)*c^5 - 6720*a^13*(-b)^(45/4)*c^4 - 10584*a^10*(-b)^(49/4)*c^4 - 44352*a^11*(-b)^(49/4)*c^3 - 2688*a^13*(-b)^(45/4)*c^5 - 11200*a^8*(-b)^(53/4)*c^3 - 30240*a^11*(-b)^(49/4)*c^4 - 21504*a^12*(-b)^(49/4)*c^3 - 336*a^13*(-b)^(45/4)*c^6 - 40320*a^9*(-b)^(53/4)*c^3 - 2520*a^11*(-b)^(49/4)*c^5 - 20160*a^12*(-b)^(49/4)*c^4 + 1120*a^9*(-b)^(53/4)*c^4 - 47040*a^10*(-b)^(53/4)*c^3 + 16800*a^10*(-b)^(53/4)*c^4 - 17920*a^11*(-b)^(53/4)*c^3 + 336*a^12*(-b)^(49/4)*c^6 + 4032*a^13*(-b)^(49/4)*c^5 + 8120*a^10*(-b)^(53/4)*c^5 + 33600*a^11*(-b)^(53/4)*c^4 + 1344*a^13*(-b)^(49/4)*c^6 + 11200*a^8*(-b)^(57/4)*c^4 + 26880*a^11*(-b)^(53/4)*c^5 + 17920*a^12*(-b)^(53/4)*c^4 + 144*a^13*(-b)^(49/4)*c^7 + 40320*a^9*(-b)^(57/4)*c^4 + 1680*a^11*(-b)^(53/4)*c^6 + 22848*a^12*(-b)^(53/4)*c^5 + 2240*a^9*(-b)^(57/4)*c^5 + 47040*a^10*(-b)^(57/4)*c^4 + 1344*a^12*(-b)^(53/4)*c^6 + 3584*a^13*(-b)^(53/4)*c^5 + 17920*a^11*(-b)^(57/4)*c^4 + 48*a^12*(-b)^(53/4)*c^7 - 1344*a^13*(-b)^(53/4)*c^6 - 3920*a^10*(-b)^(57/4)*c^6 - 9408*a^11*(-b)^(57/4)*c^5 - 384*a^13*(-b)^(53/4)*c^7 - 6720*a^8*(-b)^(61/4)*c^5 - 14784*a^11*(-b)^(57/4)*c^6 - 7168*a^12*(-b)^(57/4)*c^5 - 36*a^13*(-b)^(53/4)*c^8 - 24192*a^9*(-b)^(61/4)*c^5 - 528*a^11*(-b)^(57/4)*c^7 - 14784*a^12*(-b)^(57/4)*c^6 - 2688*a^9*(-b)^(61/4)*c^6 - 28224*a^10*(-b)^(61/4)*c^5 - 768*a^12*(-b)^(57/4)*c^7 - 3584*a^13*(-b)^(57/4)*c^6 - 6720*a^10*(-b)^(61/4)*c^6 - 10752*a^11*(-b)^(61/4)*c^5 - 60*a^12*(-b)^(57/4)*c^8 + 192*a^13*(-b)^(57/4)*c^7 + 1104*a^10*(-b)^(61/4)*c^7 - 4032*a^11*(-b)^(61/4)*c^6 + 48*a^13*(-b)^(57/4)*c^8 + 2240*a^8*(-b)^(65/4)*c^6 + 4608*a^11*(-b)^(61/4)*c^7 + 4*a^13*(-b)^(57/4)*c^9 + 8064*a^9*(-b)^(65/4)*c^6 + 36*a^11*(-b)^(61/4)*c^8 + 5184*a^12*(-b)^(61/4)*c^7 + 1216*a^9*(-b)^(65/4)*c^7 + 9408*a^10*(-b)^(65/4)*c^6 + 144*a^12*(-b)^(61/4)*c^8 + 1536*a^13*(-b)^(61/4)*c^7 + 3840*a^10*(-b)^(65/4)*c^7 + 3584*a^11*(-b)^(65/4)*c^6 + 12*a^12*(-b)^(61/4)*c^9 - 148*a^10*(-b)^(65/4)*c^8 + 3648*a^11*(-b)^(65/4)*c^7 - 320*a^8*(-b)^(69/4)*c^7 - 624*a^11*(-b)^(65/4)*c^8 + 1024*a^12*(-b)^(65/4)*c^7 - 1152*a^9*(-b)^(69/4)*c^7 + 12*a^11*(-b)^(65/4)*c^9 - 768*a^12*(-b)^(65/4)*c^8 - 208*a^9*(-b)^(69/4)*c^8 - 1344*a^10*(-b)^(69/4)*c^7 - 256*a^13*(-b)^(65/4)*c^8 - 720*a^10*(-b)^(69/4)*c^8 - 512*a^11*(-b)^(69/4)*c^7 + 4*a^10*(-b)^(69/4)*c^9 - 768*a^11*(-b)^(69/4)*c^8 - 256*a^12*(-b)^(69/4)*c^8 + 36*a^13*(-b)^(25/4)*c - 276*a^12*(-b)^(29/4)*c - 384*a^13*(-b)^(29/4)*c + 300*a^11*(-b)^(33/4)*c + 1536*a^12*(-b)^(33/4)*c + 1344*a^13*(-b)^(33/4)*c + 1380*a^10*(-b)^(37/4)*c + 2304*a^11*(-b)^(37/4)*c - 576*a^12*(-b)^(37/4)*c - 1472*a^9*(-b)^(41/4)*c - 1536*a^13*(-b)^(37/4)*c - 7680*a^10*(-b)^(41/4)*c - 11328*a^11*(-b)^(41/4)*c - 2240*a^8*(-b)^(45/4)*c - 5120*a^12*(-b)^(41/4)*c - 8064*a^9*(-b)^(45/4)*c - 9408*a^10*(-b)^(45/4)*c - 3584*a^11*(-b)^(45/4)*c))/(a^7*b^18 + 9*a^8*b^18*c + 36*a^9*b^18*c^2 + 84*a^10*b^18*c^3 + 126*a^11*b^18*c^4 + 126*a^12*b^18*c^5 + 84*a^13*b^18*c^6 + 36*a^14*b^18*c^7 + 9*a^15*b^18*c^8 + a^16*b^18*c^9) - (64*(-(b*x - 1)/(c + x))^(1/4)*(b*((-b)^(3/4) + a*((-b)^(3/4) + ((-b)^(3/4)*c)/4)) + (a*(-b)^(3/4))/4)*(16*a^13*(-b)^(11/2) - 80*a^12*(-b)^(13/2) - 80*a^11*(-b)^(15/2) - 128*a^13*(-b)^(13/2) + 400*a^10*(-b)^(17/2) + 128*a^12*(-b)^(15/2) + 896*a^9*(-b)^(19/2) + 1152*a^11*(-b)^(17/2) + 256*a^13*(-b)^(15/2) + 512*a^8*(-b)^(21/2) + 1920*a^10*(-b)^(19/2) + 768*a^12*(-b)^(17/2) + 1024*a^9*(-b)^(21/2) + 1024*a^11*(-b)^(19/2) + 512*a^10*(-b)^(21/2) + 448*a^13*(-b)^(15/2)*c^2 - 1344*a^12*(-b)^(17/2)*c^2 - 2624*a^11*(-b)^(19/2)*c^2 - 2688*a^13*(-b)^(17/2)*c^2

+ 1088*a^10*(-b)^(21/2)*c^2 + 128*a^12*(-b)^(19/2)*c^2 - 896*a^13*(-b)^(17/2)*c^3 + 9600*a^9*(-b)^(23/2)*c^2 + 9344*a^11*(-b)^(21/2)*c^2 + 1792*a^12*(-b)^(19/2)*c^3 + 4096*a^13*(-b)^(19/2)*c^2 + 7680*a^8*(-b)^(25/2)*c^2 + 21888*a^10*(-b)^(23/2)*c^2 + 4096*a^11*(-b)^(21/2)*c^3 + 8704*a^12*(-b)^(21/2)*c^2 + 4480*a^13*(-b)^(19/2)*c^3 + 15360*a^9*(-b)^(25/2)*c^2 + 3328*a^10*(-b)^(23/2)*c^3 + 12288*a^11*(-b)^(23/2)*c^2 + 128*a^12*(-b)^(21/2)*c^3 + 1120*a^13*(-b)^(19/2)*c^4 - 8320*a^9*(-b)^(25/2)*c^3 + 7680*a^10*(-b)^(25/2)*c^2 - 4992*a^11*(-b)^(23/2)*c^3 - 1120*a^12*(-b)^(21/2)*c^4 - 6656*a^13*(-b)^(21/2)*c^3 - 10240*a^8*(-b)^(27/2)*c^3 - 21120*a^10*(-b)^(25/2)*c^3 - 2400*a^11*(-b)^(23/2)*c^4 - 9216*a^12*(-b)^(23/2)*c^3 - 4480*a^13*(-b)^(21/2)*c^4 - 20480*a^9*(-b)^(27/2)*c^3 - 7200*a^10*(-b)^(25/2)*c^4 - 12800*a^11*(-b)^(25/2)*c^3 + 1920*a^12*(-b)^(23/2)*c^4 - 896*a^13*(-b)^(21/2)*c^5 + 640*a^9*(-b)^(27/2)*c^4 - 10240*a^10*(-b)^(27/2)*c^3 - 3200*a^11*(-b)^(25/2)*c^4 + 7680*a^13*(-b)^(23/2)*c^4 + 7680*a^8*(-b)^(29/2)*c^4 + 5760*a^10*(-b)^(27/2)*c^4 - 1280*a^11*(-b)^(25/2)*c^5 + 5120*a^12*(-b)^(25/2)*c^4 + 2688*a^13*(-b)^(23/2)*c^5 + 15360*a^9*(-b)^(29/2)*c^4 + 5120*a^10*(-b)^(27/2)*c^5 + 5120*a^11*(-b)^(27/2)*c^4 - 5248*a^12*(-b)^(25/2)*c^5 + 448*a^13*(-b)^(23/2)*c^6 + 4224*a^9*(-b)^(29/2)*c^5 + 7680*a^10*(-b)^(29/2)*c^4 + 3968*a^11*(-b)^(27/2)*c^5 + 448*a^12*(-b)^(25/2)*c^6 - 6656*a^13*(-b)^(25/2)*c^5 - 3072*a^8*(-b)^(31/2)*c^5 + 5760*a^10*(-b)^(29/2)*c^5 + 2752*a^11*(-b)^(27/2)*c^6 - 2048*a^12*(-b)^(27/2)*c^5 - 896*a^13*(-b)^(25/2)*c^6 - 6144*a^9*(-b)^(31/2)*c^5 - 704*a^10*(-b)^(29/2)*c^6 + 1536*a^11*(-b)^(29/2)*c^5 + 5504*a^12*(-b)^(27/2)*c^6 - 128*a^13*(-b)^(25/2)*c^7 - 2944*a^9*(-b)^(31/2)*c^6 - 3072*a^10*(-b)^(31/2)*c^5 + 384*a^11*(-b)^(29/2)*c^6 - 256*a^12*(-b)^(27/2)*c^7 + 4096*a^13*(-b)^(27/2)*c^6 + 512*a^8*(-b)^(33/2)*c^6 - 4992*a^10*(-b)^(31/2)*c^6 - 1536*a^11*(-b)^(29/2)*c^7 + 1536*a^12*(-b)^(29/2)*c^6 + 128*a^13*(-b)^(27/2)*c^7 + 1024*a^9*(-b)^(33/2)*c^6 - 768*a^10*(-b)^(31/2)*c^7 - 2048*a^11*(-b)^(31/2)*c^6 - 2688*a^12*(-b)^(29/2)*c^7 + 16*a^13*(-b)^(27/2)*c^8 + 640*a^9*(-b)^(33/2)*c^7 + 512*a^10*(-b)^(33/2)*c^6 - 1664*a^11*(-b)^(31/2)*c^7 + 48*a^12*(-b)^(29/2)*c^8 - 1536*a^13*(-b)^(29/2)*c^7 + 1152*a^10*(-b)^(33/2)*c^7 + 304*a^11*(-b)^(31/2)*c^8 - 1024*a^12*(-b)^(31/2)*c^7 + 272*a^10*(-b)^(33/2)*c^8 + 512*a^11*(-b)^(33/2)*c^7 + 512*a^12*(-b)^(31/2)*c^8 + 512*a^11*(-b)^(33/2)*c^8 + 256*a^13*(-b)^(31/2)*c^8 + 256*a^12*(-b)^(33/2)*c^8 - 128*a^13*(-b)^(13/2)*c + 512*a^12*(-b)^(15/2)*c + 768*a^11*(-b)^(17/2)*c + 896*a^13*(-b)^(15/2)*c - 1536*a^10*(-b)^(19/2)*c - 384*a^12*(-b)^(17/2)*c - 4736*a^9*(-b)^(21/2)*c - 5504*a^11*(-b)^(19/2)*c - 1536*a^13*(-b)^(17/2)*c - 3072*a^8*(-b)^(23/2)*c - 10368*a^10*(-b)^(21/2)*c - 4096*a^12*(-b)^(19/2)*c - 6144*a^9*(-b)^(23/2)*c - 5632*a^11*(-b)^(21/2)*c - 3072*a^10*(-b)^(23/2)*c))/(a^2*(-b)^(9/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^8*b^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a^12*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9)))*(b*((-b)^(3/4) + a*((-b)^(3/4) + ((-b)^(3/4)*c)/4)) + (a*(-b)^(3/4))/4)^3)/(a^6*b^6)))/(a^2*b^2) - ((b*((-b)^(3/4) + a*((-b)^(3/4) + ((-b)^(3/4)*c)/4)) + (a*(-b)^(3/4))/4)*((64*(-(b*x - 1)/(c + x))^(1/4)*(512*(-b)^(19/2) + a^5*(-b)^(9/2) + a^4*(-b)^(11/2) + 2*a^6*(-b)^(9/2) - 16*a^3*(-b)^(13/2) + 18*a^5*(-b)^(11/2) + a^7*(-b)^(9/2) - 144*a^2*(-b)^(15/2) - 176*a^4*(-b)^(13/2) + 49*a^6*(-b)^(11/2) - 576*a^3*(-b)^(15/2) - 560*a^5*(-b)^(13/2) + 48*a^7*(-b)^(11/2) + 2688*a^2*(-b)^(17/2) - 608*a^4*(-b)^(15/2) - 784*a^6*(-b)^(13/2) + 16*a^8*(-b)^(11/2) + 7680*a^3*(-b)^(17/2) + 448*a^5*(-b)^(15/2) - 512*a^7*(-b)^(13/2) + 7680*a^2*(-b)^(19/2) + 11520*a^4*(-b)^(17/2) + 1392*a^6*(-b)^(15/2) - 128*a^8*(-b)^(13/2) + 10240*a^3*(-b)^(19/2) + 9600*a^5*(-b)^(17/2) + 1024*a^7*(-b)^(15/2) + 7680*a^4*(-b)^(19/2) + 4224*a^6*(-b)^(17/2) + 256*a^8*(-b)^(15/2) + 3072*a^5*(-b)^(19/2) + 768*a^7*(-b)^(17/2) + 512*a^6*(-b)^(19/2) + 7680*(-b)^(23/2)*c^2 - 10240*(-b)^(25/2)*c^3 + 7680*(-b)^(27/2)*c^4 - 3072*(-b)^(29/2)*c^5 + 512*(-b)^(31/2)*c^6 + 384*a*(-b)^(17/2) + 3072*a*(-b)^(19/2) - 3072*(-b)^(21/2)*c + a^7*(-b)^(9/2)*c^2 - 35*a^6*(-b)^(11/2)*c^2 + 2*a^8*(-b)^(9/2)*c^2 + 265*a^5*(-b)^(13/2)*c^2 - 86*a^7*(-b)^(11/2)*c^2 + a^9*(-b)^(9/2)*c^2 - 851*a^4*(-b)^(15/2)*c^2 + 738*a^6*(-b)^(13/2)*c^2 - 10*a^7*(-b)^(11/2)*c^3 - 67*a^8*(-b)^(11/2)*c^2 + 2496*a^3*(-b)^(17/2)*c^2

- 2566*a^5*(-b)^(15/2)*c^2 + 224*a^6*(-b)^(13/2)*c^3 + 649*a^7*(-b)^(13/2)
*c^2 - 20*a^8*(-b)^(11/2)*c^3 - 16*a^9*(-b)^(11/2)*c^2 - 5184*a^2*(-b)^(19/
2)*c^2 + 10432*a^4*(-b)^(17/2)*c^2 - 1358*a^5*(-b)^(15/2)*c^3 - 1907*a^6*(-
b)^(15/2)*c^2 + 592*a^7*(-b)^(13/2)*c^3 + 144*a^8*(-b)^(13/2)*c^2 - 10*a^9*
(-b)^(11/2)*c^3 - 31104*a^3*(-b)^(19/2)*c^2 + 3784*a^4*(-b)^(17/2)*c^3 + 14
912*a^5*(-b)^(17/2)*c^2 - 4364*a^6*(-b)^(15/2)*c^3 + 45*a^7*(-b)^(13/2)*c^4
 + 1120*a^7*(-b)^(15/2)*c^2 + 512*a^8*(-b)^(13/2)*c^3 - 32*a^9*(-b)^(13/2)*
c^2 + 1152*a^2*(-b)^(21/2)*c^2 - 7552*a^3*(-b)^(19/2)*c^3 - 74624*a^4*(-b)^
(19/2)*c^2 + 14288*a^5*(-b)^(17/2)*c^3 - 771*a^6*(-b)^(15/2)*c^4 + 5824*a^6
*(-b)^(17/2)*c^2 - 4974*a^7*(-b)^(15/2)*c^3 + 90*a^8*(-b)^(13/2)*c^4 + 1952
a^8(-b)^(15/2)*c^2 + 144*a^9*(-b)^(13/2)*c^3 + 6912*a^2*(-b)^(21/2)*c^3 +
 23040*a^3*(-b)^(21/2)*c^2 - 36800*a^4*(-b)^(19/2)*c^3 + 3874*a^5*(-b)^(17/
2)*c^4 - 91136*a^5*(-b)^(19/2)*c^2 + 19144*a^6*(-b)^(17/2)*c^3 - 2118*a^7*(
-b)^(15/2)*c^4 - 5120*a^7*(-b)^(17/2)*c^2 - 2288*a^8*(-b)^(15/2)*c^3 + 45*a
^9*(-b)^(13/2)*c^4 + 640*a^9*(-b)^(15/2)*c^2 + 115200*a^2*(-b)^(23/2)*c^2 +
 49536*a^3*(-b)^(21/2)*c^3 - 8750*a^4*(-b)^(19/2)*c^4 + 57600*a^4*(-b)^(21/
2)*c^2 - 68032*a^5*(-b)^(19/2)*c^3 + 13444*a^6*(-b)^(17/2)*c^4 - 58944*a^6*
(-b)^(19/2)*c^2 - 120*a^7*(-b)^(15/2)*c^5 + 9664*a^7*(-b)^(17/2)*c^3 - 1923
a^8(-b)^(15/2)*c^4 - 5248*a^8*(-b)^(17/2)*c^2 - 320*a^9*(-b)^(15/2)*c^3 +
 44160*a^2*(-b)^(23/2)*c^3 + 11040*a^3*(-b)^(21/2)*c^4 + 153600*a^3*(-b)^(2
3/2)*c^2 + 137728*a^4*(-b)^(21/2)*c^3 - 36988*a^5*(-b)^(19/2)*c^4 + 63360*a
^5*(-b)^(21/2)*c^2 + 1644*a^6*(-b)^(17/2)*c^5 - 56512*a^6*(-b)^(19/2)*c^3 +
 17058*a^7*(-b)^(17/2)*c^4 - 18560*a^7*(-b)^(19/2)*c^2 - 240*a^8*(-b)^(15/2
)*c^5 + 128*a^8*(-b)^(17/2)*c^3 - 576*a^9*(-b)^(15/2)*c^4 - 1280*a^9*(-b)^(
17/2)*c^2 - 480*a^2*(-b)^(23/2)*c^4 + 76800*a^3*(-b)^(23/2)*c^3 + 60640*a^4
*(-b)^(21/2)*c^4 + 115200*a^4*(-b)^(23/2)*c^2 - 6776*a^5*(-b)^(19/2)*c^5 +
193792*a^5*(-b)^(21/2)*c^3 - 59150*a^6*(-b)^(19/2)*c^4 + 33408*a^6*(-b)^(21
/2)*c^2 + 4632*a^7*(-b)^(17/2)*c^5 - 16064*a^7*(-b)^(19/2)*c^3 + 9280*a^8*(
-b)^(17/2)*c^4 - 2048*a^8*(-b)^(19/2)*c^2 - 120*a^9*(-b)^(15/2)*c^5 - 896*a
^9*(-b)^(17/2)*c^3 - 153600*a^2*(-b)^(25/2)*c^3 - 23040*a^3*(-b)^(23/2)*c^4
 + 11620*a^4*(-b)^(21/2)*c^5 + 57600*a^4*(-b)^(23/2)*c^3 + 128864*a^5*(-b)^
(21/2)*c^4 + 46080*a^5*(-b)^(23/2)*c^2 - 24752*a^6*(-b)^(19/2)*c^5 + 146688
a^6(-b)^(21/2)*c^3 + 210*a^7*(-b)^(17/2)*c^6 - 43232*a^7*(-b)^(19/2)*c^4
 + 6912*a^7*(-b)^(21/2)*c^2 + 4332*a^8*(-b)^(17/2)*c^5 + 3968*a^8*(-b)^(19/2
)*c^3 + 1792*a^9*(-b)^(17/2)*c^4 - 90240*a^2*(-b)^(25/2)*c^4 - 7104*a^3*(-b
)^(23/2)*c^5 - 204800*a^3*(-b)^(25/2)*c^3 - 100160*a^4*(-b)^(23/2)*c^4 + 53
480*a^5*(-b)^(21/2)*c^5 + 9600*a^5*(-b)^(23/2)*c^3 - 2310*a^6*(-b)^(19/2)*c
^6 + 131872*a^6*(-b)^(21/2)*c^4 + 7680*a^6*(-b)^(23/2)*c^2 - 33432*a^7*(-b)
^(19/2)*c^5 + 56704*a^7*(-b)^(21/2)*c^3 + 420*a^8*(-b)^(17/2)*c^6 - 13216*a
^8*(-b)^(19/2)*c^4 + 1344*a^9*(-b)^(17/2)*c^5 + 2304*a^9*(-b)^(19/2)*c^3 -
9216*a^2*(-b)^(25/2)*c^5 - 192000*a^3*(-b)^(25/2)*c^4 - 48224*a^4*(-b)^(23/
2)*c^5 - 153600*a^4*(-b)^(25/2)*c^3 + 7546*a^5*(-b)^(21/2)*c^6 - 179840*a^5
*(-b)^(23/2)*c^4 + 94948*a^6*(-b)^(21/2)*c^5 - 9600*a^6*(-b)^(23/2)*c^3 - 6
636*a^7*(-b)^(19/2)*c^6 + 63232*a^7*(-b)^(21/2)*c^4 - 19712*a^8*(-b)^(19/2)
*c^5 + 8704*a^8*(-b)^(21/2)*c^3 + 210*a^9*(-b)^(17/2)*c^6 - 896*a^9*(-b)^(1
9/2)*c^4 + 115200*a^2*(-b)^(27/2)*c^4 - 31104*a^3*(-b)^(25/2)*c^5 - 8750*a^
4*(-b)^(23/2)*c^6 - 211200*a^4*(-b)^(25/2)*c^4 - 121120*a^5*(-b)^(23/2)*c^5
 - 61440*a^5*(-b)^(25/2)*c^3 + 28756*a^6*(-b)^(21/2)*c^6 - 161760*a^6*(-b)^
(23/2)*c^4 - 252*a^7*(-b)^(19/2)*c^7 + 80416*a^7*(-b)^(21/2)*c^5 - 3840*a^7
*(-b)^(23/2)*c^3 - 6342*a^8*(-b)^(19/2)*c^6 + 9344*a^8*(-b)^(21/2)*c^4 - 42
56*a^9*(-b)^(19/2)*c^5 + 81792*a^2*(-b)^(27/2)*c^5 - 832*a^3*(-b)^(25/2)*c^
6 + 153600*a^3*(-b)^(27/2)*c^4 - 23552*a^4*(-b)^(25/2)*c^5 - 44380*a^5*(-b)
^(23/2)*c^6 - 124800*a^5*(-b)^(25/2)*c^4 + 2184*a^6*(-b)^(21/2)*c^7 - 14633
6*a^6*(-b)^(23/2)*c^5 - 10240*a^6*(-b)^(25/2)*c^3 + 40698*a^7*(-b)^(21/2)*c
^6 - 72320*a^7*(-b)^(23/2)*c^4 - 504*a^8*(-b)^(19/2)*c^7 + 31808*a^8*(-b)^(
21/2)*c^5 - 2016*a^9*(-b)^(19/2)*c^6 - 1280*a^9*(-b)^(21/2)*c^4 + 10944*a^2
*(-b)^(27/2)*c^6 + 184320*a^3*(-b)^(27/2)*c^5 + 8896*a^4*(-b)^(25/2)*c^6 +
115200*a^4*(-b)^(27/2)*c^4 - 5300*a^5*(-b)^(23/2)*c^7 + 32512*a^5*(-b)^(25/
2)*c^5 - 86702*a^6*(-b)^(23/2)*c^6 - 36480*a^6*(-b)^(25/2)*c^4 + 6384*a^7*(

-b)^(21/2)*c^7 - 87968*a^7*(-b)^(23/2)*c^5 + 25312*a^8*(-b)^(21/2)*c^6 - 12
800*a^8*(-b)^(23/2)*c^4 - 252*a^9*(-b)^(19/2)*c^7 + 4480*a^9*(-b)^(21/2)*c^
5 - 46080*a^2*(-b)^(29/2)*c^5 + 49536*a^3*(-b)^(27/2)*c^6 + 3016*a^4*(-b)^(
25/2)*c^7 + 218880*a^4*(-b)^(27/2)*c^5 + 44864*a^5*(-b)^(25/2)*c^6 + 46080*
a^5*(-b)^(27/2)*c^4 - 21128*a^6*(-b)^(23/2)*c^7 + 66048*a^6*(-b)^(25/2)*c^5
 + 210*a^7*(-b)^(21/2)*c^8 - 81536*a^7*(-b)^(23/2)*c^6 - 3840*a^7*(-b)^(25/
2)*c^4 + 6216*a^8*(-b)^(21/2)*c^7 - 22912*a^8*(-b)^(23/2)*c^5 + 5824*a^9*(-
b)^(21/2)*c^6 - 36480*a^2*(-b)^(29/2)*c^6 + 4224*a^3*(-b)^(27/2)*c^7 - 6144
0*a^3*(-b)^(29/2)*c^5 + 86656*a^4*(-b)^(27/2)*c^6 + 18896*a^5*(-b)^(25/2)*c
^7 + 144000*a^5*(-b)^(27/2)*c^5 - 1374*a^6*(-b)^(23/2)*c^8 + 75200*a^6*(-b)
^(25/2)*c^6 + 7680*a^6*(-b)^(27/2)*c^4 - 31284*a^7*(-b)^(23/2)*c^7 + 40576*
a^7*(-b)^(25/2)*c^5 + 420*a^8*(-b)^(21/2)*c^8 - 36736*a^8*(-b)^(23/2)*c^6 +
 2016*a^9*(-b)^(21/2)*c^7 - 1280*a^9*(-b)^(23/2)*c^5 - 5376*a^2*(-b)^(29/2)
*c^7 - 84480*a^3*(-b)^(29/2)*c^6 + 13888*a^4*(-b)^(27/2)*c^7 - 46080*a^4*(-
b)^(29/2)*c^5 + 2173*a^5*(-b)^(25/2)*c^8 + 70144*a^5*(-b)^(27/2)*c^6 + 4295
2*a^6*(-b)^(25/2)*c^7 + 49536*a^6*(-b)^(27/2)*c^5 - 4092*a^7*(-b)^(23/2)*c^
8 + 57856*a^7*(-b)^(25/2)*c^6 - 20384*a^8*(-b)^(23/2)*c^7 + 8704*a^8*(-b)^(
25/2)*c^5 + 210*a^9*(-b)^(21/2)*c^8 - 6272*a^9*(-b)^(23/2)*c^6 + 7680*a^2*(
-b)^(31/2)*c^6 - 26496*a^3*(-b)^(29/2)*c^7 + 301*a^4*(-b)^(27/2)*c^8 - 1036
80*a^4*(-b)^(29/2)*c^6 + 12608*a^5*(-b)^(27/2)*c^7 - 18432*a^5*(-b)^(29/2)*
c^5 + 9274*a^6*(-b)^(25/2)*c^8 + 21696*a^6*(-b)^(27/2)*c^6 - 120*a^7*(-b)^(
23/2)*c^9 + 45760*a^7*(-b)^(25/2)*c^7 + 6912*a^7*(-b)^(27/2)*c^5 - 4062*a^8
*(-b)^(23/2)*c^8 + 20096*a^8*(-b)^(25/2)*c^6 - 4928*a^9*(-b)^(23/2)*c^7 + 6
528*a^2*(-b)^(31/2)*c^7 - 2448*a^3*(-b)^(29/2)*c^8 + 10240*a^3*(-b)^(31/2)*
c^6 - 52736*a^4*(-b)^(29/2)*c^7 - 1558*a^5*(-b)^(27/2)*c^8 - 71040*a^5*(-b)
^(29/2)*c^6 + 546*a^6*(-b)^(25/2)*c^9 - 4544*a^6*(-b)^(27/2)*c^7 - 3072*a^6
*(-b)^(29/2)*c^5 + 14589*a^7*(-b)^(25/2)*c^8 - 2432*a^7*(-b)^(27/2)*c^6 - 2
40*a^8*(-b)^(23/2)*c^9 + 23168*a^8*(-b)^(25/2)*c^7 - 1344*a^9*(-b)^(23/2)*c
^8 + 2304*a^9*(-b)^(25/2)*c^6 + 1008*a^2*(-b)^(31/2)*c^8 + 15360*a^3*(-b)^(
31/2)*c^7 - 10160*a^4*(-b)^(29/2)*c^8 + 7680*a^4*(-b)^(31/2)*c^6 - 384*a^5*
(-b)^(27/2)*c^9 - 53504*a^5*(-b)^(29/2)*c^7 - 8099*a^6*(-b)^(27/2)*c^8 - 25
728*a^6*(-b)^(29/2)*c^6 + 1668*a^7*(-b)^(25/2)*c^9 - 13760*a^7*(-b)^(27/2)*
c^7 + 10048*a^8*(-b)^(25/2)*c^8 - 2048*a^8*(-b)^(27/2)*c^6 - 120*a^9*(-b)^(
23/2)*c^9 + 4480*a^9*(-b)^(25/2)*c^7 + 5184*a^3*(-b)^(31/2)*c^8 - 570*a^4*(
-b)^(29/2)*c^9 + 19200*a^4*(-b)^(31/2)*c^7 - 16048*a^5*(-b)^(29/2)*c^8 + 30
72*a^5*(-b)^(31/2)*c^6 - 1984*a^6*(-b)^(27/2)*c^9 - 28416*a^6*(-b)^(29/2)*c
^7 + 45*a^7*(-b)^(25/2)*c^10 - 11984*a^7*(-b)^(27/2)*c^8 - 3840*a^7*(-b)^(2
9/2)*c^6 + 1698*a^8*(-b)^(25/2)*c^9 - 7552*a^8*(-b)^(27/2)*c^7 + 2560*a^9*(
-b)^(25/2)*c^8 + 480*a^3*(-b)^(31/2)*c^9 + 10912*a^4*(-b)^(31/2)*c^8 - 1732
a^5(-b)^(29/2)*c^9 + 13440*a^5*(-b)^(31/2)*c^7 - 119*a^6*(-b)^(27/2)*c^10
 - 11408*a^6*(-b)^(29/2)*c^8 + 512*a^6*(-b)^(31/2)*c^6 - 3568*a^7*(-b)^(27/
2)*c^9 - 7040*a^7*(-b)^(29/2)*c^7 + 90*a^8*(-b)^(25/2)*c^10 - 7408*a^8*(-b)
^(27/2)*c^8 + 576*a^9*(-b)^(25/2)*c^9 - 1280*a^9*(-b)^(27/2)*c^7 + 2160*a^4
*(-b)^(31/2)*c^9 - 35*a^5*(-b)^(29/2)*c^10 + 11968*a^5*(-b)^(31/2)*c^8 - 15
30*a^6*(-b)^(29/2)*c^9 + 4992*a^6*(-b)^(31/2)*c^7 - 382*a^7*(-b)^(27/2)*c^1
0 - 2816*a^7*(-b)^(29/2)*c^8 - 2720*a^8*(-b)^(27/2)*c^9 - 512*a^8*(-b)^(29/
2)*c^7 + 45*a^9*(-b)^(25/2)*c^10 - 1664*a^9*(-b)^(27/2)*c^8 + 129*a^4*(-b)^
(31/2)*c^10 + 3856*a^5*(-b)^(31/2)*c^9 + 10*a^6*(-b)^(29/2)*c^10 + 7152*a^6
*(-b)^(31/2)*c^8 - 10*a^7*(-b)^(27/2)*c^11 + 112*a^7*(-b)^(29/2)*c^9 + 768*
a^7*(-b)^(31/2)*c^7 - 407*a^8*(-b)^(27/2)*c^10 + 512*a^8*(-b)^(29/2)*c^8 -
752*a^9*(-b)^(27/2)*c^9 + 482*a^5*(-b)^(31/2)*c^10 + 8*a^6*(-b)^(29/2)*c^11
 + 3408*a^6*(-b)^(31/2)*c^9 + 221*a^7*(-b)^(29/2)*c^10 + 2176*a^7*(-b)^(31/
2)*c^8 - 20*a^8*(-b)^(27/2)*c^11 + 736*a^8*(-b)^(29/2)*c^9 - 144*a^9*(-b)^(
27/2)*c^10 + 256*a^9*(-b)^(29/2)*c^8 + 18*a^5*(-b)^(31/2)*c^11 + 673*a^6*(-
b)^(31/2)*c^10 + 32*a^7*(-b)^(29/2)*c^11 + 1488*a^7*(-b)^(31/2)*c^9 + 272*a
^8*(-b)^(29/2)*c^10 + 256*a^8*(-b)^(31/2)*c^8 - 10*a^9*(-b)^(27/2)*c^11 + 2
56*a^9*(-b)^(29/2)*c^9 + 52*a^6*(-b)^(31/2)*c^11 + a^7*(-b)^(29/2)*c^12 + 4
16*a^7*(-b)^(31/2)*c^10 + 40*a^8*(-b)^(29/2)*c^11 + 256*a^8*(-b)^(31/2)*c^9
 + 96*a^9*(-b)^(29/2)*c^10 + a^6*(-b)^(31/2)*c^12 + 50*a^7*(-b)^(31/2)*c^11

+ 2*a^8*(-b)^(29/2)*c^12 + 96*a^8*(-b)^(31/2)*c^10 + 16*a^9*(-b)^(29/2)*c^
11 + 2*a^7*(-b)^(31/2)*c^12 + 16*a^8*(-b)^(31/2)*c^11 + a^9*(-b)^(29/2)*c^1
2 + a^8*(-b)^(31/2)*c^12 - 1152*a*(-b)^(19/2)*c - 18432*a*(-b)^(21/2)*c + 2
a^6(-b)^(9/2)*c - 24*a^5*(-b)^(11/2)*c + 4*a^7*(-b)^(9/2)*c + 70*a^4*(-b)
^(13/2)*c - 48*a^6*(-b)^(11/2)*c + 2*a^8*(-b)^(9/2)*c - 288*a^3*(-b)^(15/2)
*c + 60*a^5*(-b)^(13/2)*c - 8*a^7*(-b)^(11/2)*c + 1536*a^2*(-b)^(17/2)*c -
656*a^4*(-b)^(15/2)*c - 378*a^6*(-b)^(13/2)*c + 32*a^8*(-b)^(11/2)*c + 8064
a^3(-b)^(17/2)*c + 656*a^5*(-b)^(15/2)*c - 784*a^7*(-b)^(13/2)*c + 16*a^9
*(-b)^(11/2)*c - 9600*a^2*(-b)^(19/2)*c + 16384*a^4*(-b)^(17/2)*c + 3280*a^
6*(-b)^(15/2)*c - 544*a^8*(-b)^(13/2)*c - 30720*a^3*(-b)^(19/2)*c + 15616*a
^5*(-b)^(17/2)*c + 3664*a^7*(-b)^(15/2)*c - 128*a^9*(-b)^(13/2)*c - 1152*a*
(-b)^(21/2)*c^2 - 46080*a^2*(-b)^(21/2)*c - 49920*a^4*(-b)^(19/2)*c + 6144*
a^6*(-b)^(17/2)*c + 1664*a^8*(-b)^(15/2)*c - 61440*a^3*(-b)^(21/2)*c - 4416
0*a^5*(-b)^(19/2)*c - 128*a^7*(-b)^(17/2)*c + 256*a^9*(-b)^(15/2)*c + 46080
a(-b)^(23/2)*c^2 - 46080*a^4*(-b)^(21/2)*c - 20352*a^6*(-b)^(19/2)*c - 51
2*a^8*(-b)^(17/2)*c + 9600*a*(-b)^(23/2)*c^3 - 18432*a^5*(-b)^(21/2)*c - 38
40*a^7*(-b)^(19/2)*c - 3072*a^6*(-b)^(21/2)*c - 61440*a*(-b)^(25/2)*c^3 - 1
7280*a*(-b)^(25/2)*c^4 + 46080*a*(-b)^(27/2)*c^4 + 14976*a*(-b)^(27/2)*c^5
- 18432*a*(-b)^(29/2)*c^5 - 6528*a*(-b)^(29/2)*c^6 + 3072*a*(-b)^(31/2)*c^6
+ 1152*a*(-b)^(31/2)*c^7))/((-b)^(1/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^8*b
^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a^12
*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9)) - (((64*(3
6*a^12*(-b)^(25/4) - 4*a^13*(-b)^(21/4) + 48*a^13*(-b)^(25/4) - 60*a^11*(-b
)^(29/4) - 240*a^12*(-b)^(29/4) - 192*a^13*(-b)^(29/4) - 180*a^10*(-b)^(33/
4) - 240*a^11*(-b)^(33/4) + 192*a^12*(-b)^(33/4) + 240*a^9*(-b)^(37/4) + 25
6*a^13*(-b)^(33/4) + 1200*a^10*(-b)^(37/4) + 1728*a^11*(-b)^(37/4) + 320*a^
8*(-b)^(41/4) + 768*a^12*(-b)^(37/4) + 1152*a^9*(-b)^(41/4) + 1344*a^10*(-b
)^(41/4) + 512*a^11*(-b)^(41/4) - 144*a^13*(-b)^(29/4)*c^2 + 912*a^12*(-b)^
(33/4)*c^2 + 1344*a^13*(-b)^(33/4)*c^2 + 336*a^13*(-b)^(33/4)*c^3 - 432*a^1
1*(-b)^(37/4)*c^2 - 4032*a^12*(-b)^(37/4)*c^2 - 1680*a^12*(-b)^(37/4)*c^3 -
4032*a^13*(-b)^(37/4)*c^2 - 4624*a^10*(-b)^(41/4)*c^2 - 2688*a^13*(-b)^(37
/4)*c^3 - 9408*a^11*(-b)^(41/4)*c^2 - 504*a^13*(-b)^(37/4)*c^4 - 336*a^11*(
-b)^(41/4)*c^3 - 1344*a^12*(-b)^(41/4)*c^2 + 3584*a^9*(-b)^(45/4)*c^2 + 537
6*a^12*(-b)^(41/4)*c^3 + 3584*a^13*(-b)^(41/4)*c^2 + 20160*a^10*(-b)^(45/4)
*c^2 + 1848*a^12*(-b)^(41/4)*c^4 + 6720*a^13*(-b)^(41/4)*c^3 + 8848*a^10*(-
b)^(45/4)*c^3 + 30912*a^11*(-b)^(45/4)*c^2 + 3360*a^13*(-b)^(41/4)*c^4 + 67
20*a^8*(-b)^(49/4)*c^2 + 21504*a^11*(-b)^(45/4)*c^3 + 14336*a^12*(-b)^(45/4
)*c^2 + 504*a^13*(-b)^(41/4)*c^5 + 24192*a^9*(-b)^(49/4)*c^2 + 1848*a^11*(-
b)^(45/4)*c^4 + 9408*a^12*(-b)^(45/4)*c^3 - 4032*a^9*(-b)^(49/4)*c^3 + 2822
4*a^10*(-b)^(49/4)*c^2 - 3360*a^12*(-b)^(45/4)*c^4 - 3584*a^13*(-b)^(45/4)*
c^3 - 26880*a^10*(-b)^(49/4)*c^3 + 10752*a^11*(-b)^(49/4)*c^2 - 1176*a^12*(
-b)^(45/4)*c^5 - 6720*a^13*(-b)^(45/4)*c^4 - 10584*a^10*(-b)^(49/4)*c^4 - 4
4352*a^11*(-b)^(49/4)*c^3 - 2688*a^13*(-b)^(45/4)*c^5 - 11200*a^8*(-b)^(53/
4)*c^3 - 30240*a^11*(-b)^(49/4)*c^4 - 21504*a^12*(-b)^(49/4)*c^3 - 336*a^13
*(-b)^(45/4)*c^6 - 40320*a^9*(-b)^(53/4)*c^3 - 2520*a^11*(-b)^(49/4)*c^5 -
20160*a^12*(-b)^(49/4)*c^4 + 1120*a^9*(-b)^(53/4)*c^4 - 47040*a^10*(-b)^(53
/4)*c^3 + 16800*a^10*(-b)^(53/4)*c^4 - 17920*a^11*(-b)^(53/4)*c^3 + 336*a^1
2*(-b)^(49/4)*c^6 + 4032*a^13*(-b)^(49/4)*c^5 + 8120*a^10*(-b)^(53/4)*c^5 +
33600*a^11*(-b)^(53/4)*c^4 + 1344*a^13*(-b)^(49/4)*c^6 + 11200*a^8*(-b)^(5
7/4)*c^4 + 26880*a^11*(-b)^(53/4)*c^5 + 17920*a^12*(-b)^(53/4)*c^4 + 144*a^
13*(-b)^(49/4)*c^7 + 40320*a^9*(-b)^(57/4)*c^4 + 1680*a^11*(-b)^(53/4)*c^6
+ 22848*a^12*(-b)^(53/4)*c^5 + 2240*a^9*(-b)^(57/4)*c^5 + 47040*a^10*(-b)^(
57/4)*c^4 + 1344*a^12*(-b)^(53/4)*c^6 + 3584*a^13*(-b)^(53/4)*c^5 + 17920*a
^11*(-b)^(57/4)*c^4 + 48*a^12*(-b)^(53/4)*c^7 - 1344*a^13*(-b)^(53/4)*c^6 -
3920*a^10*(-b)^(57/4)*c^6 - 9408*a^11*(-b)^(57/4)*c^5 - 384*a^13*(-b)^(53/
4)*c^7 - 6720*a^8*(-b)^(61/4)*c^5 - 14784*a^11*(-b)^(57/4)*c^6 - 7168*a^12*
(-b)^(57/4)*c^5 - 36*a^13*(-b)^(53/4)*c^8 - 24192*a^9*(-b)^(61/4)*c^5 - 528
a^11(-b)^(57/4)*c^7 - 14784*a^12*(-b)^(57/4)*c^6 - 2688*a^9*(-b)^(61/4)*c
^6 - 28224*a^10*(-b)^(61/4)*c^5 - 768*a^12*(-b)^(57/4)*c^7 - 3584*a^13*(-b)

^(57/4)*c^6 - 6720*a^10*(-b)^(61/4)*c^6 - 10752*a^11*(-b)^(61/4)*c^5 - 60*a
^12*(-b)^(57/4)*c^8 + 192*a^13*(-b)^(57/4)*c^7 + 1104*a^10*(-b)^(61/4)*c^7
- 4032*a^11*(-b)^(61/4)*c^6 + 48*a^13*(-b)^(57/4)*c^8 + 2240*a^8*(-b)^(65/4
)*c^6 + 4608*a^11*(-b)^(61/4)*c^7 + 4*a^13*(-b)^(57/4)*c^9 + 8064*a^9*(-b)^
(65/4)*c^6 + 36*a^11*(-b)^(61/4)*c^8 + 5184*a^12*(-b)^(61/4)*c^7 + 1216*a^9
*(-b)^(65/4)*c^7 + 9408*a^10*(-b)^(65/4)*c^6 + 144*a^12*(-b)^(61/4)*c^8 + 1
536*a^13*(-b)^(61/4)*c^7 + 3840*a^10*(-b)^(65/4)*c^7 + 3584*a^11*(-b)^(65/4
)*c^6 + 12*a^12*(-b)^(61/4)*c^9 - 148*a^10*(-b)^(65/4)*c^8 + 3648*a^11*(-b)
^(65/4)*c^7 - 320*a^8*(-b)^(69/4)*c^7 - 624*a^11*(-b)^(65/4)*c^8 + 1024*a^1
2*(-b)^(65/4)*c^7 - 1152*a^9*(-b)^(69/4)*c^7 + 12*a^11*(-b)^(65/4)*c^9 - 76
8*a^12*(-b)^(65/4)*c^8 - 208*a^9*(-b)^(69/4)*c^8 - 1344*a^10*(-b)^(69/4)*c^
7 - 256*a^13*(-b)^(65/4)*c^8 - 720*a^10*(-b)^(69/4)*c^8 - 512*a^11*(-b)^(69
/4)*c^7 + 4*a^10*(-b)^(69/4)*c^9 - 768*a^11*(-b)^(69/4)*c^8 - 256*a^12*(-b)
^(69/4)*c^8 + 36*a^13*(-b)^(25/4)*c - 276*a^12*(-b)^(29/4)*c - 384*a^13*(-b
)^(29/4)*c + 300*a^11*(-b)^(33/4)*c + 1536*a^12*(-b)^(33/4)*c + 1344*a^13*(
-b)^(33/4)*c + 1380*a^10*(-b)^(37/4)*c + 2304*a^11*(-b)^(37/4)*c - 576*a^12
*(-b)^(37/4)*c - 1472*a^9*(-b)^(41/4)*c - 1536*a^13*(-b)^(37/4)*c - 7680*a^
10*(-b)^(41/4)*c - 11328*a^11*(-b)^(41/4)*c - 2240*a^8*(-b)^(45/4)*c - 5120
a^12(-b)^(41/4)*c - 8064*a^9*(-b)^(45/4)*c - 9408*a^10*(-b)^(45/4)*c - 35
84*a^11*(-b)^(45/4)*c))/(a^7*b^18 + 9*a^8*b^18*c + 36*a^9*b^18*c^2 + 84*a^1
0*b^18*c^3 + 126*a^11*b^18*c^4 + 126*a^12*b^18*c^5 + 84*a^13*b^18*c^6 + 36*
a^14*b^18*c^7 + 9*a^15*b^18*c^8 + a^16*b^18*c^9) + (64*(-(b*x - 1)/(c + x))
^(1/4)*(b*((-b)^(3/4) + a*((-b)^(3/4) + ((-b)^(3/4)*c)/4)) + (a*(-b)^(3/4))
/4)*(16*a^13*(-b)^(11/2) - 80*a^12*(-b)^(13/2) - 80*a^11*(-b)^(15/2) - 128*
a^13*(-b)^(13/2) + 400*a^10*(-b)^(17/2) + 128*a^12*(-b)^(15/2) + 896*a^9*(-
b)^(19/2) + 1152*a^11*(-b)^(17/2) + 256*a^13*(-b)^(15/2) + 512*a^8*(-b)^(21
/2) + 1920*a^10*(-b)^(19/2) + 768*a^12*(-b)^(17/2) + 1024*a^9*(-b)^(21/2) +
1024*a^11*(-b)^(19/2) + 512*a^10*(-b)^(21/2) + 448*a^13*(-b)^(15/2)*c^2 -
1344*a^12*(-b)^(17/2)*c^2 - 2624*a^11*(-b)^(19/2)*c^2 - 2688*a^13*(-b)^(17/
2)*c^2 + 1088*a^10*(-b)^(21/2)*c^2 + 128*a^12*(-b)^(19/2)*c^2 - 896*a^13*(-
b)^(17/2)*c^3 + 9600*a^9*(-b)^(23/2)*c^2 + 9344*a^11*(-b)^(21/2)*c^2 + 1792
a^12(-b)^(19/2)*c^3 + 4096*a^13*(-b)^(19/2)*c^2 + 7680*a^8*(-b)^(25/2)*c^
2 + 21888*a^10*(-b)^(23/2)*c^2 + 4096*a^11*(-b)^(21/2)*c^3 + 8704*a^12*(-b)
^(21/2)*c^2 + 4480*a^13*(-b)^(19/2)*c^3 + 15360*a^9*(-b)^(25/2)*c^2 + 3328*
a^10*(-b)^(23/2)*c^3 + 12288*a^11*(-b)^(23/2)*c^2 + 128*a^12*(-b)^(21/2)*c^
3 + 1120*a^13*(-b)^(19/2)*c^4 - 8320*a^9*(-b)^(25/2)*c^3 + 7680*a^10*(-b)^(
25/2)*c^2 - 4992*a^11*(-b)^(23/2)*c^3 - 1120*a^12*(-b)^(21/2)*c^4 - 6656*a^
13*(-b)^(21/2)*c^3 - 10240*a^8*(-b)^(27/2)*c^3 - 21120*a^10*(-b)^(25/2)*c^3
 - 2400*a^11*(-b)^(23/2)*c^4 - 9216*a^12*(-b)^(23/2)*c^3 - 4480*a^13*(-b)^(
21/2)*c^4 - 20480*a^9*(-b)^(27/2)*c^3 - 7200*a^10*(-b)^(25/2)*c^4 - 12800*a
^11*(-b)^(25/2)*c^3 + 1920*a^12*(-b)^(23/2)*c^4 - 896*a^13*(-b)^(21/2)*c^5
+ 640*a^9*(-b)^(27/2)*c^4 - 10240*a^10*(-b)^(27/2)*c^3 - 3200*a^11*(-b)^(25
/2)*c^4 + 7680*a^13*(-b)^(23/2)*c^4 + 7680*a^8*(-b)^(29/2)*c^4 + 5760*a^10*
(-b)^(27/2)*c^4 - 1280*a^11*(-b)^(25/2)*c^5 + 5120*a^12*(-b)^(25/2)*c^4 + 2
688*a^13*(-b)^(23/2)*c^5 + 15360*a^9*(-b)^(29/2)*c^4 + 5120*a^10*(-b)^(27/2
)*c^5 + 5120*a^11*(-b)^(27/2)*c^4 - 5248*a^12*(-b)^(25/2)*c^5 + 448*a^13*(-
b)^(23/2)*c^6 + 4224*a^9*(-b)^(29/2)*c^5 + 7680*a^10*(-b)^(29/2)*c^4 + 3968
a^11(-b)^(27/2)*c^5 + 448*a^12*(-b)^(25/2)*c^6 - 6656*a^13*(-b)^(25/2)*c^
5 - 3072*a^8*(-b)^(31/2)*c^5 + 5760*a^10*(-b)^(29/2)*c^5 + 2752*a^11*(-b)^(
27/2)*c^6 - 2048*a^12*(-b)^(27/2)*c^5 - 896*a^13*(-b)^(25/2)*c^6 - 6144*a^9
*(-b)^(31/2)*c^5 - 704*a^10*(-b)^(29/2)*c^6 + 1536*a^11*(-b)^(29/2)*c^5 + 5
504*a^12*(-b)^(27/2)*c^6 - 128*a^13*(-b)^(25/2)*c^7 - 2944*a^9*(-b)^(31/2)*
c^6 - 3072*a^10*(-b)^(31/2)*c^5 + 384*a^11*(-b)^(29/2)*c^6 - 256*a^12*(-b)^
(27/2)*c^7 + 4096*a^13*(-b)^(27/2)*c^6 + 512*a^8*(-b)^(33/2)*c^6 - 4992*a^1
0*(-b)^(31/2)*c^6 - 1536*a^11*(-b)^(29/2)*c^7 + 1536*a^12*(-b)^(29/2)*c^6 +
128*a^13*(-b)^(27/2)*c^7 + 1024*a^9*(-b)^(33/2)*c^6 - 768*a^10*(-b)^(31/2)
*c^7 - 2048*a^11*(-b)^(31/2)*c^6 - 2688*a^12*(-b)^(29/2)*c^7 + 16*a^13*(-b)
^(27/2)*c^8 + 640*a^9*(-b)^(33/2)*c^7 + 512*a^10*(-b)^(33/2)*c^6 - 1664*a^1
1*(-b)^(31/2)*c^7 + 48*a^12*(-b)^(29/2)*c^8 - 1536*a^13*(-b)^(29/2)*c^7 + 1

```
152*a^10*(-b)^(33/2)*c^7 + 304*a^11*(-b)^(31/2)*c^8 - 1024*a^12*(-b)^(31/2)
*c^7 + 272*a^10*(-b)^(33/2)*c^8 + 512*a^11*(-b)^(33/2)*c^7 + 512*a^12*(-b)^
(31/2)*c^8 + 512*a^11*(-b)^(33/2)*c^8 + 256*a^13*(-b)^(31/2)*c^8 + 256*a^12
*(-b)^(33/2)*c^8 - 128*a^13*(-b)^(13/2)*c + 512*a^12*(-b)^(15/2)*c + 768*a^
11*(-b)^(17/2)*c + 896*a^13*(-b)^(15/2)*c - 1536*a^10*(-b)^(19/2)*c - 384*a
^12*(-b)^(17/2)*c - 4736*a^9*(-b)^(21/2)*c - 5504*a^11*(-b)^(19/2)*c - 1536
*a^13*(-b)^(17/2)*c - 3072*a^8*(-b)^(23/2)*c - 10368*a^10*(-b)^(21/2)*c - 4
096*a^12*(-b)^(19/2)*c - 6144*a^9*(-b)^(23/2)*c - 5632*a^11*(-b)^(21/2)*c -
 3072*a^10*(-b)^(23/2)*c))/(a^2*(-b)^(9/4)*(a^6*b^17 + 9*a^7*b^17*c + 36*a^
8*b^17*c^2 + 84*a^9*b^17*c^3 + 126*a^10*b^17*c^4 + 126*a^11*b^17*c^5 + 84*a
^12*b^17*c^6 + 36*a^13*b^17*c^7 + 9*a^14*b^17*c^8 + a^15*b^17*c^9)))*(b*((-
b)^(3/4) + a*((-b)^(3/4) + ((-b)^(3/4)*c)/4)) + (a*(-b)^(3/4))/4)^3)/(a^6*b
^6)))/(a^2*b^2)))*(b*((-b)^(3/4) + a*((-b)^(3/4) + ((-b)^(3/4)*c)/4)) + (a*
(-b)^(3/4))/4)*2i)/(a^2*b^2) - ((b*c + 1)*(-(b*x - 1)/(c + x))^(3/4))/(a*b^
2*((b*x - 1)/(b*(c + x)) - 1))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{ax\sqrt[4]{-\frac{bx}{c+x}+\frac{1}{c+x}}-\sqrt[4]{-\frac{bx}{c+x}+\frac{1}{c+x}}}\,dx-\int \frac{1}{ax\sqrt[4]{-\frac{bx}{c+x}+\frac{1}{c+x}}-\sqrt[4]{-\frac{bx}{c+x}+\frac{1}{c+x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-a*x+1)/((-b*x+1)/(c+x))**(1/4),x)
```

```
[Out] -Integral(x/(a*x*(-b*x/(c + x) + 1/(c + x))**(1/4) - (-b*x/(c + x) + 1/(c +
 x))**(1/4)), x) - Integral(1/(a*x*(-b*x/(c + x) + 1/(c + x))**(1/4) - (-b*
x/(c + x) + 1/(c + x))**(1/4)), x)
```

3.2347 $\int \frac{x^5\left(-7b+9ax^2\right)}{\sqrt[4]{-bx^3+ax^5}\left(1-bx^7+ax^9\right)}\,dx$

Optimal. Leaf size=413

$$-\sqrt{2}\tan^{-1}\left(\frac{x^2\sqrt[4]{ax^5-bx^3}-2^{2/3}x\sqrt[4]{ax^5-bx^3}}{2^{2/3}x\sqrt[4]{ax^5-bx^3}+x^2\left(-\sqrt[4]{ax^5-bx^3}\right)-\sqrt{2}x+2\sqrt[6]{2}}\right)+\sqrt{2}\tan^{-1}\left(\frac{x^2\sqrt[4]{ax^5-bx^3}-2^{2/3}}{2^{2/3}x\sqrt[4]{ax^5-bx^3}+x^2\left(-\sqrt[4]{ax^5}\right.}\right.$$

Rubi [F] time = 2.71, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^5\left(-7b+9ax^2\right)}{\sqrt[4]{-bx^3+ax^5}\left(1-bx^7+ax^9\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^5*(-7*b + 9*a*x^2))/((-(b*x^3) + a*x^5)^(1/4)*(1 - b*x^7 + a*x^9)),x
]
```

```
[Out] (28*b*x^(3/4)*(-b + a*x^2)^(1/4)*Defer[Subst][Defer[Int][x^20/((-b + a*x^8)
^(1/4)*(-1 + b*x^28 - a*x^36)), x], x, x^(1/4)])/(-(b*x^3) + a*x^5)^(1/4) +
 (36*a*x^(3/4)*(-b + a*x^2)^(1/4)*Defer[Subst][Defer[Int][x^28/((-b + a*x^8
)^(1/4)*(1 - b*x^28 + a*x^36)), x], x, x^(1/4)])/(-(b*x^3) + a*x^5)^(1/4)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(-7b+9ax^2\right)}{\sqrt[4]{-bx^3+ax^5}\left(1-bx^7+ax^9\right)}\,dx &= \frac{\left(x^{3/4}\sqrt[4]{-b+ax^2}\right)\int \frac{x^{17/4}\left(-7b+9ax^2\right)}{\sqrt[4]{-b+ax^2}\left(1-bx^7+ax^9\right)}\,dx}{\sqrt[4]{-bx^3+ax^5}}\\
&= \frac{\left(4x^{3/4}\sqrt[4]{-b+ax^2}\right)\operatorname{Subst}\left(\int \frac{x^{20}\left(-7b+9ax^8\right)}{\sqrt[4]{-b+ax^8}\left(1-bx^{28}+ax^{36}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx^3+ax^5}}\\
&= \frac{\left(4x^{3/4}\sqrt[4]{-b+ax^2}\right)\operatorname{Subst}\left(\int\left(\frac{7bx^{20}}{\sqrt[4]{-b+ax^8}\left(-1+bx^{28}-ax^{36}\right)}+\frac{9ax^{28}}{\sqrt[4]{-b+ax^8}\left(1-bx^{28}+ax^{3}\right.}\right.\right.}{\sqrt[4]{-bx^3+ax^5}}\\
&= \frac{\left(36ax^{3/4}\sqrt[4]{-b+ax^2}\right)\operatorname{Subst}\left(\int \frac{x^{28}}{\sqrt[4]{-b+ax^8}\left(1-bx^{28}+ax^{36}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx^3+ax^5}}+\frac{\left(28bx\right.}{}
\end{aligned}$$

Mathematica [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(-7b+9ax^2\right)}{\sqrt[4]{-bx^3+ax^5}\left(1-bx^7+ax^9\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^5*(-7*b + 9*a*x^2))/((-(b*x^3) + a*x^5)^(1/4)*(1 - b*x^7 + a*x
^9)),x]
```

```
[Out] Integrate[(x^5*(-7*b + 9*a*x^2))/((-(b*x^3) + a*x^5)^(1/4)*(1 - b*x^7 + a*x
^9)), x]
```

IntegrateAlgebraic [F] time = 78.28, size = 0, normalized size = 0.00

$$\int \frac{x^5 \left(-7b+9ax^2\right)}{\sqrt[4]{-bx^3+ax^5} \left(1-bx^7+ax^9\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(-7*b + 9*a*x^2))/((-(b*x^3) + a*x^5)^(1/4)*(1 - b*
x^7 + a*x^9)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^5*(-7*b + 9*a*x^2))/((-(b*x^3) + a*x^5)^(1/4)*
(1 - b*x^7 + a*x^9)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(9*a*x^2-7*b)/(a*x^5-b*x^3)^(1/4)/(a*x^9-b*x^7+1),x, algorith
m="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(9\,ax^2-7\,b\right)x^5}{\left(ax^9-bx^7+1\right)\left(ax^5-bx^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(9*a*x^2-7*b)/(a*x^5-b*x^3)^(1/4)/(a*x^9-b*x^7+1),x, algorith
m="giac")
```

```
[Out] integrate((9*a*x^2 - 7*b)*x^5/((a*x^9 - b*x^7 + 1)*(a*x^5 - b*x^3)^(1/4)),
x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{x^5 \left(9 a\,x^2-7 b\right)}{\left(a\,x^5-b\,x^3\right)^{\frac{1}{4}} \left(a\,x^9-b\,x^7+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(9*a*x^2-7*b)/(a*x^5-b*x^3)^(1/4)/(a*x^9-b*x^7+1),x)
```

```
[Out] int(x^5*(9*a*x^2-7*b)/(a*x^5-b*x^3)^(1/4)/(a*x^9-b*x^7+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(9\,ax^2-7\,b\right)x^5}{\left(ax^9-bx^7+1\right)\left(ax^5-bx^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(9*a*x^2-7*b)/(a*x^5-b*x^3)^(1/4)/(a*x^9-b*x^7+1),x, algorith
m="maxima")
```

```
[Out] integrate((9*a*x^2 - 7*b)*x^5/((a*x^9 - b*x^7 + 1)*(a*x^5 - b*x^3)^(1/4)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^5\,\left(7\,b-9\,a\,x^2\right)}{{\left(a\,x^5-b\,x^3\right)}^{1/4}\,\left(a\,x^9-b\,x^7+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^5*(7*b - 9*a*x^2))/((a*x^5 - b*x^3)^(1/4)*(a*x^9 - b*x^7 + 1)),x)
[Out] -int((x^5*(7*b - 9*a*x^2))/((a*x^5 - b*x^3)^(1/4)*(a*x^9 - b*x^7 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \left(9ax^2 - 7b\right)}{\sqrt[4]{x^3 \left(ax^2 - b\right)} \left(ax^9 - bx^7 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(9*a*x**2-7*b)/(a*x**5-b*x**3)**(1/4)/(a*x**9-b*x**7+1),x)
[Out] Integral(x**5*(9*a*x**2 - 7*b)/((x**3*(a*x**2 - b))**(1/4)*(a*x**9 - b*x**7
 + 1)), x)
```

3.2348 $\int \frac{b+ax^4}{\sqrt{-b+ax^4}\left(-b+c^2x^2+ax^4\right)}\,dx$

Optimal. Leaf size=415

$$\frac{i\left(\sqrt{2}c-(1-i)\sqrt{2\sqrt{a}\sqrt{b}+ic^2}\right)\sqrt{-(-1)^{3/4}c\sqrt{2\sqrt{a}\sqrt{b}+ic^2}-i\sqrt{a}\sqrt{b}+c^2}\tan^{-1}\left(\frac{\sqrt{2}x\sqrt{-(-1)^{3/4}c\sqrt{2\sqrt{a}\sqrt{b}+ic^2}-i\sqrt{a}\ldots}}{\sqrt{ax^4-b}+\sqrt{a}x^2+i\sqrt{b}}\right.}{2\sqrt{a}\sqrt{b}c}$$

Rubi [A] time = 0.15, antiderivative size = 22, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {2112, 205}

$$-\frac{\tan^{-1}\left(\frac{cx}{\sqrt{ax^4-b}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^4)/(Sqrt[-b + a*x^4]*(-b + c^2*x^2 + a*x^4)),x]
```

```
[Out] -(ArcTan[(c*x)/Sqrt[-b + a*x^4]]/c)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2112

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{b+ax^4}{\sqrt{-b+ax^4}\left(-b+c^2x^2+ax^4\right)}\,dx &= b\,\text{Subst}\left(\int \frac{1}{-b-bc^2x^2}\,dx, x, \frac{x}{\sqrt{-b+ax^4}}\right)\\ &= -\frac{\tan^{-1}\left(\frac{cx}{\sqrt{-b+ax^4}}\right)}{c}\end{aligned}$$

Mathematica [C] time = 1.08, size = 190, normalized size = 0.46

$$-\frac{i\sqrt{1-\frac{ax^4}{b}}\left(-\Pi\left(\frac{2\sqrt{a}\sqrt{b}}{\sqrt{c^4+4ab}-c^2};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{a}}{\sqrt{b}}}x\right)\middle|-1\right)-\Pi\left(-\frac{2\sqrt{a}\sqrt{b}}{c^2+\sqrt{c^4+4ab}};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{a}}{\sqrt{b}}}x\right)\middle|-1\right)+F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{a}}{\sqrt{b}}}x\right)\middle|-1\right)\right)}{\sqrt{-\frac{\sqrt{a}}{\sqrt{b}}}\sqrt{ax^4-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x^4)/(Sqrt[-b + a*x^4]*(-b + c^2*x^2 + a*x^4)),x]
```

```
[Out] ((-I)*Sqrt[1 - (a*x^4)/b]*(EllipticF[I*ArcSinh[Sqrt[-(Sqrt[a]/Sqrt[b])]*x],
 -1] - EllipticPi[(2*Sqrt[a]*Sqrt[b])/(-c^2 + Sqrt[4*a*b + c^4]), I*ArcSinh
```

[Sqrt[-(Sqrt[a]/Sqrt[b])]*x], -1] - EllipticPi[(-2*Sqrt[a]*Sqrt[b])/(c^2 + Sqrt[4*a*b + c^4]), I*ArcSinh[Sqrt[-(Sqrt[a]/Sqrt[b])]*x], -1]))/(Sqrt[-(Sqrt[a]/Sqrt[b])]*Sqrt[-b + a*x^4])

IntegrateAlgebraic [A] time = 1.23, size = 31, normalized size = 0.07

$$\frac{\tan^{-1}\left(\frac{cx\sqrt{ax^4-b}}{b-ax^4}\right)}{c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(b + a*x^4)/(Sqrt[-b + a*x^4]*(-b + c^2*x^2 + a*x^4)),x]

[Out] ArcTan[(c*x*Sqrt[-b + a*x^4])/(b - a*x^4)]/c

fricas [A] time = 1.39, size = 40, normalized size = 0.10

$$-\frac{\arctan\left(\frac{2\sqrt{ax^4-b}\,cx}{ax^4-c^2x^2-b}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^4+b)/(a*x^4-b)^(1/2)/(a*x^4+c^2*x^2-b),x, algorithm="fricas")

[Out] -1/2*arctan(2*sqrt(a*x^4 - b)*c*x/(a*x^4 - c^2*x^2 - b))/c

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4+b}{\left(ax^4+c^2x^2-b\right)\sqrt{ax^4-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^4+b)/(a*x^4-b)^(1/2)/(a*x^4+c^2*x^2-b),x, algorithm="giac")

[Out] integrate((a*x^4 + b)/((a*x^4 + c^2*x^2 - b)*sqrt(a*x^4 - b)), x)

maple [C] time = 0.07, size = 289, normalized size = 0.70

$$\frac{\sqrt{1+\frac{\sqrt{a}\,x^2}{\sqrt{b}}}\,\sqrt{1-\frac{\sqrt{a}\,x^2}{\sqrt{b}}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{\sqrt{a}}{\sqrt{b}}},i\right)}{\sqrt{-\frac{\sqrt{a}}{\sqrt{b}}}\,\sqrt{a\,x^4-b}}+\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(a\_Z^4+c^2\_Z^2-b\right)}\frac{\left(-c^2\_\alpha^2+2b\right)\left(-\frac{\operatorname{arctanh}\left(\frac{\_\alpha^2\left(-\_\alpha^2a+a\,x^2-c^2\right)}{\sqrt{-c^2\_\alpha^2}\,\sqrt{a\,x^4-b}}\right)}{\sqrt{-c^2\_\alpha^2}}-\frac{2\_\alpha\left(\_\alpha^2a+c^2\right)\sqrt{1+\frac{\sqrt{a}\,x^2}{\sqrt{b}}}\,\sqrt{1-\frac{\sqrt{a}\,x^2}{\sqrt{b}}}\,\mathrm{EllipticPi}\left(x\sqrt{-\frac{\sqrt{a}}{\sqrt{b}}},-\frac{\_\alpha^2a+c^2}{\sqrt{b}\,\sqrt{a}},\frac{\sqrt{\frac{\sqrt{a}}{\sqrt{b}}}}{\sqrt{-\frac{\sqrt{a}}{\sqrt{b}}}}\right)}{\sqrt{-\frac{\sqrt{a}}{\sqrt{b}}}\,b\sqrt{a\,x^4-b}}\right)}{\_\alpha\left(2\_\alpha^2a+c^2\right)}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^4+b)/(a*x^4-b)^(1/2)/(a*x^4+c^2*x^2-b),x)

[Out] 1/(-a^(1/2)/b^(1/2))^(1/2)*(1+a^(1/2)/b^(1/2)*x^2)^(1/2)*(1-a^(1/2)/b^(1/2)*x^2)^(1/2)/(a*x^4-b)^(1/2)*EllipticF(x*(-a^(1/2)/b^(1/2))^(1/2),I)+1/4*sum((-_alpha^2*c^2+2*b)/_alpha/(2*_alpha^2*a+c^2)*(-1/(-c^2*_alpha^2)^(1/2)*arctanh(_alpha^2*(-_alpha^2*a+a*x^2-c^2)/(-c^2*_alpha^2)^(1/2)/(a*x^4-b)^(1/2))-2/(-a^(1/2)/b^(1/2))^(1/2)*_alpha*(_alpha^2*a+c^2)/b*(1+a^(1/2)/b^(1/2)*x^2)^(1/2)*(1-a^(1/2)/b^(1/2)*x^2)^(1/2)/(a*x^4-b)^(1/2)*EllipticPi(x*(-a^(

```
1/2)/b^(1/2))^(1/2),-1/b^(1/2)/a^(1/2)*(_alpha^2*a+c^2),(a^(1/2)/b^(1/2))^(
1/2)/(-a^(1/2)/b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*a+_Z^2*c^2-b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4+b}{\left(ax^4+c^2x^2-b\right)\sqrt{ax^4-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)/(a*x^4-b)^(1/2)/(a*x^4+c^2*x^2-b),x, algorithm="maxima"
)
```

```
[Out] integrate((a*x^4 + b)/((a*x^4 + c^2*x^2 - b)*sqrt(a*x^4 - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x^4+b}{\sqrt{a\,x^4-b}\,\left(c^2\,x^2+a\,x^4-b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^4)/((a*x^4 - b)^(1/2)*(a*x^4 - b + c^2*x^2)),x)
```

```
[Out] int((b + a*x^4)/((a*x^4 - b)^(1/2)*(a*x^4 - b + c^2*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^4+b}{\sqrt{ax^4-b}\,\left(ax^4-b+c^2x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b)/(a*x**4-b)**(1/2)/(a*x**4+c**2*x**2-b),x)
```

```
[Out] Integral((a*x**4 + b)/(sqrt(a*x**4 - b)*(a*x**4 - b + c**2*x**2)), x)
```

3.2349 $\int \frac{x(-a+x)(ab-2bx+x^2)}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-b^2+2bx+(-1+a^2d)x^2-2adx^3+dx^4\right)}\,dx$

Optimal. Leaf size=415

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}}{\sqrt[6]{d}\sqrt[3]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}+2b-2x}\right)}{2d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}}{\sqrt[6]{d}\sqrt[3]{x^2(2ab+b^2)-ab^2x+x^3(-a-2b)+x^4}-2b+2x}\right)}{2d^{5/6}}+\tanh^{-}\ldots$$

Rubi [F] time = 17.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x(-a+x)(ab-2bx+x^2)}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-b^2+2bx+(-1+a^2d)x^2-2adx^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x*(-a + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2)^(1/3)*(-b^2 +
 2*b*x + (-1 + a^2*d)*x^2 - 2*a*d*x^3 + d*x^4)),x]
```

```
[Out] (6*b*x^(1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x^7*(-a
 + x^3)^(2/3))/((-b + x^3)^(2/3)*(b^2 - 2*b*x^3 + (1 - a^2*d)*x^6 + 2*a*d*x
^9 - d*x^12)), x], x, x^(1/3)])/(-((a - x)*(b - x)^2*x))^(1/3) + (3*a*b*x^(
1/3)*(-a + x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x^4*(-a + x^3)^
(2/3))/((-b + x^3)^(2/3)*(-b^2 + 2*b*x^3 - (1 - a^2*d)*x^6 - 2*a*d*x^9 + d*
x^12)), x], x, x^(1/3)])/(-((a - x)*(b - x)^2*x))^(1/3) + (3*x^(1/3)*(-a +
x)^(1/3)*(-b + x)^(2/3)*Defer[Subst][Defer[Int][(x^10*(-a + x^3)^(2/3))/((-
b + x^3)^(2/3)*(-b^2 + 2*b*x^3 - (1 - a^2*d)*x^6 - 2*a*d*x^9 + d*x^12)), x]
, x, x^(1/3)])/(-((a - x)*(b - x)^2*x))^(1/3)
```

Rubi steps

$$\int \frac{x(-a+x)(ab-2bx+x^2)}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-b^2+2bx+(-1+a^2d)x^2-2adx^3+dx^4\right)}\,dx = \frac{\left(\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\int \ldots}{\sqrt[3]{x(-a+x)(\ldots}}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\text{Subst}(\ldots}{\sqrt[3]{x}\ldots}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\text{Subst}(\ldots}{\ldots}$$

$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-a+x}(-b+x)^{2/3}\right)\text{Subst}(\ldots}{\sqrt[3]{x(\ldots}}$$

Mathematica [F] time = 1.62, size = 0, normalized size = 0.00

$$\int \frac{x(-a+x)(ab-2bx+x^2)}{\sqrt[3]{x(-a+x)(-b+x)^2}\left(-b^2+2bx+(-1+a^2d)x^2-2adx^3+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(-a + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2)^(1/3)*(
-b^2 + 2*b*x + (-1 + a^2*d)*x^2 - 2*a*d*x^3 + d*x^4)),x]
```

```
[Out] Integrate[(x*(-a + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2)^(1/3)*(
-b^2 + 2*b*x + (-1 + a^2*d)*x^2 - 2*a*d*x^3 + d*x^4)), x]
```

IntegrateAlgebraic [A] time = 2.31, size = 415, normalized size = 1.00

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}}{\sqrt[6]{d}\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}+2b-2x}\right)}{2d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}}{\sqrt[6]{d}\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}-2b+2x}\right)}{2d^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}}{b-x}\right)}{d^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[3]{x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4}\left(b\sqrt[6]{d}-\sqrt[6]{d}\,x\right)}{\sqrt[3]{d}\left(x^2\left(2ab+b^2\right)-ab^2x+x^3(-a-2b)+x^4\right)^{2/3}+b^2-2bx+x^2}\right)}{2d^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(-a + x)*(a*b - 2*b*x + x^2))/((x*(-a + x)*(-b + x)^2
)^(1/3)*(-b^2 + 2*b*x + (-1 + a^2*d)*x^2 - 2*a*d*x^3 + d*x^4)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/6)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a
 - 2*b)*x^3 + x^4)^(1/3))/(2*b - 2*x + d^(1/6)*(-(a*b^2*x) + (2*a*b + b^2)*
x^2 + (-a - 2*b)*x^3 + x^4)^(1/3))])/d^(5/6) + (Sqrt[3]*ArcTan[(Sqrt[3]*d^(
1/6)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3))/(-2*b +
 2*x + d^(1/6)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3
))])/(2*d^(5/6)) + ArcTanh[(d^(1/6)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a -
 2*b)*x^3 + x^4)^(1/3))/(b - x)]/d^(5/6) + ArcTanh[((b*d^(1/6) - d^(1/6)*x)
*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(1/3))/(b^2 - 2*b*
x + x^2 + d^(1/3)*(-(a*b^2*x) + (2*a*b + b^2)*x^2 + (-a - 2*b)*x^3 + x^4)^(
2/3))]/(2*d^(5/6))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-b^2+2*b*x+(a
^2*d-1)*x^2-2*a*d*x^3+d*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ab - 2\,bx + x^2\right)(a - x)x}{\left(2\,adx^3 - dx^4 - \left(a^2d - 1\right)x^2 + b^2 - 2\,bx\right)\left(-(a - x)(b - x)^2x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-b^2+2*b*x+(a
^2*d-1)*x^2-2*a*d*x^3+d*x^4),x, algorithm="giac")
```

```
[Out] integrate((a*b - 2*b*x + x^2)*(a - x)*x/((2*a*d*x^3 - d*x^4 - (a^2*d - 1)*x
^2 + b^2 - 2*b*x)*(-(a - x)*(b - x)^2*x)^(1/3)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x\left(-a + x\right)\left(ab - 2bx + x^2\right)}{\left(x\left(-a + x\right)\left(-b + x\right)^2\right)^{\frac{1}{3}}\left(-b^2 + 2bx + \left(a^2d - 1\right)x^2 - 2ad\,x^3 + d\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-b^2+2*b*x+(a^2*d-1
)*x^2-2*a*d*x^3+d*x^4),x)
```

```
[Out] int(x*(-a+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-b^2+2*b*x+(a^2*d-1
)*x^2-2*a*d*x^3+d*x^4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ab-2\,bx+x^2)(a-x)x}{(2\,adx^3-dx^4-(a^2d-1)x^2+b^2-2\,bx)\left(-(a-x)(b-x)^2x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+x)*(a*b-2*b*x+x^2)/(x*(-a+x)*(-b+x)^2)^(1/3)/(-b^2+2*b*x+(a
^2*d-1)*x^2-2*a*d*x^3+d*x^4),x, algorithm="maxima")
```

```
[Out] integrate((a*b - 2*b*x + x^2)*(a - x)*x/((2*a*d*x^3 - d*x^4 - (a^2*d - 1)*x
^2 + b^2 - 2*b*x)*(-(a - x)*(b - x)^2*x)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x\,(a-x)\,\left(x^2-2\,b\,x+a\,b\right)}{\left(-x\,(a-x)\,(b-x)^2\right)^{1/3}\,\left(-b^2+2\,b\,x+d\,x^4-2\,a\,d\,x^3+\left(a^2\,d-1\right)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a - x)*(a*b - 2*b*x + x^2))/((-x*(a - x)*(b - x)^2)^(1/3)*(x^2*(a^
2*d - 1) + 2*b*x + d*x^4 - b^2 - 2*a*d*x^3)),x)
```

```
[Out] int(-(x*(a - x)*(a*b - 2*b*x + x^2))/((-x*(a - x)*(b - x)^2)^(1/3)*(x^2*(a^
2*d - 1) + 2*b*x + d*x^4 - b^2 - 2*a*d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+x)*(a*b-2*b*x+x**2)/(x*(-a+x)*(-b+x)**2)**(1/3)/(-b**2+2*b*
x+(a**2*d-1)*x**2-2*a*d*x**3+d*x**4),x)
```

```
[Out] Timed out
```

3.2350 $\int \frac{b+ax}{(-b+ax)\sqrt[3]{b^2x^2+a^3x^3}}\,dx$

Optimal. Leaf size=417

$$-\frac{\log\left(\sqrt[3]{a^3x^3+b^2x^2}-ax\right)}{a}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}ax}{2\sqrt[3]{a^3x^3+b^2x^2}+ax}\right)}{a}-\frac{i\left(\sqrt{3}-i\right)\log\left(\sqrt[3]{-1}\sqrt[3]{a^3x^3+b^2x^2}+\sqrt[3]{a}\,x\sqrt[3]{a^2+b}\right)}{\sqrt[3]{a}\sqrt[3]{a^2+b}}+\cdots$$

Rubi [A] time = 0.22, antiderivative size = 453, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2056, 157, 59, 91}

$$-\frac{x^{2/3}\log(x)\sqrt[3]{a^3x+b^2}}{2a\sqrt[3]{a^3x^3+b^2x^2}}-\frac{3x^{2/3}\sqrt[3]{a^3x+b^2}\log\left(\frac{\sqrt[3]{a^3x+b^2}}{a\sqrt[3]{x}}-1\right)}{2a\sqrt[3]{a^3x^3+b^2x^2}}-\frac{\sqrt{3}x^{2/3}\sqrt[3]{a^3x+b^2}\tan^{-1}\left(\frac{2\sqrt[3]{a^3x+b^2}}{\sqrt{3}a\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{a\sqrt[3]{a^3x^3+b^2x^2}}-\frac{x^{2/3}\sqrt[3]{a^3x+b^2}\log(ax-b)}{\sqrt[3]{a}\sqrt[3]{a^2+b}\sqrt[3]{a^3x^3+b^2x^2}}+\frac{3x^{2/3}\sqrt[3]{a^3x+b^2}\log\left(\frac{\sqrt[3]{a^3x+b^2}}{\sqrt[3]{a}\sqrt[3]{a^2+b}}-\sqrt[3]{x}\right)}{\sqrt[3]{a}\sqrt[3]{a^2+b}\sqrt[3]{a^3x^3+b^2x^2}}+\frac{2\sqrt{3}x^{2/3}\sqrt[3]{a^3x+b^2}\tan^{-1}\left(\frac{2\sqrt[3]{a^3x+b^2}}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{a^2+b}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[3]{a^2+b}\sqrt[3]{a^3x^3+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x)/((-b + a*x)*(b^2*x^2 + a^3*x^3)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*x^(2/3)*(b^2 + a^3*x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(b^2 + a^3*x)^
(1/3))/(Sqrt[3]*a*x^(1/3))]])/(a*(b^2*x^2 + a^3*x^3)^(1/3))) + (2*Sqrt[3]*x^
(2/3)*(b^2 + a^3*x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(b^2 + a^3*x)^(1/3))/(Sqrt[
3]*a^(1/3)*(a^2 + b)^(1/3)*x^(1/3))]])/(a^(1/3)*(a^2 + b)^(1/3)*(b^2*x^2 + a
^3*x^3)^(1/3)) - (x^(2/3)*(b^2 + a^3*x)^(1/3)*Log[x])/(2*a*(b^2*x^2 + a^3*x
^3)^(1/3)) - (x^(2/3)*(b^2 + a^3*x)^(1/3)*Log[-b + a*x])/(a^(1/3)*(a^2 + b)
^(1/3)*(b^2*x^2 + a^3*x^3)^(1/3)) + (3*x^(2/3)*(b^2 + a^3*x)^(1/3)*Log[-x^(
1/3) + (b^2 + a^3*x)^(1/3)/(a^(1/3)*(a^2 + b)^(1/3))]])/(a^(1/3)*(a^2 + b)^(
1/3)*(b^2*x^2 + a^3*x^3)^(1/3)) - (3*x^(2/3)*(b^2 + a^3*x)^(1/3)*Log[-1 + (
b^2 + a^3*x)^(1/3)/(a*x^(1/3))]])/(2*a*(b^2*x^2 + a^3*x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
```

```
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{b+ax}{(-b+ax)\sqrt[3]{b^2x^2+a^3x^3}}\,dx = \frac{\left(x^{2/3}\sqrt[3]{b^2+a^3x}\right)\int \frac{b+ax}{x^{2/3}(-b+ax)\sqrt[3]{b^2+a^3x}}\,dx}{\sqrt[3]{b^2x^2+a^3x^3}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{b^2+a^3x}\right)\int \frac{1}{x^{2/3}\sqrt[3]{b^2+a^3x}}\,dx}{\sqrt[3]{b^2x^2+a^3x^3}} + \frac{\left(2bx^{2/3}\sqrt[3]{b^2+a^3x}\right)\int \frac{1}{x^{2/3}(-b+ax)\sqrt[3]{b^2+a^3x}}\,d}{\sqrt[3]{b^2x^2+a^3x^3}}$$

$$= -\frac{\sqrt{3}\,x^{2/3}\sqrt[3]{b^2+a^3x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b^2+a^3x}}{\sqrt{3}\,a\sqrt[3]{x}}\right)}{a\sqrt[3]{b^2x^2+a^3x^3}} + \frac{2\sqrt{3}\,x^{2/3}\sqrt[3]{b^2+a^3x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}\right.}{\sqrt[3]{a}\,\sqrt[3]{a^2+b}\,\sqrt[3]{b^2x^2}}$$

Mathematica [C] time = 0.06, size = 85, normalized size = 0.20

$$\frac{3x\sqrt[3]{\frac{a^3x}{b^2}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{a^3x}{b^2}\right)-6x\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{a\left(a^2+b\right)x}{xa^3+b^2}\right)}{\sqrt[3]{x^2\left(a^3x+b^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x)/((-b + a*x)*(b^2*x^2 + a^3*x^3)^(1/3)),x]
```

```
[Out] (3*x*(1 + (a^3*x)/b^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, -((a^3*x)/b^2
)] - 6*x*Hypergeometric2F1[1/3, 1, 4/3, (a*(a^2 + b)*x)/(b^2 + a^3*x)])/(x^
2*(b^2 + a^3*x))^(1/3)
```

IntegrateAlgebraic [A] time = 3.13, size = 466, normalized size = 1.12

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}ax}{2\sqrt[3]{a^3x^3+b^2x^2}+ax}\right)}{a}-\frac{i\left(\sqrt{3}-i\right)\log\left(2\sqrt[3]{a}\,x\sqrt[3]{a^2+b}+\left(1+i\sqrt{3}\right)\sqrt[3]{a^3x^3+b^2x^2}\right)}{\sqrt[3]{a}\sqrt[3]{a^2+b}}-\frac{\log\left(a^2x-a\sqrt[3]{a^3x^3+b^2x^2}\right)}{a}+\frac{\log\left(ax\sqrt[3]{a^3x^3+b^2x^2}+\left(a^3x^3+b^2x^2\right)^{2/3}+a^2x^2\right)}{2a}+\frac{\sqrt{6\left(-1+i\sqrt{3}\right)}\tan^{-1}\left(\frac{3\sqrt[3]{a}\,x\sqrt[3]{a^2+b}}{-\sqrt{3}\sqrt[3]{a^3x^3+b^2x^2}-3i\sqrt[3]{a^3x^3+b^2x^2}+\sqrt{3}\sqrt[3]{a}\,x\sqrt[3]{a^2+b}}\right)}{\sqrt[3]{a}\sqrt[3]{a^2+b}}+\frac{\left(1+i\sqrt{3}\right)\log\left(\left(\sqrt{3}+i\right)\left(a^3x^3+b^2x^2\right)^{2/3}-2ia^{2/3}x^2\left(a^2+b\right)^{2/3}+\sqrt[3]{a}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a^2+b}\sqrt[3]{a^3x^3+b^2x^2}\right)}{2\sqrt[3]{a}\sqrt[3]{a^2+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x)/((-b + a*x)*(b^2*x^2 + a^3*x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*a*x)/(a*x + 2*(b^2*x^2 + a^3*x^3)^(1/3))])/a + (Sq
rt[6*(-1 + I*Sqrt[3])]*ArcTan[(3*a^(1/3)*(a^2 + b)^(1/3)*x)/(Sqrt[3]*a^(1/3
)*(a^2 + b)^(1/3)*x - (3*I)*(b^2*x^2 + a^3*x^3)^(1/3) - Sqrt[3]*(b^2*x^2 +
a^3*x^3)^(1/3))])/(a^(1/3)*(a^2 + b)^(1/3)) - (I*(-I + Sqrt[3])*Log[2*a^(1/
3)*(a^2 + b)^(1/3)*x + (1 + I*Sqrt[3])*(b^2*x^2 + a^3*x^3)^(1/3)])/(a^(1/3)
*(a^2 + b)^(1/3)) - Log[a^2*x - a*(b^2*x^2 + a^3*x^3)^(1/3)]/a + Log[a^2*x^
2 + a*x*(b^2*x^2 + a^3*x^3)^(1/3) + (b^2*x^2 + a^3*x^3)^(2/3)]/(2*a) + ((1
+ I*Sqrt[3])*Log[(-2*I)*a^(2/3)*(a^2 + b)^(2/3)*x^2 + a^(1/3)*(a^2 + b)^(1/
3)*(I*x - Sqrt[3]*x)*(b^2*x^2 + a^3*x^3)^(1/3) + (I + Sqrt[3])*(b^2*x^2 + a
^3*x^3)^(2/3)])/(2*a^(1/3)*(a^2 + b)^(1/3))
```

fricas [A] time = 0.49, size = 790, normalized size = 1.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(a*x-b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(3)*(a^3 + a*b)*sqrt(-1/(a^3 + a*b)^(2/3))*log(-(2*b^2*x + (3*a
^3 + a*b)*x^2 - 3*(a^3*x^3 + b^2*x^2)^(1/3)*(a^3 + a*b)^(2/3)*x - sqrt(3)*(
```

```
(a^3 + a*b)^(4/3)*x^2 + (a^3*x^3 + b^2*x^2)^(1/3)*(a^3 + a*b)*x - 2*(a^3*x^
3 + b^2*x^2)^(2/3)*(a^3 + a*b)^(2/3))*sqrt(-1/(a^3 + a*b)^(2/3)))/(a*x^2 -
b*x)) - 2*sqrt(3)*(a^2 + b)*arctan(1/3*(sqrt(3)*a*x + 2*sqrt(3)*(a^3*x^3 +
b^2*x^2)^(1/3))/(a*x)) - 2*(a^2 + b)*log(-(a*x - (a^3*x^3 + b^2*x^2)^(1/3))
/x) + (a^2 + b)*log((a^2*x^2 + (a^3*x^3 + b^2*x^2)^(1/3)*a*x + (a^3*x^3 + b
^2*x^2)^(2/3))/x^2) + 4*(a^3 + a*b)^(2/3)*log(-((a^3 + a*b)^(1/3)*x - (a^3*
x^3 + b^2*x^2)^(1/3))/x) - 2*(a^3 + a*b)^(2/3)*log(((a^3 + a*b)^(2/3)*x^2 +
 (a^3*x^3 + b^2*x^2)^(1/3)*(a^3 + a*b)^(1/3)*x + (a^3*x^3 + b^2*x^2)^(2/3))
/x^2))/(a^3 + a*b), -1/2*(2*sqrt(3)*(a^2 + b)*arctan(1/3*(sqrt(3)*a*x + 2*s
qrt(3)*(a^3*x^3 + b^2*x^2)^(1/3))/(a*x)) - 4*sqrt(3)*(a^3 + a*b)^(2/3)*arct
an(1/3*sqrt(3)*((a^3 + a*b)^(1/3)*x + 2*(a^3*x^3 + b^2*x^2)^(1/3))/((a^3 +
a*b)^(1/3)*x)) + 2*(a^2 + b)*log(-(a*x - (a^3*x^3 + b^2*x^2)^(1/3))/x) - (a
^2 + b)*log((a^2*x^2 + (a^3*x^3 + b^2*x^2)^(1/3)*a*x + (a^3*x^3 + b^2*x^2)^
(2/3))/x^2) - 4*(a^3 + a*b)^(2/3)*log(-((a^3 + a*b)^(1/3)*x - (a^3*x^3 + b^
2*x^2)^(1/3))/x) + 2*(a^3 + a*b)^(2/3)*log(((a^3 + a*b)^(2/3)*x^2 + (a^3*x^
3 + b^2*x^2)^(1/3)*(a^3 + a*b)^(1/3)*x + (a^3*x^3 + b^2*x^2)^(2/3))/x^2))/(
a^3 + a*b)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(a*x-b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{ax+b}{(ax-b)\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)/(a*x-b)/(a^3*x^3+b^2*x^2)^(1/3),x)
[Out] int((a*x+b)/(a*x-b)/(a^3*x^3+b^2*x^2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+b}{\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}(ax-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(a*x-b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="maxima")
[Out] integrate((a*x + b)/((a^3*x^3 + b^2*x^2)^(1/3)*(a*x - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{b+a\,x}{\left(a^3\,x^3+b^2\,x^2\right)^{1/3}\,\left(b-a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x)/((a^3*x^3 + b^2*x^2)^(1/3)*(b - a*x)),x)
[Out] int(-(b + a*x)/((a^3*x^3 + b^2*x^2)^(1/3)*(b - a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+b}{\sqrt[3]{x^2\left(a^3x+b^2\right)}\,(ax-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(a*x-b)/(a**3*x**3+b**2*x**2)**(1/3),x)
```

```
[Out] Integral((a*x + b)/((x**2*(a**3*x + b**2))**(1/3)*(a*x - b)), x)
```

3.2351 $\int \frac{1}{\sqrt[3]{(-a+x)(-b+x)^2}\,(b-ad+(-1+d)x)}\,dx$

Optimal. Leaf size=423

$$\frac{\log\left(\sqrt[3]{d}\,(a-b)^{2/3}\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}-x(a-b)^{2/3}+b(a-b)^{2/3}\right)}{d^{2/3}(a-b)}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{x\left(2ab+\right.}}{\sqrt[3]{d}\,\sqrt[3]{x\left(2ab+b^2\right)-a}}\right.}{d^{2/3}(a-}$$

Rubi [A] time = 0.68, antiderivative size = 376, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2081, 2077, 91}

$$-\frac{\left((a-b)^2(b-x)\right)^{2/3}\sqrt[3]{(a-b)^2(x-a)}\,\log(-ad+b+(d-1)x)}{2d^{2/3}(a-b)^3\sqrt[3]{-ab^2+x^2(-a-2b)+bx(2a+b)+x^3}}+\frac{3\left((a-b)^2(b-x)\right)^{2/3}\sqrt[3]{(a-b)^2(x-a)}\,\log\left(-\sqrt[3]{\frac{2}{3}}\sqrt[3]{d}\sqrt[3]{(a-b)^2(x-a)}-\sqrt[3]{\frac{2}{3}}\sqrt[3]{(a-b)^2(b-x)}\right)}{2d^{2/3}(a-b)^3\sqrt[3]{-ab^2+x^2(-a-2b)+bx(2a+b)+x^3}}+\frac{\sqrt{3}\left((a-b)^2(b-x)\right)^{2/3}\sqrt[3]{(a-b)^2(x-a)}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{(a-b)^2(x-a)}}{\sqrt{3}\sqrt[3]{(a-b)^2(b-x)}}\right)}{d^{2/3}(a-b)^3\sqrt[3]{-ab^2+x^2(-a-2b)+bx(2a+b)+x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(((-a + x)*(-b + x)^2)^(1/3)*(b - a*d + (-1 + d)*x)),x]
```

```
[Out] (Sqrt[3]*((a - b)^2*(b - x))^(2/3)*((a - b)^2*(-a + x))^(1/3)*ArcTan[1/Sqrt
[3] - (2*d^(1/3)*((a - b)^2*(-a + x))^(1/3))/(Sqrt[3]*((a - b)^2*(b - x))^(
1/3))])/((a - b)^3*d^(2/3)*(-(a*b^2) + b*(2*a + b)*x + (-a - 2*b)*x^2 + x^3
)^(1/3)) - (((a - b)^2*(b - x))^(2/3)*((a - b)^2*(-a + x))^(1/3)*Log[b - a*
d + (-1 + d)*x])/(2*(a - b)^3*d^(2/3)*(-(a*b^2) + b*(2*a + b)*x + (-a - 2*b
)*x^2 + x^3)^(1/3)) + (3*((a - b)^2*(b - x))^(2/3)*((a - b)^2*(-a + x))^(1/
3)*Log[-((2/3)^(1/3)*((a - b)^2*(b - x))^(1/3)) - (2/3)^(1/3)*d^(1/3)*((a -
 b)^2*(-a + x))^(1/3)]])/(2*(a - b)^3*d^(2/3)*(-(a*b^2) + b*(2*a + b)*x + (-
a - 2*b)*x^2 + x^3)^(1/3))
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 2077

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Dist[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int
[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d, e
, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 2081

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{(-a+x)(-b+x)^2}\,(b-ad+(-1+d)x)}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{3}(-((-a-2b)(-1+d))+3(b-ad))+(-1+d)x\right.}\right.$$

$$= \frac{\left(2^{2/3}\sqrt[3]{-(a-b)^2(a-x)}\,\left((a-b)^2(b-x)\right)^{2/3}\right)\text{Subst}\left(\int \frac{}{\left(-\frac{2}{9}(a-b\right.}\right.}{3\sqrt[3]{-ab^2}}$$

$$= \frac{\sqrt{3}\,\sqrt[3]{-(a-b)^2(a-x)}\,\left((a-b)^2(b-x)\right)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\,\sqrt[3]{}}{\sqrt{3}\,\sqrt[3]{}}\right.}{(a-b)^3d^{2/3}\sqrt[3]{-ab^2+b(2a+b)x-(a+2b)x^2+x^3}}$$

Mathematica [C] time = 0.06, size = 58, normalized size = 0.14

$$-\frac{3(x-b)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{b-x}{ad-dx}\right)}{d(a-b)\sqrt[3]{(x-a)(b-x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((-a + x)*(-b + x)^2)^(1/3)*(b - a*d + (-1 + d)*x)),x]
```

```
[Out] (-3*(-b + x)*Hypergeometric2F1[1/3, 1, 4/3, (b - x)/(a*d - d*x)])/((a - b)*
d*((b - x)^2*(-a + x))^(1/3))
```

IntegrateAlgebraic [A] time = 3.19, size = 423, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{d}\,(a-b)^{2/3}\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}-x(a-b)^{2/3}+b(a-b)^{2/3}\right)}{d^{2/3}(a-b)}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}\sqrt[3]{x(2ab+b^2)-ab^2+x^2(-a-2b)+x^3}}{\sqrt[3]{d}\sqrt[3]{x(2ab+b^2)-ab^2+x^2(-a-2b)+x^3}-2b+2x}\right)}{d^{2/3}(a-b)}-\frac{\log\left(b^3\left(-\sqrt[3]{a-b}\right)+d^{2/3}(a-b)^{4/3}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}+\sqrt[3]{d}\sqrt[3]{a-b}\left(-ab+ax+b^2-bx\right)\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}+2b^2x\sqrt[3]{a-b}+ab^2\sqrt[3]{a-b}-bx^2\sqrt[3]{a-b}+ax^2\sqrt[3]{a-b}-2abx\sqrt[3]{a-b}\right)}{2d^{2/3}(a-b)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(((-a + x)*(-b + x)^2)^(1/3)*(b - a*d + (-1 + d)*x)),x
]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x
^2 + x^3)^(1/3))/(-2*b + 2*x + d^(1/3)*(-(a*b^2) + (2*a*b + b^2)*x + (-a -
2*b)*x^2 + x^3)^(1/3))])/((a - b)*d^(2/3)) + Log[(a - b)^(2/3)*b - (a - b)^
(2/3)*x + (a - b)^(2/3)*d^(1/3)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^
2 + x^3)^(1/3)]/((a - b)*d^(2/3)) - Log[a*(a - b)^(1/3)*b^2 - (a - b)^(1/3)
*b^3 - 2*a*(a - b)^(1/3)*b*x + 2*(a - b)^(1/3)*b^2*x + a*(a - b)^(1/3)*x^2
- (a - b)^(1/3)*b*x^2 + (a - b)^(1/3)*d^(1/3)*(-(a*b) + b^2 + a*x - b*x)*(-
(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3) + (a - b)^(4/3)*d^(
2/3)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(2/3)]/(2*(a - b)*
d^(2/3))
```

fricas [A] time = 0.41, size = 301, normalized size = 0.71

$$\frac{2\sqrt{3}d\sqrt{-\left(-d^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{3}\left(\left(-d^2\right)^{\frac{1}{3}}(b-x)+2\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}d\right)\sqrt{-\left(-d^2\right)^{\frac{1}{3}}}}{3(bd-dx)}\right)-\left(-d^2\right)^{\frac{2}{3}}\log\left(\frac{\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{2}{3}}d^2+\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}(bd-dx)\left(-d^2\right)^{\frac{1}{3}}+\left(b^2-2bx+x^2\right)\left(-d^2\right)^{\frac{2}{3}}}{b^2-2bx+x^2}\right)+2\left(-d^2\right)^{\frac{2}{3}}\log\left(\frac{\left(-d^2\right)^{\frac{1}{3}}(b-x)-\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}d}{b-x}\right)}{2(a-b)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-a+x)*(-b+x)^2)^(1/3)/(b-a*d+(-1+d)*x),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*d*sqrt(-(-d^2)^(1/3))*arctan(-1/3*sqrt(3)*((-d^2)^(1/3)*(b -
 x) + 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*d)*sqrt(-(-d
^2)^(1/3))/(b*d - d*x)) - (-d^2)^(2/3)*log(((-a*b^2 - (a + 2*b)*x^2 + x^3 +
```

```
 (2*a*b + b^2)*x)^(2/3)*d^2 + (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)
*x)^(1/3)*(b*d - d*x)*(-d^2)^(1/3) + (b^2 - 2*b*x + x^2)*(-d^2)^(2/3))/(b^2
 - 2*b*x + x^2)) + 2*(-d^2)^(2/3)*log(((-d^2)^(1/3)*(b - x) - (-a*b^2 - (a
+ 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*d)/(b - x)))/((a - b)*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}(ad-(d-1)x-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-a+x)*(-b+x)^2)^(1/3)/(b-a*d+(-1+d)*x),x, algorithm="giac")
[Out] integrate(-1/((-(a - x)*(b - x)^2)^(1/3)*(a*d - (d - 1)*x - b)), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((-a+x)(-b+x)^2\right)^{\frac{1}{3}}(b-ad+(-1+d)x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-a+x)*(-b+x)^2)^(1/3)/(b-a*d+(-1+d)*x),x)
[Out] int(1/((-a+x)*(-b+x)^2)^(1/3)/(b-a*d+(-1+d)*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}(ad-(d-1)x-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-a+x)*(-b+x)^2)^(1/3)/(b-a*d+(-1+d)*x),x, algorithm="maxima")
[Out] -integrate(1/((-(a - x)*(b - x)^2)^(1/3)*(a*d - (d - 1)*x - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(-(a-x)\,(b-x)^2\right)^{1/3}\,(b-a\,d+x\,(d-1))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-(a - x)*(b - x)^2)^(1/3)*(b - a*d + x*(d - 1))),x)
[Out] int(1/((-(a - x)*(b - x)^2)^(1/3)*(b - a*d + x*(d - 1))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(-a+x)(-b+x)^2}\,(-ad+b+dx-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-a+x)*(-b+x)**2)**(1/3)/(b-a*d+(-1+d)*x),x)
[Out] Integral(1/(((-a + x)*(-b + x)**2)**(1/3)*(-a*d + b + d*x - x)), x)
```

3.2352 $\int \frac{(-4-3x+2x^2)(1+x-x^2+x^4)\sqrt[3]{\frac{1+x-x^2+2x^4}{1+x-x^2+3x^4}}}{x^5(-1-x+x^2+x^4)}\,dx$

Optimal. Leaf size=423

$$\frac{\sqrt[3]{\frac{2x^4-x^2+x+1}{3x^4-x^2+x+1}}\left(-3x^4+x^2-x-1\right)}{x^4}+\frac{5}{3}\log\left(\sqrt[3]{\frac{2x^4-x^2+x+1}{3x^4-x^2+x+1}}-1\right)-\sqrt[3]{6}\log\left(6^{2/3}\sqrt[3]{\frac{2x^4-x^2+x+1}{3x^4-x^2+x+1}}-3\right)-\frac{5}{6}1$$

Rubi [F] time = 10.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(-4-3x+2x^2\right)\left(1+x-x^2+x^4\right)\sqrt[3]{\frac{1+x-x^2+2x^4}{1+x-x^2+3x^4}}}{x^5\left(-1-x+x^2+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-4 - 3*x + 2*x^2)*(1 + x - x^2 + x^4)*((1 + x - x^2 + 2*x^4)/(1 + x -
 x^2 + 3*x^4))^(1/3))/(x^5*(-1 - x + x^2 + x^4)),x]
```

```
[Out] (-2*((1 + x - x^2 + 2*x^4)/(1 + x - x^2 + 3*x^4))^(1/3)*(1 + x - x^2 + 3*x^
4)^(1/3)*Defer[Int][(1 + x - x^2 + 2*x^4)^(1/3)/((-1 + x)*(1 + x - x^2 + 3*
x^4)^(1/3)), x])/(1 + x - x^2 + 2*x^4)^(1/3) + (4*((1 + x - x^2 + 2*x^4)/(1
 + x - x^2 + 3*x^4))^(1/3)*(1 + x - x^2 + 3*x^4)^(1/3)*Defer[Int][(1 + x -
x^2 + 2*x^4)^(1/3)/(x^5*(1 + x - x^2 + 3*x^4)^(1/3)), x])/(1 + x - x^2 + 2*
x^4)^(1/3) + (3*((1 + x - x^2 + 2*x^4)/(1 + x - x^2 + 3*x^4))^(1/3)*(1 + x
- x^2 + 3*x^4)^(1/3)*Defer[Int][(1 + x - x^2 + 2*x^4)^(1/3)/(x^4*(1 + x - x
^2 + 3*x^4)^(1/3)), x])/(1 + x - x^2 + 2*x^4)^(1/3) - (2*((1 + x - x^2 + 2*
x^4)/(1 + x - x^2 + 3*x^4))^(1/3)*(1 + x - x^2 + 3*x^4)^(1/3)*Defer[Int][(1
 + x - x^2 + 2*x^4)^(1/3)/(x^3*(1 + x - x^2 + 3*x^4)^(1/3)), x])/(1 + x - x
^2 + 2*x^4)^(1/3) + (8*((1 + x - x^2 + 2*x^4)/(1 + x - x^2 + 3*x^4))^(1/3)*
(1 + x - x^2 + 3*x^4)^(1/3)*Defer[Int][(1 + x - x^2 + 2*x^4)^(1/3)/(x*(1 +
x - x^2 + 3*x^4)^(1/3)), x])/(1 + x - x^2 + 2*x^4)^(1/3) - (4*((1 + x - x^2
 + 2*x^4)/(1 + x - x^2 + 3*x^4))^(1/3)*(1 + x - x^2 + 3*x^4)^(1/3)*Defer[In
t][(1 + x - x^2 + 2*x^4)^(1/3)/((1 + 2*x + x^2 + x^3)*(1 + x - x^2 + 3*x^4)
^(1/3)), x])/(1 + x - x^2 + 2*x^4)^(1/3) - (4*((1 + x - x^2 + 2*x^4)/(1 + x
 - x^2 + 3*x^4))^(1/3)*(1 + x - x^2 + 3*x^4)^(1/3)*Defer[Int][(x*(1 + x - x
^2 + 2*x^4)^(1/3))/((1 + 2*x + x^2 + x^3)*(1 + x - x^2 + 3*x^4)^(1/3)), x])
/(1 + x - x^2 + 2*x^4)^(1/3) - (6*((1 + x - x^2 + 2*x^4)/(1 + x - x^2 + 3*x
^4))^(1/3)*(1 + x - x^2 + 3*x^4)^(1/3)*Defer[Int][(x^2*(1 + x - x^2 + 2*x^4
)^(1/3))/((1 + 2*x + x^2 + x^3)*(1 + x - x^2 + 3*x^4)^(1/3)), x])/(1 + x -
x^2 + 2*x^4)^(1/3)
```

Rubi steps

$$\int \frac{\left(-4-3x+2x^2\right)\left(1+x-x^2+x^4\right)\sqrt[3]{\frac{1+x-x^2+2x^4}{1+x-x^2+3x^4}}}{x^5\left(-1-x+x^2+x^4\right)}\,dx = \frac{\left(\sqrt[3]{\frac{1+x-x^2+2x^4}{1+x-x^2+3x^4}}\,\sqrt[3]{1+x-x^2+3x^4}\right)\int \frac{\left(-4-3x+2x^2\right)\left(1+x-x^2+x^4\right.}{x^5\left(-1-x+x^2+x^4\right)\sqrt[3]{1\cdot}}}{\sqrt[3]{1+x-x^2+2x^4}}$$

$$= \frac{\left(\sqrt[3]{\frac{1+x-x^2+2x^4}{1+x-x^2+3x^4}}\,\sqrt[3]{1+x-x^2+3x^4}\right)\int\left(-\frac{2\sqrt[3]{1+x-x^2+2x^4}}{(-1+x)\sqrt[3]{1+x-x^2+3x^4}}\right.}{}$$

$$= -\frac{\left(2\sqrt[3]{\frac{1+x-x^2+2x^4}{1+x-x^2+3x^4}}\,\sqrt[3]{1+x-x^2+3x^4}\right)\int\frac{\sqrt[3]{1+x-x^2+2x^4}}{(-1+x)\sqrt[3]{1+x-x^2+3x^4}}}{\sqrt[3]{1+x-x^2+2x^4}}$$

$$= -\frac{\left(2\sqrt[3]{\frac{1+x-x^2+2x^4}{1+x-x^2+3x^4}}\,\sqrt[3]{1+x-x^2+3x^4}\right)\int\frac{\sqrt[3]{1+x-x^2+2x^4}}{(-1+x)\sqrt[3]{1+x-x^2+3x^4}}}{\sqrt[3]{1+x-x^2+2x^4}}$$

$$= -\frac{\left(2\sqrt[3]{\frac{1+x-x^2+2x^4}{1+x-x^2+3x^4}}\,\sqrt[3]{1+x-x^2+3x^4}\right)\int\frac{\sqrt[3]{1+x-x^2+2x^4}}{(-1+x)\sqrt[3]{1+x-x^2+3x^4}}}{\sqrt[3]{1+x-x^2+2x^4}}$$

Mathematica [F] time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{\left(-4-3x+2x^2\right)\left(1+x-x^2+x^4\right)\sqrt[3]{\frac{1+x-x^2+2x^4}{1+x-x^2+3x^4}}}{x^5\left(-1-x+x^2+x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-4 - 3*x + 2*x^2)*(1 + x - x^2 + x^4)*((1 + x - x^2 + 2*x^4)/(1
 + x - x^2 + 3*x^4))^(1/3))/(x^5*(-1 - x + x^2 + x^4)),x]
```

```
[Out] Integrate[((-4 - 3*x + 2*x^2)*(1 + x - x^2 + x^4)*((1 + x - x^2 + 2*x^4)/(1
 + x - x^2 + 3*x^4))^(1/3))/(x^5*(-1 - x + x^2 + x^4)), x]
```

IntegrateAlgebraic [A] time = 1.72, size = 423, normalized size = 1.00

$$\frac{\sqrt[3]{\frac{2x^4-x^2+x+1}{3x^4-x^2+x+1}}\left(-3x^4+x^2-x-1\right)}{x^4}+\frac{5}{3}\log\left(\sqrt[3]{\frac{2x^4-x^2+x+1}{3x^4-x^2+x+1}}-1\right)-\sqrt[3]{6}\log\left(6^{2/3}\sqrt[3]{\frac{2x^4-x^2+x+1}{3x^4-x^2+x+1}}-3\right)-\frac{5}{6}\log\left(\left(\frac{2x^4-x^2+x+1}{3x^4-x^2+x+1}\right)^{2/3}+\sqrt[3]{\frac{2x^4-x^2+x+1}{3x^4-x^2+x+1}}+1\right)+\frac{\sqrt[3]{3}\log\left(2\sqrt[3]{6}\left(\frac{2x^4-x^2+x+1}{3x^4-x^2+x+1}\right)^{2/3}+6^{2/3}\sqrt[3]{\frac{2x^4-x^2+x+1}{3x^4-x^2+x+1}}+3\right)}{2^{2/3}}+\sqrt[3]{2}\,3^{5/6}\tan^{-1}\left(\frac{2\,2^{2/3}\sqrt[3]{\frac{2x^4-x^2+x+1}{3x^4-x^2+x+1}}}{3^{5/6}}+\frac{1}{\sqrt{3}}\right)-\frac{5\tan^{-1}\left(\frac{2\sqrt[3]{\frac{2x^4-x^2+x+1}{3x^4-x^2+x+1}}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-4 - 3*x + 2*x^2)*(1 + x - x^2 + x^4)*((1 + x - x^2 +
2*x^4)/(1 + x - x^2 + 3*x^4))^(1/3))/(x^5*(-1 - x + x^2 + x^4)),x]
```

```
[Out] ((-1 - x + x^2 - 3*x^4)*((1 + x - x^2 + 2*x^4)/(1 + x - x^2 + 3*x^4))^(1/3)
)/x^4 + 2^(1/3)*3^(5/6)*ArcTan[1/Sqrt[3] + (2*2^(2/3)*((1 + x - x^2 + 2*x^4
)/(1 + x - x^2 + 3*x^4))^(1/3))/3^(5/6)] - (5*ArcTan[1/Sqrt[3] + (2*((1 + x
 - x^2 + 2*x^4)/(1 + x - x^2 + 3*x^4))^(1/3))/Sqrt[3]])/Sqrt[3] + (5*Log[-1
 + ((1 + x - x^2 + 2*x^4)/(1 + x - x^2 + 3*x^4))^(1/3)])/3 - 6^(1/3)*Log[-3
 + 6^(2/3)*((1 + x - x^2 + 2*x^4)/(1 + x - x^2 + 3*x^4))^(1/3)] - (5*Log[1
+ ((1 + x - x^2 + 2*x^4)/(1 + x - x^2 + 3*x^4))^(1/3) + ((1 + x - x^2 + 2*x
^4)/(1 + x - x^2 + 3*x^4))^(2/3)])/6 + (3^(1/3)*Log[3 + 6^(2/3)*((1 + x - x
^2 + 2*x^4)/(1 + x - x^2 + 3*x^4))^(1/3) + 2*6^(1/3)*((1 + x - x^2 + 2*x^4)
/(1 + x - x^2 + 3*x^4))^(2/3)])/2^(2/3)
```

fricas [B] time = 63.51, size = 960, normalized size = 2.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-3*x-4)*(x^4-x^2+x+1)*((2*x^4-x^2+x+1)/(3*x^4-x^2+x+1))^(1/
3)/x^5/(x^4+x^2-x-1),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-6)^(1/3)*x^4*arctan(1/3*(6*sqrt(3)*(-6)^(2/3)*(1947*x^12
- 2263*x^10 + 2263*x^9 + 3128*x^8 - 1730*x^7 - 974*x^6 + 2057*x^5 + 865*x^4
 - 545*x^3 + 327*x + 109)*((2*x^4 - x^2 + x + 1)/(3*x^4 - x^2 + x + 1))^(1/
3) + 24*sqrt(3)*(-6)^(1/3)*(39*x^12 + 11*x^10 - 11*x^9 - 34*x^8 + 46*x^7 +
28*x^6 - 61*x^5 - 23*x^4 + 25*x^3 - 15*x - 5)*((2*x^4 - x^2 + x + 1)/(3*x^4
 - x^2 + x + 1))^(2/3) + sqrt(3)*(16199*x^12 - 20631*x^10 + 20631*x^9 + 292
68*x^8 - 17274*x^7 - 9826*x^6 + 20841*x^5 + 8637*x^4 - 5945*x^3 + 3567*x +
1189))/(17497*x^12 - 20409*x^10 + 20409*x^9 + 28188*x^8 - 15558*x^7 - 8750*
x^6 + 18471*x^5 + 7779*x^4 - 4855*x^3 + 2913*x + 971)) - 10*sqrt(3)*x^4*arc
tan((26407150*sqrt(3)*(3*x^4 - x^2 + x + 1)*((2*x^4 - x^2 + x + 1)/(3*x^4 -
 x^2 + x + 1))^(2/3) + 15172108*sqrt(3)*(3*x^4 - x^2 + x + 1)*((2*x^4 - x^2
 + x + 1)/(3*x^4 - x^2 + x + 1))^(1/3) + sqrt(3)*(47470762*x^4 - 20789629*x
^2 + 20789629*x + 20789629))/(29760814*x^4 - 16852563*x^2 + 16852563*x + 16
852563)) + (-6)^(1/3)*x^4*log((12*(-6)^(2/3)*(39*x^8 - 28*x^6 + 28*x^5 + 33
*x^4 - 10*x^3 - 5*x^2 + 10*x + 5)*((2*x^4 - x^2 + x + 1)/(3*x^4 - x^2 + x +
 1))^(2/3) - (-6)^(1/3)*(649*x^8 - 538*x^6 + 538*x^5 + 647*x^4 - 218*x^3 -
109*x^2 + 218*x + 109) + 18*(75*x^8 - 58*x^6 + 58*x^5 + 69*x^4 - 22*x^3 - 1
1*x^2 + 22*x + 11)*((2*x^4 - x^2 + x + 1)/(3*x^4 - x^2 + x + 1))^(1/3))/(x^
8 + 2*x^6 - 2*x^5 - x^4 - 2*x^3 - x^2 + 2*x + 1)) - 2*(-6)^(1/3)*x^4*log(((
-6)^(2/3)*(x^4 + x^2 - x - 1) + 18*(-6)^(1/3)*(3*x^4 - x^2 + x + 1)*((2*x^4
 - x^2 + x + 1)/(3*x^4 - x^2 + x + 1))^(1/3) + 36*(3*x^4 - x^2 + x + 1)*((2
*x^4 - x^2 + x + 1)/(3*x^4 - x^2 + x + 1))^(2/3))/(x^4 + x^2 - x - 1)) - 5*
x^4*log((x^4 + 3*(3*x^4 - x^2 + x + 1)*((2*x^4 - x^2 + x + 1)/(3*x^4 - x^2
 + x + 1))^(2/3) - 3*(3*x^4 - x^2 + x + 1)*((2*x^4 - x^2 + x + 1)/(3*x^4 - x
^2 + x + 1))^(1/3))/x^4) + 6*(3*x^4 - x^2 + x + 1)*((2*x^4 - x^2 + x + 1)/(
3*x^4 - x^2 + x + 1))^(1/3))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4 - x^2 + x + 1\right)\left(2\,x^2 - 3\,x - 4\right)\left(\frac{2\,x^4 - x^2 + x + 1}{3\,x^4 - x^2 + x + 1}\right)^{\frac{1}{3}}}{\left(x^4 + x^2 - x - 1\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-3*x-4)*(x^4-x^2+x+1)*((2*x^4-x^2+x+1)/(3*x^4-x^2+x+1))^(1/
3)/x^5/(x^4+x^2-x-1),x, algorithm="giac")
```

```
[Out] integrate((x^4 - x^2 + x + 1)*(2*x^2 - 3*x - 4)*((2*x^4 - x^2 + x + 1)/(3*x
^4 - x^2 + x + 1))^(1/3)/((x^4 + x^2 - x - 1)*x^5), x)
```

maple [F] time = 2.82, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2 - 3x - 4\right)\left(x^4 - x^2 + x + 1\right)\left(\frac{2x^4 - x^2 + x + 1}{3x^4 - x^2 + x + 1}\right)^{\frac{1}{3}}}{x^5\left(x^4 + x^2 - x - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-3*x-4)*(x^4-x^2+x+1)*((2*x^4-x^2+x+1)/(3*x^4-x^2+x+1))^(1/3)/x^5
/(x^4+x^2-x-1),x)
```

```
[Out] int((2*x^2-3*x-4)*(x^4-x^2+x+1)*((2*x^4-x^2+x+1)/(3*x^4-x^2+x+1))^(1/3)/x^5
/(x^4+x^2-x-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^4 - x^2 + x + 1\right)\left(2\,x^2 - 3\,x - 4\right)\left(\frac{2\,x^4 - x^2 + x + 1}{3\,x^4 - x^2 + x + 1}\right)^{\frac{1}{3}}}{\left(x^4 + x^2 - x - 1\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-3*x-4)*(x^4-x^2+x+1)*((2*x^4-x^2+x+1)/(3*x^4-x^2+x+1))^(1/
3)/x^5/(x^4+x^2-x-1),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - x^2 + x + 1)*(2*x^2 - 3*x - 4)*((2*x^4 - x^2 + x + 1)/(3*x
^4 - x^2 + x + 1))^(1/3)/((x^4 + x^2 - x - 1)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{2\,x^4 - x^2 + x + 1}{3\,x^4 - x^2 + x + 1}\right)^{1/3}\,\left(-2\,x^2 + 3\,x + 4\right)\,\left(x^4 - x^2 + x + 1\right)}{x^5\,\left(-x^4 - x^2 + x + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((x - x^2 + 2*x^4 + 1)/(x - x^2 + 3*x^4 + 1))^(1/3)*(3*x - 2*x^2 + 4)*
(x - x^2 + x^4 + 1))/(x^5*(x - x^2 - x^4 + 1)),x)
```

```
[Out] int((((x - x^2 + 2*x^4 + 1)/(x - x^2 + 3*x^4 + 1))^(1/3)*(3*x - 2*x^2 + 4)*
(x - x^2 + x^4 + 1))/(x^5*(x - x^2 - x^4 + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-3*x-4)*(x**4-x**2+x+1)*((2*x**4-x**2+x+1)/(3*x**4-x**2+x+
1))**(1/3)/x**5/(x**4+x**2-x-1),x)
```

```
[Out] Timed out
```

3.2353 $\int \frac{-b+ax}{(b+ax)\sqrt[3]{-b^2x^2+a^3x^3}}\,dx$

Optimal. Leaf size=425

$$-\frac{\log\left(\sqrt[3]{a^3x^3-b^2x^2}-ax\right)}{a}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}ax}{2\sqrt[3]{a^3x^3-b^2x^2}+ax}\right)}{a}-\frac{i\left(\sqrt{3}-i\right)\log\left(\sqrt[3]{-1}\sqrt[3]{a^3x^3-b^2x^2}+\sqrt[3]{a}\,x\sqrt[3]{a^2+b}\right)}{\sqrt[3]{a}\sqrt[3]{a^2+b}}+$$

Rubi [A] time = 0.23, antiderivative size = 477, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2056, 157, 59, 91}

$$-\frac{x^{2/3}\log(x)\sqrt[3]{a^3x-b^2}}{2a\sqrt[3]{a^3x^3-b^2x^2}}-\frac{3x^{2/3}\sqrt[3]{a^3x-b^2}\log\left(\frac{\sqrt[3]{a^3x-b^2}}{a\sqrt[3]{x}}-1\right)}{2a\sqrt[3]{a^3x^3-b^2x^2}}-\frac{\sqrt{3}x^{2/3}\sqrt[3]{a^3x-b^2}\tan^{-1}\left(\frac{2\sqrt[3]{a^3x-b^2}}{\sqrt{3}a\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{a\sqrt[3]{a^3x^3-b^2x^2}}-\frac{x^{2/3}\sqrt[3]{a^3x-b^2}\log(ax+b)}{\sqrt[3]{a}\sqrt[3]{a^2+b}\sqrt[3]{a^3x^3-b^2x^2}}+\frac{3x^{2/3}\sqrt[3]{a^3x-b^2}\log\left(\frac{\sqrt[3]{a^3x-b^2}}{\sqrt[3]{a}\sqrt[3]{a^2+b}}-\sqrt[3]{x}\right)}{\sqrt[3]{a}\sqrt[3]{a^2+b}\sqrt[3]{a^3x^3-b^2x^2}}+\frac{2\sqrt{3}x^{2/3}\sqrt[3]{a^3x-b^2}\tan^{-1}\left(\frac{2\sqrt[3]{a^3x-b^2}}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{a^2+b}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[3]{a^2+b}\sqrt[3]{a^3x^3-b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x)/((b + a*x)*(-(b^2*x^2) + a^3*x^3)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*x^(2/3)*(-b^2 + a^3*x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(-b^2 + a^3*x
)^(1/3))/(Sqrt[3]*a*x^(1/3))])/(a*(-(b^2*x^2) + a^3*x^3)^(1/3))) + (2*Sqrt[
3]*x^(2/3)*(-b^2 + a^3*x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(-b^2 + a^3*x)^(1/3))
/(Sqrt[3]*a^(1/3)*(a^2 + b)^(1/3)*x^(1/3))])/(a^(1/3)*(a^2 + b)^(1/3)*(-(b^
2*x^2) + a^3*x^3)^(1/3)) - (x^(2/3)*(-b^2 + a^3*x)^(1/3)*Log[x])/(2*a*(-(b^
2*x^2) + a^3*x^3)^(1/3)) - (x^(2/3)*(-b^2 + a^3*x)^(1/3)*Log[b + a*x])/(a^(
1/3)*(a^2 + b)^(1/3)*(-(b^2*x^2) + a^3*x^3)^(1/3)) + (3*x^(2/3)*(-b^2 + a^3
*x)^(1/3)*Log[-x^(1/3) + (-b^2 + a^3*x)^(1/3)/(a^(1/3)*(a^2 + b)^(1/3))])/(
a^(1/3)*(a^2 + b)^(1/3)*(-(b^2*x^2) + a^3*x^3)^(1/3)) - (3*x^(2/3)*(-b^2 +
a^3*x)^(1/3)*Log[-1 + (-b^2 + a^3*x)^(1/3)/(a*x^(1/3))])/(2*a*(-(b^2*x^2) +
 a^3*x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
```

```
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{-b+ax}{(b+ax)\sqrt[3]{-b^2x^2+a^3x^3}}\,dx = \frac{\left(x^{2/3}\sqrt[3]{-b^2+a^3x}\right)\int \frac{-b+ax}{x^{2/3}(b+ax)\sqrt[3]{-b^2+a^3x}}\,dx}{\sqrt[3]{-b^2x^2+a^3x^3}}$$
$$= \frac{\left(x^{2/3}\sqrt[3]{-b^2+a^3x}\right)\int \frac{1}{x^{2/3}\sqrt[3]{-b^2+a^3x}}\,dx}{\sqrt[3]{-b^2x^2+a^3x^3}} - \frac{\left(2bx^{2/3}\sqrt[3]{-b^2+a^3x}\right)\int \frac{1}{x^{2/3}(b+ax)\sqrt[3]{-b^2+a^3x}}\,d}{\sqrt[3]{-b^2x^2+a^3x^3}}$$
$$= -\frac{\sqrt{3}\,x^{2/3}\sqrt[3]{-b^2+a^3x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-b^2+a^3x}}{\sqrt{3}\,a\sqrt[3]{x}}\right)}{a\sqrt[3]{-b^2x^2+a^3x^3}} + \frac{2\sqrt{3}\,x^{2/3}\sqrt[3]{-b^2+a^3x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}\right.}{\sqrt[3]{a}\,\sqrt[3]{a^2+b}\,\sqrt[3]{-b^2x^2}}$$

Mathematica [C]   time = 0.06, size = 89, normalized size = 0.21

$$\frac{3x\sqrt[3]{1-\frac{a^3x}{b^2}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{a^3x}{b^2}\right)-6x\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{a\left(a^2+b\right)x}{a^3x-b^2}\right)}{\sqrt[3]{x^2\left(a^3x-b^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x)/((b + a*x)*(-(b^2*x^2) + a^3*x^3)^(1/3)),x]
```

```
[Out] (3*x*(1 - (a^3*x)/b^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (a^3*x)/b^2]
- 6*x*Hypergeometric2F1[1/3, 1, 4/3, (a*(a^2 + b)*x)/(-b^2 + a^3*x)])/(x^2*
(-b^2 + a^3*x))^(1/3)
```

IntegrateAlgebraic [A]   time = 3.15, size = 475, normalized size = 1.12

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}ax}{2\sqrt[3]{a^3x^3-b^2x^2}+ax}\right)}{a} - \frac{i\left(\sqrt{3}-i\right)\log\left(2\sqrt[3]{a}\,x\sqrt[3]{a^2+b}+\left(1+i\sqrt{3}\right)\sqrt[3]{a^3x^3-b^2x^2}\right)}{\sqrt[3]{a}\sqrt[3]{a^2+b}} - \frac{\log\left(a^2x-a\sqrt[3]{a^3x^3-b^2x^2}\right)}{a} + \frac{\log\left(ax\sqrt[3]{a^3x^3-b^2x^2}+\left(a^3x^3-b^2x^2\right)^{2/3}+a^2x^2\right)}{2a} + \frac{\sqrt{6\left(-1+i\sqrt{3}\right)}\tan^{-1}\left(\frac{3\sqrt[3]{a}\,x\sqrt[3]{a^2+b}}{-\sqrt{3}\sqrt[3]{a^3x^3-b^2x^2}-3i\sqrt[3]{a^3x^3-b^2x^2}+\sqrt{3}\sqrt[3]{a}\,x\sqrt[3]{a^2+b}}\right)}{\sqrt[3]{a}\sqrt[3]{a^2+b}} + \frac{\left(1+i\sqrt{3}\right)\log\left(\left(\sqrt{3}+i\right)\left(a^3x^3-b^2x^2\right)^{2/3}-2ia^{2/3}x^2\left(a^2+b\right)^{2/3}+\sqrt[3]{a}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a^2+b}\sqrt[3]{a^3x^3-b^2x^2}\right)}{2\sqrt[3]{a}\sqrt[3]{a^2+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x)/((b + a*x)*(-(b^2*x^2) + a^3*x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*a*x)/(a*x + 2*(-(b^2*x^2) + a^3*x^3)^(1/3))]])/a +
(Sqrt[6*(-1 + I*Sqrt[3])]*ArcTan[(3*a^(1/3)*(a^2 + b)^(1/3)*x)/(Sqrt[3]*a^(
1/3)*(a^2 + b)^(1/3)*x - (3*I)*(-(b^2*x^2) + a^3*x^3)^(1/3) - Sqrt[3]*(-(b^
2*x^2) + a^3*x^3)^(1/3))]])/(a^(1/3)*(a^2 + b)^(1/3)) - (I*(-I + Sqrt[3])*Lo
g[2*a^(1/3)*(a^2 + b)^(1/3)*x + (1 + I*Sqrt[3])*(-(b^2*x^2) + a^3*x^3)^(1/3
)]])/(a^(1/3)*(a^2 + b)^(1/3)) - Log[a^2*x - a*(-(b^2*x^2) + a^3*x^3)^(1/3)]
/a + Log[a^2*x^2 + a*x*(-(b^2*x^2) + a^3*x^3)^(1/3) + (-(b^2*x^2) + a^3*x^3
)^(2/3)]/(2*a) + ((1 + I*Sqrt[3])*Log[(-2*I)*a^(2/3)*(a^2 + b)^(2/3)*x^2 +
a^(1/3)*(a^2 + b)^(1/3)*(I*x - Sqrt[3]*x)*(-(b^2*x^2) + a^3*x^3)^(1/3) + (I
 + Sqrt[3])*(-(b^2*x^2) + a^3*x^3)^(2/3)]])/(2*a^(1/3)*(a^2 + b)^(1/3))
```

fricas [A]   time = 0.45, size = 806, normalized size = 1.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/(a*x+b)/(a^3*x^3-b^2*x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(3)*(a^3 + a*b)*sqrt(-1/(a^3 + a*b)^(2/3))*log((2*b^2*x - (3*a^
3 + a*b)*x^2 + 3*(a^3*x^3 - b^2*x^2)^(1/3)*(a^3 + a*b)^(2/3)*x + sqrt(3)*((
```

```
a^3 + a*b)^(4/3)*x^2 + (a^3*x^3 - b^2*x^2)^(1/3)*(a^3 + a*b)*x - 2*(a^3*x^3
 - b^2*x^2)^(2/3)*(a^3 + a*b)^(2/3))*sqrt(-1/(a^3 + a*b)^(2/3)))/(a*x^2 + b
*x)) - 2*sqrt(3)*(a^2 + b)*arctan(1/3*(sqrt(3)*a*x + 2*sqrt(3)*(a^3*x^3 - b
^2*x^2)^(1/3))/(a*x)) - 2*(a^2 + b)*log(-(a*x - (a^3*x^3 - b^2*x^2)^(1/3))/
x) + (a^2 + b)*log((a^2*x^2 + (a^3*x^3 - b^2*x^2)^(1/3)*a*x + (a^3*x^3 - b^
2*x^2)^(2/3))/x^2) + 4*(a^3 + a*b)^(2/3)*log(-((a^3 + a*b)^(1/3)*x - (a^3*x
^3 - b^2*x^2)^(1/3))/x) - 2*(a^3 + a*b)^(2/3)*log(((a^3 + a*b)^(2/3)*x^2 +
(a^3*x^3 - b^2*x^2)^(1/3)*(a^3 + a*b)^(1/3)*x + (a^3*x^3 - b^2*x^2)^(2/3))/
x^2))/(a^3 + a*b), -1/2*(2*sqrt(3)*(a^2 + b)*arctan(1/3*(sqrt(3)*a*x + 2*sq
rt(3)*(a^3*x^3 - b^2*x^2)^(1/3))/(a*x)) - 4*sqrt(3)*(a^3 + a*b)^(2/3)*arcta
n(1/3*sqrt(3)*((a^3 + a*b)^(1/3)*x + 2*(a^3*x^3 - b^2*x^2)^(1/3))/((a^3 + a
*b)^(1/3)*x)) + 2*(a^2 + b)*log(-(a*x - (a^3*x^3 - b^2*x^2)^(1/3))/x) - (a^
2 + b)*log((a^2*x^2 + (a^3*x^3 - b^2*x^2)^(1/3)*a*x + (a^3*x^3 - b^2*x^2)^(
2/3))/x^2) - 4*(a^3 + a*b)^(2/3)*log(-((a^3 + a*b)^(1/3)*x - (a^3*x^3 - b^2
*x^2)^(1/3))/x) + 2*(a^3 + a*b)^(2/3)*log(((a^3 + a*b)^(2/3)*x^2 + (a^3*x^3
 - b^2*x^2)^(1/3)*(a^3 + a*b)^(1/3)*x + (a^3*x^3 - b^2*x^2)^(2/3))/x^2))/(a
^3 + a*b)]
```

giac [A] time = 143.52, size = 255, normalized size = 0.60

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\left(a^{3}+ab\right)^{\frac{1}{3}}+2\left(a^{3}-\frac{b^{2}}{x}\right)^{\frac{1}{3}}\right)}{3\left(a^{3}+ab\right)^{\frac{1}{3}}}\right)}{\left(a^{3}+ab\right)^{\frac{1}{3}}}-\frac{\log\left(\left(a^{3}+ab\right)^{\frac{2}{3}}+\left(a^{3}+ab\right)^{\frac{1}{3}}\left(a^{3}-\frac{b^{2}}{x}\right)^{\frac{1}{3}}+\left(a^{3}-\frac{b^{2}}{x}\right)^{\frac{2}{3}}\right)}{\left(a^{3}+ab\right)^{\frac{1}{3}}}+\frac{2\log\left(\left|-\left(a^{3}+ab\right)^{\frac{1}{3}}+\left(a^{3}-\frac{b^{2}}{x}\right)^{\frac{1}{3}}\right|\right)}{\left(a^{3}+ab\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(a^{3}-\frac{b^{2}}{x}\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}+\frac{\log\left(a^{2}+\left(a^{3}-\frac{b^{2}}{x}\right)^{\frac{1}{3}}a+\left(a^{3}-\frac{b^{2}}{x}\right)^{\frac{2}{3}}\right)}{2a}-\frac{\log\left(\left|-a+\left(a^{3}-\frac{b^{2}}{x}\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/(a*x+b)/(a^3*x^3-b^2*x^2)^(1/3),x, algorithm="giac")
```

```
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*((a^3 + a*b)^(1/3) + 2*(a^3 - b^2/x)^(1/3))/(a
^3 + a*b)^(1/3))/(a^3 + a*b)^(1/3) - log((a^3 + a*b)^(2/3) + (a^3 + a*b)^(1
/3)*(a^3 - b^2/x)^(1/3) + (a^3 - b^2/x)^(2/3))/(a^3 + a*b)^(1/3) + 2*log(ab
s(-(a^3 + a*b)^(1/3) + (a^3 - b^2/x)^(1/3)))/(a^3 + a*b)^(1/3) - sqrt(3)*ar
ctan(1/3*sqrt(3)*(a + 2*(a^3 - b^2/x)^(1/3))/a)/a + 1/2*log(a^2 + (a^3 - b^
2/x)^(1/3)*a + (a^3 - b^2/x)^(2/3))/a - log(abs(-a + (a^3 - b^2/x)^(1/3)))/
a
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{ax-b}{(ax+b)\left(a^{3}x^{3}-b^{2}x^{2}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-b)/(a*x+b)/(a^3*x^3-b^2*x^2)^(1/3),x)
```

```
[Out] int((a*x-b)/(a*x+b)/(a^3*x^3-b^2*x^2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-b}{\left(a^{3}x^{3}-b^{2}x^{2}\right)^{\frac{1}{3}}(ax+b)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/(a*x+b)/(a^3*x^3-b^2*x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*x - b)/((a^3*x^3 - b^2*x^2)^(1/3)*(a*x + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{b-a\,x}{{\left(a^3\,x^3-b^2\,x^2\right)}^{1/3}\,\left(b+a\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x)/((a^3*x^3 - b^2*x^2)^(1/3)*(b + a*x)),x)
```

```
[Out] int(-(b - a*x)/((a^3*x^3 - b^2*x^2)^(1/3)*(b + a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-b}{\sqrt[3]{x^2\left(a^3x-b^2\right)}\,(ax+b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/(a*x+b)/(a**3*x**3-b**2*x**2)**(1/3),x)
```

```
[Out] Integral((a*x - b)/((x**2*(a**3*x - b**2))**(1/3)*(a*x + b)), x)
```

3.2354 $\int \frac{1-x^3+x^6}{\sqrt[3]{x^2+x^4}\left(-1+x^6\right)} dx$

Optimal. Leaf size=429

$$\frac{1}{2}\log\left(\sqrt[3]{x^4+x^2}-x\right)-\frac{1}{6}\log\left(\sqrt[3]{x^4+x^2}+x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}-2x\right)}{12\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}+2x\right)}{4\sqrt[3]{2}}+\frac{1}{12}\log\left(x\right.$$

Rubi [F] time = 2.55, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1-x^3+x^6}{\sqrt[3]{x^2+x^4}\left(-1+x^6\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - x^3 + x^6)/((x^2 + x^4)^(1/3)*(-1 + x^6)),x]
```

```
[Out] (-2*x*(1 + x^2)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6, (-2*x^2)/(1 - I*Sqrt[3]),
-x^2])/(x^2 + x^4)^(1/3) - (2*x*(1 + x^2)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6,
(-2*x^2)/(1 + I*Sqrt[3]), -x^2])/(x^2 + x^4)^(1/3) + ((I - Sqrt[3])*x^2*(1
+ x^2)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -x^2, (-2*x^2)/(1 - I*Sqrt[3])])/(4
*(I + Sqrt[3])*(x^2 + x^4)^(1/3)) + ((I + Sqrt[3])*x^2*(1 + x^2)^(1/3)*Appe
llF1[2/3, 1/3, 1, 5/3, -x^2, (-2*x^2)/(1 + I*Sqrt[3])])/(4*(I - Sqrt[3])*(x
^2 + x^4)^(1/3)) + (3*x*(1 + x^2)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -x
^2])/(x^2 + x^4)^(1/3) + (x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][1
/((-1 + x)*(1 + x^6)^(1/3)), x], x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) - (x^(2
/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][1/((1 + x)*(1 + x^6)^(1/3)), x]
, x, x^(1/3)])/(2*(x^2 + x^4)^(1/3)) + ((1 + I*Sqrt[3])*x^(2/3)*(1 + x^2)^(
1/3)*Defer[Subst][Defer[Int][1/((-1 - I*Sqrt[3] + 2*x)*(1 + x^6)^(1/3)), x]
, x, x^(1/3)])/(2*(x^2 + x^4)^(1/3)) - ((1 - I*Sqrt[3])*x^(2/3)*(1 + x^2)^(
1/3)*Defer[Subst][Defer[Int][1/((1 - I*Sqrt[3] + 2*x)*(1 + x^6)^(1/3)), x],
 x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) + ((1 - I*Sqrt[3])*x^(2/3)*(1 + x^2)^(1
/3)*Defer[Subst][Defer[Int][1/((-1 + I*Sqrt[3] + 2*x)*(1 + x^6)^(1/3)), x],
 x, x^(1/3)])/(2*(x^2 + x^4)^(1/3)) - ((1 + I*Sqrt[3])*x^(2/3)*(1 + x^2)^(1
/3)*Defer[Subst][Defer[Int][1/((1 + I*Sqrt[3] + 2*x)*(1 + x^6)^(1/3)), x],
x, x^(1/3)])/(6*(x^2 + x^4)^(1/3))
```

Rubi steps

$$\int \frac{1-x^3+x^6}{\sqrt[3]{x^2+x^4}\left(-1+x^6\right)}\,dx = \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{1-x^3+x^6}{x^{2/3}\sqrt[3]{1+x^2}\left(-1+x^6\right)}\,dx}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1-x^9+x^{18}}{\sqrt[3]{1+x^6}\left(-1+x^{18}\right)}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1+x^6}} + \frac{2-x^9}{\sqrt[3]{1+x^6}\left(-1+x^{18}\right)}\right) dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{2-x^9}{\sqrt[3]{1+x^6}\left(-1+x^{1}\right.}\right.}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int\left(-\frac{1}{2\sqrt[3]{1+x^6}\left(1-x^9\right)} - \frac{3}{2\sqrt[3]{1+x^6}\,(1}\right.\right.}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^6}\left(1-x^9\right)}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{x^2+x^4}} -$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int\left(-\frac{1}{9(-1+x)\sqrt[3]{1+x^6}} + \frac{2+}{9\left(1+x+x^2\right)}\right.\right.}{2\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{6\sqrt[3]{x^2+x^4}} -$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{6\sqrt[3]{x^2+x^4}} -$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{6\sqrt[3]{x^2+x^4}} -$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{6\sqrt[3]{x^2+x^4}} -$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{6\sqrt[3]{x^2+x^4}} -$$

$$= -\frac{2x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^2}{1-i\sqrt{3}},-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{2x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^2}{1+i\sqrt{3}},-x^2\right)}{\sqrt[3]{x^2+x^4}} +$$

$$= -\frac{2x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^2}{1-i\sqrt{3}},-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{2x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^2}{1+i\sqrt{3}},-x^2\right)}{\sqrt[3]{x^2+x^4}} +$$

Mathematica [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1-x^3+x^6}{\sqrt[3]{x^2+x^4}\left(-1+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^3 + x^6)/((x^2 + x^4)^(1/3)*(-1 + x^6)),x]
```

```
[Out] Integrate[(1 - x^3 + x^6)/((x^2 + x^4)^(1/3)*(-1 + x^6)), x]
```

IntegrateAlgebraic [A] time = 1.58, size = 429, normalized size = 1.00

$$\frac{1}{2}\log\left(\sqrt[3]{x^4+x^2}-x\right)-\frac{1}{6}\log\left(\sqrt[3]{x^4+x^2}+x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}-2x\right)}{12\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}+2x\right)}{4\sqrt[3]{2}}+\frac{1}{12}\log\left(x^2-\sqrt[3]{x^4+x^2}\,x+\left(x^4+x^2\right)^{2/3}\right)-\frac{1}{4}\log\left(x^2+\sqrt[3]{x^4+x^2}\,x+\left(x^4+x^2\right)^{2/3}\right)+\frac{\log\left(-2x^2+2^{2/3}\sqrt[3]{x^4+x^2}\,x-\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{8\sqrt[3]{2}}-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^4+x^2}\,x+\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{24\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+x^2}-x}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+x^2}+x}\right)-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}-x}\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}+x}\right)}{4\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x^3 + x^6)/((x^2 + x^4)^(1/3)*(-1 + x^6)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3]*x)/(-x + 2*(x^2 + x^4)^(1/3))]/Sqrt[3] - (Sqrt[3]*ArcT
an[(Sqrt[3]*x)/(x + 2*(x^2 + x^4)^(1/3))])/2 - (Sqrt[3]*ArcTan[(Sqrt[3]*x)/
(-x + 2^(2/3)*(x^2 + x^4)^(1/3))])/(4*2^(1/3)) - ArcTan[(Sqrt[3]*x)/(x + 2^
(2/3)*(x^2 + x^4)^(1/3))]/(4*2^(1/3)*Sqrt[3]) + Log[-x + (x^2 + x^4)^(1/3)]
/2 - Log[x + (x^2 + x^4)^(1/3)]/6 + Log[-2*x + 2^(2/3)*(x^2 + x^4)^(1/3)]/(
12*2^(1/3)) - Log[2*x + 2^(2/3)*(x^2 + x^4)^(1/3)]/(4*2^(1/3)) + Log[x^2 -
x*(x^2 + x^4)^(1/3) + (x^2 + x^4)^(2/3)]/12 - Log[x^2 + x*(x^2 + x^4)^(1/3)
 + (x^2 + x^4)^(2/3)]/4 + Log[-2*x^2 + 2^(2/3)*x*(x^2 + x^4)^(1/3) - 2^(1/3
)*(x^2 + x^4)^(2/3)]/(8*2^(1/3)) - Log[2*x^2 + 2^(2/3)*x*(x^2 + x^4)^(1/3)
+ 2^(1/3)*(x^2 + x^4)^(2/3)]/(24*2^(1/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^3+1)/(x^4+x^2)^(1/3)/(x^6-1),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 - x^3 + 1}{\left(x^6 - 1\right)\left(x^4 + x^2\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^3+1)/(x^4+x^2)^(1/3)/(x^6-1),x, algorithm="giac")
```

```
[Out] integrate((x^6 - x^3 + 1)/((x^6 - 1)*(x^4 + x^2)^(1/3)), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 - x^3 + 1}{\left(x^4 + x^2\right)^{\frac{1}{3}}\left(x^6 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-x^3+1)/(x^4+x^2)^(1/3)/(x^6-1),x)
```

```
[Out] int((x^6-x^3+1)/(x^4+x^2)^(1/3)/(x^6-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 - x^3 + 1}{\left(x^6 - 1\right)\left(x^4 + x^2\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^3+1)/(x^4+x^2)^(1/3)/(x^6-1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 - x^3 + 1)/((x^6 - 1)*(x^4 + x^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6-x^3+1}{\left(x^4+x^2\right)^{1/3}\,\left(x^6-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 - x^3 + 1)/((x^2 + x^4)^(1/3)*(x^6 - 1)),x)
```

```
[Out] int((x^6 - x^3 + 1)/((x^2 + x^4)^(1/3)*(x^6 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6-x^3+1}{\sqrt[3]{x^2\left(x^2+1\right)}\,(x-1)\,(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-x**3+1)/(x**4+x**2)**(1/3)/(x**6-1),x)
```

```
[Out] Integral((x**6 - x**3 + 1)/((x**2*(x**2 + 1))**(1/3)*(x - 1)*(x + 1)*(x**2
- x + 1)*(x**2 + x + 1)), x)
```

3.2355 $\int \frac{1+x^3+x^6}{\sqrt[3]{x^2+x^4}\left(-1+x^6\right)} dx$

Optimal. Leaf size=429

$$\frac{1}{6}\log\left(\sqrt[3]{x^4+x^2}-x\right)-\frac{1}{2}\log\left(\sqrt[3]{x^4+x^2}+x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}-2x\right)}{4\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}+2x\right)}{12\sqrt[3]{2}}+\frac{1}{4}\log\left(x^2\right.$$

Rubi [F] time = 2.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x^3+x^6}{\sqrt[3]{x^2+x^4}\left(-1+x^6\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^3 + x^6)/((x^2 + x^4)^(1/3)*(-1 + x^6)),x]
```

```
[Out] (-2*x*(1 + x^2)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6, (-2*x^2)/(1 - I*Sqrt[3]),
-x^2])/(x^2 + x^4)^(1/3) - (2*x*(1 + x^2)^(1/3)*AppellF1[1/6, 1, 1/3, 7/6,
(-2*x^2)/(1 + I*Sqrt[3]), -x^2])/(x^2 + x^4)^(1/3) - ((I - Sqrt[3])*x^2*(1
+ x^2)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -x^2, (-2*x^2)/(1 - I*Sqrt[3])])/(4
*(I + Sqrt[3])*(x^2 + x^4)^(1/3)) - ((I + Sqrt[3])*x^2*(1 + x^2)^(1/3)*Appe
llF1[2/3, 1/3, 1, 5/3, -x^2, (-2*x^2)/(1 + I*Sqrt[3])])/(4*(I - Sqrt[3])*(x
^2 + x^4)^(1/3)) + (3*x*(1 + x^2)^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -x
^2])/(x^2 + x^4)^(1/3) + (x^(2/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][1
/((-1 + x)*(1 + x^6)^(1/3)), x], x, x^(1/3)])/(2*(x^2 + x^4)^(1/3)) - (x^(2
/3)*(1 + x^2)^(1/3)*Defer[Subst][Defer[Int][1/((1 + x)*(1 + x^6)^(1/3)), x]
, x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) + ((1 + I*Sqrt[3])*x^(2/3)*(1 + x^2)^(
1/3)*Defer[Subst][Defer[Int][1/((-1 - I*Sqrt[3] + 2*x)*(1 + x^6)^(1/3)), x]
, x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) - ((1 - I*Sqrt[3])*x^(2/3)*(1 + x^2)^(
1/3)*Defer[Subst][Defer[Int][1/((1 - I*Sqrt[3] + 2*x)*(1 + x^6)^(1/3)), x],
 x, x^(1/3)])/(2*(x^2 + x^4)^(1/3)) + ((1 - I*Sqrt[3])*x^(2/3)*(1 + x^2)^(1
/3)*Defer[Subst][Defer[Int][1/((-1 + I*Sqrt[3] + 2*x)*(1 + x^6)^(1/3)), x],
 x, x^(1/3)])/(6*(x^2 + x^4)^(1/3)) - ((1 + I*Sqrt[3])*x^(2/3)*(1 + x^2)^(1
/3)*Defer[Subst][Defer[Int][1/((1 + I*Sqrt[3] + 2*x)*(1 + x^6)^(1/3)), x],
x, x^(1/3)])/(2*(x^2 + x^4)^(1/3))
```

Rubi steps

$$\int \frac{1+x^3+x^6}{\sqrt[3]{x^2+x^4}\left(-1+x^6\right)}\,dx = \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\int \frac{1+x^3+x^6}{x^{2/3}\sqrt[3]{1+x^2}\left(-1+x^6\right)}\,dx}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1+x^9+x^{18}}{\sqrt[3]{1+x^6}\left(-1+x^{18}\right)}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1+x^6}} + \frac{2+x^9}{\sqrt[3]{1+x^6}\left(-1+x^{18}\right)}\right) dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{2+x^9}{\sqrt[3]{1+x^6}\left(-1+x^{1}\right.}\right.}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} + \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int\left(-\frac{3}{2\sqrt[3]{1+x^6}\left(1-x^9\right)} - \frac{1}{2\sqrt[3]{1+x^6}\left(1\right.}\right.\right.}{\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1+x^6}\left(1+x^9\right)}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{x^2+x^4}} - \cdots$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(3x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int\left(\frac{1}{9(1+x)\sqrt[3]{1+x^6}} + \frac{2-x}{9\left(1-x+x^2\right)\sqrt[3]{1}}\right.\right.}{2\sqrt[3]{x^2+x^4}}$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{6\sqrt[3]{x^2+x^4}} - \frac{\left(x^{2}\right.}{}$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{6\sqrt[3]{x^2+x^4}} - \frac{\left(x^{2}\right.}{}$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{6\sqrt[3]{x^2+x^4}} + \frac{\left(x^{2}\right.}{}$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{6\sqrt[3]{x^2+x^4}} + \frac{\left(x^{2}\right.}{}$$

$$= \frac{3x\sqrt[3]{1+x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{\left(x^{2/3}\sqrt[3]{1+x^2}\right)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt[3]{1+x^6}}\,dx, x, \sqrt[3]{x}\right)}{6\sqrt[3]{x^2+x^4}} + \frac{\left(x^{2}\right.}{}$$

$$= -\frac{2x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^2}{1-i\sqrt{3}},-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{2x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^2}{1+i\sqrt{3}},-x^2\right)}{\sqrt[3]{x^2+x^4}} + \cdots$$

$$= -\frac{2x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^2}{1-i\sqrt{3}},-x^2\right)}{\sqrt[3]{x^2+x^4}} - \frac{2x\sqrt[3]{1+x^2}\,F_1\left(\frac{1}{6};1,\frac{1}{3};\frac{7}{6};-\frac{2x^2}{1+i\sqrt{3}},-x^2\right)}{\sqrt[3]{x^2+x^4}} - \cdots$$

Mathematica [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1+x^3+x^6}{\sqrt[3]{x^2+x^4}\left(-1+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x^3 + x^6)/((x^2 + x^4)^(1/3)*(-1 + x^6)),x]
```

```
[Out] Integrate[(1 + x^3 + x^6)/((x^2 + x^4)^(1/3)*(-1 + x^6)), x]
```

IntegrateAlgebraic [A] time = 1.42, size = 429, normalized size = 1.00

$$\frac{1}{6}\log\left(\sqrt[3]{x^4+x^2}-x\right)-\frac{1}{2}\log\left(\sqrt[3]{x^4+x^2}+x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}-2x\right)}{4\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{x^4+x^2}+2x\right)}{12\sqrt[3]{2}}+\frac{1}{4}\log\left(x^2-\sqrt[3]{x^4+x^2}x+\left(x^4+x^2\right)^{2/3}\right)-\frac{1}{12}\log\left(x^2+\sqrt[3]{x^4+x^2}x+\left(x^4+x^2\right)^{2/3}\right)+\frac{\log\left(-2x^2+2^{2/3}\sqrt[3]{x^4+x^2}x-\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{24\sqrt[3]{2}}-\frac{\log\left(2x^2+2^{2/3}\sqrt[3]{x^4+x^2}x+\sqrt[3]{2}\left(x^4+x^2\right)^{2/3}\right)}{8\sqrt[3]{2}}-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+x^2}-x}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{x^4+x^2}+x}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}-x}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{x^4+x^2}+x}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^3 + x^6)/((x^2 + x^4)^(1/3)*(-1 + x^6)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*x)/(-x + 2*(x^2 + x^4)^(1/3))]) - ArcTan[(Sqr
t[3]*x)/(x + 2*(x^2 + x^4)^(1/3))]/(2*Sqrt[3]) - ArcTan[(Sqrt[3]*x)/(-x + 2
^(2/3)*(x^2 + x^4)^(1/3))]/(4*2^(1/3)*Sqrt[3]) - (Sqrt[3]*ArcTan[(Sqrt[3]*x
)/(x + 2^(2/3)*(x^2 + x^4)^(1/3))])/(4*2^(1/3)) + Log[-x + (x^2 + x^4)^(1/3
)]/6 - Log[x + (x^2 + x^4)^(1/3)]/2 + Log[-2*x + 2^(2/3)*(x^2 + x^4)^(1/3)]
/(4*2^(1/3)) - Log[2*x + 2^(2/3)*(x^2 + x^4)^(1/3)]/(12*2^(1/3)) + Log[x^2
- x*(x^2 + x^4)^(1/3) + (x^2 + x^4)^(2/3)]/4 - Log[x^2 + x*(x^2 + x^4)^(1/3
) + (x^2 + x^4)^(2/3)]/12 + Log[-2*x^2 + 2^(2/3)*x*(x^2 + x^4)^(1/3) - 2^(1
/3)*(x^2 + x^4)^(2/3)]/(24*2^(1/3)) - Log[2*x^2 + 2^(2/3)*x*(x^2 + x^4)^(1/
3) + 2^(1/3)*(x^2 + x^4)^(2/3)]/(8*2^(1/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^4+x^2)^(1/3)/(x^6-1),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+x^3+1}{\left(x^6-1\right)\left(x^4+x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^4+x^2)^(1/3)/(x^6-1),x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^3 + 1)/((x^6 - 1)*(x^4 + x^2)^(1/3)), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+x^3+1}{\left(x^4+x^2\right)^{\frac{1}{3}}\left(x^6-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+x^3+1)/(x^4+x^2)^(1/3)/(x^6-1),x)
```

```
[Out] int((x^6+x^3+1)/(x^4+x^2)^(1/3)/(x^6-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+x^3+1}{\left(x^6-1\right)\left(x^4+x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^4+x^2)^(1/3)/(x^6-1),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + x^3 + 1)/((x^6 - 1)*(x^4 + x^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6+x^3+1}{\left(x^4+x^2\right)^{1/3}\,\left(x^6-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + x^6 + 1)/((x^2 + x^4)^(1/3)*(x^6 - 1)),x)
```

```
[Out] int((x^3 + x^6 + 1)/((x^2 + x^4)^(1/3)*(x^6 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6+x^3+1}{\sqrt[3]{x^2\left(x^2+1\right)}\,(x-1)\,(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+x**3+1)/(x**4+x**2)**(1/3)/(x**6-1),x)
```

```
[Out] Integral((x**6 + x**3 + 1)/((x**2*(x**2 + 1))**(1/3)*(x - 1)*(x + 1)*(x**2
- x + 1)*(x**2 + x + 1)), x)
```

3.2356 $\int \frac{\sqrt{-b+a^2x^2}\sqrt{ax+\sqrt{-b+a^2x^2}}}{\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=431

$$\frac{5b^2\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{c}}\right)}{16ac^{7/2}}+\frac{\sqrt{a^2x^2-b}\left(\left(-2560a^2c^4x^2-2048ac^6x-1575b^2+640bc^4\right)\sqrt{\sqrt{\sqrt{a^2x^2-b}+}}\right.}{}$$

Rubi [F] time = 1.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-b+a^2x^2}\sqrt{ax+\sqrt{-b+a^2x^2}}}{\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-b + a^2*x^2]*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/Sqrt[c + Sqrt[a*x +
 Sqrt[-b + a^2*x^2]]],x]
```

```
[Out] Defer[Int][(Sqrt[-b + a^2*x^2]*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/Sqrt[c + Sqr
t[a*x + Sqrt[-b + a^2*x^2]]], x]
```

Rubi steps

$$\int \frac{\sqrt{-b+a^2x^2}\sqrt{ax+\sqrt{-b+a^2x^2}}}{\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{\sqrt{-b+a^2x^2}\sqrt{ax+\sqrt{-b+a^2x^2}}}{\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Mathematica [F] time = 10.90, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-b+a^2x^2}\sqrt{ax+\sqrt{-b+a^2x^2}}}{\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[-b + a^2*x^2]*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/Sqrt[c + Sqrt
[a*x + Sqrt[-b + a^2*x^2]]],x]
```

```
[Out] Integrate[(Sqrt[-b + a^2*x^2]*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/Sqrt[c + Sqrt
[a*x + Sqrt[-b + a^2*x^2]]], x]
```

IntegrateAlgebraic [A] time = 1.21, size = 431, normalized size = 1.00

$$\frac{5b^2\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{c}}\right)}{16ac^{7/2}}+\frac{\sqrt{a^2x^2-b}\left(\left(-2560a^2c^4x^2-2048ac^6x-1575b^2+640bc^4\right)\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}+\sqrt{\sqrt{a^2x^2-b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}\left(2240a^2c^3x^2+1536ac^5x-10640bc^3+2048c^7\right)\right)+\sqrt{\sqrt{a^2x^2-b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}\left(2240a^3c^3x^3+1536a^2c^5x^2-11760abc^3x+2048ac^7x+1050b^2c-768bc^5\right)+\left(-2560a^3c^4x^3-2048a^2c^6x^2-1575ab^2x+1920abc^4x-840b^2c^2+1024bc^6\right)\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}}{5040ac^3\left(\sqrt{a^2x^2-b}+ax\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-b + a^2*x^2]*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/Sqrt
[c + Sqrt[a*x + Sqrt[-b + a^2*x^2]]],x]
```

```
[Out] ((-840*b^2*c^2 + 1024*b*c^6 - 1575*a*b^2*x + 1920*a*b*c^4*x - 2048*a^2*c^6*
x^2 - 2560*a^3*c^4*x^3)*Sqrt[c + Sqrt[a*x + Sqrt[-b + a^2*x^2]]] + (1050*b^
2*c - 768*b*c^5 - 11760*a*b*c^3*x + 2048*a*c^7*x + 1536*a^2*c^5*x^2 + 2240*
a^3*c^3*x^3)*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[-b + a
^2*x^2]]] + Sqrt[-b + a^2*x^2]*((-1575*b^2 + 640*b*c^4 - 2048*a*c^6*x - 256
0*a^2*c^4*x^2)*Sqrt[c + Sqrt[a*x + Sqrt[-b + a^2*x^2]]] + (-10640*b*c^3 + 2
048*c^7 + 1536*a*c^5*x + 2240*a^2*c^3*x^2)*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*S
qrt[c + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]))/(5040*a*c^3*(a*x + Sqrt[-b + a^2*
x^2])^(3/2)) + (5*b^2*ArcTanh[Sqrt[c + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]/Sqrt
[c]])/(16*a*c^(7/2))
```

fricas [A] time = 0.51, size = 564, normalized size = 1.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^2-
b)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/10080*(1575*b^2*sqrt(c)*log(2*(a*sqrt(c)*x - sqrt(a^2*x^2 - b)*sqrt(c))*
sqrt(a*x + sqrt(a^2*x^2 - b))*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 - b))) + 2*(
a*c*x - sqrt(a^2*x^2 - b)*c)*sqrt(a*x + sqrt(a^2*x^2 - b)) + b) + 2*(2048*c
^8 + 1120*a^2*c^4*x^2 - 10640*b*c^4 + 6*(128*a*c^6 + 175*a*b*c^2)*x + 2*(38
4*c^6 + 560*a*c^4*x - 525*b*c^2)*sqrt(a^2*x^2 - b) - (1024*c^7 + 1680*a^2*c
^3*x^2 - 840*b*c^3 + 5*(128*a*c^5 + 315*a*b*c)*x + 5*(128*c^5 - 336*a*c^3*x
 - 315*b*c)*sqrt(a^2*x^2 - b))*sqrt(a*x + sqrt(a^2*x^2 - b)))*sqrt(c + sqrt
(a*x + sqrt(a^2*x^2 - b))))/(a*c^4), -1/5040*(1575*b^2*sqrt(-c)*arctan(sqrt
(-c)*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 - b)))/c) - (2048*c^8 + 1120*a^2*c^4*
x^2 - 10640*b*c^4 + 6*(128*a*c^6 + 175*a*b*c^2)*x + 2*(384*c^6 + 560*a*c^4*
x - 525*b*c^2)*sqrt(a^2*x^2 - b) - (1024*c^7 + 1680*a^2*c^3*x^2 - 840*b*c^3
 + 5*(128*a*c^5 + 315*a*b*c)*x + 5*(128*c^5 - 336*a*c^3*x - 315*b*c)*sqrt(a
^2*x^2 - b))*sqrt(a*x + sqrt(a^2*x^2 - b)))*sqrt(c + sqrt(a*x + sqrt(a^2*x^
2 - b))))/(a*c^4)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^2-
b)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^{2} x^{2}-b}\,\sqrt{a x +\sqrt{a^{2} x^{2}-b}}}{\sqrt{c +\sqrt{a x +\sqrt{a^{2} x^{2}-b}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b)^(1/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/
2))^(1/2))^(1/2),x)
```

```
[Out] int((a^2*x^2-b)^(1/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/
2))^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2-b}\sqrt{ax+\sqrt{a^2x^2-b}}}{\sqrt{c+\sqrt{ax+\sqrt{a^2x^2-b}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 - b)*sqrt(a*x + sqrt(a^2*x^2 - b))/sqrt(c + sqrt(a*x + sqrt(a^2*x^2 - b))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x+\sqrt{a^2\,x^2-b}}\,\sqrt{a^2\,x^2-b}}{\sqrt{c+\sqrt{a\,x+\sqrt{a^2\,x^2-b}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (a^2*x^2 - b)^(1/2))^(1/2)*(a^2*x^2 - b)^(1/2))/(c + (a*x + (a^2*x^2 - b)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] int(((a*x + (a^2*x^2 - b)^(1/2))^(1/2)*(a^2*x^2 - b)^(1/2))/(c + (a*x + (a^2*x^2 - b)^(1/2))^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+\sqrt{a^2x^2-b}}\,\sqrt{a^2x^2-b}}{\sqrt{c+\sqrt{ax+\sqrt{a^2x^2-b}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b)**(1/2)*(a*x+(a**2*x**2-b)**(1/2))**(1/2)/(c+(a*x+(a**2*x**2-b)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a*x + sqrt(a**2*x**2 - b))*sqrt(a**2*x**2 - b)/sqrt(c + sqrt(a*x + sqrt(a**2*x**2 - b))), x)
```

3.2357 $\int \frac{(-1+ax^8)(1+ax^8)^{3/4}}{1+x^8+a^2x^{16}}\,dx$

Optimal. Leaf size=432

$$\frac{\left(1+\sqrt[4]{-1}\right)\tan^{-1}\left(\frac{(-1)^{7/8}\sqrt{2+\sqrt{2}}\,\sqrt[8]{2a-1}\,x\sqrt[4]{ax^8+1}}{\sqrt{ax^8+1}+(-1)^{3/4}\sqrt[4]{2a-1}\,x^2}\right)}{8\sqrt[8]{2a-1}}-\frac{i\left(\sqrt{2\left(3-2\sqrt{2}\right)}-i\sqrt{2}\right)\tan^{-1}\left(\frac{(-1)^{7/8}\left(\sqrt{2}-2\right)\sqrt[8]{2a-1}\,x\sqrt[4]{ax^8+1}}{\sqrt{2-\sqrt{2}}\sqrt{ax^8+1}+(-1)^{3/4}\sqrt{2-\sqrt{2}}\sqrt[4]{2a-1}\,x^2}\right)}{16\sqrt[8]{2a-1}}$$

Rubi [C] time = 0.38, antiderivative size = 160, normalized size of antiderivative = 0.37, number of steps used = 4, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {6728, 429}

$$\frac{a\left(1-\frac{2a+1}{\sqrt{1-4a^2}}\right)xF_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};-ax^8,-\frac{2a^2x^8}{1-\sqrt{1-4a^2}}\right)}{1-\sqrt{1-4a^2}}+\frac{a\left(\frac{2a+1}{\sqrt{1-4a^2}}+1\right)xF_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};-ax^8,-\frac{2a^2x^8}{\sqrt{1-4a^2}+1}\right)}{\sqrt{1-4a^2}+1}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-1 + a*x^8)*(1 + a*x^8)^(3/4))/(1 + x^8 + a^2*x^16),x]
```

```
[Out] (a*(1 - (1 + 2*a)/Sqrt[1 - 4*a^2])*x*AppellF1[1/8, -3/4, 1, 9/8, -(a*x^8),
(-2*a^2*x^8)/(1 - Sqrt[1 - 4*a^2])])/(1 - Sqrt[1 - 4*a^2]) + (a*(1 + (1 + 2
*a)/Sqrt[1 - 4*a^2])*x*AppellF1[1/8, -3/4, 1, 9/8, -(a*x^8), (-2*a^2*x^8)/(
1 + Sqrt[1 - 4*a^2])])/(1 + Sqrt[1 - 4*a^2])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-1+ax^8\right)\left(1+ax^8\right)^{3/4}}{1+x^8+a^2x^{16}}\,dx &= \int \left(\frac{\left(a-\frac{a(1+2a)}{\sqrt{1-4a^2}}\right)\left(1+ax^8\right)^{3/4}}{1-\sqrt{1-4a^2}+2a^2x^8}+\frac{\left(a+\frac{a(1+2a)}{\sqrt{1-4a^2}}\right)\left(1+ax^8\right)^{3/4}}{1+\sqrt{1-4a^2}+2a^2x^8}\right)dx\\ &= \left(a\left(1-\frac{1+2a}{\sqrt{1-4a^2}}\right)\right)\int \frac{\left(1+ax^8\right)^{3/4}}{1-\sqrt{1-4a^2}+2a^2x^8}\,dx+\left(a\left(1+\frac{1+2a}{\sqrt{1-4a^2}}\right)\right)\int \frac{\left(1+ax^8\right)^{3/4}}{1+\sqrt{1-4a^2}+2a^2x^8}\,dx\\ &= \frac{a\left(1-\frac{1+2a}{\sqrt{1-4a^2}}\right)xF_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};-ax^8,-\frac{2a^2x^8}{1-\sqrt{1-4a^2}}\right)}{1-\sqrt{1-4a^2}}+\frac{a\left(1+\frac{1+2a}{\sqrt{1-4a^2}}\right)xF_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};-ax^8,-\frac{2a^2x^8}{1+\sqrt{1-4a^2}}\right)}{1+\sqrt{1-4a^2}}\end{aligned}$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\left(-1+ax^8\right)\left(1+ax^8\right)^{3/4}}{1+x^8+a^2x^{16}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-1 + a*x^8)*(1 + a*x^8)^(3/4))/(1 + x^8 + a^2*x^16),x]
```

```
[Out] Integrate[((-1 + a*x^8)*(1 + a*x^8)^(3/4))/(1 + x^8 + a^2*x^16), x]
```

IntegrateAlgebraic [A] time = 14.88, size = 486, normalized size = 1.12

$$\frac{i\left(\sqrt{2\left(3+2\sqrt{2}\right)}+i\sqrt{2}\right)\tan^{-1}\left(\frac{(1+i)(-1)^{3/4}\sqrt{ax^8+1}+(1+i)\sqrt{ax^8+1}+\left((-1+i)-(1+i)(-1)^{3/4}\right)\sqrt[4]{2a-1}\,x^2}{2\sqrt[8]{2a-1}\,x\sqrt[4]{ax^8+1}}\right)}{16\sqrt[8]{2a-1}}+\frac{\left(\sqrt{2}+i\sqrt{2\left(3-2\sqrt{2}\right)}\right)\tan^{-1}\left(\frac{2\sqrt[8]{2a-1}\,x\sqrt[4]{ax^8+1}}{-\sqrt{2}\sqrt{ax^8+1}-(1+i)\sqrt{ax^8+1}+\left(\sqrt{2}+(1-i)\right)\sqrt[4]{2a-1}\,x^2}\right)}{16\sqrt[8]{2a-1}}+\frac{\left((-1)^{3/4}+i\right)\tanh^{-1}\left(\frac{-(2+2i)(-1)^{3/4}\sqrt{ax^8+1}-(2+2i)\sqrt{ax^8+1}+\left((-2+2i)-(2+2i)(-1)^{3/4}\right)\sqrt[4]{2a-1}\,x^2}{4\sqrt[8]{2a-1}\,x\sqrt[4]{ax^8+1}}\right)}{8\sqrt[8]{2a-1}}-\frac{i\left(\sqrt{2\left(3-2\sqrt{2}\right)}-i\sqrt{2}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{ax^8+1}+(1+i)\sqrt{ax^8+1}+\left(\sqrt{2}+(1-i)\right)\sqrt[4]{2a-1}\,x^2}{2\sqrt[8]{2a-1}\,x\sqrt[4]{ax^8+1}}\right)}{16\sqrt[8]{2a-1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-1 + a*x^8)*(1 + a*x^8)^(3/4))/(1 + x^8 + a^2*x^16),x]
```

```
[Out] ((I/16)*(I*Sqrt[2] + Sqrt[2*(3 + 2*Sqrt[2])])*ArcTan[(((-1 + I) - (1 + I)*(
-1)^(3/4))*(-1 + 2*a)^(1/4)*x^2 + (1 + I)*Sqrt[1 + a*x^8] + (1 + I)*(-1)^(3
/4)*Sqrt[1 + a*x^8])/(2*(-1 + 2*a)^(1/8)*x*(1 + a*x^8)^(1/4))])/(-1 + 2*a)^
(1/8) + ((Sqrt[2] + I*Sqrt[2*(3 - 2*Sqrt[2])])*ArcTan[(2*(-1 + 2*a)^(1/8)*x
*(1 + a*x^8)^(1/4))/(((1 - I) + Sqrt[2])*(-1 + 2*a)^(1/4)*x^2 - (1 + I)*Sqr
t[1 + a*x^8] - Sqrt[2]*Sqrt[1 + a*x^8])])/(16*(-1 + 2*a)^(1/8)) + ((I + (-1
)^(3/4))*ArcTanh[(((-2 + 2*I) - (2 + 2*I)*(-1)^(3/4))*(-1 + 2*a)^(1/4)*x^2
- (2 + 2*I)*Sqrt[1 + a*x^8] - (2 + 2*I)*(-1)^(3/4)*Sqrt[1 + a*x^8])/(4*(-1
+ 2*a)^(1/8)*x*(1 + a*x^8)^(1/4))])/(8*(-1 + 2*a)^(1/8)) - ((I/16)*((-I)*Sq
rt[2] + Sqrt[2*(3 - 2*Sqrt[2])])*ArcTanh[(((1 - I) + Sqrt[2])*(-1 + 2*a)^(1
/4)*x^2 + (1 + I)*Sqrt[1 + a*x^8] + Sqrt[2]*Sqrt[1 + a*x^8])/(2*(-1 + 2*a)^
(1/8)*x*(1 + a*x^8)^(1/4))])/(-1 + 2*a)^(1/8)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-1)*(a*x^8+1)^(3/4)/(a^2*x^16+x^8+1),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^8+1\right)^{\frac{3}{4}}\left(ax^8-1\right)}{a^2x^{16}+x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-1)*(a*x^8+1)^(3/4)/(a^2*x^16+x^8+1),x, algorithm="giac")
```

```
[Out] integrate((a*x^8 + 1)^(3/4)*(a*x^8 - 1)/(a^2*x^16 + x^8 + 1), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^8-1\right)\left(a\,x^8+1\right)^{\frac{3}{4}}}{a^2x^{16}+x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^8-1)*(a*x^8+1)^(3/4)/(a^2*x^16+x^8+1),x)
```

```
[Out] int((a*x^8-1)*(a*x^8+1)^(3/4)/(a^2*x^16+x^8+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^8+1\right)^{\frac{3}{4}}\left(ax^8-1\right)}{a^2x^{16}+x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^8-1)*(a*x^8+1)^(3/4)/(a^2*x^16+x^8+1),x, algorithm="maxima")
[Out] integrate((a*x^8 + 1)^(3/4)*(a*x^8 - 1)/(a^2*x^16 + x^8 + 1), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x^8-1\right)\,{\left(a\,x^8+1\right)}^{3/4}}{a^2\,x^{16}+x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^8 - 1)*(a*x^8 + 1)^(3/4))/(x^8 + a^2*x^16 + 1),x)
[Out] int(((a*x^8 - 1)*(a*x^8 + 1)^(3/4))/(x^8 + a^2*x^16 + 1), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**8-1)*(a*x**8+1)**(3/4)/(a**2*x**16+x**8+1),x)
[Out] Timed out
```

3.2358 $\int \frac{-b+ax}{(b+ax)\sqrt[3]{b^2x^2+a^3x^3}}\,dx$

Optimal. Leaf size=433

$$-\frac{\log\left(\sqrt[3]{a^3x^3+b^2x^2}-ax\right)}{a}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,ax}{2\sqrt[3]{a^3x^3+b^2x^2}+ax}\right)}{a}-\frac{i\left(\sqrt{3}-i\right)\log\left(\sqrt[3]{-1}\sqrt[3]{a^3x^3+b^2x^2}+\sqrt[3]{a}\,x\sqrt[3]{a^2-b}\right)}{\sqrt[3]{a}\sqrt[3]{a^2-b}}-$$

Rubi [A] time = 0.24, antiderivative size = 461, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2056, 157, 59, 91}

$$-\frac{x^{2/3}\log(x)\sqrt[3]{a^3x+b^2}}{2a\sqrt[3]{a^3x^3+b^2x^2}}-\frac{3x^{2/3}\sqrt[3]{a^3x+b^2}\log\left(\frac{\sqrt[3]{a^3x+b^2}}{a\sqrt[3]{x}}-1\right)}{2a\sqrt[3]{a^3x^3+b^2x^2}}-\frac{\sqrt{3}\,x^{2/3}\sqrt[3]{a^3x+b^2}\tan^{-1}\left(\frac{2\sqrt[3]{a^3x+b^2}}{\sqrt{3}\,a\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{a\sqrt[3]{a^3x^3+b^2x^2}}-\frac{x^{2/3}\sqrt[3]{a^3x+b^2}\log(ax+b)}{\sqrt[3]{a}\sqrt[3]{a^2-b}\sqrt[3]{a^3x^3+b^2x^2}}+\frac{3x^{2/3}\sqrt[3]{a^3x+b^2}\log\left(\frac{\sqrt[3]{a^3x+b^2}}{\sqrt[3]{a}\sqrt[3]{a^2-b}}-\sqrt[3]{x}\right)}{\sqrt[3]{a}\sqrt[3]{a^2-b}\sqrt[3]{a^3x^3+b^2x^2}}+\frac{2\sqrt{3}\,x^{2/3}\sqrt[3]{a^3x+b^2}\tan^{-1}\left(\frac{2\sqrt[3]{a^3x+b^2}}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{a^2-b}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[3]{a^2-b}\sqrt[3]{a^3x^3+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x)/((b + a*x)*(b^2*x^2 + a^3*x^3)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*x^(2/3)*(b^2 + a^3*x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(b^2 + a^3*x)^
(1/3))/(Sqrt[3]*a*x^(1/3))])/(a*(b^2*x^2 + a^3*x^3)^(1/3))) + (2*Sqrt[3]*x^
(2/3)*(b^2 + a^3*x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(b^2 + a^3*x)^(1/3))/(Sqrt[
3]*a^(1/3)*(a^2 - b)^(1/3)*x^(1/3))])/(a^(1/3)*(a^2 - b)^(1/3)*(b^2*x^2 + a
^3*x^3)^(1/3)) - (x^(2/3)*(b^2 + a^3*x)^(1/3)*Log[x])/(2*a*(b^2*x^2 + a^3*x
^3)^(1/3)) - (x^(2/3)*(b^2 + a^3*x)^(1/3)*Log[b + a*x])/(a^(1/3)*(a^2 - b)^
(1/3)*(b^2*x^2 + a^3*x^3)^(1/3)) + (3*x^(2/3)*(b^2 + a^3*x)^(1/3)*Log[-x^(1
/3) + (b^2 + a^3*x)^(1/3)/(a^(1/3)*(a^2 - b)^(1/3))])/(a^(1/3)*(a^2 - b)^(1
/3)*(b^2*x^2 + a^3*x^3)^(1/3)) - (3*x^(2/3)*(b^2 + a^3*x)^(1/3)*Log[-1 + (b
^2 + a^3*x)^(1/3)/(a*x^(1/3))])/(2*a*(b^2*x^2 + a^3*x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
```

```
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{-b+ax}{(b+ax)\sqrt[3]{b^2x^2+a^3x^3}}\,dx = \frac{\left(x^{2/3}\sqrt[3]{b^2+a^3x}\right)\int \frac{-b+ax}{x^{2/3}(b+ax)\sqrt[3]{b^2+a^3x}}\,dx}{\sqrt[3]{b^2x^2+a^3x^3}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{b^2+a^3x}\right)\int \frac{1}{x^{2/3}\sqrt[3]{b^2+a^3x}}\,dx}{\sqrt[3]{b^2x^2+a^3x^3}} - \frac{\left(2bx^{2/3}\sqrt[3]{b^2+a^3x}\right)\int \frac{1}{x^{2/3}(b+ax)\sqrt[3]{b^2+a^3x}}\,dx}{\sqrt[3]{b^2x^2+a^3x^3}}$$

$$= -\frac{\sqrt{3}\,x^{2/3}\sqrt[3]{b^2+a^3x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b^2+a^3x}}{\sqrt{3}\,a\sqrt[3]{x}}\right)}{a\sqrt[3]{b^2x^2+a^3x^3}} + \frac{2\sqrt{3}\,x^{2/3}\sqrt[3]{b^2+a^3x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+ \cdots\right.}{\sqrt[3]{a}\,\sqrt[3]{a^2-b}\,\sqrt[3]{b^2x^2+a^3 \cdots}}$$

Mathematica [C] time = 0.05, size = 87, normalized size = 0.20

$$\frac{3x\sqrt[3]{\frac{a^3x}{b^2}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{a^3x}{b^2}\right)-6x\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(a^3-ab)x}{xa^3+b^2}\right)}{\sqrt[3]{x^2\left(a^3x+b^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x)/((b + a*x)*(b^2*x^2 + a^3*x^3)^(1/3)),x]
```

```
[Out] (3*x*(1 + (a^3*x)/b^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, -((a^3*x)/b^2
)] - 6*x*Hypergeometric2F1[1/3, 1, 4/3, ((a^3 - a*b)*x)/(b^2 + a^3*x)])/(x^
2*(b^2 + a^3*x))^(1/3)
```

IntegrateAlgebraic [A] time = 3.26, size = 482, normalized size = 1.11

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}ax}{2\sqrt[3]{a^3x^3+b^2x^2}+ax}\right)}{a}-\frac{i\left(\sqrt{3}-i\right)\log\left(2\sqrt[3]{a}x\sqrt[3]{a^2-b}+\left(1+i\sqrt{3}\right)\sqrt[3]{a^3x^3+b^2x^2}\right)}{\sqrt[3]{a}\sqrt[3]{a^2-b}}-\frac{\log\left(a^2x-a\sqrt[3]{a^3x^3+b^2x^2}\right)}{a}+\frac{\log\left(ax\sqrt[3]{a^3x^3+b^2x^2}+\left(a^3x^3+b^2x^2\right)^{2/3}+a^2x^2\right)}{2a}+\frac{\sqrt{6\left(-1+i\sqrt{3}\right)}\tan^{-1}\left(\frac{3\sqrt[3]{a}x\sqrt[3]{a^2-b}}{-\sqrt{3}\sqrt[3]{a^3x^3+b^2x^2}-3i\sqrt[3]{a^3x^3+b^2x^2}+\sqrt{3}\sqrt[3]{a}x\sqrt[3]{a^2-b}}\right)}{\sqrt[3]{a}\sqrt[3]{a^2-b}}+\frac{\left(1+i\sqrt{3}\right)\log\left(\left(\sqrt{3}+i\right)\left(a^3x^3+b^2x^2\right)^{2/3}-2ia^{2/3}x^2\left(a^2-b\right)^{2/3}+\sqrt[3]{a}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a^2-b}\sqrt[3]{a^3x^3+b^2x^2}\right)}{2\sqrt[3]{a}\sqrt[3]{a^2-b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x)/((b + a*x)*(b^2*x^2 + a^3*x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*a*x)/(a*x + 2*(b^2*x^2 + a^3*x^3)^(1/3))])/a + (Sq
rt[6*(-1 + I*Sqrt[3])]*ArcTan[(3*a^(1/3)*(a^2 - b)^(1/3)*x)/(Sqrt[3]*a^(1/3
)*(a^2 - b)^(1/3)*x - (3*I)*(b^2*x^2 + a^3*x^3)^(1/3) - Sqrt[3]*(b^2*x^2 +
a^3*x^3)^(1/3))])/(a^(1/3)*(a^2 - b)^(1/3)) - (I*(-I + Sqrt[3])*Log[2*a^(1/
3)*(a^2 - b)^(1/3)*x + (1 + I*Sqrt[3])*(b^2*x^2 + a^3*x^3)^(1/3)])/(a^(1/3)
*(a^2 - b)^(1/3)) - Log[a^2*x - a*(b^2*x^2 + a^3*x^3)^(1/3)]/a + Log[a^2*x^
2 + a*x*(b^2*x^2 + a^3*x^3)^(1/3) + (b^2*x^2 + a^3*x^3)^(2/3)]/(2*a) + ((1
+ I*Sqrt[3])*Log[(-2*I)*a^(2/3)*(a^2 - b)^(2/3)*x^2 + a^(1/3)*(a^2 - b)^(1/
3)*(I*x - Sqrt[3]*x)*(b^2*x^2 + a^3*x^3)^(1/3) + (I + Sqrt[3])*(b^2*x^2 + a
^3*x^3)^(2/3)])/(2*a^(1/3)*(a^2 - b)^(1/3))
```

fricas [A] time = 0.48, size = 823, normalized size = 1.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/(a*x+b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(3)*(a^3 - a*b)*sqrt(-1/(a^3 - a*b)^(2/3))*log((2*b^2*x + (3*a^
3 - a*b)*x^2 - 3*(a^3*x^3 + b^2*x^2)^(1/3)*(a^3 - a*b)^(2/3)*x - sqrt(3)*((
```

```
a^3 - a*b)^(4/3)*x^2 + (a^3*x^3 + b^2*x^2)^(1/3)*(a^3 - a*b)*x - 2*(a^3*x^3
 + b^2*x^2)^(2/3)*(a^3 - a*b)^(2/3))*sqrt(-1/(a^3 - a*b)^(2/3)))/(a*x^2 + b
*x)) - 2*sqrt(3)*(a^2 - b)*arctan(1/3*(sqrt(3)*a*x + 2*sqrt(3)*(a^3*x^3 + b
^2*x^2)^(1/3))/(a*x)) - 2*(a^2 - b)*log(-(a*x - (a^3*x^3 + b^2*x^2)^(1/3))/
x) + (a^2 - b)*log((a^2*x^2 + (a^3*x^3 + b^2*x^2)^(1/3)*a*x + (a^3*x^3 + b^
2*x^2)^(2/3))/x^2) + 4*(a^3 - a*b)^(2/3)*log(-((a^3 - a*b)^(1/3)*x - (a^3*x
^3 + b^2*x^2)^(1/3))/x) - 2*(a^3 - a*b)^(2/3)*log(((a^3 - a*b)^(2/3)*x^2 +
(a^3*x^3 + b^2*x^2)^(1/3)*(a^3 - a*b)^(1/3)*x + (a^3*x^3 + b^2*x^2)^(2/3))/
x^2))/(a^3 - a*b), -1/2*(2*sqrt(3)*(a^2 - b)*arctan(1/3*(sqrt(3)*a*x + 2*sq
rt(3)*(a^3*x^3 + b^2*x^2)^(1/3))/(a*x)) - 4*sqrt(3)*(a^3 - a*b)^(2/3)*arcta
n(1/3*sqrt(3)*((a^3 - a*b)^(1/3)*x + 2*(a^3*x^3 + b^2*x^2)^(1/3))/((a^3 - a
*b)^(1/3)*x)) + 2*(a^2 - b)*log(-(a*x - (a^3*x^3 + b^2*x^2)^(1/3))/x) - (a^
2 - b)*log((a^2*x^2 + (a^3*x^3 + b^2*x^2)^(1/3)*a*x + (a^3*x^3 + b^2*x^2)^(
2/3))/x^2) - 4*(a^3 - a*b)^(2/3)*log(-((a^3 - a*b)^(1/3)*x - (a^3*x^3 + b^2
*x^2)^(1/3))/x) + 2*(a^3 - a*b)^(2/3)*log(((a^3 - a*b)^(2/3)*x^2 + (a^3*x^3
 + b^2*x^2)^(1/3)*(a^3 - a*b)^(1/3)*x + (a^3*x^3 + b^2*x^2)^(2/3))/x^2))/(a
^3 - a*b)]
```

giac [A] time = 141.95, size = 255, normalized size = 0.59

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\left(a^3-ab\right)^{\frac{1}{3}}+2\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}\right)}{3\left(a^3-ab\right)^{\frac{1}{3}}}\right)}{\left(a^3-ab\right)^{\frac{1}{3}}}-\frac{\log\left(\left(a^3-ab\right)^{\frac{2}{3}}+\left(a^3-ab\right)^{\frac{1}{3}}\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}+\left(a^3+\frac{b^2}{x}\right)^{\frac{2}{3}}\right)}{\left(a^3-ab\right)^{\frac{1}{3}}}+\frac{2\log\left(\left|-\left(a^3-ab\right)^{\frac{1}{3}}+\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}\right|\right)}{\left(a^3-ab\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}+\frac{\log\left(a^2+\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}a+\left(a^3+\frac{b^2}{x}\right)^{\frac{2}{3}}\right)}{2a}-\frac{\log\left(\left|-a+\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/(a*x+b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="giac")
```

```
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*((a^3 - a*b)^(1/3) + 2*(a^3 + b^2/x)^(1/3))/(a
^3 - a*b)^(1/3))/(a^3 - a*b)^(1/3) - log((a^3 - a*b)^(2/3) + (a^3 - a*b)^(1
/3)*(a^3 + b^2/x)^(1/3) + (a^3 + b^2/x)^(2/3))/(a^3 - a*b)^(1/3) + 2*log(ab
s(-(a^3 - a*b)^(1/3) + (a^3 + b^2/x)^(1/3)))/(a^3 - a*b)^(1/3) - sqrt(3)*ar
ctan(1/3*sqrt(3)*(a + 2*(a^3 + b^2/x)^(1/3))/a)/a + 1/2*log(a^2 + (a^3 + b^
2/x)^(1/3)*a + (a^3 + b^2/x)^(2/3))/a - log(abs(-a + (a^3 + b^2/x)^(1/3)))/
a
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{ax-b}{\left(ax+b\right)\left(a^{3}x^{3}+b^{2}x^{2}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-b)/(a*x+b)/(a^3*x^3+b^2*x^2)^(1/3),x)
```

```
[Out] int((a*x-b)/(a*x+b)/(a^3*x^3+b^2*x^2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-b}{\left(a^{3}x^{3}+b^{2}x^{2}\right)^{\frac{1}{3}}\left(ax+b\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/(a*x+b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*x - b)/((a^3*x^3 + b^2*x^2)^(1/3)*(a*x + b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{b-a\,x}{{\left(a^3\,x^3+b^2\,x^2\right)}^{1/3}\,\left(b+a\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x)/((a^3*x^3 + b^2*x^2)^(1/3)*(b + a*x)),x)
```

```
[Out] int(-(b - a*x)/((a^3*x^3 + b^2*x^2)^(1/3)*(b + a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax - b}{\sqrt[3]{x^2\left(a^3x + b^2\right)}\,(ax + b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-b)/(a*x+b)/(a**3*x**3+b**2*x**2)**(1/3),x)
```

```
[Out] Integral((a*x - b)/((x**2*(a**3*x + b**2))**(1/3)*(a*x + b)), x)
```

3.2359 $\int \frac{1}{(-1+x^2)^2\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=434

$$\frac{\sqrt{\sqrt{x^2+1}+x}\,x^2}{2\left(x^2-1\right)}-\frac{\sqrt{x^2+1}\,\sqrt{\sqrt{x^2+1}+x}\,x}{2\left(x^2-1\right)}+\frac{\left(\sqrt{2\left(1+\sqrt{2}\right)}\,x^2-4\sqrt{1+\sqrt{2}}\,x^2+\sqrt{2\left(1+5\sqrt{2}\right)}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{\cdots}}}{\sqrt{\cdots}}\right.}{8(x-1)(x+1)}$$

Rubi [A] time = 0.80, antiderivative size = 463, normalized size of antiderivative = 1.07, number of steps used = 34, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {6742, 2119, 1648, 12, 707, 1093, 204, 206, 207, 203, 6725, 1628, 828, 826, 1166}

$$\frac{\sqrt{\sqrt{x^2+1}+x}}{2\left(-\left(\sqrt{x^2+1}+x\right)^2-2\left(\sqrt{x^2+1}+x\right)+1\right)}+\frac{\sqrt{\sqrt{x^2+1}+x}}{2\left(-\left(\sqrt{x^2+1}+x\right)^2+2\left(\sqrt{x^2+1}+x\right)+1\right)}+\frac{\tan^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)}{2\sqrt{1+\sqrt{2}}}+\frac{1}{4}\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\tan^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)+\frac{1}{4}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)-\frac{\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)}{2\sqrt{\sqrt{2}-1}}+\frac{\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)}{2\sqrt{1+\sqrt{2}}}+\frac{1}{4}\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)+\frac{1}{4}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)-\frac{\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)}{2\sqrt{\sqrt{2}-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((-1 + x^2)^2*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] Sqrt[x + Sqrt[1 + x^2]]/(2*(1 - 2*(x + Sqrt[1 + x^2]) - (x + Sqrt[1 + x^2])
^2)) + Sqrt[x + Sqrt[1 + x^2]]/(2*(1 + 2*(x + Sqrt[1 + x^2]) - (x + Sqrt[1
+ x^2])^2)) + (Sqrt[(-1 + Sqrt[2])/2]*ArcTan[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sq
rt[1 + x^2]]])/4 + ArcTan[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]]/(2*Sq
rt[1 + Sqrt[2]]) - ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]]]/(2*Sqr
t[-1 + Sqrt[2]]) + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x +
 Sqrt[1 + x^2]]])/4 + (Sqrt[(-1 + Sqrt[2])/2]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sq
rt[x + Sqrt[1 + x^2]]])/4 + ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^
2]]]/(2*Sqrt[1 + Sqrt[2]]) - ArcTanh[Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^
2]]]/(2*Sqrt[-1 + Sqrt[2]]) + (Sqrt[(1 + Sqrt[2])/2]*ArcTanh[Sqrt[1 + Sqrt[
2]]*Sqrt[x + Sqrt[1 + x^2]]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 707

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Sym
bol] :> Dist[2*e, Subst[Int[1/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1648

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*
e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
```

```
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{\left(-1+x^2\right)^2 \sqrt{x+\sqrt{1+x^2}}}\,dx = \int \Bigg(\frac{1}{4(1-x)^2\sqrt{x+\sqrt{1+x^2}}} + \frac{1}{4(1+x)^2\sqrt{x+\sqrt{1+x^2}}} + \frac{1}{2\left(1-x^2\right)\sqrt{x+\sqrt{1}}}$$

$$= \frac{1}{4}\int \frac{1}{(1-x)^2\sqrt{x+\sqrt{1+x^2}}}\,dx + \frac{1}{4}\int \frac{1}{(1+x)^2\sqrt{x+\sqrt{1+x^2}}}\,dx + \frac{1}{2}\int \frac{}{(1-}$$

$$= \frac{1}{2}\int \Bigg(\frac{1}{2(1-x)\sqrt{x+\sqrt{1+x^2}}} + \frac{1}{2(1+x)\sqrt{x+\sqrt{1+x^2}}} \Bigg) dx + \frac{1}{2}\,\text{Subst}\Bigg(\int \frac{}{\sqrt{}}$$

$$= \frac{\sqrt{x+\sqrt{1+x^2}}}{2\Big(1-2\big(x+\sqrt{1+x^2}\big)-\big(x+\sqrt{1+x^2}\big)^2\Big)} + \frac{\sqrt{x+\sqrt{1+x^2}}}{2\Big(1+2\big(x+\sqrt{1+x^2}\big)-\big(x+\sqrt{}}$$

$$= \frac{\sqrt{x+\sqrt{1+x^2}}}{2\Big(1-2\big(x+\sqrt{1+x^2}\big)-\big(x+\sqrt{1+x^2}\big)^2\Big)} + \frac{\sqrt{x+\sqrt{1+x^2}}}{2\Big(1+2\big(x+\sqrt{1+x^2}\big)-\big(x+\sqrt{}}$$

$$= \frac{\sqrt{x+\sqrt{1+x^2}}}{2\Big(1-2\big(x+\sqrt{1+x^2}\big)-\big(x+\sqrt{1+x^2}\big)^2\Big)} + \frac{\sqrt{x+\sqrt{1+x^2}}}{2\Big(1+2\big(x+\sqrt{1+x^2}\big)-\big(x+\sqrt{}}$$

$$= \frac{\sqrt{x+\sqrt{1+x^2}}}{2\Big(1-2\big(x+\sqrt{1+x^2}\big)-\big(x+\sqrt{1+x^2}\big)^2\Big)} + \frac{\sqrt{x+\sqrt{1+x^2}}}{2\Big(1+2\big(x+\sqrt{1+x^2}\big)-\big(x+\sqrt{}}$$

$$= \frac{\sqrt{x+\sqrt{1+x^2}}}{2\Big(1-2\big(x+\sqrt{1+x^2}\big)-\big(x+\sqrt{1+x^2}\big)^2\Big)} + \frac{\sqrt{x+\sqrt{1+x^2}}}{2\Big(1+2\big(x+\sqrt{1+x^2}\big)-\big(x+\sqrt{}}$$

$$= \frac{\sqrt{x+\sqrt{1+x^2}}}{2\Big(1-2\big(x+\sqrt{1+x^2}\big)-\big(x+\sqrt{1+x^2}\big)^2\Big)} + \frac{\sqrt{x+\sqrt{1+x^2}}}{2\Big(1+2\big(x+\sqrt{1+x^2}\big)-\big(x+\sqrt{}}$$

$$= \frac{\sqrt{x+\sqrt{1+x^2}}}{2\Big(1-2\big(x+\sqrt{1+x^2}\big)-\big(x+\sqrt{1+x^2}\big)^2\Big)} + \frac{\sqrt{x+\sqrt{1+x^2}}}{2\Big(1+2\big(x+\sqrt{1+x^2}\big)-\big(x+\sqrt{}}$$

$$= \frac{\sqrt{x+\sqrt{1+x^2}}}{2\Big(1-2\big(x+\sqrt{1+x^2}\big)-\big(x+\sqrt{1+x^2}\big)^2\Big)} + \frac{\sqrt{x+\sqrt{1+x^2}}}{2\Big(1+2\big(x+\sqrt{1+x^2}\big)-\big(x+\sqrt{}}$$

Mathematica [A] time = 0.84, size = 249, normalized size = 0.57

$$\frac{1}{8}\left(-\frac{4x\left(\sqrt{x^2+1}+x\right)^{3/2}}{\left(x^2-1\right)\left(2x^2+2\sqrt{x^2+1}\,x+1\right)}+\left(\sqrt{2}-3\right)\sqrt{2\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{1}{\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}}\right)+\sqrt{2\left(\sqrt{2}-1\right)}\left(3+\sqrt{2}\right)\tan^{-1}\left(\frac{1}{\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}}\right)-\left(\sqrt{2}-3\right)\sqrt{2\left(1+\sqrt{2}\right)}\tanh^{-1}\left(\frac{1}{\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}}\right)-\sqrt{2\left(\sqrt{2}-1\right)}\left(3+\sqrt{2}\right)\tanh^{-1}\left(\frac{1}{\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-1 + x^2)^2*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] ((-4*x*(x + Sqrt[1 + x^2])^(3/2))/((-1 + x^2)*(1 + 2*x^2 + 2*x*Sqrt[1 + x^2
])) + (-3 + Sqrt[2])*Sqrt[2*(1 + Sqrt[2])]*ArcTan[1/(Sqrt[-1 + Sqrt[2]]*Sqr
t[x + Sqrt[1 + x^2]])] + Sqrt[2*(-1 + Sqrt[2])]*(3 + Sqrt[2])*ArcTan[1/(Sqr
t[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]])] - (-3 + Sqrt[2])*Sqrt[2*(1 + Sqrt[
2])]*ArcTanh[1/(Sqrt[-1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2]])] - Sqrt[2*(-1 +
 Sqrt[2])]*(3 + Sqrt[2])*ArcTanh[1/(Sqrt[1 + Sqrt[2]]*Sqrt[x + Sqrt[1 + x^2
]])])/8
```

IntegrateAlgebraic [A] time = 1.24, size = 241, normalized size = 0.56

$$-\frac{x\left(\sqrt{x^2+1}+x\right)^{3/2}}{2\left(2x^2+1\right)\left(x^2-1\right)+4x\sqrt{x^2+1}\left(x^2-1\right)}+\frac{1}{4}\sqrt{\frac{5}{\sqrt{2}}-\frac{1}{2}}\tan^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)-\frac{1}{4}\sqrt{\frac{1}{2}+\frac{5}{\sqrt{2}}}\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)+\frac{1}{4}\sqrt{\frac{5}{\sqrt{2}}-\frac{1}{2}}\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^2+1}+x}\right)-\frac{1}{4}\sqrt{\frac{1}{2}+\frac{5}{\sqrt{2}}}\tanh^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^2+1}+x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x^2)^2*Sqrt[x + Sqrt[1 + x^2]]),x]
```

```
[Out] -((x*(x + Sqrt[1 + x^2])^(3/2))/(4*x*(-1 + x^2)*Sqrt[1 + x^2] + 2*(-1 + x^2
)*(1 + 2*x^2))) + (Sqrt[-1/2 + 5/Sqrt[2]]*ArcTan[Sqrt[-1 + Sqrt[2]]*Sqrt[x
+ Sqrt[1 + x^2]]])/4 - (Sqrt[1/2 + 5/Sqrt[2]]*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt
[x + Sqrt[1 + x^2]]])/4 + (Sqrt[-1/2 + 5/Sqrt[2]]*ArcTanh[Sqrt[-1 + Sqrt[2]
]*Sqrt[x + Sqrt[1 + x^2]]])/4 - (Sqrt[1/2 + 5/Sqrt[2]]*ArcTanh[Sqrt[1 + Sqr
t[2]]*Sqrt[x + Sqrt[1 + x^2]]])/4
```

fricas [A] time = 0.47, size = 399, normalized size = 0.92

$$\frac{4\sqrt{2}\left(x^2-1\right)\sqrt{5\sqrt{2}+1}\arctan\left(\frac{1}{7}\sqrt{x+\sqrt{2}+\sqrt{x^2+1}-1}\sqrt{5\sqrt{2}+1}\left(2\sqrt{2}+1\right)-\frac{1}{7}\sqrt{x+\sqrt{x^2+1}}\sqrt{5\sqrt{2}+1}\left(2\sqrt{2}+1\right)\right)-4\sqrt{2}\left(x^2-1\right)\sqrt{5\sqrt{2}-1}\arctan\left(\frac{1}{7}\sqrt{x+\sqrt{2}+\sqrt{x^2+1}+1}\sqrt{5\sqrt{2}-1}\left(2\sqrt{2}-1\right)-\frac{1}{7}\sqrt{x+\sqrt{x^2+1}}\sqrt{5\sqrt{2}-1}\left(2\sqrt{2}-1\right)\right)+\sqrt{2}\left(x^2-1\right)\sqrt{5\sqrt{2}-1}\log\left(\sqrt{5\sqrt{2}-1}\left(\sqrt{2}+3\right)+7\sqrt{x+\sqrt{x^2+1}}\right)-\sqrt{2}\left(x^2-1\right)\sqrt{5\sqrt{2}-1}\log\left(-\sqrt{5\sqrt{2}-1}\left(\sqrt{2}+3\right)+7\sqrt{x+\sqrt{x^2+1}}\right)+\sqrt{2}\left(x^2-1\right)\sqrt{5\sqrt{2}+1}\log\left(\sqrt{5\sqrt{2}+1}\left(\sqrt{2}-3\right)+7\sqrt{x+\sqrt{x^2+1}}\right)-\sqrt{2}\left(x^2-1\right)\sqrt{5\sqrt{2}+1}\log\left(-\sqrt{5\sqrt{2}+1}\left(\sqrt{2}-3\right)+7\sqrt{x+\sqrt{x^2+1}}\right)+8\left(x^2-\sqrt{x^2+1}x\right)\sqrt{x+\sqrt{x^2+1}}}{16\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(4*sqrt(2)*(x^2 - 1)*sqrt(5*sqrt(2) + 1)*arctan(1/7*sqrt(x + sqrt(2) +
 sqrt(x^2 + 1) - 1)*sqrt(5*sqrt(2) + 1)*(2*sqrt(2) + 1) - 1/7*sqrt(x + sqrt
(x^2 + 1))*sqrt(5*sqrt(2) + 1)*(2*sqrt(2) + 1)) - 4*sqrt(2)*(x^2 - 1)*sqrt(
5*sqrt(2) - 1)*arctan(1/7*sqrt(x + sqrt(2) + sqrt(x^2 + 1) + 1)*sqrt(5*sqrt
(2) - 1)*(2*sqrt(2) - 1) - 1/7*sqrt(x + sqrt(x^2 + 1))*sqrt(5*sqrt(2) - 1)*
(2*sqrt(2) - 1)) + sqrt(2)*(x^2 - 1)*sqrt(5*sqrt(2) - 1)*log(sqrt(5*sqrt(2)
 - 1)*(sqrt(2) + 3) + 7*sqrt(x + sqrt(x^2 + 1))) - sqrt(2)*(x^2 - 1)*sqrt(5
*sqrt(2) - 1)*log(-sqrt(5*sqrt(2) - 1)*(sqrt(2) + 3) + 7*sqrt(x + sqrt(x^2
+ 1))) + sqrt(2)*(x^2 - 1)*sqrt(5*sqrt(2) + 1)*log(sqrt(5*sqrt(2) + 1)*(sqr
t(2) - 3) + 7*sqrt(x + sqrt(x^2 + 1))) - sqrt(2)*(x^2 - 1)*sqrt(5*sqrt(2) +
 1)*log(-sqrt(5*sqrt(2) + 1)*(sqrt(2) - 3) + 7*sqrt(x + sqrt(x^2 + 1))) + 8
*(x^2 - sqrt(x^2 + 1)*x)*sqrt(x + sqrt(x^2 + 1)))/(x^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-1\right)^2\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - 1)^2*sqrt(x + sqrt(x^2 + 1))), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-1\right)^2\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^2/(x+(x^2+1)^(1/2))^(1/2),x)
[Out] int(1/(x^2-1)^2/(x+(x^2+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-1\right)^2\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2/(x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 - 1)^2*sqrt(x + sqrt(x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(x^2-1\right)^2\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^2*(x + (x^2 + 1)^(1/2))^(1/2)),x)
[Out] int(1/((x^2 - 1)^2*(x + (x^2 + 1)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x-1)^2\,(x+1)^2\sqrt{x+\sqrt{x^2+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**2/(x+(x**2+1)**(1/2))**(1/2),x)
[Out] Integral(1/((x - 1)**2*(x + 1)**2*sqrt(x + sqrt(x**2 + 1))), x)
```

3.2360 $\int \frac{(-b+x)^2}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(-a^2+b^2d+2(a-bd)x+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=438

$$\frac{(x-a)^{2/3}(x-b)\left(-\sqrt[6]{d}\,\sqrt[3]{x-a}\,\sqrt[3]{b-x}+(x-a)^{2/3}+\sqrt[3]{d}\,(b-x)^{2/3}\right)\left(\sqrt[6]{d}\,\sqrt[3]{x-a}\,\sqrt[3]{b-x}+(x-a)^{2/3}+\sqrt[3]{d}\,(b-x\right.}{\left((x-a)(b-x)\right.}$$

Rubi [A] time = 1.32, antiderivative size = 513, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 5, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {6719, 911, 105, 59, 91}

$$-\frac{(x-a)^{2/3}(x-b)^{4/3}\log\left(2\left(1-\sqrt{d}\right)\left(a+b\sqrt{d}\right)-2(1-d)x\right)}{4d^{5/6}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{(x-a)^{2/3}(x-b)^{4/3}\log\left(2(1-d)x-2\left(\sqrt{d}+1\right)\left(a-b\sqrt{d}\right)\right)}{4d^{5/6}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{3(x-a)^{2/3}(x-b)^{4/3}\log\left(-\sqrt[3]{x-a}-\sqrt[6]{d}\sqrt[3]{x-b}\right)}{4d^{5/6}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}-\frac{3(x-a)^{2/3}(x-b)^{4/3}\log\left(\sqrt[6]{d}\sqrt[3]{x-b}-\sqrt[3]{x-a}\right)}{4d^{5/6}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{\sqrt{3}(x-a)^{2/3}(x-b)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[3]{x-b}}{\sqrt{3}\sqrt[3]{x-a}}\right)}{2d^{5/6}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}-\frac{\sqrt{3}(x-a)^{2/3}(x-b)^{4/3}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[3]{x-b}}{\sqrt{3}\sqrt[3]{x-a}}+\frac{1}{\sqrt{3}}\right)}{2d^{5/6}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + x)^2/(((-a + x)*(-b + x)^2)^(2/3)*(-a^2 + b^2*d + 2*(a - b*d)*x +
 (-1 + d)*x^2)),x]
```

```
[Out] (Sqrt[3]*(-a + x)^(2/3)*(-b + x)^(4/3)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(-b +
x)^(1/3))/(Sqrt[3]*(-a + x)^(1/3))])/(2*(a - b)*d^(5/6)*(-((a - x)*(b - x)^
2))^(2/3)) - (Sqrt[3]*(-a + x)^(2/3)*(-b + x)^(4/3)*ArcTan[1/Sqrt[3] + (2*d
^(1/6)*(-b + x)^(1/3))/(Sqrt[3]*(-a + x)^(1/3))])/(2*(a - b)*d^(5/6)*(-((a
- x)*(b - x)^2))^(2/3)) - ((-a + x)^(2/3)*(-b + x)^(4/3)*Log[2*(1 - Sqrt[d]
)*(a + b*Sqrt[d]) - 2*(1 - d)*x])/(4*(a - b)*d^(5/6)*(-((a - x)*(b - x)^2))
^(2/3)) + ((-a + x)^(2/3)*(-b + x)^(4/3)*Log[-2*(1 + Sqrt[d])*(a - b*Sqrt[d
]) + 2*(1 - d)*x])/(4*(a - b)*d^(5/6)*(-((a - x)*(b - x)^2))^(2/3)) + (3*(-
a + x)^(2/3)*(-b + x)^(4/3)*Log[-(-a + x)^(1/3) - d^(1/6)*(-b + x)^(1/3)])/
(4*(a - b)*d^(5/6)*(-((a - x)*(b - x)^2))^(2/3)) - (3*(-a + x)^(2/3)*(-b +
x)^(4/3)*Log[-(-a + x)^(1/3) + d^(1/6)*(-b + x)^(1/3)])/(4*(a - b)*d^(5/6)*
(-((a - x)*(b - x)^2))^(2/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{(-b+x)^2}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(-a^2+b^2d+2(a-bd)x+(-1+d)x^2\right)}\,dx = \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{(-b+}{(-a+x)^{2/3}\left(-a^2+b^2d+\right.}}{\left((-a+x)(-b+x)^2\right)^{2/3}}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int\left(\frac{(-1}{(a-b)\sqrt{d}\,(-a+x)^{2/3}(}\right.}{}$$

$$= -\frac{\left((1-d)(-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{}{(-a+x)^{2/3}(}}{(a-b)\sqrt{d}\left((-a+x)(-b\right.}$$

$$= -\frac{\left((1-d)(-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{}{(-a+x)^{2/3}}}{\left(1-\sqrt{d}\right)\sqrt{d}\left((-a+x\right.}$$

$$= \frac{\sqrt{3}\,(-a+x)^{2/3}(-b+x)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}}{\sqrt{3}}\right.}{2(a-b)d^{5/6}\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}$$

Mathematica [C] time = 0.28, size = 95, normalized size = 0.22

$$-\frac{3(b-x)^2\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{\sqrt{d}\,(b-x)}{x-a}\right)-{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{\sqrt{d}\,(x-b)}{x-a}\right)\right)}{4\sqrt{d}\,(a-b)\left((x-a)(b-x)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + x)^2/(((-a + x)*(-b + x)^2)^(2/3)*(-a^2 + b^2*d + 2*(a - b*
d)*x + (-1 + d)*x^2)),x]
```

```
[Out] (-3*(b - x)^2*(Hypergeometric2F1[2/3, 1, 5/3, (Sqrt[d]*(b - x))/(-a + x)] -
 Hypergeometric2F1[2/3, 1, 5/3, (Sqrt[d]*(-b + x))/(-a + x)]))/(4*(a - b)*S
qrt[d]*((b - x)^2*(-a + x))^(2/3))
```

IntegrateAlgebraic [A] time = 14.68, size = 438, normalized size = 1.00

$$\frac{(x-a)^{2/3}(x-b)\left(-\sqrt[6]{d}\sqrt[3]{x-a}\sqrt[3]{b-x}+(x-a)^{2/3}+\sqrt[3]{d}(b-x)^{2/3}\right)\left(\sqrt[6]{d}\sqrt[3]{x-a}\sqrt[3]{b-x}+(x-a)^{2/3}+\sqrt[3]{d}(b-x)^{2/3}\right)\left((x-a)^{2/3}\sqrt[3]{b-x}+b\left(-\sqrt[3]{d}\right)+\sqrt[3]{d}x\right)\left(\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x-a}}{\sqrt[3]{x-a}-2\sqrt[6]{d}\sqrt[3]{b-x}}\right)}{2d^{5/6}(a-b)}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x-a}}{\sqrt[3]{x-a}+2\sqrt[6]{d}\sqrt[3]{b-x}}\right)}{2d^{5/6}(a-b)}+\frac{\tanh^{-1}\left(\frac{\sqrt[3]{x-a}(b-x)^{2/3}}{\sqrt[6]{d}(x-b)}\right)}{d^{5/6}(a-b)}-\frac{\tanh^{-1}\left(\frac{\frac{(x-a)^{2/3}}{\sqrt[6]{d}}+\sqrt[6]{d}(b-x)^{2/3}}{\sqrt[3]{x-a}\sqrt[3]{b-x}}\right)}{2d^{5/6}(a-b)}\right)}{\left((x-a)(b-x)^2\right)^{2/3}\left(-a^2+2ax+b^2d-2bdx+(d-1)x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + x)^2/(((-a + x)*(-b + x)^2)^(2/3)*(-a^2 + b^2*d +
2*(a - b*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] ((-a + x)^(2/3)*(-b + x)*(d^(1/3)*(b - x)^(2/3) - d^(1/6)*(b - x)^(1/3)*(-a
 + x)^(1/3) + (-a + x)^(2/3))*(d^(1/3)*(b - x)^(2/3) + d^(1/6)*(b - x)^(1/3
)*(-a + x)^(1/3) + (-a + x)^(2/3))*(-(b*d^(1/3)) + d^(1/3)*x + (b - x)^(1/3
)*(-a + x)^(2/3))*((Sqrt[3]*ArcTan[(Sqrt[3]*(-a + x)^(1/3))/(-2*d^(1/6)*(b
- x)^(1/3) + (-a + x)^(1/3))])/(2*(a - b)*d^(5/6)) - (Sqrt[3]*ArcTan[(Sqrt[
3]*(-a + x)^(1/3))/(2*d^(1/6)*(b - x)^(1/3) + (-a + x)^(1/3))])/(2*(a - b)*
d^(5/6)) + ArcTanh[((b - x)^(2/3)*(-a + x)^(1/3))/(d^(1/6)*(-b + x))]/((a -
 b)*d^(5/6)) - ArcTanh[(d^(1/6)*(b - x)^(2/3) + (-a + x)^(2/3)/d^(1/6))/((b
 - x)^(1/3)*(-a + x)^(1/3))]/(2*(a - b)*d^(5/6))))/(((b - x)^2*(-a + x))^(2
/3)*(-a^2 + b^2*d + 2*a*x - 2*b*d*x + (-1 + d)*x^2))
```

fricas [B] time = 0.48, size = 2217, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)^2/((-a+x)*(-b+x)^2)^(2/3)/(-a^2+b^2*d+2*(-b*d+a)*x+(-1+d)*
x^2),x, algorithm="fricas")
```

```
[Out] sqrt(3)*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5
 + b^6)*d^5))^(1/6)*arctan(-1/3*(2*sqrt(3)*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10
*a^2*b^3 + 5*a*b^4 - b^5)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^
(1/3)*d^4*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b
^5 + b^6)*d^5))^(5/6) - 2*sqrt(3)*((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3
 + 5*a*b^4 - b^5)*d^4*x - (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*
a*b^5 - b^6)*d^4)*sqrt(((-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1
/3)*((a - b)*d*x - (a*b - b^2)*d)*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*
b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(1/6) + ((a^2 - 2*a*b + b^2)*d^2*x^
2 - 2*(a^2*b - 2*a*b^2 + b^3)*d^2*x + (a^2*b^2 - 2*a*b^3 + b^4)*d^2)*(1/((a
^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^
(1/3) + (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3))/(b^2 - 2*b*
x + x^2))*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b
^5 + b^6)*d^5))^(5/6) - sqrt(3)*(b - x))/(b - x)) + sqrt(3)*(1/((a^6 - 6*a^
5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(1/6)*arc
tan(-1/3*(2*sqrt(3)*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^
5)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*d^4*(1/((a^6 - 6*
a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(5/6) -
 2*sqrt(3)*((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d^4*x
 - (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*d^4)*sqrt(
-((-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*((a - b)*d*x - (a*
b - b^2)*d)*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a
*b^5 + b^6)*d^5))^(1/6) - ((a^2 - 2*a*b + b^2)*d^2*x^2 - 2*(a^2*b - 2*a*b^2
 + b^3)*d^2*x + (a^2*b^2 - 2*a*b^3 + b^4)*d^2)*(1/((a^6 - 6*a^5*b + 15*a^4*
b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(1/3) - (-a*b^2 - (a +
 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3))/(b^2 - 2*b*x + x^2))*(1/((a^6 - 6
*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(5/6)
+ sqrt(3)*(b - x))/(b - x)) + 1/4*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*
b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(1/6)*log(((-a*b^2 - (a + 2*b)*x^2
+ x^3 + (2*a*b + b^2)*x)^(1/3)*((a - b)*d*x - (a*b - b^2)*d)*(1/((a^6 - 6*a
^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(1/6) +
```

```
((a^2 - 2*a*b + b^2)*d^2*x^2 - 2*(a^2*b - 2*a*b^2 + b^3)*d^2*x + (a^2*b^2 -
 2*a*b^3 + b^4)*d^2)*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*
b^4 - 6*a*b^5 + b^6)*d^5))^(1/3) + (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b +
 b^2)*x)^(2/3))/(b^2 - 2*b*x + x^2)) - 1/4*(1/((a^6 - 6*a^5*b + 15*a^4*b^2
- 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(1/6)*log(-((-a*b^2 - (a +
 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*((a - b)*d*x - (a*b - b^2)*d)*(1/(
(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)
)^(1/6) - ((a^2 - 2*a*b + b^2)*d^2*x^2 - 2*(a^2*b - 2*a*b^2 + b^3)*d^2*x +
(a^2*b^2 - 2*a*b^3 + b^4)*d^2)*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3
 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(1/3) - (-a*b^2 - (a + 2*b)*x^2 + x^3
+ (2*a*b + b^2)*x)^(2/3))/(b^2 - 2*b*x + x^2)) + 1/2*(1/((a^6 - 6*a^5*b + 1
5*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(1/6)*log(-(((a
- b)*d*x - (a*b - b^2)*d)*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15
*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(1/6) + (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*
a*b + b^2)*x)^(1/3))/(b - x)) - 1/2*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^
3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(1/6)*log((((a - b)*d*x - (a*b -
b^2)*d)*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5
 + b^6)*d^5))^(1/6) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3
))/(b - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b-x)^2}{\left(-(a-x)(b-x)^2\right)^{\frac{2}{3}}\left(b^2d+(d-1)x^2-a^2-2\,(bd-a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)^2/((-a+x)*(-b+x)^2)^(2/3)/(-a^2+b^2*d+2*(-b*d+a)*x+(-1+d)*
x^2),x, algorithm="giac")
```

```
[Out] integrate((b - x)^2/((-(a - x)*(b - x)^2)^(2/3)*(b^2*d + (d - 1)*x^2 - a^2
- 2*(b*d - a)*x)), x)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)^2}{\left((-a+x)\,(-b+x)^2\right)^{\frac{2}{3}}\left(-a^2+b^2d+2\,(-bd+a)\,x+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b+x)^2/((-a+x)*(-b+x)^2)^(2/3)/(-a^2+b^2*d+2*(-b*d+a)*x+(-1+d)*x^2),x
)
```

```
[Out] int((-b+x)^2/((-a+x)*(-b+x)^2)^(2/3)/(-a^2+b^2*d+2*(-b*d+a)*x+(-1+d)*x^2),x
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b-x)^2}{\left(-(a-x)(b-x)^2\right)^{\frac{2}{3}}\left(b^2d+(d-1)x^2-a^2-2\,(bd-a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)^2/((-a+x)*(-b+x)^2)^(2/3)/(-a^2+b^2*d+2*(-b*d+a)*x+(-1+d)*
x^2),x, algorithm="maxima")
```

```
[Out] integrate((b - x)^2/((-(a - x)*(b - x)^2)^(2/3)*(b^2*d + (d - 1)*x^2 - a^2
- 2*(b*d - a)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b-x)^2}{\left(-(a-x)\,(b-x)^2\right)^{2/3}\,\left(b^2\,d+2\,x\,(a-b\,d)-a^2+x^2\,(d-1)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - x)^2/((-(a - x)*(b - x)^2)^(2/3)*(b^2*d + 2*x*(a - b*d) - a^2 + x^
2*(d - 1))),x)
```

```
[Out] int((b - x)^2/((-(a - x)*(b - x)^2)^(2/3)*(b^2*d + 2*x*(a - b*d) - a^2 + x^
2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)**2/((-a+x)*(-b+x)**2)**(2/3)/(-a**2+b**2*d+2*(-b*d+a)*x+(-
1+d)*x**2),x)
```

```
[Out] Timed out
```

3.2361 $\int \frac{b+ax}{(-b+ax)\sqrt[3]{-b^2x^2+a^3x^3}}\,dx$

Optimal. Leaf size=441

$$-\frac{\log\left(\sqrt[3]{a^3x^3-b^2x^2}-ax\right)}{a}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}ax}{2\sqrt[3]{a^3x^3-b^2x^2}+ax}\right)}{a}-\frac{i\left(\sqrt{3}-i\right)\log\left(\sqrt[3]{-1}\sqrt[3]{a^3x^3-b^2x^2}+\sqrt[3]{a}\,x\sqrt[3]{a^2-b}\right)}{\sqrt[3]{a}\sqrt[3]{a^2-b}}+\text{lo}$$

Rubi [A] time = 0.24, antiderivative size = 489, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2056, 157, 59, 91}

$$-\frac{x^{2/3}\log(x)\sqrt[3]{a^3x-b^2}}{2a\sqrt[3]{a^3x^3-b^2x^2}}-\frac{3x^{2/3}\sqrt[3]{a^3x-b^2}\log\left(\frac{\sqrt[3]{a^3x-b^2}}{a\sqrt[3]{x}}-1\right)}{2a\sqrt[3]{a^3x^3-b^2x^2}}-\frac{\sqrt{3}x^{2/3}\sqrt[3]{a^3x-b^2}\tan^{-1}\left(\frac{2\sqrt[3]{a^3x-b^2}}{\sqrt{3}a\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{a\sqrt[3]{a^3x^3-b^2x^2}}-\frac{x^{2/3}\sqrt[3]{a^3x-b^2}\log(ax-b)}{\sqrt[3]{a}\sqrt[3]{a^2-b}\sqrt[3]{a^3x^3-b^2x^2}}+\frac{3x^{2/3}\sqrt[3]{a^3x-b^2}\log\left(\frac{\sqrt[3]{a^3x-b^2}}{\sqrt[3]{a}\sqrt[3]{a^2-b}}-\sqrt[3]{x}\right)}{\sqrt[3]{a}\sqrt[3]{a^2-b}\sqrt[3]{a^3x^3-b^2x^2}}+\frac{2\sqrt{3}x^{2/3}\sqrt[3]{a^3x-b^2}\tan^{-1}\left(\frac{2\sqrt[3]{a^3x-b^2}}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{a^2-b}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[3]{a^2-b}\sqrt[3]{a^3x^3-b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x)/((-b + a*x)*(-(b^2*x^2) + a^3*x^3)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*x^(2/3)*(-b^2 + a^3*x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(-b^2 + a^3*x
)^(1/3))/(Sqrt[3]*a*x^(1/3))])/(a*(-(b^2*x^2) + a^3*x^3)^(1/3))) + (2*Sqrt[
3]*x^(2/3)*(-b^2 + a^3*x)^(1/3)*ArcTan[1/Sqrt[3] + (2*(-b^2 + a^3*x)^(1/3))
/(Sqrt[3]*a^(1/3)*(a^2 - b)^(1/3)*x^(1/3))])/(a^(1/3)*(a^2 - b)^(1/3)*(-(b^
2*x^2) + a^3*x^3)^(1/3)) - (x^(2/3)*(-b^2 + a^3*x)^(1/3)*Log[x])/(2*a*(-(b^
2*x^2) + a^3*x^3)^(1/3)) - (x^(2/3)*(-b^2 + a^3*x)^(1/3)*Log[-b + a*x])/(a^
(1/3)*(a^2 - b)^(1/3)*(-(b^2*x^2) + a^3*x^3)^(1/3)) + (3*x^(2/3)*(-b^2 + a^
3*x)^(1/3)*Log[-x^(1/3) + (-b^2 + a^3*x)^(1/3)/(a^(1/3)*(a^2 - b)^(1/3))])/
(a^(1/3)*(a^2 - b)^(1/3)*(-(b^2*x^2) + a^3*x^3)^(1/3)) - (3*x^(2/3)*(-b^2 +
 a^3*x)^(1/3)*Log[-1 + (-b^2 + a^3*x)^(1/3)/(a*x^(1/3))])/(2*a*(-(b^2*x^2)
+ a^3*x^3)^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
```

```
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{b+ax}{(-b+ax)\sqrt[3]{-b^2x^2+a^3x^3}}\,dx = \frac{\left(x^{2/3}\sqrt[3]{-b^2+a^3x}\right)\int \frac{b+ax}{x^{2/3}(-b+ax)\sqrt[3]{-b^2+a^3x}}\,dx}{\sqrt[3]{-b^2x^2+a^3x^3}}$$

$$= \frac{\left(x^{2/3}\sqrt[3]{-b^2+a^3x}\right)\int \frac{1}{x^{2/3}\sqrt[3]{-b^2+a^3x}}\,dx}{\sqrt[3]{-b^2x^2+a^3x^3}} + \frac{\left(2bx^{2/3}\sqrt[3]{-b^2+a^3x}\right)\int \frac{1}{x^{2/3}(-b+ax)\sqrt[3]{-b^2}}}{\sqrt[3]{-b^2x^2+a^3x^3}}$$

$$= -\frac{\sqrt{3}\,x^{2/3}\sqrt[3]{-b^2+a^3x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-b^2+a^3x}}{\sqrt{3}\,a\sqrt[3]{x}}\right)}{a\sqrt[3]{-b^2x^2+a^3x^3}} + \frac{2\sqrt{3}\,x^{2/3}\sqrt[3]{-b^2+a^3x}\,\tan^{-}}{\sqrt[3]{a}\,\sqrt[3]{a^2-b}\,\sqrt[3]{-}}$$

Mathematica [C] time = 0.06, size = 91, normalized size = 0.21

$$\frac{3x\sqrt[3]{1-\frac{a^3x}{b^2}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{a^3x}{b^2}\right)-6x\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{\left(a^3-ab\right)x}{a^3x-b^2}\right)}{\sqrt[3]{x^2\left(a^3x-b^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x)/((-b + a*x)*(-(b^2*x^2) + a^3*x^3)^(1/3)),x]
```

```
[Out] (3*x*(1 - (a^3*x)/b^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (a^3*x)/b^2]
- 6*x*Hypergeometric2F1[1/3, 1, 4/3, ((a^3 - a*b)*x)/(-b^2 + a^3*x)])/(x^2*
(-b^2 + a^3*x))^(1/3)
```

IntegrateAlgebraic [A] time = 3.37, size = 491, normalized size = 1.11

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}ax}{2\sqrt[3]{a^3x^3-b^2x^2}+ax}\right)}{a}-\frac{i\left(\sqrt{3}-i\right)\log\left(2\sqrt[3]{a}\,x\sqrt[3]{a^2-b}+\left(1+i\sqrt{3}\right)\sqrt[3]{a^3x^3-b^2x^2}\right)}{\sqrt[3]{a}\sqrt[3]{a^2-b}}-\frac{\log\left(a^2x-a\sqrt[3]{a^3x^3-b^2x^2}\right)}{a}+\frac{\log\left(ax\sqrt[3]{a^3x^3-b^2x^2}+\left(a^3x^3-b^2x^2\right)^{2/3}+a^2x^2\right)}{2a}+\frac{\sqrt{6\left(-1+i\sqrt{3}\right)}\tan^{-1}\left(\frac{3\sqrt[3]{a}\,x\sqrt[3]{a^2-b}}{-\sqrt{3}\sqrt[3]{a^3x^3-b^2x^2}-3i\sqrt[3]{a^3x^3-b^2x^2}+\sqrt{3}\sqrt[3]{a}\,x\sqrt[3]{a^2-b}}\right)}{\sqrt[3]{a}\sqrt[3]{a^2-b}}+\frac{\left(1+i\sqrt{3}\right)\log\left(\left(\sqrt{3}+i\right)\left(a^3x^3-b^2x^2\right)^{2/3}-2ia^{2/3}x^2\left(a^2-b\right)^{2/3}+\sqrt[3]{a}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a^2-b}\sqrt[3]{a^3x^3-b^2x^2}\right)}{2\sqrt[3]{a}\sqrt[3]{a^2-b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x)/((-b + a*x)*(-(b^2*x^2) + a^3*x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*a*x)/(a*x + 2*(-(b^2*x^2) + a^3*x^3)^(1/3))])/a +
(Sqrt[6*(-1 + I*Sqrt[3])]*ArcTan[(3*a^(1/3)*(a^2 - b)^(1/3)*x)/(Sqrt[3]*a^(
1/3)*(a^2 - b)^(1/3)*x - (3*I)*(-(b^2*x^2) + a^3*x^3)^(1/3) - Sqrt[3]*(-(b^
2*x^2) + a^3*x^3)^(1/3))])/(a^(1/3)*(a^2 - b)^(1/3)) - (I*(-I + Sqrt[3])*Lo
g[2*a^(1/3)*(a^2 - b)^(1/3)*x + (1 + I*Sqrt[3])*(-(b^2*x^2) + a^3*x^3)^(1/3
)])/(a^(1/3)*(a^2 - b)^(1/3)) - Log[a^2*x - a*(-(b^2*x^2) + a^3*x^3)^(1/3)]
/a + Log[a^2*x^2 + a*x*(-(b^2*x^2) + a^3*x^3)^(1/3) + (-(b^2*x^2) + a^3*x^3
)^(2/3)]/(2*a) + ((1 + I*Sqrt[3])*Log[(-2*I)*a^(2/3)*(a^2 - b)^(2/3)*x^2 +
a^(1/3)*(a^2 - b)^(1/3)*(I*x - Sqrt[3]*x)*(-(b^2*x^2) + a^3*x^3)^(1/3) + (I
 + Sqrt[3])*(-(b^2*x^2) + a^3*x^3)^(2/3)])/(2*a^(1/3)*(a^2 - b)^(1/3))
```

fricas [A] time = 0.44, size = 843, normalized size = 1.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(a*x-b)/(a^3*x^3-b^2*x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(3)*(a^3 - a*b)*sqrt(-1/(a^3 - a*b)^(2/3))*log(-(2*b^2*x - (3*a
^3 - a*b)*x^2 + 3*(a^3*x^3 - b^2*x^2)^(1/3)*(a^3 - a*b)^(2/3)*x + sqrt(3)*(
```

```
(a^3 - a*b)^(4/3)*x^2 + (a^3*x^3 - b^2*x^2)^(1/3)*(a^3 - a*b)*x - 2*(a^3*x^
3 - b^2*x^2)^(2/3)*(a^3 - a*b)^(2/3))*sqrt(-1/(a^3 - a*b)^(2/3)))/(a*x^2 -
b*x)) - 2*sqrt(3)*(a^2 - b)*arctan(1/3*(sqrt(3)*a*x + 2*sqrt(3)*(a^3*x^3 -
b^2*x^2)^(1/3))/(a*x)) - 2*(a^2 - b)*log(-(a*x - (a^3*x^3 - b^2*x^2)^(1/3))
/x) + (a^2 - b)*log((a^2*x^2 + (a^3*x^3 - b^2*x^2)^(1/3)*a*x + (a^3*x^3 - b
^2*x^2)^(2/3))/x^2) + 4*(a^3 - a*b)^(2/3)*log(-((a^3 - a*b)^(1/3)*x - (a^3*
x^3 - b^2*x^2)^(1/3))/x) - 2*(a^3 - a*b)^(2/3)*log(((a^3 - a*b)^(2/3)*x^2 +
 (a^3*x^3 - b^2*x^2)^(1/3)*(a^3 - a*b)^(1/3)*x + (a^3*x^3 - b^2*x^2)^(2/3))
/x^2))/(a^3 - a*b), -1/2*(2*sqrt(3)*(a^2 - b)*arctan(1/3*(sqrt(3)*a*x + 2*s
qrt(3)*(a^3*x^3 - b^2*x^2)^(1/3))/(a*x)) - 4*sqrt(3)*(a^3 - a*b)^(2/3)*arct
an(1/3*sqrt(3)*((a^3 - a*b)^(1/3)*x + 2*(a^3*x^3 - b^2*x^2)^(1/3))/((a^3 -
a*b)^(1/3)*x)) + 2*(a^2 - b)*log(-(a*x - (a^3*x^3 - b^2*x^2)^(1/3))/x) - (a
^2 - b)*log((a^2*x^2 + (a^3*x^3 - b^2*x^2)^(1/3)*a*x + (a^3*x^3 - b^2*x^2)^
(2/3))/x^2) - 4*(a^3 - a*b)^(2/3)*log(-((a^3 - a*b)^(1/3)*x - (a^3*x^3 - b^
2*x^2)^(1/3))/x) + 2*(a^3 - a*b)^(2/3)*log(((a^3 - a*b)^(2/3)*x^2 + (a^3*x^
3 - b^2*x^2)^(1/3)*(a^3 - a*b)^(1/3)*x + (a^3*x^3 - b^2*x^2)^(2/3))/x^2))/(
a^3 - a*b)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(a*x-b)/(a^3*x^3-b^2*x^2)^(1/3),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{ax+b}{(ax-b)\left(a^3x^3-b^2x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)/(a*x-b)/(a^3*x^3-b^2*x^2)^(1/3),x)
[Out] int((a*x+b)/(a*x-b)/(a^3*x^3-b^2*x^2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+b}{\left(a^3x^3-b^2x^2\right)^{\frac{1}{3}}(ax-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(a*x-b)/(a^3*x^3-b^2*x^2)^(1/3),x, algorithm="maxima")
[Out] integrate((a*x + b)/((a^3*x^3 - b^2*x^2)^(1/3)*(a*x - b)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{b+a\,x}{{\left(a^3\,x^3-b^2\,x^2\right)}^{1/3}\,\left(b-a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*x)/((a^3*x^3 - b^2*x^2)^(1/3)*(b - a*x)),x)
[Out] int(-(b + a*x)/((a^3*x^3 - b^2*x^2)^(1/3)*(b - a*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+b}{\sqrt[3]{x^2\left(a^3x-b^2\right)}\,(ax-b)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(a*x-b)/(a**3*x**3-b**2*x**2)**(1/3),x)
```

```
[Out] Integral((a*x + b)/((x**2*(a**3*x - b**2))**(1/3)*(a*x - b)), x)
```

3.2362 $\int \frac{1}{\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=445

$$-\frac{585b\tan^{-1}\left(\frac{\sqrt[4]{\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[4]{c}}\right)}{2048ac^{17/4}}+\frac{585b\tanh^{-1}\left(\frac{\sqrt[4]{\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[4]{c}}\right)}{2048ac^{17/4}}-\frac{585b\left(\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c\right)^{3/4}}{1024ac^4\sqrt[3]{\sqrt{a^2x^2-b}+ax}}+\frac{117b\left(\sqrt[3]{\sqrt{\cdots}}\right.}{256ac^3\left(\cdots\right.}$$

Rubi [F] time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/
3))^(1/4)),x]
```

```
[Out] Defer[Int][1/((a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x + Sqrt[-b + a^2*x^
2])^(1/3))^(1/4)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{1}{\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[1/((a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x + Sqrt[-b + a^2*x^2
])^(1/3))^(1/4)),x]
```

```
[Out] $Aborted
```

IntegrateAlgebraic [A] time = 1.44, size = 445, normalized size = 1.00

$$-\frac{585b\tan^{-1}\left(\frac{\sqrt[4]{\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[4]{c}}\right)}{2048ac^{17/4}}+\frac{585b\tanh^{-1}\left(\frac{\sqrt[4]{\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[4]{c}}\right)}{2048ac^{17/4}}-\frac{585b\left(\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c\right)^{3/4}}{1024ac^4\sqrt[3]{\sqrt{a^2x^2-b}+ax}}+\frac{117b\left(\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c\right)^{3/4}}{256ac^3\left(\sqrt{a^2x^2-b}+ax\right)^{2/3}}-\frac{13b\left(\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c\right)^{3/4}}{32ac^2\left(\sqrt{a^2x^2-b}+ax\right)}+\frac{3b\left(\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c\right)^{3/4}}{8ac\left(\sqrt{a^2x^2-b}+ax\right)^{4/3}}-\frac{8c\left(\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c\right)^{3/4}}{7a}+\frac{6\sqrt[3]{\sqrt{a^2x^2-b}+ax}\left(\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c\right)^{3/4}}{7a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x + Sqrt[-b
+ a^2*x^2])^(1/3))^(1/4)),x]
```

```
[Out] (-8*c*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(3/4))/(7*a) + (3*b*(c + (a*x
+ Sqrt[-b + a^2*x^2])^(1/3))^(3/4))/(8*a*c*(a*x + Sqrt[-b + a^2*x^2])^(4/3)
) - (13*b*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(3/4))/(32*a*c^2*(a*x + Sq
```

```
rt[-b + a^2*x^2])) + (117*b*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(3/4))/(
256*a*c^3*(a*x + Sqrt[-b + a^2*x^2])^(2/3)) - (585*b*(c + (a*x + Sqrt[-b +
a^2*x^2])^(1/3))^(3/4))/(1024*a*c^4*(a*x + Sqrt[-b + a^2*x^2])^(1/3)) + (6*
(a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(3/
4))/(7*a) - (585*b*ArcTan[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)/c^(1
/4)])/(2048*a*c^(17/4)) + (585*b*ArcTanh[(c + (a*x + Sqrt[-b + a^2*x^2])^(1
/3))^(1/4)/c^(1/4)])/(2048*a*c^(17/4))
```

fricas [A] time = 0.56, size = 469, normalized size = 1.05

$$\frac{16380\,abc^4\left(\frac{b^4}{a^4c^{17}}\right)^{\frac14}\arctan\left(-\frac{ab^3\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac13}\right)^{\frac14}c^4\left(\frac{b^4}{a^4c^{17}}\right)^{\frac14}-\sqrt{a^2b^4c^9\sqrt{\frac{b^4}{a^4c^{17}}}+b^6\sqrt{c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac13}}}\,ac^4\left(\frac{b^4}{a^4c^{17}}\right)^{\frac14}}{b^4}\right)+4095\,abc^4\left(\frac{b^4}{a^4c^{17}}\right)^{\frac14}\log\left(200201625\,a^3c^{13}\left(\frac{b^4}{a^4c^{17}}\right)^{\frac34}+200201625\,b^3\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac13}\right)^{\frac14}\right)-4095\,abc^4\left(\frac{b^4}{a^4c^{17}}\right)^{\frac14}\log\left(-200201625\,a^3c^{13}\left(\frac{b^4}{a^4c^{17}}\right)^{\frac34}+200201625\,b^3\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac13}\right)^{\frac14}\right)-4\left(8192\,bc^5+2912\,abc^2x-2912\,\sqrt{a^2x^2-b}\,bc^2-21\left(256\,a^2c^3x^2-128\,bc^3-195\,abx-\left(256\,ac^3x-195\,b\right)\sqrt{a^2x^2-b}\right)\left(ax+\sqrt{a^2x^2-b}\right)^{\frac23}-12\left(512\,bc^4+273\,abcx-273\,\sqrt{a^2x^2-b}\,bc\right)\left(ax+\sqrt{a^2x^2-b}\right)^{\frac13}\right)\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac13}\right)^{\frac34}}{28672\,abc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/3))
^(1/4),x, algorithm="fricas")
```

```
[Out] 1/28672*(16380*a*b*c^4*(b^4/(a^4*c^17))^(1/4)*arctan(-(a*b^3*(c + (a*x + sq
rt(a^2*x^2 - b))^(1/3))^(1/4)*c^4*(b^4/(a^4*c^17))^(1/4) - sqrt(a^2*b^4*c^9
*sqrt(b^4/(a^4*c^17)) + b^6*sqrt(c + (a*x + sqrt(a^2*x^2 - b))^(1/3)))*a*c^
4*(b^4/(a^4*c^17))^(1/4))/b^4) + 4095*a*b*c^4*(b^4/(a^4*c^17))^(1/4)*log(20
0201625*a^3*c^13*(b^4/(a^4*c^17))^(3/4) + 200201625*b^3*(c + (a*x + sqrt(a^
2*x^2 - b))^(1/3))^(1/4)) - 4095*a*b*c^4*(b^4/(a^4*c^17))^(1/4)*log(-200201
625*a^3*c^13*(b^4/(a^4*c^17))^(3/4) + 200201625*b^3*(c + (a*x + sqrt(a^2*x^
2 - b))^(1/3))^(1/4)) - 4*(8192*b*c^5 + 2912*a*b*c^2*x - 2912*sqrt(a^2*x^2
- b)*b*c^2 - 21*(256*a^2*c^3*x^2 - 128*b*c^3 - 195*a*b*x - (256*a*c^3*x - 1
95*b)*sqrt(a^2*x^2 - b))*(a*x + sqrt(a^2*x^2 - b))^(2/3) - 12*(512*b*c^4 +
273*a*b*c*x - 273*sqrt(a^2*x^2 - b)*b*c)*(a*x + sqrt(a^2*x^2 - b))^(1/3))*(
c + (a*x + sqrt(a^2*x^2 - b))^(1/3))^(3/4))/(a*b*c^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/3))
^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/3))^(1/4)
,x)
```

```
[Out] int(1/(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/3))^(1/4)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/3))
^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + sqrt(a^2*x^2 - b))^(1/3)*(c + (a*x + sqrt(a^2*x^2 - b))
^(1/3))^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/3}\,\left(c+\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/3}\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x + (a^2*x^2 - b)^(1/2))^(1/3)*(c + (a*x + (a^2*x^2 - b)^(1/2))^(
1/3))^(1/4)),x)
```

```
[Out] int(1/((a*x + (a^2*x^2 - b)^(1/2))^(1/3)*(c + (a*x + (a^2*x^2 - b)^(1/2))^(
1/3))^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{a^2x^2-b}}}\,\sqrt[3]{ax+\sqrt{a^2x^2-b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+(a**2*x**2-b)**(1/2))**(1/3)/(c+(a*x+(a**2*x**2-b)**(1/2))
**(1/3))**(1/4),x)
```

```
[Out] Integral(1/((c + (a*x + sqrt(a**2*x**2 - b))**(1/3))**(1/4)*(a*x + sqrt(a**
2*x**2 - b))**(1/3)), x)
```

3.2363 $\displaystyle\int \frac{x^4}{\sqrt[4]{-b+ax^4}\left(-b+2ax^4+x^8\right)}\,dx$

Optimal. Leaf size=448

$$\frac{\left(\sqrt[4]{-1}-1\right)\tan^{-1}\left(\frac{(-1)^{7/8}\sqrt{2+\sqrt{2}}\,x\,\sqrt[8]{a^2+b}\,\sqrt[4]{ax^4-b}}{(-1)^{3/4}x^2\sqrt[4]{a^2+b}+\sqrt{ax^4-b}}\right)}{8\left(a^2+b\right)^{5/8}}+\frac{i\left(\sqrt{2\left(3+2\sqrt{2}\right)}+i\sqrt{2}\right)\tan^{-1}\left(\frac{(-1)^{7/8}\left(\sqrt{2}-2\right)x\,\sqrt[8]{a^2+b}\,\sqrt[4]{ax^4-b}}{(-1)^{3/4}\sqrt{2-\sqrt{2}}\,x^2\sqrt[4]{a^2+b}+\sqrt{2-\sqrt{2}}\,\sqrt{a\cdots}}\right)}{16\left(a^2+b\right)^{5/8}}$$

Rubi [A] time = 0.71, antiderivative size = 409, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1528, 377, 212, 208, 205}

$$-\frac{\sqrt[4]{a-\sqrt{a^2+b}}\tan^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2+b}+a^2+b}}{\sqrt[4]{a-\sqrt{a^2+b}}\sqrt[4]{ax^4-b}}\right)}{4\sqrt{a^2+b}\sqrt[4]{-a\sqrt{a^2+b}+a^2+b}}+\frac{\sqrt[4]{\sqrt{a^2+b}+a}\tan^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2+b}+a^2+b}}{\sqrt[4]{\sqrt{a^2+b}+a}\sqrt[4]{ax^4-b}}\right)}{4\sqrt{a^2+b}\sqrt[4]{a\sqrt{a^2+b}+a^2+b}}-\frac{\sqrt[4]{a-\sqrt{a^2+b}}\tanh^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2+b}+a^2+b}}{\sqrt[4]{a-\sqrt{a^2+b}}\sqrt[4]{ax^4-b}}\right)}{4\sqrt{a^2+b}\sqrt[4]{-a\sqrt{a^2+b}+a^2+b}}+\frac{\sqrt[4]{\sqrt{a^2+b}+a}\tanh^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2+b}+a^2+b}}{\sqrt[4]{\sqrt{a^2+b}+a}\sqrt[4]{ax^4-b}}\right)}{4\sqrt{a^2+b}\sqrt[4]{a\sqrt{a^2+b}+a^2+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((-b + a*x^4)^(1/4)*(-b + 2*a*x^4 + x^8)),x]
```

```
[Out] -1/4*((a - Sqrt[a^2 + b])^(1/4)*ArcTan[((a^2 + b - a*Sqrt[a^2 + b])^(1/4)*x
)/((a - Sqrt[a^2 + b])^(1/4)*(-b + a*x^4)^(1/4))])/(Sqrt[a^2 + b]*(a^2 + b
- a*Sqrt[a^2 + b])^(1/4)) + ((a + Sqrt[a^2 + b])^(1/4)*ArcTan[((a^2 + b + a
*Sqrt[a^2 + b])^(1/4)*x)/((a + Sqrt[a^2 + b])^(1/4)*(-b + a*x^4)^(1/4))])/(
4*Sqrt[a^2 + b]*(a^2 + b + a*Sqrt[a^2 + b])^(1/4)) - ((a - Sqrt[a^2 + b])^(
1/4)*ArcTanh[((a^2 + b - a*Sqrt[a^2 + b])^(1/4)*x)/((a - Sqrt[a^2 + b])^(1/
4)*(-b + a*x^4)^(1/4))])/(4*Sqrt[a^2 + b]*(a^2 + b - a*Sqrt[a^2 + b])^(1/4)
) + ((a + Sqrt[a^2 + b])^(1/4)*ArcTanh[((a^2 + b + a*Sqrt[a^2 + b])^(1/4)*x
)/((a + Sqrt[a^2 + b])^(1/4)*(-b + a*x^4)^(1/4))])/(4*Sqrt[a^2 + b]*(a^2 +
b + a*Sqrt[a^2 + b])^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1528

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q, (
f*x)^m/(a + b*x^n + c*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, e, f, q, n}, x
```

```
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&  !IntegerQ[q] &&
IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{-b+ax^4}\left(-b+2ax^4+x^8\right)}\,dx = \int\left(\frac{1-\frac{a}{\sqrt{a^2+b}}}{\left(2a-2\sqrt{a^2+b}+2x^4\right)\sqrt[4]{-b+ax^4}}+\frac{1+\frac{a}{\sqrt{a^2+b}}}{\left(2a+2\sqrt{a^2+b}+2x^4\right)\sqrt[4]{-b+a\ldots}}\right.$$

$$=\left(1-\frac{a}{\sqrt{a^2+b}}\right)\int\frac{1}{\left(2a-2\sqrt{a^2+b}+2x^4\right)\sqrt[4]{-b+ax^4}}\,dx+\left(1+\frac{a}{\sqrt{a^2+b}}\right)\ldots$$

$$=\left(1-\frac{a}{\sqrt{a^2+b}}\right)\mathrm{Subst}\left(\int\frac{1}{2a-2\sqrt{a^2+b}-\left(2b+a\left(2a-2\sqrt{a^2+b}\right)\right)x^4}\,dx\ldots\right.$$

$$=-\frac{\sqrt{a-\sqrt{a^2+b}}\ \mathrm{Subst}\left(\int\frac{1}{\sqrt{a-\sqrt{a^2+b}}-\sqrt{a^2+b-a\sqrt{a^2+b}}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{-b+ax^4}}\right)}{4\sqrt{a^2+b}}-\ldots$$

$$=-\frac{\sqrt[4]{a-\sqrt{a^2+b}}\ \tan^{-1}\left(\frac{\sqrt[4]{a^2+b-a\sqrt{a^2+b}}\,x}{\sqrt[4]{a-\sqrt{a^2+b}}\,\sqrt[4]{-b+ax^4}}\right)}{4\sqrt{a^2+b}\,\sqrt[4]{a^2+b-a\sqrt{a^2+b}}}+\frac{\sqrt[4]{a+\sqrt{a^2+b}}\ \tan^{-1}\left(\frac{\sqrt[4]{a^2\ldots}}{\sqrt[4]{a+\ldots}}\right)}{4\sqrt{a^2+b}\,\sqrt[4]{a^2+b+a\ldots}}$$

Mathematica [A] time = 0.72, size = 376, normalized size = 0.84

$$\frac{-\frac{\sqrt[4]{a-\sqrt{a^2+b}}\ \tan^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2+b}+a^2+b}}{\sqrt[4]{a-\sqrt{a^2+b}}\,\sqrt[4]{ax^4-b}}\right)}{\sqrt[4]{-a\sqrt{a^2+b}+a^2+b}}+\frac{\sqrt[4]{\sqrt{a^2+b}+a}\ \tan^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2+b}+a^2+b}}{\sqrt[4]{\sqrt{a^2+b}+a}\,\sqrt[4]{ax^4-b}}\right)}{\sqrt[4]{a\sqrt{a^2+b}+a^2+b}}-\frac{\sqrt[4]{a-\sqrt{a^2+b}}\ \tanh^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2+b}+a^2+b}}{\sqrt[4]{a-\sqrt{a^2+b}}\,\sqrt[4]{ax^4-b}}\right)}{\sqrt[4]{-a\sqrt{a^2+b}+a^2+b}}+\frac{\sqrt[4]{\sqrt{a^2+b}+a}\ \tanh^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2+b}+a^2+b}}{\sqrt[4]{\sqrt{a^2+b}+a}\,\sqrt[4]{ax^4-b}}\right)}{\sqrt[4]{a\sqrt{a^2+b}+a^2+b}}}{4\sqrt{a^2+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((-b + a*x^4)^(1/4)*(-b + 2*a*x^4 + x^8)),x]
```

```
[Out] (-(((a - Sqrt[a^2 + b])^(1/4)*ArcTan[((a^2 + b - a*Sqrt[a^2 + b])^(1/4)*x)/
((a - Sqrt[a^2 + b])^(1/4)*(-b + a*x^4)^(1/4))])/(a^2 + b - a*Sqrt[a^2 + b]
)^(1/4)) + ((a + Sqrt[a^2 + b])^(1/4)*ArcTan[((a^2 + b + a*Sqrt[a^2 + b])^(
1/4)*x)/((a + Sqrt[a^2 + b])^(1/4)*(-b + a*x^4)^(1/4))])/(a^2 + b + a*Sqrt[
a^2 + b])^(1/4) - ((a - Sqrt[a^2 + b])^(1/4)*ArcTanh[((a^2 + b - a*Sqrt[a^2
 + b])^(1/4)*x)/((a - Sqrt[a^2 + b])^(1/4)*(-b + a*x^4)^(1/4))])/(a^2 + b -
 a*Sqrt[a^2 + b])^(1/4) + ((a + Sqrt[a^2 + b])^(1/4)*ArcTanh[((a^2 + b + a*
Sqrt[a^2 + b])^(1/4)*x)/((a + Sqrt[a^2 + b])^(1/4)*(-b + a*x^4)^(1/4))])/(a
^2 + b + a*Sqrt[a^2 + b])^(1/4))/(4*Sqrt[a^2 + b])
```

IntegrateAlgebraic [A] time = 41.42, size = 510, normalized size = 1.14

$$-\frac{i\left(\sqrt{2\left(3-2\sqrt{2}\right)}-i\sqrt{2}\right)\tan^{-1}\left(\frac{\left((-1+i)-(1+i)(-1)^{3/4}\right)x^2\sqrt[4]{a^2+b}+(1+i)(-1)^{3/4}\sqrt{ax^4-b}+(1+i)\sqrt{ax^4-b}}{2x\sqrt[8]{a^2+b}\,\sqrt[4]{ax^4-b}}\right)}{16\left(a^2+b\right)^{5/8}}+\frac{\left(\sqrt{2}-i\sqrt{2\left(3+2\sqrt{2}\right)}\right)\tan^{-1}\left(\frac{2x\sqrt[8]{a^2+b}\,\sqrt[4]{ax^4-b}}{\left(\sqrt{2}+(1-i)\right)x^2\sqrt[4]{a^2+b}-\sqrt{2}\sqrt{ax^4-b}-(1+i)\sqrt{ax^4-b}}\right)}{16\left(a^2+b\right)^{5/8}}+\frac{\left((-1)^{3/4}-i\right)\tanh^{-1}\left(\frac{\left((-2+2i)-(2+2i)(-1)^{3/4}\right)x^2\sqrt[4]{a^2+b}-(2+2i)(-1)^{3/4}\sqrt{ax^4-b}-(2+2i)\sqrt{ax^4-b}}{4x\sqrt[8]{a^2+b}\,\sqrt[4]{ax^4-b}}\right)}{8\left(a^2+b\right)^{5/8}}+\frac{i\left(\sqrt{2\left(3+2\sqrt{2}\right)}+i\sqrt{2}\right)\tanh^{-1}\left(\frac{\left(\sqrt{2}+(1-i)\right)x^2\sqrt[4]{a^2+b}+\sqrt{2}\sqrt{ax^4-b}+(1+i)\sqrt{ax^4-b}}{2x\sqrt[8]{a^2+b}\,\sqrt[4]{ax^4-b}}\right)}{16\left(a^2+b\right)^{5/8}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((-b + a*x^4)^(1/4)*(-b + 2*a*x^4 + x^8)),x]
```

```
[Out] ((-1/16*I)*((-I)*Sqrt[2] + Sqrt[2*(3 - 2*Sqrt[2])])*ArcTan[(((-1 + I) - (1
+ I)*(-1)^(3/4))*(a^2 + b)^(1/4)*x^2 + (1 + I)*Sqrt[-b + a*x^4] + (1 + I)*(
-1)^(3/4)*Sqrt[-b + a*x^4])/(2*(a^2 + b)^(1/8)*x*(-b + a*x^4)^(1/4))])/(a^2
 + b)^(5/8) + ((Sqrt[2] - I*Sqrt[2*(3 + 2*Sqrt[2])])*ArcTan[(2*(a^2 + b)^(1
```

```
/8)*x*(-b + a*x^4)^(1/4))/(((1 - I) + Sqrt[2])*(a^2 + b)^(1/4)*x^2 - (1 + I
)*Sqrt[-b + a*x^4] - Sqrt[2]*Sqrt[-b + a*x^4])])/(16*(a^2 + b)^(5/8)) + ((-
I + (-1)^(3/4))*ArcTanh[(((-2 + 2*I) - (2 + 2*I)*(-1)^(3/4))*(a^2 + b)^(1/4
)*x^2 - (2 + 2*I)*Sqrt[-b + a*x^4] - (2 + 2*I)*(-1)^(3/4)*Sqrt[-b + a*x^4])
/(4*(a^2 + b)^(1/8)*x*(-b + a*x^4)^(1/4))])/(8*(a^2 + b)^(5/8)) + ((I/16)*(
I*Sqrt[2] + Sqrt[2*(3 + 2*Sqrt[2])])*ArcTanh[(((1 - I) + Sqrt[2])*(a^2 + b)
^(1/4)*x^2 + (1 + I)*Sqrt[-b + a*x^4] + Sqrt[2]*Sqrt[-b + a*x^4])/(2*(a^2 +
 b)^(1/8)*x*(-b + a*x^4)^(1/4))])/(a^2 + b)^(5/8)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4-b)^(1/4)/(x^8+2*a*x^4-b),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^8 + 2\,ax^4 - b\right)\left(ax^4 - b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4-b)^(1/4)/(x^8+2*a*x^4-b),x, algorithm="giac")
[Out] integrate(x^4/((x^8 + 2*a*x^4 - b)*(a*x^4 - b)^(1/4)), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a\,x^4 - b\right)^{\frac{1}{4}}\left(x^8 + 2a\,x^4 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^4-b)^(1/4)/(x^8+2*a*x^4-b),x)
[Out] int(x^4/(a*x^4-b)^(1/4)/(x^8+2*a*x^4-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^8 + 2\,ax^4 - b\right)\left(ax^4 - b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4-b)^(1/4)/(x^8+2*a*x^4-b),x, algorithm="maxima")
[Out] integrate(x^4/((x^8 + 2*a*x^4 - b)*(a*x^4 - b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a\,x^4 - b\right)^{1/4}\,\left(x^8 + 2\,a\,x^4 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a*x^4 - b)^(1/4)*(2*a*x^4 - b + x^8)),x)
```

```
[Out] int(x^4/((a*x^4 - b)^(1/4)*(2*a*x^4 - b + x^8)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a*x**4-b)**(1/4)/(x**8+2*a*x**4-b),x)
```

```
[Out] Timed out
```

3.2364 $\int \frac{(d+cx^2)\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}{(-b+a^2x^2)^{5/2}} dx$

Optimal. Leaf size=448

$$\frac{\sqrt{a^2x^2-b}\left(\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}\right)^{3/4}(55a^4dx^3-41a^2bcx^3-87a^2bdx+9b^2cx)}{96a^2b^{13/8}\left(ax-\sqrt{b}\right)^2\left(ax+\sqrt{b}\right)^2}+\frac{(41bc-55a^2d)\tan^{-1}\left(\sqrt[4]{\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}}\right)}{64a^3b^{13/8}}+$$

Rubi [B] time = 1.87, antiderivative size = 1077, normalized size of antiderivative = 2.40, number of steps used = 38, number of rules used = 18, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6742, 2122, 288, 290, 329, 300, 297, 1162, 617, 204, 1165, 628, 298, 203, 206, 2120, 463, 457}

Antiderivative was successfully verified.

```
[In] Int[((d + c*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(3/4))/(-b + a^2*x^2)^(5/2),x]
```

```
[Out] (8*b*c*(a*x + Sqrt[-b + a^2*x^2])^(11/4))/(3*a^3*(b - (a*x + Sqrt[-b + a^2*
x^2])^2)^3) + (8*d*(a*x + Sqrt[-b + a^2*x^2])^(11/4))/(3*a*(b - (a*x + Sqrt
[-b + a^2*x^2])^2)^3) - (11*d*(a*x + Sqrt[-b + a^2*x^2])^(3/4))/(6*a*(b - (
a*x + Sqrt[-b + a^2*x^2])^2)^2) - (11*c*(a*x + Sqrt[-b + a^2*x^2])^(11/4))/
(6*a^3*(b - (a*x + Sqrt[-b + a^2*x^2])^2)^2) + (41*c*(a*x + Sqrt[-b + a^2*x
^2])^(3/4))/(48*a^3*(b - (a*x + Sqrt[-b + a^2*x^2])^2)) + (11*d*(a*x + Sqrt
[-b + a^2*x^2])^(3/4))/(16*a*b*(b - (a*x + Sqrt[-b + a^2*x^2])^2)) + (41*c*
ArcTan[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)])/(64*a^3*b^(5/8)) - (55*d*
ArcTan[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)])/(64*a*b^(13/8)) + (41*c*A
rcTan[1 - (Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/b^(1/8)])/(64*Sqrt[2]*
a^3*b^(5/8)) - (55*d*ArcTan[1 - (Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/
b^(1/8)])/(64*Sqrt[2]*a*b^(13/8)) - (41*c*ArcTan[1 + (Sqrt[2]*(a*x + Sqrt[-
b + a^2*x^2])^(1/4))/b^(1/8)])/(64*Sqrt[2]*a^3*b^(5/8)) + (55*d*ArcTan[1 +
(Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/b^(1/8)])/(64*Sqrt[2]*a*b^(13/8)
) - (41*c*ArcTanh[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)])/(64*a^3*b^(5/8
)) + (55*d*ArcTanh[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)])/(64*a*b^(13/8
)) - (41*c*Log[b^(1/4) - Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) +
 Sqrt[a*x + Sqrt[-b + a^2*x^2]]])/(128*Sqrt[2]*a^3*b^(5/8)) + (55*d*Log[b^(
1/4) - Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[-
b + a^2*x^2]]])/(128*Sqrt[2]*a*b^(13/8)) + (41*c*Log[b^(1/4) + Sqrt[2]*b^(1
/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]])/(12
8*Sqrt[2]*a^3*b^(5/8)) - (55*d*Log[b^(1/4) + Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b
 + a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]])/(128*Sqrt[2]*a*b^(13/
8))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 300

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[x^m/(r + s
*x^(n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ
[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(d+cx^2\right)\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}{\left(-b+a^2x^2\right)^{5/2}}\,dx = \int \left(\frac{d\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}{\left(-b+a^2x^2\right)^{5/2}} + \frac{cx^2\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}{\left(-b+a^2x^2\right)^{5/2}}\right) dx$$

$$= c\int \frac{x^2\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}{\left(-b+a^2x^2\right)^{5/2}}\,dx + d\int \frac{\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}{\left(-b+a^2x^2\right)^{5/2}}\,dx$$

$$= \frac{(4c)\,\text{Subst}\left(\int \frac{x^{7/4}\left(b+x^2\right)^2}{\left(-b+x^2\right)^4}\,dx, x, ax+\sqrt{-b+a^2x^2}\right)}{a^3} + \frac{(16d)\,\text{Subst}\left(\int \frac{}{(-}\right.}{}$$

$$= \frac{8bc\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a^3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} +$$

$$= \frac{8bc\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a^3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} -$$

$$= \frac{8bc\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a^3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} -$$

$$= \frac{8bc\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a^3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} -$$

$$= \frac{8bc\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a^3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} -$$

$$= \frac{8bc\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a^3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} -$$

$$= \frac{8bc\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a^3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} -$$

$$= \frac{8bc\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a^3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} -$$

$$= \frac{8bc\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a^3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}{3a\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} -$$

Mathematica [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(d+cx^2)\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}{(-b+a^2x^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + c*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(3/4))/(-b + a^2*x^2)^(5/
2),x]
```

```
[Out] Integrate[((d + c*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(3/4))/(-b + a^2*x^2)^(5/
2), x]
```

IntegrateAlgebraic [A] time = 5.52, size = 510, normalized size = 1.14

$$\frac{\sqrt{a^2x^2-b}\left(\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}\right)^{3/4}(55a^4dx^3-41a^2bcx^3-87a^2bdx+9b^2cx)}{96a^2b^{13/8}(ax-\sqrt{b})^2(ax+\sqrt{b})^2}+\frac{(41bc-55a^2d)\tan^{-1}\left(\sqrt[4]{\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}}\right)}{64a^3b^{13/8}}+\frac{(55a^2d-41bc)\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}}-1}{\sqrt{2}\sqrt[4]{\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}}}\right)}{64\sqrt{2}a^3b^{13/8}}+\frac{(55a^2d-41bc)\tanh^{-1}\left(\sqrt[4]{\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}}\right)}{64a^3b^{13/8}}-\frac{(55a^2d-41bc)\tanh^{-1}\left(\frac{\sqrt{\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}}+1}{\sqrt{2}\sqrt[4]{\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}}}\right)}{64\sqrt{2}a^3b^{13/8}}+\frac{\left(\frac{\sqrt{a^2x^2-b}+ax}{\sqrt{b}}\right)^{3/4}(-55a^4dx^2+41a^2bcx^2+43a^2bd-53b^2c)}{96a^3b^{13/8}(ax-\sqrt{b})(ax+\sqrt{b})}$$

Warning: Unable to verify antiderivative.

```
[In] IntegrateAlgebraic[((d + c*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(3/4))/(-b + a^2
*x^2)^(5/2),x]
```

```
[Out] ((-53*b^2*c + 43*a^2*b*d + 41*a^2*b*c*x^2 - 55*a^4*d*x^2)*((a*x + Sqrt[-b +
 a^2*x^2])/Sqrt[b])^(3/4))/(96*a^3*b^(13/8)*(-Sqrt[b] + a*x)*(Sqrt[b] + a*x
)) + (Sqrt[-b + a^2*x^2]*(9*b^2*c*x - 87*a^2*b*d*x - 41*a^2*b*c*x^3 + 55*a^
4*d*x^3)*((a*x + Sqrt[-b + a^2*x^2])/Sqrt[b])^(3/4))/(96*a^2*b^(13/8)*(-Sqr
t[b] + a*x)^2*(Sqrt[b] + a*x)^2) + ((41*b*c - 55*a^2*d)*ArcTan[((a*x + Sqrt
[-b + a^2*x^2])/Sqrt[b])^(1/4)])/(64*a^3*b^(13/8)) + ((-41*b*c + 55*a^2*d)*
ArcTan[(-1 + Sqrt[(a*x + Sqrt[-b + a^2*x^2])/Sqrt[b]])/(Sqrt[2]*((a*x + Sqr
t[-b + a^2*x^2])/Sqrt[b])^(1/4))])/(64*Sqrt[2]*a^3*b^(13/8)) + ((-41*b*c +
55*a^2*d)*ArcTanh[((a*x + Sqrt[-b + a^2*x^2])/Sqrt[b])^(1/4)])/(64*a^3*b^(1
3/8)) - ((-41*b*c + 55*a^2*d)*ArcTanh[(1 + Sqrt[(a*x + Sqrt[-b + a^2*x^2])/
Sqrt[b]])/(Sqrt[2]*((a*x + Sqrt[-b + a^2*x^2])/Sqrt[b])^(1/4))])/(64*Sqrt[2
]*a^3*b^(13/8))
```

fricas [B] time = 0.62, size = 4991, normalized size = 11.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+d)*(a*x+(a^2*x^2-b)^(1/2))^(3/4)/(a^2*x^2-b)^(5/2),x, algo
rithm="fricas")
```

```
[Out] -1/768*(12*sqrt(2)*(a^7*b^2*x^4 - 2*a^5*b^3*x^2 + a^3*b^4)*((83733937890625
*a^16*d^8 - 499358756875000*a^14*b*c*d^7 + 1302872392937500*a^12*b^2*c^2*d^
6 - 1942464294925000*a^10*b^3*c^3*d^5 + 1810023547543750*a^8*b^4*c^4*d^4 -
1079432224717000*a^6*b^5*c^5*d^3 + 402333829212700*a^4*b^6*c^6*d^2 - 856918
80507640*a^2*b^7*c^7*d + 7984925229121*b^8*c^8)/(a^24*b^13))^(1/8)*arctan((
sqrt(2)*sqrt(a^18*b^10*((83733937890625*a^16*d^8 - 499358756875000*a^14*b*c
*d^7 + 1302872392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*c^3*d^
5 + 1810023547543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d^3 + 4
02333829212700*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 79849252291
21*b^8*c^8)/(a^24*b^13))^(3/4) + sqrt(2)*(166375*a^15*b^5*d^3 - 372075*a^13
*b^6*c*d^2 + 277365*a^11*b^7*c^2*d - 68921*a^9*b^8*c^3)*(a*x + sqrt(a^2*x^2
 - b))^(1/4)*((83733937890625*a^16*d^8 - 499358756875000*a^14*b*c*d^7 + 130
2872392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*c^3*d^5 + 181002
3547543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d^3 + 40233382921
2700*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 7984925229121*b^8*c^8
)/(a^24*b^13))^(3/8) + (27680640625*a^12*d^6 - 123807956250*a^10*b*c*d^5 +
```

230733009375*a^8*b^2*c^2*d^4 - 229334627500*a^6*b^3*c^3*d^3 + 128218905375*
a^4*b^4*c^4*d^2 - 38232546330*a^2*b^5*c^5*d + 4750104241*b^6*c^6)*sqrt(a*x
+ sqrt(a^2*x^2 - b)))*a^15*b^8*((83733937890625*a^16*d^8 - 499358756875000*
a^14*b*c*d^7 + 1302872392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^
3*c^3*d^5 + 1810023547543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5
*d^3 + 402333829212700*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 798
4925229121*b^8*c^8)/(a^24*b^13))^(5/8) - 83733937890625*a^16*d^8 + 49935875
6875000*a^14*b*c*d^7 - 1302872392937500*a^12*b^2*c^2*d^6 + 1942464294925000
*a^10*b^3*c^3*d^5 - 1810023547543750*a^8*b^4*c^4*d^4 + 1079432224717000*a^6
*b^5*c^5*d^3 - 402333829212700*a^4*b^6*c^6*d^2 + 85691880507640*a^2*b^7*c^7
*d - 7984925229121*b^8*c^8 - sqrt(2)*(166375*a^21*b^8*d^3 - 372075*a^19*b^9
*c*d^2 + 277365*a^17*b^10*c^2*d - 68921*a^15*b^11*c^3)*(a*x + sqrt(a^2*x^2
- b))^(1/4)*((83733937890625*a^16*d^8 - 499358756875000*a^14*b*c*d^7 + 1302
872392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*c^3*d^5 + 1810023
547543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d^3 + 402333829212
700*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 7984925229121*b^8*c^8)
/(a^24*b^13))^(5/8))/(83733937890625*a^16*d^8 - 499358756875000*a^14*b*c*d^
7 + 1302872392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*c^3*d^5 +
 1810023547543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d^3 + 4023
33829212700*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 7984925229121*
b^8*c^8)) + 12*sqrt(2)*(a^7*b^2*x^4 - 2*a^5*b^3*x^2 + a^3*b^4)*((8373393789
0625*a^16*d^8 - 499358756875000*a^14*b*c*d^7 + 1302872392937500*a^12*b^2*c^
2*d^6 - 1942464294925000*a^10*b^3*c^3*d^5 + 1810023547543750*a^8*b^4*c^4*d^
4 - 1079432224717000*a^6*b^5*c^5*d^3 + 402333829212700*a^4*b^6*c^6*d^2 - 85
691880507640*a^2*b^7*c^7*d + 7984925229121*b^8*c^8)/(a^24*b^13))^(1/8)*arct
an((sqrt(2)*sqrt(a^18*b^10*((83733937890625*a^16*d^8 - 499358756875000*a^14
*b*c*d^7 + 1302872392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*c^
3*d^5 + 1810023547543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d^3
 + 402333829212700*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 7984925
229121*b^8*c^8)/(a^24*b^13))^(3/4) - sqrt(2)*(166375*a^15*b^5*d^3 - 372075*
a^13*b^6*c*d^2 + 277365*a^11*b^7*c^2*d - 68921*a^9*b^8*c^3)*(a*x + sqrt(a^2
x^2 - b))^(1/4)((83733937890625*a^16*d^8 - 499358756875000*a^14*b*c*d^7 +
 1302872392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*c^3*d^5 + 18
10023547543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d^3 + 4023338
29212700*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 7984925229121*b^8
*c^8)/(a^24*b^13))^(3/8) + (27680640625*a^12*d^6 - 123807956250*a^10*b*c*d^
5 + 230733009375*a^8*b^2*c^2*d^4 - 229334627500*a^6*b^3*c^3*d^3 + 128218905
375*a^4*b^4*c^4*d^2 - 38232546330*a^2*b^5*c^5*d + 4750104241*b^6*c^6)*sqrt(
a*x + sqrt(a^2*x^2 - b)))*a^15*b^8*((83733937890625*a^16*d^8 - 499358756875
000*a^14*b*c*d^7 + 1302872392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^1
0*b^3*c^3*d^5 + 1810023547543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5
*c^5*d^3 + 402333829212700*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d +
 7984925229121*b^8*c^8)/(a^24*b^13))^(5/8) + 83733937890625*a^16*d^8 - 4993
58756875000*a^14*b*c*d^7 + 1302872392937500*a^12*b^2*c^2*d^6 - 194246429492
5000*a^10*b^3*c^3*d^5 + 1810023547543750*a^8*b^4*c^4*d^4 - 1079432224717000
*a^6*b^5*c^5*d^3 + 402333829212700*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7
*c^7*d + 7984925229121*b^8*c^8 - sqrt(2)*(166375*a^21*b^8*d^3 - 372075*a^19
*b^9*c*d^2 + 277365*a^17*b^10*c^2*d - 68921*a^15*b^11*c^3)*(a*x + sqrt(a^2*
x^2 - b))^(1/4)*((83733937890625*a^16*d^8 - 499358756875000*a^14*b*c*d^7 +
1302872392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*c^3*d^5 + 181
0023547543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d^3 + 40233382
9212700*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 7984925229121*b^8*
c^8)/(a^24*b^13))^(5/8))/(83733937890625*a^16*d^8 - 499358756875000*a^14*b*
c*d^7 + 1302872392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*c^3*d
^5 + 1810023547543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d^3 +
402333829212700*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 7984925229
121*b^8*c^8)) + 3*sqrt(2)*(a^7*b^2*x^4 - 2*a^5*b^3*x^2 + a^3*b^4)*((8373393
7890625*a^16*d^8 - 499358756875000*a^14*b*c*d^7 + 1302872392937500*a^12*b^2
*c^2*d^6 - 1942464294925000*a^10*b^3*c^3*d^5 + 1810023547543750*a^8*b^4*c^4

```
*d^4 - 1079432224717000*a^6*b^5*c^5*d^3 + 402333829212700*a^4*b^6*c^6*d^2 -
 85691880507640*a^2*b^7*c^7*d + 7984925229121*b^8*c^8)/(a^24*b^13))^(1/8)*l
og(a^18*b^10*((83733937890625*a^16*d^8 - 499358756875000*a^14*b*c*d^7 + 130
2872392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*c^3*d^5 + 181002
3547543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d^3 + 40233382921
2700*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 7984925229121*b^8*c^8
)/(a^24*b^13))^(3/4) + sqrt(2)*(166375*a^15*b^5*d^3 - 372075*a^13*b^6*c*d^2
 + 277365*a^11*b^7*c^2*d - 68921*a^9*b^8*c^3)*(a*x + sqrt(a^2*x^2 - b))^(1/
4)*((83733937890625*a^16*d^8 - 499358756875000*a^14*b*c*d^7 + 1302872392937
500*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*c^3*d^5 + 1810023547543750
*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d^3 + 402333829212700*a^4*b
^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 7984925229121*b^8*c^8)/(a^24*b^
13))^(3/8) + (27680640625*a^12*d^6 - 123807956250*a^10*b*c*d^5 + 2307330093
75*a^8*b^2*c^2*d^4 - 229334627500*a^6*b^3*c^3*d^3 + 128218905375*a^4*b^4*c^
4*d^2 - 38232546330*a^2*b^5*c^5*d + 4750104241*b^6*c^6)*sqrt(a*x + sqrt(a^2
*x^2 - b))) - 3*sqrt(2)*(a^7*b^2*x^4 - 2*a^5*b^3*x^2 + a^3*b^4)*((837339378
90625*a^16*d^8 - 499358756875000*a^14*b*c*d^7 + 1302872392937500*a^12*b^2*c
^2*d^6 - 1942464294925000*a^10*b^3*c^3*d^5 + 1810023547543750*a^8*b^4*c^4*d
^4 - 1079432224717000*a^6*b^5*c^5*d^3 + 402333829212700*a^4*b^6*c^6*d^2 - 8
5691880507640*a^2*b^7*c^7*d + 7984925229121*b^8*c^8)/(a^24*b^13))^(1/8)*log
(a^18*b^10*((83733937890625*a^16*d^8 - 499358756875000*a^14*b*c*d^7 + 13028
72392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*c^3*d^5 + 18100235
47543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d^3 + 4023338292127
00*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 7984925229121*b^8*c^8)/
(a^24*b^13))^(3/4) - sqrt(2)*(166375*a^15*b^5*d^3 - 372075*a^13*b^6*c*d^2 +
 277365*a^11*b^7*c^2*d - 68921*a^9*b^8*c^3)*(a*x + sqrt(a^2*x^2 - b))^(1/4)
*((83733937890625*a^16*d^8 - 499358756875000*a^14*b*c*d^7 + 130287239293750
0*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*c^3*d^5 + 1810023547543750*a
^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d^3 + 402333829212700*a^4*b^6
*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 7984925229121*b^8*c^8)/(a^24*b^13
))^(3/8) + (27680640625*a^12*d^6 - 123807956250*a^10*b*c*d^5 + 230733009375
*a^8*b^2*c^2*d^4 - 229334627500*a^6*b^3*c^3*d^3 + 128218905375*a^4*b^4*c^4*
d^2 - 38232546330*a^2*b^5*c^5*d + 4750104241*b^6*c^6)*sqrt(a*x + sqrt(a^2*x
^2 - b))) - 24*(a^7*b^2*x^4 - 2*a^5*b^3*x^2 + a^3*b^4)*((83733937890625*a^1
6*d^8 - 499358756875000*a^14*b*c*d^7 + 1302872392937500*a^12*b^2*c^2*d^6 -
1942464294925000*a^10*b^3*c^3*d^5 + 1810023547543750*a^8*b^4*c^4*d^4 - 1079
432224717000*a^6*b^5*c^5*d^3 + 402333829212700*a^4*b^6*c^6*d^2 - 8569188050
7640*a^2*b^7*c^7*d + 7984925229121*b^8*c^8)/(a^24*b^13))^(1/8)*arctan((sqrt
(a^18*b^10*((83733937890625*a^16*d^8 - 499358756875000*a^14*b*c*d^7 + 13028
72392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*c^3*d^5 + 18100235
47543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d^3 + 4023338292127
00*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 7984925229121*b^8*c^8)/
(a^24*b^13))^(3/4) + (27680640625*a^12*d^6 - 123807956250*a^10*b*c*d^5 + 23
0733009375*a^8*b^2*c^2*d^4 - 229334627500*a^6*b^3*c^3*d^3 + 128218905375*a^
4*b^4*c^4*d^2 - 38232546330*a^2*b^5*c^5*d + 4750104241*b^6*c^6)*sqrt(a*x +
sqrt(a^2*x^2 - b)))*a^15*b^8*((83733937890625*a^16*d^8 - 499358756875000*a^
14*b*c*d^7 + 1302872392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*
c^3*d^5 + 1810023547543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d
^3 + 402333829212700*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 79849
25229121*b^8*c^8)/(a^24*b^13))^(5/8) - (166375*a^21*b^8*d^3 - 372075*a^19*b
^9*c*d^2 + 277365*a^17*b^10*c^2*d - 68921*a^15*b^11*c^3)*(a*x + sqrt(a^2*x^
2 - b))^(1/4)*((83733937890625*a^16*d^8 - 499358756875000*a^14*b*c*d^7 + 13
02872392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*c^3*d^5 + 18100
23547543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d^3 + 4023338292
12700*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 7984925229121*b^8*c^
8)/(a^24*b^13))^(5/8))/(83733937890625*a^16*d^8 - 499358756875000*a^14*b*c*
d^7 + 1302872392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*c^3*d^5
 + 1810023547543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d^3 + 40
2333829212700*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 798492522912
```

```
1*b^8*c^8)) - 6*(a^7*b^2*x^4 - 2*a^5*b^3*x^2 + a^3*b^4)*((83733937890625*a^
16*d^8 - 499358756875000*a^14*b*c*d^7 + 1302872392937500*a^12*b^2*c^2*d^6 -
 1942464294925000*a^10*b^3*c^3*d^5 + 1810023547543750*a^8*b^4*c^4*d^4 - 107
9432224717000*a^6*b^5*c^5*d^3 + 402333829212700*a^4*b^6*c^6*d^2 - 856918805
07640*a^2*b^7*c^7*d + 7984925229121*b^8*c^8)/(a^24*b^13))^(1/8)*log(a^9*b^5
*((83733937890625*a^16*d^8 - 499358756875000*a^14*b*c*d^7 + 130287239293750
0*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*c^3*d^5 + 1810023547543750*a
^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d^3 + 402333829212700*a^4*b^6
*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 7984925229121*b^8*c^8)/(a^24*b^13
))^(3/8) + (166375*a^6*d^3 - 372075*a^4*b*c*d^2 + 277365*a^2*b^2*c^2*d - 68
921*b^3*c^3)*(a*x + sqrt(a^2*x^2 - b))^(1/4)) + 6*(a^7*b^2*x^4 - 2*a^5*b^3*
x^2 + a^3*b^4)*((83733937890625*a^16*d^8 - 499358756875000*a^14*b*c*d^7 + 1
302872392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^10*b^3*c^3*d^5 + 1810
023547543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^5*c^5*d^3 + 402333829
212700*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d + 7984925229121*b^8*c
^8)/(a^24*b^13))^(1/8)*log(-a^9*b^5*((83733937890625*a^16*d^8 - 49935875687
5000*a^14*b*c*d^7 + 1302872392937500*a^12*b^2*c^2*d^6 - 1942464294925000*a^
10*b^3*c^3*d^5 + 1810023547543750*a^8*b^4*c^4*d^4 - 1079432224717000*a^6*b^
5*c^5*d^3 + 402333829212700*a^4*b^6*c^6*d^2 - 85691880507640*a^2*b^7*c^7*d
+ 7984925229121*b^8*c^8)/(a^24*b^13))^(3/8) + (166375*a^6*d^3 - 372075*a^4*
b*c*d^2 + 277365*a^2*b^2*c^2*d - 68921*b^3*c^3)*(a*x + sqrt(a^2*x^2 - b))^(
1/4)) + 8*(43*a^2*b^2*d + (55*a^6*d - 41*a^4*b*c)*x^4 - 53*b^3*c - 2*(49*a^
4*b*d - 47*a^2*b^2*c)*x^2 - sqrt(a^2*x^2 - b)*((55*a^5*d - 41*a^3*b*c)*x^3
- 3*(29*a^3*b*d - 3*a*b^2*c)*x))*(a*x + sqrt(a^2*x^2 - b))^(3/4))/(a^7*b^2*
x^4 - 2*a^5*b^3*x^2 + a^3*b^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+d)*(a*x+(a^2*x^2-b)^(1/2))^(3/4)/(a^2*x^2-b)^(5/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^{2}+d\right)\left(a x +\sqrt{a^{2} x^{2}-b}\right)^{\frac{3}{4}}}{\left(a^{2} x^{2}-b\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+d)*(a*x+(a^2*x^2-b)^(1/2))^(3/4)/(a^2*x^2-b)^(5/2),x)
```

```
[Out] int((c*x^2+d)*(a*x+(a^2*x^2-b)^(1/2))^(3/4)/(a^2*x^2-b)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(c x^{2} + d\right)} {\left(a x + \sqrt{a^{2} x^{2} - b}\right)}^{\frac{3}{4}}}{{\left(a^{2} x^{2} - b\right)}^{\frac{5}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+d)*(a*x+(a^2*x^2-b)^(1/2))^(3/4)/(a^2*x^2-b)^(5/2),x, algo
rithm="maxima")
```

```
[Out] integrate((c*x^2 + d)*(a*x + sqrt(a^2*x^2 - b))^(3/4)/(a^2*x^2 - b)^(5/2),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{3/4}\,\left(c\,x^2+d\right)}{\left(a^2\,x^2-b\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (a^2*x^2 - b)^(1/2))^(3/4)*(d + c*x^2))/(a^2*x^2 - b)^(5/2),x)
[Out] int(((a*x + (a^2*x^2 - b)^(1/2))^(3/4)*(d + c*x^2))/(a^2*x^2 - b)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{3}{4}}\left(cx^2+d\right)}{\left(a^2x^2-b\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+d)*(a*x+(a**2*x**2-b)**(1/2))**(3/4)/(a**2*x**2-b)**(5/2)
,x)
[Out] Integral((a*x + sqrt(a**2*x**2 - b))**(3/4)*(c*x**2 + d)/(a**2*x**2 - b)**(
5/2), x)
```

3.2365 $\int \frac{x^2}{(1+x^4)\sqrt[4]{-x^2+x^6}}\,dx$

Optimal. Leaf size=452

$$-\frac{1}{16}\sqrt{\sqrt{2}-1}\,\log\left(-2x^2+2^{3/4}\sqrt{2+\sqrt{2}}\,\sqrt[4]{x^6-x^2}\,x-\sqrt{2}\,\sqrt{x^6-x^2}\right)+\frac{1}{16}\sqrt{\sqrt{2}-1}\,\log\left(2\sqrt{2-\sqrt{2}}\,x^2+2\sqrt[4]{2}\right.$$

Rubi [C] time = 0.16, antiderivative size = 50, normalized size of antiderivative = 0.11, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2042, 466, 511, 510}

$$\frac{2x^3\sqrt[4]{1-x^4}\,F_1\left(\frac{5}{8};\frac{1}{4},1;\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^6-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2/((1 + x^4)*(-x^2 + x^6)^(1/4)),x]
```

```
[Out] (2*x^3*(1 - x^4)^(1/4)*AppellF1[5/8, 1/4, 1, 13/8, x^4, -x^4])/(5*(-x^2 + x
^6)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\int \frac{x^2}{(1+x^4)\sqrt[4]{-x^2+x^6}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{-1+x^4}\right)\int \frac{x^{3/2}}{\sqrt[4]{-1+x^4}(1+x^4)}\,dx}{\sqrt[4]{-x^2+x^6}}$$
$$= \frac{\left(2\sqrt{x}\sqrt[4]{-1+x^4}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{-1+x^8}(1+x^8)}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^6}}$$
$$= \frac{\left(2\sqrt{x}\sqrt[4]{1-x^4}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{1-x^8}(1+x^8)}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^6}}$$
$$= \frac{2x^3\sqrt[4]{1-x^4}\,F_1\left(\frac{5}{8};\frac{1}{4},1;\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{-x^2+x^6}}$$

Mathematica [C] time = 0.03, size = 50, normalized size = 0.11

$$\frac{2x^3\sqrt[4]{1-x^4}\,F_1\left(\frac{5}{8};\frac{1}{4},1;\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^2(x^4-1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((1 + x^4)*(-x^2 + x^6)^(1/4)),x]
```

```
[Out] (2*x^3*(1 - x^4)^(1/4)*AppellF1[5/8, 1/4, 1, 13/8, x^4, -x^4])/(5*(x^2*(-1
+ x^4))^(1/4))
```

IntegrateAlgebraic [C] time = 0.81, size = 153, normalized size = 0.34

$$-\frac{1}{4}\sqrt{\frac{1}{2}-\frac{i}{2}}\tan^{-1}\left(\frac{\sqrt{-1-i}\,x}{\sqrt[4]{x^6-x^2}}\right)-\frac{1}{4}\sqrt{\frac{1}{2}+\frac{i}{2}}\tan^{-1}\left(\frac{\sqrt{-1+i}\,x}{\sqrt[4]{x^6-x^2}}\right)+\frac{1}{4}\sqrt{-\frac{1}{2}-\frac{i}{2}}\tan^{-1}\left(\frac{\sqrt{1-i}\,x}{\sqrt[4]{x^6-x^2}}\right)+\frac{1}{4}\sqrt{-\frac{1}{2}+\frac{i}{2}}\tan^{-1}\left(\frac{\sqrt{1+i}\,x}{\sqrt[4]{x^6-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((1 + x^4)*(-x^2 + x^6)^(1/4)),x]
```

```
[Out] -1/4*(Sqrt[1/2 - I/2]*ArcTan[(Sqrt[-1 - I]*x)/(-x^2 + x^6)^(1/4)]) - (Sqrt[
1/2 + I/2]*ArcTan[(Sqrt[-1 + I]*x)/(-x^2 + x^6)^(1/4)])/4 + (Sqrt[-1/2 - I/
2]*ArcTan[(Sqrt[1 - I]*x)/(-x^2 + x^6)^(1/4)])/4 + (Sqrt[-1/2 + I/2]*ArcTan
[(Sqrt[1 + I]*x)/(-x^2 + x^6)^(1/4)])/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)/(x^6-x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^6-x^2)^{\frac{1}{4}}(x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)/(x^6-x^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^6 - x^2)^(1/4)*(x^4 + 1)), x)
```

maple [C] time = 91.79, size = 2929, normalized size = 6.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+1)/(x^6-x^2)^(1/4),x)
```

```
[Out] 1/4*RootOf(32*_Z^4-8*_Z^2+1)*ln(-(230912*RootOf(32*_Z^4-8*_Z^2+1)^5*x^5-865
920*RootOf(32*_Z^4-8*_Z^2+1)^5*x^3-27672*RootOf(32*_Z^4-8*_Z^2+1)^3*x^5-230
912*RootOf(32*_Z^4-8*_Z^2+1)^5*x-208080*(x^6-x^2)^(1/2)*RootOf(32*_Z^4-8*_Z
^2+1)^3*x+364448*RootOf(32*_Z^4-8*_Z^2+1)^3*x^3-5789*RootOf(32*_Z^4-8*_Z^2+
1)*x^5+73984*RootOf(32*_Z^4-8*_Z^2+1)^2*(x^6-x^2)^(3/4)-43928*RootOf(32*_Z^
4-8*_Z^2+1)^2*(x^6-x^2)^(1/4)*x^2+27672*RootOf(32*_Z^4-8*_Z^2+1)^3*x+36992*
(x^6-x^2)^(1/2)*RootOf(32*_Z^4-8*_Z^2+1)*x-38042*RootOf(32*_Z^4-8*_Z^2+1)*x
^3-5491*(x^6-x^2)^(3/4)-7514*(x^6-x^2)^(1/4)*x^2+5789*RootOf(32*_Z^4-8*_Z^2
+1)*x)/(8*RootOf(32*_Z^4-8*_Z^2+1)^2*x^2-32*RootOf(32*_Z^4-8*_Z^2+1)^2-5*x^
2+3)^2/x)+1/8*RootOf(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*ln(-(230912*RootO
f(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*RootOf(32*_Z^4-8*_Z^2+1)^4*x^5-86592
0*RootOf(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*RootOf(32*_Z^4-8*_Z^2+1)^4*x^
3-87784*RootOf(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*RootOf(32*_Z^4-8*_Z^2+1
)^2*x^5-230912*RootOf(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*RootOf(32*_Z^4-8
*_Z^2+1)^4*x+208080*RootOf(32*_Z^4-8*_Z^2+1)^2*(x^6-x^2)^(1/2)*RootOf(_Z^2+
4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*x+68512*RootOf(_Z^2+4*RootOf(32*_Z^4-8*_Z^2
+1)^2-1)*RootOf(32*_Z^4-8*_Z^2+1)^2*x^3+1725*RootOf(_Z^2+4*RootOf(32*_Z^4-8
*_Z^2+1)^2-1)*x^5-147968*RootOf(32*_Z^4-8*_Z^2+1)^2*(x^6-x^2)^(3/4)+87856*R
ootOf(32*_Z^4-8*_Z^2+1)^2*(x^6-x^2)^(1/4)*x^2+87784*RootOf(_Z^2+4*RootOf(32
*_Z^4-8*_Z^2+1)^2-1)*RootOf(32*_Z^4-8*_Z^2+1)^2*x-15028*(x^6-x^2)^(1/2)*Roo
tOf(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*x-1050*RootOf(_Z^2+4*RootOf(32*_Z^
4-8*_Z^2+1)^2-1)*x^3+26010*(x^6-x^2)^(3/4)-36992*(x^6-x^2)^(1/4)*x^2-1725*R
ootOf(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*x)/(8*RootOf(32*_Z^4-8*_Z^2+1)^2
*x^2-32*RootOf(32*_Z^4-8*_Z^2+1)^2+3*x^2+5)^2/x)+ln(-(192512*RootOf(_Z^2+4*
RootOf(32*_Z^4-8*_Z^2+1)^2-1)*RootOf(32*_Z^4-8*_Z^2+1)^4*x^5-721920*RootOf(
_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*RootOf(32*_Z^4-8*_Z^2+1)^4*x^3-122112*
RootOf(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*RootOf(32*_Z^4-8*_Z^2+1)^2*x^5-
192512*RootOf(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*RootOf(32*_Z^4-8*_Z^2+1)
^4*x-87856*RootOf(32*_Z^4-8*_Z^2+1)^2*(x^6-x^2)^(1/2)*RootOf(_Z^2+4*RootOf(
32*_Z^4-8*_Z^2+1)^2-1)*x+240592*RootOf(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)
*RootOf(32*_Z^4-8*_Z^2+1)^2*x^3+19021*RootOf(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1
)^2-1)*x^5-147968*RootOf(32*_Z^4-8*_Z^2+1)^2*(x^6-x^2)^(3/4)-87856*RootOf(3
2*_Z^4-8*_Z^2+1)^2*(x^6-x^2)^(1/4)*x^2+122112*RootOf(_Z^2+4*RootOf(32*_Z^4-
8*_Z^2+1)^2-1)*RootOf(32*_Z^4-8*_Z^2+1)^2*x+36992*(x^6-x^2)^(1/2)*RootOf(_Z
^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*x-11578*RootOf(_Z^2+4*RootOf(32*_Z^4-8*_
Z^2+1)^2-1)*x^3+26010*(x^6-x^2)^(3/4)+36992*(x^6-x^2)^(1/4)*x^2-19021*RootO
f(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*x)/(8*RootOf(32*_Z^4-8*_Z^2+1)^2*x^2
-32*RootOf(32*_Z^4-8*_Z^2+1)^2+3*x^2+5)^2/x)*RootOf(32*_Z^4-8*_Z^2+1)^2*Roo
tOf(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)-1/8*ln(-(192512*RootOf(_Z^2+4*Root
Of(32*_Z^4-8*_Z^2+1)^2-1)*RootOf(32*_Z^4-8*_Z^2+1)^4*x^5-721920*RootOf(_Z^2
+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*RootOf(32*_Z^4-8*_Z^2+1)^4*x^3-122112*Root
Of(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*RootOf(32*_Z^4-8*_Z^2+1)^2*x^5-1925
12*RootOf(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*RootOf(32*_Z^4-8*_Z^2+1)^4*x
-87856*RootOf(32*_Z^4-8*_Z^2+1)^2*(x^6-x^2)^(1/2)*RootOf(_Z^2+4*RootOf(32*_
Z^4-8*_Z^2+1)^2-1)*x+240592*RootOf(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*Roo
tOf(32*_Z^4-8*_Z^2+1)^2*x^3+19021*RootOf(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-
1)*x^5-147968*RootOf(32*_Z^4-8*_Z^2+1)^2*(x^6-x^2)^(3/4)-87856*RootOf(32*_Z
^4-8*_Z^2+1)^2*(x^6-x^2)^(1/4)*x^2+122112*RootOf(_Z^2+4*RootOf(32*_Z^4-8*_Z
^2+1)^2-1)*RootOf(32*_Z^4-8*_Z^2+1)^2*x+36992*(x^6-x^2)^(1/2)*RootOf(_Z^2+4
```

```
*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*x-11578*RootOf(_Z^2+4*RootOf(32*_Z^4-8*_Z^2+
1)^2-1)*x^3+26010*(x^6-x^2)^(3/4)+36992*(x^6-x^2)^(1/4)*x^2-19021*RootOf(_Z
^2+4*RootOf(32*_Z^4-8*_Z^2+1)^2-1)*x)/(8*RootOf(32*_Z^4-8*_Z^2+1)^2*x^2-32*
RootOf(32*_Z^4-8*_Z^2+1)^2+3*x^2+5)^2/x)*RootOf(_Z^2+4*RootOf(32*_Z^4-8*_Z^
2+1)^2-1)+2*RootOf(32*_Z^4-8*_Z^2+1)^3*ln((192512*RootOf(32*_Z^4-8*_Z^2+1)^
5*x^5-721920*RootOf(32*_Z^4-8*_Z^2+1)^5*x^3+25856*RootOf(32*_Z^4-8*_Z^2+1)^
3*x^5-192512*RootOf(32*_Z^4-8*_Z^2+1)^5*x+87856*(x^6-x^2)^(1/2)*RootOf(32*_
Z^4-8*_Z^2+1)^3*x+120368*RootOf(32*_Z^4-8*_Z^2+1)^3*x^3+525*RootOf(32*_Z^4-
8*_Z^2+1)*x^5-73984*RootOf(32*_Z^4-8*_Z^2+1)^2*(x^6-x^2)^(3/4)-43928*RootOf
(32*_Z^4-8*_Z^2+1)^2*(x^6-x^2)^(1/4)*x^2-25856*RootOf(32*_Z^4-8*_Z^2+1)^3*x
+15028*(x^6-x^2)^(1/2)*RootOf(32*_Z^4-8*_Z^2+1)*x+3450*RootOf(32*_Z^4-8*_Z^
2+1)*x^3+5491*(x^6-x^2)^(3/4)-7514*(x^6-x^2)^(1/4)*x^2-525*RootOf(32*_Z^4-8
*_Z^2+1)*x)/(8*RootOf(32*_Z^4-8*_Z^2+1)^2*x^2-32*RootOf(32*_Z^4-8*_Z^2+1)^2
-5*x^2+3)^2/x)-1/4*RootOf(32*_Z^4-8*_Z^2+1)*ln((192512*RootOf(32*_Z^4-8*_Z^
2+1)^5*x^5-721920*RootOf(32*_Z^4-8*_Z^2+1)^5*x^3+25856*RootOf(32*_Z^4-8*_Z^
2+1)^3*x^5-192512*RootOf(32*_Z^4-8*_Z^2+1)^5*x+87856*(x^6-x^2)^(1/2)*RootOf
(32*_Z^4-8*_Z^2+1)^3*x+120368*RootOf(32*_Z^4-8*_Z^2+1)^3*x^3+525*RootOf(32*
_Z^4-8*_Z^2+1)*x^5-73984*RootOf(32*_Z^4-8*_Z^2+1)^2*(x^6-x^2)^(3/4)-43928*R
ootOf(32*_Z^4-8*_Z^2+1)^2*(x^6-x^2)^(1/4)*x^2-25856*RootOf(32*_Z^4-8*_Z^2+1
)^3*x+15028*(x^6-x^2)^(1/2)*RootOf(32*_Z^4-8*_Z^2+1)*x+3450*RootOf(32*_Z^4-
8*_Z^2+1)*x^3+5491*(x^6-x^2)^(3/4)-7514*(x^6-x^2)^(1/4)*x^2-525*RootOf(32*_
Z^4-8*_Z^2+1)*x)/(8*RootOf(32*_Z^4-8*_Z^2+1)^2*x^2-32*RootOf(32*_Z^4-8*_Z^2
+1)^2-5*x^2+3)^2/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^6-x^2\right)^{\frac{1}{4}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)/(x^6-x^2)^(1/4),x, algorithm="maxima")
[Out] integrate(x^2/((x^6 - x^2)^(1/4)*(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(x^4+1\right)\,\left(x^6-x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^4 + 1)*(x^6 - x^2)^(1/4)),x)
[Out] int(x^2/((x^4 + 1)*(x^6 - x^2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[4]{x^2\,(x-1)\,(x+1)\left(x^2+1\right)}\,\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+1)/(x**6-x**2)**(1/4),x)
[Out] Integral(x**2/((x**2*(x - 1)*(x + 1)*(x**2 + 1))**(1/4)*(x**4 + 1)), x)
```

3.2366 $\int \frac{x^2}{(1+x^4)\sqrt[4]{-x^2+x^6}}\,dx$

Optimal. Leaf size=452

$$-\frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(-2x^2+2^{3/4}\sqrt{2+\sqrt{2}}\sqrt[4]{x^6-x^2}\,x-\sqrt{2}\sqrt{x^6-x^2}\right)+\frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(2\sqrt{2-\sqrt{2}}\,x^2+2\sqrt[4]{2}\right.$$

Rubi [C] time = 0.16, antiderivative size = 50, normalized size of antiderivative = 0.11, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2042, 466, 511, 510}

$$\frac{2x^3\sqrt[4]{1-x^4}\,F_1\left(\frac{5}{8};\frac{1}{4},1;\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^6-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2/((1 + x^4)*(-x^2 + x^6)^(1/4)),x]
```

```
[Out] (2*x^3*(1 - x^4)^(1/4)*AppellF1[5/8, 1/4, 1, 13/8, x^4, -x^4])/(5*(-x^2 + x
^6)^(1/4))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2042

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m]*
(a*x^j + b*x^(j + n))^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*
x^n)^FracPart[p]), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\int \frac{x^2}{(1+x^4)\sqrt[4]{-x^2+x^6}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{-1+x^4}\right)\int \frac{x^{3/2}}{\sqrt[4]{-1+x^4}(1+x^4)}\,dx}{\sqrt[4]{-x^2+x^6}}$$
$$= \frac{\left(2\sqrt{x}\sqrt[4]{-1+x^4}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{-1+x^8}(1+x^8)}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^6}}$$
$$= \frac{\left(2\sqrt{x}\sqrt[4]{1-x^4}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{1-x^8}(1+x^8)}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^6}}$$
$$= \frac{2x^3\sqrt[4]{1-x^4}\,F_1\left(\frac{5}{8};\frac{1}{4},1;\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{-x^2+x^6}}$$

Mathematica [C] time = 0.02, size = 50, normalized size = 0.11

$$\frac{2x^3\sqrt[4]{1-x^4}\,F_1\left(\frac{5}{8};\frac{1}{4},1;\frac{13}{8};x^4,-x^4\right)}{5\sqrt[4]{x^2(x^4-1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((1 + x^4)*(-x^2 + x^6)^(1/4)),x]
```

```
[Out] (2*x^3*(1 - x^4)^(1/4)*AppellF1[5/8, 1/4, 1, 13/8, x^4, -x^4])/(5*(x^2*(-1
+ x^4))^(1/4))
```

IntegrateAlgebraic [C] time = 0.00, size = 153, normalized size = 0.34

$$-\frac{1}{4}\sqrt{\frac{1}{2}-\frac{i}{2}}\tan^{-1}\left(\frac{\sqrt{-1-i}\,x}{\sqrt[4]{x^6-x^2}}\right)-\frac{1}{4}\sqrt{\frac{1}{2}+\frac{i}{2}}\tan^{-1}\left(\frac{\sqrt{-1+i}\,x}{\sqrt[4]{x^6-x^2}}\right)+\frac{1}{4}\sqrt{-\frac{1}{2}-\frac{i}{2}}\tan^{-1}\left(\frac{\sqrt{1-i}\,x}{\sqrt[4]{x^6-x^2}}\right)+\frac{1}{4}\sqrt{-\frac{1}{2}+\frac{i}{2}}\tan^{-1}\left(\frac{\sqrt{1+i}\,x}{\sqrt[4]{x^6-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((1 + x^4)*(-x^2 + x^6)^(1/4)),x]
```

```
[Out] -1/4*(Sqrt[1/2 - I/2]*ArcTan[(Sqrt[-1 - I]*x)/(-x^2 + x^6)^(1/4)]) - (Sqrt[
1/2 + I/2]*ArcTan[(Sqrt[-1 + I]*x)/(-x^2 + x^6)^(1/4)])/4 + (Sqrt[-1/2 - I/
2]*ArcTan[(Sqrt[1 - I]*x)/(-x^2 + x^6)^(1/4)])/4 + (Sqrt[-1/2 + I/2]*ArcTan
[(Sqrt[1 + I]*x)/(-x^2 + x^6)^(1/4)])/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)/(x^6-x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^6-x^2)^{\frac{1}{4}}(x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^4+1)/(x^6-x^2)^(1/4),x, algorithm="giac")

[Out] integrate(x^2/((x^6 - x^2)^(1/4)*(x^4 + 1)), x)

maple [C] time = 92.09, size = 2926, normalized size = 6.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^4+1)/(x^6-x^2)^(1/4),x)

```
[Out] -1/4*RootOf(32*_Z^4+8*_Z^2+1)*ln((230912*RootOf(32*_Z^4+8*_Z^2+1)^5*x^5-865
920*RootOf(32*_Z^4+8*_Z^2+1)^5*x^3+27672*RootOf(32*_Z^4+8*_Z^2+1)^3*x^5-230
912*RootOf(32*_Z^4+8*_Z^2+1)^5*x-208080*(x^6-x^2)^(1/2)*RootOf(32*_Z^4+8*_Z
^2+1)^3*x-364448*RootOf(32*_Z^4+8*_Z^2+1)^3*x^3-5789*RootOf(32*_Z^4+8*_Z^2+
1)*x^5+73984*RootOf(32*_Z^4+8*_Z^2+1)^2*(x^6-x^2)^(3/4)+43928*RootOf(32*_Z^
4+8*_Z^2+1)^2*(x^6-x^2)^(1/4)*x^2-27672*RootOf(32*_Z^4+8*_Z^2+1)^3*x-36992*
(x^6-x^2)^(1/2)*RootOf(32*_Z^4+8*_Z^2+1)*x-38042*RootOf(32*_Z^4+8*_Z^2+1)*x
^3+5491*(x^6-x^2)^(3/4)-7514*(x^6-x^2)^(1/4)*x^2+5789*RootOf(32*_Z^4+8*_Z^2
+1)*x)/(8*RootOf(32*_Z^4+8*_Z^2+1)^2*x^2-32*RootOf(32*_Z^4+8*_Z^2+1)^2+5*x^
2-3)^2/x)+1/8*RootOf(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*ln(-(230912*RootO
f(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*RootOf(32*_Z^4+8*_Z^2+1)^4*x^5-86592
0*RootOf(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*RootOf(32*_Z^4+8*_Z^2+1)^4*x^
3+87784*RootOf(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*RootOf(32*_Z^4+8*_Z^2+1
)^2*x^5-230912*RootOf(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*RootOf(32*_Z^4+8
*_Z^2+1)^4*x+208080*RootOf(32*_Z^4+8*_Z^2+1)^2*(x^6-x^2)^(1/2)*RootOf(_Z^2+
4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*x-68512*RootOf(_Z^2+4*RootOf(32*_Z^4+8*_Z^2
+1)^2+1)*RootOf(32*_Z^4+8*_Z^2+1)^2*x^3+1725*RootOf(_Z^2+4*RootOf(32*_Z^4+8
*_Z^2+1)^2+1)*x^5+147968*RootOf(32*_Z^4+8*_Z^2+1)^2*(x^6-x^2)^(3/4)+87856*R
ootOf(32*_Z^4+8*_Z^2+1)^2*(x^6-x^2)^(1/4)*x^2-87784*RootOf(_Z^2+4*RootOf(32
*_Z^4+8*_Z^2+1)^2+1)*RootOf(32*_Z^4+8*_Z^2+1)^2*x+15028*(x^6-x^2)^(1/2)*Roo
tOf(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*x-1050*RootOf(_Z^2+4*RootOf(32*_Z^
4+8*_Z^2+1)^2+1)*x^3+26010*(x^6-x^2)^(3/4)+36992*(x^6-x^2)^(1/4)*x^2-1725*R
ootOf(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*x)/(8*RootOf(32*_Z^4+8*_Z^2+1)^2
*x^2-32*RootOf(32*_Z^4+8*_Z^2+1)^2-3*x^2-5)^2/x)+ln((192512*RootOf(_Z^2+4*R
ootOf(32*_Z^4+8*_Z^2+1)^2+1)*RootOf(32*_Z^4+8*_Z^2+1)^4*x^5-721920*RootOf(_
Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*RootOf(32*_Z^4+8*_Z^2+1)^4*x^3+122112*R
ootOf(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*RootOf(32*_Z^4+8*_Z^2+1)^2*x^5-1
92512*RootOf(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*RootOf(32*_Z^4+8*_Z^2+1)^
4*x-87856*RootOf(32*_Z^4+8*_Z^2+1)^2*(x^6-x^2)^(1/2)*RootOf(_Z^2+4*RootOf(3
2*_Z^4+8*_Z^2+1)^2+1)*x-240592*RootOf(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*
RootOf(32*_Z^4+8*_Z^2+1)^2*x^3+19021*RootOf(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)
^2+1)*x^5-147968*RootOf(32*_Z^4+8*_Z^2+1)^2*(x^6-x^2)^(3/4)+87856*RootOf(32
*_Z^4+8*_Z^2+1)^2*(x^6-x^2)^(1/4)*x^2-122112*RootOf(_Z^2+4*RootOf(32*_Z^4+8
*_Z^2+1)^2+1)*RootOf(32*_Z^4+8*_Z^2+1)^2*x-36992*(x^6-x^2)^(1/2)*RootOf(_Z^
2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*x-11578*RootOf(_Z^2+4*RootOf(32*_Z^4+8*_Z
^2+1)^2+1)*x^3-26010*(x^6-x^2)^(3/4)+36992*(x^6-x^2)^(1/4)*x^2-19021*RootOf
(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*x)/(8*RootOf(32*_Z^4+8*_Z^2+1)^2*x^2-
32*RootOf(32*_Z^4+8*_Z^2+1)^2-3*x^2-5)^2/x)*RootOf(32*_Z^4+8*_Z^2+1)^2*Root
Of(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)+1/8*ln((192512*RootOf(_Z^2+4*RootOf
(32*_Z^4+8*_Z^2+1)^2+1)*RootOf(32*_Z^4+8*_Z^2+1)^4*x^5-721920*RootOf(_Z^2+4
*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*RootOf(32*_Z^4+8*_Z^2+1)^4*x^3+122112*RootOf
(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*RootOf(32*_Z^4+8*_Z^2+1)^2*x^5-192512
*RootOf(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*RootOf(32*_Z^4+8*_Z^2+1)^4*x-8
7856*RootOf(32*_Z^4+8*_Z^2+1)^2*(x^6-x^2)^(1/2)*RootOf(_Z^2+4*RootOf(32*_Z^
4+8*_Z^2+1)^2+1)*x-240592*RootOf(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*RootO
f(32*_Z^4+8*_Z^2+1)^2*x^3+19021*RootOf(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)
*x^5-147968*RootOf(32*_Z^4+8*_Z^2+1)^2*(x^6-x^2)^(3/4)+87856*RootOf(32*_Z^4
+8*_Z^2+1)^2*(x^6-x^2)^(1/4)*x^2-122112*RootOf(_Z^2+4*RootOf(32*_Z^4+8*_Z^2
+1)^2+1)*RootOf(32*_Z^4+8*_Z^2+1)^2*x-36992*(x^6-x^2)^(1/2)*RootOf(_Z^2+4*R
```

```
ootOf(32*_Z^4+8*_Z^2+1)^2+1)*x-11578*RootOf(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+1)
^2+1)*x^3-26010*(x^6-x^2)^(3/4)+36992*(x^6-x^2)^(1/4)*x^2-19021*RootOf(_Z^2
+4*RootOf(32*_Z^4+8*_Z^2+1)^2+1)*x)/(8*RootOf(32*_Z^4+8*_Z^2+1)^2*x^2-32*Ro
otOf(32*_Z^4+8*_Z^2+1)^2-3*x^2-5)^2/x)*RootOf(_Z^2+4*RootOf(32*_Z^4+8*_Z^2+
1)^2+1)-2*RootOf(32*_Z^4+8*_Z^2+1)^3*ln((192512*RootOf(32*_Z^4+8*_Z^2+1)^5*
x^5-721920*RootOf(32*_Z^4+8*_Z^2+1)^5*x^3-25856*RootOf(32*_Z^4+8*_Z^2+1)^3*
x^5-192512*RootOf(32*_Z^4+8*_Z^2+1)^5*x+87856*(x^6-x^2)^(1/2)*RootOf(32*_Z^
4+8*_Z^2+1)^3*x-120368*RootOf(32*_Z^4+8*_Z^2+1)^3*x^3+525*RootOf(32*_Z^4+8*
_Z^2+1)*x^5+73984*RootOf(32*_Z^4+8*_Z^2+1)^2*(x^6-x^2)^(3/4)-43928*RootOf(3
2*_Z^4+8*_Z^2+1)^2*(x^6-x^2)^(1/4)*x^2+25856*RootOf(32*_Z^4+8*_Z^2+1)^3*x-1
5028*(x^6-x^2)^(1/2)*RootOf(32*_Z^4+8*_Z^2+1)*x+3450*RootOf(32*_Z^4+8*_Z^2+
1)*x^3+5491*(x^6-x^2)^(3/4)+7514*(x^6-x^2)^(1/4)*x^2-525*RootOf(32*_Z^4+8*_
Z^2+1)*x)/(8*RootOf(32*_Z^4+8*_Z^2+1)^2*x^2-32*RootOf(32*_Z^4+8*_Z^2+1)^2+5
*x^2-3)^2/x)-1/4*RootOf(32*_Z^4+8*_Z^2+1)*ln((192512*RootOf(32*_Z^4+8*_Z^2+
1)^5*x^5-721920*RootOf(32*_Z^4+8*_Z^2+1)^5*x^3-25856*RootOf(32*_Z^4+8*_Z^2+
1)^3*x^5-192512*RootOf(32*_Z^4+8*_Z^2+1)^5*x+87856*(x^6-x^2)^(1/2)*RootOf(3
2*_Z^4+8*_Z^2+1)^3*x-120368*RootOf(32*_Z^4+8*_Z^2+1)^3*x^3+525*RootOf(32*_Z
^4+8*_Z^2+1)*x^5+73984*RootOf(32*_Z^4+8*_Z^2+1)^2*(x^6-x^2)^(3/4)-43928*Roo
tOf(32*_Z^4+8*_Z^2+1)^2*(x^6-x^2)^(1/4)*x^2+25856*RootOf(32*_Z^4+8*_Z^2+1)^
3*x-15028*(x^6-x^2)^(1/2)*RootOf(32*_Z^4+8*_Z^2+1)*x+3450*RootOf(32*_Z^4+8*
_Z^2+1)*x^3+5491*(x^6-x^2)^(3/4)+7514*(x^6-x^2)^(1/4)*x^2-525*RootOf(32*_Z^
4+8*_Z^2+1)*x)/(8*RootOf(32*_Z^4+8*_Z^2+1)^2*x^2-32*RootOf(32*_Z^4+8*_Z^2+1
)^2+5*x^2-3)^2/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^6-x^2\right)^{\frac{1}{4}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)/(x^6-x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^6 - x^2)^(1/4)*(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(x^4+1\right)\,\left(x^6-x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^4 + 1)*(x^6 - x^2)^(1/4)),x)
```

```
[Out] int(x^2/((x^4 + 1)*(x^6 - x^2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[4]{x^2\,(x-1)\,(x+1)\left(x^2+1\right)}\,\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+1)/(x**6-x**2)**(1/4),x)
```

```
[Out] Integral(x**2/((x**2*(x - 1)*(x + 1)*(x**2 + 1))**(1/4)*(x**4 + 1)), x)
```

3.2367 $\int \frac{-1+x^4}{\left(1+x^4\right)\sqrt[4]{-x^2+x^6}}\,dx$

Optimal. Leaf size=452

$$\frac{1}{8}\sqrt{1+\sqrt{2}}\,\log\left(-2x^2+2^{3/4}\sqrt{2+\sqrt{2}}\,\sqrt[4]{x^6-x^2}\,x-\sqrt{2}\sqrt{x^6-x^2}\right)-\frac{1}{8}\sqrt{1+\sqrt{2}}\,\log\left(2\sqrt{2-\sqrt{2}}\,x^2+2\sqrt[4]{2}\,\sqrt[4]{x^6\right.$$

Rubi [C] time = 0.11, antiderivative size = 46, normalized size of antiderivative = 0.10, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2056, 466, 430, 429}

$$-\frac{2x\sqrt[4]{1-x^4}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{x^6-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^4)/((1 + x^4)*(-x^2 + x^6)^(1/4)),x]
```

```
[Out] (-2*x*(1 - x^4)^(1/4)*AppellF1[1/8, -3/4, 1, 9/8, x^4, -x^4])/(-x^2 + x^6)^
(1/4)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{-1+x^4}{\left(1+x^4\right)\sqrt[4]{-x^2+x^6}}\,dx = \frac{\left(\sqrt{x}\sqrt[4]{-1+x^4}\right)\int \frac{\left(-1+x^4\right)^{3/4}}{\sqrt{x}\left(1+x^4\right)}\,dx}{\sqrt[4]{-x^2+x^6}}$$
$$= \frac{\left(2\sqrt{x}\sqrt[4]{-1+x^4}\right)\text{Subst}\left(\int \frac{\left(-1+x^8\right)^{3/4}}{1+x^8}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^6}}$$
$$= \frac{\left(2\sqrt{x}\left(-1+x^4\right)\right)\text{Subst}\left(\int \frac{\left(1-x^8\right)^{3/4}}{1+x^8}\,dx, x, \sqrt{x}\right)}{\left(1-x^4\right)^{3/4}\sqrt[4]{-x^2+x^6}}$$
$$= -\frac{2x\sqrt[4]{1-x^4}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};x^4,-x^4\right)}{\sqrt[4]{-x^2+x^6}}$$

Mathematica [C] time = 0.03, size = 48, normalized size = 0.11

$$\frac{2\left(x^2\left(x^4-1\right)\right)^{3/4}F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};x^4,-x^4\right)}{x\left(1-x^4\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-1 + x^4)/((1 + x^4)*(-x^2 + x^6)^(1/4)),x]
```

```
[Out] (2*(x^2*(-1 + x^4))^(3/4)*AppellF1[1/8, -3/4, 1, 9/8, x^4, -x^4])/(x*(1 - x
^4)^(3/4))
```

IntegrateAlgebraic [C] time = 0.78, size = 153, normalized size = 0.34

$$-\frac{1}{2}\sqrt{-\frac{1}{2}+\frac{i}{2}}\tan^{-1}\left(\frac{\sqrt{-1-i}\,x}{\sqrt[4]{x^6-x^2}}\right)-\frac{1}{2}\sqrt{-\frac{1}{2}-\frac{i}{2}}\tan^{-1}\left(\frac{\sqrt{-1+i}\,x}{\sqrt[4]{x^6-x^2}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}+\frac{i}{2}}\tan^{-1}\left(\frac{\sqrt{1-i}\,x}{\sqrt[4]{x^6-x^2}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}-\frac{i}{2}}\tan^{-1}\left(\frac{\sqrt{1+i}\,x}{\sqrt[4]{x^6-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^4)/((1 + x^4)*(-x^2 + x^6)^(1/4)),x]
```

```
[Out] -1/2*(Sqrt[-1/2 + I/2]*ArcTan[(Sqrt[-1 - I]*x)/(-x^2 + x^6)^(1/4)]) - (Sqrt
[-1/2 - I/2]*ArcTan[(Sqrt[-1 + I]*x)/(-x^2 + x^6)^(1/4)])/2 - (Sqrt[1/2 + I
/2]*ArcTan[(Sqrt[1 - I]*x)/(-x^2 + x^6)^(1/4)])/2 - (Sqrt[1/2 - I/2]*ArcTan
[(Sqrt[1 + I]*x)/(-x^2 + x^6)^(1/4)])/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)/(x^6-x^2)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\left(x^6-x^2\right)^{\frac{1}{4}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)/(x^6-x^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 1)/((x^6 - x^2)^(1/4)*(x^4 + 1)), x)
```

maple [C] time = 78.42, size = 2857, normalized size = 6.32

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/(x^4+1)/(x^6-x^2)^(1/4),x)
```

```
[Out] 1/4*RootOf(2*_Z^4+2*_Z^2+1)*ln((12032*RootOf(2*_Z^4+2*_Z^2+1)^5*x^5-45120*R
ootOf(2*_Z^4+2*_Z^2+1)^5*x^3-6464*RootOf(2*_Z^4+2*_Z^2+1)^3*x^5+21964*(x^6-
x^2)^(1/2)*RootOf(2*_Z^4+2*_Z^2+1)^3*x-12032*RootOf(2*_Z^4+2*_Z^2+1)^5*x-30
092*RootOf(2*_Z^4+2*_Z^2+1)^3*x^3+525*RootOf(2*_Z^4+2*_Z^2+1)*x^5+36992*(x^
6-x^2)^(3/4)*RootOf(2*_Z^4+2*_Z^2+1)^2-21964*(x^6-x^2)^(1/4)*RootOf(2*_Z^4+
2*_Z^2+1)^2*x^2-15028*(x^6-x^2)^(1/2)*RootOf(2*_Z^4+2*_Z^2+1)*x+6464*RootOf
(2*_Z^4+2*_Z^2+1)^3*x+3450*RootOf(2*_Z^4+2*_Z^2+1)*x^3+10982*(x^6-x^2)^(3/4
)+15028*(x^6-x^2)^(1/4)*x^2-525*RootOf(2*_Z^4+2*_Z^2+1)*x)/(2*RootOf(2*_Z^4
+2*_Z^2+1)^2*x^2-8*RootOf(2*_Z^4+2*_Z^2+1)^2+5*x^2-3)^2/x)-1/4*RootOf(RootO
f(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*ln(-(12032*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^2+_Z
^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^4*x^5-45120*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^2+
_Z^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^4*x^3+30528*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^
2+_Z^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^2*x^5-21964*(x^6-x^2)^(1/2)*RootOf(2*_Z^4
+2*_Z^2+1)^2*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*x-12032*RootOf(RootOf
(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^4*x-60148*RootOf(RootOf
(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^2*x^3+19021*RootOf(Root
Of(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*x^5+36992*(x^6-x^2)^(3/4)*RootOf(2*_Z^4+2*_Z^
2+1)^2-21964*(x^6-x^2)^(1/4)*RootOf(2*_Z^4+2*_Z^2+1)^2*x^2-36992*(x^6-x^2)^
(1/2)*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*x-30528*RootOf(RootOf(2*_Z^4
+2*_Z^2+1)^2+_Z^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^2*x-11578*RootOf(RootOf(2*_Z^4
+2*_Z^2+1)^2+_Z^2+1)*x^3+26010*(x^6-x^2)^(3/4)-36992*(x^6-x^2)^(1/4)*x^2-19
021*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*x)/(2*RootOf(2*_Z^4+2*_Z^2+1)^
2*x^2-8*RootOf(2*_Z^4+2*_Z^2+1)^2-3*x^2-5)^2/x)-1/2*RootOf(2*_Z^4+2*_Z^2+1)
^2*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*ln(-(14432*RootOf(RootOf(2*_Z^4
+2*_Z^2+1)^2+_Z^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^4*x^5-54120*RootOf(RootOf(2*_Z
^4+2*_Z^2+1)^2+_Z^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^4*x^3+21946*RootOf(RootOf(2*
_Z^4+2*_Z^2+1)^2+_Z^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^2*x^5+52020*(x^6-x^2)^(1/2
)*RootOf(2*_Z^4+2*_Z^2+1)^2*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*x-1443
2*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^4*x-1712
8*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^2*x^3+17
25*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*x^5+36992*(x^6-x^2)^(3/4)*RootO
f(2*_Z^4+2*_Z^2+1)^2+21964*(x^6-x^2)^(1/4)*RootOf(2*_Z^4+2*_Z^2+1)^2*x^2+15
028*(x^6-x^2)^(1/2)*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*x-21946*RootOf
(RootOf(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^2*x-1050*RootOf(
RootOf(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*x^3+26010*(x^6-x^2)^(3/4)+36992*(x^6-x^2)
^(1/4)*x^2-1725*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*x)/(2*RootOf(2*_Z^
4+2*_Z^2+1)^2*x^2-8*RootOf(2*_Z^4+2*_Z^2+1)^2-3*x^2-5)^2/x)-1/4*RootOf(Root
Of(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*ln(-(14432*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^2+_
Z^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^4*x^5-54120*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^2
+_Z^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^4*x^3+21946*RootOf(RootOf(2*_Z^4+2*_Z^2+1)
^2+_Z^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^2*x^5+52020*(x^6-x^2)^(1/2)*RootOf(2*_Z^
4+2*_Z^2+1)^2*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*x-14432*RootOf(RootO
f(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^4*x-17128*RootOf(RootO
f(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^2*x^3+1725*RootOf(Root
Of(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*x^5+36992*(x^6-x^2)^(3/4)*RootOf(2*_Z^4+2*_Z^
2+1)^2+21964*(x^6-x^2)^(1/4)*RootOf(2*_Z^4+2*_Z^2+1)^2*x^2+15028*(x^6-x^2)^
(1/2)*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*x-21946*RootOf(RootOf(2*_Z^4
```

```
+2*_Z^2+1)^2+_Z^2+1)*RootOf(2*_Z^4+2*_Z^2+1)^2*x-1050*RootOf(RootOf(2*_Z^4+
2*_Z^2+1)^2+_Z^2+1)*x^3+26010*(x^6-x^2)^(3/4)+36992*(x^6-x^2)^(1/4)*x^2-172
5*RootOf(RootOf(2*_Z^4+2*_Z^2+1)^2+_Z^2+1)*x)/(2*RootOf(2*_Z^4+2*_Z^2+1)^2*
x^2-8*RootOf(2*_Z^4+2*_Z^2+1)^2-3*x^2-5)^2/x)-1/2*RootOf(2*_Z^4+2*_Z^2+1)^3
*ln((14432*RootOf(2*_Z^4+2*_Z^2+1)^5*x^5-54120*RootOf(2*_Z^4+2*_Z^2+1)^5*x^
3+6918*RootOf(2*_Z^4+2*_Z^2+1)^3*x^5-52020*(x^6-x^2)^(1/2)*RootOf(2*_Z^4+2*
_Z^2+1)^3*x-14432*RootOf(2*_Z^4+2*_Z^2+1)^5*x-91112*RootOf(2*_Z^4+2*_Z^2+1)
^3*x^3-5789*RootOf(2*_Z^4+2*_Z^2+1)*x^5+36992*(x^6-x^2)^(3/4)*RootOf(2*_Z^4
+2*_Z^2+1)^2+21964*(x^6-x^2)^(1/4)*RootOf(2*_Z^4+2*_Z^2+1)^2*x^2-36992*(x^6
-x^2)^(1/2)*RootOf(2*_Z^4+2*_Z^2+1)*x-6918*RootOf(2*_Z^4+2*_Z^2+1)^3*x-3804
2*RootOf(2*_Z^4+2*_Z^2+1)*x^3+10982*(x^6-x^2)^(3/4)-15028*(x^6-x^2)^(1/4)*x
^2+5789*RootOf(2*_Z^4+2*_Z^2+1)*x)/(2*RootOf(2*_Z^4+2*_Z^2+1)^2*x^2-8*RootO
f(2*_Z^4+2*_Z^2+1)^2+5*x^2-3)^2/x)-1/4*RootOf(2*_Z^4+2*_Z^2+1)*ln((14432*Ro
otOf(2*_Z^4+2*_Z^2+1)^5*x^5-54120*RootOf(2*_Z^4+2*_Z^2+1)^5*x^3+6918*RootOf
(2*_Z^4+2*_Z^2+1)^3*x^5-52020*(x^6-x^2)^(1/2)*RootOf(2*_Z^4+2*_Z^2+1)^3*x-1
4432*RootOf(2*_Z^4+2*_Z^2+1)^5*x-91112*RootOf(2*_Z^4+2*_Z^2+1)^3*x^3-5789*R
ootOf(2*_Z^4+2*_Z^2+1)*x^5+36992*(x^6-x^2)^(3/4)*RootOf(2*_Z^4+2*_Z^2+1)^2+
21964*(x^6-x^2)^(1/4)*RootOf(2*_Z^4+2*_Z^2+1)^2*x^2-36992*(x^6-x^2)^(1/2)*R
ootOf(2*_Z^4+2*_Z^2+1)*x-6918*RootOf(2*_Z^4+2*_Z^2+1)^3*x-38042*RootOf(2*_Z
^4+2*_Z^2+1)*x^3+10982*(x^6-x^2)^(3/4)-15028*(x^6-x^2)^(1/4)*x^2+5789*RootO
f(2*_Z^4+2*_Z^2+1)*x)/(2*RootOf(2*_Z^4+2*_Z^2+1)^2*x^2-8*RootOf(2*_Z^4+2*_Z
^2+1)^2+5*x^2-3)^2/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4-1}{\left(x^6-x^2\right)^{\frac{1}{4}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/(x^4+1)/(x^6-x^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 1)/((x^6 - x^2)^(1/4)*(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4-1}{\left(x^4+1\right)\,\left(x^6-x^2\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 1)/((x^4 + 1)*(x^6 - x^2)^(1/4)),x)
```

```
[Out] int((x^4 - 1)/((x^4 + 1)*(x^6 - x^2)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)\left(x^2+1\right)}{\sqrt[4]{x^2(x-1)(x+1)\left(x^2+1\right)}\,\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/(x**4+1)/(x**6-x**2)**(1/4),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)/((x**2*(x - 1)*(x + 1)*(x**2 + 1))**(1/
4)*(x**4 + 1)), x)
```

3.2368 $\int \frac{\left(-b+ax^4\right)^{3/4}}{-b+2ax^4+x^8}\,dx$

Optimal. Leaf size=452

$$\frac{\left(-1-\sqrt[4]{-1}\right)\tan^{-1}\left(\frac{(-1)^{7/8}\sqrt{2+\sqrt{2}}\,x\sqrt[8]{a^2+b}\,\sqrt[4]{ax^4-b}}{(-1)^{3/4}x^2\sqrt[4]{a^2+b}+\sqrt{ax^4-b}}\right)}{8\sqrt[8]{a^2+b}}+\frac{\left(\sqrt{2}+i\sqrt{2\left(3-2\sqrt{2}\right)}\right)\tan^{-1}\left(\frac{(-1)^{7/8}\left(\sqrt{2}-2\right)x\sqrt[8]{a^2+b}\,\sqrt[4]{ax^4-b}}{(-1)^{3/4}\sqrt{2-\sqrt{2}}\,x^2\sqrt[4]{a^2+b}+\sqrt{2-\sqrt{2}}\,\sqrt{\cdots}}\right.}{16\sqrt[8]{a^2+b}}$$

Rubi [A] time = 0.52, antiderivative size = 409, normalized size of antiderivative = 0.90, number of steps used = 19, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1428, 408, 240, 212, 206, 203, 377, 208, 205}

$$-\frac{\left(-a\sqrt{a^2+b}+a^2+b\right)^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2+b}+a^2+b}}{\sqrt[4]{a-\sqrt{a^2+b}}\,\sqrt[4]{ax^4-b}}\right)}{4\sqrt{a^2+b}\left(a-\sqrt{a^2+b}\right)^{3/4}}+\frac{\left(a\sqrt{a^2+b}+a^2+b\right)^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2+b}+a^2+b}}{\sqrt[4]{\sqrt{a^2+b}+a}\,\sqrt[4]{ax^4-b}}\right)}{4\sqrt{a^2+b}\left(\sqrt{a^2+b}+a\right)^{3/4}}-\frac{\left(-a\sqrt{a^2+b}+a^2+b\right)^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2+b}+a^2+b}}{\sqrt[4]{a-\sqrt{a^2+b}}\,\sqrt[4]{ax^4-b}}\right)}{4\sqrt{a^2+b}\left(a-\sqrt{a^2+b}\right)^{3/4}}+\frac{\left(a\sqrt{a^2+b}+a^2+b\right)^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2+b}+a^2+b}}{\sqrt[4]{\sqrt{a^2+b}+a}\,\sqrt[4]{ax^4-b}}\right)}{4\sqrt{a^2+b}\left(\sqrt{a^2+b}+a\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^4)^(3/4)/(-b + 2*a*x^4 + x^8),x]
```

```
[Out] -1/4*((a^2 + b - a*Sqrt[a^2 + b])^(3/4)*ArcTan[((a^2 + b - a*Sqrt[a^2 + b])
^(1/4)*x)/((a - Sqrt[a^2 + b])^(1/4)*(-b + a*x^4)^(1/4))])/(Sqrt[a^2 + b]*(
a - Sqrt[a^2 + b])^(3/4)) + ((a^2 + b + a*Sqrt[a^2 + b])^(3/4)*ArcTan[((a^2
 + b + a*Sqrt[a^2 + b])^(1/4)*x)/((a + Sqrt[a^2 + b])^(1/4)*(-b + a*x^4)^(1
/4))])/(4*Sqrt[a^2 + b]*(a + Sqrt[a^2 + b])^(3/4)) - ((a^2 + b - a*Sqrt[a^2
 + b])^(3/4)*ArcTanh[((a^2 + b - a*Sqrt[a^2 + b])^(1/4)*x)/((a - Sqrt[a^2 +
 b])^(1/4)*(-b + a*x^4)^(1/4))])/(4*Sqrt[a^2 + b]*(a - Sqrt[a^2 + b])^(3/4)
) + ((a^2 + b + a*Sqrt[a^2 + b])^(3/4)*ArcTanh[((a^2 + b + a*Sqrt[a^2 + b])
^(1/4)*x)/((a + Sqrt[a^2 + b])^(1/4)*(-b + a*x^4)^(1/4))])/(4*Sqrt[a^2 + b]
*(a + Sqrt[a^2 + b])^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 408

```
Int[((a_) + (b_.)*(x_)^4)^(p_)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d
, Int[(a + b*x^4)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(p
- 1)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[p, 3/4] || EqQ[p, 5/4])
```

Rule 1428

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^
n)^q/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[(d + e*x^n)^q/(b + r + 2
*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[q]
```

Rubi steps

$$\int \frac{\left(-b+ax^4\right)^{3/4}}{-b+2ax^4+x^8}\,dx = \frac{\int \frac{\left(-b+ax^4\right)^{3/4}}{2a-2\sqrt{a^2+b}+2x^4}\,dx}{\sqrt{a^2+b}} - \frac{\int \frac{\left(-b+ax^4\right)^{3/4}}{2a+2\sqrt{a^2+b}+2x^4}\,dx}{\sqrt{a^2+b}}$$

$$= -\frac{\left(a^2+b-a\sqrt{a^2+b}\right)\int \frac{1}{\left(2a-2\sqrt{a^2+b}+2x^4\right)\sqrt[4]{-b+ax^4}}\,dx}{\sqrt{a^2+b}} + \frac{\left(b+a\left(a+\sqrt{a^2+b}\right)\right)\int \frac{}{\left(2a+2\sqrt{a^2+\right.}}}{\sqrt{a^2+b}}$$

$$= -\frac{\left(a^2+b-a\sqrt{a^2+b}\right)\text{Subst}\left(\int \frac{1}{2a-2\sqrt{a^2+b}-\left(2b+a\left(2a-2\sqrt{a^2+b}\right)\right)x^4}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{\sqrt{a^2+b}} + \frac{\left(b+a\left(\right.\right.}{}$$

$$= -\frac{\left(a^2+b-a\sqrt{a^2+b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\sqrt{a^2+b}}-\sqrt{a^2+b-a\sqrt{a^2+b}}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{4\sqrt{a^2+b}\sqrt{a-\sqrt{a^2+b}}} - \frac{\left(a^2+b-a\right.}{}$$

$$= -\frac{\left(a^2+b-a\sqrt{a^2+b}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a^2+b-a\sqrt{a^2+b}}\,x}{\sqrt[4]{a-\sqrt{a^2+b}}\sqrt[4]{-b+ax^4}}\right)}{4\sqrt{a^2+b}\left(a-\sqrt{a^2+b}\right)^{3/4}} + \frac{\left(a^2+b+a\sqrt{a^2+b}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{}}{\sqrt[4]{a}}\right.}{4\sqrt{a^2+b}\left(a+\sqrt{a^2+b}\right.}$$

Mathematica [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(-b+ax^4\right)^{3/4}}{-b+2ax^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-b + a*x^4)^(3/4)/(-b + 2*a*x^4 + x^8),x]
```

```
[Out] Integrate[(-b + a*x^4)^(3/4)/(-b + 2*a*x^4 + x^8), x]
```

IntegrateAlgebraic [A] time = 36.69, size = 510, normalized size = 1.13

$$\frac{\left(\sqrt{2}-i\sqrt{2\left(3+2\sqrt{2}\right)}\right)\tan^{-1}\left(\frac{\left((-1+i)-(1+i)(-1)^{3/4}\right)x^2\sqrt[4]{a^2+b}+(1+i)(-1)^{3/4}\sqrt{ax^4-b}+(1+i)\sqrt{ax^4-b}}{2x\sqrt[8]{a^2+b}\sqrt[4]{ax^4-b}}\right)}{16\sqrt[8]{a^2+b}}-\frac{i\left(\sqrt{2\left(3-2\sqrt{2}\right)}-i\sqrt{2}\right)\tan^{-1}\left(\frac{2x\sqrt[8]{a^2+b}\sqrt[4]{ax^4-b}}{\left(\sqrt{2}+(1-i)\right)x^2\sqrt[4]{a^2+b}-\sqrt{2}\sqrt{ax^4-b}-(1+i)\sqrt{ax^4-b}}\right)}{16\sqrt[8]{a^2+b}}+\frac{\left(-(-1)^{3/4}-i\right)\tanh^{-1}\left(\frac{\left((-2+2i)-(2+2i)(-1)^{3/4}\right)x^2\sqrt[4]{a^2+b}-(2+2i)(-1)^{3/4}\sqrt{ax^4-b}-(2+2i)\sqrt{ax^4-b}}{4x\sqrt[8]{a^2+b}\sqrt[4]{ax^4-b}}\right)}{8\sqrt[8]{a^2+b}}+\frac{\left(\sqrt{2}+i\sqrt{2\left(3-2\sqrt{2}\right)}\right)\tanh^{-1}\left(\frac{\left(\sqrt{2}+(1-i)\right)x^2\sqrt[4]{a^2+b}+\sqrt{2}\sqrt{ax^4-b}+(1+i)\sqrt{ax^4-b}}{2x\sqrt[8]{a^2+b}\sqrt[4]{ax^4-b}}\right)}{16\sqrt[8]{a^2+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^4)^(3/4)/(-b + 2*a*x^4 + x^8),x]
```

```
[Out] ((Sqrt[2] - I*Sqrt[2*(3 + 2*Sqrt[2])])*ArcTan[(((-1 + I) - (1 + I)*(-1)^(3/
4))*(a^2 + b)^(1/4)*x^2 + (1 + I)*Sqrt[-b + a*x^4] + (1 + I)*(-1)^(3/4)*Sqr
t[-b + a*x^4])/(2*(a^2 + b)^(1/8)*x*(-b + a*x^4)^(1/4))])/(16*(a^2 + b)^(1/
8)) - ((I/16)*((-I)*Sqrt[2] + Sqrt[2*(3 - 2*Sqrt[2])])*ArcTan[(2*(a^2 + b)^
(1/8)*x*(-b + a*x^4)^(1/4))/(((1 - I) + Sqrt[2])*(a^2 + b)^(1/4)*x^2 - (1 +
 I)*Sqrt[-b + a*x^4] - Sqrt[2]*Sqrt[-b + a*x^4])])/(a^2 + b)^(1/8) + ((-I -
 (-1)^(3/4))*ArcTanh[(((-2 + 2*I) - (2 + 2*I)*(-1)^(3/4))*(a^2 + b)^(1/4)*x
^2 - (2 + 2*I)*Sqrt[-b + a*x^4] - (2 + 2*I)*(-1)^(3/4)*Sqrt[-b + a*x^4])/(4
*(a^2 + b)^(1/8)*x*(-b + a*x^4)^(1/4))])/(8*(a^2 + b)^(1/8)) + ((Sqrt[2] +
I*Sqrt[2*(3 - 2*Sqrt[2])])*ArcTanh[(((1 - I) + Sqrt[2])*(a^2 + b)^(1/4)*x^2
 + (1 + I)*Sqrt[-b + a*x^4] + Sqrt[2]*Sqrt[-b + a*x^4])/(2*(a^2 + b)^(1/8)*
x*(-b + a*x^4)^(1/4))])/(16*(a^2 + b)^(1/8))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4)/(x^8+2*a*x^4-b),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4-b\right)^{\frac{3}{4}}}{x^8+2\,ax^4-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4)/(x^8+2*a*x^4-b),x, algorithm="giac")
```

```
[Out] integrate((a*x^4 - b)^(3/4)/(x^8 + 2*a*x^4 - b), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^4-b\right)^{\frac{3}{4}}}{x^8+2a\,x^4-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b)^(3/4)/(x^8+2*a*x^4-b),x)
```

```
[Out] int((a*x^4-b)^(3/4)/(x^8+2*a*x^4-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4-b\right)^{\frac{3}{4}}}{x^8+2\,ax^4-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4)/(x^8+2*a*x^4-b),x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 - b)^(3/4)/(x^8 + 2*a*x^4 - b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x^4-b\right)^{3/4}}{x^8+2\,a\,x^4-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4 - b)^(3/4)/(2*a*x^4 - b + x^8),x)
```

```
[Out] int((a*x^4 - b)^(3/4)/(2*a*x^4 - b + x^8), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b)**(3/4)/(x**8+2*a*x**4-b),x)
```

```
[Out] Timed out
```

3.2369

$$\int \sqrt{b+a^2x^2}\sqrt{ax+\sqrt{b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Optimal. Leaf size=455

$$\frac{\sqrt{\sqrt{a^2x^2+b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}\left(2240a^3c^3x^3+1536a^2c^5x^2+38640abc^3x+2048ac^7x-2310b^2c+\right.}{}$$

Rubi [F] time = 0.89, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{b+a^2x^2}\sqrt{ax+\sqrt{b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b + a^2*x^2]*Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqr
t[b + a^2*x^2]]],x]
```

```
[Out] Defer[Int][Sqrt[b + a^2*x^2]*Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*
x + Sqrt[b + a^2*x^2]]], x]
```

Rubi steps

$$\int \sqrt{b+a^2x^2}\sqrt{ax+\sqrt{b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx=\int \sqrt{b+a^2x^2}\sqrt{ax+\sqrt{b+a^2x^2}}\sqrt{c+\sqrt{ax+\cdots}}$$

Mathematica [F] time = 15.65, size = 0, normalized size = 0.00

$$\int \sqrt{b+a^2x^2}\sqrt{ax+\sqrt{b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[b + a^2*x^2]*Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x
 + Sqrt[b + a^2*x^2]]],x]
```

```
[Out] Integrate[Sqrt[b + a^2*x^2]*Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x
 + Sqrt[b + a^2*x^2]]], x]
```

IntegrateAlgebraic [A] time = 1.19, size = 455, normalized size = 1.00

$$\frac{\sqrt{\sqrt{a^2x^2+b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}\left(2240a^3c^3x^3+1536a^2c^5x^2+38640abc^3x+2048ac^7x-2310b^2c+768bc^5\right)+\sqrt{a^2x^2+b}\left(\sqrt{\sqrt{a^2x^2+b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}\left(2240a^2c^3x^2+1536ac^5x+37520bc^3+2048c^7\right)+\left(40320a^3c^2x^3-2560a^2c^4x^2+94080abc^2x-2048ac^6x+3465b^2-640bc^4\right)\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}\right)+\left(40320a^4c^2x^4-2560a^3c^4x^3+114240a^2bc^2x^2-2048a^2c^6x^2+3465ab^2x-1920abc^4x+32760b^2c^2-1024bc^6\right)\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{55440ac^2\left(\sqrt{a^2x^2+b}+ax\right)^{3/2}}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{\sqrt{c}}\right)}{16ac^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + a^2*x^2]*Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c +
 Sqrt[a*x + Sqrt[b + a^2*x^2]]],x]
```

```
[Out] ((32760*b^2*c^2 - 1024*b*c^6 + 3465*a*b^2*x - 1920*a*b*c^4*x + 114240*a^2*b
*c^2*x^2 - 2048*a^2*c^6*x^2 - 2560*a^3*c^4*x^3 + 40320*a^4*c^2*x^4)*Sqrt[c
```

```
+ Sqrt[a*x + Sqrt[b + a^2*x^2]]] + (-2310*b^2*c + 768*b*c^5 + 38640*a*b*c^3
*x + 2048*a*c^7*x + 1536*a^2*c^5*x^2 + 2240*a^3*c^3*x^3)*Sqrt[a*x + Sqrt[b
+ a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]] + Sqrt[b + a^2*x^2]*((3
465*b^2 - 640*b*c^4 + 94080*a*b*c^2*x - 2048*a*c^6*x - 2560*a^2*c^4*x^2 + 4
0320*a^3*c^2*x^3)*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]] + (37520*b*c^3 +
2048*c^7 + 1536*a*c^5*x + 2240*a^2*c^3*x^2)*Sqrt[a*x + Sqrt[b + a^2*x^2]]*S
qrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]]))/(55440*a*c^2*(a*x + Sqrt[b + a^2*x
^2])^(3/2)) - (b^2*ArcTanh[Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]]/Sqrt[c]]
)/(16*a*c^(5/2))
```

fricas [A] time = 0.50, size = 535, normalized size = 1.18

$$\left[\frac{3465\, b^{2} \sqrt{c} \log\left(2 \, {\left(a \sqrt{c} x - \sqrt{a^{2} x^{2} + b} \sqrt{c}\right)} \sqrt{a x + \sqrt{a^{2} x^{2} + b}} \sqrt{c + \sqrt{a x + \sqrt{a^{2} x^{2} + b}}} - 2 \, {\left(a c x - \sqrt{a^{2} x^{2} + b} c\right)} \sqrt{a x + \sqrt{a^{2} x^{2} + b}} + b\right) + 2 \, {\left(2048 \, c^{8} + 1120 \, a^{2} c^{4} x^{2} + 37520 \, b c^{4} + 6 \, {\left(128 \, a c^{6} + 385 \, a b c^{2}\right)} x + 2 \, {\left(384 \, c^{6} + 560 \, a c^{4} x - 1155 \, b c^{2}\right)} \sqrt{a^{2} x^{2} + b} - {\left(1024 \, c^{7} + 8400 \, a^{2} c^{3} x^{2} - 32760 \, b c^{3} + 5 \, {\left(128 \, a c^{5} + 693 \, a b c\right)} x + 5 \, {\left(128 \, c^{5} - 5712 \, a c^{3} x - 693 \, b c\right)} \sqrt{a^{2} x^{2} + b}\right)} \sqrt{a x + \sqrt{a^{2} x^{2} + b}}\right)} \sqrt{c + \sqrt{a x + \sqrt{a^{2} x^{2} + b}}}}{110880 \, a c^{3}}, \frac{3465 \, b^{2} \sqrt{-c} \arctan\left(\frac{\sqrt{-c} \sqrt{c + \sqrt{a x + \sqrt{a^{2} x^{2} + b}}}}{c}\right) + {\left(2048 \, c^{8} + 1120 \, a^{2} c^{4} x^{2} + 37520 \, b c^{4} + 6 \, {\left(128 \, a c^{6} + 385 \, a b c^{2}\right)} x + 2 \, {\left(384 \, c^{6} + 560 \, a c^{4} x - 1155 \, b c^{2}\right)} \sqrt{a^{2} x^{2} + b} - {\left(1024 \, c^{7} + 8400 \, a^{2} c^{3} x^{2} - 32760 \, b c^{3} + 5 \, {\left(128 \, a c^{5} + 693 \, a b c\right)} x + 5 \, {\left(128 \, c^{5} - 5712 \, a c^{3} x - 693 \, b c\right)} \sqrt{a^{2} x^{2} + b}\right)} \sqrt{a x + \sqrt{a^{2} x^{2} + b}}\right)} \sqrt{c + \sqrt{a x + \sqrt{a^{2} x^{2} + b}}}}{55440 \, a c^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b)^(1/2)*(a*x+(a^2*x^2+b)^(1/2))^(1/2)*(c+(a*x+(a^2*x^2+
b)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/110880*(3465*b^2*sqrt(c)*log(2*(a*sqrt(c)*x - sqrt(a^2*x^2 + b)*sqrt(c))
*sqrt(a*x + sqrt(a^2*x^2 + b))*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b))) - 2*
(a*c*x - sqrt(a^2*x^2 + b)*c)*sqrt(a*x + sqrt(a^2*x^2 + b)) + b) + 2*(2048*
c^8 + 1120*a^2*c^4*x^2 + 37520*b*c^4 + 6*(128*a*c^6 + 385*a*b*c^2)*x + 2*(3
84*c^6 + 560*a*c^4*x - 1155*b*c^2)*sqrt(a^2*x^2 + b) - (1024*c^7 + 8400*a^2
*c^3*x^2 - 32760*b*c^3 + 5*(128*a*c^5 + 693*a*b*c)*x + 5*(128*c^5 - 5712*a*
c^3*x - 693*b*c)*sqrt(a^2*x^2 + b))*sqrt(a*x + sqrt(a^2*x^2 + b)))*sqrt(c +
 sqrt(a*x + sqrt(a^2*x^2 + b))))/(a*c^3), 1/55440*(3465*b^2*sqrt(-c)*arctan
(sqrt(-c)*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b)))/c) + (2048*c^8 + 1120*a^2
*c^4*x^2 + 37520*b*c^4 + 6*(128*a*c^6 + 385*a*b*c^2)*x + 2*(384*c^6 + 560*a
*c^4*x - 1155*b*c^2)*sqrt(a^2*x^2 + b) - (1024*c^7 + 8400*a^2*c^3*x^2 - 327
60*b*c^3 + 5*(128*a*c^5 + 693*a*b*c)*x + 5*(128*c^5 - 5712*a*c^3*x - 693*b*
c)*sqrt(a^2*x^2 + b))*sqrt(a*x + sqrt(a^2*x^2 + b)))*sqrt(c + sqrt(a*x + sq
rt(a^2*x^2 + b))))/(a*c^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b)^(1/2)*(a*x+(a^2*x^2+b)^(1/2))^(1/2)*(c+(a*x+(a^2*x^2+
b)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^{2} x^{2}+b}\, \sqrt{a x +\sqrt{a^{2} x^{2}+b}}\, \sqrt{c +\sqrt{a x +\sqrt{a^{2} x^{2}+b}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2+b)^(1/2)*(a*x+(a^2*x^2+b)^(1/2))^(1/2)*(c+(a*x+(a^2*x^2+b)^(1/
2))^(1/2))^(1/2),x)
```

```
[Out] int((a^2*x^2+b)^(1/2)*(a*x+(a^2*x^2+b)^(1/2))^(1/2)*(c+(a*x+(a^2*x^2+b)^(1/
2))^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^{2} x^{2} + b} \sqrt{a x + \sqrt{a^{2} x^{2} + b}} \sqrt{c + \sqrt{a x + \sqrt{a^{2} x^{2} + b}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b)^(1/2)*(a*x+(a^2*x^2+b)^(1/2))^(1/2)*(c+(a*x+(a^2*x^2+
b)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 + b)*sqrt(a*x + sqrt(a^2*x^2 + b))*sqrt(c + sqrt(a*x
 + sqrt(a^2*x^2 + b))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\sqrt{a^2\,x^2+b}+a\,x}\,\sqrt{a^2\,x^2+b}\,\sqrt{c+\sqrt{\sqrt{a^2\,x^2+b}+a\,x}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a^2*x^2)^(1/2) + a*x)^(1/2)*(b + a^2*x^2)^(1/2)*(c + ((b + a^2*x^
2)^(1/2) + a*x)^(1/2))^(1/2),x)
```

```
[Out] int(((b + a^2*x^2)^(1/2) + a*x)^(1/2)*(b + a^2*x^2)^(1/2)*(c + ((b + a^2*x^
2)^(1/2) + a*x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}\,\sqrt{ax+\sqrt{a^2x^2+b}}\,\sqrt{a^2x^2+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2+b)**(1/2)*(a*x+(a**2*x**2+b)**(1/2))**(1/2)*(c+(a*x+(a
**2*x**2+b)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(c + sqrt(a*x + sqrt(a**2*x**2 + b)))*sqrt(a*x + sqrt(a**2*x**
2 + b))*sqrt(a**2*x**2 + b), x)
```

3.2370 $\int \frac{x^4}{\sqrt[4]{b+ax^4}\left(b+2ax^4+2x^8\right)}\,dx$

Optimal. Leaf size=456

$$\frac{\left(\sqrt[4]{-1}-1\right)\tan^{-1}\left(\frac{(-1)^{7/8}\sqrt{2+\sqrt{2}}\,x\sqrt[8]{a^2-2b}\,\sqrt[4]{ax^4+b}}{(-1)^{3/4}x^2\sqrt[4]{a^2-2b}+\sqrt{ax^4+b}}\right)}{8\left(a^2-2b\right)^{5/8}}+\frac{i\left(\sqrt{2\left(3+2\sqrt{2}\right)}+i\sqrt{2}\right)\tan^{-1}\left(\frac{(-1)^{7/8}\left(\sqrt{2}-2\right)x\sqrt[8]{a^2-2b}\,\sqrt[4]{ax^4+b}}{(-1)^{3/4}\sqrt{2-\sqrt{2}}\,x^2\sqrt[4]{a^2-2b}+\sqrt{2-\sqrt{2}}\,\sqrt{ax}\right.}}{16\left(a^2-2b\right)^{5/8}}$$

Rubi [A] time = 0.77, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1528, 377, 212, 208, 205}

$$-\frac{\sqrt[4]{a-\sqrt{a^2-2b}}\,\tan^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{a-\sqrt{a^2-2b}}\,\sqrt[4]{ax^4+b}}\right)}{4\sqrt{a^2-2b}\,\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}+\frac{\sqrt[4]{\sqrt{a^2-2b}+a}\,\tan^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{\sqrt{a^2-2b}+a}\,\sqrt[4]{ax^4+b}}\right)}{4\sqrt{a^2-2b}\,\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}-\frac{\sqrt[4]{a-\sqrt{a^2-2b}}\,\tanh^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{a-\sqrt{a^2-2b}}\,\sqrt[4]{ax^4+b}}\right)}{4\sqrt{a^2-2b}\,\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}+\frac{\sqrt[4]{\sqrt{a^2-2b}+a}\,\tanh^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{\sqrt{a^2-2b}+a}\,\sqrt[4]{ax^4+b}}\right)}{4\sqrt{a^2-2b}\,\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((b + a*x^4)^(1/4)*(b + 2*a*x^4 + 2*x^8)),x]
```

```
[Out] -1/4*((a - Sqrt[a^2 - 2*b])^(1/4)*ArcTan[((a^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(
1/4)*x)/((a - Sqrt[a^2 - 2*b])^(1/4)*(b + a*x^4)^(1/4))])/(Sqrt[a^2 - 2*b]*
(a^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)) + ((a + Sqrt[a^2 - 2*b])^(1/4)*ArcTa
n[((a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)*x)/((a + Sqrt[a^2 - 2*b])^(1/4)*(b
 + a*x^4)^(1/4))])/(4*Sqrt[a^2 - 2*b]*(a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)
) - ((a - Sqrt[a^2 - 2*b])^(1/4)*ArcTanh[((a^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(
1/4)*x)/((a - Sqrt[a^2 - 2*b])^(1/4)*(b + a*x^4)^(1/4))])/(4*Sqrt[a^2 - 2*b
]*(a^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)) + ((a + Sqrt[a^2 - 2*b])^(1/4)*Arc
Tanh[((a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)*x)/((a + Sqrt[a^2 - 2*b])^(1/4)
*(b + a*x^4)^(1/4))])/(4*Sqrt[a^2 - 2*b]*(a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(1
/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1528

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q, (
```

```
f*x)^m/(a + b*x^n + c*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, e, f, q, n}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && !IntegerQ[q] &&
IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{b+ax^4}\left(b+2ax^4+2x^8\right)}\,dx = \int\left(\frac{1-\frac{a}{\sqrt{a^2-2b}}}{\left(2a-2\sqrt{a^2-2b}+4x^4\right)\sqrt[4]{b+ax^4}}+\frac{1+\frac{a}{\sqrt{a^2-2b}}}{\left(2a+2\sqrt{a^2-2b}+4x^4\right)\sqrt[4]{b+a\cdots}}\cdots\right.$$

$$=\left(1-\frac{a}{\sqrt{a^2-2b}}\right)\int\frac{1}{\left(2a-2\sqrt{a^2-2b}+4x^4\right)\sqrt[4]{b+ax^4}}\,dx+\left(1+\frac{a}{\sqrt{a^2-2\cdots}}\right.\cdots$$

$$=\left(1-\frac{a}{\sqrt{a^2-2b}}\right)\mathrm{Subst}\left(\int\frac{1}{2a-2\sqrt{a^2-2b}-\left(a\left(2a-2\sqrt{a^2-2b}\right)-4b\right)x\cdots}\right.\cdots$$

$$=-\frac{\sqrt{a-\sqrt{a^2-2b}}\ \mathrm{Subst}\left(\int\frac{1}{\sqrt{a-\sqrt{a^2-2b}}-\sqrt{a^2-a\sqrt{a^2-2b}-2b}\,x^2}dx,x,\frac{x}{\sqrt[4]{b+ax^4}}\right)}{4\sqrt{a^2-2b}}-\cdots$$

$$=-\frac{\sqrt[4]{a-\sqrt{a^2-2b}}\ \tan^{-1}\left(\frac{\sqrt[4]{a^2-a\sqrt{a^2-2b}-2b}\,x}{\sqrt[4]{a-\sqrt{a^2-2b}}\ \sqrt[4]{b+ax^4}}\right)}{4\sqrt{a^2-2b}\ \sqrt[4]{a^2-a\sqrt{a^2-2b}-2b}}+\frac{\sqrt[4]{a+\sqrt{a^2-2b}}\ \tan^{-1}\left(\cdots\right)}{4\sqrt{a^2-2b}\ \sqrt[4]{a^2+a\sqrt{a\cdots}}}\cdots$$

Mathematica [A] time = 0.86, size = 418, normalized size = 0.92

$$\frac{-\frac{\sqrt[4]{a-\sqrt{a^2-2b}}\ \tan^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{a-\sqrt{a^2-2b}}\ \sqrt[4]{ax^4+b}}\right)}{\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}+\frac{\sqrt[4]{\sqrt{a^2-2b}+a}\ \tan^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{\sqrt{a^2-2b}+a}\ \sqrt[4]{ax^4+b}}\right)}{\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}-\frac{\sqrt[4]{a-\sqrt{a^2-2b}}\ \tanh^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{a-\sqrt{a^2-2b}}\ \sqrt[4]{ax^4+b}}\right)}{\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}+\frac{\sqrt[4]{\sqrt{a^2-2b}+a}\ \tanh^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{\sqrt{a^2-2b}+a}\ \sqrt[4]{ax^4+b}}\right)}{\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}}{4\sqrt{a^2-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((b + a*x^4)^(1/4)*(b + 2*a*x^4 + 2*x^8)),x]
```

```
[Out] (-(((a - Sqrt[a^2 - 2*b])^(1/4)*ArcTan[((a^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(1/
4)*x)/((a - Sqrt[a^2 - 2*b])^(1/4)*(b + a*x^4)^(1/4))])/(a^2 - a*Sqrt[a^2 -
 2*b] - 2*b)^(1/4)) + ((a + Sqrt[a^2 - 2*b])^(1/4)*ArcTan[((a^2 + a*Sqrt[a^
2 - 2*b] - 2*b)^(1/4)*x)/((a + Sqrt[a^2 - 2*b])^(1/4)*(b + a*x^4)^(1/4))])/
(a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(1/4) - ((a - Sqrt[a^2 - 2*b])^(1/4)*ArcTan
h[((a^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)*x)/((a - Sqrt[a^2 - 2*b])^(1/4)*(b
 + a*x^4)^(1/4))])/(a^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(1/4) + ((a + Sqrt[a^2 -
 2*b])^(1/4)*ArcTanh[((a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)*x)/((a + Sqrt[a
^2 - 2*b])^(1/4)*(b + a*x^4)^(1/4))])/(a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)
)/(4*Sqrt[a^2 - 2*b])
```

IntegrateAlgebraic [A] time = 40.40, size = 510, normalized size = 1.12

$$-\frac{i\left(\sqrt{2\left(3-2\sqrt{2}\right)}-i\sqrt{2}\right)\tan^{-1}\left(\frac{\left((-1+i)-(1+i)(-1)^{3/4}\right)x^2\sqrt[4]{a^2-2b}+(1+i)(-1)^{3/4}\sqrt{ax^4+b}+(1+i)\sqrt{ax^4+b}}{2x\sqrt[8]{a^2-2b}\ \sqrt[4]{ax^4+b}}\right)}{16\left(a^2-2b\right)^{5/8}}+\frac{\left(\sqrt{2}-i\sqrt{2\left(3+2\sqrt{2}\right)}\right)\tan^{-1}\left(\frac{2x\sqrt[8]{a^2-2b}\ \sqrt[4]{ax^4+b}}{\left(\sqrt{2}+(1-i)\right)x^2\sqrt[4]{a^2-2b}-\sqrt{2}\sqrt{ax^4+b}-(1+i)\sqrt{ax^4+b}}\right)}{16\left(a^2-2b\right)^{5/8}}+\frac{\left((-1)^{3/4}-i\right)\tanh^{-1}\left(\frac{\left((-2+2i)-(2+2i)(-1)^{3/4}\right)x^2\sqrt[4]{a^2-2b}-(2+2i)(-1)^{3/4}\sqrt{ax^4+b}-(2+2i)\sqrt{ax^4+b}}{4x\sqrt[8]{a^2-2b}\ \sqrt[4]{ax^4+b}}\right)}{8\left(a^2-2b\right)^{5/8}}+\frac{i\left(\sqrt{2\left(3+2\sqrt{2}\right)}+i\sqrt{2}\right)\tanh^{-1}\left(\frac{\left(\sqrt{2}+(1-i)\right)x^2\sqrt[4]{a^2-2b}+\sqrt{2}\sqrt{ax^4+b}+(1+i)\sqrt{ax^4+b}}{2x\sqrt[8]{a^2-2b}\ \sqrt[4]{ax^4+b}}\right)}{16\left(a^2-2b\right)^{5/8}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((b + a*x^4)^(1/4)*(b + 2*a*x^4 + 2*x^8)),x]
```

```
[Out] ((-1/16*I)*((-I)*Sqrt[2] + Sqrt[2*(3 - 2*Sqrt[2])])*ArcTan[(((-1 + I) - (1
+ I)*(-1)^(3/4))*(a^2 - 2*b)^(1/4)*x^2 + (1 + I)*Sqrt[b + a*x^4] + (1 + I)*
```

```
(-1)^(3/4)*Sqrt[b + a*x^4])/(2*(a^2 - 2*b)^(1/8)*x*(b + a*x^4)^(1/4))])/(a^
2 - 2*b)^(5/8) + ((Sqrt[2] - I*Sqrt[2*(3 + 2*Sqrt[2])])*ArcTan[(2*(a^2 - 2*
b)^(1/8)*x*(b + a*x^4)^(1/4))/(((1 - I) + Sqrt[2])*(a^2 - 2*b)^(1/4)*x^2 -
(1 + I)*Sqrt[b + a*x^4] - Sqrt[2]*Sqrt[b + a*x^4])])/(16*(a^2 - 2*b)^(5/8))
 + ((-I + (-1)^(3/4))*ArcTanh[(((-2 + 2*I) - (2 + 2*I)*(-1)^(3/4))*(a^2 - 2
*b)^(1/4)*x^2 - (2 + 2*I)*Sqrt[b + a*x^4] - (2 + 2*I)*(-1)^(3/4)*Sqrt[b + a
*x^4])/(4*(a^2 - 2*b)^(1/8)*x*(b + a*x^4)^(1/4))])/(8*(a^2 - 2*b)^(5/8)) +
((I/16)*(I*Sqrt[2] + Sqrt[2*(3 + 2*Sqrt[2])])*ArcTanh[(((1 - I) + Sqrt[2])*
(a^2 - 2*b)^(1/4)*x^2 + (1 + I)*Sqrt[b + a*x^4] + Sqrt[2]*Sqrt[b + a*x^4])/
(2*(a^2 - 2*b)^(1/8)*x*(b + a*x^4)^(1/4))])/(a^2 - 2*b)^(5/8)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+b)^(1/4)/(2*x^8+2*a*x^4+b),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(2\,x^8+2\,ax^4+b\right)\left(ax^4+b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+b)^(1/4)/(2*x^8+2*a*x^4+b),x, algorithm="giac")
[Out] integrate(x^4/((2*x^8 + 2*a*x^4 + b)*(a*x^4 + b)^(1/4)), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a\,x^4+b\right)^{\frac{1}{4}}\left(2x^8+2a\,x^4+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^4+b)^(1/4)/(2*x^8+2*a*x^4+b),x)
[Out] int(x^4/(a*x^4+b)^(1/4)/(2*x^8+2*a*x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(2\,x^8+2\,ax^4+b\right)\left(ax^4+b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+b)^(1/4)/(2*x^8+2*a*x^4+b),x, algorithm="maxima")
[Out] integrate(x^4/((2*x^8 + 2*a*x^4 + b)*(a*x^4 + b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(a\,x^4+b\right)}^{1/4}\,\left(2\,x^8+2\,a\,x^4+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((b + a*x^4)^(1/4)*(b + 2*a*x^4 + 2*x^8)),x)
[Out] int(x^4/((b + a*x^4)^(1/4)*(b + 2*a*x^4 + 2*x^8)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a*x**4+b)**(1/4)/(2*x**8+2*a*x**4+b),x)
[Out] Timed out
```

3.2371 $\int \frac{x^4\left(-q+px^4\right)\sqrt{q+px^4}}{bx^8+a\left(q+px^4\right)^4}\,dx$

Optimal. Leaf size=457

$$-\frac{\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{x\left(\sqrt{\frac{2}{2-\sqrt{2}}}\,\sqrt[8]{a}\,\sqrt[8]{b}-\frac{2\sqrt[8]{a}\,\sqrt[8]{b}}{\sqrt{2-\sqrt{2}}}\right)\sqrt{px^4+q}}{-\sqrt[4]{a}\,px^4-\sqrt[4]{a}\,q+\sqrt[4]{b}\,x^2}\right)}{8a^{3/8}b^{5/8}}+\frac{\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{b}\,x\sqrt{px^4+q}}{\sqrt[4]{a}\,px^4+\sqrt[4]{a}\,q-\sqrt[4]{b}\,x^2}\right)}{8a^{3/8}b^{5/8}}-\frac{\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\cdots\right.}{\cdots}$$

Rubi [F] time = 2.62, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4\left(-q+px^4\right)\sqrt{q+px^4}}{bx^8+a\left(q+px^4\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(-q + p*x^4)*Sqrt[q + p*x^4])/(b*x^8 + a*(q + p*x^4)^4),x]
```

```
[Out] q*Defer[Int][(x^4*Sqrt[q + p*x^4])/(-(b*x^8) - a*(q + p*x^4)^4), x] + p*Def
er[Int][(x^8*Sqrt[q + p*x^4])/(b*x^8 + a*(q + p*x^4)^4), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(-q+px^4\right)\sqrt{q+px^4}}{bx^8+a\left(q+px^4\right)^4}\,dx &= \int \left(\frac{qx^4\sqrt{q+px^4}}{-aq^4-4apq^3x^4-b\left(1+\frac{6ap^2q^2}{b}\right)x^8-4ap^3qx^{12}-ap^4x^{16}}+\frac{\cdots}{aq^4+4apq^3x^4+\cdots}\right.\\
&= p\int \frac{x^8\sqrt{q+px^4}}{aq^4+4apq^3x^4+b\left(1+\frac{6ap^2q^2}{b}\right)x^8+4ap^3qx^{12}+ap^4x^{16}}\,dx+q\int \frac{\cdots}{-aq^4-4a\cdots}\\
&= p\int \frac{x^8\sqrt{q+px^4}}{bx^8+a\left(q+px^4\right)^4}\,dx+q\int \frac{x^4\sqrt{q+px^4}}{-bx^8-a\left(q+px^4\right)^4}\,dx
\end{aligned}$$

Mathematica [C] time = 7.80, size = 15065, normalized size = 32.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(-q + p*x^4)*Sqrt[q + p*x^4])/(b*x^8 + a*(q + p*x^4)^4),x]
```

```
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 24.81, size = 425, normalized size = 0.93

$$-\frac{\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{b}\,x\sqrt{px^4+q}}{\sqrt[4]{a}\,px^4+\sqrt[4]{a}\,q-\sqrt[4]{b}\,x^2}\right)}{8a^{3/8}b^{5/8}}+\frac{\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,\sqrt[8]{a}\,\sqrt[8]{b}\,x\sqrt{px^4+q}}{\sqrt[4]{a}\,px^4+\sqrt[4]{a}\,q-\sqrt[4]{b}\,x^2}\right)}{8a^{3/8}b^{5/8}}+\frac{\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\,\sqrt[8]{a}\,px^4}{\sqrt[8]{b}}+\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\,\sqrt[8]{a}\,q}{\sqrt[8]{b}}+\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\,\sqrt[8]{b}\,x^2}{\sqrt[8]{a}}}{x\sqrt{px^4+q}}\right)}{8a^{3/8}b^{5/8}}-\frac{\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\,\sqrt[8]{a}\,px^4}{\sqrt[8]{b}}+\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\,\sqrt[8]{a}\,q}{\sqrt[8]{b}}+\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\,\sqrt[8]{b}\,x^2}{\sqrt[8]{a}}}{x\sqrt{px^4+q}}\right)}{8a^{3/8}b^{5/8}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(-q + p*x^4)*Sqrt[q + p*x^4])/(b*x^8 + a*(q + p*x^4
)^4),x]
```

```
[Out] -1/8*(Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]]*a^(1/8)*b^(1/8)*x*Sqrt[q
+ p*x^4])/(a^(1/4)*q - b^(1/4)*x^2 + a^(1/4)*p*x^4)])/(a^(3/8)*b^(5/8)) + (
Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]]*a^(1/8)*b^(1/8)*x*Sqrt[q + p*x^
4])/(a^(1/4)*q - b^(1/4)*x^2 + a^(1/4)*p*x^4)])/(8*a^(3/8)*b^(5/8)) + (Sqrt
[2 - Sqrt[2]]*ArcTanh[((Sqrt[1 - 1/Sqrt[2]]*a^(1/8)*q)/b^(1/8) + (Sqrt[1 -
1/Sqrt[2]]*b^(1/8)*x^2)/a^(1/8) + (Sqrt[1 - 1/Sqrt[2]]*a^(1/8)*p*x^4)/b^(1/
8))/(x*Sqrt[q + p*x^4])])/(8*a^(3/8)*b^(5/8)) - (Sqrt[2 + Sqrt[2]]*ArcTanh[
((Sqrt[1 + 1/Sqrt[2]]*a^(1/8)*q)/b^(1/8) + (Sqrt[1 + 1/Sqrt[2]]*b^(1/8)*x^2
)/a^(1/8) + (Sqrt[1 + 1/Sqrt[2]]*a^(1/8)*p*x^4)/b^(1/8))/(x*Sqrt[q + p*x^4]
)])/(8*a^(3/8)*b^(5/8))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(p*x^4-q)*(p*x^4+q)^(1/2)/(b*x^8+a*(p*x^4+q)^4),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(p*x^4-q)*(p*x^4+q)^(1/2)/(b*x^8+a*(p*x^4+q)^4),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [C] time = 0.09, size = 47, normalized size = 0.10

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(16\_Z^{8}a+b\right)}\frac{\ln\left(\frac{\sqrt{p\,x^{4}+q}\,\sqrt{2}}{2x}-\_R\right)}{\_R^{5}}\right)\sqrt{2}}{64a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(p*x^4-q)*(p*x^4+q)^(1/2)/(b*x^8+a*(p*x^4+q)^4),x)
```

```
[Out] 1/64/a*sum(1/_R^5*ln(1/2*(p*x^4+q)^(1/2)*2^(1/2)/x-_R),_R=RootOf(16*_Z^8*a+
b))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^4+q}\left(px^4-q\right)x^4}{bx^8+\left(px^4+q\right)^4a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(p*x^4-q)*(p*x^4+q)^(1/2)/(b*x^8+a*(p*x^4+q)^4),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(p*x^4 + q)*(p*x^4 - q)*x^4/(b*x^8 + (p*x^4 + q)^4*a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^4\,\sqrt{p\,x^4+q}\,\left(q-p\,x^4\right)}{a\,{\left(p\,x^4+q\right)}^4+b\,x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(q + p*x^4)^(1/2)*(q - p*x^4))/(a*(q + p*x^4)^4 + b*x^8),x)
[Out] -int((x^4*(q + p*x^4)^(1/2)*(q - p*x^4))/(a*(q + p*x^4)^4 + b*x^8), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(p*x**4-q)*(p*x**4+q)**(1/2)/(b*x**8+a*(p*x**4+q)**4),x)
[Out] Timed out
```

3.2372 $\int \frac{\left(b+ax^4\right)^{3/4}}{b+2ax^4+2x^8}\,dx$

Optimal. Leaf size=460

$$\frac{\left(-1-\sqrt[4]{-1}\right)\tan^{-1}\left(\frac{(-1)^{7/8}\sqrt{2+\sqrt{2}}\,x\,\sqrt[8]{a^2-2b}\,\sqrt[4]{ax^4+b}}{(-1)^{3/4}x^2\sqrt[4]{a^2-2b}+\sqrt{ax^4+b}}\right)}{8\sqrt[8]{a^2-2b}}+\frac{\left(\sqrt{2}+i\sqrt{2\left(3-2\sqrt{2}\right)}\right)\tan^{-1}\left(\frac{(-1)^{7/8}\left(\sqrt{2}-2\right)x\,\sqrt[8]{a^2-2b}\,\sqrt[4]{ax^4+}}{(-1)^{3/4}\sqrt{2-\sqrt{2}}\,x^2\sqrt[4]{a^2-2b}+\sqrt{2-\sqrt{2}}}\right)}{16\sqrt[8]{a^2-2b}}$$

Rubi [A] time = 0.59, antiderivative size = 457, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1428, 408, 240, 212, 206, 203, 377, 208, 205}

$$-\frac{\left(-a\sqrt{a^2-2b}+a^2-2b\right)^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{a-\sqrt{a^2-2b}}\,\sqrt[4]{ax^4+b}}\right)}{4\left(a-\sqrt{a^2-2b}\right)^{3/4}\sqrt{a^2-2b}}+\frac{\left(a\sqrt{a^2-2b}+a^2-2b\right)^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{\sqrt{a^2-2b}+a}\,\sqrt[4]{ax^4+b}}\right)}{4\left(\sqrt{a^2-2b}+a\right)^{3/4}\sqrt{a^2-2b}}-\frac{\left(-a\sqrt{a^2-2b}+a^2-2b\right)^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{a-\sqrt{a^2-2b}}\,\sqrt[4]{ax^4+b}}\right)}{4\left(a-\sqrt{a^2-2b}\right)^{3/4}\sqrt{a^2-2b}}+\frac{\left(a\sqrt{a^2-2b}+a^2-2b\right)^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{\sqrt{a^2-2b}+a}\,\sqrt[4]{ax^4+b}}\right)}{4\left(\sqrt{a^2-2b}+a\right)^{3/4}\sqrt{a^2-2b}}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x^4)^(3/4)/(b + 2*a*x^4 + 2*x^8),x]
```

```
[Out] -1/4*((a^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(3/4)*ArcTan[((a^2 - a*Sqrt[a^2 - 2*b
] - 2*b)^(1/4)*x)/((a - Sqrt[a^2 - 2*b])^(1/4)*(b + a*x^4)^(1/4))])/((a - S
qrt[a^2 - 2*b])^(3/4)*Sqrt[a^2 - 2*b]) + ((a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(
3/4)*ArcTan[((a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)*x)/((a + Sqrt[a^2 - 2*b]
)^(1/4)*(b + a*x^4)^(1/4))])/(4*(a + Sqrt[a^2 - 2*b])^(3/4)*Sqrt[a^2 - 2*b]
) - ((a^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(3/4)*ArcTanh[((a^2 - a*Sqrt[a^2 - 2*b
] - 2*b)^(1/4)*x)/((a - Sqrt[a^2 - 2*b])^(1/4)*(b + a*x^4)^(1/4))])/(4*(a -
 Sqrt[a^2 - 2*b])^(3/4)*Sqrt[a^2 - 2*b]) + ((a^2 + a*Sqrt[a^2 - 2*b] - 2*b)
^(3/4)*ArcTanh[((a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)*x)/((a + Sqrt[a^2 - 2
*b])^(1/4)*(b + a*x^4)^(1/4))])/(4*(a + Sqrt[a^2 - 2*b])^(3/4)*Sqrt[a^2 - 2
*b])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 408

```
Int[((a_) + (b_.)*(x_)^4)^(p_)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d
, Int[(a + b*x^4)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(p
- 1)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[p, 3/4] || EqQ[p, 5/4])
```

Rule 1428

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^
n)^q/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[(d + e*x^n)^q/(b + r + 2
*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{\left(b+ax^4\right)^{3/4}}{b+2ax^4+2x^8}\,dx = \frac{2\int \frac{\left(b+ax^4\right)^{3/4}}{2a-2\sqrt{a^2-2b}+4x^4}\,dx}{\sqrt{a^2-2b}} - \frac{2\int \frac{\left(b+ax^4\right)^{3/4}}{2a+2\sqrt{a^2-2b}+4x^4}\,dx}{\sqrt{a^2-2b}}$$

$$= \frac{\left(a\left(a+\sqrt{a^2-2b}\right)-2b\right)\int \frac{1}{\left(2a+2\sqrt{a^2-2b}+4x^4\right)\sqrt[4]{b+ax^4}}\,dx}{\sqrt{a^2-2b}} - \frac{\left(a^2-a\sqrt{a^2-2b}-2b\right)\int \frac{1}{\left(2a-2\sqrt{a\cdots}\right.}\cdots}{\sqrt{a^2-2b}}$$

$$= \frac{\left(a\left(a+\sqrt{a^2-2b}\right)-2b\right)\mathrm{Subst}\left(\int \frac{1}{2a+2\sqrt{a^2-2b}-\left(a\left(2a+2\sqrt{a^2-2b}\right)-4b\right)x^4}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{\sqrt{a^2-2b}} - \frac{\left(a^2-\cdots\right.}{\cdots}$$

$$= \frac{\left(a\left(a+\sqrt{a^2-2b}\right)-2b\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\sqrt{a^2-2b}}-\sqrt{a^2+a\sqrt{a^2-2b}-2b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right)}{4\sqrt{a+\sqrt{a^2-2b}}\,\sqrt{a^2-2b}} + \frac{\left(a\left(a+\cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(a^2-a\sqrt{a^2-2b}-2b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a^2-a\sqrt{a^2-2b}-2b}\,x}{\sqrt[4]{a-\sqrt{a^2-2b}}\,\sqrt[4]{b+ax^4}}\right)}{4\left(a-\sqrt{a^2-2b}\right)^{3/4}\sqrt{a^2-2b}} + \frac{\left(a^2+a\sqrt{a^2-2b}-2b\right)^{3/4}\tan^{-1}\left(\cdots\right.}{4\left(a+\sqrt{a^2-2b}\right)^{3/4}\sqrt{\cdots}}$$

Mathematica [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(b+ax^4\right)^{3/4}}{b+2ax^4+2x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(b + a*x^4)^(3/4)/(b + 2*a*x^4 + 2*x^8),x]
```

```
[Out] Integrate[(b + a*x^4)^(3/4)/(b + 2*a*x^4 + 2*x^8), x]
```

IntegrateAlgebraic [A] time = 37.48, size = 510, normalized size = 1.11

$$\frac{\left(\sqrt{2}-i\sqrt{2\left(3+2\sqrt{2}\right)}\right)\tan^{-1}\left(\frac{\left((-1+i)-(1+i)(-1)^{3/4}\right)x^2\sqrt[4]{a^2-2b}+(1+i)(-1)^{3/4}\sqrt{ax^4+b}+(1+i)\sqrt{ax^4+b}}{2x\sqrt[8]{a^2-2b}\sqrt[4]{ax^4+b}}\right)}{16\sqrt[8]{a^2-2b}}-\frac{i\left(\sqrt{2\left(3-2\sqrt{2}\right)}-i\sqrt{2}\right)\tan^{-1}\left(\frac{2x\sqrt[8]{a^2-2b}\sqrt[4]{ax^4+b}}{\left(\sqrt{2}+(1-i)\right)x^2\sqrt[4]{a^2-2b}-\sqrt{2}\sqrt{ax^4+b}-(1+i)\sqrt{ax^4+b}}\right)}{16\sqrt[8]{a^2-2b}}+\frac{\left(-(-1)^{3/4}-i\right)\tanh^{-1}\left(\frac{\left((-2+2i)-(2+2i)(-1)^{3/4}\right)x^2\sqrt[4]{a^2-2b}-(2+2i)(-1)^{3/4}\sqrt{ax^4+b}-(2+2i)\sqrt{ax^4+b}}{4x\sqrt[8]{a^2-2b}\sqrt[4]{ax^4+b}}\right)}{8\sqrt[8]{a^2-2b}}+\frac{\left(\sqrt{2}+i\sqrt{2\left(3-2\sqrt{2}\right)}\right)\tanh^{-1}\left(\frac{\left(\sqrt{2}+(1-i)\right)x^2\sqrt[4]{a^2-2b}+\sqrt{2}\sqrt{ax^4+b}+(1+i)\sqrt{ax^4+b}}{2x\sqrt[8]{a^2-2b}\sqrt[4]{ax^4+b}}\right)}{16\sqrt[8]{a^2-2b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a*x^4)^(3/4)/(b + 2*a*x^4 + 2*x^8),x]
```

```
[Out] ((Sqrt[2] - I*Sqrt[2*(3 + 2*Sqrt[2])])*ArcTan[(((-1 + I) - (1 + I)*(-1)^(3/
4))*(a^2 - 2*b)^(1/4)*x^2 + (1 + I)*Sqrt[b + a*x^4] + (1 + I)*(-1)^(3/4)*Sq
rt[b + a*x^4])/(2*(a^2 - 2*b)^(1/8)*x*(b + a*x^4)^(1/4))])/(16*(a^2 - 2*b)^
(1/8)) - ((I/16)*((-I)*Sqrt[2] + Sqrt[2*(3 - 2*Sqrt[2])])*ArcTan[(2*(a^2 -
2*b)^(1/8)*x*(b + a*x^4)^(1/4))/(((1 - I) + Sqrt[2])*(a^2 - 2*b)^(1/4)*x^2
- (1 + I)*Sqrt[b + a*x^4] - Sqrt[2]*Sqrt[b + a*x^4])])/(a^2 - 2*b)^(1/8) +
((-I - (-1)^(3/4))*ArcTanh[(((-2 + 2*I) - (2 + 2*I)*(-1)^(3/4))*(a^2 - 2*b)
^(1/4)*x^2 - (2 + 2*I)*Sqrt[b + a*x^4] - (2 + 2*I)*(-1)^(3/4)*Sqrt[b + a*x^
4])/(4*(a^2 - 2*b)^(1/8)*x*(b + a*x^4)^(1/4))])/(8*(a^2 - 2*b)^(1/8)) + ((S
qrt[2] + I*Sqrt[2*(3 - 2*Sqrt[2])])*ArcTanh[(((1 - I) + Sqrt[2])*(a^2 - 2*b
)^(1/4)*x^2 + (1 + I)*Sqrt[b + a*x^4] + Sqrt[2]*Sqrt[b + a*x^4])/(2*(a^2 -
2*b)^(1/8)*x*(b + a*x^4)^(1/4))])/(16*(a^2 - 2*b)^(1/8))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4)/(2*x^8+2*a*x^4+b),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4+b\right)^{\frac{3}{4}}}{2\,x^8+2\,ax^4+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4)/(2*x^8+2*a*x^4+b),x, algorithm="giac")
```

```
[Out] integrate((a*x^4 + b)^(3/4)/(2*x^8 + 2*a*x^4 + b), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^4+b\right)^{\frac{3}{4}}}{2x^8+2a\,x^4+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b)^(3/4)/(2*x^8+2*a*x^4+b),x)
```

```
[Out] int((a*x^4+b)^(3/4)/(2*x^8+2*a*x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4+b\right)^{\frac{3}{4}}}{2\,x^8+2\,ax^4+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)^(3/4)/(2*x^8+2*a*x^4+b),x, algorithm="maxima")
```

```
[Out] integrate((a*x^4 + b)^(3/4)/(2*x^8 + 2*a*x^4 + b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x^4+b\right)}^{3/4}}{2\,x^8+2\,a\,x^4+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x^4)^(3/4)/(b + 2*a*x^4 + 2*x^8),x)
```

```
[Out] int((b + a*x^4)^(3/4)/(b + 2*a*x^4 + 2*x^8), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b)**(3/4)/(2*x**8+2*a*x**4+b),x)
```

```
[Out] Timed out
```

3.2373 $\int \sqrt{\frac{-1+ax-2x^2+2ax^3-x^4+ax^5}{1+ax-2x^2-2ax^3+x^4+ax^5}}\,dx$

Optimal. Leaf size=466

$$\frac{2\sqrt{-(a+1)^2}\tan^{-1}\left(\frac{\frac{(a+1)^2x^2}{\sqrt{-a^2-2a-1}\sqrt{a^2-1}}+\frac{(a+1)^2}{\sqrt{-a^2-2a-1}\sqrt{a^2-1}}}{(x-1)(x+1)\sqrt{\frac{ax^5+2ax^3+ax-x^4-2x^2-1}{ax^5-2ax^3+ax+x^4-2x^2+1}}}\right)}{\sqrt{a^2-1}}-\frac{2\sqrt{-(a-1)^2}\tan^{-1}\left(\frac{\frac{(a-1)^2x^2}{\sqrt{-a^2+2a-1}\sqrt{a^2-1}}+\frac{(a-1)^2}{\sqrt{-a^2+2a-1}\sqrt{a^2-1}}}{(x-1)(x+1)\sqrt{\frac{ax^5+2ax^3+ax-x^4-2x^2-1}{ax^5-2ax^3+ax+x^4-2x^2+1}}}\right)}{\sqrt{a^2-1}}+\sqrt{\frac{ax^5+}{ax^5-}}$$

Rubi [F] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{\frac{-1+ax-2x^2+2ax^3-x^4+ax^5}{1+ax-2x^2-2ax^3+x^4+ax^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[(-1 + a*x - 2*x^2 + 2*a*x^3 - x^4 + a*x^5)/(1 + a*x - 2*x^2 - 2*a*
x^3 + x^4 + a*x^5)],x]
```

```
[Out] Defer[Int][Sqrt[((-1 + a*x)*(1 + x^2)^2)/((1 + a*x)*(-1 + x^2)^2)], x]
```

Rubi steps

$$\int \sqrt{\frac{-1+ax-2x^2+2ax^3-x^4+ax^5}{1+ax-2x^2-2ax^3+x^4+ax^5}}\,dx=\int \sqrt{\frac{(-1+ax)\left(1+x^2\right)^2}{(1+ax)\left(-1+x^2\right)^2}}\,dx$$

Mathematica [A] time = 0.65, size = 238, normalized size = 0.51

$$\frac{\left(x^2-1\right)\sqrt{ax+1}\sqrt{\frac{\left(x^2+1\right)^2(ax-1)}{\left(x^2-1\right)^2(ax+1)}}\left((a+1)\left(\sqrt{\frac{a-1}{a+1}}\left(\sqrt{ax+1}\,(ax-1)^{3/2}+2\sqrt{-(ax-1)^2}\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)+2a(ax-1)\tanh^{-1}\left(\sqrt{\frac{a-1}{a+1}}\sqrt{\frac{ax-1}{ax+1}}\right)\right)-2(a-1)a(ax-1)\tanh^{-1}\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{\frac{a-1}{a+1}}}\right)\right)}{a\sqrt{\frac{a-1}{a+1}}(a+1)\left(x^2+1\right)(ax-1)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[(-1 + a*x - 2*x^2 + 2*a*x^3 - x^4 + a*x^5)/(1 + a*x - 2*x^2
- 2*a*x^3 + x^4 + a*x^5)],x]
```

```
[Out] (Sqrt[1 + a*x]*(-1 + x^2)*Sqrt[((-1 + a*x)*(1 + x^2)^2)/((1 + a*x)*(-1 + x^
2)^2)]*(-2*(-1 + a)*a*(-1 + a*x)*ArcTanh[Sqrt[(-1 + a*x)/(1 + a*x)]/Sqrt[(-
1 + a)/(1 + a)]] + (1 + a)*(Sqrt[(-1 + a)/(1 + a)]*((-1 + a*x)^(3/2)*Sqrt[1
 + a*x] + 2*Sqrt[-(-1 + a*x)^2]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]) + 2*a*(-1 +
a*x)*ArcTanh[Sqrt[(-1 + a)/(1 + a)]*Sqrt[(-1 + a*x)/(1 + a*x)]])))/(a*Sqrt[
(-1 + a)/(1 + a)]*(1 + a)*(-1 + a*x)^(3/2)*(1 + x^2))
```

IntegrateAlgebraic [A] time = 1.96, size = 511, normalized size = 1.10

$$\frac{2(a-1)\tan^{-1}\left(\frac{(a-1)\left(x^2+1\right)}{\sqrt{1-a}\sqrt{a+1}(x-1)(x+1)\sqrt{\frac{ax^5+2ax^3+ax-x^4-2x^2-1}{ax^5-2ax^3+ax+x^4-2x^2+1}}}\right)}{\sqrt{1-a^2}}+\frac{\sqrt{\frac{ax^5+2ax^3+ax-x^4-2x^2-1}{ax^5-2ax^3+ax+x^4-2x^2+1}}\left(ax^3-ax+x^2-1\right)}{a\left(x^2+1\right)}+\frac{\log\left(\left(x^2-1\right)\sqrt{\frac{ax^5+2ax^3+ax-x^4-2x^2-1}{ax^5-2ax^3+ax+x^4-2x^2+1}}-x^2-1\right)}{a}-\frac{\log\left(ax^2+\left(ax^2-a\right)\sqrt{\frac{ax^5+2ax^3+ax-x^4-2x^2-1}{ax^5-2ax^3+ax+x^4-2x^2+1}}+a\right)}{a}+\frac{2\sqrt{-a-1}\tan^{-1}\left(\frac{\frac{ax^2}{\sqrt{-a-1}\sqrt{a-1}}+\frac{x^2}{\sqrt{-a-1}\sqrt{a-1}}+\frac{a}{\sqrt{-a-1}\sqrt{a-1}}+\frac{1}{\sqrt{-a-1}\sqrt{a-1}}}{(x-1)(x+1)\sqrt{\frac{ax^5+2ax^3+ax-x^4-2x^2-1}{ax^5-2ax^3+ax+x^4-2x^2+1}}}\right)}{\sqrt{a-1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(-1 + a*x - 2*x^2 + 2*a*x^3 - x^4 + a*x^5)/(1 + a*x
 - 2*x^2 - 2*a*x^3 + x^4 + a*x^5)],x]
```

[Out] ((-1 - a*x + x^2 + a*x^3)*Sqrt[(-1 + a*x - 2*x^2 + 2*a*x^3 - x^4 + a*x^5)/(
1 + a*x - 2*x^2 - 2*a*x^3 + x^4 + a*x^5)])/(a*(1 + x^2)) + (2*(-1 + a)*ArcT
an[((-1 + a)*(1 + x^2))/(Sqrt[1 - a]*Sqrt[1 + a]*(-1 + x)*(1 + x)*Sqrt[(-1
+ a*x - 2*x^2 + 2*a*x^3 - x^4 + a*x^5)/(1 + a*x - 2*x^2 - 2*a*x^3 + x^4 + a
*x^5)])])/Sqrt[1 - a^2] + (2*Sqrt[-1 - a]*ArcTan[(1/(Sqrt[-1 - a]*Sqrt[-1 +
 a]) + a/(Sqrt[-1 - a]*Sqrt[-1 + a]) + x^2/(Sqrt[-1 - a]*Sqrt[-1 + a]) + (a
*x^2)/(Sqrt[-1 - a]*Sqrt[-1 + a]))/((-1 + x)*(1 + x)*Sqrt[(-1 + a*x - 2*x^2
 + 2*a*x^3 - x^4 + a*x^5)/(1 + a*x - 2*x^2 - 2*a*x^3 + x^4 + a*x^5)])])/Sqr
t[-1 + a] + Log[-1 - x^2 + (-1 + x^2)*Sqrt[(-1 + a*x - 2*x^2 + 2*a*x^3 - x^
4 + a*x^5)/(1 + a*x - 2*x^2 - 2*a*x^3 + x^4 + a*x^5)]]/a - Log[a + a*x^2 +
(-a + a*x^2)*Sqrt[(-1 + a*x - 2*x^2 + 2*a*x^3 - x^4 + a*x^5)/(1 + a*x - 2*x
^2 - 2*a*x^3 + x^4 + a*x^5)]]/a

fricas [A] time = 0.65, size = 1023, normalized size = 2.20

$$\left[\frac{(ax^2+a)\sqrt{\frac{a+1}{a-1}}\log\left(-\frac{a^2x^3+a^2x+x^2+((a^2-a)x^3+(a-1)x^2-(a^2-a)x-a+1)\sqrt{\frac{ax^5+2ax^3-x^4+ax-2x^2-1}{ax^5-2ax^3+x^4+ax-2x^2+1}}\sqrt{\frac{a+1}{a-1}}+1}{x^3+x^2+x+1}\right)+(ax^2+a)\sqrt{\frac{a-1}{a+1}}\log\left(\frac{a^2x^3+a^2x-x^2-((a^2+a)x^3+(a+1)x^2-(a^2+a)x-a-1)\sqrt{\frac{ax^5+2ax^3-x^4+ax-2x^2-1}{ax^5-2ax^3+x^4+ax-2x^2+1}}\sqrt{\frac{a-1}{a+1}}-1}{x^3-x^2+x-1}\right)-(x^2+1)\log\left(\frac{x^2+(x^2-1)\sqrt{\frac{ax^5+2ax^3-x^4+ax-2x^2-1}{ax^5-2ax^3+x^4+ax-2x^2+1}}+1}{x^2+1}\right)+(x^2+1)\log\left(-\frac{x^2-(x^2-1)\sqrt{\frac{ax^5+2ax^3-x^4+ax-2x^2-1}{ax^5-2ax^3+x^4+ax-2x^2+1}}+1}{x^2+1}\right)+(ax^3-ax+x^2-1)\sqrt{\frac{ax^5+2ax^3-x^4+ax-2x^2-1}{ax^5-2ax^3+x^4+ax-2x^2+1}}}{ax^2+a},\ -\frac{2(ax^2+a)\sqrt{-\frac{a+1}{a-1}}\arctan\left(\frac{((a-1)x^2-a+1)\sqrt{\frac{ax^5+2ax^3-x^4+ax-2x^2-1}{ax^5-2ax^3+x^4+ax-2x^2+1}}\sqrt{-\frac{a+1}{a-1}}}{(a+1)x^2+a+1}\right)-2(ax^2+a)\sqrt{-\frac{a-1}{a+1}}\arctan\left(\frac{((a+1)x^2-a-1)\sqrt{\frac{ax^5+2ax^3-x^4+ax-2x^2-1}{ax^5-2ax^3+x^4+ax-2x^2+1}}\sqrt{-\frac{a-1}{a+1}}}{(a-1)x^2+a-1}\right)+(x^2+1)\log\left(\frac{x^2+(x^2-1)\sqrt{\frac{ax^5+2ax^3-x^4+ax-2x^2-1}{ax^5-2ax^3+x^4+ax-2x^2+1}}+1}{x^2+1}\right)-(x^2+1)\log\left(-\frac{x^2-(x^2-1)\sqrt{\frac{ax^5+2ax^3-x^4+ax-2x^2-1}{ax^5-2ax^3+x^4+ax-2x^2+1}}+1}{x^2+1}\right)-(ax^3-ax+x^2-1)\sqrt{\frac{ax^5+2ax^3-x^4+ax-2x^2-1}{ax^5-2ax^3+x^4+ax-2x^2+1}}}{ax^2+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x^5+2*a*x^3-x^4+a*x-2*x^2-1)/(a*x^5-2*a*x^3+x^4+a*x-2*x^2+1))
^(1/2),x, algorithm="fricas")

[Out] [((a*x^2 + a)*sqrt((a + 1)/(a - 1))*log(-(a^2*x^3 + a^2*x + x^2 + ((a^2 - a
)*x^3 + (a - 1)*x^2 - (a^2 - a)*x - a + 1)*sqrt((a*x^5 + 2*a*x^3 - x^4 + a*
x - 2*x^2 - 1)/(a*x^5 - 2*a*x^3 + x^4 + a*x - 2*x^2 + 1))*sqrt((a + 1)/(a -
 1)) + 1)/(x^3 + x^2 + x + 1)) + (a*x^2 + a)*sqrt((a - 1)/(a + 1))*log((a^2
*x^3 + a^2*x - x^2 - ((a^2 + a)*x^3 + (a + 1)*x^2 - (a^2 + a)*x - a - 1)*sq
rt((a*x^5 + 2*a*x^3 - x^4 + a*x - 2*x^2 - 1)/(a*x^5 - 2*a*x^3 + x^4 + a*x -
 2*x^2 + 1))*sqrt((a - 1)/(a + 1)) - 1)/(x^3 - x^2 + x - 1)) - (x^2 + 1)*lo
g((x^2 + (x^2 - 1)*sqrt((a*x^5 + 2*a*x^3 - x^4 + a*x - 2*x^2 - 1)/(a*x^5 -
2*a*x^3 + x^4 + a*x - 2*x^2 + 1)) + 1)/(x^2 + 1)) + (x^2 + 1)*log(-(x^2 - (
x^2 - 1)*sqrt((a*x^5 + 2*a*x^3 - x^4 + a*x - 2*x^2 - 1)/(a*x^5 - 2*a*x^3 +
x^4 + a*x - 2*x^2 + 1)) + 1)/(x^2 + 1)) + (a*x^3 - a*x + x^2 - 1)*sqrt((a*x
^5 + 2*a*x^3 - x^4 + a*x - 2*x^2 - 1)/(a*x^5 - 2*a*x^3 + x^4 + a*x - 2*x^2
+ 1)))/(a*x^2 + a), -(2*(a*x^2 + a)*sqrt(-(a + 1)/(a - 1))*arctan(((a - 1)*
x^2 - a + 1)*sqrt((a*x^5 + 2*a*x^3 - x^4 + a*x - 2*x^2 - 1)/(a*x^5 - 2*a*x^
3 + x^4 + a*x - 2*x^2 + 1))*sqrt(-(a + 1)/(a - 1))/((a + 1)*x^2 + a + 1)) -
 2*(a*x^2 + a)*sqrt(-(a - 1)/(a + 1))*arctan(((a + 1)*x^2 - a - 1)*sqrt((a*
x^5 + 2*a*x^3 - x^4 + a*x - 2*x^2 - 1)/(a*x^5 - 2*a*x^3 + x^4 + a*x - 2*x^2
 + 1))*sqrt(-(a - 1)/(a + 1))/((a - 1)*x^2 + a - 1)) + (x^2 + 1)*log((x^2 +
 (x^2 - 1)*sqrt((a*x^5 + 2*a*x^3 - x^4 + a*x - 2*x^2 - 1)/(a*x^5 - 2*a*x^3
+ x^4 + a*x - 2*x^2 + 1)) + 1)/(x^2 + 1)) - (x^2 + 1)*log(-(x^2 - (x^2 - 1)
*sqrt((a*x^5 + 2*a*x^3 - x^4 + a*x - 2*x^2 - 1)/(a*x^5 - 2*a*x^3 + x^4 + a*
x - 2*x^2 + 1)) + 1)/(x^2 + 1)) - (a*x^3 - a*x + x^2 - 1)*sqrt((a*x^5 + 2*a
*x^3 - x^4 + a*x - 2*x^2 - 1)/(a*x^5 - 2*a*x^3 + x^4 + a*x - 2*x^2 + 1)))/(
a*x^2 + a)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x^5+2*a*x^3-x^4+a*x-2*x^2-1)/(a*x^5-2*a*x^3+x^4+a*x-2*x^2+1))
^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^4
-1),abs(a*x^5-2*a*x^3+a*x+x^4-2*x^2+1)]sym2poly/r2sym(const gen & e,const i
ndex_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.07, size = 475, normalized size = 1.02

$$-\frac{\sqrt{\frac{a\,x^{5}+2a\,x^{3}-x^{4}+ax-2x^{2}-1}{a\,x^{5}-2a\,x^{3}+x^{4}+ax-2x^{2}+1}}\,\left(x^{2}-1\right)\left(ax+1\right)\left(\ln\left(\frac{2a^{2}x+2\sqrt{a^{2}-1}\,\sqrt{a^{2}x^{2}-1}-2}{-1+x}\right)\sqrt{a^{2}-1}\,\sqrt{a^{2}}\,a^{2}+\ln\left(\frac{-2a^{2}x+2\sqrt{a^{2}-1}\,\sqrt{a^{2}x^{2}-1}-2}{1+x}\right)\sqrt{a^{2}-1}\,\sqrt{a^{2}}\,a^{2}+\sqrt{\left(ax+1\right)\left(ax-1\right)}\,\sqrt{a^{2}}\,a^{2}+2\ln\left(\frac{a^{2}x+\sqrt{a^{2}x^{2}-1}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{3}-\ln\left(\frac{2a^{2}x+2\sqrt{a^{2}-1}\,\sqrt{a^{2}x^{2}-1}-2}{-1+x}\right)\sqrt{a^{2}-1}\,\sqrt{a^{2}}\,a+\ln\left(\frac{-2a^{2}x+2\sqrt{a^{2}-1}\,\sqrt{a^{2}x^{2}-1}-2}{1+x}\right)\sqrt{a^{2}-1}\,\sqrt{a^{2}}\,a-\ln\left(\frac{a^{2}x+\sqrt{(ax+1)(ax-1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)a^{3}-2\sqrt{a^{2}}\,\sqrt{a^{2}x^{2}-1}\,a^{2}+\sqrt{\left(ax+1\right)\left(ax-1\right)}\,\sqrt{a^{2}}-a\ln\left(\frac{a^{2}x+\sqrt{(ax+1)(ax-1)}\,\sqrt{a^{2}}}{\sqrt{a^{2}}}\right)\right)}{\left(x^{2}+1\right)\sqrt{\left(ax+1\right)\left(ax-1\right)}\,\left(1+a\right)\left(-1+a\right)a\sqrt{a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^5+2*a*x^3-x^4+a*x-2*x^2-1)/(a*x^5-2*a*x^3+x^4+a*x-2*x^2+1))^(1/2)
,x)
```

```
[Out] -((a*x^5+2*a*x^3-x^4+a*x-2*x^2-1)/(a*x^5-2*a*x^3+x^4+a*x-2*x^2+1))^(1/2)/(x
^2+1)*(x^2-1)*(a*x+1)*(ln(2*(a^2*x+(a^2-1)^(1/2)*(a^2*x^2-1)^(1/2)-1)/(-1+x
))*(a^2-1)^(1/2)*(a^2)^(1/2)*a^2+ln(2*(-a^2*x+(a^2-1)^(1/2)*(a^2*x^2-1)^(1/
2)-1)/(1+x))*(a^2-1)^(1/2)*(a^2)^(1/2)*a^2+((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1
/2)*a^2+2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^3-ln(2*(a
^2*x+(a^2-1)^(1/2)*(a^2*x^2-1)^(1/2)-1)/(-1+x))*(a^2-1)^(1/2)*(a^2)^(1/2)*a
+ln(2*(-a^2*x+(a^2-1)^(1/2)*(a^2*x^2-1)^(1/2)-1)/(1+x))*(a^2-1)^(1/2)*(a^2)
^(1/2)*a-ln((a^2*x+((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^3-2*
(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a^2+((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)-a*ln(
(a^2*x+((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x+1)*(a*x-1))
^(1/2)/(1+a)/(-1+a)/a/(a^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax^{5}+2\,ax^{3}-x^{4}+ax-2\,x^{2}-1}{ax^{5}-2\,ax^{3}+x^{4}+ax-2\,x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x^5+2*a*x^3-x^4+a*x-2*x^2-1)/(a*x^5-2*a*x^3+x^4+a*x-2*x^2+1))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x^5 + 2*a*x^3 - x^4 + a*x - 2*x^2 - 1)/(a*x^5 - 2*a*x^3 +
 x^4 + a*x - 2*x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\frac{a\,x^{5}-x^{4}+2\,a\,x^{3}-2\,x^{2}+a\,x-1}{a\,x^{5}+x^{4}-2\,a\,x^{3}-2\,x^{2}+a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + 2*a*x^3 + a*x^5 - 2*x^2 - x^4 - 1)/(a*x - 2*a*x^3 + a*x^5 - 2*x
^2 + x^4 + 1))^(1/2),x)
```

```
[Out] int(((a*x + 2*a*x^3 + a*x^5 - 2*x^2 - x^4 - 1)/(a*x - 2*a*x^3 + a*x^5 - 2*x
^2 + x^4 + 1))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x**5+2*a*x**3-x**4+a*x-2*x**2-1)/(a*x**5-2*a*x**3+x**4+a*x-2*
x**2+1))**(1/2),x)
```

```
[Out] Timed out
```

3.2374 $\int \frac{1+x}{(-3+x^2)\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=468

$$\frac{\sqrt[3]{\frac{1}{2}\left(9+5\sqrt{3}\right)}\log\left(-6\sqrt[3]{x^2+1}+2^{2/3}\sqrt{3}\,x+3\,2^{2/3}\right)}{6\,3^{2/3}}+\frac{\sqrt[3]{\frac{1}{2}\left(9-5\sqrt{3}\right)}\log\left(6\sqrt[3]{x^2+1}+2^{2/3}\sqrt{3}\,x-3\,2^{2/3}\right)}{6\,3^{2/3}}-\frac{\sqrt[3]{\frac{1}{2}(}}{}$$

Rubi [A] time = 0.07, antiderivative size = 191, normalized size of antiderivative = 0.41, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1010, 392, 444, 55, 617, 204, 31}

$$-\frac{\log\left(3-x^2\right)}{4\,2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{x^2+1}\right)}{4\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\,2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}{\sqrt{3}}\right)}{2\,2^{2/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\tan^{-1}(x)}{6\,2^{2/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-3 + x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] ArcTan[x]/(6*2^(2/3)) - ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(2/3))
 + (Sqrt[3]*ArcTan[(1 + 2^(1/3)*(1 + x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) + Ar
cTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*(1 + x
^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - Log[3 - x^2]/(4*2^(2/3)) + (3*Log[2^(2
/3) - (1 + x^2)^(1/3)])/(4*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 392

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTanh[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1
/3)*d), x] + (-Simp[(q*ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] + Simp[(q*ArcTan[q*x])/(6*2^(2/3)*a^(1/3
)*d), x] + Simp[(q*ArcTanh[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/
(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{1+x}{\left(-3+x^2\right)\sqrt[3]{1+x^2}}\,dx = \int \frac{1}{\left(-3+x^2\right)\sqrt[3]{1+x^2}}\,dx + \int \frac{x}{\left(-3+x^2\right)\sqrt[3]{1+x^2}}\,dx$$

$$= \frac{\tan^{-1}(x)}{6\ 2^{2/3}} - \frac{\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{1}{2}\,S$$

$$= \frac{\tan^{-1}(x)}{6\ 2^{2/3}} - \frac{\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\log}{}$$

$$= \frac{\tan^{-1}(x)}{6\ 2^{2/3}} - \frac{\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\log}{}$$

$$= \frac{\tan^{-1}(x)}{6\ 2^{2/3}} - \frac{\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\tanh}{}$$

Mathematica [C] time = 0.27, size = 151, normalized size = 0.32

$$\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)}{\left(x^2-3\right)\sqrt[3]{x^2+1}\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)\right)} - \frac{1}{6}x^2F_1\left(1;\frac{1}{3},1;2;-x^2,\frac{x^2}{3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x)/((-3 + x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -1/6*(x^2*AppellF1[1, 1/3, 1, 2, -x^2, x^2/3]) + (9*x*AppellF1[1/2, 1/3, 1,
 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)*(9*AppellF1[1/2, 1/3, 1, 3/
2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -x^2, x^2/3] - AppellF1
[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

IntegrateAlgebraic [A] time = 9.35, size = 626, normalized size = 1.34

$$\frac{\sqrt[3]{\frac{1}{2}\left(9+5\sqrt{3}\right)}\log\left(-6\sqrt[3]{x^2+1}+2^{2/3}\sqrt{3}x+3\ 2^{2/3}\right)}{6\ 3^{2/3}} - \frac{\log\left(6\sqrt[3]{x^2+1}+2^{2/3}\sqrt{3}x-3\ 2^{2/3}\right)}{6\ 2^{2/3}\sqrt{3}} + \frac{\log\left(6\sqrt[3]{x^2+1}+2^{2/3}\sqrt{3}x-3\ 2^{2/3}\right)}{6\ 2^{2/3}} + \frac{\log\left(-\sqrt[3]{2}x^2+2^{2/3}\sqrt{3}\sqrt[3]{x^2+1}x-6\left(x^2+1\right)^{2/3}-3\ 2^{2/3}\sqrt[3]{x^2+1}+2\sqrt[3]{2}\sqrt{3}x-3\sqrt[3]{2}\right)}{12\ 2^{2/3}\sqrt{3}} - \frac{\log\left(-\sqrt[3]{2}x^2+2^{2/3}\sqrt{3}\sqrt[3]{x^2+1}x-6\left(x^2+1\right)^{2/3}-3\ 2^{2/3}\sqrt[3]{x^2+1}+2\sqrt[3]{2}\sqrt{3}x-3\sqrt[3]{2}\right)}{12\ 2^{2/3}} - \frac{\sqrt[3]{\frac{1}{2}\left(9+5\sqrt{3}\right)}\log\left(\sqrt[3]{2}x^2+2^{2/3}\sqrt{3}\sqrt[3]{x^2+1}x+6\left(x^2+1\right)^{2/3}+3\ 2^{2/3}\sqrt[3]{x^2+1}+2\sqrt[3]{2}\sqrt{3}x+3\sqrt[3]{2}\right)}{12\ 3^{2/3}} + \frac{\tan^{-1}\left(\frac{-\frac{\sqrt[3]{x^2+1}}{\sqrt{3}}+\frac{1}{3}2^{2/3}x-\frac{2^{2/3}}{\sqrt{3}}}{\sqrt[3]{x^2+1}}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{-\frac{\sqrt[3]{x^2+1}}{\sqrt{3}}+\frac{1}{3}2^{2/3}x-\frac{2^{2/3}}{\sqrt{3}}}{\sqrt[3]{x^2+1}}\right)}{6\ 2^{2/3}} - \frac{1}{6}\sqrt[3]{\frac{1}{2}\left(5+3\sqrt{3}\right)}\tan^{-1}\left(\frac{\frac{\sqrt[3]{x^2+1}}{\sqrt{3}}+\frac{1}{3}2^{2/3}x+\frac{2^{2/3}}{\sqrt{3}}}{\sqrt[3]{x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/((-3 + x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -1/6*ArcTan[(-(2^(2/3)/Sqrt[3]) + (2^(2/3)*x)/3 - (1 + x^2)^(1/3)/Sqrt[3])/
(1 + x^2)^(1/3)]/2^(2/3) + ArcTan[(-(2^(2/3)/Sqrt[3]) + (2^(2/3)*x)/3 - (1
+ x^2)^(1/3)/Sqrt[3])/(1 + x^2)^(1/3)]/(2*2^(2/3)*Sqrt[3]) - (((5 + 3*Sqrt[
3])/2)^(1/3)*ArcTan[(2^(2/3)/Sqrt[3] + (2^(2/3)*x)/3 + (1 + x^2)^(1/3)/Sqrt
[3])/(1 + x^2)^(1/3)])/6 + (((9 + 5*Sqrt[3])/2)^(1/3)*Log[3*2^(2/3) + 2^(2/
3)*Sqrt[3]*x - 6*(1 + x^2)^(1/3)])/(6*3^(2/3)) + Log[-3*2^(2/3) + 2^(2/3)*S
qrt[3]*x + 6*(1 + x^2)^(1/3)]/(6*2^(2/3)) - Log[-3*2^(2/3) + 2^(2/3)*Sqrt[3
]*x + 6*(1 + x^2)^(1/3)]/(6*2^(2/3)*Sqrt[3]) - Log[-3*2^(1/3) + 2*2^(1/3)*S
qrt[3]*x - 2^(1/3)*x^2 - 3*2^(2/3)*(1 + x^2)^(1/3) + 2^(2/3)*Sqrt[3]*x*(1 +
 x^2)^(1/3) - 6*(1 + x^2)^(2/3)]/(12*2^(2/3)) + Log[-3*2^(1/3) + 2*2^(1/3)*
Sqrt[3]*x - 2^(1/3)*x^2 - 3*2^(2/3)*(1 + x^2)^(1/3) + 2^(2/3)*Sqrt[3]*x*(1
+ x^2)^(1/3) - 6*(1 + x^2)^(2/3)]/(12*2^(2/3)*Sqrt[3]) - (((9 + 5*Sqrt[3])/
2)^(1/3)*Log[3*2^(1/3) + 2*2^(1/3)*Sqrt[3]*x + 2^(1/3)*x^2 + 3*2^(2/3)*(1 +
 x^2)^(1/3) + 2^(2/3)*Sqrt[3]*x*(1 + x^2)^(1/3) + 6*(1 + x^2)^(2/3)])/(12*3
^(2/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-3)/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-3)/(x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1+x}{\left(x^2-3\right)\left(x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2-3)/(x^2+1)^(1/3),x)
```

```
[Out] int((1+x)/(x^2-3)/(x^2+1)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-3)/(x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x+1}{\left(x^2+1\right)^{1/3}\,\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^2 + 1)^(1/3)*(x^2 - 3)),x)
[Out] int((x + 1)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(x^2-3\right)\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2-3)/(x**2+1)**(1/3),x)
[Out] Integral((x + 1)/((x**2 - 3)*(x**2 + 1)**(1/3)), x)
```

3.2375 $\int \frac{1-x^4}{(1+x^4)\sqrt[4]{-x^3+x^5}}\,dx$

Optimal. Leaf size=469

$$\frac{1}{2}\sqrt[4]{3\sqrt{2}-4}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x}{2^{7/8}\sqrt[4]{x^5-x^3}-\sqrt{2+\sqrt{2}}\,x}\right)+\frac{1}{2}\sqrt[4]{3\sqrt{2}-4}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,x}{2^{7/8}\sqrt[4]{x^5-x^3}+\sqrt{2+\sqrt{2}}\,x}\right)-\frac{1}{4}\sqrt[4]{4+3\sqrt{}}$$

Rubi [C] time = 0.65, antiderivative size = 101, normalized size of antiderivative = 0.22, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2056, 1586, 6715, 6725, 430, 429}

$$\frac{(2-2i)x\sqrt[4]{1-x^2}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};x^2,ix^2\right)}{\sqrt[4]{x^5-x^3}}+\frac{(2+2i)x\sqrt[4]{1-x^2}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};-ix^2,x^2\right)}{\sqrt[4]{x^5-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x^4)/((1 + x^4)*(-x^3 + x^5)^(1/4)),x]
```

```
[Out] ((2 - 2*I)*x*(1 - x^2)^(1/4)*AppellF1[1/8, -3/4, 1, 9/8, x^2, I*x^2])/(-x^3
 + x^5)^(1/4) + ((2 + 2*I)*x*(1 - x^2)^(1/4)*AppellF1[1/8, 1, -3/4, 9/8, (-
I)*x^2, x^2])/(-x^3 + x^5)^(1/4)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1-x^4}{\left(1+x^4\right)\sqrt[4]{-x^3+x^5}}\,dx = \frac{\left(x^{3/4}\sqrt[4]{-1+x^2}\right)\int \frac{1-x^4}{x^{3/4}\sqrt[4]{-1+x^2}\left(1+x^4\right)}\,dx}{\sqrt[4]{-x^3+x^5}}$$

$$= \frac{\left(x^{3/4}\sqrt[4]{-1+x^2}\right)\int \frac{\left(-1-x^2\right)\left(-1+x^2\right)^{3/4}}{x^{3/4}\left(1+x^4\right)}\,dx}{\sqrt[4]{-x^3+x^5}}$$

$$= \frac{\left(4x^{3/4}\sqrt[4]{-1+x^2}\right)\mathrm{Subst}\left(\int \frac{\left(-1-x^8\right)\left(-1+x^8\right)^{3/4}}{1+x^{16}}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-x^3+x^5}}$$

$$= \frac{\left(4x^{3/4}\sqrt[4]{-1+x^2}\right)\mathrm{Subst}\left(\int \left(\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(-1+x^8\right)^{3/4}}{i-x^8}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-1+x^8\right)^{3/4}}{i+x^8}\right)dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-x^3+x^5}}$$

$$= -\frac{\left((2+2i)x^{3/4}\sqrt[4]{-1+x^2}\right)\mathrm{Subst}\left(\int \frac{\left(-1+x^8\right)^{3/4}}{i+x^8}\,dx, x, \sqrt[4]{x}\right)}{\sqrt[4]{-x^3+x^5}} + \frac{\left((2-2i)x^{3/4}\sqrt[4]{-1+x^2}\right.}{\sqrt[4]{}}$$

$$= -\frac{\left((2+2i)x^{3/4}\left(-1+x^2\right)\right)\mathrm{Subst}\left(\int \frac{\left(1-x^8\right)^{3/4}}{i+x^8}\,dx, x, \sqrt[4]{x}\right)}{\left(1-x^2\right)^{3/4}\sqrt[4]{-x^3+x^5}} + \frac{\left((2-2i)x^{3/4}\left(-1+x^2\right)\right)}{\left(1-x^2\right)^{3}}$$

$$= \frac{(2-2i)x\sqrt[4]{1-x^2}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};x^2,ix^2\right)}{\sqrt[4]{-x^3+x^5}} + \frac{(2+2i)x\sqrt[4]{1-x^2}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};-ix^2,\right.}{\sqrt[4]{-x^3+x^5}}$$

Mathematica [F] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{1-x^4}{\left(1+x^4\right)\sqrt[4]{-x^3+x^5}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^4)/((1 + x^4)*(-x^3 + x^5)^(1/4)),x]
```

```
[Out] Integrate[(1 - x^4)/((1 + x^4)*(-x^3 + x^5)^(1/4)), x]
```

IntegrateAlgebraic [A] time = 24.18, size = 394, normalized size = 0.84

$$-\frac{1}{4}\sqrt[4]{4+3\sqrt{2}}\log\left(2x^2-2\sqrt[4]{4+3\sqrt{2}}\sqrt[4]{x^5-x^3}\,x+2^{3/4}\sqrt{x^5-x^3}\right)+\frac{1}{4}\sqrt[4]{4+3\sqrt{2}}\log\left(\sqrt{2-\sqrt{2}}\,x^2+2^{3/8}\sqrt[4]{x^5-x^3}\,x+\sqrt{\sqrt{2}-1}\sqrt{x^5-x^3}\right)-\frac{1}{2}\sqrt[4]{3\sqrt{2}-4}\tan^{-1}\left(\frac{\sqrt[4]{6\sqrt{2}-8}\,x\sqrt[4]{x^5-x^3}}{\sqrt[4]{2}x^2-\sqrt{x^5-x^3}}\right)-\frac{1}{2}\sqrt[4]{4+3\sqrt{2}}\tan^{-1}\left(\frac{\sqrt[4]{8+6\sqrt{2}}\,x\sqrt[4]{x^5-x^3}}{\sqrt[4]{2}x^2-\sqrt{x^5-x^3}}\right)+\frac{1}{2}\sqrt[4]{3\sqrt{2}-4}\tanh^{-1}\left(\frac{2\sqrt[4]{\frac{1}{8}+\frac{3}{16\sqrt{2}}}\,x^2+2^{3/4}\sqrt[4]{\frac{1}{8}+\frac{3}{16\sqrt{2}}}\sqrt{x^5-x^3}}{x\sqrt[4]{x^5-x^3}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] IntegrateAlgebraic[(1 - x^4)/((1 + x^4)*(-x^3 + x^5)^(1/4)),x]
```

```
[Out] -1/2*((-4 + 3*Sqrt[2])^(1/4)*ArcTan[((-8 + 6*Sqrt[2])^(1/4)*x*(-x^3 + x^5)^
(1/4))/(2^(1/4)*x^2 - Sqrt[-x^3 + x^5])]) - ((4 + 3*Sqrt[2])^(1/4)*ArcTan[(
(8 + 6*Sqrt[2])^(1/4)*x*(-x^3 + x^5)^(1/4))/(2^(1/4)*x^2 - Sqrt[-x^3 + x^5]
```

```
)])/2 + ((-4 + 3*Sqrt[2])^(1/4)*ArcTanh[(2*(1/8 + 3/(16*Sqrt[2]))^(1/4)*x^2
 + 2^(3/4)*(1/8 + 3/(16*Sqrt[2]))^(1/4)*Sqrt[-x^3 + x^5])/(x*(-x^3 + x^5)^(
1/4))])/2 - ((4 + 3*Sqrt[2])^(1/4)*Log[2*x^2 - 2*(4 + 3*Sqrt[2])^(1/4)*x*(-
x^3 + x^5)^(1/4) + 2^(3/4)*Sqrt[-x^3 + x^5]])/4 + ((4 + 3*Sqrt[2])^(1/4)*Lo
g[Sqrt[2 - Sqrt[2]]*x^2 + 2^(3/8)*x*(-x^3 + x^5)^(1/4) + Sqrt[-1 + Sqrt[2]]
*Sqrt[-x^3 + x^5]])/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^4+1)/(x^5-x^3)^(1/4),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^4-1}{\left(x^5-x^3\right)^{\frac{1}{4}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^4+1)/(x^5-x^3)^(1/4),x, algorithm="giac")
[Out] integrate(-(x^4 - 1)/((x^5 - x^3)^(1/4)*(x^4 + 1)), x)
```

maple [C] time = 80.87, size = 4192, normalized size = 8.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^4+1)/(x^5-x^3)^(1/4),x)
[Out] -1/32*ln((-3072*RootOf(_Z^8+128)*x^4-28*(x^5-x^3)^(1/2)*RootOf(-_Z*RootOf(_
Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*RootOf(_Z^8+128)^6*x+56*RootOf(_Z^8
+128)^4*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*x^3+26*R
ootOf(_Z^8+128)^9*x^4-39*RootOf(_Z^8+128)^9*x^3-56*RootOf(_Z^8+128)^5*x^4-8
00*RootOf(_Z^8+128)^5*x^3+56*RootOf(_Z^8+128)^5*x^2+2496*RootOf(-_Z*RootOf(
_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*x^4+3328*RootOf(-_Z*RootOf(_Z^8+12
8)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*x^3-2496*RootOf(-_Z*RootOf(_Z^8+128)^5-8*
RootOf(_Z^8+128)^2+8*_Z^2)*x^2+16*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z
^8+128)^2+8*_Z^2)*RootOf(_Z^8+128)^8*x^4-24*RootOf(-_Z*RootOf(_Z^8+128)^5-8
*RootOf(_Z^8+128)^2+8*_Z^2)*RootOf(_Z^8+128)^8*x^3-4096*RootOf(_Z^8+128)*x^
3+3072*RootOf(_Z^8+128)*x^2+400*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8
+128)^2+8*_Z^2)*RootOf(_Z^8+128)^4*x^2-400*RootOf(-_Z*RootOf(_Z^8+128)^5-8*
RootOf(_Z^8+128)^2+8*_Z^2)*RootOf(_Z^8+128)^4*x^4-16*RootOf(-_Z*RootOf(_Z^8
+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*RootOf(_Z^8+128)^8*x^2-26*RootOf(_Z^8+
128)^9*x^2-56*RootOf(_Z^8+128)^6*(x^5-x^3)^(1/4)*x^2-6400*RootOf(_Z^8+128)^
2*(x^5-x^3)^(1/4)*x^2+1792*(x^5-x^3)^(3/4)-800*RootOf(_Z^8+128)^5*(x^5-x^3)
^(1/4)*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*x^2-3200*
(x^5-x^3)^(1/2)*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*
RootOf(_Z^8+128)^2*x+896*RootOf(_Z^8+128)*(x^5-x^3)^(1/4)*RootOf(-_Z*RootOf
(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*x^2-1600*RootOf(_Z^8+128)^4*(x^5-
x^3)^(3/4))/x^2/(2*x^2*RootOf(_Z^8+128)^4-3*x*RootOf(_Z^8+128)^4-2*RootOf(_
Z^8+128)^4+24*x^2+32*x-24))*RootOf(_Z^8+128)^5+1/4*ln((-3072*RootOf(_Z^8+12
8)*x^4-28*(x^5-x^3)^(1/2)*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^
2+8*_Z^2)*RootOf(_Z^8+128)^6*x+56*RootOf(_Z^8+128)^4*RootOf(-_Z*RootOf(_Z^8
+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*x^3+26*RootOf(_Z^8+128)^9*x^4-39*RootO
```

f(_Z^8+128)^9*x^3-56*RootOf(_Z^8+128)^5*x^4-800*RootOf(_Z^8+128)^5*x^3+56*R
ootOf(_Z^8+128)^5*x^2+2496*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)
^2+8*_Z^2)*x^4+3328*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z
^2)*x^3-2496*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*x^2
+16*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*RootOf(_Z^8+
128)^8*x^4-24*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*Ro
otOf(_Z^8+128)^8*x^3-4096*RootOf(_Z^8+128)*x^3+3072*RootOf(_Z^8+128)*x^2+40
0*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*RootOf(_Z^8+12
8)^4*x^2-400*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*Roo
tOf(_Z^8+128)^4*x^4-16*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8
*_Z^2)*RootOf(_Z^8+128)^8*x^2-26*RootOf(_Z^8+128)^9*x^2-56*RootOf(_Z^8+128)
^6*(x^5-x^3)^(1/4)*x^2-6400*RootOf(_Z^8+128)^2*(x^5-x^3)^(1/4)*x^2+1792*(x^
5-x^3)^(3/4)-800*RootOf(_Z^8+128)^5*(x^5-x^3)^(1/4)*RootOf(-_Z*RootOf(_Z^8+
128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*x^2-3200*(x^5-x^3)^(1/2)*RootOf(-_Z*Roo
tOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*RootOf(_Z^8+128)^2*x+896*RootO
f(_Z^8+128)*(x^5-x^3)^(1/4)*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128
)^2+8*_Z^2)*x^2-1600*RootOf(_Z^8+128)^4*(x^5-x^3)^(3/4))/x^2/(2*x^2*RootOf(
_Z^8+128)^4-3*x*RootOf(_Z^8+128)^4-2*RootOf(_Z^8+128)^4+24*x^2+32*x-24))*Ro
otOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)+1/32*ln((2496*Root
Of(_Z^8+128)*x^4-400*(x^5-x^3)^(1/2)*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf
(_Z^8+128)^2+8*_Z^2)*RootOf(_Z^8+128)^6*x+800*RootOf(_Z^8+128)^4*RootOf(-_Z
*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*x^3+16*RootOf(_Z^8+128)^9*
x^4-24*RootOf(_Z^8+128)^9*x^3+400*RootOf(_Z^8+128)^5*x^4-56*RootOf(_Z^8+128
)^5*x^3-400*RootOf(_Z^8+128)^5*x^2-3072*RootOf(-_Z*RootOf(_Z^8+128)^5-8*Roo
tOf(_Z^8+128)^2+8*_Z^2)*x^4-4096*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^
8+128)^2+8*_Z^2)*x^3+3072*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^
2+8*_Z^2)*x^2+26*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)
*RootOf(_Z^8+128)^8*x^4-39*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)
^2+8*_Z^2)*RootOf(_Z^8+128)^8*x^3+3328*RootOf(_Z^8+128)*x^3-2496*RootOf(_Z^
8+128)*x^2-56*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*Ro
otOf(_Z^8+128)^4*x^2+56*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+
8*_Z^2)*RootOf(_Z^8+128)^4*x^4-26*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z
^8+128)^2+8*_Z^2)*RootOf(_Z^8+128)^8*x^2-16*RootOf(_Z^8+128)^9*x^2+56*RootO
f(_Z^8+128)^6*(x^5-x^3)^(1/4)*x^2-6400*RootOf(_Z^8+128)^2*(x^5-x^3)^(1/4)*x
^2+28*(x^5-x^3)^(1/2)*RootOf(_Z^8+128)^7*x-3200*(x^5-x^3)^(1/2)*RootOf(_Z^8
+128)^3*x+1792*(x^5-x^3)^(3/4)-448*(x^5-x^3)^(1/2)*RootOf(-_Z*RootOf(_Z^8+1
28)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*RootOf(_Z^8+128)^2*x+1600*RootOf(_Z^8+12
8)^4*(x^5-x^3)^(3/4))/x^2/(2*x^2*RootOf(_Z^8+128)^4-3*x*RootOf(_Z^8+128)^4-
2*RootOf(_Z^8+128)^4-24*x^2-32*x+24))*RootOf(_Z^8+128)^4*RootOf(-_Z*RootOf(
_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)+1/4*ln((2496*RootOf(_Z^8+128)*x^4-
400*(x^5-x^3)^(1/2)*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z
^2)*RootOf(_Z^8+128)^6*x+800*RootOf(_Z^8+128)^4*RootOf(-_Z*RootOf(_Z^8+128)
^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*x^3+16*RootOf(_Z^8+128)^9*x^4-24*RootOf(_Z^
8+128)^9*x^3+400*RootOf(_Z^8+128)^5*x^4-56*RootOf(_Z^8+128)^5*x^3-400*RootO
f(_Z^8+128)^5*x^2-3072*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8
*_Z^2)*x^4-4096*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*
x^3+3072*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*x^2+26*
RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*RootOf(_Z^8+128)
^8*x^4-39*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*RootOf
(_Z^8+128)^8*x^3+3328*RootOf(_Z^8+128)*x^3-2496*RootOf(_Z^8+128)*x^2-56*Roo
tOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*RootOf(_Z^8+128)^4*
x^2+56*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*RootOf(_Z
^8+128)^4*x^4-26*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)
*RootOf(_Z^8+128)^8*x^2-16*RootOf(_Z^8+128)^9*x^2+56*RootOf(_Z^8+128)^6*(x^
5-x^3)^(1/4)*x^2-6400*RootOf(_Z^8+128)^2*(x^5-x^3)^(1/4)*x^2+28*(x^5-x^3)^(
1/2)*RootOf(_Z^8+128)^7*x-3200*(x^5-x^3)^(1/2)*RootOf(_Z^8+128)^3*x+1792*(x
^5-x^3)^(3/4)-448*(x^5-x^3)^(1/2)*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z
^8+128)^2+8*_Z^2)*RootOf(_Z^8+128)^2*x+1600*RootOf(_Z^8+128)^4*(x^5-x^3)^(3
/4))/x^2/(2*x^2*RootOf(_Z^8+128)^4-3*x*RootOf(_Z^8+128)^4-2*RootOf(_Z^8+128

```
)^4-24*x^2-32*x+24))*RootOf(_Z^8+128)+1/4*RootOf(-_Z*RootOf(_Z^8+128)^5-8*R
ootOf(_Z^8+128)^2+8*_Z^2)*ln((4*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8
+128)^2+8*_Z^2)*RootOf(_Z^8+128)^8*x^4-6*RootOf(-_Z*RootOf(_Z^8+128)^5-8*Ro
otOf(_Z^8+128)^2+8*_Z^2)*RootOf(_Z^8+128)^8*x^3-4*RootOf(-_Z*RootOf(_Z^8+12
8)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*RootOf(_Z^8+128)^8*x^2+100*(x^5-x^3)^(1/2
)*RootOf(_Z^8+128)^7*x-7*(x^5-x^3)^(1/2)*RootOf(-_Z*RootOf(_Z^8+128)^5-8*Ro
otOf(_Z^8+128)^2+8*_Z^2)*RootOf(_Z^8+128)^6*x+14*RootOf(_Z^8+128)^6*(x^5-x^
3)^(1/4)*x^2+200*RootOf(_Z^8+128)^5*(x^5-x^3)^(1/4)*RootOf(-_Z*RootOf(_Z^8+
128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*x^2-100*RootOf(-_Z*RootOf(_Z^8+128)^5-8
*RootOf(_Z^8+128)^2+8*_Z^2)*RootOf(_Z^8+128)^4*x^4+14*RootOf(_Z^8+128)^4*Ro
otOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)*x^3-400*RootOf(_Z^
8+128)^4*(x^5-x^3)^(3/4)+100*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+12
8)^2+8*_Z^2)*RootOf(_Z^8+128)^4*x^2-112*(x^5-x^3)^(1/2)*RootOf(_Z^8+128)^3*
x-800*(x^5-x^3)^(1/2)*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*
_Z^2)*RootOf(_Z^8+128)^2*x+1600*RootOf(_Z^8+128)^2*(x^5-x^3)^(1/4)*x^2-224*
RootOf(_Z^8+128)*(x^5-x^3)^(1/4)*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^
8+128)^2+8*_Z^2)*x^2+624*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2
+8*_Z^2)*x^4+832*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+128)^2+8*_Z^2)
*x^3+448*(x^5-x^3)^(3/4)-624*RootOf(-_Z*RootOf(_Z^8+128)^5-8*RootOf(_Z^8+12
8)^2+8*_Z^2)*x^2)/x^2/(2*x^2*RootOf(_Z^8+128)^4-3*x*RootOf(_Z^8+128)^4-2*Ro
otOf(_Z^8+128)^4+24*x^2+32*x-24))+1/4*RootOf(_Z^8+128)*ln((4*RootOf(_Z^8+12
8)^9*x^4-6*RootOf(_Z^8+128)^9*x^3-4*RootOf(_Z^8+128)^9*x^2-7*(x^5-x^3)^(1/2
)*RootOf(_Z^8+128)^7*x-14*RootOf(_Z^8+128)^6*(x^5-x^3)^(1/4)*x^2+100*RootOf
(_Z^8+128)^5*x^4-14*RootOf(_Z^8+128)^5*x^3-100*RootOf(_Z^8+128)^5*x^2+400*R
ootOf(_Z^8+128)^4*(x^5-x^3)^(3/4)+800*(x^5-x^3)^(1/2)*RootOf(_Z^8+128)^3*x+
1600*RootOf(_Z^8+128)^2*(x^5-x^3)^(1/4)*x^2+624*RootOf(_Z^8+128)*x^4+832*Ro
otOf(_Z^8+128)*x^3-624*RootOf(_Z^8+128)*x^2+448*(x^5-x^3)^(3/4))/x^2/(2*x^2
*RootOf(_Z^8+128)^4-3*x*RootOf(_Z^8+128)^4-2*RootOf(_Z^8+128)^4-24*x^2-32*x
+24))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4-1}{\left(x^5-x^3\right)^{\frac{1}{4}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^4+1)/(x^5-x^3)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/((x^5 - x^3)^(1/4)*(x^4 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^4-1}{\left(x^4+1\right)\,\left(x^5-x^3\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/((x^4 + 1)*(x^5 - x^3)^(1/4)),x)
```

```
[Out] -int((x^4 - 1)/((x^4 + 1)*(x^5 - x^3)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{x^4\sqrt[4]{x^5-x^3}+\sqrt[4]{x^5-x^3}}\,dx-\int \left(-\frac{1}{x^4\sqrt[4]{x^5-x^3}+\sqrt[4]{x^5-x^3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**4+1)/(x**5-x**3)**(1/4),x)
```

```
[Out] -Integral(x**4/(x**4*(x**5 - x**3)**(1/4) + (x**5 - x**3)**(1/4)), x) - Int
egral(-1/(x**4*(x**5 - x**3)**(1/4) + (x**5 - x**3)**(1/4)), x)
```

3.2376 $\int \frac{-1+x^8}{\sqrt[4]{-x^2+x^6}\left(1+x^8\right)}\,dx$

Optimal. Leaf size=469

$$\frac{1}{8}\sqrt[4]{4+3\sqrt{2}}\,\log\left(-2x^2+2^{7/8}\sqrt{2+\sqrt{2}}\,\sqrt[4]{x^6-x^2}\,x-2^{3/4}\sqrt{x^6-x^2}\right)-\frac{1}{8}\sqrt[4]{4+3\sqrt{2}}\,\log\left(2\sqrt{2-\sqrt{2}}\,x^2+2\,2^{3/8}\right.$$

Rubi [C] time = 0.56, antiderivative size = 101, normalized size of antiderivative = 0.22, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2056, 1586, 6715, 6725, 430, 429}

$$-\frac{(1-i)x\sqrt[4]{1-x^4}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};x^4,ix^4\right)}{\sqrt[4]{x^6-x^2}}-\frac{(1+i)x\sqrt[4]{1-x^4}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};-ix^4,x^4\right)}{\sqrt[4]{x^6-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^8)/((-x^2 + x^6)^(1/4)*(1 + x^8)),x]
```

```
[Out] ((-1 + I)*x*(1 - x^4)^(1/4)*AppellF1[1/8, -3/4, 1, 9/8, x^4, I*x^4])/(-x^2
+ x^6)^(1/4) - ((1 + I)*x*(1 - x^4)^(1/4)*AppellF1[1/8, 1, -3/4, 9/8, (-I)*
x^4, x^4])/(-x^2 + x^6)^(1/4)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function0
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^8}{\sqrt[4]{-x^2+x^6}\left(1+x^8\right)}\,dx &= \frac{\left(\sqrt{x}\sqrt[4]{-1+x^4}\right)\int \frac{-1+x^8}{\sqrt{x}\sqrt[4]{-1+x^4}\left(1+x^8\right)}\,dx}{\sqrt[4]{-x^2+x^6}} \\
&= \frac{\left(\sqrt{x}\sqrt[4]{-1+x^4}\right)\int \frac{\left(-1+x^4\right)^{3/4}\left(1+x^4\right)}{\sqrt{x}\left(1+x^8\right)}\,dx}{\sqrt[4]{-x^2+x^6}} \\
&= \frac{\left(2\sqrt{x}\sqrt[4]{-1+x^4}\right)\mathrm{Subst}\left(\int \frac{\left(-1+x^8\right)^{3/4}\left(1+x^8\right)}{1+x^{16}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^6}} \\
&= \frac{\left(2\sqrt{x}\sqrt[4]{-1+x^4}\right)\mathrm{Subst}\left(\int \left(-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(-1+x^8\right)^{3/4}}{i-x^8}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-1+x^8\right)^{3/4}}{i+x^8}\right)dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^6}} \\
&= -\frac{\left((1-i)\sqrt{x}\sqrt[4]{-1+x^4}\right)\mathrm{Subst}\left(\int \frac{\left(-1+x^8\right)^{3/4}}{i-x^8}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^6}}+\frac{\left((1+i)\sqrt{x}\sqrt[4]{-1+x^4}\right)\mathrm{Subs}}{\sqrt[4]{-x^2}} \\
&= -\frac{\left((1-i)\sqrt{x}\left(-1+x^4\right)\right)\mathrm{Subst}\left(\int \frac{\left(1-x^8\right)^{3/4}}{i-x^8}\,dx, x, \sqrt{x}\right)}{\left(1-x^4\right)^{3/4}\sqrt[4]{-x^2+x^6}}+\frac{\left((1+i)\sqrt{x}\left(-1+x^4\right)\right)\mathrm{Subst}}{\left(1-x^4\right)^{3/4}\sqrt[4]{-}} \\
&= -\frac{(1-i)x\sqrt[4]{1-x^4}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};x^4,ix^4\right)}{\sqrt[4]{-x^2+x^6}}-\frac{(1+i)x\sqrt[4]{1-x^4}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};-ix^4,x^4\right)}{\sqrt[4]{-x^2+x^6}}
\end{aligned}$$

Mathematica [F] time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{-1+x^8}{\sqrt[4]{-x^2+x^6}\left(1+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^8)/((-x^2 + x^6)^(1/4)*(1 + x^8)),x]
```

```
[Out] Integrate[(-1 + x^8)/((-x^2 + x^6)^(1/4)*(1 + x^8)), x]
```

IntegrateAlgebraic [A] time = 27.55, size = 394, normalized size = 0.84

$$\frac{1}{8}\sqrt[4]{4+3\sqrt{2}}\log\left(2x^2-2\sqrt[4]{4+3\sqrt{2}}\sqrt[4]{x^6-x^2}\,x+2^{3/4}\sqrt{x^6-x^2}\right)-\frac{1}{8}\sqrt[4]{4+3\sqrt{2}}\log\left(\sqrt{2-\sqrt{2}}\,x^2+2^{3/8}\sqrt[4]{x^6-x^2}\,x+\sqrt{\sqrt{2}-1}\sqrt{x^6-x^2}\right)+\frac{1}{4}\sqrt[4]{3\sqrt{2}-4}\tan^{-1}\left(\frac{\sqrt[4]{6\sqrt{2}-8}\,x\sqrt[4]{x^6-x^2}}{\sqrt[4]{2}x^2-\sqrt{x^6-x^2}}\right)+\frac{1}{4}\sqrt[4]{4+3\sqrt{2}}\tan^{-1}\left(\frac{\sqrt[4]{8+6\sqrt{2}}\,x\sqrt[4]{x^6-x^2}}{\sqrt[4]{2}x^2-\sqrt{x^6-x^2}}\right)-\frac{1}{4}\sqrt[4]{3\sqrt{2}-4}\tanh^{-1}\left(\frac{2\sqrt[4]{\frac{1}{8}+\frac{3}{16\sqrt{2}}}\,x^2+2^{3/4}\sqrt[4]{\frac{1}{8}+\frac{3}{16\sqrt{2}}}\sqrt{x^6-x^2}}{x\sqrt[4]{x^6-x^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] IntegrateAlgebraic[(-1 + x^8)/((-x^2 + x^6)^(1/4)*(1 + x^8)),x]
```

```
[Out] ((-4 + 3*Sqrt[2])^(1/4)*ArcTan[((-8 + 6*Sqrt[2])^(1/4)*x*(-x^2 + x^6)^(1/4)
)/(2^(1/4)*x^2 - Sqrt[-x^2 + x^6])])/4 + ((4 + 3*Sqrt[2])^(1/4)*ArcTan[((8
+ 6*Sqrt[2])^(1/4)*x*(-x^2 + x^6)^(1/4))/(2^(1/4)*x^2 - Sqrt[-x^2 + x^6])])
```

```
/4 - ((-4 + 3*Sqrt[2])^(1/4)*ArcTanh[(2*(1/8 + 3/(16*Sqrt[2]))^(1/4)*x^2 +
2^(3/4)*(1/8 + 3/(16*Sqrt[2]))^(1/4)*Sqrt[-x^2 + x^6])/(x*(-x^2 + x^6)^(1/4
))])/4 + ((4 + 3*Sqrt[2])^(1/4)*Log[2*x^2 - 2*(4 + 3*Sqrt[2])^(1/4)*x*(-x^2
 + x^6)^(1/4) + 2^(3/4)*Sqrt[-x^2 + x^6]])/8 - ((4 + 3*Sqrt[2])^(1/4)*Log[S
qrt[2 - Sqrt[2]]*x^2 + 2^(3/8)*x*(-x^2 + x^6)^(1/4) + Sqrt[-1 + Sqrt[2]]*Sq
rt[-x^2 + x^6]])/8
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/(x^6-x^2)^(1/4)/(x^8+1),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8-1}{\left(x^8+1\right)\left(x^6-x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/(x^6-x^2)^(1/4)/(x^8+1),x, algorithm="giac")
[Out] integrate((x^8 - 1)/((x^8 + 1)*(x^6 - x^2)^(1/4)), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^8-1}{\left(x^6-x^2\right)^{\frac{1}{4}}\left(x^8+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-1)/(x^6-x^2)^(1/4)/(x^8+1),x)
[Out] int((x^8-1)/(x^6-x^2)^(1/4)/(x^8+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8-1}{\left(x^8+1\right)\left(x^6-x^2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/(x^6-x^2)^(1/4)/(x^8+1),x, algorithm="maxima")
[Out] integrate((x^8 - 1)/((x^8 + 1)*(x^6 - x^2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^8-1}{\left(x^8+1\right)\,{\left(x^6-x^2\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 - 1)/((x^8 + 1)*(x^6 - x^2)^(1/4)),x)
[Out] int((x^8 - 1)/((x^8 + 1)*(x^6 - x^2)^(1/4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-1)/(x**6-x**2)**(1/4)/(x**8+1),x)
```

```
[Out] Timed out
```

3.2377 $\int \frac{-1+x^8}{\sqrt[4]{-x^2+x^6}\left(1+x^8\right)} dx$

Optimal. Leaf size=469

$$\frac{1}{8}\sqrt[4]{4+3\sqrt{2}}\ \log\left(-2x^2+2^{7/8}\sqrt{2+\sqrt{2}}\ \sqrt[4]{x^6-x^2}\,x-2^{3/4}\sqrt{x^6-x^2}\right)-\frac{1}{8}\sqrt[4]{4+3\sqrt{2}}\ \log\left(2\sqrt{2-\sqrt{2}}\,x^2+2\,2^{3/8}\right.$$

Rubi [C] time = 0.48, antiderivative size = 101, normalized size of antiderivative = 0.22, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2056, 1586, 6715, 6725, 430, 429}

$$-\frac{(1-i)x\sqrt[4]{1-x^4}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};x^4,ix^4\right)}{\sqrt[4]{x^6-x^2}}-\frac{(1+i)x\sqrt[4]{1-x^4}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};-ix^4,x^4\right)}{\sqrt[4]{x^6-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-1 + x^8)/((-x^2 + x^6)^(1/4)*(1 + x^8)),x]
```

```
[Out] ((-1 + I)*x*(1 - x^4)^(1/4)*AppellF1[1/8, -3/4, 1, 9/8, x^4, I*x^4])/(-x^2
+ x^6)^(1/4) - ((1 + I)*x*(1 - x^4)^(1/4)*AppellF1[1/8, 1, -3/4, 9/8, (-I)*
x^4, x^4])/(-x^2 + x^6)^(1/4)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-1+x^8}{\sqrt[4]{-x^2+x^6}\,(1+x^8)}\,dx = \frac{\left(\sqrt{x}\,\sqrt[4]{-1+x^4}\right)\int \frac{-1+x^8}{\sqrt{x}\,\sqrt[4]{-1+x^4}\,(1+x^8)}\,dx}{\sqrt[4]{-x^2+x^6}}$$

$$= \frac{\left(\sqrt{x}\,\sqrt[4]{-1+x^4}\right)\int \frac{(-1+x^4)^{3/4}(1+x^4)}{\sqrt{x}\,(1+x^8)}\,dx}{\sqrt[4]{-x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{-1+x^4}\right)\text{Subst}\left(\int \frac{(-1+x^8)^{3/4}(1+x^8)}{1+x^{16}}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^6}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt[4]{-1+x^4}\right)\text{Subst}\left(\int \left(-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)(-1+x^8)^{3/4}}{i-x^8}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-1+x^8)^{3/4}}{i+x^8}\right)dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^6}}$$

$$= -\frac{\left((1-i)\sqrt{x}\,\sqrt[4]{-1+x^4}\right)\text{Subst}\left(\int \frac{(-1+x^8)^{3/4}}{i-x^8}\,dx, x, \sqrt{x}\right)}{\sqrt[4]{-x^2+x^6}} + \frac{\left((1+i)\sqrt{x}\,\sqrt[4]{-1+x^4}\right)\text{Subs}}{\sqrt[4]{-x^2}}$$

$$= -\frac{\left((1-i)\sqrt{x}\,(-1+x^4)\right)\text{Subst}\left(\int \frac{(1-x^8)^{3/4}}{i-x^8}\,dx, x, \sqrt{x}\right)}{(1-x^4)^{3/4}\,\sqrt[4]{-x^2+x^6}} + \frac{\left((1+i)\sqrt{x}\,(-1+x^4)\right)\text{Subst}}{(1-x^4)^{3/4}\,\sqrt[4]{-}}$$

$$= -\frac{(1-i)x\sqrt[4]{1-x^4}\,F_1\left(\frac{1}{8};-\frac{3}{4},1;\frac{9}{8};x^4,ix^4\right)}{\sqrt[4]{-x^2+x^6}} - \frac{(1+i)x\sqrt[4]{1-x^4}\,F_1\left(\frac{1}{8};1,-\frac{3}{4};\frac{9}{8};-ix^4,x^4\right)}{\sqrt[4]{-x^2+x^6}}$$

Mathematica [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{-1+x^8}{\sqrt[4]{-x^2+x^6}\,(1+x^8)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-1 + x^8)/((-x^2 + x^6)^(1/4)*(1 + x^8)),x]
```

```
[Out] Integrate[(-1 + x^8)/((-x^2 + x^6)^(1/4)*(1 + x^8)), x]
```

IntegrateAlgebraic [A] time = 0.00, size = 394, normalized size = 0.84

$$\frac{1}{8}\sqrt[4]{4+3\sqrt{2}}\log\left(2x^2-2\sqrt[4]{4+3\sqrt{2}}\,\sqrt[4]{x^6-x^2}\,x+2^{3/4}\sqrt{x^6-x^2}\right)-\frac{1}{8}\sqrt[4]{4+3\sqrt{2}}\log\left(\sqrt{2-\sqrt{2}}\,x^2+2^{3/8}\sqrt[4]{x^6-x^2}\,x+\sqrt{\sqrt{2}-1}\,\sqrt{x^6-x^2}\right)+\frac{1}{4}\sqrt[4]{3\sqrt{2}-4}\tan^{-1}\left(\frac{\sqrt[4]{6\sqrt{2}-8}\,x\sqrt[4]{x^6-x^2}}{\sqrt[4]{2}x^2-\sqrt{x^6-x^2}}\right)+\frac{1}{4}\sqrt[4]{4+3\sqrt{2}}\tan^{-1}\left(\frac{\sqrt[4]{8+6\sqrt{2}}\,x\sqrt[4]{x^6-x^2}}{\sqrt[4]{2}x^2-\sqrt{x^6-x^2}}\right)-\frac{1}{4}\sqrt[4]{3\sqrt{2}-4}\tanh^{-1}\left(\frac{2\sqrt[4]{\frac{1}{8}+\frac{3}{16\sqrt{2}}}\,x^2+2^{3/4}\sqrt[4]{\frac{1}{8}+\frac{3}{16\sqrt{2}}}\,\sqrt{x^6-x^2}}{x\sqrt[4]{x^6-x^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] IntegrateAlgebraic[(-1 + x^8)/((-x^2 + x^6)^(1/4)*(1 + x^8)),x]
```

```
[Out] ((-4 + 3*Sqrt[2])^(1/4)*ArcTan[((-8 + 6*Sqrt[2])^(1/4)*x*(-x^2 + x^6)^(1/4)
)/(2^(1/4)*x^2 - Sqrt[-x^2 + x^6])])/4 + ((4 + 3*Sqrt[2])^(1/4)*ArcTan[((8
+ 6*Sqrt[2])^(1/4)*x*(-x^2 + x^6)^(1/4))/(2^(1/4)*x^2 - Sqrt[-x^2 + x^6])])
```

```
/4 - ((-4 + 3*Sqrt[2])^(1/4)*ArcTanh[(2*(1/8 + 3/(16*Sqrt[2]))^(1/4)*x^2 +
2^(3/4)*(1/8 + 3/(16*Sqrt[2]))^(1/4)*Sqrt[-x^2 + x^6])/(x*(-x^2 + x^6)^(1/4
))])/4 + ((4 + 3*Sqrt[2])^(1/4)*Log[2*x^2 - 2*(4 + 3*Sqrt[2])^(1/4)*x*(-x^2
 + x^6)^(1/4) + 2^(3/4)*Sqrt[-x^2 + x^6]])/8 - ((4 + 3*Sqrt[2])^(1/4)*Log[S
qrt[2 - Sqrt[2]]*x^2 + 2^(3/8)*x*(-x^2 + x^6)^(1/4) + Sqrt[-1 + Sqrt[2]]*Sq
rt[-x^2 + x^6]])/8
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/(x^6-x^2)^(1/4)/(x^8+1),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 - 1}{\left(x^8 + 1\right)\left(x^6 - x^2\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/(x^6-x^2)^(1/4)/(x^8+1),x, algorithm="giac")
[Out] integrate((x^8 - 1)/((x^8 + 1)*(x^6 - x^2)^(1/4)), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 - 1}{\left(x^6 - x^2\right)^{\frac{1}{4}} \left(x^8 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8-1)/(x^6-x^2)^(1/4)/(x^8+1),x)
[Out] int((x^8-1)/(x^6-x^2)^(1/4)/(x^8+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 - 1}{\left(x^8 + 1\right)\left(x^6 - x^2\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8-1)/(x^6-x^2)^(1/4)/(x^8+1),x, algorithm="maxima")
[Out] integrate((x^8 - 1)/((x^8 + 1)*(x^6 - x^2)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^8 - 1}{\left(x^8 + 1\right)\,\left(x^6 - x^2\right)^{1/4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 - 1)/((x^8 + 1)*(x^6 - x^2)^(1/4)),x)
[Out] int((x^8 - 1)/((x^8 + 1)*(x^6 - x^2)^(1/4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8-1)/(x**6-x**2)**(1/4)/(x**8+1),x)
```

```
[Out] Timed out
```

3.2378 $\int \frac{\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=470

$$-\frac{15b\tan^{-1}\left(\frac{\sqrt[4]{\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[4]{c}}\right)}{32ac^{9/4}}+\frac{15b\tanh^{-1}\left(\frac{\sqrt[4]{\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[4]{c}}\right)}{32ac^{9/4}}+\frac{\left(3072ac^4x+4620bc\right)\left(\sqrt[3]{\sqrt{a^2x^2-b}+ax}+\right.}{}$$

Rubi [F] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*x + Sqrt[-b + a^2*x^2])^(1/3)/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))
^(1/4),x]
```

```
[Out] Defer[Int][(a*x + Sqrt[-b + a^2*x^2])^(1/3)/(c + (a*x + Sqrt[-b + a^2*x^2])
^(1/3))^(1/4), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Mathematica [F] time = 174.92, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*x + Sqrt[-b + a^2*x^2])^(1/3)/(c + (a*x + Sqrt[-b + a^2*x^2])^
(1/3))^(1/4),x]
```

```
[Out] Integrate[(a*x + Sqrt[-b + a^2*x^2])^(1/3)/(c + (a*x + Sqrt[-b + a^2*x^2])^
(1/3))^(1/4), x]
```

IntegrateAlgebraic [A] time = 1.03, size = 470, normalized size = 1.00

$$-\frac{15b\tan^{-1}\left(\frac{\sqrt[4]{\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[4]{c}}\right)}{32ac^{9/4}}+\frac{15b\tanh^{-1}\left(\frac{\sqrt[4]{\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[4]{c}}\right)}{32ac^{9/4}}+\frac{\left(3072ac^4x+4620bc\right)\left(\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c\right)^{3/4}+\sqrt[3]{\sqrt{a^2x^2-b}+ax}\left(-2688ac^3x-5775b\right)\left(\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c\right)^{3/4}+\left(\sqrt{a^2x^2-b}+ax\right)^{2/3}\left(2464ac^2x-4096c^5\right)\left(\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c\right)^{3/4}+\sqrt{a^2x^2-b}\left(3072c^4\left(\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c\right)^{3/4}-2688c^3\sqrt[3]{\sqrt{a^2x^2-b}+ax}\left(\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c\right)^{3/4}+2464c^2\left(\sqrt{a^2x^2-b}+ax\right)^{2/3}\left(\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c\right)^{3/4}\right)}{6160ac^2\left(\sqrt{a^2x^2-b}+ax\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x + Sqrt[-b + a^2*x^2])^(1/3)/(c + (a*x + Sqrt[-b + a
^2*x^2])^(1/3))^(1/4),x]
```

```
[Out] ((4620*b*c + 3072*a*c^4*x)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(3/4) + (
-5775*b - 2688*a*c^3*x)*(a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x + Sqrt[-
b + a^2*x^2])^(1/3))^(3/4) + (-4096*c^5 + 2464*a*c^2*x)*(a*x + Sqrt[-b + a^
2*x^2])^(2/3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(3/4) + Sqrt[-b + a^2*
x^2]*(3072*c^4*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(3/4) - 2688*c^3*(a*x
 + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(3/4) +
 2464*c^2*(a*x + Sqrt[-b + a^2*x^2])^(2/3)*(c + (a*x + Sqrt[-b + a^2*x^2])^
(1/3))^(3/4)))/(6160*a*c^2*(a*x + Sqrt[-b + a^2*x^2])^(2/3)) - (15*b*ArcTan
[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)/c^(1/4)])/(32*a*c^(9/4)) + (1
5*b*ArcTanh[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)/c^(1/4)])/(32*a*c^
(9/4))
```

fricas [A] time = 1.02, size = 433, normalized size = 0.92

$$\frac{23100\, a c^{2} \left(\frac{b^{4}}{a^{4} c^{9}}\right)^{\frac{1}{4}} \arctan\left(-\frac{a b^{3} \left(c + \left(a x + \sqrt{a^{2} x^{2} - b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}} c^{2} \left(\frac{b^{4}}{a^{4} c^{9}}\right)^{\frac{1}{4}} - \sqrt{a^{2} b^{4} c^{5} \sqrt{\frac{b^{4}}{a^{4} c^{9}}} + b^{6} \sqrt{c + \left(a x + \sqrt{a^{2} x^{2} - b}\right)^{\frac{1}{3}}}}\, a c^{2} \left(\frac{b^{4}}{a^{4} c^{9}}\right)^{\frac{1}{4}}}{b^{4}}\right) + 5775\, a c^{2} \left(\frac{b^{4}}{a^{4} c^{9}}\right)^{\frac{1}{4}} \log\left(3375\, a^{3} c^{7} \left(\frac{b^{4}}{a^{4} c^{9}}\right)^{\frac{3}{4}} + 3375\, b^{3} \left(c + \left(a x + \sqrt{a^{2} x^{2} - b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}\right) - 5775\, a c^{2} \left(\frac{b^{4}}{a^{4} c^{9}}\right)^{\frac{1}{4}} \log\left(-3375\, a^{3} c^{7} \left(\frac{b^{4}}{a^{4} c^{9}}\right)^{\frac{3}{4}} + 3375\, b^{3} \left(c + \left(a x + \sqrt{a^{2} x^{2} - b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}\right) - 4 \left(4096\, c^{5} - 2464\, a c^{2} x - 2464 \sqrt{a^{2} x^{2} - b}\, c^{2} + 21 \left(128\, c^{3} + 275\, a x - 275 \sqrt{a^{2} x^{2} - b}\right) \left(a x + \sqrt{a^{2} x^{2} - b}\right)^{\frac{2}{3}} - 12 \left(256\, c^{4} + 385\, a c x - 385 \sqrt{a^{2} x^{2} - b}\, c\right) \left(a x + \sqrt{a^{2} x^{2} - b}\right)^{\frac{1}{3}}\right) \left(c + \left(a x + \sqrt{a^{2} x^{2} - b}\right)^{\frac{1}{3}}\right)^{\frac{3}{4}}}{24640\, a c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/3))^(
1/4),x, algorithm="fricas")
```

```
[Out] 1/24640*(23100*a*c^2*(b^4/(a^4*c^9))^(1/4)*arctan(-(a*b^3*(c + (a*x + sqrt(
a^2*x^2 - b))^(1/3))^(1/4)*c^2*(b^4/(a^4*c^9))^(1/4) - sqrt(a^2*b^4*c^5*sqr
t(b^4/(a^4*c^9)) + b^6*sqrt(c + (a*x + sqrt(a^2*x^2 - b))^(1/3)))*a*c^2*(b^
4/(a^4*c^9))^(1/4))/b^4) + 5775*a*c^2*(b^4/(a^4*c^9))^(1/4)*log(3375*a^3*c^
7*(b^4/(a^4*c^9))^(3/4) + 3375*b^3*(c + (a*x + sqrt(a^2*x^2 - b))^(1/3))^(1
/4)) - 5775*a*c^2*(b^4/(a^4*c^9))^(1/4)*log(-3375*a^3*c^7*(b^4/(a^4*c^9))^(
3/4) + 3375*b^3*(c + (a*x + sqrt(a^2*x^2 - b))^(1/3))^(1/4)) - 4*(4096*c^5
- 2464*a*c^2*x - 2464*sqrt(a^2*x^2 - b)*c^2 + 21*(128*c^3 + 275*a*x - 275*s
qrt(a^2*x^2 - b))*(a*x + sqrt(a^2*x^2 - b))^(2/3) - 12*(256*c^4 + 385*a*c*x
 - 385*sqrt(a^2*x^2 - b)*c)*(a*x + sqrt(a^2*x^2 - b))^(1/3))*(c + (a*x + sq
rt(a^2*x^2 - b))^(1/3))^(3/4))/(a*c^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/3))^(
1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a x + \sqrt{a^{2} x^{2} - b}\right)^{\frac{1}{3}}}{\left(c + \left(a x + \sqrt{a^{2} x^{2} - b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/3))^(1/4),x
)
```

```
[Out] int((a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/3))^(1/4),x
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}}{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/3))^(
1/4),x, algorithm="maxima")
```

```
[Out] integrate((a*x + sqrt(a^2*x^2 - b))^(1/3)/(c + (a*x + sqrt(a^2*x^2 - b))^(1
/3))^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/3}}{\left(c+\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/3}\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + (a^2*x^2 - b)^(1/2))^(1/3)/(c + (a*x + (a^2*x^2 - b)^(1/2))^(1/3
))^(1/4),x)
```

```
[Out] int((a*x + (a^2*x^2 - b)^(1/2))^(1/3)/(c + (a*x + (a^2*x^2 - b)^(1/2))^(1/3
))^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{ax+\sqrt{a^2x^2-b}}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{a^2x^2-b}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a**2*x**2-b)**(1/2))**(1/3)/(c+(a*x+(a**2*x**2-b)**(1/2))**
(1/3))**(1/4),x)
```

```
[Out] Integral((a*x + sqrt(a**2*x**2 - b))**(1/3)/(c + (a*x + sqrt(a**2*x**2 - b)
)**(1/3))**(1/4), x)
```

3.2379 $\int \frac{b^2+ax}{\left(-b^2+ax\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$

Optimal. Leaf size=471

$$\frac{2\left(\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,\sqrt{\sqrt{a+b^2}+b}+\sqrt{2}\,\sqrt{b}\,\sqrt{a+b^2}\,\sqrt{\sqrt{a+b^2}+b}-\sqrt{2}\,b^{3/2}\sqrt{\sqrt{a+b^2}+b}\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{\sqrt{ax^2+b^2}-\sqrt{a}\,x}}{\sqrt{b}\,\sqrt{\sqrt{a+b^2}+b}}\right.}{a^{5/4}}$$

Rubi [F] time = 0.67, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{b^2+ax}{\left(-b^2+ax\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(b^2 + a*x)/((-b^2 + a*x)*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

```
[Out] Defer[Int][1/Sqrt[b + Sqrt[b^2 + a*x^2]], x] - 2*b^2*Defer[Int][1/((b^2 - a
*x)*Sqrt[b + Sqrt[b^2 + a*x^2]]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{b^2+ax}{\left(-b^2+ax\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx &= \int \left(\frac{1}{\sqrt{b+\sqrt{b^2+ax^2}}}-\frac{2b^2}{\left(b^2-ax\right)\sqrt{b+\sqrt{b^2+ax^2}}}\right)dx\\ &= -\left(\left(2b^2\right)\int \frac{1}{\left(b^2-ax\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx\right)+\int \frac{1}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx\end{aligned}$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{b^2+ax}{\left(-b^2+ax\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(b^2 + a*x)/((-b^2 + a*x)*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

```
[Out] Integrate[(b^2 + a*x)/((-b^2 + a*x)*Sqrt[b + Sqrt[b^2 + a*x^2]]), x]
```

IntegrateAlgebraic [C] time = 4.58, size = 1138, normalized size = 2.42

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^2 + a*x)/((-b^2 + a*x)*Sqrt[b + Sqrt[b^2 + a*x^2]]),x
]
```

```
[Out] (2*x)/Sqrt[b + Sqrt[b^2 + a*x^2]] + (2*Sqrt[2]*Sqrt[b]*ArcTan[(Sqrt[a]*x)/(
Sqrt[2]*Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]]) - Sqrt[b + Sqrt[b^2 + a*x^2]]/
```

```
(Sqrt[2]*Sqrt[b])])/Sqrt[a] - (2*I)*Sqrt[a]*b^3*RootSum[16*a^4*b^4 - 8*a^2*
b^2*#1^4 - 16*a*b^4*#1^4 + #1^8 & , (Log[(I*a*x)/Sqrt[b + Sqrt[b^2 + a*x^2]
] - I*Sqrt[a]*Sqrt[b + Sqrt[b^2 + a*x^2]] - #1]*#1)/(-4*a^2*b^2 - 8*a*b^4 +
 #1^4) & ] - (I*b^2*RootSum[16*a^4*b^4 - 8*a^2*b^2*#1^4 - 16*a*b^4*#1^4 + #
1^8 & , (Log[(I*a*x)/Sqrt[b + Sqrt[b^2 + a*x^2]] - I*Sqrt[a]*Sqrt[b + Sqrt[
b^2 + a*x^2]] - #1]*#1^3)/(-4*a^2*b^2 - 8*a*b^4 + #1^4) & ])/Sqrt[a] + (4*I
)*a^(3/2)*b^4*RootSum[16*a^4*b^4 - 8*a^2*b^2*#1^4 - 16*a*b^4*#1^4 + #1^8 &
, Log[(I*a*x)/Sqrt[b + Sqrt[b^2 + a*x^2]] - I*Sqrt[a]*Sqrt[b + Sqrt[b^2 + a
*x^2]] - #1]/(4*a^2*b^2*#1 + 8*a*b^4*#1 - #1^5) & ] + (8*I)*a^(5/2)*b^5*Roo
tSum[16*a^4*b^4 - 8*a^2*b^2*#1^4 - 16*a*b^4*#1^4 + #1^8 & , Log[(I*a*x)/Sqr
t[b + Sqrt[b^2 + a*x^2]] - I*Sqrt[a]*Sqrt[b + Sqrt[b^2 + a*x^2]] - #1]/(4*a
^2*b^2*#1^3 + 8*a*b^4*#1^3 - #1^7) & ] - I*b*RootSum[16*a^4*b^4 - 8*a^2*b^2
*#1^4 - 16*a*b^4*#1^4 + #1^8 & , (8*a^3*b^3*Log[(I*a*x)/Sqrt[b + Sqrt[b^2 +
 a*x^2]] - I*Sqrt[a]*Sqrt[b + Sqrt[b^2 + a*x^2]] - #1] + 4*a^2*b^2*Log[(I*a
*x)/Sqrt[b + Sqrt[b^2 + a*x^2]] - I*Sqrt[a]*Sqrt[b + Sqrt[b^2 + a*x^2]] - #
1]*#1^2 + 8*a*b^4*Log[(I*a*x)/Sqrt[b + Sqrt[b^2 + a*x^2]] - I*Sqrt[a]*Sqrt[
b + Sqrt[b^2 + a*x^2]] - #1]*#1^2 - 2*a*b*Log[(I*a*x)/Sqrt[b + Sqrt[b^2 + a
*x^2]] - I*Sqrt[a]*Sqrt[b + Sqrt[b^2 + a*x^2]] - #1]*#1^4 - 4*b^3*Log[(I*a*
x)/Sqrt[b + Sqrt[b^2 + a*x^2]] - I*Sqrt[a]*Sqrt[b + Sqrt[b^2 + a*x^2]] - #1
]*#1^4 - Log[(I*a*x)/Sqrt[b + Sqrt[b^2 + a*x^2]] - I*Sqrt[a]*Sqrt[b + Sqrt[
b^2 + a*x^2]] - #1]*#1^6)/(4*a^2*b^2*#1^3 + 8*a*b^4*#1^3 - #1^7) & ]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b^2)/(a*x-b^2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b^2 + ax}{\left(b^2 - ax\right)\sqrt{b + \sqrt{ax^2 + b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b^2)/(a*x-b^2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(-(b^2 + a*x)/((b^2 - a*x)*sqrt(b + sqrt(a*x^2 + b^2))), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{ax + b^2}{\left(ax - b^2\right)\sqrt{b + \sqrt{a\,x^2 + b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b^2)/(a*x-b^2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x+b^2)/(a*x-b^2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{b^2 + ax}{\left(b^2 - ax\right)\sqrt{b + \sqrt{ax^2 + b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b^2)/(a*x-b^2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm="max
ima")
```

```
[Out] -integrate((b^2 + a*x)/((b^2 - a*x)*sqrt(b + sqrt(a*x^2 + b^2))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b^2 + a\,x}{\left(a\,x - b^2\right)\,\sqrt{b + \sqrt{b^2 + a\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b^2)/((a*x - b^2)*(b + (a*x^2 + b^2)^(1/2))^(1/2)),x)
```

```
[Out] int((a*x + b^2)/((a*x - b^2)*(b + (a*x^2 + b^2)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax + b^2}{\sqrt{b + \sqrt{ax^2 + b^2}}\,\left(ax - b^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b**2)/(a*x-b**2)/(b+(a*x**2+b**2)**(1/2))**(1/2),x)
```

```
[Out] Integral((a*x + b**2)/(sqrt(b + sqrt(a*x**2 + b**2))*(a*x - b**2)), x)
```

3.2380 $\int \frac{\left(-b+ax^4\right)^{3/4}}{b-2ax^4+2x^8}\,dx$

Optimal. Leaf size=472

$$\frac{\left(1+\sqrt[4]{-1}\right)\tan^{-1}\left(\frac{(-1)^{7/8}\sqrt{2+\sqrt{2}}\,x\sqrt[8]{a^2-2b}\,\sqrt[4]{ax^4-b}}{(-1)^{3/4}x^2\sqrt[4]{a^2-2b}+\sqrt{ax^4-b}}\right)}{8\sqrt[8]{a^2-2b}}-\frac{i\left(\sqrt{2\left(3-2\sqrt{2}\right)}-i\sqrt{2}\right)\tan^{-1}\left(\frac{(-1)^{7/8}\left(\sqrt{2}-2\right)x\sqrt[8]{a^2-2b}\,\sqrt[4]{ax^4-\ldots}}{(-1)^{3/4}\sqrt{2-\sqrt{2}}\,x^2\sqrt[4]{a^2-2b}+\sqrt{2-\sqrt{2}}\,\ldots}\right.}{16\sqrt[8]{a^2-2b}}$$

Rubi [A] time = 0.60, antiderivative size = 465, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {1428, 408, 240, 212, 206, 203, 377, 208, 205}

$$\frac{\left(-a\sqrt{a^2-2b}+a^2-2b\right)^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{a-\sqrt{a^2-2b}}\,\sqrt[4]{ax^4-b}}\right)}{4\left(a-\sqrt{a^2-2b}\right)^{3/4}\sqrt{a^2-2b}}-\frac{\left(a\sqrt{a^2-2b}+a^2-2b\right)^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{\sqrt{a^2-2b}+a}\,\sqrt[4]{ax^4-b}}\right)}{4\left(\sqrt{a^2-2b}+a\right)^{3/4}\sqrt{a^2-2b}}+\frac{\left(-a\sqrt{a^2-2b}+a^2-2b\right)^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{a-\sqrt{a^2-2b}}\,\sqrt[4]{ax^4-b}}\right)}{4\left(a-\sqrt{a^2-2b}\right)^{3/4}\sqrt{a^2-2b}}-\frac{\left(a\sqrt{a^2-2b}+a^2-2b\right)^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{\sqrt{a^2-2b}+a}\,\sqrt[4]{ax^4-b}}\right)}{4\left(\sqrt{a^2-2b}+a\right)^{3/4}\sqrt{a^2-2b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^4)^(3/4)/(b - 2*a*x^4 + 2*x^8),x]
```

```
[Out] ((a^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(3/4)*ArcTan[((a^2 - a*Sqrt[a^2 - 2*b] - 2
*b)^(1/4)*x)/((a - Sqrt[a^2 - 2*b])^(1/4)*(-b + a*x^4)^(1/4))])/(4*(a - Sqr
t[a^2 - 2*b])^(3/4)*Sqrt[a^2 - 2*b]) - ((a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(3/
4)*ArcTan[((a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)*x)/((a + Sqrt[a^2 - 2*b])^
(1/4)*(-b + a*x^4)^(1/4))])/(4*(a + Sqrt[a^2 - 2*b])^(3/4)*Sqrt[a^2 - 2*b])
 + ((a^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(3/4)*ArcTanh[((a^2 - a*Sqrt[a^2 - 2*b]
 - 2*b)^(1/4)*x)/((a - Sqrt[a^2 - 2*b])^(1/4)*(-b + a*x^4)^(1/4))])/(4*(a -
 Sqrt[a^2 - 2*b])^(3/4)*Sqrt[a^2 - 2*b]) - ((a^2 + a*Sqrt[a^2 - 2*b] - 2*b)
^(3/4)*ArcTanh[((a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)*x)/((a + Sqrt[a^2 - 2
*b])^(1/4)*(-b + a*x^4)^(1/4))])/(4*(a + Sqrt[a^2 - 2*b])^(3/4)*Sqrt[a^2 -
2*b])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 408

```
Int[((a_) + (b_.)*(x_)^4)^(p_)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d
, Int[(a + b*x^4)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(p
- 1)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[p, 3/4] || EqQ[p, 5/4])
```

Rule 1428

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^
n)^q/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[(d + e*x^n)^q/(b + r + 2
*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{\left(-b+ax^4\right)^{3/4}}{b-2ax^4+2x^8}\,dx = \frac{2\int \frac{\left(-b+ax^4\right)^{3/4}}{-2a-2\sqrt{a^2-2b}+4x^4}\,dx}{\sqrt{a^2-2b}} - \frac{2\int \frac{\left(-b+ax^4\right)^{3/4}}{-2a+2\sqrt{a^2-2b}+4x^4}\,dx}{\sqrt{a^2-2b}}$$

$$= \frac{\left(a\left(a+\sqrt{a^2-2b}\right)-2b\right)\int \frac{1}{\left(-2a-2\sqrt{a^2-2b}+4x^4\right)\sqrt[4]{-b+ax^4}}\,dx}{\sqrt{a^2-2b}} + \frac{\left(a\left(-2a+2\sqrt{a^2-2b}\right)+4b\right)\int}{2\sqrt{a^2}}$$

$$= \frac{\left(a\left(a+\sqrt{a^2-2b}\right)-2b\right)\text{Subst}\left(\int \frac{1}{-2a-2\sqrt{a^2-2b}-\left(a\left(-2a-2\sqrt{a^2-2b}\right)+4b\right)x^4}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{\sqrt{a^2-2b}} + \frac{(a}{}$$

$$= -\frac{\left(a\left(a+\sqrt{a^2-2b}\right)-2b\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\sqrt{a^2-2b}}-\sqrt{a^2+a\sqrt{a^2-2b}-2b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{4\sqrt{a+\sqrt{a^2-2b}}\,\sqrt{a^2-2b}} - \frac{(a(a}{}$$

$$= \frac{\left(a^2-a\sqrt{a^2-2b}-2b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a^2-a\sqrt{a^2-2b}-2b}\,x}{\sqrt[4]{a-\sqrt{a^2-2b}}\,\sqrt[4]{-b+ax^4}}\right)}{4\left(a-\sqrt{a^2-2b}\right)^{3/4}\sqrt{a^2-2b}} - \frac{\left(a^2+a\sqrt{a^2-2b}-2b\right)^{3/4}\tan^{-1}\left(\right.}{4\left(a+\sqrt{a^2-2b}\right)^{3/4}\sqrt{}}$$

Mathematica [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(-b+ax^4\right)^{3/4}}{b-2ax^4+2x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(-b + a*x^4)^(3/4)/(b - 2*a*x^4 + 2*x^8),x]
```

```
[Out] Integrate[(-b + a*x^4)^(3/4)/(b - 2*a*x^4 + 2*x^8), x]
```

IntegrateAlgebraic [A] time = 42.14, size = 534, normalized size = 1.13

$$\frac{i\left(\sqrt{2\left(3+2\sqrt{2}\right)}+i\sqrt{2}\right)\tan^{-1}\left(\frac{\left((-1+i)-(1+i)(-1)^{3/4}\right)x^2\sqrt[4]{a^2-2b}+(1+i)(-1)^{3/4}\sqrt{ax^4-b}+(1+i)\sqrt{ax^4-b}}{2x\sqrt[8]{a^2-2b}\sqrt[4]{ax^4-b}}\right)}{16\sqrt[8]{a^2-2b}}+\frac{\left(\sqrt{2}+i\sqrt{2\left(3-2\sqrt{2}\right)}\right)\tan^{-1}\left(\frac{2x\sqrt[8]{a^2-2b}\sqrt[4]{ax^4-b}}{\left(\sqrt{2}+(1-i)\right)x^2\sqrt[4]{a^2-2b}-\sqrt{2}\sqrt{ax^4-b}-(1+i)\sqrt{ax^4-b}}\right)}{16\sqrt[8]{a^2-2b}}+\frac{\left((-1)^{3/4}+i\right)\tanh^{-1}\left(\frac{\left((-2+2i)-(2+2i)(-1)^{3/4}\right)x^2\sqrt[4]{a^2-2b}-(2+2i)(-1)^{3/4}\sqrt{ax^4-b}-(2+2i)\sqrt{ax^4-b}}{4x\sqrt[8]{a^2-2b}\sqrt[4]{ax^4-b}}\right)}{8\sqrt[8]{a^2-2b}}-\frac{i\left(\sqrt{2\left(3-2\sqrt{2}\right)}-i\sqrt{2}\right)\tanh^{-1}\left(\frac{\left(\sqrt{2}+(1-i)\right)x^2\sqrt[4]{a^2-2b}+\sqrt{2}\sqrt{ax^4-b}+(1+i)\sqrt{ax^4-b}}{2x\sqrt[8]{a^2-2b}\sqrt[4]{ax^4-b}}\right)}{16\sqrt[8]{a^2-2b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + a*x^4)^(3/4)/(b - 2*a*x^4 + 2*x^8),x]
```

```
[Out] ((I/16)*(I*Sqrt[2] + Sqrt[2*(3 + 2*Sqrt[2])])*ArcTan[(((-1 + I) - (1 + I)*(
-1)^(3/4))*(a^2 - 2*b)^(1/4)*x^2 + (1 + I)*Sqrt[-b + a*x^4] + (1 + I)*(-1)^
(3/4)*Sqrt[-b + a*x^4])/(2*(a^2 - 2*b)^(1/8)*x*(-b + a*x^4)^(1/4))])/(a^2 -
 2*b)^(1/8) + ((Sqrt[2] + I*Sqrt[2*(3 - 2*Sqrt[2])])*ArcTan[(2*(a^2 - 2*b)^
(1/8)*x*(-b + a*x^4)^(1/4))/(((1 - I) + Sqrt[2])*(a^2 - 2*b)^(1/4)*x^2 - (1
 + I)*Sqrt[-b + a*x^4] - Sqrt[2]*Sqrt[-b + a*x^4])])/(16*(a^2 - 2*b)^(1/8))
 + ((I + (-1)^(3/4))*ArcTanh[(((-2 + 2*I) - (2 + 2*I)*(-1)^(3/4))*(a^2 - 2*
b)^(1/4)*x^2 - (2 + 2*I)*Sqrt[-b + a*x^4] - (2 + 2*I)*(-1)^(3/4)*Sqrt[-b +
a*x^4])/(4*(a^2 - 2*b)^(1/8)*x*(-b + a*x^4)^(1/4))])/(8*(a^2 - 2*b)^(1/8))
- ((I/16)*((-I)*Sqrt[2] + Sqrt[2*(3 - 2*Sqrt[2])])*ArcTanh[(((1 - I) + Sqrt
[2])*(a^2 - 2*b)^(1/4)*x^2 + (1 + I)*Sqrt[-b + a*x^4] + Sqrt[2]*Sqrt[-b + a
*x^4])/(2*(a^2 - 2*b)^(1/8)*x*(-b + a*x^4)^(1/4))])/(a^2 - 2*b)^(1/8)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4)/(2*x^8-2*a*x^4+b),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^4-b\right)^{\frac{3}{4}}}{2\,x^8-2\,ax^4+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4)/(2*x^8-2*a*x^4+b),x, algorithm="giac")
```

```
[Out] integrate((a*x^4 - b)^(3/4)/(2*x^8 - 2*a*x^4 + b), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^4-b\right)^{\frac{3}{4}}}{2x^8-2a\,x^4+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4-b)^(3/4)/(2*x^8-2*a*x^4+b),x)
```

```
[Out] int((a*x^4-b)^(3/4)/(2*x^8-2*a*x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{4}-b\right)^{\frac{3}{4}}}{2\,x^{8}-2\,ax^{4}+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4-b)^(3/4)/(2*x^8-2*a*x^4+b),x, algorithm="maxima")
[Out] integrate((a*x^4 - b)^(3/4)/(2*x^8 - 2*a*x^4 + b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x^{4}-b\right)^{3/4}}{2\,x^{8}-2\,a\,x^{4}+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4 - b)^(3/4)/(b - 2*a*x^4 + 2*x^8),x)
[Out] int((a*x^4 - b)^(3/4)/(b - 2*a*x^4 + 2*x^8), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4-b)**(3/4)/(2*x**8-2*a*x**4+b),x)
[Out] Timed out
```

3.2381 $\int \frac{x^4}{\sqrt[4]{-b+ax^4}\left(b-2ax^4+2x^8\right)}\,dx$

Optimal. Leaf size=476

$$\frac{\left(1-\sqrt[4]{-1}\right)\tan^{-1}\left(\frac{(-1)^{7/8}\sqrt{2+\sqrt{2}}\,x\sqrt[8]{a^2-2b}\,\sqrt[4]{ax^4-b}}{(-1)^{3/4}x^2\sqrt[4]{a^2-2b}+\sqrt{ax^4-b}}\right)}{8\left(a^2-2b\right)^{5/8}}+\frac{\left(\sqrt{2}-i\sqrt{2\left(3+2\sqrt{2}\right)}\right)\tan^{-1}\left(\frac{(-1)^{7/8}\left(\sqrt{2}-2\right)x\sqrt[8]{a^2-2b}\,\sqrt[4]{ax^4-b}}{(-1)^{3/4}\sqrt{2-\sqrt{2}}\,x^2\sqrt[4]{a^2-2b}+\sqrt{2-\sqrt{2}}\,\sqrt{\ldots}}\right)}{16\left(a^2-2b\right)^{5/8}}$$

Rubi [A] time = 0.38, antiderivative size = 465, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1528, 377, 212, 208, 205}

$$\frac{\sqrt[4]{a-\sqrt{a^2-2b}}\tan^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{a-\sqrt{a^2-2b}}\sqrt[4]{ax^4-b}}\right)}{4\sqrt{a^2-2b}\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}-\frac{\sqrt[4]{\sqrt{a^2-2b}+a}\tan^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{\sqrt{a^2-2b}+a}\sqrt[4]{ax^4-b}}\right)}{4\sqrt{a^2-2b}\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}+\frac{\sqrt[4]{a-\sqrt{a^2-2b}}\tanh^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{a-\sqrt{a^2-2b}}\sqrt[4]{ax^4-b}}\right)}{4\sqrt{a^2-2b}\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}-\frac{\sqrt[4]{\sqrt{a^2-2b}+a}\tanh^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{\sqrt{a^2-2b}+a}\sqrt[4]{ax^4-b}}\right)}{4\sqrt{a^2-2b}\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((-b + a*x^4)^(1/4)*(b - 2*a*x^4 + 2*x^8)),x]
```

```
[Out] ((a - Sqrt[a^2 - 2*b])^(1/4)*ArcTan[((a^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)*
x)/((a - Sqrt[a^2 - 2*b])^(1/4)*(-b + a*x^4)^(1/4))]])/(4*Sqrt[a^2 - 2*b]*(a
^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)) - ((a + Sqrt[a^2 - 2*b])^(1/4)*ArcTan[
((a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)*x)/((a + Sqrt[a^2 - 2*b])^(1/4)*(-b
+ a*x^4)^(1/4))]])/(4*Sqrt[a^2 - 2*b]*(a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(1/4))
 + ((a - Sqrt[a^2 - 2*b])^(1/4)*ArcTanh[((a^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(1
/4)*x)/((a - Sqrt[a^2 - 2*b])^(1/4)*(-b + a*x^4)^(1/4))]])/(4*Sqrt[a^2 - 2*b
]*(a^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)) - ((a + Sqrt[a^2 - 2*b])^(1/4)*Arc
Tanh[((a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)*x)/((a + Sqrt[a^2 - 2*b])^(1/4)
*(-b + a*x^4)^(1/4))]])/(4*Sqrt[a^2 - 2*b]*(a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(
1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1528

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q, (
```

```
f*x)^m/(a + b*x^n + c*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, e, f, q, n}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&  !IntegerQ[q] &&
IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{-b+ax^4}\left(b-2ax^4+2x^8\right)}\,dx = \int \left(\frac{1+\frac{a}{\sqrt{a^2-2b}}}{\left(-2a-2\sqrt{a^2-2b}+4x^4\right)\sqrt[4]{-b+ax^4}} + \frac{1-\frac{a}{\sqrt{a^2-2b}}}{\left(-2a+2\sqrt{a^2-2b}+4x^4\right)\sqrt[4]{-\cdots}} \right.$$

$$= \left(1-\frac{a}{\sqrt{a^2-2b}}\right)\int \frac{1}{\left(-2a+2\sqrt{a^2-2b}+4x^4\right)\sqrt[4]{-b+ax^4}}\,dx + \left(1+\frac{a}{\sqrt{a^2-\cdots}}\right.$$

$$= \left(1-\frac{a}{\sqrt{a^2-2b}}\right)\text{Subst}\left(\int \frac{1}{-2a+2\sqrt{a^2-2b}-\left(a\left(-2a+2\sqrt{a^2-2b}\right)+4b\cdots\right.}\right.$$

$$= \frac{\sqrt{a-\sqrt{a^2-2b}}\ \text{Subst}\left(\int \frac{1}{\sqrt{a-\sqrt{a^2-2b}}-\sqrt{a^2-a\sqrt{a^2-2b}-2b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{-b+ax^4}}\right)}{4\sqrt{a^2-2b}} + \cdots$$

$$= \frac{\sqrt[4]{a-\sqrt{a^2-2b}}\ \tan^{-1}\left(\frac{\sqrt[4]{a^2-a\sqrt{a^2-2b}-2b}\,x}{\sqrt[4]{a-\sqrt{a^2-2b}}\ \sqrt[4]{-b+ax^4}}\right)}{4\sqrt{a^2-2b}\ \sqrt[4]{a^2-a\sqrt{a^2-2b}-2b}} - \frac{\sqrt[4]{a+\sqrt{a^2-2b}}\ \tan^{-1}\left(\frac{\sqrt[4]{a^2\cdots}}{\sqrt[4]{a+\cdots}}\right.}{4\sqrt{a^2-2b}\ \sqrt[4]{a^2+a\sqrt{a^2\cdots}}}$$

Mathematica [A] time = 0.73, size = 426, normalized size = 0.89

$$\frac{\frac{\sqrt[4]{a-\sqrt{a^2-2b}}\ \tan^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{a-\sqrt{a^2-2b}}\ \sqrt[4]{ax^4-b}}\right)}{\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}} - \frac{\sqrt[4]{\sqrt{a^2-2b}+a}\ \tan^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{\sqrt{a^2-2b}+a}\ \sqrt[4]{ax^4-b}}\right)}{\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}} + \frac{\sqrt[4]{a-\sqrt{a^2-2b}}\ \tanh^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{a-\sqrt{a^2-2b}}\ \sqrt[4]{ax^4-b}}\right)}{\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}} - \frac{\sqrt[4]{\sqrt{a^2-2b}+a}\ \tanh^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{\sqrt{a^2-2b}+a}\ \sqrt[4]{ax^4-b}}\right)}{\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}}{4\sqrt{a^2-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((-b + a*x^4)^(1/4)*(b - 2*a*x^4 + 2*x^8)),x]
```

```
[Out] (((a - Sqrt[a^2 - 2*b])^(1/4)*ArcTan[((a^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)
*x)/((a - Sqrt[a^2 - 2*b])^(1/4)*(-b + a*x^4)^(1/4))])/(a^2 - a*Sqrt[a^2 -
2*b] - 2*b)^(1/4) - ((a + Sqrt[a^2 - 2*b])^(1/4)*ArcTan[((a^2 + a*Sqrt[a^2
- 2*b] - 2*b)^(1/4)*x)/((a + Sqrt[a^2 - 2*b])^(1/4)*(-b + a*x^4)^(1/4))])/(
a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(1/4) + ((a - Sqrt[a^2 - 2*b])^(1/4)*ArcTanh
[((a^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)*x)/((a - Sqrt[a^2 - 2*b])^(1/4)*(-b
 + a*x^4)^(1/4))])/(a^2 - a*Sqrt[a^2 - 2*b] - 2*b)^(1/4) - ((a + Sqrt[a^2 -
 2*b])^(1/4)*ArcTanh[((a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(1/4)*x)/((a + Sqrt[a
^2 - 2*b])^(1/4)*(-b + a*x^4)^(1/4))])/(a^2 + a*Sqrt[a^2 - 2*b] - 2*b)^(1/4
))/(4*Sqrt[a^2 - 2*b])
```

IntegrateAlgebraic [A] time = 45.77, size = 534, normalized size = 1.12

$$\frac{\left(\sqrt{2}+i\sqrt{2\left(3-2\sqrt{2}\right)}\right)\tan^{-1}\left(\frac{\left((-1+i)-(1+i)(-1)^{3/4}\right)x^2\sqrt[4]{a^2-2b}+(1+i)(-1)^{3/4}\sqrt{ax^4-b}+(1+i)\sqrt{ax^4-b}}{2x\sqrt[8]{a^2-2b}\ \sqrt[4]{ax^4-b}}\right)}{16\left(a^2-2b\right)^{5/8}} + \frac{i\left(\sqrt{2\left(3+2\sqrt{2}\right)}+i\sqrt{2}\right)\tan^{-1}\left(\frac{2x\sqrt[8]{a^2-2b}\ \sqrt[4]{ax^4-b}}{\left(\sqrt{2}+(1-i)\right)x^2\sqrt[4]{a^2-2b}-\sqrt{2}\sqrt{ax^4-b}-(1+i)\sqrt{ax^4-b}}\right)}{16\left(a^2-2b\right)^{5/8}} + \frac{\left(-(-1)^{3/4}+i\right)\tanh^{-1}\left(\frac{\left((-2+2i)-(2+2i)(-1)^{3/4}\right)x^2\sqrt[4]{a^2-2b}-(2+2i)(-1)^{3/4}\sqrt{ax^4-b}-(2+2i)\sqrt{ax^4-b}}{4x\sqrt[8]{a^2-2b}\ \sqrt[4]{ax^4-b}}\right)}{8\left(a^2-2b\right)^{5/8}} + \frac{\left(\sqrt{2}-i\sqrt{2\left(3+2\sqrt{2}\right)}\right)\tanh^{-1}\left(\frac{\left(\sqrt{2}+(1-i)\right)x^2\sqrt[4]{a^2-2b}+\sqrt{2}\sqrt{ax^4-b}+(1+i)\sqrt{ax^4-b}}{2x\sqrt[8]{a^2-2b}\ \sqrt[4]{ax^4-b}}\right)}{16\left(a^2-2b\right)^{5/8}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((-b + a*x^4)^(1/4)*(b - 2*a*x^4 + 2*x^8)),x]
```

```
[Out] ((Sqrt[2] + I*Sqrt[2*(3 - 2*Sqrt[2])])*ArcTan[(((-1 + I) - (1 + I)*(-1)^(3/
4))*(a^2 - 2*b)^(1/4)*x^2 + (1 + I)*Sqrt[-b + a*x^4] + (1 + I)*(-1)^(3/4)*S
```

```
qrt[-b + a*x^4])/(2*(a^2 - 2*b)^(1/8)*x*(-b + a*x^4)^(1/4))])/(16*(a^2 - 2*
b)^(5/8)) + ((I/16)*(I*Sqrt[2] + Sqrt[2*(3 + 2*Sqrt[2])])*ArcTan[(2*(a^2 -
2*b)^(1/8)*x*(-b + a*x^4)^(1/4))/(((1 - I) + Sqrt[2])*(a^2 - 2*b)^(1/4)*x^2
 - (1 + I)*Sqrt[-b + a*x^4] - Sqrt[2]*Sqrt[-b + a*x^4])])/(a^2 - 2*b)^(5/8)
 + ((I - (-1)^(3/4))*ArcTanh[(((-2 + 2*I) - (2 + 2*I)*(-1)^(3/4))*(a^2 - 2*
b)^(1/4)*x^2 - (2 + 2*I)*Sqrt[-b + a*x^4] - (2 + 2*I)*(-1)^(3/4)*Sqrt[-b +
a*x^4])/(4*(a^2 - 2*b)^(1/8)*x*(-b + a*x^4)^(1/4))])/(8*(a^2 - 2*b)^(5/8))
+ ((Sqrt[2] - I*Sqrt[2*(3 + 2*Sqrt[2])])*ArcTanh[(((1 - I) + Sqrt[2])*(a^2
- 2*b)^(1/4)*x^2 + (1 + I)*Sqrt[-b + a*x^4] + Sqrt[2]*Sqrt[-b + a*x^4])/(2*
(a^2 - 2*b)^(1/8)*x*(-b + a*x^4)^(1/4))])/(16*(a^2 - 2*b)^(5/8))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4-b)^(1/4)/(2*x^8-2*a*x^4+b),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(2\,x^8-2\,ax^4+b\right)\left(ax^4-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4-b)^(1/4)/(2*x^8-2*a*x^4+b),x, algorithm="giac")
[Out] integrate(x^4/((2*x^8 - 2*a*x^4 + b)*(a*x^4 - b)^(1/4)), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a\,x^4-b\right)^{\frac{1}{4}}\left(2x^8-2a\,x^4+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^4-b)^(1/4)/(2*x^8-2*a*x^4+b),x)
[Out] int(x^4/(a*x^4-b)^(1/4)/(2*x^8-2*a*x^4+b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(2\,x^8-2\,ax^4+b\right)\left(ax^4-b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4-b)^(1/4)/(2*x^8-2*a*x^4+b),x, algorithm="maxima")
[Out] integrate(x^4/((2*x^8 - 2*a*x^4 + b)*(a*x^4 - b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(a\,x^4-b\right)}^{1/4}\,\left(2\,x^8-2\,a\,x^4+b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a*x^4 - b)^(1/4)*(b - 2*a*x^4 + 2*x^8)),x)
```

```
[Out] int(x^4/((a*x^4 - b)^(1/4)*(b - 2*a*x^4 + 2*x^8)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a*x**4-b)**(1/4)/(2*x**8-2*a*x**4+b),x)
```

```
[Out] Timed out
```

3.2382 $\int \frac{x^3(-5b+6ax)}{\sqrt[4]{-bx+ax^2}\left(c-bx^5+ax^6\right)}\,dx$

Optimal. Leaf size=477

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[3]{c}\,x^2\sqrt[4]{ax^2-bx}-2^{2/3}x\sqrt[4]{ax^2-bx}}{-\sqrt[3]{c}\,x^2\sqrt[4]{ax^2-bx}+2^{2/3}x\sqrt[4]{ax^2-bx}-\sqrt{2}\,c^{7/12}x+2\sqrt[6]{2}\sqrt[4]{c}}\right)}{\sqrt[4]{c}}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[3]{c}\,x^2\sqrt[4]{ax^2-bx}-2^{2/3}x\sqrt[4]{ax^2-bx}}{-\sqrt[3]{c}\,x^2\sqrt[4]{ax^2-bx}+2^{2/3}x\sqrt[4]{ax^2-bx}+\sqrt{2}\,c^{7/12}x-2\sqrt[6]{2}\sqrt[4]{c}}\right.}{\sqrt[4]{c}}$$

Rubi [F] time = 3.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3(-5b+6ax)}{\sqrt[4]{-bx+ax^2}\left(c-bx^5+ax^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^3*(-5*b + 6*a*x))/((-(b*x) + a*x^2)^(1/4)*(c - b*x^5 + a*x^6)),x]
```

```
[Out] (20*b*x^(1/4)*(-b + a*x)^(1/4)*Defer[Subst][Defer[Int][x^14/((-b + a*x^4)^(
1/4)*(-c + b*x^20 - a*x^24)), x], x, x^(1/4)])/(-(b*x) + a*x^2)^(1/4) + (24
*a*x^(1/4)*(-b + a*x)^(1/4)*Defer[Subst][Defer[Int][x^18/((-b + a*x^4)^(1/4
)*(c - b*x^20 + a*x^24)), x], x, x^(1/4)])/(-(b*x) + a*x^2)^(1/4)
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(-5b+6ax)}{\sqrt[4]{-bx+ax^2}\left(c-bx^5+ax^6\right)}\,dx &= \frac{\left(\sqrt[4]{x}\sqrt[4]{-b+ax}\right)\int \frac{x^{11/4}(-5b+6ax)}{\sqrt[4]{-b+ax}\left(c-bx^5+ax^6\right)}\,dx}{\sqrt[4]{-bx+ax^2}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax}\right)\mathrm{Subst}\left(\int \frac{x^{14}\left(-5b+6ax^4\right)}{\sqrt[4]{-b+ax^4}\left(c-bx^{20}+ax^{24}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^2}}\\
&= \frac{\left(4\sqrt[4]{x}\sqrt[4]{-b+ax}\right)\mathrm{Subst}\left(\int \left(\frac{5bx^{14}}{\sqrt[4]{-b+ax^4}\left(-c+bx^{20}-ax^{24}\right)}+\frac{6ax^{18}}{\sqrt[4]{-b+ax^4}\left(c-bx^{20}+ax^{24}\right)}\right)\right.}{\sqrt[4]{-bx+ax^2}}\\
&= \frac{\left(24a\sqrt[4]{x}\sqrt[4]{-b+ax}\right)\mathrm{Subst}\left(\int \frac{x^{18}}{\sqrt[4]{-b+ax^4}\left(c-bx^{20}+ax^{24}\right)}\,dx,x,\sqrt[4]{x}\right)}{\sqrt[4]{-bx+ax^2}}+\frac{\left(20b\sqrt[4]{x}\right.}{}
\end{aligned}$$

Mathematica [F] time = 2.09, size = 0, normalized size = 0.00

$$\int \frac{x^3(-5b+6ax)}{\sqrt[4]{-bx+ax^2}\left(c-bx^5+ax^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^3*(-5*b + 6*a*x))/((-(b*x) + a*x^2)^(1/4)*(c - b*x^5 + a*x^6))
,x]
```

```
[Out] Integrate[(x^3*(-5*b + 6*a*x))/((-(b*x) + a*x^2)^(1/4)*(c - b*x^5 + a*x^6))
, x]
```

IntegrateAlgebraic [A] time = 8.25, size = 477, normalized size = 1.00

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[3]{c}\,x^2\sqrt[4]{ax^2-bx}-2^{2/3}x\sqrt[4]{ax^2-bx}}{-\sqrt[3]{c}\,x^2\sqrt[4]{ax^2-bx}+2^{2/3}x\sqrt[4]{ax^2-bx}-\sqrt{2}\,c^{7/12}x+2\sqrt[6]{2}\,\sqrt[4]{c}}\right)}{\sqrt[4]{c}}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[3]{c}\,x^2\sqrt[4]{ax^2-bx}-2^{2/3}x\sqrt[4]{ax^2-bx}}{-\sqrt[3]{c}\,x^2\sqrt[4]{ax^2-bx}+2^{2/3}x\sqrt[4]{ax^2-bx}+\sqrt{2}\,c^{7/12}x-2\sqrt[6]{2}\,\sqrt[4]{c}}\right)}{\sqrt[4]{c}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{-4\sqrt[6]{2}\,c^{7/12}x^2\sqrt[4]{ax^2-bx}+\sqrt{2}\,c^{11/12}x^3\sqrt[4]{ax^2-bx}+2\,2^{5/6}\sqrt[4]{c}\,x\sqrt[4]{ax^2-bx}}{c^{2/3}x^4\sqrt{ax^2-bx}-2\,2^{2/3}\sqrt[3]{c}\,x^3\sqrt{ax^2-bx}+2\sqrt[3]{2}\,x^2\sqrt{ax^2-bx}+c^{7/6}x^2-2\,2^{2/3}c^{5/6}x+2\sqrt[3]{2}\,\sqrt{c}}\right)}{\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(-5*b + 6*a*x))/((-(b*x) + a*x^2)^(1/4)*(c - b*x^5
+ a*x^6)),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(-(2^(2/3)*x*(-(b*x) + a*x^2)^(1/4)) + c^(1/3)*x^2*(-(b*x
) + a*x^2)^(1/4))/(2*2^(1/6)*c^(1/4) - Sqrt[2]*c^(7/12)*x + 2^(2/3)*x*(-(b*
x) + a*x^2)^(1/4) - c^(1/3)*x^2*(-(b*x) + a*x^2)^(1/4))])/c^(1/4)) + (Sqrt[
2]*ArcTan[(-(2^(2/3)*x*(-(b*x) + a*x^2)^(1/4)) + c^(1/3)*x^2*(-(b*x) + a*x^
2)^(1/4))/(-2*2^(1/6)*c^(1/4) + Sqrt[2]*c^(7/12)*x + 2^(2/3)*x*(-(b*x) + a*
x^2)^(1/4) - c^(1/3)*x^2*(-(b*x) + a*x^2)^(1/4))])/c^(1/4) - (Sqrt[2]*ArcTa
nh[(2*2^(5/6)*c^(1/4)*x*(-(b*x) + a*x^2)^(1/4) - 4*2^(1/6)*c^(7/12)*x^2*(-(
b*x) + a*x^2)^(1/4) + Sqrt[2]*c^(11/12)*x^3*(-(b*x) + a*x^2)^(1/4))/(2*2^(1
/3)*Sqrt[c] - 2*2^(2/3)*c^(5/6)*x + c^(7/6)*x^2 + 2*2^(1/3)*x^2*Sqrt[-(b*x)
 + a*x^2] - 2*2^(2/3)*c^(1/3)*x^3*Sqrt[-(b*x) + a*x^2] + c^(2/3)*x^4*Sqrt[-
(b*x) + a*x^2])])/c^(1/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(6*a*x-5*b)/(a*x^2-b*x)^(1/4)/(a*x^6-b*x^5+c),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(6\,ax-5\,b)x^3}{\left(ax^6-bx^5+c\right)\left(ax^2-bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(6*a*x-5*b)/(a*x^2-b*x)^(1/4)/(a*x^6-b*x^5+c),x, algorithm="g
iac")
```

```
[Out] integrate((6*a*x - 5*b)*x^3/((a*x^6 - b*x^5 + c)*(a*x^2 - b*x)^(1/4)), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{x^3\,(6ax-5b)}{\left(a\,x^2-bx\right)^{\frac{1}{4}}\left(a\,x^6-b\,x^5+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(6*a*x-5*b)/(a*x^2-b*x)^(1/4)/(a*x^6-b*x^5+c),x)
```

```
[Out] int(x^3*(6*a*x-5*b)/(a*x^2-b*x)^(1/4)/(a*x^6-b*x^5+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(6\,ax-5\,b)x^3}{\left(ax^6-bx^5+c\right)\left(ax^2-bx\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(6*a*x-5*b)/(a*x^2-b*x)^(1/4)/(a*x^6-b*x^5+c),x, algorithm="m
axima")
```

```
[Out] integrate((6*a*x - 5*b)*x^3/((a*x^6 - b*x^5 + c)*(a*x^2 - b*x)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x^3\,\left(5\,b-6\,a\,x\right)}{{\left(a\,x^2-b\,x\right)}^{1/4}\,\left(a\,x^6-b\,x^5+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(5*b - 6*a*x))/((a*x^2 - b*x)^(1/4)*(c + a*x^6 - b*x^5)),x)
```

```
[Out] int(-(x^3*(5*b - 6*a*x))/((a*x^2 - b*x)^(1/4)*(c + a*x^6 - b*x^5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(6 a x - 5 b\right)}{\sqrt[4]{x \left(a x - b\right)} \left(a x^{6} - b x^{5} + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(6*a*x-5*b)/(a*x**2-b*x)**(1/4)/(a*x**6-b*x**5+c),x)
```

```
[Out] Integral(x**3*(6*a*x - 5*b)/((x*(a*x - b))**(1/4)*(a*x**6 - b*x**5 + c)), x
)
```

3.2383 $$\int \frac{-a+x}{\sqrt[3]{(-a+x)(-b+x)^2}\left(-b^2+a^2d+2(b-ad)x+(-1+d)x^2\right)}\,dx$$

Optimal. Leaf size=481

$$\frac{\left(1+i\sqrt{3}\right)(x-b)^{2/3}\left(-\sqrt[6]{d}\sqrt[3]{a-x}\sqrt[3]{x-b}+\sqrt[3]{d}(a-x)^{2/3}+(x-b)^{2/3}\right)\left(\sqrt[6]{d}\sqrt[3]{a-x}\sqrt[3]{x-b}+\sqrt[3]{d}(a-x)^{2/3}+(x-\right.}{2\sqrt[3]{(}}$$

Rubi [A] time = 1.06, antiderivative size = 513, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 5, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.096, Rules used = {6719, 911, 105, 59, 91}

$$-\frac{\sqrt[3]{x-a}(x-b)^{2/3}\log\left(2\left(\sqrt{d}+1\right)\left(b-a\sqrt{d}\right)-2(1-d)x\right)}{4d^{5/6}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}+\frac{\sqrt[3]{x-a}(x-b)^{2/3}\log\left(2(1-d)x-2\left(1-\sqrt{d}\right)\left(a\sqrt{d}+b\right)\right)}{4d^{5/6}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{3\sqrt[3]{x-a}(x-b)^{2/3}\log\left(-\sqrt[6]{d}\sqrt[3]{x-a}-\sqrt[3]{x-b}\right)}{4d^{5/6}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}+\frac{3\sqrt[3]{x-a}(x-b)^{2/3}\log\left(\sqrt[6]{d}\sqrt[3]{x-a}-\sqrt[3]{x-b}\right)}{4d^{5/6}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{\sqrt{3}\sqrt[3]{x-a}(x-b)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{x-b}}\right)}{2d^{5/6}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}+\frac{\sqrt{3}\sqrt[3]{x-a}(x-b)^{2/3}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{x-b}}+\frac{1}{\sqrt{3}}\right)}{2d^{5/6}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + x)/(((-a + x)*(-b + x)^2)^(1/3)*(-b^2 + a^2*d + 2*(b - a*d)*x + (
-1 + d)*x^2)),x]
```

```
[Out] -1/2*(Sqrt[3]*(-a + x)^(1/3)*(-b + x)^(2/3)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(
-a + x)^(1/3))/(Sqrt[3]*(-b + x)^(1/3))])/((a - b)*d^(5/6)*(-((a - x)*(b -
x)^2))^(1/3)) + (Sqrt[3]*(-a + x)^(1/3)*(-b + x)^(2/3)*ArcTan[1/Sqrt[3] + (
2*d^(1/6)*(-a + x)^(1/3))/(Sqrt[3]*(-b + x)^(1/3))])/(2*(a - b)*d^(5/6)*(-(
(a - x)*(b - x)^2))^(1/3)) - ((-a + x)^(1/3)*(-b + x)^(2/3)*Log[2*(1 + Sqrt
[d])*(b - a*Sqrt[d]) - 2*(1 - d)*x])/(4*(a - b)*d^(5/6)*(-((a - x)*(b - x)^
2))^(1/3)) + ((-a + x)^(1/3)*(-b + x)^(2/3)*Log[-2*(1 - Sqrt[d])*(b + a*Sqr
t[d]) + 2*(1 - d)*x])/(4*(a - b)*d^(5/6)*(-((a - x)*(b - x)^2))^(1/3)) - (3
*(-a + x)^(1/3)*(-b + x)^(2/3)*Log[-(d^(1/6)*(-a + x)^(1/3)) - (-b + x)^(1/
3)])/(4*(a - b)*d^(5/6)*(-((a - x)*(b - x)^2))^(1/3)) + (3*(-a + x)^(1/3)*(
-b + x)^(2/3)*Log[d^(1/6)*(-a + x)^(1/3) - (-b + x)^(1/3)])/(4*(a - b)*d^(5
/6)*(-((a - x)*(b - x)^2))^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{-a+x}{\sqrt[3]{(-a+x)(-b+x)^2}\left(-b^2+a^2d+2(b-ad)x+(-1+d)x^2\right)}dx = \frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{(-a+x \cdots}{(-b+x)^{2/3}\left(-b^2+a^2d+2 \cdots\right.}}{\sqrt[3]{(-a+x)(-b+x)^2}}$$

$$= \frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int\left(\frac{(-1+ \cdots}{(a-b)\sqrt{d}\,(-b+x)^{2/3}(2 \cdots}\right.}{\cdots}$$

$$= -\frac{\left((1-d)\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{\cdots}{(-b+x)^{2/3}(2 \cdots}}{(a-b)\sqrt{d}\,\sqrt[3]{(-a+x)(- \cdots}}$$

$$= \frac{\left((1-d)\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{\cdots}{\sqrt[3]{-a+x}\,(-b+ \cdots}}{\left(1-\sqrt{d}\right)\sqrt{d}\,\sqrt[3]{(-a+x \cdots}}$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{-a+x}\,(-b+x)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{\cdots}}{\sqrt{3 \cdots}}\right.}{2(a-b)d^{5/6}\sqrt[3]{-\left((a-x)(b-x)^2\right)}}$$

Mathematica [C] time = 0.28, size = 89, normalized size = 0.19

$$-\frac{3(x-b)\left({}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{b-x}{\sqrt{d}\,(x-a)}\right)+{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{x-b}{\sqrt{d}\,(x-a)}\right)\right)}{2d(a-b)\sqrt[3]{(x-a)(b-x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + x)/(((-a + x)*(-b + x)^2)^(1/3)*(-b^2 + a^2*d + 2*(b - a*d)
*x + (-1 + d)*x^2)),x]
```

```
[Out] (-3*(-b + x)*(Hypergeometric2F1[1/3, 1, 4/3, (b - x)/(Sqrt[d]*(-a + x))] +
Hypergeometric2F1[1/3, 1, 4/3, (-b + x)/(Sqrt[d]*(-a + x))]))/(2*(a - b)*d*
((b - x)^2*(-a + x))^(1/3))
```

IntegrateAlgebraic [A] time = 15.05, size = 481, normalized size = 1.00

$$\frac{\left(1+i\sqrt{3}\right)(x-b)^{2/3}\left(-\sqrt[6]{d}\sqrt[3]{a-x}\sqrt[3]{x-b}+\sqrt[3]{d}(a-x)^{2/3}+(x-b)^{2/3}\right)\left(\sqrt[6]{d}\sqrt[3]{a-x}\sqrt[3]{x-b}+\sqrt[3]{d}(a-x)^{2/3}+(x-b)^{2/3}\right)\left(\sqrt[3]{a-x}(x-b)^{2/3}+a\left(-\sqrt[3]{d}\right)+\sqrt[3]{d}x\right)\left(\frac{\left(\sqrt{3}-3i\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x-b}}{\sqrt[3]{x-b}-2\sqrt[6]{d}\sqrt[3]{a-x}}\right)}{4d^{5/6}(a-b)}+\frac{\left(-\sqrt{3}+3i\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x-b}}{2\sqrt[6]{d}\sqrt[3]{a-x}+\sqrt[3]{x-b}}\right)}{4d^{5/6}(a-b)}+\frac{\left(1-i\sqrt{3}\right)\tanh^{-1}\left(\frac{(a-x)^{2/3}\sqrt[3]{x-b}}{\sqrt[6]{d}(x-a)}\right)}{2d^{5/6}(a-b)}+\frac{i\left(\sqrt{3}+i\right)\tanh^{-1}\left(\frac{\sqrt[3]{d}(a-x)^{2/3}+(x-b)^{2/3}}{\sqrt[6]{d}\sqrt[3]{a-x}\sqrt[3]{x-b}}\right)}{4d^{5/6}(a-b)}\right)}{2\sqrt[3]{(x-a)(b-x)^2}\left(a^2(-d)+2adx+b^2-2bx-(d-1)x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + x)/(((-a + x)*(-b + x)^2)^(1/3)*(-b^2 + a^2*d + 2*
(b - a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] ((1 + I*Sqrt[3])*(-b + x)^(2/3)*(d^(1/3)*(a - x)^(2/3) - d^(1/6)*(a - x)^(1
/3)*(-b + x)^(1/3) + (-b + x)^(2/3))*(d^(1/3)*(a - x)^(2/3) + d^(1/6)*(a -
x)^(1/3)*(-b + x)^(1/3) + (-b + x)^(2/3))*(-(a*d^(1/3)) + d^(1/3)*x + (a -
x)^(1/3)*(-b + x)^(2/3))*(((-3*I + Sqrt[3])*ArcTan[(Sqrt[3]*(-b + x)^(1/3))
/(-2*d^(1/6)*(a - x)^(1/3) + (-b + x)^(1/3))])/(4*(a - b)*d^(5/6)) + ((3*I
- Sqrt[3])*ArcTan[(Sqrt[3]*(-b + x)^(1/3))/(2*d^(1/6)*(a - x)^(1/3) + (-b +
 x)^(1/3))])/(4*(a - b)*d^(5/6)) + ((1 - I*Sqrt[3])*ArcTanh[((a - x)^(2/3)*
(-b + x)^(1/3))/(d^(1/6)*(-a + x))])/(2*(a - b)*d^(5/6)) + ((I/4)*(I + Sqrt
[3])*ArcTanh[(d^(1/3)*(a - x)^(2/3) + (-b + x)^(2/3))/(d^(1/6)*(a - x)^(1/3
)*(-b + x)^(1/3))])/((a - b)*d^(5/6))))/(2*((b - x)^2*(-a + x))^(1/3)*(b^2
- a^2*d - 2*b*x + 2*a*d*x - (-1 + d)*x^2))
```

fricas [B] time = 0.62, size = 2613, normalized size = 5.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)/((-a+x)*(-b+x)^2)^(1/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-1+d)*x^
2),x, algorithm="fricas")
```

```
[Out] sqrt(3)*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5
 + b^6)*d^5))^(1/6)*arctan(-1/3*(2*sqrt(3)*(-a*b^2 - (a + 2*b)*x^2 + x^3 +
(2*a*b + b^2)*x)^(1/3)*(a - b)*d*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b
^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(1/6) - 2*sqrt(3)*((a - b)*d*x - (a*
b - b^2)*d)*sqrt((((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
)*d^4*x - (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*d^4
)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(1/((a^6 - 6*a^5*b
 + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(5/6) + ((a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d^3*x^2 - 2*(a^4*b - 4*a^3*b^2 + 6
*a^2*b^3 - 4*a*b^4 + b^5)*d^3*x + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^
5 + b^6)*d^3)*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6
*a*b^5 + b^6)*d^5))^(2/3) + (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x
)^(2/3))/(b^2 - 2*b*x + x^2))*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3
+ 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(1/6) - sqrt(3)*(b - x))/(b - x)) + sqr
t(3)*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 +
b^6)*d^5))^(1/6)*arctan(-1/3*(2*sqrt(3)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*
a*b + b^2)*x)^(1/3)*(a - b)*d*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3
+ 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(1/6) - 2*sqrt(3)*((a - b)*d*x - (a*b -
 b^2)*d)*sqrt(-(((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*
d^4*x - (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*d^4)*
(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(1/((a^6 - 6*a^5*b +
 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(5/6) - ((a^4
- 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d^3*x^2 - 2*(a^4*b - 4*a^3*b^2 + 6*a
^2*b^3 - 4*a*b^4 + b^5)*d^3*x + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5
+ b^6)*d^3)*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a
*b^5 + b^6)*d^5))^(2/3) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^
(2/3))/(b^2 - 2*b*x + x^2))*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 +
15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(1/6) + sqrt(3)*(b - x))/(b - x)) - 1/4*(
1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d
^5))^(1/6)*log((((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*
d^4*x - (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*d^4)*
(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(1/((a^6 - 6*a^5*b +
 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(5/6) + ((a^4
- 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d^3*x^2 - 2*(a^4*b - 4*a^3*b^2 + 6*a
^2*b^3 - 4*a*b^4 + b^5)*d^3*x + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5
+ b^6)*d^3)*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a
*b^5 + b^6)*d^5))^(2/3) + (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^
(2/3))/(b^2 - 2*b*x + x^2)) + 1/4*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*
```

```
b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(1/6)*log(-(((a^5 - 5*a^4*b + 10*a^
3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d^4*x - (a^5*b - 5*a^4*b^2 + 10*a^3*b^3
 - 10*a^2*b^4 + 5*a*b^5 - b^6)*d^4)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b
+ b^2)*x)^(1/3)*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 -
 6*a*b^5 + b^6)*d^5))^(5/6) - ((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*
d^3*x^2 - 2*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d^3*x + (a^4*b^
2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*d^3)*(1/((a^6 - 6*a^5*b + 15*a^4
*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))^(2/3) - (-a*b^2 - (a
+ 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3))/(b^2 - 2*b*x + x^2)) - 1/2*(1/((
a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))
^(1/6)*log(-(((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d^4
*x - (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*d^4)*(1/
((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5
))^(5/6) + (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3))/(b - x))
 + 1/2*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5
+ b^6)*d^5))^(1/6)*log((((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
 - b^5)*d^4*x - (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^
6)*d^4)*(1/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5
 + b^6)*d^5))^(5/6) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3
))/(b - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{a-x}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}\left(a^2d+(d-1)x^2-b^2-2\,(ad-b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)/((-a+x)*(-b+x)^2)^(1/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-1+d)*x^
2),x, algorithm="giac")
```

```
[Out] integrate(-(a - x)/((-(a - x)*(b - x)^2)^(1/3)*(a^2*d + (d - 1)*x^2 - b^2 -
 2*(a*d - b)*x)), x)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{-a+x}{\left((-a+x)\,(-b+x)^2\right)^{\frac{1}{3}}\left(-b^2+a^2d+2\,(-ad+b)\,x+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x)/((-a+x)*(-b+x)^2)^(1/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-1+d)*x^2),x)
```

```
[Out] int((-a+x)/((-a+x)*(-b+x)^2)^(1/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-1+d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a-x}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}\left(a^2d+(d-1)x^2-b^2-2\,(ad-b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)/((-a+x)*(-b+x)^2)^(1/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-1+d)*x^
2),x, algorithm="maxima")
```

```
[Out] -integrate((a - x)/((-(a - x)*(b - x)^2)^(1/3)*(a^2*d + (d - 1)*x^2 - b^2 -
 2*(a*d - b)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{a-x}{\left(-\left(a-x\right)\,{\left(b-x\right)}^2\right)^{1/3}\,\left(a^2\,d+2\,x\,\left(b-a\,d\right)-b^2+x^2\,\left(d-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a - x)/((-(a - x)*(b - x)^2)^(1/3)*(a^2*d + 2*x*(b - a*d) - b^2 + x^2
*(d - 1))),x)
```

```
[Out] int(-(a - x)/((-(a - x)*(b - x)^2)^(1/3)*(a^2*d + 2*x*(b - a*d) - b^2 + x^2
*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)/((-a+x)*(-b+x)**2)**(1/3)/(-b**2+a**2*d+2*(-a*d+b)*x+(-1+d
)*x**2),x)
```

```
[Out] Timed out
```

3.2384 $\int \frac{b-ax^4+2x^8}{\sqrt[4]{b+ax^4}\left(-b-2ax^4+x^8\right)}\,dx$

Optimal. Leaf size=482

$$\frac{3\left(1+\sqrt[4]{-1}\right)\tan^{-1}\left(\frac{(-1)^{7/8}\sqrt{2+\sqrt{2}}\,x\,\sqrt[8]{a^2+b}\,\sqrt[4]{ax^4+b}}{(-1)^{3/4}x^2\sqrt[4]{a^2+b}+\sqrt{ax^4+b}}\right)}{8\sqrt[8]{a^2+b}}-\frac{3i\left(\sqrt{2\left(3-2\sqrt{2}\right)}-i\sqrt{2}\right)\tan^{-1}\left(\frac{(-1)^{7/8}\left(\sqrt{2}-2\right)x\,\sqrt[8]{a^2+b}\,\sqrt[4]{ax^4}}{(-1)^{3/4}\sqrt{2-\sqrt{2}}\,x^2\sqrt[4]{a^2+b}+\sqrt{2-\sqrt{2}}}\right.}{16\sqrt[8]{a^2+b}}$$

Rubi [A] time = 1.05, antiderivative size = 451, normalized size of antiderivative = 0.94, number of steps used = 25, number of rules used = 10, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6728, 240, 212, 206, 203, 1428, 408, 377, 208, 205}

$$\frac{3\left(-a\sqrt{a^2+b}+a^2+b\right)^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2+b}+a^2+b}}{\sqrt[4]{a-\sqrt{a^2+b}}\,\sqrt[4]{ax^4+b}}\right)}{4\sqrt{a^2+b}\left(a-\sqrt{a^2+b}\right)^{3/4}}-\frac{3\left(a\sqrt{a^2+b}+a^2+b\right)^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2+b}+a^2+b}}{\sqrt[4]{\sqrt{a^2+b}+a}\,\sqrt[4]{ax^4+b}}\right)}{4\sqrt{a^2+b}\left(\sqrt{a^2+b}+a\right)^{3/4}}+\frac{3\left(-a\sqrt{a^2+b}+a^2+b\right)^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2+b}+a^2+b}}{\sqrt[4]{a-\sqrt{a^2+b}}\,\sqrt[4]{ax^4+b}}\right)}{4\sqrt{a^2+b}\left(a-\sqrt{a^2+b}\right)^{3/4}}-\frac{3\left(a\sqrt{a^2+b}+a^2+b\right)^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2+b}+a^2+b}}{\sqrt[4]{\sqrt{a^2+b}+a}\,\sqrt[4]{ax^4+b}}\right)}{4\sqrt{a^2+b}\left(\sqrt{a^2+b}+a\right)^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(b - a*x^4 + 2*x^8)/((b + a*x^4)^(1/4)*(-b - 2*a*x^4 + x^8)),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)]/a^(1/4) + (3*(a^2 + b - a*Sqrt[a^2 +
b])^(3/4)*ArcTan[((a^2 + b - a*Sqrt[a^2 + b])^(1/4)*x)/((a - Sqrt[a^2 + b])
^(1/4)*(b + a*x^4)^(1/4))])/(4*Sqrt[a^2 + b]*(a - Sqrt[a^2 + b])^(3/4)) - (
3*(a^2 + b + a*Sqrt[a^2 + b])^(3/4)*ArcTan[((a^2 + b + a*Sqrt[a^2 + b])^(1/
4)*x)/((a + Sqrt[a^2 + b])^(1/4)*(b + a*x^4)^(1/4))])/(4*Sqrt[a^2 + b]*(a +
 Sqrt[a^2 + b])^(3/4)) + ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4)]/a^(1/4) + (
3*(a^2 + b - a*Sqrt[a^2 + b])^(3/4)*ArcTanh[((a^2 + b - a*Sqrt[a^2 + b])^(1
/4)*x)/((a - Sqrt[a^2 + b])^(1/4)*(b + a*x^4)^(1/4))])/(4*Sqrt[a^2 + b]*(a
- Sqrt[a^2 + b])^(3/4)) - (3*(a^2 + b + a*Sqrt[a^2 + b])^(3/4)*ArcTanh[((a^
2 + b + a*Sqrt[a^2 + b])^(1/4)*x)/((a + Sqrt[a^2 + b])^(1/4)*(b + a*x^4)^(1
/4))])/(4*Sqrt[a^2 + b]*(a + Sqrt[a^2 + b])^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 408

```
Int[((a_) + (b_.)*(x_)^4)^(p_)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d
, Int[(a + b*x^4)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(p
- 1)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[p, 3/4] || EqQ[p, 5/4])
```

Rule 1428

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^
n)^q/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[(d + e*x^n)^q/(b + r + 2
*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{b-ax^4+2x^8}{\sqrt[4]{b+ax^4}\left(-b-2ax^4+x^8\right)}\,dx = \int \left(\frac{2}{\sqrt[4]{b+ax^4}} + \frac{3\left(b+ax^4\right)^{3/4}}{-b-2ax^4+x^8}\right)dx$$

$$= 2\int \frac{1}{\sqrt[4]{b+ax^4}}\,dx + 3\int \frac{\left(b+ax^4\right)^{3/4}}{-b-2ax^4+x^8}\,dx$$

$$= 2\,\text{Subst}\left(\int \frac{1}{1-ax^4}\,dx, x, \frac{x}{\sqrt[4]{b+ax^4}}\right) + \frac{3\int \frac{\left(b+ax^4\right)^{3/4}}{-2a-2\sqrt{a^2+b}+2x^4}\,dx}{\sqrt{a^2+b}} - \frac{3\int \frac{(\ldots}{-2a+\ldots}}{\sqrt{\ldots}}$$

$$= -\frac{\left(3\left(a^2+b-a\sqrt{a^2+b}\right)\right)\int \frac{1}{\left(-2a+2\sqrt{a^2+b}+2x^4\right)\sqrt[4]{b+ax^4}}\,dx}{\sqrt{a^2+b}} + \frac{\left(3\left(b+a\left(a+\sqrt{\ldots}\right.\right.\right.}{\ldots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{\sqrt[4]{a}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{\sqrt[4]{a}} - \frac{\left(3\left(a^2+b-a\sqrt{a^2+b}\right)\right)\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{\sqrt[4]{a}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{\sqrt[4]{a}} + \frac{\left(3\left(a^2+b-a\sqrt{a^2+b}\right)\right)\text{Subst}\left(\int \ldots\right.}{4\sqrt{a^2+b}\ldots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b+ax^4}}\right)}{\sqrt[4]{a}} + \frac{3\left(a^2+b-a\sqrt{a^2+b}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a^2+b-a\sqrt{a^2+b}}\,x}{\sqrt[4]{a-\sqrt{a^2+b}}\,\sqrt[4]{b+ax^4}}\right)}{4\sqrt{a^2+b}\left(a-\sqrt{a^2+b}\right)^{3/4}} - \frac{3\ldots}{\ldots}$$

Mathematica [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{b-ax^4+2x^8}{\sqrt[4]{b+ax^4}\left(-b-2ax^4+x^8\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(b - a*x^4 + 2*x^8)/((b + a*x^4)^(1/4)*(-b - 2*a*x^4 + x^8)),x]
[Out] Integrate[(b - a*x^4 + 2*x^8)/((b + a*x^4)^(1/4)*(-b - 2*a*x^4 + x^8)), x]
```

IntegrateAlgebraic [A] time = 136.81, size = 536, normalized size = 1.11

$$\frac{3i\left(\sqrt{2\left(3+2\sqrt{2}\right)}+i\sqrt{2}\right)\tan^{-1}\left(\frac{\left((-1+i)-(1+i)(-1)^{3/4}\right)x^2\sqrt[4]{a^2+b}+(1+i)(-1)^{3/4}\sqrt{ax^4+b}+(1+i)\sqrt{ax^4+b}}{2x\sqrt[8]{a^2+b}\sqrt[4]{ax^4+b}}\right)}{16\sqrt[8]{a^2+b}} + \frac{3\left(\sqrt{2}+i\sqrt{2\left(3-2\sqrt{2}\right)}\right)\tan^{-1}\left(\frac{2x\sqrt[8]{a^2+b}\sqrt[4]{ax^4+b}}{\left(\sqrt{2}+(1-i)\right)x^2\sqrt[4]{a^2+b}-\sqrt{2}\sqrt{ax^4+b}-(1+i)\sqrt{ax^4+b}}\right)}{16\sqrt[8]{a^2+b}} + \frac{3\left((-1)^{3/4}+i\right)\tanh^{-1}\left(\frac{\left((-2+2i)-(2+2i)(-1)^{3/4}\right)x^2\sqrt[4]{a^2+b}-(2+2i)(-1)^{3/4}\sqrt{ax^4+b}-(2+2i)\sqrt{ax^4+b}}{4x\sqrt[8]{a^2+b}\sqrt[4]{ax^4+b}}\right)}{8\sqrt[8]{a^2+b}} - \frac{3i\left(\sqrt{2\left(3-2\sqrt{2}\right)}-i\sqrt{2}\right)\tanh^{-1}\left(\frac{\left(\sqrt{2}+(1-i)\right)x^2\sqrt[4]{a^2+b}+\sqrt{2}\sqrt{ax^4+b}+(1+i)\sqrt{ax^4+b}}{2x\sqrt[8]{a^2+b}\sqrt[4]{ax^4+b}}\right)}{16\sqrt[8]{a^2+b}} + \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{\sqrt[4]{a}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{ax^4+b}}\right)}{\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b - a*x^4 + 2*x^8)/((b + a*x^4)^(1/4)*(-b - 2*a*x^4 + x
^8)),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)]/a^(1/4) + (((3*I)/16)*(I*Sqrt[2] + Sq
rt[2*(3 + 2*Sqrt[2])])*ArcTan[(((-1 + I) - (1 + I)*(-1)^(3/4))*(a^2 + b)^(1
/4)*x^2 + (1 + I)*Sqrt[b + a*x^4] + (1 + I)*(-1)^(3/4)*Sqrt[b + a*x^4])/(2*
(a^2 + b)^(1/8)*x*(b + a*x^4)^(1/4))])/(a^2 + b)^(1/8) + (3*(Sqrt[2] + I*Sq
rt[2*(3 - 2*Sqrt[2])])*ArcTan[(2*(a^2 + b)^(1/8)*x*(b + a*x^4)^(1/4))/(((1
- I) + Sqrt[2])*(a^2 + b)^(1/4)*x^2 - (1 + I)*Sqrt[b + a*x^4] - Sqrt[2]*Sqr
t[b + a*x^4])])/(16*(a^2 + b)^(1/8)) + ArcTanh[(a^(1/4)*x)/(b + a*x^4)^(1/4
)]/a^(1/4) + (3*(I + (-1)^(3/4))*ArcTanh[(((-2 + 2*I) - (2 + 2*I)*(-1)^(3/4
))*(a^2 + b)^(1/4)*x^2 - (2 + 2*I)*Sqrt[b + a*x^4] - (2 + 2*I)*(-1)^(3/4)*S
```

```
qrt[b + a*x^4])/(4*(a^2 + b)^(1/8)*x*(b + a*x^4)^(1/4))])/(8*(a^2 + b)^(1/8
)) - (((3*I)/16)*((-I)*Sqrt[2] + Sqrt[2*(3 - 2*Sqrt[2])])*ArcTanh[(((1 - I)
 + Sqrt[2])*(a^2 + b)^(1/4)*x^2 + (1 + I)*Sqrt[b + a*x^4] + Sqrt[2]*Sqrt[b
+ a*x^4])/(2*(a^2 + b)^(1/8)*x*(b + a*x^4)^(1/4))])/(a^2 + b)^(1/8)
```

fricas [A] time = 0.64, size = 567, normalized size = 1.18

$$-\frac{3\sqrt{2}\arctan\left(\frac{\frac{\sqrt{2}x\sqrt{\frac{(a^2+b)^{\frac{1}{4}}x^2+\sqrt{2}(ax^4+b)^{\frac{1}{4}}(a^2+b)^{\frac{1}{8}}x+\sqrt{ax^4+b}}{x^2}}}{(a^2+b)^{\frac{1}{8}}}-x-\frac{\sqrt{2}(ax^4+b)^{\frac{1}{4}}}{(a^2+b)^{\frac{1}{8}}}}{x}\right)}{4(a^2+b)^{\frac{1}{8}}}-\frac{3\sqrt{2}\arctan\left(\frac{\frac{\sqrt{2}x\sqrt{\frac{(a^2+b)^{\frac{1}{4}}x^2-\sqrt{2}(ax^4+b)^{\frac{1}{4}}(a^2+b)^{\frac{1}{8}}x+\sqrt{ax^4+b}}{x^2}}}{(a^2+b)^{\frac{1}{8}}}+x-\frac{\sqrt{2}(ax^4+b)^{\frac{1}{4}}}{(a^2+b)^{\frac{1}{8}}}}{x}\right)}{4(a^2+b)^{\frac{1}{8}}}-\frac{3\sqrt{2}\log\left(\frac{(a^2+b)^{\frac{1}{4}}x^2+\sqrt{2}(ax^4+b)^{\frac{1}{4}}(a^2+b)^{\frac{1}{8}}x+\sqrt{ax^4+b}}{x^2}\right)}{16(a^2+b)^{\frac{1}{8}}}+\frac{3\sqrt{2}\log\left(\frac{(a^2+b)^{\frac{1}{4}}x^2-\sqrt{2}(ax^4+b)^{\frac{1}{4}}(a^2+b)^{\frac{1}{8}}x+\sqrt{ax^4+b}}{x^2}\right)}{16(a^2+b)^{\frac{1}{8}}}-\frac{3\arctan\left(\frac{\frac{x\sqrt{\frac{(a^2+b)^{\frac{1}{4}}x^2+\sqrt{ax^4+b}}{x^2}}}{(a^2+b)^{\frac{1}{8}}}-\frac{(ax^4+b)^{\frac{1}{4}}}{(a^2+b)^{\frac{1}{8}}}}{x}\right)}{2(a^2+b)^{\frac{1}{8}}}-\frac{3\log\left(\frac{(a^2+b)^{\frac{1}{8}}x+(ax^4+b)^{\frac{1}{4}}}{x}\right)}{8(a^2+b)^{\frac{1}{8}}}+\frac{3\log\left(-\frac{(a^2+b)^{\frac{1}{8}}x-(ax^4+b)^{\frac{1}{4}}}{x}\right)}{8(a^2+b)^{\frac{1}{8}}}+\frac{2\arctan\left(\frac{\frac{x\sqrt{\frac{\sqrt{a}x^2+\sqrt{ax^4+b}}{x^2}}}{a^{\frac{1}{4}}}-\frac{(ax^4+b)^{\frac{1}{4}}}{a^{\frac{1}{4}}}}{x}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{a^{\frac{1}{4}}x+(ax^4+b)^{\frac{1}{4}}}{x}\right)}{2a^{\frac{1}{4}}}-\frac{\log\left(-\frac{a^{\frac{1}{4}}x-(ax^4+b)^{\frac{1}{4}}}{x}\right)}{2a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-a*x^4+b)/(a*x^4+b)^(1/4)/(x^8-2*a*x^4-b),x, algorithm="fri
cas")
```

```
[Out] -3/4*sqrt(2)*arctan((sqrt(2)*x*sqrt(((a^2 + b)^(1/4)*x^2 + sqrt(2)*(a*x^4 +
 b)^(1/4)*(a^2 + b)^(1/8)*x + sqrt(a*x^4 + b))/x^2)/(a^2 + b)^(1/8) - x - s
qrt(2)*(a*x^4 + b)^(1/4)/(a^2 + b)^(1/8))/x)/(a^2 + b)^(1/8) - 3/4*sqrt(2)*
arctan((sqrt(2)*x*sqrt(((a^2 + b)^(1/4)*x^2 - sqrt(2)*(a*x^4 + b)^(1/4)*(a^
2 + b)^(1/8)*x + sqrt(a*x^4 + b))/x^2)/(a^2 + b)^(1/8) + x - sqrt(2)*(a*x^4
 + b)^(1/4)/(a^2 + b)^(1/8))/x)/(a^2 + b)^(1/8) - 3/16*sqrt(2)*log(((a^2 +
b)^(1/4)*x^2 + sqrt(2)*(a*x^4 + b)^(1/4)*(a^2 + b)^(1/8)*x + sqrt(a*x^4 + b
))/x^2)/(a^2 + b)^(1/8) + 3/16*sqrt(2)*log(((a^2 + b)^(1/4)*x^2 - sqrt(2)*(
a*x^4 + b)^(1/4)*(a^2 + b)^(1/8)*x + sqrt(a*x^4 + b))/x^2)/(a^2 + b)^(1/8)
- 3/2*arctan((x*sqrt(((a^2 + b)^(1/4)*x^2 + sqrt(a*x^4 + b))/x^2)/(a^2 + b)
^(1/8) - (a*x^4 + b)^(1/4)/(a^2 + b)^(1/8))/x)/(a^2 + b)^(1/8) - 3/8*log(((
a^2 + b)^(1/8)*x + (a*x^4 + b)^(1/4))/x)/(a^2 + b)^(1/8) + 3/8*log(-((a^2 +
 b)^(1/8)*x - (a*x^4 + b)^(1/4))/x)/(a^2 + b)^(1/8) + 2*arctan((x*sqrt((sqr
t(a)*x^2 + sqrt(a*x^4 + b))/x^2)/a^(1/4) - (a*x^4 + b)^(1/4)/a^(1/4))/x)/a^
(1/4) + 1/2*log((a^(1/4)*x + (a*x^4 + b)^(1/4))/x)/a^(1/4) - 1/2*log(-(a^(1
/4)*x - (a*x^4 + b)^(1/4))/x)/a^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^8 - ax^4 + b}{\left(x^8 - 2\,ax^4 - b\right)\left(ax^4 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-a*x^4+b)/(a*x^4+b)^(1/4)/(x^8-2*a*x^4-b),x, algorithm="gia
c")
```

```
[Out] integrate((2*x^8 - a*x^4 + b)/((x^8 - 2*a*x^4 - b)*(a*x^4 + b)^(1/4)), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{2x^8 - a\,x^4 + b}{\left(a\,x^4 + b\right)^{\frac{1}{4}}\left(x^8 - 2a\,x^4 - b\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8-a*x^4+b)/(a*x^4+b)^(1/4)/(x^8-2*a*x^4-b),x)
```

```
[Out] int((2*x^8-a*x^4+b)/(a*x^4+b)^(1/4)/(x^8-2*a*x^4-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^8 - ax^4 + b}{\left(x^8 - 2\,ax^4 - b\right)\left(ax^4 + b\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8-a*x^4+b)/(a*x^4+b)^(1/4)/(x^8-2*a*x^4-b),x, algorithm="max
ima")
```

```
[Out] integrate((2*x^8 - a*x^4 + b)/((x^8 - 2*a*x^4 - b)*(a*x^4 + b)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{2\,x^8-a\,x^4+b}{{\left(a\,x^4+b\right)}^{1/4}\,\left(-x^8+2\,a\,x^4+b\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - a*x^4 + 2*x^8)/((b + a*x^4)^(1/4)*(b + 2*a*x^4 - x^8)),x)
```

```
[Out] int(-(b - a*x^4 + 2*x^8)/((b + a*x^4)^(1/4)*(b + 2*a*x^4 - x^8)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8-a*x**4+b)/(a*x**4+b)**(1/4)/(x**8-2*a*x**4-b),x)
```

```
[Out] Timed out
```

3.2385 $\int \frac{\sqrt{-b+a^2x^2}}{\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=495

$$-\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{c}}\right)}{128ac^{9/2}}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{\sqrt{a^2x^2-b}\left(\left(-9216a^2c^5x^2-12288ac^7x-245\ldots\right.\right.}{}$$

Rubi [F] time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-b+a^2x^2}}{\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[-b + a^2*x^2]/Sqrt[c + Sqrt[a*x + Sqrt[-b + a^2*x^2]]],x]
```

```
[Out] Defer[Int][Sqrt[-b + a^2*x^2]/Sqrt[c + Sqrt[a*x + Sqrt[-b + a^2*x^2]]], x]
```

Rubi steps

$$\int \frac{\sqrt{-b+a^2x^2}}{\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{\sqrt{-b+a^2x^2}}{\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Mathematica [A] time = 3.32, size = 536, normalized size = 1.08

$$\frac{\left(2ax\left(\sqrt{a^2x^2-b}+ax\right)-2b\right)\left(\sqrt{\sqrt{a^2x^2-b}+ax}+c\right)^{3/2}\left(\frac{\sqrt{c}\left(35b^2\left(56c^2\sqrt{\sqrt{a^2x^2-b}+ax}-70c\left(\sqrt{a^2x^2-b}+ax\right)+105\left(\sqrt{a^2x^2-b}+ax\right)^{3/2}-48c^3\right)+1536ac^4x\left(\sqrt{a^2x^2-b}+ax\right)\left(4c^2\sqrt{\sqrt{a^2x^2-b}+ax}+5ax\sqrt{\sqrt{a^2x^2-b}+ax}-6acx-8c^3\right)-384bc^4\left(8c^2\sqrt{\sqrt{a^2x^2-b}+ax}-6c\left(\sqrt{a^2x^2-b}+3ax\right)+5\sqrt{\sqrt{a^2x^2-b}+ax}\left(\sqrt{a^2x^2-b}+3ax\right)-16c^3\right)\right)}{\left(\sqrt{\sqrt{a^2x^2-b}+ax}+c\right)^7}-\frac{105b\left(35b-256c^4\right)\left(\sqrt{a^2x^2-b}+ax\right)^2\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}}\right)}{\left(\sqrt{\sqrt{a^2x^2-b}+ax}+c\right)^{15/2}}\right)}{13440ac^{9/2}\sqrt{4a^2x^2-4b}\left(\frac{c}{\sqrt{\sqrt{a^2x^2-b}+ax}+c}-1\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-b + a^2*x^2]/Sqrt[c + Sqrt[a*x + Sqrt[-b + a^2*x^2]]],x]
```

```
[Out] ((c + Sqrt[a*x + Sqrt[-b + a^2*x^2]])^(3/2)*(-2*b + 2*a*x*(a*x + Sqrt[-b +
a^2*x^2]))*((Sqrt[c]*(1536*a*c^4*x*(a*x + Sqrt[-b + a^2*x^2])*(-8*c^3 - 6*a
*c*x + 4*c^2*Sqrt[a*x + Sqrt[-b + a^2*x^2]] + 5*a*x*Sqrt[a*x + Sqrt[-b + a^
2*x^2]]) + 35*b^2*(-48*c^3 + 56*c^2*Sqrt[a*x + Sqrt[-b + a^2*x^2]] - 70*c*(
a*x + Sqrt[-b + a^2*x^2]) + 105*(a*x + Sqrt[-b + a^2*x^2])^(3/2)) - 384*b*c
^4*(-16*c^3 + 8*c^2*Sqrt[a*x + Sqrt[-b + a^2*x^2]] - 6*c*(3*a*x + Sqrt[-b +
 a^2*x^2]) + 5*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*(3*a*x + Sqrt[-b + a^2*x^2]))
))/(c + Sqrt[a*x + Sqrt[-b + a^2*x^2]])^7 - (105*b*(35*b - 256*c^4)*(a*x +
Sqrt[-b + a^2*x^2])^2*ArcTanh[Sqrt[c]/Sqrt[c + Sqrt[a*x + Sqrt[-b + a^2*x^2
]]]])/(c + Sqrt[a*x + Sqrt[-b + a^2*x^2]])^(15/2)))/(13440*a*c^(9/2)*Sqrt[-
4*b + 4*a^2*x^2]*(-1 + c/(c + Sqrt[a*x + Sqrt[-b + a^2*x^2]]))^6)
```

IntegrateAlgebraic [A] time = 1.05, size = 495, normalized size = 1.00

$$-\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{c}}\right)}{128ac^{9/2}}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{\sqrt{a^2x^2-b}\left(\left(-9216a^2c^5x^2-12288ac^7x-2450b^2c+2304bc^5\right)\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}+\sqrt{\sqrt{a^2x^2-b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}\left(7680a^2c^4x^2+6144ac^6x+3675b^2-1920bc^4\right)\right)+\left(-9216a^3c^5x^3-12288a^2c^7x^2-2450ab^2cx+6912abc^5x-1680b^2c^3+6144bc^7\right)\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}+\sqrt{\sqrt{a^2x^2-b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}\left(7680a^3c^4x^3+6144a^2c^6x^2+3675ab^2x-5760abc^4x+1960b^2c^2-3072bc^6\right)}{13440ac^4\left(2a^2x^2-b\right)+26880a^2c^4x\sqrt{a^2x^2-b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-b + a^2*x^2]/Sqrt[c + Sqrt[a*x + Sqrt[-b + a^2*x^2
]]],x]
```

```
[Out] ((-1680*b^2*c^3 + 6144*b*c^7 - 2450*a*b^2*c*x + 6912*a*b*c^5*x - 12288*a^2*
c^7*x^2 - 9216*a^3*c^5*x^3)*Sqrt[c + Sqrt[a*x + Sqrt[-b + a^2*x^2]]] + (196
0*b^2*c^2 - 3072*b*c^6 + 3675*a*b^2*x - 5760*a*b*c^4*x + 6144*a^2*c^6*x^2 +
 7680*a^3*c^4*x^3)*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[
-b + a^2*x^2]]] + Sqrt[-b + a^2*x^2]*((-2450*b^2*c + 2304*b*c^5 - 12288*a*c
^7*x - 9216*a^2*c^5*x^2)*Sqrt[c + Sqrt[a*x + Sqrt[-b + a^2*x^2]]] + (3675*b
^2 - 1920*b*c^4 + 6144*a*c^6*x + 7680*a^2*c^4*x^2)*Sqrt[a*x + Sqrt[-b + a^2
*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]))/(26880*a^2*c^4*x*Sqrt[-b
+ a^2*x^2] + 13440*a*c^4*(-b + 2*a^2*x^2)) - (35*b^2*ArcTanh[Sqrt[c + Sqrt[
a*x + Sqrt[-b + a^2*x^2]]]/Sqrt[c]])/(128*a*c^(9/2)) + (2*b*ArcTanh[Sqrt[c
+ Sqrt[a*x + Sqrt[-b + a^2*x^2]]]/Sqrt[c]])/(a*Sqrt[c])
```

fricas [A] time = 0.72, size = 581, normalized size = 1.17

$$\left[\frac{105\,(256\,bc^4-35\,b^2)\sqrt{c}\log\left(2\left(a\sqrt{c}x-\sqrt{a^2x^2-b}\sqrt{c}\right)\sqrt{ax+\sqrt{a^2x^2-b}}\sqrt{c+\sqrt{ax+\sqrt{a^2x^2-b}}}+2\left(acx-\sqrt{a^2x^2-b}\,c\right)\sqrt{ax+\sqrt{a^2x^2-b}}+b\right)-2\left(6144\,c^8+3360\,a^2c^4x^2-1680\,bc^4+2\,(1152\,ac^6+1225\,abc^2)x+2\,(1152\,c^6-1680\,ac^4x-1225\,bc^2)\sqrt{a^2x^2-b}-\left(3072\,c^7+3920\,a^2c^3x^2-1960\,bc^3+15\,(128\,ac^5+245\,abc)x+5\,(384\,c^5-784\,ac^3x-735\,bc)\sqrt{a^2x^2-b}\right)\sqrt{ax+\sqrt{a^2x^2-b}}\right)\sqrt{c+\sqrt{ax+\sqrt{a^2x^2-b}}}}{26880\,ac^5},\ -\frac{105\,(256\,bc^4-35\,b^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{c+\sqrt{ax+\sqrt{a^2x^2-b}}}}{c}\right)+\left(6144\,c^8+3360\,a^2c^4x^2-1680\,bc^4+2\,(1152\,ac^6+1225\,abc^2)x+2\,(1152\,c^6-1680\,ac^4x-1225\,bc^2)\sqrt{a^2x^2-b}-\left(3072\,c^7+3920\,a^2c^3x^2-1960\,bc^3+15\,(128\,ac^5+245\,abc)x+5\,(384\,c^5-784\,ac^3x-735\,bc)\sqrt{a^2x^2-b}\right)\sqrt{ax+\sqrt{a^2x^2-b}}\right)\sqrt{c+\sqrt{ax+\sqrt{a^2x^2-b}}}}{13440\,ac^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x, algo
rithm="fricas")
```

```
[Out] [1/26880*(105*(256*b*c^4 - 35*b^2)*sqrt(c)*log(2*(a*sqrt(c)*x - sqrt(a^2*x^
2 - b)*sqrt(c))*sqrt(a*x + sqrt(a^2*x^2 - b))*sqrt(c + sqrt(a*x + sqrt(a^2*
x^2 - b))) + 2*(a*c*x - sqrt(a^2*x^2 - b)*c)*sqrt(a*x + sqrt(a^2*x^2 - b))
+ b) - 2*(6144*c^8 + 3360*a^2*c^4*x^2 - 1680*b*c^4 + 2*(1152*a*c^6 + 1225*a
*b*c^2)*x + 2*(1152*c^6 - 1680*a*c^4*x - 1225*b*c^2)*sqrt(a^2*x^2 - b) - (3
072*c^7 + 3920*a^2*c^3*x^2 - 1960*b*c^3 + 15*(128*a*c^5 + 245*a*b*c)*x + 5*
(384*c^5 - 784*a*c^3*x - 735*b*c)*sqrt(a^2*x^2 - b))*sqrt(a*x + sqrt(a^2*x^
2 - b)))*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 - b))))/(a*c^5), -1/13440*(105*(2
56*b*c^4 - 35*b^2)*sqrt(-c)*arctan(sqrt(-c)*sqrt(c + sqrt(a*x + sqrt(a^2*x^
2 - b)))/c) + (6144*c^8 + 3360*a^2*c^4*x^2 - 1680*b*c^4 + 2*(1152*a*c^6 + 1
225*a*b*c^2)*x + 2*(1152*c^6 - 1680*a*c^4*x - 1225*b*c^2)*sqrt(a^2*x^2 - b)
 - (3072*c^7 + 3920*a^2*c^3*x^2 - 1960*b*c^3 + 15*(128*a*c^5 + 245*a*b*c)*x
 + 5*(384*c^5 - 784*a*c^3*x - 735*b*c)*sqrt(a^2*x^2 - b))*sqrt(a*x + sqrt(a
^2*x^2 - b)))*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 - b))))/(a*c^5)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2-b}}{\sqrt{c+\sqrt{ax+\sqrt{a^2x^2-b}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] int((a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2 - b}}{\sqrt{c + \sqrt{ax + \sqrt{a^2x^2 - b}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x, algo
rithm="maxima")
[Out] integrate(sqrt(a^2*x^2 - b)/sqrt(c + sqrt(a*x + sqrt(a^2*x^2 - b))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a^2\,x^2 - b}}{\sqrt{c + \sqrt{a\,x + \sqrt{a^2\,x^2 - b}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 - b)^(1/2)/(c + (a*x + (a^2*x^2 - b)^(1/2))^(1/2))^(1/2),x)
[Out] int((a^2*x^2 - b)^(1/2)/(c + (a*x + (a^2*x^2 - b)^(1/2))^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2 - b}}{\sqrt{c + \sqrt{ax + \sqrt{a^2x^2 - b}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b)**(1/2)/(c+(a*x+(a**2*x**2-b)**(1/2))**(1/2))**(1/2)
,x)
[Out] Integral(sqrt(a**2*x**2 - b)/sqrt(c + sqrt(a*x + sqrt(a**2*x**2 - b))), x)
```

3.2386 $\int \frac{1+x}{\sqrt[3]{27+189x+522x^2+784x^3+825x^4+679x^5+338x^6+84x^7+8x^8}}\,dx$

Optimal. Leaf size=496

$$-\frac{\log\left(2x^2+7x+3\right)}{5\sqrt[3]{10}}+\frac{\log\left(4x^4+28x^3+61x^2+42x+9\right)}{10\sqrt[3]{10}}+\frac{\log\left(2\sqrt[3]{10}\,x^3+3\sqrt[3]{10}\,x^2+5\sqrt[3]{8x^8+84x^7+338x^6}\right.}{}$$

Rubi [F] time = 0.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x}{\sqrt[3]{27+189x+522x^2+784x^3+825x^4+679x^5+338x^6+84x^7+8x^8}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x)/(27 + 189*x + 522*x^2 + 784*x^3 + 825*x^4 + 679*x^5 + 338*x^6 +
 84*x^7 + 8*x^8)^(1/3),x]
```

```
[Out] ((1 + x^2)^(1/3)*(3 + 7*x + 2*x^2)*Defer[Int][(1 + x)/((1 + x^2)^(1/3)*(3 +
 7*x + 2*x^2)), x])/((1 + x^2)*(3 + 7*x + 2*x^2)^3)^(1/3)
```

Rubi steps

$$\int \frac{1+x}{\sqrt[3]{27+189x+522x^2+784x^3+825x^4+679x^5+338x^6+84x^7+8x^8}}\,dx=\int \frac{1+x}{\sqrt[3]{\left(1+x^2\right)\left(3+7x+2x^2\right)^3}}$$

$$=\frac{\left(\sqrt[3]{1+x^2}\,\left(3+7x+2x^2\right)\right)\int}{\sqrt[3]{\left(1+x^2\right)\left(3+7x+\right.}}$$

Mathematica [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{1+x}{\sqrt[3]{27+189x+522x^2+784x^3+825x^4+679x^5+338x^6+84x^7+8x^8}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x)/(27 + 189*x + 522*x^2 + 784*x^3 + 825*x^4 + 679*x^5 + 338
*x^6 + 84*x^7 + 8*x^8)^(1/3),x]
```

```
[Out] Integrate[(1 + x)/(27 + 189*x + 522*x^2 + 784*x^3 + 825*x^4 + 679*x^5 + 338
*x^6 + 84*x^7 + 8*x^8)^(1/3), x]
```

IntegrateAlgebraic [A] time = 1.06, size = 496, normalized size = 1.00

$$-\frac{\log\left(2x^2+7x+3\right)}{5\sqrt[3]{10}}+\frac{\log\left(4x^4+28x^3+61x^2+42x+9\right)}{10\sqrt[3]{10}}+\frac{\log\left(2\sqrt[3]{10}x^3+3\sqrt[3]{10}x^2+5\sqrt[3]{8x^8+84x^7+338x^6+679x^5+825x^4+784x^3+522x^2+189x+27}-11\sqrt[3]{10}x-6\sqrt[3]{10}\right)}{5\sqrt[3]{10}}-\frac{\log\left(4\,10^{2/3}x^6+12\,10^{2/3}x^5-35\,10^{2/3}x^4-90\,10^{2/3}x^3+85\,10^{2/3}x^2+25\left(8x^8+84x^7+338x^6+679x^5+825x^4+784x^3+522x^2+189x+27\right)^{2/3}+\left(-10\sqrt[3]{10}x^3-15\sqrt[3]{10}x^2+55\sqrt[3]{10}x+30\sqrt[3]{10}\right)\sqrt[3]{8x^8+84x^7+338x^6+679x^5+825x^4+784x^3+522x^2+189x+27}+132\,10^{2/3}x+36\,10^{2/3}\right)}{10\sqrt[3]{10}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{5\sqrt{3}\sqrt[3]{8x^8+84x^7+338x^6+679x^5+825x^4+784x^3+522x^2+189x+27}}{-4\sqrt[3]{10}x^3-6\sqrt[3]{10}x^2+5\sqrt[3]{8x^8+84x^7+338x^6+679x^5+825x^4+784x^3+522x^2+189x+27}+22\sqrt[3]{10}x+12\sqrt[3]{10}}\right)}{5\sqrt[3]{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)/(27 + 189*x + 522*x^2 + 784*x^3 + 825*x^4 + 679*
x^5 + 338*x^6 + 84*x^7 + 8*x^8)^(1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(5*Sqrt[3]*(27 + 189*x + 522*x^2 + 784*x^3 + 825*x^4 + 679*
x^5 + 338*x^6 + 84*x^7 + 8*x^8)^(1/3))/(12*10^(1/3) + 22*10^(1/3)*x - 6*10^
(1/3)*x^2 - 4*10^(1/3)*x^3 + 5*(27 + 189*x + 522*x^2 + 784*x^3 + 825*x^4 +
```

```
679*x^5 + 338*x^6 + 84*x^7 + 8*x^8)^(1/3))])/(5*10^(1/3)) - Log[3 + 7*x + 2
*x^2]/(5*10^(1/3)) + Log[9 + 42*x + 61*x^2 + 28*x^3 + 4*x^4]/(10*10^(1/3))
+ Log[-6*10^(1/3) - 11*10^(1/3)*x + 3*10^(1/3)*x^2 + 2*10^(1/3)*x^3 + 5*(27
 + 189*x + 522*x^2 + 784*x^3 + 825*x^4 + 679*x^5 + 338*x^6 + 84*x^7 + 8*x^8
)^(1/3)]/(5*10^(1/3)) - Log[36*10^(2/3) + 132*10^(2/3)*x + 85*10^(2/3)*x^2
- 90*10^(2/3)*x^3 - 35*10^(2/3)*x^4 + 12*10^(2/3)*x^5 + 4*10^(2/3)*x^6 + (3
0*10^(1/3) + 55*10^(1/3)*x - 15*10^(1/3)*x^2 - 10*10^(1/3)*x^3)*(27 + 189*x
 + 522*x^2 + 784*x^3 + 825*x^4 + 679*x^5 + 338*x^6 + 84*x^7 + 8*x^8)^(1/3)
+ 25*(27 + 189*x + 522*x^2 + 784*x^3 + 825*x^4 + 679*x^5 + 338*x^6 + 84*x^7
 + 8*x^8)^(2/3)]/(10*10^(1/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x
+27)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(8\,x^{8}+84\,x^{7}+338\,x^{6}+679\,x^{5}+825\,x^{4}+784\,x^{3}+522\,x^{2}+189\,x+27\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x
+27)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(8*x^8 + 84*x^7 + 338*x^6 + 679*x^5 + 825*x^4 + 784*x^3 +
 522*x^2 + 189*x + 27)^(1/3), x)
```

maple [C] time = 19.66, size = 3644, normalized size = 7.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(
1/3),x)
```

```
[Out] 1/9*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*ln((127
9821560256*RootOf(_Z^3-100)^2*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3
+522*x^2+189*x+27)^(1/3)*x^4-639910780128*RootOf(_Z^3-100)^2*(8*x^8+84*x^7+
338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(1/3)*x^3-3555059889600*R
ootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-
100)^2*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(2/3
)-10878483262176*RootOf(_Z^3-100)^2*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+7
84*x^3+522*x^2+189*x+27)^(1/3)*x^2+10238572482048*RootOf(_Z^3-100)^2*(8*x^8
+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(1/3)*x+280692190
56600*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(1/3)
*RootOf(_Z^3-100)*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500
*_Z^2)-21887249427900*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2
+189*x+27)^(2/3)-1607334566400*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_
Z^3-100)+2500*_Z^2)*x^7+49877600763600*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*
RootOf(_Z^3-100)+2500*_Z^2)*x^6+384102732164400*RootOf(81*RootOf(_Z^3-100)^
```

2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x^5+5755429762500*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)^2*RootOf(_Z^3-100)^2*x^5-3549117144375*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^3*x^5+17440696250000*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)^2*RootOf(_Z^3-100)^2*x^4-10754900437500*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^3*x^4+28777148812500*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)^2*RootOf(_Z^3-100)^2*x^3-17745585721875*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^3*x^3+18208086885000*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)^2*RootOf(_Z^3-100)^2*x^2-11228116056750*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^3*x^2+3767190390000*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)^2*RootOf(_Z^3-100)^2*x-2323058494500*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^3*x-489933036490050*RootOf(_Z^3-100)*x^3-565014287032290*RootOf(_Z^3-100)*x^2-389855676546990*RootOf(_Z^3-100)*x-520868432108475*RootOf(_Z^3-100)*x^4+144773126687700*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)+37425625408800*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(1/3)*RootOf(_Z^3-100)*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x-89275137943635*RootOf(_Z^3-100)+632209891249800*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x-236859044099220*RootOf(_Z^3-100)*x^5+139525570000*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)^2*RootOf(_Z^3-100)^2*x^7-86039203500*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^3*x^7+1255730130000*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)^2*RootOf(_Z^3-100)^2*x^6-774352831500*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^3*x^6+844666871944500*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x^4+794500453251000*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x^3+916256046655800*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x^2-30757294467180*RootOf(_Z^3-100)*x^6+991171624320*RootOf(_Z^3-100)*x^7+4678203176100*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(1/3)*RootOf(_Z^3-100)*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x^4+1777529944800*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^2*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(2/3)*x-2339101588050*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(1/3)*RootOf(_Z^3-100)*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x^3-39764726996850*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(1/3)*RootOf(_Z^3-100)*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x^2+10943624713950*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(2/3)*x+7678929361536*RootOf(_Z^3-100)^2*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(1/3))/(1+2*x)^3/(3+x)^4)+1/50*RootOf(_Z^3-100)*ln((-639910780128*RootOf(_Z^3-100)^2*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(1/3)*x^4+319955390064*RootOf(_Z^3-100)^2*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(1/3)*x^3+1777529944800*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^2*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(2/3)+5439241631088*RootOf(_Z^3-100)^2*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(1/3)*x^2-5119286241024*RootOf(_Z^3-100)^2*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(1/3)*x-7295749809300*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(1/3)*RootOf(_Z^3-100)*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)+21051914292450*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(2/3)+7055214687000*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x^7-46719287500500*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x^6-480485931945750*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x^5+9858658734375*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)^2*RootOf(

```
_Z^3-100)^2*x^5-517988678625*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^
3-100)+2500*_Z^2)*RootOf(_Z^3-100)^3*x^5+29874723437500*RootOf(81*RootOf(_Z
^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)^2*RootOf(_Z^3-100)^2*x^4-15696
62662500*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*Ro
otOf(_Z^3-100)^3*x^4+49293293671875*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*Roo
tOf(_Z^3-100)+2500*_Z^2)^2*RootOf(_Z^3-100)^2*x^3-2589943393125*RootOf(81*R
ootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^3*x^3
+31189211268750*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_
Z^2)^2*RootOf(_Z^3-100)^2*x^2-1638727819650*RootOf(81*RootOf(_Z^3-100)^2+45
0*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^3*x^2+6452940262500*RootO
f(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)^2*RootOf(_Z^3-10
0)^2*x-339047135100*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+25
00*_Z^2)*RootOf(_Z^3-100)^3*x+24886059716340*RootOf(_Z^3-100)*x^3+529659434
45322*RootOf(_Z^3-100)*x^2+50796041780682*RootOf(_Z^3-100)*x+47766090550005
*RootOf(_Z^3-100)*x^4-247986494287875*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*R
ootOf(_Z^3-100)+2500*_Z^2)-9727666412400*(8*x^8+84*x^7+338*x^6+679*x^5+825*
x^4+784*x^3+522*x^2+189*x+27)^(1/3)*RootOf(_Z^3-100)*RootOf(81*RootOf(_Z^3-
100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x+13029581401893*RootOf(_Z^3-100)
-966779510127750*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*
_Z^2)*x+25245449679546*RootOf(_Z^3-100)*x^5+238997787500*RootOf(81*RootOf(_
Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)^2*RootOf(_Z^3-100)^2*x^7-1255
7301300*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*Roo
tOf(_Z^3-100)^3*x^7+2150980087500*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootO
f(_Z^3-100)+2500*_Z^2)^2*RootOf(_Z^3-100)^2*x^6-113015711700*RootOf(81*Root
Of(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^3*x^6-90
9111733981875*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^
2)*x^4-473645815267500*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)
+2500*_Z^2)*x^3-1008078327807750*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf
(_Z^3-100)+2500*_Z^2)*x^2+2454701258124*RootOf(_Z^3-100)*x^6-370691534376*R
ootOf(_Z^3-100)*x^7-1215958301550*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784
*x^3+522*x^2+189*x+27)^(1/3)*RootOf(_Z^3-100)*RootOf(81*RootOf(_Z^3-100)^2+
450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x^4-888764972400*RootOf(81*RootOf(_Z^3-1
00)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*RootOf(_Z^3-100)^2*(8*x^8+84*x^7+3
38*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(2/3)*x+607979150775*(8*x^
8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(1/3)*RootOf(_Z^
3-100)*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-100)+2500*_Z^2)*x^3+
10335645563175*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+
27)^(1/3)*RootOf(_Z^3-100)*RootOf(81*RootOf(_Z^3-100)^2+450*_Z*RootOf(_Z^3-
100)+2500*_Z^2)*x^2-10525957146225*(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+78
4*x^3+522*x^2+189*x+27)^(2/3)*x-3839464680768*RootOf(_Z^3-100)^2*(8*x^8+84*
x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x+27)^(1/3))/(1+2*x)^3/(3+x
)^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(8\,x^8+84\,x^7+338\,x^6+679\,x^5+825\,x^4+784\,x^3+522\,x^2+189\,x+27\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(8*x^8+84*x^7+338*x^6+679*x^5+825*x^4+784*x^3+522*x^2+189*x
+27)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(8*x^8 + 84*x^7 + 338*x^6 + 679*x^5 + 825*x^4 + 784*x^3 +
 522*x^2 + 189*x + 27)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x+1}{{\left(8\,x^8+84\,x^7+338\,x^6+679\,x^5+825\,x^4+784\,x^3+522\,x^2+189\,x+27\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(189*x + 522*x^2 + 784*x^3 + 825*x^4 + 679*x^5 + 338*x^6 + 84*x
^7 + 8*x^8 + 27)^(1/3),x)
```

```
[Out] int((x + 1)/(189*x + 522*x^2 + 784*x^3 + 825*x^4 + 679*x^5 + 338*x^6 + 84*x
^7 + 8*x^8 + 27)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt[3]{(x+3)^3 (2x+1)^3 \left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(8*x**8+84*x**7+338*x**6+679*x**5+825*x**4+784*x**3+522*x**
2+189*x+27)**(1/3),x)
```

```
[Out] Integral((x + 1)/((x + 3)**3*(2*x + 1)**3*(x**2 + 1))**(1/3), x)
```

3.2387 $\int \frac{-1+k^2x^4}{\sqrt{(1-x^2)(1-k^2x^2)}\,(1+k^2x^4)}\,dx$

Optimal. Leaf size=497

$$\frac{i\left(i\sqrt{k^2+2i\sqrt{2}\sqrt{k^2+1}\sqrt{k}-2k+1}\,k^2+\sqrt{2}\sqrt{k^2+1}\sqrt{k^2+2i\sqrt{2}\sqrt{k^2+1}\sqrt{k}-2k+1}\sqrt{k}+i\sqrt{k^2+2i\sqrt{2}\sqrt{k^2+\cdots}}\right.}{\left(\sqrt{k}-i\right)^2\left(\sqrt{k}+i\right)^2(k-i)(k+i)}$$

Rubi [A] time = 0.16, antiderivative size = 45, normalized size of antiderivative = 0.09, number of steps used = 2, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2112, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{k^2+1}\,x}{\sqrt{(1-x^2)(1-k^2x^2)}}\right)}{\sqrt{k^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + k^2*x^4)/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(1 + k^2*x^4)),x]
```

```
[Out] -(ArcTan[(Sqrt[1 + k^2]*x)/Sqrt[(1 - x^2)*(1 - k^2*x^2)]]/Sqrt[1 + k^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2112

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+k^2x^4}{\sqrt{(1-x^2)(1-k^2x^2)}\,(1+k^2x^4)}\,dx &= -\text{Subst}\left(\int \frac{1}{1-(-1-k^2)x^2}\,dx, x, \frac{x}{\sqrt{(1-x^2)(1-k^2x^2)}}\right)\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{1+k^2}\,x}{\sqrt{(1-x^2)(1-k^2x^2)}}\right)}{\sqrt{1+k^2}}\end{aligned}$$

Mathematica [C] time = 0.47, size = 78, normalized size = 0.16

$$\frac{\sqrt{1-x^2}\sqrt{1-k^2x^2}\left(F\left(\sin^{-1}(x)|k^2\right)-\Pi\left(-ik;\sin^{-1}(x)|k^2\right)-\Pi\left(ik;\sin^{-1}(x)|k^2\right)\right)}{\sqrt{(x^2-1)(k^2x^2-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + k^2*x^4)/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(1 + k^2*x^4)),x]
```

```
[Out] (Sqrt[1 - x^2]*Sqrt[1 - k^2*x^2]*(EllipticF[ArcSin[x], k^2] - EllipticPi[(-
I)*k, ArcSin[x], k^2] - EllipticPi[I*k, ArcSin[x], k^2]))/Sqrt[(-1 + x^2)*(
-1 + k^2*x^2)]
```

IntegrateAlgebraic [A] time = 2.89, size = 497, normalized size = 1.00

$$\frac{i\left(i\sqrt{k^2+2i\sqrt{2}\sqrt{k^2+1}\sqrt{k}-2k+1}\,k^2+\sqrt{2}\sqrt{k^2+1}\sqrt{k^2+2i\sqrt{2}\sqrt{k^2+1}\sqrt{k}-2k+1}\sqrt{k}+i\sqrt{k^2+2i\sqrt{2}\sqrt{k^2+1}\sqrt{k}-2k+1}\right)\tan^{-1}\left(\frac{\sqrt{k^2+2i\sqrt{2}\sqrt{k^2+1}\sqrt{k}-2k+1}\,x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2+1}\right)}{\left(\sqrt{k}-i\right)^2\left(\sqrt{k}+i\right)^2(k-i)(k+i)}-\frac{i\left(-i\sqrt{k^2-2i\sqrt{2}\sqrt{k^2+1}\sqrt{k}-2k+1}\,k^2+\sqrt{2}\sqrt{k^2+1}\sqrt{k^2-2i\sqrt{2}\sqrt{k^2+1}\sqrt{k}-2k+1}\sqrt{k}-i\sqrt{k^2-2i\sqrt{2}\sqrt{k^2+1}\sqrt{k}-2k+1}\right)\tan^{-1}\left(\frac{\sqrt{k^2-2i\sqrt{2}\sqrt{k^2+1}\sqrt{k}-2k+1}\,x}{\sqrt{k^2x^4+\left(-k^2-1\right)x^2+1}+kx^2+1}\right)}{\left(\sqrt{k}-i\right)^2\left(\sqrt{k}+i\right)^2(k-i)(k+i)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + k^2*x^4)/(Sqrt[(1 - x^2)*(1 - k^2*x^2)]*(1 + k^2*x
^4)),x]
```

```
[Out] ((-I)*((-I)*Sqrt[1 - 2*k + k^2 - (2*I)*Sqrt[2]*Sqrt[k]*Sqrt[1 + k^2]] - I*k
^2*Sqrt[1 - 2*k + k^2 - (2*I)*Sqrt[2]*Sqrt[k]*Sqrt[1 + k^2]] + Sqrt[2]*Sqrt
[k]*Sqrt[1 + k^2]*Sqrt[1 - 2*k + k^2 - (2*I)*Sqrt[2]*Sqrt[k]*Sqrt[1 + k^2]]
)*ArcTan[(Sqrt[1 - 2*k + k^2 - (2*I)*Sqrt[2]*Sqrt[k]*Sqrt[1 + k^2]]*x)/(1 +
 k*x^2 + Sqrt[1 + (-1 - k^2)*x^2 + k^2*x^4]))/((-I + Sqrt[k])^2*(I + Sqrt[
k])^2*(-I + k)*(I + k)) + (I*(I*Sqrt[1 - 2*k + k^2 + (2*I)*Sqrt[2]*Sqrt[k]*
Sqrt[1 + k^2]] + I*k^2*Sqrt[1 - 2*k + k^2 + (2*I)*Sqrt[2]*Sqrt[k]*Sqrt[1 +
k^2]] + Sqrt[2]*Sqrt[k]*Sqrt[1 + k^2]*Sqrt[1 - 2*k + k^2 + (2*I)*Sqrt[2]*Sq
rt[k]*Sqrt[1 + k^2]])*ArcTan[(Sqrt[1 - 2*k + k^2 + (2*I)*Sqrt[2]*Sqrt[k]*Sq
rt[1 + k^2]]*x)/(1 + k*x^2 + Sqrt[1 + (-1 - k^2)*x^2 + k^2*x^4])])/((-I + S
qrt[k])^2*(I + Sqrt[k])^2*(-I + k)*(I + k))
```

fricas [A] time = 0.46, size = 62, normalized size = 0.12

$$-\frac{\arctan\left(\frac{2\sqrt{k^2x^4-\left(k^2+1\right)x^2+1}\,\sqrt{k^2+1}\,x}{k^2x^4-2\left(k^2+1\right)x^2+1}\right)}{2\sqrt{k^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^4-1)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^4+1),x, algorith
m="fricas")
```

```
[Out] -1/2*arctan(2*sqrt(k^2*x^4 - (k^2 + 1)*x^2 + 1)*sqrt(k^2 + 1)*x/(k^2*x^4 -
2*(k^2 + 1)*x^2 + 1))/sqrt(k^2 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^4-1}{\left(k^2x^4+1\right)\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^4-1)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^4+1),x, algorith
m="giac")
```

```
[Out] integrate((k^2*x^4 - 1)/((k^2*x^4 + 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))), x)
```

maple [C] time = 0.07, size = 269, normalized size = 0.54

$$\frac{\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\operatorname{EllipticF}(x,k)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(k^2\_Z^4+1\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{\left(2k^2\_\alpha^2-k^2-1\right)\left(\_\alpha^2k^4+k^4x^2-2k^2\_\alpha^2+6k^2x^2+\_\alpha^2-4k^2+x^2-4\right)}{2\left(k^4+6k^2+1\right)\sqrt{-k^2\_\alpha^2-\_\alpha^2}\,\sqrt{k^2x^4-k^2x^2-x^2+1}}\right)}{\sqrt{-k^2\_\alpha^2-\_\alpha^2}}+\frac{2\sqrt{-x^2+1}\,\sqrt{-k^2x^2+1}\,\_\alpha^3k^2\operatorname{EllipticPi}\left(x,-k^2\_\alpha^2,k\right)}{\sqrt{k^2x^4-k^2x^2-x^2+1}}}{\_\alpha^3}}{4k^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^4-1)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^4+1),x)
```

```
[Out] (-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1)^(1/2)*EllipticF(x
,k)-1/4/k^2*sum(1/_alpha^3*(-1/(-_alpha^2*k^2-_alpha^2)^(1/2)*arctanh(1/2*(
2*_alpha^2*k^2-k^2-1)/(k^4+6*k^2+1)*(_alpha^2*k^4+k^4*x^2-2*_alpha^2*k^2+6*
k^2*x^2+_alpha^2-4*k^2+x^2-4)/(-_alpha^2*k^2-_alpha^2)^(1/2)/(k^2*x^4-k^2*x
^2-x^2+1)^(1/2))+2*(-x^2+1)^(1/2)*(-k^2*x^2+1)^(1/2)/(k^2*x^4-k^2*x^2-x^2+1
)^(1/2)*_alpha^3*k^2*EllipticPi(x,-k^2*_alpha^2,k)),_alpha=RootOf(_Z^4*k^2+
1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{k^2x^4-1}{\left(k^2x^4+1\right)\sqrt{\left(k^2x^2-1\right)\left(x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k^2*x^4-1)/((-x^2+1)*(-k^2*x^2+1))^(1/2)/(k^2*x^4+1),x, algorith
m="maxima")
```

```
[Out] integrate((k^2*x^4 - 1)/((k^2*x^4 + 1)*sqrt((k^2*x^2 - 1)*(x^2 - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{k^2\,x^4-1}{\left(k^2\,x^4+1\right)\,\sqrt{\left(x^2-1\right)\,\left(k^2\,x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k^2*x^4 - 1)/((k^2*x^4 + 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)),x)
```

```
[Out] int((k^2*x^4 - 1)/((k^2*x^4 + 1)*((x^2 - 1)*(k^2*x^2 - 1))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(kx^2-1\right)\left(kx^2+1\right)}{\sqrt{(x-1)\,(x+1)\,(kx-1)\,(kx+1)}\,\left(k^2x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k**2*x**4-1)/((-x**2+1)*(-k**2*x**2+1))**(1/2)/(k**2*x**4+1),x)
```

```
[Out] Integral((k*x**2 - 1)*(k*x**2 + 1)/(sqrt((x - 1)*(x + 1)*(k*x - 1)*(k*x + 1
))*(k**2*x**4 + 1)), x)
```

3.2388 $\int \frac{1+2x}{\sqrt[3]{-1+x^2}\,(3+x^2)}\,dx$

Optimal. Leaf size=499

$$\frac{(-1)^{2/3}\sqrt[3]{-54+35i\sqrt{3}}\,\log\left(6\sqrt[3]{x^2-1}-i2^{2/3}\sqrt{3}\,x+3\,2^{2/3}\right)}{6\,6^{2/3}}-\frac{\sqrt[3]{-1}\,\sqrt[3]{-54-35i\sqrt{3}}\,\log\left(6\sqrt[3]{x^2-1}+i2^{2/3}\sqrt{3}\,x\right.}{6\,6^{2/3}}$$

Rubi [A] time = 0.09, antiderivative size = 213, normalized size of antiderivative = 0.43, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1010, 393, 444, 56, 617, 204, 31}

$$\frac{\log\left(x^2+3\right)}{2\,2^{2/3}}-\frac{3\log\left(\sqrt[3]{x^2-1}+2^{2/3}\right)}{2\,2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\sqrt[3]{2}\sqrt[3]{x^2-1}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left((-1)^{2/3}\sqrt[3]{2}\sqrt[3]{x^2-1}+1\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{1}{2}\left(-\frac{1}{2}\right)^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{-1}\,x}{\sqrt[3]{2}\sqrt[3]{x^2-1}+\sqrt[3]{-1}}\right)-\frac{(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{1}{6}\left(-\frac{1}{2}\right)^{2/3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/((-1 + x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -1/2*((-1)^(2/3)*ArcTan[Sqrt[3]/x])/(2^(2/3)*Sqrt[3]) - (Sqrt[3]*ArcTan[(1
- 2^(1/3)*(-1 + x^2)^(1/3))/Sqrt[3]])/2^(2/3) - ((-1)^(2/3)*ArcTan[(Sqrt[3]
*(1 + (-1)^(2/3)*2^(1/3)*(-1 + x^2)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) + ((-1/
2)^(2/3)*ArcTanh[x])/6 - ((-1/2)^(2/3)*ArcTanh[((-1)^(1/3)*x)/((-1)^(1/3) +
 2^(1/3)*(-1 + x^2)^(1/3))])/2 + Log[3 + x^2]/(2*2^(2/3)) - (3*Log[2^(2/3)
+ (-1 + x^2)^(1/3)])/(2*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{1+2x}{\sqrt[3]{-1+x^2}\left(3+x^2\right)}\,dx = 2\int \frac{x}{\sqrt[3]{-1+x^2}\left(3+x^2\right)}\,dx + \int \frac{1}{\sqrt[3]{-1+x^2}\left(3+x^2\right)}\,dx$$

$$= -\frac{(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1+(-1)^{2/3}\sqrt[3]{2}\sqrt[3]{-1+x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{1}{6}\left(-\frac{1}{2}\right)^{2/3}\tanh^{-1}(x)$$

$$= -\frac{(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1+(-1)^{2/3}\sqrt[3]{2}\sqrt[3]{-1+x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{1}{6}\left(-\frac{1}{2}\right)^{2/3}\tanh^{-1}(x)$$

$$= -\frac{(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1+(-1)^{2/3}\sqrt[3]{2}\sqrt[3]{-1+x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{1}{6}\left(-\frac{1}{2}\right)^{2/3}\tanh^{-1}(x)$$

$$= -\frac{(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\sqrt[3]{2}\sqrt[3]{-1+x^2}}{\sqrt{3}}\right)}{2^{2/3}} - \frac{(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1+(-1)^{2/3}\sqrt[3]{2}\sqrt[3]{-1+x}\right.}{x}\right.}{2\,2^{2/3}\sqrt{3}}$$

Mathematica [C] time = 0.17, size = 151, normalized size = 0.30

$$\frac{x\left(x\sqrt[3]{1-x^2}\,F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right) - \frac{27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\left(x^2+3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}\right)}{3\sqrt[3]{x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + 2*x)/((-1 + x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (x*(x*(1 - x^2)^(1/3)*AppellF1[1, 1/3, 1, 2, x^2, -1/3*x^2] - (27*AppellF1[
1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((3 + x^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2,
 x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2] - Appell
F1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2]))))))/(3*(-1 + x^2)^(1/3))
```

IntegrateAlgebraic [A] time = 7.99, size = 499, normalized size = 1.00

$$\frac{(-1)^{2/3}\sqrt[3]{-54+35i\sqrt{3}}\,\log\left(6\sqrt[3]{x^2-1}-i2^{2/3}\sqrt{3}x+3\,2^{2/3}\right)}{6\,6^{2/3}} - \frac{\sqrt[3]{-1}\sqrt[3]{-54-35i\sqrt{3}}\,\log\left(6\sqrt[3]{x^2-1}+i2^{2/3}\sqrt{3}x+3\,2^{2/3}\right)}{6\,6^{2/3}} + \frac{\sqrt[3]{54+35i\sqrt{3}}\,\log\left(-\sqrt[3]{2}x^2-i2^{2/3}\sqrt{3}\sqrt[3]{x^2-1}\,x+6\left(x^2-1\right)^{2/3}-3\,2^{2/3}\sqrt[3]{x^2-1}+2i\sqrt[3]{2}\sqrt{3}x+3\sqrt[3]{2}\right)}{12\,6^{2/3}} + \frac{\sqrt[3]{54-35i\sqrt{3}}\,\log\left(-\sqrt[3]{2}x^2+i2^{2/3}\sqrt{3}\sqrt[3]{x^2-1}\,x+6\left(x^2-1\right)^{2/3}-3\,2^{2/3}\sqrt[3]{x^2-1}-2i\sqrt[3]{2}\sqrt{3}x+3\sqrt[3]{2}\right)}{12\,6^{2/3}} - \frac{\sqrt[6]{-253+1260i\sqrt{3}}\,\tan^{-1}\left(\frac{3\sqrt[3]{x^2-1}}{\sqrt{3}\sqrt[3]{x^2-1}-i2^{2/3}x-2^{2/3}\sqrt{3}}\right)}{6\,2^{2/3}} - \frac{\sqrt[6]{-253-1260i\sqrt{3}}\,\tan^{-1}\left(\frac{3\sqrt[3]{x^2-1}}{\sqrt{3}\sqrt[3]{x^2-1}+i2^{2/3}x-2^{2/3}\sqrt{3}}\right)}{6\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*x)/((-1 + x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -1/6*((-253 + (1260*I)*Sqrt[3])^(1/6)*ArcTan[(3*(-1 + x^2)^(1/3))/(-(2^(2/3
)*Sqrt[3]) - I*2^(2/3)*x + Sqrt[3]*(-1 + x^2)^(1/3))])/2^(2/3) - ((-253 - (
1260*I)*Sqrt[3])^(1/6)*ArcTan[(3*(-1 + x^2)^(1/3))/(-(2^(2/3)*Sqrt[3]) + I*
2^(2/3)*x + Sqrt[3]*(-1 + x^2)^(1/3))])/(6*2^(2/3)) + ((-1)^(2/3)*(-54 + (3
5*I)*Sqrt[3])^(1/3)*Log[3*2^(2/3) - I*2^(2/3)*Sqrt[3]*x + 6*(-1 + x^2)^(1/3
)])/(6*6^(2/3)) - ((-1)^(1/3)*(-54 - (35*I)*Sqrt[3])^(1/3)*Log[3*2^(2/3) +
I*2^(2/3)*Sqrt[3]*x + 6*(-1 + x^2)^(1/3)])/(6*6^(2/3)) + ((54 + (35*I)*Sqrt
[3])^(1/3)*Log[3*2^(1/3) + (2*I)*2^(1/3)*Sqrt[3]*x - 2^(1/3)*x^2 - 3*2^(2/3
)*(-1 + x^2)^(1/3) - I*2^(2/3)*Sqrt[3]*x*(-1 + x^2)^(1/3) + 6*(-1 + x^2)^(2
/3)])/(12*6^(2/3)) + ((54 - (35*I)*Sqrt[3])^(1/3)*Log[3*2^(1/3) - (2*I)*2^(
1/3)*Sqrt[3]*x - 2^(1/3)*x^2 - 3*2^(2/3)*(-1 + x^2)^(1/3) + I*2^(2/3)*Sqrt[
3]*x*(-1 + x^2)^(1/3) + 6*(-1 + x^2)^(2/3)])/(12*6^(2/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2-1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x+1}{\left(x^2+3\right)\left(x^2-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2-1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate((2*x + 1)/((x^2 + 3)*(x^2 - 1)^(1/3)), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1+2x}{\left(x^2-1\right)^{\frac{1}{3}}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(x^2-1)^(1/3)/(x^2+3),x)
```

```
[Out] int((1+2*x)/(x^2-1)^(1/3)/(x^2+3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x+1}{\left(x^2+3\right)\left(x^2-1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2-1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)/((x^2 + 3)*(x^2 - 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{2\,x+1}{{\left(x^2-1\right)}^{1/3}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/((x^2 - 1)^(1/3)*(x^2 + 3)),x)
```

```
[Out] int((2*x + 1)/((x^2 - 1)^(1/3)*(x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\sqrt[3]{(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2-1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral((2*x + 1)/(((x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.2389 $\int \frac{x}{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}\,dx$

Optimal. Leaf size=501

$$\frac{-291\left(1-\sqrt{1-\sqrt{1-\frac{1}{x}}}\right)^{15/2}+2275\left(1-\sqrt{1-\sqrt{1-\frac{1}{x}}}\right)^{13/2}-6611\left(1-\sqrt{1-\sqrt{1-\frac{1}{x}}}\right)^{11/2}+8403\left(1-\sqrt{1-\cdots\right.}{384\left(\left(1-\sqrt{1-\sqrt{1-\frac{1}{x}}}\right.\right.\cdots}$$

Rubi [A] time = 1.50, antiderivative size = 758, normalized size of antiderivative = 1.51, number of steps used = 16, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1586, 1692, 207, 1178, 1166, 203}

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Int[x/Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]],x]
```

```
[Out] 1/(128*(1 - Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]])^4) + 5/(192*(1 - Sqrt[1 -
 Sqrt[1 - Sqrt[1 - x^(-1)]]])^3) + 23/(256*(1 - Sqrt[1 - Sqrt[1 - Sqrt[1 -
x^(-1)]]])^2) + 59/(256*(1 - Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]])) - 1/(12
8*(1 + Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]])^4) - 5/(192*(1 + Sqrt[1 - Sqrt
[1 - Sqrt[1 - x^(-1)]]])^3) - 23/(256*(1 + Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1
)]]])^2) - 59/(256*(1 + Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]])) - (Sqrt[1 -
Sqrt[1 - Sqrt[1 - x^(-1)]]]*(1 + Sqrt[1 - Sqrt[1 - x^(-1)]]))/(8*(1 + Sqrt[
1 - x^(-1)])^2) - (Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]]*(1 + Sqrt[1 - Sqrt[
1 - x^(-1)]]))/(8*(1 + Sqrt[1 - x^(-1)])) - (Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(
-1)]]]*(12 + 11*Sqrt[1 - Sqrt[1 - x^(-1)]]))/(64*(1 + Sqrt[1 - x^(-1)])) -
((1 + Sqrt[2])^(3/2)*ArcTan[Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]]/Sqrt[-1 +
Sqrt[2]]])/(16*Sqrt[2]) - (Sqrt[527 + 373*Sqrt[2]]*ArcTan[Sqrt[1 - Sqrt[1 -
 Sqrt[1 - x^(-1)]]]/Sqrt[-1 + Sqrt[2]]])/128 + (59*ArcTanh[Sqrt[1 - Sqrt[1
- Sqrt[1 - x^(-1)]]]])/128 + ((-1 + Sqrt[2])^(3/2)*ArcTanh[Sqrt[1 - Sqrt[1
- Sqrt[1 - x^(-1)]]]/Sqrt[1 + Sqrt[2]]])/(16*Sqrt[2]) + (Sqrt[-527 + 373*Sq
rt[2]]*ArcTanh[Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]]/Sqrt[1 + Sqrt[2]]])/128
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{x}{\sqrt{1-\sqrt{1-x}}\,\left(-1+x^2\right)^3}\,dx, x, \sqrt{1-\frac{1}{x}}\right)\right)$$

$$= 4\,\text{Subst}\left(\int \frac{1-x^2}{\sqrt{1-x}\,x^5\left(-2+x^2\right)^3}\,dx, x, \sqrt{1-\sqrt{1-\frac{1}{x}}}\right)$$

$$= 4\,\text{Subst}\left(\int \frac{\sqrt{1-x}\,(1+x)}{x^5\left(-2+x^2\right)^3}\,dx, x, \sqrt{1-\sqrt{1-\frac{1}{x}}}\right)$$

$$= -\left(8\,\text{Subst}\left(\int \frac{x^2\left(-2+x^2\right)}{\left(-1+x^2\right)^5\left(-1-2x^2+x^4\right)^3}\,dx, x, \sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)\right)$$

$$= -\left(8\,\text{Subst}\left(\int\left(\frac{1}{256(-1+x)^5} - \frac{5}{512(-1+x)^4} + \frac{23}{1024(-1+x)^3} - \frac{59}{2048(-1+x)^2}\right.\right.\right.$$

$$= \frac{1}{128\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^4} + \frac{5}{192\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^3} + \frac{}{256\left(1-\right.}$$

$$= \frac{1}{128\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^4} + \frac{5}{192\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^3} + \frac{}{256\left(1-\right.}$$

$$= \frac{1}{128\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^4} + \frac{5}{192\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^3} + \frac{}{256\left(1-\right.}$$

$$= \frac{1}{128\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^4} + \frac{5}{192\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^3} + \frac{}{256\left(1-\right.}$$

$$= \frac{1}{128\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^4} + \frac{5}{192\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^3} + \frac{}{256\left(1-\right.}$$

Mathematica [A] time = 1.40, size = 467, normalized size = 0.93

$$\frac{32\sqrt{1-\sqrt{\frac{x-1}{x}}}\,x^2+32\sqrt{1-\sqrt{\frac{x-1}{x}}}\sqrt{\frac{x-1}{x}}\,x^2+384x^2+114\sqrt{1-\sqrt{\frac{x-1}{x}}}\,x+106\sqrt{1-\sqrt{\frac{x-1}{x}}}\sqrt{\frac{x-1}{x}}\,x+4\sqrt{\frac{x-1}{x}}\,x+52x-177\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\log\left(1-\frac{1}{\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}}\right)+177\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\log\left(\frac{1}{\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}}+1\right)+6\sqrt{\sqrt{2}-1}\left(41+30\sqrt{2}\right)\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\tan^{-1}\left(\frac{1}{\sqrt{1+\sqrt{2}}\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}}\right)+6\sqrt{1+\sqrt{2}}\left(30\sqrt{2}-41\right)\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\tanh^{-1}\left(\frac{1}{\sqrt{\sqrt{2}-1}\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}}\right)-582}{768\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]],x]
```

```
[Out] (-582 + 52*x + 114*Sqrt[1 - Sqrt[(-1 + x)/x]]*x + 4*Sqrt[(-1 + x)/x]*x + 10
6*Sqrt[1 - Sqrt[(-1 + x)/x]]*Sqrt[(-1 + x)/x]*x + 384*x^2 + 32*Sqrt[1 - Sqr
t[(-1 + x)/x]]*x^2 + 32*Sqrt[1 - Sqrt[(-1 + x)/x]]*Sqrt[(-1 + x)/x]*x^2 + 6
*Sqrt[-1 + Sqrt[2]]*(41 + 30*Sqrt[2])*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]]*
ArcTan[1/(Sqrt[1 + Sqrt[2]]*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]])] + 6*Sqrt
[1 + Sqrt[2]]*(-41 + 30*Sqrt[2])*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]]*ArcTa
nh[1/(Sqrt[-1 + Sqrt[2]]*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]])] - 177*Sqrt[
1 - Sqrt[1 - Sqrt[(-1 + x)/x]]]*Log[1 - 1/Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x
]]]] + 177*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]]*Log[1 + 1/Sqrt[1 - Sqrt[1 -
 Sqrt[(-1 + x)/x]]]])/(768*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]])
```

IntegrateAlgebraic [F] time = 3.70, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]],x]
```

```
[Out] Defer[IntegrateAlgebraic][x/Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]], x]
```

fricas [A] time = 0.45, size = 344, normalized size = 0.69

$$\frac{1}{384}\left(\left(16x^2+\left(208x^2+291x\right)\sqrt{\frac{x-1}{x}}+55x\right)\sqrt{-\sqrt{\frac{x-1}{x}}+1}+2\left(96x^2+119x\right)\sqrt{\frac{x-1}{x}}-2x\right)\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}+\frac{1}{64}\sqrt{1021\sqrt{2}+1439}\arctan\left(-\frac{1}{119}\sqrt{1021\sqrt{2}+1439}\left(11\sqrt{2}-19\right)\sqrt{\sqrt{2}-\sqrt{-\sqrt{\frac{x-1}{x}}+1}}+\frac{1}{119}\sqrt{1021\sqrt{2}+1439}\left(11\sqrt{2}-19\right)\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}\right)+\frac{1}{256}\sqrt{1021\sqrt{2}-1439}\log\left(\sqrt{1021\sqrt{2}-1439}\left(30\sqrt{2}+41\right)+119\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}\right)-\frac{1}{256}\sqrt{1021\sqrt{2}-1439}\log\left(-\sqrt{1021\sqrt{2}-1439}\left(30\sqrt{2}+41\right)+119\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}\right)+\frac{59}{256}\log\left(\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}+1\right)-\frac{59}{256}\log\left(\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-(1-(1-1/x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*((16*x^2 + (208*x^2 + 291*x)*sqrt((x - 1)/x) + 55*x)*sqrt(-sqrt((x -
1)/x) + 1) + 2*(96*x^2 + 119*x)*sqrt((x - 1)/x) - 2*x)*sqrt(-sqrt(-sqrt((x
- 1)/x) + 1) + 1) + 1/64*sqrt(1021*sqrt(2) + 1439)*arctan(-1/119*sqrt(1021*
sqrt(2) + 1439)*(11*sqrt(2) - 19)*sqrt(sqrt(2) - sqrt(-sqrt((x - 1)/x) + 1)
) + 1/119*sqrt(1021*sqrt(2) + 1439)*(11*sqrt(2) - 19)*sqrt(-sqrt(-sqrt((x -
 1)/x) + 1) + 1)) + 1/256*sqrt(1021*sqrt(2) - 1439)*log(sqrt(1021*sqrt(2) -
 1439)*(30*sqrt(2) + 41) + 119*sqrt(-sqrt(-sqrt((x - 1)/x) + 1) + 1)) - 1/2
56*sqrt(1021*sqrt(2) - 1439)*log(-sqrt(1021*sqrt(2) - 1439)*(30*sqrt(2) + 4
1) + 119*sqrt(-sqrt(-sqrt((x - 1)/x) + 1) + 1)) + 59/256*log(sqrt(-sqrt(-sq
rt((x - 1)/x) + 1) + 1) + 1) - 59/256*log(sqrt(-sqrt(-sqrt((x - 1)/x) + 1)
+ 1) - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-(1-(1-1/x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1-(1-(1-1/x)^(1/2))^(1/2))^(1/2),x)
[Out] int(x/(1-(1-(1-1/x)^(1/2))^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\sqrt{-\sqrt{-\frac{1}{x}+1}+1}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-(1-(1-1/x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(-sqrt(-sqrt(-1/x + 1) + 1) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1 - (1 - (1 - 1/x)^(1/2))^(1/2))^(1/2),x)
[Out] int(x/(1 - (1 - (1 - 1/x)^(1/2))^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-(1-(1-1/x)**(1/2))**(1/2))**(1/2),x)
[Out] Integral(x/sqrt(1 - sqrt(1 - sqrt(1 - 1/x))), x)
```

3.2390 $\int \frac{\sqrt[4]{-x^2+x^6}\left(1-x^4+x^8\right)}{x^4\left(1+x^4\right)}\,dx$

Optimal. Leaf size=501

$$-\frac{3}{8}\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\log\left(-2x^2+2^{3/4}\sqrt{2+\sqrt{2}}\sqrt[4]{x^6-x^2}\,x-\sqrt{2}\sqrt{x^6-x^2}\right)+\frac{3}{8}\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\log\left(2\sqrt{2-\sqrt{2}}\,x^2+2\right.$$

Rubi [C] time = 0.55, antiderivative size = 131, normalized size of antiderivative = 0.26, number of steps used = 14, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2056, 6725, 277, 329, 365, 364, 279, 466, 511, 510}

$$-\frac{6\sqrt[4]{x^6-x^2}\,F_1\left(-\frac{5}{8};-\frac{1}{4},1;\frac{3}{8};x^4,-x^4\right)}{5\sqrt[4]{1-x^4}\,x^3}+\frac{4\sqrt[4]{x^6-x^2}\,x\,{}_2F_1\left(\frac{3}{8},\frac{3}{4};\frac{11}{8};x^4\right)}{5\sqrt[4]{1-x^4}}+\frac{2}{5}\sqrt[4]{x^6-x^2}\,x+\frac{4\sqrt[4]{x^6-x^2}}{5x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-x^2 + x^6)^(1/4)*(1 - x^4 + x^8))/(x^4*(1 + x^4)),x]
```

```
[Out] (4*(-x^2 + x^6)^(1/4))/(5*x^3) + (2*x*(-x^2 + x^6)^(1/4))/5 - (6*(-x^2 + x^
6)^(1/4)*AppellF1[-5/8, -1/4, 1, 3/8, x^4, -x^4])/(5*x^3*(1 - x^4)^(1/4)) +
 (4*x*(-x^2 + x^6)^(1/4)*Hypergeometric2F1[3/8, 3/4, 11/8, x^4])/(5*(1 - x^
4)^(1/4))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{-x^2+x^6}\left(1-x^4+x^8\right)}{x^4\left(1+x^4\right)}dx = \frac{\sqrt[4]{-x^2+x^6}\int \frac{\sqrt[4]{-1+x^4}\left(1-x^4+x^8\right)}{x^{7/2}\left(1+x^4\right)}dx}{\sqrt{x}\sqrt[4]{-1+x^4}}$$

$$= \frac{\sqrt[4]{-x^2+x^6}\int \left(-\frac{2\sqrt[4]{-1+x^4}}{x^{7/2}}+\sqrt{x}\sqrt[4]{-1+x^4}+\frac{3\sqrt[4]{-1+x^4}}{x^{7/2}\left(1+x^4\right)}\right)dx}{\sqrt{x}\sqrt[4]{-1+x^4}}$$

$$= \frac{\sqrt[4]{-x^2+x^6}\int \sqrt{x}\sqrt[4]{-1+x^4}\,dx}{\sqrt{x}\sqrt[4]{-1+x^4}} - \frac{\left(2\sqrt[4]{-x^2+x^6}\right)\int \frac{\sqrt[4]{-1+x^4}}{x^{7/2}}dx}{\sqrt{x}\sqrt[4]{-1+x^4}} + \frac{\left(3\sqrt[4]{-x^2+x^6}\right)}{\sqrt{x}\sqrt[4]{-}}$$

$$= \frac{4\sqrt[4]{-x^2+x^6}}{5x^3} + \frac{2}{5}x\sqrt[4]{-x^2+x^6} - \frac{\left(2\sqrt[4]{-x^2+x^6}\right)\int \frac{\sqrt{x}}{\left(-1+x^4\right)^{3/4}}dx}{5\sqrt{x}\sqrt[4]{-1+x^4}} - \frac{\left(4\sqrt[4]{-x^2+x^6}\right)}{5\sqrt{x}\sqrt[4]{}}$$

$$= \frac{4\sqrt[4]{-x^2+x^6}}{5x^3} + \frac{2}{5}x\sqrt[4]{-x^2+x^6} + \frac{\left(6\sqrt[4]{-x^2+x^6}\right)\text{Subst}\left(\int \frac{\sqrt[4]{1-x^8}}{x^6\left(1+x^8\right)}dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt[4]{1-x^4}} -$$

$$= \frac{4\sqrt[4]{-x^2+x^6}}{5x^3} + \frac{2}{5}x\sqrt[4]{-x^2+x^6} - \frac{6\sqrt[4]{-x^2+x^6}F_1\left(-\frac{5}{8};-\frac{1}{4},1;\frac{3}{8};x^4,-x^4\right)}{5x^3\sqrt[4]{1-x^4}} - \frac{\left(4\left(1-\right.\right.}{}$$

$$= \frac{4\sqrt[4]{-x^2+x^6}}{5x^3} + \frac{2}{5}x\sqrt[4]{-x^2+x^6} - \frac{6\sqrt[4]{-x^2+x^6}F_1\left(-\frac{5}{8};-\frac{1}{4},1;\frac{3}{8};x^4,-x^4\right)}{5x^3\sqrt[4]{1-x^4}} + \frac{4x\sqrt[4]{-}}{}$$

Mathematica [C] time = 0.11, size = 69, normalized size = 0.14

$$\frac{2\sqrt[4]{x^2\left(x^4-1\right)}\left(-5x^4F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};x^4,-x^4\right)-\left(1-x^4\right)^{5/4}\right)}{5x^3\sqrt[4]{1-x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-x^2 + x^6)^(1/4)*(1 - x^4 + x^8))/(x^4*(1 + x^4)),x]
```

```
[Out] (2*(x^2*(-1 + x^4))^(1/4)*(-(1 - x^4)^(5/4) - 5*x^4*AppellF1[3/8, -1/4, 1,
11/8, x^4, -x^4]))/(5*x^3*(1 - x^4)^(1/4))
```

IntegrateAlgebraic [C] time = 1.02, size = 162, normalized size = 0.32

$$\frac{3}{4}\sqrt{1+i}\tan^{-1}\left(\frac{\sqrt{-1-i}x}{\sqrt[4]{x^6-x^2}}\right)+\frac{3}{4}\sqrt{1-i}\tan^{-1}\left(\frac{\sqrt{-1+i}x}{\sqrt[4]{x^6-x^2}}\right)-\frac{3}{4}\sqrt{-1+i}\tan^{-1}\left(\frac{\sqrt{1-i}x}{\sqrt[4]{x^6-x^2}}\right)-\frac{3}{4}\sqrt{-1-i}\tan^{-1}\left(\frac{\sqrt{1+i}x}{\sqrt[4]{x^6-x^2}}\right)+\frac{2\sqrt[4]{x^6-x^2}\left(x^4-1\right)}{5x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-x^2 + x^6)^(1/4)*(1 - x^4 + x^8))/(x^4*(1 + x^4)),x]
```

```
[Out] (2*(-1 + x^4)*(-x^2 + x^6)^(1/4))/(5*x^3) + (3*Sqrt[1 + I]*ArcTan[(Sqrt[-1
- I]*x)/(-x^2 + x^6)^(1/4)])/4 + (3*Sqrt[1 - I]*ArcTan[(Sqrt[-1 + I]*x)/(-x
^2 + x^6)^(1/4)])/4 - (3*Sqrt[-1 + I]*ArcTan[(Sqrt[1 - I]*x)/(-x^2 + x^6)^(
1/4)])/4 - (3*Sqrt[-1 - I]*ArcTan[(Sqrt[1 + I]*x)/(-x^2 + x^6)^(1/4)])/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2)^(1/4)*(x^8-x^4+1)/x^4/(x^4+1),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^8 - x^4 + 1)(x^6 - x^2)^{\frac{1}{4}}}{(x^4 + 1)x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2)^(1/4)*(x^8-x^4+1)/x^4/(x^4+1),x, algorithm="giac")
[Out] integrate((x^8 - x^4 + 1)*(x^6 - x^2)^(1/4)/((x^4 + 1)*x^4), x)
```

maple [C] time = 130.13, size = 7288, normalized size = 14.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-x^2)^(1/4)*(x^8-x^4+1)/x^4/(x^4+1),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^8 - x^4 + 1)(x^6 - x^2)^{\frac{1}{4}}}{(x^4 + 1)x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2)^(1/4)*(x^8-x^4+1)/x^4/(x^4+1),x, algorithm="maxima")
[Out] integrate((x^8 - x^4 + 1)*(x^6 - x^2)^(1/4)/((x^4 + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x^6 - x^2)^{1/4}\,(x^8 - x^4 + 1)}{x^4\,(x^4 + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - x^2)^(1/4)*(x^8 - x^4 + 1))/(x^4*(x^4 + 1)),x)
[Out] int(((x^6 - x^2)^(1/4)*(x^8 - x^4 + 1))/(x^4*(x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\,(x-1)\,(x+1)\,(x^2+1)}\,(x^8 - x^4 + 1)}{x^4\,(x^4 + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-x**2)**(1/4)*(x**8-x**4+1)/x**4/(x**4+1),x)
[Out] Integral((x**2*(x - 1)*(x + 1)*(x**2 + 1))**(1/4)*(x**8 - x**4 + 1)/(x**4*(
x**4 + 1)), x)
```

3.2391 $\int \frac{\sqrt[4]{-x^2+x^6}\left(1-x^4+x^8\right)}{x^4\left(1+x^4\right)}\,dx$

Optimal. Leaf size=501

$$-\frac{3}{8}\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\log\left(-2x^2+2^{3/4}\sqrt{2+\sqrt{2}}\sqrt[4]{x^6-x^2}\,x-\sqrt{2}\sqrt{x^6-x^2}\right)+\frac{3}{8}\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\log\left(2\sqrt{2-\sqrt{2}}\,x^2+2\right.$$

Rubi [C] time = 0.45, antiderivative size = 131, normalized size of antiderivative = 0.26, number of steps used = 14, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2056, 6725, 277, 329, 365, 364, 279, 466, 511, 510}

$$-\frac{6\sqrt[4]{x^6-x^2}\,F_1\left(-\frac{5}{8};-\frac{1}{4},1;\frac{3}{8};x^4,-x^4\right)}{5\sqrt[4]{1-x^4}\,x^3}+\frac{4\sqrt[4]{x^6-x^2}\,x\,{}_2F_1\left(\frac{3}{8},\frac{3}{4};\frac{11}{8};x^4\right)}{5\sqrt[4]{1-x^4}}+\frac{2}{5}\sqrt[4]{x^6-x^2}\,x+\frac{4\sqrt[4]{x^6-x^2}}{5x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-x^2 + x^6)^(1/4)*(1 - x^4 + x^8))/(x^4*(1 + x^4)),x]
```

```
[Out] (4*(-x^2 + x^6)^(1/4))/(5*x^3) + (2*x*(-x^2 + x^6)^(1/4))/5 - (6*(-x^2 + x^
6)^(1/4)*AppellF1[-5/8, -1/4, 1, 3/8, x^4, -x^4])/(5*x^3*(1 - x^4)^(1/4)) +
 (4*x*(-x^2 + x^6)^(1/4)*Hypergeometric2F1[3/8, 3/4, 11/8, x^4])/(5*(1 - x^
4)^(1/4))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{-x^2+x^6}\left(1-x^4+x^8\right)}{x^4\left(1+x^4\right)}dx = \frac{\sqrt[4]{-x^2+x^6}\int \frac{\sqrt[4]{-1+x^4}\left(1-x^4+x^8\right)}{x^{7/2}\left(1+x^4\right)}dx}{\sqrt{x}\sqrt[4]{-1+x^4}}$$

$$= \frac{\sqrt[4]{-x^2+x^6}\int \left(-\frac{2\sqrt[4]{-1+x^4}}{x^{7/2}}+\sqrt{x}\sqrt[4]{-1+x^4}+\frac{3\sqrt[4]{-1+x^4}}{x^{7/2}\left(1+x^4\right)}\right)dx}{\sqrt{x}\sqrt[4]{-1+x^4}}$$

$$= \frac{\sqrt[4]{-x^2+x^6}\int \sqrt{x}\sqrt[4]{-1+x^4}\,dx}{\sqrt{x}\sqrt[4]{-1+x^4}} - \frac{\left(2\sqrt[4]{-x^2+x^6}\right)\int \frac{\sqrt[4]{-1+x^4}}{x^{7/2}}dx}{\sqrt{x}\sqrt[4]{-1+x^4}} + \frac{\left(3\sqrt[4]{-x^2+x^6}\right)}{\sqrt{x}\sqrt[4]{-}}$$

$$= \frac{4\sqrt[4]{-x^2+x^6}}{5x^3} + \frac{2}{5}x\sqrt[4]{-x^2+x^6} - \frac{\left(2\sqrt[4]{-x^2+x^6}\right)\int \frac{\sqrt{x}}{\left(-1+x^4\right)^{3/4}}dx}{5\sqrt{x}\sqrt[4]{-1+x^4}} - \frac{\left(4\sqrt[4]{-x^2+x^6}\right)}{5\sqrt{x}\sqrt[4]{}}$$

$$= \frac{4\sqrt[4]{-x^2+x^6}}{5x^3} + \frac{2}{5}x\sqrt[4]{-x^2+x^6} + \frac{\left(6\sqrt[4]{-x^2+x^6}\right)\text{Subst}\left(\int \frac{\sqrt[4]{1-x^8}}{x^6\left(1+x^8\right)}dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt[4]{1-x^4}} -$$

$$= \frac{4\sqrt[4]{-x^2+x^6}}{5x^3} + \frac{2}{5}x\sqrt[4]{-x^2+x^6} - \frac{6\sqrt[4]{-x^2+x^6}\,F_1\left(-\frac{5}{8};-\frac{1}{4},1;\frac{3}{8};x^4,-x^4\right)}{5x^3\sqrt[4]{1-x^4}} - \frac{\left(4\left(1-\right.\right.}{}$$

$$= \frac{4\sqrt[4]{-x^2+x^6}}{5x^3} + \frac{2}{5}x\sqrt[4]{-x^2+x^6} - \frac{6\sqrt[4]{-x^2+x^6}\,F_1\left(-\frac{5}{8};-\frac{1}{4},1;\frac{3}{8};x^4,-x^4\right)}{5x^3\sqrt[4]{1-x^4}} + \frac{4x\sqrt[4]{-}}{}$$

Mathematica [C] time = 0.06, size = 69, normalized size = 0.14

$$\frac{2\sqrt[4]{x^2\left(x^4-1\right)}\left(-5x^4F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};x^4,-x^4\right)-\left(1-x^4\right)^{5/4}\right)}{5x^3\sqrt[4]{1-x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-x^2 + x^6)^(1/4)*(1 - x^4 + x^8))/(x^4*(1 + x^4)),x]
```

```
[Out] (2*(x^2*(-1 + x^4))^(1/4)*(-(1 - x^4)^(5/4) - 5*x^4*AppellF1[3/8, -1/4, 1,
11/8, x^4, -x^4]))/(5*x^3*(1 - x^4)^(1/4))
```

IntegrateAlgebraic [C] time = 0.00, size = 162, normalized size = 0.32

$$\frac{3}{4}\sqrt{1+i}\tan^{-1}\left(\frac{\sqrt{-1-i}\,x}{\sqrt[4]{x^6-x^2}}\right)+\frac{3}{4}\sqrt{1-i}\tan^{-1}\left(\frac{\sqrt{-1+i}\,x}{\sqrt[4]{x^6-x^2}}\right)-\frac{3}{4}\sqrt{-1+i}\tan^{-1}\left(\frac{\sqrt{1-i}\,x}{\sqrt[4]{x^6-x^2}}\right)-\frac{3}{4}\sqrt{-1-i}\tan^{-1}\left(\frac{\sqrt{1+i}\,x}{\sqrt[4]{x^6-x^2}}\right)+\frac{2\sqrt[4]{x^6-x^2}\left(x^4-1\right)}{5x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-x^2 + x^6)^(1/4)*(1 - x^4 + x^8))/(x^4*(1 + x^4)),x]
```

```
[Out] (2*(-1 + x^4)*(-x^2 + x^6)^(1/4))/(5*x^3) + (3*Sqrt[1 + I]*ArcTan[(Sqrt[-1
- I]*x)/(-x^2 + x^6)^(1/4)])/4 + (3*Sqrt[1 - I]*ArcTan[(Sqrt[-1 + I]*x)/(-x
^2 + x^6)^(1/4)])/4 - (3*Sqrt[-1 + I]*ArcTan[(Sqrt[1 - I]*x)/(-x^2 + x^6)^(
1/4)])/4 - (3*Sqrt[-1 - I]*ArcTan[(Sqrt[1 + I]*x)/(-x^2 + x^6)^(1/4)])/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2)^(1/4)*(x^8-x^4+1)/x^4/(x^4+1),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8 - x^4 + 1\right)\left(x^6 - x^2\right)^{\frac{1}{4}}}{\left(x^4 + 1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2)^(1/4)*(x^8-x^4+1)/x^4/(x^4+1),x, algorithm="giac")
[Out] integrate((x^8 - x^4 + 1)*(x^6 - x^2)^(1/4)/((x^4 + 1)*x^4), x)
```

maple [C] time = 131.38, size = 7288, normalized size = 14.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-x^2)^(1/4)*(x^8-x^4+1)/x^4/(x^4+1),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8 - x^4 + 1\right)\left(x^6 - x^2\right)^{\frac{1}{4}}}{\left(x^4 + 1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2)^(1/4)*(x^8-x^4+1)/x^4/(x^4+1),x, algorithm="maxima")
[Out] integrate((x^8 - x^4 + 1)*(x^6 - x^2)^(1/4)/((x^4 + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^6 - x^2\right)^{1/4}\,\left(x^8 - x^4 + 1\right)}{x^4\,\left(x^4 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - x^2)^(1/4)*(x^8 - x^4 + 1))/(x^4*(x^4 + 1)),x)
[Out] int(((x^6 - x^2)^(1/4)*(x^8 - x^4 + 1))/(x^4*(x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\,(x-1)\,(x+1)\left(x^2+1\right)}\,\left(x^8 - x^4 + 1\right)}{x^4\left(x^4 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-x**2)**(1/4)*(x**8-x**4+1)/x**4/(x**4+1),x)
[Out] Integral((x**2*(x - 1)*(x + 1)*(x**2 + 1))**(1/4)*(x**8 - x**4 + 1)/(x**4*(
x**4 + 1)), x)
```

3.2392 $\int \frac{\sqrt[4]{-x^2+x^6}\left(1+x^4+x^8\right)}{x^4\left(1+x^4\right)}\,dx$

Optimal. Leaf size=501

$$-\frac{1}{8}\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\log\left(-2x^2+2^{3/4}\sqrt{2+\sqrt{2}}\sqrt[4]{x^6-x^2}\,x-\sqrt{2}\sqrt{x^6-x^2}\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\log\left(2\sqrt{2-\sqrt{2}}\,x^2+2\right.$$

Rubi [C] time = 0.44, antiderivative size = 111, normalized size of antiderivative = 0.22, number of steps used = 10, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2056, 6725, 279, 329, 365, 364, 466, 511, 510}

$$-\frac{2\sqrt[4]{x^6-x^2}\,F_1\left(-\frac{5}{8};-\frac{1}{4},1;\frac{3}{8};x^4,-x^4\right)}{5\sqrt[4]{1-x^4}\,x^3}+\frac{4\sqrt[4]{x^6-x^2}\,x\,{}_2F_1\left(\frac{3}{8},\frac{3}{4};\frac{11}{8};x^4\right)}{15\sqrt[4]{1-x^4}}+\frac{2}{5}\sqrt[4]{x^6-x^2}\,x$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-x^2 + x^6)^(1/4)*(1 + x^4 + x^8))/(x^4*(1 + x^4)),x]
```

```
[Out] (2*x*(-x^2 + x^6)^(1/4))/5 - (2*(-x^2 + x^6)^(1/4)*AppellF1[-5/8, -1/4, 1, 
3/8, x^4, -x^4])/(5*x^3*(1 - x^4)^(1/4)) + (4*x*(-x^2 + x^6)^(1/4)*Hypergeo
metric2F1[3/8, 3/4, 11/8, x^4])/(15*(1 - x^4)^(1/4))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p + 
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
```

```
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{-x^2+x^6}\left(1+x^4+x^8\right)}{x^4\left(1+x^4\right)}\,dx = \frac{\sqrt[4]{-x^2+x^6}\int \frac{\sqrt[4]{-1+x^4}\left(1+x^4+x^8\right)}{x^{7/2}\left(1+x^4\right)}\,dx}{\sqrt{x}\sqrt[4]{-1+x^4}}$$

$$= \frac{\sqrt[4]{-x^2+x^6}\int\left(\sqrt{x}\sqrt[4]{-1+x^4}+\frac{\sqrt[4]{-1+x^4}}{x^{7/2}\left(1+x^4\right)}\right)dx}{\sqrt{x}\sqrt[4]{-1+x^4}}$$

$$= \frac{\sqrt[4]{-x^2+x^6}\int\sqrt{x}\sqrt[4]{-1+x^4}\,dx}{\sqrt{x}\sqrt[4]{-1+x^4}} + \frac{\sqrt[4]{-x^2+x^6}\int\frac{\sqrt[4]{-1+x^4}}{x^{7/2}\left(1+x^4\right)}\,dx}{\sqrt{x}\sqrt[4]{-1+x^4}}$$

$$= \frac{2}{5}x\sqrt[4]{-x^2+x^6} - \frac{\left(2\sqrt[4]{-x^2+x^6}\right)\int\frac{\sqrt{x}}{\left(-1+x^4\right)^{3/4}}\,dx}{5\sqrt{x}\sqrt[4]{-1+x^4}} + \frac{\left(2\sqrt[4]{-x^2+x^6}\right)\text{Subst}\left(\int\frac{\sqrt[4]{-1+}}{x^6(1+}\right.}{\sqrt{x}\sqrt[4]{-1+x^4}}$$

$$= \frac{2}{5}x\sqrt[4]{-x^2+x^6} + \frac{\left(2\sqrt[4]{-x^2+x^6}\right)\text{Subst}\left(\int\frac{\sqrt[4]{1-x^8}}{x^6\left(1+x^8\right)}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt[4]{1-x^4}} - \frac{\left(4\sqrt[4]{-x^2+x^6}\right)}{5}$$

$$= \frac{2}{5}x\sqrt[4]{-x^2+x^6} - \frac{2\sqrt[4]{-x^2+x^6}\,F_1\left(-\frac{5}{8};-\frac{1}{4},1;\frac{3}{8};x^4,-x^4\right)}{5x^3\sqrt[4]{1-x^4}} - \frac{\left(4\left(1-x^4\right)^{3/4}\sqrt[4]{-x^2+}\right.}{5}$$

$$= \frac{2}{5}x\sqrt[4]{-x^2+x^6} - \frac{2\sqrt[4]{-x^2+x^6}\,F_1\left(-\frac{5}{8};-\frac{1}{4},1;\frac{3}{8};x^4,-x^4\right)}{5x^3\sqrt[4]{1-x^4}} + \frac{4x\sqrt[4]{-x^2+x^6}\,{}_2F_1\left(\frac{3}{8},\right.}{15\sqrt[4]{1-x^4}}$$

Mathematica [C] time = 0.07, size = 69, normalized size = 0.14

$$\frac{2\sqrt[4]{x^2\left(x^4-1\right)}\left(-5x^4F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};x^4,-x^4\right)-3\left(1-x^4\right)^{5/4}\right)}{15x^3\sqrt[4]{1-x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-x^2 + x^6)^(1/4)*(1 + x^4 + x^8))/(x^4*(1 + x^4)),x]
```

```
[Out] (2*(x^2*(-1 + x^4))^(1/4)*(-3*(1 - x^4)^(5/4) - 5*x^4*AppellF1[3/8, -1/4, 1
, 11/8, x^4, -x^4]))/(15*x^3*(1 - x^4)^(1/4))
```

IntegrateAlgebraic [C] time = 0.95, size = 162, normalized size = 0.32

$$\frac{1}{4}\sqrt{1+i}\tan^{-1}\left(\frac{\sqrt{-1-i}\,x}{\sqrt[4]{x^6-x^2}}\right)+\frac{1}{4}\sqrt{1-i}\tan^{-1}\left(\frac{\sqrt{-1+i}\,x}{\sqrt[4]{x^6-x^2}}\right)-\frac{1}{4}\sqrt{-1+i}\tan^{-1}\left(\frac{\sqrt{1-i}\,x}{\sqrt[4]{x^6-x^2}}\right)-\frac{1}{4}\sqrt{-1-i}\tan^{-1}\left(\frac{\sqrt{1+i}\,x}{\sqrt[4]{x^6-x^2}}\right)+\frac{2\sqrt[4]{x^6-x^2}\left(x^4-1\right)}{5x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-x^2 + x^6)^(1/4)*(1 + x^4 + x^8))/(x^4*(1 + x^4)),x]
```

```
[Out] (2*(-1 + x^4)*(-x^2 + x^6)^(1/4))/(5*x^3) + (Sqrt[1 + I]*ArcTan[(Sqrt[-1 -
I]*x)/(-x^2 + x^6)^(1/4)])/4 + (Sqrt[1 - I]*ArcTan[(Sqrt[-1 + I]*x)/(-x^2 +
 x^6)^(1/4)])/4 - (Sqrt[-1 + I]*ArcTan[(Sqrt[1 - I]*x)/(-x^2 + x^6)^(1/4)])
/4 - (Sqrt[-1 - I]*ArcTan[(Sqrt[1 + I]*x)/(-x^2 + x^6)^(1/4)])/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2)^(1/4)*(x^8+x^4+1)/x^4/(x^4+1),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+x^4+1\right)\left(x^6-x^2\right)^{\frac{1}{4}}}{\left(x^4+1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2)^(1/4)*(x^8+x^4+1)/x^4/(x^4+1),x, algorithm="giac")
[Out] integrate((x^8 + x^4 + 1)*(x^6 - x^2)^(1/4)/((x^4 + 1)*x^4), x)
```

maple [C] time = 118.48, size = 7288, normalized size = 14.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-x^2)^(1/4)*(x^8+x^4+1)/x^4/(x^4+1),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^8+x^4+1\right)\left(x^6-x^2\right)^{\frac{1}{4}}}{\left(x^4+1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2)^(1/4)*(x^8+x^4+1)/x^4/(x^4+1),x, algorithm="maxima")
[Out] integrate((x^8 + x^4 + 1)*(x^6 - x^2)^(1/4)/((x^4 + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(x^6-x^2\right)}^{1/4}\,\left(x^8+x^4+1\right)}{x^4\,\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - x^2)^(1/4)*(x^4 + x^8 + 1))/(x^4*(x^4 + 1)),x)
[Out] int(((x^6 - x^2)^(1/4)*(x^4 + x^8 + 1))/(x^4*(x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^{2}\left(x-1\right)\left(x+1\right)\left(x^{2}+1\right)}\left(x^{2}-x+1\right)\left(x^{2}+x+1\right)\left(x^{4}-x^{2}+1\right)}{x^{4}\left(x^{4}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-x**2)**(1/4)*(x**8+x**4+1)/x**4/(x**4+1),x)
[Out] Integral((x**2*(x - 1)*(x + 1)*(x**2 + 1))**(1/4)*(x**2 - x + 1)*(x**2 + x
+ 1)*(x**4 - x**2 + 1)/(x**4*(x**4 + 1)), x)
```

3.2393 $\int \frac{\sqrt[4]{-x^2+x^6}\left(1+x^4+x^8\right)}{x^4\left(1+x^4\right)}\,dx$

Optimal. Leaf size=501

$$-\frac{1}{8}\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\log\left(-2x^2+2^{3/4}\sqrt{2+\sqrt{2}}\sqrt[4]{x^6-x^2}\,x-\sqrt{2}\sqrt{x^6-x^2}\right)+\frac{1}{8}\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\log\left(2\sqrt{2-\sqrt{2}}\,x^2+2\right.$$

Rubi [C] time = 0.40, antiderivative size = 111, normalized size of antiderivative = 0.22, number of steps used = 10, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2056, 6725, 279, 329, 365, 364, 466, 511, 510}

$$-\frac{2\sqrt[4]{x^6-x^2}\,F_1\left(-\frac{5}{8};-\frac{1}{4},1;\frac{3}{8};x^4,-x^4\right)}{5\sqrt[4]{1-x^4}\,x^3}+\frac{4\sqrt[4]{x^6-x^2}\,x\,{}_2F_1\left(\frac{3}{8},\frac{3}{4};\frac{11}{8};x^4\right)}{15\sqrt[4]{1-x^4}}+\frac{2}{5}\sqrt[4]{x^6-x^2}\,x$$

Warning: Unable to verify antiderivative.

```
[In] Int[((-x^2 + x^6)^(1/4)*(1 + x^4 + x^8))/(x^4*(1 + x^4)),x]
```

```
[Out] (2*x*(-x^2 + x^6)^(1/4))/5 - (2*(-x^2 + x^6)^(1/4)*AppellF1[-5/8, -1/4, 1, 
3/8, x^4, -x^4])/(5*x^3*(1 - x^4)^(1/4)) + (4*x*(-x^2 + x^6)^(1/4)*Hypergeo
metric2F1[3/8, 3/4, 11/8, x^4])/(15*(1 - x^4)^(1/4))
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p + 
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
```

```
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{-x^2+x^6}\left(1+x^4+x^8\right)}{x^4\left(1+x^4\right)}dx = \frac{\sqrt[4]{-x^2+x^6}\int \frac{\sqrt[4]{-1+x^4}\left(1+x^4+x^8\right)}{x^{7/2}\left(1+x^4\right)}dx}{\sqrt{x}\sqrt[4]{-1+x^4}}$$

$$= \frac{\sqrt[4]{-x^2+x^6}\int\left(\sqrt{x}\sqrt[4]{-1+x^4}+\frac{\sqrt[4]{-1+x^4}}{x^{7/2}\left(1+x^4\right)}\right)dx}{\sqrt{x}\sqrt[4]{-1+x^4}}$$

$$= \frac{\sqrt[4]{-x^2+x^6}\int\sqrt{x}\sqrt[4]{-1+x^4}\,dx}{\sqrt{x}\sqrt[4]{-1+x^4}} + \frac{\sqrt[4]{-x^2+x^6}\int\frac{\sqrt[4]{-1+x^4}}{x^{7/2}\left(1+x^4\right)}dx}{\sqrt{x}\sqrt[4]{-1+x^4}}$$

$$= \frac{2}{5}x\sqrt[4]{-x^2+x^6} - \frac{\left(2\sqrt[4]{-x^2+x^6}\right)\int\frac{\sqrt{x}}{\left(-1+x^4\right)^{3/4}}dx}{5\sqrt{x}\sqrt[4]{-1+x^4}} + \frac{\left(2\sqrt[4]{-x^2+x^6}\right)\text{Subst}\left(\int\frac{\sqrt[4]{-1+}}{x^6(1+}\right.}{\sqrt{x}\sqrt[4]{-1+x^4}}$$

$$= \frac{2}{5}x\sqrt[4]{-x^2+x^6} + \frac{\left(2\sqrt[4]{-x^2+x^6}\right)\text{Subst}\left(\int\frac{\sqrt[4]{1-x^8}}{x^6\left(1+x^8\right)}dx,x,\sqrt{x}\right)}{\sqrt{x}\sqrt[4]{1-x^4}} - \frac{\left(4\sqrt[4]{-x^2+x^6}\right)}{5}$$

$$= \frac{2}{5}x\sqrt[4]{-x^2+x^6} - \frac{2\sqrt[4]{-x^2+x^6}\,F_1\left(-\frac{5}{8};-\frac{1}{4},1;\frac{3}{8};x^4,-x^4\right)}{5x^3\sqrt[4]{1-x^4}} - \frac{\left(4\left(1-x^4\right)^{3/4}\sqrt[4]{-x^2+}\right.}{5}$$

$$= \frac{2}{5}x\sqrt[4]{-x^2+x^6} - \frac{2\sqrt[4]{-x^2+x^6}\,F_1\left(-\frac{5}{8};-\frac{1}{4},1;\frac{3}{8};x^4,-x^4\right)}{5x^3\sqrt[4]{1-x^4}} + \frac{4x\sqrt[4]{-x^2+x^6}\,{}_2F_1\left(\frac{3}{8},\right.}{15\sqrt[4]{1-x^4}}$$

Mathematica [C] time = 0.03, size = 69, normalized size = 0.14

$$\frac{2\sqrt[4]{x^2\left(x^4-1\right)}\left(-5x^4F_1\left(\frac{3}{8};-\frac{1}{4},1;\frac{11}{8};x^4,-x^4\right)-3\left(1-x^4\right)^{5/4}\right)}{15x^3\sqrt[4]{1-x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-x^2 + x^6)^(1/4)*(1 + x^4 + x^8))/(x^4*(1 + x^4)),x]
```

```
[Out] (2*(x^2*(-1 + x^4))^(1/4)*(-3*(1 - x^4)^(5/4) - 5*x^4*AppellF1[3/8, -1/4, 1
, 11/8, x^4, -x^4]))/(15*x^3*(1 - x^4)^(1/4))
```

IntegrateAlgebraic [C] time = 0.00, size = 162, normalized size = 0.32

$$\frac{1}{4}\sqrt{1+i}\tan^{-1}\left(\frac{\sqrt{-1-i}\,x}{\sqrt[4]{x^6-x^2}}\right)+\frac{1}{4}\sqrt{1-i}\tan^{-1}\left(\frac{\sqrt{-1+i}\,x}{\sqrt[4]{x^6-x^2}}\right)-\frac{1}{4}\sqrt{-1+i}\tan^{-1}\left(\frac{\sqrt{1-i}\,x}{\sqrt[4]{x^6-x^2}}\right)-\frac{1}{4}\sqrt{-1-i}\tan^{-1}\left(\frac{\sqrt{1+i}\,x}{\sqrt[4]{x^6-x^2}}\right)+\frac{2\sqrt[4]{x^6-x^2}\left(x^4-1\right)}{5x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-x^2 + x^6)^(1/4)*(1 + x^4 + x^8))/(x^4*(1 + x^4)),x]
```

```
[Out] (2*(-1 + x^4)*(-x^2 + x^6)^(1/4))/(5*x^3) + (Sqrt[1 + I]*ArcTan[(Sqrt[-1 -
I]*x)/(-x^2 + x^6)^(1/4)])/4 + (Sqrt[1 - I]*ArcTan[(Sqrt[-1 + I]*x)/(-x^2 +
 x^6)^(1/4)])/4 - (Sqrt[-1 + I]*ArcTan[(Sqrt[1 - I]*x)/(-x^2 + x^6)^(1/4)])
/4 - (Sqrt[-1 - I]*ArcTan[(Sqrt[1 + I]*x)/(-x^2 + x^6)^(1/4)])/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2)^(1/4)*(x^8+x^4+1)/x^4/(x^4+1),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^8 + x^4 + 1)(x^6 - x^2)^{\frac{1}{4}}}{(x^4 + 1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2)^(1/4)*(x^8+x^4+1)/x^4/(x^4+1),x, algorithm="giac")
[Out] integrate((x^8 + x^4 + 1)*(x^6 - x^2)^(1/4)/((x^4 + 1)*x^4), x)
```

maple [C] time = 118.87, size = 7288, normalized size = 14.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-x^2)^(1/4)*(x^8+x^4+1)/x^4/(x^4+1),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^8 + x^4 + 1)(x^6 - x^2)^{\frac{1}{4}}}{(x^4 + 1)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-x^2)^(1/4)*(x^8+x^4+1)/x^4/(x^4+1),x, algorithm="maxima")
[Out] integrate((x^8 + x^4 + 1)*(x^6 - x^2)^(1/4)/((x^4 + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x^6 - x^2)^{1/4}\,(x^8 + x^4 + 1)}{x^4\,(x^4 + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6 - x^2)^(1/4)*(x^4 + x^8 + 1))/(x^4*(x^4 + 1)),x)
[Out] int(((x^6 - x^2)^(1/4)*(x^4 + x^8 + 1))/(x^4*(x^4 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{x^2\,(x-1)\,(x+1)\,(x^2+1)}\,(x^2 - x + 1)\,(x^2 + x + 1)\,(x^4 - x^2 + 1)}{x^4\,(x^4 + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-x**2)**(1/4)*(x**8+x**4+1)/x**4/(x**4+1),x)
[Out] Integral((x**2*(x - 1)*(x + 1)*(x**2 + 1))**(1/4)*(x**2 - x + 1)*(x**2 + x
+ 1)*(x**4 - x**2 + 1)/(x**4*(x**4 + 1)), x)
```

3.2394 $\int \frac{\sqrt[3]{b^2x^2+a^3x^3}}{-b+ax}\,dx$

Optimal. Leaf size=506

$$\frac{\sqrt[3]{a^3x^3+b^2x^2}}{a}+\frac{\left(-3a^2b-b^2\right)\log\left(\sqrt[3]{a^3x^3+b^2x^2}-ax\right)}{3a^3}+\frac{\left(3a^2b+b^2\right)\log\left(ax\sqrt[3]{a^3x^3+b^2x^2}+\left(a^3x^3+b^2x^2\right)^{2/3}+a$$

Rubi [A] time = 0.27, antiderivative size = 510, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2056, 101, 157, 59, 91}

$$\frac{\sqrt[3]{a^3x^3+b^2x^2}}{a}-\frac{b\left(3a^2+b\right)\sqrt[3]{a^3x^3+b^2x^2}\log\left(a^3x+b^2\right)}{6a^3x^{2/3}\sqrt[3]{a^3x+b^2}}-\frac{b\left(3a^2+b\right)\sqrt[3]{a^3x^3+b^2x^2}\log\left(\frac{a\sqrt[3]{x}}{\sqrt[3]{a^3x+b^2}}-1\right)}{2a^3x^{2/3}\sqrt[3]{a^3x+b^2}}-\frac{b\left(3a^2+b\right)\sqrt[3]{a^3x^3+b^2x^2}\tan^{-1}\left(\frac{2a\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a^3x+b^2}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}a^3x^{2/3}\sqrt[3]{a^3x+b^2}}-\frac{b\sqrt[3]{a^2+b}\sqrt[3]{a^3x^3+b^2x^2}\log(ax-b)}{2a^{5/3}x^{2/3}\sqrt[3]{a^3x+b^2}}+\frac{3b\sqrt[3]{a^2+b}\sqrt[3]{a^3x^3+b^2x^2}\log\left(\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{a^2+b}-\sqrt[3]{a^3x+b^2}\right)}{2a^{5/3}x^{2/3}\sqrt[3]{a^3x+b^2}}+\frac{\sqrt{3}b\sqrt[3]{a^2+b}\sqrt[3]{a^3x^3+b^2x^2}\tan^{-1}\left(\frac{2\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{a^2+b}}{\sqrt{3}\sqrt[3]{a^3x+b^2}}+\frac{1}{\sqrt{3}}\right)}{a^{5/3}x^{2/3}\sqrt[3]{a^3x+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2*x^2 + a^3*x^3)^(1/3)/(-b + a*x),x]
```

```
[Out] (b^2*x^2 + a^3*x^3)^(1/3)/a - (b*(3*a^2 + b)*(b^2*x^2 + a^3*x^3)^(1/3)*ArcT
an[1/Sqrt[3] + (2*a*x^(1/3))/(Sqrt[3]*(b^2 + a^3*x)^(1/3))])/(Sqrt[3]*a^3*x
^(2/3)*(b^2 + a^3*x)^(1/3)) + (Sqrt[3]*b*(a^2 + b)^(1/3)*(b^2*x^2 + a^3*x^3
)^(1/3)*ArcTan[1/Sqrt[3] + (2*a^(1/3)*(a^2 + b)^(1/3)*x^(1/3))/(Sqrt[3]*(b^
2 + a^3*x)^(1/3))])/(a^(5/3)*x^(2/3)*(b^2 + a^3*x)^(1/3)) - (b*(a^2 + b)^(1
/3)*(b^2*x^2 + a^3*x^3)^(1/3)*Log[-b + a*x])/(2*a^(5/3)*x^(2/3)*(b^2 + a^3*
x)^(1/3)) - (b*(3*a^2 + b)*(b^2*x^2 + a^3*x^3)^(1/3)*Log[b^2 + a^3*x])/(6*a
^3*x^(2/3)*(b^2 + a^3*x)^(1/3)) - (b*(3*a^2 + b)*(b^2*x^2 + a^3*x^3)^(1/3)*
Log[-1 + (a*x^(1/3))/(b^2 + a^3*x)^(1/3)])/(2*a^3*x^(2/3)*(b^2 + a^3*x)^(1/
3)) + (3*b*(a^2 + b)^(1/3)*(b^2*x^2 + a^3*x^3)^(1/3)*Log[a^(1/3)*(a^2 + b)^
(1/3)*x^(1/3) - (b^2 + a^3*x)^(1/3)])/(2*a^(5/3)*x^(2/3)*(b^2 + a^3*x)^(1/3
))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\int \frac{\sqrt[3]{b^2x^2+a^3x^3}}{-b+ax}\,dx = \frac{\sqrt[3]{b^2x^2+a^3x^3}\int \frac{x^{2/3}\sqrt[3]{b^2+a^3x}}{-b+ax}\,dx}{x^{2/3}\sqrt[3]{b^2+a^3x}}$$
$$= \frac{\sqrt[3]{b^2x^2+a^3x^3}}{a} - \frac{\sqrt[3]{b^2x^2+a^3x^3}\int \frac{-\frac{2b^3}{3}-\frac{1}{3}ab\left(3a^2+b\right)x}{\sqrt[3]{x}(-b+ax)\left(b^2+a^3x\right)^{2/3}}\,dx}{ax^{2/3}\sqrt[3]{b^2+a^3x}}$$
$$= \frac{\sqrt[3]{b^2x^2+a^3x^3}}{a} + \frac{\left(b^2\left(a^2+b\right)\sqrt[3]{b^2x^2+a^3x^3}\right)\int \frac{1}{\sqrt[3]{x}(-b+ax)\left(b^2+a^3x\right)^{2/3}}\,dx}{ax^{2/3}\sqrt[3]{b^2+a^3x}} + \frac{\left(b\left(3a^2+b\right)\sqrt[3]{\cdots}\right.}{\cdots}$$
$$= \frac{\sqrt[3]{b^2x^2+a^3x^3}}{a} - \frac{b\left(3a^2+b\right)\sqrt[3]{b^2x^2+a^3x^3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2a\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{b^2+a^3x}}\right)}{\sqrt{3}a^3x^{2/3}\sqrt[3]{b^2+a^3x}} + \frac{\sqrt{3}\,b\sqrt[3]{a^2+b}\,\sqrt[3]{\cdots}}{\cdots}$$

Mathematica [C] time = 0.14, size = 158, normalized size = 0.31

$$\frac{3x^2\left(\left(a^3bx+b^3\right){}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{a^3x}{b^2}\right)+a^2\left(a^3x+b^2\right){}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{a^3x}{b^2}\right)-b^2\left(a^2+b\right)\sqrt[3]{\frac{a^3x}{b^2}+1}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{a\left(a^2+b\right)x}{xa^3+b^2}\right)\right)}{2ab\left(x^2\left(a^3x+b^2\right)\right)^{2/3}\sqrt[3]{\frac{a^3x}{b^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2*x^2 + a^3*x^3)^(1/3)/(-b + a*x),x]
```

```
[Out] (3*x^2*((b^3 + a^3*b*x)*Hypergeometric2F1[-1/3, 2/3, 5/3, -((a^3*x)/b^2)] +
 a^2*(b^2 + a^3*x)*Hypergeometric2F1[2/3, 2/3, 5/3, -((a^3*x)/b^2)] - b^2*(
a^2 + b)*(1 + (a^3*x)/b^2)^(1/3)*Hypergeometric2F1[2/3, 1, 5/3, (a*(a^2 + b
)*x)/(b^2 + a^3*x)]))/(2*a*b*(x^2*(b^2 + a^3*x))^(2/3)*(1 + (a^3*x)/b^2)^(1
/3))
```

IntegrateAlgebraic [A] time = 4.05, size = 551, normalized size = 1.09

$$\frac{\sqrt[3]{a^3x^3+b^2x^2}}{a}+\frac{\left(-3a^2b-b^2\right)\log\left(\sqrt[3]{a^3x^3+b^2x^2}-ax\right)}{3a^3}+\frac{\left(3a^2b+b^2\right)\log\left(ax\sqrt[3]{a^3x^3+b^2x^2}+\left(a^3x^3+b^2x^2\right)^{2/3}+a^2x^2\right)}{6a^3}-\frac{\left(3a^2b+b^2\right)\tan^{-1}\left(\frac{\sqrt{3}ax}{2\sqrt[3]{a^3x^3+b^2x^2}+ax}\right)}{\sqrt{3}a^3}+\frac{i\left(\sqrt{3}b\sqrt[3]{a^2+b}+ib\sqrt[3]{a^2+b}\right)\log\left(2\sqrt[3]{a}x\sqrt[3]{a^2+b}+\left(1+i\sqrt{3}\right)\sqrt[3]{a^3x^3+b^2x^2}\right)}{2a^{5/3}}+\frac{\left(b\sqrt[3]{a^2+b}-i\sqrt{3}b\sqrt[3]{a^2+b}\right)\log\left(\left(\sqrt{3}+i\right)\left(a^3x^3+b^2x^2\right)^{2/3}-2ia^{2/3}x^2\left(a^2+b\right)^{2/3}+\sqrt[3]{a}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a^2+b}\sqrt[3]{a^3x^3+b^2x^2}\right)}{4a^{5/3}}-\frac{\sqrt{-3-3i\sqrt{3}}\,b\sqrt[3]{a^2+b}\tan^{-1}\left(\frac{3\sqrt[3]{a}x\sqrt[3]{a^2+b}}{-\sqrt{3}\sqrt[3]{a^3x^3+b^2x^2}-3i\sqrt[3]{a^3x^3+b^2x^2}+\sqrt{3}\sqrt[3]{a}x\sqrt[3]{a^2+b}}\right)}{\sqrt{2}a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^2*x^2 + a^3*x^3)^(1/3)/(-b + a*x),x]
```

```
[Out] (b^2*x^2 + a^3*x^3)^(1/3)/a - ((3*a^2*b + b^2)*ArcTan[(Sqrt[3]*a*x)/(a*x +
2*(b^2*x^2 + a^3*x^3)^(1/3))])/(Sqrt[3]*a^3) - (Sqrt[-3 - (3*I)*Sqrt[3]]*b*
(a^2 + b)^(1/3)*ArcTan[(3*a^(1/3)*(a^2 + b)^(1/3)*x)/(Sqrt[3]*a^(1/3)*(a^2
+ b)^(1/3)*x - (3*I)*(b^2*x^2 + a^3*x^3)^(1/3) - Sqrt[3]*(b^2*x^2 + a^3*x^3
```

```
)^(1/3))])/(Sqrt[2]*a^(5/3)) + ((-3*a^2*b - b^2)*Log[-(a*x) + (b^2*x^2 + a^
3*x^3)^(1/3)])/(3*a^3) + ((I/2)*(I*b*(a^2 + b)^(1/3) + Sqrt[3]*b*(a^2 + b)^
(1/3))*Log[2*a^(1/3)*(a^2 + b)^(1/3)*x + (1 + I*Sqrt[3])*(b^2*x^2 + a^3*x^3
)^(1/3)])/a^(5/3) + ((3*a^2*b + b^2)*Log[a^2*x^2 + a*x*(b^2*x^2 + a^3*x^3)^
(1/3) + (b^2*x^2 + a^3*x^3)^(2/3)])/(6*a^3) + ((b*(a^2 + b)^(1/3) - I*Sqrt[
3]*b*(a^2 + b)^(1/3))*Log[(-2*I)*a^(2/3)*(a^2 + b)^(2/3)*x^2 + a^(1/3)*(a^2
 + b)^(1/3)*(I*x - Sqrt[3]*x)*(b^2*x^2 + a^3*x^3)^(1/3) + (I + Sqrt[3])*(b^
2*x^2 + a^3*x^3)^(2/3)])/(4*a^(5/3))
```

fricas [A] time = 0.68, size = 408, normalized size = 0.81

$$-\frac{6\sqrt{3}a^2b\left(\frac{a^2+b}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(a^2+b\right)x+2\sqrt{3}\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}a\left(\frac{a^2+b}{a^2}\right)^{\frac{2}{3}}}{3\left(a^2+b\right)x}\right)-6a^2b\left(\frac{a^2+b}{a^2}\right)^{\frac{1}{3}}\log\left(-\frac{ax\left(\frac{a^2+b}{a^2}\right)^{\frac{1}{3}}-\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}}{x}\right)+3a^2b\left(\frac{a^2+b}{a^2}\right)^{\frac{1}{3}}\log\left(\frac{a^2x^2\left(\frac{a^2+b}{a^2}\right)^{\frac{2}{3}}+\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}ax\left(\frac{a^2+b}{a^2}\right)^{\frac{1}{3}}+\left(a^3x^3+b^2x^2\right)^{\frac{2}{3}}}{x^2}\right)-2\sqrt{3}\left(3a^2b+b^2\right)\arctan\left(\frac{\sqrt{3}ax+2\sqrt{3}\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}}{3ax}\right)-6\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}a^2+2\left(3a^2b+b^2\right)\log\left(-\frac{ax-\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}}{x}\right)-\left(3a^2b+b^2\right)\log\left(\frac{a^2x^2+\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}ax+\left(a^3x^3+b^2x^2\right)^{\frac{2}{3}}}{x^2}\right)}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3+b^2*x^2)^(1/3)/(a*x-b),x, algorithm="fricas")
```

```
[Out] -1/6*(6*sqrt(3)*a^2*b*((a^2 + b)/a^2)^(1/3)*arctan(1/3*(sqrt(3)*(a^2 + b)*x
 + 2*sqrt(3)*(a^3*x^3 + b^2*x^2)^(1/3)*a*((a^2 + b)/a^2)^(2/3))/((a^2 + b)*
x)) - 6*a^2*b*((a^2 + b)/a^2)^(1/3)*log(-(a*x*((a^2 + b)/a^2)^(1/3) - (a^3*
x^3 + b^2*x^2)^(1/3))/x) + 3*a^2*b*((a^2 + b)/a^2)^(1/3)*log((a^2*x^2*((a^2
 + b)/a^2)^(2/3) + (a^3*x^3 + b^2*x^2)^(1/3)*a*x*((a^2 + b)/a^2)^(1/3) + (a
^3*x^3 + b^2*x^2)^(2/3))/x^2) - 2*sqrt(3)*(3*a^2*b + b^2)*arctan(1/3*(sqrt(
3)*a*x + 2*sqrt(3)*(a^3*x^3 + b^2*x^2)^(1/3))/(a*x)) - 6*(a^3*x^3 + b^2*x^2
)^(1/3)*a^2 + 2*(3*a^2*b + b^2)*log(-(a*x - (a^3*x^3 + b^2*x^2)^(1/3))/x) -
 (3*a^2*b + b^2)*log((a^2*x^2 + (a^3*x^3 + b^2*x^2)^(1/3)*a*x + (a^3*x^3 +
b^2*x^2)^(2/3))/x^2))/a^3
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3+b^2*x^2)^(1/3)/(a*x-b),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{3} x^{3}+b^{2} x^{2}\right)^{\frac{1}{3}}}{a x -b}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^3*x^3+b^2*x^2)^(1/3)/(a*x-b),x)
```

```
[Out] int((a^3*x^3+b^2*x^2)^(1/3)/(a*x-b),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{3} x^{3}+b^{2} x^{2}\right)^{\frac{1}{3}}}{a x -b}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3+b^2*x^2)^(1/3)/(a*x-b),x, algorithm="maxima")
```

```
[Out] integrate((a^3*x^3 + b^2*x^2)^(1/3)/(a*x - b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(a^3\,x^3+b^2\,x^2\right)^{1/3}}{b-a\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a^3*x^3 + b^2*x^2)^(1/3)/(b - a*x),x)
```

```
[Out] -int((a^3*x^3 + b^2*x^2)^(1/3)/(b - a*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x^2\left(a^3x+b^2\right)}}{ax-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**3*x**3+b**2*x**2)**(1/3)/(a*x-b),x)
```

```
[Out] Integral((x**2*(a**3*x + b**2))**(1/3)/(a*x - b), x)
```

3.2395 $\int \frac{\sqrt{-b+a^2x^2}}{\sqrt{ax+\sqrt{-b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=507

$$\frac{63b^2\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{c}}\right)}{256ac^{11/2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{\sqrt{a^2x^2-b}\left(\sqrt{\sqrt{a^2x^2-b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2-b}+\cdots}}\right.}{\cdots}$$

Rubi [F] time = 1.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-b+a^2x^2}}{\sqrt{ax+\sqrt{-b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[-b + a^2*x^2]/(Sqrt[a*x + Sqrt[-b + a^2*x^2]]*Sqrt[c + Sqrt[a*x +
Sqrt[-b + a^2*x^2]]]),x]
```

```
[Out] Defer[Int][Sqrt[-b + a^2*x^2]/(Sqrt[a*x + Sqrt[-b + a^2*x^2]]*Sqrt[c + Sqrt
[a*x + Sqrt[-b + a^2*x^2]]]), x]
```

Rubi steps

$$\int \frac{\sqrt{-b+a^2x^2}}{\sqrt{ax+\sqrt{-b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{\sqrt{-b+a^2x^2}}{\sqrt{ax+\sqrt{-b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Mathematica [F] time = 20.90, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-b+a^2x^2}}{\sqrt{ax+\sqrt{-b+a^2x^2}}\sqrt{c+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[-b + a^2*x^2]/(Sqrt[a*x + Sqrt[-b + a^2*x^2]]*Sqrt[c + Sqrt[
a*x + Sqrt[-b + a^2*x^2]]]),x]
```

```
[Out] Integrate[Sqrt[-b + a^2*x^2]/(Sqrt[a*x + Sqrt[-b + a^2*x^2]]*Sqrt[c + Sqrt[
a*x + Sqrt[-b + a^2*x^2]]]), x]
```

IntegrateAlgebraic [A] time = 1.45, size = 507, normalized size = 1.00

$$\frac{63b^2\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{c}}\right)}{256ac^{11/2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{\sqrt{a^2x^2-b}\left(\sqrt{\sqrt{a^2x^2-b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}\left(3072a^2c^5x^2+4096ac^7x+630b^2c-768bc^5\right)+\left(-4096a^2c^6x^2-1890ab^2x+7680abc^4x-504b^2c^2+1024bc^6\right)\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}\right)+\sqrt{\sqrt{a^2x^2-b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}\left(3072a^3c^5x^3+4096a^2c^7x^2+630ab^2cx-2304abc^5x+432b^2c^3-2048bc^7\right)+\left(-4096a^3c^6x^3-1890a^2b^2x^2+7680a^2bc^4x^2-504ab^2c^2x+3072abc^6x+945b^3-4224b^2c^4\right)\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+c}}{3840ac^5\left(\sqrt{a^2x^2-b}+ax\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-b + a^2*x^2]/(Sqrt[a*x + Sqrt[-b + a^2*x^2]]*Sqrt[
c + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]),x]
```

```
[Out] ((945*b^3 - 4224*b^2*c^4 - 504*a*b^2*c^2*x + 3072*a*b*c^6*x - 1890*a^2*b^2*
x^2 + 7680*a^2*b*c^4*x^2 - 4096*a^3*c^6*x^3)*Sqrt[c + Sqrt[a*x + Sqrt[-b +
a^2*x^2]]] + (432*b^2*c^3 - 2048*b*c^7 + 630*a*b^2*c*x - 2304*a*b*c^5*x + 4
096*a^2*c^7*x^2 + 3072*a^3*c^5*x^3)*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*Sqrt[c +
 Sqrt[a*x + Sqrt[-b + a^2*x^2]]] + Sqrt[-b + a^2*x^2]*((-504*b^2*c^2 + 1024
*b*c^6 - 1890*a*b^2*x + 7680*a*b*c^4*x - 4096*a^2*c^6*x^2)*Sqrt[c + Sqrt[a*
x + Sqrt[-b + a^2*x^2]]] + (630*b^2*c - 768*b*c^5 + 4096*a*c^7*x + 3072*a^2
*c^5*x^2)*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[-b + a^2*
x^2]]]))/(3840*a*c^5*(a*x + Sqrt[-b + a^2*x^2])^(5/2)) + (63*b^2*ArcTanh[Sq
rt[c + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]/Sqrt[c]])/(256*a*c^(11/2)) - (b*ArcT
anh[Sqrt[c + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]/Sqrt[c]])/(a*c^(3/2))
```

fricas [A] time = 1.09, size = 679, normalized size = 1.34

$$\left[\frac{15\left(256b^2c^4-63b^3\right)\sqrt{c}\log\left(-2\left(a\sqrt{c}x-\sqrt{a^2x^2-b}\sqrt{c}\right)\sqrt{ax+\sqrt{a^2x^2-b}}\sqrt{c+\sqrt{ax+\sqrt{a^2x^2-b}}}+2\left(acx-\sqrt{a^2x^2-b}\,c\right)\sqrt{ax+\sqrt{a^2x^2-b}}+b\right)+2\left(2048bc^8+864a^2bc^4x^2-432b^2c^4+6\left(128abc^6+105ab^2c^2\right)x+6\left(128bc^6-144abc^4x-105b^2c^2\right)\sqrt{a^2x^2-b}-\left(1536a^3c^5x^3+1024bc^7+1008a^2bc^3x^2-504b^2c^3-3\left(1664abc^5-315ab^2c\right)x-3\left(512a^2c^5x^2-1408bc^5+336abc^3x+315b^2c\right)\sqrt{a^2x^2-b}\right)\sqrt{ax+\sqrt{a^2x^2-b}}\right)\sqrt{c+\sqrt{ax+\sqrt{a^2x^2-b}}}}{7680\,abc^6},\ \frac{15\left(256b^2c^4-63b^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{c+\sqrt{ax+\sqrt{a^2x^2-b}}}}{c}\right)+\left(2048bc^8+864a^2bc^4x^2-432b^2c^4+6\left(128abc^6+105ab^2c^2\right)x+6\left(128bc^6-144abc^4x-105b^2c^2\right)\sqrt{a^2x^2-b}-\left(1536a^3c^5x^3+1024bc^7+1008a^2bc^3x^2-504b^2c^3-3\left(1664abc^5-315ab^2c\right)x-3\left(512a^2c^5x^2-1408bc^5+336abc^3x+315b^2c\right)\sqrt{a^2x^2-b}\right)\sqrt{ax+\sqrt{a^2x^2-b}}\right)\sqrt{c+\sqrt{ax+\sqrt{a^2x^2-b}}}}{3840\,abc^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^2-
b)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(256*b^2*c^4 - 63*b^3)*sqrt(c)*log(-2*(a*sqrt(c)*x - sqrt(a^2*x
^2 - b)*sqrt(c))*sqrt(a*x + sqrt(a^2*x^2 - b))*sqrt(c + sqrt(a*x + sqrt(a^2
*x^2 - b))) + 2*(a*c*x - sqrt(a^2*x^2 - b)*c)*sqrt(a*x + sqrt(a^2*x^2 - b))
 + b) + 2*(2048*b*c^8 + 864*a^2*b*c^4*x^2 - 432*b^2*c^4 + 6*(128*a*b*c^6 +
105*a*b^2*c^2)*x + 6*(128*b*c^6 - 144*a*b*c^4*x - 105*b^2*c^2)*sqrt(a^2*x^2
 - b) - (1536*a^3*c^5*x^3 + 1024*b*c^7 + 1008*a^2*b*c^3*x^2 - 504*b^2*c^3 -
 3*(1664*a*b*c^5 - 315*a*b^2*c)*x - 3*(512*a^2*c^5*x^2 - 1408*b*c^5 + 336*a
*b*c^3*x + 315*b^2*c)*sqrt(a^2*x^2 - b))*sqrt(a*x + sqrt(a^2*x^2 - b)))*sqr
t(c + sqrt(a*x + sqrt(a^2*x^2 - b))))/(a*b*c^6), 1/3840*(15*(256*b^2*c^4 -
63*b^3)*sqrt(-c)*arctan(sqrt(-c)*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 - b)))/c)
 + (2048*b*c^8 + 864*a^2*b*c^4*x^2 - 432*b^2*c^4 + 6*(128*a*b*c^6 + 105*a*b
^2*c^2)*x + 6*(128*b*c^6 - 144*a*b*c^4*x - 105*b^2*c^2)*sqrt(a^2*x^2 - b) -
 (1536*a^3*c^5*x^3 + 1024*b*c^7 + 1008*a^2*b*c^3*x^2 - 504*b^2*c^3 - 3*(166
4*a*b*c^5 - 315*a*b^2*c)*x - 3*(512*a^2*c^5*x^2 - 1408*b*c^5 + 336*a*b*c^3*
x + 315*b^2*c)*sqrt(a^2*x^2 - b))*sqrt(a*x + sqrt(a^2*x^2 - b)))*sqrt(c + s
qrt(a*x + sqrt(a^2*x^2 - b))))/(a*b*c^6)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^2-
b)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^{2} x^{2}-b}}{\sqrt{a x +\sqrt{a^{2} x^{2}-b}}\,\sqrt{c +\sqrt{a x +\sqrt{a^{2} x^{2}-b}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/
2))^(1/2))^(1/2),x)
```

```
[Out] int((a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/
2))^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2 - b}}{\sqrt{ax + \sqrt{a^2x^2 - b}}\,\sqrt{c + \sqrt{ax + \sqrt{a^2x^2 - b}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/2)/(c+(a*x+(a^2*x^2-
b)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 - b)/(sqrt(a*x + sqrt(a^2*x^2 - b))*sqrt(c + sqrt(a*
x + sqrt(a^2*x^2 - b)))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a^2\,x^2 - b}}{\sqrt{a\,x + \sqrt{a^2\,x^2 - b}}\,\sqrt{c + \sqrt{a\,x + \sqrt{a^2\,x^2 - b}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 - b)^(1/2)/((a*x + (a^2*x^2 - b)^(1/2))^(1/2)*(c + (a*x + (a^2
*x^2 - b)^(1/2))^(1/2))^(1/2)),x)
```

```
[Out] int((a^2*x^2 - b)^(1/2)/((a*x + (a^2*x^2 - b)^(1/2))^(1/2)*(c + (a*x + (a^2
*x^2 - b)^(1/2))^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2 - b}}{\sqrt{c + \sqrt{ax + \sqrt{a^2x^2 - b}}}\,\sqrt{ax + \sqrt{a^2x^2 - b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b)**(1/2)/(a*x+(a**2*x**2-b)**(1/2))**(1/2)/(c+(a*x+(a
**2*x**2-b)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a**2*x**2 - b)/(sqrt(c + sqrt(a*x + sqrt(a**2*x**2 - b)))*sqr
t(a*x + sqrt(a**2*x**2 - b))), x)
```

3.2396 $\int \frac{(-3+x^2)(1-2x^2+x^4+x^6)}{x^{10}\sqrt[4]{\frac{-a+ax^2+bx^3}{-c+cx^2+dx^3}}}\,dx$

Optimal. Leaf size=514

$$\left(\frac{ax^2-a+bx^3}{cx^2-c+dx^3}\right)^{3/4}\left(-96a^2c^3x^8+64a^2c^3x^6+96a^2c^3x^4-96a^2c^3x^2+32a^2c^3-96a^2c^2dx^9-36a^2c^2dx^7+72a^2c^2dx^5\right.$$

Rubi [F] time = 5.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(-3+x^2)(1-2x^2+x^4+x^6)}{x^{10}\sqrt[4]{\frac{-a+ax^2+bx^3}{-c+cx^2+dx^3}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((-3 + x^2)*(1 - 2*x^2 + x^4 + x^6))/(x^10*((-a + a*x^2 + b*x^3)/(-c +
c*x^2 + d*x^3))^(1/4)),x]
```

```
[Out] (-3*(-a + a*x^2 + b*x^3)^(1/4)*Defer[Int][(-c + c*x^2 + d*x^3)^(1/4)/(x^10*
(-a + a*x^2 + b*x^3)^(1/4)), x])/(((a - a*x^2 - b*x^3)/(c - c*x^2 - d*x^3))
^(1/4)*(-c + c*x^2 + d*x^3)^(1/4)) + (7*(-a + a*x^2 + b*x^3)^(1/4)*Defer[In
t][(-c + c*x^2 + d*x^3)^(1/4)/(x^8*(-a + a*x^2 + b*x^3)^(1/4)), x])/(((a -
a*x^2 - b*x^3)/(c - c*x^2 - d*x^3))^(1/4)*(-c + c*x^2 + d*x^3)^(1/4)) - (5*
(-a + a*x^2 + b*x^3)^(1/4)*Defer[Int][(-c + c*x^2 + d*x^3)^(1/4)/(x^6*(-a +
 a*x^2 + b*x^3)^(1/4)), x])/(((a - a*x^2 - b*x^3)/(c - c*x^2 - d*x^3))^(1/4
)*(-c + c*x^2 + d*x^3)^(1/4)) - (2*(-a + a*x^2 + b*x^3)^(1/4)*Defer[Int][(-
c + c*x^2 + d*x^3)^(1/4)/(x^4*(-a + a*x^2 + b*x^3)^(1/4)), x])/(((a - a*x^2
 - b*x^3)/(c - c*x^2 - d*x^3))^(1/4)*(-c + c*x^2 + d*x^3)^(1/4)) + ((-a + a
*x^2 + b*x^3)^(1/4)*Defer[Int][(-c + c*x^2 + d*x^3)^(1/4)/(x^2*(-a + a*x^2
+ b*x^3)^(1/4)), x])/(((a - a*x^2 - b*x^3)/(c - c*x^2 - d*x^3))^(1/4)*(-c +
 c*x^2 + d*x^3)^(1/4))
```

Rubi steps

$$\begin{aligned}
\int \frac{(-3+x^2)(1-2x^2+x^4+x^6)}{x^{10}\sqrt[4]{\frac{-a+ax^2+bx^3}{-c+cx^2+dx^3}}}\,dx &= \frac{\sqrt[4]{-a+ax^2+bx^3}\int \frac{(-3+x^2)\sqrt[4]{-c+cx^2+dx^3}(1-2x^2+x^4+x^6)}{x^{10}\sqrt[4]{-a+ax^2+bx^3}}\,dx}{\sqrt[4]{\frac{-a+ax^2+bx^3}{-c+cx^2+dx^3}}\sqrt[4]{-c+cx^2+dx^3}}\\
&= \frac{\sqrt[4]{-a+ax^2+bx^3}\int \left(-\frac{3\sqrt[4]{-c+cx^2+dx^3}}{x^{10}\sqrt[4]{-a+ax^2+bx^3}}+\frac{7\sqrt[4]{-c+cx^2+dx^3}}{x^8\sqrt[4]{-a+ax^2+bx^3}}-\frac{5\sqrt[4]{-c+cx^2+dx^3}}{x^6\sqrt[4]{-a+ax^2+bx^3}}\right.}{\sqrt[4]{\frac{-a+ax^2+bx^3}{-c+cx^2+dx^3}}\sqrt[4]{-c+cx^2+dx^3}}\\
&= \frac{\sqrt[4]{-a+ax^2+bx^3}\int \frac{\sqrt[4]{-c+cx^2+dx^3}}{x^2\sqrt[4]{-a+ax^2+bx^3}}\,dx}{\sqrt[4]{\frac{-a+ax^2+bx^3}{-c+cx^2+dx^3}}\sqrt[4]{-c+cx^2+dx^3}}-\frac{\left(2\sqrt[4]{-a+ax^2+bx^3}\right)\int \frac{\sqrt[4]{-c+cx}}{x^4\sqrt[4]{-a+a}}}{\sqrt[4]{\frac{-a+ax^2+bx^3}{-c+cx^2+dx^3}}\sqrt[4]{-c+cx^2+}}
\end{aligned}$$

Mathematica [F] time = 1.82, size = 0, normalized size = 0.00

$$\int \frac{(-3+x^2)(1-2x^2+x^4+x^6)}{x^{10}\sqrt[4]{\frac{-a+ax^2+bx^3}{-c+cx^2+dx^3}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((-3 + x^2)*(1 - 2*x^2 + x^4 + x^6))/(x^10*((-a + a*x^2 + b*x^3)/
(-c + c*x^2 + d*x^3))^(1/4)),x]
```

```
[Out] Integrate[((-3 + x^2)*(1 - 2*x^2 + x^4 + x^6))/(x^10*((-a + a*x^2 + b*x^3)/
(-c + c*x^2 + d*x^3))^(1/4)), x]
```

IntegrateAlgebraic [A] time = 2.14, size = 514, normalized size = 1.00

$$\frac{\left(\frac{ax^2-a+bx^3}{cx^2-c+dx^3}\right)^{3/4}\left(-96a^2c^3x^8+64a^2c^3x^6+96a^2c^3x^4-96a^2c^3x^2+32a^2c^3-96a^2c^2dx^9-36a^2c^2dx^7+72a^2c^2dx^5-36a^2c^2dx^3+3a^2cd^2x^8-3a^2cd^2x^6+7a^2d^3x^9+36abc^3x^7-72abc^3x^5+36abc^3x^3+42abc^2dx^8-42abc^2dx^6+6abcd^2x^9-45b^2c^3x^8+45b^2c^3x^6-45b^2c^2dx^9\right)}{96a^3c^2x^9}+\frac{\left(32a^3c^2d+7a^3d^3-32a^2bc^3+3a^2bcd^2+5ab^2c^2d-15b^3c^3\right)\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt[4]{\frac{ax^2-a+bx^3}{cx^2-c+dx^3}}}{\sqrt[4]{a}}\right)}{64a^{13/4}c^{11/4}}+\frac{\left(-32a^3c^2d-7a^3d^3+32a^2bc^3-3a^2bcd^2-5ab^2c^2d+15b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt[4]{\frac{ax^2-a+bx^3}{cx^2-c+dx^3}}}{\sqrt[4]{a}}\right)}{64a^{13/4}c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-3 + x^2)*(1 - 2*x^2 + x^4 + x^6))/(x^10*((-a + a*x^2
+ b*x^3)/(-c + c*x^2 + d*x^3))^(1/4)),x]
```

```
[Out] (((-a + a*x^2 + b*x^3)/(-c + c*x^2 + d*x^3))^(3/4)*(32*a^2*c^3 - 96*a^2*c^3
*x^2 + 36*a*b*c^3*x^3 - 36*a^2*c^2*d*x^3 + 96*a^2*c^3*x^4 - 72*a*b*c^3*x^5
+ 72*a^2*c^2*d*x^5 + 64*a^2*c^3*x^6 + 45*b^2*c^3*x^6 - 42*a*b*c^2*d*x^6 - 3
*a^2*c*d^2*x^6 + 36*a*b*c^3*x^7 - 36*a^2*c^2*d*x^7 - 96*a^2*c^3*x^8 - 45*b^
2*c^3*x^8 + 42*a*b*c^2*d*x^8 + 3*a^2*c*d^2*x^8 - 96*a^2*c^2*d*x^9 - 45*b^2*
c^2*d*x^9 + 6*a*b*c*d^2*x^9 + 7*a^2*d^3*x^9))/(96*a^3*c^2*x^9) + ((-32*a^2*
b*c^3 - 15*b^3*c^3 + 32*a^3*c^2*d + 5*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 7*a^3*d
^3)*ArcTan[(c^(1/4)*((-a + a*x^2 + b*x^3)/(-c + c*x^2 + d*x^3))^(1/4))/a^(1
/4)])/(64*a^(13/4)*c^(11/4)) + ((32*a^2*b*c^3 + 15*b^3*c^3 - 32*a^3*c^2*d -
 5*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 7*a^3*d^3)*ArcTanh[(c^(1/4)*((-a + a*x^2 +
 b*x^3)/(-c + c*x^2 + d*x^3))^(1/4))/a^(1/4)])/(64*a^(13/4)*c^(11/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)*(x^6+x^4-2*x^2+1)/x^10/((b*x^3+a*x^2-a)/(d*x^3+c*x^2-c))^
(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{6}+x^{4}-2\,x^{2}+1\right)\left(x^{2}-3\right)}{x^{10}\left(\frac{bx^{3}+ax^{2}-a}{dx^{3}+cx^{2}-c}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)*(x^6+x^4-2*x^2+1)/x^10/((b*x^3+a*x^2-a)/(d*x^3+c*x^2-c))^
(1/4),x, algorithm="giac")
```

```
[Out] integrate((x^6 + x^4 - 2*x^2 + 1)*(x^2 - 3)/(x^10*((b*x^3 + a*x^2 - a)/(d*x
^3 + c*x^2 - c))^(1/4)), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}-3\right)\left(x^{6}+x^{4}-2x^{2}+1\right)}{x^{10}\left(\frac{b\,x^{3}+a\,x^{2}-a}{d\,x^{3}+c\,x^{2}-c}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3)*(x^6+x^4-2*x^2+1)/x^10/((b*x^3+a*x^2-a)/(d*x^3+c*x^2-c))^(1/4),
x)
```

```
[Out] int((x^2-3)*(x^6+x^4-2*x^2+1)/x^10/((b*x^3+a*x^2-a)/(d*x^3+c*x^2-c))^(1/4),
x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^6 + x^4 - 2\,x^2 + 1\right)\left(x^2 - 3\right)}{x^{10} \left(\frac{bx^3+ax^2-a}{dx^3+cx^2-c}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3)*(x^6+x^4-2*x^2+1)/x^10/((b*x^3+a*x^2-a)/(d*x^3+c*x^2-c))^
(1/4),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + x^4 - 2*x^2 + 1)*(x^2 - 3)/(x^10*((b*x^3 + a*x^2 - a)/(d*x
^3 + c*x^2 - c))^(1/4)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 3)*(x^4 - 2*x^2 + x^6 + 1))/(x^10*((a*x^2 - a + b*x^3)/(c*x^2 -
 c + d*x^3))^(1/4)),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3)*(x**6+x**4-2*x**2+1)/x**10/((b*x**3+a*x**2-a)/(d*x**3+c*
x**2-c))**(1/4),x)
```

```
[Out] Timed out
```

3.2397 $\int \sqrt{b+a^2x^2}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$

Optimal. Leaf size=515

$$\frac{5b^2\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{\sqrt{c}}\right)}{128ac^{7/2}}-\frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{\sqrt{c}}\right)}{a}+\frac{\sqrt{\sqrt{a^2x^2+b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}\,(}{}$$

Rubi [F] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{b+a^2x^2}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[b + a^2*x^2]*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]],x]
```

```
[Out] Defer[Int][Sqrt[b + a^2*x^2]*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]], x]
```

Rubi steps

$$\int \sqrt{b+a^2x^2}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx=\int \sqrt{b+a^2x^2}\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Mathematica [A] time = 1.26, size = 392, normalized size = 0.76

$$\frac{\left(\left(\sqrt{a^2x^2+b}+ax\right)^2+b\right)\left(\sqrt{c}\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}\left(-105b^2\left(8c^2\sqrt{\sqrt{a^2x^2+b}+ax}-10c\left(\sqrt{a^2x^2+b}+ax\right)+15\left(\sqrt{a^2x^2+b}+ax\right)^{3/2}+48c^3\right)+80640bc^3\left(\sqrt{a^2x^2+b}+ax\right)^2-128c^3\left(\sqrt{a^2x^2+b}+ax\right)^2\left(\sqrt{\sqrt{a^2x^2+b}+ax}+c\right)\left(-24c^2\sqrt{\sqrt{a^2x^2+b}+ax}+30c\left(\sqrt{a^2x^2+b}+ax\right)-35\left(\sqrt{a^2x^2+b}+ax\right)^{3/2}+16c^3\right)\right)+315b\left(5b-256c^4\right)\left(\sqrt{a^2x^2+b}+ax\right)^2\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{\sqrt{c}}\right)\right)}{80640ac^{7/2}\sqrt{a^2x^2+b}\left(\sqrt{a^2x^2+b}+ax\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b + a^2*x^2]*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]],x]
```

```
[Out] ((b + (a*x + Sqrt[b + a^2*x^2])^2)*(Sqrt[c]*Sqrt[c + Sqrt[a*x + Sqrt[b + a^
2*x^2]]]*(80640*b*c^3*(a*x + Sqrt[b + a^2*x^2])^2 - 128*c^3*(a*x + Sqrt[b +
 a^2*x^2])^2*(c + Sqrt[a*x + Sqrt[b + a^2*x^2]])*(16*c^3 - 24*c^2*Sqrt[a*x
+ Sqrt[b + a^2*x^2]] + 30*c*(a*x + Sqrt[b + a^2*x^2]) - 35*(a*x + Sqrt[b +
a^2*x^2])^(3/2)) - 105*b^2*(48*c^3 + 8*c^2*Sqrt[a*x + Sqrt[b + a^2*x^2]] -
10*c*(a*x + Sqrt[b + a^2*x^2]) + 15*(a*x + Sqrt[b + a^2*x^2])^(3/2))) + 315
*b*(5*b - 256*c^4)*(a*x + Sqrt[b + a^2*x^2])^2*ArcTanh[Sqrt[c + Sqrt[a*x +
Sqrt[b + a^2*x^2]]]/Sqrt[c]]))/(80640*a*c^(7/2)*Sqrt[b + a^2*x^2]*(a*x + Sq
rt[b + a^2*x^2])^3)
```

IntegrateAlgebraic [A] time = 1.36, size = 515, normalized size = 1.00

$$\frac{5b^2\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{\sqrt{c}}\right)}{128ac^{7/2}}-\frac{2b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{\sqrt{c}}\right)}{a}+\frac{\sqrt{\sqrt{a^2x^2+b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}\left(2560a^3c^4x^3+2048a^2c^6x^2-1575ab^2x+1920abc^4x-840b^2c^2+1024bc^6\right)+\sqrt{a^2x^2+b}\left(\sqrt{\sqrt{a^2x^2+b}+ax}\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}\left(2560a^2c^4x^2+2048ac^6x-1575b^2+640bc^4\right)+\left(35840a^3c^3x^3-3072a^2c^5x^2+179200abc^3x-4096ac^7x+1050b^2c-768bc^5\right)\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}\right)+\left(35840a^4c^3x^4-3072a^3c^5x^3+197120a^2bc^3x^2-4096a^2c^7x^2+1050ab^2cx-2304abc^5x+80080b^2c^3-2048bc^7\right)\sqrt{\sqrt{\sqrt{a^2x^2+b}+ax}+c}}{40320ac^3\left(2a^2x^2+b\right)+80640a^2c^3x\sqrt{a^2x^2+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[b + a^2*x^2]*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]
],x]
```

```
[Out] ((80080*b^2*c^3 - 2048*b*c^7 + 1050*a*b^2*c*x - 2304*a*b*c^5*x + 197120*a^2
*b*c^3*x^2 - 4096*a^2*c^7*x^2 - 3072*a^3*c^5*x^3 + 35840*a^4*c^3*x^4)*Sqrt[
```

```
c + Sqrt[a*x + Sqrt[b + a^2*x^2]]] + (-840*b^2*c^2 + 1024*b*c^6 - 1575*a*b^
2*x + 1920*a*b*c^4*x + 2048*a^2*c^6*x^2 + 2560*a^3*c^4*x^3)*Sqrt[a*x + Sqrt
[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]] + Sqrt[b + a^2*x^2]*
((1050*b^2*c - 768*b*c^5 + 179200*a*b*c^3*x - 4096*a*c^7*x - 3072*a^2*c^5*x
^2 + 35840*a^3*c^3*x^3)*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]] + (-1575*b^
2 + 640*b*c^4 + 2048*a*c^6*x + 2560*a^2*c^4*x^2)*Sqrt[a*x + Sqrt[b + a^2*x^
2]]*Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]]))/(80640*a^2*c^3*x*Sqrt[b + a^2
*x^2] + 40320*a*c^3*(b + 2*a^2*x^2)) + (5*b^2*ArcTanh[Sqrt[c + Sqrt[a*x + S
qrt[b + a^2*x^2]]]/Sqrt[c]])/(128*a*c^(7/2)) - (2*b*Sqrt[c]*ArcTanh[Sqrt[c
+ Sqrt[a*x + Sqrt[b + a^2*x^2]]]/Sqrt[c]])/a
```

fricas [A] time = 0.49, size = 554, normalized size = 1.08

$$\left[\frac{315\left(256bc^4-5b^2\right)\sqrt{c}\log\left(2\left(a\sqrt{c}x-\sqrt{a^2x^2+b}\sqrt{c}\right)\sqrt{ax+\sqrt{a^2x^2+b}}\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}-2\left(acx-\sqrt{a^2x^2+b}\,c\right)\sqrt{ax+\sqrt{a^2x^2+b}}+b\right)-2\left(2048c^8+1120a^2c^4x^2-80080bc^4+6\left(128ac^6+175abc^2\right)x+2\left(384c^6-9520ac^4x-525bc^2\right)\sqrt{a^2x^2+b}-\left(1024c^7-1680a^2c^3x^2-840bc^3+5\left(128ac^5+315abc\right)x+5\left(128c^5+336ac^3x-315bc\right)\sqrt{a^2x^2+b}\right)\sqrt{ax+\sqrt{a^2x^2+b}}\right)\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{80640\,ac^4},\ \frac{315\left(256bc^4-5b^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{c}\right)-\left(2048c^8+1120a^2c^4x^2-80080bc^4+6\left(128ac^6+175abc^2\right)x+2\left(384c^6-9520ac^4x-525bc^2\right)\sqrt{a^2x^2+b}-\left(1024c^7-1680a^2c^3x^2-840bc^3+5\left(128ac^5+315abc\right)x+5\left(128c^5+336ac^3x-315bc\right)\sqrt{a^2x^2+b}\right)\sqrt{ax+\sqrt{a^2x^2+b}}\right)\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{40320\,ac^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b)^(1/2)*(c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2),x, algo
rithm="fricas")
```

```
[Out] [1/80640*(315*(256*b*c^4 - 5*b^2)*sqrt(c)*log(2*(a*sqrt(c)*x - sqrt(a^2*x^2
 + b)*sqrt(c))*sqrt(a*x + sqrt(a^2*x^2 + b))*sqrt(c + sqrt(a*x + sqrt(a^2*x
^2 + b))) - 2*(a*c*x - sqrt(a^2*x^2 + b)*c)*sqrt(a*x + sqrt(a^2*x^2 + b)) +
 b) - 2*(2048*c^8 + 1120*a^2*c^4*x^2 - 80080*b*c^4 + 6*(128*a*c^6 + 175*a*b
*c^2)*x + 2*(384*c^6 - 9520*a*c^4*x - 525*b*c^2)*sqrt(a^2*x^2 + b) - (1024*
c^7 - 1680*a^2*c^3*x^2 - 840*b*c^3 + 5*(128*a*c^5 + 315*a*b*c)*x + 5*(128*c
^5 + 336*a*c^3*x - 315*b*c)*sqrt(a^2*x^2 + b))*sqrt(a*x + sqrt(a^2*x^2 + b)
))*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b))))/(a*c^4), 1/40320*(315*(256*b*c^
4 - 5*b^2)*sqrt(-c)*arctan(sqrt(-c)*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b)))
/c) - (2048*c^8 + 1120*a^2*c^4*x^2 - 80080*b*c^4 + 6*(128*a*c^6 + 175*a*b*c
^2)*x + 2*(384*c^6 - 9520*a*c^4*x - 525*b*c^2)*sqrt(a^2*x^2 + b) - (1024*c^
7 - 1680*a^2*c^3*x^2 - 840*b*c^3 + 5*(128*a*c^5 + 315*a*b*c)*x + 5*(128*c^5
 + 336*a*c^3*x - 315*b*c)*sqrt(a^2*x^2 + b))*sqrt(a*x + sqrt(a^2*x^2 + b)))
*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b))))/(a*c^4)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b)^(1/2)*(c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \sqrt{a^2x^2+b}\,\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2+b)^(1/2)*(c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] int((a^2*x^2+b)^(1/2)*(c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2x^2+b}\,\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b)^(1/2)*(c+(a*x+(a^2*x^2+b)^(1/2))^(1/2))^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 + b)*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a^2\,x^2+b}\,\sqrt{c+\sqrt{\sqrt{a^2\,x^2+b}+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a^2*x^2)^(1/2)*(c + ((b + a^2*x^2)^(1/2) + a*x)^(1/2))^(1/2),x)
```

```
[Out] int((b + a^2*x^2)^(1/2)*(c + ((b + a^2*x^2)^(1/2) + a*x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}\,\sqrt{a^2x^2+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2+b)**(1/2)*(c+(a*x+(a**2*x**2+b)**(1/2))**(1/2))**(1/2)
,x)
```

```
[Out] Integral(sqrt(c + sqrt(a*x + sqrt(a**2*x**2 + b)))*sqrt(a**2*x**2 + b), x)
```

3.2398 $\int \frac{\sqrt{\left(-81+27x+135x^2-150x^3+65x^4-13x^5+x^6\right)^3}}{-1+x}\,dx$

Optimal. Leaf size=518

$$-\frac{19451047\log\left(x^8-19x^7+152x^6-657x^5+1620x^4-2133x^3+972x^2+729x-729\right)}{65536}+\frac{\sqrt{x^{18}-39x^{17}+702x}}{}$$

Rubi [A] time = 0.93, antiderivative size = 459, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 9, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {6688, 6719, 1653, 814, 843, 621, 206, 724, 204}

$$-\frac{\left(-x^2+x+1\right)\sqrt{-(3-x)^{12}\left(-x^2+x+1\right)^3}(1-x)^4}{9(3-x)^6}-\frac{229\left(-x^2+x+1\right)\sqrt{-(3-x)^{12}\left(-x^2+x+1\right)^3}(1-x)^3}{144(3-x)^6}-\frac{19927\left(-x^2+x+1\right)\sqrt{-(3-x)^{12}\left(-x^2+x+1\right)^3}(1-x)^2}{2016(3-x)^6}-\frac{281233\left(-x^2+x+1\right)\sqrt{-(3-x)^{12}\left(-x^2+x+1\right)^3}(1-x)}{8064(3-x)^6}-\frac{6158183\left(-x^2+x+1\right)\sqrt{-(3-x)^{12}\left(-x^2+x+1\right)^3}}{80640(3-x)^6}+\frac{(5567931-6941558x)\sqrt{-(3-x)^{12}\left(-x^2+x+1\right)^3}}{32768(3-x)^6\left(-x^2+x+1\right)}+\frac{(903871-1283454x)\sqrt{-(3-x)^{12}\left(-x^2+x+1\right)^3}}{12288(3-x)^6}-\frac{64\sqrt{-(3-x)^{12}\left(-x^2+x+1\right)^3}\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)}{(3-x)^6\left(x^2-x-1\right)^{3/2}}+\frac{19451047\sqrt{-(3-x)^{12}\left(-x^2+x+1\right)^3}\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)}{65536(3-x)^6\left(x^2-x-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-81 + 27*x + 135*x^2 - 150*x^3 + 65*x^4 - 13*x^5 + x^6)^3]/(-1 + x),x]
```

```
[Out] ((903871 - 1283454*x)*Sqrt[-((3 - x)^12*(1 + x - x^2)^3)])/(12288*(3 - x)^6
) + ((5567931 - 6941558*x)*Sqrt[-((3 - x)^12*(1 + x - x^2)^3)])/(32768*(3 -
 x)^6*(1 + x - x^2)) - (6158183*(1 + x - x^2)*Sqrt[-((3 - x)^12*(1 + x - x^
2)^3)])/(80640*(3 - x)^6) - (281233*(1 - x)*(1 + x - x^2)*Sqrt[-((3 - x)^12
*(1 + x - x^2)^3)])/(8064*(3 - x)^6) - (19927*(1 - x)^2*(1 + x - x^2)*Sqrt[
-((3 - x)^12*(1 + x - x^2)^3)])/(2016*(3 - x)^6) - (229*(1 - x)^3*(1 + x -
x^2)*Sqrt[-((3 - x)^12*(1 + x - x^2)^3)])/(144*(3 - x)^6) - ((1 - x)^4*(1 +
 x - x^2)*Sqrt[-((3 - x)^12*(1 + x - x^2)^3)])/(9*(3 - x)^6) - (64*Sqrt[-((
3 - x)^12*(1 + x - x^2)^3)]*ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])])/((3 - x
)^6*(-1 - x + x^2)^(3/2)) + (19451047*Sqrt[-((3 - x)^12*(1 + x - x^2)^3)]*A
rcTanh[(1 - 2*x)/(2*Sqrt[-1 - x + x^2])])/(65536*(3 - x)^6*(-1 - x + x^2)^(
3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\sqrt{\left(-81+27x+135x^2-150x^3+65x^4-13x^5+x^6\right)^3}}{-1+x}\,dx = \int \frac{\sqrt{(-3+x)^{12}\left(-1-x+x^2\right)^3}}{-1+x}\,dx$$

$$= \frac{\sqrt{(-3+x)^{12}\left(-1-x+x^2\right)^3}\int \frac{(-3+x)^6\left(-1-x+x^2\right)^{3/2}}{-1+x}}{(-3+x)^6\left(-1-x+x^2\right)^{3/2}}$$

$$= -\frac{(1-x)^4\left(1+x-x^2\right)\sqrt{-(3-x)^{12}\left(1+x-x^2\right)^3}}{9(3-x)^6}$$

$$= -\frac{229(1-x)^3\left(1+x-x^2\right)\sqrt{-(3-x)^{12}\left(1+x-\right.}}{144(3-x)^6}$$

$$= -\frac{19927(1-x)^2\left(1+x-x^2\right)\sqrt{-(3-x)^{12}\left(1+x\right.}}{2016(3-x)^6}$$

$$= -\frac{281233(1-x)\left(1+x-x^2\right)\sqrt{-(3-x)^{12}\left(1+x\right.}}{8064(3-x)^6}$$

$$= -\frac{6158183\left(1+x-x^2\right)\sqrt{-(3-x)^{12}\left(1+x-x^2\right)}}{80640(3-x)^6}$$

$$= \frac{(903871-1283454x)\sqrt{-(3-x)^{12}\left(1+x-x^2\right)^3}}{12288(3-x)^6}$$

$$= \frac{(903871-1283454x)\sqrt{-(3-x)^{12}\left(1+x-x^2\right)^3}}{12288(3-x)^6}$$

$$= \frac{(903871-1283454x)\sqrt{-(3-x)^{12}\left(1+x-x^2\right)^3}}{12288(3-x)^6}$$

$$= \frac{(903871-1283454x)\sqrt{-(3-x)^{12}\left(1+x-x^2\right)^3}}{12288(3-x)^6}$$

$$= \frac{(903871-1283454x)\sqrt{-(3-x)^{12}\left(1+x-x^2\right)^3}}{12288(3-x)^6}$$

Mathematica [A] time = 0.40, size = 144, normalized size = 0.28

$$\frac{(x-3)^6\left(x^2-x-1\right)^{3/2}\left(-1321205760\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)+6127079805\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)+2\sqrt{x^2-x-1}\left(1146880x^8-23296000x^7+199009280x^6-910869760x^5+2304529024x^4-2700564848x^3-508033624x^2+4423205098x-1245336401\right)\right)}{20643840\sqrt{(x-3)^{12}\left(x^2-x-1\right)^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-81 + 27*x + 135*x^2 - 150*x^3 + 65*x^4 - 13*x^5 + x^6)^3]/
(-1 + x),x]
```

```
[Out] ((-3 + x)^6*(-1 - x + x^2)^(3/2)*(2*Sqrt[-1 - x + x^2]*(-1245336401 + 44232
05098*x - 508033624*x^2 - 2700564848*x^3 + 2304529024*x^4 - 910869760*x^5 +
 199009280*x^6 - 23296000*x^7 + 1146880*x^8) - 1321205760*ArcTan[(3 - x)/(2
*Sqrt[-1 - x + x^2])] + 6127079805*ArcTanh[(1 - 2*x)/(2*Sqrt[-1 - x + x^2])
]))/(20643840*Sqrt[(-3 + x)^12*(-1 - x + x^2)^3])
```

IntegrateAlgebraic [A] time = 0.47, size = 518, normalized size = 1.00

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[(-81 + 27*x + 135*x^2 - 150*x^3 + 65*x^4 - 13*x^5 +
 x^6)^3]/(-1 + x),x]
```

```
[Out] ((-1245336401 + 4423205098*x - 508033624*x^2 - 2700564848*x^3 + 2304529024*
x^4 - 910869760*x^5 + 199009280*x^6 - 23296000*x^7 + 1146880*x^8)*Sqrt[-531
441 + 531441*x + 2480058*x^2 - 4704237*x^3 - 885735*x^4 + 9880866*x^5 - 102
19851*x^6 + 592677*x^7 + 8764767*x^8 - 10819710*x^9 + 7498953*x^10 - 355416
3*x^11 + 1221371*x^12 - 309774*x^13 + 57735*x^14 - 7717*x^15 + 702*x^16 - 3
9*x^17 + x^18])/(10321920*(-3 + x)^6*(-1 - x + x^2)) + 128*ArcTan[((-1 - x
+ x^2)*(729 - 1458*x + 1215*x^2 - 540*x^3 + 135*x^4 - 18*x^5 + x^6))/(729 -
 1458*x - 243*x^2 + 3105*x^3 - 3753*x^4 + 2277*x^5 - 809*x^6 + 171*x^7 - 20
*x^8 + x^9 - Sqrt[-531441 + 531441*x + 2480058*x^2 - 4704237*x^3 - 885735*x
^4 + 9880866*x^5 - 10219851*x^6 + 592677*x^7 + 8764767*x^8 - 10819710*x^9 +
 7498953*x^10 - 3554163*x^11 + 1221371*x^12 - 309774*x^13 + 57735*x^14 - 77
17*x^15 + 702*x^16 - 39*x^17 + x^18])] - (19451047*Log[-729 + 729*x + 972*x
^2 - 2133*x^3 + 1620*x^4 - 657*x^5 + 152*x^6 - 19*x^7 + x^8])/65536 + (1945
1047*Log[-729 + 2187*x - 486*x^2 - 4077*x^3 + 5886*x^4 - 3897*x^5 + 1466*x^
6 - 323*x^7 + 39*x^8 - 2*x^9 + 2*Sqrt[-531441 + 531441*x + 2480058*x^2 - 47
04237*x^3 - 885735*x^4 + 9880866*x^5 - 10219851*x^6 + 592677*x^7 + 8764767*
x^8 - 10819710*x^9 + 7498953*x^10 - 3554163*x^11 + 1221371*x^12 - 309774*x^
13 + 57735*x^14 - 7717*x^15 + 702*x^16 - 39*x^17 + x^18]])/65536
```

fricas [A] time = 0.42, size = 652, normalized size = 1.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^6-13*x^5+65*x^4-150*x^3+135*x^2+27*x-81)^3)^(1/2)/(-1+x),x, a
lgorithm="fricas")
```

```
[Out] 1/330301440*(4819349233*x^8 - 91567635427*x^7 + 732541083416*x^6 - 31663124
46081*x^5 + 7807345757460*x^4 - 10279671913989*x^3 + 4684407454476*x^2 + 42
278584320*(x^8 - 19*x^7 + 152*x^6 - 657*x^5 + 1620*x^4 - 2133*x^3 + 972*x^2
 + 729*x - 729)*arctan(-(x^9 - 20*x^8 + 171*x^7 - 809*x^6 + 2277*x^5 - 3753
*x^4 + 3105*x^3 - 243*x^2 - 1458*x - sqrt(x^18 - 39*x^17 + 702*x^16 - 7717*
x^15 + 57735*x^14 - 309774*x^13 + 1221371*x^12 - 3554163*x^11 + 7498953*x^1
0 - 10819710*x^9 + 8764767*x^8 + 592677*x^7 - 10219851*x^6 + 9880866*x^5 -
885735*x^4 - 4704237*x^3 + 2480058*x^2 + 531441*x - 531441) + 729)/(x^8 - 1
9*x^7 + 152*x^6 - 657*x^5 + 1620*x^4 - 2133*x^3 + 972*x^2 + 729*x - 729)) +
 98033276880*(x^8 - 19*x^7 + 152*x^6 - 657*x^5 + 1620*x^4 - 2133*x^3 + 972*
x^2 + 729*x - 729)*log(-(2*x^9 - 39*x^8 + 323*x^7 - 1466*x^6 + 3897*x^5 - 5
886*x^4 + 4077*x^3 + 486*x^2 - 2187*x - 2*sqrt(x^18 - 39*x^17 + 702*x^16 -
7717*x^15 + 57735*x^14 - 309774*x^13 + 1221371*x^12 - 3554163*x^11 + 749895
3*x^10 - 10819710*x^9 + 8764767*x^8 + 592677*x^7 - 10219851*x^6 + 9880866*x
^5 - 885735*x^4 - 4704237*x^3 + 2480058*x^2 + 531441*x - 531441) + 729)/(x^
8 - 19*x^7 + 152*x^6 - 657*x^5 + 1620*x^4 - 2133*x^3 + 972*x^2 + 729*x - 72
9)) + 32*sqrt(x^18 - 39*x^17 + 702*x^16 - 7717*x^15 + 57735*x^14 - 309774*x
^13 + 1221371*x^12 - 3554163*x^11 + 7498953*x^10 - 10819710*x^9 + 8764767*x
```

```
^8 + 592677*x^7 - 10219851*x^6 + 9880866*x^5 - 885735*x^4 - 4704237*x^3 + 2
480058*x^2 + 531441*x - 531441)*(1146880*x^8 - 23296000*x^7 + 199009280*x^6
 - 910869760*x^5 + 2304529024*x^4 - 2700564848*x^3 - 508033624*x^2 + 442320
5098*x - 1245336401) + 3513305590857*x - 3513305590857)/(x^8 - 19*x^7 + 152
*x^6 - 657*x^5 + 1620*x^4 - 2133*x^3 + 972*x^2 + 729*x - 729)
```

giac [A] time = 0.33, size = 92, normalized size = 0.18

$$\frac{1}{10321920} \left(2 \left(4 \left(2 \left(8 \left(10 \left(4 \left(14 \left(16 x - 325\right)x + 38869\right)x - 711617\right)x + 18004133\right)x - 168785303\right)x - 63504203\right)x + 2211602549\right)x - 1245336401\right)\sqrt{x^2 - x - 1} + 128 \arctan\left(-x + \sqrt{x^2 - x - 1} + 1\right) + \frac{19451047}{65536} \log\left(\left|-2 x + 2 \sqrt{x^2 - x - 1} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^6-13*x^5+65*x^4-150*x^3+135*x^2+27*x-81)^3)^(1/2)/(-1+x),x, a
lgorithm="giac")
```

```
[Out] 1/10321920*(2*(4*(2*(8*(10*(4*(14*(16*x - 325)*x + 38869)*x - 711617)*x + 1
8004133)*x - 168785303)*x - 63504203)*x + 2211602549)*x - 1245336401)*sqrt(
x^2 - x - 1) + 128*arctan(-x + sqrt(x^2 - x - 1) + 1) + 19451047/65536*log(
abs(-2*x + 2*sqrt(x^2 - x - 1) + 1))
```

maple [A] time = 0.04, size = 205, normalized size = 0.40

$$\frac{\sqrt{\left(x^6 - 13x^5 + 65x^4 - 150x^3 + 135x^2 + 27x - 81\right)^3} \left(2293760x^4 \left(x^2 - x - 1\right)^{\frac{5}{2}} - 42004480x^3 \left(x^2 - x - 1\right)^{\frac{5}{2}} + 316303360x^2 \left(x^2 - x - 1\right)^{\frac{5}{2}} - 1235724800x \left(x^2 - x - 1\right)^{\frac{5}{2}} + 2535627008 \left(x^2 - x - 1\right)^{\frac{5}{2}} - 2156202720x \left(x^2 - x - 1\right)^{\frac{3}{2}} + 1518503280 \left(x^2 - x - 1\right)^{\frac{3}{2}} + 4373181540x\sqrt{x^2 - x - 1} - 3507796530\sqrt{x^2 - x - 1} - 6127079805 \ln\left(-\frac{1}{2} + x + \sqrt{x^2 - x - 1}\right) + 1321205760 \arctan\left(\frac{-3+x}{2\sqrt{x^2-x-1}}\right)\right)}{20643840 \left(x^2 - x - 1\right)^{\frac{3}{2}} \left(-3 + x\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6-13*x^5+65*x^4-150*x^3+135*x^2+27*x-81)^3)^(1/2)/(-1+x),x)
```

```
[Out] 1/20643840*((x^6-13*x^5+65*x^4-150*x^3+135*x^2+27*x-81)^3)^(1/2)*(2293760*x
^4*(x^2-x-1)^(5/2)-42004480*x^3*(x^2-x-1)^(5/2)+316303360*x^2*(x^2-x-1)^(5/
2)-1235724800*x*(x^2-x-1)^(5/2)+2535627008*(x^2-x-1)^(5/2)-2156202720*x*(x^
2-x-1)^(3/2)+1518503280*(x^2-x-1)^(3/2)+4373181540*x*(x^2-x-1)^(1/2)-350779
6530*(x^2-x-1)^(1/2)-6127079805*ln(-1/2+x+(x^2-x-1)^(1/2))+1321205760*arcta
n(1/2*(-3+x)/(x^2-x-1)^(1/2)))/(x^2-x-1)^(3/2)/(-3+x)^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(x^6 - 13\,x^5 + 65\,x^4 - 150\,x^3 + 135\,x^2 + 27\,x - 81\right)^3}}{x - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^6-13*x^5+65*x^4-150*x^3+135*x^2+27*x-81)^3)^(1/2)/(-1+x),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt((x^6 - 13*x^5 + 65*x^4 - 150*x^3 + 135*x^2 + 27*x - 81)^3)/(
x - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\left(x^6 - 13\,x^5 + 65\,x^4 - 150\,x^3 + 135\,x^2 + 27\,x - 81\right)^3}}{x - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((27*x + 135*x^2 - 150*x^3 + 65*x^4 - 13*x^5 + x^6 - 81)^3)^(1/2)/(x -
1),x)
```

```
[Out] int(((27*x + 135*x^2 - 150*x^3 + 65*x^4 - 13*x^5 + x^6 - 81)^3)^(1/2)/(x -
1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-3)^{12}\left(x^2-x-1\right)^3}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**6-13*x**5+65*x**4-150*x**3+135*x**2+27*x-81)**3)**(1/2)/(-1+
x),x)
```

```
[Out] Integral(sqrt((x - 3)**12*(x**2 - x - 1)**3)/(x - 1), x)
```

3.2399 $\int \frac{\left(-q+px^4\right)\sqrt{q+px^4}}{x^2\left(aq+bx^2+apx^4\right)}\,dx$

Optimal. Leaf size=524

$$\frac{\left(\sqrt{2}\, b\sqrt{i\sqrt{b}\,\sqrt{2a\sqrt{p}\,\sqrt{q}-b}+a\left(-\sqrt{p}\right)\sqrt{q}+b}-i\sqrt{2}\,\sqrt{b}\,\sqrt{i\sqrt{b}\,\sqrt{2a\sqrt{p}\,\sqrt{q}-b}+a\left(-\sqrt{p}\right)\sqrt{q}+b}\,\sqrt{2a\sqrt{p}\cdots}{2a^{5/2}\sqrt{p}\,\sqrt{q}}$$

Rubi [C] time = 3.47, antiderivative size = 1029, normalized size of antiderivative = 1.96, number of steps used = 22, number of rules used = 9, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.220, Rules used = {6728, 277, 305, 220, 1196, 1209, 1198, 1217, 1707}

Antiderivative was successfully verified.

```
[In] Int[((-q + p*x^4)*Sqrt[q + p*x^4])/(x^2*(a*q + b*x^2 + a*p*x^4)),x]
```

```
[Out] Sqrt[q + p*x^4]/(a*x) + (Sqrt[b]*ArcTan[(Sqrt[b]*x)/(Sqrt[a]*Sqrt[q + p*x^4
])])/a^(3/2) - (p^(1/4)*q^(1/4)*(Sqrt[q] + Sqrt[p]*x^2)*Sqrt[(q + p*x^4)/(S
qrt[q] + Sqrt[p]*x^2)^2]*EllipticF[2*ArcTan[(p^(1/4)*x)/q^(1/4)], 1/2])/(a*
Sqrt[q + p*x^4]) + (b*(b - Sqrt[b^2 - 4*a^2*p*q])*(Sqrt[q] + Sqrt[p]*x^2)*S
qrt[(q + p*x^4)/(Sqrt[q] + Sqrt[p]*x^2)^2]*EllipticF[2*ArcTan[(p^(1/4)*x)/q
^(1/4)], 1/2])/(2*a^2*p^(1/4)*q^(1/4)*(b - 2*a*Sqrt[p]*Sqrt[q] - Sqrt[b^2 -
 4*a^2*p*q])*Sqrt[q + p*x^4]) - ((b - 2*a*Sqrt[p]*Sqrt[q] - Sqrt[b^2 - 4*a^
2*p*q])*(Sqrt[q] + Sqrt[p]*x^2)*Sqrt[(q + p*x^4)/(Sqrt[q] + Sqrt[p]*x^2)^2]
*EllipticF[2*ArcTan[(p^(1/4)*x)/q^(1/4)], 1/2])/(4*a^2*p^(1/4)*q^(1/4)*Sqrt
[q + p*x^4]) + (b*(b + Sqrt[b^2 - 4*a^2*p*q])*(Sqrt[q] + Sqrt[p]*x^2)*Sqrt[
(q + p*x^4)/(Sqrt[q] + Sqrt[p]*x^2)^2]*EllipticF[2*ArcTan[(p^(1/4)*x)/q^(1/
4)], 1/2])/(2*a^2*p^(1/4)*q^(1/4)*(b - 2*a*Sqrt[p]*Sqrt[q] + Sqrt[b^2 - 4*a
^2*p*q])*Sqrt[q + p*x^4]) - ((b - 2*a*Sqrt[p]*Sqrt[q] + Sqrt[b^2 - 4*a^2*p*
q])*(Sqrt[q] + Sqrt[p]*x^2)*Sqrt[(q + p*x^4)/(Sqrt[q] + Sqrt[p]*x^2)^2]*Ell
ipticF[2*ArcTan[(p^(1/4)*x)/q^(1/4)], 1/2])/(4*a^2*p^(1/4)*q^(1/4)*Sqrt[q +
 p*x^4]) - (b*(b + 2*a*Sqrt[p]*Sqrt[q] - Sqrt[b^2 - 4*a^2*p*q])*(Sqrt[q] +
Sqrt[p]*x^2)*Sqrt[(q + p*x^4)/(Sqrt[q] + Sqrt[p]*x^2)^2]*EllipticPi[(2 - b/
(a*Sqrt[p]*Sqrt[q]))/4, 2*ArcTan[(p^(1/4)*x)/q^(1/4)], 1/2])/(4*a^2*p^(1/4)
*q^(1/4)*(b - 2*a*Sqrt[p]*Sqrt[q] - Sqrt[b^2 - 4*a^2*p*q])*Sqrt[q + p*x^4])
 - (b*(b + 2*a*Sqrt[p]*Sqrt[q] + Sqrt[b^2 - 4*a^2*p*q])*(Sqrt[q] + Sqrt[p]*
x^2)*Sqrt[(q + p*x^4)/(Sqrt[q] + Sqrt[p]*x^2)^2]*EllipticPi[(2 - b/(a*Sqrt[
p]*Sqrt[q]))/4, 2*ArcTan[(p^(1/4)*x)/q^(1/4)], 1/2])/(4*a^2*p^(1/4)*q^(1/4)
*(b - 2*a*Sqrt[p]*Sqrt[q] + Sqrt[b^2 - 4*a^2*p*q])*Sqrt[q + p*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1209

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> -Dist[(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(-q+px^4\right)\sqrt{q+px^4}}{x^2\left(aq+bx^2+apx^4\right)}\,dx = \int \left(-\frac{\sqrt{q+px^4}}{ax^2}+\frac{\left(b+2apx^2\right)\sqrt{q+px^4}}{a\left(aq+bx^2+apx^4\right)}\right)dx$$

$$= -\frac{\int \frac{\sqrt{q+px^4}}{x^2}\,dx}{a}+\frac{\int \frac{\left(b+2apx^2\right)\sqrt{q+px^4}}{aq+bx^2+apx^4}\,dx}{a}$$

$$= \frac{\sqrt{q+px^4}}{ax}+\frac{\int \left(\frac{2ap\sqrt{q+px^4}}{b-\sqrt{b^2-4a^2pq}+2apx^2}+\frac{2ap\sqrt{q+px^4}}{b+\sqrt{b^2-4a^2pq}+2apx^2}\right)dx}{a}-\frac{(2p)\int \frac{x^2}{\sqrt{q+px^4}}\,dx}{a}$$

$$= \frac{\sqrt{q+px^4}}{ax}+(2p)\int \frac{\sqrt{q+px^4}}{b-\sqrt{b^2-4a^2pq}+2apx^2}\,dx+(2p)\int \frac{\sqrt{q+px^4}}{b+\sqrt{b^2-4a^2pq}+ \cdots}$$

$$= \frac{\sqrt{q+px^4}}{ax}-\frac{2\sqrt{p}\,x\sqrt{q+px^4}}{a\left(\sqrt{q}+\sqrt{p}\,x^2\right)}+\frac{2\sqrt[4]{p}\,\sqrt[4]{q}\left(\sqrt{q}+\sqrt{p}\,x^2\right)\sqrt{\frac{q+px^4}{\left(\sqrt{q}+\sqrt{p}\,x^2\right)^2}}\,E\left(2\tan^{-1} \cdots\right.}{a\sqrt{q+px^4}}$$

$$= \frac{\sqrt{q+px^4}}{ax}-\frac{2\sqrt{p}\,x\sqrt{q+px^4}}{a\left(\sqrt{q}+\sqrt{p}\,x^2\right)}+\frac{2\sqrt[4]{p}\,\sqrt[4]{q}\left(\sqrt{q}+\sqrt{p}\,x^2\right)\sqrt{\frac{q+px^4}{\left(\sqrt{q}+\sqrt{p}\,x^2\right)^2}}\,E\left(2\tan^{-1} \cdots\right.}{a\sqrt{q+px^4}}$$

$$= \frac{\sqrt{q+px^4}}{ax}-\frac{2\sqrt{p}\,x\sqrt{q+px^4}}{a\left(\sqrt{q}+\sqrt{p}\,x^2\right)}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}\sqrt{q+px^4}}\right)}{a^{3/2}}+\frac{2\sqrt[4]{p}\,\sqrt[4]{q}\left(\sqrt{q}+\sqrt{p}\,x^2 \cdots\right.}{\cdots}$$

Mathematica [C] time = 1.51, size = 293, normalized size = 0.56

$$\frac{-ibx\sqrt{\frac{px^4}{q}+1}\,\Pi\left(\frac{2ia\sqrt{p}\sqrt{q}}{\sqrt{b^2-4a^2pq}-b};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{p}}{\sqrt{q}}}x\right)\middle|-1\right)-ibx\sqrt{\frac{px^4}{q}+1}\,\Pi\left(-\frac{2ia\sqrt{p}\sqrt{q}}{b+\sqrt{b^2-4a^2pq}};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{p}}{\sqrt{q}}}x\right)\middle|-1\right)+apx^4\sqrt{\frac{i\sqrt{p}}{\sqrt{q}}}+aq\sqrt{\frac{i\sqrt{p}}{\sqrt{q}}}+ibx\sqrt{\frac{px^4}{q}+1}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{p}}{\sqrt{q}}}x\right)\middle|-1\right)}{a^2x\sqrt{\frac{i\sqrt{p}}{\sqrt{q}}}\sqrt{px^4+q}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-q + p*x^4)*Sqrt[q + p*x^4])/(x^2*(a*q + b*x^2 + a*p*x^4)),x]
```

```
[Out] (a*Sqrt[(I*Sqrt[p])/Sqrt[q]]*q + a*p*Sqrt[(I*Sqrt[p])/Sqrt[q]]*x^4 + I*b*x*
Sqrt[1 + (p*x^4)/q]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[p])/Sqrt[q]]*x], -1] -
 I*b*x*Sqrt[1 + (p*x^4)/q]*EllipticPi[((2*I)*a*Sqrt[p]*Sqrt[q])/(-b + Sqrt[
b^2 - 4*a^2*p*q]), I*ArcSinh[Sqrt[(I*Sqrt[p])/Sqrt[q]]*x], -1] - I*b*x*Sqrt
[1 + (p*x^4)/q]*EllipticPi[((-2*I)*a*Sqrt[p]*Sqrt[q])/(b + Sqrt[b^2 - 4*a^2
*p*q]), I*ArcSinh[Sqrt[(I*Sqrt[p])/Sqrt[q]]*x], -1])/(a^2*Sqrt[(I*Sqrt[p])/
Sqrt[q]]*x*Sqrt[q + p*x^4])
```

IntegrateAlgebraic [A] time = 0.92, size = 54, normalized size = 0.10

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}\sqrt{px^4+q}}\right)}{a^{3/2}}+\frac{\sqrt{px^4+q}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-q + p*x^4)*Sqrt[q + p*x^4])/(x^2*(a*q + b*x^2 + a*p*x
^4)),x]
```

```
[Out] Sqrt[q + p*x^4]/(a*x) + (Sqrt[b]*ArcTan[(Sqrt[b]*x)/(Sqrt[a]*Sqrt[q + p*x^4
])])/a^(3/2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^4-q)*(p*x^4+q)^(1/2)/x^2/(a*p*x^4+b*x^2+a*q),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^4+q}\left(px^4-q\right)}{\left(apx^4+bx^2+aq\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^4-q)*(p*x^4+q)^(1/2)/x^2/(a*p*x^4+b*x^2+a*q),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(p*x^4 + q)*(p*x^4 - q)/((a*p*x^4 + b*x^2 + a*q)*x^2), x)
```

maple [C] time = 0.08, size = 528, normalized size = 1.01

$$-\frac{-\frac{\sqrt{px^4+q}}{x}+\frac{2i\sqrt{p}\sqrt{q}\sqrt{1-\frac{i\sqrt{p}\,x^2}{\sqrt{q}}}\sqrt{1+\frac{i\sqrt{p}\,x^2}{\sqrt{q}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{p}}{\sqrt{q}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{p}}{\sqrt{q}}},i\right)\right)}{\sqrt{\frac{i\sqrt{p}}{\sqrt{q}}}\sqrt{px^4+q}}}{a}+\frac{-\frac{b\sqrt{1-\frac{i\sqrt{p}\,x^2}{\sqrt{q}}}\sqrt{1+\frac{i\sqrt{p}\,x^2}{\sqrt{q}}}\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{p}}{\sqrt{q}}},i\right)}{a\sqrt{\frac{i\sqrt{p}}{\sqrt{q}}}\sqrt{px^4+q}}+\frac{2i\sqrt{p}\sqrt{q}\sqrt{1-\frac{i\sqrt{p}\,x^2}{\sqrt{q}}}\sqrt{1+\frac{i\sqrt{p}\,x^2}{\sqrt{q}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{p}}{\sqrt{q}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{p}}{\sqrt{q}}},i\right)\right)}{\sqrt{\frac{i\sqrt{p}}{\sqrt{q}}}\sqrt{px^4+q}}+\frac{b\left(\sum_{\_\alpha=\operatorname{RootOf}\left(ap\_Z^4+b\_Z^2+aq\right)}\frac{\left(\_\alpha^2 b+2aq\right)\left(-\frac{\operatorname{arctanh}\left(\frac{\_\alpha^2\left(-\_\alpha^2 ap+apx^2-b\right)}{a\sqrt{-\frac{b\_\alpha^2}{a}}\sqrt{px^4+q}}\right)}{\sqrt{-\frac{b\_\alpha^2}{a}}}+\frac{2\_\alpha\left(\_\alpha^2 ap+b\right)\sqrt{1-\frac{i\sqrt{p}\,x^2}{\sqrt{q}}}\sqrt{1+\frac{i\sqrt{p}\,x^2}{\sqrt{q}}}\operatorname{EllipticPi}\left(x\sqrt{\frac{i\sqrt{p}}{\sqrt{q}}},\frac{i\left(\_\alpha^2 ap+b\right)}{\sqrt{q}\sqrt{p}\,a},\frac{\sqrt{-\frac{i\sqrt{p}}{\sqrt{q}}}}{\sqrt{\frac{i\sqrt{p}}{\sqrt{q}}}}\right)}{\sqrt{\frac{i\sqrt{p}}{\sqrt{q}}}\,aq\sqrt{px^4+q}}\right)}{\_\alpha\left(2\_\alpha^2 ap+b\right)}\right)}{4a}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^4-q)*(p*x^4+q)^(1/2)/x^2/(a*p*x^4+b*x^2+a*q),x)
```

```
[Out] -1/a*(-(p*x^4+q)^(1/2)/x+2*I*p^(1/2)*q^(1/2)/(I/q^(1/2)*p^(1/2))^(1/2)*(1-I
/q^(1/2)*p^(1/2)*x^2)^(1/2)*(1+I/q^(1/2)*p^(1/2)*x^2)^(1/2)/(p*x^4+q)^(1/2)
*(EllipticF(x*(I/q^(1/2)*p^(1/2))^(1/2),I)-EllipticE(x*(I/q^(1/2)*p^(1/2))^
(1/2),I)))+1/a*(-b/a/(I/q^(1/2)*p^(1/2))^(1/2)*(1-I/q^(1/2)*p^(1/2)*x^2)^(1
/2)*(1+I/q^(1/2)*p^(1/2)*x^2)^(1/2)/(p*x^4+q)^(1/2)*EllipticF(x*(I/q^(1/2)*
p^(1/2))^(1/2),I)+2*I*p^(1/2)*q^(1/2)/(I/q^(1/2)*p^(1/2))^(1/2)*(1-I/q^(1/2
)*p^(1/2)*x^2)^(1/2)*(1+I/q^(1/2)*p^(1/2)*x^2)^(1/2)/(p*x^4+q)^(1/2)*(Ellip
ticF(x*(I/q^(1/2)*p^(1/2))^(1/2),I)-EllipticE(x*(I/q^(1/2)*p^(1/2))^(1/2),I
))+1/4*b/a*sum((_alpha^2*b+2*a*q)/_alpha/(2*_alpha^2*a*p+b)*(-1/(-1/a*b*_al
pha^2)^(1/2)*arctanh(_alpha^2/a*(-_alpha^2*a*p+a*p*x^2-b)/(-1/a*b*_alpha^2)
^(1/2)/(p*x^4+q)^(1/2))+2/(I/q^(1/2)*p^(1/2))^(1/2)*_alpha*(_alpha^2*a*p+b)
/a/q*(1-I/q^(1/2)*p^(1/2)*x^2)^(1/2)*(1+I/q^(1/2)*p^(1/2)*x^2)^(1/2)/(p*x^4
+q)^(1/2)*EllipticPi(x*(I/q^(1/2)*p^(1/2))^(1/2),I/q^(1/2)/p^(1/2)*(_alpha^
2*a*p+b)/a,(-I/q^(1/2)*p^(1/2))^(1/2)/(I/q^(1/2)*p^(1/2))^(1/2))),_alpha=Ro
otOf(_Z^4*a*p+_Z^2*b+a*q)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^4+q}\left(px^4-q\right)}{\left(apx^4+bx^2+aq\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x^4-q)*(p*x^4+q)^(1/2)/x^2/(a*p*x^4+b*x^2+a*q),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(p*x^4 + q)*(p*x^4 - q)/((a*p*x^4 + b*x^2 + a*q)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\sqrt{p\,x^4+q}\,\left(q-p\,x^4\right)}{x^2\,\left(a\,p\,x^4+b\,x^2+a\,q\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((q + p*x^4)^(1/2)*(q - p*x^4))/(x^2*(a*q + b*x^2 + a*p*x^4)),x)
[Out] int(-((q + p*x^4)^(1/2)*(q - p*x^4))/(x^2*(a*q + b*x^2 + a*p*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(px^4-q\right)\sqrt{px^4+q}}{x^2\left(apx^4+aq+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x**4-q)*(p*x**4+q)**(1/2)/x**2/(a*p*x**4+b*x**2+a*q),x)
[Out] Integral((p*x**4 - q)*sqrt(p*x**4 + q)/(x**2*(a*p*x**4 + a*q + b*x**2)), x)
```

3.2400 $\int \frac{\sqrt[3]{b^2x^2+a^3x^3}}{b+ax}\,dx$

Optimal. Leaf size=526

$$\frac{\sqrt[3]{a^3x^3+b^2x^2}}{a}+\frac{(3a^2b-b^2)\log\left(\sqrt[3]{a^3x^3+b^2x^2}-ax\right)}{3a^3}+\frac{(b^2-3a^2b)\log\left(ax\sqrt[3]{a^3x^3+b^2x^2}+\left(a^3x^3+b^2x^2\right)^{2/3}+a^2\right.}{6a^3}$$

Rubi [A] time = 0.27, antiderivative size = 524, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2056, 101, 157, 59, 91}

$$\frac{\sqrt[3]{a^3x^3+b^2x^2}}{a}+\frac{b(3a^2-b)\sqrt[3]{a^3x^3+b^2x^2}\log\left(a^3x+b^2\right)}{6a^3x^{2/3}\sqrt[3]{a^3x+b^2}}+\frac{b(3a^2-b)\sqrt[3]{a^3x^3+b^2x^2}\log\left(\frac{a\sqrt[3]{x}}{\sqrt[3]{a^3x+b^2}}-1\right)}{2a^3x^{2/3}\sqrt[3]{a^3x+b^2}}+\frac{b(3a^2-b)\sqrt[3]{a^3x^3+b^2x^2}\tan^{-1}\left(\frac{2a\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a^3x+b^2}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}a^3x^{2/3}\sqrt[3]{a^3x+b^2}}+\frac{b\sqrt[3]{a^2-b}\sqrt[3]{a^3x^3+b^2x^2}\log(ax+b)}{2a^{5/3}x^{2/3}\sqrt[3]{a^3x+b^2}}-\frac{3b\sqrt[3]{a^2-b}\sqrt[3]{a^3x^3+b^2x^2}\log\left(\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{a^2-b}-\sqrt[3]{a^3x+b^2}\right)}{2a^{5/3}x^{2/3}\sqrt[3]{a^3x+b^2}}-\frac{\sqrt{3}b\sqrt[3]{a^2-b}\sqrt[3]{a^3x^3+b^2x^2}\tan^{-1}\left(\frac{2\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{a^2-b}}{\sqrt{3}\sqrt[3]{a^3x+b^2}}+\frac{1}{\sqrt{3}}\right)}{a^{5/3}x^{2/3}\sqrt[3]{a^3x+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2*x^2 + a^3*x^3)^(1/3)/(b + a*x),x]
```

```
[Out] (b^2*x^2 + a^3*x^3)^(1/3)/a + ((3*a^2 - b)*b*(b^2*x^2 + a^3*x^3)^(1/3)*ArcT
an[1/Sqrt[3] + (2*a*x^(1/3))/(Sqrt[3]*(b^2 + a^3*x)^(1/3))])/(Sqrt[3]*a^3*x
^(2/3)*(b^2 + a^3*x)^(1/3)) - (Sqrt[3]*(a^2 - b)^(1/3)*b*(b^2*x^2 + a^3*x^3
)^(1/3)*ArcTan[1/Sqrt[3] + (2*a^(1/3)*(a^2 - b)^(1/3)*x^(1/3))/(Sqrt[3]*(b^
2 + a^3*x)^(1/3))])/(a^(5/3)*x^(2/3)*(b^2 + a^3*x)^(1/3)) + ((a^2 - b)^(1/3
)*b*(b^2*x^2 + a^3*x^3)^(1/3)*Log[b + a*x])/(2*a^(5/3)*x^(2/3)*(b^2 + a^3*x
)^(1/3)) + ((3*a^2 - b)*b*(b^2*x^2 + a^3*x^3)^(1/3)*Log[b^2 + a^3*x])/(6*a^
3*x^(2/3)*(b^2 + a^3*x)^(1/3)) + ((3*a^2 - b)*b*(b^2*x^2 + a^3*x^3)^(1/3)*L
og[-1 + (a*x^(1/3))/(b^2 + a^3*x)^(1/3)])/(2*a^3*x^(2/3)*(b^2 + a^3*x)^(1/3
)) - (3*(a^2 - b)^(1/3)*b*(b^2*x^2 + a^3*x^3)^(1/3)*Log[a^(1/3)*(a^2 - b)^(
1/3)*x^(1/3) - (b^2 + a^3*x)^(1/3)])/(2*a^(5/3)*x^(2/3)*(b^2 + a^3*x)^(1/3)
)
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{b^2x^2+a^3x^3}}{b+ax}\,dx &= \frac{\sqrt[3]{b^2x^2+a^3x^3}\int \frac{x^{2/3}\sqrt[3]{b^2+a^3x}}{b+ax}\,dx}{x^{2/3}\sqrt[3]{b^2+a^3x}}\\
&= \frac{\sqrt[3]{b^2x^2+a^3x^3}}{a} - \frac{\sqrt[3]{b^2x^2+a^3x^3}\int \frac{\frac{2b^3}{3}+\frac{1}{3}a\left(3a^2-b\right)bx}{\sqrt[3]{x}\,(b+ax)\left(b^2+a^3x\right)^{2/3}}\,dx}{ax^{2/3}\sqrt[3]{b^2+a^3x}}\\
&= \frac{\sqrt[3]{b^2x^2+a^3x^3}}{a} - \frac{\left(\left(3a^2-b\right)b\sqrt[3]{b^2x^2+a^3x^3}\right)\int \frac{1}{\sqrt[3]{x}\left(b^2+a^3x\right)^{2/3}}\,dx}{3ax^{2/3}\sqrt[3]{b^2+a^3x}} + \frac{\left(\left(a^2-b\right)b^2\sqrt[3]{b^2x^2+\ldots}\right.}{ax^{2/\ldots}}\\
&= \frac{\sqrt[3]{b^2x^2+a^3x^3}}{a} + \frac{\left(3a^2-b\right)b\sqrt[3]{b^2x^2+a^3x^3}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2a\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{b^2+a^3x}}\right)}{\sqrt{3}\,a^3x^{2/3}\sqrt[3]{b^2+a^3x}} - \frac{\sqrt{3}\sqrt[3]{a^2-b}\,b\sqrt[3]{\ldots}}{\ldots}
\end{aligned}$$

Mathematica [C] time = 0.15, size = 162, normalized size = 0.31

$$\frac{3x^2\left(\left(a^3bx+b^3\right){}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{a^3x}{b^2}\right)-\left(a^2\left(a^3x+b^2\right){}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{a^3x}{b^2}\right)\right)+b^2\left(a^2-b\right)\sqrt[3]{\frac{a^3x}{b^2}+1}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{\left(a^3-ab\right)x}{xa^3+b^2}\right)\right)}{2ab\left(x^2\left(a^3x+b^2\right)\right)^{2/3}\sqrt[3]{\frac{a^3x}{b^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2*x^2 + a^3*x^3)^(1/3)/(b + a*x),x]
```

```
[Out] (3*x^2*((b^3 + a^3*b*x)*Hypergeometric2F1[-1/3, 2/3, 5/3, -((a^3*x)/b^2)] -
 a^2*(b^2 + a^3*x)*Hypergeometric2F1[2/3, 2/3, 5/3, -((a^3*x)/b^2)] + (a^2
- b)*b^2*(1 + (a^3*x)/b^2)^(1/3)*Hypergeometric2F1[2/3, 1, 5/3, ((a^3 - a*b
)*x)/(b^2 + a^3*x)]))/(2*a*b*(x^2*(b^2 + a^3*x))^(2/3)*(1 + (a^3*x)/b^2)^(1
/3))
```

IntegrateAlgebraic [A] time = 4.20, size = 571, normalized size = 1.09

$$\frac{\sqrt[3]{a^3x^3+b^2x^2}}{a}+\frac{\left(3a^2b-b^2\right)\log\left(\sqrt[3]{a^3x^3+b^2x^2}-ax\right)}{3a^3}+\frac{\left(b^2-3a^2b\right)\log\left(ax\sqrt[3]{a^3x^3+b^2x^2}+\left(a^3x^3+b^2x^2\right)^{2/3}+a^2x^2\right)}{6a^3}+\frac{\left(3a^2b-b^2\right)\tan^{-1}\left(\frac{\sqrt{3}ax}{2\sqrt[3]{a^3x^3+b^2x^2}+ax}\right)}{\sqrt{3}a^3}+\frac{\left(b\sqrt[3]{a^2-b}-i\sqrt{3}b\sqrt[3]{a^2-b}\right)\log\left(2\sqrt[3]{a}\,x\sqrt[3]{a^2-b}+\left(1+i\sqrt{3}\right)\sqrt[3]{a^3x^3+b^2x^2}\right)}{2a^{5/3}}+\frac{i\left(\sqrt{3}b\sqrt[3]{a^2-b}+ib\sqrt[3]{a^2-b}\right)\log\left(\left(\sqrt{3}+i\right)\left(a^3x^3+b^2x^2\right)^{2/3}-2ia^{2/3}x^2\left(a^2-b\right)^{2/3}+\sqrt[3]{a}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a^2-b}\sqrt[3]{a^3x^3+b^2x^2}\right)}{4a^{5/3}}+\frac{\sqrt{-3-3i\sqrt{3}}\,b\sqrt[3]{a^2-b}\tan^{-1}\left(\frac{3\sqrt[3]{a}x\sqrt[3]{a^2-b}}{-\sqrt{3}\sqrt[3]{a^3x^3+b^2x^2}-3i\sqrt[3]{a^3x^3+b^2x^2}+\sqrt{3}\sqrt[3]{a}x\sqrt[3]{a^2-b}}\right)}{\sqrt{2}a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^2*x^2 + a^3*x^3)^(1/3)/(b + a*x),x]
```

```
[Out] (b^2*x^2 + a^3*x^3)^(1/3)/a + ((3*a^2*b - b^2)*ArcTan[(Sqrt[3]*a*x)/(a*x +
2*(b^2*x^2 + a^3*x^3)^(1/3))])/(Sqrt[3]*a^3) + (Sqrt[-3 - (3*I)*Sqrt[3]]*(a
^2 - b)^(1/3)*b*ArcTan[(3*a^(1/3)*(a^2 - b)^(1/3)*x)/(Sqrt[3]*a^(1/3)*(a^2
- b)^(1/3)*x - (3*I)*(b^2*x^2 + a^3*x^3)^(1/3) - Sqrt[3]*(b^2*x^2 + a^3*x^3
```

```
)^(1/3))])/(Sqrt[2]*a^(5/3)) + ((3*a^2*b - b^2)*Log[-(a*x) + (b^2*x^2 + a^3
*x^3)^(1/3)])/(3*a^3) + (((a^2 - b)^(1/3)*b - I*Sqrt[3]*(a^2 - b)^(1/3)*b)*
Log[2*a^(1/3)*(a^2 - b)^(1/3)*x + (1 + I*Sqrt[3])*(b^2*x^2 + a^3*x^3)^(1/3)
])/(2*a^(5/3)) + ((-3*a^2*b + b^2)*Log[a^2*x^2 + a*x*(b^2*x^2 + a^3*x^3)^(1
/3) + (b^2*x^2 + a^3*x^3)^(2/3)])/(6*a^3) + ((I/4)*(I*(a^2 - b)^(1/3)*b + S
qrt[3]*(a^2 - b)^(1/3)*b)*Log[(-2*I)*a^(2/3)*(a^2 - b)^(2/3)*x^2 + a^(1/3)*
(a^2 - b)^(1/3)*(I*x - Sqrt[3]*x)*(b^2*x^2 + a^3*x^3)^(1/3) + (I + Sqrt[3])
*(b^2*x^2 + a^3*x^3)^(2/3)])/a^(5/3)
```

fricas [A]   time = 0.45, size = 437, normalized size = 0.83

$$\frac{6\sqrt{3}a^2b\left(-\frac{a^2-b}{a^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(a^2-b)x+2\sqrt{3}\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}a\left(-\frac{a^2-b}{a^2}\right)^{\frac{2}{3}}}{3(a^2-b)x}\right)+6a^2b\left(-\frac{a^2-b}{a^2}\right)^{\frac{1}{3}}\log\left(\frac{ax\left(-\frac{a^2-b}{a^2}\right)^{\frac{1}{3}}+\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}}{x}\right)-3a^2b\left(-\frac{a^2-b}{a^2}\right)^{\frac{1}{3}}\log\left(\frac{a^2x^2\left(-\frac{a^2-b}{a^2}\right)^{\frac{2}{3}}-\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}ax\left(-\frac{a^2-b}{a^2}\right)^{\frac{1}{3}}+\left(a^3x^3+b^2x^2\right)^{\frac{2}{3}}}{x^2}\right)-2\sqrt{3}\left(3a^2b-b^2\right)\arctan\left(\frac{\sqrt{3}ax+2\sqrt{3}\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}}{3ax}\right)+6\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}a^2+2\left(3a^2b-b^2\right)\log\left(-\frac{ax-\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}}{x}\right)-\left(3a^2b-b^2\right)\log\left(\frac{a^2x^2+\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}ax+\left(a^3x^3+b^2x^2\right)^{\frac{2}{3}}}{x^2}\right)}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3+b^2*x^2)^(1/3)/(a*x+b),x, algorithm="fricas")
```

```
[Out] 1/6*(6*sqrt(3)*a^2*b*(-(a^2 - b)/a^2)^(1/3)*arctan(-1/3*(sqrt(3)*(a^2 - b)*
x + 2*sqrt(3)*(a^3*x^3 + b^2*x^2)^(1/3)*a*(-(a^2 - b)/a^2)^(2/3))/((a^2 - b
)*x)) + 6*a^2*b*(-(a^2 - b)/a^2)^(1/3)*log((a*x*(-(a^2 - b)/a^2)^(1/3) + (a
^3*x^3 + b^2*x^2)^(1/3))/x) - 3*a^2*b*(-(a^2 - b)/a^2)^(1/3)*log((a^2*x^2*(
-(a^2 - b)/a^2)^(2/3) - (a^3*x^3 + b^2*x^2)^(1/3)*a*x*(-(a^2 - b)/a^2)^(1/3
) + (a^3*x^3 + b^2*x^2)^(2/3))/x^2) - 2*sqrt(3)*(3*a^2*b - b^2)*arctan(1/3*
(sqrt(3)*a*x + 2*sqrt(3)*(a^3*x^3 + b^2*x^2)^(1/3))/(a*x)) + 6*(a^3*x^3 + b
^2*x^2)^(1/3)*a^2 + 2*(3*a^2*b - b^2)*log(-(a*x - (a^3*x^3 + b^2*x^2)^(1/3)
)/x) - (3*a^2*b - b^2)*log((a^2*x^2 + (a^3*x^3 + b^2*x^2)^(1/3)*a*x + (a^3*
x^3 + b^2*x^2)^(2/3))/x^2))/a^3
```

giac [A]   time = 126.67, size = 339, normalized size = 0.64

$$-\frac{\left(a^3-ab\right)^{\frac{1}{3}}\left(a^2b-b^2\right)\log\left(\left|-\left(a^3-ab\right)^{\frac{1}{3}}+\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}\right|\right)}{a^4-a^2b}+\frac{\sqrt{3}\left(a^3-ab\right)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(\left(a^3-ab\right)^{\frac{1}{3}}+2\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}\right)}{3\left(a^3-ab\right)^{\frac{1}{3}}}\right)}{a^2}+\frac{\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}x}{a}+\frac{\left(a^3-ab\right)^{\frac{1}{3}}b\log\left(\left(a^3-ab\right)^{\frac{2}{3}}+\left(a^3-ab\right)^{\frac{1}{3}}\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}+\left(a^3+\frac{b^2}{x}\right)^{\frac{2}{3}}\right)}{2a^2}-\frac{\sqrt{3}\left(3a^2b-b^2\right)\arctan\left(\frac{\sqrt{3}\left(a+2\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}\right)}{3a}\right)}{3a^3}-\frac{\left(3a^2b-b^2\right)\log\left(a^2+\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}a+\left(a^3+\frac{b^2}{x}\right)^{\frac{2}{3}}\right)}{6a^3}+\frac{\left(3a^2b-b^2\right)\log\left(\left|-a+\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}\right|\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3+b^2*x^2)^(1/3)/(a*x+b),x, algorithm="giac")
```

```
[Out] -(a^3 - a*b)^(1/3)*(a^2*b - b^2)*log(abs(-(a^3 - a*b)^(1/3) + (a^3 + b^2/x)
^(1/3)))/(a^4 - a^2*b) + sqrt(3)*(a^3 - a*b)^(1/3)*b*arctan(1/3*sqrt(3)*((a
^3 - a*b)^(1/3) + 2*(a^3 + b^2/x)^(1/3))/(a^3 - a*b)^(1/3))/a^2 + (a^3 + b^
2/x)^(1/3)*x/a + 1/2*(a^3 - a*b)^(1/3)*b*log((a^3 - a*b)^(2/3) + (a^3 - a*b
)^(1/3)*(a^3 + b^2/x)^(1/3) + (a^3 + b^2/x)^(2/3))/a^2 - 1/3*sqrt(3)*(3*a^2
*b - b^2)*arctan(1/3*sqrt(3)*(a + 2*(a^3 + b^2/x)^(1/3))/a)/a^3 - 1/6*(3*a^
2*b - b^2)*log(a^2 + (a^3 + b^2/x)^(1/3)*a + (a^3 + b^2/x)^(2/3))/a^3 + 1/3
*(3*a^2*b - b^2)*log(abs(-a + (a^3 + b^2/x)^(1/3)))/a^3
```

maple [F]   time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}}{ax+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^3*x^3+b^2*x^2)^(1/3)/(a*x+b),x)
```

```
[Out] int((a^3*x^3+b^2*x^2)^(1/3)/(a*x+b),x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^3x^3+b^2x^2\right)^{\frac{1}{3}}}{ax+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^3*x^3+b^2*x^2)^(1/3)/(a*x+b),x, algorithm="maxima")
```

```
[Out] integrate((a^3*x^3 + b^2*x^2)^(1/3)/(a*x + b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^3\,x^3+b^2\,x^2\right)^{1/3}}{b+a\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^3*x^3 + b^2*x^2)^(1/3)/(b + a*x),x)
```

```
[Out] int((a^3*x^3 + b^2*x^2)^(1/3)/(b + a*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x^{2}\left(a^{3}x+b^{2}\right)}}{ax+b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**3*x**3+b**2*x**2)**(1/3)/(a*x+b),x)
```

```
[Out] Integral((x**2*(a**3*x + b**2))**(1/3)/(a*x + b), x)
```

3.2401 $\int \frac{\left(-b+a^2x^2\right)^{3/2} \sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{x}\,dx$

Optimal. Leaf size=526

$$\sqrt{2+\sqrt{2}}\, b^{13/8} \tan^{-1}\left(\frac{\left(\sqrt{\frac{2}{2-\sqrt{2}}}\sqrt[8]{b}-\frac{2\sqrt[8]{b}}{\sqrt{2-\sqrt{2}}}\right)\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)+\sqrt{2-\sqrt{2}}\, b^{13/8} \tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{b}\sqrt[4]{\sqrt{a^2x^2}}}{\sqrt{\sqrt{a^2x^2-b}+ax}}\right.$$

Rubi [A] time = 0.66, antiderivative size = 468, normalized size of antiderivative = 0.89, number of steps used = 17, number of rules used = 13, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2120, 466, 461, 301, 211, 1165, 628, 1162, 617, 204, 212, 206, 203}

$$-\frac{b^3}{22\left(\sqrt{a^2x^2-b}+ax\right)^{11/4}}+\frac{5b^2}{6\left(\sqrt{a^2x^2-b}+ax\right)^{3/4}}+\frac{1}{26}\left(\sqrt{a^2x^2-b}+ax\right)^{13/4}-\frac{1}{2}b\left(\sqrt{a^2x^2-b}+ax\right)^{5/4}-\frac{(-b)^{13/8}\log\left(\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt{2}\sqrt[8]{-b}\sqrt[4]{\sqrt{a^2x^2-b}+ax}+\sqrt[4]{-b}\right)}{\sqrt{2}}+\frac{(-b)^{13/8}\log\left(\sqrt{\sqrt{a^2x^2-b}+ax}+\sqrt{2}\sqrt[8]{-b}\sqrt[4]{\sqrt{a^2x^2-b}+ax}+\sqrt[4]{-b}\right)}{\sqrt{2}}-2(-b)^{13/8}\tan^{-1}\left(\frac{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{-b}}\right)-\sqrt{2}(-b)^{13/8}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{-b}}\right)+\sqrt{2}(-b)^{13/8}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{-b}}+1\right)-2(-b)^{13/8}\tanh^{-1}\left(\frac{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a^2*x^2)^(3/2)*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/x,x]
```

```
[Out] -1/22*b^3/(a*x + Sqrt[-b + a^2*x^2])^(11/4) + (5*b^2)/(6*(a*x + Sqrt[-b + a
^2*x^2])^(3/4)) - (b*(a*x + Sqrt[-b + a^2*x^2])^(5/4))/2 + (a*x + Sqrt[-b +
 a^2*x^2])^(13/4)/26 - 2*(-b)^(13/8)*ArcTan[(a*x + Sqrt[-b + a^2*x^2])^(1/4
)/(-b)^(1/8)] - Sqrt[2]*(-b)^(13/8)*ArcTan[1 - (Sqrt[2]*(a*x + Sqrt[-b + a^
2*x^2])^(1/4))/(-b)^(1/8)] + Sqrt[2]*(-b)^(13/8)*ArcTan[1 + (Sqrt[2]*(a*x +
 Sqrt[-b + a^2*x^2])^(1/4))/(-b)^(1/8)] - 2*(-b)^(13/8)*ArcTanh[(a*x + Sqrt
[-b + a^2*x^2])^(1/4)/(-b)^(1/8)] - ((-b)^(13/8)*Log[(-b)^(1/4) - Sqrt[2]*(
-b)^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]
])/Sqrt[2] + ((-b)^(13/8)*Log[(-b)^(1/4) + Sqrt[2]*(-b)^(1/8)*(a*x + Sqrt[-
b + a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]])/Sqrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
```

```
1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(-b+a^2x^2\right)^{3/2}\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{x}\,dx = \frac{1}{8}\,\text{Subst}\left(\int \frac{\left(-b+x^2\right)^4}{x^{15/4}\left(b+x^2\right)}\,dx, x, ax+\sqrt{-b+a^2x^2}\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(-b+x^8\right)^4}{x^{12}\left(b+x^8\right)}\,dx, x, \sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b^3}{x^{12}}-\frac{5b^2}{x^4}-5bx^4+x^{12}+\frac{16b^2x^4}{b+x^8}\right)dx, x, \sqrt[4]{ax+\sqrt{-b+}}\right.$$

$$= -\frac{b^3}{22\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}+\frac{5b^2}{6\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}-\frac{1}{2}b\left(ax+\sqrt{}\right.$$

$$= -\frac{b^3}{22\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}+\frac{5b^2}{6\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}-\frac{1}{2}b\left(ax+\sqrt{}\right.$$

$$= -\frac{b^3}{22\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}+\frac{5b^2}{6\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}-\frac{1}{2}b\left(ax+\sqrt{}\right.$$

$$= -\frac{b^3}{22\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}+\frac{5b^2}{6\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}-\frac{1}{2}b\left(ax+\sqrt{}\right.$$

$$= -\frac{b^3}{22\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}+\frac{5b^2}{6\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}-\frac{1}{2}b\left(ax+\sqrt{}\right.$$

$$= -\frac{b^3}{22\left(ax+\sqrt{-b+a^2x^2}\right)^{11/4}}+\frac{5b^2}{6\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}-\frac{1}{2}b\left(ax+\sqrt{}\right.$$

Mathematica [B] time = 23.49, size = 14841, normalized size = 28.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-b + a^2*x^2)^(3/2)*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/x,x]
[Out] Result too large to show
```

IntegrateAlgebraic [A] time = 1.78, size = 501, normalized size = 0.95

$$-\sqrt{2+\sqrt{2}}\,b^{13/8}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{b}\,\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)+\sqrt{2-\sqrt{2}}\,b^{13/8}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,\sqrt[8]{b}\,\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)-\sqrt{2-\sqrt{2}}\,b^{13/8}\tanh^{-1}\left(\frac{\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\,\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{b}}+\sqrt{1-\frac{1}{\sqrt{2}}}\,\sqrt[8]{b}}{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}\right)+\sqrt{2+\sqrt{2}}\,b^{13/8}\tanh^{-1}\left(\frac{\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\,\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{b}}+\sqrt{1+\frac{1}{\sqrt{2}}}\,\sqrt[8]{b}}{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}\right)+\frac{4\left(132a^6x^6-627a^4bx^4+682a^2b^2x^2-152b^3\right)+4\sqrt{a^2x^2-b}\left(132a^5x^5-561a^3bx^3+418ab^2x\right)}{429\left(\sqrt{a^2x^2-b}+ax\right)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a^2*x^2)^(3/2)*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/
x,x]
```

```
[Out] (4*Sqrt[-b + a^2*x^2]*(418*a*b^2*x - 561*a^3*b*x^3 + 132*a^5*x^5) + 4*(-152
*b^3 + 682*a^2*b^2*x^2 - 627*a^4*b*x^4 + 132*a^6*x^6))/(429*(a*x + Sqrt[-b
+ a^2*x^2])^(11/4)) - Sqrt[2 + Sqrt[2]]*b^(13/8)*ArcTan[(Sqrt[2 - Sqrt[2]]*
b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-b^(1/4) + Sqrt[a*x + Sqrt[-b +
a^2*x^2]])] + Sqrt[2 - Sqrt[2]]*b^(13/8)*ArcTan[(Sqrt[2 + Sqrt[2]]*b^(1/8)*
(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-b^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]
])] - Sqrt[2 - Sqrt[2]]*b^(13/8)*ArcTanh[(Sqrt[1 - 1/Sqrt[2]]*b^(1/8) + (Sq
rt[1 - 1/Sqrt[2]]*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/b^(1/8))/(a*x + Sqrt[-b +
 a^2*x^2])^(1/4)] + Sqrt[2 + Sqrt[2]]*b^(13/8)*ArcTanh[(Sqrt[1 + 1/Sqrt[2]]
*b^(1/8) + (Sqrt[1 + 1/Sqrt[2]]*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/b^(1/8))/(a
*x + Sqrt[-b + a^2*x^2])^(1/4)]
```

fricas [A] time = 0.44, size = 683, normalized size = 1.30

$$2\sqrt{2}\left(-b^{13}\right)^{\frac{1}{8}}\arctan\left(-\frac{\left(b^{13}+\sqrt{2}\left(-b^{13}\right)^{\frac{3}{8}}\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}b^{8}-\sqrt{2}\sqrt{\sqrt{ax+\sqrt{a^2x^2-b}}\,b^{16}-\left(-b^{13}\right)^{\frac{1}{4}}b^{13}-\sqrt{2}\left(-b^{13}\right)^{\frac{5}{8}}\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}b^{8}}\right)\left(-b^{13}\right)^{\frac{3}{8}}}{b^{13}}\right)+2\sqrt{2}\left(-b^{13}\right)^{\frac{1}{8}}\arctan\left(\frac{\left(b^{13}-\sqrt{2}\left(-b^{13}\right)^{\frac{3}{8}}\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}b^{8}+\sqrt{2}\sqrt{\sqrt{ax+\sqrt{a^2x^2-b}}\,b^{16}-\left(-b^{13}\right)^{\frac{1}{4}}b^{13}+\sqrt{2}\left(-b^{13}\right)^{\frac{5}{8}}\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}b^{8}}\right)\left(-b^{13}\right)^{\frac{3}{8}}}{b^{13}}\right)+\frac{1}{2}\sqrt{2}\left(-b^{13}\right)^{\frac{1}{8}}\log\left(4\sqrt{ax+\sqrt{a^2x^2-b}}\,b^{16}-4\left(-b^{13}\right)^{\frac{1}{4}}b^{13}+4\sqrt{2}\left(-b^{13}\right)^{\frac{5}{8}}\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}b^{8}\right)-\frac{1}{2}\sqrt{2}\left(-b^{13}\right)^{\frac{1}{8}}\log\left(4\sqrt{ax+\sqrt{a^2x^2-b}}\,b^{16}-4\left(-b^{13}\right)^{\frac{1}{4}}b^{13}-4\sqrt{2}\left(-b^{13}\right)^{\frac{5}{8}}\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}b^{8}\right)-\frac{4}{429}\left(3a^3x^3-38abx-4\left(9a^2x^2-38b\right)\sqrt{a^2x^2-b}\right)\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}-4\left(-b^{13}\right)^{\frac{1}{8}}\arctan\left(-\frac{\left(\left(-b^{13}\right)^{\frac{3}{8}}\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}b^{8}-\sqrt{\sqrt{ax+\sqrt{a^2x^2-b}}\,b^{16}-\left(-b^{13}\right)^{\frac{1}{4}}b^{13}}\right)\left(-b^{13}\right)^{\frac{3}{8}}}{b^{13}}\right)-\left(-b^{13}\right)^{\frac{1}{8}}\log\left(\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}b^{8}+\left(-b^{13}\right)^{\frac{5}{8}}\right)+\left(-b^{13}\right)^{\frac{1}{8}}\log\left(\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}b^{8}-\left(-b^{13}\right)^{\frac{5}{8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(3/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/4)/x,x, algorithm="f
ricas")
```

```
[Out] 2*sqrt(2)*(-b^13)^(1/8)*arctan(-(b^13 + sqrt(2)*(-b^13)^(3/8)*(a*x + sqrt(a
^2*x^2 - b))^(1/4)*b^8 - sqrt(2)*sqrt(sqrt(a*x + sqrt(a^2*x^2 - b))*b^16 -
(-b^13)^(1/4)*b^13 - sqrt(2)*(-b^13)^(5/8)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*
b^8)*(-b^13)^(3/8))/b^13) + 2*sqrt(2)*(-b^13)^(1/8)*arctan((b^13 - sqrt(2)*
(-b^13)^(3/8)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*b^8 + sqrt(2)*sqrt(sqrt(a*x +
 sqrt(a^2*x^2 - b))*b^16 - (-b^13)^(1/4)*b^13 + sqrt(2)*(-b^13)^(5/8)*(a*x
+ sqrt(a^2*x^2 - b))^(1/4)*b^8)*(-b^13)^(3/8))/b^13) + 1/2*sqrt(2)*(-b^13)^
(1/8)*log(4*sqrt(a*x + sqrt(a^2*x^2 - b))*b^16 - 4*(-b^13)^(1/4)*b^13 + 4*s
qrt(2)*(-b^13)^(5/8)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*b^8) - 1/2*sqrt(2)*(-b
^13)^(1/8)*log(4*sqrt(a*x + sqrt(a^2*x^2 - b))*b^16 - 4*(-b^13)^(1/4)*b^13
- 4*sqrt(2)*(-b^13)^(5/8)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*b^8) - 4/429*(3*a
^3*x^3 - 38*a*b*x - 4*(9*a^2*x^2 - 38*b)*sqrt(a^2*x^2 - b))*(a*x + sqrt(a^2
*x^2 - b))^(1/4) - 4*(-b^13)^(1/8)*arctan(-((-b^13)^(3/8)*(a*x + sqrt(a^2*x
^2 - b))^(1/4)*b^8 - sqrt(sqrt(a*x + sqrt(a^2*x^2 - b))*b^16 - (-b^13)^(1/4
)*b^13)*(-b^13)^(3/8))/b^13) - (-b^13)^(1/8)*log((a*x + sqrt(a^2*x^2 - b))^
(1/4)*b^8 + (-b^13)^(5/8)) + (-b^13)^(1/8)*log((a*x + sqrt(a^2*x^2 - b))^(1
/4)*b^8 - (-b^13)^(5/8))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(3/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/4)/x,x, algorithm="g
iac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{2} x^{2}-b\right)^{\frac{3}{2}} \left(a x +\sqrt{a^{2} x^{2}-b}\right)^{\frac{1}{4}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b)^(3/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/4)/x,x)
```

```
[Out] int((a^2*x^2-b)^(3/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/4)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2x^2-b\right)^{\frac{3}{2}}\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(3/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/4)/x,x, algorithm="m
axima")
```

```
[Out] integrate((a^2*x^2 - b)^(3/2)*(a*x + sqrt(a^2*x^2 - b))^(1/4)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/4}\,\left(a^2\,x^2-b\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (a^2*x^2 - b)^(1/2))^(1/4)*(a^2*x^2 - b)^(3/2))/x,x)
```

```
[Out] int(((a*x + (a^2*x^2 - b)^(1/2))^(1/4)*(a^2*x^2 - b)^(3/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{ax+\sqrt{a^2x^2-b}}\,\left(a^2x^2-b\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b)**(3/2)*(a*x+(a**2*x**2-b)**(1/2))**(1/4)/x,x)
```

```
[Out] Integral((a*x + sqrt(a**2*x**2 - b))**(1/4)*(a**2*x**2 - b)**(3/2)/x, x)
```

3.2402 $\int \frac{(b^2+ax^2)\sqrt{b+\sqrt{b^2+ax^2}}}{-b^2+ax^2}\,dx$

Optimal. Leaf size=530

$$\frac{4bx}{3\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{2x\sqrt{ax^2+b^2}}{3\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{2\sqrt{\sqrt{2}-1}\,b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2(1+\sqrt{2})}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{\sqrt{2(1+\sqrt{2})}\sqrt{b}}\right)}{\sqrt{a}}+\frac{2i\left(\cdots\right.}{}$$

Rubi [F] time = 0.92, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(b^2+ax^2)\sqrt{b+\sqrt{b^2+ax^2}}}{-b^2+ax^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]])/(-b^2 + a*x^2),x]
```

```
[Out] (2*a*x^3)/(3*(b + Sqrt[b^2 + a*x^2])^(3/2)) + (2*b*x)/Sqrt[b + Sqrt[b^2 + a
*x^2]] - b*Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b - Sqrt[a]*x), x] - b*D
efer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b + Sqrt[a]*x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b^2+ax^2)\sqrt{b+\sqrt{b^2+ax^2}}}{-b^2+ax^2}\,dx &= \int \left(\sqrt{b+\sqrt{b^2+ax^2}}+\frac{2b^2\sqrt{b+\sqrt{b^2+ax^2}}}{-b^2+ax^2}\right)dx\\
&= (2b^2)\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{-b^2+ax^2}\,dx+\int \sqrt{b+\sqrt{b^2+ax^2}}\,dx\\
&= \frac{2ax^3}{3\left(b+\sqrt{b^2+ax^2}\right)^{3/2}}+\frac{2bx}{\sqrt{b+\sqrt{b^2+ax^2}}}+(2b^2)\int \left(-\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{2b\left(b-\sqrt{a}\,x\right)}\cdots\right.\\
&= \frac{2ax^3}{3\left(b+\sqrt{b^2+ax^2}\right)^{3/2}}+\frac{2bx}{\sqrt{b+\sqrt{b^2+ax^2}}}-b\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b-\sqrt{a}\,x}\,dx-\cdots
\end{aligned}$$

Mathematica [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(b^2+ax^2)\sqrt{b+\sqrt{b^2+ax^2}}}{-b^2+ax^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]])/(-b^2 + a*x^2),x]
```

```
[Out] Integrate[((b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]])/(-b^2 + a*x^2), x]
```

IntegrateAlgebraic [A] time = 0.53, size = 195, normalized size = 0.37

$$\frac{4bx}{3\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{2x\sqrt{ax^2+b^2}}{3\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{2\sqrt{\sqrt{2}-1}\,b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{2}-1}\sqrt{a}x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}-\frac{2\sqrt{1+\sqrt{2}}\,b^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{2}}\sqrt{a}x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]])/(-b^2 + a*x^
2),x]
```

```
[Out] (4*b*x)/(3*Sqrt[b + Sqrt[b^2 + a*x^2]]) + (2*x*Sqrt[b^2 + a*x^2])/(3*Sqrt[b
 + Sqrt[b^2 + a*x^2]]) + (2*Sqrt[-1 + Sqrt[2]]*b^(3/2)*ArcTan[(Sqrt[-1 + Sq
rt[2]]*Sqrt[a]*x)/(Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/Sqrt[a] - (2*Sqrt
[1 + Sqrt[2]]*b^(3/2)*ArcTanh[(Sqrt[1 + Sqrt[2]]*Sqrt[a]*x)/(Sqrt[b]*Sqrt[b
 + Sqrt[b^2 + a*x^2]])])/Sqrt[a]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2-b^2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^2+b^2)\sqrt{b+\sqrt{ax^2+b^2}}}{ax^2-b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2-b^2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x^2 + b^2)*sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 - b^2), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{(a\,x^2+b^2)\sqrt{b+\sqrt{a\,x^2+b^2}}}{a\,x^2-b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b^2)*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2-b^2),x)
```

```
[Out] int((a*x^2+b^2)*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2-b^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^2+b^2)\sqrt{b+\sqrt{ax^2+b^2}}}{ax^2-b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2-b^2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x^2 + b^2)*sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 - b^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b^2+a\,x^2\right)\,\sqrt{b+\sqrt{b^2+a\,x^2}}}{a\,x^2-b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^2 + b^2)*(b + (a*x^2 + b^2)^(1/2))^(1/2))/(a*x^2 - b^2),x)
[Out] int(((a*x^2 + b^2)*(b + (a*x^2 + b^2)^(1/2))^(1/2))/(a*x^2 - b^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}\,\left(ax^2+b^2\right)}{ax^2-b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b**2)*(b+(a*x**2+b**2)**(1/2))**(1/2)/(a*x**2-b**2),x)
[Out] Integral(sqrt(b + sqrt(a*x**2 + b**2))*(a*x**2 + b**2)/(a*x**2 - b**2), x)
```

3.2403 $\int \frac{d+cx^4}{x\sqrt{-b+a^2x^2}\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx$

Optimal. Leaf size=535

$$\frac{\sqrt{2+\sqrt{2}}\,d\tan^{-1}\left(\frac{\left(\sqrt{\frac{2}{2-\sqrt{2}}}\sqrt[8]{b}-\frac{2\sqrt[8]{b}}{\sqrt{2-\sqrt{2}}}\right)\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)}{b^{5/8}}+\frac{\sqrt{2-\sqrt{2}}\,d\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{b}\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)}{b^{5/8}}-\frac{\sqrt{2+\sqrt{2}}\,d\tan$$

Rubi [A] time = 2.11, antiderivative size = 490, normalized size of antiderivative = 0.92, number of steps used = 20, number of rules used = 14, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6742, 2120, 329, 300, 297, 1162, 617, 204, 1165, 628, 298, 203, 206, 270}

$$-\frac{d\log\left(\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt{2}\sqrt[8]{-b}\sqrt[4]{\sqrt{a^2x^2-b}+ax}+\sqrt[4]{-b}\right)}{\sqrt{2}(-b)^{5/8}}+\frac{d\log\left(\sqrt{\sqrt{a^2x^2-b}+ax}+\sqrt{2}\sqrt[8]{-b}\sqrt[4]{\sqrt{a^2x^2-b}+ax}+\sqrt[4]{-b}\right)}{\sqrt{2}(-b)^{5/8}}+\frac{2d\tan^{-1}\left(\frac{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{-b}}\right)}{(-b)^{5/8}}+\frac{\sqrt{2}d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{-b}}\right)}{(-b)^{5/8}}-\frac{\sqrt{2}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{-b}}+1\right)}{(-b)^{5/8}}-\frac{2d\tanh^{-1}\left(\frac{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{-b}}\right)}{(-b)^{5/8}}-\frac{b^3c}{26a^4\left(\sqrt{a^2x^2-b}+ax\right)^{13/4}}-\frac{3b^2c}{10a^4\left(\sqrt{a^2x^2-b}+ax\right)^{5/4}}+\frac{bc\left(\sqrt{a^2x^2-b}+ax\right)^{3/4}}{2a^4}+\frac{c\left(\sqrt{a^2x^2-b}+ax\right)^{11/4}}{22a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*x^4)/(x*Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4)),x]
```

```
[Out] -1/26*(b^3*c)/(a^4*(a*x + Sqrt[-b + a^2*x^2])^(13/4)) - (3*b^2*c)/(10*a^4*(
a*x + Sqrt[-b + a^2*x^2])^(5/4)) + (b*c*(a*x + Sqrt[-b + a^2*x^2])^(3/4))/(
2*a^4) + (c*(a*x + Sqrt[-b + a^2*x^2])^(11/4))/(22*a^4) + (2*d*ArcTan[(a*x
+ Sqrt[-b + a^2*x^2])^(1/4)/(-b)^(1/8)])/(-b)^(5/8) + (Sqrt[2]*d*ArcTan[1 -
 (Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-b)^(1/8)])/(-b)^(5/8) - (Sqrt
[2]*d*ArcTan[1 + (Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-b)^(1/8)])/(-
b)^(5/8) - (2*d*ArcTanh[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/(-b)^(1/8)])/(-b)^
(5/8) - (d*Log[(-b)^(1/4) - Sqrt[2]*(-b)^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(
1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]])/(Sqrt[2]*(-b)^(5/8)) + (d*Log[(-b)^
(1/4) + Sqrt[2]*(-b)^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a*x + Sq
rt[-b + a^2*x^2]]])/(Sqrt[2]*(-b)^(5/8))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 300

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[x^m/(r + s
*x^(n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ
[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
```

```
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{d+cx^4}{x\sqrt{-b+a^2x^2}\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx = \int \left(\frac{d}{x\sqrt{-b+a^2x^2}\sqrt[4]{ax+\sqrt{-b+a^2x^2}}} + \frac{cx^3}{\sqrt{-b+a^2x^2}\sqrt[4]{ax+\sqrt{-b+a^2}}}\right.$$

$$= c\int \frac{x^3}{\sqrt{-b+a^2x^2}\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx + d\int \frac{1}{x\sqrt{-b+a^2x^2}\sqrt[4]{ax+\sqrt{}}}$$

$$= \frac{c\,\mathrm{Subst}\left(\int \frac{(b+x^2)^3}{x^{17/4}}\,dx, x, ax+\sqrt{-b+a^2x^2}\right)}{8a^4} + (2d)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{x}\,(b+}\right.$$

$$= \frac{c\,\mathrm{Subst}\left(\int \left(\frac{b^3}{x^{17/4}} + \frac{3b^2}{x^{9/4}} + \frac{3b}{\sqrt[4]{x}} + x^{7/4}\right) dx, x, ax+\sqrt{-b+a^2x^2}\right)}{8a^4} + (8d)\,\mathrm{S}$$

$$= -\frac{b^3c}{26a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{13/4}} - \frac{3b^2c}{10a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}} + \frac{bc\left(ax\right.}{}$$

$$= -\frac{b^3c}{26a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{13/4}} - \frac{3b^2c}{10a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}} + \frac{bc\left(ax\right.}{}$$

$$= -\frac{b^3c}{26a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{13/4}} - \frac{3b^2c}{10a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}} + \frac{bc\left(ax\right.}{}$$

$$= -\frac{b^3c}{26a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{13/4}} - \frac{3b^2c}{10a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}} + \frac{bc\left(ax\right.}{}$$

$$= -\frac{b^3c}{26a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{13/4}} - \frac{3b^2c}{10a^4\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}} + \frac{bc\left(ax\right.}{}$$

Mathematica [C] time = 5.18, size = 742, normalized size = 1.39

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + c*x^4)/(x*Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4
)),x]
```

```
[Out] (4*((3*c*Sqrt[-b + a^2*x^2]*(b - 2*a*x*(a*x + Sqrt[-b + a^2*x^2]))^4*(2048*
b^4 + 5720*a^7*x^7*(a*x + Sqrt[-b + a^2*x^2]) + 260*a^5*b*x^5*(-6*a*x + 5*S
qrt[-b + a^2*x^2]) - 832*a*b^3*x*(13*a*x + 8*Sqrt[-b + a^2*x^2]) + 455*a^3*
b^2*x^3*(13*a*x + 16*Sqrt[-b + a^2*x^2])))/(b^6 + 1024*a^11*x^11*(a*x + Sqr
t[-b + a^2*x^2]) - 256*a^9*b*x^9*(13*a*x + 11*Sqrt[-b + a^2*x^2]) + 256*a^7
*b^2*x^7*(16*a*x + 11*Sqrt[-b + a^2*x^2]) - 112*a^5*b^3*x^5*(21*a*x + 11*Sq
rt[-b + a^2*x^2]) + 20*a^3*b^4*x^3*(31*a*x + 11*Sqrt[-b + a^2*x^2]) - a*b^5
*x*(61*a*x + 11*Sqrt[-b + a^2*x^2])) - (3*c*Sqrt[-b + a^2*x^2]*(-384*b^4 +
5720*a^7*x^7*(a*x + Sqrt[-b + a^2*x^2]) + 156*a*b^3*x*(13*a*x + 8*Sqrt[-b +
 a^2*x^2]) - 260*a^5*b*x^5*(25*a*x + 14*Sqrt[-b + a^2*x^2]) - 65*a^3*b^2*x^
3*(4*a*x + 21*Sqrt[-b + a^2*x^2])))/((a*x + Sqrt[-b + a^2*x^2])^2*(-b + a*x
*(a*x + Sqrt[-b + a^2*x^2]))) + (13585*a^4*d*Sqrt[-b + a^2*x^2]*(a*x + Sqrt
[-b + a^2*x^2])^2*(-1 + 2*Hypergeometric2F1[3/8, 1, 11/8, -((a*x + Sqrt[-b
+ a^2*x^2])^2/b)]))/(-b + a*x*(a*x + Sqrt[-b + a^2*x^2])) + (13585*a^5*d*(a
*x + Sqrt[-b + a^2*x^2])*Sqrt[Sign[a]^2])/(Sqrt[a^2]*Sign[a])))/(40755*a^4*
b*(a*x + Sqrt[-b + a^2*x^2])^(1/4))
```

IntegrateAlgebraic [A] time = 2.00, size = 510, normalized size = 0.95

$$-\frac{\sqrt{2+\sqrt{2}}\, d \tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{b}\,\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)}{b^{5/8}}+\frac{\sqrt{2-\sqrt{2}}\, d \tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,\sqrt[8]{b}\,\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)}{b^{5/8}}+\frac{\sqrt{2-\sqrt{2}}\, d \tanh^{-1}\left(\frac{\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{b}}+\sqrt{1-\frac{1}{\sqrt{2}}}\,\sqrt[8]{b}}{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}\right)}{b^{5/8}}-\frac{\sqrt{2+\sqrt{2}}\, d \tanh^{-1}\left(\frac{\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{b}}+\sqrt{1+\frac{1}{\sqrt{2}}}\,\sqrt[8]{b}}{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}\right)}{b^{5/8}}+\frac{4c\left(260a^6x^6+325a^4bx^4-676a^2b^2x^2+128b^3\right)+4c\sqrt{a^2x^2-b}\left(260a^5x^5+455a^3bx^3-416ab^2x\right)}{715a^4\left(\sqrt{a^2x^2-b}+ax\right)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + c*x^4)/(x*Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x
^2])^(1/4)),x]
```

```
[Out] (4*c*Sqrt[-b + a^2*x^2]*(-416*a*b^2*x + 455*a^3*b*x^3 + 260*a^5*x^5) + 4*c*
(128*b^3 - 676*a^2*b^2*x^2 + 325*a^4*b*x^4 + 260*a^6*x^6))/(715*a^4*(a*x +
Sqrt[-b + a^2*x^2])^(13/4)) - (Sqrt[2 + Sqrt[2]]*d*ArcTan[(Sqrt[2 - Sqrt[2]
]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-b^(1/4) + Sqrt[a*x + Sqrt[-b
+ a^2*x^2]])])/b^(5/8) + (Sqrt[2 - Sqrt[2]]*d*ArcTan[(Sqrt[2 + Sqrt[2]]*b^(
1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-b^(1/4) + Sqrt[a*x + Sqrt[-b + a^2
*x^2]])])/b^(5/8) + (Sqrt[2 - Sqrt[2]]*d*ArcTanh[(Sqrt[1 - 1/Sqrt[2]]*b^(1/
8) + (Sqrt[1 - 1/Sqrt[2]]*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/b^(1/8))/(a*x + S
qrt[-b + a^2*x^2])^(1/4)])/b^(5/8) - (Sqrt[2 + Sqrt[2]]*d*ArcTanh[(Sqrt[1 +
 1/Sqrt[2]]*b^(1/8) + (Sqrt[1 + 1/Sqrt[2]]*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/
b^(1/8))/(a*x + Sqrt[-b + a^2*x^2])^(1/4)])/b^(5/8)
```

fricas [A] time = 0.46, size = 842, normalized size = 1.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+d)/x/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4),x, al
gorithm="fricas")
```

```
[Out] -1/1430*(2860*sqrt(2)*(-d^8/b^5)^(1/8)*a^4*b*arctan(-(d^8 + sqrt(2)*(-d^8/b
^5)^(5/8)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*b^3*d^3 - sqrt(2)*sqrt(sqrt(a*x +
 sqrt(a^2*x^2 - b))*d^6 - sqrt(2)*(-d^8/b^5)^(3/8)*(a*x + sqrt(a^2*x^2 - b)
)^(1/4)*b^2*d^3 + (-d^8/b^5)^(3/4)*b^4)*(-d^8/b^5)^(5/8)*b^3)/d^8) + 2860*s
qrt(2)*(-d^8/b^5)^(1/8)*a^4*b*arctan((d^8 - sqrt(2)*(-d^8/b^5)^(5/8)*(a*x +
 sqrt(a^2*x^2 - b))^(1/4)*b^3*d^3 + sqrt(2)*sqrt(sqrt(a*x + sqrt(a^2*x^2 -
b))*d^6 + sqrt(2)*(-d^8/b^5)^(3/8)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*b^2*d^3
+ (-d^8/b^5)^(3/4)*b^4)*(-d^8/b^5)^(5/8)*b^3)/d^8) - 715*sqrt(2)*(-d^8/b^5)
^(1/8)*a^4*b*log(4*sqrt(a*x + sqrt(a^2*x^2 - b))*d^6 + 4*sqrt(2)*(-d^8/b^5)
^(3/8)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*b^2*d^3 + 4*(-d^8/b^5)^(3/4)*b^4) +
715*sqrt(2)*(-d^8/b^5)^(1/8)*a^4*b*log(4*sqrt(a*x + sqrt(a^2*x^2 - b))*d^6
```

```
- 4*sqrt(2)*(-d^8/b^5)^(3/8)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*b^2*d^3 + 4*(-
d^8/b^5)^(3/4)*b^4) - 5720*(-d^8/b^5)^(1/8)*a^4*b*arctan(-((-d^8/b^5)^(5/8)
*(a*x + sqrt(a^2*x^2 - b))^(1/4)*b^3*d^3 - sqrt(sqrt(a*x + sqrt(a^2*x^2 - b
))*d^6 + (-d^8/b^5)^(3/4)*b^4)*(-d^8/b^5)^(5/8)*b^3)/d^8) + 1430*(-d^8/b^5)
^(1/8)*a^4*b*log((a*x + sqrt(a^2*x^2 - b))^(1/4)*d^3 + (-d^8/b^5)^(3/8)*b^2
) - 1430*(-d^8/b^5)^(1/8)*a^4*b*log((a*x + sqrt(a^2*x^2 - b))^(1/4)*d^3 - (
-d^8/b^5)^(3/8)*b^2) + 8*(55*a^4*c*x^4 + 36*a^2*b*c*x^2 - 128*b^2*c - (55*a
^3*c*x^3 + 96*a*b*c*x)*sqrt(a^2*x^2 - b))*(a*x + sqrt(a^2*x^2 - b))^(3/4))/
(a^4*b)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+d)/x/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4),x, al
gorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{c\,x^4+d}{x\sqrt{a^2x^2-b}\,\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+d)/x/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4),x)
```

```
[Out] int((c*x^4+d)/x/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^4+d}{\sqrt{a^2x^2-b}\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+d)/x/(a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/4),x, al
gorithm="maxima")
```

```
[Out] integrate((c*x^4 + d)/(sqrt(a^2*x^2 - b)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*x)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,x^4+d}{x\,\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/4}\,\sqrt{a^2\,x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + c*x^4)/(x*(a*x + (a^2*x^2 - b)^(1/2))^(1/4)*(a^2*x^2 - b)^(1/2)),x
)
```

```
[Out] int((d + c*x^4)/(x*(a*x + (a^2*x^2 - b)^(1/2))^(1/4)*(a^2*x^2 - b)^(1/2)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^4 + d}{x\sqrt[4]{ax + \sqrt{a^2x^2 - b}}\,\sqrt{a^2x^2 - b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+d)/x/(a**2*x**2-b)**(1/2)/(a*x+(a**2*x**2-b)**(1/2))**(1/
4),x)
```

```
[Out] Integral((c*x**4 + d)/(x*(a*x + sqrt(a**2*x**2 - b))**(1/4)*sqrt(a**2*x**2
- b)), x)
```

3.2404 $\int \frac{\left(-b+a^2x^2\right)^{3/2} \sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=540

$$\frac{1}{4}\sqrt{2-\sqrt{2}}\,ab^{9/8}\tan^{-1}\left(\frac{\left(\sqrt{\frac{2}{2-\sqrt{2}}}\sqrt[8]{b}-\frac{2\sqrt[8]{b}}{\sqrt{2-\sqrt{2}}}\right)\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)-\frac{1}{4}\sqrt{2+\sqrt{2}}\,ab^{9/8}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{b}\sqrt[4]{\sqrt{a}}}{\sqrt{\sqrt{a^2x^2-b}+}}\right.$$

Rubi [A] time = 0.70, antiderivative size = 539, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2120, 466, 468, 570, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$-\frac{11ab^2}{28\left(\sqrt{a^2x^2-b}+ax\right)^{7/4}}+\frac{a\left(b-\left(\sqrt{a^2x^2-b}+ax\right)^2\right)^3}{4\left(\sqrt{a^2x^2-b}+ax\right)^{7/4}\left(\left(\sqrt{a^2x^2-b}+ax\right)^2+b\right)}+\frac{13}{36}a\left(\sqrt{a^2x^2-b}+ax\right)^{9/4}-7ab\sqrt[4]{\sqrt{a^2x^2-b}+ax}+\frac{a(-b)^{9/8}\log\left(\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt{2}\sqrt[8]{-b}\sqrt[4]{\sqrt{a^2x^2-b}+ax}+\sqrt[4]{-b}\right)}{4\sqrt{2}}-\frac{a(-b)^{9/8}\log\left(\sqrt{\sqrt{a^2x^2-b}+ax}+\sqrt{2}\sqrt[8]{-b}\sqrt[4]{\sqrt{a^2x^2-b}+ax}+\sqrt[4]{-b}\right)}{4\sqrt{2}}-\frac{1}{2}a(-b)^{9/8}\tan^{-1}\left(\frac{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{-b}}\right)+\frac{a(-b)^{9/8}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{-b}}\right)}{2\sqrt{2}}-\frac{a(-b)^{9/8}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{-b}}+1\right)}{2\sqrt{2}}-\frac{1}{2}a(-b)^{9/8}\tanh^{-1}\left(\frac{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{-b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-b + a^2*x^2)^(3/2)*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/x^2,x]
```

```
[Out] (-11*a*b^2)/(28*(a*x + Sqrt[-b + a^2*x^2])^(7/4)) - 7*a*b*(a*x + Sqrt[-b +
a^2*x^2])^(1/4) + (13*a*(a*x + Sqrt[-b + a^2*x^2])^(9/4))/36 + (a*(b - (a*x
 + Sqrt[-b + a^2*x^2])^2)^3)/(4*(a*x + Sqrt[-b + a^2*x^2])^(7/4)*(b + (a*x
+ Sqrt[-b + a^2*x^2])^2)) - (a*(-b)^(9/8)*ArcTan[(a*x + Sqrt[-b + a^2*x^2])
^(1/4)/(-b)^(1/8)])/2 + (a*(-b)^(9/8)*ArcTan[1 - (Sqrt[2]*(a*x + Sqrt[-b +
a^2*x^2])^(1/4))/(-b)^(1/8)])/(2*Sqrt[2]) - (a*(-b)^(9/8)*ArcTan[1 + (Sqrt[
2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-b)^(1/8)])/(2*Sqrt[2]) - (a*(-b)^(9/
8)*ArcTanh[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/(-b)^(1/8)])/2 + (a*(-b)^(9/8)*
Log[(-b)^(1/4) - Sqrt[2]*(-b)^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt
[a*x + Sqrt[-b + a^2*x^2]]])/(4*Sqrt[2]) - (a*(-b)^(9/8)*Log[(-b)^(1/4) + S
qrt[2]*(-b)^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a
^2*x^2]]])/(4*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(-b+a^2x^2\right)^{3/2}\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{x^2}\,dx = \frac{1}{4}a\,\text{Subst}\left(\int \frac{\left(-b+x^2\right)^4}{x^{11/4}\left(b+x^2\right)^2}\,dx, x, ax+\sqrt{-b+a^2x^2}\right)$$

$$= a\,\text{Subst}\left(\int \frac{\left(-b+x^8\right)^4}{x^8\left(b+x^8\right)^2}\,dx, x, \sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)$$

$$= \frac{a\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3}{4\left(ax+\sqrt{-b+a^2x^2}\right)^{7/4}\left(b+\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} - \frac{a\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{a\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3}{4\left(ax+\sqrt{-b+a^2x^2}\right)^{7/4}\left(b+\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} - \frac{a\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{11ab^2}{28\left(ax+\sqrt{-b+a^2x^2}\right)^{7/4}} - 7ab\sqrt[4]{ax+\sqrt{-b+a^2x^2}} + \frac{13}{36}a\left(ax+\right.$$

$$= -\frac{11ab^2}{28\left(ax+\sqrt{-b+a^2x^2}\right)^{7/4}} - 7ab\sqrt[4]{ax+\sqrt{-b+a^2x^2}} + \frac{13}{36}a\left(ax+\right.$$

$$= -\frac{11ab^2}{28\left(ax+\sqrt{-b+a^2x^2}\right)^{7/4}} - 7ab\sqrt[4]{ax+\sqrt{-b+a^2x^2}} + \frac{13}{36}a\left(ax+\right.$$

$$= -\frac{11ab^2}{28\left(ax+\sqrt{-b+a^2x^2}\right)^{7/4}} - 7ab\sqrt[4]{ax+\sqrt{-b+a^2x^2}} + \frac{13}{36}a\left(ax+\right.$$

$$= -\frac{11ab^2}{28\left(ax+\sqrt{-b+a^2x^2}\right)^{7/4}} - 7ab\sqrt[4]{ax+\sqrt{-b+a^2x^2}} + \frac{13}{36}a\left(ax+\right.$$

$$= -\frac{11ab^2}{28\left(ax+\sqrt{-b+a^2x^2}\right)^{7/4}} - 7ab\sqrt[4]{ax+\sqrt{-b+a^2x^2}} + \frac{13}{36}a\left(ax+\right.$$

Mathematica [C] time = 3.90, size = 226, normalized size = 0.42

$$\frac{\left(a^2x^2-b\right)\sqrt[4]{\sqrt{a^2x^2-b}+ax}\left(56a^5x^5-812a^3bx^3-63b^2\sqrt{a^2x^2-b}+126abx\left(2ax\left(\sqrt{a^2x^2-b}+ax\right)-b\right){}_2F_1\left(\frac{1}{8},1;\frac{9}{8};-\frac{\left(ax+\sqrt{a^2x^2-b}\right)^2}{b}\right)-784a^2bx^2\sqrt{a^2x^2-b}+56a^4x^4\sqrt{a^2x^2-b}+313ab^2x\right)}{63x\left(b-ax\left(\sqrt{a^2x^2-b}+ax\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + a^2*x^2)^(3/2)*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/x^2,x]
```

```
[Out] ((-b + a^2*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*(313*a*b^2*x - 812*a^3*b*x
^3 + 56*a^5*x^5 - 63*b^2*Sqrt[-b + a^2*x^2] - 784*a^2*b*x^2*Sqrt[-b + a^2*x
^2] + 56*a^4*x^4*Sqrt[-b + a^2*x^2] + 126*a*b*x*(-b + 2*a*x*(a*x + Sqrt[-b
+ a^2*x^2]))*Hypergeometric2F1[1/8, 1, 9/8, -((a*x + Sqrt[-b + a^2*x^2])^2/
b)]))/(63*x*(b - a*x*(a*x + Sqrt[-b + a^2*x^2]))^2)
```

IntegrateAlgebraic [A] time = 2.08, size = 514, normalized size = 0.95

$$-\frac{1}{4}\sqrt{2-\sqrt{2}}\,ab^{9/8}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{b}\,\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)-\frac{1}{4}\sqrt{2+\sqrt{2}}\,ab^{9/8}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\,\sqrt[8]{b}\,\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\,ab^{9/8}\tanh^{-1}\left(\frac{\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{b}}+\sqrt{1-\frac{1}{\sqrt{2}}}\,\sqrt[8]{b}}{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}\right)+\frac{1}{4}\sqrt{2-\sqrt{2}}\,ab^{9/8}\tanh^{-1}\left(\frac{\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{b}}+\sqrt{1+\frac{1}{\sqrt{2}}}\,\sqrt[8]{b}}{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}\right)+\frac{112a^6x^6-1652a^4bx^4+1034a^2b^2x^2+\sqrt{a^2x^2-b}\left(112a^5x^5-1596a^3bx^3+250ab^2x\right)+63b^3}{63x\left(\sqrt{a^2x^2-b}+ax\right)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + a^2*x^2)^(3/2)*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/
x^2,x]
```

```
[Out] (63*b^3 + 1034*a^2*b^2*x^2 - 1652*a^4*b*x^4 + 112*a^6*x^6 + Sqrt[-b + a^2*x
^2]*(250*a*b^2*x - 1596*a^3*b*x^3 + 112*a^5*x^5))/(63*x*(a*x + Sqrt[-b + a^
2*x^2])^(11/4)) - (Sqrt[2 - Sqrt[2]]*a*b^(9/8)*ArcTan[(Sqrt[2 - Sqrt[2]]*b^
(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-b^(1/4) + Sqrt[a*x + Sqrt[-b + a^
2*x^2]])])/4 - (Sqrt[2 + Sqrt[2]]*a*b^(9/8)*ArcTan[(Sqrt[2 + Sqrt[2]]*b^(1/
8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-b^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x
^2]])])/4 + (Sqrt[2 + Sqrt[2]]*a*b^(9/8)*ArcTanh[(Sqrt[1 - 1/Sqrt[2]]*b^(1/
8) + (Sqrt[1 - 1/Sqrt[2]]*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/b^(1/8))/(a*x + S
qrt[-b + a^2*x^2])^(1/4)])/4 + (Sqrt[2 - Sqrt[2]]*a*b^(9/8)*ArcTanh[(Sqrt[1
 + 1/Sqrt[2]]*b^(1/8) + (Sqrt[1 + 1/Sqrt[2]]*Sqrt[a*x + Sqrt[-b + a^2*x^2]]
)/b^(1/8))/(a*x + Sqrt[-b + a^2*x^2])^(1/4)])/4
```

fricas [A] time = 0.48, size = 770, normalized size = 1.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(3/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/4)/x^2,x, algorithm=
"fricas")
```

```
[Out] 1/504*(252*sqrt(2)*(-a^8*b^9)^(1/8)*x*arctan(-(a^8*b^9 + sqrt(2)*(-a^8*b^9)
^(7/8)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*a*b - sqrt(2)*(-a^8*b^9)^(7/8)*sqrt(
sqrt(a*x + sqrt(a^2*x^2 - b))*a^2*b^2 - sqrt(2)*(-a^8*b^9)^(1/8)*(a*x + sqr
t(a^2*x^2 - b))^(1/4)*a*b + (-a^8*b^9)^(1/4)))/(a^8*b^9)) + 252*sqrt(2)*(-a
^8*b^9)^(1/8)*x*arctan((a^8*b^9 - sqrt(2)*(-a^8*b^9)^(7/8)*(a*x + sqrt(a^2*
x^2 - b))^(1/4)*a*b + sqrt(2)*(-a^8*b^9)^(7/8)*sqrt(sqrt(a*x + sqrt(a^2*x^2
 - b))*a^2*b^2 + sqrt(2)*(-a^8*b^9)^(1/8)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*a
*b + (-a^8*b^9)^(1/4)))/(a^8*b^9)) + 63*sqrt(2)*(-a^8*b^9)^(1/8)*x*log(4*sq
rt(a*x + sqrt(a^2*x^2 - b))*a^2*b^2 + 4*sqrt(2)*(-a^8*b^9)^(1/8)*(a*x + sqr
t(a^2*x^2 - b))^(1/4)*a*b + 4*(-a^8*b^9)^(1/4)) - 63*sqrt(2)*(-a^8*b^9)^(1/
8)*x*log(4*sqrt(a*x + sqrt(a^2*x^2 - b))*a^2*b^2 - 4*sqrt(2)*(-a^8*b^9)^(1/
8)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*a*b + 4*(-a^8*b^9)^(1/4)) + 504*(-a^8*b^
9)^(1/8)*x*arctan(-((-a^8*b^9)^(7/8)*(a*x + sqrt(a^2*x^2 - b))^(1/4)*a*b -
(-a^8*b^9)^(7/8)*sqrt(sqrt(a*x + sqrt(a^2*x^2 - b))*a^2*b^2 + (-a^8*b^9)^(1
/4)))/(a^8*b^9)) + 126*(-a^8*b^9)^(1/8)*x*log((a*x + sqrt(a^2*x^2 - b))^(1/
4)*a*b + (-a^8*b^9)^(1/8)) - 126*(-a^8*b^9)^(1/8)*x*log((a*x + sqrt(a^2*x^2
 - b))^(1/4)*a*b - (-a^8*b^9)^(1/8)) - 8*(4*a^3*x^3 + 439*a*b*x - (32*a^2*x
^2 + 63*b)*sqrt(a^2*x^2 - b))*(a*x + sqrt(a^2*x^2 - b))^(1/4))/x
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(3/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/4)/x^2,x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{2} x^{2}-b\right)^{\frac{3}{2}} \left(a x +\sqrt{a^{2} x^{2}-b}\right)^{\frac{1}{4}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b)^(3/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/4)/x^2,x)
```

```
[Out] int((a^2*x^2-b)^(3/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/4)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^{2} x^{2}-b\right)^{\frac{3}{2}} \left(a x +\sqrt{a^{2} x^{2}-b}\right)^{\frac{1}{4}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(3/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/4)/x^2,x, algorithm=
"maxima")
```

```
[Out] integrate((a^2*x^2 - b)^(3/2)*(a*x + sqrt(a^2*x^2 - b))^(1/4)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{1/4}\,{\left(a^2\,x^2-b\right)}^{3/2}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (a^2*x^2 - b)^(1/2))^(1/4)*(a^2*x^2 - b)^(3/2))/x^2,x)
```

```
[Out] int(((a*x + (a^2*x^2 - b)^(1/2))^(1/4)*(a^2*x^2 - b)^(3/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a x + \sqrt{a^{2} x^{2} - b}} \left(a^{2} x^{2} - b\right)^{\frac{3}{2}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b)**(3/2)*(a*x+(a**2*x**2-b)**(1/2))**(1/4)/x**2,x)
```

```
[Out] Integral((a*x + sqrt(a**2*x**2 - b))**(1/4)*(a**2*x**2 - b)**(3/2)/x**2, x)
```

3.2405 $\int \frac{b-x}{\sqrt[3]{(-a+x)(-b+x)^2}\left(a^2-b^2d-2(a-bd)x+(1-d)x^2\right)}\,dx$

Optimal. Leaf size=541

$$\log\left(a^3\sqrt[3]{b-a}-2a^2x\sqrt[3]{b-a}-a^2b\sqrt[3]{b-a}+\left(bd^{2/3}\sqrt[3]{a-b}-ad^{2/3}\sqrt[3]{a-b}\right)\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right.\right.$$

Rubi [A] time = 1.08, antiderivative size = 513, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 6, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}} = 0.113$, Rules used = $\{6719, 21, 911, 105, 59, 91\}$

$$\frac{\sqrt[3]{x-a}(x-b)^{2/3}\log\left(2\left(1-\sqrt{d}\right)\left(a+b\sqrt{d}\right)-2(1-d)x\right)}{4d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}+\frac{\sqrt[3]{x-a}(x-b)^{2/3}\log\left(2(1-d)x-2\left(\sqrt{d}+1\right)\left(a-b\sqrt{d}\right)\right)}{4d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{3\sqrt[3]{x-a}(x-b)^{2/3}\log\left(-\frac{\sqrt[3]{x-a}}{\sqrt[6]{d}}-\sqrt[3]{x-b}\right)}{4d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{3\sqrt[3]{x-a}(x-b)^{2/3}\log\left(\frac{\sqrt[3]{x-a}}{\sqrt[6]{d}}-\sqrt[3]{x-b}\right)}{4d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{\sqrt{3}\sqrt[3]{x-a}(x-b)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-b}}\right)}{2d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{\sqrt{3}\sqrt[3]{x-a}(x-b)^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-b}}+\frac{1}{\sqrt{3}}\right)}{2d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b - x)/(((-a + x)*(-b + x)^2)^(1/3)*(a^2 - b^2*d - 2*(a - b*d)*x + (1
- d)*x^2)),x]
```

```
[Out] -1/2*(Sqrt[3]*(-a + x)^(1/3)*(-b + x)^(2/3)*ArcTan[1/Sqrt[3] - (2*(-a + x)^
(1/3))/(Sqrt[3]*d^(1/6)*(-b + x)^(1/3))]])/((a - b)*d^(2/3)*(-((a - x)*(b -
x)^2))^(1/3)) - (Sqrt[3]*(-a + x)^(1/3)*(-b + x)^(2/3)*ArcTan[1/Sqrt[3] + (
2*(-a + x)^(1/3))/(Sqrt[3]*d^(1/6)*(-b + x)^(1/3))]])/(2*(a - b)*d^(2/3)*(-(
(a - x)*(b - x)^2))^(1/3)) + ((-a + x)^(1/3)*(-b + x)^(2/3)*Log[2*(1 - Sqrt
[d])*(a + b*Sqrt[d]) - 2*(1 - d)*x])/(4*(a - b)*d^(2/3)*(-((a - x)*(b - x)^
2))^(1/3)) + ((-a + x)^(1/3)*(-b + x)^(2/3)*Log[-2*(1 + Sqrt[d])*(a - b*Sqr
t[d]) + 2*(1 - d)*x])/(4*(a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(1/3)) - (3
*(-a + x)^(1/3)*(-b + x)^(2/3)*Log[-((-a + x)^(1/3)/d^(1/6)) - (-b + x)^(1/
3)])/(4*(a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(1/3)) - (3*(-a + x)^(1/3)*(
-b + x)^(2/3)*Log[(-a + x)^(1/3)/d^(1/6) - (-b + x)^(1/3)])/(4*(a - b)*d^(2
/3)*(-((a - x)*(b - x)^2))^(1/3))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{b-x}{\sqrt[3]{(-a+x)(-b+x)^2}\left(a^2-b^2d-2(a-bd)x+(1-d)x^2\right)}\,dx = \frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{b-x}{\sqrt[3]{-a+x}\,(-b+x)^{2/3}\left(a^2-b^2d-\cdots\right.}}{\sqrt[3]{(-a+x)(-b+x)^2}}$$

$$= -\frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{\sqrt[3]{-b+x}}{\sqrt[3]{-a+x}\left(a^2-b^2d-2(a-bd\cdots\right.}}{\sqrt[3]{(-a+x)(-b+x)^2}}$$

$$= -\frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int\left(\frac{(1-d)\cdots}{(a-b)\sqrt{d}\,\sqrt[3]{-a+x}\left(2a-2\cdots\right.}\right.}{\cdots}$$

$$= -\frac{\left((1-d)\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{\cdots}{\sqrt[3]{-a+x}\left(2a-2(a\cdots\right.}}{(a-b)\sqrt{d}\,\sqrt[3]{(-a+x)(-b+\cdots}}$$

$$= -\frac{\left((1-d)\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{\cdots}{\sqrt[3]{-a+x}\,(-b+x)^{2\cdots}}}{\left(1-\sqrt{d}\right)\sqrt{d}\,\sqrt[3]{(-a+x)\cdots}}$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{-a+x}\,(-b+x)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{-\cdots}}{\sqrt{3}\,\sqrt[6]{d}\cdots}\right.}{2(a-b)d^{2/3}\sqrt[3]{-\left((a-x)(b-x)^2\right)}}$$

Mathematica [C] time = 0.30, size = 93, normalized size = 0.17

$$-\frac{3(x-b)\left({}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{\sqrt{d}\,(b-x)}{x-a}\right)-{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{\sqrt{d}\,(x-b)}{x-a}\right)\right)}{2\sqrt{d}\,(a-b)\sqrt[3]{(x-a)(b-x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b - x)/(((-a + x)*(-b + x)^2)^(1/3)*(a^2 - b^2*d - 2*(a - b*d)*x
 + (1 - d)*x^2)),x]
```

```
[Out] (-3*(-b + x)*(Hypergeometric2F1[1/3, 1, 4/3, (Sqrt[d]*(b - x))/(-a + x)] -
Hypergeometric2F1[1/3, 1, 4/3, (Sqrt[d]*(-b + x))/(-a + x)]))/(2*(a - b)*Sq
rt[d]*((b - x)^2*(-a + x))^(1/3))
```

IntegrateAlgebraic [A] time = 3.77, size = 541, normalized size = 1.00

$$\frac{\log\left(a^3\sqrt[3]{b-a}-2a^2x\sqrt[3]{b-a}-a^2b\sqrt[3]{b-a}+\left(bd^{2/3}\sqrt[3]{a-b}-ad^{2/3}\sqrt[3]{a-b}\right)\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}+\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}\left(a\sqrt[3]{d}(a-b)^{2/3}(b-a)^{2/3}-\sqrt[3]{d}x(a-b)^{2/3}(b-a)^{2/3}\right)+ax^2\sqrt[3]{b-a}-bx^2\sqrt[3]{b-a}+2abx\sqrt[3]{b-a}\right)}{4d^{2/3}\sqrt[3]{a-b}(b-a)^{2/3}}-\frac{\log\left(\sqrt[3]{d}(a-b)^{2/3}\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}-x(b-a)^{2/3}+a(b-a)^{2/3}\right)}{2d^{2/3}\sqrt[3]{a-b}(b-a)^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{d}(a-b)^{2/3}\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}}{\sqrt[3]{d}(a-b)^{2/3}\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}+2x(b-a)^{2/3}-2a(b-a)^{2/3}}\right)}{2d^{2/3}\sqrt[3]{a-b}(b-a)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b - x)/(((-a + x)*(-b + x)^2)^(1/3)*(a^2 - b^2*d - 2*(a
 - b*d)*x + (1 - d)*x^2)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*(a - b)^(2/3)*d^(1/3)*(-(a*b^2) + (2*a*b + b^
2)*x + (-a - 2*b)*x^2 + x^3)^(1/3))/(-2*a*(-a + b)^(2/3) + 2*(-a + b)^(2/3)
*x + (a - b)^(2/3)*d^(1/3)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x
^3)^(1/3))])/((a - b)^(1/3)*(-a + b)^(2/3)*d^(2/3)) - Log[a*(-a + b)^(2/3)
- (-a + b)^(2/3)*x + (a - b)^(2/3)*d^(1/3)*(-(a*b^2) + (2*a*b + b^2)*x + (-
a - 2*b)*x^2 + x^3)^(1/3)]/(2*(a - b)^(1/3)*(-a + b)^(2/3)*d^(2/3)) + Log[a
^3*(-a + b)^(1/3) - a^2*b*(-a + b)^(1/3) - 2*a^2*(-a + b)^(1/3)*x + 2*a*b*(
-a + b)^(1/3)*x + a*(-a + b)^(1/3)*x^2 - b*(-a + b)^(1/3)*x^2 + (a*(a - b)^
(2/3)*(-a + b)^(2/3)*d^(1/3) - (a - b)^(2/3)*(-a + b)^(2/3)*d^(1/3)*x)*(-(a
*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3) + (-(a*(a - b)^(1/3)*
d^(2/3)) + (a - b)^(1/3)*b*d^(2/3))*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b
)*x^2 + x^3)^(2/3)]/(4*(a - b)^(1/3)*(-a + b)^(2/3)*d^(2/3))
```

fricas [A] time = 0.41, size = 323, normalized size = 0.60

$$\frac{2\sqrt{3}\left(d^2\right)^{\frac{1}{6}}d\arctan\left(\frac{\sqrt{3}\left(d^2\right)^{\frac{1}{6}}\left(\left(b^2d-2bdx+dx^2\right)\left(d^2\right)^{\frac{1}{3}}+2\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{2}{3}}\left(d^2\right)^{\frac{2}{3}}\right)}{3\left(b^2d^2-2bd^2x+d^2x^2\right)}\right)-2\left(d^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(b^2-2bx+x^2\right)\left(d^2\right)^{\frac{2}{3}}-\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{2}{3}}d}{b^2-2bx+x^2}\right)+\left(d^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}(ad-dx)-\left(b^2d-2bdx+dx^2\right)\left(d^2\right)^{\frac{1}{3}}-\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{2}{3}}\left(d^2\right)^{\frac{2}{3}}}{b^2-2bx+x^2}\right)}{4(a-b)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-x)/((-a+x)*(-b+x)^2)^(1/3)/(a^2-b^2*d-2*(-b*d+a)*x+(1-d)*x^2),
x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(1/3*sqrt(3)*(d^2)^(1/6)*((b^2*d - 2*b*d
*x + d*x^2)*(d^2)^(1/3) + 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x
)^(2/3)*(d^2)^(2/3))/(b^2*d^2 - 2*b*d^2*x + d^2*x^2)) - 2*(d^2)^(2/3)*log(-
((b^2 - 2*b*x + x^2)*(d^2)^(2/3) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b +
 b^2)*x)^(2/3)*d)/(b^2 - 2*b*x + x^2)) + (d^2)^(2/3)*log(-((-a*b^2 - (a + 2
*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(a*d - d*x) - (b^2*d - 2*b*d*x + d*x
^2)*(d^2)^(1/3) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*(d
^2)^(2/3))/(b^2 - 2*b*x + x^2)))/((a - b)*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b-x}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}\left(b^2d+(d-1)x^2-a^2-2(bd-a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-x)/((-a+x)*(-b+x)^2)^(1/3)/(a^2-b^2*d-2*(-b*d+a)*x+(1-d)*x^2),
x, algorithm="giac")
```

```
[Out] integrate(-(b - x)/((-(a - x)*(b - x)^2)^(1/3)*(b^2*d + (d - 1)*x^2 - a^2 -
 2*(b*d - a)*x)), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{b-x}{\left(\left(-a+x\right)\left(-b+x\right)^{2}\right)^{\frac{1}{3}}\left(a^{2}-b^{2}d-2\left(-bd+a\right)x+\left(1-d\right)x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-x)/((-a+x)*(-b+x)^2)^(1/3)/(a^2-b^2*d-2*(-b*d+a)*x+(1-d)*x^2),x)
```

```
[Out] int((b-x)/((-a+x)*(-b+x)^2)^(1/3)/(a^2-b^2*d-2*(-b*d+a)*x+(1-d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{b-x}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}\left(b^2d+(d-1)x^2-a^2-2(bd-a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-x)/((-a+x)*(-b+x)^2)^(1/3)/(a^2-b^2*d-2*(-b*d+a)*x+(1-d)*x^2),
x, algorithm="maxima")
```

```
[Out] -integrate((b - x)/((-(a - x)*(b - x)^2)^(1/3)*(b^2*d + (d - 1)*x^2 - a^2 -
 2*(b*d - a)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{b-x}{\left(-(a-x)\,(b-x)^2\right)^{1/3}\left(b^2\,d+2\,x\,(a-b\,d)-a^2+x^2\,(d-1)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - x)/((-(a - x)*(b - x)^2)^(1/3)*(b^2*d + 2*x*(a - b*d) - a^2 + x^2
*(d - 1))),x)
```

```
[Out] int(-(b - x)/((-(a - x)*(b - x)^2)^(1/3)*(b^2*d + 2*x*(a - b*d) - a^2 + x^2
*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-x)/((-a+x)*(-b+x)**2)**(1/3)/(a**2-b**2*d-2*(-b*d+a)*x+(1-d)*x
**2),x)
```

```
[Out] Timed out
```

3.2406 $\int \frac{-b+x}{\sqrt[3]{(-a+x)(-b+x)^2}\left(-a^2+b^2d+2(a-bd)x+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=541

$$\log\left(a^3\sqrt[3]{b-a}-2a^2x\sqrt[3]{b-a}-a^2b\sqrt[3]{b-a}+\left(bd^{2/3}\sqrt[3]{a-b}-ad^{2/3}\sqrt[3]{a-b}\right)\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x\right.\right.$$

Rubi [A] time = 0.96, antiderivative size = 513, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 5, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.096, Rules used = {6719, 911, 105, 59, 91}

$$\frac{\sqrt[3]{x-a}(x-b)^{2/3}\log\left(2\left(1-\sqrt{d}\right)\left(a+b\sqrt{d}\right)-2(1-d)x\right)}{4d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}+\frac{\sqrt[3]{x-a}(x-b)^{2/3}\log\left(2(1-d)x-2\left(\sqrt{d}+1\right)\left(a-b\sqrt{d}\right)\right)}{4d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{3\sqrt[3]{x-a}(x-b)^{2/3}\log\left(-\frac{\sqrt[3]{x-a}}{\sqrt[6]{d}}-\sqrt[3]{x-b}\right)}{4d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{3\sqrt[3]{x-a}(x-b)^{2/3}\log\left(\frac{\sqrt[3]{x-a}}{\sqrt[6]{d}}-\sqrt[3]{x-b}\right)}{4d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{\sqrt{3}\sqrt[3]{x-a}(x-b)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-b}}\right)}{2d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{\sqrt{3}\sqrt[3]{x-a}(x-b)^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-b}}+\frac{1}{\sqrt{3}}\right)}{2d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + x)/(((-a + x)*(-b + x)^2)^(1/3)*(-a^2 + b^2*d + 2*(a - b*d)*x + (
-1 + d)*x^2)),x]
```

```
[Out] -1/2*(Sqrt[3]*(-a + x)^(1/3)*(-b + x)^(2/3)*ArcTan[1/Sqrt[3] - (2*(-a + x)^
(1/3))/(Sqrt[3]*d^(1/6)*(-b + x)^(1/3))]])/((a - b)*d^(2/3)*(-((a - x)*(b -
x)^2))^(1/3)) - (Sqrt[3]*(-a + x)^(1/3)*(-b + x)^(2/3)*ArcTan[1/Sqrt[3] + (
2*(-a + x)^(1/3))/(Sqrt[3]*d^(1/6)*(-b + x)^(1/3))]])/(2*(a - b)*d^(2/3)*(-(
(a - x)*(b - x)^2))^(1/3)) + ((-a + x)^(1/3)*(-b + x)^(2/3)*Log[2*(1 - Sqrt
[d])*(a + b*Sqrt[d]) - 2*(1 - d)*x]])/(4*(a - b)*d^(2/3)*(-((a - x)*(b - x)^
2))^(1/3)) + ((-a + x)^(1/3)*(-b + x)^(2/3)*Log[-2*(1 + Sqrt[d])*(a - b*Sqr
t[d]) + 2*(1 - d)*x]])/(4*(a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(1/3)) - (3
*(-a + x)^(1/3)*(-b + x)^(2/3)*Log[-((-a + x)^(1/3)/d^(1/6)) - (-b + x)^(1/
3)]])/(4*(a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(1/3)) - (3*(-a + x)^(1/3)*(
-b + x)^(2/3)*Log[(-a + x)^(1/3)/d^(1/6) - (-b + x)^(1/3)]])/(4*(a - b)*d^(2
/3)*(-((a - x)*(b - x)^2))^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[m] && !
IntegerQ[n]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{-b+x}{\sqrt[3]{(-a+x)(-b+x)^2}\left(-a^2+b^2d+2(a-bd)x+(-1+d)x^2\right)}\,dx = \frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{\sqrt[3]{-b+}}{\sqrt[3]{-a+x}\left(-a^2+b^2d+2(\right.}}{\sqrt[3]{(-a+x)(-b+x)^2}}$$

$$= \frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int\left(\frac{(-1+}{(a-b)\sqrt{d}\,\sqrt[3]{-a+x}\left(2a\right.}\right.}{}$$

$$= -\frac{\left((1-d)\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{}{\sqrt[3]{-a+x}\,(2a}}{(a-b)\sqrt{d}\,\sqrt[3]{(-a+x)(-}}$$

$$= -\frac{\left((1-d)\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{}{\sqrt[3]{-a+x}\,(-b}}{\left(1-\sqrt{d}\right)\sqrt{d}\,\sqrt[3]{(-a+}}$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{-a+x}\,(-b+x)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2}{\sqrt{3}}\right.}{2(a-b)d^{2/3}\sqrt[3]{-\left((a-x)(b-x)^2\right.}}$$

Mathematica [C] time = 0.15, size = 93, normalized size = 0.17

$$-\frac{3(x-b)\left({}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{\sqrt{d}\,(b-x)}{x-a}\right)-{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{\sqrt{d}\,(x-b)}{x-a}\right)\right)}{2\sqrt{d}\,(a-b)\sqrt[3]{(x-a)(b-x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + x)/(((-a + x)*(-b + x)^2)^(1/3)*(-a^2 + b^2*d + 2*(a - b*d)
*x + (-1 + d)*x^2)),x]
```

```
[Out] (-3*(-b + x)*(Hypergeometric2F1[1/3, 1, 4/3, (Sqrt[d]*(b - x))/(-a + x)] -
Hypergeometric2F1[1/3, 1, 4/3, (Sqrt[d]*(-b + x))/(-a + x)]))/(2*(a - b)*Sq
rt[d]*((b - x)^2*(-a + x))^(1/3))
```

IntegrateAlgebraic [A] time = 3.65, size = 541, normalized size = 1.00

$$\frac{\log\left(a^3\sqrt[3]{b-a}-2a^2x\sqrt[3]{b-a}-a^2b\sqrt[3]{b-a}+\left(bd^{2/3}\sqrt[3]{a-b}-ad^{2/3}\sqrt[3]{a-b}\right)\left(x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3\right)^{2/3}+\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}\left(a\sqrt[3]{d}\,(a-b)^{2/3}(b-a)^{2/3}-\sqrt[3]{d}\,x(a-b)^{2/3}(b-a)^{2/3}\right)+ax^2\sqrt[3]{b-a}-bx^2\sqrt[3]{b-a}+2abx\sqrt[3]{b-a}\right)}{4d^{2/3}\sqrt[3]{a-b}\,(b-a)^{2/3}}-\frac{\log\left(\sqrt[3]{d}\,(a-b)^{2/3}\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}-x(b-a)^{2/3}+a(b-a)^{2/3}\right)}{2d^{2/3}\sqrt[3]{a-b}\,(b-a)^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{d}\,(a-b)^{2/3}\sqrt[3]{x(2ab+b^2)-ab^2+x^2(-a-2b)+x^3}}{\sqrt[3]{d}\,(a-b)^{2/3}\sqrt[3]{x(2ab+b^2)-ab^2+x^2(-a-2b)+x^3}+2x(b-a)^{2/3}-2a(b-a)^{2/3}}\right)}{2d^{2/3}\sqrt[3]{a-b}\,(b-a)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + x)/(((-a + x)*(-b + x)^2)^(1/3)*(-a^2 + b^2*d + 2*
(a - b*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(Sqrt[3]*(a - b)^(2/3)*d^(1/3)*(-(a*b^2) + (2*a*b + b^
2)*x + (-a - 2*b)*x^2 + x^3)^(1/3))/(-2*a*(-a + b)^(2/3) + 2*(-a + b)^(2/3)
*x + (a - b)^(2/3)*d^(1/3)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x
^3)^(1/3))])/((a - b)^(1/3)*(-a + b)^(2/3)*d^(2/3)) - Log[a*(-a + b)^(2/3)
 - (-a + b)^(2/3)*x + (a - b)^(2/3)*d^(1/3)*(-(a*b^2) + (2*a*b + b^2)*x + (-
a - 2*b)*x^2 + x^3)^(1/3)]/(2*(a - b)^(1/3)*(-a + b)^(2/3)*d^(2/3)) + Log[a
^3*(-a + b)^(1/3) - a^2*b*(-a + b)^(1/3) - 2*a^2*(-a + b)^(1/3)*x + 2*a*b*(
-a + b)^(1/3)*x + a*(-a + b)^(1/3)*x^2 - b*(-a + b)^(1/3)*x^2 + (a*(a - b)^
(2/3)*(-a + b)^(2/3)*d^(1/3) - (a - b)^(2/3)*(-a + b)^(2/3)*d^(1/3)*x)*(-(a
*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3) + (-(a*(a - b)^(1/3)*
d^(2/3)) + (a - b)^(1/3)*b*d^(2/3))*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b
)*x^2 + x^3)^(2/3)]/(4*(a - b)^(1/3)*(-a + b)^(2/3)*d^(2/3))
```

fricas [A] time = 0.43, size = 323, normalized size = 0.60

$$\frac{2\sqrt{3}\left(d^{2}\right)^{\frac{1}{6}}d\arctan\left(\frac{\sqrt{3}\left(d^{2}\right)^{\frac{1}{6}}\left(\left(b^{2}d-2\,bdx+dx^{2}\right)\left(d^{2}\right)^{\frac{1}{3}}+2\left(-ab^{2}-(a+2\,b)x^{2}+x^{3}+\left(2\,ab+b^{2}\right)x\right)^{\frac{2}{3}}\left(d^{2}\right)^{\frac{2}{3}}\right)}{3\left(b^{2}d^{2}-2\,bd^{2}x+d^{2}x^{2}\right)}\right)-2\left(d^{2}\right)^{\frac{2}{3}}\log\left(-\frac{\left(b^{2}-2\,bx+x^{2}\right)\left(d^{2}\right)^{\frac{2}{3}}-\left(-ab^{2}-(a+2\,b)x^{2}+x^{3}+\left(2\,ab+b^{2}\right)x\right)^{\frac{2}{3}}d}{b^{2}-2\,bx+x^{2}}\right)+\left(d^{2}\right)^{\frac{2}{3}}\log\left(-\frac{\left(-ab^{2}-(a+2\,b)x^{2}+x^{3}+\left(2\,ab+b^{2}\right)x\right)^{\frac{1}{3}}(ad-dx)-\left(b^{2}d-2\,bdx+dx^{2}\right)\left(d^{2}\right)^{\frac{1}{3}}-\left(-ab^{2}-(a+2\,b)x^{2}+x^{3}+\left(2\,ab+b^{2}\right)x\right)^{\frac{2}{3}}\left(d^{2}\right)^{\frac{2}{3}}}{b^{2}-2\,bx+x^{2}}\right)}{4\,(a-b)d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)/((-a+x)*(-b+x)^2)^(1/3)/(-a^2+b^2*d+2*(-b*d+a)*x+(-1+d)*x^
2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(1/3*sqrt(3)*(d^2)^(1/6)*((b^2*d - 2*b*d
*x + d*x^2)*(d^2)^(1/3) + 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x
)^(2/3)*(d^2)^(2/3))/(b^2*d^2 - 2*b*d^2*x + d^2*x^2)) - 2*(d^2)^(2/3)*log(-
((b^2 - 2*b*x + x^2)*(d^2)^(2/3) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b +
 b^2)*x)^(2/3)*d)/(b^2 - 2*b*x + x^2)) + (d^2)^(2/3)*log(-((-a*b^2 - (a + 2
*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(a*d - d*x) - (b^2*d - 2*b*d*x + d*x
^2)*(d^2)^(1/3) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*(d
^2)^(2/3))/(b^2 - 2*b*x + x^2)))/((a - b)*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b-x}{\left(-(a-x)(b-x)^{2}\right)^{\frac{1}{3}}\left(b^{2}d+(d-1)x^{2}-a^{2}-2\,(bd-a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)/((-a+x)*(-b+x)^2)^(1/3)/(-a^2+b^2*d+2*(-b*d+a)*x+(-1+d)*x^
2),x, algorithm="giac")
```

```
[Out] integrate(-(b - x)/((-(a - x)*(b - x)^2)^(1/3)*(b^2*d + (d - 1)*x^2 - a^2 -
 2*(b*d - a)*x)), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{-b+x}{\left(\left(-a+x\right)\left(-b+x\right)^{2}\right)^{\frac{1}{3}}\left(-a^{2}+b^{2}d+2\left(-bd+a\right)x+\left(-1+d\right)x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b+x)/((-a+x)*(-b+x)^2)^(1/3)/(-a^2+b^2*d+2*(-b*d+a)*x+(-1+d)*x^2),x)
```

```
[Out] int((-b+x)/((-a+x)*(-b+x)^2)^(1/3)/(-a^2+b^2*d+2*(-b*d+a)*x+(-1+d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{b-x}{\left(-(a-x)(b-x)^{2}\right)^{\frac{1}{3}}\left(b^{2}d+(d-1)x^{2}-a^{2}-2\,(bd-a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)/((-a+x)*(-b+x)^2)^(1/3)/(-a^2+b^2*d+2*(-b*d+a)*x+(-1+d)*x^
2),x, algorithm="maxima")
```

```
[Out] -integrate((b - x)/((-(a - x)*(b - x)^2)^(1/3)*(b^2*d + (d - 1)*x^2 - a^2 -
 2*(b*d - a)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{b-x}{\left(-(a-x)\,(b-x)^2\right)^{1/3}\,\left(b^2\,d+2\,x\,(a-b\,d)-a^2+x^2\,(d-1)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - x)/((-(a - x)*(b - x)^2)^(1/3)*(b^2*d + 2*x*(a - b*d) - a^2 + x^2
*(d - 1))),x)
```

```
[Out] int(-(b - x)/((-(a - x)*(b - x)^2)^(1/3)*(b^2*d + 2*x*(a - b*d) - a^2 + x^2
*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)/((-a+x)*(-b+x)**2)**(1/3)/(-a**2+b**2*d+2*(-b*d+a)*x+(-1+d
)*x**2),x)
```

```
[Out] Timed out
```

3.2407 $\int \frac{b^2+ax^2}{\left(-b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$

Optimal. Leaf size=541

$$\frac{2x}{\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{2\sqrt{2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{a}}-\frac{2\sqrt{1+\sqrt{2}}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2\left(\sqrt{2}-1\right)}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}}}\right.}{\sqrt{a}}$$

Rubi [F] time = 0.97, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{b^2+ax^2}{\left(-b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(b^2 + a*x^2)/((-b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

```
[Out] Defer[Int][1/Sqrt[b + Sqrt[b^2 + a*x^2]], x] - b*Defer[Int][1/((b - Sqrt[a]
*x)*Sqrt[b + Sqrt[b^2 + a*x^2]]), x] - b*Defer[Int][1/((b + Sqrt[a]*x)*Sqrt
[b + Sqrt[b^2 + a*x^2]]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{b^2+ax^2}{\left(-b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx &= \int \left(\frac{1}{\sqrt{b+\sqrt{b^2+ax^2}}}+\frac{2b^2}{\left(-b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}}\right)dx \\
&= \left(2b^2\right)\int \frac{1}{\left(-b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx+\int \frac{1}{\sqrt{b+\sqrt{b^2+ax^2}}}\,dx \\
&= \left(2b^2\right)\int \left(-\frac{1}{2b\left(b-\sqrt{a}\,x\right)\sqrt{b+\sqrt{b^2+ax^2}}}-\frac{1}{2b\left(b+\sqrt{a}\,x\right)\sqrt{b+\sqrt{b^2+a}}}\right. \\
&= -\left(b\int \frac{1}{\left(b-\sqrt{a}\,x\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx\right)-b\int \frac{1}{\left(b+\sqrt{a}\,x\right)\sqrt{b+\sqrt{b^2+ax}}}
\end{aligned}$$

Mathematica [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{b^2+ax^2}{\left(-b^2+ax^2\right)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(b^2 + a*x^2)/((-b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

```
[Out] Integrate[(b^2 + a*x^2)/((-b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]]), x]
```

IntegrateAlgebraic [A] time = 0.77, size = 251, normalized size = 0.46

$$\frac{2x}{\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{\sqrt{2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}-\frac{2\left(\sqrt{2\left(\sqrt{2}-1\right)}\sqrt{b}+\sqrt{\sqrt{2}-1}\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{2}-1}\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}-\frac{2\left(\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{b}-\sqrt{1+\sqrt{2}}\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{2}}\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^2 + a*x^2)/((-b^2 + a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]
]),x]
```

```
[Out] (2*x)/Sqrt[b + Sqrt[b^2 + a*x^2]] + (Sqrt[2]*Sqrt[b]*ArcTan[(Sqrt[a]*x)/(Sq
rt[2]*Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/Sqrt[a] - (2*(Sqrt[-1 + Sqrt[2
]]*Sqrt[b] + Sqrt[2*(-1 + Sqrt[2])]*Sqrt[b])*ArcTan[(Sqrt[-1 + Sqrt[2]]*Sqr
t[a]*x)/(Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/Sqrt[a] - (2*(-(Sqrt[1 + Sq
rt[2]]*Sqrt[b]) + Sqrt[2*(1 + Sqrt[2])]*Sqrt[b])*ArcTanh[(Sqrt[1 + Sqrt[2]]
*Sqrt[a]*x)/(Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/Sqrt[a]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)/(a*x^2-b^2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + b^2}{\left(ax^2 - b^2\right)\sqrt{b + \sqrt{ax^2 + b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)/(a*x^2-b^2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((a*x^2 + b^2)/((a*x^2 - b^2)*sqrt(b + sqrt(a*x^2 + b^2))), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{a\,x^2 + b^2}{\left(a\,x^2 - b^2\right)\sqrt{b + \sqrt{a\,x^2 + b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b^2)/(a*x^2-b^2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x^2+b^2)/(a*x^2-b^2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + b^2}{\left(ax^2 - b^2\right)\sqrt{b + \sqrt{ax^2 + b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)/(a*x^2-b^2)/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x^2 + b^2)/((a*x^2 - b^2)*sqrt(b + sqrt(a*x^2 + b^2))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b^2 + a\,x^2}{\left(a\,x^2 - b^2\right)\sqrt{b + \sqrt{b^2 + a\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b^2)/((a*x^2 - b^2)*(b + (a*x^2 + b^2)^(1/2))^(1/2)),x)
```

```
[Out] int((a*x^2 + b^2)/((a*x^2 - b^2)*(b + (a*x^2 + b^2)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + b^2}{\sqrt{b + \sqrt{ax^2 + b^2}}\,(ax^2 - b^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b**2)/(a*x**2-b**2)/(b+(a*x**2+b**2)**(1/2))**(1/2),x)
```

```
[Out] Integral((a*x**2 + b**2)/(sqrt(b + sqrt(a*x**2 + b**2))*(a*x**2 - b**2)), x
)
```

3.2408 $\int \frac{1}{\sqrt[3]{(-a+x)(-b+x)^2}\,(a-bd+(-1+d)x)}\,dx$

Optimal. Leaf size=543

$$\frac{\left(\sqrt[3]{d}-i\sqrt{3}\sqrt[3]{d}\right)\log\left(\sqrt[3]{-1}\,(b-x)(bd-ad)^{2/3}-\sqrt[3]{d}\,(a-b)^{2/3}\sqrt[3]{x\left(2ab+b^2\right)-ab^2+x^2(-a-2b)+x^3}\right)}{2\sqrt[3]{a-b}\,(-(d(a-b)))^{2/3}}+\frac{\sqrt{-3}\cdots}{}$$

Rubi [A] time = 0.67, antiderivative size = 377, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2081, 2077, 91}

$$\frac{\left((a-b)^2(b-x)\right)^{2/3}\sqrt[3]{(a-b)^2(x-a)}\log(a-bd+(d-1)x)}{2\sqrt[3]{d}\,(a-b)^3\sqrt[3]{-ab^2+x^2(-a-2b)+bx(2a+b)+x^3}}-\frac{3\left((a-b)^2(b-x)\right)^{2/3}\sqrt[3]{(a-b)^2(x-a)}\log\left(-\frac{\sqrt[3]{\frac{2}{3}}\sqrt[3]{(a-b)^2(x-a)}}{\sqrt[3]{d}}-\sqrt[3]{\frac{2}{3}}\sqrt[3]{(a-b)^2(b-x)}\right)}{2\sqrt[3]{d}\,(a-b)^3\sqrt[3]{-ab^2+x^2(-a-2b)+bx(2a+b)+x^3}}-\frac{\sqrt{3}\left((a-b)^2(b-x)\right)^{2/3}\sqrt[3]{(a-b)^2(x-a)}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{(a-b)^2(x-a)}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{(a-b)^2(b-x)}}\right)}{\sqrt[3]{d}\,(a-b)^3\sqrt[3]{-ab^2+x^2(-a-2b)+bx(2a+b)+x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(((-a + x)*(-b + x)^2)^(1/3)*(a - b*d + (-1 + d)*x)),x]
```

```
[Out] -((Sqrt[3]*((a - b)^2*(b - x))^(2/3)*((a - b)^2*(-a + x))^(1/3)*ArcTan[1/Sq
rt[3] - (2*((a - b)^2*(-a + x))^(1/3))/(Sqrt[3]*d^(1/3)*((a - b)^2*(b - x))
^(1/3))])/((a - b)^3*d^(1/3)*(-(a*b^2) + b*(2*a + b)*x + (-a - 2*b)*x^2 + x
^3)^(1/3))) + (((a - b)^2*(b - x))^(2/3)*((a - b)^2*(-a + x))^(1/3)*Log[a -
 b*d + (-1 + d)*x])/(2*(a - b)^3*d^(1/3)*(-(a*b^2) + b*(2*a + b)*x + (-a -
2*b)*x^2 + x^3)^(1/3)) - (3*((a - b)^2*(b - x))^(2/3)*((a - b)^2*(-a + x))^
(1/3)*Log[-((2/3)^(1/3)*((a - b)^2*(b - x))^(1/3)) - ((2/3)^(1/3)*((a - b)^
2*(-a + x))^(1/3))/d^(1/3)])/(2*(a - b)^3*d^(1/3)*(-(a*b^2) + b*(2*a + b)*x
 + (-a - 2*b)*x^2 + x^3)^(1/3))
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 2077

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Dist[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int
[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d, e
, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 2081

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{(-a+x)(-b+x)^2}\,(a-bd+(-1+d)x)}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{3}(-((-a-2b)(-1+d))+3(a-bd))+(-1+d)x\right)\sqrt[3]{\cdots}}\right.$$

$$= \frac{\left(2^{2/3}\sqrt[3]{-(a-b)^2(a-x)}\,\left((a-b)^2(b-x)\right)^{2/3}\right)\text{Subst}\left(\int \frac{\cdots}{\left(-\frac{2}{9}(a-b)^3-\cdots\right.}\right.}{3\sqrt[3]{-ab^2+\cdots}}$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{-(a-b)^2(a-x)}\,\left((a-b)^2(b-x)\right)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{-(a\cdots}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{\cdots}}\right.}{(a-b)^3\sqrt[3]{d}\,\sqrt[3]{-ab^2+b(2a+b)x-(a+2b)x^2+x^3}}$$

Mathematica [C] time = 0.04, size = 53, normalized size = 0.10

$$\frac{3(x-b)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{d(b-x)}{a-x}\right)}{(a-b)\sqrt[3]{(x-a)(b-x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((((-a + x)*(-b + x)^2)^(1/3)*(a - b*d + (-1 + d)*x)),x]
```

```
[Out] (3*(-b + x)*Hypergeometric2F1[1/3, 1, 4/3, (d*(b - x))/(a - x)])/((a - b)*(
(b - x)^2*(-a + x))^(1/3))
```

IntegrateAlgebraic [A] time = 7.72, size = 703, normalized size = 1.29

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((((-a + x)*(-b + x)^2)^(1/3)*(a - b*d + (-1 + d)*x)),x
]
```

```
[Out] (Sqrt[-3 - (3*I)*Sqrt[3]]*d^(1/3)*ArcTan[(Sqrt[3]*(a - b)^(2/3)*d^(1/3)*(-(
a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3))/(b*(-(a*d) + b*d)^(
2/3) + I*Sqrt[3]*b*(-(a*d) + b*d)^(2/3) + (-(-(a*d) + b*d)^(2/3) - I*Sqrt[3
]*(-(a*d) + b*d)^(2/3))*x + (a - b)^(2/3)*d^(1/3)*(-(a*b^2) + (2*a*b + b^2)
*x + (-a - 2*b)*x^2 + x^3)^(1/3))])/(Sqrt[2]*(a - b)^(1/3)*(-((a - b)*d))^(
2/3)) + ((d^(1/3) - I*Sqrt[3]*d^(1/3))*Log[(-(a*d) + b*d)^(2/3)*(-b + Sqrt[
3]*((-I)*b + I*x) + x) + 2*(a - b)^(2/3)*d^(1/3)*(-(a*b^2) + (2*a*b + b^2)*
x + (-a - 2*b)*x^2 + x^3)^(1/3)])/(2*(a - b)^(1/3)*(-((a - b)*d))^(2/3)) +
((I/4)*(I*d^(1/3) + Sqrt[3]*d^(1/3))*Log[2*(a - b)^(4/3)*d^(2/3)*(-(a*b^2)
+ (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(2/3) + (-(a*d) + b*d)^(1/3)*(d*(
a*b^2 - b^3 - 2*a*b*x + 2*b^2*x + a*x^2 - b*x^2) + Sqrt[3]*d*((-I)*a*b^2 +
I*b^3 + (2*I)*a*b*x - (2*I)*b^2*x - I*a*x^2 + I*b*x^2)) + (-(a*d) + b*d)^(2
/3)*((a - b)^(2/3)*d^(1/3)*(b - x)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)
*x^2 + x^3)^(1/3) + Sqrt[3]*(a - b)^(2/3)*d^(1/3)*(I*b - I*x)*(-(a*b^2) + (
2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3))])/((a - b)^(1/3)*(-((a - b)*d
))^(2/3))
```

fricas [A] time = 0.45, size = 662, normalized size = 1.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-a+x)*(-b+x)^2)^(1/3)/(a-b*d+(-1+d)*x),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(3)*d*sqrt(-1/d^(2/3))*log(-(b^2*d + (d + 2)*x^2 + 2*a*b + 3*(-a
*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(b - x)*d^(2/3) - 2*(b*
d + a + b)*x + sqrt(3)*((-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1
/3)*(b*d - d*x) - (b^2*d - 2*b*d*x + d*x^2)*d^(1/3) + 2*(-a*b^2 - (a + 2*b)
*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*d^(2/3))*sqrt(-1/d^(2/3)))/(b^2*d + (d
- 1)*x^2 - a*b - (2*b*d - a - b)*x)) - d^(2/3)*log(-((-a*b^2 - (a + 2*b)*x^
2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(b - x)*d^(1/3) - (b^2 - 2*b*x + x^2)*d^(2
/3) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3))/(b^2 - 2*b*x
+ x^2)) + 2*d^(2/3)*log(-((b - x)*d^(1/3) + (-a*b^2 - (a + 2*b)*x^2 + x^3 +
 (2*a*b + b^2)*x)^(1/3))/(b - x)))/((a - b)*d), -1/2*(2*sqrt(3)*d^(2/3)*arc
tan(1/3*sqrt(3)*((b - x)*d^(1/3) - 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b
 + b^2)*x)^(1/3))/((b - x)*d^(1/3))) - d^(2/3)*log(-((-a*b^2 - (a + 2*b)*x^
2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(b - x)*d^(1/3) - (b^2 - 2*b*x + x^2)*d^(2
/3) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3))/(b^2 - 2*b*x
+ x^2)) + 2*d^(2/3)*log(-((b - x)*d^(1/3) + (-a*b^2 - (a + 2*b)*x^2 + x^3 +
 (2*a*b + b^2)*x)^(1/3))/(b - x)))/((a - b)*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}(bd-(d-1)x-a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-a+x)*(-b+x)^2)^(1/3)/(a-b*d+(-1+d)*x),x, algorithm="giac")
```

```
[Out] integrate(-1/((-(a - x)*(b - x)^2)^(1/3)*(b*d - (d - 1)*x - a)), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((-a+x)(-b+x)^2\right)^{\frac{1}{3}}(a-bd+(-1+d)x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-a+x)*(-b+x)^2)^(1/3)/(a-b*d+(-1+d)*x),x)
```

```
[Out] int(1/((-a+x)*(-b+x)^2)^(1/3)/(a-b*d+(-1+d)*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}(bd-(d-1)x-a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-a+x)*(-b+x)^2)^(1/3)/(a-b*d+(-1+d)*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/((-(a - x)*(b - x)^2)^(1/3)*(b*d - (d - 1)*x - a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(-(a-x)\,(b-x)^2\right)^{1/3}\,(a-b\,d+x\,(d-1))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-(a - x)*(b - x)^2)^(1/3)*(a - b*d + x*(d - 1))),x)
```

```
[Out] int(1/((-(a - x)*(b - x)^2)^(1/3)*(a - b*d + x*(d - 1))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(-a+x)(-b+x)^2}\,(a-bd+dx-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-a+x)*(-b+x)**2)**(1/3)/(a-b*d+(-1+d)*x),x)
```

```
[Out] Integral(1/(((-a + x)*(-b + x)**2)**(1/3)*(a - b*d + d*x - x)), x)
```

3.2409 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+ax}\,dx$

Optimal. Leaf size=553

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)}{a^2}+\frac{\left(a^4+\sqrt{a^4+1}+1\right)\tan^{-1}\left(\frac{a\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{-\sqrt{a^4+1}-1}}\right)}{a^2\sqrt{a^4+1}\sqrt{-\sqrt{a^4+1}-1}}+\frac{\left(-a^4+\sqrt{a^4+1}-1\right)\tan^{-1}\left(\frac{a\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{\sqrt{a^4+1}-1}}\right)}{a^2\sqrt{a^4+1}\sqrt{\sqrt{a^4+1}-1}}$$

Rubi [F] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+ax}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/(1 + a*x),x]
[Out] Defer[Int][Sqrt[x^2 + Sqrt[1 + x^4]]/(1 + a*x), x]
```

Rubi steps

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+ax}\,dx=\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+ax}\,dx$$

Mathematica [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{1+ax}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/(1 + a*x),x]
[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/(1 + a*x), x]
```

IntegrateAlgebraic [A] time = 5.65, size = 553, normalized size = 1.00

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)}{a^2}+\frac{\left(a^4+\sqrt{a^4+1}+1\right)\tan^{-1}\left(\frac{a\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{-\sqrt{a^4+1}-1}}\right)}{a^2\sqrt{a^4+1}\sqrt{-\sqrt{a^4+1}-1}}+\frac{\left(-a^4+\sqrt{a^4+1}-1\right)\tan^{-1}\left(\frac{a\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{\sqrt{a^4+1}-1}}\right)}{a^2\sqrt{a^4+1}\sqrt{\sqrt{a^4+1}-1}}+\frac{\left(-\sqrt{2}\sqrt{-\sqrt{a^4+1}-a^2}\,a^2+\sqrt{2}\sqrt{a^4+1}\sqrt{-\sqrt{a^4+1}-a^2}+\sqrt{2}\sqrt{-\sqrt{a^4+1}-a^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{-\sqrt{a^4+1}-a^2}\left(\sqrt{x^4+1}+x^2-1\right)}\right)}{2a^2}+\frac{\left(\sqrt{2}\sqrt{\sqrt{a^4+1}-a^2}\,a^2+\sqrt{2}\sqrt{a^4+1}\sqrt{\sqrt{a^4+1}-a^2}-\sqrt{2}\sqrt{\sqrt{a^4+1}-a^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{a^4+1}-a^2}\,x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)}{2a^2}+\frac{\sqrt{\sqrt{x^4+1}+x^2}}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x^2 + Sqrt[1 + x^4]]/(1 + a*x),x]
[Out] Sqrt[x^2 + Sqrt[1 + x^4]]/a + ((1 + a^4 + Sqrt[1 + a^4])*ArcTan[(a*Sqrt[x^2
 + Sqrt[1 + x^4]])/Sqrt[-1 - Sqrt[1 + a^4]]])/(a^2*Sqrt[1 + a^4]*Sqrt[-1 -
Sqrt[1 + a^4]]) + ((-1 - a^4 + Sqrt[1 + a^4])*ArcTan[(a*Sqrt[x^2 + Sqrt[1 +
 x^4]])/Sqrt[-1 + Sqrt[1 + a^4]]])/(a^2*Sqrt[1 + a^4]*Sqrt[-1 + Sqrt[1 + a^
4]]) + ((Sqrt[2]*Sqrt[-a^2 - Sqrt[1 + a^4]] - Sqrt[2]*a^2*Sqrt[-a^2 - Sqrt[
1 + a^4]] + Sqrt[2]*Sqrt[1 + a^4]*Sqrt[-a^2 - Sqrt[1 + a^4]])*ArcTan[(Sqrt[
2]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(Sqrt[-a^2 - Sqrt[1 + a^4]]*(-1 + x^2 + Sqr
t[1 + x^4]))])/(2*a^2) + ((-(Sqrt[2]*Sqrt[-a^2 + Sqrt[1 + a^4]]) + Sqrt[2]*
a^2*Sqrt[-a^2 + Sqrt[1 + a^4]] + Sqrt[2]*Sqrt[1 + a^4]*Sqrt[-a^2 + Sqrt[1 +
```

```
a^4]])*ArcTan[(Sqrt[2]*Sqrt[-a^2 + Sqrt[1 + a^4]]*x*Sqrt[x^2 + Sqrt[1 + x^
4]])/(1 + x^2 + Sqrt[1 + x^4])])/(2*a^2) - (Sqrt[2]*ArcTanh[(Sqrt[2]*x*Sqrt
[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])])/a^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(a*x+1),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{ax + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(a*x+1),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(a*x + 1), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{ax + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(a*x+1),x)
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(a*x+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{ax + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(a*x+1),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(a*x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\sqrt{x^4 + 1} + x^2}}{a\,x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(a*x + 1),x)
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(a*x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 + \sqrt{x^4 + 1}}}{ax + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(a*x+1),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/(a*x + 1), x)
```

3.2410 $\int \frac{-b+x}{\sqrt[3]{(-a+x)(-b+x)^2}\left(-b^2+a^2d+2(b-ad)x+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=569

$$-\frac{d^{4/3}(a-b)^{2/3}\log\left(a^3d\sqrt[3]{ad-bd}-2a^2dx\sqrt[3]{ad-bd}-a^2bd\sqrt[3]{ad-bd}+d^{2/3}(a-b)^{4/3}\left(x\left(2ab+b^2\right)-ab^2+x^2(-a\right.\right.}{}$$

Rubi [A] time = 1.05, antiderivative size = 513, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 5, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.096, Rules used = {6719, 911, 105, 59, 91}

$$-\frac{\sqrt[3]{x-a}(x-b)^{2/3}\log\left(2\left(\sqrt{d}+1\right)\left(b-a\sqrt{d}\right)-2(1-d)x\right)}{4\sqrt[3]{d}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{\sqrt[3]{x-a}(x-b)^{2/3}\log\left(2(1-d)x-2\left(1-\sqrt{d}\right)\left(a\sqrt{d}+b\right)\right)}{4\sqrt[3]{d}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}+\frac{3\sqrt[3]{x-a}(x-b)^{2/3}\log\left(-\sqrt[6]{d}\sqrt[3]{x-a}-\sqrt[3]{x-b}\right)}{4\sqrt[3]{d}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}+\frac{3\sqrt[3]{x-a}(x-b)^{2/3}\log\left(\sqrt[6]{d}\sqrt[3]{x-a}-\sqrt[3]{x-b}\right)}{4\sqrt[3]{d}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}+\frac{\sqrt{3}\sqrt[3]{x-a}(x-b)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{x-b}}\right)}{2\sqrt[3]{d}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}+\frac{\sqrt{3}\sqrt[3]{x-a}(x-b)^{2/3}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{x-b}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{d}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + x)/(((-a + x)*(-b + x)^2)^(1/3)*(-b^2 + a^2*d + 2*(b - a*d)*x + (
-1 + d)*x^2)),x]
```

```
[Out] (Sqrt[3]*(-a + x)^(1/3)*(-b + x)^(2/3)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(-a +
x)^(1/3))/(Sqrt[3]*(-b + x)^(1/3))]])/(2*(a - b)*d^(1/3)*(-((a - x)*(b - x)^
2))^(1/3)) + (Sqrt[3]*(-a + x)^(1/3)*(-b + x)^(2/3)*ArcTan[1/Sqrt[3] + (2*d
^(1/6)*(-a + x)^(1/3))/(Sqrt[3]*(-b + x)^(1/3))]])/(2*(a - b)*d^(1/3)*(-((a
- x)*(b - x)^2))^(1/3)) - ((-a + x)^(1/3)*(-b + x)^(2/3)*Log[2*(1 + Sqrt[d]
)*(b - a*Sqrt[d]) - 2*(1 - d)*x]])/(4*(a - b)*d^(1/3)*(-((a - x)*(b - x)^2))
^(1/3)) - ((-a + x)^(1/3)*(-b + x)^(2/3)*Log[-2*(1 - Sqrt[d])*(b + a*Sqrt[d
]) + 2*(1 - d)*x]])/(4*(a - b)*d^(1/3)*(-((a - x)*(b - x)^2))^(1/3)) + (3*(-
a + x)^(1/3)*(-b + x)^(2/3)*Log[-(d^(1/6)*(-a + x)^(1/3)) - (-b + x)^(1/3)]
)/(4*(a - b)*d^(1/3)*(-((a - x)*(b - x)^2))^(1/3)) + (3*(-a + x)^(1/3)*(-b
+ x)^(2/3)*Log[d^(1/6)*(-a + x)^(1/3) - (-b + x)^(1/3)]])/(4*(a - b)*d^(1/3)
*(-((a - x)*(b - x)^2))^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1]])/(2*d), x] - Simp[(q*Log[c + d*x]])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))]])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x]])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)]])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 911

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[m] && !
IntegerQ[n]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{-b+x}{\sqrt[3]{(-a+x)(-b+x)^2}\left(-b^2+a^2d+2(b-ad)x+(-1+d)x^2\right)}\,dx = \frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{\sqrt[3]{-b+}}{\sqrt[3]{-a+x}\left(-b^2+a^2d+2(b\right.}}{\sqrt[3]{(-a+x)(-b+x)^2}}$$

$$= \frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int\left(\frac{(-1+}{(a-b)\sqrt{d}\,\sqrt[3]{-a+x}\,(2b\cdot}\right.}{}$$

$$= -\frac{\left((1-d)\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{}{\sqrt[3]{-a+x}\,(2b}}{(a-b)\sqrt{d}\,\sqrt[3]{(-a+x)(-}}$$

$$= \frac{\left((1-d)\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{}{\sqrt[3]{-a+x}\,(-b+}}{\left(1-\sqrt{d}\right)\sqrt[3]{(-a+x)(}}$$

$$= \frac{\sqrt{3}\,\sqrt[3]{-a+x}\,(-b+x)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}}{\sqrt{3}\,\sqrt[3]{}}\right.}{2(a-b)\sqrt[3]{d}\,\sqrt[3]{-\left((a-x)(b-x)^2\right)}}$$

Mathematica [C] time = 0.29, size = 93, normalized size = 0.16

$$\frac{3(x-b)\left({}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{b-x}{\sqrt{d}\,(x-a)}\right)-{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{x-b}{\sqrt{d}\,(x-a)}\right)\right)}{2\sqrt{d}\,(a-b)\sqrt[3]{(x-a)(b-x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + x)/(((-a + x)*(-b + x)^2)^(1/3)*(-b^2 + a^2*d + 2*(b - a*d)
*x + (-1 + d)*x^2)),x]
```

```
[Out] (3*(-b + x)*(Hypergeometric2F1[1/3, 1, 4/3, (b - x)/(Sqrt[d]*(-a + x))] - H
ypergeometric2F1[1/3, 1, 4/3, (-b + x)/(Sqrt[d]*(-a + x))]))/(2*(a - b)*Sqr
t[d]*((b - x)^2*(-a + x))^(1/3))
```

IntegrateAlgebraic [A] time = 3.83, size = 569, normalized size = 1.00

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-b + x)/(((-a + x)*(-b + x)^2)^(1/3)*(-b^2 + a^2*d + 2*
(b - a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] (Sqrt[3]*(a - b)^(2/3)*d^(4/3)*ArcTan[(Sqrt[3]*(a*d - b*d)^(1/3)*(-(a*b^2)
+ (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3))/(-2*a*(a - b)^(1/3)*d^(2/3
) + 2*(a - b)^(1/3)*d^(2/3)*x + (a*d - b*d)^(1/3)*(-(a*b^2) + (2*a*b + b^2)
*x + (-a - 2*b)*x^2 + x^3)^(1/3))])/(2*((a - b)*d)^(5/3)) + ((a - b)^(2/3)*
d^(4/3)*Log[a*(a*d - b*d)^(2/3) - (a*d - b*d)^(2/3)*x + (a - b)^(2/3)*d^(1/
3)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3)])/(2*((a - b)*
d)^(5/3)) - ((a - b)^(2/3)*d^(4/3)*Log[a^3*d*(a*d - b*d)^(1/3) - a^2*b*d*(a
*d - b*d)^(1/3) - 2*a^2*d*(a*d - b*d)^(1/3)*x + 2*a*b*d*(a*d - b*d)^(1/3)*x
 + a*d*(a*d - b*d)^(1/3)*x^2 - b*d*(a*d - b*d)^(1/3)*x^2 + (-(a*(a - b)^(2/
3)*d^(1/3)*(a*d - b*d)^(2/3)) + (a - b)^(2/3)*d^(1/3)*(a*d - b*d)^(2/3)*x)*
(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(1/3) + (a - b)^(4/3)*d
^(2/3)*(-(a*b^2) + (2*a*b + b^2)*x + (-a - 2*b)*x^2 + x^3)^(2/3)])/(4*((a -
 b)*d)^(5/3))
```

fricas [A] time = 0.81, size = 755, normalized size = 1.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)/((-a+x)*(-b+x)^2)^(1/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-1+d)*x^
2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(3)*d*sqrt((-d)^(1/3)/d)*log((2*a^2*d + (2*d + 1)*x^2 + b^2 - 2*
(2*a*d + b)*x - sqrt(3)*(2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)
^(1/3)*(a*d - d*x) - (b^2 - 2*b*x + x^2)*(-d)^(1/3) + (-a*b^2 - (a + 2*b)*x
^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*(-d)^(2/3))*sqrt((-d)^(1/3)/d) + 3*(-a*b^
2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*(-d)^(1/3))/(a^2*d + (d -
1)*x^2 - b^2 - 2*(a*d - b)*x)) - 2*(-d)^(2/3)*log(-((b^2 - 2*b*x + x^2)*(-d
)^(2/3) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*d)/(b^2 -
2*b*x + x^2)) + (-d)^(2/3)*log(-((-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b
^2)*x)^(1/3)*(a*d - d*x) + (b^2 - 2*b*x + x^2)*(-d)^(1/3) - (-a*b^2 - (a +
2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*(-d)^(2/3))/(b^2 - 2*b*x + x^2)))/(
(a - b)*d), -1/4*(2*sqrt(3)*d*sqrt(-(-d)^(1/3)/d)*arctan(-1/3*sqrt(3)*((b^2
 - 2*b*x + x^2)*(-d)^(1/3) - 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2
)*x)^(2/3)*(-d)^(2/3))*sqrt(-(-d)^(1/3)/d)/(b^2 - 2*b*x + x^2)) - 2*(-d)^(2
/3)*log(-((b^2 - 2*b*x + x^2)*(-d)^(2/3) - (-a*b^2 - (a + 2*b)*x^2 + x^3 +
(2*a*b + b^2)*x)^(2/3)*d)/(b^2 - 2*b*x + x^2)) + (-d)^(2/3)*log(-((-a*b^2 -
 (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(a*d - d*x) + (b^2 - 2*b*x +
x^2)*(-d)^(1/3) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*(-
d)^(2/3))/(b^2 - 2*b*x + x^2)))/((a - b)*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b-x}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}\left(a^2d+(d-1)x^2-b^2-2\,(ad-b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)/((-a+x)*(-b+x)^2)^(1/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-1+d)*x^
2),x, algorithm="giac")
```

```
[Out] integrate(-(b - x)/((-(a - x)*(b - x)^2)^(1/3)*(a^2*d + (d - 1)*x^2 - b^2 -
 2*(a*d - b)*x)), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{-b+x}{\left((-a+x)\,(-b+x)^2\right)^{\frac{1}{3}}\left(-b^2+a^2d+2\,(-ad+b)\,x+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b+x)/((-a+x)*(-b+x)^2)^(1/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-1+d)*x^2),x)
```

```
[Out] int((-b+x)/((-a+x)*(-b+x)^2)^(1/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-1+d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{b-x}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}\left(a^2d+(d-1)x^2-b^2-2\,(ad-b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)/((-a+x)*(-b+x)^2)^(1/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-1+d)*x^
2),x, algorithm="maxima")
```

```
[Out] -integrate((b - x)/((-(a - x)*(b - x)^2)^(1/3)*(a^2*d + (d - 1)*x^2 - b^2 -
 2*(a*d - b)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{b-x}{\left(-\left(a-x\right)\,{\left(b-x\right)}^2\right)^{1/3}\,\left(a^2\,d+2\,x\,\left(b-a\,d\right)-b^2+x^2\,\left(d-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b - x)/((-(a - x)*(b - x)^2)^(1/3)*(a^2*d + 2*x*(b - a*d) - b^2 + x^2
*(d - 1))),x)
```

```
[Out] int(-(b - x)/((-(a - x)*(b - x)^2)^(1/3)*(a^2*d + 2*x*(b - a*d) - b^2 + x^2
*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)/((-a+x)*(-b+x)**2)**(1/3)/(-b**2+a**2*d+2*(-a*d+b)*x+(-1+d
)*x**2),x)
```

```
[Out] Timed out
```

3.2411 $\int \frac{(b^2+ax^2)^2}{(-b^2+ax^2)^2\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$

Optimal. Leaf size=569

$$\frac{2x(ax^2-2b^2)}{(ax^2-b^2)\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{2\sqrt{2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}-\frac{\sqrt{\sqrt{ax^2+b^2}+b}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{a}}-\frac{\sqrt{\frac{1}{2}(\sqrt{2}-1)}\sqrt{b}\tan^{-1}\left(\frac{\cdots}{\sqrt{2(\sqrt{2}-\cdots}}\right)}{\sqrt{\cdots}}$$

Rubi [F] time = 2.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(b^2+ax^2)^2}{(-b^2+ax^2)^2\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(b^2 + a*x^2)^2/((-b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

```
[Out] Defer[Int][1/Sqrt[b + Sqrt[b^2 + a*x^2]], x] - b*Defer[Int][1/((b - Sqrt[a]
*x)*Sqrt[b + Sqrt[b^2 + a*x^2]]), x] - b*Defer[Int][1/((b + Sqrt[a]*x)*Sqrt
[b + Sqrt[b^2 + a*x^2]]), x] + a*b^2*Defer[Int][1/((Sqrt[a]*b - a*x)^2*Sqrt
[b + Sqrt[b^2 + a*x^2]]), x] + a*b^2*Defer[Int][1/((Sqrt[a]*b + a*x)^2*Sqrt
[b + Sqrt[b^2 + a*x^2]]), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b^2+ax^2)^2}{(-b^2+ax^2)^2\sqrt{b+\sqrt{b^2+ax^2}}}\,dx &= \int \left(\frac{1}{\sqrt{b+\sqrt{b^2+ax^2}}}+\frac{4b^4}{(b^2-ax^2)^2\sqrt{b+\sqrt{b^2+ax^2}}}-\frac{4b^2}{(b^2-ax^2)\sqrt{b+\cdots}}\right.\\
&= -\left((4b^2)\int \frac{1}{(b^2-ax^2)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx\right)+(4b^4)\int \frac{1}{(b^2-ax^2)^2\sqrt{b+\cdots}}\\
&= -\left((4b^2)\int \left(\frac{1}{2b(b-\sqrt{a}\,x)\sqrt{b+\sqrt{b^2+ax^2}}}+\frac{1}{2b(b+\sqrt{a}\,x)\sqrt{b+\sqrt{b^2+\cdots}}}\right.\right.\\
&= -\left((2b)\int \frac{1}{(b-\sqrt{a}\,x)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx\right)-(2b)\int \frac{1}{(b+\sqrt{a}\,x)\sqrt{b+\sqrt{\cdots}}}\\
&= -\left((2b)\int \frac{1}{(b-\sqrt{a}\,x)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx\right)-(2b)\int \frac{1}{(b+\sqrt{a}\,x)\sqrt{b+\sqrt{\cdots}}}\\
&= b\int \frac{1}{(b-\sqrt{a}\,x)\sqrt{b+\sqrt{b^2+ax^2}}}\,dx+b\int \frac{1}{(b+\sqrt{a}\,x)\sqrt{b+\sqrt{b^2+ax^2}}}\,\cdots
\end{aligned}$$

Mathematica [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2+ax^2\right)^2}{\left(-b^2+ax^2\right)^2\sqrt{b+\sqrt{b^2+ax^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(b^2 + a*x^2)^2/((-b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]]),x]
```

```
[Out] Integrate[(b^2 + a*x^2)^2/((-b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]]), x
]
```

IntegrateAlgebraic [A] time = 1.53, size = 241, normalized size = 0.42

$$\frac{\sqrt{2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}{\sqrt{a}\,x}\right)}{\sqrt{a}}+\frac{\sqrt{\frac{1}{\sqrt{2}}-\frac{1}{2}}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{1+\sqrt{2}}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}{\sqrt{a}\,x}\right)}{\sqrt{a}}-\frac{\sqrt{\frac{1}{2}+\frac{1}{\sqrt{2}}}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{2}-1}\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}{\sqrt{a}\,x}\right)}{\sqrt{a}}+\frac{2\left(2b^2x-ax^3\right)}{\left(b^2-ax^2\right)\sqrt{\sqrt{ax^2+b^2}+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b^2 + a*x^2)^2/((-b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*
x^2]]),x]
```

```
[Out] (2*(2*b^2*x - a*x^3))/((b^2 - a*x^2)*Sqrt[b + Sqrt[b^2 + a*x^2]]) + (Sqrt[2
]*Sqrt[b]*ArcTan[(Sqrt[2]*Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])/(Sqrt[a]*x)]
)/Sqrt[a] + (Sqrt[-1/2 + 1/Sqrt[2]]*Sqrt[b]*ArcTan[(Sqrt[1 + Sqrt[2]]*Sqrt[
b]*Sqrt[b + Sqrt[b^2 + a*x^2]])/(Sqrt[a]*x)])/Sqrt[a] - (Sqrt[1/2 + 1/Sqrt[
2]]*Sqrt[b]*ArcTanh[(Sqrt[-1 + Sqrt[2]]*Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]]
)/(Sqrt[a]*x)])/Sqrt[a]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^2/(a*x^2-b^2)^2/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2+b^2\right)^2}{\left(ax^2-b^2\right)^2\sqrt{b+\sqrt{ax^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^2/(a*x^2-b^2)^2/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((a*x^2 + b^2)^2/((a*x^2 - b^2)^2*sqrt(b + sqrt(a*x^2 + b^2))), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^2+b^2\right)^2}{\left(a\,x^2-b^2\right)^2\sqrt{b+\sqrt{a\,x^2+b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b^2)^2/(a*x^2-b^2)^2/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x^2+b^2)^2/(a*x^2-b^2)^2/(b+(a*x^2+b^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^2 + b^2)^2}{(ax^2 - b^2)^2 \sqrt{b + \sqrt{ax^2 + b^2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^2/(a*x^2-b^2)^2/(b+(a*x^2+b^2)^(1/2))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((a*x^2 + b^2)^2/((a*x^2 - b^2)^2*sqrt(b + sqrt(a*x^2 + b^2))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b^2 + a\,x^2)^2}{(a\,x^2 - b^2)^2 \sqrt{b + \sqrt{b^2 + a\,x^2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b^2)^2/((a*x^2 - b^2)^2*(b + (a*x^2 + b^2)^(1/2))^(1/2)),x)
```

```
[Out] int((a*x^2 + b^2)^2/((a*x^2 - b^2)^2*(b + (a*x^2 + b^2)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^2 + b^2)^2}{\sqrt{b + \sqrt{ax^2 + b^2}}\, (ax^2 - b^2)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b**2)**2/(a*x**2-b**2)**2/(b+(a*x**2+b**2)**(1/2))**(1/2)
,x)
```

```
[Out] Integral((a*x**2 + b**2)**2/(sqrt(b + sqrt(a*x**2 + b**2))*(a*x**2 - b**2)*
*2), x)
```

3.2412 $\int \frac{\sqrt{1+x^4}}{(1+x)^3\sqrt{x^2+\sqrt{1+x^4}}}\,dx$

Optimal. Leaf size=604

$$-\frac{5\tan^{-1}\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{\sqrt{2}-1}}\right)}{2\sqrt{2\left(\sqrt{2}-1\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{\sqrt{2}-1}}\right)}{\sqrt{\sqrt{2}-1}}-3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(73+53\sqrt{2}\right)}\tan^{-1}\left(\right.$$

Rubi [F] time = 0.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1+x^4}}{(1+x)^3\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + x^4]/((1 + x)^3*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] Defer[Int][Sqrt[1 + x^4]/((1 + x)^3*Sqrt[x^2 + Sqrt[1 + x^4]]), x]
```

Rubi steps

$$\int \frac{\sqrt{1+x^4}}{(1+x)^3\sqrt{x^2+\sqrt{1+x^4}}}\,dx=\int \frac{\sqrt{1+x^4}}{(1+x)^3\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Mathematica [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+x^4}}{(1+x)^3\sqrt{x^2+\sqrt{1+x^4}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 + x^4]/((1 + x)^3*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] Integrate[Sqrt[1 + x^4]/((1 + x)^3*Sqrt[x^2 + Sqrt[1 + x^4]]), x]
```

IntegrateAlgebraic [A] time = 8.37, size = 600, normalized size = 0.99

$$-\frac{5\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^4+1}+x^2}\right)}{2\sqrt{2\sqrt{2}-2}}-\frac{\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{\sqrt{x^4+1}+x^2}\right)}{\sqrt{\sqrt{2}-1}}-3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)+\frac{1}{2}\sqrt{\frac{73}{2}+\frac{53}{\sqrt{2}}}\tan^{-1}\left(\frac{\sqrt{2\sqrt{2}-2}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)-\frac{5\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^4+1}+x^2}\right)}{2\sqrt{2+2\sqrt{2}}}+\frac{\tanh^{-1}\left(\sqrt{\sqrt{2}-1}\sqrt{\sqrt{x^4+1}+x^2}\right)}{\sqrt{1+\sqrt{2}}}+\frac{1}{2}\sqrt{\frac{53}{\sqrt{2}}-\frac{73}{2}}\tanh^{-1}\left(\frac{\sqrt{2+2\sqrt{2}}x\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}+x^2+1}\right)+\frac{x\sqrt{\sqrt{x^4+1}+x^2}\left(-48x^{12}+72x^{10}-96x^8+72x^6-52x^4+9x^2-5\right)+\sqrt{x^4+1}\left(x\sqrt{\sqrt{x^4+1}+x^2}\left(-48x^{10}+72x^8-72x^6+36x^4-22x^2\right)+\left(16x^{12}-16x^{10}+28x^8-4x^6+9x^4+x^2\right)\sqrt{\sqrt{x^4+1}+x^2}\right)+\left(16x^{14}-16x^{12}+36x^{10}-12x^8+21x^6+x^4+2x^2\right)\sqrt{\sqrt{x^4+1}+x^2}}{2x\left(x^2-1\right)^2\left(\sqrt{x^4+1}+x^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x^4]/((1 + x)^3*Sqrt[x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] (x*(-5 + 9*x^2 - 52*x^4 + 72*x^6 - 96*x^8 + 72*x^10 - 48*x^12)*Sqrt[x^2 + S
qrt[1 + x^4]] + (2*x^2 + x^4 + 21*x^6 - 12*x^8 + 36*x^10 - 16*x^12 + 16*x^1
4)*Sqrt[x^2 + Sqrt[1 + x^4]] + Sqrt[1 + x^4]*(x*(-22*x^2 + 36*x^4 - 72*x^6
+ 72*x^8 - 48*x^10)*Sqrt[x^2 + Sqrt[1 + x^4]] + (x^2 + 9*x^4 - 4*x^6 + 28*x
^8 - 16*x^10 + 16*x^12)*Sqrt[x^2 + Sqrt[1 + x^4]]))/(2*x*(-1 + x^2)^2*(x^2
+ Sqrt[1 + x^4])^5) - ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]]/S
qrt[-1 + Sqrt[2]] - (5*ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]])
```

```
/(2*Sqrt[-2 + 2*Sqrt[2]]) - 3*Sqrt[2]*ArcTan[(Sqrt[2]*x*Sqrt[x^2 + Sqrt[1 +
 x^4]])/(1 + x^2 + Sqrt[1 + x^4])] + (Sqrt[73/2 + 53/Sqrt[2]]*ArcTan[(Sqrt[
-2 + 2*Sqrt[2]]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])])/2
+ ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[1 + Sqrt[2]] -
 (5*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]])/(2*Sqrt[2 + 2*Sq
rt[2]]) + (Sqrt[-73/2 + 53/Sqrt[2]]*ArcTanh[(Sqrt[2 + 2*Sqrt[2]]*x*Sqrt[x^2
 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])])/2
```

fricas [A] time = 5.76, size = 562, normalized size = 0.93

$$-\frac{4\sqrt{2}\left(x^{2}+2x+1\right)\sqrt{53\sqrt{2}+73}\arctan\left(\frac{\sqrt{2}\left(14x^{2}-9\sqrt{2}\left(x^{2}+1\right)+2\sqrt{x^{4}+1}\left(7\sqrt{2}-9\right)+14\right)\sqrt{53\sqrt{2}+73}\sqrt{\sqrt{2}+1}+2\left(9x^{3}+5x^{2}-\sqrt{2}\left(7x^{3}+2x^{2}-2x+7\right)+\sqrt{x^{4}+1}\left(\sqrt{2}(7x+2)-9x-5\right)-5x+9\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}\sqrt{53\sqrt{2}+73}}{34\left(x^{2}-2x+1\right)}\right)-\sqrt{2}\left(x^{2}+2x+1\right)\sqrt{53\sqrt{2}-73}\log\left(-\frac{17\left(2x^{3}-\sqrt{2}\left(x^{3}-x^{2}-x-1\right)+\sqrt{x^{4}+1}\left(\sqrt{2}(x-1)-2x\right)-2\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}+\left(4x^{2}+5\sqrt{2}\left(x^{2}+1\right)+2\sqrt{x^{4}+1}\left(2\sqrt{2}+5\right)+4\right)\sqrt{53\sqrt{2}-73}}{x^{2}+2x+1}\right)+\sqrt{2}\left(x^{2}+2x+1\right)\sqrt{53\sqrt{2}-73}\log\left(-\frac{17\left(2x^{3}-\sqrt{2}\left(x^{3}-x^{2}-x-1\right)+\sqrt{x^{4}+1}\left(\sqrt{2}(x-1)-2x\right)-2\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}-\left(4x^{2}+5\sqrt{2}\left(x^{2}+1\right)+2\sqrt{x^{4}+1}\left(2\sqrt{2}+5\right)+4\right)\sqrt{53\sqrt{2}-73}}{x^{2}+2x+1}\right)-24\sqrt{2}\left(x^{2}+2x+1\right)\arctan\left(-\frac{\left(\sqrt{2}x^{2}-\sqrt{2}\sqrt{x^{4}+1}\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}}{2x}\right)-8\left(2x^{4}+8x^{3}+5x^{2}-\sqrt{x^{4}+1}\left(2x^{2}+8x+5\right)+x\right)\sqrt{x^{2}+\sqrt{x^{4}+1}}}{16\left(x^{2}+2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(1+x)^3/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/16*(4*sqrt(2)*(x^2 + 2*x + 1)*sqrt(53*sqrt(2) + 73)*arctan(1/34*(sqrt(2)
*(14*x^2 - 9*sqrt(2)*(x^2 + 1) + 2*sqrt(x^4 + 1)*(7*sqrt(2) - 9) + 14)*sqrt
(53*sqrt(2) + 73)*sqrt(sqrt(2) + 1) + 2*(9*x^3 + 5*x^2 - sqrt(2)*(7*x^3 + 2
*x^2 - 2*x + 7) + sqrt(x^4 + 1)*(sqrt(2)*(7*x + 2) - 9*x - 5) - 5*x + 9)*sq
rt(x^2 + sqrt(x^4 + 1))*sqrt(53*sqrt(2) + 73))/(x^2 - 2*x + 1)) - sqrt(2)*(
x^2 + 2*x + 1)*sqrt(53*sqrt(2) - 73)*log(-(17*(2*x^3 - sqrt(2)*(x^3 - x^2 -
 x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 +
1)) + (4*x^2 + 5*sqrt(2)*(x^2 + 1) + 2*sqrt(x^4 + 1)*(2*sqrt(2) + 5) + 4)*s
qrt(53*sqrt(2) - 73))/(x^2 + 2*x + 1)) + sqrt(2)*(x^2 + 2*x + 1)*sqrt(53*sq
rt(2) - 73)*log(-(17*(2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(
sqrt(2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - (4*x^2 + 5*sqrt(2)*
(x^2 + 1) + 2*sqrt(x^4 + 1)*(2*sqrt(2) + 5) + 4)*sqrt(53*sqrt(2) - 73))/(x^
2 + 2*x + 1)) - 24*sqrt(2)*(x^2 + 2*x + 1)*arctan(-1/2*(sqrt(2)*x^2 - sqrt(
2)*sqrt(x^4 + 1))*sqrt(x^2 + sqrt(x^4 + 1))/x) - 8*(2*x^4 + 8*x^3 + 5*x^2 -
 sqrt(x^4 + 1)*(2*x^2 + 8*x + 5) + x)*sqrt(x^2 + sqrt(x^4 + 1)))/(x^2 + 2*x
 + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{4}+1}}{\sqrt{x^{2}+\sqrt{x^{4}+1}}\,(x+1)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(1+x)^3/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(x^4 + 1)/(sqrt(x^2 + sqrt(x^4 + 1))*(x + 1)^3), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{4}+1}}{(1+x)^{3}\sqrt{x^{2}+\sqrt{x^{4}+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(1/2)/(1+x)^3/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int((x^4+1)^(1/2)/(1+x)^3/(x^2+(x^4+1)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{4}+1}}{\sqrt{x^{2}+\sqrt{x^{4}+1}}\,(x+1)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(1+x)^3/(x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x^4 + 1)/(sqrt(x^2 + sqrt(x^4 + 1))*(x + 1)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x^4+1}}{\sqrt{\sqrt{x^4+1}+x^2}\,(x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)^(1/2)/(((x^4 + 1)^(1/2) + x^2)^(1/2)*(x + 1)^3),x)
```

```
[Out] int((x^4 + 1)^(1/2)/(((x^4 + 1)^(1/2) + x^2)^(1/2)*(x + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4+1}}{(x+1)^3\sqrt{x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(1/2)/(1+x)**3/(x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x**4 + 1)/((x + 1)**3*sqrt(x**2 + sqrt(x**4 + 1))), x)
```

3.2413 $\int \frac{x^2}{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}\,dx$

Optimal. Leaf size=617

$$\frac{-17265\left(1-\sqrt{1-\sqrt{1-\frac{1}{x}}}\right)^{23/2}+204165\left(1-\sqrt{1-\sqrt{1-\frac{1}{x}}}\right)^{21/2}-1002573\left(1-\sqrt{1-\sqrt{1-\frac{1}{x}}}\right)^{19/2}+2613381}{}$$

Rubi [A] time = 2.03, antiderivative size = 1229, normalized size of antiderivative = 1.99, number of steps used = 22, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1586, 1692, 207, 1178, 1166, 203}

Warning: Unable to verify antiderivative.

```
[In] Int[x^2/Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]],x]
```

```
[Out] 1/(1536*(1 - Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]])^6) + 7/(2560*(1 - Sqrt[1
 - Sqrt[1 - Sqrt[1 - x^(-1)]]])^5) + 11/(1024*(1 - Sqrt[1 - Sqrt[1 - Sqrt[1
 - x^(-1)]]])^4) + 41/(1536*(1 - Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]])^3) +
 289/(4096*(1 - Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]])^2) + 703/(4096*(1 - S
qrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]])) - 1/(1536*(1 + Sqrt[1 - Sqrt[1 - Sqrt
[1 - x^(-1)]]])^6) - 7/(2560*(1 + Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]])^5)
- 11/(1024*(1 + Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]])^4) - 41/(1536*(1 + Sq
rt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]])^3) - 289/(4096*(1 + Sqrt[1 - Sqrt[1 - S
qrt[1 - x^(-1)]]])^2) - 703/(4096*(1 + Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]]
)) - (Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]]*(1 + Sqrt[1 - Sqrt[1 - x^(-1)]])
)/(24*(1 + Sqrt[1 - x^(-1)])^3) - (Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]]*(1
+ Sqrt[1 - Sqrt[1 - x^(-1)]]))/(16*(1 + Sqrt[1 - x^(-1)])^2) - (Sqrt[1 - Sq
rt[1 - Sqrt[1 - x^(-1)]]]*(20 + 19*Sqrt[1 - Sqrt[1 - x^(-1)]]))/(384*(1 + S
qrt[1 - x^(-1)])^2) - (Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]]*(1 + Sqrt[1 - S
qrt[1 - x^(-1)]]))/(16*(1 + Sqrt[1 - x^(-1)])) - (Sqrt[1 - Sqrt[1 - Sqrt[1
- x^(-1)]]]*(12 + 11*Sqrt[1 - Sqrt[1 - x^(-1)]]))/(128*(1 + Sqrt[1 - x^(-1)
])) - (Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]]*(121 + 108*Sqrt[1 - Sqrt[1 - x^
(-1)]]))/(1536*(1 + Sqrt[1 - x^(-1)])) - ((1 + Sqrt[2])^(3/2)*ArcTan[Sqrt[1
 - Sqrt[1 - Sqrt[1 - x^(-1)]]]/Sqrt[-1 + Sqrt[2]]])/(32*Sqrt[2]) - (Sqrt[52
7 + 373*Sqrt[2]]*ArcTan[Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]]/Sqrt[-1 + Sqrt
[2]]])/256 - (Sqrt[(12049 + 8521*Sqrt[2])/2]*ArcTan[Sqrt[1 - Sqrt[1 - Sqrt[
1 - x^(-1)]]]/Sqrt[-1 + Sqrt[2]]])/1024 + (703*ArcTanh[Sqrt[1 - Sqrt[1 - Sq
rt[1 - x^(-1)]]]])/2048 + ((-1 + Sqrt[2])^(3/2)*ArcTanh[Sqrt[1 - Sqrt[1 - S
qrt[1 - x^(-1)]]]/Sqrt[1 + Sqrt[2]]])/(32*Sqrt[2]) + (Sqrt[-527 + 373*Sqrt[
2]]*ArcTanh[Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]]/Sqrt[1 + Sqrt[2]]])/256 +
(Sqrt[(-12049 + 8521*Sqrt[2])/2]*ArcTanh[Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]
]]/Sqrt[1 + Sqrt[2]]])/1024
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}\,dx = 2\,\text{Subst}\left(\int \frac{x}{\sqrt{1-\sqrt{1-x}}\,\left(-1+x^2\right)^4}\,dx, x, \sqrt{1-\frac{1}{x}}\right)$$

$$= -\left(4\,\text{Subst}\left(\int \frac{1-x^2}{\sqrt{1-x}\,x^7\left(-2+x^2\right)^4}\,dx, x, \sqrt{1-\sqrt{1-\frac{1}{x}}}\right)\right)$$

$$= -\left(4\,\text{Subst}\left(\int \frac{\sqrt{1-x}\,(1+x)}{x^7\left(-2+x^2\right)^4}\,dx, x, \sqrt{1-\sqrt{1-\frac{1}{x}}}\right)\right)$$

$$= 8\,\text{Subst}\left(\int \frac{x^2\left(-2+x^2\right)}{\left(-1+x^2\right)^7\left(-1-2x^2+x^4\right)^4}\,dx, x, \sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)$$

$$= 8\,\text{Subst}\left(\int\left(-\frac{1}{2048(-1+x)^7}+\frac{7}{4096(-1+x)^6}-\frac{11}{2048(-1+x)^5}+\frac{41}{4096(-1+x)^4}\right.\right.$$

$$= \frac{1}{1536\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^6}+\frac{7}{2560\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^5}+\frac{}{1024\left(1-\right.}$$

$$= \frac{1}{1536\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^6}+\frac{7}{2560\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^5}+\frac{}{1024\left(1-\right.}$$

$$= \frac{1}{1536\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^6}+\frac{7}{2560\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^5}+\frac{}{1024\left(1-\right.}$$

$$= \frac{1}{1536\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^6}+\frac{7}{2560\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^5}+\frac{}{1024\left(1-\right.}$$

$$= \frac{1}{1536\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^6}+\frac{7}{2560\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^5}+\frac{}{1024\left(1-\right.}$$

$$= \frac{1}{1536\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^6}+\frac{7}{2560\left(1-\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}\right)^5}+\frac{}{1024\left(1-\right.}$$

Mathematica [A] time = 1.98, size = 547, normalized size = 0.89

$$\frac{1024\sqrt{1-\sqrt{\frac{x-1}{x}}}\,x^3+1024\sqrt{1-\sqrt{\frac{x-1}{x}}}\sqrt{\frac{x-1}{x}}\,x^3+20480x^3+1888\sqrt{1-\sqrt{\frac{x-1}{x}}}\,x^2+1760\sqrt{1-\sqrt{\frac{x-1}{x}}}\sqrt{\frac{x-1}{x}}\,x^2+64\sqrt{\frac{x-1}{x}}\,x^2+1344x^2+6702\sqrt{1-\sqrt{\frac{x-1}{x}}}\,x+6030\sqrt{1-\sqrt{\frac{x-1}{x}}}\sqrt{\frac{x-1}{x}}\,x+324\sqrt{\frac{x-1}{x}}\,x+3092x-10545\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\log\left(1-\frac{1}{\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}}\right)+10545\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\log\left(\frac{1}{\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}}+1\right)+30\sqrt{\sqrt{2}-1}\left(498+361\sqrt{2}\right)\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\tan^{-1}\left(\frac{1}{\sqrt{1+\sqrt{2}}\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}}\right)+30\sqrt{1+\sqrt{2}}\left(361\sqrt{2}-498\right)\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}\tanh^{-1}\left(\frac{1}{\sqrt{\sqrt{2}-1}\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}}\right)-34530}{61440\sqrt{1-\sqrt{1-\sqrt{\frac{x-1}{x}}}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]],x]
```

```
[Out] (-34530 + 3092*x + 6702*Sqrt[1 - Sqrt[(-1 + x)/x]]*x + 324*Sqrt[(-1 + x)/x]
*x + 6030*Sqrt[1 - Sqrt[(-1 + x)/x]]*Sqrt[(-1 + x)/x]*x + 1344*x^2 + 1888*S
qrt[1 - Sqrt[(-1 + x)/x]]*x^2 + 64*Sqrt[(-1 + x)/x]*x^2 + 1760*Sqrt[1 - Sqr
t[(-1 + x)/x]]*Sqrt[(-1 + x)/x]*x^2 + 20480*x^3 + 1024*Sqrt[1 - Sqrt[(-1 +
x)/x]]*x^3 + 1024*Sqrt[1 - Sqrt[(-1 + x)/x]]*Sqrt[(-1 + x)/x]*x^3 + 30*Sqrt
[-1 + Sqrt[2]]*(498 + 361*Sqrt[2])*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]]*Arc
Tan[1/(Sqrt[1 + Sqrt[2]]*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]])] + 30*Sqrt[1
 + Sqrt[2]]*(-498 + 361*Sqrt[2])*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]]*ArcTa
nh[1/(Sqrt[-1 + Sqrt[2]]*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]])] - 10545*Sqr
t[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]]*Log[1 - 1/Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)
/x]]]] + 10545*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]]*Log[1 + 1/Sqrt[1 - Sqrt
[1 - Sqrt[(-1 + x)/x]]]])/(61440*Sqrt[1 - Sqrt[1 - Sqrt[(-1 + x)/x]]])
```

IntegrateAlgebraic [F] time = 3.70, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]],x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2/Sqrt[1 - Sqrt[1 - Sqrt[1 - x^(-1)]]], x]
```

fricas [A] time = 0.46, size = 374, normalized size = 0.61

$$-\frac{1}{1024}\sqrt{2}\sqrt{74545\sqrt{2}+105233}\arctan\left(-\frac{1}{6319}\sqrt{74545\sqrt{2}+105233}\left(112\sqrt{2}-137\right)\sqrt{\sqrt{2}-\sqrt{-\sqrt{\frac{x-1}{x}}+1}}+\frac{1}{6319}\sqrt{74545\sqrt{2}+105233}\left(112\sqrt{2}-137\right)\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}\right)+\frac{1}{4096}\sqrt{2}\sqrt{74545\sqrt{2}-105233}\log\left(\sqrt{74545\sqrt{2}-105233}\left(249\sqrt{2}+361\right)+6319\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}\right)-\frac{1}{4096}\sqrt{2}\sqrt{74545\sqrt{2}-105233}\log\left(-\sqrt{74545\sqrt{2}-105233}\left(249\sqrt{2}+361\right)+6319\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}\right)-\frac{1}{30720}\left(32x^2-\left(512x^3+912x^2+\left(10752x^3+12368x^2+17265x\right)\sqrt{\frac{x-1}{x}}+3177x\right)\sqrt{-\sqrt{\frac{x-1}{x}}+1}-2\left(5120x^3+5744x^2+7125x\right)\sqrt{\frac{x-1}{x}}+174x\right)\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}+\frac{703}{4096}\log\left(\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}+1\right)-\frac{703}{4096}\log\left(\sqrt{-\sqrt{-\sqrt{\frac{x-1}{x}}+1}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(1-(1-1/x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/1024*sqrt(2)*sqrt(74545*sqrt(2) + 105233)*arctan(-1/6319*sqrt(74545*sqrt
(2) + 105233)*(112*sqrt(2) - 137)*sqrt(sqrt(2) - sqrt(-sqrt((x - 1)/x) + 1)
) + 1/6319*sqrt(74545*sqrt(2) + 105233)*(112*sqrt(2) - 137)*sqrt(-sqrt(-sqr
t((x - 1)/x) + 1) + 1)) + 1/4096*sqrt(2)*sqrt(74545*sqrt(2) - 105233)*log(s
qrt(74545*sqrt(2) - 105233)*(249*sqrt(2) + 361) + 6319*sqrt(-sqrt(-sqrt((x
- 1)/x) + 1) + 1)) - 1/4096*sqrt(2)*sqrt(74545*sqrt(2) - 105233)*log(-sqrt(
74545*sqrt(2) - 105233)*(249*sqrt(2) + 361) + 6319*sqrt(-sqrt(-sqrt((x - 1)
/x) + 1) + 1)) - 1/30720*(32*x^2 - (512*x^3 + 912*x^2 + (10752*x^3 + 12368*
x^2 + 17265*x)*sqrt((x - 1)/x) + 3177*x)*sqrt(-sqrt((x - 1)/x) + 1) - 2*(51
20*x^3 + 5744*x^2 + 7125*x)*sqrt((x - 1)/x) + 174*x)*sqrt(-sqrt(-sqrt((x -
1)/x) + 1) + 1) + 703/4096*log(sqrt(-sqrt(-sqrt((x - 1)/x) + 1) + 1) + 1) -
 703/4096*log(sqrt(-sqrt(-sqrt((x - 1)/x) + 1) + 1) - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(1-(1-1/x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1-(1-(1-1/x)^(1/2))^(1/2))^(1/2),x)
[Out] int(x^2/(1-(1-(1-1/x)^(1/2))^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-\sqrt{-\sqrt{-\frac{1}{x}+1}+1}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(1-(1-1/x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(-sqrt(-sqrt(-1/x + 1) + 1) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1 - (1 - (1 - 1/x)^(1/2))^(1/2))^(1/2),x)
[Out] int(x^2/(1 - (1 - (1 - 1/x)^(1/2))^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{1-\sqrt{1-\sqrt{1-\frac{1}{x}}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1-(1-(1-1/x)**(1/2))**(1/2))**(1/2),x)
[Out] Integral(x**2/sqrt(1 - sqrt(1 - sqrt(1 - 1/x))), x)
```

3.2414 $\int \frac{x^2}{\sqrt{1+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=650

$$\frac{231b^3\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}\right)}{2048a^3}+\frac{3b^2\tanh^{-1}\left(\sqrt{\sqrt{\sqrt{a^2x^2-b}+ax}+1}\right)}{16a^3}+\frac{\sqrt{a^2x^2-b}\left(\sqrt{\sqrt{\sqrt{a^2x^2-\cdots}}}\right.}{\cdots}$$

Rubi [F] time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\sqrt{1+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]],x]
```

```
[Out] Defer[Int][x^2/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]], x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{x^2}{\sqrt{1+\sqrt{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Mathematica [A] time = 1.83, size = 655, normalized size = 1.01

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]],x]
```

```
[Out] -1/2*((1 + b)*Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]] + (b^2*(640 + 793*b)
*Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]])/(1024*Sqrt[a*x + Sqrt[-b + a^2*x
^2]]) - ((5 + b)*(1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]])^(3/2))/3 - (b^2*(384
+ 2279*b)*(1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]])^(3/2))/(1536*(a*x + Sqrt[-b
+ a^2*x^2])) + 2*(1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]])^(5/2) + (3481*b^3*(1
+ Sqrt[a*x + Sqrt[-b + a^2*x^2]])^(5/2))/(1920*(a*x + Sqrt[-b + a^2*x^2])^(
3/2)) - (10*(1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]])^(7/2))/7 - (417*b^3*(1 + S
qrt[a*x + Sqrt[-b + a^2*x^2]])^(7/2))/(320*(a*x + Sqrt[-b + a^2*x^2])^2) +
(5*(1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]])^(9/2))/9 + (61*b^3*(1 + Sqrt[a*x +
Sqrt[-b + a^2*x^2]])^(9/2))/(120*(a*x + Sqrt[-b + a^2*x^2])^(5/2)) - (1 + S
qrt[a*x + Sqrt[-b + a^2*x^2]])^(11/2)/11 - (b^3*(1 + Sqrt[a*x + Sqrt[-b + a
^2*x^2]])^(11/2))/(12*(a*x + Sqrt[-b + a^2*x^2])^3) + (3*b^2*(128 + 77*b)*L
og[1 - 1/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]])/2048 - (3*b^2*(128 + 77
*b)*Log[1 + 1/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]])/2048)/a^3
```

IntegrateAlgebraic [A] time = 2.21, size = 650, normalized size = 1.00

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]],x]
```

```
[Out] ((-491520*b^2 - 591360*b^3 - 533610*b^4 + 3932160*a*b*x + 5304320*a*b^2*x +
 365904*a*b^3*x + 3932160*a^2*b*x^2 + 1774080*a^2*b^2*x^2 + 1067220*a^2*b^3
*x^2 - 5242880*a^3*x^3 - 2293760*a^3*b*x^3 - 3932160*a^4*x^4 - 5734400*a^5*
x^5)*Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]] + (409600*b^2 + 1005312*b^3 +
 800415*b^4 - 1966080*a*b*x - 1935360*a*b^2*x - 426888*a*b^3*x - 3276800*a^
2*b*x^2 - 2661120*a^2*b^2*x^2 - 1600830*a^2*b^3*x^2 + 2621440*a^3*x^3 - 172
0320*a^3*b*x^3 + 3276800*a^4*x^4 + 5160960*a^5*x^5)*Sqrt[a*x + Sqrt[-b + a^
2*x^2]]*Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]] + Sqrt[-b + a^2*x^2]*((131
0720*b + 2007040*b^2 + 365904*b^3 + 1966080*a*b*x + 1774080*a*b^2*x + 10672
20*a*b^3*x - 5242880*a^2*x^2 - 5160960*a^2*b*x^2 - 3932160*a^3*x^3 - 573440
0*a^4*x^4)*Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]] + (-655360*b - 860160*b
^2 - 426888*b^3 - 1638400*a*b*x - 2661120*a*b^2*x - 1600830*a*b^3*x + 26214
40*a^2*x^2 + 860160*a^2*b*x^2 + 3276800*a^3*x^3 + 5160960*a^4*x^4)*Sqrt[a*x
 + Sqrt[-b + a^2*x^2]]*Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]))/(7096320*
a^3*Sqrt[-b + a^2*x^2]*(-b + 4*a^2*x^2) + 7096320*a^3*(-3*a*b*x + 4*a^3*x^3
)) + (3*b^2*ArcTanh[Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]])/(16*a^3) + (
231*b^3*ArcTanh[Sqrt[1 + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]])/(2048*a^3)
```

fricas [A] time = 0.43, size = 317, normalized size = 0.49

$$\frac{10395\left(77\,b^{3}+128\,b^{2}\right)\log\left(\sqrt{\sqrt{ax+\sqrt{a^{2}x^{2}-b}}+1}+1\right)-10395\left(77\,b^{3}+128\,b^{2}\right)\log\left(\sqrt{\sqrt{ax+\sqrt{a^{2}x^{2}-b}}+1}-1\right)+2\left(1182720\,a^{3}x^{3}+224\left(3267\,a^{2}b-3200\,a^{2}\right)x^{2}-365904\,b^{2}+30\left(17787\,ab^{2}-16384\,a\right)x-2\left(591360\,a^{2}x^{2}+266805\,b^{2}+112\left(3267\,ab+3200\,a\right)x+295680\,b+245760\right)\sqrt{a^{2}x^{2}-b}-\left(1300992\,a^{3}x^{3}+1008\left(847\,a^{2}b-640\,a^{2}\right)x^{2}-426888\,b^{2}+\left(800415\,ab^{2}+354816\,ab-409600\,a\right)x-\left(1300992\,a^{2}x^{2}+800415\,b^{2}+1008\left(847\,ab+640\,a\right)x+1005312\,b+409600\right)\sqrt{a^{2}x^{2}-b}-860160\,b-655360\right)\sqrt{ax+\sqrt{a^{2}x^{2}-b}}-2007040\,b-1310720\right)\sqrt{\sqrt{ax+\sqrt{a^{2}x^{2}-b}}+1}}{14192640\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/14192640*(10395*(77*b^3 + 128*b^2)*log(sqrt(sqrt(a*x + sqrt(a^2*x^2 - b))
 + 1) + 1) - 10395*(77*b^3 + 128*b^2)*log(sqrt(sqrt(a*x + sqrt(a^2*x^2 - b)
) + 1) - 1) + 2*(1182720*a^3*x^3 + 224*(3267*a^2*b - 3200*a^2)*x^2 - 365904
*b^2 + 30*(17787*a*b^2 - 16384*a)*x - 2*(591360*a^2*x^2 + 266805*b^2 + 112*
(3267*a*b + 3200*a)*x + 295680*b + 245760)*sqrt(a^2*x^2 - b) - (1300992*a^3
*x^3 + 1008*(847*a^2*b - 640*a^2)*x^2 - 426888*b^2 + (800415*a*b^2 + 354816
*a*b - 409600*a)*x - (1300992*a^2*x^2 + 800415*b^2 + 1008*(847*a*b + 640*a)
*x + 1005312*b + 409600)*sqrt(a^2*x^2 - b) - 860160*b - 655360)*sqrt(a*x +
sqrt(a^2*x^2 - b)) - 2007040*b - 1310720)*sqrt(sqrt(a*x + sqrt(a^2*x^2 - b)
) + 1))/a^3
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\sqrt{1+\sqrt{ax+\sqrt{a^{2}x^{2}-b}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x)
```

```
[Out] int(x^2/(1+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\sqrt{ax + \sqrt{a^2x^2 - b}} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(a*x+(a^2*x^2-b)^(1/2))^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x^2/sqrt(sqrt(a*x + sqrt(a^2*x^2 - b)) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{\sqrt{a\,x + \sqrt{a^2\,x^2 - b}} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x + (a^2*x^2 - b)^(1/2))^(1/2) + 1)^(1/2),x)
```

```
[Out] int(x^2/((a*x + (a^2*x^2 - b)^(1/2))^(1/2) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\sqrt{ax + \sqrt{a^2x^2 - b}} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+(a*x+(a**2*x**2-b)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(sqrt(a*x + sqrt(a**2*x**2 - b)) + 1), x)
```

3.2415 $\int \sqrt[3]{\frac{x}{-1-ax+3x^2+3ax^3-3x^4-3ax^5+x^6+ax^7}}\,dx$

Optimal. Leaf size=669

$$\frac{\log\left(\sqrt[3]{\frac{x}{ax^7-3ax^5+3ax^3-ax+x^6-3x^4+3x^2-1}}\left(\sqrt[3]{1-a}\,x^2-\sqrt[3]{1-a}\right)+1\right)}{2\sqrt[3]{1-a}}+\frac{\log\left(\left(\sqrt[3]{a+1}\,x^2-\sqrt[3]{a+1}\right)\sqrt[3]{\frac{x}{ax^7-3ax^5+3ax^3-ax+x^6-}}\right.}{2\sqrt[3]{a+1}}$$

Rubi [A] time = 0.20, antiderivative size = 473, normalized size of antiderivative = 0.71, number of steps used = 10, number of rules used = 6, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6688, 6718, 912, 105, 59, 91}

$$\frac{(1-x^2)\sqrt[3]{ax+1}\sqrt[3]{-\frac{x}{(1-x^2)^3(ax+1)}}\log(1-x)}{4\sqrt[3]{a+1}\sqrt[3]{x}}-\frac{(1-x^2)\sqrt[3]{ax+1}\sqrt[3]{-\frac{x}{(1-x^2)^3(ax+1)}}\log(x+1)}{4\sqrt[3]{a-1}\sqrt[3]{x}}+\frac{3(1-x^2)\sqrt[3]{ax+1}\sqrt[3]{-\frac{x}{(1-x^2)^3(ax+1)}}\log\left(\frac{\sqrt[3]{ax+1}}{\sqrt[3]{a-1}}-\sqrt[3]{x}\right)}{4\sqrt[3]{a-1}\sqrt[3]{x}}-\frac{3(1-x^2)\sqrt[3]{ax+1}\sqrt[3]{-\frac{x}{(1-x^2)^3(ax+1)}}\log\left(\frac{\sqrt[3]{ax+1}}{\sqrt[3]{a+1}}-\sqrt[3]{x}\right)}{4\sqrt[3]{a+1}\sqrt[3]{x}}+\frac{\sqrt{3}(1-x^2)\sqrt[3]{ax+1}\sqrt[3]{-\frac{x}{(1-x^2)^3(ax+1)}}\tan^{-1}\left(\frac{2\sqrt[3]{ax+1}}{\sqrt{3}\sqrt[3]{a-1}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{a-1}\sqrt[3]{x}}-\frac{\sqrt{3}(1-x^2)\sqrt[3]{ax+1}\sqrt[3]{-\frac{x}{(1-x^2)^3(ax+1)}}\tan^{-1}\left(\frac{2\sqrt[3]{ax+1}}{\sqrt{3}\sqrt[3]{a+1}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{a+1}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(x/(-1 - a*x + 3*x^2 + 3*a*x^3 - 3*x^4 - 3*a*x^5 + x^6 + a*x^7))^(1/3),
x]
```

```
[Out] (Sqrt[3]*(1 + a*x)^(1/3)*(-(x/((1 + a*x)*(1 - x^2)^3)))^(1/3)*(1 - x^2)*Arc
Tan[1/Sqrt[3] + (2*(1 + a*x)^(1/3))/(Sqrt[3]*(-1 + a)^(1/3)*x^(1/3))]])/(2*(
-1 + a)^(1/3)*x^(1/3)) - (Sqrt[3]*(1 + a*x)^(1/3)*(-(x/((1 + a*x)*(1 - x^2)
^3)))^(1/3)*(1 - x^2)*ArcTan[1/Sqrt[3] + (2*(1 + a*x)^(1/3))/(Sqrt[3]*(1 +
a)^(1/3)*x^(1/3))]])/(2*(1 + a)^(1/3)*x^(1/3)) + ((1 + a*x)^(1/3)*(-(x/((1 +
 a*x)*(1 - x^2)^3)))^(1/3)*(1 - x^2)*Log[1 - x])/(4*(1 + a)^(1/3)*x^(1/3))
 - ((1 + a*x)^(1/3)*(-(x/((1 + a*x)*(1 - x^2)^3)))^(1/3)*(1 - x^2)*Log[1 + x
])/(4*(-1 + a)^(1/3)*x^(1/3)) + (3*(1 + a*x)^(1/3)*(-(x/((1 + a*x)*(1 - x^2
)^3)))^(1/3)*(1 - x^2)*Log[-x^(1/3) + (1 + a*x)^(1/3)/(-1 + a)^(1/3)]])/(4*(
-1 + a)^(1/3)*x^(1/3)) - (3*(1 + a*x)^(1/3)*(-(x/((1 + a*x)*(1 - x^2)^3)))^
(1/3)*(1 - x^2)*Log[-x^(1/3) + (1 + a*x)^(1/3)/(1 + a)^(1/3)]])/(4*(1 + a)^(
1/3)*x^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !FreeQ[w, x] &&  !Fr
eeQ[z, x]
```

Rubi steps

$$\int \sqrt[3]{\frac{x}{-1-ax+3x^2+3ax^3-3x^4-3ax^5+x^6+ax^7}}\,dx = \int \sqrt[3]{\frac{x}{(1+ax)\left(-1+x^2\right)^3}}\,dx$$

$$= \frac{\left(\sqrt[3]{1+ax}\sqrt[3]{\frac{x}{(1+ax)\left(-1+x^2\right)^3}}\left(-1+x^2\right)\right)\int \frac{\sqrt[3]{x}}{\sqrt[3]{1+ax}\left(-1+x^2\right.}}{\sqrt[3]{x}}$$

$$= \frac{\left(\sqrt[3]{1+ax}\sqrt[3]{\frac{x}{(1+ax)\left(-1+x^2\right)^3}}\left(-1+x^2\right)\right)\int \left(-\frac{\sqrt[3]{x}}{2(1-x)\sqrt[3]{1+}}\right.}{\sqrt[3]{x}}$$

$$= -\frac{\left(\sqrt[3]{1+ax}\sqrt[3]{\frac{x}{(1+ax)\left(-1+x^2\right)^3}}\left(-1+x^2\right)\right)\int \frac{\sqrt[3]{x}}{(1-x)\sqrt[3]{1+ax}}}{2\sqrt[3]{x}}$$

$$= -\frac{\left(\sqrt[3]{1+ax}\sqrt[3]{\frac{x}{(1+ax)\left(-1+x^2\right)^3}}\left(-1+x^2\right)\right)\int \frac{1}{(1-x)x^{2/3}\sqrt[3]{1-}}}{2\sqrt[3]{x}}$$

$$= \frac{\sqrt{3}\sqrt[3]{1+ax}\sqrt[3]{-\frac{x}{(1+ax)\left(1-x^2\right)^3}}\left(1-x^2\right)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\right.}{2\sqrt[3]{-1+a}\sqrt[3]{x}}$$

Mathematica [C] time = 0.05, size = 72, normalized size = 0.11

$$\frac{3}{2}\left(x^2-1\right)\sqrt[3]{\frac{x}{\left(x^2-1\right)^3(ax+1)}}\left({}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(a-1)x}{ax+1}\right)-{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(a+1)x}{ax+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x/(-1 - a*x + 3*x^2 + 3*a*x^3 - 3*x^4 - 3*a*x^5 + x^6 + a*x^7))^
(1/3),x]
```

```
[Out] (3*(x/((1 + a*x)*(-1 + x^2)^3))^(1/3)*(-1 + x^2)*(Hypergeometric2F1[1/3, 1,
 4/3, ((-1 + a)*x)/(1 + a*x)] - Hypergeometric2F1[1/3, 1, 4/3, ((1 + a)*x)/
(1 + a*x)]))/2
```

<u>**IntegrateAlgebraic**</u> **[A]** time = 2.05, size = 669, normalized size = 1.00

$$\frac{\log\left(\sqrt[3]{\frac{x}{ax^7-3ax^5+3ax^3-ax+x^6-3x^4+3x^2-1}}\left(\sqrt[3]{1-a}\,x^2-\sqrt[3]{1-a}\right)+1\right)}{2\sqrt[3]{1-a}}+\frac{\log\left(\left(\sqrt[3]{a+1}\,x^2-\sqrt[3]{a+1}\right)\sqrt[3]{\frac{x}{ax^7-3ax^5+3ax^3-ax+x^6-3x^4+3x^2-1}}-1\right)}{2\sqrt[3]{a+1}}-\frac{\log\left(\sqrt[3]{\frac{x}{ax^7-3ax^5+3ax^3-ax+x^6-3x^4+3x^2-1}}\left(\sqrt[3]{1-a}-\sqrt[3]{1-a}\,x^2\right)+\left((1-a)^{2/3}x^4-2(1-a)^{2/3}x^2+(1-a)^{2/3}\right)\left(\frac{x}{ax^7-3ax^5+3ax^3-ax+x^6-3x^4+3x^2-1}\right)^{2/3}+1\right)}{4\sqrt[3]{1-a}}-\frac{\log\left(\sqrt[3]{\frac{x}{ax^7-3ax^5+3ax^3-ax+x^6-3x^4+3x^2-1}}\left(\sqrt[3]{a+1}\,x^2-\sqrt[3]{a+1}\right)+\left((a+1)^{2/3}x^4-2(a+1)^{2/3}x^2+(a+1)^{2/3}\right)\left(\frac{x}{ax^7-3ax^5+3ax^3-ax+x^6-3x^4+3x^2-1}\right)^{2/3}+1\right)}{4\sqrt[3]{a+1}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{1-a}\left(2x^2-2\right)\sqrt[3]{\frac{x}{ax^7-3ax^5+3ax^3-ax+x^6-3x^4+3x^2-1}}}{\sqrt{3}}\right)}{2\sqrt[3]{1-a}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a+1}\left(2x^2-2\right)\sqrt[3]{\frac{x}{ax^7-3ax^5+3ax^3-ax+x^6-3x^4+3x^2-1}}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{a+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x/(-1 - a*x + 3*x^2 + 3*a*x^3 - 3*x^4 - 3*a*x^5 + x^6 +
 a*x^7))^(1/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[1/Sqrt[3] - ((1 - a)^(1/3)*(-2 + 2*x^2)*(x/(-1 - a*x +
 3*x^2 + 3*a*x^3 - 3*x^4 - 3*a*x^5 + x^6 + a*x^7))^(1/3))/Sqrt[3]])/(1 - a)
^(1/3) - (Sqrt[3]*ArcTan[1/Sqrt[3] + ((1 + a)^(1/3)*(-2 + 2*x^2)*(x/(-1 - a
*x + 3*x^2 + 3*a*x^3 - 3*x^4 - 3*a*x^5 + x^6 + a*x^7))^(1/3))/Sqrt[3]])/(2*
(1 + a)^(1/3)) + Log[1 + (-(1 - a)^(1/3) + (1 - a)^(1/3)*x^2)*(x/(-1 - a*x
+ 3*x^2 + 3*a*x^3 - 3*x^4 - 3*a*x^5 + x^6 + a*x^7))^(1/3)]/(2*(1 - a)^(1/3)
) + Log[-1 + (-(1 + a)^(1/3) + (1 + a)^(1/3)*x^2)*(x/(-1 - a*x + 3*x^2 + 3*
a*x^3 - 3*x^4 - 3*a*x^5 + x^6 + a*x^7))^(1/3)]/(2*(1 + a)^(1/3)) - Log[1 +
((1 - a)^(1/3) - (1 - a)^(1/3)*x^2)*(x/(-1 - a*x + 3*x^2 + 3*a*x^3 - 3*x^4
- 3*a*x^5 + x^6 + a*x^7))^(1/3) + ((1 - a)^(2/3) - 2*(1 - a)^(2/3)*x^2 + (1
 - a)^(2/3)*x^4)*(x/(-1 - a*x + 3*x^2 + 3*a*x^3 - 3*x^4 - 3*a*x^5 + x^6 + a
*x^7))^(2/3)]/(4*(1 - a)^(1/3)) - Log[1 + (-(1 + a)^(1/3) + (1 + a)^(1/3)*x
^2)*(x/(-1 - a*x + 3*x^2 + 3*a*x^3 - 3*x^4 - 3*a*x^5 + x^6 + a*x^7))^(1/3)
+ ((1 + a)^(2/3) - 2*(1 + a)^(2/3)*x^2 + (1 + a)^(2/3)*x^4)*(x/(-1 - a*x +
3*x^2 + 3*a*x^3 - 3*x^4 - 3*a*x^5 + x^6 + a*x^7))^(2/3)]/(4*(1 + a)^(1/3))
```

<u>**fricas**</u> **[A]** time = 0.50, size = 3267, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(a*x^7-3*a*x^5+x^6+3*a*x^3-3*x^4-a*x+3*x^2-1))^(1/3),x, algori
thm="fricas")
```

```
[Out] [1/4*(sqrt(3)*(a^2 - 1)*sqrt((-a + 1)^(1/3)/(a - 1))*log(-((3*a - 2)*x - sq
rt(3)*((a*x^3 - a*x + x^2 - 1)*(-a + 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3
*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) - 2*((a^2 - a)*x^5 + (a - 1)*x^4 -
 2*(a^2 - a)*x^3 - 2*(a - 1)*x^2 + (a^2 - a)*x + a - 1)*(x/(a*x^7 - 3*a*x^5
 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(2/3) - (a*x + 1)*(-a + 1)^(1/
3))*sqrt((-a + 1)^(1/3)/(a - 1)) + 3*(a*x^3 - a*x + x^2 - 1)*(-a + 1)^(1/3)
*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) + 1)
/(x + 1)) + sqrt(3)*(a^2 - 1)*sqrt(-1/(a + 1)^(2/3))*log(((3*a + 2)*x + sqr
t(3)*((a*x^3 - a*x + x^2 - 1)*(a + 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a
*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) - 2*((a^2 + a)*x^5 + (a + 1)*x^4 - 2
*(a^2 + a)*x^3 - 2*(a + 1)*x^2 + (a^2 + a)*x + a + 1)*(x/(a*x^7 - 3*a*x^5 +
 x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(2/3) + (a*x + 1)*(a + 1)^(1/3))
*sqrt(-1/(a + 1)^(2/3)) - 3*(a*x^3 - a*x + x^2 - 1)*(a + 1)^(1/3)*(x/(a*x^7
 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) + 1)/(x - 1))
- (a + 1)^(2/3)*(a - 1)*log((x^2 - 1)*(a + 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x
^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) + ((a + 1)*x^4 - 2*(a + 1)*x
^2 + a + 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))
^(2/3) + (a + 1)^(1/3)) + (a + 1)*(-a + 1)^(2/3)*log((x^2 - 1)*(-a + 1)^(2/
3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) +
((a - 1)*x^4 - 2*(a - 1)*x^2 + a - 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 -
 3*x^4 - a*x + 3*x^2 - 1))^(2/3) - (-a + 1)^(1/3)) + 2*(a + 1)^(2/3)*(a - 1
)*log(((a + 1)*x^2 - a - 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a
*x + 3*x^2 - 1))^(1/3) - (a + 1)^(2/3)) - 2*(a + 1)*(-a + 1)^(2/3)*log(((a
- 1)*x^2 - a + 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2
 - 1))^(1/3) - (-a + 1)^(2/3)))/(a^2 - 1), 1/4*(2*sqrt(3)*(a^2 - 1)*sqrt(-(
-a + 1)^(1/3)/(a - 1))*arctan(1/3*sqrt(3)*(2*(x^2 - 1)*(-a + 1)^(2/3)*(x/(a
*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) - (-a + 1)
^(1/3))*sqrt(-(-a + 1)^(1/3)/(a - 1))) + sqrt(3)*(a^2 - 1)*sqrt(-1/(a + 1)^
```

```
(2/3))*log(((3*a + 2)*x + sqrt(3)*((a*x^3 - a*x + x^2 - 1)*(a + 1)^(2/3)*(x
/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) - 2*((a
^2 + a)*x^5 + (a + 1)*x^4 - 2*(a^2 + a)*x^3 - 2*(a + 1)*x^2 + (a^2 + a)*x +
 a + 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(2/
3) + (a*x + 1)*(a + 1)^(1/3))*sqrt(-1/(a + 1)^(2/3)) - 3*(a*x^3 - a*x + x^2
 - 1)*(a + 1)^(1/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x
^2 - 1))^(1/3) + 1)/(x - 1)) - (a + 1)^(2/3)*(a - 1)*log((x^2 - 1)*(a + 1)^
(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3)
 + ((a + 1)*x^4 - 2*(a + 1)*x^2 + a + 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^
3 - 3*x^4 - a*x + 3*x^2 - 1))^(2/3) + (a + 1)^(1/3)) + (a + 1)*(-a + 1)^(2/
3)*log((x^2 - 1)*(-a + 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4
 - a*x + 3*x^2 - 1))^(1/3) + ((a - 1)*x^4 - 2*(a - 1)*x^2 + a - 1)*(x/(a*x^
7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(2/3) - (-a + 1)^(1
/3)) + 2*(a + 1)^(2/3)*(a - 1)*log(((a + 1)*x^2 - a - 1)*(x/(a*x^7 - 3*a*x^
5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) - (a + 1)^(2/3)) - 2*(a
 + 1)*(-a + 1)^(2/3)*log(((a - 1)*x^2 - a + 1)*(x/(a*x^7 - 3*a*x^5 + x^6 +
3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) - (-a + 1)^(2/3)))/(a^2 - 1), 1/4
*(sqrt(3)*(a^2 - 1)*sqrt((-a + 1)^(1/3)/(a - 1))*log(-((3*a - 2)*x - sqrt(3
)*((a*x^3 - a*x + x^2 - 1)*(-a + 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x
^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) - 2*((a^2 - a)*x^5 + (a - 1)*x^4 - 2*(
a^2 - a)*x^3 - 2*(a - 1)*x^2 + (a^2 - a)*x + a - 1)*(x/(a*x^7 - 3*a*x^5 + x
^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(2/3) - (a*x + 1)*(-a + 1)^(1/3))*
sqrt((-a + 1)^(1/3)/(a - 1)) + 3*(a*x^3 - a*x + x^2 - 1)*(-a + 1)^(1/3)*(x/
(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) + 1)/(x
+ 1)) - (a + 1)^(2/3)*(a - 1)*log((x^2 - 1)*(a + 1)^(2/3)*(x/(a*x^7 - 3*a*x
^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) + ((a + 1)*x^4 - 2*(a
+ 1)*x^2 + a + 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2
 - 1))^(2/3) + (a + 1)^(1/3)) + (a + 1)*(-a + 1)^(2/3)*log((x^2 - 1)*(-a +
1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1
/3) + ((a - 1)*x^4 - 2*(a - 1)*x^2 + a - 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a
*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(2/3) - (-a + 1)^(1/3)) + 2*(a + 1)^(2/3)*
(a - 1)*log(((a + 1)*x^2 - a - 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x
^4 - a*x + 3*x^2 - 1))^(1/3) - (a + 1)^(2/3)) - 2*(a + 1)*(-a + 1)^(2/3)*lo
g(((a - 1)*x^2 - a + 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x +
 3*x^2 - 1))^(1/3) - (-a + 1)^(2/3)) - 2*sqrt(3)*(a^2 - 1)*arctan(1/3*sqrt(
3)*(2*(x^2 - 1)*(a + 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 -
 a*x + 3*x^2 - 1))^(1/3) + (a + 1)^(1/3))/(a + 1)^(1/3))/(a + 1)^(1/3))/(a^
2 - 1), 1/4*(2*sqrt(3)*(a^2 - 1)*sqrt(-(-a + 1)^(1/3)/(a - 1))*arctan(1/3*s
qrt(3)*(2*(x^2 - 1)*(-a + 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*
x^4 - a*x + 3*x^2 - 1))^(1/3) - (-a + 1)^(1/3))*sqrt(-(-a + 1)^(1/3)/(a - 1
))) - (a + 1)^(2/3)*(a - 1)*log((x^2 - 1)*(a + 1)^(2/3)*(x/(a*x^7 - 3*a*x^5
 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) + ((a + 1)*x^4 - 2*(a +
1)*x^2 + a + 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 -
 1))^(2/3) + (a + 1)^(1/3)) + (a + 1)*(-a + 1)^(2/3)*log((x^2 - 1)*(-a + 1)
^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3
) + ((a - 1)*x^4 - 2*(a - 1)*x^2 + a - 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x
^3 - 3*x^4 - a*x + 3*x^2 - 1))^(2/3) - (-a + 1)^(1/3)) + 2*(a + 1)^(2/3)*(a
 - 1)*log(((a + 1)*x^2 - a - 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4
 - a*x + 3*x^2 - 1))^(1/3) - (a + 1)^(2/3)) - 2*(a + 1)*(-a + 1)^(2/3)*log(
((a - 1)*x^2 - a + 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3
*x^2 - 1))^(1/3) - (-a + 1)^(2/3)) - 2*sqrt(3)*(a^2 - 1)*arctan(1/3*sqrt(3)
*(2*(x^2 - 1)*(a + 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a
*x + 3*x^2 - 1))^(1/3) + (a + 1)^(1/3))/(a + 1)^(1/3))/(a + 1)^(1/3))/(a^2
- 1)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{x}{ax^7 - 3\,ax^5 + x^6 + 3\,ax^3 - 3\,x^4 - ax + 3\,x^2 - 1}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(a*x^7-3*a*x^5+x^6+3*a*x^3-3*x^4-a*x+3*x^2-1))^(1/3),x, algori
thm="giac")
```

```
[Out] integrate((x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(
1/3), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\frac{x}{a\,x^{7}-3 a\,x^{5}+x^{6}+3 a\,x^{3}-3 x^{4}-a x+3 x^{2}-1}\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(a*x^7-3*a*x^5+x^6+3*a*x^3-3*x^4-a*x+3*x^2-1))^(1/3),x)
```

```
[Out] int((x/(a*x^7-3*a*x^5+x^6+3*a*x^3-3*x^4-a*x+3*x^2-1))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{x}{ax^{7}-3\,ax^{5}+x^{6}+3\,ax^{3}-3\,x^{4}-ax+3\,x^{2}-1}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(a*x^7-3*a*x^5+x^6+3*a*x^3-3*x^4-a*x+3*x^2-1))^(1/3),x, algori
thm="maxima")
```

```
[Out] integrate((x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(
1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(-\frac{x}{-a\,x^7-x^6+3\,a\,x^5+3\,x^4-3\,a\,x^3-3\,x^2+a\,x+1}\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x/(a*x - 3*a*x^3 + 3*a*x^5 - a*x^7 - 3*x^2 + 3*x^4 - x^6 + 1))^(1/3),
x)
```

```
[Out] int((-x/(a*x - 3*a*x^3 + 3*a*x^5 - a*x^7 - 3*x^2 + 3*x^4 - x^6 + 1))^(1/3),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{\frac{x}{ax^{7} - 3ax^{5} + 3ax^{3} - ax + x^{6} - 3x^{4} + 3x^{2} - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(a*x**7-3*a*x**5+x**6+3*a*x**3-3*x**4-a*x+3*x**2-1))**(1/3),x)
```

```
[Out] Integral((x/(a*x**7 - 3*a*x**5 + 3*a*x**3 - a*x + x**6 - 3*x**4 + 3*x**2 -
1))**(1/3), x)
```

3.2416 $$\int \frac{\sqrt{-b+a^2x^2}\,\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Optimal. Leaf size=674

$$-\frac{1989b^2\tan^{-1}\left(\frac{\sqrt[4]{\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[4]{c}}\right)}{16384ac^{21/4}}+\frac{1989b^2\tanh^{-1}\left(\frac{\sqrt[4]{\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[4]{c}}\right)}{16384ac^{21/4}}+\frac{\sqrt{a^2x^2-b}\left(\left(6055526400a^2c^7x^2-40\right.\right.}{}$$

Rubi [F] time = 1.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-b+a^2x^2}\,\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/3))/(c + (a*x + Sqrt[
-b + a^2*x^2])^(1/3))^(1/4),x]
```

```
[Out] Defer[Int][(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/3))/(c + (a*x
+ Sqrt[-b + a^2*x^2])^(1/3))^(1/4), x]
```

Rubi steps

$$\int \frac{\sqrt{-b+a^2x^2}\,\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{\sqrt{-b+a^2x^2}\,\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Mathematica [F] time = 157.13, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-b+a^2x^2}\,\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/3))/(c + (a*x +
 Sqrt[-b + a^2*x^2])^(1/3))^(1/4),x]
```

```
[Out] Integrate[(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/3))/(c + (a*x +
 Sqrt[-b + a^2*x^2])^(1/3))^(1/4), x]
```

IntegrateAlgebraic [A] time = 1.94, size = 674, normalized size = 1.00

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(1/3))/(c
 + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4),x]
```

```
[Out] ((-1860655104*b^2*c^4 + 2013265920*b*c^10 + 2409402996*a*b^2*c*x - 45416448
00*a*b*c^7*x - 4026531840*a^2*c^10*x^2 + 6055526400*a^3*c^7*x^3)*(c + (a*x
+ Sqrt[-b + a^2*x^2])^(1/3))^(3/4) + (1976946048*b^2*c^3 - 1761607680*b*c^9
 - 3011753745*a*b^2*x + 4314562560*a*b*c^6*x + 3523215360*a^2*c^9*x^2 - 575
2750080*a^3*c^6*x^3)*(a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x + Sqrt[-b +
 a^2*x^2])^(1/3))^(3/4) + (-2141691552*b^2*c^2 + 1614807040*b*c^8 - 2894352
3840*a*b*c^5*x + 2684354560*a*c^11*x - 3229614080*a^2*c^8*x^2 + 5513052160*
a^3*c^5*x^3)*(a*x + Sqrt[-b + a^2*x^2])^(2/3)*(c + (a*x + Sqrt[-b + a^2*x^2
])^(1/3))^(3/4) + Sqrt[-b + a^2*x^2]*((2409402996*b^2*c - 1513881600*b*c^7
- 4026531840*a*c^10*x + 6055526400*a^2*c^7*x^2)*(c + (a*x + Sqrt[-b + a^2*x
^2])^(1/3))^(3/4) + (-3011753745*b^2 + 1438187520*b*c^6 + 3523215360*a*c^9*
x - 5752750080*a^2*c^6*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x + Sq
rt[-b + a^2*x^2])^(1/3))^(3/4) + (-26186997760*b*c^5 + 2684354560*c^11 - 32
29614080*a*c^8*x + 5513052160*a^2*c^5*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(2/3)
*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(3/4)))/(12404367360*a*c^5*(a*x + S
qrt[-b + a^2*x^2])^(5/3)) - (1989*b^2*ArcTan[(c + (a*x + Sqrt[-b + a^2*x^2]
)^(1/3))^(1/4)/c^(1/4)])/(16384*a*c^(21/4)) + (1989*b^2*ArcTanh[(c + (a*x +
 Sqrt[-b + a^2*x^2])^(1/3))^(1/4)/c^(1/4)])/(16384*a*c^(21/4))
```

fricas [A] time = 0.54, size = 562, normalized size = 0.83

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-
b)^(1/2))^(1/3))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/49617469440*(12047014980*a*c^5*(b^8/(a^4*c^21))^(1/4)*arctan(-(a*b^6*(c +
 (a*x + sqrt(a^2*x^2 - b))^(1/3))^(1/4)*c^5*(b^8/(a^4*c^21))^(1/4) - sqrt(a
^2*b^8*c^11*sqrt(b^8/(a^4*c^21)) + b^12*sqrt(c + (a*x + sqrt(a^2*x^2 - b))^
(1/3)))*a*c^5*(b^8/(a^4*c^21))^(1/4))/b^8) + 3011753745*a*c^5*(b^8/(a^4*c^2
1))^(1/4)*log(7868724669*a^3*c^16*(b^8/(a^4*c^21))^(3/4) + 7868724669*b^6*(
c + (a*x + sqrt(a^2*x^2 - b))^(1/3))^(1/4)) - 3011753745*a*c^5*(b^8/(a^4*c^
21))^(1/4)*log(-7868724669*a^3*c^16*(b^8/(a^4*c^21))^(3/4) + 7868724669*b^6
*(c + (a*x + sqrt(a^2*x^2 - b))^(1/3))^(1/4)) + 4*(2684354560*c^11 + 275652
6080*a^2*c^5*x^2 - 26186997760*b*c^5 - 2464*(655360*a*c^8 + 869193*a*b*c^2)
*x + 21*(83886080*c^9 + 188280576*a^2*c^3*x^2 - 94140288*b*c^3 - 1045*(6553
6*a*c^6 + 137241*a*b)*x - 209*(327680*c^6 + 900864*a*c^3*x - 686205*b)*sqrt
(a^2*x^2 - b))*(a*x + sqrt(a^2*x^2 - b))^(2/3) - 2464*(655360*c^8 - 1118720
*a*c^5*x - 869193*b*c^2)*sqrt(a^2*x^2 - b) - 12*(167772160*c^10 + 310109184
*a^2*c^4*x^2 - 155054592*b*c^4 - 77*(1638400*a*c^7 + 2607579*a*b*c)*x - 77*
(1638400*c^7 + 4027392*a*c^4*x - 2607579*b*c)*sqrt(a^2*x^2 - b))*(a*x + sqr
t(a^2*x^2 - b))^(1/3))*(c + (a*x + sqrt(a^2*x^2 - b))^(1/3))^(3/4))/(a*c^5)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-
b)^(1/2))^(1/3))^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2-b}\,\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}}{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b)^(1/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-b)^(1/
2))^(1/3))^(1/4),x)
```

```
[Out] int((a^2*x^2-b)^(1/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-b)^(1/
2))^(1/3))^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2-b}\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}}{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-
b)^(1/2))^(1/3))^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 - b)*(a*x + sqrt(a^2*x^2 - b))^(1/3)/(c + (a*x + sqr
t(a^2*x^2 - b))^(1/3))^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/3}\,\sqrt{a^2\,x^2-b}}{\left(c+\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/3}\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (a^2*x^2 - b)^(1/2))^(1/3)*(a^2*x^2 - b)^(1/2))/(c + (a*x + (a^
2*x^2 - b)^(1/2))^(1/3))^(1/4),x)
```

```
[Out] int(((a*x + (a^2*x^2 - b)^(1/2))^(1/3)*(a^2*x^2 - b)^(1/2))/(c + (a*x + (a^
2*x^2 - b)^(1/2))^(1/3))^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{ax+\sqrt{a^2x^2-b}}\,\sqrt{a^2x^2-b}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{a^2x^2-b}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b)**(1/2)*(a*x+(a**2*x**2-b)**(1/2))**(1/3)/(c+(a*x+(a
**2*x**2-b)**(1/2))**(1/3))**(1/4),x)
```

```
[Out] Integral((a*x + sqrt(a**2*x**2 - b))**(1/3)*sqrt(a**2*x**2 - b)/(c + (a*x +
 sqrt(a**2*x**2 - b))**(1/3))**(1/4), x)
```

3.2417 $\int \frac{\sqrt[6]{ax+\sqrt{-b+a^2x^2}}}{x^3\sqrt{-b+a^2x^2}}\,dx$

Optimal. Leaf size=678

$$-\frac{35\sqrt{2+\sqrt{3}}\,a^2\tan^{-1}\left(\frac{\left(\sqrt{\frac{3}{2}}\sqrt[12]{b}-\frac{\sqrt[12]{b}}{\sqrt{2}}\right)\sqrt[6]{\sqrt{a^2x^2-b}+ax}}{\sqrt[3]{\sqrt{a^2x^2-b}+ax}-\sqrt[6]{b}}\right)}{72b^{17/12}}-\frac{35\sqrt{2-\sqrt{3}}\,a^2\tan^{-1}\left(\frac{\left(\frac{\sqrt[12]{b}}{\sqrt{2}}+\sqrt{\frac{3}{2}}\sqrt[12]{b}\right)\sqrt[6]{\sqrt{a^2x^2-b}+ax}}{\sqrt[3]{\sqrt{a^2x^2-b}+ax}-\sqrt[6]{b}}\right)}{72b^{17/12}}+\frac{35a^2\tan}{}$$

Rubi [A] time = 1.26, antiderivative size = 760, normalized size of antiderivative = 1.12, number of steps used = 25, number of rules used = 13, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2120, 288, 290, 329, 301, 209, 634, 618, 204, 628, 203, 210, 206}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(a*x + Sqrt[-b + a^2*x^2])^(1/6)/(x^3*Sqrt[-b + a^2*x^2]),x]
```

```
[Out] (-2*a^2*(a*x + Sqrt[-b + a^2*x^2])^(7/6))/(b + (a*x + Sqrt[-b + a^2*x^2])^2
)^2 + (7*a^2*(a*x + Sqrt[-b + a^2*x^2])^(7/6))/(6*b*(b + (a*x + Sqrt[-b + a
^2*x^2])^2)) - (35*a^2*ArcTan[(a*x + Sqrt[-b + a^2*x^2])^(1/6)/(-b)^(1/12)]
)/(36*(-b)^(17/12)) - (35*a^2*ArcTan[(1 - (2*(a*x + Sqrt[-b + a^2*x^2])^(1/
6))/(-b)^(1/12))/Sqrt[3]])/(24*Sqrt[3]*(-b)^(17/12)) + (35*a^2*ArcTan[Sqrt[
3] - (2*(a*x + Sqrt[-b + a^2*x^2])^(1/6))/(-b)^(1/12)])/(72*(-b)^(17/12)) +
 (35*a^2*ArcTan[(1 + (2*(a*x + Sqrt[-b + a^2*x^2])^(1/6))/(-b)^(1/12))/Sqrt
[3]])/(24*Sqrt[3]*(-b)^(17/12)) - (35*a^2*ArcTan[Sqrt[3] + (2*(a*x + Sqrt[-
b + a^2*x^2])^(1/6))/(-b)^(1/12)])/(72*(-b)^(17/12)) + (35*a^2*ArcTanh[(a*x
 + Sqrt[-b + a^2*x^2])^(1/6)/(-b)^(1/12)])/(36*(-b)^(17/12)) - (35*a^2*Log[
(-b)^(1/6) - (-b)^(1/12)*(a*x + Sqrt[-b + a^2*x^2])^(1/6) + (a*x + Sqrt[-b
+ a^2*x^2])^(1/3)])/(144*(-b)^(17/12)) + (35*a^2*Log[(-b)^(1/6) + (-b)^(1/1
2)*(a*x + Sqrt[-b + a^2*x^2])^(1/6) + (a*x + Sqrt[-b + a^2*x^2])^(1/3)])/(1
44*(-b)^(17/12)) + (35*a^2*Log[(-b)^(1/6) - Sqrt[3]*(-b)^(1/12)*(a*x + Sqrt
[-b + a^2*x^2])^(1/6) + (a*x + Sqrt[-b + a^2*x^2])^(1/3)])/(48*Sqrt[3]*(-b)
^(17/12)) - (35*a^2*Log[(-b)^(1/6) + Sqrt[3]*(-b)^(1/12)*(a*x + Sqrt[-b + a
^2*x^2])^(1/6) + (a*x + Sqrt[-b + a^2*x^2])^(1/3)])/(48*Sqrt[3]*(-b)^(17/12
))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\sqrt[6]{ax+\sqrt{-b+a^2x^2}}}{x^3\sqrt{-b+a^2x^2}}\,dx = \left(8a^2\right)\text{Subst}\left(\int \frac{x^{13/6}}{\left(b+x^2\right)^3}\,dx, x, ax+\sqrt{-b+a^2x^2}\right)$$

$$= -\frac{2a^2\left(ax+\sqrt{-b+a^2x^2}\right)^{7/6}}{\left(b+\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^2} + \frac{1}{3}\left(7a^2\right)\text{Subst}\left(\int \frac{\sqrt[6]{x}}{\left(b+x^2\right)^2}\,dx, x, ax+\sqrt{-b+a\right.$$

$$= -\frac{2a^2\left(ax+\sqrt{-b+a^2x^2}\right)^{7/6}}{\left(b+\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^2} + \frac{7a^2\left(ax+\sqrt{-b+a^2x^2}\right)^{7/6}}{6b\left(b+\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} + \frac{\left(35a^2\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{2a^2\left(ax+\sqrt{-b+a^2x^2}\right)^{7/6}}{\left(b+\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^2} + \frac{7a^2\left(ax+\sqrt{-b+a^2x^2}\right)^{7/6}}{6b\left(b+\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} + \frac{\left(35a^2\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{2a^2\left(ax+\sqrt{-b+a^2x^2}\right)^{7/6}}{\left(b+\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^2} + \frac{7a^2\left(ax+\sqrt{-b+a^2x^2}\right)^{7/6}}{6b\left(b+\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} - \frac{\left(35a^2\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{2a^2\left(ax+\sqrt{-b+a^2x^2}\right)^{7/6}}{\left(b+\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^2} + \frac{7a^2\left(ax+\sqrt{-b+a^2x^2}\right)^{7/6}}{6b\left(b+\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} + \frac{\left(35a^2\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{2a^2\left(ax+\sqrt{-b+a^2x^2}\right)^{7/6}}{\left(b+\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^2} + \frac{7a^2\left(ax+\sqrt{-b+a^2x^2}\right)^{7/6}}{6b\left(b+\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} - \frac{35a^2\tan^{-1}\left(\frac{\sqrt[6]{a}}{}\right.}{36(-b)}$$

$$= -\frac{2a^2\left(ax+\sqrt{-b+a^2x^2}\right)^{7/6}}{\left(b+\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^2} + \frac{7a^2\left(ax+\sqrt{-b+a^2x^2}\right)^{7/6}}{6b\left(b+\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} - \frac{35a^2\tan^{-1}\left(\frac{\sqrt[6]{a}}{}\right.}{36(-b)}$$

$$= -\frac{2a^2\left(ax+\sqrt{-b+a^2x^2}\right)^{7/6}}{\left(b+\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^2} + \frac{7a^2\left(ax+\sqrt{-b+a^2x^2}\right)^{7/6}}{6b\left(b+\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} - \frac{35a^2\tan^{-1}\left(\frac{\sqrt[6]{a}}{}\right.}{36(-b)}$$

Mathematica [C] time = 2.36, size = 317, normalized size = 0.47

$$\frac{12\sqrt{a^2x^2-b}\left(b-2ax\left(\sqrt{a^2x^2-b}+ax\right)\right)^4\left(4a^2x^2\left(2ax\left(\sqrt{a^2x^2-b}+ax\right)-b\right){}_2F_1\left(\frac{7}{12},3;\frac{19}{12};-\frac{\left(ax+\sqrt{a^2x^2-b}\right)^2}{b}\right)-b^2\right)}{17b^2x^2\left(\sqrt{a^2x^2-b}+ax\right)^{5/6}\left(128a^9x^9-320a^7bx^7+272a^5b^2x^5-88a^3b^3x^3+b^4\sqrt{a^2x^2-b}-32a^2b^3x^2\sqrt{a^2x^2-b}+128a^8x^8\sqrt{a^2x^2-b}-256a^6bx^6\sqrt{a^2x^2-b}+160a^4b^2x^4\sqrt{a^2x^2-b}+8ab^4x\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + Sqrt[-b + a^2*x^2])^(1/6)/(x^3*Sqrt[-b + a^2*x^2]),x]
```

```
[Out] (12*Sqrt[-b + a^2*x^2]*(b - 2*a*x*(a*x + Sqrt[-b + a^2*x^2]))^4*(-b^2 + 4*a
^2*x^2*(-b + 2*a*x*(a*x + Sqrt[-b + a^2*x^2]))*Hypergeometric2F1[7/12, 3, 1
9/12, -((a*x + Sqrt[-b + a^2*x^2])^2/b)]))/(17*b^2*x^2*(a*x + Sqrt[-b + a^2
*x^2])^(5/6)*(8*a*b^4*x - 88*a^3*b^3*x^3 + 272*a^5*b^2*x^5 - 320*a^7*b*x^7
+ 128*a^9*x^9 + b^4*Sqrt[-b + a^2*x^2] - 32*a^2*b^3*x^2*Sqrt[-b + a^2*x^2]
+ 160*a^4*b^2*x^4*Sqrt[-b + a^2*x^2] - 256*a^6*b*x^6*Sqrt[-b + a^2*x^2] + 1
28*a^8*x^8*Sqrt[-b + a^2*x^2]))
```

IntegrateAlgebraic [A] time = 7.51, size = 1211, normalized size = 1.79

$$\frac{35\tan^{-1}\left(\frac{\sqrt{2}\sqrt[12]{b}\sqrt[6]{ax+\sqrt{a^2x^2-b}}}{\sqrt[3]{ax+\sqrt{a^2x^2-b}}-\sqrt[6]{b}}\right)a^2}{36\sqrt{2}b^{17/12}}-\frac{35\sqrt{2+\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}\sqrt[12]{b}\sqrt[6]{ax+\sqrt{a^2x^2-b}}}{\sqrt[3]{ax+\sqrt{a^2x^2-b}}-\sqrt[6]{b}}\right)a^2}{144b^{17/12}}-\frac{35\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}\sqrt[12]{b}\sqrt[6]{ax+\sqrt{a^2x^2-b}}}{\sqrt[3]{ax+\sqrt{a^2x^2-b}}-\sqrt[6]{b}}\right)a^2}{48\sqrt{6}b^{17/12}}-\frac{35\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}\sqrt[12]{b}\sqrt[6]{ax+\sqrt{a^2x^2-b}}}{\sqrt[3]{ax+\sqrt{a^2x^2-b}}-\sqrt[6]{b}}\right)a^2}{144\sqrt{2}b^{17/12}}-\frac{35\sqrt{2-\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}\sqrt[12]{b}\sqrt[6]{ax+\sqrt{a^2x^2-b}}}{\sqrt[3]{ax+\sqrt{a^2x^2-b}}-\sqrt[6]{b}}\right)a^2}{144b^{17/12}}-\frac{35\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}\sqrt[12]{b}\sqrt[6]{ax+\sqrt{a^2x^2-b}}}{\sqrt[3]{ax+\sqrt{a^2x^2-b}}-\sqrt[6]{b}}\right)a^2}{48\sqrt{6}b^{17/12}}+\frac{35\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}\sqrt[12]{b}\sqrt[6]{ax+\sqrt{a^2x^2-b}}}{\sqrt[3]{ax+\sqrt{a^2x^2-b}}-\sqrt[6]{b}}\right)a^2}{144\sqrt{2}b^{17/12}}+\frac{35\tanh^{-1}\left(\frac{\frac{\sqrt[3]{ax+\sqrt{a^2x^2-b}}}{\sqrt{2}\sqrt[12]{b}}+\frac{\sqrt[12]{b}}{\sqrt{2}}}{\sqrt[6]{ax+\sqrt{a^2x^2-b}}}\right)a^2}{36\sqrt{2}b^{17/12}}-\frac{35\tanh^{-1}\left(\frac{\frac{\sqrt{2-\sqrt{3}}\sqrt[3]{ax+\sqrt{a^2x^2-b}}}{\sqrt[12]{b}}+\sqrt{2-\sqrt{3}}\sqrt[12]{b}}{\sqrt[6]{ax+\sqrt{a^2x^2-b}}}\right)a^2}{24\sqrt{6}b^{17/12}}+\frac{35\tanh^{-1}\left(\frac{\frac{\sqrt{2-\sqrt{3}}\sqrt[3]{ax+\sqrt{a^2x^2-b}}}{\sqrt[12]{b}}+\sqrt{2-\sqrt{3}}\sqrt[12]{b}}{\sqrt[6]{ax+\sqrt{a^2x^2-b}}}\right)a^2}{72\sqrt{2}b^{17/12}}-\frac{35\tanh^{-1}\left(\frac{\frac{\sqrt{2+\sqrt{3}}\sqrt[3]{ax+\sqrt{a^2x^2-b}}}{\sqrt[12]{b}}+\sqrt{2+\sqrt{3}}\sqrt[12]{b}}{\sqrt[6]{ax+\sqrt{a^2x^2-b}}}\right)a^2}{24\sqrt{6}b^{17/12}}-\frac{35\tanh^{-1}\left(\frac{\frac{\sqrt{2+\sqrt{3}}\sqrt[3]{ax+\sqrt{a^2x^2-b}}}{\sqrt[12]{b}}+\sqrt{2+\sqrt{3}}\sqrt[12]{b}}{\sqrt[6]{ax+\sqrt{a^2x^2-b}}}\right)a^2}{72\sqrt{2}b^{17/12}}+\frac{7\left(ax+\sqrt{a^2x^2-b}\right)^{7/6}}{24bx^2}-\frac{5}{24x^2\left(ax+\sqrt{a^2x^2-b}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x + Sqrt[-b + a^2*x^2])^(1/6)/(x^3*Sqrt[-b + a^2*x^2]
),x]
```

```
[Out] -5/(24*x^2*(a*x + Sqrt[-b + a^2*x^2])^(5/6)) + (7*(a*x + Sqrt[-b + a^2*x^2]
)^(7/6))/(24*b*x^2) + (35*a^2*ArcTan[(Sqrt[2]*b^(1/12)*(a*x + Sqrt[-b + a^2
*x^2])^(1/6))/(-b^(1/6) + (a*x + Sqrt[-b + a^2*x^2])^(1/3))])/(36*Sqrt[2]*b
^(17/12)) - (35*a^2*ArcTan[(Sqrt[2 - Sqrt[3]]*b^(1/12)*(a*x + Sqrt[-b + a^2
*x^2])^(1/6))/(-b^(1/6) + (a*x + Sqrt[-b + a^2*x^2])^(1/3))])/(144*Sqrt[2]*
b^(17/12)) - (35*a^2*ArcTan[(Sqrt[2 - Sqrt[3]]*b^(1/12)*(a*x + Sqrt[-b + a^
2*x^2])^(1/6))/(-b^(1/6) + (a*x + Sqrt[-b + a^2*x^2])^(1/3))])/(48*Sqrt[6]*
b^(17/12)) - (35*Sqrt[2 + Sqrt[3]]*a^2*ArcTan[(Sqrt[2 - Sqrt[3]]*b^(1/12)*(
a*x + Sqrt[-b + a^2*x^2])^(1/6))/(-b^(1/6) + (a*x + Sqrt[-b + a^2*x^2])^(1/
3))])/(144*b^(17/12)) + (35*a^2*ArcTan[(Sqrt[2 + Sqrt[3]]*b^(1/12)*(a*x + S
qrt[-b + a^2*x^2])^(1/6))/(-b^(1/6) + (a*x + Sqrt[-b + a^2*x^2])^(1/3))])/(
144*Sqrt[2]*b^(17/12)) - (35*a^2*ArcTan[(Sqrt[2 + Sqrt[3]]*b^(1/12)*(a*x +
Sqrt[-b + a^2*x^2])^(1/6))/(-b^(1/6) + (a*x + Sqrt[-b + a^2*x^2])^(1/3))])/
(48*Sqrt[6]*b^(17/12)) - (35*Sqrt[2 - Sqrt[3]]*a^2*ArcTan[(Sqrt[2 + Sqrt[3]
]*b^(1/12)*(a*x + Sqrt[-b + a^2*x^2])^(1/6))/(-b^(1/6) + (a*x + Sqrt[-b + a
^2*x^2])^(1/3))])/(144*b^(17/12)) + (35*a^2*ArcTanh[(b^(1/12)/Sqrt[2] + (a*
x + Sqrt[-b + a^2*x^2])^(1/3)/(Sqrt[2]*b^(1/12)))/(a*x + Sqrt[-b + a^2*x^2]
)^(1/6)])/(36*Sqrt[2]*b^(17/12)) + (35*a^2*ArcTanh[(Sqrt[2 - Sqrt[3]]*b^(1/
12) + (Sqrt[2 - Sqrt[3]]*(a*x + Sqrt[-b + a^2*x^2])^(1/3))/b^(1/12))/(a*x +
 Sqrt[-b + a^2*x^2])^(1/6)])/(72*Sqrt[2]*b^(17/12)) - (35*a^2*ArcTanh[(Sqrt
[2 - Sqrt[3]]*b^(1/12) + (Sqrt[2 - Sqrt[3]]*(a*x + Sqrt[-b + a^2*x^2])^(1/3
))/b^(1/12))/(a*x + Sqrt[-b + a^2*x^2])^(1/6)])/(24*Sqrt[6]*b^(17/12)) - (3
5*a^2*ArcTanh[(Sqrt[2 + Sqrt[3]]*b^(1/12) + (Sqrt[2 + Sqrt[3]]*(a*x + Sqrt[
-b + a^2*x^2])^(1/3))/b^(1/12))/(a*x + Sqrt[-b + a^2*x^2])^(1/6)])/(72*Sqrt
[2]*b^(17/12)) - (35*a^2*ArcTanh[(Sqrt[2 + Sqrt[3]]*b^(1/12) + (Sqrt[2 + Sq
rt[3]]*(a*x + Sqrt[-b + a^2*x^2])^(1/3))/b^(1/12))/(a*x + Sqrt[-b + a^2*x^2
])^(1/6)])/(24*Sqrt[6]*b^(17/12))
```

fricas [C] time = 1.19, size = 1065, normalized size = 1.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2-b)^(1/2))^(1/6)/x^3/(a^2*x^2-b)^(1/2),x, algorithm=
"fricas")
```

```
[Out] -1/144*(70*(-a^24/b^17)^(1/12)*b*x^2*(1/2*I*sqrt(3) + 1/2)^(3/2)*log(643392
96875*(a*x + sqrt(a^2*x^2 - b))^(1/6)*a^14 + 64339296875*(-a^24/b^17)^(7/12
)*b^10*(1/2*I*sqrt(3) + 1/2)^(3/2)) - 70*(-a^24/b^17)^(1/12)*b*x^2*(1/2*I*s
qrt(3) + 1/2)^(3/2)*log(64339296875*(a*x + sqrt(a^2*x^2 - b))^(1/6)*a^14 -
64339296875*(-a^24/b^17)^(7/12)*b^10*(1/2*I*sqrt(3) + 1/2)^(3/2)) - 35*(-a^
24/b^17)^(1/12)*b*x^2*(-I*sqrt(3) - 1)*log(64339296875*(a*x + sqrt(a^2*x^2
- b))^(1/6)*a^14 + 64339296875/2*(-a^24/b^17)^(7/12)*b^10*(-I*sqrt(3) - 1))
 + 35*(-a^24/b^17)^(1/12)*b*x^2*(-I*sqrt(3) - 1)*log(64339296875*(a*x + sqr
```

```
t(a^2*x^2 - b))^(1/6)*a^14 - 64339296875/2*(-a^24/b^17)^(7/12)*b^10*(-I*sqr
t(3) - 1)) + 70*(-a^24/b^17)^(1/12)*b*x^2*sqrt(1/2*I*sqrt(3) + 1/2)*log(643
39296875*(a*x + sqrt(a^2*x^2 - b))^(1/6)*a^14 + 64339296875*(-a^24/b^17)^(7
/12)*b^10*sqrt(1/2*I*sqrt(3) + 1/2)) - 70*(-a^24/b^17)^(1/12)*b*x^2*sqrt(1/
2*I*sqrt(3) + 1/2)*log(64339296875*(a*x + sqrt(a^2*x^2 - b))^(1/6)*a^14 - 6
4339296875*(-a^24/b^17)^(7/12)*b^10*sqrt(1/2*I*sqrt(3) + 1/2)) - 70*(-a^24/
b^17)^(1/12)*b*x^2*log(64339296875*(a*x + sqrt(a^2*x^2 - b))^(1/6)*a^14 + 6
4339296875*(-a^24/b^17)^(7/12)*b^10) + 70*(-a^24/b^17)^(1/12)*b*x^2*log(643
39296875*(a*x + sqrt(a^2*x^2 - b))^(1/6)*a^14 - 64339296875*(-a^24/b^17)^(7
/12)*b^10) + 70*((-a^24/b^17)^(1/12)*b*x^2*(1/2*I*sqrt(3) + 1/2)^(3/2) - (-
a^24/b^17)^(1/12)*b*x^2*sqrt(1/2*I*sqrt(3) + 1/2))*log(64339296875*(a*x + s
qrt(a^2*x^2 - b))^(1/6)*a^14 + 64339296875*(-a^24/b^17)^(7/12)*b^10*(1/2*I*
sqrt(3) + 1/2)^(3/2) - 64339296875*(-a^24/b^17)^(7/12)*b^10*sqrt(1/2*I*sqrt
(3) + 1/2)) - 70*((-a^24/b^17)^(1/12)*b*x^2*(1/2*I*sqrt(3) + 1/2)^(3/2) - (
-a^24/b^17)^(1/12)*b*x^2*sqrt(1/2*I*sqrt(3) + 1/2))*log(64339296875*(a*x +
sqrt(a^2*x^2 - b))^(1/6)*a^14 - 64339296875*(-a^24/b^17)^(7/12)*b^10*(1/2*I
*sqrt(3) + 1/2)^(3/2) + 64339296875*(-a^24/b^17)^(7/12)*b^10*sqrt(1/2*I*sqr
t(3) + 1/2)) - 35*((-a^24/b^17)^(1/12)*b*x^2*(-I*sqrt(3) - 1) + 2*(-a^24/b^
17)^(1/12)*b*x^2)*log(64339296875*(a*x + sqrt(a^2*x^2 - b))^(1/6)*a^14 + 64
339296875/2*(-a^24/b^17)^(7/12)*b^10*(-I*sqrt(3) - 1) + 64339296875*(-a^24/
b^17)^(7/12)*b^10) + 35*((-a^24/b^17)^(1/12)*b*x^2*(-I*sqrt(3) - 1) + 2*(-a
^24/b^17)^(1/12)*b*x^2)*log(64339296875*(a*x + sqrt(a^2*x^2 - b))^(1/6)*a^1
4 - 64339296875/2*(-a^24/b^17)^(7/12)*b^10*(-I*sqrt(3) - 1) - 64339296875*(
-a^24/b^17)^(7/12)*b^10) - 12*(a*x + 6*sqrt(a^2*x^2 - b))*(a*x + sqrt(a^2*x
^2 - b))^(1/6))/(b*x^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2-b)^(1/2))^(1/6)/x^3/(a^2*x^2-b)^(1/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{6}}}{x^{3}\sqrt{a^{2}x^{2}-b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+(a^2*x^2-b)^(1/2))^(1/6)/x^3/(a^2*x^2-b)^(1/2),x)
```

```
[Out] int((a*x+(a^2*x^2-b)^(1/2))^(1/6)/x^3/(a^2*x^2-b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{6}}}{\sqrt{a^{2}x^{2}-b}\,x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2-b)^(1/2))^(1/6)/x^3/(a^2*x^2-b)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + sqrt(a^2*x^2 - b))^(1/6)/(sqrt(a^2*x^2 - b)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/6}}{x^3\,\sqrt{a^2\,x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + (a^2*x^2 - b)^(1/2))^(1/6)/(x^3*(a^2*x^2 - b)^(1/2)),x)
```

```
[Out] int((a*x + (a^2*x^2 - b)^(1/2))^(1/6)/(x^3*(a^2*x^2 - b)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{ax+\sqrt{a^2x^2-b}}}{x^3\sqrt{a^2x^2-b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a**2*x**2-b)**(1/2))**(1/6)/x**3/(a**2*x**2-b)**(1/2),x)
```

```
[Out] Integral((a*x + sqrt(a**2*x**2 - b))**(1/6)/(x**3*sqrt(a**2*x**2 - b)), x)
```

3.2418 $\int \frac{1}{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\,\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=697

$$\frac{182b\log\left(\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}-\sqrt[3]{c}\right)}{729ac^{16/3}}-\frac{91b\log\left(\sqrt[3]{c}\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^2\right.}{729ac^{16/3}}$$

Rubi [F] time = 0.36, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\,\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((a*x + Sqrt[-b + a^2*x^2])^(1/4)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/
4))^(1/3)),x]
```

```
[Out] Defer[Int][1/((a*x + Sqrt[-b + a^2*x^2])^(1/4)*(c + (a*x + Sqrt[-b + a^2*x^
2])^(1/4))^(1/3)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\,\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{1}{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\,\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[1/((a*x + Sqrt[-b + a^2*x^2])^(1/4)*(c + (a*x + Sqrt[-b + a^2*x^2
])^(1/4))^(1/3)),x]
```

```
[Out] $Aborted
```

IntegrateAlgebraic [A] time = 1.55, size = 697, normalized size = 1.00

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a*x + Sqrt[-b + a^2*x^2])^(1/4)*(c + (a*x + Sqrt[-b
+ a^2*x^2])^(1/4))^(1/3)),x]
```

```
[Out] ((1944*b*c^4 + 3640*a*b*x)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3) + (
-2106*b*c^3 + 6561*a*c^7*x)*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*(c + (a*x + Sq
rt[-b + a^2*x^2])^(1/4))^(2/3) + (2340*b*c^2 - 4374*a*c^6*x)*Sqrt[a*x + Sqr
t[-b + a^2*x^2]]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3) + (-2730*b*c
```

```
+ 3645*a*c^5*x)*(a*x + Sqrt[-b + a^2*x^2])^(3/4)*(c + (a*x + Sqrt[-b + a^2*
x^2])^(1/4))^(2/3) + Sqrt[-b + a^2*x^2]*(3640*b*(c + (a*x + Sqrt[-b + a^2*x
^2])^(1/4))^(2/3) + 6561*c^7*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*(c + (a*x + S
qrt[-b + a^2*x^2])^(1/4))^(2/3) - 4374*c^6*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*(
c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3) + 3645*c^5*(a*x + Sqrt[-b + a^2
*x^2])^(3/4)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3)))/(4860*a*c^5*(a*
x + Sqrt[-b + a^2*x^2])^(5/4)) + (182*b*ArcTan[1/Sqrt[3] + (2*(c + (a*x + S
qrt[-b + a^2*x^2])^(1/4))^(1/3))/(Sqrt[3]*c^(1/3))]])/(243*Sqrt[3]*a*c^(16/3
)) + (182*b*Log[-c^(1/3) + (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)])/(
729*a*c^(16/3)) - (91*b*Log[c^(2/3) + c^(1/3)*(c + (a*x + Sqrt[-b + a^2*x^2
])^(1/4))^(1/3) + (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3)])/(729*a*c^(
16/3))
```

fricas [A] time = 0.59, size = 1036, normalized size = 1.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+(a^2*x^2-b)^(1/2))^(1/4)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))
^(1/3),x, algorithm="fricas")
```

```
[Out] [1/14580*(5460*sqrt(1/3)*b^2*c*sqrt(-1/c^(2/3))*log(6*sqrt(1/3)*(a*c^(2/3)*
x - sqrt(a^2*x^2 - b)*c^(2/3))*(a*x + sqrt(a^2*x^2 - b))^(3/4)*(c + (a*x +
sqrt(a^2*x^2 - b))^(1/4))^(2/3)*sqrt(-1/c^(2/3)) - 3*(a*c^(2/3)*x + sqrt(1/
3)*(a*c*x - sqrt(a^2*x^2 - b)*c)*sqrt(-1/c^(2/3)) - sqrt(a^2*x^2 - b)*c^(2/
3))*(a*x + sqrt(a^2*x^2 - b))^(3/4)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(
1/3) + 3*(a*c*x - sqrt(1/3)*(a*c^(4/3)*x - sqrt(a^2*x^2 - b)*c^(4/3))*sqrt(
-1/c^(2/3)) - sqrt(a^2*x^2 - b)*c)*(a*x + sqrt(a^2*x^2 - b))^(3/4) + 2*b) -
 1820*b^2*c^(2/3)*log((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3) + (c + (a
*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)*c^(1/3) + c^(2/3)) + 3640*b^2*c^(2/3)*
log((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3) - c^(1/3)) + 3*(6561*b*c^8
- 2106*a*b*c^4*x + 2106*sqrt(a^2*x^2 - b)*b*c^4 + 8*(486*a^2*c^5*x^2 - 243*
b*c^5 + 455*a*b*c*x - (486*a*c^5*x + 455*b*c)*sqrt(a^2*x^2 - b))*(a*x + sqr
t(a^2*x^2 - b))^(3/4) + 15*(243*b*c^6 - 182*a*b*c^2*x + 182*sqrt(a^2*x^2 -
b)*b*c^2)*sqrt(a*x + sqrt(a^2*x^2 - b)) - 18*(243*b*c^7 - 130*a*b*c^3*x + 1
30*sqrt(a^2*x^2 - b)*b*c^3)*(a*x + sqrt(a^2*x^2 - b))^(1/4))*(c + (a*x + sq
rt(a^2*x^2 - b))^(1/4))^(2/3))/(a*b*c^6), 1/14580*(10920*sqrt(1/3)*b^2*c^(2
/3)*arctan(sqrt(1/3) + 2*sqrt(1/3)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1
/3)/c^(1/3)) - 1820*b^2*c^(2/3)*log((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(
2/3) + (c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)*c^(1/3) + c^(2/3)) + 364
0*b^2*c^(2/3)*log((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3) - c^(1/3)) +
3*(6561*b*c^8 - 2106*a*b*c^4*x + 2106*sqrt(a^2*x^2 - b)*b*c^4 + 8*(486*a^2*
c^5*x^2 - 243*b*c^5 + 455*a*b*c*x - (486*a*c^5*x + 455*b*c)*sqrt(a^2*x^2 -
b))*(a*x + sqrt(a^2*x^2 - b))^(3/4) + 15*(243*b*c^6 - 182*a*b*c^2*x + 182*s
qrt(a^2*x^2 - b)*b*c^2)*sqrt(a*x + sqrt(a^2*x^2 - b)) - 18*(243*b*c^7 - 130
*a*b*c^3*x + 130*sqrt(a^2*x^2 - b)*b*c^3)*(a*x + sqrt(a^2*x^2 - b))^(1/4))*
(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3))/(a*b*c^6)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+(a^2*x^2-b)^(1/2))^(1/4)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))
^(1/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+(a^2*x^2-b)^(1/2))^(1/4)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3)
,x)
```

```
[Out] int(1/(a*x+(a^2*x^2-b)^(1/2))^(1/4)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+(a^2*x^2-b)^(1/2))^(1/4)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))
^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + sqrt(a^2*x^2 - b))^(1/4)*(c + (a*x + sqrt(a^2*x^2 - b))
^(1/4))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{1/4}\,{\left(c+{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{1/4}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x + (a^2*x^2 - b)^(1/2))^(1/4)*(c + (a*x + (a^2*x^2 - b)^(1/2))^(
1/4))^(1/3)),x)
```

```
[Out] int(1/((a*x + (a^2*x^2 - b)^(1/2))^(1/4)*(c + (a*x + (a^2*x^2 - b)^(1/2))^(
1/4))^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}\,\sqrt[4]{ax+\sqrt{a^2x^2-b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+(a**2*x**2-b)**(1/2))**(1/4)/(c+(a*x+(a**2*x**2-b)**(1/2))
**(1/4))**(1/3),x)
```

```
[Out] Integral(1/((c + (a*x + sqrt(a**2*x**2 - b))**(1/4))**(1/3)*(a*x + sqrt(a**
2*x**2 - b))**(1/4)), x)
```

3.2419 $\int \frac{1}{\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=699

$$-\frac{70b\log\left(\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}-\sqrt[3]{c}\right)}{243ac^{13/3}}+\frac{35b\log\left(\sqrt[3]{c}\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}\right.}{243ac^{13/3}}$$

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(-1/3),x]
```

```
[Out] Defer[Int][(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(-1/3), x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{1}{\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Mathematica [A] time = 1.76, size = 575, normalized size = 0.82

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(-1/3),x]
```

```
[Out] (6*(-1/2*(c^3*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3)) - (104*b*(c + (
a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3))/(243*c^4*(a*x + Sqrt[-b + a^2*x^2])
^(1/4)) + (3*c^2*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(5/3))/5 + (44*b*(c
 + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(5/3))/(81*c^4*Sqrt[a*x + Sqrt[-b + a^
2*x^2]]) - (3*c*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(8/3))/8 - (37*b*(c
+ (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(8/3))/(108*c^4*(a*x + Sqrt[-b + a^2*x^
2])^(3/4)) + (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(11/3)/11 + (b*(c + (a*
x + Sqrt[-b + a^2*x^2])^(1/4))^(11/3))/(12*c^4*(a*x + Sqrt[-b + a^2*x^2]))
+ (35*b*ArcTan[(1 + (2*c^(1/3))/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3
))/Sqrt[3]])/(243*Sqrt[3]*c^(13/3)) - (35*b*Log[1 - c^(1/3)/(c + (a*x + Sqr
t[-b + a^2*x^2])^(1/4))^(1/3)])/(729*c^(13/3)) + (35*b*Log[1 + c^(2/3)/(c +
 (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3) + c^(1/3)/(c + (a*x + Sqrt[-b + a^
2*x^2])^(1/4))^(1/3)])/(1458*c^(13/3))))/a
```

IntegrateAlgebraic [A] time = 1.19, size = 699, normalized size = 1.00

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(-1/3),x]
```

```
[Out] ((8910*b*c^3 - 19683*a*c^7*x)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3)
+ (-9900*b*c^2 + 13122*a*c^6*x)*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*(c + (a*x
+ Sqrt[-b + a^2*x^2])^(1/4))^(2/3) + (11550*b*c - 10935*a*c^5*x)*Sqrt[a*x +
 Sqrt[-b + a^2*x^2]]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3) + (-15400
*b + 9720*a*c^4*x)*(a*x + Sqrt[-b + a^2*x^2])^(3/4)*(c + (a*x + Sqrt[-b + a
^2*x^2])^(1/4))^(2/3) + Sqrt[-b + a^2*x^2]*(-19683*c^7*(c + (a*x + Sqrt[-b
+ a^2*x^2])^(1/4))^(2/3) + 13122*c^6*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*(c +
(a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3) - 10935*c^5*Sqrt[a*x + Sqrt[-b + a^
2*x^2]]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3) + 9720*c^4*(a*x + Sqrt
[-b + a^2*x^2])^(3/4)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3)))/(17820
*a^2*c^4*x + 17820*a*c^4*Sqrt[-b + a^2*x^2]) - (70*b*ArcTan[1/Sqrt[3] + (2*
(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3))/(Sqrt[3]*c^(1/3))])/(81*Sqrt[
3]*a*c^(13/3)) - (70*b*Log[-c^(1/3) + (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4)
)^(1/3)])/(243*a*c^(13/3)) + (35*b*Log[c^(2/3) + c^(1/3)*(c + (a*x + Sqrt[-
b + a^2*x^2])^(1/4))^(1/3) + (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3)])
/(243*a*c^(13/3))
```

fricas [A] time = 0.58, size = 1039, normalized size = 1.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3),x, algorithm="fricas")
```

```
[Out] [1/53460*(23100*sqrt(1/3)*b*c*sqrt((-c)^(1/3)/c)*log(-6*sqrt(1/3)*(a*(-c)^(
2/3)*x - sqrt(a^2*x^2 - b)*(-c)^(2/3))*(a*x + sqrt(a^2*x^2 - b))^(3/4)*(c +
 (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3)*sqrt((-c)^(1/3)/c) - 3*(a*(-c)^(2/3
)*x - sqrt(1/3)*(a*c*x - sqrt(a^2*x^2 - b)*c)*sqrt((-c)^(1/3)/c) - sqrt(a^2
*x^2 - b)*(-c)^(2/3))*(a*x + sqrt(a^2*x^2 - b))^(3/4)*(c + (a*x + sqrt(a^2*
x^2 - b))^(1/4))^(1/3) + 3*(a*c*x - sqrt(1/3)*(a*(-c)^(1/3)*c*x - sqrt(a^2*
x^2 - b)*(-c)^(1/3)*c)*sqrt((-c)^(1/3)/c) - sqrt(a^2*x^2 - b)*c)*(a*x + sqr
t(a^2*x^2 - b))^(3/4) + 2*b) + 7700*b*(-c)^(2/3)*log((-c)^(2/3) - (-c)^(1/3
)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3) + (c + (a*x + sqrt(a^2*x^2 -
b))^(1/4))^(2/3)) - 15400*b*(-c)^(2/3)*log((-c)^(1/3) + (c + (a*x + sqrt(a^
2*x^2 - b))^(1/4))^(1/3)) - 3*(19683*c^8 - 8910*a*c^4*x + 8910*sqrt(a^2*x^2
 - b)*c^4 - 40*(243*c^5 - 385*a*c*x + 385*sqrt(a^2*x^2 - b)*c)*(a*x + sqrt(
a^2*x^2 - b))^(3/4) + 15*(729*c^6 - 770*a*c^2*x + 770*sqrt(a^2*x^2 - b)*c^2
)*sqrt(a*x + sqrt(a^2*x^2 - b)) - 18*(729*c^7 - 550*a*c^3*x + 550*sqrt(a^2*
x^2 - b)*c^3)*(a*x + sqrt(a^2*x^2 - b))^(1/4))*(c + (a*x + sqrt(a^2*x^2 - b
))^(1/4))^(2/3))/(a*c^5), -1/53460*(46200*sqrt(1/3)*b*c*sqrt(-(-c)^(1/3)/c)
*arctan(-sqrt(1/3)*(-c)^(1/3)*sqrt(-(-c)^(1/3)/c) + 2*sqrt(1/3)*(c + (a*x +
 sqrt(a^2*x^2 - b))^(1/4))^(1/3)*sqrt(-(-c)^(1/3)/c)) - 7700*b*(-c)^(2/3)*l
og((-c)^(2/3) - (-c)^(1/3)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3) + (c
 + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3)) + 15400*b*(-c)^(2/3)*log((-c)^(1
/3) + (c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)) + 3*(19683*c^8 - 8910*a*
c^4*x + 8910*sqrt(a^2*x^2 - b)*c^4 - 40*(243*c^5 - 385*a*c*x + 385*sqrt(a^2
*x^2 - b)*c)*(a*x + sqrt(a^2*x^2 - b))^(3/4) + 15*(729*c^6 - 770*a*c^2*x +
770*sqrt(a^2*x^2 - b)*c^2)*sqrt(a*x + sqrt(a^2*x^2 - b)) - 18*(729*c^7 - 55
0*a*c^3*x + 550*sqrt(a^2*x^2 - b)*c^3)*(a*x + sqrt(a^2*x^2 - b))^(1/4))*(c
+ (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3))/(a*c^5)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3),x)
[Out] int(1/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3),x, algorithm="maxima")
[Out] integrate((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c+{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{1/4}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(1/3),x)
[Out] int(1/(c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(a*x+(a**2*x**2-b)**(1/2))**(1/4))**(1/3),x)
[Out] Integral((c + (a*x + sqrt(a**2*x**2 - b))**(1/4))**(-1/3), x)
```

3.2420 $\int \sqrt[4]{\frac{1+ax-4x^2-4ax^3+6x^4+6ax^5-4x^6-4ax^7+x^8+ax^9}{-c+bx}}\,dx$

Optimal. Leaf size=708

$$\frac{32a^3b^2x^9-192a^3b^2x^7+384a^3b^2x^5-320a^3b^2x^3+96a^3b^2x+36a^3bcx^8-108a^3bcx^6+108a^3bcx^4-36a^3bcx^2 \cdots}{\cdots}$$

Rubi [A] time = 0.41, antiderivative size = 471, normalized size of antiderivative = 0.67, number of steps used = 10, number of rules used = 10, integrand size = 60, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6688, 6718, 952, 80, 50, 63, 240, 212, 208, 205}

$$-\frac{(ax+1)(5b-3ac)(c-bx)\sqrt[4]{-\frac{(1-x^2)^4(ax+1)}{c-bx}}}{8a^2b^2(1-x^2)}+\frac{((7-32a^2)b^2+15a^2c^2-10abc)(c-bx)\sqrt[4]{-\frac{(1-x^2)^4(ax+1)}{c-bx}}}{32a^2b^3(1-x^2)}+\frac{(ax+1)^2(c-bx)\sqrt[4]{-\frac{(1-x^2)^4(ax+1)}{c-bx}}}{3a^2b(1-x^2)}-\frac{(ac+b)((7-32a^2)b^2+15a^2c^2-10abc)\sqrt[4]{bx-c}\sqrt[4]{-\frac{(1-x^2)^4(ax+1)}{c-bx}}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{ax+1}}{\sqrt[4]{a}\sqrt[4]{bx-c}}\right)}{64a^{11/4}b^{13/4}(1-x^2)\sqrt[4]{ax+1}}-\frac{(ac+b)((7-32a^2)b^2+15a^2c^2-10abc)\sqrt[4]{bx-c}\sqrt[4]{-\frac{(1-x^2)^4(ax+1)}{c-bx}}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{ax+1}}{\sqrt[4]{a}\sqrt[4]{bx-c}}\right)}{64a^{11/4}b^{13/4}(1-x^2)\sqrt[4]{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + a*x - 4*x^2 - 4*a*x^3 + 6*x^4 + 6*a*x^5 - 4*x^6 - 4*a*x^7 + x^8 +
 a*x^9)/(-c + b*x))^(1/4),x]
```

```
[Out] (((7 - 32*a^2)*b^2 - 10*a*b*c + 15*a^2*c^2)*(c - b*x)*(-(((1 + a*x)*(1 - x^
2)^4)/(c - b*x)))^(1/4))/(32*a^2*b^3*(1 - x^2)) - ((5*b - 3*a*c)*(1 + a*x)*
(c - b*x)*(-(((1 + a*x)*(1 - x^2)^4)/(c - b*x)))^(1/4))/(8*a^2*b^2*(1 - x^2
)) + ((1 + a*x)^2*(c - b*x)*(-(((1 + a*x)*(1 - x^2)^4)/(c - b*x)))^(1/4))/(
3*a^2*b*(1 - x^2)) - ((b + a*c)*((7 - 32*a^2)*b^2 - 10*a*b*c + 15*a^2*c^2)*
(-c + b*x)^(1/4)*(-(((1 + a*x)*(1 - x^2)^4)/(c - b*x)))^(1/4)*ArcTan[(b^(1/
4)*(1 + a*x)^(1/4))/(a^(1/4)*(-c + b*x)^(1/4))])/(64*a^(11/4)*b^(13/4)*(1 +
 a*x)^(1/4)*(1 - x^2)) - ((b + a*c)*((7 - 32*a^2)*b^2 - 10*a*b*c + 15*a^2*c
^2)*(-c + b*x)^(1/4)*(-(((1 + a*x)*(1 - x^2)^4)/(c - b*x)))^(1/4)*ArcTanh[(
b^(1/4)*(1 + a*x)^(1/4))/(a^(1/4)*(-c + b*x)^(1/4))])/(64*a^(11/4)*b^(13/4)
*(1 + a*x)^(1/4)*(1 - x^2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 952

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x)^(n + 1))/(g*
e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !FreeQ[w, x] &&  !Fr
eeQ[z, x]
```

Rubi steps

$$\int \sqrt[4]{\frac{1+ax-4x^2-4ax^3+6x^4+6ax^5-4x^6-4ax^7+x^8+ax^9}{-c+bx}}\,dx = \int \sqrt[4]{\frac{(1+ax)\left(-1+x^2\right)^4}{-c+bx}}\,dx$$

$$= \frac{\left(\sqrt[4]{-c+bx}\sqrt[4]{\frac{(1+ax)\left(-1+x^2\right)^4}{-c+bx}}\right)\int \frac{\sqrt[4]{1+ax}\,(-1}{\sqrt[4]{-c+b}}}{\sqrt[4]{1+ax}\left(-1+x^2\right)}$$

$$= \frac{(1+ax)^2(c-bx)\sqrt[4]{-\frac{(1+ax)\left(1-x^2\right)^4}{c-bx}}}{3a^2b\left(1-x^2\right)} + \frac{\left(\sqrt[4]{\right.}}{}$$

$$= -\frac{(5b-3ac)(1+ax)(c-bx)\sqrt[4]{-\frac{(1+ax)(1-}{c-bx}}}{8a^2b^2\left(1-x^2\right)}$$

$$= \frac{\left(\left(7-32a^2\right)b^2-10abc+15a^2c^2\right)(c-b}{32a^2b^3\left(1-x^2\right)}$$

$$= \frac{\left(\left(7-32a^2\right)b^2-10abc+15a^2c^2\right)(c-b}{32a^2b^3\left(1-x^2\right)}$$

$$= \frac{\left(\left(7-32a^2\right)b^2-10abc+15a^2c^2\right)(c-b}{32a^2b^3\left(1-x^2\right)}$$

$$= \frac{\left(\left(7-32a^2\right)b^2-10abc+15a^2c^2\right)(c-b}{32a^2b^3\left(1-x^2\right)}$$

$$= \frac{\left(\left(7-32a^2\right)b^2-10abc+15a^2c^2\right)(c-b}{32a^2b^3\left(1-x^2\right)}$$

<u>Mathematica</u> [C] time = 0.24, size = 171, normalized size = 0.24

$$\frac{4(ax+1)\sqrt[4]{\frac{a(c-bx)}{ac+b}}\sqrt[4]{\frac{\left(x^2-1\right)^4(ax+1)}{bx-c}}\left((ac+b)^2\,{}_2F_1\left(-\frac{7}{4},\frac{5}{4};\frac{9}{4};\frac{axb+b}{b+ac}\right)-a\left(a\left(b^2-c^2\right)\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\frac{axb+b}{b+ac}\right)+2c(ac+b)\,{}_2F_1\left(-\frac{3}{4},\frac{5}{4};\frac{9}{4};\frac{axb+b}{b+ac}\right)\right)\right)}{5a^3b^2\left(x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + a*x - 4*x^2 - 4*a*x^3 + 6*x^4 + 6*a*x^5 - 4*x^6 - 4*a*x^7 +
 x^8 + a*x^9)/(-c + b*x))^(1/4),x]
```

```
[Out] (4*(1 + a*x)*((a*(c - b*x))/(b + a*c))^(1/4)*(((1 + a*x)*(-1 + x^2)^4)/(-c
+ b*x))^(1/4)*((b + a*c)^2*Hypergeometric2F1[-7/4, 5/4, 9/4, (b + a*b*x)/(b
 + a*c)] - a*(2*c*(b + a*c)*Hypergeometric2F1[-3/4, 5/4, 9/4, (b + a*b*x)/(
b + a*c)] + a*(b^2 - c^2)*Hypergeometric2F1[1/4, 5/4, 9/4, (b + a*b*x)/(b +
 a*c)])))/(5*a^3*b^2*(-1 + x^2))
```

IntegrateAlgebraic [A] time = 2.02, size = 708, normalized size = 1.00

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + a*x - 4*x^2 - 4*a*x^3 + 6*x^4 + 6*a*x^5 - 4*x^6 -
4*a*x^7 + x^8 + a*x^9)/(-c + b*x))^(1/4),x]
```

```
[Out] (7*b^2 + 96*a^2*b^2 - 6*a*b*c - 45*a^2*c^2 + 3*a*b^2*x + 96*a^3*b^2*x - 42*
a^2*b*c*x - 45*a^3*c^2*x - 21*b^2*x^2 - 324*a^2*b^2*x^2 + 18*a*b*c*x^2 - 36
*a^3*b*c*x^2 + 135*a^2*c^2*x^2 - 9*a*b^2*x^3 - 320*a^3*b^2*x^3 + 126*a^2*b*
c*x^3 + 135*a^3*c^2*x^3 + 21*b^2*x^4 + 396*a^2*b^2*x^4 - 18*a*b*c*x^4 + 108
*a^3*b*c*x^4 - 135*a^2*c^2*x^4 + 9*a*b^2*x^5 + 384*a^3*b^2*x^5 - 126*a^2*b*
c*x^5 - 135*a^3*c^2*x^5 - 7*b^2*x^6 - 204*a^2*b^2*x^6 + 6*a*b*c*x^6 - 108*a
^3*b*c*x^6 + 45*a^2*c^2*x^6 - 3*a*b^2*x^7 - 192*a^3*b^2*x^7 + 42*a^2*b*c*x^
7 + 45*a^3*c^2*x^7 + 36*a^2*b^2*x^8 + 36*a^3*b*c*x^8 + 32*a^3*b^2*x^9)/(96*
a^2*b^3*((1 + a*x - 4*x^2 - 4*a*x^3 + 6*x^4 + 6*a*x^5 - 4*x^6 - 4*a*x^7 + x
^8 + a*x^9)/(-c + b*x))^(3/4)) + ((-7*b^3 + 32*a^2*b^3 + 3*a*b^2*c + 32*a^3
*b^2*c - 5*a^2*b*c^2 - 15*a^3*c^3)*ArcTan[(a^(1/4)*(-1 + x)*(1 + x))/(b^(1/
4)*((1 + a*x - 4*x^2 - 4*a*x^3 + 6*x^4 + 6*a*x^5 - 4*x^6 - 4*a*x^7 + x^8 +
a*x^9)/(-c + b*x))^(1/4))])/(64*a^(11/4)*b^(13/4)) + ((7*b^3 - 32*a^2*b^3 -
 3*a*b^2*c - 32*a^3*b^2*c + 5*a^2*b*c^2 + 15*a^3*c^3)*ArcTanh[(a^(1/4)*(-1
+ x)*(1 + x))/(b^(1/4)*((1 + a*x - 4*x^2 - 4*a*x^3 + 6*x^4 + 6*a*x^5 - 4*x^
6 - 4*a*x^7 + x^8 + a*x^9)/(-c + b*x))^(1/4))])/(64*a^(11/4)*b^(13/4))
```

fricas [B] time = 0.77, size = 4114, normalized size = 5.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x^9-4*a*x^7+x^8+6*a*x^5-4*x^6-4*a*x^3+6*x^4+a*x-4*x^2+1)/(b*x
-c))^(1/4),x, algorithm="fricas")
```

```
[Out] -1/384*(12*(a^2*b^3*x^2 - a^2*b^3)*((50625*a^12*c^12 + 67500*a^11*b*c^11 +
(1048576*a^8 - 917504*a^6 + 301056*a^4 - 43904*a^2 + 2401)*b^12 + 4*(104857
6*a^9 - 589824*a^7 + 86016*a^5 + 3136*a^3 - 1029*a)*b^11*c + 2*(3145728*a^1
0 - 1114112*a^8 + 135168*a^6 - 30912*a^4 + 4753*a^2)*b^10*c^2 + 4*(1048576*
a^11 - 917504*a^9 + 454656*a^7 - 79104*a^5 + 2751*a^3)*b^9*c^3 + (1048576*a
^12 - 7471104*a^10 + 2297856*a^8 - 40704*a^6 - 15249*a^4)*b^8*c^4 - 200*(32
768*a^11 - 6144*a^9 + 1344*a^7 - 243*a^5)*b^7*c^5 - 20*(98304*a^12 - 141312
*a^10 + 43712*a^8 - 1579*a^6)*b^6*c^6 + 200*(18432*a^11 - 1664*a^9 - 93*a^7
)*b^5*c^7 + 25*(55296*a^12 - 12672*a^10 + 3751*a^8)*b^4*c^8 - 1500*(576*a^1
1 - 41*a^9)*b^3*c^9 - 6750*(64*a^12 + a^10)*b^2*c^10)/(a^11*b^13))^(1/4)*ar
ctan(-((a^8*b^10*x^2 - a^8*b^10)*sqrt(((225*a^6*c^6 + 150*a^5*b*c^5 + (1024
*a^4 - 448*a^2 + 49)*b^6 + 2*(1024*a^5 - 128*a^3 - 21*a)*b^5*c + (1024*a^6
- 128*a^4 + 79*a^2)*b^4*c^2 - 20*(64*a^5 - 9*a^3)*b^3*c^3 - 5*(192*a^6 + 13
*a^4)*b^2*c^4)*sqrt((a*x^9 - 4*a*x^7 + x^8 + 6*a*x^5 - 4*x^6 - 4*a*x^3 + 6*
x^4 + a*x - 4*x^2 + 1)/(b*x - c)) + (a^6*b^6*x^4 - 2*a^6*b^6*x^2 + a^6*b^6)
*sqrt((50625*a^12*c^12 + 67500*a^11*b*c^11 + (1048576*a^8 - 917504*a^6 + 30
1056*a^4 - 43904*a^2 + 2401)*b^12 + 4*(1048576*a^9 - 589824*a^7 + 86016*a^5
 + 3136*a^3 - 1029*a)*b^11*c + 2*(3145728*a^10 - 1114112*a^8 + 135168*a^6 -
 30912*a^4 + 4753*a^2)*b^10*c^2 + 4*(1048576*a^11 - 917504*a^9 + 454656*a^7
 - 79104*a^5 + 2751*a^3)*b^9*c^3 + (1048576*a^12 - 7471104*a^10 + 2297856*a
^8 - 40704*a^6 - 15249*a^4)*b^8*c^4 - 200*(32768*a^11 - 6144*a^9 + 1344*a^7
 - 243*a^5)*b^7*c^5 - 20*(98304*a^12 - 141312*a^10 + 43712*a^8 - 1579*a^6)*
b^6*c^6 + 200*(18432*a^11 - 1664*a^9 - 93*a^7)*b^5*c^7 + 25*(55296*a^12 - 1
2672*a^10 + 3751*a^8)*b^4*c^8 - 1500*(576*a^11 - 41*a^9)*b^3*c^9 - 6750*(64
*a^12 + a^10)*b^2*c^10)/(a^11*b^13)))/(x^4 - 2*x^2 + 1))*((50625*a^12*c^12
+ 67500*a^11*b*c^11 + (1048576*a^8 - 917504*a^6 + 301056*a^4 - 43904*a^2 +
2401)*b^12 + 4*(1048576*a^9 - 589824*a^7 + 86016*a^5 + 3136*a^3 - 1029*a)*b
```

```
^11*c + 2*(3145728*a^10 - 1114112*a^8 + 135168*a^6 - 30912*a^4 + 4753*a^2)*
b^10*c^2 + 4*(1048576*a^11 - 917504*a^9 + 454656*a^7 - 79104*a^5 + 2751*a^3
)*b^9*c^3 + (1048576*a^12 - 7471104*a^10 + 2297856*a^8 - 40704*a^6 - 15249*
a^4)*b^8*c^4 - 200*(32768*a^11 - 6144*a^9 + 1344*a^7 - 243*a^5)*b^7*c^5 - 2
0*(98304*a^12 - 141312*a^10 + 43712*a^8 - 1579*a^6)*b^6*c^6 + 200*(18432*a^
11 - 1664*a^9 - 93*a^7)*b^5*c^7 + 25*(55296*a^12 - 12672*a^10 + 3751*a^8)*b
^4*c^8 - 1500*(576*a^11 - 41*a^9)*b^3*c^9 - 6750*(64*a^12 + a^10)*b^2*c^10)
/(a^11*b^13))^(3/4) - (15*a^11*b^10*c^3 + 5*a^10*b^11*c^2 - (32*a^10 - 7*a^
8)*b^13 - (32*a^11 + 3*a^9)*b^12*c)*((a*x^9 - 4*a*x^7 + x^8 + 6*a*x^5 - 4*x
^6 - 4*a*x^3 + 6*x^4 + a*x - 4*x^2 + 1)/(b*x - c))^(1/4)*((50625*a^12*c^12
+ 67500*a^11*b*c^11 + (1048576*a^8 - 917504*a^6 + 301056*a^4 - 43904*a^2 +
2401)*b^12 + 4*(1048576*a^9 - 589824*a^7 + 86016*a^5 + 3136*a^3 - 1029*a)*b
^11*c + 2*(3145728*a^10 - 1114112*a^8 + 135168*a^6 - 30912*a^4 + 4753*a^2)*
b^10*c^2 + 4*(1048576*a^11 - 917504*a^9 + 454656*a^7 - 79104*a^5 + 2751*a^3
)*b^9*c^3 + (1048576*a^12 - 7471104*a^10 + 2297856*a^8 - 40704*a^6 - 15249*
a^4)*b^8*c^4 - 200*(32768*a^11 - 6144*a^9 + 1344*a^7 - 243*a^5)*b^7*c^5 - 2
0*(98304*a^12 - 141312*a^10 + 43712*a^8 - 1579*a^6)*b^6*c^6 + 200*(18432*a^
11 - 1664*a^9 - 93*a^7)*b^5*c^7 + 25*(55296*a^12 - 12672*a^10 + 3751*a^8)*b
^4*c^8 - 1500*(576*a^11 - 41*a^9)*b^3*c^9 - 6750*(64*a^12 + a^10)*b^2*c^10)
/(a^11*b^13))^(3/4))/(50625*a^12*c^12 + 67500*a^11*b*c^11 + (1048576*a^8 -
917504*a^6 + 301056*a^4 - 43904*a^2 + 2401)*b^12 + 4*(1048576*a^9 - 589824*
a^7 + 86016*a^5 + 3136*a^3 - 1029*a)*b^11*c + 2*(3145728*a^10 - 1114112*a^8
 + 135168*a^6 - 30912*a^4 + 4753*a^2)*b^10*c^2 + 4*(1048576*a^11 - 917504*a
^9 + 454656*a^7 - 79104*a^5 + 2751*a^3)*b^9*c^3 + (1048576*a^12 - 7471104*a
^10 + 2297856*a^8 - 40704*a^6 - 15249*a^4)*b^8*c^4 - 200*(32768*a^11 - 6144
*a^9 + 1344*a^7 - 243*a^5)*b^7*c^5 - 20*(98304*a^12 - 141312*a^10 + 43712*a
^8 - 1579*a^6)*b^6*c^6 + 200*(18432*a^11 - 1664*a^9 - 93*a^7)*b^5*c^7 + 25*
(55296*a^12 - 12672*a^10 + 3751*a^8)*b^4*c^8 - 1500*(576*a^11 - 41*a^9)*b^3
*c^9 - 6750*(64*a^12 + a^10)*b^2*c^10 - (50625*a^12*c^12 + 67500*a^11*b*c^1
1 + (1048576*a^8 - 917504*a^6 + 301056*a^4 - 43904*a^2 + 2401)*b^12 + 4*(10
48576*a^9 - 589824*a^7 + 86016*a^5 + 3136*a^3 - 1029*a)*b^11*c + 2*(3145728
*a^10 - 1114112*a^8 + 135168*a^6 - 30912*a^4 + 4753*a^2)*b^10*c^2 + 4*(1048
576*a^11 - 917504*a^9 + 454656*a^7 - 79104*a^5 + 2751*a^3)*b^9*c^3 + (10485
76*a^12 - 7471104*a^10 + 2297856*a^8 - 40704*a^6 - 15249*a^4)*b^8*c^4 - 200
*(32768*a^11 - 6144*a^9 + 1344*a^7 - 243*a^5)*b^7*c^5 - 20*(98304*a^12 - 14
1312*a^10 + 43712*a^8 - 1579*a^6)*b^6*c^6 + 200*(18432*a^11 - 1664*a^9 - 93
*a^7)*b^5*c^7 + 25*(55296*a^12 - 12672*a^10 + 3751*a^8)*b^4*c^8 - 1500*(576
*a^11 - 41*a^9)*b^3*c^9 - 6750*(64*a^12 + a^10)*b^2*c^10)*x^2)) - 3*(a^2*b^
3*x^2 - a^2*b^3)*((50625*a^12*c^12 + 67500*a^11*b*c^11 + (1048576*a^8 - 917
504*a^6 + 301056*a^4 - 43904*a^2 + 2401)*b^12 + 4*(1048576*a^9 - 589824*a^7
 + 86016*a^5 + 3136*a^3 - 1029*a)*b^11*c + 2*(3145728*a^10 - 1114112*a^8 +
135168*a^6 - 30912*a^4 + 4753*a^2)*b^10*c^2 + 4*(1048576*a^11 - 917504*a^9
+ 454656*a^7 - 79104*a^5 + 2751*a^3)*b^9*c^3 + (1048576*a^12 - 7471104*a^10
 + 2297856*a^8 - 40704*a^6 - 15249*a^4)*b^8*c^4 - 200*(32768*a^11 - 6144*a^
9 + 1344*a^7 - 243*a^5)*b^7*c^5 - 20*(98304*a^12 - 141312*a^10 + 43712*a^8
- 1579*a^6)*b^6*c^6 + 200*(18432*a^11 - 1664*a^9 - 93*a^7)*b^5*c^7 + 25*(55
296*a^12 - 12672*a^10 + 3751*a^8)*b^4*c^8 - 1500*(576*a^11 - 41*a^9)*b^3*c^
9 - 6750*(64*a^12 + a^10)*b^2*c^10)/(a^11*b^13))^(1/4)*log(((15*a^3*c^3 + 5
*a^2*b*c^2 - (32*a^2 - 7)*b^3 - (32*a^3 + 3*a)*b^2*c)*((a*x^9 - 4*a*x^7 + x
^8 + 6*a*x^5 - 4*x^6 - 4*a*x^3 + 6*x^4 + a*x - 4*x^2 + 1)/(b*x - c))^(1/4)
+ (a^3*b^3*x^2 - a^3*b^3)*((50625*a^12*c^12 + 67500*a^11*b*c^11 + (1048576*
a^8 - 917504*a^6 + 301056*a^4 - 43904*a^2 + 2401)*b^12 + 4*(1048576*a^9 - 5
89824*a^7 + 86016*a^5 + 3136*a^3 - 1029*a)*b^11*c + 2*(3145728*a^10 - 11141
12*a^8 + 135168*a^6 - 30912*a^4 + 4753*a^2)*b^10*c^2 + 4*(1048576*a^11 - 91
7504*a^9 + 454656*a^7 - 79104*a^5 + 2751*a^3)*b^9*c^3 + (1048576*a^12 - 747
1104*a^10 + 2297856*a^8 - 40704*a^6 - 15249*a^4)*b^8*c^4 - 200*(32768*a^11
- 6144*a^9 + 1344*a^7 - 243*a^5)*b^7*c^5 - 20*(98304*a^12 - 141312*a^10 + 4
3712*a^8 - 1579*a^6)*b^6*c^6 + 200*(18432*a^11 - 1664*a^9 - 93*a^7)*b^5*c^7
 + 25*(55296*a^12 - 12672*a^10 + 3751*a^8)*b^4*c^8 - 1500*(576*a^11 - 41*a^
```

```
9)*b^3*c^9 - 6750*(64*a^12 + a^10)*b^2*c^10)/(a^11*b^13))^(1/4))/(x^2 - 1))
 + 3*(a^2*b^3*x^2 - a^2*b^3)*((50625*a^12*c^12 + 67500*a^11*b*c^11 + (10485
76*a^8 - 917504*a^6 + 301056*a^4 - 43904*a^2 + 2401)*b^12 + 4*(1048576*a^9
- 589824*a^7 + 86016*a^5 + 3136*a^3 - 1029*a)*b^11*c + 2*(3145728*a^10 - 11
14112*a^8 + 135168*a^6 - 30912*a^4 + 4753*a^2)*b^10*c^2 + 4*(1048576*a^11 -
 917504*a^9 + 454656*a^7 - 79104*a^5 + 2751*a^3)*b^9*c^3 + (1048576*a^12 -
7471104*a^10 + 2297856*a^8 - 40704*a^6 - 15249*a^4)*b^8*c^4 - 200*(32768*a^
11 - 6144*a^9 + 1344*a^7 - 243*a^5)*b^7*c^5 - 20*(98304*a^12 - 141312*a^10
+ 43712*a^8 - 1579*a^6)*b^6*c^6 + 200*(18432*a^11 - 1664*a^9 - 93*a^7)*b^5*
c^7 + 25*(55296*a^12 - 12672*a^10 + 3751*a^8)*b^4*c^8 - 1500*(576*a^11 - 41
*a^9)*b^3*c^9 - 6750*(64*a^12 + a^10)*b^2*c^10)/(a^11*b^13))^(1/4)*log(((15
*a^3*c^3 + 5*a^2*b*c^2 - (32*a^2 - 7)*b^3 - (32*a^3 + 3*a)*b^2*c)*((a*x^9 -
 4*a*x^7 + x^8 + 6*a*x^5 - 4*x^6 - 4*a*x^3 + 6*x^4 + a*x - 4*x^2 + 1)/(b*x
- c))^(1/4) - (a^3*b^3*x^2 - a^3*b^3)*((50625*a^12*c^12 + 67500*a^11*b*c^11
 + (1048576*a^8 - 917504*a^6 + 301056*a^4 - 43904*a^2 + 2401)*b^12 + 4*(104
8576*a^9 - 589824*a^7 + 86016*a^5 + 3136*a^3 - 1029*a)*b^11*c + 2*(3145728*
a^10 - 1114112*a^8 + 135168*a^6 - 30912*a^4 + 4753*a^2)*b^10*c^2 + 4*(10485
76*a^11 - 917504*a^9 + 454656*a^7 - 79104*a^5 + 2751*a^3)*b^9*c^3 + (104857
6*a^12 - 7471104*a^10 + 2297856*a^8 - 40704*a^6 - 15249*a^4)*b^8*c^4 - 200*
(32768*a^11 - 6144*a^9 + 1344*a^7 - 243*a^5)*b^7*c^5 - 20*(98304*a^12 - 141
312*a^10 + 43712*a^8 - 1579*a^6)*b^6*c^6 + 200*(18432*a^11 - 1664*a^9 - 93*
a^7)*b^5*c^7 + 25*(55296*a^12 - 12672*a^10 + 3751*a^8)*b^4*c^8 - 1500*(576*
a^11 - 41*a^9)*b^3*c^9 - 6750*(64*a^12 + a^10)*b^2*c^10)/(a^11*b^13))^(1/4)
)/(x^2 - 1)) - 4*(32*a^2*b^3*x^3 - 45*a^2*c^3 + (96*a^2 + 7)*b^2*c - 6*a*b*
c^2 + 4*(a^2*b^2*c + a*b^3)*x^2 + (9*a^2*b*c^2 - (96*a^2 + 7)*b^3 + 2*a*b^2
*c)*x)*((a*x^9 - 4*a*x^7 + x^8 + 6*a*x^5 - 4*x^6 - 4*a*x^3 + 6*x^4 + a*x -
4*x^2 + 1)/(b*x - c))^(1/4))/(a^2*b^3*x^2 - a^2*b^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax^9 - 4\,ax^7 + x^8 + 6\,ax^5 - 4\,x^6 - 4\,ax^3 + 6\,x^4 + ax - 4\,x^2 + 1}{bx - c}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x^9-4*a*x^7+x^8+6*a*x^5-4*x^6-4*a*x^3+6*x^4+a*x-4*x^2+1)/(b*x
-c))^(1/4),x, algorithm="giac")
```

```
[Out] integrate(((a*x^9 - 4*a*x^7 + x^8 + 6*a*x^5 - 4*x^6 - 4*a*x^3 + 6*x^4 + a*x
 - 4*x^2 + 1)/(b*x - c))^(1/4), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\frac{a\,x^9 - 4a\,x^7 + x^8 + 6a\,x^5 - 4x^6 - 4a\,x^3 + 6x^4 + ax - 4x^2 + 1}{bx - c}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^9-4*a*x^7+x^8+6*a*x^5-4*x^6-4*a*x^3+6*x^4+a*x-4*x^2+1)/(b*x-c))^(
1/4),x)
```

```
[Out] int(((a*x^9-4*a*x^7+x^8+6*a*x^5-4*x^6-4*a*x^3+6*x^4+a*x-4*x^2+1)/(b*x-c))^(
1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax^9 - 4\,ax^7 + x^8 + 6\,ax^5 - 4\,x^6 - 4\,ax^3 + 6\,x^4 + ax - 4\,x^2 + 1}{bx - c}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x^9-4*a*x^7+x^8+6*a*x^5-4*x^6-4*a*x^3+6*x^4+a*x-4*x^2+1)/(b*x
-c))^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(((a*x^9 - 4*a*x^7 + x^8 + 6*a*x^5 - 4*x^6 - 4*a*x^3 + 6*x^4 + a*x
 - 4*x^2 + 1)/(b*x - c))^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(-\frac{a\,x^9+x^8-4\,a\,x^7-4\,x^6+6\,a\,x^5+6\,x^4-4\,a\,x^3-4\,x^2+a\,x+1}{c-b\,x}\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a*x - 4*a*x^3 + 6*a*x^5 - 4*a*x^7 + a*x^9 - 4*x^2 + 6*x^4 - 4*x^6 +
x^8 + 1)/(c - b*x))^(1/4),x)
```

```
[Out] int((-(a*x - 4*a*x^3 + 6*a*x^5 - 4*a*x^7 + a*x^9 - 4*x^2 + 6*x^4 - 4*x^6 +
x^8 + 1)/(c - b*x))^(1/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x**9-4*a*x**7+x**8+6*a*x**5-4*x**6-4*a*x**3+6*x**4+a*x-4*x**2
+1)/(b*x-c))**(1/4),x)
```

```
[Out] Timed out
```

3.2421 $\int \sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx$

Optimal. Leaf size=719

$$\frac{20b\log\left(\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}-\sqrt[3]{c}\right)}{243ac^{11/3}}-\frac{10b\log\left(\sqrt[3]{c}\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}+\right.}{243ac^{11/3}}$$

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3),x]
[Out] Defer[Int][(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3), x]
```

Rubi steps

$$\int \sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx=\int \sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx$$

Mathematica [A] time = 0.86, size = 569, normalized size = 0.79

$$\frac{6\left(-\frac{10b\log\left(\sqrt[3]{c}-\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}\right)}{729c^{11/3}}+\frac{5b\log\left(\sqrt[3]{c}\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}+\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{2/3}+c^{2/3}\right)}{729c^{11/3}}+\frac{10b\tan^{-1}\left(\frac{\frac{2\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[3]{c}}+1}{\sqrt{3}}\right)}{243\sqrt{3}c^{11/3}}+\frac{1}{4}c^3\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{4/3}-\frac{5b\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{243c^3\sqrt[4]{\sqrt{a^2x^2-b}+ax}}-\frac{3}{7}c^2\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{7/3}+\frac{b\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{81c^2\sqrt{\sqrt{a^2x^2-b}+ax}}-\frac{1}{13}\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{13/3}+\frac{3}{10}c\left(\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c\right)^{10/3}-\frac{b\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{108c\left(\sqrt{a^2x^2-b}+ax\right)^{3/4}}-\frac{b\sqrt[3]{\sqrt[4]{\sqrt{a^2x^2-b}+ax}+c}}{12\left(\sqrt{a^2x^2-b}+ax\right)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3),x]
[Out] (-6*(-1/12*(b*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3))/(a*x + Sqrt[-b
+ a^2*x^2]) - (b*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3))/(108*c*(a*x
+ Sqrt[-b + a^2*x^2])^(3/4)) + (b*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1
/3))/(81*c^2*Sqrt[a*x + Sqrt[-b + a^2*x^2]]) - (5*b*(c + (a*x + Sqrt[-b + a
^2*x^2])^(1/4))^(1/3))/(243*c^3*(a*x + Sqrt[-b + a^2*x^2])^(1/4)) + (c^3*(c
 + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(4/3))/4 - (3*c^2*(c + (a*x + Sqrt[-b
+ a^2*x^2])^(1/4))^(7/3))/7 + (3*c*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(
10/3))/10 - (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(13/3)/13 + (10*b*ArcTan
[(1 + (2*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3))/c^(1/3))/Sqrt[3]])/(
243*Sqrt[3]*c^(11/3)) - (10*b*Log[c^(1/3) - (c + (a*x + Sqrt[-b + a^2*x^2])
^(1/4))^(1/3)])/(729*c^(11/3)) + (5*b*Log[c^(2/3) + c^(1/3)*(c + (a*x + Sqr
t[-b + a^2*x^2])^(1/4))^(1/3) + (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3
)])/(729*c^(11/3))))/a
```

IntegrateAlgebraic [A] time = 1.24, size = 719, normalized size = 1.00

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3),x]
```

```
[Out] ((2835*b*c^3 - 19683*a*c^7*x + 68040*a^2*c^3*x^2)*(c + (a*x + Sqrt[-b + a^2
*x^2])^(1/4))^(1/3) + (4095*b*c^2 + 6561*a*c^6*x)*(a*x + Sqrt[-b + a^2*x^2]
)^(1/4)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + (-5460*b*c - 4374*a*
c^5*x)*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4)
)^(1/3) + (9100*b + 3402*a*c^4*x)*(a*x + Sqrt[-b + a^2*x^2])^(3/4)*(c + (a*
x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + Sqrt[-b + a^2*x^2]*((-19683*c^7 + 68
040*a*c^3*x)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + 6561*c^6*(a*x +
 Sqrt[-b + a^2*x^2])^(1/4)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) - 4
374*c^5*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4
))^(1/3) + 3402*c^4*(a*x + Sqrt[-b + a^2*x^2])^(3/4)*(c + (a*x + Sqrt[-b +
a^2*x^2])^(1/4))^(1/3)))/(73710*a^2*c^3*x + 73710*a*c^3*Sqrt[-b + a^2*x^2])
 - (20*b*ArcTan[1/Sqrt[3] + (2*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)
)/(Sqrt[3]*c^(1/3))])/(81*Sqrt[3]*a*c^(11/3)) + (20*b*Log[-c^(1/3) + (c + (
a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)])/(243*a*c^(11/3)) - (10*b*Log[c^(2/
3) + c^(1/3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + (c + (a*x + Sqr
t[-b + a^2*x^2])^(1/4))^(2/3)])/(243*a*c^(11/3))
```

fricas [A] time = 0.57, size = 396, normalized size = 0.55

$$-\frac{18200 \sqrt{3} b \left(c^{2}\right)^{\frac{1}{6}} c \arctan\left(\frac{\sqrt{3} \sqrt{c^{2}} c + 2 \sqrt{3} \left(c^{2}\right)^{\frac{5}{6}} \left(c + \left(ax + \sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}{3 c^{2}}\right) + 9100 b \left(c^{2}\right)^{\frac{2}{3}} \log\left(\left(c + \left(ax + \sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{2}{3}} c + \left(c^{2}\right)^{\frac{1}{3}} c + \left(c^{2}\right)^{\frac{2}{3}} \left(c + \left(ax + \sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}\right) - 18200 b \left(c^{2}\right)^{\frac{2}{3}} \log\left(\left(c + \left(ax + \sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}} c - \left(c^{2}\right)^{\frac{2}{3}}\right) + 3 \left(19683 c^{9} - 70875 a c^{5} x + 2835 \sqrt{a^{2}x^{2}-b}\, c^{5} - 14 \left(243 c^{6} + 650 a c^{2} x - 650 \sqrt{a^{2}x^{2}-b}\, c^{2}\right) \left(ax + \sqrt{a^{2}x^{2}-b}\right)^{\frac{3}{4}} + 6 \left(729 c^{7} + 910 a c^{3} x - 910 \sqrt{a^{2}x^{2}-b}\, c^{3}\right) \sqrt{ax + \sqrt{a^{2}x^{2}-b}} - 9 \left(729 c^{8} + 455 a c^{4} x - 455 \sqrt{a^{2}x^{2}-b}\, c^{4}\right) \left(ax + \sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{4}}\right) \left(c + \left(ax + \sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}{221130\, a c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3),x, algorithm="fricas")
```

```
[Out] -1/221130*(18200*sqrt(3)*b*(c^2)^(1/6)*c*arctan(1/3*(sqrt(3)*sqrt(c^2)*c +
2*sqrt(3)*(c^2)^(5/6)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3))/c^2) + 9
100*b*(c^2)^(2/3)*log((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3)*c + (c^2)
^(1/3)*c + (c^2)^(2/3)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)) - 18200
*b*(c^2)^(2/3)*log((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)*c - (c^2)^(2
/3)) + 3*(19683*c^9 - 70875*a*c^5*x + 2835*sqrt(a^2*x^2 - b)*c^5 - 14*(243*
c^6 + 650*a*c^2*x - 650*sqrt(a^2*x^2 - b)*c^2)*(a*x + sqrt(a^2*x^2 - b))^(3
/4) + 6*(729*c^7 + 910*a*c^3*x - 910*sqrt(a^2*x^2 - b)*c^3)*sqrt(a*x + sqrt
(a^2*x^2 - b)) - 9*(729*c^8 + 455*a*c^4*x - 455*sqrt(a^2*x^2 - b)*c^4)*(a*x
 + sqrt(a^2*x^2 - b))^(1/4))*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3))/(
a*c^5)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \left(c + \left(ax + \sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3),x)
```

```
[Out] int((c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + \left(ax + \sqrt{a^2x^2 - b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3),x, algorithm="maxima")
[Out] integrate((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c + \left(a\,x + \sqrt{a^2\,x^2 - b}\right)^{1/4}\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(1/3),x)
[Out] int((c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{c + \sqrt[4]{ax + \sqrt{a^2x^2 - b}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+(a*x+(a**2*x**2-b)**(1/2))**(1/4))**(1/3),x)
[Out] Integral((c + (a*x + sqrt(a**2*x**2 - b))**(1/4))**(1/3), x)
```

3.2422 $\int \frac{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{x\left(-b+a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=725

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{b}}\right)}{2b^{11/8}}-\frac{\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\left(\sqrt{\frac{2}{2-\sqrt{2}}}\,\sqrt[8]{b}-\frac{2\sqrt[8]{b}}{\sqrt{2-\sqrt{2}}}\right)\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax-\sqrt[4]{b}}}\right)}{b^{11/8}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[8]{b}\,\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax-\sqrt[4]{b}}}\right)}{2\sqrt{2}\,b^{11/8}}-\ldots$$

Rubi [A] time = 0.91, antiderivative size = 746, normalized size of antiderivative = 1.03, number of steps used = 31, number of rules used = 14, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2120, 466, 471, 584, 301, 211, 1165, 628, 1162, 617, 204, 212, 206, 203}

Antiderivative was successfully verified.

```
[In] Int[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/(x*(-b + a^2*x^2)^(3/2)),x]
```

```
[Out] (2*(a*x + Sqrt[-b + a^2*x^2])^(5/4))/(b*(b - (a*x + Sqrt[-b + a^2*x^2])^2))
 - (2*ArcTan[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/(-b)^(1/8)])/(-b)^(11/8) - Ar
cTan[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)]/(2*b^(11/8)) - (Sqrt[2]*ArcT
an[1 - (Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-b)^(1/8)])/(-b)^(11/8)
+ (Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-b)^(1/8)
])/(-b)^(11/8) - ArcTan[1 - (Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/b^(1
/8)]/(2*Sqrt[2]*b^(11/8)) + ArcTan[1 + (Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^
(1/4))/b^(1/8)]/(2*Sqrt[2]*b^(11/8)) - (2*ArcTanh[(a*x + Sqrt[-b + a^2*x^2]
)^(1/4)/(-b)^(1/8)])/(-b)^(11/8) - ArcTanh[(a*x + Sqrt[-b + a^2*x^2])^(1/4)
/b^(1/8)]/(2*b^(11/8)) - Log[(-b)^(1/4) - Sqrt[2]*(-b)^(1/8)*(a*x + Sqrt[-b
 + a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]/(Sqrt[2]*(-b)^(11/8))
+ Log[(-b)^(1/4) + Sqrt[2]*(-b)^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sq
rt[a*x + Sqrt[-b + a^2*x^2]]]/(Sqrt[2]*(-b)^(11/8)) - Log[b^(1/4) - Sqrt[2]
*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]
/(4*Sqrt[2]*b^(11/8)) + Log[b^(1/4) + Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*
x^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]]/(4*Sqrt[2]*b^(11/8))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{x\left(-b+a^2x^2\right)^{3/2}}\,dx = 8\,\text{Subst}\left(\int \frac{x^{9/4}}{\left(-b+x^2\right)^2\left(b+x^2\right)}\,dx, x, ax+\sqrt{-b+a^2x^2}\right)$$

$$= 32\,\text{Subst}\left(\int \frac{x^{12}}{\left(-b+x^8\right)^2\left(b+x^8\right)}\,dx, x, \sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)$$

$$= \frac{2\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{b\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} + \frac{2\,\text{Subst}\left(\int \frac{x^4(5b-3x^8)}{(-b+x^8)(b+x^8)}\,dx, x, \sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)}{b}$$

$$= \frac{2\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{b\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} + \frac{2\,\text{Subst}\left(\int \left(\frac{x^4}{-b+x^8}-\frac{4x^4}{b+x^8}\right)dx, x, \sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right.}{b}$$

$$= \frac{2\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{b\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} + \frac{2\,\text{Subst}\left(\int \frac{x^4}{-b+x^8}\,dx, x, \sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)}{b} - \frac{8\,\text{Su}}{}$$

$$= \frac{2\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{b\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{b}-x^4}\,dx, x, \sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)}{b} + \frac{\text{Subs}}{}$$

$$= \frac{2\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{b\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-b}-x^2}\,dx, x, \sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)}{(-b)^{5/4}} - \frac{2\,\text{S}}{}$$

$$= \frac{2\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{b\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} - \frac{2\tan^{-1}\left(\frac{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[8]{-b}}\right)}{(-b)^{11/8}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[8]{b}}\right)}{2b^{11/8}} -$$

$$= \frac{2\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{b\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} - \frac{2\tan^{-1}\left(\frac{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[8]{-b}}\right)}{(-b)^{11/8}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[8]{b}}\right)}{2b^{11/8}} -$$

$$= \frac{2\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{b\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)} - \frac{2\tan^{-1}\left(\frac{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[8]{-b}}\right)}{(-b)^{11/8}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[8]{b}}\right)}{2b^{11/8}} -$$

Mathematica [B] time = 3.95, size = 2000, normalized size = 2.76

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/(x*(-b + a^2*x^2)^(3/2)),x]
```

```
[Out] (-16*b^(3/8)*(a*x + Sqrt[-b + a^2*x^2])^(5/4) - 4*(-b + (a*x + Sqrt[-b + a^
2*x^2])^2)*ArcTan[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)] - 2*Sqrt[2]*(-b
 + (a*x + Sqrt[-b + a^2*x^2])^2)*ArcTan[1 - (Sqrt[2]*(a*x + Sqrt[-b + a^2*x
```

```
^2])^(1/4))/b^(1/8)] - 2*Sqrt[2]*b*ArcTan[1 + (Sqrt[2]*(a*x + Sqrt[-b + a^2
*x^2])^(1/4))/b^(1/8)] + 2*Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^2*ArcTan[1 +
(Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/b^(1/8)] - 16*b*ArcTan[Cot[Pi/8]
 - ((a*x + Sqrt[-b + a^2*x^2])^(1/4)*Csc[Pi/8])/b^(1/8)]*Cos[Pi/8] + 16*(a*
x + Sqrt[-b + a^2*x^2])^2*ArcTan[Cot[Pi/8] - ((a*x + Sqrt[-b + a^2*x^2])^(1
/4)*Csc[Pi/8])/b^(1/8)]*Cos[Pi/8] + 16*b*ArcTan[Cot[Pi/8] + ((a*x + Sqrt[-b
 + a^2*x^2])^(1/4)*Csc[Pi/8])/b^(1/8)]*Cos[Pi/8] - 16*(a*x + Sqrt[-b + a^2*
x^2])^2*ArcTan[Cot[Pi/8] + ((a*x + Sqrt[-b + a^2*x^2])^(1/4)*Csc[Pi/8])/b^(
1/8)]*Cos[Pi/8] - 2*b*Log[b^(1/8) - (a*x + Sqrt[-b + a^2*x^2])^(1/4)] + 2*(
a*x + Sqrt[-b + a^2*x^2])^2*Log[b^(1/8) - (a*x + Sqrt[-b + a^2*x^2])^(1/4)]
 + 2*b*Log[b^(1/8) + (a*x + Sqrt[-b + a^2*x^2])^(1/4)] - 2*(a*x + Sqrt[-b +
 a^2*x^2])^2*Log[b^(1/8) + (a*x + Sqrt[-b + a^2*x^2])^(1/4)] + Sqrt[2]*b*Lo
g[b^(1/4) - Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a*x + S
qrt[-b + a^2*x^2]]] - Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^2*Log[b^(1/4) - Sq
rt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x
^2]]] - Sqrt[2]*b*Log[b^(1/4) + Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^
(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]] + Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2]
)^2*Log[b^(1/4) + Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a
*x + Sqrt[-b + a^2*x^2]]] - 8*b*Cos[Pi/8]*Log[b^(1/4) + Sqrt[a*x + Sqrt[-b
+ a^2*x^2]] - 2*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*Sin[Pi/8]] + 8*(a*
x + Sqrt[-b + a^2*x^2])^2*Cos[Pi/8]*Log[b^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*
x^2]] - 2*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*Sin[Pi/8]] + 8*b*Cos[Pi/
8]*Log[b^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]] + 2*b^(1/8)*(a*x + Sqrt[-b
+ a^2*x^2])^(1/4)*Sin[Pi/8]] - 8*(a*x + Sqrt[-b + a^2*x^2])^2*Cos[Pi/8]*Log
[b^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]] + 2*b^(1/8)*(a*x + Sqrt[-b + a^2*
x^2])^(1/4)*Sin[Pi/8]] - 16*b*ArcTan[((a*x + Sqrt[-b + a^2*x^2])^(1/4)*Sec[
Pi/8])/b^(1/8) - Tan[Pi/8]]*Sin[Pi/8] + 16*(a*x + Sqrt[-b + a^2*x^2])^2*Arc
Tan[((a*x + Sqrt[-b + a^2*x^2])^(1/4)*Sec[Pi/8])/b^(1/8) - Tan[Pi/8]]*Sin[P
i/8] - 16*b*ArcTan[((a*x + Sqrt[-b + a^2*x^2])^(1/4)*Sec[Pi/8])/b^(1/8) + T
an[Pi/8]]*Sin[Pi/8] + 16*(a*x + Sqrt[-b + a^2*x^2])^2*ArcTan[((a*x + Sqrt[-
b + a^2*x^2])^(1/4)*Sec[Pi/8])/b^(1/8) + Tan[Pi/8]]*Sin[Pi/8] + 8*b*Log[b^(
1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]] - 2*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2]
)^(1/4)*Cos[Pi/8]]*Sin[Pi/8] - 8*(a*x + Sqrt[-b + a^2*x^2])^2*Log[b^(1/4) +
 Sqrt[a*x + Sqrt[-b + a^2*x^2]] - 2*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4
)*Cos[Pi/8]]*Sin[Pi/8] - 8*b*Log[b^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]] +
 2*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*Cos[Pi/8]]*Sin[Pi/8] + 8*(a*x +
 Sqrt[-b + a^2*x^2])^2*Log[b^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]] + 2*b^(
1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*Cos[Pi/8]]*Sin[Pi/8])/(8*b^(11/8)*Sqr
t[-4*b + 4*a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2]))
```

IntegrateAlgebraic [A] time = 3.23, size = 698, normalized size = 0.96

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{b}}\right)}{2b^{11/8}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[8]{b}\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)}{2\sqrt{2}b^{11/8}}+\frac{\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{b}\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)}{b^{11/8}}-\frac{\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{b}\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)}{b^{11/8}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{b}}\right)}{2b^{11/8}}+\frac{\tanh^{-1}\left(\frac{\frac{\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt{2}\sqrt[8]{b}}+\frac{\sqrt[8]{b}}{\sqrt{2}}}{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}\right)}{2\sqrt{2}b^{11/8}}+\frac{\sqrt{2-\sqrt{2}}\tanh^{-1}\left(\frac{\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{b}}+\sqrt{1-\frac{1}{\sqrt{2}}}\sqrt[8]{b}}{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}\right)}{b^{11/8}}-\frac{\sqrt{2+\sqrt{2}}\tanh^{-1}\left(\frac{\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{b}}+\sqrt{1+\frac{1}{\sqrt{2}}}\sqrt[8]{b}}{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}\right)}{b^{11/8}}+\frac{\left(\sqrt{a^2x^2-b}+ax\right)^{5/4}}{b\left(-ax\sqrt{a^2x^2-b}-a^2x^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/(x*(-b + a^2*x^2)^(3/2)
),x]
```

```
[Out] (a*x + Sqrt[-b + a^2*x^2])^(5/4)/(b*(b - a^2*x^2 - a*x*Sqrt[-b + a^2*x^2]))
 - ArcTan[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)]/(2*b^(11/8)) - ArcTan[(
Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-b^(1/4) + Sqrt[a*x + Sq
rt[-b + a^2*x^2]])]/(2*Sqrt[2]*b^(11/8)) + (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[
2 - Sqrt[2]]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-b^(1/4) + Sqrt[a*x
 + Sqrt[-b + a^2*x^2]])])/b^(11/8) - (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sq
rt[2]]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-b^(1/4) + Sqrt[a*x + Sqr
t[-b + a^2*x^2]])])/b^(11/8) - ArcTanh[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/b^(
1/8)]/(2*b^(11/8)) + ArcTanh[(b^(1/8)/Sqrt[2] + Sqrt[a*x + Sqrt[-b + a^2*x^
2]]/(Sqrt[2]*b^(1/8)))/(a*x + Sqrt[-b + a^2*x^2])^(1/4)]/(2*Sqrt[2]*b^(11/8
)) + (Sqrt[2 - Sqrt[2]]*ArcTanh[(Sqrt[1 - 1/Sqrt[2]]*b^(1/8) + (Sqrt[1 - 1/
```

```
Sqrt[2]]*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/b^(1/8))/(a*x + Sqrt[-b + a^2*x^2]
)^(1/4)])/b^(11/8) - (Sqrt[2 + Sqrt[2]]*ArcTanh[(Sqrt[1 + 1/Sqrt[2]]*b^(1/8
) + (Sqrt[1 + 1/Sqrt[2]]*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/b^(1/8))/(a*x + Sq
rt[-b + a^2*x^2])^(1/4)])/b^(11/8)
```

fricas [A] time = 0.43, size = 50, normalized size = 0.07

$$-\frac{\sqrt{a^2x^2-b}\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}}{a^2bx^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2-b)^(1/2))^(1/4)/x/(a^2*x^2-b)^(3/2),x, algorithm="f
ricas")
[Out] -sqrt(a^2*x^2 - b)*(a*x + sqrt(a^2*x^2 - b))^(1/4)/(a^2*b*x^2 - b^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2-b)^(1/2))^(1/4)/x/(a^2*x^2-b)^(3/2),x, algorithm="g
iac")
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}}{x\left(a^2x^2-b\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+(a^2*x^2-b)^(1/2))^(1/4)/x/(a^2*x^2-b)^(3/2),x)
[Out] int((a*x+(a^2*x^2-b)^(1/2))^(1/4)/x/(a^2*x^2-b)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}}{\left(a^2x^2-b\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2-b)^(1/2))^(1/4)/x/(a^2*x^2-b)^(3/2),x, algorithm="m
axima")
[Out] integrate((a*x + sqrt(a^2*x^2 - b))^(1/4)/((a^2*x^2 - b)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x+\sqrt{a^2\,x^2-b}\right)^{1/4}}{x\,\left(a^2\,x^2-b\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + (a^2*x^2 - b)^(1/2))^(1/4)/(x*(a^2*x^2 - b)^(3/2)),x)
[Out] int((a*x + (a^2*x^2 - b)^(1/2))^(1/4)/(x*(a^2*x^2 - b)^(3/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{ax + \sqrt{a^2x^2 - b}}}{x\left(a^2x^2 - b\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a**2*x**2-b)**(1/2))**(1/4)/x/(a**2*x**2-b)**(3/2),x)
[Out] Integral((a*x + sqrt(a**2*x**2 - b))**(1/4)/(x*(a**2*x**2 - b)**(3/2)), x)
```

3.2423 $\int \frac{(-d+cx)\sqrt{ax+\sqrt{b^2+a^2x^2}}}{d+cx}\,dx$

Optimal. Leaf size=747

$$-\frac{4b^2\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}\right)}{\sqrt{a^2d^2+b^2c^2}\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}+\frac{4b^2\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}\right)}{\sqrt{a^2d^2+b^2c^2}\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}+\frac{4ad^2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}\right)}{c^{3/2}\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}+\frac{4ad^2}{c^{3/}}$$

Rubi [A] time = 0.89, antiderivative size = 279, normalized size of antiderivative = 0.37, number of steps used = 12, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {6742, 2117, 14, 2119, 1628, 826, 1166, 205}

$$\frac{4d\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}\right)}{c^{3/2}}+\frac{4d\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}\right)}{c^{3/2}}-\frac{4d\sqrt{\sqrt{a^2x^2+b^2}+ax}}{c}-\frac{b^2}{a\sqrt{\sqrt{a^2x^2+b^2}+ax}}+\frac{\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[((-d + c*x)*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/(d + c*x),x]
```

```
[Out] -(b^2/(a*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])) - (4*d*Sqrt[a*x + Sqrt[b^2 + a^2
*x^2]])/c + (a*x + Sqrt[b^2 + a^2*x^2])^(3/2)/(3*a) + (4*d*Sqrt[a*d - Sqrt[
b^2*c^2 + a^2*d^2]]*ArcTan[(Sqrt[c]*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/Sqrt[a
*d - Sqrt[b^2*c^2 + a^2*d^2]]])/c^(3/2) + (4*d*Sqrt[a*d + Sqrt[b^2*c^2 + a^
2*d^2]]*ArcTan[(Sqrt[c]*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/Sqrt[a*d + Sqrt[b^
2*c^2 + a^2*d^2]]])/c^(3/2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(-d+cx)\sqrt{ax+\sqrt{b^2+a^2x^2}}}{d+cx}\,dx = \int \left(\sqrt{ax+\sqrt{b^2+a^2x^2}} - \frac{2d\sqrt{ax+\sqrt{b^2+a^2x^2}}}{d+cx}\right) dx$$

$$= -\left((2d)\int \frac{\sqrt{ax+\sqrt{b^2+a^2x^2}}}{d+cx}\,dx\right) + \int \sqrt{ax+\sqrt{b^2+a^2x^2}}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{b^2+x^2}{x^{3/2}}\,dx, x, ax+\sqrt{b^2+a^2x^2}\right)}{2a} - (2d)\,\text{Subst}\left(\int \frac{b^2+x}{\sqrt{x}\left(-b^2c+2a\right.}\right.$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{x^{3/2}}+\sqrt{x}\right) dx, x, ax+\sqrt{b^2+a^2x^2}\right)}{2a} - (2d)\,\text{Subst}\left(\int\left(\frac{1}{c\sqrt{x}}\right.\right.$$

$$= -\frac{b^2}{a\sqrt{ax+\sqrt{b^2+a^2x^2}}} - \frac{4d\sqrt{ax+\sqrt{b^2+a^2x^2}}}{c} + \frac{\left(ax+\sqrt{b^2+a^2x^2}\right)^{3/}}{3a}$$

$$= -\frac{b^2}{a\sqrt{ax+\sqrt{b^2+a^2x^2}}} - \frac{4d\sqrt{ax+\sqrt{b^2+a^2x^2}}}{c} + \frac{\left(ax+\sqrt{b^2+a^2x^2}\right)^{3/}}{3a}$$

$$= -\frac{b^2}{a\sqrt{ax+\sqrt{b^2+a^2x^2}}} - \frac{4d\sqrt{ax+\sqrt{b^2+a^2x^2}}}{c} + \frac{\left(ax+\sqrt{b^2+a^2x^2}\right)^{3/}}{3a}$$

$$= -\frac{b^2}{a\sqrt{ax+\sqrt{b^2+a^2x^2}}} - \frac{4d\sqrt{ax+\sqrt{b^2+a^2x^2}}}{c} + \frac{\left(ax+\sqrt{b^2+a^2x^2}\right)^{3/}}{3a}$$

Mathematica [A] time = 0.55, size = 388, normalized size = 0.52

$$\frac{-\frac{4ad\left(ad\left(ad-\sqrt{a^2d^2+b^2c^2}\right)+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}\right)}{c^{3/2}\sqrt{a^2d^2+b^2c^2}\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}+\frac{4ad\left(ad\left(\sqrt{a^2d^2+b^2c^2}+ad\right)+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}\right)}{c^{3/2}\sqrt{a^2d^2+b^2c^2}\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}-\frac{4ad\sqrt{\sqrt{a^2x^2+b^2}+ax}}{c}-\frac{b^2}{\sqrt{\sqrt{a^2x^2+b^2}+ax}}+\frac{1}{3}\left(\sqrt{a^2x^2+b^2}+ax\right)^{3/2}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-d + c*x)*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/(d + c*x),x]
```

```
[Out] (-(b^2/Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]) - (4*a*d*Sqrt[a*x + Sqrt[b^2 + a^2*
x^2]])/c + (a*x + Sqrt[b^2 + a^2*x^2])^(3/2)/3 - (4*a*d*(b^2*c^2 + a*d*(a*d
 - Sqrt[b^2*c^2 + a^2*d^2]))*ArcTan[(Sqrt[c]*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]
])/Sqrt[a*d - Sqrt[b^2*c^2 + a^2*d^2]]])/(c^(3/2)*Sqrt[b^2*c^2 + a^2*d^2]*S
qrt[a*d - Sqrt[b^2*c^2 + a^2*d^2]]) + (4*a*d*(b^2*c^2 + a*d*(a*d + Sqrt[b^2
*c^2 + a^2*d^2]))*ArcTan[(Sqrt[c]*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/Sqrt[a*d
 + Sqrt[b^2*c^2 + a^2*d^2]]])/(c^(3/2)*Sqrt[b^2*c^2 + a^2*d^2]*Sqrt[a*d + S
qrt[b^2*c^2 + a^2*d^2]]))/a
```

IntegrateAlgebraic [A] time = 1.89, size = 747, normalized size = 1.00

$$-\frac{4b^2\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}\right)}{\sqrt{a^2d^2+b^2c^2}\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}+\frac{4b^2\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}\right)}{\sqrt{a^2d^2+b^2c^2}\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}+\frac{4ad^2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}\right)}{c^{3/2}\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}+\frac{4ad^2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}\right)}{c^{3/2}\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}-\frac{4a^2d^3\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}\right)}{c^{3/2}\sqrt{a^2d^2+b^2c^2}\sqrt{ad-\sqrt{a^2d^2+b^2c^2}}}+\frac{4a^2d^3\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sqrt{a^2x^2+b^2}+ax}}{\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}\right)}{c^{3/2}\sqrt{a^2d^2+b^2c^2}\sqrt{\sqrt{a^2d^2+b^2c^2}+ad}}+\frac{2\sqrt{a^2x^2+b^2}\,(acx-6ad)+2\left(a^2cx^2-6a^2dx-b^2c\right)}{3ac\sqrt{\sqrt{a^2x^2+b^2}+ax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-d + c*x)*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/(d + c*x),x
]
```

```
[Out] (2*(-6*a*d + a*c*x)*Sqrt[b^2 + a^2*x^2] + 2*(-(b^2*c) - 6*a^2*d*x + a^2*c*x
^2))/(3*a*c*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]]) + (4*a*d^2*ArcTan[(Sqrt[c]*Sqr
t[a*x + Sqrt[b^2 + a^2*x^2]])/Sqrt[a*d - Sqrt[b^2*c^2 + a^2*d^2]]])/(c^(3/2
)*Sqrt[a*d - Sqrt[b^2*c^2 + a^2*d^2]]) - (4*b^2*Sqrt[c]*d*ArcTan[(Sqrt[c]*S
qrt[a*x + Sqrt[b^2 + a^2*x^2]])/Sqrt[a*d - Sqrt[b^2*c^2 + a^2*d^2]]])/(Sqrt
[b^2*c^2 + a^2*d^2]*Sqrt[a*d - Sqrt[b^2*c^2 + a^2*d^2]]) - (4*a^2*d^3*ArcTa
n[(Sqrt[c]*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/Sqrt[a*d - Sqrt[b^2*c^2 + a^2*d
^2]]])/(c^(3/2)*Sqrt[b^2*c^2 + a^2*d^2]*Sqrt[a*d - Sqrt[b^2*c^2 + a^2*d^2]]
) + (4*a*d^2*ArcTan[(Sqrt[c]*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/Sqrt[a*d + Sq
rt[b^2*c^2 + a^2*d^2]]])/(c^(3/2)*Sqrt[a*d + Sqrt[b^2*c^2 + a^2*d^2]]) + (4
*b^2*Sqrt[c]*d*ArcTan[(Sqrt[c]*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])/Sqrt[a*d +
Sqrt[b^2*c^2 + a^2*d^2]]])/(Sqrt[b^2*c^2 + a^2*d^2]*Sqrt[a*d + Sqrt[b^2*c^2
 + a^2*d^2]]) + (4*a^2*d^3*ArcTan[(Sqrt[c]*Sqrt[a*x + Sqrt[b^2 + a^2*x^2]])
/Sqrt[a*d + Sqrt[b^2*c^2 + a^2*d^2]]])/(c^(3/2)*Sqrt[b^2*c^2 + a^2*d^2]*Sqr
t[a*d + Sqrt[b^2*c^2 + a^2*d^2]])
```

fricas [A] time = 0.43, size = 511, normalized size = 0.68

$$\frac{2\left(3ac\sqrt{-\frac{ad^3+c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{c^6}}}{c^3}}\log\left(4\sqrt{ax+\sqrt{a^2x^2+b^2}}\,d+4c\sqrt{-\frac{ad^3+c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{c^6}}}{c^3}}\right)-3ac\sqrt{-\frac{ad^3+c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{c^6}}}{c^3}}\log\left(4\sqrt{ax+\sqrt{a^2x^2+b^2}}\,d-4c\sqrt{-\frac{ad^3+c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{c^6}}}{c^3}}\right)+3ac\sqrt{-\frac{ad^3-c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{c^6}}}{c^3}}\log\left(4\sqrt{ax+\sqrt{a^2x^2+b^2}}\,d+4c\sqrt{-\frac{ad^3-c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{c^6}}}{c^3}}\right)-3ac\sqrt{-\frac{ad^3-c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{c^6}}}{c^3}}\log\left(4\sqrt{ax+\sqrt{a^2x^2+b^2}}\,d-4c\sqrt{-\frac{ad^3-c^3\sqrt{\frac{b^2c^2d^4+a^2d^6}{c^6}}}{c^3}}\right)+\left(2acx-6ad-\sqrt{a^2x^2+b^2}\,c\right)\sqrt{ax+\sqrt{a^2x^2+b^2}}\right)}{3ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x-d)*(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)/(c*x+d),x, algorithm="fri
cas")
```

```
[Out] 2/3*(3*a*c*sqrt(-(a*d^3 + c^3*sqrt((b^2*c^2*d^4 + a^2*d^6)/c^6))/c^3)*log(4
*sqrt(a*x + sqrt(a^2*x^2 + b^2))*d + 4*c*sqrt(-(a*d^3 + c^3*sqrt((b^2*c^2*d
^4 + a^2*d^6)/c^6))/c^3)) - 3*a*c*sqrt(-(a*d^3 + c^3*sqrt((b^2*c^2*d^4 + a^
2*d^6)/c^6))/c^3)*log(4*sqrt(a*x + sqrt(a^2*x^2 + b^2))*d - 4*c*sqrt(-(a*d^
3 + c^3*sqrt((b^2*c^2*d^4 + a^2*d^6)/c^6))/c^3)) + 3*a*c*sqrt(-(a*d^3 - c^3
*sqrt((b^2*c^2*d^4 + a^2*d^6)/c^6))/c^3)*log(4*sqrt(a*x + sqrt(a^2*x^2 + b^
2))*d + 4*c*sqrt(-(a*d^3 - c^3*sqrt((b^2*c^2*d^4 + a^2*d^6)/c^6))/c^3)) - 3
*a*c*sqrt(-(a*d^3 - c^3*sqrt((b^2*c^2*d^4 + a^2*d^6)/c^6))/c^3)*log(4*sqrt(
```

```
a*x + sqrt(a^2*x^2 + b^2))*d - 4*c*sqrt(-(a*d^3 - c^3*sqrt((b^2*c^2*d^4 + a
^2*d^6)/c^6))/c^3)) + (2*a*c*x - 6*a*d - sqrt(a^2*x^2 + b^2)*c)*sqrt(a*x +
sqrt(a^2*x^2 + b^2)))/(a*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+\sqrt{a^2x^2+b^2}}\,(cx-d)}{cx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x-d)*(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)/(c*x+d),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(a*x + sqrt(a^2*x^2 + b^2))*(c*x - d)/(c*x + d), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(cx-d)\sqrt{ax+\sqrt{a^2x^2+b^2}}}{cx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x-d)*(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)/(c*x+d),x)
```

```
[Out] int((c*x-d)*(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)/(c*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+\sqrt{a^2x^2+b^2}}\,(cx-d)}{cx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x-d)*(a*x+(a^2*x^2+b^2)^(1/2))^(1/2)/(c*x+d),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(a*x + sqrt(a^2*x^2 + b^2))*(c*x - d)/(c*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\sqrt{a\,x+\sqrt{a^2\,x^2+b^2}}\,\left(d-c\,x\right)}{d+c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*x + (b^2 + a^2*x^2)^(1/2))^(1/2)*(d - c*x))/(d + c*x),x)
```

```
[Out] int(-((a*x + (b^2 + a^2*x^2)^(1/2))^(1/2)*(d - c*x))/(d + c*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+\sqrt{a^2x^2+b^2}}\,(cx-d)}{cx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x-d)*(a*x+(a**2*x**2+b**2)**(1/2))**(1/2)/(c*x+d),x)
```

```
[Out] Integral(sqrt(a*x + sqrt(a**2*x**2 + b**2))*(c*x - d)/(c*x + d), x)
```

3.2424 $\int \frac{\sqrt[3]{\frac{x}{-1-ax+3x^2+3ax^3-3x^4-3ax^5+x^6+ax^7}}}{x^3}\,dx$

Optimal. Leaf size=752

$$-\frac{3\sqrt[3]{\frac{x}{ax^7-3ax^5+3ax^3-ax+x^6-3x^4+3x^2-1}}\left(3a^2x^4-3a^2x^2+ax^3-ax-2x^2+2\right)}{10x^2}-\frac{\log\left(\sqrt[3]{\frac{x}{ax^7-3ax^5+3ax^3-ax+x^6-3x^4+3x^2-1}}\left(\sqrt[3]{1\right.\right.}{2\sqrt[3]{1-a}}$$

Rubi [A] time = 0.54, antiderivative size = 609, normalized size of antiderivative = 0.81, number of steps used = 12, number of rules used = 7, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {6688, 6718, 912, 129, 155, 12, 91}

$$-\frac{3(5-3a)\left(1-x^2\right)(ax+1)\sqrt[3]{-\frac{x}{\left(1-x^2\right)^3(ax+1)}}}{20x}+\frac{3(3a+5)\left(1-x^2\right)(ax+1)\sqrt[3]{-\frac{x}{\left(1-x^2\right)^3(ax+1)}}}{20x}-\frac{3\left(1-x^2\right)(ax+1)\sqrt[3]{-\frac{x}{\left(1-x^2\right)^3(ax+1)}}}{5x^2}+\frac{\left(1-x^2\right)\sqrt[3]{ax+1}\sqrt[3]{-\frac{x}{\left(1-x^2\right)^3(ax+1)}}\log(1-x)}{4\sqrt[3]{a+1}\sqrt[3]{x}}+\frac{\left(1-x^2\right)\sqrt[3]{ax+1}\sqrt[3]{-\frac{x}{\left(1-x^2\right)^3(ax+1)}}\log(x+1)}{4\sqrt[3]{a-1}\sqrt[3]{x}}-\frac{3\left(1-x^2\right)\sqrt[3]{ax+1}\sqrt[3]{-\frac{x}{\left(1-x^2\right)^3(ax+1)}}\log\left(\frac{\sqrt[3]{ax+1}}{\sqrt[3]{a-1}}-\sqrt[3]{x}\right)}{4\sqrt[3]{a-1}\sqrt[3]{x}}-\frac{3\left(1-x^2\right)\sqrt[3]{ax+1}\sqrt[3]{-\frac{x}{\left(1-x^2\right)^3(ax+1)}}\log\left(\frac{\sqrt[3]{ax+1}}{\sqrt[3]{a+1}}-\sqrt[3]{x}\right)}{4\sqrt[3]{a+1}\sqrt[3]{x}}-\frac{\sqrt{3}\left(1-x^2\right)\sqrt[3]{ax+1}\sqrt[3]{-\frac{x}{\left(1-x^2\right)^3(ax+1)}}\tan^{-1}\left(\frac{2\sqrt[3]{ax+1}}{\sqrt{3}\sqrt[3]{a-1}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{a-1}\sqrt[3]{x}}-\frac{\sqrt{3}\left(1-x^2\right)\sqrt[3]{ax+1}\sqrt[3]{-\frac{x}{\left(1-x^2\right)^3(ax+1)}}\tan^{-1}\left(\frac{2\sqrt[3]{ax+1}}{\sqrt{3}\sqrt[3]{a+1}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{a+1}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(x/(-1 - a*x + 3*x^2 + 3*a*x^3 - 3*x^4 - 3*a*x^5 + x^6 + a*x^7))^(1/3)/
x^3,x]
```

```
[Out] (-3*(1 + a*x)*(-(x/((1 + a*x)*(1 - x^2)^3)))^(1/3)*(1 - x^2))/(5*x^2) - (3*
(5 - 3*a)*(1 + a*x)*(-(x/((1 + a*x)*(1 - x^2)^3)))^(1/3)*(1 - x^2))/(20*x)
+ (3*(5 + 3*a)*(1 + a*x)*(-(x/((1 + a*x)*(1 - x^2)^3)))^(1/3)*(1 - x^2))/(2
0*x) - (Sqrt[3]*(1 + a*x)^(1/3)*(-(x/((1 + a*x)*(1 - x^2)^3)))^(1/3)*(1 - x
^2)*ArcTan[1/Sqrt[3] + (2*(1 + a*x)^(1/3))/(Sqrt[3]*(-1 + a)^(1/3)*x^(1/3))
])/(2*(-1 + a)^(1/3)*x^(1/3)) - (Sqrt[3]*(1 + a*x)^(1/3)*(-(x/((1 + a*x)*(1
 - x^2)^3)))^(1/3)*(1 - x^2)*ArcTan[1/Sqrt[3] + (2*(1 + a*x)^(1/3))/(Sqrt[3
]*(1 + a)^(1/3)*x^(1/3))])/(2*(1 + a)^(1/3)*x^(1/3)) + ((1 + a*x)^(1/3)*(-(
x/((1 + a*x)*(1 - x^2)^3)))^(1/3)*(1 - x^2)*Log[1 - x])/(4*(1 + a)^(1/3)*x^
(1/3)) + ((1 + a*x)^(1/3)*(-(x/((1 + a*x)*(1 - x^2)^3)))^(1/3)*(1 - x^2)*Lo
g[1 + x])/(4*(-1 + a)^(1/3)*x^(1/3)) - (3*(1 + a*x)^(1/3)*(-(x/((1 + a*x)*(
1 - x^2)^3)))^(1/3)*(1 - x^2)*Log[-x^(1/3) + (1 + a*x)^(1/3)/(-1 + a)^(1/3)
])/(4*(-1 + a)^(1/3)*x^(1/3)) - (3*(1 + a*x)^(1/3)*(-(x/((1 + a*x)*(1 - x^2
)^3)))^(1/3)*(1 - x^2)*Log[-x^(1/3) + (1 + a*x)^(1/3)/(1 + a)^(1/3)])/(4*(1
 + a)^(1/3)*x^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
```

```
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1]))
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 912

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6718

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart
[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !FreeQ[w, x] &&  !Fr
eeQ[z, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{\frac{x}{-1-ax+3x^2+3ax^3-3x^4-3ax^5+x^6+ax^7}}}{x^3}\,dx = \int \frac{\sqrt[3]{\frac{x}{(1+ax)\left(-1+x^2\right)^3}}}{x^3}\,dx$$

$$= \frac{\left(\sqrt[3]{1+ax}\,\sqrt[3]{\frac{x}{(1+ax)\left(-1+x^2\right)^3}}\,\left(-1+x^2\right)\right)\int \frac{1}{x^{8/3}\sqrt[3]{1+ax}\,\left(-1+x^2\right)}\,dx}{\sqrt[3]{x}}$$

$$= \frac{\left(\sqrt[3]{1+ax}\,\sqrt[3]{\frac{x}{(1+ax)\left(-1+x^2\right)^3}}\,\left(-1+x^2\right)\right)\int \left(-\frac{1}{2(1-x)x^{8/3}\sqrt[3]{1+ax}} - \frac{1}{2x^{8/3}(1+x)\sqrt[3]{1+a}}\right.}{\sqrt[3]{x}}$$

$$= -\frac{\left(\sqrt[3]{1+ax}\,\sqrt[3]{\frac{x}{(1+ax)\left(-1+x^2\right)^3}}\,\left(-1+x^2\right)\right)\int \frac{1}{(1-x)x^{8/3}\sqrt[3]{1+ax}}\,dx}{2\sqrt[3]{x}} - \frac{\left(\sqrt[3]{1+ax}\ \sqrt[3]{}\right.}{}$$

$$= -\frac{3(1+ax)\sqrt[3]{-\frac{x}{(1+ax)\left(1-x^2\right)^3}}\,\left(1-x^2\right)}{5x^2} + \frac{\left(3\sqrt[3]{1+ax}\,\sqrt[3]{\frac{x}{(1+ax)\left(-1+x^2\right)^3}}\,\left(-1+\right.\right.}{10\sqrt[3]{x}}$$

$$= -\frac{3(1+ax)\sqrt[3]{-\frac{x}{(1+ax)\left(1-x^2\right)^3}}\,\left(1-x^2\right)}{5x^2} - \frac{3(5-3a)(1+ax)\sqrt[3]{-\frac{x}{(1+ax)\left(1-x^2\right)^3}}}{20x}$$

$$= -\frac{3(1+ax)\sqrt[3]{-\frac{x}{(1+ax)\left(1-x^2\right)^3}}\,\left(1-x^2\right)}{5x^2} - \frac{3(5-3a)(1+ax)\sqrt[3]{-\frac{x}{(1+ax)\left(1-x^2\right)^3}}}{20x}$$

$$= -\frac{3(1+ax)\sqrt[3]{-\frac{x}{(1+ax)\left(1-x^2\right)^3}}\,\left(1-x^2\right)}{5x^2} - \frac{3(5-3a)(1+ax)\sqrt[3]{-\frac{x}{(1+ax)\left(1-x^2\right)^3}}}{20x}$$

Mathematica [C] time = 0.07, size = 95, normalized size = 0.13

$$-\frac{3\left(x^2-1\right)\sqrt[3]{\frac{x}{\left(x^2-1\right)^3(ax+1)}}\left(3a^2x^2+5x^2\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(a-1)x}{ax+1}\right)+5x^2\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(a+1)x}{ax+1}\right)+ax-2\right)}{10x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x/(-1 - a*x + 3*x^2 + 3*a*x^3 - 3*x^4 - 3*a*x^5 + x^6 + a*x^7))^
(1/3)/x^3,x]
```

```
[Out] (-3*(x/((1 + a*x)*(-1 + x^2)^3))^(1/3)*(-1 + x^2)*(-2 + a*x + 3*a^2*x^2 + 5
*x^2*Hypergeometric2F1[1/3, 1, 4/3, ((-1 + a)*x)/(1 + a*x)] + 5*x^2*Hyperge
ometric2F1[1/3, 1, 4/3, ((1 + a)*x)/(1 + a*x)]))/(10*x^2)
```

IntegrateAlgebraic [A] time = 3.77, size = 752, normalized size = 1.00

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x/(-1 - a*x + 3*x^2 + 3*a*x^3 - 3*x^4 - 3*a*x^5 + x^6 +
 a*x^7))^(1/3)/x^3,x]
```

```
[Out] (-3*(2 - a*x - 2*x^2 - 3*a^2*x^2 + a*x^3 + 3*a^2*x^4)*(x/(-1 - a*x + 3*x^2
+ 3*a*x^3 - 3*x^4 - 3*a*x^5 + x^6 + a*x^7))^(1/3))/(10*x^2) + (Sqrt[3]*ArcT
```

```
an[1/Sqrt[3] - ((1 - a)^(1/3)*(-2 + 2*x^2)*(x/(-1 - a*x + 3*x^2 + 3*a*x^3 -
 3*x^4 - 3*a*x^5 + x^6 + a*x^7))^(1/3))/Sqrt[3]])/(2*(1 - a)^(1/3)) - (Sqrt
[3]*ArcTan[1/Sqrt[3] + ((1 + a)^(1/3)*(-2 + 2*x^2)*(x/(-1 - a*x + 3*x^2 + 3
*a*x^3 - 3*x^4 - 3*a*x^5 + x^6 + a*x^7))^(1/3))/Sqrt[3]])/(2*(1 + a)^(1/3))
 - Log[1 + (-(1 - a)^(1/3) + (1 - a)^(1/3)*x^2)*(x/(-1 - a*x + 3*x^2 + 3*a*
x^3 - 3*x^4 - 3*a*x^5 + x^6 + a*x^7))^(1/3)]/(2*(1 - a)^(1/3)) + Log[-1 + (
-(1 + a)^(1/3) + (1 + a)^(1/3)*x^2)*(x/(-1 - a*x + 3*x^2 + 3*a*x^3 - 3*x^4
- 3*a*x^5 + x^6 + a*x^7))^(1/3)]/(2*(1 + a)^(1/3)) + Log[1 + ((1 - a)^(1/3)
 - (1 - a)^(1/3)*x^2)*(x/(-1 - a*x + 3*x^2 + 3*a*x^3 - 3*x^4 - 3*a*x^5 + x^
6 + a*x^7))^(1/3) + ((1 - a)^(2/3) - 2*(1 - a)^(2/3)*x^2 + (1 - a)^(2/3)*x^
4)*(x/(-1 - a*x + 3*x^2 + 3*a*x^3 - 3*x^4 - 3*a*x^5 + x^6 + a*x^7))^(2/3)]/
(4*(1 - a)^(1/3)) - Log[1 + (-(1 + a)^(1/3) + (1 + a)^(1/3)*x^2)*(x/(-1 - a
*x + 3*x^2 + 3*a*x^3 - 3*x^4 - 3*a*x^5 + x^6 + a*x^7))^(1/3) + ((1 + a)^(2/
3) - 2*(1 + a)^(2/3)*x^2 + (1 + a)^(2/3)*x^4)*(x/(-1 - a*x + 3*x^2 + 3*a*x^
3 - 3*x^4 - 3*a*x^5 + x^6 + a*x^7))^(2/3)]/(4*(1 + a)^(1/3))
```

fricas [A] time = 0.50, size = 3627, normalized size = 4.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(a*x^7-3*a*x^5+x^6+3*a*x^3-3*x^4-a*x+3*x^2-1))^(1/3)/x^3,x, al
gorithm="fricas")
```

```
[Out] [1/20*(5*sqrt(3)*(a^2 - 1)*x^2*sqrt(-1/(a - 1)^(2/3))*log(-((3*a - 2)*x + s
qrt(3)*((a*x^3 - a*x + x^2 - 1)*(a - 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3
*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) - 2*((a^2 - a)*x^5 + (a - 1)*x^4 -
 2*(a^2 - a)*x^3 - 2*(a - 1)*x^2 + (a^2 - a)*x + a - 1)*(x/(a*x^7 - 3*a*x^5
 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(2/3) + (a*x + 1)*(a - 1)^(1/3
))*sqrt(-1/(a - 1)^(2/3)) - 3*(a*x^3 - a*x + x^2 - 1)*(a - 1)^(1/3)*(x/(a*x
^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) + 1)/(x + 1)
) + 5*sqrt(3)*(a^2 - 1)*x^2*sqrt(-1/(a + 1)^(2/3))*log(((3*a + 2)*x + sqrt(
3)*((a*x^3 - a*x + x^2 - 1)*(a + 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x
^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) - 2*((a^2 + a)*x^5 + (a + 1)*x^4 - 2*(
a^2 + a)*x^3 - 2*(a + 1)*x^2 + (a^2 + a)*x + a + 1)*(x/(a*x^7 - 3*a*x^5 + x
^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(2/3) + (a*x + 1)*(a + 1)^(1/3))*s
qrt(-1/(a + 1)^(2/3)) - 3*(a*x^3 - a*x + x^2 - 1)*(a + 1)^(1/3)*(x/(a*x^7 -
 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) + 1)/(x - 1)) -
5*(a + 1)^(2/3)*(a - 1)*x^2*log((x^2 - 1)*(a + 1)^(2/3)*(x/(a*x^7 - 3*a*x^5
 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) + ((a + 1)*x^4 - 2*(a +
1)*x^2 + a + 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 -
 1))^(2/3) + (a + 1)^(1/3)) - 5*(a + 1)*(a - 1)^(2/3)*x^2*log((x^2 - 1)*(a
- 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^
(1/3) + ((a - 1)*x^4 - 2*(a - 1)*x^2 + a - 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3
*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(2/3) + (a - 1)^(1/3)) + 10*(a + 1)^(2/3
)*(a - 1)*x^2*log(((a + 1)*x^2 - a - 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3
 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) - (a + 1)^(2/3)) + 10*(a + 1)*(a - 1)^(2
/3)*x^2*log(((a - 1)*x^2 - a + 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x
^4 - a*x + 3*x^2 - 1))^(1/3) - (a - 1)^(2/3)) - 6*(3*(a^4 - a^2)*x^4 + (a^3
 - a)*x^3 - (3*a^4 - a^2 - 2)*x^2 + 2*a^2 - (a^3 - a)*x - 2)*(x/(a*x^7 - 3*
a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3))/((a^2 - 1)*x^2), 1
/20*(5*sqrt(3)*(a^2 - 1)*x^2*sqrt(-1/(a - 1)^(2/3))*log(-((3*a - 2)*x + sqr
t(3)*((a*x^3 - a*x + x^2 - 1)*(a - 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a
*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) - 2*((a^2 - a)*x^5 + (a - 1)*x^4 - 2
*(a^2 - a)*x^3 - 2*(a - 1)*x^2 + (a^2 - a)*x + a - 1)*(x/(a*x^7 - 3*a*x^5 +
 x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(2/3) + (a*x + 1)*(a - 1)^(1/3))
*sqrt(-1/(a - 1)^(2/3)) - 3*(a*x^3 - a*x + x^2 - 1)*(a - 1)^(1/3)*(x/(a*x^7
 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) + 1)/(x + 1))
- 5*(a + 1)^(2/3)*(a - 1)*x^2*log((x^2 - 1)*(a + 1)^(2/3)*(x/(a*x^7 - 3*a*x
^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) + ((a + 1)*x^4 - 2*(a
```

```
+ 1)*x^2 + a + 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2
 - 1))^(2/3) + (a + 1)^(1/3)) - 5*(a + 1)*(a - 1)^(2/3)*x^2*log((x^2 - 1)*(
a - 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1)
)^(1/3) + ((a - 1)*x^4 - 2*(a - 1)*x^2 + a - 1)*(x/(a*x^7 - 3*a*x^5 + x^6 +
 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(2/3) + (a - 1)^(1/3)) + 10*(a + 1)^(2
/3)*(a - 1)*x^2*log(((a + 1)*x^2 - a - 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x
^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) - (a + 1)^(2/3)) + 10*(a + 1)*(a - 1)^
(2/3)*x^2*log(((a - 1)*x^2 - a + 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3
*x^4 - a*x + 3*x^2 - 1))^(1/3) - (a - 1)^(2/3)) - 10*sqrt(3)*(a^2 - 1)*x^2*
arctan(1/3*sqrt(3)*(2*(x^2 - 1)*(a + 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3
*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) + (a + 1)^(1/3))/(a + 1)^(1/3))/(a
 + 1)^(1/3) - 6*(3*(a^4 - a^2)*x^4 + (a^3 - a)*x^3 - (3*a^4 - a^2 - 2)*x^2
+ 2*a^2 - (a^3 - a)*x - 2)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*
x + 3*x^2 - 1))^(1/3))/((a^2 - 1)*x^2), 1/20*(5*sqrt(3)*(a^2 - 1)*x^2*sqrt(
-1/(a + 1)^(2/3))*log(((3*a + 2)*x + sqrt(3)*((a*x^3 - a*x + x^2 - 1)*(a +
1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1
/3) - 2*((a^2 + a)*x^5 + (a + 1)*x^4 - 2*(a^2 + a)*x^3 - 2*(a + 1)*x^2 + (a
^2 + a)*x + a + 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^
2 - 1))^(2/3) + (a*x + 1)*(a + 1)^(1/3))*sqrt(-1/(a + 1)^(2/3)) - 3*(a*x^3
- a*x + x^2 - 1)*(a + 1)^(1/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4
- a*x + 3*x^2 - 1))^(1/3) + 1)/(x - 1)) - 5*(a + 1)^(2/3)*(a - 1)*x^2*log((
x^2 - 1)*(a + 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x +
3*x^2 - 1))^(1/3) + ((a + 1)*x^4 - 2*(a + 1)*x^2 + a + 1)*(x/(a*x^7 - 3*a*x
^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(2/3) + (a + 1)^(1/3)) - 5*(
a + 1)*(a - 1)^(2/3)*x^2*log((x^2 - 1)*(a - 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 +
x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) + ((a - 1)*x^4 - 2*(a - 1)*
x^2 + a - 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1)
)^(2/3) + (a - 1)^(1/3)) + 10*(a + 1)^(2/3)*(a - 1)*x^2*log(((a + 1)*x^2 -
a - 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3
) - (a + 1)^(2/3)) + 10*(a + 1)*(a - 1)^(2/3)*x^2*log(((a - 1)*x^2 - a + 1)
*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) - (a
 - 1)^(2/3)) - 10*sqrt(3)*(a^2 - 1)*x^2*arctan(1/3*sqrt(3)*(2*(x^2 - 1)*(a
- 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^
(1/3) + (a - 1)^(1/3))/(a - 1)^(1/3))/(a - 1)^(1/3) - 6*(3*(a^4 - a^2)*x^4
+ (a^3 - a)*x^3 - (3*a^4 - a^2 - 2)*x^2 + 2*a^2 - (a^3 - a)*x - 2)*(x/(a*x^
7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3))/((a^2 - 1)*x
^2), -1/20*(5*(a + 1)^(2/3)*(a - 1)*x^2*log((x^2 - 1)*(a + 1)^(2/3)*(x/(a*x
^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) + ((a + 1)*x
^4 - 2*(a + 1)*x^2 + a + 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a
*x + 3*x^2 - 1))^(2/3) + (a + 1)^(1/3)) + 5*(a + 1)*(a - 1)^(2/3)*x^2*log((
x^2 - 1)*(a - 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x +
3*x^2 - 1))^(1/3) + ((a - 1)*x^4 - 2*(a - 1)*x^2 + a - 1)*(x/(a*x^7 - 3*a*x
^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(2/3) + (a - 1)^(1/3)) - 10*
(a + 1)^(2/3)*(a - 1)*x^2*log(((a + 1)*x^2 - a - 1)*(x/(a*x^7 - 3*a*x^5 + x
^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) - (a + 1)^(2/3)) - 10*(a + 1
)*(a - 1)^(2/3)*x^2*log(((a - 1)*x^2 - a + 1)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3
*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) - (a - 1)^(2/3)) + 10*sqrt(3)*(a^2
 - 1)*x^2*arctan(1/3*sqrt(3)*(2*(x^2 - 1)*(a + 1)^(2/3)*(x/(a*x^7 - 3*a*x^5
 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3) + (a + 1)^(1/3))/(a + 1)
^(1/3))/(a + 1)^(1/3) + 10*sqrt(3)*(a^2 - 1)*x^2*arctan(1/3*sqrt(3)*(2*(x^2
 - 1)*(a - 1)^(2/3)*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x
^2 - 1))^(1/3) + (a - 1)^(1/3))/(a - 1)^(1/3))/(a - 1)^(1/3) + 6*(3*(a^4 -
a^2)*x^4 + (a^3 - a)*x^3 - (3*a^4 - a^2 - 2)*x^2 + 2*a^2 - (a^3 - a)*x - 2)
*(x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(1/3))/((a
^2 - 1)*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x}{ax^7-3\,ax^5+x^6+3\,ax^3-3\,x^4-ax+3\,x^2-1}\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(a*x^7-3*a*x^5+x^6+3*a*x^3-3*x^4-a*x+3*x^2-1))^(1/3)/x^3,x, al
gorithm="giac")
```

```
[Out] integrate((x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(
1/3)/x^3, x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x}{a\,x^7-3a\,x^5+x^6+3a\,x^3-3x^4-ax+3x^2-1}\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(a*x^7-3*a*x^5+x^6+3*a*x^3-3*x^4-a*x+3*x^2-1))^(1/3)/x^3,x)
```

```
[Out] int((x/(a*x^7-3*a*x^5+x^6+3*a*x^3-3*x^4-a*x+3*x^2-1))^(1/3)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{x}{ax^7-3\,ax^5+x^6+3\,ax^3-3\,x^4-ax+3\,x^2-1}\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(a*x^7-3*a*x^5+x^6+3*a*x^3-3*x^4-a*x+3*x^2-1))^(1/3)/x^3,x, al
gorithm="maxima")
```

```
[Out] integrate((x/(a*x^7 - 3*a*x^5 + x^6 + 3*a*x^3 - 3*x^4 - a*x + 3*x^2 - 1))^(
1/3)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(-\frac{x}{-a\,x^7-x^6+3\,a\,x^5+3\,x^4-3\,a\,x^3-3\,x^2+a\,x+1}\right)}^{1/3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x/(a*x - 3*a*x^3 + 3*a*x^5 - a*x^7 - 3*x^2 + 3*x^4 - x^6 + 1))^(1/3)/
x^3,x)
```

```
[Out] int((-x/(a*x - 3*a*x^3 + 3*a*x^5 - a*x^7 - 3*x^2 + 3*x^4 - x^6 + 1))^(1/3)/
x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\frac{x}{ax^7-3ax^5+3ax^3-ax+x^6-3x^4+3x^2-1}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(a*x**7-3*a*x**5+x**6+3*a*x**3-3*x**4-a*x+3*x**2-1))**(1/3)/x*
*3,x)
```

```
[Out] Integral((x/(a*x**7 - 3*a*x**5 + 3*a*x**3 - a*x + x**6 - 3*x**4 + 3*x**2 -
1))**(1/3)/x**3, x)
```

3.2425 $\int \frac{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{x^2\left(-b+a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=757

$$-\frac{a\tan^{-1}\left(\frac{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{b}}\right)}{2b^{15/8}}-\frac{\sqrt{2-\sqrt{2}}\,a\tan^{-1}\left(\frac{\left(\sqrt{\frac{2}{2-\sqrt{2}}}\,\sqrt[8]{b}-\frac{2\sqrt[8]{b}}{\sqrt{2-\sqrt{2}}}\right)\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)}{4b^{15/8}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[8]{b}\,\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)}{2\sqrt{2}\,b^{15/8}}+\frac{\sqrt{2}}{}$$

Rubi [A] time = 0.87, antiderivative size = 772, normalized size of antiderivative = 1.02, number of steps used = 31, number of rules used = 14, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2120, 259, 288, 329, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/(x^2*(-b + a^2*x^2)^(3/2)),x]
```

```
[Out] (4*a*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(b^2 - (a*x + Sqrt[-b + a^2*x^2])^4)
 - (a*ArcTan[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/(-b)^(1/8)])/(2*(-b)^(15/8))
- (a*ArcTan[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)])/(2*b^(15/8)) + (a*Ar
cTan[1 - (Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-b)^(1/8)])/(2*Sqrt[2]
*(-b)^(15/8)) - (a*ArcTan[1 + (Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-
b)^(1/8)])/(2*Sqrt[2]*(-b)^(15/8)) + (a*ArcTan[1 - (Sqrt[2]*(a*x + Sqrt[-b
+ a^2*x^2])^(1/4))/b^(1/8)])/(2*Sqrt[2]*b^(15/8)) - (a*ArcTan[1 + (Sqrt[2]*
(a*x + Sqrt[-b + a^2*x^2])^(1/4))/b^(1/8)])/(2*Sqrt[2]*b^(15/8)) - (a*ArcTa
nh[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/(-b)^(1/8)])/(2*(-b)^(15/8)) - (a*ArcTa
nh[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)])/(2*b^(15/8)) + (a*Log[(-b)^(1
/4) - Sqrt[2]*(-b)^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt
[-b + a^2*x^2]]])/(4*Sqrt[2]*(-b)^(15/8)) - (a*Log[(-b)^(1/4) + Sqrt[2]*(-b
)^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]])
/(4*Sqrt[2]*(-b)^(15/8)) + (a*Log[b^(1/4) - Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b
+ a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]])/(4*Sqrt[2]*b^(15/8)) -
 (a*Log[b^(1/4) + Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a
*x + Sqrt[-b + a^2*x^2]]])/(4*Sqrt[2]*b^(15/8))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 259

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{x^2\left(-b+a^2x^2\right)^{3/2}}\,dx = (16a)\,\text{Subst}\left(\int \frac{x^{13/4}}{\left(-b+x^2\right)^2\left(b+x^2\right)^2}\,dx, x, ax+\sqrt{-b+a^2x^2}\right)$$

$$= (16a)\,\text{Subst}\left(\int \frac{x^{13/4}}{\left(-b^2+x^4\right)^2}\,dx, x, ax+\sqrt{-b+a^2x^2}\right)$$

$$= \frac{4a\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{b^2-\left(ax+\sqrt{-b+a^2x^2}\right)^4} + a\,\text{Subst}\left(\int \frac{1}{x^{3/4}\left(-b^2+x^4\right)}\,dx, x, ax+\sqrt{-b+a^2x^2}\right)$$

$$= \frac{4a\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{b^2-\left(ax+\sqrt{-b+a^2x^2}\right)^4} + (4a)\,\text{Subst}\left(\int \frac{1}{-b^2+x^{16}}\,dx, x, \sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)$$

$$= \frac{4a\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{b^2-\left(ax+\sqrt{-b+a^2x^2}\right)^4} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{b-x^8}\,dx, x, \sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)}{b} - \frac{(2a}{}$$

$$= \frac{4a\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{b^2-\left(ax+\sqrt{-b+a^2x^2}\right)^4} - \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{-b}-x^4}\,dx, x, \sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)}{(-b)^{3/2}} - \frac{a\,\text{S}}{}$$

$$= \frac{4a\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{b^2-\left(ax+\sqrt{-b+a^2x^2}\right)^4} - \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-b}-x^2}\,dx, x, \sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)}{2(-b)^{7/4}} - \frac{a\,\text{S}}{}$$

$$= \frac{4a\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{b^2-\left(ax+\sqrt{-b+a^2x^2}\right)^4} - \frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[8]{-b}}\right)}{2(-b)^{15/8}} - \frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[8]{b}}\right)}{2b^{15/8}}$$

$$= \frac{4a\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{b^2-\left(ax+\sqrt{-b+a^2x^2}\right)^4} - \frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[8]{-b}}\right)}{2(-b)^{15/8}} - \frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[8]{b}}\right)}{2b^{15/8}}$$

$$= \frac{4a\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{b^2-\left(ax+\sqrt{-b+a^2x^2}\right)^4} - \frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[8]{-b}}\right)}{2(-b)^{15/8}} - \frac{a\tan^{-1}\left(\frac{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}{\sqrt[8]{b}}\right)}{2b^{15/8}}$$

Mathematica [B] time = 2.35, size = 2041, normalized size = 2.70

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/(x^2*(-b + a^2*x^2)^(3/2)),x]
```

```
[Out] (-32*b^(15/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) - 4*(-b^2 + (a*x + Sqrt[-b +
 a^2*x^2])^4)*ArcTan[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)] + 2*Sqrt[2]*
(-b^2 + (a*x + Sqrt[-b + a^2*x^2])^4)*ArcTan[1 - (Sqrt[2]*(a*x + Sqrt[-b +
a^2*x^2])^(1/4))/b^(1/8)] + 2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a*x + Sqrt[-
b + a^2*x^2])^(1/4))/b^(1/8)] - 2*Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^4*ArcT
```

```
an[1 + (Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/b^(1/8)] + 4*b^2*ArcTan[(
(a*x + Sqrt[-b + a^2*x^2])^(1/4)*Sec[Pi/8])/b^(1/8) - Tan[Pi/8]]*Cos[Pi/8]
- 4*(a*x + Sqrt[-b + a^2*x^2])^4*ArcTan[((a*x + Sqrt[-b + a^2*x^2])^(1/4)*S
ec[Pi/8])/b^(1/8) - Tan[Pi/8]]*Cos[Pi/8] + 4*b^2*ArcTan[((a*x + Sqrt[-b + a
^2*x^2])^(1/4)*Sec[Pi/8])/b^(1/8) + Tan[Pi/8]]*Cos[Pi/8] - 4*(a*x + Sqrt[-b
 + a^2*x^2])^4*ArcTan[((a*x + Sqrt[-b + a^2*x^2])^(1/4)*Sec[Pi/8])/b^(1/8)
+ Tan[Pi/8]]*Cos[Pi/8] - 2*b^2*Log[b^(1/8) - (a*x + Sqrt[-b + a^2*x^2])^(1/
4)] + 2*(a*x + Sqrt[-b + a^2*x^2])^4*Log[b^(1/8) - (a*x + Sqrt[-b + a^2*x^2
])^(1/4)] + 2*b^2*Log[b^(1/8) + (a*x + Sqrt[-b + a^2*x^2])^(1/4)] - 2*(a*x
+ Sqrt[-b + a^2*x^2])^4*Log[b^(1/8) + (a*x + Sqrt[-b + a^2*x^2])^(1/4)] - S
qrt[2]*b^2*Log[b^(1/4) - Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) +
 Sqrt[a*x + Sqrt[-b + a^2*x^2]]] + Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^4*Log
[b^(1/4) - Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a*x + Sq
rt[-b + a^2*x^2]]] + Sqrt[2]*b^2*Log[b^(1/4) + Sqrt[2]*b^(1/8)*(a*x + Sqrt[
-b + a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]] - Sqrt[2]*(a*x + Sqr
t[-b + a^2*x^2])^4*Log[b^(1/4) + Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])
^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]] - 2*b^2*Cos[Pi/8]*Log[b^(1/4) + Sq
rt[a*x + Sqrt[-b + a^2*x^2]] - 2*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*C
os[Pi/8]] + 2*(a*x + Sqrt[-b + a^2*x^2])^4*Cos[Pi/8]*Log[b^(1/4) + Sqrt[a*x
 + Sqrt[-b + a^2*x^2]] - 2*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*Cos[Pi/
8]] + 2*b^2*Cos[Pi/8]*Log[b^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]] + 2*b^(1
/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*Cos[Pi/8]] - 2*(a*x + Sqrt[-b + a^2*x^
2])^4*Cos[Pi/8]*Log[b^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]] + 2*b^(1/8)*(a
*x + Sqrt[-b + a^2*x^2])^(1/4)*Cos[Pi/8]] - 4*b^2*ArcTan[Cot[Pi/8] - ((a*x
+ Sqrt[-b + a^2*x^2])^(1/4)*Csc[Pi/8])/b^(1/8)]*Sin[Pi/8] + 4*(a*x + Sqrt[-
b + a^2*x^2])^4*ArcTan[Cot[Pi/8] - ((a*x + Sqrt[-b + a^2*x^2])^(1/4)*Csc[Pi
/8])/b^(1/8)]*Sin[Pi/8] + 4*b^2*ArcTan[Cot[Pi/8] + ((a*x + Sqrt[-b + a^2*x^
2])^(1/4)*Csc[Pi/8])/b^(1/8)]*Sin[Pi/8] - 4*(a*x + Sqrt[-b + a^2*x^2])^4*Ar
cTan[Cot[Pi/8] + ((a*x + Sqrt[-b + a^2*x^2])^(1/4)*Csc[Pi/8])/b^(1/8)]*Sin[
Pi/8] - 2*b^2*Log[b^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]] - 2*b^(1/8)*(a*x
 + Sqrt[-b + a^2*x^2])^(1/4)*Sin[Pi/8]]*Sin[Pi/8] + 2*(a*x + Sqrt[-b + a^2*
x^2])^4*Log[b^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]] - 2*b^(1/8)*(a*x + Sqr
t[-b + a^2*x^2])^(1/4)*Sin[Pi/8]]*Sin[Pi/8] + 2*b^2*Log[b^(1/4) + Sqrt[a*x
+ Sqrt[-b + a^2*x^2]] + 2*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*Sin[Pi/8
]]*Sin[Pi/8] - 2*(a*x + Sqrt[-b + a^2*x^2])^4*Log[b^(1/4) + Sqrt[a*x + Sqrt
[-b + a^2*x^2]] + 2*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*Sin[Pi/8]]*Sin
[Pi/8])/(16*b^(15/8)*x*Sqrt[-4*b + 4*a^2*x^2]*(-b + 2*a*x*(a*x + Sqrt[-b +
a^2*x^2])))
```

IntegrateAlgebraic [A] time = 5.80, size = 731, normalized size = 0.97

$$-\frac{a\tan^{-1}\left(\frac{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{b}}\right)}{2b^{15/8}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt[8]{b}\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)}{2\sqrt{2}b^{15/8}}+\frac{\sqrt{2-\sqrt{2}}\,a\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{b}\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)}{4b^{15/8}}+\frac{\sqrt{2+\sqrt{2}}\,a\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{b}\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt{\sqrt{a^2x^2-b}+ax}-\sqrt[4]{b}}\right)}{4b^{15/8}}-\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{b}}\right)}{2b^{15/8}}-\frac{a\tanh^{-1}\left(\frac{\frac{\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt{2}\sqrt[8]{b}}+\frac{\sqrt[8]{b}}{\sqrt{2}}}{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}\right)}{2\sqrt{2}b^{15/8}}-\frac{\sqrt{2+\sqrt{2}}\,a\tanh^{-1}\left(\frac{\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{b}}+\sqrt{1-\frac{1}{\sqrt{2}}}\sqrt[8]{b}}{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}\right)}{4b^{15/8}}-\frac{\sqrt{2-\sqrt{2}}\,a\tanh^{-1}\left(\frac{\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\sqrt{\sqrt{a^2x^2-b}+ax}}{\sqrt[8]{b}}+\sqrt{1+\frac{1}{\sqrt{2}}}\sqrt[8]{b}}{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}\right)}{4b^{15/8}}-\frac{\sqrt[4]{\sqrt{a^2x^2-b}+ax}}{x\left(2a^3x^3-2abx\right)+x\sqrt{a^2x^2-b}\left(2a^2x^2-b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/(x^2*(-b + a^2*x^2)^(3/
2)),x]
```

```
[Out] -((a*x + Sqrt[-b + a^2*x^2])^(1/4)/(x*Sqrt[-b + a^2*x^2]*(-b + 2*a^2*x^2) +
 x*(-2*a*b*x + 2*a^3*x^3))) - (a*ArcTan[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/b^
(1/8)])/(2*b^(15/8)) + (a*ArcTan[(Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2]
)^(1/4))/(-b^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]])])/(2*Sqrt[2]*b^(15/8))
 + (Sqrt[2 - Sqrt[2]]*a*ArcTan[(Sqrt[2 - Sqrt[2]]*b^(1/8)*(a*x + Sqrt[-b +
a^2*x^2])^(1/4))/(-b^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]])])/(4*b^(15/8))
 + (Sqrt[2 + Sqrt[2]]*a*ArcTan[(Sqrt[2 + Sqrt[2]]*b^(1/8)*(a*x + Sqrt[-b +
a^2*x^2])^(1/4))/(-b^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]])])/(4*b^(15/8))
 - (a*ArcTanh[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)])/(2*b^(15/8)) - (a*
ArcTanh[(b^(1/8)/Sqrt[2] + Sqrt[a*x + Sqrt[-b + a^2*x^2]]/(Sqrt[2]*b^(1/8))
)/(a*x + Sqrt[-b + a^2*x^2])^(1/4)])/(2*Sqrt[2]*b^(15/8)) - (Sqrt[2 + Sqrt[
2]]*a*ArcTanh[(Sqrt[1 - 1/Sqrt[2]]*b^(1/8) + (Sqrt[1 - 1/Sqrt[2]]*Sqrt[a*x
```

```
+ Sqrt[-b + a^2*x^2]])/b^(1/8))/(a*x + Sqrt[-b + a^2*x^2])^(1/4)])/(4*b^(15
/8)) - (Sqrt[2 - Sqrt[2]]*a*ArcTanh[(Sqrt[1 + 1/Sqrt[2]]*b^(1/8) + (Sqrt[1
+ 1/Sqrt[2]]*Sqrt[a*x + Sqrt[-b + a^2*x^2]])/b^(1/8))/(a*x + Sqrt[-b + a^2*
x^2])^(1/4)])/(4*b^(15/8))
```

fricas [A] time = 0.42, size = 80, normalized size = 0.11

$$\frac{\left(2\,a^3x^3-2\,abx-\left(2\,a^2x^2-b\right)\sqrt{a^2x^2-b}\right)\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}}{a^2b^2x^3-b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2-b)^(1/2))^(1/4)/x^2/(a^2*x^2-b)^(3/2),x, algorithm=
"fricas")
```

```
[Out] (2*a^3*x^3 - 2*a*b*x - (2*a^2*x^2 - b)*sqrt(a^2*x^2 - b))*(a*x + sqrt(a^2*x
^2 - b))^(1/4)/(a^2*b^2*x^3 - b^3*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2-b)^(1/2))^(1/4)/x^2/(a^2*x^2-b)^(3/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}}{x^2\left(a^2x^2-b\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+(a^2*x^2-b)^(1/2))^(1/4)/x^2/(a^2*x^2-b)^(3/2),x)
```

```
[Out] int((a*x+(a^2*x^2-b)^(1/2))^(1/4)/x^2/(a^2*x^2-b)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}}{\left(a^2x^2-b\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a^2*x^2-b)^(1/2))^(1/4)/x^2/(a^2*x^2-b)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate((a*x + sqrt(a^2*x^2 - b))^(1/4)/((a^2*x^2 - b)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{1/4}}{x^2\,{\left(a^2\,x^2-b\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + (a^2*x^2 - b)^(1/2))^(1/4)/(x^2*(a^2*x^2 - b)^(3/2)),x)
```

```
[Out] int((a*x + (a^2*x^2 - b)^(1/2))^(1/4)/(x^2*(a^2*x^2 - b)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{ax + \sqrt{a^2x^2 - b}}}{x^2 \left(a^2x^2 - b\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+(a**2*x**2-b)**(1/2))**(1/4)/x**2/(a**2*x**2-b)**(3/2),x)
```

```
[Out] Integral((a*x + sqrt(a**2*x**2 - b))**(1/4)/(x**2*(a**2*x**2 - b)**(3/2)),
x)
```

3.2426 $\int \frac{\sqrt{-b+a^2x^2}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=787

$$\frac{13923b^2\tan^{-1}\left(\frac{\sqrt[4]{\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[4]{c}}\right)}{131072ac^{25/4}}-\frac{13923b^2\tanh^{-1}\left(\frac{\sqrt[4]{\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[4]{c}}\right)}{131072ac^{25/4}}-\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{\sqrt[3]{\sqrt{a^2x^2-b}+ax}+c}}{\sqrt[4]{c}}\right)}{a\sqrt[4]{c}}+\frac{3b\tan}{}$$

Rubi [F] time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-b+a^2x^2}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[-b + a^2*x^2]/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4),x]
```

```
[Out] Defer[Int][Sqrt[-b + a^2*x^2]/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4),
 x]
```

Rubi steps

$$\int \frac{\sqrt{-b+a^2x^2}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{\sqrt{-b+a^2x^2}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Mathematica [B] time = 7.38, size = 2441, normalized size = 3.10

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-b + a^2*x^2]/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4),x
]
```

```
[Out] -1/44104417280*((c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(35/4)*(1 - b/(c + (
a*x + Sqrt[-b + a^2*x^2])^(1/3))^6 + c^6/(c + (a*x + Sqrt[-b + a^2*x^2])^(1
/3))^6 - (6*c^5)/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^5 + (15*c^4)/(c + (
a*x + Sqrt[-b + a^2*x^2])^(1/3))^4 - (20*c^3)/(c + (a*x + Sqrt[-b + a^2*x^2
])^(1/3))^3 + (15*c^2)/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^2 - (6*c)/(c
+ (a*x + Sqrt[-b + a^2*x^2])^(1/3)))*(5752750080*c^(25/4) - (40981117100*b^
2*c^(21/4))/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^11 - (44104417280*c^(69/
4))/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^11 + (114912007980*b^2*c^(17/4))
/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^10 + (359135969280*c^(65/4))/(c + (
a*x + Sqrt[-b + a^2*x^2])^(1/3))^10 - (164814147960*b^2*c^(13/4))/(c + (a*x
 + Sqrt[-b + a^2*x^2])^(1/3))^9 - (1348907827200*c^(61/4))/(c + (a*x + Sqrt
[-b + a^2*x^2])^(1/3))^9 + (130345727512*b^2*c^(9/4))/(c + (a*x + Sqrt[-b +
 a^2*x^2])^(1/3))^8 + (3109647810560*c^(57/4))/(c + (a*x + Sqrt[-b + a^2*x^
2])^(1/3))^8 - (54345423132*b^2*c^(5/4))/(c + (a*x + Sqrt[-b + a^2*x^2])^(1
/3))^7 - (4920099471360*c^(53/4))/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^7
```

```
+ (9369900540*b^2*c^(1/4))/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^6 + (5625
764904960*c^(49/4))/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^6 - (47364151705
60*c^(45/4))/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^5 + (2922027417600*c^(4
1/4))/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^4 - (1286881935360*c^(37/4))/(
c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^3 + (383415746560*c^(33/4))/(c + (a*x
 + Sqrt[-b + a^2*x^2])^(1/3))^2 - (69335777280*c^(29/4))/(c + (a*x + Sqrt[-
b + a^2*x^2])^(1/3)) - (1009470*b*(4641*b - 131072*c^6)*(-1 + c/(c + (a*x +
 Sqrt[-b + a^2*x^2])^(1/3)))^6*ArcTan[c^(1/4)/(c + (a*x + Sqrt[-b + a^2*x^2
])^(1/3))^(1/4)])/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(23/4) + (504735*b
*(4641*b - 131072*c^6)*(-1 + c/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3)))^6*Lo
g[1 - c^(1/4)/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)])/(c + (a*x + Sq
rt[-b + a^2*x^2])^(1/3))^(23/4) - (2342475135*b^2*c^6*Log[1 + c^(1/4)/(c +
(a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)])/(c + (a*x + Sqrt[-b + a^2*x^2])^(
1/3))^(47/4) + (66156625920*b*c^12*Log[1 + c^(1/4)/(c + (a*x + Sqrt[-b + a^
2*x^2])^(1/3))^(1/4)])/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(47/4) + (140
54850810*b^2*c^5*Log[1 + c^(1/4)/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/
4)])/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(43/4) - (396939755520*b*c^11*L
og[1 + c^(1/4)/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)])/(c + (a*x + S
qrt[-b + a^2*x^2])^(1/3))^(43/4) - (35137127025*b^2*c^4*Log[1 + c^(1/4)/(c
+ (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)])/(c + (a*x + Sqrt[-b + a^2*x^2])
^(1/3))^(39/4) + (992349388800*b*c^10*Log[1 + c^(1/4)/(c + (a*x + Sqrt[-b +
 a^2*x^2])^(1/3))^(1/4)])/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(39/4) + (
46849502700*b^2*c^3*Log[1 + c^(1/4)/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^
(1/4)])/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(35/4) - (1323132518400*b*c^
9*Log[1 + c^(1/4)/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)])/(c + (a*x
+ Sqrt[-b + a^2*x^2])^(1/3))^(35/4) - (35137127025*b^2*c^2*Log[1 + c^(1/4)/
(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)])/(c + (a*x + Sqrt[-b + a^2*x^
2])^(1/3))^(31/4) + (992349388800*b*c^8*Log[1 + c^(1/4)/(c + (a*x + Sqrt[-b
 + a^2*x^2])^(1/3))^(1/4)])/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(31/4) +
 (14054850810*b^2*c*Log[1 + c^(1/4)/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^
(1/4)])/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(27/4) - (396939755520*b*c^7
*Log[1 + c^(1/4)/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)])/(c + (a*x +
 Sqrt[-b + a^2*x^2])^(1/3))^(27/4) - (2342475135*b^2*Log[1 + c^(1/4)/(c + (
a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)])/(c + (a*x + Sqrt[-b + a^2*x^2])^(1
/3))^(23/4) + (66156625920*b*c^6*Log[1 + c^(1/4)/(c + (a*x + Sqrt[-b + a^2*
x^2])^(1/3))^(1/4)])/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(23/4)))/(a*c^(
25/4)*Sqrt[((c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^6*(1 - b/(c + (a*x + Sqr
t[-b + a^2*x^2])^(1/3))^6 + c^6/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^6 -
(6*c^5)/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^5 + (15*c^4)/(c + (a*x + Sqr
t[-b + a^2*x^2])^(1/3))^4 - (20*c^3)/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))
^3 + (15*c^2)/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^2 - (6*c)/(c + (a*x +
Sqrt[-b + a^2*x^2])^(1/3)))^2)/(-1 + c/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3
)))^6]*(-1 + c/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3)))^9)
```

IntegrateAlgebraic [A] time = 1.94, size = 787, normalized size = 1.00

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-b + a^2*x^2]/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3)
)^(1/4),x]
```

```
[Out] ((-1378263040*b^2*c^5 + 2684354560*b*c^11 + 1665760096*a*b^2*c^2*x - 484442
1120*a*b*c^8*x - 5368709120*a^2*c^11*x^2 + 6459228160*a^3*c^8*x^3)*(c + (a*
x + Sqrt[-b + a^2*x^2])^(1/3))^(3/4) + (1447176192*b^2*c^4 - 2013265920*b*c
^10 - 1873980108*a*b^2*c*x + 4541644800*a*b*c^7*x + 4026531840*a^2*c^10*x^2
 - 6055526400*a^3*c^7*x^3)*(a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x + Sqr
t[-b + a^2*x^2])^(1/3))^(3/4) + (-1537624704*b^2*c^3 + 1761607680*b*c^9 + 2
342475135*a*b^2*x - 4314562560*a*b*c^6*x - 3523215360*a^2*c^9*x^2 + 5752750
080*a^3*c^6*x^3)*(a*x + Sqrt[-b + a^2*x^2])^(2/3)*(c + (a*x + Sqrt[-b + a^2
```

```
*x^2])^(1/3))^(3/4) + Sqrt[-b + a^2*x^2]*((1665760096*b^2*c^2 - 1614807040*
b*c^8 - 5368709120*a*c^11*x + 6459228160*a^2*c^8*x^2)*(c + (a*x + Sqrt[-b +
 a^2*x^2])^(1/3))^(3/4) + (-1873980108*b^2*c + 1513881600*b*c^7 + 402653184
0*a*c^10*x - 6055526400*a^2*c^7*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c +
(a*x + Sqrt[-b + a^2*x^2])^(1/3))^(3/4) + (2342475135*b^2 - 1438187520*b*c^
6 - 3523215360*a*c^9*x + 5752750080*a^2*c^6*x^2)*(a*x + Sqrt[-b + a^2*x^2])
^(2/3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(3/4)))/(22052208640*a^2*c^6*
x*Sqrt[-b + a^2*x^2] + 11026104320*a*c^6*(-b + 2*a^2*x^2)) + (13923*b^2*Arc
Tan[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)/c^(1/4)])/(131072*a*c^(25/
4)) - (3*b*ArcTan[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)/c^(1/4)])/(a
*c^(1/4)) - (13923*b^2*ArcTanh[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)
/c^(1/4)])/(131072*a*c^(25/4)) + (3*b*ArcTanh[(c + (a*x + Sqrt[-b + a^2*x^2
])^(1/3))^(1/4)/c^(1/4)])/(a*c^(1/4))
```

fricas [A] time = 0.58, size = 1060, normalized size = 1.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/3))^(1/4),x, algo
rithm="fricas")
```

```
[Out] 1/44104417280*(2018940*a*c^6*((295147905179352825856*b^4*c^24 - 41802411741
252943872*b^5*c^18 + 2220210947698458624*b^6*c^12 - 52408849122459648*b^7*c
^6 + 463923394732161*b^8)/(a^4*c^25))^(1/4)*arctan((sqrt((50706024009129176
0598681282150 4*b^6*c^36 - 10772399479356468479637816606 72*b^7*c^30 + 953573
341058604625968916 07040*b^8*c^24 - 4501885860039249744793436160*b^9*c^18 +
119552148493435810464399360*b^10*c^12 - 1693241946893419178360832*b^11*c^6
+ 9992390792252042651841*b^12)*sqrt(c + (a*x + sqrt(a^2*x^2 - b))^(1/3)) +
(295147905179352825856*a^2*b^4*c^37 - 41802411741252943872*a^2*b^5*c^31 + 2
220210947698458624*a^2*b^6*c^25 - 52408849122459648*a^2*b^7*c^19 + 46392339
4732161*a^2*b^8*c^13)*sqrt((295147905179352825856*b^4*c^24 - 41802411741252
943872*b^5*c^18 + 2220210947698458624*b^6*c^12 - 52408849122459648*b^7*c^6
+ 463923394732161*b^8)/(a^4*c^25)))*a*c^6*((295147905179352825856*b^4*c^24
- 41802411741252943872*b^5*c^18 + 2220210947698458624*b^6*c^12 - 5240884912
2459648*b^7*c^6 + 463923394732161*b^8)/(a^4*c^25))^(1/4) - (225179981368524
8*a*b^3*c^24 - 239195318648832*a*b^4*c^18 + 8469432631296*a*b^5*c^12 - 9996
1946721*a*b^6*c^6)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/3))^(1/4)*((2951479051
79352825856*b^4*c^24 - 41802411741252943872*b^5*c^18 + 2220210947698458624*
b^6*c^12 - 52408849122459648*b^7*c^6 + 463923394732161*b^8)/(a^4*c^25))^(1/
4))/(295147905179352825856*b^4*c^24 - 41802411741252943872*b^5*c^18 + 22202
10947698458624*b^6*c^12 - 52408849122459648*b^7*c^6 + 463923394732161*b^8))
 + 504735*a*c^6*((295147905179352825856*b^4*c^24 - 41802411741252943872*b^5
*c^18 + 2220210947698458624*b^6*c^12 - 52408849122459648*b^7*c^6 + 46392339
4732161*b^8)/(a^4*c^25))^(1/4)*log(27*a^3*c^19*((295147905179352825856*b^4*
c^24 - 41802411741252943872*b^5*c^18 + 2220210947698458624*b^6*c^12 - 52408
849122459648*b^7*c^6 + 463923394732161*b^8)/(a^4*c^25))^(3/4) + 27*(2251799
813685248*b^3*c^18 - 239195318648832*b^4*c^12 + 8469432631296*b^5*c^6 - 999
61946721*b^6)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/3))^(1/4)) - 504735*a*c^6*(
(295147905179352825856*b^4*c^24 - 41802411741252943872*b^5*c^18 + 222021094
7698458624*b^6*c^12 - 52408849122459648*b^7*c^6 + 463923394732161*b^8)/(a^4
*c^25))^(1/4)*log(-27*a^3*c^19*((295147905179352825856*b^4*c^24 - 418024117
41252943872*b^5*c^18 + 2220210947698458624*b^6*c^12 - 52408849122459648*b^7
*c^6 + 463923394732161*b^8)/(a^4*c^25))^(3/4) + 27*(2251799813685248*b^3*c^
18 - 239195318648832*b^4*c^12 + 8469432631296*b^5*c^6 - 99961946721*b^6)*(c
 + (a*x + sqrt(a^2*x^2 - b))^(1/3))^(1/4)) - 4*(2684354560*c^11 + 275652608
0*a^2*c^5*x^2 - 1378263040*b*c^5 - 2464*(655360*a*c^8 + 676039*a*b*c^2)*x +
 21*(83886080*c^9 + 146440448*a^2*c^3*x^2 - 73220224*b*c^3 - 1045*(65536*a*
c^6 + 106743*a*b)*x - 209*(327680*c^6 + 700672*a*c^3*x - 533715*b)*sqrt(a^2
*x^2 - b))*(a*x + sqrt(a^2*x^2 - b))^(2/3) - 2464*(655360*c^8 + 1118720*a*c
```

```
^5*x - 676039*b*c^2)*sqrt(a^2*x^2 - b) - 12*(167772160*c^10 + 241196032*a^2
*c^4*x^2 - 120598016*b*c^4 - 77*(1638400*a*c^7 + 2028117*a*b*c)*x - 77*(163
8400*c^7 + 3132416*a*c^4*x - 2028117*b*c)*sqrt(a^2*x^2 - b))*(a*x + sqrt(a^
2*x^2 - b))^(1/3))*(c + (a*x + sqrt(a^2*x^2 - b))^(1/3))^(3/4))/(a*c^6)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/3))^(1/4),x, algo
rithm="giac")
[Out] Timed out
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^{2}x^{2}-b}}{\left(c+\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/3))^(1/4),x)
[Out] int((a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/3))^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^{2}x^{2}-b}}{\left(c+\left(ax+\sqrt{a^{2}x^{2}-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/3))^(1/4),x, algo
rithm="maxima")
[Out] integrate(sqrt(a^2*x^2 - b)/(c + (a*x + sqrt(a^2*x^2 - b))^(1/3))^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a^2\,x^2-b}}{{\left(c+{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{1/3}\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 - b)^(1/2)/(c + (a*x + (a^2*x^2 - b)^(1/2))^(1/3))^(1/4),x)
[Out] int((a^2*x^2 - b)^(1/2)/(c + (a*x + (a^2*x^2 - b)^(1/2))^(1/3))^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^{2}x^{2}-b}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{a^{2}x^{2}-b}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b)**(1/2)/(c+(a*x+(a**2*x**2-b)**(1/2))**(1/3))**(1/4)
,x)
```

```
[Out] Integral(sqrt(a**2*x**2 - b)/(c + (a*x + sqrt(a**2*x**2 - b))**(1/3))**(1/4
), x)
```

3.2427 $\int \frac{\sqrt{-b+a^2x^2}}{\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$

Optimal. Leaf size=803

$$-\frac{49725\tan^{-1}\left(\frac{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{a^2x^2-b}}}}{\sqrt[4]{c}}\right)b^2}{524288ac^{29/4}}+\frac{49725\tanh^{-1}\left(\frac{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{a^2x^2-b}}}}{\sqrt[4]{c}}\right)b^2}{524288ac^{29/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{a^2x^2-b}}}}{\sqrt[4]{c}}\right)b}{4ac^{5/4}}-\frac{3\tanh}{}$$

Rubi [F] time = 1.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-b+a^2x^2}}{\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[-b + a^2*x^2]/((a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x + Sqrt[-
b + a^2*x^2])^(1/3))^(1/4)),x]
```

```
[Out] Defer[Int][Sqrt[-b + a^2*x^2]/((a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x +
 Sqrt[-b + a^2*x^2])^(1/3))^(1/4)), x]
```

Rubi steps

$$\int \frac{\sqrt{-b+a^2x^2}}{\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx=\int \frac{\sqrt{-b+a^2x^2}}{\sqrt[3]{ax+\sqrt{-b+a^2x^2}}\sqrt[4]{c+\sqrt[3]{ax+\sqrt{-b+a^2x^2}}}}\,dx$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[-b + a^2*x^2]/((a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x +
Sqrt[-b + a^2*x^2])^(1/3))^(1/4)),x]
```

```
[Out] $Aborted
```

IntegrateAlgebraic [A] time = 2.56, size = 803, normalized size = 1.00

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-b + a^2*x^2]/((a*x + Sqrt[-b + a^2*x^2])^(1/3)*(c
+ (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)),x]
```

```
[Out] ((363738375*b^3 - 3081830400*b^2*c^6 + 238761600*a*b^2*c^3*x - 1056964608*a
*b*c^9*x - 727476750*a^2*b^2*x^2 + 5752750080*a^2*b*c^6*x^2 + 1409286144*a^
3*c^9*x^3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(3/4) + (214016000*b^2*c^
```

```
5 - 536870912*b*c^11 - 258658400*a*b^2*c^2*x + 968884224*a*b*c^8*x + 107374
1824*a^2*c^11*x^2 - 1291845632*a^3*c^8*x^3)*(a*x + Sqrt[-b + a^2*x^2])^(1/3
)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(3/4) + (-224716800*b^2*c^4 + 4026
53184*b*c^10 + 290990700*a*b^2*c*x - 908328960*a*b*c^7*x - 805306368*a^2*c^
10*x^2 + 1211105280*a^3*c^7*x^3)*(a*x + Sqrt[-b + a^2*x^2])^(2/3)*(c + (a*x
 + Sqrt[-b + a^2*x^2])^(1/3))^(3/4) + Sqrt[-b + a^2*x^2]*((238761600*b^2*c^
3 - 352321536*b*c^9 - 727476750*a*b^2*x + 5752750080*a*b*c^6*x + 1409286144
*a^2*c^9*x^2)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(3/4) + (-258658400*b^
2*c^2 + 322961408*b*c^8 + 1073741824*a*c^11*x - 1291845632*a^2*c^8*x^2)*(a*
x + Sqrt[-b + a^2*x^2])^(1/3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(3/4)
+ (290990700*b^2*c - 302776320*b*c^7 - 805306368*a*c^10*x + 1211105280*a^2*
c^7*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(2/3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(
1/3))^(3/4)))/(1917583360*a*c^7*(a*x + Sqrt[-b + a^2*x^2])^(7/3)) - (49725*
b^2*ArcTan[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)/c^(1/4)])/(524288*a
*c^(29/4)) + (3*b*ArcTan[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/3))^(1/4)/c^(1/
4)])/(4*a*c^(5/4)) + (49725*b^2*ArcTanh[(c + (a*x + Sqrt[-b + a^2*x^2])^(1/
3))^(1/4)/c^(1/4)])/(524288*a*c^(29/4)) - (3*b*ArcTanh[(c + (a*x + Sqrt[-b
+ a^2*x^2])^(1/3))^(1/4)/c^(1/4)])/(4*a*c^(5/4))
```

fricas [A] time = 0.56, size = 1121, normalized size = 1.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-
b)^(1/2))^(1/3))^(1/4),x, algorithm="fricas")
```

```
[Out] -1/7670333440*(87780*a*b*c^7*((2951479051793528258 56*b^4*c^24 - 14929432764
7331942400*b^5*c^18 + 283190171900313600 00*b^6*c^12 - 2387429351424000000*b
^7*c^6 + 75476916312890625*b^8)/(a^4*c^29))^(1/4)*arctan((sqrt((50706024009
1291760598681282150 4*b^6*c^36 - 38472855283415958855849345024 00*b^7*c^30 +
12162925268604650841440256000 00*b^8*c^24 - 20507861971753142059008000000*b
^9*c^18 + 19450253230007648256000000000*b^10*c^12 - 98384993679009024000000
0000*b^11*c^6 + 20735820391713136962890625*b^12)*sqrt(c + (a*x + sqrt(a^2*x
^2 - b))^(1/3)) + (295147905179352825856*a^2*b^4*c^39 - 1492943276473319424
00*a^2*b^5*c^33 + 28319017190031360000*a^2*b^6*c^27 - 2387429351424000000*a
^2*b^7*c^21 + 75476916312890625*a^2*b^8*c^15)*sqrt((295147905179352825856*b
^4*c^24 - 149294327647331942400*b^5*c^18 + 28319017190031360000*b^6*c^12 -
2387429351424000000*b^7*c^6 + 75476916312890625*b^8)/(a^4*c^29)))*a*c^7*((2
95147905179352825856*b^4*c^24 - 149294327647331942400*b^5*c^18 + 2831901719
0031360000*b^6*c^12 - 2387429351424000000*b^7*c^6 + 75476916312890625*b^8)/
(a^4*c^29))^(1/4) - (2251799813685248*a*b^3*c^25 - 854268995174400*a*b^4*c^
19 + 108028477440000*a*b^5*c^13 - 4553660109375*a*b^6*c^7)*(c + (a*x + sqrt
(a^2*x^2 - b))^(1/3))^(1/4)*((295147905179352825856*b^4*c^24 - 149294327647
331942400*b^5*c^18 + 28319017190031360000*b^6*c^12 - 2387429351424000000*b^
7*c^6 + 75476916312890625*b^8)/(a^4*c^29))^(1/4))/(295147905179352825856*b^
4*c^24 - 149294327647331942400*b^5*c^18 + 28319017190031360000*b^6*c^12 - 2
387429351424000000*b^7*c^6 + 75476916312890625*b^8)) + 21945*a*b*c^7*((2951
47905179352825856*b^4*c^24 - 149294327647331942400*b^5*c^18 + 2831901719003
1360000*b^6*c^12 - 2387429351424000000*b^7*c^6 + 75476916312890625*b^8)/(a^
4*c^29))^(1/4)*log(27*a^3*c^22*((295147905179352825856*b^4*c^24 - 149294327
647331942400*b^5*c^18 + 28319017190031360000*b^6*c^12 - 2387429351424000000
*b^7*c^6 + 75476916312890625*b^8)/(a^4*c^29))^(3/4) + 27*(2251799813685248*
b^3*c^18 - 854268995174400*b^4*c^12 + 108028477440000*b^5*c^6 - 45536601093
75*b^6)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/3))^(1/4)) - 21945*a*b*c^7*((2951
47905179352825856*b^4*c^24 - 149294327647331942400*b^5*c^18 + 2831901719003
1360000*b^6*c^12 - 2387429351424000000*b^7*c^6 + 75476916312890625*b^8)/(a^
4*c^29))^(1/4)*log(-27*a^3*c^22*((295147905179352825856*b^4*c^24 - 14929432
7647331942400*b^5*c^18 + 28319017190031360000*b^6*c^12 - 238742935142400000
0*b^7*c^6 + 75476916312890625*b^8)/(a^4*c^29))^(3/4) + 27*(2251799813685248
```

```
*b^3*c^18 - 854268995174400*b^4*c^12 + 108028477440000*b^5*c^6 - 4553660109
375*b^6)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/3))^(1/4)) - 4*(536870912*b*c^11
 + 428032000*a^2*b*c^5*x^2 - 214016000*b^2*c^5 - 2464*(131072*a*b*c^8 + 104
975*a*b^2*c^2)*x - 3*(273940480*a^3*c^6*x^3 - 117440512*b*c^9 - 159174400*a
^2*b*c^3*x^2 + 79587200*b^2*c^3 - 17765*(65536*a*b*c^6 - 6825*a*b^2)*x - 10
45*(262144*a^2*c^6*x^2 - 983040*b*c^6 - 152320*a*b*c^3*x + 116025*b^2)*sqrt
(a^2*x^2 - b))*(a*x + sqrt(a^2*x^2 - b))^(2/3) - 352*(917504*b*c^8 + 121600
0*a*b*c^5*x - 734825*b^2*c^2)*sqrt(a^2*x^2 - b) - 12*(33554432*b*c^10 + 374
52800*a^2*b*c^4*x^2 - 18726400*b^2*c^4 - 385*(65536*a*b*c^7 + 62985*a*b^2*c
)*x - 385*(65536*b*c^7 + 97280*a*b*c^4*x - 62985*b^2*c)*sqrt(a^2*x^2 - b))*
(a*x + sqrt(a^2*x^2 - b))^(1/3))*(c + (a*x + sqrt(a^2*x^2 - b))^(1/3))^(3/4
))/(a*b*c^7)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-
b)^(1/2))^(1/3))^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^{2} x^{2}-b}}{\left(a x +\sqrt{a^{2} x^{2}-b}\right)^{\frac{1}{3}} \left(c +\left(a x +\sqrt{a^{2} x^{2}-b}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-b)^(1/
2))^(1/3))^(1/4),x)
```

```
[Out] int((a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-b)^(1/
2))^(1/3))^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^{2} x^{2} - b}}{{\left(a x + \sqrt{a^{2} x^{2} - b}\right)}^{\frac{1}{3}} {\left(c + {\left(a x + \sqrt{a^{2} x^{2} - b}\right)}^{\frac{1}{3}}\right)}^{\frac{1}{4}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(a*x+(a^2*x^2-b)^(1/2))^(1/3)/(c+(a*x+(a^2*x^2-
b)^(1/2))^(1/3))^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 - b)/((a*x + sqrt(a^2*x^2 - b))^(1/3)*(c + (a*x + sq
rt(a^2*x^2 - b))^(1/3))^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a^2\,x^2-b}}{{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{1/3}\,{\left(c+{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{1/3}\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 - b)^(1/2)/((a*x + (a^2*x^2 - b)^(1/2))^(1/3)*(c + (a*x + (a^2
*x^2 - b)^(1/2))^(1/3))^(1/4)),x)
```

```
[Out] int((a^2*x^2 - b)^(1/2)/((a*x + (a^2*x^2 - b)^(1/2))^(1/3)*(c + (a*x + (a^2
*x^2 - b)^(1/2))^(1/3))^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2-b}}{\sqrt[4]{c+\sqrt[3]{ax+\sqrt{a^2x^2-b}}}\,\sqrt[3]{ax+\sqrt{a^2x^2-b}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b)**(1/2)/(a*x+(a**2*x**2-b)**(1/2))**(1/3)/(c+(a*x+(a
**2*x**2-b)**(1/2))**(1/3))**(1/4),x)
```

```
[Out] Integral(sqrt(a**2*x**2 - b)/((c + (a*x + sqrt(a**2*x**2 - b))**(1/3))**(1/
4)*(a*x + sqrt(a**2*x**2 - b))**(1/3)), x)
```

3.2428 $\int \frac{(b+ax^4)\sqrt{-b-cx^2+ax^4}}{(-b+ax^4)^2}\,dx$

Optimal. Leaf size=827

$$\frac{x\left(ia^{3/2}x^6 - i\sqrt{a}\,cx^4 - a\sqrt{b}\,x^4 + ia\sqrt{ax^4-cx^2-b}\,x^4 - i\sqrt{a}\,bx^2 + \sqrt{b}\,cx^2 - \frac{1}{2}ic\sqrt{ax^4-cx^2-b}\,x^2 - \sqrt{a}\,\sqrt{b}\,\sqrt{ax^4-c\right.}{\left(ax^4-b\right)\left(-2iax^4 + icx^2 + 2\sqrt{a}\,\sqrt{b}\,x^2 - 2i\sqrt{a}\,\sqrt{ax^4-cx^2-b}\,x^2 + 2ib + 2\sqrt{b}\,\sqrt{ax^4-\right.}$$

Rubi [C] time = 5.57, antiderivative size = 908, normalized size of antiderivative = 1.10, number of steps used = 50, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6742, 1226, 1202, 524, 424, 419, 1220, 537, 6725, 1208}

$$\frac{\sqrt{ax^4-cx^2-b}\,x}{4\sqrt{b}\left(\sqrt{b}-\sqrt{a}\,x^2\right)} + \frac{\sqrt{ax^4-cx^2-b}\,x}{4\sqrt{b}\left(\sqrt{a}\,x^2+\sqrt{b}\right)} + \frac{\left(c+2\sqrt{a}\sqrt{b}-\sqrt{c^2+4ab}\right)\sqrt{c+\sqrt{c^2+4ab}}\sqrt{1-\frac{2ax^2}{c-\sqrt{c^2+4ab}}}\sqrt{1-\frac{2ax^2}{c+\sqrt{c^2+4ab}}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x}{\sqrt{c+\sqrt{c^2+4ab}}}\right)|\frac{c+\sqrt{c^2+4ab}}{c-\sqrt{c^2+4ab}}\right)}{8\sqrt{2}\,a\sqrt{b}\sqrt{ax^4-cx^2-b}} + \frac{\left(-c+2\sqrt{a}\sqrt{b}+\sqrt{c^2+4ab}\right)\sqrt{c+\sqrt{c^2+4ab}}\sqrt{1-\frac{2ax^2}{c-\sqrt{c^2+4ab}}}\sqrt{1-\frac{2ax^2}{c+\sqrt{c^2+4ab}}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x}{\sqrt{c+\sqrt{c^2+4ab}}}\right)|\frac{c+\sqrt{c^2+4ab}}{c-\sqrt{c^2+4ab}}\right)}{8\sqrt{2}\,a\sqrt{b}\sqrt{ax^4-cx^2-b}} - \frac{\sqrt{c+\sqrt{c^2+4ab}}\sqrt{1-\frac{2ax^2}{c-\sqrt{c^2+4ab}}}\sqrt{1-\frac{2ax^2}{c+\sqrt{c^2+4ab}}}\,\Pi\left(-\frac{c+\sqrt{c^2+4ab}}{2\sqrt{a}\sqrt{b}};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x}{\sqrt{c+\sqrt{c^2+4ab}}}\right)|\frac{c+\sqrt{c^2+4ab}}{c-\sqrt{c^2+4ab}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{ax^4-cx^2-b}} - \frac{\sqrt{c+\sqrt{c^2+4ab}}\sqrt{1-\frac{2ax^2}{c-\sqrt{c^2+4ab}}}\sqrt{1-\frac{2ax^2}{c+\sqrt{c^2+4ab}}}\,\Pi\left(\frac{c+\sqrt{c^2+4ab}}{2\sqrt{a}\sqrt{b}};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x}{\sqrt{c+\sqrt{c^2+4ab}}}\right)|\frac{c+\sqrt{c^2+4ab}}{c-\sqrt{c^2+4ab}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{ax^4-cx^2-b}}$$

Antiderivative was successfully verified.

```
[In] Int[((b + a*x^4)*Sqrt[-b - c*x^2 + a*x^4])/(-b + a*x^4)^2,x]
```

```
[Out] (x*Sqrt[-b - c*x^2 + a*x^4])/(4*Sqrt[b]*(Sqrt[b] - Sqrt[a]*x^2)) + (x*Sqrt[
-b - c*x^2 + a*x^4])/(4*Sqrt[b]*(Sqrt[b] + Sqrt[a]*x^2)) + ((2*Sqrt[a]*Sqrt
[b] + c - Sqrt[4*a*b + c^2])*Sqrt[c + Sqrt[4*a*b + c^2]]*Sqrt[1 - (2*a*x^2)
/(c - Sqrt[4*a*b + c^2])]*Sqrt[1 - (2*a*x^2)/(c + Sqrt[4*a*b + c^2])]*Ellip
ticF[ArcSin[(Sqrt[2]*Sqrt[a]*x)/Sqrt[c + Sqrt[4*a*b + c^2]]], (c + Sqrt[4*a
*b + c^2])/(c - Sqrt[4*a*b + c^2])])/(8*Sqrt[2]*a*Sqrt[b]*Sqrt[-b - c*x^2 +
 a*x^4]) + ((2*Sqrt[a]*Sqrt[b] - c + Sqrt[4*a*b + c^2])*Sqrt[c + Sqrt[4*a*b
 + c^2]]*Sqrt[1 - (2*a*x^2)/(c - Sqrt[4*a*b + c^2])]*Sqrt[1 - (2*a*x^2)/(c
+ Sqrt[4*a*b + c^2])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[a]*x)/Sqrt[c + Sqrt[4*
a*b + c^2]]], (c + Sqrt[4*a*b + c^2])/(c - Sqrt[4*a*b + c^2])])/(8*Sqrt[2]*
a*Sqrt[b]*Sqrt[-b - c*x^2 + a*x^4]) - (Sqrt[c + Sqrt[4*a*b + c^2]]*Sqrt[1 -
 (2*a*x^2)/(c - Sqrt[4*a*b + c^2])]*Sqrt[1 - (2*a*x^2)/(c + Sqrt[4*a*b + c^
2])]*EllipticPi[-1/2*(c + Sqrt[4*a*b + c^2])/(Sqrt[a]*Sqrt[b]), ArcSin[(Sqr
t[2]*Sqrt[a]*x)/Sqrt[c + Sqrt[4*a*b + c^2]]], (c + Sqrt[4*a*b + c^2])/(c -
Sqrt[4*a*b + c^2])])/(2*Sqrt[2]*Sqrt[a]*Sqrt[-b - c*x^2 + a*x^4]) - (Sqrt[c
 + Sqrt[4*a*b + c^2]]*Sqrt[1 - (2*a*x^2)/(c - Sqrt[4*a*b + c^2])]*Sqrt[1 -
(2*a*x^2)/(c + Sqrt[4*a*b + c^2])]*EllipticPi[(c + Sqrt[4*a*b + c^2])/(2*Sq
rt[a]*Sqrt[b]), ArcSin[(Sqrt[2]*Sqrt[a]*x)/Sqrt[c + Sqrt[4*a*b + c^2]]], (c
 + Sqrt[4*a*b + c^2])/(c - Sqrt[4*a*b + c^2])])/(2*Sqrt[2]*Sqrt[a]*Sqrt[-b
- c*x^2 + a*x^4])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1202

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt
[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[(d + e*x^2)/(Sqrt[1 +
 (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1220

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*
Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/((d + e*x^2)*Sq
rt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1226

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[(x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(d + e*x^2)), x] + (Dist[c/(2
*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*
e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(b+ax^4)\sqrt{-b-cx^2+ax^4}}{(-b+ax^4)^2}\,dx = \int \left(\frac{2b\sqrt{-b-cx^2+ax^4}}{(-b+ax^4)^2} + \frac{\sqrt{-b-cx^2+ax^4}}{-b+ax^4}\right) dx$$

$$= (2b)\int \frac{\sqrt{-b-cx^2+ax^4}}{(-b+ax^4)^2}\,dx + \int \frac{\sqrt{-b-cx^2+ax^4}}{-b+ax^4}\,dx$$

$$= (2b)\int \left(\frac{a\sqrt{-b-cx^2+ax^4}}{4b\left(\sqrt{a}\sqrt{b}-ax^2\right)^2} + \frac{a\sqrt{-b-cx^2+ax^4}}{4b\left(\sqrt{a}\sqrt{b}+ax^2\right)^2} + \frac{a\sqrt{-b-cx^2+ax^4}}{2b\left(ab-a^2x^4\right)}\right) dx$$

$$= \frac{1}{2}a\int \frac{\sqrt{-b-cx^2+ax^4}}{\left(\sqrt{a}\sqrt{b}-ax^2\right)^2}\,dx + \frac{1}{2}a\int \frac{\sqrt{-b-cx^2+ax^4}}{\left(\sqrt{a}\sqrt{b}+ax^2\right)^2}\,dx + a\int \frac{\sqrt{-b-cx^2+}}{ab-a^2x^4}$$

$$= \frac{x\sqrt{-b-cx^2+ax^4}}{4\sqrt{b}\left(\sqrt{b}-\sqrt{a}x^2\right)} + \frac{x\sqrt{-b-cx^2+ax^4}}{4\sqrt{b}\left(\sqrt{b}+\sqrt{a}x^2\right)} + a\int \left(\frac{\sqrt{-b-cx^2+ax^4}}{2a\sqrt{b}\left(\sqrt{b}-\sqrt{a}x^2\right)} + \frac{}{2a}\right.$$

$$= \frac{x\sqrt{-b-cx^2+ax^4}}{4\sqrt{b}\left(\sqrt{b}-\sqrt{a}x^2\right)} + \frac{x\sqrt{-b-cx^2+ax^4}}{4\sqrt{b}\left(\sqrt{b}+\sqrt{a}x^2\right)} + \frac{\int \frac{\sqrt{-b-cx^2+ax^4}}{\sqrt{b}-\sqrt{a}x^2}\,dx}{2\sqrt{b}} + \frac{\int \frac{\sqrt{-b-cx^2+a}}{\sqrt{b}+\sqrt{a}x^2}}{2\sqrt{b}}$$

$$= \frac{x\sqrt{-b-cx^2+ax^4}}{4\sqrt{b}\left(\sqrt{b}-\sqrt{a}x^2\right)} + \frac{x\sqrt{-b-cx^2+ax^4}}{4\sqrt{b}\left(\sqrt{b}+\sqrt{a}x^2\right)} - \frac{\sqrt{c+\sqrt{4ab+c^2}}\sqrt{1-\frac{2ax^2}{c-\sqrt{4ab+c^2}}}}{}$$

$$= \frac{x\sqrt{-b-cx^2+ax^4}}{4\sqrt{b}\left(\sqrt{b}-\sqrt{a}x^2\right)} + \frac{x\sqrt{-b-cx^2+ax^4}}{4\sqrt{b}\left(\sqrt{b}+\sqrt{a}x^2\right)} + \frac{\left(2\sqrt{a}\sqrt{b}-c-\sqrt{4ab+c^2}\right)\sqrt{c}}{}$$

$$= \frac{x\sqrt{-b-cx^2+ax^4}}{4\sqrt{b}\left(\sqrt{b}-\sqrt{a}x^2\right)} + \frac{x\sqrt{-b-cx^2+ax^4}}{4\sqrt{b}\left(\sqrt{b}+\sqrt{a}x^2\right)} + \frac{\left(2\sqrt{a}\sqrt{b}-c-\sqrt{4ab+c^2}\right)\sqrt{c}}{}$$

$$= \frac{x\sqrt{-b-cx^2+ax^4}}{4\sqrt{b}\left(\sqrt{b}-\sqrt{a}x^2\right)} + \frac{x\sqrt{-b-cx^2+ax^4}}{4\sqrt{b}\left(\sqrt{b}+\sqrt{a}x^2\right)} + \frac{\left(2\sqrt{a}\sqrt{b}+c-\sqrt{4ab+c^2}\right)\sqrt{c}}{}$$

Mathematica [C] time = 2.96, size = 416, normalized size = 0.50

$$\frac{1}{2}\sqrt{ax^4-b-cx^2}\left(\frac{x}{b-ax^4} - \frac{i\sqrt{\frac{4ax^2}{\sqrt{4ab+c^2}-c}+2}\sqrt{1-\frac{2ax^2}{\sqrt{4ab+c^2}+c}}\left(F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{a}{\sqrt{c^2+4ab}-c}}x\right)\middle|\frac{c-\sqrt{c^2+4ab}}{c+\sqrt{c^2+4ab}}\right) - \Pi\left(\frac{c-\sqrt{c^2+4ab}}{2\sqrt{a}\sqrt{b}};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{a}{\sqrt{c^2+4ab}-c}}x\right)\middle|\frac{c-\sqrt{c^2+4ab}}{c+\sqrt{c^2+4ab}}\right) - \Pi\left(\frac{\sqrt{c^2+4ab}-c}{2\sqrt{a}\sqrt{b}};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{a}{\sqrt{c^2+4ab}-c}}x\right)\middle|\frac{c-\sqrt{c^2+4ab}}{c+\sqrt{c^2+4ab}}\right)\right)}{2\sqrt{\frac{a}{\sqrt{4ab+c^2}-c}}\left(ax^4-b-cx^2\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((b + a*x^4)*Sqrt[-b - c*x^2 + a*x^4])/(-b + a*x^4)^2,x]
```

```
[Out] (Sqrt[-b - c*x^2 + a*x^4]*(x/(b - a*x^4) - ((I/2)*Sqrt[2 + (4*a*x^2)/(-c +
Sqrt[4*a*b + c^2])]*Sqrt[1 - (2*a*x^2)/(c + Sqrt[4*a*b + c^2])]*(EllipticF[
I*ArcSinh[Sqrt[2]*Sqrt[a/(-c + Sqrt[4*a*b + c^2])]*x], (c - Sqrt[4*a*b + c^
2])/(c + Sqrt[4*a*b + c^2])] - EllipticPi[(c - Sqrt[4*a*b + c^2])/(2*Sqrt[a
]*Sqrt[b]), I*ArcSinh[Sqrt[2]*Sqrt[a/(-c + Sqrt[4*a*b + c^2])]*x], (c - Sqr
t[4*a*b + c^2])/(c + Sqrt[4*a*b + c^2])] - EllipticPi[(-c + Sqrt[4*a*b + c^
2])/(2*Sqrt[a]*Sqrt[b]), I*ArcSinh[Sqrt[2]*Sqrt[a/(-c + Sqrt[4*a*b + c^2])]
```

```
*x], (c - Sqrt[4*a*b + c^2])/(c + Sqrt[4*a*b + c^2])]))/(Sqrt[a/(-c + Sqrt[
4*a*b + c^2])]*(-b - c*x^2 + a*x^4))))/2
```

IntegrateAlgebraic [A] time = 0.88, size = 87, normalized size = 0.11

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x\sqrt{ax^4-b-cx^2}}{-ax^4+b+cx^2}\right)}{2\sqrt{c}}-\frac{x\sqrt{ax^4-b-cx^2}}{2\left(ax^4-b\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b + a*x^4)*Sqrt[-b - c*x^2 + a*x^4])/(-b + a*x^4)^2,x]
```

```
[Out] -1/2*(x*Sqrt[-b - c*x^2 + a*x^4])/(-b + a*x^4) + ArcTan[(Sqrt[c]*x*Sqrt[-b
- c*x^2 + a*x^4])/(b + c*x^2 - a*x^4)]/(2*Sqrt[c])
```

fricas [A] time = 0.52, size = 246, normalized size = 0.30

$$\left[-\frac{4\sqrt{ax^4-cx^2-b}\,cx+\left(ax^4-b\right)\sqrt{-c}\log\left(-\frac{a^2x^8-8\,acx^6-2\left(ab-4\,c^2\right)x^4+8\,bcx^2+b^2-4\left(ax^5-2\,cx^3-bx\right)\sqrt{ax^4-cx^2-b}\,\sqrt{-c}}{a^2x^8-2\,abx^4+b^2}\right)}{8\left(acx^4-bc\right)},\ -\frac{2\sqrt{ax^4-cx^2-b}\,cx+\left(ax^4-b\right)\sqrt{c}\arctan\left(\frac{2\sqrt{ax^4-cx^2-b}\,\sqrt{c}\,x}{ax^4-2\,cx^2-b}\right)}{4\left(acx^4-bc\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)*(a*x^4-c*x^2-b)^(1/2)/(a*x^4-b)^2,x, algorithm="fricas"
)
```

```
[Out] [-1/8*(4*sqrt(a*x^4 - c*x^2 - b)*c*x + (a*x^4 - b)*sqrt(-c)*log(-(a^2*x^8 -
 8*a*c*x^6 - 2*(a*b - 4*c^2)*x^4 + 8*b*c*x^2 + b^2 - 4*(a*x^5 - 2*c*x^3 - b
*x)*sqrt(a*x^4 - c*x^2 - b)*sqrt(-c))/(a^2*x^8 - 2*a*b*x^4 + b^2)))/(a*c*x^
4 - b*c), -1/4*(2*sqrt(a*x^4 - c*x^2 - b)*c*x + (a*x^4 - b)*sqrt(c)*arctan(
2*sqrt(a*x^4 - c*x^2 - b)*sqrt(c)*x/(a*x^4 - 2*c*x^2 - b)))/(a*c*x^4 - b*c)
]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ax^4-cx^2-b}\left(ax^4+b\right)}{\left(ax^4-b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)*(a*x^4-c*x^2-b)^(1/2)/(a*x^4-b)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^4 - c*x^2 - b)*(a*x^4 + b)/(a*x^4 - b)^2, x)
```

maple [C] time = 0.08, size = 899, normalized size = 1.09

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4+b)*(a*x^4-c*x^2-b)^(1/2)/(a*x^4-b)^2,x)
```

```
[Out] 2*b*(-1/4/b*x*(a*x^4-c*x^2-b)^(1/2)/(a*x^4-b)-1/8/b/(-2*(c+(4*a*b+c^2)^(1/2
))/b)^(1/2)*(4+2*(c+(4*a*b+c^2)^(1/2))/b*x^2)^(1/2)*(4-2*(-c+(4*a*b+c^2)^(1
/2))/b*x^2)^(1/2)/(a*x^4-c*x^2-b)^(1/2)*EllipticF(1/2*x*(-2*(c+(4*a*b+c^2)^
(1/2))/b)^(1/2),1/2*(-4+2*c*(-c+(4*a*b+c^2)^(1/2))/b/a)^(1/2))-1/16/a/b*sum
((-_alpha^2*c-b)/_alpha^3*(-1/(-c*_alpha^2)^(1/2)*arctanh(1/2*(2*_alpha^2*a
*x^2-_alpha^2*c-c*x^2-2*b)/(-c*_alpha^2)^(1/2)/(a*x^4-c*x^2-b)^(1/2))-1/b*a
*2^(1/2)*_alpha^3/(-(c+(4*a*b+c^2)^(1/2))/b)^(1/2)*(2+1/b*c*x^2+1/b*x^2*(4*
```

```
a*b+c^2)^(1/2))^(1/2)*(2+1/b*c*x^2-1/b*x^2*(4*a*b+c^2)^(1/2))^(1/2)/(a*x^4-
c*x^2-b)^(1/2)*EllipticPi((-1/2*(c+(4*a*b+c^2)^(1/2))/b)^(1/2)*x,1/2*_alpha
^2*(c-(4*a*b+c^2)^(1/2))/b,1/2*2^(1/2)*((-c+(4*a*b+c^2)^(1/2))/b)^(1/2)/(-1
/2*(c+(4*a*b+c^2)^(1/2))/b)^(1/2))),_alpha=RootOf(_Z^4*a-b)))+1/2/(-2*(c+(4
*a*b+c^2)^(1/2))/b)^(1/2)*(4+2*(c+(4*a*b+c^2)^(1/2))/b*x^2)^(1/2)*(4-2*(-c+
(4*a*b+c^2)^(1/2))/b*x^2)^(1/2)/(a*x^4-c*x^2-b)^(1/2)*EllipticF(1/2*x*(-2*(
c+(4*a*b+c^2)^(1/2))/b)^(1/2),1/2*(-4+2*c*(-c+(4*a*b+c^2)^(1/2))/b/a)^(1/2)
)-1/8*c/a*sum(1/_alpha*(-1/(-c*_alpha^2)^(1/2)*arctanh(1/2*(2*_alpha^2*a*x^
2-_alpha^2*c-c*x^2-2*b)/(-c*_alpha^2)^(1/2)/(a*x^4-c*x^2-b)^(1/2))-1/b*a*2^
(1/2)*_alpha^3/(-(c+(4*a*b+c^2)^(1/2))/b)^(1/2)*(2+1/b*c*x^2+1/b*x^2*(4*a*b
+c^2)^(1/2))^(1/2)*(2+1/b*c*x^2-1/b*x^2*(4*a*b+c^2)^(1/2))^(1/2)/(a*x^4-c*x
^2-b)^(1/2)*EllipticPi((-1/2*(c+(4*a*b+c^2)^(1/2))/b)^(1/2)*x,1/2*_alpha^2*
(c-(4*a*b+c^2)^(1/2))/b,1/2*2^(1/2)*((-c+(4*a*b+c^2)^(1/2))/b)^(1/2)/(-1/2*
(c+(4*a*b+c^2)^(1/2))/b)^(1/2))),_alpha=RootOf(_Z^4*a-b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^4 - cx^2 - b}\left(ax^4 + b\right)}{\left(ax^4 - b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4+b)*(a*x^4-c*x^2-b)^(1/2)/(a*x^4-b)^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*x^4 - c*x^2 - b)*(a*x^4 + b)/(a*x^4 - b)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x^4 + b\right)\,\sqrt{a\,x^4 - c\,x^2 - b}}{\left(b - a\,x^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a*x^4)*(a*x^4 - b - c*x^2)^(1/2))/(b - a*x^4)^2,x)
```

```
[Out] int(((b + a*x^4)*(a*x^4 - b - c*x^2)^(1/2))/(b - a*x^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4+b)*(a*x**4-c*x**2-b)**(1/2)/(a*x**4-b)**2,x)
```

```
[Out] Timed out
```

3.2429 $$\int \frac{-a-bc+(1+c)x}{\left((-a+x)(-b+x)^2\right)^{2/3}(a-bd+(-1+d)x)}\,dx$$

Optimal. Leaf size=849

$$\frac{(b-x)^{4/3}(x-a)^{2/3}\left(\frac{\sqrt{3}(d-1)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x-a}}{\sqrt[3]{x-a}-2\sqrt[3]{d}\sqrt[3]{b-x}}\right)a}{(a-b)^2d^{2/3}}-\frac{(d-1)\log\left(\sqrt[3]{d}\sqrt[3]{b-x}+\sqrt[3]{x-a}\right)a}{(a-b)^2d^{2/3}}+\frac{(d-1)\log\left(d^{2/3}(b-x)^{2/3}-\sqrt[3]{d}\sqrt[3]{x-a}\sqrt[3]{b-x}+(x-\right.}{2(a-b)^2d^{2/3}}\right.}{}$$

Rubi [A] time = 0.88, antiderivative size = 286, normalized size of antiderivative = 0.34, number of steps used = 4, number of rules used = 4, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6719, 155, 12, 91}

$$\frac{(x-a)^{2/3}(x-b)^{4/3}(c+d)\log(a-bd-(1-d)x)}{2d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}-\frac{3(x-a)^{2/3}(x-b)^{4/3}(c+d)\log\left(\sqrt[3]{d}\sqrt[3]{x-b}-\sqrt[3]{x-a}\right)}{2d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}-\frac{\sqrt{3}(x-a)^{2/3}(x-b)^{4/3}(c+d)\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{x-b}}{\sqrt{3}\sqrt[3]{x-a}}+\frac{1}{\sqrt{3}}\right)}{d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}-\frac{3(a-x)(b-x)}{(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a - b*c + (1 + c)*x)/(((-a + x)*(-b + x)^2)^(2/3)*(a - b*d + (-1 + d)
*x)),x]
```

```
[Out] (-3*(a - x)*(b - x))/((a - b)*(-((a - x)*(b - x)^2))^(2/3)) - (Sqrt[3]*(c +
 d)*(-a + x)^(2/3)*(-b + x)^(4/3)*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(-b + x)^(1
/3))/(Sqrt[3]*(-a + x)^(1/3))])/((a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(2/
3)) + ((c + d)*(-a + x)^(2/3)*(-b + x)^(4/3)*Log[a - b*d - (1 - d)*x])/(2*(
a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(2/3)) - (3*(c + d)*(-a + x)^(2/3)*(-
b + x)^(4/3)*Log[-(-a + x)^(1/3) + d^(1/3)*(-b + x)^(1/3)])/(2*(a - b)*d^(2
/3)*(-((a - x)*(b - x)^2))^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
```

```
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{-a-bc+(1+c)x}{\left((-a+x)(-b+x)^2\right)^{2/3}(a-bd+(-1+d)x)}\,dx = \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{-a-bc+(1+c)x}{(-a+x)^{2/3}(-b+x)^{4/3}(a-bd+(-1+d)x)}\,dx}{\left((-a+x)(-b+x)^2\right)^{2/3}}$$

$$= -\frac{3(a-x)(b-x)}{(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}} + \frac{\left(3(-a+x)^{2/3}(-b+x)^{4/3}\right)\ldots}{(a-b)^2\left((-a \ldots\right.}$$

$$= -\frac{3(a-x)(b-x)}{(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}} + \frac{\left((c+d)(-a+x)^{2/3}(-b+x \ldots\right.}{\left((-a+\ldots\right.}$$

$$= -\frac{3(a-x)(b-x)}{(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}} - \frac{\sqrt{3}\,(c+d)(-a+x)^{2/3}(-b+\ldots}{(a-b)d^{2/3}\left(-\left(\ldots\right.\right.}$$

Mathematica [C] time = 0.07, size = 72, normalized size = 0.08

$$\frac{3(x-b)\left((x-b)(c+d)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d(b-x)}{a-x}\right)+2(a-x)\right)}{2(a-b)\left((x-a)(b-x)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a - b*c + (1 + c)*x)/(((-a + x)*(-b + x)^2)^(2/3)*(a - b*d + (-
1 + d)*x)),x]
```

```
[Out] (3*(-b + x)*(2*(a - x) + (c + d)*(-b + x)*Hypergeometric2F1[2/3, 1, 5/3, (d
*(b - x))/(a - x)]))/(2*(a - b)*((b - x)^2*(-a + x))^(2/3))
```

IntegrateAlgebraic [F] time = 180.04, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-a - b*c + (1 + c)*x)/(((-a + x)*(-b + x)^2)^(2/3)*(a -
 b*d + (-1 + d)*x)),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.42, size = 389, normalized size = 0.46

$$-\frac{2\sqrt{3}\left(bcd+bd^2-\left(cd+d^2\right)x\right)\left(d^2\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(d^2\right)^{\frac{1}{6}}\left((bd-dx)\left(d^2\right)^{\frac{1}{3}}-2\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}\left(d^2\right)^{\frac{2}{3}}\right)}{3\left(bd^2-d^2x\right)}\right)+(bc+bd-(c+d)x)\left(d^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}\left(d^2\right)^{\frac{2}{3}}(b-x)-\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{2}{3}}d-\left(b^2d-2bdx+dx^2\right)\left(d^2\right)^{\frac{1}{3}}}{b^2-2bx+x^2}\right)-2(bc+bd-(c+d)x)\left(d^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(d^2\right)^{\frac{2}{3}}(b-x)+\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}d}{b-x}\right)+6\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}d^2}{2\left((a-b)d^2x-\left(ab-b^2\right)d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a-b*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(2/3)/(a-b*d+(-1+d)*x),x, algo
rithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(b*c*d + b*d^2 - (c*d + d^2)*x)*(d^2)^(1/6)*arctan(1/3*sqrt
(3)*(d^2)^(1/6)*((b*d - d*x)*(d^2)^(1/3) - 2*(-a*b^2 - (a + 2*b)*x^2 + x^3
+ (2*a*b + b^2)*x)^(1/3)*(d^2)^(2/3))/(b*d^2 - d^2*x)) + (b*c + b*d - (c +
```

```
d)*x)*(d^2)^(2/3)*log(-((-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1
/3)*(d^2)^(2/3)*(b - x) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^
(2/3)*d - (b^2*d - 2*b*d*x + d*x^2)*(d^2)^(1/3))/(b^2 - 2*b*x + x^2)) - 2*(
b*c + b*d - (c + d)*x)*(d^2)^(2/3)*log(-((d^2)^(2/3)*(b - x) + (-a*b^2 - (a
 + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*d)/(b - x)) + 6*(-a*b^2 - (a + 2
*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*d^2)/((a - b)*d^2*x - (a*b - b^2)*d^
2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bc - (c + 1)x + a}{\left(-(a - x)(b - x)^2\right)^{\frac{2}{3}} (bd - (d - 1)x - a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a-b*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(2/3)/(a-b*d+(-1+d)*x),x, algo
rithm="giac")
```

```
[Out] integrate((b*c - (c + 1)*x + a)/((-(a - x)*(b - x)^2)^(2/3)*(b*d - (d - 1)*
x - a)), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{-a - bc + (1 + c)\, x}{\left((-a + x)\,(-b + x)^2\right)^{\frac{2}{3}} (a - bd + (-1 + d)\, x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a-b*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(2/3)/(a-b*d+(-1+d)*x),x)
```

```
[Out] int((-a-b*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(2/3)/(a-b*d+(-1+d)*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bc - (c + 1)x + a}{\left(-(a - x)(b - x)^2\right)^{\frac{2}{3}} (bd - (d - 1)x - a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a-b*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(2/3)/(a-b*d+(-1+d)*x),x, algo
rithm="maxima")
```

```
[Out] integrate((b*c - (c + 1)*x + a)/((-(a - x)*(b - x)^2)^(2/3)*(b*d - (d - 1)*
x - a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{a + b\,c - x\,(c + 1)}{{\left(-(a - x)\,{(b - x)}^2\right)}^{2/3}\,(a - b\,d + x\,(d - 1))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*c - x*(c + 1))/((-(a - x)*(b - x)^2)^(2/3)*(a - b*d + x*(d - 1)
)),x)
```

```
[Out] int(-(a + b*c - x*(c + 1))/((-(a - x)*(b - x)^2)^(2/3)*(a - b*d + x*(d - 1)
)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-a-bc+cx+x}{\left((-a+x)\,(-b+x)^2\right)^{\frac{2}{3}}\,(a-bd+dx-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a-b*c+(1+c)*x)/((-a+x)*(-b+x)**2)**(2/3)/(a-b*d+(-1+d)*x),x)
```

```
[Out] Integral((-a - b*c + c*x + x)/(((-a + x)*(-b + x)**2)**(2/3)*(a - b*d + d*x
 - x)), x)
```

3.2430 $\int \frac{-a-bc+(1+c)x}{(-b+x)\sqrt[3]{(-a+x)(-b+x)^2}\,(a-bd+(-1+d)x)}\,dx$

Optimal. Leaf size=857

$$\frac{(b-x)^{2/3}\sqrt[3]{x-a}\left(-\frac{\sqrt{3}(d-1)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x-a}}{\sqrt[3]{x-a}-2\sqrt[3]{d}\sqrt[3]{b-x}}\right)a}{(a-b)^2\sqrt[3]{d}}-\frac{(d-1)\log\left(\sqrt[3]{d}\sqrt[3]{b-x}+\sqrt[3]{x-a}\right)a}{(a-b)^2\sqrt[3]{d}}+\frac{(d-1)\log\left(d^{2/3}(b-x)^{2/3}-\sqrt[3]{d}\sqrt[3]{x-a}\sqrt[3]{b-x}+(x-\right.}{2(a-b)^2\sqrt[3]{d}}\right.}{}$$

Rubi [A] time = 1.44, antiderivative size = 283, normalized size of antiderivative = 0.33, number of steps used = 4, number of rules used = 4, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.078, Rules used = {6719, 155, 12, 91}

$$\frac{\sqrt[3]{x-a}(x-b)^{2/3}(c+d)\log(a-bd-(1-d)x)}{2\sqrt[3]{d}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{3\sqrt[3]{x-a}(x-b)^{2/3}(c+d)\log\left(\frac{\sqrt[3]{x-a}}{\sqrt[3]{d}}-\sqrt[3]{x-b}\right)}{2\sqrt[3]{d}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{\sqrt{3}\sqrt[3]{x-a}(x-b)^{2/3}(c+d)\tan^{-1}\left(\frac{2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{x-b}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{d}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}+\frac{3(a-x)}{2(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a - b*c + (1 + c)*x)/((-b + x)*((-a + x)*(-b + x)^2)^(1/3)*(a - b*d +
 (-1 + d)*x)),x]
```

```
[Out] (3*(a - x))/(2*(a - b)*(-((a - x)*(b - x)^2))^(1/3)) - (Sqrt[3]*(c + d)*(-a
 + x)^(1/3)*(-b + x)^(2/3)*ArcTan[1/Sqrt[3] + (2*(-a + x)^(1/3))/(Sqrt[3]*d
^(1/3)*(-b + x)^(1/3))])/((a - b)*d^(1/3)*(-((a - x)*(b - x)^2))^(1/3)) + (
(c + d)*(-a + x)^(1/3)*(-b + x)^(2/3)*Log[a - b*d - (1 - d)*x])/(2*(a - b)*
d^(1/3)*(-((a - x)*(b - x)^2))^(1/3)) - (3*(c + d)*(-a + x)^(1/3)*(-b + x)^
(2/3)*Log[(-a + x)^(1/3)/d^(1/3) - (-b + x)^(1/3)])/(2*(a - b)*d^(1/3)*(-((
a - x)*(b - x)^2))^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
```

```
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{-a-bc+(1+c)x}{(-b+x)\sqrt[3]{(-a+x)(-b+x)^2}\,(a-bd+(-1+d)x)}\,dx = \frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{-a-bc+(1+c)x}{\sqrt[3]{-a+x}\,(-b+x)^{5/3}(a-bd+(-1+d)x)}\,dx}{\sqrt[3]{(-a+x)(-b+x)^2}}$$

$$= \frac{3(a-x)}{2(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}} + \frac{\left(3\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right.}{2(a-b)^2}$$

$$= \frac{3(a-x)}{2(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}} + \frac{\left((c+d)\sqrt[3]{-a+x}\,(-b+\right.}{\sqrt[3]{(}}$$

$$= \frac{3(a-x)}{2(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}} - \frac{\sqrt{3}\,(c+d)\sqrt[3]{-a+x}\,(-b}{(a-b)\sqrt[3]{d}}$$

Mathematica [C] time = 0.05, size = 65, normalized size = 0.08

$$\frac{3\left(2(x-b)(c+d)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{d(b-x)}{a-x}\right)+a-x\right)}{2(a-b)\sqrt[3]{(x-a)(b-x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a - b*c + (1 + c)*x)/((-b + x)*((-a + x)*(-b + x)^2)^(1/3)*(a -
 b*d + (-1 + d)*x)),x]
```

```
[Out] (3*(a - x + 2*(c + d)*(-b + x)*Hypergeometric2F1[1/3, 1, 4/3, (d*(b - x))/(
a - x)]))/(2*(a - b)*((b - x)^2*(-a + x))^(1/3))
```

IntegrateAlgebraic [A] time = 11.63, size = 242, normalized size = 0.28

$$\frac{\sqrt[3]{x-a}\,(b-x)^{2/3}\left(\frac{(c+d)\log\left(\frac{d^{2/3}(b-x)^{2/3}}{(x-a)^{2/3}}-\frac{\sqrt[3]{d}\,\sqrt[3]{b-x}}{\sqrt[3]{x-a}}+1\right)}{2\sqrt[3]{d}\,(a-b)}+\frac{(-c-d)\log\left(\frac{\sqrt[3]{d}\,\sqrt[3]{b-x}}{\sqrt[3]{x-a}}+1\right)}{\sqrt[3]{d}\,(a-b)}+\frac{\sqrt{3}\,(c+d)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\,\sqrt[3]{b-x}}{\sqrt{3}\,\sqrt[3]{x-a}}\right)}{\sqrt[3]{d}\,(a-b)}-\frac{3(x-a)^{2/3}}{2(a-b)(b-x)^{2/3}}\right)}{\sqrt[3]{(x-a)(b-x)^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a - b*c + (1 + c)*x)/((-b + x)*((-a + x)*(-b + x)^2)^(
1/3)*(a - b*d + (-1 + d)*x)),x]
```

```
[Out] ((b - x)^(2/3)*(-a + x)^(1/3)*((-3*(-a + x)^(2/3))/(2*(a - b)*(b - x)^(2/3)
) + (Sqrt[3]*(c + d)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(b - x)^(1/3))/(Sqrt[3]*
(-a + x)^(1/3))])/((a - b)*d^(1/3)) + ((c + d)*Log[1 + (d^(2/3)*(b - x)^(2/
3))/(-a + x)^(2/3) - (d^(1/3)*(b - x)^(1/3))/(-a + x)^(1/3)])/(2*(a - b)*d^
(1/3)) + ((-c - d)*Log[1 + (d^(1/3)*(b - x)^(1/3))/(-a + x)^(1/3)])/((a - b
)*d^(1/3))))/((b - x)^2*(-a + x))^(1/3)
```

fricas [A] time = 0.46, size = 976, normalized size = 1.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a-b*c+(1+c)*x)/(-b+x)/((-a+x)*(-b+x)^2)^(1/3)/(a-b*d+(-1+d)*x),
x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(3)*(b^2*c*d + b^2*d^2 + (c*d + d^2)*x^2 - 2*(b*c*d + b*d^2)*x)*
sqrt(-1/d^(2/3))*log(-(b^2*d + (d + 2)*x^2 + 2*a*b + 3*(-a*b^2 - (a + 2*b)*
x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(b - x)*d^(2/3) - 2*(b*d + a + b)*x + sq
rt(3)*((-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(b*d - d*x) -
 (b^2*d - 2*b*d*x + d*x^2)*d^(1/3) + 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a
*b + b^2)*x)^(2/3)*d^(2/3))*sqrt(-1/d^(2/3))))/(b^2*d + (d - 1)*x^2 - a*b -
(2*b*d - a - b)*x)) - (b^2*c + b^2*d + (c + d)*x^2 - 2*(b*c + b*d)*x)*d^(2/
3)*log(-((-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(b - x)*d^(
1/3) - (b^2 - 2*b*x + x^2)*d^(2/3) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b
 + b^2)*x)^(2/3))/(b^2 - 2*b*x + x^2)) + 2*(b^2*c + b^2*d + (c + d)*x^2 - 2
*(b*c + b*d)*x)*d^(2/3)*log(-((b - x)*d^(1/3) + (-a*b^2 - (a + 2*b)*x^2 + x
^3 + (2*a*b + b^2)*x)^(1/3))/(b - x)) + 3*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (
2*a*b + b^2)*x)^(2/3)*d)/((a - b)*d*x^2 - 2*(a*b - b^2)*d*x + (a*b^2 - b^3)
*d), 1/2*((b^2*c + b^2*d + (c + d)*x^2 - 2*(b*c + b*d)*x)*d^(2/3)*log(-((-a
*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(b - x)*d^(1/3) - (b^2
- 2*b*x + x^2)*d^(2/3) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(
2/3))/(b^2 - 2*b*x + x^2)) - 2*(b^2*c + b^2*d + (c + d)*x^2 - 2*(b*c + b*d)
*x)*d^(2/3)*log(-((b - x)*d^(1/3) + (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b
+ b^2)*x)^(1/3))/(b - x)) - 2*sqrt(3)*(b^2*c*d + b^2*d^2 + (c*d + d^2)*x^2
- 2*(b*c*d + b*d^2)*x)*arctan(1/3*sqrt(3)*((b - x)*d^(1/3) - 2*(-a*b^2 - (a
 + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3))/((b - x)*d^(1/3)))/d^(1/3) - 3*
(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*d)/((a - b)*d*x^2 -
2*(a*b - b^2)*d*x + (a*b^2 - b^3)*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bc - (c+1)x + a}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}} (bd - (d-1)x - a)(b-x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a-b*c+(1+c)*x)/(-b+x)/((-a+x)*(-b+x)^2)^(1/3)/(a-b*d+(-1+d)*x),
x, algorithm="giac")
```

```
[Out] integrate(-(b*c - (c + 1)*x + a)/((-(a - x)*(b - x)^2)^(1/3)*(b*d - (d - 1)
*x - a)*(b - x)), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{-a - bc + (1+c)\,x}{(-b+x)\left((-a+x)\,(-b+x)^2\right)^{\frac{1}{3}} (a - bd + (-1+d)\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a-b*c+(1+c)*x)/(-b+x)/((-a+x)*(-b+x)^2)^(1/3)/(a-b*d+(-1+d)*x),x)
```

```
[Out] int((-a-b*c+(1+c)*x)/(-b+x)/((-a+x)*(-b+x)^2)^(1/3)/(a-b*d+(-1+d)*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{bc - (c+1)x + a}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}} (bd - (d-1)x - a)(b-x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a-b*c+(1+c)*x)/(-b+x)/((-a+x)*(-b+x)^2)^(1/3)/(a-b*d+(-1+d)*x),
x, algorithm="maxima")
```

```
[Out] -integrate((b*c - (c + 1)*x + a)/((-(a - x)*(b - x)^2)^(1/3)*(b*d - (d - 1)
*x - a)*(b - x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int -\frac{a+b\,c-x\,\left(c+1\right)}{\left(b-x\right)\,{\left(-\left(a-x\right)\,{\left(b-x\right)}^2\right)}^{1/3}\,\left(a-b\,d+x\,\left(d-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*c - x*(c + 1))/((b - x)*(-(a - x)*(b - x)^2)^(1/3)*(a - b*d + x*
(d - 1))),x)
```

```
[Out] -int(-(a + b*c - x*(c + 1))/((b - x)*(-(a - x)*(b - x)^2)^(1/3)*(a - b*d +
x*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a-b*c+(1+c)*x)/(-b+x)/((-a+x)*(-b+x)**2)**(1/3)/(a-b*d+(-1+d)*x
),x)
```

```
[Out] Timed out
```

3.2431 $\int \frac{(d+cx^2)\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{x\left(-b+a^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=876

$$\frac{\sqrt[4]{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}\left(-51dx^2a^4+45bcx^2a^2-bda^2-97b^2c\right)}{96a^2b^{15/8}\left(ax-\sqrt{b}\right)\left(ax+\sqrt{b}\right)}+\frac{5\left(29a^2d-3bc\right)\tan^{-1}\left(\sqrt[4]{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}\right)}{64a^2b^{15/8}}-\frac{\sqrt{2-\sqrt{2}}\,d\tan}{}$$

Rubi [A] time = 3.19, antiderivative size = 1407, normalized size of antiderivative = 1.61, number of steps used = 52, number of rules used = 20, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.408, Rules used = {6742, 2120, 466, 470, 578, 527, 522, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204, 457, 288, 329}

Warning: Unable to verify antiderivative.

```
[In] Int[((d + c*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(5/4))/(x*(-b + a^2*x^2)^(5/2))
,x]
```

```
[Out] (8*d*(a*x + Sqrt[-b + a^2*x^2])^(9/4))/(3*(b - (a*x + Sqrt[-b + a^2*x^2])^2
)^3) + (8*c*(a*x + Sqrt[-b + a^2*x^2])^(17/4))/(3*a^2*(b - (a*x + Sqrt[-b +
 a^2*x^2])^2)^3) - (7*d*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(2*(b - (a*x + Sq
rt[-b + a^2*x^2])^2)^2) - (5*c*(a*x + Sqrt[-b + a^2*x^2])^(9/4))/(6*a^2*(b
- (a*x + Sqrt[-b + a^2*x^2])^2)^2) + (15*c*(a*x + Sqrt[-b + a^2*x^2])^(1/4)
)/(16*a^2*(b - (a*x + Sqrt[-b + a^2*x^2])^2)) + (39*d*(a*x + Sqrt[-b + a^2*
x^2])^(1/4))/(16*b*(b - (a*x + Sqrt[-b + a^2*x^2])^2)) - (2*d*ArcTan[(a*x +
 Sqrt[-b + a^2*x^2])^(1/4)/(-b)^(1/8)])/(-b)^(15/8) - (15*c*ArcTan[(a*x + S
qrt[-b + a^2*x^2])^(1/4)/b^(1/8)])/(64*a^2*b^(7/8)) + (145*d*ArcTan[(a*x +
Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)])/(64*b^(15/8)) + (Sqrt[2]*d*ArcTan[1 - (
Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-b)^(1/8)])/(-b)^(15/8) - (Sqrt[
2]*d*ArcTan[1 + (Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/(-b)^(1/8)])/(-b
)^(15/8) + (15*c*ArcTan[1 - (Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/b^(1
/8)])/(64*Sqrt[2]*a^2*b^(7/8)) - (145*d*ArcTan[1 - (Sqrt[2]*(a*x + Sqrt[-b
+ a^2*x^2])^(1/4))/b^(1/8)])/(64*Sqrt[2]*b^(15/8)) - (15*c*ArcTan[1 + (Sqrt
[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/b^(1/8)])/(64*Sqrt[2]*a^2*b^(7/8)) +
(145*d*ArcTan[1 + (Sqrt[2]*(a*x + Sqrt[-b + a^2*x^2])^(1/4))/b^(1/8)])/(64*
Sqrt[2]*b^(15/8)) - (2*d*ArcTanh[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/(-b)^(1/8
)])/(-b)^(15/8) - (15*c*ArcTanh[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)])/
(64*a^2*b^(7/8)) + (145*d*ArcTanh[(a*x + Sqrt[-b + a^2*x^2])^(1/4)/b^(1/8)]
)/(64*b^(15/8)) + (d*Log[(-b)^(1/4) - Sqrt[2]*(-b)^(1/8)*(a*x + Sqrt[-b + a
^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]])/(Sqrt[2]*(-b)^(15/8)) - (
d*Log[(-b)^(1/4) + Sqrt[2]*(-b)^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sq
rt[a*x + Sqrt[-b + a^2*x^2]]])/(Sqrt[2]*(-b)^(15/8)) + (15*c*Log[b^(1/4) -
Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a^2
*x^2]]])/(128*Sqrt[2]*a^2*b^(7/8)) - (145*d*Log[b^(1/4) - Sqrt[2]*b^(1/8)*(
a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]])/(128*Sqr
t[2]*b^(15/8)) - (15*c*Log[b^(1/4) + Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x
^2])^(1/4) + Sqrt[a*x + Sqrt[-b + a^2*x^2]]])/(128*Sqrt[2]*a^2*b^(7/8)) + (
145*d*Log[b^(1/4) + Sqrt[2]*b^(1/8)*(a*x + Sqrt[-b + a^2*x^2])^(1/4) + Sqrt
[a*x + Sqrt[-b + a^2*x^2]]])/(128*Sqrt[2]*b^(15/8))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
```

```
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(d+cx^2\right)\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{x\left(-b+a^2x^2\right)^{5/2}}\,dx = \int \left(\frac{d\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{x\left(-b+a^2x^2\right)^{5/2}} + \frac{cx\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{\left(-b+a^2x^2\right)^{5/2}}\right)dx$$

$$= c\int \frac{x\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{\left(-b+a^2x^2\right)^{5/2}}\,dx + d\int \frac{\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{x\left(-b+a^2x^2\right)^{5/2}}\,dx$$

$$= \frac{(8c)\,\mathrm{Subst}\left(\int \frac{x^{13/4}\left(b+x^2\right)}{\left(-b+x^2\right)^4}\,dx, x, ax+\sqrt{-b+a^2x^2}\right)}{a^2} + (32d)\,\mathrm{Subst}\left(\int \cdots\right.$$

$$= \frac{8c\left(ax+\sqrt{-b+a^2x^2}\right)^{17/4}}{3a^2\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{(10c)\,\mathrm{Subst}\left(\int \frac{x^{13/4}}{\left(-b+x^2\right)^3}\,dx, x, ax+\cdots\right.}{3a^2}$$

$$= \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{9/4}}{3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8c\left(ax+\sqrt{-b+a^2x^2}\right)^{17/4}}{3a^2\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} - \frac{\cdots}{6\cdots}$$

$$= \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{9/4}}{3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8c\left(ax+\sqrt{-b+a^2x^2}\right)^{17/4}}{3a^2\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} - \frac{\cdots}{2\cdots}$$

$$= \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{9/4}}{3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8c\left(ax+\sqrt{-b+a^2x^2}\right)^{17/4}}{3a^2\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} - \frac{\cdots}{2\cdots}$$

$$= \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{9/4}}{3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8c\left(ax+\sqrt{-b+a^2x^2}\right)^{17/4}}{3a^2\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} - \frac{\cdots}{2\cdots}$$

$$= \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{9/4}}{3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8c\left(ax+\sqrt{-b+a^2x^2}\right)^{17/4}}{3a^2\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} - \frac{\cdots}{2\cdots}$$

$$= \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{9/4}}{3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8c\left(ax+\sqrt{-b+a^2x^2}\right)^{17/4}}{3a^2\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} - \frac{\cdots}{2\cdots}$$

$$= \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{9/4}}{3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8c\left(ax+\sqrt{-b+a^2x^2}\right)^{17/4}}{3a^2\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} - \frac{\cdots}{2\cdots}$$

$$= \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{9/4}}{3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8c\left(ax+\sqrt{-b+a^2x^2}\right)^{17/4}}{3a^2\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} - \frac{\cdots}{2\cdots}$$

$$= \frac{8d\left(ax+\sqrt{-b+a^2x^2}\right)^{9/4}}{3\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} + \frac{8c\left(ax+\sqrt{-b+a^2x^2}\right)^{17/4}}{3a^2\left(b-\left(ax+\sqrt{-b+a^2x^2}\right)^2\right)^3} - \cdots$$

Mathematica [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\left(d+cx^2\right)\left(ax+\sqrt{-b+a^2x^2}\right)^{5/4}}{x\left(-b+a^2x^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + c*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(5/4))/(x*(-b + a^2*x^2)^
(5/2)),x]
```

```
[Out] Integrate[((d + c*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(5/4))/(x*(-b + a^2*x^2)^
(5/2)), x]
```

IntegrateAlgebraic [A] time = 15.50, size = 938, normalized size = 1.07

$$\frac{\sqrt[4]{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}\left(-51dx^2a^4+45bcx^2a^2-bda^2-97b^2c\right)}{96a^2b^{15/8}\left(ax-\sqrt{b}\right)\left(ax+\sqrt{b}\right)}+\frac{5\left(29a^2d-3bc\right)\tan^{-1}\left(\sqrt[4]{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}\right)}{64a^2b^{15/8}}+\frac{5\left(29a^2d-3bc\right)\tan^{-1}\left(\frac{\sqrt{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}-1}{\sqrt{2}\sqrt[4]{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}}\right)}{64\sqrt{2}a^2b^{15/8}}-\frac{\sqrt{2+\sqrt{2}}\,d\tan^{-1}\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\sqrt{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}-\sqrt{1-\frac{1}{\sqrt{2}}}}{\sqrt[4]{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}}\right)}{b^{15/8}}-\frac{\sqrt{2-\sqrt{2}}\,d\tan^{-1}\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\sqrt{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}-\sqrt{1+\frac{1}{\sqrt{2}}}}{\sqrt[4]{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}}\right)}{b^{15/8}}+\frac{5\left(29a^2d-3bc\right)\tanh^{-1}\left(\sqrt[4]{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}\right)}{64a^2b^{15/8}}+\frac{5\left(29a^2d-3bc\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}+1}{\sqrt{2}\sqrt[4]{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}}\right)}{64\sqrt{2}a^2b^{15/8}}-\frac{\sqrt{2+\sqrt{2}}\,d\tanh^{-1}\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\sqrt{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}+\sqrt{1-\frac{1}{\sqrt{2}}}}{\sqrt[4]{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}}\right)}{b^{15/8}}-\frac{\sqrt{2-\sqrt{2}}\,d\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\sqrt{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}+\sqrt{1+\frac{1}{\sqrt{2}}}}{\sqrt[4]{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}}\right)}{b^{15/8}}+\frac{\sqrt{a^2x^2-b}\left(51dx^3a^4-45bcx^3a^2-83bdxa^2+13b^2cx\right)\sqrt[4]{\frac{ax+\sqrt{a^2x^2-b}}{\sqrt{b}}}}{96ab^{15/8}\left(ax-\sqrt{b}\right)^2\left(ax+\sqrt{b}\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] IntegrateAlgebraic[((d + c*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(5/4))/(x*(-b +
a^2*x^2)^(5/2)),x]
```

```
[Out] ((-97*b^2*c - a^2*b*d + 45*a^2*b*c*x^2 - 51*a^4*d*x^2)*((a*x + Sqrt[-b + a^
2*x^2])/Sqrt[b])^(1/4))/(96*a^2*b^(15/8)*(-Sqrt[b] + a*x)*(Sqrt[b] + a*x))
+ (Sqrt[-b + a^2*x^2]*(13*b^2*c*x - 83*a^2*b*d*x - 45*a^2*b*c*x^3 + 51*a^4*
d*x^3)*((a*x + Sqrt[-b + a^2*x^2])/Sqrt[b])^(1/4))/(96*a*b^(15/8)*(-Sqrt[b]
 + a*x)^2*(Sqrt[b] + a*x)^2) + (5*(-3*b*c + 29*a^2*d)*ArcTan[((a*x + Sqrt[-
b + a^2*x^2])/Sqrt[b])^(1/4)])/(64*a^2*b^(15/8)) + (5*(-3*b*c + 29*a^2*d)*A
rcTan[(-1 + Sqrt[(a*x + Sqrt[-b + a^2*x^2])/Sqrt[b]])/(Sqrt[2]*((a*x + Sqrt
[-b + a^2*x^2])/Sqrt[b])^(1/4))])/(64*Sqrt[2]*a^2*b^(15/8)) - (Sqrt[2 + Sqr
t[2]]*d*ArcTan[(-Sqrt[1 - 1/Sqrt[2]] + Sqrt[1 - 1/Sqrt[2]]*Sqrt[(a*x + Sqrt
[-b + a^2*x^2])/Sqrt[b]])/((a*x + Sqrt[-b + a^2*x^2])/Sqrt[b])^(1/4)])/b^(1
5/8) - (Sqrt[2 - Sqrt[2]]*d*ArcTan[(-Sqrt[1 + 1/Sqrt[2]] + Sqrt[1 + 1/Sqrt[
2]]*Sqrt[(a*x + Sqrt[-b + a^2*x^2])/Sqrt[b]])/((a*x + Sqrt[-b + a^2*x^2])/S
qrt[b])^(1/4)])/b^(15/8) + (5*(-3*b*c + 29*a^2*d)*ArcTanh[((a*x + Sqrt[-b +
 a^2*x^2])/Sqrt[b])^(1/4)])/(64*a^2*b^(15/8)) + (5*(-3*b*c + 29*a^2*d)*ArcT
anh[(1 + Sqrt[(a*x + Sqrt[-b + a^2*x^2])/Sqrt[b]])/(Sqrt[2]*((a*x + Sqrt[-b
 + a^2*x^2])/Sqrt[b])^(1/4))])/(64*Sqrt[2]*a^2*b^(15/8)) - (Sqrt[2 + Sqrt[2
]]*d*ArcTanh[(Sqrt[1 - 1/Sqrt[2]] + Sqrt[1 - 1/Sqrt[2]]*Sqrt[(a*x + Sqrt[-b
 + a^2*x^2])/Sqrt[b]])/((a*x + Sqrt[-b + a^2*x^2])/Sqrt[b])^(1/4)])/b^(15/8
) - (Sqrt[2 - Sqrt[2]]*d*ArcTanh[(Sqrt[1 + 1/Sqrt[2]] + Sqrt[1 + 1/Sqrt[2]]
*Sqrt[(a*x + Sqrt[-b + a^2*x^2])/Sqrt[b]])/((a*x + Sqrt[-b + a^2*x^2])/Sqrt
[b])^(1/4)])/b^(15/8)
```

fricas [A] time = 24.53, size = 160, normalized size = 0.18

$$\frac{\left(a^2b^2d-3\left(17a^6d-15a^4bc\right)x^4+97b^3c+2\left(25a^4bd-71a^2b^2c\right)x^2+\sqrt{a^2x^2-b}\left(3\left(17a^5d-15a^3bc\right)x^3-\left(83a^3bd-13ab^2c\right)x\right)\right)\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}}{96\left(a^6b^2x^4-2a^4b^3x^2+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+d)*(a*x+(a^2*x^2-b)^(1/2))^(5/4)/x/(a^2*x^2-b)^(5/2),x, al
gorithm="fricas")
```

```
[Out] 1/96*(a^2*b^2*d - 3*(17*a^6*d - 15*a^4*b*c)*x^4 + 97*b^3*c + 2*(25*a^4*b*d
- 71*a^2*b^2*c)*x^2 + sqrt(a^2*x^2 - b)*(3*(17*a^5*d - 15*a^3*b*c)*x^3 - (8
3*a^3*b*d - 13*a*b^2*c)*x))*(a*x + sqrt(a^2*x^2 - b))^(1/4)/(a^6*b^2*x^4 -
2*a^4*b^3*x^2 + a^2*b^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+d)*(a*x+(a^2*x^2-b)^(1/2))^(5/4)/x/(a^2*x^2-b)^(5/2),x, al
gorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^{2}+d\right)\left(a x +\sqrt{a^{2} x^{2}-b}\right)^{\frac{5}{4}}}{x \left(a^{2} x^{2}-b \right)^{\frac{5}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+d)*(a*x+(a^2*x^2-b)^(1/2))^(5/4)/x/(a^2*x^2-b)^(5/2),x)
```

```
[Out] int((c*x^2+d)*(a*x+(a^2*x^2-b)^(1/2))^(5/4)/x/(a^2*x^2-b)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(c x^{2} + d\right)} {\left(a x + \sqrt{a^{2} x^{2} - b}\right)}^{\frac{5}{4}}}{{\left(a^{2} x^{2} - b\right)}^{\frac{5}{2}} x}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+d)*(a*x+(a^2*x^2-b)^(1/2))^(5/4)/x/(a^2*x^2-b)^(5/2),x, al
gorithm="maxima")
```

```
[Out] integrate((c*x^2 + d)*(a*x + sqrt(a^2*x^2 - b))^(5/4)/((a^2*x^2 - b)^(5/2)*
x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{5/4}\,\left(c\,x^2+d\right)}{x\,{\left(a^2\,x^2-b\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (a^2*x^2 - b)^(1/2))^(5/4)*(d + c*x^2))/(x*(a^2*x^2 - b)^(5/2))
,x)
```

```
[Out] int(((a*x + (a^2*x^2 - b)^(1/2))^(5/4)*(d + c*x^2))/(x*(a^2*x^2 - b)^(5/2))
, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+d)*(a*x+(a**2*x**2-b)**(1/2))**(5/4)/x/(a**2*x**2-b)**(5/
2),x)
```

```
[Out] Timed out
```

3.2432 $\int \frac{\sqrt[6]{\frac{1-bx}{c+x}}\left(1+dx^2\right)}{(1+bx)(1+cx)}\,dx$

Optimal. Leaf size=887

$$\frac{d\sqrt[6]{\frac{1-bx}{c+x}}(c+x)}{bc}-\frac{\left(bc^2+7c+6b\right)d\tan^{-1}\left(\frac{\sqrt[6]{\frac{1-bx}{c+x}}}{\sqrt[6]{b}}\right)}{3b^{11/6}c^2}-\frac{2\sqrt[6]{b+c}\left(c^2+d\right)\tan^{-1}\left(\frac{\sqrt[6]{1-c^2}\sqrt[6]{\frac{1-bx}{c+x}}}{\sqrt[6]{b+c}}\right)}{c^2(c-b)\sqrt[6]{1-c^2}}-\frac{\sqrt[6]{2}\sqrt{3}\left(b^2+d\right)\tan^{-1}\left(\right.}{b^{11/6}(b-c)}$$

Rubi [A] time = 5.28, antiderivative size = 1549, normalized size of antiderivative = 1.75, number of steps used = 45, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6, 6725, 199, 209, 634, 618, 204, 628, 203, 205}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(((1 - b*x)/(c + x))^(1/6)*(1 + d*x^2))/((1 + b*x)*(1 + c*x)),x]
```

```
[Out] (d*(c + x)*((1 - b*x)/(c + x))^(1/6))/(b*c) + (5*(1 + b*c)*d*ArcTan[((1 - b
*x)/(c + x))^(1/6)/b^(1/6)])/(3*b^(11/6)*c) - (2*(b + 2*c + b*c^2)*d*ArcTan
[((1 - b*x)/(c + x))^(1/6)/b^(1/6)])/(b^(11/6)*c^2) - (2*2^(1/6)*(b^2 + d)*
ArcTan[((1 - b*c)^(1/6)*((1 - b*x)/(c + x))^(1/6))/(2^(1/6)*b^(1/6))])/(b^(
11/6)*(b - c)*(1 - b*c)^(1/6)) + (2*(b + c)^(1/6)*(c^2 + d)*ArcTan[((1 - c^
2)^(1/6)*((1 - b*x)/(c + x))^(1/6))/(b + c)^(1/6)])/((b - c)*c^2*(1 - c^2)^
(1/6)) - (5*(1 + b*c)*d*ArcTan[Sqrt[3] - (2*((1 - b*x)/(c + x))^(1/6))/b^(1
/6)])/(6*b^(11/6)*c) + ((b + 2*c + b*c^2)*d*ArcTan[Sqrt[3] - (2*((1 - b*x)/
(c + x))^(1/6))/b^(1/6)])/(b^(11/6)*c^2) + (5*(1 + b*c)*d*ArcTan[Sqrt[3] +
(2*((1 - b*x)/(c + x))^(1/6))/b^(1/6)])/(6*b^(11/6)*c) - ((b + 2*c + b*c^2)
*d*ArcTan[Sqrt[3] + (2*((1 - b*x)/(c + x))^(1/6))/b^(1/6)])/(b^(11/6)*c^2)
+ (2^(1/6)*(b^2 + d)*ArcTan[Sqrt[3] - (2^(5/6)*(1 - b*c)^(1/6)*((1 - b*x)/(
c + x))^(1/6))/b^(1/6)])/(b^(11/6)*(b - c)*(1 - b*c)^(1/6)) - (2^(1/6)*(b^2
 + d)*ArcTan[Sqrt[3] + (2^(5/6)*(1 - b*c)^(1/6)*((1 - b*x)/(c + x))^(1/6))/
b^(1/6)])/(b^(11/6)*(b - c)*(1 - b*c)^(1/6)) - ((b + c)^(1/6)*(c^2 + d)*Arc
Tan[Sqrt[3] - (2*(1 - c^2)^(1/6)*((1 - b*x)/(c + x))^(1/6))/(b + c)^(1/6)])
/((b - c)*c^2*(1 - c^2)^(1/6)) + ((b + c)^(1/6)*(c^2 + d)*ArcTan[Sqrt[3] +
(2*(1 - c^2)^(1/6)*((1 - b*x)/(c + x))^(1/6))/(b + c)^(1/6)])/((b - c)*c^2*
(1 - c^2)^(1/6)) - (5*(1 + b*c)*d*Log[b^(1/3) - Sqrt[3]*b^(1/6)*((1 - b*x)/
(c + x))^(1/6) + ((1 - b*x)/(c + x))^(1/3)])/(4*Sqrt[3]*b^(11/6)*c) + (Sqrt
[3]*(b + 2*c + b*c^2)*d*Log[b^(1/3) - Sqrt[3]*b^(1/6)*((1 - b*x)/(c + x))^(
1/6) + ((1 - b*x)/(c + x))^(1/3)])/(2*b^(11/6)*c^2) + (5*(1 + b*c)*d*Log[b^
(1/3) + Sqrt[3]*b^(1/6)*((1 - b*x)/(c + x))^(1/6) + ((1 - b*x)/(c + x))^(1/
3)])/(4*Sqrt[3]*b^(11/6)*c) - (Sqrt[3]*(b + 2*c + b*c^2)*d*Log[b^(1/3) + Sq
rt[3]*b^(1/6)*((1 - b*x)/(c + x))^(1/6) + ((1 - b*x)/(c + x))^(1/3)])/(2*b^
(11/6)*c^2) + (Sqrt[3]*(b^2 + d)*Log[2^(1/3)*b^(1/3) - 2^(1/6)*Sqrt[3]*b^(1
/6)*(1 - b*c)^(1/6)*((1 - b*x)/(c + x))^(1/6) + (1 - b*c)^(1/3)*((1 - b*x)/
(c + x))^(1/3)])/(2^(5/6)*b^(11/6)*(b - c)*(1 - b*c)^(1/6)) - (Sqrt[3]*(b^2
 + d)*Log[2^(1/3)*b^(1/3) + 2^(1/6)*Sqrt[3]*b^(1/6)*(1 - b*c)^(1/6)*((1 - b
*x)/(c + x))^(1/6) + (1 - b*c)^(1/3)*((1 - b*x)/(c + x))^(1/3)])/(2^(5/6)*b
^(11/6)*(b - c)*(1 - b*c)^(1/6)) - (Sqrt[3]*(b + c)^(1/6)*(c^2 + d)*Log[(b
+ c)^(1/3) - Sqrt[3]*(b + c)^(1/6)*(1 - c^2)^(1/6)*((1 - b*x)/(c + x))^(1/6
) + (1 - c^2)^(1/3)*((1 - b*x)/(c + x))^(1/3)])/(2*(b - c)*c^2*(1 - c^2)^(1
/6)) + (Sqrt[3]*(b + c)^(1/6)*(c^2 + d)*Log[(b + c)^(1/3) + Sqrt[3]*(b + c)
^(1/6)*(1 - c^2)^(1/6)*((1 - b*x)/(c + x))^(1/6) + (1 - c^2)^(1/3)*((1 - b*
x)/(c + x))^(1/3)])/(2*(b - c)*c^2*(1 - c^2)^(1/6))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt[6]{\frac{1-bx}{c+x}}\left(1+dx^2\right)}{(1+bx)(1+cx)}\,dx = (6(1+bc))\,\text{Subst}\left(\int \frac{x^6\left(b^2+2bx^6+x^{12}+d\left(-1+cx^6\right)^2\right)}{\left(b+x^6\right)^2\left(b+c+x^6-c^2x^6\right)\left(-x^6+b\left(-2+cx^6\right)\right)}\,dx, x, \sqrt[6]{\frac{1-bx}{c+x}}\right.$$

$$= (6(1+bc))\,\text{Subst}\left(\int \frac{x^6\left(b^2+2bx^6+x^{12}+d\left(-1+cx^6\right)^2\right)}{\left(b+x^6\right)^2\left(b+c+\left(1-c^2\right)x^6\right)\left(-x^6+b\left(-2+cx^6\right)\right)}\,dx, x, \sqrt[6]{\frac{1-b}{c+}}\right.$$

$$= (6(1+bc))\,\text{Subst}\left(\int\left(\frac{d}{c\left(b+x^6\right)^2}-\frac{\left(b+2c+bc^2\right)d}{bc^2(1+bc)\left(b+x^6\right)}+\frac{2\left(-b^2-d\right)}{b(b-c)(1+bc)\left(2b+(1-b\right.}\right.\right.$$

$$= \frac{(6(1+bc)d)\,\text{Subst}\left(\int \frac{1}{\left(b+x^6\right)^2}\,dx, x, \sqrt[6]{\frac{1-bx}{c+x}}\right)}{c} - \frac{\left(6\left(b+2c+bc^2\right)d\right)\text{Subst}\left(\int \frac{1}{b+x^6}\,dx, x, \right.}{bc^2}$$

$$= \frac{d(c+x)\sqrt[6]{\frac{1-bx}{c+x}}}{bc} + \frac{(5(1+bc)d)\,\text{Subst}\left(\int \frac{1}{b+x^6}\,dx, x, \sqrt[6]{\frac{1-bx}{c+x}}\right)}{bc} - \frac{\left(2\left(b+2c+bc^2\right)d\right)\text{Subs}}{}$$

$$= \frac{d(c+x)\sqrt[6]{\frac{1-bx}{c+x}}}{bc} - \frac{2\left(b+2c+bc^2\right)d\tan^{-1}\left(\frac{\sqrt[6]{\frac{1-bx}{c+x}}}{\sqrt[6]{b}}\right)}{b^{11/6}c^2} - \frac{2\sqrt[6]{2}\left(b^2+d\right)\tan^{-1}\left(\frac{\sqrt[6]{1-bc}\sqrt[6]{\frac{1-bx}{c+x}}}{\sqrt[6]{2}\sqrt[6]{b}}\right.}{b^{11/6}(b-c)\sqrt[6]{1-bc}}$$

$$= \frac{d(c+x)\sqrt[6]{\frac{1-bx}{c+x}}}{bc} + \frac{5(1+bc)d\tan^{-1}\left(\frac{\sqrt[6]{\frac{1-bx}{c+x}}}{\sqrt[6]{b}}\right)}{3b^{11/6}c} - \frac{2\left(b+2c+bc^2\right)d\tan^{-1}\left(\frac{\sqrt[6]{\frac{1-bx}{c+x}}}{\sqrt[6]{b}}\right)}{b^{11/6}c^2} - \frac{2\sqrt[6]{2}}{}$$

$$= \frac{d(c+x)\sqrt[6]{\frac{1-bx}{c+x}}}{bc} + \frac{5(1+bc)d\tan^{-1}\left(\frac{\sqrt[6]{\frac{1-bx}{c+x}}}{\sqrt[6]{b}}\right)}{3b^{11/6}c} - \frac{2\left(b+2c+bc^2\right)d\tan^{-1}\left(\frac{\sqrt[6]{\frac{1-bx}{c+x}}}{\sqrt[6]{b}}\right)}{b^{11/6}c^2} - \frac{2\sqrt[6]{2}}{}$$

$$= \frac{d(c+x)\sqrt[6]{\frac{1-bx}{c+x}}}{bc} + \frac{5(1+bc)d\tan^{-1}\left(\frac{\sqrt[6]{\frac{1-bx}{c+x}}}{\sqrt[6]{b}}\right)}{3b^{11/6}c} - \frac{2\left(b+2c+bc^2\right)d\tan^{-1}\left(\frac{\sqrt[6]{\frac{1-bx}{c+x}}}{\sqrt[6]{b}}\right)}{b^{11/6}c^2} - \frac{2\sqrt[6]{2}}{}$$

Mathematica [C] time = 0.48, size = 322, normalized size = 0.36

$$\frac{6}{5}(c+x)\sqrt[6]{\frac{1-bx}{c+x}}\left(\frac{\left(b^2+d\right)\left(\frac{1-bx}{bc+1}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{5}{6};\frac{11}{6};\frac{b(c+x)}{bc+1}\right)}{b(b-c)(bx-1)}+\frac{2\left(b^2+d\right){}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\frac{2b(c+x)}{(bc-1)(bx-1)}\right)}{b(b-c)(bc-1)(bx-1)}+\frac{b\left(c^2+d\right){}_2F_1\left(\frac{5}{6},\frac{5}{6};\frac{11}{6};\frac{b(c+x)}{bc+1}\right)}{c^2(b-c)(bc+1)\sqrt[6]{\frac{1-bx}{bc+1}}}-\frac{(b+c)\left(c^2+d\right){}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\frac{(b+c)(c+x)}{\left(c^2-1\right)(bx-1)}\right)}{c^2\left(c^2-1\right)(b-c)(bx-1)}+\frac{d\,{}_2F_1\left(-\frac{1}{6},\frac{5}{6};\frac{11}{6};\frac{b(c+x)}{bc+1}\right)}{bc\sqrt[6]{\frac{1-bx}{bc+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(((1 - b*x)/(c + x))^(1/6)*(1 + d*x^2))/((1 + b*x)*(1 + c*x)),x]
```

```
[Out] (6*(c + x)*((1 - b*x)/(c + x))^(1/6)*((d*Hypergeometric2F1[-1/6, 5/6, 11/6,
 (b*(c + x))/(1 + b*c)])/(b*c*((1 - b*x)/(1 + b*c))^(1/6)) + (b*(c^2 + d)*H
ypergeometric2F1[5/6, 5/6, 11/6, (b*(c + x))/(1 + b*c)])/((b - c)*c^2*(1 +
b*c)*((1 - b*x)/(1 + b*c))^(1/6)) + ((b^2 + d)*((1 - b*x)/(1 + b*c))^(5/6)*
Hypergeometric2F1[5/6, 5/6, 11/6, (b*(c + x))/(1 + b*c)])/(b*(b - c)*(-1 +
b*x)) + (2*(b^2 + d)*Hypergeometric2F1[5/6, 1, 11/6, (-2*b*(c + x))/((-1 +
b*c)*(-1 + b*x))])/(b*(b - c)*(-1 + b*c)*(-1 + b*x)) - ((b + c)*(c^2 + d)*H
ypergeometric2F1[5/6, 1, 11/6, -(((b + c)*(c + x))/((-1 + c^2)*(-1 + b*x)))
])/((b - c)*c^2*(-1 + c^2)*(-1 + b*x))))/5
```

IntegrateAlgebraic [A] time = 3.51, size = 898, normalized size = 1.01

$$-\frac{\left(bc^2+7c+6b\right)\tan^{-1}\left(\frac{\sqrt[6]{\frac{1-bx}{c+x}}}{\sqrt[6]{b}}\right)d}{3b^{11/6}c^2}+\frac{\left(bc^2+7c+6b\right)\tan^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{\frac{1-bx}{c+x}}}{\sqrt[6]{b}\sqrt[6]{\frac{1-bx}{c+x}}}\right)d}{6b^{11/6}c^2}-\frac{\left(bc^2+7c+6b\right)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[6]{\frac{1-bx}{c+x}}}{\sqrt[3]{b}+\sqrt[3]{\frac{1-bx}{c+x}}}\right)d}{2\sqrt{3}b^{11/6}c^2}+\frac{(bc+1)\sqrt[6]{\frac{1-bx}{c+x}}d}{bc\left(b+\frac{1-bx}{c+x}\right)}-\frac{2\sqrt[6]{b+c}\left(c^2+d\right)\tan^{-1}\left(\frac{\sqrt[6]{1-c^2}\sqrt[6]{\frac{1-bx}{c+x}}}{\sqrt[6]{b+c}}\right)}{c^2(c-b)\sqrt[6]{1-c^2}}+\frac{\sqrt[6]{2}\sqrt{3}\left(b^2+d\right)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{5/6}\sqrt[6]{bc-1}\sqrt[6]{\frac{1-bx}{c+x}}}{\sqrt{3}\sqrt[6]{b}}\right)}{b^{11/6}(b-c)\sqrt[6]{bc-1}}-\frac{\sqrt[6]{2}\sqrt{3}\left(b^2+d\right)\tan^{-1}\left(\frac{2^{5/6}\sqrt[6]{bc-1}\sqrt[6]{\frac{1-bx}{c+x}}}{\sqrt{3}\sqrt[6]{b}}+\frac{1}{\sqrt{3}}\right)}{b^{11/6}(b-c)\sqrt[6]{bc-1}}+\frac{\sqrt[6]{b+c}\left(c^2+d\right)\tan^{-1}\left(\frac{\sqrt[3]{b+c}-\sqrt[3]{1-c^2}\sqrt[3]{\frac{1-bx}{c+x}}}{\sqrt[6]{b+c}\sqrt[6]{1-c^2}\sqrt[6]{\frac{1-bx}{c+x}}}\right)}{c^2(c-b)\sqrt[6]{1-c^2}}-\frac{2\sqrt[6]{2}\left(b^2+d\right)\tanh^{-1}\left(\frac{\sqrt[6]{bc-1}\sqrt[6]{\frac{1-bx}{c+x}}}{\sqrt[6]{2}\sqrt[6]{b}}\right)}{b^{11/6}(b-c)\sqrt[6]{bc-1}}-\frac{\sqrt[6]{2}\left(b^2+d\right)\tanh^{-1}\left(\frac{2^{5/6}\sqrt[6]{b}\sqrt[6]{bc-1}\sqrt[6]{\frac{1-bx}{c+x}}}{2\sqrt[3]{b}+2^{2/3}\sqrt[3]{bc-1}\sqrt[3]{\frac{1-bx}{c+x}}}\right)}{b^{11/6}(b-c)\sqrt[6]{bc-1}}-\frac{\sqrt{3}\sqrt[6]{b+c}\left(c^2+d\right)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b+c}\sqrt[6]{1-c^2}\sqrt[6]{\frac{1-bx}{c+x}}}{\sqrt[3]{b+c}+\sqrt[3]{1-c^2}\sqrt[3]{\frac{1-bx}{c+x}}}\right)}{c^2(c-b)\sqrt[6]{1-c^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(((1 - b*x)/(c + x))^(1/6)*(1 + d*x^2))/((1 + b*x)*(1 +
c*x)),x]
```

```
[Out] ((1 + b*c)*d*((1 - b*x)/(c + x))^(1/6))/(b*c*(b + (1 - b*x)/(c + x))) - ((6
*b + 7*c + b*c^2)*d*ArcTan[((1 - b*x)/(c + x))^(1/6)/b^(1/6)])/(3*b^(11/6)*
c^2) - (2*(b + c)^(1/6)*(c^2 + d)*ArcTan[((1 - c^2)^(1/6)*((1 - b*x)/(c + x
))^(1/6))/(b + c)^(1/6)])/(c^2*(-b + c)*(1 - c^2)^(1/6)) + (2^(1/6)*Sqrt[3]
*(b^2 + d)*ArcTan[1/Sqrt[3] - (2^(5/6)*(-1 + b*c)^(1/6)*((1 - b*x)/(c + x))
^(1/6))/(Sqrt[3]*b^(1/6))])/(b^(11/6)*(b - c)*(-1 + b*c)^(1/6)) - (2^(1/6)*
Sqrt[3]*(b^2 + d)*ArcTan[1/Sqrt[3] + (2^(5/6)*(-1 + b*c)^(1/6)*((1 - b*x)/(
c + x))^(1/6))/(Sqrt[3]*b^(1/6))])/(b^(11/6)*(b - c)*(-1 + b*c)^(1/6)) + ((
6*b + 7*c + b*c^2)*d*ArcTan[(b^(1/3) - ((1 - b*x)/(c + x))^(1/3))/(b^(1/6)*
((1 - b*x)/(c + x))^(1/6))])/(6*b^(11/6)*c^2) + ((b + c)^(1/6)*(c^2 + d)*Ar
cTan[((b + c)^(1/3) - (1 - c^2)^(1/3)*((1 - b*x)/(c + x))^(1/3))/((b + c)^(
1/6)*(1 - c^2)^(1/6)*((1 - b*x)/(c + x))^(1/6))])/(c^2*(-b + c)*(1 - c^2)^(
1/6)) - (2*2^(1/6)*(b^2 + d)*ArcTanh[((-1 + b*c)^(1/6)*((1 - b*x)/(c + x))^
(1/6))/(2^(1/6)*b^(1/6))])/(b^(11/6)*(b - c)*(-1 + b*c)^(1/6)) - ((6*b + 7*
c + b*c^2)*d*ArcTanh[(Sqrt[3]*b^(1/6)*((1 - b*x)/(c + x))^(1/6))/(b^(1/3) +
 ((1 - b*x)/(c + x))^(1/3))])/(2*Sqrt[3]*b^(11/6)*c^2) - (2^(1/6)*(b^2 + d)
*ArcTanh[(2^(5/6)*b^(1/6)*(-1 + b*c)^(1/6)*((1 - b*x)/(c + x))^(1/6))/(2*b^
(1/3) + 2^(2/3)*(-1 + b*c)^(1/3)*((1 - b*x)/(c + x))^(1/3))])/(b^(11/6)*(b
- c)*(-1 + b*c)^(1/6)) - (Sqrt[3]*(b + c)^(1/6)*(c^2 + d)*ArcTanh[(Sqrt[3]*
(b + c)^(1/6)*(1 - c^2)^(1/6)*((1 - b*x)/(c + x))^(1/6))/((b + c)^(1/3) + (
1 - c^2)^(1/3)*((1 - b*x)/(c + x))^(1/3))])/(c^2*(-b + c)*(1 - c^2)^(1/6))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-b*x+1)/(c+x))^(1/6)*(d*x^2+1)/(b*x+1)/(c*x+1),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-b*x+1)/(c+x))^(1/6)*(d*x^2+1)/(b*x+1)/(c*x+1),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{-bx+1}{c+x}\right)^{\frac{1}{6}} \left(d\,x^2+1\right)}{(bx+1)\,(cx+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-b*x+1)/(c+x))^(1/6)*(d*x^2+1)/(b*x+1)/(c*x+1),x)
```

```
[Out] int(((-b*x+1)/(c+x))^(1/6)*(d*x^2+1)/(b*x+1)/(c*x+1),x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-b*x+1)/(c+x))^(1/6)*(d*x^2+1)/(b*x+1)/(c*x+1),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*c-1>0)', see `assume?` for more d
etails)Is b*c-1 positive or negative?
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-(b*x - 1)/(c + x))^(1/6)*(d*x^2 + 1))/((b*x + 1)*(c*x + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-b*x+1)/(c+x))**(1/6)*(d*x**2+1)/(b*x+1)/(c*x+1),x)
```

```
[Out] Timed out
```

3.2433 $\int \frac{\sqrt{-b+a^2x^2}\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}{\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx$

Optimal. Leaf size=963

$$\frac{308\tan^{-1}\left(\frac{2\sqrt[3]{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}}{\sqrt{3}\sqrt[3]{c}}+\frac{1}{\sqrt{3}}\right)b^2}{243\sqrt{3}\,ac^{17/3}}-\frac{308\log\left(\sqrt[3]{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}-\sqrt[3]{c}\right)b^2}{729ac^{17/3}}+\frac{154\log\left(c^{2/3}+\sqrt[3]{c+\ldots}\right.}{}$$

Rubi [F] time = 1.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-b+a^2x^2}\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}{\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(3/4))/(c + (a*x + Sqrt[
-b + a^2*x^2])^(1/4))^(2/3),x]
```

```
[Out] Defer[Int][(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(3/4))/(c + (a*x
+ Sqrt[-b + a^2*x^2])^(1/4))^(2/3), x]
```

Rubi steps

$$\int \frac{\sqrt{-b+a^2x^2}\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}{\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx=\int \frac{\sqrt{-b+a^2x^2}\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}{\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx$$

Mathematica [F] time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-b+a^2x^2}\left(ax+\sqrt{-b+a^2x^2}\right)^{3/4}}{\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(3/4))/(c + (a*x +
 Sqrt[-b + a^2*x^2])^(1/4))^(2/3),x]
```

```
[Out] Integrate[(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(3/4))/(c + (a*x +
 Sqrt[-b + a^2*x^2])^(1/4))^(2/3), x]
```

IntegrateAlgebraic [A] time = 5.96, size = 1246, normalized size = 1.29

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-b + a^2*x^2]*(a*x + Sqrt[-b + a^2*x^2])^(3/4))/(c
 + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3),x]
```

```
[Out] -1/2865402540*(4964339380*b^2*c^4*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1
/3) - 17192415240*b*c^12*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + 859
6207620*c^20*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) - 12835352530*b^2
*c^3*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(4/3) + 94558283820*b*c^11*(c +
 (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(4/3) - 64471557150*c^19*(c + (a*x + Sqr
t[-b + a^2*x^2])^(1/4))^(4/3) + 14981456490*b^2*c^2*(c + (a*x + Sqrt[-b + a
^2*x^2])^(1/4))^(7/3) - 217361249820*b*c^10*(c + (a*x + Sqrt[-b + a^2*x^2])
^(1/4))^(7/3) + 248676006150*c^18*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(7
/3) - 8353296952*b^2*c*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(10/3) + 2701
66525200*b*c^9*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(10/3) - 680328431640
*c^17*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(10/3) + 1815934120*b^2*(c + (
a*x + Sqrt[-b + a^2*x^2])^(1/4))^(13/3) - 196484745600*b*c^8*(c + (a*x + Sq
rt[-b + a^2*x^2])^(1/4))^(13/3) + 1465133848200*c^16*(c + (a*x + Sqrt[-b +
a^2*x^2])^(1/4))^(13/3) + 84734046540*b*c^7*(c + (a*x + Sqrt[-b + a^2*x^2])
^(1/4))^(16/3) - 2529906411285*c^15*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^
(16/3) - 20876504220*b*c^6*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(19/3) +
3489922326825*c^14*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(19/3) + 24560593
20*b*c^5*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(22/3) - 3835496630250*c^13
*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(22/3) + 3351388626306*c^12*(c + (a
*x + Sqrt[-b + a^2*x^2])^(1/4))^(25/3) - 2315240318205*c^11*(c + (a*x + Sqr
t[-b + a^2*x^2])^(1/4))^(28/3) + 1249687305945*c^10*(c + (a*x + Sqrt[-b + a
^2*x^2])^(1/4))^(31/3) - 516271705560*c^9*(c + (a*x + Sqrt[-b + a^2*x^2])^(
1/4))^(34/3) + 157727985480*c^8*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(37/
3) - 33596514666*c^7*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(40/3) + 445655
9250*c^6*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(43/3) - 277297020*c^5*(c +
 (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(46/3))/(a*c^5*(a*x + Sqrt[-b + a^2*x^2]
)^(5/4)) + (308*b^2*ArcTan[1/Sqrt[3] + (2*(c + (a*x + Sqrt[-b + a^2*x^2])^(
1/4))^(1/3))/(Sqrt[3]*c^(1/3))])/(243*Sqrt[3]*a*c^(17/3)) - (308*b^2*Log[c^
(1/3) - (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)])/(729*a*c^(17/3)) + (
154*b^2*Log[c^(2/3) + c^(1/3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)
+ (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3)])/(729*a*c^(17/3))
```

fricas [A] time = 0.58, size = 617, normalized size = 0.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(a*x+(a^2*x^2-b)^(1/2))^(3/4)/(c+(a*x+(a^2*x^2-
b)^(1/2))^(1/4))^(2/3),x, algorithm="fricas")
```

```
[Out] 1/8596207620*(3631868240*sqrt(3)*b^2*c*sqrt(-(-c^2)^(1/3))*arctan(-1/3*(sqr
t(3)*(-c^2)^(1/3)*c*sqrt(-(-c^2)^(1/3)) - 2*sqrt(3)*(-c^2)^(2/3)*(c + (a*x
+ sqrt(a^2*x^2 - b))^(1/4))^(1/3)*sqrt(-(-c^2)^(1/3)))/c^2) + 1815934120*(-
c^2)^(2/3)*b^2*log((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3)*c - (-c^2)^(
1/3)*c + (-c^2)^(2/3)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)) - 363186
8240*(-c^2)^(2/3)*b^2*log((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)*c - (
-c^2)^(2/3)) + 3*(3486784401*c^17 + 641744532*a^2*c^9*x^2 - 11373139206*b*c
^9 + 567*(885735*a*c^13 + 1179178*a*b*c^5)*x - 2*(301327047*c^14 + 57308050
8*a^2*c^6*x^2 - 286540254*b*c^6 + 988*(177147*a*c^10 + 918995*a*b*c^2)*x +
988*(177147*c^10 - 580041*a*c^6*x - 918995*b*c^2)*sqrt(a^2*x^2 - b))*(a*x +
 sqrt(a^2*x^2 - b))^(3/4) + 81*(6200145*c^13 + 7922772*a*c^9*x - 8254246*b*
c^5)*sqrt(a^2*x^2 - b) + 6*(129140163*c^15 + 92432340*a^2*c^7*x^2 - 4555593
90*b*c^7 + 364*(177147*a*c^11 + 498883*a*b*c^3)*x + 364*(177147*c^11 + 2539
35*a*c^7*x - 498883*b*c^3)*sqrt(a^2*x^2 - b))*sqrt(a*x + sqrt(a^2*x^2 - b))
 - 9*(129140163*c^16 + 66023100*a^2*c^8*x^2 - 442354770*b*c^8 + 91*(531441*
```

```
a*c^12 + 997766*a*b*c^4)*x + 13*(3720087*c^12 + 5078700*a*c^8*x - 6984362*b
*c^4)*sqrt(a^2*x^2 - b))*(a*x + sqrt(a^2*x^2 - b))^(1/4))*(c + (a*x + sqrt(
a^2*x^2 - b))^(1/4))^(1/3))/(a*c^7)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(a*x+(a^2*x^2-b)^(1/2))^(3/4)/(c+(a*x+(a^2*x^2-
b)^(1/2))^(1/4))^(2/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^{2} x^{2}-b}\,\left(a x +\sqrt{a^{2} x^{2}-b}\right)^{\frac{3}{4}}}{\left(c +\left(a x +\sqrt{a^{2} x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b)^(1/2)*(a*x+(a^2*x^2-b)^(1/2))^(3/4)/(c+(a*x+(a^2*x^2-b)^(1/
2))^(1/4))^(2/3),x)
```

```
[Out] int((a^2*x^2-b)^(1/2)*(a*x+(a^2*x^2-b)^(1/2))^(3/4)/(c+(a*x+(a^2*x^2-b)^(1/
2))^(1/4))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^{2} x^{2} - b} {\left(a x + \sqrt{a^{2} x^{2} - b}\right)}^{\frac{3}{4}}}{{\left(c + {\left(a x + \sqrt{a^{2} x^{2} - b}\right)}^{\frac{1}{4}}\right)}^{\frac{2}{3}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(a*x+(a^2*x^2-b)^(1/2))^(3/4)/(c+(a*x+(a^2*x^2-
b)^(1/2))^(1/4))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 - b)*(a*x + sqrt(a^2*x^2 - b))^(3/4)/(c + (a*x + sqr
t(a^2*x^2 - b))^(1/4))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{3/4}\,\sqrt{a^2\,x^2-b}}{{\left(c+{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{1/4}\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (a^2*x^2 - b)^(1/2))^(3/4)*(a^2*x^2 - b)^(1/2))/(c + (a*x + (a^
2*x^2 - b)^(1/2))^(1/4))^(2/3),x)
```

```
[Out] int(((a*x + (a^2*x^2 - b)^(1/2))^(3/4)*(a^2*x^2 - b)^(1/2))/(c + (a*x + (a^
2*x^2 - b)^(1/2))^(1/4))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{3}{4}}\sqrt{a^2x^2-b}}{\left(c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b)**(1/2)*(a*x+(a**2*x**2-b)**(1/2))**(3/4)/(c+(a*x+(a
**2*x**2-b)**(1/2))**(1/4))**(2/3),x)
```

```
[Out] Integral((a*x + sqrt(a**2*x**2 - b))**(3/4)*sqrt(a**2*x**2 - b)/(c + (a*x +
 sqrt(a**2*x**2 - b))**(1/4))**(2/3), x)
```

3.2434 $\int \frac{\sqrt{-b+a^2x^2}}{\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx$

Optimal. Leaf size=1186

$$-\frac{21505\tan^{-1}\left(\frac{2\sqrt[3]{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}}{\sqrt{3}\sqrt[3]{c}}+\frac{1}{\sqrt{3}}\right)b^2}{19683\sqrt{3}\,ac^{26/3}}+\frac{21505\log\left(\sqrt[3]{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}-\sqrt[3]{c}\right)b^2}{59049ac^{26/3}}-\frac{21505\log\left(c^{2/3}+\right.}{}$$

Rubi [F] time = 0.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-b+a^2x^2}}{\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[-b + a^2*x^2]/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3),x]
```

```
[Out] Defer[Int][Sqrt[-b + a^2*x^2]/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3),
 x]
```

Rubi steps

$$\int \frac{\sqrt{-b+a^2x^2}}{\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx=\int \frac{\sqrt{-b+a^2x^2}}{\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx$$

Mathematica [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-b+a^2x^2}}{\left(c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[-b + a^2*x^2]/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3),x
]
```

```
[Out] Integrate[Sqrt[-b + a^2*x^2]/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3),
x]
```

IntegrateAlgebraic [A] time = 3.31, size = 1186, normalized size = 1.00

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-b + a^2*x^2]/(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4)
)^(2/3),x]
```

```
[Out] ((-748701954*b^2*c^7 + 8135830269*b*c^15 - 1204701498*a*b^2*c^3*x + 3515482
215*a*b*c^11*x - 16271660538*a^2*c^15*x^2 - 4687309620*a^3*c^11*x^3)*(c + (
a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + (820006902*b^2*c^6 - 2711943423*b*
c^14 + 1472412942*a*b^2*c^2*x - 3046751253*a*b*c^10*x + 5423886846*a^2*c^14
*x^2 + 4062335004*a^3*c^10*x^3)*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*(c + (a*x
+ Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + (-911118780*b^2*c^5 + 1807962282*b*c^1
3 - 1963217256*a*b^2*c*x + 2708223336*a*b*c^9*x - 3615924564*a^2*c^13*x^2 -
 3610964448*a^3*c^9*x^3)*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*(c + (a*x + Sqrt[-b
 + a^2*x^2])^(1/4))^(1/3) + (1032601284*b^2*c^4 - 1406192886*b*c^12 + 32720
28760*a*b^2*x - 2450297304*a*b*c^8*x + 2812385772*a^2*c^12*x^2 + 3267063072
*a^3*c^8*x^3)*(a*x + Sqrt[-b + a^2*x^2])^(3/4)*(c + (a*x + Sqrt[-b + a^2*x^
2])^(1/4))^(1/3) + Sqrt[-b + a^2*x^2]*((-1204701498*b^2*c^3 + 1171827405*b*
c^11 - 16271660538*a*c^15*x - 4687309620*a^2*c^11*x^2)*(c + (a*x + Sqrt[-b
+ a^2*x^2])^(1/4))^(1/3) + (1472412942*b^2*c^2 - 1015583751*b*c^10 + 542388
6846*a*c^14*x + 4062335004*a^2*c^10*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*(
c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + (-1963217256*b^2*c + 90274111
2*b*c^9 - 3615924564*a*c^13*x - 3610964448*a^2*c^9*x^2)*Sqrt[a*x + Sqrt[-b
+ a^2*x^2]]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + (3272028760*b^2
- 816765768*b*c^8 + 2812385772*a*c^12*x + 3267063072*a^2*c^8*x^2)*(a*x + Sq
rt[-b + a^2*x^2])^(3/4)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)))/(119
79231264*a^2*c^8*x*Sqrt[-b + a^2*x^2] + 5989615632*a*c^8*(-b + 2*a^2*x^2))
- (21505*b^2*ArcTan[1/Sqrt[3] + (2*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(
1/3))/(Sqrt[3]*c^(1/3))]])/(19683*Sqrt[3]*a*c^(26/3)) + (2*Sqrt[3]*b*ArcTan[
1/Sqrt[3] + (2*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3))/(Sqrt[3]*c^(1/
3))]])/(a*c^(2/3)) + (21505*b^2*Log[-c^(1/3) + (c + (a*x + Sqrt[-b + a^2*x^2
])^(1/4))^(1/3)]])/(59049*a*c^(26/3)) - (2*b*Log[-c^(1/3) + (c + (a*x + Sqrt
[-b + a^2*x^2])^(1/4))^(1/3)]])/(a*c^(2/3)) - (21505*b^2*Log[c^(2/3) + c^(1/
3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + (c + (a*x + Sqrt[-b + a^2
*x^2])^(1/4))^(2/3)]])/(118098*a*c^(26/3)) + (b*Log[c^(2/3) + c^(1/3)*(c + (
a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + (c + (a*x + Sqrt[-b + a^2*x^2])^(1
/4))^(2/3)]])/(a*c^(2/3))
```

fricas [A] time = 0.59, size = 644, normalized size = 0.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(2/3),x, algo
rithm="fricas")
```

```
[Out] 1/17968846896*(304304*sqrt(3)*(118098*b*c^9 - 21505*b^2*c)*sqrt(-(-c^2)^(1/
3))*arctan(-1/3*(sqrt(3)*(-c^2)^(1/3)*c*sqrt(-(-c^2)^(1/3)) - 2*sqrt(3)*(-c
^2)^(2/3)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)*sqrt(-(-c^2)^(1/3)))/
c^2) + 152152*(118098*b*c^8 - 21505*b^2)*(-c^2)^(2/3)*log((c + (a*x + sqrt(
a^2*x^2 - b))^(1/4))^(2/3)*c - (-c^2)^(1/3)*c + (-c^2)^(2/3)*(c + (a*x + sq
rt(a^2*x^2 - b))^(1/4))^(1/3)) - 304304*(118098*b*c^8 - 21505*b^2)*(-c^2)^(
2/3)*log((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)*c - (-c^2)^(2/3)) - 3*
(8135830269*c^17 + 1497403908*a^2*c^9*x^2 - 748701954*b*c^9 + 567*(2066715*
a*c^13 + 2124694*a*b*c^5)*x - 2*(703096443*c^14 + 1032601284*a^2*c^6*x^2 -
516300642*b*c^6 + 6916*(59049*a*c^10 + 236555*a*b*c^2)*x + 988*(413343*c^10
 - 1045143*a*c^6*x - 1655885*b*c^2)*sqrt(a^2*x^2 - b))*(a*x + sqrt(a^2*x^2
- b))^(3/4) + 567*(2066715*c^13 - 2640924*a*c^9*x - 2124694*b*c^5)*sqrt(a^2
*x^2 - b) + 6*(301327047*c^15 + 303706260*a^2*c^7*x^2 - 151853130*b*c^7 + 3
64*(413343*a*c^11 + 898909*a*b*c^3)*x + 52*(2893401*c^11 - 5840505*a*c^7*x
- 6292363*b*c^3)*sqrt(a^2*x^2 - b))*sqrt(a*x + sqrt(a^2*x^2 - b)) - 9*(3013
27047*c^16 + 182223756*a^2*c^8*x^2 - 91111878*b*c^8 + 91*(1240029*a*c^12 +
1797818*a*b*c^4)*x + 13*(8680203*c^12 - 14017212*a*c^8*x - 12584726*b*c^4)*
sqrt(a^2*x^2 - b))*(a*x + sqrt(a^2*x^2 - b))^(1/4))*(c + (a*x + sqrt(a^2*x^
2 - b))^(1/4))^(1/3))/(a*c^10)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(2/3),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2-b}}{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(2/3),x)
```

```
[Out] int((a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2-b}}{\left(c+\left(ax+\sqrt{a^2x^2-b}\right)^{\frac{1}{4}}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)/(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(2/3),x, algo
rithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 - b)/(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a^2\,x^2-b}}{{\left(c+{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{1/4}\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2 - b)^(1/2)/(c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(2/3),x)
```

```
[Out] int((a^2*x^2 - b)^(1/2)/(c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2x^2-b}}{\left(c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b)**(1/2)/(c+(a*x+(a**2*x**2-b)**(1/2))**(1/4))**(2/3)
,x)
```

```
[Out] Integral(sqrt(a**2*x**2 - b)/(c + (a*x + sqrt(a**2*x**2 - b))**(1/4))**(2/3
), x)
```

3.2435

$$\int \left(b+a^2x^2\right)^{3/2} \sqrt{ax+\sqrt{b+a^2x^2}}\,\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Optimal. Leaf size=1202

$$-\frac{33\tanh^{-1}\left(\frac{\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{\sqrt{c}}\right)b^4}{8192ac^{13/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}}{\sqrt{c}}\right)b^3}{16ac^{5/2}}+\frac{\sqrt{ax+\sqrt{a^2x^2+b}}\,\sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}\,(1}{}$$

Rubi [F] time = 0.88, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(b+a^2x^2\right)^{3/2} \sqrt{ax+\sqrt{b+a^2x^2}}\,\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(b + a^2*x^2)^(3/2)*Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + S
qrt[b + a^2*x^2]]],x]
```

```
[Out] Defer[Int][(b + a^2*x^2)^(3/2)*Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[
a*x + Sqrt[b + a^2*x^2]]], x]
```

Rubi steps

$$\int \left(b+a^2x^2\right)^{3/2} \sqrt{ax+\sqrt{b+a^2x^2}}\,\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx=\int \left(b+a^2x^2\right)^{3/2} \sqrt{ax+\sqrt{b+a^2x^2}}\,\sqrt{c+\sqrt{ax+}}$$

Mathematica [F] time = 22.26, size = 0, normalized size = 0.00

$$\int \left(b+a^2x^2\right)^{3/2} \sqrt{ax+\sqrt{b+a^2x^2}}\,\sqrt{c+\sqrt{ax+\sqrt{b+a^2x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(b + a^2*x^2)^(3/2)*Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a
*x + Sqrt[b + a^2*x^2]]],x]
```

```
[Out] Integrate[(b + a^2*x^2)^(3/2)*Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a
*x + Sqrt[b + a^2*x^2]]], x]
```

IntegrateAlgebraic [A] time = 4.57, size = 1202, normalized size = 1.00

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + a^2*x^2)^(3/2)*Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c
 + Sqrt[a*x + Sqrt[b + a^2*x^2]]],x]
```

```
[Out] ((256071816*b^5*c^2 + 50005263360*b^4*c^6 - 2317090816*b^3*c^10 - 234881024
*b^2*c^14 + 1440403965*a*b^5*x + 22543188096*a*b^4*c^4*x - 7568457728*a*b^3
```

```
*c^8*x - 734003200*a*b^2*c^12*x + 512143632*a^2*b^4*c^2*x^2 + 682252247040*
a^2*b^3*c^6*x^2 - 19692781568*a^2*b^2*c^10*x^2 - 1879048192*a^2*b*c^14*x^2
+ 1920538620*a^3*b^4*x^3 + 29797447680*a^3*b^3*c^4*x^3 - 33029095424*a^3*b^
2*c^8*x^3 - 2936012800*a^3*b*c^12*x^3 + 1247785943040*a^4*b^2*c^6*x^4 - 231
60946688*a^4*b*c^10*x^4 - 1879048192*a^4*c^14*x^4 - 33035911168*a^5*b*c^8*x
^5 - 2348810240*a^5*c^12*x^5 + 748215336960*a^6*b*c^6*x^6 - 3699376128*a^6*
c^10*x^6 - 6297747456*a^7*c^8*x^7 + 200740700160*a^8*c^6*x^8)*Sqrt[c + Sqrt
[a*x + Sqrt[b + a^2*x^2]]] + (-320089770*b^5*c - 5143607040*b^4*c^5 + 17570
85696*b^3*c^9 + 176160768*b^2*c^13 - 219490128*a*b^4*c^3*x + 185450137600*a
*b^3*c^7*x + 13851164672*a*b^2*c^11*x + 1409286144*a*b*c^15*x - 640179540*a
^2*b^4*c*x^2 - 9932482560*a^2*b^3*c^5*x^2 + 15116402688*a^2*b^2*c^9*x^2 + 1
409286144*a^2*b*c^13*x^2 + 267624448000*a^3*b^2*c^7*x^3 + 20180893696*a^3*b
*c^11*x^3 + 1879048192*a^3*c^15*x^3 + 18295554048*a^4*b*c^9*x^4 + 140928614
4*a^4*c^13*x^4 + 29595238400*a^5*b*c^7*x^5 + 2055208960*a^5*c^11*x^5 + 3391
094784*a^6*c^9*x^6 + 5904138240*a^7*c^7*x^7)*Sqrt[a*x + Sqrt[b + a^2*x^2]]*
Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]] + Sqrt[b + a^2*x^2]*((480134655*b^5
 + 7644464256*b^4*c^4 - 1474330624*b^3*c^8 - 146800640*b^2*c^12 + 512143632
*a*b^4*c^2*x + 276208558080*a*b^3*c^6*x - 9499574272*a*b^2*c^10*x - 9395240
96*a*b*c^14*x + 1920538620*a^2*b^4*x^2 + 29797447680*a^2*b^3*c^4*x^2 - 1887
2795136*a^2*b^2*c^8*x^2 - 1761607680*a^2*b*c^12*x^2 + 948956037120*a^3*b^2*
c^6*x^3 - 21311258624*a^3*b*c^10*x^3 - 1879048192*a^3*c^14*x^3 - 2988703744
0*a^4*b*c^8*x^4 - 2348810240*a^4*c^12*x^4 + 647844986880*a^5*b*c^6*x^5 - 36
99376128*a^5*c^10*x^5 - 6297747456*a^6*c^8*x^6 + 200740700160*a^7*c^6*x^7)*
Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]] + (-219490128*b^4*c^3 + 60891084800
*b^3*c^7 + 4531421184*b^2*c^11 + 469762048*b*c^15 - 640179540*a*b^4*c*x - 9
932482560*a*b^3*c^5*x + 7240286208*a*b^2*c^9*x + 704643072*a*b*c^13*x + 255
040880640*a^2*b^2*c^7*x^2 + 19153289216*a^2*b*c^11*x^2 + 1879048192*a^2*c^1
5*x^2 + 16600006656*a^3*b*c^9*x^3 + 1409286144*a^3*c^13*x^3 + 26643169280*a
^4*b*c^7*x^4 + 2055208960*a^4*c^11*x^4 + 3391094784*a^5*c^9*x^5 + 590413824
0*a^6*c^7*x^6)*Sqrt[a*x + Sqrt[b + a^2*x^2]]*Sqrt[c + Sqrt[a*x + Sqrt[b + a
^2*x^2]]]))/(119189790720*a*c^6*(a*x + Sqrt[b + a^2*x^2])^(7/2)) - (33*b^4*
ArcTanh[Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]]/Sqrt[c]])/(8192*a*c^(13/2))
 - (b^3*ArcTanh[Sqrt[c + Sqrt[a*x + Sqrt[b + a^2*x^2]]]/Sqrt[c]])/(16*a*c^(
5/2))
```

fricas [A] time = 0.52, size = 1193, normalized size = 0.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b)^(3/2)*(a*x+(a^2*x^2+b)^(1/2))^(1/2)*(c+(a*x+(a^2*x^2+
b)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/238379581440*(14549535*(512*b^3*c^4 + 33*b^4)*sqrt(c)*log(2*(a*sqrt(c)*x
 - sqrt(a^2*x^2 + b)*sqrt(c))*sqrt(a*x + sqrt(a^2*x^2 + b))*sqrt(c + sqrt(a
*x + sqrt(a^2*x^2 + b))) - 2*(a*c*x - sqrt(a^2*x^2 + b)*c)*sqrt(a*x + sqrt(
a^2*x^2 + b)) + b) + 2*(469762048*c^16 + 738017280*a^4*c^8*x^4 + 4531421184
*b*c^12 + 60891084800*b^2*c^8 - 219490128*b^3*c^4 + 33792*(12544*a^3*c^10 +
 20995*a^3*b*c^6)*x^3 + 32*(8028160*a^2*c^12 + 98309120*a^2*b*c^8 - 1371813
3*a^2*b^2*c^4)*x^2 + 6*(29360128*a*c^14 + 328171520*a*b*c^10 + 916389760*a*
b^2*c^6 + 53348295*a*b^3*c^2)*x + 2*(88080384*c^14 + 369008640*a^3*c^8*x^3
+ 878542848*b*c^10 - 2571803520*b^2*c^6 - 160044885*b^3*c^2 + 16896*(12544*
a^2*c^10 - 20995*a^2*b*c^6)*x^2 + 16*(8028160*a*c^12 + 86777600*a*b*c^8 + 1
3718133*a*b^2*c^4)*x)*sqrt(a^2*x^2 + b) - (234881024*c^15 + 4480819200*a^4*
c^7*x^4 + 2317090816*b*c^11 - 50005263360*b^2*c^7 - 256071816*b^3*c^3 + 219
648*(1792*a^3*c^9 + 3553*a^3*b*c^5)*x^3 + 48*(4816896*a^2*c^11 + 469580800*
a^2*b*c^7 - 10669659*a^2*b^2*c^3)*x^2 + (146800640*a*c^13 + 1671135232*a*b*
c^9 + 8034668928*a*b^2*c^5 + 480134655*a*b^3*c)*x + (146800640*c^13 - 29573
406720*a^3*c^7*x^3 + 1474330624*b*c^9 - 7644464256*b^2*c^5 - 480134655*b^3*
c + 219648*(1792*a^2*c^9 - 3553*a^2*b*c^5)*x^2 + 48*(4816896*a*c^11 - 15872
```

```
39680*a*b*c^7 + 10669659*a*b^2*c^3)*x)*sqrt(a^2*x^2 + b))*sqrt(a*x + sqrt(a
^2*x^2 + b)))*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b))))/(a*c^7), 1/119189790
720*(14549535*(512*b^3*c^4 + 33*b^4)*sqrt(-c)*arctan(sqrt(-c)*sqrt(c + sqrt
(a*x + sqrt(a^2*x^2 + b)))/c) + (469762048*c^16 + 738017280*a^4*c^8*x^4 + 4
531421184*b*c^12 + 60891084800*b^2*c^8 - 219490128*b^3*c^4 + 33792*(12544*a
^3*c^10 + 20995*a^3*b*c^6)*x^3 + 32*(8028160*a^2*c^12 + 98309120*a^2*b*c^8
- 13718133*a^2*b^2*c^4)*x^2 + 6*(29360128*a*c^14 + 328171520*a*b*c^10 + 916
389760*a*b^2*c^6 + 53348295*a*b^3*c^2)*x + 2*(88080384*c^14 + 369008640*a^3
*c^8*x^3 + 878542848*b*c^10 - 2571803520*b^2*c^6 - 160044885*b^3*c^2 + 1689
6*(12544*a^2*c^10 - 20995*a^2*b*c^6)*x^2 + 16*(8028160*a*c^12 + 86777600*a*
b*c^8 + 13718133*a*b^2*c^4)*x)*sqrt(a^2*x^2 + b) - (234881024*c^15 + 448081
9200*a^4*c^7*x^4 + 2317090816*b*c^11 - 50005263360*b^2*c^7 - 256071816*b^3*
c^3 + 219648*(1792*a^3*c^9 + 3553*a^3*b*c^5)*x^3 + 48*(4816896*a^2*c^11 + 4
69580800*a^2*b*c^7 - 10669659*a^2*b^2*c^3)*x^2 + (146800640*a*c^13 + 167113
5232*a*b*c^9 + 8034668928*a*b^2*c^5 + 480134655*a*b^3*c)*x + (146800640*c^1
3 - 29573406720*a^3*c^7*x^3 + 1474330624*b*c^9 - 7644464256*b^2*c^5 - 48013
4655*b^3*c + 219648*(1792*a^2*c^9 - 3553*a^2*b*c^5)*x^2 + 48*(4816896*a*c^1
1 - 1587239680*a*b*c^7 + 10669659*a*b^2*c^3)*x)*sqrt(a^2*x^2 + b))*sqrt(a*x
 + sqrt(a^2*x^2 + b)))*sqrt(c + sqrt(a*x + sqrt(a^2*x^2 + b))))/(a*c^7)]
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b)^(3/2)*(a*x+(a^2*x^2+b)^(1/2))^(1/2)*(c+(a*x+(a^2*x^2+
b)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} x^{2}+b\right)^{\frac{3}{2}} \sqrt{a x +\sqrt{a^{2} x^{2}+b}}\, \sqrt{c +\sqrt{a x +\sqrt{a^{2} x^{2}+b}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2+b)^(3/2)*(a*x+(a^2*x^2+b)^(1/2))^(1/2)*(c+(a*x+(a^2*x^2+b)^(1/
2))^(1/2))^(1/2),x)
```

```
[Out] int((a^2*x^2+b)^(3/2)*(a*x+(a^2*x^2+b)^(1/2))^(1/2)*(c+(a*x+(a^2*x^2+b)^(1/
2))^(1/2))^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(a^{2} x^{2} + b\right)}^{\frac{3}{2}} \sqrt{a x + \sqrt{a^{2} x^{2} + b}} \sqrt{c + \sqrt{a x + \sqrt{a^{2} x^{2} + b}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2+b)^(3/2)*(a*x+(a^2*x^2+b)^(1/2))^(1/2)*(c+(a*x+(a^2*x^2+
b)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 + b)^(3/2)*sqrt(a*x + sqrt(a^2*x^2 + b))*sqrt(c + sqrt(a
*x + sqrt(a^2*x^2 + b))), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\sqrt{a^2\,x^2+b}+a\,x}\,{\left(a^2\,x^2+b\right)}^{3/2}\,\sqrt{c+\sqrt{\sqrt{a^2\,x^2+b}+a\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b + a^2*x^2)^(1/2) + a*x)^(1/2)*(b + a^2*x^2)^(3/2)*(c + ((b + a^2*x^
2)^(1/2) + a*x)^(1/2))^(1/2),x)
```

```
[Out] int(((b + a^2*x^2)^(1/2) + a*x)^(1/2)*(b + a^2*x^2)^(3/2)*(c + ((b + a^2*x^
2)^(1/2) + a*x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+\sqrt{ax+\sqrt{a^2x^2+b}}}\,\sqrt{ax+\sqrt{a^2x^2+b}}\,\left(a^2x^2+b\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2+b)**(3/2)*(a*x+(a**2*x**2+b)**(1/2))**(1/2)*(c+(a*x+(a
**2*x**2+b)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(c + sqrt(a*x + sqrt(a**2*x**2 + b)))*sqrt(a*x + sqrt(a**2*x**
2 + b))*(a**2*x**2 + b)**(3/2), x)
```

3.2436 $\int \frac{\sqrt{-b+a^2x^2}\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx$

Optimal. Leaf size=1225

$$-\frac{21505\tan^{-1}\left(\frac{2\sqrt[3]{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}}{\sqrt{3}\sqrt[3]{c}}+\frac{1}{\sqrt{3}}\right)b^2}{531441\sqrt{3}\,ac^{26/3}}+\frac{21505\log\left(\sqrt[3]{c+\sqrt[4]{ax+\sqrt{a^2x^2-b}}}-\sqrt[3]{c}\right)b^2}{1594323ac^{26/3}}-\frac{21505\log\left(c^{2/3}+\sqrt[3]{\ldots}\right.}{\ldots}$$

Rubi [F] time = 0.95, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{-b+a^2x^2}\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[-b + a^2*x^2]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3))/(a*x
+ Sqrt[-b + a^2*x^2])^(1/4),x]
```

```
[Out] Defer[Int][(Sqrt[-b + a^2*x^2]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)
)/(a*x + Sqrt[-b + a^2*x^2])^(1/4), x]
```

Rubi steps

$$\int \frac{\sqrt{-b+a^2x^2}\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx=\int \frac{\sqrt{-b+a^2x^2}\sqrt[3]{c+\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}}{\sqrt[4]{ax+\sqrt{-b+a^2x^2}}}\,dx$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[-b + a^2*x^2]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3))
/(a*x + Sqrt[-b + a^2*x^2])^(1/4),x]
```

```
[Out] $Aborted
```

IntegrateAlgebraic [A] time = 4.27, size = 1225, normalized size = 1.00

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-b + a^2*x^2]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4
))^(1/3))/(a*x + Sqrt[-b + a^2*x^2])^(1/4),x]
```

```
[Out] ((-3272028760*b^3 - 319355415288*b^2*c^8 + 1032601284*a*b^2*c^4*x + 5423886
846*a*b*c^12*x + 6544057520*a^2*b^2*x^2 + 470457082368*a^2*b*c^8*x^2 - 7231
```

```
849128*a^3*c^12*x^3 + 176421405888*a^4*c^8*x^4)*(c + (a*x + Sqrt[-b + a^2*x
^2])^(1/4))^(1/3) + (-748701954*b^2*c^7 - 10460353203*b*c^15 - 1204701498*a
*b^2*c^3*x - 4519905705*a*b*c^11*x + 20920706406*a^2*c^15*x^2 + 6026540940*
a^3*c^11*x^3)*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*(c + (a*x + Sqrt[-b + a^2*x^
2])^(1/4))^(1/3) + (820006902*b^2*c^6 + 3486784401*b*c^14 + 1472412942*a*b^
2*c^2*x + 3917251611*a*b*c^10*x - 6973568802*a^2*c^14*x^2 - 5223002148*a^3*
c^10*x^3)*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*(c + (a*x + Sqrt[-b + a^2*x^2])^(1
/4))^(1/3) + (-911118780*b^2*c^5 - 2324522934*b*c^13 - 1963217256*a*b^2*c*x
 - 3482001432*a*b*c^9*x + 4649045868*a^2*c^13*x^2 + 4642668576*a^3*c^9*x^3)
*(a*x + Sqrt[-b + a^2*x^2])^(3/4)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1
/3) + Sqrt[-b + a^2*x^2]*((1032601284*b^2*c^4 + 1807962282*b*c^12 + 6544057
520*a*b^2*x + 558667785312*a*b*c^8*x - 7231849128*a^2*c^12*x^2 + 1764214058
88*a^3*c^8*x^3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + (-1204701498
*b^2*c^3 - 1506635235*b*c^11 + 20920706406*a*c^15*x + 6026540940*a^2*c^11*x
^2)*(a*x + Sqrt[-b + a^2*x^2])^(1/4)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))
^(1/3) + (1472412942*b^2*c^2 + 1305750537*b*c^10 - 6973568802*a*c^14*x - 52
23002148*a^2*c^10*x^2)*Sqrt[a*x + Sqrt[-b + a^2*x^2]]*(c + (a*x + Sqrt[-b +
 a^2*x^2])^(1/4))^(1/3) + (-1963217256*b^2*c - 1160667144*b*c^9 + 464904586
8*a*c^13*x + 4642668576*a^2*c^9*x^2)*(a*x + Sqrt[-b + a^2*x^2])^(3/4)*(c +
(a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)))/(161719622064*a*c^8*(a*x + Sqrt[-
b + a^2*x^2])^(9/4)) - (21505*b^2*ArcTan[1/Sqrt[3] + (2*(c + (a*x + Sqrt[-b
 + a^2*x^2])^(1/4))^(1/3))/(Sqrt[3]*c^(1/3))]])/(531441*Sqrt[3]*a*c^(26/3))
+ (2*b*ArcTan[1/Sqrt[3] + (2*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3))/
(Sqrt[3]*c^(1/3))]])/(Sqrt[3]*a*c^(2/3)) + (21505*b^2*Log[-c^(1/3) + (c + (a
*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)]])/(1594323*a*c^(26/3)) - (2*b*Log[-c^
(1/3) + (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3)]])/(3*a*c^(2/3)) - (215
05*b^2*Log[c^(2/3) + c^(1/3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) +
 (c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3)]])/(3188646*a*c^(26/3)) + (b*L
og[c^(2/3) + c^(1/3)*(c + (a*x + Sqrt[-b + a^2*x^2])^(1/4))^(1/3) + (c + (a
*x + Sqrt[-b + a^2*x^2])^(1/4))^(2/3)]])/(3*a*c^(2/3))
```

fricas [A] time = 0.60, size = 719, normalized size = 0.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3)/(a*x+(a
^2*x^2-b)^(1/2))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/485158866192*(304304*sqrt(3)*(1062882*b^2*c^9 - 21505*b^3*c)*sqrt(-(-c^2)
^(1/3))*arctan(-1/3*(sqrt(3)*(-c^2)^(1/3)*c*sqrt(-(-c^2)^(1/3)) - 2*sqrt(3)
*(-c^2)^(2/3)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)*sqrt(-(-c^2)^(1/3
)))/c^2) + 152152*(1062882*b^2*c^8 - 21505*b^3)*(-c^2)^(2/3)*log((c + (a*x
+ sqrt(a^2*x^2 - b))^(1/4))^(2/3)*c - (-c^2)^(1/3)*c + (-c^2)^(2/3)*(c + (a
*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)) - 304304*(1062882*b^2*c^8 - 21505*b^3
)*(-c^2)^(2/3)*log((c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)*c - (-c^2)^(
2/3)) + 3*(10460353203*b*c^17 - 1497403908*a^2*b*c^9*x^2 + 748701954*b^2*c^
9 + 567*(2657205*a*b*c^13 - 2124694*a*b^2*c^5)*x - 2*(35937693792*a^3*c^10*
x^3 + 903981141*b*c^14 - 1032601284*a^2*b*c^6*x^2 + 516300642*b^2*c^6 - 691
6*(28874961*a*b*c^10 + 236555*a*b^2*c^2)*x - 988*(36374184*a^2*c^10*x^2 - 1
61617113*b*c^10 - 1045143*a*b*c^6*x - 1655885*b^2*c^2)*sqrt(a^2*x^2 - b))*(
a*x + sqrt(a^2*x^2 - b))^(3/4) + 567*(2657205*b*c^13 + 2640924*a*b*c^9*x +
2124694*b^2*c^5)*sqrt(a^2*x^2 - b) + 6*(387420489*b*c^15 - 303706260*a^2*b*
c^7*x^2 + 151853130*b^2*c^7 + 364*(531441*a*b*c^11 - 898909*a*b^2*c^3)*x +
52*(3720087*b*c^11 + 5840505*a*b*c^7*x + 6292363*b^2*c^3)*sqrt(a^2*x^2 - b)
)*sqrt(a*x + sqrt(a^2*x^2 - b)) - 9*(387420489*b*c^16 - 182223756*a^2*b*c^8
*x^2 + 91111878*b^2*c^8 + 91*(1594323*a*b*c^12 - 1797818*a*b^2*c^4)*x + 13*
(11160261*b*c^12 + 14017212*a*b*c^8*x + 12584726*b^2*c^4)*sqrt(a^2*x^2 - b)
)*(a*x + sqrt(a^2*x^2 - b))^(1/4))*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1
/3))/(a*b*c^10)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3)/(a*x+(a
^2*x^2-b)^(1/2))^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^{2} x^{2}-b}\,\left(c+\left(a x+\sqrt{a^{2} x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}{\left(a x+\sqrt{a^{2} x^{2}-b}\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-b)^(1/2)*(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3)/(a*x+(a^2*x^2
-b)^(1/2))^(1/4),x)
```

```
[Out] int((a^2*x^2-b)^(1/2)*(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3)/(a*x+(a^2*x^2
-b)^(1/2))^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^{2} x^{2}-b}\left(c+\left(a x+\sqrt{a^{2} x^{2}-b}\right)^{\frac{1}{4}}\right)^{\frac{1}{3}}}{\left(a x+\sqrt{a^{2} x^{2}-b}\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-b)^(1/2)*(c+(a*x+(a^2*x^2-b)^(1/2))^(1/4))^(1/3)/(a*x+(a
^2*x^2-b)^(1/2))^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2*x^2 - b)*(c + (a*x + sqrt(a^2*x^2 - b))^(1/4))^(1/3)/(a*
x + sqrt(a^2*x^2 - b))^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{1/4}\right)}^{1/3}\,\sqrt{a^2\,x^2-b}}{{\left(a\,x+\sqrt{a^2\,x^2-b}\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(1/3)*(a^2*x^2 - b)^(1/2))/(a*
x + (a^2*x^2 - b)^(1/2))^(1/4),x)
```

```
[Out] int(((c + (a*x + (a^2*x^2 - b)^(1/2))^(1/4))^(1/3)*(a^2*x^2 - b)^(1/2))/(a*
x + (a^2*x^2 - b)^(1/2))^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c + \sqrt[4]{a x + \sqrt{a^{2} x^{2} - b}}} \sqrt{a^{2} x^{2} - b}}{\sqrt[4]{a x + \sqrt{a^{2} x^{2} - b}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-b)**(1/2)*(c+(a*x+(a**2*x**2-b)**(1/2))**(1/4))**(1/3)
/(a*x+(a**2*x**2-b)**(1/2))**(1/4),x)
```

```
[Out] Integral((c + (a*x + sqrt(a**2*x**2 - b))**(1/4))**(1/3)*sqrt(a**2*x**2 - b
)/(a*x + sqrt(a**2*x**2 - b))**(1/4), x)
```

3.2437 $\int \frac{(b^2+ax^2)^2\sqrt{b+\sqrt{b^2+ax^2}}}{(-b^2+ax^2)^2}\,dx$

Optimal. Leaf size=1310

$$\frac{5\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{b}\sqrt{b+\sqrt{b^2+ax^2}}}-\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{b}}\right)b^{3/2}}{\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{a}}-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{b}\sqrt{b+\sqrt{b^2+ax^2}}}-\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{b}}\right)b^{3/2}}{\sqrt{-1+\sqrt{2}}\sqrt{a}}+$$

Rubi [F] time = 2.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(b^2+ax^2)^2\sqrt{b+\sqrt{b^2+ax^2}}}{(-b^2+ax^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]])/(-b^2 + a*x^2)^2,x]
```

```
[Out] (2*a*x^3)/(3*(b + Sqrt[b^2 + a*x^2])^(3/2)) + (2*b*x)/Sqrt[b + Sqrt[b^2 + a
*x^2]] - b*Defer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b - Sqrt[a]*x), x] - b*D
efer[Int][Sqrt[b + Sqrt[b^2 + a*x^2]]/(b + Sqrt[a]*x), x] + a*b^2*Defer[Int
][Sqrt[b + Sqrt[b^2 + a*x^2]]/(Sqrt[a]*b - a*x)^2, x] + a*b^2*Defer[Int][Sq
rt[b + Sqrt[b^2 + a*x^2]]/(Sqrt[a]*b + a*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b^2+ax^2)^2\sqrt{b+\sqrt{b^2+ax^2}}}{(-b^2+ax^2)^2}\,dx &= \int \left(\sqrt{b+\sqrt{b^2+ax^2}}+\frac{4b^4\sqrt{b+\sqrt{b^2+ax^2}}}{(b^2-ax^2)^2}-\frac{4b^2\sqrt{b+\sqrt{b^2+ax^2}}}{b^2-ax^2}\right)dx\\
&= -\left((4b^2)\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b^2-ax^2}\,dx\right)+(4b^4)\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{(b^2-ax^2)^2}\,dx+\int \sqrt{b\\
&= \frac{2ax^3}{3\left(b+\sqrt{b^2+ax^2}\right)^{3/2}}+\frac{2bx}{\sqrt{b+\sqrt{b^2+ax^2}}}-(4b^2)\int\left(\frac{\sqrt{b+\sqrt{b^2+ax^2}}}{2b\left(b-\sqrt{a}\,x\right)}+\right.\\
&= \frac{2ax^3}{3\left(b+\sqrt{b^2+ax^2}\right)^{3/2}}+\frac{2bx}{\sqrt{b+\sqrt{b^2+ax^2}}}-(2b)\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b-\sqrt{a}\,x}\,dx-\\
&= \frac{2ax^3}{3\left(b+\sqrt{b^2+ax^2}\right)^{3/2}}+\frac{2bx}{\sqrt{b+\sqrt{b^2+ax^2}}}-(2b)\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b-\sqrt{a}\,x}\,dx-\\
&= \frac{2ax^3}{3\left(b+\sqrt{b^2+ax^2}\right)^{3/2}}+\frac{2bx}{\sqrt{b+\sqrt{b^2+ax^2}}}+b\int \frac{\sqrt{b+\sqrt{b^2+ax^2}}}{b-\sqrt{a}\,x}\,dx+b
\end{aligned}$$

Mathematica [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b^2+ax^2\right)^2\sqrt{b+\sqrt{b^2+ax^2}}}{\left(-b^2+ax^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]])/(-b^2 + a*x^2)^2,x]
```

```
[Out] Integrate[((b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]])/(-b^2 + a*x^2)^2, x
]
```

IntegrateAlgebraic [A] time = 1.03, size = 262, normalized size = 0.20

$$\frac{2x\left(ax^2-4b^2\right)\sqrt{ax^2+b^2}}{3\left(ax^2-b^2\right)\sqrt{\sqrt{ax^2+b^2}+b}}+\frac{\sqrt{\sqrt{2}-1}\left(6+\sqrt{2}\right)b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{2}-1}\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{2\sqrt{a}}+\frac{\left(\sqrt{2}-6\right)\sqrt{1+\sqrt{2}}\,b^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\sqrt{2}}\sqrt{a}\,x}{\sqrt{b}\sqrt{\sqrt{ax^2+b^2}+b}}\right)}{2\sqrt{a}}+\frac{2x\left(2abx^2-5b^3\right)}{3\left(ax^2-b^2\right)\sqrt{\sqrt{ax^2+b^2}+b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((b^2 + a*x^2)^2*Sqrt[b + Sqrt[b^2 + a*x^2]])/(-b^2 + a*
x^2)^2,x]
```

```
[Out] (2*x*(-4*b^2 + a*x^2)*Sqrt[b^2 + a*x^2])/(3*(-b^2 + a*x^2)*Sqrt[b + Sqrt[b^
2 + a*x^2]]) + (2*x*(-5*b^3 + 2*a*b*x^2))/(3*(-b^2 + a*x^2)*Sqrt[b + Sqrt[b
^2 + a*x^2]]) + (Sqrt[-1 + Sqrt[2]]*(6 + Sqrt[2])*b^(3/2)*ArcTan[(Sqrt[-1 +
 Sqrt[2]]*Sqrt[a]*x)/(Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/(2*Sqrt[a]) +
((-6 + Sqrt[2])*Sqrt[1 + Sqrt[2]]*b^(3/2)*ArcTanh[(Sqrt[1 + Sqrt[2]]*Sqrt[a
]*x)/(Sqrt[b]*Sqrt[b + Sqrt[b^2 + a*x^2]])])/(2*Sqrt[a])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^2*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2-b^2)^2,x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2+b^2\right)^2\sqrt{b+\sqrt{ax^2+b^2}}}{\left(ax^2-b^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^2*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2-b^2)^2,x, algori
thm="giac")
```

```
[Out] integrate((a*x^2 + b^2)^2*sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 - b^2)^2, x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,x^2+b^2\right)^2\sqrt{b+\sqrt{a\,x^2+b^2}}}{\left(a\,x^2-b^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b^2)^2*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2-b^2)^2,x)
```

```
[Out] int((a*x^2+b^2)^2*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2-b^2)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^2+b^2\right)^2\sqrt{b+\sqrt{ax^2+b^2}}}{\left(ax^2-b^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b^2)^2*(b+(a*x^2+b^2)^(1/2))^(1/2)/(a*x^2-b^2)^2,x, algori
thm="maxima")
```

```
[Out] integrate((a*x^2 + b^2)^2*sqrt(b + sqrt(a*x^2 + b^2))/(a*x^2 - b^2)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b^2+a\,x^2\right)^2\sqrt{b+\sqrt{b^2+a\,x^2}}}{\left(a\,x^2-b^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^2 + b^2)^2*(b + (a*x^2 + b^2)^(1/2))^(1/2))/(a*x^2 - b^2)^2,x)
```

```
[Out] int(((a*x^2 + b^2)^2*(b + (a*x^2 + b^2)^(1/2))^(1/2))/(a*x^2 - b^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b+\sqrt{ax^2+b^2}}\left(ax^2+b^2\right)^2}{\left(ax^2-b^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b**2)**2*(b+(a*x**2+b**2)**(1/2))**(1/2)/(a*x**2-b**2)**2
,x)
```

```
[Out] Integral(sqrt(b + sqrt(a*x**2 + b**2))*(a*x**2 + b**2)**2/(a*x**2 - b**2)**
2, x)
```

3.2438 $\int \frac{-b-ac+(1+c)x}{(-a+x)\sqrt[3]{(-a+x)(-b+x)^2}\,(b-ad+(-1+d)x)}\,dx$

Optimal. Leaf size=1387

$$\frac{\left(1+i\sqrt{3}\right)\sqrt[3]{a-x}\,(x-b)^{2/3}\left(\sqrt[3]{d}\,\sqrt[3]{a-x}+\sqrt[3]{x-b}\right)\left(d^{2/3}(a-x)^{2/3}-\sqrt[3]{d}\,\sqrt[3]{x-b}\,\sqrt[3]{a-x}+(x-b)^{2/3}\right)\left(\frac{3\left(a\sqrt[3]{x-b}-i\right.}{2(a-b)}\right.}{}$$

Rubi [A] time = 1.47, antiderivative size = 280, normalized size of antiderivative = 0.20, number of steps used = 4, number of rules used = 4, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.078$, Rules used = {6719, 155, 12, 91}

$$-\frac{\sqrt[3]{x-a}\,(x-b)^{2/3}(c+d)\log(-ad+b-(1-d)x)}{2d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}+\frac{3\sqrt[3]{x-a}\,(x-b)^{2/3}(c+d)\log\left(\sqrt[3]{d}\,\sqrt[3]{x-a}-\sqrt[3]{x-b}\right)}{2d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}+\frac{\sqrt{3}\,\sqrt[3]{x-a}\,(x-b)^{2/3}(c+d)\tan^{-1}\left(\frac{2\sqrt[3]{d}\,\sqrt[3]{x-a}}{\sqrt{3}\,\sqrt[3]{x-b}}+\frac{1}{\sqrt{3}}\right)}{d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{3(b-x)}{(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b - a*c + (1 + c)*x)/((-a + x)*((-a + x)*(-b + x)^2)^(1/3)*(b - a*d +
 (-1 + d)*x)),x]
```

```
[Out] (-3*(b - x))/((a - b)*(-((a - x)*(b - x)^2))^(1/3)) + (Sqrt[3]*(c + d)*(-a
+ x)^(1/3)*(-b + x)^(2/3)*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(-a + x)^(1/3))/(Sq
rt[3]*(-b + x)^(1/3))])/((a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(1/3)) - ((
c + d)*(-a + x)^(1/3)*(-b + x)^(2/3)*Log[b - a*d - (1 - d)*x])/(2*(a - b)*d
^(2/3)*(-((a - x)*(b - x)^2))^(1/3)) + (3*(c + d)*(-a + x)^(1/3)*(-b + x)^(
2/3)*Log[d^(1/3)*(-a + x)^(1/3) - (-b + x)^(1/3)])/(2*(a - b)*d^(2/3)*(-((a
 - x)*(b - x)^2))^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
```

```
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{-b-ac+(1+c)x}{(-a+x)\sqrt[3]{(-a+x)(-b+x)^2}\,(b-ad+(-1+d)x)}\,dx = \frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{-b-ac+(1+c)x}{(-a+x)^{4/3}(-b+x)^{2/3}(b-ad+(-1+d)x)}\,dx}{\sqrt[3]{(-a+x)(-b+x)^2}}$$

$$= -\frac{3(b-x)}{(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}} + \frac{\left(3\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right.}{(a-b)^2}$$

$$= -\frac{3(b-x)}{(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}} + \frac{\left((c+d)\sqrt[3]{-a+x}\,(-b+\right.}{\sqrt[3]{}}$$

$$= -\frac{3(b-x)}{(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}} + \frac{\sqrt{3}\,(c+d)\sqrt[3]{-a+x}\,(-}{(a-b)d^{2/3}}$$

Mathematica [C] time = 0.08, size = 66, normalized size = 0.05

$$-\frac{3(x-b)\left((c+d)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{b-x}{ad-dx}\right)-d\right)}{d(a-b)\sqrt[3]{(x-a)(b-x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b - a*c + (1 + c)*x)/((-a + x)*((-a + x)*(-b + x)^2)^(1/3)*(b -
 a*d + (-1 + d)*x)),x]
```

```
[Out] (-3*(-b + x)*(-d + (c + d)*Hypergeometric2F1[1/3, 1, 4/3, (b - x)/(a*d - d*
x)]))/((a - b)*d*((b - x)^2*(-a + x))^(1/3))
```

IntegrateAlgebraic [F] time = 180.69, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-b - a*c + (1 + c)*x)/((-a + x)*((-a + x)*(-b + x)^2)^(
1/3)*(b - a*d + (-1 + d)*x)),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.78, size = 473, normalized size = 0.34

$$\frac{2\sqrt{3}\left(abcd+abd^2+\left(cd+d^2\right)x^2-\left((a+b)cd+(a+b)d^2\right)x\right)\sqrt{-\left(-d^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{3}\left(\left(-d^2\right)^{\frac{1}{3}}(b-x)+2\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}d\right)\sqrt{-\left(-d^2\right)^{\frac{1}{3}}}}{3(bd-dx)}\right)+6\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{2}{3}}d^2-\left(abc+abd+(c+d)x^2-((a+b)c+(a+b)d)x\right)\left(-d^2\right)^{\frac{2}{3}}\log\left(\frac{\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{2}{3}}d^2+\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}(bd-dx)\left(-d^2\right)^{\frac{1}{3}}+\left(b^2-2bx+x^2\right)\left(-d^2\right)^{\frac{2}{3}}}{b^2-2bx+x^2}\right)+2\left(abc+abd+(c+d)x^2-((a+b)c+(a+b)d)x\right)\left(-d^2\right)^{\frac{2}{3}}\log\left(\frac{\left(-d^2\right)^{\frac{1}{3}}(b-x)-\left(-ab^2-(a+2b)x^2+x^3+\left(2ab+b^2\right)x\right)^{\frac{1}{3}}d}{b-x}\right)}{2\left((a-b)d^2x^2-\left(a^2-b^2\right)d^2x+\left(a^2b-ab^2\right)d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b-a*c+(1+c)*x)/(-a+x)/((-a+x)*(-b+x)^2)^(1/3)/(b-a*d+(-1+d)*x),
x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*(a*b*c*d + a*b*d^2 + (c*d + d^2)*x^2 - ((a + b)*c*d + (a + b
)*d^2)*x)*sqrt(-(-d^2)^(1/3))*arctan(-1/3*sqrt(3)*((-d^2)^(1/3)*(b - x) + 2
*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*d)*sqrt(-(-d^2)^(1/
3))/(b*d - d*x)) + 6*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)
```

```
*d^2 - (a*b*c + a*b*d + (c + d)*x^2 - ((a + b)*c + (a + b)*d)*x)*(-d^2)^(2/
3)*log(((-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3)*d^2 + (-a*b^
2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(b*d - d*x)*(-d^2)^(1/3) +
 (b^2 - 2*b*x + x^2)*(-d^2)^(2/3))/(b^2 - 2*b*x + x^2)) + 2*(a*b*c + a*b*d
+ (c + d)*x^2 - ((a + b)*c + (a + b)*d)*x)*(-d^2)^(2/3)*log(((-d^2)^(1/3)*(
b - x) - (-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*d)/(b - x))
)/((a - b)*d^2*x^2 - (a^2 - b^2)*d^2*x + (a^2*b - a*b^2)*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ac-(c+1)x+b}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}(ad-(d-1)x-b)(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b-a*c+(1+c)*x)/(-a+x)/((-a+x)*(-b+x)^2)^(1/3)/(b-a*d+(-1+d)*x),
x, algorithm="giac")
```

```
[Out] integrate(-(a*c - (c + 1)*x + b)/((-(a - x)*(b - x)^2)^(1/3)*(a*d - (d - 1)
*x - b)*(a - x)), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{-b-ac+(1+c)\,x}{(-a+x)\left((-a+x)\,(-b+x)^2\right)^{\frac{1}{3}}(b-ad+(-1+d)\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b-a*c+(1+c)*x)/(-a+x)/((-a+x)*(-b+x)^2)^(1/3)/(b-a*d+(-1+d)*x),x)
```

```
[Out] int((-b-a*c+(1+c)*x)/(-a+x)/((-a+x)*(-b+x)^2)^(1/3)/(b-a*d+(-1+d)*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ac-(c+1)x+b}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}(ad-(d-1)x-b)(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b-a*c+(1+c)*x)/(-a+x)/((-a+x)*(-b+x)^2)^(1/3)/(b-a*d+(-1+d)*x),
x, algorithm="maxima")
```

```
[Out] -integrate((a*c - (c + 1)*x + b)/((-(a - x)*(b - x)^2)^(1/3)*(a*d - (d - 1)
*x - b)*(a - x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int -\frac{b+a\,c-x\,(c+1)}{(a-x)\,\left(-(a-x)\,(b-x)^2\right)^{1/3}\,(b-a\,d+x\,(d-1))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*c - x*(c + 1))/((a - x)*(-(a - x)*(b - x)^2)^(1/3)*(b - a*d + x*
(d - 1))),x)
```

```
[Out] -int(-(b + a*c - x*(c + 1))/((a - x)*(-(a - x)*(b - x)^2)^(1/3)*(b - a*d +
x*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b-a*c+(1+c)*x)/(-a+x)/((-a+x)*(-b+x)**2)**(1/3)/(b-a*d+(-1+d)*x
),x)
```

```
[Out] Timed out
```

3.2439 $$\int \frac{-a-bc+(1+c)x}{\sqrt[3]{(-a+x)(-b+x)^2}\left(-a^2+b^2d+2(a-bd)x+(-1+d)x^2\right)}\,dx$$

Optimal. Leaf size=1655

$$(b-x)^{2/3}\sqrt[3]{x-a}\left(-\frac{\sqrt{3}bc\left(\sqrt{d}-1\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x-a}}{\sqrt[3]{x-a}-2\sqrt[6]{d}\sqrt[3]{b-x}}\right)}{2(a-b)^2d^{2/3}}-\frac{\sqrt{3}a\left(\sqrt{d}-1\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x-a}}{\sqrt[3]{x-a}-2\sqrt[6]{d}\sqrt[3]{b-x}}\right)}{2(a-b)^2d^{2/3}}-\frac{\sqrt{3}c\left(a-b\sqrt{d}\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x-a}}{\sqrt[3]{x-a}-2\sqrt[6]{d}\sqrt[3]{\ldots}}\right)}{2(a-b)^2d^{2/3}}\right.$$

Rubi [A] time = 1.96, antiderivative size = 561, normalized size of antiderivative = 0.34, number of steps used = 5, number of rules used = 3, integrand size = 60, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {6719, 6728, 91}

$$\frac{\sqrt[3]{x-a}(x-b)^{2/3}\left(c+\sqrt{d}\right)\log\left(2\left(\sqrt{d}+1\right)\left(a-b\sqrt{d}\right)-2(1-d)x\right)}{4d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}+\frac{\sqrt[3]{x-a}(x-b)^{2/3}\left(c-\sqrt{d}\right)\log\left(2\left(1-\sqrt{d}\right)\left(a+b\sqrt{d}\right)-2(1-d)x\right)}{4d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{3\sqrt[3]{x-a}(x-b)^{2/3}\left(c-\sqrt{d}\right)\log\left(-\frac{\sqrt[3]{x-a}}{\sqrt[6]{d}}-\sqrt[3]{x-b}\right)}{4d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{3\sqrt[3]{x-a}(x-b)^{2/3}\left(c+\sqrt{d}\right)\log\left(\frac{\sqrt[3]{x-a}}{\sqrt[6]{d}}-\sqrt[3]{x-b}\right)}{4d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{\sqrt{3}\sqrt[3]{x-a}(x-b)^{2/3}\left(c-\sqrt{d}\right)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-b}}\right)}{2d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{\sqrt{3}\sqrt[3]{x-a}(x-b)^{2/3}\left(c+\sqrt{d}\right)\tan^{-1}\left(\frac{2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-b}}+\frac{1}{\sqrt{3}}\right)}{2d^{2/3}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a - b*c + (1 + c)*x)/(((-a + x)*(-b + x)^2)^(1/3)*(-a^2 + b^2*d + 2*(
a - b*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] -1/2*(Sqrt[3]*(c - Sqrt[d])*(-a + x)^(1/3)*(-b + x)^(2/3)*ArcTan[1/Sqrt[3]
- (2*(-a + x)^(1/3))/(Sqrt[3]*d^(1/6)*(-b + x)^(1/3))])/((a - b)*d^(2/3)*(-
((a - x)*(b - x)^2))^(1/3)) - (Sqrt[3]*(c + Sqrt[d])*(-a + x)^(1/3)*(-b + x
)^(2/3)*ArcTan[1/Sqrt[3] + (2*(-a + x)^(1/3))/(Sqrt[3]*d^(1/6)*(-b + x)^(1/
3))])/(2*(a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(1/3)) + ((c + Sqrt[d])*(-a
 + x)^(1/3)*(-b + x)^(2/3)*Log[2*(1 + Sqrt[d])*(a - b*Sqrt[d]) - 2*(1 - d)*
x])/(4*(a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(1/3)) + ((c - Sqrt[d])*(-a +
 x)^(1/3)*(-b + x)^(2/3)*Log[2*(1 - Sqrt[d])*(a + b*Sqrt[d]) - 2*(1 - d)*x]
)/(4*(a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(1/3)) - (3*(c - Sqrt[d])*(-a +
 x)^(1/3)*(-b + x)^(2/3)*Log[-((-a + x)^(1/3)/d^(1/6)) - (-b + x)^(1/3)])/(
4*(a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(1/3)) - (3*(c + Sqrt[d])*(-a + x)
^(1/3)*(-b + x)^(2/3)*Log[(-a + x)^(1/3)/d^(1/6) - (-b + x)^(1/3)])/(4*(a -
 b)*d^(2/3)*(-((a - x)*(b - x)^2))^(1/3))
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-a-bc+(1+c)x}{\sqrt[3]{(-a+x)(-b+x)^2}\left(-a^2+b^2d+2(a-bd)x+(-1+d)x^2\right)}\,dx = \frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{-a-bc+(}{\sqrt[3]{-a+x}\,(-b+x)^{2/3}\left(-a^2+b^2\right.}}{\sqrt[3]{(-a+x)(-b+x)^2}}$$

$$= \frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int\left(\frac{1+c}{\sqrt[3]{-a+x}\,(-b+x)^{2/3}\left(-2(a-\right.}\right.}{}$$

$$= \frac{\left(\left(1+c-\frac{-c-d}{\sqrt{d}}\right)\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{}{\sqrt[3]{-a+}}}{\sqrt[3]{(-a+x)(-b}}$$

$$= -\frac{\sqrt{3}\left(c-\sqrt{d}\right)\sqrt[3]{-a+x}\,(-b+x)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{}}\right.}{2(a-b)d^{2/3}\sqrt[3]{-\left((a-x)(b-x\right.}}$$

Mathematica [C] time = 0.36, size = 109, normalized size = 0.07

$$\frac{3(x-b)\left(\left(\sqrt{d}-c\right){}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{\sqrt{d}\,(b-x)}{x-a}\right)+\left(c+\sqrt{d}\right){}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{\sqrt{d}\,(x-b)}{x-a}\right)\right)}{2\sqrt{d}\,(a-b)\sqrt[3]{(x-a)(b-x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a - b*c + (1 + c)*x)/(((-a + x)*(-b + x)^2)^(1/3)*(-a^2 + b^2*d
 + 2*(a - b*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] (3*(-b + x)*((-c + Sqrt[d])*Hypergeometric2F1[1/3, 1, 4/3, (Sqrt[d]*(b - x)
)/(-a + x)] + (c + Sqrt[d])*Hypergeometric2F1[1/3, 1, 4/3, (Sqrt[d]*(-b + x
))/(-a + x)]))/(2*(a - b)*Sqrt[d]*((b - x)^2*(-a + x))^(1/3))
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-a - b*c + (1 + c)*x)/(((-a + x)*(-b + x)^2)^(1/3)*(-a^
2 + b^2*d + 2*(a - b*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 1.67, size = 11598, normalized size = 7.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a-b*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(1/3)/(-a^2+b^2*d+2*(-b*d+a)*x
+(-1+d)*x^2),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2
)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d
^3)) + c^3 + 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*arctan(1/3
*(2*(sqrt(3)*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c*d^2*x - (a^4*b
- 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c*d^2)*sqrt((9*c^4 + 6*c^2*d + d^2
)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d
^3)) + sqrt(3)*(3*(a*b - b^2)*c^2*d + (a*b - b^2)*d^2 - (3*(a - b)*c^2*d +
(a - b)*d^2)*x))*sqrt(((18*(a^2*b - 2*a*b^2 + b^3)*c^9*d^2 - 24*(a^2*b - 2*
a*b^2 + b^3)*c^7*d^3 - 4*(a^2*b - 2*a*b^2 + b^3)*c^5*d^4 + 8*(a^2*b - 2*a*b
```

^2 + b^3)*c^3*d^5 + 2*(a^2*b - 2*a*b^2 + b^3)*c*d^6 - 2*(9*(a^2 - 2*a*b + b
^2)*c^9*d^2 - 12*(a^2 - 2*a*b + b^2)*c^7*d^3 - 2*(a^2 - 2*a*b + b^2)*c^5*d^
4 + 4*(a^2 - 2*a*b + b^2)*c^3*d^5 + (a^2 - 2*a*b + b^2)*c*d^6)*x - (3*(a^5*
b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^8*d^3 - 2*(a^5*b
 - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^6*d^4 - 4*(a^5*b
- 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^4*d^5 + 2*(a^5*b -
 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^2*d^6 + (a^5*b - 5*
a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*d^7 - (3*(a^5 - 5*a^4*b
+ 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^8*d^3 - 2*(a^5 - 5*a^4*b + 10*
a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^6*d^4 - 4*(a^5 - 5*a^4*b + 10*a^3*b
^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^4*d^5 + 2*(a^5 - 5*a^4*b + 10*a^3*b^2 -
10*a^2*b^3 + 5*a*b^4 - b^5)*c^2*d^6 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*
b^3 + 5*a*b^4 - b^5)*d^7)*x)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b +
 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)))*(-a*b^2 - (a
+ 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^
3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) + c^3 + 3*c*d)/((a^3 - 3*a^2*b + 3*a*
b^2 - b^3)*d^2))^(2/3) + (9*c^12 - 30*c^10*d + 31*c^8*d^2 - 4*c^6*d^3 - 9*c
^4*d^4 + 2*c^2*d^5 + d^6)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^
(2/3) - (9*(a*b^2 - b^3)*c^11*d - 21*(a*b^2 - b^3)*c^9*d^2 + 10*(a*b^2 - b^
3)*c^7*d^3 + 6*(a*b^2 - b^3)*c^5*d^4 - 3*(a*b^2 - b^3)*c^3*d^5 - (a*b^2 - b
^3)*c*d^6 + (9*(a - b)*c^11*d - 21*(a - b)*c^9*d^2 + 10*(a - b)*c^7*d^3 + 6
*(a - b)*c^5*d^4 - 3*(a - b)*c^3*d^5 - (a - b)*c*d^6)*x^2 - 2*(9*(a*b - b^2
)*c^11*d - 21*(a*b - b^2)*c^9*d^2 + 10*(a*b - b^2)*c^7*d^3 + 6*(a*b - b^2)*
c^5*d^4 - 3*(a*b - b^2)*c^3*d^5 - (a*b - b^2)*c*d^6)*x - (3*(a^4*b^2 - 4*a^
3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^8*d^3 - 8*(a^4*b^2 - 4*a^3*b^3 + 6*a^2
*b^4 - 4*a*b^5 + b^6)*c^6*d^4 + 6*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^
5 + b^6)*c^4*d^5 - (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*d^7 +
(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^8*d^3 - 8*(a^4 - 4*a^3*b +
 6*a^2*b^2 - 4*a*b^3 + b^4)*c^6*d^4 + 6*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^
3 + b^4)*c^4*d^5 - (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d^7)*x^2 - 2
(3(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^8*d^3 - 8*(a^4*b - 4*
a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^6*d^4 + 6*(a^4*b - 4*a^3*b^2 + 6*a^2
*b^3 - 4*a*b^4 + b^5)*c^4*d^5 - (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 +
b^5)*d^7)*x)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20
*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)))*(-((a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^
3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) + c^3 + 3*c*d)/((a^3 - 3*a^2*b +
3*a*b^2 - b^3)*d^2))^(1/3))/(b^2 - 2*b*x + x^2))*(-((a^3 - 3*a^2*b + 3*a*b^
2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20
*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) + c^3 + 3*c*d)/((a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*d^2))^(1/3) - 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b +
b^2)*x)^(1/3)*(sqrt(3)*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^7*d
^2 - 5*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^5*d^3 + (a^4 - 4*a^3*b
 + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^3*d^4 + (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^
3 + b^4)*c*d^5)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 -
 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) - sqrt(3)*(9*(a - b)*c^8*d
- 12*(a - b)*c^6*d^2 - 2*(a - b)*c^4*d^3 + 4*(a - b)*c^2*d^4 + (a - b)*d^5)
)*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6
 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) +
c^3 + 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3) - sqrt(3)*(9*b*c^
10 - 21*b*c^8*d + 10*b*c^6*d^2 + 6*b*c^4*d^3 - 3*b*c^2*d^4 - b*d^5 - (9*c^1
0 - 21*c^8*d + 10*c^6*d^2 + 6*c^4*d^3 - 3*c^2*d^4 - d^5)*x))/(9*b*c^10 - 21
*b*c^8*d + 10*b*c^6*d^2 + 6*b*c^4*d^3 - 3*b*c^2*d^4 - b*d^5 - (9*c^10 - 21*
c^8*d + 10*c^6*d^2 + 6*c^4*d^3 - 3*c^2*d^4 - d^5)*x)) + sqrt(3)*(((a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 1
5*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) - c^3 - 3*c*d)/(
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*arctan(1/3*(2*(sqrt(3)*((a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c*d^2*x - (a^4*b - 4*a^3*b^2 + 6*a^2*b

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^3 - 4*a*b^4 + b^5)*c*d^2)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 1
5*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) - sqrt(3)*(3*(a*
b - b^2)*c^2*d + (a*b - b^2)*d^2 - (3*(a - b)*c^2*d + (a - b)*d^2)*x))*sqrt
(((18*(a^2*b - 2*a*b^2 + b^3)*c^9*d^2 - 24*(a^2*b - 2*a*b^2 + b^3)*c^7*d^3
- 4*(a^2*b - 2*a*b^2 + b^3)*c^5*d^4 + 8*(a^2*b - 2*a*b^2 + b^3)*c^3*d^5 + 2
*(a^2*b - 2*a*b^2 + b^3)*c*d^6 - 2*(9*(a^2 - 2*a*b + b^2)*c^9*d^2 - 12*(a^2
 - 2*a*b + b^2)*c^7*d^3 - 2*(a^2 - 2*a*b + b^2)*c^5*d^4 + 4*(a^2 - 2*a*b +
b^2)*c^3*d^5 + (a^2 - 2*a*b + b^2)*c*d^6)*x + (3*(a^5*b - 5*a^4*b^2 + 10*a^
3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^8*d^3 - 2*(a^5*b - 5*a^4*b^2 + 10*a^3
*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^6*d^4 - 4*(a^5*b - 5*a^4*b^2 + 10*a^3*
b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^4*d^5 + 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b
^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^2*d^6 + (a^5*b - 5*a^4*b^2 + 10*a^3*b^3
- 10*a^2*b^4 + 5*a*b^5 - b^6)*d^7 - (3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2
*b^3 + 5*a*b^4 - b^5)*c^8*d^3 - 2*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3
+ 5*a*b^4 - b^5)*c^6*d^4 - 4*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a
*b^4 - b^5)*c^4*d^5 + 2*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
- b^5)*c^2*d^6 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*
d^7)*x)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*
b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)))*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2
*a*b + b^2)*x)^(1/3)*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*
c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b
^5 + b^6)*d^3)) - c^3 - 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3)
 + (9*c^12 - 30*c^10*d + 31*c^8*d^2 - 4*c^6*d^3 - 9*c^4*d^4 + 2*c^2*d^5 + d
^6)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3) - (9*(a*b^2 - b^
3)*c^11*d - 21*(a*b^2 - b^3)*c^9*d^2 + 10*(a*b^2 - b^3)*c^7*d^3 + 6*(a*b^2
- b^3)*c^5*d^4 - 3*(a*b^2 - b^3)*c^3*d^5 - (a*b^2 - b^3)*c*d^6 + (9*(a - b)
*c^11*d - 21*(a - b)*c^9*d^2 + 10*(a - b)*c^7*d^3 + 6*(a - b)*c^5*d^4 - 3*(
a - b)*c^3*d^5 - (a - b)*c*d^6)*x^2 - 2*(9*(a*b - b^2)*c^11*d - 21*(a*b - b
^2)*c^9*d^2 + 10*(a*b - b^2)*c^7*d^3 + 6*(a*b - b^2)*c^5*d^4 - 3*(a*b - b^2
)*c^3*d^5 - (a*b - b^2)*c*d^6)*x + (3*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*
a*b^5 + b^6)*c^8*d^3 - 8*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*
c^6*d^4 + 6*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^4*d^5 - (a^
4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*d^7 + (3*(a^4 - 4*a^3*b + 6*
a^2*b^2 - 4*a*b^3 + b^4)*c^8*d^3 - 8*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
 b^4)*c^6*d^4 + 6*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^4*d^5 - (a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d^7)*x^2 - 2*(3*(a^4*b - 4*a^3*b^2
 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^8*d^3 - 8*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 -
4*a*b^4 + b^5)*c^6*d^4 + 6*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*
c^4*d^5 - (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d^7)*x)*sqrt((9*c
^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4
- 6*a*b^5 + b^6)*d^3)))*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 +
 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*
a*b^5 + b^6)*d^3)) - c^3 - 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1
/3))/(b^2 - 2*b*x + x^2))*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4
 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 -
6*a*b^5 + b^6)*d^3)) - c^3 - 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^
(1/3) - 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(sqrt(3)*(
3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^7*d^2 - 5*(a^4 - 4*a^3*b +
6*a^2*b^2 - 4*a*b^3 + b^4)*c^5*d^3 + (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
 b^4)*c^3*d^4 + (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c*d^5)*sqrt((9*
c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4
 - 6*a*b^5 + b^6)*d^3)) + sqrt(3)*(9*(a - b)*c^8*d - 12*(a - b)*c^6*d^2 - 2
*(a - b)*c^4*d^3 + 4*(a - b)*c^2*d^4 + (a - b)*d^5))*(((a^3 - 3*a^2*b + 3*a
*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 -
 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) - c^3 - 3*c*d)/((a^3 - 3*a^
2*b + 3*a*b^2 - b^3)*d^2))^(1/3) + sqrt(3)*(9*b*c^10 - 21*b*c^8*d + 10*b*c^
6*d^2 + 6*b*c^4*d^3 - 3*b*c^2*d^4 - b*d^5 - (9*c^10 - 21*c^8*d + 10*c^6*d^2
 + 6*c^4*d^3 - 3*c^2*d^4 - d^5)*x))/(9*b*c^10 - 21*b*c^8*d + 10*b*c^6*d^2 +
 6*b*c^4*d^3 - 3*b*c^2*d^4 - b*d^5 - (9*c^10 - 21*c^8*d + 10*c^6*d^2 + 6*c^
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4*d^3 - 3*c^2*d^4 - d^5)*x)) - 1/4*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*s
qrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*
a^2*b^4 - 6*a*b^5 + b^6)*d^3)) + c^3 + 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b
^3)*d^2))^(1/3)*log(((18*(a^2*b - 2*a*b^2 + b^3)*c^9*d^2 - 24*(a^2*b - 2*a*
b^2 + b^3)*c^7*d^3 - 4*(a^2*b - 2*a*b^2 + b^3)*c^5*d^4 + 8*(a^2*b - 2*a*b^2
 + b^3)*c^3*d^5 + 2*(a^2*b - 2*a*b^2 + b^3)*c*d^6 - 2*(9*(a^2 - 2*a*b + b^2
)*c^9*d^2 - 12*(a^2 - 2*a*b + b^2)*c^7*d^3 - 2*(a^2 - 2*a*b + b^2)*c^5*d^4
+ 4*(a^2 - 2*a*b + b^2)*c^3*d^5 + (a^2 - 2*a*b + b^2)*c*d^6)*x - (3*(a^5*b
- 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^8*d^3 - 2*(a^5*b -
 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^6*d^4 - 4*(a^5*b -
5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^4*d^5 + 2*(a^5*b - 5
*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^2*d^6 + (a^5*b - 5*a^
4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*d^7 - (3*(a^5 - 5*a^4*b +
10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^8*d^3 - 2*(a^5 - 5*a^4*b + 10*a^
3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^6*d^4 - 4*(a^5 - 5*a^4*b + 10*a^3*b^2
 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^4*d^5 + 2*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10
*a^2*b^3 + 5*a*b^4 - b^5)*c^2*d^6 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^
3 + 5*a*b^4 - b^5)*d^7)*x)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 1
5*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)))*(-a*b^2 - (a +
2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*
d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3
+ 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) + c^3 + 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^
2 - b^3)*d^2))^(2/3) + (9*c^12 - 30*c^10*d + 31*c^8*d^2 - 4*c^6*d^3 - 9*c^4
*d^4 + 2*c^2*d^5 + d^6)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2
/3) - (9*(a*b^2 - b^3)*c^11*d - 21*(a*b^2 - b^3)*c^9*d^2 + 10*(a*b^2 - b^3)
*c^7*d^3 + 6*(a*b^2 - b^3)*c^5*d^4 - 3*(a*b^2 - b^3)*c^3*d^5 - (a*b^2 - b^3
)*c*d^6 + (9*(a - b)*c^11*d - 21*(a - b)*c^9*d^2 + 10*(a - b)*c^7*d^3 + 6*(
a - b)*c^5*d^4 - 3*(a - b)*c^3*d^5 - (a - b)*c*d^6)*x^2 - 2*(9*(a*b - b^2)*
c^11*d - 21*(a*b - b^2)*c^9*d^2 + 10*(a*b - b^2)*c^7*d^3 + 6*(a*b - b^2)*c^
5*d^4 - 3*(a*b - b^2)*c^3*d^5 - (a*b - b^2)*c*d^6)*x - (3*(a^4*b^2 - 4*a^3*
b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^8*d^3 - 8*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b
^4 - 4*a*b^5 + b^6)*c^6*d^4 + 6*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5
+ b^6)*c^4*d^5 - (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*d^7 + (3
*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^8*d^3 - 8*(a^4 - 4*a^3*b + 6
*a^2*b^2 - 4*a*b^3 + b^4)*c^6*d^4 + 6*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3
+ b^4)*c^4*d^5 - (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d^7)*x^2 - 2*(
3*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^8*d^3 - 8*(a^4*b - 4*a^
3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^6*d^4 + 6*(a^4*b - 4*a^3*b^2 + 6*a^2*b
^3 - 4*a*b^4 + b^5)*c^4*d^5 - (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^
5)*d^7)*x)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a
^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)))*(-((a^3 - 3*a^2*b + 3*a*b^2 - b
^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*
b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) + c^3 + 3*c*d)/((a^3 - 3*a^2*b + 3*
a*b^2 - b^3)*d^2))^(1/3))/(b^2 - 2*b*x + x^2)) - 1/4*(((a^3 - 3*a^2*b + 3*a
*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 -
 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) - c^3 - 3*c*d)/((a^3 - 3*a^
2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*log(((18*(a^2*b - 2*a*b^2 + b^3)*c^9*d^2 -
 24*(a^2*b - 2*a*b^2 + b^3)*c^7*d^3 - 4*(a^2*b - 2*a*b^2 + b^3)*c^5*d^4 + 8
*(a^2*b - 2*a*b^2 + b^3)*c^3*d^5 + 2*(a^2*b - 2*a*b^2 + b^3)*c*d^6 - 2*(9*(
a^2 - 2*a*b + b^2)*c^9*d^2 - 12*(a^2 - 2*a*b + b^2)*c^7*d^3 - 2*(a^2 - 2*a*
b + b^2)*c^5*d^4 + 4*(a^2 - 2*a*b + b^2)*c^3*d^5 + (a^2 - 2*a*b + b^2)*c*d^
6)*x + (3*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^8
*d^3 - 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^6*
d^4 - 4*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^4*d
^5 + 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^2*d^
6 + (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*d^7 - (3*
(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^8*d^3 - 2*(a^5
- 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^6*d^4 - 4*(a^5 - 5*a
^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^4*d^5 + 2*(a^5 - 5*a^4*b
```

```
+ 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^2*d^6 + (a^5 - 5*a^4*b + 10*a^
3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d^7)*x)*sqrt((9*c^4 + 6*c^2*d + d^2)/((
a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3))
)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(((a^3 - 3*a^2*b +
 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b
^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) - c^3 - 3*c*d)/((a^3 -
3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3) + (9*c^12 - 30*c^10*d + 31*c^8*d^2 - 4
*c^6*d^3 - 9*c^4*d^4 + 2*c^2*d^5 + d^6)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*
a*b + b^2)*x)^(2/3) - (9*(a*b^2 - b^3)*c^11*d - 21*(a*b^2 - b^3)*c^9*d^2 +
10*(a*b^2 - b^3)*c^7*d^3 + 6*(a*b^2 - b^3)*c^5*d^4 - 3*(a*b^2 - b^3)*c^3*d^
5 - (a*b^2 - b^3)*c*d^6 + (9*(a - b)*c^11*d - 21*(a - b)*c^9*d^2 + 10*(a -
b)*c^7*d^3 + 6*(a - b)*c^5*d^4 - 3*(a - b)*c^3*d^5 - (a - b)*c*d^6)*x^2 - 2
*(9*(a*b - b^2)*c^11*d - 21*(a*b - b^2)*c^9*d^2 + 10*(a*b - b^2)*c^7*d^3 +
6*(a*b - b^2)*c^5*d^4 - 3*(a*b - b^2)*c^3*d^5 - (a*b - b^2)*c*d^6)*x + (3*(
a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^8*d^3 - 8*(a^4*b^2 - 4*a
^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^6*d^4 + 6*(a^4*b^2 - 4*a^3*b^3 + 6*a^
2*b^4 - 4*a*b^5 + b^6)*c^4*d^5 - (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5
 + b^6)*d^7 + (3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^8*d^3 - 8*(a
^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^6*d^4 + 6*(a^4 - 4*a^3*b + 6*a^
2*b^2 - 4*a*b^3 + b^4)*c^4*d^5 - (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4
)*d^7)*x^2 - 2*(3*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^8*d^3 -
 8*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^6*d^4 + 6*(a^4*b - 4*a
^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^4*d^5 - (a^4*b - 4*a^3*b^2 + 6*a^2*b^
3 - 4*a*b^4 + b^5)*d^7)*x)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 1
5*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)))*(((a^3 - 3*a^2*
b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^
4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) - c^3 - 3*c*d)/((a^3
 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3))/(b^2 - 2*b*x + x^2)) + 1/2*(-((a^3
 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*
b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) + c^3 + 3*c
*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*log(((6*(a^2*b - 2*a*b^2 +
 b^3)*c^3*d^2 + 2*(a^2*b - 2*a*b^2 + b^3)*c*d^3 - 2*(3*(a^2 - 2*a*b + b^2)*
c^3*d^2 + (a^2 - 2*a*b + b^2)*c*d^3)*x - ((a^5*b - 5*a^4*b^2 + 10*a^3*b^3 -
 10*a^2*b^4 + 5*a*b^5 - b^6)*c^2*d^3 + (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10
*a^2*b^4 + 5*a*b^5 - b^6)*d^4 - ((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 +
 5*a*b^4 - b^5)*c^2*d^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^
4 - b^5)*d^4)*x)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2
- 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)))*(-((a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 2
0*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) + c^3 + 3*c*d)/((a^3 - 3*a^2*
b + 3*a*b^2 - b^3)*d^2))^(2/3) - (3*c^6 - 5*c^4*d + c^2*d^2 + d^3)*(-a*b^2
- (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3))/(b - x)) + 1/2*(((a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15
*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) - c^3 - 3*c*d)/((
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*log(((6*(a^2*b - 2*a*b^2 + b^3)*
c^3*d^2 + 2*(a^2*b - 2*a*b^2 + b^3)*c*d^3 - 2*(3*(a^2 - 2*a*b + b^2)*c^3*d^
2 + (a^2 - 2*a*b + b^2)*c*d^3)*x + ((a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^
2*b^4 + 5*a*b^5 - b^6)*c^2*d^3 + (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b
^4 + 5*a*b^5 - b^6)*d^4 - ((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b
^4 - b^5)*c^2*d^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^
5)*d^4)*x)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a
^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)))*(((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b
^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) - c^3 - 3*c*d)/((a^3 - 3*a^2*b + 3*a
*b^2 - b^3)*d^2))^(2/3) - (3*c^6 - 5*c^4*d + c^2*d^2 + d^3)*(-a*b^2 - (a +
2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3))/(b - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bc-(c+1)x+a}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}\left(b^2d+(d-1)x^2-a^2-2\,(bd-a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a-b*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(1/3)/(-a^2+b^2*d+2*(-b*d+a)*x
+(-1+d)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-(b*c - (c + 1)*x + a)/((-(a - x)*(b - x)^2)^(1/3)*(b^2*d + (d -
1)*x^2 - a^2 - 2*(b*d - a)*x)), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{-a-bc+(1+c)\,x}{\left((-a+x)\,(-b+x)^2\right)^{\frac{1}{3}}\left(-a^2+b^2d+2\,(-bd+a)\,x+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a-b*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(1/3)/(-a^2+b^2*d+2*(-b*d+a)*x+(-1+d
)*x^2),x)
```

```
[Out] int((-a-b*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(1/3)/(-a^2+b^2*d+2*(-b*d+a)*x+(-1+d
)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{bc-(c+1)x+a}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}\left(b^2d+(d-1)x^2-a^2-2\,(bd-a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a-b*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(1/3)/(-a^2+b^2*d+2*(-b*d+a)*x
+(-1+d)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((b*c - (c + 1)*x + a)/((-(a - x)*(b - x)^2)^(1/3)*(b^2*d + (d -
1)*x^2 - a^2 - 2*(b*d - a)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{a+b\,c-x\,(c+1)}{\left(-(a-x)\,(b-x)^2\right)^{1/3}\left(b^2\,d+2\,x\,(a-b\,d)-a^2+x^2\,(d-1)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*c - x*(c + 1))/((-(a - x)*(b - x)^2)^(1/3)*(b^2*d + 2*x*(a - b*
d) - a^2 + x^2*(d - 1))),x)
```

```
[Out] -int((a + b*c - x*(c + 1))/((-(a - x)*(b - x)^2)^(1/3)*(b^2*d + 2*x*(a - b*
d) - a^2 + x^2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a-b*c+(1+c)*x)/((-a+x)*(-b+x)**2)**(1/3)/(-a**2+b**2*d+2*(-b*d+
a)*x+(-1+d)*x**2),x)
```

```
[Out] Timed out
```

3.2440 $\int \frac{-b-ac+(1+c)x}{\sqrt[3]{(-a+x)(-b+x)^2}\left(-b^2+a^2d+2(b-ad)x+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=1707

$$(b-x)^{2/3}\sqrt[3]{x-a}\left(-\frac{\sqrt{3}\,b\left(\sqrt{d}-1\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt[6]{d}\sqrt[3]{x-a}-2\sqrt[3]{b-x}}\right)}{2(a-b)^2d^{5/6}}-\frac{\sqrt{3}\,ac\left(\sqrt{d}-1\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt[6]{d}\sqrt[3]{x-a}-2\sqrt[3]{b-x}}\right)}{2(a-b)^2d^{5/6}}+\frac{\sqrt{3}\,c\left(a\sqrt{d}-b\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt[6]{d}\sqrt[3]{x-a}-2\sqrt[3]{b-x}}\right)}{2(a-b)^2d^{5/6}}\right.$$

Rubi [A] time = 1.87, antiderivative size = 561, normalized size of antiderivative = 0.33, number of steps used = 5, number of rules used = 3, integrand size = 60, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6719, 6728, 91}

$$-\frac{\sqrt[3]{x-a}(x-b)^{2/3}\left(c+\sqrt{d}\right)\log\left(2\left(\sqrt{d}+1\right)\left(b-a\sqrt{d}\right)-2(1-d)x\right)}{4d^{5/6}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}+\frac{\sqrt[3]{x-a}(x-b)^{2/3}\left(c-\sqrt{d}\right)\log\left(2\left(1-\sqrt{d}\right)\left(a\sqrt{d}+b\right)-2(1-d)x\right)}{4d^{5/6}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{3\sqrt[3]{x-a}(x-b)^{2/3}\left(c-\sqrt{d}\right)\log\left(-\sqrt[6]{d}\sqrt[3]{x-a}-\sqrt[3]{x-b}\right)}{4d^{5/6}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}+\frac{3\sqrt[3]{x-a}(x-b)^{2/3}\left(c+\sqrt{d}\right)\log\left(\sqrt[6]{d}\sqrt[3]{x-a}-\sqrt[3]{x-b}\right)}{4d^{5/6}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}-\frac{\sqrt{3}\sqrt[3]{x-a}(x-b)^{2/3}\left(c-\sqrt{d}\right)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{x-b}}\right)}{2d^{5/6}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}+\frac{\sqrt{3}\sqrt[3]{x-a}(x-b)^{2/3}\left(c+\sqrt{d}\right)\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{x-b}}+\frac{1}{\sqrt{3}}\right)}{2d^{5/6}(a-b)\sqrt[3]{-\left((a-x)(b-x)^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b - a*c + (1 + c)*x)/(((-a + x)*(-b + x)^2)^(1/3)*(-b^2 + a^2*d + 2*(
b - a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] -1/2*(Sqrt[3]*(c - Sqrt[d])*(-a + x)^(1/3)*(-b + x)^(2/3)*ArcTan[1/Sqrt[3]
- (2*d^(1/6)*(-a + x)^(1/3))/(Sqrt[3]*(-b + x)^(1/3))])/((a - b)*d^(5/6)*(-
((a - x)*(b - x)^2))^(1/3)) + (Sqrt[3]*(c + Sqrt[d])*(-a + x)^(1/3)*(-b + x
)^(2/3)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(-a + x)^(1/3))/(Sqrt[3]*(-b + x)^(1/
3))])/(2*(a - b)*d^(5/6)*(-((a - x)*(b - x)^2))^(1/3)) - ((c + Sqrt[d])*(-a
 + x)^(1/3)*(-b + x)^(2/3)*Log[2*(1 + Sqrt[d])*(b - a*Sqrt[d]) - 2*(1 - d)*
x])/(4*(a - b)*d^(5/6)*(-((a - x)*(b - x)^2))^(1/3)) + ((c - Sqrt[d])*(-a +
 x)^(1/3)*(-b + x)^(2/3)*Log[2*(1 - Sqrt[d])*(b + a*Sqrt[d]) - 2*(1 - d)*x]
)/(4*(a - b)*d^(5/6)*(-((a - x)*(b - x)^2))^(1/3)) - (3*(c - Sqrt[d])*(-a +
 x)^(1/3)*(-b + x)^(2/3)*Log[-(d^(1/6)*(-a + x)^(1/3)) - (-b + x)^(1/3)])/(
4*(a - b)*d^(5/6)*(-((a - x)*(b - x)^2))^(1/3)) + (3*(c + Sqrt[d])*(-a + x)
^(1/3)*(-b + x)^(2/3)*Log[d^(1/6)*(-a + x)^(1/3) - (-b + x)^(1/3)])/(4*(a -
 b)*d^(5/6)*(-((a - x)*(b - x)^2))^(1/3))
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-b-ac+(1+c)x}{\sqrt[3]{(-a+x)(-b+x)^2}\left(-b^2+a^2d+2(b-ad)x+(-1+d)x^2\right)}dx = \frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{-b-}{\sqrt[3]{-a+x}\,(-b+x)^{2/3}(-b^2}}{\sqrt[3]{(-a+x)(-b+x)}}$$

$$= \frac{\left(\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int\left(\frac{}{\sqrt[3]{-a+x}\,(-b+x)^{2/3}(-2}\right.}{}$$

$$= \frac{\left(\left(1+c-\frac{c+d}{\sqrt{d}}\right)\sqrt[3]{-a+x}\,(-b+x)^{2/3}\right)\int \frac{}{\sqrt[3]{-}}}{\sqrt[3]{(-a+x)(-}}$$

$$= -\frac{\sqrt{3}\left(c-\sqrt{d}\right)\sqrt[3]{-a+x}\,(-b+x)^{2/3}\tan^{-1}}{2(a-b)d^{5/6}\sqrt[3]{-\left((a-x)(b-\right.}}$$

Mathematica [C] time = 0.36, size = 107, normalized size = 0.06

$$-\frac{3(x-b)\left(\left(c-\sqrt{d}\right){}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{b-x}{\sqrt{d}\,(x-a)}\right)+\left(c+\sqrt{d}\right){}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{x-b}{\sqrt{d}\,(x-a)}\right)\right)}{2d(a-b)\sqrt[3]{(x-a)(b-x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b - a*c + (1 + c)*x)/(((-a + x)*(-b + x)^2)^(1/3)*(-b^2 + a^2*d
 + 2*(b - a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] (-3*(-b + x)*((c - Sqrt[d])*Hypergeometric2F1[1/3, 1, 4/3, (b - x)/(Sqrt[d]
*(-a + x))] + (c + Sqrt[d])*Hypergeometric2F1[1/3, 1, 4/3, (-b + x)/(Sqrt[d
]*(-a + x))]))/(2*(a - b)*d*((b - x)^2*(-a + x))^(1/3))
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-b - a*c + (1 + c)*x)/(((-a + x)*(-b + x)^2)^(1/3)*(-b^
2 + a^2*d + 2*(b - a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] $Aborted
```

fricas [B] time = 1.86, size = 11788, normalized size = 6.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b-a*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(1/3)/(-b^2+a^2*d+2*(-a*d+b)*x
+(-1+d)*x^2),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*
d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6
)*d^5)) + 3*c^2 + d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*arctan(1/
3*(2*(sqrt(3)*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d^4*x - (a^4*b -
 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d^4)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^
2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*
d^5)) + sqrt(3)*((a*b - b^2)*c^4*d + 3*(a*b - b^2)*c^2*d^2 - ((a - b)*c^4*d
 + 3*(a - b)*c^2*d^2)*x))*sqrt(((2*(a^2*b - 2*a*b^2 + b^3)*c^11*d^2 + 8*(a^
```

2*b - 2*a*b^2 + b^3)*c^9*d^3 - 4*(a^2*b - 2*a*b^2 + b^3)*c^7*d^4 - 24*(a^2*
b - 2*a*b^2 + b^3)*c^5*d^5 + 18*(a^2*b - 2*a*b^2 + b^3)*c^3*d^6 - 2*((a^2 -
 2*a*b + b^2)*c^11*d^2 + 4*(a^2 - 2*a*b + b^2)*c^9*d^3 - 2*(a^2 - 2*a*b + b
^2)*c^7*d^4 - 12*(a^2 - 2*a*b + b^2)*c^5*d^5 + 9*(a^2 - 2*a*b + b^2)*c^3*d^
6)*x - ((a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^9*d
^4 + 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^7*d^
5 - 4*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^5*d^6
 - 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^3*d^7
+ 3*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c*d^8 - (
(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^9*d^4 + 2*(a^5
- 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^7*d^5 - 4*(a^5 - 5*a
^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^5*d^6 - 2*(a^5 - 5*a^4*b
+ 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^3*d^7 + 3*(a^5 - 5*a^4*b + 10*
a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c*d^8)*x)*sqrt((c^6 + 6*c^4*d + 9*c^2
*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^
6)*d^5)))*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(((a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*
b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) + 3*c^2 + d
)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3) + (c^14 + 2*c^12*d - 9*c^10*
d^2 - 4*c^8*d^3 + 31*c^6*d^4 - 30*c^4*d^5 + 9*c^2*d^6)*(-a*b^2 - (a + 2*b)*
x^2 + x^3 + (2*a*b + b^2)*x)^(2/3) - ((a*b^2 - b^3)*c^12*d + 3*(a*b^2 - b^3
)*c^10*d^2 - 6*(a*b^2 - b^3)*c^8*d^3 - 10*(a*b^2 - b^3)*c^6*d^4 + 21*(a*b^2
 - b^3)*c^4*d^5 - 9*(a*b^2 - b^3)*c^2*d^6 + ((a - b)*c^12*d + 3*(a - b)*c^1
0*d^2 - 6*(a - b)*c^8*d^3 - 10*(a - b)*c^6*d^4 + 21*(a - b)*c^4*d^5 - 9*(a
- b)*c^2*d^6)*x^2 - 2*((a*b - b^2)*c^12*d + 3*(a*b - b^2)*c^10*d^2 - 6*(a*b
 - b^2)*c^8*d^3 - 10*(a*b - b^2)*c^6*d^4 + 21*(a*b - b^2)*c^4*d^5 - 9*(a*b
- b^2)*c^2*d^6)*x - ((a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^10
d^3 - 6(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^6*d^5 + 8*(a^4
*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^4*d^6 - 3*(a^4*b^2 - 4*a^3*
b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^2*d^7 + ((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4)*c^10*d^3 - 6*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^6*d
^5 + 8*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^4*d^6 - 3*(a^4 - 4*a^3
*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^2*d^7)*x^2 - 2*((a^4*b - 4*a^3*b^2 + 6*a^
2*b^3 - 4*a*b^4 + b^5)*c^10*d^3 - 6*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^
4 + b^5)*c^6*d^5 + 8*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^4*d^
6 - 3*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^2*d^7)*x)*sqrt((c^6
 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*
b^4 - 6*a*b^5 + b^6)*d^5)))*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6
 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*
b^4 - 6*a*b^5 + b^6)*d^5)) + 3*c^2 + d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^
2))^(1/3))/(b^2 - 2*b*x + x^2))*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt(
(c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*
a^2*b^4 - 6*a*b^5 + b^6)*d^5)) + 3*c^2 + d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*d^2))^(1/3) - 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(s
qrt(3)*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^7*d^4 + (a^4 - 4*a^3*
b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^5*d^5 - 5*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a
*b^3 + b^4)*c^3*d^6 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c*d^7)*
sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3
+ 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - sqrt(3)*((a - b)*c^11*d + 4*(a - b)*c
^9*d^2 - 2*(a - b)*c^7*d^3 - 12*(a - b)*c^5*d^4 + 9*(a - b)*c^3*d^5))*(((a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6
*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) + 3*c^
2 + d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3) - sqrt(3)*(b*c^12 + 3*b
*c^10*d - 6*b*c^8*d^2 - 10*b*c^6*d^3 + 21*b*c^4*d^4 - 9*b*c^2*d^5 - (c^12 +
 3*c^10*d - 6*c^8*d^2 - 10*c^6*d^3 + 21*c^4*d^4 - 9*c^2*d^5)*x))/(b*c^12 +
3*b*c^10*d - 6*b*c^8*d^2 - 10*b*c^6*d^3 + 21*b*c^4*d^4 - 9*b*c^2*d^5 - (c^1
2 + 3*c^10*d - 6*c^8*d^2 - 10*c^6*d^3 + 21*c^4*d^4 - 9*c^2*d^5)*x)) + sqrt(
3)*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/
((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5

```
)) - 3*c^2 - d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*arctan(1/3*(2*
(sqrt(3)*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d^4*x - (a^4*b - 4*a^
3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d^4)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((
a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5))
 - sqrt(3)*((a*b - b^2)*c^4*d + 3*(a*b - b^2)*c^2*d^2 - ((a - b)*c^4*d + 3*
(a - b)*c^2*d^2)*x))*sqrt(((2*(a^2*b - 2*a*b^2 + b^3)*c^11*d^2 + 8*(a^2*b -
 2*a*b^2 + b^3)*c^9*d^3 - 4*(a^2*b - 2*a*b^2 + b^3)*c^7*d^4 - 24*(a^2*b - 2
*a*b^2 + b^3)*c^5*d^5 + 18*(a^2*b - 2*a*b^2 + b^3)*c^3*d^6 - 2*((a^2 - 2*a*
b + b^2)*c^11*d^2 + 4*(a^2 - 2*a*b + b^2)*c^9*d^3 - 2*(a^2 - 2*a*b + b^2)*c
^7*d^4 - 12*(a^2 - 2*a*b + b^2)*c^5*d^5 + 9*(a^2 - 2*a*b + b^2)*c^3*d^6)*x
+ ((a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^9*d^4 +
2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^7*d^5 - 4
*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^5*d^6 - 2*
(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^3*d^7 + 3*(
a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c*d^8 - ((a^5
- 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^9*d^4 + 2*(a^5 - 5*a
^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^7*d^5 - 4*(a^5 - 5*a^4*b
+ 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^5*d^6 - 2*(a^5 - 5*a^4*b + 10*
a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^3*d^7 + 3*(a^5 - 5*a^4*b + 10*a^3*b
^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c*d^8)*x)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)
/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^
5)))*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(-((a^3 - 3*a^2
*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b +
15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - 3*c^2 - d)/((
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3) + (c^14 + 2*c^12*d - 9*c^10*d^2
- 4*c^8*d^3 + 31*c^6*d^4 - 30*c^4*d^5 + 9*c^2*d^6)*(-a*b^2 - (a + 2*b)*x^2
+ x^3 + (2*a*b + b^2)*x)^(2/3) - ((a*b^2 - b^3)*c^12*d + 3*(a*b^2 - b^3)*c^
10*d^2 - 6*(a*b^2 - b^3)*c^8*d^3 - 10*(a*b^2 - b^3)*c^6*d^4 + 21*(a*b^2 - b
^3)*c^4*d^5 - 9*(a*b^2 - b^3)*c^2*d^6 + ((a - b)*c^12*d + 3*(a - b)*c^10*d^
2 - 6*(a - b)*c^8*d^3 - 10*(a - b)*c^6*d^4 + 21*(a - b)*c^4*d^5 - 9*(a - b)
*c^2*d^6)*x^2 - 2*((a*b - b^2)*c^12*d + 3*(a*b - b^2)*c^10*d^2 - 6*(a*b - b
^2)*c^8*d^3 - 10*(a*b - b^2)*c^6*d^4 + 21*(a*b - b^2)*c^4*d^5 - 9*(a*b - b^
2)*c^2*d^6)*x + ((a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^10*d^3
 - 6*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^6*d^5 + 8*(a^4*b^2
 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^4*d^6 - 3*(a^4*b^2 - 4*a^3*b^3
+ 6*a^2*b^4 - 4*a*b^5 + b^6)*c^2*d^7 + ((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^
3 + b^4)*c^10*d^3 - 6*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^6*d^5 +
 8*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^4*d^6 - 3*(a^4 - 4*a^3*b +
 6*a^2*b^2 - 4*a*b^3 + b^4)*c^2*d^7)*x^2 - 2*((a^4*b - 4*a^3*b^2 + 6*a^2*b^
3 - 4*a*b^4 + b^5)*c^10*d^3 - 6*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 +
b^5)*c^6*d^5 + 8*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^4*d^6 -
3*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^2*d^7)*x)*sqrt((c^6 + 6
*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4
- 6*a*b^5 + b^6)*d^5)))*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 +
6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4
 - 6*a*b^5 + b^6)*d^5)) - 3*c^2 - d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))
^(1/3))/(b^2 - 2*b*x + x^2))*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c
^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^
2*b^4 - 6*a*b^5 + b^6)*d^5)) - 3*c^2 - d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*
d^2))^(1/3) - 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(sqr
t(3)*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^7*d^4 + (a^4 - 4*a^3*b
+ 6*a^2*b^2 - 4*a*b^3 + b^4)*c^5*d^5 - 5*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b
^3 + b^4)*c^3*d^6 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c*d^7)*sq
rt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 +
15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) + sqrt(3)*((a - b)*c^11*d + 4*(a - b)*c^9
*d^2 - 2*(a - b)*c^7*d^3 - 12*(a - b)*c^5*d^4 + 9*(a - b)*c^3*d^5))*(-((a^3
 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*
a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - 3*c^2
 - d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3) + sqrt(3)*(b*c^12 + 3*b*
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c^10*d - 6*b*c^8*d^2 - 10*b*c^6*d^3 + 21*b*c^4*d^4 - 9*b*c^2*d^5 - (c^12 + 3*c^10*d - 6*c^8*d^2 - 10*c^6*d^3 + 21*c^4*d^4 - 9*c^2*d^5)*x))/(b*c^12 + 3*b*c^10*d - 6*b*c^8*d^2 - 10*b*c^6*d^3 + 21*b*c^4*d^4 - 9*b*c^2*d^5 - (c^12 + 3*c^10*d - 6*c^8*d^2 - 10*c^6*d^3 + 21*c^4*d^4 - 9*c^2*d^5)*x)) - 1/4*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) + 3*c^2 + d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*log(((2*(a^2*b - 2*a*b^2 + b^3)*c^11*d^2 + 8*(a^2*b - 2*a*b^2 + b^3)*c^9*d^3 - 4*(a^2*b - 2*a*b^2 + b^3)*c^7*d^4 - 24*(a^2*b - 2*a*b^2 + b^3)*c^5*d^5 + 18*(a^2*b - 2*a*b^2 + b^3)*c^3*d^6 - 2*((a^2 - 2*a*b + b^2)*c^11*d^2 + 4*(a^2 - 2*a*b + b^2)*c^9*d^3 - 2*(a^2 - 2*a*b + b^2)*c^7*d^4 - 12*(a^2 - 2*a*b + b^2)*c^5*d^5 + 9*(a^2 - 2*a*b + b^2)*c^3*d^6)*x - ((a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^9*d^4 + 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^7*d^5 - 4*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^5*d^6 - 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^3*d^7 + 3*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c*d^8 - ((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^9*d^4 + 2*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^7*d^5 - 4*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^5*d^6 - 2*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^3*d^7 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c*d^8)*x)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)))*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) + 3*c^2 + d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3) + (c^14 + 2*c^12*d - 9*c^10*d^2 - 4*c^8*d^3 + 31*c^6*d^4 - 30*c^4*d^5 + 9*c^2*d^6)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3) - ((a*b^2 - b^3)*c^12*d + 3*(a*b^2 - b^3)*c^10*d^2 - 6*(a*b^2 - b^3)*c^8*d^3 - 10*(a*b^2 - b^3)*c^6*d^4 + 21*(a*b^2 - b^3)*c^4*d^5 - 9*(a*b^2 - b^3)*c^2*d^6 + ((a - b)*c^12*d + 3*(a - b)*c^10*d^2 - 6*(a - b)*c^8*d^3 - 10*(a - b)*c^6*d^4 + 21*(a - b)*c^4*d^5 - 9*(a - b)*c^2*d^6)*x^2 - 2*((a*b - b^2)*c^12*d + 3*(a*b - b^2)*c^10*d^2 - 6*(a*b - b^2)*c^8*d^3 - 10*(a*b - b^2)*c^6*d^4 + 21*(a*b - b^2)*c^4*d^5 - 9*(a*b - b^2)*c^2*d^6)*x - ((a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^10*d^3 - 6*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^6*d^5 + 8*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^4*d^6 - 3*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^2*d^7 + ((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^10*d^3 - 6*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^6*d^5 + 8*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^4*d^6 - 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^2*d^7)*x^2 - 2*((a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^10*d^3 - 6*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^6*d^5 + 8*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^4*d^6 - 3*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^2*d^7)*x)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)))*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) + 3*c^2 + d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3))/(b^2 - 2*b*x + x^2)) - 1/4*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - 3*c^2 - d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*log(((2*(a^2*b - 2*a*b^2 + b^3)*c^11*d^2 + 8*(a^2*b - 2*a*b^2 + b^3)*c^9*d^3 - 4*(a^2*b - 2*a*b^2 + b^3)*c^7*d^4 - 24*(a^2*b - 2*a*b^2 + b^3)*c^5*d^5 + 18*(a^2*b - 2*a*b^2 + b^3)*c^3*d^6 - 2*((a^2 - 2*a*b + b^2)*c^11*d^2 + 4*(a^2 - 2*a*b + b^2)*c^9*d^3 - 2*(a^2 - 2*a*b + b^2)*c^7*d^4 - 12*(a^2 - 2*a*b + b^2)*c^5*d^5 + 9*(a^2 - 2*a*b + b^2)*c^3*d^6)*x + ((a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^9*d^4 + 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^7*d^5 - 4*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^5*d^6 - 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5

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*a*b^5 - b^6)*c^3*d^7 + 3*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*
a*b^5 - b^6)*c*d^8 - ((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 -
b^5)*c^9*d^4 + 2*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*
c^7*d^5 - 4*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^5*d
^6 - 2*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^3*d^7 +
3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c*d^8)*x)*sqrt(
(c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*
a^2*b^4 - 6*a*b^5 + b^6)*d^5)))*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^
2)*x)^(1/3)*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*
c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 +
 b^6)*d^5)) - 3*c^2 - d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3) + (c^
14 + 2*c^12*d - 9*c^10*d^2 - 4*c^8*d^3 + 31*c^6*d^4 - 30*c^4*d^5 + 9*c^2*d^
6)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3) - ((a*b^2 - b^3)*
c^12*d + 3*(a*b^2 - b^3)*c^10*d^2 - 6*(a*b^2 - b^3)*c^8*d^3 - 10*(a*b^2 - b
^3)*c^6*d^4 + 21*(a*b^2 - b^3)*c^4*d^5 - 9*(a*b^2 - b^3)*c^2*d^6 + ((a - b)
*c^12*d + 3*(a - b)*c^10*d^2 - 6*(a - b)*c^8*d^3 - 10*(a - b)*c^6*d^4 + 21*
(a - b)*c^4*d^5 - 9*(a - b)*c^2*d^6)*x^2 - 2*((a*b - b^2)*c^12*d + 3*(a*b -
 b^2)*c^10*d^2 - 6*(a*b - b^2)*c^8*d^3 - 10*(a*b - b^2)*c^6*d^4 + 21*(a*b -
 b^2)*c^4*d^5 - 9*(a*b - b^2)*c^2*d^6)*x + ((a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^
4 - 4*a*b^5 + b^6)*c^10*d^3 - 6*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5
+ b^6)*c^6*d^5 + 8*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^4*d^
6 - 3*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*c^2*d^7 + ((a^4 - 4
*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^10*d^3 - 6*(a^4 - 4*a^3*b + 6*a^2*b^2
 - 4*a*b^3 + b^4)*c^6*d^5 + 8*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c
^4*d^6 - 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^2*d^7)*x^2 - 2*((a
^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^10*d^3 - 6*(a^4*b - 4*a^3*b
^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^6*d^5 + 8*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3
- 4*a*b^4 + b^5)*c^4*d^6 - 3*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5
)*c^2*d^7)*x)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2
 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)))*(-((a^3 - 3*a^2*b + 3*a*
b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^
2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - 3*c^2 - d)/((a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*d^2))^(1/3))/(b^2 - 2*b*x + x^2)) + 1/2*(((a^3 - 3*a^
2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b +
 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) + 3*c^2 + d)/(
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*log(((2*(a^2*b - 2*a*b^2 + b^3)
*c^4*d^2 + 6*(a^2*b - 2*a*b^2 + b^3)*c^2*d^3 - 2*((a^2 - 2*a*b + b^2)*c^4*d
^2 + 3*(a^2 - 2*a*b + b^2)*c^2*d^3)*x - ((a^5*b - 5*a^4*b^2 + 10*a^3*b^3 -
10*a^2*b^4 + 5*a*b^5 - b^6)*c^2*d^4 + (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*
a^2*b^4 + 5*a*b^5 - b^6)*d^5 - ((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 +
5*a*b^4 - b^5)*c^2*d^4 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
 - b^5)*d^5)*x)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b
^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)))*(((a^3 - 3*a^2*b + 3*a
*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b
^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) + 3*c^2 + d)/((a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3) - (c^7 + c^5*d - 5*c^3*d^2 + 3*c*d^3)*(-
a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3))/(b - x)) + 1/2*(-((a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6
*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - 3*c^
2 - d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*log(((2*(a^2*b - 2*a*b^
2 + b^3)*c^4*d^2 + 6*(a^2*b - 2*a*b^2 + b^3)*c^2*d^3 - 2*((a^2 - 2*a*b + b^
2)*c^4*d^2 + 3*(a^2 - 2*a*b + b^2)*c^2*d^3)*x + ((a^5*b - 5*a^4*b^2 + 10*a^
3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^2*d^4 + (a^5*b - 5*a^4*b^2 + 10*a^3*b
^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*d^5 - ((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^
2*b^3 + 5*a*b^4 - b^5)*c^2*d^4 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 +
 5*a*b^4 - b^5)*d^5)*x)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b +
15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)))*(-((a^3 - 3*a^
2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b +
 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - 3*c^2 - d)/(
```

```
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3) - (c^7 + c^5*d - 5*c^3*d^2 + 3*
c*d^3)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3))/(b - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{ac-(c+1)x+b}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}\left(a^2d+(d-1)x^2-b^2-2\,(ad-b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b-a*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(1/3)/(-b^2+a^2*d+2*(-a*d+b)*x
+(-1+d)*x^2),x, algorithm="giac")
```

```
[Out] integrate(-(a*c - (c + 1)*x + b)/((-(a - x)*(b - x)^2)^(1/3)*(a^2*d + (d -
1)*x^2 - b^2 - 2*(a*d - b)*x)), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{-b-ac+(1+c)\,x}{\left((-a+x)\,(-b+x)^2\right)^{\frac{1}{3}}\left(-b^2+a^2d+2\,(-ad+b)\,x+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b-a*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(1/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-1+d
)*x^2),x)
```

```
[Out] int((-b-a*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(1/3)/(-b^2+a^2*d+2*(-a*d+b)*x+(-1+d
)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ac-(c+1)x+b}{\left(-(a-x)(b-x)^2\right)^{\frac{1}{3}}\left(a^2d+(d-1)x^2-b^2-2\,(ad-b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b-a*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(1/3)/(-b^2+a^2*d+2*(-a*d+b)*x
+(-1+d)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((a*c - (c + 1)*x + b)/((-(a - x)*(b - x)^2)^(1/3)*(a^2*d + (d -
1)*x^2 - b^2 - 2*(a*d - b)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{b+a\,c-x\,(c+1)}{\left(-(a-x)\,(b-x)^2\right)^{1/3}\,\left(a^2\,d+2\,x\,(b-a\,d)-b^2+x^2\,(d-1)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b + a*c - x*(c + 1))/((-(a - x)*(b - x)^2)^(1/3)*(a^2*d + 2*x*(b - a*
d) - b^2 + x^2*(d - 1))),x)
```

```
[Out] -int((b + a*c - x*(c + 1))/((-(a - x)*(b - x)^2)^(1/3)*(a^2*d + 2*x*(b - a*
d) - b^2 + x^2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b-a*c+(1+c)*x)/((-a+x)*(-b+x)**2)**(1/3)/(-b**2+a**2*d+2*(-a*d+
b)*x+(-1+d)*x**2),x)
```

```
[Out] Timed out
```

3.2441 $\int \frac{(-b+x)(-a-bc+(1+c)x)}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(-a^2+b^2d+2(a-bd)x+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=1835

$$(b-x)^{4/3}(x-a)^{2/3}\left(\sqrt[6]{d}\,\sqrt[3]{b-x}-\sqrt[3]{x-a}\right)\left(\sqrt[6]{d}\,\sqrt[3]{b-x}+\sqrt[3]{x-a}\right)\left(\sqrt[3]{d}\,(b-x)^{2/3}-\sqrt[6]{d}\,\sqrt[3]{x-a}\,\sqrt[3]{b-x}+(x-a$$

Rubi [A] time = 2.28, antiderivative size = 561, normalized size of antiderivative = 0.31, number of steps used = 5, number of rules used = 3, integrand size = 65, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.046, Rules used = {6719, 6728, 91}

$$\frac{(x-a)^{2/3}(x-b)^{4/3}\left(c+\sqrt{d}\right)\log\left(2\left(\sqrt{d}+1\right)\left(a-b\sqrt{d}\right)-2(1-d)x\right)}{4d^{5/6}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}-\frac{(x-a)^{2/3}(x-b)^{4/3}\left(c-\sqrt{d}\right)\log\left(2\left(1-\sqrt{d}\right)\left(a+b\sqrt{d}\right)-2(1-d)x\right)}{4d^{5/6}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{3(x-a)^{2/3}(x-b)^{4/3}\left(c-\sqrt{d}\right)\log\left(-\sqrt[3]{x-a}-\sqrt[6]{d}\,\sqrt[3]{x-b}\right)}{4d^{5/6}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}-\frac{3(x-a)^{2/3}(x-b)^{4/3}\left(c+\sqrt{d}\right)\log\left(\sqrt[6]{d}\,\sqrt[3]{x-b}-\sqrt[3]{x-a}\right)}{4d^{5/6}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{\sqrt{3}\,(x-a)^{2/3}(x-b)^{4/3}\left(c-\sqrt{d}\right)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[3]{x-b}}{\sqrt{3}\,\sqrt[3]{x-a}}\right)}{2d^{5/6}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}-\frac{\sqrt{3}\,(x-a)^{2/3}(x-b)^{4/3}\left(c+\sqrt{d}\right)\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[3]{x-b}}{\sqrt{3}\,\sqrt[3]{x-a}}+\frac{1}{\sqrt{3}}\right)}{2d^{5/6}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + x)*(-a - b*c + (1 + c)*x))/(((-a + x)*(-b + x)^2)^(2/3)*(-a^2 +
b^2*d + 2*(a - b*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] (Sqrt[3]*(c - Sqrt[d])*(-a + x)^(2/3)*(-b + x)^(4/3)*ArcTan[1/Sqrt[3] - (2*
d^(1/6)*(-b + x)^(1/3))/(Sqrt[3]*(-a + x)^(1/3))])/(2*(a - b)*d^(5/6)*(-((a
 - x)*(b - x)^2))^(2/3)) - (Sqrt[3]*(c + Sqrt[d])*(-a + x)^(2/3)*(-b + x)^(
4/3)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(-b + x)^(1/3))/(Sqrt[3]*(-a + x)^(1/3))
])/(2*(a - b)*d^(5/6)*(-((a - x)*(b - x)^2))^(2/3)) + ((c + Sqrt[d])*(-a +
x)^(2/3)*(-b + x)^(4/3)*Log[2*(1 + Sqrt[d])*(a - b*Sqrt[d]) - 2*(1 - d)*x])
/(4*(a - b)*d^(5/6)*(-((a - x)*(b - x)^2))^(2/3)) - ((c - Sqrt[d])*(-a + x)
^(2/3)*(-b + x)^(4/3)*Log[2*(1 - Sqrt[d])*(a + b*Sqrt[d]) - 2*(1 - d)*x])/(
4*(a - b)*d^(5/6)*(-((a - x)*(b - x)^2))^(2/3)) + (3*(c - Sqrt[d])*(-a + x)
^(2/3)*(-b + x)^(4/3)*Log[-(-a + x)^(1/3) - d^(1/6)*(-b + x)^(1/3)])/(4*(a
- b)*d^(5/6)*(-((a - x)*(b - x)^2))^(2/3)) - (3*(c + Sqrt[d])*(-a + x)^(2/3
)*(-b + x)^(4/3)*Log[-(-a + x)^(1/3) + d^(1/6)*(-b + x)^(1/3)])/(4*(a - b)*
d^(5/6)*(-((a - x)*(b - x)^2))^(2/3))
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(-b+x)(-a-bc+(1+c)x)}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(-a^2+b^2d+2(a-bd)x+(-1+d)x^2\right)}\,dx = \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{-a\cdots}{(-a+x)^{2/3}\sqrt[3]{-b+x}\,(-a\cdots}}{\left((-a+x)(-b+x)^2\right)\cdots}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int\left(\frac{\cdots}{(-a+x)^{2/3}\sqrt[3]{-b+x}\,(\cdots}\right.}{\cdots}$$

$$= \frac{\left(\left(1+c-\frac{-c-d}{\sqrt{d}}\right)(-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{\cdots}{(\cdots}}{\left((-a+x)(-\cdots\right.}$$

$$= \frac{\sqrt{3}\left(c-\sqrt{d}\right)(-a+x)^{2/3}(-b+x)^{4/3}\tan^{-1}\left(\cdots\right.}{2(a-b)d^{5/6}\left(-\left((a-x)(b-x)^{\cdots}\right.\right.}$$

Mathematica [C] time = 0.34, size = 111, normalized size = 0.06

$$\frac{3(b-x)^2\left(\left(\sqrt{d}-c\right){}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{\sqrt{d}(b-x)}{x-a}\right)+\left(c+\sqrt{d}\right){}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{\sqrt{d}(x-b)}{x-a}\right)\right)}{4\sqrt{d}(a-b)\left((x-a)(b-x)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + x)*(-a - b*c + (1 + c)*x))/(((-a + x)*(-b + x)^2)^(2/3)*(-
a^2 + b^2*d + 2*(a - b*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] (3*(b - x)^2*((-c + Sqrt[d])*Hypergeometric2F1[2/3, 1, 5/3, (Sqrt[d]*(b - x
))/(-a + x)] + (c + Sqrt[d])*Hypergeometric2F1[2/3, 1, 5/3, (Sqrt[d]*(-b +
x))/(-a + x)]))/(4*(a - b)*Sqrt[d]*((b - x)^2*(-a + x))^(2/3))
```

IntegrateAlgebraic [A] time = 18.97, size = 571, normalized size = 0.31

$$\frac{(x-a)^{2/3}(b-x)^{4/3}\left(\sqrt[6]{d}\sqrt[3]{b-x}-\sqrt[3]{x-a}\right)\left(\sqrt[3]{x-a}+\sqrt[6]{d}\sqrt[3]{b-x}\right)\left(\sqrt[3]{d}(x-a)^{2/3}(b-x)^{2/3}+(x-a)^{4/3}-d^{2/3}x\sqrt[3]{b-x}+bd^{2/3}\sqrt[3]{b-x}\right)\left(\frac{\left(c-\sqrt{d}\right)\log\left(\sqrt[6]{d}-\frac{\sqrt[3]{x-a}}{\sqrt[3]{b-x}}\right)}{2d^{5/6}(a-b)}+\frac{\left(-c-\sqrt{d}\right)\log\left(\frac{\sqrt[3]{x-a}}{\sqrt[3]{b-x}}+\sqrt[6]{d}\right)}{2d^{5/6}(a-b)}+\frac{\left(c+\sqrt{d}\right)\log\left(-\frac{\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt[3]{b-x}}+\frac{(x-a)^{2/3}}{(b-x)^{2/3}}+\sqrt[3]{d}\right)}{4d^{5/6}(a-b)}+\frac{\left(\sqrt{d}-c\right)\log\left(\frac{\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt[3]{b-x}}+\frac{(x-a)^{2/3}}{(b-x)^{2/3}}+\sqrt[3]{d}\right)}{4d^{5/6}(a-b)}+\frac{\sqrt{3}\left(c+\sqrt{d}\right)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{b-x}}\right)}{2d^{5/6}(a-b)}-\frac{\sqrt{3}\left(c-\sqrt{d}\right)\tan^{-1}\left(\frac{2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{b-x}}+\frac{1}{\sqrt{3}}\right)}{2d^{5/6}(a-b)}\right)}{\left((x-a)(b-x)^2\right)^{2/3}\left(-a^2+2ax+b^2d-2bdx+(d-1)x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + x)*(-a - b*c + (1 + c)*x))/(((-a + x)*(-b + x)^2)
^(2/3)*(-a^2 + b^2*d + 2*(a - b*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] ((b - x)^(4/3)*(-a + x)^(2/3)*(d^(1/6)*(b - x)^(1/3) - (-a + x)^(1/3))*(d^(
1/6)*(b - x)^(1/3) + (-a + x)^(1/3))*(b*d^(2/3)*(b - x)^(1/3) - d^(2/3)*(b
- x)^(1/3)*x + d^(1/3)*(b - x)^(2/3)*(-a + x)^(2/3) + (-a + x)^(4/3))*((Sqr
t[3]*(c + Sqrt[d])*ArcTan[1/Sqrt[3] - (2*(-a + x)^(1/3))/(Sqrt[3]*d^(1/6)*(
b - x)^(1/3))])/(2*(a - b)*d^(5/6)) - (Sqrt[3]*(c - Sqrt[d])*ArcTan[1/Sqrt[
3] + (2*(-a + x)^(1/3))/(Sqrt[3]*d^(1/6)*(b - x)^(1/3))])/(2*(a - b)*d^(5/6
)) + ((c - Sqrt[d])*Log[d^(1/6) - (-a + x)^(1/3)/(b - x)^(1/3)])/(2*(a - b)
*d^(5/6)) + ((-c - Sqrt[d])*Log[d^(1/6) + (-a + x)^(1/3)/(b - x)^(1/3)])/(2
*(a - b)*d^(5/6)) + ((c + Sqrt[d])*Log[d^(1/3) - (d^(1/6)*(-a + x)^(1/3))/(
b - x)^(1/3) + (-a + x)^(2/3)/(b - x)^(2/3)])/(4*(a - b)*d^(5/6)) + ((-c +
Sqrt[d])*Log[d^(1/3) + (d^(1/6)*(-a + x)^(1/3))/(b - x)^(1/3) + (-a + x)^(2
/3)/(b - x)^(2/3)])/(4*(a - b)*d^(5/6))))/(((b - x)^2*(-a + x))^(2/3)*(-a^2
 + b^2*d + 2*a*x - 2*b*d*x + (-1 + d)*x^2))
```

fricas [B] time = 1.09, size = 9684, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-a-b*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(2/3)/(-a^2+b^2*d+2*(-b
*d+a)*x+(-1+d)*x^2),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2
*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^
6)*d^5)) + 3*c^2 + d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*arctan(1
/3*(2*(sqrt(3)*((a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^
6)*c^2*d^4 + (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*
d^5 - ((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^2*d^4 +
(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d^5)*x)*sqrt((c^6
 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*
b^4 - 6*a*b^5 + b^6)*d^5)) - 2*sqrt(3)*((a^2*b - 2*a*b^2 + b^3)*c^4*d^2 + 3
*(a^2*b - 2*a*b^2 + b^3)*c^2*d^3 - ((a^2 - 2*a*b + b^2)*c^4*d^2 + 3*(a^2 -
2*a*b + b^2)*c^2*d^3)*x))*sqrt(((c^10 + 4*c^8*d - 2*c^6*d^2 - 12*c^4*d^3 +
9*c^2*d^4)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3) + ((a*b -
 b^2)*c^9*d + 5*(a*b - b^2)*c^7*d^2 + 3*(a*b - b^2)*c^5*d^3 - 9*(a*b - b^2)
*c^3*d^4 - ((a - b)*c^9*d + 5*(a - b)*c^7*d^2 + 3*(a - b)*c^5*d^3 - 9*(a -
b)*c^3*d^4)*x - ((a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^5*d^4 +
2*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^3*d^5 - 3*(a^4*b - 4*a^
3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c*d^6 - ((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4)*c^5*d^4 + 2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^3*d^
5 - 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c*d^6)*x)*sqrt((c^6 + 6*c
^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 -
6*a*b^5 + b^6)*d^5)))*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3
)*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/(
(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)
) + 3*c^2 + d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3) + ((a^2*b^2 - 2
*a*b^3 + b^4)*c^8*d^2 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*c^6*d^3 + 15*(a^2*b^2 -
 2*a*b^3 + b^4)*c^4*d^4 + 9*(a^2*b^2 - 2*a*b^3 + b^4)*c^2*d^5 + ((a^2 - 2*a
*b + b^2)*c^8*d^2 + 7*(a^2 - 2*a*b + b^2)*c^6*d^3 + 15*(a^2 - 2*a*b + b^2)*
c^4*d^4 + 9*(a^2 - 2*a*b + b^2)*c^2*d^5)*x^2 - 2*((a^2*b - 2*a*b^2 + b^3)*c
^8*d^2 + 7*(a^2*b - 2*a*b^2 + b^3)*c^6*d^3 + 15*(a^2*b - 2*a*b^2 + b^3)*c^4
*d^4 + 9*(a^2*b - 2*a*b^2 + b^3)*c^2*d^5)*x - 2*((a^5*b^2 - 5*a^4*b^3 + 10*
a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*c^4*d^5 + 3*(a^5*b^2 - 5*a^4*b^3 + 10
*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*c^2*d^6 + ((a^5 - 5*a^4*b + 10*a^3*b
^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^4*d^5 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 -
10*a^2*b^3 + 5*a*b^4 - b^5)*c^2*d^6)*x^2 - 2*((a^5*b - 5*a^4*b^2 + 10*a^3*b
^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^4*d^5 + 3*(a^5*b - 5*a^4*b^2 + 10*a^3*b^
3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^2*d^6)*x)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2
)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d
^5)))*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^
2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*
d^5)) + 3*c^2 + d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3))/(b^2 - 2*b
*x + x^2))*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c
^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 +
b^6)*d^5)) + 3*c^2 + d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3) - 2*(-
a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(sqrt(3)*((a^5 - 5*a^4
*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^7*d^4 + 3*(a^5 - 5*a^4*b +
10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^5*d^5 - (a^5 - 5*a^4*b + 10*a^3*
b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^3*d^6 - 3*(a^5 - 5*a^4*b + 10*a^3*b^2 -
 10*a^2*b^3 + 5*a*b^4 - b^5)*c*d^7)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6
- 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - 2
*sqrt(3)*((a^2 - 2*a*b + b^2)*c^9*d^2 + 5*(a^2 - 2*a*b + b^2)*c^7*d^3 + 3*(
a^2 - 2*a*b + b^2)*c^5*d^4 - 9*(a^2 - 2*a*b + b^2)*c^3*d^5))*(-((a^3 - 3*a^
2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b +
 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) + 3*c^2 + d)/(
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3) + sqrt(3)*(b*c^12 + 3*b*c^10*d
- 6*b*c^8*d^2 - 10*b*c^6*d^3 + 21*b*c^4*d^4 - 9*b*c^2*d^5 - (c^12 + 3*c^10*
```

d - 6*c^8*d^2 - 10*c^6*d^3 + 21*c^4*d^4 - 9*c^2*d^5)*x))/(b*c^12 + 3*b*c^10
*d - 6*b*c^8*d^2 - 10*b*c^6*d^3 + 21*b*c^4*d^4 - 9*b*c^2*d^5 - (c^12 + 3*c^
10*d - 6*c^8*d^2 - 10*c^6*d^3 + 21*c^4*d^4 - 9*c^2*d^5)*x)) + sqrt(3)*(((a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6
*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - 3*c^
2 - d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*arctan(1/3*(2*(sqrt(3)*
((a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^2*d^4 + (a
^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*d^5 - ((a^5 - 5
*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^2*d^4 + (a^5 - 5*a^4*b
+ 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d^5)*x)*sqrt((c^6 + 6*c^4*d + 9*
c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 +
b^6)*d^5)) + 2*sqrt(3)*((a^2*b - 2*a*b^2 + b^3)*c^4*d^2 + 3*(a^2*b - 2*a*b
^2 + b^3)*c^2*d^3 - ((a^2 - 2*a*b + b^2)*c^4*d^2 + 3*(a^2 - 2*a*b + b^2)*c^
2*d^3)*x))*sqrt(((c^10 + 4*c^8*d - 2*c^6*d^2 - 12*c^4*d^3 + 9*c^2*d^4)*(-a*
b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3) + ((a*b - b^2)*c^9*d + 5
*(a*b - b^2)*c^7*d^2 + 3*(a*b - b^2)*c^5*d^3 - 9*(a*b - b^2)*c^3*d^4 - ((a
- b)*c^9*d + 5*(a - b)*c^7*d^2 + 3*(a - b)*c^5*d^3 - 9*(a - b)*c^3*d^4)*x +
((a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^5*d^4 + 2*(a^4*b - 4*a^
3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^3*d^5 - 3*(a^4*b - 4*a^3*b^2 + 6*a^2*b
^3 - 4*a*b^4 + b^5)*c*d^6 - ((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^
5*d^4 + 2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^3*d^5 - 3*(a^4 - 4*
a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c*d^6)*x)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^
2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*
d^5)))*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(((a^3 - 3*a^
2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b +
15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - 3*c^2 - d)/(
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3) + ((a^2*b^2 - 2*a*b^3 + b^4)*c^
8*d^2 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*c^6*d^3 + 15*(a^2*b^2 - 2*a*b^3 + b^4)*
c^4*d^4 + 9*(a^2*b^2 - 2*a*b^3 + b^4)*c^2*d^5 + ((a^2 - 2*a*b + b^2)*c^8*d^
2 + 7*(a^2 - 2*a*b + b^2)*c^6*d^3 + 15*(a^2 - 2*a*b + b^2)*c^4*d^4 + 9*(a^2
- 2*a*b + b^2)*c^2*d^5)*x^2 - 2*((a^2*b - 2*a*b^2 + b^3)*c^8*d^2 + 7*(a^2*
b - 2*a*b^2 + b^3)*c^6*d^3 + 15*(a^2*b - 2*a*b^2 + b^3)*c^4*d^4 + 9*(a^2*b
- 2*a*b^2 + b^3)*c^2*d^5)*x + 2*((a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2
*b^5 + 5*a*b^6 - b^7)*c^4*d^5 + 3*(a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^
2*b^5 + 5*a*b^6 - b^7)*c^2*d^6 + ((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3
+ 5*a*b^4 - b^5)*c^4*d^5 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a
*b^4 - b^5)*c^2*d^6)*x^2 - 2*((a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4
+ 5*a*b^5 - b^6)*c^4*d^5 + 3*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 +
5*a*b^5 - b^6)*c^2*d^6)*x)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*
b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)))*(((a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*
b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - 3*c^2 - d
)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3))/(b^2 - 2*b*x + x^2))*(((a^3
- 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*
a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - 3*c^2
- d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3) - 2*(-a*b^2 - (a + 2*b)*
x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(sqrt(3)*((a^5 - 5*a^4*b + 10*a^3*b^2 -
10*a^2*b^3 + 5*a*b^4 - b^5)*c^7*d^4 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^
2*b^3 + 5*a*b^4 - b^5)*c^5*d^5 - (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 +
5*a*b^4 - b^5)*c^3*d^6 - 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*
b^4 - b^5)*c*d^7)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4
*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) + 2*sqrt(3)*((a^2 - 2
*a*b + b^2)*c^9*d^2 + 5*(a^2 - 2*a*b + b^2)*c^7*d^3 + 3*(a^2 - 2*a*b + b^2)
*c^5*d^4 - 9*(a^2 - 2*a*b + b^2)*c^3*d^5))*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^
3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - 3*c^2 - d)/((a^3 - 3*a^2*b + 3*
a*b^2 - b^3)*d^2))^(2/3) - sqrt(3)*(b*c^12 + 3*b*c^10*d - 6*b*c^8*d^2 - 10*
b*c^6*d^3 + 21*b*c^4*d^4 - 9*b*c^2*d^5 - (c^12 + 3*c^10*d - 6*c^8*d^2 - 10*
c^6*d^3 + 21*c^4*d^4 - 9*c^2*d^5)*x))/(b*c^12 + 3*b*c^10*d - 6*b*c^8*d^2 -

10*b*c^6*d^3 + 21*b*c^4*d^4 - 9*b*c^2*d^5 - (c^12 + 3*c^10*d - 6*c^8*d^2 - 10*c^6*d^3 + 21*c^4*d^4 - 9*c^2*d^5)*x)) - 1/4*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) + 3*c^2 + d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*log(((c^10 + 4*c^8*d - 2*c^6*d^2 - 12*c^4*d^3 + 9*c^2*d^4)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3) + ((a*b - b^2)*c^9*d + 5*(a*b - b^2)*c^7*d^2 + 3*(a*b - b^2)*c^5*d^3 - 9*(a*b - b^2)*c^3*d^4 - ((a - b)*c^9*d + 5*(a - b)*c^7*d^2 + 3*(a - b)*c^5*d^3 - 9*(a - b)*c^3*d^4)*x - ((a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^5*d^4 + 2*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^3*d^5 - 3*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c*d^6 - ((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^5*d^4 + 2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^3*d^5 - 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c*d^6)*x)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)))*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) + 3*c^2 + d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3) + ((a^2*b^2 - 2*a*b^3 + b^4)*c^8*d^2 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*c^6*d^3 + 15*(a^2*b^2 - 2*a*b^3 + b^4)*c^4*d^4 + 9*(a^2*b^2 - 2*a*b^3 + b^4)*c^2*d^5 + ((a^2 - 2*a*b + b^2)*c^8*d^2 + 7*(a^2 - 2*a*b + b^2)*c^6*d^3 + 15*(a^2 - 2*a*b + b^2)*c^4*d^4 + 9*(a^2 - 2*a*b + b^2)*c^2*d^5)*x^2 - 2*((a^2*b - 2*a*b^2 + b^3)*c^8*d^2 + 7*(a^2*b - 2*a*b^2 + b^3)*c^6*d^3 + 15*(a^2*b - 2*a*b^2 + b^3)*c^4*d^4 + 9*(a^2*b - 2*a*b^2 + b^3)*c^2*d^5)*x - 2*((a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*c^4*d^5 + 3*(a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*c^2*d^6 + ((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^4*d^5 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^2*d^6)*x^2 - 2*((a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^4*d^5 + 3*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^2*d^6)*x)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)))*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) + 3*c^2 + d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3))/(b^2 - 2*b*x + x^2)) - 1/4*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - 3*c^2 - d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*log(((c^10 + 4*c^8*d - 2*c^6*d^2 - 12*c^4*d^3 + 9*c^2*d^4)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3) + ((a*b - b^2)*c^9*d + 5*(a*b - b^2)*c^7*d^2 + 3*(a*b - b^2)*c^5*d^3 - 9*(a*b - b^2)*c^3*d^4 - ((a - b)*c^9*d + 5*(a - b)*c^7*d^2 + 3*(a - b)*c^5*d^3 - 9*(a - b)*c^3*d^4)*x + ((a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^5*d^4 + 2*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^3*d^5 - 3*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c*d^6 - ((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^5*d^4 + 2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^3*d^5 - 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c*d^6)*x)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)))*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - 3*c^2 - d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3) + ((a^2*b^2 - 2*a*b^3 + b^4)*c^8*d^2 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*c^6*d^3 + 15*(a^2*b^2 - 2*a*b^3 + b^4)*c^4*d^4 + 9*(a^2*b^2 - 2*a*b^3 + b^4)*c^2*d^5 + ((a^2 - 2*a*b + b^2)*c^8*d^2 + 7*(a^2 - 2*a*b + b^2)*c^6*d^3 + 15*(a^2 - 2*a*b + b^2)*c^4*d^4 + 9*(a^2 - 2*a*b + b^2)*c^2*d^5)*x^2 - 2*((a^2*b - 2*a*b^2 + b^3)*c^8*d^2 + 7*(a^2*b - 2*a*b^2 + b^3)*c^6*d^3 + 15*(a^2*b - 2*a*b^2 + b^3)*c^4*d^4 + 9*(a^2*b - 2*a*b^2 + b^3)*c^2*d^5)*x + 2*((a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*c^4*d^5 + 3*(a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*c^2*d^6 + ((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4

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- b^5)*c^4*d^5 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
)*c^2*d^6)*x^2 - 2*((a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5
- b^6)*c^4*d^5 + 3*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 -
 b^6)*c^2*d^6)*x)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4
*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)))*(((a^3 - 3*a^2*b + 3
*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b + 15*a^4
*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - 3*c^2 - d)/((a^3 -
3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3))/(b^2 - 2*b*x + x^2)) + 1/2*(-((a^3 -
3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5
*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) + 3*c^2 +
d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*log(((c^5 + 2*c^3*d - 3*c*d
^2)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3) - ((a*b - b^2)*c
^4*d + 3*(a*b - b^2)*c^2*d^2 - ((a - b)*c^4*d + 3*(a - b)*c^2*d^2)*x + ((a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d^4*x - (a^4*b - 4*a^3*b^2 + 6*a^2
*b^3 - 4*a*b^4 + b^5)*d^4)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b
 + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)))*(-((a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*
b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) + 3*c^2 + d
)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3))/(b - x)) + 1/2*(((a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b
+ 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - 3*c^2 - d)/
((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*log(((c^5 + 2*c^3*d - 3*c*d^2)
*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3) - ((a*b - b^2)*c^4*
d + 3*(a*b - b^2)*c^2*d^2 - ((a - b)*c^4*d + 3*(a - b)*c^2*d^2)*x - ((a^4 -
 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d^4*x - (a^4*b - 4*a^3*b^2 + 6*a^2*b^
3 - 4*a*b^4 + b^5)*d^4)*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b +
15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)))*(((a^3 - 3*a^2
*b + 3*a*b^2 - b^3)*d^2*sqrt((c^6 + 6*c^4*d + 9*c^2*d^2)/((a^6 - 6*a^5*b +
15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^5)) - 3*c^2 - d)/((
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3))/(b - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bc-(c+1)x+a)(b-x)}{\left(-(a-x)(b-x)^2\right)^{\frac{2}{3}}\left(b^2d+(d-1)x^2-a^2-2\,(bd-a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-a-b*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(2/3)/(-a^2+b^2*d+2*(-b
*d+a)*x+(-1+d)*x^2),x, algorithm="giac")
```

```
[Out] integrate((b*c - (c + 1)*x + a)*(b - x)/((-(a - x)*(b - x)^2)^(2/3)*(b^2*d
+ (d - 1)*x^2 - a^2 - 2*(b*d - a)*x)), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)\,(-a-bc+(1+c)\,x)}{\left((-a+x)\,(-b+x)^2\right)^{\frac{2}{3}}\left(-a^2+b^2d+2\,(-bd+a)\,x+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b+x)*(-a-b*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(2/3)/(-a^2+b^2*d+2*(-b*d+a)*
x+(-1+d)*x^2),x)
```

```
[Out] int((-b+x)*(-a-b*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(2/3)/(-a^2+b^2*d+2*(-b*d+a)*
x+(-1+d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bc-(c+1)x+a)(b-x)}{\left(-(a-x)(b-x)^2\right)^{\frac{2}{3}}\left(b^2d+(d-1)x^2-a^2-2\,(bd-a)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-a-b*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(2/3)/(-a^2+b^2*d+2*(-b
*d+a)*x+(-1+d)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((b*c - (c + 1)*x + a)*(b - x)/((-(a - x)*(b - x)^2)^(2/3)*(b^2*d
+ (d - 1)*x^2 - a^2 - 2*(b*d - a)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int -\frac{(b-x)\,(a+b\,c-x\,(c+1))}{\left(-(a-x)\,(b-x)^2\right)^{2/3}\,\left(b^2\,d+2\,x\,(a-b\,d)-a^2+x^2\,(d-1)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b - x)*(a + b*c - x*(c + 1)))/((-(a - x)*(b - x)^2)^(2/3)*(b^2*d + 2*
x*(a - b*d) - a^2 + x^2*(d - 1))),x)
```

```
[Out] -int(-((b - x)*(a + b*c - x*(c + 1)))/((-(a - x)*(b - x)^2)^(2/3)*(b^2*d +
2*x*(a - b*d) - a^2 + x^2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-a-b*c+(1+c)*x)/((-a+x)*(-b+x)**2)**(2/3)/(-a**2+b**2*d+2
*(-b*d+a)*x+(-1+d)*x**2),x)
```

```
[Out] Timed out
```

3.2442 $\int \frac{(-b+x)(-b-ac+(1+c)x)}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(-b^2+a^2d+2(b-ad)x+(-1+d)x^2\right)}\,dx$

Optimal. Leaf size=1886

$$(b-x)^{4/3}(x-a)^{2/3}\left(\sqrt[3]{b-x}-\sqrt[6]{d}\,\sqrt[3]{x-a}\right)\left(\sqrt[3]{b-x}+\sqrt[6]{d}\,\sqrt[3]{x-a}\right)\left((b-x)^{2/3}-\sqrt[6]{d}\,\sqrt[3]{x-a}\,\sqrt[3]{b-x}+\sqrt[3]{d}\,(x-a)^{2/3}\right.$$

Rubi [A] time = 2.16, antiderivative size = 561, normalized size of antiderivative = 0.30, number of steps used = 5, number of rules used = 3, integrand size = 65, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.046, Rules used = {6719, 6728, 91}

$$-\frac{(x-a)^{2/3}(x-b)^{4/3}\left(c+\sqrt{d}\right)\log\left(2\left(\sqrt{d}+1\right)\left(b-a\sqrt{d}\right)-2(1-d)x\right)}{4d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}-\frac{(x-a)^{2/3}(x-b)^{4/3}\left(c-\sqrt{d}\right)\log\left(2\left(1-\sqrt{d}\right)\left(a\sqrt{d}+b\right)-2(1-d)x\right)}{4d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{3(x-a)^{2/3}(x-b)^{4/3}\left(c-\sqrt{d}\right)\log\left(-\sqrt[3]{x-a}-\frac{\sqrt[3]{x-b}}{\sqrt[6]{d}}\right)}{4d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{3(x-a)^{2/3}(x-b)^{4/3}\left(c+\sqrt{d}\right)\log\left(\frac{\sqrt[3]{x-b}}{\sqrt[6]{d}}-\sqrt[3]{x-a}\right)}{4d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{\sqrt{3}(x-a)^{2/3}(x-b)^{4/3}\left(c-\sqrt{d}\right)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x-b}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-a}}\right)}{2d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}+\frac{\sqrt{3}(x-a)^{2/3}(x-b)^{4/3}\left(c+\sqrt{d}\right)\tan^{-1}\left(\frac{2\sqrt[3]{x-b}}{\sqrt{3}\sqrt[6]{d}\sqrt[3]{x-a}}+\frac{1}{\sqrt{3}}\right)}{2d^{2/3}(a-b)\left(-\left((a-x)(b-x)^2\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((-b + x)*(-b - a*c + (1 + c)*x))/(((-a + x)*(-b + x)^2)^(2/3)*(-b^2 +
a^2*d + 2*(b - a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] (Sqrt[3]*(c - Sqrt[d])*(-a + x)^(2/3)*(-b + x)^(4/3)*ArcTan[1/Sqrt[3] - (2*
(-b + x)^(1/3))/(Sqrt[3]*d^(1/6)*(-a + x)^(1/3))]])/(2*(a - b)*d^(2/3)*(-((a
 - x)*(b - x)^2))^(2/3)) + (Sqrt[3]*(c + Sqrt[d])*(-a + x)^(2/3)*(-b + x)^(
4/3)*ArcTan[1/Sqrt[3] + (2*(-b + x)^(1/3))/(Sqrt[3]*d^(1/6)*(-a + x)^(1/3))
]])/(2*(a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(2/3)) - ((c + Sqrt[d])*(-a +
x)^(2/3)*(-b + x)^(4/3)*Log[2*(1 + Sqrt[d])*(b - a*Sqrt[d]) - 2*(1 - d)*x])
/(4*(a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(2/3)) - ((c - Sqrt[d])*(-a + x)
^(2/3)*(-b + x)^(4/3)*Log[2*(1 - Sqrt[d])*(b + a*Sqrt[d]) - 2*(1 - d)*x])/(
4*(a - b)*d^(2/3)*(-((a - x)*(b - x)^2))^(2/3)) + (3*(c - Sqrt[d])*(-a + x)
^(2/3)*(-b + x)^(4/3)*Log[-(-a + x)^(1/3) - (-b + x)^(1/3)/d^(1/6)])/(4*(a
- b)*d^(2/3)*(-((a - x)*(b - x)^2))^(2/3)) + (3*(c + Sqrt[d])*(-a + x)^(2/3
)*(-b + x)^(4/3)*Log[-(-a + x)^(1/3) + (-b + x)^(1/3)/d^(1/6)])/(4*(a - b)*
d^(2/3)*(-((a - x)*(b - x)^2))^(2/3))
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(-b+x)(-b-ac+(1+c)x)}{\left((-a+x)(-b+x)^2\right)^{2/3}\left(-b^2+a^2d+2(b-ad)x+(-1+d)x^2\right)}dx = \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \frac{}{(-a+x)^{2/3}\sqrt[3]{-b+x}}}{\left((-a+x)(-b+x\right.}$$

$$= \frac{\left((-a+x)^{2/3}(-b+x)^{4/3}\right)\int \left(\frac{}{(-a+x)^{2/3}\sqrt[3]{-b+}}\right.}{}$$

$$= \frac{\left(\left(1+c-\frac{c+d}{\sqrt{d}}\right)(-a+x)^{2/3}(-b+x)^{4/3}\right)\int}{\left((-a+x)\right.}$$

$$= \frac{\sqrt{3}\left(c-\sqrt{d}\right)(-a+x)^{2/3}(-b+x)^{4/3}\tan}{2(a-b)d^{2/3}\left(-\left((a-x)(b-\right.\right.}$$

Mathematica [C] time = 0.83, size = 109, normalized size = 0.06

$$-\frac{3(b-x)^2\left(\left(c-\sqrt{d}\right){}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{b-x}{\sqrt{d}\,(x-a)}\right)+\left(c+\sqrt{d}\right){}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x-b}{\sqrt{d}\,(x-a)}\right)\right)}{4d(a-b)\left((x-a)(b-x)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-b + x)*(-b - a*c + (1 + c)*x))/(((-a + x)*(-b + x)^2)^(2/3)*(-
b^2 + a^2*d + 2*(b - a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] (-3*(b - x)^2*((c - Sqrt[d])*Hypergeometric2F1[2/3, 1, 5/3, (b - x)/(Sqrt[d
]*(-a + x))] + (c + Sqrt[d])*Hypergeometric2F1[2/3, 1, 5/3, (-b + x)/(Sqrt[
d]*(-a + x))]))/(4*(a - b)*d*((b - x)^2*(-a + x))^(2/3))
```

IntegrateAlgebraic [A] time = 22.36, size = 587, normalized size = 0.31

$$\frac{(x-a)^{2/3}(b-x)^{4/3}\left(\sqrt[3]{b-x}-\sqrt[6]{d}\sqrt[3]{x-a}\right)\left(\sqrt[6]{d}\sqrt[3]{x-a}+\sqrt[3]{b-x}\right)\left(\sqrt[3]{d}(x-a)^{2/3}(b-x)^{2/3}+a\left(-d^{2/3}\right)\sqrt[3]{x-a}+d^{2/3}x\sqrt[3]{x-a}-x\sqrt[3]{b-x}+b\sqrt[3]{b-x}\right)\left(\frac{\left(c-\sqrt{d}\right)\log\left(\frac{\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt[3]{b-x}}-1\right)}{2d^{2/3}(a-b)}+\frac{\left(c+\sqrt{d}\right)\log\left(\frac{\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt[3]{b-x}}+1\right)}{2d^{2/3}(a-b)}+\frac{\left(-c-\sqrt{d}\right)\log\left(\frac{\sqrt[3]{d}(x-a)^{2/3}}{(b-x)^{2/3}}-\frac{\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt[3]{b-x}}+1\right)}{4d^{2/3}(a-b)}+\frac{\left(\sqrt{d}-c\right)\log\left(\frac{\sqrt[3]{d}(x-a)^{2/3}}{(b-x)^{2/3}}+\frac{\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt[3]{b-x}}+1\right)}{4d^{2/3}(a-b)}-\frac{\sqrt{3}\left(c+\sqrt{d}\right)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{b-x}}\right)}{2d^{2/3}(a-b)}-\frac{\sqrt{3}\left(c-\sqrt{d}\right)\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{b-x}}+\frac{1}{\sqrt{3}}\right)}{2d^{2/3}(a-b)}\right)}{\left((x-a)(b-x)^2\right)^{2/3}\left(a^2(-d)+2adx+b^2-2bx-(d-1)x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((-b + x)*(-b - a*c + (1 + c)*x))/(((-a + x)*(-b + x)^2)
^(2/3)*(-b^2 + a^2*d + 2*(b - a*d)*x + (-1 + d)*x^2)),x]
```

```
[Out] ((b - x)^(4/3)*(-a + x)^(2/3)*((b - x)^(1/3) - d^(1/6)*(-a + x)^(1/3))*((b
- x)^(1/3) + d^(1/6)*(-a + x)^(1/3))*(b*(b - x)^(1/3) - (b - x)^(1/3)*x - a
*d^(2/3)*(-a + x)^(1/3) + d^(2/3)*x*(-a + x)^(1/3) + d^(1/3)*(b - x)^(2/3)*
(-a + x)^(2/3))*(-1/2*(Sqrt[3]*(c + Sqrt[d])*ArcTan[1/Sqrt[3] - (2*d^(1/6)*
(-a + x)^(1/3))/(Sqrt[3]*(b - x)^(1/3))])/((a - b)*d^(2/3)) - (Sqrt[3]*(c -
 Sqrt[d])*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(-a + x)^(1/3))/(Sqrt[3]*(b - x)^(1
/3))])/(2*(a - b)*d^(2/3)) + ((c - Sqrt[d])*Log[-1 + (d^(1/6)*(-a + x)^(1/3
))/(b - x)^(1/3)])/(2*(a - b)*d^(2/3)) + ((c + Sqrt[d])*Log[1 + (d^(1/6)*(-
a + x)^(1/3))/(b - x)^(1/3)])/(2*(a - b)*d^(2/3)) + ((-c - Sqrt[d])*Log[1 -
 (d^(1/6)*(-a + x)^(1/3))/(b - x)^(1/3) + (d^(1/3)*(-a + x)^(2/3))/(b - x)^
(2/3)])/(4*(a - b)*d^(2/3)) + ((-c + Sqrt[d])*Log[1 + (d^(1/6)*(-a + x)^(1/
3))/(b - x)^(1/3) + (d^(1/3)*(-a + x)^(2/3))/(b - x)^(2/3)])/(4*(a - b)*d^(
2/3))))/(((b - x)^2*(-a + x))^(2/3)*(b^2 - a^2*d - 2*b*x + 2*a*d*x - (-1 +
d)*x^2))
```

fricas [B] time = 1.01, size = 9468, normalized size = 5.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-b-a*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(2/3)/(-b^2+a^2*d+2*(-a
*d+b)*x+(-1+d)*x^2),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)
/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^
3)) + c^3 + 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*arctan(1/3*
(2*(sqrt(3)*((a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*
c^2*d^3 + (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*d^4
 - ((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^2*d^3 + (a^
5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d^4)*x)*sqrt((9*c^4
+ 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6
*a*b^5 + b^6)*d^3)) - 2*sqrt(3)*(3*(a^2*b - 2*a*b^2 + b^3)*c^3*d^2 + (a^2*b
 - 2*a*b^2 + b^3)*c*d^3 - (3*(a^2 - 2*a*b + b^2)*c^3*d^2 + (a^2 - 2*a*b + b
^2)*c*d^3)*x))*sqrt(((9*c^8 - 12*c^6*d - 2*c^4*d^2 + 4*c^2*d^3 + d^4)*(-a*b
^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3) + (9*(a*b - b^2)*c^6*d -
3*(a*b - b^2)*c^4*d^2 - 5*(a*b - b^2)*c^2*d^3 - (a*b - b^2)*d^4 - (9*(a - b
)*c^6*d - 3*(a - b)*c^4*d^2 - 5*(a - b)*c^2*d^3 - (a - b)*d^4)*x - (3*(a^4*
b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^5*d^2 - 2*(a^4*b - 4*a^3*b^2 +
 6*a^2*b^3 - 4*a*b^4 + b^5)*c^3*d^3 - (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*
b^4 + b^5)*c*d^4 - (3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^5*d^2 -
 2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^3*d^3 - (a^4 - 4*a^3*b + 6
*a^2*b^2 - 4*a*b^3 + b^4)*c*d^4)*x)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*
a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)))*(-a*b^
2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(((a^3 - 3*a^2*b + 3*a*b^2
 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*
a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) + c^3 + 3*c*d)/((a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*d^2))^(1/3) + (9*(a^2*b^2 - 2*a*b^3 + b^4)*c^6*d + 15*(a^2
*b^2 - 2*a*b^3 + b^4)*c^4*d^2 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*c^2*d^3 + (a^2*
b^2 - 2*a*b^3 + b^4)*d^4 + (9*(a^2 - 2*a*b + b^2)*c^6*d + 15*(a^2 - 2*a*b +
 b^2)*c^4*d^2 + 7*(a^2 - 2*a*b + b^2)*c^2*d^3 + (a^2 - 2*a*b + b^2)*d^4)*x^
2 - 2*(9*(a^2*b - 2*a*b^2 + b^3)*c^6*d + 15*(a^2*b - 2*a*b^2 + b^3)*c^4*d^2
 + 7*(a^2*b - 2*a*b^2 + b^3)*c^2*d^3 + (a^2*b - 2*a*b^2 + b^3)*d^4)*x - 2*(
3*(a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*c^3*d^3 +
 (a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*c*d^4 + (3
*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^3*d^3 + (a^5 -
 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c*d^4)*x^2 - 2*(3*(a^5*
b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^3*d^3 + (a^5*b -
 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c*d^4)*x)*sqrt((9*c^4
 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 -
6*a*b^5 + b^6)*d^3)))*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6
*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*
b^5 + b^6)*d^3)) + c^3 + 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3
))/(b^2 - 2*b*x + x^2))*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 +
 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*
a*b^5 + b^6)*d^3)) + c^3 + 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2
/3) - 2*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(sqrt(3)*(3*
(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^6*d^3 + (a^5 -
5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^4*d^4 - 3*(a^5 - 5*a^4
*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^2*d^5 - (a^5 - 5*a^4*b + 10
*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d^6)*sqrt((9*c^4 + 6*c^2*d + d^2)/((
a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3))
 - 2*sqrt(3)*(9*(a^2 - 2*a*b + b^2)*c^7*d^2 - 3*(a^2 - 2*a*b + b^2)*c^5*d^3
 - 5*(a^2 - 2*a*b + b^2)*c^3*d^4 - (a^2 - 2*a*b + b^2)*c*d^5))*(((a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15
*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) + c^3 + 3*c*d)/((
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3) + sqrt(3)*(9*b*c^10 - 21*b*c^8*d
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+ 10*b*c^6*d^2 + 6*b*c^4*d^3 - 3*b*c^2*d^4 - b*d^5 - (9*c^10 - 21*c^8*d +
10*c^6*d^2 + 6*c^4*d^3 - 3*c^2*d^4 - d^5)*x))/(9*b*c^10 - 21*b*c^8*d + 10*b
*c^6*d^2 + 6*b*c^4*d^3 - 3*b*c^2*d^4 - b*d^5 - (9*c^10 - 21*c^8*d + 10*c^6*
d^2 + 6*c^4*d^3 - 3*c^2*d^4 - d^5)*x)) + sqrt(3)*(-((a^3 - 3*a^2*b + 3*a*b^
2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20
*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) - c^3 - 3*c*d)/((a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*d^2))^(1/3)*arctan(1/3*(2*(sqrt(3)*((a^5*b - 5*a^4*b^2 +
10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^2*d^3 + (a^5*b - 5*a^4*b^2 + 10*
a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*d^4 - ((a^5 - 5*a^4*b + 10*a^3*b^2 -
10*a^2*b^3 + 5*a*b^4 - b^5)*c^2*d^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*
b^3 + 5*a*b^4 - b^5)*d^4)*x)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b +
 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) + 2*sqrt(3)*(3
*(a^2*b - 2*a*b^2 + b^3)*c^3*d^2 + (a^2*b - 2*a*b^2 + b^3)*c*d^3 - (3*(a^2
- 2*a*b + b^2)*c^3*d^2 + (a^2 - 2*a*b + b^2)*c*d^3)*x))*sqrt(((9*c^8 - 12*c
^6*d - 2*c^4*d^2 + 4*c^2*d^3 + d^4)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b
+ b^2)*x)^(2/3) + (9*(a*b - b^2)*c^6*d - 3*(a*b - b^2)*c^4*d^2 - 5*(a*b - b
^2)*c^2*d^3 - (a*b - b^2)*d^4 - (9*(a - b)*c^6*d - 3*(a - b)*c^4*d^2 - 5*(a
 - b)*c^2*d^3 - (a - b)*d^4)*x + (3*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^
4 + b^5)*c^5*d^2 - 2*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^3*d^
3 - (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c*d^4 - (3*(a^4 - 4*a^3
*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^5*d^2 - 2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4)*c^3*d^3 - (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c*d^4)*x
)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 +
15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)))*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b +
 b^2)*x)^(1/3)*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d
 + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 +
b^6)*d^3)) - c^3 - 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3) + (9
*(a^2*b^2 - 2*a*b^3 + b^4)*c^6*d + 15*(a^2*b^2 - 2*a*b^3 + b^4)*c^4*d^2 + 7
*(a^2*b^2 - 2*a*b^3 + b^4)*c^2*d^3 + (a^2*b^2 - 2*a*b^3 + b^4)*d^4 + (9*(a^
2 - 2*a*b + b^2)*c^6*d + 15*(a^2 - 2*a*b + b^2)*c^4*d^2 + 7*(a^2 - 2*a*b +
b^2)*c^2*d^3 + (a^2 - 2*a*b + b^2)*d^4)*x^2 - 2*(9*(a^2*b - 2*a*b^2 + b^3)*
c^6*d + 15*(a^2*b - 2*a*b^2 + b^3)*c^4*d^2 + 7*(a^2*b - 2*a*b^2 + b^3)*c^2*
d^3 + (a^2*b - 2*a*b^2 + b^3)*d^4)*x + 2*(3*(a^5*b^2 - 5*a^4*b^3 + 10*a^3*b
^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*c^3*d^3 + (a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^
4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*c*d^4 + (3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10
*a^2*b^3 + 5*a*b^4 - b^5)*c^3*d^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^
3 + 5*a*b^4 - b^5)*c*d^4)*x^2 - 2*(3*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a
^2*b^4 + 5*a*b^5 - b^6)*c^3*d^3 + (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*
b^4 + 5*a*b^5 - b^6)*c*d^4)*x)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b
 + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)))*(-((a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b +
15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) - c^3 - 3*c*d)/
((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3))/(b^2 - 2*b*x + x^2))*(-((a^3
- 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b
 + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) - c^3 - 3*c*
d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3) - 2*(-a*b^2 - (a + 2*b)*x^2
 + x^3 + (2*a*b + b^2)*x)^(1/3)*(sqrt(3)*(3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 1
0*a^2*b^3 + 5*a*b^4 - b^5)*c^6*d^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b
^3 + 5*a*b^4 - b^5)*c^4*d^4 - 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 +
5*a*b^4 - b^5)*c^2*d^5 - (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
 - b^5)*d^6)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20
*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) + 2*sqrt(3)*(9*(a^2 - 2*a*b +
b^2)*c^7*d^2 - 3*(a^2 - 2*a*b + b^2)*c^5*d^3 - 5*(a^2 - 2*a*b + b^2)*c^3*d^
4 - (a^2 - 2*a*b + b^2)*c*d^5))*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt
((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2
*b^4 - 6*a*b^5 + b^6)*d^3)) - c^3 - 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)
*d^2))^(2/3) - sqrt(3)*(9*b*c^10 - 21*b*c^8*d + 10*b*c^6*d^2 + 6*b*c^4*d^3
- 3*b*c^2*d^4 - b*d^5 - (9*c^10 - 21*c^8*d + 10*c^6*d^2 + 6*c^4*d^3 - 3*c^2
*d^4 - d^5)*x))/(9*b*c^10 - 21*b*c^8*d + 10*b*c^6*d^2 + 6*b*c^4*d^3 - 3*b*c
```

^2*d^4 - b*d^5 - (9*c^10 - 21*c^8*d + 10*c^6*d^2 + 6*c^4*d^3 - 3*c^2*d^4 - d^5)*x)) - 1/4*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) + c^3 + 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*log(((9*c^8 - 12*c^6*d - 2*c^4*d^2 + 4*c^2*d^3 + d^4)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3) + (9*(a*b - b^2)*c^6*d - 3*(a*b - b^2)*c^4*d^2 - 5*(a*b - b^2)*c^2*d^3 - (a*b - b^2)*d^4 - (9*(a - b)*c^6*d - 3*(a - b)*c^4*d^2 - 5*(a - b)*c^2*d^3 - (a - b)*d^4)*x - (3*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^5*d^2 - 2*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^3*d^3 - (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c*d^4 - (3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^5*d^2 - 2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^3*d^3 - (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c*d^4)*x)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)))*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) + c^3 + 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3) + (9*(a^2*b^2 - 2*a*b^3 + b^4)*c^6*d + 15*(a^2*b^2 - 2*a*b^3 + b^4)*c^4*d^2 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*c^2*d^3 + (a^2*b^2 - 2*a*b^3 + b^4)*d^4 + (9*(a^2 - 2*a*b + b^2)*c^6*d + 15*(a^2 - 2*a*b + b^2)*c^4*d^2 + 7*(a^2 - 2*a*b + b^2)*c^2*d^3 + (a^2 - 2*a*b + b^2)*d^4)*x^2 - 2*(9*(a^2*b - 2*a*b^2 + b^3)*c^6*d + 15*(a^2*b - 2*a*b^2 + b^3)*c^4*d^2 + 7*(a^2*b - 2*a*b^2 + b^3)*c^2*d^3 + (a^2*b - 2*a*b^2 + b^3)*d^4)*x - 2*(3*(a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*c^3*d^3 + (a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*c*d^4 + (3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^3*d^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c*d^4)*x^2 - 2*(3*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^3*d^3 + (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c*d^4)*x)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)))*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) + c^3 + 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3))/(b^2 - 2*b*x + x^2)) - 1/4*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) - c^3 - 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*log(((9*c^8 - 12*c^6*d - 2*c^4*d^2 + 4*c^2*d^3 + d^4)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(2/3) + (9*(a*b - b^2)*c^6*d - 3*(a*b - b^2)*c^4*d^2 - 5*(a*b - b^2)*c^2*d^3 - (a*b - b^2)*d^4 - (9*(a - b)*c^6*d - 3*(a - b)*c^4*d^2 - 5*(a - b)*c^2*d^3 - (a - b)*d^4)*x + (3*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^5*d^2 - 2*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c^3*d^3 - (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*c*d^4 - (3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^5*d^2 - 2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c^3*d^3 - (a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c*d^4)*x)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)))*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3)*(-((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) - c^3 - 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3) + (9*(a^2*b^2 - 2*a*b^3 + b^4)*c^6*d + 15*(a^2*b^2 - 2*a*b^3 + b^4)*c^4*d^2 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*c^2*d^3 + (a^2*b^2 - 2*a*b^3 + b^4)*d^4 + (9*(a^2 - 2*a*b + b^2)*c^6*d + 15*(a^2 - 2*a*b + b^2)*c^4*d^2 + 7*(a^2 - 2*a*b + b^2)*c^2*d^3 + (a^2 - 2*a*b + b^2)*d^4)*x^2 - 2*(9*(a^2*b - 2*a*b^2 + b^3)*c^6*d + 15*(a^2*b - 2*a*b^2 + b^3)*c^4*d^2 + 7*(a^2*b - 2*a*b^2 + b^3)*c^2*d^3 + (a^2*b - 2*a*b^2 + b^3)*d^4)*x + 2*(3*(a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*c^3*d^3 + (a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*c*d^4 + (3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c^3*d^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*c*d^4)*x^2 - 2*(3*(a^5*b - 5*a^4*b^2 + 10*a^3*b^

```
3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*c^3*d^3 + (a^5*b - 5*a^4*b^2 + 10*a^3*b^3 -
 10*a^2*b^4 + 5*a*b^5 - b^6)*c*d^4)*x)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 -
 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)))*(-(
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*
a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) - c^3 -
 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(2/3))/(b^2 - 2*b*x + x^2))
+ 1/2*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((
a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3))
 + c^3 + 3*c*d)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*log(((3*c^4 -
2*c^2*d - d^2)*(-a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3) - (3*
(a*b - b^2)*c^2*d + (a*b - b^2)*d^2 - (3*(a - b)*c^2*d + (a - b)*d^2)*x + (
(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*c*d^2*x - (a^4*b - 4*a^3*b^2 +
6*a^2*b^3 - 4*a*b^4 + b^5)*c*d^2)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^
5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)))*(((a^3 -
 3*a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b
+ 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) + c^3 + 3*c*d
)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3))/(b - x)) + 1/2*(-((a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 1
5*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) - c^3 - 3*c*d)/(
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2))^(1/3)*log(((3*c^4 - 2*c^2*d - d^2)*(-
a*b^2 - (a + 2*b)*x^2 + x^3 + (2*a*b + b^2)*x)^(1/3) - (3*(a*b - b^2)*c^2*d
 + (a*b - b^2)*d^2 - (3*(a - b)*c^2*d + (a - b)*d^2)*x - ((a^4 - 4*a^3*b +
6*a^2*b^2 - 4*a*b^3 + b^4)*c*d^2*x - (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b
^4 + b^5)*c*d^2)*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2
- 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)))*(-((a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*d^2*sqrt((9*c^4 + 6*c^2*d + d^2)/((a^6 - 6*a^5*b + 15*a^4*b^2 - 2
0*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*d^3)) - c^3 - 3*c*d)/((a^3 - 3*a^2*
b + 3*a*b^2 - b^3)*d^2))^(1/3))/(b - x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ac-(c+1)x+b)(b-x)}{\left(-(a-x)(b-x)^2\right)^{\frac{2}{3}}\left(a^2d+(d-1)x^2-b^2-2\,(ad-b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-b-a*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(2/3)/(-b^2+a^2*d+2*(-a
*d+b)*x+(-1+d)*x^2),x, algorithm="giac")
```

```
[Out] integrate((a*c - (c + 1)*x + b)*(b - x)/((-(a - x)*(b - x)^2)^(2/3)*(a^2*d
+ (d - 1)*x^2 - b^2 - 2*(a*d - b)*x)), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{(-b+x)\,(-b-ac+(1+c)\,x)}{\left((-a+x)\,(-b+x)^2\right)^{\frac{2}{3}}\left(-b^2+a^2d+2\,(-ad+b)\,x+(-1+d)\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b+x)*(-b-a*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(2/3)/(-b^2+a^2*d+2*(-a*d+b)*
x+(-1+d)*x^2),x)
```

```
[Out] int((-b+x)*(-b-a*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(2/3)/(-b^2+a^2*d+2*(-a*d+b)*
x+(-1+d)*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ac-(c+1)x+b)(b-x)}{\left(-(a-x)(b-x)^2\right)^{\frac{2}{3}}\left(a^2d+(d-1)x^2-b^2-2\,(ad-b)x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-b-a*c+(1+c)*x)/((-a+x)*(-b+x)^2)^(2/3)/(-b^2+a^2*d+2*(-a
*d+b)*x+(-1+d)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((a*c - (c + 1)*x + b)*(b - x)/((-(a - x)*(b - x)^2)^(2/3)*(a^2*d
+ (d - 1)*x^2 - b^2 - 2*(a*d - b)*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int -\frac{(b-x)\,(b+a\,c-x\,(c+1))}{\left(-(a-x)\,(b-x)^2\right)^{2/3}\,\left(a^2\,d+2\,x\,(b-a\,d)-b^2+x^2\,(d-1)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b - x)*(b + a*c - x*(c + 1)))/((-(a - x)*(b - x)^2)^(2/3)*(a^2*d + 2*
x*(b - a*d) - b^2 + x^2*(d - 1))),x)
```

```
[Out] -int(-((b - x)*(b + a*c - x*(c + 1)))/((-(a - x)*(b - x)^2)^(2/3)*(a^2*d +
2*x*(b - a*d) - b^2 + x^2*(d - 1))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+x)*(-b-a*c+(1+c)*x)/((-a+x)*(-b+x)**2)**(2/3)/(-b**2+a**2*d+2
*(-a*d+b)*x+(-1+d)*x**2),x)
```

```
[Out] Timed out
```

3.2443 $\int \frac{x^2-cx^2\left(\frac{b+ax}{d+cx}\right)^{3/2}}{a-b\sqrt{\frac{b+ax}{d+cx}}}\,dx$

Optimal. Leaf size=1916

$$-3a^2c^2x^2b^6+6c^3xb^6+6a^2cdxb^6-2a^3c^2x^3b^5-3ac^3x^2b^5+3a^3cdx^2b^5-6a^3d^2xb^5-6ac^2dxb^5+2a^2c^3x^3b^4+9$$

Rubi [A] time = 4.68, antiderivative size = 1063, normalized size of antiderivative = 0.55, number of steps used = 9, number of rules used = 5, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.096, Rules used = {1647, 1629, 635, 208, 260}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(x^2 - c*x^2*((b + a*x)/(d + c*x))^(3/2))/(a - b*Sqrt[(b + a*x)/(d + c*
x)]),x]
```

```
[Out] ((d + c*x)*(8*a*(6*a^3*b^4*c^2*d - 6*a^4*b^2*c^3*d + 3*a^3*b^5*d^2 + 6*a^5*
b*c^2*d^2 - a^4*c^3*d^2 - b^6*(c^3 + 2*a^2*c*d) + a^2*b^3*c*(a*c^3 + 2*c^2*
d - 8*a^2*d^2)) - c*(5*b^7*c^2 - 29*a^6*c^2*d^2 + 2*a*b^5*c*(2*c^2 + b*d) +
 2*a^5*b*c*d*(11*c^2 + 16*b*d) - 2*a^3*b^2*c*(4*b^2*c^2 - b^3*d + c^2*d + 2
*b*d^2) + a^2*b^3*(b^3*c^2 - c^4 - 16*b*c^2*d + b^2*d^2) - a^4*b*(b*c^4 + 8
*b^2*c^2*d + 11*b^3*d^2 - 11*c^2*d^2))*Sqrt[(b + a*x)/(d + c*x)]))/(8*a^3*c
^3*(b^2 - a*c)^3) - ((b*c - a*d)^3*(a*(a*b - c) + (a^2 - b)*c*Sqrt[(b + a*x
)/(d + c*x)]))/(3*a*c^3*(b^2 - a*c)*(a - (c*(b + a*x))/(d + c*x))^3) + (6*a
*(b*c - a*d)^2*(b^3*c^2 + 4*a^3*b*c*d + a*b^2*c*(b^2 + d) - a^2*(2*b^2*c^2
+ 3*b^3*d + 2*c^2*d)) + c*(b*c - a*d)^2*(5*b^4*c + a^2*b*c*(b^2 - 13*d) + 1
9*a^4*c*d + a*b^2*(c^2 + 7*b*d) - a^3*b*(7*c^2 + 13*b*d))*Sqrt[(b + a*x)/(d
 + c*x)])/(12*a^2*c^3*(b^2 - a*c)^2*(a - (c*(b + a*x))/(d + c*x))^2) + ((b*
c - a*d)*(5*b^9*c^2 - 35*a^7*c^3*d^2 + a*b^7*c*(15*c^2 + 2*b*d) + 5*a^6*b*c
^2*d*(2*c^2 + 7*b*d) + a^2*b^5*(b^3*c^2 - 5*c^4 - 18*b*c^2*d + b^2*d^2) - a
^3*b^3*c*(9*b^3*c^2 - c^4 - 2*b^4*d + 18*b*c^2*d + 5*b^2*d^2) + a^5*b*c*(b*
c^4 + 30*b^2*c^2*d - 21*b^3*d^2 + 5*c^2*d^2) - a^4*b^2*(9*b^2*c^4 + 10*b^3*
c^2*d - 2*c^4*d - 5*b^4*d^2 - 15*b*c^2*d^2))*ArcTanh[(Sqrt[c]*Sqrt[(b + a*x
)/(d + c*x)])/Sqrt[a]])/(8*a^(7/2)*c^(5/2)*(b^2 - a*c)^4) - (b*(b*c - a*d)*
(b^7*c^3 + a^3*b^6*d^2 + 6*a^5*b^2*c^2*d^2 - 4*a^6*c^3*d^2 + a^4*b*c^3*d*(2
*a*c + d) - a^2*b^4*c*(a*c^3 + 2*c^2*d + 4*a^2*d^2))*Log[(d + c*x)^(-1)])/(
a^3*c^3*(b^2 - a*c)^4) + (2*(b^3 - a^3*c)*(b*c - a*d)*(b^3 - a^2*d)^2*Log[a
 - b*Sqrt[(b + a*x)/(d + c*x)]])/(a^3*b*(b^2 - a*c)^4)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2-cx^2\left(\frac{b+ax}{d+cx}\right)^{3/2}}{a-b\sqrt{\frac{b+ax}{d+cx}}}\,dx=(2(bc-ad))\,\text{Subst}\left(\int \frac{x\left(b-dx^2\right)^2\left(-1+cx^3\right)}{(a-bx)\left(a-cx^2\right)^4}\,dx,x,\sqrt{\frac{b+ax}{d+cx}}\right)$$

$$=-\frac{(bc-ad)^3\left(a(ab-c)+\left(a^2-b\right)c\sqrt{\frac{b+ax}{d+cx}}\right)}{3ac^3\left(b^2-ac\right)\left(a-\frac{c(b+ax)}{d+cx}\right)^3}+\frac{(bc-ad)\,\text{Subst}\left(\int \frac{\frac{a\left(a^2-b\right)(bc-ad)^2}{c\left(b^2-ac\right)}-\frac{\left(5b^4c^2+2\right.}{}}{}\right.}{}$$

$$=-\frac{(bc-ad)^3\left(a(ab-c)+\left(a^2-b\right)c\sqrt{\frac{b+ax}{d+cx}}\right)}{3ac^3\left(b^2-ac\right)\left(a-\frac{c(b+ax)}{d+cx}\right)^3}+\frac{(bc-ad)^2\left(6a\left(b^3c^2+4a^3bcd+ab^2c\left(\right.\right.\right.}{}$$

$$=\frac{(d+cx)\left(8a\left(6a^3b^4c^2d-6a^4b^2c^3d+3a^3b^5d^2+6a^5bc^2d^2-a^4c^3d^2-b^6\left(c^3+2a^2cd\right)\right.\right.}{}$$

$$=\frac{(d+cx)\left(8a\left(6a^3b^4c^2d-6a^4b^2c^3d+3a^3b^5d^2+6a^5bc^2d^2-a^4c^3d^2-b^6\left(c^3+2a^2cd\right)\right.\right.}{}$$

$$=\frac{(d+cx)\left(8a\left(6a^3b^4c^2d-6a^4b^2c^3d+3a^3b^5d^2+6a^5bc^2d^2-a^4c^3d^2-b^6\left(c^3+2a^2cd\right)\right.\right.}{}$$

$$=\frac{(d+cx)\left(8a\left(6a^3b^4c^2d-6a^4b^2c^3d+3a^3b^5d^2+6a^5bc^2d^2-a^4c^3d^2-b^6\left(c^3+2a^2cd\right)\right.\right.}{}$$

$$=\frac{(d+cx)\left(8a\left(6a^3b^4c^2d-6a^4b^2c^3d+3a^3b^5d^2+6a^5bc^2d^2-a^4c^3d^2-b^6\left(c^3+2a^2cd\right)\right.\right.}{}$$

Mathematica [A] time = 5.33, size = 1379, normalized size = 0.72

Antiderivative was successfully verified.

```
[In] Integrate[(x^2 - c*x^2*((b + a*x)/(d + c*x))^(3/2))/(a - b*Sqrt[(b + a*x)/(
d + c*x)]),x]
```

```
[Out] ((-(b*c) + a*d)*((24*(-6*a^3*b^4*c^2*d + 6*a^4*b^2*c^3*d - 3*a^3*b^5*d^2 -
6*a^5*b*c^2*d^2 + a^4*c^3*d^2 + b^6*(c^3 + 2*a^2*c*d) + a^2*b^3*c*(-(a*c^3)
 - 2*c^2*d + 8*a^2*d^2))*(d + c*x))/(a^2*c^3*(-b^2 + a*c)^3*(-(b*c) + a*d))
 + (24*(-3*a^3*b^4*d^2 - 6*a^5*c^2*d^2 - 3*a^2*b^3*c*d*(2*a*c + d) + 3*a^3*
b*c^2*d*(2*a*c + d) + b^5*(c^3 + 2*a^2*c*d + a*d^2) + a^2*b^2*c*(-(a*c^3) -
 2*c^2*d + 8*a^2*d^2))*Sqrt[(b + a*x)/(d + c*x)]*(d + c*x))/(a^2*c^2*(-b^2
```

```
+ a*c)^3*(-(b*c) + a*d)) + (12*(b^3*c^2 + 4*a^3*b*c*d + a*b^2*c*(b^2 + d) -
 a^2*(2*b^2*c^2 + 3*b^3*d + 2*c^2*d))*(d + c*x)^2)/(a*c^3*(b^2 - a*c)^2*(-(
b*c) + a*d)) + (12*(a*b*c*(b^2 - 3*d) + 4*a^3*c*d + b^2*(c^2 + 2*b*d) - a^2
*b*(2*c^2 + 3*b*d))*Sqrt[(b + a*x)/(d + c*x)]*(d + c*x)^2)/(a*c^2*(b^2 - a*
c)^2*(-(b*c) + a*d)) + (8*(a*b - c)*(d + c*x)^3)/(c^3*(-b^2 + a*c)*(b*c - a
*d)) - (8*(a^2 - b)*Sqrt[(b + a*x)/(d + c*x)]*(d + c*x)^3)/(a*c^2*(-b^2 + a
*c)*(-(b*c) + a*d)) + (24*(3*a^3*b^4*d^2 + 6*a^5*c^2*d^2 + 3*a^2*b^3*c*d*(2
*a*c + d) - 3*a^3*b*c^2*d*(2*a*c + d) - b^5*(c^3 + 2*a^2*c*d + a*d^2) + a^2
*b^2*c*(a*c^3 + 2*c^2*d - 8*a^2*d^2))*ArcTanh[(Sqrt[c]*Sqrt[(b + a*x)/(d +
c*x)])/Sqrt[a]])/(a^(5/2)*c^(5/2)*(b^2 - a*c)^3) - (5*(a^2 - b)*(Sqrt[a]*Sq
rt[c]*Sqrt[(b + a*x)/(d + c*x)]*(d + c*x)*(-3*b*c + 5*a*d + 2*a*c*x) + 3*(b
*c - a*d)^2*ArcTanh[(Sqrt[c]*Sqrt[(b + a*x)/(d + c*x)])/Sqrt[a]]))/(a^(7/2)
*c^(5/2)*(-b^2 + a*c)) + (18*(a*b*c*(b^2 - 3*d) + 4*a^3*c*d + b^2*(c^2 + 2*
b*d) - a^2*b*(2*c^2 + 3*b*d))*(Sqrt[a]*Sqrt[c]*Sqrt[(b + a*x)/(d + c*x)]*(d
 + c*x) + (-(b*c) + a*d)*ArcTanh[(Sqrt[c]*Sqrt[(b + a*x)/(d + c*x)])/Sqrt[a
]]))/(a^(5/2)*c^(5/2)*(b^2 - a*c)^2) - (48*(b^3 - a^3*c)*(b^3 - a^2*d)^2*Lo
g[a - b*Sqrt[(b + a*x)/(d + c*x)]])/(a^3*b*(b^2 - a*c)^4) + (24*(b^7*c^3 +
a^3*b^6*d^2 + 6*a^5*b^2*c^2*d^2 - 4*a^6*c^3*d^2 + a^4*b*c^3*d*(2*a*c + d) -
 a^2*b^4*c*(a*c^3 + 2*c^2*d + 4*a^2*d^2))*Log[Sqrt[a] - Sqrt[c]*Sqrt[(b + a
*x)/(d + c*x)]])/(a^3*(b - Sqrt[a]*Sqrt[c])^4*(b + Sqrt[a]*Sqrt[c])^3*c^3)
- (24*(b^7*c^3 + a^3*b^6*d^2 + 6*a^5*b^2*c^2*d^2 - 4*a^6*c^3*d^2 + a^4*b*c^
3*d*(2*a*c + d) - a^2*b^4*c*(a*c^3 + 2*c^2*d + 4*a^2*d^2))*Log[Sqrt[a] + Sq
rt[c]*Sqrt[(b + a*x)/(d + c*x)]])/((b + Sqrt[a]*Sqrt[c])^4*(-(a*b*c) + a^(3
/2)*c^(3/2))^3)))/24
```

IntegrateAlgebraic [B] time = 3.44, size = 4104, normalized size = 2.14

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2 - c*x^2*((b + a*x)/(d + c*x))^(3/2))/(a - b*Sqrt[(b
 + a*x)/(d + c*x)]),x]
```

```
[Out] (-4*a^4*b^8*c^3 + 20*a^5*b^6*c^4 - 44*a^3*b^7*c^4 + 8*a^6*b^4*c^5 + 28*a^4*
b^5*c^5 - 8*a^5*b^3*c^6 - 12*a^5*b^7*c^2*d + 36*a^6*b^5*c^3*d + 60*a^4*b^6*
c^3*d - 96*a^7*b^3*c^4*d + 12*a^5*b^4*c^4*d + 60*a^6*b^6*c*d^2 - 180*a^7*b^
4*c^2*d^2 - 12*a^5*b^5*c^2*d^2 + 192*a^8*b^2*c^3*d^2 - 60*a^6*b^3*c^3*d^2 -
 44*a^7*b^5*d^3 + 124*a^8*b^3*c*d^3 - 4*a^6*b^4*c*d^3 - 104*a^9*b*c^2*d^3 +
 20*a^7*b^2*c^2*d^3 + 8*a^8*c^3*d^3 + 3*a^4*b^7*c^4*Sqrt[(b + a*x)/(d + c*x
)] - 33*a^2*b^8*c^4*Sqrt[(b + a*x)/(d + c*x)] + 24*a^5*b^5*c^5*Sqrt[(b + a*
x)/(d + c*x)] + 12*a^3*b^6*c^5*Sqrt[(b + a*x)/(d + c*x)] - 3*a^6*b^3*c^6*Sq
rt[(b + a*x)/(d + c*x)] - 3*a^4*b^4*c^6*Sqrt[(b + a*x)/(d + c*x)] + 3*a^5*b
^6*c^3*d*Sqrt[(b + a*x)/(d + c*x)] + 39*a^3*b^7*c^3*d*Sqrt[(b + a*x)/(d + c
*x)] - 48*a^6*b^4*c^4*d*Sqrt[(b + a*x)/(d + c*x)] + 36*a^4*b^5*c^4*d*Sqrt[(
b + a*x)/(d + c*x)] - 27*a^7*b^2*c^5*d*Sqrt[(b + a*x)/(d + c*x)] - 3*a^5*b^
3*c^5*d*Sqrt[(b + a*x)/(d + c*x)] + 9*a^6*b^5*c^2*d^2*Sqrt[(b + a*x)/(d + c
*x)] - 3*a^4*b^6*c^2*d^2*Sqrt[(b + a*x)/(d + c*x)] - 24*a^7*b^3*c^3*d^2*Sqr
t[(b + a*x)/(d + c*x)] - 60*a^5*b^4*c^3*d^2*Sqrt[(b + a*x)/(d + c*x)] + 87*
a^8*b*c^4*d^2*Sqrt[(b + a*x)/(d + c*x)] - 9*a^6*b^2*c^4*d^2*Sqrt[(b + a*x)/
(d + c*x)] - 15*a^7*b^4*c*d^3*Sqrt[(b + a*x)/(d + c*x)] - 3*a^5*b^5*c*d^3*S
qrt[(b + a*x)/(d + c*x)] + 48*a^8*b^2*c^2*d^3*Sqrt[(b + a*x)/(d + c*x)] + 1
2*a^6*b^3*c^2*d^3*Sqrt[(b + a*x)/(d + c*x)] - 57*a^9*c^3*d^3*Sqrt[(b + a*x)
/(d + c*x)] + 15*a^7*b*c^3*d^3*Sqrt[(b + a*x)/(d + c*x)] + 8*a^3*b^7*c^5*((
b + a*x)/(d + c*x))^(3/2) + 40*a*b^8*c^5*((b + a*x)/(d + c*x))^(3/2) - 64*a
^4*b^5*c^6*((b + a*x)/(d + c*x))^(3/2) + 16*a^2*b^6*c^6*((b + a*x)/(d + c*x
))^(3/2) + 8*a^5*b^3*c^7*((b + a*x)/(d + c*x))^(3/2) - 8*a^3*b^4*c^7*((b +
a*x)/(d + c*x))^(3/2) - 24*a^4*b^6*c^4*d*((b + a*x)/(d + c*x))^(3/2) - 24*a
^2*b^7*c^4*d*((b + a*x)/(d + c*x))^(3/2) + 96*a^5*b^4*c^5*d*((b + a*x)/(d +
 c*x))^(3/2) - 144*a^3*b^5*c^5*d*((b + a*x)/(d + c*x))^(3/2) + 72*a^6*b^2*c
^6*d*((b + a*x)/(d + c*x))^(3/2) + 24*a^4*b^3*c^6*d*((b + a*x)/(d + c*x))^(
```

```
3/2) - 24*a^5*b^5*c^3*d^2*((b + a*x)/(d + c*x))^(3/2) - 24*a^3*b^6*c^3*d^2*
((b + a*x)/(d + c*x))^(3/2) + 96*a^6*b^3*c^4*d^2*((b + a*x)/(d + c*x))^(3/2
) + 144*a^4*b^4*c^4*d^2*((b + a*x)/(d + c*x))^(3/2) - 216*a^7*b*c^5*d^2*((b
 + a*x)/(d + c*x))^(3/2) + 24*a^5*b^2*c^5*d^2*((b + a*x)/(d + c*x))^(3/2) +
 40*a^6*b^4*c^2*d^3*((b + a*x)/(d + c*x))^(3/2) + 8*a^4*b^5*c^2*d^3*((b + a
*x)/(d + c*x))^(3/2) - 128*a^7*b^2*c^3*d^3*((b + a*x)/(d + c*x))^(3/2) - 16
*a^5*b^3*c^3*d^3*((b + a*x)/(d + c*x))^(3/2) + 136*a^8*c^4*d^3*((b + a*x)/(
d + c*x))^(3/2) - 40*a^6*b*c^4*d^3*((b + a*x)/(d + c*x))^(3/2) - 3*a^2*b^7*
c^6*((b + a*x)/(d + c*x))^(5/2) - 15*b^8*c^6*((b + a*x)/(d + c*x))^(5/2) +
24*a^3*b^5*c^7*((b + a*x)/(d + c*x))^(5/2) - 12*a*b^6*c^7*((b + a*x)/(d + c
*x))^(5/2) + 3*a^4*b^3*c^8*((b + a*x)/(d + c*x))^(5/2) + 3*a^2*b^4*c^8*((b
+ a*x)/(d + c*x))^(5/2) - 3*a^3*b^6*c^5*d*((b + a*x)/(d + c*x))^(5/2) + 9*a
*b^7*c^5*d*((b + a*x)/(d + c*x))^(5/2) + 60*a^2*b^5*c^6*d*((b + a*x)/(d + c
*x))^(5/2) - 69*a^5*b^2*c^7*d*((b + a*x)/(d + c*x))^(5/2) + 3*a^3*b^3*c^7*d
*((b + a*x)/(d + c*x))^(5/2) + 39*a^4*b^5*c^4*d^2*((b + a*x)/(d + c*x))^(5/
2) + 3*a^2*b^6*c^4*d^2*((b + a*x)/(d + c*x))^(5/2) - 120*a^5*b^3*c^5*d^2*((
b + a*x)/(d + c*x))^(5/2) - 36*a^3*b^4*c^5*d^2*((b + a*x)/(d + c*x))^(5/2)
+ 153*a^6*b*c^6*d^2*((b + a*x)/(d + c*x))^(5/2) - 39*a^4*b^2*c^6*d^2*((b +
a*x)/(d + c*x))^(5/2) - 33*a^5*b^4*c^3*d^3*((b + a*x)/(d + c*x))^(5/2) + 3*
a^3*b^5*c^3*d^3*((b + a*x)/(d + c*x))^(5/2) + 96*a^6*b^2*c^4*d^3*((b + a*x)
/(d + c*x))^(5/2) - 12*a^4*b^3*c^4*d^3*((b + a*x)/(d + c*x))^(5/2) - 87*a^7
*c^5*d^3*((b + a*x)/(d + c*x))^(5/2) + 33*a^5*b*c^5*d^3*((b + a*x)/(d + c*x
))^(5/2) - (24*a*b^7*c^6*(b + a*x)^2)/(d + c*x)^2 + (24*a^4*b^4*c^7*(b + a*
x)^2)/(d + c*x)^2 - (48*a^3*b^7*c^4*d*(b + a*x)^2)/(d + c*x)^2 + (144*a^4*b
^5*c^5*d*(b + a*x)^2)/(d + c*x)^2 + (24*a^2*b^6*c^5*d*(b + a*x)^2)/(d + c*x
)^2 - (168*a^5*b^3*c^6*d*(b + a*x)^2)/(d + c*x)^2 + (48*a^3*b^4*c^6*d*(b +
a*x)^2)/(d + c*x)^2 + (120*a^4*b^6*c^3*d^2*(b + a*x)^2)/(d + c*x)^2 - (336*
a^5*b^4*c^4*d^2*(b + a*x)^2)/(d + c*x)^2 + (288*a^6*b^2*c^5*d^2*(b + a*x)^2
)/(d + c*x)^2 - (48*a^4*b^3*c^5*d^2*(b + a*x)^2)/(d + c*x)^2 - (24*a^5*b*c^
6*d^2*(b + a*x)^2)/(d + c*x)^2 - (72*a^5*b^5*c^2*d^3*(b + a*x)^2)/(d + c*x)
^2 + (192*a^6*b^3*c^3*d^3*(b + a*x)^2)/(d + c*x)^2 - (144*a^7*b*c^4*d^3*(b
+ a*x)^2)/(d + c*x)^2 + (24*a^6*c^5*d^3*(b + a*x)^2)/(d + c*x)^2 + (12*a^3*
b^8*c^4*(b + a*x))/(d + c*x) - (36*a^4*b^6*c^5*(b + a*x))/(d + c*x) + (60*a
^2*b^7*c^5*(b + a*x))/(d + c*x) - (24*a^5*b^4*c^6*(b + a*x))/(d + c*x) - (1
2*a^3*b^5*c^6*(b + a*x))/(d + c*x) + (36*a^4*b^7*c^3*d*(b + a*x))/(d + c*x)
 - (132*a^5*b^5*c^4*d*(b + a*x))/(d + c*x) - (60*a^3*b^6*c^4*d*(b + a*x))/(
d + c*x) + (240*a^6*b^3*c^5*d*(b + a*x))/(d + c*x) - (108*a^4*b^4*c^5*d*(b
+ a*x))/(d + c*x) + (24*a^5*b^2*c^6*d*(b + a*x))/(d + c*x) - (156*a^5*b^6*c
^2*d^2*(b + a*x))/(d + c*x) + (468*a^6*b^4*c^3*d^2*(b + a*x))/(d + c*x) - (
12*a^4*b^5*c^3*d^2*(b + a*x))/(d + c*x) - (456*a^7*b^2*c^4*d^2*(b + a*x))/(
d + c*x) + (156*a^5*b^3*c^4*d^2*(b + a*x))/(d + c*x) + (108*a^6*b^5*c*d^3*(
b + a*x))/(d + c*x) - (300*a^7*b^3*c^2*d^3*(b + a*x))/(d + c*x) + (12*a^5*b
^4*c^2*d^3*(b + a*x))/(d + c*x) + (240*a^8*b*c^3*d^3*(b + a*x))/(d + c*x) -
 (36*a^6*b^2*c^3*d^3*(b + a*x))/(d + c*x) - (24*a^7*c^4*d^3*(b + a*x))/(d +
 c*x))/(24*a^3*c^3*(-b^2 + a*c)^3*(a - (c*(b + a*x))/(d + c*x))^3) + ((a^2*
b^9*c^3 + 5*b^10*c^3 - 9*a^3*b^7*c^4 + 15*a*b^8*c^4 - 9*a^4*b^5*c^5 - 5*a^2
*b^6*c^5 + a^5*b^3*c^6 + a^3*b^4*c^6 + a^3*b^8*c^2*d - 3*a*b^9*c^2*d - a^4*
b^6*c^3*d - 33*a^2*b^7*c^3*d + 39*a^5*b^4*c^4*d - 13*a^3*b^5*c^4*d + 9*a^6*
b^2*c^5*d + a^4*b^3*c^5*d + 3*a^4*b^7*c*d^2 - a^2*b^8*c*d^2 - 11*a^5*b^5*c^
2*d^2 + 13*a^3*b^6*c^2*d^2 + 5*a^6*b^3*c^3*d^2 + 33*a^4*b^4*c^3*d^2 - 45*a^
7*b*c^4*d^2 + 3*a^5*b^2*c^4*d^2 - 5*a^5*b^6*d^3 - a^3*b^7*d^3 + 21*a^6*b^4*
c*d^3 + 5*a^4*b^5*c*d^3 - 35*a^7*b^2*c^2*d^3 - 15*a^5*b^3*c^2*d^3 + 35*a^8*
c^3*d^3 - 5*a^6*b*c^3*d^3)*ArcTanh[(Sqrt[c]*Sqrt[(b + a*x)/(d + c*x)])/Sqrt
[a]])/(8*a^(7/2)*c^(5/2)*(-b^2 + a*c)^4) + (2*(-b^3 + a^2*d)^2*(b^4*c - a^3
*b*c^2 - a*b^3*d + a^4*c*d)*Log[a - b*Sqrt[(b + a*x)/(d + c*x)]])/(a^3*b*(-
b^2 + a*c)^4) + ((-(b^9*c^4) + a^3*b^6*c^5 + a*b^8*c^3*d - a^4*b^5*c^4*d +
2*a^2*b^6*c^4*d - 2*a^5*b^3*c^5*d - a^3*b^8*c*d^2 + 4*a^4*b^6*c^2*d^2 - 6*a
^5*b^4*c^3*d^2 - 2*a^3*b^5*c^3*d^2 + 6*a^6*b^2*c^4*d^2 - a^4*b^3*c^4*d^2 +
a^4*b^7*d^3 - 4*a^5*b^5*c*d^3 + 6*a^6*b^3*c^2*d^3 - 4*a^7*b*c^3*d^3 + a^5*b
```

```
^2*c^3*d^3)*Log[a - (c*(b + a*x))/(d + c*x)])/(a^3*c^3*(-b^2 + a*c)^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-c*x^2*((a*x+b)/(c*x+d))^(3/2))/(a-b*((a*x+b)/(c*x+d))^(1/2))
,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-c*x^2*((a*x+b)/(c*x+d))^(3/2))/(a-b*((a*x+b)/(c*x+d))^(1/2))
,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x*c
+d)]Evaluation time: 0.42Error: Bad Argument Type
```

maple [B] time = 0.48, size = 384279, normalized size = 200.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-c*x^2*((a*x+b)/(c*x+d))^(3/2))/(a-b*((a*x+b)/(c*x+d))^(1/2)),x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.97, size = 2606, normalized size = 1.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-c*x^2*((a*x+b)/(c*x+d))^(3/2))/(a-b*((a*x+b)/(c*x+d))^(1/2))
,x, algorithm="maxima")
```

```
[Out] -2*(a^3*b^7*c^2 - b^10*c - (a^8*c - a^5*b^3)*d^3 + (a^7*b*c^2 - 2*a^3*b^6 +
 (2*a^6*b^3 - a^4*b^4)*c)*d^2 - (2*a^5*b^4*c^2 - a*b^9 + (a^4*b^6 - 2*a^2*b
^7)*c)*d)*log(b*sqrt((a*x + b)/(c*x + d)) - a)/(a^3*b^9 - 4*a^4*b^7*c + 6*a
^5*b^5*c^2 - 4*a^6*b^3*c^3 + a^7*b*c^4) + (a^3*b^6*c^5 - b^9*c^4 + (a^4*b^7
 - 4*a^5*b^5*c + 6*a^6*b^3*c^2 - (4*a^7*b - a^5*b^2)*c^3)*d^3 - (a^3*b^8*c
- 4*a^4*b^6*c^2 - (6*a^6*b^2 - a^4*b^3)*c^4 + 2*(3*a^5*b^4 + a^3*b^5)*c^3)*
d^2 - (2*a^5*b^3*c^5 - a*b^8*c^3 + (a^4*b^5 - 2*a^2*b^6)*c^4)*d)*log(-a + (
a*x + b)*c/(c*x + d))/(a^3*b^8*c^3 - 4*a^4*b^6*c^4 + 6*a^5*b^4*c^5 - 4*a^6*
b^2*c^6 + a^7*c^7) - 1/16*((a^5*b^3 + a^3*b^4)*c^6 - (9*a^4*b^5 + 5*a^2*b^6
)*c^5 - 3*(3*a^3*b^7 - 5*a*b^8)*c^4 + (a^2*b^9 + 5*b^10)*c^3 - (5*a^5*b^6 +
 a^3*b^7 - 5*(7*a^8 - a^6*b)*c^3 + 5*(7*a^7*b^2 + 3*a^5*b^3)*c^2 - (21*a^6*
b^4 + 5*a^4*b^5)*c)*d^3 - (3*(15*a^7*b - a^5*b^2)*c^4 - (5*a^6*b^3 + 33*a^4
*b^4)*c^3 + (11*a^5*b^5 - 13*a^3*b^6)*c^2 - (3*a^4*b^7 - a^2*b^8)*c)*d^2 +
((9*a^6*b^2 + a^4*b^3)*c^5 + 13*(3*a^5*b^4 - a^3*b^5)*c^4 - (a^4*b^6 + 33*a
^2*b^7)*c^3 + (a^3*b^8 - 3*a*b^9)*c^2)*d)*log((c*sqrt((a*x + b)/(c*x + d))
- sqrt(a*c))/(c*sqrt((a*x + b)/(c*x + d)) + sqrt(a*c)))/((a^3*b^8*c^2 - 4*a
^4*b^6*c^3 + 6*a^5*b^4*c^4 - 4*a^6*b^2*c^5 + a^7*c^6)*sqrt(a*c)) + 1/24*(4*
a^4*b^8*c^3 + 8*a^5*b^3*c^6 - 4*(2*a^6*b^4 + 7*a^4*b^5)*c^5 - 4*(5*a^5*b^6
```

```
- 11*a^3*b^7)*c^4 + 4*(11*a^7*b^5 - 2*a^8*c^3 + (26*a^9*b - 5*a^7*b^2)*c^2
- (31*a^8*b^3 - a^6*b^4)*c)*d^3 - 3*((a^4*b^3 + a^2*b^4)*c^8 + 4*(2*a^3*b^5
 - a*b^6)*c^7 - (a^2*b^7 + 5*b^8)*c^6 - ((29*a^7 - 11*a^5*b)*c^5 - 4*(8*a^6
*b^2 - a^4*b^3)*c^4 + (11*a^5*b^4 - a^3*b^5)*c^3)*d^3 + ((51*a^6*b - 13*a^4
*b^2)*c^6 - 4*(10*a^5*b^3 + 3*a^3*b^4)*c^5 + (13*a^4*b^5 + a^2*b^6)*c^4)*d^
2 + (20*a^2*b^5*c^6 - (23*a^5*b^2 - a^3*b^3)*c^7 - (a^3*b^6 - 3*a*b^7)*c^5)
*d)*((a*x + b)/(c*x + d))^(5/2) - 12*(5*a^6*b^6*c + (16*a^8*b^2 - 5*a^6*b^3
)*c^3 - (15*a^7*b^4 + a^5*b^5)*c^2)*d^2 - 8*((a^5*b^3 - a^3*b^4)*c^7 - 2*(4
*a^4*b^5 - a^2*b^6)*c^6 + (a^3*b^7 + 5*a*b^8)*c^5 + ((17*a^8 - 5*a^6*b)*c^4
 - 2*(8*a^7*b^2 + a^5*b^3)*c^3 + (5*a^6*b^4 + a^4*b^5)*c^2)*d^3 - 3*((9*a^7
*b - a^5*b^2)*c^5 - 2*(2*a^6*b^3 + 3*a^4*b^4)*c^4 + (a^5*b^5 + a^3*b^6)*c^3
)*d^2 + 3*((3*a^6*b^2 + a^4*b^3)*c^6 + 2*(2*a^5*b^4 - 3*a^3*b^5)*c^5 - (a^4
*b^6 + a^2*b^7)*c^4)*d)*((a*x + b)/(c*x + d))^(3/2) + 12*(a^5*b^7*c^2 + (8*
a^7*b^3 - a^5*b^4)*c^4 - (3*a^6*b^5 + 5*a^4*b^6)*c^3)*d + 3*((a^6*b^3 + a^4
*b^4)*c^6 - 4*(2*a^5*b^5 + a^3*b^6)*c^5 - (a^4*b^7 - 11*a^2*b^8)*c^4 + ((19
*a^9 - 5*a^7*b)*c^3 - 4*(4*a^8*b^2 + a^6*b^3)*c^2 + (5*a^7*b^4 + a^5*b^5)*c
)*d^3 - ((29*a^8*b - 3*a^6*b^2)*c^4 - 4*(2*a^7*b^3 + 5*a^5*b^4)*c^3 + (3*a^
6*b^5 - a^4*b^6)*c^2)*d^2 + ((9*a^7*b^2 + a^5*b^3)*c^5 + 4*(4*a^6*b^4 - 3*a
^4*b^5)*c^4 - (a^5*b^6 + 13*a^3*b^7)*c^3)*d)*sqrt((a*x + b)/(c*x + d)) - 24
*(a^4*b^4*c^7 - a*b^7*c^6 - (3*a^5*b^5*c^2 - 8*a^6*b^3*c^3 + 6*a^7*b*c^4 -
a^6*c^5)*d^3 + (5*a^4*b^6*c^3 - 14*a^5*b^4*c^4 - a^5*b*c^6 + 2*(6*a^6*b^2 -
 a^4*b^3)*c^5)*d^2 - (2*a^3*b^7*c^4 + (7*a^5*b^3 - 2*a^3*b^4)*c^6 - (6*a^4*
b^5 + a^2*b^6)*c^5)*d)*(a*x + b)^2/(c*x + d)^2 - 12*(a^3*b^8*c^4 - (2*a^5*b
^4 + a^3*b^5)*c^6 - (3*a^4*b^6 - 5*a^2*b^7)*c^5 + (9*a^6*b^5*c - 2*a^7*c^4
+ (20*a^8*b - 3*a^6*b^2)*c^3 - (25*a^7*b^3 - a^5*b^4)*c^2)*d^3 - (13*a^5*b^
6*c^2 + (38*a^7*b^2 - 13*a^5*b^3)*c^4 - (39*a^6*b^4 - a^4*b^5)*c^3)*d^2 + (
3*a^4*b^7*c^3 + 2*a^5*b^2*c^6 + (20*a^6*b^3 - 9*a^4*b^4)*c^5 - (11*a^5*b^5
+ 5*a^3*b^6)*c^4)*d)*(a*x + b)/(c*x + d))/(a^6*b^6*c^3 - 3*a^7*b^4*c^4 + 3*
a^8*b^2*c^5 - a^9*c^6 - (a^3*b^6*c^6 - 3*a^4*b^4*c^7 + 3*a^5*b^2*c^8 - a^6*
c^9)*(a*x + b)^3/(c*x + d)^3 + 3*(a^4*b^6*c^5 - 3*a^5*b^4*c^6 + 3*a^6*b^2*c
^7 - a^7*c^8)*(a*x + b)^2/(c*x + d)^2 - 3*(a^5*b^6*c^4 - 3*a^6*b^4*c^5 + 3*
a^7*b^2*c^6 - a^8*c^7)*(a*x + b)/(c*x + d))
```

mupad **[F(-1)]** time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - c*x^2*((b + a*x)/(d + c*x))^(3/2))/(a - b*((b + a*x)/(d + c*x))^
(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-c*x**2*((a*x+b)/(c*x+d))**(3/2))/(a-b*((a*x+b)/(c*x+d))**(1
/2)),x)
```

```
[Out] Timed out
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
```

```
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
```

```
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
```

```
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
```

```
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
```

```
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```